Computer algebra independent integration tests

1-Algebraic-functions/1.1-Binomial-products/1.1.3-General/1.1.3.2-c-x-^m-a+b-x^n-^p

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3.262 $\int x^5\left(a+bx^3\right)^5\,dx$ . . . 1185
3.263 $\int x^2\left(a+bx^3\right)^5\,dx$ . . . 1188
3.264 $\int \frac{\left(a+bx^3\right)^5}{x}\,dx$ . . . 1190
3.265 $\int \frac{\left(a+bx^3\right)^5}{x^4}\,dx$ . . . 1193
3.266 $\int \frac{\left(a+bx^3\right)^5}{x^7}\,dx$ . . . 1196
3.267 $\int \frac{\left(a+bx^3\right)^5}{x^{10}}\,dx$ . . . 1199
3.268 $\int \frac{\left(a+bx^3\right)^5}{x^{13}}\,dx$ . . . 1202
3.269 $\int \frac{\left(a+bx^3\right)^5}{x^{16}}\,dx$ . . . 1205
3.270 $\int \frac{\left(a+bx^3\right)^5}{x^{19}}\,dx$ . . . 1208
3.271 $\int \frac{\left(a+bx^3\right)^5}{x^{22}}\,dx$ . . . 1210
3.272 $\int \frac{\left(a+bx^3\right)^5}{x^{25}}\,dx$ . . . 1213
3.273 $\int \frac{\left(a+bx^3\right)^5}{x^{28}}\,dx$ . . . 1216
3.274 $\int \frac{\left(a+bx^3\right)^5}{x^{31}}\,dx$ . . . 1219
3.275 $\int x^4\left(a+bx^3\right)^5\,dx$ . . . 1222
3.276 $\int x^3\left(a+bx^3\right)^5\,dx$ . . . 1224
3.277 $\int x\left(a+bx^3\right)^5\,dx$ . . . 1226
3.278 $\int \left(a+bx^3\right)^5\,dx$ . . . 1228
3.279 $\int \frac{\left(a+bx^3\right)^5}{x^2}\,dx$ . . . 1230
3.280 $\int \frac{\left(a+bx^3\right)^5}{x^3}\,dx$ . . . 1232
3.281 $\int \frac{\left(a+bx^3\right)^5}{x^5}\,dx$ . . . 1234
3.282 $\int \frac{\left(a+bx^3\right)^5}{x^6}\,dx$ . . . 1236
3.283 $\int \frac{\left(a+bx^3\right)^5}{x^8}\,dx$ . . . 1238

3.284 $\int \frac{\left(a+bx^3\right)^5}{x^9}\,dx$ 1240
3.285 $\int x^{20}\left(a+bx^3\right)^8\,dx$ 1242
3.286 $\int x^{17}\left(a+bx^3\right)^8\,dx$ 1245
3.287 $\int x^{14}\left(a+bx^3\right)^8\,dx$ 1248
3.288 $\int x^{11}\left(a+bx^3\right)^8\,dx$ 1251
3.289 $\int x^8\left(a+bx^3\right)^8\,dx$ 1254
3.290 $\int x^5\left(a+bx^3\right)^8\,dx$ 1257
3.291 $\int x^2\left(a+bx^3\right)^8\,dx$ 1260
3.292 $\int \frac{\left(a+bx^3\right)^8}{x}\,dx$ 1262
3.293 $\int \frac{\left(a+bx^3\right)^8}{x^4}\,dx$ 1265
3.294 $\int \frac{\left(a+bx^3\right)^8}{x^7}\,dx$ 1268
3.295 $\int \frac{\left(a+bx^3\right)^8}{x^{10}}\,dx$ 1271
3.296 $\int \frac{\left(a+bx^3\right)^8}{x^{13}}\,dx$ 1274
3.297 $\int \frac{\left(a+bx^3\right)^8}{x^{16}}\,dx$ 1277
3.298 $\int \frac{\left(a+bx^3\right)^8}{x^{19}}\,dx$ 1280
3.299 $\int \frac{\left(a+bx^3\right)^8}{x^{22}}\,dx$ 1283
3.300 $\int \frac{\left(a+bx^3\right)^8}{x^{25}}\,dx$ 1286
3.301 $\int \frac{\left(a+bx^3\right)^8}{x^{28}}\,dx$ 1289
3.302 $\int \frac{\left(a+bx^3\right)^8}{x^{31}}\,dx$ 1291
3.303 $\int \frac{\left(a+bx^3\right)^8}{x^{34}}\,dx$ 1294
3.304 $\int \frac{\left(a+bx^3\right)^8}{x^{37}}\,dx$ 1297
3.305 $\int \frac{\left(a+bx^3\right)^8}{x^{40}}\,dx$ 1300
3.306 $\int \frac{\left(a+bx^3\right)^8}{x^{43}}\,dx$ 1303
3.307 $\int \frac{\left(a+bx^3\right)^8}{x^{46}}\,dx$ 1306
3.308 $\int x^4\left(a+bx^3\right)^8\,dx$ 1309
3.309 $\int x^3\left(a+bx^3\right)^8\,dx$ 1311
3.310 $\int x\left(a+bx^3\right)^8\,dx$ 1313
3.311 $\int \left(a+bx^3\right)^8\,dx$ 1315
3.312 $\int \frac{\left(a+bx^3\right)^8}{x^2}\,dx$ 1317
3.313 $\int \frac{\left(a+bx^3\right)^8}{x^3}\,dx$ 1319
3.314 $\int \frac{\left(a+bx^3\right)^8}{x^5}\,dx$ 1321
3.315 $\int \frac{\left(a+bx^3\right)^8}{x^6}\,dx$ 1324
3.316 $\int \frac{\left(a+bx^3\right)^8}{x^8}\,dx$ 1327

3.317 $\int \frac{\left(a+bx^3\right)^8}{x^9}\,dx$ 1329
3.318 $\int \frac{x^8}{a+bx^3}\,dx$ 1332
3.319 $\int \frac{x^5}{a+bx^3}\,dx$ 1334
3.320 $\int \frac{x^2}{a+bx^3}\,dx$ 1336
3.321 $\int \frac{1}{x\left(a+bx^3\right)}\,dx$ 1338
3.322 $\int \frac{1}{x^4\left(a+bx^3\right)}\,dx$ 1341
3.323 $\int \frac{x^4}{a+bx^3}\,dx$ 1343
3.324 $\int \frac{x^3}{a+bx^3}\,dx$ 1347
3.325 $\int \frac{x}{a+bx^3}\,dx$ 1351
3.326 $\int \frac{1}{a+bx^3}\,dx$ 1355
3.327 $\int \frac{1}{x^2\left(a+bx^3\right)}\,dx$ 1359
3.328 $\int \frac{1}{x^3\left(a+bx^3\right)}\,dx$ 1363
3.329 $\int \frac{x^8}{\left(a+bx^3\right)^2}\,dx$ 1367
3.330 $\int \frac{x^5}{\left(a+bx^3\right)^2}\,dx$ 1370
3.331 $\int \frac{x^2}{\left(a+bx^3\right)^2}\,dx$ 1373
3.332 $\int \frac{1}{x\left(a+bx^3\right)^2}\,dx$ 1375
3.333 $\int \frac{1}{x^4\left(a+bx^3\right)^2}\,dx$ 1378
3.334 $\int \frac{x^4}{\left(a+bx^3\right)^2}\,dx$ 1381
3.335 $\int \frac{x^3}{\left(a+bx^3\right)^2}\,dx$ 1385
3.336 $\int \frac{x}{\left(a+bx^3\right)^2}\,dx$ 1389
3.337 $\int \frac{1}{\left(a+bx^3\right)^2}\,dx$ 1393
3.338 $\int \frac{1}{x^2\left(a+bx^3\right)^2}\,dx$ 1397
3.339 $\int \frac{1}{x^3\left(a+bx^3\right)^2}\,dx$ 1401
3.340 $\int \frac{x^{11}}{\left(a+bx^3\right)^3}\,dx$ 1405
3.341 $\int \frac{x^8}{\left(a+bx^3\right)^3}\,dx$ 1408
3.342 $\int \frac{x^5}{\left(a+bx^3\right)^3}\,dx$ 1411
3.343 $\int \frac{x^2}{\left(a+bx^3\right)^3}\,dx$ 1413
3.344 $\int \frac{1}{x\left(a+bx^3\right)^3}\,dx$ 1415
3.345 $\int \frac{1}{x^4\left(a+bx^3\right)^3}\,dx$ 1418
3.346 $\int \frac{x^7}{\left(a+bx^3\right)^3}\,dx$ 1421
3.347 $\int \frac{x^6}{\left(a+bx^3\right)^3}\,dx$ 1425

3.348 $\int \frac{x^4}{\left(a+bx^3\right)^3}\,dx$ 1429
3.349 $\int \frac{x^3}{\left(a+bx^3\right)^3}\,dx$ 1434
3.350 $\int \frac{x}{\left(a+bx^3\right)^3}\,dx$ 1439
3.351 $\int \frac{1}{\left(a+bx^3\right)^3}\,dx$ 1443
3.352 $\int \frac{x^8}{a-bx^3}\,dx$ 1447
3.353 $\int \frac{x^5}{a-bx^3}\,dx$ 1449
3.354 $\int \frac{x^2}{a-bx^3}\,dx$ 1451
3.355 $\int \frac{1}{x\left(a-bx^3\right)}\,dx$ 1453
3.356 $\int \frac{1}{x^4\left(a-bx^3\right)}\,dx$ 1456
3.357 $\int \frac{x^4}{a-bx^3}\,dx$ 1458
3.358 $\int \frac{x^3}{a-bx^3}\,dx$ 1462
3.359 $\int \frac{x}{a-bx^3}\,dx$ 1466
3.360 $\int \frac{1}{a-bx^3}\,dx$ 1470
3.361 $\int \frac{1}{x^2\left(a-bx^3\right)}\,dx$ 1474
3.362 $\int \frac{1}{x^3\left(a-bx^3\right)}\,dx$ 1478
3.363 $\int \frac{1}{1+a+bx^3}\,dx$ 1482
3.364 $\int \frac{1}{1+a-bx^3}\,dx$ 1486
3.365 $\int \frac{1}{-1+a+bx^3}\,dx$ 1490
3.366 $\int \frac{1}{-1+a-bx^3}\,dx$ 1494
3.367 $\int \frac{\sqrt{x}}{1+x^3}\,dx$ 1498
3.368 $\int x^{11}\sqrt{a+bx^3}\,dx$ 1500
3.369 $\int x^8\sqrt{a+bx^3}\,dx$ 1503
3.370 $\int x^5\sqrt{a+bx^3}\,dx$ 1506
3.371 $\int x^2\sqrt{a+bx^3}\,dx$ 1508
3.372 $\int \frac{\sqrt{a+bx^3}}{x}\,dx$ 1510
3.373 $\int \frac{\sqrt{a+bx^3}}{x^4}\,dx$ 1513
3.374 $\int \frac{\sqrt{a+bx^3}}{x^7}\,dx$ 1516
3.375 $\int \frac{\sqrt{a+bx^3}}{x^{10}}\,dx$ 1519
3.376 $\int x^6\sqrt{a+bx^3}\,dx$ 1523
3.377 $\int x^3\sqrt{a+bx^3}\,dx$ 1526
3.378 $\int \sqrt{a+bx^3}\,dx$ 1529
3.379 $\int \frac{\sqrt{a+bx^3}}{x^3}\,dx$ 1532
3.380 $\int \frac{\sqrt{a+bx^3}}{x^6}\,dx$ 1535
3.381 $\int \frac{\sqrt{a+bx^3}}{x^9}\,dx$ 1538
3.382 $\int x^7\sqrt{a+bx^3}\,dx$ 1541
3.383 $\int x^4\sqrt{a+bx^3}\,dx$ 1545
3.384 $\int x\sqrt{a+bx^3}\,dx$ 1549
3.385 $\int \frac{\sqrt{a+bx^3}}{x^2}\,dx$ 1553

3.386 $\int \frac{\sqrt{a+bx^3}}{x^5}\,dx$ . . . 1557
3.387 $\int x^{11}\left(a+bx^3\right)^{3/2}\,dx$ . . . 1561
3.388 $\int x^8\left(a+bx^3\right)^{3/2}\,dx$ . . . 1564
3.389 $\int x^5\left(a+bx^3\right)^{3/2}\,dx$ . . . 1567
3.390 $\int x^2\left(a+bx^3\right)^{3/2}\,dx$ . . . 1569
3.391 $\int \frac{\left(a+bx^3\right)^{3/2}}{x}\,dx$ . . . 1571
3.392 $\int \frac{\left(a+bx^3\right)^{3/2}}{x^4}\,dx$ . . . 1574
3.393 $\int \frac{\left(a+bx^3\right)^{3/2}}{x^7}\,dx$ . . . 1577
3.394 $\int x^6\left(a+bx^3\right)^{3/2}\,dx$ . . . 1580
3.395 $\int x^3\left(a+bx^3\right)^{3/2}\,dx$ . . . 1583
3.396 $\int \left(a+bx^3\right)^{3/2}\,dx$ . . . 1586
3.397 $\int \frac{\left(a+bx^3\right)^{3/2}}{x^3}\,dx$ . . . 1589
3.398 $\int \frac{\left(a+bx^3\right)^{3/2}}{x^6}\,dx$ . . . 1592
3.399 $\int x^7\left(a+bx^3\right)^{3/2}\,dx$ . . . 1595
3.400 $\int x^4\left(a+bx^3\right)^{3/2}\,dx$ . . . 1599
3.401 $\int x\left(a+bx^3\right)^{3/2}\,dx$ . . . 1603
3.402 $\int \frac{\left(a+bx^3\right)^{3/2}}{x^2}\,dx$ . . . 1607
3.403 $\int \frac{\left(a+bx^3\right)^{3/2}}{x^5}\,dx$ . . . 1611
3.404 $\int \frac{x^{11}}{\sqrt{a+bx^3}}\,dx$ . . . 1615
3.405 $\int \frac{x^8}{\sqrt{a+bx^3}}\,dx$ . . . 1618
3.406 $\int \frac{x^5}{\sqrt{a+bx^3}}\,dx$ . . . 1621
3.407 $\int \frac{x^2}{\sqrt{a+bx^3}}\,dx$ . . . 1624
3.408 $\int \frac{1}{x\sqrt{a+bx^3}}\,dx$ . . . 1626
3.409 $\int \frac{1}{x^4\sqrt{a+bx^3}}\,dx$ . . . 1629
3.410 $\int \frac{1}{x^7\sqrt{a+bx^3}}\,dx$ . . . 1632
3.411 $\int \frac{x^6}{\sqrt{a+bx^3}}\,dx$ . . . 1635
3.412 $\int \frac{x^3}{\sqrt{a+bx^3}}\,dx$ . . . 1638
3.413 $\int \frac{1}{\sqrt{a+bx^3}}\,dx$ . . . 1641
3.414 $\int \frac{1}{x^3\sqrt{a+bx^3}}\,dx$ . . . 1644
3.415 $\int \frac{1}{x^6\sqrt{a+bx^3}}\,dx$ . . . 1647
3.416 $\int \frac{x^7}{\sqrt{a+bx^3}}\,dx$ . . . 1650
3.417 $\int \frac{x^4}{\sqrt{a+bx^3}}\,dx$ . . . 1654
3.418 $\int \frac{x}{\sqrt{a+bx^3}}\,dx$ . . . 1658
3.419 $\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx$ . . . 1662
3.420 $\int \frac{1}{x^5\sqrt{a+bx^3}}\,dx$ . . . 1666

3.421 $\int \frac{x^{11}}{\left(a+bx^3\right)^{3/2}}\,dx$ 1670

3.422 $\int \frac{x^{8}}{\left(a+bx^3\right)^{3/2}}\,dx$ 1673

3.423 $\int \frac{x^{5}}{\left(a+bx^3\right)^{3/2}}\,dx$ 1676

3.424 $\int \frac{x^{2}}{\left(a+bx^3\right)^{3/2}}\,dx$ 1679

3.425 $\int \frac{1}{x\left(a+bx^3\right)^{3/2}}\,dx$ 1681

3.426 $\int \frac{1}{x^4\left(a+bx^3\right)^{3/2}}\,dx$ 1684

3.427 $\int \frac{1}{x^7\left(a+bx^3\right)^{3/2}}\,dx$ 1687

3.428 $\int \frac{x^{6}}{\left(a+bx^3\right)^{3/2}}\,dx$ 1690

3.429 $\int \frac{x^{3}}{\left(a+bx^3\right)^{3/2}}\,dx$ 1693

3.430 $\int \frac{1}{\left(a+bx^3\right)^{3/2}}\,dx$ 1696

3.431 $\int \frac{1}{x^3\left(a+bx^3\right)^{3/2}}\,dx$ 1699

3.432 $\int \frac{1}{x^6\left(a+bx^3\right)^{3/2}}\,dx$ 1702

3.433 $\int \frac{x^{7}}{\left(a+bx^3\right)^{3/2}}\,dx$ 1706

3.434 $\int \frac{x^{4}}{\left(a+bx^3\right)^{3/2}}\,dx$ 1710

3.435 $\int \frac{x}{\left(a+bx^3\right)^{3/2}}\,dx$ 1714

3.436 $\int \frac{1}{x^2\left(a+bx^3\right)^{3/2}}\,dx$ 1718

3.437 $\int \frac{1}{x^5\left(a+bx^3\right)^{3/2}}\,dx$ 1722

3.438 $\int \frac{x^{11}}{\sqrt{1+x^3}}\,dx$ 1726

3.439 $\int \frac{x^{8}}{\sqrt{1+x^3}}\,dx$ 1728

3.440 $\int \frac{x^{5}}{\sqrt{1+x^3}}\,dx$ 1730

3.441 $\int \frac{x^{2}}{\sqrt{1+x^3}}\,dx$ 1732

3.442 $\int \frac{1}{x\sqrt{1+x^3}}\,dx$ 1734

3.443 $\int \frac{1}{x^4\sqrt{1+x^3}}\,dx$ 1737

3.444 $\int \frac{1}{x^7\sqrt{1+x^3}}\,dx$ 1740

3.445 $\int \frac{1}{x^{10}\sqrt{1+x^3}}\,dx$ 1743

3.446 $\int \frac{x^{6}}{\sqrt{1+x^3}}\,dx$ 1746

3.447 $\int \frac{x^{3}}{\sqrt{1+x^3}}\,dx$ 1749

3.448 $\int \frac{1}{\sqrt{1+x^3}}\,dx$ 1752

3.449 $\int \frac{1}{x^3\sqrt{1+x^3}}\,dx$ 1755

3.450 $\int \frac{1}{x^6\sqrt{1+x^3}}\,dx$ 1758

3.451 $\int \frac{x^{7}}{\sqrt{1+x^3}}\,dx$ 1761

3.452 $\int \frac{x^{4}}{\sqrt{1+x^3}}\,dx$ 1765

3.453 $\int \frac{x}{\sqrt{1+x^3}}\,dx$ . . . 1769
3.454 $\int \frac{1}{x^2\sqrt{1+x^3}}\,dx$ . . . 1772
3.455 $\int \frac{1}{x^5\sqrt{1+x^3}}\,dx$ . . . 1776
3.456 $\int \frac{x^{11}}{\sqrt{1-x^3}}\,dx$ . . . 1780
3.457 $\int \frac{x^8}{\sqrt{1-x^3}}\,dx$ . . . 1782
3.458 $\int \frac{x^5}{\sqrt{1-x^3}}\,dx$ . . . 1784
3.459 $\int \frac{x^2}{\sqrt{1-x^3}}\,dx$ . . . 1786
3.460 $\int \frac{1}{x\sqrt{1-x^3}}\,dx$ . . . 1788
3.461 $\int \frac{1}{x^4\sqrt{1-x^3}}\,dx$ . . . 1791
3.462 $\int \frac{1}{x^7\sqrt{1-x^3}}\,dx$ . . . 1794
3.463 $\int \frac{1}{x^{10}\sqrt{1-x^3}}\,dx$ . . . 1797
3.464 $\int \frac{x^6}{\sqrt{1-x^3}}\,dx$ . . . 1800
3.465 $\int \frac{x^3}{\sqrt{1-x^3}}\,dx$ . . . 1803
3.466 $\int \frac{1}{\sqrt{1-x^3}}\,dx$ . . . 1806
3.467 $\int \frac{1}{x^3\sqrt{1-x^3}}\,dx$ . . . 1809
3.468 $\int \frac{1}{x^6\sqrt{1-x^3}}\,dx$ . . . 1812
3.469 $\int \frac{x^7}{\sqrt{1-x^3}}\,dx$ . . . 1815
3.470 $\int \frac{x^4}{\sqrt{1-x^3}}\,dx$ . . . 1819
3.471 $\int \frac{x}{\sqrt{1-x^3}}\,dx$ . . . 1823
3.472 $\int \frac{1}{x^2\sqrt{1-x^3}}\,dx$ . . . 1826
3.473 $\int \frac{1}{x^5\sqrt{1-x^3}}\,dx$ . . . 1830
3.474 $\int \frac{x^{11}}{\sqrt{-1+x^3}}\,dx$ . . . 1834
3.475 $\int \frac{x^8}{\sqrt{-1+x^3}}\,dx$ . . . 1836
3.476 $\int \frac{x^5}{\sqrt{-1+x^3}}\,dx$ . . . 1838
3.477 $\int \frac{x^2}{\sqrt{-1+x^3}}\,dx$ . . . 1840
3.478 $\int \frac{1}{x\sqrt{-1+x^3}}\,dx$ . . . 1842
3.479 $\int \frac{1}{x^4\sqrt{-1+x^3}}\,dx$ . . . 1845
3.480 $\int \frac{1}{x^7\sqrt{-1+x^3}}\,dx$ . . . 1848
3.481 $\int \frac{1}{x^{10}\sqrt{-1+x^3}}\,dx$ . . . 1851
3.482 $\int \frac{x^6}{\sqrt{-1+x^3}}\,dx$ . . . 1854
3.483 $\int \frac{x^3}{\sqrt{-1+x^3}}\,dx$ . . . 1857
3.484 $\int \frac{1}{\sqrt{-1+x^3}}\,dx$ . . . 1860
3.485 $\int \frac{1}{x^3\sqrt{-1+x^3}}\,dx$ . . . 1863
3.486 $\int \frac{1}{x^6\sqrt{-1+x^3}}\,dx$ . . . 1866
3.487 $\int \frac{x^7}{\sqrt{-1+x^3}}\,dx$ . . . 1869
3.488 $\int \frac{x^4}{\sqrt{-1+x^3}}\,dx$ . . . 1873

3.489 $\int \frac{x}{\sqrt{-1+x^3}}\,dx$ . . . 1877
3.490 $\int \frac{1}{x^2\sqrt{-1+x^3}}\,dx$ . . . 1880
3.491 $\int \frac{1}{x^5\sqrt{-1+x^3}}\,dx$ . . . 1883
3.492 $\int \frac{x^{11}}{\sqrt{-1-x^3}}\,dx$ . . . 1887
3.493 $\int \frac{x^8}{\sqrt{-1-x^3}}\,dx$ . . . 1889
3.494 $\int \frac{x^5}{\sqrt{-1-x^3}}\,dx$ . . . 1891
3.495 $\int \frac{x^2}{\sqrt{-1-x^3}}\,dx$ . . . 1893
3.496 $\int \frac{1}{x\sqrt{-1-x^3}}\,dx$ . . . 1895
3.497 $\int \frac{1}{x^4\sqrt{-1-x^3}}\,dx$ . . . 1898
3.498 $\int \frac{1}{x^7\sqrt{-1-x^3}}\,dx$ . . . 1901
3.499 $\int \frac{1}{x^{10}\sqrt{-1-x^3}}\,dx$ . . . 1904
3.500 $\int \frac{x^6}{\sqrt{-1-x^3}}\,dx$ . . . 1907
3.501 $\int \frac{x^3}{\sqrt{-1-x^3}}\,dx$ . . . 1910
3.502 $\int \frac{1}{\sqrt{-1-x^3}}\,dx$ . . . 1913
3.503 $\int \frac{1}{x^3\sqrt{-1-x^3}}\,dx$ . . . 1916
3.504 $\int \frac{1}{x^6\sqrt{-1-x^3}}\,dx$ . . . 1919
3.505 $\int \frac{x^7}{\sqrt{-1-x^3}}\,dx$ . . . 1922
3.506 $\int \frac{x^4}{\sqrt{-1-x^3}}\,dx$ . . . 1926
3.507 $\int \frac{x}{\sqrt{-1-x^3}}\,dx$ . . . 1930
3.508 $\int \frac{1}{x^2\sqrt{-1-x^3}}\,dx$ . . . 1933
3.509 $\int \frac{1}{x^5\sqrt{-1-x^3}}\,dx$ . . . 1937
3.510 $\int x^{11}\sqrt[3]{a+bx^3}\,dx$ . . . 1941
3.511 $\int x^8\sqrt[3]{a+bx^3}\,dx$ . . . 1944
3.512 $\int x^5\sqrt[3]{a+bx^3}\,dx$ . . . 1947
3.513 $\int x^2\sqrt[3]{a+bx^3}\,dx$ . . . 1950
3.514 $\int \frac{\sqrt[3]{a+bx^3}}{x}\,dx$ . . . 1952
3.515 $\int \frac{\sqrt[3]{a+bx^3}}{x^4}\,dx$ . . . 1956
3.516 $\int x^4\sqrt[3]{a+bx^3}\,dx$ . . . 1960
3.517 $\int x\sqrt[3]{a+bx^3}\,dx$ . . . 1964
3.518 $\int \frac{\sqrt[3]{a+bx^3}}{x^2}\,dx$ . . . 1968
3.519 $\int \frac{\sqrt[3]{a+bx^3}}{x^5}\,dx$ . . . 1972
3.520 $\int \frac{\sqrt[3]{a+bx^3}}{x^8}\,dx$ . . . 1974
3.521 $\int \frac{\sqrt[3]{a+bx^3}}{x^{11}}\,dx$ . . . 1976
3.522 $\int \frac{\sqrt[3]{a+bx^3}}{x^{14}}\,dx$ . . . 1979
3.523 $\int x^3\sqrt[3]{a+bx^3}\,dx$ . . . 1982
3.524 $\int \sqrt[3]{a+bx^3}\,dx$ . . . 1985
3.525 $\int \frac{\sqrt[3]{a+bx^3}}{x^3}\,dx$ . . . 1988

3.526 $\int \frac{\sqrt[3]{a+bx^3}}{x^6}\,dx$ . . . 1991
3.527 $\int x^{11}\left(a+bx^3\right)^{2/3}\,dx$ . . . 1994
3.528 $\int x^8\left(a+bx^3\right)^{2/3}\,dx$ . . . 1997
3.529 $\int x^5\left(a+bx^3\right)^{2/3}\,dx$ . . . 2000
3.530 $\int x^2\left(a+bx^3\right)^{2/3}\,dx$ . . . 2003
3.531 $\int \frac{\left(a+bx^3\right)^{2/3}}{x}\,dx$ . . . 2005
3.532 $\int \frac{\left(a+bx^3\right)^{2/3}}{x^4}\,dx$ . . . 2009
3.533 $\int x^4\left(a+bx^3\right)^{2/3}\,dx$ . . . 2013
3.534 $\int x\left(a+bx^3\right)^{2/3}\,dx$ . . . 2016
3.535 $\int \frac{\left(a+bx^3\right)^{2/3}}{x^2}\,dx$ . . . 2019
3.536 $\int \frac{\left(a+bx^3\right)^{2/3}}{x^5}\,dx$ . . . 2022
3.537 $\int x^3\left(a+bx^3\right)^{2/3}\,dx$ . . . 2025
3.538 $\int \left(a+bx^3\right)^{2/3}\,dx$ . . . 2028
3.539 $\int \frac{\left(a+bx^3\right)^{2/3}}{x^3}\,dx$ . . . 2031
3.540 $\int \frac{\left(a+bx^3\right)^{2/3}}{x^6}\,dx$ . . . 2034
3.541 $\int \frac{\left(a+bx^3\right)^{2/3}}{x^9}\,dx$ . . . 2036
3.542 $\int \frac{\left(a+bx^3\right)^{2/3}}{x^{12}}\,dx$ . . . 2038
3.543 $\int x^8\left(1-x^3\right)^{6/5}\,dx$ . . . 2041
3.544 $\int \frac{x^{11}}{\sqrt[3]{a+bx^3}}\,dx$ . . . 2043
3.545 $\int \frac{x^8}{\sqrt[3]{a+bx^3}}\,dx$ . . . 2046
3.546 $\int \frac{x^5}{\sqrt[3]{a+bx^3}}\,dx$ . . . 2049
3.547 $\int \frac{x^2}{\sqrt[3]{a+bx^3}}\,dx$ . . . 2052
3.548 $\int \frac{1}{x\sqrt[3]{a+bx^3}}\,dx$ . . . 2054
3.549 $\int \frac{1}{x^4\sqrt[3]{a+bx^3}}\,dx$ . . . 2057
3.550 $\int \frac{x^7}{\sqrt[3]{a+bx^3}}\,dx$ . . . 2061
3.551 $\int \frac{x^4}{\sqrt[3]{a+bx^3}}\,dx$ . . . 2064
3.552 $\int \frac{x}{\sqrt[3]{a+bx^3}}\,dx$ . . . 2067
3.553 $\int \frac{1}{x^2\sqrt[3]{a+bx^3}}\,dx$ . . . 2070
3.554 $\int \frac{1}{x^5\sqrt[3]{a+bx^3}}\,dx$ . . . 2073
3.555 $\int \frac{x^3}{\sqrt[3]{a+bx^3}}\,dx$ . . . 2076
3.556 $\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx$ . . . 2079
3.557 $\int \frac{1}{x^3\sqrt[3]{a+bx^3}}\,dx$ . . . 2082
3.558 $\int \frac{1}{x^6\sqrt[3]{a+bx^3}}\,dx$ . . . 2084
3.559 $\int \frac{1}{x^9\sqrt[3]{a+bx^3}}\,dx$ . . . 2086
3.560 $\int \frac{1}{x^{12}\sqrt[3]{a+bx^3}}\,dx$ . . . 2089

3.561 $\int \frac{x^{11}}{\left(a+bx^3\right)^{2/3}}\,dx$ . . . 2092
3.562 $\int \frac{x^8}{\left(a+bx^3\right)^{2/3}}\,dx$ . . . 2095
3.563 $\int \frac{x^5}{\left(a+bx^3\right)^{2/3}}\,dx$ . . . 2098
3.564 $\int \frac{x^2}{\left(a+bx^3\right)^{2/3}}\,dx$ . . . 2101
3.565 $\int \frac{1}{x\left(a+bx^3\right)^{2/3}}\,dx$ . . . 2103
3.566 $\int \frac{1}{x^4\left(a+bx^3\right)^{2/3}}\,dx$ . . . 2106
3.567 $\int \frac{x^7}{\left(a+bx^3\right)^{2/3}}\,dx$ . . . 2110
3.568 $\int \frac{x^4}{\left(a+bx^3\right)^{2/3}}\,dx$ . . . 2114
3.569 $\int \frac{x}{\left(a+bx^3\right)^{2/3}}\,dx$ . . . 2118
3.570 $\int \frac{1}{x^2\left(a+bx^3\right)^{2/3}}\,dx$ . . . 2122
3.571 $\int \frac{1}{x^5\left(a+bx^3\right)^{2/3}}\,dx$ . . . 2124
3.572 $\int \frac{1}{x^8\left(a+bx^3\right)^{2/3}}\,dx$ . . . 2126
3.573 $\int \frac{1}{x^{11}\left(a+bx^3\right)^{2/3}}\,dx$ . . . 2129
3.574 $\int \frac{x^6}{\left(a+bx^3\right)^{2/3}}\,dx$ . . . 2132
3.575 $\int \frac{x^3}{\left(a+bx^3\right)^{2/3}}\,dx$ . . . 2135
3.576 $\int \frac{1}{\left(a+bx^3\right)^{2/3}}\,dx$ . . . 2138
3.577 $\int \frac{1}{x^3\left(a+bx^3\right)^{2/3}}\,dx$ . . . 2141
3.578 $\int \frac{1}{x^6\left(a+bx^3\right)^{2/3}}\,dx$ . . . 2144
3.579 $\int \frac{1}{\sqrt[3]{a-bx^3}}\,dx$ . . . 2147
3.580 $\int \frac{1}{\sqrt[3]{2+x^3}}\,dx$ . . . 2150
3.581 $\int \frac{x}{\left(1-x^3\right)^{2/3}}\,dx$ . . . 2153
3.582 $\int \frac{x^2}{\sqrt[4]{2+x^3}}\,dx$ . . . 2156
3.583 $\int x^m\left(a+bx^3\right)^8\,dx$ . . . 2158
3.584 $\int x^m\left(a+bx^3\right)^5\,dx$ . . . 2166
3.585 $\int x^m\left(a+bx^3\right)^3\,dx$ . . . 2170
3.586 $\int x^m\left(a+bx^3\right)^2\,dx$ . . . 2173
3.587 $\int x^m\left(a+bx^3\right)\,dx$ . . . 2176
3.588 $\int \frac{x^m}{a+bx^3}\,dx$ . . . 2178
3.589 $\int \frac{x^m}{\left(a+bx^3\right)^2}\,dx$ . . . 2180
3.590 $\int \frac{x^m}{\left(a+bx^3\right)^3}\,dx$ . . . 2183
3.591 $\int x^m\left(a+bx^3\right)^{3/2}\,dx$ . . . 2185
3.592 $\int x^m\sqrt{a+bx^3}\,dx$ . . . 2188

3.593 $\int \frac{x^m}{\sqrt{a+bx^3}}\,dx$ . . . 2191
3.594 $\int \frac{x^m}{\left(a+bx^3\right)^{3/2}}\,dx$ . . . 2194
3.595 $\int (cx)^m \left(a+bx^3\right)^{4/3}\,dx$ . . . 2197
3.596 $\int (cx)^m \left(a+bx^3\right)^{2/3}\,dx$ . . . 2200
3.597 $\int (cx)^m \sqrt[3]{a+bx^3}\,dx$ . . . 2203
3.598 $\int (cx)^m \left(a+bx^3\right)^p\,dx$ . . . 2206
3.599 $\int x^2 \left(a+bx^3\right)^p\,dx$ . . . 2208
3.600 $\int x^5 \left(a+bx^3\right)^p\,dx$ . . . 2211
3.601 $\int x^8 \left(a+bx^3\right)^p\,dx$ . . . 2214
3.602 $\int x^{11} \left(a+bx^3\right)^p\,dx$ . . . 2217
3.603 $\int x^m \left(a+bx^4\right)\,dx$ . . . 2221
3.604 $\int x^5 \left(a+bx^4\right)\,dx$ . . . 2223
3.605 $\int x^4 \left(a+bx^4\right)\,dx$ . . . 2225
3.606 $\int x^3 \left(a+bx^4\right)\,dx$ . . . 2227
3.607 $\int x^2 \left(a+bx^4\right)\,dx$ . . . 2229
3.608 $\int x \left(a+bx^4\right)\,dx$ . . . 2231
3.609 $\int \left(a+bx^4\right)\,dx$ . . . 2233
3.610 $\int \frac{a+bx^4}{x}\,dx$ . . . 2235
3.611 $\int \frac{a+bx^4}{x^2}\,dx$ . . . 2237
3.612 $\int \frac{a+bx^4}{x^3}\,dx$ . . . 2239
3.613 $\int \frac{a+bx^4}{x^4}\,dx$ . . . 2241
3.614 $\int \frac{a+bx^4}{x^5}\,dx$ . . . 2243
3.615 $\int \frac{a+bx^4}{x^6}\,dx$ . . . 2245
3.616 $\int \frac{a+bx^4}{x^7}\,dx$ . . . 2247
3.617 $\int \frac{a+bx^4}{x^8}\,dx$ . . . 2249
3.618 $\int \frac{a+bx^4}{x^9}\,dx$ . . . 2251
3.619 $\int \frac{a+bx^4}{x^{10}}\,dx$ . . . 2253
3.620 $\int x^m \left(a+bx^4\right)^2\,dx$ . . . 2255
3.621 $\int x^5 \left(a+bx^4\right)^2\,dx$ . . . 2258
3.622 $\int x^4 \left(a+bx^4\right)^2\,dx$ . . . 2260
3.623 $\int x^3 \left(a+bx^4\right)^2\,dx$ . . . 2262
3.624 $\int x^2 \left(a+bx^4\right)^2\,dx$ . . . 2264
3.625 $\int x \left(a+bx^4\right)^2\,dx$ . . . 2266
3.626 $\int \left(a+bx^4\right)^2\,dx$ . . . 2268
3.627 $\int \frac{\left(a+bx^4\right)^2}{x}\,dx$ . . . 2270
3.628 $\int \frac{\left(a+bx^4\right)^2}{x^2}\,dx$ . . . 2272
3.629 $\int \frac{\left(a+bx^4\right)^2}{x^3}\,dx$ . . . 2274
3.630 $\int \frac{\left(a+bx^4\right)^2}{x^4}\,dx$ . . . 2276
3.631 $\int \frac{\left(a+bx^4\right)^2}{x^5}\,dx$ . . . 2278

3.632 $\int x^m \left(a+bx^4\right)^3\,dx$ . . . 2280
3.633 $\int x^5 \left(a+bx^4\right)^3\,dx$ . . . 2283
3.634 $\int x^4 \left(a+bx^4\right)^3\,dx$ . . . 2285
3.635 $\int x^3 \left(a+bx^4\right)^3\,dx$ . . . 2287
3.636 $\int x^2 \left(a+bx^4\right)^3\,dx$ . . . 2289
3.637 $\int x \left(a+bx^4\right)^3\,dx$ . . . 2291
3.638 $\int \left(a+bx^4\right)^3\,dx$ . . . 2293
3.639 $\int \frac{\left(a+bx^4\right)^3}{x}\,dx$ . . . 2295
3.640 $\int \frac{\left(a+bx^4\right)^3}{x^2}\,dx$ . . . 2297
3.641 $\int \frac{\left(a+bx^4\right)^3}{x^3}\,dx$ . . . 2299
3.642 $\int \frac{\left(a+bx^4\right)^3}{x^4}\,dx$ . . . 2301
3.643 $\int \frac{\left(a+bx^4\right)^3}{x^5}\,dx$ . . . 2303
3.644 $\int \frac{x^9}{a+cx^4}\,dx$ . . . 2305
3.645 $\int \frac{x^7}{a+cx^4}\,dx$ . . . 2308
3.646 $\int \frac{x^5}{a+cx^4}\,dx$ . . . 2310
3.647 $\int \frac{x^3}{a+cx^4}\,dx$ . . . 2313
3.648 $\int \frac{x}{a+cx^4}\,dx$ . . . 2315
3.649 $\int \frac{1}{x\left(a+cx^4\right)}\,dx$ . . . 2317
3.650 $\int \frac{1}{x^3\left(a+cx^4\right)}\,dx$ . . . 2320
3.651 $\int \frac{1}{x^5\left(a+cx^4\right)}\,dx$ . . . 2323
3.652 $\int \frac{1}{x^7\left(a+cx^4\right)}\,dx$ . . . 2325
3.653 $\int \frac{x^4}{a+cx^4}\,dx$ . . . 2328
3.654 $\int \frac{x^2}{a+cx^4}\,dx$ . . . 2332
3.655 $\int \frac{1}{a+cx^4}\,dx$ . . . 2336
3.656 $\int \frac{1}{x^2\left(a+cx^4\right)}\,dx$ . . . 2340
3.657 $\int \frac{1}{x^4\left(a+cx^4\right)}\,dx$ . . . 2344
3.658 $\int \frac{x^{11}}{\left(a+cx^4\right)^2}\,dx$ . . . 2348
3.659 $\int \frac{x^9}{\left(a+cx^4\right)^2}\,dx$ . . . 2351
3.660 $\int \frac{x^7}{\left(a+cx^4\right)^2}\,dx$ . . . 2354
3.661 $\int \frac{x^5}{\left(a+cx^4\right)^2}\,dx$ . . . 2357
3.662 $\int \frac{x^3}{\left(a+cx^4\right)^2}\,dx$ . . . 2360
3.663 $\int \frac{x}{\left(a+cx^4\right)^2}\,dx$ . . . 2362
3.664 $\int \frac{1}{x\left(a+cx^4\right)^2}\,dx$ . . . 2365
3.665 $\int \frac{1}{x^3\left(a+cx^4\right)^2}\,dx$ . . . 2368

3.666 $\int \frac{x^6}{\left(a+cx^4\right)^2}\,dx$ 2371
3.667 $\int \frac{x^4}{\left(a+cx^4\right)^2}\,dx$ 2375
3.668 $\int \frac{x^2}{\left(a+cx^4\right)^2}\,dx$ 2379
3.669 $\int \frac{1}{\left(a+cx^4\right)^2}\,dx$ 2383
3.670 $\int \frac{1}{x^2\left(a+cx^4\right)^2}\,dx$ 2387
3.671 $\int \frac{1}{x^4\left(a+cx^4\right)^2}\,dx$ 2391
3.672 $\int \frac{x^{11}}{\left(a+cx^4\right)^3}\,dx$ 2395
3.673 $\int \frac{x^9}{\left(a+cx^4\right)^3}\,dx$ 2398
3.674 $\int \frac{x^7}{\left(a+cx^4\right)^3}\,dx$ 2401
3.675 $\int \frac{x^5}{\left(a+cx^4\right)^3}\,dx$ 2403
3.676 $\int \frac{x^3}{\left(a+cx^4\right)^3}\,dx$ 2406
3.677 $\int \frac{x}{\left(a+cx^4\right)^3}\,dx$ 2408
3.678 $\int \frac{1}{x\left(a+cx^4\right)^3}\,dx$ 2411
3.679 $\int \frac{1}{x^3\left(a+cx^4\right)^3}\,dx$ 2414
3.680 $\int \frac{x^{10}}{\left(a+cx^4\right)^3}\,dx$ 2417
3.681 $\int \frac{x^8}{\left(a+cx^4\right)^3}\,dx$ 2421
3.682 $\int \frac{x^6}{\left(a+cx^4\right)^3}\,dx$ 2425
3.683 $\int \frac{x^4}{\left(a+cx^4\right)^3}\,dx$ 2429
3.684 $\int \frac{x^2}{\left(a+cx^4\right)^3}\,dx$ 2433
3.685 $\int \frac{1}{\left(a+cx^4\right)^3}\,dx$ 2437
3.686 $\int \frac{1}{x^2\left(a+cx^4\right)^3}\,dx$ 2441
3.687 $\int \frac{x^9}{2+3x^4}\,dx$ 2446
3.688 $\int \frac{x^7}{2+3x^4}\,dx$ 2449
3.689 $\int \frac{x^5}{2+3x^4}\,dx$ 2451
3.690 $\int \frac{x^3}{2+3x^4}\,dx$ 2454
3.691 $\int \frac{x}{2+3x^4}\,dx$ 2456
3.692 $\int \frac{1}{x\left(2+3x^4\right)}\,dx$ 2458
3.693 $\int \frac{1}{x^3\left(2+3x^4\right)}\,dx$ 2460
3.694 $\int \frac{x^6}{2+3x^4}\,dx$ 2463
3.695 $\int \frac{x^4}{2+3x^4}\,dx$ 2467
3.696 $\int \frac{x^2}{2+3x^4}\,dx$ 2471
3.697 $\int \frac{1}{2+3x^4}\,dx$ 2474

3.698 $\int \frac{1}{x^2\left(2+3x^4\right)}\,dx$ 2477
3.699 $\int \frac{x^3}{\left(2+3x^4\right)^2}\,dx$ 2481
3.700 $\int \frac{x}{\left(2+3x^4\right)^2}\,dx$ 2483
3.701 $\int \frac{1}{x\left(2+3x^4\right)^2}\,dx$ 2486
3.702 $\int \frac{1}{x^3\left(2+3x^4\right)^2}\,dx$ 2488
3.703 $\int \frac{x^4}{\left(2+3x^4\right)^2}\,dx$ 2491
3.704 $\int \frac{x^2}{\left(2+3x^4\right)^2}\,dx$ 2495
3.705 $\int \frac{1}{\left(2+3x^4\right)^2}\,dx$ 2499
3.706 $\int \frac{1}{x^2\left(2+3x^4\right)^2}\,dx$ 2503
3.707 $\int \frac{x^2}{3+x^4}\,dx$ 2507
3.708 $\int \frac{1}{1+a+(-1+a)x^4}\,dx$ 2510
3.709 $\int \frac{1}{2a+2b+x^4}\,dx$ 2514
3.710 $\int \frac{1}{2(a+b)+x^4}\,dx$ 2517
3.711 $\int \frac{x}{2a+2b+x^4}\,dx$ 2520
3.712 $\int \frac{x}{2(a+b)+x^4}\,dx$ 2522
3.713 $\int \frac{x^2}{2a+2b+x^4}\,dx$ 2524
3.714 $\int \frac{x^2}{2(a+b)+x^4}\,dx$ 2527
3.715 $\int \frac{x^3}{2a+2b+x^4}\,dx$ 2530
3.716 $\int \frac{x^3}{2(a+b)+x^4}\,dx$ 2532
3.717 $\int x^{5/2}\left(a+cx^4\right)\,dx$ 2534
3.718 $\int x^{3/2}\left(a+cx^4\right)\,dx$ 2536
3.719 $\int \sqrt{x}\left(a+cx^4\right)\,dx$ 2538
3.720 $\int \frac{a+cx^4}{\sqrt{x}}\,dx$ 2540
3.721 $\int \frac{a+cx^4}{x^{3/2}}\,dx$ 2542
3.722 $\int \frac{a+cx^4}{x^{5/2}}\,dx$ 2544
3.723 $\int \frac{a+cx^4}{x^{7/2}}\,dx$ 2546
3.724 $\int x^{5/2}\left(a+cx^4\right)^2\,dx$ 2548
3.725 $\int x^{3/2}\left(a+cx^4\right)^2\,dx$ 2550
3.726 $\int \sqrt{x}\left(a+cx^4\right)^2\,dx$ 2552
3.727 $\int \frac{\left(a+cx^4\right)^2}{\sqrt{x}}\,dx$ 2554
3.728 $\int \frac{\left(a+cx^4\right)^2}{x^{3/2}}\,dx$ 2556
3.729 $\int \frac{\left(a+cx^4\right)^2}{x^{5/2}}\,dx$ 2558
3.730 $\int \frac{\left(a+cx^4\right)^2}{x^{7/2}}\,dx$ 2560
3.731 $\int x^{5/2}\left(a+cx^4\right)^3\,dx$ 2562
3.732 $\int x^{3/2}\left(a+cx^4\right)^3\,dx$ 2564

3.733 $\int \sqrt{x}\left(a+cx^4\right)^3\,dx$ . . . 2566
3.734 $\int \frac{\left(a+cx^4\right)^3}{\sqrt{x}}\,dx$ . . . 2568
3.735 $\int \frac{\left(a+cx^4\right)^3}{x^{3/2}}\,dx$ . . . 2570
3.736 $\int \frac{\left(a+cx^4\right)^3}{x^{5/2}}\,dx$ . . . 2572
3.737 $\int \frac{\left(a+cx^4\right)^3}{x^{7/2}}\,dx$ . . . 2574
3.738 $\int \frac{x^{9/2}}{a+cx^4}\,dx$ . . . 2576
3.739 $\int \frac{x^{7/2}}{a+cx^4}\,dx$ . . . 2581
3.740 $\int \frac{x^{5/2}}{a+cx^4}\,dx$ . . . 2586
3.741 $\int \frac{x^{3/2}}{a+cx^4}\,dx$ . . . 2591
3.742 $\int \frac{\sqrt{x}}{a+cx^4}\,dx$ . . . 2596
3.743 $\int \frac{1}{\sqrt{x}\left(a+cx^4\right)}\,dx$ . . . 2601
3.744 $\int \frac{1}{x^{3/2}\left(a+cx^4\right)}\,dx$ . . . 2606
3.745 $\int \frac{1}{x^{5/2}\left(a+cx^4\right)}\,dx$ . . . 2611
3.746 $\int \frac{x^{13/2}}{\left(a+cx^4\right)^2}\,dx$ . . . 2616
3.747 $\int \frac{x^{11/2}}{\left(a+cx^4\right)^2}\,dx$ . . . 2621
3.748 $\int \frac{x^{9/2}}{\left(a+cx^4\right)^2}\,dx$ . . . 2626
3.749 $\int \frac{x^{7/2}}{\left(a+cx^4\right)^2}\,dx$ . . . 2631
3.750 $\int \frac{x^{5/2}}{\left(a+cx^4\right)^2}\,dx$ . . . 2636
3.751 $\int \frac{x^{3/2}}{\left(a+cx^4\right)^2}\,dx$ . . . 2641
3.752 $\int \frac{\sqrt{x}}{\left(a+cx^4\right)^2}\,dx$ . . . 2646
3.753 $\int \frac{1}{\sqrt{x}\left(a+cx^4\right)^2}\,dx$ . . . 2651
3.754 $\int \frac{1}{x^{3/2}\left(a+cx^4\right)^2}\,dx$ . . . 2656
3.755 $\int \frac{x^{15/2}}{\left(a+cx^4\right)^3}\,dx$ . . . 2661
3.756 $\int \frac{x^{13/2}}{\left(a+cx^4\right)^3}\,dx$ . . . 2666
3.757 $\int \frac{x^{11/2}}{\left(a+cx^4\right)^3}\,dx$ . . . 2671
3.758 $\int \frac{x^{9/2}}{\left(a+cx^4\right)^3}\,dx$ . . . 2676
3.759 $\int \frac{x^{7/2}}{\left(a+cx^4\right)^3}\,dx$ . . . 2681
3.760 $\int \frac{x^{5/2}}{\left(a+cx^4\right)^3}\,dx$ . . . 2686
3.761 $\int \frac{x^{3/2}}{\left(a+cx^4\right)^3}\,dx$ . . . 2691
3.762 $\int \frac{\sqrt{x}}{\left(a+cx^4\right)^3}\,dx$ . . . 2696

3.763 $\int \frac{1}{\sqrt{x}\left(a+cx^4\right)^3}\,dx$ 2701
3.764 $\int x^{11}\sqrt{a+cx^4}\,dx$ 2706
3.765 $\int x^7\sqrt{a+cx^4}\,dx$ 2709
3.766 $\int x^3\sqrt{a+cx^4}\,dx$ 2711
3.767 $\int \frac{\sqrt{a+cx^4}}{x}\,dx$ 2713
3.768 $\int \frac{\sqrt{a+cx^4}}{x^5}\,dx$ 2716
3.769 $\int \frac{\sqrt{a+cx^4}}{x^9}\,dx$ 2719
3.770 $\int x^5\sqrt{a+cx^4}\,dx$ 2722
3.771 $\int x\sqrt{a+cx^4}\,dx$ 2725
3.772 $\int \frac{\sqrt{a+cx^4}}{x^3}\,dx$ 2728
3.773 $\int \frac{\sqrt{a+cx^4}}{x^7}\,dx$ 2731
3.774 $\int \frac{\sqrt{a+cx^4}}{x^{11}}\,dx$ 2733
3.775 $\int \frac{\sqrt{a+cx^4}}{x^{15}}\,dx$ 2735
3.776 $\int x^4\sqrt{a+cx^4}\,dx$ 2738
3.777 $\int \sqrt{a+cx^4}\,dx$ 2741
3.778 $\int \frac{\sqrt{a+cx^4}}{x^4}\,dx$ 2744
3.779 $\int \frac{\sqrt{a+cx^4}}{x^8}\,dx$ 2747
3.780 $\int x^2\sqrt{a+cx^4}\,dx$ 2750
3.781 $\int \frac{\sqrt{a+cx^4}}{x^2}\,dx$ 2753
3.782 $\int \frac{\sqrt{a+cx^4}}{x^6}\,dx$ 2756
3.783 $\int x^{11}\left(a+cx^4\right)^{3/2}\,dx$ 2759
3.784 $\int x^7\left(a+cx^4\right)^{3/2}\,dx$ 2762
3.785 $\int x^3\left(a+cx^4\right)^{3/2}\,dx$ 2764
3.786 $\int \frac{\left(a+cx^4\right)^{3/2}}{x}\,dx$ 2766
3.787 $\int \frac{\left(a+cx^4\right)^{3/2}}{x^5}\,dx$ 2769
3.788 $\int \frac{\left(a+cx^4\right)^{3/2}}{x^9}\,dx$ 2772
3.789 $\int x^5\left(a+cx^4\right)^{3/2}\,dx$ 2775
3.790 $\int x\left(a+cx^4\right)^{3/2}\,dx$ 2778
3.791 $\int \frac{\left(a+cx^4\right)^{3/2}}{x^3}\,dx$ 2781
3.792 $\int \frac{\left(a+cx^4\right)^{3/2}}{x^7}\,dx$ 2784
3.793 $\int \frac{\left(a+cx^4\right)^{3/2}}{x^{11}}\,dx$ 2787
3.794 $\int \frac{\left(a+cx^4\right)^{3/2}}{x^{15}}\,dx$ 2789
3.795 $\int \frac{\left(a+cx^4\right)^{3/2}}{x^{19}}\,dx$ 2792
3.796 $\int x^4\left(a+cx^4\right)^{3/2}\,dx$ 2795
3.797 $\int \left(a+cx^4\right)^{3/2}\,dx$ 2798
3.798 $\int \frac{\left(a+cx^4\right)^{3/2}}{x^4}\,dx$ 2801

3.799 $\int \frac{\left(a+cx^4\right)^{3/2}}{x^8}\,dx$ 2804
3.800 $\int x^2\left(a+cx^4\right)^{3/2}\,dx$ 2807
3.801 $\int \frac{\left(a+cx^4\right)^{3/2}}{x^2}\,dx$ 2810
3.802 $\int \frac{\left(a+cx^4\right)^{3/2}}{x^6}\,dx$ 2813
3.803 $\int \left(1+x^4\right)^{3/2}\,dx$ 2816
3.804 $\int \left(1-x^4\right)^{3/2}\,dx$ 2819
3.805 $\int x^7\sqrt{5+3x^4}\,dx$ 2822
3.806 $\int x^3\sqrt{5+x^4}\,dx$ 2824
3.807 $\int x\sqrt{3+2x^4}\,dx$ 2826
3.808 $\int x\sqrt{-2+x^4}\,dx$ 2829
3.809 $\int \sqrt{1+x^4}\,dx$ 2832
3.810 $\int \sqrt{1-x^4}\,dx$ 2835
3.811 $\int \frac{x^{11}}{\sqrt{a+bx^4}}\,dx$ 2837
3.812 $\int \frac{x^7}{\sqrt{a+bx^4}}\,dx$ 2840
3.813 $\int \frac{x^3}{\sqrt{a+bx^4}}\,dx$ 2843
3.814 $\int \frac{1}{x\sqrt{a+bx^4}}\,dx$ 2845
3.815 $\int \frac{1}{x^5\sqrt{a+bx^4}}\,dx$ 2848
3.816 $\int \frac{x^5}{\sqrt{a+bx^4}}\,dx$ 2851
3.817 $\int \frac{x}{\sqrt{a+bx^4}}\,dx$ 2854
3.818 $\int \frac{1}{x^3\sqrt{a+bx^4}}\,dx$ 2857
3.819 $\int \frac{1}{x^7\sqrt{a+bx^4}}\,dx$ 2859
3.820 $\int \frac{1}{x^{11}\sqrt{a+bx^4}}\,dx$ 2861
3.821 $\int \frac{x^8}{\sqrt{a+bx^4}}\,dx$ 2864
3.822 $\int \frac{x^4}{\sqrt{a+bx^4}}\,dx$ 2867
3.823 $\int \frac{1}{\sqrt{a+bx^4}}\,dx$ 2870
3.824 $\int \frac{1}{x^4\sqrt{a+bx^4}}\,dx$ 2873
3.825 $\int \frac{1}{x^8\sqrt{a+bx^4}}\,dx$ 2876
3.826 $\int \frac{x^{10}}{\sqrt{a+bx^4}}\,dx$ 2879
3.827 $\int \frac{x^6}{\sqrt{a+bx^4}}\,dx$ 2882
3.828 $\int \frac{x^2}{\sqrt{a+bx^4}}\,dx$ 2885
3.829 $\int \frac{1}{x^2\sqrt{a+bx^4}}\,dx$ 2888
3.830 $\int \frac{1}{x^6\sqrt{a+bx^4}}\,dx$ 2891
3.831 $\int \frac{x^{11}}{\sqrt{a-bx^4}}\,dx$ 2894
3.832 $\int \frac{x^7}{\sqrt{a-bx^4}}\,dx$ 2897
3.833 $\int \frac{x^3}{\sqrt{a-bx^4}}\,dx$ 2900
3.834 $\int \frac{1}{x\sqrt{a-bx^4}}\,dx$ 2902
3.835 $\int \frac{1}{x^5\sqrt{a-bx^4}}\,dx$ 2905

3.836 $\int \frac{x^5}{\sqrt{a-bx^4}}\,dx$ . . . 2908
3.837 $\int \frac{x}{\sqrt{a-bx^4}}\,dx$ . . . 2911
3.838 $\int \frac{1}{x^3\sqrt{a-bx^4}}\,dx$ . . . 2914
3.839 $\int \frac{1}{x^7\sqrt{a-bx^4}}\,dx$ . . . 2916
3.840 $\int \frac{1}{x^{11}\sqrt{a-bx^4}}\,dx$ . . . 2919
3.841 $\int \frac{x^8}{\sqrt{a-bx^4}}\,dx$ . . . 2922
3.842 $\int \frac{x^4}{\sqrt{a-bx^4}}\,dx$ . . . 2925
3.843 $\int \frac{1}{\sqrt{a-bx^4}}\,dx$ . . . 2928
3.844 $\int \frac{1}{x^4\sqrt{a-bx^4}}\,dx$ . . . 2931
3.845 $\int \frac{1}{x^8\sqrt{a-bx^4}}\,dx$ . . . 2934
3.846 $\int \frac{x^{10}}{\sqrt{a-bx^4}}\,dx$ . . . 2937
3.847 $\int \frac{x^6}{\sqrt{a-bx^4}}\,dx$ . . . 2941
3.848 $\int \frac{x^2}{\sqrt{a-bx^4}}\,dx$ . . . 2945
3.849 $\int \frac{1}{x^2\sqrt{a-bx^4}}\,dx$ . . . 2948
3.850 $\int \frac{1}{x^6\sqrt{a-bx^4}}\,dx$ . . . 2952
3.851 $\int \frac{x^{11}}{\left(a+bx^4\right)^{3/2}}\,dx$ . . . 2956
3.852 $\int \frac{x^7}{\left(a+bx^4\right)^{3/2}}\,dx$ . . . 2959
3.853 $\int \frac{x^3}{\left(a+bx^4\right)^{3/2}}\,dx$ . . . 2962
3.854 $\int \frac{1}{x\left(a+bx^4\right)^{3/2}}\,dx$ . . . 2964
3.855 $\int \frac{1}{x^5\left(a+bx^4\right)^{3/2}}\,dx$ . . . 2967
3.856 $\int \frac{x^9}{\left(a+bx^4\right)^{3/2}}\,dx$ . . . 2970
3.857 $\int \frac{x^5}{\left(a+bx^4\right)^{3/2}}\,dx$ . . . 2973
3.858 $\int \frac{x}{\left(a+bx^4\right)^{3/2}}\,dx$ . . . 2976
3.859 $\int \frac{1}{x^3\left(a+bx^4\right)^{3/2}}\,dx$ . . . 2978
3.860 $\int \frac{1}{x^7\left(a+bx^4\right)^{3/2}}\,dx$ . . . 2980
3.861 $\int \frac{x^{12}}{\left(a+bx^4\right)^{3/2}}\,dx$ . . . 2983
3.862 $\int \frac{x^8}{\left(a+bx^4\right)^{3/2}}\,dx$ . . . 2986
3.863 $\int \frac{x^4}{\left(a+bx^4\right)^{3/2}}\,dx$ . . . 2989
3.864 $\int \frac{1}{\left(a+bx^4\right)^{3/2}}\,dx$ . . . 2992
3.865 $\int \frac{1}{x^4\left(a+bx^4\right)^{3/2}}\,dx$ . . . 2995
3.866 $\int \frac{1}{x^8\left(a+bx^4\right)^{3/2}}\,dx$ . . . 2998
3.867 $\int \frac{x^{14}}{\left(a+bx^4\right)^{3/2}}\,dx$ . . . 3001

3.868 $\int \frac{x^{10}}{\left(a+bx^4\right)^{3/2}}\,dx$ 3005
3.869 $\int \frac{x^6}{\left(a+bx^4\right)^{3/2}}\,dx$ 3008
3.870 $\int \frac{x^2}{\left(a+bx^4\right)^{3/2}}\,dx$ 3011
3.871 $\int \frac{1}{x^2\left(a+bx^4\right)^{3/2}}\,dx$ 3014
3.872 $\int \frac{1}{x^6\left(a+bx^4\right)^{3/2}}\,dx$ 3017
3.873 $\int \frac{1}{\left(a+bx^4\right)^{5/2}}\,dx$ 3021
3.874 $\int \frac{x^{11}}{\sqrt{1-x^4}}\,dx$ 3024
3.875 $\int \frac{x^7}{\sqrt{1-x^4}}\,dx$ 3026
3.876 $\int \frac{x^3}{\sqrt{1-x^4}}\,dx$ 3028
3.877 $\int \frac{1}{x\sqrt{1-x^4}}\,dx$ 3030
3.878 $\int \frac{1}{x^5\sqrt{1-x^4}}\,dx$ 3033
3.879 $\int \frac{x^5}{\sqrt{1-x^4}}\,dx$ 3036
3.880 $\int \frac{x}{\sqrt{1-x^4}}\,dx$ 3039
3.881 $\int \frac{1}{x^3\sqrt{1-x^4}}\,dx$ 3041
3.882 $\int \frac{1}{x^7\sqrt{1-x^4}}\,dx$ 3043
3.883 $\int \frac{1}{x^{11}\sqrt{1-x^4}}\,dx$ 3046
3.884 $\int \frac{x^8}{\sqrt{1-x^4}}\,dx$ 3049
3.885 $\int \frac{x^4}{\sqrt{1-x^4}}\,dx$ 3052
3.886 $\int \frac{1}{\sqrt{1-x^4}}\,dx$ 3054
3.887 $\int \frac{1}{x^4\sqrt{1-x^4}}\,dx$ 3056
3.888 $\int \frac{1}{x^8\sqrt{1-x^4}}\,dx$ 3058
3.889 $\int \frac{x^{10}}{\sqrt{1-x^4}}\,dx$ 3061
3.890 $\int \frac{x^6}{\sqrt{1-x^4}}\,dx$ 3064
3.891 $\int \frac{x^2}{\sqrt{1-x^4}}\,dx$ 3067
3.892 $\int \frac{1}{x^2\sqrt{1-x^4}}\,dx$ 3070
3.893 $\int \frac{1}{x^6\sqrt{1-x^4}}\,dx$ 3073
3.894 $\int \frac{x^{11}}{\left(1-x^4\right)^{3/2}}\,dx$ 3076
3.895 $\int \frac{x^7}{\left(1-x^4\right)^{3/2}}\,dx$ 3078
3.896 $\int \frac{x^3}{\left(1-x^4\right)^{3/2}}\,dx$ 3080
3.897 $\int \frac{1}{x\left(1-x^4\right)^{3/2}}\,dx$ 3082
3.898 $\int \frac{1}{x^5\left(1-x^4\right)^{3/2}}\,dx$ 3085
3.899 $\int \frac{x^9}{\left(1-x^4\right)^{3/2}}\,dx$ 3088
3.900 $\int \frac{x^5}{\left(1-x^4\right)^{3/2}}\,dx$ 3091

3.901 $\int \frac{x}{\left(1-x^4\right)^{3/2}}\,dx$ 3094
3.902 $\int \frac{1}{x^3\left(1-x^4\right)^{3/2}}\,dx$ 3096
3.903 $\int \frac{1}{x^7\left(1-x^4\right)^{3/2}}\,dx$ 3099
3.904 $\int \frac{x^{12}}{\left(1-x^4\right)^{3/2}}\,dx$ 3102
3.905 $\int \frac{x^8}{\left(1-x^4\right)^{3/2}}\,dx$ 3105
3.906 $\int \frac{x^4}{\left(1-x^4\right)^{3/2}}\,dx$ 3108
3.907 $\int \frac{1}{\left(1-x^4\right)^{3/2}}\,dx$ 3111
3.908 $\int \frac{1}{x^4\left(1-x^4\right)^{3/2}}\,dx$ 3113
3.909 $\int \frac{1}{x^8\left(1-x^4\right)^{3/2}}\,dx$ 3116
3.910 $\int \frac{x^{14}}{\left(1-x^4\right)^{3/2}}\,dx$ 3119
3.911 $\int \frac{x^{10}}{\left(1-x^4\right)^{3/2}}\,dx$ 3122
3.912 $\int \frac{x^6}{\left(1-x^4\right)^{3/2}}\,dx$ 3125
3.913 $\int \frac{x^2}{\left(1-x^4\right)^{3/2}}\,dx$ 3128
3.914 $\int \frac{1}{x^2\left(1-x^4\right)^{3/2}}\,dx$ 3131
3.915 $\int \frac{1}{x^6\left(1-x^4\right)^{3/2}}\,dx$ 3134
3.916 $\int \frac{1}{\left(1-x^4\right)^{5/2}}\,dx$ 3137
3.917 $\int \frac{x^{11}}{\sqrt{1+x^4}}\,dx$ 3140
3.918 $\int \frac{x^7}{\sqrt{1+x^4}}\,dx$ 3142
3.919 $\int \frac{x^3}{\sqrt{1+x^4}}\,dx$ 3144
3.920 $\int \frac{1}{x\sqrt{1+x^4}}\,dx$ 3146
3.921 $\int \frac{1}{x^5\sqrt{1+x^4}}\,dx$ 3148
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3.923 $\int \frac{x}{\sqrt{1+x^4}}\,dx$ 3153
3.924 $\int \frac{1}{x^3\sqrt{1+x^4}}\,dx$ 3155
3.925 $\int \frac{1}{x^7\sqrt{1+x^4}}\,dx$ 3157
3.926 $\int \frac{1}{x^{11}\sqrt{1+x^4}}\,dx$ 3159
3.927 $\int \frac{x^8}{\sqrt{1+x^4}}\,dx$ 3162
3.928 $\int \frac{x^4}{\sqrt{1+x^4}}\,dx$ 3165
3.929 $\int \frac{1}{\sqrt{1+x^4}}\,dx$ 3168
3.930 $\int \frac{1}{x^4\sqrt{1+x^4}}\,dx$ 3170
3.931 $\int \frac{1}{x^8\sqrt{1+x^4}}\,dx$ 3173
3.932 $\int \frac{x^{10}}{\sqrt{1+x^4}}\,dx$ 3176

3.933 $\int \frac{x^6}{\sqrt{1+x^4}}\,dx$ . . . 3179
3.934 $\int \frac{x^2}{\sqrt{1+x^4}}\,dx$ . . . 3182
3.935 $\int \frac{1}{x^2\sqrt{1+x^4}}\,dx$ . . . 3185
3.936 $\int \frac{1}{x^6\sqrt{1+x^4}}\,dx$ . . . 3188
3.937 $\int \frac{x^{11}}{\left(1+x^4\right)^{3/2}}\,dx$ . . . 3191
3.938 $\int \frac{x^7}{\left(1+x^4\right)^{3/2}}\,dx$ . . . 3193
3.939 $\int \frac{x^3}{\left(1+x^4\right)^{3/2}}\,dx$ . . . 3195
3.940 $\int \frac{1}{x\left(1+x^4\right)^{3/2}}\,dx$ . . . 3197
3.941 $\int \frac{1}{x^5\left(1+x^4\right)^{3/2}}\,dx$ . . . 3200
3.942 $\int \frac{x^9}{\left(1+x^4\right)^{3/2}}\,dx$ . . . 3203
3.943 $\int \frac{x^5}{\left(1+x^4\right)^{3/2}}\,dx$ . . . 3206
3.944 $\int \frac{x}{\left(1+x^4\right)^{3/2}}\,dx$ . . . 3209
3.945 $\int \frac{1}{x^3\left(1+x^4\right)^{3/2}}\,dx$ . . . 3211
3.946 $\int \frac{1}{x^7\left(1+x^4\right)^{3/2}}\,dx$ . . . 3213
3.947 $\int \frac{x^{12}}{\left(1+x^4\right)^{3/2}}\,dx$ . . . 3216
3.948 $\int \frac{x^8}{\left(1+x^4\right)^{3/2}}\,dx$ . . . 3219
3.949 $\int \frac{x^4}{\left(1+x^4\right)^{3/2}}\,dx$ . . . 3222
3.950 $\int \frac{1}{\left(1+x^4\right)^{3/2}}\,dx$ . . . 3225
3.951 $\int \frac{1}{x^4\left(1+x^4\right)^{3/2}}\,dx$ . . . 3228
3.952 $\int \frac{1}{x^8\left(1+x^4\right)^{3/2}}\,dx$ . . . 3231
3.953 $\int \frac{x^{14}}{\left(1+x^4\right)^{3/2}}\,dx$ . . . 3234
3.954 $\int \frac{x^{10}}{\left(1+x^4\right)^{3/2}}\,dx$ . . . 3237
3.955 $\int \frac{x^6}{\left(1+x^4\right)^{3/2}}\,dx$ . . . 3240
3.956 $\int \frac{x^2}{\left(1+x^4\right)^{3/2}}\,dx$ . . . 3243
3.957 $\int \frac{1}{x^2\left(1+x^4\right)^{3/2}}\,dx$ . . . 3246
3.958 $\int \frac{1}{x^6\left(1+x^4\right)^{3/2}}\,dx$ . . . 3249
3.959 $\int \frac{1}{\left(1+x^4\right)^{5/2}}\,dx$ . . . 3252
3.960 $\int \frac{x^7}{\sqrt{16-x^4}}\,dx$ . . . 3255
3.961 $\int \frac{x^5}{\sqrt{16-x^4}}\,dx$ . . . 3257
3.962 $\int \frac{x^3}{\sqrt{16-x^4}}\,dx$ . . . 3260

3.963 $\int \frac{x}{\sqrt{16-x^4}}\,dx$ . . . 3262

3.964 $\int \frac{1}{x\sqrt{16-x^4}}\,dx$ . . . 3264

3.965 $\int \frac{1}{x^3\sqrt{16-x^4}}\,dx$ . . . 3267

3.966 $\int \frac{1}{x^5\sqrt{16-x^4}}\,dx$ . . . 3269

3.967 $\int \frac{1}{x^7\sqrt{16-x^4}}\,dx$ . . . 3272

3.968 $\int \frac{x^6}{\sqrt{16-x^4}}\,dx$ . . . 3275

3.969 $\int \frac{x^4}{\sqrt{16-x^4}}\,dx$ . . . 3278

3.970 $\int \frac{x^2}{\sqrt{16-x^4}}\,dx$ . . . 3280

3.971 $\int \frac{1}{\sqrt{16-x^4}}\,dx$ . . . 3283

3.972 $\int \frac{1}{x^2\sqrt{16-x^4}}\,dx$ . . . 3285

3.973 $\int \frac{1}{x^4\sqrt{16-x^4}}\,dx$ . . . 3288

3.974 $\int \frac{x}{\sqrt{-4+x^4}}\,dx$ . . . 3290

3.975 $\int \frac{x}{\sqrt{4+x^4}}\,dx$ . . . 3293

3.976 $\int \frac{1}{x\sqrt{-1+x^4}}\,dx$ . . . 3295

3.977 $\int \frac{x^4}{\sqrt{-1+x^4}}\,dx$ . . . 3298

3.978 $\int \frac{1}{\sqrt{-1+x^4}}\,dx$ . . . 3301

3.979 $\int \frac{1}{x^4\sqrt{-1+x^4}}\,dx$ . . . 3303

3.980 $\int \frac{x^6}{\sqrt{-1+x^4}}\,dx$ . . . 3306

3.981 $\int \frac{x^2}{\sqrt{-1+x^4}}\,dx$ . . . 3309

3.982 $\int \frac{1}{x^2\sqrt{-1+x^4}}\,dx$ . . . 3312

3.983 $\int \frac{x^2}{\sqrt{3-2x^4}}\,dx$ . . . 3315

3.984 $\int \frac{x^2}{\sqrt{3-bx^4}}\,dx$ . . . 3318

3.985 $\int x^7\sqrt[3]{1+x^4}\,dx$ . . . 3321

3.986 $\int \frac{x^3}{\left(1+x^4\right)^{4/3}}\,dx$ . . . 3323

3.987 $\int \frac{x^3}{\sqrt[3]{1+x^4}}\,dx$ . . . 3325

3.988 $\int x^{19}\sqrt[4]{a+bx^4}\,dx$ . . . 3327

3.989 $\int x^{15}\sqrt[4]{a+bx^4}\,dx$ . . . 3330

3.990 $\int x^{11}\sqrt[4]{a+bx^4}\,dx$ . . . 3333

3.991 $\int x^7\sqrt[4]{a+bx^4}\,dx$ . . . 3336

3.992 $\int x^3\sqrt[4]{a+bx^4}\,dx$ . . . 3339

3.993 $\int \frac{\sqrt[4]{a+bx^4}}{x}\,dx$ . . . 3341

3.994 $\int \frac{\sqrt[4]{a+bx^4}}{x^5}\,dx$ . . . 3344

3.995 $\int \frac{\sqrt[4]{a+bx^4}}{x^9}\,dx$ . . . 3348

3.996 $\int x^9\sqrt[4]{a+bx^4}\,dx$ . . . 3352

3.997 $\int x^5\sqrt[4]{a+bx^4}\,dx$ . . . 3355

3.998 $\int x\sqrt[4]{a+bx^4}\,dx$ . . . 3358

3.999 $\int \frac{\sqrt[4]{a+bx^4}}{x^3}\,dx$ . . . 3361

3.1000 $\int \frac{\sqrt[4]{a+bx^4}}{x^7}\,dx$ . . . 3364
3.1001 $\int \frac{\sqrt[4]{a+bx^4}}{x^{11}}\,dx$ . . . 3367
3.1002 $\int x^6\sqrt[4]{a+bx^4}\,dx$ . . . 3370
3.1003 $\int x^2\sqrt[4]{a+bx^4}\,dx$ . . . 3374
3.1004 $\int \frac{\sqrt[4]{a+bx^4}}{x^2}\,dx$ . . . 3377
3.1005 $\int \frac{\sqrt[4]{a+bx^4}}{x^6}\,dx$ . . . 3380
3.1006 $\int \frac{\sqrt[4]{a+bx^4}}{x^{10}}\,dx$ . . . 3382
3.1007 $\int \frac{\sqrt[4]{a+bx^4}}{x^{14}}\,dx$ . . . 3384
3.1008 $\int \frac{\sqrt[4]{a+bx^4}}{x^{18}}\,dx$ . . . 3387
3.1009 $\int x^{12}\sqrt[4]{a+bx^4}\,dx$ . . . 3390
3.1010 $\int x^8\sqrt[4]{a+bx^4}\,dx$ . . . 3394
3.1011 $\int x^4\sqrt[4]{a+bx^4}\,dx$ . . . 3398
3.1012 $\int \sqrt[4]{a+bx^4}\,dx$ . . . 3401
3.1013 $\int \frac{\sqrt[4]{a+bx^4}}{x^4}\,dx$ . . . 3404
3.1014 $\int \frac{\sqrt[4]{a+bx^4}}{x^8}\,dx$ . . . 3407
3.1015 $\int \frac{\sqrt[4]{a+bx^4}}{x^{12}}\,dx$ . . . 3410
3.1016 $\int \frac{\sqrt[4]{a+bx^4}}{x^{16}}\,dx$ . . . 3414
3.1017 $\int x^{19}\left(a+bx^4\right)^{3/4}\,dx$ . . . 3418
3.1018 $\int x^{15}\left(a+bx^4\right)^{3/4}\,dx$ . . . 3421
3.1019 $\int x^{11}\left(a+bx^4\right)^{3/4}\,dx$ . . . 3424
3.1020 $\int x^7\left(a+bx^4\right)^{3/4}\,dx$ . . . 3427
3.1021 $\int x^3\left(a+bx^4\right)^{3/4}\,dx$ . . . 3430
3.1022 $\int \frac{\left(a+bx^4\right)^{3/4}}{x}\,dx$ . . . 3432
3.1023 $\int \frac{\left(a+bx^4\right)^{3/4}}{x^5}\,dx$ . . . 3436
3.1024 $\int \frac{\left(a+bx^4\right)^{3/4}}{x^9}\,dx$ . . . 3440
3.1025 $\int x^9\left(a+bx^4\right)^{3/4}\,dx$ . . . 3444
3.1026 $\int x^5\left(a+bx^4\right)^{3/4}\,dx$ . . . 3447
3.1027 $\int x\left(a+bx^4\right)^{3/4}\,dx$ . . . 3450
3.1028 $\int \frac{\left(a+bx^4\right)^{3/4}}{x^3}\,dx$ . . . 3453
3.1029 $\int \frac{\left(a+bx^4\right)^{3/4}}{x^7}\,dx$ . . . 3456
3.1030 $\int \frac{\left(a+bx^4\right)^{3/4}}{x^{11}}\,dx$ . . . 3459
3.1031 $\int x^{12}\left(a+bx^4\right)^{3/4}\,dx$ . . . 3462
3.1032 $\int x^8\left(a+bx^4\right)^{3/4}\,dx$ . . . 3466
3.1033 $\int x^4\left(a+bx^4\right)^{3/4}\,dx$ . . . 3470
3.1034 $\int \left(a+bx^4\right)^{3/4}\,dx$ . . . 3474
3.1035 $\int \frac{\left(a+bx^4\right)^{3/4}}{x^4}\,dx$ . . . 3477

3.1036 $\int \frac{\left(a+bx^4\right)^{3/4}}{x^8}\,dx$ . . . 3480
3.1037 $\int \frac{\left(a+bx^4\right)^{3/4}}{x^{12}}\,dx$ . . . 3482
3.1038 $\int \frac{\left(a+bx^4\right)^{3/4}}{x^{16}}\,dx$ . . . 3484
3.1039 $\int \frac{\left(a+bx^4\right)^{3/4}}{x^{20}}\,dx$ . . . 3487
3.1040 $\int x^{10}\left(a+bx^4\right)^{3/4}\,dx$ . . . 3490
3.1041 $\int x^6\left(a+bx^4\right)^{3/4}\,dx$ . . . 3494
3.1042 $\int x^2\left(a+bx^4\right)^{3/4}\,dx$ . . . 3498
3.1043 $\int \frac{\left(a+bx^4\right)^{3/4}}{x^2}\,dx$ . . . 3501
3.1044 $\int \frac{\left(a+bx^4\right)^{3/4}}{x^6}\,dx$ . . . 3504
3.1045 $\int \frac{\left(a+bx^4\right)^{3/4}}{x^{10}}\,dx$ . . . 3507
3.1046 $\int \frac{\left(a+bx^4\right)^{3/4}}{x^{14}}\,dx$ . . . 3511
3.1047 $\int x^{19}\left(a+bx^4\right)^{5/4}\,dx$ . . . 3515
3.1048 $\int x^{15}\left(a+bx^4\right)^{5/4}\,dx$ . . . 3518
3.1049 $\int x^{11}\left(a+bx^4\right)^{5/4}\,dx$ . . . 3521
3.1050 $\int x^7\left(a+bx^4\right)^{5/4}\,dx$ . . . 3524
3.1051 $\int x^3\left(a+bx^4\right)^{5/4}\,dx$ . . . 3527
3.1052 $\int \frac{\left(a+bx^4\right)^{5/4}}{x}\,dx$ . . . 3529
3.1053 $\int \frac{\left(a+bx^4\right)^{5/4}}{x^5}\,dx$ . . . 3533
3.1054 $\int \frac{\left(a+bx^4\right)^{5/4}}{x^9}\,dx$ . . . 3537
3.1055 $\int x^9\left(a+bx^4\right)^{5/4}\,dx$ . . . 3541
3.1056 $\int x^5\left(a+bx^4\right)^{5/4}\,dx$ . . . 3544
3.1057 $\int x\left(a+bx^4\right)^{5/4}\,dx$ . . . 3547
3.1058 $\int \frac{\left(a+bx^4\right)^{5/4}}{x^3}\,dx$ . . . 3550
3.1059 $\int \frac{\left(a+bx^4\right)^{5/4}}{x^7}\,dx$ . . . 3553
3.1060 $\int \frac{\left(a+bx^4\right)^{5/4}}{x^{11}}\,dx$ . . . 3556
3.1061 $\int \frac{\left(a+bx^4\right)^{5/4}}{x^{15}}\,dx$ . . . 3559
3.1062 $\int x^{10}\left(a+bx^4\right)^{5/4}\,dx$ . . . 3562
3.1063 $\int x^6\left(a+bx^4\right)^{5/4}\,dx$ . . . 3566
3.1064 $\int x^2\left(a+bx^4\right)^{5/4}\,dx$ . . . 3570
3.1065 $\int \frac{\left(a+bx^4\right)^{5/4}}{x^2}\,dx$ . . . 3573
3.1066 $\int \frac{\left(a+bx^4\right)^{5/4}}{x^6}\,dx$ . . . 3576
3.1067 $\int \frac{\left(a+bx^4\right)^{5/4}}{x^{10}}\,dx$ . . . 3579
3.1068 $\int \frac{\left(a+bx^4\right)^{5/4}}{x^{14}}\,dx$ . . . 3581
3.1069 $\int \frac{\left(a+bx^4\right)^{5/4}}{x^{18}}\,dx$ . . . 3584

3.1070 $\int \frac{\left(a+bx^4\right)^{5/4}}{x^{22}}\,dx$ . . . 3587
3.1071 $\int x^{12}\left(a+bx^4\right)^{5/4}\,dx$ . . . 3590
3.1072 $\int x^{8}\left(a+bx^4\right)^{5/4}\,dx$ . . . 3594
3.1073 $\int x^{4}\left(a+bx^4\right)^{5/4}\,dx$ . . . 3598
3.1074 $\int \left(a+bx^4\right)^{5/4}\,dx$ . . . 3601
3.1075 $\int \frac{\left(a+bx^4\right)^{5/4}}{x^{4}}\,dx$ . . . 3604
3.1076 $\int \frac{\left(a+bx^4\right)^{5/4}}{x^{8}}\,dx$ . . . 3607
3.1077 $\int \frac{\left(a+bx^4\right)^{5/4}}{x^{12}}\,dx$ . . . 3610
3.1078 $\int \frac{\left(a+bx^4\right)^{5/4}}{x^{16}}\,dx$ . . . 3614
3.1079 $\int \left(a+bx^4\right)^{7/4}\,dx$ . . . 3618
3.1080 $\int \frac{x^{19}}{\sqrt[4]{a+bx^4}}\,dx$ . . . 3621
3.1081 $\int \frac{x^{15}}{\sqrt[4]{a+bx^4}}\,dx$ . . . 3624
3.1082 $\int \frac{x^{11}}{\sqrt[4]{a+bx^4}}\,dx$ . . . 3627
3.1083 $\int \frac{x^{7}}{\sqrt[4]{a+bx^4}}\,dx$ . . . 3630
3.1084 $\int \frac{x^{3}}{\sqrt[4]{a+bx^4}}\,dx$ . . . 3633
3.1085 $\int \frac{1}{x\sqrt[4]{a+bx^4}}\,dx$ . . . 3635
3.1086 $\int \frac{1}{x^5\sqrt[4]{a+bx^4}}\,dx$ . . . 3638
3.1087 $\int \frac{1}{x^9\sqrt[4]{a+bx^4}}\,dx$ . . . 3642
3.1088 $\int \frac{x^{13}}{\sqrt[4]{a+bx^4}}\,dx$ . . . 3646
3.1089 $\int \frac{x^{9}}{\sqrt[4]{a+bx^4}}\,dx$ . . . 3649
3.1090 $\int \frac{x^{5}}{\sqrt[4]{a+bx^4}}\,dx$ . . . 3652
3.1091 $\int \frac{x}{\sqrt[4]{a+bx^4}}\,dx$ . . . 3655
3.1092 $\int \frac{1}{x^3\sqrt[4]{a+bx^4}}\,dx$ . . . 3658
3.1093 $\int \frac{1}{x^7\sqrt[4]{a+bx^4}}\,dx$ . . . 3661
3.1094 $\int \frac{1}{x^{11}\sqrt[4]{a+bx^4}}\,dx$ . . . 3664
3.1095 $\int \frac{x^{8}}{\sqrt[4]{a+bx^4}}\,dx$ . . . 3667
3.1096 $\int \frac{x^{4}}{\sqrt[4]{a+bx^4}}\,dx$ . . . 3670
3.1097 $\int \frac{1}{\sqrt[4]{a+bx^4}}\,dx$ . . . 3673
3.1098 $\int \frac{1}{x^4\sqrt[4]{a+bx^4}}\,dx$ . . . 3676
3.1099 $\int \frac{1}{x^8\sqrt[4]{a+bx^4}}\,dx$ . . . 3678
3.1100 $\int \frac{1}{x^{12}\sqrt[4]{a+bx^4}}\,dx$ . . . 3680
3.1101 $\int \frac{1}{x^{16}\sqrt[4]{a+bx^4}}\,dx$ . . . 3683
3.1102 $\int \frac{1}{x^{20}\sqrt[4]{a+bx^4}}\,dx$ . . . 3686
3.1103 $\int \frac{x^{10}}{\sqrt[4]{a+bx^4}}\,dx$ . . . 3689

3.1104 $\int \frac{x^6}{\sqrt[4]{a+bx^4}}\,dx$ . . . 3692
3.1105 $\int \frac{x^2}{\sqrt[4]{a+bx^4}}\,dx$ . . . 3695
3.1106 $\int \frac{1}{x^2\sqrt[4]{a+bx^4}}\,dx$ . . . 3698
3.1107 $\int \frac{1}{x^6\sqrt[4]{a+bx^4}}\,dx$ . . . 3701
3.1108 $\int \frac{1}{x^{10}\sqrt[4]{a+bx^4}}\,dx$ . . . 3704
3.1109 $\int \frac{1}{x^{14}\sqrt[4]{a+bx^4}}\,dx$ . . . 3707
3.1110 $\int \frac{x^{19}}{\left(a+bx^4\right)^{3/4}}\,dx$ . . . 3711
3.1111 $\int \frac{x^{15}}{\left(a+bx^4\right)^{3/4}}\,dx$ . . . 3714
3.1112 $\int \frac{x^{11}}{\left(a+bx^4\right)^{3/4}}\,dx$ . . . 3717
3.1113 $\int \frac{x^7}{\left(a+bx^4\right)^{3/4}}\,dx$ . . . 3720
3.1114 $\int \frac{x^3}{\left(a+bx^4\right)^{3/4}}\,dx$ . . . 3723
3.1115 $\int \frac{1}{x\left(a+bx^4\right)^{3/4}}\,dx$ . . . 3725
3.1116 $\int \frac{1}{x^5\left(a+bx^4\right)^{3/4}}\,dx$ . . . 3728
3.1117 $\int \frac{1}{x^9\left(a+bx^4\right)^{3/4}}\,dx$ . . . 3732
3.1118 $\int \frac{x^{13}}{\left(a+bx^4\right)^{3/4}}\,dx$ . . . 3736
3.1119 $\int \frac{x^9}{\left(a+bx^4\right)^{3/4}}\,dx$ . . . 3739
3.1120 $\int \frac{x^5}{\left(a+bx^4\right)^{3/4}}\,dx$ . . . 3742
3.1121 $\int \frac{x}{\left(a+bx^4\right)^{3/4}}\,dx$ . . . 3745
3.1122 $\int \frac{1}{x^3\left(a+bx^4\right)^{3/4}}\,dx$ . . . 3748
3.1123 $\int \frac{1}{x^7\left(a+bx^4\right)^{3/4}}\,dx$ . . . 3751
3.1124 $\int \frac{1}{x^{11}\left(a+bx^4\right)^{3/4}}\,dx$ . . . 3754
3.1125 $\int \frac{x^{10}}{\left(a+bx^4\right)^{3/4}}\,dx$ . . . 3757
3.1126 $\int \frac{x^6}{\left(a+bx^4\right)^{3/4}}\,dx$ . . . 3761
3.1127 $\int \frac{x^2}{\left(a+bx^4\right)^{3/4}}\,dx$ . . . 3764
3.1128 $\int \frac{1}{x^2\left(a+bx^4\right)^{3/4}}\,dx$ . . . 3767
3.1129 $\int \frac{1}{x^6\left(a+bx^4\right)^{3/4}}\,dx$ . . . 3769
3.1130 $\int \frac{1}{x^{10}\left(a+bx^4\right)^{3/4}}\,dx$ . . . 3771
3.1131 $\int \frac{1}{x^{14}\left(a+bx^4\right)^{3/4}}\,dx$ . . . 3774
3.1132 $\int \frac{x^{12}}{\left(a+bx^4\right)^{3/4}}\,dx$ . . . 3777
3.1133 $\int \frac{x^8}{\left(a+bx^4\right)^{3/4}}\,dx$ . . . 3780

3.1134 $\int \frac{x^4}{\left(a+bx^4\right)^{3/4}}\,dx$ . . . 3783
3.1135 $\int \frac{1}{\left(a+bx^4\right)^{3/4}}\,dx$ . . . 3786
3.1136 $\int \frac{1}{x^4\left(a+bx^4\right)^{3/4}}\,dx$ . . . 3789
3.1137 $\int \frac{1}{x^8\left(a+bx^4\right)^{3/4}}\,dx$ . . . 3792
3.1138 $\int \frac{1}{x^{12}\left(a+bx^4\right)^{3/4}}\,dx$ . . . 3795
3.1139 $\int \frac{x^{19}}{\left(a+bx^4\right)^{5/4}}\,dx$ . . . 3798
3.1140 $\int \frac{x^{15}}{\left(a+bx^4\right)^{5/4}}\,dx$ . . . 3801
3.1141 $\int \frac{x^{11}}{\left(a+bx^4\right)^{5/4}}\,dx$ . . . 3804
3.1142 $\int \frac{x^7}{\left(a+bx^4\right)^{5/4}}\,dx$ . . . 3807
3.1143 $\int \frac{x^3}{\left(a+bx^4\right)^{5/4}}\,dx$ . . . 3810
3.1144 $\int \frac{1}{x\left(a+bx^4\right)^{5/4}}\,dx$ . . . 3812
3.1145 $\int \frac{1}{x^5\left(a+bx^4\right)^{5/4}}\,dx$ . . . 3816
3.1146 $\int \frac{1}{x^9\left(a+bx^4\right)^{5/4}}\,dx$ . . . 3820
3.1147 $\int \frac{x^{13}}{\left(a+bx^4\right)^{5/4}}\,dx$ . . . 3824
3.1148 $\int \frac{x^9}{\left(a+bx^4\right)^{5/4}}\,dx$ . . . 3827
3.1149 $\int \frac{x^5}{\left(a+bx^4\right)^{5/4}}\,dx$ . . . 3830
3.1150 $\int \frac{x}{\left(a+bx^4\right)^{5/4}}\,dx$ . . . 3833
3.1151 $\int \frac{1}{x^3\left(a+bx^4\right)^{5/4}}\,dx$ . . . 3836
3.1152 $\int \frac{1}{x^7\left(a+bx^4\right)^{5/4}}\,dx$ . . . 3839
3.1153 $\int \frac{1}{x^{11}\left(a+bx^4\right)^{5/4}}\,dx$ . . . 3842
3.1154 $\int \frac{x^{12}}{\left(a+bx^4\right)^{5/4}}\,dx$ . . . 3845
3.1155 $\int \frac{x^8}{\left(a+bx^4\right)^{5/4}}\,dx$ . . . 3849
3.1156 $\int \frac{x^4}{\left(a+bx^4\right)^{5/4}}\,dx$ . . . 3853
3.1157 $\int \frac{1}{\left(a+bx^4\right)^{5/4}}\,dx$ . . . 3856
3.1158 $\int \frac{1}{x^4\left(a+bx^4\right)^{5/4}}\,dx$ . . . 3858
3.1159 $\int \frac{1}{x^8\left(a+bx^4\right)^{5/4}}\,dx$ . . . 3860
3.1160 $\int \frac{1}{x^{12}\left(a+bx^4\right)^{5/4}}\,dx$ . . . 3863
3.1161 $\int \frac{1}{x^{16}\left(a+bx^4\right)^{5/4}}\,dx$ . . . 3866
3.1162 $\int \frac{x^{14}}{\left(a+bx^4\right)^{5/4}}\,dx$ . . . 3869

3.1163 $\int \frac{x^{10}}{\left(a+bx^4\right)^{5/4}}\,dx$ 3872
3.1164 $\int \frac{x^6}{\left(a+bx^4\right)^{5/4}}\,dx$ 3875
3.1165 $\int \frac{x^2}{\left(a+bx^4\right)^{5/4}}\,dx$ 3878
3.1166 $\int \frac{1}{x^2\left(a+bx^4\right)^{5/4}}\,dx$ 3881
3.1167 $\int \frac{1}{x^6\left(a+bx^4\right)^{5/4}}\,dx$ 3884
3.1168 $\int \frac{1}{x^{10}\left(a+bx^4\right)^{5/4}}\,dx$ 3887
3.1169 $\int \frac{1}{x^{14}\left(a+bx^4\right)^{5/4}}\,dx$ 3890
3.1170 $\int \frac{1}{\left(a+bx^4\right)^{7/4}}\,dx$ 3893
3.1171 $\int \frac{1}{\left(a+bx^4\right)^{9/4}}\,dx$ 3896
3.1172 $\int \frac{1}{\left(a+bx^4\right)^{11/4}}\,dx$ 3899
3.1173 $\int \frac{1}{\left(a+bx^4\right)^{13/4}}\,dx$ 3902
3.1174 $\int \frac{1}{\left(a+bx^4\right)^{17/4}}\,dx$ 3905
3.1175 $\int x^{19}\sqrt[4]{a-bx^4}\,dx$ 3908
3.1176 $\int x^{15}\sqrt[4]{a-bx^4}\,dx$ 3911
3.1177 $\int x^{11}\sqrt[4]{a-bx^4}\,dx$ 3914
3.1178 $\int x^7\sqrt[4]{a-bx^4}\,dx$ 3917
3.1179 $\int x^3\sqrt[4]{a-bx^4}\,dx$ 3920
3.1180 $\int \frac{\sqrt[4]{a-bx^4}}{x}\,dx$ 3922
3.1181 $\int \frac{\sqrt[4]{a-bx^4}}{x^5}\,dx$ 3925
3.1182 $\int \frac{\sqrt[4]{a-bx^4}}{x^9}\,dx$ 3929
3.1183 $\int x^9\sqrt[4]{a-bx^4}\,dx$ 3933
3.1184 $\int x^5\sqrt[4]{a-bx^4}\,dx$ 3936
3.1185 $\int x\sqrt[4]{a-bx^4}\,dx$ 3939
3.1186 $\int \frac{\sqrt[4]{a-bx^4}}{x^3}\,dx$ 3942
3.1187 $\int \frac{\sqrt[4]{a-bx^4}}{x^7}\,dx$ 3945
3.1188 $\int \frac{\sqrt[4]{a-bx^4}}{x^{11}}\,dx$ 3948
3.1189 $\int x^6\sqrt[4]{a-bx^4}\,dx$ 3951
3.1190 $\int x^2\sqrt[4]{a-bx^4}\,dx$ 3956
3.1191 $\int \frac{\sqrt[4]{a-bx^4}}{x^2}\,dx$ 3960
3.1192 $\int \frac{\sqrt[4]{a-bx^4}}{x^6}\,dx$ 3964
3.1193 $\int \frac{\sqrt[4]{a-bx^4}}{x^{10}}\,dx$ 3966
3.1194 $\int \frac{\sqrt[4]{a-bx^4}}{x^{14}}\,dx$ 3969
3.1195 $\int \frac{\sqrt[4]{a-bx^4}}{x^{18}}\,dx$ 3972
3.1196 $\int x^{12}\sqrt[4]{a-bx^4}\,dx$ 3975
3.1197 $\int x^8\sqrt[4]{a-bx^4}\,dx$ 3979
3.1198 $\int x^4\sqrt[4]{a-bx^4}\,dx$ 3983

3.1199 $\int \sqrt[4]{a-bx^4}\,dx$ . . . 3986
3.1200 $\int \frac{\sqrt[4]{a-bx^4}}{x^4}\,dx$ . . . 3989
3.1201 $\int \frac{\sqrt[4]{a-bx^4}}{x^8}\,dx$ . . . 3992
3.1202 $\int \frac{\sqrt[4]{a-bx^4}}{x^{12}}\,dx$ . . . 3995
3.1203 $\int \frac{\sqrt[4]{a-bx^4}}{x^{16}}\,dx$ . . . 3999
3.1204 $\int \frac{x^{19}}{\sqrt[4]{a-bx^4}}\,dx$ . . . 4003
3.1205 $\int \frac{x^{15}}{\sqrt[4]{a-bx^4}}\,dx$ . . . 4006
3.1206 $\int \frac{x^{11}}{\sqrt[4]{a-bx^4}}\,dx$ . . . 4009
3.1207 $\int \frac{x^7}{\sqrt[4]{a-bx^4}}\,dx$ . . . 4012
3.1208 $\int \frac{x^3}{\sqrt[4]{a-bx^4}}\,dx$ . . . 4015
3.1209 $\int \frac{1}{x\sqrt[4]{a-bx^4}}\,dx$ . . . 4017
3.1210 $\int \frac{1}{x^5\sqrt[4]{a-bx^4}}\,dx$ . . . 4020
3.1211 $\int \frac{1}{x^9\sqrt[4]{a-bx^4}}\,dx$ . . . 4024
3.1212 $\int \frac{x^{13}}{\sqrt[4]{a-bx^4}}\,dx$ . . . 4028
3.1213 $\int \frac{x^9}{\sqrt[4]{a-bx^4}}\,dx$ . . . 4031
3.1214 $\int \frac{x^5}{\sqrt[4]{a-bx^4}}\,dx$ . . . 4034
3.1215 $\int \frac{x}{\sqrt[4]{a-bx^4}}\,dx$ . . . 4037
3.1216 $\int \frac{1}{x^3\sqrt[4]{a-bx^4}}\,dx$ . . . 4040
3.1217 $\int \frac{1}{x^7\sqrt[4]{a-bx^4}}\,dx$ . . . 4043
3.1218 $\int \frac{1}{x^{11}\sqrt[4]{a-bx^4}}\,dx$ . . . 4046
3.1219 $\int \frac{x^8}{\sqrt[4]{a-bx^4}}\,dx$ . . . 4049
3.1220 $\int \frac{x^4}{\sqrt[4]{a-bx^4}}\,dx$ . . . 4053
3.1221 $\int \frac{1}{\sqrt[4]{a-bx^4}}\,dx$ . . . 4057
3.1222 $\int \frac{1}{x^4\sqrt[4]{a-bx^4}}\,dx$ . . . 4061
3.1223 $\int \frac{1}{x^8\sqrt[4]{a-bx^4}}\,dx$ . . . 4063
3.1224 $\int \frac{1}{x^{12}\sqrt[4]{a-bx^4}}\,dx$ . . . 4066
3.1225 $\int \frac{1}{x^{16}\sqrt[4]{a-bx^4}}\,dx$ . . . 4069
3.1226 $\int \frac{1}{x^{20}\sqrt[4]{a-bx^4}}\,dx$ . . . 4072
3.1227 $\int \frac{x^{10}}{\sqrt[4]{a-bx^4}}\,dx$ . . . 4076
3.1228 $\int \frac{x^6}{\sqrt[4]{a-bx^4}}\,dx$ . . . 4079
3.1229 $\int \frac{x^2}{\sqrt[4]{a-bx^4}}\,dx$ . . . 4082
3.1230 $\int \frac{1}{x^2\sqrt[4]{a-bx^4}}\,dx$ . . . 4085
3.1231 $\int \frac{1}{x^6\sqrt[4]{a-bx^4}}\,dx$ . . . 4088
3.1232 $\int \frac{1}{x^{10}\sqrt[4]{a-bx^4}}\,dx$ . . . 4091
3.1233 $\int \frac{1}{x^{14}\sqrt[4]{a-bx^4}}\,dx$ . . . 4094

3.1234 $\int \frac{x^{19}}{\left(a-bx^4\right)^{3/4}}\,dx$ 4097
3.1235 $\int \frac{x^{15}}{\left(a-bx^4\right)^{3/4}}\,dx$ 4100
3.1236 $\int \frac{x^{11}}{\left(a-bx^4\right)^{3/4}}\,dx$ 4103
3.1237 $\int \frac{x^{7}}{\left(a-bx^4\right)^{3/4}}\,dx$ 4106
3.1238 $\int \frac{x^{3}}{\left(a-bx^4\right)^{3/4}}\,dx$ 4109
3.1239 $\int \frac{1}{x\left(a-bx^4\right)^{3/4}}\,dx$ 4111
3.1240 $\int \frac{1}{x^5\left(a-bx^4\right)^{3/4}}\,dx$ 4114
3.1241 $\int \frac{1}{x^9\left(a-bx^4\right)^{3/4}}\,dx$ 4118
3.1242 $\int \frac{x^{13}}{\left(a-bx^4\right)^{3/4}}\,dx$ 4122
3.1243 $\int \frac{x^{9}}{\left(a-bx^4\right)^{3/4}}\,dx$ 4125
3.1244 $\int \frac{x^{5}}{\left(a-bx^4\right)^{3/4}}\,dx$ 4128
3.1245 $\int \frac{x}{\left(a-bx^4\right)^{3/4}}\,dx$ 4131
3.1246 $\int \frac{1}{x^3\left(a-bx^4\right)^{3/4}}\,dx$ 4134
3.1247 $\int \frac{1}{x^7\left(a-bx^4\right)^{3/4}}\,dx$ 4137
3.1248 $\int \frac{1}{x^{11}\left(a-bx^4\right)^{3/4}}\,dx$ 4140
3.1249 $\int \frac{x^{10}}{\left(a-bx^4\right)^{3/4}}\,dx$ 4143
3.1250 $\int \frac{x^{6}}{\left(a-bx^4\right)^{3/4}}\,dx$ 4148
3.1251 $\int \frac{x^{2}}{\left(a-bx^4\right)^{3/4}}\,dx$ 4152
3.1252 $\int \frac{1}{x^2\left(a-bx^4\right)^{3/4}}\,dx$ 4156
3.1253 $\int \frac{1}{x^6\left(a-bx^4\right)^{3/4}}\,dx$ 4158
3.1254 $\int \frac{1}{x^{10}\left(a-bx^4\right)^{3/4}}\,dx$ 4161
3.1255 $\int \frac{1}{x^{14}\left(a-bx^4\right)^{3/4}}\,dx$ 4164
3.1256 $\int \frac{x^{12}}{\left(a-bx^4\right)^{3/4}}\,dx$ 4168
3.1257 $\int \frac{x^{8}}{\left(a-bx^4\right)^{3/4}}\,dx$ 4171
3.1258 $\int \frac{x^{4}}{\left(a-bx^4\right)^{3/4}}\,dx$ 4174
3.1259 $\int \frac{1}{\left(a-bx^4\right)^{3/4}}\,dx$ 4177
3.1260 $\int \frac{1}{x^4\left(a-bx^4\right)^{3/4}}\,dx$ 4180
3.1261 $\int \frac{1}{x^8\left(a-bx^4\right)^{3/4}}\,dx$ 4183
3.1262 $\int \frac{1}{x^{12}\left(a-bx^4\right)^{3/4}}\,dx$ 4186

3.1263 $\int \frac{x^2}{\left(a-bx^4\right)^{5/4}}\,dx$ 4189
3.1264 $\int x^7\left(a+bx^4\right)^p\,dx$ 4192
3.1265 $\int x^3\left(a+bx^4\right)^p\,dx$ 4195
3.1266 $\int \frac{\left(a+bx^4\right)^p}{x}\,dx$ 4198
3.1267 $\int \frac{x^{24}}{a+bx^5}\,dx$ 4201
3.1268 $\int \frac{x^{19}}{a+bx^5}\,dx$ 4204
3.1269 $\int \frac{x^{14}}{a+bx^5}\,dx$ 4207
3.1270 $\int \frac{x^9}{a+bx^5}\,dx$ 4209
3.1271 $\int \frac{x^4}{a+bx^5}\,dx$ 4211
3.1272 $\int \frac{1}{x\left(a+bx^5\right)}\,dx$ 4213
3.1273 $\int \frac{1}{x^6\left(a+bx^5\right)}\,dx$ 4216
3.1274 $\int \frac{1}{x^{11}\left(a+bx^5\right)}\,dx$ 4218
3.1275 $\int \frac{1}{x^{16}\left(a+bx^5\right)}\,dx$ 4221
3.1276 $\int \frac{1}{a+bx^5}\,dx$ 4224
3.1277 $\int \frac{x^{24}}{\left(a+bx^5\right)^2}\,dx$ 4228
3.1278 $\int \frac{x^{19}}{\left(a+bx^5\right)^2}\,dx$ 4231
3.1279 $\int \frac{x^{14}}{\left(a+bx^5\right)^2}\,dx$ 4234
3.1280 $\int \frac{x^9}{\left(a+bx^5\right)^2}\,dx$ 4237
3.1281 $\int \frac{x^4}{\left(a+bx^5\right)^2}\,dx$ 4240
3.1282 $\int \frac{1}{x\left(a+bx^5\right)^2}\,dx$ 4242
3.1283 $\int \frac{1}{x^6\left(a+bx^5\right)^2}\,dx$ 4245
3.1284 $\int \frac{1}{x^{11}\left(a+bx^5\right)^2}\,dx$ 4248
3.1285 $\int \frac{x^{14}}{2b+bx^5}\,dx$ 4251
3.1286 $\int \frac{x^9}{2b+bx^5}\,dx$ 4253
3.1287 $\int \frac{x^4}{2b+bx^5}\,dx$ 4255
3.1288 $\int \frac{1}{x\left(2b+bx^5\right)}\,dx$ 4257
3.1289 $\int \frac{1}{x^6\left(2b+bx^5\right)}\,dx$ 4260
3.1290 $\int \frac{x^{14}}{3+bx^5}\,dx$ 4262
3.1291 $\int \frac{x^9}{3+bx^5}\,dx$ 4264
3.1292 $\int \frac{x^4}{3+bx^5}\,dx$ 4266
3.1293 $\int \frac{1}{x\left(3+bx^5\right)}\,dx$ 4268
3.1294 $\int \frac{1}{x^6\left(3+bx^5\right)}\,dx$ 4270
3.1295 $\int \frac{x^{14}}{1+x^5}\,dx$ 4272
3.1296 $\int \frac{x^9}{1+x^5}\,dx$ 4274

3.1297 $\int \frac{x^4}{1+x^5}\,dx$ 4276
3.1298 $\int \frac{1}{x\left(1+x^5\right)}\,dx$ 4278
3.1299 $\int \frac{1}{x^6\left(1+x^5\right)}\,dx$ 4280
3.1300 $\int \frac{x^5}{1+x^5}\,dx$ 4282
3.1301 $\int \frac{x^3}{1+x^5}\,dx$ 4286
3.1302 $\int \frac{x^2}{1+x^5}\,dx$ 4290
3.1303 $\int \frac{x}{1+x^5}\,dx$ 4294
3.1304 $\int \frac{1}{1+x^5}\,dx$ 4298
3.1305 $\int \frac{1}{x^2\left(1+x^5\right)}\,dx$ 4302
3.1306 $\int \frac{1}{x^3\left(1+x^5\right)}\,dx$ 4306
3.1307 $\int \frac{1}{x^4\left(1+x^5\right)}\,dx$ 4310
3.1308 $\int \frac{x^{23/2}}{\sqrt{a+bx^5}}\,dx$ 4314
3.1309 $\int \frac{x^{13/2}}{\sqrt{a+bx^5}}\,dx$ 4317
3.1310 $\int \frac{x^{3/2}}{\sqrt{a+bx^5}}\,dx$ 4320
3.1311 $\int \frac{1}{x^{7/2}\sqrt{a+bx^5}}\,dx$ 4323
3.1312 $\int \frac{1}{x^{17/2}\sqrt{a+bx^5}}\,dx$ 4325
3.1313 $\int \frac{x^{23/2}}{\sqrt{1+x^5}}\,dx$ 4327
3.1314 $\int \frac{x^{13/2}}{\sqrt{1+x^5}}\,dx$ 4330
3.1315 $\int \frac{x^{3/2}}{\sqrt{1+x^5}}\,dx$ 4333
3.1316 $\int \frac{1}{x^{7/2}\sqrt{1+x^5}}\,dx$ 4335
3.1317 $\int \frac{1}{x^{17/2}\sqrt{1+x^5}}\,dx$ 4337
3.1318 $\int \frac{x^8}{a+bx^6}\,dx$ 4339
3.1319 $\int \frac{x^7}{a+bx^6}\,dx$ 4342
3.1320 $\int \frac{x^6}{a+bx^6}\,dx$ 4346
3.1321 $\int \frac{x^5}{a+bx^6}\,dx$ 4350
3.1322 $\int \frac{x^4}{a+bx^6}\,dx$ 4352
3.1323 $\int \frac{x^3}{a+bx^6}\,dx$ 4356
3.1324 $\int \frac{x^2}{a+bx^6}\,dx$ 4360
3.1325 $\int \frac{x}{a+bx^6}\,dx$ 4362
3.1326 $\int \frac{1}{a+bx^6}\,dx$ 4366
3.1327 $\int \frac{1}{x\left(a+bx^6\right)}\,dx$ 4370
3.1328 $\int \frac{1}{x^2\left(a+bx^6\right)}\,dx$ 4373
3.1329 $\int \frac{1}{x^3\left(a+bx^6\right)}\,dx$ 4377
3.1330 $\int \frac{1}{x^4\left(a+bx^6\right)}\,dx$ 4381
3.1331 $\int \frac{x^8}{\left(a+bx^6\right)^2}\,dx$ 4384
3.1332 $\int \frac{x^7}{\left(a+bx^6\right)^2}\,dx$ 4387

3.1333 $\int \frac{x^6}{\left(a+bx^6\right)^2}\,dx$ 4391
3.1334 $\int \frac{x^5}{\left(a+bx^6\right)^2}\,dx$ 4395
3.1335 $\int \frac{x^4}{\left(a+bx^6\right)^2}\,dx$ 4397
3.1336 $\int \frac{x^3}{\left(a+bx^6\right)^2}\,dx$ 4401
3.1337 $\int \frac{x^2}{\left(a+bx^6\right)^2}\,dx$ 4405
3.1338 $\int \frac{x}{\left(a+bx^6\right)^2}\,dx$ 4408
3.1339 $\int \frac{1}{\left(a+bx^6\right)^2}\,dx$ 4412
3.1340 $\int \frac{1}{x\left(a+bx^6\right)^2}\,dx$ 4416
3.1341 $\int \frac{1}{x^2\left(a+bx^6\right)^2}\,dx$ 4419
3.1342 $\int \frac{1}{x^3\left(a+bx^6\right)^2}\,dx$ 4424
3.1343 $\int \frac{1}{x^4\left(a+bx^6\right)^2}\,dx$ 4429
3.1344 $\int \frac{x^8}{1-x^6}\,dx$ 4432
3.1345 $\int \frac{x^7}{1-x^6}\,dx$ 4434
3.1346 $\int \frac{x^6}{1-x^6}\,dx$ 4437
3.1347 $\int \frac{x^5}{1-x^6}\,dx$ 4440
3.1348 $\int \frac{x^4}{1-x^6}\,dx$ 4442
3.1349 $\int \frac{x^3}{1-x^6}\,dx$ 4445
3.1350 $\int \frac{x^2}{1-x^6}\,dx$ 4448
3.1351 $\int \frac{x}{1-x^6}\,dx$ 4450
3.1352 $\int \frac{1}{1-x^6}\,dx$ 4453
3.1353 $\int \frac{1}{x\left(1-x^6\right)}\,dx$ 4456
3.1354 $\int \frac{1}{x^2\left(1-x^6\right)}\,dx$ 4458
3.1355 $\int \frac{1}{x^3\left(1-x^6\right)}\,dx$ 4461
3.1356 $\int \frac{1}{x^4\left(1-x^6\right)}\,dx$ 4464
3.1357 $\int \frac{1}{x^5\left(1-x^6\right)}\,dx$ 4466
3.1358 $\int \frac{1}{x^6\left(1-x^6\right)}\,dx$ 4469
3.1359 $\int \frac{1}{x^7\left(1-x^6\right)}\,dx$ 4472
3.1360 $\int \frac{1}{x^8\left(1-x^6\right)}\,dx$ 4474
3.1361 $\int \frac{x^8}{1+x^6}\,dx$ 4478
3.1362 $\int \frac{x^7}{1+x^6}\,dx$ 4480
3.1363 $\int \frac{x^6}{1+x^6}\,dx$ 4483
3.1364 $\int \frac{x^5}{1+x^6}\,dx$ 4486
3.1365 $\int \frac{x^4}{1+x^6}\,dx$ 4488
3.1366 $\int \frac{x^3}{1+x^6}\,dx$ 4491

3.1367 $\int \frac{x^2}{1+x^6}\,dx$ 4494
3.1368 $\int \frac{x}{1+x^6}\,dx$ 4496
3.1369 $\int \frac{1}{1+x^6}\,dx$ 4499
3.1370 $\int \frac{1}{x\left(1+x^6\right)}\,dx$ 4502
3.1371 $\int \frac{1}{x^2\left(1+x^6\right)}\,dx$ 4504
3.1372 $\int \frac{1}{x^3\left(1+x^6\right)}\,dx$ 4507
3.1373 $\int \frac{1}{x^4\left(1+x^6\right)}\,dx$ 4510
3.1374 $\int \frac{1}{x^5\left(1+x^6\right)}\,dx$ 4512
3.1375 $\int \frac{1}{x^6\left(1+x^6\right)}\,dx$ 4515
3.1376 $\int \frac{1}{x^7\left(1+x^6\right)}\,dx$ 4518
3.1377 $\int \frac{1}{x^8\left(1+x^6\right)}\,dx$ 4520
3.1378 $\int \frac{1}{2-3x^6}\,dx$ 4524
3.1379 $\int \frac{\sqrt[3]{x}}{1-x^6}\,dx$ 4528
3.1380 $\int x^8\sqrt{-1+4x^6}\,dx$ 4532
3.1381 $\int x^5\sqrt{a^6-x^6}\,dx$ 4535
3.1382 $\int x^2\sqrt{-2+x^6}\,dx$ 4537
3.1383 $\int \frac{x^{23}}{\sqrt{2+x^6}}\,dx$ 4540
3.1384 $\int \frac{x^{17}}{\sqrt{2+x^6}}\,dx$ 4542
3.1385 $\int \frac{x^{11}}{\sqrt{2+x^6}}\,dx$ 4544
3.1386 $\int \frac{x^5}{\sqrt{2+x^6}}\,dx$ 4546
3.1387 $\int \frac{1}{x\sqrt{2+x^6}}\,dx$ 4548
3.1388 $\int \frac{1}{x^7\sqrt{2+x^6}}\,dx$ 4551
3.1389 $\int \frac{1}{x^{13}\sqrt{2+x^6}}\,dx$ 4554
3.1390 $\int \frac{x^{14}}{\sqrt{2+x^6}}\,dx$ 4557
3.1391 $\int \frac{x^8}{\sqrt{2+x^6}}\,dx$ 4560
3.1392 $\int \frac{x^2}{\sqrt{2+x^6}}\,dx$ 4563
3.1393 $\int \frac{1}{x^4\sqrt{2+x^6}}\,dx$ 4565
3.1394 $\int \frac{1}{x^{10}\sqrt{2+x^6}}\,dx$ 4567
3.1395 $\int \frac{1}{x^{16}\sqrt{2+x^6}}\,dx$ 4569
3.1396 $\int \frac{x^7}{\sqrt{2+x^6}}\,dx$ 4571
3.1397 $\int \frac{x}{\sqrt{2+x^6}}\,dx$ 4574
3.1398 $\int \frac{1}{x^5\sqrt{2+x^6}}\,dx$ 4577
3.1399 $\int \frac{x^6}{\sqrt{2+x^6}}\,dx$ 4580
3.1400 $\int \frac{1}{\sqrt{2+x^6}}\,dx$ 4583
3.1401 $\int \frac{1}{x^6\sqrt{2+x^6}}\,dx$ 4586
3.1402 $\int \frac{x^9}{\sqrt{2+x^6}}\,dx$ 4589

3.1403 $\int \frac{x^3}{\sqrt{2+x^6}}\,dx$ 4592
3.1404 $\int \frac{1}{x^3\sqrt{2+x^6}}\,dx$ 4595
3.1405 $\int \frac{x^{10}}{\sqrt{2+x^6}}\,dx$ 4598
3.1406 $\int \frac{x^4}{\sqrt{2+x^6}}\,dx$ 4601
3.1407 $\int \frac{1}{x^2\sqrt{2+x^6}}\,dx$ 4604
3.1408 $\int \frac{x^{23}}{\left(2+x^6\right)^{3/2}}\,dx$ 4607
3.1409 $\int \frac{x^{17}}{\left(2+x^6\right)^{3/2}}\,dx$ 4610
3.1410 $\int \frac{x^{11}}{\left(2+x^6\right)^{3/2}}\,dx$ 4612
3.1411 $\int \frac{x^5}{\left(2+x^6\right)^{3/2}}\,dx$ 4614
3.1412 $\int \frac{1}{x\left(2+x^6\right)^{3/2}}\,dx$ 4616
3.1413 $\int \frac{1}{x^7\left(2+x^6\right)^{3/2}}\,dx$ 4619
3.1414 $\int \frac{1}{x^{13}\left(2+x^6\right)^{3/2}}\,dx$ 4622
3.1415 $\int \frac{x^{14}}{\left(2+x^6\right)^{3/2}}\,dx$ 4625
3.1416 $\int \frac{x^8}{\left(2+x^6\right)^{3/2}}\,dx$ 4628
3.1417 $\int \frac{x^2}{\left(2+x^6\right)^{3/2}}\,dx$ 4631
3.1418 $\int \frac{1}{x^4\left(2+x^6\right)^{3/2}}\,dx$ 4633
3.1419 $\int \frac{1}{x^{10}\left(2+x^6\right)^{3/2}}\,dx$ 4635
3.1420 $\int \frac{x^{13}}{\left(2+x^6\right)^{3/2}}\,dx$ 4638
3.1421 $\int \frac{x^7}{\left(2+x^6\right)^{3/2}}\,dx$ 4641
3.1422 $\int \frac{x}{\left(2+x^6\right)^{3/2}}\,dx$ 4644
3.1423 $\int \frac{1}{x^5\left(2+x^6\right)^{3/2}}\,dx$ 4647
3.1424 $\int \frac{x^{12}}{\left(2+x^6\right)^{3/2}}\,dx$ 4650
3.1425 $\int \frac{x^6}{\left(2+x^6\right)^{3/2}}\,dx$ 4653
3.1426 $\int \frac{1}{\left(2+x^6\right)^{3/2}}\,dx$ 4656
3.1427 $\int \frac{1}{x^6\left(2+x^6\right)^{3/2}}\,dx$ 4659
3.1428 $\int \frac{x^{15}}{\left(2+x^6\right)^{3/2}}\,dx$ 4662
3.1429 $\int \frac{x^9}{\left(2+x^6\right)^{3/2}}\,dx$ 4666
3.1430 $\int \frac{x^3}{\left(2+x^6\right)^{3/2}}\,dx$ 4670
3.1431 $\int \frac{1}{x^3\left(2+x^6\right)^{3/2}}\,dx$ 4673
3.1432 $\int \frac{x^{10}}{\left(2+x^6\right)^{3/2}}\,dx$ 4677

3.1433 $\int \frac{x^4}{\left(2+x^6\right)^{3/2}}\,dx$ 4680
3.1434 $\int \frac{1}{x^2\left(2+x^6\right)^{3/2}}\,dx$ 4683
3.1435 $\int x^m\left(a+bx^7\right)\,dx$ 4687
3.1436 $\int x^8\left(a+bx^7\right)\,dx$ 4689
3.1437 $\int \frac{a+bx^7}{x^8}\,dx$ 4691
3.1438 $\int x^m\left(a+bx^7\right)^2\,dx$ 4693
3.1439 $\int x^8\left(a+bx^7\right)^2\,dx$ 4696
3.1440 $\int \frac{\left(a+bx^7\right)^2}{x^8}\,dx$ 4698
3.1441 $\int \frac{x^m}{a+bx^7}\,dx$ 4700
3.1442 $\int \frac{x^6}{a+bx^7}\,dx$ 4702
3.1443 $\int \frac{1}{a+bx^7}\,dx$ 4704
3.1444 $\int \frac{1}{x\left(a+bx^7\right)}\,dx$ 4708
3.1445 $\int \frac{x^m}{a-bx^7}\,dx$ 4711
3.1446 $\int \frac{x^6}{a-bx^7}\,dx$ 4713
3.1447 $\int \frac{1}{a-bx^7}\,dx$ 4715
3.1448 $\int \frac{1}{x\left(a-bx^7\right)}\,dx$ 4719
3.1449 $\int \frac{1}{1-x^7}\,dx$ 4722
3.1450 $\int \frac{1}{1+x^7}\,dx$ 4725
3.1451 $\int \frac{x^9}{a+bx^8}\,dx$ 4728
3.1452 $\int \frac{x^7}{a+bx^8}\,dx$ 4732
3.1453 $\int \frac{x^5}{a+bx^8}\,dx$ 4734
3.1454 $\int \frac{x^3}{a+bx^8}\,dx$ 4738
3.1455 $\int \frac{x}{a+bx^8}\,dx$ 4740
3.1456 $\int \frac{1}{x\left(a+bx^8\right)}\,dx$ 4744
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3.1458 $\int \frac{1}{x^5\left(a+bx^8\right)}\,dx$ 4751
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3.1460 $\int \frac{1}{x^9\left(a+bx^8\right)}\,dx$ 4758
3.1461 $\int \frac{x^8}{a+bx^8}\,dx$ 4760
3.1462 $\int \frac{x^6}{a+bx^8}\,dx$ 4765
3.1463 $\int \frac{x^4}{a+bx^8}\,dx$ 4770
3.1464 $\int \frac{x^2}{a+bx^8}\,dx$ 4775
3.1465 $\int \frac{1}{a+bx^8}\,dx$ 4780
3.1466 $\int \frac{1}{x^2\left(a+bx^8\right)}\,dx$ 4785
3.1467 $\int \frac{1}{x^4\left(a+bx^8\right)}\,dx$ 4790
3.1468 $\int \frac{1}{x^6\left(a+bx^8\right)}\,dx$ 4795
3.1469 $\int \frac{1}{x^8\left(a+bx^8\right)}\,dx$ 4800

3.1470 $\int \frac{1}{a-bx^8}\,dx$ 4805
3.1471 $\int \frac{x^9}{1-x^8}\,dx$ 4810
3.1472 $\int \frac{x^7}{1-x^8}\,dx$ 4813
3.1473 $\int \frac{x^5}{1-x^8}\,dx$ 4815
3.1474 $\int \frac{x^3}{1-x^8}\,dx$ 4818
3.1475 $\int \frac{x}{1-x^8}\,dx$ 4820
3.1476 $\int \frac{1}{x\left(1-x^8\right)}\,dx$ 4823
3.1477 $\int \frac{1}{x^3\left(1-x^8\right)}\,dx$ 4825
3.1478 $\int \frac{1}{x^5\left(1-x^8\right)}\,dx$ 4828
3.1479 $\int \frac{1}{x^7\left(1-x^8\right)}\,dx$ 4830
3.1480 $\int \frac{1}{x^9\left(1-x^8\right)}\,dx$ 4833
3.1481 $\int \frac{x^8}{1-x^8}\,dx$ 4835
3.1482 $\int \frac{x^6}{1-x^8}\,dx$ 4839
3.1483 $\int \frac{x^4}{1-x^8}\,dx$ 4843
3.1484 $\int \frac{x^2}{1-x^8}\,dx$ 4847
3.1485 $\int \frac{1}{1-x^8}\,dx$ 4851
3.1486 $\int \frac{1}{x^2\left(1-x^8\right)}\,dx$ 4855
3.1487 $\int \frac{1}{x^4\left(1-x^8\right)}\,dx$ 4859
3.1488 $\int \frac{1}{x^6\left(1-x^8\right)}\,dx$ 4863
3.1489 $\int \frac{1}{x^8\left(1-x^8\right)}\,dx$ 4867
3.1490 $\int \frac{x^9}{1+x^8}\,dx$ 4871
3.1491 $\int \frac{x^7}{1+x^8}\,dx$ 4875
3.1492 $\int \frac{x^5}{1+x^8}\,dx$ 4877
3.1493 $\int \frac{x^3}{1+x^8}\,dx$ 4881
3.1494 $\int \frac{x}{1+x^8}\,dx$ 4883
3.1495 $\int \frac{1}{x\left(1+x^8\right)}\,dx$ 4886
3.1496 $\int \frac{1}{x^3\left(1+x^8\right)}\,dx$ 4888
3.1497 $\int \frac{1}{x^5\left(1+x^8\right)}\,dx$ 4892
3.1498 $\int \frac{1}{x^7\left(1+x^8\right)}\,dx$ 4894
3.1499 $\int \frac{1}{x^9\left(1+x^8\right)}\,dx$ 4898
3.1500 $\int \frac{x^8}{1+x^8}\,dx$ 4900
3.1501 $\int \frac{x^6}{1+x^8}\,dx$ 4904
3.1502 $\int \frac{x^4}{1+x^8}\,dx$ 4909
3.1503 $\int \frac{x^2}{1+x^8}\,dx$ 4913
3.1504 $\int \frac{1}{1+x^8}\,dx$ 4918
3.1505 $\int \frac{1}{x^2\left(1+x^8\right)}\,dx$ 4922
3.1506 $\int \frac{1}{x^4\left(1+x^8\right)}\,dx$ 4928

3.1507 $\int \frac{1}{x^6\left(1+x^8\right)}\,dx$ 4933
3.1508 $\int \frac{1}{x^8\left(1+x^8\right)}\,dx$ 4938
3.1509 $\int x^3\sqrt{1+x^8}\,dx$ 4942
3.1510 $\int x\sqrt{1+x^8}\,dx$ 4944
3.1511 $\int \frac{\sqrt{1+x^8}}{x}\,dx$ 4947
3.1512 $\int \frac{\sqrt{1+x^8}}{x^3}\,dx$ 4950
3.1513 $\int x^3\sqrt{-2+x^8}\,dx$ 4953
3.1514 $\int \frac{x^{19}}{\sqrt{1+x^8}}\,dx$ 4956
3.1515 $\int \frac{x^{15}}{\sqrt{1+x^8}}\,dx$ 4959
3.1516 $\int \frac{x^{11}}{\sqrt{1+x^8}}\,dx$ 4961
3.1517 $\int \frac{x^7}{\sqrt{1+x^8}}\,dx$ 4963
3.1518 $\int \frac{x^3}{\sqrt{1+x^8}}\,dx$ 4965
3.1519 $\int \frac{1}{x\sqrt{1+x^8}}\,dx$ 4967
3.1520 $\int \frac{1}{x^5\sqrt{1+x^8}}\,dx$ 4970
3.1521 $\int \frac{1}{x^9\sqrt{1+x^8}}\,dx$ 4972
3.1522 $\int \frac{1}{x^{13}\sqrt{1+x^8}}\,dx$ 4975
3.1523 $\int \frac{1}{x^{17}\sqrt{1+x^8}}\,dx$ 4977
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3.1526 $\int \frac{x^5}{\sqrt{1+x^8}}\,dx$ 4986
3.1527 $\int \frac{x}{\sqrt{1+x^8}}\,dx$ 4989
3.1528 $\int \frac{1}{x^3\sqrt{1+x^8}}\,dx$ 4992
3.1529 $\int \frac{1}{x^7\sqrt{1+x^8}}\,dx$ 4995
3.1530 $\int \frac{x^{10}}{\sqrt{1+x^8}}\,dx$ 4998
3.1531 $\int \frac{x^8}{\sqrt{1+x^8}}\,dx$ 5001
3.1532 $\int \frac{x^6}{\sqrt{1+x^8}}\,dx$ 5004
3.1533 $\int \frac{x^4}{\sqrt{1+x^8}}\,dx$ 5006
3.1534 $\int \frac{x^2}{\sqrt{1+x^8}}\,dx$ 5008
3.1535 $\int \frac{1}{\sqrt{1+x^8}}\,dx$ 5011
3.1536 $\int \frac{1}{x^2\sqrt{1+x^8}}\,dx$ 5014
3.1537 $\int \frac{1}{x^4\sqrt{1+x^8}}\,dx$ 5017
3.1538 $\int \frac{1}{x^6\sqrt{1+x^8}}\,dx$ 5020
3.1539 $\int \frac{1}{x^8\sqrt{1+x^8}}\,dx$ 5023
3.1540 $\int \frac{1}{1-x^{10}}\,dx$ 5026
3.1541 $\int \frac{x^4}{\sqrt{1-x^{10}}}\,dx$ 5031
3.1542 $\int \frac{x^4}{\sqrt{-2+x^{10}}}\,dx$ 5033

3.1543 $\int \frac{x^5}{9+x^{12}}\,dx$ . . . 5036
3.1544 $\int \frac{x^5}{9-x^{12}}\,dx$ . . . 5038
3.1545 $\int x^5\sqrt{9+x^{12}}\,dx$ . . . 5040
3.1546 $\int \left(a+\frac{b}{x}\right)x^6\,dx$ . . . 5042
3.1547 $\int \left(a+\frac{b}{x}\right)x^5\,dx$ . . . 5044
3.1548 $\int \left(a+\frac{b}{x}\right)x^4\,dx$ . . . 5046
3.1549 $\int \left(a+\frac{b}{x}\right)x^3\,dx$ . . . 5048
3.1550 $\int \left(a+\frac{b}{x}\right)x^2\,dx$ . . . 5050
3.1551 $\int \left(a+\frac{b}{x}\right)x\,dx$ . . . 5052
3.1552 $\int \left(a+\frac{b}{x}\right)\,dx$ . . . 5054
3.1553 $\int \frac{a+\frac{b}{x}}{x}\,dx$ . . . 5056
3.1554 $\int \frac{a+\frac{b}{x}}{x^2}\,dx$ . . . 5058
3.1555 $\int \frac{a+\frac{b}{x}}{x^3}\,dx$ . . . 5060
3.1556 $\int \frac{a+\frac{b}{x}}{x^4}\,dx$ . . . 5062
3.1557 $\int \frac{a+\frac{b}{x}}{x^5}\,dx$ . . . 5064
3.1558 $\int \frac{a+\frac{b}{x}}{x^6}\,dx$ . . . 5066
3.1559 $\int \left(a+\frac{b}{x}\right)^2 x^5\,dx$ . . . 5068
3.1560 $\int \left(a+\frac{b}{x}\right)^2 x^4\,dx$ . . . 5070
3.1561 $\int \left(a+\frac{b}{x}\right)^2 x^3\,dx$ . . . 5072
3.1562 $\int \left(a+\frac{b}{x}\right)^2 x^2\,dx$ . . . 5074
3.1563 $\int \left(a+\frac{b}{x}\right)^2 x\,dx$ . . . 5076
3.1564 $\int \left(a+\frac{b}{x}\right)^2\,dx$ . . . 5078
3.1565 $\int \frac{\left(a+\frac{b}{x}\right)^2}{x}\,dx$ . . . 5080
3.1566 $\int \frac{\left(a+\frac{b}{x}\right)^2}{x^2}\,dx$ . . . 5082
3.1567 $\int \frac{\left(a+\frac{b}{x}\right)^2}{x^3}\,dx$ . . . 5084
3.1568 $\int \frac{\left(a+\frac{b}{x}\right)^2}{x^4}\,dx$ . . . 5086
3.1569 $\int \frac{\left(a+\frac{b}{x}\right)^2}{x^5}\,dx$ . . . 5088
3.1570 $\int \left(a+\frac{b}{x}\right)^3 x^6\,dx$ . . . 5090
3.1571 $\int \left(a+\frac{b}{x}\right)^3 x^5\,dx$ . . . 5092
3.1572 $\int \left(a+\frac{b}{x}\right)^3 x^4\,dx$ . . . 5094
3.1573 $\int \left(a+\frac{b}{x}\right)^3 x^3\,dx$ . . . 5096

3.1574 $\int \left(a+\frac{b}{x}\right)^3 x^2\,dx$ 5098
3.1575 $\int \left(a+\frac{b}{x}\right)^3 x\,dx$ 5100
3.1576 $\int \left(a+\frac{b}{x}\right)^3 dx$ 5102
3.1577 $\int \frac{\left(a+\frac{b}{x}\right)^3}{x}\,dx$ 5104
3.1578 $\int \frac{\left(a+\frac{b}{x}\right)^3}{x^2}\,dx$ 5106
3.1579 $\int \frac{\left(a+\frac{b}{x}\right)^3}{x^3}\,dx$ 5108
3.1580 $\int \frac{\left(a+\frac{b}{x}\right)^3}{x^4}\,dx$ 5111
3.1581 $\int \frac{\left(a+\frac{b}{x}\right)^3}{x^5}\,dx$ 5113
3.1582 $\int \frac{\left(a+\frac{b}{x}\right)^3}{x^6}\,dx$ 5115
3.1583 $\int \left(a+\frac{b}{x}\right)^8 x^{16}\,dx$ 5117
3.1584 $\int \left(a+\frac{b}{x}\right)^8 x^{15}\,dx$ 5120
3.1585 $\int \left(a+\frac{b}{x}\right)^8 x^{13}\,dx$ 5123
3.1586 $\int \left(a+\frac{b}{x}\right)^8 x^{12}\,dx$ 5126
3.1587 $\int \left(a+\frac{b}{x}\right)^8 x^{11}\,dx$ 5129
3.1588 $\int \left(a+\frac{b}{x}\right)^8 x^{10}\,dx$ 5132
3.1589 $\int \left(a+\frac{b}{x}\right)^8 x^9\,dx$ 5135
3.1590 $\int \left(a+\frac{b}{x}\right)^8 x^8\,dx$ 5137
3.1591 $\int \left(a+\frac{b}{x}\right)^8 x^7\,dx$ 5139
3.1592 $\int \left(a+\frac{b}{x}\right)^8 x^6\,dx$ 5142
3.1593 $\int \left(a+\frac{b}{x}\right)^8 x^5\,dx$ 5145
3.1594 $\int \left(a+\frac{b}{x}\right)^8 x^4\,dx$ 5148
3.1595 $\int \left(a+\frac{b}{x}\right)^8 x^3\,dx$ 5151
3.1596 $\int \left(a+\frac{b}{x}\right)^8 x^2\,dx$ 5154
3.1597 $\int \left(a+\frac{b}{x}\right)^8 x\,dx$ 5157
3.1598 $\int \left(a+\frac{b}{x}\right)^8 dx$ 5160
3.1599 $\int \frac{\left(a+\frac{b}{x}\right)^8}{x}\,dx$ 5163
3.1600 $\int \frac{\left(a+\frac{b}{x}\right)^8}{x^2}\,dx$ 5166
3.1601 $\int \frac{\left(a+\frac{b}{x}\right)^8}{x^3}\,dx$ 5168

3.1602 $\int \frac{\left(a+\frac{b}{x}\right)^8}{x^4}\,dx$ 5171
3.1603 $\int \frac{\left(a+\frac{b}{x}\right)^8}{x^5}\,dx$ 5174
3.1604 $\int \frac{\left(a+\frac{b}{x}\right)^8}{x^6}\,dx$ 5177
3.1605 $\int \frac{\left(a+\frac{b}{x}\right)^8}{x^7}\,dx$ 5180
3.1606 $\int \frac{\left(a+\frac{b}{x}\right)^8}{x^8}\,dx$ 5183
3.1607 $\int \frac{x^4}{a+\frac{b}{x}}\,dx$ 5186
3.1608 $\int \frac{x^3}{a+\frac{b}{x}}\,dx$ 5189
3.1609 $\int \frac{x^2}{a+\frac{b}{x}}\,dx$ 5192
3.1610 $\int \frac{x}{a+\frac{b}{x}}\,dx$ 5194
3.1611 $\int \frac{1}{a+\frac{b}{x}}\,dx$ 5196
3.1612 $\int \frac{1}{\left(a+\frac{b}{x}\right)x}\,dx$ 5198
3.1613 $\int \frac{1}{\left(a+\frac{b}{x}\right)x^2}\,dx$ 5200
3.1614 $\int \frac{1}{\left(a+\frac{b}{x}\right)x^3}\,dx$ 5202
3.1615 $\int \frac{1}{\left(a+\frac{b}{x}\right)x^4}\,dx$ 5204
3.1616 $\int \frac{1}{\left(a+\frac{b}{x}\right)x^5}\,dx$ 5206
3.1617 $\int \frac{1}{\left(a+\frac{b}{x}\right)x^6}\,dx$ 5209
3.1618 $\int \frac{1}{\left(a+\frac{b}{x}\right)x^7}\,dx$ 5212
3.1619 $\int \frac{x^5}{\left(a+\frac{b}{x}\right)^2}\,dx$ 5215
3.1620 $\int \frac{x^4}{\left(a+\frac{b}{x}\right)^2}\,dx$ 5218
3.1621 $\int \frac{x^3}{\left(a+\frac{b}{x}\right)^2}\,dx$ 5221
3.1622 $\int \frac{x^2}{\left(a+\frac{b}{x}\right)^2}\,dx$ 5224
3.1623 $\int \frac{x}{\left(a+\frac{b}{x}\right)^2}\,dx$ 5227
3.1624 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2}\,dx$ 5230
3.1625 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x}\,dx$ 5233
3.1626 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^2}\,dx$ 5235
3.1627 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^3}\,dx$ 5237
3.1628 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^4}\,dx$ 5239

3.1629 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^5}\,dx$ 5241
3.1630 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^6}\,dx$ 5244
3.1631 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^7}\,dx$ 5247
3.1632 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^8}\,dx$ 5250
3.1633 $\int \frac{x^4}{\left(a+\frac{b}{x}\right)^3}\,dx$ 5253
3.1634 $\int \frac{x^3}{\left(a+\frac{b}{x}\right)^3}\,dx$ 5256
3.1635 $\int \frac{x^2}{\left(a+\frac{b}{x}\right)^3}\,dx$ 5259
3.1636 $\int \frac{x}{\left(a+\frac{b}{x}\right)^3}\,dx$ 5262
3.1637 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3}\,dx$ 5265
3.1638 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x}\,dx$ 5268
3.1639 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^2}\,dx$ 5271
3.1640 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^3}\,dx$ 5273
3.1641 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^4}\,dx$ 5275
3.1642 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^5}\,dx$ 5278
3.1643 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^6}\,dx$ 5281
3.1644 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^7}\,dx$ 5284
3.1645 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^8}\,dx$ 5287
3.1646 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^9}\,dx$ 5290
3.1647 $\int \left(a+\frac{b}{x}\right) x^{5/2}\,dx$ 5293
3.1648 $\int \left(a+\frac{b}{x}\right) x^{3/2}\,dx$ 5295
3.1649 $\int \left(a+\frac{b}{x}\right) \sqrt{x}\,dx$ 5297
3.1650 $\int \frac{a+\frac{b}{x}}{\sqrt{x}}\,dx$ 5299
3.1651 $\int \frac{a+\frac{b}{x}}{x^{3/2}}\,dx$ 5301
3.1652 $\int \frac{a+\frac{b}{x}}{x^{5/2}}\,dx$ 5303
3.1653 $\int \left(a+\frac{b}{x}\right)^2 x^{5/2}\,dx$ 5305
3.1654 $\int \left(a+\frac{b}{x}\right)^2 x^{3/2}\,dx$ 5307
3.1655 $\int \left(a+\frac{b}{x}\right)^2 \sqrt{x}\,dx$ 5309

3.1656 $\int \frac{\left(a+\frac{b}{x}\right)^2}{\sqrt{x}}\,dx$ 5311
3.1657 $\int \frac{\left(a+\frac{b}{x}\right)^2}{x^{3/2}}\,dx$ 5313
3.1658 $\int \frac{\left(a+\frac{b}{x}\right)^2}{x^{5/2}}\,dx$ 5315
3.1659 $\int \left(a+\frac{b}{x}\right)^3 x^{5/2}\,dx$ 5317
3.1660 $\int \left(a+\frac{b}{x}\right)^3 x^{3/2}\,dx$ 5319
3.1661 $\int \left(a+\frac{b}{x}\right)^3 \sqrt{x}\,dx$ 5321
3.1662 $\int \frac{\left(a+\frac{b}{x}\right)^3}{\sqrt{x}}\,dx$ 5323
3.1663 $\int \frac{\left(a+\frac{b}{x}\right)^3}{x^{3/2}}\,dx$ 5325
3.1664 $\int \frac{\left(a+\frac{b}{x}\right)^3}{x^{5/2}}\,dx$ 5327
3.1665 $\int \frac{x^{5/2}}{a+\frac{b}{x}}\,dx$ 5329
3.1666 $\int \frac{x^{3/2}}{a+\frac{b}{x}}\,dx$ 5332
3.1667 $\int \frac{\sqrt{x}}{a+\frac{b}{x}}\,dx$ 5335
3.1668 $\int \frac{1}{\left(a+\frac{b}{x}\right)\sqrt{x}}\,dx$ 5338
3.1669 $\int \frac{1}{\left(a+\frac{b}{x}\right)x^{3/2}}\,dx$ 5341
3.1670 $\int \frac{1}{\left(a+\frac{b}{x}\right)x^{5/2}}\,dx$ 5344
3.1671 $\int \frac{1}{\left(a+\frac{b}{x}\right)x^{7/2}}\,dx$ 5347
3.1672 $\int \frac{1}{\left(a+\frac{b}{x}\right)x^{9/2}}\,dx$ 5350
3.1673 $\int \frac{x^{5/2}}{\left(a+\frac{b}{x}\right)^2}\,dx$ 5353
3.1674 $\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^2}\,dx$ 5356
3.1675 $\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^2}\,dx$ 5360
3.1676 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2\sqrt{x}}\,dx$ 5364
3.1677 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{3/2}}\,dx$ 5368
3.1678 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{5/2}}\,dx$ 5371
3.1679 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{7/2}}\,dx$ 5374
3.1680 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{9/2}}\,dx$ 5377
3.1681 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{11/2}}\,dx$ 5380

3.1682 $\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^3}\,dx$ 5383
3.1683 $\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^3}\,dx$ 5387
3.1684 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3\sqrt{x}}\,dx$ 5391
3.1685 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{3/2}}\,dx$ 5395
3.1686 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{5/2}}\,dx$ 5399
3.1687 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{7/2}}\,dx$ 5402
3.1688 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{9/2}}\,dx$ 5405
3.1689 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{11/2}}\,dx$ 5408
3.1690 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{13/2}}\,dx$ 5411
3.1691 $\int \sqrt{a+\frac{b}{x}}\,x^3\,dx$ 5414
3.1692 $\int \sqrt{a+\frac{b}{x}}\,x^2\,dx$ 5418
3.1693 $\int \sqrt{a+\frac{b}{x}}\,x\,dx$ 5421
3.1694 $\int \sqrt{a+\frac{b}{x}}\,dx$ 5424
3.1695 $\int \frac{\sqrt{a+\frac{b}{x}}}{x}\,dx$ 5427
3.1696 $\int \frac{\sqrt{a+\frac{b}{x}}}{x^2}\,dx$ 5430
3.1697 $\int \frac{\sqrt{a+\frac{b}{x}}}{x^3}\,dx$ 5432
3.1698 $\int \frac{\sqrt{a+\frac{b}{x}}}{x^4}\,dx$ 5435
3.1699 $\int \frac{\sqrt{a+\frac{b}{x}}}{x^5}\,dx$ 5438
3.1700 $\int \frac{\sqrt{a+\frac{b}{x}}}{x^6}\,dx$ 5442
3.1701 $\int \left(a+\frac{b}{x}\right)^{3/2} x^3\,dx$ 5447
3.1702 $\int \left(a+\frac{b}{x}\right)^{3/2} x^2\,dx$ 5451
3.1703 $\int \left(a+\frac{b}{x}\right)^{3/2} x\,dx$ 5454
3.1704 $\int \left(a+\frac{b}{x}\right)^{3/2}\,dx$ 5457
3.1705 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x}\,dx$ 5461
3.1706 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^2}\,dx$ 5464
3.1707 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^3}\,dx$ 5466
3.1708 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^4}\,dx$ 5469
3.1709 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^5}\,dx$ 5472

3.1710 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^6}\,dx$ 5476
3.1711 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^7}\,dx$ 5482
3.1712 $\int \left(a+\frac{b}{x}\right)^{5/2} x^3\,dx$ 5490
3.1713 $\int \left(a+\frac{b}{x}\right)^{5/2} x^2\,dx$ 5494
3.1714 $\int \left(a+\frac{b}{x}\right)^{5/2} x\,dx$ 5497
3.1715 $\int \left(a+\frac{b}{x}\right)^{5/2}\,dx$ 5501
3.1716 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x}\,dx$ 5505
3.1717 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^2}\,dx$ 5509
3.1718 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^3}\,dx$ 5512
3.1719 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^4}\,dx$ 5515
3.1720 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^5}\,dx$ 5518
3.1721 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^6}\,dx$ 5522
3.1722 $\int \frac{x^3}{\sqrt{a+\frac{b}{x}}}\,dx$ 5528
3.1723 $\int \frac{x^2}{\sqrt{a+\frac{b}{x}}}\,dx$ 5532
3.1724 $\int \frac{x}{\sqrt{a+\frac{b}{x}}}\,dx$ 5535
3.1725 $\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx$ 5538
3.1726 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\,x}\,dx$ 5541
3.1727 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\,x^2}\,dx$ 5544
3.1728 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\,x^3}\,dx$ 5546
3.1729 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\,x^4}\,dx$ 5549
3.1730 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\,x^5}\,dx$ 5552
3.1731 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\,x^6}\,dx$ 5556
3.1732 $\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$ 5561
3.1733 $\int \frac{x}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$ 5565
3.1734 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$ 5569
3.1735 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x}\,dx$ 5572
3.1736 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^2}\,dx$ 5575

3.1737 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^3}\,dx$ 5577
3.1738 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^4}\,dx$ 5580
3.1739 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^5}\,dx$ 5583
3.1740 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^6}\,dx$ 5587
3.1741 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^7}\,dx$ 5592
3.1742 $\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$ 5600
3.1743 $\int \frac{x}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$ 5604
3.1744 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$ 5608
3.1745 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x}\,dx$ 5612
3.1746 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^2}\,dx$ 5616
3.1747 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^3}\,dx$ 5618
3.1748 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^4}\,dx$ 5621
3.1749 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^5}\,dx$ 5624
3.1750 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^6}\,dx$ 5627
3.1751 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^7}\,dx$ 5631
3.1752 $\int \sqrt{a+\frac{b}{x}}\, x^{7/2}\,dx$ 5638
3.1753 $\int \sqrt{a+\frac{b}{x}}\, x^{5/2}\,dx$ 5641
3.1754 $\int \sqrt{a+\frac{b}{x}}\, x^{3/2}\,dx$ 5644
3.1755 $\int \sqrt{a+\frac{b}{x}}\, \sqrt{x}\,dx$ 5647
3.1756 $\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{x}}\,dx$ 5649
3.1757 $\int \frac{\sqrt{a+\frac{b}{x}}}{x^{3/2}}\,dx$ 5652
3.1758 $\int \frac{\sqrt{a+\frac{b}{x}}}{x^{5/2}}\,dx$ 5655
3.1759 $\int \frac{\sqrt{a+\frac{b}{x}}}{x^{7/2}}\,dx$ 5658
3.1760 $\int \left(a+\frac{b}{x}\right)^{3/2} x^{9/2}\,dx$ 5662
3.1761 $\int \left(a+\frac{b}{x}\right)^{3/2} x^{7/2}\,dx$ 5665
3.1762 $\int \left(a+\frac{b}{x}\right)^{3/2} x^{5/2}\,dx$ 5668
3.1763 $\int \left(a+\frac{b}{x}\right)^{3/2} x^{3/2}\,dx$ 5671
3.1764 $\int \left(a+\frac{b}{x}\right)^{3/2} \sqrt{x}\,dx$ 5673

3.1765 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\sqrt{x}}\,dx$ 5676
3.1766 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^{3/2}}\,dx$ 5679
3.1767 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^{5/2}}\,dx$ 5682
3.1768 $\int \left(a+\frac{b}{x}\right)^{5/2} x^{11/2}\,dx$ 5686
3.1769 $\int \left(a+\frac{b}{x}\right)^{5/2} x^{9/2}\,dx$ 5689
3.1770 $\int \left(a+\frac{b}{x}\right)^{5/2} x^{7/2}\,dx$ 5692
3.1771 $\int \left(a+\frac{b}{x}\right)^{5/2} x^{5/2}\,dx$ 5695
3.1772 $\int \left(a+\frac{b}{x}\right)^{5/2} x^{3/2}\,dx$ 5697
3.1773 $\int \left(a+\frac{b}{x}\right)^{5/2} \sqrt{x}\,dx$ 5700
3.1774 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\sqrt{x}}\,dx$ 5703
3.1775 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^{3/2}}\,dx$ 5707
3.1776 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^{5/2}}\,dx$ 5710
3.1777 $\int \frac{x^{7/2}}{\sqrt{a+\frac{b}{x}}}\,dx$ 5714
3.1778 $\int \frac{x^{5/2}}{\sqrt{a+\frac{b}{x}}}\,dx$ 5717
3.1779 $\int \frac{x^{3/2}}{\sqrt{a+\frac{b}{x}}}\,dx$ 5720
3.1780 $\int \frac{\sqrt{x}}{\sqrt{a+\frac{b}{x}}}\,dx$ 5723
3.1781 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\,dx$ 5725
3.1782 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\,x^{3/2}}\,dx$ 5727
3.1783 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\,x^{5/2}}\,dx$ 5730
3.1784 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\,x^{7/2}}\,dx$ 5733
3.1785 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\,x^{9/2}}\,dx$ 5736
3.1786 $\int \frac{x^{5/2}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$ 5739
3.1787 $\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$ 5743
3.1788 $\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$ 5746
3.1789 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\sqrt{x}}\,dx$ 5749
3.1790 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}x^{3/2}}\,dx$ 5752
3.1791 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}x^{5/2}}\,dx$ 5754

3.1792 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{7/2}}\,dx$ 5757
3.1793 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{9/2}}\,dx$ 5761
3.1794 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{11/2}}\,dx$ 5765
3.1795 $\int \frac{x^{5/2}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$ 5769
3.1796 $\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$ 5773
3.1797 $\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$ 5777
3.1798 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} \sqrt{x}}\,dx$ 5780
3.1799 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{3/2}}\,dx$ 5783
3.1800 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{5/2}}\,dx$ 5786
3.1801 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{7/2}}\,dx$ 5788
3.1802 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{9/2}}\,dx$ 5792
3.1803 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{11/2}}\,dx$ 5796
3.1804 $\int \left(a+\frac{b}{x^2}\right) x^6\,dx$ 5800
3.1805 $\int \left(a+\frac{b}{x^2}\right) x^5\,dx$ 5802
3.1806 $\int \left(a+\frac{b}{x^2}\right) x^4\,dx$ 5804
3.1807 $\int \left(a+\frac{b}{x^2}\right) x^3\,dx$ 5806
3.1808 $\int \left(a+\frac{b}{x^2}\right) x^2\,dx$ 5808
3.1809 $\int \left(a+\frac{b}{x^2}\right) x\,dx$ 5810
3.1810 $\int \left(a+\frac{b}{x^2}\right)\,dx$ 5812
3.1811 $\int \frac{a+\frac{b}{x^2}}{x}\,dx$ 5814
3.1812 $\int \frac{a+\frac{b}{x^2}}{x^2}\,dx$ 5816
3.1813 $\int \frac{a+\frac{b}{x^2}}{x^3}\,dx$ 5818
3.1814 $\int \frac{a+\frac{b}{x^2}}{x^4}\,dx$ 5820
3.1815 $\int \frac{a+\frac{b}{x^2}}{x^5}\,dx$ 5822
3.1816 $\int \frac{a+\frac{b}{x^2}}{x^6}\,dx$ 5824
3.1817 $\int \left(a+\frac{b}{x^2}\right)^2 x^6\,dx$ 5826
3.1818 $\int \left(a+\frac{b}{x^2}\right)^2 x^5\,dx$ 5828
3.1819 $\int \left(a+\frac{b}{x^2}\right)^2 x^4\,dx$ 5830

3.1820 $\int \left(a+\frac{b}{x^2}\right)^2 x^3\,dx$ 5832
3.1821 $\int \left(a+\frac{b}{x^2}\right)^2 x^2\,dx$ 5834
3.1822 $\int \left(a+\frac{b}{x^2}\right)^2 x\,dx$ 5836
3.1823 $\int \left(a+\frac{b}{x^2}\right)^2 dx$ 5838
3.1824 $\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x}\,dx$ 5840
3.1825 $\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^2}\,dx$ 5843
3.1826 $\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^3}\,dx$ 5845
3.1827 $\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^4}\,dx$ 5847
3.1828 $\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^5}\,dx$ 5849
3.1829 $\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^6}\,dx$ 5852
3.1830 $\int \left(a+\frac{b}{x^2}\right)^3 x^6\,dx$ 5854
3.1831 $\int \left(a+\frac{b}{x^2}\right)^3 x^5\,dx$ 5856
3.1832 $\int \left(a+\frac{b}{x^2}\right)^3 x^4\,dx$ 5858
3.1833 $\int \left(a+\frac{b}{x^2}\right)^3 x^3\,dx$ 5860
3.1834 $\int \left(a+\frac{b}{x^2}\right)^3 x^2\,dx$ 5862
3.1835 $\int \left(a+\frac{b}{x^2}\right)^3 x\,dx$ 5864
3.1836 $\int \left(a+\frac{b}{x^2}\right)^3 dx$ 5866
3.1837 $\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x}\,dx$ 5868
3.1838 $\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^2}\,dx$ 5871
3.1839 $\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^3}\,dx$ 5873
3.1840 $\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^4}\,dx$ 5875
3.1841 $\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^5}\,dx$ 5877
3.1842 $\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^6}\,dx$ 5880
3.1843 $\int \frac{x^6}{a+\frac{b}{x^2}}\,dx$ 5882
3.1844 $\int \frac{x^5}{a+\frac{b}{x^2}}\,dx$ 5885
3.1845 $\int \frac{x^4}{a+\frac{b}{x^2}}\,dx$ 5888
3.1846 $\int \frac{x^3}{a+\frac{b}{x^2}}\,dx$ 5891
3.1847 $\int \frac{x^2}{a+\frac{b}{x^2}}\,dx$ 5894

3.1848 $\int \frac{x}{a+\frac{b}{x^2}}\,dx$ 5897
3.1849 $\int \frac{1}{a+\frac{b}{x^2}}\,dx$ 5900
3.1850 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)x}\,dx$ 5903
3.1851 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^2}\,dx$ 5905
3.1852 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^3}\,dx$ 5907
3.1853 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^4}\,dx$ 5909
3.1854 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^5}\,dx$ 5912
3.1855 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^6}\,dx$ 5915
3.1856 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^7}\,dx$ 5918
3.1857 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^8}\,dx$ 5921
3.1858 $\int \frac{x^6}{\left(a+\frac{b}{x^2}\right)^2}\,dx$ 5924
3.1859 $\int \frac{x^5}{\left(a+\frac{b}{x^2}\right)^2}\,dx$ 5927
3.1860 $\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^2}\,dx$ 5930
3.1861 $\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^2}\,dx$ 5933
3.1862 $\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^2}\,dx$ 5936
3.1863 $\int \frac{x}{\left(a+\frac{b}{x^2}\right)^2}\,dx$ 5939
3.1864 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2}\,dx$ 5942
3.1865 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x}\,dx$ 5945
3.1866 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^2}\,dx$ 5948
3.1867 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^3}\,dx$ 5951
3.1868 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^4}\,dx$ 5953
3.1869 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^5}\,dx$ 5956
3.1870 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^6}\,dx$ 5959
3.1871 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^7}\,dx$ 5962
3.1872 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^8}\,dx$ 5965
3.1873 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^9}\,dx$ 5968

3.1874 $\int \frac{x^5}{\left(a+\frac{b}{x^2}\right)^3}\,dx$ . . . 5971
3.1875 $\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^3}\,dx$ . . . 5974
3.1876 $\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^3}\,dx$ . . . 5977
3.1877 $\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^3}\,dx$ . . . 5980
3.1878 $\int \frac{x}{\left(a+\frac{b}{x^2}\right)^3}\,dx$ . . . 5983
3.1879 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3}\,dx$ . . . 5986
3.1880 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x}\,dx$ . . . 5989
3.1881 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^2}\,dx$ . . . 5992
3.1882 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^3}\,dx$ . . . 5995
3.1883 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^4}\,dx$ . . . 5997
3.1884 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^5}\,dx$ . . . 6000
3.1885 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^6}\,dx$ . . . 6002
3.1886 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^7}\,dx$ . . . 6005
3.1887 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^8}\,dx$ . . . 6008
3.1888 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^9}\,dx$ . . . 6011
3.1889 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^{10}}\,dx$ . . . 6014
3.1890 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^{11}}\,dx$ . . . 6017
3.1891 $\int \sqrt{a+\frac{b}{x^2}}\,x^3\,dx$ . . . 6020
3.1892 $\int \sqrt{a+\frac{b}{x^2}}\,x^2\,dx$ . . . 6024
3.1893 $\int \sqrt{a+\frac{b}{x^2}}\,x\,dx$ . . . 6026
3.1894 $\int \sqrt{a+\frac{b}{x^2}}\,dx$ . . . 6029
3.1895 $\int \frac{\sqrt{a+\frac{b}{x^2}}}{x}\,dx$ . . . 6032
3.1896 $\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^2}\,dx$ . . . 6035
3.1897 $\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^3}\,dx$ . . . 6038
3.1898 $\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^4}\,dx$ . . . 6040
3.1899 $\int \left(a+\frac{b}{x^2}\right)^{3/2} x^3\,dx$ . . . 6043
3.1900 $\int \left(a+\frac{b}{x^2}\right)^{3/2} x^2\,dx$ . . . 6046

3.1901 $\int \left(a+\frac{b}{x^2}\right)^{3/2} x\,dx$ 6049
3.1902 $\int \left(a+\frac{b}{x^2}\right)^{3/2} dx$ 6052
3.1903 $\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x} dx$ 6055
3.1904 $\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^2} dx$ 6058
3.1905 $\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^3} dx$ 6061
3.1906 $\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^4} dx$ 6063
3.1907 $\int \left(a+\frac{b}{x^2}\right)^{5/2} x^3\,dx$ 6067
3.1908 $\int \left(a+\frac{b}{x^2}\right)^{5/2} x^2\,dx$ 6070
3.1909 $\int \left(a+\frac{b}{x^2}\right)^{5/2} x\,dx$ 6073
3.1910 $\int \left(a+\frac{b}{x^2}\right)^{5/2} dx$ 6077
3.1911 $\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x} dx$ 6081
3.1912 $\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^2} dx$ 6084
3.1913 $\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^3} dx$ 6088
3.1914 $\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^4} dx$ 6091
3.1915 $\int \frac{x^3}{\sqrt{a+\frac{b}{x^2}}} dx$ 6095
3.1916 $\int \frac{x}{\sqrt{a+\frac{b}{x^2}}} dx$ 6098
3.1917 $\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x} dx$ 6101
3.1918 $\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x^3} dx$ 6104
3.1919 $\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x^5} dx$ 6106
3.1920 $\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x^7} dx$ 6109
3.1921 $\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x^9} dx$ 6112
3.1922 $\int \frac{x^4}{\sqrt{a+\frac{b}{x^2}}} dx$ 6116
3.1923 $\int \frac{x^2}{\sqrt{a+\frac{b}{x^2}}} dx$ 6119
3.1924 $\int \frac{1}{\sqrt{a+\frac{b}{x^2}}} dx$ 6122
3.1925 $\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x^2} dx$ 6124
3.1926 $\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x^4} dx$ 6127

3.1927 $\int \frac{1}{\sqrt{-a+\frac{b}{x^2}}\,x}\,dx$ 6130
3.1928 $\int \frac{1}{\sqrt{2+\frac{b}{x^2}}\,x^2}\,dx$ 6133
3.1929 $\int \frac{1}{\sqrt{2-\frac{b}{x^2}}\,x^2}\,dx$ 6136
3.1930 $\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx$ 6139
3.1931 $\int \frac{x}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx$ 6143
3.1932 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}\,x}\,dx$ 6147
3.1933 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}\,x^3}\,dx$ 6151
3.1934 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}\,x^5}\,dx$ 6153
3.1935 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}\,x^7}\,dx$ 6156
3.1936 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}\,x^9}\,dx$ 6159
3.1937 $\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx$ 6162
3.1938 $\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx$ 6166
3.1939 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx$ 6169
3.1940 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}\,x^2}\,dx$ 6172
3.1941 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}\,x^4}\,dx$ 6174
3.1942 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}\,x^6}\,dx$ 6177
3.1943 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}\,x^8}\,dx$ 6181
3.1944 $\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx$ 6185
3.1945 $\int \frac{x}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx$ 6189
3.1946 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}\,x}\,dx$ 6193
3.1947 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}\,x^3}\,dx$ 6197
3.1948 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}\,x^5}\,dx$ 6200
3.1949 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}\,x^7}\,dx$ 6203
3.1950 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}\,x^9}\,dx$ 6206
3.1951 $\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx$ 6209

3.1952 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx$ 6213
3.1953 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^2}\,dx$ 6216
3.1954 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^4}\,dx$ 6219
3.1955 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^6}\,dx$ 6222
3.1956 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^8}\,dx$ 6226
3.1957 $\int \frac{\sqrt[3]{1+\frac{1}{x^2}}}{x^3}\,dx$ 6230
3.1958 $\int \frac{\left(1+\frac{1}{x^2}\right)^{5/3}}{x^3}\,dx$ 6232
3.1959 $\int \left(1+\frac{b}{x^2}\right)^{3/2} (cx)^m\,dx$ 6234
3.1960 $\int \sqrt{1+\frac{b}{x^2}}\,(cx)^m\,dx$ 6237
3.1961 $\int \frac{(cx)^m}{\sqrt{1+\frac{b}{x^2}}}\,dx$ 6240
3.1962 $\int \frac{(cx)^m}{\left(1+\frac{b}{x^2}\right)^{3/2}}\,dx$ 6243
3.1963 $\int \left(1+\frac{b}{x^2}\right)^p (cx)^m\,dx$ 6246
3.1964 $\int \left(a+\frac{b}{x^2}\right)^p (cx)^m\,dx$ 6249
3.1965 $\int \frac{x^5}{a+\frac{b}{x^3}}\,dx$ 6252
3.1966 $\int \frac{x^4}{a+\frac{b}{x^3}}\,dx$ 6255
3.1967 $\int \frac{x^3}{a+\frac{b}{x^3}}\,dx$ 6259
3.1968 $\int \frac{x^2}{a+\frac{b}{x^3}}\,dx$ 6263
3.1969 $\int \frac{x}{a+\frac{b}{x^3}}\,dx$ 6266
3.1970 $\int \frac{1}{a+\frac{b}{x^3}}\,dx$ 6270
3.1971 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)x}\,dx$ 6274
3.1972 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^2}\,dx$ 6276
3.1973 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^3}\,dx$ 6280
3.1974 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^4}\,dx$ 6284
3.1975 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^5}\,dx$ 6286
3.1976 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^6}\,dx$ 6290
3.1977 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^7}\,dx$ 6294
3.1978 $\int \frac{x^5}{\left(a+\frac{b}{x^3}\right)^2}\,dx$ 6297

3.1979 $\int \frac{x^4}{\left(a+\frac{b}{x^3}\right)^2}\,dx$ 6300
3.1980 $\int \frac{x^3}{\left(a+\frac{b}{x^3}\right)^2}\,dx$ 6305
3.1981 $\int \frac{x^2}{\left(a+\frac{b}{x^3}\right)^2}\,dx$ 6310
3.1982 $\int \frac{x}{\left(a+\frac{b}{x^3}\right)^2}\,dx$ 6313
3.1983 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2}\,dx$ 6317
3.1984 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x}\,dx$ 6321
3.1985 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^2}\,dx$ 6324
3.1986 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^3}\,dx$ 6328
3.1987 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^4}\,dx$ 6332
3.1988 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^5}\,dx$ 6334
3.1989 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^6}\,dx$ 6338
3.1990 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^7}\,dx$ 6342
3.1991 $\int \sqrt{a+\frac{b}{x^3}}\,x^5\,dx$ 6345
3.1992 $\int \sqrt{a+\frac{b}{x^3}}\,x^2\,dx$ 6350
3.1993 $\int \frac{\sqrt{a+\frac{b}{x^3}}}{x}\,dx$ 6354
3.1994 $\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^4}\,dx$ 6358
3.1995 $\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^7}\,dx$ 6360
3.1996 $\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^{10}}\,dx$ 6363
3.1997 $\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^{13}}\,dx$ 6366
3.1998 $\int \sqrt{a+\frac{b}{x^3}}\,x^7\,dx$ 6370
3.1999 $\int \sqrt{a+\frac{b}{x^3}}\,x^4\,dx$ 6374
3.2000 $\int \sqrt{a+\frac{b}{x^3}}\,x\,dx$ 6378
3.2001 $\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^2}\,dx$ 6382
3.2002 $\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^5}\,dx$ 6386
3.2003 $\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^8}\,dx$ 6390
3.2004 $\int \sqrt{a+\frac{b}{x^3}}\,x^6\,dx$ 6394
3.2005 $\int \sqrt{a+\frac{b}{x^3}}\,x^3\,dx$ 6399

3.2006 $\int \sqrt{a+\frac{b}{x^3}}\,dx$ 6404
3.2007 $\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^3}\,dx$ 6409
3.2008 $\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^6}\,dx$ 6414
3.2009 $\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^9}\,dx$ 6420
3.2010 $\int \left(a+\frac{b}{x^3}\right)^{3/2} x^5\,dx$ 6426
3.2011 $\int \left(a+\frac{b}{x^3}\right)^{3/2} x^2\,dx$ 6431
3.2012 $\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x}\,dx$ 6436
3.2013 $\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^4}\,dx$ 6441
3.2014 $\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^7}\,dx$ 6443
3.2015 $\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^{10}}\,dx$ 6446
3.2016 $\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^{13}}\,dx$ 6449
3.2017 $\int \frac{x^5}{\sqrt{a+\frac{b}{x^3}}}\,dx$ 6453
3.2018 $\int \frac{x^2}{\sqrt{a+\frac{b}{x^3}}}\,dx$ 6458
3.2019 $\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x}\,dx$ 6463
3.2020 $\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^4}\,dx$ 6466
3.2021 $\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^7}\,dx$ 6468
3.2022 $\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^{10}}\,dx$ 6471
3.2023 $\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^{13}}\,dx$ 6474
3.2024 $\int \frac{x^7}{\sqrt{a+\frac{b}{x^3}}}\,dx$ 6478
3.2025 $\int \frac{x^4}{\sqrt{a+\frac{b}{x^3}}}\,dx$ 6482
3.2026 $\int \frac{x}{\sqrt{a+\frac{b}{x^3}}}\,dx$ 6486
3.2027 $\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^2}\,dx$ 6490
3.2028 $\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^5}\,dx$ 6493
3.2029 $\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^8}\,dx$ 6497
3.2030 $\int \frac{x^6}{\sqrt{a+\frac{b}{x^3}}}\,dx$ 6501

3.2031 $\int \frac{x^3}{\sqrt{a+\frac{b}{x^3}}}\,dx$ 6506
3.2032 $\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}\,dx$ 6511
3.2033 $\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^3}\,dx$ 6516
3.2034 $\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^6}\,dx$ 6521
3.2035 $\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^9}\,dx$ 6526
3.2036 $\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^{12}}\,dx$ 6532
3.2037 $\int \frac{x^5}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$ 6538
3.2038 $\int \frac{x^2}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$ 6543
3.2039 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}\,x}\,dx$ 6548
3.2040 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}\,x^4}\,dx$ 6553
3.2041 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}\,x^7}\,dx$ 6555
3.2042 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}\,x^{10}}\,dx$ 6558
3.2043 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}\,x^{13}}\,dx$ 6561
3.2044 $\int \frac{x^7}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$ 6565
3.2045 $\int \frac{x^4}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$ 6570
3.2046 $\int \frac{x}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$ 6574
3.2047 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}\,x^2}\,dx$ 6578
3.2048 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}\,x^5}\,dx$ 6582
3.2049 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}\,x^8}\,dx$ 6586
3.2050 $\int \frac{x^6}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$ 6590
3.2051 $\int \frac{x^3}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$ 6596
3.2052 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$ 6601
3.2053 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}\,x^3}\,dx$ 6606
3.2054 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}\,x^6}\,dx$ 6611
3.2055 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}\,x^9}\,dx$ 6616

3.2056 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^{12}}\,dx$ 6622
3.2057 $\int \frac{1}{a+\frac{b}{x^4}}\,dx$ 6628
3.2058 $\int \sqrt{a+\frac{b}{x^4}}\,x^3\,dx$ 6632
3.2059 $\int \sqrt{a+\frac{b}{x^4}}\,x\,dx$ 6635
3.2060 $\int \frac{\sqrt{a+\frac{b}{x^4}}}{x}\,dx$ 6638
3.2061 $\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^3}\,dx$ 6641
3.2062 $\int \sqrt{a+\frac{b}{x^4}}\,x^2\,dx$ 6644
3.2063 $\int \sqrt{a+\frac{b}{x^4}}\,dx$ 6647
3.2064 $\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^2}\,dx$ 6650
3.2065 $\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^4}\,dx$ 6653
3.2066 $\int \left(a+\frac{b}{x^4}\right)^{3/2} x^3\,dx$ 6657
3.2067 $\int \left(a+\frac{b}{x^4}\right)^{3/2} x\,dx$ 6660
3.2068 $\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x}\,dx$ 6663
3.2069 $\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^3}\,dx$ 6666
3.2070 $\int \left(a+\frac{b}{x^4}\right)^{3/2} x^2\,dx$ 6670
3.2071 $\int \left(a+\frac{b}{x^4}\right)^{3/2}\,dx$ 6673
3.2072 $\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^2}\,dx$ 6677
3.2073 $\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^4}\,dx$ 6680
3.2074 $\int \left(a+\frac{b}{x^4}\right)^{5/2} x^3\,dx$ 6684
3.2075 $\int \left(a+\frac{b}{x^4}\right)^{5/2} x\,dx$ 6688
3.2076 $\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x}\,dx$ 6692
3.2077 $\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^3}\,dx$ 6695
3.2078 $\int \left(a+\frac{b}{x^4}\right)^{5/2} x^2\,dx$ 6699
3.2079 $\int \left(a+\frac{b}{x^4}\right)^{5/2}\,dx$ 6702
3.2080 $\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^2}\,dx$ 6706
3.2081 $\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^4}\,dx$ 6709
3.2082 $\int \frac{x^3}{\sqrt{a+\frac{b}{x^4}}}\,dx$ 6713
3.2083 $\int \frac{x}{\sqrt{a+\frac{b}{x^4}}}\,dx$ 6716

3.2084 $\int \frac{1}{\sqrt{a+\frac{b}{x^4}}\,x}\,dx$ 6718
3.2085 $\int \frac{1}{\sqrt{a+\frac{b}{x^4}}\,x^3}\,dx$ 6721
3.2086 $\int \frac{x^2}{\sqrt{a+\frac{b}{x^4}}}\,dx$ 6724
3.2087 $\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}\,dx$ 6727
3.2088 $\int \frac{1}{\sqrt{a+\frac{b}{x^4}}\,x^2}\,dx$ 6731
3.2089 $\int \frac{1}{\sqrt{a+\frac{b}{x^4}}\,x^4}\,dx$ 6734
3.2090 $\int \frac{x^3}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx$ 6737
3.2091 $\int \frac{x}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx$ 6741
3.2092 $\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2}\,x}\,dx$ 6744
3.2093 $\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2}\,x^3}\,dx$ 6747
3.2094 $\int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx$ 6749
3.2095 $\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx$ 6752
3.2096 $\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2}\,x^2}\,dx$ 6756
3.2097 $\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2}\,x^4}\,dx$ 6759
3.2098 $\int \frac{x^3}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx$ 6763
3.2099 $\int \frac{x}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx$ 6767
3.2100 $\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2}\,x}\,dx$ 6770
3.2101 $\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2}\,x^3}\,dx$ 6774
3.2102 $\int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx$ 6777
3.2103 $\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx$ 6781
3.2104 $\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2}\,x^2}\,dx$ 6785
3.2105 $\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2}\,x^4}\,dx$ 6788
3.2106 $\int \frac{1}{a+\frac{b}{x^5}}\,dx$ 6792
3.2107 $\int \frac{1}{\sqrt{a+\frac{b}{x^5}}\,x}\,dx$ 6797
3.2108 $\int \frac{1}{\sqrt{-a+\frac{b}{x^5}}\,x}\,dx$ 6800

3.2109 $\int \frac{1}{a+\frac{b}{x^6}}\,dx$ 6803
3.2110 $\int \frac{1}{a+\frac{b}{x^8}}\,dx$ 6807
3.2111 $\int \left(a+b\sqrt{x}\right)x^4\,dx$ 6812
3.2112 $\int \left(a+b\sqrt{x}\right)x^3\,dx$ 6814
3.2113 $\int \left(a+b\sqrt{x}\right)x^2\,dx$ 6816
3.2114 $\int \left(a+b\sqrt{x}\right)x\,dx$ 6818
3.2115 $\int \left(a+b\sqrt{x}\right)\,dx$ 6820
3.2116 $\int \frac{a+b\sqrt{x}}{x}\,dx$ 6822
3.2117 $\int \frac{a+b\sqrt{x}}{x^2}\,dx$ 6824
3.2118 $\int \frac{a+b\sqrt{x}}{x^3}\,dx$ 6826
3.2119 $\int \frac{a+b\sqrt{x}}{x^4}\,dx$ 6828
3.2120 $\int \left(a+b\sqrt{x}\right)^2 x^4\,dx$ 6830
3.2121 $\int \left(a+b\sqrt{x}\right)^2 x^3\,dx$ 6832
3.2122 $\int \left(a+b\sqrt{x}\right)^2 x^2\,dx$ 6834
3.2123 $\int \left(a+b\sqrt{x}\right)^2 x\,dx$ 6836
3.2124 $\int \left(a+b\sqrt{x}\right)^2\,dx$ 6838
3.2125 $\int \frac{\left(a+b\sqrt{x}\right)^2}{x}\,dx$ 6840
3.2126 $\int \frac{\left(a+b\sqrt{x}\right)^2}{x^2}\,dx$ 6842
3.2127 $\int \frac{\left(a+b\sqrt{x}\right)^2}{x^3}\,dx$ 6844
3.2128 $\int \frac{\left(a+b\sqrt{x}\right)^2}{x^4}\,dx$ 6846
3.2129 $\int \frac{\left(a+b\sqrt{x}\right)^2}{x^5}\,dx$ 6848
3.2130 $\int \left(a+b\sqrt{x}\right)^3 x^4\,dx$ 6850
3.2131 $\int \left(a+b\sqrt{x}\right)^3 x^3\,dx$ 6852
3.2132 $\int \left(a+b\sqrt{x}\right)^3 x^2\,dx$ 6854
3.2133 $\int \left(a+b\sqrt{x}\right)^3 x\,dx$ 6856
3.2134 $\int \left(a+b\sqrt{x}\right)^3\,dx$ 6858
3.2135 $\int \frac{\left(a+b\sqrt{x}\right)^3}{x}\,dx$ 6860
3.2136 $\int \frac{\left(a+b\sqrt{x}\right)^3}{x^2}\,dx$ 6862
3.2137 $\int \frac{\left(a+b\sqrt{x}\right)^3}{x^3}\,dx$ 6864
3.2138 $\int \frac{\left(a+b\sqrt{x}\right)^3}{x^4}\,dx$ 6866
3.2139 $\int \frac{\left(a+b\sqrt{x}\right)^3}{x^5}\,dx$ 6868
3.2140 $\int \frac{\left(a+b\sqrt{x}\right)^3}{x^6}\,dx$ 6870
3.2141 $\int \left(a+b\sqrt{x}\right)^5 x^4\,dx$ 6872
3.2142 $\int \left(a+b\sqrt{x}\right)^5 x^3\,dx$ 6875
3.2143 $\int \left(a+b\sqrt{x}\right)^5 x^2\,dx$ 6878
3.2144 $\int \left(a+b\sqrt{x}\right)^5 x\,dx$ 6881

3.2145 $\int \left(a+b\sqrt{x}\right)^5\,dx$ . . . 6884
3.2146 $\int \frac{\left(a+b\sqrt{x}\right)^5}{x}\,dx$ . . . 6886
3.2147 $\int \frac{\left(a+b\sqrt{x}\right)^5}{x^2}\,dx$ . . . 6889
3.2148 $\int \frac{\left(a+b\sqrt{x}\right)^5}{x^3}\,dx$ . . . 6892
3.2149 $\int \frac{\left(a+b\sqrt{x}\right)^5}{x^4}\,dx$ . . . 6895
3.2150 $\int \frac{\left(a+b\sqrt{x}\right)^5}{x^5}\,dx$ . . . 6897
3.2151 $\int \frac{\left(a+b\sqrt{x}\right)^5}{x^6}\,dx$ . . . 6900
3.2152 $\int \frac{\left(a+b\sqrt{x}\right)^5}{x^7}\,dx$ . . . 6903
3.2153 $\int \left(a+b\sqrt{x}\right)^{10} x^4\,dx$ . . . 6906
3.2154 $\int \left(a+b\sqrt{x}\right)^{10} x^3\,dx$ . . . 6909
3.2155 $\int \left(a+b\sqrt{x}\right)^{10} x^2\,dx$ . . . 6912
3.2156 $\int \left(a+b\sqrt{x}\right)^{10} x\,dx$ . . . 6915
3.2157 $\int \left(a+b\sqrt{x}\right)^{10}\,dx$ . . . 6918
3.2158 $\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x}\,dx$ . . . 6921
3.2159 $\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^2}\,dx$ . . . 6924
3.2160 $\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^3}\,dx$ . . . 6927
3.2161 $\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^4}\,dx$ . . . 6930
3.2162 $\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^5}\,dx$ . . . 6933
3.2163 $\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^6}\,dx$ . . . 6936
3.2164 $\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^7}\,dx$ . . . 6939
3.2165 $\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^8}\,dx$ . . . 6942
3.2166 $\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^9}\,dx$ . . . 6945
3.2167 $\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^{10}}\,dx$ . . . 6948
3.2168 $\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^{11}}\,dx$ . . . 6951
3.2169 $\int \left(a+b\sqrt{x}\right)^{15} x^5\,dx$ . . . 6954
3.2170 $\int \left(a+b\sqrt{x}\right)^{15} x^4\,dx$ . . . 6957
3.2171 $\int \left(a+b\sqrt{x}\right)^{15} x^3\,dx$ . . . 6960
3.2172 $\int \left(a+b\sqrt{x}\right)^{15} x^2\,dx$ . . . 6963
3.2173 $\int \left(a+b\sqrt{x}\right)^{15} x\,dx$ . . . 6966
3.2174 $\int \left(a+b\sqrt{x}\right)^{15}\,dx$ . . . 6969
3.2175 $\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x}\,dx$ . . . 6972
3.2176 $\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^2}\,dx$ . . . 6975
3.2177 $\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^3}\,dx$ . . . 6978

3.2178 $\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^4}\,dx$ 6981
3.2179 $\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^6}\,dx$ 6984
3.2180 $\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^7}\,dx$ 6987
3.2181 $\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^8}\,dx$ 6990
3.2182 $\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^9}\,dx$ 6993
3.2183 $\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{10}}\,dx$ 6996
3.2184 $\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{11}}\,dx$ 6999
3.2185 $\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{12}}\,dx$ 7002
3.2186 $\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{13}}\,dx$ 7006
3.2187 $\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{14}}\,dx$ 7010
3.2188 $\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{15}}\,dx$ 7014
3.2189 $\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{16}}\,dx$ 7017
3.2190 $\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{17}}\,dx$ 7020
3.2191 $\int \frac{x^3}{a+b\sqrt{x}}\,dx$ 7023
3.2192 $\int \frac{x^2}{a+b\sqrt{x}}\,dx$ 7026
3.2193 $\int \frac{x}{a+b\sqrt{x}}\,dx$ 7029
3.2194 $\int \frac{1}{a+b\sqrt{x}}\,dx$ 7032
3.2195 $\int \frac{1}{\left(a+b\sqrt{x}\right)x}\,dx$ 7034
3.2196 $\int \frac{1}{\left(a+b\sqrt{x}\right)x^2}\,dx$ 7037
3.2197 $\int \frac{1}{\left(a+b\sqrt{x}\right)x^3}\,dx$ 7040
3.2198 $\int \frac{1}{\left(a+b\sqrt{x}\right)x^4}\,dx$ 7043
3.2199 $\int \frac{x^3}{\left(a+b\sqrt{x}\right)^2}\,dx$ 7046
3.2200 $\int \frac{x^2}{\left(a+b\sqrt{x}\right)^2}\,dx$ 7049
3.2201 $\int \frac{x}{\left(a+b\sqrt{x}\right)^2}\,dx$ 7052
3.2202 $\int \frac{1}{\left(a+b\sqrt{x}\right)^2}\,dx$ 7055
3.2203 $\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x}\,dx$ 7058
3.2204 $\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x^2}\,dx$ 7061
3.2205 $\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x^3}\,dx$ 7064
3.2206 $\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x^4}\,dx$ 7067
3.2207 $\int \frac{x^3}{\left(a+b\sqrt{x}\right)^3}\,dx$ 7070
3.2208 $\int \frac{x^2}{\left(a+b\sqrt{x}\right)^3}\,dx$ 7073

3.2209 $\int \frac{x}{\left(a+b\sqrt{x}\right)^3}\,dx$ 7076
3.2210 $\int \frac{1}{\left(a+b\sqrt{x}\right)^3}\,dx$ 7079
3.2211 $\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x}\,dx$ 7081
3.2212 $\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x^2}\,dx$ 7084
3.2213 $\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x^3}\,dx$ 7087
3.2214 $\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x^4}\,dx$ 7090
3.2215 $\int \frac{x^4}{\left(a+b\sqrt{x}\right)^5}\,dx$ 7093
3.2216 $\int \frac{x^3}{\left(a+b\sqrt{x}\right)^5}\,dx$ 7096
3.2217 $\int \frac{x^2}{\left(a+b\sqrt{x}\right)^5}\,dx$ 7099
3.2218 $\int \frac{x}{\left(a+b\sqrt{x}\right)^5}\,dx$ 7102
3.2219 $\int \frac{1}{\left(a+b\sqrt{x}\right)^5}\,dx$ 7105
3.2220 $\int \frac{1}{\left(a+b\sqrt{x}\right)^5 x}\,dx$ 7108
3.2221 $\int \frac{1}{\left(a+b\sqrt{x}\right)^5 x^2}\,dx$ 7111
3.2222 $\int \frac{1}{\left(a+b\sqrt{x}\right)^5 x^3}\,dx$ 7114
3.2223 $\int \frac{x^5}{\left(a+b\sqrt{x}\right)^8}\,dx$ 7117
3.2224 $\int \frac{x^4}{\left(a+b\sqrt{x}\right)^8}\,dx$ 7121
3.2225 $\int \frac{x^3}{\left(a+b\sqrt{x}\right)^8}\,dx$ 7125
3.2226 $\int \frac{x^2}{\left(a+b\sqrt{x}\right)^8}\,dx$ 7129
3.2227 $\int \frac{x}{\left(a+b\sqrt{x}\right)^8}\,dx$ 7132
3.2228 $\int \frac{1}{\left(a+b\sqrt{x}\right)^8}\,dx$ 7135
3.2229 $\int \frac{1}{\left(a+b\sqrt{x}\right)^8 x}\,dx$ 7138
3.2230 $\int \frac{1}{\left(a+b\sqrt{x}\right)^8 x^2}\,dx$ 7142
3.2231 $\int \frac{1}{\left(a+b\sqrt{x}\right)^8 x^3}\,dx$ 7146
3.2232 $\int \frac{1}{\left(2+b\sqrt{x}\right)x}\,dx$ 7151
3.2233 $\int \sqrt{a+b\sqrt{x}}\,x^2\,dx$ 7153
3.2234 $\int \sqrt{a+b\sqrt{x}}\,x\,dx$ 7160
3.2235 $\int \sqrt{a+b\sqrt{x}}\,dx$ 7164
3.2236 $\int \frac{\sqrt{a+b\sqrt{x}}}{x}\,dx$ 7167
3.2237 $\int \frac{\sqrt{a+b\sqrt{x}}}{x^2}\,dx$ 7170
3.2238 $\int \frac{\sqrt{a+b\sqrt{x}}}{x^3}\,dx$ 7173

3.2239 $\int \frac{x^2}{\sqrt{a+b\sqrt{x}}}\,dx$ 7177
3.2240 $\int \frac{x}{\sqrt{a+b\sqrt{x}}}\,dx$ 7184
3.2241 $\int \frac{1}{\sqrt{a+b\sqrt{x}}}\,dx$ 7187
3.2242 $\int \frac{1}{\sqrt{a+b\sqrt{x}}\,x}\,dx$ 7190
3.2243 $\int \frac{1}{\sqrt{a+b\sqrt{x}}\,x^2}\,dx$ 7193
3.2244 $\int \frac{1}{\sqrt{a+b\sqrt{x}}\,x^3}\,dx$ 7196
3.2245 $\int \left(a+b\sqrt{x}\right)^n \sqrt{x}\,dx$ 7200
3.2246 $\int \frac{\left(a+b\sqrt{x}\right)^n}{\sqrt{x}}\,dx$ 7205
3.2247 $\int \frac{1+\sqrt{x}}{\sqrt{x}}\,dx$ 7208
3.2248 $\int \frac{\left(1+\sqrt{x}\right)^2}{\sqrt{x}}\,dx$ 7210
3.2249 $\int \frac{\left(1+\sqrt{x}\right)^3}{\sqrt{x}}\,dx$ 7212
3.2250 $\int \frac{\sqrt{x}}{1+\sqrt{x}}\,dx$ 7214
3.2251 $\int \frac{1}{\left(1+\sqrt{x}\right)\sqrt{x}}\,dx$ 7216
3.2252 $\int \frac{1}{\left(1+\sqrt{x}\right)^2\sqrt{x}}\,dx$ 7218
3.2253 $\int \frac{1}{\left(1+\sqrt{x}\right)^3\sqrt{x}}\,dx$ 7220
3.2254 $\int \sqrt{1+\sqrt{x}}\,\sqrt{x}\,dx$ 7222
3.2255 $\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx$ 7225
3.2256 $\int \frac{\sqrt[3]{x}}{1+\sqrt{x}}\,dx$ 7227
3.2257 $\int \left(a+b\sqrt{x}\right)^4 x^m\,dx$ 7230
3.2258 $\int \left(a+b\sqrt{x}\right)^3 x^m\,dx$ 7233
3.2259 $\int \left(a+b\sqrt{x}\right)^2 x^m\,dx$ 7236
3.2260 $\int \left(a+b\sqrt{x}\right) x^m\,dx$ 7238
3.2261 $\int \frac{x^m}{a+b\sqrt{x}}\,dx$ 7240
3.2262 $\int \frac{x^m}{\left(a+b\sqrt{x}\right)^2}\,dx$ 7242
3.2263 $\int \left(a+b\sqrt{x}\right)^p x^m\,dx$ 7245
3.2264 $\int \left(a+b\sqrt{x}\right)^p x^3\,dx$ 7248
3.2265 $\int \left(a+b\sqrt{x}\right)^p x^2\,dx$ 7252
3.2266 $\int \left(a+b\sqrt{x}\right)^p x\,dx$ 7255
3.2267 $\int \left(a+b\sqrt{x}\right)^p\,dx$ 7258
3.2268 $\int \frac{\left(a+b\sqrt{x}\right)^p}{x}\,dx$ 7261
3.2269 $\int \frac{\left(a+b\sqrt{x}\right)^p}{x^2}\,dx$ 7264
3.2270 $\int \frac{\sqrt{x}}{1+x^{3/2}}\,dx$ 7267
3.2271 $\int \frac{x^3}{\left(a+bx^{3/2}\right)^{2/3}}\,dx$ 7269

3.2272 $\int \frac{1}{\left(a+bx^{3/2}\right)^{2/3}}\,dx$ . . . 7274
3.2273 $\int \frac{1}{x^3\left(a+bx^{3/2}\right)^{2/3}}\,dx$ . . . 7278
3.2274 $\int \frac{1}{x^6\left(a+bx^{3/2}\right)^{2/3}}\,dx$ . . . 7281
3.2275 $\int \frac{1}{x^9\left(a+bx^{3/2}\right)^{2/3}}\,dx$ . . . 7284
3.2276 $\int \frac{x^8}{\left(a+bx^{3/2}\right)^{2/3}}\,dx$ . . . 7288
3.2277 $\int \frac{x^5}{\left(a+bx^{3/2}\right)^{2/3}}\,dx$ . . . 7291
3.2278 $\int \frac{x^2}{\left(a+bx^{3/2}\right)^{2/3}}\,dx$ . . . 7294
3.2279 $\int \frac{1}{x\left(a+bx^{3/2}\right)^{2/3}}\,dx$ . . . 7297
3.2280 $\int \frac{1}{x^4\left(a+bx^{3/2}\right)^{2/3}}\,dx$ . . . 7301
3.2281 $\int \frac{x^4}{\left(a+bx^{3/2}\right)^{2/3}}\,dx$ . . . 7305
3.2282 $\int \frac{x}{\left(a+bx^{3/2}\right)^{2/3}}\,dx$ . . . 7308
3.2283 $\int \frac{1}{x^2\left(a+bx^{3/2}\right)^{2/3}}\,dx$ . . . 7311
3.2284 $\int \frac{1}{x^5\left(a+bx^{3/2}\right)^{2/3}}\,dx$ . . . 7314
3.2285 $\int \left(a+b\sqrt[3]{x}\right)x^4\,dx$ . . . 7317
3.2286 $\int \left(a+b\sqrt[3]{x}\right)x^3\,dx$ . . . 7319
3.2287 $\int \left(a+b\sqrt[3]{x}\right)x^2\,dx$ . . . 7321
3.2288 $\int \left(a+b\sqrt[3]{x}\right)x\,dx$ . . . 7323
3.2289 $\int \left(a+b\sqrt[3]{x}\right)\,dx$ . . . 7325
3.2290 $\int \frac{a+b\sqrt[3]{x}}{x}\,dx$ . . . 7327
3.2291 $\int \frac{a+b\sqrt[3]{x}}{x^2}\,dx$ . . . 7329
3.2292 $\int \frac{a+b\sqrt[3]{x}}{x^3}\,dx$ . . . 7331
3.2293 $\int \frac{a+b\sqrt[3]{x}}{x^4}\,dx$ . . . 7333
3.2294 $\int \left(a+b\sqrt[3]{x}\right)^2 x^4\,dx$ . . . 7335
3.2295 $\int \left(a+b\sqrt[3]{x}\right)^2 x^3\,dx$ . . . 7337
3.2296 $\int \left(a+b\sqrt[3]{x}\right)^2 x^2\,dx$ . . . 7339
3.2297 $\int \left(a+b\sqrt[3]{x}\right)^2 x\,dx$ . . . 7341
3.2298 $\int \left(a+b\sqrt[3]{x}\right)^2\,dx$ . . . 7343
3.2299 $\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x}\,dx$ . . . 7345
3.2300 $\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x^2}\,dx$ . . . 7347
3.2301 $\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x^3}\,dx$ . . . 7349
3.2302 $\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x^4}\,dx$ . . . 7351
3.2303 $\int \left(a+b\sqrt[3]{x}\right)^3 x^4\,dx$ . . . 7353
3.2304 $\int \left(a+b\sqrt[3]{x}\right)^3 x^3\,dx$ . . . 7355
3.2305 $\int \left(a+b\sqrt[3]{x}\right)^3 x^2\,dx$ . . . 7357

3.2306 $\int \left(a+b\sqrt[3]{x}\right)^3 x\,dx$ 7359
3.2307 $\int \left(a+b\sqrt[3]{x}\right)^3\,dx$ 7361
3.2308 $\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x}\,dx$ 7363
3.2309 $\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^2}\,dx$ 7365
3.2310 $\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^3}\,dx$ 7367
3.2311 $\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^4}\,dx$ 7369
3.2312 $\int \left(a+b\sqrt[3]{x}\right)^5 x^4\,dx$ 7371
3.2313 $\int \left(a+b\sqrt[3]{x}\right)^5 x^3\,dx$ 7374
3.2314 $\int \left(a+b\sqrt[3]{x}\right)^5 x^2\,dx$ 7377
3.2315 $\int \left(a+b\sqrt[3]{x}\right)^5 x\,dx$ 7380
3.2316 $\int \left(a+b\sqrt[3]{x}\right)^5\,dx$ 7383
3.2317 $\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x}\,dx$ 7386
3.2318 $\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^2}\,dx$ 7389
3.2319 $\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^3}\,dx$ 7392
3.2320 $\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^4}\,dx$ 7394
3.2321 $\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^5}\,dx$ 7397
3.2322 $\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^6}\,dx$ 7400
3.2323 $\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^7}\,dx$ 7403
3.2324 $\int \left(a+b\sqrt[3]{x}\right)^{10} x^4\,dx$ 7406
3.2325 $\int \left(a+b\sqrt[3]{x}\right)^{10} x^3\,dx$ 7409
3.2326 $\int \left(a+b\sqrt[3]{x}\right)^{10} x^2\,dx$ 7412
3.2327 $\int \left(a+b\sqrt[3]{x}\right)^{10} x\,dx$ 7415
3.2328 $\int \left(a+b\sqrt[3]{x}\right)^{10}\,dx$ 7418
3.2329 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x}\,dx$ 7421
3.2330 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^2}\,dx$ 7424
3.2331 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^3}\,dx$ 7427
3.2332 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^4}\,dx$ 7430
3.2333 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^5}\,dx$ 7433
3.2334 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^6}\,dx$ 7436
3.2335 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^7}\,dx$ 7439
3.2336 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^8}\,dx$ 7442
3.2337 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^9}\,dx$ 7445
3.2338 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^{10}}\,dx$ 7448

3.2339 $\int \left(a+b\sqrt[3]{x}\right)^{15} x^5\,dx$ . . . 7451
3.2340 $\int \left(a+b\sqrt[3]{x}\right)^{15} x^4\,dx$ . . . 7454
3.2341 $\int \left(a+b\sqrt[3]{x}\right)^{15} x^3\,dx$ . . . 7457
3.2342 $\int \left(a+b\sqrt[3]{x}\right)^{15} x^2\,dx$ . . . 7460
3.2343 $\int \left(a+b\sqrt[3]{x}\right)^{15} x\,dx$ . . . 7463
3.2344 $\int \left(a+b\sqrt[3]{x}\right)^{15}\,dx$ . . . 7466
3.2345 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x}\,dx$ . . . 7469
3.2346 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^2}\,dx$ . . . 7472
3.2347 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^3}\,dx$ . . . 7475
3.2348 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^4}\,dx$ . . . 7478
3.2349 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^6}\,dx$ . . . 7481
3.2350 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^7}\,dx$ . . . 7484
3.2351 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^8}\,dx$ . . . 7487
3.2352 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^9}\,dx$ . . . 7491
3.2353 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{10}}\,dx$ . . . 7495
3.2354 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{11}}\,dx$ . . . 7498
3.2355 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{12}}\,dx$ . . . 7501
3.2356 $\int \frac{x^3}{a+b\sqrt[3]{x}}\,dx$ . . . 7504
3.2357 $\int \frac{x^2}{a+b\sqrt[3]{x}}\,dx$ . . . 7507
3.2358 $\int \frac{x}{a+b\sqrt[3]{x}}\,dx$ . . . 7510
3.2359 $\int \frac{1}{a+b\sqrt[3]{x}}\,dx$ . . . 7513
3.2360 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x}\,dx$ . . . 7516
3.2361 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x^2}\,dx$ . . . 7519
3.2362 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x^3}\,dx$ . . . 7522
3.2363 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x^4}\,dx$ . . . 7525
3.2364 $\int \frac{1}{\left(2+b\sqrt[3]{x}\right)x}\,dx$ . . . 7528
3.2365 $\int \frac{x^3}{\left(a+b\sqrt[3]{x}\right)^2}\,dx$ . . . 7531
3.2366 $\int \frac{x^2}{\left(a+b\sqrt[3]{x}\right)^2}\,dx$ . . . 7534
3.2367 $\int \frac{x}{\left(a+b\sqrt[3]{x}\right)^2}\,dx$ . . . 7537
3.2368 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2}\,dx$ . . . 7540
3.2369 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x}\,dx$ . . . 7543
3.2370 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x^2}\,dx$ . . . 7546

3.2371 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x^3}\,dx$ 7549
3.2372 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x^4}\,dx$ 7552
3.2373 $\int \frac{x^3}{\left(a+b\sqrt[3]{x}\right)^3}\,dx$ 7555
3.2374 $\int \frac{x^2}{\left(a+b\sqrt[3]{x}\right)^3}\,dx$ 7559
3.2375 $\int \frac{x}{\left(a+b\sqrt[3]{x}\right)^3}\,dx$ 7562
3.2376 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3}\,dx$ 7565
3.2377 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x}\,dx$ 7568
3.2378 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x^2}\,dx$ 7571
3.2379 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x^3}\,dx$ 7574
3.2380 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x^4}\,dx$ 7577
3.2381 $\int \frac{1}{\sqrt{1+\sqrt[3]{x}}}\,dx$ 7581
3.2382 $\int \frac{1}{\left(1+\sqrt[3]{x}\right)x^{3/2}}\,dx$ 7584
3.2383 $\int \frac{x^{2/3}}{1+\sqrt[3]{x}}\,dx$ 7587
3.2384 $\int \frac{1}{1+x^{2/3}}\,dx$ 7589
3.2385 $\int \frac{1}{\left(1+x^{2/3}\right)\sqrt[3]{x}}\,dx$ 7591
3.2386 $\int \frac{1}{\left(1+x^{2/3}\right)x^{2/3}}\,dx$ 7593
3.2387 $\int \frac{\sqrt{-1+x^{2/3}}}{\sqrt[3]{x}}\,dx$ 7595
3.2388 $\int \frac{\left(1+x^{2/3}\right)^{3/2}}{\sqrt[3]{x}}\,dx$ 7597
3.2389 $\int \frac{\sqrt{x}}{1+x^{2/3}}\,dx$ 7599
3.2390 $\int \frac{\sqrt[3]{x}}{-1+x^{5/6}}\,dx$ 7603
3.2391 $\int \sqrt{3-\frac{1}{\sqrt{x}}}\,dx$ 7608
3.2392 $\int \frac{1}{\sqrt{1+\frac{1}{\sqrt{x}}}}\,dx$ 7612
3.2393 $\int \left(a+\frac{b}{x^{3/2}}\right)^{2/3}\,dx$ 7615
3.2394 $\int \left(a+\frac{b}{\sqrt[3]{x}}\right)x^4\,dx$ 7618
3.2395 $\int \left(a+\frac{b}{\sqrt[3]{x}}\right)x^3\,dx$ 7620
3.2396 $\int \left(a+\frac{b}{\sqrt[3]{x}}\right)x^2\,dx$ 7622
3.2397 $\int \left(a+\frac{b}{\sqrt[3]{x}}\right)x\,dx$ 7624
3.2398 $\int \left(a+\frac{b}{\sqrt[3]{x}}\right)\,dx$ 7626
3.2399 $\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x}\,dx$ 7628
3.2400 $\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x^2}\,dx$ 7630

3.2401 $\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x^3}\,dx$ 7632
3.2402 $\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x^4}\,dx$ 7634
3.2403 $\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^4\,dx$ 7636
3.2404 $\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^3\,dx$ 7638
3.2405 $\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^2\,dx$ 7640
3.2406 $\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x\,dx$ 7642
3.2407 $\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 dx$ 7644
3.2408 $\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x}\,dx$ 7646
3.2409 $\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x^2}\,dx$ 7649
3.2410 $\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x^3}\,dx$ 7652
3.2411 $\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x^4}\,dx$ 7655
3.2412 $\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^4\,dx$ 7658
3.2413 $\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^3\,dx$ 7660
3.2414 $\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^2\,dx$ 7662
3.2415 $\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x\,dx$ 7664
3.2416 $\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 dx$ 7666
3.2417 $\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x}\,dx$ 7668
3.2418 $\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x^2}\,dx$ 7671
3.2419 $\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x^3}\,dx$ 7674
3.2420 $\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x^4}\,dx$ 7677
3.2421 $\int \frac{x^2}{a+\frac{b}{\sqrt[3]{x}}}\,dx$ 7680
3.2422 $\int \frac{x}{a+\frac{b}{\sqrt[3]{x}}}\,dx$ 7683
3.2423 $\int \frac{1}{a+\frac{b}{\sqrt[3]{x}}}\,dx$ 7686
3.2424 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x}\,dx$ 7689
3.2425 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x^2}\,dx$ 7691
3.2426 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x^3}\,dx$ 7694

3.2427 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x^4}\,dx$ 7697
3.2428 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x^5}\,dx$ 7700
3.2429 $\int \frac{x^2}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}\,dx$ 7704
3.2430 $\int \frac{x}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}\,dx$ 7708
3.2431 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}\,dx$ 7711
3.2432 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x}\,dx$ 7714
3.2433 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^2}\,dx$ 7717
3.2434 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^3}\,dx$ 7720
3.2435 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^4}\,dx$ 7723
3.2436 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^5}\,dx$ 7727
3.2437 $\int \frac{x^2}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}\,dx$ 7731
3.2438 $\int \frac{x}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}\,dx$ 7735
3.2439 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}\,dx$ 7739
3.2440 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x}\,dx$ 7742
3.2441 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^2}\,dx$ 7745
3.2442 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^3}\,dx$ 7748
3.2443 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^4}\,dx$ 7751
3.2444 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^5}\,dx$ 7755
3.2445 $\int \frac{1}{1+\frac{b}{\sqrt[3]{x}}}\,dx$ 7759
3.2446 $\int x^{2/3}\left(1+x^{5/3}\right)^{2/3}\,dx$ 7761
3.2447 $\int x^{7/3}\left(a^{10/3}-x^{10/3}\right)^{19/7}\,dx$ 7763
3.2448 $\int \frac{1}{1+\sqrt[5]{x}}\,dx$ 7765
3.2449 $\int \frac{1}{\sqrt{1+x^{4/5}}\,\sqrt[5]{x}}\,dx$ 7767
3.2450 $\int \left(a+\frac{b}{x^{3/5}}\right)^{2/3}\,dx$ 7769
3.2451 $\int x^3\left(a+bx^n\right)\,dx$ 7771

3.2452 $\int x^2 (a + bx^n)\, dx$ . . . 7773
3.2453 $\int x (a + bx^n)\, dx$ . . . 7775
3.2454 $\int (a + bx^n)\, dx$ . . . 7777
3.2455 $\int \frac{a+bx^n}{x}\, dx$ . . . 7779
3.2456 $\int \frac{a+bx^n}{x^2}\, dx$ . . . 7781
3.2457 $\int \frac{a+bx^n}{x^3}\, dx$ . . . 7783
3.2458 $\int x^3 (a + bx^n)^2\, dx$ . . . 7785
3.2459 $\int x^2 (a + bx^n)^2\, dx$ . . . 7787
3.2460 $\int x (a + bx^n)^2\, dx$ . . . 7789
3.2461 $\int (a + bx^n)^2\, dx$ . . . 7791
3.2462 $\int \frac{(a+bx^n)^2}{x}\, dx$ . . . 7793
3.2463 $\int \frac{(a+bx^n)^2}{x^2}\, dx$ . . . 7795
3.2464 $\int \frac{(a+bx^n)^2}{x^3}\, dx$ . . . 7797
3.2465 $\int x^3 (a + bx^n)^3\, dx$ . . . 7799
3.2466 $\int x^2 (a + bx^n)^3\, dx$ . . . 7802
3.2467 $\int x (a + bx^n)^3\, dx$ . . . 7805
3.2468 $\int (a + bx^n)^3\, dx$ . . . 7808
3.2469 $\int \frac{(a+bx^n)^3}{x}\, dx$ . . . 7811
3.2470 $\int \frac{(a+bx^n)^3}{x^2}\, dx$ . . . 7813
3.2471 $\int \frac{(a+bx^n)^3}{x^3}\, dx$ . . . 7816
3.2472 $\int \frac{x^3}{a+bx^n}\, dx$ . . . 7819
3.2473 $\int \frac{1}{a+bx^n}\, dx$ . . . 7821
3.2474 $\int \frac{1}{x(a+bx^n)}\, dx$ . . . 7823
3.2475 $\int \frac{1}{x^2(a+bx^n)}\, dx$ . . . 7826
3.2476 $\int \frac{1}{x^3(a+bx^n)}\, dx$ . . . 7828
3.2477 $\int \frac{x}{(a+bx^n)^2}\, dx$ . . . 7830
3.2478 $\int \frac{1}{(a+bx^n)^2}\, dx$ . . . 7832
3.2479 $\int \frac{1}{x(a+bx^n)^2}\, dx$ . . . 7834
3.2480 $\int \frac{1}{x^2(a+bx^n)^2}\, dx$ . . . 7837
3.2481 $\int \frac{1}{x^3(a+bx^n)^2}\, dx$ . . . 7839
3.2482 $\int \frac{x}{(a+bx^n)^3}\, dx$ . . . 7841
3.2483 $\int \frac{1}{(a+bx^n)^3}\, dx$ . . . 7844
3.2484 $\int \frac{1}{x(a+bx^n)^3}\, dx$ . . . 7847
3.2485 $\int \frac{1}{x^2(a+bx^n)^3}\, dx$ . . . 7850
3.2486 $\int \frac{1}{x^3(a+bx^n)^3}\, dx$ . . . 7853
3.2487 $\int x\sqrt{a + bx^n}\, dx$ . . . 7856
3.2488 $\int \sqrt{a + bx^n}\, dx$ . . . 7859
3.2489 $\int \frac{\sqrt{a+bx^n}}{x}\, dx$ . . . 7862
3.2490 $\int \frac{\sqrt{a+bx^n}}{x^2}\, dx$ . . . 7865
3.2491 $\int \frac{\sqrt{a+bx^n}}{x^3}\, dx$ . . . 7868
3.2492 $\int x (a + bx^n)^{3/2}\, dx$ . . . 7871

3.2493 $\int (a+bx^n)^{3/2}\,dx$ . . . 7874
3.2494 $\int \frac{(a+bx^n)^{3/2}}{x}\,dx$ . . . 7877
3.2495 $\int \frac{(a+bx^n)^{3/2}}{x^2}\,dx$ . . . 7880
3.2496 $\int \frac{(a+bx^n)^{3/2}}{x^3}\,dx$ . . . 7883
3.2497 $\int x\,(a+bx^n)^{5/2}\,dx$ . . . 7886
3.2498 $\int (a+bx^n)^{5/2}\,dx$ . . . 7889
3.2499 $\int \frac{(a+bx^n)^{5/2}}{x}\,dx$ . . . 7892
3.2500 $\int \frac{(a+bx^n)^{5/2}}{x^2}\,dx$ . . . 7895
3.2501 $\int \frac{(a+bx^n)^{5/2}}{x^3}\,dx$ . . . 7898
3.2502 $\int \frac{x}{\sqrt{a+bx^n}}\,dx$ . . . 7901
3.2503 $\int \frac{1}{\sqrt{a+bx^n}}\,dx$ . . . 7904
3.2504 $\int \frac{1}{x\sqrt{a+bx^n}}\,dx$ . . . 7907
3.2505 $\int \frac{1}{x^2\sqrt{a+bx^n}}\,dx$ . . . 7910
3.2506 $\int \frac{1}{x^3\sqrt{a+bx^n}}\,dx$ . . . 7913
3.2507 $\int \frac{x}{(a+bx^n)^{3/2}}\,dx$ . . . 7916
3.2508 $\int \frac{1}{(a+bx^n)^{3/2}}\,dx$ . . . 7919
3.2509 $\int \frac{1}{x(a+bx^n)^{3/2}}\,dx$ . . . 7922
3.2510 $\int \frac{1}{x^2(a+bx^n)^{3/2}}\,dx$ . . . 7925
3.2511 $\int \frac{1}{x^3(a+bx^n)^{3/2}}\,dx$ . . . 7928
3.2512 $\int \frac{x}{(a+bx^n)^{5/2}}\,dx$ . . . 7931
3.2513 $\int \frac{1}{(a+bx^n)^{5/2}}\,dx$ . . . 7934
3.2514 $\int \frac{1}{x(a+bx^n)^{5/2}}\,dx$ . . . 7937
3.2515 $\int \frac{1}{x^2(a+bx^n)^{5/2}}\,dx$ . . . 7941
3.2516 $\int \frac{1}{x^3(a+bx^n)^{5/2}}\,dx$ . . . 7944
3.2517 $\int \frac{\sqrt[3]{a+bx^n}}{x}\,dx$ . . . 7947
3.2518 $\int x^{-1+4n}\,(a+bx^n)\,dx$ . . . 7951
3.2519 $\int x^{-1+3n}\,(a+bx^n)\,dx$ . . . 7953
3.2520 $\int x^{-1+2n}\,(a+bx^n)\,dx$ . . . 7955
3.2521 $\int x^{-1+n}\,(a+bx^n)\,dx$ . . . 7957
3.2522 $\int \frac{a+bx^n}{x}\,dx$ . . . 7959
3.2523 $\int x^{-1-n}\,(a+bx^n)\,dx$ . . . 7961
3.2524 $\int x^{-1-2n}\,(a+bx^n)\,dx$ . . . 7963
3.2525 $\int x^{-1-3n}\,(a+bx^n)\,dx$ . . . 7965
3.2526 $\int x^{-1-4n}\,(a+bx^n)\,dx$ . . . 7967
3.2527 $\int x^{-1-5n}\,(a+bx^n)\,dx$ . . . 7969
3.2528 $\int x^{-1+4n}\,(a+bx^n)^2\,dx$ . . . 7971
3.2529 $\int x^{-1+3n}\,(a+bx^n)^2\,dx$ . . . 7973
3.2530 $\int x^{-1+2n}\,(a+bx^n)^2\,dx$ . . . 7975
3.2531 $\int x^{-1+n}\,(a+bx^n)^2\,dx$ . . . 7977
3.2532 $\int \frac{(a+bx^n)^2}{x}\,dx$ . . . 7979
3.2533 $\int x^{-1-n}\,(a+bx^n)^2\,dx$ . . . 7981

3.2534 $\int x^{-1-2n}\left(a+bx^n\right)^2\,dx$ . . . 7983
3.2535 $\int x^{-1-3n}\left(a+bx^n\right)^2\,dx$ . . . 7986
3.2536 $\int x^{-1-4n}\left(a+bx^n\right)^2\,dx$ . . . 7988
3.2537 $\int x^{-1-5n}\left(a+bx^n\right)^2\,dx$ . . . 7990
3.2538 $\int x^{-1-6n}\left(a+bx^n\right)^2\,dx$ . . . 7992
3.2539 $\int x^{-1+4n}\left(a+bx^n\right)^3\,dx$ . . . 7994
3.2540 $\int x^{-1+3n}\left(a+bx^n\right)^3\,dx$ . . . 7997
3.2541 $\int x^{-1+2n}\left(a+bx^n\right)^3\,dx$ . . . 8000
3.2542 $\int x^{-1+n}\left(a+bx^n\right)^3\,dx$ . . . 8002
3.2543 $\int \frac{\left(a+bx^n\right)^3}{x}\,dx$ . . . 8004
3.2544 $\int x^{-1-n}\left(a+bx^n\right)^3\,dx$ . . . 8006
3.2545 $\int x^{-1-2n}\left(a+bx^n\right)^3\,dx$ . . . 8009
3.2546 $\int x^{-1-3n}\left(a+bx^n\right)^3\,dx$ . . . 8012
3.2547 $\int x^{-1-4n}\left(a+bx^n\right)^3\,dx$ . . . 8014
3.2548 $\int x^{-1-5n}\left(a+bx^n\right)^3\,dx$ . . . 8016
3.2549 $\int x^{-1-6n}\left(a+bx^n\right)^3\,dx$ . . . 8019
3.2550 $\int x^{-1-7n}\left(a+bx^n\right)^3\,dx$ . . . 8022
3.2551 $\int x^{-1+4n}\left(a+bx^n\right)^5\,dx$ . . . 8025
3.2552 $\int x^{-1+3n}\left(a+bx^n\right)^5\,dx$ . . . 8028
3.2553 $\int x^{-1+2n}\left(a+bx^n\right)^5\,dx$ . . . 8031
3.2554 $\int x^{-1+n}\left(a+bx^n\right)^5\,dx$ . . . 8033
3.2555 $\int \frac{\left(a+bx^n\right)^5}{x}\,dx$ . . . 8035
3.2556 $\int x^{-1-n}\left(a+bx^n\right)^5\,dx$ . . . 8038
3.2557 $\int x^{-1-2n}\left(a+bx^n\right)^5\,dx$ . . . 8041
3.2558 $\int x^{-1-3n}\left(a+bx^n\right)^5\,dx$ . . . 8044
3.2559 $\int x^{-1-4n}\left(a+bx^n\right)^5\,dx$ . . . 8047
3.2560 $\int x^{-1-5n}\left(a+bx^n\right)^5\,dx$ . . . 8050
3.2561 $\int x^{-1-6n}\left(a+bx^n\right)^5\,dx$ . . . 8053
3.2562 $\int x^{-1-7n}\left(a+bx^n\right)^5\,dx$ . . . 8055
3.2563 $\int x^{-1-8n}\left(a+bx^n\right)^5\,dx$ . . . 8058
3.2564 $\int x^{-1-9n}\left(a+bx^n\right)^5\,dx$ . . . 8061
3.2565 $\int x^{-1-10n}\left(a+bx^n\right)^5\,dx$ . . . 8064
3.2566 $\int x^{-1+9n}\left(a+bx^n\right)^8\,dx$ . . . 8067
3.2567 $\int x^{-1+8n}\left(a+bx^n\right)^8\,dx$ . . . 8070
3.2568 $\int x^{-1+7n}\left(a+bx^n\right)^8\,dx$ . . . 8073
3.2569 $\int x^{-1+6n}\left(a+bx^n\right)^8\,dx$ . . . 8076
3.2570 $\int x^{-1+5n}\left(a+bx^n\right)^8\,dx$ . . . 8079
3.2571 $\int x^{-1+4n}\left(a+bx^n\right)^8\,dx$ . . . 8082
3.2572 $\int x^{-1+3n}\left(a+bx^n\right)^8\,dx$ . . . 8085
3.2573 $\int x^{-1+2n}\left(a+bx^n\right)^8\,dx$ . . . 8088
3.2574 $\int x^{-1+n}\left(a+bx^n\right)^8\,dx$ . . . 8090
3.2575 $\int \frac{\left(a+bx^n\right)^8}{x}\,dx$ . . . 8092
3.2576 $\int x^{-1-n}\left(a+bx^n\right)^8\,dx$ . . . 8095
3.2577 $\int x^{-1-2n}\left(a+bx^n\right)^8\,dx$ . . . 8098
3.2578 $\int x^{-1-3n}\left(a+bx^n\right)^8\,dx$ . . . 8101
3.2579 $\int x^{-1-4n}\left(a+bx^n\right)^8\,dx$ . . . 8104
3.2580 $\int x^{-1-5n}\left(a+bx^n\right)^8\,dx$ . . . 8107

3.2581 $\int x^{-1-6n}\left(a+bx^n\right)^8\,dx$ 8110
3.2582 $\int x^{-1-7n}\left(a+bx^n\right)^8\,dx$ 8113
3.2583 $\int x^{-1-8n}\left(a+bx^n\right)^8\,dx$ 8116
3.2584 $\int x^{-1-9n}\left(a+bx^n\right)^8\,dx$ 8119
3.2585 $\int x^{-1-10n}\left(a+bx^n\right)^8\,dx$ 8121
3.2586 $\int x^{-1-11n}\left(a+bx^n\right)^8\,dx$ 8124
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3.2588 $\int x^{-1-13n}\left(a+bx^n\right)^8\,dx$ 8130
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3.2591 $\int x^{-1+n}\left(a+bx^n\right)^{16}\,dx$ 8139
3.2592 $\int x^{12}\left(a+bx^{13}\right)^{12}\,dx$ 8141
3.2593 $\int x^{24}\left(a+bx^{25}\right)^{12}\,dx$ 8143
3.2594 $\int x^{36}\left(a+bx^{37}\right)^{12}\,dx$ 8145
3.2595 $\int x^{12m}\left(a+bx^{1+12m}\right)^{12}\,dx$ 8147
3.2596 $\int x^{12+12(-1+m)}\left(a+bx^{1+12m}\right)^{12}\,dx$ 8150
3.2597 $\int \frac{x^{-1+5n}}{a+bx^n}\,dx$ 8153
3.2598 $\int \frac{x^{-1+4n}}{a+bx^n}\,dx$ 8156
3.2599 $\int \frac{x^{-1+3n}}{a+bx^n}\,dx$ 8159
3.2600 $\int \frac{x^{-1+2n}}{a+bx^n}\,dx$ 8162
3.2601 $\int \frac{x^{-1+n}}{a+bx^n}\,dx$ 8164
3.2602 $\int \frac{1}{x(a+bx^n)}\,dx$ 8166
3.2603 $\int \frac{x^{-1-n}}{a+bx^n}\,dx$ 8169
3.2604 $\int \frac{x^{-1-2n}}{a+bx^n}\,dx$ 8172
3.2605 $\int \frac{x^{-1-3n}}{a+bx^n}\,dx$ 8175
3.2606 $\int \frac{x^{4+5(-1+n)}}{a+bx^n}\,dx$ 8178
3.2607 $\int \frac{x^{3+4(-1+n)}}{a+bx^n}\,dx$ 8181
3.2608 $\int \frac{x^{2+3(-1+n)}}{a+bx^n}\,dx$ 8184
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3.2611 $\int \frac{1}{x(a+bx^n)}\,dx$ 8191
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3.2684 $\int \frac{x^{m-n}}{\sqrt{a+bx^n}}\,dx$ 8411
3.2685 $\int \frac{x^{m-2n}}{\sqrt{a+bx^n}}\,dx$ 8414
3.2686 $\int \left(-\frac{bnx^{-1+m+n}}{2\left(a+bx^n\right)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^n}}\right)\,dx$ 8417
3.2687 $\int \frac{x^{-1+\frac{7n}{2}}}{\sqrt{a+bx^n}}\,dx$ 8420
3.2688 $\int \frac{x^{-1+\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx$ 8423
3.2689 $\int \frac{x^{-1+\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx$ 8426
3.2690 $\int \frac{x^{-1+\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx$ 8429
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3.2692 $\int \frac{x^{-1-\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx$ . . . 8434
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3.2694 $\int \frac{x^{-1-\frac{7n}{2}}}{\sqrt{a+bx^n}}\,dx$ . . . 8439
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3.2699 $\int \left(-\frac{bnx^{-1+m+n}}{2(a+bx^n)^{3/2}} + \frac{mx^{-1+m}}{\sqrt{a+bx^n}}\right)dx$ . . . 8454
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3.2702 $\int \frac{x^{-1+\frac{n}{3}}}{\sqrt[3]{a+bx^n}}\,dx$ . . . 8463
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3.2715 $\int (bx^n)^p\,dx$ . . . 8497
3.2716 $\int \frac{(bx^n)^p}{x}\,dx$ . . . 8499
3.2717 $\int \frac{(bx^n)^p}{x^2}\,dx$ . . . 8501
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3.2720 $\int x^{-1+n}\,(a + bx^n)^p\,dx$ . . . 8507
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3.2723 $\int x^{-1+4n}\,(a + bx^n)^p\,dx$ . . . 8514
3.2724 $\int x^{-1-n-np}\,(a + bx^n)^p\,dx$ . . . 8517
3.2725 $\int x^{-1-9n}\,(a + bx^n)^8\,dx$ . . . 8519
3.2726 $\int x^{-4-3p}\left(a + bx^3\right)^p\,dx$ . . . 8521
3.2727 $\int \frac{\left(a+bx^3\right)^8}{x^{28}}\,dx$ . . . 8523
3.2728 $\int \frac{1}{x(a+bx^n)}\,dx$ . . . 8525
3.2729 $\int \frac{1}{x\left(a+bx^3\right)}\,dx$ . . . 8528

3.2730 $\int (a+bx^n)^{-\frac{1+4n}{n}}\,dx$ . . . 8531
3.2731 $\int (a+bx^n)^{-\frac{1+3n}{n}}\,dx$ . . . 8534
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3.2739 $\int x^m \left(a+bx^{1+m}\right)^n\,dx$ . . . 8554
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3.2741 $\int x^m \left(a+bx^{2+2m}\right)^2\,dx$ . . . 8558
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3.2743 $\int \frac{x^m}{a+bx^{2+2m}}\,dx$ . . . 8562
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3.2756 $\int (cx)^m \left(a+bx^3\right)^2\,dx$ . . . 8599
3.2757 $\int (cx)^m \left(a+bx^2\right)^2\,dx$ . . . 8602
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3.2839 $\int \frac{1}{\left(\frac{c}{(a+bx)^{3/2}}\right)^{2/3}}\,dx$ 8839
3.2840 $\int \frac{1}{\left(\frac{c}{(a+bx)^{2/3}}\right)^{3/2}}\,dx$ 8842
3.2841 $\int (c+dx)^3\left(a+b(c+dx)^2\right)\,dx$ 8845
3.2842 $\int (c+dx)^3\left(a+b(c+dx)^2\right)^2\,dx$ 8848
3.2843 $\int (c+dx)^3\left(a+b(c+dx)^2\right)^3\,dx$ 8851
3.2844 $\int \frac{2+x}{1+(2+x)^2}\,dx$ 8854
3.2845 $\int \frac{2+x}{\left(1+(2+x)^2\right)^2}\,dx$ 8856
3.2846 $\int \frac{2+x}{\left(1+(2+x)^2\right)^3}\,dx$ 8858
3.2847 $\int (c+dx)^5\left(a+b(c+dx)^2\right)^p\,dx$ 8860
3.2848 $\int (c+dx)^4\left(a+b(c+dx)^2\right)^p\,dx$ 8864
3.2849 $\int (c+dx)^3\left(a+b(c+dx)^2\right)^p\,dx$ 8867
3.2850 $\int (c+dx)^2\left(a+b(c+dx)^2\right)^p\,dx$ 8870
3.2851 $\int (c+dx)\left(a+b(c+dx)^2\right)^p\,dx$ 8873
3.2852 $\int \frac{\left(a+b(c+dx)^2\right)^p}{c+dx}\,dx$ 8875
3.2853 $\int (c+dx)^3\left(a+b(c+dx)^3\right)\,dx$ 8878
3.2854 $\int (c+dx)^3\left(a+b(c+dx)^3\right)^2\,dx$ 8881
3.2855 $\int (c+dx)^3\left(a+b(c+dx)^3\right)^3\,dx$ 8884
3.2856 $\int (ce+dex)^3\left(a+b(c+dx)^3\right)\,dx$ 8888
3.2857 $\int (ce+dex)^3\left(a+b(c+dx)^3\right)^2\,dx$ 8891
3.2858 $\int (ce+dex)^3\left(a+b(c+dx)^3\right)^3\,dx$ 8894
3.2859 $\int \frac{(c+dx)^4}{a+b(c+dx)^3}\,dx$ 8898
3.2860 $\int \frac{(c+dx)^3}{a+b(c+dx)^3}\,dx$ 8902
3.2861 $\int \frac{(c+dx)^2}{a+b(c+dx)^3}\,dx$ 8906
3.2862 $\int \frac{c+dx}{a+b(c+dx)^3}\,dx$ 8908
3.2863 $\int \frac{1}{a+b(c+dx)^3}\,dx$ 8912
3.2864 $\int \frac{1}{(c+dx)\left(a+b(c+dx)^3\right)}\,dx$ 8916
3.2865 $\int \frac{1}{(c+dx)^2\left(a+b(c+dx)^3\right)}\,dx$ 8919
3.2866 $\int \frac{1}{(c+dx)^3\left(a+b(c+dx)^3\right)}\,dx$ 8923
3.2867 $\int \frac{1}{(c+dx)^4\left(a+b(c+dx)^3\right)}\,dx$ 8927
3.2868 $\int \frac{(c+dx)^4}{\left(a+b(c+dx)^3\right)^2}\,dx$ 8930
3.2869 $\int \frac{(c+dx)^3}{\left(a+b(c+dx)^3\right)^2}\,dx$ 8934
3.2870 $\int \frac{(c+dx)^2}{\left(a+b(c+dx)^3\right)^2}\,dx$ 8938

3.2871 $\int \frac{c+dx}{\left(a+b(c+dx)^3\right)^2}\,dx$ 8940
3.2872 $\int \frac{1}{\left(a+b(c+dx)^3\right)^2}\,dx$ 8944
3.2873 $\int \frac{1}{(c+dx)\left(a+b(c+dx)^3\right)^2}\,dx$ 8948
3.2874 $\int \frac{1}{(c+dx)^2\left(a+b(c+dx)^3\right)^2}\,dx$ 8951
3.2875 $\int \frac{1}{(c+dx)^3\left(a+b(c+dx)^3\right)^2}\,dx$ 8956
3.2876 $\int \frac{1}{(c+dx)^4\left(a+b(c+dx)^3\right)^2}\,dx$ 8961
3.2877 $\int \frac{(c+dx)^4}{\left(a+b(c+dx)^3\right)^3}\,dx$ 8964
3.2878 $\int \frac{(c+dx)^3}{\left(a+b(c+dx)^3\right)^3}\,dx$ 8969
3.2879 $\int \frac{(c+dx)^2}{\left(a+b(c+dx)^3\right)^3}\,dx$ 8974
3.2880 $\int \frac{c+dx}{\left(a+b(c+dx)^3\right)^3}\,dx$ 8977
3.2881 $\int \frac{1}{\left(a+b(c+dx)^3\right)^3}\,dx$ 8982
3.2882 $\int \frac{1}{(c+dx)\left(a+b(c+dx)^3\right)^3}\,dx$ 8987
3.2883 $\int \frac{1}{(c+dx)^2\left(a+b(c+dx)^3\right)^3}\,dx$ 8991
3.2884 $\int \frac{1}{(c+dx)^3\left(a+b(c+dx)^3\right)^3}\,dx$ 8997
3.2885 $\int \frac{1}{(c+dx)^4\left(a+b(c+dx)^3\right)^3}\,dx$ 9003
3.2886 $\int \frac{(ce+dex)^4}{a+b(c+dx)^3}\,dx$ 9007
3.2887 $\int \frac{(ce+dex)^3}{a+b(c+dx)^3}\,dx$ 9011
3.2888 $\int \frac{(ce+dex)^2}{a+b(c+dx)^3}\,dx$ 9015
3.2889 $\int \frac{ce+dex}{a+b(c+dx)^3}\,dx$ 9017
3.2890 $\int \frac{1}{(ce+dex)\left(a+b(c+dx)^3\right)}\,dx$ 9021
3.2891 $\int \frac{1}{(ce+dex)^2\left(a+b(c+dx)^3\right)}\,dx$ 9024
3.2892 $\int \frac{1}{(ce+dex)^3\left(a+b(c+dx)^3\right)}\,dx$ 9028
3.2893 $\int \frac{1}{(ce+dex)^4\left(a+b(c+dx)^3\right)}\,dx$ 9032
3.2894 $\int \frac{(ce+dex)^4}{\left(a+b(c+dx)^3\right)^2}\,dx$ 9035
3.2895 $\int \frac{(ce+dex)^3}{\left(a+b(c+dx)^3\right)^2}\,dx$ 9040
3.2896 $\int \frac{(ce+dex)^2}{\left(a+b(c+dx)^3\right)^2}\,dx$ 9045
3.2897 $\int \frac{ce+dex}{\left(a+b(c+dx)^3\right)^2}\,dx$ 9047
3.2898 $\int \frac{1}{(ce+dex)\left(a+b(c+dx)^3\right)^2}\,dx$ 9051
3.2899 $\int \frac{1}{(ce+dex)^2\left(a+b(c+dx)^3\right)^2}\,dx$ 9054
3.2900 $\int \frac{1}{(ce+dex)^3\left(a+b(c+dx)^3\right)^2}\,dx$ 9059
3.2901 $\int \frac{1}{(ce+dex)^4\left(a+b(c+dx)^3\right)^2}\,dx$ 9064

3.2902 $\int \frac{(ce+dex)^4}{\left(a+b(c+dx)^3\right)^3}\,dx$ 9068
3.2903 $\int \frac{(ce+dex)^3}{\left(a+b(c+dx)^3\right)^3}\,dx$ 9074
3.2904 $\int \frac{(ce+dex)^2}{\left(a+b(c+dx)^3\right)^3}\,dx$ 9080
3.2905 $\int \frac{ce+dex}{\left(a+b(c+dx)^3\right)^3}\,dx$ 9083
3.2906 $\int \frac{1}{(ce+dex)\left(a+b(c+dx)^3\right)^3}\,dx$ 9088
3.2907 $\int \frac{1}{(ce+dex)^2\left(a+b(c+dx)^3\right)^3}\,dx$ 9092
3.2908 $\int \frac{1}{(ce+dex)^3\left(a+b(c+dx)^3\right)^3}\,dx$ 9098
3.2909 $\int \frac{1}{(ce+dex)^4\left(a+b(c+dx)^3\right)^3}\,dx$ 9104
3.2910 $\int (c+dx)^3\left(a+b(c+dx)^4\right)^p\,dx$ 9108
3.2911 $\int (c+dx)^3\left(a+b(c+dx)^4\right)\,dx$ 9110
3.2912 $\int (c+dx)^3\left(a+b(c+dx)^4\right)^2\,dx$ 9113
3.2913 $\int (c+dx)^3\left(a+b(c+dx)^4\right)^3\,dx$ 9116
3.2914 $\int \frac{(c+dx)^3}{a+b(c+dx)^4}\,dx$ 9120
3.2915 $\int \frac{(c+dx)^3}{\left(a+b(c+dx)^4\right)^2}\,dx$ 9122
3.2916 $\int \frac{(c+dx)^3}{\left(a+b(c+dx)^4\right)^3}\,dx$ 9124
3.2917 $\int \frac{1}{\sqrt{a+b(c+dx)^4}}\,dx$ 9127
3.2918 $\int \frac{x}{\sqrt{a+b(c+dx)^4}}\,dx$ 9130
3.2919 $\int (dx)^m\sqrt{a+b(cx)^{3/2}}\,dx$ 9134
3.2920 $\int (dx)^m\sqrt{a+b\sqrt{cx}}\,dx$ 9137
3.2921 $\int (dx)^m\sqrt{a+\frac{b}{\sqrt{cx}}}\,dx$ 9140
3.2922 $\int (dx)^m\sqrt{a+\frac{b}{(cx)^{3/2}}}\,dx$ 9143
3.2923 $\int (dx)^m\,(a+b(cx)^n)^p\,dx$ 9146
3.2924 $\int x^2\,(a+b(cx)^n)^p\,dx$ 9149
3.2925 $\int x\,(a+b(cx)^n)^p\,dx$ 9152
3.2926 $\int (a+b(cx)^n)^p\,dx$ 9155
3.2927 $\int \frac{(a+b(cx)^n)^p}{x}\,dx$ 9158
3.2928 $\int \frac{(a+b(cx)^n)^p}{x^2}\,dx$ 9161
3.2929 $\int \frac{1}{1+\left(x^2\right)^{3/2}}\,dx$ 9164
3.2930 $\int x^5\sqrt{a+b\sqrt{cx^2}}\,dx$ 9168
3.2931 $\int x^3\sqrt{a+b\sqrt{cx^2}}\,dx$ 9171
3.2932 $\int x\sqrt{a+b\sqrt{cx^2}}\,dx$ 9174
3.2933 $\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x}\,dx$ 9177
3.2934 $\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^3}\,dx$ 9180
3.2935 $\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^5}\,dx$ 9184

3.2936 $\int x^4\sqrt{a+b\sqrt{cx^2}}\,dx$ 9188
3.2937 $\int x^2\sqrt{a+b\sqrt{cx^2}}\,dx$ 9195
3.2938 $\int \sqrt{a+b\sqrt{cx^2}}\,dx$ 9198
3.2939 $\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^2}\,dx$ 9201
3.2940 $\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^4}\,dx$ 9204
3.2941 $\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^6}\,dx$ 9208
3.2942 $\int x^8\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx$ 9212
3.2943 $\int x^5\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx$ 9215
3.2944 $\int x^2\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx$ 9218
3.2945 $\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x}\,dx$ 9221
3.2946 $\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^4}\,dx$ 9224
3.2947 $\int x^3\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx$ 9227
3.2948 $\int \sqrt{a+b\left(cx^2\right)^{3/2}}\,dx$ 9230
3.2949 $\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^3}\,dx$ 9233
3.2950 $\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^6}\,dx$ 9236
3.2951 $\int x^4\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx$ 9239
3.2952 $\int x\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx$ 9243
3.2953 $\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^2}\,dx$ 9247
3.2954 $\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^5}\,dx$ 9251
3.2955 $\int (dx)^m\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx$ 9255
3.2956 $\int (dx)^m\sqrt{a+b\sqrt{cx^2}}\,dx$ 9258
3.2957 $\int (dx)^m\sqrt{a+\frac{b}{\sqrt{cx^2}}}\,dx$ 9261
3.2958 $\int (dx)^m\sqrt{a+\frac{b}{\left(cx^2\right)^{3/2}}}\,dx$ 9264
3.2959 $\int \frac{1}{1+\left(x^3\right)^{2/3}}\,dx$ 9267
3.2960 $\int x^5\sqrt{a+b\sqrt{cx^3}}\,dx$ 9269
3.2961 $\int x^2\sqrt{a+b\sqrt{cx^3}}\,dx$ 9272
3.2962 $\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x}\,dx$ 9275
3.2963 $\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^4}\,dx$ 9278
3.2964 $\int x\sqrt{a+b\sqrt{cx^3}}\,dx$ 9282
3.2965 $\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^2}\,dx$ 9286

3.2966 $\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^5}\,dx$ . . . 9290
3.2967 $\int x^3\sqrt{a+b\sqrt{cx^3}}\,dx$ . . . 9295
3.2968 $\int \sqrt{a+b\sqrt{cx^3}}\,dx$ . . . 9300
3.2969 $\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^3}\,dx$ . . . 9305
3.2970 $\int x^{17}\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx$ . . . 9310
3.2971 $\int x^8\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx$ . . . 9313
3.2972 $\int \frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{x}\,dx$ . . . 9316
3.2973 $\int \frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{x^{10}}\,dx$ . . . 9319
3.2974 $\int x^2\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx$ . . . 9323
3.2975 $\int x^9\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx$ . . . 9328
3.2976 $\int \sqrt{a+b\left(cx^3\right)^{3/2}}\,dx$ . . . 9331
3.2977 $\int \frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{x^9}\,dx$ . . . 9334
3.2978 $\int (dx)^m\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx$ . . . 9337
3.2979 $\int (dx)^m\sqrt{a+b\sqrt{cx^3}}\,dx$ . . . 9340
3.2980 $\int (dx)^m\sqrt{a+\frac{b}{\sqrt{cx^3}}}\,dx$ . . . 9343
3.2981 $\int (dx)^m\sqrt{a+\frac{b}{\left(cx^3\right)^{3/2}}}\,dx$ . . . 9346
3.2982 $\int \sqrt{a+b\sqrt{\frac{c}{x}}}\,x\,dx$ . . . 9349
3.2983 $\int \sqrt{a+b\sqrt{\frac{c}{x}}}\,dx$ . . . 9358
3.2984 $\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x}\,dx$ . . . 9362
3.2985 $\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^2}\,dx$ . . . 9366
3.2986 $\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^3}\,dx$ . . . 9369
3.2987 $\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^4}\,dx$ . . . 9372
3.2988 $\int \frac{x}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx$ . . . 9376
3.2989 $\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx$ . . . 9380
3.2990 $\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}\,x}\,dx$ . . . 9384
3.2991 $\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}\,x^2}\,dx$ . . . 9387
3.2992 $\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}\,x^3}\,dx$ . . . 9390
3.2993 $\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}\,x^4}\,dx$ . . . 9393

3.2994 $\int \frac{1}{\sqrt{1+\sqrt{\frac{1}{x}}}}\,dx$ 9396

3.2995 $\int \sqrt{a+b\left(\frac{c}{x}\right)^{3/2}}\,(dx)^m\,dx$ 9400

3.2996 $\int \sqrt{a+b\sqrt{\frac{c}{x}}}\,(dx)^m\,dx$ 9403

3.2997 $\int \sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}\,(dx)^m\,dx$ 9406

3.2998 $\int \sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{3/2}}}\,(dx)^m\,dx$ 9409

3.2999 $\int \frac{(dx)^m}{\sqrt{a+b\left(\frac{c}{x}\right)^{3/2}}}\,dx$ 9412

3.3000 $\int \frac{(dx)^m}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx$ 9415

3.3001 $\int \frac{(dx)^m}{\sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}}\,dx$ 9418

3.3002 $\int \frac{(dx)^m}{\sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{3/2}}}}\,dx$ 9421

3.3003 $\int \left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)\,dx$ 9424

3.3004 $\int \left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2\,dx$ 9426

3.3005 $\int \left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^3\,dx$ 9428

3.3006 $\int \frac{x^3}{a+b\left(cx^n\right)^{\frac{1}{n}}}\,dx$ 9430

3.3007 $\int \frac{x^2}{a+b\left(cx^n\right)^{\frac{1}{n}}}\,dx$ 9433

3.3008 $\int \frac{x}{a+b\left(cx^n\right)^{\frac{1}{n}}}\,dx$ 9436

3.3009 $\int \frac{1}{a+b\left(cx^n\right)^{\frac{1}{n}}}\,dx$ 9439

3.3010 $\int \frac{1}{x\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}\,dx$ 9441

3.3011 $\int \frac{1}{x^2\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}\,dx$ 9444

3.3012 $\int \frac{1}{x^3\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}\,dx$ 9447

3.3013 $\int \frac{x^3}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx$ 9450

3.3014 $\int \frac{x^2}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx$ 9453

3.3015 $\int \frac{x}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx$ 9456

3.3016 $\int \frac{1}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx$ 9459

3.3017 $\int \frac{1}{x\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx$ 9462

3.3018 $\int \frac{1}{x^2\left(a+b(cx^n)^{\frac{1}{n}}\right)^2}\,dx$ 9465
3.3019 $\int \frac{1}{x^3\left(a+b(cx^n)^{\frac{1}{n}}\right)^2}\,dx$ 9468
3.3020 $\int \frac{1}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^3}\,dx$ 9471
3.3021 $\int \frac{x}{\left(1+(x^n)^{\frac{1}{n}}\right)^2}\,dx$ 9474
3.3022 $\int x^3\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^p\,dx$ 9477
3.3023 $\int x^2\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^p\,dx$ 9480
3.3024 $\int x\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^p\,dx$ 9483
3.3025 $\int \left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^p\,dx$ 9486
3.3026 $\int \frac{\left(a+b(cx^n)^{\frac{1}{n}}\right)^p}{x}\,dx$ 9489
3.3027 $\int \frac{\left(a+b(cx^n)^{\frac{1}{n}}\right)^p}{x^2}\,dx$ 9492
3.3028 $\int \left(a+b\left(cx^n\right)^{2/n}\right)^3\,dx$ 9495
3.3029 $\int \left(a+b\left(cx^n\right)^{2/n}\right)^2\,dx$ 9497
3.3030 $\int \left(a+b\left(cx^n\right)^{2/n}\right)\,dx$ 9499
3.3031 $\int \frac{1}{a+b(cx^n)^{2/n}}\,dx$ 9501
3.3032 $\int \frac{1}{\left(a+b(cx^n)^{2/n}\right)^2}\,dx$ 9504
3.3033 $\int \frac{1}{\left(a+b(cx^n)^{2/n}\right)^3}\,dx$ 9507
3.3034 $\int \frac{1}{1+4\sqrt{x^4}}\,dx$ 9510
3.3035 $\int \frac{1}{1-4\sqrt{x^4}}\,dx$ 9512
3.3036 $\int \frac{1}{1+4\sqrt[3]{x^6}}\,dx$ 9514
3.3037 $\int \frac{1}{1-4\sqrt[3]{x^6}}\,dx$ 9516
3.3038 $\int \frac{1}{1+4\left(x^{2n}\right)^{\frac{1}{n}}}\,dx$ 9518
3.3039 $\int \frac{1}{1-4\left(x^{2n}\right)^{\frac{1}{n}}}\,dx$ 9520
3.3040 $\int \left(a+b\left(cx^n\right)^{3/n}\right)^3\,dx$ 9522
3.3041 $\int \left(a+b\left(cx^n\right)^{3/n}\right)^2\,dx$ 9524
3.3042 $\int \left(a+b\left(cx^n\right)^{3/n}\right)\,dx$ 9526
3.3043 $\int \frac{1}{a+b(cx^n)^{3/n}}\,dx$ 9528
3.3044 $\int \frac{1}{\left(a+b(cx^n)^{3/n}\right)^2}\,dx$ 9532
3.3045 $\int \frac{1}{\left(a+b(cx^n)^{3/n}\right)^3}\,dx$ 9537
3.3046 $\int (dx)^m\left(a+b\left(cx^q\right)^n\right)^p\,dx$ 9542
3.3047 $\int x^2\left(a+b\left(cx^q\right)^n\right)^p\,dx$ 9545
3.3048 $\int x\left(a+b\left(cx^q\right)^n\right)^p\,dx$ 9547

3.3049 $\int \left(a+b\left(cx^q\right)^n\right)^p\,dx$ 9549
3.3050 $\int \frac{\left(a+b\left(cx^q\right)^n\right)^p}{x}\,dx$ 9551
3.3051 $\int \frac{\left(a+b\left(cx^q\right)^n\right)^p}{x^2}\,dx$ 9554
3.3052 $\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^m\,dx$ 9557
3.3053 $\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^2\,dx$ 9560
3.3054 $\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x\,dx$ 9565
3.3055 $\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,dx$ 9569
3.3056 $\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x}\,dx$ 9572
3.3057 $\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^2}\,dx$ 9576
3.3058 $\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^3}\,dx$ 9580
3.3059 $\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^4}\,dx$ 9585
3.3060 $\int \frac{x^m}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$ 9590
3.3061 $\int \frac{x^2}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$ 9593
3.3062 $\int \frac{x}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$ 9598
3.3063 $\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$ 9603
3.3064 $\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x}\,dx$ 9607
3.3065 $\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^2}\,dx$ 9610
3.3066 $\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^3}\,dx$ 9613
3.3067 $\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^4}\,dx$ 9617
3.3068 $\int \sqrt{\sqrt{\frac{1}{x}}+\frac{1}{x}}\,dx$ 9622
3.3069 $\int \sqrt{2+\sqrt{\frac{1}{x}}+\frac{1}{x}}\,dx$ 9625
3.3070 $\int \left(cx^n\right)^{\frac{1}{n}}\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^p\,dx$ 9628
3.3071 $\int \left(cx^n\right)^{\frac{1}{n}}\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^3\,dx$ 9631
3.3072 $\int \left(cx^n\right)^{\frac{1}{n}}\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2\,dx$ 9634
3.3073 $\int \left(cx^n\right)^{\frac{1}{n}}\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)\,dx$ 9637
3.3074 $\int \frac{\left(cx^n\right)^{\frac{1}{n}}}{a+b\left(cx^n\right)^{\frac{1}{n}}}\,dx$ 9640

3.3075 $\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^2}\,dx$ 9643
3.3076 $\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^3}\,dx$ 9646
3.3077 $\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^4}\,dx$ 9649
3.3078 $\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^5}\,dx$ 9652

4 Listing of Grading functions **9655**
4.0.1 Mathematica and Rubi grading function 9655
4.0.2 Maple grading function 9657
4.0.3 Sympy grading function 9660
4.0.4 SageMath grading function 9662

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [3078]. This is test number [25].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (3078)	% 0.00 (0)
Mathematica	% 98.90 (3044)	% 1.10 (34)
Maple	% 84.18 (2591)	% 15.82 (487)
Maxima	% 71.35 (2196)	% 28.65 (882)
Fricas	% 77.00 (2370)	% 23.00 (708)
Sympy	% 87.33 (2688)	% 12.67 (390)
Giac	% 64.65 (1990)	% 35.35 (1088)
Mupad	% 72.38 (2228)	% 27.62 (850)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

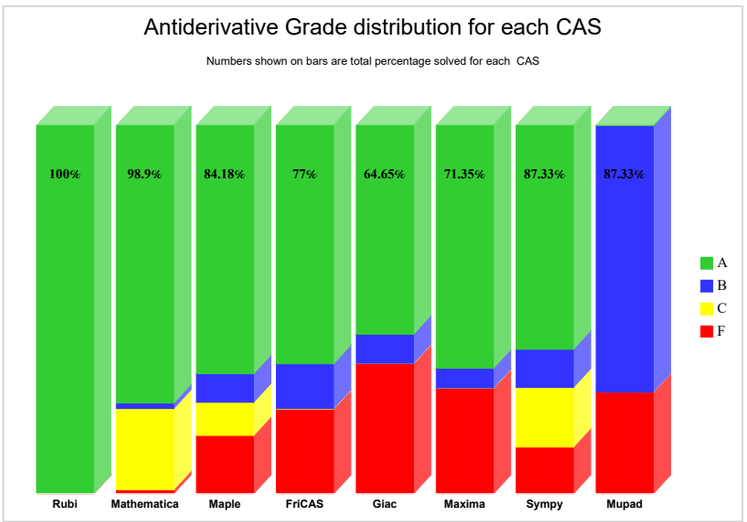
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

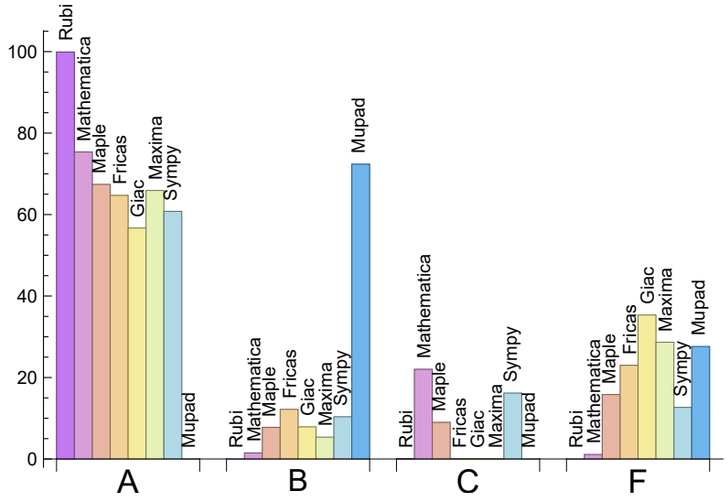
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.90	0.00	0.10	0.00
Mathematica	75.37	1.49	22.03	1.10
Maple	67.41	7.76	9.00	15.82
Maxima	65.92	5.36	0.06	28.65
Fricas	64.72	12.18	0.10	23.00
Sympy	60.79	10.36	16.18	12.67
Giac	56.69	7.89	0.06	35.35
Mupad	0.00	72.38	0.00	27.62

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	34	100.00 %	0.00 %	0.00 %
Maple	487	99.59 %	0.41 %	0.00 %
Maxima	882	95.46 %	0.00 %	4.54 %
Fricas	708	77.82 %	5.51 %	16.67 %
Sympy	390	62.05 %	36.15 %	1.79 %
Giac	1088	94.30 %	0.55 %	5.15 %
Mupad	850	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

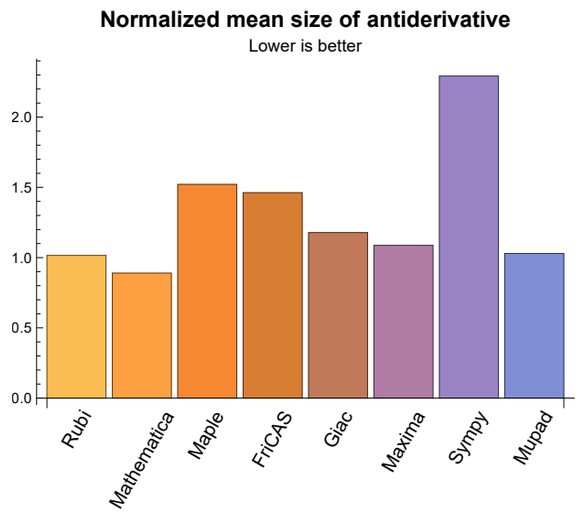
1.3 Performance

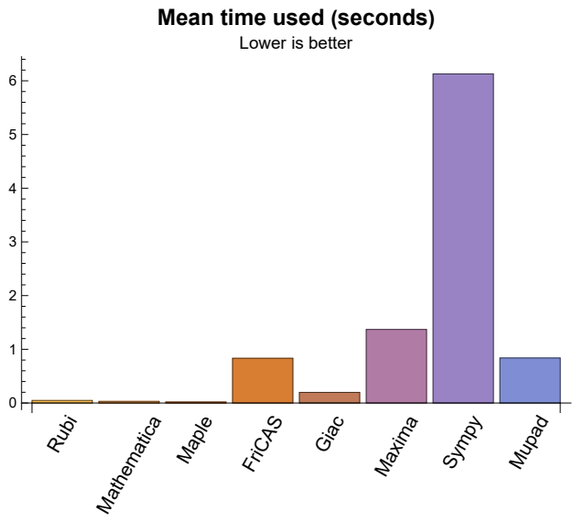
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.05	87.35	1.02	59.00	1.00
Mathematica	0.03	56.02	0.89	43.00	0.90
Maple	0.02	131.85	1.52	47.00	0.87
Maxima	1.37	63.98	1.09	47.00	0.89
Fricas	0.83	112.07	1.46	56.50	1.02
Sympy	6.13	153.83	2.29	42.00	0.96
Giac	0.20	79.32	1.18	47.00	0.90
Mupad	0.84	64.40	1.03	39.00	0.85

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {1276, 1304}

Mathematica {524, 538, 576, 708, 1302, 1303, 1306, 1307}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
```

```
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

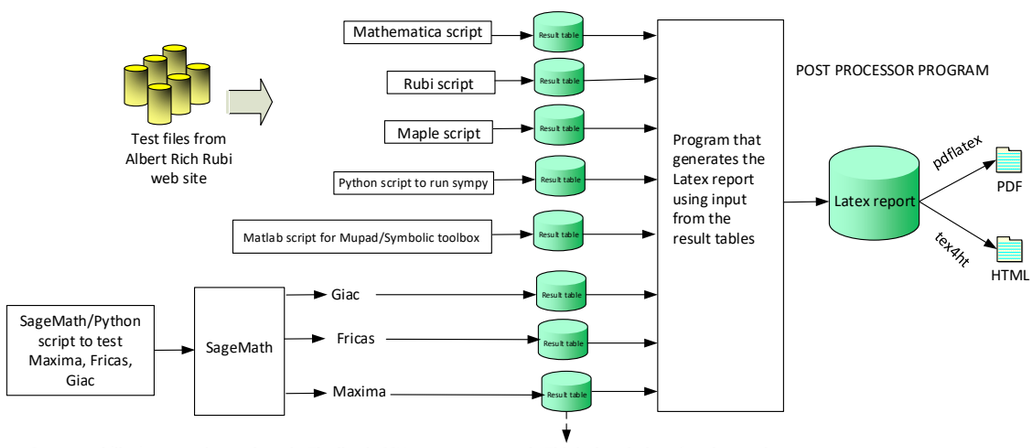
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"

The following field present only in Rubi and Mathematica Tables

13. integer. 1 if result was verified or 0 if not verified.

The following fields present only in Rubi Tables

14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,..}" which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

Nasser M. Abbasi
May 11, 2021

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864,

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B grade: { }

C grade: { 2686, 2697, 2699 }

F grade: { }

2.1.2 Mathematica

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B grade: { 240, 245, 262, 263, 270, 289, 290, 291, 301, 302, 635, 1330, 1350, 1458, 1474, 1496, 1578, 1588, 1589, 1600, 1601, 1839, 1917, 1927, 1928, 1929, 2019, 2084, 2107, 2108, 2592, 2593, 2594, 2727, 2841, 2842, 2843, 2853, 2854, 2855, 2856, 2857, 2858, 2911, 2912, 2913 }

C grade: { 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 392, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 515, 516, 517, 518, 524, 532, 537, 538, 539, 549, 566, 567, 568, 569, 576, 581, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 769, 776, 777, 778, 779, 780, 781, 782, 787, 791, 792, 796, 797, 798, 799, 800, 801, 802, 803, 804, 809, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 854, 855, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 884, 885, 887, 888, 889, 890, 891, 892, 893, 897, 898, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 940, 941, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 968, 969, 970, 972, 973, 977, 979, 980, 981, 982, 983, 984, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1009, 1010,

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F grade: { 2955, 2958, 2963, 2964, 2965, 2966, 2967, 2968, 2969, 2973, 2975, 2976, 2977, 2978, 2981, 2995, 2998, 2999, 3002, 3052, 3053, 3054, 3055, 3056, 3057, 3058, 3059, 3060, 3061, 3062, 3063, 3065, 3066, 3067 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 184, 185, 186, 189, 190, 191, 192, 193, 194, 195, 196, 199, 200, 201, 202, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 241, 242, 243, 244, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 264, 265, 266, 267, 268, 269, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 292, 293, 294, 295, 296, 297, 298, 299, 300, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 519, 520, 521, 522, 527, 528, 529, 530, 540, 541, 542, 543, 544, 545, 546, 547, 557, 558, 559, 560, 561, 562, 563, 564, 570, 571, 572, 573, 582, 587, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 633, 634, 636, 637, 638, 639, 640, 641, 642, 643, 644,

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1966, 1967, 1968, 1969, 1970, 1971, 1972, 1973, 1974, 1975, 1976, 1977, 1978, 1979, 1980, 1981, 1982, 1983, 1984, 1985, 1986, 1987, 1988, 1989, 1990, 1994, 1995, 1996, 1997, 2013, 2014, 2015, 2016, 2020, 2021, 2022, 2023, 2040, 2041, 2042, 2043, 2057, 2058, 2059, 2066, 2067, 2068, 2069, 2074, 2075, 2076, 2077, 2082, 2083, 2090, 2091, 2092, 2093, 2099, 2101, 2109, 2111, 2112, 2113, 2114, 2115, 2116, 2117, 2118, 2119, 2120, 2121, 2122, 2123, 2124, 2125, 2126, 2127, 2128, 2129, 2130, 2131, 2132, 2133, 2134, 2135, 2136, 2138, 2139, 2140, 2141, 2142, 2143, 2144, 2145, 2146, 2147, 2148, 2150, 2151, 2152, 2153, 2154, 2155, 2156, 2158, 2159, 2160, 2161, 2162, 2163, 2165, 2166, 2167, 2168, 2169, 2170, 2171, 2172, 2175, 2176, 2177, 2178, 2179, 2180, 2181, 2184, 2185, 2186,

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B grade: { 240, 245, 263, 270, 290, 291, 301, 302, 441, 583, 584, 585, 586, 620, 632, 635, 708, 709, 710, 713, 714, 810, 885, 886, 887, 891, 892, 897, 898, 899, 900, 906, 907, 916, 969, 970, 971, 973, 984, 1276, 1298, 1316, 1335, 1339, 1350, 1353, 1356, 1359, 1370, 1373, 1381, 1438, 1472, 1473, 1474, 1475, 1476, 1478, 1511, 1540, 1544, 1578, 1588, 1589, 1600, 1601, 1694, 1695, 1704, 1705, 1715, 1716, 1723, 1724, 1726, 1732, 1733, 1734, 1735, 1742, 1743, 1744, 1745, 1839, 1882, 1895, 1896, 1901, 1903, 1904, 1909, 1910, 1911, 1912, 1914, 1917, 1925, 1927, 1928, 1929, 1933, 1957, 1958, 1998, 1999, 2000, 2001, 2002, 2003, 2004, 2005, 2006, 2007, 2008, 2009, 2024, 2025, 2026, 2027, 2028, 2029, 2030, 2031, 2032, 2033, 2034, 2035, 2036, 2044, 2045, 2046, 2047, 2048, 2049, 2050, 2051, 2052, 2053, 2054, 2055, 2056, 2060, 2061, 2084, 2085, 2098, 2100, 2106, 2137, 2149, 2157, 2164, 2173, 2174, 2182, 2183, 2194, 2202, 2210, 2218, 2219, 2226, 2228, 2249, 2319, 2328, 2333, 2344, 2350, 2359, 2368, 2376, 2384, 2440, 2448, 2531, 2542, 2547, 2553, 2554, 2561, 2572, 2573, 2574, 2584, 2585, 2591, 2592, 2593, 2594, 2595, 2596, 2708, 2725, 2727, 2739, 2743, 2802, 2803, 2830, 2841, 2842, 2843, 2847, 2853, 2854, 2855, 2856, 2857, 2858, 2861, 2870, 2879, 2882, 2885, 2906, 2909, 2910, 2911, 2912, 2913, 2914, 2915, 2916, 2983, 2984, 2988, 2989, 2990, 2994, 3053, 3054, 3055, 3056, 3057, 3058, 3059, 3064, 3069 }

C grade: { 580, 581, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 776, 777, 778, 779, 780, 781, 782, 796, 797, 798, 799, 800, 801, 802, 803, 809, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 977, 978, 979, 980, 981, 982, 1379, 1380, 1382, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1434, 1443, 1447, 1449, 1450, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1468, 1469, 1470, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1508, 1510, 1512, 1513, 1524, 1525, 1526, 1527, 1528, 1529, 1530, 1531, 1534, 1535, 1538, 1539, 1542, 1991, 1992, 1993, 2010, 2011, 2012, 2017, 2018, 2019, 2037, 2038, 2039, 2062, 2063, 2064, 2065, 2070, 2071, 2072, 2073, 2078, 2079, 2080, 2081, 2086, 2087, 2088, 2089, 2094, 2095, 2096, 2097,

2102, 2103, 2104, 2105, 2110, 2640, 2641, 2644, 2645, 2647, 2648, 2755, 2780, 2859, 2860, 2862, 2863, 2865, 2866, 2868, 2869, 2871, 2872, 2874, 2875, 2877, 2878, 2880, 2881, 2883, 2884, 2886, 2887, 2889, 2891, 2892, 2894, 2895, 2897, 2899, 2900, 2902, 2903, 2905, 2907, 2908, 2917, 2918, 2991, 2992, 2993, 3006, 3007, 3008, 3009, 3011, 3012, 3013, 3014, 3015, 3016, 3018, 3019, 3020, 3022, 3023, 3024, 3025, 3031, 3032, 3033, 3043, 3044, 3045, 3070, 3074, 3075, 3076, 3077, 3078 }

F grade: { 183, 187, 188, 197, 198, 203, 204, 514, 515, 516, 517, 518, 523, 524, 525, 526, 531, 532, 533, 534, 535, 536, 537, 538, 539, 548, 549, 550, 551, 552, 553, 554, 555, 556, 565, 566, 567, 568, 569, 574, 575, 576, 577, 578, 579, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1172, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1266, 1308, 1309, 1310, 1441, 1445, 1959, 1960, 1961, 1962, 1963, 1964, 2107, 2108, 2245, 2257, 2258, 2259, 2260, 2261, 2262, 2263, 2264, 2265, 2266, 2267, 2268, 2269, 2271, 2272, 2273, 2274, 2275, 2276, 2277, 2280, 2281, 2282, 2283, 2284, 2393, 2450, 2472, 2473, 2475, 2476, 2477, 2478, 2480, 2481, 2482, 2483, 2485, 2486, 2487, 2488, 2490, 2491, 2492, 2493, 2495, 2496, 2497, 2498, 2500, 2501, 2502, 2503, 2505, 2506, 2507, 2508, 2510, 2511, 2512, 2513, 2515, 2516, 2654, 2655, 2656, 2657, 2663, 2664, 2665, 2666, 2670, 2671, 2672, 2673, 2674, 2675, 2676, 2677, 2678, 2679, 2680, 2681, 2682, 2683, 2684, 2685, 2686, 2690, 2691, 2692, 2693, 2694, 2695, 2696, 2698, 2699, 2700, 2701, 2702, 2703, 2704, 2705, 2706, 2707, 2709, 2710, 2711, 2724, 2730, 2731, 2732, 2734, 2735, 2736, 2746, 2747, 2748, 2749, 2750, 2751, 2752, 2762, 2763, 2764, 2765, 2766, 2767, 2768, 2769, 2770, 2771, 2772, 2773, 2774, 2775, 2776, 2777, 2778, 2779, 2781, 2782, 2783, 2784, 2785, 2786, 2787, 2788, 2789, 2790, 2791, 2792, 2793, 2794, 2795, 2796, 2797, 2798, 2799, 2800, 2801, 2848, 2850, 2852, 2919, 2920, 2921, 2922, 2923, 2924, 2925, 2926, 2927, 2928, 2942, 2943, 2945, 2946, 2947, 2948, 2949, 2950, 2951, 2953, 2954, 2955, 2956, 2957, 2958, 2970, 2971, 2972, 2973, 2975, 2976, 2977, 2978, 2979, 2980, 2981, 2995, 2996, 2997, 2998, 2999, 3000, 3001, 3002, 3004, 3005, 3026, 3027, 3028, 3029, 3040, 3041, 3046, 3047, 3048, 3049, 3050, 3051, 3052, 3060, 3071, 3072, 3073 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 11, 12, 13, 14, 15, 22, 23, 24, 25, 26, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 117, 118, 119, 120, 121, 122, 123, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 137, 138, 139, 146, 147, 148, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 185, 189, 190, 191, 192, 193, 194, 195, 196, 197, 199, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 367, 368, 369, 370, 371, 372, 373, 374, 375, 387, 388, 389, 390, 391, 392, 393, 404, 405, 406, 407, 408, 409, 410, 421, 422, 423, 424, 425, 426, 427, 438, 439, 440, 441, 443, 444, 445, 456, 457, 458, 459, 461, 462, 463, 474, 475, 476, 477, 478, 479, 480, 481, 492, 493, 494, 495, 496, 497, 498, 499, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 527, 528, 529, 530, 531, 532, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 579, 580, 581, 582, 583, 584, 585, 586, 587, 599, 600, 601, 602, 603, 604, 605,

606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 711, 712, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 764, 765, 766, 767, 768, 769, 772, 773, 774, 775, 783, 784, 785, 786, 787, 788, 791, 792, 793, 794, 795, 805, 806, 811, 812, 813, 814, 815, 816, 818, 819, 820, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 874, 875, 876, 878, 881, 882, 883, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 917, 918, 919, 921, 924, 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B grade: { 245, 270, 290, 301, 302, 342, 442, 460, 537, 674, 708, 709, 710, 713, 714, 770, 771, 789, 790, 807, 808, 817, 877, 879, 880, 920, 922, 923, 942, 943, 964, 974, 975, 1308, 1310, 1313, 1314, 1315, 1350, 1380, 1382, 1390, 1391, 1392, 1415, 1474, 1509, 1513, 1514, 1516, 1518, 1519, 1541, 1542, 1544, 1545, 1573, 1588, 1589, 1590, 1601, 1757, 1758, 1759, 1766, 1767, 1775, 1776, 1904, 1906, 1912, 1914, 1917, 1928, 2061, 2069, 2077, 2085, 2111, 2112, 2113, 2114, 2120, 2121, 2122, 2123, 2130, 2131, 2132, 2141, 2142, 2149, 2164, 2182, 2183, 2210, 2218, 2226, 2285, 2286, 2287, 2288, 2294, 2295, 2296, 2297, 2303, 2304, 2305, 2306, 2312, 2313, 2314, 2319, 2324, 2333, 2350, 2390, 2400, 2401, 2402, 2410, 2411, 2419, 2420, 2547, 2553, 2561, 2572, 2573, 2584, 2585, 2725, 2727, 2737, 2802, 2803, 2804, 2813, 2814, 2818, 2830, 2841, 2842, 2843, 2847, 2849, 2853, 2854, 2855, 2856, 2857, 2858, 2876, 2882, 2885, 2896, 2901, 2904, 2906, 2909, 2936, 2937, 3020, 3078 }

C grade: { 2247, 2812 }

F grade: { 6, 7, 8, 9, 10, 16, 17, 18, 19, 20, 21, 27, 28, 29, 30, 31, 32, 33, 116, 124, 135, 136, 140, 141, 142, 143, 144, 145, 149, 150, 151, 168, 169, 170, 171, 184, 186, 187, 188, 198, 200, 201, 202, 203, 204, 205, 365, 366, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 523, 524, 525, 526, 533, 534, 535, 536, 550, 551, 552, 553, 554, 574, 575, 576, 577, 578, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 776, 777, 778, 779, 780, 781, 782,

796, 797, 798, 799, 800, 801, 802, 803, 804, 809, 810, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 968, 969, 970, 971, 972, 973, 977, 978, 979, 980, 981, 982, 983, 984, 996, 997, 998, 999, 1000, 1001, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1025, 1026, 1027, 1028, 1029, 1030, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1172, 1183, 1184, 1185, 1186, 1187, 1188, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1266, 1379, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1434, 1441, 1443, 1445, 1447, 1449, 1450, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1468, 1469, 1470, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1508, 1510, 1512, 1524, 1525, 1526, 1527, 1528, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1540, 1959, 1960, 1961, 1962, 1963, 1964, 1998, 1999, 2000, 2001, 2002, 2003, 2004, 2005, 2006, 2007, 2008, 2009, 2024, 2025, 2026, 2027, 2028, 2029, 2030, 2031, 2032, 2033, 2034, 2035, 2036, 2044, 2045, 2046, 2047, 2048, 2049, 2050, 2051, 2052, 2053, 2054, 2055, 2056, 2062, 2063, 2064, 2065, 2070, 2071, 2072, 2073, 2078, 2079, 2080, 2081, 2086, 2087, 2088, 2089, 2094, 2095, 2096, 2097, 2102, 2103, 2104, 2105, 2110, 2261, 2262, 2263, 2268, 2269, 2281, 2282, 2283, 2284, 2456, 2457, 2463, 2464, 2470, 2471, 2472, 2473, 2475, 2476, 2477, 2478, 2480, 2481, 2482, 2483, 2485, 2486, 2487, 2488, 2490, 2491, 2492, 2493, 2495, 2496, 2497, 2498, 2500, 2501, 2502, 2503, 2505, 2506, 2507, 2508, 2510, 2511, 2512, 2513, 2515, 2516, 2630, 2639, 2640, 2641, 2643, 2644, 2645, 2646, 2647, 2648, 2654, 2655, 2656, 2657, 2663, 2664, 2665, 2666, 2670, 2671, 2672, 2673, 2674, 2675, 2676, 2677, 2678, 2679, 2680, 2681, 2682, 2683, 2684, 2685, 2687, 2688, 2689, 2690, 2691, 2692, 2693, 2694, 2695, 2696, 2700, 2701, 2702, 2703, 2704, 2705, 2706, 2707, 2708, 2709, 2710, 2711, 2724, 2730, 2731, 2732, 2733, 2734, 2735, 2736, 2743, 2744, 2745, 2746, 2747, 2748, 2749, 2750, 2751, 2752, 2762, 2763, 2764, 2766, 2767, 2768, 2769, 2770, 2771, 2772, 2773, 2774, 2775, 2776, 2778, 2779, 2781, 2782, 2783, 2784, 2785, 2786, 2787, 2788, 2789, 2790, 2791, 2792, 2793, 2794, 2795, 2796, 2797, 2798, 2799, 2800, 2801, 2848, 2850, 2852, 2859, 2860, 2862, 2863, 2865, 2866, 2868, 2869, 2871, 2872, 2874, 2875, 2877, 2878, 2880, 2881, 2883, 2884, 2886, 2887, 2889, 2891, 2892, 2894, 2895, 2897, 2899, 2900, 2902, 2903, 2905, 2907, 2908, 2917, 2918, 2919, 2920, 2921, 2922, 2923, 2924, 2925, 2926, 2927, 2928, 2939, 2940, 2941, 2942, 2944, 2946, 2947, 2948, 2949, 2950, 2951, 2952, 2953, 2954, 2955, 2956, 2957, 2958, 2964, 2965, 2966, 2967, 2968, 2969, 2974, 2975, 2976, 2977, 2978, 2979, 2980, 2981, 2995, 2996, 2997, 2998, 2999, 3000, 3001, 3002, 3003, 3004, 3005, 3006, 3007, 3008, 3009, 3011, 3012, 3013, 3014, 3015, 3018, 3019, 3021, 3022, 3023, 3024, 3025, 3026, 3027, 3028, 3029, 3030, 3031, 3032, 3033, 3038, 3039, 3040, 3041, 3042, 3043, 3044, 3045, 3046, 3047, 3048, 3049, 3050, 3051, 3052, 3053, 3054, 3055, 3056, 3057, 3058, 3059, 3060, 3061, 3062, 3063, 3064, 3065, 3066, 3067, 3068, 3069, 3070, 3071, 3072, 3073, 3074, 3075 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 105, 106, 107, 108, 110, 111, 112, 113, 114, 115, 117, 118, 119, 120, 121, 122, 123, 125, 126, 127, 128, 129, 130, 134, 139, 146, 152, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 241, 242, 243, 244, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 264, 265, 266, 267, 268, 269, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 292, 293, 294, 295, 296, 297, 298, 299, 300, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 343, 344, 345, 348, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 365, 367, 368, 369, 370, 371, 372, 373,

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B grade: { 168, 240, 245, 263, 270, 290, 291, 301, 302, 342, 346, 347, 349, 350, 351, 363, 364, 366, 390, 442, 460, 517, 538, 556, 565, 566, 567, 568, 569, 579, 583, 584, 585, 632, 635, 674, 694, 695, 696, 697, 698, 703, 704, 705, 706, 708, 709, 710, 713, 714, 738, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 785, 793, 877, 880, 897, 898, 900, 920, 923, 940, 941, 943, 944, 963, 964, 994, 995, 1002, 1003, 1022, 1023, 1024, 1031, 1032, 1033, 1034, 1051, 1052, 1053, 1054, 1062, 1063, 1064, 1067, 1079, 1085, 1086, 1087, 1095, 1096, 1097, 1115, 1116, 1117, 1125, 1126, 1127, 1144, 1145, 1146, 1154, 1155, 1156, 1181, 1182, 1209, 1210, 1211, 1239, 1240, 1241, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1315, 1320, 1322, 1326, 1328, 1333, 1335, 1339, 1341, 1350, 1356, 1379, 1417, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1468, 1469, 1470, 1474, 1477, 1478, 1479, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1508, 1518, 1519, 1540, 1541, 1544, 1573, 1578, 1588, 1589, 1590, 1600, 1601, 1639, 1706, 1717, 1745, 1746, 1763, 1771, 1800, 1839, 1882, 1905, 1913, 1917, 1928, 1929, 1932, 1933, 1946, 1947, 1954, 1957, 1958, 2013, 2019, 2039, 2040, 2092, 2100, 2107, 2109, 2110, 2149, 2157, 2164, 2173, 2174, 2182, 2183, 2210, 2211, 2212, 2216, 2217, 2218, 2219, 2220, 2221, 2222, 2223, 2224, 2225, 2226, 2227, 2228, 2229, 2230, 2231, 2253, 2257, 2258, 2264, 2265, 2319, 2328, 2333, 2344, 2350, 2369, 2375, 2376, 2377, 2378, 2388, 2390, 2433, 2440, 2441, 2442, 2465, 2466, 2467, 2468, 2470, 2471, 2542, 2553, 2554, 2561, 2572, 2573, 2574, 2584, 2585, 2591, 2592, 2593, 2594, 2595, 2596, 2667, 2668, 2725, 2727, 2755, 2802, 2803, 2806, 2807, 2813, 2814, 2815, 2816, 2817, 2818, 2824, 2825,

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C grade: { 1449, 1450, 2812 }

F grade: { 101, 109, 116, 124, 131, 132, 133, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 518, 523, 524, 525, 526, 533, 534, 535, 536, 539, 550, 551, 552, 553, 554, 574, 575, 576, 577, 578, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 776, 777, 778, 779, 780, 781, 782, 796, 797, 798, 799, 800, 801, 802, 803, 804, 809, 810, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 968, 969, 970, 971, 972, 973, 977, 978, 979, 980, 981, 982, 983, 984, 996, 997, 998, 999, 1000, 1001, 1004, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1025, 1026, 1027, 1028, 1029, 1030, 1035, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1065, 1066, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1172, 1183, 1184, 1185, 1186, 1187, 1188, 1191, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1266, 1276, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1434, 1441, 1443, 1445, 1447, 1510, 1512, 1524, 1525, 1526, 1527, 1528, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1959, 1960, 1961, 1962, 1963, 1964, 1998, 1999, 2000, 2001, 2002, 2003, 2004, 2005, 2006, 2007, 2008, 2009, 2024, 2025, 2026, 2027, 2028, 2029, 2030, 2031, 2032, 2033, 2034, 2035, 2036, 2044, 2045, 2046, 2047, 2048, 2049, 2050, 2051, 2052, 2053, 2054, 2055, 2056, 2062, 2063, 2064, 2065, 2070, 2071, 2072, 2073, 2078, 2079, 2080, 2081, 2086, 2087, 2088, 2089, 2094, 2095, 2096, 2097, 2102, 2103, 2104, 2105, 2106, 2261, 2262, 2263, 2268, 2269, 2271, 2272, 2279, 2280, 2281, 2282, 2283, 2284, 2393, 2446, 2447, 2449, 2450, 2472, 2473, 2475, 2476, 2477, 2478, 2480, 2481, 2482, 2483, 2485, 2486, 2487, 2488, 2490, 2491, 2492, 2493, 2495, 2496, 2497, 2498, 2500, 2501, 2502, 2503, 2505, 2506, 2507, 2508, 2510, 2511, 2512, 2513, 2515, 2516, 2670, 2671, 2672, 2673, 2674, 2675, 2676, 2677, 2678, 2679, 2680, 2681, 2682, 2683, 2684, 2685, 2686, 2690, 2691, 2692, 2693, 2694, 2695, 2696, 2697, 2699, 2700, 2701, 2702, 2703, 2704, 2709, 2710, 2711, 2734, 2735, 2736, 2746, 2747, 2748, 2749, 2772, 2773, 2774, 2775, 2776, 2778, 2779, 2784, 2785, 2786, 2787, 2788, 2789, 2790, 2791, 2792, 2793, 2794, 2795, 2796, 2797, 2848, 2850, 2852, 2917, 2918, 2919, 2920, 2921, 2922, 2923, 2924, 2925, 2926, 2927, 2928, 2947, 2948, 2949, 2950, 2951, 2952, 2953, 2954, 2955, 2956, 2957, 2958, 2962, 2963, 2964, 2965, 2966, 2967, 2968, 2969, 2972, 2973, 2974, 2975, 2976, 2977, 2978, 2979, 2980, 2981, 2995, 2996, 2997, 2998, 2999, 3000, 3001, 3002, 3026, 3027, 3046, 3047, 3048, 3049, 3050, 3051, 3052, 3053, 3054, 3055, 3056, 3057, 3058, 3059, 3060, 3061, 3062, 3063, 3064, 3065, 3066, 3067 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 69, 71, 73, 75, 77, 78, 79, 80, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 105, 106, 108, 110, 111, 112, 113, 114, 115, 117, 119, 120, 121, 122, 123, 125, 126, 127, 128, 129, 130, 134, 139, 146, 152, 177, 182, 183, 185, 186, 193, 196, 197, 199, 200, 203, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 241, 242, 243, 244, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 264, 265, 266, 267, 268, 269, 271, 272, 273, 274, 275, 276,

277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 292, 293, 294, 295, 296, 297, 298, 299, 300, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 368, 369, 370, 371, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 527, 528, 529, 530, 544, 545, 546, 547, 557, 558, 561, 562, 563, 564, 571, 582, 583, 584, 585, 586, 587, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 636, 637, 638, 639, 640, 641, 642, 643, 645, 646, 647, 649, 650, 651, 653, 654, 655, 656, 657, 658, 659, 660, 662, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 739, 740, 741, 742, 743, 744, 764, 765, 766, 767, 768, 769, 770, 771, 772, 774, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 804, 805, 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1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1309, 1310, 1311, 1314, 1315, 1316, 1318, 1319, 1320, 1321, 1322, 1323, 1325, 1326, 1327, 1328, 1329, 1330, 1332, 1333, 1334, 1335, 1336, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1347, 1349, 1351, 1353, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1380, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1402, 1403, 1404, 1408, 1409, 1410, 1411, 1413, 1414, 1415, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1423, 1428, 1429, 1430, 1431, 1435, 1436, 1437, 1438, 1439, 1440, 1442, 1443, 1444, 1446, 1447, 1448, 1449, 1450, 1451, 1452, 1453, 1455, 1456, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1468, 1469, 1470, 1471, 1472, 1473, 1475, 1476, 1477, 1478, 1479, 1480, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1508, 1509, 1511, 1513, 1514, 1515, 1516, 1517, 1518, 1519, 1520, 1521, 1522, 1523, 1540, 1541, 1542, 1543, 1545, 1546, 1547, 1548, 1549, 1550, 1551, 1552, 1553, 1554, 1555, 1556, 1557, 1558, 1559, 1560, 1561, 1563, 1564, 1565, 1567, 1568, 1569, 1570, 1571, 1572, 1574, 1575, 1576, 1577, 1579, 1580, 1581, 1582, 1583, 1584, 1585, 1586, 1587, 1591, 1592, 1593, 1594, 1595, 1596, 1597, 1598, 1599, 1603, 1604, 1605, 1606, 1607, 1608, 1609, 1610, 1611, 1612, 1613, 1614, 1615, 1616, 1617, 1618, 1619, 1620, 1621, 1622, 1623, 1624, 1625, 1626, 1627, 1628, 1629, 1630, 1631, 1632, 1633, 1634, 1635, 1636, 1637, 1638, 1641, 1642, 1643, 1644, 1645, 1646, 1647, 1648, 1649, 1650, 1651, 1652, 1653, 1654, 1655, 1656, 1657, 1658, 1659, 1660, 1661, 1662, 1663, 1664, 1665, 1666, 1667, 1668, 1669, 1670, 1671, 1672, 1674, 1675, 1676, 1677, 1678, 1683, 1684, 1685, 1691, 1692, 1693, 1694, 1701, 1702, 1703, 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1928, 1929, 1930, 1931, 1933, 1934, 1939, 1940, 1942, 1943, 1947, 1948, 1949, 1950, 1965, 1966, 1967, 1968, 1969, 1970, 1971, 1972, 1973, 1974, 1975, 1976, 1977, 1978, 1979, 1980, 1981, 1982, 1983, 1984, 1985, 1986, 1987, 1988, 1989, 1990, 1991, 1992, 1998, 1999, 2000, 2001, 2002, 2003, 2004, 2005, 2006, 2007, 2008, 2009, 2010, 2012, 2017, 2018, 2019, 2020, 2024, 2025, 2026, 2027, 2028, 2029, 2030, 2031, 2032, 2033, 2034, 2035, 2036, 2037, 2038, 2040, 2041, 2044, 2045, 2046, 2047, 2048, 2049, 2050, 2051, 2052, 2053, 2054, 2055, 2056, 2057, 2058, 2059, 2060, 2061, 2066, 2067, 2068, 2069, 2074, 2075, 2076, 2077, 2082, 2083, 2084, 2085, 2090, 2091, 2093, 2106, 2107, 2108, 2109, 2110, 2111, 2112, 2113, 2114, 2115, 2116, 2117, 2118, 2119, 2120, 2121, 2122, 2123, 2124, 2125, 2126, 2127, 2128, 2129, 2130, 2131, 2132, 2133, 2134, 2135, 2136, 2138, 2139, 2140, 2141, 2142, 2143, 2144, 2146, 2147, 2148, 2150, 2151, 2152, 2153, 2154, 2155, 2156, 2158, 2159, 2160, 2161, 2162, 2163, 2165, 2166, 2167, 2168, 2169, 2170, 2171, 2172, 2175, 2176, 2177, 2178, 2179, 2180, 2181, 2184, 2185, 2186, 2187, 2188, 2189, 2190, 2191, 2192, 2193, 2194, 2195, 2196, 2197, 2198, 2199, 2200, 2201, 2202, 2203, 2204, 2205, 2206, 2207, 2208, 2209, 2210, 2211, 2212, 2213, 2214, 2215, 2216, 2217, 2218, 2219, 2220, 2221, 2222, 2223, 2224, 2225, 2226, 2227, 2228, 2229, 2230, 2231, 2232, 2237, 2238, 2242, 2243, 2244, 2246, 2247, 2248, 2250, 2251, 2252, 2253, 2257, 2258, 2259, 2260, 2273, 2277, 2278, 2285, 2286, 2287, 2288, 2289, 2290, 2291, 2292, 2293, 2294, 2295, 2296, 2297, 2298, 2299, 2300, 2301, 2302, 2303, 2304, 2305, 2306, 2307, 2308, 2309, 2310, 2311, 2312, 2313, 2314, 2315, 2316, 2317, 2318, 2320, 2321, 2322, 2323, 2324, 2325, 2326, 2327, 2329, 2330, 2331, 2332, 2334, 2335, 2336, 2337, 2338, 2339, 2340, 2341, 2342, 2343, 2345, 2346, 2347, 2348, 2349, 2351, 2352, 2353, 2354, 2355, 2357, 2358, 2359, 2360, 2361, 2362, 2363, 2364, 2368, 2369, 2370, 2371, 2372, 2373, 2374, 2375, 2376, 2377, 2378, 2379, 2380, 2382, 2383, 2384, 2385, 2386, 2391, 2392, 2394, 2395, 2396, 2397, 2398, 2399, 2400, 2401, 2402, 2403, 2404, 2405, 2406, 2407, 2408, 2409, 2410, 2411, 2412, 2413, 2414, 2415, 2416, 2417, 2418, 2419, 2420, 2421, 2422, 2423, 2424, 2425, 2426, 2427, 2428, 2429, 2430, 2431, 2432, 2433, 2434, 2435, 2436, 2437, 2438, 2439, 2440, 2441, 2442, 2443, 2444, 2445, 2448, 2449, 2451, 2452, 2453, 2454, 2455, 2456, 2457, 2458, 2459, 2460, 2461, 2462, 2463, 2464, 2465, 2466, 2467, 2468, 2469, 2470, 2471, 2474, 2479, 2484, 2494, 2499, 2504, 2518, 2519, 2520, 2521, 2522, 2523, 2524, 2525, 2526, 2527, 2528, 2529, 2530, 2531, 2532, 2533, 2534, 2535, 2536, 2537, 2538, 2539, 2540, 2541, 2542, 2543, 2544, 2545, 2547, 2548, 2549, 2550, 2551, 2552, 2553, 2554, 2555, 2561, 2564, 2565, 2574, 2575, 2595, 2596, 2597, 2598, 2599, 2600, 2601, 2602, 2603, 2604, 2606, 2607, 2608, 2609, 2610, 2611, 2612, 2613, 2615, 2616, 2617, 2618, 2619, 2620, 2621, 2622, 2625, 2626, 2627, 2628, 2629, 2634, 2635, 2636, 2642, 2646, 2654, 2655, 2661, 2662, 2663, 2664, 2668, 2669, 2687, 2688, 2689, 2690, 2691, 2692, 2708, 2716, 2720, 2728, 2729, 2737, 2738, 2739, 2741, 2742, 2753, 2754, 2755, 2756, 2757, 2758, 2759, 2760, 2761, 2762, 2765, 2766, 2769, 2777, 2780, 2781, 2782, 2783, 2784, 2785, 2786, 2808, 2809, 2810, 2811, 2819, 2820, 2821, 2822, 2823, 2824, 2831, 2832, 2833, 2838, 2840, 2844, 2845, 2846, 2859, 2860, 2862, 2863, 2864, 2865, 2866, 2868, 2869, 2871, 2872, 2874, 2875, 2877, 2878, 2880, 2881, 2886, 2887, 2889, 2890, 2891, 2892, 2894, 2895, 2897, 2899, 2900, 2902, 2903, 2905, 2929, 2959, 3003, 3004, 3010, 3016, 3021, 3028, 3029, 3030, 3034, 3035, 3036, 3037, 3038, 3039, 3040, 3041, 3042, 3071, 3072, 3073, 3076 }

B grade: { 220, 240, 245, 262, 263, 270, 289, 290, 291, 301, 302, 342, 367, 372, 392, 425, 519, 520, 521, 522, 540, 541, 542, 543, 559, 560, 570, 572, 573, 623, 635, 644, 648, 652, 661, 663, 674, 675, 711, 712, 773, 775, 793, 794, 795, 806, 820, 840, 854, 860, 885, 886, 891, 892, 906, 940, 970, 971, 1005, 1006, 1007, 1008, 1036, 1037, 1038, 1039, 1067, 1068, 1069, 1070, 1100, 1101, 1102, 1128, 1130, 1131, 1157, 1159, 1160, 1161, 1171, 1173, 1174, 1192, 1193, 1194, 1195, 1252, 1324, 1331, 1337, 1346, 1348, 1350, 1352, 1354, 1381, 1412, 1454, 1474, 1544, 1562, 1566, 1573, 1578, 1588, 1589, 1590, 1600, 1601, 1602, 1639, 1640, 1695, 1696, 1697, 1698, 1699, 1700, 1704, 1706, 1707, 1708, 1709, 1710, 1711, 1718, 1719, 1720, 1721, 1728, 1729, 1730, 1731, 1735, 1738, 1739, 1740, 1741, 1742, 1743, 1744, 1745, 1750, 1751, 1752, 1753, 1755, 1760, 1761, 1762, 1763, 1770, 1771, 1777, 1778, 1779, 1786, 1787, 1788, 1791, 1795, 1796, 1797, 1798, 1799, 1800, 1801, 1802, 1818, 1826, 1839, 1847, 1849, 1851, 1853, 1855, 1857, 1866, 1868, 1882, 1883, 1892, 1897, 1905, 1919, 1920, 1921, 1922, 1932, 1935, 1936, 1937, 1938, 1941, 1944, 1945, 1946, 1951, 1952, 1953, 1954, 1955, 1956, 1957, 1958, 1993, 1994, 1995, 1996, 1997, 2011, 2013, 2014, 2015, 2016, 2021, 2022, 2023, 2039, 2042, 2043, 2092, 2098, 2099, 2100, 2101, 2137, 2145, 2149, 2157, 2164, 2173, 2174, 2182, 2183, 2233, 2234, 2235, 2236, 2239, 2240, 2241, 2245, 2249, 2254, 2255, 2267, 2270, 2274, 2275, 2319, 2328, 2333, 2344, 2350, 2366, 2367, 2381, 2387, 2388, 2390, 2446, 2450, 2489, 2509, 2514, 2592, 2593, 2594, 2649, 2650, 2651, 2652, 2653, 2658, 2659, 2660, 2693, 2694, 2727, 2787, 2788, 2841, 2842, 2843, 2853, 2854, 2855, 2856, 2857, 2858, 2861, 2867, 2870, 2873, 2876, 2879, 2882, 2883, 2884, 2885, 2888, 2893, 2896, 2898, 2901, 2904, 2906, 2907, 2908, 2909, 2911, 2912, 2913, 2914, 2915, 2916, 3005 }

C grade: { 496, 497, 498, 499, 514, 515, 516, 517, 518, 523, 524, 525, 526, 531, 532, 533, 534, 535, 536, 537, 538, 539, 548, 549, 550, 551, 552, 553, 554, 555, 556, 565, 566, 567, 568, 569, 574, 575, 576, 577, 578, 579, 580, 581, 588, 589, 591, 592, 593, 594, 595, 596, 597, 776, 777, 778, 779, 780, 781, 782, 796, 797, 798, 799, 800, 801, 802, 803, 809, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 897, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 977, 978, 979, 980, 981, 982, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1172, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1266, 1399, 1400, 1401, 1405, 1406, 1407, 1424, 1425, 1426, 1427, 1432, 1433, 1434, 1481, 1482, 1488, 1489, 1510, 1512, 1524, 1525, 1526, 1527, 1528, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1959, 1960, 1961, 1962, 1963, 1964, 2062, 2063, 2064, 2065, 2070, 2071, 2072, 2073, 2078, 2079, 2080, 2081, 2086, 2087, 2088, 2089, 2094, 2095, 2096, 2097, 2102, 2103, 2104, 2105, 2256, 2261, 2262, 2263, 2268, 2269, 2271, 2272, 2279, 2280, 2281, 2282, 2283, 2284, 2389, 2393, 2472, 2473, 2475, 2476, 2477, 2478, 2480, 2481, 2482, 2483, 2485, 2486, 2487, 2488, 2490, 2491, 2492, 2493, 2495, 2496, 2497, 2498, 2500, 2501, 2502, 2503, 2505, 2506, 2507, 2508, 2510, 2511, 2512, 2513, 2515, 2516, 2517, 2640, 2641, 2647, 2648, 2670, 2671, 2672, 2673, 2674, 2675, 2676, 2677, 2678, 2679, 2680, 2681, 2682, 2683, 2684, 2685, 2686, 2695, 2696, 2697, 2698, 2699, 2700, 2702, 2703, 2704, 2709, 2710, 2711, 2734, 2735, 2736, 2743, 2744, 2748, 2749, 2750, 2763, 2764, 2767, 2768, 2770, 2771, 2772, 2773, 2774, 2775, 2776, 2778, 2779, 2789, 2790, 2791, 2792, 2793, 2794, 2795, 2796, 2797 }

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A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 52, 53, 54, 55, 56, 57, 58,

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1504, 1505, 1506, 1507, 1508, 1509, 1511, 1513, 1514, 1515, 1516, 1517, 1520, 1521, 1522, 1523, 1540, 1541, 1542, 1543, 1545, 1546, 1547, 1548, 1549, 1550, 1551, 1552, 1553, 1554, 1555, 1556, 1557, 1558, 1559, 1560, 1561, 1562, 1563, 1564, 1565, 1566, 1567, 1568, 1569, 1570, 1571, 1572, 1574, 1575, 1576, 1577, 1578, 1579, 1580, 1581, 1582, 1583, 1584, 1585, 1586, 1587, 1591, 1592, 1593, 1594, 1595, 1596, 1597, 1598, 1599, 1600, 1602, 1603, 1604, 1605, 1606, 1607, 1608, 1609, 1610, 1611, 1612, 1613, 1614, 1615, 1616, 1617, 1618, 1619, 1620, 1621, 1622, 1623, 1624, 1625, 1626, 1627, 1628, 1629, 1630, 1631, 1632, 1633, 1634, 1635, 1636, 1637, 1638, 1639, 1640, 1641, 1642, 1643, 1644, 1645, 1646, 1647, 1648, 1649, 1650, 1651, 1652, 1653, 1654, 1655, 1656, 1657, 1658, 1659, 1660, 1661, 1662, 1663, 1664, 1665, 1666, 1667, 1668, 1669, 1670, 1671, 1672, 1673, 1674, 1675, 1676, 1677, 1678, 1679, 1680, 1681, 1682, 1683, 1684, 1685, 1686, 1687, 1688, 1689, 1690, 1691, 1692, 1693, 1701, 1702, 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1881, 1882, 1883, 1884, 1885, 1886, 1887, 1888, 1889, 1890, 1891, 1892, 1893, 1896, 1898, 1899, 1900, 1901, 1902, 1904, 1906, 1907, 1908, 1910, 1912, 1914, 1915, 1916, 1918, 1919, 1920, 1921, 1924, 1934, 1950, 1965, 1966, 1967, 1968, 1969, 1970, 1971, 1972, 1973, 1974, 1975, 1976, 1977, 1978, 1979, 1980, 1981, 1982, 1983, 1984, 1985, 1986, 1987, 1988, 1989, 1990, 1991, 1992, 1993, 1994, 1995, 1996, 1997, 2010, 2011, 2012, 2013, 2014, 2015, 2016, 2017, 2018, 2020, 2037, 2038, 2057, 2058, 2059, 2060, 2061, 2066, 2067, 2069, 2075, 2077, 2082, 2083, 2084, 2085, 2090, 2091, 2092, 2093, 2098, 2099, 2100, 2101, 2106, 2109, 2111, 2112, 2113, 2114, 2115, 2116, 2117, 2118, 2119, 2120, 2121, 2122, 2123, 2124, 2125, 2126, 2127, 2128, 2129, 2130, 2131, 2132, 2133, 2134, 2135, 2136, 2137, 2138, 2139, 2140, 2141, 2142, 2143, 2144, 2145, 2146, 2147, 2148, 2150, 2151, 2152, 2153, 2154, 2155, 2156, 2158, 2159, 2160, 2161, 2162, 2163, 2165, 2166, 2167, 2168, 2169, 2170, 2171, 2172, 2175, 2176, 2177, 2178, 2179, 2180, 2181, 2184, 2185, 2186, 2187, 2188, 2189, 2190, 2191, 2192, 2193, 2194, 2195, 2196, 2197, 2198, 2199, 2200, 2201, 2202, 2203, 2204, 2205, 2206, 2207, 2208, 2209, 2210, 2211, 2212, 2213, 2214, 2215, 2216, 2217, 2219, 2220, 2221, 2222, 2223, 2224, 2225, 2226, 2227, 2228, 2229, 2230, 2231, 2232, 2233, 2236, 2237, 2238, 2239, 2240, 2241, 2242, 2243, 2244, 2246, 2247, 2248, 2250, 2251, 2252, 2253, 2254, 2255, 2256, 2257, 2258, 2259, 2260, 2276, 2277, 2278, 2279, 2285, 2286, 2287, 2288, 2289, 2290, 2291, 2292, 2293, 2294, 2295, 2296, 2297, 2298, 2299, 2300, 2301, 2302, 2303, 2304, 2305, 2306, 2307, 2308, 2309, 2310, 2311, 2312, 2313, 2314, 2315, 2316, 2317, 2318, 2320, 2321, 2322, 2323, 2324, 2325, 2326, 2327, 2329, 2330, 2331, 2332, 2334, 2335, 2336, 2337, 2338, 2339, 2340, 2341, 2342, 2343, 2345, 2346, 2347, 2348, 2349, 2351, 2352, 2353, 2354, 2355, 2356, 2357, 2358, 2359, 2360, 2361, 2362, 2363, 2364, 2365, 2366, 2367, 2368, 2369, 2370, 2371, 2372, 2373, 2374, 2375, 2376, 2377, 2378, 2379, 2380, 2381, 2382, 2383, 2384, 2385, 2386, 2387, 2388, 2389, 2390, 2391, 2392, 2394, 2395, 2396, 2397, 2398, 2399, 2400, 2401, 2402, 2403, 2404, 2405, 2406, 2407, 2408, 2409, 2410, 2411, 2412, 2413, 2414, 2415, 2416, 2417, 2418, 2419, 2420, 2421, 2422, 2423, 2424, 2425, 2426, 2427, 2428, 2429, 2430, 2431, 2432, 2433, 2434, 2435, 2436, 2437, 2438, 2439, 2440, 2441, 2442, 2443, 2444, 2445, 2446, 2448, 2449, 2451, 2452, 2453, 2454, 2461, 2521, 2523, 2524, 2525, 2526, 2527, 2531, 2533, 2534, 2535, 2536, 2537, 2538, 2544, 2545, 2546, 2547, 2548, 2549, 2550, 2556, 2557, 2558, 2559, 2560, 2562, 2563, 2564, 2565, 2576, 2577, 2578, 2579, 2580, 2581, 2582, 2583, 2586, 2587, 2588, 2589, 2590, 2592, 2593, 2594, 2601, 2610, 2619, 2627, 2635, 2652, 2661, 2712, 2713, 2714, 2715, 2716, 2720, 2729, 2738, 2739, 2740, 2741, 2742, 2753, 2754, 2803, 2804, 2805, 2808, 2809, 2810, 2811, 2816, 2819, 2820, 2821, 2823, 2824, 2825, 2826, 2827, 2828, 2831, 2837, 2838, 2840, 2844, 2845, 2846, 2851, 2859, 2860, 2861, 2862, 2863, 2864, 2865, 2866, 2867, 2868, 2869, 2870, 2871, 2872, 2873, 2874, 2875, 2876, 2877, 2878, 2879, 2880, 2881, 2882, 2883, 2885, 2886, 2887, 2888, 2889, 2890, 2891, 2892, 2893, 2894, 2895, 2896, 2897, 2898, 2899, 2900, 2901, 2902, 2903, 2904, 2905, 2906, 2907, 2908, 2911, 2912, 2913, 2914, 2915, 2916, 2930, 2931, 2933, 2934, 2935, 2936, 2937, 2938, 2939, 2940, 2941, 2942, 2943, 2944, 2945, 2946, 2959, 2960, 2961, 2962, 2963, 2970, 2971, 2972, 2973, 2983, 2989, 2991, 2992, 2994, 3003, 3004, 3005, 3021, 3023, 3024, 3025, 3028, 3029, 3030, 3034, 3035, 3036, 3037, 3038, 3039, 3040, 3041, 3042, 3063, 3068, 3069, 3070, 3071, 3072, 3073 }

B grade: { 245, 270, 290, 301, 302, 442, 460, 583, 584, 585, 586, 600, 601, 602, 620, 632, 708, 709, 710, 713, 714, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 773, 774, 775, 792, 793, 794, 795, 860, 877, 881, 882, 883, 903, 920, 923, 964, 965, 967, 993, 994, 995, 1017, 1018, 1019, 1020, 1022, 1023, 1024, 1052, 1053, 1054, 1085, 1086, 1087, 1115, 1116, 1117, 1144, 1145, 1146, 1180, 1181, 1182, 1209, 1210, 1211, 1239, 1240, 1241, 1264, 1315, 1350, 1387, 1438, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1468, 1469, 1470, 1473, 1474, 1492, 1518, 1519, 1544, 1573, 1588, 1589, 1590, 1601, 1694, 1696, 1697, 1698, 1699, 1700, 1706, 1707, 1708, 1709, 1710, 1711, 1717, 1718, 1719, 1720, 1721, 1725, 1761, 1762, 1763, 1768, 1769, 1770, 1771, 1839, 1894, 1895, 1897, 1903, 1905, 1909, 1911, 1913, 1917, 1927, 1933, 1944, 1945, 1946, 1947, 1948, 1957, 1958, 2068, 2074, 2076, 2110, 2149, 2157, 2164, 2173, 2174, 2182, 2183, 2218, 2234, 2235, 2245, 2249, 2264, 2265, 2266, 2267, 2270, 2319, 2328, 2333, 2344, 2350, 2447, 2458, 2459, 2460, 2465, 2466, 2467, 2468, 2542, 2554, 2561, 2574, 2584, 2585, 2591, 2595, 2596, 2667, 2668, 2669, 2725, 2727, 2755, 2756, 2757, 2758, 2802, 2813, 2814, 2815, 2822, 2832, 2833, 2834, 2841, 2842, 2843, 2847, 2849, 2853, 2854, 2855, 2856, 2857, 2858, 2884, 2909, 2910, 2932, 3022, 3055 }

C grade: { 2812, 2929 }

F grade: { 41, 50, 51, 59, 60, 61, 62, 63, 64, 65, 68, 74, 75, 76, 95, 96, 100, 101, 106, 107, 108, 109, 113, 114, 115, 116, 117, 122, 123, 124, 129, 130, 131, 132, 133, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163,

164, 165, 166, 167, 168, 169, 171, 178, 179, 186, 187, 188, 194, 195, 200, 204, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 776, 777, 778, 779, 780, 781, 782, 796, 797, 798, 799, 800, 801, 802, 803, 804, 809, 810, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 968, 969, 970, 971, 972, 973, 977, 978, 979, 980, 981, 982, 983, 984, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1266, 1317, 1380, 1390, 1391, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1415, 1416, 1418, 1419, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1434, 1441, 1445, 1510, 1512, 1524, 1525, 1526, 1527, 1528, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1695, 1704, 1705, 1714, 1715, 1716, 1922, 1923, 1925, 1926, 1928, 1929, 1930, 1931, 1932, 1935, 1936, 1937, 1938, 1939, 1940, 1941, 1942, 1943, 1949, 1951, 1952, 1953, 1954, 1955, 1956, 1959, 1960, 1961, 1962, 1963, 1964, 1998, 1999, 2000, 2001, 2002, 2003, 2004, 2005, 2006, 2007, 2008, 2009, 2019, 2021, 2022, 2023, 2024, 2025, 2026, 2027, 2028, 2029, 2030, 2031, 2032, 2033, 2034, 2035, 2036, 2039, 2040, 2041, 2042, 2043, 2044, 2045, 2046, 2047, 2048, 2049, 2050, 2051, 2052, 2053, 2054, 2055, 2056, 2062, 2063, 2064, 2065, 2070, 2071, 2072, 2073, 2078, 2079, 2080, 2081, 2086, 2087, 2088, 2089, 2094, 2095, 2096, 2097, 2102, 2103, 2104, 2105, 2107, 2108, 2261, 2262, 2263, 2268, 2269, 2271, 2272, 2273, 2274, 2275, 2280, 2281, 2282, 2283, 2284, 2393, 2450, 2455, 2456, 2457, 2462, 2463, 2464, 2469, 2470, 2471, 2472, 2473, 2474, 2475, 2476, 2477, 2478, 2479, 2480, 2481, 2482, 2483, 2484, 2485, 2486, 2487, 2488, 2489, 2490, 2491, 2492, 2493, 2494, 2495, 2496, 2497, 2498, 2499, 2500, 2501, 2502, 2503, 2504, 2505, 2506, 2507, 2508, 2509, 2510, 2511, 2512, 2513, 2514, 2515, 2516, 2517, 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2768, 2769, 2770, 2771, 2772, 2773, 2774, 2775, 2776, 2777, 2778, 2779, 2780, 2781, 2782, 2783, 2784, 2785, 2786, 2787, 2788, 2789, 2790, 2791, 2792, 2793, 2794, 2795, 2796, 2797, 2798, 2799, 2800, 2801, 2806, 2807, 2817, 2818, 2829, 2830, 2835, 2836, 2839, 2848, 2850, 2852, 2917, 2918, 2919, 2920, 2921, 2922, 2923, 2924, 2925, 2926, 2927, 2928, 2947, 2948, 2949, 2950, 2951, 2952, 2953, 2954, 2955, 2956, 2957, 2958, 2964, 2965, 2966, 2967, 2968, 2969, 2974, 2975, 2976, 2977, 2978, 2979, 2980, 2981, 2982, 2984, 2985, 2986, 2987, 2988, 2990, 2993, 2995, 2996, 2997, 2998, 2999, 3000, 3001, 3002, 3006, 3007, 3008, 3009, 3010, 3011, 3012, 3013, 3014, 3015, 3016, 3017, 3018, 3019, 3020, 3026, 3027, 3031, 3032, 3033, 3043, 3044, 3045, 3046, 3047, 3048, 3049, 3050, 3051, 3052, 3053, 3054,

3056, 3057, 3058, 3059, 3060, 3061, 3062, 3064, 3065, 3066, 3067, 3074, 3075, 3076, 3077, 3078 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 4, 5, 6, 8, 9, 10, 13, 15, 16, 17, 19, 20, 21, 23, 25, 26, 27, 28, 29, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 152, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 172, 173, 174, 175, 176, 180, 181, 182, 183, 185, 189, 190, 191, 192, 196, 197, 199, 201, 202, 203, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 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2930, 2931, 2933, 2934, 2935, 2936, 2937, 2938, 2939, 2940, 2941, 2942, 2943, 2944, 2945, 2946, 2947, 2948, 2949, 2950, 2951, 2953, 2954, 2955, 2956, 2957, 2958, 2959, 2960, 2962, 2963, 2964, 2965, 2966, 2967, 2968, 2969, 2970, 2971, 2972, 2973, 2975, 2976, 2977, 2978, 2979, 2980, 2981, 2982, 2983, 2984, 2986, 2987, 2988, 2989, 2990, 2992, 2993, 2995, 2996, 2997, 2998, 2999, 3000, 3001, 3002, 3006, 3007, 3008, 3009, 3011, 3012, 3013, 3014, 3015, 3018, 3019, 3021, 3022, 3023, 3024, 3025, 3026, 3027, 3031, 3032, 3033, 3034, 3035, 3036, 3037, 3038, 3039, 3043, 3044, 3045, 3046, 3047, 3048, 3049, 3050, 3051, 3052, 3053, 3054, 3055, 3056, 3057, 3058, 3059, 3060, 3061, 3062, 3063, 3064, 3065, 3066, 3067, 3068, 3069, 3070, 3074, 3075 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	18	19	24	29	60	26	18
normalized size	1	1.00	0.82	0.86	1.09	1.32	2.73	1.18	0.82
time (sec)	N/A	0.007	0.004	0.003	1.407	0.703	5.131	0.182	1.059

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	15	12	15	10	10
normalized size	1	1.00	1.00	0.81	0.94	0.75	0.94	0.62	0.62
time (sec)	N/A	0.002	0.002	0.003	1.290	0.606	0.304	0.147	1.033

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	13	12	15	10	-1
normalized size	1	1.00	1.00	0.81	0.81	0.75	0.94	0.62	-0.06
time (sec)	N/A	0.002	0.001	0.003	1.321	0.677	0.242	0.173	0.000

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	15	10	10
normalized size	1	1.00	1.00	0.81	0.75	0.75	0.94	0.62	0.62
time (sec)	N/A	0.002	0.001	0.004	1.310	0.627	0.194	0.150	1.022

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	14	10	8
normalized size	1	1.00	1.00	0.79	0.71	0.71	1.00	0.71	0.57
time (sec)	N/A	0.002	0.001	0.002	1.254	0.687	0.166	0.162	1.244

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	14	8	0	7	10	7	6
normalized size	1	1.00	1.56	0.89	0.00	0.78	1.11	0.78	0.67
time (sec)	N/A	0.001	0.002	0.002	0.000	0.720	0.170	0.208	0.926

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	14	14	0	13	0	9	-1
normalized size	1	1.00	0.93	0.93	0.00	0.87	0.00	0.60	-0.07
time (sec)	N/A	0.001	0.001	0.001	0.000	0.539	0.000	0.160	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	0	12	15	10	10
normalized size	1	1.00	1.00	0.93	0.00	0.86	1.07	0.71	0.71
time (sec)	N/A	0.002	0.002	0.002	0.000	0.871	0.394	0.170	0.902

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	0	12	17	10	13
normalized size	1	1.00	1.00	0.81	0.00	0.75	1.06	0.62	0.81
time (sec)	N/A	0.001	0.001	0.003	0.000	0.773	0.468	0.153	0.938

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	0	12	17	10	10
normalized size	1	1.00	1.00	0.81	0.00	0.75	1.06	0.62	0.62
time (sec)	N/A	0.002	0.001	0.001	0.000	0.555	0.600	0.182	0.947

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	11	8	5	3	7	-1
normalized size	1	1.00	0.86	0.79	0.57	0.36	0.21	0.50	-0.07
time (sec)	N/A	0.001	0.004	0.002	1.344	0.557	0.067	0.152	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	16	13	13	13	15	10	-1
normalized size	1	1.00	0.94	0.76	0.76	0.76	0.88	0.59	-0.06
time (sec)	N/A	0.002	0.002	0.001	1.357	0.522	0.686	0.150	0.000

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	16	13	12	13	15	10	10
normalized size	1	1.00	0.94	0.76	0.71	0.76	0.88	0.59	0.59
time (sec)	N/A	0.002	0.001	0.002	1.326	0.752	0.512	0.170	0.960

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	14	11	10	13	14	10	-1
normalized size	1	1.00	0.82	0.65	0.59	0.76	0.82	0.59	-0.06
time (sec)	N/A	0.002	0.001	0.002	1.282	0.697	0.376	0.152	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	10	9	13	12	10	10
normalized size	1	1.00	1.00	0.59	0.53	0.76	0.71	0.59	0.59
time (sec)	N/A	0.002	0.001	0.003	1.284	0.553	0.395	0.178	0.959

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	13	0	11	14	10	8
normalized size	1	1.00	1.00	0.87	0.00	0.73	0.93	0.67	0.53
time (sec)	N/A	0.002	0.001	0.002	0.000	0.633	0.417	0.155	0.970

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	13	12	0	9	14	7	6
normalized size	1	1.00	1.18	1.09	0.00	0.82	1.27	0.64	0.55
time (sec)	N/A	0.001	0.001	0.002	0.000	0.605	0.546	0.183	0.903

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	15	14	0	14	0	9	-1
normalized size	1	1.00	0.94	0.88	0.00	0.88	0.00	0.56	-0.06
time (sec)	N/A	0.001	0.002	0.003	0.000	0.538	0.000	0.187	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	14	13	0	13	15	10	10
normalized size	1	1.00	0.93	0.87	0.00	0.87	1.00	0.67	0.67
time (sec)	N/A	0.002	0.002	0.002	0.000	0.559	0.851	0.158	0.904

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	16	13	0	13	17	10	13
normalized size	1	1.00	0.94	0.76	0.00	0.76	1.00	0.59	0.76
time (sec)	N/A	0.002	0.001	0.001	0.000	0.482	1.023	0.177	0.904

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	16	13	0	13	17	10	10
normalized size	1	1.00	0.94	0.76	0.00	0.76	1.00	0.59	0.59
time (sec)	N/A	0.002	0.001	0.003	0.000	0.555	1.293	0.155	0.908

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	9	8	5	3	7	-1
normalized size	1	1.00	0.86	0.64	0.57	0.36	0.21	0.50	-0.07
time (sec)	N/A	0.001	0.004	0.002	1.263	0.761	0.070	0.151	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	13	12	15	15	10	10
normalized size	1	1.00	0.84	0.68	0.63	0.79	0.79	0.53	0.53
time (sec)	N/A	0.002	0.002	0.001	1.328	0.592	1.299	0.147	0.936

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	14	11	10	15	14	10	-1
normalized size	1	1.00	0.74	0.58	0.53	0.79	0.74	0.53	-0.05
time (sec)	N/A	0.002	0.001	0.003	1.340	0.670	1.005	0.172	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	17	10	9	15	12	10	10
normalized size	1	1.00	0.89	0.53	0.47	0.79	0.63	0.53	0.53
time (sec)	N/A	0.002	0.002	0.003	1.333	0.653	1.017	0.152	0.902

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	15	13	12	15	14	10	13
normalized size	1	1.00	0.79	0.68	0.63	0.79	0.74	0.53	0.68
time (sec)	N/A	0.002	0.001	0.003	1.312	0.584	1.063	0.166	0.970

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	13	0	15	15	10	10
normalized size	1	1.00	0.84	0.68	0.00	0.79	0.79	0.53	0.53
time (sec)	N/A	0.002	0.002	0.001	0.000	0.780	1.220	0.178	0.931

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	13	0	13	15	10	8
normalized size	1	1.00	1.00	0.76	0.00	0.76	0.88	0.59	0.47
time (sec)	N/A	0.002	0.002	0.002	0.000	0.746	1.231	0.153	0.936

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	0	11	14	7	6
normalized size	1	1.00	1.00	0.92	0.00	0.85	1.08	0.54	0.46
time (sec)	N/A	0.001	0.001	0.001	0.000	0.522	1.239	0.170	0.947

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	15	14	0	16	0	9	-1
normalized size	1	1.00	0.83	0.78	0.00	0.89	0.00	0.50	-0.06
time (sec)	N/A	0.002	0.002	0.003	0.000	0.611	0.000	0.174	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	14	13	0	15	15	10	10
normalized size	1	1.00	0.82	0.76	0.00	0.88	0.88	0.59	0.59
time (sec)	N/A	0.002	0.002	0.002	0.000	0.751	1.866	0.154	0.926

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	13	0	15	17	10	13
normalized size	1	1.00	0.84	0.68	0.00	0.79	0.89	0.53	0.68
time (sec)	N/A	0.002	0.001	0.002	0.000	0.609	2.239	0.155	0.933

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	13	0	15	17	10	10
normalized size	1	1.00	0.84	0.68	0.00	0.79	0.89	0.53	0.53
time (sec)	N/A	0.002	0.001	0.003	0.000	0.543	2.671	0.153	0.906

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	9	8	5	3	7	-1
normalized size	1	1.00	0.86	0.64	0.57	0.36	0.21	0.50	-0.07
time (sec)	N/A	0.001	0.003	0.002	1.296	0.495	0.070	0.413	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	15	15	15	15	10
normalized size	1	1.00	1.00	0.81	0.94	0.94	0.94	0.94	0.62
time (sec)	N/A	0.002	0.002	0.003	1.348	0.509	0.488	0.155	0.970

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	14	11	6
normalized size	1	1.00	1.00	0.92	0.85	0.85	1.08	0.85	0.46
time (sec)	N/A	0.001	0.001	0.001	1.336	0.509	0.399	0.151	0.963

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	15	10	8	15	14	9	10
normalized size	1	1.00	1.36	0.91	0.73	1.36	1.27	0.82	0.91
time (sec)	N/A	0.001	0.002	0.001	1.368	0.624	0.459	0.154	0.962

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	8	15	19	12	10
normalized size	1	1.00	1.00	0.81	0.50	0.94	1.19	0.75	0.62
time (sec)	N/A	0.001	0.002	0.004	1.334	0.592	0.624	0.165	0.969

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	8	13	15	13	11
normalized size	1	1.00	1.00	0.81	0.50	0.81	0.94	0.81	0.69
time (sec)	N/A	0.002	0.002	0.001	1.339	0.677	0.450	0.220	0.988

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	6	16	0	19	10
normalized size	1	1.00	1.00	0.92	0.46	1.23	0.00	1.46	0.77
time (sec)	N/A	0.001	0.000	0.003	1.324	0.582	0.000	0.203	1.089

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	15	13	8	15	17	0	13
normalized size	1	1.00	0.94	0.81	0.50	0.94	1.06	0.00	0.81
time (sec)	N/A	0.001	0.002	0.002	1.329	0.542	0.502	0.000	0.974

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	13	15	15	15	15	10
normalized size	1	1.00	0.84	0.68	0.79	0.79	0.79	0.79	0.53
time (sec)	N/A	0.002	0.001	0.003	1.299	0.701	0.708	0.154	1.018

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	13	12	14	11	14	11	6
normalized size	1	1.00	0.81	0.75	0.88	0.69	0.88	0.69	0.38
time (sec)	N/A	0.001	0.001	0.002	1.301	0.749	0.513	0.151	0.987

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	15	15	12	10
normalized size	1	1.00	1.00	0.93	0.86	1.07	1.07	0.86	0.71
time (sec)	N/A	0.002	0.001	0.002	1.327	0.723	0.519	0.154	0.931

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	17	10	8	15	15	15	10
normalized size	1	1.00	0.89	0.53	0.42	0.79	0.79	0.79	0.53
time (sec)	N/A	0.002	0.003	0.002	1.275	0.538	0.592	0.162	0.961

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	13	8	15	19	15	10
normalized size	1	1.00	0.84	0.68	0.42	0.79	1.00	0.79	0.53
time (sec)	N/A	0.002	0.003	0.003	1.347	0.576	0.863	0.150	0.953

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	13	15	15	15	15	-1
normalized size	1	1.00	0.84	0.68	0.79	0.79	0.79	0.79	-0.05
time (sec)	N/A	0.002	0.002	0.003	1.341	0.635	0.851	0.167	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	13	15	13	15	13	-1
normalized size	1	1.00	0.84	0.68	0.79	0.68	0.79	0.68	-0.05
time (sec)	N/A	0.002	0.002	0.003	1.296	0.628	0.584	0.214	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	15	14	6	16	0	21	37
normalized size	1	1.00	0.94	0.88	0.38	1.00	0.00	1.31	2.31
time (sec)	N/A	0.002	0.001	0.003	1.324	0.769	0.000	0.175	0.071

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	14	11	8	15	15	0	13
normalized size	1	1.00	0.74	0.58	0.42	0.79	0.79	0.00	0.68
time (sec)	N/A	0.002	0.000	0.001	1.387	0.527	0.503	0.000	0.913

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	15	13	8	15	17	0	13
normalized size	1	1.00	0.79	0.68	0.42	0.79	0.89	0.00	0.68
time (sec)	N/A	0.002	0.002	0.003	1.287	0.579	0.732	0.000	0.930

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	13	15	15	15	15	10
normalized size	1	1.00	0.84	0.68	0.79	0.79	0.79	0.79	0.53
time (sec)	N/A	0.002	0.001	0.002	1.360	0.505	1.202	0.154	0.984

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	13	12	14	11	14	11	6
normalized size	1	1.00	0.81	0.75	0.88	0.69	0.88	0.69	0.38
time (sec)	N/A	0.001	0.002	0.002	1.290	0.599	0.825	0.165	0.959

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	15	15	15	12	10
normalized size	1	1.00	1.00	0.93	1.07	1.07	1.07	0.86	0.71
time (sec)	N/A	0.002	0.003	0.003	1.334	0.434	0.858	0.181	0.984

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	13	12	15	17	15	10
normalized size	1	1.00	0.84	0.68	0.63	0.79	0.89	0.79	0.53
time (sec)	N/A	0.002	0.001	0.003	1.354	0.523	0.849	0.156	0.949

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	17	10	8	15	15	15	10
normalized size	1	1.00	0.89	0.53	0.42	0.79	0.79	0.79	0.53
time (sec)	N/A	0.002	0.004	0.002	1.357	0.652	1.031	0.152	0.921

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	13	15	13	15	13	-1
normalized size	1	1.00	0.84	0.68	0.79	0.68	0.79	0.68	-0.05
time (sec)	N/A	0.002	0.002	0.003	1.289	0.769	1.020	0.169	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	15	14	6	16	0	21	-1
normalized size	1	1.00	0.94	0.88	0.38	1.00	0.00	1.31	-0.06
time (sec)	N/A	0.002	0.001	0.003	1.344	0.650	0.000	0.222	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	13	8	15	17	0	13
normalized size	1	1.00	0.84	0.68	0.42	0.79	0.89	0.00	0.68
time (sec)	N/A	0.002	0.001	0.003	1.282	0.556	0.851	0.000	0.925

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	14	11	8	15	15	0	13
normalized size	1	1.00	0.74	0.58	0.42	0.79	0.79	0.00	0.68
time (sec)	N/A	0.002	0.000	0.003	1.304	0.527	0.843	0.000	0.909

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	15	13	8	15	17	0	13
normalized size	1	1.00	0.79	0.68	0.42	0.79	0.89	0.00	0.68
time (sec)	N/A	0.002	0.002	0.003	1.320	0.647	1.253	0.000	0.919

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	21	20	18	22	0	0	22
normalized size	1	1.00	0.72	0.69	0.62	0.76	0.00	0.00	0.76
time (sec)	N/A	0.010	0.005	0.003	1.392	0.686	0.000	0.000	0.960

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	21	20	18	19	0	0	20
normalized size	1	1.00	0.75	0.71	0.64	0.68	0.00	0.00	0.71
time (sec)	N/A	0.009	0.004	0.002	1.431	0.667	0.000	0.000	0.943

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	13	22	0	0	18
normalized size	1	1.00	1.00	0.95	0.68	1.16	0.00	0.00	0.95
time (sec)	N/A	0.005	0.003	0.001	1.416	0.587	0.000	0.000	0.959

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	21	20	18	29	0	0	22
normalized size	1	1.00	0.66	0.62	0.56	0.91	0.00	0.00	0.69
time (sec)	N/A	0.012	0.005	0.001	1.417	0.619	0.000	0.000	0.999

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	24	24	0	24	21
normalized size	1	1.00	1.00	1.05	1.14	1.14	0.00	1.14	1.00
time (sec)	N/A	0.006	0.004	0.002	1.329	0.741	0.000	0.169	1.070

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	23	27	32	0	29	22
normalized size	1	1.00	1.00	1.05	1.23	1.45	0.00	1.32	1.00
time (sec)	N/A	0.005	0.003	0.001	1.372	0.739	0.000	0.165	0.962

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	21	2	2	0	0	-1
normalized size	1	1.00	1.00	1.62	0.15	0.15	0.00	0.00	-0.08
time (sec)	N/A	0.002	0.006	0.027	1.284	0.767	0.000	0.000	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	18	18	17	24	17	18
normalized size	1	1.00	0.95	0.95	0.95	0.89	1.26	0.89	0.95
time (sec)	N/A	0.005	0.003	0.002	1.368	0.828	0.424	0.153	0.965

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	19	18	0	18	18
normalized size	1	1.00	1.00	1.06	1.06	1.00	0.00	1.00	1.00
time (sec)	N/A	0.004	0.002	0.003	1.364	0.833	0.000	0.179	0.979

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	18	19	17	22	19	18
normalized size	1	1.00	0.95	0.95	1.00	0.89	1.16	1.00	0.95
time (sec)	N/A	0.005	0.003	0.001	1.323	0.807	0.236	0.157	0.986

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	17	16	0	16	16
normalized size	1	1.00	1.00	1.06	1.06	1.00	0.00	1.00	1.00
time (sec)	N/A	0.004	0.002	0.003	1.340	0.831	0.000	0.191	1.097

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	13	12	14	12	12
normalized size	1	1.00	1.00	0.93	0.93	0.86	1.00	0.86	0.86
time (sec)	N/A	0.003	0.001	0.001	1.312	0.646	0.200	0.154	0.949

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	19	19	18	0	0	18
normalized size	1	1.00	0.95	1.00	1.00	0.95	0.00	0.00	0.95
time (sec)	N/A	0.006	0.004	0.002	1.364	0.834	0.000	0.000	0.928

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	18	18	18	17	24	0	18
normalized size	1	1.00	0.86	0.86	0.86	0.81	1.14	0.00	0.86
time (sec)	N/A	0.005	0.003	0.002	1.362	0.652	0.479	0.000	0.979

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	19	19	18	0	0	18
normalized size	1	1.00	0.95	1.00	1.00	0.95	0.00	0.00	0.95
time (sec)	N/A	0.006	0.002	0.002	1.333	0.832	0.000	0.000	0.965

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	15	12	10
normalized size	1	1.00	1.00	0.79	0.71	0.71	1.07	0.86	0.71
time (sec)	N/A	0.002	0.001	0.003	1.343	0.685	0.169	0.165	1.359

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	14	10	8
normalized size	1	1.00	1.00	0.79	0.71	0.71	1.00	0.71	0.57
time (sec)	N/A	0.001	0.001	0.000	1.321	0.823	0.164	0.150	0.002

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	9	10	8	10	8	10
normalized size	1	1.00	0.86	0.64	0.71	0.57	0.71	0.57	0.71
time (sec)	N/A	0.001	0.001	0.001	1.299	0.817	0.061	0.144	0.022

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	14	9	10
normalized size	1	1.00	1.00	0.92	0.83	0.83	1.17	0.75	0.83
time (sec)	N/A	0.001	0.001	0.002	1.281	0.879	0.167	0.151	0.967

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	0	9	-1
normalized size	1	1.00	1.00	0.92	0.85	0.85	0.00	0.69	-0.08
time (sec)	N/A	0.003	0.002	0.004	1.327	0.842	0.000	0.169	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	17	8	10
normalized size	1	1.00	1.00	0.92	0.83	0.83	1.42	0.67	0.83
time (sec)	N/A	0.002	0.001	0.002	1.281	0.842	0.299	0.153	0.976

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	14	11	10	13	15	13	10
normalized size	1	1.00	0.82	0.65	0.59	0.76	0.88	0.76	0.59
time (sec)	N/A	0.002	0.001	0.003	1.291	0.864	0.386	0.150	0.951

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	14	11	10	13	14	10	-1
normalized size	1	1.00	0.82	0.65	0.59	0.76	0.82	0.59	-0.06
time (sec)	N/A	0.002	0.000	0.000	1.309	0.720	0.393	0.166	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	9	10	11	10	11	10
normalized size	1	1.00	0.86	0.64	0.71	0.79	0.71	0.79	0.71
time (sec)	N/A	0.001	0.001	0.001	1.313	0.635	0.063	0.176	0.015

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	15	12	10
normalized size	1	1.00	1.00	0.92	0.83	0.83	1.25	1.00	0.83
time (sec)	N/A	0.002	0.001	0.001	1.320	0.813	0.420	0.157	0.931

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	14	11	10	13	17	10	13
normalized size	1	1.00	0.82	0.65	0.59	0.76	1.00	0.59	0.76
time (sec)	N/A	0.002	0.001	0.003	1.302	0.796	0.531	0.149	0.979

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	14	11	10	13	19	13	13
normalized size	1	1.00	0.82	0.65	0.59	0.76	1.12	0.76	0.76
time (sec)	N/A	0.002	0.001	0.004	1.344	0.778	0.539	0.154	0.968

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	15	15	11	15
normalized size	1	1.00	1.00	0.92	0.83	1.25	1.25	0.92	1.25
time (sec)	N/A	0.001	0.002	0.003	1.317	0.828	0.402	0.153	0.940

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	6	16	0	19	10
normalized size	1	1.00	1.00	0.92	0.46	1.23	0.00	1.46	0.77
time (sec)	N/A	0.001	0.001	0.000	1.305	0.812	0.000	0.198	0.002

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	10	9	10	10	8	10	10
normalized size	1	1.00	0.83	0.75	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.001	0.001	0.003	1.311	0.802	0.062	0.222	0.013

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	15	15	15	15
normalized size	1	1.00	1.00	0.79	0.71	1.07	1.07	1.07	1.07
time (sec)	N/A	0.002	0.002	0.002	1.329	0.879	0.393	0.155	0.952

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	15	15	12	13
normalized size	1	1.00	1.00	0.79	0.71	1.07	1.07	0.86	0.93
time (sec)	N/A	0.001	0.002	0.003	1.295	0.720	0.394	0.191	0.926

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	15	17	13	15
normalized size	1	1.00	1.00	0.79	0.71	1.07	1.21	0.93	1.07
time (sec)	N/A	0.001	0.002	0.003	1.260	0.861	0.407	0.171	0.948

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	14	11	10	15	17	0	15
normalized size	1	1.00	0.74	0.58	0.53	0.79	0.89	0.00	0.79
time (sec)	N/A	0.002	0.002	0.001	1.321	0.889	0.512	0.000	0.937

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	14	11	8	15	15	0	13
normalized size	1	1.00	0.74	0.58	0.42	0.79	0.79	0.00	0.68
time (sec)	N/A	0.002	0.001	0.000	1.224	0.761	0.507	0.000	0.002

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	10	9	10	13	10	10	10
normalized size	1	1.00	0.83	0.75	0.83	1.08	0.83	0.83	0.83
time (sec)	N/A	0.001	0.001	0.001	1.305	0.947	0.061	0.148	0.017

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	14	11	10	15	15	15	15
normalized size	1	1.00	0.74	0.58	0.53	0.79	0.79	0.79	0.79
time (sec)	N/A	0.002	0.002	0.003	1.324	0.915	0.503	0.171	0.938

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	14	11	10	15	15	8	13
normalized size	1	1.00	0.74	0.58	0.53	0.79	0.79	0.42	0.68
time (sec)	N/A	0.002	0.002	0.003	1.316	0.656	0.505	0.156	0.926

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	14	11	10	15	17	0	15
normalized size	1	1.00	0.74	0.58	0.53	0.79	0.89	0.00	0.79
time (sec)	N/A	0.002	0.002	0.002	1.276	0.817	0.510	0.000	0.946

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	0	0	0	15
normalized size	1	1.00	1.00	0.94	0.88	0.00	0.00	0.00	0.88
time (sec)	N/A	0.005	0.003	0.003	1.330	0.000	0.000	0.000	0.996

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	14	8	-1
normalized size	1	1.00	1.00	0.79	0.71	0.71	1.00	0.57	-0.07
time (sec)	N/A	0.001	0.001	0.002	1.313	0.876	0.191	0.148	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	15	11	11
normalized size	1	1.00	1.00	0.79	0.71	0.71	1.07	0.79	0.79
time (sec)	N/A	0.002	0.001	0.003	1.326	0.852	0.196	0.161	0.917

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	9	10	8	10	8	10
normalized size	1	1.00	0.86	0.64	0.71	0.57	0.71	0.57	0.71
time (sec)	N/A	0.001	0.001	0.000	1.320	0.890	0.064	0.149	0.015

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	15	10	10
normalized size	1	1.00	1.00	0.79	0.71	0.71	1.07	0.71	0.71
time (sec)	N/A	0.002	0.001	0.001	1.301	0.881	0.191	0.150	0.921

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	15	0	11
normalized size	1	1.00	1.00	0.92	0.83	0.83	1.25	0.00	0.92
time (sec)	N/A	0.002	0.001	0.003	1.244	0.743	0.322	0.000	1.033

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	0	0	-1
normalized size	1	1.00	1.00	0.92	0.85	0.85	0.00	0.00	-0.08
time (sec)	N/A	0.003	0.001	0.015	1.294	0.639	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	17	0	11
normalized size	1	1.00	1.00	0.92	0.83	0.83	1.42	0.00	0.92
time (sec)	N/A	0.002	0.008	0.003	1.321	0.821	0.325	0.000	1.024

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	20	18	17	0	0	0	17
normalized size	1	1.00	1.05	0.95	0.89	0.00	0.00	0.00	0.89
time (sec)	N/A	0.005	0.003	0.001	1.339	0.000	0.000	0.000	0.977

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	15	13	11
normalized size	1	1.00	1.00	0.79	0.71	0.71	1.07	0.93	0.79
time (sec)	N/A	0.001	0.001	0.001	1.329	0.581	0.299	0.160	1.001

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	9	10	8	10	8	10
normalized size	1	1.00	0.86	0.64	0.71	0.57	0.71	0.57	0.71
time (sec)	N/A	0.001	0.001	0.001	1.315	0.814	0.062	0.156	0.016

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	14	10	10
normalized size	1	1.00	1.00	0.92	0.83	0.83	1.17	0.83	0.83
time (sec)	N/A	0.001	0.002	0.002	1.352	0.859	0.328	0.153	0.918

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	17	0	11
normalized size	1	1.00	1.00	0.92	0.83	0.83	1.42	0.00	0.92
time (sec)	N/A	0.001	0.005	0.002	1.361	0.863	0.480	0.000	0.930

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	15	0	10
normalized size	1	1.00	1.00	0.92	0.83	0.83	1.25	0.00	0.83
time (sec)	N/A	0.001	0.001	0.002	1.308	0.988	0.505	0.000	0.918

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	19	0	11
normalized size	1	1.00	1.00	0.79	0.71	0.71	1.36	0.00	0.79
time (sec)	N/A	0.002	0.001	0.001	1.287	1.060	0.443	0.000	0.916

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	17	16	0	0	0	0	24
normalized size	1	1.00	0.89	0.84	0.00	0.00	0.00	0.00	1.26
time (sec)	N/A	0.006	0.005	0.002	0.000	0.000	0.000	0.000	0.977

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	15	15	0	13
normalized size	1	1.00	1.00	0.92	0.83	1.25	1.25	0.00	1.08
time (sec)	N/A	0.001	0.002	0.003	1.285	0.798	0.592	0.000	0.929

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	16	0	7	-1
normalized size	1	1.00	1.00	0.92	0.85	1.23	0.00	0.54	-0.08
time (sec)	N/A	0.001	0.002	0.010	1.329	0.819	0.000	0.160	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	15	14	9	13
normalized size	1	1.00	1.00	0.92	0.83	1.25	1.17	0.75	1.08
time (sec)	N/A	0.001	0.002	0.002	1.327	0.746	0.830	0.197	0.935

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	9	10	10	10	10	10
normalized size	1	1.00	0.86	0.64	0.71	0.71	0.71	0.71	0.71
time (sec)	N/A	0.001	0.002	0.001	1.290	0.749	0.071	0.160	0.015

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	15	15	10	15
normalized size	1	1.00	1.00	0.79	0.71	1.07	1.07	0.71	1.07
time (sec)	N/A	0.002	0.002	0.004	1.325	0.770	0.429	0.198	0.923

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	15	17	0	13
normalized size	1	1.00	1.00	0.79	0.71	1.07	1.21	0.00	0.93
time (sec)	N/A	0.001	0.002	0.002	1.377	0.779	0.436	0.000	0.928

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	15	15	0	15
normalized size	1	1.00	1.00	0.79	0.71	1.07	1.07	0.00	1.07
time (sec)	N/A	0.001	0.002	0.000	1.309	0.844	0.448	0.000	0.941

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	20	18	0	0	0	0	26
normalized size	1	1.00	1.05	0.95	0.00	0.00	0.00	0.00	1.37
time (sec)	N/A	0.005	0.006	0.002	0.000	0.000	0.000	0.000	0.984

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	15	14	8	15
normalized size	1	1.00	1.00	0.92	0.83	1.25	1.17	0.67	1.25
time (sec)	N/A	0.001	0.002	0.002	1.292	0.743	0.543	0.154	0.927

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	15	15	12	13
normalized size	1	1.00	1.00	0.92	0.83	1.25	1.25	1.00	1.08
time (sec)	N/A	0.001	0.002	0.002	1.339	0.839	0.728	0.156	0.912

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	10	9	10	10	8	10	10
normalized size	1	1.00	0.83	0.75	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.001	0.001	0.003	1.301	0.865	0.104	0.175	0.014

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	15	15	10	15
normalized size	1	1.00	1.00	0.79	0.71	1.07	1.07	0.71	1.07
time (sec)	N/A	0.001	0.002	0.003	1.286	0.718	0.959	0.171	0.906

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	15	17	0	13
normalized size	1	1.00	1.00	0.79	0.71	1.07	1.21	0.00	0.93
time (sec)	N/A	0.001	0.002	0.001	1.306	0.767	0.679	0.000	0.920

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	15	15	0	15
normalized size	1	1.00	1.00	0.79	0.71	1.07	1.07	0.00	1.07
time (sec)	N/A	0.001	0.002	0.004	1.321	0.836	0.725	0.000	0.900

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	0	0	0	17
normalized size	1	1.00	1.00	0.95	0.89	0.00	0.00	0.00	0.89
time (sec)	N/A	0.005	0.004	0.003	1.244	0.000	0.000	0.000	0.995

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	0	0	0	17
normalized size	1	1.00	1.00	0.95	0.89	0.00	0.00	0.00	0.89
time (sec)	N/A	0.005	0.018	0.001	1.279	0.000	0.000	0.000	0.947

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	0	0	0	15
normalized size	1	1.00	1.00	0.94	0.88	0.00	0.00	0.00	0.88
time (sec)	N/A	0.004	0.002	0.003	1.305	0.000	0.000	0.000	1.481

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	22	12	12
normalized size	1	1.00	1.00	0.93	0.86	0.86	1.57	0.86	0.86
time (sec)	N/A	0.003	0.002	0.001	1.321	0.562	0.378	0.176	0.926

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	18	0	0	0	0	-1
normalized size	1	1.00	0.90	0.86	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.005	0.002	0.001	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	18	0	0	0	0	-1
normalized size	1	1.00	0.90	0.86	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.005	0.009	0.002	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	20	19	0	0	0	22
normalized size	1	1.00	0.92	0.83	0.79	0.00	0.00	0.00	0.92
time (sec)	N/A	0.006	0.006	0.002	1.309	0.000	0.000	0.000	0.982

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	20	18	17	0	0	0	22
normalized size	1	1.00	0.83	0.75	0.71	0.00	0.00	0.00	0.92
time (sec)	N/A	0.005	0.004	0.001	1.369	0.000	0.000	0.000	0.965

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	16	13	12	16	24	0	16
normalized size	1	1.00	0.80	0.65	0.60	0.80	1.20	0.00	0.80
time (sec)	N/A	0.003	0.002	0.001	1.324	0.856	6.694	0.000	0.931

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	20	0	0	0	0	22
normalized size	1	1.00	0.92	0.83	0.00	0.00	0.00	0.00	0.92
time (sec)	N/A	0.007	0.007	0.003	0.000	0.000	0.000	0.000	0.934

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	20	0	0	0	0	22
normalized size	1	1.00	0.92	0.83	0.00	0.00	0.00	0.00	0.92
time (sec)	N/A	0.007	0.006	0.000	0.000	0.000	0.000	0.000	0.962

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	22	18	0	0	0	0	22
normalized size	1	1.00	0.85	0.69	0.00	0.00	0.00	0.00	0.85
time (sec)	N/A	0.005	0.006	0.003	0.000	0.000	0.000	0.000	0.958

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	18	0	0	0	0	24
normalized size	1	1.00	0.90	0.86	0.00	0.00	0.00	0.00	1.14
time (sec)	N/A	0.005	0.005	0.003	0.000	0.000	0.000	0.000	1.037

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	18	0	0	0	0	24
normalized size	1	1.00	0.90	0.86	0.00	0.00	0.00	0.00	1.14
time (sec)	N/A	0.004	0.004	0.003	0.000	0.000	0.000	0.000	1.037

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	17	16	0	0	0	0	24
normalized size	1	1.00	0.89	0.84	0.00	0.00	0.00	0.00	1.26
time (sec)	N/A	0.004	0.003	0.002	0.000	0.000	0.000	0.000	0.968

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	20	24	0	20
normalized size	1	1.00	1.00	0.93	0.86	1.43	1.71	0.00	1.43
time (sec)	N/A	0.003	0.002	0.002	1.288	0.865	2.043	0.000	0.968

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	0	0	0	24
normalized size	1	1.00	1.00	0.95	0.89	0.00	0.00	0.00	1.26
time (sec)	N/A	0.004	0.021	0.001	1.332	0.000	0.000	0.000	0.974

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	0	0	0	24
normalized size	1	1.00	1.00	0.95	0.89	0.00	0.00	0.00	1.26
time (sec)	N/A	0.004	0.004	0.002	1.308	0.000	0.000	0.000	0.980

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	22	18	0	0	0	0	-1
normalized size	1	1.00	0.73	0.60	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.008	0.008	0.002	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	22	20	0	0	0	0	-1
normalized size	1	1.00	0.79	0.71	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.008	0.007	0.003	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	20	18	0	0	0	0	-1
normalized size	1	1.00	0.71	0.64	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.006	0.005	0.003	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	16	13	12	22	26	0	12
normalized size	1	1.00	0.67	0.54	0.50	0.92	1.08	0.00	0.50
time (sec)	N/A	0.004	0.003	0.002	1.333	0.888	3.306	0.000	0.930

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	22	20	19	0	0	0	-1
normalized size	1	1.00	0.79	0.71	0.68	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.007	0.007	0.003	1.336	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	22	20	19	0	0	0	-1
normalized size	1	1.00	0.79	0.71	0.68	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.007	0.007	0.002	1.325	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	22	18	17	0	0	0	-1
normalized size	1	1.00	0.79	0.64	0.61	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.006	0.008	0.001	1.352	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	25	25	24	0	0	0	-1
normalized size	1	1.00	0.78	0.78	0.75	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.010	0.010	0.003	1.491	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	26	26	27	0	0	0	34
normalized size	1	1.00	0.72	0.72	0.75	0.00	0.00	0.00	0.94
time (sec)	N/A	0.007	0.006	0.002	1.623	0.000	0.000	0.000	1.038

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	25	25	24	0	0	0	26
normalized size	1	1.00	0.89	0.89	0.86	0.00	0.00	0.00	0.93
time (sec)	N/A	0.008	0.019	0.003	1.394	0.000	0.000	0.000	1.062

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	25	23	22	0	0	0	22
normalized size	1	1.00	1.04	0.96	0.92	0.00	0.00	0.00	0.92
time (sec)	N/A	0.007	0.004	0.002	1.516	0.000	0.000	0.000	1.042

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	25	25	24	0	0	0	30
normalized size	1	1.00	0.96	0.96	0.92	0.00	0.00	0.00	1.15
time (sec)	N/A	0.008	0.006	0.001	1.572	0.000	0.000	0.000	1.092

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	25	25	24	0	0	0	-1
normalized size	1	1.00	0.78	0.78	0.75	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.006	0.006	0.002	1.519	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	36	26	26	27	0	0	0	34
normalized size	1	1.16	0.84	0.84	0.87	0.00	0.00	0.00	1.10
time (sec)	N/A	0.013	0.010	0.003	1.538	0.000	0.000	0.000	1.052

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	26	26	27	0	0	0	30
normalized size	1	1.00	0.81	0.81	0.84	0.00	0.00	0.00	0.94
time (sec)	N/A	0.007	0.006	0.003	1.507	0.000	0.000	0.000	0.998

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	25	26	24	25	0	0	0	23
normalized size	1	0.86	0.90	0.83	0.86	0.00	0.00	0.00	0.79
time (sec)	N/A	0.006	0.005	0.002	1.474	0.000	0.000	0.000	1.013

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	26	27	0	0	0	34
normalized size	1	1.00	0.96	0.96	1.00	0.00	0.00	0.00	1.26
time (sec)	N/A	0.006	0.005	0.001	1.504	0.000	0.000	0.000	1.032

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	26	26	27	0	0	0	34
normalized size	1	1.00	0.72	0.72	0.75	0.00	0.00	0.00	0.94
time (sec)	N/A	0.007	0.005	0.002	1.519	0.000	0.000	0.000	1.034

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	26	26	27	0	0	0	34
normalized size	1	1.00	0.72	0.72	0.75	0.00	0.00	0.00	0.94
time (sec)	N/A	0.013	0.019	0.003	1.567	0.000	0.000	0.000	1.053

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	19	23	0	110	0	0	-1
normalized size	1	1.00	0.95	1.15	0.00	5.50	0.00	0.00	-0.05
time (sec)	N/A	0.002	0.005	0.041	0.000	9.710	0.000	0.000	0.000

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	22	0	27	0	0	-1
normalized size	1	1.00	1.00	1.16	0.00	1.42	0.00	0.00	-0.05
time (sec)	N/A	0.003	0.004	0.032	0.000	0.817	0.000	0.000	0.000

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	0	30	0	14	-1
normalized size	1	1.00	1.00	1.05	0.00	1.58	0.00	0.74	-0.05
time (sec)	N/A	0.003	0.003	0.032	0.000	0.882	0.000	0.213	0.000

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	19	23	0	20	0	0	-1
normalized size	1	1.00	0.86	1.05	0.00	0.91	0.00	0.00	-0.05
time (sec)	N/A	0.002	0.003	0.028	0.000	0.539	0.000	0.000	0.000

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	25	24	0	24	21
normalized size	1	1.00	1.00	1.05	1.19	1.14	0.00	1.14	1.00
time (sec)	N/A	0.006	0.005	0.002	1.478	0.724	0.000	0.172	1.145

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	22	22	23	28	29	0	29	22
normalized size	1	0.85	0.85	0.88	1.08	1.12	0.00	1.12	0.85
time (sec)	N/A	0.005	0.004	0.002	1.481	0.699	0.000	0.209	1.008

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	19	22	0	22	18
normalized size	1	1.00	1.00	1.06	1.06	1.22	0.00	1.22	1.00
time (sec)	N/A	0.005	0.003	0.002	1.478	0.903	0.000	0.187	1.084

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	19	22	0	22	18
normalized size	1	1.00	1.00	1.06	1.06	1.22	0.00	1.22	1.00
time (sec)	N/A	0.005	0.009	0.002	1.422	0.857	0.000	0.167	1.060

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	17	20	0	20	16
normalized size	1	1.00	1.00	1.06	1.06	1.25	0.00	1.25	1.00
time (sec)	N/A	0.005	0.002	0.001	1.465	0.935	0.000	0.213	1.194

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	15	18	22	14	-1
normalized size	1	1.00	1.00	1.07	1.07	1.29	1.57	1.00	-0.07
time (sec)	N/A	0.004	0.002	0.002	1.422	0.648	0.316	0.152	0.000

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	18	19	19	22	0	0	-1
normalized size	1	1.00	0.90	0.95	0.95	1.10	0.00	0.00	-0.05
time (sec)	N/A	0.007	0.003	0.003	1.496	0.934	0.000	0.000	0.000

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	18	19	19	22	0	0	-1
normalized size	1	1.00	0.90	0.95	0.95	1.10	0.00	0.00	-0.05
time (sec)	N/A	0.006	0.003	0.001	1.464	0.871	0.000	0.000	0.000

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	27	14	0	14	20
normalized size	1	1.00	1.00	1.05	1.35	0.70	0.00	0.70	1.00
time (sec)	N/A	0.003	0.004	0.001	1.475	0.812	0.000	0.212	1.131

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	30	21	0	21	21
normalized size	1	1.00	1.00	1.05	1.43	1.00	0.00	1.00	1.00
time (sec)	N/A	0.005	0.002	0.002	1.520	0.680	0.000	0.222	0.990

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	21	12	58	12	16
normalized size	1	1.00	1.00	0.94	1.17	0.67	3.22	0.67	0.89
time (sec)	N/A	0.002	0.002	0.002	1.473	0.816	3.564	0.192	1.124

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	0	20	9	56	9	15
normalized size	1	1.00	1.00	0.00	1.33	0.60	3.73	0.60	1.00
time (sec)	N/A	0.001	0.001	0.020	1.464	0.786	2.015	0.173	1.062

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	29	0	10	0	10	-1
normalized size	1	1.00	1.00	1.93	0.00	0.67	0.00	0.67	-0.07
time (sec)	N/A	0.002	0.001	0.021	0.000	0.850	0.000	0.180	0.000

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	18	12	32	13	13
normalized size	1	1.00	1.00	1.08	1.38	0.92	2.46	1.00	1.00
time (sec)	N/A	0.002	0.001	0.003	1.505	0.757	1.309	0.157	1.015

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	0	12	60	0	-1
normalized size	1	1.00	1.00	0.94	0.00	0.67	3.33	0.00	-0.06
time (sec)	N/A	0.002	0.002	0.003	0.000	0.601	2.033	0.000	0.000

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	0	0	14	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.64	0.00	0.00	-0.05
time (sec)	N/A	0.003	0.008	0.046	0.000	0.988	0.000	0.000	0.000

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	0	0	6	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.38	0.00	0.00	-0.06
time (sec)	N/A	0.003	0.010	0.042	0.000	0.821	0.000	0.000	0.000

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	27	33	31	0	31	26
normalized size	1	1.00	1.00	1.04	1.27	1.19	0.00	1.19	1.00
time (sec)	N/A	0.045	0.008	0.003	2.636	0.700	0.000	0.233	1.229

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	24	27	29	0	29	23
normalized size	1	1.00	1.00	1.04	1.17	1.26	0.00	1.26	1.00
time (sec)	N/A	0.046	0.005	0.001	2.677	0.708	0.000	0.262	1.023

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	24	27	29	0	29	23
normalized size	1	1.00	1.00	1.04	1.17	1.26	0.00	1.26	1.00
time (sec)	N/A	0.029	0.005	0.001	2.696	0.601	0.000	0.237	0.989

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	25	27	0	27	21
normalized size	1	1.00	1.00	1.05	1.19	1.29	0.00	1.29	1.00
time (sec)	N/A	0.014	0.003	0.002	2.668	0.585	0.000	0.192	0.979

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	25	27	41	21	-1
normalized size	1	1.00	1.00	1.05	1.19	1.29	1.95	1.00	-0.05
time (sec)	N/A	0.045	0.002	0.003	2.730	0.657	0.722	0.163	0.000

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	24	27	29	0	0	-1
normalized size	1	1.00	0.92	0.96	1.08	1.16	0.00	0.00	-0.04
time (sec)	N/A	0.045	0.006	0.003	2.801	0.557	0.000	0.000	0.000

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	24	27	29	0	0	-1
normalized size	1	1.00	0.92	0.96	1.08	1.16	0.00	0.00	-0.04
time (sec)	N/A	0.045	0.004	0.002	2.815	0.712	0.000	0.000	0.000

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	25	38	21	248	21	24
normalized size	1	1.00	1.00	1.00	1.52	0.84	9.92	0.84	0.96
time (sec)	N/A	0.050	0.010	0.001	3.048	0.699	20.010	0.236	1.011

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	0	37	18	245	18	23
normalized size	1	1.00	1.00	0.00	1.68	0.82	11.14	0.82	1.05
time (sec)	N/A	0.029	0.002	0.109	3.131	0.671	10.352	0.234	0.983

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	0	0	19	0	19	-1
normalized size	1	1.00	1.00	0.00	0.00	0.86	0.00	0.86	-0.05
time (sec)	N/A	0.011	0.002	0.046	0.000	0.595	0.000	0.203	0.000

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	22	35	21	153	21	21
normalized size	1	1.00	1.00	1.10	1.75	1.05	7.65	1.05	1.05
time (sec)	N/A	0.044	0.002	0.002	3.111	0.727	5.883	0.169	0.965

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	25	0	21	252	0	-1
normalized size	1	1.00	1.00	1.00	0.00	0.84	10.08	0.00	-0.04
time (sec)	N/A	0.043	0.003	0.003	0.000	0.824	11.057	0.000	0.000

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	0	16	0	18	22
normalized size	1	1.00	1.00	0.96	0.00	0.67	0.00	0.75	0.92
time (sec)	N/A	0.043	0.010	0.001	0.000	0.679	0.000	0.277	1.021

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	0	16	0	16	22
normalized size	1	1.00	1.00	0.96	0.00	0.67	0.00	0.67	0.92
time (sec)	N/A	0.042	0.004	0.002	0.000	0.600	0.000	0.278	1.005

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	0	0	13	22	27	21
normalized size	1	1.00	1.00	0.00	0.00	0.62	1.05	1.29	1.00
time (sec)	N/A	0.043	0.008	0.207	0.000	0.738	2.633	0.426	0.997

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	0	0	14	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.67	0.00	0.00	-0.05
time (sec)	N/A	0.044	0.006	0.172	0.000	0.707	0.000	0.000	0.000

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	23	0	16	24	18	23
normalized size	1	1.00	1.00	1.05	0.00	0.73	1.09	0.82	1.05
time (sec)	N/A	0.042	0.006	0.003	0.000	0.762	93.319	0.280	1.006

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.005	0.001	0.001	1.313	0.471	0.072	0.149	0.021

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	14	13	12	13	13
normalized size	1	1.00	1.00	0.82	0.82	0.76	0.71	0.76	0.76
time (sec)	N/A	0.004	0.001	0.000	1.358	0.496	0.064	0.165	0.019

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.005	0.001	0.000	1.295	0.633	0.078	0.152	0.019

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	10
normalized size	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.002	0.000	0.000	1.322	0.500	0.089	0.152	0.018

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	14	11	10	12	11
normalized size	1	1.00	1.00	0.92	1.08	0.85	0.77	0.92	0.85
time (sec)	N/A	0.004	0.002	0.002	1.254	0.511	0.139	0.160	0.022

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	14	8	13	13
normalized size	1	1.00	1.00	0.93	0.87	0.93	0.53	0.87	0.87
time (sec)	N/A	0.005	0.001	0.003	1.277	0.715	0.092	0.150	0.024

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	15	8	10	10
normalized size	1	1.00	1.00	0.92	0.83	1.25	0.67	0.83	0.83
time (sec)	N/A	0.004	0.001	0.003	1.335	0.583	0.096	0.162	0.023

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	14	17	10	18	11
normalized size	1	1.00	1.00	0.92	1.08	1.31	0.77	1.38	0.85
time (sec)	N/A	0.004	0.002	0.005	1.308	0.714	0.139	0.149	0.954

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	14	13	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.93	0.87	0.87
time (sec)	N/A	0.004	0.002	0.005	1.236	0.718	0.156	0.177	0.023

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	15	15	15	15	15
normalized size	1	1.00	1.00	0.82	0.88	0.88	0.88	0.88	0.88
time (sec)	N/A	0.005	0.002	0.006	1.289	0.587	0.173	0.147	0.025

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	14	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.82	0.76	0.76
time (sec)	N/A	0.005	0.002	0.005	1.316	0.826	0.156	0.146	0.025

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	15	15	15	15	15
normalized size	1	1.00	1.00	0.82	0.88	0.88	0.88	0.88	0.88
time (sec)	N/A	0.005	0.002	0.005	1.279	0.935	0.159	0.179	0.026

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	24	24
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.010	0.001	0.000	1.367	0.456	0.074	0.149	0.036

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.009	0.001	0.002	1.341	0.391	0.079	0.181	0.030

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	30	25	14	24	24	14	24
normalized size	1	1.00	1.88	1.56	0.88	1.50	1.50	0.88	1.50
time (sec)	N/A	0.003	0.001	0.001	1.257	0.659	0.078	0.158	0.032

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.009	0.001	0.001	1.267	0.715	0.086	0.167	0.031

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	20	21	21
normalized size	1	1.00	1.00	0.88	0.84	0.84	0.80	0.84	0.84
time (sec)	N/A	0.007	0.001	0.001	1.355	0.390	0.085	0.150	0.028

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	25	22	24	23	22
normalized size	1	1.00	1.00	0.88	0.96	0.85	0.92	0.88	0.85
time (sec)	N/A	0.012	0.001	0.003	1.267	0.806	0.119	0.146	0.028

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	23	25	19	23	24
normalized size	1	1.00	1.00	0.96	0.92	1.00	0.76	0.92	0.96
time (sec)	N/A	0.009	0.001	0.004	1.284	0.836	0.115	0.149	0.031

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	22	25	22	22	22
normalized size	1	1.00	1.00	0.88	0.85	0.96	0.85	0.85	0.85
time (sec)	N/A	0.009	0.001	0.003	1.270	0.832	0.110	0.164	0.031

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	25	27	24	32	23
normalized size	1	1.00	1.00	0.89	0.93	1.00	0.89	1.19	0.85
time (sec)	N/A	0.013	0.001	0.006	1.321	0.839	0.215	0.186	0.029

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	25	26	26	25	27
normalized size	1	1.00	1.00	0.89	0.89	0.93	0.93	0.89	0.96
time (sec)	N/A	0.009	0.001	0.005	1.369	0.692	0.179	0.152	0.027

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	22	26	22	22	23
normalized size	1	1.00	1.00	0.96	0.96	1.13	0.96	0.96	1.00
time (sec)	N/A	0.009	0.001	0.007	1.344	0.828	0.179	0.159	0.026

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	26	28	24	32	25
normalized size	1	1.00	1.00	0.88	1.00	1.08	0.92	1.23	0.96
time (sec)	N/A	0.013	0.001	0.004	1.329	0.557	0.250	0.147	0.043

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	26	26	27	26	25
normalized size	1	1.00	1.00	0.89	0.93	0.93	0.96	0.93	0.89
time (sec)	N/A	0.010	0.001	0.005	1.314	0.742	0.223	0.234	0.033

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	26	26	27	26	26
normalized size	1	1.00	1.00	0.83	0.87	0.87	0.90	0.87	0.87
time (sec)	N/A	0.010	0.001	0.006	1.318	0.486	0.233	0.171	0.035

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	30	25	24	24	26	24	26
normalized size	1	1.00	1.58	1.32	1.26	1.26	1.37	1.26	1.37
time (sec)	N/A	0.003	0.001	0.005	1.398	0.800	0.268	0.149	0.035

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	26	26	27	26	26
normalized size	1	1.00	1.00	0.83	0.87	0.87	0.90	0.87	0.87
time (sec)	N/A	0.010	0.001	0.005	1.315	0.848	0.276	0.175	0.950

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	26	26	27	26	26
normalized size	1	1.00	1.00	0.83	0.87	0.87	0.90	0.87	0.87
time (sec)	N/A	0.010	0.001	0.005	1.314	0.879	0.274	0.149	0.036

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	26	26	27	26	26
normalized size	1	1.00	1.00	0.83	0.87	0.87	0.90	0.87	0.87
time (sec)	N/A	0.014	0.001	0.004	1.258	0.651	0.282	0.151	0.037

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	36	35	35
normalized size	1	1.00	1.00	0.84	0.81	0.81	0.84	0.81	0.81
time (sec)	N/A	0.027	0.002	0.000	1.296	0.716	0.073	0.150	0.040

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	36	35	35
normalized size	1	1.00	1.00	0.84	0.81	0.81	0.84	0.81	0.81
time (sec)	N/A	0.025	0.002	0.000	1.294	0.715	0.074	0.265	0.041

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	36	35	35
normalized size	1	1.00	1.00	0.84	0.81	0.81	0.84	0.81	0.81
time (sec)	N/A	0.025	0.002	0.001	1.307	0.411	0.080	0.155	0.040

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	43	36	35	35	36	35	35
normalized size	1	1.00	1.26	1.06	1.03	1.03	1.06	1.03	1.03
time (sec)	N/A	0.030	0.002	0.001	1.293	0.469	0.085	0.162	0.041

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	43	36	14	35	36	14	35
normalized size	1	1.00	2.69	2.25	0.88	2.19	2.25	0.88	2.19
time (sec)	N/A	0.003	0.002	0.000	1.272	0.782	0.077	0.149	0.039

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	33	35	32	32	33	32
normalized size	1	1.00	1.00	0.92	0.97	0.89	0.89	0.92	0.89
time (sec)	N/A	0.019	0.007	0.001	1.373	0.843	0.133	0.150	0.034

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	34	34	38	34	44	33
normalized size	1	1.00	1.00	0.92	0.92	1.03	0.92	1.19	0.89
time (sec)	N/A	0.020	0.005	0.006	1.358	0.540	0.171	0.165	0.033

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	35	36	39	37	45	36
normalized size	1	1.00	1.00	0.92	0.95	1.03	0.97	1.18	0.95
time (sec)	N/A	0.018	0.005	0.006	1.346	0.833	0.248	0.187	0.945

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	34	39	39	37	45	35
normalized size	1	1.00	1.00	0.92	1.05	1.05	1.00	1.22	0.95
time (sec)	N/A	0.018	0.004	0.007	1.332	0.817	0.324	0.155	0.947

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	43	36	35	35	37	35	37
normalized size	1	1.00	2.26	1.89	1.84	1.84	1.95	1.84	1.95
time (sec)	N/A	0.003	0.005	0.004	1.349	0.775	0.380	0.154	0.027

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	43	36	37	37	39	37	37
normalized size	1	1.00	1.08	0.90	0.92	0.92	0.98	0.92	0.92
time (sec)	N/A	0.017	0.004	0.006	1.306	0.785	0.505	0.168	0.030

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	37	37	39	37	37
normalized size	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86
time (sec)	N/A	0.018	0.004	0.005	1.347	0.549	0.571	0.151	0.032

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	37	37	39	37	37
normalized size	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86
time (sec)	N/A	0.019	0.004	0.007	1.346	0.820	0.487	0.151	0.031

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	39	35	35
normalized size	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.81
time (sec)	N/A	0.015	0.002	0.000	1.329	0.579	0.130	0.229	0.040

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	39	35	35
normalized size	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.81
time (sec)	N/A	0.014	0.002	0.002	1.293	0.711	0.083	0.178	0.040

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	39	35	35
normalized size	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.81
time (sec)	N/A	0.014	0.002	0.000	1.374	0.752	0.072	0.152	0.039

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	33	32	32	36	32	32
normalized size	1	1.00	1.00	0.87	0.84	0.84	0.95	0.84	0.84
time (sec)	N/A	0.011	0.001	0.001	1.352	0.742	0.076	0.159	0.038

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	36	35	37	36	35	35
normalized size	1	1.00	1.00	0.88	0.85	0.90	0.88	0.85	0.85
time (sec)	N/A	0.014	0.008	0.004	1.323	0.797	0.125	0.193	0.042

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	34	33	37	36	33	33
normalized size	1	1.00	1.00	0.87	0.85	0.95	0.92	0.85	0.85
time (sec)	N/A	0.013	0.006	0.003	1.320	0.874	0.148	0.154	0.043

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	36	36	37	39	36	38
normalized size	1	1.00	1.00	0.88	0.88	0.90	0.95	0.88	0.93
time (sec)	N/A	0.015	0.004	0.005	1.314	0.849	0.194	0.178	0.997

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	34	36	37	37	36	36
normalized size	1	1.00	1.00	0.87	0.92	0.95	0.95	0.92	0.92
time (sec)	N/A	0.013	0.006	0.005	1.317	0.794	0.202	0.153	0.038

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	36	38	37	39	38	38
normalized size	1	1.00	1.00	0.88	0.93	0.90	0.95	0.93	0.93
time (sec)	N/A	0.013	0.004	0.006	1.286	0.818	0.276	0.161	0.975

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	57	57	65	57	57
normalized size	1	1.00	1.00	0.84	0.83	0.83	0.94	0.83	0.83
time (sec)	N/A	0.046	0.003	0.002	1.351	0.774	0.095	0.208	0.025

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	57	57	66	57	57
normalized size	1	1.00	1.00	0.84	0.83	0.83	0.96	0.83	0.83
time (sec)	N/A	0.042	0.003	0.001	1.356	0.746	0.095	0.167	0.024

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	69	58	57	57	65	57	57
normalized size	1	1.00	0.96	0.81	0.79	0.79	0.90	0.79	0.79
time (sec)	N/A	0.091	0.002	0.001	1.326	0.710	0.083	0.154	0.024

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	69	58	57	57	66	57	57
normalized size	1	1.00	1.30	1.09	1.08	1.08	1.25	1.08	1.08
time (sec)	N/A	0.064	0.002	0.000	1.330	0.713	0.083	0.153	0.026

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	69	58	57	57	66	57	57
normalized size	1	1.00	2.03	1.71	1.68	1.68	1.94	1.68	1.68
time (sec)	N/A	0.036	0.002	0.001	1.333	0.698	0.093	0.153	0.024

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	69	58	14	57	65	14	57
normalized size	1	1.00	4.31	3.62	0.88	3.56	4.06	0.88	3.56
time (sec)	N/A	0.003	0.002	0.001	1.375	0.694	0.092	0.159	0.025

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	56	58	55	65	56	55
normalized size	1	1.00	1.00	0.86	0.89	0.85	1.00	0.86	0.85
time (sec)	N/A	0.033	0.004	0.003	1.280	0.821	0.163	0.152	0.028

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	57	58	61	65	67	56
normalized size	1	1.00	1.00	0.86	0.88	0.92	0.98	1.02	0.85
time (sec)	N/A	0.036	0.005	0.007	1.366	0.820	0.217	0.150	0.030

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	57	59	61	65	69	59
normalized size	1	1.00	1.00	0.86	0.89	0.92	0.98	1.05	0.89
time (sec)	N/A	0.034	0.005	0.007	1.286	0.785	0.332	0.152	0.029

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	57	61	61	65	71	59
normalized size	1	1.00	1.00	0.86	0.92	0.92	0.98	1.08	0.89
time (sec)	N/A	0.033	0.007	0.007	1.321	0.806	0.407	0.191	0.038

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	57	61	61	63	69	59
normalized size	1	1.00	1.00	0.86	0.92	0.92	0.95	1.05	0.89
time (sec)	N/A	0.031	0.005	0.008	1.305	0.874	0.566	0.155	0.044

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	56	61	61	61	67	58
normalized size	1	1.00	1.00	0.86	0.94	0.94	0.94	1.03	0.89
time (sec)	N/A	0.031	0.005	0.006	1.353	0.695	0.677	0.161	1.012

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	69	58	57	57	61	57	59
normalized size	1	1.00	3.63	3.05	3.00	3.00	3.21	3.00	3.11
time (sec)	N/A	0.003	0.004	0.006	1.304	1.062	0.761	0.256	0.040

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	69	58	59	59	63	59	59
normalized size	1	1.00	1.72	1.45	1.48	1.48	1.58	1.48	1.48
time (sec)	N/A	0.017	0.021	0.004	1.333	1.028	0.733	0.174	0.953

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	69	58	59	59	63	59	59
normalized size	1	1.00	1.11	0.94	0.95	0.95	1.02	0.95	0.95
time (sec)	N/A	0.028	0.007	0.004	1.320	0.842	0.880	0.151	0.950

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	59	59	63	59	59
normalized size	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86
time (sec)	N/A	0.031	0.005	0.005	1.331	0.843	0.787	0.151	0.044

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	59	59	63	59	59
normalized size	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86
time (sec)	N/A	0.030	0.008	0.005	1.345	0.841	0.996	0.171	0.044

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	57	57	66	57	57
normalized size	1	1.00	1.00	0.84	0.83	0.83	0.96	0.83	0.83
time (sec)	N/A	0.025	0.002	0.001	1.348	0.771	0.097	0.149	0.024

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	57	56	56	63	56	56
normalized size	1	1.00	1.00	0.86	0.85	0.85	0.95	0.85	0.85
time (sec)	N/A	0.022	0.002	0.001	1.324	0.705	0.091	0.182	0.024

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	57	56	56	63	56	56
normalized size	1	1.00	1.00	0.86	0.85	0.85	0.95	0.85	0.85
time (sec)	N/A	0.023	0.002	0.000	1.326	0.562	0.086	0.208	0.023

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	54	53	53	60	53	53
normalized size	1	1.00	1.00	0.89	0.87	0.87	0.98	0.87	0.87
time (sec)	N/A	0.019	0.001	0.000	1.286	0.725	0.106	0.147	0.022

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	58	57	59	61	57	57
normalized size	1	1.00	1.00	0.89	0.88	0.91	0.94	0.88	0.88
time (sec)	N/A	0.022	0.004	0.003	1.286	0.759	0.146	0.181	0.025

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	56	55	59	61	55	55
normalized size	1	1.00	1.00	0.86	0.85	0.91	0.94	0.85	0.85
time (sec)	N/A	0.022	0.004	0.004	1.320	0.495	0.162	0.366	0.026

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	58	58	59	63	58	60
normalized size	1	1.00	1.00	0.92	0.92	0.94	1.00	0.92	0.95
time (sec)	N/A	0.023	0.007	0.005	1.353	0.849	0.213	0.154	0.025

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	56	58	59	65	58	58
normalized size	1	1.00	1.00	0.86	0.89	0.91	1.00	0.89	0.89
time (sec)	N/A	0.021	0.004	0.005	1.304	0.806	0.256	0.175	0.024

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	57	59	59	61	59	59
normalized size	1	1.00	1.00	0.92	0.95	0.95	0.98	0.95	0.95
time (sec)	N/A	0.023	0.017	0.006	1.304	0.668	0.284	0.151	0.045

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	56	56	59	61	56	57
normalized size	1	1.00	1.00	0.92	0.92	0.97	1.00	0.92	0.93
time (sec)	N/A	0.022	0.004	0.005	1.347	0.619	0.294	0.152	0.050

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	108	91	90	90	105	90	90
normalized size	1	1.00	0.84	0.71	0.70	0.70	0.81	0.70	0.70
time (sec)	N/A	0.216	0.004	0.000	1.291	0.609	0.109	0.149	1.026

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	108	91	90	90	107	90	90
normalized size	1	1.00	0.98	0.83	0.82	0.82	0.97	0.82	0.82
time (sec)	N/A	0.173	0.003	0.001	1.340	0.739	0.114	0.152	0.089

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	108	91	90	90	107	90	90
normalized size	1	1.00	1.19	1.00	0.99	0.99	1.18	0.99	0.99
time (sec)	N/A	0.140	0.003	0.001	1.339	0.783	0.107	0.207	0.088

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	108	91	90	90	107	90	90
normalized size	1	1.00	1.50	1.26	1.25	1.25	1.49	1.25	1.25
time (sec)	N/A	0.110	0.024	0.003	1.347	0.677	0.159	0.151	1.002

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	108	91	90	90	107	90	90
normalized size	1	1.00	2.04	1.72	1.70	1.70	2.02	1.70	1.70
time (sec)	N/A	0.079	0.003	0.001	1.321	0.679	0.106	0.157	0.086

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	108	91	90	90	107	90	90
normalized size	1	1.00	3.18	2.68	2.65	2.65	3.15	2.65	2.65
time (sec)	N/A	0.043	0.003	0.000	1.327	0.720	0.109	0.149	0.087

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	108	91	14	90	105	14	14
normalized size	1	1.00	6.75	5.69	0.88	5.62	6.56	0.88	0.88
time (sec)	N/A	0.003	0.003	0.000	1.265	0.735	0.094	0.177	0.091

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	104	89	91	88	105	89	88
normalized size	1	1.00	1.00	0.86	0.88	0.85	1.01	0.86	0.85
time (sec)	N/A	0.055	0.005	0.003	1.282	0.813	0.202	0.163	0.055

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	105	90	91	94	105	100	89
normalized size	1	1.00	1.00	0.86	0.87	0.90	1.00	0.95	0.85
time (sec)	N/A	0.058	0.010	0.007	1.303	0.615	0.300	0.148	0.056

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	105	90	92	94	105	102	92
normalized size	1	1.00	1.00	0.86	0.88	0.90	1.00	0.97	0.88
time (sec)	N/A	0.058	0.005	0.008	1.351	0.559	0.373	0.187	0.053

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	105	90	92	94	104	102	92
normalized size	1	1.00	1.00	0.86	0.88	0.90	0.99	0.97	0.88
time (sec)	N/A	0.056	0.011	0.008	1.299	0.772	0.475	0.154	0.960

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	105	90	94	94	104	104	92
normalized size	1	1.00	1.00	0.86	0.90	0.90	0.99	0.99	0.88
time (sec)	N/A	0.056	0.005	0.007	1.305	0.853	0.542	0.154	0.965

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	105	90	94	94	102	104	92
normalized size	1	1.00	1.00	0.86	0.90	0.90	0.97	0.99	0.88
time (sec)	N/A	0.060	0.008	0.010	1.322	0.533	0.808	0.174	0.050

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	105	90	94	94	100	104	92
normalized size	1	1.00	1.00	0.86	0.90	0.90	0.95	0.99	0.88
time (sec)	N/A	0.055	0.006	0.009	1.359	0.657	0.825	0.160	0.976

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	105	90	94	94	99	102	94
normalized size	1	1.00	1.00	0.86	0.90	0.90	0.94	0.97	0.90
time (sec)	N/A	0.054	0.011	0.010	1.388	0.683	1.179	0.157	0.992

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	104	89	94	94	97	100	91
normalized size	1	1.00	1.00	0.86	0.90	0.90	0.93	0.96	0.88
time (sec)	N/A	0.051	0.005	0.009	1.326	0.539	1.205	0.155	0.072

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	108	91	90	90	97	90	92
normalized size	1	1.00	5.68	4.79	4.74	4.74	5.11	4.74	4.84
time (sec)	N/A	0.003	0.009	0.006	1.348	0.695	2.455	0.151	0.997

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	108	91	92	92	99	92	92
normalized size	1	1.00	2.70	2.28	2.30	2.30	2.48	2.30	2.30
time (sec)	N/A	0.018	0.005	0.007	1.291	0.625	1.365	0.154	0.982

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	108	91	92	92	99	92	92
normalized size	1	1.00	1.74	1.47	1.48	1.48	1.60	1.48	1.48
time (sec)	N/A	0.027	0.010	0.007	1.368	0.603	1.553	0.164	0.078

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	108	91	92	92	99	92	92
normalized size	1	1.00	1.29	1.08	1.10	1.10	1.18	1.10	1.10
time (sec)	N/A	0.039	0.005	0.006	1.368	0.786	1.492	0.173	0.077

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	108	91	92	92	99	92	92
normalized size	1	1.00	1.02	0.86	0.87	0.87	0.93	0.87	0.87
time (sec)	N/A	0.052	0.007	0.007	1.356	0.699	2.959	0.154	0.995

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	108	91	92	92	99	92	92
normalized size	1	1.00	1.00	0.84	0.85	0.85	0.92	0.85	0.85
time (sec)	N/A	0.051	0.005	0.006	1.363	0.606	3.016	0.157	0.082

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	108	91	92	92	99	92	92
normalized size	1	1.00	1.00	0.84	0.85	0.85	0.92	0.85	0.85
time (sec)	N/A	0.049	0.010	0.008	1.294	0.593	1.917	0.188	0.997

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	103	90	89	89	102	89	89
normalized size	1	1.00	1.00	0.87	0.86	0.86	0.99	0.86	0.86
time (sec)	N/A	0.045	0.003	0.001	1.300	0.458	0.105	0.207	1.013

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	108	91	90	90	107	90	90
normalized size	1	1.00	1.00	0.84	0.83	0.83	0.99	0.83	0.83
time (sec)	N/A	0.039	0.003	0.000	1.296	0.386	0.101	0.150	0.086

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	106	91	90	90	105	90	90
normalized size	1	1.00	1.00	0.86	0.85	0.85	0.99	0.85	0.85
time (sec)	N/A	0.039	0.003	0.001	1.326	0.479	0.093	0.170	0.983

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	99	88	87	87	100	87	87
normalized size	1	1.00	1.00	0.89	0.88	0.88	1.01	0.88	0.88
time (sec)	N/A	0.037	0.002	0.002	1.264	0.432	0.092	0.154	0.040

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	100	91	90	92	99	90	90
normalized size	1	1.00	1.00	0.91	0.90	0.92	0.99	0.90	0.90
time (sec)	N/A	0.040	0.010	0.004	1.330	0.557	0.207	0.157	0.053

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	98	89	88	92	99	88	88
normalized size	1	1.00	1.00	0.91	0.90	0.94	1.01	0.90	0.90
time (sec)	N/A	0.037	0.005	0.004	1.340	0.527	0.206	0.153	0.052

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	102	91	91	92	104	91	93
normalized size	1	1.00	1.00	0.89	0.89	0.90	1.02	0.89	0.91
time (sec)	N/A	0.038	0.005	0.006	1.332	0.614	0.295	0.164	0.048

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	98	89	89	92	99	89	91
normalized size	1	1.00	1.00	0.91	0.91	0.94	1.01	0.91	0.93
time (sec)	N/A	0.036	0.010	0.005	1.360	0.537	0.279	0.150	0.049

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	98	91	91	92	100	91	93
normalized size	1	1.00	1.00	0.93	0.93	0.94	1.02	0.93	0.95
time (sec)	N/A	0.037	0.007	0.006	1.284	0.585	0.335	0.153	0.952

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	100	89	91	92	102	91	91
normalized size	1	1.00	1.00	0.89	0.91	0.92	1.02	0.91	0.91
time (sec)	N/A	0.036	0.005	0.006	1.319	0.682	0.363	0.206	0.048

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	35	34	33	32	35	33
normalized size	1	1.00	1.00	0.88	0.85	0.82	0.80	0.88	0.82
time (sec)	N/A	0.026	0.006	0.003	1.320	0.724	0.375	0.180	0.052

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	23	22	20	24	22
normalized size	1	1.00	1.00	0.89	0.85	0.81	0.74	0.89	0.81
time (sec)	N/A	0.017	0.004	0.004	1.346	0.614	0.197	0.183	0.947

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	14	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87
time (sec)	N/A	0.004	0.003	0.001	1.322	0.644	0.167	0.155	0.031

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	23	18	15	22	18
normalized size	1	1.00	1.00	0.95	1.05	0.82	0.68	1.00	0.82
time (sec)	N/A	0.010	0.005	0.005	1.323	0.607	0.291	0.152	0.997

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	33	33	31	42	31
normalized size	1	1.00	1.00	0.91	0.94	0.94	0.89	1.20	0.89
time (sec)	N/A	0.021	0.007	0.006	1.293	0.584	0.695	0.157	0.076

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	111	102	109	123	32	114	120
normalized size	1	1.00	0.90	0.82	0.88	0.99	0.26	0.92	0.97
time (sec)	N/A	0.083	0.061	0.005	2.885	0.573	0.206	0.193	0.214

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	108	99	106	106	22	111	114
normalized size	1	1.00	0.91	0.83	0.89	0.89	0.18	0.93	0.96
time (sec)	N/A	0.068	0.016	0.003	2.917	0.646	0.233	0.180	1.161

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	89	91	98	304	24	112	111
normalized size	1	1.00	0.77	0.79	0.85	2.64	0.21	0.97	0.97
time (sec)	N/A	0.050	0.011	0.002	3.030	0.808	0.154	0.204	1.174

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	89	91	98	299	20	112	99
normalized size	1	1.00	0.77	0.79	0.85	2.60	0.17	0.97	0.86
time (sec)	N/A	0.051	0.012	0.002	2.928	0.518	0.167	0.163	1.139

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	114	99	106	103	29	121	102
normalized size	1	1.00	0.93	0.81	0.87	0.84	0.24	0.99	0.84
time (sec)	N/A	0.062	0.019	0.005	3.004	0.484	0.204	0.179	1.161

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	119	99	106	143	32	115	128
normalized size	1	1.00	0.96	0.80	0.85	1.15	0.26	0.93	1.03
time (sec)	N/A	0.065	0.021	0.005	3.050	0.590	0.319	0.166	0.216

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	38	41	43	56	42	49	45
normalized size	1	1.00	0.83	0.89	0.93	1.22	0.91	1.07	0.98
time (sec)	N/A	0.030	0.018	0.010	1.371	0.615	0.374	0.174	0.049

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	27	30	32	35	29	48	29
normalized size	1	1.00	0.82	0.91	0.97	1.06	0.88	1.45	0.88
time (sec)	N/A	0.025	0.009	0.008	1.339	0.585	0.325	0.162	0.043

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	15	15	14	14
normalized size	1	1.00	1.00	0.94	0.88	0.94	0.94	0.88	0.88
time (sec)	N/A	0.003	0.005	0.000	1.303	0.592	0.248	0.170	0.966

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	33	35	37	47	34	45	34
normalized size	1	1.00	0.87	0.92	0.97	1.24	0.89	1.18	0.89
time (sec)	N/A	0.026	0.016	0.010	1.322	0.775	0.432	0.155	1.005

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	41	47	53	73	54	51	53
normalized size	1	1.00	0.79	0.90	1.02	1.40	1.04	0.98	1.02
time (sec)	N/A	0.036	0.042	0.013	1.335	0.805	0.563	0.178	0.074

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	119	108	116	400	44	132	138
normalized size	1	1.00	0.88	0.79	0.85	2.94	0.32	0.97	1.01
time (sec)	N/A	0.065	0.080	0.007	2.989	0.592	0.283	0.163	0.225

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	118	106	114	391	39	130	108
normalized size	1	1.00	0.88	0.79	0.85	2.92	0.29	0.97	0.81
time (sec)	N/A	0.064	0.065	0.008	2.843	0.606	0.284	0.243	1.148

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	119	117	124	402	44	129	138
normalized size	1	1.00	0.88	0.86	0.91	2.96	0.32	0.95	1.01
time (sec)	N/A	0.063	0.062	0.005	3.003	0.548	0.287	0.204	1.197

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	118	115	122	389	39	127	128
normalized size	1	1.00	0.88	0.86	0.91	2.90	0.29	0.95	0.96
time (sec)	N/A	0.063	0.060	0.004	2.960	0.825	0.309	0.171	1.105

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	131	117	126	146	56	139	120
normalized size	1	1.00	0.90	0.80	0.86	1.00	0.38	0.95	0.82
time (sec)	N/A	0.077	0.089	0.010	2.976	0.580	0.380	0.172	0.241

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	129	115	128	187	58	131	146
normalized size	1	1.00	0.88	0.79	0.88	1.28	0.40	0.90	1.00
time (sec)	N/A	0.078	0.091	0.010	3.009	0.794	0.457	0.191	1.338

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	48	58	66	91	65	62	67
normalized size	1	1.00	0.79	0.95	1.08	1.49	1.07	1.02	1.10
time (sec)	N/A	0.048	0.058	0.012	1.329	0.645	0.525	0.166	1.022

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	39	47	55	69	53	42	53
normalized size	1	1.00	0.75	0.90	1.06	1.33	1.02	0.81	1.02
time (sec)	N/A	0.037	0.016	0.010	1.348	0.652	0.457	0.204	0.059

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	24	31	36	36	36	22	37
normalized size	1	1.00	1.26	1.63	1.89	1.89	1.89	1.16	1.95
time (sec)	N/A	0.003	0.007	0.008	1.335	0.506	0.386	0.169	0.033

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	26	27	14	28
normalized size	1	1.00	1.00	0.94	0.88	1.62	1.69	0.88	1.75
time (sec)	N/A	0.003	0.005	0.000	1.353	0.454	0.320	0.190	0.940

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	43	49	60	90	56	57	56
normalized size	1	1.00	0.80	0.91	1.11	1.67	1.04	1.06	1.04
time (sec)	N/A	0.036	0.032	0.013	1.348	0.670	0.540	0.159	0.072

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	59	61	76	119	76	80	73
normalized size	1	1.00	0.89	0.92	1.15	1.80	1.15	1.21	1.11
time (sec)	N/A	0.049	0.064	0.013	1.356	0.573	0.679	0.194	0.090

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	140	119	139	510	70	142	161
normalized size	1	1.00	0.90	0.77	0.90	3.29	0.45	0.92	1.04
time (sec)	N/A	0.079	0.074	0.011	3.058	0.636	0.513	0.177	1.149

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	136	117	137	503	68	140	127
normalized size	1	1.00	0.89	0.76	0.90	3.29	0.44	0.92	0.83
time (sec)	N/A	0.076	0.072	0.010	2.988	0.663	0.474	0.180	0.309

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	141	127	149	512	70	145	168
normalized size	1	1.00	0.89	0.80	0.94	3.24	0.44	0.92	1.06
time (sec)	N/A	0.077	0.106	0.012	2.972	0.665	0.468	0.175	1.166

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	138	125	146	503	65	142	145
normalized size	1	1.00	0.90	0.81	0.95	3.27	0.42	0.92	0.94
time (sec)	N/A	0.077	0.077	0.010	2.950	0.613	0.454	0.176	1.128

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	139	134	147	514	70	139	163
normalized size	1	1.00	0.90	0.86	0.95	3.32	0.45	0.90	1.05
time (sec)	N/A	0.076	0.074	0.003	2.981	0.792	0.399	0.172	0.231

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	135	130	145	499	63	137	142
normalized size	1	1.00	0.89	0.86	0.96	3.30	0.42	0.91	0.94
time (sec)	N/A	0.078	0.070	0.005	2.900	0.774	0.452	0.161	1.112

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	37	36	35	34	37	35
normalized size	1	1.00	1.00	0.90	0.88	0.85	0.83	0.90	0.85
time (sec)	N/A	0.029	0.008	0.004	1.337	0.589	0.230	0.155	0.056

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	26	25	23	22	26	23
normalized size	1	1.00	1.00	0.93	0.89	0.82	0.79	0.93	0.82
time (sec)	N/A	0.019	0.005	0.004	1.301	0.528	0.216	0.193	0.039

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	16	15	15	12	16	15
normalized size	1	1.00	1.00	1.00	0.94	0.94	0.75	1.00	0.94
time (sec)	N/A	0.003	0.003	0.001	1.344	0.645	0.162	0.157	0.036

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	23	25	20	15	24	21
normalized size	1	1.00	1.00	1.00	1.09	0.87	0.65	1.04	0.91
time (sec)	N/A	0.011	0.007	0.004	1.342	0.590	0.275	0.162	0.096

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	33	35	33	31	41	31
normalized size	1	1.00	1.00	0.94	1.00	0.94	0.89	1.17	0.89
time (sec)	N/A	0.022	0.009	0.006	1.321	0.848	0.425	0.209	0.074

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	111	103	108	130	34	106	134
normalized size	1	1.00	0.89	0.82	0.86	1.04	0.27	0.85	1.07
time (sec)	N/A	0.066	0.039	0.005	2.999	0.764	0.196	0.176	1.155

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	108	101	106	103	24	104	116
normalized size	1	1.00	0.90	0.84	0.88	0.86	0.20	0.87	0.97
time (sec)	N/A	0.060	0.017	0.003	3.022	0.704	0.242	0.167	1.117

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	89	92	97	307	26	104	109
normalized size	1	1.00	0.77	0.80	0.84	2.67	0.23	0.90	0.95
time (sec)	N/A	0.050	0.011	0.002	2.886	0.829	0.173	0.189	1.113

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	89	92	97	320	22	104	115
normalized size	1	1.00	0.78	0.81	0.85	2.81	0.19	0.91	1.01
time (sec)	N/A	0.047	0.011	0.002	2.971	0.951	0.255	0.158	1.229

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	114	100	105	111	31	113	127
normalized size	1	1.00	0.93	0.81	0.85	0.90	0.25	0.92	1.03
time (sec)	N/A	0.061	0.044	0.006	2.848	0.775	0.252	0.191	1.154

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	119	100	105	141	34	107	127
normalized size	1	1.00	0.96	0.81	0.85	1.14	0.27	0.86	1.02
time (sec)	N/A	0.063	0.032	0.005	2.952	0.916	0.589	0.165	1.105

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	101	105	114	446	32	143	137
normalized size	1	1.00	0.81	0.84	0.91	3.57	0.26	1.14	1.10
time (sec)	N/A	0.086	0.054	0.004	2.921	0.807	0.319	0.160	1.210

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	124	106	113	467	34	131	165
normalized size	1	1.00	1.00	0.85	0.91	3.77	0.27	1.06	1.33
time (sec)	N/A	0.058	0.087	0.005	3.003	0.700	0.418	0.159	1.323

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	101	105	0	446	32	142	137
normalized size	1	1.00	0.73	0.76	0.00	3.21	0.23	1.02	0.99
time (sec)	N/A	0.098	0.036	0.004	0.000	0.940	0.368	0.161	1.226

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	124	106	0	467	34	138	165
normalized size	1	1.00	0.90	0.77	0.00	3.38	0.25	1.00	1.20
time (sec)	N/A	0.068	0.088	0.005	0.000	0.923	0.284	0.205	1.336

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	6	6	42	6	6
normalized size	1	1.00	1.00	0.70	0.60	0.60	4.20	0.60	0.60
time (sec)	N/A	0.006	0.003	0.006	2.987	0.581	1.223	0.151	0.149

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	64	57	114	57	90
normalized size	1	1.00	0.62	0.59	0.80	0.71	1.42	0.71	1.12
time (sec)	N/A	0.045	0.052	0.008	1.275	0.757	4.985	0.157	1.120

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	47	46	90	43	70
normalized size	1	1.00	0.66	0.61	0.80	0.78	1.53	0.73	1.19
time (sec)	N/A	0.036	0.019	0.006	1.287	0.752	2.390	0.188	1.091

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	25	30	34	66	29	29
normalized size	1	1.00	0.74	0.66	0.79	0.89	1.74	0.76	0.76
time (sec)	N/A	0.022	0.014	0.005	1.292	0.801	0.827	0.151	1.088

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	42	14	14
normalized size	1	1.00	1.00	0.83	0.78	0.78	2.33	0.78	0.78
time (sec)	N/A	0.004	0.021	0.006	1.278	1.000	0.222	0.148	1.088

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	32	49	84	76	36	52
normalized size	1	1.00	1.00	0.74	1.14	1.95	1.77	0.84	1.21
time (sec)	N/A	0.026	0.010	0.276	2.946	0.905	2.075	0.161	1.209

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	59	36	53	108	49	46	56
normalized size	1	1.00	1.26	0.77	1.13	2.30	1.04	0.98	1.19
time (sec)	N/A	0.027	0.048	0.023	2.941	0.952	2.842	0.159	1.257

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	39	56	100	133	100	72	76
normalized size	1	1.00	0.55	0.79	1.41	1.87	1.41	1.01	1.07
time (sec)	N/A	0.037	0.009	0.026	2.898	0.935	6.010	0.157	1.345

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	39	76	137	159	129	92	96
normalized size	1	1.00	0.41	0.80	1.44	1.67	1.36	0.97	1.01
time (sec)	N/A	0.053	0.031	0.026	2.944	0.893	7.145	0.180	1.498

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	94	337	0	0	39	0	-1
normalized size	1	1.00	0.34	1.23	0.00	0.00	0.14	0.00	-0.00
time (sec)	N/A	0.124	0.074	0.020	0.000	0.887	1.074	0.000	0.000

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	62	317	0	0	39	0	-1
normalized size	1	1.00	0.25	1.26	0.00	0.00	0.16	0.00	-0.00
time (sec)	N/A	0.070	0.061	0.022	0.000	0.632	0.938	0.000	0.000

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	46	297	0	0	37	0	37
normalized size	1	1.00	0.20	1.31	0.00	0.00	0.16	0.00	0.16
time (sec)	N/A	0.040	0.005	0.018	0.000	0.801	0.871	0.000	1.015

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	51	295	0	0	42	0	-1
normalized size	1	1.00	0.22	1.29	0.00	0.00	0.18	0.00	-0.00
time (sec)	N/A	0.042	0.009	0.022	0.000	0.925	1.540	0.000	0.000

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	51	317	0	0	46	0	-1
normalized size	1	1.00	0.20	1.25	0.00	0.00	0.18	0.00	-0.00
time (sec)	N/A	0.069	0.009	0.023	0.000	0.867	1.186	0.000	0.000

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	51	339	0	0	46	0	-1
normalized size	1	1.00	0.18	1.22	0.00	0.00	0.17	0.00	-0.00
time (sec)	N/A	0.098	0.009	0.027	0.000	0.883	1.390	0.000	0.000

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	535	535	96	491	0	0	39	0	-1
normalized size	1	1.00	0.18	0.92	0.00	0.00	0.07	0.00	-0.00
time (sec)	N/A	0.298	0.079	0.020	0.000	0.674	2.375	0.000	0.000

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	511	511	64	471	0	0	39	0	-1
normalized size	1	1.00	0.13	0.92	0.00	0.00	0.08	0.00	-0.00
time (sec)	N/A	0.201	0.077	0.022	0.000	0.846	1.237	0.000	0.000

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	487	487	51	451	0	0	39	0	-1
normalized size	1	1.00	0.10	0.93	0.00	0.00	0.08	0.00	-0.00
time (sec)	N/A	0.147	0.008	0.019	0.000	0.804	1.236	0.000	0.000

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	479	479	49	447	0	0	41	0	40
normalized size	1	1.00	0.10	0.93	0.00	0.00	0.09	0.00	0.08
time (sec)	N/A	0.144	0.008	0.023	0.000	0.751	1.700	0.000	1.268

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	511	511	51	469	0	0	46	0	-1
normalized size	1	1.00	0.10	0.92	0.00	0.00	0.09	0.00	-0.00
time (sec)	N/A	0.201	0.009	0.024	0.000	0.728	3.424	0.000	0.000

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	64	68	136	57	108
normalized size	1	1.00	0.62	0.59	0.80	0.85	1.70	0.71	1.35
time (sec)	N/A	0.047	0.030	0.008	1.260	0.780	9.733	0.160	1.117

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	47	57	112	43	88
normalized size	1	1.00	0.66	0.61	0.80	0.97	1.90	0.73	1.49
time (sec)	N/A	0.035	0.020	0.006	1.378	0.866	4.807	0.158	1.073

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	25	30	45	88	29	68
normalized size	1	1.00	0.74	0.66	0.79	1.18	2.32	0.76	1.79
time (sec)	N/A	0.024	0.016	0.005	1.343	0.816	4.021	0.173	1.102

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	32	65	14	14
normalized size	1	1.00	1.00	0.83	0.78	1.78	3.61	0.78	0.78
time (sec)	N/A	0.005	0.006	0.006	1.355	0.869	1.971	0.145	1.094

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	51	48	61	103	83	50	68
normalized size	1	1.00	0.86	0.81	1.03	1.75	1.41	0.85	1.15
time (sec)	N/A	0.034	0.032	0.021	2.887	0.956	6.240	0.171	1.182

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	37	49	66	121	100	63	69
normalized size	1	1.00	0.64	0.84	1.14	2.09	1.72	1.09	1.19
time (sec)	N/A	0.036	0.009	0.026	2.989	0.819	5.249	0.183	1.306

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	76	54	98	138	78	70	74
normalized size	1	1.00	1.12	0.79	1.44	2.03	1.15	1.03	1.09
time (sec)	N/A	0.039	0.041	0.027	2.904	0.771	7.535	0.160	1.435

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	79	355	0	0	39	0	-1
normalized size	1	1.00	0.27	1.20	0.00	0.00	0.13	0.00	-0.00
time (sec)	N/A	0.128	0.076	0.022	0.000	0.657	1.842	0.000	0.000

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	67	335	0	0	39	0	-1
normalized size	1	1.00	0.25	1.23	0.00	0.00	0.14	0.00	-0.00
time (sec)	N/A	0.094	0.057	0.022	0.000	0.824	2.565	0.000	0.000

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	47	315	0	0	37	0	37
normalized size	1	1.00	0.19	1.28	0.00	0.00	0.15	0.00	0.15
time (sec)	N/A	0.063	0.005	0.020	0.000	0.819	1.328	0.000	1.018

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	52	310	0	0	42	0	-1
normalized size	1	1.00	0.21	1.26	0.00	0.00	0.17	0.00	-0.00
time (sec)	N/A	0.063	0.017	0.022	0.000	0.565	1.283	0.000	0.000

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	52	312	0	0	46	0	-1
normalized size	1	1.00	0.21	1.26	0.00	0.00	0.19	0.00	-0.00
time (sec)	N/A	0.065	0.010	0.026	0.000	0.645	1.567	0.000	0.000

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	556	556	81	509	0	0	39	0	-1
normalized size	1	1.00	0.15	0.92	0.00	0.00	0.07	0.00	-0.00
time (sec)	N/A	0.304	0.113	0.022	0.000	0.794	3.754	0.000	0.000

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	532	532	69	489	0	0	39	0	-1
normalized size	1	1.00	0.13	0.92	0.00	0.00	0.07	0.00	-0.00
time (sec)	N/A	0.244	0.078	0.022	0.000	1.084	1.943	0.000	0.000

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	508	508	52	469	0	0	39	0	-1
normalized size	1	1.00	0.10	0.92	0.00	0.00	0.08	0.00	-0.00
time (sec)	N/A	0.188	0.008	0.019	0.000	0.904	2.348	0.000	0.000

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	504	504	50	464	0	0	41	0	40
normalized size	1	1.00	0.10	0.92	0.00	0.00	0.08	0.00	0.08
time (sec)	N/A	0.192	0.009	0.023	0.000	0.868	1.369	0.000	1.575

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	505	505	52	464	0	0	46	0	-1
normalized size	1	1.00	0.10	0.92	0.00	0.00	0.09	0.00	-0.00
time (sec)	N/A	0.187	0.010	0.026	0.000	0.918	2.641	0.000	0.000

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	64	46	94	61	73
normalized size	1	1.00	0.62	0.59	0.80	0.58	1.18	0.76	0.91
time (sec)	N/A	0.044	0.029	0.007	1.343	0.858	3.617	0.188	1.129

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	47	35	70	47	35
normalized size	1	1.00	0.66	0.61	0.80	0.59	1.19	0.80	0.59
time (sec)	N/A	0.034	0.024	0.007	1.363	0.834	2.848	0.157	1.106

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	25	30	23	46	30	24
normalized size	1	1.00	0.71	0.66	0.79	0.61	1.21	0.79	0.63
time (sec)	N/A	0.023	0.014	0.004	1.324	0.815	1.728	0.147	1.090

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	24	14	14
normalized size	1	1.00	1.00	0.83	0.78	0.78	1.33	0.78	0.78
time (sec)	N/A	0.005	0.004	0.004	1.295	0.806	0.844	0.157	1.096

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	20	37	63	26	23	40
normalized size	1	1.00	1.00	0.74	1.37	2.33	0.96	0.85	1.48
time (sec)	N/A	0.017	0.005	0.021	2.884	0.930	3.239	0.152	1.228

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	64	39	68	107	49	51	59
normalized size	1	1.00	1.28	0.78	1.36	2.14	0.98	1.02	1.18
time (sec)	N/A	0.026	0.051	0.024	2.880	0.852	5.729	0.195	1.281

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	39	59	104	137	104	75	79
normalized size	1	1.00	0.53	0.80	1.41	1.85	1.41	1.01	1.07
time (sec)	N/A	0.038	0.008	0.025	3.014	0.847	10.875	0.166	1.319

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	79	320	0	0	37	0	-1
normalized size	1	1.00	0.31	1.26	0.00	0.00	0.15	0.00	-0.00
time (sec)	N/A	0.067	0.039	0.021	0.000	0.862	2.006	0.000	0.000

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	62	300	0	0	37	0	-1
normalized size	1	1.00	0.27	1.30	0.00	0.00	0.16	0.00	-0.00
time (sec)	N/A	0.041	0.019	0.023	0.000	0.818	1.451	0.000	0.000

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	46	283	0	0	36	0	37
normalized size	1	1.00	0.22	1.37	0.00	0.00	0.17	0.00	0.18
time (sec)	N/A	0.018	0.008	0.017	0.000	0.911	1.644	0.000	1.021

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	51	301	0	0	41	0	-1
normalized size	1	1.00	0.22	1.29	0.00	0.00	0.18	0.00	-0.00
time (sec)	N/A	0.042	0.009	0.022	0.000	0.861	1.590	0.000	0.000

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	51	320	0	0	44	0	-1
normalized size	1	1.00	0.20	1.25	0.00	0.00	0.17	0.00	-0.00
time (sec)	N/A	0.067	0.009	0.024	0.000	0.828	2.592	0.000	0.000

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	514	514	80	474	0	0	37	0	-1
normalized size	1	1.00	0.16	0.92	0.00	0.00	0.07	0.00	-0.00
time (sec)	N/A	0.196	0.031	0.022	0.000	0.936	2.309	0.000	0.000

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	490	490	64	454	0	0	37	0	-1
normalized size	1	1.00	0.13	0.93	0.00	0.00	0.08	0.00	-0.00
time (sec)	N/A	0.146	0.023	0.021	0.000	0.891	0.964	0.000	0.000

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	462	462	51	435	0	0	37	0	-1
normalized size	1	1.00	0.11	0.94	0.00	0.00	0.08	0.00	-0.00
time (sec)	N/A	0.100	0.008	0.020	0.000	0.852	0.970	0.000	0.000

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	484	484	49	453	0	0	39	0	40
normalized size	1	1.00	0.10	0.94	0.00	0.00	0.08	0.00	0.08
time (sec)	N/A	0.142	0.010	0.020	0.000	0.656	1.776	0.000	1.283

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	514	514	51	472	0	0	44	0	-1
normalized size	1	1.00	0.10	0.92	0.00	0.00	0.09	0.00	-0.00
time (sec)	N/A	0.191	0.009	0.023	0.000	0.893	1.430	0.000	0.000

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	49	46	64	57	94	69	70
normalized size	1	1.00	0.63	0.59	0.82	0.73	1.21	0.88	0.90
time (sec)	N/A	0.044	0.034	0.008	1.331	0.904	4.620	0.154	1.336

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	38	36	47	46	70	52	41
normalized size	1	1.00	0.64	0.61	0.80	0.78	1.19	0.88	0.69
time (sec)	N/A	0.034	0.039	0.006	1.316	0.849	4.924	0.156	1.183

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	24	30	35	46	33	24
normalized size	1	1.00	0.71	0.63	0.79	0.92	1.21	0.87	0.63
time (sec)	N/A	0.023	0.014	0.005	1.325	0.666	1.157	0.157	1.138

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	24	26	14	14
normalized size	1	1.00	1.00	0.83	0.78	1.33	1.44	0.78	0.78
time (sec)	N/A	0.005	0.004	0.005	1.358	0.818	0.671	0.156	1.095

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	36	39	52	129	184	41	55
normalized size	1	1.00	0.78	0.85	1.13	2.80	4.00	0.89	1.20
time (sec)	N/A	0.026	0.007	0.029	2.925	0.866	2.530	0.156	1.230

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	66	37	57	86	173	75	72	74
normalized size	1	1.03	0.58	0.89	1.34	2.70	1.17	1.12	1.16
time (sec)	N/A	0.038	0.008	0.027	2.977	0.791	4.176	0.177	1.337

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	39	80	122	203	112	88	96
normalized size	1	1.00	0.41	0.84	1.28	2.14	1.18	0.93	1.01
time (sec)	N/A	0.051	0.026	0.029	2.989	0.776	7.236	0.161	1.465

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	65	320	0	0	37	0	-1
normalized size	1	1.00	0.26	1.27	0.00	0.00	0.15	0.00	-0.00
time (sec)	N/A	0.067	0.050	0.023	0.000	0.700	2.736	0.000	0.000

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	55	303	0	0	37	0	-1
normalized size	1	1.00	0.24	1.32	0.00	0.00	0.16	0.00	-0.00
time (sec)	N/A	0.043	0.018	0.023	0.000	0.711	2.001	0.000	0.000

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	55	306	0	0	36	0	37
normalized size	1	1.00	0.24	1.32	0.00	0.00	0.16	0.00	0.16
time (sec)	N/A	0.040	0.013	0.020	0.000	0.883	1.921	0.000	1.130

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	54	321	0	0	41	0	-1
normalized size	1	1.00	0.21	1.26	0.00	0.00	0.16	0.00	-0.00
time (sec)	N/A	0.067	0.009	0.026	0.000	0.657	1.293	0.000	0.000

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	54	342	0	0	44	0	-1
normalized size	1	1.00	0.19	1.23	0.00	0.00	0.16	0.00	-0.00
time (sec)	N/A	0.094	0.010	0.028	0.000	0.823	2.988	0.000	0.000

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	511	511	66	476	0	0	37	0	-1
normalized size	1	1.00	0.13	0.93	0.00	0.00	0.07	0.00	-0.00
time (sec)	N/A	0.195	0.032	0.026	0.000	0.915	2.427	0.000	0.000

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	487	487	55	457	0	0	37	0	-1
normalized size	1	1.00	0.11	0.94	0.00	0.00	0.08	0.00	-0.00
time (sec)	N/A	0.149	0.020	0.025	0.000	0.667	2.098	0.000	0.000

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	489	489	54	460	0	0	37	0	-1
normalized size	1	1.00	0.11	0.94	0.00	0.00	0.08	0.00	-0.00
time (sec)	N/A	0.148	0.009	0.021	0.000	0.566	1.931	0.000	0.000

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	513	513	52	475	0	0	39	0	40
normalized size	1	1.00	0.10	0.93	0.00	0.00	0.08	0.00	0.08
time (sec)	N/A	0.192	0.010	0.025	0.000	0.871	1.990	0.000	1.370

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	535	535	54	496	0	0	44	0	-1
normalized size	1	1.00	0.10	0.93	0.00	0.00	0.08	0.00	-0.00
time (sec)	N/A	0.247	0.011	0.028	0.000	0.887	2.300	0.000	0.000

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	30	38	37	26	56	37	46
normalized size	1	1.00	0.57	0.72	0.70	0.49	1.06	0.70	0.87
time (sec)	N/A	0.018	0.010	0.005	1.357	0.719	2.915	0.156	1.004

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	25	33	28	21	41	28	21
normalized size	1	1.00	0.62	0.82	0.70	0.52	1.02	0.70	0.52
time (sec)	N/A	0.015	0.007	0.005	1.316	0.828	1.467	0.156	0.035

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	18	26	19	14	26	19	14
normalized size	1	1.00	0.67	0.96	0.70	0.52	0.96	0.70	0.52
time (sec)	N/A	0.010	0.005	0.005	1.316	0.671	0.393	0.174	0.024

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	21	9	9	10	9	9
normalized size	1	1.00	1.00	1.62	0.69	0.69	0.77	0.69	0.69
time (sec)	N/A	0.003	0.002	0.004	1.401	0.871	0.350	0.146	0.023

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	25	25	12	26	164
normalized size	1	1.00	1.00	0.79	1.79	1.79	0.86	1.86	11.71
time (sec)	N/A	0.007	0.002	0.161	1.320	0.614	1.267	0.150	1.144

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	24	37	44	26	38	176
normalized size	1	1.00	1.00	0.77	1.19	1.42	0.84	1.23	5.68
time (sec)	N/A	0.011	0.008	0.023	1.299	0.737	2.487	0.226	0.989

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	26	36	64	52	65	50	189
normalized size	1	1.00	0.55	0.77	1.36	1.11	1.38	1.06	4.02
time (sec)	N/A	0.016	0.005	0.022	1.357	0.798	6.951	0.154	0.061

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	26	48	80	57	85	59	201
normalized size	1	1.00	0.41	0.76	1.27	0.90	1.35	0.94	3.19
time (sec)	N/A	0.020	0.005	0.024	1.326	0.914	5.976	0.173	0.039

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	40	139	0	0	29	0	179
normalized size	1	1.00	0.29	1.02	0.00	0.00	0.21	0.00	1.32
time (sec)	N/A	0.034	0.012	0.018	0.000	0.939	2.111	0.000	0.984

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	32	127	0	0	29	0	167
normalized size	1	1.00	0.27	1.06	0.00	0.00	0.24	0.00	1.39
time (sec)	N/A	0.017	0.006	0.019	0.000	0.797	1.645	0.000	0.045

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	17	116	0	0	27	0	155
normalized size	1	1.00	0.17	1.13	0.00	0.00	0.26	0.00	1.50
time (sec)	N/A	0.007	0.002	0.020	0.000	0.870	1.502	0.000	0.036

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	22	129	0	0	32	0	169
normalized size	1	1.00	0.18	1.06	0.00	0.00	0.26	0.00	1.39
time (sec)	N/A	0.017	0.003	0.021	0.000	0.767	1.705	0.000	0.994

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	22	141	0	0	36	0	181
normalized size	1	1.00	0.16	1.02	0.00	0.00	0.26	0.00	1.31
time (sec)	N/A	0.027	0.003	0.022	0.000	0.804	1.971	0.000	1.000

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	42	198	0	0	29	0	239
normalized size	1	1.00	0.16	0.76	0.00	0.00	0.11	0.00	0.91
time (sec)	N/A	0.070	0.013	0.018	0.000	0.814	1.438	0.000	0.992

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	34	186	0	0	29	0	227
normalized size	1	1.00	0.14	0.76	0.00	0.00	0.12	0.00	0.92
time (sec)	N/A	0.049	0.005	0.018	0.000	0.624	0.796	0.000	1.003

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	22	173	0	0	29	0	214
normalized size	1	1.00	0.10	0.77	0.00	0.00	0.13	0.00	0.96
time (sec)	N/A	0.033	0.002	0.018	0.000	0.652	1.511	0.000	0.046

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	20	185	0	0	31	0	227
normalized size	1	1.00	0.08	0.78	0.00	0.00	0.13	0.00	0.95
time (sec)	N/A	0.053	0.003	0.020	0.000	0.884	1.020	0.000	0.071

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	22	198	0	0	36	0	239
normalized size	1	1.00	0.08	0.76	0.00	0.00	0.14	0.00	0.91
time (sec)	N/A	0.066	0.014	0.022	0.000	0.733	1.422	0.000	1.008

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	32	38	45	28	58	59	54
normalized size	1	1.00	0.52	0.62	0.74	0.46	0.95	0.97	0.89
time (sec)	N/A	0.024	0.011	0.007	1.330	0.882	1.911	0.183	0.037

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	27	33	34	23	42	41	23
normalized size	1	1.00	0.59	0.72	0.74	0.50	0.91	0.89	0.50
time (sec)	N/A	0.019	0.009	0.007	1.323	0.539	0.784	0.154	0.028

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	20	26	23	16	27	23	16
normalized size	1	1.00	0.65	0.84	0.74	0.52	0.87	0.74	0.52
time (sec)	N/A	0.013	0.006	0.005	1.295	0.837	0.686	0.209	0.028

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	21	11	11	12	11	11
normalized size	1	1.00	1.00	1.40	0.73	0.73	0.80	0.73	0.73
time (sec)	N/A	0.003	0.005	0.006	1.339	0.862	0.337	0.152	0.022

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	29	29	31	30	180
normalized size	1	1.00	1.00	0.81	1.81	1.81	1.94	1.88	11.25
time (sec)	N/A	0.008	0.003	0.055	1.298	0.863	1.951	0.153	0.186

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	43	50	82	44	195
normalized size	1	1.00	1.00	0.80	1.23	1.43	2.34	1.26	5.57
time (sec)	N/A	0.014	0.011	0.027	1.344	0.877	3.793	0.160	0.066

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	30	42	70	58	138	58	209
normalized size	1	1.00	0.57	0.79	1.32	1.09	2.60	1.09	3.94
time (sec)	N/A	0.020	0.005	0.028	1.339	0.545	5.109	0.154	0.042

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	30	56	90	63	182	76	223
normalized size	1	1.00	0.42	0.79	1.27	0.89	2.56	1.07	3.14
time (sec)	N/A	0.028	0.006	0.029	1.400	0.824	6.895	0.171	0.030

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	40	134	0	0	31	0	199
normalized size	1	1.00	0.26	0.88	0.00	0.00	0.20	0.00	1.31
time (sec)	N/A	0.038	0.033	0.024	0.000	0.824	1.687	0.000	0.039

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	32	120	0	0	31	0	185
normalized size	1	1.00	0.24	0.90	0.00	0.00	0.23	0.00	1.38
time (sec)	N/A	0.021	0.006	0.023	0.000	0.891	2.080	0.000	0.993

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	15	107	0	0	29	0	172
normalized size	1	1.00	0.13	0.93	0.00	0.00	0.25	0.00	1.50
time (sec)	N/A	0.009	0.002	0.021	0.000	0.806	1.243	0.000	0.032

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	20	122	0	0	34	0	187
normalized size	1	1.00	0.15	0.90	0.00	0.00	0.25	0.00	1.38
time (sec)	N/A	0.021	0.003	0.023	0.000	0.582	1.563	0.000	0.055

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	20	136	0	0	37	0	201
normalized size	1	1.00	0.13	0.88	0.00	0.00	0.24	0.00	1.31
time (sec)	N/A	0.033	0.003	0.027	0.000	0.732	1.641	0.000	0.041

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	42	187	0	0	31	0	260
normalized size	1	1.00	0.14	0.64	0.00	0.00	0.11	0.00	0.88
time (sec)	N/A	0.085	0.018	0.024	0.000	0.826	1.798	0.000	0.046

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	34	173	0	0	31	0	246
normalized size	1	1.00	0.12	0.63	0.00	0.00	0.11	0.00	0.89
time (sec)	N/A	0.061	0.006	0.023	0.000	0.901	1.017	0.000	0.059

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	20	158	0	0	31	0	231
normalized size	1	1.00	0.08	0.63	0.00	0.00	0.12	0.00	0.92
time (sec)	N/A	0.041	0.003	0.022	0.000	0.885	0.828	0.000	0.039

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	18	173	0	0	32	0	245
normalized size	1	1.00	0.07	0.64	0.00	0.00	0.12	0.00	0.91
time (sec)	N/A	0.062	0.003	0.023	0.000	0.892	1.357	0.000	1.021

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	20	187	0	0	37	0	260
normalized size	1	1.00	0.07	0.64	0.00	0.00	0.13	0.00	0.88
time (sec)	N/A	0.085	0.003	0.027	0.000	0.796	1.576	0.000	0.045

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	30	36	37	26	56	37	46
normalized size	1	1.00	0.57	0.68	0.70	0.49	1.06	0.70	0.87
time (sec)	N/A	0.017	0.013	0.005	1.291	0.887	3.080	0.167	0.037

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	25	31	28	21	41	28	21
normalized size	1	1.00	0.62	0.78	0.70	0.52	1.02	0.70	0.52
time (sec)	N/A	0.014	0.007	0.004	1.404	0.828	1.378	0.153	0.026

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	18	24	19	14	26	19	14
normalized size	1	1.00	0.67	0.89	0.70	0.52	0.96	0.70	0.52
time (sec)	N/A	0.010	0.006	0.006	1.267	0.571	0.508	0.163	0.021

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	19	9	9	10	9	9
normalized size	1	1.00	1.00	1.46	0.69	0.69	0.77	0.69	0.69
time (sec)	N/A	0.002	0.002	0.004	1.272	0.935	0.223	0.149	0.022

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	31	10	164
normalized size	1	1.00	1.00	0.79	0.71	0.71	2.21	0.71	11.71
time (sec)	N/A	0.006	0.003	0.099	2.904	0.746	1.350	0.152	0.041

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	41	24	23	25	82	23	177
normalized size	1	1.00	1.32	0.77	0.74	0.81	2.65	0.74	5.71
time (sec)	N/A	0.011	0.034	0.022	2.995	0.795	3.640	0.150	1.000

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	28	36	48	34	138	35	189
normalized size	1	1.00	0.60	0.77	1.02	0.72	2.94	0.74	4.02
time (sec)	N/A	0.014	0.004	0.024	2.907	0.540	3.735	0.152	0.999

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	28	48	66	39	182	44	201
normalized size	1	1.00	0.44	0.76	1.05	0.62	2.89	0.70	3.19
time (sec)	N/A	0.019	0.015	0.023	3.032	0.827	5.568	0.152	0.038

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	52	139	0	0	27	0	179
normalized size	1	1.00	0.34	0.91	0.00	0.00	0.18	0.00	1.17
time (sec)	N/A	0.037	0.013	0.019	0.000	0.885	0.990	0.000	0.047

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	44	127	0	0	27	0	167
normalized size	1	1.00	0.32	0.93	0.00	0.00	0.20	0.00	1.22
time (sec)	N/A	0.020	0.009	0.019	0.000	0.919	1.755	0.000	0.045

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	35	116	0	0	26	0	156
normalized size	1	1.00	0.29	0.97	0.00	0.00	0.22	0.00	1.30
time (sec)	N/A	0.009	0.003	0.015	0.000	0.797	0.995	0.000	0.036

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	40	129	0	0	31	0	169
normalized size	1	1.00	0.29	0.93	0.00	0.00	0.22	0.00	1.22
time (sec)	N/A	0.020	0.006	0.020	0.000	0.640	1.579	0.000	0.055

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	40	141	0	0	34	0	181
normalized size	1	1.00	0.26	0.91	0.00	0.00	0.22	0.00	1.17
time (sec)	N/A	0.033	0.006	0.022	0.000	0.774	2.202	0.000	0.987

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	54	198	0	0	27	0	240
normalized size	1	1.00	0.18	0.67	0.00	0.00	0.09	0.00	0.82
time (sec)	N/A	0.082	0.011	0.018	0.000	0.873	1.644	0.000	0.972

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	46	186	0	0	27	0	228
normalized size	1	1.00	0.17	0.67	0.00	0.00	0.10	0.00	0.82
time (sec)	N/A	0.058	0.010	0.020	0.000	0.854	1.280	0.000	0.972

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	40	173	0	0	27	0	215
normalized size	1	1.00	0.16	0.68	0.00	0.00	0.11	0.00	0.84
time (sec)	N/A	0.038	0.004	0.017	0.000	0.799	1.121	0.000	0.045

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	38	185	0	0	29	0	226
normalized size	1	1.00	0.14	0.69	0.00	0.00	0.11	0.00	0.84
time (sec)	N/A	0.058	0.006	0.022	0.000	0.896	0.937	0.000	1.001

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	40	198	0	0	34	0	240
normalized size	1	1.00	0.14	0.67	0.00	0.00	0.12	0.00	0.82
time (sec)	N/A	0.078	0.006	0.022	0.000	0.818	0.997	0.000	1.000

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	32	40	45	28	63	59	54
normalized size	1	1.00	0.52	0.66	0.74	0.46	1.03	0.97	0.89
time (sec)	N/A	0.025	0.011	0.006	1.316	0.811	1.902	0.173	0.032

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	27	35	34	23	46	41	23
normalized size	1	1.00	0.59	0.76	0.74	0.50	1.00	0.89	0.50
time (sec)	N/A	0.019	0.009	0.005	1.338	0.609	0.811	0.155	0.029

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	20	28	23	16	29	23	16
normalized size	1	1.00	0.65	0.90	0.74	0.52	0.94	0.74	0.52
time (sec)	N/A	0.014	0.006	0.006	1.322	0.731	0.664	0.150	0.027

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	23	11	11	14	11	11
normalized size	1	1.00	1.00	1.53	0.73	0.73	0.93	0.73	0.73
time (sec)	N/A	0.003	0.002	0.006	1.239	0.839	0.207	0.150	0.022

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	12	12	180
normalized size	1	1.00	1.00	0.81	0.75	0.75	0.75	0.75	11.25
time (sec)	N/A	0.008	0.003	0.026	2.934	0.765	1.591	0.156	0.031

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	40	28	27	31	29	27	194
normalized size	1	1.00	1.14	0.80	0.77	0.89	0.83	0.77	5.54
time (sec)	N/A	0.015	0.024	0.026	2.984	0.935	2.146	0.240	0.980

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	28	42	56	39	66	41	209
normalized size	1	1.00	0.53	0.79	1.06	0.74	1.25	0.77	3.94
time (sec)	N/A	0.020	0.005	0.029	3.016	0.850	4.739	0.150	0.038

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	28	56	74	44	90	59	223
normalized size	1	1.00	0.39	0.79	1.04	0.62	1.27	0.83	3.14
time (sec)	N/A	0.028	0.006	0.027	3.000	0.854	8.482	0.174	0.030

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	54	134	0	0	32	0	199
normalized size	1	1.00	0.36	0.90	0.00	0.00	0.21	0.00	1.34
time (sec)	N/A	0.037	0.013	0.022	0.000	0.771	1.196	0.000	0.040

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	47	120	0	0	32	0	185
normalized size	1	1.00	0.36	0.92	0.00	0.00	0.24	0.00	1.41
time (sec)	N/A	0.018	0.019	0.023	0.000	0.985	1.514	0.000	0.049

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	37	107	0	0	31	0	172
normalized size	1	1.00	0.33	0.96	0.00	0.00	0.28	0.00	1.54
time (sec)	N/A	0.008	0.004	0.022	0.000	0.703	1.195	0.000	0.030

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	42	122	0	0	36	0	187
normalized size	1	1.00	0.32	0.92	0.00	0.00	0.27	0.00	1.41
time (sec)	N/A	0.018	0.005	0.025	0.000	0.777	0.958	0.000	0.993

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	42	136	0	0	39	0	201
normalized size	1	1.00	0.28	0.90	0.00	0.00	0.26	0.00	1.33
time (sec)	N/A	0.031	0.029	0.027	0.000	0.879	1.996	0.000	0.040

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	56	189	0	0	32	0	259
normalized size	1	1.00	0.20	0.67	0.00	0.00	0.11	0.00	0.92
time (sec)	N/A	0.076	0.014	0.025	0.000	0.915	1.272	0.000	0.044

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	49	175	0	0	32	0	245
normalized size	1	1.00	0.19	0.66	0.00	0.00	0.12	0.00	0.93
time (sec)	N/A	0.054	0.011	0.023	0.000	0.875	1.323	0.000	0.058

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	42	160	0	0	32	0	230
normalized size	1	1.00	0.18	0.67	0.00	0.00	0.13	0.00	0.96
time (sec)	N/A	0.036	0.005	0.020	0.000	0.760	0.842	0.000	0.038

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	40	174	0	0	34	0	244
normalized size	1	1.00	0.16	0.68	0.00	0.00	0.13	0.00	0.95
time (sec)	N/A	0.054	0.006	0.024	0.000	0.783	1.720	0.000	1.029

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	42	189	0	0	39	0	259
normalized size	1	1.00	0.15	0.67	0.00	0.00	0.14	0.00	0.92
time (sec)	N/A	0.074	0.006	0.027	0.000	0.856	1.689	0.000	0.047

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	64	57	110	57	55
normalized size	1	1.00	0.62	0.59	0.80	0.71	1.38	0.71	0.69
time (sec)	N/A	0.044	0.040	0.007	1.336	0.826	5.548	0.187	1.106

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	47	46	87	43	44
normalized size	1	1.00	0.66	0.61	0.80	0.78	1.47	0.73	0.75
time (sec)	N/A	0.032	0.032	0.007	1.317	0.828	4.466	0.157	1.050

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	25	30	34	63	29	33
normalized size	1	1.00	0.74	0.66	0.79	0.89	1.66	0.76	0.87
time (sec)	N/A	0.022	0.019	0.006	1.375	0.576	1.821	0.156	1.032

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	39	14	14
normalized size	1	1.00	1.00	0.83	0.78	0.78	2.17	0.78	0.78
time (sec)	N/A	0.004	0.004	0.004	1.298	0.828	0.491	0.150	1.034

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	124	0	95	100	42	96	112
normalized size	1	1.00	1.31	0.00	1.00	1.05	0.44	1.01	1.18
time (sec)	N/A	0.069	0.067	0.126	2.878	0.780	2.309	0.672	1.078

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	37	0	103	155	41	115	122
normalized size	1	1.00	0.35	0.00	0.96	1.45	0.38	1.07	1.14
time (sec)	N/A	0.064	0.009	0.145	2.970	0.809	2.623	0.694	1.209

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	173	64	0	181	181	39	0	-1
normalized size	1	1.44	0.53	0.00	1.51	1.51	0.32	0.00	-0.01
time (sec)	N/A	0.100	0.088	0.123	2.985	0.855	1.624	0.000	0.000

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	145	51	0	134	193	39	0	-1
normalized size	1	1.54	0.54	0.00	1.43	2.05	0.41	0.00	-0.01
time (sec)	N/A	0.067	0.012	0.110	2.976	0.857	1.209	0.000	0.000

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	F(-1)	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	138	49	0	114	0	41	0	40
normalized size	1	1.57	0.56	0.00	1.30	0.00	0.47	0.00	0.45
time (sec)	N/A	0.070	0.009	0.128	2.867	0.000	2.211	0.000	1.210

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	68	0	17
normalized size	1	1.00	1.00	0.86	0.81	0.81	3.24	0.00	0.81
time (sec)	N/A	0.004	0.005	0.004	1.321	0.883	1.470	0.000	1.106

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	31	28	35	38	109	0	32
normalized size	1	1.00	0.70	0.64	0.80	0.86	2.48	0.00	0.73
time (sec)	N/A	0.011	0.009	0.006	1.336	1.087	1.231	0.000	1.233

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	42	39	52	49	520	0	73
normalized size	1	1.00	0.62	0.57	0.76	0.72	7.65	0.00	1.07
time (sec)	N/A	0.019	0.018	0.006	1.338	0.879	3.234	0.000	1.352

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	53	50	69	60	847	0	93
normalized size	1	1.00	0.58	0.54	0.75	0.65	9.21	0.00	1.01
time (sec)	N/A	0.030	0.037	0.006	1.373	0.698	2.814	0.000	1.502

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	51	51	0	0	0	39	0	-1
normalized size	1	1.34	1.34	0.00	0.00	0.00	1.03	0.00	-0.03
time (sec)	N/A	0.016	0.012	0.114	0.000	0.574	1.829	0.000	0.000

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	33	46	196	0	0	0	37	0	37
normalized size	1	1.39	5.94	0.00	0.00	0.00	1.12	0.00	1.12
time (sec)	N/A	0.009	0.249	0.134	0.000	0.685	0.981	0.000	1.007

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	51	51	0	0	0	42	0	-1
normalized size	1	1.34	1.34	0.00	0.00	0.00	1.11	0.00	-0.03
time (sec)	N/A	0.015	0.010	0.109	0.000	0.590	1.518	0.000	0.000

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	51	51	0	0	0	42	0	-1
normalized size	1	1.34	1.34	0.00	0.00	0.00	1.11	0.00	-0.03
time (sec)	N/A	0.017	0.010	0.120	0.000	0.594	1.174	0.000	0.000

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	64	57	110	57	55
normalized size	1	1.00	0.62	0.59	0.80	0.71	1.38	0.71	0.69
time (sec)	N/A	0.049	0.026	0.007	1.354	0.592	12.615	0.189	1.106

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	47	46	87	43	44
normalized size	1	1.00	0.66	0.61	0.80	0.78	1.47	0.73	0.75
time (sec)	N/A	0.034	0.042	0.006	1.307	0.556	4.212	0.158	1.075

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	25	30	35	63	29	33
normalized size	1	1.00	0.74	0.66	0.79	0.92	1.66	0.76	0.87
time (sec)	N/A	0.023	0.017	0.005	1.324	0.731	1.938	0.155	1.109

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	39	14	14
normalized size	1	1.00	1.00	0.83	0.78	0.78	2.17	0.78	0.78
time (sec)	N/A	0.005	0.004	0.006	1.332	0.540	0.931	0.169	1.047

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	90	0	97	122	44	98	122
normalized size	1	1.00	0.92	0.00	0.99	1.24	0.45	1.00	1.24
time (sec)	N/A	0.066	0.083	0.115	2.947	0.776	2.191	0.689	1.080

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	37	0	103	290	39	116	137
normalized size	1	1.00	0.35	0.00	0.96	2.71	0.36	1.08	1.28
time (sec)	N/A	0.064	0.008	0.115	2.943	0.670	1.348	0.709	1.189

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	51	51	0	0	0	39	0	-1
normalized size	1	1.34	1.34	0.00	0.00	0.00	1.03	0.00	-0.03
time (sec)	N/A	0.015	0.013	0.122	0.000	0.738	2.049	0.000	0.000

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	51	51	0	0	0	39	0	-1
normalized size	1	1.34	1.34	0.00	0.00	0.00	1.03	0.00	-0.03
time (sec)	N/A	0.013	0.009	0.114	0.000	0.682	1.371	0.000	0.000

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	49	49	0	0	0	41	0	39
normalized size	1	1.36	1.36	0.00	0.00	0.00	1.14	0.00	1.08
time (sec)	N/A	0.016	0.018	0.114	0.000	0.706	1.556	0.000	1.258

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	51	51	0	0	0	46	0	-1
normalized size	1	1.34	1.34	0.00	0.00	0.00	1.21	0.00	-0.03
time (sec)	N/A	0.016	0.010	0.116	0.000	0.865	1.324	0.000	0.000

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	62	0	181	345	39	0	-1
normalized size	1	1.00	0.53	0.00	1.55	2.95	0.33	0.00	-0.01
time (sec)	N/A	0.036	0.057	0.113	2.898	0.673	2.495	0.000	0.000

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	196	0	134	362	37	0	37
normalized size	1	1.00	2.15	0.00	1.47	3.98	0.41	0.00	0.41
time (sec)	N/A	0.017	0.221	0.131	3.007	0.656	2.194	0.000	1.016

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	51	0	114	0	42	0	-1
normalized size	1	1.00	0.58	0.00	1.30	0.00	0.48	0.00	-0.01
time (sec)	N/A	0.019	0.012	0.112	3.025	0.000	2.097	0.000	0.000

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	68	0	17
normalized size	1	1.00	1.00	0.86	0.81	0.81	3.24	0.00	0.81
time (sec)	N/A	0.005	0.005	0.005	1.342	0.883	1.274	0.000	1.115

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	31	28	35	38	110	0	38
normalized size	1	1.00	0.70	0.64	0.80	0.86	2.50	0.00	0.86
time (sec)	N/A	0.011	0.010	0.005	1.241	0.871	1.992	0.000	1.208

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	42	39	52	49	520	0	73
normalized size	1	1.00	0.62	0.57	0.76	0.72	7.65	0.00	1.07
time (sec)	N/A	0.019	0.012	0.007	1.313	0.979	2.232	0.000	1.346

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	27	33	34	33	71	55	33
normalized size	1	1.00	0.59	0.72	0.74	0.72	1.54	1.20	0.72
time (sec)	N/A	0.020	0.015	0.006	1.322	0.802	16.997	0.158	1.085

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	64	46	92	61	48
normalized size	1	1.00	0.62	0.59	0.80	0.58	1.15	0.76	0.60
time (sec)	N/A	0.045	0.025	0.007	1.381	0.735	3.306	0.175	1.112

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	47	35	68	47	36
normalized size	1	1.00	0.66	0.61	0.80	0.59	1.15	0.80	0.61
time (sec)	N/A	0.035	0.019	0.006	1.333	0.844	2.729	0.155	1.079

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	25	30	24	44	30	24
normalized size	1	1.00	0.74	0.66	0.79	0.63	1.16	0.79	0.63
time (sec)	N/A	0.022	0.012	0.006	1.323	0.692	1.397	0.167	1.067

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	22	14	14
normalized size	1	1.00	1.00	0.83	0.78	0.78	1.22	0.78	0.78
time (sec)	N/A	0.004	0.003	0.005	1.308	0.457	0.489	0.167	1.079

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	70	0	86	236	37	87	100
normalized size	1	1.00	0.84	0.00	1.04	2.84	0.45	1.05	1.20
time (sec)	N/A	0.051	0.020	0.115	3.022	0.636	1.107	0.719	1.202

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	37	0	118	344	39	119	138
normalized size	1	1.00	0.34	0.00	1.07	3.13	0.35	1.08	1.25
time (sec)	N/A	0.069	0.007	0.114	3.035	0.709	1.808	0.710	1.335

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	51	51	0	0	0	37	0	-1
normalized size	1	1.34	1.34	0.00	0.00	0.00	0.97	0.00	-0.03
time (sec)	N/A	0.014	0.010	0.115	0.000	0.610	1.852	0.000	0.000

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	51	51	0	0	0	37	0	-1
normalized size	1	1.34	1.34	0.00	0.00	0.00	0.97	0.00	-0.03
time (sec)	N/A	0.016	0.009	0.123	0.000	0.570	1.417	0.000	0.000

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	51	51	0	0	0	37	0	-1
normalized size	1	1.34	1.34	0.00	0.00	0.00	0.97	0.00	-0.03
time (sec)	N/A	0.012	0.008	0.118	0.000	0.687	1.423	0.000	0.000

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	49	49	0	0	0	39	0	40
normalized size	1	1.36	1.36	0.00	0.00	0.00	1.08	0.00	1.11
time (sec)	N/A	0.016	0.009	0.112	0.000	0.635	1.545	0.000	1.220

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	51	51	0	0	0	44	0	-1
normalized size	1	1.34	1.34	0.00	0.00	0.00	1.16	0.00	-0.03
time (sec)	N/A	0.015	0.009	0.112	0.000	0.619	1.227	0.000	0.000

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	134	0	137	309	37	0	-1
normalized size	1	1.00	1.43	0.00	1.46	3.29	0.39	0.00	-0.01
time (sec)	N/A	0.019	0.107	0.110	2.964	0.674	1.326	0.000	0.000

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	110	0	100	330	36	0	37
normalized size	1	1.00	1.57	0.00	1.43	4.71	0.51	0.00	0.53
time (sec)	N/A	0.008	0.022	0.125	3.033	0.677	1.799	0.000	1.031

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	31	0	17
normalized size	1	1.00	1.00	0.86	0.81	0.81	1.48	0.00	0.81
time (sec)	N/A	0.005	0.004	0.005	1.338	0.659	1.288	0.000	1.028

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	31	28	35	27	70	0	27
normalized size	1	1.00	0.70	0.64	0.80	0.61	1.59	0.00	0.61
time (sec)	N/A	0.011	0.012	0.005	1.354	0.629	2.033	0.000	1.083

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	42	39	52	38	406	0	38
normalized size	1	1.00	0.62	0.57	0.76	0.56	5.97	0.00	0.56
time (sec)	N/A	0.018	0.033	0.006	1.312	0.765	2.585	0.000	1.137

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	53	50	69	49	692	0	76
normalized size	1	1.00	0.58	0.54	0.75	0.53	7.52	0.00	0.83
time (sec)	N/A	0.029	0.024	0.006	1.292	0.606	3.439	0.000	1.203

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	50	47	64	46	92	61	48
normalized size	1	1.00	0.64	0.60	0.82	0.59	1.18	0.78	0.62
time (sec)	N/A	0.044	0.035	0.007	1.351	0.625	3.359	0.155	1.100

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	39	36	46	35	68	46	36
normalized size	1	1.00	0.70	0.64	0.82	0.62	1.21	0.82	0.64
time (sec)	N/A	0.033	0.022	0.007	1.278	0.555	2.118	0.154	1.077

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	27	25	30	23	44	30	24
normalized size	1	1.00	0.75	0.69	0.83	0.64	1.22	0.83	0.67
time (sec)	N/A	0.022	0.033	0.005	1.391	0.775	0.900	0.178	1.071

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	20	13	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	1.33	0.87	0.87
time (sec)	N/A	0.004	0.003	0.004	1.269	0.681	0.758	0.155	1.055

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	101	0	86	126	39	87	102
normalized size	1	1.00	1.20	0.00	1.02	1.50	0.46	1.04	1.21
time (sec)	N/A	0.049	0.046	0.112	2.985	0.465	1.764	0.705	1.235

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	34	0	118	182	39	118	128
normalized size	1	1.00	0.31	0.00	1.07	1.65	0.35	1.07	1.16
time (sec)	N/A	0.067	0.007	0.125	2.959	0.609	2.262	0.700	1.319

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	176	69	0	184	212	37	0	-1
normalized size	1	1.43	0.56	0.00	1.50	1.72	0.30	0.00	-0.01
time (sec)	N/A	0.091	0.020	0.115	3.005	0.659	2.844	0.000	0.000

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	148	53	0	137	165	37	0	-1
normalized size	1	1.53	0.55	0.00	1.41	1.70	0.38	0.00	-0.01
time (sec)	N/A	0.069	0.011	0.114	2.897	0.559	2.764	0.000	0.000

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	122	40	0	100	173	37	0	-1
normalized size	1	1.69	0.56	0.00	1.39	2.40	0.51	0.00	-0.01
time (sec)	N/A	0.053	0.006	0.112	2.879	0.694	2.518	0.000	0.000

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	17	31	0	17
normalized size	1	1.00	1.00	0.95	0.89	0.89	1.63	0.00	0.89
time (sec)	N/A	0.005	0.023	0.005	1.338	0.569	1.151	0.000	1.035

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	29	26	35	27	68	0	25
normalized size	1	1.00	0.66	0.59	0.80	0.61	1.55	0.00	0.57
time (sec)	N/A	0.011	0.012	0.005	1.319	0.507	1.542	0.000	1.080

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	42	39	52	38	406	0	38
normalized size	1	1.00	0.62	0.57	0.76	0.56	5.97	0.00	0.56
time (sec)	N/A	0.020	0.023	0.003	1.280	0.590	3.048	0.000	1.152

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	53	50	69	49	692	0	76
normalized size	1	1.00	0.58	0.54	0.75	0.53	7.52	0.00	0.83
time (sec)	N/A	0.031	0.025	0.006	1.269	0.593	3.183	0.000	1.199

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	51	51	0	0	0	37	0	-1
normalized size	1	1.34	1.34	0.00	0.00	0.00	0.97	0.00	-0.03
time (sec)	N/A	0.016	0.009	0.115	0.000	0.582	1.483	0.000	0.000

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	51	51	0	0	0	37	0	-1
normalized size	1	1.34	1.34	0.00	0.00	0.00	0.97	0.00	-0.03
time (sec)	N/A	0.017	0.015	0.116	0.000	0.592	1.383	0.000	0.000

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	33	46	177	0	0	0	36	0	37
normalized size	1	1.39	5.36	0.00	0.00	0.00	1.09	0.00	1.12
time (sec)	N/A	0.009	0.221	0.135	0.000	0.666	0.902	0.000	1.025

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	51	51	0	0	0	41	0	-1
normalized size	1	1.34	1.34	0.00	0.00	0.00	1.08	0.00	-0.03
time (sec)	N/A	0.017	0.010	0.132	0.000	0.896	1.180	0.000	0.000

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	51	51	0	0	0	44	0	-1
normalized size	1	1.34	1.34	0.00	0.00	0.00	1.16	0.00	-0.03
time (sec)	N/A	0.016	0.010	0.115	0.000	0.719	1.591	0.000	0.000

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	116	0	103	284	37	0	38
normalized size	1	1.00	1.61	0.00	1.43	3.94	0.51	0.00	0.53
time (sec)	N/A	0.010	0.112	0.145	2.949	0.595	1.727	0.000	1.067

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	78	18	69	76	34	0	16
normalized size	1	1.00	1.70	0.39	1.50	1.65	0.74	0.00	0.35
time (sec)	N/A	0.005	0.052	0.155	2.888	0.622	1.805	0.000	1.037

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	87	37	15	78	82	31	0	-1
normalized size	1	1.64	0.70	0.28	1.47	1.55	0.58	0.00	-0.02
time (sec)	N/A	0.041	0.009	0.113	2.895	0.705	2.014	0.000	0.000

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	10	9	9
normalized size	1	1.00	1.00	0.77	0.69	0.69	0.77	0.69	0.69
time (sec)	N/A	0.003	0.003	0.004	1.267	0.742	0.718	0.153	1.132

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	136	1023	151	847	5902	1269	860
normalized size	1	1.00	0.90	6.77	1.00	5.61	39.09	8.40	5.70
time (sec)	N/A	0.076	0.106	0.011	1.356	0.731	47.374	0.317	2.181

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	88	432	97	367	2006	540	389
normalized size	1	1.00	0.91	4.45	1.00	3.78	20.68	5.57	4.01
time (sec)	N/A	0.037	0.053	0.007	1.342	0.568	14.931	0.218	1.553

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	56	178	61	157	666	224	167
normalized size	1	1.00	0.92	2.92	1.00	2.57	10.92	3.67	2.74
time (sec)	N/A	0.022	0.035	0.005	1.360	0.679	4.186	0.245	1.350

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	40	93	43	85	313	117	93
normalized size	1	1.00	0.93	2.16	1.00	1.98	7.28	2.72	2.16
time (sec)	N/A	0.014	0.023	0.006	1.329	0.802	2.719	0.169	1.306

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	35	25	33	94	43	34
normalized size	1	1.00	1.00	1.40	1.00	1.32	3.76	1.72	1.36
time (sec)	N/A	0.007	0.014	0.003	1.300	0.566	1.092	0.162	1.211

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	41	0	0	0	88	0	-1
normalized size	1	1.00	1.05	0.00	0.00	0.00	2.26	0.00	-0.03
time (sec)	N/A	0.008	0.012	0.104	0.000	0.570	8.746	0.000	0.000

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	41	0	0	0	515	0	-1
normalized size	1	1.00	1.05	0.00	0.00	0.00	13.21	0.00	-0.03
time (sec)	N/A	0.008	0.007	0.107	0.000	0.681	88.024	0.000	0.000

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	41	0	0	0	0	0	-1
normalized size	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.008	0.007	0.101	0.000	0.646	0.000	0.000	0.000

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	66	0	0	0	54	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.84	0.00	-0.02
time (sec)	N/A	0.020	0.026	0.107	0.000	0.820	4.350	0.000	0.000

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	65	0	0	0	54	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.86	0.00	-0.02
time (sec)	N/A	0.018	0.014	0.107	0.000	0.645	1.840	0.000	0.000

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	63	65	0	0	0	53	0	-1
normalized size	1	1.21	1.25	0.00	0.00	0.00	1.02	0.00	-0.02
time (sec)	N/A	0.018	0.017	0.102	0.000	0.595	2.030	0.000	0.000

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	68	0	0	0	53	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.80	0.00	-0.02
time (sec)	N/A	0.019	0.018	0.103	0.000	0.718	2.062	0.000	0.000

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	67	0	0	0	58	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.84	0.00	-0.01
time (sec)	N/A	0.023	0.023	0.106	0.000	0.678	12.598	0.000	0.000

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	68	66	0	0	0	58	0	-1
normalized size	1	1.11	1.08	0.00	0.00	0.00	0.95	0.00	-0.02
time (sec)	N/A	0.022	0.020	0.108	0.000	0.674	2.786	0.000	0.000

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	68	66	0	0	0	58	0	-1
normalized size	1	1.11	1.08	0.00	0.00	0.00	0.95	0.00	-0.02
time (sec)	N/A	0.019	0.029	0.101	0.000	0.663	2.553	0.000	0.000

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	64	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.018	0.014	0.105	0.000	0.586	0.000	0.000	0.000

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	21	25	134	21	21
normalized size	1	1.00	1.00	0.96	0.91	1.09	5.83	0.91	0.91
time (sec)	N/A	0.006	0.006	0.004	1.362	0.631	2.125	0.153	1.087

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	40	42	47	58	524	94	68
normalized size	1	1.00	0.83	0.88	0.98	1.21	10.92	1.96	1.42
time (sec)	N/A	0.029	0.031	0.005	1.331	0.892	8.075	0.171	1.076

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	64	80	73	98	1370	231	118
normalized size	1	1.00	0.86	1.08	0.99	1.32	18.51	3.12	1.59
time (sec)	N/A	0.044	0.038	0.007	1.437	0.839	32.724	0.163	1.091

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	132	106	148	2773	410	183
normalized size	1	1.00	1.00	1.39	1.12	1.56	29.19	4.32	1.93
time (sec)	N/A	0.060	0.047	0.009	1.433	0.708	80.687	0.193	1.116

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	35	25	33	94	43	34
normalized size	1	1.00	1.00	1.40	1.00	1.32	3.76	1.72	1.36
time (sec)	N/A	0.008	0.014	0.004	1.286	0.859	1.304	0.170	1.236

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.005	0.001	0.001	1.287	0.459	0.084	0.146	0.021

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.005	0.001	0.001	1.322	0.467	0.077	0.145	0.019

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	14	13	12	13	13
normalized size	1	1.00	1.00	0.82	0.82	0.76	0.71	0.76	0.76
time (sec)	N/A	0.005	0.001	0.001	1.290	0.629	0.177	0.150	0.020

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.005	0.001	0.000	1.322	0.859	0.066	0.151	0.019

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.005	0.001	0.000	1.333	0.475	0.078	0.148	0.021

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	10
normalized size	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.002	0.000	0.000	1.284	0.526	0.101	0.147	0.016

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	14	11	10	14	11
normalized size	1	1.00	1.00	0.92	1.08	0.85	0.77	1.08	0.85
time (sec)	N/A	0.004	0.002	0.003	1.321	0.649	0.095	0.195	0.021

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	14	8	13	13
normalized size	1	1.00	1.00	0.93	0.87	0.93	0.53	0.87	0.87
time (sec)	N/A	0.005	0.002	0.004	1.371	0.646	0.135	0.149	0.022

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	14	12	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.82	0.71	0.76	0.76
time (sec)	N/A	0.005	0.001	0.005	1.333	0.754	0.165	0.175	0.022

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	15	8	10	10
normalized size	1	1.00	1.00	0.92	0.83	1.25	0.67	0.83	0.83
time (sec)	N/A	0.004	0.001	0.003	1.344	0.754	0.220	0.147	0.022

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	14	17	10	20	11
normalized size	1	1.00	1.00	0.92	1.08	1.31	0.77	1.54	0.85
time (sec)	N/A	0.005	0.003	0.005	1.316	0.592	0.255	0.148	0.040

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	14	13	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.93	0.87	0.87
time (sec)	N/A	0.005	0.002	0.003	1.342	0.610	0.386	0.150	0.024

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	14	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.82	0.76	0.76
time (sec)	N/A	0.005	0.002	0.005	1.257	0.549	0.374	0.144	0.025

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	15	15	15	15	15
normalized size	1	1.00	1.00	0.82	0.88	0.88	0.88	0.88	0.88
time (sec)	N/A	0.005	0.002	0.004	1.340	0.654	0.318	0.148	0.026

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	14	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.82	0.76	0.76
time (sec)	N/A	0.005	0.002	0.004	1.286	0.531	0.181	0.176	0.026

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	15	15	15	15	15
normalized size	1	1.00	1.00	0.82	0.88	0.88	0.88	0.88	0.88
time (sec)	N/A	0.005	0.002	0.006	1.275	0.785	0.208	0.148	0.027

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	40	93	43	85	309	117	93
normalized size	1	1.00	0.93	2.16	1.00	1.98	7.19	2.72	2.16
time (sec)	N/A	0.015	0.025	0.006	1.318	0.636	3.936	0.163	1.315

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	24	24
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.010	0.001	0.001	1.348	0.532	0.101	0.151	0.034

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.010	0.001	0.002	1.326	0.616	0.071	0.154	0.030

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	30	25	14	24	24	14	24
normalized size	1	1.00	1.88	1.56	0.88	1.50	1.50	0.88	1.50
time (sec)	N/A	0.003	0.001	0.000	1.346	0.639	0.198	0.154	0.031

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.010	0.001	0.001	1.338	0.500	0.084	0.147	0.031

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	24	24
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.009	0.002	0.000	1.319	0.483	0.089	0.150	0.031

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	22	21	21
normalized size	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.84
time (sec)	N/A	0.007	0.001	0.001	1.328	0.419	0.092	0.153	0.028

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	25	22	22	25	22
normalized size	1	1.00	1.00	0.88	0.96	0.85	0.85	0.96	0.85
time (sec)	N/A	0.012	0.001	0.003	1.316	0.553	0.182	0.149	0.962

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	24	26	22	24	25
normalized size	1	1.00	1.00	0.89	0.86	0.93	0.79	0.86	0.89
time (sec)	N/A	0.009	0.001	0.002	1.297	0.757	0.196	0.149	0.033

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	23	25	22	23	25
normalized size	1	1.00	1.00	0.89	0.85	0.93	0.81	0.85	0.93
time (sec)	N/A	0.009	0.001	0.004	1.322	0.807	0.162	0.151	0.031

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	22	26	22	22	22
normalized size	1	1.00	1.00	0.88	0.85	1.00	0.85	0.85	0.85
time (sec)	N/A	0.009	0.001	0.003	1.355	0.721	0.192	0.148	0.031

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	25	27	24	33	23
normalized size	1	1.00	1.00	0.89	0.93	1.00	0.89	1.22	0.85
time (sec)	N/A	0.013	0.001	0.007	1.278	0.595	0.195	0.155	0.030

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	56	178	61	157	683	224	167
normalized size	1	1.00	0.92	2.92	1.00	2.57	11.20	3.67	2.74
time (sec)	N/A	0.020	0.031	0.006	1.336	0.579	7.362	0.175	1.332

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	39	35	35
normalized size	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.81
time (sec)	N/A	0.015	0.002	0.000	1.341	0.523	0.321	0.182	0.039

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	37	35	35
normalized size	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.81
time (sec)	N/A	0.014	0.002	0.001	1.340	0.564	0.127	0.152	0.039

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	43	36	14	35	37	14	35
normalized size	1	1.00	2.69	2.25	0.88	2.19	2.31	0.88	2.19
time (sec)	N/A	0.003	0.002	0.001	1.322	0.488	0.089	0.148	0.039

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	39	35	35
normalized size	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.81
time (sec)	N/A	0.014	0.002	0.002	1.244	0.650	0.083	0.152	0.042

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	37	35	35
normalized size	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.81
time (sec)	N/A	0.013	0.002	0.000	1.337	0.453	0.087	0.158	0.039

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	33	32	32	34	32	32
normalized size	1	1.00	1.00	0.87	0.84	0.84	0.89	0.84	0.84
time (sec)	N/A	0.011	0.001	0.002	1.350	0.394	0.083	0.171	0.038

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	34	36	33	37	36	33
normalized size	1	1.00	1.00	0.87	0.92	0.85	0.95	0.92	0.85
time (sec)	N/A	0.018	0.004	0.003	1.343	0.627	0.140	0.153	0.035

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	35	34	37	32	34	34
normalized size	1	1.00	1.00	0.92	0.89	0.97	0.84	0.89	0.89
time (sec)	N/A	0.013	0.004	0.004	1.272	0.565	0.153	0.231	0.041

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	36	37	35	35
normalized size	1	1.00	1.00	0.84	0.81	0.84	0.86	0.81	0.81
time (sec)	N/A	0.014	0.006	0.004	1.314	0.681	0.310	0.154	0.042

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	34	33	37	36	33	33
normalized size	1	1.00	1.00	0.87	0.85	0.95	0.92	0.85	0.85
time (sec)	N/A	0.014	0.004	0.004	1.342	0.699	0.303	0.151	0.040

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	35	36	38	37	46	34
normalized size	1	1.00	1.00	0.88	0.90	0.95	0.92	1.15	0.85
time (sec)	N/A	0.021	0.005	0.008	1.313	0.592	0.317	0.152	0.035

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	48	43	42	107	87	45	37
normalized size	1	1.00	0.94	0.84	0.82	2.10	1.71	0.88	0.73
time (sec)	N/A	0.031	0.032	0.005	2.938	0.693	0.282	0.158	0.085

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	23	22	20	24	22
normalized size	1	1.00	1.00	0.89	0.85	0.81	0.74	0.89	0.81
time (sec)	N/A	0.019	0.006	0.004	1.269	0.504	0.239	0.156	0.041

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	32	31	89	63	31	28
normalized size	1	1.00	1.00	0.80	0.78	2.22	1.58	0.78	0.70
time (sec)	N/A	0.020	0.013	0.001	2.968	0.629	0.320	0.155	0.041

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	14	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87
time (sec)	N/A	0.003	0.004	0.001	1.356	0.491	0.190	0.166	0.030

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	19	18	72	56	18	19
normalized size	1	1.00	1.00	0.66	0.62	2.48	1.93	0.62	0.66
time (sec)	N/A	0.010	0.006	0.003	2.945	0.543	0.223	0.183	0.050

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	23	18	15	24	18
normalized size	1	1.00	1.00	0.95	1.05	0.82	0.68	1.09	0.82
time (sec)	N/A	0.011	0.006	0.006	1.290	0.522	0.610	0.155	0.998

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	79	32	31	94	71	31	28
normalized size	1	1.00	1.98	0.80	0.78	2.35	1.78	0.78	0.70
time (sec)	N/A	0.017	0.034	0.003	2.983	0.533	0.465	0.157	0.049

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	33	33	31	43	31
normalized size	1	1.00	1.00	0.91	0.94	0.94	0.89	1.23	0.89
time (sec)	N/A	0.023	0.008	0.006	1.295	0.539	0.834	0.153	0.074

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	88	43	43	112	90	43	40
normalized size	1	1.00	1.73	0.84	0.84	2.20	1.76	0.84	0.78
time (sec)	N/A	0.026	0.039	0.007	3.086	0.655	0.525	0.179	0.061

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	173	133	179	119	22	172	48
normalized size	1	1.00	0.91	0.70	0.94	0.63	0.12	0.91	0.25
time (sec)	N/A	0.142	0.033	0.009	2.917	0.661	0.231	0.178	1.038

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	134	128	169	122	26	179	35
normalized size	1	1.00	0.72	0.69	0.91	0.66	0.14	0.97	0.19
time (sec)	N/A	0.107	0.021	0.003	3.016	0.803	0.224	0.163	1.036

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	134	128	169	121	20	179	33
normalized size	1	1.00	0.72	0.69	0.91	0.65	0.11	0.97	0.18
time (sec)	N/A	0.100	0.018	0.002	2.929	0.523	0.238	0.172	1.014

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	179	136	183	130	29	187	51
normalized size	1	1.00	0.93	0.70	0.95	0.67	0.15	0.97	0.26
time (sec)	N/A	0.121	0.028	0.006	2.974	0.647	0.269	0.186	0.101

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	188	139	184	162	32	175	51
normalized size	1	1.00	0.96	0.71	0.94	0.83	0.16	0.90	0.26
time (sec)	N/A	0.119	0.032	0.006	2.997	0.690	0.336	0.159	0.099

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	38	41	43	56	41	49	45
normalized size	1	1.00	0.83	0.89	0.93	1.22	0.89	1.07	0.98
time (sec)	N/A	0.033	0.020	0.013	1.342	0.632	0.521	0.188	0.994

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	60	50	52	144	92	49	50
normalized size	1	1.00	1.02	0.85	0.88	2.44	1.56	0.83	0.85
time (sec)	N/A	0.031	0.046	0.012	2.945	0.593	0.853	0.159	1.071

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	27	30	32	35	29	48	29
normalized size	1	1.00	0.82	0.91	0.97	1.06	0.88	1.45	0.88
time (sec)	N/A	0.024	0.010	0.011	1.275	0.642	0.337	0.199	0.046

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	40	40	127	83	39	37
normalized size	1	1.00	1.00	0.82	0.82	2.59	1.69	0.80	0.76
time (sec)	N/A	0.024	0.023	0.010	2.971	0.615	0.483	0.158	0.074

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	15	15	14	14
normalized size	1	1.00	1.00	0.94	0.88	0.94	0.94	0.88	0.88
time (sec)	N/A	0.003	0.004	0.001	1.327	0.732	0.646	0.163	0.955

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	40	39	129	83	39	37
normalized size	1	1.00	1.00	0.82	0.80	2.63	1.69	0.80	0.76
time (sec)	N/A	0.021	0.026	0.006	2.894	0.628	0.531	0.158	0.045

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	33	35	37	47	34	47	34
normalized size	1	1.00	0.87	0.92	0.97	1.24	0.89	1.24	0.89
time (sec)	N/A	0.026	0.016	0.016	1.266	0.683	0.570	0.154	0.065

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	94	50	53	152	97	51	50
normalized size	1	1.00	1.59	0.85	0.90	2.58	1.64	0.86	0.85
time (sec)	N/A	0.031	0.054	0.015	2.988	0.722	0.872	0.193	1.023

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	185	145	192	181	44	196	60
normalized size	1	1.00	0.91	0.71	0.94	0.89	0.22	0.96	0.29
time (sec)	N/A	0.126	0.116	0.010	3.014	0.729	0.371	0.184	1.076

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	182	152	190	181	39	194	58
normalized size	1	1.00	0.90	0.75	0.94	0.90	0.19	0.96	0.29
time (sec)	N/A	0.120	0.115	0.007	2.963	0.797	0.536	0.161	1.061

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	184	154	191	178	46	196	60
normalized size	1	1.00	0.90	0.75	0.94	0.87	0.23	0.96	0.29
time (sec)	N/A	0.120	0.106	0.005	3.078	0.659	0.644	0.182	0.114

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	183	143	189	173	39	194	58
normalized size	1	1.00	0.91	0.71	0.94	0.86	0.19	0.96	0.29
time (sec)	N/A	0.119	0.100	0.004	2.974	0.694	0.403	0.185	1.046

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	196	154	203	192	56	205	69
normalized size	1	1.00	0.92	0.72	0.95	0.90	0.26	0.96	0.32
time (sec)	N/A	0.137	0.133	0.012	2.945	0.705	0.713	0.199	1.033

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	194	155	206	221	58	191	73
normalized size	1	1.00	0.91	0.72	0.96	1.03	0.27	0.89	0.34
time (sec)	N/A	0.137	0.138	0.012	3.029	0.621	0.884	0.168	0.109

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	39	47	55	69	53	42	53
normalized size	1	1.00	0.75	0.90	1.06	1.33	1.02	0.81	1.02
time (sec)	N/A	0.036	0.019	0.010	1.344	0.561	0.743	0.268	0.073

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	58	52	63	196	116	49	60
normalized size	1	1.00	0.85	0.76	0.93	2.88	1.71	0.72	0.88
time (sec)	N/A	0.036	0.051	0.012	3.031	0.555	1.039	0.165	1.044

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	24	31	36	36	36	22	37
normalized size	1	1.00	1.26	1.63	1.89	1.89	1.89	1.16	1.95
time (sec)	N/A	0.004	0.010	0.010	1.324	0.552	0.504	0.164	0.032

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	62	54	66	199	116	54	58
normalized size	1	1.00	0.87	0.76	0.93	2.80	1.63	0.76	0.82
time (sec)	N/A	0.035	0.035	0.010	2.922	0.809	0.806	0.166	1.036

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	26	27	14	25
normalized size	1	1.00	1.00	0.94	0.88	1.62	1.69	0.88	1.56
time (sec)	N/A	0.003	0.004	0.002	1.334	0.559	0.481	0.179	0.028

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	58	57	62	196	110	49	59
normalized size	1	1.00	0.85	0.84	0.91	2.88	1.62	0.72	0.87
time (sec)	N/A	0.031	0.049	0.007	2.979	0.647	0.582	0.161	1.004

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	43	49	60	90	56	59	56
normalized size	1	1.00	0.80	0.91	1.11	1.67	1.04	1.09	1.04
time (sec)	N/A	0.037	0.043	0.017	1.356	0.505	0.779	0.160	1.036

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	105	70	75	218	121	61	72
normalized size	1	1.00	1.35	0.90	0.96	2.79	1.55	0.78	0.92
time (sec)	N/A	0.043	0.091	0.015	2.950	0.566	0.821	0.160	1.052

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	205	156	215	240	70	206	83
normalized size	1	1.00	0.92	0.70	0.96	1.08	0.31	0.92	0.37
time (sec)	N/A	0.142	0.098	0.011	3.026	0.662	0.684	0.174	1.070

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	201	163	213	245	68	204	81
normalized size	1	1.00	0.91	0.74	0.96	1.11	0.31	0.92	0.37
time (sec)	N/A	0.138	0.098	0.013	3.030	0.655	0.959	0.170	1.080

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	207	164	219	252	71	209	81
normalized size	1	1.00	0.92	0.73	0.97	1.12	0.31	0.92	0.36
time (sec)	N/A	0.140	0.108	0.011	3.027	0.639	0.976	0.172	1.075

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	203	162	216	249	66	206	80
normalized size	1	1.00	0.91	0.73	0.97	1.12	0.30	0.93	0.36
time (sec)	N/A	0.141	0.096	0.013	3.013	0.806	0.848	0.170	0.118

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	204	171	214	242	71	206	82
normalized size	1	1.00	0.91	0.77	0.96	1.09	0.32	0.92	0.37
time (sec)	N/A	0.144	0.094	0.004	3.039	0.707	0.575	0.192	1.075

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	200	158	212	232	63	204	80
normalized size	1	1.00	0.91	0.72	0.97	1.06	0.29	0.93	0.37
time (sec)	N/A	0.136	0.090	0.006	2.947	0.667	0.575	0.160	1.084

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	216	174	225	247	80	217	91
normalized size	1	1.00	0.93	0.75	0.97	1.06	0.34	0.93	0.39
time (sec)	N/A	0.161	0.104	0.015	3.038	0.564	1.038	0.249	0.112

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	34	26	25	31	27	25	25
normalized size	1	1.00	0.89	0.68	0.66	0.82	0.71	0.66	0.66
time (sec)	N/A	0.022	0.016	0.004	2.889	0.768	0.242	0.167	0.039

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	21	17	16	16	14	16	14
normalized size	1	1.00	1.05	0.85	0.80	0.80	0.70	0.80	0.70
time (sec)	N/A	0.013	0.005	0.003	1.301	0.534	0.245	0.168	0.029

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	21	20	20	22	20	20
normalized size	1	1.00	1.00	0.72	0.69	0.69	0.76	0.69	0.69
time (sec)	N/A	0.013	0.010	0.003	2.929	0.618	0.159	0.195	1.005

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	8
normalized size	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.67
time (sec)	N/A	0.003	0.003	0.000	1.298	0.476	0.095	0.174	0.981

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	15	14	14	17	14	14
normalized size	1	1.00	1.00	0.71	0.67	0.67	0.81	0.67	0.67
time (sec)	N/A	0.008	0.014	0.003	2.953	0.566	0.122	0.165	0.027

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	17	15	14	17	13
normalized size	1	1.00	1.00	0.84	0.89	0.79	0.74	0.89	0.68
time (sec)	N/A	0.008	0.003	0.005	1.342	0.611	0.119	0.176	0.066

Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	48	21	20	29	26	20	20
normalized size	1	1.00	1.55	0.68	0.65	0.94	0.84	0.65	0.65
time (sec)	N/A	0.012	0.023	0.004	2.885	0.549	0.149	0.177	1.005

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	98	116	126	164	92	100	38
normalized size	1	1.00	0.94	1.12	1.21	1.58	0.88	0.96	0.37
time (sec)	N/A	0.095	0.027	0.007	2.939	0.645	0.466	0.189	0.123

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	96	114	124	166	90	98	36
normalized size	1	1.00	0.94	1.12	1.22	1.63	0.88	0.96	0.35
time (sec)	N/A	0.075	0.025	0.003	2.996	0.702	0.495	0.186	1.092

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	77	111	121	159	87	95	33
normalized size	1	1.00	0.79	1.14	1.25	1.64	0.90	0.98	0.34
time (sec)	N/A	0.068	0.015	0.002	3.061	0.777	0.477	0.349	0.108

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	77	111	121	163	87	95	33
normalized size	1	1.00	0.79	1.14	1.25	1.68	0.90	0.98	0.34
time (sec)	N/A	0.065	0.015	0.002	2.936	0.747	0.471	0.201	1.058

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	101	116	126	174	92	100	38
normalized size	1	1.00	0.81	0.94	1.02	1.40	0.74	0.81	0.31
time (sec)	N/A	0.076	0.028	0.006	3.012	0.803	0.474	0.193	0.108

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	8	11	11
normalized size	1	1.00	1.00	0.92	0.85	0.85	0.62	0.85	0.85
time (sec)	N/A	0.003	0.003	0.001	1.370	0.572	0.179	0.207	0.999

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	30	29	37	27	29	28
normalized size	1	1.00	1.00	0.79	0.76	0.97	0.71	0.76	0.74
time (sec)	N/A	0.013	0.017	0.007	2.914	0.683	0.174	0.164	0.037

Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	28	40	22	35	22
normalized size	1	1.00	1.00	0.84	0.88	1.25	0.69	1.09	0.69
time (sec)	N/A	0.020	0.010	0.013	1.313	0.509	0.169	0.167	0.065

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	59	33	37	53	37	37	41
normalized size	1	1.00	1.26	0.70	0.79	1.13	0.79	0.79	0.87
time (sec)	N/A	0.020	0.047	0.010	2.882	0.646	0.298	0.174	1.003

Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	105	121	133	204	95	107	45
normalized size	1	1.00	0.95	1.09	1.20	1.84	0.86	0.96	0.41
time (sec)	N/A	0.076	0.117	0.007	3.001	0.575	0.786	0.203	1.165

Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	107	125	135	202	97	109	46
normalized size	1	1.00	0.95	1.11	1.19	1.79	0.86	0.96	0.41
time (sec)	N/A	0.076	0.094	0.005	3.166	0.627	0.476	0.188	0.121

Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	105	123	133	248	95	107	44
normalized size	1	1.00	0.80	0.94	1.02	1.89	0.73	0.82	0.34
time (sec)	N/A	0.074	0.079	0.004	3.083	0.715	0.599	0.198	1.061

Problem 706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	113	128	141	220	110	115	51
normalized size	1	1.00	0.81	0.91	1.01	1.57	0.79	0.82	0.36
time (sec)	N/A	0.085	0.119	0.010	3.063	0.724	0.668	0.187	1.097

Problem 707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	101	85	107	141	124	95	45
normalized size	1	1.00	0.76	0.64	0.80	1.06	0.93	0.71	0.34
time (sec)	N/A	0.079	0.049	0.003	3.014	0.815	0.472	0.180	0.114

Problem 708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	160	170	318	216	32	267	543
normalized size	1	1.00	1.93	2.05	3.83	2.60	0.39	3.22	6.54
time (sec)	N/A	0.072	0.081	0.010	3.157	0.733	0.494	0.165	1.183

Problem 709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	128	137	179	294	42	219	121
normalized size	1	1.00	1.62	1.73	2.27	3.72	0.53	2.77	1.53
time (sec)	N/A	0.060	0.099	0.009	3.012	0.716	0.432	0.171	0.123

Problem 710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	128	137	179	294	42	219	121
normalized size	1	1.00	1.62	1.73	2.27	3.72	0.53	2.77	1.53
time (sec)	N/A	0.030	0.018	0.000	3.023	0.658	0.548	0.209	0.002

Problem 711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	25	91	110	24	32
normalized size	1	1.00	1.00	0.79	0.76	2.76	3.33	0.73	0.97
time (sec)	N/A	0.022	0.009	0.003	3.027	0.640	0.491	0.155	1.039

Problem 712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	25	91	110	24	32
normalized size	1	1.00	1.00	0.79	0.76	2.76	3.33	0.73	0.97
time (sec)	N/A	0.012	0.006	0.000	2.944	0.599	0.292	0.154	0.002

Problem 713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	128	137	179	136	29	219	53
normalized size	1	1.00	1.62	1.73	2.27	1.72	0.37	2.77	0.67
time (sec)	N/A	0.026	0.033	0.003	3.037	0.691	0.284	0.167	0.130

Problem 714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	128	137	179	136	29	219	53
normalized size	1	1.00	1.62	1.73	2.27	1.72	0.37	2.77	0.67
time (sec)	N/A	0.024	0.012	0.000	3.114	0.664	0.491	0.187	0.002

Problem 715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	15	14	13	13	12	14	13
normalized size	1	1.00	1.07	1.00	0.93	0.93	0.86	1.00	0.93
time (sec)	N/A	0.003	0.004	0.002	1.404	0.527	0.460	0.154	0.038

Problem 716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	15	14	13	13	12	14	13
normalized size	1	1.00	1.07	1.00	0.93	0.93	0.86	1.00	0.93
time (sec)	N/A	0.003	0.004	0.000	1.337	0.741	0.207	0.173	0.002

Problem 717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	13	18	19	13	15
normalized size	1	1.00	1.00	0.76	0.62	0.86	0.90	0.62	0.71
time (sec)	N/A	0.004	0.005	0.003	1.360	0.484	7.941	0.173	1.046

Problem 718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	13	18	19	13	15
normalized size	1	1.00	1.00	0.76	0.62	0.86	0.90	0.62	0.71
time (sec)	N/A	0.004	0.005	0.004	1.370	0.677	4.008	0.151	0.027

Problem 719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	13	16	19	13	15
normalized size	1	1.00	1.00	0.76	0.62	0.76	0.90	0.62	0.71
time (sec)	N/A	0.004	0.013	0.003	1.340	0.721	2.779	0.148	0.026

Problem 720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	15	13	14	17	13	14
normalized size	1	1.00	1.00	0.79	0.68	0.74	0.89	0.68	0.74
time (sec)	N/A	0.004	0.005	0.003	1.327	0.628	1.691	0.149	0.028

Problem 721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	13	14	17	13	15
normalized size	1	1.00	1.00	0.84	0.68	0.74	0.89	0.68	0.79
time (sec)	N/A	0.004	0.005	0.003	1.383	0.763	1.658	0.148	0.027

Problem 722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	13	15	19	13	15
normalized size	1	1.00	1.00	0.76	0.62	0.71	0.90	0.62	0.71
time (sec)	N/A	0.004	0.030	0.003	1.346	0.622	1.556	0.188	0.028

Problem 723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	13	15	19	13	15
normalized size	1	1.00	1.00	0.76	0.62	0.71	0.90	0.62	0.71
time (sec)	N/A	0.004	0.006	0.002	1.370	0.447	3.379	0.158	0.028

Problem 724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	27	24	29	34	24	25
normalized size	1	1.00	0.83	0.75	0.67	0.81	0.94	0.67	0.69
time (sec)	N/A	0.009	0.015	0.006	1.303	0.612	24.247	0.151	1.021

Problem 725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	27	24	29	34	24	26
normalized size	1	1.00	0.83	0.75	0.67	0.81	0.94	0.67	0.72
time (sec)	N/A	0.008	0.008	0.005	1.311	0.681	17.038	0.150	0.035

Problem 726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	27	24	27	34	24	26
normalized size	1	1.00	0.83	0.75	0.67	0.75	0.94	0.67	0.72
time (sec)	N/A	0.008	0.016	0.006	1.342	0.565	2.961	0.180	1.057

Problem 727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	27	24	26	32	24	26
normalized size	1	1.00	0.88	0.79	0.71	0.76	0.94	0.71	0.76
time (sec)	N/A	0.008	0.008	0.006	1.457	0.699	7.336	0.150	0.035

Problem 728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	27	24	26	32	24	25
normalized size	1	1.00	0.88	0.79	0.71	0.76	0.94	0.71	0.74
time (sec)	N/A	0.008	0.031	0.006	1.346	0.798	5.887	0.173	0.036

Problem 729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	27	24	26	34	24	26
normalized size	1	1.00	0.83	0.75	0.67	0.72	0.94	0.67	0.72
time (sec)	N/A	0.009	0.009	0.005	1.369	0.640	7.431	0.152	0.036

Problem 730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	27	24	26	34	24	26
normalized size	1	1.00	0.83	0.75	0.67	0.72	0.94	0.67	0.72
time (sec)	N/A	0.008	0.009	0.007	1.352	0.629	8.958	0.191	0.036

Problem 731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	38	35	40	49	35	35
normalized size	1	1.00	1.00	0.75	0.69	0.78	0.96	0.69	0.69
time (sec)	N/A	0.013	0.021	0.007	1.347	0.646	70.178	0.150	0.044

Problem 732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	38	35	40	49	35	35
normalized size	1	1.00	1.00	0.75	0.69	0.78	0.96	0.69	0.69
time (sec)	N/A	0.013	0.012	0.006	1.385	0.550	48.124	0.153	0.042

Problem 733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	38	35	38	49	35	35
normalized size	1	1.00	0.80	0.75	0.69	0.75	0.96	0.69	0.69
time (sec)	N/A	0.012	0.010	0.006	1.315	0.690	5.952	0.173	0.043

Problem 734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	41	38	35	37	48	35	35
normalized size	1	1.00	0.84	0.78	0.71	0.76	0.98	0.71	0.71
time (sec)	N/A	0.012	0.012	0.007	1.354	0.564	24.752	0.152	0.043

Problem 735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	41	38	35	37	48	35	35
normalized size	1	1.00	0.84	0.78	0.71	0.76	0.98	0.71	0.71
time (sec)	N/A	0.013	0.020	0.006	1.388	0.618	27.824	0.214	0.046

Problem 736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	38	35	37	49	35	35
normalized size	1	1.00	0.80	0.75	0.69	0.73	0.96	0.69	0.69
time (sec)	N/A	0.012	0.012	0.008	1.159	0.608	37.073	0.213	0.045

Problem 737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	41	38	35	37	48	35	35
normalized size	1	1.00	0.84	0.78	0.71	0.76	0.98	0.71	0.71
time (sec)	N/A	0.012	0.013	0.006	1.270	0.517	37.210	0.191	0.045

Problem 738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	31	39	0	491	0	453	126
normalized size	1	1.00	0.10	0.13	0.00	1.64	0.00	1.52	0.42
time (sec)	N/A	0.376	0.008	0.032	0.000	0.630	0.000	0.641	1.107

Problem 739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	29	39	0	392	459	453	126
normalized size	1	1.00	0.10	0.13	0.00	1.32	1.55	1.53	0.42
time (sec)	N/A	0.289	0.020	0.019	0.000	0.711	168.072	0.452	1.100

Problem 740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	29	29	0	426	452	445	118
normalized size	1	1.00	0.10	0.10	0.00	1.48	1.57	1.55	0.41
time (sec)	N/A	0.235	0.007	0.017	0.000	0.625	94.125	0.528	1.113

Problem 741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	29	29	0	446	454	445	118
normalized size	1	1.00	0.10	0.10	0.00	1.55	1.58	1.55	0.41
time (sec)	N/A	0.233	0.007	0.017	0.000	0.723	74.187	0.546	1.109

Problem 742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	29	29	0	460	442	445	118
normalized size	1	1.00	0.10	0.10	0.00	1.60	1.54	1.55	0.41
time (sec)	N/A	0.239	0.006	0.007	0.000	0.791	40.999	0.513	0.129

Problem 743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	27	29	0	415	440	445	118
normalized size	1	1.00	0.09	0.10	0.00	1.45	1.53	1.55	0.41
time (sec)	N/A	0.224	0.006	0.005	0.000	0.757	58.917	0.360	1.098

Problem 744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	27	38	0	446	450	461	126
normalized size	1	1.00	0.09	0.13	0.00	1.50	1.52	1.55	0.42
time (sec)	N/A	0.274	0.006	0.010	0.000	0.685	148.289	0.553	1.090

Problem 745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	29	38	0	553	0	461	126
normalized size	1	1.00	0.10	0.13	0.00	1.85	0.00	1.54	0.42
time (sec)	N/A	0.286	0.009	0.010	0.000	0.848	0.000	0.551	1.082

Problem 746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	43	47	0	535	0	486	135
normalized size	1	1.00	0.14	0.15	0.00	1.74	0.00	1.58	0.44
time (sec)	N/A	0.269	0.040	0.015	0.000	0.646	0.000	0.736	0.145

Problem 747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	54	47	0	553	0	486	135
normalized size	1	1.00	0.18	0.15	0.00	1.80	0.00	1.58	0.44
time (sec)	N/A	0.251	0.012	0.015	0.000	0.609	0.000	0.753	1.088

Problem 748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	54	47	0	570	0	486	135
normalized size	1	1.00	0.18	0.15	0.00	1.85	0.00	1.58	0.44
time (sec)	N/A	0.254	0.013	0.017	0.000	0.781	0.000	0.733	1.057

Problem 749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	54	47	0	545	0	486	135
normalized size	1	1.00	0.18	0.15	0.00	1.77	0.00	1.58	0.44
time (sec)	N/A	0.247	0.014	0.016	0.000	0.731	0.000	0.577	1.071

Problem 750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	29	50	0	542	0	462	135
normalized size	1	1.00	0.09	0.16	0.00	1.76	0.00	1.50	0.44
time (sec)	N/A	0.251	0.006	0.014	0.000	0.695	0.000	0.727	0.134

Problem 751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	29	50	0	557	0	462	135
normalized size	1	1.00	0.09	0.16	0.00	1.81	0.00	1.50	0.44
time (sec)	N/A	0.261	0.005	0.014	0.000	0.794	0.000	0.774	1.093

Problem 752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	29	50	0	550	0	462	135
normalized size	1	1.00	0.09	0.16	0.00	1.79	0.00	1.50	0.44
time (sec)	N/A	0.294	0.006	0.013	0.000	0.679	0.000	0.722	0.140

Problem 753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	51	50	0	517	0	462	135
normalized size	1	1.00	0.17	0.16	0.00	1.68	0.00	1.50	0.44
time (sec)	N/A	0.266	0.013	0.013	0.000	0.709	0.000	0.567	0.129

Problem 754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	27	56	0	575	0	481	148
normalized size	1	1.00	0.08	0.18	0.00	1.80	0.00	1.50	0.46
time (sec)	N/A	0.299	0.006	0.018	0.000	0.683	0.000	0.685	1.145

Problem 755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	66	59	0	672	0	496	158
normalized size	1	1.00	0.20	0.18	0.00	2.04	0.00	1.51	0.48
time (sec)	N/A	0.283	0.020	0.022	0.000	0.722	0.000	0.702	1.136

Problem 756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	45	61	0	688	0	499	156
normalized size	1	1.00	0.14	0.18	0.00	2.07	0.00	1.50	0.47
time (sec)	N/A	0.286	0.018	0.020	0.000	0.841	0.000	0.875	0.147

Problem 757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	45	61	0	700	0	499	156
normalized size	1	1.00	0.14	0.18	0.00	2.11	0.00	1.50	0.47
time (sec)	N/A	0.298	0.024	0.020	0.000	0.641	0.000	0.874	1.127

Problem 758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	45	61	0	693	0	499	156
normalized size	1	1.00	0.14	0.18	0.00	2.09	0.00	1.50	0.47
time (sec)	N/A	0.304	0.016	0.022	0.000	0.923	0.000	0.856	1.082

Problem 759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	64	61	0	679	0	498	156
normalized size	1	1.00	0.19	0.18	0.00	2.05	0.00	1.50	0.47
time (sec)	N/A	0.290	0.018	0.022	0.000	0.764	0.000	0.726	1.099

Problem 760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	29	62	0	672	0	472	157
normalized size	1	1.00	0.09	0.19	0.00	2.04	0.00	1.43	0.48
time (sec)	N/A	0.308	0.005	0.022	0.000	0.568	0.000	0.858	1.096

Problem 761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	29	62	0	674	0	472	157
normalized size	1	1.00	0.09	0.19	0.00	2.05	0.00	1.43	0.48
time (sec)	N/A	0.303	0.006	0.020	0.000	0.724	0.000	0.856	0.140

Problem 762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	29	62	0	665	0	472	157
normalized size	1	1.00	0.09	0.19	0.00	2.02	0.00	1.43	0.48
time (sec)	N/A	0.313	0.006	0.021	0.000	0.835	0.000	0.826	1.078

Problem 763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	62	62	0	631	0	472	157
normalized size	1	1.00	0.19	0.19	0.00	1.92	0.00	1.43	0.48
time (sec)	N/A	0.303	0.016	0.020	0.000	0.815	0.000	0.651	0.143

Problem 764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	47	46	87	43	44
normalized size	1	1.00	0.66	0.61	0.80	0.78	1.47	0.73	0.75
time (sec)	N/A	0.035	0.036	0.007	1.368	0.700	3.789	0.168	1.172

Problem 765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	25	30	34	61	29	33
normalized size	1	1.00	0.74	0.66	0.79	0.89	1.61	0.76	0.87
time (sec)	N/A	0.023	0.026	0.005	1.363	0.624	2.899	0.151	1.134

Problem 766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	39	14	14
normalized size	1	1.00	1.00	0.83	0.78	0.78	2.17	0.78	0.78
time (sec)	N/A	0.004	0.003	0.003	1.215	0.832	0.396	0.183	1.118

Problem 767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	41	49	84	66	36	31
normalized size	1	1.00	1.00	0.95	1.14	1.95	1.53	0.84	0.72
time (sec)	N/A	0.026	0.010	0.013	3.042	0.770	2.359	0.154	1.203

Problem 768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	59	63	53	108	46	46	35
normalized size	1	1.00	1.26	1.34	1.13	2.30	0.98	0.98	0.74
time (sec)	N/A	0.026	0.035	0.011	2.985	0.642	4.063	0.159	1.283

Problem 769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	39	85	100	133	95	72	54
normalized size	1	1.00	0.55	1.20	1.41	1.87	1.34	1.01	0.76
time (sec)	N/A	0.039	0.009	0.014	2.972	0.556	5.094	0.160	1.439

Problem 770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	63	120	127	95	54	-1
normalized size	1	1.00	1.00	0.85	1.62	1.72	1.28	0.73	-0.01
time (sec)	N/A	0.046	0.144	0.013	2.987	0.580	5.167	0.176	0.000

Problem 771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	72	40	78	102	44	41	-1
normalized size	1	1.00	1.44	0.80	1.56	2.04	0.88	0.82	-0.02
time (sec)	N/A	0.025	0.068	0.009	3.026	0.784	3.535	0.194	0.000

Problem 772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	69	60	60	96	66	60	-1
normalized size	1	1.00	1.41	1.22	1.22	1.96	1.35	1.22	-0.02
time (sec)	N/A	0.026	0.111	0.010	2.928	0.530	3.178	0.208	0.000

Problem 773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	42	63	17
normalized size	1	1.00	1.00	0.86	0.81	0.81	2.00	3.00	0.81
time (sec)	N/A	0.004	0.005	0.003	1.359	0.580	1.625	0.182	1.219

Problem 774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	31	28	35	38	66	120	37
normalized size	1	1.00	0.70	0.64	0.80	0.86	1.50	2.73	0.84
time (sec)	N/A	0.011	0.011	0.004	1.362	0.645	2.347	0.207	1.321

Problem 775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	42	39	52	49	359	148	73
normalized size	1	1.00	0.62	0.57	0.76	0.72	5.28	2.18	1.07
time (sec)	N/A	0.019	0.011	0.004	1.398	0.629	3.434	0.195	1.515

Problem 776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	62	108	0	0	39	0	-1
normalized size	1	1.00	0.49	0.85	0.00	0.00	0.31	0.00	-0.01
time (sec)	N/A	0.041	0.061	0.048	0.000	0.622	2.034	0.000	0.000

Problem 777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	89	85	0	0	37	0	37
normalized size	1	1.00	0.85	0.81	0.00	0.00	0.35	0.00	0.35
time (sec)	N/A	0.018	0.158	0.006	0.000	0.733	1.817	0.000	1.048

Problem 778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	51	87	0	0	42	0	-1
normalized size	1	1.00	0.48	0.81	0.00	0.00	0.39	0.00	-0.01
time (sec)	N/A	0.022	0.010	0.013	0.000	0.582	1.624	0.000	0.000

Problem 779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	51	110	0	0	46	0	-1
normalized size	1	1.00	0.40	0.85	0.00	0.00	0.36	0.00	-0.01
time (sec)	N/A	0.035	0.022	0.016	0.000	0.862	1.387	0.000	0.000

Problem 780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	51	112	0	0	39	0	-1
normalized size	1	1.00	0.22	0.48	0.00	0.00	0.17	0.00	-0.00
time (sec)	N/A	0.072	0.010	0.008	0.000	0.763	1.765	0.000	0.000

Problem 781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	49	112	0	0	41	0	39
normalized size	1	1.00	0.22	0.50	0.00	0.00	0.18	0.00	0.17
time (sec)	N/A	0.066	0.010	0.010	0.000	1.483	1.440	0.000	1.281

Problem 782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	51	130	0	0	46	0	-1
normalized size	1	1.00	0.20	0.50	0.00	0.00	0.18	0.00	-0.00
time (sec)	N/A	0.090	0.009	0.014	0.000	0.820	2.208	0.000	0.000

Problem 783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	47	57	109	43	53
normalized size	1	1.00	0.66	0.61	0.80	0.97	1.85	0.73	0.90
time (sec)	N/A	0.034	0.026	0.007	1.404	0.905	11.028	0.165	1.176

Problem 784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	25	30	45	83	29	42
normalized size	1	1.00	0.74	0.66	0.79	1.18	2.18	0.76	1.11
time (sec)	N/A	0.024	0.016	0.004	1.318	0.499	5.423	0.155	1.141

Problem 785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	32	60	14	14
normalized size	1	1.00	1.00	0.83	0.78	1.78	3.33	0.78	0.78
time (sec)	N/A	0.005	0.008	0.005	1.328	0.796	1.208	0.153	1.142

Problem 786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	51	57	61	103	80	50	43
normalized size	1	1.00	0.86	0.97	1.03	1.75	1.36	0.85	0.73
time (sec)	N/A	0.035	0.022	0.016	3.000	0.654	3.642	0.160	1.169

Problem 787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	37	58	66	121	95	63	48
normalized size	1	1.00	0.59	0.92	1.05	1.92	1.51	1.00	0.76
time (sec)	N/A	0.038	0.010	0.016	2.895	0.845	4.611	0.174	1.361

Problem 788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	76	63	98	138	75	70	52
normalized size	1	1.00	1.12	0.93	1.44	2.03	1.10	1.03	0.76
time (sec)	N/A	0.039	0.047	0.017	2.944	0.762	4.985	0.162	1.495

Problem 789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	87	78	160	153	122	126	-1
normalized size	1	1.00	0.92	0.82	1.68	1.61	1.28	1.33	-0.01
time (sec)	N/A	0.059	0.137	0.020	2.977	0.641	6.345	0.223	0.000

Problem 790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	69	58	119	132	73	99	-1
normalized size	1	1.00	0.97	0.82	1.68	1.86	1.03	1.39	-0.01
time (sec)	N/A	0.036	0.120	0.011	2.990	0.587	4.593	0.214	0.000

Problem 791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	52	56	94	120	95	78	-1
normalized size	1	1.00	0.75	0.81	1.36	1.74	1.38	1.13	-0.01
time (sec)	N/A	0.039	0.012	0.016	2.994	0.638	2.880	0.218	0.000

Problem 792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	52	55	75	116	80	122	-1
normalized size	1	1.00	0.76	0.81	1.10	1.71	1.18	1.79	-0.01
time (sec)	N/A	0.041	0.012	0.017	3.041	0.627	4.477	0.230	0.000

Problem 793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	35	66	92	17
normalized size	1	1.00	1.00	0.86	0.81	1.67	3.14	4.38	0.81
time (sec)	N/A	0.005	0.006	0.005	1.348	0.874	2.496	0.228	1.523

Problem 794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	31	28	35	49	92	178	71
normalized size	1	1.00	0.70	0.64	0.80	1.11	2.09	4.05	1.61
time (sec)	N/A	0.011	0.021	0.005	1.400	1.154	2.347	0.236	1.763

Problem 795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	42	39	52	60	420	206	91
normalized size	1	1.00	0.62	0.57	0.76	0.88	6.18	3.03	1.34
time (sec)	N/A	0.019	0.013	0.006	1.330	0.560	5.409	0.253	2.073

Problem 796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	67	126	0	0	39	0	-1
normalized size	1	1.00	0.45	0.85	0.00	0.00	0.26	0.00	-0.01
time (sec)	N/A	0.048	0.099	0.010	0.000	0.627	1.768	0.000	0.000

Problem 797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	47	103	0	0	37	0	37
normalized size	1	1.00	0.39	0.84	0.00	0.00	0.30	0.00	0.30
time (sec)	N/A	0.028	0.022	0.005	0.000	0.507	1.245	0.000	1.044

Problem 798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	52	102	0	0	42	0	-1
normalized size	1	1.00	0.42	0.82	0.00	0.00	0.34	0.00	-0.01
time (sec)	N/A	0.030	0.010	0.013	0.000	0.777	2.122	0.000	0.000

Problem 799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	52	105	0	0	46	0	-1
normalized size	1	1.00	0.41	0.83	0.00	0.00	0.37	0.00	-0.01
time (sec)	N/A	0.032	0.010	0.014	0.000	0.634	2.471	0.000	0.000

Problem 800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	52	128	0	0	39	0	-1
normalized size	1	1.00	0.20	0.50	0.00	0.00	0.15	0.00	-0.00
time (sec)	N/A	0.086	0.011	0.009	0.000	0.594	1.870	0.000	0.000

Problem 801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	50	128	0	0	41	0	40
normalized size	1	1.00	0.20	0.51	0.00	0.00	0.16	0.00	0.16
time (sec)	N/A	0.088	0.010	0.013	0.000	0.689	1.652	0.000	1.501

Problem 802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	52	128	0	0	46	0	-1
normalized size	1	1.00	0.21	0.51	0.00	0.00	0.18	0.00	-0.00
time (sec)	N/A	0.086	0.017	0.016	0.000	0.528	1.435	0.000	0.000

Problem 803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	17	84	0	0	29	0	12
normalized size	1	1.00	0.24	1.17	0.00	0.00	0.40	0.00	0.17
time (sec)	N/A	0.011	0.002	0.046	0.000	0.908	1.007	0.000	1.019

Problem 804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	15	59	0	0	31	0	10
normalized size	1	1.00	0.37	1.44	0.00	0.00	0.76	0.00	0.24
time (sec)	N/A	0.006	0.003	0.007	0.000	0.560	1.313	0.000	1.016

Problem 805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	22	19	23	23	42	23	18
normalized size	1	1.00	0.71	0.61	0.74	0.74	1.35	0.74	0.58
time (sec)	N/A	0.015	0.008	0.003	1.335	1.005	0.854	0.150	1.127

Problem 806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	24	9	9
normalized size	1	1.00	1.00	0.77	0.69	0.69	1.85	0.69	0.69
time (sec)	N/A	0.002	0.003	0.005	1.350	0.847	0.325	0.157	1.115

Problem 807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	30	75	45	51	39	-1
normalized size	1	1.00	1.00	0.75	1.88	1.12	1.28	0.98	-0.02
time (sec)	N/A	0.014	0.031	0.009	2.981	1.081	2.764	0.179	0.000

Problem 808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	50	28	58	29	90	29	-1
normalized size	1	1.00	1.43	0.80	1.66	0.83	2.57	0.83	-0.03
time (sec)	N/A	0.011	0.030	0.008	1.333	0.899	2.964	0.179	0.000

Problem 809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	48	72	0	0	29	0	12
normalized size	1	1.00	0.83	1.24	0.00	0.00	0.50	0.00	0.21
time (sec)	N/A	0.007	0.028	0.006	0.000	0.635	1.434	0.000	0.043

Problem 810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	39	45	0	0	31	0	10
normalized size	1	1.00	1.56	1.80	0.00	0.00	1.24	0.00	0.40
time (sec)	N/A	0.003	0.019	0.006	0.000	0.659	1.561	0.000	1.010

Problem 811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	47	35	68	47	36
normalized size	1	1.00	0.66	0.61	0.80	0.59	1.15	0.80	0.61
time (sec)	N/A	0.033	0.022	0.007	1.368	0.822	5.089	0.150	1.195

Problem 812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	25	30	23	42	30	24
normalized size	1	1.00	0.71	0.66	0.79	0.61	1.11	0.79	0.63
time (sec)	N/A	0.022	0.013	0.007	1.348	1.077	1.938	0.216	1.146

Problem 813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	22	14	14
normalized size	1	1.00	1.00	0.83	0.78	0.78	1.22	0.78	0.78
time (sec)	N/A	0.004	0.018	0.003	1.306	1.075	0.837	0.149	1.146

Problem 814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	29	37	63	22	23	19
normalized size	1	1.00	1.00	1.07	1.37	2.33	0.81	0.85	0.70
time (sec)	N/A	0.017	0.006	0.013	3.032	1.129	1.596	0.168	1.224

Problem 815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	64	48	68	107	46	51	38
normalized size	1	1.00	1.28	0.96	1.36	2.14	0.92	1.02	0.76
time (sec)	N/A	0.026	0.091	0.013	2.914	0.998	4.217	0.170	1.311

Problem 816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	43	81	101	46	44	-1
normalized size	1	1.00	1.00	0.81	1.53	1.91	0.87	0.83	-0.02
time (sec)	N/A	0.028	0.035	0.012	2.905	0.783	4.174	0.189	0.000

Problem 817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	24	45	63	20	25	-1
normalized size	1	1.00	1.00	0.80	1.50	2.10	0.67	0.83	-0.03
time (sec)	N/A	0.015	0.005	0.007	3.033	0.845	1.939	0.176	0.000

Problem 818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	20	31	17
normalized size	1	1.00	1.00	0.86	0.81	0.81	0.95	1.48	0.81
time (sec)	N/A	0.004	0.004	0.004	1.268	0.700	0.976	0.186	1.136

Problem 819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	29	26	35	27	44	59	25
normalized size	1	1.00	0.66	0.59	0.80	0.61	1.00	1.34	0.57
time (sec)	N/A	0.010	0.007	0.006	1.456	0.893	1.333	0.202	1.186

Problem 820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	42	39	52	38	298	83	38
normalized size	1	1.00	0.62	0.57	0.76	0.56	4.38	1.22	0.56
time (sec)	N/A	0.019	0.022	0.006	1.336	0.631	2.915	0.212	1.275

Problem 821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	79	111	0	0	37	0	-1
normalized size	1	1.00	0.61	0.85	0.00	0.00	0.28	0.00	-0.01
time (sec)	N/A	0.041	0.021	0.046	0.000	0.784	1.343	0.000	0.000

Problem 822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	62	91	0	0	37	0	-1
normalized size	1	1.00	0.57	0.84	0.00	0.00	0.34	0.00	-0.01
time (sec)	N/A	0.021	0.050	0.010	0.000	0.862	1.595	0.000	0.000

Problem 823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	74	70	0	0	36	0	37
normalized size	1	1.00	0.84	0.80	0.00	0.00	0.41	0.00	0.42
time (sec)	N/A	0.010	0.035	0.005	0.000	0.875	1.495	0.000	1.077

Problem 824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	51	93	0	0	41	0	-1
normalized size	1	1.00	0.46	0.85	0.00	0.00	0.37	0.00	-0.01
time (sec)	N/A	0.025	0.010	0.011	0.000	0.941	2.045	0.000	0.000

Problem 825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	51	113	0	0	44	0	-1
normalized size	1	1.00	0.39	0.86	0.00	0.00	0.33	0.00	-0.01
time (sec)	N/A	0.036	0.041	0.015	0.000	0.949	2.281	0.000	0.000

Problem 826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	80	133	0	0	37	0	-1
normalized size	1	1.00	0.31	0.51	0.00	0.00	0.14	0.00	-0.00
time (sec)	N/A	0.092	0.031	0.010	0.000	0.745	1.380	0.000	0.000

Problem 827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	64	115	0	0	37	0	-1
normalized size	1	1.00	0.27	0.49	0.00	0.00	0.16	0.00	-0.00
time (sec)	N/A	0.067	0.020	0.009	0.000	0.814	1.708	0.000	0.000

Problem 828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	51	97	0	0	37	0	-1
normalized size	1	1.00	0.24	0.46	0.00	0.00	0.18	0.00	-0.00
time (sec)	N/A	0.046	0.009	0.008	0.000	0.922	1.488	0.000	0.000

Problem 829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	49	115	0	0	39	0	40
normalized size	1	1.00	0.21	0.50	0.00	0.00	0.17	0.00	0.17
time (sec)	N/A	0.067	0.010	0.011	0.000	0.806	2.126	0.000	1.258

Problem 830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	51	133	0	0	44	0	-1
normalized size	1	1.00	0.20	0.51	0.00	0.00	0.17	0.00	-0.00
time (sec)	N/A	0.092	0.009	0.014	0.000	0.853	2.312	0.000	0.000

Problem 831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	40	37	50	36	70	61	38
normalized size	1	1.00	0.65	0.60	0.81	0.58	1.13	0.98	0.61
time (sec)	N/A	0.035	0.024	0.006	1.342	0.807	3.251	0.173	1.209

Problem 832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	28	25	32	24	44	32	24
normalized size	1	1.00	0.70	0.62	0.80	0.60	1.10	0.80	0.60
time (sec)	N/A	0.024	0.015	0.007	1.299	0.866	1.605	0.153	1.168

Problem 833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	15	24	15	15
normalized size	1	1.00	1.00	0.84	0.79	0.79	1.26	0.79	0.79
time (sec)	N/A	0.004	0.005	0.004	1.301	0.828	1.029	0.189	1.189

Problem 834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	30	39	65	53	24	20
normalized size	1	1.00	1.00	1.07	1.39	2.32	1.89	0.86	0.71
time (sec)	N/A	0.018	0.006	0.013	2.947	0.780	2.209	0.156	1.211

Problem 835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	61	50	71	112	129	54	40
normalized size	1	1.00	1.17	0.96	1.37	2.15	2.48	1.04	0.77
time (sec)	N/A	0.028	0.076	0.013	3.023	0.851	5.231	0.163	1.287

Problem 836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	44	62	116	128	53	-1
normalized size	1	1.00	1.00	0.80	1.13	2.11	2.33	0.96	-0.02
time (sec)	N/A	0.030	0.025	0.013	3.038	0.888	4.381	0.211	0.000

Problem 837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	24	23	76	53	30	-1
normalized size	1	1.00	1.00	0.77	0.74	2.45	1.71	0.97	-0.03
time (sec)	N/A	0.014	0.005	0.008	3.035	0.855	1.640	0.172	0.000

Problem 838	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	18	51	36	18
normalized size	1	1.00	1.00	0.86	0.82	0.82	2.32	1.64	0.82
time (sec)	N/A	0.004	0.004	0.004	1.405	0.914	1.771	0.176	1.145

Problem 839	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	30	27	36	26	189	68	26
normalized size	1	1.00	0.65	0.59	0.78	0.57	4.11	1.48	0.57
time (sec)	N/A	0.011	0.008	0.006	1.370	0.785	1.740	0.181	1.192

Problem 840	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	43	40	55	39	609	97	39
normalized size	1	1.00	0.61	0.56	0.77	0.55	8.58	1.37	0.55
time (sec)	N/A	0.019	0.010	0.009	1.347	1.030	5.277	0.184	1.311

Problem 841	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	80	107	0	0	39	0	-1
normalized size	1	1.00	0.80	1.07	0.00	0.00	0.39	0.00	-0.01
time (sec)	N/A	0.034	0.021	0.021	0.000	0.586	2.105	0.000	0.000

Problem 842	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	64	86	0	0	39	0	-1
normalized size	1	1.00	0.83	1.12	0.00	0.00	0.51	0.00	-0.01
time (sec)	N/A	0.020	0.023	0.009	0.000	0.707	1.784	0.000	0.000

Problem 843	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	72	64	0	0	37	0	38
normalized size	1	1.00	1.36	1.21	0.00	0.00	0.70	0.00	0.72
time (sec)	N/A	0.011	0.045	0.006	0.000	0.546	1.003	0.000	1.090

Problem 844	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	52	88	0	0	42	0	-1
normalized size	1	1.00	0.66	1.11	0.00	0.00	0.53	0.00	-0.01
time (sec)	N/A	0.021	0.025	0.013	0.000	0.660	1.981	0.000	0.000

Problem 845	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	52	109	0	0	46	0	-1
normalized size	1	1.00	0.51	1.07	0.00	0.00	0.45	0.00	-0.01
time (sec)	N/A	0.033	0.011	0.016	0.000	0.938	1.402	0.000	0.000

Problem 846	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	81	126	0	0	39	0	-1
normalized size	1	1.00	0.51	0.80	0.00	0.00	0.25	0.00	-0.01
time (sec)	N/A	0.095	0.041	0.010	0.000	0.904	2.250	0.000	0.000

Problem 847	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	66	107	0	0	39	0	-1
normalized size	1	1.00	0.49	0.79	0.00	0.00	0.29	0.00	-0.01
time (sec)	N/A	0.077	0.025	0.010	0.000	0.859	2.032	0.000	0.000

Problem 848	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	52	88	0	0	39	0	-1
normalized size	1	1.00	0.48	0.81	0.00	0.00	0.36	0.00	-0.01
time (sec)	N/A	0.064	0.009	0.009	0.000	0.850	0.855	0.000	0.000

Problem 849	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	50	106	0	0	41	0	41
normalized size	1	1.00	0.39	0.83	0.00	0.00	0.32	0.00	0.32
time (sec)	N/A	0.077	0.010	0.010	0.000	0.932	2.035	0.000	1.281

Problem 850	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	52	126	0	0	39	0	-1
normalized size	1	1.00	0.33	0.80	0.00	0.00	0.25	0.00	-0.01
time (sec)	N/A	0.091	0.011	0.013	0.000	0.862	1.480	0.000	0.000

Problem 851	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	38	36	47	46	68	52	41
normalized size	1	1.00	0.67	0.63	0.82	0.81	1.19	0.91	0.72
time (sec)	N/A	0.033	0.037	0.007	1.305	0.893	3.567	0.198	1.302

Problem 852	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	24	30	35	41	33	21
normalized size	1	1.00	0.71	0.63	0.79	0.92	1.08	0.87	0.55
time (sec)	N/A	0.022	0.017	0.006	1.303	0.978	2.476	0.167	1.205

Problem 853	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	24	26	14	14
normalized size	1	1.00	1.00	0.83	0.78	1.33	1.44	0.78	0.78
time (sec)	N/A	0.005	0.005	0.005	1.296	0.744	1.117	0.151	1.132

Problem 854	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	36	44	52	129	184	41	34
normalized size	1	1.00	0.78	0.96	1.13	2.80	4.00	0.89	0.74
time (sec)	N/A	0.027	0.015	0.013	3.013	0.845	3.788	0.156	1.248

Problem 855	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	71	37	63	86	173	76	72	53
normalized size	1	1.03	0.54	0.91	1.25	2.51	1.10	1.04	0.77
time (sec)	N/A	0.038	0.009	0.015	2.985	0.973	6.882	0.166	1.372

Problem 856	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	75	61	103	167	75	55	-1
normalized size	1	1.00	1.01	0.82	1.39	2.26	1.01	0.74	-0.01
time (sec)	N/A	0.046	0.065	0.013	3.010	0.598	4.270	0.187	0.000

Problem 857	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	66	42	63	138	44	43	-1
normalized size	1	1.00	1.27	0.81	1.21	2.65	0.85	0.83	-0.02
time (sec)	N/A	0.029	0.089	0.013	2.882	0.900	3.319	0.193	0.000

Problem 858	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	26	20	17	17
normalized size	1	1.00	1.00	0.86	0.81	1.24	0.95	0.81	0.81
time (sec)	N/A	0.004	0.019	0.003	1.302	1.140	1.039	0.172	1.031

Problem 859	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	29	26	36	37	46	53	25
normalized size	1	1.00	0.69	0.62	0.86	0.88	1.10	1.26	0.60
time (sec)	N/A	0.010	0.008	0.005	1.322	0.886	1.337	0.189	1.155

Problem 860	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	40	37	56	50	233	115	70
normalized size	1	1.00	0.59	0.54	0.82	0.74	3.43	1.69	1.03
time (sec)	N/A	0.018	0.011	0.007	1.338	0.839	2.660	0.220	1.280

Problem 861	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	79	133	0	0	37	0	-1
normalized size	1	1.00	0.52	0.88	0.00	0.00	0.25	0.00	-0.01
time (sec)	N/A	0.052	0.039	0.019	0.000	0.860	2.467	0.000	0.000

Problem 862	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	66	111	0	0	37	0	-1
normalized size	1	1.00	0.51	0.86	0.00	0.00	0.29	0.00	-0.01
time (sec)	N/A	0.035	0.044	0.013	0.000	0.694	2.213	0.000	0.000

Problem 863	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	55	94	0	0	37	0	-1
normalized size	1	1.00	0.51	0.87	0.00	0.00	0.34	0.00	-0.01
time (sec)	N/A	0.021	0.024	0.013	0.000	0.813	1.993	0.000	0.000

Problem 864	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	55	94	0	0	36	0	37
normalized size	1	1.00	0.51	0.87	0.00	0.00	0.33	0.00	0.34
time (sec)	N/A	0.019	0.018	0.010	0.000	0.913	0.896	0.000	1.114

Problem 865	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	54	113	0	0	41	0	-1
normalized size	1	1.00	0.41	0.86	0.00	0.00	0.31	0.00	-0.01
time (sec)	N/A	0.035	0.010	0.017	0.000	0.886	1.912	0.000	0.000

Problem 866	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	54	135	0	0	44	0	-1
normalized size	1	1.00	0.35	0.88	0.00	0.00	0.29	0.00	-0.01
time (sec)	N/A	0.050	0.010	0.022	0.000	0.902	1.562	0.000	0.000

Problem 867	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	80	157	0	0	37	0	-1
normalized size	1	1.00	0.28	0.56	0.00	0.00	0.13	0.00	-0.00
time (sec)	N/A	0.115	0.031	0.013	0.000	0.779	1.881	0.000	0.000

Problem 868	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	66	137	0	0	37	0	-1
normalized size	1	1.00	0.26	0.53	0.00	0.00	0.14	0.00	-0.00
time (sec)	N/A	0.088	0.028	0.012	0.000	0.872	1.632	0.000	0.000

Problem 869	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	57	119	0	0	37	0	-1
normalized size	1	1.00	0.24	0.50	0.00	0.00	0.16	0.00	-0.00
time (sec)	N/A	0.067	0.017	0.010	0.000	0.803	1.225	0.000	0.000

Problem 870	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	54	119	0	0	37	0	-1
normalized size	1	1.00	0.23	0.50	0.00	0.00	0.15	0.00	-0.00
time (sec)	N/A	0.069	0.009	0.012	0.000	1.021	0.994	0.000	0.000

Problem 871	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	52	137	0	0	39	0	40
normalized size	1	1.00	0.20	0.53	0.00	0.00	0.15	0.00	0.15
time (sec)	N/A	0.089	0.009	0.015	0.000	0.678	1.794	0.000	1.352

Problem 872	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	54	157	0	0	44	0	-1
normalized size	1	1.00	0.19	0.56	0.00	0.00	0.16	0.00	-0.00
time (sec)	N/A	0.116	0.009	0.021	0.000	0.873	1.799	0.000	0.000

Problem 873	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	72	123	0	0	36	0	37
normalized size	1	1.00	0.57	0.97	0.00	0.00	0.28	0.00	0.29
time (sec)	N/A	0.028	0.038	0.012	0.000	0.893	1.333	0.000	1.155

Problem 874	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	27	35	34	23	41	41	23
normalized size	1	1.00	0.59	0.76	0.74	0.50	0.89	0.89	0.50
time (sec)	N/A	0.020	0.009	0.007	1.385	0.708	1.564	0.171	1.200

Problem 875	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	20	28	23	16	24	23	16
normalized size	1	1.00	0.65	0.90	0.74	0.52	0.77	0.74	0.52
time (sec)	N/A	0.015	0.006	0.004	1.187	0.637	1.131	0.153	1.130

Problem 876	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	23	11	11	10	11	11
normalized size	1	1.00	1.00	1.53	0.73	0.73	0.67	0.73	0.73
time (sec)	N/A	0.003	0.002	0.006	1.248	0.765	0.341	0.148	1.224

Problem 877	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	29	29	24	31	12
normalized size	1	1.00	1.00	0.81	1.81	1.81	1.50	1.94	0.75
time (sec)	N/A	0.009	0.003	0.010	1.359	0.677	1.918	0.150	1.314

Problem 878	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	43	50	73	45	27
normalized size	1	1.00	1.00	0.80	1.23	1.43	2.09	1.29	0.77
time (sec)	N/A	0.014	0.009	0.013	1.333	0.843	2.197	0.150	1.214

Problem 879	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	22	44	33	61	21	-1
normalized size	1	1.00	1.00	0.81	1.63	1.22	2.26	0.78	-0.04
time (sec)	N/A	0.012	0.007	0.012	2.973	0.862	3.177	0.180	0.000

Problem 880	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	16	18	19	6	16
normalized size	1	1.00	1.00	0.88	2.00	2.25	2.38	0.75	2.00
time (sec)	N/A	0.003	0.003	0.009	2.864	0.818	1.803	0.162	0.159

Problem 881	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	26	14	14	34	35	14
normalized size	1	1.00	1.00	1.44	0.78	0.78	1.89	1.94	0.78
time (sec)	N/A	0.003	0.003	0.006	1.333	0.713	1.183	0.170	1.121

Problem 882	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	25	33	29	21	63	73	21
normalized size	1	1.00	0.68	0.89	0.78	0.57	1.70	1.97	0.57
time (sec)	N/A	0.007	0.005	0.005	1.326	0.880	2.245	0.196	1.155

Problem 883	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	30	38	43	26	104	109	26
normalized size	1	1.00	0.55	0.69	0.78	0.47	1.89	1.98	0.47
time (sec)	N/A	0.012	0.008	0.005	1.260	0.850	1.689	0.174	1.189

Problem 884	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	42	59	0	0	31	0	-1
normalized size	1	1.00	0.98	1.37	0.00	0.00	0.72	0.00	-0.02
time (sec)	N/A	0.009	0.014	0.010	0.000	0.828	1.011	0.000	0.000

Problem 885	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	32	45	0	0	31	0	-1
normalized size	1	1.00	1.28	1.80	0.00	0.00	1.24	0.00	-0.04
time (sec)	N/A	0.005	0.007	0.008	0.000	0.807	1.165	0.000	0.000

Problem 886	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	31	0	0	29	0	10
normalized size	1	1.00	1.00	7.75	0.00	0.00	7.25	0.00	2.50
time (sec)	N/A	0.001	0.012	0.005	0.000	1.226	0.830	0.000	1.066

Problem 887	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	20	47	0	0	34	0	-1
normalized size	1	1.00	0.74	1.74	0.00	0.00	1.26	0.00	-0.04
time (sec)	N/A	0.005	0.003	0.012	0.000	0.937	0.965	0.000	0.000

Problem 888	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	20	61	0	0	37	0	-1
normalized size	1	1.00	0.44	1.36	0.00	0.00	0.82	0.00	-0.02
time (sec)	N/A	0.009	0.003	0.013	0.000	0.911	1.963	0.000	0.000

Problem 889	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	43	68	0	0	31	0	-1
normalized size	1	1.00	0.81	1.28	0.00	0.00	0.58	0.00	-0.02
time (sec)	N/A	0.021	0.030	0.010	0.000	0.822	1.210	0.000	0.000

Problem 890	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	34	54	0	0	31	0	-1
normalized size	1	1.00	0.97	1.54	0.00	0.00	0.89	0.00	-0.03
time (sec)	N/A	0.016	0.007	0.010	0.000	0.861	1.545	0.000	0.000

Problem 891	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	20	39	0	0	31	0	-1
normalized size	1	1.00	1.82	3.55	0.00	0.00	2.82	0.00	-0.09
time (sec)	N/A	0.012	0.003	0.007	0.000	0.764	1.537	0.000	0.000

Problem 892	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	18	53	0	0	32	0	13
normalized size	1	1.00	0.67	1.96	0.00	0.00	1.19	0.00	0.48
time (sec)	N/A	0.016	0.003	0.010	0.000	0.847	1.128	0.000	1.124

Problem 893	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	20	68	0	0	37	0	-1
normalized size	1	1.00	0.38	1.28	0.00	0.00	0.70	0.00	-0.02
time (sec)	N/A	0.020	0.003	0.013	0.000	0.785	1.013	0.000	0.000

Problem 894	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	27	33	32	28	39	32	25
normalized size	1	1.00	0.64	0.79	0.76	0.67	0.93	0.76	0.60
time (sec)	N/A	0.018	0.009	0.006	1.329	0.852	1.722	0.170	1.187

Problem 895	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	22	28	23	23	22	23	16
normalized size	1	1.00	0.71	0.90	0.74	0.74	0.71	0.74	0.52
time (sec)	N/A	0.014	0.007	0.004	1.296	0.839	1.202	0.149	1.166

Problem 896	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	23	11	18	10	11	11
normalized size	1	1.00	1.00	1.53	0.73	1.20	0.67	0.73	0.73
time (sec)	N/A	0.004	0.003	0.006	1.364	0.780	0.585	0.186	1.192

Problem 897	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	30	68	40	58	228	42	24
normalized size	1	1.00	0.94	2.12	1.25	1.81	7.12	1.31	0.75
time (sec)	N/A	0.014	0.006	0.020	1.307	0.987	2.698	0.152	1.223

Problem 898	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	53	30	82	61	78	95	63	38
normalized size	1	1.06	0.60	1.64	1.22	1.56	1.90	1.26	0.76
time (sec)	N/A	0.022	0.005	0.017	1.367	0.635	4.870	0.172	1.226

Problem 899	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	41	76	60	52	82	33	-1
normalized size	1	1.00	0.91	1.69	1.33	1.16	1.82	0.73	-0.02
time (sec)	N/A	0.019	0.022	0.017	2.954	0.862	4.162	0.187	0.000

Problem 900	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	62	31	46	46	28	-1
normalized size	1	1.00	0.96	2.30	1.15	1.70	1.70	1.04	-0.04
time (sec)	N/A	0.012	0.024	0.017	2.911	1.088	2.294	0.177	0.000

Problem 901	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	26	14	21	32	21	14
normalized size	1	1.00	1.00	1.44	0.78	1.17	1.78	1.17	0.78
time (sec)	N/A	0.003	0.003	0.005	1.276	0.863	1.405	0.182	1.050

Problem 902	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	25	33	29	29	90	56	18
normalized size	1	1.00	0.74	0.97	0.85	0.85	2.65	1.65	0.53
time (sec)	N/A	0.007	0.005	0.005	1.320	0.877	1.863	0.186	1.141

Problem 903	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	30	38	43	34	151	94	30
normalized size	1	1.00	0.55	0.69	0.78	0.62	2.75	1.71	0.55
time (sec)	N/A	0.012	0.006	0.005	1.341	1.118	2.360	0.170	1.199

Problem 904	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	54	71	0	0	31	0	-1
normalized size	1	1.00	0.89	1.16	0.00	0.00	0.51	0.00	-0.02
time (sec)	N/A	0.015	0.011	0.010	0.000	0.860	1.736	0.000	0.000

Problem 905	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	49	57	0	0	31	0	-1
normalized size	1	1.00	1.14	1.33	0.00	0.00	0.72	0.00	-0.02
time (sec)	N/A	0.010	0.012	0.013	0.000	0.620	1.143	0.000	0.000

Problem 906	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	32	45	0	0	31	0	-1
normalized size	1	1.00	1.28	1.80	0.00	0.00	1.24	0.00	-0.04
time (sec)	N/A	0.005	0.009	0.012	0.000	1.120	1.278	0.000	0.000

Problem 907	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	30	45	0	0	29	0	10
normalized size	1	1.00	1.20	1.80	0.00	0.00	1.16	0.00	0.40
time (sec)	N/A	0.003	0.018	0.010	0.000	0.789	1.018	0.000	1.072

Problem 908	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	20	59	0	0	34	0	-1
normalized size	1	1.00	0.44	1.31	0.00	0.00	0.76	0.00	-0.02
time (sec)	N/A	0.010	0.003	0.016	0.000	0.843	1.393	0.000	0.000

Problem 909	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	20	73	0	0	37	0	-1
normalized size	1	1.00	0.32	1.16	0.00	0.00	0.59	0.00	-0.02
time (sec)	N/A	0.015	0.003	0.017	0.000	0.912	1.500	0.000	0.000

Problem 910	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	56	82	0	0	31	0	-1
normalized size	1	1.00	0.79	1.15	0.00	0.00	0.44	0.00	-0.01
time (sec)	N/A	0.027	0.022	0.013	0.000	0.884	1.658	0.000	0.000

Problem 911	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	49	68	0	0	31	0	-1
normalized size	1	1.00	0.92	1.28	0.00	0.00	0.58	0.00	-0.02
time (sec)	N/A	0.021	0.027	0.013	0.000	0.914	1.631	0.000	0.000

Problem 912	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	31	54	0	0	31	0	-1
normalized size	1	1.00	0.89	1.54	0.00	0.00	0.89	0.00	-0.03
time (sec)	N/A	0.017	0.009	0.013	0.000	0.828	1.986	0.000	0.000

Problem 913	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	20	54	0	0	31	0	-1
normalized size	1	1.00	0.57	1.54	0.00	0.00	0.89	0.00	-0.03
time (sec)	N/A	0.016	0.003	0.010	0.000	0.703	0.857	0.000	0.000

Problem 914	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	18	68	0	0	32	0	13
normalized size	1	1.00	0.34	1.28	0.00	0.00	0.60	0.00	0.25
time (sec)	N/A	0.020	0.003	0.015	0.000	0.882	1.262	0.000	1.156

Problem 915	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	20	82	0	0	37	0	-1
normalized size	1	1.00	0.28	1.15	0.00	0.00	0.52	0.00	-0.01
time (sec)	N/A	0.026	0.007	0.017	0.000	1.214	1.480	0.000	0.000

Problem 916	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	51	64	0	0	29	0	10
normalized size	1	1.00	1.24	1.56	0.00	0.00	0.71	0.00	0.24
time (sec)	N/A	0.006	0.018	0.010	0.000	0.896	1.365	0.000	1.075

Problem 917	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	25	22	28	21	39	28	20
normalized size	1	1.00	0.62	0.55	0.70	0.52	0.98	0.70	0.50
time (sec)	N/A	0.014	0.008	0.005	1.352	0.850	2.189	0.151	1.163

Problem 918	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	18	15	19	14	22	19	14
normalized size	1	1.00	0.67	0.56	0.70	0.52	0.81	0.70	0.52
time (sec)	N/A	0.010	0.006	0.005	1.347	0.802	0.827	0.149	1.154

Problem 919	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	8	9	9
normalized size	1	1.00	1.00	0.77	0.69	0.69	0.62	0.69	0.69
time (sec)	N/A	0.002	0.002	0.003	1.307	0.808	0.192	0.183	1.044

Problem 920	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	25	25	8	25	10
normalized size	1	1.00	1.00	0.79	1.79	1.79	0.57	1.79	0.71
time (sec)	N/A	0.007	0.003	0.010	1.286	0.908	1.110	0.149	0.054

Problem 921	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	24	37	44	22	37	23
normalized size	1	1.00	1.00	0.77	1.19	1.42	0.71	1.19	0.74
time (sec)	N/A	0.011	0.016	0.010	1.383	0.722	2.102	0.150	1.213

Problem 922	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	58	29	19	29	-1
normalized size	1	1.00	1.00	0.80	2.32	1.16	0.76	1.16	-0.04
time (sec)	N/A	0.009	0.006	0.011	1.419	0.889	2.117	0.163	0.000

Problem 923	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	33	16	5	16	6
normalized size	1	1.00	1.00	0.88	4.12	2.00	0.62	2.00	0.75
time (sec)	N/A	0.003	0.002	0.007	1.366	0.871	1.022	0.163	1.018

Problem 924	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	16	12	19	12
normalized size	1	1.00	1.00	0.81	0.75	1.00	0.75	1.19	0.75
time (sec)	N/A	0.003	0.026	0.004	1.346	0.721	0.791	0.156	1.113

Problem 925	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	23	20	25	26	26	40	19
normalized size	1	1.00	0.70	0.61	0.76	0.79	0.79	1.21	0.58
time (sec)	N/A	0.006	0.005	0.004	1.324	0.767	1.068	0.164	1.154

Problem 926	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	28	25	37	31	44	57	24
normalized size	1	1.00	0.57	0.51	0.76	0.63	0.90	1.16	0.49
time (sec)	N/A	0.010	0.006	0.006	1.318	0.608	1.544	0.183	1.220

Problem 927	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	40	84	0	0	29	0	-1
normalized size	1	1.00	0.54	1.14	0.00	0.00	0.39	0.00	-0.01
time (sec)	N/A	0.014	0.012	0.009	0.000	0.798	1.540	0.000	0.000

Problem 928	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	32	72	0	0	29	0	-1
normalized size	1	1.00	0.55	1.24	0.00	0.00	0.50	0.00	-0.02
time (sec)	N/A	0.009	0.006	0.007	0.000	0.850	0.770	0.000	0.000

Problem 929	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	21	60	0	0	27	0	12
normalized size	1	1.00	0.49	1.40	0.00	0.00	0.63	0.00	0.28
time (sec)	N/A	0.003	0.025	0.005	0.000	0.957	0.792	0.000	1.051

Problem 930	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	22	74	0	0	32	0	-1
normalized size	1	1.00	0.37	1.23	0.00	0.00	0.53	0.00	-0.02
time (sec)	N/A	0.008	0.003	0.013	0.000	0.773	0.901	0.000	0.000

Problem 931	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	22	86	0	0	36	0	-1
normalized size	1	1.00	0.29	1.13	0.00	0.00	0.47	0.00	-0.01
time (sec)	N/A	0.016	0.003	0.012	0.000	0.852	1.676	0.000	0.000

Problem 932	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	42	107	0	0	29	0	-1
normalized size	1	1.00	0.30	0.76	0.00	0.00	0.21	0.00	-0.01
time (sec)	N/A	0.031	0.023	0.006	0.000	0.871	1.161	0.000	0.000

Problem 933	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	34	95	0	0	29	0	-1
normalized size	1	1.00	0.27	0.77	0.00	0.00	0.23	0.00	-0.01
time (sec)	N/A	0.022	0.007	0.008	0.000	0.593	0.908	0.000	0.000

Problem 934	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	22	82	0	0	29	0	-1
normalized size	1	1.00	0.21	0.80	0.00	0.00	0.28	0.00	-0.01
time (sec)	N/A	0.013	0.003	0.006	0.000	0.845	0.938	0.000	0.000

Problem 935	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	20	95	0	0	31	0	15
normalized size	1	1.00	0.17	0.81	0.00	0.00	0.26	0.00	0.13
time (sec)	N/A	0.015	0.008	0.008	0.000	0.678	1.279	0.000	1.139

Problem 936	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	22	107	0	0	36	0	-1
normalized size	1	1.00	0.16	0.76	0.00	0.00	0.26	0.00	-0.01
time (sec)	N/A	0.024	0.003	0.012	0.000	0.897	1.123	0.000	0.000

Problem 937	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	23	20	28	19	39	28	25
normalized size	1	1.00	0.61	0.53	0.74	0.50	1.03	0.74	0.66
time (sec)	N/A	0.014	0.024	0.005	1.274	0.807	2.261	0.152	1.317

Problem 938	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	18	15	19	14	22	19	14
normalized size	1	1.00	0.67	0.56	0.70	0.52	0.81	0.70	0.52
time (sec)	N/A	0.010	0.006	0.005	1.310	0.868	1.172	0.151	1.158

Problem 939	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	12	9	9
normalized size	1	1.00	1.00	0.77	0.69	0.69	0.92	0.69	0.69
time (sec)	N/A	0.003	0.002	0.004	1.279	0.657	0.434	0.154	1.175

Problem 940	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	21	34	52	87	34	20
normalized size	1	1.00	0.93	0.75	1.21	1.86	3.11	1.21	0.71
time (sec)	N/A	0.011	0.005	0.011	1.284	0.871	1.890	0.167	1.190

Problem 941	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	47	26	33	53	66	42	53	32
normalized size	1	1.07	0.59	0.75	1.20	1.50	0.95	1.20	0.73
time (sec)	N/A	0.015	0.005	0.013	1.319	0.829	3.071	0.151	1.237

Problem 942	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	37	32	73	54	36	34	-1
normalized size	1	1.00	0.90	0.78	1.78	1.32	0.88	0.83	-0.02
time (sec)	N/A	0.015	0.011	0.013	1.378	0.952	4.012	0.165	0.000

Problem 943	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	20	45	45	19	29	-1
normalized size	1	1.00	0.92	0.80	1.80	1.80	0.76	1.16	-0.04
time (sec)	N/A	0.009	0.020	0.011	1.303	0.947	2.297	0.180	0.000

Problem 944	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	25	12	12	12
normalized size	1	1.00	1.00	0.81	0.75	1.56	0.75	0.75	0.75
time (sec)	N/A	0.002	0.003	0.003	1.281	0.839	0.659	0.159	1.049

Problem 945	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	23	20	25	37	42	32	17
normalized size	1	1.00	0.74	0.65	0.81	1.19	1.35	1.03	0.55
time (sec)	N/A	0.006	0.005	0.005	1.361	0.853	0.896	0.173	1.136

Problem 946	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	28	25	36	42	70	70	28
normalized size	1	1.00	0.57	0.51	0.73	0.86	1.43	1.43	0.57
time (sec)	N/A	0.010	0.006	0.003	1.256	0.872	2.251	0.179	1.230

Problem 947	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	52	94	0	0	29	0	-1
normalized size	1	1.00	0.58	1.04	0.00	0.00	0.32	0.00	-0.01
time (sec)	N/A	0.020	0.022	0.007	0.000	1.095	1.942	0.000	0.000

Problem 948	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	47	82	0	0	29	0	-1
normalized size	1	1.00	0.64	1.11	0.00	0.00	0.39	0.00	-0.01
time (sec)	N/A	0.014	0.009	0.010	0.000	0.960	1.400	0.000	0.000

Problem 949	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	32	72	0	0	29	0	-1
normalized size	1	1.00	0.55	1.24	0.00	0.00	0.50	0.00	-0.02
time (sec)	N/A	0.009	0.020	0.007	0.000	0.857	0.825	0.000	0.000

Problem 950	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	30	72	0	0	27	0	12
normalized size	1	1.00	0.52	1.24	0.00	0.00	0.47	0.00	0.21
time (sec)	N/A	0.007	0.023	0.005	0.000	0.731	0.856	0.000	0.081

Problem 951	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	22	84	0	0	32	0	-1
normalized size	1	1.00	0.29	1.11	0.00	0.00	0.42	0.00	-0.01
time (sec)	N/A	0.015	0.003	0.013	0.000	0.923	1.394	0.000	0.000

Problem 952	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	22	96	0	0	36	0	-1
normalized size	1	1.00	0.24	1.04	0.00	0.00	0.39	0.00	-0.01
time (sec)	N/A	0.021	0.003	0.018	0.000	0.711	2.060	0.000	0.000

Problem 953	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	54	119	0	0	29	0	-1
normalized size	1	1.00	0.35	0.76	0.00	0.00	0.19	0.00	-0.01
time (sec)	N/A	0.040	0.015	0.010	0.000	0.896	2.241	0.000	0.000

Problem 954	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	47	107	0	0	29	0	-1
normalized size	1	1.00	0.34	0.76	0.00	0.00	0.21	0.00	-0.01
time (sec)	N/A	0.032	0.018	0.009	0.000	0.889	1.195	0.000	0.000

Problem 955	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	31	95	0	0	29	0	-1
normalized size	1	1.00	0.25	0.77	0.00	0.00	0.23	0.00	-0.01
time (sec)	N/A	0.021	0.009	0.008	0.000	0.780	0.969	0.000	0.000

Problem 956	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	22	95	0	0	29	0	-1
normalized size	1	1.00	0.18	0.77	0.00	0.00	0.23	0.00	-0.01
time (sec)	N/A	0.021	0.003	0.007	0.000	0.822	0.890	0.000	0.000

Problem 957	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	20	107	0	0	31	0	15
normalized size	1	1.00	0.14	0.76	0.00	0.00	0.22	0.00	0.11
time (sec)	N/A	0.025	0.003	0.013	0.000	0.849	1.462	0.000	1.143

Problem 958	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	22	119	0	0	36	0	-1
normalized size	1	1.00	0.14	0.76	0.00	0.00	0.23	0.00	-0.01
time (sec)	N/A	0.034	0.003	0.014	0.000	0.831	1.822	0.000	0.000

Problem 959	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	49	82	0	0	27	0	12
normalized size	1	1.00	0.68	1.14	0.00	0.00	0.38	0.00	0.17
time (sec)	N/A	0.010	0.019	0.008	0.000	0.892	1.028	0.000	1.069

Problem 960	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	20	28	23	16	26	23	16
normalized size	1	1.00	0.69	0.97	0.79	0.55	0.90	0.79	0.55
time (sec)	N/A	0.014	0.008	0.007	1.281	0.741	0.658	0.149	1.187

Problem 961	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	44	33	80	23	-1
normalized size	1	1.00	1.00	0.83	1.52	1.14	2.76	0.79	-0.03
time (sec)	N/A	0.014	0.028	0.015	2.967	0.785	2.894	0.166	0.000

Problem 962	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	23	11	11	10	11	11
normalized size	1	1.00	1.00	1.53	0.73	0.73	0.67	0.73	0.73
time (sec)	N/A	0.003	0.003	0.007	1.296	0.848	0.321	0.158	1.228

Problem 963	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	16	18	24	8	16
normalized size	1	1.00	1.00	0.75	1.33	1.50	2.00	0.67	1.33
time (sec)	N/A	0.004	0.002	0.010	2.957	0.863	1.195	0.170	0.167

Problem 964	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	29	29	24	31	14
normalized size	1	1.00	1.00	0.75	1.45	1.45	1.20	1.55	0.70
time (sec)	N/A	0.010	0.003	0.010	1.308	0.843	1.874	0.147	1.277

Problem 965	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	26	14	14	32	35	14
normalized size	1	1.00	1.00	1.44	0.78	0.78	1.78	1.94	0.78
time (sec)	N/A	0.003	0.008	0.005	1.346	0.889	0.872	0.169	1.140

Problem 966	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	37	30	43	50	73	45	29
normalized size	1	1.00	0.95	0.77	1.10	1.28	1.87	1.15	0.74
time (sec)	N/A	0.016	0.011	0.012	1.321	0.877	3.011	0.150	1.195

Problem 967	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	25	31	29	19	65	73	19
normalized size	1	1.00	0.68	0.84	0.78	0.51	1.76	1.97	0.51
time (sec)	N/A	0.008	0.011	0.005	1.314	0.827	1.584	0.170	1.137

Problem 968	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	38	58	0	0	32	0	-1
normalized size	1	1.00	0.88	1.35	0.00	0.00	0.74	0.00	-0.02
time (sec)	N/A	0.020	0.006	0.010	0.000	0.868	0.987	0.000	0.000

Problem 969	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	36	47	0	0	32	0	-1
normalized size	1	1.00	1.24	1.62	0.00	0.00	1.10	0.00	-0.03
time (sec)	N/A	0.006	0.006	0.007	0.000	0.833	0.985	0.000	0.000

Problem 970	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	24	43	0	0	32	0	-1
normalized size	1	1.00	1.14	2.05	0.00	0.00	1.52	0.00	-0.05
time (sec)	N/A	0.015	0.003	0.007	0.000	0.876	1.314	0.000	0.000

Problem 971	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	34	0	0	31	0	13
normalized size	1	1.00	1.00	2.83	0.00	0.00	2.58	0.00	1.08
time (sec)	N/A	0.002	0.014	0.006	0.000	0.939	0.919	0.000	0.073

Problem 972	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	24	58	0	0	34	0	33
normalized size	1	1.00	0.56	1.35	0.00	0.00	0.79	0.00	0.77
time (sec)	N/A	0.020	0.003	0.010	0.000	0.844	0.981	0.000	1.250

Problem 973	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	24	49	0	0	36	0	-1
normalized size	1	1.00	0.77	1.58	0.00	0.00	1.16	0.00	-0.03
time (sec)	N/A	0.006	0.003	0.012	0.000	0.929	1.112	0.000	0.000

Problem 974	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	33	16	24	16	14
normalized size	1	1.00	1.00	0.83	1.83	0.89	1.33	0.89	0.78
time (sec)	N/A	0.007	0.003	0.010	1.348	1.139	1.957	0.156	0.084

Problem 975	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	33	16	7	16	8
normalized size	1	1.00	1.00	0.75	2.75	1.33	0.58	1.33	0.67
time (sec)	N/A	0.004	0.003	0.010	1.360	0.982	1.476	0.196	0.035

Problem 976	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	24	10	10
normalized size	1	1.00	1.00	0.79	0.71	0.71	1.71	0.71	0.71
time (sec)	N/A	0.007	0.003	0.011	2.995	1.056	1.558	0.192	0.152

Problem 977	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	44	45	0	0	27	0	-1
normalized size	1	1.00	0.61	0.62	0.00	0.00	0.38	0.00	-0.01
time (sec)	N/A	0.009	0.009	0.010	0.000	0.593	1.463	0.000	0.000

Problem 978	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	25	34	0	0	26	0	26
normalized size	1	1.00	0.46	0.63	0.00	0.00	0.48	0.00	0.48
time (sec)	N/A	0.004	0.027	0.006	0.000	0.549	0.774	0.000	1.110

Problem 979	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	40	47	0	0	31	0	-1
normalized size	1	1.00	0.54	0.64	0.00	0.00	0.42	0.00	-0.01
time (sec)	N/A	0.009	0.006	0.012	0.000	0.660	1.576	0.000	0.000

Problem 980	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	46	57	0	0	27	0	-1
normalized size	1	1.00	0.31	0.38	0.00	0.00	0.18	0.00	-0.01
time (sec)	N/A	0.023	0.036	0.010	0.000	0.670	1.007	0.000	0.000

Problem 981	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	40	44	0	0	27	0	-1
normalized size	1	1.00	0.32	0.35	0.00	0.00	0.21	0.00	-0.01
time (sec)	N/A	0.013	0.005	0.007	0.000	0.841	0.937	0.000	0.000

Problem 982	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	38	56	0	0	29	0	18
normalized size	1	1.00	0.27	0.40	0.00	0.00	0.21	0.00	0.13
time (sec)	N/A	0.016	0.006	0.010	0.000	0.968	0.967	0.000	1.212

Problem 983	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	29	69	0	0	39	0	-1
normalized size	1	1.00	0.60	1.44	0.00	0.00	0.81	0.00	-0.02
time (sec)	N/A	0.034	0.009	0.044	0.000	0.818	1.284	0.000	0.000

Problem 984	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	30	94	0	0	39	0	-1
normalized size	1	1.00	0.56	1.74	0.00	0.00	0.72	0.00	-0.02
time (sec)	N/A	0.047	0.007	0.014	0.000	0.966	1.386	0.000	0.000

Problem 985	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	20	17	19	19	41	19	16
normalized size	1	1.00	0.74	0.63	0.70	0.70	1.52	0.70	0.59
time (sec)	N/A	0.010	0.006	0.004	1.359	0.734	1.099	0.149	1.100

Problem 986	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	12	9	9
normalized size	1	1.00	1.00	0.77	0.69	0.69	0.92	0.69	0.69
time (sec)	N/A	0.003	0.003	0.004	1.354	1.054	0.562	0.162	1.095

Problem 987	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	10	9	9
normalized size	1	1.00	1.00	0.77	0.69	0.69	0.77	0.69	0.69
time (sec)	N/A	0.003	0.017	0.004	1.305	0.581	0.197	0.151	0.068

Problem 988	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	61	58	81	68	134	71	66
normalized size	1	1.00	0.60	0.57	0.80	0.67	1.33	0.70	0.65
time (sec)	N/A	0.059	0.034	0.010	1.338	0.862	22.687	0.165	1.148

Problem 989	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	64	57	110	57	55
normalized size	1	1.00	0.62	0.59	0.80	0.71	1.38	0.71	0.69
time (sec)	N/A	0.044	0.044	0.007	1.296	0.848	15.769	0.158	1.130

Problem 990	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	47	46	87	43	44
normalized size	1	1.00	0.66	0.61	0.80	0.78	1.47	0.73	0.75
time (sec)	N/A	0.034	0.020	0.006	1.367	0.471	5.619	0.164	1.121

Problem 991	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	25	30	34	63	29	33
normalized size	1	1.00	0.74	0.66	0.79	0.89	1.66	0.76	0.87
time (sec)	N/A	0.023	0.015	0.006	1.357	0.866	2.884	0.165	1.102

Problem 992	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	39	14	14
normalized size	1	1.00	1.00	0.83	0.78	0.78	2.17	0.78	0.78
time (sec)	N/A	0.004	0.004	0.006	1.337	0.756	0.902	0.174	1.134

Problem 993	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	0	66	93	42	183	48
normalized size	1	1.00	1.00	0.00	1.00	1.41	0.64	2.77	0.73
time (sec)	N/A	0.043	0.033	0.149	2.920	0.829	2.246	0.165	1.180

Problem 994	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	37	0	73	180	41	207	55
normalized size	1	1.00	0.49	0.00	0.97	2.40	0.55	2.76	0.73
time (sec)	N/A	0.044	0.025	0.181	3.017	0.705	2.855	0.176	1.277

Problem 995	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	39	0	129	209	41	243	79
normalized size	1	1.00	0.39	0.00	1.28	2.07	0.41	2.41	0.78
time (sec)	N/A	0.059	0.010	0.149	2.932	0.694	2.655	0.178	1.386

Problem 996	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	95	0	0	0	29	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.23	0.00	-0.01
time (sec)	N/A	0.084	0.085	0.149	0.000	0.655	1.784	0.000	0.000

Problem 997	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	64	0	0	0	29	0	-1
normalized size	1	1.00	0.63	0.00	0.00	0.00	0.29	0.00	-0.01
time (sec)	N/A	0.062	0.092	0.139	0.000	0.581	1.322	0.000	0.000

Problem 998	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	51	0	0	0	29	0	-1
normalized size	1	1.00	0.65	0.00	0.00	0.00	0.37	0.00	-0.01
time (sec)	N/A	0.041	0.020	0.138	0.000	0.802	1.850	0.000	0.000

Problem 999	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	51	0	0	0	32	0	-1
normalized size	1	1.00	0.65	0.00	0.00	0.00	0.41	0.00	-0.01
time (sec)	N/A	0.047	0.019	0.138	0.000	0.732	1.096	0.000	0.000

Problem 1000	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	51	0	0	0	34	0	-1
normalized size	1	1.00	0.50	0.00	0.00	0.00	0.34	0.00	-0.01
time (sec)	N/A	0.064	0.029	0.135	0.000	0.658	2.780	0.000	0.000

Problem 1001	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	51	0	0	0	34	0	-1
normalized size	1	1.00	0.41	0.00	0.00	0.00	0.27	0.00	-0.01
time (sec)	N/A	0.083	0.032	0.135	0.000	0.733	3.449	0.000	0.000

Problem 1002	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	64	0	152	220	39	0	-1
normalized size	1	1.00	0.62	0.00	1.48	2.14	0.38	0.00	-0.01
time (sec)	N/A	0.038	0.061	0.138	2.976	0.689	3.099	0.000	0.000

Problem 1003	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	51	0	101	191	39	0	-1
normalized size	1	1.00	0.66	0.00	1.31	2.48	0.51	0.00	-0.01
time (sec)	N/A	0.025	0.052	0.138	2.996	0.786	1.653	0.000	0.000

Problem 1004	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	F(-1)	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	49	0	82	0	41	0	40
normalized size	1	1.00	0.67	0.00	1.12	0.00	0.56	0.00	0.55
time (sec)	N/A	0.024	0.009	0.146	2.871	0.000	1.949	0.000	1.229

Problem 1005	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	68	0	17
normalized size	1	1.00	1.00	0.86	0.81	0.81	3.24	0.00	0.81
time (sec)	N/A	0.005	0.005	0.004	1.326	1.135	1.775	0.000	1.288

Problem 1006	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	31	28	35	38	109	0	37
normalized size	1	1.00	0.70	0.64	0.80	0.86	2.48	0.00	0.84
time (sec)	N/A	0.010	0.010	0.005	1.343	1.035	2.838	0.000	1.300

Problem 1007	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	42	39	52	49	520	0	73
normalized size	1	1.00	0.62	0.57	0.76	0.72	7.65	0.00	1.07
time (sec)	N/A	0.019	0.011	0.005	1.326	0.979	3.261	0.000	1.478

Problem 1008	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	53	50	69	60	847	0	93
normalized size	1	1.00	0.58	0.54	0.75	0.65	9.21	0.00	1.01
time (sec)	N/A	0.027	0.014	0.007	1.359	0.827	5.853	0.000	1.695

Problem 1009	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	105	0	0	0	39	0	-1
normalized size	1	1.00	0.70	0.00	0.00	0.00	0.26	0.00	-0.01
time (sec)	N/A	0.074	0.078	0.138	0.000	0.946	2.914	0.000	0.000

Problem 1010	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	93	0	0	0	39	0	-1
normalized size	1	1.00	0.74	0.00	0.00	0.00	0.31	0.00	-0.01
time (sec)	N/A	0.056	0.056	0.142	0.000	0.900	2.827	0.000	0.000

Problem 1011	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	62	0	0	0	39	0	-1
normalized size	1	1.00	0.61	0.00	0.00	0.00	0.38	0.00	-0.01
time (sec)	N/A	0.045	0.074	0.159	0.000	0.928	2.042	0.000	0.000

Problem 1012	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	46	0	0	0	37	0	37
normalized size	1	1.00	0.58	0.00	0.00	0.00	0.46	0.00	0.46
time (sec)	N/A	0.032	0.006	0.154	0.000	0.860	1.447	0.000	1.032

Problem 1013	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	51	0	0	0	31	0	-1
normalized size	1	1.00	0.62	0.00	0.00	0.00	0.38	0.00	-0.01
time (sec)	N/A	0.034	0.009	0.138	0.000	0.914	1.790	0.000	0.000

Problem 1014	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	51	0	0	0	31	0	-1
normalized size	1	1.00	0.49	0.00	0.00	0.00	0.30	0.00	-0.01
time (sec)	N/A	0.047	0.010	0.138	0.000	1.021	2.435	0.000	0.000

Problem 1015	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	51	0	0	0	46	0	-1
normalized size	1	1.00	0.40	0.00	0.00	0.00	0.36	0.00	-0.01
time (sec)	N/A	0.060	0.010	0.135	0.000	0.864	2.839	0.000	0.000

Problem 1016	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	51	0	0	0	46	0	-1
normalized size	1	1.00	0.34	0.00	0.00	0.00	0.30	0.00	-0.01
time (sec)	N/A	0.078	0.012	0.133	0.000	0.914	4.380	0.000	0.000

Problem 1017	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	61	58	81	68	136	163	66
normalized size	1	1.00	0.60	0.57	0.80	0.67	1.35	1.61	0.65
time (sec)	N/A	0.057	0.047	0.010	1.355	0.557	39.716	0.166	1.126

Problem 1018	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	64	57	110	134	55
normalized size	1	1.00	0.62	0.59	0.80	0.71	1.38	1.68	0.69
time (sec)	N/A	0.047	0.025	0.008	1.333	0.650	22.762	0.198	1.091

Problem 1019	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	47	46	87	106	44
normalized size	1	1.00	0.66	0.61	0.80	0.78	1.47	1.80	0.75
time (sec)	N/A	0.034	0.035	0.006	1.301	0.773	15.893	0.161	1.076

Problem 1020	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	25	30	35	65	78	33
normalized size	1	1.00	0.74	0.66	0.79	0.92	1.71	2.05	0.87
time (sec)	N/A	0.023	0.023	0.005	1.359	0.805	8.263	0.156	1.085

Problem 1021	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	39	14	14
normalized size	1	1.00	1.00	0.83	0.78	0.78	2.17	0.78	0.78
time (sec)	N/A	0.005	0.004	0.003	1.274	0.889	1.797	0.182	1.074

Problem 1022	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	0	71	132	44	185	50
normalized size	1	1.00	1.00	0.00	1.01	1.89	0.63	2.64	0.71
time (sec)	N/A	0.046	0.025	0.154	2.921	0.925	1.991	0.171	1.137

Problem 1023	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	37	0	74	185	39	209	55
normalized size	1	1.00	0.49	0.00	0.99	2.47	0.52	2.79	0.73
time (sec)	N/A	0.047	0.027	0.138	3.013	0.655	3.048	0.211	1.261

Problem 1024	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	39	0	125	209	41	243	79
normalized size	1	1.00	0.39	0.00	1.24	2.07	0.41	2.41	0.78
time (sec)	N/A	0.059	0.010	0.145	2.995	0.937	2.981	0.182	1.421

Problem 1025	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	95	0	0	0	29	0	-1
normalized size	1	1.00	0.64	0.00	0.00	0.00	0.19	0.00	-0.01
time (sec)	N/A	0.098	0.088	0.151	0.000	0.894	2.523	0.000	0.000

Problem 1026	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	64	0	0	0	29	0	-1
normalized size	1	1.00	0.51	0.00	0.00	0.00	0.23	0.00	-0.01
time (sec)	N/A	0.075	0.087	0.138	0.000	0.880	3.895	0.000	0.000

Problem 1027	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	51	0	0	0	29	0	-1
normalized size	1	1.00	0.52	0.00	0.00	0.00	0.30	0.00	-0.01
time (sec)	N/A	0.050	0.009	0.134	0.000	0.763	1.899	0.000	0.000

Problem 1028	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	51	0	0	0	32	0	-1
normalized size	1	1.00	0.52	0.00	0.00	0.00	0.33	0.00	-0.01
time (sec)	N/A	0.054	0.011	0.135	0.000	0.989	1.246	0.000	0.000

Problem 1029	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	51	0	0	0	34	0	-1
normalized size	1	1.00	0.41	0.00	0.00	0.00	0.27	0.00	-0.01
time (sec)	N/A	0.073	0.016	0.138	0.000	0.917	3.167	0.000	0.000

Problem 1030	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	51	0	0	0	34	0	-1
normalized size	1	1.00	0.34	0.00	0.00	0.00	0.23	0.00	-0.01
time (sec)	N/A	0.094	0.012	0.140	0.000	0.914	2.217	0.000	0.000

Problem 1031	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	105	0	225	250	39	0	-1
normalized size	1	1.00	0.70	0.00	1.51	1.68	0.26	0.00	-0.01
time (sec)	N/A	0.061	0.102	0.136	3.080	0.900	10.400	0.000	0.000

Problem 1032	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	94	0	189	239	39	0	-1
normalized size	1	1.00	0.75	0.00	1.51	1.91	0.31	0.00	-0.01
time (sec)	N/A	0.045	0.053	0.144	2.951	0.937	4.691	0.000	0.000

Problem 1033	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	62	0	148	218	39	0	-1
normalized size	1	1.00	0.61	0.00	1.47	2.16	0.39	0.00	-0.01
time (sec)	N/A	0.032	0.069	0.137	3.028	0.937	4.294	0.000	0.000

Problem 1034	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	46	0	102	192	37	0	37
normalized size	1	1.00	0.61	0.00	1.36	2.56	0.49	0.00	0.49
time (sec)	N/A	0.019	0.006	0.154	2.895	0.937	1.786	0.000	1.058

Problem 1035	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	51	0	85	0	42	0	-1
normalized size	1	1.00	0.68	0.00	1.13	0.00	0.56	0.00	-0.01
time (sec)	N/A	0.022	0.010	0.142	3.028	0.000	3.342	0.000	0.000

Problem 1036	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	68	0	17
normalized size	1	1.00	1.00	0.86	0.81	0.81	3.24	0.00	0.81
time (sec)	N/A	0.004	0.005	0.004	1.362	1.226	3.180	0.000	1.247

Problem 1037	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	31	28	35	38	110	0	56
normalized size	1	1.00	0.70	0.64	0.80	0.86	2.50	0.00	1.27
time (sec)	N/A	0.011	0.010	0.005	1.339	1.166	3.086	0.000	1.334

Problem 1038	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	42	39	52	49	520	0	73
normalized size	1	1.00	0.62	0.57	0.76	0.72	7.65	0.00	1.07
time (sec)	N/A	0.020	0.011	0.007	1.296	1.046	7.157	0.000	1.547

Problem 1039	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	53	50	69	60	847	0	93
normalized size	1	1.00	0.58	0.54	0.75	0.65	9.21	0.00	1.01
time (sec)	N/A	0.028	0.014	0.007	1.368	1.059	14.531	0.000	1.811

Problem 1040	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	96	0	0	0	39	0	-1
normalized size	1	1.00	0.64	0.00	0.00	0.00	0.26	0.00	-0.01
time (sec)	N/A	0.071	0.066	0.141	0.000	0.854	4.844	0.000	0.000

Problem 1041	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	64	0	0	0	39	0	-1
normalized size	1	1.00	0.51	0.00	0.00	0.00	0.31	0.00	-0.01
time (sec)	N/A	0.057	0.059	0.143	0.000	1.094	3.856	0.000	0.000

Problem 1042	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	51	0	0	0	39	0	-1
normalized size	1	1.00	0.52	0.00	0.00	0.00	0.39	0.00	-0.01
time (sec)	N/A	0.047	0.010	0.141	0.000	0.675	2.502	0.000	0.000

Problem 1043	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	49	0	0	0	41	0	40
normalized size	1	1.00	0.51	0.00	0.00	0.00	0.42	0.00	0.41
time (sec)	N/A	0.047	0.011	0.155	0.000	0.908	2.044	0.000	1.350

Problem 1044	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	51	0	0	0	31	0	-1
normalized size	1	1.00	0.52	0.00	0.00	0.00	0.31	0.00	-0.01
time (sec)	N/A	0.047	0.010	0.141	0.000	0.819	3.581	0.000	0.000

Problem 1045	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	51	0	0	0	31	0	-1
normalized size	1	1.00	0.40	0.00	0.00	0.00	0.25	0.00	-0.01
time (sec)	N/A	0.058	0.010	0.142	0.000	0.871	4.519	0.000	0.000

Problem 1046	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	51	0	0	0	46	0	-1
normalized size	1	1.00	0.34	0.00	0.00	0.00	0.31	0.00	-0.01
time (sec)	N/A	0.071	0.011	0.141	0.000	0.761	4.450	0.000	0.000

Problem 1047	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	61	58	81	79	156	71	75
normalized size	1	1.00	0.60	0.57	0.80	0.78	1.54	0.70	0.74
time (sec)	N/A	0.054	0.039	0.008	1.351	0.871	87.308	0.203	1.181

Problem 1048	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	64	68	134	57	64
normalized size	1	1.00	0.62	0.59	0.80	0.85	1.68	0.71	0.80
time (sec)	N/A	0.044	0.030	0.007	1.314	0.711	38.259	0.178	1.142

Problem 1049	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	47	57	110	43	53
normalized size	1	1.00	0.66	0.61	0.80	0.97	1.86	0.73	0.90
time (sec)	N/A	0.032	0.020	0.005	1.293	0.775	25.288	0.162	1.137

Problem 1050	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	25	30	45	85	29	42
normalized size	1	1.00	0.74	0.66	0.79	1.18	2.24	0.76	1.11
time (sec)	N/A	0.022	0.015	0.005	1.408	0.883	18.342	0.159	1.115

Problem 1051	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	32	61	14	14
normalized size	1	1.00	1.00	0.83	0.78	1.78	3.39	0.78	0.78
time (sec)	N/A	0.004	0.007	0.004	1.616	0.775	11.152	0.173	1.111

Problem 1052	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	77	0	79	130	48	200	64
normalized size	1	1.00	0.93	0.00	0.95	1.57	0.58	2.41	0.77
time (sec)	N/A	0.052	0.043	0.151	2.949	0.548	2.763	0.178	1.161

Problem 1053	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	37	0	89	169	42	221	70
normalized size	1	1.00	0.41	0.00	0.98	1.86	0.46	2.43	0.77
time (sec)	N/A	0.053	0.009	0.161	2.972	0.884	3.891	0.184	1.403

Problem 1054	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	39	0	120	200	41	232	77
normalized size	1	1.00	0.40	0.00	1.22	2.04	0.42	2.37	0.79
time (sec)	N/A	0.057	0.010	0.179	2.887	0.810	4.185	0.190	1.499

Problem 1055	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	81	0	0	0	29	0	-1
normalized size	1	1.00	0.55	0.00	0.00	0.00	0.20	0.00	-0.01
time (sec)	N/A	0.103	0.105	0.141	0.000	0.606	4.834	0.000	0.000

Problem 1056	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	69	0	0	0	29	0	-1
normalized size	1	1.00	0.57	0.00	0.00	0.00	0.24	0.00	-0.01
time (sec)	N/A	0.079	0.099	0.140	0.000	0.891	4.293	0.000	0.000

Problem 1057	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	52	0	0	0	29	0	-1
normalized size	1	1.00	0.53	0.00	0.00	0.00	0.30	0.00	-0.01
time (sec)	N/A	0.055	0.010	0.142	0.000	0.866	2.251	0.000	0.000

Problem 1058	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	52	0	0	0	32	0	-1
normalized size	1	1.00	0.53	0.00	0.00	0.00	0.33	0.00	-0.01
time (sec)	N/A	0.060	0.039	0.143	0.000	0.888	1.483	0.000	0.000

Problem 1059	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	52	0	0	0	34	0	-1
normalized size	1	1.00	0.53	0.00	0.00	0.00	0.35	0.00	-0.01
time (sec)	N/A	0.061	0.012	0.145	0.000	1.624	3.131	0.000	0.000

Problem 1060	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	52	0	0	0	34	0	-1
normalized size	1	1.00	0.43	0.00	0.00	0.00	0.28	0.00	-0.01
time (sec)	N/A	0.078	0.012	0.143	0.000	0.911	6.724	0.000	0.000

Problem 1061	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	52	0	0	0	34	0	-1
normalized size	1	1.00	0.36	0.00	0.00	0.00	0.23	0.00	-0.01
time (sec)	N/A	0.100	0.015	0.148	0.000	0.579	5.554	0.000	0.000

Problem 1062	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	81	0	229	249	39	0	-1
normalized size	1	1.00	0.55	0.00	1.55	1.68	0.26	0.00	-0.01
time (sec)	N/A	0.064	0.107	0.142	2.974	1.024	11.436	0.000	0.000

Problem 1063	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	69	0	191	232	39	0	-1
normalized size	1	1.00	0.56	0.00	1.54	1.87	0.31	0.00	-0.01
time (sec)	N/A	0.050	0.082	0.153	3.073	0.915	5.830	0.000	0.000

Problem 1064	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	52	0	144	209	39	0	-1
normalized size	1	1.00	0.52	0.00	1.44	2.09	0.39	0.00	-0.01
time (sec)	N/A	0.037	0.009	0.144	2.956	0.835	1.937	0.000	0.000

Problem 1065	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	F(-1)	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	50	0	119	0	41	0	40
normalized size	1	1.00	0.53	0.00	1.27	0.00	0.44	0.00	0.43
time (sec)	N/A	0.036	0.010	0.143	3.020	0.000	3.852	0.000	1.529

Problem 1066	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	52	0	97	0	46	0	-1
normalized size	1	1.00	0.57	0.00	1.05	0.00	0.50	0.00	-0.01
time (sec)	N/A	0.035	0.010	0.140	2.951	0.000	4.834	0.000	0.000

Problem 1067	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	35	105	0	17
normalized size	1	1.00	1.00	0.86	0.81	1.67	5.00	0.00	0.81
time (sec)	N/A	0.005	0.014	0.006	1.382	1.196	3.354	0.000	1.460

Problem 1068	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	31	28	35	49	148	0	71
normalized size	1	1.00	0.70	0.64	0.80	1.11	3.36	0.00	1.61
time (sec)	N/A	0.011	0.011	0.006	1.309	0.880	5.332	0.000	1.743

Problem 1069	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	42	39	52	60	609	0	91
normalized size	1	1.00	0.62	0.57	0.76	0.88	8.96	0.00	1.34
time (sec)	N/A	0.020	0.012	0.007	1.328	0.798	11.921	0.000	2.066

Problem 1070	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	53	50	69	71	954	0	111
normalized size	1	1.00	0.58	0.54	0.75	0.77	10.37	0.00	1.21
time (sec)	N/A	0.028	0.015	0.009	1.374	0.822	15.290	0.000	2.518

Problem 1071	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	89	0	0	0	39	0	-1
normalized size	1	1.00	0.52	0.00	0.00	0.00	0.23	0.00	-0.01
time (sec)	N/A	0.088	0.131	0.140	0.000	0.920	3.992	0.000	0.000

Problem 1072	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	76	0	0	0	39	0	-1
normalized size	1	1.00	0.52	0.00	0.00	0.00	0.27	0.00	-0.01
time (sec)	N/A	0.069	0.116	0.143	0.000	0.839	3.373	0.000	0.000

Problem 1073	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	67	0	0	0	39	0	-1
normalized size	1	1.00	0.54	0.00	0.00	0.00	0.32	0.00	-0.01
time (sec)	N/A	0.056	0.087	0.142	0.000	0.819	1.708	0.000	0.000

Problem 1074	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	47	0	0	0	37	0	37
normalized size	1	1.00	0.48	0.00	0.00	0.00	0.38	0.00	0.38
time (sec)	N/A	0.038	0.005	0.161	0.000	0.897	2.698	0.000	1.068

Problem 1075	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	52	0	0	0	42	0	-1
normalized size	1	1.00	0.53	0.00	0.00	0.00	0.42	0.00	-0.01
time (sec)	N/A	0.041	0.033	0.153	0.000	0.896	2.590	0.000	0.000

Problem 1076	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	52	0	0	0	31	0	-1
normalized size	1	1.00	0.51	0.00	0.00	0.00	0.31	0.00	-0.01
time (sec)	N/A	0.044	0.012	0.139	0.000	1.027	3.392	0.000	0.000

Problem 1077	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	52	0	0	0	31	0	-1
normalized size	1	1.00	0.42	0.00	0.00	0.00	0.25	0.00	-0.01
time (sec)	N/A	0.055	0.010	0.143	0.000	1.007	2.397	0.000	0.000

Problem 1078	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	52	0	0	0	46	0	-1
normalized size	1	1.00	0.35	0.00	0.00	0.00	0.31	0.00	-0.01
time (sec)	N/A	0.070	0.011	0.148	0.000	0.983	6.850	0.000	0.000

Problem 1079	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	47	0	143	202	37	0	37
normalized size	1	1.00	0.49	0.00	1.49	2.10	0.39	0.00	0.39
time (sec)	N/A	0.026	0.010	0.160	3.025	0.908	2.005	0.000	1.058

Problem 1080	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	61	58	81	57	116	71	58
normalized size	1	1.00	0.60	0.57	0.80	0.56	1.15	0.70	0.57
time (sec)	N/A	0.053	0.047	0.010	1.381	0.955	24.291	0.156	1.161

Problem 1081	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	64	46	92	57	48
normalized size	1	1.00	0.62	0.59	0.80	0.58	1.15	0.71	0.60
time (sec)	N/A	0.043	0.025	0.007	1.341	0.862	14.847	0.174	1.128

Problem 1082	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	47	35	68	43	36
normalized size	1	1.00	0.66	0.61	0.80	0.59	1.15	0.73	0.61
time (sec)	N/A	0.033	0.029	0.006	1.318	0.656	6.349	0.174	1.134

Problem 1083	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	25	30	24	44	29	26
normalized size	1	1.00	0.74	0.66	0.79	0.63	1.16	0.76	0.68
time (sec)	N/A	0.021	0.018	0.007	1.341	0.882	2.270	0.152	1.107

Problem 1084	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	22	14	14
normalized size	1	1.00	1.00	0.83	0.78	0.78	1.22	0.78	0.78
time (sec)	N/A	0.004	0.004	0.005	1.307	0.784	0.895	0.167	1.099

Problem 1085	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	48	0	57	81	37	186	36
normalized size	1	1.00	0.87	0.00	1.04	1.47	0.67	3.38	0.65
time (sec)	N/A	0.032	0.029	0.141	2.886	0.974	1.246	0.171	1.159

Problem 1086	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	37	0	92	195	39	219	58
normalized size	1	1.00	0.47	0.00	1.18	2.50	0.50	2.81	0.74
time (sec)	N/A	0.045	0.008	0.138	2.944	0.871	2.720	0.199	1.281

Problem 1087	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	39	0	128	219	39	244	82
normalized size	1	1.00	0.38	0.00	1.23	2.11	0.38	2.35	0.79
time (sec)	N/A	0.061	0.008	0.139	2.995	0.752	3.379	0.238	1.360

Problem 1088	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	91	0	0	0	27	0	-1
normalized size	1	1.00	0.60	0.00	0.00	0.00	0.18	0.00	-0.01
time (sec)	N/A	0.099	0.037	0.140	0.000	1.177	1.632	0.000	0.000

Problem 1089	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	80	0	0	0	27	0	-1
normalized size	1	1.00	0.62	0.00	0.00	0.00	0.21	0.00	-0.01
time (sec)	N/A	0.074	0.032	0.139	0.000	0.852	1.212	0.000	0.000

Problem 1090	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	64	0	0	0	27	0	-1
normalized size	1	1.00	0.62	0.00	0.00	0.00	0.26	0.00	-0.01
time (sec)	N/A	0.056	0.020	0.141	0.000	0.903	1.547	0.000	0.000

Problem 1091	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	51	0	0	0	27	0	-1
normalized size	1	1.00	0.69	0.00	0.00	0.00	0.36	0.00	-0.01
time (sec)	N/A	0.038	0.009	0.142	0.000	0.731	1.543	0.000	0.000

Problem 1092	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	51	0	0	0	31	0	-1
normalized size	1	1.00	0.49	0.00	0.00	0.00	0.30	0.00	-0.01
time (sec)	N/A	0.054	0.012	0.140	0.000	0.904	1.769	0.000	0.000

Problem 1093	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	51	0	0	0	32	0	-1
normalized size	1	1.00	0.40	0.00	0.00	0.00	0.25	0.00	-0.01
time (sec)	N/A	0.074	0.011	0.160	0.000	0.816	2.481	0.000	0.000

Problem 1094	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	51	0	0	0	32	0	-1
normalized size	1	1.00	0.34	0.00	0.00	0.00	0.21	0.00	-0.01
time (sec)	N/A	0.095	0.012	0.144	0.000	0.645	2.415	0.000	0.000

Problem 1095	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	87	0	151	228	37	0	-1
normalized size	1	1.00	0.84	0.00	1.45	2.19	0.36	0.00	-0.01
time (sec)	N/A	0.031	0.059	0.140	3.129	0.874	2.173	0.000	0.000

Problem 1096	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	73	0	108	206	37	0	-1
normalized size	1	1.00	0.94	0.00	1.38	2.64	0.47	0.00	-0.01
time (sec)	N/A	0.022	0.035	0.155	2.935	1.059	1.548	0.000	0.000

Problem 1097	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	76	0	68	106	36	0	37
normalized size	1	1.00	1.33	0.00	1.19	1.86	0.63	0.00	0.65
time (sec)	N/A	0.013	0.031	0.150	2.985	0.613	1.230	0.000	1.070

Problem 1098	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	31	0	17
normalized size	1	1.00	1.00	0.86	0.81	0.81	1.48	0.00	0.81
time (sec)	N/A	0.005	0.004	0.004	1.268	0.770	1.060	0.000	1.089

Problem 1099	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	31	28	35	27	70	0	36
normalized size	1	1.00	0.70	0.64	0.80	0.61	1.59	0.00	0.82
time (sec)	N/A	0.010	0.012	0.005	1.344	0.560	2.768	0.000	1.157

Problem 1100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	42	39	52	38	406	0	56
normalized size	1	1.00	0.62	0.57	0.76	0.56	5.97	0.00	0.82
time (sec)	N/A	0.018	0.019	0.007	1.327	0.811	3.071	0.000	1.237

Problem 1101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	53	50	69	49	692	0	76
normalized size	1	1.00	0.58	0.54	0.75	0.53	7.52	0.00	0.83
time (sec)	N/A	0.028	0.033	0.007	1.346	0.792	5.364	0.000	1.352

Problem 1102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	64	61	86	60	1046	0	96
normalized size	1	1.00	0.55	0.53	0.74	0.52	9.02	0.00	0.83
time (sec)	N/A	0.043	0.049	0.008	1.329	0.811	8.483	0.000	1.486

Problem 1103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	80	0	0	0	37	0	-1
normalized size	1	1.00	0.62	0.00	0.00	0.00	0.29	0.00	-0.01
time (sec)	N/A	0.060	0.038	0.147	0.000	1.175	1.413	0.000	0.000

Problem 1104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	64	0	0	0	37	0	-1
normalized size	1	1.00	0.61	0.00	0.00	0.00	0.35	0.00	-0.01
time (sec)	N/A	0.046	0.018	0.145	0.000	0.899	1.767	0.000	0.000

Problem 1105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	51	0	0	0	37	0	-1
normalized size	1	1.00	0.64	0.00	0.00	0.00	0.46	0.00	-0.01
time (sec)	N/A	0.041	0.009	0.145	0.000	0.774	1.449	0.000	0.000

Problem 1106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	49	0	0	0	39	0	40
normalized size	1	1.00	0.65	0.00	0.00	0.00	0.52	0.00	0.53
time (sec)	N/A	0.037	0.010	0.145	0.000	0.905	1.548	0.000	1.459

Problem 1107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	51	0	0	0	29	0	-1
normalized size	1	1.00	0.49	0.00	0.00	0.00	0.28	0.00	-0.01
time (sec)	N/A	0.049	0.010	0.141	0.000	0.989	1.734	0.000	0.000

Problem 1108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	51	0	0	0	44	0	-1
normalized size	1	1.00	0.40	0.00	0.00	0.00	0.34	0.00	-0.01
time (sec)	N/A	0.059	0.010	0.145	0.000	0.801	1.825	0.000	0.000

Problem 1109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	51	0	0	0	44	0	-1
normalized size	1	1.00	0.33	0.00	0.00	0.00	0.29	0.00	-0.01
time (sec)	N/A	0.074	0.010	0.145	0.000	0.827	3.609	0.000	0.000

Problem 1110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	61	58	80	57	116	74	58
normalized size	1	1.00	0.62	0.59	0.82	0.58	1.18	0.76	0.59
time (sec)	N/A	0.054	0.038	0.008	1.362	0.652	25.210	0.158	1.170

Problem 1111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	50	47	64	46	92	61	48
normalized size	1	1.00	0.64	0.60	0.82	0.59	1.18	0.78	0.62
time (sec)	N/A	0.045	0.024	0.007	1.299	0.874	10.225	0.171	1.176

Problem 1112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	39	36	46	35	68	46	36
normalized size	1	1.00	0.70	0.64	0.82	0.62	1.21	0.82	0.64
time (sec)	N/A	0.032	0.044	0.005	1.331	0.867	4.242	0.154	1.138

Problem 1113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	27	25	30	23	44	30	24
normalized size	1	1.00	0.75	0.69	0.83	0.64	1.22	0.83	0.67
time (sec)	N/A	0.021	0.013	0.005	1.270	1.123	2.276	0.153	1.133

Problem 1114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	20	13	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	1.33	0.87	0.87
time (sec)	N/A	0.004	0.003	0.004	1.348	0.893	0.927	0.152	1.118

Problem 1115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	46	0	57	109	39	186	34
normalized size	1	1.00	0.84	0.00	1.04	1.98	0.71	3.38	0.62
time (sec)	N/A	0.031	0.010	0.142	3.010	0.909	2.088	0.160	1.157

Problem 1116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	34	0	94	194	39	221	58
normalized size	1	1.00	0.44	0.00	1.21	2.49	0.50	2.83	0.74
time (sec)	N/A	0.045	0.007	0.165	3.028	0.889	2.466	0.164	1.291

Problem 1117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	37	0	132	216	39	244	82
normalized size	1	1.00	0.36	0.00	1.27	2.08	0.38	2.35	0.79
time (sec)	N/A	0.061	0.007	0.158	3.102	0.788	3.507	0.179	1.363

Problem 1118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	91	0	0	0	27	0	-1
normalized size	1	1.00	0.71	0.00	0.00	0.00	0.21	0.00	-0.01
time (sec)	N/A	0.082	0.046	0.143	0.000	0.926	2.716	0.000	0.000

Problem 1119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	79	0	0	0	27	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.26	0.00	-0.01
time (sec)	N/A	0.064	0.046	0.145	0.000	0.784	2.321	0.000	0.000

Problem 1120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	64	0	0	0	27	0	-1
normalized size	1	1.00	0.78	0.00	0.00	0.00	0.33	0.00	-0.01
time (sec)	N/A	0.045	0.022	0.146	0.000	0.890	1.297	0.000	0.000

Problem 1121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	51	0	0	0	27	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.47	0.00	-0.02
time (sec)	N/A	0.031	0.010	0.141	0.000	0.975	1.211	0.000	0.000

Problem 1122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	51	0	0	0	31	0	-1
normalized size	1	1.00	0.62	0.00	0.00	0.00	0.38	0.00	-0.01
time (sec)	N/A	0.045	0.012	0.148	0.000	0.895	1.469	0.000	0.000

Problem 1123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	51	0	0	0	32	0	-1
normalized size	1	1.00	0.49	0.00	0.00	0.00	0.31	0.00	-0.01
time (sec)	N/A	0.063	0.012	0.148	0.000	1.505	2.203	0.000	0.000

Problem 1124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	51	0	0	0	32	0	-1
normalized size	1	1.00	0.40	0.00	0.00	0.00	0.25	0.00	-0.01
time (sec)	N/A	0.083	0.012	0.142	0.000	0.865	2.712	0.000	0.000

Problem 1125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	89	0	155	227	37	0	-1
normalized size	1	1.00	0.84	0.00	1.46	2.14	0.35	0.00	-0.01
time (sec)	N/A	0.037	0.044	0.148	2.996	1.013	2.354	0.000	0.000

Problem 1126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	75	0	110	204	37	0	-1
normalized size	1	1.00	0.94	0.00	1.38	2.55	0.46	0.00	-0.01
time (sec)	N/A	0.025	0.015	0.159	3.000	0.533	1.640	0.000	0.000

Problem 1127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	50	0	68	134	37	0	-1
normalized size	1	1.00	0.88	0.00	1.19	2.35	0.65	0.00	-0.02
time (sec)	N/A	0.021	0.007	0.144	2.898	0.915	1.623	0.000	0.000

Problem 1128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	17	31	0	17
normalized size	1	1.00	1.00	0.95	0.89	0.89	1.63	0.00	0.89
time (sec)	N/A	0.006	0.016	0.005	1.331	1.091	1.212	0.000	1.100

Problem 1129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	29	26	35	27	68	0	25
normalized size	1	1.00	0.66	0.59	0.80	0.61	1.55	0.00	0.57
time (sec)	N/A	0.012	0.010	0.006	1.330	0.913	1.324	0.000	1.166

Problem 1130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	42	39	52	38	406	0	38
normalized size	1	1.00	0.62	0.57	0.76	0.56	5.97	0.00	0.56
time (sec)	N/A	0.021	0.028	0.006	1.365	0.925	2.631	0.000	1.265

Problem 1131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	53	50	69	49	692	0	76
normalized size	1	1.00	0.58	0.54	0.75	0.53	7.52	0.00	0.83
time (sec)	N/A	0.029	0.031	0.008	1.357	1.021	3.258	0.000	1.357

Problem 1132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	90	0	0	0	37	0	-1
normalized size	1	1.00	0.70	0.00	0.00	0.00	0.29	0.00	-0.01
time (sec)	N/A	0.059	0.025	0.142	0.000	0.730	2.142	0.000	0.000

Problem 1133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	79	0	0	0	37	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.35	0.00	-0.01
time (sec)	N/A	0.044	0.020	0.145	0.000	0.861	1.553	0.000	0.000

Problem 1134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	62	0	0	0	37	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.45	0.00	-0.01
time (sec)	N/A	0.033	0.018	0.143	0.000	0.998	0.973	0.000	0.000

Problem 1135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	46	0	0	0	36	0	37
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.59	0.00	0.61
time (sec)	N/A	0.024	0.007	0.167	0.000	0.874	0.915	0.000	1.078

Problem 1136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	51	0	0	0	41	0	-1
normalized size	1	1.00	0.60	0.00	0.00	0.00	0.48	0.00	-0.01
time (sec)	N/A	0.034	0.010	0.148	0.000	1.219	1.555	0.000	0.000

Problem 1137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	51	0	0	0	44	0	-1
normalized size	1	1.00	0.48	0.00	0.00	0.00	0.41	0.00	-0.01
time (sec)	N/A	0.045	0.010	0.150	0.000	0.863	1.991	0.000	0.000

Problem 1138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	51	0	0	0	44	0	-1
normalized size	1	1.00	0.39	0.00	0.00	0.00	0.34	0.00	-0.01
time (sec)	N/A	0.057	0.011	0.144	0.000	1.180	2.569	0.000	0.000

Problem 1139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	61	58	81	69	116	87	69
normalized size	1	1.00	0.62	0.59	0.82	0.70	1.17	0.88	0.70
time (sec)	N/A	0.054	0.047	0.010	1.117	0.850	18.125	0.223	1.290

Problem 1140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	50	47	62	58	92	69	55
normalized size	1	1.00	0.68	0.64	0.84	0.78	1.24	0.93	0.74
time (sec)	N/A	0.044	0.022	0.007	1.364	1.040	12.755	0.161	1.235

Problem 1141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	39	36	47	47	68	53	39
normalized size	1	1.00	0.68	0.63	0.82	0.82	1.19	0.93	0.68
time (sec)	N/A	0.033	0.017	0.007	1.203	0.956	5.034	0.182	1.211

Problem 1142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	27	24	29	35	44	34	23
normalized size	1	1.00	0.77	0.69	0.83	1.00	1.26	0.97	0.66
time (sec)	N/A	0.021	0.012	0.006	1.317	0.843	2.096	0.187	1.154

Problem 1143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	24	24	14	14
normalized size	1	1.00	1.00	0.94	0.88	1.50	1.50	0.88	0.88
time (sec)	N/A	0.004	0.004	0.006	1.304	0.835	1.299	0.162	1.100

Problem 1144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	33	0	75	164	39	199	52
normalized size	1	1.00	0.47	0.00	1.07	2.34	0.56	2.84	0.74
time (sec)	N/A	0.044	0.007	0.150	3.092	0.895	1.821	0.171	1.226

Problem 1145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	96	35	0	110	259	39	234	87
normalized size	1	0.99	0.36	0.00	1.13	2.67	0.40	2.41	0.90
time (sec)	N/A	0.060	0.009	0.165	2.978	0.921	1.828	0.179	1.391

Problem 1146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	122	36	0	146	272	39	255	123
normalized size	1	0.98	0.29	0.00	1.17	2.18	0.31	2.04	0.98
time (sec)	N/A	0.076	0.009	0.169	2.993	0.874	3.206	0.184	1.512

Problem 1147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	79	0	0	0	27	0	-1
normalized size	1	1.00	0.62	0.00	0.00	0.00	0.21	0.00	-0.01
time (sec)	N/A	0.085	0.045	0.164	0.000	0.965	2.288	0.000	0.000

Problem 1148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	66	0	0	0	27	0	-1
normalized size	1	1.00	0.63	0.00	0.00	0.00	0.26	0.00	-0.01
time (sec)	N/A	0.065	0.024	0.168	0.000	0.906	1.729	0.000	0.000

Problem 1149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	54	0	0	0	27	0	-1
normalized size	1	1.00	0.70	0.00	0.00	0.00	0.35	0.00	-0.01
time (sec)	N/A	0.047	0.021	0.171	0.000	0.913	1.243	0.000	0.000

Problem 1150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	0	0	0	27	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.47	0.00	-0.02
time (sec)	N/A	0.030	0.015	0.153	0.000	0.832	0.942	0.000	0.000

Problem 1151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	54	0	0	0	31	0	-1
normalized size	1	1.00	0.66	0.00	0.00	0.00	0.38	0.00	-0.01
time (sec)	N/A	0.047	0.011	0.169	0.000	0.954	1.697	0.000	0.000

Problem 1152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	54	0	0	0	32	0	-1
normalized size	1	1.00	0.52	0.00	0.00	0.00	0.31	0.00	-0.01
time (sec)	N/A	0.064	0.011	0.168	0.000	0.924	1.613	0.000	0.000

Problem 1153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	54	0	0	0	32	0	-1
normalized size	1	1.00	0.42	0.00	0.00	0.00	0.25	0.00	-0.01
time (sec)	N/A	0.082	0.012	0.165	0.000	1.015	3.155	0.000	0.000

Problem 1154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	67	0	172	281	37	0	-1
normalized size	1	1.00	0.54	0.00	1.40	2.28	0.30	0.00	-0.01
time (sec)	N/A	0.045	0.026	0.152	3.192	0.714	2.759	0.000	0.000

Problem 1155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	60	0	130	269	37	0	-1
normalized size	1	1.00	0.62	0.00	1.34	2.77	0.38	0.00	-0.01
time (sec)	N/A	0.031	0.016	0.163	2.881	0.867	1.575	0.000	0.000

Problem 1156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	54	0	88	193	37	0	-1
normalized size	1	1.00	0.73	0.00	1.19	2.61	0.50	0.00	-0.01
time (sec)	N/A	0.022	0.011	0.148	3.197	0.702	1.774	0.000	0.000

Problem 1157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	23	29	0	14
normalized size	1	1.00	1.00	0.94	0.88	1.44	1.81	0.00	0.88
time (sec)	N/A	0.002	0.006	0.001	1.358	0.818	1.076	0.000	1.019

Problem 1158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	29	26	34	37	68	0	25
normalized size	1	1.00	0.69	0.62	0.81	0.88	1.62	0.00	0.60
time (sec)	N/A	0.008	0.013	0.005	1.425	0.841	1.670	0.000	1.160

Problem 1159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	42	39	53	50	323	0	70
normalized size	1	1.00	0.64	0.59	0.80	0.76	4.89	0.00	1.06
time (sec)	N/A	0.016	0.008	0.005	1.587	0.892	2.041	0.000	1.308

Problem 1160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	53	50	71	61	592	0	74
normalized size	1	1.00	0.59	0.56	0.79	0.68	6.58	0.00	0.82
time (sec)	N/A	0.026	0.010	0.006	1.288	0.660	4.093	0.000	1.458

Problem 1161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	64	61	87	72	928	0	93
normalized size	1	1.00	0.56	0.54	0.76	0.63	8.14	0.00	0.82
time (sec)	N/A	0.040	0.010	0.008	1.363	0.753	6.407	0.000	1.654

Problem 1162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	80	0	0	0	37	0	-1
normalized size	1	1.00	0.62	0.00	0.00	0.00	0.29	0.00	-0.01
time (sec)	N/A	0.062	0.036	0.170	0.000	0.726	2.268	0.000	0.000

Problem 1163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	67	0	0	0	37	0	-1
normalized size	1	1.00	0.64	0.00	0.00	0.00	0.35	0.00	-0.01
time (sec)	N/A	0.049	0.022	0.173	0.000	0.934	1.687	0.000	0.000

Problem 1164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	60	0	0	0	37	0	-1
normalized size	1	1.00	0.72	0.00	0.00	0.00	0.45	0.00	-0.01
time (sec)	N/A	0.037	0.016	0.156	0.000	0.777	1.328	0.000	0.000

Problem 1165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	54	0	0	0	37	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.63	0.00	-0.02
time (sec)	N/A	0.028	0.009	0.148	0.000	0.868	0.968	0.000	0.000

Problem 1166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	52	0	0	0	39	0	40
normalized size	1	1.00	0.66	0.00	0.00	0.00	0.49	0.00	0.51
time (sec)	N/A	0.037	0.010	0.166	0.000	0.817	1.597	0.000	1.351

Problem 1167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	54	0	0	0	44	0	-1
normalized size	1	1.00	0.51	0.00	0.00	0.00	0.42	0.00	-0.01
time (sec)	N/A	0.048	0.017	0.171	0.000	0.634	2.057	0.000	0.000

Problem 1168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	54	0	0	0	44	0	-1
normalized size	1	1.00	0.42	0.00	0.00	0.00	0.34	0.00	-0.01
time (sec)	N/A	0.063	0.016	0.165	0.000	0.901	3.163	0.000	0.000

Problem 1169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	54	0	0	0	44	0	-1
normalized size	1	1.00	0.35	0.00	0.00	0.00	0.29	0.00	-0.01
time (sec)	N/A	0.081	0.016	0.168	0.000	0.808	3.964	0.000	0.000

Problem 1170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	56	0	0	0	36	0	37
normalized size	1	1.00	0.67	0.00	0.00	0.00	0.43	0.00	0.45
time (sec)	N/A	0.031	0.038	0.168	0.000	0.894	2.119	0.000	1.135

Problem 1171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	29	26	31	47	126	0	28
normalized size	1	1.00	0.74	0.67	0.79	1.21	3.23	0.00	0.72
time (sec)	N/A	0.005	0.011	0.003	1.445	0.839	2.527	0.000	1.062

Problem 1172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	72	0	0	0	36	0	37
normalized size	1	1.00	0.71	0.00	0.00	0.00	0.35	0.00	0.36
time (sec)	N/A	0.040	0.051	0.177	0.000	0.829	3.532	0.000	1.127

Problem 1173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	40	37	50	69	515	0	44
normalized size	1	1.00	0.69	0.64	0.86	1.19	8.88	0.00	0.76
time (sec)	N/A	0.010	0.014	0.005	1.396	0.789	5.222	0.000	1.069

Problem 1174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	51	48	67	91	1550	0	61
normalized size	1	1.00	0.66	0.62	0.87	1.18	20.13	0.00	0.79
time (sec)	N/A	0.016	0.017	0.006	1.329	0.873	7.864	0.000	1.078

Problem 1175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	62	59	86	69	134	120	68
normalized size	1	1.00	0.58	0.56	0.81	0.65	1.26	1.13	0.64
time (sec)	N/A	0.057	0.050	0.009	1.330	0.800	31.775	0.185	1.148

Problem 1176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	51	48	68	58	110	94	57
normalized size	1	1.00	0.61	0.57	0.81	0.69	1.31	1.12	0.68
time (sec)	N/A	0.046	0.025	0.007	1.411	0.872	13.329	0.163	1.120

Problem 1177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	40	37	50	47	87	68	46
normalized size	1	1.00	0.65	0.60	0.81	0.76	1.40	1.10	0.74
time (sec)	N/A	0.034	0.022	0.006	1.349	0.853	6.248	0.159	1.128

Problem 1178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	29	26	32	36	63	42	35
normalized size	1	1.00	0.72	0.65	0.80	0.90	1.58	1.05	0.88
time (sec)	N/A	0.023	0.027	0.006	1.296	0.825	1.835	0.170	1.124

Problem 1179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	24	39	15	15
normalized size	1	1.00	1.00	0.84	0.79	1.26	2.05	0.79	0.79
time (sec)	N/A	0.005	0.005	0.005	1.277	1.040	0.689	0.151	1.114

Problem 1180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	0	70	98	44	190	51
normalized size	1	1.00	1.00	0.00	1.01	1.42	0.64	2.75	0.74
time (sec)	N/A	0.043	0.036	0.161	3.140	0.687	1.519	0.167	1.213

Problem 1181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	39	0	77	185	42	220	58
normalized size	1	1.00	0.50	0.00	0.99	2.37	0.54	2.82	0.74
time (sec)	N/A	0.043	0.011	0.089	3.014	0.891	1.954	0.182	1.286

Problem 1182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	41	0	138	214	42	251	83
normalized size	1	1.00	0.39	0.00	1.31	2.04	0.40	2.39	0.79
time (sec)	N/A	0.058	0.010	0.058	2.965	1.000	2.086	0.207	1.451

Problem 1183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	98	0	0	0	31	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.24	0.00	-0.01
time (sec)	N/A	0.083	0.089	0.053	0.000	0.938	1.944	0.000	0.000

Problem 1184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	66	0	0	0	31	0	-1
normalized size	1	1.00	0.63	0.00	0.00	0.00	0.30	0.00	-0.01
time (sec)	N/A	0.064	0.111	0.034	0.000	0.969	1.904	0.000	0.000

Problem 1185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	52	0	0	0	31	0	-1
normalized size	1	1.00	0.63	0.00	0.00	0.00	0.38	0.00	-0.01
time (sec)	N/A	0.043	0.010	0.033	0.000	0.798	1.289	0.000	0.000

Problem 1186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	52	0	0	0	34	0	-1
normalized size	1	1.00	0.63	0.00	0.00	0.00	0.41	0.00	-0.01
time (sec)	N/A	0.047	0.011	0.036	0.000	0.846	1.662	0.000	0.000

Problem 1187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	52	0	0	0	36	0	-1
normalized size	1	1.00	0.50	0.00	0.00	0.00	0.34	0.00	-0.01
time (sec)	N/A	0.063	0.012	0.041	0.000	0.923	1.848	0.000	0.000

Problem 1188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	52	0	0	0	36	0	-1
normalized size	1	1.00	0.40	0.00	0.00	0.00	0.28	0.00	-0.01
time (sec)	N/A	0.085	0.028	0.048	0.000	0.794	2.388	0.000	0.000

Problem 1189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	66	0	272	234	41	0	-1
normalized size	1	1.00	0.25	0.00	1.03	0.89	0.16	0.00	-0.00
time (sec)	N/A	0.158	0.078	0.043	3.092	0.902	3.081	0.000	0.000

Problem 1190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	52	0	214	204	41	0	-1
normalized size	1	1.00	0.22	0.00	0.92	0.88	0.18	0.00	-0.00
time (sec)	N/A	0.111	0.013	0.039	2.961	0.851	1.853	0.000	0.000

Problem 1191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	F(-1)	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	50	0	191	0	42	0	41
normalized size	1	1.00	0.22	0.00	0.85	0.00	0.19	0.00	0.18
time (sec)	N/A	0.111	0.030	0.042	3.052	0.000	1.859	0.000	1.282

Problem 1192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	27	158	0	18
normalized size	1	1.00	1.00	0.86	0.82	1.23	7.18	0.00	0.82
time (sec)	N/A	0.004	0.025	0.004	1.417	0.635	1.307	0.000	1.209

Problem 1193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	32	29	37	38	406	0	38
normalized size	1	1.00	0.70	0.63	0.80	0.83	8.83	0.00	0.83
time (sec)	N/A	0.011	0.008	0.006	1.342	0.850	2.058	0.000	1.302

Problem 1194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	43	40	55	50	1090	0	77
normalized size	1	1.00	0.61	0.56	0.77	0.70	15.35	0.00	1.08
time (sec)	N/A	0.019	0.011	0.006	1.417	0.903	4.231	0.000	1.481

Problem 1195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	54	51	73	61	1756	0	98
normalized size	1	1.00	0.56	0.53	0.76	0.64	18.29	0.00	1.02
time (sec)	N/A	0.029	0.023	0.008	1.320	1.186	4.921	0.000	1.757

Problem 1196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	108	0	0	0	41	0	-1
normalized size	1	1.00	0.69	0.00	0.00	0.00	0.26	0.00	-0.01
time (sec)	N/A	0.077	0.087	0.040	0.000	0.585	2.251	0.000	0.000

Problem 1197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	96	0	0	0	41	0	-1
normalized size	1	1.00	0.73	0.00	0.00	0.00	0.31	0.00	-0.01
time (sec)	N/A	0.064	0.073	0.036	0.000	0.731	2.046	0.000	0.000

Problem 1198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	64	0	0	0	41	0	-1
normalized size	1	1.00	0.60	0.00	0.00	0.00	0.39	0.00	-0.01
time (sec)	N/A	0.047	0.052	0.036	0.000	0.725	1.369	0.000	0.000

Problem 1199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	47	0	0	0	39	0	38
normalized size	1	1.00	0.57	0.00	0.00	0.00	0.47	0.00	0.46
time (sec)	N/A	0.033	0.006	0.163	0.000	0.538	0.970	0.000	1.070

Problem 1200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	52	0	0	0	34	0	-1
normalized size	1	1.00	0.61	0.00	0.00	0.00	0.40	0.00	-0.01
time (sec)	N/A	0.034	0.009	0.038	0.000	0.528	1.995	0.000	0.000

Problem 1201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	52	0	0	0	34	0	-1
normalized size	1	1.00	0.48	0.00	0.00	0.00	0.31	0.00	-0.01
time (sec)	N/A	0.046	0.010	0.047	0.000	0.583	2.392	0.000	0.000

Problem 1202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	52	0	0	0	34	0	-1
normalized size	1	1.00	0.39	0.00	0.00	0.00	0.26	0.00	-0.01
time (sec)	N/A	0.061	0.011	0.047	0.000	0.636	1.877	0.000	0.000

Problem 1203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	52	0	0	0	34	0	-1
normalized size	1	1.00	0.33	0.00	0.00	0.00	0.22	0.00	-0.01
time (sec)	N/A	0.078	0.010	0.052	0.000	0.663	3.133	0.000	0.000

Problem 1204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	62	59	86	58	117	109	60
normalized size	1	1.00	0.58	0.56	0.81	0.55	1.10	1.03	0.57
time (sec)	N/A	0.057	0.034	0.009	1.379	0.563	16.938	0.165	1.197

Problem 1205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	51	48	68	47	94	83	49
normalized size	1	1.00	0.61	0.57	0.81	0.56	1.12	0.99	0.58
time (sec)	N/A	0.046	0.027	0.008	1.335	0.604	9.897	0.156	1.169

Problem 1206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	40	37	50	36	70	57	38
normalized size	1	1.00	0.65	0.60	0.81	0.58	1.13	0.92	0.61
time (sec)	N/A	0.034	0.020	0.004	1.364	0.725	4.229	0.153	1.160

Problem 1207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	29	26	32	25	46	31	27
normalized size	1	1.00	0.72	0.65	0.80	0.62	1.15	0.78	0.68
time (sec)	N/A	0.023	0.013	0.006	1.295	0.679	1.875	0.153	1.152

Problem 1208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	15	24	15	15
normalized size	1	1.00	1.00	0.84	0.79	0.79	1.26	0.79	0.79
time (sec)	N/A	0.004	0.004	0.003	1.245	0.569	0.579	0.155	1.131

Problem 1209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	50	0	60	85	39	192	38
normalized size	1	1.00	0.88	0.00	1.05	1.49	0.68	3.37	0.67
time (sec)	N/A	0.035	0.009	0.151	2.849	0.786	1.334	0.166	1.179

Problem 1210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	39	0	96	200	41	226	61
normalized size	1	1.00	0.48	0.00	1.19	2.47	0.51	2.79	0.75
time (sec)	N/A	0.047	0.008	0.146	2.974	0.559	1.723	0.178	1.302

Problem 1211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	41	0	137	224	41	252	86
normalized size	1	1.00	0.38	0.00	1.27	2.07	0.38	2.33	0.80
time (sec)	N/A	0.061	0.009	0.145	2.413	0.569	2.119	0.179	1.390

Problem 1212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	91	0	0	0	29	0	-1
normalized size	1	1.00	0.68	0.00	0.00	0.00	0.22	0.00	-0.01
time (sec)	N/A	0.085	0.067	0.154	0.000	0.453	1.677	0.000	0.000

Problem 1213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	80	0	0	0	29	0	-1
normalized size	1	1.00	0.74	0.00	0.00	0.00	0.27	0.00	-0.01
time (sec)	N/A	0.065	0.035	0.149	0.000	0.699	1.538	0.000	0.000

Problem 1214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	66	0	0	0	29	0	-1
normalized size	1	1.00	0.78	0.00	0.00	0.00	0.34	0.00	-0.01
time (sec)	N/A	0.047	0.021	0.149	0.000	0.671	1.700	0.000	0.000

Problem 1215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	52	0	0	0	29	0	-1
normalized size	1	1.00	0.88	0.00	0.00	0.00	0.49	0.00	-0.02
time (sec)	N/A	0.030	0.011	0.146	0.000	0.654	1.280	0.000	0.000

Problem 1216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	52	0	0	0	32	0	-1
normalized size	1	1.00	0.61	0.00	0.00	0.00	0.38	0.00	-0.01
time (sec)	N/A	0.050	0.012	0.151	0.000	0.590	1.412	0.000	0.000

Problem 1217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	52	0	0	0	34	0	-1
normalized size	1	1.00	0.48	0.00	0.00	0.00	0.31	0.00	-0.01
time (sec)	N/A	0.066	0.027	0.146	0.000	0.776	1.906	0.000	0.000

Problem 1218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	52	0	0	0	34	0	-1
normalized size	1	1.00	0.39	0.00	0.00	0.00	0.26	0.00	-0.01
time (sec)	N/A	0.085	0.013	0.150	0.000	0.838	4.300	0.000	0.000

Problem 1219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	245	0	265	243	39	0	-1
normalized size	1	1.00	0.93	0.00	1.00	0.92	0.15	0.00	-0.00
time (sec)	N/A	0.135	0.177	0.145	2.462	0.801	2.296	0.000	0.000

Problem 1220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	213	0	219	221	39	0	-1
normalized size	1	1.00	0.91	0.00	0.94	0.95	0.17	0.00	-0.00
time (sec)	N/A	0.109	0.085	0.145	2.418	0.700	1.369	0.000	0.000

Problem 1221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	173	0	176	148	37	0	38
normalized size	1	1.00	0.83	0.00	0.84	0.71	0.18	0.00	0.18
time (sec)	N/A	0.087	0.035	0.170	2.323	0.626	1.578	0.000	1.101

Problem 1222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	18	78	0	18
normalized size	1	1.00	1.00	0.86	0.82	0.82	3.55	0.00	0.82
time (sec)	N/A	0.005	0.004	0.006	0.988	0.517	1.553	0.000	1.095

Problem 1223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	32	29	37	28	303	0	38
normalized size	1	1.00	0.70	0.63	0.80	0.61	6.59	0.00	0.83
time (sec)	N/A	0.011	0.020	0.005	0.992	0.649	1.997	0.000	1.170

Problem 1224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	43	40	55	39	1068	0	59
normalized size	1	1.00	0.61	0.56	0.77	0.55	15.04	0.00	0.83
time (sec)	N/A	0.019	0.040	0.004	1.067	0.513	3.664	0.000	1.261

Problem 1225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	54	51	73	50	1821	0	80
normalized size	1	1.00	0.56	0.53	0.76	0.52	18.97	0.00	0.83
time (sec)	N/A	0.028	0.020	0.008	0.972	0.453	6.840	0.000	1.364

Problem 1226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	65	62	91	61	2788	0	101
normalized size	1	1.00	0.54	0.51	0.75	0.50	23.04	0.00	0.83
time (sec)	N/A	0.041	0.025	0.008	1.065	0.593	9.876	0.000	1.438

Problem 1227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	80	0	0	0	39	0	-1
normalized size	1	1.00	0.60	0.00	0.00	0.00	0.29	0.00	-0.01
time (sec)	N/A	0.061	0.030	0.155	0.000	0.441	2.456	0.000	0.000

Problem 1228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	66	0	0	0	39	0	-1
normalized size	1	1.00	0.61	0.00	0.00	0.00	0.36	0.00	-0.01
time (sec)	N/A	0.047	0.020	0.155	0.000	0.777	1.855	0.000	0.000

Problem 1229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	52	0	0	0	39	0	-1
normalized size	1	1.00	0.60	0.00	0.00	0.00	0.45	0.00	-0.01
time (sec)	N/A	0.038	0.070	0.171	0.000	0.614	1.333	0.000	0.000

Problem 1230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	50	0	0	0	31	0	41
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.51	0.00	0.67
time (sec)	N/A	0.028	0.010	0.156	0.000	0.577	1.658	0.000	1.340

Problem 1231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	52	0	0	0	34	0	-1
normalized size	1	1.00	0.60	0.00	0.00	0.00	0.40	0.00	-0.01
time (sec)	N/A	0.037	0.011	0.157	0.000	0.574	2.555	0.000	0.000

Problem 1232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	52	0	0	0	31	0	-1
normalized size	1	1.00	0.48	0.00	0.00	0.00	0.28	0.00	-0.01
time (sec)	N/A	0.049	0.013	0.163	0.000	0.781	2.907	0.000	0.000

Problem 1233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	52	0	0	0	34	0	-1
normalized size	1	1.00	0.39	0.00	0.00	0.00	0.25	0.00	-0.01
time (sec)	N/A	0.064	0.010	0.150	0.000	0.787	5.428	0.000	0.000

Problem 1234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	62	59	86	58	117	113	60
normalized size	1	1.00	0.60	0.57	0.83	0.56	1.12	1.09	0.58
time (sec)	N/A	0.055	0.032	0.009	0.972	0.534	26.214	0.162	1.219

Problem 1235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	51	48	68	47	94	87	49
normalized size	1	1.00	0.62	0.59	0.83	0.57	1.15	1.06	0.60
time (sec)	N/A	0.046	0.043	0.007	1.076	0.579	12.757	0.159	1.170

Problem 1236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	40	37	50	36	70	61	38
normalized size	1	1.00	0.67	0.62	0.83	0.60	1.17	1.02	0.63
time (sec)	N/A	0.034	0.019	0.007	1.059	0.694	5.674	0.156	1.152

Problem 1237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	25	32	24	46	32	24
normalized size	1	1.00	0.74	0.66	0.84	0.63	1.21	0.84	0.63
time (sec)	N/A	0.023	0.014	0.005	1.034	0.584	1.229	0.159	1.149

Problem 1238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	22	15	15
normalized size	1	1.00	1.00	0.94	0.88	0.88	1.29	0.88	0.88
time (sec)	N/A	0.004	0.004	0.005	0.983	0.531	0.824	0.187	1.131

Problem 1239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	48	0	60	113	42	192	36
normalized size	1	1.00	0.84	0.00	1.05	1.98	0.74	3.37	0.63
time (sec)	N/A	0.033	0.009	0.154	2.284	0.655	2.394	0.162	1.159

Problem 1240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	37	0	98	199	39	228	61
normalized size	1	1.00	0.46	0.00	1.21	2.46	0.48	2.81	0.75
time (sec)	N/A	0.045	0.008	0.060	2.417	0.734	2.307	0.168	1.305

Problem 1241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	39	0	141	221	42	252	86
normalized size	1	1.00	0.36	0.00	1.31	2.05	0.39	2.33	0.80
time (sec)	N/A	0.059	0.017	0.058	2.384	0.695	3.257	0.170	1.393

Problem 1242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	92	0	0	0	29	0	-1
normalized size	1	1.00	0.69	0.00	0.00	0.00	0.22	0.00	-0.01
time (sec)	N/A	0.086	0.037	0.037	0.000	0.598	1.761	0.000	0.000

Problem 1243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	79	0	0	0	29	0	-1
normalized size	1	1.00	0.73	0.00	0.00	0.00	0.27	0.00	-0.01
time (sec)	N/A	0.065	0.032	0.036	0.000	0.714	1.414	0.000	0.000

Problem 1244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	66	0	0	0	29	0	-1
normalized size	1	1.00	0.78	0.00	0.00	0.00	0.34	0.00	-0.01
time (sec)	N/A	0.049	0.021	0.036	0.000	0.535	1.880	0.000	0.000

Problem 1245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	52	0	0	0	29	0	-1
normalized size	1	1.00	0.88	0.00	0.00	0.00	0.49	0.00	-0.02
time (sec)	N/A	0.032	0.011	0.152	0.000	0.681	1.452	0.000	0.000

Problem 1246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	52	0	0	0	32	0	-1
normalized size	1	1.00	0.61	0.00	0.00	0.00	0.38	0.00	-0.01
time (sec)	N/A	0.046	0.012	0.040	0.000	0.630	2.067	0.000	0.000

Problem 1247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	52	0	0	0	34	0	-1
normalized size	1	1.00	0.48	0.00	0.00	0.00	0.31	0.00	-0.01
time (sec)	N/A	0.068	0.013	0.043	0.000	0.615	2.771	0.000	0.000

Problem 1248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	52	0	0	0	34	0	-1
normalized size	1	1.00	0.39	0.00	0.00	0.00	0.26	0.00	-0.01
time (sec)	N/A	0.086	0.012	0.046	0.000	0.663	2.178	0.000	0.000

Problem 1249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	104	0	274	240	39	0	-1
normalized size	1	1.00	0.39	0.00	1.03	0.90	0.15	0.00	-0.00
time (sec)	N/A	0.140	0.051	0.042	2.302	0.779	1.772	0.000	0.000

Problem 1250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	90	0	222	217	39	0	-1
normalized size	1	1.00	0.38	0.00	0.94	0.92	0.17	0.00	-0.00
time (sec)	N/A	0.110	0.034	0.037	2.476	0.685	1.946	0.000	0.000

Problem 1251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	58	0	176	154	39	0	-1
normalized size	1	1.00	0.28	0.00	0.84	0.74	0.19	0.00	-0.00
time (sec)	N/A	0.088	0.027	0.149	2.408	0.852	2.000	0.000	0.000

Problem 1252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	18	18	80	0	18
normalized size	1	1.00	1.00	0.95	0.90	0.90	4.00	0.00	0.90
time (sec)	N/A	0.005	0.006	0.005	1.177	0.688	1.523	0.000	1.107

Problem 1253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	30	27	36	26	311	0	26
normalized size	1	1.00	0.65	0.59	0.78	0.57	6.76	0.00	0.57
time (sec)	N/A	0.012	0.013	0.005	1.150	0.635	2.354	0.000	1.188

Problem 1254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	43	40	55	39	1110	0	39
normalized size	1	1.00	0.61	0.56	0.77	0.55	15.63	0.00	0.55
time (sec)	N/A	0.019	0.016	0.005	1.109	0.619	2.357	0.000	1.286

Problem 1255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	54	51	73	50	1928	0	80
normalized size	1	1.00	0.56	0.53	0.76	0.52	20.08	0.00	0.83
time (sec)	N/A	0.030	0.032	0.006	1.049	0.462	5.654	0.000	1.372

Problem 1256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	91	0	0	0	39	0	-1
normalized size	1	1.00	0.68	0.00	0.00	0.00	0.29	0.00	-0.01
time (sec)	N/A	0.061	0.028	0.039	0.000	0.701	2.673	0.000	0.000

Problem 1257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	80	0	0	0	39	0	-1
normalized size	1	1.00	0.73	0.00	0.00	0.00	0.36	0.00	-0.01
time (sec)	N/A	0.048	0.022	0.039	0.000	0.763	1.925	0.000	0.000

Problem 1258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	64	0	0	0	39	0	-1
normalized size	1	1.00	0.74	0.00	0.00	0.00	0.45	0.00	-0.01
time (sec)	N/A	0.036	0.031	0.034	0.000	0.738	1.563	0.000	0.000

Problem 1259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	47	0	0	0	37	0	38
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.59	0.00	0.60
time (sec)	N/A	0.026	0.007	0.161	0.000	0.562	1.572	0.000	1.097

Problem 1260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	52	0	0	0	34	0	-1
normalized size	1	1.00	0.59	0.00	0.00	0.00	0.39	0.00	-0.01
time (sec)	N/A	0.037	0.011	0.039	0.000	0.620	2.297	0.000	0.000

Problem 1261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	52	0	0	0	31	0	-1
normalized size	1	1.00	0.47	0.00	0.00	0.00	0.28	0.00	-0.01
time (sec)	N/A	0.049	0.011	0.042	0.000	0.575	2.606	0.000	0.000

Problem 1262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	52	0	0	0	34	0	-1
normalized size	1	1.00	0.38	0.00	0.00	0.00	0.25	0.00	-0.01
time (sec)	N/A	0.060	0.011	0.046	0.000	0.657	2.936	0.000	0.000

Problem 1263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	55	0	0	0	39	0	-1
normalized size	1	1.00	0.68	0.00	0.00	0.00	0.48	0.00	-0.01
time (sec)	N/A	0.039	0.041	0.162	0.000	0.635	2.273	0.000	0.000

Problem 1264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	40	42	47	58	495	94	68
normalized size	1	1.00	0.83	0.88	0.98	1.21	10.31	1.96	1.42
time (sec)	N/A	0.027	0.021	0.006	1.038	0.505	8.387	0.170	1.139

Problem 1265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	21	25	129	21	21
normalized size	1	1.00	1.00	0.96	0.91	1.09	5.61	0.91	0.91
time (sec)	N/A	0.004	0.005	0.002	1.154	0.912	3.363	0.153	1.122

Problem 1266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	0	0	0	39	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.95	0.00	-0.02
time (sec)	N/A	0.020	0.020	0.137	0.000	0.657	7.825	0.000	0.000

Problem 1267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	57	57	56	56	58	56
normalized size	1	1.00	1.00	0.86	0.86	0.85	0.85	0.88	0.85
time (sec)	N/A	0.046	0.008	0.002	1.019	0.591	0.324	0.170	0.077

Problem 1268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	46	46	45	44	47	45
normalized size	1	1.00	1.00	0.87	0.87	0.85	0.83	0.89	0.85
time (sec)	N/A	0.035	0.006	0.004	1.010	0.596	0.669	0.208	0.049

Problem 1269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	35	34	33	32	35	33
normalized size	1	1.00	1.00	0.88	0.85	0.82	0.80	0.88	0.82
time (sec)	N/A	0.025	0.006	0.003	1.060	0.548	0.599	0.168	1.062

Problem 1270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	23	22	20	24	22
normalized size	1	1.00	1.00	0.89	0.85	0.81	0.74	0.89	0.81
time (sec)	N/A	0.018	0.005	0.002	1.076	0.672	0.412	0.182	1.050

Problem 1271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	14	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87
time (sec)	N/A	0.003	0.003	0.000	1.114	0.709	0.290	0.166	0.032

Problem 1272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	23	18	15	22	18
normalized size	1	1.00	1.00	0.95	1.05	0.82	0.68	1.00	0.82
time (sec)	N/A	0.010	0.005	0.005	0.970	0.795	0.532	0.207	0.086

Problem 1273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	33	33	31	42	31
normalized size	1	1.00	1.00	0.91	0.94	0.94	0.89	1.20	0.89
time (sec)	N/A	0.022	0.007	0.007	0.967	0.679	0.697	0.170	1.070

Problem 1274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	44	47	45	42	55	46
normalized size	1	1.00	1.00	0.90	0.96	0.92	0.86	1.12	0.94
time (sec)	N/A	0.027	0.007	0.007	1.147	0.799	1.259	0.172	0.089

Problem 1275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	56	58	58	56	69	58
normalized size	1	1.00	1.00	0.89	0.92	0.92	0.89	1.10	0.92
time (sec)	N/A	0.033	0.007	0.009	1.090	0.599	1.028	0.167	1.087

Problem 1276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F(-1)	A	A	B
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	305	311	895	239	0	20	275	235
normalized size	1	1.01	1.03	2.97	0.79	0.00	0.07	0.91	0.78
time (sec)	N/A	0.638	0.228	0.148	2.490	0.000	0.260	0.171	1.528

Problem 1277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	60	63	65	81	68	80	68
normalized size	1	1.00	0.83	0.88	0.90	1.12	0.94	1.11	0.94
time (sec)	N/A	0.058	0.028	0.011	1.021	0.797	1.019	0.172	0.089

Problem 1278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	49	52	54	70	56	67	57
normalized size	1	1.00	0.83	0.88	0.92	1.19	0.95	1.14	0.97
time (sec)	N/A	0.045	0.030	0.011	1.146	0.604	0.966	0.195	0.088

Problem 1279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	38	41	43	56	42	49	45
normalized size	1	1.00	0.83	0.89	0.93	1.22	0.91	1.07	0.98
time (sec)	N/A	0.034	0.032	0.011	1.071	0.669	0.776	0.171	1.051

Problem 1280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	27	30	32	35	29	48	29
normalized size	1	1.00	0.82	0.91	0.97	1.06	0.88	1.45	0.88
time (sec)	N/A	0.025	0.012	0.008	0.984	0.814	0.559	0.172	0.048

Problem 1281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	15	15	14	14
normalized size	1	1.00	1.00	0.94	0.88	0.94	0.94	0.88	0.88
time (sec)	N/A	0.004	0.005	0.000	1.007	0.861	0.602	0.166	0.027

Problem 1282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	33	35	37	47	34	45	34
normalized size	1	1.00	0.87	0.92	0.97	1.24	0.89	1.18	0.89
time (sec)	N/A	0.028	0.030	0.012	1.118	0.826	0.572	0.166	0.065

Problem 1283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	41	47	53	73	54	51	53
normalized size	1	1.00	0.79	0.90	1.02	1.40	1.04	0.98	1.02
time (sec)	N/A	0.037	0.058	0.014	1.124	0.678	1.271	0.239	1.069

Problem 1284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	57	62	70	90	68	85	67
normalized size	1	1.00	0.83	0.90	1.01	1.30	0.99	1.23	0.97
time (sec)	N/A	0.047	0.084	0.016	1.024	0.915	1.604	0.177	1.070

Problem 1285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	25	29	26	22	26	30	22
normalized size	1	1.00	0.74	0.85	0.76	0.65	0.76	0.88	0.65
time (sec)	N/A	0.020	0.011	0.003	1.042	0.697	0.434	0.198	0.056

Problem 1286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	20	17	17	21	19
normalized size	1	1.00	1.00	0.88	0.83	0.71	0.71	0.88	0.79
time (sec)	N/A	0.014	0.004	0.003	0.971	0.767	0.280	0.227	0.035

Problem 1287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	17	12	15	11	8	16	11
normalized size	1	1.00	1.31	0.92	1.15	0.85	0.62	1.23	0.85
time (sec)	N/A	0.003	0.003	0.002	1.057	0.801	0.367	0.197	0.039

Problem 1288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	21	16	15	21	19
normalized size	1	1.00	1.00	0.87	0.91	0.70	0.65	0.91	0.83
time (sec)	N/A	0.009	0.006	0.004	1.096	0.838	0.832	0.239	1.095

Problem 1289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	29	27	24	34	27
normalized size	1	1.00	1.00	0.85	0.88	0.82	0.73	1.03	0.82
time (sec)	N/A	0.018	0.004	0.006	1.068	0.892	0.416	0.231	1.069

Problem 1290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	31	30	29	31	31	29
normalized size	1	1.00	1.00	0.86	0.83	0.81	0.86	0.86	0.81
time (sec)	N/A	0.025	0.020	0.003	0.964	0.853	0.439	0.674	0.053

Problem 1291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	22	21	20	23	22
normalized size	1	1.00	1.00	0.88	0.85	0.81	0.77	0.88	0.85
time (sec)	N/A	0.016	0.004	0.002	1.007	0.788	0.254	0.667	1.018

Problem 1292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	14	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87
time (sec)	N/A	0.003	0.003	0.002	0.993	0.912	0.404	0.670	1.020

Problem 1293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	17	15	14	17	16
normalized size	1	1.00	1.00	0.84	0.89	0.79	0.74	0.89	0.84
time (sec)	N/A	0.009	0.004	0.004	0.954	0.781	0.491	0.336	0.063

Problem 1294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	24	28	24	31	22
normalized size	1	1.00	1.00	0.82	0.86	1.00	0.86	1.11	0.79
time (sec)	N/A	0.019	0.005	0.007	1.024	0.818	0.880	0.522	0.060

Problem 1295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	22	20	19	19	17	20	19
normalized size	1	1.00	0.88	0.80	0.76	0.76	0.68	0.80	0.76
time (sec)	N/A	0.012	0.005	0.004	1.079	0.807	0.193	0.157	0.036

Problem 1296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	12	15	14
normalized size	1	1.00	1.00	0.83	0.78	0.78	0.67	0.83	0.78
time (sec)	N/A	0.008	0.003	0.002	1.023	0.860	0.214	0.157	0.029

Problem 1297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	7	9	8
normalized size	1	1.00	1.00	0.90	0.80	0.80	0.70	0.90	0.80
time (sec)	N/A	0.002	0.002	0.001	1.030	0.837	0.169	0.205	0.030

Problem 1298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	29	15	11	10	13	11
normalized size	1	1.00	1.00	2.23	1.15	0.85	0.77	1.00	0.85
time (sec)	N/A	0.005	0.003	0.009	1.082	0.882	0.173	0.157	1.053

Problem 1299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	36	20	24	17	25	18
normalized size	1	1.00	1.00	1.64	0.91	1.09	0.77	1.14	0.82
time (sec)	N/A	0.010	0.004	0.007	0.982	0.765	0.215	0.157	0.052

Problem 1300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	147	219	145	502	36	128	181
normalized size	1	1.00	0.79	1.18	0.78	2.70	0.19	0.69	0.97
time (sec)	N/A	0.229	0.142	0.048	2.399	2.583	1.066	0.176	0.291

Problem 1301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	144	216	144	905	36	112	196
normalized size	1	1.00	0.78	1.17	0.78	4.89	0.19	0.61	1.06
time (sec)	N/A	0.274	0.099	0.021	2.277	3.082	1.044	0.182	1.268

Problem 1302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	144	156	124	637	36	112	197
normalized size	1	1.00	0.78	0.84	0.67	3.44	0.19	0.61	1.06
time (sec)	N/A	0.294	0.102	0.020	2.282	2.880	2.249	0.178	1.907

Problem 1303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	144	156	124	980	36	112	200
normalized size	1	1.00	0.78	0.84	0.67	5.30	0.19	0.61	1.08
time (sec)	N/A	0.222	0.069	0.023	2.320	2.702	2.131	0.176	0.256

Problem 1304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	185	144	216	144	539	34	112	182
normalized size	1	1.10	0.86	1.29	0.86	3.21	0.20	0.67	1.08
time (sec)	N/A	0.193	0.071	0.020	2.343	2.878	1.925	0.163	1.170

Problem 1305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	149	223	149	1073	39	132	203
normalized size	1	1.00	0.78	1.17	0.78	5.65	0.21	0.69	1.07
time (sec)	N/A	0.294	0.103	0.022	2.406	3.010	2.825	0.180	1.251

Problem 1306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	149	161	129	692	42	132	200
normalized size	1	1.00	0.78	0.84	0.67	3.60	0.22	0.69	1.04
time (sec)	N/A	0.276	0.119	0.022	2.274	2.926	1.996	0.180	1.700

Problem 1307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	150	161	129	1174	42	132	207
normalized size	1	1.00	0.78	0.84	0.67	6.11	0.22	0.69	1.08
time (sec)	N/A	0.227	0.143	0.023	2.288	2.648	3.062	0.179	0.256

Problem 1308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	70	0	124	172	0	58	-1
normalized size	1	1.00	0.84	0.00	1.49	2.07	0.00	0.70	-0.01
time (sec)	N/A	0.046	0.033	0.154	2.346	1.770	0.000	0.298	0.000

Problem 1309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	0	81	136	49	44	-1
normalized size	1	1.00	1.00	0.00	1.42	2.39	0.86	0.77	-0.02
time (sec)	N/A	0.036	0.023	0.145	2.346	1.974	165.093	0.295	0.000

Problem 1310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	0	45	97	24	25	-1
normalized size	1	1.00	1.00	0.00	1.41	3.03	0.75	0.78	-0.03
time (sec)	N/A	0.029	0.006	0.141	2.274	1.887	2.320	0.175	0.000

Problem 1311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	17	17	22	23	-1
normalized size	1	1.00	1.00	0.78	0.74	0.74	0.96	1.00	-0.04
time (sec)	N/A	0.005	0.005	0.005	1.013	0.870	7.206	0.205	0.000

Problem 1312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	31	26	35	27	0	38	-1
normalized size	1	1.00	0.65	0.54	0.73	0.56	0.00	0.79	-0.02
time (sec)	N/A	0.012	0.010	0.005	1.098	1.069	0.000	0.209	0.000

Problem 1313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	35	46	86	46	0	36	-1
normalized size	1	1.00	0.74	0.98	1.83	0.98	0.00	0.77	-0.02
time (sec)	N/A	0.015	0.014	0.143	1.118	0.968	0.000	0.218	0.000

Problem 1314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	39	58	35	22	29	-1
normalized size	1	1.00	1.00	1.34	2.00	1.21	0.76	1.00	-0.03
time (sec)	N/A	0.010	0.006	0.131	1.049	0.906	145.061	0.255	0.000

Problem 1315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	33	22	8	16	-1
normalized size	1	1.00	1.00	0.70	3.30	2.20	0.80	1.60	-0.10
time (sec)	N/A	0.007	0.003	0.132	0.927	0.883	2.375	0.165	0.000

Problem 1316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	32	12	12	14	11	-1
normalized size	1	1.00	1.00	1.78	0.67	0.67	0.78	0.61	-0.06
time (sec)	N/A	0.003	0.004	0.006	1.054	0.849	11.347	0.180	0.000

Problem 1317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	25	39	25	19	0	0	-1
normalized size	1	1.00	0.68	1.05	0.68	0.51	0.00	0.00	-0.03
time (sec)	N/A	0.007	0.006	0.004	0.946	0.845	0.000	0.000	0.000

Problem 1318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	32	31	89	63	31	28
normalized size	1	1.00	1.00	0.80	0.78	2.22	1.58	0.78	0.70
time (sec)	N/A	0.021	0.015	0.003	2.265	0.838	0.337	0.155	1.017

Problem 1319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	186	108	115	114	27	120	130
normalized size	1	1.00	1.40	0.81	0.86	0.86	0.20	0.90	0.98
time (sec)	N/A	0.122	0.055	0.005	2.468	0.881	0.432	0.164	1.246

Problem 1320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	182	164	194	314	22	180	227
normalized size	1	1.00	0.83	0.75	0.88	1.43	0.10	0.82	1.03
time (sec)	N/A	0.423	0.036	0.097	2.430	0.906	0.235	0.159	1.166

Problem 1321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	14	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87
time (sec)	N/A	0.004	0.004	0.002	0.985	0.849	0.237	0.156	1.028

Problem 1322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	154	159	184	348	26	187	134
normalized size	1	1.00	0.72	0.74	0.86	1.62	0.12	0.87	0.62
time (sec)	N/A	0.485	0.021	0.065	2.378	0.931	0.380	0.273	1.142

Problem 1323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	154	97	104	318	26	118	118
normalized size	1	1.00	1.25	0.79	0.85	2.59	0.21	0.96	0.96
time (sec)	N/A	0.092	0.020	0.002	2.328	0.850	0.216	0.171	1.312

Problem 1324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	19	18	72	56	18	19
normalized size	1	1.00	1.00	0.66	0.62	2.48	1.93	0.62	0.66
time (sec)	N/A	0.012	0.006	0.003	2.315	0.888	0.255	0.160	0.054

Problem 1325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	154	97	104	313	22	118	105
normalized size	1	1.00	1.25	0.79	0.85	2.54	0.18	0.96	0.85
time (sec)	N/A	0.087	0.023	0.002	2.343	0.987	0.240	0.192	0.244

Problem 1326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	154	159	184	334	20	190	219
normalized size	1	1.00	0.72	0.74	0.86	1.55	0.09	0.88	1.02
time (sec)	N/A	0.398	0.020	0.050	2.456	0.840	0.424	0.174	1.120

Problem 1327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	23	18	15	24	18
normalized size	1	1.00	1.00	0.95	1.05	0.82	0.68	1.09	0.82
time (sec)	N/A	0.011	0.006	0.004	1.072	0.801	0.611	0.159	1.073

Problem 1328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	189	169	198	338	29	196	149
normalized size	1	1.00	0.85	0.76	0.89	1.52	0.13	0.88	0.67
time (sec)	N/A	0.480	0.046	0.050	2.484	0.906	0.464	0.265	0.184

Problem 1329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	203	105	112	115	34	127	116
normalized size	1	1.00	1.53	0.79	0.84	0.86	0.26	0.95	0.87
time (sec)	N/A	0.105	0.031	0.004	2.312	0.565	0.327	0.182	1.172

Problem 1330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	101	32	31	90	71	31	28
normalized size	1	1.00	2.52	0.80	0.78	2.25	1.78	0.78	0.70
time (sec)	N/A	0.019	0.021	0.006	2.292	0.836	0.561	0.182	0.045

Problem 1331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	40	40	128	83	39	37
normalized size	1	1.00	1.00	0.82	0.82	2.61	1.69	0.80	0.76
time (sec)	N/A	0.024	0.026	0.011	2.399	0.728	0.611	0.166	0.091

Problem 1332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	197	114	122	409	42	138	118
normalized size	1	1.00	1.39	0.80	0.86	2.88	0.30	0.97	0.83
time (sec)	N/A	0.106	0.128	0.010	2.364	0.915	0.778	0.201	1.185

Problem 1333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	191	189	205	443	39	205	242
normalized size	1	1.00	0.82	0.81	0.88	1.91	0.17	0.88	1.04
time (sec)	N/A	0.419	0.124	0.057	2.296	0.881	0.724	0.207	1.154

Problem 1334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	15	15	14	14
normalized size	1	1.00	1.00	0.94	0.88	0.94	0.94	0.88	0.88
time (sec)	N/A	0.003	0.004	0.001	1.006	0.863	0.513	0.185	1.009

Problem 1335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	193	346	206	450	46	204	160
normalized size	1	1.00	0.82	1.48	0.88	1.92	0.20	0.87	0.68
time (sec)	N/A	0.483	0.114	0.435	2.459	0.692	0.659	0.330	1.183

Problem 1336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	195	123	130	416	46	135	146
normalized size	1	1.00	1.37	0.87	0.92	2.93	0.32	0.95	1.03
time (sec)	N/A	0.109	0.110	0.007	2.389	0.889	0.669	0.184	1.344

Problem 1337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	40	39	128	83	39	37
normalized size	1	1.00	1.00	0.82	0.80	2.61	1.69	0.80	0.76
time (sec)	N/A	0.022	0.027	0.007	2.377	0.814	0.835	0.178	1.063

Problem 1338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	197	123	130	407	41	135	128
normalized size	1	1.00	1.39	0.87	0.92	2.87	0.29	0.95	0.90
time (sec)	N/A	0.104	0.116	0.007	2.310	1.126	0.792	0.202	1.191

Problem 1339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	192	343	204	423	39	205	242
normalized size	1	1.00	0.83	1.48	0.88	1.82	0.17	0.88	1.04
time (sec)	N/A	0.397	0.119	0.454	2.331	0.870	0.718	0.158	1.144

Problem 1340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	33	35	37	47	34	47	34
normalized size	1	1.00	0.87	0.92	0.97	1.24	0.89	1.24	0.89
time (sec)	N/A	0.028	0.018	0.011	1.088	0.986	0.604	0.158	0.067

Problem 1341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	205	187	218	443	56	216	169
normalized size	1	1.00	0.84	0.77	0.89	1.82	0.23	0.89	0.69
time (sec)	N/A	0.498	0.145	0.062	2.414	0.903	0.515	0.247	1.179

Problem 1342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	208	123	134	160	60	147	140
normalized size	1	1.00	1.37	0.81	0.88	1.05	0.39	0.97	0.92
time (sec)	N/A	0.129	0.138	0.013	2.320	0.888	0.962	0.204	1.212

Problem 1343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	114	50	53	148	94	51	50
normalized size	1	1.00	1.93	0.85	0.90	2.51	1.59	0.86	0.85
time (sec)	N/A	0.032	0.090	0.013	2.362	0.952	0.973	0.162	0.088

Problem 1344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	30	23	22	22	20	24	12
normalized size	1	1.00	1.88	1.44	1.38	1.38	1.25	1.50	0.75
time (sec)	N/A	0.008	0.005	0.004	0.981	0.853	0.127	0.152	0.060

Problem 1345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	78	71	43	43	51	44	57
normalized size	1	1.00	1.39	1.27	0.77	0.77	0.91	0.79	1.02
time (sec)	N/A	0.041	0.021	0.014	2.444	0.896	0.176	0.181	1.095

Problem 1346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	76	78	69	68	68	85	70	93
normalized size	1	1.52	1.56	1.38	1.36	1.36	1.70	1.40	1.86
time (sec)	N/A	0.109	0.013	0.009	2.214	0.738	0.519	0.159	1.063

Problem 1347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	18	8	8	8	9	8
normalized size	1	1.00	1.00	1.50	0.67	0.67	0.67	0.75	0.67
time (sec)	N/A	0.003	0.003	0.003	1.109	0.807	0.100	0.162	0.041

Problem 1348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	73	75	66	65	65	83	67	53
normalized size	1	1.55	1.60	1.40	1.38	1.38	1.77	1.43	1.13
time (sec)	N/A	0.149	0.012	0.010	2.312	0.908	0.521	0.189	0.057

Problem 1349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	73	66	38	38	46	39	52
normalized size	1	1.00	1.49	1.35	0.78	0.78	0.94	0.80	1.06
time (sec)	N/A	0.037	0.009	0.009	2.348	0.872	0.275	0.155	0.120

Problem 1350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	23	18	17	17	15	19	6
normalized size	1	1.00	2.88	2.25	2.12	2.12	1.88	2.38	0.75
time (sec)	N/A	0.004	0.004	0.002	1.009	0.586	0.116	0.150	1.068

Problem 1351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	73	66	38	38	46	39	52
normalized size	1	1.00	1.49	1.35	0.78	0.78	0.94	0.80	1.06
time (sec)	N/A	0.036	0.010	0.010	2.345	0.850	0.264	0.152	0.061

Problem 1352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	73	75	66	65	65	83	67	86
normalized size	1	1.55	1.60	1.40	1.38	1.38	1.77	1.43	1.83
time (sec)	N/A	0.104	0.010	0.008	2.363	0.899	0.479	0.208	0.034

Problem 1353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	36	15	11	10	16	11
normalized size	1	1.00	1.00	2.40	1.00	0.73	0.67	1.07	0.73
time (sec)	N/A	0.007	0.004	0.008	1.044	0.851	0.177	0.153	1.054

Problem 1354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	78	86	71	70	76	87	72	64
normalized size	1	1.50	1.65	1.37	1.35	1.46	1.67	1.38	1.23
time (sec)	N/A	0.149	0.022	0.010	2.415	0.677	0.552	0.155	0.056

Problem 1355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	78	71	43	53	53	44	57
normalized size	1	1.00	1.39	1.27	0.77	0.95	0.95	0.79	1.02
time (sec)	N/A	0.040	0.024	0.011	2.349	0.901	0.205	0.151	1.058

Problem 1356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	30	39	22	28	22	24	12
normalized size	1	1.00	1.88	2.44	1.38	1.75	1.38	1.50	0.75
time (sec)	N/A	0.007	0.004	0.008	1.126	0.772	0.237	0.156	0.034

Problem 1357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	78	71	43	52	53	44	57
normalized size	1	1.00	1.39	1.27	0.77	0.93	0.95	0.79	1.02
time (sec)	N/A	0.038	0.026	0.010	2.220	0.845	0.211	0.165	0.079

Problem 1358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	82	71	70	89	90	72	95
normalized size	1	1.00	1.02	0.89	0.88	1.11	1.12	0.90	1.19
time (sec)	N/A	0.110	0.026	0.010	2.252	0.577	0.630	0.158	1.045

Problem 1359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	41	20	24	17	26	16
normalized size	1	1.00	1.00	1.86	0.91	1.09	0.77	1.18	0.73
time (sec)	N/A	0.012	0.004	0.010	1.079	0.764	0.242	0.171	1.023

Problem 1360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	87	76	77	94	95	79	69
normalized size	1	1.00	1.02	0.89	0.91	1.11	1.12	0.93	0.81
time (sec)	N/A	0.147	0.029	0.013	2.253	0.991	0.552	0.157	0.060

Problem 1361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	10	12	12
normalized size	1	1.00	1.00	0.81	0.75	0.75	0.62	0.75	0.75
time (sec)	N/A	0.006	0.006	0.002	2.460	0.785	0.209	0.152	1.026

Problem 1362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	79	46	45	45	51	45	57
normalized size	1	1.00	1.41	0.82	0.80	0.80	0.91	0.80	1.02
time (sec)	N/A	0.043	0.017	0.007	2.297	0.840	0.208	0.151	1.076

Problem 1363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	76	62	61	90	70	61	88
normalized size	1	1.00	0.94	0.77	0.75	1.11	0.86	0.75	1.09
time (sec)	N/A	0.172	0.019	0.036	2.379	0.893	0.424	0.162	0.111

Problem 1364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	7	8	8
normalized size	1	1.00	1.00	0.90	0.80	0.80	0.70	0.80	0.80
time (sec)	N/A	0.002	0.003	0.001	0.972	0.839	0.169	0.143	0.025

Problem 1365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	73	61	60	89	68	60	53
normalized size	1	1.00	0.91	0.76	0.75	1.11	0.85	0.75	0.66
time (sec)	N/A	0.245	0.035	0.018	2.485	0.661	0.371	0.174	1.051

Problem 1366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	74	41	40	40	46	40	52
normalized size	1	1.00	1.51	0.84	0.82	0.82	0.94	0.82	1.06
time (sec)	N/A	0.039	0.010	0.007	2.374	0.873	0.171	0.153	0.118

Problem 1367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	6	6
normalized size	1	1.00	1.00	0.88	0.75	0.75	0.62	0.75	0.75
time (sec)	N/A	0.003	0.004	0.001	2.337	0.973	0.147	0.148	0.038

Problem 1368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	78	41	40	40	46	40	52
normalized size	1	1.00	1.59	0.84	0.82	0.82	0.94	0.82	1.06
time (sec)	N/A	0.037	0.010	0.005	2.425	0.602	0.318	0.150	1.063

Problem 1369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	73	61	60	89	68	60	85
normalized size	1	1.00	0.91	0.76	0.75	1.11	0.85	0.75	1.06
time (sec)	N/A	0.171	0.012	0.015	2.323	0.615	0.421	0.179	0.042

Problem 1370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	25	15	11	10	15	11
normalized size	1	1.00	1.00	1.92	1.15	0.85	0.77	1.15	0.85
time (sec)	N/A	0.005	0.003	0.007	1.032	0.750	0.158	0.216	0.035

Problem 1371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	82	66	65	99	71	65	56
normalized size	1	1.00	0.96	0.78	0.76	1.16	0.84	0.76	0.66
time (sec)	N/A	0.250	0.041	0.017	2.412	0.900	0.499	0.160	0.060

Problem 1372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	83	46	45	54	53	45	57
normalized size	1	1.00	1.48	0.82	0.80	0.96	0.95	0.80	1.02
time (sec)	N/A	0.041	0.076	0.010	2.358	0.811	0.203	0.184	0.072

Problem 1373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	33	12	15	14	12	12
normalized size	1	1.00	1.00	2.06	0.75	0.94	0.88	0.75	0.75
time (sec)	N/A	0.006	0.006	0.016	2.355	0.858	0.278	0.150	1.018

Problem 1374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	79	46	45	55	53	45	57
normalized size	1	1.00	1.41	0.82	0.80	0.98	0.95	0.80	1.02
time (sec)	N/A	0.041	0.022	0.008	2.334	0.874	0.196	0.169	1.066

Problem 1375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	79	66	65	110	75	65	92
normalized size	1	1.00	0.91	0.76	0.75	1.26	0.86	0.75	1.06
time (sec)	N/A	0.174	0.029	0.015	2.351	0.909	0.485	0.208	0.051

Problem 1376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	32	20	24	17	25	18
normalized size	1	1.00	1.00	1.45	0.91	1.09	0.77	1.14	0.82
time (sec)	N/A	0.010	0.004	0.010	0.979	0.874	0.224	0.152	0.038

Problem 1377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	84	69	72	115	80	39	62
normalized size	1	1.00	0.93	0.77	0.80	1.28	0.89	0.43	0.69
time (sec)	N/A	0.247	0.032	0.016	2.393	0.908	0.455	0.158	1.047

Problem 1378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	167	162	228	224	169	15	114	164
normalized size	1	0.93	0.90	1.27	1.24	0.94	0.08	0.63	0.91
time (sec)	N/A	0.280	0.075	0.224	2.341	0.959	0.954	0.175	1.155

Problem 1379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	20	162	0	555	0	217	186
normalized size	1	1.00	0.08	0.66	0.00	2.27	0.00	0.89	0.76
time (sec)	N/A	0.304	0.007	0.038	0.000	1.065	0.000	3.591	1.098

Problem 1380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	53	97	41	119	0	-1
normalized size	1	1.00	0.97	0.91	1.67	0.71	2.05	0.00	-0.02
time (sec)	N/A	0.029	0.030	0.159	1.059	0.575	5.615	0.000	0.000

Problem 1381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	43	13	13	29	13	13
normalized size	1	1.00	1.00	2.53	0.76	0.76	1.71	0.76	0.76
time (sec)	N/A	0.003	0.005	0.008	1.137	0.583	0.422	0.151	1.232

Problem 1382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	50	47	58	29	90	30	-1
normalized size	1	1.00	1.43	1.34	1.66	0.83	2.57	0.86	-0.03
time (sec)	N/A	0.012	0.020	0.159	0.987	0.910	3.144	0.177	0.000

Problem 1383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	30	27	37	26	54	37	25
normalized size	1	1.00	0.57	0.51	0.70	0.49	1.02	0.70	0.47
time (sec)	N/A	0.021	0.011	0.006	1.013	0.547	24.025	0.151	1.154

Problem 1384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	25	22	28	21	39	28	20
normalized size	1	1.00	0.62	0.55	0.70	0.52	0.98	0.70	0.50
time (sec)	N/A	0.015	0.008	0.004	0.981	0.580	9.141	0.150	1.148

Problem 1385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	18	15	19	14	24	19	14
normalized size	1	1.00	0.67	0.56	0.70	0.52	0.89	0.70	0.52
time (sec)	N/A	0.011	0.005	0.005	0.994	0.807	2.382	0.152	1.158

Problem 1386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	8	9	9
normalized size	1	1.00	1.00	0.77	0.69	0.69	0.62	0.69	0.69
time (sec)	N/A	0.003	0.002	0.005	1.115	0.692	0.529	0.148	1.154

Problem 1387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	26	34	27	17	37	18
normalized size	1	1.00	1.00	1.04	1.36	1.08	0.68	1.48	0.72
time (sec)	N/A	0.012	0.006	0.006	2.366	0.843	1.691	0.183	0.074

Problem 1388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	39	47	44	31	47	31
normalized size	1	1.00	1.00	0.93	1.12	1.05	0.74	1.12	0.74
time (sec)	N/A	0.017	0.008	0.024	2.438	0.837	2.647	0.163	1.260

Problem 1389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	30	51	74	52	66	59	57
normalized size	1	1.00	0.52	0.88	1.28	0.90	1.14	1.02	0.98
time (sec)	N/A	0.023	0.007	0.025	2.302	0.647	4.149	0.160	1.278

Problem 1390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	35	30	86	35	53	0	-1
normalized size	1	1.00	0.74	0.64	1.83	0.74	1.13	0.00	-0.02
time (sec)	N/A	0.020	0.012	0.137	1.004	0.713	6.295	0.000	0.000

Problem 1391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	25	58	29	39	0	-1
normalized size	1	1.00	1.00	0.81	1.87	0.94	1.26	0.00	-0.03
time (sec)	N/A	0.012	0.007	0.135	0.993	0.860	3.324	0.000	0.000

Problem 1392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	12	33	16	12	16	-1
normalized size	1	1.00	1.00	0.86	2.36	1.14	0.86	1.14	-0.07
time (sec)	N/A	0.006	0.002	0.137	1.046	0.886	1.748	0.209	0.000

Problem 1393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	16	12	20	12
normalized size	1	1.00	1.00	0.81	0.75	1.00	0.75	1.25	0.75
time (sec)	N/A	0.003	0.003	0.004	0.920	0.883	1.174	0.196	1.124

Problem 1394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	21	18	25	22	26	0	17
normalized size	1	1.00	0.64	0.55	0.76	0.67	0.79	0.00	0.52
time (sec)	N/A	0.006	0.005	0.004	0.989	0.835	1.981	0.000	1.168

Problem 1395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	28	25	37	31	41	0	24
normalized size	1	1.00	0.57	0.51	0.76	0.63	0.84	0.00	0.49
time (sec)	N/A	0.010	0.007	0.005	1.026	0.829	3.710	0.000	1.238

Problem 1396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	41	33	0	0	36	0	-1
normalized size	1	1.00	0.22	0.18	0.00	0.00	0.19	0.00	-0.01
time (sec)	N/A	0.129	0.009	0.139	0.000	0.882	0.927	0.000	0.000

Problem 1397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	29	20	0	0	36	0	-1
normalized size	1	1.00	0.17	0.12	0.00	0.00	0.22	0.00	-0.01
time (sec)	N/A	0.070	0.005	0.138	0.000	0.813	1.166	0.000	0.000

Problem 1398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	29	33	0	0	39	0	-1
normalized size	1	1.00	0.16	0.18	0.00	0.00	0.21	0.00	-0.01
time (sec)	N/A	0.091	0.006	0.138	0.000	0.818	0.946	0.000	0.000

Problem 1399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	39	29	0	0	36	0	-1
normalized size	1	1.00	0.22	0.16	0.00	0.00	0.20	0.00	-0.01
time (sec)	N/A	0.047	0.008	0.141	0.000	0.907	1.452	0.000	0.000

Problem 1400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	24	18	0	0	34	0	16
normalized size	1	1.00	0.15	0.11	0.00	0.00	0.21	0.00	0.10
time (sec)	N/A	0.014	0.003	0.132	0.000	0.868	1.098	0.000	0.081

Problem 1401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	29	31	0	0	39	0	-1
normalized size	1	1.00	0.16	0.17	0.00	0.00	0.22	0.00	-0.01
time (sec)	N/A	0.030	0.005	0.133	0.000	0.981	1.702	0.000	0.000

Problem 1402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	41	33	0	0	36	0	-1
normalized size	1	1.00	0.11	0.09	0.00	0.00	0.10	0.00	-0.00
time (sec)	N/A	0.211	0.027	0.147	0.000	0.893	1.016	0.000	0.000

Problem 1403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	354	29	20	0	0	36	0	-1
normalized size	1	1.00	0.08	0.06	0.00	0.00	0.10	0.00	-0.00
time (sec)	N/A	0.176	0.005	0.149	0.000	0.860	0.723	0.000	0.000

Problem 1404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	29	33	0	0	39	0	-1
normalized size	1	1.00	0.08	0.09	0.00	0.00	0.10	0.00	-0.00
time (sec)	N/A	0.193	0.006	0.136	0.000	0.973	1.322	0.000	0.000

Problem 1405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	41	33	0	0	36	0	-1
normalized size	1	1.00	0.10	0.08	0.00	0.00	0.09	0.00	-0.00
time (sec)	N/A	0.131	0.019	0.142	0.000	0.876	2.147	0.000	0.000

Problem 1406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	29	20	0	0	36	0	-1
normalized size	1	1.00	0.08	0.05	0.00	0.00	0.10	0.00	-0.00
time (sec)	N/A	0.080	0.005	0.136	0.000	0.644	1.172	0.000	0.000

Problem 1407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	392	392	27	33	0	0	37	0	31
normalized size	1	1.00	0.07	0.08	0.00	0.00	0.09	0.00	0.08
time (sec)	N/A	0.080	0.005	0.016	0.000	0.828	1.317	0.000	1.208

Problem 1408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	28	25	37	24	54	37	32
normalized size	1	1.00	0.55	0.49	0.73	0.47	1.06	0.73	0.63
time (sec)	N/A	0.018	0.011	0.006	1.020	0.568	19.149	0.152	1.216

Problem 1409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	23	20	28	19	39	28	25
normalized size	1	1.00	0.58	0.50	0.70	0.48	0.98	0.70	0.62
time (sec)	N/A	0.015	0.010	0.006	0.981	0.559	8.230	0.150	1.182

Problem 1410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	18	15	19	14	24	19	14
normalized size	1	1.00	0.67	0.56	0.70	0.52	0.89	0.70	0.52
time (sec)	N/A	0.011	0.007	0.006	1.029	0.516	2.823	0.163	1.155

Problem 1411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	12	9	9
normalized size	1	1.00	1.00	0.77	0.69	0.69	0.92	0.69	0.69
time (sec)	N/A	0.003	0.002	0.003	1.044	0.893	0.611	0.175	1.149

Problem 1412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	30	36	44	50	194	44	28
normalized size	1	1.00	0.77	0.92	1.13	1.28	4.97	1.13	0.72
time (sec)	N/A	0.016	0.006	0.017	2.347	0.881	1.625	0.153	1.218

Problem 1413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	58	30	46	63	66	49	63	40
normalized size	1	1.05	0.55	0.84	1.15	1.20	0.89	1.15	0.73
time (sec)	N/A	0.023	0.006	0.021	2.389	0.748	2.839	0.170	1.301

Problem 1414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	74	30	51	81	71	68	68	65
normalized size	1	1.04	0.42	0.72	1.14	1.00	0.96	0.96	0.92
time (sec)	N/A	0.030	0.007	0.029	2.271	0.752	6.640	0.152	1.527

Problem 1415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	43	30	73	54	36	0	-1
normalized size	1	1.00	0.96	0.67	1.62	1.20	0.80	0.00	-0.02
time (sec)	N/A	0.019	0.020	0.139	1.005	0.838	5.511	0.000	0.000

Problem 1416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	29	25	45	45	26	0	-1
normalized size	1	1.00	0.94	0.81	1.45	1.45	0.84	0.00	-0.03
time (sec)	N/A	0.012	0.017	0.015	0.999	0.842	2.826	0.000	0.000

Problem 1417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	25	12	12	12
normalized size	1	1.00	1.00	0.81	0.75	1.56	0.75	0.75	0.75
time (sec)	N/A	0.003	0.003	0.003	1.078	0.682	1.045	0.193	1.139

Problem 1418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	23	18	25	35	31	0	17
normalized size	1	1.00	0.70	0.55	0.76	1.06	0.94	0.00	0.52
time (sec)	N/A	0.006	0.005	0.003	1.030	0.560	1.499	0.000	1.147

Problem 1419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	28	25	37	44	70	0	28
normalized size	1	1.00	0.57	0.51	0.76	0.90	1.43	0.00	0.57
time (sec)	N/A	0.010	0.006	0.004	1.113	0.890	2.532	0.000	1.241

Problem 1420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	56	40	0	0	36	0	-1
normalized size	1	1.00	0.28	0.20	0.00	0.00	0.18	0.00	-0.00
time (sec)	N/A	0.103	0.014	0.150	0.000	0.651	2.411	0.000	0.000

Problem 1421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	42	33	0	0	36	0	-1
normalized size	1	1.00	0.23	0.18	0.00	0.00	0.19	0.00	-0.01
time (sec)	N/A	0.086	0.013	0.133	0.000	0.858	1.609	0.000	0.000

Problem 1422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	42	33	0	0	36	0	-1
normalized size	1	1.00	0.23	0.18	0.00	0.00	0.19	0.00	-0.01
time (sec)	N/A	0.083	0.010	0.156	0.000	0.835	1.420	0.000	0.000

Problem 1423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	29	40	0	0	39	0	-1
normalized size	1	1.00	0.14	0.20	0.00	0.00	0.19	0.00	-0.00
time (sec)	N/A	0.102	0.006	0.135	0.000	0.768	1.971	0.000	0.000

Problem 1424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	54	36	0	0	36	0	-1
normalized size	1	1.00	0.28	0.18	0.00	0.00	0.18	0.00	-0.01
time (sec)	N/A	0.045	0.014	0.135	0.000	0.870	2.350	0.000	0.000

Problem 1425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	42	29	0	0	36	0	-1
normalized size	1	1.00	0.23	0.16	0.00	0.00	0.20	0.00	-0.01
time (sec)	N/A	0.030	0.011	0.135	0.000	0.968	1.564	0.000	0.000

Problem 1426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	38	18	0	0	34	0	16
normalized size	1	1.00	0.21	0.10	0.00	0.00	0.19	0.00	0.09
time (sec)	N/A	0.027	0.018	0.141	0.000	0.898	0.742	0.000	0.082

Problem 1427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	29	38	0	0	39	0	-1
normalized size	1	1.00	0.15	0.19	0.00	0.00	0.20	0.00	-0.01
time (sec)	N/A	0.046	0.005	0.134	0.000	0.934	2.005	0.000	0.000

Problem 1428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	54	40	0	0	36	0	-1
normalized size	1	1.00	0.14	0.10	0.00	0.00	0.09	0.00	-0.00
time (sec)	N/A	0.222	0.013	0.135	0.000	0.903	2.820	0.000	0.000

Problem 1429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	41	33	0	0	36	0	-1
normalized size	1	1.00	0.11	0.09	0.00	0.00	0.10	0.00	-0.00
time (sec)	N/A	0.191	0.013	0.136	0.000	0.799	1.746	0.000	0.000

Problem 1430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	29	33	0	0	36	0	-1
normalized size	1	1.00	0.08	0.09	0.00	0.00	0.10	0.00	-0.00
time (sec)	N/A	0.187	0.006	0.137	0.000	0.707	1.386	0.000	0.000

Problem 1431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	29	40	0	0	39	0	-1
normalized size	1	1.00	0.07	0.10	0.00	0.00	0.10	0.00	-0.00
time (sec)	N/A	0.221	0.029	0.135	0.000	0.670	1.685	0.000	0.000

Problem 1432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	392	392	43	33	0	0	36	0	-1
normalized size	1	1.00	0.11	0.08	0.00	0.00	0.09	0.00	-0.00
time (sec)	N/A	0.111	0.015	0.134	0.000	0.876	1.975	0.000	0.000

Problem 1433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	29	33	0	0	36	0	-1
normalized size	1	1.00	0.07	0.08	0.00	0.00	0.09	0.00	-0.00
time (sec)	N/A	0.111	0.005	0.138	0.000	0.978	1.358	0.000	0.000

Problem 1434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	408	408	29	40	0	0	37	0	31
normalized size	1	1.00	0.07	0.10	0.00	0.00	0.09	0.00	0.08
time (sec)	N/A	0.113	0.005	0.138	0.000	0.871	1.541	0.000	1.261

Problem 1435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	35	25	33	94	43	34
normalized size	1	1.00	1.00	1.40	1.00	1.32	3.76	1.72	1.36
time (sec)	N/A	0.008	0.019	0.005	1.019	0.851	2.973	0.160	1.322

Problem 1436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.005	0.001	0.000	0.961	0.466	0.072	0.190	0.021

Problem 1437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	14	17	10	18	11
normalized size	1	1.00	1.00	0.92	1.08	1.31	0.77	1.38	0.85
time (sec)	N/A	0.005	0.003	0.005	1.017	0.807	0.335	0.147	1.029

Problem 1438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	40	93	43	85	313	117	93
normalized size	1	1.00	0.93	2.16	1.00	1.98	7.28	2.72	2.16
time (sec)	N/A	0.017	0.030	0.008	1.116	0.916	13.851	0.173	1.374

Problem 1439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	24	24
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.012	0.001	0.001	0.941	0.675	0.070	0.149	0.036

Problem 1440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	25	27	24	32	23
normalized size	1	1.00	1.00	0.89	0.93	1.00	0.89	1.19	0.85
time (sec)	N/A	0.014	0.001	0.007	1.061	0.789	0.279	0.154	1.022

Problem 1441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	41	0	0	0	0	0	-1
normalized size	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.009	0.010	0.145	0.000	0.764	0.000	0.000	0.000

Problem 1442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	14	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87
time (sec)	N/A	0.004	0.003	0.001	1.007	0.774	0.330	0.165	0.039

Problem 1443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	262	27	0	0	20	310	196
normalized size	1	1.00	0.78	0.08	0.00	0.00	0.06	0.93	0.59
time (sec)	N/A	0.536	0.389	0.444	0.000	0.000	0.275	0.247	1.845

Problem 1444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	23	18	15	22	18
normalized size	1	1.00	1.00	0.95	1.05	0.82	0.68	1.00	0.82
time (sec)	N/A	0.012	0.006	0.004	1.067	0.584	0.522	0.159	0.091

Problem 1445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	40	0	0	0	0	0	-1
normalized size	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.008	0.008	0.141	0.000	0.649	0.000	0.000	0.000

Problem 1446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	16	15	15	12	16	15
normalized size	1	1.00	1.00	1.00	0.94	0.94	0.75	1.00	0.94
time (sec)	N/A	0.004	0.003	0.001	0.982	0.785	0.380	0.163	0.046

Problem 1447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	263	29	0	0	22	290	246
normalized size	1	1.00	0.79	0.09	0.00	0.00	0.07	0.87	0.73
time (sec)	N/A	0.438	0.298	0.063	0.000	0.000	0.216	0.176	1.894

Problem 1448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	23	25	20	15	24	21
normalized size	1	1.00	1.00	1.00	1.09	0.87	0.65	1.04	0.91
time (sec)	N/A	0.014	0.007	0.005	1.013	0.477	0.628	0.155	0.097

Problem 1449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	166	89	0	65	46	127	103
normalized size	1	1.00	1.00	0.54	0.00	0.39	0.28	0.77	0.62
time (sec)	N/A	0.142	0.005	0.013	0.000	10.963	0.347	0.159	1.375

Problem 1450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	166	97	0	65	44	129	103
normalized size	1	1.00	1.01	0.59	0.00	0.39	0.27	0.78	0.62
time (sec)	N/A	0.139	0.004	0.010	0.000	11.362	0.347	0.157	1.343

Problem 1451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	361	144	190	127	27	183	55
normalized size	1	1.00	1.78	0.71	0.94	0.63	0.13	0.90	0.27
time (sec)	N/A	0.194	0.327	0.007	2.369	0.818	0.380	0.193	1.141

Problem 1452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	14	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87
time (sec)	N/A	0.003	0.004	0.002	1.004	0.548	0.398	0.182	0.036

Problem 1453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	279	136	177	128	27	199	39
normalized size	1	1.00	1.45	0.70	0.92	0.66	0.14	1.03	0.20
time (sec)	N/A	0.156	0.074	0.003	2.281	0.648	0.278	0.205	1.105

Problem 1454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	19	18	72	56	18	19
normalized size	1	1.00	1.00	0.66	0.62	2.48	1.93	0.62	0.66
time (sec)	N/A	0.012	0.007	0.003	2.338	0.601	0.425	0.190	0.057

Problem 1455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	279	136	177	128	22	187	37
normalized size	1	1.00	1.45	0.70	0.92	0.66	0.11	0.97	0.19
time (sec)	N/A	0.154	0.043	0.003	2.338	0.587	0.219	0.219	1.104

Problem 1456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	23	18	15	24	18
normalized size	1	1.00	1.00	0.95	1.05	0.82	0.68	1.09	0.82
time (sec)	N/A	0.011	0.006	0.006	1.058	0.573	0.649	0.200	1.104

Problem 1457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	385	144	191	141	34	195	55
normalized size	1	1.00	1.90	0.71	0.94	0.69	0.17	0.96	0.27
time (sec)	N/A	0.175	0.079	0.006	2.429	0.733	0.512	0.194	1.114

Problem 1458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	164	32	31	94	71	31	28
normalized size	1	1.00	4.10	0.80	0.78	2.35	1.78	0.78	0.70
time (sec)	N/A	0.021	0.050	0.007	2.208	0.542	0.694	0.187	0.053

Problem 1459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	387	147	192	168	34	183	55
normalized size	1	1.00	1.91	0.72	0.95	0.83	0.17	0.90	0.27
time (sec)	N/A	0.182	0.101	0.005	2.491	0.726	0.392	0.225	1.127

Problem 1460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	33	33	31	43	31
normalized size	1	1.00	1.00	0.91	0.94	0.94	0.89	1.23	0.89
time (sec)	N/A	0.024	0.007	0.006	1.051	0.600	1.675	0.205	0.084

Problem 1461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	367	34	0	385	22	442	115
normalized size	1	1.00	1.35	0.12	0.00	1.42	0.08	1.62	0.42
time (sec)	N/A	0.297	0.097	0.018	0.000	0.738	0.419	0.227	0.148

Problem 1462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	324	27	0	418	26	437	110
normalized size	1	1.00	1.21	0.10	0.00	1.57	0.10	1.64	0.41
time (sec)	N/A	0.203	0.075	0.013	0.000	0.896	0.339	0.396	1.174

Problem 1463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	324	27	0	438	29	437	110
normalized size	1	1.00	1.21	0.10	0.00	1.64	0.11	1.64	0.41
time (sec)	N/A	0.189	0.099	0.013	0.000	0.704	0.341	0.366	1.174

Problem 1464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	324	27	0	453	27	437	110
normalized size	1	1.00	1.21	0.10	0.00	1.70	0.10	1.64	0.41
time (sec)	N/A	0.194	0.087	0.013	0.000	0.583	0.347	0.350	1.172

Problem 1465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	324	27	0	408	20	437	110
normalized size	1	1.00	1.21	0.10	0.00	1.53	0.07	1.64	0.41
time (sec)	N/A	0.184	0.104	0.003	0.000	0.668	0.393	0.227	0.152

Problem 1466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	377	36	0	408	29	453	118
normalized size	1	1.00	1.37	0.13	0.00	1.48	0.11	1.65	0.43
time (sec)	N/A	0.228	0.132	0.006	0.000	0.738	0.560	0.364	0.147

Problem 1467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	395	36	0	516	36	453	118
normalized size	1	1.00	1.43	0.13	0.00	1.86	0.13	1.64	0.43
time (sec)	N/A	0.227	0.092	0.005	0.000	0.698	0.737	0.457	1.155

Problem 1468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	395	36	0	524	36	453	118
normalized size	1	1.00	1.43	0.13	0.00	1.89	0.13	1.64	0.43
time (sec)	N/A	0.233	0.100	0.006	0.000	0.737	0.575	0.360	0.146

Problem 1469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	395	36	0	514	32	453	118
normalized size	1	1.00	1.43	0.13	0.00	1.86	0.12	1.64	0.43
time (sec)	N/A	0.242	0.213	0.004	0.000	0.857	0.402	0.228	1.141

Problem 1470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	198	29	0	384	22	461	94
normalized size	1	1.00	0.83	0.12	0.00	1.61	0.09	1.93	0.39
time (sec)	N/A	0.169	0.076	0.013	0.000	0.750	0.405	0.208	0.138

Problem 1471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	38	33	28	28	27	29	18
normalized size	1	1.00	1.58	1.38	1.17	1.17	1.12	1.21	0.75
time (sec)	N/A	0.011	0.009	0.006	2.491	0.539	0.300	0.153	0.058

Problem 1472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	30	8	8	8	9	8
normalized size	1	1.00	1.00	2.50	0.67	0.67	0.67	0.75	0.67
time (sec)	N/A	0.003	0.003	0.007	1.063	0.508	0.288	0.164	0.037

Problem 1473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	31	28	23	23	22	55	13
normalized size	1	1.00	1.82	1.65	1.35	1.35	1.29	3.24	0.76
time (sec)	N/A	0.009	0.008	0.005	2.412	0.553	0.419	0.163	1.041

Problem 1474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	23	30	17	17	15	18	6
normalized size	1	1.00	2.88	3.75	2.12	2.12	1.88	2.25	0.75
time (sec)	N/A	0.004	0.005	0.004	1.057	0.516	0.336	0.154	0.068

Problem 1475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	31	28	23	23	22	24	13
normalized size	1	1.00	1.82	1.65	1.35	1.35	1.29	1.41	0.76
time (sec)	N/A	0.006	0.005	0.005	2.417	0.596	0.279	0.154	0.026

Problem 1476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	32	15	11	10	16	11
normalized size	1	1.00	1.00	2.13	1.00	0.73	0.67	1.07	0.73
time (sec)	N/A	0.007	0.003	0.011	1.090	0.715	0.289	0.153	1.069

Problem 1477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	38	33	28	37	29	29	18
normalized size	1	1.00	1.58	1.38	1.17	1.54	1.21	1.21	0.75
time (sec)	N/A	0.011	0.005	0.012	2.377	0.586	0.200	0.160	1.045

Problem 1478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	30	35	22	28	22	23	12
normalized size	1	1.00	1.88	2.19	1.38	1.75	1.38	1.44	0.75
time (sec)	N/A	0.007	0.004	0.009	1.009	0.758	0.269	0.152	1.050

Problem 1479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	38	33	28	38	29	29	18
normalized size	1	1.00	1.58	1.38	1.17	1.58	1.21	1.21	0.75
time (sec)	N/A	0.010	0.006	0.010	2.353	0.519	0.211	0.316	0.049

Problem 1480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	37	20	24	17	26	16
normalized size	1	1.00	1.00	1.68	0.91	1.09	0.77	1.18	0.73
time (sec)	N/A	0.011	0.004	0.013	1.067	0.546	0.270	0.157	0.035

Problem 1481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	101	77	91	114	46	93	48
normalized size	1	1.00	1.01	0.77	0.91	1.14	0.46	0.93	0.48
time (sec)	N/A	0.052	0.038	0.006	2.307	0.686	176.278	0.155	0.074

Problem 1482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	98	74	88	111	46	90	45
normalized size	1	1.00	1.01	0.76	0.91	1.14	0.47	0.93	0.46
time (sec)	N/A	0.053	0.019	0.007	2.324	0.636	171.714	0.152	0.062

Problem 1483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	98	74	88	111	0	90	45
normalized size	1	1.00	1.01	0.76	0.91	1.14	0.00	0.93	0.46
time (sec)	N/A	0.053	0.027	0.005	2.321	0.697	0.000	0.171	0.062

Problem 1484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	98	74	88	111	0	90	45
normalized size	1	1.00	1.01	0.76	0.91	1.14	0.00	0.93	0.46
time (sec)	N/A	0.052	0.016	0.007	2.345	0.625	0.000	0.151	1.056

Problem 1485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	98	66	88	111	0	90	45
normalized size	1	1.00	1.01	0.68	0.91	1.14	0.00	0.93	0.46
time (sec)	N/A	0.050	0.028	0.002	2.267	0.576	0.000	0.148	0.041

Problem 1486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	109	79	93	123	0	95	50
normalized size	1	1.00	1.07	0.77	0.91	1.21	0.00	0.93	0.49
time (sec)	N/A	0.056	0.043	0.010	2.354	0.609	0.000	0.170	1.036

Problem 1487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	104	79	93	138	0	95	50
normalized size	1	1.00	1.00	0.76	0.89	1.33	0.00	0.91	0.48
time (sec)	N/A	0.052	0.051	0.012	2.346	0.710	0.000	0.154	1.044

Problem 1488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	104	79	93	138	53	95	50
normalized size	1	1.00	1.00	0.76	0.89	1.33	0.51	0.91	0.48
time (sec)	N/A	0.053	0.051	0.011	2.320	0.604	171.554	0.179	0.043

Problem 1489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	104	79	93	138	51	95	50
normalized size	1	1.00	1.00	0.76	0.89	1.33	0.49	0.91	0.48
time (sec)	N/A	0.052	0.061	0.010	2.309	0.615	122.760	0.153	1.023

Problem 1490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	191	71	85	112	85	85	42
normalized size	1	1.00	1.91	0.71	0.85	1.12	0.85	0.85	0.42
time (sec)	N/A	0.065	0.116	0.002	2.265	0.676	0.180	0.169	0.065

Problem 1491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	7	8	8
normalized size	1	1.00	1.00	0.90	0.80	0.80	0.70	0.80	0.80
time (sec)	N/A	0.002	0.003	0.001	0.998	0.610	0.106	0.169	1.014

Problem 1492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	149	66	80	107	80	199	37
normalized size	1	1.00	1.60	0.71	0.86	1.15	0.86	2.14	0.40
time (sec)	N/A	0.063	0.052	0.002	2.324	0.586	0.184	0.307	0.052

Problem 1493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	6	6
normalized size	1	1.00	1.00	0.88	0.75	0.75	0.62	0.75	0.75
time (sec)	N/A	0.003	0.005	0.001	2.220	0.646	0.116	0.170	1.036

Problem 1494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	149	66	80	107	80	80	37
normalized size	1	1.00	1.60	0.71	0.86	1.15	0.86	0.86	0.40
time (sec)	N/A	0.062	0.039	0.002	2.240	0.643	0.178	0.170	0.038

Problem 1495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	15	11	10	15	11
normalized size	1	1.00	1.00	0.92	1.15	0.85	0.77	1.15	0.85
time (sec)	N/A	0.005	0.003	0.005	0.902	0.755	0.136	0.168	0.035

Problem 1496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	208	71	85	124	87	97	42
normalized size	1	1.00	2.08	0.71	0.85	1.24	0.87	0.97	0.42
time (sec)	N/A	0.066	0.038	0.004	2.327	0.866	0.222	0.236	1.055

Problem 1497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	15	14	12	12
normalized size	1	1.00	1.00	0.81	0.75	0.94	0.88	0.75	0.75
time (sec)	N/A	0.006	0.006	0.005	2.404	0.810	0.147	0.166	0.025

Problem 1498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	193	71	85	125	87	85	42
normalized size	1	1.00	1.93	0.71	0.85	1.25	0.87	0.85	0.42
time (sec)	N/A	0.067	0.082	0.003	2.403	0.912	0.222	0.171	1.055

Problem 1499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	20	24	17	25	18
normalized size	1	1.00	1.00	0.86	0.91	1.09	0.77	1.14	0.82
time (sec)	N/A	0.010	0.004	0.006	1.011	0.840	0.169	0.172	0.040

Problem 1500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	210	24	0	1028	15	240	289
normalized size	1	1.00	0.62	0.07	0.00	3.02	0.04	0.71	0.85
time (sec)	N/A	0.319	0.007	0.006	0.000	0.932	3.146	0.204	1.251

Problem 1501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	209	22	0	2513	15	239	256
normalized size	1	1.00	0.62	0.06	0.00	7.41	0.04	0.71	0.76
time (sec)	N/A	0.207	0.007	0.006	0.000	0.974	2.961	0.311	1.083

Problem 1502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	209	22	0	1027	15	239	295
normalized size	1	1.00	0.60	0.06	0.00	2.96	0.04	0.69	0.85
time (sec)	N/A	0.187	0.007	0.005	0.000	0.879	2.989	0.311	1.082

Problem 1503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	209	22	0	2513	15	239	193
normalized size	1	1.00	0.62	0.06	0.00	7.41	0.04	0.71	0.57
time (sec)	N/A	0.200	0.007	0.007	0.000	0.564	3.134	0.333	0.094

Problem 1504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	209	22	0	1027	14	239	288
normalized size	1	1.00	0.62	0.06	0.00	3.03	0.04	0.71	0.85
time (sec)	N/A	0.193	0.007	0.003	0.000	1.181	2.973	0.169	0.068

Problem 1505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	214	28	0	2541	19	244	261
normalized size	1	1.00	0.62	0.08	0.00	7.39	0.06	0.71	0.76
time (sec)	N/A	0.193	0.008	0.007	0.000	0.704	3.178	0.331	0.063

Problem 1506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	354	216	28	0	1105	22	244	298
normalized size	1	1.00	0.61	0.08	0.00	3.12	0.06	0.69	0.84
time (sec)	N/A	0.187	0.008	0.007	0.000	0.893	3.169	0.309	0.064

Problem 1507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	216	28	0	2591	22	244	196
normalized size	1	1.00	0.62	0.08	0.00	7.49	0.06	0.71	0.57
time (sec)	N/A	0.166	0.011	0.007	0.000	0.994	3.181	0.341	1.058

Problem 1508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	216	28	0	1105	20	244	293
normalized size	1	1.00	0.62	0.08	0.00	3.19	0.06	0.71	0.85
time (sec)	N/A	0.186	0.008	0.006	0.000	0.852	3.240	0.188	0.039

Problem 1509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	22	20	58	29	19	29	-1
normalized size	1	1.00	0.88	0.80	2.32	1.16	0.76	1.16	-0.04
time (sec)	N/A	0.008	0.007	0.146	1.100	0.851	1.569	0.162	0.000

Problem 1510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	22	30	0	0	31	0	-1
normalized size	1	1.00	0.35	0.48	0.00	0.00	0.50	0.00	-0.02
time (sec)	N/A	0.025	0.003	0.154	0.000	0.855	0.757	0.000	0.000

Problem 1511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	56	34	34	39	34	20
normalized size	1	1.00	1.00	2.00	1.21	1.21	1.39	1.21	0.71
time (sec)	N/A	0.011	0.004	0.169	1.046	0.775	1.303	0.152	1.138

Problem 1512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	22	30	0	0	34	0	-1
normalized size	1	1.00	0.18	0.24	0.00	0.00	0.27	0.00	-0.01
time (sec)	N/A	0.055	0.003	0.144	0.000	0.898	0.761	0.000	0.000

Problem 1513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	50	47	58	29	90	29	-1
normalized size	1	1.00	1.43	1.34	1.66	0.83	2.57	0.83	-0.03
time (sec)	N/A	0.012	0.036	0.161	1.078	0.876	1.770	0.158	0.000

Problem 1514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	31	27	86	37	49	36	-1
normalized size	1	1.00	0.76	0.66	2.10	0.90	1.20	0.88	-0.02
time (sec)	N/A	0.016	0.012	0.144	1.061	0.962	4.296	0.166	0.000

Problem 1515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	18	15	19	14	22	19	14
normalized size	1	1.00	0.67	0.56	0.70	0.52	0.81	0.70	0.52
time (sec)	N/A	0.010	0.005	0.004	1.017	0.789	2.829	0.150	1.164

Problem 1516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	58	29	19	29	-1
normalized size	1	1.00	1.00	0.80	2.32	1.16	0.76	1.16	-0.04
time (sec)	N/A	0.010	0.005	0.142	1.067	0.781	2.279	0.163	0.000

Problem 1517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	8	9	9
normalized size	1	1.00	1.00	0.77	0.69	0.69	0.62	0.69	0.69
time (sec)	N/A	0.003	0.002	0.005	0.973	0.857	0.491	0.147	0.085

Problem 1518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	33	16	5	16	-1
normalized size	1	1.00	1.00	0.88	4.12	2.00	0.62	2.00	-0.12
time (sec)	N/A	0.003	0.002	0.138	1.057	0.801	1.062	0.159	0.000

Problem 1519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	19	25	25	8	25	10
normalized size	1	1.00	1.00	1.36	1.79	1.79	0.57	1.79	0.71
time (sec)	N/A	0.007	0.002	0.006	1.013	0.640	1.022	0.214	0.053

Problem 1520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	16	12	21	12
normalized size	1	1.00	1.00	0.81	0.75	1.00	0.75	1.31	0.75
time (sec)	N/A	0.003	0.003	0.005	1.108	0.829	0.744	0.167	1.130

Problem 1521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	32	37	44	22	37	23
normalized size	1	1.00	1.00	1.03	1.19	1.42	0.71	1.19	0.74
time (sec)	N/A	0.011	0.016	0.025	1.050	0.721	2.292	0.147	1.411

Problem 1522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	23	20	25	26	26	40	19
normalized size	1	1.00	0.70	0.61	0.76	0.79	0.79	1.21	0.58
time (sec)	N/A	0.006	0.004	0.005	1.079	0.795	1.567	0.168	1.192

Problem 1523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	26	42	64	52	60	49	49
normalized size	1	1.00	0.55	0.89	1.36	1.11	1.28	1.04	1.04
time (sec)	N/A	0.016	0.005	0.026	0.990	0.736	4.663	0.149	1.464

Problem 1524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	34	30	0	0	29	0	-1
normalized size	1	1.00	0.26	0.23	0.00	0.00	0.22	0.00	-0.01
time (sec)	N/A	0.053	0.007	0.146	0.000	0.828	1.293	0.000	0.000

Problem 1525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	34	30	0	0	29	0	-1
normalized size	1	1.00	0.55	0.48	0.00	0.00	0.47	0.00	-0.02
time (sec)	N/A	0.024	0.008	0.144	0.000	0.914	0.967	0.000	0.000

Problem 1526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	22	17	0	0	29	0	-1
normalized size	1	1.00	0.19	0.15	0.00	0.00	0.25	0.00	-0.01
time (sec)	N/A	0.041	0.004	0.146	0.000	1.131	0.774	0.000	0.000

Problem 1527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	22	17	0	0	29	0	-1
normalized size	1	1.00	0.49	0.38	0.00	0.00	0.64	0.00	-0.02
time (sec)	N/A	0.015	0.003	0.142	0.000	0.957	0.643	0.000	0.000

Problem 1528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	22	30	0	0	32	0	-1
normalized size	1	1.00	0.17	0.23	0.00	0.00	0.25	0.00	-0.01
time (sec)	N/A	0.046	0.004	0.016	0.000	0.883	0.803	0.000	0.000

Problem 1529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	22	30	0	0	32	0	-1
normalized size	1	1.00	0.35	0.48	0.00	0.00	0.52	0.00	-0.02
time (sec)	N/A	0.024	0.012	0.019	0.000	0.807	0.990	0.000	0.000

Problem 1530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	34	30	0	0	29	0	-1
normalized size	1	1.00	0.14	0.13	0.00	0.00	0.12	0.00	-0.00
time (sec)	N/A	0.067	0.008	0.151	0.000	1.002	1.045	0.000	0.000

Problem 1531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	32	26	0	0	29	0	-1
normalized size	1	1.00	0.14	0.11	0.00	0.00	0.12	0.00	-0.00
time (sec)	N/A	0.061	0.007	0.146	0.000	1.069	0.870	0.000	0.000

Problem 1532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	0	0	29	0	-1
normalized size	1	1.00	1.00	0.77	0.00	0.00	1.32	0.00	-0.05
time (sec)	N/A	0.004	0.003	0.147	0.000	1.045	0.856	0.000	0.000

Problem 1533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	0	0	29	0	-1
normalized size	1	1.00	1.00	0.77	0.00	0.00	1.32	0.00	-0.05
time (sec)	N/A	0.005	0.003	0.150	0.000	0.729	0.736	0.000	0.000

Problem 1534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	22	17	0	0	29	0	-1
normalized size	1	1.00	0.10	0.08	0.00	0.00	0.13	0.00	-0.00
time (sec)	N/A	0.048	0.003	0.142	0.000	1.150	0.677	0.000	0.000

Problem 1535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	17	14	0	0	27	0	12
normalized size	1	1.00	0.08	0.06	0.00	0.00	0.12	0.00	0.05
time (sec)	N/A	0.046	0.002	0.146	0.000	0.959	0.668	0.000	0.074

Problem 1536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	20	30	0	0	31	0	15
normalized size	1	1.00	0.54	0.81	0.00	0.00	0.84	0.00	0.41
time (sec)	N/A	0.009	0.003	0.145	0.000	0.901	0.810	0.000	1.121

Problem 1537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	22	30	0	0	32	0	-1
normalized size	1	1.00	0.56	0.77	0.00	0.00	0.82	0.00	-0.03
time (sec)	N/A	0.008	0.003	0.148	0.000	0.950	0.841	0.000	0.000

Problem 1538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	22	30	0	0	32	0	-1
normalized size	1	1.00	0.09	0.13	0.00	0.00	0.13	0.00	-0.00
time (sec)	N/A	0.064	0.003	0.144	0.000	1.086	0.946	0.000	0.000

Problem 1539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	22	28	0	0	32	0	-1
normalized size	1	1.00	0.09	0.12	0.00	0.00	0.13	0.00	-0.00
time (sec)	N/A	0.061	0.003	0.143	0.000	1.064	1.046	0.000	0.000

Problem 1540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	325	289	426	0	1039	70	223	852
normalized size	1	1.99	1.77	2.61	0.00	6.37	0.43	1.37	5.23
time (sec)	N/A	0.209	0.255	0.037	0.000	4.323	7.280	0.203	1.294

Problem 1541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	16	18	19	6	-1
normalized size	1	1.00	1.00	0.88	2.00	2.25	2.38	0.75	-0.12
time (sec)	N/A	0.004	0.003	0.152	2.231	0.968	1.120	0.196	0.000

Problem 1542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	34	33	16	34	17	-1
normalized size	1	1.00	1.00	1.89	1.83	0.89	1.89	0.94	-0.06
time (sec)	N/A	0.008	0.003	0.168	1.093	0.861	1.174	0.195	0.000

Problem 1543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	8	8	7	8	8
normalized size	1	1.00	1.00	0.75	0.67	0.67	0.58	0.67	0.67
time (sec)	N/A	0.005	0.005	0.003	2.302	1.081	0.136	0.173	0.059

Problem 1544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	23	18	17	17	15	18	8
normalized size	1	1.00	1.92	1.50	1.42	1.42	1.25	1.50	0.67
time (sec)	N/A	0.005	0.004	0.006	1.009	0.892	0.143	0.199	1.081

Problem 1545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	28	22	58	29	37	29	-1
normalized size	1	1.00	0.97	0.76	2.00	1.00	1.28	1.00	-0.03
time (sec)	N/A	0.009	0.007	0.152	0.971	0.974	1.643	0.194	0.000

Problem 1546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.011	0.002	0.002	1.008	0.923	0.062	0.163	0.023

Problem 1547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.010	0.001	0.001	1.058	0.935	0.063	0.186	0.021

Problem 1548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.009	0.001	0.000	1.022	0.892	0.063	0.152	0.021

Problem 1549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.008	0.001	0.001	0.988	0.967	0.065	0.164	0.022

Problem 1550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.006	0.001	0.001	1.024	0.766	0.064	0.147	0.020

Problem 1551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	10
normalized size	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.004	0.001	0.001	1.000	0.968	0.061	0.164	0.018

Problem 1552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	7	9	8
normalized size	1	1.00	1.00	1.12	1.00	1.00	0.88	1.12	1.00
time (sec)	N/A	0.002	0.001	0.002	0.964	0.853	0.087	0.169	0.020

Problem 1553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	13	7	12	11
normalized size	1	1.00	1.00	1.09	1.00	1.18	0.64	1.09	1.00
time (sec)	N/A	0.005	0.002	0.007	1.027	0.993	0.106	0.180	0.034

Problem 1554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	14	11	12	13	11
normalized size	1	1.00	1.00	0.93	0.93	0.73	0.80	0.87	0.73
time (sec)	N/A	0.005	0.002	0.004	1.112	0.849	0.119	0.148	0.023

Problem 1555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	14	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.82	0.76	0.76
time (sec)	N/A	0.005	0.002	0.005	1.000	0.539	0.121	0.155	0.024

Problem 1556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	14	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.82	0.76	0.76
time (sec)	N/A	0.005	0.002	0.005	1.053	0.904	0.126	0.148	0.024

Problem 1557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	14	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.82	0.76	0.76
time (sec)	N/A	0.005	0.002	0.003	1.100	0.974	0.148	0.174	0.026

Problem 1558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	14	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.82	0.76	0.76
time (sec)	N/A	0.005	0.002	0.005	0.968	0.883	0.146	0.158	0.026

Problem 1559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.014	0.002	0.001	1.009	0.995	0.084	0.160	0.042

Problem 1560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	24	24
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.013	0.002	0.001	1.049	0.886	0.070	0.151	0.032

Problem 1561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.012	0.001	0.000	1.043	1.023	0.068	0.182	0.032

Problem 1562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	20	20	19	20	20
normalized size	1	1.00	1.00	0.93	1.43	1.43	1.36	1.43	1.43
time (sec)	N/A	0.004	0.002	0.001	0.998	0.931	0.070	0.155	0.029

Problem 1563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	20	20	21	20
normalized size	1	1.00	1.00	0.95	0.91	0.91	0.91	0.95	0.91
time (sec)	N/A	0.008	0.001	0.003	1.054	0.898	0.106	0.159	0.029

Problem 1564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	20	24	17	21	20
normalized size	1	1.00	1.00	1.05	1.00	1.20	0.85	1.05	1.00
time (sec)	N/A	0.009	0.004	0.004	1.052	0.754	0.119	0.171	0.029

Problem 1565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	21	26	22	22	23
normalized size	1	1.00	1.00	0.96	0.88	1.08	0.92	0.92	0.96
time (sec)	N/A	0.011	0.004	0.007	1.019	0.842	0.165	0.153	0.043

Problem 1566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	26	25	14	22	24	14	22
normalized size	1	1.00	1.62	1.56	0.88	1.38	1.50	0.88	1.38
time (sec)	N/A	0.003	0.007	0.006	0.999	0.928	0.172	0.149	0.036

Problem 1567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.011	0.004	0.006	1.020	1.108	0.182	0.149	0.036

Problem 1568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.011	0.007	0.005	1.000	0.893	0.188	0.175	0.034

Problem 1569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.011	0.004	0.005	0.965	0.893	0.219	0.149	0.036

Problem 1570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	37	35	35
normalized size	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.81
time (sec)	N/A	0.020	0.002	0.002	1.114	0.971	0.075	0.163	0.042

Problem 1571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	39	35	35
normalized size	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.81
time (sec)	N/A	0.018	0.002	0.001	1.109	0.971	0.071	0.170	0.043

Problem 1572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	40	35	34	34	36	34	34
normalized size	1	1.00	1.33	1.17	1.13	1.13	1.20	1.13	1.13
time (sec)	N/A	0.010	0.002	0.002	1.147	0.849	0.079	0.148	0.040

Problem 1573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	31	31	32	31	31
normalized size	1	1.00	1.00	0.93	2.21	2.21	2.29	2.21	2.21
time (sec)	N/A	0.004	0.002	0.000	1.102	1.125	0.070	0.146	0.040

Problem 1574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	31	31	34	32	31
normalized size	1	1.00	1.00	0.91	0.89	0.89	0.97	0.91	0.89
time (sec)	N/A	0.012	0.003	0.003	1.001	1.026	0.124	0.152	0.037

Problem 1575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	33	32	36	31	33	32
normalized size	1	1.00	1.00	0.97	0.94	1.06	0.91	0.97	0.94
time (sec)	N/A	0.014	0.005	0.006	1.038	0.891	0.141	0.147	0.039

Problem 1576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	32	31	37	32	31	32
normalized size	1	1.00	1.00	0.97	0.94	1.12	0.97	0.94	0.97
time (sec)	N/A	0.013	0.004	0.005	1.026	0.914	0.175	0.150	0.031

Problem 1577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	34	34	37	36	35	34
normalized size	1	1.00	1.00	0.92	0.92	1.00	0.97	0.95	0.92
time (sec)	N/A	0.015	0.004	0.006	1.011	0.944	0.208	0.190	0.038

Problem 1578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	39	36	14	33	36	14	33
normalized size	1	1.00	2.44	2.25	0.88	2.06	2.25	0.88	2.06
time (sec)	N/A	0.003	0.010	0.005	1.019	1.074	0.219	0.150	0.028

Problem 1579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	41	36	35	35	37	35	34
normalized size	1	1.00	1.14	1.00	0.97	0.97	1.03	0.97	0.94
time (sec)	N/A	0.007	0.006	0.004	1.035	1.142	0.244	0.149	0.028

Problem 1580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	37	35	35
normalized size	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.81
time (sec)	N/A	0.015	0.004	0.005	0.979	1.005	0.258	0.149	0.030

Problem 1581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	37	35	35
normalized size	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.81
time (sec)	N/A	0.015	0.004	0.005	1.032	0.625	0.279	0.155	0.030

Problem 1582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	37	35	35
normalized size	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.81
time (sec)	N/A	0.015	0.006	0.004	1.032	1.009	0.319	0.152	0.031

Problem 1583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	106	91	90	90	104	90	90
normalized size	1	1.00	1.00	0.86	0.85	0.85	0.98	0.85	0.85
time (sec)	N/A	0.057	0.003	0.000	0.951	0.674	0.092	0.154	0.058

Problem 1584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	106	91	90	90	105	90	90
normalized size	1	1.00	1.00	0.86	0.85	0.85	0.99	0.85	0.85
time (sec)	N/A	0.048	0.003	0.002	0.983	0.952	0.093	0.160	1.071

Problem 1585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	106	91	90	90	105	90	90
normalized size	1	1.00	1.08	0.93	0.92	0.92	1.07	0.92	0.92
time (sec)	N/A	0.044	0.003	0.001	1.034	1.028	0.094	0.152	0.053

Problem 1586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	104	91	90	90	104	90	90
normalized size	1	1.00	1.28	1.12	1.11	1.11	1.28	1.11	1.11
time (sec)	N/A	0.038	0.003	0.002	0.990	0.934	0.093	0.157	1.065

Problem 1587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	106	91	90	90	105	90	90
normalized size	1	1.00	1.66	1.42	1.41	1.41	1.64	1.41	1.41
time (sec)	N/A	0.033	0.003	0.000	1.057	1.054	0.095	0.150	0.054

Problem 1588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	102	91	90	90	102	90	90
normalized size	1	1.00	2.17	1.94	1.91	1.91	2.17	1.91	1.91
time (sec)	N/A	0.028	0.003	0.001	0.961	0.898	0.091	0.165	1.063

Problem 1589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	104	91	90	90	104	90	90
normalized size	1	1.00	3.47	3.03	3.00	3.00	3.47	3.00	3.00
time (sec)	N/A	0.010	0.003	0.003	1.056	0.961	0.105	0.168	1.061

Problem 1590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	86	86	94	86	86
normalized size	1	1.00	1.00	0.93	6.14	6.14	6.71	6.14	6.14
time (sec)	N/A	0.004	0.001	0.000	0.968	0.857	0.094	0.151	0.051

Problem 1591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	98	87	86	86	100	87	86
normalized size	1	1.00	1.00	0.89	0.88	0.88	1.02	0.89	0.88
time (sec)	N/A	0.036	0.005	0.003	0.966	1.089	0.208	0.181	1.059

Problem 1592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	88	87	92	95	88	87
normalized size	1	1.00	1.00	0.93	0.92	0.97	1.00	0.93	0.92
time (sec)	N/A	0.040	0.009	0.005	1.015	0.853	0.221	0.153	0.057

Problem 1593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	88	86	92	97	87	88
normalized size	1	1.00	1.00	0.93	0.91	0.97	1.02	0.92	0.93
time (sec)	N/A	0.039	0.005	0.006	0.988	0.629	0.263	0.153	0.052

Problem 1594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	93	88	86	92	95	87	88
normalized size	1	1.00	1.00	0.95	0.92	0.99	1.02	0.94	0.95
time (sec)	N/A	0.040	0.009	0.006	0.939	1.027	0.321	0.154	0.051

Problem 1595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	88	88	92	97	89	88
normalized size	1	1.00	1.00	0.93	0.93	0.97	1.02	0.94	0.93
time (sec)	N/A	0.039	0.005	0.007	1.057	0.661	0.369	0.169	0.053

Problem 1596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	93	88	88	92	95	89	88
normalized size	1	1.00	1.00	0.95	0.95	0.99	1.02	0.96	0.95
time (sec)	N/A	0.043	0.010	0.008	0.950	1.016	0.455	0.193	1.053

Problem 1597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	88	88	92	95	89	88
normalized size	1	1.00	1.00	0.93	0.93	0.97	1.00	0.94	0.93
time (sec)	N/A	0.039	0.006	0.010	0.922	0.875	0.519	0.155	1.061

Problem 1598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	94	87	87	92	94	88	92
normalized size	1	1.00	1.00	0.93	0.93	0.98	1.00	0.94	0.98
time (sec)	N/A	0.040	0.005	0.008	1.044	0.859	0.559	0.174	1.070

Problem 1599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	100	89	89	92	95	90	89
normalized size	1	1.00	1.00	0.89	0.89	0.92	0.95	0.90	0.89
time (sec)	N/A	0.042	0.005	0.007	1.065	1.064	0.640	0.151	0.066

Problem 1600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	96	91	14	88	95	14	88
normalized size	1	1.00	6.00	5.69	0.88	5.50	5.94	0.88	5.50
time (sec)	N/A	0.003	0.010	0.006	1.100	0.992	0.685	0.148	1.099

Problem 1601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	104	91	90	90	97	90	90
normalized size	1	1.00	2.89	2.53	2.50	2.50	2.69	2.50	2.50
time (sec)	N/A	0.007	0.004	0.005	0.976	0.999	0.705	0.214	0.072

Problem 1602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	102	91	90	90	97	90	90
normalized size	1	1.00	1.82	1.62	1.61	1.61	1.73	1.61	1.61
time (sec)	N/A	0.012	0.009	0.006	1.051	1.242	0.765	0.165	1.084

Problem 1603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	106	91	90	90	97	90	90
normalized size	1	1.00	1.39	1.20	1.18	1.18	1.28	1.18	1.18
time (sec)	N/A	0.019	0.005	0.006	1.158	0.863	0.852	0.174	0.073

Problem 1604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	104	91	90	90	97	90	90
normalized size	1	1.00	1.08	0.95	0.94	0.94	1.01	0.94	0.94
time (sec)	N/A	0.027	0.024	0.006	1.009	1.033	0.886	0.151	1.076

Problem 1605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	106	91	90	90	97	90	90
normalized size	1	1.00	1.00	0.86	0.85	0.85	0.92	0.85	0.85
time (sec)	N/A	0.043	0.013	0.007	1.079	0.840	0.925	0.166	0.075

Problem 1606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	106	91	90	90	97	90	89
normalized size	1	1.00	1.00	0.86	0.85	0.85	0.92	0.85	0.84
time (sec)	N/A	0.041	0.012	0.006	1.006	0.982	0.920	0.154	0.072

Problem 1607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	63	64	63	61	65	62
normalized size	1	1.00	1.00	0.90	0.91	0.90	0.87	0.93	0.89
time (sec)	N/A	0.037	0.005	0.003	1.089	0.958	0.155	0.173	1.036

Problem 1608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	52	52	52	49	53	51
normalized size	1	1.00	1.00	0.91	0.91	0.91	0.86	0.93	0.89
time (sec)	N/A	0.027	0.005	0.004	1.078	1.186	0.145	0.173	0.029

Problem 1609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	41	42	41	37	43	40
normalized size	1	1.00	1.00	0.93	0.95	0.93	0.84	0.98	0.91
time (sec)	N/A	0.022	0.005	0.003	0.998	0.715	0.136	0.147	0.040

Problem 1610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	30	29	29	26	30	29
normalized size	1	1.00	1.00	0.97	0.94	0.94	0.84	0.97	0.94
time (sec)	N/A	0.016	0.004	0.003	1.103	0.680	0.120	0.149	0.039

Problem 1611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	17	14	19	18
normalized size	1	1.00	1.00	1.06	1.00	0.94	0.78	1.06	1.00
time (sec)	N/A	0.010	0.003	0.003	0.947	1.046	0.116	0.175	1.048

Problem 1612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	7	11	10
normalized size	1	1.00	1.00	1.10	1.00	1.00	0.70	1.10	1.00
time (sec)	N/A	0.004	0.001	0.000	0.992	0.719	0.072	0.148	0.021

Problem 1613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	18	19	13	16	10	14	15
normalized size	1	1.00	1.38	1.46	1.00	1.23	0.77	1.08	1.15
time (sec)	N/A	0.004	0.004	0.004	1.099	0.962	0.153	0.232	0.045

Problem 1614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	28	26	19	30	25
normalized size	1	1.00	1.00	1.04	1.00	0.93	0.68	1.07	0.89
time (sec)	N/A	0.016	0.006	0.006	1.035	0.900	0.197	0.173	0.057

Problem 1615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	41	40	41	31	45	38
normalized size	1	1.00	1.00	0.98	0.95	0.98	0.74	1.07	0.90
time (sec)	N/A	0.020	0.005	0.009	1.116	0.890	0.232	0.169	0.059

Problem 1616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	53	51	54	44	56	48
normalized size	1	1.00	1.00	0.95	0.91	0.96	0.79	1.00	0.86
time (sec)	N/A	0.023	0.006	0.009	1.033	0.983	0.244	0.152	1.083

Problem 1617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	63	62	65	56	67	60
normalized size	1	1.00	1.00	0.93	0.91	0.96	0.82	0.99	0.88
time (sec)	N/A	0.030	0.006	0.008	0.938	0.689	0.262	0.213	1.094

Problem 1618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	75	73	76	68	78	70
normalized size	1	1.00	1.00	0.91	0.89	0.93	0.83	0.95	0.85
time (sec)	N/A	0.034	0.008	0.010	1.069	1.129	0.296	0.168	1.098

Problem 1619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	88	89	92	107	99	95	94
normalized size	1	1.00	0.90	0.91	0.94	1.09	1.01	0.97	0.96
time (sec)	N/A	0.066	0.023	0.008	1.061	0.836	0.289	0.159	0.060

Problem 1620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	77	78	82	96	78	85	83
normalized size	1	1.00	0.95	0.96	1.01	1.19	0.96	1.05	1.02
time (sec)	N/A	0.053	0.025	0.009	1.178	1.144	0.257	0.152	0.051

Problem 1621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	66	67	70	85	71	73	72
normalized size	1	1.00	0.92	0.93	0.97	1.18	0.99	1.01	1.00
time (sec)	N/A	0.048	0.018	0.008	1.088	0.970	0.260	0.154	0.049

Problem 1622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	54	57	59	73	54	62	62
normalized size	1	1.00	0.93	0.98	1.02	1.26	0.93	1.07	1.07
time (sec)	N/A	0.037	0.027	0.009	1.053	0.885	0.221	0.181	1.053

Problem 1623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	45	47	62	44	48	50
normalized size	1	1.00	0.93	0.98	1.02	1.35	0.96	1.04	1.09
time (sec)	N/A	0.028	0.015	0.008	1.091	1.058	0.198	0.149	0.045

Problem 1624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	29	34	36	47	31	34	36
normalized size	1	1.00	0.85	1.00	1.06	1.38	0.91	1.00	1.06
time (sec)	N/A	0.014	0.012	0.005	1.249	1.067	0.174	0.149	1.060

Problem 1625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	24	26	28	20	24	23
normalized size	1	1.00	0.87	1.04	1.13	1.22	0.87	1.04	1.00
time (sec)	N/A	0.015	0.007	0.007	1.040	0.848	0.158	0.194	1.063

Problem 1626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	12	13	13	13	10	13	12
normalized size	1	1.00	0.92	1.00	1.00	1.00	0.77	1.00	0.92
time (sec)	N/A	0.004	0.003	0.000	1.047	0.798	0.141	0.146	0.031

Problem 1627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	24	30	28	39	22	31	26
normalized size	1	1.00	0.83	1.03	0.97	1.34	0.76	1.07	0.90
time (sec)	N/A	0.017	0.013	0.008	1.080	1.026	0.215	0.154	0.045

Problem 1628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	35	43	45	63	37	45	41
normalized size	1	1.00	0.83	1.02	1.07	1.50	0.88	1.07	0.98
time (sec)	N/A	0.024	0.041	0.010	1.141	0.905	0.277	0.151	1.068

Problem 1629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	53	57	64	86	54	64	57
normalized size	1	1.00	0.91	0.98	1.10	1.48	0.93	1.10	0.98
time (sec)	N/A	0.031	0.055	0.010	1.027	0.908	0.322	0.149	1.084

Problem 1630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	66	68	73	95	66	73	69
normalized size	1	1.00	0.96	0.99	1.06	1.38	0.96	1.06	1.00
time (sec)	N/A	0.038	0.062	0.010	1.061	0.823	0.390	0.155	0.081

Problem 1631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	79	79	86	108	80	86	79
normalized size	1	1.00	0.94	0.94	1.02	1.29	0.95	1.02	0.94
time (sec)	N/A	0.047	0.045	0.011	1.012	1.011	0.374	0.209	1.100

Problem 1632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	90	91	97	119	92	97	91
normalized size	1	1.00	0.96	0.97	1.03	1.27	0.98	1.03	0.97
time (sec)	N/A	0.055	0.074	0.012	1.094	0.977	0.427	0.151	1.101

Problem 1633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	85	94	103	129	109	95	104
normalized size	1	1.00	0.86	0.95	1.04	1.30	1.10	0.96	1.05
time (sec)	N/A	0.070	0.060	0.008	1.024	0.874	0.405	0.155	1.086

Problem 1634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	73	83	91	117	92	83	92
normalized size	1	1.00	0.85	0.97	1.06	1.36	1.07	0.97	1.07
time (sec)	N/A	0.056	0.058	0.009	1.075	0.757	0.355	0.178	0.055

Problem 1635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	63	72	81	107	85	73	82
normalized size	1	1.00	0.82	0.94	1.05	1.39	1.10	0.95	1.06
time (sec)	N/A	0.049	0.063	0.010	1.120	0.742	0.328	0.150	0.050

Problem 1636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	50	61	69	95	70	61	70
normalized size	1	1.00	0.78	0.95	1.08	1.48	1.09	0.95	1.09
time (sec)	N/A	0.037	0.036	0.008	1.069	1.055	0.299	0.155	1.072

Problem 1637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	40	49	57	83	58	44	59
normalized size	1	1.00	0.75	0.92	1.08	1.57	1.09	0.83	1.11
time (sec)	N/A	0.020	0.042	0.007	1.040	0.740	0.273	0.149	0.061

Problem 1638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	33	40	48	61	46	37	46
normalized size	1	1.00	0.80	0.98	1.17	1.49	1.12	0.90	1.12
time (sec)	N/A	0.022	0.014	0.007	0.983	0.817	0.223	0.154	1.075

Problem 1639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	20	27	14	32	32	14	32
normalized size	1	1.00	1.25	1.69	0.88	2.00	2.00	0.88	2.00
time (sec)	N/A	0.004	0.006	0.005	1.017	1.014	0.209	0.150	1.057

Problem 1640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	24	24	26	12	26
normalized size	1	1.00	1.00	0.93	1.71	1.71	1.86	0.86	1.86
time (sec)	N/A	0.004	0.003	0.001	1.017	1.099	0.206	0.152	1.049

Problem 1641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	37	42	51	80	46	43	47
normalized size	1	1.00	0.86	0.98	1.19	1.86	1.07	1.00	1.09
time (sec)	N/A	0.023	0.033	0.007	1.024	0.799	0.313	0.152	1.076

Problem 1642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	53	56	69	109	66	60	63
normalized size	1	1.00	0.93	0.98	1.21	1.91	1.16	1.05	1.11
time (sec)	N/A	0.032	0.053	0.010	1.017	1.023	0.366	0.155	0.080

Problem 1643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	68	73	86	130	78	73	79
normalized size	1	1.00	0.89	0.96	1.13	1.71	1.03	0.96	1.04
time (sec)	N/A	0.040	0.052	0.011	1.105	0.905	0.422	0.157	0.087

Problem 1644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	79	84	97	141	92	86	91
normalized size	1	1.00	0.89	0.94	1.09	1.58	1.03	0.97	1.02
time (sec)	N/A	0.049	0.078	0.011	1.065	1.006	0.445	0.167	0.091

Problem 1645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	90	94	108	152	102	97	101
normalized size	1	1.00	0.93	0.97	1.11	1.57	1.05	1.00	1.04
time (sec)	N/A	0.057	0.058	0.011	1.144	0.991	0.509	0.154	1.132

Problem 1646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	101	106	119	163	116	108	113
normalized size	1	1.00	0.91	0.95	1.07	1.47	1.05	0.97	1.02
time (sec)	N/A	0.067	0.095	0.010	1.012	0.949	0.506	0.153	1.176

Problem 1647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	15	18	19	13	13
normalized size	1	1.00	0.81	0.67	0.71	0.86	0.90	0.62	0.62
time (sec)	N/A	0.004	0.005	0.005	1.007	1.160	1.771	0.149	1.074

Problem 1648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	15	16	19	13	13
normalized size	1	1.00	0.81	0.67	0.71	0.76	0.90	0.62	0.62
time (sec)	N/A	0.004	0.006	0.003	1.105	1.231	0.717	0.162	0.027

Problem 1649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	13	13	12	17	13	12
normalized size	1	1.00	0.84	0.68	0.68	0.63	0.89	0.68	0.63
time (sec)	N/A	0.004	0.005	0.003	0.991	0.998	0.222	0.167	0.027

Problem 1650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	14	13	13	12	15	13	11
normalized size	1	1.00	0.82	0.76	0.76	0.71	0.88	0.76	0.65
time (sec)	N/A	0.004	0.005	0.003	1.071	0.645	0.356	0.152	0.030

Problem 1651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	15	12	13	11	19	11	13
normalized size	1	1.00	0.79	0.63	0.68	0.58	1.00	0.58	0.68
time (sec)	N/A	0.004	0.005	0.001	1.074	0.936	0.504	0.150	0.028

Problem 1652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	13	13	20	13	13
normalized size	1	1.00	0.81	0.67	0.62	0.62	0.95	0.62	0.62
time (sec)	N/A	0.004	0.005	0.003	1.019	0.978	0.935	0.152	0.029

Problem 1653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	25	26	27	34	24	24
normalized size	1	1.00	0.78	0.69	0.72	0.75	0.94	0.67	0.67
time (sec)	N/A	0.010	0.008	0.005	1.054	1.057	2.562	0.150	0.051

Problem 1654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	28	25	26	24	32	24	24
normalized size	1	1.00	0.82	0.74	0.76	0.71	0.94	0.71	0.71
time (sec)	N/A	0.009	0.008	0.004	1.024	1.011	1.157	0.153	0.037

Problem 1655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	27	24	25	23	31	24	24
normalized size	1	1.00	0.84	0.75	0.78	0.72	0.97	0.75	0.75
time (sec)	N/A	0.009	0.009	0.005	0.982	1.075	0.503	0.147	0.038

Problem 1656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	26	25	24	24	31	23	24
normalized size	1	1.00	0.81	0.78	0.75	0.75	0.97	0.72	0.75
time (sec)	N/A	0.009	0.009	0.005	1.037	1.024	0.541	0.150	1.082

Problem 1657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	28	25	24	24	34	24	24
normalized size	1	1.00	0.82	0.74	0.71	0.71	1.00	0.71	0.71
time (sec)	N/A	0.009	0.008	0.004	1.069	0.898	0.851	0.155	0.028

Problem 1658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	25	24	24	36	24	24
normalized size	1	1.00	0.78	0.69	0.67	0.67	1.00	0.67	0.67
time (sec)	N/A	0.009	0.009	0.006	1.166	1.048	1.481	0.154	0.031

Problem 1659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	39	36	37	35	46	35	35
normalized size	1	1.00	0.83	0.77	0.79	0.74	0.98	0.74	0.74
time (sec)	N/A	0.013	0.011	0.005	1.065	0.907	3.881	0.150	0.046

Problem 1660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	38	35	36	34	44	35	35
normalized size	1	1.00	0.84	0.78	0.80	0.76	0.98	0.78	0.78
time (sec)	N/A	0.012	0.020	0.006	1.021	0.787	1.814	0.151	0.047

Problem 1661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	38	35	36	34	46	34	35
normalized size	1	1.00	0.81	0.74	0.77	0.72	0.98	0.72	0.74
time (sec)	N/A	0.013	0.011	0.005	0.861	0.864	0.799	0.177	1.083

Problem 1662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	39	36	35	35	44	34	35
normalized size	1	1.00	0.87	0.80	0.78	0.78	0.98	0.76	0.78
time (sec)	N/A	0.013	0.011	0.006	1.046	1.029	0.853	0.162	1.072

Problem 1663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	39	36	35	35	48	35	35
normalized size	1	1.00	0.83	0.77	0.74	0.74	1.02	0.74	0.74
time (sec)	N/A	0.013	0.021	0.005	1.068	0.774	1.221	0.165	0.033

Problem 1664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	39	36	35	35	51	35	35
normalized size	1	1.00	0.76	0.71	0.69	0.69	1.00	0.69	0.69
time (sec)	N/A	0.013	0.012	0.005	1.034	0.779	2.037	0.158	0.037

Problem 1665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	72	65	65	153	136	70	59
normalized size	1	1.00	0.87	0.78	0.78	1.84	1.64	0.84	0.71
time (sec)	N/A	0.036	0.044	0.010	2.293	1.060	22.577	0.153	0.037

Problem 1666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	61	54	54	132	121	59	48
normalized size	1	1.00	0.90	0.79	0.79	1.94	1.78	0.87	0.71
time (sec)	N/A	0.023	0.032	0.007	2.369	0.717	5.749	0.210	0.052

Problem 1667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	49	43	41	103	105	45	37
normalized size	1	1.00	0.92	0.81	0.77	1.94	1.98	0.85	0.70
time (sec)	N/A	0.018	0.019	0.008	2.329	0.864	1.367	0.174	0.054

Problem 1668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	32	31	85	92	31	28
normalized size	1	1.00	1.00	0.80	0.78	2.12	2.30	0.78	0.70
time (sec)	N/A	0.014	0.010	0.006	2.652	0.941	1.353	0.150	0.042

Problem 1669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	19	18	68	94	18	19
normalized size	1	1.00	1.00	0.66	0.62	2.34	3.24	0.62	0.66
time (sec)	N/A	0.010	0.005	0.006	2.329	1.038	3.293	0.150	1.064

Problem 1670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	25	32	31	93	102	31	28
normalized size	1	1.00	0.62	0.80	0.78	2.32	2.55	0.78	0.70
time (sec)	N/A	0.015	0.012	0.009	2.484	1.017	12.593	0.173	0.056

Problem 1671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	27	43	43	118	121	41	38
normalized size	1	1.00	0.51	0.81	0.81	2.23	2.28	0.77	0.72
time (sec)	N/A	0.019	0.005	0.010	2.308	0.892	57.640	0.175	0.064

Problem 1672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	27	54	54	144	139	52	49
normalized size	1	1.00	0.40	0.79	0.79	2.12	2.04	0.76	0.72
time (sec)	N/A	0.025	0.006	0.010	2.290	0.786	149.236	0.159	0.057

Problem 1673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	27	83	88	209	0	88	80
normalized size	1	1.00	0.28	0.85	0.90	2.13	0.00	0.90	0.82
time (sec)	N/A	0.041	0.005	0.013	2.388	1.010	0.000	0.155	1.079

Problem 1674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	27	71	77	188	542	76	68
normalized size	1	1.00	0.32	0.84	0.91	2.21	6.38	0.89	0.80
time (sec)	N/A	0.033	0.005	0.011	2.558	0.628	57.535	0.159	0.038

Problem 1675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	27	61	66	161	479	65	58
normalized size	1	1.00	0.39	0.87	0.94	2.30	6.84	0.93	0.83
time (sec)	N/A	0.024	0.005	0.013	2.348	0.921	9.641	0.184	0.059

Problem 1676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	27	47	52	134	411	46	46
normalized size	1	1.00	0.47	0.82	0.91	2.35	7.21	0.81	0.81
time (sec)	N/A	0.019	0.004	0.012	2.350	0.914	12.339	0.164	1.100

Problem 1677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	37	39	115	337	36	34
normalized size	1	1.00	1.00	0.80	0.85	2.50	7.33	0.78	0.74
time (sec)	N/A	0.016	0.030	0.012	2.347	1.124	28.925	0.153	0.043

Problem 1678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	39	116	328	35	33
normalized size	1	1.00	1.00	0.80	0.87	2.58	7.29	0.78	0.73
time (sec)	N/A	0.016	0.032	0.008	2.334	0.909	102.665	0.161	1.096

Problem 1679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	25	48	52	147	0	49	48
normalized size	1	1.00	0.45	0.86	0.93	2.62	0.00	0.88	0.86
time (sec)	N/A	0.019	0.005	0.013	2.350	0.723	0.000	0.155	1.126

Problem 1680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	27	60	65	184	0	58	58
normalized size	1	1.00	0.39	0.87	0.94	2.67	0.00	0.84	0.84
time (sec)	N/A	0.024	0.005	0.015	2.237	0.783	0.000	0.174	1.136

Problem 1681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	27	72	77	210	0	70	70
normalized size	1	1.00	0.32	0.86	0.92	2.50	0.00	0.83	0.83
time (sec)	N/A	0.032	0.006	0.016	2.304	0.985	0.000	0.162	1.151

Problem 1682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	27	90	99	254	0	88	91
normalized size	1	1.00	0.25	0.82	0.90	2.31	0.00	0.80	0.83
time (sec)	N/A	0.041	0.012	0.014	2.363	0.781	0.000	0.196	1.084

Problem 1683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	27	79	88	227	906	77	81
normalized size	1	1.00	0.28	0.83	0.93	2.39	9.54	0.81	0.85
time (sec)	N/A	0.034	0.005	0.015	2.367	1.155	43.537	0.191	0.069

Problem 1684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	27	66	75	200	816	59	69
normalized size	1	1.00	0.33	0.80	0.91	2.44	9.95	0.72	0.84
time (sec)	N/A	0.025	0.005	0.015	2.411	1.025	39.807	0.155	0.083

Problem 1685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	59	50	62	185	726	47	58
normalized size	1	1.00	0.84	0.71	0.89	2.64	10.37	0.67	0.83
time (sec)	N/A	0.022	0.048	0.011	2.327	1.006	110.844	0.157	1.120

Problem 1686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	27	52	66	186	0	52	57
normalized size	1	1.00	0.37	0.71	0.90	2.55	0.00	0.71	0.78
time (sec)	N/A	0.022	0.015	0.011	2.202	0.832	0.000	0.156	1.125

Problem 1687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	25	53	63	186	0	47	57
normalized size	1	1.00	0.36	0.76	0.90	2.66	0.00	0.67	0.81
time (sec)	N/A	0.023	0.005	0.008	2.372	1.034	0.000	0.152	1.104

Problem 1688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	25	66	75	214	0	59	70
normalized size	1	1.00	0.30	0.80	0.91	2.61	0.00	0.72	0.85
time (sec)	N/A	0.029	0.005	0.017	2.416	1.089	0.000	0.159	1.132

Problem 1689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	27	79	89	250	0	71	80
normalized size	1	1.00	0.28	0.83	0.94	2.63	0.00	0.75	0.84
time (sec)	N/A	0.032	0.005	0.017	2.361	1.087	0.000	0.153	0.104

Problem 1690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	27	90	99	276	0	80	92
normalized size	1	1.00	0.25	0.82	0.90	2.51	0.00	0.73	0.84
time (sec)	N/A	0.043	0.006	0.018	2.307	1.016	0.000	0.160	1.177

Problem 1691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	39	135	166	173	153	108	91
normalized size	1	1.00	0.33	1.15	1.42	1.48	1.31	0.92	0.78
time (sec)	N/A	0.055	0.016	0.016	2.368	1.023	10.572	0.219	1.706

Problem 1692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	39	115	137	152	122	94	74
normalized size	1	1.00	0.42	1.24	1.47	1.63	1.31	1.01	0.80
time (sec)	N/A	0.039	0.015	0.007	2.346	1.094	6.167	0.304	1.348

Problem 1693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	39	96	100	125	97	78	54
normalized size	1	1.00	0.57	1.39	1.45	1.81	1.41	1.13	0.78
time (sec)	N/A	0.027	0.014	0.006	2.394	1.058	4.179	0.207	1.199

Problem 1694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	74	50	99	42	64	58
normalized size	1	1.00	1.00	1.90	1.28	2.54	1.08	1.64	1.49
time (sec)	N/A	0.017	0.024	0.006	2.380	1.049	2.013	0.205	0.093

Problem 1695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	103	49	85	68	0	31
normalized size	1	1.00	1.00	2.64	1.26	2.18	1.74	0.00	0.79
time (sec)	N/A	0.019	0.011	0.007	2.387	1.013	1.823	0.000	1.497

Problem 1696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	25	14	24	41	83	22
normalized size	1	1.00	1.00	1.39	0.78	1.33	2.28	4.61	1.22
time (sec)	N/A	0.005	0.007	0.005	1.074	1.194	0.939	0.217	1.371

Problem 1697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	34	33	30	38	304	115	35
normalized size	1	1.00	0.89	0.87	0.79	1.00	8.00	3.03	0.92
time (sec)	N/A	0.017	0.016	0.005	0.961	0.901	1.427	0.226	1.460

Problem 1698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	45	44	47	49	899	146	70
normalized size	1	1.00	0.76	0.75	0.80	0.83	15.24	2.47	1.19
time (sec)	N/A	0.025	0.021	0.004	1.063	1.060	2.191	0.247	1.535

Problem 1699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	56	55	64	60	2297	177	90
normalized size	1	1.00	0.70	0.69	0.80	0.75	28.71	2.21	1.12
time (sec)	N/A	0.033	0.029	0.006	0.953	1.030	3.005	0.252	1.610

Problem 1700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	67	66	81	71	5095	208	110
normalized size	1	1.00	0.66	0.65	0.80	0.70	50.45	2.06	1.09
time (sec)	N/A	0.043	0.063	0.006	1.039	0.840	5.619	0.338	1.705

Problem 1701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	39	135	166	174	153	106	89
normalized size	1	1.00	0.34	1.18	1.46	1.53	1.34	0.93	0.78
time (sec)	N/A	0.052	0.017	0.008	2.276	0.889	8.937	0.260	1.357

Problem 1702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	39	115	135	151	124	93	72
normalized size	1	1.00	0.43	1.28	1.50	1.68	1.38	1.03	0.80
time (sec)	N/A	0.039	0.014	0.008	2.335	1.167	9.202	0.219	1.361

Problem 1703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	76	96	98	130	75	79	52
normalized size	1	1.00	1.15	1.45	1.48	1.97	1.14	1.20	0.79
time (sec)	N/A	0.028	0.055	0.008	2.228	1.039	4.109	0.222	1.171

Problem 1704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	46	100	63	100	92	0	34
normalized size	1	1.00	0.85	1.85	1.17	1.85	1.70	0.00	0.63
time (sec)	N/A	0.024	0.033	0.010	2.359	1.067	3.747	0.000	1.253

Problem 1705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	50	123	61	110	71	0	43
normalized size	1	1.00	0.91	2.24	1.11	2.00	1.29	0.00	0.78
time (sec)	N/A	0.027	0.056	0.010	2.307	0.745	2.310	0.000	1.507

Problem 1706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	25	14	35	65	145	24
normalized size	1	1.00	1.00	1.39	0.78	1.94	3.61	8.06	1.33
time (sec)	N/A	0.006	0.009	0.005	0.972	1.029	1.362	0.245	1.594

Problem 1707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	33	30	49	360	177	68
normalized size	1	1.00	1.00	0.87	0.79	1.29	9.47	4.66	1.79
time (sec)	N/A	0.018	0.029	0.005	1.049	0.697	1.472	0.254	1.731

Problem 1708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	47	44	47	60	986	208	88
normalized size	1	1.00	0.80	0.75	0.80	1.02	16.71	3.53	1.49
time (sec)	N/A	0.026	0.040	0.007	0.987	1.379	2.401	0.321	1.892

Problem 1709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	58	55	64	71	2297	239	108
normalized size	1	1.00	0.72	0.69	0.80	0.89	28.71	2.99	1.35
time (sec)	N/A	0.034	0.031	0.005	1.053	0.914	3.507	0.283	2.095

Problem 1710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	69	66	81	82	5289	270	128
normalized size	1	1.00	0.68	0.65	0.80	0.81	52.37	2.67	1.27
time (sec)	N/A	0.040	0.047	0.006	1.058	1.086	6.155	0.333	2.309

Problem 1711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	80	77	98	93	10344	301	148
normalized size	1	1.00	0.66	0.63	0.80	0.76	84.79	2.47	1.21
time (sec)	N/A	0.051	0.053	0.006	0.968	0.946	11.853	0.336	2.517

Problem 1712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	39	135	164	173	155	107	89
normalized size	1	1.00	0.35	1.22	1.48	1.56	1.40	0.96	0.80
time (sec)	N/A	0.052	0.019	0.008	2.290	0.964	8.593	0.226	1.366

Problem 1713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	115	131	152	102	93	72
normalized size	1	1.00	1.00	1.32	1.51	1.75	1.17	1.07	0.83
time (sec)	N/A	0.038	0.044	0.008	2.310	1.205	5.577	0.254	1.350

Problem 1714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	39	125	115	130	126	0	72
normalized size	1	1.00	0.46	1.49	1.37	1.55	1.50	0.00	0.86
time (sec)	N/A	0.035	0.036	0.010	2.527	0.788	4.201	0.000	1.317

Problem 1715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	64	120	78	139	99	0	34
normalized size	1	1.00	0.90	1.69	1.10	1.96	1.39	0.00	0.48
time (sec)	N/A	0.031	0.066	0.010	2.429	0.768	3.902	0.000	1.393

Problem 1716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	63	145	75	142	97	0	60
normalized size	1	1.00	0.86	1.99	1.03	1.95	1.33	0.00	0.82
time (sec)	N/A	0.034	0.077	0.011	2.434	1.096	4.424	0.000	1.749

Problem 1717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	25	14	46	80	207	68
normalized size	1	1.00	1.00	1.39	0.78	2.56	4.44	11.50	3.78
time (sec)	N/A	0.006	0.012	0.005	1.076	0.828	3.527	0.273	1.884

Problem 1718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	33	30	60	416	239	88
normalized size	1	1.00	1.00	0.87	0.79	1.58	10.95	6.29	2.32
time (sec)	N/A	0.017	0.039	0.006	0.994	0.957	1.762	0.317	2.115

Problem 1719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	47	44	47	71	1073	270	108
normalized size	1	1.00	0.80	0.75	0.80	1.20	18.19	4.58	1.83
time (sec)	N/A	0.024	0.030	0.005	0.990	0.906	2.763	0.293	2.406

Problem 1720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	58	55	64	82	2562	301	128
normalized size	1	1.00	0.72	0.69	0.80	1.02	32.02	3.76	1.60
time (sec)	N/A	0.032	0.042	0.007	1.073	0.555	3.356	0.470	2.714

Problem 1721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	69	66	81	93	5482	332	148
normalized size	1	1.00	0.68	0.65	0.80	0.92	54.28	3.29	1.47
time (sec)	N/A	0.040	0.043	0.007	1.049	0.991	5.709	0.380	3.008

Problem 1722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	37	184	166	174	155	141	94
normalized size	1	1.00	0.31	1.53	1.38	1.45	1.29	1.18	0.78
time (sec)	N/A	0.051	0.012	0.013	2.416	1.083	11.844	0.183	1.427

Problem 1723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	37	164	137	151	128	115	77
normalized size	1	1.00	0.39	1.71	1.43	1.57	1.33	1.20	0.80
time (sec)	N/A	0.039	0.012	0.010	2.345	0.752	7.921	0.170	1.395

Problem 1724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	37	142	104	130	100	89	57
normalized size	1	1.00	0.51	1.97	1.44	1.81	1.39	1.24	0.79
time (sec)	N/A	0.027	0.037	0.007	2.335	1.047	4.482	0.162	1.210

Problem 1725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	70	67	98	44	71	66
normalized size	1	1.00	1.00	1.63	1.56	2.28	1.02	1.65	1.53
time (sec)	N/A	0.017	0.021	0.005	2.228	0.975	2.630	0.231	1.201

Problem 1726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	121	37	60	22	25	19
normalized size	1	1.00	1.00	4.84	1.48	2.40	0.88	1.00	0.76
time (sec)	N/A	0.014	0.008	0.012	2.337	1.112	1.200	0.178	1.406

Problem 1727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	25	14	16	22	16	14
normalized size	1	1.00	1.00	1.56	0.88	1.00	1.38	1.00	0.88
time (sec)	N/A	0.005	0.007	0.004	1.144	1.023	0.959	0.149	1.322

Problem 1728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	29	33	30	27	248	41	23
normalized size	1	1.00	0.81	0.92	0.83	0.75	6.89	1.14	0.64
time (sec)	N/A	0.016	0.036	0.005	0.905	0.864	1.451	0.192	1.301

Problem 1729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	40	44	47	38	813	67	54
normalized size	1	1.00	0.70	0.77	0.82	0.67	14.26	1.18	0.95
time (sec)	N/A	0.023	0.036	0.005	1.045	0.979	2.253	0.165	1.299

Problem 1730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	51	55	64	49	2164	93	73
normalized size	1	1.00	0.67	0.72	0.84	0.64	28.47	1.22	0.96
time (sec)	N/A	0.032	0.041	0.005	0.981	0.570	3.948	0.200	1.284

Problem 1731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	62	66	81	60	4901	119	93
normalized size	1	1.00	0.63	0.67	0.82	0.61	49.51	1.20	0.94
time (sec)	N/A	0.039	0.049	0.004	0.957	1.285	5.829	0.167	1.281

Problem 1732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	115	37	458	154	207	133	131	93
normalized size	1	0.98	0.32	3.91	1.32	1.77	1.14	1.12	0.79
time (sec)	N/A	0.053	0.012	0.016	2.440	1.062	9.825	0.186	1.569

Problem 1733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	91	37	395	122	186	105	105	73
normalized size	1	0.98	0.40	4.25	1.31	2.00	1.13	1.13	0.78
time (sec)	N/A	0.038	0.011	0.013	2.371	0.963	5.968	0.174	1.349

Problem 1734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	61	36	198	85	156	71	86	34
normalized size	1	1.02	0.60	3.30	1.42	2.60	1.18	1.43	0.57
time (sec)	N/A	0.026	0.038	0.010	2.308	1.129	5.493	0.164	1.279

Problem 1735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	34	198	52	128	148	45	34
normalized size	1	1.00	0.81	4.71	1.24	3.05	3.52	1.07	0.81
time (sec)	N/A	0.021	0.017	0.007	2.445	1.091	3.819	0.196	1.416

Problem 1736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	25	14	24	20	16	14
normalized size	1	1.00	1.00	1.56	0.88	1.50	1.25	1.00	0.88
time (sec)	N/A	0.006	0.008	0.004	0.993	1.014	1.801	0.156	1.310

Problem 1737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	25	31	30	31	42	30	36
normalized size	1	1.00	0.74	0.91	0.88	0.91	1.24	0.88	1.06
time (sec)	N/A	0.017	0.043	0.005	0.984	0.616	3.375	0.162	1.258

Problem 1738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	40	44	47	48	457	57	43
normalized size	1	1.00	0.73	0.80	0.85	0.87	8.31	1.04	0.78
time (sec)	N/A	0.025	0.025	0.006	0.974	0.963	2.386	0.174	1.316

Problem 1739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	49	53	64	59	2032	82	52
normalized size	1	1.00	0.66	0.72	0.86	0.80	27.46	1.11	0.70
time (sec)	N/A	0.032	0.044	0.006	1.051	1.064	3.606	0.213	1.371

Problem 1740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	62	66	81	72	4707	109	91
normalized size	1	1.00	0.65	0.69	0.85	0.76	49.55	1.15	0.96
time (sec)	N/A	0.040	0.037	0.007	0.948	1.150	6.287	0.191	1.447

Problem 1741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	73	77	98	83	9534	135	112
normalized size	1	1.00	0.63	0.66	0.84	0.72	82.19	1.16	0.97
time (sec)	N/A	0.049	0.076	0.005	0.999	0.699	11.380	0.180	1.444

Problem 1742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	134	39	616	171	273	532	134	113
normalized size	1	0.97	0.28	4.46	1.24	1.98	3.86	0.97	0.82
time (sec)	N/A	0.066	0.017	0.015	2.329	1.097	17.636	0.179	1.793

Problem 1743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	39	531	139	252	464	125	93
normalized size	1	1.00	0.34	4.66	1.22	2.21	4.07	1.10	0.82
time (sec)	N/A	0.050	0.011	0.011	2.542	1.003	10.574	0.192	1.539

Problem 1744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	82	38	271	101	225	774	98	34
normalized size	1	1.04	0.48	3.43	1.28	2.85	9.80	1.24	0.43
time (sec)	N/A	0.036	0.019	0.010	2.318	0.975	6.573	0.170	1.441

Problem 1745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	36	274	62	197	700	65	49
normalized size	1	1.00	0.60	4.57	1.03	3.28	11.67	1.08	0.82
time (sec)	N/A	0.029	0.029	0.010	2.213	1.036	3.559	0.166	1.582

Problem 1746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	25	14	37	39	24	14
normalized size	1	1.00	1.00	1.39	0.78	2.06	2.17	1.33	0.78
time (sec)	N/A	0.006	0.010	0.005	1.039	1.154	2.878	0.161	1.403

Problem 1747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	33	33	30	47	82	36	25
normalized size	1	1.00	0.92	0.92	0.83	1.31	2.28	1.00	0.69
time (sec)	N/A	0.017	0.026	0.005	1.089	1.208	4.037	0.164	1.304

Problem 1748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	44	44	47	55	136	54	40
normalized size	1	1.00	0.80	0.80	0.85	1.00	2.47	0.98	0.73
time (sec)	N/A	0.025	0.038	0.006	1.001	0.942	4.271	0.171	1.348

Problem 1749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	58	55	64	70	187	79	54
normalized size	1	1.00	0.76	0.72	0.84	0.92	2.46	1.04	0.71
time (sec)	N/A	0.032	0.042	0.007	1.054	0.923	5.249	0.240	1.315

Problem 1750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	69	66	81	83	2032	105	65
normalized size	1	1.00	0.71	0.68	0.84	0.86	20.95	1.08	0.67
time (sec)	N/A	0.039	0.059	0.007	1.035	0.882	6.360	0.171	1.377

Problem 1751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	80	77	98	94	9263	131	110
normalized size	1	1.00	0.69	0.66	0.84	0.81	79.85	1.13	0.95
time (sec)	N/A	0.048	0.076	0.006	0.964	0.829	11.993	0.175	1.422

Problem 1752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	58	55	69	60	518	61	58
normalized size	1	1.00	0.58	0.55	0.69	0.60	5.18	0.61	0.58
time (sec)	N/A	0.031	0.017	0.006	1.088	0.860	38.789	0.170	1.668

Problem 1753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	47	44	52	49	314	50	47
normalized size	1	1.00	0.64	0.59	0.70	0.66	4.24	0.68	0.64
time (sec)	N/A	0.022	0.016	0.005	1.108	0.777	16.439	0.162	1.398

Problem 1754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	36	33	35	37	65	37	36
normalized size	1	1.00	0.75	0.69	0.73	0.77	1.35	0.77	0.75
time (sec)	N/A	0.012	0.039	0.004	1.165	0.700	5.238	0.161	1.396

Problem 1755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	25	17	24	39	24	22
normalized size	1	1.00	1.00	1.09	0.74	1.04	1.70	1.04	0.96
time (sec)	N/A	0.005	0.011	0.003	1.011	0.852	2.063	0.195	1.310

Problem 1756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	78	50	59	101	68	62	-1
normalized size	1	1.00	1.59	1.02	1.20	2.06	1.39	1.27	-0.02
time (sec)	N/A	0.025	0.124	0.011	2.424	0.888	2.409	0.177	0.000

Problem 1757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	74	54	77	121	44	43	-1
normalized size	1	1.00	1.48	1.08	1.54	2.42	0.88	0.86	-0.02
time (sec)	N/A	0.026	0.229	0.015	2.343	0.945	3.478	0.256	0.000

Problem 1758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	77	73	118	147	97	68	-1
normalized size	1	1.00	0.96	0.91	1.48	1.84	1.21	0.85	-0.01
time (sec)	N/A	0.039	0.171	0.017	2.304	0.881	8.938	0.286	0.000

Problem 1759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	89	92	161	173	122	86	-1
normalized size	1	1.00	0.84	0.87	1.52	1.63	1.15	0.81	-0.01
time (sec)	N/A	0.055	0.272	0.016	2.345	0.727	20.965	0.323	0.000

Problem 1760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	60	55	69	71	585	138	67
normalized size	1	1.00	0.60	0.55	0.69	0.71	5.85	1.38	0.67
time (sec)	N/A	0.034	0.025	0.004	0.989	0.681	171.286	0.201	1.461

Problem 1761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	49	44	52	60	369	114	56
normalized size	1	1.00	0.66	0.59	0.70	0.81	4.99	1.54	0.76
time (sec)	N/A	0.023	0.018	0.005	1.077	0.867	64.429	0.228	1.465

Problem 1762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	40	33	35	48	88	90	45
normalized size	1	1.00	0.83	0.69	0.73	1.00	1.83	1.88	0.94
time (sec)	N/A	0.014	0.016	0.004	1.054	0.810	27.922	0.179	1.427

Problem 1763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	25	17	35	63	64	34
normalized size	1	1.00	1.00	1.09	0.74	1.52	2.74	2.78	1.48
time (sec)	N/A	0.006	0.016	0.002	1.081	0.794	9.457	0.174	1.380

Problem 1764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	54	63	74	116	71	44	-1
normalized size	1	1.00	0.77	0.90	1.06	1.66	1.01	0.63	-0.01
time (sec)	N/A	0.036	0.013	0.011	2.289	0.844	6.738	0.208	0.000

Problem 1765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	52	70	93	130	92	56	-1
normalized size	1	1.00	0.75	1.01	1.35	1.88	1.33	0.81	-0.01
time (sec)	N/A	0.035	0.015	0.018	2.326	0.925	9.084	0.275	0.000

Problem 1766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	73	74	117	152	76	64	-1
normalized size	1	1.00	0.95	0.96	1.52	1.97	0.99	0.83	-0.01
time (sec)	N/A	0.032	0.192	0.016	2.323	0.947	9.026	0.343	0.000

Problem 1767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	90	92	159	173	124	84	-1
normalized size	1	1.00	0.87	0.89	1.54	1.68	1.20	0.82	-0.01
time (sec)	N/A	0.053	0.186	0.018	2.406	0.820	14.318	0.366	0.000

Problem 1768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	60	55	69	82	0	229	78
normalized size	1	1.00	0.60	0.55	0.69	0.82	0.00	2.29	0.78
time (sec)	N/A	0.035	0.021	0.006	1.049	0.792	0.000	0.296	1.535

Problem 1769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	51	44	52	71	0	194	67
normalized size	1	1.00	0.69	0.59	0.70	0.96	0.00	2.62	0.91
time (sec)	N/A	0.023	0.037	0.004	1.118	0.625	0.000	0.210	1.459

Problem 1770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	38	33	35	59	114	157	56
normalized size	1	1.00	0.79	0.69	0.73	1.23	2.38	3.27	1.17
time (sec)	N/A	0.013	0.029	0.003	1.097	0.800	98.843	0.197	1.446

Problem 1771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	25	17	46	88	120	45
normalized size	1	1.00	1.00	1.09	0.74	2.00	3.83	5.22	1.96
time (sec)	N/A	0.006	0.027	0.003	1.051	0.831	40.239	0.204	1.384

Problem 1772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	56	81	91	142	97	56	-1
normalized size	1	1.00	0.60	0.87	0.98	1.53	1.04	0.60	-0.01
time (sec)	N/A	0.049	0.016	0.011	2.345	0.899	26.953	0.261	0.000

Problem 1773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	56	91	111	154	99	74	-1
normalized size	1	1.00	0.60	0.97	1.18	1.64	1.05	0.79	-0.01
time (sec)	N/A	0.048	0.032	0.016	2.256	0.875	17.417	0.317	0.000

Problem 1774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	54	93	137	161	126	80	-1
normalized size	1	1.00	0.56	0.96	1.41	1.66	1.30	0.82	-0.01
time (sec)	N/A	0.046	0.027	0.016	2.355	0.855	13.986	0.304	0.000

Problem 1775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	85	92	156	174	104	79	-1
normalized size	1	1.00	0.85	0.92	1.56	1.74	1.04	0.79	-0.01
time (sec)	N/A	0.045	0.224	0.018	2.478	0.694	14.402	0.337	0.000

Problem 1776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	101	110	194	195	155	99	-1
normalized size	1	1.00	0.80	0.87	1.54	1.55	1.23	0.79	-0.01
time (sec)	N/A	0.068	0.218	0.019	2.358	0.883	17.397	0.372	0.000

Problem 1777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	64	66	86	60	692	73	61
normalized size	1	1.00	0.51	0.52	0.68	0.48	5.49	0.58	0.48
time (sec)	N/A	0.044	0.041	0.007	1.006	0.799	31.826	0.165	1.488

Problem 1778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	53	55	69	49	452	61	50
normalized size	1	1.00	0.53	0.55	0.69	0.49	4.52	0.61	0.50
time (sec)	N/A	0.031	0.022	0.005	1.071	0.582	11.926	0.166	1.432

Problem 1779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	42	44	52	38	260	49	39
normalized size	1	1.00	0.57	0.59	0.70	0.51	3.51	0.66	0.53
time (sec)	N/A	0.021	0.019	0.005	1.050	0.782	3.828	0.163	1.445

Problem 1780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	30	32	34	26	42	34	28
normalized size	1	1.00	0.62	0.67	0.71	0.54	0.88	0.71	0.58
time (sec)	N/A	0.013	0.017	0.004	1.047	0.841	1.407	0.192	1.417

Problem 1781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	25	17	19	17	21	17
normalized size	1	1.00	1.00	1.19	0.81	0.90	0.81	1.00	0.81
time (sec)	N/A	0.005	0.010	0.003	1.030	0.582	1.722	0.177	1.409

Problem 1782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	54	39	44	70	24	39	-1
normalized size	1	1.00	1.80	1.30	1.47	2.33	0.80	1.30	-0.03
time (sec)	N/A	0.015	0.019	0.010	2.373	0.768	3.238	0.166	0.000

Problem 1783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	77	55	80	121	44	47	-1
normalized size	1	1.00	1.48	1.06	1.54	2.33	0.85	0.90	-0.02
time (sec)	N/A	0.026	0.061	0.013	2.424	0.951	8.685	0.207	0.000

Problem 1784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	93	74	122	151	102	69	-1
normalized size	1	1.00	1.12	0.89	1.47	1.82	1.23	0.83	-0.01
time (sec)	N/A	0.039	0.072	0.013	2.308	0.806	22.609	0.320	0.000

Problem 1785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	106	92	161	173	129	84	-1
normalized size	1	1.00	0.97	0.84	1.48	1.59	1.18	0.77	-0.01
time (sec)	N/A	0.054	0.082	0.012	2.375	0.898	56.433	0.231	0.000

Problem 1786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	64	66	90	70	614	85	67
normalized size	1	1.00	0.51	0.52	0.71	0.56	4.87	0.67	0.53
time (sec)	N/A	0.045	0.016	0.006	0.989	0.590	13.998	0.178	1.631

Problem 1787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	52	54	72	58	320	69	59
normalized size	1	1.00	0.52	0.54	0.72	0.58	3.20	0.69	0.59
time (sec)	N/A	0.033	0.014	0.006	1.037	0.827	5.101	0.208	1.524

Problem 1788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	41	43	55	47	206	54	49
normalized size	1	1.00	0.55	0.58	0.74	0.64	2.78	0.73	0.66
time (sec)	N/A	0.023	0.012	0.006	1.168	0.731	2.680	0.174	1.489

Problem 1789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	28	32	36	36	39	38	37
normalized size	1	1.00	0.64	0.73	0.82	0.82	0.89	0.86	0.84
time (sec)	N/A	0.013	0.012	0.005	1.108	0.645	3.819	0.170	1.440

Problem 1790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	25	17	27	19	21	25
normalized size	1	1.00	1.00	1.19	0.81	1.29	0.90	1.00	1.19
time (sec)	N/A	0.006	0.012	0.003	1.098	0.783	5.402	0.165	1.426

Problem 1791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	73	52	62	138	146	67	-1
normalized size	1	1.00	1.40	1.00	1.19	2.65	2.81	1.29	-0.02
time (sec)	N/A	0.028	0.086	0.013	2.387	1.181	13.988	0.173	0.000

Problem 1792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	56	61	101	179	73	64	-1
normalized size	1	1.00	0.76	0.82	1.36	2.42	0.99	0.86	-0.01
time (sec)	N/A	0.040	0.017	0.021	2.404	1.071	38.023	0.268	0.000

Problem 1793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	56	78	143	217	107	80	-1
normalized size	1	1.00	0.54	0.75	1.38	2.09	1.03	0.77	-0.01
time (sec)	N/A	0.054	0.017	0.018	2.404	0.949	85.831	0.239	0.000

Problem 1794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	56	89	181	239	0	95	-1
normalized size	1	1.00	0.43	0.68	1.39	1.84	0.00	0.73	-0.01
time (sec)	N/A	0.071	0.024	0.019	2.260	0.619	0.000	0.257	0.000

Problem 1795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	82	77	106	92	799	98	78
normalized size	1	1.00	0.54	0.51	0.70	0.61	5.26	0.64	0.51
time (sec)	N/A	0.062	0.022	0.006	1.127	0.872	14.648	0.189	1.674

Problem 1796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	71	66	89	81	536	83	81
normalized size	1	1.00	0.56	0.52	0.71	0.64	4.25	0.66	0.64
time (sec)	N/A	0.047	0.029	0.005	1.036	0.917	8.805	0.184	1.568

Problem 1797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	59	54	71	69	320	67	71
normalized size	1	1.00	0.61	0.56	0.74	0.72	3.33	0.70	0.74
time (sec)	N/A	0.035	0.018	0.005	0.970	1.203	6.454	0.186	1.520

Problem 1798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	49	44	52	59	151	47	43
normalized size	1	1.00	0.70	0.63	0.74	0.84	2.16	0.67	0.61
time (sec)	N/A	0.022	0.029	0.007	1.056	0.943	7.543	0.200	1.544

Problem 1799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	38	33	31	48	94	29	49
normalized size	1	1.00	0.83	0.72	0.67	1.04	2.04	0.63	1.07
time (sec)	N/A	0.013	0.026	0.003	1.068	0.683	14.289	0.187	1.494

Problem 1800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	25	17	38	42	21	38
normalized size	1	1.00	1.00	1.09	0.74	1.65	1.83	0.91	1.65
time (sec)	N/A	0.006	0.017	0.003	0.899	0.978	20.020	0.175	1.487

Problem 1801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	94	85	74	205	697	78	-1
normalized size	1	1.00	1.25	1.13	0.99	2.73	9.29	1.04	-0.01
time (sec)	N/A	0.040	0.179	0.017	2.344	0.691	51.156	0.188	0.000

Problem 1802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	56	102	119	249	818	65	-1
normalized size	1	1.00	0.57	1.03	1.20	2.52	8.26	0.66	-0.01
time (sec)	N/A	0.054	0.019	0.020	2.373	0.890	133.442	0.254	0.000

Problem 1803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	56	117	163	283	0	93	-1
normalized size	1	1.00	0.43	0.91	1.26	2.19	0.00	0.72	-0.01
time (sec)	N/A	0.071	0.018	0.020	2.504	0.805	0.000	0.279	0.000

Problem 1804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.005	0.002	0.001	1.130	0.895	0.064	0.181	0.023

Problem 1805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.005	0.001	0.000	1.017	0.602	0.064	0.155	0.020

Problem 1806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.005	0.001	0.001	1.060	0.692	0.063	0.153	0.020

Problem 1807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.005	0.001	0.002	1.026	0.753	0.063	0.177	0.020

Problem 1808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	10
normalized size	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.003	0.000	0.001	1.038	0.841	0.062	0.152	0.018

Problem 1809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	14	11	10	12	11
normalized size	1	1.00	1.00	0.92	1.08	0.85	0.77	0.92	0.85
time (sec)	N/A	0.003	0.003	0.002	0.999	0.816	0.091	0.171	0.024

Problem 1810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	13	5	10	10
normalized size	1	1.00	1.00	1.10	1.00	1.30	0.50	1.00	1.00
time (sec)	N/A	0.002	0.000	0.001	1.056	0.892	0.084	0.178	0.023

Problem 1811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	14	17	10	20	11
normalized size	1	1.00	1.00	0.92	1.08	1.31	0.77	1.54	0.85
time (sec)	N/A	0.005	0.002	0.004	1.098	0.861	0.112	0.150	0.039

Problem 1812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	14	13	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.93	0.87	0.87
time (sec)	N/A	0.005	0.002	0.006	1.027	0.740	0.122	0.148	0.023

Problem 1813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	14	13	14	13	13
normalized size	1	1.00	1.00	0.82	0.82	0.76	0.82	0.76	0.76
time (sec)	N/A	0.005	0.002	0.004	1.031	0.847	0.124	0.146	0.026

Problem 1814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	15	15	15	15	15
normalized size	1	1.00	1.00	0.82	0.88	0.88	0.88	0.88	0.88
time (sec)	N/A	0.004	0.002	0.006	1.035	0.786	0.142	0.153	0.026

Problem 1815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	15	15	15	15	15
normalized size	1	1.00	1.00	0.82	0.88	0.88	0.88	0.88	0.88
time (sec)	N/A	0.005	0.002	0.005	1.081	0.609	0.141	0.199	0.026

Problem 1816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	15	15	15	15	15
normalized size	1	1.00	1.00	0.82	0.88	0.88	0.88	0.88	0.88
time (sec)	N/A	0.005	0.002	0.006	0.998	0.811	0.157	0.158	0.028

Problem 1817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.012	0.001	0.000	0.800	0.771	0.069	0.219	0.044

Problem 1818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	25	24	24	24	24	24
normalized size	1	1.00	1.00	1.56	1.50	1.50	1.50	1.50	1.50
time (sec)	N/A	0.004	0.002	0.002	0.879	0.756	0.068	0.161	0.033

Problem 1819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	22	21	21
normalized size	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.84
time (sec)	N/A	0.009	0.001	0.000	0.848	0.608	0.066	0.184	0.031

Problem 1820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	24	21	20	22	21
normalized size	1	1.00	1.00	0.96	1.04	0.91	0.87	0.96	0.91
time (sec)	N/A	0.014	0.001	0.002	0.916	0.647	0.108	0.169	1.036

Problem 1821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	22	25	19	22	22
normalized size	1	1.00	1.00	0.96	0.92	1.04	0.79	0.92	0.92
time (sec)	N/A	0.011	0.001	0.003	0.896	0.694	0.103	0.149	0.034

Problem 1822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	24	27	24	24	23
normalized size	1	1.00	1.00	0.89	0.89	1.00	0.89	0.89	0.85
time (sec)	N/A	0.014	0.005	0.006	0.881	0.832	0.138	0.165	1.058

Problem 1823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	21	26	22	22	24
normalized size	1	1.00	1.00	0.96	0.91	1.13	0.96	0.96	1.04
time (sec)	N/A	0.010	0.005	0.006	0.859	0.736	0.144	0.153	0.026

Problem 1824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	26	28	24	34	24
normalized size	1	1.00	1.00	0.96	1.08	1.17	1.00	1.42	1.00
time (sec)	N/A	0.015	0.001	0.006	0.876	0.750	0.180	0.157	0.045

Problem 1825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	26	26	27	26	25
normalized size	1	1.00	1.00	0.89	0.93	0.93	0.96	0.93	0.89
time (sec)	N/A	0.012	0.001	0.006	0.832	0.770	0.187	0.148	0.036

Problem 1826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	30	25	14	24	26	24	26
normalized size	1	1.00	1.88	1.56	0.88	1.50	1.62	1.50	1.62
time (sec)	N/A	0.004	0.001	0.005	0.835	0.743	0.194	0.151	0.036

Problem 1827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	26	26	27	26	26
normalized size	1	1.00	1.00	0.83	0.87	0.87	0.90	0.87	0.87
time (sec)	N/A	0.012	0.001	0.005	0.876	0.657	0.207	0.169	0.036

Problem 1828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	26	26	27	26	26
normalized size	1	1.00	1.00	0.83	0.87	0.87	0.90	0.87	0.87
time (sec)	N/A	0.015	0.001	0.005	0.885	0.840	0.223	0.155	0.038

Problem 1829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	26	26	27	26	26
normalized size	1	1.00	1.00	0.83	0.87	0.87	0.90	0.87	0.87
time (sec)	N/A	0.012	0.001	0.005	0.877	0.634	0.233	0.149	0.037

Problem 1830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	31	31	32	31	31
normalized size	1	1.00	1.00	0.91	0.89	0.89	0.91	0.89	0.89
time (sec)	N/A	0.014	0.001	0.001	0.871	1.308	0.071	0.151	0.042

Problem 1831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	34	36	33	37	36	33
normalized size	1	1.00	1.00	0.87	0.92	0.85	0.95	0.92	0.85
time (sec)	N/A	0.020	0.004	0.002	0.868	0.889	0.121	0.162	0.040

Problem 1832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	33	32	36	29	32	32
normalized size	1	1.00	1.00	0.97	0.94	1.06	0.85	0.94	0.94
time (sec)	N/A	0.015	0.004	0.005	0.907	0.527	0.117	0.167	0.042

Problem 1833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	35	36	38	37	35	34
normalized size	1	1.00	1.00	0.88	0.90	0.95	0.92	0.88	0.85
time (sec)	N/A	0.022	0.007	0.007	0.876	0.955	0.154	0.153	0.036

Problem 1834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	34	34	36	36	34	36
normalized size	1	1.00	1.00	0.92	0.92	0.97	0.97	0.92	0.97
time (sec)	N/A	0.015	0.004	0.006	0.818	0.764	0.160	0.189	1.068

Problem 1835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	35	37	39	37	36	37
normalized size	1	1.00	1.00	0.88	0.92	0.98	0.92	0.90	0.92
time (sec)	N/A	0.020	0.005	0.007	0.879	0.915	0.204	0.151	0.036

Problem 1836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	33	32	37	34	33	34
normalized size	1	1.00	1.00	0.97	0.94	1.09	1.00	0.97	1.00
time (sec)	N/A	0.015	0.005	0.006	0.877	1.178	0.206	0.165	0.029

Problem 1837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	34	39	39	37	47	36
normalized size	1	1.00	1.00	0.87	1.00	1.00	0.95	1.21	0.92
time (sec)	N/A	0.021	0.004	0.007	0.891	0.572	0.263	0.183	0.048

Problem 1838	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	36	37	37	39	37	35
normalized size	1	1.00	1.00	0.92	0.95	0.95	1.00	0.95	0.90
time (sec)	N/A	0.016	0.004	0.005	0.897	0.772	0.264	0.150	0.029

Problem 1839	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	43	36	14	35	37	35	37
normalized size	1	1.00	2.69	2.25	0.88	2.19	2.31	2.19	2.31
time (sec)	N/A	0.004	0.007	0.004	0.855	0.781	0.295	0.152	0.029

Problem 1840	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	37	37	39	37	37
normalized size	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86
time (sec)	N/A	0.015	0.004	0.005	0.873	0.834	0.292	0.155	0.029

Problem 1841	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	43	36	37	37	39	37	37
normalized size	1	1.00	1.08	0.90	0.92	0.92	0.98	0.92	0.92
time (sec)	N/A	0.018	0.005	0.004	0.866	1.070	0.314	0.189	0.031

Problem 1842	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	37	37	39	37	37
normalized size	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86
time (sec)	N/A	0.015	0.008	0.005	0.832	0.761	0.316	0.159	0.032

Problem 1843	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	60	60	148	107	65	54
normalized size	1	1.00	1.00	0.88	0.88	2.18	1.57	0.96	0.79
time (sec)	N/A	0.028	0.028	0.004	1.843	0.830	0.206	0.161	1.100

Problem 1844	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	46	46	45	44	47	45
normalized size	1	1.00	1.00	0.87	0.87	0.85	0.83	0.89	0.85
time (sec)	N/A	0.034	0.006	0.005	0.875	0.864	0.167	0.165	0.048

Problem 1845	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	49	50	126	95	55	43
normalized size	1	1.00	1.00	0.89	0.91	2.29	1.73	1.00	0.78
time (sec)	N/A	0.024	0.025	0.004	1.999	0.983	0.192	0.183	0.063

Problem 1846	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	35	34	33	32	35	33
normalized size	1	1.00	1.00	0.88	0.85	0.82	0.80	0.88	0.82
time (sec)	N/A	0.026	0.006	0.003	0.825	0.779	0.160	0.153	1.091

Problem 1847	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	38	37	99	80	40	32
normalized size	1	1.00	1.00	0.90	0.88	2.36	1.90	0.95	0.76
time (sec)	N/A	0.019	0.022	0.003	1.850	0.684	0.179	0.156	0.048

Problem 1848	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	23	22	20	24	22
normalized size	1	1.00	1.00	0.89	0.85	0.81	0.74	0.89	0.81
time (sec)	N/A	0.017	0.005	0.003	0.786	0.981	0.146	0.184	0.037

Problem 1849	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	27	26	82	56	26	23
normalized size	1	1.00	1.00	0.87	0.84	2.65	1.81	0.84	0.74
time (sec)	N/A	0.011	0.009	0.003	1.935	0.757	0.160	0.151	1.085

Problem 1850	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	14	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87
time (sec)	N/A	0.005	0.003	0.001	0.862	0.798	0.119	0.149	1.064

Problem 1851	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	15	67	53	15	16
normalized size	1	1.00	1.00	0.67	0.62	2.79	2.21	0.62	0.67
time (sec)	N/A	0.007	0.005	0.003	1.931	0.775	0.145	0.189	0.046

Problem 1852	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	22	21	13	18	15	24	18
normalized size	1	1.00	1.47	1.40	0.87	1.20	1.00	1.60	1.20
time (sec)	N/A	0.004	0.006	0.004	0.773	0.656	0.218	0.166	0.065

Problem 1853	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	30	29	82	65	29	26
normalized size	1	1.00	1.00	0.88	0.85	2.41	1.91	0.85	0.76
time (sec)	N/A	0.012	0.013	0.004	1.845	0.830	0.188	0.148	0.045

Problem 1854	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	33	33	31	43	31
normalized size	1	1.00	1.00	0.91	0.94	0.94	0.89	1.23	0.89
time (sec)	N/A	0.023	0.007	0.006	0.852	0.880	0.278	0.165	0.074

Problem 1855	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	39	40	106	87	40	37
normalized size	1	1.00	1.00	0.91	0.93	2.47	2.02	0.93	0.86
time (sec)	N/A	0.017	0.023	0.007	1.907	1.042	0.233	0.161	1.079

Problem 1856	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	44	47	45	42	57	46
normalized size	1	1.00	1.00	0.90	0.96	0.92	0.86	1.16	0.94
time (sec)	N/A	0.030	0.008	0.007	0.857	1.013	0.324	0.152	0.075

Problem 1857	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	52	52	132	100	52	48
normalized size	1	1.00	1.00	0.90	0.90	2.28	1.72	0.90	0.83
time (sec)	N/A	0.026	0.027	0.008	1.887	0.888	0.279	0.153	0.054

Problem 1858	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	82	78	82	212	134	84	77
normalized size	1	1.00	0.89	0.85	0.89	2.30	1.46	0.91	0.84
time (sec)	N/A	0.037	0.058	0.008	1.997	0.856	0.346	0.165	1.096

Problem 1859	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	60	63	65	81	66	80	68
normalized size	1	1.00	0.86	0.90	0.93	1.16	0.94	1.14	0.97
time (sec)	N/A	0.052	0.023	0.012	0.877	0.943	0.292	0.158	0.041

Problem 1860	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	71	68	71	190	124	73	66
normalized size	1	1.00	0.90	0.86	0.90	2.41	1.57	0.92	0.84
time (sec)	N/A	0.033	0.052	0.009	1.883	0.888	0.324	0.159	0.046

Problem 1861	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	49	52	54	70	53	55	57
normalized size	1	1.00	0.86	0.91	0.95	1.23	0.93	0.96	1.00
time (sec)	N/A	0.042	0.017	0.010	0.784	0.959	0.271	0.170	0.050

Problem 1862	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	60	57	59	164	107	61	56
normalized size	1	1.00	0.91	0.86	0.89	2.48	1.62	0.92	0.85
time (sec)	N/A	0.026	0.046	0.009	1.934	0.787	0.301	0.153	0.068

Problem 1863	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	38	41	43	56	39	41	45
normalized size	1	1.00	0.86	0.93	0.98	1.27	0.89	0.93	1.02
time (sec)	N/A	0.031	0.019	0.011	0.896	0.676	0.251	0.152	1.079

Problem 1864	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	51	43	45	136	83	42	43
normalized size	1	1.00	0.93	0.78	0.82	2.47	1.51	0.76	0.78
time (sec)	N/A	0.018	0.034	0.010	1.983	0.900	0.276	0.177	1.101

Problem 1865	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	27	30	32	35	29	32	29
normalized size	1	1.00	0.82	0.91	0.97	1.06	0.88	0.97	0.88
time (sec)	N/A	0.024	0.009	0.007	0.882	0.796	0.206	0.151	0.043

Problem 1866	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	36	120	78	35	33
normalized size	1	1.00	1.00	0.80	0.80	2.67	1.73	0.78	0.73
time (sec)	N/A	0.013	0.023	0.007	1.987	0.909	0.222	0.149	0.039

Problem 1867	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	15	15	14	14
normalized size	1	1.00	1.00	0.94	0.88	0.94	0.94	0.88	0.88
time (sec)	N/A	0.004	0.002	0.001	0.837	0.580	0.169	0.153	0.024

Problem 1868	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	35	120	78	35	33
normalized size	1	1.00	1.00	0.80	0.78	2.67	1.73	0.78	0.73
time (sec)	N/A	0.011	0.027	0.005	1.989	1.511	0.227	0.161	0.039

Problem 1869	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	33	35	37	47	34	47	34
normalized size	1	1.00	0.87	0.92	0.97	1.24	0.89	1.24	0.89
time (sec)	N/A	0.025	0.017	0.010	0.812	0.822	0.306	0.150	0.060

Problem 1870	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	54	46	49	136	92	47	44
normalized size	1	1.00	0.95	0.81	0.86	2.39	1.61	0.82	0.77
time (sec)	N/A	0.018	0.045	0.010	1.950	1.144	0.312	0.152	1.115

Problem 1871	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	41	46	52	73	51	51	51
normalized size	1	1.00	0.84	0.94	1.06	1.49	1.04	1.04	1.04
time (sec)	N/A	0.033	0.043	0.012	0.911	0.944	0.391	0.164	1.120

Problem 1872	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	67	59	64	172	114	59	58
normalized size	1	1.00	0.99	0.87	0.94	2.53	1.68	0.87	0.85
time (sec)	N/A	0.025	0.054	0.012	1.894	0.847	0.371	0.169	1.169

Problem 1873	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	57	61	70	90	68	86	67
normalized size	1	1.00	0.86	0.92	1.06	1.36	1.03	1.30	1.02
time (sec)	N/A	0.042	0.059	0.012	0.882	0.825	0.459	0.152	0.076

Problem 1874	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	71	80	89	115	92	92	90
normalized size	1	1.00	0.82	0.92	1.02	1.32	1.06	1.06	1.03
time (sec)	N/A	0.064	0.074	0.013	0.883	0.725	0.466	0.166	0.065

Problem 1875	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	88	88	93	256	144	84	87
normalized size	1	1.00	0.90	0.90	0.95	2.61	1.47	0.86	0.89
time (sec)	N/A	0.041	0.051	0.011	1.868	0.843	0.485	0.156	1.106

Problem 1876	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	58	69	77	103	78	69	78
normalized size	1	1.00	0.78	0.93	1.04	1.39	1.05	0.93	1.05
time (sec)	N/A	0.056	0.046	0.010	0.834	0.750	0.441	0.159	1.132

Problem 1877	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	77	77	82	230	133	73	77
normalized size	1	1.00	0.91	0.91	0.96	2.71	1.56	0.86	0.91
time (sec)	N/A	0.036	0.047	0.010	1.758	0.827	0.477	0.156	1.137

Problem 1878	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	48	58	66	91	68	53	68
normalized size	1	1.00	0.74	0.89	1.02	1.40	1.05	0.82	1.05
time (sec)	N/A	0.046	0.067	0.010	0.857	0.990	0.404	0.159	1.110

Problem 1879	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	66	63	68	202	107	54	64
normalized size	1	1.00	0.89	0.85	0.92	2.73	1.45	0.73	0.86
time (sec)	N/A	0.025	0.047	0.012	1.971	0.830	0.425	0.151	1.362

Problem 1880	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	39	46	55	69	53	42	52
normalized size	1	1.00	0.80	0.94	1.12	1.41	1.08	0.86	1.06
time (sec)	N/A	0.035	0.016	0.007	0.858	1.271	0.346	0.153	1.113

Problem 1881	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	55	47	59	188	110	45	56
normalized size	1	1.00	0.86	0.73	0.92	2.94	1.72	0.70	0.88
time (sec)	N/A	0.020	0.043	0.009	1.922	0.748	0.367	0.166	1.131

Problem 1882	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	24	31	14	36	36	22	37
normalized size	1	1.00	1.50	1.94	0.88	2.25	2.25	1.38	2.31
time (sec)	N/A	0.004	0.007	0.007	0.868	0.923	0.288	0.157	1.080

Problem 1883	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	58	49	62	190	110	50	55
normalized size	1	1.00	0.89	0.75	0.95	2.92	1.69	0.77	0.85
time (sec)	N/A	0.020	0.031	0.008	1.863	0.770	0.334	0.154	1.116

Problem 1884	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	26	26	27	14	28
normalized size	1	1.00	1.00	0.94	1.62	1.62	1.69	0.88	1.75
time (sec)	N/A	0.005	0.002	0.000	0.674	1.070	0.260	0.155	1.074

Problem 1885	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	55	51	58	188	105	45	55
normalized size	1	1.00	0.89	0.82	0.94	3.03	1.69	0.73	0.89
time (sec)	N/A	0.017	0.048	0.005	1.953	0.653	0.336	0.220	0.066

Problem 1886	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	43	49	60	90	56	59	56
normalized size	1	1.00	0.80	0.91	1.11	1.67	1.04	1.09	1.04
time (sec)	N/A	0.036	0.036	0.012	0.891	0.787	0.434	0.154	1.136

Problem 1887	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	68	66	71	202	116	57	66
normalized size	1	1.00	0.89	0.87	0.93	2.66	1.53	0.75	0.87
time (sec)	N/A	0.027	0.043	0.011	1.956	0.811	0.432	0.158	1.186

Problem 1888	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	59	62	77	119	80	82	75
normalized size	1	1.00	0.88	0.93	1.15	1.78	1.19	1.22	1.12
time (sec)	N/A	0.046	0.060	0.012	0.898	0.873	0.530	0.156	0.079

Problem 1889	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	79	79	86	238	138	71	80
normalized size	1	1.00	0.91	0.91	0.99	2.74	1.59	0.82	0.92
time (sec)	N/A	0.035	0.045	0.015	2.026	0.846	0.486	0.155	1.172

Problem 1890	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	74	79	92	134	90	80	88
normalized size	1	1.00	0.86	0.92	1.07	1.56	1.05	0.93	1.02
time (sec)	N/A	0.054	0.050	0.014	0.926	0.812	0.572	0.154	1.146

Problem 1891	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	88	82	100	152	92	69	54
normalized size	1	1.00	1.24	1.15	1.41	2.14	1.30	0.97	0.76
time (sec)	N/A	0.036	0.042	0.015	1.972	0.885	3.929	0.183	1.607

Problem 1892	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	26	27	17	27	41	27	23
normalized size	1	1.00	1.24	1.29	0.81	1.29	1.95	1.29	1.10
time (sec)	N/A	0.006	0.005	0.004	0.882	0.889	0.851	0.153	1.240

Problem 1893	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	58	62	53	127	41	52	35
normalized size	1	1.00	1.23	1.32	1.13	2.70	0.87	1.11	0.74
time (sec)	N/A	0.022	0.029	0.005	2.049	0.833	2.056	0.192	1.385

Problem 1894	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	62	63	53	108	56	69	55
normalized size	1	1.00	1.48	1.50	1.26	2.57	1.33	1.64	1.31
time (sec)	N/A	0.018	0.025	0.007	1.974	0.920	1.583	0.160	1.139

Problem 1895	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	69	83	49	109	56	61	30
normalized size	1	1.00	1.82	2.18	1.29	2.87	1.47	1.61	0.79
time (sec)	N/A	0.023	0.072	0.007	1.935	0.822	1.585	0.217	1.300

Problem 1896	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	68	85	73	129	42	50	39
normalized size	1	1.00	1.36	1.70	1.46	2.58	0.84	1.00	0.78
time (sec)	N/A	0.020	0.030	0.006	1.892	0.859	2.245	0.209	1.414

Problem 1897	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	29	14	28	42	63	24
normalized size	1	1.00	1.00	1.61	0.78	1.56	2.33	3.50	1.33
time (sec)	N/A	0.006	0.012	0.004	0.859	0.988	1.097	0.203	1.254

Problem 1898	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	47	106	114	158	92	78	-1
normalized size	1	1.00	0.64	1.43	1.54	2.14	1.24	1.05	-0.01
time (sec)	N/A	0.037	0.011	0.007	1.975	0.973	4.079	0.225	0.000

Problem 1899	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	66	84	98	157	70	68	52
normalized size	1	1.00	0.97	1.24	1.44	2.31	1.03	1.00	0.76
time (sec)	N/A	0.035	0.155	0.005	1.927	0.756	3.526	0.186	1.650

Problem 1900	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	74	76	68	129	78	89	-1
normalized size	1	1.00	1.21	1.25	1.11	2.11	1.28	1.46	-0.02
time (sec)	N/A	0.033	0.059	0.006	1.864	0.943	2.423	0.181	0.000

Problem 1901	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	47	107	66	129	88	79	48
normalized size	1	1.00	0.75	1.70	1.05	2.05	1.40	1.25	0.76
time (sec)	N/A	0.031	0.013	0.007	1.972	0.835	2.718	0.280	1.562

Problem 1902	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	47	100	86	142	88	69	36
normalized size	1	1.00	0.73	1.56	1.34	2.22	1.38	1.08	0.56
time (sec)	N/A	0.026	0.012	0.007	1.941	0.997	2.807	0.209	1.314

Problem 1903	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	52	125	61	141	78	122	42
normalized size	1	1.00	0.96	2.31	1.13	2.61	1.44	2.26	0.78
time (sec)	N/A	0.030	0.030	0.007	1.944	0.867	2.341	0.492	1.407

Problem 1904	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	85	125	113	163	71	76	39
normalized size	1	1.00	1.20	1.76	1.59	2.30	1.00	1.07	0.55
time (sec)	N/A	0.028	0.049	0.007	1.941	0.902	3.645	0.272	1.532

Problem 1905	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	28	29	14	39	68	92	26
normalized size	1	1.00	1.56	1.61	0.78	2.17	3.78	5.11	1.44
time (sec)	N/A	0.006	0.025	0.004	0.863	1.017	1.176	0.222	1.464

Problem 1906	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	49	145	155	184	119	100	-1
normalized size	1	1.00	0.52	1.53	1.63	1.94	1.25	1.05	-0.01
time (sec)	N/A	0.046	0.014	0.010	2.004	0.856	5.974	0.242	0.000

Problem 1907	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	49	127	115	157	117	95	72
normalized size	1	1.00	0.57	1.48	1.34	1.83	1.36	1.10	0.84
time (sec)	N/A	0.042	0.014	0.009	1.799	0.888	4.667	0.251	1.841

Problem 1908	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	47	122	105	165	112	90	-1
normalized size	1	1.00	0.53	1.39	1.19	1.88	1.27	1.02	-0.01
time (sec)	N/A	0.043	0.015	0.009	1.804	0.920	3.996	0.223	0.000

Problem 1909	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	54	149	81	170	112	142	66
normalized size	1	1.00	0.68	1.86	1.01	2.12	1.40	1.78	0.82
time (sec)	N/A	0.040	0.013	0.009	1.873	0.603	4.005	0.501	1.754

Problem 1910	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	49	144	130	172	117	96	36
normalized size	1	1.00	0.57	1.67	1.51	2.00	1.36	1.12	0.42
time (sec)	N/A	0.036	0.015	0.007	1.900	0.985	4.435	0.222	1.487

Problem 1911	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	54	166	75	169	105	180	60
normalized size	1	1.00	0.75	2.31	1.04	2.35	1.46	2.50	0.83
time (sec)	N/A	0.039	0.014	0.012	1.831	0.898	4.103	1.058	1.653

Problem 1912	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	96	166	152	185	99	95	39
normalized size	1	1.00	1.04	1.80	1.65	2.01	1.08	1.03	0.42
time (sec)	N/A	0.037	0.042	0.012	1.994	0.813	4.822	0.227	1.785

Problem 1913	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	28	29	14	50	88	121	68
normalized size	1	1.00	1.56	1.61	0.78	2.78	4.89	6.72	3.78
time (sec)	N/A	0.006	0.011	0.005	0.875	1.048	3.632	0.259	1.749

Problem 1914	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	49	186	190	206	150	119	-1
normalized size	1	1.00	0.42	1.60	1.64	1.78	1.29	1.03	-0.01
time (sec)	N/A	0.062	0.016	0.020	1.971	0.979	8.186	0.247	0.000

Problem 1915	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	87	87	104	157	95	84	57
normalized size	1	1.00	1.18	1.18	1.41	2.12	1.28	1.14	0.77
time (sec)	N/A	0.037	0.033	0.009	1.888	1.209	4.866	0.242	1.581

Problem 1916	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	71	66	68	126	42	68	38
normalized size	1	1.00	1.42	1.32	1.36	2.52	0.84	1.36	0.76
time (sec)	N/A	0.023	0.024	0.009	1.980	0.901	2.739	0.248	1.433

Problem 1917	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	50	46	37	80	17	48	18
normalized size	1	1.00	2.08	1.92	1.54	3.33	0.71	2.00	0.75
time (sec)	N/A	0.017	0.015	0.004	1.901	0.850	1.287	0.247	1.310

Problem 1918	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	29	14	18	26	18	14
normalized size	1	1.00	1.00	1.81	0.88	1.12	1.62	1.12	0.88
time (sec)	N/A	0.006	0.010	0.005	0.862	0.757	1.391	0.189	1.208

Problem 1919	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	31	39	29	31	231	57	25
normalized size	1	1.00	0.89	1.11	0.83	0.89	6.60	1.63	0.71
time (sec)	N/A	0.020	0.015	0.006	0.868	0.817	1.619	0.205	1.238

Problem 1920	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	42	50	47	42	750	90	56
normalized size	1	1.00	0.74	0.88	0.82	0.74	13.16	1.58	0.98
time (sec)	N/A	0.029	0.020	0.008	0.875	1.129	2.312	0.222	1.284

Problem 1921	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	53	61	63	53	1969	123	73
normalized size	1	1.00	0.71	0.81	0.84	0.71	26.25	1.64	0.97
time (sec)	N/A	0.039	0.018	0.007	0.903	1.415	3.555	0.226	1.324

Problem 1922	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	40	50	50	40	279	0	38
normalized size	1	1.00	0.61	0.76	0.76	0.61	4.23	0.00	0.58
time (sec)	N/A	0.019	0.027	0.006	0.790	0.543	1.253	0.000	1.558

Problem 1923	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	28	38	32	28	46	0	25
normalized size	1	1.00	0.67	0.90	0.76	0.67	1.10	0.00	0.60
time (sec)	N/A	0.009	0.022	0.006	0.874	0.871	0.864	0.000	1.404

Problem 1924	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	28	14	18	17	28	39
normalized size	1	1.00	1.00	1.75	0.88	1.12	1.06	1.75	2.44
time (sec)	N/A	0.002	0.006	0.002	0.862	0.866	0.783	0.155	1.168

Problem 1925	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	50	52	41	79	19	0	23
normalized size	1	1.00	1.79	1.86	1.46	2.82	0.68	0.00	0.82
time (sec)	N/A	0.014	0.016	0.006	1.836	0.808	1.465	0.000	1.399

Problem 1926	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	71	73	76	128	42	0	60
normalized size	1	1.00	1.34	1.38	1.43	2.42	0.79	0.00	1.13
time (sec)	N/A	0.023	0.051	0.007	1.992	0.867	2.678	0.000	1.572

Problem 1927	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	56	50	21	88	46	55	21
normalized size	1	1.00	2.07	1.85	0.78	3.26	1.70	2.04	0.78
time (sec)	N/A	0.018	0.036	0.012	1.985	1.028	1.306	0.254	1.347

Problem 1928	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	50	52	41	75	20	0	17
normalized size	1	1.00	2.50	2.60	2.05	3.75	1.00	0.00	0.85
time (sec)	N/A	0.008	0.018	0.013	1.957	1.017	1.318	0.000	1.271

Problem 1929	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	54	64	20	84	49	0	21
normalized size	1	1.00	2.70	3.20	1.00	4.20	2.45	0.00	1.05
time (sec)	N/A	0.008	0.019	0.011	1.970	0.779	1.326	0.000	1.250

Problem 1930	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	93	86	87	122	221	100	0	75
normalized size	1	0.98	0.91	0.92	1.28	2.33	1.05	0.00	0.79
time (sec)	N/A	0.045	0.057	0.010	1.959	0.899	6.327	0.000	1.826

Problem 1931	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	74	73	86	192	71	0	53
normalized size	1	1.00	1.07	1.06	1.25	2.78	1.03	0.00	0.77
time (sec)	N/A	0.033	0.040	0.009	1.816	0.627	3.575	0.000	1.613

Problem 1932	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	62	63	52	163	187	0	33
normalized size	1	1.00	1.51	1.54	1.27	3.98	4.56	0.00	0.80
time (sec)	N/A	0.024	0.059	0.007	1.976	0.826	2.180	0.000	1.324

Problem 1933	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	28	13	29	26	44	18
normalized size	1	1.00	1.00	1.87	0.87	1.93	1.73	2.93	1.20
time (sec)	N/A	0.006	0.007	0.003	0.876	0.987	2.368	0.191	1.247

Problem 1934	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	28	37	30	37	48	28	36
normalized size	1	1.00	0.82	1.09	0.88	1.09	1.41	0.82	1.06
time (sec)	N/A	0.022	0.009	0.004	0.886	0.754	3.383	0.212	1.225

Problem 1935	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	42	50	46	54	423	0	47
normalized size	1	1.00	0.78	0.93	0.85	1.00	7.83	0.00	0.87
time (sec)	N/A	0.030	0.010	0.007	0.883	0.839	2.604	0.000	1.338

Problem 1936	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	53	59	63	63	1844	0	56
normalized size	1	1.00	0.73	0.81	0.86	0.86	25.26	0.00	0.77
time (sec)	N/A	0.038	0.013	0.007	0.872	0.839	4.251	0.000	1.408

Problem 1937	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	52	60	69	62	337	0	48
normalized size	1	1.00	0.59	0.68	0.78	0.70	3.83	0.00	0.55
time (sec)	N/A	0.027	0.026	0.007	0.865	0.851	1.644	0.000	1.719

Problem 1938	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	41	49	53	51	219	0	38
normalized size	1	1.00	0.66	0.79	0.85	0.82	3.53	0.00	0.61
time (sec)	N/A	0.016	0.019	0.006	0.823	0.776	1.279	0.000	1.593

Problem 1939	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	27	37	32	39	42	0	36
normalized size	1	1.00	0.77	1.06	0.91	1.11	1.20	0.00	1.03
time (sec)	N/A	0.006	0.013	0.004	0.894	0.786	0.988	0.000	1.192

Problem 1940	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	29	17	29	20	0	24
normalized size	1	1.00	1.00	1.53	0.89	1.53	1.05	0.00	1.26
time (sec)	N/A	0.007	0.005	0.004	0.899	0.831	1.015	0.000	1.199

Problem 1941	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	36	67	58	155	184	0	40
normalized size	1	1.00	0.77	1.43	1.23	3.30	3.91	0.00	0.85
time (sec)	N/A	0.023	0.010	0.008	1.791	1.083	2.371	0.000	1.373

Problem 1942	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	38	81	97	190	73	0	-1
normalized size	1	1.00	0.54	1.14	1.37	2.68	1.03	0.00	-0.01
time (sec)	N/A	0.034	0.011	0.009	1.957	0.955	4.141	0.000	0.000

Problem 1943	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	39	94	139	228	102	0	-1
normalized size	1	1.00	0.41	0.99	1.46	2.40	1.07	0.00	-0.01
time (sec)	N/A	0.048	0.011	0.010	1.989	0.913	6.998	0.000	0.000

Problem 1944	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	112	98	139	287	432	196	95
normalized size	1	1.00	0.97	0.84	1.20	2.47	3.72	1.69	0.82
time (sec)	N/A	0.059	0.226	0.015	1.900	1.043	8.922	0.370	1.920

Problem 1945	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	92	101	85	101	260	819	178	73
normalized size	1	1.05	1.15	0.97	1.15	2.95	9.31	2.02	0.83
time (sec)	N/A	0.043	0.168	0.010	1.889	0.807	5.724	0.319	1.641

Problem 1946	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	91	73	62	232	743	154	47
normalized size	1	1.00	1.54	1.24	1.05	3.93	12.59	2.61	0.80
time (sec)	N/A	0.033	0.175	0.009	1.987	1.132	3.500	0.333	1.389

Problem 1947	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	28	29	14	41	48	110	14
normalized size	1	1.00	1.56	1.61	0.78	2.28	2.67	6.11	0.78
time (sec)	N/A	0.006	0.009	0.006	0.755	0.778	3.779	0.204	1.249

Problem 1948	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	37	39	29	53	94	78	27
normalized size	1	1.00	1.06	1.11	0.83	1.51	2.69	2.23	0.77
time (sec)	N/A	0.021	0.020	0.003	0.918	0.792	5.697	0.240	1.259

Problem 1949	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	48	50	47	61	153	0	42
normalized size	1	1.00	0.87	0.91	0.85	1.11	2.78	0.00	0.76
time (sec)	N/A	0.029	0.021	0.006	0.923	1.324	8.359	0.000	1.292

Problem 1950	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	62	61	64	76	201	58	58
normalized size	1	1.00	0.82	0.80	0.84	1.00	2.64	0.76	0.76
time (sec)	N/A	0.040	0.014	0.006	0.888	0.631	12.148	0.220	1.359

Problem 1951	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	61	60	71	73	337	0	81
normalized size	1	1.00	0.74	0.73	0.87	0.89	4.11	0.00	0.99
time (sec)	N/A	0.022	0.030	0.008	0.912	1.037	1.876	0.000	1.729

Problem 1952	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	51	50	51	63	163	0	43
normalized size	1	1.00	0.88	0.86	0.88	1.09	2.81	0.00	0.74
time (sec)	N/A	0.011	0.028	0.006	0.815	0.642	1.745	0.000	1.332

Problem 1953	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	38	39	33	52	105	0	34
normalized size	1	1.00	0.90	0.93	0.79	1.24	2.50	0.00	0.81
time (sec)	N/A	0.014	0.024	0.007	0.874	0.950	1.856	0.000	1.267

Problem 1954	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	28	29	17	40	48	0	17
normalized size	1	1.00	1.33	1.38	0.81	1.90	2.29	0.00	0.81
time (sec)	N/A	0.006	0.008	0.003	0.924	0.688	1.957	0.000	1.199

Problem 1955	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	46	77	73	226	740	0	-1
normalized size	1	1.00	0.68	1.13	1.07	3.32	10.88	0.00	-0.01
time (sec)	N/A	0.035	0.011	0.010	1.984	0.963	4.589	0.000	0.000

Problem 1956	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	47	92	117	260	864	0	-1
normalized size	1	1.00	0.49	0.97	1.23	2.74	9.09	0.00	-0.01
time (sec)	N/A	0.049	0.012	0.009	1.896	1.160	7.013	0.000	0.000

Problem 1957	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	22	9	21	31	21	17
normalized size	1	1.00	1.00	1.69	0.69	1.62	2.38	1.62	1.31
time (sec)	N/A	0.003	0.007	0.003	0.834	0.990	1.096	0.163	1.353

Problem 1958	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	21	22	9	26	48	23	19
normalized size	1	1.00	1.62	1.69	0.69	2.00	3.69	1.77	1.46
time (sec)	N/A	0.004	0.008	0.003	0.850	0.642	2.288	0.173	1.415

Problem 1959	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	63	0	0	0	56	0	-1
normalized size	1	1.00	1.43	0.00	0.00	0.00	1.27	0.00	-0.02
time (sec)	N/A	0.022	0.032	0.017	0.000	0.833	5.294	0.000	0.000

Problem 1960	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	58	0	0	0	48	0	-1
normalized size	1	1.00	1.32	0.00	0.00	0.00	1.09	0.00	-0.02
time (sec)	N/A	0.017	0.017	0.011	0.000	0.775	2.040	0.000	0.000

Problem 1961	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	64	0	0	0	54	0	-1
normalized size	1	1.00	1.45	0.00	0.00	0.00	1.23	0.00	-0.02
time (sec)	N/A	0.017	0.023	0.014	0.000	0.902	1.406	0.000	0.000

Problem 1962	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	67	0	0	0	54	0	-1
normalized size	1	1.00	1.52	0.00	0.00	0.00	1.23	0.00	-0.02
time (sec)	N/A	0.018	0.040	0.011	0.000	0.888	2.475	0.000	0.000

Problem 1963	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	71	0	0	0	54	0	-1
normalized size	1	1.00	1.61	0.00	0.00	0.00	1.23	0.00	-0.02
time (sec)	N/A	0.017	0.020	0.131	0.000	0.765	17.754	0.000	0.000

Problem 1964	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	73	0	0	0	60	0	-1
normalized size	1	1.00	1.04	0.00	0.00	0.00	0.86	0.00	-0.01
time (sec)	N/A	0.030	0.020	0.093	0.000	0.891	22.270	0.000	0.000

Problem 1965	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	35	34	33	32	35	33
normalized size	1	1.00	1.00	0.88	0.85	0.82	0.80	0.88	0.82
time (sec)	N/A	0.026	0.007	0.004	0.877	0.879	0.230	0.158	0.065

Problem 1966	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	122	117	125	140	44	132	143
normalized size	1	1.00	0.90	0.86	0.92	1.03	0.32	0.97	1.05
time (sec)	N/A	0.095	0.059	0.006	1.987	0.829	0.264	0.168	1.316

Problem 1967	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	120	115	122	108	37	129	119
normalized size	1	1.00	0.91	0.87	0.92	0.82	0.28	0.98	0.90
time (sec)	N/A	0.082	0.052	0.004	1.934	0.949	0.212	0.172	0.196

Problem 1968	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	23	22	20	24	22
normalized size	1	1.00	1.00	0.89	0.85	0.81	0.74	0.89	0.81
time (sec)	N/A	0.021	0.005	0.002	0.856	0.821	0.236	0.203	1.125

Problem 1969	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	111	102	109	123	32	114	120
normalized size	1	1.00	0.90	0.82	0.88	0.99	0.26	0.92	0.97
time (sec)	N/A	0.066	0.056	0.003	1.922	0.873	0.189	0.166	0.190

Problem 1970	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	108	99	106	106	22	111	114
normalized size	1	1.00	0.91	0.83	0.89	0.89	0.18	0.93	0.96
time (sec)	N/A	0.066	0.021	0.003	1.808	0.849	0.177	0.159	1.322

Problem 1971	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	14	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87
time (sec)	N/A	0.006	0.004	0.002	0.891	0.871	0.160	0.152	0.034

Problem 1972	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	89	91	98	304	24	112	111
normalized size	1	1.00	0.77	0.79	0.85	2.64	0.21	0.97	0.97
time (sec)	N/A	0.051	0.014	0.002	1.898	0.868	0.164	0.215	1.350

Problem 1973	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	89	91	98	299	20	112	99
normalized size	1	1.00	0.77	0.79	0.85	2.60	0.17	0.97	0.86
time (sec)	N/A	0.051	0.014	0.002	1.982	0.849	0.189	0.168	1.304

Problem 1974	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	22	21	13	18	15	22	18
normalized size	1	1.00	1.47	1.40	0.87	1.20	1.00	1.47	1.20
time (sec)	N/A	0.004	0.014	0.003	0.886	0.850	0.316	0.153	1.180

Problem 1975	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	114	99	106	103	29	121	102
normalized size	1	1.00	0.93	0.81	0.87	0.84	0.24	0.99	0.84
time (sec)	N/A	0.065	0.021	0.004	1.947	0.778	0.228	0.211	0.251

Problem 1976	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	119	99	106	143	32	115	128
normalized size	1	1.00	0.96	0.80	0.85	1.15	0.26	0.93	1.03
time (sec)	N/A	0.065	0.029	0.005	1.994	0.764	0.320	0.164	1.340

Problem 1977	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	33	33	31	42	31
normalized size	1	1.00	1.00	0.91	0.94	0.94	0.89	1.20	0.89
time (sec)	N/A	0.023	0.008	0.007	0.822	0.830	0.439	0.186	1.159

Problem 1978	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	49	51	53	70	53	54	56
normalized size	1	1.00	0.88	0.91	0.95	1.25	0.95	0.96	1.00
time (sec)	N/A	0.040	0.019	0.007	0.848	0.822	0.398	0.162	1.128

Problem 1979	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	144	137	147	190	70	151	172
normalized size	1	1.00	0.92	0.87	0.94	1.21	0.45	0.96	1.10
time (sec)	N/A	0.097	0.109	0.010	1.804	0.629	0.414	0.197	1.289

Problem 1980	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	140	133	143	158	65	147	152
normalized size	1	1.00	0.90	0.86	0.92	1.02	0.42	0.95	0.98
time (sec)	N/A	0.094	0.106	0.008	2.007	0.741	0.389	0.237	0.216

Problem 1981	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	38	41	43	56	42	41	45
normalized size	1	1.00	0.83	0.89	0.93	1.22	0.91	0.89	0.98
time (sec)	N/A	0.032	0.023	0.007	0.900	0.972	0.368	0.214	0.054

Problem 1982	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	131	120	130	163	58	132	139
normalized size	1	1.00	0.90	0.82	0.89	1.12	0.40	0.90	0.95
time (sec)	N/A	0.076	0.093	0.008	1.966	0.922	0.417	0.167	1.247

Problem 1983	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	127	115	125	146	48	127	132
normalized size	1	1.00	0.88	0.80	0.87	1.01	0.33	0.88	0.92
time (sec)	N/A	0.074	0.115	0.009	1.975	1.059	0.337	0.208	1.320

Problem 1984	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	27	30	32	35	29	32	29
normalized size	1	1.00	0.82	0.91	0.97	1.06	0.88	0.97	0.88
time (sec)	N/A	0.024	0.010	0.004	0.890	0.663	0.378	0.169	0.048

Problem 1985	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	119	108	116	400	44	132	138
normalized size	1	1.00	0.88	0.79	0.85	2.94	0.32	0.97	1.01
time (sec)	N/A	0.068	0.077	0.009	1.978	1.194	0.319	0.198	0.246

Problem 1986	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	118	106	114	391	39	130	108
normalized size	1	1.00	0.88	0.79	0.85	2.92	0.29	0.97	0.81
time (sec)	N/A	0.067	0.069	0.010	1.977	1.266	0.308	0.172	1.329

Problem 1987	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	15	15	14	14
normalized size	1	1.00	1.00	0.94	0.88	0.94	0.94	0.88	0.88
time (sec)	N/A	0.004	0.005	0.002	0.852	0.843	0.272	0.164	1.104

Problem 1988	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	119	117	124	402	44	129	138
normalized size	1	1.00	0.88	0.86	0.91	2.96	0.32	0.95	1.01
time (sec)	N/A	0.069	0.069	0.005	1.856	0.898	0.499	0.182	1.318

Problem 1989	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	118	115	122	389	39	127	128
normalized size	1	1.00	0.88	0.86	0.91	2.90	0.29	0.95	0.96
time (sec)	N/A	0.063	0.067	0.005	1.891	0.846	0.332	0.165	0.202

Problem 1990	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	33	35	37	47	34	45	34
normalized size	1	1.00	0.87	0.92	0.97	1.24	0.89	1.18	0.89
time (sec)	N/A	0.025	0.019	0.009	0.887	0.839	0.426	0.158	0.066

Problem 1991	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	83	3560	100	175	100	55	76
normalized size	1	1.00	1.17	50.14	1.41	2.46	1.41	0.77	1.07
time (sec)	N/A	0.036	0.167	0.676	1.913	1.166	4.530	0.264	1.771

Problem 1992	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	68	3340	53	150	48	39	56
normalized size	1	1.00	1.45	71.06	1.13	3.19	1.02	0.83	1.19
time (sec)	N/A	0.024	0.114	0.030	1.922	0.856	2.441	0.264	1.600

Problem 1993	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	64	3339	49	131	76	36	31
normalized size	1	1.00	1.49	77.65	1.14	3.05	1.77	0.84	0.72
time (sec)	N/A	0.023	0.096	0.030	1.861	1.169	1.747	0.244	1.375

Problem 1994	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	29	14	28	46	14	24
normalized size	1	1.00	1.00	1.61	0.78	1.56	2.56	0.78	1.33
time (sec)	N/A	0.006	0.012	0.004	0.787	0.869	1.228	0.176	1.435

Problem 1995	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	39	30	42	313	29	37
normalized size	1	1.00	1.00	1.03	0.79	1.11	8.24	0.76	0.97
time (sec)	N/A	0.021	0.011	0.005	0.919	1.454	1.795	0.185	1.673

Problem 1996	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	49	50	47	53	913	43	70
normalized size	1	1.00	0.83	0.85	0.80	0.90	15.47	0.73	1.19
time (sec)	N/A	0.030	0.013	0.006	0.892	0.959	2.914	0.222	1.937

Problem 1997	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	60	61	64	64	2317	57	90
normalized size	1	1.00	0.75	0.76	0.80	0.80	28.96	0.71	1.12
time (sec)	N/A	0.041	0.014	0.006	0.814	1.241	4.284	0.196	2.338

Problem 1998	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	96	2226	0	0	48	0	-1
normalized size	1	1.00	0.33	7.65	0.00	0.00	0.16	0.00	-0.00
time (sec)	N/A	0.191	0.076	0.028	0.000	0.916	1.587	0.000	0.000

Problem 1999	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	80	2010	0	0	48	0	-1
normalized size	1	1.00	0.30	7.53	0.00	0.00	0.18	0.00	-0.00
time (sec)	N/A	0.121	0.058	0.030	0.000	0.868	1.291	0.000	0.000

Problem 2000	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	49	1786	0	0	44	0	-1
normalized size	1	1.00	0.20	7.38	0.00	0.00	0.18	0.00	-0.00
time (sec)	N/A	0.086	0.010	0.030	0.000	0.958	1.091	0.000	0.000

Problem 2001	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	51	1785	0	0	39	0	40
normalized size	1	1.00	0.21	7.35	0.00	0.00	0.16	0.00	0.16
time (sec)	N/A	0.088	0.013	0.032	0.000	0.921	1.090	0.000	1.414

Problem 2002	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	51	2002	0	0	41	0	-1
normalized size	1	1.00	0.19	7.50	0.00	0.00	0.15	0.00	-0.00
time (sec)	N/A	0.123	0.018	0.037	0.000	0.819	1.337	0.000	0.000

Problem 2003	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	51	2222	0	0	41	0	-1
normalized size	1	1.00	0.18	7.64	0.00	0.00	0.14	0.00	-0.00
time (sec)	N/A	0.155	0.014	0.040	0.000	0.835	1.723	0.000	0.000

Problem 2004	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	563	563	80	2799	0	0	48	0	-1
normalized size	1	1.00	0.14	4.97	0.00	0.00	0.09	0.00	-0.00
time (sec)	N/A	0.389	0.050	0.030	0.000	0.945	1.613	0.000	0.000

Problem 2005	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	539	539	51	2579	0	0	48	0	-1
normalized size	1	1.00	0.09	4.78	0.00	0.00	0.09	0.00	-0.00
time (sec)	N/A	0.286	0.012	0.029	0.000	0.872	1.364	0.000	0.000

Problem 2006	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	507	507	47	2864	0	0	42	0	38
normalized size	1	1.00	0.09	5.65	0.00	0.00	0.08	0.00	0.07
time (sec)	N/A	0.233	0.011	0.028	0.000	0.828	0.997	0.000	1.286

Problem 2007	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	517	517	51	3309	0	0	41	0	-1
normalized size	1	1.00	0.10	6.40	0.00	0.00	0.08	0.00	-0.00
time (sec)	N/A	0.245	0.014	0.036	0.000	0.751	1.268	0.000	0.000

Problem 2008	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	541	541	51	3556	0	0	41	0	-1
normalized size	1	1.00	0.09	6.57	0.00	0.00	0.08	0.00	-0.00
time (sec)	N/A	0.299	0.014	0.038	0.000	0.827	1.458	0.000	0.000

Problem 2009	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	565	565	51	3788	0	0	41	0	-1
normalized size	1	1.00	0.09	6.70	0.00	0.00	0.07	0.00	-0.00
time (sec)	N/A	0.356	0.014	0.043	0.000	0.861	1.803	0.000	0.000

Problem 2010	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	80	3569	98	179	76	51	74
normalized size	1	1.00	1.18	52.49	1.44	2.63	1.12	0.75	1.09
time (sec)	N/A	0.036	0.188	0.014	1.858	1.180	3.873	0.318	1.858

Problem 2011	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	49	3559	66	151	100	55	69
normalized size	1	1.00	0.84	61.36	1.14	2.60	1.72	0.95	1.19
time (sec)	N/A	0.034	0.018	0.016	1.959	1.171	2.997	0.353	1.788

Problem 2012	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	52	3535	61	164	83	50	43
normalized size	1	1.00	0.88	59.92	1.03	2.78	1.41	0.85	0.73
time (sec)	N/A	0.033	0.023	0.046	1.966	1.082	2.378	0.275	1.623

Problem 2013	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	28	29	14	39	71	14	26
normalized size	1	1.00	1.56	1.61	0.78	2.17	3.94	0.78	1.44
time (sec)	N/A	0.006	0.010	0.005	0.847	1.053	1.358	0.185	1.886

Problem 2014	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	40	39	30	53	371	29	68
normalized size	1	1.00	1.05	1.03	0.79	1.39	9.76	0.76	1.79
time (sec)	N/A	0.023	0.012	0.006	0.865	0.843	2.054	0.199	2.356

Problem 2015	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	51	50	47	64	1001	43	88
normalized size	1	1.00	0.86	0.85	0.80	1.08	16.97	0.73	1.49
time (sec)	N/A	0.032	0.014	0.008	0.821	0.819	3.237	0.204	2.846

Problem 2016	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	62	61	64	75	2317	57	108
normalized size	1	1.00	0.78	0.76	0.80	0.94	28.96	0.71	1.35
time (sec)	N/A	0.042	0.016	0.007	0.823	0.787	4.751	0.205	3.432

Problem 2017	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	97	3567	104	179	102	99	79
normalized size	1	1.00	1.31	48.20	1.41	2.42	1.38	1.34	1.07
time (sec)	N/A	0.037	0.043	0.031	2.006	1.136	5.786	0.231	1.744

Problem 2018	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	81	3347	68	149	49	67	59
normalized size	1	1.00	1.62	66.94	1.36	2.98	0.98	1.34	1.18
time (sec)	N/A	0.026	0.027	0.011	1.934	1.247	2.632	0.247	1.654

Problem 2019	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	59	480	37	102	24	0	19
normalized size	1	1.00	2.19	17.78	1.37	3.78	0.89	0.00	0.70
time (sec)	N/A	0.018	0.016	0.010	1.911	1.181	1.368	0.000	1.305

Problem 2020	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	29	14	18	29	18	14
normalized size	1	1.00	1.00	1.61	0.78	1.00	1.61	1.00	0.78
time (sec)	N/A	0.006	0.008	0.006	0.790	0.827	2.039	0.152	1.183

Problem 2021	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	31	39	30	31	255	0	25
normalized size	1	1.00	0.82	1.03	0.79	0.82	6.71	0.00	0.66
time (sec)	N/A	0.020	0.016	0.005	0.838	0.814	1.962	0.000	1.252

Problem 2022	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	42	50	47	42	824	0	56
normalized size	1	1.00	0.71	0.85	0.80	0.71	13.97	0.00	0.95
time (sec)	N/A	0.031	0.018	0.005	0.885	0.797	3.193	0.000	1.351

Problem 2023	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	53	61	64	53	2183	0	73
normalized size	1	1.00	0.66	0.76	0.80	0.66	27.29	0.00	0.91
time (sec)	N/A	0.040	0.020	0.007	0.819	0.919	4.743	0.000	1.439

Problem 2024	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	91	2233	0	0	46	0	-1
normalized size	1	1.00	0.31	7.60	0.00	0.00	0.16	0.00	-0.00
time (sec)	N/A	0.150	0.029	0.033	0.000	0.828	1.462	0.000	0.000

Problem 2025	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	80	2017	0	0	46	0	-1
normalized size	1	1.00	0.30	7.47	0.00	0.00	0.17	0.00	-0.00
time (sec)	N/A	0.119	0.026	0.012	0.000	0.839	1.199	0.000	0.000

Problem 2026	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	64	1793	0	0	42	0	-1
normalized size	1	1.00	0.26	7.23	0.00	0.00	0.17	0.00	-0.00
time (sec)	N/A	0.088	0.021	0.010	0.000	0.947	1.030	0.000	0.000

Problem 2027	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	49	437	0	0	37	0	40
normalized size	1	1.00	0.22	1.98	0.00	0.00	0.17	0.00	0.18
time (sec)	N/A	0.062	0.012	0.014	0.000	0.968	1.149	0.000	1.280

Problem 2028	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	51	1795	0	0	39	0	-1
normalized size	1	1.00	0.21	7.30	0.00	0.00	0.16	0.00	-0.00
time (sec)	N/A	0.090	0.014	0.016	0.000	0.831	1.346	0.000	0.000

Problem 2029	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	51	2009	0	0	39	0	-1
normalized size	1	1.00	0.19	7.44	0.00	0.00	0.14	0.00	-0.00
time (sec)	N/A	0.120	0.013	0.018	0.000	0.795	1.676	0.000	0.000

Problem 2030	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	566	566	79	2806	0	0	46	0	-1
normalized size	1	1.00	0.14	4.96	0.00	0.00	0.08	0.00	-0.00
time (sec)	N/A	0.343	0.026	0.035	0.000	1.108	1.315	0.000	0.000

Problem 2031	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	542	542	62	2586	0	0	46	0	-1
normalized size	1	1.00	0.11	4.77	0.00	0.00	0.08	0.00	-0.00
time (sec)	N/A	0.286	0.024	0.014	0.000	0.844	1.138	0.000	0.000

Problem 2032	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	513	513	49	2374	0	0	41	0	38
normalized size	1	1.00	0.10	4.63	0.00	0.00	0.08	0.00	0.07
time (sec)	N/A	0.227	0.012	0.011	0.000	0.603	1.042	0.000	1.162

Problem 2033	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	491	491	49	2860	0	0	39	0	-1
normalized size	1	1.00	0.10	5.82	0.00	0.00	0.08	0.00	-0.00
time (sec)	N/A	0.192	0.014	0.016	0.000	1.056	1.197	0.000	0.000

Problem 2034	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	520	520	51	3300	0	0	39	0	-1
normalized size	1	1.00	0.10	6.35	0.00	0.00	0.08	0.00	-0.00
time (sec)	N/A	0.237	0.014	0.018	0.000	1.110	1.466	0.000	0.000

Problem 2035	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	544	544	51	3547	0	0	39	0	-1
normalized size	1	1.00	0.09	6.52	0.00	0.00	0.07	0.00	-0.00
time (sec)	N/A	0.295	0.021	0.019	0.000	1.075	1.793	0.000	0.000

Problem 2036	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	568	568	51	3779	0	0	39	0	-1
normalized size	1	1.00	0.09	6.65	0.00	0.00	0.07	0.00	-0.00
time (sec)	N/A	0.349	0.015	0.020	0.000	1.151	2.253	0.000	0.000

Problem 2037	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	96	3910	122	243	110	117	96
normalized size	1	1.00	1.01	41.16	1.28	2.56	1.16	1.23	1.01
time (sec)	N/A	0.047	0.055	0.046	1.983	1.178	6.164	0.252	1.927

Problem 2038	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	66	84	3664	86	215	73	97	73
normalized size	1	1.03	1.31	57.25	1.34	3.36	1.14	1.52	1.14
time (sec)	N/A	0.035	0.045	0.016	1.955	1.178	3.582	0.241	1.758

Problem 2039	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	75	3438	52	186	187	0	34
normalized size	1	1.00	1.63	74.74	1.13	4.04	4.07	0.00	0.74
time (sec)	N/A	0.025	0.076	0.014	1.921	1.393	2.222	0.000	1.388

Problem 2040	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	29	14	30	27	0	14
normalized size	1	1.00	1.00	1.61	0.78	1.67	1.50	0.00	0.78
time (sec)	N/A	0.006	0.007	0.006	0.832	0.879	3.197	0.000	1.221

Problem 2041	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	29	37	30	37	51	0	31
normalized size	1	1.00	0.76	0.97	0.79	0.97	1.34	0.00	0.82
time (sec)	N/A	0.021	0.011	0.006	0.874	0.962	6.154	0.000	1.249

Problem 2042	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	42	50	47	54	466	0	62
normalized size	1	1.00	0.71	0.85	0.80	0.92	7.90	0.00	1.05
time (sec)	N/A	0.031	0.015	0.007	0.840	0.872	3.480	0.000	1.362

Problem 2043	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	51	59	64	63	2048	0	82
normalized size	1	1.00	0.65	0.76	0.82	0.81	26.26	0.00	1.05
time (sec)	N/A	0.040	0.016	0.007	0.899	0.950	5.174	0.000	1.516

Problem 2044	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	91	2540	0	0	46	0	-1
normalized size	1	1.00	0.29	8.06	0.00	0.00	0.15	0.00	-0.00
time (sec)	N/A	0.180	0.032	0.039	0.000	0.965	1.589	0.000	0.000

Problem 2045	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	80	2302	0	0	46	0	-1
normalized size	1	1.00	0.27	7.91	0.00	0.00	0.16	0.00	-0.00
time (sec)	N/A	0.145	0.036	0.017	0.000	1.178	1.379	0.000	0.000

Problem 2046	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	67	2052	0	0	42	0	-1
normalized size	1	1.00	0.25	7.63	0.00	0.00	0.16	0.00	-0.00
time (sec)	N/A	0.115	0.037	0.013	0.000	0.906	1.198	0.000	0.000

Problem 2047	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	57	1822	0	0	37	0	40
normalized size	1	1.00	0.23	7.35	0.00	0.00	0.15	0.00	0.16
time (sec)	N/A	0.088	0.024	0.011	0.000	0.799	1.416	0.000	1.445

Problem 2048	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	58	1825	0	0	39	0	-1
normalized size	1	1.00	0.24	7.45	0.00	0.00	0.16	0.00	-0.00
time (sec)	N/A	0.087	0.044	0.020	0.000	0.645	2.150	0.000	0.000

Problem 2049	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	54	2056	0	0	39	0	-1
normalized size	1	1.00	0.20	7.70	0.00	0.00	0.15	0.00	-0.00
time (sec)	N/A	0.121	0.016	0.019	0.000	1.091	2.247	0.000	0.000

Problem 2050	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	587	587	78	3182	0	0	46	0	-1
normalized size	1	1.00	0.13	5.42	0.00	0.00	0.08	0.00	-0.00
time (sec)	N/A	0.394	0.052	0.040	0.000	0.855	1.558	0.000	0.000

Problem 2051	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	563	563	65	2936	0	0	46	0	-1
normalized size	1	1.00	0.12	5.21	0.00	0.00	0.08	0.00	-0.00
time (sec)	N/A	0.337	0.028	0.016	0.000	0.853	1.447	0.000	0.000

Problem 2052	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	539	539	53	2700	0	0	41	0	38
normalized size	1	1.00	0.10	5.01	0.00	0.00	0.08	0.00	0.07
time (sec)	N/A	0.283	0.031	0.016	0.000	1.086	1.243	0.000	1.265

Problem 2053	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	520	520	52	2703	0	0	39	0	-1
normalized size	1	1.00	0.10	5.20	0.00	0.00	0.08	0.00	-0.00
time (sec)	N/A	0.238	0.014	0.013	0.000	0.872	1.444	0.000	0.000

Problem 2054	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	517	517	52	2867	0	0	39	0	-1
normalized size	1	1.00	0.10	5.55	0.00	0.00	0.08	0.00	-0.00
time (sec)	N/A	0.237	0.029	0.020	0.000	0.919	1.839	0.000	0.000

Problem 2055	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	541	541	54	3307	0	0	39	0	-1
normalized size	1	1.00	0.10	6.11	0.00	0.00	0.07	0.00	-0.00
time (sec)	N/A	0.294	0.016	0.021	0.000	0.567	2.547	0.000	0.000

Problem 2056	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	565	565	54	3554	0	0	39	0	-1
normalized size	1	1.00	0.10	6.29	0.00	0.00	0.07	0.00	-0.00
time (sec)	N/A	0.344	0.021	0.022	0.000	1.079	3.060	0.000	0.000

Problem 2057	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	173	133	179	119	22	172	48
normalized size	1	1.00	0.91	0.70	0.94	0.63	0.12	0.91	0.25
time (sec)	N/A	0.132	0.061	0.008	2.003	0.855	0.211	0.156	0.100

Problem 2058	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	62	68	53	127	44	41	35
normalized size	1	1.00	1.32	1.45	1.13	2.70	0.94	0.87	0.74
time (sec)	N/A	0.026	0.144	0.018	1.949	0.751	2.501	0.167	1.568

Problem 2059	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	66	64	60	112	66	36	-1
normalized size	1	1.00	1.35	1.31	1.22	2.29	1.35	0.73	-0.02
time (sec)	N/A	0.037	0.057	0.013	2.043	1.188	1.795	0.152	0.000

Problem 2060	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	60	80	49	109	66	60	31
normalized size	1	1.00	1.40	1.86	1.14	2.53	1.53	1.40	0.72
time (sec)	N/A	0.024	0.123	0.012	1.989	0.891	1.777	0.195	1.337

Problem 2061	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	68	90	79	130	46	46	-1
normalized size	1	1.00	1.36	1.80	1.58	2.60	0.92	0.92	-0.02
time (sec)	N/A	0.036	0.034	0.012	1.870	0.861	2.450	0.154	0.000

Problem 2062	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	93	130	0	0	44	0	-1
normalized size	1	1.00	0.87	1.21	0.00	0.00	0.41	0.00	-0.01
time (sec)	N/A	0.048	0.246	0.047	0.000	1.162	1.298	0.000	0.000

Problem 2063	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	47	202	0	0	42	0	38
normalized size	1	1.00	0.21	0.90	0.00	0.00	0.19	0.00	0.17
time (sec)	N/A	0.108	0.009	0.013	0.000	1.407	1.171	0.000	1.232

Problem 2064	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	51	132	0	0	39	0	39
normalized size	1	1.00	0.48	1.23	0.00	0.00	0.36	0.00	0.36
time (sec)	N/A	0.043	0.010	0.016	0.000	1.246	1.212	0.000	1.343

Problem 2065	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	51	235	0	0	41	0	-1
normalized size	1	1.00	0.22	1.00	0.00	0.00	0.17	0.00	-0.00
time (sec)	N/A	0.115	0.010	0.018	0.000	0.840	1.438	0.000	0.000

Problem 2066	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	49	82	66	129	95	78	48
normalized size	1	1.00	0.78	1.30	1.05	2.05	1.51	1.24	0.76
time (sec)	N/A	0.034	0.015	0.022	2.019	0.926	3.275	0.222	1.739

Problem 2067	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	47	86	95	143	95	63	-1
normalized size	1	1.00	0.68	1.25	1.38	2.07	1.38	0.91	-0.01
time (sec)	N/A	0.055	0.014	0.019	1.974	0.726	3.076	0.162	0.000

Problem 2068	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	52	78	61	141	80	122	43
normalized size	1	1.00	0.88	1.32	1.03	2.39	1.36	2.07	0.73
time (sec)	N/A	0.034	0.015	0.019	1.953	0.839	2.476	0.254	1.503

Problem 2069	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	85	93	119	160	75	70	-1
normalized size	1	1.00	1.20	1.31	1.68	2.25	1.06	0.99	-0.01
time (sec)	N/A	0.052	0.052	0.018	1.876	1.036	3.634	0.164	0.000

Problem 2070	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	52	138	0	0	44	0	-1
normalized size	1	1.00	0.41	1.10	0.00	0.00	0.35	0.00	-0.01
time (sec)	N/A	0.059	0.013	0.014	0.000	0.688	1.609	0.000	0.000

Problem 2071	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	52	228	0	0	42	0	38
normalized size	1	1.00	0.21	0.91	0.00	0.00	0.17	0.00	0.15
time (sec)	N/A	0.132	0.009	0.017	0.000	0.868	1.379	0.000	1.422

Problem 2072	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	52	158	0	0	39	0	39
normalized size	1	1.00	0.41	1.25	0.00	0.00	0.31	0.00	0.31
time (sec)	N/A	0.055	0.028	0.019	0.000	0.886	1.418	0.000	1.575

Problem 2073	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	52	257	0	0	41	0	-1
normalized size	1	1.00	0.20	1.00	0.00	0.00	0.16	0.00	-0.00
time (sec)	N/A	0.136	0.030	0.023	0.000	1.184	1.506	0.000	0.000

Problem 2074	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	54	103	81	170	112	142	66
normalized size	1	1.00	0.68	1.29	1.01	2.12	1.40	1.78	0.82
time (sec)	N/A	0.044	0.016	0.020	1.923	0.567	4.603	0.271	2.036

Problem 2075	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	49	108	139	169	124	88	-1
normalized size	1	1.00	0.54	1.19	1.53	1.86	1.36	0.97	-0.01
time (sec)	N/A	0.073	0.016	0.017	1.959	0.879	4.969	0.191	0.000

Problem 2076	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	54	99	75	169	107	180	60
normalized size	1	1.00	0.70	1.29	0.97	2.19	1.39	2.34	0.78
time (sec)	N/A	0.041	0.036	0.023	2.013	1.271	6.300	0.284	1.874

Problem 2077	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	96	113	158	182	102	87	-1
normalized size	1	1.00	1.04	1.23	1.72	1.98	1.11	0.95	-0.01
time (sec)	N/A	0.067	0.053	0.023	1.987	1.381	6.228	0.166	0.000

Problem 2078	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	54	181	0	0	44	0	-1
normalized size	1	1.00	0.37	1.24	0.00	0.00	0.30	0.00	-0.01
time (sec)	N/A	0.070	0.034	0.020	0.000	1.139	2.240	0.000	0.000

Problem 2079	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	54	251	0	0	42	0	38
normalized size	1	1.00	0.20	0.92	0.00	0.00	0.15	0.00	0.14
time (sec)	N/A	0.154	0.014	0.020	0.000	0.716	1.976	0.000	1.705

Problem 2080	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	54	180	0	0	39	0	39
normalized size	1	1.00	0.37	1.22	0.00	0.00	0.27	0.00	0.27
time (sec)	N/A	0.070	0.015	0.025	0.000	0.868	2.206	0.000	2.000

Problem 2081	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	54	279	0	0	41	0	-1
normalized size	1	1.00	0.19	1.00	0.00	0.00	0.15	0.00	-0.00
time (sec)	N/A	0.164	0.014	0.026	0.000	1.102	1.895	0.000	0.000

Problem 2082	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	75	70	68	126	46	53	38
normalized size	1	1.00	1.50	1.40	1.36	2.52	0.92	1.06	0.76
time (sec)	N/A	0.026	0.043	0.016	1.965	1.135	2.758	0.200	1.575

Problem 2083	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	29	17	21	19	23	17
normalized size	1	1.00	1.00	1.38	0.81	1.00	0.90	1.10	0.81
time (sec)	N/A	0.005	0.010	0.006	0.868	1.333	0.870	0.186	1.320

Problem 2084	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	55	49	37	80	20	34	19
normalized size	1	1.00	2.04	1.81	1.37	2.96	0.74	1.26	0.70
time (sec)	N/A	0.018	0.029	0.008	1.965	1.121	1.453	0.210	1.299

Problem 2085	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	52	52	45	77	22	41	-1
normalized size	1	1.00	1.73	1.73	1.50	2.57	0.73	1.37	-0.03
time (sec)	N/A	0.024	0.030	0.012	1.969	1.047	1.633	0.173	0.000

Problem 2086	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	64	124	0	0	42	0	-1
normalized size	1	1.00	0.58	1.13	0.00	0.00	0.38	0.00	-0.01
time (sec)	N/A	0.045	0.020	0.012	0.000	0.857	1.301	0.000	0.000

Problem 2087	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	49	113	0	0	41	0	43
normalized size	1	1.00	0.21	0.49	0.00	0.00	0.18	0.00	0.19
time (sec)	N/A	0.102	0.010	0.010	0.000	0.892	1.113	0.000	1.189

Problem 2088	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	77	86	0	0	37	0	39
normalized size	1	1.00	0.88	0.98	0.00	0.00	0.42	0.00	0.44
time (sec)	N/A	0.028	0.059	0.009	0.000	0.839	1.239	0.000	1.260

Problem 2089	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	49	199	0	0	39	0	-1
normalized size	1	1.00	0.23	0.94	0.00	0.00	0.18	0.00	-0.00
time (sec)	N/A	0.084	0.010	0.013	0.000	0.510	1.494	0.000	0.000

Problem 2090	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	71	78	79	86	192	75	64	53
normalized size	1	1.03	1.13	1.14	1.25	2.78	1.09	0.93	0.77
time (sec)	N/A	0.036	0.055	0.017	1.958	1.104	4.007	0.247	1.757

Problem 2091	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	30	38	36	42	42	42	24
normalized size	1	1.00	0.75	0.95	0.90	1.05	1.05	1.05	0.60
time (sec)	N/A	0.011	0.020	0.004	0.817	0.842	1.206	0.217	1.395

Problem 2092	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	69	67	52	163	187	51	34
normalized size	1	1.00	1.50	1.46	1.13	3.54	4.07	1.11	0.74
time (sec)	N/A	0.025	0.090	0.014	1.934	0.819	2.331	0.224	1.360

Problem 2093	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	29	17	31	22	23	26
normalized size	1	1.00	1.00	1.38	0.81	1.48	1.05	1.10	1.24
time (sec)	N/A	0.007	0.013	0.005	0.880	0.839	1.397	0.173	1.217

Problem 2094	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	67	133	0	0	42	0	-1
normalized size	1	1.00	0.51	1.02	0.00	0.00	0.32	0.00	-0.01
time (sec)	N/A	0.061	0.021	0.019	0.000	1.012	1.285	0.000	0.000

Problem 2095	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	53	187	0	0	41	0	43
normalized size	1	1.00	0.21	0.72	0.00	0.00	0.16	0.00	0.17
time (sec)	N/A	0.130	0.033	0.014	0.000	0.884	1.300	0.000	1.232

Problem 2096	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	57	113	0	0	37	0	39
normalized size	1	1.00	0.52	1.03	0.00	0.00	0.34	0.00	0.35
time (sec)	N/A	0.041	0.019	0.013	0.000	1.140	1.505	0.000	1.369

Problem 2097	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	52	187	0	0	39	0	-1
normalized size	1	1.00	0.22	0.78	0.00	0.00	0.16	0.00	-0.00
time (sec)	N/A	0.109	0.026	0.013	0.000	1.090	1.710	0.000	0.000

Problem 2098	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	92	103	282	101	260	819	78	73
normalized size	1	1.05	1.17	3.20	1.15	2.95	9.31	0.89	0.83
time (sec)	N/A	0.047	0.230	0.066	1.978	0.854	6.204	0.234	1.886

Problem 2099	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	51	50	54	65	163	53	38
normalized size	1	1.00	0.80	0.78	0.84	1.02	2.55	0.83	0.59
time (sec)	N/A	0.019	0.024	0.006	0.875	0.672	1.664	0.195	1.477

Problem 2100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	93	221	62	232	743	63	48
normalized size	1	1.00	1.45	3.45	0.97	3.62	11.61	0.98	0.75
time (sec)	N/A	0.033	0.243	0.026	1.921	0.804	3.694	0.261	1.383

Problem 2101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	40	39	33	54	105	33	36
normalized size	1	1.00	0.91	0.89	0.75	1.23	2.39	0.75	0.82
time (sec)	N/A	0.014	0.019	0.006	0.868	0.783	1.921	0.193	1.271

Problem 2102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	94	304	0	0	42	0	-1
normalized size	1	1.00	0.62	2.00	0.00	0.00	0.28	0.00	-0.01
time (sec)	N/A	0.077	0.065	0.023	0.000	0.839	1.513	0.000	0.000

Problem 2103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	81	503	0	0	41	0	43
normalized size	1	1.00	0.29	1.82	0.00	0.00	0.15	0.00	0.16
time (sec)	N/A	0.160	0.047	0.022	0.000	0.969	1.548	0.000	1.299

Problem 2104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	82	279	0	0	37	0	39
normalized size	1	1.00	0.63	2.13	0.00	0.00	0.28	0.00	0.30
time (sec)	N/A	0.056	0.060	0.022	0.000	1.144	1.850	0.000	1.436

Problem 2105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	77	503	0	0	39	0	-1
normalized size	1	1.00	0.29	1.92	0.00	0.00	0.15	0.00	-0.00
time (sec)	N/A	0.133	0.057	0.021	0.000	1.090	2.108	0.000	0.000

Problem 2106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F(-1)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	267	911	250	0	22	268	290
normalized size	1	1.00	0.86	2.94	0.81	0.00	0.07	0.86	0.94
time (sec)	N/A	0.658	0.291	0.092	2.028	0.000	0.204	0.195	1.626

Problem 2107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	59	0	37	102	24	0	19
normalized size	1	1.00	2.19	0.00	1.37	3.78	0.89	0.00	0.70
time (sec)	N/A	0.019	0.022	0.066	1.928	2.136	1.442	0.000	1.350

Problem 2108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	65	0	21	111	60	0	21
normalized size	1	1.00	2.24	0.00	0.72	3.83	2.07	0.00	0.72
time (sec)	N/A	0.020	0.037	0.062	1.958	2.777	1.577	0.000	1.308

Problem 2109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	182	164	194	314	22	180	227
normalized size	1	1.00	0.83	0.75	0.88	1.43	0.10	0.82	1.03
time (sec)	N/A	0.436	0.080	0.095	1.767	1.299	0.211	0.172	1.234

Problem 2110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	367	34	0	385	22	442	115
normalized size	1	1.00	1.35	0.12	0.00	1.42	0.08	1.62	0.42
time (sec)	N/A	0.271	0.342	0.019	0.000	1.005	0.222	0.213	1.209

Problem 2111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	166	13	15	13	13
normalized size	1	1.00	1.00	0.74	8.74	0.68	0.79	0.68	0.68
time (sec)	N/A	0.005	0.007	0.003	0.784	1.168	1.475	0.151	0.028

Problem 2112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	132	13	15	13	13
normalized size	1	1.00	1.00	0.74	6.95	0.68	0.79	0.68	0.68
time (sec)	N/A	0.005	0.006	0.002	0.920	1.236	1.410	0.153	0.026

Problem 2113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	98	13	15	13	13
normalized size	1	1.00	1.00	0.74	5.16	0.68	0.79	0.68	0.68
time (sec)	N/A	0.005	0.006	0.002	0.894	0.679	1.355	0.149	0.024

Problem 2114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	64	13	15	13	13
normalized size	1	1.00	1.00	0.74	3.37	0.68	0.79	0.68	0.68
time (sec)	N/A	0.005	0.006	0.003	0.922	1.076	1.189	0.146	0.024

Problem 2115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	12	10	10
normalized size	1	1.00	1.00	0.79	0.71	0.71	0.86	0.71	0.71
time (sec)	N/A	0.002	0.001	0.001	0.864	1.201	0.063	0.156	0.025

Problem 2116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	14	12	12	11
normalized size	1	1.00	1.00	0.92	0.85	1.08	0.92	0.92	0.85
time (sec)	N/A	0.005	0.005	0.001	0.917	1.066	0.184	0.181	0.052

Problem 2117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	12	13	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.80	0.87	0.87
time (sec)	N/A	0.005	0.006	0.000	0.788	1.011	0.438	0.152	0.036

Problem 2118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	15	15	17	15	13
normalized size	1	1.00	1.00	0.74	0.79	0.79	0.89	0.79	0.68
time (sec)	N/A	0.005	0.007	0.001	0.838	0.976	0.665	0.166	0.027

Problem 2119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	15	15	17	15	15
normalized size	1	1.00	1.00	0.74	0.79	0.79	0.89	0.79	0.79
time (sec)	N/A	0.005	0.007	0.000	0.880	1.099	0.973	0.159	0.029

Problem 2120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	28	25	166	24	27	24	24
normalized size	1	1.00	0.88	0.78	5.19	0.75	0.84	0.75	0.75
time (sec)	N/A	0.026	0.022	0.002	0.851	0.871	1.459	0.151	1.104

Problem 2121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	25	132	24	27	24	24
normalized size	1	1.00	1.00	0.78	4.12	0.75	0.84	0.75	0.75
time (sec)	N/A	0.023	0.018	0.002	0.866	1.011	0.900	0.154	0.034

Problem 2122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	28	25	98	24	27	24	24
normalized size	1	1.00	0.88	0.78	3.06	0.75	0.84	0.75	0.75
time (sec)	N/A	0.021	0.017	0.002	0.812	0.777	0.495	0.150	1.087

Problem 2123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	28	25	64	24	27	24	24
normalized size	1	1.00	0.88	0.78	2.00	0.75	0.84	0.75	0.75
time (sec)	N/A	0.018	0.014	0.001	0.895	1.079	0.303	0.157	0.034

Problem 2124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	22	21	21	24	21	21
normalized size	1	1.00	1.00	0.81	0.78	0.78	0.89	0.78	0.78
time (sec)	N/A	0.016	0.013	0.001	0.893	1.468	0.179	0.150	0.032

Problem 2125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	20	19	22	20	20	22
normalized size	1	1.00	1.00	0.95	0.90	1.05	0.95	0.95	1.05
time (sec)	N/A	0.012	0.035	0.002	0.875	0.996	0.309	0.153	1.089

Problem 2126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	23	27	20	24	26
normalized size	1	1.00	1.00	0.96	0.96	1.12	0.83	1.00	1.08
time (sec)	N/A	0.013	0.017	0.002	0.975	1.236	0.449	0.155	1.082

Problem 2127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.014	0.014	0.003	0.812	1.140	0.701	0.155	0.025

Problem 2128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	28	25	24	24	29	24	24
normalized size	1	1.00	0.88	0.78	0.75	0.75	0.91	0.75	0.75
time (sec)	N/A	0.014	0.014	0.000	0.841	0.842	1.056	0.156	0.026

Problem 2129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	28	25	24	24	29	24	24
normalized size	1	1.00	0.88	0.78	0.75	0.75	0.91	0.75	0.75
time (sec)	N/A	0.014	0.039	0.002	0.855	0.903	1.356	0.160	0.027

Problem 2130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	36	166	41	42	35	35
normalized size	1	1.00	1.00	0.77	3.53	0.87	0.89	0.74	0.74
time (sec)	N/A	0.030	0.049	0.002	0.917	0.925	2.092	0.152	0.042

Problem 2131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	36	132	41	44	35	35
normalized size	1	1.00	1.00	0.77	2.81	0.87	0.94	0.74	0.74
time (sec)	N/A	0.028	0.029	0.001	0.941	1.001	1.919	0.163	0.041

Problem 2132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	36	98	41	44	35	35
normalized size	1	1.00	1.00	0.77	2.09	0.87	0.94	0.74	0.74
time (sec)	N/A	0.028	0.019	0.002	0.927	0.689	1.942	0.185	0.042

Problem 2133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	35	64	40	41	34	34
normalized size	1	1.00	1.00	0.80	1.45	0.91	0.93	0.77	0.77
time (sec)	N/A	0.025	0.018	0.002	0.841	1.151	1.859	0.164	0.044

Problem 2134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	33	32	35	39	32	32
normalized size	1	1.00	0.74	0.87	0.84	0.92	1.03	0.84	0.84
time (sec)	N/A	0.015	0.018	0.002	0.915	0.683	1.747	0.149	0.039

Problem 2135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	32	31	34	37	32	34
normalized size	1	1.00	1.00	0.86	0.84	0.92	1.00	0.86	0.92
time (sec)	N/A	0.018	0.014	0.002	0.885	1.007	0.263	0.156	0.037

Problem 2136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	35	35	38	36	36	37
normalized size	1	1.00	1.00	0.92	0.92	1.00	0.95	0.95	0.97
time (sec)	N/A	0.020	0.024	0.002	0.888	0.943	0.487	0.155	1.097

Problem 2137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	36	33	32	39	33	33
normalized size	1	1.00	1.00	1.71	1.57	1.52	1.86	1.57	1.57
time (sec)	N/A	0.003	0.004	0.001	0.812	1.076	0.686	0.152	0.031

Problem 2138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	41	36	35	36	46	35	35
normalized size	1	1.00	0.87	0.77	0.74	0.77	0.98	0.74	0.74
time (sec)	N/A	0.020	0.018	0.001	0.836	1.046	1.105	0.151	1.072

Problem 2139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	41	36	35	36	42	35	35
normalized size	1	1.00	0.91	0.80	0.78	0.80	0.93	0.78	0.78
time (sec)	N/A	0.020	0.018	0.003	0.796	1.021	1.549	0.154	0.030

Problem 2140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	41	36	35	36	46	35	35
normalized size	1	1.00	0.87	0.77	0.74	0.77	0.98	0.74	0.74
time (sec)	N/A	0.020	0.019	0.001	0.904	0.881	2.223	0.160	0.033

Problem 2141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	58	166	63	73	57	57
normalized size	1	1.00	1.00	0.77	2.21	0.84	0.97	0.76	0.76
time (sec)	N/A	0.045	0.034	0.004	0.935	0.950	2.731	0.152	0.027

Problem 2142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	58	132	63	71	57	57
normalized size	1	1.00	1.00	0.79	1.81	0.86	0.97	0.78	0.78
time (sec)	N/A	0.041	0.030	0.003	0.895	1.002	2.556	0.156	0.027

Problem 2143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	57	98	62	70	56	56
normalized size	1	1.00	1.00	0.79	1.36	0.86	0.97	0.78	0.78
time (sec)	N/A	0.040	0.026	0.002	0.895	0.968	2.337	0.157	0.025

Problem 2144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	73	58	64	63	71	57	57
normalized size	1	1.00	0.91	0.72	0.80	0.79	0.89	0.71	0.71
time (sec)	N/A	0.037	0.025	0.001	0.838	1.259	2.340	0.162	0.025

Problem 2145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	55	54	58	66	54	54
normalized size	1	1.00	0.74	1.45	1.42	1.53	1.74	1.42	1.42
time (sec)	N/A	0.013	0.021	0.001	0.844	0.903	2.238	0.157	0.025

Problem 2146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	54	53	57	66	54	56
normalized size	1	1.00	1.00	0.83	0.82	0.88	1.02	0.83	0.86
time (sec)	N/A	0.031	0.021	0.003	0.875	1.186	0.457	0.180	0.030

Problem 2147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	55	55	61	61	56	57
normalized size	1	1.00	1.00	0.89	0.89	0.98	0.98	0.90	0.92
time (sec)	N/A	0.033	0.031	0.002	0.915	0.833	0.646	0.162	0.031

Problem 2148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	57	57	62	65	58	59
normalized size	1	1.00	1.00	0.86	0.86	0.94	0.98	0.88	0.89
time (sec)	N/A	0.031	0.034	0.003	0.784	0.888	0.738	0.210	0.049

Problem 2149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	58	55	56	66	55	55
normalized size	1	1.00	1.00	2.76	2.62	2.67	3.14	2.62	2.62
time (sec)	N/A	0.003	0.004	0.001	0.866	0.887	1.110	0.153	1.111

Problem 2150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	65	58	57	58	73	57	57
normalized size	1	1.00	0.93	0.83	0.81	0.83	1.04	0.81	0.81
time (sec)	N/A	0.022	0.026	0.003	0.859	0.955	1.638	0.210	1.098

Problem 2151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	65	58	57	58	75	57	57
normalized size	1	1.00	0.87	0.77	0.76	0.77	1.00	0.76	0.76
time (sec)	N/A	0.032	0.027	0.003	0.878	1.317	2.209	0.180	0.040

Problem 2152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	65	58	57	58	73	57	57
normalized size	1	1.00	0.89	0.79	0.78	0.79	1.00	0.78	0.78
time (sec)	N/A	0.032	0.028	0.002	0.876	1.023	3.214	0.164	0.043

Problem 2153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	140	113	166	118	139	112	112
normalized size	1	1.00	1.00	0.81	1.19	0.84	0.99	0.80	0.80
time (sec)	N/A	0.097	0.070	0.003	0.926	1.034	7.819	0.160	1.089

Problem 2154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	136	113	132	118	136	112	112
normalized size	1	1.00	0.84	0.70	0.81	0.73	0.84	0.69	0.69
time (sec)	N/A	0.080	0.055	0.002	0.875	0.958	6.048	0.155	0.058

Problem 2155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	74	113	98	118	139	112	112
normalized size	1	1.00	0.61	0.93	0.80	0.97	1.14	0.92	0.92
time (sec)	N/A	0.064	0.052	0.001	0.879	0.874	4.009	0.159	0.056

Problem 2156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	50	113	64	118	136	112	112
normalized size	1	1.00	0.62	1.41	0.80	1.48	1.70	1.40	1.40
time (sec)	N/A	0.045	0.039	0.002	0.846	0.782	2.756	0.170	0.057

Problem 2157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	110	30	113	133	109	109
normalized size	1	1.00	0.74	2.89	0.79	2.97	3.50	2.87	2.87
time (sec)	N/A	0.015	0.029	0.002	0.867	1.248	1.825	0.153	0.056

Problem 2158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	128	109	108	112	131	109	111
normalized size	1	1.00	1.00	0.85	0.84	0.88	1.02	0.85	0.87
time (sec)	N/A	0.066	0.044	0.004	0.849	1.089	2.121	0.163	0.061

Problem 2159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	123	110	110	117	124	111	112
normalized size	1	1.00	1.00	0.89	0.89	0.95	1.01	0.90	0.91
time (sec)	N/A	0.069	0.065	0.003	0.808	0.629	2.176	0.174	0.059

Problem 2160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	127	110	110	117	128	111	112
normalized size	1	1.00	1.00	0.87	0.87	0.92	1.01	0.87	0.88
time (sec)	N/A	0.070	0.062	0.004	0.879	0.963	1.954	0.167	0.049

Problem 2161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	124	110	110	117	128	111	112
normalized size	1	1.00	0.98	0.87	0.87	0.92	1.01	0.87	0.88
time (sec)	N/A	0.069	0.074	0.005	0.866	1.071	2.004	0.248	1.097

Problem 2162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	122	109	109	117	124	110	111
normalized size	1	1.00	1.00	0.89	0.89	0.96	1.02	0.90	0.91
time (sec)	N/A	0.067	0.087	0.004	0.887	1.042	2.012	0.163	0.054

Problem 2163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	130	111	111	117	131	112	114
normalized size	1	1.00	1.00	0.85	0.85	0.90	1.01	0.86	0.88
time (sec)	N/A	0.068	0.081	0.002	0.887	0.915	2.729	0.282	1.142

Problem 2164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	32	113	112	113	134	112	111
normalized size	1	1.00	0.70	2.46	2.43	2.46	2.91	2.43	2.41
time (sec)	N/A	0.013	0.008	0.003	0.911	1.196	3.635	0.164	1.132

Problem 2165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	54	113	112	113	138	112	112
normalized size	1	1.00	0.56	1.18	1.17	1.18	1.44	1.17	1.17
time (sec)	N/A	0.034	0.012	0.002	0.866	0.927	4.892	0.165	1.132

Problem 2166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	140	113	112	113	141	112	112
normalized size	1	1.00	0.96	0.77	0.77	0.77	0.97	0.77	0.77
time (sec)	N/A	0.057	0.030	0.003	0.936	0.807	7.272	0.176	0.088

Problem 2167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	136	113	112	113	138	112	112
normalized size	1	1.00	1.00	0.83	0.82	0.83	1.01	0.82	0.82
time (sec)	N/A	0.068	0.064	0.004	0.904	0.747	9.419	0.167	0.091

Problem 2168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	140	113	112	113	141	112	112
normalized size	1	1.00	1.00	0.81	0.80	0.81	1.01	0.80	0.80
time (sec)	N/A	0.066	0.061	0.003	0.884	0.777	12.380	0.192	1.117

Problem 2169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	211	168	200	173	214	167	167
normalized size	1	1.00	0.87	0.69	0.83	0.71	0.88	0.69	0.69
time (sec)	N/A	0.162	0.123	0.003	0.908	0.571	7.544	0.213	0.158

Problem 2170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	122	168	166	173	211	167	167
normalized size	1	1.00	0.60	0.83	0.82	0.86	1.04	0.83	0.83
time (sec)	N/A	0.112	0.098	0.003	0.926	1.134	6.728	0.172	0.161

Problem 2171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	98	168	132	173	209	167	167
normalized size	1	1.00	0.60	1.04	0.81	1.07	1.29	1.03	1.03
time (sec)	N/A	0.092	0.077	0.004	0.893	1.082	6.355	0.161	0.159

Problem 2172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	74	168	98	173	212	167	167
normalized size	1	1.00	0.61	1.38	0.80	1.42	1.74	1.37	1.37
time (sec)	N/A	0.074	0.067	0.003	0.906	0.662	6.042	0.161	0.154

Problem 2173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	50	168	64	173	204	167	167
normalized size	1	1.00	0.62	2.10	0.80	2.16	2.55	2.09	2.09
time (sec)	N/A	0.057	0.057	0.004	0.893	1.142	5.786	0.162	0.162

Problem 2174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	165	30	167	197	164	164
normalized size	1	1.00	0.74	4.34	0.79	4.39	5.18	4.32	4.32
time (sec)	N/A	0.014	0.044	0.002	0.855	1.170	5.428	0.157	0.162

Problem 2175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	205	164	163	167	211	164	166
normalized size	1	1.00	1.00	0.80	0.80	0.81	1.03	0.80	0.81
time (sec)	N/A	0.112	0.106	0.003	0.859	1.209	6.027	0.170	0.164

Problem 2176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	192	165	165	172	197	166	167
normalized size	1	1.00	1.00	0.86	0.86	0.90	1.03	0.86	0.87
time (sec)	N/A	0.117	0.131	0.003	0.870	1.184	5.807	0.189	0.142

Problem 2177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	190	165	163	172	196	164	167
normalized size	1	1.00	1.00	0.87	0.86	0.91	1.03	0.86	0.88
time (sec)	N/A	0.115	0.139	0.005	0.903	0.880	5.627	0.178	0.104

Problem 2178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	196	165	165	171	201	166	167
normalized size	1	1.00	1.00	0.84	0.84	0.87	1.03	0.85	0.85
time (sec)	N/A	0.116	0.144	0.004	0.898	1.054	5.372	0.187	1.158

Problem 2179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	194	165	165	172	199	166	167
normalized size	1	1.00	1.00	0.85	0.85	0.89	1.03	0.86	0.86
time (sec)	N/A	0.117	0.151	0.004	0.906	0.924	5.046	0.174	1.116

Problem 2180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	196	165	165	172	201	166	167
normalized size	1	1.00	1.00	0.84	0.84	0.88	1.03	0.85	0.85
time (sec)	N/A	0.114	0.159	0.004	0.905	0.819	5.038	0.175	1.118

Problem 2181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	198	167	167	172	202	168	168
normalized size	1	1.00	1.00	0.84	0.84	0.87	1.02	0.85	0.85
time (sec)	N/A	0.110	0.181	0.005	0.872	1.092	5.872	0.178	1.270

Problem 2182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	168	165	164	197	165	167
normalized size	1	1.00	1.00	8.00	7.86	7.81	9.38	7.86	7.95
time (sec)	N/A	0.003	0.007	0.005	0.958	0.919	7.555	0.189	0.199

Problem 2183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	41	168	167	168	209	167	167
normalized size	1	1.00	0.59	2.40	2.39	2.40	2.99	2.39	2.39
time (sec)	N/A	0.024	0.011	0.003	0.907	0.983	10.073	0.170	1.236

Problem 2184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	65	168	167	168	211	167	167
normalized size	1	1.00	0.54	1.40	1.39	1.40	1.76	1.39	1.39
time (sec)	N/A	0.045	0.042	0.004	0.867	1.006	13.034	0.181	0.189

Problem 2185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	89	168	167	168	214	167	167
normalized size	1	1.00	0.52	0.99	0.98	0.99	1.26	0.98	0.98
time (sec)	N/A	0.072	0.019	0.004	0.939	0.737	16.781	0.174	1.265

Problem 2186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	209	168	167	168	214	167	167
normalized size	1	1.00	0.95	0.76	0.76	0.76	0.97	0.76	0.76
time (sec)	N/A	0.108	0.065	0.004	0.828	1.044	21.119	0.176	1.269

Problem 2187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	207	168	167	168	212	167	167
normalized size	1	1.00	0.77	0.62	0.62	0.62	0.79	0.62	0.62
time (sec)	N/A	0.151	0.149	0.004	0.963	1.354	27.382	0.195	1.273

Problem 2188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	211	168	167	168	216	167	167
normalized size	1	1.00	1.00	0.80	0.79	0.80	1.02	0.79	0.79
time (sec)	N/A	0.116	0.134	0.004	0.901	0.885	34.854	0.175	0.197

Problem 2189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	211	168	167	168	216	167	167
normalized size	1	1.00	1.00	0.80	0.79	0.80	1.02	0.79	0.79
time (sec)	N/A	0.113	0.140	0.004	0.836	0.826	48.089	0.171	1.247

Problem 2190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	207	168	167	168	212	167	167
normalized size	1	1.00	1.00	0.81	0.81	0.81	1.02	0.81	0.81
time (sec)	N/A	0.111	0.139	0.004	0.865	0.965	51.039	0.176	0.197

Problem 2191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	99	88	129	88	109	89	87
normalized size	1	1.00	0.93	0.82	1.21	0.82	1.02	0.83	0.81
time (sec)	N/A	0.070	0.096	0.003	0.877	0.924	1.088	0.154	0.037

Problem 2192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	75	66	95	66	82	67	65
normalized size	1	1.00	0.95	0.84	1.20	0.84	1.04	0.85	0.82
time (sec)	N/A	0.046	0.066	0.004	0.856	0.924	0.654	0.148	0.029

Problem 2193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	44	61	43	54	45	43
normalized size	1	1.00	1.00	0.86	1.20	0.84	1.06	0.88	0.84
time (sec)	N/A	0.028	0.030	0.004	0.772	0.742	0.311	0.167	0.040

Problem 2194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	57	27	22	27	24	23
normalized size	1	1.00	1.00	2.11	1.00	0.81	1.00	0.89	0.85
time (sec)	N/A	0.014	0.035	0.012	0.925	1.059	0.206	0.185	0.039

Problem 2195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	20	37	22	17
normalized size	1	1.00	1.00	0.95	0.91	0.91	1.68	1.00	0.77
time (sec)	N/A	0.009	0.004	0.007	0.888	1.024	0.377	0.169	0.044

Problem 2196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	44	44	43	43	68	48	39
normalized size	1	1.00	0.94	0.94	0.91	0.91	1.45	1.02	0.83
time (sec)	N/A	0.029	0.032	0.008	0.927	1.129	1.784	0.192	0.062

Problem 2197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	69	66	64	69	99	69	60
normalized size	1	1.00	0.92	0.88	0.85	0.92	1.32	0.92	0.80
time (sec)	N/A	0.038	0.075	0.010	0.852	0.754	4.527	0.161	0.066

Problem 2198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	93	88	86	92	126	91	82
normalized size	1	1.00	0.90	0.85	0.83	0.89	1.22	0.88	0.80
time (sec)	N/A	0.052	0.088	0.010	0.917	1.077	10.186	0.155	1.110

Problem 2199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	102	94	129	128	272	100	99
normalized size	1	1.00	0.92	0.85	1.16	1.15	2.45	0.90	0.89
time (sec)	N/A	0.083	0.108	0.012	0.911	0.959	2.613	0.209	1.111

Problem 2200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	78	72	95	106	212	78	77
normalized size	1	1.00	0.94	0.87	1.14	1.28	2.55	0.94	0.93
time (sec)	N/A	0.060	0.063	0.007	0.892	1.082	1.261	0.213	0.035

Problem 2201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	50	49	60	82	134	52	54
normalized size	1	1.00	0.93	0.91	1.11	1.52	2.48	0.96	1.00
time (sec)	N/A	0.036	0.053	0.007	0.852	0.733	0.477	0.150	0.044

Problem 2202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	29	96	29	50	80	30	29
normalized size	1	1.00	0.88	2.91	0.88	1.52	2.42	0.91	0.88
time (sec)	N/A	0.018	0.021	0.015	0.880	0.668	0.628	0.165	1.091

Problem 2203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	33	35	34	67	151	36	32
normalized size	1	1.00	0.87	0.92	0.89	1.76	3.97	0.95	0.84
time (sec)	N/A	0.022	0.045	0.009	0.881	1.391	1.470	0.217	0.049

Problem 2204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	60	62	63	105	238	67	57
normalized size	1	1.00	0.90	0.93	0.94	1.57	3.55	1.00	0.85
time (sec)	N/A	0.042	0.127	0.010	0.912	1.272	1.823	0.214	1.140

Problem 2205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	91	84	88	133	333	90	81
normalized size	1	1.00	0.96	0.88	0.93	1.40	3.51	0.95	0.85
time (sec)	N/A	0.061	0.136	0.012	0.905	1.000	4.074	0.157	1.172

Problem 2206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	115	106	110	155	400	112	103
normalized size	1	1.00	0.93	0.86	0.89	1.26	3.25	0.91	0.84
time (sec)	N/A	0.079	0.179	0.014	0.848	0.813	5.649	0.161	0.083

Problem 2207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	107	99	128	157	413	101	108
normalized size	1	1.00	0.94	0.87	1.12	1.38	3.62	0.89	0.95
time (sec)	N/A	0.087	0.133	0.010	0.892	1.252	3.160	0.172	1.129

Problem 2208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	83	77	94	135	333	79	86
normalized size	1	1.00	0.94	0.88	1.07	1.53	3.78	0.90	0.98
time (sec)	N/A	0.063	0.083	0.009	0.854	0.905	1.457	0.157	0.038

Problem 2209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	57	57	60	101	233	53	66
normalized size	1	1.00	0.89	0.89	0.94	1.58	3.64	0.83	1.03
time (sec)	N/A	0.037	0.077	0.007	0.850	0.889	0.950	0.154	1.148

Problem 2210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	131	29	47	63	22	34
normalized size	1	1.00	1.00	8.19	1.81	2.94	3.94	1.38	2.12
time (sec)	N/A	0.005	0.025	0.025	0.759	0.627	1.150	0.186	1.116

Problem 2211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	44	48	54	115	364	48	52
normalized size	1	1.00	0.83	0.91	1.02	2.17	6.87	0.91	0.98
time (sec)	N/A	0.031	0.064	0.008	0.890	0.985	2.594	0.158	0.068

Problem 2212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	77	78	85	155	481	74	79
normalized size	1	1.00	0.91	0.92	1.00	1.82	5.66	0.87	0.93
time (sec)	N/A	0.057	0.137	0.011	0.886	0.802	4.984	0.159	1.175

Problem 2213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	104	100	110	182	612	101	102
normalized size	1	1.00	0.94	0.90	0.99	1.64	5.51	0.91	0.92
time (sec)	N/A	0.071	0.185	0.013	0.880	0.852	11.058	0.161	0.108

Problem 2214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	128	122	132	207	707	123	125
normalized size	1	1.00	0.92	0.88	0.95	1.49	5.09	0.88	0.90
time (sec)	N/A	0.099	0.220	0.013	0.966	1.249	19.335	0.178	1.216

Problem 2215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	150	134	163	245	949	121	148
normalized size	1	1.00	0.97	0.86	1.05	1.58	6.12	0.78	0.95
time (sec)	N/A	0.140	0.192	0.012	0.854	1.099	5.948	0.189	0.057

Problem 2216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	126	112	129	223	818	99	126
normalized size	1	1.00	0.96	0.85	0.98	1.70	6.24	0.76	0.96
time (sec)	N/A	0.105	0.119	0.011	0.898	0.669	3.027	0.160	1.196

Problem 2217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	100	92	95	191	687	73	106
normalized size	1	1.00	0.93	0.86	0.89	1.79	6.42	0.68	0.99
time (sec)	N/A	0.068	0.090	0.010	0.765	0.842	2.166	0.154	1.172

Problem 2218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	65	64	108	253	42	77
normalized size	1	1.00	1.00	3.10	3.05	5.14	12.05	2.00	3.67
time (sec)	N/A	0.003	0.004	0.007	0.925	1.130	1.848	0.155	1.190

Problem 2219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	200	30	96	121	22	57
normalized size	1	1.00	0.74	5.26	0.79	2.53	3.18	0.58	1.50
time (sec)	N/A	0.021	0.019	0.042	0.862	1.011	1.809	0.156	1.201

Problem 2220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	71	76	97	227	1049	69	94
normalized size	1	1.00	0.80	0.85	1.09	2.55	11.79	0.78	1.06
time (sec)	N/A	0.052	0.118	0.011	0.929	1.048	5.828	0.163	0.079

Problem 2221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	104	113	130	267	1232	101	123
normalized size	1	1.00	0.83	0.90	1.03	2.12	9.78	0.80	0.98
time (sec)	N/A	0.085	0.175	0.012	0.929	0.725	12.840	0.170	0.124

Problem 2222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	128	135	154	295	1380	119	147
normalized size	1	1.00	0.82	0.87	0.99	1.89	8.85	0.76	0.94
time (sec)	N/A	0.109	0.210	0.014	0.931	1.183	22.061	0.168	1.268

Problem 2223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	174	174	197	370	2048	143	202
normalized size	1	1.00	0.86	0.86	0.97	1.82	10.09	0.70	1.00
time (sec)	N/A	0.199	0.267	0.013	0.856	0.822	14.039	0.183	1.201

Problem 2224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	150	151	162	348	1839	119	179
normalized size	1	1.00	0.87	0.88	0.94	2.02	10.69	0.69	1.04
time (sec)	N/A	0.157	0.178	0.011	0.877	1.088	6.981	0.167	0.091

Problem 2225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	102	132	131	315	1629	95	159
normalized size	1	1.00	0.65	0.84	0.83	2.01	10.38	0.61	1.01
time (sec)	N/A	0.106	0.141	0.010	0.859	1.149	6.907	0.159	1.231

Problem 2226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	32	99	98	188	619	64	130
normalized size	1	1.00	0.74	2.30	2.28	4.37	14.40	1.49	3.02
time (sec)	N/A	0.014	0.009	0.006	0.904	1.080	6.492	0.191	0.173

Problem 2227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	50	65	64	177	410	42	110
normalized size	1	1.00	0.64	0.83	0.82	2.27	5.26	0.54	1.41
time (sec)	N/A	0.049	0.052	0.007	0.851	0.896	6.165	0.160	1.215

Problem 2228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	399	30	164	199	22	90
normalized size	1	1.00	0.74	10.50	0.79	4.32	5.24	0.58	2.37
time (sec)	N/A	0.021	0.021	0.067	0.862	0.940	5.668	0.184	1.221

Problem 2229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	106	118	163	398	2581	102	160
normalized size	1	1.00	0.74	0.83	1.14	2.78	18.05	0.71	1.12
time (sec)	N/A	0.089	0.181	0.010	1.009	1.018	11.354	0.169	1.197

Problem 2230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	139	163	196	435	2854	134	189
normalized size	1	1.00	0.76	0.89	1.07	2.36	15.51	0.73	1.03
time (sec)	N/A	0.158	0.270	0.014	0.978	1.478	20.425	0.206	1.264

Problem 2231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	163	186	220	463	3077	156	213
normalized size	1	1.00	0.75	0.86	1.01	2.13	14.18	0.72	0.98
time (sec)	N/A	0.191	0.364	0.016	1.115	1.024	30.479	0.168	1.602

Problem 2232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	15	19	17	10
normalized size	1	1.00	1.00	0.84	0.79	0.79	1.00	0.89	0.53
time (sec)	N/A	0.008	0.006	0.005	0.890	1.140	0.468	0.166	0.048

Problem 2233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	78	85	98	77	8588	191	98
normalized size	1	1.00	0.59	0.64	0.74	0.58	65.06	1.45	0.74
time (sec)	N/A	0.062	0.070	0.005	0.803	1.018	10.054	0.157	1.834

Problem 2234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	54	58	64	55	1987	134	64
normalized size	1	1.00	0.61	0.66	0.73	0.62	22.58	1.52	0.73
time (sec)	N/A	0.037	0.028	0.003	0.881	0.956	2.823	0.184	1.162

Problem 2235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	32	30	30	32	272	76	37
normalized size	1	1.00	0.76	0.71	0.71	0.76	6.48	1.81	0.88
time (sec)	N/A	0.017	0.018	0.003	0.920	1.671	1.235	0.174	1.153

Problem 2236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	32	49	88	75	41	31
normalized size	1	1.00	1.00	0.74	1.14	2.05	1.74	0.95	0.72
time (sec)	N/A	0.022	0.011	0.005	2.038	0.902	1.571	0.173	1.887

Problem 2237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	43	59	100	133	105	72	54
normalized size	1	1.00	0.56	0.77	1.30	1.73	1.36	0.94	0.70
time (sec)	N/A	0.034	0.010	0.008	2.037	1.421	4.235	0.163	1.567

Problem 2238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	43	87	166	184	170	109	91
normalized size	1	1.00	0.32	0.65	1.25	1.38	1.28	0.82	0.68
time (sec)	N/A	0.061	0.009	0.010	1.897	1.104	10.263	0.170	1.423

Problem 2239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	78	86	98	67	8356	85	98
normalized size	1	1.00	0.60	0.66	0.75	0.52	64.28	0.65	0.75
time (sec)	N/A	0.059	0.056	0.003	0.875	0.907	9.636	0.157	1.464

Problem 2240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	54	57	64	45	1872	57	64
normalized size	1	1.00	0.64	0.68	0.76	0.54	22.29	0.68	0.76
time (sec)	N/A	0.038	0.038	0.003	0.840	0.810	2.955	0.164	1.225

Problem 2241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	31	30	30	23	219	27	37
normalized size	1	1.00	0.78	0.75	0.75	0.58	5.48	0.68	0.92
time (sec)	N/A	0.017	0.029	0.005	0.809	1.183	1.187	0.161	1.195

Problem 2242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	20	37	67	24	23	19
normalized size	1	1.00	1.00	0.74	1.37	2.48	0.89	0.85	0.70
time (sec)	N/A	0.016	0.004	0.003	1.860	1.221	1.266	0.156	1.510

Problem 2243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	41	72	104	137	110	75	57
normalized size	1	1.00	0.51	0.90	1.30	1.71	1.38	0.94	0.71
time (sec)	N/A	0.035	0.008	0.006	1.924	0.961	4.860	0.167	1.509

Problem 2244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	41	124	166	184	173	109	94
normalized size	1	1.00	0.30	0.91	1.22	1.35	1.27	0.80	0.69
time (sec)	N/A	0.059	0.012	0.006	2.000	1.284	11.846	0.160	1.465

Problem 2245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	64	0	71	98	5039	231	-1
normalized size	1	1.00	0.86	0.00	0.96	1.32	68.09	3.12	-0.01
time (sec)	N/A	0.040	0.063	0.178	0.935	1.114	4.319	0.175	0.000

Problem 2246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	21	25	182	21	21
normalized size	1	1.00	1.00	0.96	0.91	1.09	7.91	0.91	0.91
time (sec)	N/A	0.005	0.005	0.003	0.866	0.806	1.013	0.158	1.330

Problem 2247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	7	7	7	7
normalized size	1	1.00	1.00	0.89	0.78	0.78	0.78	0.78	0.78
time (sec)	N/A	0.003	0.002	0.001	0.864	0.875	0.147	0.167	0.021

Problem 2248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	20	15	9	12	17	14	13
normalized size	1	1.00	1.54	1.15	0.69	0.92	1.31	1.08	1.00
time (sec)	N/A	0.002	0.002	0.002	0.827	1.034	0.154	0.154	0.030

Problem 2249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	20	9	17	20	19	19
normalized size	1	1.00	1.00	1.54	0.69	1.31	1.54	1.46	1.46
time (sec)	N/A	0.002	0.003	0.002	0.747	0.834	0.193	0.189	0.031

Problem 2250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	22	15	17	15	15
normalized size	1	1.00	1.00	0.84	1.16	0.79	0.89	0.79	0.79
time (sec)	N/A	0.011	0.009	0.004	0.871	0.959	0.154	0.174	0.048

Problem 2251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	8	8	8
normalized size	1	1.00	1.00	0.90	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.002	0.001	0.002	0.856	0.784	0.154	0.174	0.048

Problem 2252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	12	8	9	9
normalized size	1	1.00	1.00	0.91	0.82	1.09	0.73	0.82	0.82
time (sec)	N/A	0.002	0.008	0.003	0.807	0.960	0.356	0.155	0.028

Problem 2253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	20	12	9	9
normalized size	1	1.00	1.00	0.91	0.82	1.82	1.09	0.82	0.82
time (sec)	N/A	0.002	0.002	0.003	0.846	0.861	0.443	0.150	0.030

Problem 2254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	27	29	28	23	398	28	-1
normalized size	1	1.00	0.59	0.63	0.61	0.50	8.65	0.61	-0.02
time (sec)	N/A	0.012	0.009	0.006	0.855	0.932	1.389	0.153	0.000

Problem 2255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	10	9	9	31	9	9
normalized size	1	1.00	1.00	0.67	0.60	0.60	2.07	0.60	0.60
time (sec)	N/A	0.002	0.003	0.002	0.881	1.062	0.230	0.186	1.670

Problem 2256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	35	49	48	50	138	48	78
normalized size	1	1.00	0.60	0.84	0.83	0.86	2.38	0.83	1.34
time (sec)	N/A	0.030	0.008	0.010	1.984	0.887	1.210	0.160	1.125

Problem 2257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	78	0	83	260	117	106	292
normalized size	1	1.00	0.90	0.00	0.95	2.99	1.34	1.22	3.36
time (sec)	N/A	0.043	0.100	0.181	0.900	1.371	16.334	0.229	1.700

Problem 2258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	63	0	66	167	94	84	184
normalized size	1	1.00	0.90	0.00	0.94	2.39	1.34	1.20	2.63
time (sec)	N/A	0.027	0.080	0.172	0.899	1.411	15.172	0.183	1.430

Problem 2259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	42	0	45	89	63	58	103
normalized size	1	1.00	0.89	0.00	0.96	1.89	1.34	1.23	2.19
time (sec)	N/A	0.017	0.062	0.170	0.896	0.856	3.197	0.181	1.278

Problem 2260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	0	28	37	41	36	39
normalized size	1	1.00	1.00	0.00	0.93	1.23	1.37	1.20	1.30
time (sec)	N/A	0.008	0.023	0.004	0.887	0.928	2.542	0.187	1.206

Problem 2261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	36	0	0	0	82	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	2.22	0.00	-0.03
time (sec)	N/A	0.015	0.012	0.180	0.000	0.903	1.108	0.000	0.000

Problem 2262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	39	0	0	0	473	0	-1
normalized size	1	1.00	1.05	0.00	0.00	0.00	12.78	0.00	-0.03
time (sec)	N/A	0.014	0.021	0.178	0.000	0.693	2.234	0.000	0.000

Problem 2263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	63	65	0	0	0	46	0	-1
normalized size	1	1.21	1.25	0.00	0.00	0.00	0.88	0.00	-0.02
time (sec)	N/A	0.026	0.030	0.178	0.000	0.000	36.185	0.000	0.000

Problem 2264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	168	0	286	458	0	1642	590
normalized size	1	1.00	0.82	0.00	1.40	2.25	0.00	8.05	2.89
time (sec)	N/A	0.118	0.222	0.181	0.909	1.036	0.000	0.245	2.108

Problem 2265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	126	0	185	281	0	922	355
normalized size	1	1.00	0.83	0.00	1.22	1.85	0.00	6.07	2.34
time (sec)	N/A	0.081	0.131	0.181	0.972	1.003	0.000	0.191	1.698

Problem 2266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	95	0	104	148	0	410	180
normalized size	1	1.00	0.95	0.00	1.04	1.48	0.00	4.10	1.80
time (sec)	N/A	0.051	0.079	0.178	0.960	0.950	0.000	0.194	1.598

Problem 2267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	42	0	45	56	823	94	41
normalized size	1	1.00	0.88	0.00	0.94	1.17	17.15	1.96	0.85
time (sec)	N/A	0.022	0.024	0.180	0.935	1.001	1.832	0.157	1.176

Problem 2268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	0	0	0	41	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.95	0.00	-0.02
time (sec)	N/A	0.016	0.009	0.184	0.000	0.924	2.038	0.000	0.000

Problem 2269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	42	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.91	0.00	-0.02
time (sec)	N/A	0.019	0.011	0.177	0.000	0.990	5.345	0.000	0.000

Problem 2270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	8	8	24	20	8
normalized size	1	1.00	1.00	0.75	0.67	0.67	2.00	1.67	0.67
time (sec)	N/A	0.003	0.002	0.001	0.830	0.879	0.216	0.155	0.097

Problem 2271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	F(-1)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	198	75	0	184	0	41	0	-1
normalized size	1	1.42	0.54	0.00	1.32	0.00	0.29	0.00	-0.01
time (sec)	N/A	0.162	0.031	0.197	1.941	0.000	2.121	0.000	0.000

Problem 2272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	F(-1)	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	140	52	0	100	0	39	0	37
normalized size	1	1.71	0.63	0.00	1.22	0.00	0.48	0.00	0.45
time (sec)	N/A	0.095	0.010	0.178	1.887	0.000	1.130	0.000	1.206

Problem 2273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	33	0	35	27	76	0	-1
normalized size	1	1.00	0.66	0.00	0.70	0.54	1.52	0.00	-0.02
time (sec)	N/A	0.013	0.019	0.184	0.819	1.612	2.679	0.000	0.000

Problem 2274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	59	0	69	53	736	0	-1
normalized size	1	1.00	0.57	0.00	0.66	0.51	7.08	0.00	-0.01
time (sec)	N/A	0.033	0.053	0.185	0.850	1.588	14.493	0.000	0.000

Problem 2275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	83	0	103	75	1554	0	-1
normalized size	1	1.00	0.53	0.00	0.65	0.47	9.84	0.00	-0.01
time (sec)	N/A	0.061	0.045	0.178	0.833	1.334	55.104	0.000	0.000

Problem 2276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	80	0	98	72	0	89	98
normalized size	1	1.00	0.62	0.00	0.75	0.55	0.00	0.68	0.75
time (sec)	N/A	0.061	0.062	0.180	0.902	1.249	0.000	0.230	1.385

Problem 2277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	56	0	64	50	102	61	64
normalized size	1	1.00	0.65	0.00	0.74	0.58	1.19	0.71	0.74
time (sec)	N/A	0.041	0.028	0.180	0.903	1.294	39.171	0.208	1.420

Problem 2278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	31	30	30	23	49	30	29
normalized size	1	1.00	0.78	0.75	0.75	0.58	1.22	0.75	0.72
time (sec)	N/A	0.021	0.015	0.003	0.904	1.052	1.582	0.205	1.390

Problem 2279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	109	85	86	0	41	87	102
normalized size	1	1.00	1.28	1.00	1.01	0.00	0.48	1.02	1.20
time (sec)	N/A	0.054	0.086	0.009	1.853	0.000	1.408	2.782	1.547

Problem 2280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	F(-1)	C	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	41	0	158	0	42	0	205
normalized size	1	1.00	0.28	0.00	1.07	0.00	0.28	0.00	1.39
time (sec)	N/A	0.088	0.010	0.184	1.932	0.000	5.300	0.000	1.615

Problem 2281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	57	57	0	0	0	41	0	-1
normalized size	1	1.36	1.36	0.00	0.00	0.00	0.98	0.00	-0.02
time (sec)	N/A	0.034	0.013	0.184	0.000	2.953	2.800	0.000	0.000

Problem 2282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	57	57	0	0	0	41	0	-1
normalized size	1	1.36	1.36	0.00	0.00	0.00	0.98	0.00	-0.02
time (sec)	N/A	0.030	0.012	0.182	0.000	3.010	0.882	0.000	0.000

Problem 2283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	55	55	0	0	0	42	0	-1
normalized size	1	1.38	1.38	0.00	0.00	0.00	1.05	0.00	-0.02
time (sec)	N/A	0.033	0.012	0.184	0.000	2.489	1.734	0.000	0.000

Problem 2284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	0	0	0	48	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.84	0.00	-0.02
time (sec)	N/A	0.033	0.012	0.183	0.000	2.402	8.193	0.000	0.000

Problem 2285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	251	13	15	13	13
normalized size	1	1.00	1.00	0.74	13.21	0.68	0.79	0.68	0.68
time (sec)	N/A	0.005	0.007	0.002	0.824	0.488	1.746	0.153	0.028

Problem 2286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	200	13	15	13	13
normalized size	1	1.00	1.00	0.74	10.53	0.68	0.79	0.68	0.68
time (sec)	N/A	0.005	0.006	0.003	0.936	0.670	1.519	0.148	0.026

Problem 2287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	149	13	15	13	13
normalized size	1	1.00	1.00	0.74	7.84	0.68	0.79	0.68	0.68
time (sec)	N/A	0.005	0.007	0.001	0.899	0.520	1.349	0.151	0.025

Problem 2288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	98	13	15	13	13
normalized size	1	1.00	1.00	0.74	5.16	0.68	0.79	0.68	0.68
time (sec)	N/A	0.005	0.006	0.002	0.823	0.702	1.242	0.167	0.025

Problem 2289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	12	10	10
normalized size	1	1.00	1.00	0.79	0.71	0.71	0.86	0.71	0.71
time (sec)	N/A	0.002	0.002	0.002	0.871	0.616	0.060	0.159	0.025

Problem 2290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	14	12	12	11
normalized size	1	1.00	1.00	0.92	0.85	1.08	0.92	0.92	0.85
time (sec)	N/A	0.005	0.005	0.002	0.896	0.682	0.270	0.152	1.101

Problem 2291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	15	15	14	15	13
normalized size	1	1.00	1.00	0.82	0.88	0.88	0.82	0.88	0.76
time (sec)	N/A	0.005	0.007	0.003	0.905	0.616	0.727	0.192	0.026

Problem 2292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	15	15	17	15	15
normalized size	1	1.00	1.00	0.74	0.79	0.79	0.89	0.79	0.79
time (sec)	N/A	0.005	0.022	0.006	0.837	0.735	1.271	0.194	0.026

Problem 2293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	15	15	17	15	15
normalized size	1	1.00	1.00	0.74	0.79	0.79	0.89	0.79	0.79
time (sec)	N/A	0.005	0.007	0.004	0.871	0.659	2.148	0.165	0.028

Problem 2294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	25	250	24	31	24	24
normalized size	1	1.00	1.00	0.74	7.35	0.71	0.91	0.71	0.71
time (sec)	N/A	0.030	0.036	0.002	0.916	0.680	2.025	0.200	0.041

Problem 2295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	25	200	24	31	24	24
normalized size	1	1.00	1.00	0.74	5.88	0.71	0.91	0.71	0.71
time (sec)	N/A	0.029	0.034	0.001	0.919	0.522	1.728	0.152	0.036

Problem 2296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	25	148	24	31	24	24
normalized size	1	1.00	1.00	0.74	4.35	0.71	0.91	0.71	0.71
time (sec)	N/A	0.025	0.035	0.002	0.893	0.542	1.607	0.153	0.039

Problem 2297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	25	98	24	31	24	24
normalized size	1	1.00	1.00	0.74	2.88	0.71	0.91	0.71	0.71
time (sec)	N/A	0.020	0.031	0.000	0.921	0.623	1.513	0.158	0.035

Problem 2298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	22	21	21	27	21	21
normalized size	1	1.00	1.00	0.76	0.72	0.72	0.93	0.72	0.72
time (sec)	N/A	0.016	0.018	0.002	0.881	0.549	1.386	0.151	1.122

Problem 2299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	22	25	27	23	25
normalized size	1	1.00	1.00	0.82	0.79	0.89	0.96	0.82	0.89
time (sec)	N/A	0.011	0.011	0.004	0.891	0.642	0.237	0.169	1.111

Problem 2300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	25	24	24	26	24	24
normalized size	1	1.00	1.00	1.32	1.26	1.26	1.37	1.26	1.26
time (sec)	N/A	0.003	0.004	0.005	0.932	0.578	0.629	0.173	0.034

Problem 2301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	25	26	26	32	26	26
normalized size	1	1.00	1.00	0.74	0.76	0.76	0.94	0.76	0.76
time (sec)	N/A	0.015	0.013	0.006	0.917	0.520	1.071	0.175	0.029

Problem 2302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	25	26	26	32	26	26
normalized size	1	1.00	1.00	0.74	0.76	0.76	0.94	0.76	0.76
time (sec)	N/A	0.014	0.014	0.007	0.917	0.650	1.881	0.154	0.028

Problem 2303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	36	251	35	42	35	35
normalized size	1	1.00	1.00	0.77	5.34	0.74	0.89	0.74	0.74
time (sec)	N/A	0.035	0.032	0.000	0.849	0.553	2.325	0.151	0.042

Problem 2304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	36	200	35	42	35	35
normalized size	1	1.00	1.00	0.77	4.26	0.74	0.89	0.74	0.74
time (sec)	N/A	0.033	0.024	0.003	0.902	0.567	2.095	0.160	0.041

Problem 2305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	41	36	149	35	42	35	35
normalized size	1	1.00	0.87	0.77	3.17	0.74	0.89	0.74	0.74
time (sec)	N/A	0.030	0.025	0.002	0.903	0.675	1.877	0.163	0.043

Problem 2306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	36	98	35	42	35	35
normalized size	1	1.00	1.00	0.77	2.09	0.74	0.89	0.74	0.74
time (sec)	N/A	0.026	0.020	0.001	0.902	0.537	1.729	0.161	0.043

Problem 2307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	33	32	32	39	32	32
normalized size	1	1.00	1.00	0.79	0.76	0.76	0.93	0.76	0.76
time (sec)	N/A	0.022	0.018	0.000	0.817	0.769	1.598	0.158	0.041

Problem 2308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	31	30	33	36	31	33
normalized size	1	1.00	1.00	0.86	0.83	0.92	1.00	0.86	0.92
time (sec)	N/A	0.017	0.013	0.003	0.882	0.448	0.251	0.153	0.035

Problem 2309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	34	36	39	36	37	37
normalized size	1	1.00	1.00	0.87	0.92	1.00	0.92	0.95	0.95
time (sec)	N/A	0.019	0.026	0.007	0.901	0.644	0.653	0.186	0.037

Problem 2310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	41	36	35	35	41	35	35
normalized size	1	1.00	0.91	0.80	0.78	0.78	0.91	0.78	0.78
time (sec)	N/A	0.019	0.017	0.006	0.863	0.580	1.163	0.208	1.105

Problem 2311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	41	36	35	35	44	35	35
normalized size	1	1.00	0.87	0.77	0.74	0.74	0.94	0.74	0.74
time (sec)	N/A	0.021	0.018	0.006	0.808	0.708	1.951	0.157	1.106

Problem 2312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	58	251	69	75	57	57
normalized size	1	1.00	1.00	0.75	3.26	0.90	0.97	0.74	0.74
time (sec)	N/A	0.051	0.056	0.002	0.884	0.519	2.900	0.153	0.027

Problem 2313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	58	200	69	73	57	57
normalized size	1	1.00	1.00	0.77	2.67	0.92	0.97	0.76	0.76
time (sec)	N/A	0.045	0.052	0.002	0.886	0.556	2.613	0.167	0.026

Problem 2314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	58	149	69	75	57	57
normalized size	1	1.00	1.00	0.75	1.94	0.90	0.97	0.74	0.74
time (sec)	N/A	0.045	0.046	0.001	0.892	0.676	2.384	0.162	0.025

Problem 2315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	58	98	69	75	57	57
normalized size	1	1.00	1.00	0.75	1.27	0.90	0.97	0.74	0.74
time (sec)	N/A	0.041	0.044	0.002	0.803	0.515	2.243	0.164	0.025

Problem 2316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	41	55	54	61	68	54	54
normalized size	1	1.00	0.69	0.93	0.92	1.03	1.15	0.92	0.92
time (sec)	N/A	0.039	0.024	0.003	0.879	0.604	2.082	0.169	0.026

Problem 2317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	54	53	56	66	54	56
normalized size	1	1.00	1.00	0.83	0.82	0.86	1.02	0.83	0.86
time (sec)	N/A	0.029	0.047	0.005	0.912	0.513	29.951	0.153	0.030

Problem 2318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	57	59	62	66	60	59
normalized size	1	1.00	1.00	0.84	0.87	0.91	0.97	0.88	0.87
time (sec)	N/A	0.032	0.027	0.007	0.882	0.693	0.713	0.170	0.041

Problem 2319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	58	55	57	70	55	55
normalized size	1	1.00	1.00	2.76	2.62	2.71	3.33	2.62	2.62
time (sec)	N/A	0.003	0.004	0.007	0.927	0.699	1.239	0.195	1.122

Problem 2320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	67	58	57	59	73	57	57
normalized size	1	1.00	0.92	0.79	0.78	0.81	1.00	0.78	0.78
time (sec)	N/A	0.033	0.027	0.005	0.939	0.680	2.148	0.160	0.042

Problem 2321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	67	58	57	58	75	57	57
normalized size	1	1.00	0.89	0.77	0.76	0.77	1.00	0.76	0.76
time (sec)	N/A	0.033	0.031	0.007	0.871	0.835	3.531	0.159	1.128

Problem 2322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	67	58	57	59	76	57	57
normalized size	1	1.00	0.87	0.75	0.74	0.77	0.99	0.74	0.74
time (sec)	N/A	0.034	0.034	0.007	0.820	0.667	5.349	0.158	0.044

Problem 2323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	67	58	57	59	75	57	57
normalized size	1	1.00	0.89	0.77	0.76	0.79	1.00	0.76	0.76
time (sec)	N/A	0.035	0.030	0.006	0.879	0.828	8.509	0.156	0.044

Problem 2324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	144	113	251	124	144	112	112
normalized size	1	1.00	1.00	0.78	1.74	0.86	1.00	0.78	0.78
time (sec)	N/A	0.095	0.107	0.003	0.925	0.814	4.665	0.160	1.153

Problem 2325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	144	113	200	124	144	112	112
normalized size	1	1.00	1.00	0.78	1.39	0.86	1.00	0.78	0.78
time (sec)	N/A	0.085	0.081	0.001	0.900	0.660	4.203	0.161	0.060

Problem 2326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	140	113	149	124	141	112	112
normalized size	1	1.00	0.78	0.63	0.83	0.69	0.79	0.63	0.63
time (sec)	N/A	0.092	0.086	0.002	0.905	0.834	3.874	0.159	0.061

Problem 2327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	76	113	98	124	143	112	112
normalized size	1	1.00	0.63	0.94	0.82	1.03	1.19	0.93	0.93
time (sec)	N/A	0.065	0.085	0.002	0.987	0.890	3.563	0.188	0.059

Problem 2328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	41	110	47	117	136	109	109
normalized size	1	1.00	0.69	1.86	0.80	1.98	2.31	1.85	1.85
time (sec)	N/A	0.038	0.064	0.002	0.831	0.833	3.417	0.156	0.059

Problem 2329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	136	109	108	113	139	109	111
normalized size	1	1.00	1.00	0.80	0.79	0.83	1.02	0.80	0.82
time (sec)	N/A	0.066	0.083	0.004	0.902	0.934	66.359	0.188	0.064

Problem 2330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	125	110	110	116	131	111	112
normalized size	1	1.00	1.00	0.88	0.88	0.93	1.05	0.89	0.90
time (sec)	N/A	0.071	0.077	0.006	0.928	0.825	15.710	0.163	0.052

Problem 2331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	131	110	110	117	136	111	112
normalized size	1	1.00	1.00	0.84	0.84	0.89	1.04	0.85	0.85
time (sec)	N/A	0.072	0.060	0.008	0.907	0.785	15.849	0.196	1.130

Problem 2332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	131	112	112	118	133	113	113
normalized size	1	1.00	1.00	0.85	0.85	0.90	1.02	0.86	0.86
time (sec)	N/A	0.071	0.082	0.010	0.919	0.869	2.419	0.168	1.165

Problem 2333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	32	113	112	114	139	112	112
normalized size	1	1.00	0.70	2.46	2.43	2.48	3.02	2.43	2.43
time (sec)	N/A	0.014	0.009	0.010	0.909	0.753	3.876	0.181	1.177

Problem 2334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	67	113	112	114	143	112	112
normalized size	1	1.00	0.55	0.93	0.92	0.93	1.17	0.92	0.92
time (sec)	N/A	0.047	0.014	0.007	0.908	0.763	6.119	0.190	1.168

Problem 2335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	144	113	112	114	146	112	112
normalized size	1	1.00	1.00	0.78	0.78	0.79	1.01	0.78	0.78
time (sec)	N/A	0.074	0.068	0.007	0.853	0.714	9.502	0.174	1.165

Problem 2336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	144	113	112	114	146	112	112
normalized size	1	1.00	1.00	0.78	0.78	0.79	1.01	0.78	0.78
time (sec)	N/A	0.068	0.067	0.008	0.911	0.943	13.721	0.172	0.089

Problem 2337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	144	113	112	114	146	112	112
normalized size	1	1.00	1.00	0.78	0.78	0.79	1.01	0.78	0.78
time (sec)	N/A	0.071	0.064	0.007	0.939	0.750	19.826	0.167	0.094

Problem 2338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	142	113	112	114	144	112	112
normalized size	1	1.00	1.00	0.80	0.79	0.80	1.01	0.79	0.79
time (sec)	N/A	0.069	0.064	0.009	0.842	0.932	27.836	0.198	1.194

Problem 2339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	217	168	302	179	218	167	167
normalized size	1	1.00	1.00	0.77	1.39	0.82	1.00	0.77	0.77
time (sec)	N/A	0.172	0.126	0.003	0.935	0.798	7.376	0.167	0.159

Problem 2340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	217	168	251	179	218	167	167
normalized size	1	1.00	1.00	0.77	1.16	0.82	1.00	0.77	0.77
time (sec)	N/A	0.133	0.104	0.003	0.915	0.816	6.796	0.463	0.155

Problem 2341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	215	168	200	179	216	167	167
normalized size	1	1.00	0.88	0.69	0.82	0.73	0.89	0.68	0.68
time (sec)	N/A	0.136	0.091	0.002	0.878	0.882	6.212	0.189	0.156

Problem 2342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	113	168	149	179	214	167	167
normalized size	1	1.00	0.62	0.92	0.81	0.98	1.17	0.91	0.91
time (sec)	N/A	0.101	0.080	0.004	0.918	0.541	5.631	0.174	0.160

Problem 2343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	76	168	98	179	214	167	167
normalized size	1	1.00	0.62	1.38	0.80	1.47	1.75	1.37	1.37
time (sec)	N/A	0.075	0.065	0.002	0.877	0.766	5.100	0.178	0.157

Problem 2344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	41	165	47	172	207	164	164
normalized size	1	1.00	0.69	2.80	0.80	2.92	3.51	2.78	2.78
time (sec)	N/A	0.047	0.045	0.006	0.852	0.550	4.785	0.174	0.153

Problem 2345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	209	164	163	168	212	164	166
normalized size	1	1.00	1.00	0.78	0.78	0.80	1.01	0.78	0.79
time (sec)	N/A	0.105	0.073	0.006	0.880	0.938	4.720	0.213	0.159

Problem 2346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	202	165	167	173	204	168	167
normalized size	1	1.00	1.00	0.82	0.83	0.86	1.01	0.83	0.83
time (sec)	N/A	0.118	0.104	0.010	0.907	0.837	4.852	0.181	1.193

Problem 2347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	200	165	165	173	202	166	167
normalized size	1	1.00	1.00	0.82	0.82	0.86	1.01	0.83	0.84
time (sec)	N/A	0.116	0.109	0.010	0.911	1.059	4.632	0.189	0.080

Problem 2348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	200	165	165	173	202	166	167
normalized size	1	1.00	1.00	0.82	0.82	0.86	1.01	0.83	0.84
time (sec)	N/A	0.122	0.114	0.010	0.900	0.765	4.614	0.203	1.179

Problem 2349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	211	166	166	173	212	167	168
normalized size	1	1.00	1.00	0.79	0.79	0.82	1.00	0.79	0.80
time (sec)	N/A	0.116	0.138	0.013	0.895	0.748	7.895	0.184	1.280

Problem 2350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	45	168	167	169	209	167	166
normalized size	1	1.00	0.62	2.33	2.32	2.35	2.90	2.32	2.31
time (sec)	N/A	0.023	0.011	0.008	0.918	0.823	10.485	0.180	0.193

Problem 2351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	80	168	167	169	216	167	167
normalized size	1	1.00	0.54	1.14	1.13	1.14	1.46	1.13	1.13
time (sec)	N/A	0.061	0.017	0.007	0.868	0.723	15.179	0.183	1.343

Problem 2352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	213	168	167	169	216	167	167
normalized size	1	1.00	0.95	0.75	0.75	0.75	0.96	0.75	0.75
time (sec)	N/A	0.111	0.046	0.008	0.947	0.827	21.797	0.183	0.197

Problem 2353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	215	168	167	169	218	167	167
normalized size	1	1.00	1.00	0.78	0.78	0.79	1.01	0.78	0.78
time (sec)	N/A	0.113	0.113	0.009	0.788	0.623	30.918	0.181	0.195

Problem 2354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	217	168	167	169	219	167	167
normalized size	1	1.00	1.00	0.77	0.77	0.78	1.01	0.77	0.77
time (sec)	N/A	0.112	0.103	0.007	0.950	0.789	42.990	0.179	0.188

Problem 2355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	217	168	167	169	219	167	167
normalized size	1	1.00	1.00	0.77	0.77	0.78	1.01	0.77	0.77
time (sec)	N/A	0.118	0.104	0.009	0.884	0.860	57.156	0.203	1.276

Problem 2356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	166	131	197	133	0	133	130
normalized size	1	1.00	1.00	0.79	1.19	0.80	0.00	0.80	0.78
time (sec)	N/A	0.109	0.137	0.005	0.920	0.815	0.000	0.186	0.071

Problem 2357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	124	99	146	100	122	100	98
normalized size	1	1.00	1.00	0.80	1.18	0.81	0.98	0.81	0.79
time (sec)	N/A	0.070	0.080	0.005	0.879	0.794	103.585	0.161	0.041

Problem 2358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	77	65	95	66	80	67	64
normalized size	1	1.00	0.96	0.81	1.19	0.82	1.00	0.84	0.80
time (sec)	N/A	0.046	0.038	0.004	0.813	0.635	8.889	0.208	0.031

Problem 2359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	79	44	33	42	35	34
normalized size	1	1.00	1.00	1.88	1.05	0.79	1.00	0.83	0.81
time (sec)	N/A	0.021	0.023	0.016	0.449	0.532	0.194	0.161	0.043

Problem 2360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	20	37	22	17
normalized size	1	1.00	1.00	0.95	0.91	0.91	1.68	1.00	0.77
time (sec)	N/A	0.009	0.004	0.001	0.626	0.734	0.627	0.203	1.210

Problem 2361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	56	56	56	83	61	50
normalized size	1	1.00	1.00	0.89	0.89	0.89	1.32	0.97	0.79
time (sec)	N/A	0.034	0.031	0.007	0.558	0.531	2.788	0.156	0.067

Problem 2362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	95	87	86	93	129	91	83
normalized size	1	1.00	0.91	0.84	0.83	0.89	1.24	0.88	0.80
time (sec)	N/A	0.054	0.067	0.010	0.543	0.912	8.703	0.157	0.063

Problem 2363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	138	122	120	126	172	125	116
normalized size	1	1.00	0.93	0.82	0.81	0.85	1.15	0.84	0.78
time (sec)	N/A	0.079	0.066	0.010	0.546	0.695	22.369	0.173	1.165

Problem 2364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	15	17	22	17	10
normalized size	1	1.00	1.00	0.76	0.71	0.81	1.05	0.81	0.48
time (sec)	N/A	0.008	0.005	0.002	0.546	0.665	0.627	0.157	0.047

Problem 2365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	171	138	197	181	0	144	143
normalized size	1	1.00	1.00	0.81	1.15	1.06	0.00	0.84	0.84
time (sec)	N/A	0.135	0.160	0.011	0.486	0.600	0.000	0.167	0.079

Problem 2366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	122	105	146	147	343	111	110
normalized size	1	1.00	1.00	0.86	1.20	1.20	2.81	0.91	0.90
time (sec)	N/A	0.089	0.108	0.009	0.556	0.697	56.225	0.173	0.047

Problem 2367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	80	72	95	114	243	78	77
normalized size	1	1.00	0.94	0.85	1.12	1.34	2.86	0.92	0.91
time (sec)	N/A	0.055	0.061	0.007	0.485	0.535	8.521	0.158	1.125

Problem 2368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	42	160	44	68	109	41	43
normalized size	1	1.00	0.91	3.48	0.96	1.48	2.37	0.89	0.93
time (sec)	N/A	0.026	0.039	0.046	0.531	0.595	0.450	0.152	0.036

Problem 2369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	33	35	34	70	160	36	32
normalized size	1	1.00	0.87	0.92	0.89	1.84	4.21	0.95	0.84
time (sec)	N/A	0.022	0.027	0.008	0.543	0.632	1.381	0.160	0.052

Problem 2370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	77	73	73	122	272	77	67
normalized size	1	1.00	0.96	0.91	0.91	1.52	3.40	0.96	0.84
time (sec)	N/A	0.051	0.103	0.011	0.573	0.739	5.574	0.169	1.159

Problem 2371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	117	106	110	164	405	112	103
normalized size	1	1.00	0.94	0.85	0.88	1.31	3.24	0.90	0.82
time (sec)	N/A	0.084	0.137	0.012	0.528	0.615	14.130	0.163	1.164

Problem 2372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	154	139	143	196	505	145	136
normalized size	1	1.00	0.95	0.86	0.88	1.21	3.12	0.90	0.84
time (sec)	N/A	0.115	0.230	0.014	0.531	0.602	33.659	0.187	0.090

Problem 2373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	157	144	197	225	624	145	154
normalized size	1	1.00	0.92	0.84	1.15	1.32	3.65	0.85	0.90
time (sec)	N/A	0.148	0.169	0.010	0.508	0.762	5.872	0.165	0.070

Problem 2374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	120	111	146	192	493	112	120
normalized size	1	1.00	0.90	0.83	1.09	1.43	3.68	0.84	0.90
time (sec)	N/A	0.100	0.109	0.010	0.510	0.647	2.248	0.208	0.046

Problem 2375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	83	77	94	159	362	79	87
normalized size	1	1.00	0.92	0.86	1.04	1.77	4.02	0.88	0.97
time (sec)	N/A	0.061	0.071	0.004	0.632	0.754	0.871	0.154	0.036

Problem 2376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	45	237	46	113	228	44	53
normalized size	1	1.00	0.83	4.39	0.85	2.09	4.22	0.81	0.98
time (sec)	N/A	0.030	0.040	0.076	0.539	0.554	0.666	0.164	1.139

Problem 2377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	50	49	57	129	386	49	54
normalized size	1	1.00	0.89	0.88	1.02	2.30	6.89	0.88	0.96
time (sec)	N/A	0.032	0.058	0.008	0.493	0.629	1.913	0.167	1.138

Problem 2378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	93	90	97	191	561	90	89
normalized size	1	1.00	0.90	0.87	0.94	1.85	5.45	0.87	0.86
time (sec)	N/A	0.066	0.149	0.010	0.636	0.670	4.867	0.166	1.163

Problem 2379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	130	123	132	230	706	123	125
normalized size	1	1.00	0.89	0.84	0.90	1.58	4.84	0.84	0.86
time (sec)	N/A	0.100	0.175	0.002	0.526	0.513	11.460	0.171	1.213

Problem 2380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	167	156	165	263	847	156	159
normalized size	1	1.00	0.91	0.85	0.90	1.44	4.63	0.85	0.87
time (sec)	N/A	0.140	0.284	0.015	0.559	0.668	22.572	0.166	1.286

Problem 2381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	31	29	28	21	359	28	12
normalized size	1	1.00	0.74	0.69	0.67	0.50	8.55	0.67	0.29
time (sec)	N/A	0.011	0.011	0.008	0.586	0.616	1.362	0.150	1.205

Problem 2382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	22	18	19	23	20	19	18
normalized size	1	1.00	0.96	0.78	0.83	1.00	0.87	0.83	0.78
time (sec)	N/A	0.008	0.005	0.007	1.217	0.699	1.365	0.155	0.041

Problem 2383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	28	42	25	34	27	27
normalized size	1	1.00	1.00	0.72	1.08	0.64	0.87	0.69	0.69
time (sec)	N/A	0.016	0.014	0.003	0.564	0.648	0.246	0.159	0.024

Problem 2384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	41	12	12	14	12	12
normalized size	1	1.00	1.00	2.56	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.005	0.004	0.038	1.199	0.624	0.173	0.154	1.116

Problem 2385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	8	8	10	8	8
normalized size	1	1.00	1.00	0.75	0.67	0.67	0.83	0.67	0.67
time (sec)	N/A	0.003	0.002	0.001	0.514	0.646	0.196	0.149	0.151

Problem 2386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
normalized size	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.004	0.002	0.003	1.114	0.725	0.292	0.165	0.152

Problem 2387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	7	7	24	7	7
normalized size	1	1.00	1.00	0.73	0.64	0.64	2.18	0.64	0.64
time (sec)	N/A	0.003	0.003	0.008	0.524	1.915	0.239	0.184	2.192

Problem 2388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	10	9	19	49	9	9
normalized size	1	1.00	1.00	0.67	0.60	1.27	3.27	0.60	0.60
time (sec)	N/A	0.003	0.022	0.006	0.446	2.201	1.452	0.154	2.089

Problem 2389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	76	88	125	187	88	47
normalized size	1	1.00	1.00	0.65	0.75	1.07	1.60	0.75	0.40
time (sec)	N/A	0.077	0.042	0.006	1.255	0.570	1.380	0.167	1.193

Problem 2390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	200	22	175	272	638	462	139	223
normalized size	1	1.06	0.12	0.93	1.45	3.39	2.46	0.74	1.19
time (sec)	N/A	0.351	0.007	0.033	1.188	2.757	135.827	0.468	0.120

Problem 2391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	36	91	78	63	165	59	31
normalized size	1	1.00	0.54	1.36	1.16	0.94	2.46	0.88	0.46
time (sec)	N/A	0.025	0.032	0.017	1.209	0.558	2.960	0.842	1.266

Problem 2392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	28	65	62	55	60	36	27
normalized size	1	1.00	0.56	1.30	1.24	1.10	1.20	0.72	0.54
time (sec)	N/A	0.015	0.011	0.017	0.483	0.796	3.238	0.313	1.187

Problem 2393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	F(-1)	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	52	0	109	0	46	0	37
normalized size	1	1.00	0.55	0.00	1.15	0.00	0.48	0.00	0.39
time (sec)	N/A	0.090	0.029	0.200	1.276	0.000	4.318	0.000	1.312

Problem 2394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	15	13	15	13	13
normalized size	1	1.00	1.00	0.74	0.79	0.68	0.79	0.68	0.68
time (sec)	N/A	0.006	0.009	0.002	0.467	0.959	2.269	0.157	0.032

Problem 2395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	15	13	15	13	13
normalized size	1	1.00	1.00	0.74	0.79	0.68	0.79	0.68	0.68
time (sec)	N/A	0.005	0.003	0.001	0.561	1.041	1.243	0.174	0.028

Problem 2396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	15	13	15	13	13
normalized size	1	1.00	1.00	0.74	0.79	0.68	0.79	0.68	0.68
time (sec)	N/A	0.005	0.020	0.000	0.498	0.839	0.551	0.151	0.028

Problem 2397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	15	13	15	13	13
normalized size	1	1.00	1.00	0.74	0.79	0.68	0.79	0.68	0.68
time (sec)	N/A	0.005	0.003	0.000	0.595	0.755	0.237	0.185	0.028

Problem 2398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	12	10	10
normalized size	1	1.00	1.00	0.79	0.71	0.71	0.86	0.71	0.71
time (sec)	N/A	0.002	0.002	0.000	0.661	0.867	0.083	0.148	0.028

Problem 2399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	19	12	12	14
normalized size	1	1.00	1.00	0.92	0.85	1.46	0.92	0.92	1.08
time (sec)	N/A	0.005	0.013	0.006	0.602	0.793	0.399	0.149	0.040

Problem 2400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	47	16	14	15	13
normalized size	1	1.00	1.00	0.82	2.76	0.94	0.82	0.88	0.76
time (sec)	N/A	0.005	0.008	0.006	0.522	0.675	0.737	0.174	1.131

Problem 2401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	98	16	17	15	13
normalized size	1	1.00	1.00	0.74	5.16	0.84	0.89	0.79	0.68
time (sec)	N/A	0.005	0.008	0.005	0.525	0.786	1.323	0.154	1.126

Problem 2402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	149	16	17	15	13
normalized size	1	1.00	1.00	0.74	7.84	0.84	0.89	0.79	0.68
time (sec)	N/A	0.005	0.009	0.006	0.667	0.770	2.402	0.192	1.114

Problem 2403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	25	26	24	31	24	24
normalized size	1	1.00	1.00	0.74	0.76	0.71	0.91	0.71	0.71
time (sec)	N/A	0.031	0.030	0.002	0.543	0.958	3.172	0.152	0.046

Problem 2404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	25	26	24	31	24	24
normalized size	1	1.00	1.00	0.74	0.76	0.71	0.91	0.71	0.71
time (sec)	N/A	0.028	0.012	0.002	0.591	0.770	1.787	0.148	1.139

Problem 2405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	25	26	24	31	24	24
normalized size	1	1.00	1.00	0.74	0.76	0.71	0.91	0.71	0.71
time (sec)	N/A	0.024	0.024	0.002	0.438	0.528	0.895	0.157	0.038

Problem 2406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	25	26	24	31	24	24
normalized size	1	1.00	1.00	0.74	0.76	0.71	0.91	0.71	0.71
time (sec)	N/A	0.020	0.024	0.002	0.515	0.689	0.402	0.184	0.040

Problem 2407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	25	14	21	21	24	21	21
normalized size	1	1.00	1.56	0.88	1.31	1.31	1.50	1.31	1.31
time (sec)	N/A	0.007	0.012	0.000	0.612	0.912	0.187	0.159	0.039

Problem 2408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	22	31	27	24	28
normalized size	1	1.00	1.00	0.82	0.79	1.11	0.96	0.86	1.00
time (sec)	N/A	0.015	0.019	0.007	0.575	0.678	0.510	0.149	0.038

Problem 2409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	25	47	27	29	26	24
normalized size	1	1.00	1.00	0.78	1.47	0.84	0.91	0.81	0.75
time (sec)	N/A	0.016	0.023	0.004	0.474	0.674	0.955	0.171	0.048

Problem 2410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	25	97	27	32	26	24
normalized size	1	1.00	1.00	0.74	2.85	0.79	0.94	0.76	0.71
time (sec)	N/A	0.017	0.035	0.005	0.541	0.969	1.848	0.189	1.136

Problem 2411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	25	149	27	32	26	24
normalized size	1	1.00	1.00	0.74	4.38	0.79	0.94	0.76	0.71
time (sec)	N/A	0.016	0.040	0.006	0.535	0.793	2.977	0.204	1.132

Problem 2412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	36	37	35	42	35	35
normalized size	1	1.00	1.00	0.77	0.79	0.74	0.89	0.74	0.74
time (sec)	N/A	0.037	0.032	0.002	0.695	0.681	3.961	0.160	0.050

Problem 2413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	41	36	37	35	42	35	35
normalized size	1	1.00	0.87	0.77	0.79	0.74	0.89	0.74	0.74
time (sec)	N/A	0.034	0.025	0.001	0.573	0.617	2.275	0.152	0.044

Problem 2414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	36	37	35	42	35	35
normalized size	1	1.00	1.00	0.77	0.79	0.74	0.89	0.74	0.74
time (sec)	N/A	0.030	0.020	0.002	0.567	0.892	1.218	0.194	0.045

Problem 2415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	33	37	32	39	32	32
normalized size	1	1.00	1.00	0.79	0.88	0.76	0.93	0.76	0.76
time (sec)	N/A	0.024	0.017	0.003	0.548	0.764	0.560	0.160	0.048

Problem 2416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	31	30	33	36	31	33
normalized size	1	1.00	1.00	0.86	0.83	0.92	1.00	0.86	0.92
time (sec)	N/A	0.020	0.029	0.003	0.490	1.126	0.257	0.173	0.038

Problem 2417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	34	33	39	36	37	37
normalized size	1	1.00	1.00	0.87	0.85	1.00	0.92	0.95	0.95
time (sec)	N/A	0.023	0.041	0.008	0.526	0.687	0.657	0.156	0.041

Problem 2418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	41	36	47	35	41	35	35
normalized size	1	1.00	0.91	0.80	1.04	0.78	0.91	0.78	0.78
time (sec)	N/A	0.022	0.024	0.006	0.525	0.724	1.184	0.158	1.120

Problem 2419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	41	36	98	35	44	35	35
normalized size	1	1.00	0.87	0.77	2.09	0.74	0.94	0.74	0.74
time (sec)	N/A	0.022	0.028	0.007	0.456	0.721	2.038	0.183	1.118

Problem 2420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	41	36	149	35	44	35	35
normalized size	1	1.00	0.87	0.77	3.17	0.74	0.94	0.74	0.74
time (sec)	N/A	0.021	0.034	0.005	0.489	1.142	3.470	0.155	0.037

Problem 2421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	146	109	122	111	143	111	108
normalized size	1	1.00	1.07	0.80	0.90	0.82	1.05	0.82	0.79
time (sec)	N/A	0.089	0.139	0.006	0.493	1.028	4.142	0.151	0.054

Problem 2422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	95	77	88	77	100	78	76
normalized size	1	1.00	1.01	0.82	0.94	0.82	1.06	0.83	0.81
time (sec)	N/A	0.054	0.042	0.004	0.563	0.847	1.262	0.207	0.034

Problem 2423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	50	43	54	43	58	45	42
normalized size	1	1.00	0.83	0.72	0.90	0.72	0.97	0.75	0.70
time (sec)	N/A	0.037	0.027	0.004	0.452	0.687	0.411	0.156	0.038

Problem 2424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	20	13	20	14	13
normalized size	1	1.00	1.00	0.93	1.33	0.87	1.33	0.93	0.87
time (sec)	N/A	0.006	0.003	0.003	0.496	0.893	0.631	0.156	0.038

Problem 2425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	48	44	44	47	73	49	41
normalized size	1	1.00	0.94	0.86	0.86	0.92	1.43	0.96	0.80
time (sec)	N/A	0.031	0.041	0.008	0.566	0.871	1.815	0.158	1.163

Problem 2426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	84	78	95	85	116	81	71
normalized size	1	1.00	0.90	0.84	1.02	0.91	1.25	0.87	0.76
time (sec)	N/A	0.051	0.085	0.010	0.562	1.065	5.504	0.186	1.176

Problem 2427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	121	109	146	118	158	113	105
normalized size	1	1.00	0.90	0.81	1.09	0.88	1.18	0.84	0.78
time (sec)	N/A	0.076	0.120	0.010	0.638	0.599	13.976	0.179	0.074

Problem 2428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	158	144	197	151	201	147	138
normalized size	1	1.00	0.88	0.80	1.10	0.84	1.12	0.82	0.77
time (sec)	N/A	0.094	0.196	0.013	0.538	0.897	31.456	0.160	0.076

Problem 2429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	148	127	145	169	367	133	132
normalized size	1	1.00	0.99	0.85	0.97	1.13	2.45	0.89	0.88
time (sec)	N/A	0.132	0.197	0.010	0.475	0.661	7.047	0.170	0.068

Problem 2430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	111	94	112	137	277	100	99
normalized size	1	1.00	0.98	0.83	0.99	1.21	2.45	0.88	0.88
time (sec)	N/A	0.085	0.141	0.009	0.554	0.909	2.196	0.158	1.125

Problem 2431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	63	60	76	100	165	65	65
normalized size	1	1.00	0.82	0.78	0.99	1.30	2.14	0.84	0.84
time (sec)	N/A	0.052	0.055	0.008	0.463	0.782	0.669	0.176	1.126

Problem 2432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	30	34	56	99	30	29
normalized size	1	1.00	1.00	0.91	1.03	1.70	3.00	0.91	0.88
time (sec)	N/A	0.021	0.032	0.007	0.583	0.563	1.081	0.157	0.044

Problem 2433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	46	47	44	98	211	51	49
normalized size	1	1.00	0.88	0.90	0.85	1.88	4.06	0.98	0.94
time (sec)	N/A	0.035	0.051	0.010	0.482	0.823	2.793	0.171	0.076

Problem 2434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	82	84	95	148	340	90	81
normalized size	1	1.00	0.85	0.87	0.98	1.53	3.51	0.93	0.84
time (sec)	N/A	0.065	0.102	0.013	0.646	0.861	7.658	0.164	1.168

Problem 2435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	123	116	146	179	440	123	115
normalized size	1	1.00	0.92	0.87	1.10	1.35	3.31	0.92	0.86
time (sec)	N/A	0.103	0.127	0.011	0.502	1.205	18.162	0.167	1.193

Problem 2436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	173	150	197	214	541	156	147
normalized size	1	1.00	0.95	0.82	1.08	1.17	2.96	0.85	0.80
time (sec)	N/A	0.132	0.164	0.015	0.489	0.732	39.255	0.193	0.101

Problem 2437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	157	144	167	225	624	145	154
normalized size	1	1.00	0.92	0.84	0.98	1.32	3.65	0.85	0.90
time (sec)	N/A	0.152	0.164	0.010	0.566	0.853	6.312	0.167	1.146

Problem 2438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	132	111	134	192	493	112	120
normalized size	1	1.00	0.99	0.83	1.00	1.43	3.68	0.84	0.90
time (sec)	N/A	0.106	0.156	0.010	0.497	0.846	2.233	0.157	0.050

Problem 2439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	83	77	101	159	362	79	87
normalized size	1	1.00	0.83	0.77	1.01	1.59	3.62	0.79	0.87
time (sec)	N/A	0.068	0.082	0.009	0.567	0.813	0.895	0.163	0.040

Problem 2440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	45	237	57	113	240	44	53
normalized size	1	1.00	0.83	4.39	1.06	2.09	4.44	0.81	0.98
time (sec)	N/A	0.032	0.041	0.080	0.479	0.584	1.893	0.171	1.138

Problem 2441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	50	49	46	129	406	49	54
normalized size	1	1.00	0.89	0.88	0.82	2.30	7.25	0.88	0.96
time (sec)	N/A	0.034	0.061	0.009	0.491	0.843	4.626	0.166	1.139

Problem 2442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	93	90	95	191	561	90	89
normalized size	1	1.00	0.90	0.87	0.92	1.85	5.45	0.87	0.86
time (sec)	N/A	0.068	0.156	0.012	0.513	0.918	11.105	0.195	0.098

Problem 2443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	130	123	146	230	707	123	125
normalized size	1	1.00	0.89	0.84	1.00	1.58	4.84	0.84	0.86
time (sec)	N/A	0.106	0.172	0.014	0.493	1.013	22.226	0.208	1.212

Problem 2444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	167	156	197	263	848	156	159
normalized size	1	1.00	0.91	0.85	1.08	1.44	4.63	0.85	0.87
time (sec)	N/A	0.144	0.282	0.013	0.493	0.708	40.814	0.172	1.314

Problem 2445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	35	28	40	27	34	28	27
normalized size	1	1.00	0.80	0.64	0.91	0.61	0.77	0.64	0.61
time (sec)	N/A	0.024	0.020	0.004	0.533	0.723	0.149	0.166	0.036

Problem 2446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	10	9	0	31	9	9
normalized size	1	1.00	1.00	0.67	0.60	0.00	2.07	0.60	0.60
time (sec)	N/A	0.003	0.005	0.001	0.559	0.000	1.901	0.189	1.542

Problem 2447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	13	0	0	81	13
normalized size	1	1.00	1.00	0.67	0.62	0.00	0.00	3.86	0.62
time (sec)	N/A	0.004	0.013	0.001	0.471	0.000	0.000	0.710	2.042

Problem 2448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	79	42	29	41	29	29
normalized size	1	1.00	1.00	1.76	0.93	0.64	0.91	0.64	0.64
time (sec)	N/A	0.015	0.019	0.062	0.504	1.610	4.079	0.187	0.024

Problem 2449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	10	9	0	12	9	9
normalized size	1	1.00	1.00	0.67	0.60	0.00	0.80	0.60	0.60
time (sec)	N/A	0.003	0.004	0.005	0.519	0.000	0.640	0.159	1.406

Problem 2450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	0	14	0	78	0	38
normalized size	1	1.00	1.00	0.00	0.78	0.00	4.33	0.00	2.11
time (sec)	N/A	0.003	0.019	0.204	0.562	0.000	8.204	0.000	1.286

Problem 2451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	23	19	28	51	29	20
normalized size	1	1.00	1.00	1.10	0.90	1.33	2.43	1.38	0.95
time (sec)	N/A	0.009	0.017	0.011	0.742	1.419	0.496	0.172	1.335

Problem 2452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	23	19	28	51	29	20
normalized size	1	1.00	1.00	1.10	0.90	1.33	2.43	1.38	0.95
time (sec)	N/A	0.008	0.015	0.010	0.476	1.296	0.374	0.187	1.264

Problem 2453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	23	19	28	51	29	20
normalized size	1	1.00	1.00	1.10	0.90	1.33	2.43	1.38	0.95
time (sec)	N/A	0.007	0.015	0.009	0.485	0.740	0.288	0.155	1.240

Problem 2454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	16	20	17	16	15
normalized size	1	1.00	1.00	1.06	1.00	1.25	1.06	1.00	0.94
time (sec)	N/A	0.004	0.006	0.000	0.523	0.772	0.062	0.154	1.281

Problem 2455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	19	18	15	17	0	13
normalized size	1	1.00	1.00	1.46	1.38	1.15	1.31	0.00	1.00
time (sec)	N/A	0.006	0.010	0.008	0.498	0.742	0.197	0.000	1.187

Problem 2456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	0	23	32	0	20
normalized size	1	1.00	1.00	0.95	0.00	1.05	1.45	0.00	0.91
time (sec)	N/A	0.009	0.011	0.010	0.000	0.606	0.428	0.000	1.186

Problem 2457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	20	21	0	23	60	0	20
normalized size	1	1.00	0.83	0.88	0.00	0.96	2.50	0.00	0.83
time (sec)	N/A	0.009	0.023	0.012	0.000	0.702	0.570	0.000	1.149

Problem 2458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	38	47	40	74	202	88	43
normalized size	1	1.00	0.86	1.07	0.91	1.68	4.59	2.00	0.98
time (sec)	N/A	0.018	0.039	0.015	0.613	0.603	0.968	0.185	1.313

Problem 2459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	40	48	41	78	211	91	43
normalized size	1	1.00	0.93	1.12	0.95	1.81	4.91	2.12	1.00
time (sec)	N/A	0.020	0.038	0.012	0.635	0.896	1.923	0.166	1.310

Problem 2460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	37	47	40	72	201	87	43
normalized size	1	1.00	0.84	1.07	0.91	1.64	4.57	1.98	0.98
time (sec)	N/A	0.016	0.034	0.010	0.460	0.840	0.527	0.180	1.243

Problem 2461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	34	41	38	65	182	73	36
normalized size	1	1.00	0.89	1.08	1.00	1.71	4.79	1.92	0.95
time (sec)	N/A	0.015	0.033	0.010	0.495	1.043	0.466	0.166	1.183

Problem 2462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	27	36	34	30	36	0	30
normalized size	1	1.00	0.84	1.12	1.06	0.94	1.12	0.00	0.94
time (sec)	N/A	0.014	0.019	0.003	0.514	0.832	0.279	0.000	1.175

Problem 2463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	38	43	0	68	190	0	43
normalized size	1	1.00	0.86	0.98	0.00	1.55	4.32	0.00	0.98
time (sec)	N/A	0.022	0.041	0.015	0.000	0.936	0.752	0.000	1.165

Problem 2464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	39	42	0	66	245	0	43
normalized size	1	1.00	0.78	0.84	0.00	1.32	4.90	0.00	0.86
time (sec)	N/A	0.021	0.041	0.016	0.000	0.702	0.724	0.000	1.150

Problem 2465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	58	65	61	145	507	188	66
normalized size	1	1.00	0.89	1.00	0.94	2.23	7.80	2.89	1.02
time (sec)	N/A	0.032	0.045	0.013	0.444	0.738	10.032	0.187	1.341

Problem 2466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	57	72	62	144	500	188	66
normalized size	1	1.00	0.86	1.09	0.94	2.18	7.58	2.85	1.00
time (sec)	N/A	0.027	0.039	0.018	0.529	1.661	3.276	0.263	1.322

Problem 2467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	58	71	61	145	500	188	66
normalized size	1	1.00	0.89	1.09	0.94	2.23	7.69	2.89	1.02
time (sec)	N/A	0.026	0.039	0.014	0.482	0.822	1.778	0.185	1.278

Problem 2468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	54	64	60	130	469	159	57
normalized size	1	1.00	0.90	1.07	1.00	2.17	7.82	2.65	0.95
time (sec)	N/A	0.022	0.034	0.013	0.461	0.542	0.820	0.162	1.192

Problem 2469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	41	52	48	44	53	0	46
normalized size	1	1.00	0.82	1.04	0.96	0.88	1.06	0.00	0.92
time (sec)	N/A	0.019	0.026	0.003	0.453	0.658	0.423	0.000	1.162

Problem 2470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	58	65	0	131	508	0	66
normalized size	1	1.00	0.88	0.98	0.00	1.98	7.70	0.00	1.00
time (sec)	N/A	0.032	0.054	0.017	0.000	1.058	1.287	0.000	1.171

Problem 2471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	60	65	0	134	627	0	66
normalized size	1	1.00	0.83	0.90	0.00	1.86	8.71	0.00	0.92
time (sec)	N/A	0.033	0.053	0.021	0.000	0.669	1.723	0.000	1.180

Problem 2472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	0	0	0	36	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	1.09	0.00	-0.03
time (sec)	N/A	0.008	0.003	0.226	0.000	0.778	0.835	0.000	0.000

Problem 2473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	0	0	0	32	0	25
normalized size	1	1.00	1.00	0.00	0.00	0.00	1.33	0.00	1.04
time (sec)	N/A	0.005	0.002	0.201	0.000	0.552	0.797	0.000	1.237

Problem 2474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	22	29	28	22	41	0	22
normalized size	1	1.00	0.96	1.26	1.22	0.96	1.78	0.00	0.96
time (sec)	N/A	0.011	0.007	0.002	0.507	1.933	0.740	0.000	1.162

Problem 2475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	31	0	0	0	39	0	-1
normalized size	1	1.00	0.91	0.00	0.00	0.00	1.15	0.00	-0.03
time (sec)	N/A	0.007	0.003	0.201	0.000	0.538	0.916	0.000	0.000

Problem 2476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	33	0	0	0	44	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	1.22	0.00	-0.03
time (sec)	N/A	0.007	0.003	0.241	0.000	0.624	1.003	0.000	0.000

Problem 2477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	0	0	0	274	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	8.30	0.00	-0.03
time (sec)	N/A	0.006	0.003	0.356	0.000	0.622	1.221	0.000	0.000

Problem 2478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	0	0	0	257	0	25
normalized size	1	1.00	1.00	0.00	0.00	0.00	10.71	0.00	1.04
time (sec)	N/A	0.005	0.002	0.259	0.000	0.821	1.164	0.000	1.168

Problem 2479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	33	45	43	50	160	0	39
normalized size	1	1.00	0.85	1.15	1.10	1.28	4.10	0.00	1.00
time (sec)	N/A	0.023	0.062	0.003	0.584	0.662	1.554	0.000	1.138

Problem 2480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	31	0	0	0	289	0	-1
normalized size	1	1.00	0.91	0.00	0.00	0.00	8.50	0.00	-0.03
time (sec)	N/A	0.007	0.004	0.234	0.000	0.665	1.499	0.000	0.000

Problem 2481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	33	0	0	0	321	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	8.92	0.00	-0.03
time (sec)	N/A	0.007	0.003	0.217	0.000	0.768	1.727	0.000	0.000

Problem 2482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	0	0	0	1930	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	58.48	0.00	-0.03
time (sec)	N/A	0.006	0.003	0.209	0.000	0.636	1.774	0.000	0.000

Problem 2483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	0	0	0	1953	0	25
normalized size	1	1.00	1.00	0.00	0.00	0.00	81.38	0.00	1.04
time (sec)	N/A	0.005	0.002	0.207	0.000	0.749	1.708	0.000	1.181

Problem 2484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	47	62	71	106	406	0	69
normalized size	1	1.00	0.81	1.07	1.22	1.83	7.00	0.00	1.19
time (sec)	N/A	0.032	0.123	0.001	0.509	0.649	2.798	0.000	1.153

Problem 2485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	31	0	0	0	2118	0	-1
normalized size	1	1.00	0.91	0.00	0.00	0.00	62.29	0.00	-0.03
time (sec)	N/A	0.007	0.004	0.210	0.000	0.903	2.378	0.000	0.000

Problem 2486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	33	0	0	0	2207	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	61.31	0.00	-0.03
time (sec)	N/A	0.007	0.004	0.214	0.000	0.713	2.796	0.000	0.000

Problem 2487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	57	57	0	0	0	42	0	-1
normalized size	1	1.19	1.19	0.00	0.00	0.00	0.88	0.00	-0.02
time (sec)	N/A	0.014	0.019	0.213	0.000	0.000	1.256	0.000	0.000

Problem 2488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	48	48	0	0	0	41	0	43
normalized size	1	1.23	1.23	0.00	0.00	0.00	1.05	0.00	1.10
time (sec)	N/A	0.012	0.008	0.211	0.000	0.000	1.147	0.000	1.460

Problem 2489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	43	36	54	91	76	0	-1
normalized size	1	1.00	0.96	0.80	1.20	2.02	1.69	0.00	-0.02
time (sec)	N/A	0.023	0.012	0.003	1.187	0.580	1.707	0.000	0.000

Problem 2490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	58	55	0	0	0	44	0	-1
normalized size	1	1.18	1.12	0.00	0.00	0.00	0.90	0.00	-0.02
time (sec)	N/A	0.019	0.013	0.214	0.000	0.000	1.247	0.000	0.000

Problem 2491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	60	57	0	0	0	46	0	-1
normalized size	1	1.18	1.12	0.00	0.00	0.00	0.90	0.00	-0.02
time (sec)	N/A	0.019	0.014	0.212	0.000	0.000	1.430	0.000	0.000

Problem 2492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	58	58	0	0	0	42	0	-1
normalized size	1	1.21	1.21	0.00	0.00	0.00	0.88	0.00	-0.02
time (sec)	N/A	0.015	0.030	0.208	0.000	0.000	2.399	0.000	0.000

Problem 2493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	49	49	0	0	0	41	0	43
normalized size	1	1.26	1.26	0.00	0.00	0.00	1.05	0.00	1.10
time (sec)	N/A	0.012	0.007	0.210	0.000	0.000	1.915	0.000	1.193

Problem 2494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	55	48	67	114	87	0	-1
normalized size	1	1.00	0.86	0.75	1.05	1.78	1.36	0.00	-0.02
time (sec)	N/A	0.031	0.025	0.004	1.244	1.334	3.021	0.000	0.000

Problem 2495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	59	56	0	0	0	44	0	-1
normalized size	1	1.20	1.14	0.00	0.00	0.00	0.90	0.00	-0.02
time (sec)	N/A	0.019	0.014	0.215	0.000	0.000	1.959	0.000	0.000

Problem 2496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	61	58	0	0	0	46	0	-1
normalized size	1	1.20	1.14	0.00	0.00	0.00	0.90	0.00	-0.02
time (sec)	N/A	0.020	0.012	0.218	0.000	0.000	1.989	0.000	0.000

Problem 2497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	60	60	0	0	0	42	0	-1
normalized size	1	1.25	1.25	0.00	0.00	0.00	0.88	0.00	-0.02
time (sec)	N/A	0.016	0.015	0.211	0.000	0.000	6.902	0.000	0.000

Problem 2498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	51	51	0	0	0	41	0	43
normalized size	1	1.31	1.31	0.00	0.00	0.00	1.05	0.00	1.10
time (sec)	N/A	0.011	0.008	0.208	0.000	0.000	5.244	0.000	1.228

Problem 2499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	69	62	83	144	117	0	-1
normalized size	1	1.00	0.81	0.73	0.98	1.69	1.38	0.00	-0.01
time (sec)	N/A	0.041	0.045	0.004	1.312	1.136	7.983	0.000	0.000

Problem 2500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	61	58	0	0	0	44	0	-1
normalized size	1	1.24	1.18	0.00	0.00	0.00	0.90	0.00	-0.02
time (sec)	N/A	0.021	0.037	0.211	0.000	0.000	5.344	0.000	0.000

Problem 2501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	63	60	0	0	0	46	0	-1
normalized size	1	1.24	1.18	0.00	0.00	0.00	0.90	0.00	-0.02
time (sec)	N/A	0.019	0.013	0.210	0.000	0.000	5.360	0.000	0.000

Problem 2502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	57	57	0	0	0	41	0	-1
normalized size	1	1.19	1.19	0.00	0.00	0.00	0.85	0.00	-0.02
time (sec)	N/A	0.015	0.016	0.205	0.000	0.000	1.121	0.000	0.000

Problem 2503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	48	48	0	0	0	39	0	43
normalized size	1	1.23	1.23	0.00	0.00	0.00	1.00	0.00	1.10
time (sec)	N/A	0.011	0.009	0.211	0.000	0.000	1.087	0.000	1.385

Problem 2504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	39	70	26	0	-1
normalized size	1	1.00	1.00	0.82	1.39	2.50	0.93	0.00	-0.04
time (sec)	N/A	0.019	0.005	0.005	1.126	1.152	1.327	0.000	0.000

Problem 2505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	58	55	0	0	0	42	0	-1
normalized size	1	1.18	1.12	0.00	0.00	0.00	0.86	0.00	-0.02
time (sec)	N/A	0.019	0.023	0.205	0.000	0.000	1.334	0.000	0.000

Problem 2506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	60	57	0	0	0	44	0	-1
normalized size	1	1.18	1.12	0.00	0.00	0.00	0.86	0.00	-0.02
time (sec)	N/A	0.018	0.016	0.204	0.000	0.000	1.582	0.000	0.000

Problem 2507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	60	60	0	0	0	41	0	-1
normalized size	1	1.25	1.25	0.00	0.00	0.00	0.85	0.00	-0.02
time (sec)	N/A	0.017	0.016	0.204	0.000	0.000	1.414	0.000	0.000

Problem 2508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	51	51	0	0	0	39	0	43
normalized size	1	1.31	1.31	0.00	0.00	0.00	1.00	0.00	1.10
time (sec)	N/A	0.011	0.014	0.199	0.000	0.000	1.366	0.000	1.281

Problem 2509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	37	39	57	144	184	0	-1
normalized size	1	1.00	0.77	0.81	1.19	3.00	3.83	0.00	-0.02
time (sec)	N/A	0.026	0.010	0.005	1.451	0.677	2.491	0.000	0.000

Problem 2510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	61	58	0	0	0	42	0	-1
normalized size	1	1.24	1.18	0.00	0.00	0.00	0.86	0.00	-0.02
time (sec)	N/A	0.020	0.016	0.208	0.000	0.000	1.913	0.000	0.000

Problem 2511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	63	60	0	0	0	44	0	-1
normalized size	1	1.24	1.18	0.00	0.00	0.00	0.86	0.00	-0.02
time (sec)	N/A	0.019	0.015	0.206	0.000	0.000	2.399	0.000	0.000

Problem 2512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	60	60	0	0	0	41	0	-1
normalized size	1	1.25	1.25	0.00	0.00	0.00	0.85	0.00	-0.02
time (sec)	N/A	0.017	0.016	0.206	0.000	0.000	2.453	0.000	0.000

Problem 2513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	51	51	0	0	0	39	0	43
normalized size	1	1.31	1.31	0.00	0.00	0.00	1.00	0.00	1.10
time (sec)	N/A	0.011	0.024	0.205	0.000	0.000	2.373	0.000	1.361

Problem 2514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	39	53	67	230	860	0	-1
normalized size	1	1.00	0.57	0.77	0.97	3.33	12.46	0.00	-0.01
time (sec)	N/A	0.036	0.011	0.009	1.180	1.437	5.322	0.000	0.000

Problem 2515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	61	58	0	0	0	42	0	-1
normalized size	1	1.24	1.18	0.00	0.00	0.00	0.86	0.00	-0.02
time (sec)	N/A	0.020	0.027	0.219	0.000	0.000	3.999	0.000	0.000

Problem 2516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	63	60	0	0	0	44	0	-1
normalized size	1	1.24	1.18	0.00	0.00	0.00	0.86	0.00	-0.02
time (sec)	N/A	0.020	0.030	0.201	0.000	0.000	5.226	0.000	0.000

Problem 2517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	129	107	108	106	46	0	-1
normalized size	1	1.00	1.22	1.01	1.02	1.00	0.43	0.00	-0.01
time (sec)	N/A	0.067	0.073	0.006	1.320	0.703	1.339	0.000	0.000

Problem 2518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	28	23	22	26	0	20
normalized size	1	1.00	0.81	1.04	0.85	0.81	0.96	0.00	0.74
time (sec)	N/A	0.008	0.013	0.022	0.666	0.614	4.851	0.000	1.265

Problem 2519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	28	23	22	26	0	20
normalized size	1	1.00	0.81	1.04	0.85	0.81	0.96	0.00	0.74
time (sec)	N/A	0.007	0.010	0.021	0.490	0.710	4.696	0.000	1.270

Problem 2520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	28	23	22	26	0	20
normalized size	1	1.00	0.81	1.04	0.85	0.81	0.96	0.00	0.74
time (sec)	N/A	0.007	0.010	0.017	0.517	0.500	4.745	0.000	1.256

Problem 2521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	19	25	17	19	22	19	15
normalized size	1	1.00	0.86	1.14	0.77	0.86	1.00	0.86	0.68
time (sec)	N/A	0.006	0.008	0.015	0.584	0.755	1.588	0.149	1.237

Problem 2522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	19	18	15	17	0	13
normalized size	1	1.00	1.00	1.46	1.38	1.15	1.31	0.00	1.00
time (sec)	N/A	0.006	0.008	0.000	0.505	0.596	0.203	0.000	0.002

Problem 2523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	25	16	21	107	21	29
normalized size	1	1.00	1.00	1.56	1.00	1.31	6.69	1.31	1.81
time (sec)	N/A	0.006	0.010	0.014	0.667	0.569	12.856	0.180	1.412

Problem 2524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	20	27	25	20	24	20	20
normalized size	1	1.00	0.80	1.08	1.00	0.80	0.96	0.80	0.80
time (sec)	N/A	0.007	0.011	0.018	0.607	0.648	4.946	0.179	1.207

Problem 2525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	27	27	22	27	22	22
normalized size	1	1.00	0.81	1.00	1.00	0.81	1.00	0.81	0.81
time (sec)	N/A	0.007	0.035	0.017	0.650	0.660	4.944	0.190	1.204

Problem 2526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	27	27	22	27	22	22
normalized size	1	1.00	0.81	1.00	1.00	0.81	1.00	0.81	0.81
time (sec)	N/A	0.007	0.011	0.020	0.639	0.700	4.949	0.164	1.229

Problem 2527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	27	27	22	27	22	22
normalized size	1	1.00	0.81	1.00	1.00	0.81	1.00	0.81	0.81
time (sec)	N/A	0.008	0.011	0.021	0.541	0.764	5.040	0.163	1.221

Problem 2528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	35	40	39	35	44	0	33
normalized size	1	1.00	0.78	0.89	0.87	0.78	0.98	0.00	0.73
time (sec)	N/A	0.019	0.023	0.020	0.683	0.516	11.973	0.000	1.337

Problem 2529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	35	46	39	35	42	0	33
normalized size	1	1.00	0.78	1.02	0.87	0.78	0.93	0.00	0.73
time (sec)	N/A	0.017	0.041	0.023	0.521	0.962	11.758	0.000	1.315

Problem 2530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	35	46	39	35	44	0	33
normalized size	1	1.00	0.78	1.02	0.87	0.78	0.98	0.00	0.73
time (sec)	N/A	0.017	0.019	0.020	0.665	1.078	11.698	0.000	1.290

Problem 2531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	42	17	32	37	32	27
normalized size	1	1.00	1.00	2.21	0.89	1.68	1.95	1.68	1.42
time (sec)	N/A	0.004	0.005	0.017	0.674	1.434	3.145	0.151	1.273

Problem 2532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	27	36	34	30	36	0	30
normalized size	1	1.00	0.84	1.12	1.06	0.94	1.12	0.00	0.94
time (sec)	N/A	0.013	0.023	0.000	0.565	0.790	0.285	0.000	0.002

Problem 2533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	29	43	30	34	175	34	30
normalized size	1	1.00	0.97	1.43	1.00	1.13	5.83	1.13	1.00
time (sec)	N/A	0.015	0.043	0.015	0.585	0.947	28.503	0.179	1.319

Problem 2534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	28	43	34	38	235	38	34
normalized size	1	1.00	0.82	1.26	1.00	1.12	6.91	1.12	1.00
time (sec)	N/A	0.016	0.038	0.015	0.541	0.593	57.237	0.183	1.333

Problem 2535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	45	43	33	39	33	33
normalized size	1	1.00	1.00	1.88	1.79	1.38	1.62	1.38	1.38
time (sec)	N/A	0.005	0.007	0.018	0.619	0.477	11.446	0.189	1.335

Problem 2536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	35	45	45	35	46	35	35
normalized size	1	1.00	0.78	1.00	1.00	0.78	1.02	0.78	0.78
time (sec)	N/A	0.017	0.015	0.019	0.530	0.586	11.469	0.187	1.296

Problem 2537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	35	45	45	35	44	35	35
normalized size	1	1.00	0.78	1.00	1.00	0.78	0.98	0.78	0.78
time (sec)	N/A	0.018	0.015	0.024	0.462	0.634	11.329	0.183	1.278

Problem 2538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	35	40	45	35	46	35	35
normalized size	1	1.00	0.78	0.89	1.00	0.78	1.02	0.78	0.78
time (sec)	N/A	0.017	0.017	0.023	0.475	0.783	11.684	0.191	1.293

Problem 2539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	48	56	55	48	60	0	55
normalized size	1	1.00	0.76	0.89	0.87	0.76	0.95	0.00	0.87
time (sec)	N/A	0.027	0.024	0.020	0.557	0.672	26.846	0.000	1.359

Problem 2540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	48	56	55	48	61	0	55
normalized size	1	1.00	0.76	0.89	0.87	0.76	0.97	0.00	0.87
time (sec)	N/A	0.026	0.029	0.020	0.542	0.734	26.439	0.000	1.342

Problem 2541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	27	63	54	48	58	0	54
normalized size	1	1.00	0.68	1.58	1.35	1.20	1.45	0.00	1.35
time (sec)	N/A	0.018	0.027	0.021	0.463	0.506	26.600	0.000	1.331

Problem 2542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	60	17	45	54	45	51
normalized size	1	1.00	1.00	3.16	0.89	2.37	2.84	2.37	2.68
time (sec)	N/A	0.005	0.004	0.022	0.609	0.656	5.481	0.154	1.297

Problem 2543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	41	52	48	44	53	0	46
normalized size	1	1.00	0.82	1.04	0.96	0.88	1.06	0.00	0.92
time (sec)	N/A	0.020	0.043	0.000	0.597	0.568	0.445	0.000	0.002

Problem 2544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	45	62	47	48	277	48	47
normalized size	1	1.00	0.92	1.27	0.96	0.98	5.65	0.98	0.96
time (sec)	N/A	0.022	0.031	0.017	0.687	0.689	61.805	0.191	1.402

Problem 2545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	44	61	48	51	318	51	48
normalized size	1	1.00	0.92	1.27	1.00	1.06	6.62	1.06	1.00
time (sec)	N/A	0.021	0.046	0.017	0.653	0.783	117.454	0.214	1.364

Problem 2546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	43	61	52	51	0	51	52
normalized size	1	1.00	0.83	1.17	1.00	0.98	0.00	0.98	1.00
time (sec)	N/A	0.022	0.058	0.019	0.514	0.667	0.000	0.199	1.344

Problem 2547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	63	61	46	56	46	61
normalized size	1	1.00	1.00	2.62	2.54	1.92	2.33	1.92	2.54
time (sec)	N/A	0.005	0.007	0.021	0.446	0.662	26.982	0.212	1.357

Problem 2548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	48	63	63	48	60	48	63
normalized size	1	1.00	0.96	1.26	1.26	0.96	1.20	0.96	1.26
time (sec)	N/A	0.016	0.022	0.024	0.585	0.831	26.466	0.202	1.313

Problem 2549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	48	56	63	48	63	48	63
normalized size	1	1.00	0.76	0.89	1.00	0.76	1.00	0.76	1.00
time (sec)	N/A	0.023	0.021	0.022	0.488	0.636	26.874	0.231	1.372

Problem 2550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	48	56	63	48	61	48	63
normalized size	1	1.00	0.76	0.89	1.00	0.76	0.97	0.76	1.00
time (sec)	N/A	0.024	0.039	0.022	0.543	0.654	27.126	0.212	1.365

Problem 2551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	82	87	86	74	92	0	86
normalized size	1	1.00	0.98	1.04	1.02	0.88	1.10	0.00	1.02
time (sec)	N/A	0.047	0.060	0.023	0.535	0.796	117.297	0.000	1.417

Problem 2552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	40	88	87	74	94	0	87
normalized size	1	1.00	0.65	1.42	1.40	1.19	1.52	0.00	1.40
time (sec)	N/A	0.035	0.044	0.023	0.713	0.557	117.785	0.000	1.349

Problem 2553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	27	88	87	74	94	0	87
normalized size	1	1.00	0.68	2.20	2.18	1.85	2.35	0.00	2.18
time (sec)	N/A	0.019	0.029	0.025	0.587	0.765	117.653	0.000	1.357

Problem 2554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	84	17	71	88	71	83
normalized size	1	1.00	1.00	4.42	0.89	3.74	4.63	3.74	4.37
time (sec)	N/A	0.005	0.005	0.028	0.544	0.835	15.829	0.174	1.359

Problem 2555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	74	84	74	70	85	0	78
normalized size	1	1.00	0.88	1.00	0.88	0.83	1.01	0.00	0.93
time (sec)	N/A	0.035	0.072	0.003	0.490	0.616	0.944	0.000	1.221

Problem 2556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	73	98	79	75	0	75	79
normalized size	1	1.00	0.88	1.18	0.95	0.90	0.00	0.90	0.95
time (sec)	N/A	0.037	0.057	0.026	0.566	0.750	0.000	0.283	1.520

Problem 2557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	72	98	81	77	0	77	81
normalized size	1	1.00	0.85	1.15	0.95	0.91	0.00	0.91	0.95
time (sec)	N/A	0.037	0.080	0.023	0.579	0.619	0.000	0.242	1.437

Problem 2558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	72	98	83	77	0	77	83
normalized size	1	1.00	0.85	1.15	0.98	0.91	0.00	0.91	0.98
time (sec)	N/A	0.036	0.066	0.023	0.514	0.610	0.000	0.243	1.435

Problem 2559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	72	97	84	77	0	77	84
normalized size	1	1.00	0.88	1.18	1.02	0.94	0.00	0.94	1.02
time (sec)	N/A	0.035	0.055	0.025	0.711	0.952	0.000	0.241	1.415

Problem 2560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	69	97	88	77	0	77	88
normalized size	1	1.00	0.80	1.13	1.02	0.90	0.00	0.90	1.02
time (sec)	N/A	0.037	0.084	0.024	0.594	0.573	0.000	0.248	1.428

Problem 2561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	88	97	72	90	72	97
normalized size	1	1.00	1.00	3.67	4.04	3.00	3.75	3.00	4.04
time (sec)	N/A	0.005	0.009	0.021	0.615	0.701	124.556	0.239	1.371

Problem 2562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	33	88	99	74	0	74	99
normalized size	1	1.00	0.66	1.76	1.98	1.48	0.00	1.48	1.98
time (sec)	N/A	0.017	0.025	0.024	0.515	0.522	0.000	0.241	1.341

Problem 2563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	74	88	99	74	0	74	99
normalized size	1	1.00	0.96	1.14	1.29	0.96	0.00	0.96	1.29
time (sec)	N/A	0.027	0.039	0.023	0.480	0.591	0.000	0.244	1.359

Problem 2564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	74	88	99	74	94	74	99
normalized size	1	1.00	0.76	0.91	1.02	0.76	0.97	0.76	1.02
time (sec)	N/A	0.039	0.046	0.023	0.553	0.771	124.330	0.324	1.341

Problem 2565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	74	88	99	74	97	74	99
normalized size	1	1.00	0.75	0.89	1.00	0.75	0.98	0.75	1.00
time (sec)	N/A	0.040	0.048	0.025	0.577	0.598	118.317	0.246	1.337

Problem 2566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	128	136	135	113	0	0	135
normalized size	1	1.00	0.85	0.90	0.89	0.75	0.00	0.00	0.89
time (sec)	N/A	0.078	0.093	0.028	0.575	0.522	0.000	0.000	1.500

Problem 2567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	128	136	135	113	0	0	135
normalized size	1	1.00	0.85	0.90	0.89	0.75	0.00	0.00	0.89
time (sec)	N/A	0.072	0.081	0.025	0.536	0.583	0.000	0.000	1.486

Problem 2568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	127	135	134	113	0	0	134
normalized size	1	1.00	0.85	0.90	0.89	0.75	0.00	0.00	0.89
time (sec)	N/A	0.078	0.085	0.026	0.666	0.578	0.000	0.000	1.472

Problem 2569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	79	136	135	113	0	0	135
normalized size	1	1.00	0.62	1.06	1.05	0.88	0.00	0.00	1.05
time (sec)	N/A	0.068	0.100	0.028	0.579	0.788	0.000	0.000	1.451

Problem 2570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	66	136	135	113	0	0	135
normalized size	1	1.00	0.62	1.28	1.27	1.07	0.00	0.00	1.27
time (sec)	N/A	0.058	0.061	0.026	0.590	0.616	0.000	0.000	1.456

Problem 2571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	53	136	135	113	0	0	135
normalized size	1	1.00	0.63	1.62	1.61	1.35	0.00	0.00	1.61
time (sec)	N/A	0.050	0.059	0.028	0.473	0.720	0.000	0.000	1.457

Problem 2572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	40	136	135	113	0	0	135
normalized size	1	1.00	0.65	2.19	2.18	1.82	0.00	0.00	2.18
time (sec)	N/A	0.039	0.035	0.026	0.463	0.841	0.000	0.000	1.449

Problem 2573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	27	136	135	113	0	0	135
normalized size	1	1.00	0.68	3.40	3.38	2.82	0.00	0.00	3.38
time (sec)	N/A	0.019	0.041	0.027	0.499	0.680	0.000	0.000	1.445

Problem 2574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	132	17	110	133	110	131
normalized size	1	1.00	1.00	6.95	0.89	5.79	7.00	5.79	6.89
time (sec)	N/A	0.005	0.011	0.026	0.459	0.656	64.514	0.166	1.461

Problem 2575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	119	132	113	109	136	0	126
normalized size	1	1.00	0.86	0.96	0.82	0.79	0.99	0.00	0.91
time (sec)	N/A	0.055	0.084	0.005	0.540	0.636	4.052	0.000	1.287

Problem 2576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	116	128	127	114	0	114	127
normalized size	1	1.00	0.86	0.95	0.94	0.84	0.00	0.84	0.94
time (sec)	N/A	0.057	0.100	0.029	0.504	0.609	0.000	0.277	1.717

Problem 2577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	116	128	129	116	0	116	129
normalized size	1	1.00	0.86	0.95	0.96	0.86	0.00	0.86	0.96
time (sec)	N/A	0.057	0.108	0.029	0.517	0.874	0.000	0.289	1.600

Problem 2578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	114	128	131	116	0	116	131
normalized size	1	1.00	0.86	0.96	0.98	0.87	0.00	0.87	0.98
time (sec)	N/A	0.058	0.079	0.029	0.613	0.489	0.000	0.294	1.599

Problem 2579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	116	128	133	116	0	116	133
normalized size	1	1.00	0.86	0.95	0.99	0.86	0.00	0.86	0.99
time (sec)	N/A	0.059	0.094	0.030	0.530	0.516	0.000	0.296	1.541

Problem 2580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	114	128	135	116	0	116	135
normalized size	1	1.00	0.86	0.96	1.02	0.87	0.00	0.87	1.02
time (sec)	N/A	0.058	0.101	0.029	0.652	0.766	0.000	0.305	1.551

Problem 2581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	116	128	137	116	0	116	137
normalized size	1	1.00	0.86	0.95	1.01	0.86	0.00	0.86	1.01
time (sec)	N/A	0.061	0.136	0.029	0.543	0.553	0.000	0.296	1.503

Problem 2582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	115	127	138	116	0	116	138
normalized size	1	1.00	0.86	0.95	1.03	0.87	0.00	0.87	1.03
time (sec)	N/A	0.061	0.144	0.028	0.588	0.550	0.000	0.309	1.462

Problem 2583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	108	129	142	116	0	116	142
normalized size	1	1.00	0.77	0.92	1.01	0.83	0.00	0.83	1.01
time (sec)	N/A	0.059	0.136	0.030	0.468	0.722	0.000	0.302	1.487

Problem 2584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	136	151	111	0	111	151
normalized size	1	1.00	1.00	5.67	6.29	4.62	0.00	4.62	6.29
time (sec)	N/A	0.006	0.011	0.027	0.496	0.557	0.000	0.286	1.462

Problem 2585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	33	136	153	113	0	113	153
normalized size	1	1.00	0.66	2.72	3.06	2.26	0.00	2.26	3.06
time (sec)	N/A	0.019	0.039	0.027	0.583	0.610	0.000	0.313	1.456

Problem 2586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	46	136	153	113	0	113	153
normalized size	1	1.00	0.60	1.77	1.99	1.47	0.00	1.47	1.99
time (sec)	N/A	0.029	0.026	0.025	0.605	0.590	0.000	0.329	1.422

Problem 2587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	59	136	153	113	0	113	153
normalized size	1	1.00	0.57	1.31	1.47	1.09	0.00	1.09	1.47
time (sec)	N/A	0.040	0.021	0.028	0.531	0.597	0.000	0.450	1.432

Problem 2588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	113	136	153	113	0	113	153
normalized size	1	1.00	0.86	1.04	1.17	0.86	0.00	0.86	1.17
time (sec)	N/A	0.054	0.061	0.026	0.506	0.710	0.000	0.309	1.437

Problem 2589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	113	136	153	113	0	113	153
normalized size	1	1.00	0.75	0.90	1.01	0.75	0.00	0.75	1.01
time (sec)	N/A	0.061	0.054	0.027	0.562	0.643	0.000	0.293	1.514

Problem 2590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	113	136	153	113	0	113	153
normalized size	1	1.00	0.75	0.90	1.01	0.75	0.00	0.75	1.01
time (sec)	N/A	0.061	0.059	0.027	0.540	0.538	0.000	0.290	1.430

Problem 2591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	260	17	214	0	214	259
normalized size	1	1.00	1.00	13.68	0.89	11.26	0.00	11.26	13.63
time (sec)	N/A	0.004	0.008	0.036	0.555	0.700	0.000	0.178	1.867

Problem 2592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	14	134	160	14	14
normalized size	1	1.00	10.00	8.44	0.88	8.38	10.00	0.88	0.88
time (sec)	N/A	0.003	0.006	0.002	0.521	0.529	0.107	0.155	0.122

Problem 2593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	14	134	160	14	14
normalized size	1	1.00	10.00	8.44	0.88	8.38	10.00	0.88	0.88
time (sec)	N/A	0.003	0.007	0.003	0.469	0.583	0.110	0.151	0.107

Problem 2594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	14	134	160	14	14
normalized size	1	1.00	10.00	8.44	0.88	8.38	10.00	0.88	0.88
time (sec)	N/A	0.003	0.032	0.001	0.492	0.500	0.113	0.151	1.184

Problem 2595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	24	311	25	194	284	205	285
normalized size	1	1.00	0.89	11.52	0.93	7.19	10.52	7.59	10.56
time (sec)	N/A	0.007	0.016	0.030	0.440	0.696	159.992	0.513	2.095

Problem 2596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	24	311	25	194	284	205	285
normalized size	1	1.00	0.89	11.52	0.93	7.19	10.52	7.59	10.56
time (sec)	N/A	0.006	0.003	0.000	0.592	0.761	161.661	0.509	0.002

Problem 2597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	65	87	72	65	87	0	-1
normalized size	1	1.00	0.79	1.06	0.88	0.79	1.06	0.00	-0.01
time (sec)	N/A	0.045	0.077	0.035	0.567	0.548	95.403	0.000	0.000

Problem 2598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	52	69	60	52	71	0	-1
normalized size	1	1.00	0.81	1.08	0.94	0.81	1.11	0.00	-0.02
time (sec)	N/A	0.034	0.039	0.029	0.502	0.791	45.921	0.000	0.000

Problem 2599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	38	51	45	38	56	0	-1
normalized size	1	1.00	0.83	1.11	0.98	0.83	1.22	0.00	-0.02
time (sec)	N/A	0.025	0.048	0.024	0.533	0.828	25.360	0.000	0.000

Problem 2600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	33	32	24	41	0	-1
normalized size	1	1.00	0.93	1.18	1.14	0.86	1.46	0.00	-0.04
time (sec)	N/A	0.017	0.017	0.020	0.596	0.657	12.228	0.000	0.000

Problem 2601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	18	15	15	27	16	15
normalized size	1	1.00	1.00	1.20	1.00	1.00	1.80	1.07	1.00
time (sec)	N/A	0.004	0.002	0.015	0.635	0.707	2.542	0.151	1.218

Problem 2602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	22	29	28	22	41	0	22
normalized size	1	1.00	0.96	1.26	1.22	0.96	1.78	0.00	0.96
time (sec)	N/A	0.010	0.011	0.000	0.598	0.689	0.736	0.000	1.178

Problem 2603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	32	50	42	37	48	0	-1
normalized size	1	1.00	0.84	1.32	1.11	0.97	1.26	0.00	-0.03
time (sec)	N/A	0.022	0.053	0.020	0.480	0.807	21.081	0.000	0.000

Problem 2604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	49	69	58	59	85	0	-1
normalized size	1	1.00	0.86	1.21	1.02	1.04	1.49	0.00	-0.02
time (sec)	N/A	0.029	0.068	0.023	0.669	0.800	141.582	0.000	0.000

Problem 2605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	62	88	71	72	0	0	-1
normalized size	1	1.00	0.82	1.16	0.93	0.95	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.111	0.029	0.508	0.713	0.000	0.000	0.000

Problem 2606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	65	87	72	65	87	0	-1
normalized size	1	1.00	0.79	1.06	0.88	0.79	1.06	0.00	-0.01
time (sec)	N/A	0.040	0.034	0.000	0.566	0.824	95.823	0.000	0.000

Problem 2607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	52	69	60	52	71	0	-1
normalized size	1	1.00	0.81	1.08	0.94	0.81	1.11	0.00	-0.02
time (sec)	N/A	0.031	0.042	0.000	0.625	0.573	48.258	0.000	0.000

Problem 2608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	38	51	45	38	56	0	-1
normalized size	1	1.00	0.83	1.11	0.98	0.83	1.22	0.00	-0.02
time (sec)	N/A	0.024	0.014	0.000	0.502	0.680	26.721	0.000	0.000

Problem 2609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	33	32	24	41	0	-1
normalized size	1	1.00	0.93	1.18	1.14	0.86	1.46	0.00	-0.04
time (sec)	N/A	0.017	0.012	0.000	0.631	0.797	13.294	0.000	0.000

Problem 2610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	18	15	15	27	16	15
normalized size	1	1.00	1.00	1.20	1.00	1.00	1.80	1.07	1.00
time (sec)	N/A	0.005	0.002	0.000	0.528	0.617	2.702	0.151	0.002

Problem 2611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	22	29	28	22	41	0	22
normalized size	1	1.00	0.96	1.26	1.22	0.96	1.78	0.00	0.96
time (sec)	N/A	0.010	0.023	0.000	0.736	0.689	0.744	0.000	0.002

Problem 2612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	32	50	42	37	48	0	-1
normalized size	1	1.00	0.84	1.32	1.11	0.97	1.26	0.00	-0.03
time (sec)	N/A	0.020	0.010	0.000	0.693	0.836	22.380	0.000	0.000

Problem 2613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	49	69	58	59	85	0	-1
normalized size	1	1.00	0.86	1.21	1.02	1.04	1.49	0.00	-0.02
time (sec)	N/A	0.028	0.017	0.000	0.656	0.799	145.964	0.000	0.000

Problem 2614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	62	88	71	72	0	0	-1
normalized size	1	1.00	0.82	1.16	0.93	0.95	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.017	0.000	0.528	0.584	0.000	0.000	0.000

Problem 2615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	54	78	63	55	78	0	-1
normalized size	1	1.00	0.76	1.10	0.89	0.77	1.10	0.00	-0.01
time (sec)	N/A	0.034	0.047	0.032	0.493	0.472	76.518	0.000	0.000

Problem 2616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	43	63	52	44	63	0	-1
normalized size	1	1.00	0.77	1.12	0.93	0.79	1.12	0.00	-0.02
time (sec)	N/A	0.026	0.025	0.027	0.619	0.614	36.356	0.000	0.000

Problem 2617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	33	48	42	34	53	0	-1
normalized size	1	1.00	0.77	1.12	0.98	0.79	1.23	0.00	-0.02
time (sec)	N/A	0.021	0.019	0.023	0.535	0.583	19.572	0.000	0.000

Problem 2618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	25	32	31	23	39	0	-1
normalized size	1	1.00	0.93	1.19	1.15	0.85	1.44	0.00	-0.04
time (sec)	N/A	0.016	0.012	0.020	0.663	0.662	9.717	0.000	0.000

Problem 2619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	18	15	15	27	16	15
normalized size	1	1.00	1.00	1.20	1.00	1.00	1.80	1.07	1.00
time (sec)	N/A	0.005	0.002	0.016	0.704	0.528	2.380	0.158	1.209

Problem 2620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	24	23	20	29	0	18
normalized size	1	1.00	1.00	1.09	1.05	0.91	1.32	0.00	0.82
time (sec)	N/A	0.009	0.006	0.002	0.692	0.704	0.586	0.000	1.170

Problem 2621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	31	42	34	34	29	0	-1
normalized size	1	1.00	0.86	1.17	0.94	0.94	0.81	0.00	-0.03
time (sec)	N/A	0.019	0.045	0.018	0.477	0.547	18.257	0.000	0.000

Problem 2622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	42	59	47	50	63	0	-1
normalized size	1	1.00	0.79	1.11	0.89	0.94	1.19	0.00	-0.02
time (sec)	N/A	0.023	0.053	0.023	0.551	0.619	132.458	0.000	0.000

Problem 2623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	53	74	58	61	0	0	-1
normalized size	1	1.00	0.78	1.09	0.85	0.90	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.082	0.029	0.507	0.779	0.000	0.000	0.000

Problem 2624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	54	78	78	76	0	0	-1
normalized size	1	1.00	0.82	1.18	1.18	1.15	0.00	0.00	-0.02
time (sec)	N/A	0.041	0.073	0.029	0.596	0.671	0.000	0.000	0.000

Problem 2625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	38	59	61	59	129	0	-1
normalized size	1	1.00	0.79	1.23	1.27	1.23	2.69	0.00	-0.02
time (sec)	N/A	0.032	0.050	0.026	0.506	0.864	157.496	0.000	0.000

Problem 2626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	27	38	39	36	95	0	-1
normalized size	1	1.00	0.82	1.15	1.18	1.09	2.88	0.00	-0.03
time (sec)	N/A	0.023	0.028	0.020	0.485	0.886	99.174	0.000	0.000

Problem 2627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	24	17	17	37	17	21
normalized size	1	1.00	1.00	1.41	1.00	1.00	2.18	1.00	1.24
time (sec)	N/A	0.005	0.006	0.020	0.497	0.629	9.896	0.151	1.302

Problem 2628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	33	45	43	50	160	0	39
normalized size	1	1.00	0.85	1.15	1.10	1.28	4.10	0.00	1.00
time (sec)	N/A	0.024	0.070	0.000	0.546	0.526	1.451	0.000	1.189

Problem 2629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	45	97	62	82	172	0	-1
normalized size	1	1.00	0.79	1.70	1.09	1.44	3.02	0.00	-0.02
time (sec)	N/A	0.033	0.153	0.026	0.617	0.742	125.743	0.000	0.000

Problem 2630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	65	117	0	105	0	0	-1
normalized size	1	1.00	0.83	1.50	0.00	1.35	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.210	0.029	0.000	0.669	0.000	0.000	0.000

Problem 2631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	78	135	94	116	0	0	-1
normalized size	1	1.00	0.83	1.44	1.00	1.23	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.221	0.034	0.550	0.635	0.000	0.000	0.000

Problem 2632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	51	75	91	102	0	0	-1
normalized size	1	1.00	0.73	1.07	1.30	1.46	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.181	0.039	0.540	0.596	0.000	0.000	0.000

Problem 2633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	42	57	66	76	0	0	-1
normalized size	1	1.00	0.75	1.02	1.18	1.36	0.00	0.00	-0.02
time (sec)	N/A	0.033	0.048	0.032	0.580	0.718	0.000	0.000	0.000

Problem 2634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	36	41	41	61	0	37
normalized size	1	1.00	1.00	1.50	1.71	1.71	2.54	0.00	1.54
time (sec)	N/A	0.005	0.007	0.032	0.503	0.749	158.689	0.000	1.384

Problem 2635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	17	31	56	17	33
normalized size	1	1.00	1.00	1.05	0.89	1.63	2.95	0.89	1.74
time (sec)	N/A	0.005	0.005	0.031	0.582	0.754	30.215	0.153	1.383

Problem 2636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	47	62	71	106	406	0	69
normalized size	1	1.00	0.81	1.07	1.22	1.83	7.00	0.00	1.19
time (sec)	N/A	0.031	0.122	0.000	0.595	0.561	2.671	0.000	1.213

Problem 2637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	58	132	91	139	0	0	-1
normalized size	1	1.00	0.75	1.71	1.18	1.81	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.216	0.039	0.556	1.167	0.000	0.000	0.000

Problem 2638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	79	152	110	160	0	0	-1
normalized size	1	1.00	0.78	1.50	1.09	1.58	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.266	0.049	0.486	0.558	0.000	0.000	0.000

Problem 2639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	32	79	0	130	0	0	-1
normalized size	1	1.00	0.64	1.58	0.00	2.60	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.007	0.093	0.000	0.784	0.000	0.000	0.000

Problem 2640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	34	54	0	178	192	0	-1
normalized size	1	1.00	0.21	0.34	0.00	1.11	1.20	0.00	-0.01
time (sec)	N/A	0.114	0.018	0.165	0.000	0.918	5.470	0.000	0.000

Problem 2641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	34	54	0	261	267	0	-1
normalized size	1	1.00	0.14	0.23	0.00	1.11	1.13	0.00	-0.00
time (sec)	N/A	0.200	0.008	0.076	0.000	1.088	5.728	0.000	0.000

Problem 2642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	32	50	42	37	48	0	-1
normalized size	1	1.00	0.84	1.32	1.11	0.97	1.26	0.00	-0.03
time (sec)	N/A	0.021	0.028	0.000	0.614	0.625	25.359	0.000	0.000

Problem 2643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	32	79	0	130	0	0	-1
normalized size	1	1.00	0.64	1.58	0.00	2.60	0.00	0.00	-0.02
time (sec)	N/A	0.023	0.004	0.000	0.000	0.675	0.000	0.000	0.000

Problem 2644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	32	57	0	145	0	0	-1
normalized size	1	1.00	0.20	0.36	0.00	0.92	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.007	0.074	0.000	0.793	0.000	0.000	0.000

Problem 2645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	32	56	0	211	0	0	-1
normalized size	1	1.00	0.14	0.24	0.00	0.90	0.00	0.00	-0.00
time (sec)	N/A	0.184	0.007	0.077	0.000	0.648	0.000	0.000	0.000

Problem 2646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	34	97	0	251	56	0	-1
normalized size	1	1.00	0.50	1.43	0.00	3.69	0.82	0.00	-0.01
time (sec)	N/A	0.036	0.008	0.092	0.000	0.727	4.656	0.000	0.000

Problem 2647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	34	73	0	180	230	0	-1
normalized size	1	1.00	0.19	0.41	0.00	1.02	1.31	0.00	-0.01
time (sec)	N/A	0.112	0.008	0.083	0.000	0.756	4.341	0.000	0.000

Problem 2648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	34	73	0	270	309	0	-1
normalized size	1	1.00	0.13	0.29	0.00	1.07	1.23	0.00	-0.00
time (sec)	N/A	0.197	0.008	0.091	0.000	1.150	4.749	0.000	0.000

Problem 2649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	57	67	66	66	2572	0	-1
normalized size	1	1.00	0.62	0.73	0.72	0.72	27.96	0.00	-0.01
time (sec)	N/A	0.043	0.049	0.025	0.572	0.584	36.972	0.000	0.000

Problem 2650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	44	54	53	53	1015	0	-1
normalized size	1	1.00	0.65	0.79	0.78	0.78	14.93	0.00	-0.01
time (sec)	N/A	0.033	0.037	0.021	0.598	0.805	21.958	0.000	0.000

Problem 2651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	31	41	39	39	338	0	-1
normalized size	1	1.00	0.70	0.93	0.89	0.89	7.68	0.00	-0.02
time (sec)	N/A	0.022	0.034	0.021	0.617	0.742	13.986	0.000	0.000

Problem 2652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	48	17	17
normalized size	1	1.00	1.00	0.86	0.81	0.81	2.29	0.81	0.81
time (sec)	N/A	0.006	0.012	0.022	0.514	0.674	2.661	0.163	1.311

Problem 2653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	43	36	54	91	76	0	-1
normalized size	1	1.00	0.96	0.80	1.20	2.02	1.69	0.00	-0.02
time (sec)	N/A	0.024	0.013	0.000	1.261	0.726	1.664	0.000	0.000

Problem 2654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	62	0	0	119	49	0	-1
normalized size	1	1.00	1.22	0.00	0.00	2.33	0.96	0.00	-0.02
time (sec)	N/A	0.024	0.056	0.206	0.000	0.876	25.075	0.000	0.000

Problem 2655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	42	0	0	153	112	0	-1
normalized size	1	1.00	0.50	0.00	0.00	1.82	1.33	0.00	-0.01
time (sec)	N/A	0.037	0.013	0.205	0.000	0.794	54.197	0.000	0.000

Problem 2656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	42	0	0	183	0	0	-1
normalized size	1	1.00	0.37	0.00	0.00	1.62	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.012	0.207	0.000	0.723	0.000	0.000	0.000

Problem 2657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	42	0	0	209	0	0	-1
normalized size	1	1.00	0.30	0.00	0.00	1.47	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.012	0.214	0.000	0.586	0.000	0.000	0.000

Problem 2658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	57	54	66	53	2422	0	-1
normalized size	1	1.00	0.65	0.61	0.75	0.60	27.52	0.00	-0.01
time (sec)	N/A	0.041	0.051	0.026	0.784	0.562	28.398	0.000	0.000

Problem 2659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	44	41	53	40	916	0	-1
normalized size	1	1.00	0.67	0.62	0.80	0.61	13.88	0.00	-0.02
time (sec)	N/A	0.030	0.025	0.025	0.730	0.563	16.534	0.000	0.000

Problem 2660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	30	28	39	26	275	0	-1
normalized size	1	1.00	0.71	0.67	0.93	0.62	6.55	0.00	-0.02
time (sec)	N/A	0.020	0.041	0.023	0.595	0.534	9.485	0.000	0.000

Problem 2661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	17	41	17	17
normalized size	1	1.00	1.00	0.95	0.89	0.89	2.16	0.89	0.89
time (sec)	N/A	0.006	0.004	0.020	0.638	0.493	7.715	0.158	1.274

Problem 2662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	39	70	26	0	-1
normalized size	1	1.00	1.00	0.82	1.39	2.50	0.93	0.00	-0.04
time (sec)	N/A	0.016	0.010	0.000	1.380	0.638	1.696	0.000	0.000

Problem 2663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	67	0	0	119	49	0	-1
normalized size	1	1.00	1.26	0.00	0.00	2.25	0.92	0.00	-0.02
time (sec)	N/A	0.025	0.089	0.210	0.000	0.742	39.241	0.000	0.000

Problem 2664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	40	0	0	157	117	0	-1
normalized size	1	1.00	0.46	0.00	0.00	1.80	1.34	0.00	-0.01
time (sec)	N/A	0.036	0.015	0.202	0.000	0.676	85.035	0.000	0.000

Problem 2665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	40	0	0	183	0	0	-1
normalized size	1	1.00	0.34	0.00	0.00	1.58	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.009	0.211	0.000	0.640	0.000	0.000	0.000

Problem 2666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	40	0	0	209	0	0	-1
normalized size	1	1.00	0.28	0.00	0.00	1.44	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.010	0.211	0.000	0.647	0.000	0.000	0.000

Problem 2667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	67	92	75	362	0	622	77
normalized size	1	1.00	0.89	1.23	1.00	4.83	0.00	8.29	1.03
time (sec)	N/A	0.034	0.062	0.023	0.490	0.546	0.000	0.210	1.530

Problem 2668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	46	63	51	151	1280	238	52
normalized size	1	1.00	0.90	1.24	1.00	2.96	25.10	4.67	1.02
time (sec)	N/A	0.018	0.039	0.019	0.581	0.606	59.344	0.302	1.465

Problem 2669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	34	27	43	175	55	27
normalized size	1	1.00	1.00	1.26	1.00	1.59	6.48	2.04	1.00
time (sec)	N/A	0.008	0.021	0.014	0.613	0.722	3.421	0.159	1.403

Problem 2670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	41	0	0	0	95	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	2.38	0.00	-0.02
time (sec)	N/A	0.010	0.008	0.214	0.000	0.574	1.169	0.000	0.000

Problem 2671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	41	0	0	0	840	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	21.00	0.00	-0.02
time (sec)	N/A	0.010	0.015	0.220	0.000	0.726	2.151	0.000	0.000

Problem 2672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	41	0	0	0	6803	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	170.08	0.00	-0.02
time (sec)	N/A	0.009	0.007	0.217	0.000	0.777	4.753	0.000	0.000

Problem 2673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	65	66	0	0	0	58	0	-1
normalized size	1	1.18	1.20	0.00	0.00	0.00	1.05	0.00	-0.02
time (sec)	N/A	0.023	0.022	0.209	0.000	0.000	15.810	0.000	0.000

Problem 2674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	64	65	0	0	0	58	0	-1
normalized size	1	1.16	1.18	0.00	0.00	0.00	1.05	0.00	-0.02
time (sec)	N/A	0.020	0.016	0.211	0.000	0.000	2.043	0.000	0.000

Problem 2675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	64	65	0	0	0	56	0	-1
normalized size	1	1.16	1.18	0.00	0.00	0.00	1.02	0.00	-0.02
time (sec)	N/A	0.020	0.024	0.205	0.000	0.000	1.556	0.000	0.000

Problem 2676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	67	68	0	0	0	56	0	-1
normalized size	1	1.22	1.24	0.00	0.00	0.00	1.02	0.00	-0.02
time (sec)	N/A	0.022	0.035	0.210	0.000	0.000	2.963	0.000	0.000

Problem 2677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	67	68	0	0	0	56	0	-1
normalized size	1	1.22	1.24	0.00	0.00	0.00	1.02	0.00	-0.02
time (sec)	N/A	0.024	0.021	0.208	0.000	0.000	15.544	0.000	0.000

Problem 2678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	70	68	0	0	0	49	0	-1
normalized size	1	1.19	1.15	0.00	0.00	0.00	0.83	0.00	-0.02
time (sec)	N/A	0.024	0.031	0.230	0.000	0.000	29.263	0.000	0.000

Problem 2679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	65	65	0	0	0	48	0	-1
normalized size	1	1.16	1.16	0.00	0.00	0.00	0.86	0.00	-0.02
time (sec)	N/A	0.022	0.042	0.223	0.000	0.000	4.291	0.000	0.000

Problem 2680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	65	64	0	0	0	44	0	-1
normalized size	1	1.16	1.14	0.00	0.00	0.00	0.79	0.00	-0.02
time (sec)	N/A	0.024	0.029	0.212	0.000	0.000	40.983	0.000	0.000

Problem 2681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	72	68	0	0	0	49	0	-1
normalized size	1	1.18	1.11	0.00	0.00	0.00	0.80	0.00	-0.02
time (sec)	N/A	0.026	0.024	0.211	0.000	0.000	37.274	0.000	0.000

Problem 2682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	75	77	0	0	0	65	0	-1
normalized size	1	1.14	1.17	0.00	0.00	0.00	0.98	0.00	-0.02
time (sec)	N/A	0.026	0.023	0.206	0.000	0.000	3.286	0.000	0.000

Problem 2683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	69	69	0	0	0	63	0	-1
normalized size	1	1.15	1.15	0.00	0.00	0.00	1.05	0.00	-0.02
time (sec)	N/A	0.025	0.028	0.206	0.000	0.000	4.496	0.000	0.000

Problem 2684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	72	72	0	0	0	60	0	-1
normalized size	1	1.14	1.14	0.00	0.00	0.00	0.95	0.00	-0.02
time (sec)	N/A	0.027	0.025	0.205	0.000	0.000	9.555	0.000	0.000

Problem 2685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	75	77	0	0	0	65	0	-1
normalized size	1	1.14	1.17	0.00	0.00	0.00	0.98	0.00	-0.02
time (sec)	N/A	0.027	0.029	0.203	0.000	0.000	16.600	0.000	0.000

Problem 2686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	A	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	126	111	0	13	0	94	0	-1
normalized size	1	8.40	7.40	0.00	0.87	0.00	6.27	0.00	-0.07
time (sec)	N/A	0.078	0.227	0.234	0.891	0.000	64.200	0.000	0.000

Problem 2687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	100	98	0	175	148	0	-1
normalized size	1	1.00	0.78	0.76	0.00	1.36	1.15	0.00	-0.01
time (sec)	N/A	0.059	0.226	0.109	0.000	0.628	16.506	0.000	0.000

Problem 2688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	100	82	0	150	116	0	-1
normalized size	1	1.00	1.02	0.84	0.00	1.53	1.18	0.00	-0.01
time (sec)	N/A	0.042	0.068	0.036	0.000	0.835	12.737	0.000	0.000

Problem 2689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	81	64	0	114	49	0	-1
normalized size	1	1.00	1.31	1.03	0.00	1.84	0.79	0.00	-0.02
time (sec)	N/A	0.027	0.055	0.034	0.000	0.724	9.863	0.000	0.000

Problem 2690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	59	0	0	0	24	0	-1
normalized size	1	1.00	1.69	0.00	0.00	0.00	0.69	0.00	-0.03
time (sec)	N/A	0.019	0.019	0.215	0.000	0.000	8.138	0.000	0.000

Problem 2691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	0	0	22	0	24
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.85	0.00	0.92
time (sec)	N/A	0.006	0.007	0.215	0.000	0.000	34.933	0.000	1.312

Problem 2692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	36	0	0	0	51	0	-1
normalized size	1	1.00	0.62	0.00	0.00	0.00	0.88	0.00	-0.02
time (sec)	N/A	0.015	0.013	0.208	0.000	0.000	5.960	0.000	0.000

Problem 2693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	51	0	0	0	354	0	-1
normalized size	1	1.00	0.57	0.00	0.00	0.00	3.98	0.00	-0.01
time (sec)	N/A	0.027	0.018	0.211	0.000	0.000	6.250	0.000	0.000

Problem 2694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	64	0	0	0	605	0	-1
normalized size	1	1.00	0.53	0.00	0.00	0.00	5.04	0.00	-0.01
time (sec)	N/A	0.039	0.023	0.214	0.000	0.000	6.513	0.000	0.000

Problem 2695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	80	110	0	0	0	94	0	-1
normalized size	1	1.23	1.69	0.00	0.00	0.00	1.45	0.00	-0.02
time (sec)	N/A	0.032	0.177	0.225	0.000	0.000	7.845	0.000	0.000

Problem 2696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	81	79	0	0	0	95	0	-1
normalized size	1	1.21	1.18	0.00	0.00	0.00	1.42	0.00	-0.01
time (sec)	N/A	0.034	0.050	0.214	0.000	0.000	17.104	0.000	0.000

Problem 2697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	A	A	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	160	26	40	37	0	170	0	-1
normalized size	1	6.15	1.00	1.54	1.42	0.00	6.54	0.00	-0.04
time (sec)	N/A	0.086	0.198	0.047	0.849	0.000	25.958	0.000	0.000

Problem 2698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	111	0	13	16	100	0	13
normalized size	1	1.00	7.40	0.00	0.87	1.07	6.67	0.00	0.87
time (sec)	N/A	0.018	0.132	0.217	1.021	0.774	101.859	0.000	1.418

Problem 2699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	A	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	126	111	0	13	0	94	0	-1
normalized size	1	8.40	7.40	0.00	0.87	0.00	6.27	0.00	-0.07
time (sec)	N/A	0.072	0.096	0.001	0.853	0.000	47.509	0.000	0.000

Problem 2700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	67	0	0	0	117	0	-1
normalized size	1	1.00	0.69	0.00	0.00	0.00	1.21	0.00	-0.01
time (sec)	N/A	0.047	0.039	0.220	0.000	0.000	2.482	0.000	0.000

Problem 2701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	73	0	0	0	0	0	-1
normalized size	1	1.00	0.53	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.079	0.238	0.000	0.000	0.000	0.000	0.000

Problem 2702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	56	0	0	0	39	0	-1
normalized size	1	1.00	0.63	0.00	0.00	0.00	0.44	0.00	-0.01
time (sec)	N/A	0.040	0.019	0.214	0.000	0.000	7.814	0.000	0.000

Problem 2703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	58	0	0	0	46	0	-1
normalized size	1	1.00	0.51	0.00	0.00	0.00	0.40	0.00	-0.01
time (sec)	N/A	0.054	0.028	0.215	0.000	0.000	123.911	0.000	0.000

Problem 2704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	62	63	0	0	0	54	0	-1
normalized size	1	1.15	1.17	0.00	0.00	0.00	1.00	0.00	-0.02
time (sec)	N/A	0.018	0.033	0.212	0.000	0.569	10.683	0.000	0.000

Problem 2705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	55	0	0	192	0	0	64
normalized size	1	1.00	0.38	0.00	0.00	1.32	0.00	0.00	0.44
time (sec)	N/A	0.088	0.074	0.217	0.000	0.636	0.000	0.000	1.556

Problem 2706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	55	0	0	126	0	0	64
normalized size	1	1.00	0.57	0.00	0.00	1.31	0.00	0.00	0.67
time (sec)	N/A	0.033	0.041	0.217	0.000	0.850	0.000	0.000	1.426

Problem 2707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	55	0	0	68	0	0	64
normalized size	1	1.00	1.10	0.00	0.00	1.36	0.00	0.00	1.28
time (sec)	N/A	0.012	0.046	0.216	0.000	0.781	0.000	0.000	1.435

Problem 2708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	53	0	31	211	0	75
normalized size	1	1.00	1.00	2.94	0.00	1.72	11.72	0.00	4.17
time (sec)	N/A	0.003	0.064	0.029	0.000	0.529	28.867	0.000	1.329

Problem 2709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	0	0	0	39	0	51
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.78	0.00	1.02
time (sec)	N/A	0.013	0.014	0.218	0.000	0.862	10.711	0.000	1.476

Problem 2710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	0	0	0	42	0	54
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.79	0.00	1.02
time (sec)	N/A	0.011	0.012	0.292	0.000	0.669	7.410	0.000	1.461

Problem 2711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	0	0	0	44	0	56
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.80	0.00	1.02
time (sec)	N/A	0.013	0.008	0.293	0.000	0.678	90.916	0.000	1.313

Problem 2712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	25	24	0	24	21
normalized size	1	1.00	1.00	1.05	1.19	1.14	0.00	1.14	1.00
time (sec)	N/A	0.006	0.004	0.000	0.649	0.925	0.000	0.169	1.302

Problem 2713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	19	22	0	22	18
normalized size	1	1.00	1.00	1.06	1.06	1.22	0.00	1.22	1.00
time (sec)	N/A	0.006	0.003	0.003	0.574	0.813	0.000	0.195	1.220

Problem 2714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	19	22	0	22	18
normalized size	1	1.00	1.00	1.06	1.06	1.22	0.00	1.22	1.00
time (sec)	N/A	0.005	0.020	0.002	0.566	0.875	0.000	0.174	1.199

Problem 2715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	17	20	0	20	16
normalized size	1	1.00	1.00	1.06	1.06	1.25	0.00	1.25	1.00
time (sec)	N/A	0.005	0.002	0.002	0.604	1.009	0.000	0.157	1.757

Problem 2716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	15	18	22	14	-1
normalized size	1	1.00	1.00	1.07	1.07	1.29	1.57	1.00	-0.07
time (sec)	N/A	0.004	0.001	0.001	0.717	0.845	0.303	0.153	0.000

Problem 2717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	18	19	19	22	0	0	-1
normalized size	1	1.00	0.90	0.95	0.95	1.10	0.00	0.00	-0.05
time (sec)	N/A	0.008	0.004	0.002	0.585	0.589	0.000	0.000	0.000

Problem 2718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	18	19	19	22	0	0	-1
normalized size	1	1.00	0.90	0.95	0.95	1.10	0.00	0.00	-0.05
time (sec)	N/A	0.007	0.003	0.001	0.600	0.674	0.000	0.000	0.000

Problem 2719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	18	19	19	22	0	0	-1
normalized size	1	1.00	0.90	0.95	0.95	1.10	0.00	0.00	-0.05
time (sec)	N/A	0.007	0.003	0.003	0.534	0.709	0.000	0.000	0.000

Problem 2720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	29	23	27	75	23	23
normalized size	1	1.00	1.00	1.26	1.00	1.17	3.26	1.00	1.00
time (sec)	N/A	0.008	0.007	0.053	0.494	0.637	46.607	0.157	1.587

Problem 2721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	40	61	51	62	0	0	-1
normalized size	1	1.00	0.82	1.24	1.04	1.27	0.00	0.00	-0.02
time (sec)	N/A	0.032	0.024	0.057	0.579	0.586	0.000	0.000	0.000

Problem 2722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	66	105	79	108	0	0	-1
normalized size	1	1.00	0.88	1.40	1.05	1.44	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.046	0.071	0.602	0.741	0.000	0.000	0.000

Problem 2723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	78	171	114	158	0	0	-1
normalized size	1	1.00	0.76	1.66	1.11	1.53	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.088	0.076	0.584	0.629	0.000	0.000	0.000

Problem 2724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	0	0	53	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	1.66	0.00	0.00	-0.03
time (sec)	N/A	0.009	0.023	0.212	0.000	0.635	0.000	0.000	0.000

Problem 2725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	136	151	111	0	111	151
normalized size	1	1.00	1.00	5.67	6.29	4.62	0.00	4.62	6.29
time (sec)	N/A	0.005	0.011	0.002	0.540	0.595	0.000	0.291	0.002

Problem 2726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	29	29	37	34	0	0	52
normalized size	1	1.00	0.97	0.97	1.23	1.13	0.00	0.00	1.73
time (sec)	N/A	0.008	0.015	0.005	0.537	0.634	0.000	0.000	1.422

Problem 2727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	108	91	90	90	97	90	92
normalized size	1	1.00	5.68	4.79	4.74	4.74	5.11	4.74	4.84
time (sec)	N/A	0.003	0.010	0.009	0.451	0.756	1.648	0.198	1.229

Problem 2728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	22	29	28	22	41	0	22
normalized size	1	1.00	0.96	1.26	1.22	0.96	1.78	0.00	0.96
time (sec)	N/A	0.010	0.005	0.001	0.503	0.763	0.996	0.000	0.002

Problem 2729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	23	18	15	22	18
normalized size	1	1.00	1.00	0.95	1.05	0.82	0.68	1.00	0.82
time (sec)	N/A	0.012	0.006	0.004	0.488	0.543	0.325	0.157	1.184

Problem 2730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	55	0	0	192	0	0	64
normalized size	1	1.00	0.37	0.00	0.00	1.31	0.00	0.00	0.44
time (sec)	N/A	0.103	0.045	0.214	0.000	0.676	0.000	0.000	1.528

Problem 2731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	55	0	0	126	0	0	64
normalized size	1	1.00	0.57	0.00	0.00	1.30	0.00	0.00	0.66
time (sec)	N/A	0.053	0.037	0.218	0.000	0.668	0.000	0.000	1.341

Problem 2732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	55	0	0	68	0	0	64
normalized size	1	1.00	1.08	0.00	0.00	1.33	0.00	0.00	1.25
time (sec)	N/A	0.019	0.048	0.213	0.000	0.679	0.000	0.000	1.324

Problem 2733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	35	0	31	0	0	75
normalized size	1	1.00	1.00	1.94	0.00	1.72	0.00	0.00	4.17
time (sec)	N/A	0.003	0.042	0.027	0.000	0.693	0.000	0.000	1.304

Problem 2734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	0	0	0	39	0	51
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.78	0.00	1.02
time (sec)	N/A	0.011	0.003	0.000	0.000	0.624	11.536	0.000	0.002

Problem 2735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	53	0	0	0	42	0	54
normalized size	1	1.00	0.90	0.00	0.00	0.00	0.71	0.00	0.92
time (sec)	N/A	0.013	0.005	0.214	0.000	0.738	12.215	0.000	1.508

Problem 2736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	55	0	0	0	44	0	56
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.79	0.00	1.00
time (sec)	N/A	0.013	0.003	0.214	0.000	0.546	109.599	0.000	1.320

Problem 2737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	32	15	39	0	22
normalized size	1	1.00	1.00	1.07	2.13	1.00	2.60	0.00	1.47
time (sec)	N/A	0.008	0.004	0.003	0.458	0.848	1.302	0.000	1.179

Problem 2738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	21	19	18	37	19	19
normalized size	1	1.00	1.00	1.11	1.00	0.95	1.95	1.00	1.00
time (sec)	N/A	0.005	0.006	0.015	0.565	0.667	1.460	0.152	1.252

Problem 2739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	60	27	35	100	26	27
normalized size	1	1.00	1.00	2.22	1.00	1.30	3.70	0.96	1.00
time (sec)	N/A	0.010	0.012	0.037	0.500	0.585	134.229	0.197	1.494

Problem 2740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	57	70	67	58	0	58	72
normalized size	1	1.00	0.80	0.99	0.94	0.82	0.00	0.82	1.01
time (sec)	N/A	0.036	0.058	0.022	0.633	0.656	0.000	0.235	1.451

Problem 2741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	42	50	48	42	61	42	38
normalized size	1	1.00	0.81	0.96	0.92	0.81	1.17	0.81	0.73
time (sec)	N/A	0.021	0.025	0.020	0.592	0.889	113.142	0.179	1.364

Problem 2742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	26	33	28	25	36	25	23
normalized size	1	1.00	0.87	1.10	0.93	0.83	1.20	0.83	0.77
time (sec)	N/A	0.008	0.014	0.021	0.649	0.647	9.913	0.161	1.286

Problem 2743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	61	0	96	197	0	-1
normalized size	1	1.00	1.00	1.85	0.00	2.91	5.97	0.00	-0.03
time (sec)	N/A	0.017	0.014	0.063	0.000	0.717	2.570	0.000	0.000

Problem 2744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	53	95	0	212	865	0	-1
normalized size	1	1.00	0.79	1.42	0.00	3.16	12.91	0.00	-0.01
time (sec)	N/A	0.026	0.015	0.051	0.000	0.597	8.623	0.000	0.000

Problem 2745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	35	110	0	340	0	0	-1
normalized size	1	1.00	0.36	1.13	0.00	3.51	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.009	0.052	0.000	0.633	0.000	0.000	0.000

Problem 2746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	88	0	0	0	0	0	-1
normalized size	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.094	0.229	0.000	0.000	0.000	0.000	0.000

Problem 2747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	86	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.048	0.221	0.000	0.000	0.000	0.000	0.000

Problem 2748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	85	0	0	0	121	0	-1
normalized size	1	1.00	1.18	0.00	0.00	0.00	1.68	0.00	-0.01
time (sec)	N/A	0.028	0.030	0.226	0.000	0.000	9.589	0.000	0.000

Problem 2749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	66	0	0	0	117	0	-1
normalized size	1	1.00	1.74	0.00	0.00	0.00	3.08	0.00	-0.03
time (sec)	N/A	0.020	0.039	0.205	0.000	0.000	3.423	0.000	0.000

Problem 2750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	0	0	47	121	0	27
normalized size	1	1.00	1.00	0.00	0.00	1.62	4.17	0.00	0.93
time (sec)	N/A	0.008	0.013	0.215	0.000	0.581	21.633	0.000	1.220

Problem 2751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	46	0	0	93	0	0	39
normalized size	1	1.00	0.71	0.00	0.00	1.43	0.00	0.00	0.60
time (sec)	N/A	0.025	0.056	0.219	0.000	0.567	0.000	0.000	1.240

Problem 2752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	61	0	0	135	0	0	54
normalized size	1	1.00	0.60	0.00	0.00	1.32	0.00	0.00	0.53
time (sec)	N/A	0.054	0.059	0.218	0.000	0.800	0.000	0.000	1.284

Problem 2753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	48	16	18
normalized size	1	1.00	1.00	0.85	0.80	0.80	2.40	0.80	0.90
time (sec)	N/A	0.005	0.015	0.032	0.547	0.652	4.445	0.155	1.245

Problem 2754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	18	58	18	20
normalized size	1	1.00	1.00	0.86	0.82	0.82	2.64	0.82	0.91
time (sec)	N/A	0.005	0.013	0.030	0.493	0.625	4.387	0.163	1.230

Problem 2755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	47	234	71	172	1290	613	89
normalized size	1	1.00	0.73	3.66	1.11	2.69	20.16	9.58	1.39
time (sec)	N/A	0.033	0.046	0.055	0.482	0.637	25.848	0.205	1.236

Problem 2756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	41	94	56	87	352	135	95
normalized size	1	1.00	0.71	1.62	0.97	1.50	6.07	2.33	1.64
time (sec)	N/A	0.030	0.034	0.008	0.532	0.689	1.388	0.190	1.216

Problem 2757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	41	94	56	87	345	135	95
normalized size	1	1.00	0.71	1.62	0.97	1.50	5.95	2.33	1.64
time (sec)	N/A	0.027	0.072	0.004	0.611	0.634	0.896	0.203	1.200

Problem 2758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	39	88	56	87	338	135	95
normalized size	1	1.00	0.67	1.52	0.97	1.50	5.83	2.33	1.64
time (sec)	N/A	0.023	0.061	0.004	0.510	0.637	0.550	0.158	1.370

Problem 2759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	36	68	51	63	202	0	45
normalized size	1	1.00	0.69	1.31	0.98	1.21	3.88	0.00	0.87
time (sec)	N/A	0.022	0.051	0.006	0.582	0.738	0.575	0.000	1.210

Problem 2760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	64	90	56	81	401	0	89
normalized size	1	1.00	1.05	1.48	0.92	1.33	6.57	0.00	1.46
time (sec)	N/A	0.029	0.034	0.005	0.503	0.780	1.109	0.000	1.228

Problem 2761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	67	96	56	87	464	0	97
normalized size	1	1.00	1.06	1.52	0.89	1.38	7.37	0.00	1.54
time (sec)	N/A	0.029	0.037	0.006	0.629	0.560	1.606	0.000	1.224

Problem 2762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	37	0	0	225	54	0	-1
normalized size	1	1.00	0.50	0.00	0.00	3.04	0.73	0.00	-0.01
time (sec)	N/A	0.037	0.012	0.221	0.000	0.788	3.836	0.000	0.000

Problem 2763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	39	0	0	329	226	0	-1
normalized size	1	1.00	0.18	0.00	0.00	1.48	1.02	0.00	-0.00
time (sec)	N/A	0.138	0.012	0.223	0.000	1.037	4.287	0.000	0.000

Problem 2764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	39	0	0	426	309	0	-1
normalized size	1	1.00	0.12	0.00	0.00	1.34	0.97	0.00	-0.00
time (sec)	N/A	0.243	0.011	0.216	0.000	1.401	4.551	0.000	0.000

Problem 2765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	42	0	63	59	34	0	-1
normalized size	1	1.00	0.61	0.00	0.91	0.86	0.49	0.00	-0.01
time (sec)	N/A	0.038	0.044	0.212	0.623	0.640	4.106	0.000	0.000

Problem 2766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	37	0	0	225	54	0	-1
normalized size	1	1.00	0.50	0.00	0.00	3.04	0.73	0.00	-0.01
time (sec)	N/A	0.036	0.003	0.000	0.000	0.652	3.779	0.000	0.000

Problem 2767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	37	0	0	255	216	0	-1
normalized size	1	1.00	0.17	0.00	0.00	1.16	0.98	0.00	-0.00
time (sec)	N/A	0.137	0.011	0.214	0.000	0.560	4.228	0.000	0.000

Problem 2768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	37	0	0	321	299	0	-1
normalized size	1	1.00	0.12	0.00	0.00	1.02	0.95	0.00	-0.00
time (sec)	N/A	0.228	0.012	0.219	0.000	0.829	4.656	0.000	0.000

Problem 2769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	39	0	0	492	82	0	-1
normalized size	1	1.00	0.39	0.00	0.00	4.92	0.82	0.00	-0.01
time (sec)	N/A	0.052	0.012	0.228	0.000	1.035	4.650	0.000	0.000

Problem 2770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	39	0	0	351	272	0	-1
normalized size	1	1.00	0.16	0.00	0.00	1.43	1.11	0.00	-0.00
time (sec)	N/A	0.151	0.015	0.211	0.000	1.259	5.170	0.000	0.000

Problem 2771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	39	0	0	448	360	0	-1
normalized size	1	1.00	0.11	0.00	0.00	1.31	1.06	0.00	-0.00
time (sec)	N/A	0.236	0.012	0.217	0.000	3.030	6.097	0.000	0.000

Problem 2772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	97	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	2.20	0.00	-0.02
time (sec)	N/A	0.018	0.013	0.217	0.000	0.663	8.244	0.000	0.000

Problem 2773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	44	0	0	0	97	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	2.11	0.00	-0.02
time (sec)	N/A	0.013	0.020	0.212	0.000	0.806	5.524	0.000	0.000

Problem 2774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	97	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	2.20	0.00	-0.02
time (sec)	N/A	0.013	0.012	0.211	0.000	0.455	3.632	0.000	0.000

Problem 2775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	45	0	0	0	94	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	2.14	0.00	-0.02
time (sec)	N/A	0.013	0.011	0.217	0.000	0.617	2.646	0.000	0.000

Problem 2776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	36	0	0	0	41	0	-1
normalized size	1	1.00	0.90	0.00	0.00	0.00	1.02	0.00	-0.02
time (sec)	N/A	0.012	0.009	0.217	0.000	0.623	1.219	0.000	0.000

Problem 2777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	29	0	24	20	17	0	-1
normalized size	1	1.00	1.04	0.00	0.86	0.71	0.61	0.00	-0.04
time (sec)	N/A	0.011	0.006	0.293	0.700	0.602	4.801	0.000	0.000

Problem 2778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	44	0	0	0	41	0	-1
normalized size	1	1.00	0.88	0.00	0.00	0.00	0.82	0.00	-0.02
time (sec)	N/A	0.013	0.013	0.228	0.000	0.602	11.521	0.000	0.000

Problem 2779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	44	0	0	0	44	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.85	0.00	-0.02
time (sec)	N/A	0.013	0.013	0.230	0.000	0.680	27.442	0.000	0.000

Problem 2780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	31	99	22	22	330	0	29
normalized size	1	1.00	1.29	4.12	0.92	0.92	13.75	0.00	1.21
time (sec)	N/A	0.006	0.015	0.026	0.550	0.742	43.303	0.000	1.163

Problem 2781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	133	0	0	231	190	0	-1
normalized size	1	1.00	0.75	0.00	0.00	1.30	1.07	0.00	-0.01
time (sec)	N/A	0.082	0.268	0.270	0.000	0.621	15.874	0.000	0.000

Problem 2782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	121	0	0	192	150	0	-1
normalized size	1	1.00	0.88	0.00	0.00	1.40	1.09	0.00	-0.01
time (sec)	N/A	0.057	0.118	0.231	0.000	0.743	11.852	0.000	0.000

Problem 2783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	110	0	0	142	66	0	-1
normalized size	1	1.00	1.21	0.00	0.00	1.56	0.73	0.00	-0.01
time (sec)	N/A	0.040	0.061	0.217	0.000	0.810	8.785	0.000	0.000

Problem 2784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	78	0	0	0	31	0	-1
normalized size	1	1.00	1.44	0.00	0.00	0.00	0.57	0.00	-0.02
time (sec)	N/A	0.030	0.032	0.216	0.000	0.000	7.112	0.000	0.000

Problem 2785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	0	0	0	29	0	29
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.94	0.00	0.94
time (sec)	N/A	0.009	0.012	0.213	0.000	0.000	5.455	0.000	1.214

Problem 2786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	41	0	0	0	68	0	-1
normalized size	1	1.00	0.63	0.00	0.00	0.00	1.05	0.00	-0.02
time (sec)	N/A	0.017	0.017	0.221	0.000	0.000	5.052	0.000	0.000

Problem 2787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	56	0	0	0	396	0	-1
normalized size	1	1.00	0.57	0.00	0.00	0.00	4.04	0.00	-0.01
time (sec)	N/A	0.031	0.020	0.218	0.000	0.000	5.377	0.000	0.000

Problem 2788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	69	0	0	0	665	0	-1
normalized size	1	1.00	0.53	0.00	0.00	0.00	5.16	0.00	-0.01
time (sec)	N/A	0.050	0.027	0.209	0.000	0.000	5.908	0.000	0.000

Problem 2789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	67	64	0	0	0	58	0	-1
normalized size	1	1.14	1.08	0.00	0.00	0.00	0.98	0.00	-0.02
time (sec)	N/A	0.023	0.017	0.217	0.000	0.582	9.774	0.000	0.000

Problem 2790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	66	62	0	0	0	51	0	-1
normalized size	1	1.18	1.11	0.00	0.00	0.00	0.91	0.00	-0.02
time (sec)	N/A	0.025	0.017	0.217	0.000	0.625	7.632	0.000	0.000

Problem 2791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	66	62	0	0	0	51	0	-1
normalized size	1	1.18	1.11	0.00	0.00	0.00	0.91	0.00	-0.02
time (sec)	N/A	0.022	0.017	0.223	0.000	0.894	8.080	0.000	0.000

Problem 2792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	62	58	0	0	0	48	0	-1
normalized size	1	1.19	1.12	0.00	0.00	0.00	0.92	0.00	-0.02
time (sec)	N/A	0.021	0.014	0.215	0.000	0.656	7.319	0.000	0.000

Problem 2793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	46	46	0	0	0	37	0	47
normalized size	1	1.21	1.21	0.00	0.00	0.00	0.97	0.00	1.24
time (sec)	N/A	0.011	0.006	0.218	0.000	0.667	1.640	0.000	1.937

Problem 2794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	64	59	0	0	0	44	0	-1
normalized size	1	1.21	1.11	0.00	0.00	0.00	0.83	0.00	-0.02
time (sec)	N/A	0.023	0.016	0.220	0.000	0.665	7.575	0.000	0.000

Problem 2795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	66	63	0	0	0	51	0	-1
normalized size	1	1.18	1.12	0.00	0.00	0.00	0.91	0.00	-0.02
time (sec)	N/A	0.024	0.017	0.216	0.000	0.829	7.668	0.000	0.000

Problem 2796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	66	63	0	0	0	51	0	-1
normalized size	1	1.18	1.12	0.00	0.00	0.00	0.91	0.00	-0.02
time (sec)	N/A	0.025	0.017	0.216	0.000	0.645	7.626	0.000	0.000

Problem 2797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	0	0	44	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.70	0.00	-0.02
time (sec)	N/A	0.024	0.014	0.217	0.000	0.782	119.276	0.000	0.000

Problem 2798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	0	75	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	2.03	0.00	0.00	-0.03
time (sec)	N/A	0.012	0.028	0.217	0.000	0.571	0.000	0.000	0.000

Problem 2799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	69	0	0	144	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	1.82	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.049	0.216	0.000	0.539	0.000	0.000	0.000

Problem 2800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	69	0	0	215	0	0	-1
normalized size	1	1.00	0.54	0.00	0.00	1.69	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.037	0.223	0.000	0.593	0.000	0.000	0.000

Problem 2801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	69	0	0	294	0	0	-1
normalized size	1	1.00	0.39	0.00	0.00	1.64	0.00	0.00	-0.01
time (sec)	N/A	0.130	0.028	0.217	0.000	0.774	0.000	0.000	0.000

Problem 2802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	25	73	54	103	0	98	36
normalized size	1	1.00	0.83	2.43	1.80	3.43	0.00	3.27	1.20
time (sec)	N/A	0.011	0.023	0.007	0.589	0.927	0.000	0.172	1.259

Problem 2803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	25	51	54	67	0	21	36
normalized size	1	1.00	0.89	1.82	1.93	2.39	0.00	0.75	1.29
time (sec)	N/A	0.011	0.010	0.004	0.535	0.807	0.000	0.149	1.197

Problem 2804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	31	29	54	41	0	36	21
normalized size	1	1.00	1.24	1.16	2.16	1.64	0.00	1.44	0.84
time (sec)	N/A	0.010	0.014	0.001	0.644	0.550	0.000	0.168	1.474

Problem 2805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	27	15	42	0	34	29
normalized size	1	1.00	1.00	0.96	0.54	1.50	0.00	1.21	1.04
time (sec)	N/A	0.012	0.007	0.010	0.532	0.809	0.000	0.153	1.334

Problem 2806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	25	22	17	69	0	0	26
normalized size	1	1.00	0.83	0.73	0.57	2.30	0.00	0.00	0.87
time (sec)	N/A	0.011	0.009	0.003	0.550	0.837	0.000	0.000	1.245

Problem 2807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	25	22	17	97	0	0	26
normalized size	1	1.00	0.83	0.73	0.57	3.23	0.00	0.00	0.87
time (sec)	N/A	0.013	0.013	0.004	0.555	0.817	0.000	0.000	1.278

Problem 2808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	25	25	30	9	8	26	13
normalized size	1	1.00	1.25	1.25	1.50	0.45	0.40	1.30	0.65
time (sec)	N/A	0.005	0.009	0.006	1.380	0.624	0.070	0.151	0.111

Problem 2809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	25	24	29	9	7	25	13
normalized size	1	1.00	1.25	1.20	1.45	0.45	0.35	1.25	0.65
time (sec)	N/A	0.005	0.006	0.002	1.232	0.769	0.072	0.152	0.081

Problem 2810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	6	8	7	15	14
normalized size	1	1.00	1.00	0.88	0.23	0.31	0.27	0.58	0.54
time (sec)	N/A	0.005	0.009	0.010	1.259	0.839	0.075	0.152	1.380

Problem 2811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	26	6	8	7	15	14
normalized size	1	1.00	1.00	1.00	0.23	0.31	0.27	0.58	0.54
time (sec)	N/A	0.005	0.007	0.004	1.126	0.644	0.075	0.172	1.257

Problem 2812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	C	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	6	6	0	23	15
normalized size	1	1.00	1.00	0.89	0.21	0.21	0.00	0.82	0.54
time (sec)	N/A	0.008	0.006	0.008	1.292	0.807	0.000	0.243	1.360

Problem 2813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	25	22	108	135	0	851	110
normalized size	1	1.00	0.83	0.73	3.60	4.50	0.00	28.37	3.67
time (sec)	N/A	0.015	0.033	0.003	0.637	0.887	0.000	0.252	1.390

Problem 2814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	25	22	66	83	0	408	58
normalized size	1	1.00	0.89	0.79	2.36	2.96	0.00	14.57	2.07
time (sec)	N/A	0.012	0.015	0.003	0.691	0.603	0.000	0.197	1.246

Problem 2815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	24	44	0	121	22
normalized size	1	1.00	1.00	0.88	0.96	1.76	0.00	4.84	0.88
time (sec)	N/A	0.011	0.008	0.002	0.817	0.573	0.000	0.179	1.154

Problem 2816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	27	61	0	23	26
normalized size	1	1.00	1.00	0.96	1.17	2.65	0.00	1.00	1.13
time (sec)	N/A	0.011	0.010	0.003	0.678	0.790	0.000	0.156	1.175

Problem 2817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	25	22	43	109	0	0	26
normalized size	1	1.00	0.83	0.73	1.43	3.63	0.00	0.00	0.87
time (sec)	N/A	0.013	0.012	0.003	0.670	0.889	0.000	0.000	1.241

Problem 2818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	25	22	85	151	0	0	26
normalized size	1	1.00	0.83	0.73	2.83	5.03	0.00	0.00	0.87
time (sec)	N/A	0.013	0.018	0.003	0.787	0.671	0.000	0.000	1.320

Problem 2819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	21	22	17	27	51	24	17
normalized size	1	1.00	0.70	0.73	0.57	0.90	1.70	0.80	0.57
time (sec)	N/A	0.008	0.013	0.004	0.619	0.750	3.593	0.156	1.184

Problem 2820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	22	17	17	48	24	17
normalized size	1	1.00	1.00	1.16	0.89	0.89	2.53	1.26	0.89
time (sec)	N/A	0.006	0.006	0.003	0.538	0.927	1.234	0.200	1.139

Problem 2821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	19	22	17	21	46	21	22
normalized size	1	1.00	0.83	0.96	0.74	0.91	2.00	0.91	0.96
time (sec)	N/A	0.006	0.013	0.004	0.561	0.568	0.341	0.155	1.161

Problem 2822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	21	22	17	35	49	62	40
normalized size	1	1.00	0.84	0.88	0.68	1.40	1.96	2.48	1.60
time (sec)	N/A	0.006	0.013	0.003	0.628	0.764	1.624	0.157	1.191

Problem 2823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	21	22	17	46	49	26	54
normalized size	1	1.00	0.70	0.73	0.57	1.53	1.63	0.87	1.80
time (sec)	N/A	0.008	0.015	0.003	0.499	0.931	1.688	0.172	1.228

Problem 2824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	21	22	17	57	49	26	26
normalized size	1	1.00	0.70	0.73	0.57	1.90	1.63	0.87	0.87
time (sec)	N/A	0.009	0.018	0.003	0.621	0.836	2.572	0.176	1.282

Problem 2825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	25	22	49	60	0	21	26
normalized size	1	1.00	0.83	0.73	1.63	2.00	0.00	0.70	0.87
time (sec)	N/A	0.009	0.012	0.004	0.666	0.824	0.000	0.201	1.227

Problem 2826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	25	22	27	36	0	21	24
normalized size	1	1.00	0.89	0.79	0.96	1.29	0.00	0.75	0.86
time (sec)	N/A	0.008	0.008	0.003	0.659	0.538	0.000	0.158	1.166

Problem 2827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	27	13	37	0	20	-1
normalized size	1	1.00	1.00	0.96	0.46	1.32	0.00	0.71	-0.04
time (sec)	N/A	0.010	0.008	0.006	0.665	0.717	0.000	0.170	0.000

Problem 2828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	32	29	15	48	0	23	42
normalized size	1	1.00	1.28	1.16	0.60	1.92	0.00	0.92	1.68
time (sec)	N/A	0.007	0.014	0.003	0.612	0.853	0.000	0.252	1.220

Problem 2829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	25	51	37	70	0	0	70
normalized size	1	1.00	0.83	1.70	1.23	2.33	0.00	0.00	2.33
time (sec)	N/A	0.008	0.015	0.003	0.583	0.901	0.000	0.000	1.292

Problem 2830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	25	73	59	92	0	0	98
normalized size	1	1.00	0.83	2.43	1.97	3.07	0.00	0.00	3.27
time (sec)	N/A	0.009	0.018	0.004	0.709	0.821	0.000	0.000	1.332

Problem 2831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	25	22	24	93	1294	52	55
normalized size	1	1.00	0.83	0.73	0.80	3.10	43.13	1.73	1.83
time (sec)	N/A	0.009	0.014	0.002	0.689	0.671	113.936	0.183	1.243

Problem 2832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	25	22	24	58	357	52	24
normalized size	1	1.00	0.89	0.79	0.86	2.07	12.75	1.86	0.86
time (sec)	N/A	0.009	0.011	0.003	0.602	0.849	8.922	0.185	1.199

Problem 2833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	24	43	97	50	52
normalized size	1	1.00	1.00	0.96	1.04	1.87	4.22	2.17	2.26
time (sec)	N/A	0.008	0.009	0.001	0.566	0.829	0.870	0.385	1.162

Problem 2834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	27	79	0	144	26
normalized size	1	1.00	1.00	0.88	1.08	3.16	0.00	5.76	1.04
time (sec)	N/A	0.008	0.019	0.001	0.564	0.824	0.000	0.358	1.246

Problem 2835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	25	22	27	112	0	0	110
normalized size	1	1.00	0.83	0.73	0.90	3.73	0.00	0.00	3.67
time (sec)	N/A	0.009	0.024	0.003	0.725	0.809	0.000	0.000	1.377

Problem 2836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	25	22	27	145	0	0	152
normalized size	1	1.00	0.83	0.73	0.90	4.83	0.00	0.00	5.07
time (sec)	N/A	0.010	0.018	0.002	0.573	0.884	0.000	0.000	1.489

Problem 2837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	34	29	21	37	0	15	-1
normalized size	1	1.00	1.26	1.07	0.78	1.37	0.00	0.56	-0.04
time (sec)	N/A	0.010	0.017	0.002	0.504	0.862	0.000	0.240	0.000

Problem 2838	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	29	15	17	65	15	-1
normalized size	1	1.00	1.06	0.91	0.47	0.53	2.03	0.47	-0.03
time (sec)	N/A	0.010	0.019	0.003	0.524	0.797	4.604	0.150	0.000

Problem 2839	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	34	29	21	51	0	0	31
normalized size	1	1.00	1.26	1.07	0.78	1.89	0.00	0.00	1.15
time (sec)	N/A	0.010	0.026	0.002	0.667	1.259	0.000	0.000	1.254

Problem 2840	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	29	15	15	76	15	42
normalized size	1	1.00	1.00	0.85	0.44	0.44	2.24	0.44	1.24
time (sec)	N/A	0.011	0.026	0.003	0.478	0.741	3.152	0.152	1.256

Problem 2841	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	77	112	86	93	99	86	86
normalized size	1	1.00	2.48	3.61	2.77	3.00	3.19	2.77	2.77
time (sec)	N/A	0.026	0.025	0.000	0.535	0.658	0.112	0.165	0.055

Problem 2842	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	172	324	176	207	209	173	162
normalized size	1	1.00	3.37	6.35	3.45	4.06	4.10	3.39	3.18
time (sec)	N/A	0.067	0.035	0.001	0.505	0.777	0.155	0.161	1.201

Problem 2843	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	249	960	284	355	357	288	237
normalized size	1	1.00	5.19	20.00	5.92	7.40	7.44	6.00	4.94
time (sec)	N/A	0.218	0.050	0.000	0.638	0.738	0.190	0.180	1.226

Problem 2844	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	12	10	11	10	11	10
normalized size	1	1.00	1.00	1.00	0.83	0.92	0.83	0.92	0.83
time (sec)	N/A	0.006	0.003	0.000	0.529	0.807	0.109	0.150	0.056

Problem 2845	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	13	11	12	12	12	13
normalized size	1	1.00	1.00	1.00	0.85	0.92	0.92	0.92	1.00
time (sec)	N/A	0.005	0.003	0.000	0.576	0.648	0.120	0.199	1.138

Problem 2846	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	13	11	22	22	12	11
normalized size	1	1.00	1.00	1.00	0.85	1.69	1.69	0.92	0.85
time (sec)	N/A	0.006	0.003	0.000	0.474	0.562	0.143	0.172	1.143

Problem 2847	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	73	289	300	438	0	1290	401
normalized size	1	1.00	0.78	3.11	3.23	4.71	0.00	13.87	4.31
time (sec)	N/A	0.095	0.096	0.016	0.990	0.708	0.000	0.255	1.568

Problem 2848	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	68	68	0	0	0	0	0	-1
normalized size	1	1.24	1.24	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.052	0.039	0.452	0.000	0.883	0.000	0.000	0.000

Problem 2849	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	51	91	140	183	0	345	168
normalized size	1	1.00	0.82	1.47	2.26	2.95	0.00	5.56	2.71
time (sec)	N/A	0.059	0.054	0.009	1.017	0.876	0.000	0.196	1.336

Problem 2850	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	68	68	0	0	0	0	0	-1
normalized size	1	1.24	1.24	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.047	0.018	0.278	0.000	0.950	0.000	0.000	0.000

Problem 2851	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	39	28	56	0	38	55
normalized size	1	1.00	1.00	1.30	0.93	1.87	0.00	1.27	1.83
time (sec)	N/A	0.021	0.009	0.003	0.529	0.905	0.000	0.166	1.205

Problem 2852	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.053	0.038	0.273	0.000	0.782	0.000	0.000	0.000

Problem 2853	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	98	124	95	105	107	105	93
normalized size	1	1.00	3.16	4.00	3.06	3.39	3.45	3.39	3.00
time (sec)	N/A	0.029	0.014	0.000	0.515	0.768	0.089	0.148	0.052

Problem 2854	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	203	470	210	248	252	248	191
normalized size	1	1.00	3.98	9.22	4.12	4.86	4.94	4.86	3.75
time (sec)	N/A	0.063	0.030	0.002	0.634	0.630	0.127	0.161	1.230

Problem 2855	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	323	1948	359	442	437	442	309
normalized size	1	1.00	4.55	27.44	5.06	6.23	6.15	6.23	4.35
time (sec)	N/A	0.108	0.081	0.002	0.580	0.746	0.170	0.161	1.279

Problem 2856	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	102	154	116	138	144	127	113
normalized size	1	1.00	2.76	4.16	3.14	3.73	3.89	3.43	3.05
time (sec)	N/A	0.029	0.006	0.001	0.521	0.636	0.105	0.152	1.188

Problem 2857	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	207	536	240	311	323	290	221
normalized size	1	1.00	3.45	8.93	4.00	5.18	5.38	4.83	3.68
time (sec)	N/A	0.062	0.012	0.000	0.548	0.478	0.143	0.156	1.242

Problem 2858	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	327	2050	398	544	552	510	348
normalized size	1	1.00	3.94	24.70	4.80	6.55	6.65	6.14	4.19
time (sec)	N/A	0.099	0.018	0.002	0.489	0.781	0.187	0.182	1.246

Problem 2859	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	159	93	0	164	46	160	169
normalized size	1	1.00	1.02	0.60	0.00	1.05	0.29	1.03	1.08
time (sec)	N/A	0.167	0.053	0.007	0.000	0.875	0.368	0.216	1.398

Problem 2860	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	142	78	0	135	27	180	158
normalized size	1	1.00	0.99	0.54	0.00	0.94	0.19	1.25	1.10
time (sec)	N/A	0.135	0.022	0.004	0.000	0.687	0.312	0.252	0.238

Problem 2861	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	43	20	42	42	21	42
normalized size	1	1.00	1.00	1.95	0.91	1.91	1.91	0.95	1.91
time (sec)	N/A	0.021	0.008	0.003	0.503	0.761	0.272	0.183	1.191

Problem 2862	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	114	76	0	452	29	141	150
normalized size	1	1.00	0.81	0.54	0.00	3.23	0.21	1.01	1.07
time (sec)	N/A	0.114	0.014	0.003	0.000	0.682	0.258	0.216	1.437

Problem 2863	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	116	71	0	442	26	160	144
normalized size	1	1.00	0.83	0.51	0.00	3.16	0.19	1.14	1.03
time (sec)	N/A	0.110	0.012	0.003	0.000	0.991	0.262	0.167	1.402

Problem 2864	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	57	56	51	49	58	56
normalized size	1	1.00	1.00	1.58	1.56	1.42	1.36	1.61	1.56
time (sec)	N/A	0.028	0.009	0.006	0.546	0.614	0.472	0.208	1.284

Problem 2865	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	140	92	0	150	44	175	138
normalized size	1	1.00	0.91	0.60	0.00	0.97	0.29	1.14	0.90
time (sec)	N/A	0.125	0.048	0.005	0.000	0.832	0.660	0.203	0.375

Problem 2866	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	139	87	0	236	61	193	201
normalized size	1	1.00	0.89	0.56	0.00	1.51	0.39	1.24	1.29
time (sec)	N/A	0.123	0.037	0.003	0.000	0.631	0.870	0.212	0.331

Problem 2867	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	44	75	98	158	100	41	104
normalized size	1	1.00	0.79	1.34	1.75	2.82	1.79	0.73	1.86
time (sec)	N/A	0.041	0.024	0.009	0.590	0.816	1.280	0.209	1.423

Problem 2868	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	152	141	0	838	109	205	240
normalized size	1	1.00	0.88	0.82	0.00	4.87	0.63	1.19	1.40
time (sec)	N/A	0.136	0.105	0.013	0.000	0.738	1.110	0.223	1.498

Problem 2869	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	151	124	0	810	92	214	185
normalized size	1	1.00	0.89	0.73	0.00	4.76	0.54	1.26	1.09
time (sec)	N/A	0.141	0.069	0.011	0.000	0.708	1.031	0.249	0.292

Problem 2870	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	44	21	52	58	21	43
normalized size	1	1.00	1.00	1.91	0.91	2.26	2.52	0.91	1.87
time (sec)	N/A	0.019	0.014	0.002	0.625	0.552	0.961	0.192	1.206

Problem 2871	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	152	144	0	852	105	205	216
normalized size	1	1.00	0.88	0.84	0.00	4.95	0.61	1.19	1.26
time (sec)	N/A	0.136	0.089	0.011	0.000	0.835	1.001	0.286	1.423

Problem 2872	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	151	127	0	808	92	214	207
normalized size	1	1.00	0.89	0.75	0.00	4.75	0.54	1.26	1.22
time (sec)	N/A	0.133	0.059	0.010	0.000	0.978	0.945	0.234	1.456

Problem 2873	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	48	100	104	169	110	101	105
normalized size	1	1.00	0.81	1.69	1.76	2.86	1.86	1.71	1.78
time (sec)	N/A	0.057	0.026	0.018	0.558	0.808	1.483	0.192	0.220

Problem 2874	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	168	227	0	384	170	204	231
normalized size	1	1.00	0.89	1.20	0.00	2.03	0.90	1.08	1.22
time (sec)	N/A	0.160	0.093	0.017	0.000	0.821	1.954	0.236	1.811

Problem 2875	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	166	174	0	509	199	244	255
normalized size	1	1.00	0.88	0.92	0.00	2.69	1.05	1.29	1.35
time (sec)	N/A	0.153	0.086	0.015	0.000	0.878	2.500	0.225	1.978

Problem 2876	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	60	119	222	431	250	65	211
normalized size	1	1.00	0.75	1.49	2.78	5.39	3.12	0.81	2.64
time (sec)	N/A	0.070	0.078	0.020	0.681	0.805	3.578	0.217	1.936

Problem 2877	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	182	214	0	1706	287	274	399
normalized size	1	1.00	0.89	1.04	0.00	8.32	1.40	1.34	1.95
time (sec)	N/A	0.170	0.183	0.020	0.000	0.815	3.442	0.263	1.809

Problem 2878	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	179	186	0	1650	260	268	347
normalized size	1	1.00	0.89	0.93	0.00	8.21	1.29	1.33	1.73
time (sec)	N/A	0.164	0.122	0.020	0.000	0.899	3.314	0.266	1.659

Problem 2879	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	44	21	133	153	21	131
normalized size	1	1.00	1.00	1.91	0.91	5.78	6.65	0.91	5.70
time (sec)	N/A	0.019	0.014	0.002	0.508	0.803	3.055	0.230	1.352

Problem 2880	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	180	214	0	1708	296	268	403
normalized size	1	1.00	0.89	1.06	0.00	8.46	1.47	1.33	2.00
time (sec)	N/A	0.166	0.129	0.019	0.000	0.903	3.277	0.252	0.498

Problem 2881	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	176	185	0	1646	267	264	354
normalized size	1	1.00	0.89	0.93	0.00	8.31	1.35	1.33	1.79
time (sec)	N/A	0.156	0.106	0.018	0.000	1.092	3.272	0.186	1.609

Problem 2882	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	63	283	245	467	269	144	230
normalized size	1	1.00	0.77	3.45	2.99	5.70	3.28	1.76	2.80
time (sec)	N/A	0.075	0.056	0.027	0.656	1.141	4.762	0.239	1.871

Problem 2883	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	196	524	0	855	391	223	431
normalized size	1	1.00	0.89	2.39	0.00	3.90	1.79	1.02	1.97
time (sec)	N/A	0.187	0.156	0.026	0.000	1.178	5.969	0.249	2.578

Problem 2884	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	192	419	0	1024	435	365	520
normalized size	1	1.00	0.88	1.91	0.00	4.68	1.99	1.67	2.37
time (sec)	N/A	0.179	0.158	0.027	0.000	1.280	6.632	0.273	2.688

Problem 2885	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	80	311	438	889	500	80	438
normalized size	1	1.00	0.79	3.08	4.34	8.80	4.95	0.79	4.34
time (sec)	N/A	0.095	0.119	0.031	0.785	1.199	7.805	0.253	3.199

Problem 2886	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	163	102	0	179	66	170	151
normalized size	1	1.00	0.97	0.61	0.00	1.07	0.39	1.01	0.90
time (sec)	N/A	0.134	0.016	0.003	0.000	0.995	0.543	0.236	1.696

Problem 2887	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	145	84	0	147	44	188	162
normalized size	1	1.00	0.93	0.54	0.00	0.94	0.28	1.21	1.04
time (sec)	N/A	0.129	0.020	0.003	0.000	1.009	0.496	0.219	1.516

Problem 2888	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	46	45	45	46	45	45
normalized size	1	1.00	1.00	1.84	1.80	1.80	1.84	1.80	1.80
time (sec)	N/A	0.020	0.009	0.002	0.515	0.762	0.410	0.187	1.411

Problem 2889	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	115	77	0	458	41	147	127
normalized size	1	1.00	0.80	0.54	0.00	3.20	0.29	1.03	0.89
time (sec)	N/A	0.119	0.014	0.001	0.000	0.970	0.408	0.224	0.521

Problem 2890	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	63	62	54	53	62	62
normalized size	1	1.00	1.00	1.50	1.48	1.29	1.26	1.48	1.48
time (sec)	N/A	0.033	0.010	0.005	0.498	1.090	0.751	0.197	0.139

Problem 2891	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	143	98	0	156	54	226	154
normalized size	1	1.00	0.86	0.59	0.00	0.94	0.33	1.36	0.93
time (sec)	N/A	0.130	0.038	0.004	0.000	1.101	0.992	0.214	1.585

Problem 2892	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	142	93	0	245	75	197	200
normalized size	1	1.00	0.85	0.55	0.00	1.46	0.45	1.17	1.19
time (sec)	N/A	0.126	0.039	0.006	0.000	0.980	1.355	0.222	0.421

Problem 2893	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	47	84	116	170	121	82	118
normalized size	1	1.00	0.72	1.29	1.78	2.62	1.86	1.26	1.82
time (sec)	N/A	0.045	0.018	0.008	0.688	1.060	2.101	0.194	1.443

Problem 2894	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	155	221	0	940	133	218	266
normalized size	1	1.00	0.84	1.20	0.00	5.11	0.72	1.18	1.45
time (sec)	N/A	0.142	0.091	0.013	0.000	0.977	1.674	0.252	1.559

Problem 2895	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	154	166	0	906	112	225	194
normalized size	1	1.00	0.85	0.91	0.00	4.98	0.62	1.24	1.07
time (sec)	N/A	0.140	0.068	0.011	0.000	1.041	1.548	0.248	0.340

Problem 2896	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	47	55	55	60	45	46
normalized size	1	1.00	1.00	1.81	2.12	2.12	2.31	1.73	1.77
time (sec)	N/A	0.022	0.014	0.001	0.545	0.946	1.374	0.194	1.254

Problem 2897	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	153	216	0	894	122	218	192
normalized size	1	1.00	0.87	1.23	0.00	5.08	0.69	1.24	1.09
time (sec)	N/A	0.145	0.013	0.011	0.000	1.061	1.373	0.227	0.321

Problem 2898	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	51	109	114	173	122	107	116
normalized size	1	1.00	0.75	1.60	1.68	2.54	1.79	1.57	1.71
time (sec)	N/A	0.058	0.023	0.016	0.572	0.786	2.168	0.236	1.386

Problem 2899	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	171	242	0	399	197	269	261
normalized size	1	1.00	0.84	1.19	0.00	1.96	0.97	1.32	1.28
time (sec)	N/A	0.151	0.086	0.017	0.000	1.181	2.773	0.226	1.850

Problem 2900	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	169	186	0	527	233	250	291
normalized size	1	1.00	0.83	0.91	0.00	2.58	1.14	1.23	1.43
time (sec)	N/A	0.150	0.065	0.016	0.000	1.063	4.105	0.232	2.100

Problem 2901	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	63	131	249	452	294	158	250
normalized size	1	1.00	0.68	1.42	2.71	4.91	3.20	1.72	2.72
time (sec)	N/A	0.067	0.060	0.018	0.646	0.922	5.098	0.205	1.977

Problem 2902	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	185	521	0	1886	332	298	430
normalized size	1	1.00	0.84	2.37	0.00	8.57	1.51	1.35	1.95
time (sec)	N/A	0.160	0.016	0.022	0.000	1.094	3.848	0.287	1.857

Problem 2903	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	182	414	0	1822	298	288	349
normalized size	1	1.00	0.84	1.92	0.00	8.44	1.38	1.33	1.62
time (sec)	N/A	0.165	0.019	0.021	0.000	1.196	3.665	0.275	1.673

Problem 2904	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	47	136	136	155	45	134
normalized size	1	1.00	1.00	1.81	5.23	5.23	5.96	1.73	5.15
time (sec)	N/A	0.021	0.014	0.002	0.576	0.915	3.373	0.241	1.357

Problem 2905	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	181	507	0	1780	323	292	357
normalized size	1	1.00	0.87	2.45	0.00	8.60	1.56	1.41	1.72
time (sec)	N/A	0.174	0.013	0.019	0.000	1.268	3.590	0.275	0.500

Problem 2906	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	66	304	258	474	292	150	249
normalized size	1	1.00	0.70	3.23	2.74	5.04	3.11	1.60	2.65
time (sec)	N/A	0.083	0.038	0.028	0.610	1.098	5.060	0.242	1.911

Problem 2907	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	199	557	0	879	445	295	485
normalized size	1	1.00	0.84	2.35	0.00	3.71	1.88	1.24	2.05
time (sec)	N/A	0.181	0.098	0.026	0.000	1.050	6.126	0.269	2.600

Problem 2908	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	195	446	0	1051	500	369	563
normalized size	1	1.00	0.82	1.88	0.00	4.43	2.11	1.56	2.38
time (sec)	N/A	0.184	0.059	0.025	0.000	1.030	7.361	0.320	3.460

Problem 2909	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	83	335	474	919	578	259	507
normalized size	1	1.00	0.72	2.89	4.09	7.92	4.98	2.23	4.37
time (sec)	N/A	0.089	0.056	0.030	0.712	1.157	8.761	0.247	3.567

Problem 2910	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	63	28	104	0	62	88
normalized size	1	1.00	1.00	2.10	0.93	3.47	0.00	2.07	2.93
time (sec)	N/A	0.030	0.019	0.007	0.442	1.067	0.000	0.175	1.412

Problem 2911	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	31	80	136	21	117	126	25	107
normalized size	1	1.35	3.48	5.91	0.91	5.09	5.48	1.09	4.65
time (sec)	N/A	0.031	0.020	0.000	0.476	0.824	0.095	0.162	0.055

Problem 2912	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	172	622	21	289	299	21	229
normalized size	1	1.00	7.48	27.04	0.91	12.57	13.00	0.91	9.96
time (sec)	N/A	0.093	0.040	0.001	0.549	0.893	0.144	0.183	0.150

Problem 2913	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	308	3262	21	529	541	21	376
normalized size	1	1.00	13.39	141.83	0.91	23.00	23.52	0.91	16.35
time (sec)	N/A	0.166	0.075	0.002	0.642	0.650	0.233	0.219	0.291

Problem 2914	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	55	20	54	56	20	54
normalized size	1	1.00	1.00	2.50	0.91	2.45	2.55	0.91	2.45
time (sec)	N/A	0.024	0.009	0.002	0.552	0.997	0.437	0.194	1.264

Problem 2915	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	56	21	66	73	21	55
normalized size	1	1.00	1.00	2.43	0.91	2.87	3.17	0.91	2.39
time (sec)	N/A	0.026	0.012	0.000	0.463	0.706	2.025	0.200	0.190

Problem 2916	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	56	21	172	197	21	171
normalized size	1	1.00	1.00	2.43	0.91	7.48	8.57	0.91	7.43
time (sec)	N/A	0.026	0.013	0.001	0.559	1.008	5.422	0.251	2.353

Problem 2917	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	90	1036	0	0	0	0	56
normalized size	1	1.00	0.81	9.33	0.00	0.00	0.00	0.00	0.50
time (sec)	N/A	0.101	0.060	0.440	0.000	0.959	0.000	0.000	1.670

Problem 2918	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	330	1528	0	0	0	0	-1
normalized size	1	1.00	2.14	9.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.169	1.009	0.028	0.000	1.221	0.000	0.000	0.000

Problem 2919	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	79	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.111	0.255	0.000	0.000	0.000	0.000	0.000

Problem 2920	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	112	0	0	0	0	0	-1
normalized size	1	1.00	1.51	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.224	0.249	0.000	0.000	0.000	0.000	0.000

Problem 2921	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	135	0	0	0	0	0	-1
normalized size	1	1.00	1.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.344	0.238	0.000	0.000	0.000	0.000	0.000

Problem 2922	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	84	0	0	0	0	0	-1
normalized size	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.163	0.240	0.000	0.000	0.000	0.000	0.000

Problem 2923	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	70	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.123	0.355	0.000	1.035	0.000	0.000	0.000

Problem 2924	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.036	0.061	0.237	0.000	0.892	0.000	0.000	0.000

Problem 2925	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.033	0.041	0.237	0.000	1.007	0.000	0.000	0.000

Problem 2926	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	0	0	0	0	53
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.02
time (sec)	N/A	0.026	0.007	0.255	0.000	0.911	0.000	0.000	2.798

Problem 2927	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.041	0.042	0.236	0.000	0.913	0.000	0.000	0.000

Problem 2928	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	59	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.035	0.054	0.239	0.000	0.935	0.000	0.000	0.000

Problem 2929	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	67	108	34	48	41	108	-1
normalized size	1	1.00	0.81	1.30	0.41	0.58	0.49	1.30	-0.01
time (sec)	N/A	0.039	0.029	0.012	1.239	0.919	0.192	0.201	0.000

Problem 2930	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	103	92	127	103	0	231	-1
normalized size	1	1.00	0.59	0.53	0.73	0.59	0.00	1.33	-0.01
time (sec)	N/A	0.081	0.089	0.008	0.555	0.827	0.000	0.361	0.000

Problem 2931	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	72	63	85	75	0	164	-1
normalized size	1	1.00	0.62	0.54	0.73	0.65	0.00	1.41	-0.01
time (sec)	N/A	0.052	0.058	0.004	0.634	0.933	0.000	0.169	0.000

Problem 2932	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	43	36	43	46	0	90	52
normalized size	1	1.00	0.77	0.64	0.77	0.82	0.00	1.61	0.93
time (sec)	N/A	0.025	0.027	0.008	0.583	0.780	0.000	0.162	1.422

Problem 2933	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	40	60	114	0	38	-1
normalized size	1	1.00	1.00	0.78	1.18	2.24	0.00	0.75	-0.02
time (sec)	N/A	0.024	0.032	0.006	1.218	0.684	0.000	0.158	0.000

Problem 2934	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	52	72	126	174	0	90	-1
normalized size	1	1.00	0.54	0.74	1.30	1.79	0.00	0.93	-0.01
time (sec)	N/A	0.043	0.019	0.012	1.212	1.108	0.000	0.165	0.000

Problem 2935	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	54	114	211	237	0	135	-1
normalized size	1	1.00	0.32	0.67	1.23	1.39	0.00	0.79	-0.01
time (sec)	N/A	0.078	0.020	0.013	1.197	0.950	0.000	0.206	0.000

Problem 2936	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	96	84	2192	98	0	201	-1
normalized size	1	1.00	0.50	0.44	11.48	0.51	0.00	1.05	-0.01
time (sec)	N/A	0.068	0.052	0.006	1.259	0.929	0.000	0.222	0.000

Problem 2937	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	64	55	387	70	0	135	-1
normalized size	1	1.00	0.57	0.49	3.42	0.62	0.00	1.19	-0.01
time (sec)	N/A	0.040	0.031	0.005	0.709	0.914	0.000	0.166	0.000

Problem 2938	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	27	42	40	0	18	-1
normalized size	1	1.00	1.00	0.79	1.24	1.18	0.00	0.53	-0.03
time (sec)	N/A	0.011	0.008	0.001	0.677	0.957	0.000	0.159	0.000

Problem 2939	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	87	54	0	180	0	54	-1
normalized size	1	1.00	1.30	0.81	0.00	2.69	0.00	0.81	-0.01
time (sec)	N/A	0.029	0.052	0.010	0.000	1.106	0.000	0.164	0.000

Problem 2940	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	63	97	0	252	0	114	-1
normalized size	1	1.00	0.44	0.67	0.00	1.75	0.00	0.79	-0.01
time (sec)	N/A	0.059	0.015	0.014	0.000	0.738	0.000	0.166	0.000

Problem 2941	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	63	133	0	314	0	156	-1
normalized size	1	1.00	0.29	0.61	0.00	1.43	0.00	0.71	-0.00
time (sec)	N/A	0.094	0.014	0.015	0.000	1.045	0.000	0.172	0.000

Problem 2942	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	67	0	0	78	0	64	-1
normalized size	1	1.00	0.59	0.00	0.00	0.69	0.00	0.57	-0.01
time (sec)	N/A	0.067	0.043	0.243	0.000	0.882	0.000	0.178	0.000

Problem 2943	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	43	0	43	56	0	38	-1
normalized size	1	1.00	0.77	0.00	0.77	1.00	0.00	0.68	-0.02
time (sec)	N/A	0.045	0.024	0.227	0.534	1.089	0.000	0.178	0.000

Problem 2944	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	29	0	46	0	20	-1
normalized size	1	1.00	1.00	0.81	0.00	1.28	0.00	0.56	-0.03
time (sec)	N/A	0.017	0.011	0.005	0.000	1.046	0.000	0.156	0.000

Problem 2945	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	0	61	133	0	42	-1
normalized size	1	1.00	1.00	0.00	1.11	2.42	0.00	0.76	-0.02
time (sec)	N/A	0.043	0.025	0.227	1.152	0.798	0.000	0.166	0.000

Problem 2946	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	93	0	0	206	0	61	-1
normalized size	1	1.00	1.31	0.00	0.00	2.90	0.00	0.86	-0.01
time (sec)	N/A	0.064	0.088	0.223	0.000	1.081	0.000	0.169	0.000

Problem 2947	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	109	0	0	0	0	0	-1
normalized size	1	1.00	0.32	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.247	0.112	0.224	0.000	1.002	0.000	0.000	0.000

Problem 2948	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	64	0	0	0	0	0	-1
normalized size	1	1.00	0.21	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.143	0.013	0.224	0.000	0.919	0.000	0.000	0.000

Problem 2949	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	69	0	0	0	0	0	-1
normalized size	1	1.00	0.23	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.143	0.020	0.223	0.000	1.063	0.000	0.000	0.000

Problem 2950	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	69	0	0	0	0	0	-1
normalized size	1	1.00	0.20	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.186	0.017	0.235	0.000	1.258	0.000	0.000	0.000

Problem 2951	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	709	709	109	0	0	0	0	0	-1
normalized size	1	1.00	0.15	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.490	0.095	0.229	0.000	1.299	0.000	0.000	0.000

Problem 2952	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	642	642	69	495	0	0	0	0	55
normalized size	1	1.00	0.11	0.77	0.00	0.00	0.00	0.00	0.09
time (sec)	N/A	0.361	0.019	0.012	0.000	1.379	0.000	0.000	1.545

Problem 2953	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	661	661	67	0	0	0	0	0	-1
normalized size	1	1.00	0.10	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.347	0.019	0.221	0.000	8.807	0.000	0.000	0.000

Problem 2954	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	681	681	69	0	0	0	0	0	-1
normalized size	1	1.00	0.10	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.417	0.042	0.223	0.000	1.117	0.000	0.000	0.000

Problem 2955	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.081	0.223	0.000	1.017	0.000	0.000	0.000

Problem 2956	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	74	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.081	0.221	0.000	0.960	0.000	0.000	0.000

Problem 2957	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	81	0	0	0	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.123	0.226	0.000	1.213	0.000	0.000	0.000

Problem 2958	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.162	0.227	0.000	1.130	0.000	0.000	0.000

Problem 2959	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	2	6	2	2	-1
normalized size	1	1.00	1.00	0.82	0.12	0.35	0.12	0.12	-0.06
time (sec)	N/A	0.005	0.011	0.220	1.171	0.907	0.133	0.164	0.000

Problem 2960	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	72	103	85	75	0	111	-1
normalized size	1	1.00	0.62	0.89	0.73	0.65	0.00	0.96	-0.01
time (sec)	N/A	0.069	0.098	0.434	0.638	1.304	0.000	0.204	0.000

Problem 2961	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	43	65	43	46	0	57	52
normalized size	1	1.00	0.77	1.16	0.77	0.82	0.00	1.02	0.93
time (sec)	N/A	0.035	0.036	0.290	0.454	1.021	0.000	0.183	1.383

Problem 2962	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	40	61	0	0	67	-1
normalized size	1	1.00	1.00	0.73	1.11	0.00	0.00	1.22	-0.02
time (sec)	N/A	0.035	0.046	0.289	1.264	0.000	0.000	0.206	0.000

Problem 2963	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	A	A	F(-1)	F	A	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	0	81	126	0	0	122	-1
normalized size	1	1.00	0.00	0.84	1.30	0.00	0.00	1.26	-0.01
time (sec)	N/A	0.055	0.048	0.303	1.165	0.000	0.000	0.211	0.000

Problem 2964	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	A	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	400	400	0	350	0	0	0	0	-1
normalized size	1	1.00	0.00	0.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.248	0.044	0.434	0.000	6.831	0.000	0.000	0.000

Problem 2965	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	A	F	F	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	0	306	0	0	0	0	-1
normalized size	1	1.00	0.00	0.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.149	0.043	0.289	0.000	6.022	0.000	0.000	0.000

Problem 2966	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	A	F	F	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	434	434	0	346	0	0	0	0	-1
normalized size	1	1.00	0.00	0.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.268	0.044	0.300	0.000	4.687	0.000	0.000	0.000

Problem 2967	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	A	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	843	843	0	932	0	0	0	0	-1
normalized size	1	1.00	0.00	1.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.636	0.058	0.288	0.000	5.837	0.000	0.000	0.000

Problem 2968	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	A	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	770	770	0	854	0	0	0	0	-1
normalized size	1	1.00	0.00	1.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.394	0.013	0.353	0.000	5.779	0.000	0.000	0.000

Problem 2969	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	A	F	F	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	810	810	0	863	0	0	0	0	-1
normalized size	1	1.00	0.00	1.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.459	0.060	0.298	0.000	6.101	0.000	0.000	0.000

Problem 2970	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	80	0	85	87	0	121	-1
normalized size	1	1.00	0.69	0.00	0.73	0.75	0.00	1.04	-0.01
time (sec)	N/A	0.076	0.150	0.247	0.598	5.273	0.000	0.284	0.000

Problem 2971	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	43	0	43	56	0	63	-1
normalized size	1	1.00	0.77	0.00	0.77	1.00	0.00	1.12	-0.02
time (sec)	N/A	0.037	0.076	0.257	0.620	2.858	0.000	0.199	0.000

Problem 2972	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	0	61	0	0	74	-1
normalized size	1	1.00	1.00	0.00	1.11	0.00	0.00	1.35	-0.02
time (sec)	N/A	0.058	0.071	0.230	1.245	0.000	0.000	0.224	0.000

Problem 2973	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	A	F(-1)	F	A	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	104	0	0	128	0	0	130	-1
normalized size	1	1.03	0.00	0.00	1.27	0.00	0.00	1.29	-0.01
time (sec)	N/A	0.065	0.065	0.238	1.207	0.000	0.000	0.247	0.000

Problem 2974	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	642	642	69	495	0	0	0	0	52
normalized size	1	1.00	0.11	0.77	0.00	0.00	0.00	0.00	0.08
time (sec)	N/A	0.735	0.053	0.013	0.000	2.401	0.000	0.000	1.480

Problem 2975	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.068	0.232	0.000	0.000	0.000	0.000	0.000

Problem 2976	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.017	0.231	0.000	0.000	0.000	0.000	0.000

Problem 2977	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	141	0	0	0	0	0	0	-1
normalized size	1	1.01	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.056	0.228	0.000	0.000	0.000	0.000	0.000

Problem 2978	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.085	0.232	0.000	0.000	0.000	0.000	0.000

Problem 2979	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	84	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.096	0.231	0.000	0.000	0.000	0.000	0.000

Problem 2980	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	89	0	0	0	0	0	-1
normalized size	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.151	0.231	0.000	0.000	0.000	0.000	0.000

Problem 2981	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.189	0.228	0.000	0.000	0.000	0.000	0.000

Problem 2982	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	172	111	211	211	224	0	0	-1
normalized size	1	1.02	0.66	1.25	1.25	1.33	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.161	0.030	1.349	1.300	0.000	0.000	0.000

Problem 2983	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	79	147	126	161	0	130	-1
normalized size	1	1.00	0.86	1.60	1.37	1.75	0.00	1.41	-0.01
time (sec)	N/A	0.049	0.074	0.022	1.323	1.361	0.000	0.363	0.000

Problem 2984	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	150	61	110	0	0	-1
normalized size	1	1.00	1.00	2.94	1.20	2.16	0.00	0.00	-0.02
time (sec)	N/A	0.035	0.049	0.023	1.290	0.935	0.000	0.000	0.000

Problem 2985	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	43	70	43	48	0	0	52
normalized size	1	1.00	0.77	1.25	0.77	0.86	0.00	0.00	0.93
time (sec)	N/A	0.038	0.036	0.023	0.537	0.656	0.000	0.000	1.392

Problem 2986	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	75	97	85	77	0	0	-1
normalized size	1	1.00	0.65	0.84	0.73	0.66	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.054	0.020	0.451	0.915	0.000	0.000	0.000

Problem 2987	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	111	133	127	105	0	0	-1
normalized size	1	1.00	0.64	0.76	0.73	0.60	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.065	0.025	0.591	0.898	0.000	0.000	0.000

Problem 2988	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	175	126	298	211	224	0	0	-1
normalized size	1	1.02	0.73	1.73	1.23	1.30	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.162	0.047	1.339	0.959	0.000	0.000	0.000

Problem 2989	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	89	229	131	165	0	139	-1
normalized size	1	1.00	0.94	2.41	1.38	1.74	0.00	1.46	-0.01
time (sec)	N/A	0.052	0.101	0.038	1.163	0.966	0.000	0.408	0.000

Problem 2990	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	200	45	81	0	0	-1
normalized size	1	1.00	1.00	6.45	1.45	2.61	0.00	0.00	-0.03
time (sec)	N/A	0.033	0.046	0.043	1.114	0.964	0.000	0.000	0.000

Problem 2991	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	42	274	42	34	0	60	52
normalized size	1	1.00	0.78	5.07	0.78	0.63	0.00	1.11	0.96
time (sec)	N/A	0.044	0.036	0.045	0.557	0.743	0.000	0.177	1.440

Problem 2992	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	75	336	85	61	0	152	-1
normalized size	1	1.00	0.67	3.00	0.76	0.54	0.00	1.36	-0.01
time (sec)	N/A	0.071	0.078	0.046	0.533	1.081	0.000	0.252	0.000

Problem 2993	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	111	400	127	89	0	0	-1
normalized size	1	1.00	0.65	2.33	0.74	0.52	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.092	0.056	0.478	0.909	0.000	0.000	0.000

Problem 2994	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	70	92	62	55	0	36	37
normalized size	1	1.00	1.21	1.59	1.07	0.95	0.00	0.62	0.64
time (sec)	N/A	0.016	0.143	0.024	0.552	0.729	0.000	0.240	1.880

Problem 2995	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.151	0.236	0.000	0.000	0.000	0.000	0.000

Problem 2996	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	80	78	0	0	0	0	0	-1
normalized size	1	1.33	1.30	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.090	0.097	0.235	0.000	0.000	0.000	0.000	0.000

Problem 2997	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	85	0	0	0	0	0	-1
normalized size	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.118	0.227	0.000	0.000	0.000	0.000	0.000

Problem 2998	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.222	0.233	0.000	0.000	0.000	0.000	0.000

Problem 2999	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.247	0.230	0.000	0.000	0.000	0.000	0.000

Problem 3000	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	78	96	0	0	0	0	0	-1
normalized size	1	1.34	1.66	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.088	0.248	0.247	0.000	0.000	0.000	0.000	0.000

Problem 3001	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	116	0	0	0	0	0	-1
normalized size	1	1.00	1.53	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.435	0.233	0.000	0.000	0.000	0.000	0.000

Problem 3002	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.291	0.229	0.000	0.000	0.000	0.000	0.000

Problem 3003	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	22	0	15	19	15	17
normalized size	1	1.00	1.00	1.16	0.00	0.79	1.00	0.79	0.89
time (sec)	N/A	0.005	0.006	0.023	0.000	0.850	0.238	0.162	1.195

Problem 3004	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	38	0	0	32	39	32	36
normalized size	1	1.00	1.12	0.00	0.00	0.94	1.15	0.94	1.06
time (sec)	N/A	0.009	0.020	0.207	0.000	0.889	0.431	0.189	1.234

Problem 3005	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	60	0	0	50	63	50	56
normalized size	1	1.00	1.76	0.00	0.00	1.47	1.85	1.47	1.65
time (sec)	N/A	0.008	0.037	0.204	0.000	0.824	0.781	0.212	1.275

Problem 3006	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	87	387	0	74	0	0	-1
normalized size	1	1.00	0.86	3.83	0.00	0.73	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.063	0.197	0.000	0.932	0.000	0.000	0.000

Problem 3007	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	67	310	0	55	0	0	-1
normalized size	1	1.00	0.87	4.03	0.00	0.71	0.00	0.00	-0.01
time (sec)	N/A	0.027	0.044	0.115	0.000	1.003	0.000	0.000	0.000

Problem 3008	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	49	233	0	36	0	0	-1
normalized size	1	1.00	0.92	4.40	0.00	0.68	0.00	0.00	-0.02
time (sec)	N/A	0.018	0.018	0.117	0.000	0.779	0.000	0.000	0.000

Problem 3009	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	139	0	22	0	0	-1
normalized size	1	1.00	1.00	4.63	0.00	0.73	0.00	0.00	-0.03
time (sec)	N/A	0.008	0.004	0.060	0.000	0.871	0.000	0.000	0.000

Problem 3010	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	23	35	40	21	56	0	24
normalized size	1	1.00	0.88	1.35	1.54	0.81	2.15	0.00	0.92
time (sec)	N/A	0.009	0.010	0.016	0.649	0.886	1.553	0.000	1.223

Problem 3011	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	49	220	0	41	0	0	-1
normalized size	1	1.00	0.82	3.67	0.00	0.68	0.00	0.00	-0.02
time (sec)	N/A	0.023	0.025	0.160	0.000	0.934	0.000	0.000	0.000

Problem 3012	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	75	302	0	65	0	0	-1
normalized size	1	1.00	0.86	3.47	0.00	0.75	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.052	0.180	0.000	0.872	0.000	0.000	0.000

Problem 3013	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	89	463	0	105	0	0	-1
normalized size	1	1.00	0.78	4.06	0.00	0.92	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.080	0.128	0.000	0.907	0.000	0.000	0.000

Problem 3014	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	67	386	0	83	0	0	-1
normalized size	1	1.00	0.74	4.29	0.00	0.92	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.066	0.133	0.000	0.956	0.000	0.000	0.000

Problem 3015	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	50	310	0	52	0	0	-1
normalized size	1	1.00	0.75	4.63	0.00	0.78	0.00	0.00	-0.01
time (sec)	N/A	0.026	0.036	0.113	0.000	0.917	0.000	0.000	0.000

Problem 3016	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	32	33	74	23	25	80	0	20
normalized size	1	1.60	1.65	3.70	1.15	1.25	4.00	0.00	1.00
time (sec)	N/A	0.008	0.018	0.026	0.540	0.860	169.225	0.000	1.191

Problem 3017	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	40	54	61	57	0	0	44
normalized size	1	1.00	0.89	1.20	1.36	1.27	0.00	0.00	0.98
time (sec)	N/A	0.020	0.039	0.009	0.528	0.930	0.000	0.000	1.187

Problem 3018	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	71	296	0	99	0	0	-1
normalized size	1	1.00	0.76	3.15	0.00	1.05	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.131	0.143	0.000	0.925	0.000	0.000	0.000

Problem 3019	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	99	379	0	131	0	0	-1
normalized size	1	1.00	0.79	3.03	0.00	1.05	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.212	0.161	0.000	0.925	0.000	0.000	0.000

Problem 3020	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	143	69	43	0	0	36
normalized size	1	1.00	1.00	4.21	2.03	1.26	0.00	0.00	1.06
time (sec)	N/A	0.008	0.015	0.030	0.562	0.887	0.000	0.000	1.199

Problem 3021	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	35	63	0	16	19	11	-1
normalized size	1	1.00	0.73	1.31	0.00	0.33	0.40	0.23	-0.02
time (sec)	N/A	0.011	0.021	0.044	0.000	0.966	0.111	0.169	0.000

Problem 3022	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	113	1127	0	183	0	384	-1
normalized size	1	1.00	0.66	6.59	0.00	1.07	0.00	2.25	-0.01
time (sec)	N/A	0.064	0.131	0.368	0.000	0.926	0.000	0.469	0.000

Problem 3023	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	95	787	0	129	0	243	-1
normalized size	1	1.00	0.75	6.25	0.00	1.02	0.00	1.93	-0.01
time (sec)	N/A	0.045	0.053	0.314	0.000	0.904	0.000	0.455	0.000

Problem 3024	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	63	304	0	79	0	136	-1
normalized size	1	1.00	0.76	3.66	0.00	0.95	0.00	1.64	-0.01
time (sec)	N/A	0.030	0.030	0.183	0.000	0.925	0.000	0.458	0.000

Problem 3025	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	64	147	0	37	0	54	-1
normalized size	1	1.00	1.68	3.87	0.00	0.97	0.00	1.42	-0.03
time (sec)	N/A	0.012	0.147	0.163	0.000	0.902	0.000	0.306	0.000

Problem 3026	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.016	0.017	180.000	0.000	0.903	0.000	0.000	0.000

Problem 3027	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.012	1.043	0.000	0.834	0.000	0.000	0.000

Problem 3028	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	0	0	52	63	52	58
normalized size	1	1.00	1.00	0.00	0.00	0.84	1.02	0.84	0.94
time (sec)	N/A	0.023	0.042	0.326	0.000	0.700	0.995	0.201	1.246

Problem 3029	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	0	0	35	42	35	39
normalized size	1	1.00	1.00	0.00	0.00	0.81	0.98	0.81	0.91
time (sec)	N/A	0.017	0.016	0.313	0.000	0.918	0.571	0.194	1.216

Problem 3030	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	23	0	17	19	17	19
normalized size	1	1.00	1.00	1.10	0.00	0.81	0.90	0.81	0.90
time (sec)	N/A	0.006	0.002	0.049	0.000	0.878	0.312	0.163	1.169

Problem 3031	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	222	0	127	0	0	-1
normalized size	1	1.00	1.00	5.05	0.00	2.89	0.00	0.00	-0.02
time (sec)	N/A	0.012	0.010	0.467	0.000	0.658	0.000	0.000	0.000

Problem 3032	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	305	0	218	0	0	-1
normalized size	1	1.00	1.00	4.18	0.00	2.99	0.00	0.00	-0.01
time (sec)	N/A	0.024	0.064	0.416	0.000	0.899	0.000	0.000	0.000

Problem 3033	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	91	378	0	328	0	0	-1
normalized size	1	1.00	0.93	3.86	0.00	3.35	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.137	0.434	0.000	0.976	0.000	0.000	0.000

Problem 3034	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	29	6	6	5	6	-1
normalized size	1	1.00	1.00	1.32	0.27	0.27	0.23	0.27	-0.05
time (sec)	N/A	0.006	0.010	0.048	1.357	0.946	0.137	0.147	0.000

Problem 3035	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	29	17	17	15	15	-1
normalized size	1	1.00	1.00	1.32	0.77	0.77	0.68	0.68	-0.05
time (sec)	N/A	0.005	0.005	0.043	0.537	0.678	0.145	0.148	0.000

Problem 3036	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	6	6	5	6	-1
normalized size	1	1.00	1.00	0.77	0.27	0.27	0.23	0.27	-0.05
time (sec)	N/A	0.006	0.017	0.329	1.250	0.754	0.126	0.149	0.000

Problem 3037	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	17	17	15	15	-1
normalized size	1	1.00	1.00	0.77	0.77	0.77	0.68	0.68	-0.05
time (sec)	N/A	0.005	0.019	0.319	0.472	0.906	0.130	0.148	0.000

Problem 3038	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	42	0	6	5	6	-1
normalized size	1	1.00	1.00	1.24	0.00	0.18	0.15	0.18	-0.03
time (sec)	N/A	0.007	0.009	0.165	0.000	0.872	0.127	0.165	0.000

Problem 3039	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	42	0	17	15	15	-1
normalized size	1	1.00	1.00	1.24	0.00	0.50	0.44	0.44	-0.03
time (sec)	N/A	0.006	0.005	0.134	0.000	0.876	0.135	0.151	0.000

Problem 3040	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	0	0	53	66	53	59
normalized size	1	1.00	1.00	0.00	0.00	0.82	1.02	0.82	0.91
time (sec)	N/A	0.023	0.024	0.335	0.000	0.944	1.057	0.398	1.250

Problem 3041	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	0	0	35	41	35	39
normalized size	1	1.00	1.00	0.00	0.00	0.81	0.95	0.81	0.91
time (sec)	N/A	0.015	0.014	0.320	0.000	0.882	0.563	0.198	1.217

Problem 3042	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	23	0	17	19	17	19
normalized size	1	1.00	1.00	1.10	0.00	0.81	0.90	0.81	0.90
time (sec)	N/A	0.006	0.002	0.050	0.000	0.903	0.297	0.166	1.169

Problem 3043	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	133	821	0	549	0	0	-1
normalized size	1	1.00	0.73	4.49	0.00	3.00	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.090	0.440	0.000	0.791	0.000	0.000	0.000

Problem 3044	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	158	908	0	705	0	0	-1
normalized size	1	1.00	0.75	4.32	0.00	3.36	0.00	0.00	-0.00
time (sec)	N/A	0.095	0.336	0.385	0.000	1.043	0.000	0.000	0.000

Problem 3045	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	215	981	0	885	0	0	-1
normalized size	1	1.00	0.91	4.17	0.00	3.77	0.00	0.00	-0.00
time (sec)	N/A	0.116	0.226	0.395	0.000	0.807	0.000	0.000	0.000

Problem 3046	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	82	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.229	2.611	0.000	0.656	0.000	0.000	0.000

Problem 3047	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.101	0.895	0.000	0.659	0.000	0.000	0.000

Problem 3048	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.024	0.086	0.820	0.000	0.936	0.000	0.000	0.000

Problem 3049	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.072	0.937	0.000	0.880	0.000	0.000	0.000

Problem 3050	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.034	0.059	180.000	0.000	0.809	0.000	0.000	0.000

Problem 3051	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.108	1.191	0.000	0.947	0.000	0.000	0.000

Problem 3052	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.719	0.183	0.215	0.000	0.000	0.000	0.000	0.000

Problem 3053	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	0	655	0	0	0	0	-1
normalized size	1	1.00	0.00	1.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.600	0.214	0.187	0.000	0.000	0.000	0.000	0.000

Problem 3054	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	0	398	0	0	0	0	-1
normalized size	1	1.00	0.00	1.90	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.284	0.330	0.164	0.000	0.000	0.000	0.000	0.000

Problem 3055	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	F(-1)	F	B	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	0	213	0	0	0	213	-1
normalized size	1	1.00	0.00	1.88	0.00	0.00	0.00	1.88	-0.01
time (sec)	N/A	0.118	0.172	0.145	0.000	0.000	0.000	0.987	0.000

Problem 3056	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	0	237	0	0	0	0	-1
normalized size	1	1.00	0.00	1.63	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.208	0.148	0.155	0.000	0.000	0.000	0.000	0.000

Problem 3057	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	0	331	0	0	0	0	-1
normalized size	1	1.00	0.00	2.14	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.184	0.134	0.130	0.000	0.000	0.000	0.000	0.000

Problem 3058	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	0	615	0	0	0	0	-1
normalized size	1	1.00	0.00	2.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.308	0.132	0.139	0.000	0.000	0.000	0.000	0.000

Problem 3059	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	0	979	0	0	0	0	-1
normalized size	1	1.00	0.00	2.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.665	0.141	0.149	0.000	0.000	0.000	0.000	0.000

Problem 3060	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.475	0.257	0.051	0.000	0.000	0.000	0.000	0.000

Problem 3061	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	A	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	386	386	0	655	0	0	0	0	-1
normalized size	1	1.00	0.00	1.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.729	0.414	0.156	0.000	0.000	0.000	0.000	0.000

Problem 3062	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	A	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	0	398	0	0	0	0	-1
normalized size	1	1.00	0.00	1.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.430	0.329	0.139	0.000	0.000	0.000	0.000	0.000

Problem 3063	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	A	F	F(-1)	F	A	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	0	213	0	0	0	218	-1
normalized size	1	1.00	0.00	1.58	0.00	0.00	0.00	1.61	-0.01
time (sec)	N/A	0.180	0.378	0.140	0.000	0.000	0.000	0.592	0.000

Problem 3064	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	94	0	0	0	0	-1
normalized size	1	1.00	1.00	1.74	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.098	0.385	0.136	0.000	0.000	0.000	0.000	0.000

Problem 3065	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	A	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	0	120	0	0	0	0	-1
normalized size	1	1.00	0.00	1.29	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.200	0.144	0.000	0.000	0.000	0.000	0.000

Problem 3066	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	A	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	0	267	0	0	0	0	-1
normalized size	1	1.00	0.00	1.62	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.228	0.201	0.160	0.000	0.000	0.000	0.000	0.000

Problem 3067	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	A	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	0	487	0	0	0	0	-1
normalized size	1	1.00	0.00	1.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.539	0.210	0.164	0.000	0.000	0.000	0.000	0.000

Problem 3068	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	32	0	18	0	11	-1
normalized size	1	1.00	1.00	1.23	0.00	0.69	0.00	0.42	-0.04
time (sec)	N/A	0.026	0.018	0.107	0.000	0.931	0.000	0.167	0.000

Problem 3069	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	123	0	94	0	74	-1
normalized size	1	1.00	1.00	1.64	0.00	1.25	0.00	0.99	-0.01
time (sec)	N/A	0.042	0.083	0.114	0.000	3.688	0.000	0.200	0.000

Problem 3070	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	61	377	0	77	0	130	-1
normalized size	1	1.00	0.77	4.77	0.00	0.97	0.00	1.65	-0.01
time (sec)	N/A	0.041	0.054	0.819	0.000	0.881	0.000	0.504	0.000

Problem 3071	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	48	0	0	60	76	60	68
normalized size	1	1.00	0.69	0.00	0.00	0.86	1.09	0.86	0.97
time (sec)	N/A	0.031	0.039	0.365	0.000	0.466	1.837	0.269	1.255

Problem 3072	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	49	0	0	43	56	43	47
normalized size	1	1.00	0.89	0.00	0.00	0.78	1.02	0.78	0.85
time (sec)	N/A	0.025	0.020	0.389	0.000	0.435	0.997	0.233	1.217

Problem 3073	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	30	0	0	25	32	25	28
normalized size	1	1.00	0.91	0.00	0.00	0.76	0.97	0.76	0.85
time (sec)	N/A	0.014	0.012	0.059	0.000	0.420	0.499	0.191	1.182

Problem 3074	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	35	147	0	34	0	0	-1
normalized size	1	1.00	0.92	3.87	0.00	0.89	0.00	0.00	-0.03
time (sec)	N/A	0.024	0.032	0.187	0.000	0.429	0.000	0.000	0.000

Problem 3075	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	48	214	0	50	0	0	-1
normalized size	1	1.00	0.76	3.40	0.00	0.79	0.00	0.00	-0.02
time (sec)	N/A	0.032	0.037	0.164	0.000	0.424	0.000	0.000	0.000

Problem 3076	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	137	60	56	160	0	30
normalized size	1	1.00	1.00	4.28	1.88	1.75	5.00	0.00	0.94
time (sec)	N/A	0.013	0.009	0.072	0.675	0.425	178.513	0.000	1.183

Problem 3077	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	48	205	109	74	0	0	127
normalized size	1	1.00	0.69	2.93	1.56	1.06	0.00	0.00	1.81
time (sec)	N/A	0.035	0.045	0.188	0.783	0.440	0.000	0.000	1.182

Problem 3078	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	48	279	158	92	0	0	208
normalized size	1	1.00	0.69	3.99	2.26	1.31	0.00	0.00	2.97
time (sec)	N/A	0.035	0.034	0.185	0.654	0.424	0.000	0.000	1.202

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [2110] had the largest ratio of [1.333]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	11	0.182
2	A	2	2	1.00	13	0.154
3	A	2	2	1.00	13	0.154
4	A	2	2	1.00	11	0.182
5	A	2	2	1.00	9	0.222
6	A	2	2	1.00	13	0.154
7	A	2	2	1.00	13	0.154
8	A	2	2	1.00	13	0.154
9	A	2	2	1.00	13	0.154
10	A	2	2	1.00	13	0.154
11	A	2	2	1.00	7	0.286
12	A	2	2	1.00	13	0.154
13	A	2	2	1.00	11	0.182
14	A	2	2	1.00	9	0.222
15	A	2	2	1.00	13	0.154
16	A	2	2	1.00	13	0.154
17	A	2	2	1.00	13	0.154
18	A	2	2	1.00	13	0.154
19	A	2	2	1.00	13	0.154
20	A	2	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
21	A	2	2	1.00	13	0.154
22	A	2	2	1.00	7	0.286
23	A	2	2	1.00	11	0.182
24	A	2	2	1.00	9	0.222
25	A	2	2	1.00	13	0.154
26	A	2	2	1.00	13	0.154
27	A	2	2	1.00	13	0.154
28	A	2	2	1.00	13	0.154
29	A	2	2	1.00	13	0.154
30	A	2	2	1.00	13	0.154
31	A	2	2	1.00	13	0.154
32	A	2	2	1.00	13	0.154
33	A	2	2	1.00	13	0.154
34	A	2	2	1.00	7	0.286
35	A	2	2	1.00	13	0.154
36	A	2	2	1.00	11	0.182
37	A	2	2	1.00	13	0.154
38	A	2	2	1.00	13	0.154
39	A	2	2	1.00	13	0.154
40	A	2	2	1.00	9	0.222
41	A	2	2	1.00	13	0.154
42	A	2	2	1.00	13	0.154
43	A	2	2	1.00	13	0.154
44	A	2	2	1.00	11	0.182
45	A	2	2	1.00	13	0.154
46	A	2	2	1.00	13	0.154
47	A	2	2	1.00	13	0.154
48	A	2	2	1.00	13	0.154
49	A	2	2	1.00	13	0.154
50	A	2	2	1.00	9	0.222
51	A	2	2	1.00	13	0.154
52	A	2	2	1.00	13	0.154
53	A	2	2	1.00	13	0.154
54	A	2	2	1.00	13	0.154
55	A	2	2	1.00	11	0.182
56	A	2	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	2	2	1.00	13	0.154
58	A	2	2	1.00	13	0.154
59	A	2	2	1.00	13	0.154
60	A	2	2	1.00	9	0.222
61	A	2	2	1.00	13	0.154
62	A	3	3	1.00	15	0.200
63	A	3	3	1.00	15	0.200
64	A	3	3	1.00	15	0.200
65	A	3	3	1.00	15	0.200
66	A	2	2	1.00	11	0.182
67	A	3	3	1.00	13	0.231
68	A	2	2	1.00	13	0.154
69	A	2	2	1.00	11	0.182
70	A	2	2	1.00	11	0.182
71	A	2	2	1.00	9	0.222
72	A	2	2	1.00	7	0.286
73	A	2	2	1.00	11	0.182
74	A	2	2	1.00	11	0.182
75	A	2	2	1.00	11	0.182
76	A	2	2	1.00	11	0.182
77	A	2	2	1.00	9	0.222
78	A	2	2	1.00	9	0.222
79	A	1	1	1.00	7	0.143
80	A	2	2	1.00	9	0.222
81	A	2	2	1.00	9	0.222
82	A	2	2	1.00	9	0.222
83	A	2	2	1.00	9	0.222
84	A	2	2	1.00	9	0.222
85	A	1	1	1.00	7	0.143
86	A	2	2	1.00	9	0.222
87	A	2	2	1.00	9	0.222
88	A	2	2	1.00	9	0.222
89	A	2	2	1.00	9	0.222
90	A	2	2	1.00	9	0.222
91	A	1	1	1.00	7	0.143
92	A	2	2	1.00	9	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
93	A	2	2	1.00	9	0.222
94	A	2	2	1.00	9	0.222
95	A	2	2	1.00	9	0.222
96	A	2	2	1.00	9	0.222
97	A	1	1	1.00	7	0.143
98	A	2	2	1.00	9	0.222
99	A	2	2	1.00	9	0.222
100	A	2	2	1.00	9	0.222
101	A	2	2	1.00	9	0.222
102	A	2	2	1.00	9	0.222
103	A	2	2	1.00	9	0.222
104	A	1	1	1.00	7	0.143
105	A	2	2	1.00	9	0.222
106	A	2	2	1.00	9	0.222
107	A	2	2	1.00	9	0.222
108	A	2	2	1.00	9	0.222
109	A	2	2	1.00	9	0.222
110	A	2	2	1.00	9	0.222
111	A	1	1	1.00	7	0.143
112	A	2	2	1.00	9	0.222
113	A	2	2	1.00	9	0.222
114	A	2	2	1.00	9	0.222
115	A	2	2	1.00	9	0.222
116	A	2	2	1.00	9	0.222
117	A	2	2	1.00	9	0.222
118	A	2	2	1.00	9	0.222
119	A	2	2	1.00	9	0.222
120	A	1	1	1.00	7	0.143
121	A	2	2	1.00	9	0.222
122	A	2	2	1.00	9	0.222
123	A	2	2	1.00	9	0.222
124	A	2	2	1.00	9	0.222
125	A	2	2	1.00	9	0.222
126	A	2	2	1.00	9	0.222
127	A	1	1	1.00	7	0.143
128	A	2	2	1.00	9	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
129	A	2	2	1.00	9	0.222
130	A	2	2	1.00	9	0.222
131	A	2	2	1.00	13	0.154
132	A	2	2	1.00	11	0.182
133	A	2	2	1.00	9	0.222
134	A	2	2	1.00	13	0.154
135	A	2	2	1.00	13	0.154
136	A	2	2	1.00	13	0.154
137	A	2	2	1.00	11	0.182
138	A	2	2	1.00	9	0.222
139	A	2	2	1.00	13	0.154
140	A	2	2	1.00	13	0.154
141	A	2	2	1.00	13	0.154
142	A	2	2	1.00	13	0.154
143	A	2	2	1.00	13	0.154
144	A	2	2	1.00	11	0.182
145	A	2	2	1.00	9	0.222
146	A	2	2	1.00	13	0.154
147	A	2	2	1.00	13	0.154
148	A	2	2	1.00	13	0.154
149	A	2	2	1.00	13	0.154
150	A	2	2	1.00	11	0.182
151	A	2	2	1.00	9	0.222
152	A	2	2	1.00	13	0.154
153	A	2	2	1.00	13	0.154
154	A	2	2	1.00	13	0.154
155	A	2	2	1.00	13	0.154
156	A	2	2	1.00	13	0.154
157	A	3	3	1.00	15	0.200
158	A	2	2	1.00	13	0.154
159	A	2	2	1.00	13	0.154
160	A	2	2	1.00	13	0.154
161	A	2	2	1.00	13	0.154
162	A	3	3	1.16	15	0.200
163	A	3	3	1.00	15	0.200
164	A	3	3	0.86	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	3	3	1.00	15	0.200
166	A	3	3	1.00	15	0.200
167	A	3	3	1.00	15	0.200
168	A	2	2	1.00	19	0.105
169	A	2	2	1.00	19	0.105
170	A	2	2	1.00	19	0.105
171	A	2	2	1.00	19	0.105
172	A	2	2	1.00	11	0.182
173	A	3	3	0.85	13	0.231
174	A	2	2	1.00	11	0.182
175	A	2	2	1.00	9	0.222
176	A	2	2	1.00	7	0.286
177	A	2	2	1.00	11	0.182
178	A	2	2	1.00	11	0.182
179	A	2	2	1.00	11	0.182
180	A	2	2	1.00	15	0.133
181	A	3	3	1.00	17	0.176
182	A	2	2	1.00	15	0.133
183	A	2	2	1.00	13	0.154
184	A	2	2	1.00	11	0.182
185	A	2	2	1.00	15	0.133
186	A	2	2	1.00	15	0.133
187	A	2	2	1.00	18	0.111
188	A	2	2	1.00	16	0.125
189	A	2	2	1.00	15	0.133
190	A	2	2	1.00	15	0.133
191	A	2	2	1.00	13	0.154
192	A	2	2	1.00	11	0.182
193	A	2	2	1.00	15	0.133
194	A	2	2	1.00	15	0.133
195	A	2	2	1.00	15	0.133
196	A	2	2	1.00	22	0.091
197	A	2	2	1.00	20	0.100
198	A	2	2	1.00	18	0.111
199	A	2	2	1.00	22	0.091
200	A	2	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	2	2	1.00	21	0.095
202	A	2	2	1.00	21	0.095
203	A	2	2	1.00	19	0.105
204	A	2	2	1.00	21	0.095
205	A	2	2	1.00	21	0.095
206	A	2	1	1.00	11	0.091
207	A	2	1	1.00	11	0.091
208	A	2	1	1.00	9	0.111
209	A	1	0	1.00	7	0.000
210	A	2	1	1.00	11	0.091
211	A	2	1	1.00	11	0.091
212	A	2	1	1.00	11	0.091
213	A	2	1	1.00	11	0.091
214	A	2	1	1.00	11	0.091
215	A	2	1	1.00	11	0.091
216	A	2	1	1.00	11	0.091
217	A	2	1	1.00	11	0.091
218	A	2	1	1.00	13	0.077
219	A	2	1	1.00	13	0.077
220	A	1	1	1.00	13	0.077
221	A	2	1	1.00	11	0.091
222	A	2	1	1.00	9	0.111
223	A	3	2	1.00	13	0.154
224	A	2	1	1.00	13	0.077
225	A	2	1	1.00	13	0.077
226	A	3	2	1.00	13	0.154
227	A	2	1	1.00	13	0.077
228	A	2	1	1.00	13	0.077
229	A	3	2	1.00	13	0.154
230	A	2	1	1.00	13	0.077
231	A	2	1	1.00	13	0.077
232	A	1	1	1.00	13	0.077
233	A	2	1	1.00	13	0.077
234	A	2	1	1.00	13	0.077
235	A	3	2	1.00	13	0.154
236	A	3	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
237	A	3	2	1.00	13	0.154
238	A	3	2	1.00	13	0.154
239	A	3	2	1.00	13	0.154
240	A	1	1	1.00	13	0.077
241	A	3	2	1.00	13	0.154
242	A	3	2	1.00	13	0.154
243	A	3	2	1.00	13	0.154
244	A	3	2	1.00	13	0.154
245	A	1	1	1.00	13	0.077
246	A	3	3	1.00	13	0.231
247	A	3	2	1.00	13	0.154
248	A	3	2	1.00	13	0.154
249	A	2	1	1.00	13	0.077
250	A	2	1	1.00	13	0.077
251	A	2	1	1.00	11	0.091
252	A	2	1	1.00	9	0.111
253	A	2	1	1.00	13	0.077
254	A	2	1	1.00	13	0.077
255	A	2	1	1.00	13	0.077
256	A	2	1	1.00	13	0.077
257	A	2	1	1.00	13	0.077
258	A	3	2	1.00	13	0.154
259	A	3	2	1.00	13	0.154
260	A	3	2	1.00	13	0.154
261	A	3	2	1.00	13	0.154
262	A	3	2	1.00	13	0.154
263	A	1	1	1.00	13	0.077
264	A	3	2	1.00	13	0.154
265	A	3	2	1.00	13	0.154
266	A	3	2	1.00	13	0.154
267	A	3	2	1.00	13	0.154
268	A	3	2	1.00	13	0.154
269	A	3	2	1.00	13	0.154
270	A	1	1	1.00	13	0.077
271	A	3	3	1.00	13	0.231
272	A	4	3	1.00	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
273	A	3	2	1.00	13	0.154
274	A	3	2	1.00	13	0.154
275	A	2	1	1.00	13	0.077
276	A	2	1	1.00	13	0.077
277	A	2	1	1.00	11	0.091
278	A	2	1	1.00	9	0.111
279	A	2	1	1.00	13	0.077
280	A	2	1	1.00	13	0.077
281	A	2	1	1.00	13	0.077
282	A	2	1	1.00	13	0.077
283	A	2	1	1.00	13	0.077
284	A	2	1	1.00	13	0.077
285	A	3	2	1.00	13	0.154
286	A	3	2	1.00	13	0.154
287	A	3	2	1.00	13	0.154
288	A	3	2	1.00	13	0.154
289	A	3	2	1.00	13	0.154
290	A	3	2	1.00	13	0.154
291	A	1	1	1.00	13	0.077
292	A	3	2	1.00	13	0.154
293	A	3	2	1.00	13	0.154
294	A	3	2	1.00	13	0.154
295	A	3	2	1.00	13	0.154
296	A	3	2	1.00	13	0.154
297	A	3	2	1.00	13	0.154
298	A	3	2	1.00	13	0.154
299	A	3	2	1.00	13	0.154
300	A	3	2	1.00	13	0.154
301	A	1	1	1.00	13	0.077
302	A	3	3	1.00	13	0.231
303	A	4	3	1.00	13	0.231
304	A	5	3	1.00	13	0.231
305	A	6	3	1.00	13	0.231
306	A	3	2	1.00	13	0.154
307	A	3	2	1.00	13	0.154
308	A	2	1	1.00	13	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
309	A	2	1	1.00	13	0.077
310	A	2	1	1.00	11	0.091
311	A	2	1	1.00	9	0.111
312	A	2	1	1.00	13	0.077
313	A	2	1	1.00	13	0.077
314	A	2	1	1.00	13	0.077
315	A	2	1	1.00	13	0.077
316	A	2	1	1.00	13	0.077
317	A	2	1	1.00	13	0.077
318	A	3	2	1.00	13	0.154
319	A	3	2	1.00	13	0.154
320	A	1	1	1.00	13	0.077
321	A	4	4	1.00	13	0.308
322	A	3	2	1.00	13	0.154
323	A	7	7	1.00	13	0.538
324	A	7	7	1.00	13	0.538
325	A	6	6	1.00	11	0.546
326	A	6	6	1.00	9	0.667
327	A	7	7	1.00	13	0.538
328	A	7	7	1.00	13	0.538
329	A	3	2	1.00	13	0.154
330	A	3	2	1.00	13	0.154
331	A	1	1	1.00	13	0.077
332	A	3	2	1.00	13	0.154
333	A	3	2	1.00	13	0.154
334	A	7	7	1.00	13	0.538
335	A	7	7	1.00	13	0.538
336	A	7	7	1.00	11	0.636
337	A	7	7	1.00	9	0.778
338	A	8	8	1.00	13	0.615
339	A	8	8	1.00	13	0.615
340	A	3	2	1.00	13	0.154
341	A	3	2	1.00	13	0.154
342	A	1	1	1.00	13	0.077
343	A	1	1	1.00	13	0.077
344	A	3	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
345	A	3	2	1.00	13	0.154
346	A	8	7	1.00	13	0.538
347	A	8	7	1.00	13	0.538
348	A	8	8	1.00	13	0.615
349	A	8	8	1.00	13	0.615
350	A	8	7	1.00	11	0.636
351	A	8	7	1.00	9	0.778
352	A	3	2	1.00	14	0.143
353	A	3	2	1.00	14	0.143
354	A	1	1	1.00	14	0.071
355	A	4	4	1.00	14	0.286
356	A	3	2	1.00	14	0.143
357	A	7	7	1.00	14	0.500
358	A	7	7	1.00	14	0.500
359	A	6	6	1.00	12	0.500
360	A	6	6	1.00	10	0.600
361	A	7	7	1.00	14	0.500
362	A	7	7	1.00	14	0.500
363	A	6	6	1.00	10	0.600
364	A	6	6	1.00	11	0.546
365	A	6	6	1.00	10	0.600
366	A	6	6	1.00	11	0.546
367	A	3	3	1.00	13	0.231
368	A	3	2	1.00	15	0.133
369	A	3	2	1.00	15	0.133
370	A	3	2	1.00	15	0.133
371	A	1	1	1.00	15	0.067
372	A	4	4	1.00	15	0.267
373	A	4	4	1.00	15	0.267
374	A	5	5	1.00	15	0.333
375	A	6	5	1.00	15	0.333
376	A	4	3	1.00	15	0.200
377	A	3	3	1.00	15	0.200
378	A	2	2	1.00	11	0.182
379	A	2	2	1.00	15	0.133
380	A	3	3	1.00	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
381	A	4	3	1.00	15	0.200
382	A	6	5	1.00	15	0.333
383	A	5	5	1.00	15	0.333
384	A	4	4	1.00	13	0.308
385	A	4	4	1.00	15	0.267
386	A	5	5	1.00	15	0.333
387	A	3	2	1.00	15	0.133
388	A	3	2	1.00	15	0.133
389	A	3	2	1.00	15	0.133
390	A	1	1	1.00	15	0.067
391	A	5	4	1.00	15	0.267
392	A	5	5	1.00	15	0.333
393	A	5	4	1.00	15	0.267
394	A	5	3	1.00	15	0.200
395	A	4	3	1.00	15	0.200
396	A	3	2	1.00	11	0.182
397	A	3	3	1.00	15	0.200
398	A	3	2	1.00	15	0.133
399	A	7	5	1.00	15	0.333
400	A	6	5	1.00	15	0.333
401	A	5	4	1.00	13	0.308
402	A	5	5	1.00	15	0.333
403	A	5	4	1.00	15	0.267
404	A	3	2	1.00	15	0.133
405	A	3	2	1.00	15	0.133
406	A	3	2	1.00	15	0.133
407	A	1	1	1.00	15	0.067
408	A	3	3	1.00	15	0.200
409	A	4	4	1.00	15	0.267
410	A	5	4	1.00	15	0.267
411	A	3	2	1.00	15	0.133
412	A	2	2	1.00	15	0.133
413	A	1	1	1.00	11	0.091
414	A	2	2	1.00	15	0.133
415	A	3	2	1.00	15	0.133
416	A	5	4	1.00	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
417	A	4	4	1.00	15	0.267
418	A	3	3	1.00	13	0.231
419	A	4	4	1.00	15	0.267
420	A	5	4	1.00	15	0.267
421	A	3	2	1.00	15	0.133
422	A	3	2	1.00	15	0.133
423	A	3	2	1.00	15	0.133
424	A	1	1	1.00	15	0.067
425	A	4	4	1.00	15	0.267
426	A	5	4	1.03	15	0.267
427	A	6	4	1.00	15	0.267
428	A	3	3	1.00	15	0.200
429	A	2	2	1.00	15	0.133
430	A	2	2	1.00	11	0.182
431	A	3	3	1.00	15	0.200
432	A	4	3	1.00	15	0.200
433	A	5	5	1.00	15	0.333
434	A	4	4	1.00	15	0.267
435	A	4	4	1.00	13	0.308
436	A	5	5	1.00	15	0.333
437	A	6	5	1.00	15	0.333
438	A	3	2	1.00	13	0.154
439	A	3	2	1.00	13	0.154
440	A	3	2	1.00	13	0.154
441	A	1	1	1.00	13	0.077
442	A	3	3	1.00	13	0.231
443	A	4	4	1.00	13	0.308
444	A	5	4	1.00	13	0.308
445	A	6	4	1.00	13	0.308
446	A	3	2	1.00	13	0.154
447	A	2	2	1.00	13	0.154
448	A	1	1	1.00	9	0.111
449	A	2	2	1.00	13	0.154
450	A	3	2	1.00	13	0.154
451	A	5	4	1.00	13	0.308
452	A	4	4	1.00	13	0.308

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
453	A	3	3	1.00	11	0.273
454	A	4	4	1.00	13	0.308
455	A	5	4	1.00	13	0.308
456	A	3	2	1.00	15	0.133
457	A	3	2	1.00	15	0.133
458	A	3	2	1.00	15	0.133
459	A	1	1	1.00	15	0.067
460	A	3	3	1.00	15	0.200
461	A	4	4	1.00	15	0.267
462	A	5	4	1.00	15	0.267
463	A	6	4	1.00	15	0.267
464	A	3	2	1.00	15	0.133
465	A	2	2	1.00	15	0.133
466	A	1	1	1.00	11	0.091
467	A	2	2	1.00	15	0.133
468	A	3	2	1.00	15	0.133
469	A	5	4	1.00	15	0.267
470	A	4	4	1.00	15	0.267
471	A	3	3	1.00	13	0.231
472	A	4	4	1.00	15	0.267
473	A	5	4	1.00	15	0.267
474	A	3	2	1.00	13	0.154
475	A	3	2	1.00	13	0.154
476	A	3	2	1.00	13	0.154
477	A	1	1	1.00	13	0.077
478	A	3	3	1.00	13	0.231
479	A	4	4	1.00	13	0.308
480	A	5	4	1.00	13	0.308
481	A	6	4	1.00	13	0.308
482	A	3	2	1.00	13	0.154
483	A	2	2	1.00	13	0.154
484	A	1	1	1.00	9	0.111
485	A	2	2	1.00	13	0.154
486	A	3	2	1.00	13	0.154
487	A	5	4	1.00	13	0.308
488	A	4	4	1.00	13	0.308

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
489	A	3	3	1.00	11	0.273
490	A	4	4	1.00	13	0.308
491	A	5	4	1.00	13	0.308
492	A	3	2	1.00	15	0.133
493	A	3	2	1.00	15	0.133
494	A	3	2	1.00	15	0.133
495	A	1	1	1.00	15	0.067
496	A	3	3	1.00	15	0.200
497	A	4	4	1.00	15	0.267
498	A	5	4	1.00	15	0.267
499	A	6	4	1.00	15	0.267
500	A	3	2	1.00	15	0.133
501	A	2	2	1.00	15	0.133
502	A	1	1	1.00	11	0.091
503	A	2	2	1.00	15	0.133
504	A	3	2	1.00	15	0.133
505	A	5	4	1.00	15	0.267
506	A	4	4	1.00	15	0.267
507	A	3	3	1.00	13	0.231
508	A	4	4	1.00	15	0.267
509	A	5	4	1.00	15	0.267
510	A	3	2	1.00	15	0.133
511	A	3	2	1.00	15	0.133
512	A	3	2	1.00	15	0.133
513	A	1	1	1.00	15	0.067
514	A	6	6	1.00	15	0.400
515	A	6	6	1.00	15	0.400
516	A	9	9	1.44	15	0.600
517	A	8	8	1.54	13	0.615
518	A	8	8	1.57	15	0.533
519	A	1	1	1.00	15	0.067
520	A	2	2	1.00	15	0.133
521	A	3	2	1.00	15	0.133
522	A	4	2	1.00	15	0.133
523	A	2	2	1.34	15	0.133
524	A	2	2	1.39	11	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
525	A	2	2	1.34	15	0.133
526	A	2	2	1.34	15	0.133
527	A	3	2	1.00	15	0.133
528	A	3	2	1.00	15	0.133
529	A	3	2	1.00	15	0.133
530	A	1	1	1.00	15	0.067
531	A	6	6	1.00	15	0.400
532	A	6	6	1.00	15	0.400
533	A	2	2	1.34	15	0.133
534	A	2	2	1.34	13	0.154
535	A	2	2	1.36	15	0.133
536	A	2	2	1.34	15	0.133
537	A	3	3	1.00	15	0.200
538	A	2	2	1.00	11	0.182
539	A	2	2	1.00	15	0.133
540	A	1	1	1.00	15	0.067
541	A	2	2	1.00	15	0.133
542	A	3	2	1.00	15	0.133
543	A	3	2	1.00	15	0.133
544	A	3	2	1.00	15	0.133
545	A	3	2	1.00	15	0.133
546	A	3	2	1.00	15	0.133
547	A	1	1	1.00	15	0.067
548	A	5	5	1.00	15	0.333
549	A	6	6	1.00	15	0.400
550	A	2	2	1.34	15	0.133
551	A	2	2	1.34	15	0.133
552	A	2	2	1.34	13	0.154
553	A	2	2	1.36	15	0.133
554	A	2	2	1.34	15	0.133
555	A	2	2	1.00	15	0.133
556	A	1	1	1.00	11	0.091
557	A	1	1	1.00	15	0.067
558	A	2	2	1.00	15	0.133
559	A	3	2	1.00	15	0.133
560	A	4	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
561	A	3	2	1.00	15	0.133
562	A	3	2	1.00	15	0.133
563	A	3	2	1.00	15	0.133
564	A	1	1	1.00	15	0.067
565	A	5	5	1.00	15	0.333
566	A	6	6	1.00	15	0.400
567	A	9	8	1.43	15	0.533
568	A	8	8	1.53	15	0.533
569	A	7	7	1.69	13	0.538
570	A	1	1	1.00	15	0.067
571	A	2	2	1.00	15	0.133
572	A	3	2	1.00	15	0.133
573	A	4	2	1.00	15	0.133
574	A	2	2	1.34	15	0.133
575	A	2	2	1.34	15	0.133
576	A	2	2	1.39	11	0.182
577	A	2	2	1.34	15	0.133
578	A	2	2	1.34	15	0.133
579	A	1	1	1.00	12	0.083
580	A	1	1	1.00	9	0.111
581	A	7	7	1.64	13	0.538
582	A	1	1	1.00	13	0.077
583	A	2	1	1.00	13	0.077
584	A	2	1	1.00	13	0.077
585	A	2	1	1.00	13	0.077
586	A	2	1	1.00	13	0.077
587	A	2	1	1.00	11	0.091
588	A	1	1	1.00	13	0.077
589	A	1	1	1.00	13	0.077
590	A	1	1	1.00	13	0.077
591	A	2	2	1.00	15	0.133
592	A	2	2	1.00	15	0.133
593	A	2	2	1.21	15	0.133
594	A	2	2	1.00	15	0.133
595	A	2	2	1.00	17	0.118
596	A	2	2	1.11	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
597	A	2	2	1.11	17	0.118
598	A	2	2	1.00	15	0.133
599	A	1	1	1.00	13	0.077
600	A	3	2	1.00	13	0.154
601	A	3	2	1.00	13	0.154
602	A	3	2	1.00	13	0.154
603	A	2	1	1.00	11	0.091
604	A	2	1	1.00	11	0.091
605	A	2	1	1.00	11	0.091
606	A	2	1	1.00	11	0.091
607	A	2	1	1.00	11	0.091
608	A	2	1	1.00	9	0.111
609	A	1	0	1.00	7	0.000
610	A	2	1	1.00	11	0.091
611	A	2	1	1.00	11	0.091
612	A	2	1	1.00	11	0.091
613	A	2	1	1.00	11	0.091
614	A	2	1	1.00	11	0.091
615	A	2	1	1.00	11	0.091
616	A	2	1	1.00	11	0.091
617	A	2	1	1.00	11	0.091
618	A	2	1	1.00	11	0.091
619	A	2	1	1.00	11	0.091
620	A	2	1	1.00	13	0.077
621	A	2	1	1.00	13	0.077
622	A	2	1	1.00	13	0.077
623	A	1	1	1.00	13	0.077
624	A	2	1	1.00	13	0.077
625	A	2	1	1.00	11	0.091
626	A	2	1	1.00	9	0.111
627	A	3	2	1.00	13	0.154
628	A	2	1	1.00	13	0.077
629	A	2	1	1.00	13	0.077
630	A	2	1	1.00	13	0.077
631	A	3	2	1.00	13	0.154
632	A	2	1	1.00	13	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
633	A	2	1	1.00	13	0.077
634	A	2	1	1.00	13	0.077
635	A	1	1	1.00	13	0.077
636	A	2	1	1.00	13	0.077
637	A	2	1	1.00	11	0.091
638	A	2	1	1.00	9	0.111
639	A	3	2	1.00	13	0.154
640	A	2	1	1.00	13	0.077
641	A	2	1	1.00	13	0.077
642	A	2	1	1.00	13	0.077
643	A	3	2	1.00	13	0.154
644	A	4	3	1.00	13	0.231
645	A	3	2	1.00	13	0.154
646	A	3	3	1.00	13	0.231
647	A	1	1	1.00	13	0.077
648	A	2	2	1.00	11	0.182
649	A	4	4	1.00	13	0.308
650	A	3	3	1.00	13	0.231
651	A	3	2	1.00	13	0.154
652	A	4	3	1.00	13	0.231
653	A	10	7	1.00	13	0.538
654	A	9	6	1.00	13	0.462
655	A	9	6	1.00	9	0.667
656	A	10	7	1.00	13	0.538
657	A	10	7	1.00	13	0.538
658	A	3	2	1.00	13	0.154
659	A	4	4	1.00	13	0.308
660	A	3	2	1.00	13	0.154
661	A	3	3	1.00	13	0.231
662	A	1	1	1.00	13	0.077
663	A	3	3	1.00	11	0.273
664	A	3	2	1.00	13	0.154
665	A	4	4	1.00	13	0.308
666	A	10	7	1.00	13	0.538
667	A	10	7	1.00	13	0.538
668	A	10	7	1.00	13	0.538

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
669	A	10	7	1.00	9	0.778
670	A	11	8	1.00	13	0.615
671	A	11	8	1.00	13	0.615
672	A	3	2	1.00	13	0.154
673	A	4	3	1.00	13	0.231
674	A	1	1	1.00	13	0.077
675	A	4	4	1.00	13	0.308
676	A	1	1	1.00	13	0.077
677	A	4	3	1.00	11	0.273
678	A	3	2	1.00	13	0.154
679	A	5	4	1.00	13	0.308
680	A	11	7	1.00	13	0.538
681	A	11	7	1.00	13	0.538
682	A	11	8	1.00	13	0.615
683	A	11	8	1.00	13	0.615
684	A	11	7	1.00	13	0.538
685	A	11	7	1.00	9	0.778
686	A	12	8	1.00	13	0.615
687	A	4	3	1.00	13	0.231
688	A	3	2	1.00	13	0.154
689	A	3	3	1.00	13	0.231
690	A	1	1	1.00	13	0.077
691	A	2	2	1.00	11	0.182
692	A	4	4	1.00	13	0.308
693	A	3	3	1.00	13	0.231
694	A	10	7	1.00	13	0.538
695	A	10	7	1.00	13	0.538
696	A	9	6	1.00	13	0.462
697	A	9	6	1.00	9	0.667
698	A	10	7	1.00	13	0.538
699	A	1	1	1.00	13	0.077
700	A	3	3	1.00	11	0.273
701	A	3	2	1.00	13	0.154
702	A	4	4	1.00	13	0.308
703	A	10	7	1.00	13	0.538
704	A	10	7	1.00	13	0.538

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
705	A	10	7	1.00	9	0.778
706	A	11	8	1.00	13	0.615
707	A	9	6	1.00	11	0.546
708	A	3	3	1.00	12	0.250
709	A	3	3	1.00	12	0.250
710	A	3	3	1.00	11	0.273
711	A	2	2	1.00	14	0.143
712	A	2	2	1.00	13	0.154
713	A	3	3	1.00	16	0.188
714	A	3	3	1.00	15	0.200
715	A	1	1	1.00	16	0.062
716	A	1	1	1.00	15	0.067
717	A	2	1	1.00	13	0.077
718	A	2	1	1.00	13	0.077
719	A	2	1	1.00	13	0.077
720	A	2	1	1.00	13	0.077
721	A	2	1	1.00	13	0.077
722	A	2	1	1.00	13	0.077
723	A	2	1	1.00	13	0.077
724	A	2	1	1.00	15	0.067
725	A	2	1	1.00	15	0.067
726	A	2	1	1.00	15	0.067
727	A	2	1	1.00	15	0.067
728	A	2	1	1.00	15	0.067
729	A	2	1	1.00	15	0.067
730	A	2	1	1.00	15	0.067
731	A	2	1	1.00	15	0.067
732	A	2	1	1.00	15	0.067
733	A	2	1	1.00	15	0.067
734	A	2	1	1.00	15	0.067
735	A	2	1	1.00	15	0.067
736	A	2	1	1.00	15	0.067
737	A	2	1	1.00	15	0.067
738	A	15	12	1.00	15	0.800
739	A	15	12	1.00	15	0.800
740	A	14	11	1.00	15	0.733

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
741	A	14	11	1.00	15	0.733
742	A	14	11	1.00	15	0.733
743	A	14	11	1.00	15	0.733
744	A	15	12	1.00	15	0.800
745	A	15	12	1.00	15	0.800
746	A	15	12	1.00	15	0.800
747	A	15	12	1.00	15	0.800
748	A	15	12	1.00	15	0.800
749	A	15	12	1.00	15	0.800
750	A	15	12	1.00	15	0.800
751	A	15	12	1.00	15	0.800
752	A	15	12	1.00	15	0.800
753	A	15	12	1.00	15	0.800
754	A	16	13	1.00	15	0.867
755	A	16	12	1.00	15	0.800
756	A	16	13	1.00	15	0.867
757	A	16	13	1.00	15	0.867
758	A	16	13	1.00	15	0.867
759	A	16	13	1.00	15	0.867
760	A	16	12	1.00	15	0.800
761	A	16	12	1.00	15	0.800
762	A	16	12	1.00	15	0.800
763	A	16	12	1.00	15	0.800
764	A	3	2	1.00	15	0.133
765	A	3	2	1.00	15	0.133
766	A	1	1	1.00	15	0.067
767	A	4	4	1.00	15	0.267
768	A	4	4	1.00	15	0.267
769	A	5	5	1.00	15	0.333
770	A	5	5	1.00	15	0.333
771	A	4	4	1.00	13	0.308
772	A	4	4	1.00	15	0.267
773	A	1	1	1.00	15	0.067
774	A	2	2	1.00	15	0.133
775	A	3	2	1.00	15	0.133
776	A	3	3	1.00	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
777	A	2	2	1.00	11	0.182
778	A	2	2	1.00	15	0.133
779	A	3	3	1.00	15	0.200
780	A	4	4	1.00	15	0.267
781	A	4	4	1.00	15	0.267
782	A	5	5	1.00	15	0.333
783	A	3	2	1.00	15	0.133
784	A	3	2	1.00	15	0.133
785	A	1	1	1.00	15	0.067
786	A	5	4	1.00	15	0.267
787	A	5	5	1.00	15	0.333
788	A	5	4	1.00	15	0.267
789	A	6	5	1.00	15	0.333
790	A	5	4	1.00	13	0.308
791	A	5	5	1.00	15	0.333
792	A	5	4	1.00	15	0.267
793	A	1	1	1.00	15	0.067
794	A	2	2	1.00	15	0.133
795	A	3	2	1.00	15	0.133
796	A	4	3	1.00	15	0.200
797	A	3	2	1.00	11	0.182
798	A	3	3	1.00	15	0.200
799	A	3	2	1.00	15	0.133
800	A	5	4	1.00	15	0.267
801	A	5	5	1.00	15	0.333
802	A	5	4	1.00	15	0.267
803	A	3	2	1.00	9	0.222
804	A	3	2	1.00	11	0.182
805	A	3	2	1.00	15	0.133
806	A	1	1	1.00	13	0.077
807	A	3	3	1.00	13	0.231
808	A	4	4	1.00	11	0.364
809	A	2	2	1.00	9	0.222
810	A	2	2	1.00	11	0.182
811	A	3	2	1.00	15	0.133
812	A	3	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
813	A	1	1	1.00	15	0.067
814	A	3	3	1.00	15	0.200
815	A	4	4	1.00	15	0.267
816	A	4	4	1.00	15	0.267
817	A	3	3	1.00	13	0.231
818	A	1	1	1.00	15	0.067
819	A	2	2	1.00	15	0.133
820	A	3	2	1.00	15	0.133
821	A	3	2	1.00	15	0.133
822	A	2	2	1.00	15	0.133
823	A	1	1	1.00	11	0.091
824	A	2	2	1.00	15	0.133
825	A	3	2	1.00	15	0.133
826	A	5	4	1.00	15	0.267
827	A	4	4	1.00	15	0.267
828	A	3	3	1.00	15	0.200
829	A	4	4	1.00	15	0.267
830	A	5	4	1.00	15	0.267
831	A	3	2	1.00	16	0.125
832	A	3	2	1.00	16	0.125
833	A	1	1	1.00	16	0.062
834	A	3	3	1.00	16	0.188
835	A	4	4	1.00	16	0.250
836	A	4	4	1.00	16	0.250
837	A	3	3	1.00	14	0.214
838	A	1	1	1.00	16	0.062
839	A	2	2	1.00	16	0.125
840	A	3	2	1.00	16	0.125
841	A	4	3	1.00	16	0.188
842	A	3	3	1.00	16	0.188
843	A	2	2	1.00	12	0.167
844	A	3	3	1.00	16	0.188
845	A	4	3	1.00	16	0.188
846	A	8	7	1.00	16	0.438
847	A	7	7	1.00	16	0.438
848	A	6	6	1.00	16	0.375

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
849	A	7	7	1.00	16	0.438
850	A	8	7	1.00	16	0.438
851	A	3	2	1.00	15	0.133
852	A	3	2	1.00	15	0.133
853	A	1	1	1.00	15	0.067
854	A	4	4	1.00	15	0.267
855	A	5	4	1.03	15	0.267
856	A	5	5	1.00	15	0.333
857	A	4	4	1.00	15	0.267
858	A	1	1	1.00	13	0.077
859	A	2	2	1.00	15	0.133
860	A	3	2	1.00	15	0.133
861	A	4	3	1.00	15	0.200
862	A	3	3	1.00	15	0.200
863	A	2	2	1.00	15	0.133
864	A	2	2	1.00	11	0.182
865	A	3	3	1.00	15	0.200
866	A	4	3	1.00	15	0.200
867	A	6	5	1.00	15	0.333
868	A	5	5	1.00	15	0.333
869	A	4	4	1.00	15	0.267
870	A	4	4	1.00	15	0.267
871	A	5	5	1.00	15	0.333
872	A	6	5	1.00	15	0.333
873	A	3	2	1.00	11	0.182
874	A	3	2	1.00	15	0.133
875	A	3	2	1.00	15	0.133
876	A	1	1	1.00	15	0.067
877	A	3	3	1.00	15	0.200
878	A	4	4	1.00	15	0.267
879	A	3	3	1.00	15	0.200
880	A	2	2	1.00	13	0.154
881	A	1	1	1.00	15	0.067
882	A	2	2	1.00	15	0.133
883	A	3	2	1.00	15	0.133
884	A	3	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
885	A	2	2	1.00	15	0.133
886	A	1	1	1.00	11	0.091
887	A	2	2	1.00	15	0.133
888	A	3	2	1.00	15	0.133
889	A	6	5	1.00	15	0.333
890	A	5	5	1.00	15	0.333
891	A	4	4	1.00	15	0.267
892	A	5	5	1.00	15	0.333
893	A	6	5	1.00	15	0.333
894	A	3	2	1.00	15	0.133
895	A	3	2	1.00	15	0.133
896	A	1	1	1.00	15	0.067
897	A	4	4	1.00	15	0.267
898	A	5	4	1.06	15	0.267
899	A	4	4	1.00	15	0.267
900	A	3	3	1.00	15	0.200
901	A	1	1	1.00	13	0.077
902	A	2	2	1.00	15	0.133
903	A	3	2	1.00	15	0.133
904	A	4	3	1.00	15	0.200
905	A	3	3	1.00	15	0.200
906	A	2	2	1.00	15	0.133
907	A	2	2	1.00	11	0.182
908	A	3	3	1.00	15	0.200
909	A	4	3	1.00	15	0.200
910	A	7	6	1.00	15	0.400
911	A	6	6	1.00	15	0.400
912	A	5	5	1.00	15	0.333
913	A	5	5	1.00	15	0.333
914	A	6	6	1.00	15	0.400
915	A	7	6	1.00	15	0.400
916	A	3	2	1.00	11	0.182
917	A	3	2	1.00	13	0.154
918	A	3	2	1.00	13	0.154
919	A	1	1	1.00	13	0.077
920	A	3	3	1.00	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
921	A	4	4	1.00	13	0.308
922	A	3	3	1.00	13	0.231
923	A	2	2	1.00	11	0.182
924	A	1	1	1.00	13	0.077
925	A	2	2	1.00	13	0.154
926	A	3	2	1.00	13	0.154
927	A	3	2	1.00	13	0.154
928	A	2	2	1.00	13	0.154
929	A	1	1	1.00	9	0.111
930	A	2	2	1.00	13	0.154
931	A	3	2	1.00	13	0.154
932	A	5	4	1.00	13	0.308
933	A	4	4	1.00	13	0.308
934	A	3	3	1.00	13	0.231
935	A	4	4	1.00	13	0.308
936	A	5	4	1.00	13	0.308
937	A	3	2	1.00	13	0.154
938	A	3	2	1.00	13	0.154
939	A	1	1	1.00	13	0.077
940	A	4	4	1.00	13	0.308
941	A	5	4	1.07	13	0.308
942	A	4	4	1.00	13	0.308
943	A	3	3	1.00	13	0.231
944	A	1	1	1.00	11	0.091
945	A	2	2	1.00	13	0.154
946	A	3	2	1.00	13	0.154
947	A	4	3	1.00	13	0.231
948	A	3	3	1.00	13	0.231
949	A	2	2	1.00	13	0.154
950	A	2	2	1.00	9	0.222
951	A	3	3	1.00	13	0.231
952	A	4	3	1.00	13	0.231
953	A	6	5	1.00	13	0.385
954	A	5	5	1.00	13	0.385
955	A	4	4	1.00	13	0.308
956	A	4	4	1.00	13	0.308

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
957	A	5	5	1.00	13	0.385
958	A	6	5	1.00	13	0.385
959	A	3	2	1.00	9	0.222
960	A	3	2	1.00	15	0.133
961	A	3	3	1.00	15	0.200
962	A	1	1	1.00	15	0.067
963	A	2	2	1.00	13	0.154
964	A	3	3	1.00	15	0.200
965	A	1	1	1.00	15	0.067
966	A	4	4	1.00	15	0.267
967	A	2	2	1.00	15	0.133
968	A	6	6	1.00	15	0.400
969	A	2	2	1.00	15	0.133
970	A	5	5	1.00	15	0.333
971	A	1	1	1.00	11	0.091
972	A	6	6	1.00	15	0.400
973	A	2	2	1.00	15	0.133
974	A	3	3	1.00	11	0.273
975	A	2	2	1.00	11	0.182
976	A	3	3	1.00	13	0.231
977	A	2	2	1.00	13	0.154
978	A	1	1	1.00	9	0.111
979	A	2	2	1.00	13	0.154
980	A	4	4	1.00	13	0.308
981	A	3	3	1.00	13	0.231
982	A	4	4	1.00	13	0.308
983	A	5	5	1.00	15	0.333
984	A	4	4	1.00	16	0.250
985	A	3	2	1.00	13	0.154
986	A	1	1	1.00	13	0.077
987	A	1	1	1.00	13	0.077
988	A	3	2	1.00	15	0.133
989	A	3	2	1.00	15	0.133
990	A	3	2	1.00	15	0.133
991	A	3	2	1.00	15	0.133
992	A	1	1	1.00	15	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
993	A	6	6	1.00	15	0.400
994	A	6	6	1.00	15	0.400
995	A	7	7	1.00	15	0.467
996	A	6	5	1.00	15	0.333
997	A	5	5	1.00	15	0.333
998	A	4	4	1.00	13	0.308
999	A	4	4	1.00	15	0.267
1000	A	5	5	1.00	15	0.333
1001	A	6	5	1.00	15	0.333
1002	A	6	6	1.00	15	0.400
1003	A	5	5	1.00	15	0.333
1004	A	5	5	1.00	15	0.333
1005	A	1	1	1.00	15	0.067
1006	A	2	2	1.00	15	0.133
1007	A	3	2	1.00	15	0.133
1008	A	4	2	1.00	15	0.133
1009	A	8	6	1.00	15	0.400
1010	A	7	6	1.00	15	0.400
1011	A	6	6	1.00	15	0.400
1012	A	5	5	1.00	11	0.454
1013	A	5	5	1.00	15	0.333
1014	A	6	6	1.00	15	0.400
1015	A	7	6	1.00	15	0.400
1016	A	8	6	1.00	15	0.400
1017	A	3	2	1.00	15	0.133
1018	A	3	2	1.00	15	0.133
1019	A	3	2	1.00	15	0.133
1020	A	3	2	1.00	15	0.133
1021	A	1	1	1.00	15	0.067
1022	A	6	6	1.00	15	0.400
1023	A	6	6	1.00	15	0.400
1024	A	7	7	1.00	15	0.467
1025	A	7	6	1.00	15	0.400
1026	A	6	6	1.00	15	0.400
1027	A	5	5	1.00	13	0.385
1028	A	5	5	1.00	15	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1029	A	6	6	1.00	15	0.400
1030	A	7	6	1.00	15	0.400
1031	A	8	6	1.00	15	0.400
1032	A	7	6	1.00	15	0.400
1033	A	6	6	1.00	15	0.400
1034	A	5	5	1.00	11	0.454
1035	A	5	5	1.00	15	0.333
1036	A	1	1	1.00	15	0.067
1037	A	2	2	1.00	15	0.133
1038	A	3	2	1.00	15	0.133
1039	A	4	2	1.00	15	0.133
1040	A	8	7	1.00	15	0.467
1041	A	7	7	1.00	15	0.467
1042	A	6	6	1.00	15	0.400
1043	A	6	6	1.00	15	0.400
1044	A	6	6	1.00	15	0.400
1045	A	7	7	1.00	15	0.467
1046	A	8	7	1.00	15	0.467
1047	A	3	2	1.00	15	0.133
1048	A	3	2	1.00	15	0.133
1049	A	3	2	1.00	15	0.133
1050	A	3	2	1.00	15	0.133
1051	A	1	1	1.00	15	0.067
1052	A	7	6	1.00	15	0.400
1053	A	7	7	1.00	15	0.467
1054	A	7	6	1.00	15	0.400
1055	A	7	5	1.00	15	0.333
1056	A	6	5	1.00	15	0.333
1057	A	5	4	1.00	13	0.308
1058	A	5	5	1.00	15	0.333
1059	A	5	4	1.00	15	0.267
1060	A	6	5	1.00	15	0.333
1061	A	7	5	1.00	15	0.333
1062	A	8	6	1.00	15	0.400
1063	A	7	6	1.00	15	0.400
1064	A	6	5	1.00	15	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1065	A	6	6	1.00	15	0.400
1066	A	6	5	1.00	15	0.333
1067	A	1	1	1.00	15	0.067
1068	A	2	2	1.00	15	0.133
1069	A	3	2	1.00	15	0.133
1070	A	4	2	1.00	15	0.133
1071	A	9	6	1.00	15	0.400
1072	A	8	6	1.00	15	0.400
1073	A	7	6	1.00	15	0.400
1074	A	6	5	1.00	11	0.454
1075	A	6	6	1.00	15	0.400
1076	A	6	5	1.00	15	0.333
1077	A	7	6	1.00	15	0.400
1078	A	8	6	1.00	15	0.400
1079	A	6	5	1.00	11	0.454
1080	A	3	2	1.00	15	0.133
1081	A	3	2	1.00	15	0.133
1082	A	3	2	1.00	15	0.133
1083	A	3	2	1.00	15	0.133
1084	A	1	1	1.00	15	0.067
1085	A	5	5	1.00	15	0.333
1086	A	6	6	1.00	15	0.400
1087	A	7	6	1.00	15	0.400
1088	A	7	5	1.00	15	0.333
1089	A	6	5	1.00	15	0.333
1090	A	5	5	1.00	15	0.333
1091	A	4	4	1.00	13	0.308
1092	A	5	5	1.00	15	0.333
1093	A	6	5	1.00	15	0.333
1094	A	7	5	1.00	15	0.333
1095	A	6	5	1.00	15	0.333
1096	A	5	5	1.00	15	0.333
1097	A	4	4	1.00	11	0.364
1098	A	1	1	1.00	15	0.067
1099	A	2	2	1.00	15	0.133
1100	A	3	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1101	A	4	2	1.00	15	0.133
1102	A	5	2	1.00	15	0.133
1103	A	7	6	1.00	15	0.400
1104	A	6	6	1.00	15	0.400
1105	A	5	5	1.00	15	0.333
1106	A	5	5	1.00	15	0.333
1107	A	6	6	1.00	15	0.400
1108	A	7	6	1.00	15	0.400
1109	A	8	6	1.00	15	0.400
1110	A	3	2	1.00	15	0.133
1111	A	3	2	1.00	15	0.133
1112	A	3	2	1.00	15	0.133
1113	A	3	2	1.00	15	0.133
1114	A	1	1	1.00	15	0.067
1115	A	5	5	1.00	15	0.333
1116	A	6	6	1.00	15	0.400
1117	A	7	6	1.00	15	0.400
1118	A	6	4	1.00	15	0.267
1119	A	5	4	1.00	15	0.267
1120	A	4	4	1.00	15	0.267
1121	A	3	3	1.00	13	0.231
1122	A	4	4	1.00	15	0.267
1123	A	5	4	1.00	15	0.267
1124	A	6	4	1.00	15	0.267
1125	A	6	5	1.00	15	0.333
1126	A	5	5	1.00	15	0.333
1127	A	4	4	1.00	15	0.267
1128	A	1	1	1.00	15	0.067
1129	A	2	2	1.00	15	0.133
1130	A	3	2	1.00	15	0.133
1131	A	4	2	1.00	15	0.133
1132	A	7	5	1.00	15	0.333
1133	A	6	5	1.00	15	0.333
1134	A	5	5	1.00	15	0.333
1135	A	4	4	1.00	11	0.364
1136	A	5	5	1.00	15	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1137	A	6	5	1.00	15	0.333
1138	A	7	5	1.00	15	0.333
1139	A	3	2	1.00	15	0.133
1140	A	3	2	1.00	15	0.133
1141	A	3	2	1.00	15	0.133
1142	A	3	2	1.00	15	0.133
1143	A	1	1	1.00	15	0.067
1144	A	6	6	1.00	15	0.400
1145	A	7	6	0.99	15	0.400
1146	A	8	6	0.98	15	0.400
1147	A	6	4	1.00	15	0.267
1148	A	5	4	1.00	15	0.267
1149	A	4	4	1.00	15	0.267
1150	A	3	3	1.00	13	0.231
1151	A	4	4	1.00	15	0.267
1152	A	5	4	1.00	15	0.267
1153	A	6	4	1.00	15	0.267
1154	A	7	6	1.00	15	0.400
1155	A	6	6	1.00	15	0.400
1156	A	5	5	1.00	15	0.333
1157	A	1	1	1.00	11	0.091
1158	A	2	2	1.00	15	0.133
1159	A	3	2	1.00	15	0.133
1160	A	4	2	1.00	15	0.133
1161	A	5	2	1.00	15	0.133
1162	A	7	5	1.00	15	0.333
1163	A	6	5	1.00	15	0.333
1164	A	5	5	1.00	15	0.333
1165	A	4	4	1.00	15	0.267
1166	A	5	5	1.00	15	0.333
1167	A	6	5	1.00	15	0.333
1168	A	7	5	1.00	15	0.333
1169	A	8	5	1.00	15	0.333
1170	A	5	5	1.00	11	0.454
1171	A	2	2	1.00	11	0.182
1172	A	6	5	1.00	11	0.454

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1173	A	3	2	1.00	11	0.182
1174	A	4	2	1.00	11	0.182
1175	A	3	2	1.00	16	0.125
1176	A	3	2	1.00	16	0.125
1177	A	3	2	1.00	16	0.125
1178	A	3	2	1.00	16	0.125
1179	A	1	1	1.00	16	0.062
1180	A	6	6	1.00	16	0.375
1181	A	6	6	1.00	16	0.375
1182	A	7	7	1.00	16	0.438
1183	A	6	5	1.00	16	0.312
1184	A	5	5	1.00	16	0.312
1185	A	4	4	1.00	14	0.286
1186	A	4	4	1.00	16	0.250
1187	A	5	5	1.00	16	0.312
1188	A	6	5	1.00	16	0.312
1189	A	12	9	1.00	16	0.562
1190	A	11	8	1.00	16	0.500
1191	A	11	8	1.00	16	0.500
1192	A	1	1	1.00	16	0.062
1193	A	2	2	1.00	16	0.125
1194	A	3	2	1.00	16	0.125
1195	A	4	2	1.00	16	0.125
1196	A	8	6	1.00	16	0.375
1197	A	7	6	1.00	16	0.375
1198	A	6	6	1.00	16	0.375
1199	A	5	5	1.00	12	0.417
1200	A	5	5	1.00	16	0.312
1201	A	6	6	1.00	16	0.375
1202	A	7	6	1.00	16	0.375
1203	A	8	6	1.00	16	0.375
1204	A	3	2	1.00	16	0.125
1205	A	3	2	1.00	16	0.125
1206	A	3	2	1.00	16	0.125
1207	A	3	2	1.00	16	0.125
1208	A	1	1	1.00	16	0.062

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1209	A	5	5	1.00	16	0.312
1210	A	6	6	1.00	16	0.375
1211	A	7	6	1.00	16	0.375
1212	A	6	4	1.00	16	0.250
1213	A	5	4	1.00	16	0.250
1214	A	4	4	1.00	16	0.250
1215	A	3	3	1.00	14	0.214
1216	A	4	4	1.00	16	0.250
1217	A	5	4	1.00	16	0.250
1218	A	6	4	1.00	16	0.250
1219	A	12	8	1.00	16	0.500
1220	A	11	8	1.00	16	0.500
1221	A	10	7	1.00	12	0.583
1222	A	1	1	1.00	16	0.062
1223	A	2	2	1.00	16	0.125
1224	A	3	2	1.00	16	0.125
1225	A	4	2	1.00	16	0.125
1226	A	5	2	1.00	16	0.125
1227	A	7	6	1.00	16	0.375
1228	A	6	6	1.00	16	0.375
1229	A	5	5	1.00	16	0.312
1230	A	4	4	1.00	16	0.250
1231	A	5	5	1.00	16	0.312
1232	A	6	5	1.00	16	0.312
1233	A	7	5	1.00	16	0.312
1234	A	3	2	1.00	16	0.125
1235	A	3	2	1.00	16	0.125
1236	A	3	2	1.00	16	0.125
1237	A	3	2	1.00	16	0.125
1238	A	1	1	1.00	16	0.062
1239	A	5	5	1.00	16	0.312
1240	A	6	6	1.00	16	0.375
1241	A	7	6	1.00	16	0.375
1242	A	6	4	1.00	16	0.250
1243	A	5	4	1.00	16	0.250
1244	A	4	4	1.00	16	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1245	A	3	3	1.00	14	0.214
1246	A	4	4	1.00	16	0.250
1247	A	5	4	1.00	16	0.250
1248	A	6	4	1.00	16	0.250
1249	A	12	8	1.00	16	0.500
1250	A	11	8	1.00	16	0.500
1251	A	10	7	1.00	16	0.438
1252	A	1	1	1.00	16	0.062
1253	A	2	2	1.00	16	0.125
1254	A	3	2	1.00	16	0.125
1255	A	4	2	1.00	16	0.125
1256	A	7	5	1.00	16	0.312
1257	A	6	5	1.00	16	0.312
1258	A	5	5	1.00	16	0.312
1259	A	4	4	1.00	12	0.333
1260	A	5	5	1.00	16	0.312
1261	A	6	5	1.00	16	0.312
1262	A	7	5	1.00	16	0.312
1263	A	5	5	1.00	16	0.312
1264	A	3	2	1.00	13	0.154
1265	A	1	1	1.00	13	0.077
1266	A	2	2	1.00	13	0.154
1267	A	3	2	1.00	13	0.154
1268	A	3	2	1.00	13	0.154
1269	A	3	2	1.00	13	0.154
1270	A	3	2	1.00	13	0.154
1271	A	1	1	1.00	13	0.077
1272	A	4	4	1.00	13	0.308
1273	A	3	2	1.00	13	0.154
1274	A	3	2	1.00	13	0.154
1275	A	3	2	1.00	13	0.154
1276	A	6	6	1.01	9	0.667
1277	A	3	2	1.00	13	0.154
1278	A	3	2	1.00	13	0.154
1279	A	3	2	1.00	13	0.154
1280	A	3	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1281	A	1	1	1.00	13	0.077
1282	A	3	2	1.00	13	0.154
1283	A	3	2	1.00	13	0.154
1284	A	3	2	1.00	13	0.154
1285	A	3	2	1.00	15	0.133
1286	A	3	2	1.00	15	0.133
1287	A	1	1	1.00	15	0.067
1288	A	4	4	1.00	15	0.267
1289	A	3	2	1.00	15	0.133
1290	A	3	2	1.00	13	0.154
1291	A	3	2	1.00	13	0.154
1292	A	1	1	1.00	13	0.077
1293	A	4	4	1.00	13	0.308
1294	A	3	2	1.00	13	0.154
1295	A	3	2	1.00	11	0.182
1296	A	3	2	1.00	11	0.182
1297	A	1	1	1.00	11	0.091
1298	A	4	4	1.00	11	0.364
1299	A	3	2	1.00	11	0.182
1300	A	7	7	1.00	11	0.636
1301	A	6	6	1.00	11	0.546
1302	A	6	6	1.00	11	0.546
1303	A	6	6	1.00	9	0.667
1304	A	6	6	1.10	7	0.857
1305	A	7	7	1.00	11	0.636
1306	A	7	7	1.00	11	0.636
1307	A	7	7	1.00	11	0.636
1308	A	6	5	1.00	17	0.294
1309	A	5	5	1.00	17	0.294
1310	A	4	4	1.00	17	0.235
1311	A	1	1	1.00	17	0.059
1312	A	2	2	1.00	17	0.118
1313	A	5	4	1.00	15	0.267
1314	A	4	4	1.00	15	0.267
1315	A	3	3	1.00	15	0.200
1316	A	1	1	1.00	15	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1317	A	2	2	1.00	15	0.133
1318	A	3	3	1.00	13	0.231
1319	A	8	8	1.00	13	0.615
1320	A	11	7	1.00	13	0.538
1321	A	1	1	1.00	13	0.077
1322	A	10	6	1.00	13	0.462
1323	A	7	7	1.00	13	0.538
1324	A	2	2	1.00	13	0.154
1325	A	7	7	1.00	11	0.636
1326	A	10	6	1.00	9	0.667
1327	A	4	4	1.00	13	0.308
1328	A	11	7	1.00	13	0.538
1329	A	8	8	1.00	13	0.615
1330	A	3	3	1.00	13	0.231
1331	A	3	3	1.00	13	0.231
1332	A	8	8	1.00	13	0.615
1333	A	11	7	1.00	13	0.538
1334	A	1	1	1.00	13	0.077
1335	A	11	7	1.00	13	0.538
1336	A	8	8	1.00	13	0.615
1337	A	3	3	1.00	13	0.231
1338	A	8	8	1.00	11	0.727
1339	A	11	7	1.00	9	0.778
1340	A	3	2	1.00	13	0.154
1341	A	12	8	1.00	13	0.615
1342	A	9	9	1.00	13	0.692
1343	A	4	4	1.00	13	0.308
1344	A	3	3	1.00	13	0.231
1345	A	8	8	1.00	13	0.615
1346	A	11	7	1.52	13	0.538
1347	A	1	1	1.00	13	0.077
1348	A	10	6	1.55	13	0.462
1349	A	7	7	1.00	13	0.538
1350	A	2	2	1.00	13	0.154
1351	A	7	7	1.00	11	0.636
1352	A	10	6	1.55	9	0.667

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1353	A	4	4	1.00	13	0.308
1354	A	11	7	1.50	13	0.538
1355	A	8	8	1.00	13	0.615
1356	A	3	3	1.00	13	0.231
1357	A	8	8	1.00	13	0.615
1358	A	11	7	1.00	13	0.538
1359	A	3	2	1.00	13	0.154
1360	A	12	7	1.00	13	0.538
1361	A	3	3	1.00	11	0.273
1362	A	8	8	1.00	11	0.727
1363	A	11	7	1.00	11	0.636
1364	A	1	1	1.00	11	0.091
1365	A	10	6	1.00	11	0.546
1366	A	7	7	1.00	11	0.636
1367	A	2	2	1.00	11	0.182
1368	A	7	7	1.00	9	0.778
1369	A	10	6	1.00	7	0.857
1370	A	4	4	1.00	11	0.364
1371	A	11	7	1.00	11	0.636
1372	A	8	8	1.00	11	0.727
1373	A	3	3	1.00	11	0.273
1374	A	8	8	1.00	11	0.727
1375	A	11	7	1.00	11	0.636
1376	A	3	2	1.00	11	0.182
1377	A	12	7	1.00	11	0.636
1378	A	10	6	0.93	9	0.667
1379	A	8	8	1.00	15	0.533
1380	A	5	5	1.00	15	0.333
1381	A	1	1	1.00	17	0.059
1382	A	4	4	1.00	13	0.308
1383	A	3	2	1.00	13	0.154
1384	A	3	2	1.00	13	0.154
1385	A	3	2	1.00	13	0.154
1386	A	1	1	1.00	13	0.077
1387	A	3	3	1.00	13	0.231
1388	A	4	4	1.00	13	0.308

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1389	A	5	4	1.00	13	0.308
1390	A	4	3	1.00	13	0.231
1391	A	3	3	1.00	13	0.231
1392	A	2	2	1.00	13	0.154
1393	A	1	1	1.00	13	0.077
1394	A	2	2	1.00	13	0.154
1395	A	3	2	1.00	13	0.154
1396	A	3	3	1.00	13	0.231
1397	A	2	2	1.00	11	0.182
1398	A	3	3	1.00	13	0.231
1399	A	2	2	1.00	13	0.154
1400	A	1	1	1.00	9	0.111
1401	A	2	2	1.00	13	0.154
1402	A	5	5	1.00	13	0.385
1403	A	4	4	1.00	13	0.308
1404	A	5	5	1.00	13	0.385
1405	A	4	4	1.00	13	0.308
1406	A	3	3	1.00	13	0.231
1407	A	4	4	1.00	13	0.308
1408	A	3	2	1.00	13	0.154
1409	A	3	2	1.00	13	0.154
1410	A	3	2	1.00	13	0.154
1411	A	1	1	1.00	13	0.077
1412	A	4	4	1.00	13	0.308
1413	A	5	4	1.05	13	0.308
1414	A	6	4	1.04	13	0.308
1415	A	4	4	1.00	13	0.308
1416	A	3	3	1.00	13	0.231
1417	A	1	1	1.00	13	0.077
1418	A	2	2	1.00	13	0.154
1419	A	3	2	1.00	13	0.154
1420	A	4	4	1.00	13	0.308
1421	A	3	3	1.00	13	0.231
1422	A	3	3	1.00	11	0.273
1423	A	4	4	1.00	13	0.308
1424	A	3	3	1.00	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1425	A	2	2	1.00	13	0.154
1426	A	2	2	1.00	9	0.222
1427	A	3	3	1.00	13	0.231
1428	A	6	6	1.00	13	0.462
1429	A	5	5	1.00	13	0.385
1430	A	5	5	1.00	13	0.385
1431	A	6	6	1.00	13	0.462
1432	A	4	4	1.00	13	0.308
1433	A	4	4	1.00	13	0.308
1434	A	5	5	1.00	13	0.385
1435	A	2	1	1.00	11	0.091
1436	A	2	1	1.00	11	0.091
1437	A	2	1	1.00	11	0.091
1438	A	2	1	1.00	13	0.077
1439	A	2	1	1.00	13	0.077
1440	A	3	2	1.00	13	0.154
1441	A	1	1	1.00	13	0.077
1442	A	1	1	1.00	13	0.077
1443	A	6	6	1.00	9	0.667
1444	A	4	4	1.00	13	0.308
1445	A	1	1	1.00	14	0.071
1446	A	1	1	1.00	14	0.071
1447	A	6	6	1.00	10	0.600
1448	A	4	4	1.00	14	0.286
1449	A	6	6	1.00	9	0.667
1450	A	6	6	1.00	7	0.857
1451	A	11	8	1.00	13	0.615
1452	A	1	1	1.00	13	0.077
1453	A	10	7	1.00	13	0.538
1454	A	2	2	1.00	13	0.154
1455	A	10	7	1.00	11	0.636
1456	A	4	4	1.00	13	0.308
1457	A	11	8	1.00	13	0.615
1458	A	3	3	1.00	13	0.231
1459	A	11	8	1.00	13	0.615
1460	A	3	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1461	A	14	11	1.00	13	0.846
1462	A	13	10	1.00	13	0.769
1463	A	13	10	1.00	13	0.769
1464	A	13	10	1.00	13	0.769
1465	A	13	10	1.00	9	1.111
1466	A	14	11	1.00	13	0.846
1467	A	14	11	1.00	13	0.846
1468	A	14	11	1.00	13	0.846
1469	A	14	11	1.00	13	0.846
1470	A	13	10	1.00	10	1.000
1471	A	5	5	1.00	13	0.385
1472	A	1	1	1.00	13	0.077
1473	A	4	4	1.00	13	0.308
1474	A	2	2	1.00	13	0.154
1475	A	4	4	1.00	11	0.364
1476	A	4	4	1.00	13	0.308
1477	A	5	5	1.00	13	0.385
1478	A	3	3	1.00	13	0.231
1479	A	5	5	1.00	13	0.385
1480	A	3	2	1.00	13	0.154
1481	A	14	11	1.00	13	0.846
1482	A	13	10	1.00	13	0.769
1483	A	13	10	1.00	13	0.769
1484	A	13	10	1.00	13	0.769
1485	A	13	10	1.00	9	1.111
1486	A	14	11	1.00	13	0.846
1487	A	14	11	1.00	13	0.846
1488	A	14	11	1.00	13	0.846
1489	A	14	11	1.00	13	0.846
1490	A	11	8	1.00	11	0.727
1491	A	1	1	1.00	11	0.091
1492	A	10	7	1.00	11	0.636
1493	A	2	2	1.00	11	0.182
1494	A	10	7	1.00	9	0.778
1495	A	4	4	1.00	11	0.364
1496	A	11	8	1.00	11	0.727

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1497	A	3	3	1.00	11	0.273
1498	A	11	8	1.00	11	0.727
1499	A	3	2	1.00	11	0.182
1500	A	20	7	1.00	11	0.636
1501	A	21	7	1.00	11	0.636
1502	A	19	7	1.00	11	0.636
1503	A	19	6	1.00	11	0.546
1504	A	19	6	1.00	7	0.857
1505	A	22	8	1.00	11	0.727
1506	A	20	8	1.00	11	0.727
1507	A	20	7	1.00	11	0.636
1508	A	20	7	1.00	11	0.636
1509	A	3	3	1.00	13	0.231
1510	A	3	3	1.00	11	0.273
1511	A	4	4	1.00	13	0.308
1512	A	5	5	1.00	13	0.385
1513	A	4	4	1.00	13	0.308
1514	A	4	3	1.00	13	0.231
1515	A	3	2	1.00	13	0.154
1516	A	3	3	1.00	13	0.231
1517	A	1	1	1.00	13	0.077
1518	A	2	2	1.00	13	0.154
1519	A	3	3	1.00	13	0.231
1520	A	1	1	1.00	13	0.077
1521	A	4	4	1.00	13	0.308
1522	A	2	2	1.00	13	0.154
1523	A	5	4	1.00	13	0.308
1524	A	5	5	1.00	13	0.385
1525	A	3	3	1.00	13	0.231
1526	A	4	4	1.00	13	0.308
1527	A	2	2	1.00	11	0.182
1528	A	5	5	1.00	13	0.385
1529	A	3	3	1.00	13	0.231
1530	A	4	3	1.00	13	0.231
1531	A	4	3	1.00	13	0.231
1532	A	1	1	1.00	13	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1533	A	1	1	1.00	13	0.077
1534	A	3	2	1.00	13	0.154
1535	A	3	2	1.00	9	0.222
1536	A	2	2	1.00	13	0.154
1537	A	2	2	1.00	13	0.154
1538	A	4	3	1.00	13	0.231
1539	A	4	3	1.00	13	0.231
1540	A	10	6	1.99	9	0.667
1541	A	2	2	1.00	15	0.133
1542	A	3	3	1.00	13	0.231
1543	A	2	2	1.00	11	0.182
1544	A	2	2	1.00	13	0.154
1545	A	3	3	1.00	13	0.231
1546	A	2	1	1.00	11	0.091
1547	A	2	1	1.00	11	0.091
1548	A	2	1	1.00	11	0.091
1549	A	2	1	1.00	11	0.091
1550	A	2	1	1.00	11	0.091
1551	A	2	1	1.00	9	0.111
1552	A	1	0	1.00	7	0.000
1553	A	2	1	1.00	11	0.091
1554	A	2	1	1.00	11	0.091
1555	A	2	1	1.00	11	0.091
1556	A	2	1	1.00	11	0.091
1557	A	2	1	1.00	11	0.091
1558	A	2	1	1.00	11	0.091
1559	A	3	2	1.00	13	0.154
1560	A	3	2	1.00	13	0.154
1561	A	3	2	1.00	13	0.154
1562	A	2	2	1.00	13	0.154
1563	A	3	2	1.00	11	0.182
1564	A	3	2	1.00	9	0.222
1565	A	3	2	1.00	13	0.154
1566	A	1	1	1.00	13	0.077
1567	A	3	2	1.00	13	0.154
1568	A	3	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1569	A	3	2	1.00	13	0.154
1570	A	3	2	1.00	13	0.154
1571	A	3	2	1.00	13	0.154
1572	A	3	2	1.00	13	0.154
1573	A	2	2	1.00	13	0.154
1574	A	3	2	1.00	13	0.154
1575	A	3	2	1.00	11	0.182
1576	A	3	2	1.00	9	0.222
1577	A	3	2	1.00	13	0.154
1578	A	1	1	1.00	13	0.077
1579	A	3	3	1.00	13	0.231
1580	A	3	2	1.00	13	0.154
1581	A	3	2	1.00	13	0.154
1582	A	3	2	1.00	13	0.154
1583	A	3	2	1.00	13	0.154
1584	A	3	2	1.00	13	0.154
1585	A	3	2	1.00	13	0.154
1586	A	3	2	1.00	13	0.154
1587	A	3	2	1.00	13	0.154
1588	A	3	2	1.00	13	0.154
1589	A	3	2	1.00	13	0.154
1590	A	2	2	1.00	13	0.154
1591	A	3	2	1.00	13	0.154
1592	A	3	2	1.00	13	0.154
1593	A	3	2	1.00	13	0.154
1594	A	3	2	1.00	13	0.154
1595	A	3	2	1.00	13	0.154
1596	A	3	2	1.00	13	0.154
1597	A	3	2	1.00	11	0.182
1598	A	3	2	1.00	9	0.222
1599	A	3	2	1.00	13	0.154
1600	A	1	1	1.00	13	0.077
1601	A	3	3	1.00	13	0.231
1602	A	4	3	1.00	13	0.231
1603	A	5	3	1.00	13	0.231
1604	A	6	3	1.00	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1605	A	3	2	1.00	13	0.154
1606	A	3	2	1.00	13	0.154
1607	A	3	2	1.00	13	0.154
1608	A	3	2	1.00	13	0.154
1609	A	3	2	1.00	13	0.154
1610	A	3	2	1.00	11	0.182
1611	A	3	2	1.00	9	0.222
1612	A	2	2	1.00	13	0.154
1613	A	1	1	1.00	13	0.077
1614	A	3	2	1.00	13	0.154
1615	A	3	2	1.00	13	0.154
1616	A	3	2	1.00	13	0.154
1617	A	3	2	1.00	13	0.154
1618	A	3	2	1.00	13	0.154
1619	A	3	2	1.00	13	0.154
1620	A	3	2	1.00	13	0.154
1621	A	3	2	1.00	13	0.154
1622	A	3	2	1.00	13	0.154
1623	A	3	2	1.00	11	0.182
1624	A	4	3	1.00	9	0.333
1625	A	3	2	1.00	13	0.154
1626	A	1	1	1.00	13	0.077
1627	A	3	2	1.00	13	0.154
1628	A	3	2	1.00	13	0.154
1629	A	3	2	1.00	13	0.154
1630	A	3	2	1.00	13	0.154
1631	A	3	2	1.00	13	0.154
1632	A	3	2	1.00	13	0.154
1633	A	3	2	1.00	13	0.154
1634	A	3	2	1.00	13	0.154
1635	A	3	2	1.00	13	0.154
1636	A	3	2	1.00	11	0.182
1637	A	5	3	1.00	9	0.333
1638	A	3	2	1.00	13	0.154
1639	A	1	1	1.00	13	0.077
1640	A	2	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1641	A	3	2	1.00	13	0.154
1642	A	3	2	1.00	13	0.154
1643	A	3	2	1.00	13	0.154
1644	A	3	2	1.00	13	0.154
1645	A	3	2	1.00	13	0.154
1646	A	3	2	1.00	13	0.154
1647	A	2	1	1.00	13	0.077
1648	A	2	1	1.00	13	0.077
1649	A	2	1	1.00	13	0.077
1650	A	2	1	1.00	13	0.077
1651	A	2	1	1.00	13	0.077
1652	A	2	1	1.00	13	0.077
1653	A	3	2	1.00	15	0.133
1654	A	3	2	1.00	15	0.133
1655	A	3	2	1.00	15	0.133
1656	A	3	2	1.00	15	0.133
1657	A	3	2	1.00	15	0.133
1658	A	3	2	1.00	15	0.133
1659	A	3	2	1.00	15	0.133
1660	A	3	2	1.00	15	0.133
1661	A	3	2	1.00	15	0.133
1662	A	3	2	1.00	15	0.133
1663	A	3	2	1.00	15	0.133
1664	A	3	2	1.00	15	0.133
1665	A	7	4	1.00	15	0.267
1666	A	6	4	1.00	15	0.267
1667	A	5	4	1.00	15	0.267
1668	A	4	4	1.00	15	0.267
1669	A	3	3	1.00	15	0.200
1670	A	4	4	1.00	15	0.267
1671	A	5	4	1.00	15	0.267
1672	A	6	4	1.00	15	0.267
1673	A	8	5	1.00	15	0.333
1674	A	7	5	1.00	15	0.333
1675	A	6	5	1.00	15	0.333
1676	A	5	5	1.00	15	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1677	A	4	4	1.00	15	0.267
1678	A	4	4	1.00	15	0.267
1679	A	5	4	1.00	15	0.267
1680	A	6	4	1.00	15	0.267
1681	A	7	4	1.00	15	0.267
1682	A	8	5	1.00	15	0.333
1683	A	7	5	1.00	15	0.333
1684	A	6	5	1.00	15	0.333
1685	A	5	4	1.00	15	0.267
1686	A	5	5	1.00	15	0.333
1687	A	5	4	1.00	15	0.267
1688	A	6	4	1.00	15	0.267
1689	A	7	4	1.00	15	0.267
1690	A	8	4	1.00	15	0.267
1691	A	7	5	1.00	15	0.333
1692	A	6	5	1.00	15	0.333
1693	A	5	5	1.00	13	0.385
1694	A	4	4	1.00	11	0.364
1695	A	4	4	1.00	15	0.267
1696	A	1	1	1.00	15	0.067
1697	A	3	2	1.00	15	0.133
1698	A	3	2	1.00	15	0.133
1699	A	3	2	1.00	15	0.133
1700	A	3	2	1.00	15	0.133
1701	A	7	5	1.00	15	0.333
1702	A	6	5	1.00	15	0.333
1703	A	5	4	1.00	13	0.308
1704	A	5	5	1.00	11	0.454
1705	A	5	4	1.00	15	0.267
1706	A	1	1	1.00	15	0.067
1707	A	3	2	1.00	15	0.133
1708	A	3	2	1.00	15	0.133
1709	A	3	2	1.00	15	0.133
1710	A	3	2	1.00	15	0.133
1711	A	3	2	1.00	15	0.133
1712	A	7	5	1.00	15	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1713	A	6	4	1.00	15	0.267
1714	A	6	5	1.00	13	0.385
1715	A	6	5	1.00	11	0.454
1716	A	6	4	1.00	15	0.267
1717	A	1	1	1.00	15	0.067
1718	A	3	2	1.00	15	0.133
1719	A	3	2	1.00	15	0.133
1720	A	3	2	1.00	15	0.133
1721	A	3	2	1.00	15	0.133
1722	A	7	4	1.00	15	0.267
1723	A	6	4	1.00	15	0.267
1724	A	5	4	1.00	13	0.308
1725	A	4	4	1.00	11	0.364
1726	A	3	3	1.00	15	0.200
1727	A	1	1	1.00	15	0.067
1728	A	3	2	1.00	15	0.133
1729	A	3	2	1.00	15	0.133
1730	A	3	2	1.00	15	0.133
1731	A	3	2	1.00	15	0.133
1732	A	7	4	0.98	15	0.267
1733	A	6	4	0.98	13	0.308
1734	A	5	4	1.02	11	0.364
1735	A	4	4	1.00	15	0.267
1736	A	1	1	1.00	15	0.067
1737	A	3	2	1.00	15	0.133
1738	A	3	2	1.00	15	0.133
1739	A	3	2	1.00	15	0.133
1740	A	3	2	1.00	15	0.133
1741	A	3	2	1.00	15	0.133
1742	A	8	4	0.97	15	0.267
1743	A	7	4	1.00	13	0.308
1744	A	6	4	1.04	11	0.364
1745	A	5	4	1.00	15	0.267
1746	A	1	1	1.00	15	0.067
1747	A	3	2	1.00	15	0.133
1748	A	3	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1749	A	3	2	1.00	15	0.133
1750	A	3	2	1.00	15	0.133
1751	A	3	2	1.00	15	0.133
1752	A	4	2	1.00	17	0.118
1753	A	3	2	1.00	17	0.118
1754	A	2	2	1.00	17	0.118
1755	A	1	1	1.00	17	0.059
1756	A	4	4	1.00	17	0.235
1757	A	4	4	1.00	17	0.235
1758	A	5	5	1.00	17	0.294
1759	A	6	5	1.00	17	0.294
1760	A	4	2	1.00	17	0.118
1761	A	3	2	1.00	17	0.118
1762	A	2	2	1.00	17	0.118
1763	A	1	1	1.00	17	0.059
1764	A	5	4	1.00	17	0.235
1765	A	5	5	1.00	17	0.294
1766	A	5	4	1.00	17	0.235
1767	A	6	5	1.00	17	0.294
1768	A	4	2	1.00	17	0.118
1769	A	3	2	1.00	17	0.118
1770	A	2	2	1.00	17	0.118
1771	A	1	1	1.00	17	0.059
1772	A	6	4	1.00	17	0.235
1773	A	6	5	1.00	17	0.294
1774	A	6	5	1.00	17	0.294
1775	A	6	4	1.00	17	0.235
1776	A	7	5	1.00	17	0.294
1777	A	5	2	1.00	17	0.118
1778	A	4	2	1.00	17	0.118
1779	A	3	2	1.00	17	0.118
1780	A	2	2	1.00	17	0.118
1781	A	1	1	1.00	17	0.059
1782	A	3	3	1.00	17	0.176
1783	A	4	4	1.00	17	0.235
1784	A	5	4	1.00	17	0.235

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1785	A	6	4	1.00	17	0.235
1786	A	5	2	1.00	17	0.118
1787	A	4	2	1.00	17	0.118
1788	A	3	2	1.00	17	0.118
1789	A	2	2	1.00	17	0.118
1790	A	1	1	1.00	17	0.059
1791	A	4	4	1.00	17	0.235
1792	A	5	5	1.00	17	0.294
1793	A	6	5	1.00	17	0.294
1794	A	7	5	1.00	17	0.294
1795	A	6	2	1.00	17	0.118
1796	A	5	2	1.00	17	0.118
1797	A	4	2	1.00	17	0.118
1798	A	3	2	1.00	17	0.118
1799	A	2	2	1.00	17	0.118
1800	A	1	1	1.00	17	0.059
1801	A	5	4	1.00	17	0.235
1802	A	6	5	1.00	17	0.294
1803	A	7	5	1.00	17	0.294
1804	A	2	1	1.00	11	0.091
1805	A	2	1	1.00	11	0.091
1806	A	2	1	1.00	11	0.091
1807	A	2	1	1.00	11	0.091
1808	A	2	1	1.00	11	0.091
1809	A	2	1	1.00	9	0.111
1810	A	1	0	1.00	7	0.000
1811	A	2	1	1.00	11	0.091
1812	A	2	1	1.00	11	0.091
1813	A	2	1	1.00	11	0.091
1814	A	2	1	1.00	11	0.091
1815	A	2	1	1.00	11	0.091
1816	A	2	1	1.00	11	0.091
1817	A	3	2	1.00	13	0.154
1818	A	2	2	1.00	13	0.154
1819	A	3	2	1.00	13	0.154
1820	A	4	3	1.00	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1821	A	3	2	1.00	13	0.154
1822	A	4	3	1.00	11	0.273
1823	A	3	2	1.00	9	0.222
1824	A	4	3	1.00	13	0.231
1825	A	3	2	1.00	13	0.154
1826	A	1	1	1.00	13	0.077
1827	A	3	2	1.00	13	0.154
1828	A	4	3	1.00	13	0.231
1829	A	3	2	1.00	13	0.154
1830	A	3	2	1.00	13	0.154
1831	A	4	3	1.00	13	0.231
1832	A	3	2	1.00	13	0.154
1833	A	4	3	1.00	13	0.231
1834	A	3	2	1.00	13	0.154
1835	A	4	3	1.00	11	0.273
1836	A	3	2	1.00	9	0.222
1837	A	4	3	1.00	13	0.231
1838	A	3	2	1.00	13	0.154
1839	A	1	1	1.00	13	0.077
1840	A	3	2	1.00	13	0.154
1841	A	4	4	1.00	13	0.308
1842	A	3	2	1.00	13	0.154
1843	A	4	3	1.00	13	0.231
1844	A	4	3	1.00	13	0.231
1845	A	4	3	1.00	13	0.231
1846	A	4	3	1.00	13	0.231
1847	A	4	3	1.00	13	0.231
1848	A	4	3	1.00	11	0.273
1849	A	3	3	1.00	9	0.333
1850	A	2	2	1.00	13	0.154
1851	A	2	2	1.00	13	0.154
1852	A	1	1	1.00	13	0.077
1853	A	3	3	1.00	13	0.231
1854	A	4	3	1.00	13	0.231
1855	A	4	3	1.00	13	0.231
1856	A	4	3	1.00	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1857	A	5	3	1.00	13	0.231
1858	A	5	4	1.00	13	0.308
1859	A	4	3	1.00	13	0.231
1860	A	5	4	1.00	13	0.308
1861	A	4	3	1.00	13	0.231
1862	A	5	4	1.00	13	0.308
1863	A	4	3	1.00	11	0.273
1864	A	4	4	1.00	9	0.444
1865	A	4	3	1.00	13	0.231
1866	A	3	3	1.00	13	0.231
1867	A	1	1	1.00	13	0.077
1868	A	3	3	1.00	13	0.231
1869	A	4	3	1.00	13	0.231
1870	A	4	4	1.00	13	0.308
1871	A	4	3	1.00	13	0.231
1872	A	5	4	1.00	13	0.308
1873	A	4	3	1.00	13	0.231
1874	A	4	3	1.00	13	0.231
1875	A	6	4	1.00	13	0.308
1876	A	4	3	1.00	13	0.231
1877	A	6	4	1.00	13	0.308
1878	A	4	3	1.00	11	0.273
1879	A	5	4	1.00	9	0.444
1880	A	4	3	1.00	13	0.231
1881	A	4	3	1.00	13	0.231
1882	A	1	1	1.00	13	0.077
1883	A	4	4	1.00	13	0.308
1884	A	2	2	1.00	13	0.154
1885	A	4	3	1.00	13	0.231
1886	A	4	3	1.00	13	0.231
1887	A	5	4	1.00	13	0.308
1888	A	4	3	1.00	13	0.231
1889	A	6	4	1.00	13	0.308
1890	A	4	3	1.00	13	0.231
1891	A	5	5	1.00	15	0.333
1892	A	1	1	1.00	15	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1893	A	4	4	1.00	13	0.308
1894	A	4	4	1.00	11	0.364
1895	A	4	4	1.00	15	0.267
1896	A	4	4	1.00	15	0.267
1897	A	1	1	1.00	15	0.067
1898	A	5	5	1.00	15	0.333
1899	A	5	4	1.00	15	0.267
1900	A	5	4	1.00	15	0.267
1901	A	5	5	1.00	13	0.385
1902	A	5	5	1.00	11	0.454
1903	A	5	4	1.00	15	0.267
1904	A	5	4	1.00	15	0.267
1905	A	1	1	1.00	15	0.067
1906	A	6	5	1.00	15	0.333
1907	A	6	5	1.00	15	0.333
1908	A	6	5	1.00	15	0.333
1909	A	6	5	1.00	13	0.385
1910	A	6	5	1.00	11	0.454
1911	A	6	4	1.00	15	0.267
1912	A	6	4	1.00	15	0.267
1913	A	1	1	1.00	15	0.067
1914	A	7	5	1.00	15	0.333
1915	A	5	4	1.00	15	0.267
1916	A	4	4	1.00	13	0.308
1917	A	3	3	1.00	15	0.200
1918	A	1	1	1.00	15	0.067
1919	A	3	2	1.00	15	0.133
1920	A	3	2	1.00	15	0.133
1921	A	3	2	1.00	15	0.133
1922	A	3	2	1.00	15	0.133
1923	A	2	2	1.00	15	0.133
1924	A	1	1	1.00	11	0.091
1925	A	3	3	1.00	15	0.200
1926	A	4	4	1.00	15	0.267
1927	A	3	3	1.00	17	0.176
1928	A	2	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1929	A	2	2	1.00	16	0.125
1930	A	6	4	0.98	15	0.267
1931	A	5	4	1.00	13	0.308
1932	A	4	4	1.00	15	0.267
1933	A	1	1	1.00	15	0.067
1934	A	3	2	1.00	15	0.133
1935	A	3	2	1.00	15	0.133
1936	A	3	2	1.00	15	0.133
1937	A	4	3	1.00	15	0.200
1938	A	3	3	1.00	15	0.200
1939	A	2	2	1.00	11	0.182
1940	A	1	1	1.00	15	0.067
1941	A	4	4	1.00	15	0.267
1942	A	5	5	1.00	15	0.333
1943	A	6	5	1.00	15	0.333
1944	A	7	4	1.00	15	0.267
1945	A	6	4	1.05	13	0.308
1946	A	5	4	1.00	15	0.267
1947	A	1	1	1.00	15	0.067
1948	A	3	2	1.00	15	0.133
1949	A	3	2	1.00	15	0.133
1950	A	3	2	1.00	15	0.133
1951	A	4	3	1.00	15	0.200
1952	A	3	2	1.00	11	0.182
1953	A	2	2	1.00	15	0.133
1954	A	1	1	1.00	15	0.067
1955	A	5	4	1.00	15	0.267
1956	A	6	5	1.00	15	0.333
1957	A	1	1	1.00	13	0.077
1958	A	1	1	1.00	13	0.077
1959	A	2	2	1.00	17	0.118
1960	A	2	2	1.00	17	0.118
1961	A	2	2	1.00	17	0.118
1962	A	2	2	1.00	17	0.118
1963	A	2	2	1.00	15	0.133
1964	A	3	3	1.00	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1965	A	4	3	1.00	13	0.231
1966	A	9	8	1.00	13	0.615
1967	A	9	8	1.00	13	0.615
1968	A	4	3	1.00	13	0.231
1969	A	8	8	1.00	11	0.727
1970	A	8	8	1.00	9	0.889
1971	A	2	2	1.00	13	0.154
1972	A	7	7	1.00	13	0.538
1973	A	7	7	1.00	13	0.538
1974	A	1	1	1.00	13	0.077
1975	A	8	8	1.00	13	0.615
1976	A	8	8	1.00	13	0.615
1977	A	4	3	1.00	13	0.231
1978	A	4	3	1.00	13	0.231
1979	A	10	9	1.00	13	0.692
1980	A	10	9	1.00	13	0.692
1981	A	4	3	1.00	13	0.231
1982	A	9	9	1.00	11	0.818
1983	A	9	9	1.00	9	1.000
1984	A	4	3	1.00	13	0.231
1985	A	8	8	1.00	13	0.615
1986	A	8	8	1.00	13	0.615
1987	A	1	1	1.00	13	0.077
1988	A	8	8	1.00	13	0.615
1989	A	8	8	1.00	13	0.615
1990	A	4	3	1.00	13	0.231
1991	A	5	5	1.00	15	0.333
1992	A	4	4	1.00	15	0.267
1993	A	4	4	1.00	15	0.267
1994	A	1	1	1.00	15	0.067
1995	A	3	2	1.00	15	0.133
1996	A	3	2	1.00	15	0.133
1997	A	3	2	1.00	15	0.133
1998	A	5	4	1.00	15	0.267
1999	A	4	4	1.00	15	0.267
2000	A	3	3	1.00	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
2001	A	3	3	1.00	15	0.200
2002	A	4	4	1.00	15	0.267
2003	A	5	4	1.00	15	0.267
2004	A	7	6	1.00	15	0.400
2005	A	6	6	1.00	15	0.400
2006	A	5	5	1.00	11	0.454
2007	A	5	5	1.00	15	0.333
2008	A	6	6	1.00	15	0.400
2009	A	7	6	1.00	15	0.400
2010	A	5	4	1.00	15	0.267
2011	A	5	5	1.00	15	0.333
2012	A	5	4	1.00	15	0.267
2013	A	1	1	1.00	15	0.067
2014	A	3	2	1.00	15	0.133
2015	A	3	2	1.00	15	0.133
2016	A	3	2	1.00	15	0.133
2017	A	5	4	1.00	15	0.267
2018	A	4	4	1.00	15	0.267
2019	A	3	3	1.00	15	0.200
2020	A	1	1	1.00	15	0.067
2021	A	3	2	1.00	15	0.133
2022	A	3	2	1.00	15	0.133
2023	A	3	2	1.00	15	0.133
2024	A	5	3	1.00	15	0.200
2025	A	4	3	1.00	15	0.200
2026	A	3	3	1.00	13	0.231
2027	A	2	2	1.00	15	0.133
2028	A	3	3	1.00	15	0.200
2029	A	4	3	1.00	15	0.200
2030	A	7	5	1.00	15	0.333
2031	A	6	5	1.00	15	0.333
2032	A	5	5	1.00	11	0.454
2033	A	4	4	1.00	15	0.267
2034	A	5	5	1.00	15	0.333
2035	A	6	5	1.00	15	0.333
2036	A	7	5	1.00	15	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
2037	A	6	4	1.00	15	0.267
2038	A	5	4	1.03	15	0.267
2039	A	4	4	1.00	15	0.267
2040	A	1	1	1.00	15	0.067
2041	A	3	2	1.00	15	0.133
2042	A	3	2	1.00	15	0.133
2043	A	3	2	1.00	15	0.133
2044	A	6	4	1.00	15	0.267
2045	A	5	4	1.00	15	0.267
2046	A	4	4	1.00	13	0.308
2047	A	3	3	1.00	15	0.200
2048	A	3	3	1.00	15	0.200
2049	A	4	4	1.00	15	0.267
2050	A	8	6	1.00	15	0.400
2051	A	7	6	1.00	15	0.400
2052	A	6	6	1.00	11	0.546
2053	A	5	5	1.00	15	0.333
2054	A	5	5	1.00	15	0.333
2055	A	6	6	1.00	15	0.400
2056	A	7	6	1.00	15	0.400
2057	A	11	8	1.00	9	0.889
2058	A	4	4	1.00	15	0.267
2059	A	5	5	1.00	13	0.385
2060	A	4	4	1.00	15	0.267
2061	A	5	5	1.00	15	0.333
2062	A	3	3	1.00	15	0.200
2063	A	5	5	1.00	11	0.454
2064	A	3	3	1.00	15	0.200
2065	A	5	5	1.00	15	0.333
2066	A	5	5	1.00	15	0.333
2067	A	6	6	1.00	13	0.462
2068	A	5	4	1.00	15	0.267
2069	A	6	5	1.00	15	0.333
2070	A	4	4	1.00	15	0.267
2071	A	6	6	1.00	11	0.546
2072	A	4	3	1.00	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
2073	A	6	5	1.00	15	0.333
2074	A	6	5	1.00	15	0.333
2075	A	7	6	1.00	13	0.462
2076	A	6	4	1.00	15	0.267
2077	A	7	5	1.00	15	0.333
2078	A	5	4	1.00	15	0.267
2079	A	7	6	1.00	11	0.546
2080	A	5	3	1.00	15	0.200
2081	A	7	5	1.00	15	0.333
2082	A	4	4	1.00	15	0.267
2083	A	1	1	1.00	13	0.077
2084	A	3	3	1.00	15	0.200
2085	A	4	4	1.00	15	0.267
2086	A	3	3	1.00	15	0.200
2087	A	5	5	1.00	11	0.454
2088	A	2	2	1.00	15	0.133
2089	A	4	4	1.00	15	0.267
2090	A	5	4	1.03	15	0.267
2091	A	2	2	1.00	13	0.154
2092	A	4	4	1.00	15	0.267
2093	A	1	1	1.00	15	0.067
2094	A	4	4	1.00	15	0.267
2095	A	6	6	1.00	11	0.546
2096	A	3	3	1.00	15	0.200
2097	A	5	5	1.00	15	0.333
2098	A	6	4	1.05	15	0.267
2099	A	3	2	1.00	13	0.154
2100	A	5	4	1.00	15	0.267
2101	A	2	2	1.00	15	0.133
2102	A	5	4	1.00	15	0.267
2103	A	7	6	1.00	11	0.546
2104	A	4	3	1.00	15	0.200
2105	A	6	5	1.00	15	0.333
2106	A	8	8	1.00	9	0.889
2107	A	3	3	1.00	15	0.200
2108	A	3	3	1.00	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
2109	A	12	8	1.00	9	0.889
2110	A	15	12	1.00	9	1.333
2111	A	2	1	1.00	13	0.077
2112	A	2	1	1.00	13	0.077
2113	A	2	1	1.00	13	0.077
2114	A	2	1	1.00	11	0.091
2115	A	1	0	1.00	9	0.000
2116	A	2	1	1.00	13	0.077
2117	A	2	1	1.00	13	0.077
2118	A	2	1	1.00	13	0.077
2119	A	2	1	1.00	13	0.077
2120	A	3	2	1.00	15	0.133
2121	A	3	2	1.00	15	0.133
2122	A	3	2	1.00	15	0.133
2123	A	3	2	1.00	13	0.154
2124	A	3	2	1.00	11	0.182
2125	A	3	2	1.00	15	0.133
2126	A	3	2	1.00	15	0.133
2127	A	3	2	1.00	15	0.133
2128	A	3	2	1.00	15	0.133
2129	A	3	2	1.00	15	0.133
2130	A	3	2	1.00	15	0.133
2131	A	3	2	1.00	15	0.133
2132	A	3	2	1.00	15	0.133
2133	A	3	2	1.00	13	0.154
2134	A	3	2	1.00	11	0.182
2135	A	3	2	1.00	15	0.133
2136	A	3	2	1.00	15	0.133
2137	A	1	1	1.00	15	0.067
2138	A	3	2	1.00	15	0.133
2139	A	3	2	1.00	15	0.133
2140	A	3	2	1.00	15	0.133
2141	A	3	2	1.00	15	0.133
2142	A	3	2	1.00	15	0.133
2143	A	3	2	1.00	15	0.133
2144	A	3	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
2145	A	3	2	1.00	11	0.182
2146	A	3	2	1.00	15	0.133
2147	A	3	2	1.00	15	0.133
2148	A	3	2	1.00	15	0.133
2149	A	1	1	1.00	15	0.067
2150	A	4	3	1.00	15	0.200
2151	A	3	2	1.00	15	0.133
2152	A	3	2	1.00	15	0.133
2153	A	3	2	1.00	15	0.133
2154	A	3	2	1.00	15	0.133
2155	A	3	2	1.00	15	0.133
2156	A	3	2	1.00	13	0.154
2157	A	3	2	1.00	11	0.182
2158	A	3	2	1.00	15	0.133
2159	A	3	2	1.00	15	0.133
2160	A	3	2	1.00	15	0.133
2161	A	3	2	1.00	15	0.133
2162	A	3	2	1.00	15	0.133
2163	A	3	2	1.00	15	0.133
2164	A	3	3	1.00	15	0.200
2165	A	5	3	1.00	15	0.200
2166	A	7	3	1.00	15	0.200
2167	A	3	2	1.00	15	0.133
2168	A	3	2	1.00	15	0.133
2169	A	3	2	1.00	15	0.133
2170	A	3	2	1.00	15	0.133
2171	A	3	2	1.00	15	0.133
2172	A	3	2	1.00	15	0.133
2173	A	3	2	1.00	13	0.154
2174	A	3	2	1.00	11	0.182
2175	A	3	2	1.00	15	0.133
2176	A	3	2	1.00	15	0.133
2177	A	3	2	1.00	15	0.133
2178	A	3	2	1.00	15	0.133
2179	A	3	2	1.00	15	0.133
2180	A	3	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
2181	A	3	2	1.00	15	0.133
2182	A	1	1	1.00	15	0.067
2183	A	4	3	1.00	15	0.200
2184	A	6	3	1.00	15	0.200
2185	A	8	3	1.00	15	0.200
2186	A	10	3	1.00	15	0.200
2187	A	12	3	1.00	15	0.200
2188	A	3	2	1.00	15	0.133
2189	A	3	2	1.00	15	0.133
2190	A	3	2	1.00	15	0.133
2191	A	3	2	1.00	15	0.133
2192	A	3	2	1.00	15	0.133
2193	A	3	2	1.00	13	0.154
2194	A	3	2	1.00	11	0.182
2195	A	4	4	1.00	15	0.267
2196	A	3	2	1.00	15	0.133
2197	A	3	2	1.00	15	0.133
2198	A	3	2	1.00	15	0.133
2199	A	3	2	1.00	15	0.133
2200	A	3	2	1.00	15	0.133
2201	A	3	2	1.00	13	0.154
2202	A	3	2	1.00	11	0.182
2203	A	3	2	1.00	15	0.133
2204	A	3	2	1.00	15	0.133
2205	A	3	2	1.00	15	0.133
2206	A	3	2	1.00	15	0.133
2207	A	3	2	1.00	15	0.133
2208	A	3	2	1.00	15	0.133
2209	A	3	2	1.00	13	0.154
2210	A	2	2	1.00	11	0.182
2211	A	3	2	1.00	15	0.133
2212	A	3	2	1.00	15	0.133
2213	A	3	2	1.00	15	0.133
2214	A	3	2	1.00	15	0.133
2215	A	3	2	1.00	15	0.133
2216	A	3	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
2217	A	3	2	1.00	15	0.133
2218	A	1	1	1.00	13	0.077
2219	A	3	2	1.00	11	0.182
2220	A	3	2	1.00	15	0.133
2221	A	3	2	1.00	15	0.133
2222	A	3	2	1.00	15	0.133
2223	A	3	2	1.00	15	0.133
2224	A	3	2	1.00	15	0.133
2225	A	3	2	1.00	15	0.133
2226	A	3	3	1.00	15	0.200
2227	A	3	2	1.00	13	0.154
2228	A	3	2	1.00	11	0.182
2229	A	3	2	1.00	15	0.133
2230	A	3	2	1.00	15	0.133
2231	A	3	2	1.00	15	0.133
2232	A	4	4	1.00	15	0.267
2233	A	3	2	1.00	17	0.118
2234	A	3	2	1.00	15	0.133
2235	A	3	2	1.00	13	0.154
2236	A	4	4	1.00	17	0.235
2237	A	5	5	1.00	17	0.294
2238	A	7	5	1.00	17	0.294
2239	A	3	2	1.00	17	0.118
2240	A	3	2	1.00	15	0.133
2241	A	3	2	1.00	13	0.154
2242	A	3	3	1.00	17	0.176
2243	A	5	4	1.00	17	0.235
2244	A	7	4	1.00	17	0.235
2245	A	3	2	1.00	17	0.118
2246	A	1	1	1.00	17	0.059
2247	A	2	1	1.00	13	0.077
2248	A	1	1	1.00	15	0.067
2249	A	1	1	1.00	15	0.067
2250	A	3	2	1.00	15	0.133
2251	A	1	1	1.00	15	0.067
2252	A	1	1	1.00	15	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
2253	A	1	1	1.00	15	0.067
2254	A	3	2	1.00	17	0.118
2255	A	1	1	1.00	17	0.059
2256	A	7	6	1.00	15	0.400
2257	A	2	1	1.00	15	0.067
2258	A	2	1	1.00	15	0.067
2259	A	2	1	1.00	15	0.067
2260	A	2	1	1.00	13	0.077
2261	A	2	2	1.00	15	0.133
2262	A	2	2	1.00	15	0.133
2263	A	3	3	1.21	15	0.200
2264	A	3	2	1.00	15	0.133
2265	A	3	2	1.00	15	0.133
2266	A	3	2	1.00	13	0.154
2267	A	3	2	1.00	11	0.182
2268	A	2	2	1.00	15	0.133
2269	A	2	2	1.00	15	0.133
2270	A	1	1	1.00	15	0.067
2271	A	10	9	1.42	17	0.529
2272	A	8	8	1.71	13	0.615
2273	A	2	2	1.00	17	0.118
2274	A	4	2	1.00	17	0.118
2275	A	6	2	1.00	17	0.118
2276	A	3	2	1.00	17	0.118
2277	A	3	2	1.00	17	0.118
2278	A	3	2	1.00	17	0.118
2279	A	5	5	1.00	17	0.294
2280	A	7	6	1.00	17	0.353
2281	A	3	3	1.36	17	0.176
2282	A	3	3	1.36	15	0.200
2283	A	3	3	1.38	17	0.176
2284	A	3	3	1.00	17	0.176
2285	A	2	1	1.00	13	0.077
2286	A	2	1	1.00	13	0.077
2287	A	2	1	1.00	13	0.077
2288	A	2	1	1.00	11	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
2289	A	1	0	1.00	9	0.000
2290	A	2	1	1.00	13	0.077
2291	A	2	1	1.00	13	0.077
2292	A	2	1	1.00	13	0.077
2293	A	2	1	1.00	13	0.077
2294	A	3	2	1.00	15	0.133
2295	A	3	2	1.00	15	0.133
2296	A	3	2	1.00	15	0.133
2297	A	3	2	1.00	13	0.154
2298	A	3	2	1.00	11	0.182
2299	A	3	2	1.00	15	0.133
2300	A	1	1	1.00	15	0.067
2301	A	3	2	1.00	15	0.133
2302	A	3	2	1.00	15	0.133
2303	A	3	2	1.00	15	0.133
2304	A	3	2	1.00	15	0.133
2305	A	3	2	1.00	15	0.133
2306	A	3	2	1.00	13	0.154
2307	A	3	2	1.00	11	0.182
2308	A	3	2	1.00	15	0.133
2309	A	3	2	1.00	15	0.133
2310	A	3	2	1.00	15	0.133
2311	A	3	2	1.00	15	0.133
2312	A	3	2	1.00	15	0.133
2313	A	3	2	1.00	15	0.133
2314	A	3	2	1.00	15	0.133
2315	A	3	2	1.00	13	0.154
2316	A	3	2	1.00	11	0.182
2317	A	3	2	1.00	15	0.133
2318	A	3	2	1.00	15	0.133
2319	A	1	1	1.00	15	0.067
2320	A	3	2	1.00	15	0.133
2321	A	3	2	1.00	15	0.133
2322	A	3	2	1.00	15	0.133
2323	A	3	2	1.00	15	0.133
2324	A	3	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
2325	A	3	2	1.00	15	0.133
2326	A	3	2	1.00	15	0.133
2327	A	3	2	1.00	13	0.154
2328	A	3	2	1.00	11	0.182
2329	A	3	2	1.00	15	0.133
2330	A	3	2	1.00	15	0.133
2331	A	3	2	1.00	15	0.133
2332	A	3	2	1.00	15	0.133
2333	A	3	3	1.00	15	0.200
2334	A	6	3	1.00	15	0.200
2335	A	3	2	1.00	15	0.133
2336	A	3	2	1.00	15	0.133
2337	A	3	2	1.00	15	0.133
2338	A	3	2	1.00	15	0.133
2339	A	3	2	1.00	15	0.133
2340	A	3	2	1.00	15	0.133
2341	A	3	2	1.00	15	0.133
2342	A	3	2	1.00	15	0.133
2343	A	3	2	1.00	13	0.154
2344	A	3	2	1.00	11	0.182
2345	A	3	2	1.00	15	0.133
2346	A	3	2	1.00	15	0.133
2347	A	3	2	1.00	15	0.133
2348	A	3	2	1.00	15	0.133
2349	A	3	2	1.00	15	0.133
2350	A	4	3	1.00	15	0.200
2351	A	7	3	1.00	15	0.200
2352	A	10	3	1.00	15	0.200
2353	A	3	2	1.00	15	0.133
2354	A	3	2	1.00	15	0.133
2355	A	3	2	1.00	15	0.133
2356	A	3	2	1.00	15	0.133
2357	A	3	2	1.00	15	0.133
2358	A	3	2	1.00	13	0.154
2359	A	3	2	1.00	11	0.182
2360	A	4	4	1.00	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
2361	A	3	2	1.00	15	0.133
2362	A	3	2	1.00	15	0.133
2363	A	3	2	1.00	15	0.133
2364	A	4	4	1.00	15	0.267
2365	A	3	2	1.00	15	0.133
2366	A	3	2	1.00	15	0.133
2367	A	3	2	1.00	13	0.154
2368	A	3	2	1.00	11	0.182
2369	A	3	2	1.00	15	0.133
2370	A	3	2	1.00	15	0.133
2371	A	3	2	1.00	15	0.133
2372	A	3	2	1.00	15	0.133
2373	A	3	2	1.00	15	0.133
2374	A	3	2	1.00	15	0.133
2375	A	3	2	1.00	13	0.154
2376	A	3	2	1.00	11	0.182
2377	A	3	2	1.00	15	0.133
2378	A	3	2	1.00	15	0.133
2379	A	3	2	1.00	15	0.133
2380	A	3	2	1.00	15	0.133
2381	A	3	2	1.00	11	0.182
2382	A	5	4	1.00	15	0.267
2383	A	3	2	1.00	15	0.133
2384	A	3	3	1.00	9	0.333
2385	A	1	1	1.00	15	0.067
2386	A	2	2	1.00	15	0.133
2387	A	1	1	1.00	17	0.059
2388	A	1	1	1.00	17	0.059
2389	A	13	9	1.00	15	0.600
2390	A	8	8	1.06	15	0.533
2391	A	5	5	1.00	13	0.385
2392	A	5	4	1.00	11	0.364
2393	A	4	4	1.00	13	0.308
2394	A	2	1	1.00	13	0.077
2395	A	2	1	1.00	13	0.077
2396	A	2	1	1.00	13	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
2397	A	2	1	1.00	11	0.091
2398	A	1	0	1.00	9	0.000
2399	A	2	1	1.00	13	0.077
2400	A	2	1	1.00	13	0.077
2401	A	2	1	1.00	13	0.077
2402	A	2	1	1.00	13	0.077
2403	A	4	3	1.00	15	0.200
2404	A	4	3	1.00	15	0.200
2405	A	4	3	1.00	15	0.200
2406	A	4	3	1.00	13	0.231
2407	A	2	2	1.00	11	0.182
2408	A	4	3	1.00	15	0.200
2409	A	4	3	1.00	15	0.200
2410	A	4	3	1.00	15	0.200
2411	A	4	3	1.00	15	0.200
2412	A	4	3	1.00	15	0.200
2413	A	4	3	1.00	15	0.200
2414	A	4	3	1.00	15	0.200
2415	A	4	3	1.00	13	0.231
2416	A	3	2	1.00	11	0.182
2417	A	4	3	1.00	15	0.200
2418	A	4	3	1.00	15	0.200
2419	A	4	3	1.00	15	0.200
2420	A	4	3	1.00	15	0.200
2421	A	4	3	1.00	15	0.200
2422	A	4	3	1.00	13	0.231
2423	A	3	2	1.00	11	0.182
2424	A	2	2	1.00	15	0.133
2425	A	4	3	1.00	15	0.200
2426	A	4	3	1.00	15	0.200
2427	A	4	3	1.00	15	0.200
2428	A	4	3	1.00	15	0.200
2429	A	4	3	1.00	15	0.200
2430	A	4	3	1.00	13	0.231
2431	A	3	2	1.00	11	0.182
2432	A	4	3	1.00	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
2433	A	4	3	1.00	15	0.200
2434	A	4	3	1.00	15	0.200
2435	A	4	3	1.00	15	0.200
2436	A	4	3	1.00	15	0.200
2437	A	4	3	1.00	15	0.200
2438	A	4	3	1.00	13	0.231
2439	A	3	2	1.00	11	0.182
2440	A	4	3	1.00	15	0.200
2441	A	4	3	1.00	15	0.200
2442	A	4	3	1.00	15	0.200
2443	A	4	3	1.00	15	0.200
2444	A	4	3	1.00	15	0.200
2445	A	3	2	1.00	11	0.182
2446	A	1	1	1.00	17	0.059
2447	A	1	1	1.00	23	0.043
2448	A	3	2	1.00	9	0.222
2449	A	1	1	1.00	17	0.059
2450	A	1	1	1.00	13	0.077
2451	A	2	1	1.00	11	0.091
2452	A	2	1	1.00	11	0.091
2453	A	2	1	1.00	9	0.111
2454	A	1	0	1.00	7	0.000
2455	A	2	1	1.00	11	0.091
2456	A	2	1	1.00	11	0.091
2457	A	2	1	1.00	11	0.091
2458	A	2	1	1.00	13	0.077
2459	A	2	1	1.00	13	0.077
2460	A	2	1	1.00	11	0.091
2461	A	2	1	1.00	9	0.111
2462	A	3	2	1.00	13	0.154
2463	A	2	1	1.00	13	0.077
2464	A	2	1	1.00	13	0.077
2465	A	2	1	1.00	13	0.077
2466	A	2	1	1.00	13	0.077
2467	A	2	1	1.00	11	0.091
2468	A	2	1	1.00	9	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
2469	A	3	2	1.00	13	0.154
2470	A	2	1	1.00	13	0.077
2471	A	2	1	1.00	13	0.077
2472	A	1	1	1.00	11	0.091
2473	A	1	1	1.00	9	0.111
2474	A	4	4	1.00	13	0.308
2475	A	1	1	1.00	13	0.077
2476	A	1	1	1.00	13	0.077
2477	A	1	1	1.00	11	0.091
2478	A	1	1	1.00	9	0.111
2479	A	3	2	1.00	13	0.154
2480	A	1	1	1.00	13	0.077
2481	A	1	1	1.00	13	0.077
2482	A	1	1	1.00	11	0.091
2483	A	1	1	1.00	9	0.111
2484	A	3	2	1.00	13	0.154
2485	A	1	1	1.00	13	0.077
2486	A	1	1	1.00	13	0.077
2487	A	2	2	1.19	13	0.154
2488	A	2	2	1.23	11	0.182
2489	A	4	4	1.00	15	0.267
2490	A	2	2	1.18	15	0.133
2491	A	2	2	1.18	15	0.133
2492	A	2	2	1.21	13	0.154
2493	A	2	2	1.26	11	0.182
2494	A	5	4	1.00	15	0.267
2495	A	2	2	1.20	15	0.133
2496	A	2	2	1.20	15	0.133
2497	A	2	2	1.25	13	0.154
2498	A	2	2	1.31	11	0.182
2499	A	6	4	1.00	15	0.267
2500	A	2	2	1.24	15	0.133
2501	A	2	2	1.24	15	0.133
2502	A	2	2	1.19	13	0.154
2503	A	2	2	1.23	11	0.182
2504	A	3	3	1.00	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
2505	A	2	2	1.18	15	0.133
2506	A	2	2	1.18	15	0.133
2507	A	2	2	1.25	13	0.154
2508	A	2	2	1.31	11	0.182
2509	A	4	4	1.00	15	0.267
2510	A	2	2	1.24	15	0.133
2511	A	2	2	1.24	15	0.133
2512	A	2	2	1.25	13	0.154
2513	A	2	2	1.31	11	0.182
2514	A	5	4	1.00	15	0.267
2515	A	2	2	1.24	15	0.133
2516	A	2	2	1.24	15	0.133
2517	A	6	6	1.00	15	0.400
2518	A	2	1	1.00	15	0.067
2519	A	2	1	1.00	15	0.067
2520	A	2	1	1.00	15	0.067
2521	A	2	1	1.00	13	0.077
2522	A	2	1	1.00	11	0.091
2523	A	2	1	1.00	15	0.067
2524	A	2	1	1.00	15	0.067
2525	A	2	1	1.00	15	0.067
2526	A	2	1	1.00	15	0.067
2527	A	2	1	1.00	15	0.067
2528	A	3	2	1.00	17	0.118
2529	A	3	2	1.00	17	0.118
2530	A	3	2	1.00	17	0.118
2531	A	1	1	1.00	15	0.067
2532	A	3	2	1.00	13	0.154
2533	A	3	2	1.00	17	0.118
2534	A	3	2	1.00	17	0.118
2535	A	1	1	1.00	17	0.059
2536	A	3	2	1.00	17	0.118
2537	A	3	2	1.00	17	0.118
2538	A	3	2	1.00	17	0.118
2539	A	3	2	1.00	17	0.118
2540	A	3	2	1.00	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
2541	A	3	2	1.00	17	0.118
2542	A	1	1	1.00	15	0.067
2543	A	3	2	1.00	13	0.154
2544	A	3	2	1.00	17	0.118
2545	A	3	2	1.00	17	0.118
2546	A	3	2	1.00	17	0.118
2547	A	1	1	1.00	17	0.059
2548	A	3	3	1.00	17	0.176
2549	A	3	2	1.00	17	0.118
2550	A	3	2	1.00	17	0.118
2551	A	3	2	1.00	17	0.118
2552	A	3	2	1.00	17	0.118
2553	A	3	2	1.00	17	0.118
2554	A	1	1	1.00	15	0.067
2555	A	3	2	1.00	13	0.154
2556	A	3	2	1.00	17	0.118
2557	A	3	2	1.00	17	0.118
2558	A	3	2	1.00	17	0.118
2559	A	3	2	1.00	17	0.118
2560	A	3	2	1.00	17	0.118
2561	A	1	1	1.00	17	0.059
2562	A	3	3	1.00	17	0.176
2563	A	4	3	1.00	17	0.176
2564	A	3	2	1.00	17	0.118
2565	A	3	2	1.00	17	0.118
2566	A	3	2	1.00	17	0.118
2567	A	3	2	1.00	17	0.118
2568	A	3	2	1.00	17	0.118
2569	A	3	2	1.00	17	0.118
2570	A	3	2	1.00	17	0.118
2571	A	3	2	1.00	17	0.118
2572	A	3	2	1.00	17	0.118
2573	A	3	2	1.00	17	0.118
2574	A	1	1	1.00	15	0.067
2575	A	3	2	1.00	13	0.154
2576	A	3	2	1.00	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
2577	A	3	2	1.00	17	0.118
2578	A	3	2	1.00	17	0.118
2579	A	3	2	1.00	17	0.118
2580	A	3	2	1.00	17	0.118
2581	A	3	2	1.00	17	0.118
2582	A	3	2	1.00	17	0.118
2583	A	3	2	1.00	17	0.118
2584	A	1	1	1.00	17	0.059
2585	A	3	3	1.00	17	0.176
2586	A	4	3	1.00	17	0.176
2587	A	5	3	1.00	17	0.176
2588	A	6	3	1.00	17	0.176
2589	A	3	2	1.00	17	0.118
2590	A	3	2	1.00	17	0.118
2591	A	1	1	1.00	15	0.067
2592	A	1	1	1.00	13	0.077
2593	A	1	1	1.00	13	0.077
2594	A	1	1	1.00	13	0.077
2595	A	1	1	1.00	19	0.053
2596	A	1	1	1.00	23	0.043
2597	A	3	2	1.00	17	0.118
2598	A	3	2	1.00	17	0.118
2599	A	3	2	1.00	17	0.118
2600	A	3	2	1.00	17	0.118
2601	A	1	1	1.00	15	0.067
2602	A	4	4	1.00	13	0.308
2603	A	3	2	1.00	17	0.118
2604	A	3	2	1.00	17	0.118
2605	A	3	2	1.00	17	0.118
2606	A	3	2	1.00	19	0.105
2607	A	3	2	1.00	19	0.105
2608	A	3	2	1.00	19	0.105
2609	A	3	2	1.00	19	0.105
2610	A	1	1	1.00	15	0.067
2611	A	4	4	1.00	13	0.308
2612	A	3	2	1.00	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
2613	A	3	2	1.00	19	0.105
2614	A	3	2	1.00	19	0.105
2615	A	3	2	1.00	17	0.118
2616	A	3	2	1.00	17	0.118
2617	A	3	2	1.00	17	0.118
2618	A	3	2	1.00	17	0.118
2619	A	1	1	1.00	15	0.067
2620	A	4	4	1.00	13	0.308
2621	A	3	2	1.00	17	0.118
2622	A	3	2	1.00	17	0.118
2623	A	3	2	1.00	17	0.118
2624	A	3	2	1.00	17	0.118
2625	A	3	2	1.00	17	0.118
2626	A	3	2	1.00	17	0.118
2627	A	1	1	1.00	15	0.067
2628	A	3	2	1.00	13	0.154
2629	A	3	2	1.00	17	0.118
2630	A	3	2	1.00	17	0.118
2631	A	3	2	1.00	17	0.118
2632	A	3	2	1.00	17	0.118
2633	A	3	2	1.00	17	0.118
2634	A	1	1	1.00	17	0.059
2635	A	1	1	1.00	15	0.067
2636	A	3	2	1.00	13	0.154
2637	A	3	2	1.00	17	0.118
2638	A	3	2	1.00	17	0.118
2639	A	4	4	1.00	19	0.210
2640	A	8	8	1.00	19	0.421
2641	A	11	8	1.00	19	0.421
2642	A	3	2	1.00	17	0.118
2643	A	4	4	1.00	19	0.210
2644	A	9	9	1.00	19	0.474
2645	A	12	9	1.00	19	0.474
2646	A	5	5	1.00	19	0.263
2647	A	10	10	1.00	19	0.526
2648	A	13	10	1.00	19	0.526

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
2649	A	3	2	1.00	19	0.105
2650	A	3	2	1.00	19	0.105
2651	A	3	2	1.00	19	0.105
2652	A	1	1	1.00	17	0.059
2653	A	4	4	1.00	15	0.267
2654	A	4	4	1.00	19	0.210
2655	A	5	5	1.00	19	0.263
2656	A	6	5	1.00	19	0.263
2657	A	7	5	1.00	19	0.263
2658	A	3	2	1.00	19	0.105
2659	A	3	2	1.00	19	0.105
2660	A	3	2	1.00	19	0.105
2661	A	1	1	1.00	17	0.059
2662	A	3	3	1.00	15	0.200
2663	A	4	4	1.00	19	0.210
2664	A	5	4	1.00	19	0.210
2665	A	6	4	1.00	19	0.210
2666	A	7	4	1.00	19	0.210
2667	A	2	1	1.00	13	0.077
2668	A	2	1	1.00	13	0.077
2669	A	2	1	1.00	11	0.091
2670	A	1	1	1.00	13	0.077
2671	A	1	1	1.00	13	0.077
2672	A	1	1	1.00	13	0.077
2673	A	2	2	1.18	15	0.133
2674	A	2	2	1.16	15	0.133
2675	A	2	2	1.16	15	0.133
2676	A	2	2	1.22	15	0.133
2677	A	2	2	1.22	15	0.133
2678	A	2	2	1.19	19	0.105
2679	A	2	2	1.16	17	0.118
2680	A	2	2	1.16	19	0.105
2681	A	2	2	1.18	19	0.105
2682	A	2	2	1.14	19	0.105
2683	A	2	2	1.15	17	0.118
2684	A	2	2	1.14	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
2685	A	2	2	1.14	19	0.105
2686	C	5	2	8.40	42	0.048
2687	A	5	3	1.00	21	0.143
2688	A	4	3	1.00	21	0.143
2689	A	3	3	1.00	21	0.143
2690	A	3	3	1.00	21	0.143
2691	A	1	1	1.00	21	0.048
2692	A	2	2	1.00	21	0.095
2693	A	3	2	1.00	21	0.095
2694	A	4	2	1.00	21	0.095
2695	A	2	2	1.23	17	0.118
2696	A	2	2	1.21	19	0.105
2697	C	5	2	6.15	45	0.044
2698	A	2	2	1.00	37	0.054
2699	C	5	2	8.40	42	0.048
2700	A	2	2	1.00	19	0.105
2701	A	3	3	1.00	21	0.143
2702	A	2	2	1.00	21	0.095
2703	A	3	3	1.00	21	0.143
2704	A	2	2	1.15	13	0.154
2705	A	4	2	1.00	15	0.133
2706	A	3	2	1.00	15	0.133
2707	A	2	2	1.00	15	0.133
2708	A	1	1	1.00	15	0.067
2709	A	2	2	1.00	13	0.154
2710	A	2	2	1.00	15	0.133
2711	A	2	2	1.00	15	0.133
2712	A	2	2	1.00	11	0.182
2713	A	2	2	1.00	11	0.182
2714	A	2	2	1.00	9	0.222
2715	A	2	2	1.00	7	0.286
2716	A	2	2	1.00	11	0.182
2717	A	2	2	1.00	11	0.182
2718	A	2	2	1.00	11	0.182
2719	A	2	2	1.00	11	0.182
2720	A	1	1	1.00	15	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
2721	A	3	2	1.00	17	0.118
2722	A	3	2	1.00	17	0.118
2723	A	3	2	1.00	17	0.118
2724	A	1	1	1.00	21	0.048
2725	A	1	1	1.00	17	0.059
2726	A	1	1	1.00	17	0.059
2727	A	1	1	1.00	13	0.077
2728	A	4	4	1.00	13	0.308
2729	A	4	4	1.00	13	0.308
2730	A	4	2	1.00	18	0.111
2731	A	3	2	1.00	18	0.111
2732	A	2	2	1.00	18	0.111
2733	A	1	1	1.00	16	0.062
2734	A	2	2	1.00	13	0.154
2735	A	2	2	1.00	18	0.111
2736	A	2	2	1.00	18	0.111
2737	A	2	2	1.00	15	0.133
2738	A	1	1	1.00	15	0.067
2739	A	1	1	1.00	15	0.067
2740	A	2	1	1.00	17	0.059
2741	A	2	1	1.00	17	0.059
2742	A	2	1	1.00	15	0.067
2743	A	2	2	1.00	17	0.118
2744	A	3	3	1.00	17	0.176
2745	A	4	3	1.00	17	0.176
2746	A	6	4	1.00	19	0.210
2747	A	5	4	1.00	19	0.210
2748	A	4	4	1.00	19	0.210
2749	A	3	3	1.00	19	0.158
2750	A	1	1	1.00	19	0.053
2751	A	2	2	1.00	19	0.105
2752	A	3	2	1.00	19	0.105
2753	A	1	1	1.00	15	0.067
2754	A	1	1	1.00	17	0.059
2755	A	6	3	1.00	15	0.200
2756	A	2	1	1.00	15	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
2757	A	2	1	1.00	15	0.067
2758	A	2	1	1.00	13	0.077
2759	A	2	1	1.00	15	0.067
2760	A	2	1	1.00	15	0.067
2761	A	2	1	1.00	15	0.067
2762	A	5	5	1.00	21	0.238
2763	A	9	9	1.00	21	0.429
2764	A	12	9	1.00	21	0.429
2765	A	4	3	1.00	19	0.158
2766	A	5	5	1.00	21	0.238
2767	A	10	10	1.00	21	0.476
2768	A	13	10	1.00	21	0.476
2769	A	6	6	1.00	21	0.286
2770	A	11	11	1.00	21	0.524
2771	A	14	11	1.00	21	0.524
2772	A	1	1	1.00	17	0.059
2773	A	1	1	1.00	17	0.059
2774	A	1	1	1.00	17	0.059
2775	A	1	1	1.00	17	0.059
2776	A	1	1	1.00	15	0.067
2777	A	2	2	1.00	17	0.118
2778	A	1	1	1.00	17	0.059
2779	A	1	1	1.00	17	0.059
2780	A	1	1	1.00	17	0.059
2781	A	6	4	1.00	23	0.174
2782	A	5	4	1.00	23	0.174
2783	A	4	4	1.00	23	0.174
2784	A	4	4	1.00	23	0.174
2785	A	1	1	1.00	23	0.043
2786	A	2	2	1.00	23	0.087
2787	A	3	2	1.00	23	0.087
2788	A	4	2	1.00	23	0.087
2789	A	2	2	1.14	15	0.133
2790	A	2	2	1.18	17	0.118
2791	A	2	2	1.18	17	0.118
2792	A	2	2	1.19	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
2793	A	2	2	1.21	9	0.222
2794	A	2	2	1.21	17	0.118
2795	A	2	2	1.18	17	0.118
2796	A	2	2	1.18	17	0.118
2797	A	2	2	1.00	20	0.100
2798	A	1	1	1.00	23	0.043
2799	A	2	2	1.00	23	0.087
2800	A	3	2	1.00	23	0.087
2801	A	4	2	1.00	23	0.087
2802	A	3	3	1.00	13	0.231
2803	A	3	3	1.00	13	0.231
2804	A	3	3	1.00	13	0.231
2805	A	3	3	1.00	13	0.231
2806	A	3	3	1.00	13	0.231
2807	A	3	3	1.00	13	0.231
2808	A	3	3	1.00	11	0.273
2809	A	3	3	1.00	11	0.273
2810	A	3	3	1.00	11	0.273
2811	A	3	3	1.00	11	0.273
2812	A	3	3	1.00	13	0.231
2813	A	3	3	1.00	13	0.231
2814	A	3	3	1.00	13	0.231
2815	A	3	3	1.00	13	0.231
2816	A	3	3	1.00	13	0.231
2817	A	3	3	1.00	13	0.231
2818	A	3	3	1.00	13	0.231
2819	A	3	3	1.00	13	0.231
2820	A	3	3	1.00	13	0.231
2821	A	3	3	1.00	13	0.231
2822	A	3	3	1.00	13	0.231
2823	A	3	3	1.00	13	0.231
2824	A	3	3	1.00	13	0.231
2825	A	3	3	1.00	13	0.231
2826	A	3	3	1.00	13	0.231
2827	A	3	3	1.00	13	0.231
2828	A	3	3	1.00	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
2829	A	3	3	1.00	13	0.231
2830	A	3	3	1.00	13	0.231
2831	A	3	3	1.00	13	0.231
2832	A	3	3	1.00	13	0.231
2833	A	3	3	1.00	13	0.231
2834	A	3	3	1.00	13	0.231
2835	A	3	3	1.00	13	0.231
2836	A	3	3	1.00	13	0.231
2837	A	3	3	1.00	15	0.200
2838	A	3	3	1.00	15	0.200
2839	A	3	3	1.00	15	0.200
2840	A	3	3	1.00	15	0.200
2841	A	3	2	1.00	19	0.105
2842	A	4	3	1.00	21	0.143
2843	A	4	3	1.00	21	0.143
2844	A	2	2	1.00	13	0.154
2845	A	2	2	1.00	13	0.154
2846	A	2	2	1.00	13	0.154
2847	A	4	3	1.00	21	0.143
2848	A	3	3	1.24	21	0.143
2849	A	4	3	1.00	21	0.143
2850	A	3	3	1.24	21	0.143
2851	A	2	2	1.00	19	0.105
2852	A	3	3	1.00	21	0.143
2853	A	3	2	1.00	19	0.105
2854	A	3	2	1.00	21	0.095
2855	A	3	2	1.00	21	0.095
2856	A	3	2	1.00	22	0.091
2857	A	3	2	1.00	24	0.083
2858	A	3	2	1.00	24	0.083
2859	A	8	8	1.00	21	0.381
2860	A	8	8	1.00	21	0.381
2861	A	2	2	1.00	21	0.095
2862	A	7	7	1.00	19	0.368
2863	A	7	7	1.00	13	0.538
2864	A	5	5	1.00	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
2865	A	8	8	1.00	21	0.381
2866	A	8	8	1.00	21	0.381
2867	A	4	3	1.00	21	0.143
2868	A	8	8	1.00	21	0.381
2869	A	8	8	1.00	21	0.381
2870	A	2	2	1.00	21	0.095
2871	A	8	8	1.00	19	0.421
2872	A	8	8	1.00	13	0.615
2873	A	4	3	1.00	21	0.143
2874	A	9	9	1.00	21	0.429
2875	A	9	9	1.00	21	0.429
2876	A	4	3	1.00	21	0.143
2877	A	9	9	1.00	21	0.429
2878	A	9	9	1.00	21	0.429
2879	A	2	2	1.00	21	0.095
2880	A	9	8	1.00	19	0.421
2881	A	9	8	1.00	13	0.615
2882	A	4	3	1.00	21	0.143
2883	A	10	9	1.00	21	0.429
2884	A	10	9	1.00	21	0.429
2885	A	4	3	1.00	21	0.143
2886	A	8	8	1.00	24	0.333
2887	A	8	8	1.00	24	0.333
2888	A	2	2	1.00	24	0.083
2889	A	7	7	1.00	22	0.318
2890	A	5	5	1.00	24	0.208
2891	A	8	8	1.00	24	0.333
2892	A	8	8	1.00	24	0.333
2893	A	4	3	1.00	24	0.125
2894	A	8	8	1.00	24	0.333
2895	A	8	8	1.00	24	0.333
2896	A	2	2	1.00	24	0.083
2897	A	8	8	1.00	22	0.364
2898	A	4	3	1.00	24	0.125
2899	A	9	9	1.00	24	0.375
2900	A	9	9	1.00	24	0.375

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
2901	A	4	3	1.00	24	0.125
2902	A	9	9	1.00	24	0.375
2903	A	9	9	1.00	24	0.375
2904	A	2	2	1.00	24	0.083
2905	A	9	8	1.00	22	0.364
2906	A	4	3	1.00	24	0.125
2907	A	10	9	1.00	24	0.375
2908	A	10	9	1.00	24	0.375
2909	A	4	3	1.00	24	0.125
2910	A	2	2	1.00	21	0.095
2911	A	3	2	1.35	19	0.105
2912	A	2	2	1.00	21	0.095
2913	A	2	2	1.00	21	0.095
2914	A	2	2	1.00	21	0.095
2915	A	2	2	1.00	21	0.095
2916	A	2	2	1.00	21	0.095
2917	A	2	2	1.00	15	0.133
2918	A	7	6	1.00	17	0.353
2919	A	5	5	1.00	21	0.238
2920	A	5	5	1.00	21	0.238
2921	A	6	6	1.00	21	0.286
2922	A	6	6	1.00	21	0.286
2923	A	3	3	1.00	17	0.176
2924	A	4	4	1.00	15	0.267
2925	A	4	4	1.00	13	0.308
2926	A	3	3	1.00	11	0.273
2927	A	4	4	1.00	15	0.267
2928	A	4	4	1.00	15	0.267
2929	A	7	7	1.00	11	0.636
2930	A	3	2	1.00	21	0.095
2931	A	3	2	1.00	21	0.095
2932	A	3	2	1.00	19	0.105
2933	A	4	4	1.00	21	0.190
2934	A	5	5	1.00	21	0.238
2935	A	7	5	1.00	21	0.238
2936	A	3	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
2937	A	3	2	1.00	21	0.095
2938	A	2	2	1.00	17	0.118
2939	A	4	4	1.00	21	0.190
2940	A	6	5	1.00	21	0.238
2941	A	8	5	1.00	21	0.238
2942	A	4	3	1.00	21	0.143
2943	A	4	3	1.00	21	0.143
2944	A	2	2	1.00	21	0.095
2945	A	5	5	1.00	21	0.238
2946	A	5	5	1.00	21	0.238
2947	A	4	4	1.00	21	0.190
2948	A	3	3	1.00	17	0.176
2949	A	3	3	1.00	21	0.143
2950	A	4	4	1.00	21	0.190
2951	A	6	6	1.00	21	0.286
2952	A	5	5	1.00	19	0.263
2953	A	5	5	1.00	21	0.238
2954	A	6	6	1.00	21	0.286
2955	A	3	3	1.00	23	0.130
2956	A	3	3	1.00	23	0.130
2957	A	4	4	1.00	23	0.174
2958	A	4	4	1.00	23	0.174
2959	A	2	2	1.00	11	0.182
2960	A	4	3	1.00	21	0.143
2961	A	4	3	1.00	21	0.143
2962	A	5	5	1.00	21	0.238
2963	A	6	6	1.00	21	0.286
2964	A	5	5	1.00	19	0.263
2965	A	4	4	1.00	21	0.190
2966	A	6	5	1.00	21	0.238
2967	A	8	7	1.00	21	0.333
2968	A	6	6	1.00	17	0.353
2969	A	7	7	1.00	21	0.333
2970	A	4	3	1.00	21	0.143
2971	A	4	3	1.00	21	0.143
2972	A	5	5	1.00	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
2973	A	6	6	1.03	21	0.286
2974	A	7	7	1.00	21	0.333
2975	A	7	6	1.00	21	0.286
2976	A	5	5	1.00	17	0.294
2977	A	6	6	1.01	21	0.286
2978	A	5	5	1.00	23	0.217
2979	A	5	5	1.00	23	0.217
2980	A	6	6	1.00	23	0.261
2981	A	6	6	1.00	23	0.261
2982	A	8	6	1.02	19	0.316
2983	A	6	6	1.00	17	0.353
2984	A	5	5	1.00	21	0.238
2985	A	4	3	1.00	21	0.143
2986	A	4	3	1.00	21	0.143
2987	A	4	3	1.00	21	0.143
2988	A	8	5	1.02	19	0.263
2989	A	6	5	1.00	17	0.294
2990	A	4	4	1.00	21	0.190
2991	A	4	3	1.00	21	0.143
2992	A	4	3	1.00	21	0.143
2993	A	4	3	1.00	21	0.143
2994	A	6	5	1.00	13	0.385
2995	A	6	6	1.00	23	0.261
2996	A	6	6	1.33	23	0.261
2997	A	5	5	1.00	23	0.217
2998	A	5	5	1.00	23	0.217
2999	A	6	6	1.00	23	0.261
3000	A	6	6	1.34	23	0.261
3001	A	5	5	1.00	23	0.217
3002	A	5	5	1.00	23	0.217
3003	A	3	2	1.00	13	0.154
3004	A	2	2	1.00	15	0.133
3005	A	2	2	1.00	15	0.133
3006	A	3	2	1.00	19	0.105
3007	A	3	2	1.00	19	0.105
3008	A	3	2	1.00	17	0.118

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
3009	A	2	2	1.00	15	0.133
3010	A	4	4	1.00	19	0.210
3011	A	3	2	1.00	19	0.105
3012	A	3	2	1.00	19	0.105
3013	A	3	2	1.00	19	0.105
3014	A	3	2	1.00	19	0.105
3015	A	3	2	1.00	17	0.118
3016	A	2	2	1.60	15	0.133
3017	A	3	2	1.00	19	0.105
3018	A	3	2	1.00	19	0.105
3019	A	3	2	1.00	19	0.105
3020	A	2	2	1.00	15	0.133
3021	A	3	2	1.00	13	0.154
3022	A	3	2	1.00	19	0.105
3023	A	3	2	1.00	19	0.105
3024	A	3	2	1.00	17	0.118
3025	A	2	2	1.00	15	0.133
3026	A	2	2	1.00	19	0.105
3027	A	2	2	1.00	19	0.105
3028	A	3	2	1.00	17	0.118
3029	A	3	2	1.00	17	0.118
3030	A	3	2	1.00	15	0.133
3031	A	2	2	1.00	17	0.118
3032	A	3	3	1.00	17	0.176
3033	A	4	3	1.00	17	0.176
3034	A	2	2	1.00	13	0.154
3035	A	2	2	1.00	13	0.154
3036	A	2	2	1.00	13	0.154
3037	A	2	2	1.00	13	0.154
3038	A	2	2	1.00	15	0.133
3039	A	2	2	1.00	15	0.133
3040	A	3	2	1.00	17	0.118
3041	A	3	2	1.00	17	0.118
3042	A	3	2	1.00	15	0.133
3043	A	7	7	1.00	17	0.412
3044	A	8	8	1.00	17	0.471

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
3045	A	9	8	1.00	17	0.471
3046	A	3	3	1.00	19	0.158
3047	A	3	3	1.00	17	0.176
3048	A	3	3	1.00	15	0.200
3049	A	3	3	1.00	13	0.231
3050	A	3	3	1.00	17	0.176
3051	A	3	3	1.00	17	0.176
3052	A	4	4	1.00	26	0.154
3053	A	9	8	1.00	26	0.308
3054	A	7	7	1.00	24	0.292
3055	A	5	5	1.00	22	0.227
3056	A	8	7	1.00	26	0.269
3057	A	6	6	1.00	26	0.231
3058	A	7	7	1.00	26	0.269
3059	A	9	8	1.00	26	0.308
3060	A	4	4	1.00	26	0.154
3061	A	10	7	1.00	26	0.269
3062	A	8	7	1.00	24	0.292
3063	A	6	6	1.00	22	0.273
3064	A	4	4	1.00	26	0.154
3065	A	5	5	1.00	26	0.192
3066	A	6	6	1.00	26	0.231
3067	A	8	7	1.00	26	0.269
3068	A	2	2	1.00	15	0.133
3069	A	5	5	1.00	16	0.312
3070	A	4	3	1.00	25	0.120
3071	A	4	3	1.00	25	0.120
3072	A	4	3	1.00	25	0.120
3073	A	5	3	1.00	23	0.130
3074	A	4	3	1.00	25	0.120
3075	A	4	3	1.00	25	0.120
3076	A	3	3	1.00	25	0.120
3077	A	4	3	1.00	25	0.120
3078	A	4	3	1.00	25	0.120

Chapter 3

Listing of integrals

3.1 $\int (bx)^p (cx)^m \, dx$

Optimal. Leaf size=22

$$\frac{(bx)^{p+1}(cx)^m}{b(m+p+1)}$$

```
[Out] (b*x)^(1+p)*(c*x)^m/b/(1+m+p)
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {20, 32}

$$\frac{(bx)^{p+1}(cx)^m}{b(m+p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x)^p*(c*x)^m,x]
```

```
[Out] ((b*x)^(1 + p)*(c*x)^m)/(b*(1 + m + p))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (bx)^p (cx)^m \, dx &= ((bx)^{-m}(cx)^m) \int (bx)^{m+p} \, dx \\ &= \frac{(bx)^{1+p}(cx)^m}{b(1+m+p)}\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 0.82

$$\frac{x(bx)^p (cx)^m}{m+p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x)^p*(c*x)^m,x]
[Out] (x*(b*x)^p*(c*x)^m)/(1 + m + p)
```

fricas [A] time = 0.70, size = 29, normalized size = 1.32

$$\frac{(bx)^p\, xe^{\left(m\log(bx)+m\log\left(\frac{c}{b}\right)\right)}}{m+p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^p*(c*x)^m,x, algorithm="fricas")
[Out] (b*x)^p*x*e^(m*log(b*x) + m*log(c/b))/(m + p + 1)
```

giac [A] time = 0.18, size = 26, normalized size = 1.18

$$\frac{xe^{\left(p\log(b)+m\log(c)+m\log(x)+p\log(x)\right)}}{m+p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^p*(c*x)^m,x, algorithm="giac")
[Out] x*e^(p*log(b) + m*log(c) + m*log(x) + p*log(x))/(m + p + 1)
```

maple [A] time = 0.00, size = 19, normalized size = 0.86

$$\frac{x\,(bx)^p\,(cx)^m}{m+p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^p*(c*x)^m,x)
[Out] x/(1+m+p)*(b*x)^p*(c*x)^m
```

maxima [A] time = 1.41, size = 24, normalized size = 1.09

$$\frac{b^p c^m xe^{\left(m\log(x)+p\log(x)\right)}}{m+p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^p*(c*x)^m,x, algorithm="maxima")
[Out] b^p*c^m*x*e^(m*log(x) + p*log(x))/(m + p + 1)
```

mupad [B] time = 1.06, size = 18, normalized size = 0.82

$$\frac{x\,(b\,x)^p\,(c\,x)^m}{m+p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^p*(c*x)^m,x)
[Out] (x*(b*x)^p*(c*x)^m)/(m + p + 1)
```

sympy [**A**] time = 5.13, size = 60, normalized size = 2.73

$$\begin{cases} \frac{b^p c^m x x^m x^p}{m+p+1} & \text{for } m \neq -p-1 \\ \begin{cases} \frac{b^p c^{-p} \log(x)}{c} & \text{for } |x| < 1 \\ -\frac{b^p c^{-p} G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix} \middle| x\right)}{c} + \frac{b^p c^{-p} G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix} \middle| x\right)}{c} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)**p*(c*x)**m,x)
```

```
[Out] Piecewise((b**p*c**m*x*x**m*x**p/(m + p + 1), Ne(m, -p - 1)), (Piecewise((b
**p*c**(-p)*log(x)/c, Abs(x) < 1), (-b**p*c**(-p)*meijerg(((), (1, 1)), ((0
, 0), ()), x)/c + b**p*c**(-p)*meijerg(((1, 1), ()), ((), (0, 0)), x)/c, Tr
ue)), True))
```

3.2 $\int x^3\sqrt{bx^2}\,dx$

Optimal. Leaf size=16

$$\frac{1}{5}x^4\sqrt{bx^2}$$

```
[Out] 1/5*x^4*(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$\frac{1}{5}x^4\sqrt{bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[b*x^2],x]
```

```
[Out] (x^4*Sqrt[b*x^2])/5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{bx^2}\,dx &= \frac{\sqrt{bx^2}\int x^4\,dx}{x}\\ &= \frac{1}{5}x^4\sqrt{bx^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{1}{5}x^4\sqrt{bx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[b*x^2],x]
```

```
[Out] (x^4*Sqrt[b*x^2])/5
```

fricas [A] time = 0.61, size = 12, normalized size = 0.75

$$\frac{1}{5}\sqrt{bx^2}\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(b*x^2)*x^4
```

giac [A] time = 0.15, size = 10, normalized size = 0.62

$$\frac{1}{5}\sqrt{b}\,x^5\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2)^(1/2),x, algorithm="giac")
[Out] 1/5*sqrt(b)*x^5*sgn(x)
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{\sqrt{b\,x^2}\;x^4}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2)^(1/2),x)
[Out] 1/5*x^4*(b*x^2)^(1/2)
```

maxima [A] time = 1.29, size = 15, normalized size = 0.94

$$\frac{\left(bx^2\right)^{\frac{3}{2}}\,x^2}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/5*(b*x^2)^(3/2)*x^2/b
```

mupad [B] time = 1.03, size = 10, normalized size = 0.62

$$\frac{\sqrt{b}\;\sqrt{x^{10}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2)^(1/2),x)
[Out] (b^(1/2)*(x^10)^(1/2))/5
```

sympy [A] time = 0.30, size = 15, normalized size = 0.94

$$\frac{\sqrt{b}\,x^4\sqrt{x^2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2)**(1/2),x)
[Out] sqrt(b)*x**4*sqrt(x**2)/5
```

3.3 $\int x^2\sqrt{bx^2}\,dx$

Optimal. Leaf size=16

$$\frac{1}{4}x^3\sqrt{bx^2}$$

```
[Out] 1/4*x^3*(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$\frac{1}{4}x^3\sqrt{bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[b*x^2],x]
```

```
[Out] (x^3*Sqrt[b*x^2])/4
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{bx^2}\,dx &= \frac{\sqrt{bx^2}\int x^3\,dx}{x}\\ &= \frac{1}{4}x^3\sqrt{bx^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{1}{4}x^3\sqrt{bx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[b*x^2],x]
```

```
[Out] (x^3*Sqrt[b*x^2])/4
```

fricas [A] time = 0.68, size = 12, normalized size = 0.75

$$\frac{1}{4}\sqrt{bx^2}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(b*x^2)*x^3
```

giac [A] time = 0.17, size = 10, normalized size = 0.62

$$\frac{1}{4}\sqrt{b}\,x^4\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2)^(1/2),x, algorithm="giac")
[Out] 1/4*sqrt(b)*x^4*sgn(x)
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{\sqrt{b\,x^2}\;x^3}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2)^(1/2),x)
[Out] 1/4*x^3*(b*x^2)^(1/2)
```

maxima [A] time = 1.32, size = 13, normalized size = 0.81

$$\frac{\left(bx^2\right)^{\frac{3}{2}}x}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/4*(b*x^2)^(3/2)*x/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int x^2\,\sqrt{b\,x^2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2)^(1/2),x)
[Out] int(x^2*(b*x^2)^(1/2), x)
```

sympy [A] time = 0.24, size = 15, normalized size = 0.94

$$\frac{\sqrt{b}\,x^3\sqrt{x^2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2)**(1/2),x)
[Out] sqrt(b)*x**3*sqrt(x**2)/4
```

3.4 $\int x\sqrt{bx^2}\,dx$

Optimal. Leaf size=16

$$\frac{1}{3}x^2\sqrt{bx^2}$$

```
[Out] 1/3*x^2*(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 30}

$$\frac{1}{3}x^2\sqrt{bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[b*x^2],x]
```

```
[Out] (x^2*Sqrt[b*x^2])/3
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{bx^2}\,dx &= \frac{\sqrt{bx^2}\int x^2\,dx}{x}\\ &= \frac{1}{3}x^2\sqrt{bx^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{1}{3}x^2\sqrt{bx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[b*x^2],x]
```

```
[Out] (x^2*Sqrt[b*x^2])/3
```

fricas [A] time = 0.63, size = 12, normalized size = 0.75

$$\frac{1}{3}\sqrt{bx^2}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(b*x^2)*x^2
```

giac [A] time = 0.15, size = 10, normalized size = 0.62

$$\frac{1}{3}\sqrt{b}\,x^3\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2)^(1/2),x, algorithm="giac")
[Out] 1/3*sqrt(b)*x^3*sgn(x)
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{\sqrt{b\,x^2}\;x^2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2)^(1/2),x)
[Out] 1/3*x^2*(b*x^2)^(1/2)
```

maxima [A] time = 1.31, size = 12, normalized size = 0.75

$$\frac{\left(bx^2\right)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/3*(b*x^2)^(3/2)/b
```

mupad [B] time = 1.02, size = 10, normalized size = 0.62

$$\frac{\sqrt{b}\;\sqrt{x^6}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2)^(1/2),x)
[Out] (b^(1/2)*(x^6)^(1/2))/3
```

sympy [A] time = 0.19, size = 15, normalized size = 0.94

$$\frac{\sqrt{b}\,x^2\sqrt{x^2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2)**(1/2),x)
[Out] sqrt(b)*x**2*sqrt(x**2)/3
```

3.5 $\int \sqrt{bx^2}\, dx$

Optimal. Leaf size=14

$$\frac{1}{2}x\sqrt{bx^2}$$

```
[Out] 1/2*x*(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$\frac{1}{2}x\sqrt{bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2],x]
```

```
[Out] (x*Sqrt[b*x^2])/2
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt{bx^2}\, dx &= \frac{\sqrt{bx^2}\int x\, dx}{x}\\ &= \frac{1}{2}x\sqrt{bx^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{1}{2}x\sqrt{bx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2],x]
```

```
[Out] (x*Sqrt[b*x^2])/2
```

fricas [A] time = 0.69, size = 10, normalized size = 0.71

$$\frac{1}{2}\sqrt{bx^2}\, x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(b*x^2)*x
```

giac [A] time = 0.16, size = 10, normalized size = 0.71

$$\frac{1}{2}\sqrt{b}\,x^2\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(b)*x^2*sgn(x)
```

maple [A] time = 0.00, size = 11, normalized size = 0.79

$$\frac{\sqrt{b\,x^2}\;x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(1/2),x)
[Out] 1/2*x*(b*x^2)^(1/2)
```

maxima [A] time = 1.25, size = 10, normalized size = 0.71

$$\frac{1}{2}\sqrt{bx^2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(b*x^2)*x
```

mupad [B] time = 1.24, size = 8, normalized size = 0.57

$$\frac{\sqrt{b}\;x\,|x|}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(1/2),x)
[Out] (b^(1/2)*x*abs(x))/2
```

sympy [A] time = 0.17, size = 14, normalized size = 1.00

$$\frac{\sqrt{b}\,x\sqrt{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2)**(1/2),x)
[Out] sqrt(b)*x*sqrt(x**2)/2
```

3.6 $\int \frac{\sqrt{bx^2}}{x}\,dx$

Optimal. Leaf size=9

$$\sqrt{bx^2}$$

```
[Out] (b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {15, 8}

$$\sqrt{bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2]/x,x]
```

```
[Out] Sqrt[b*x^2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{bx^2}}{x}\,dx &= \frac{\sqrt{bx^2}\int 1\,dx}{x}\\ &= \sqrt{bx^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.56

$$\frac{bx^2}{\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2]/x,x]
```

```
[Out] (b*x^2)/Sqrt[b*x^2]
```

fricas [A] time = 0.72, size = 7, normalized size = 0.78

$$\sqrt{bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] sqrt(b*x^2)
```

giac [A] time = 0.21, size = 7, normalized size = 0.78

$$\sqrt{b}\,x\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] sqrt(b)*x*sgn(x)
```

maple [A] time = 0.00, size = 8, normalized size = 0.89

$$\sqrt{b\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(1/2)/x,x)
```

```
[Out] (b*x^2)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 0.93, size = 6, normalized size = 0.67

$$\sqrt{b}\,|x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(1/2)/x,x)
```

```
[Out] b^(1/2)*abs(x)
```

sympy [A] time = 0.17, size = 10, normalized size = 1.11

$$\sqrt{b}\,\sqrt{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2)**(1/2)/x,x)
```

```
[Out] sqrt(b)*sqrt(x**2)
```

3.7 $\int \frac{\sqrt{bx^2}}{x^2}\,dx$

Optimal. Leaf size=15

$$\frac{\sqrt{bx^2}\,\log(x)}{x}$$

```
[Out] ln(x)*(b*x^2)^(1/2)/x
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 29}

$$\frac{\sqrt{bx^2}\,\log(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2]/x^2,x]
```

```
[Out] (Sqrt[b*x^2]*Log[x])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{bx^2}}{x^2}\,dx &= \frac{\sqrt{bx^2}\int \frac{1}{x}\,dx}{x}\\ &= \frac{\sqrt{bx^2}\,\log(x)}{x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{bx\log(x)}{\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2]/x^2,x]
```

```
[Out] (b*x*Log[x])/Sqrt[b*x^2]
```

fricas [A] time = 0.54, size = 13, normalized size = 0.87

$$\frac{\sqrt{bx^2}\,\log(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] sqrt(b*x^2)*log(x)/x
```

giac [**A**] time = 0.16, size = 9, normalized size = 0.60

$$\sqrt{b}\,\log\left(|x|\right)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] sqrt(b)*log(abs(x))*sgn(x)
```

maple [**A**] time = 0.00, size = 14, normalized size = 0.93

$$\frac{\sqrt{b\,x^2}\,\ln(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(1/2)/x^2,x)
```

```
[Out] ln(x)*(b*x^2)^(1/2)/x
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\sqrt{b\,x^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(1/2)/x^2,x)
```

```
[Out] int((b*x^2)^(1/2)/x^2, x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(b*x**2)/x**2, x)
```

3.8 $\int \frac{\sqrt{bx^2}}{x^3}\,dx$

Optimal. Leaf size=14

$$-\frac{\sqrt{bx^2}}{x^2}$$

```
[Out] -(b*x^2)^(1/2)/x^2
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$-\frac{\sqrt{bx^2}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2]/x^3,x]
```

```
[Out] -(Sqrt[b*x^2]/x^2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{bx^2}}{x^3}\,dx &= \frac{\sqrt{bx^2}\int \frac{1}{x^2}\,dx}{x}\\ &= -\frac{\sqrt{bx^2}}{x^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{\sqrt{bx^2}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2]/x^3,x]
```

```
[Out] -(Sqrt[b*x^2]/x^2)
```

fricas [A] time = 0.87, size = 12, normalized size = 0.86

$$-\frac{\sqrt{bx^2}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] -sqrt(b*x^2)/x^2
```

giac [**A**] time = 0.17, size = 10, normalized size = 0.71

$$-\frac{\sqrt{b}\,\mathrm{sgn}(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -sqrt(b)*sgn(x)/x
```

maple [**A**] time = 0.00, size = 13, normalized size = 0.93

$$-\frac{\sqrt{b\,x^2}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(1/2)/x^3,x)
```

```
[Out] -(b*x^2)^(1/2)/x^2
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [**B**] time = 0.90, size = 10, normalized size = 0.71

$$-\frac{\sqrt{b}}{\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(1/2)/x^3,x)
```

```
[Out] -b^(1/2)/(x^2)^(1/2)
```

sympy [**A**] time = 0.39, size = 15, normalized size = 1.07

$$-\frac{\sqrt{b}\,\sqrt{x^2}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2)**(1/2)/x**3,x)
```

```
[Out] -sqrt(b)*sqrt(x**2)/x**2
```

3.9 $\int \frac{\sqrt{bx^2}}{x^4}\,dx$

Optimal. Leaf size=16

$$-\frac{\sqrt{bx^2}}{2x^3}$$

```
[Out] -1/2*(b*x^2)^(1/2)/x^3
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$-\frac{\sqrt{bx^2}}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2]/x^4,x]
```

```
[Out] -Sqrt[b*x^2]/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{bx^2}}{x^4}\,dx &= \frac{\sqrt{bx^2}\int \frac{1}{x^3}\,dx}{x}\\ &= -\frac{\sqrt{bx^2}}{2x^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{\sqrt{bx^2}}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2]/x^4,x]
```

```
[Out] -1/2*Sqrt[b*x^2]/x^3
```

fricas [A] time = 0.77, size = 12, normalized size = 0.75

$$-\frac{\sqrt{bx^2}}{2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(b*x^2)/x^3
```

giac [A] time = 0.15, size = 10, normalized size = 0.62

$$-\frac{\sqrt{b}\,\mathrm{sgn}(x)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(b)*sgn(x)/x^2
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$-\frac{\sqrt{b\,x^2}}{2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(1/2)/x^4,x)
```

```
[Out] -1/2*(b*x^2)^(1/2)/x^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 0.94, size = 13, normalized size = 0.81

$$-\frac{\sqrt{b}}{2\,x\,\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(1/2)/x^4,x)
```

```
[Out] -b^(1/2)/(2*x*(x^2)^(1/2))
```

sympy [A] time = 0.47, size = 17, normalized size = 1.06

$$-\frac{\sqrt{b}\,\sqrt{x^2}}{2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2)**(1/2)/x**4,x)
```

```
[Out] -sqrt(b)*sqrt(x**2)/(2*x**3)
```

3.10 $\int \frac{\sqrt{bx^2}}{x^5}\,dx$

Optimal. Leaf size=16

$$-\frac{\sqrt{bx^2}}{3x^4}$$

```
[Out] -1/3*(b*x^2)^(1/2)/x^4
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$-\frac{\sqrt{bx^2}}{3x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2]/x^5,x]
```

```
[Out] -Sqrt[b*x^2]/(3*x^4)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{bx^2}}{x^5}\,dx &= \frac{\sqrt{bx^2}\int \frac{1}{x^4}\,dx}{x}\\ &= -\frac{\sqrt{bx^2}}{3x^4}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{\sqrt{bx^2}}{3x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2]/x^5,x]
```

```
[Out] -1/3*Sqrt[b*x^2]/x^4
```

fricas [A] time = 0.56, size = 12, normalized size = 0.75

$$-\frac{\sqrt{bx^2}}{3\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(b*x^2)/x^4
```

giac [A] time = 0.18, size = 10, normalized size = 0.62

$$-\frac{\sqrt{b}\,\mathrm{sgn}(x)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/3*sqrt(b)*sgn(x)/x^3
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$-\frac{\sqrt{b\,x^2}}{3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(1/2)/x^5,x)
```

```
[Out] -1/3*(b*x^2)^(1/2)/x^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 0.95, size = 10, normalized size = 0.62

$$-\frac{\sqrt{b}}{3\left(x^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(1/2)/x^5,x)
```

```
[Out] -b^(1/2)/(3*(x^2)^(3/2))
```

sympy [A] time = 0.60, size = 17, normalized size = 1.06

$$-\frac{\sqrt{b}\,\sqrt{x^2}}{3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2)**(1/2)/x**5,x)
```

```
[Out] -sqrt(b)*sqrt(x**2)/(3*x**4)
```

3.11 $\int \left(x^2\right)^{3/2}\, dx$

Optimal. Leaf size=14

$$\frac{1}{4}x^3\sqrt{x^2}$$

```
[Out] 1/4*x^3*(x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {15, 30}

$$\frac{1}{4}x^3\sqrt{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2)^(3/2),x]
```

```
[Out] (x^3*Sqrt[x^2])/4
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \left(x^2\right)^{3/2}\, dx &= \frac{\sqrt{x^2}\int x^3\, dx}{x}\\ &= \frac{1}{4}x^3\sqrt{x^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 0.86

$$\frac{1}{4}x\left(x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2)^(3/2),x]
```

```
[Out] (x*(x^2)^(3/2))/4
```

fricas [A] time = 0.56, size = 5, normalized size = 0.36

$$\frac{1}{4}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*x^4
```

giac [A] time = 0.15, size = 7, normalized size = 0.50

$$\frac{1}{4}\, x^4 \mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2)^(1/2),x, algorithm="giac")
[Out] 1/4*x^4*sgn(x)
```

maple [A] time = 0.00, size = 11, normalized size = 0.79

$$\frac{\sqrt{x^2}\, x^3}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^2)^(1/2),x)
[Out] 1/4*x^3*(x^2)^(1/2)
```

maxima [A] time = 1.34, size = 8, normalized size = 0.57

$$\frac{1}{4}\left(x^2\right)^{\frac{3}{2}} x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2)^(1/2),x, algorithm="maxima")
[Out] 1/4*(x^2)^(3/2)*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.07

$$\int x^2 \sqrt{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^2)^(1/2),x)
[Out] int(x^2*(x^2)^(1/2), x)
```

sympy [A] time = 0.07, size = 3, normalized size = 0.21

$$\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**2)**(1/2),x)
[Out] x**4/4
```

3.12 $\int x^2 \left(bx^2\right)^{3/2}\, dx$

Optimal. Leaf size=17

$$\frac{1}{6} bx^5 \sqrt{bx^2}$$

```
[Out] 1/6*b*x^5*(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$\frac{1}{6} bx^5 \sqrt{bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b*x^2)^(3/2),x]
```

```
[Out] (b*x^5*Sqrt[b*x^2])/6
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(bx^2\right)^{3/2}\, dx &= \frac{\left(b\sqrt{bx^2}\right) \int x^5\, dx}{x} \\ &= \frac{1}{6} bx^5 \sqrt{bx^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 0.94

$$\frac{1}{6} x^3 \left(bx^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b*x^2)^(3/2),x]
```

```
[Out] (x^3*(b*x^2)^(3/2))/6
```

fricas [A] time = 0.52, size = 13, normalized size = 0.76

$$\frac{1}{6} \sqrt{bx^2}\, bx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(b*x^2)*b*x^5
```

giac [A] time = 0.15, size = 10, normalized size = 0.59

$$\frac{1}{6} b^{\frac{3}{2}} x^6 \operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/6*b^(3/2)*x^6*sgn(x)
```

maple [A] time = 0.00, size = 13, normalized size = 0.76

$$\frac{\left(b\,x^2\right)^{\frac{3}{2}} x^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2)^(3/2),x)
```

```
[Out] 1/6*x^3*(b*x^2)^(3/2)
```

maxima [A] time = 1.36, size = 13, normalized size = 0.76

$$\frac{\left(bx^2\right)^{\frac{5}{2}} x}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2)^(5/2)*x/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int x^2 \left(b\,x^2\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2)^(3/2),x)
```

```
[Out] int(x^2*(b*x^2)^(3/2), x)
```

sympy [A] time = 0.69, size = 15, normalized size = 0.88

$$\frac{b^{\frac{3}{2}} x^3 \left(x^2\right)^{\frac{3}{2}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2)**(3/2),x)
```

```
[Out] b**(3/2)*x**3*(x**2)**(3/2)/6
```

3.13 $\int x\left(bx^2\right)^{3/2}\,dx$

Optimal. Leaf size=17

$$\frac{1}{5}bx^4\sqrt{bx^2}$$

```
[Out] 1/5*b*x^4*(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 30}

$$\frac{1}{5}bx^4\sqrt{bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b*x^2)^(3/2),x]
```

```
[Out] (b*x^4*Sqrt[b*x^2])/5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(bx^2\right)^{3/2}\,dx &= \frac{\left(b\sqrt{bx^2}\right)\int x^4\,dx}{x}\\ &= \frac{1}{5}bx^4\sqrt{bx^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 0.94

$$\frac{1}{5}x^2\left(bx^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b*x^2)^(3/2),x]
```

```
[Out] (x^2*(b*x^2)^(3/2))/5
```

fricas [A] time = 0.75, size = 13, normalized size = 0.76

$$\frac{1}{5}\sqrt{bx^2}\,bx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(b*x^2)*b*x^4
```

giac [A] time = 0.17, size = 10, normalized size = 0.59

$$\frac{1}{5}\, b^{\frac{3}{2}} x^5 \operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/5*b^(3/2)*x^5*sgn(x)
```

maple [A] time = 0.00, size = 13, normalized size = 0.76

$$\frac{\left(b\, x^2\right)^{\frac{3}{2}} x^2}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2)^(3/2),x)
```

```
[Out] 1/5*x^2*(b*x^2)^(3/2)
```

maxima [A] time = 1.33, size = 12, normalized size = 0.71

$$\frac{\left(bx^2\right)^{\frac{5}{2}}}{5\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/5*(b*x^2)^(5/2)/b
```

mupad [B] time = 0.96, size = 10, normalized size = 0.59

$$\frac{b^{3/2}\sqrt{x^{10}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2)^(3/2),x)
```

```
[Out] (b^(3/2)*(x^10)^(1/2))/5
```

sympy [A] time = 0.51, size = 15, normalized size = 0.88

$$\frac{b^{\frac{3}{2}} x^2 \left(x^2\right)^{\frac{3}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2)**(3/2),x)
```

```
[Out] b**(3/2)*x**2*(x**2)**(3/2)/5
```

3.14 $\int \left(bx^2\right)^{3/2}\,dx$

Optimal. Leaf size=17

$$\frac{1}{4}bx^3\sqrt{bx^2}$$

```
[Out] 1/4*b*x^3*(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$\frac{1}{4}bx^3\sqrt{bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2)^(3/2),x]
```

```
[Out] (b*x^3*Sqrt[b*x^2])/4
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \left(bx^2\right)^{3/2}\,dx &= \frac{\left(b\sqrt{bx^2}\right)\int x^3\,dx}{x}\\ &= \frac{1}{4}bx^3\sqrt{bx^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{4}x\left(bx^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2)^(3/2),x]
```

```
[Out] (x*(b*x^2)^(3/2))/4
```

fricas [A] time = 0.70, size = 13, normalized size = 0.76

$$\frac{1}{4}\sqrt{bx^2}\,bx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(b*x^2)*b*x^3
```

giac [A] time = 0.15, size = 10, normalized size = 0.59

$$\frac{1}{4} b^{\frac{3}{2}} x^4 \text{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(3/2),x, algorithm="giac")
[Out] 1/4*b^(3/2)*x^4*sgn(x)
```

maple [A] time = 0.00, size = 11, normalized size = 0.65

$$\frac{\left(b\,x^2\right)^{\frac{3}{2}} x}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(3/2),x)
[Out] 1/4*x*(b*x^2)^(3/2)
```

maxima [A] time = 1.28, size = 10, normalized size = 0.59

$$\frac{1}{4}\left(bx^2\right)^{\frac{3}{2}} x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(3/2),x, algorithm="maxima")
[Out] 1/4*(b*x^2)^(3/2)*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \left(b\,x^2\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(3/2),x)
[Out] int((b*x^2)^(3/2), x)
```

sympy [A] time = 0.38, size = 14, normalized size = 0.82

$$\frac{b^{\frac{3}{2}} x \left(x^2\right)^{\frac{3}{2}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2)**(3/2),x)
[Out] b**(3/2)*x*(x**2)**(3/2)/4
```

3.15 $\int \frac{\left(bx^2\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=17

$$\frac{1}{3}bx^2\sqrt{bx^2}$$

```
[Out] 1/3*b*x^2*(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$\frac{1}{3}bx^2\sqrt{bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2)^(3/2)/x,x]
```

```
[Out] (b*x^2*Sqrt[b*x^2])/3
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2\right)^{3/2}}{x}\,dx &= \frac{\left(b\sqrt{bx^2}\right)\int x^2\,dx}{x}\\ &= \frac{1}{3}bx^2\sqrt{bx^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{1}{3}bx^2\sqrt{bx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2)^(3/2)/x,x]
```

```
[Out] (b*x^2*Sqrt[b*x^2])/3
```

fricas [A] time = 0.55, size = 13, normalized size = 0.76

$$\frac{1}{3}\sqrt{bx^2}\,bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(b*x^2)*b*x^2
```

giac [A] time = 0.18, size = 10, normalized size = 0.59

$$\frac{1}{3}\, b^{\frac{3}{2}} x^3 \operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(3/2)/x,x, algorithm="giac")
[Out] 1/3*b^(3/2)*x^3*sgn(x)
```

maple [A] time = 0.00, size = 10, normalized size = 0.59

$$\frac{\left(b\, x^2\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(3/2)/x,x)
[Out] 1/3*(b*x^2)^(3/2)
```

maxima [A] time = 1.28, size = 9, normalized size = 0.53

$$\frac{1}{3}\, \left(b x^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(3/2)/x,x, algorithm="maxima")
[Out] 1/3*(b*x^2)^(3/2)
```

mupad [B] time = 0.96, size = 10, normalized size = 0.59

$$\frac{b^{3/2}\, \sqrt{x^6}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(3/2)/x,x)
[Out] (b^(3/2)*(x^6)^(1/2))/3
```

sympy [A] time = 0.40, size = 12, normalized size = 0.71

$$\frac{b^{\frac{3}{2}}\, \left(x^2\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2)**(3/2)/x,x)
[Out] b**(3/2)*(x**2)**(3/2)/3
```

3.16 $\int \frac{\left(bx^2\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=15

$$\frac{1}{2}bx\sqrt{bx^2}$$

```
[Out] 1/2*b*x*(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$\frac{1}{2}bx\sqrt{bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2)^(3/2)/x^2,x]
```

```
[Out] (b*x*Sqrt[b*x^2])/2
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2\right)^{3/2}}{x^2}\,dx &= \frac{\left(b\sqrt{bx^2}\right)\int x\,dx}{x}\\ &= \frac{1}{2}bx\sqrt{bx^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{1}{2}bx\sqrt{bx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2)^(3/2)/x^2,x]
```

```
[Out] (b*x*Sqrt[b*x^2])/2
```

fricas [A] time = 0.63, size = 11, normalized size = 0.73

$$\frac{1}{2}\sqrt{bx^2}\,bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(b*x^2)*b*x
```

giac [A] time = 0.15, size = 10, normalized size = 0.67

$$\frac{1}{2} b^{\frac{3}{2}} x^2 \operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*b^(3/2)*x^2*sgn(x)
```

maple [A] time = 0.00, size = 13, normalized size = 0.87

$$\frac{\left(b\,x^2\right)^{\frac{3}{2}}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(3/2)/x^2,x)
```

```
[Out] 1/2*(b*x^2)^(3/2)/x
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 0.97, size = 8, normalized size = 0.53

$$\frac{b^{3/2}\,x\,|x|}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(3/2)/x^2,x)
```

```
[Out] (b^(3/2)*x*abs(x))/2
```

sympy [A] time = 0.42, size = 14, normalized size = 0.93

$$\frac{b^{\frac{3}{2}} \left(x^2\right)^{\frac{3}{2}}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2)**(3/2)/x**2,x)
```

```
[Out] b**(3/2)*(x**2)**(3/2)/(2*x)
```

3.17 $\int \frac{\left(bx^2\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=11

$$b\sqrt{bx^2}$$

```
[Out] b*(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 8}

$$b\sqrt{bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2)^(3/2)/x^3,x]
```

```
[Out] b*Sqrt[b*x^2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2\right)^{3/2}}{x^3}\,dx &= \frac{\left(b\sqrt{bx^2}\right)\int 1\,dx}{x}\\ &= b\sqrt{bx^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.18

$$\frac{\left(bx^2\right)^{3/2}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2)^(3/2)/x^3,x]
```

```
[Out] (b*x^2)^(3/2)/x^2
```

fricas [A] time = 0.61, size = 9, normalized size = 0.82

$$\sqrt{bx^2}\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] sqrt(b*x^2)*b
```

giac [**A**] time = 0.18, size = 7, normalized size = 0.64

$$b^{\frac{3}{2}} x \operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(3/2)/x^3,x, algorithm="giac")
[Out] b^(3/2)*x*sgn(x)
```

maple [**A**] time = 0.00, size = 12, normalized size = 1.09

$$\frac{\left(b\,x^2\right)^{\frac{3}{2}}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(3/2)/x^3,x)
[Out] (b*x^2)^(3/2)/x^2
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(3/2)/x^3,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [**B**] time = 0.90, size = 6, normalized size = 0.55

$$b^{3/2}\,|x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(3/2)/x^3,x)
[Out] b^(3/2)*abs(x)
```

sympy [**A**] time = 0.55, size = 14, normalized size = 1.27

$$\frac{b^{\frac{3}{2}} \left(x^2\right)^{\frac{3}{2}}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2)**(3/2)/x**3,x)
[Out] b**(3/2)*(x**2)**(3/2)/x**2
```

3.18 $\int \frac{\left(bx^2\right)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=16

$$\frac{b\sqrt{bx^2}\,\log(x)}{x}$$

```
[Out] b*ln(x)*(b*x^2)^(1/2)/x
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 29}

$$\frac{b\sqrt{bx^2}\,\log(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2)^(3/2)/x^4,x]
```

```
[Out] (b*Sqrt[b*x^2]*Log[x])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2\right)^{3/2}}{x^4}\,dx &= \frac{\left(b\sqrt{bx^2}\right)\int \frac{1}{x}\,dx}{x}\\ &= \frac{b\sqrt{bx^2}\,\log(x)}{x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 0.94

$$\frac{\left(bx^2\right)^{3/2}\log(x)}{x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2)^(3/2)/x^4,x]
```

```
[Out] ((b*x^2)^(3/2)*Log[x])/x^3
```

fricas [A] time = 0.54, size = 14, normalized size = 0.88

$$\frac{\sqrt{bx^2}\,b\log(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] sqrt(b*x^2)*b*log(x)/x
```

giac [A] time = 0.19, size = 9, normalized size = 0.56

$$b^{\frac{3}{2}} \log\left(|x|\right) \operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] b^(3/2)*log(abs(x))*sgn(x)
```

maple [A] time = 0.00, size = 14, normalized size = 0.88

$$\frac{\left(b\,x^2\right)^{\frac{3}{2}} \ln(x)}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(3/2)/x^4,x)
```

```
[Out] (b*x^2)^(3/2)/x^3*ln(x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\left(b\,x^2\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(3/2)/x^4,x)
```

```
[Out] int((b*x^2)^(3/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2)**(3/2)/x**4,x)
```

```
[Out] Integral((b*x**2)**(3/2)/x**4, x)
```

3.19 $\int \frac{\left(bx^2\right)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=15

$$-\frac{b\sqrt{bx^2}}{x^2}$$

```
[Out] -b*(b*x^2)^(1/2)/x^2
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$-\frac{b\sqrt{bx^2}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2)^(3/2)/x^5,x]
```

```
[Out] -((b*Sqrt[b*x^2])/x^2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2\right)^{3/2}}{x^5}\,dx &= \frac{\left(b\sqrt{bx^2}\right)\int \frac{1}{x^2}\,dx}{x}\\ &= -\frac{b\sqrt{bx^2}}{x^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 0.93

$$-\frac{\left(bx^2\right)^{3/2}}{x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2)^(3/2)/x^5,x]
```

```
[Out] -((b*x^2)^(3/2)/x^4)
```

fricas [A] time = 0.56, size = 13, normalized size = 0.87

$$-\frac{\sqrt{bx^2}\,b}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(3/2)/x^5,x, algorithm="fricas")
[Out] -sqrt(b*x^2)*b/x^2
```

giac [A] time = 0.16, size = 10, normalized size = 0.67

$$-\frac{b^{\frac{3}{2}}\operatorname{sgn}(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(3/2)/x^5,x, algorithm="giac")
[Out] -b^(3/2)*sgn(x)/x
```

maple [A] time = 0.00, size = 13, normalized size = 0.87

$$-\frac{\left(b\,x^2\right)^{\frac{3}{2}}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(3/2)/x^5,x)
[Out] -(b*x^2)^(3/2)/x^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(3/2)/x^5,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 0.90, size = 10, normalized size = 0.67

$$-\frac{b^{3/2}}{\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(3/2)/x^5,x)
[Out] -b^(3/2)/(x^2)^(1/2)
```

sympy [A] time = 0.85, size = 15, normalized size = 1.00

$$-\frac{b^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2)**(3/2)/x**5,x)
[Out] -b**(3/2)*(x**2)**(3/2)/x**4
```

3.20 $\int \frac{\left(bx^2\right)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=17

$$-\frac{b\sqrt{bx^2}}{2x^3}$$

```
[Out] -1/2*b*(b*x^2)^(1/2)/x^3
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$-\frac{b\sqrt{bx^2}}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2)^(3/2)/x^6,x]
```

```
[Out] -(b*Sqrt[b*x^2])/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2\right)^{3/2}}{x^6}\,dx &= \frac{\left(b\sqrt{bx^2}\right)\int \frac{1}{x^3}\,dx}{x}\\ &= -\frac{b\sqrt{bx^2}}{2x^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 0.94

$$-\frac{\left(bx^2\right)^{3/2}}{2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2)^(3/2)/x^6,x]
```

```
[Out] -1/2*(b*x^2)^(3/2)/x^5
```

fricas [A] time = 0.48, size = 13, normalized size = 0.76

$$-\frac{\sqrt{bx^2}\,b}{2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(b*x^2)*b/x^3
```

giac [A] time = 0.18, size = 10, normalized size = 0.59

$$-\frac{b^{\frac{3}{2}}\operatorname{sgn}(x)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] -1/2*b^(3/2)*sgn(x)/x^2
```

maple [A] time = 0.00, size = 13, normalized size = 0.76

$$-\frac{\left(b\,x^2\right)^{\frac{3}{2}}}{2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(3/2)/x^6,x)
```

```
[Out] -1/2*(b*x^2)^(3/2)/x^5
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 0.90, size = 13, normalized size = 0.76

$$-\frac{b^{3/2}}{2\,x\,\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(3/2)/x^6,x)
```

```
[Out] -b^(3/2)/(2*x*(x^2)^(1/2))
```

sympy [A] time = 1.02, size = 17, normalized size = 1.00

$$-\frac{b^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}{2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2)**(3/2)/x**6,x)
```

```
[Out] -b**(3/2)*(x**2)**(3/2)/(2*x**5)
```

3.21 $\int \frac{\left(bx^2\right)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=17

$$-\frac{b\sqrt{bx^2}}{3x^4}$$

```
[Out] -1/3*b*(b*x^2)^(1/2)/x^4
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$-\frac{b\sqrt{bx^2}}{3x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2)^(3/2)/x^7,x]
```

```
[Out] -(b*Sqrt[b*x^2])/(3*x^4)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2\right)^{3/2}}{x^7}\,dx &= \frac{\left(b\sqrt{bx^2}\right)\int \frac{1}{x^4}\,dx}{x}\\ &= -\frac{b\sqrt{bx^2}}{3x^4}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 0.94

$$-\frac{\left(bx^2\right)^{3/2}}{3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2)^(3/2)/x^7,x]
```

```
[Out] -1/3*(b*x^2)^(3/2)/x^6
```

fricas [A] time = 0.56, size = 13, normalized size = 0.76

$$-\frac{\sqrt{bx^2}\,b}{3\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(b*x^2)*b/x^4
```

giac [A] time = 0.16, size = 10, normalized size = 0.59

$$-\frac{b^{\frac{3}{2}}\operatorname{sgn}(x)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/3*b^(3/2)*sgn(x)/x^3
```

maple [A] time = 0.00, size = 13, normalized size = 0.76

$$-\frac{\left(b\,x^2\right)^{\frac{3}{2}}}{3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(3/2)/x^7,x)
```

```
[Out] -1/3*(b*x^2)^(3/2)/x^6
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 0.91, size = 10, normalized size = 0.59

$$-\frac{b^{3/2}}{3\left(x^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(3/2)/x^7,x)
```

```
[Out] -b^(3/2)/(3*(x^2)^(3/2))
```

sympy [A] time = 1.29, size = 17, normalized size = 1.00

$$-\frac{b^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}{3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2)**(3/2)/x**7,x)
```

```
[Out] -b**(3/2)*(x**2)**(3/2)/(3*x**6)
```

3.22 $\int \left(x^2\right)^{5/2}\, dx$

Optimal. Leaf size=14

$$\frac{1}{6}x^5\sqrt{x^2}$$

```
[Out] 1/6*x^5*(x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {15, 30}

$$\frac{1}{6}x^5\sqrt{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2)^(5/2),x]
```

```
[Out] (x^5*Sqrt[x^2])/6
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \left(x^2\right)^{5/2}\, dx &= \frac{\sqrt{x^2}\int x^5\, dx}{x}\\ &= \frac{1}{6}x^5\sqrt{x^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 0.86

$$\frac{1}{6}x\left(x^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2)^(5/2),x]
```

```
[Out] (x*(x^2)^(5/2))/6
```

fricas [A] time = 0.76, size = 5, normalized size = 0.36

$$\frac{1}{6}\,x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*x^6
```

giac [**A**] time = 0.15, size = 7, normalized size = 0.50

$$\frac{1}{6}x^6\text{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2)^(5/2),x, algorithm="giac")
[Out] 1/6*x^6*sgn(x)
```

maple [**A**] time = 0.00, size = 9, normalized size = 0.64

$$\frac{\left(x^2\right)^{\frac{5}{2}}x}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2)^(5/2),x)
[Out] 1/6*x*(x^2)^(5/2)
```

maxima [**A**] time = 1.26, size = 8, normalized size = 0.57

$$\frac{1}{6}\left(x^2\right)^{\frac{5}{2}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2)^(5/2),x, algorithm="maxima")
[Out] 1/6*(x^2)^(5/2)*x
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.07

$$\int \left(x^2\right)^{5/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2)^(5/2),x)
[Out] int((x^2)^(5/2), x)
```

sympy [**A**] time = 0.07, size = 3, normalized size = 0.21

$$\frac{x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2)**(5/2),x)
[Out] x**6/6
```

3.23 $\int x \left(bx^2\right)^{5/2}\, dx$

Optimal. Leaf size=19

$$\frac{1}{7} b^2 x^6 \sqrt{bx^2}$$

```
[Out] 1/7*b^2*x^6*(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 30}

$$\frac{1}{7} b^2 x^6 \sqrt{bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b*x^2)^(5/2),x]
```

```
[Out] (b^2*x^6*Sqrt[b*x^2])/7
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x \left(bx^2\right)^{5/2}\, dx &= \frac{\left(b^2\sqrt{bx^2}\right)\int x^6\, dx}{x} \\ &= \frac{1}{7} b^2 x^6 \sqrt{bx^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 0.84

$$\frac{1}{7} x^2 \left(bx^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b*x^2)^(5/2),x]
```

```
[Out] (x^2*(b*x^2)^(5/2))/7
```

fricas [A] time = 0.59, size = 15, normalized size = 0.79

$$\frac{1}{7} \sqrt{bx^2}\, b^2 x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/7*sqrt(b*x^2)*b^2*x^6
```

giac [A] time = 0.15, size = 10, normalized size = 0.53

$$\frac{1}{7} b^{\frac{5}{2}} x^7 \operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2)^(5/2),x, algorithm="giac")
[Out] 1/7*b^(5/2)*x^7*sgn(x)
```

maple [A] time = 0.00, size = 13, normalized size = 0.68

$$\frac{\left(b\,x^2\right)^{\frac{5}{2}} x^2}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2)^(5/2),x)
[Out] 1/7*x^2*(b*x^2)^(5/2)
```

maxima [A] time = 1.33, size = 12, normalized size = 0.63

$$\frac{\left(bx^2\right)^{\frac{7}{2}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2)^(5/2),x, algorithm="maxima")
[Out] 1/7*(b*x^2)^(7/2)/b
```

mupad [B] time = 0.94, size = 10, normalized size = 0.53

$$\frac{b^{5/2}\sqrt{x^{14}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2)^(5/2),x)
[Out] (b^(5/2)*(x^14)^(1/2))/7
```

sympy [A] time = 1.30, size = 15, normalized size = 0.79

$$\frac{b^{\frac{5}{2}} x^2 \left(x^2\right)^{\frac{5}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2)**(5/2),x)
[Out] b**(5/2)*x**2*(x**2)**(5/2)/7
```

3.24 $\int \left(bx^2\right)^{5/2}\, dx$

Optimal. Leaf size=19

$$\frac{1}{6}b^2x^5\sqrt{bx^2}$$

```
[Out] 1/6*b^2*x^5*(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$\frac{1}{6}b^2x^5\sqrt{bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2)^(5/2),x]
```

```
[Out] (b^2*x^5*Sqrt[b*x^2])/6
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \left(bx^2\right)^{5/2}\, dx &= \frac{\left(b^2\sqrt{bx^2}\right)\int x^5\, dx}{x}\\ &= \frac{1}{6}b^2x^5\sqrt{bx^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 0.74

$$\frac{1}{6}x\left(bx^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2)^(5/2),x]
```

```
[Out] (x*(b*x^2)^(5/2))/6
```

fricas [A] time = 0.67, size = 15, normalized size = 0.79

$$\frac{1}{6}\sqrt{bx^2}\, b^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(b*x^2)*b^2*x^5
```

giac [**A**] time = 0.17, size = 10, normalized size = 0.53

$$\frac{1}{6} b^{\frac{5}{2}} x^6 \text{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(5/2),x, algorithm="giac")
[Out] 1/6*b^(5/2)*x^6*sgn(x)
```

maple [**A**] time = 0.00, size = 11, normalized size = 0.58

$$\frac{\left(b\,x^2\right)^{\frac{5}{2}} x}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(5/2),x)
[Out] 1/6*x*(b*x^2)^(5/2)
```

maxima [**A**] time = 1.34, size = 10, normalized size = 0.53

$$\frac{1}{6} \left(bx^2\right)^{\frac{5}{2}} x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(5/2),x, algorithm="maxima")
[Out] 1/6*(b*x^2)^(5/2)*x
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.05

$$\int \left(b\,x^2\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(5/2),x)
[Out] int((b*x^2)^(5/2), x)
```

sympy [**A**] time = 1.00, size = 14, normalized size = 0.74

$$\frac{b^{\frac{5}{2}} x \left(x^2\right)^{\frac{5}{2}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2)**(5/2),x)
[Out] b**(5/2)*x*(x**2)**(5/2)/6
```

3.25 $\int \frac{\left(bx^2\right)^{5/2}}{x}\,dx$

Optimal. Leaf size=19

$$\frac{1}{5}b^2x^4\sqrt{bx^2}$$

```
[Out] 1/5*b^2*x^4*(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$\frac{1}{5}b^2x^4\sqrt{bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2)^(5/2)/x,x]
```

```
[Out] (b^2*x^4*Sqrt[b*x^2])/5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2\right)^{5/2}}{x}\,dx &= \frac{\left(b^2\sqrt{bx^2}\right)\int x^4\,dx}{x}\\ &= \frac{1}{5}b^2x^4\sqrt{bx^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 0.89

$$\frac{1}{5}bx^2\left(bx^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2)^(5/2)/x,x]
```

```
[Out] (b*x^2*(b*x^2)^(3/2))/5
```

fricas [A] time = 0.65, size = 15, normalized size = 0.79

$$\frac{1}{5}\sqrt{bx^2}\,b^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(b*x^2)*b^2*x^4
```

giac [A] time = 0.15, size = 10, normalized size = 0.53

$$\frac{1}{5} b^{\frac{5}{2}} x^5 \operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(5/2)/x,x, algorithm="giac")
[Out] 1/5*b^(5/2)*x^5*sgn(x)
```

maple [A] time = 0.00, size = 10, normalized size = 0.53

$$\frac{\left(b\,x^2\right)^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(5/2)/x,x)
[Out] 1/5*(b*x^2)^(5/2)
```

maxima [A] time = 1.33, size = 9, normalized size = 0.47

$$\frac{1}{5} \left(bx^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(5/2)/x,x, algorithm="maxima")
[Out] 1/5*(b*x^2)^(5/2)
```

mupad [B] time = 0.90, size = 10, normalized size = 0.53

$$\frac{b^{5/2}\sqrt{x^{10}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(5/2)/x,x)
[Out] (b^(5/2)*(x^10)^(1/2))/5
```

sympy [A] time = 1.02, size = 12, normalized size = 0.63

$$\frac{b^{\frac{5}{2}} \left(x^2\right)^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2)**(5/2)/x,x)
[Out] b**(5/2)*(x**2)**(5/2)/5
```

3.26 $\int \frac{\left(bx^2\right)^{5/2}}{x^2}\,dx$

Optimal. Leaf size=19

$$\frac{1}{4}b^2x^3\sqrt{bx^2}$$

```
[Out] 1/4*b^2*x^3*(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$\frac{1}{4}b^2x^3\sqrt{bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2)^(5/2)/x^2,x]
```

```
[Out] (b^2*x^3*Sqrt[b*x^2])/4
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2\right)^{5/2}}{x^2}\,dx &= \frac{\left(b^2\sqrt{bx^2}\right)\int x^3\,dx}{x}\\ &= \frac{1}{4}b^2x^3\sqrt{bx^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 0.79

$$\frac{1}{4}bx\left(bx^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2)^(5/2)/x^2,x]
```

```
[Out] (b*x*(b*x^2)^(3/2))/4
```

fricas [A] time = 0.58, size = 15, normalized size = 0.79

$$\frac{1}{4}\sqrt{bx^2}\,b^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(b*x^2)*b^2*x^3
```

giac [A] time = 0.17, size = 10, normalized size = 0.53

$$\frac{1}{4}\, b^{\frac{5}{2}} x^4 \mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/4*b^(5/2)*x^4*sgn(x)
```

maple [A] time = 0.00, size = 13, normalized size = 0.68

$$\frac{\left(b\, x^2\right)^{\frac{5}{2}}}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(5/2)/x^2,x)
```

```
[Out] 1/4*(b*x^2)^(5/2)/x
```

maxima [A] time = 1.31, size = 12, normalized size = 0.63

$$\frac{\left(bx^2\right)^{\frac{5}{2}}}{4\, x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^2)^(5/2)/x
```

mupad [B] time = 0.97, size = 13, normalized size = 0.68

$$\frac{b^{5/2}\, x^3\, \sqrt{x^2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(5/2)/x^2,x)
```

```
[Out] (b^(5/2)*x^3*(x^2)^(1/2))/4
```

sympy [A] time = 1.06, size = 14, normalized size = 0.74

$$\frac{b^{\frac{5}{2}} \left(x^2\right)^{\frac{5}{2}}}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2)**(5/2)/x**2,x)
```

```
[Out] b**(5/2)*(x**2)**(5/2)/(4*x)
```

3.27 $\int \frac{\left(bx^2\right)^{5/2}}{x^3}\,dx$

Optimal. Leaf size=19

$$\frac{1}{3}b^2x^2\sqrt{bx^2}$$

```
[Out] 1/3*b^2*x^2*(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$\frac{1}{3}b^2x^2\sqrt{bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2)^(5/2)/x^3,x]
```

```
[Out] (b^2*x^2*Sqrt[b*x^2])/3
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2\right)^{5/2}}{x^3}\,dx &= \frac{\left(b^2\sqrt{bx^2}\right)\int x^2\,dx}{x}\\ &= \frac{1}{3}b^2x^2\sqrt{bx^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 0.84

$$\frac{\left(bx^2\right)^{5/2}}{3x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2)^(5/2)/x^3,x]
```

```
[Out] (b*x^2)^(5/2)/(3*x^2)
```

fricas [A] time = 0.78, size = 15, normalized size = 0.79

$$\frac{1}{3}\sqrt{bx^2}\,b^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(b*x^2)*b^2*x^2
```

giac [**A**] time = 0.18, size = 10, normalized size = 0.53

$$\frac{1}{3}\, b^{\frac{5}{2}} x^3 \operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/3*b^(5/2)*x^3*sgn(x)
```

maple [**A**] time = 0.00, size = 13, normalized size = 0.68

$$\frac{\left(b\, x^2\right)^{\frac{5}{2}}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(5/2)/x^3,x)
```

```
[Out] 1/3*(b*x^2)^(5/2)/x^2
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [**B**] time = 0.93, size = 10, normalized size = 0.53

$$\frac{b^{5/2}\sqrt{x^6}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(5/2)/x^3,x)
```

```
[Out] (b^(5/2)*(x^6)^(1/2))/3
```

sympy [**A**] time = 1.22, size = 15, normalized size = 0.79

$$\frac{b^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2)**(5/2)/x**3,x)
```

```
[Out] b**(5/2)*(x**2)**(5/2)/(3*x**2)
```

3.28 $\int \frac{\left(bx^2\right)^{5/2}}{x^4}\,dx$

Optimal. Leaf size=17

$$\frac{1}{2}b^2x\sqrt{bx^2}$$

```
[Out] 1/2*b^2*x*(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$\frac{1}{2}b^2x\sqrt{bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2)^(5/2)/x^4,x]
```

```
[Out] (b^2*x*Sqrt[b*x^2])/2
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2\right)^{5/2}}{x^4}\,dx &= \frac{\left(b^2\sqrt{bx^2}\right)\int x\,dx}{x}\\ &= \frac{1}{2}b^2x\sqrt{bx^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{1}{2}b^2x\sqrt{bx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2)^(5/2)/x^4,x]
```

```
[Out] (b^2*x*Sqrt[b*x^2])/2
```

fricas [A] time = 0.75, size = 13, normalized size = 0.76

$$\frac{1}{2}\sqrt{bx^2}\,b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(b*x^2)*b^2*x
```

giac [A] time = 0.15, size = 10, normalized size = 0.59

$$\frac{1}{2}\, b^{\frac{5}{2}} x^2 \operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*b^(5/2)*x^2*sgn(x)
```

maple [A] time = 0.00, size = 13, normalized size = 0.76

$$\frac{\left(b\, x^2\right)^{\frac{5}{2}}}{2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(5/2)/x^4,x)
```

```
[Out] 1/2*(b*x^2)^(5/2)/x^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 0.94, size = 8, normalized size = 0.47

$$\frac{b^{5/2}\, x\, |x|}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(5/2)/x^4,x)
```

```
[Out] (b^(5/2)*x*abs(x))/2
```

sympy [A] time = 1.23, size = 15, normalized size = 0.88

$$\frac{b^{\frac{5}{2}} \left(x^2\right)^{\frac{5}{2}}}{2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2)**(5/2)/x**4,x)
```

```
[Out] b**(5/2)*(x**2)**(5/2)/(2*x**3)
```

3.29 $\int \frac{\left(bx^2\right)^{5/2}}{x^5}\,dx$

Optimal. Leaf size=13

$$b^2\sqrt{bx^2}$$

```
[Out] b^2*(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 8}

$$b^2\sqrt{bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2)^(5/2)/x^5,x]
```

```
[Out] b^2*Sqrt[b*x^2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2\right)^{5/2}}{x^5}\,dx &= \frac{\left(b^2\sqrt{bx^2}\right)\int 1\,dx}{x}\\ &= b^2\sqrt{bx^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{\left(bx^2\right)^{5/2}}{x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2)^(5/2)/x^5,x]
```

```
[Out] (b*x^2)^(5/2)/x^4
```

fricas [A] time = 0.52, size = 11, normalized size = 0.85

$$\sqrt{bx^2}\,b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] sqrt(b*x^2)*b^2
```

giac [**A**] time = 0.17, size = 7, normalized size = 0.54

$$b^{\frac{5}{2}} x \operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(5/2)/x^5,x, algorithm="giac")
[Out] b^(5/2)*x*sgn(x)
```

maple [**A**] time = 0.00, size = 12, normalized size = 0.92

$$\frac{\left(b\,x^2\right)^{\frac{5}{2}}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(5/2)/x^5,x)
[Out] (b*x^2)^(5/2)/x^4
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(5/2)/x^5,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [**B**] time = 0.95, size = 6, normalized size = 0.46

$$b^{5/2}\,|x|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(5/2)/x^5,x)
[Out] b^(5/2)*abs(x)
```

sympy [**A**] time = 1.24, size = 14, normalized size = 1.08

$$\frac{b^{\frac{5}{2}} \left(x^2\right)^{\frac{5}{2}}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2)**(5/2)/x**5,x)
[Out] b**(5/2)*(x**2)**(5/2)/x**4
```

3.30 $\int \frac{\left(bx^2\right)^{5/2}}{x^6}\,dx$

Optimal. Leaf size=18

$$\frac{b^2\sqrt{bx^2}\,\log(x)}{x}$$

```
[Out] b^2*ln(x)*(b*x^2)^(1/2)/x
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 29}

$$\frac{b^2\sqrt{bx^2}\,\log(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2)^(5/2)/x^6,x]
```

```
[Out] (b^2*Sqrt[b*x^2]*Log[x])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2\right)^{5/2}}{x^6}\,dx &= \frac{\left(b^2\sqrt{bx^2}\right)\int \frac{1}{x}\,dx}{x}\\ &= \frac{b^2\sqrt{bx^2}\,\log(x)}{x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 0.83

$$\frac{\left(bx^2\right)^{5/2}\log(x)}{x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2)^(5/2)/x^6,x]
```

```
[Out] ((b*x^2)^(5/2)*Log[x])/x^5
```

fricas [A] time = 0.61, size = 16, normalized size = 0.89

$$\frac{\sqrt{bx^2}\,b^2\log(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(5/2)/x^6,x, algorithm="fricas")
```

```
[Out] sqrt(b*x^2)*b^2*log(x)/x
```

giac [A] time = 0.17, size = 9, normalized size = 0.50

$$b^{\frac{5}{2}} \log(|x|) \operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(5/2)/x^6,x, algorithm="giac")
```

```
[Out] b^(5/2)*log(abs(x))*sgn(x)
```

maple [A] time = 0.00, size = 14, normalized size = 0.78

$$\frac{\left(b\,x^2\right)^{\frac{5}{2}} \ln(x)}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(5/2)/x^6,x)
```

```
[Out] (b*x^2)^(5/2)/x^5*ln(x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(5/2)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\left(b\,x^2\right)^{5/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(5/2)/x^6,x)
```

```
[Out] int((b*x^2)^(5/2)/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2\right)^{\frac{5}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2)**(5/2)/x**6,x)
```

```
[Out] Integral((b*x**2)**(5/2)/x**6, x)
```

3.31 $\int \frac{\left(bx^2\right)^{5/2}}{x^7}\,dx$

Optimal. Leaf size=17

$$-\frac{b^2\sqrt{bx^2}}{x^2}$$

```
[Out] -b^2*(b*x^2)^(1/2)/x^2
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$-\frac{b^2\sqrt{bx^2}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2)^(5/2)/x^7,x]
```

```
[Out] -((b^2*Sqrt[b*x^2])/x^2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\left(bx^2\right)^{5/2}}{x^7}\,dx = \frac{\left(b^2\sqrt{bx^2}\right)\int \frac{1}{x^2}\,dx}{x}$$
$$= -\frac{b^2\sqrt{bx^2}}{x^2}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 0.82

$$-\frac{\left(bx^2\right)^{5/2}}{x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2)^(5/2)/x^7,x]
```

```
[Out] -((b*x^2)^(5/2)/x^6)
```

fricas [A] time = 0.75, size = 15, normalized size = 0.88

$$-\frac{\sqrt{bx^2}\,b^2}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(5/2)/x^7,x, algorithm="fricas")
```

```
[Out] -sqrt(b*x^2)*b^2/x^2
```

giac [A] time = 0.15, size = 10, normalized size = 0.59

$$-\frac{b^{\frac{5}{2}}\operatorname{sgn}(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(5/2)/x^7,x, algorithm="giac")
```

```
[Out] -b^(5/2)*sgn(x)/x
```

maple [A] time = 0.00, size = 13, normalized size = 0.76

$$-\frac{\left(b\,x^2\right)^{\frac{5}{2}}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(5/2)/x^7,x)
```

```
[Out] -(b*x^2)^(5/2)/x^6
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(5/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 0.93, size = 10, normalized size = 0.59

$$-\frac{b^{5/2}}{\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(5/2)/x^7,x)
```

```
[Out] -b^(5/2)/(x^2)^(1/2)
```

sympy [A] time = 1.87, size = 15, normalized size = 0.88

$$-\frac{b^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2)**(5/2)/x**7,x)
```

```
[Out] -b**(5/2)*(x**2)**(5/2)/x**6
```

3.32 $\int \frac{\left(bx^2\right)^{5/2}}{x^8}\,dx$

Optimal. Leaf size=19

$$-\frac{b^2\sqrt{bx^2}}{2x^3}$$

```
[Out] -1/2*b^2*(b*x^2)^(1/2)/x^3
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$-\frac{b^2\sqrt{bx^2}}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2)^(5/2)/x^8,x]
```

```
[Out] -(b^2*Sqrt[b*x^2])/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2\right)^{5/2}}{x^8}\,dx &= \frac{\left(b^2\sqrt{bx^2}\right)\int \frac{1}{x^3}\,dx}{x}\\ &= -\frac{b^2\sqrt{bx^2}}{2x^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 0.84

$$-\frac{\left(bx^2\right)^{5/2}}{2x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2)^(5/2)/x^8,x]
```

```
[Out] -1/2*(b*x^2)^(5/2)/x^7
```

fricas [A] time = 0.61, size = 15, normalized size = 0.79

$$-\frac{\sqrt{bx^2}\,b^2}{2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(5/2)/x^8,x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(b*x^2)*b^2/x^3
```

giac [A] time = 0.16, size = 10, normalized size = 0.53

$$-\frac{b^{\frac{5}{2}}\operatorname{sgn}(x)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(5/2)/x^8,x, algorithm="giac")
```

```
[Out] -1/2*b^(5/2)*sgn(x)/x^2
```

maple [A] time = 0.00, size = 13, normalized size = 0.68

$$-\frac{\left(b\,x^2\right)^{\frac{5}{2}}}{2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(5/2)/x^8,x)
```

```
[Out] -1/2*(b*x^2)^(5/2)/x^7
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(5/2)/x^8,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 0.93, size = 13, normalized size = 0.68

$$-\frac{b^{5/2}}{2\,x\,\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(5/2)/x^8,x)
```

```
[Out] -b^(5/2)/(2*x*(x^2)^(1/2))
```

sympy [A] time = 2.24, size = 17, normalized size = 0.89

$$-\frac{b^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2)**(5/2)/x**8,x)
```

```
[Out] -b**(5/2)*(x**2)**(5/2)/(2*x**7)
```

3.33 $\int \frac{\left(bx^2\right)^{5/2}}{x^9}\,dx$

Optimal. Leaf size=19

$$-\frac{b^2\sqrt{bx^2}}{3x^4}$$

```
[Out] -1/3*b^2*(b*x^2)^(1/2)/x^4
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$-\frac{b^2\sqrt{bx^2}}{3x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2)^(5/2)/x^9,x]
```

```
[Out] -(b^2*Sqrt[b*x^2])/(3*x^4)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2\right)^{5/2}}{x^9}\,dx &= \frac{\left(b^2\sqrt{bx^2}\right)\int \frac{1}{x^4}\,dx}{x}\\ &= -\frac{b^2\sqrt{bx^2}}{3x^4}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 0.84

$$-\frac{\left(bx^2\right)^{5/2}}{3x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2)^(5/2)/x^9,x]
```

```
[Out] -1/3*(b*x^2)^(5/2)/x^8
```

fricas [A] time = 0.54, size = 15, normalized size = 0.79

$$-\frac{\sqrt{bx^2}\,b^2}{3\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(5/2)/x^9,x, algorithm="fricas")
[Out] -1/3*sqrt(b*x^2)*b^2/x^4
```

giac [**A**] time = 0.15, size = 10, normalized size = 0.53

$$-\frac{b^{\frac{5}{2}}\operatorname{sgn}(x)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(5/2)/x^9,x, algorithm="giac")
[Out] -1/3*b^(5/2)*sgn(x)/x^3
```

maple [**A**] time = 0.00, size = 13, normalized size = 0.68

$$-\frac{\left(b\,x^2\right)^{\frac{5}{2}}}{3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(5/2)/x^9,x)
[Out] -1/3*(b*x^2)^(5/2)/x^8
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(5/2)/x^9,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [**B**] time = 0.91, size = 10, normalized size = 0.53

$$-\frac{b^{5/2}}{3\left(x^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(5/2)/x^9,x)
[Out] -b^(5/2)/(3*(x^2)^(3/2))
```

sympy [**A**] time = 2.67, size = 17, normalized size = 0.89

$$-\frac{b^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2)**(5/2)/x**9,x)
[Out] -b**(5/2)*(x**2)**(5/2)/(3*x**8)
```

3.34 $\int \left(x^2\right)^{7/2}\, dx$

Optimal. Leaf size=14

$$\frac{1}{8}x^7\sqrt{x^2}$$

```
[Out] 1/8*x^7*(x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {15, 30}

$$\frac{1}{8}x^7\sqrt{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2)^(7/2),x]
```

```
[Out] (x^7*Sqrt[x^2])/8
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \left(x^2\right)^{7/2}\, dx &= \frac{\sqrt{x^2}\int x^7\, dx}{x}\\ &= \frac{1}{8}x^7\sqrt{x^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 0.86

$$\frac{1}{8}x\left(x^2\right)^{7/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2)^(7/2),x]
```

```
[Out] (x*(x^2)^(7/2))/8
```

fricas [A] time = 0.49, size = 5, normalized size = 0.36

$$\frac{1}{8}x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/8*x^8
```

giac [**A**] time = 0.41, size = 7, normalized size = 0.50

$$\frac{1}{8}x^8\text{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2)^(7/2),x, algorithm="giac")
[Out] 1/8*x^8*sgn(x)
```

maple [**A**] time = 0.00, size = 9, normalized size = 0.64

$$\frac{\left(x^2\right)^{\frac{7}{2}}x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2)^(7/2),x)
[Out] 1/8*x*(x^2)^(7/2)
```

maxima [**A**] time = 1.30, size = 8, normalized size = 0.57

$$\frac{1}{8}\left(x^2\right)^{\frac{7}{2}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2)^(7/2),x, algorithm="maxima")
[Out] 1/8*(x^2)^(7/2)*x
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.07

$$\int \left(x^2\right)^{7/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2)^(7/2),x)
[Out] int((x^2)^(7/2), x)
```

sympy [**A**] time = 0.07, size = 3, normalized size = 0.21

$$\frac{x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2)**(7/2),x)
[Out] x**8/8
```

3.35 $\int \frac{x^3}{\sqrt{bx^2}}\,dx$

Optimal. Leaf size=16

$$\frac{x^4}{3\sqrt{bx^2}}$$

```
[Out] 1/3*x^4/(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$\frac{x^4}{3\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[b*x^2],x]
```

```
[Out] x^4/(3*Sqrt[b*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{bx^2}}\,dx &= \frac{x\int x^2\,dx}{\sqrt{bx^2}}\\ &= \frac{x^4}{3\sqrt{bx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{x^4}{3\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[b*x^2],x]
```

```
[Out] x^4/(3*Sqrt[b*x^2])
```

fricas [A] time = 0.51, size = 15, normalized size = 0.94

$$\frac{\sqrt{bx^2}\,x^2}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(b*x^2)*x^2/b
```

giac [A] time = 0.16, size = 15, normalized size = 0.94

$$\frac{\sqrt{bx^2}\,x^2}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2)^(1/2),x, algorithm="giac")
[Out] 1/3*sqrt(b*x^2)*x^2/b
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{x^4}{3\sqrt{b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2)^(1/2),x)
[Out] 1/3*x^4/(b*x^2)^(1/2)
```

maxima [A] time = 1.35, size = 15, normalized size = 0.94

$$\frac{\sqrt{bx^2}\,x^2}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/3*sqrt(b*x^2)*x^2/b
```

mupad [B] time = 0.97, size = 10, normalized size = 0.62

$$\frac{\sqrt{x^6}}{3\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2)^(1/2),x)
[Out] (x^6)^(1/2)/(3*b^(1/2))
```

sympy [A] time = 0.49, size = 15, normalized size = 0.94

$$\frac{x^4}{3\sqrt{b}\,\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2)**(1/2),x)
[Out] x**4/(3*sqrt(b)*sqrt(x**2))
```

3.36 $\int \frac{x}{\sqrt{bx^2}}\,dx$

Optimal. Leaf size=13

$$\frac{x^2}{\sqrt{bx^2}}$$

```
[Out] x^2/(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 8}

$$\frac{x^2}{\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[b*x^2],x]
```

```
[Out] x^2/Sqrt[b*x^2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{bx^2}}\,dx &= \frac{x\int 1\,dx}{\sqrt{bx^2}}\\ &= \frac{x^2}{\sqrt{bx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{x^2}{\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[b*x^2],x]
```

```
[Out] x^2/Sqrt[b*x^2]
```

fricas [A] time = 0.51, size = 11, normalized size = 0.85

$$\frac{\sqrt{bx^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*x^2)/b
```

giac [A] time = 0.15, size = 11, normalized size = 0.85

$$\frac{\sqrt{bx^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2)^(1/2),x, algorithm="giac")
[Out] sqrt(b*x^2)/b
```

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$\frac{x^2}{\sqrt{b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2)^(1/2),x)
[Out] x^2/(b*x^2)^(1/2)
```

maxima [A] time = 1.34, size = 11, normalized size = 0.85

$$\frac{\sqrt{bx^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2)^(1/2),x, algorithm="maxima")
[Out] sqrt(b*x^2)/b
```

mupad [B] time = 0.96, size = 6, normalized size = 0.46

$$\frac{|x|}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2)^(1/2),x)
[Out] abs(x)/b^(1/2)
```

sympy [A] time = 0.40, size = 14, normalized size = 1.08

$$\frac{x^2}{\sqrt{b}\,\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2)**(1/2),x)
[Out] x**2/(sqrt(b)*sqrt(x**2))
```

3.37 $\int \frac{1}{x\sqrt{bx^2}}\,dx$

Optimal. Leaf size=11

$$-\frac{1}{\sqrt{bx^2}}$$

```
[Out] -1/(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$-\frac{1}{\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[b*x^2]),x]
```

```
[Out] -(1/Sqrt[b*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{bx^2}}\,dx &= \frac{x\int \frac{1}{x^2}\,dx}{\sqrt{bx^2}}\\ &= -\frac{1}{\sqrt{bx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.36

$$-\frac{bx^2}{\left(bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[b*x^2]),x]
```

```
[Out] -((b*x^2)/(b*x^2)^(3/2))
```

fricas [A] time = 0.62, size = 15, normalized size = 1.36

$$-\frac{\sqrt{bx^2}}{bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2)^(1/2),x, algorithm="fricas")
[Out] -sqrt(b*x^2)/(b*x^2)
```

giac [A] time = 0.15, size = 9, normalized size = 0.82

$$-\frac{1}{\sqrt{bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2)^(1/2),x, algorithm="giac")
[Out] -1/sqrt(b*x^2)
```

maple [A] time = 0.00, size = 10, normalized size = 0.91

$$-\frac{1}{\sqrt{b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2)^(1/2),x)
[Out] -1/(b*x^2)^(1/2)
```

maxima [A] time = 1.37, size = 8, normalized size = 0.73

$$-\frac{1}{\sqrt{b}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2)^(1/2),x, algorithm="maxima")
[Out] -1/(sqrt(b)*x)
```

mupad [B] time = 0.96, size = 10, normalized size = 0.91

$$-\frac{1}{\sqrt{b}\,\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^2)^(1/2)),x)
[Out] -1/(b^(1/2)*(x^2)^(1/2))
```

sympy [A] time = 0.46, size = 14, normalized size = 1.27

$$-\frac{1}{\sqrt{b}\,\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2)**(1/2),x)
[Out] -1/(sqrt(b)*sqrt(x**2))
```

3.38 $\int \frac{1}{x^3\sqrt{bx^2}}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{3x^2\sqrt{bx^2}}$$

```
[Out] -1/3/x^2/(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$-\frac{1}{3x^2\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[b*x^2]),x]
```

```
[Out] -1/(3*x^2*Sqrt[b*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{bx^2}}\,dx &= \frac{x\int \frac{1}{x^4}\,dx}{\sqrt{bx^2}}\\ &= -\frac{1}{3x^2\sqrt{bx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{3x^2\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[b*x^2]),x]
```

```
[Out] -1/3*1/(x^2*Sqrt[b*x^2])
```

fricas [A] time = 0.59, size = 15, normalized size = 0.94

$$-\frac{\sqrt{bx^2}}{3\,bx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2)^(1/2),x, algorithm="fricas")
[Out] -1/3*sqrt(b*x^2)/(b*x^4)
```

giac [A] time = 0.17, size = 12, normalized size = 0.75

$$-\frac{1}{3\sqrt{bx^2}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2)^(1/2),x, algorithm="giac")
[Out] -1/3/(sqrt(b*x^2)*x^2)
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$-\frac{1}{3\sqrt{b\,x^2}\;x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2)^(1/2),x)
[Out] -1/3/x^2/(b*x^2)^(1/2)
```

maxima [A] time = 1.33, size = 8, normalized size = 0.50

$$-\frac{1}{3\sqrt{b}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2)^(1/2),x, algorithm="maxima")
[Out] -1/3/(sqrt(b)*x^3)
```

mupad [B] time = 0.97, size = 10, normalized size = 0.62

$$-\frac{1}{3\sqrt{b}\,\left(x^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b*x^2)^(1/2)),x)
[Out] -1/(3*b^(1/2)*(x^2)^(3/2))
```

sympy [A] time = 0.62, size = 19, normalized size = 1.19

$$-\frac{1}{3\sqrt{b}\,x^2\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2)**(1/2),x)
[Out] -1/(3*sqrt(b)*x**2*sqrt(x**2))
```

3.39 $\int \frac{x^2}{\sqrt{bx^2}}\,dx$

Optimal. Leaf size=16

$$\frac{x^3}{2\sqrt{bx^2}}$$

```
[Out] 1/2*x^3/(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$\frac{x^3}{2\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[b*x^2],x]
```

```
[Out] x^3/(2*Sqrt[b*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{bx^2}}\,dx &= \frac{x\int x\,dx}{\sqrt{bx^2}}\\ &= \frac{x^3}{2\sqrt{bx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{x^3}{2\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[b*x^2],x]
```

```
[Out] x^3/(2*Sqrt[b*x^2])
```

fricas [A] time = 0.68, size = 13, normalized size = 0.81

$$\frac{\sqrt{bx^2}\,x}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(b*x^2)*x/b
```

giac [A] time = 0.22, size = 13, normalized size = 0.81

$$\frac{\sqrt{bx^2}\,x}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(b*x^2)*x/b
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{x^3}{2\sqrt{b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2)^(1/2)*x^2,x)
[Out] 1/2*x^3/(b*x^2)^(1/2)
```

maxima [A] time = 1.34, size = 8, normalized size = 0.50

$$\frac{x^2}{2\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*x^2/sqrt(b)
```

mupad [B] time = 0.99, size = 11, normalized size = 0.69

$$\frac{x\,\sqrt{x^2}}{2\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2)^(1/2),x)
[Out] (x*(x^2)^(1/2))/(2*b^(1/2))
```

sympy [A] time = 0.45, size = 15, normalized size = 0.94

$$\frac{x^3}{2\sqrt{b}\,\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2)**(1/2),x)
[Out] x**3/(2*sqrt(b)*sqrt(x**2))
```

3.40 $\int \frac{1}{\sqrt{bx^2}}\,dx$

Optimal. Leaf size=13

$$\frac{x\log(x)}{\sqrt{bx^2}}$$

```
[Out] x*ln(x)/(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 29}

$$\frac{x\log(x)}{\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*x^2],x]
```

```
[Out] (x*Log[x])/Sqrt[b*x^2]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{bx^2}}\,dx &= \frac{x\int \frac{1}{x}\,dx}{\sqrt{bx^2}}\\ &= \frac{x\log(x)}{\sqrt{bx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{x\log(x)}{\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*x^2],x]
```

```
[Out] (x*Log[x])/Sqrt[b*x^2]
```

fricas [A] time = 0.58, size = 16, normalized size = 1.23

$$\frac{\sqrt{bx^2}\,\log(x)}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*x^2)*log(x)/(b*x)
```

giac [**A**] time = 0.20, size = 19, normalized size = 1.46

$$\frac{\log\left(\sqrt{|b|}\,|x|\left|\operatorname{sgn}(x)\right|\right)}{\sqrt{b}\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2)^(1/2),x, algorithm="giac")
[Out] log(sqrt(abs(b))*abs(x)*abs(sgn(x)))/(sqrt(b)*sgn(x))
```

maple [**A**] time = 0.00, size = 12, normalized size = 0.92

$$\frac{x\ln(x)}{\sqrt{b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2)^(1/2),x)
[Out] x*ln(x)/(b*x^2)^(1/2)
```

maxima [**A**] time = 1.32, size = 6, normalized size = 0.46

$$\frac{\log(x)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2)^(1/2),x, algorithm="maxima")
[Out] log(x)/sqrt(b)
```

mupad [**B**] time = 1.09, size = 10, normalized size = 0.77

$$\frac{\ln(b\,x)\ \operatorname{sign}(x)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2)^(1/2),x)
[Out] (log(b*x)*sign(x))/b^(1/2)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2)**(1/2),x)
[Out] Integral(1/sqrt(b*x**2), x)
```

3.41 $\int \frac{1}{x^2\sqrt{bx^2}}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{2x\sqrt{bx^2}}$$

```
[Out] -1/2/x/(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$-\frac{1}{2x\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[b*x^2]),x]
```

```
[Out] -1/(2*x*Sqrt[b*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{bx^2}}\,dx &= \frac{x\int \frac{1}{x^3}\,dx}{\sqrt{bx^2}}\\ &= -\frac{1}{2x\sqrt{bx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 0.94

$$-\frac{bx}{2\left(bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[b*x^2]),x]
```

```
[Out] -1/2*(b*x)/(b*x^2)^(3/2)
```

fricas [A] time = 0.54, size = 15, normalized size = 0.94

$$-\frac{\sqrt{bx^2}}{2\,bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(b*x^2)/(b*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$-\frac{1}{2\sqrt{b\,x^2}\;x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2)^(1/2)/x^2,x)
[Out] -1/2/x/(b*x^2)^(1/2)
```

maxima [A] time = 1.33, size = 8, normalized size = 0.50

$$-\frac{1}{2\,\sqrt{b}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2)^(1/2),x, algorithm="maxima")
[Out] -1/2/(sqrt(b)*x^2)
```

mupad [B] time = 0.97, size = 13, normalized size = 0.81

$$-\frac{1}{2\,\sqrt{b}\;x\,\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x^2)^(1/2)),x)
[Out] -1/(2*b^(1/2)*x*(x^2)^(1/2))
```

sympy [A] time = 0.50, size = 17, normalized size = 1.06

$$-\frac{1}{2\sqrt{b}\,x\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2)**(1/2),x)
[Out] -1/(2*sqrt(b)*x*sqrt(x**2))
```

3.42 $\int \frac{x^5}{\left(bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=19

$$\frac{x^4}{3b\sqrt{bx^2}}$$

```
[Out] 1/3*x^4/b/(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$\frac{x^4}{3b\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(b*x^2)^(3/2),x]
```

```
[Out] x^4/(3*b*Sqrt[b*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\left(bx^2\right)^{3/2}}\,dx &= \frac{x\int x^2\,dx}{b\sqrt{bx^2}}\\ &= \frac{x^4}{3b\sqrt{bx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 0.84

$$\frac{x^6}{3\left(bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(b*x^2)^(3/2),x]
```

```
[Out] x^6/(3*(b*x^2)^(3/2))
```

fricas [A] time = 0.70, size = 15, normalized size = 0.79

$$\frac{\sqrt{bx^2}\,x^2}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2)^(3/2),x, algorithm="fricas")
[Out] 1/3*sqrt(b*x^2)*x^2/b^2
```

giac [A] time = 0.15, size = 15, normalized size = 0.79

$$\frac{\sqrt{bx^2}\,x^2}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2)^(3/2),x, algorithm="giac")
[Out] 1/3*sqrt(b*x^2)*x^2/b^2
```

maple [A] time = 0.00, size = 13, normalized size = 0.68

$$\frac{x^6}{3\left(b\,x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2)^(3/2),x)
[Out] 1/3*x^6/(b*x^2)^(3/2)
```

maxima [A] time = 1.30, size = 15, normalized size = 0.79

$$\frac{x^4}{3\,\sqrt{bx^2}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2)^(3/2),x, algorithm="maxima")
[Out] 1/3*x^4/(sqrt(b*x^2)*b)
```

mupad [B] time = 1.02, size = 10, normalized size = 0.53

$$\frac{\sqrt{x^6}}{3\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2)^(3/2),x)
[Out] (x^6)^(1/2)/(3*b^(3/2))
```

sympy [A] time = 0.71, size = 15, normalized size = 0.79

$$\frac{x^6}{3b^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2)**(3/2),x)
[Out] x**6/(3*b**(3/2)*(x**2)**(3/2))
```

3.43 $\int \frac{x^3}{\left(bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=16

$$\frac{x^2}{b\sqrt{bx^2}}$$

```
[Out] x^2/b/(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 8}

$$\frac{x^2}{b\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(b*x^2)^(3/2),x]
```

```
[Out] x^2/(b*Sqrt[b*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(bx^2\right)^{3/2}}\,dx &= \frac{x\int 1\,dx}{b\sqrt{bx^2}}\\ &= \frac{x^2}{b\sqrt{bx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{x^4}{\left(bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(b*x^2)^(3/2),x]
```

```
[Out] x^4/(b*x^2)^(3/2)
```

fricas [A] time = 0.75, size = 11, normalized size = 0.69

$$\frac{\sqrt{bx^2}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2)^(3/2),x, algorithm="fricas")
[Out] sqrt(b*x^2)/b^2
```

giac [A] time = 0.15, size = 11, normalized size = 0.69

$$\frac{\sqrt{bx^2}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2)^(3/2),x, algorithm="giac")
[Out] sqrt(b*x^2)/b^2
```

maple [A] time = 0.00, size = 12, normalized size = 0.75

$$\frac{x^4}{\left(b\,x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2)^(3/2),x)
[Out] x^4/(b*x^2)^(3/2)
```

maxima [A] time = 1.30, size = 14, normalized size = 0.88

$$\frac{x^2}{\sqrt{bx^2}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2)^(3/2),x, algorithm="maxima")
[Out] x^2/(sqrt(b*x^2)*b)
```

mupad [B] time = 0.99, size = 6, normalized size = 0.38

$$\frac{|x|}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2)^(3/2),x)
[Out] abs(x)/b^(3/2)
```

sympy [A] time = 0.51, size = 14, normalized size = 0.88

$$\frac{x^4}{b^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2)**(3/2),x)
[Out] x**4/(b**(3/2)*(x**2)**(3/2))
```

3.44 $\int \frac{x}{\left(bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{b\sqrt{bx^2}}$$

```
[Out] -1/b/(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 30}

$$-\frac{1}{b\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(b*x^2)^(3/2),x]
```

```
[Out] -(1/(b*Sqrt[b*x^2]))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(bx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{1}{x^2}\,dx}{b\sqrt{bx^2}}\\ &= -\frac{1}{b\sqrt{bx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{x^2}{\left(bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(b*x^2)^(3/2),x]
```

```
[Out] -(x^2/(b*x^2)^(3/2))
```

fricas [A] time = 0.72, size = 15, normalized size = 1.07

$$-\frac{\sqrt{bx^2}}{b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2)^(3/2),x, algorithm="fricas")
[Out] -sqrt(b*x^2)/(b^2*x^2)
```

giac [A] time = 0.15, size = 12, normalized size = 0.86

$$-\frac{1}{\sqrt{bx^2}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2)^(3/2),x, algorithm="giac")
[Out] -1/(sqrt(b*x^2)*b)
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$-\frac{x^2}{\left(b\,x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2)^(3/2),x)
[Out] -x^2/(b*x^2)^(3/2)
```

maxima [A] time = 1.33, size = 12, normalized size = 0.86

$$-\frac{1}{\sqrt{bx^2}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2)^(3/2),x, algorithm="maxima")
[Out] -1/(sqrt(b*x^2)*b)
```

mupad [B] time = 0.93, size = 10, normalized size = 0.71

$$-\frac{1}{b^{3/2}\,\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2)^(3/2),x)
[Out] -1/(b^(3/2)*(x^2)^(1/2))
```

sympy [A] time = 0.52, size = 15, normalized size = 1.07

$$-\frac{x^2}{b^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2)**(3/2),x)
[Out] -x**2/(b**(3/2)*(x**2)**(3/2))
```

3.45 $\int \frac{1}{x\left(bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=19

$$-\frac{1}{3bx^2\sqrt{bx^2}}$$

```
[Out] -1/3/b/x^2/(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$-\frac{1}{3bx^2\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(b*x^2)^(3/2)),x]
```

```
[Out] -1/(3*b*x^2*Sqrt[b*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(bx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{1}{x^4}\,dx}{b\sqrt{bx^2}}\\ &= -\frac{1}{3bx^2\sqrt{bx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 0.89

$$-\frac{bx^2}{3\left(bx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(b*x^2)^(3/2)),x]
```

```
[Out] -1/3*(b*x^2)/(b*x^2)^(5/2)
```

fricas [A] time = 0.54, size = 15, normalized size = 0.79

$$-\frac{\sqrt{bx^2}}{3\,b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2)^(3/2),x, algorithm="fricas")
[Out] -1/3*sqrt(b*x^2)/(b^2*x^4)
```

giac [A] time = 0.16, size = 15, normalized size = 0.79

$$-\frac{1}{3\sqrt{bx^2}\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2)^(3/2),x, algorithm="giac")
[Out] -1/3/(sqrt(b*x^2)*b*x^2)
```

maple [A] time = 0.00, size = 10, normalized size = 0.53

$$-\frac{1}{3\left(b\,x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2)^(3/2),x)
[Out] -1/3/(b*x^2)^(3/2)
```

maxima [A] time = 1.27, size = 8, normalized size = 0.42

$$-\frac{1}{3\,b^{\frac{3}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2)^(3/2),x, algorithm="maxima")
[Out] -1/3/(b^(3/2)*x^3)
```

mupad [B] time = 0.96, size = 10, normalized size = 0.53

$$-\frac{1}{3\,b^{3/2}\left(x^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^2)^(3/2)),x)
[Out] -1/(3*b^(3/2)*(x^2)^(3/2))
```

sympy [A] time = 0.59, size = 15, normalized size = 0.79

$$-\frac{1}{3b^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2)**(3/2),x)
[Out] -1/(3*b**(3/2)*(x**2)**(3/2))
```

3.46 $\int \frac{1}{x^3\left(bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=19

$$-\frac{1}{5bx^4\sqrt{bx^2}}$$

```
[Out] -1/5/b/x^4/(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$-\frac{1}{5bx^4\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(b*x^2)^(3/2)),x]
```

```
[Out] -1/(5*b*x^4*Sqrt[b*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(bx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{1}{x^6}\,dx}{b\sqrt{bx^2}}\\ &= -\frac{1}{5bx^4\sqrt{bx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 0.84

$$-\frac{1}{5x^2\left(bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(b*x^2)^(3/2)),x]
```

```
[Out] -1/5*1/(x^2*(b*x^2)^(3/2))
```

fricas [A] time = 0.58, size = 15, normalized size = 0.79

$$-\frac{\sqrt{bx^2}}{5\,b^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2)^(3/2),x, algorithm="fricas")
[Out] -1/5*sqrt(b*x^2)/(b^2*x^6)
```

giac [A] time = 0.15, size = 15, normalized size = 0.79

$$-\frac{1}{5\sqrt{bx^2}\,bx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2)^(3/2),x, algorithm="giac")
[Out] -1/5/(sqrt(b*x^2)*b*x^4)
```

maple [A] time = 0.00, size = 13, normalized size = 0.68

$$-\frac{1}{5\left(b\,x^2\right)^{\frac{3}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2)^(3/2),x)
[Out] -1/5/x^2/(b*x^2)^(3/2)
```

maxima [A] time = 1.35, size = 8, normalized size = 0.42

$$-\frac{1}{5\,b^{\frac{3}{2}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2)^(3/2),x, algorithm="maxima")
[Out] -1/5/(b^(3/2)*x^5)
```

mupad [B] time = 0.95, size = 10, normalized size = 0.53

$$-\frac{1}{5\,b^{3/2}\left(x^2\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b*x^2)^(3/2)),x)
[Out] -1/(5*b^(3/2)*(x^2)^(5/2))
```

sympy [A] time = 0.86, size = 19, normalized size = 1.00

$$-\frac{1}{5b^{\frac{3}{2}}x^2\left(x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2)**(3/2),x)
[Out] -1/(5*b**(3/2)*x**2*(x**2)**(3/2))
```

3.47 $\int \frac{x^6}{\left(bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=19

$$\frac{x^5}{4b\sqrt{bx^2}}$$

```
[Out] 1/4*x^5/b/(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$\frac{x^5}{4b\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(b*x^2)^(3/2),x]
```

```
[Out] x^5/(4*b*Sqrt[b*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{\left(bx^2\right)^{3/2}}\,dx &= \frac{x\int x^3\,dx}{b\sqrt{bx^2}}\\ &= \frac{x^5}{4b\sqrt{bx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 0.84

$$\frac{x^7}{4\left(bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(b*x^2)^(3/2),x]
```

```
[Out] x^7/(4*(b*x^2)^(3/2))
```

fricas [A] time = 0.63, size = 15, normalized size = 0.79

$$\frac{\sqrt{bx^2}\,x^3}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2)^(3/2),x, algorithm="fricas")
[Out] 1/4*sqrt(b*x^2)*x^3/b^2
```

giac [**A**] time = 0.17, size = 15, normalized size = 0.79

$$\frac{\sqrt{bx^2}\,x^3}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2)^(3/2),x, algorithm="giac")
[Out] 1/4*sqrt(b*x^2)*x^3/b^2
```

maple [**A**] time = 0.00, size = 13, normalized size = 0.68

$$\frac{x^7}{4\left(b\,x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2)^(3/2)*x^6,x)
[Out] 1/4*x^7/(b*x^2)^(3/2)
```

maxima [**A**] time = 1.34, size = 15, normalized size = 0.79

$$\frac{x^5}{4\,\sqrt{bx^2}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2)^(3/2),x, algorithm="maxima")
[Out] 1/4*x^5/(sqrt(b*x^2)*b)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^6}{\left(b\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2)^(3/2),x)
[Out] int(x^6/(b*x^2)^(3/2), x)
```

sympy [**A**] time = 0.85, size = 15, normalized size = 0.79

$$\frac{x^7}{4b^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2)**(3/2),x)
[Out] x**7/(4*b**(3/2)*(x**2)**(3/2))
```

3.48 $\int \frac{x^4}{\left(bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=19

$$\frac{x^3}{2b\sqrt{bx^2}}$$

```
[Out] 1/2*x^3/b/(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$\frac{x^3}{2b\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(b*x^2)^(3/2),x]
```

```
[Out] x^3/(2*b*Sqrt[b*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\left(bx^2\right)^{3/2}}\,dx &= \frac{x\int x\,dx}{b\sqrt{bx^2}}\\ &= \frac{x^3}{2b\sqrt{bx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 0.84

$$\frac{x^5}{2\left(bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(b*x^2)^(3/2),x]
```

```
[Out] x^5/(2*(b*x^2)^(3/2))
```

fricas [A] time = 0.63, size = 13, normalized size = 0.68

$$\frac{\sqrt{bx^2}\,x}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2)^(3/2),x, algorithm="fricas")
[Out] 1/2*sqrt(b*x^2)*x/b^2
```

giac [**A**] time = 0.21, size = 13, normalized size = 0.68

$$\frac{\sqrt{bx^2}\,x}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2)^(3/2),x, algorithm="giac")
[Out] 1/2*sqrt(b*x^2)*x/b^2
```

maple [**A**] time = 0.00, size = 13, normalized size = 0.68

$$\frac{x^5}{2\left(b\,x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2)^(3/2)*x^4,x)
[Out] 1/2*x^5/(b*x^2)^(3/2)
```

maxima [**A**] time = 1.30, size = 15, normalized size = 0.79

$$\frac{x^3}{2\sqrt{bx^2}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2)^(3/2),x, algorithm="maxima")
[Out] 1/2*x^3/(sqrt(b*x^2)*b)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^4}{\left(b\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2)^(3/2),x)
[Out] int(x^4/(b*x^2)^(3/2), x)
```

sympy [**A**] time = 0.58, size = 15, normalized size = 0.79

$$\frac{x^5}{2b^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2)**(3/2),x)
[Out] x**5/(2*b**(3/2)*(x**2)**(3/2))
```

3.49 $\int \frac{x^2}{\left(bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=16

$$\frac{x\log(x)}{b\sqrt{bx^2}}$$

```
[Out] x*ln(x)/b/(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 29}

$$\frac{x\log(x)}{b\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(b*x^2)^(3/2),x]
```

```
[Out] (x*Log[x])/(b*Sqrt[b*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(bx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{1}{x}\,dx}{b\sqrt{bx^2}}\\ &= \frac{x\log(x)}{b\sqrt{bx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 0.94

$$\frac{x^3\log(x)}{\left(bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(b*x^2)^(3/2),x]
```

```
[Out] (x^3*Log[x])/(b*x^2)^(3/2)
```

fricas [A] time = 0.77, size = 16, normalized size = 1.00

$$\frac{\sqrt{bx^2}\,\log(x)}{b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*x^2)*log(x)/(b^2*x)
```

giac [A] time = 0.17, size = 21, normalized size = 1.31

$$-\frac{\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2}\right|\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -log(abs(-sqrt(b)*x + sqrt(b*x^2)))/b^(3/2)
```

maple [A] time = 0.00, size = 14, normalized size = 0.88

$$\frac{x^3\ln(x)}{\left(b\,x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2)^(3/2)*x^2,x)
```

```
[Out] 1/(b*x^2)^(3/2)*x^3*ln(x)
```

maxima [A] time = 1.32, size = 6, normalized size = 0.38

$$\frac{\log(x)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] log(x)/b^(3/2)
```

mupad [B] time = 0.07, size = 37, normalized size = 2.31

$$-\frac{x-\ln\left(2\sqrt{b}\,\sqrt{x^2}+2\sqrt{b}\,x\right)\sqrt{x^2}}{b^{3/2}\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2)^(3/2),x)
```

```
[Out] -(x - log(2*b^(1/2)*(x^2)^(1/2) + 2*b^(1/2)*x)*(x^2)^(1/2))/(b^(3/2)*(x^2)^
(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2)**(3/2),x)
```

```
[Out] Integral(x**2/(b*x**2)**(3/2), x)
```

3.50 $\int \frac{1}{\left(bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=19

$$-\frac{1}{2bx\sqrt{bx^2}}$$

```
[Out] -1/2/b/x/(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$-\frac{1}{2bx\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2)^(-3/2),x]
```

```
[Out] -1/(2*b*x*Sqrt[b*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(bx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{1}{x^3}\,dx}{b\sqrt{bx^2}}\\ &= -\frac{1}{2bx\sqrt{bx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 0.74

$$-\frac{x}{2\left(bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2)^(-3/2),x]
```

```
[Out] -1/2*x/(b*x^2)^(3/2)
```

fricas [A] time = 0.53, size = 15, normalized size = 0.79

$$-\frac{\sqrt{bx^2}}{2\,b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2)^(3/2),x, algorithm="fricas")
[Out] -1/2*sqrt(b*x^2)/(b^2*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.00, size = 11, normalized size = 0.58

$$-\frac{x}{2\left(b\,x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2)^(3/2),x)
[Out] -1/2*x/(b*x^2)^(3/2)
```

maxima [A] time = 1.39, size = 8, normalized size = 0.42

$$-\frac{1}{2\,b^{\frac{3}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2)^(3/2),x, algorithm="maxima")
[Out] -1/2/(b^(3/2)*x^2)
```

mupad [B] time = 0.91, size = 13, normalized size = 0.68

$$-\frac{1}{2\,b^{3/2}\,x\,\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2)^(3/2),x)
[Out] -1/(2*b^(3/2)*x*(x^2)^(1/2))
```

sympy [A] time = 0.50, size = 15, normalized size = 0.79

$$-\frac{x}{2b^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2)**(3/2),x)
[Out] -x/(2*b**(3/2)*(x**2)**(3/2))
```

3.51 $\int \frac{1}{x^2\left(bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=19

$$-\frac{1}{4bx^3\sqrt{bx^2}}$$

```
[Out] -1/4/b/x^3/(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$-\frac{1}{4bx^3\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(b*x^2)^(3/2)),x]
```

```
[Out] -1/(4*b*x^3*Sqrt[b*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(bx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{1}{x^5}\,dx}{b\sqrt{bx^2}}\\ &= -\frac{1}{4bx^3\sqrt{bx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 0.79

$$-\frac{bx}{4\left(bx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(b*x^2)^(3/2)),x]
```

```
[Out] -1/4*(b*x)/(b*x^2)^(5/2)
```

fricas [A] time = 0.58, size = 15, normalized size = 0.79

$$-\frac{\sqrt{bx^2}}{4\,b^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2)^(3/2),x, algorithm="fricas")
[Out] -1/4*sqrt(b*x^2)/(b^2*x^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.00, size = 13, normalized size = 0.68

$$-\frac{1}{4\left(b\,x^2\right)^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2)^(3/2)/x^2,x)
[Out] -1/4/x/(b*x^2)^(3/2)
```

maxima [A] time = 1.29, size = 8, normalized size = 0.42

$$-\frac{1}{4\,b^{\frac{3}{2}}x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2)^(3/2),x, algorithm="maxima")
[Out] -1/4/(b^(3/2)*x^4)
```

mupad [B] time = 0.93, size = 13, normalized size = 0.68

$$-\frac{1}{4\,b^{3/2}\,x\left(x^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x^2)^(3/2)),x)
[Out] -1/(4*b^(3/2)*x*(x^2)^(3/2))
```

sympy [A] time = 0.73, size = 17, normalized size = 0.89

$$-\frac{1}{4b^{\frac{3}{2}}x\left(x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2)**(3/2),x)
[Out] -1/(4*b**(3/2)*x*(x**2)**(3/2))
```

3.52 $\int \frac{x^7}{\left(bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=19

$$\frac{x^4}{3b^2\sqrt{bx^2}}$$

```
[Out] 1/3*x^4/b^2/(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$\frac{x^4}{3b^2\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(b*x^2)^(5/2),x]
```

```
[Out] x^4/(3*b^2*Sqrt[b*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{\left(bx^2\right)^{5/2}}\,dx &= \frac{x\int x^2\,dx}{b^2\sqrt{bx^2}}\\ &= \frac{x^4}{3b^2\sqrt{bx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 0.84

$$\frac{x^8}{3\left(bx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(b*x^2)^(5/2),x]
```

```
[Out] x^8/(3*(b*x^2)^(5/2))
```

fricas [A] time = 0.51, size = 15, normalized size = 0.79

$$\frac{\sqrt{bx^2}\,x^2}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2)^(5/2),x, algorithm="fricas")
[Out] 1/3*sqrt(b*x^2)*x^2/b^3
```

giac [A] time = 0.15, size = 15, normalized size = 0.79

$$\frac{\sqrt{bx^2}\,x^2}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2)^(5/2),x, algorithm="giac")
[Out] 1/3*sqrt(b*x^2)*x^2/b^3
```

maple [A] time = 0.00, size = 13, normalized size = 0.68

$$\frac{x^8}{3\left(b\,x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2)^(5/2),x)
[Out] 1/3*x^8/(b*x^2)^(5/2)
```

maxima [A] time = 1.36, size = 15, normalized size = 0.79

$$\frac{x^6}{3\left(bx^2\right)^{\frac{3}{2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2)^(5/2),x, algorithm="maxima")
[Out] 1/3*x^6/((b*x^2)^(3/2)*b)
```

mupad [B] time = 0.98, size = 10, normalized size = 0.53

$$\frac{\sqrt{x^6}}{3\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2)^(5/2),x)
[Out] (x^6)^(1/2)/(3*b^(5/2))
```

sympy [A] time = 1.20, size = 15, normalized size = 0.79

$$\frac{x^8}{3b^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**2)**(5/2),x)
[Out] x**8/(3*b**(5/2)*(x**2)**(5/2))
```

3.53 $\int \frac{x^5}{\left(bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=16

$$\frac{x^2}{b^2\sqrt{bx^2}}$$

```
[Out] x^2/b^2/(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 8}

$$\frac{x^2}{b^2\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(b*x^2)^(5/2),x]
```

```
[Out] x^2/(b^2*Sqrt[b*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\left(bx^2\right)^{5/2}}\,dx &= \frac{x\int 1\,dx}{b^2\sqrt{bx^2}}\\ &= \frac{x^2}{b^2\sqrt{bx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{x^6}{\left(bx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(b*x^2)^(5/2),x]
```

```
[Out] x^6/(b*x^2)^(5/2)
```

fricas [A] time = 0.60, size = 11, normalized size = 0.69

$$\frac{\sqrt{bx^2}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2)^(5/2),x, algorithm="fricas")
[Out] sqrt(b*x^2)/b^3
```

giac [A] time = 0.17, size = 11, normalized size = 0.69

$$\frac{\sqrt{bx^2}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2)^(5/2),x, algorithm="giac")
[Out] sqrt(b*x^2)/b^3
```

maple [A] time = 0.00, size = 12, normalized size = 0.75

$$\frac{x^6}{\left(b\,x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2)^(5/2),x)
[Out] x^6/(b*x^2)^(5/2)
```

maxima [A] time = 1.29, size = 14, normalized size = 0.88

$$\frac{x^4}{\left(bx^2\right)^{\frac{3}{2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2)^(5/2),x, algorithm="maxima")
[Out] x^4/((b*x^2)^(3/2)*b)
```

mupad [B] time = 0.96, size = 6, normalized size = 0.38

$$\frac{|x|}{b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2)^(5/2),x)
[Out] abs(x)/b^(5/2)
```

sympy [A] time = 0.83, size = 14, normalized size = 0.88

$$\frac{x^6}{b^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2)**(5/2),x)
[Out] x**6/(b**(5/2)*(x**2)**(5/2))
```

3.54 $\int \frac{x^3}{\left(bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{b^2\sqrt{bx^2}}$$

```
[Out] -1/b^2/(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$-\frac{1}{b^2\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(b*x^2)^(5/2),x]
```

```
[Out] -(1/(b^2*Sqrt[b*x^2]))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(bx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{1}{x^2}\,dx}{b^2\sqrt{bx^2}}\\ &= -\frac{1}{b^2\sqrt{bx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{x^4}{\left(bx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(b*x^2)^(5/2),x]
```

```
[Out] -(x^4/(b*x^2)^(5/2))
```

fricas [A] time = 0.43, size = 15, normalized size = 1.07

$$-\frac{\sqrt{bx^2}}{b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2)^(5/2),x, algorithm="fricas")
[Out] -sqrt(b*x^2)/(b^3*x^2)
```

giac [A] time = 0.18, size = 12, normalized size = 0.86

$$-\frac{1}{\sqrt{bx^2}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2)^(5/2),x, algorithm="giac")
[Out] -1/(sqrt(b*x^2)*b^2)
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$-\frac{x^4}{\left(b\,x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2)^(5/2),x)
[Out] -x^4/(b*x^2)^(5/2)
```

maxima [A] time = 1.33, size = 15, normalized size = 1.07

$$-\frac{x^2}{\left(bx^2\right)^{\frac{3}{2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2)^(5/2),x, algorithm="maxima")
[Out] -x^2/((b*x^2)^(3/2)*b)
```

mupad [B] time = 0.98, size = 10, normalized size = 0.71

$$-\frac{1}{b^{5/2}\,\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2)^(5/2),x)
[Out] -1/(b^(5/2)*(x^2)^(1/2))
```

sympy [A] time = 0.86, size = 15, normalized size = 1.07

$$-\frac{x^4}{b^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2)**(5/2),x)
[Out] -x**4/(b**(5/2)*(x**2)**(5/2))
```

3.55 $\int \frac{x}{\left(bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=19

$$-\frac{1}{3b^2x^2\sqrt{bx^2}}$$

```
[Out] -1/3/b^2/x^2/(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 30}

$$-\frac{1}{3b^2x^2\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(b*x^2)^(5/2),x]
```

```
[Out] -1/(3*b^2*x^2*Sqrt[b*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(bx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{1}{x^4}\,dx}{b^2\sqrt{bx^2}}\\ &= -\frac{1}{3b^2x^2\sqrt{bx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 0.84

$$-\frac{x^2}{3\left(bx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(b*x^2)^(5/2),x]
```

```
[Out] -1/3*x^2/(b*x^2)^(5/2)
```

fricas [A] time = 0.52, size = 15, normalized size = 0.79

$$-\frac{\sqrt{bx^2}}{3\,b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2)^(5/2),x, algorithm="fricas")
[Out] -1/3*sqrt(b*x^2)/(b^3*x^4)
```

giac [A] time = 0.16, size = 15, normalized size = 0.79

$$-\frac{1}{3\sqrt{bx^2}\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2)^(5/2),x, algorithm="giac")
[Out] -1/3/(sqrt(b*x^2)*b^2*x^2)
```

maple [A] time = 0.00, size = 13, normalized size = 0.68

$$-\frac{x^2}{3\left(b\,x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2)^(5/2),x)
[Out] -1/3*x^2/(b*x^2)^(5/2)
```

maxima [A] time = 1.35, size = 12, normalized size = 0.63

$$-\frac{1}{3\left(bx^2\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2)^(5/2),x, algorithm="maxima")
[Out] -1/3/((b*x^2)^(3/2)*b)
```

mupad [B] time = 0.95, size = 10, normalized size = 0.53

$$-\frac{1}{3\,b^{5/2}\left(x^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2)^(5/2),x)
[Out] -1/(3*b^(5/2)*(x^2)^(3/2))
```

sympy [A] time = 0.85, size = 17, normalized size = 0.89

$$-\frac{x^2}{3b^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2)**(5/2),x)
[Out] -x**2/(3*b**(5/2)*(x**2)**(5/2))
```

3.56 $\int \frac{1}{x\left(bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=19

$$-\frac{1}{5b^2x^4\sqrt{bx^2}}$$

```
[Out] -1/5/b^2/x^4/(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$-\frac{1}{5b^2x^4\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(b*x^2)^(5/2)),x]
```

```
[Out] -1/(5*b^2*x^4*Sqrt[b*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(bx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{1}{x^6}\,dx}{b^2\sqrt{bx^2}}\\ &= -\frac{1}{5b^2x^4\sqrt{bx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 0.89

$$-\frac{bx^2}{5\left(bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(b*x^2)^(5/2)),x]
```

```
[Out] -1/5*(b*x^2)/(b*x^2)^(7/2)
```

fricas [A] time = 0.65, size = 15, normalized size = 0.79

$$-\frac{\sqrt{bx^2}}{5\,b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2)^(5/2),x, algorithm="fricas")
[Out] -1/5*sqrt(b*x^2)/(b^3*x^6)
```

giac [A] time = 0.15, size = 15, normalized size = 0.79

$$-\frac{1}{5\sqrt{bx^2}\,b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2)^(5/2),x, algorithm="giac")
[Out] -1/5/(sqrt(b*x^2)*b^2*x^4)
```

maple [A] time = 0.00, size = 10, normalized size = 0.53

$$-\frac{1}{5\left(b\,x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2)^(5/2),x)
[Out] -1/5/(b*x^2)^(5/2)
```

maxima [A] time = 1.36, size = 8, normalized size = 0.42

$$-\frac{1}{5\,b^{\frac{5}{2}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2)^(5/2),x, algorithm="maxima")
[Out] -1/5/(b^(5/2)*x^5)
```

mupad [B] time = 0.92, size = 10, normalized size = 0.53

$$-\frac{1}{5\,b^{5/2}\left(x^2\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^2)^(5/2)),x)
[Out] -1/(5*b^(5/2)*(x^2)^(5/2))
```

sympy [A] time = 1.03, size = 15, normalized size = 0.79

$$-\frac{1}{5b^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2)**(5/2),x)
[Out] -1/(5*b**(5/2)*(x**2)**(5/2))
```

3.57 $\int \frac{x^6}{\left(bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=19

$$\frac{x^3}{2b^2\sqrt{bx^2}}$$

```
[Out] 1/2*x^3/b^2/(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$\frac{x^3}{2b^2\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(b*x^2)^(5/2),x]
```

```
[Out] x^3/(2*b^2*Sqrt[b*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{\left(bx^2\right)^{5/2}}\,dx &= \frac{x\int x\,dx}{b^2\sqrt{bx^2}}\\ &= \frac{x^3}{2b^2\sqrt{bx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 0.84

$$\frac{x^7}{2\left(bx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(b*x^2)^(5/2),x]
```

```
[Out] x^7/(2*(b*x^2)^(5/2))
```

fricas [A] time = 0.77, size = 13, normalized size = 0.68

$$\frac{\sqrt{bx^2}\,x}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2)^(5/2),x, algorithm="fricas")
[Out] 1/2*sqrt(b*x^2)*x/b^3
```

giac [**A**] time = 0.17, size = 13, normalized size = 0.68

$$\frac{\sqrt{bx^2}\,x}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2)^(5/2),x, algorithm="giac")
[Out] 1/2*sqrt(b*x^2)*x/b^3
```

maple [**A**] time = 0.00, size = 13, normalized size = 0.68

$$\frac{x^7}{2\left(b\,x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2)^(5/2)*x^6,x)
[Out] 1/2*x^7/(b*x^2)^(5/2)
```

maxima [**A**] time = 1.29, size = 15, normalized size = 0.79

$$\frac{x^5}{2\left(bx^2\right)^{\frac{3}{2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2)^(5/2),x, algorithm="maxima")
[Out] 1/2*x^5/((b*x^2)^(3/2)*b)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^6}{\left(b\,x^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2)^(5/2),x)
[Out] int(x^6/(b*x^2)^(5/2), x)
```

sympy [**A**] time = 1.02, size = 15, normalized size = 0.79

$$\frac{x^7}{2b^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**2)**(5/2),x)
[Out] x**7/(2*b**(5/2)*(x**2)**(5/2))
```

3.58 $\int \frac{x^4}{\left(bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=16

$$\frac{x\log(x)}{b^2\sqrt{bx^2}}$$

```
[Out] x*ln(x)/b^2/(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 29}

$$\frac{x\log(x)}{b^2\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(b*x^2)^(5/2),x]
```

```
[Out] (x*Log[x])/(b^2*Sqrt[b*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\left(bx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{1}{x}\,dx}{b^2\sqrt{bx^2}}\\ &= \frac{x\log(x)}{b^2\sqrt{bx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 0.94

$$\frac{x^5\log(x)}{\left(bx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(b*x^2)^(5/2),x]
```

```
[Out] (x^5*Log[x])/(b*x^2)^(5/2)
```

fricas [A] time = 0.65, size = 16, normalized size = 1.00

$$\frac{\sqrt{bx^2}\,\log(x)}{b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2)^(5/2),x, algorithm="fricas")
[Out] sqrt(b*x^2)*log(x)/(b^3*x)
```

giac [A] time = 0.22, size = 21, normalized size = 1.31

$$-\frac{\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2}\right|\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2)^(5/2),x, algorithm="giac")
[Out] -log(abs(-sqrt(b)*x + sqrt(b*x^2)))/b^(5/2)
```

maple [A] time = 0.00, size = 14, normalized size = 0.88

$$\frac{x^5\ln(x)}{\left(b\,x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2)^(5/2)*x^4,x)
[Out] 1/(b*x^2)^(5/2)*x^5*ln(x)
```

maxima [A] time = 1.34, size = 6, normalized size = 0.38

$$\frac{\log(x)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2)^(5/2),x, algorithm="maxima")
[Out] log(x)/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{x^4}{\left(b\,x^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2)^(5/2),x)
[Out] int(x^4/(b*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2)**(5/2),x)
[Out] Integral(x**4/(b*x**2)**(5/2), x)
```

3.59 $\int \frac{x^2}{\left(bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=19

$$-\frac{1}{2b^2x\sqrt{bx^2}}$$

```
[Out] -1/2/b^2/x/(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$-\frac{1}{2b^2x\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(b*x^2)^(5/2),x]
```

```
[Out] -1/(2*b^2*x*Sqrt[b*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(bx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{1}{x^3}\,dx}{b^2\sqrt{bx^2}}\\ &= -\frac{1}{2b^2x\sqrt{bx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 0.84

$$-\frac{x^3}{2\left(bx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(b*x^2)^(5/2),x]
```

```
[Out] -1/2*x^3/(b*x^2)^(5/2)
```

fricas [A] time = 0.56, size = 15, normalized size = 0.79

$$-\frac{\sqrt{bx^2}}{2\,b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2)^(5/2),x, algorithm="fricas")
[Out] -1/2*sqrt(b*x^2)/(b^3*x^3)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

maple [**A**] time = 0.00, size = 13, normalized size = 0.68

$$-\frac{x^3}{2\left(b\,x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2)^(5/2)*x^2,x)
[Out] -1/2*x^3/(b*x^2)^(5/2)
```

maxima [**A**] time = 1.28, size = 8, normalized size = 0.42

$$-\frac{1}{2\,b^{\frac{5}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2)^(5/2),x, algorithm="maxima")
[Out] -1/2/(b^(5/2)*x^2)
```

mupad [**B**] time = 0.92, size = 13, normalized size = 0.68

$$-\frac{1}{2\,b^{5/2}\,x\,\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2)^(5/2),x)
[Out] -1/(2*b^(5/2)*x*(x^2)^(1/2))
```

sympy [**A**] time = 0.85, size = 17, normalized size = 0.89

$$-\frac{x^3}{2b^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2)**(5/2),x)
[Out] -x**3/(2*b**(5/2)*(x**2)**(5/2))
```

3.60 $\int \frac{1}{\left(bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=19

$$-\frac{1}{4b^2x^3\sqrt{bx^2}}$$

```
[Out] -1/4/b^2/x^3/(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$-\frac{1}{4b^2x^3\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2)^(-5/2),x]
```

```
[Out] -1/(4*b^2*x^3*Sqrt[b*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(bx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{1}{x^5}\,dx}{b^2\sqrt{bx^2}}\\ &= -\frac{1}{4b^2x^3\sqrt{bx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 0.74

$$-\frac{x}{4\left(bx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2)^(-5/2),x]
```

```
[Out] -1/4*x/(b*x^2)^(5/2)
```

fricas [A] time = 0.53, size = 15, normalized size = 0.79

$$-\frac{\sqrt{bx^2}}{4\,b^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2)^(5/2),x, algorithm="fricas")
[Out] -1/4*sqrt(b*x^2)/(b^3*x^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.00, size = 11, normalized size = 0.58

$$-\frac{x}{4\left(b\,x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2)^(5/2),x)
[Out] -1/4*x/(b*x^2)^(5/2)
```

maxima [A] time = 1.30, size = 8, normalized size = 0.42

$$-\frac{1}{4\,b^{\frac{5}{2}}x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2)^(5/2),x, algorithm="maxima")
[Out] -1/4/(b^(5/2)*x^4)
```

mupad [B] time = 0.91, size = 13, normalized size = 0.68

$$-\frac{1}{4\,b^{5/2}\,x\left(x^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2)^(5/2),x)
[Out] -1/(4*b^(5/2)*x*(x^2)^(3/2))
```

sympy [A] time = 0.84, size = 15, normalized size = 0.79

$$-\frac{x}{4b^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2)**(5/2),x)
[Out] -x/(4*b**(5/2)*(x**2)**(5/2))
```

3.61 $\int \frac{1}{x^2\left(bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=19

$$-\frac{1}{6b^2x^5\sqrt{bx^2}}$$

```
[Out] -1/6/b^2/x^5/(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$-\frac{1}{6b^2x^5\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(b*x^2)^(5/2)),x]
```

```
[Out] -1/(6*b^2*x^5*Sqrt[b*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(bx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{1}{x^7}\,dx}{b^2\sqrt{bx^2}}\\ &= -\frac{1}{6b^2x^5\sqrt{bx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 0.79

$$-\frac{bx}{6\left(bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(b*x^2)^(5/2)),x]
```

```
[Out] -1/6*(b*x)/(b*x^2)^(7/2)
```

fricas [A] time = 0.65, size = 15, normalized size = 0.79

$$-\frac{\sqrt{bx^2}}{6\,b^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2)^(5/2),x, algorithm="fricas")
[Out] -1/6*sqrt(b*x^2)/(b^3*x^7)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.00, size = 13, normalized size = 0.68

$$-\frac{1}{6\left(b\,x^2\right)^{\frac{5}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2)^(5/2),x)
[Out] -1/6/x/(b*x^2)^(5/2)
```

maxima [A] time = 1.32, size = 8, normalized size = 0.42

$$-\frac{1}{6\,b^{\frac{5}{2}}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2)^(5/2),x, algorithm="maxima")
[Out] -1/6/(b^(5/2)*x^6)
```

mupad [B] time = 0.92, size = 13, normalized size = 0.68

$$-\frac{1}{6\,b^{5/2}\,x\left(x^2\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x^2)^(5/2)),x)
[Out] -1/(6*b^(5/2)*x*(x^2)^(5/2))
```

sympy [A] time = 1.25, size = 17, normalized size = 0.89

$$-\frac{1}{6b^{\frac{5}{2}}x\left(x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2)**(5/2),x)
[Out] -1/(6*b**(5/2)*x*(x**2)**(5/2))
```

3.62 $\int (cx)^m \left(bx^2\right)^{3/2}\, dx$

Optimal. Leaf size=29

$$\frac{b\sqrt{bx^2}\,(cx)^{m+4}}{c^4(m+4)x}$$

```
[Out] b*(c*x)^(4+m)*(b*x^2)^(1/2)/c^4/(4+m)/x
```

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {15, 16, 32}

$$\frac{b\sqrt{bx^2}\,(cx)^{m+4}}{c^4(m+4)x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*(b*x^2)^(3/2),x]
```

```
[Out] (b*(c*x)^(4 + m)*Sqrt[b*x^2])/(c^4*(4 + m)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (cx)^m \left(bx^2\right)^{3/2}\, dx &= \frac{\left(b\sqrt{bx^2}\right)\int x^3(cx)^m\, dx}{x}\\ &= \frac{\left(b\sqrt{bx^2}\right)\int (cx)^{3+m}\, dx}{c^3x}\\ &= \frac{b(cx)^{4+m}\sqrt{bx^2}}{c^4(4+m)x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 0.72

$$\frac{x\left(bx^2\right)^{3/2}(cx)^m}{m+4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*(b*x^2)^(3/2),x]
```

```
[Out] (x*(c*x)^m*(b*x^2)^(3/2))/(4 + m)
```

fricas [**A**] time = 0.69, size = 22, normalized size = 0.76

$$\frac{\sqrt{bx^2}\,(cx)^m\,bx^3}{m+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*x^2)*(c*x)^m*b*x^3/(m + 4)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Undef/Unsigned Inf encountered in limitLimit: Max order reached or unable t
o make series expansion Error: Bad Argument Value
```

maple [**A**] time = 0.00, size = 20, normalized size = 0.69

$$\frac{\left(b\,x^2\right)^{\frac{3}{2}}\,x\,(cx)^m}{m+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(b*x^2)^(3/2),x)
```

```
[Out] x/(4+m)*(c*x)^m*(b*x^2)^(3/2)
```

maxima [**A**] time = 1.39, size = 18, normalized size = 0.62

$$\frac{b^{\frac{3}{2}}c^m x^4 x^m}{m+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] b^(3/2)*c^m*x^4*x^m/(m + 4)
```

mupad [**B**] time = 0.96, size = 22, normalized size = 0.76

$$\frac{b^{3/2}\,x^3\,(c\,x)^m\,\sqrt{x^2}}{m+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(b*x^2)^(3/2),x)
```

```
[Out] (b^(3/2)*x^3*(c*x)^m*(x^2)^(1/2))/(m + 4)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{b^{\frac{3}{2}} c^m x x^m \left(x^2\right)^{\frac{3}{2}}}{m+4} & \text{for } m \neq -4 \\ \frac{\int \frac{\left(bx^2\right)^{\frac{3}{2}}}{x^4}\, dx}{c^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(b*x**2)**(3/2),x)
```

```
[Out] Piecewise((b**(3/2)*c**m*x*x**m*(x**2)**(3/2)/(m + 4), Ne(m, -4)), (Integra
l((b*x**2)**(3/2)/x**4, x)/c**4, True))
```

3.63 $\int (cx)^m \sqrt{bx^2}\, dx$

Optimal. Leaf size=28

$$\frac{\sqrt{bx^2}\,(cx)^{m+2}}{c^2(m+2)x}$$

```
[Out] (c*x)^(2+m)*(b*x^2)^(1/2)/c^2/(2+m)/x
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {15, 16, 32}

$$\frac{\sqrt{bx^2}\,(cx)^{m+2}}{c^2(m+2)x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*Sqrt[b*x^2],x]
```

```
[Out] ((c*x)^(2 + m)*Sqrt[b*x^2])/(c^2*(2 + m)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (cx)^m \sqrt{bx^2}\, dx &= \frac{\sqrt{bx^2}\int x(cx)^m\, dx}{x}\\ &= \frac{\sqrt{bx^2}\int (cx)^{1+m}\, dx}{cx}\\ &= \frac{(cx)^{2+m}\sqrt{bx^2}}{c^2(2+m)x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 0.75

$$\frac{x\sqrt{bx^2}\,(cx)^m}{m+2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*Sqrt[b*x^2],x]
```

```
[Out] (x*(c*x)^m*Sqrt[b*x^2])/(2 + m)
```

fricas [**A**] time = 0.67, size = 19, normalized size = 0.68

$$\frac{\sqrt{bx^2}\,(cx)^m\,x}{m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*x^2)*(c*x)^m*x/(m + 2)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Undef/Unsigned Inf encountered in limitLimit: Max order reached or unable t
o make series expansion Error: Bad Argument Value
```

maple [**A**] time = 0.00, size = 20, normalized size = 0.71

$$\frac{\sqrt{b\,x^2}\;x\,(cx)^m}{m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(b*x^2)^(1/2),x)
```

```
[Out] x/(2+m)*(c*x)^m*(b*x^2)^(1/2)
```

maxima [**A**] time = 1.43, size = 18, normalized size = 0.64

$$\frac{\sqrt{b}\,c^m x^2 x^m}{m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(b)*c^m*x^2*x^m/(m + 2)
```

mupad [**B**] time = 0.94, size = 20, normalized size = 0.71

$$\frac{\sqrt{b}\;x\,(c\,x)^m\,\sqrt{x^2}}{m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(b*x^2)^(1/2),x)
```

```
[Out] (b^(1/2)*x*(c*x)^m*(x^2)^(1/2))/(m + 2)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{\sqrt{b}\,c^m x x^m \sqrt{x^2}}{m+2} & \text{for } m \neq -2 \\ \frac{\int \frac{\sqrt{bx^2}}{x^2}\,dx}{c^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(b*x**2)**(1/2),x)
```

```
[Out] Piecewise((sqrt(b)*c**m*x*x**m*sqrt(x**2)/(m + 2), Ne(m, -2)), (Integral(sq
rt(b*x**2)/x**2, x)/c**2, True))
```

3.64 $\int \frac{(cx)^m}{\sqrt{bx^2}}\,dx$

Optimal. Leaf size=19

$$\frac{x(cx)^m}{m\sqrt{bx^2}}$$

```
[Out] x*(c*x)^m/m/(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {15, 16, 32}

$$\frac{x(cx)^m}{m\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m/Sqrt[b*x^2],x]
```

```
[Out] (x*(c*x)^m)/(m*Sqrt[b*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx)^m}{\sqrt{bx^2}}\,dx &= \frac{x\int \frac{(cx)^m}{x}\,dx}{\sqrt{bx^2}}\\ &= \frac{(cx)\int (cx)^{-1+m}\,dx}{\sqrt{bx^2}}\\ &= \frac{x(cx)^m}{m\sqrt{bx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{x(cx)^m}{m\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m/Sqrt[b*x^2],x]
```

```
[Out] (x*(c*x)^m)/(m*Sqrt[b*x^2])
```

fricas [A] time = 0.59, size = 22, normalized size = 1.16

$$\frac{\sqrt{bx^2}\,(cx)^m}{bmx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(b*x^2)^(1/2),x, algorithm="fricas")
[Out] sqrt(b*x^2)*(c*x)^m/(b*m*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{\sqrt{bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(b*x^2)^(1/2),x, algorithm="giac")
[Out] integrate((c*x)^m/sqrt(b*x^2), x)
```

maple [A] time = 0.00, size = 18, normalized size = 0.95

$$\frac{x\,(cx)^m}{\sqrt{b\,x^2}\;m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(b*x^2)^(1/2),x)
[Out] x*(c*x)^m/m/(b*x^2)^(1/2)
```

maxima [A] time = 1.42, size = 13, normalized size = 0.68

$$\frac{c^m x^m}{\sqrt{b}\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(b*x^2)^(1/2),x, algorithm="maxima")
[Out] c^m*x^m/(sqrt(b)*m)
```

mupad [B] time = 0.96, size = 18, normalized size = 0.95

$$\frac{x\,(c\,x)^m}{\sqrt{b}\;m\,\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(b*x^2)^(1/2),x)
[Out] (x*(c*x)^m)/(b^(1/2)*m*(x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{c^m x x^m}{\sqrt{b}\,m\sqrt{x^2}} & \text{for } m \neq 0 \\ \int \frac{1}{\sqrt{bx^2}}\,dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m/(b*x**2)**(1/2),x)
[Out] Piecewise((c**m*x*x**m/(sqrt(b)*m*sqrt(x**2)), Ne(m, 0)), (Integral(1/sqrt(
b*x**2), x), True))
```

3.65 $\int \frac{(cx)^m}{\left(bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=32

$$-\frac{c^2 x (cx)^{m-2}}{b(2-m)\sqrt{bx^2}}$$

```
[Out] -c^2*x*(c*x)^(-2+m)/b/(2-m)/(b*x^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {15, 16, 32}

$$-\frac{c^2 x (cx)^{m-2}}{b(2-m)\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m/(b*x^2)^(3/2),x]
```

```
[Out] -((c^2*x*(c*x)^(-2 + m))/(b*(2 - m)*Sqrt[b*x^2]))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx)^m}{\left(bx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{(cx)^m}{x^3}\,dx}{b\sqrt{bx^2}}\\ &= \frac{\left(c^3 x\right)\int (cx)^{-3+m}\,dx}{b\sqrt{bx^2}}\\ &= -\frac{c^2 x (cx)^{-2+m}}{b(2-m)\sqrt{bx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 0.66

$$\frac{x(cx)^m}{(m-2)\left(bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m/(b*x^2)^(3/2),x]
[Out] (x*(c*x)^m)/((-2 + m)*(b*x^2)^(3/2))
```

fricas [A] time = 0.62, size = 29, normalized size = 0.91

$$\frac{\sqrt{bx^2}\,(cx)^m}{\left(b^2m - 2\,b^2\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(b*x^2)^(3/2),x, algorithm="fricas")
[Out] sqrt(b*x^2)*(c*x)^m/((b^2*m - 2*b^2)*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{\left(bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(b*x^2)^(3/2),x, algorithm="giac")
[Out] integrate((c*x)^m/(b*x^2)^(3/2), x)
```

maple [A] time = 0.00, size = 20, normalized size = 0.62

$$\frac{x\,(cx)^m}{(m-2)\left(b\,x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(b*x^2)^(3/2),x)
[Out] x/(-2+m)*(c*x)^m/(b*x^2)^(3/2)
```

maxima [A] time = 1.42, size = 18, normalized size = 0.56

$$\frac{c^m x^m}{b^{\frac{3}{2}}(m-2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(b*x^2)^(3/2),x, algorithm="maxima")
[Out] c^m*x^m/(b^(3/2)*(m - 2)*x^2)
```

mupad [B] time = 1.00, size = 22, normalized size = 0.69

$$\frac{(c\,x)^m}{b^{3/2}\,x\,(m-2)\,\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(b*x^2)^(3/2),x)
[Out] (c*x)^m/(b^(3/2)*x*(m - 2)*(x^2)^(1/2))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \dfrac{c^m x x^m}{b^{\frac{3}{2}} m \left(x^2\right)^{\frac{3}{2}} - 2b^{\frac{3}{2}} \left(x^2\right)^{\frac{3}{2}}} & \text{for } m \neq 2 \\ c^2 \displaystyle\int \frac{x^2}{\left(bx^2\right)^{\frac{3}{2}}}\, dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m/(b*x**2)**(3/2),x)
```

```
[Out] Piecewise((c**m*x*x**m/(b**(3/2)*m*(x**2)**(3/2) - 2*b**(3/2)*(x**2)**(3/2)
), Ne(m, 2)), (c**2*Integral(x**2/(b*x**2)**(3/2), x), True))
```

3.66 $\int x^m \left(bx^2\right)^p \, dx$

Optimal. Leaf size=21

$$\frac{x^{m+1}\left(bx^2\right)^p}{m+2p+1}$$

```
[Out] x^(1+m)*(b*x^2)^p/(1+m+2*p)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 30}

$$\frac{x^{m+1}\left(bx^2\right)^p}{m+2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(b*x^2)^p,x]
```

```
[Out] (x^(1 + m)*(b*x^2)^p)/(1 + m + 2*p)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^m \left(bx^2\right)^p \, dx &= \left(x^{-2p}\left(bx^2\right)^p\right)\int x^{m+2p}\, dx \\ &= \frac{x^{1+m}\left(bx^2\right)^p}{1+m+2p}\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{x^{m+1}\left(bx^2\right)^p}{m+2p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(b*x^2)^p,x]
```

```
[Out] (x^(1 + m)*(b*x^2)^p)/(1 + m + 2*p)
```

fricas [A] time = 0.74, size = 24, normalized size = 1.14

$$\frac{x x^m e^{\left(p\log(b)+2\,p\log(x)\right)}}{m+2\,p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2)^p,x, algorithm="fricas")
[Out] x*x^m*e^(p*log(b) + 2*p*log(x))/(m + 2*p + 1)
```

giac [A] time = 0.17, size = 24, normalized size = 1.14

$$\frac{x x^m e^{\left(p \log(b)+2 p \log(x)\right)}}{m+2 p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2)^p,x, algorithm="giac")
[Out] x*x^m*e^(p*log(b) + 2*p*log(x))/(m + 2*p + 1)
```

maple [A] time = 0.00, size = 22, normalized size = 1.05

$$\frac{x^{m+1}\left(b\,x^2\right)^p}{m+2p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2)^p,x)
[Out] x^(m+1)*(b*x^2)^p/(1+m+2*p)
```

maxima [A] time = 1.33, size = 24, normalized size = 1.14

$$\frac{b^p x e^{\left(m \log(x)+2 p \log(x)\right)}}{m+2 p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2)^p,x, algorithm="maxima")
[Out] b^p*x*e^(m*log(x) + 2*p*log(x))/(m + 2*p + 1)
```

mupad [B] time = 1.07, size = 21, normalized size = 1.00

$$\frac{x^{m+1}\left(b\,x^2\right)^p}{m+2\,p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2)^p,x)
[Out] (x^(m + 1)*(b*x^2)^p)/(m + 2*p + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{b^p x x^m \left(x^2\right)^p}{m+2p+1} & \text{for } m \neq -2p-1 \\ \int x^{-2p-1}\left(bx^2\right)^p\,dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2)**p,x)
[Out] Piecewise((b**p*x*x**m*(x**2)**p/(m + 2*p + 1), Ne(m, -2*p - 1)), (Integral
(x**(-2*p - 1)*(b*x**2)**p, x), True))
```

3.67 $\int (cx)^m \left(bx^2\right)^p \, dx$

Optimal. Leaf size=22

$$\frac{x \left(bx^2\right)^p (cx)^m}{m+2p+1}$$

```
[Out] x*(c*x)^m*(b*x^2)^p/(1+m+2*p)
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {15, 20, 30}

$$\frac{x \left(bx^2\right)^p (cx)^m}{m+2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*(b*x^2)^p,x]
```

```
[Out] (x*(c*x)^m*(b*x^2)^p)/(1 + m + 2*p)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (cx)^m \left(bx^2\right)^p \, dx &= \left(x^{-2p} \left(bx^2\right)^p\right) \int x^{2p} (cx)^m \, dx \\
&= \left(x^{-m-2p} (cx)^m \left(bx^2\right)^p\right) \int x^{m+2p} \, dx \\
&= \frac{x (cx)^m \left(bx^2\right)^p}{1+m+2p}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$\frac{x \left(bx^2\right)^p (cx)^m}{m+2p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*(b*x^2)^p,x]
```

```
[Out] (x*(c*x)^m*(b*x^2)^p)/(1 + m + 2*p)
```

fricas [A] time = 0.74, size = 32, normalized size = 1.45

$$\frac{(cx)^m\, x e^{\left(2p\log(cx)+p\log\left(\frac{b}{c^2}\right)\right)}}{m+2p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^2)^p,x, algorithm="fricas")
[Out] (c*x)^m*x*e^(2*p*log(c*x) + p*log(b/c^2))/(m + 2*p + 1)
```

giac [A] time = 0.17, size = 29, normalized size = 1.32

$$\frac{x e^{\left(p\log(b)+m\log(c)+m\log(x)+2p\log(x)\right)}}{m+2p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^2)^p,x, algorithm="giac")
[Out] x*e^(p*log(b) + m*log(c) + m*log(x) + 2*p*log(x))/(m + 2*p + 1)
```

maple [A] time = 0.00, size = 23, normalized size = 1.05

$$\frac{x\left(b\,x^2\right)^p (cx)^m}{m+2p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(b*x^2)^p,x)
[Out] x*(c*x)^m*(b*x^2)^p/(m+2*p+1)
```

maxima [A] time = 1.37, size = 27, normalized size = 1.23

$$\frac{b^p c^m x e^{\left(m\log(x)+2p\log(x)\right)}}{m+2p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^2)^p,x, algorithm="maxima")
[Out] b^p*c^m*x*e^(m*log(x) + 2*p*log(x))/(m + 2*p + 1)
```

mupad [B] time = 0.96, size = 22, normalized size = 1.00

$$\frac{x\,(c\,x)^m\left(b\,x^2\right)^p}{m+2p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(b*x^2)^p,x)
[Out] (x*(c*x)^m*(b*x^2)^p)/(m + 2*p + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{b^p c^m x x^m \left(x^2\right)^p}{m+2p+1} & \text{for } m \neq -2p-1 \\ \int \left(bx^2\right)^p (cx)^{-2p-1}\, dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(b*x**2)**p,x)
```

```
[Out] Piecewise((b**p*c**m*x*x**m*(x**2)**p/(m + 2*p + 1), Ne(m, -2*p - 1)), (Int
egral((b*x**2)**p*(c*x)**(-2*p - 1), x), True))
```

3.68 $\int x^{-1-2p}\left(x^2\right)^p\,dx$

Optimal. Leaf size=13

$$x^{-2p}\left(x^2\right)^p\log(x)$$

```
[Out] (x^2)^p*ln(x)/(x^(2*p))
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 29}

$$x^{-2p}\left(x^2\right)^p\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 2*p)*(x^2)^p,x]
```

```
[Out] ((x^2)^p*Log[x])/x^(2*p)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int x^{-1-2p}\left(x^2\right)^p\,dx &= \left(x^{-2p}\left(x^2\right)^p\right)\int\frac{1}{x}\,dx\\ &= x^{-2p}\left(x^2\right)^p\log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$x^{-2p}\left(x^2\right)^p\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 2*p)*(x^2)^p,x]
```

```
[Out] ((x^2)^p*Log[x])/x^(2*p)
```

fricas [A] time = 0.77, size = 2, normalized size = 0.15

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*p)*(x^2)^p,x, algorithm="fricas")
```

```
[Out] log(x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x^2\right)^p x^{-2p-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*p)*(x^2)^p,x, algorithm="giac")
[Out] integrate((x^2)^p*x^(-2*p - 1), x)
```

maple [A] time = 0.03, size = 21, normalized size = 1.62

$$x\, e^{p \ln\left(x^2\right)} e^{\left(-2p-1\right)\ln(x)} \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-2*p)*(x^2)^p,x)
[Out] x*ln(x)*exp(p*ln(x^2))*exp((-1-2*p)*ln(x))
```

maxima [A] time = 1.28, size = 2, normalized size = 0.15

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*p)*(x^2)^p,x, algorithm="maxima")
[Out] log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\left(x^2\right)^p}{x^{2\,p+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2)^p/x^(2*p + 1),x)
[Out] int((x^2)^p/x^(2*p + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-2p-1} \left(x^2\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-2*p)*(x**2)**p,x)
[Out] Integral(x**(-2*p - 1)*(x**2)**p, x)
```

3.69 $\int x^3 \left(bx^2\right)^p \, dx$

Optimal. Leaf size=19

$$\frac{x^4 \left(bx^2\right)^p}{2(p+2)}$$

```
[Out] 1/2*x^4*(b*x^2)^p/(2+p)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 30}

$$\frac{x^4 \left(bx^2\right)^p}{2(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(b*x^2)^p,x]
```

```
[Out] (x^4*(b*x^2)^p)/(2*(2 + p))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^3 \left(bx^2\right)^p \, dx &= \left(x^{-2p} \left(bx^2\right)^p\right) \int x^{3+2p} \, dx \\ &= \frac{x^4 \left(bx^2\right)^p}{2(2+p)}\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 0.95

$$\frac{x^4 \left(bx^2\right)^p}{2p+4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(b*x^2)^p,x]
```

```
[Out] (x^4*(b*x^2)^p)/(4 + 2*p)
```

fricas [A] time = 0.83, size = 17, normalized size = 0.89

$$\frac{\left(bx^2\right)^p x^4}{2\left(p+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2)^p,x, algorithm="fricas")
[Out] 1/2*(b*x^2)^p*x^4/(p + 2)
```

giac [A] time = 0.15, size = 17, normalized size = 0.89

$$\frac{\left(bx^2\right)^p x^4}{2\left(p+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2)^p,x, algorithm="giac")
[Out] 1/2*(b*x^2)^p*x^4/(p + 2)
```

maple [A] time = 0.00, size = 18, normalized size = 0.95

$$\frac{x^4\left(b\,x^2\right)^p}{4+2p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2)^p,x)
[Out] 1/2*x^4*(b*x^2)^p/(2+p)
```

maxima [A] time = 1.37, size = 18, normalized size = 0.95

$$\frac{b^p\left(x^2\right)^p x^4}{2\left(p+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2)^p,x, algorithm="maxima")
[Out] 1/2*b^p*(x^2)^p*x^4/(p + 2)
```

mupad [B] time = 0.96, size = 18, normalized size = 0.95

$$\frac{x^4\left(b\,x^2\right)^p}{2\,\left(p+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2)^p,x)
[Out] (x^4*(b*x^2)^p)/(2*(p + 2))
```

sympy [A] time = 0.42, size = 24, normalized size = 1.26

$$\begin{cases} \frac{b^p x^4\left(x^2\right)^p}{2p+4} & \text{for } p \neq -2 \\ \frac{\log(x)}{b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2)**p,x)
[Out] Piecewise((b**p*x**4*(x**2)**p/(2*p + 4), Ne(p, -2)), (log(x)/b**2, True))
```

3.70 $\int x^2 \left(bx^2\right)^p \, dx$

Optimal. Leaf size=18

$$\frac{x^3 \left(bx^2\right)^p}{2p+3}$$

```
[Out] x^3*(b*x^2)^p/(3+2*p)
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 30}

$$\frac{x^3 \left(bx^2\right)^p}{2p+3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b*x^2)^p,x]
```

```
[Out] (x^3*(b*x^2)^p)/(3 + 2*p)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(bx^2\right)^p \, dx &= \left(x^{-2p}\left(bx^2\right)^p\right) \int x^{2+2p} \, dx \\ &= \frac{x^3 \left(bx^2\right)^p}{3+2p}\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{x^3 \left(bx^2\right)^p}{2p+3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b*x^2)^p,x]
```

```
[Out] (x^3*(b*x^2)^p)/(3 + 2*p)
```

fricas [A] time = 0.83, size = 18, normalized size = 1.00

$$\frac{\left(bx^2\right)^p x^3}{2\,p+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2)^p,x, algorithm="fricas")
```

```
[Out] (b*x^2)^p*x^3/(2*p + 3)
```

giac [A] time = 0.18, size = 18, normalized size = 1.00

$$\frac{\left(bx^2\right)^p x^3}{2\,p+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2)^p,x, algorithm="giac")
```

```
[Out] (b*x^2)^p*x^3/(2*p + 3)
```

maple [A] time = 0.00, size = 19, normalized size = 1.06

$$\frac{x^3\left(b\,x^2\right)^p}{2p+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2)^p,x)
```

```
[Out] x^3*(b*x^2)^p/(3+2*p)
```

maxima [A] time = 1.36, size = 19, normalized size = 1.06

$$\frac{b^p x^3 x^{2\,p}}{2\,p+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2)^p,x, algorithm="maxima")
```

```
[Out] b^p*x^3*x^(2*p)/(2*p + 3)
```

mupad [B] time = 0.98, size = 18, normalized size = 1.00

$$\frac{x^3\left(b\,x^2\right)^p}{2\,p+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2)^p,x)
```

```
[Out] (x^3*(b*x^2)^p)/(2*p + 3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{b^p x^3 \left(x^2\right)^p}{2p+3} & \text{for } p \neq -\frac{3}{2} \\ \int \frac{x^2}{\left(bx^2\right)^{\frac{3}{2}}}\,dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2)**p,x)
```

```
[Out] Piecewise((b**p*x**3*(x**2)**p/(2*p + 3), Ne(p, -3/2)), (Integral(x**2/(b*x
**2)**(3/2), x), True))
```

3.71 $\int x \left(bx^2\right)^p \, dx$

Optimal. Leaf size=19

$$\frac{x^2 \left(bx^2\right)^p}{2(p+1)}$$

```
[Out] 1/2*x^2*(b*x^2)^p/(1+p)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$\frac{x^2 \left(bx^2\right)^p}{2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b*x^2)^p,x]
```

```
[Out] (x^2*(b*x^2)^p)/(2*(1 + p))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x \left(bx^2\right)^p \, dx &= \left(x^{-2p} \left(bx^2\right)^p\right) \int x^{1+2p} \, dx \\ &= \frac{x^2 \left(bx^2\right)^p}{2(1+p)}\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 0.95

$$\frac{x^2 \left(bx^2\right)^p}{2p+2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b*x^2)^p,x]
```

```
[Out] (x^2*(b*x^2)^p)/(2 + 2*p)
```

fricas [A] time = 0.81, size = 17, normalized size = 0.89

$$\frac{\left(bx^2\right)^p x^2}{2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2)^p,x, algorithm="fricas")
[Out] 1/2*(b*x^2)^p*x^2/(p + 1)
```

giac [A] time = 0.16, size = 19, normalized size = 1.00

$$\frac{\left(bx^2\right)^{p+1}}{2\,b\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2)^p,x, algorithm="giac")
[Out] 1/2*(b*x^2)^(p + 1)/(b*(p + 1))
```

maple [A] time = 0.00, size = 18, normalized size = 0.95

$$\frac{x^2\left(b\,x^2\right)^p}{2+2p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2)^p,x)
[Out] 1/2*x^2*(b*x^2)^p/(1+p)
```

maxima [A] time = 1.32, size = 19, normalized size = 1.00

$$\frac{\left(bx^2\right)^{p+1}}{2\,b\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2)^p,x, algorithm="maxima")
[Out] 1/2*(b*x^2)^(p + 1)/(b*(p + 1))
```

mupad [B] time = 0.99, size = 18, normalized size = 0.95

$$\frac{x^2\left(b\,x^2\right)^p}{2\,\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2)^p,x)
[Out] (x^2*(b*x^2)^p)/(2*(p + 1))
```

sympy [A] time = 0.24, size = 22, normalized size = 1.16

$$\begin{cases} \frac{b^p x^2 \left(x^2\right)^p}{2p+2} & \text{for } p \neq -1 \\ \frac{\log(x)}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2)**p,x)
[Out] Piecewise((b**p*x**2*(x**2)**p/(2*p + 2), Ne(p, -1)), (log(x)/b, True))
```

3.72 $\int \left(bx^2\right)^p \, dx$

Optimal. Leaf size=16

$$\frac{x\left(bx^2\right)^p}{2p+1}$$

```
[Out] x*(b*x^2)^p/(1+2*p)
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {15, 30}

$$\frac{x\left(bx^2\right)^p}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2)^p,x]
```

```
[Out] (x*(b*x^2)^p)/(1 + 2*p)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \left(bx^2\right)^p \, dx &= \left(x^{-2p}\left(bx^2\right)^p\right)\int x^{2p}\, dx \\ &= \frac{x\left(bx^2\right)^p}{1+2p}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{x\left(bx^2\right)^p}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2)^p,x]
```

```
[Out] (x*(b*x^2)^p)/(1 + 2*p)
```

fricas [A] time = 0.83, size = 16, normalized size = 1.00

$$\frac{\left(bx^2\right)^p x}{2\,p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^p,x, algorithm="fricas")
```

```
[Out] (b*x^2)^p*x/(2*p + 1)
```

giac [A] time = 0.19, size = 16, normalized size = 1.00

$$\frac{\left(bx^2\right)^p x}{2p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^p,x, algorithm="giac")
```

```
[Out] (b*x^2)^p*x/(2*p + 1)
```

maple [A] time = 0.00, size = 17, normalized size = 1.06

$$\frac{x\left(b\,x^2\right)^p}{2p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^p,x)
```

```
[Out] x*(b*x^2)^p/(1+2*p)
```

maxima [A] time = 1.34, size = 17, normalized size = 1.06

$$\frac{b^p x x^{2\,p}}{2\,p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^p,x, algorithm="maxima")
```

```
[Out] b^p*x*x^(2*p)/(2*p + 1)
```

mupad [B] time = 1.10, size = 16, normalized size = 1.00

$$\frac{x\left(b\,x^2\right)^p}{2\,p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^p,x)
```

```
[Out] (x*(b*x^2)^p)/(2*p + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{b^p x\left(x^2\right)^p}{2p+1} & \text{for } p \neq -\frac{1}{2} \\ \int \frac{1}{\sqrt{bx^2}}\,dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2)**p,x)
```

```
[Out] Piecewise((b**p*x*(x**2)**p/(2*p + 1), Ne(p, -1/2)), (Integral(1/sqrt(b*x**
2), x), True))
```

3.73 $\int \frac{\left(bx^2\right)^p}{x}\,dx$

Optimal. Leaf size=14

$$\frac{\left(bx^2\right)^p}{2p}$$

```
[Out] 1/2*(b*x^2)^p/p
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 30}

$$\frac{\left(bx^2\right)^p}{2p}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2)^p/x,x]
```

```
[Out] (b*x^2)^p/(2*p)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2\right)^p}{x}\,dx &= \left(x^{-2p}\left(bx^2\right)^p\right)\int x^{-1+2p}\,dx\\ &= \frac{\left(bx^2\right)^p}{2p}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{\left(bx^2\right)^p}{2p}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2)^p/x,x]
```

```
[Out] (b*x^2)^p/(2*p)
```

fricas [A] time = 0.65, size = 12, normalized size = 0.86

$$\frac{\left(bx^2\right)^p}{2\,p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^p/x,x, algorithm="fricas")
[Out] 1/2*(b*x^2)^p/p
```

giac [A] time = 0.15, size = 12, normalized size = 0.86

$$\frac{\left(bx^2\right)^p}{2\,p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^p/x,x, algorithm="giac")
[Out] 1/2*(b*x^2)^p/p
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$\frac{\left(b\,x^2\right)^p}{2p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^p/x,x)
[Out] 1/2*(b*x^2)^p/p
```

maxima [A] time = 1.31, size = 13, normalized size = 0.93

$$\frac{b^p\left(x^2\right)^p}{2\,p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^p/x,x, algorithm="maxima")
[Out] 1/2*b^p*(x^2)^p/p
```

mupad [B] time = 0.95, size = 12, normalized size = 0.86

$$\frac{\left(b\,x^2\right)^p}{2\,p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^p/x,x)
[Out] (b*x^2)^p/(2*p)
```

sympy [A] time = 0.20, size = 14, normalized size = 1.00

$$\begin{cases} \frac{b^p\left(x^2\right)^p}{2p} & \text{for } p \neq 0 \\ \log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2)**p/x,x)
[Out] Piecewise((b**p*(x**2)**p/(2*p), Ne(p, 0)), (log(x), True))
```

3.74 $\int \frac{\left(bx^2\right)^p}{x^2}\,dx$

Optimal. Leaf size=19

$$-\frac{\left(bx^2\right)^p}{(1-2p)x}$$

```
[Out] -(b*x^2)^p/(1-2*p)/x
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 30}

$$-\frac{\left(bx^2\right)^p}{(1-2p)x}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2)^p/x^2,x]
```

```
[Out] -((b*x^2)^p/((1 - 2*p)*x))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2\right)^p}{x^2}\,dx &= \left(x^{-2p}\left(bx^2\right)^p\right)\int x^{-2+2p}\,dx\\ &= -\frac{\left(bx^2\right)^p}{(1-2p)x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 0.95

$$\frac{\left(bx^2\right)^p}{(2p-1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2)^p/x^2,x]
```

```
[Out] (b*x^2)^p/((-1 + 2*p)*x)
```

fricas [A] time = 0.83, size = 18, normalized size = 0.95

$$\frac{\left(bx^2\right)^p}{\left(2\,p-1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^p/x^2,x, algorithm="fricas")
[Out] (b*x^2)^p/((2*p - 1)*x)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^p/x^2,x, algorithm="giac")
[Out] integrate((b*x^2)^p/x^2, x)
```

maple [**A**] time = 0.00, size = 19, normalized size = 1.00

$$\frac{\left(b\,x^2\right)^p}{\left(2p-1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^p/x^2,x)
[Out] 1/x/(2*p-1)*(b*x^2)^p
```

maxima [**A**] time = 1.36, size = 19, normalized size = 1.00

$$\frac{b^p x^{2\,p}}{\left(2\,p-1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^p/x^2,x, algorithm="maxima")
[Out] b^p*x^(2*p)/((2*p - 1)*x)
```

mupad [**B**] time = 0.93, size = 18, normalized size = 0.95

$$\frac{\left(b\,x^2\right)^p}{x\,\left(2\,p-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^p/x^2,x)
[Out] (b*x^2)^p/(x*(2*p - 1))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{b^p\left(x^2\right)^p}{2px-x} & \text{for } p \neq \frac{1}{2} \\ \int \frac{\sqrt{bx^2}}{x^2}\,dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2)**p/x**2,x)
[Out] Piecewise((b**p*(x**2)**p/(2*p*x - x), Ne(p, 1/2)), (Integral(sqrt(b*x**2)/
x**2, x), True))
```

3.75 $\int \frac{\left(bx^2\right)^p}{x^3}\,dx$

Optimal. Leaf size=21

$$-\frac{\left(bx^2\right)^p}{2(1-p)x^2}$$

```
[Out] -1/2*(b*x^2)^p/(-p+1)/x^2
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 30}

$$-\frac{\left(bx^2\right)^p}{2(1-p)x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2)^p/x^3,x]
```

```
[Out] -(b*x^2)^p/(2*(1 - p)*x^2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2\right)^p}{x^3}\,dx &= \left(x^{-2p}\left(bx^2\right)^p\right)\int x^{-3+2p}\,dx\\ &= -\frac{\left(bx^2\right)^p}{2(1-p)x^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 0.86

$$\frac{\left(bx^2\right)^p}{(2p-2)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2)^p/x^3,x]
```

```
[Out] (b*x^2)^p/((-2 + 2*p)*x^2)
```

fricas [A] time = 0.65, size = 17, normalized size = 0.81

$$\frac{\left(bx^2\right)^p}{2\left(p-1\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^p/x^3,x, algorithm="fricas")
[Out] 1/2*(b*x^2)^p/((p - 1)*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^p/x^3,x, algorithm="giac")
[Out] integrate((b*x^2)^p/x^3, x)
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$\frac{\left(b\,x^2\right)^p}{2\left(p-1\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^p/x^3,x)
[Out] 1/2/x^2/(p-1)*(b*x^2)^p
```

maxima [A] time = 1.36, size = 18, normalized size = 0.86

$$\frac{b^p\left(x^2\right)^p}{2\left(p-1\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^p/x^3,x, algorithm="maxima")
[Out] 1/2*b^p*(x^2)^p/((p - 1)*x^2)
```

mupad [B] time = 0.98, size = 18, normalized size = 0.86

$$\frac{\left(b\,x^2\right)^p}{x^2\,\left(2\,p-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^p/x^3,x)
[Out] (b*x^2)^p/(x^2*(2*p - 2))
```

sympy [A] time = 0.48, size = 24, normalized size = 1.14

$$\begin{cases} \frac{b^p\left(x^2\right)^p}{2px^2-2x^2} & \text{for } p \neq 1 \\ b\log\left(x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2)**p/x**3,x)
[Out] Piecewise((b**p*(x**2)**p/(2*p*x**2 - 2*x**2), Ne(p, 1)), (b*log(x), True))
```

3.76 $\int \frac{\left(bx^2\right)^p}{x^4}\,dx$

Optimal. Leaf size=19

$$-\frac{\left(bx^2\right)^p}{(3-2p)x^3}$$

```
[Out] -(b*x^2)^p/(3-2*p)/x^3
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 30}

$$-\frac{\left(bx^2\right)^p}{(3-2p)x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2)^p/x^4,x]
```

```
[Out] -((b*x^2)^p/((3 - 2*p)*x^3))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^2\right)^p}{x^4}\,dx &= \left(x^{-2p}\left(bx^2\right)^p\right)\int x^{-4+2p}\,dx\\ &= -\frac{\left(bx^2\right)^p}{(3-2p)x^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 0.95

$$\frac{\left(bx^2\right)^p}{(2p-3)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2)^p/x^4,x]
```

```
[Out] (b*x^2)^p/((-3 + 2*p)*x^3)
```

fricas [A] time = 0.83, size = 18, normalized size = 0.95

$$\frac{\left(bx^2\right)^p}{\left(2\,p-3\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^p/x^4,x, algorithm="fricas")
[Out] (b*x^2)^p/((2*p - 3)*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^2\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^p/x^4,x, algorithm="giac")
[Out] integrate((b*x^2)^p/x^4, x)
```

maple [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{\left(b\,x^2\right)^p}{\left(2p-3\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^p/x^4,x)
[Out] 1/x^3/(2*p-3)*(b*x^2)^p
```

maxima [A] time = 1.33, size = 19, normalized size = 1.00

$$\frac{b^p x^{2\,p}}{\left(2\,p-3\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^p/x^4,x, algorithm="maxima")
[Out] b^p*x^(2*p)/((2*p - 3)*x^3)
```

mupad [B] time = 0.97, size = 18, normalized size = 0.95

$$\frac{\left(b\,x^2\right)^p}{x^3\,\left(2\,p-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^p/x^4,x)
[Out] (b*x^2)^p/(x^3*(2*p - 3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{b^p\left(x^2\right)^p}{2px^3-3x^3} & \text{for } p \neq \frac{3}{2} \\ \int \frac{\left(bx^2\right)^{\frac{3}{2}}}{x^4}\,dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2)**p/x**4,x)
[Out] Piecewise((b**p*(x**2)**p/(2*p*x**3 - 3*x**3), Ne(p, 3/2)), (Integral((b*x*
**2)**(3/2)/x**4, x), True))
```

3.77 $\int \sqrt{bx^3}\, dx$

Optimal. Leaf size=14

$$\frac{2}{5}x\sqrt{bx^3}$$

```
[Out] 2/5*x*(b*x^3)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$\frac{2}{5}x\sqrt{bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^3],x]
```

```
[Out] (2*x*Sqrt[b*x^3])/5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt{bx^3}\, dx &= \frac{\sqrt{bx^3}\int x^{3/2}\, dx}{x^{3/2}} \\ &= \frac{2}{5}x\sqrt{bx^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{2}{5}x\sqrt{bx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^3],x]
```

```
[Out] (2*x*Sqrt[b*x^3])/5
```

fricas [A] time = 0.69, size = 10, normalized size = 0.71

$$\frac{2}{5}\sqrt{bx^3}\, x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*sqrt(b*x^3)*x
```

giac [A] time = 0.17, size = 12, normalized size = 0.86

$$\frac{2}{5}\sqrt{bx}\,x^2\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3)^(1/2),x, algorithm="giac")
[Out] 2/5*sqrt(b*x)*x^2*sgn(x)
```

maple [A] time = 0.00, size = 11, normalized size = 0.79

$$\frac{2\sqrt{b\,x^3}\;x}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3)^(1/2),x)
[Out] 2/5*x*(b*x^3)^(1/2)
```

maxima [A] time = 1.34, size = 10, normalized size = 0.71

$$\frac{2}{5}\sqrt{bx^3}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3)^(1/2),x, algorithm="maxima")
[Out] 2/5*sqrt(b*x^3)*x
```

mupad [B] time = 1.36, size = 10, normalized size = 0.71

$$\frac{2\,x\,\sqrt{b\,x^3}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3)^(1/2),x)
[Out] (2*x*(b*x^3)^(1/2))/5
```

sympy [A] time = 0.17, size = 15, normalized size = 1.07

$$\frac{2\sqrt{b}\,x\sqrt{x^3}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3)**(1/2),x)
[Out] 2*sqrt(b)*x*sqrt(x**3)/5
```

3.78 $\int \sqrt{bx^2}\, dx$

Optimal. Leaf size=14

$$\frac{1}{2}x\sqrt{bx^2}$$

```
[Out] 1/2*x*(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$\frac{1}{2}x\sqrt{bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2],x]
```

```
[Out] (x*Sqrt[b*x^2])/2
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt{bx^2}\, dx &= \frac{\sqrt{bx^2}\int x\, dx}{x}\\ &= \frac{1}{2}x\sqrt{bx^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{1}{2}x\sqrt{bx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2],x]
```

```
[Out] (x*Sqrt[b*x^2])/2
```

fricas [A] time = 0.82, size = 10, normalized size = 0.71

$$\frac{1}{2}\sqrt{bx^2}\, x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(b*x^2)*x
```

giac [A] time = 0.15, size = 10, normalized size = 0.71

$$\frac{1}{2}\sqrt{b}\,x^2\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(b)*x^2*sgn(x)
```

maple [A] time = 0.00, size = 11, normalized size = 0.79

$$\frac{\sqrt{b\,x^2}\;x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(1/2),x)
[Out] 1/2*(b*x^2)^(1/2)*x
```

maxima [A] time = 1.32, size = 10, normalized size = 0.71

$$\frac{1}{2}\sqrt{bx^2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(b*x^2)*x
```

mupad [B] time = 0.00, size = 8, normalized size = 0.57

$$\frac{\sqrt{b}\;x\,|x|}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(1/2),x)
[Out] (b^(1/2)*x*abs(x))/2
```

sympy [A] time = 0.16, size = 14, normalized size = 1.00

$$\frac{\sqrt{b}\,x\sqrt{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2)**(1/2),x)
[Out] sqrt(b)*x*sqrt(x**2)/2
```

3.79 $\int \sqrt{bx}\, dx$

Optimal. Leaf size=14

$$\frac{2(bx)^{3/2}}{3b}$$

```
[Out] 2/3*(b*x)^(3/2)/b
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{2(bx)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x],x]
```

```
[Out] (2*(b*x)^(3/2))/(3*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{bx}\, dx = \frac{2(bx)^{3/2}}{3b}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 0.86

$$\frac{2}{3}x\sqrt{bx}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x],x]
```

```
[Out] (2*x*Sqrt[b*x])/3
```

fricas [A] time = 0.82, size = 8, normalized size = 0.57

$$\frac{2}{3}\sqrt{bx}\, x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*x)*x
```

giac [A] time = 0.14, size = 8, normalized size = 0.57

$$\frac{2}{3}\sqrt{bx}\, x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(b*x)*x
```

maple [A] time = 0.00, size = 9, normalized size = 0.64

$$\frac{2\sqrt{bx}\,x}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^(1/2),x)
```

```
[Out] 2/3*x*(b*x)^(1/2)
```

maxima [A] time = 1.30, size = 10, normalized size = 0.71

$$\frac{2\,(bx)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b*x)^(3/2)/b
```

mupad [B] time = 0.02, size = 10, normalized size = 0.71

$$\frac{2\,(b\,x)^{3/2}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^(1/2),x)
```

```
[Out] (2*(b*x)^(3/2))/(3*b)
```

sympy [A] time = 0.06, size = 10, normalized size = 0.71

$$\frac{2\,(bx)^{\frac{3}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)**(1/2),x)
```

```
[Out] 2*(b*x)**(3/2)/(3*b)
```

3.80 $\int \sqrt{\frac{b}{x}}\, dx$

Optimal. Leaf size=12

$$2x\sqrt{\frac{b}{x}}$$

```
[Out] 2*x*(b/x)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$2x\sqrt{\frac{b}{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b/x],x]
```

```
[Out] 2*Sqrt[b/x]*x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{b}{x}}\, dx &= \left(\sqrt{\frac{b}{x}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}}\, dx \\ &= 2\sqrt{\frac{b}{x}}\, x\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$2x\sqrt{\frac{b}{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b/x],x]
```

```
[Out] 2*Sqrt[b/x]*x
```

fricas [A] time = 0.88, size = 10, normalized size = 0.83

$$2\,x\sqrt{\frac{b}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*x*sqrt(b/x)
```

giac [A] time = 0.15, size = 9, normalized size = 0.75

$$2\sqrt{bx}\,\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x)^(1/2),x, algorithm="giac")
[Out] 2*sqrt(b*x)*sgn(x)
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$2\sqrt{\frac{b}{x}}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x)^(1/2),x)
[Out] 2*x*(b/x)^(1/2)
```

maxima [A] time = 1.28, size = 10, normalized size = 0.83

$$2\,x\sqrt{\frac{b}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x)^(1/2),x, algorithm="maxima")
[Out] 2*x*sqrt(b/x)
```

mupad [B] time = 0.97, size = 10, normalized size = 0.83

$$2\,x\,\sqrt{\frac{b}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x)^(1/2),x)
[Out] 2*x*(b/x)^(1/2)
```

sympy [A] time = 0.17, size = 14, normalized size = 1.17

$$2\sqrt{b}\,x\sqrt{\frac{1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x)**(1/2),x)
[Out] 2*sqrt(b)*x*sqrt(1/x)
```

3.81 $\int \sqrt{\frac{b}{x^2}}\, dx$

Optimal. Leaf size=13

$$x\sqrt{\frac{b}{x^2}}\,\log(x)$$

```
[Out] x*ln(x)*(b/x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 29}

$$x\sqrt{\frac{b}{x^2}}\,\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b/x^2],x]
```

```
[Out] Sqrt[b/x^2]*x*Log[x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{b}{x^2}}\, dx &= \left(\sqrt{\frac{b}{x^2}}\, x\right)\int \frac{1}{x}\, dx\\ &= \sqrt{\frac{b}{x^2}}\, x\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$x\sqrt{\frac{b}{x^2}}\,\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b/x^2],x]
```

```
[Out] Sqrt[b/x^2]*x*Log[x]
```

fricas [A] time = 0.84, size = 11, normalized size = 0.85

$$x\sqrt{\frac{b}{x^2}}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] x*sqrt(b/x^2)*log(x)
```

giac [A] time = 0.17, size = 9, normalized size = 0.69

$$\sqrt{b}\,\log\left(|x|\right)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x^2)^(1/2),x, algorithm="giac")
[Out] sqrt(b)*log(abs(x))*sgn(x)
```

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$\sqrt{\frac{b}{x^2}}\,x\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2)^(1/2),x)
[Out] x*ln(x)*(b/x^2)^(1/2)
```

maxima [A] time = 1.33, size = 11, normalized size = 0.85

$$x\sqrt{\frac{b}{x^2}}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x^2)^(1/2),x, algorithm="maxima")
[Out] x*sqrt(b/x^2)*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.08

$$\int \sqrt{\frac{b}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2)^(1/2),x)
[Out] int((b/x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{b}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x**2)**(1/2),x)
[Out] Integral(sqrt(b/x**2), x)
```

3.82 $\int \sqrt{\frac{b}{x^3}}\, dx$

Optimal. Leaf size=12

$$-2x\sqrt{\frac{b}{x^3}}$$

```
[Out] -2*x*(b/x^3)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$-2x\sqrt{\frac{b}{x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b/x^3],x]
```

```
[Out] -2*Sqrt[b/x^3]*x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{b}{x^3}}\, dx &= \left(\sqrt{\frac{b}{x^3}}\, x^{3/2}\right) \int \frac{1}{x^{3/2}}\, dx \\ &= -2\sqrt{\frac{b}{x^3}}\, x\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$-2x\sqrt{\frac{b}{x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b/x^3],x]
```

```
[Out] -2*Sqrt[b/x^3]*x
```

fricas [A] time = 0.84, size = 10, normalized size = 0.83

$$-2\,x\sqrt{\frac{b}{x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*x*sqrt(b/x^3)
```

giac [A] time = 0.15, size = 8, normalized size = 0.67

$$-\frac{2\,b}{\sqrt{bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x^3)^(1/2),x, algorithm="giac")
[Out] -2*b/sqrt(b*x)
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$-2\sqrt{\frac{b}{x^3}}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^3)^(1/2),x)
[Out] -2*x*(b/x^3)^(1/2)
```

maxima [A] time = 1.28, size = 10, normalized size = 0.83

$$-2\,x\sqrt{\frac{b}{x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x^3)^(1/2),x, algorithm="maxima")
[Out] -2*x*sqrt(b/x^3)
```

mupad [B] time = 0.98, size = 10, normalized size = 0.83

$$-2\,x\,\sqrt{\frac{b}{x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^3)^(1/2),x)
[Out] -2*x*(b/x^3)^(1/2)
```

sympy [A] time = 0.30, size = 17, normalized size = 1.42

$$-2\sqrt{b}\,x\sqrt{\frac{1}{x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x**3)**(1/2),x)
[Out] -2*sqrt(b)*x*sqrt(x**(-3))
```

3.83 $\int \left(bx^3\right)^{3/2}\, dx$

Optimal. Leaf size=17

$$\frac{2}{11} bx^4 \sqrt{bx^3}$$

```
[Out] 2/11*b*x^4*(b*x^3)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$\frac{2}{11} bx^4 \sqrt{bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^3)^(3/2),x]
```

```
[Out] (2*b*x^4*Sqrt[b*x^3])/11
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \left(bx^3\right)^{3/2}\, dx &= \frac{\left(b\sqrt{bx^3}\right)\int x^{9/2}\, dx}{x^{3/2}} \\ &= \frac{2}{11} bx^4 \sqrt{bx^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{2}{11} x \left(bx^3\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^3)^(3/2),x]
```

```
[Out] (2*x*(b*x^3)^(3/2))/11
```

fricas [A] time = 0.86, size = 13, normalized size = 0.76

$$\frac{2}{11} \sqrt{bx^3}\, bx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/11*sqrt(b*x^3)*b*x^4
```

giac [A] time = 0.15, size = 13, normalized size = 0.76

$$\frac{2}{11}\sqrt{bx}\,bx^5\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3)^(3/2),x, algorithm="giac")
[Out] 2/11*sqrt(b*x)*b*x^5*sgn(x)
```

maple [A] time = 0.00, size = 11, normalized size = 0.65

$$\frac{2\left(b\,x^3\right)^{\frac{3}{2}}x}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3)^(3/2),x)
[Out] 2/11*x*(b*x^3)^(3/2)
```

maxima [A] time = 1.29, size = 10, normalized size = 0.59

$$\frac{2}{11}\left(bx^3\right)^{\frac{3}{2}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3)^(3/2),x, algorithm="maxima")
[Out] 2/11*(b*x^3)^(3/2)*x
```

mupad [B] time = 0.95, size = 10, normalized size = 0.59

$$\frac{2\,x\left(b\,x^3\right)^{3/2}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3)^(3/2),x)
[Out] (2*x*(b*x^3)^(3/2))/11
```

sympy [A] time = 0.39, size = 15, normalized size = 0.88

$$\frac{2b^{\frac{3}{2}}x\left(x^3\right)^{\frac{3}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3)**(3/2),x)
[Out] 2*b**(3/2)*x*(x**3)**(3/2)/11
```

3.84 $\int \left(bx^2\right)^{3/2}\, dx$

Optimal. Leaf size=17

$$\frac{1}{4} bx^3 \sqrt{bx^2}$$

```
[Out] 1/4*b*x^3*(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$\frac{1}{4} bx^3 \sqrt{bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2)^(3/2),x]
```

```
[Out] (b*x^3*Sqrt[b*x^2])/4
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \left(bx^2\right)^{3/2}\, dx &= \frac{\left(b\sqrt{bx^2}\right)\int x^3\, dx}{x} \\ &= \frac{1}{4} bx^3 \sqrt{bx^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{4} x \left(bx^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2)^(3/2),x]
```

```
[Out] (x*(b*x^2)^(3/2))/4
```

fricas [A] time = 0.72, size = 13, normalized size = 0.76

$$\frac{1}{4} \sqrt{bx^2}\, bx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(b*x^2)*b*x^3
```

giac [A] time = 0.17, size = 10, normalized size = 0.59

$$\frac{1}{4} b^{\frac{3}{2}} x^4 \text{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(3/2),x, algorithm="giac")
[Out] 1/4*b^(3/2)*x^4*sgn(x)
```

maple [A] time = 0.00, size = 11, normalized size = 0.65

$$\frac{\left(b\,x^2\right)^{\frac{3}{2}} x}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(3/2),x)
[Out] 1/4*(b*x^2)^(3/2)*x
```

maxima [A] time = 1.31, size = 10, normalized size = 0.59

$$\frac{1}{4} \left(bx^2\right)^{\frac{3}{2}} x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(3/2),x, algorithm="maxima")
[Out] 1/4*(b*x^2)^(3/2)*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \left(b\,x^2\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(3/2),x)
[Out] int((b*x^2)^(3/2), x)
```

sympy [A] time = 0.39, size = 14, normalized size = 0.82

$$\frac{b^{\frac{3}{2}} x \left(x^2\right)^{\frac{3}{2}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2)**(3/2),x)
[Out] b**(3/2)*x*(x**2)**(3/2)/4
```

3.85 $\int (bx)^{3/2}\,dx$

Optimal. Leaf size=14

$$\frac{2(bx)^{5/2}}{5b}$$

```
[Out] 2/5*(b*x)^(5/2)/b
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{2(bx)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x)^(3/2),x]
```

```
[Out] (2*(b*x)^(5/2))/(5*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (bx)^{3/2}\,dx = \frac{2(bx)^{5/2}}{5b}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 0.86

$$\frac{2}{5}x(bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x)^(3/2),x]
```

```
[Out] (2*x*(b*x)^(3/2))/5
```

fricas [A] time = 0.63, size = 11, normalized size = 0.79

$$\frac{2}{5}\sqrt{bx}\,bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*sqrt(b*x)*b*x^2
```

giac [A] time = 0.18, size = 11, normalized size = 0.79

$$\frac{2}{5}\sqrt{bx}\,bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^(3/2),x, algorithm="giac")
```

```
[Out] 2/5*sqrt(b*x)*b*x^2
```

maple [A] time = 0.00, size = 9, normalized size = 0.64

$$\frac{2\,(bx)^{\frac{3}{2}}\,x}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^(3/2),x)
```

```
[Out] 2/5*x*(b*x)^(3/2)
```

maxima [A] time = 1.31, size = 10, normalized size = 0.71

$$\frac{2\,(bx)^{\frac{5}{2}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b*x)^(5/2)/b
```

mupad [B] time = 0.01, size = 10, normalized size = 0.71

$$\frac{2\,(b\,x)^{5/2}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^(3/2),x)
```

```
[Out] (2*(b*x)^(5/2))/(5*b)
```

sympy [A] time = 0.06, size = 10, normalized size = 0.71

$$\frac{2\,(bx)^{\frac{5}{2}}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)**(3/2),x)
```

```
[Out] 2*(b*x)**(5/2)/(5*b)
```

3.86 $\int \left(\frac{b}{x}\right)^{3/2} dx$

Optimal. Leaf size=12

$$-2b\sqrt{\frac{b}{x}}$$

```
[Out] -2*b*(b/x)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$-2b\sqrt{\frac{b}{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(b/x)^(3/2),x]
```

```
[Out] -2*b*Sqrt[b/x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \left(\frac{b}{x}\right)^{3/2} dx = \left(b\sqrt{\frac{b}{x}}\,\sqrt{x}\right)\int \frac{1}{x^{3/2}}\,dx$$
$$= -2b\sqrt{\frac{b}{x}}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$-2x\left(\frac{b}{x}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x)^(3/2),x]
```

```
[Out] -2*(b/x)^(3/2)*x
```

fricas [A] time = 0.81, size = 10, normalized size = 0.83

$$-2\,b\sqrt{\frac{b}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x)^(3/2),x, algorithm="fricas")
[Out] -2*b*sqrt(b/x)
```

giac [A] time = 0.16, size = 12, normalized size = 1.00

$$-\frac{2\,b^2\mathrm{sgn}(x)}{\sqrt{bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x)^(3/2),x, algorithm="giac")
[Out] -2*b^2*sgn(x)/sqrt(b*x)
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$-2\left(\frac{b}{x}\right)^{\frac{3}{2}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x)^(3/2),x)
[Out] -2*x*(b/x)^(3/2)
```

maxima [A] time = 1.32, size = 10, normalized size = 0.83

$$-2\,x\left(\frac{b}{x}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x)^(3/2),x, algorithm="maxima")
[Out] -2*x*(b/x)^(3/2)
```

mupad [B] time = 0.93, size = 10, normalized size = 0.83

$$-2\,b\sqrt{\frac{b}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x)^(3/2),x)
[Out] -2*b*(b/x)^(1/2)
```

sympy [A] time = 0.42, size = 15, normalized size = 1.25

$$-2b^{\frac{3}{2}}x\left(\frac{1}{x}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x)**(3/2),x)
[Out] -2*b**(3/2)*x*(1/x)**(3/2)
```

3.87 $\int \left(\frac{b}{x^2}\right)^{3/2} dx$

Optimal. Leaf size=17

$$-\frac{b\sqrt{\frac{b}{x^2}}}{2x}$$

```
[Out] -1/2*b*(b/x^2)^(1/2)/x
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$-\frac{b\sqrt{\frac{b}{x^2}}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(b/x^2)^(3/2),x]
```

```
[Out] -(b*Sqrt[b/x^2])/(2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \left(\frac{b}{x^2}\right)^{3/2} dx = \left(b\sqrt{\frac{b}{x^2}}\, x\right) \int \frac{1}{x^3}\, dx$$
$$= -\frac{b\sqrt{\frac{b}{x^2}}}{2x}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 0.82

$$-\frac{1}{2} x \left(\frac{b}{x^2}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x^2)^(3/2),x]
```

```
[Out] -1/2*((b/x^2)^(3/2)*x)
```

fricas [A] time = 0.80, size = 13, normalized size = 0.76

$$-\frac{b\sqrt{\frac{b}{x^2}}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x^2)^(3/2),x, algorithm="fricas")
[Out] -1/2*b*sqrt(b/x^2)/x
```

giac [A] time = 0.15, size = 10, normalized size = 0.59

$$-\frac{b^{\frac{3}{2}}\operatorname{sgn}(x)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x^2)^(3/2),x, algorithm="giac")
[Out] -1/2*b^(3/2)*sgn(x)/x^2
```

maple [A] time = 0.00, size = 11, normalized size = 0.65

$$-\frac{\left(\frac{b}{x^2}\right)^{\frac{3}{2}} x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2)^(3/2),x)
[Out] -1/2*x*(b/x^2)^(3/2)
```

maxima [A] time = 1.30, size = 10, normalized size = 0.59

$$-\frac{1}{2}\,x\left(\frac{b}{x^2}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x^2)^(3/2),x, algorithm="maxima")
[Out] -1/2*x*(b/x^2)^(3/2)
```

mupad [B] time = 0.98, size = 13, normalized size = 0.76

$$-\frac{b^{3/2}\sqrt{\frac{1}{x^2}}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2)^(3/2),x)
[Out] -(b^(3/2)*(1/x^2)^(1/2))/(2*x)
```

sympy [A] time = 0.53, size = 17, normalized size = 1.00

$$-\frac{b^{\frac{3}{2}}x\left(\frac{1}{x^2}\right)^{\frac{3}{2}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x**2)**(3/2),x)
[Out] -b**(3/2)*x*(x**(-2))**(3/2)/2
```

3.88 $\int \left(\frac{b}{x^3}\right)^{3/2} dx$

Optimal. Leaf size=17

$$-\frac{2b\sqrt{\frac{b}{x^3}}}{7x^2}$$

```
[Out] -2/7*b*(b/x^3)^(1/2)/x^2
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$-\frac{2b\sqrt{\frac{b}{x^3}}}{7x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b/x^3)^(3/2),x]
```

```
[Out] (-2*b*Sqrt[b/x^3])/(7*x^2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \left(\frac{b}{x^3}\right)^{3/2} dx = \left(b\sqrt{\frac{b}{x^3}}\, x^{3/2}\right) \int \frac{1}{x^{9/2}}\, dx$$
$$= -\frac{2b\sqrt{\frac{b}{x^3}}}{7x^2}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 0.82

$$-\frac{2}{7}x\left(\frac{b}{x^3}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x^3)^(3/2),x]
```

```
[Out] (-2*(b/x^3)^(3/2)*x)/7
```

fricas [A] time = 0.78, size = 13, normalized size = 0.76

$$-\frac{2\,b\sqrt{\frac{b}{x^3}}}{7\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x^3)^(3/2),x, algorithm="fricas")
[Out] -2/7*b*sqrt(b/x^3)/x^2
```

giac [A] time = 0.15, size = 13, normalized size = 0.76

$$-\frac{2\,b^2}{7\sqrt{bx}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x^3)^(3/2),x, algorithm="giac")
[Out] -2/7*b^2/(sqrt(b*x)*x^3)
```

maple [A] time = 0.00, size = 11, normalized size = 0.65

$$-\frac{2\left(\frac{b}{x^3}\right)^{\frac{3}{2}}x}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^3)^(3/2),x)
[Out] -2/7*x*(b/x^3)^(3/2)
```

maxima [A] time = 1.34, size = 10, normalized size = 0.59

$$-\frac{2}{7}\,x\left(\frac{b}{x^3}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x^3)^(3/2),x, algorithm="maxima")
[Out] -2/7*x*(b/x^3)^(3/2)
```

mupad [B] time = 0.97, size = 13, normalized size = 0.76

$$-\frac{2\,b\sqrt{\frac{b}{x^3}}}{7\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^3)^(3/2),x)
[Out] -(2*b*(b/x^3)^(1/2))/(7*x^2)
```

sympy [A] time = 0.54, size = 19, normalized size = 1.12

$$-\frac{2b^{\frac{3}{2}}x\left(\frac{1}{x^3}\right)^{\frac{3}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x**3)**(3/2),x)
[Out] -2*b**(3/2)*x*(x**(-3))**(3/2)/7
```

3.89 $\int \frac{1}{\sqrt{bx^3}}\,dx$

Optimal. Leaf size=12

$$-\frac{2x}{\sqrt{bx^3}}$$

```
[Out] -2*x/(b*x^3)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$-\frac{2x}{\sqrt{bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*x^3],x]
```

```
[Out] (-2*x)/Sqrt[b*x^3]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{bx^3}}\,dx &= \frac{x^{3/2}\int \frac{1}{x^{3/2}}\,dx}{\sqrt{bx^3}}\\ &= -\frac{2x}{\sqrt{bx^3}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$-\frac{2x}{\sqrt{bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*x^3],x]
```

```
[Out] (-2*x)/Sqrt[b*x^3]
```

fricas [A] time = 0.83, size = 15, normalized size = 1.25

$$-\frac{2\sqrt{bx^3}}{bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(b*x^3)/(b*x^2)
```

giac [A] time = 0.15, size = 11, normalized size = 0.92

$$-\frac{2}{\sqrt{bx}\,\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3)^(1/2),x, algorithm="giac")
[Out] -2/(sqrt(b*x)*sgn(x))
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$-\frac{2x}{\sqrt{b\,x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3)^(1/2),x)
[Out] -2*x/(b*x^3)^(1/2)
```

maxima [A] time = 1.32, size = 10, normalized size = 0.83

$$-\frac{2\,x}{\sqrt{bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3)^(1/2),x, algorithm="maxima")
[Out] -2*x/sqrt(b*x^3)
```

mupad [B] time = 0.94, size = 15, normalized size = 1.25

$$-\frac{2\sqrt{b\,x^3}}{b\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3)^(1/2),x)
[Out] -(2*(b*x^3)^(1/2))/(b*x^2)
```

sympy [A] time = 0.40, size = 15, normalized size = 1.25

$$-\frac{2x}{\sqrt{b}\,\sqrt{x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3)**(1/2),x)
[Out] -2*x/(sqrt(b)*sqrt(x**3))
```

3.90 $\int \frac{1}{\sqrt{bx^2}}\,dx$

Optimal. Leaf size=13

$$\frac{x\log(x)}{\sqrt{bx^2}}$$

```
[Out] x*ln(x)/(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 29}

$$\frac{x\log(x)}{\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*x^2],x]
```

```
[Out] (x*Log[x])/Sqrt[b*x^2]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{bx^2}}\,dx &= \frac{x\int \frac{1}{x}\,dx}{\sqrt{bx^2}}\\ &= \frac{x\log(x)}{\sqrt{bx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{x\log(x)}{\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*x^2],x]
```

```
[Out] (x*Log[x])/Sqrt[b*x^2]
```

fricas [A] time = 0.81, size = 16, normalized size = 1.23

$$\frac{\sqrt{bx^2}\,\log(x)}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*x^2)*log(x)/(b*x)
```

giac [A] time = 0.20, size = 19, normalized size = 1.46

$$\frac{\log\left(\sqrt{|b|}\,|x|\left|\operatorname{sgn}(x)\right|\right)}{\sqrt{b}\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2)^(1/2),x, algorithm="giac")
[Out] log(sqrt(abs(b))*abs(x)*abs(sgn(x)))/(sqrt(b)*sgn(x))
```

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$\frac{x\ln(x)}{\sqrt{b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2)^(1/2),x)
[Out] 1/(b*x^2)^(1/2)*x*ln(x)
```

maxima [A] time = 1.31, size = 6, normalized size = 0.46

$$\frac{\log(x)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2)^(1/2),x, algorithm="maxima")
[Out] log(x)/sqrt(b)
```

mupad [B] time = 0.00, size = 10, normalized size = 0.77

$$\frac{\ln(b\,x)\ \operatorname{sign}(x)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2)^(1/2),x)
[Out] (log(b*x)*sign(x))/b^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2)**(1/2),x)
[Out] Integral(1/sqrt(b*x**2), x)
```

3.91 $\int \frac{1}{\sqrt{bx}}\,dx$

Optimal. Leaf size=12

$$\frac{2\sqrt{bx}}{b}$$

```
[Out] 2*(b*x)^(1/2)/b
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{2\sqrt{bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*x],x]
```

```
[Out] (2*Sqrt[b*x])/b
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{bx}}\,dx = \frac{2\sqrt{bx}}{b}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 0.83

$$\frac{2x}{\sqrt{bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*x],x]
```

```
[Out] (2*x)/Sqrt[b*x]
```

fricas [A] time = 0.80, size = 10, normalized size = 0.83

$$\frac{2\sqrt{bx}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(b*x)/b
```

giac [A] time = 0.22, size = 10, normalized size = 0.83

$$\frac{2\sqrt{bx}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(1/2),x, algorithm="giac")
[Out] 2*sqrt(b*x)/b
```

maple [A] time = 0.00, size = 9, normalized size = 0.75

$$\frac{2x}{\sqrt{bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x)^(1/2),x)
[Out] 2*x/(b*x)^(1/2)
```

maxima [A] time = 1.31, size = 10, normalized size = 0.83

$$\frac{2\sqrt{bx}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(b*x)/b
```

mupad [B] time = 0.01, size = 10, normalized size = 0.83

$$\frac{2\sqrt{b\,x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x)^(1/2),x)
[Out] (2*(b*x)^(1/2))/b
```

sympy [A] time = 0.06, size = 8, normalized size = 0.67

$$\frac{2\sqrt{bx}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)**(1/2),x)
[Out] 2*sqrt(b*x)/b
```

3.92 $\int \frac{1}{\sqrt{\frac{b}{x}}}\,dx$

Optimal. Leaf size=14

$$\frac{2x}{3\sqrt{\frac{b}{x}}}$$

```
[Out] 2/3*x/(b/x)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$\frac{2x}{3\sqrt{\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b/x],x]
```

```
[Out] (2*x)/(3*Sqrt[b/x])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\frac{b}{x}}}\,dx &= \frac{\int \sqrt{x}\,dx}{\sqrt{\frac{b}{x}}\,\sqrt{x}}\\ &= \frac{2x}{3\sqrt{\frac{b}{x}}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{2x}{3\sqrt{\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b/x],x]
```

```
[Out] (2*x)/(3*Sqrt[b/x])
```

fricas [A] time = 0.88, size = 15, normalized size = 1.07

$$\frac{2\,x^2\sqrt{\frac{b}{x}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x)^(1/2),x, algorithm="fricas")
[Out] 2/3*x^2*sqrt(b/x)/b
```

giac [A] time = 0.15, size = 15, normalized size = 1.07

$$\frac{2\sqrt{bx}\,x}{3\,b\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x)^(1/2),x, algorithm="giac")
[Out] 2/3*sqrt(b*x)*x/(b*sgn(x))
```

maple [A] time = 0.00, size = 11, normalized size = 0.79

$$\frac{2x}{3\sqrt{\frac{b}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x)^(1/2),x)
[Out] 2/3*x/(b/x)^(1/2)
```

maxima [A] time = 1.33, size = 10, normalized size = 0.71

$$\frac{2\,x}{3\,\sqrt{\frac{b}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x)^(1/2),x, algorithm="maxima")
[Out] 2/3*x/sqrt(b/x)
```

mupad [B] time = 0.95, size = 15, normalized size = 1.07

$$\frac{2\,x^2\,\sqrt{\frac{b}{x}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x)^(1/2),x)
[Out] (2*x^2*(b/x)^(1/2))/(3*b)
```

sympy [A] time = 0.39, size = 15, normalized size = 1.07

$$\frac{2x}{3\sqrt{b}\,\sqrt{\frac{1}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x)**(1/2),x)
[Out] 2*x/(3*sqrt(b)*sqrt(1/x))
```

3.93 $\int \frac{1}{\sqrt{\frac{b}{x^2}}}\,dx$

Optimal. Leaf size=14

$$\frac{x}{2\sqrt{\frac{b}{x^2}}}$$

```
[Out] 1/2*x/(b/x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$\frac{x}{2\sqrt{\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b/x^2],x]
```

```
[Out] x/(2*Sqrt[b/x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\frac{b}{x^2}}}\,dx &= \frac{\int x\,dx}{\sqrt{\frac{b}{x^2}}\,x}\\ &= \frac{x}{2\sqrt{\frac{b}{x^2}}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{x}{2\sqrt{\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b/x^2],x]
```

```
[Out] x/(2*Sqrt[b/x^2])
```

fricas [A] time = 0.72, size = 15, normalized size = 1.07

$$\frac{x^3\sqrt{\frac{b}{x^2}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*x^3*sqrt(b/x^2)/b
```

giac [A] time = 0.19, size = 12, normalized size = 0.86

$$\frac{x^2}{2\sqrt{b}\,\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^2)^(1/2),x, algorithm="giac")
[Out] 1/2*x^2/(sqrt(b)*sgn(x))
```

maple [A] time = 0.00, size = 11, normalized size = 0.79

$$\frac{x}{2\sqrt{\frac{b}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x^2)^(1/2),x)
[Out] 1/2*x/(b/x^2)^(1/2)
```

maxima [A] time = 1.30, size = 10, normalized size = 0.71

$$\frac{x}{2\sqrt{\frac{b}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*x/sqrt(b/x^2)
```

mupad [B] time = 0.93, size = 13, normalized size = 0.93

$$\frac{x^3\sqrt{\frac{1}{x^2}}}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x^2)^(1/2),x)
[Out] (x^3*(1/x^2)^(1/2))/(2*b^(1/2))
```

sympy [A] time = 0.39, size = 15, normalized size = 1.07

$$\frac{x}{2\sqrt{b}\sqrt{\frac{1}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x**2)**(1/2),x)
[Out] x/(2*sqrt(b)*sqrt(x**(-2)))
```

3.94 $\int \frac{1}{\sqrt{\frac{b}{x^3}}}\,dx$

Optimal. Leaf size=14

$$\frac{2x}{5\sqrt{\frac{b}{x^3}}}$$

```
[Out] 2/5*x/(b/x^3)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$\frac{2x}{5\sqrt{\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b/x^3],x]
```

```
[Out] (2*x)/(5*Sqrt[b/x^3])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\frac{b}{x^3}}}\,dx &= \frac{\int x^{3/2}\,dx}{\sqrt{\frac{b}{x^3}}\,x^{3/2}}\\ &= \frac{2x}{5\sqrt{\frac{b}{x^3}}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{2x}{5\sqrt{\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b/x^3],x]
```

```
[Out] (2*x)/(5*Sqrt[b/x^3])
```

fricas [A] time = 0.86, size = 15, normalized size = 1.07

$$\frac{2\,x^4\sqrt{\frac{b}{x^3}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^3)^(1/2),x, algorithm="fricas")
[Out] 2/5*x^4*sqrt(b/x^3)/b
```

giac [A] time = 0.17, size = 13, normalized size = 0.93

$$\frac{2\sqrt{bx}\,x^2}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^3)^(1/2),x, algorithm="giac")
[Out] 2/5*sqrt(b*x)*x^2/b
```

maple [A] time = 0.00, size = 11, normalized size = 0.79

$$\frac{2x}{5\sqrt{\frac{b}{x^3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x^3)^(1/2),x)
[Out] 2/5*x/(b/x^3)^(1/2)
```

maxima [A] time = 1.26, size = 10, normalized size = 0.71

$$\frac{2\,x}{5\,\sqrt{\frac{b}{x^3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^3)^(1/2),x, algorithm="maxima")
[Out] 2/5*x/sqrt(b/x^3)
```

mupad [B] time = 0.95, size = 15, normalized size = 1.07

$$\frac{2\,x^4\,\sqrt{\frac{b}{x^3}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x^3)^(1/2),x)
[Out] (2*x^4*(b/x^3)^(1/2))/(5*b)
```

sympy [A] time = 0.41, size = 17, normalized size = 1.21

$$\frac{2x}{5\sqrt{b}\,\sqrt{\frac{1}{x^3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x**3)**(1/2),x)
[Out] 2*x/(5*sqrt(b)*sqrt(x**(-3)))
```

3.95 $\int \frac{1}{\left(bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=19

$$-\frac{2}{7bx^2\sqrt{bx^3}}$$

```
[Out] -2/7/b/x^2/(b*x^3)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$-\frac{2}{7bx^2\sqrt{bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^3)^(-3/2),x]
```

```
[Out] -2/(7*b*x^2*Sqrt[b*x^3])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(bx^3\right)^{3/2}}\,dx &= \frac{x^{3/2}\int \frac{1}{x^{9/2}}\,dx}{b\sqrt{bx^3}}\\ &= -\frac{2}{7bx^2\sqrt{bx^3}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 0.74

$$-\frac{2x}{7\left(bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^3)^(-3/2),x]
```

```
[Out] (-2*x)/(7*(b*x^3)^(3/2))
```

fricas [A] time = 0.89, size = 15, normalized size = 0.79

$$-\frac{2\sqrt{bx^3}}{7b^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3)^(3/2),x, algorithm="fricas")
[Out] -2/7*sqrt(b*x^3)/(b^2*x^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.00, size = 11, normalized size = 0.58

$$-\frac{2x}{7\left(b\,x^3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3)^(3/2),x)
[Out] -2/7*x/(b*x^3)^(3/2)
```

maxima [A] time = 1.32, size = 10, normalized size = 0.53

$$-\frac{2\,x}{7\,\left(bx^3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3)^(3/2),x, algorithm="maxima")
[Out] -2/7*x/(b*x^3)^(3/2)
```

mupad [B] time = 0.94, size = 15, normalized size = 0.79

$$-\frac{2\,\sqrt{b\,x^3}}{7\,b^2\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3)^(3/2),x)
[Out] -(2*(b*x^3)^(1/2))/(7*b^2*x^5)
```

sympy [A] time = 0.51, size = 17, normalized size = 0.89

$$-\frac{2x}{7b^{\frac{3}{2}}\left(x^3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3)**(3/2),x)
[Out] -2*x/(7*b**(3/2)*(x**3)**(3/2))
```

3.96 $\int \frac{1}{\left(bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=19

$$-\frac{1}{2bx\sqrt{bx^2}}$$

```
[Out] -1/2/b/x/(b*x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$-\frac{1}{2bx\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2)^(-3/2),x]
```

```
[Out] -1/(2*b*x*Sqrt[b*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(bx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{1}{x^3}\,dx}{b\sqrt{bx^2}}\\ &= -\frac{1}{2bx\sqrt{bx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 0.74

$$-\frac{x}{2\left(bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2)^(-3/2),x]
```

```
[Out] -1/2*x/(b*x^2)^(3/2)
```

fricas [A] time = 0.76, size = 15, normalized size = 0.79

$$-\frac{\sqrt{bx^2}}{2\,b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2)^(3/2),x, algorithm="fricas")
[Out] -1/2*sqrt(b*x^2)/(b^2*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.00, size = 11, normalized size = 0.58

$$-\frac{x}{2\left(b\,x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2)^(3/2),x)
[Out] -1/2/(b*x^2)^(3/2)*x
```

maxima [A] time = 1.22, size = 8, normalized size = 0.42

$$-\frac{1}{2\,b^{\frac{3}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2)^(3/2),x, algorithm="maxima")
[Out] -1/2/(b^(3/2)*x^2)
```

mupad [B] time = 0.00, size = 13, normalized size = 0.68

$$-\frac{1}{2\,b^{3/2}\,x\,\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2)^(3/2),x)
[Out] -1/(2*b^(3/2)*x*(x^2)^(1/2))
```

sympy [A] time = 0.51, size = 15, normalized size = 0.79

$$-\frac{x}{2b^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2)**(3/2),x)
[Out] -x/(2*b**(3/2)*(x**2)**(3/2))
```

3.97 $\int \frac{1}{(bx)^{3/2}}\,dx$

Optimal. Leaf size=12

$$-\frac{2}{b\sqrt{bx}}$$

```
[Out] -2/b/(b*x)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$-\frac{2}{b\sqrt{bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x)^(-3/2),x]
```

```
[Out] -2/(b*Sqrt[b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(bx)^{3/2}}\,dx = -\frac{2}{b\sqrt{bx}}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 0.83

$$-\frac{2x}{(bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x)^(-3/2),x]
```

```
[Out] (-2*x)/(b*x)^(3/2)
```

fricas [A] time = 0.95, size = 13, normalized size = 1.08

$$-\frac{2\sqrt{bx}}{b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(b*x)/(b^2*x)
```

giac [A] time = 0.15, size = 10, normalized size = 0.83

$$-\frac{2}{\sqrt{bx}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(3/2),x, algorithm="giac")
[Out] -2/(sqrt(b*x)*b)
```

maple [A] time = 0.00, size = 9, normalized size = 0.75

$$-\frac{2x}{(bx)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x)^(3/2),x)
[Out] -2*x/(b*x)^(3/2)
```

maxima [A] time = 1.30, size = 10, normalized size = 0.83

$$-\frac{2}{\sqrt{bx}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(3/2),x, algorithm="maxima")
[Out] -2/(sqrt(b*x)*b)
```

mupad [B] time = 0.02, size = 10, normalized size = 0.83

$$-\frac{2}{b\sqrt{b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x)^(3/2),x)
[Out] -2/(b*(b*x)^(1/2))
```

sympy [A] time = 0.06, size = 10, normalized size = 0.83

$$-\frac{2}{b\sqrt{bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)**(3/2),x)
[Out] -2/(b*sqrt(b*x))
```

3.98 $\int \frac{1}{\left(\frac{b}{x}\right)^{3/2}}\,dx$

Optimal. Leaf size=19

$$\frac{2x^2}{5b\sqrt{\frac{b}{x}}}$$

```
[Out] 2/5*x^2/b/(b/x)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$\frac{2x^2}{5b\sqrt{\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(b/x)^(-3/2),x]
```

```
[Out] (2*x^2)/(5*b*Sqrt[b/x])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{b}{x}\right)^{3/2}}\,dx &= \frac{\int x^{3/2}\,dx}{b\sqrt{\frac{b}{x}}\,\sqrt{x}}\\ &= \frac{2x^2}{5b\sqrt{\frac{b}{x}}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 0.74

$$\frac{2x}{5\left(\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x)^(-3/2),x]
```

```
[Out] (2*x)/(5*(b/x)^(3/2))
```

fricas [A] time = 0.92, size = 15, normalized size = 0.79

$$\frac{2\,x^3\sqrt{\frac{b}{x}}}{5\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x)^(3/2),x, algorithm="fricas")
[Out] 2/5*x^3*sqrt(b/x)/b^2
```

giac [A] time = 0.17, size = 15, normalized size = 0.79

$$\frac{2\,x^2}{5\,b\sqrt{\frac{b}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x)^(3/2),x, algorithm="giac")
[Out] 2/5*x^2/(b*sqrt(b/x))
```

maple [A] time = 0.00, size = 11, normalized size = 0.58

$$\frac{2x}{5\left(\frac{b}{x}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x)^(3/2),x)
[Out] 2/5*x/(b/x)^(3/2)
```

maxima [A] time = 1.32, size = 10, normalized size = 0.53

$$\frac{2\,x}{5\,\left(\frac{b}{x}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x)^(3/2),x, algorithm="maxima")
[Out] 2/5*x/(b/x)^(3/2)
```

mupad [B] time = 0.94, size = 15, normalized size = 0.79

$$\frac{2\,x^3\,\sqrt{\frac{b}{x}}}{5\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x)^(3/2),x)
[Out] (2*x^3*(b/x)^(1/2))/(5*b^2)
```

sympy [A] time = 0.50, size = 15, normalized size = 0.79

$$\frac{2x}{5b^{\frac{3}{2}}\left(\frac{1}{x}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x)**(3/2),x)
[Out] 2*x/(5*b**(3/2)*(1/x)**(3/2))
```

3.99 $\int \frac{1}{\left(\frac{b}{x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=19

$$\frac{x^3}{4b\sqrt{\frac{b}{x^2}}}$$

```
[Out] 1/4*x^3/b/(b/x^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$\frac{x^3}{4b\sqrt{\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(b/x^2)^(-3/2),x]
```

```
[Out] x^3/(4*b*Sqrt[b/x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{b}{x^2}\right)^{3/2}}\,dx &= \frac{\int x^3\,dx}{b\sqrt{\frac{b}{x^2}}\,x}\\ &= \frac{x^3}{4b\sqrt{\frac{b}{x^2}}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 0.74

$$\frac{x}{4\left(\frac{b}{x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x^2)^(-3/2),x]
```

```
[Out] x/(4*(b/x^2)^(3/2))
```

fricas [A] time = 0.66, size = 15, normalized size = 0.79

$$\frac{x^5\sqrt{\frac{b}{x^2}}}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^2)^(3/2),x, algorithm="fricas")
[Out] 1/4*x^5*sqrt(b/x^2)/b^2
```

giac [A] time = 0.16, size = 8, normalized size = 0.42

$$\frac{x^4}{4\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^2)^(3/2),x, algorithm="giac")
[Out] 1/4*x^4/b^(3/2)
```

maple [A] time = 0.00, size = 11, normalized size = 0.58

$$\frac{x}{4\left(\frac{b}{x^2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x^2)^(3/2),x)
[Out] 1/4*x/(b/x^2)^(3/2)
```

maxima [A] time = 1.32, size = 10, normalized size = 0.53

$$\frac{x}{4\left(\frac{b}{x^2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^2)^(3/2),x, algorithm="maxima")
[Out] 1/4*x/(b/x^2)^(3/2)
```

mupad [B] time = 0.93, size = 13, normalized size = 0.68

$$\frac{x^5\sqrt{\frac{1}{x^2}}}{4\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x^2)^(3/2),x)
[Out] (x^5*(1/x^2)^(1/2))/(4*b^(3/2))
```

sympy [A] time = 0.51, size = 15, normalized size = 0.79

$$\frac{x}{4b^{\frac{3}{2}}\left(\frac{1}{x^2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x**2)**(3/2),x)
[Out] x/(4*b**(3/2)*(x**(-2))**(3/2))
```

3.100 $\int \frac{1}{\left(\frac{b}{x^3}\right)^{3/2}}\,dx$

Optimal. Leaf size=19

$$\frac{2x^4}{11b\sqrt{\frac{b}{x^3}}}$$

```
[Out] 2/11*x^4/b/(b/x^3)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$\frac{2x^4}{11b\sqrt{\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(b/x^3)^(-3/2),x]
```

```
[Out] (2*x^4)/(11*b*Sqrt[b/x^3])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{b}{x^3}\right)^{3/2}}\,dx &= \frac{\int x^{9/2}\,dx}{b\sqrt{\frac{b}{x^3}}\,x^{3/2}}\\ &= \frac{2x^4}{11b\sqrt{\frac{b}{x^3}}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 0.74

$$\frac{2x}{11\left(\frac{b}{x^3}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x^3)^(-3/2),x]
```

```
[Out] (2*x)/(11*(b/x^3)^(3/2))
```

fricas [A] time = 0.82, size = 15, normalized size = 0.79

$$\frac{2\,x^7\sqrt{\frac{b}{x^3}}}{11\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^3)^(3/2),x, algorithm="fricas")
[Out] 2/11*x^7*sqrt(b/x^3)/b^2
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{b}{x^3}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^3)^(3/2),x, algorithm="giac")
[Out] integrate((b/x^3)^(-3/2), x)
```

maple [**A**] time = 0.00, size = 11, normalized size = 0.58

$$\frac{2x}{11\left(\frac{b}{x^3}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x^3)^(3/2),x)
[Out] 2/11*x/(b/x^3)^(3/2)
```

maxima [**A**] time = 1.28, size = 10, normalized size = 0.53

$$\frac{2\,x}{11\,\left(\frac{b}{x^3}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^3)^(3/2),x, algorithm="maxima")
[Out] 2/11*x/(b/x^3)^(3/2)
```

mupad [**B**] time = 0.95, size = 15, normalized size = 0.79

$$\frac{2\,x^7\,\sqrt{\frac{b}{x^3}}}{11\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x^3)^(3/2),x)
[Out] (2*x^7*(b/x^3)^(1/2))/(11*b^2)
```

sympy [**A**] time = 0.51, size = 17, normalized size = 0.89

$$\frac{2x}{11b^{\frac{3}{2}}\left(\frac{1}{x^3}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x**3)**(3/2),x)
[Out] 2*x/(11*b**(3/2)*(x**(-3))**(3/2))
```

3.101 $\int \sqrt[3]{bx^n}\, dx$

Optimal. Leaf size=17

$$\frac{3x\sqrt[3]{bx^n}}{n+3}$$

```
[Out] 3*x*(b*x^n)^(1/3)/(3+n)
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$\frac{3x\sqrt[3]{bx^n}}{n+3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^n)^(1/3),x]
```

```
[Out] (3*x*(b*x^n)^(1/3))/(3 + n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{bx^n}\, dx &= \left(x^{-n/3}\sqrt[3]{bx^n}\right)\int x^{n/3}\, dx \\ &= \frac{3x\sqrt[3]{bx^n}}{3+n}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{3x\sqrt[3]{bx^n}}{n+3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^n)^(1/3),x]
```

```
[Out] (3*x*(b*x^n)^(1/3))/(3 + n)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^(1/3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^(1/3),x, algorithm="giac")
[Out] integrate((b*x^n)^(1/3), x)
```

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$\frac{3\,(b\,x^n)^{\frac{1}{3}}\,x}{n+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^(1/3),x)
[Out] 3*x*(b*x^n)^(1/3)/(3+n)
```

maxima [A] time = 1.33, size = 15, normalized size = 0.88

$$\frac{3\ (bx^n)^{\frac{1}{3}}\,x}{n+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^(1/3),x, algorithm="maxima")
[Out] 3*(b*x^n)^(1/3)*x/(n + 3)
```

mupad [B] time = 1.00, size = 15, normalized size = 0.88

$$\frac{3\,x\,(b\,x^n)^{1/3}}{n+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^(1/3),x)
[Out] (3*x*(b*x^n)^(1/3))/(n + 3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{3\sqrt[3]{b}\,x\,\sqrt[3]{x^n}}{n+3} & \text{for } n \neq -3 \\ \int \sqrt[3]{\frac{b}{x^3}}\, dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**n)**(1/3),x)
[Out] Piecewise((3*b**(1/3)*x*(x**n)**(1/3)/(n + 3), Ne(n, -3)), (Integral((b/x**
3)**(1/3), x), True))
```

3.102 $\int \sqrt[3]{bx^3}\, dx$

Optimal. Leaf size=14

$$\frac{1}{2}x\sqrt[3]{bx^3}$$

```
[Out] 1/2*x*(b*x^3)^(1/3)
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$\frac{1}{2}x\sqrt[3]{bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^3)^(1/3),x]
```

```
[Out] (x*(b*x^3)^(1/3))/2
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{bx^3}\, dx &= \frac{\sqrt[3]{bx^3}\int x\, dx}{x}\\ &= \frac{1}{2}x\sqrt[3]{bx^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{1}{2}x\sqrt[3]{bx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^3)^(1/3),x]
```

```
[Out] (x*(b*x^3)^(1/3))/2
```

fricas [A] time = 0.88, size = 10, normalized size = 0.71

$$\frac{1}{2}\left(bx^3\right)^{\frac{1}{3}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^3)^(1/3)*x
```

giac [A] time = 0.15, size = 8, normalized size = 0.57

$$\frac{1}{2} b^{\frac{1}{3}} x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3)^(1/3),x, algorithm="giac")
[Out] 1/2*b^(1/3)*x^2
```

maple [A] time = 0.00, size = 11, normalized size = 0.79

$$\frac{\left(b\,x^3\right)^{\frac{1}{3}} x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3)^(1/3),x)
[Out] 1/2*x*(b*x^3)^(1/3)
```

maxima [A] time = 1.31, size = 10, normalized size = 0.71

$$\frac{1}{2}\left(bx^3\right)^{\frac{1}{3}} x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3)^(1/3),x, algorithm="maxima")
[Out] 1/2*(b*x^3)^(1/3)*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.07

$$\int \left(b\,x^3\right)^{1/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3)^(1/3),x)
[Out] int((b*x^3)^(1/3), x)
```

sympy [A] time = 0.19, size = 14, normalized size = 1.00

$$\frac{\sqrt[3]{b}\, x \sqrt[3]{x^3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3)**(1/3),x)
[Out] b**(1/3)*x*(x**3)**(1/3)/2
```

3.103 $\int \sqrt[3]{bx^2}\, dx$

Optimal. Leaf size=14

$$\frac{3}{5}x\sqrt[3]{bx^2}$$

```
[Out] 3/5*x*(b*x^2)^(1/3)
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$\frac{3}{5}x\sqrt[3]{bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2)^(1/3),x]
```

```
[Out] (3*x*(b*x^2)^(1/3))/5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{bx^2}\, dx &= \frac{\sqrt[3]{bx^2}\int x^{2/3}\, dx}{x^{2/3}}\\ &= \frac{3}{5}x\sqrt[3]{bx^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{3}{5}x\sqrt[3]{bx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2)^(1/3),x]
```

```
[Out] (3*x*(b*x^2)^(1/3))/5
```

fricas [A] time = 0.85, size = 10, normalized size = 0.71

$$\frac{3}{5}\left(bx^2\right)^{\frac{1}{3}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/5*(b*x^2)^(1/3)*x
```

giac [A] time = 0.16, size = 11, normalized size = 0.79

$$\frac{3\,bx}{5\left(\frac{b}{x}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(1/3),x, algorithm="giac")
[Out] 3/5*b*x/(b/x)^(2/3)
```

maple [A] time = 0.00, size = 11, normalized size = 0.79

$$\frac{3\left(b\,x^2\right)^{\frac{1}{3}}x}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(1/3),x)
[Out] 3/5*x*(b*x^2)^(1/3)
```

maxima [A] time = 1.33, size = 10, normalized size = 0.71

$$\frac{3}{5}\left(bx^2\right)^{\frac{1}{3}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(1/3),x, algorithm="maxima")
[Out] 3/5*(b*x^2)^(1/3)*x
```

mupad [B] time = 0.92, size = 11, normalized size = 0.79

$$\frac{3\,b^{1/3}\,x\left(x^2\right)^{1/3}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(1/3),x)
[Out] (3*b^(1/3)*x*(x^2)^(1/3))/5
```

sympy [A] time = 0.20, size = 15, normalized size = 1.07

$$\frac{3\sqrt[3]{b}\,x\sqrt[3]{x^2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2)**(1/3),x)
[Out] 3*b**(1/3)*x*(x**2)**(1/3)/5
```

3.104 $\int \sqrt[3]{bx}\, dx$

Optimal. Leaf size=14

$$\frac{3(bx)^{4/3}}{4b}$$

```
[Out] 3/4*(b*x)^(4/3)/b
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{3(bx)^{4/3}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x)^(1/3),x]
```

```
[Out] (3*(b*x)^(4/3))/(4*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt[3]{bx}\, dx = \frac{3(bx)^{4/3}}{4b}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 0.86

$$\frac{3}{4} x \sqrt[3]{bx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x)^(1/3),x]
```

```
[Out] (3*x*(b*x)^(1/3))/4
```

fricas [A] time = 0.89, size = 8, normalized size = 0.57

$$\frac{3}{4} (bx)^{\frac{1}{3}} x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/4*(b*x)^(1/3)*x
```

giac [A] time = 0.15, size = 8, normalized size = 0.57

$$\frac{3}{4} (bx)^{\frac{1}{3}} x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^(1/3),x, algorithm="giac")
```

```
[Out] 3/4*(b*x)^(1/3)*x
```

maple [A] time = 0.00, size = 9, normalized size = 0.64

$$\frac{3\,(bx)^{\frac{1}{3}}\,x}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^(1/3),x)
```

```
[Out] 3/4*x*(b*x)^(1/3)
```

maxima [A] time = 1.32, size = 10, normalized size = 0.71

$$\frac{3\,(bx)^{\frac{4}{3}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/4*(b*x)^(4/3)/b
```

mupad [B] time = 0.01, size = 10, normalized size = 0.71

$$\frac{3\,(b\,x)^{4/3}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^(1/3),x)
```

```
[Out] (3*(b*x)^(4/3))/(4*b)
```

sympy [A] time = 0.06, size = 10, normalized size = 0.71

$$\frac{3\,(bx)^{\frac{4}{3}}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)**(1/3),x)
```

```
[Out] 3*(b*x)**(4/3)/(4*b)
```

3.105 $\int \sqrt[3]{\frac{b}{x}}\, dx$

Optimal. Leaf size=14

$$\frac{3}{2} x \sqrt[3]{\frac{b}{x}}$$

```
[Out] 3/2*(b/x)^(1/3)*x
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$\frac{3}{2} x \sqrt[3]{\frac{b}{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(b/x)^(1/3),x]
```

```
[Out] (3*(b/x)^(1/3)*x)/2
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{\frac{b}{x}}\, dx &= \left(\sqrt[3]{\frac{b}{x}}\, \sqrt[3]{x}\right) \int \frac{1}{\sqrt[3]{x}}\, dx \\ &= \frac{3}{2} \sqrt[3]{\frac{b}{x}}\, x\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{3}{2} x \sqrt[3]{\frac{b}{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x)^(1/3),x]
```

```
[Out] (3*(b/x)^(1/3)*x)/2
```

fricas [A] time = 0.88, size = 10, normalized size = 0.71

$$\frac{3}{2}\, x \left(\frac{b}{x}\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x)^(1/3),x, algorithm="fricas")
[Out] 3/2*x*(b/x)^(1/3)
```

giac [A] time = 0.15, size = 10, normalized size = 0.71

$$\frac{3\,b}{2\left(\frac{b}{x}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x)^(1/3),x, algorithm="giac")
[Out] 3/2*b/(b/x)^(2/3)
```

maple [A] time = 0.00, size = 11, normalized size = 0.79

$$\frac{3\left(\frac{b}{x}\right)^{\frac{1}{3}}x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x)^(1/3),x)
[Out] 3/2*(b/x)^(1/3)*x
```

maxima [A] time = 1.30, size = 10, normalized size = 0.71

$$\frac{3}{2}\,x\left(\frac{b}{x}\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x)^(1/3),x, algorithm="maxima")
[Out] 3/2*x*(b/x)^(1/3)
```

mupad [B] time = 0.92, size = 10, normalized size = 0.71

$$\frac{3\,x\left(\frac{b}{x}\right)^{1/3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x)^(1/3),x)
[Out] (3*x*(b/x)^(1/3))/2
```

sympy [A] time = 0.19, size = 15, normalized size = 1.07

$$\frac{3\sqrt[3]{b}\,x\sqrt[3]{\frac{1}{x}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x)**(1/3),x)
[Out] 3*b**(1/3)*x*(1/x)**(1/3)/2
```

3.106 $\int \sqrt[3]{\frac{b}{x^2}}\, dx$

Optimal. Leaf size=12

$$3x\sqrt[3]{\frac{b}{x^2}}$$

```
[Out] 3*(b/x^2)^(1/3)*x
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$3x\sqrt[3]{\frac{b}{x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b/x^2)^(1/3),x]
```

```
[Out] 3*(b/x^2)^(1/3)*x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{\frac{b}{x^2}}\, dx &= \left(\sqrt[3]{\frac{b}{x^2}}\, x^{2/3}\right)\int \frac{1}{x^{2/3}}\, dx \\ &= 3\sqrt[3]{\frac{b}{x^2}}\, x\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$3x\sqrt[3]{\frac{b}{x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x^2)^(1/3),x]
```

```
[Out] 3*(b/x^2)^(1/3)*x
```

fricas [A] time = 0.74, size = 10, normalized size = 0.83

$$3\, x\left(\frac{b}{x^2}\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x^2)^(1/3),x, algorithm="fricas")
[Out] 3*x*(b/x^2)^(1/3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{b}{x^2}\right)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x^2)^(1/3),x, algorithm="giac")
[Out] integrate((b/x^2)^(1/3), x)
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$3\left(\frac{b}{x^2}\right)^{\frac{1}{3}} x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2)^(1/3),x)
[Out] 3*(b/x^2)^(1/3)*x
```

maxima [A] time = 1.24, size = 10, normalized size = 0.83

$$3\,x\left(\frac{b}{x^2}\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x^2)^(1/3),x, algorithm="maxima")
[Out] 3*x*(b/x^2)^(1/3)
```

mupad [B] time = 1.03, size = 11, normalized size = 0.92

$$3\,b^{1/3}\,x\left(\frac{1}{x^2}\right)^{1/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2)^(1/3),x)
[Out] 3*b^(1/3)*x*(1/x^2)^(1/3)
```

sympy [A] time = 0.32, size = 15, normalized size = 1.25

$$3\sqrt[3]{b}\,x\sqrt[3]{\frac{1}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x**2)**(1/3),x)
[Out] 3*b**(1/3)*x*(x**(-2))**(1/3)
```

3.107 $\int \sqrt[3]{\frac{b}{x^3}}\, dx$

Optimal. Leaf size=13

$$x\sqrt[3]{\frac{b}{x^3}}\,\log(x)$$

```
[Out] (b/x^3)^(1/3)*x*ln(x)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 29}

$$x\sqrt[3]{\frac{b}{x^3}}\,\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(b/x^3)^(1/3),x]
```

```
[Out] (b/x^3)^(1/3)*x*Log[x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{\frac{b}{x^3}}\, dx &= \left(\sqrt[3]{\frac{b}{x^3}}\, x\right)\int \frac{1}{x}\, dx\\ &= \sqrt[3]{\frac{b}{x^3}}\, x\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$x\sqrt[3]{\frac{b}{x^3}}\,\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x^3)^(1/3),x]
```

```
[Out] (b/x^3)^(1/3)*x*Log[x]
```

fricas [A] time = 0.64, size = 11, normalized size = 0.85

$$x\left(\frac{b}{x^3}\right)^{\frac{1}{3}}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x^3)^(1/3),x, algorithm="fricas")
```

```
[Out] x*(b/x^3)^(1/3)*log(x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{b}{x^3}\right)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x^3)^(1/3),x, algorithm="giac")
[Out] integrate((b/x^3)^(1/3), x)
```

maple [A] time = 0.02, size = 12, normalized size = 0.92

$$\left(\frac{b}{x^3}\right)^{\frac{1}{3}} x \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^3)^(1/3),x)
[Out] (b/x^3)^(1/3)*x*ln(x)
```

maxima [A] time = 1.29, size = 11, normalized size = 0.85

$$x\left(\frac{b}{x^3}\right)^{\frac{1}{3}} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x^3)^(1/3),x, algorithm="maxima")
[Out] x*(b/x^3)^(1/3)*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.08

$$\int \left(\frac{b}{x^3}\right)^{1/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^3)^(1/3),x)
[Out] int((b/x^3)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{\frac{b}{x^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x**3)**(1/3),x)
[Out] Integral((b/x**3)**(1/3), x)
```

3.108 $\int \sqrt[3]{\frac{b}{x^4}}\, dx$

Optimal. Leaf size=12

$$-3x\sqrt[3]{\frac{b}{x^4}}$$

```
[Out] -3*(b/x^4)^(1/3)*x
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$-3x\sqrt[3]{\frac{b}{x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(b/x^4)^(1/3),x]
```

```
[Out] -3*(b/x^4)^(1/3)*x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{\frac{b}{x^4}}\, dx &= \left(\sqrt[3]{\frac{b}{x^4}}\, x^{4/3}\right)\int \frac{1}{x^{4/3}}\, dx \\ &= -3\sqrt[3]{\frac{b}{x^4}}\, x\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$-3x\sqrt[3]{\frac{b}{x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x^4)^(1/3),x]
```

```
[Out] -3*(b/x^4)^(1/3)*x
```

fricas [A] time = 0.82, size = 10, normalized size = 0.83

$$-3\, x\left(\frac{b}{x^4}\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x^4)^(1/3),x, algorithm="fricas")
[Out] -3*x*(b/x^4)^(1/3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{b}{x^4}\right)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x^4)^(1/3),x, algorithm="giac")
[Out] integrate((b/x^4)^(1/3), x)
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$-3\left(\frac{b}{x^4}\right)^{\frac{1}{3}} x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^4)^(1/3),x)
[Out] -3*(b/x^4)^(1/3)*x
```

maxima [A] time = 1.32, size = 10, normalized size = 0.83

$$-3\,x\left(\frac{b}{x^4}\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x^4)^(1/3),x, algorithm="maxima")
[Out] -3*x*(b/x^4)^(1/3)
```

mupad [B] time = 1.02, size = 11, normalized size = 0.92

$$-3\,b^{1/3}\,x\left(\frac{1}{x^4}\right)^{1/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^4)^(1/3),x)
[Out] -3*b^(1/3)*x*(1/x^4)^(1/3)
```

sympy [A] time = 0.33, size = 17, normalized size = 1.42

$$-3\sqrt[3]{b}\,x\sqrt[3]{\frac{1}{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x**4)**(1/3),x)
[Out] -3*b**(1/3)*x*(x**(-4))**(1/3)
```

3.109 $\int (bx^n)^{2/3}\,dx$

Optimal. Leaf size=19

$$\frac{3x\,(bx^n)^{2/3}}{2n+3}$$

```
[Out] 3*x*(b*x^n)^(2/3)/(3+2*n)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$\frac{3x\,(bx^n)^{2/3}}{2n+3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^n)^(2/3),x]
```

```
[Out] (3*x*(b*x^n)^(2/3))/(3 + 2*n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (bx^n)^{2/3}\,dx &= \left(x^{-2n/3}\,(bx^n)^{2/3}\right)\int x^{2n/3}\,dx\\ &= \frac{3x\,(bx^n)^{2/3}}{3+2n}\end{aligned}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.05

$$\frac{x\,(bx^n)^{2/3}}{\frac{2n}{3}+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^n)^(2/3),x]
```

```
[Out] (x*(b*x^n)^(2/3))/(1 + (2*n)/3)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^(2/3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^(2/3),x, algorithm="giac")
[Out] integrate((b*x^n)^(2/3), x)
```

maple [A] time = 0.00, size = 18, normalized size = 0.95

$$\frac{3\,(b\,x^n)^{\frac{2}{3}}\,x}{2n+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^(2/3),x)
[Out] 3*x*(b*x^n)^(2/3)/(3+2*n)
```

maxima [A] time = 1.34, size = 17, normalized size = 0.89

$$\frac{3\,(bx^n)^{\frac{2}{3}}\,x}{2\,n+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^(2/3),x, algorithm="maxima")
[Out] 3*(b*x^n)^(2/3)*x/(2*n + 3)
```

mupad [B] time = 0.98, size = 17, normalized size = 0.89

$$\frac{3\,x\,(b\,x^n)^{2/3}}{2\,n+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^(2/3),x)
[Out] (3*x*(b*x^n)^(2/3))/(2*n + 3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{3b^{\frac{2}{3}}x(x^n)^{\frac{2}{3}}}{2n+3} & \text{for } n \neq -\frac{3}{2} \\ \int \left(\frac{b}{x^{\frac{3}{2}}}\right)^{\frac{2}{3}} dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**n)**(2/3),x)
[Out] Piecewise((3*b**(2/3)*x*(x**n)**(2/3)/(2*n + 3), Ne(n, -3/2)), (Integral((b
/x**(3/2))**(2/3), x), True))
```

3.110 $\int \left(bx^2\right)^{2/3}\, dx$

Optimal. Leaf size=14

$$\frac{3}{7}x\left(bx^2\right)^{2/3}$$

```
[Out] 3/7*x*(b*x^2)^(2/3)
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$\frac{3}{7}x\left(bx^2\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2)^(2/3),x]
```

```
[Out] (3*x*(b*x^2)^(2/3))/7
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \left(bx^2\right)^{2/3}\, dx &= \frac{\left(bx^2\right)^{2/3}\int x^{4/3}\, dx}{x^{4/3}}\\ &= \frac{3}{7}x\left(bx^2\right)^{2/3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{3}{7}x\left(bx^2\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2)^(2/3),x]
```

```
[Out] (3*x*(b*x^2)^(2/3))/7
```

fricas [A] time = 0.58, size = 10, normalized size = 0.71

$$\frac{3}{7}\left(bx^2\right)^{\frac{2}{3}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/7*(b*x^2)^(2/3)*x
```

giac [A] time = 0.16, size = 13, normalized size = 0.93

$$\frac{3\,bx^2}{7\left(\frac{b}{x}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(2/3),x, algorithm="giac")
[Out] 3/7*b*x^2/(b/x)^(1/3)
```

maple [A] time = 0.00, size = 11, normalized size = 0.79

$$\frac{3\left(b\,x^2\right)^{\frac{2}{3}}x}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(2/3),x)
[Out] 3/7*x*(b*x^2)^(2/3)
```

maxima [A] time = 1.33, size = 10, normalized size = 0.71

$$\frac{3}{7}\left(bx^2\right)^{\frac{2}{3}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2)^(2/3),x, algorithm="maxima")
[Out] 3/7*(b*x^2)^(2/3)*x
```

mupad [B] time = 1.00, size = 11, normalized size = 0.79

$$\frac{3\,b^{2/3}\,x\left(x^4\right)^{1/3}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2)^(2/3),x)
[Out] (3*b^(2/3)*x*(x^4)^(1/3))/7
```

sympy [A] time = 0.30, size = 15, normalized size = 1.07

$$\frac{3b^{\frac{2}{3}}x\left(x^2\right)^{\frac{2}{3}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2)**(2/3),x)
[Out] 3*b**(2/3)*x*(x**2)**(2/3)/7
```

3.111 $\int (bx)^{2/3}\,dx$

Optimal. Leaf size=14

$$\frac{3(bx)^{5/3}}{5b}$$

```
[Out] 3/5*(b*x)^(5/3)/b
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{3(bx)^{5/3}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x)^(2/3),x]
```

```
[Out] (3*(b*x)^(5/3))/(5*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (bx)^{2/3}\,dx = \frac{3(bx)^{5/3}}{5b}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 0.86

$$\frac{3}{5}x(bx)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x)^(2/3),x]
```

```
[Out] (3*x*(b*x)^(2/3))/5
```

fricas [A] time = 0.81, size = 8, normalized size = 0.57

$$\frac{3}{5}\,(bx)^{\frac{2}{3}}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/5*(b*x)^(2/3)*x
```

giac [A] time = 0.16, size = 8, normalized size = 0.57

$$\frac{3}{5}\,(bx)^{\frac{2}{3}}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^(2/3),x, algorithm="giac")
```

```
[Out] 3/5*(b*x)^(2/3)*x
```

maple [A] time = 0.00, size = 9, normalized size = 0.64

$$\frac{3\,(bx)^{\frac{2}{3}}\,x}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^(2/3),x)
```

```
[Out] 3/5*x*(b*x)^(2/3)
```

maxima [A] time = 1.32, size = 10, normalized size = 0.71

$$\frac{3\,(bx)^{\frac{5}{3}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/5*(b*x)^(5/3)/b
```

mupad [B] time = 0.02, size = 10, normalized size = 0.71

$$\frac{3\,(b\,x)^{5/3}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^(2/3),x)
```

```
[Out] (3*(b*x)^(5/3))/(5*b)
```

sympy [A] time = 0.06, size = 10, normalized size = 0.71

$$\frac{3\,(bx)^{\frac{5}{3}}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)**(2/3),x)
```

```
[Out] 3*(b*x)**(5/3)/(5*b)
```

3.112 $\int \left(\frac{b}{x}\right)^{2/3} dx$

Optimal. Leaf size=12

$$3x\left(\frac{b}{x}\right)^{2/3}$$

```
[Out] 3*(b/x)^(2/3)*x
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$3x\left(\frac{b}{x}\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[(b/x)^(2/3),x]
```

```
[Out] 3*(b/x)^(2/3)*x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{b}{x}\right)^{2/3} dx &= \left(\left(\frac{b}{x}\right)^{2/3} x^{2/3}\right) \int \frac{1}{x^{2/3}}\, dx \\ &= 3\left(\frac{b}{x}\right)^{2/3} x\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$3x\left(\frac{b}{x}\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x)^(2/3),x]
```

```
[Out] 3*(b/x)^(2/3)*x
```

fricas [A] time = 0.86, size = 10, normalized size = 0.83

$$3\,x\left(\frac{b}{x}\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x)^(2/3),x, algorithm="fricas")
[Out] 3*x*(b/x)^(2/3)
```

giac [A] time = 0.15, size = 10, normalized size = 0.83

$$\frac{3\,b}{\left(\frac{b}{x}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x)^(2/3),x, algorithm="giac")
[Out] 3*b/(b/x)^(1/3)
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$3\left(\frac{b}{x}\right)^{\frac{2}{3}} x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x)^(2/3),x)
[Out] 3*(b/x)^(2/3)*x
```

maxima [A] time = 1.35, size = 10, normalized size = 0.83

$$3\,x\left(\frac{b}{x}\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x)^(2/3),x, algorithm="maxima")
[Out] 3*x*(b/x)^(2/3)
```

mupad [B] time = 0.92, size = 10, normalized size = 0.83

$$3\,x\left(\frac{b}{x}\right)^{2/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x)^(2/3),x)
[Out] 3*x*(b/x)^(2/3)
```

sympy [A] time = 0.33, size = 14, normalized size = 1.17

$$3b^{\frac{2}{3}}x\left(\frac{1}{x}\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x)**(2/3),x)
[Out] 3*b**(2/3)*x*(1/x)**(2/3)
```

3.113 $\int \left(\frac{b}{x^2}\right)^{2/3} dx$

Optimal. Leaf size=12

$$-3x\left(\frac{b}{x^2}\right)^{2/3}$$

```
[Out] -3*(b/x^2)^(2/3)*x
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$-3x\left(\frac{b}{x^2}\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[(b/x^2)^(2/3),x]
```

```
[Out] -3*(b/x^2)^(2/3)*x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{b}{x^2}\right)^{2/3} dx &= \left(\left(\frac{b}{x^2}\right)^{2/3} x^{4/3}\right) \int \frac{1}{x^{4/3}}\, dx \\ &= -3\left(\frac{b}{x^2}\right)^{2/3} x\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$-3x\left(\frac{b}{x^2}\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x^2)^(2/3),x]
```

```
[Out] -3*(b/x^2)^(2/3)*x
```

fricas [A] time = 0.86, size = 10, normalized size = 0.83

$$-3\,x\left(\frac{b}{x^2}\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x^2)^(2/3),x, algorithm="fricas")
[Out] -3*x*(b/x^2)^(2/3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{b}{x^2}\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x^2)^(2/3),x, algorithm="giac")
[Out] integrate((b/x^2)^(2/3), x)
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$-3\left(\frac{b}{x^2}\right)^{\frac{2}{3}} x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2)^(2/3),x)
[Out] -3*(b/x^2)^(2/3)*x
```

maxima [A] time = 1.36, size = 10, normalized size = 0.83

$$-3\,x\left(\frac{b}{x^2}\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x^2)^(2/3),x, algorithm="maxima")
[Out] -3*x*(b/x^2)^(2/3)
```

mupad [B] time = 0.93, size = 11, normalized size = 0.92

$$-3\,b^{2/3}\,x\left(\frac{1}{x^2}\right)^{2/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2)^(2/3),x)
[Out] -3*b^(2/3)*x*(1/x^2)^(2/3)
```

sympy [A] time = 0.48, size = 17, normalized size = 1.42

$$-3b^{\frac{2}{3}}x\left(\frac{1}{x^2}\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x**2)**(2/3),x)
[Out] -3*b**(2/3)*x*(x**(-2))**(2/3)
```

3.114 $\int \left(\frac{b}{x^3}\right)^{2/3} dx$

Optimal. Leaf size=12

$$x\left(-\left(\frac{b}{x^3}\right)^{2/3}\right)$$

```
[Out] -(b/x^3)^(2/3)*x
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$x\left(-\left(\frac{b}{x^3}\right)^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(b/x^3)^(2/3),x]
```

```
[Out] -((b/x^3)^(2/3)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{b}{x^3}\right)^{2/3} dx &= \left(\left(\frac{b}{x^3}\right)^{2/3} x^2\right) \int \frac{1}{x^2}\, dx \\ &= -\left(\frac{b}{x^3}\right)^{2/3} x\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$x\left(-\left(\frac{b}{x^3}\right)^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x^3)^(2/3),x]
```

```
[Out] -((b/x^3)^(2/3)*x)
```

fricas [A] time = 0.99, size = 10, normalized size = 0.83

$$-x\left(\frac{b}{x^3}\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x^3)^(2/3),x, algorithm="fricas")
[Out] -x*(b/x^3)^(2/3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{b}{x^3}\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x^3)^(2/3),x, algorithm="giac")
[Out] integrate((b/x^3)^(2/3), x)
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$-\left(\frac{b}{x^3}\right)^{\frac{2}{3}} x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^3)^(2/3),x)
[Out] -(b/x^3)^(2/3)*x
```

maxima [A] time = 1.31, size = 10, normalized size = 0.83

$$-x\left(\frac{b}{x^3}\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x^3)^(2/3),x, algorithm="maxima")
[Out] -x*(b/x^3)^(2/3)
```

mupad [B] time = 0.92, size = 10, normalized size = 0.83

$$-x\left(\frac{b}{x^3}\right)^{2/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^3)^(2/3),x)
[Out] -x*(b/x^3)^(2/3)
```

sympy [A] time = 0.51, size = 15, normalized size = 1.25

$$-b^{\frac{2}{3}} x\left(\frac{1}{x^3}\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x**3)**(2/3),x)
[Out] -b**(2/3)*x*(x**(-3))**(2/3)
```

3.115 $\int \left(\frac{b}{x^4}\right)^{2/3} dx$

Optimal. Leaf size=14

$$-\frac{3}{5}x\left(\frac{b}{x^4}\right)^{2/3}$$

```
[Out] -3/5*(b/x^4)^(2/3)*x
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$-\frac{3}{5}x\left(\frac{b}{x^4}\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[(b/x^4)^(2/3),x]
```

```
[Out] (-3*(b/x^4)^(2/3)*x)/5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{b}{x^4}\right)^{2/3} dx &= \left(\left(\frac{b}{x^4}\right)^{2/3} x^{8/3}\right) \int \frac{1}{x^{8/3}}\, dx \\ &= -\frac{3}{5}\left(\frac{b}{x^4}\right)^{2/3} x\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{3}{5}x\left(\frac{b}{x^4}\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x^4)^(2/3),x]
```

```
[Out] (-3*(b/x^4)^(2/3)*x)/5
```

fricas [A] time = 1.06, size = 10, normalized size = 0.71

$$-\frac{3}{5}\,x\left(\frac{b}{x^4}\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x^4)^(2/3),x, algorithm="fricas")
[Out] -3/5*x*(b/x^4)^(2/3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{b}{x^4}\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x^4)^(2/3),x, algorithm="giac")
[Out] integrate((b/x^4)^(2/3), x)
```

maple [A] time = 0.00, size = 11, normalized size = 0.79

$$-\frac{3\left(\frac{b}{x^4}\right)^{\frac{2}{3}} x}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^4)^(2/3),x)
[Out] -3/5*(b/x^4)^(2/3)*x
```

maxima [A] time = 1.29, size = 10, normalized size = 0.71

$$-\frac{3}{5} x \left(\frac{b}{x^4}\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x^4)^(2/3),x, algorithm="maxima")
[Out] -3/5*x*(b/x^4)^(2/3)
```

mupad [B] time = 0.92, size = 11, normalized size = 0.79

$$-\frac{3\, b^{2/3}\, x \left(\frac{1}{x^4}\right)^{2/3}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^4)^(2/3),x)
[Out] -(3*b^(2/3)*x*(1/x^4)^(2/3))/5
```

sympy [A] time = 0.44, size = 19, normalized size = 1.36

$$-\frac{3 b^{\frac{2}{3}} x \left(\frac{1}{x^4}\right)^{\frac{2}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b/x**4)**(2/3),x)
[Out] -3*b**(2/3)*x*(x**(-4))**(2/3)/5
```

3.116 $\int \frac{1}{\sqrt[3]{bx^n}}\,dx$

Optimal. Leaf size=19

$$\frac{3x}{(3-n)\sqrt[3]{bx^n}}$$

```
[Out] 3*x/(3-n)/(b*x^n)^(1/3)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$\frac{3x}{(3-n)\sqrt[3]{bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^n)^(-1/3),x]
```

```
[Out] (3*x)/((3 - n)*(b*x^n)^(1/3))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[3]{bx^n}}\,dx &= \frac{x^{n/3}\int x^{-n/3}\,dx}{\sqrt[3]{bx^n}}\\ &= \frac{3x}{(3-n)\sqrt[3]{bx^n}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 0.89

$$-\frac{3x}{(n-3)\sqrt[3]{bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^n)^(-1/3),x]
```

```
[Out] (-3*x)/((-3 + n)*(b*x^n)^(1/3))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^n)^(1/3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^n)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^n)^(-1/3), x)
```

maple [**A**] time = 0.00, size = 16, normalized size = 0.84

$$-\frac{3x}{(n-3)\,(b\,x^n)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^n)^(1/3),x)
```

```
[Out] -3*x/(-3+n)/(b*x^n)^(1/3)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^n)^(1/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-n/3>0)', see `assume?` for more de
tails)Is -n/3 equal to -1?
```

mupad [**B**] time = 0.98, size = 24, normalized size = 1.26

$$-\frac{3\,x^{1-n}\,(b\,x^n)^{2/3}}{b\,(n-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^n)^(1/3),x)
```

```
[Out] -(3*x^(1 - n)*(b*x^n)^(2/3))/(b*(n - 3))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**n)**(1/3),x)
```

```
[Out] Timed out
```

3.117 $\int \frac{1}{\sqrt[3]{bx^4}}\,dx$

Optimal. Leaf size=12

$$-\frac{3x}{\sqrt[3]{bx^4}}$$

```
[Out] -3*x/(b*x^4)^(1/3)
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$-\frac{3x}{\sqrt[3]{bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^4)^(-1/3),x]
```

```
[Out] (-3*x)/(b*x^4)^(1/3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[3]{bx^4}}\,dx &= \frac{x^{4/3}\int \frac{1}{x^{4/3}}\,dx}{\sqrt[3]{bx^4}}\\ &= -\frac{3x}{\sqrt[3]{bx^4}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$-\frac{3x}{\sqrt[3]{bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^4)^(-1/3),x]
```

```
[Out] (-3*x)/(b*x^4)^(1/3)
```

fricas [A] time = 0.80, size = 15, normalized size = 1.25

$$-\frac{3\left(bx^4\right)^{\frac{2}{3}}}{bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4)^(1/3),x, algorithm="fricas")
```

```
[Out] -3*(b*x^4)^(2/3)/(b*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4)^(1/3),x, algorithm="giac")
[Out] integrate((b*x^4)^(-1/3), x)
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$-\frac{3x}{\left(b\,x^4\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4)^(1/3),x)
[Out] -3*x/(b*x^4)^(1/3)
```

maxima [A] time = 1.29, size = 10, normalized size = 0.83

$$-\frac{3\,x}{\left(bx^4\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4)^(1/3),x, algorithm="maxima")
[Out] -3*x/(b*x^4)^(1/3)
```

mupad [B] time = 0.93, size = 13, normalized size = 1.08

$$-\frac{3\left(x^8\right)^{1/3}}{b^{1/3}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4)^(1/3),x)
[Out] -(3*(x^8)^(1/3))/(b^(1/3)*x^3)
```

sympy [A] time = 0.59, size = 15, normalized size = 1.25

$$-\frac{3x}{\sqrt[3]{b}\,\sqrt[3]{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4)**(1/3),x)
[Out] -3*x/(b**(1/3)*(x**4)**(1/3))
```

3.118 $\int \frac{1}{\sqrt[3]{bx^3}}\,dx$

Optimal. Leaf size=13

$$\frac{x\log(x)}{\sqrt[3]{bx^3}}$$

```
[Out] x*ln(x)/(b*x^3)^(1/3)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 29}

$$\frac{x\log(x)}{\sqrt[3]{bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^3)^(-1/3),x]
```

```
[Out] (x*Log[x])/(b*x^3)^(1/3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[3]{bx^3}}\,dx &= \frac{x\int \frac{1}{x}\,dx}{\sqrt[3]{bx^3}}\\ &= \frac{x\log(x)}{\sqrt[3]{bx^3}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{x\log(x)}{\sqrt[3]{bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^3)^(-1/3),x]
```

```
[Out] (x*Log[x])/(b*x^3)^(1/3)
```

fricas [A] time = 0.82, size = 16, normalized size = 1.23

$$\frac{\left(bx^3\right)^{\frac{2}{3}}\log(x)}{bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3)^(1/3),x, algorithm="fricas")
```

```
[Out] (b*x^3)^(2/3)*log(x)/(b*x^2)
```

giac [A] time = 0.16, size = 7, normalized size = 0.54

$$\frac{\log(|x|)}{b^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3)^(1/3),x, algorithm="giac")
[Out] log(abs(x))/b^(1/3)
```

maple [A] time = 0.01, size = 12, normalized size = 0.92

$$\frac{x\ln(x)}{\left(b\,x^3\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3)^(1/3),x)
[Out] x*ln(x)/(b*x^3)^(1/3)
```

maxima [A] time = 1.33, size = 11, normalized size = 0.85

$$\frac{x\log(x)}{\left(bx^3\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3)^(1/3),x, algorithm="maxima")
[Out] x*log(x)/(b*x^3)^(1/3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{\left(b\,x^3\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3)^(1/3),x)
[Out] int(1/(b*x^3)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3)**(1/3),x)
[Out] Integral((b*x**3)**(-1/3), x)
```

3.119 $\int \frac{1}{\sqrt[3]{bx^2}}\,dx$

Optimal. Leaf size=12

$$\frac{3x}{\sqrt[3]{bx^2}}$$

```
[Out] 3*x/(b*x^2)^(1/3)
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$\frac{3x}{\sqrt[3]{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2)^(-1/3),x]
```

```
[Out] (3*x)/(b*x^2)^(1/3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[3]{bx^2}}\,dx &= \frac{x^{2/3}\int \frac{1}{x^{2/3}}\,dx}{\sqrt[3]{bx^2}}\\ &= \frac{3x}{\sqrt[3]{bx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\frac{3x}{\sqrt[3]{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2)^(-1/3),x]
```

```
[Out] (3*x)/(b*x^2)^(1/3)
```

fricas [A] time = 0.75, size = 15, normalized size = 1.25

$$\frac{3\left(bx^2\right)^{\frac{2}{3}}}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2)^(1/3),x, algorithm="fricas")
```

```
[Out] 3*(b*x^2)^(2/3)/(b*x)
```

giac [A] time = 0.20, size = 9, normalized size = 0.75

$$\frac{3}{\left(\frac{b}{x}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2)^(1/3),x, algorithm="giac")
[Out] 3/(b/x)^(1/3)
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$\frac{3x}{\left(b\,x^2\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2)^(1/3),x)
[Out] 3*x/(b*x^2)^(1/3)
```

maxima [A] time = 1.33, size = 10, normalized size = 0.83

$$\frac{3\,x}{\left(bx^2\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2)^(1/3),x, algorithm="maxima")
[Out] 3*x/(b*x^2)^(1/3)
```

mupad [B] time = 0.94, size = 13, normalized size = 1.08

$$\frac{3\left(x^4\right)^{1/3}}{b^{1/3}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2)^(1/3),x)
[Out] (3*(x^4)^(1/3))/(b^(1/3)*x)
```

sympy [A] time = 0.83, size = 14, normalized size = 1.17

$$\frac{3x}{\sqrt[3]{b}\,\sqrt[3]{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2)**(1/3),x)
[Out] 3*x/(b**(1/3)*(x**2)**(1/3))
```

3.120 $\int \frac{1}{\sqrt[3]{bx}}\,dx$

Optimal. Leaf size=14

$$\frac{3(bx)^{2/3}}{2b}$$

```
[Out] 3/2*(b*x)^(2/3)/b
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{3(bx)^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x)^(-1/3),x]
```

```
[Out] (3*(b*x)^(2/3))/(2*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{bx}}\,dx = \frac{3(bx)^{2/3}}{2b}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 0.86

$$\frac{3x}{2\sqrt[3]{bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x)^(-1/3),x]
```

```
[Out] (3*x)/(2*(b*x)^(1/3))
```

fricas [A] time = 0.75, size = 10, normalized size = 0.71

$$\frac{3\ (bx)^{\frac{2}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/2*(b*x)^(2/3)/b
```

giac [A] time = 0.16, size = 10, normalized size = 0.71

$$\frac{3\ (bx)^{\frac{2}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(1/3),x, algorithm="giac")
[Out] 3/2*(b*x)^(2/3)/b
```

maple [A] time = 0.00, size = 9, normalized size = 0.64

$$\frac{3x}{2\,(bx)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x)^(1/3),x)
[Out] 3/2*x/(b*x)^(1/3)
```

maxima [A] time = 1.29, size = 10, normalized size = 0.71

$$\frac{3\,(bx)^{\frac{2}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(1/3),x, algorithm="maxima")
[Out] 3/2*(b*x)^(2/3)/b
```

mupad [B] time = 0.01, size = 10, normalized size = 0.71

$$\frac{3\,(b\,x)^{2/3}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x)^(1/3),x)
[Out] (3*(b*x)^(2/3))/(2*b)
```

sympy [A] time = 0.07, size = 10, normalized size = 0.71

$$\frac{3\,(bx)^{\frac{2}{3}}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)**(1/3),x)
[Out] 3*(b*x)**(2/3)/(2*b)
```

3.121 $\int \frac{1}{\sqrt[3]{\frac{b}{x}}}\,dx$

Optimal. Leaf size=14

$$\frac{3x}{4\sqrt[3]{\frac{b}{x}}}$$

```
[Out] 3/4*x/(b/x)^(1/3)
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$\frac{3x}{4\sqrt[3]{\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(b/x)^(-1/3),x]
```

```
[Out] (3*x)/(4*(b/x)^(1/3))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[3]{\frac{b}{x}}}\,dx &= \frac{\int \sqrt[3]{x}\,dx}{\sqrt[3]{\frac{b}{x}}\,\sqrt[3]{x}}\\ &= \frac{3x}{4\sqrt[3]{\frac{b}{x}}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{3x}{4\sqrt[3]{\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x)^(-1/3),x]
```

```
[Out] (3*x)/(4*(b/x)^(1/3))
```

fricas [A] time = 0.77, size = 15, normalized size = 1.07

$$\frac{3\,x^2\left(\frac{b}{x}\right)^{\frac{2}{3}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x)^(1/3),x, algorithm="fricas")
[Out] 3/4*x^2*(b/x)^(2/3)/b
```

giac [A] time = 0.20, size = 10, normalized size = 0.71

$$\frac{3\,x}{4\left(\frac{b}{x}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x)^(1/3),x, algorithm="giac")
[Out] 3/4*x/(b/x)^(1/3)
```

maple [A] time = 0.00, size = 11, normalized size = 0.79

$$\frac{3x}{4\left(\frac{b}{x}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x)^(1/3),x)
[Out] 3/4*x/(b/x)^(1/3)
```

maxima [A] time = 1.33, size = 10, normalized size = 0.71

$$\frac{3\,x}{4\left(\frac{b}{x}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x)^(1/3),x, algorithm="maxima")
[Out] 3/4*x/(b/x)^(1/3)
```

mupad [B] time = 0.92, size = 15, normalized size = 1.07

$$\frac{3\,x^2\left(\frac{b}{x}\right)^{2/3}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x)^(1/3),x)
[Out] (3*x^2*(b/x)^(2/3))/(4*b)
```

sympy [A] time = 0.43, size = 15, normalized size = 1.07

$$\frac{3x}{4\sqrt[3]{b}\,\sqrt[3]{\frac{1}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x)**(1/3),x)
[Out] 3*x/(4*b**(1/3)*(1/x)**(1/3))
```

3.122 $\int \frac{1}{\sqrt[3]{\frac{b}{x^2}}}\,dx$

Optimal. Leaf size=14

$$\frac{3x}{5\sqrt[3]{\frac{b}{x^2}}}$$

```
[Out] 3/5*x/(b/x^2)^(1/3)
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$\frac{3x}{5\sqrt[3]{\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(b/x^2)^(-1/3),x]
```

```
[Out] (3*x)/(5*(b/x^2)^(1/3))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[3]{\frac{b}{x^2}}}\,dx &= \frac{\int x^{2/3}\,dx}{\sqrt[3]{\frac{b}{x^2}}\,x^{2/3}}\\ &= \frac{3x}{5\sqrt[3]{\frac{b}{x^2}}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{3x}{5\sqrt[3]{\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x^2)^(-1/3),x]
```

```
[Out] (3*x)/(5*(b/x^2)^(1/3))
```

fricas [A] time = 0.78, size = 15, normalized size = 1.07

$$\frac{3\,x^3\left(\frac{b}{x^2}\right)^{\frac{2}{3}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^2)^(1/3),x, algorithm="fricas")
[Out] 3/5*x^3*(b/x^2)^(2/3)/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{b}{x^2}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^2)^(1/3),x, algorithm="giac")
[Out] integrate((b/x^2)^(-1/3), x)
```

maple [A] time = 0.00, size = 11, normalized size = 0.79

$$\frac{3x}{5\left(\frac{b}{x^2}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x^2)^(1/3),x)
[Out] 3/5*x/(b/x^2)^(1/3)
```

maxima [A] time = 1.38, size = 10, normalized size = 0.71

$$\frac{3\,x}{5\,\left(\frac{b}{x^2}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^2)^(1/3),x, algorithm="maxima")
[Out] 3/5*x/(b/x^2)^(1/3)
```

mupad [B] time = 0.93, size = 13, normalized size = 0.93

$$\frac{3\,x^3\,\left(\frac{1}{x^2}\right)^{2/3}}{5\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x^2)^(1/3),x)
[Out] (3*x^3*(1/x^2)^(2/3))/(5*b^(1/3))
```

sympy [A] time = 0.44, size = 17, normalized size = 1.21

$$\frac{3x}{5\sqrt[3]{b}\,\sqrt[3]{\frac{1}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x**2)**(1/3),x)
[Out] 3*x/(5*b**(1/3)*(x**(-2))**(1/3))
```

3.123 $\int \frac{1}{\sqrt[3]{\frac{b}{x^3}}}\,dx$

Optimal. Leaf size=14

$$\frac{x}{2\sqrt[3]{\frac{b}{x^3}}}$$

```
[Out] 1/2*x/(b/x^3)^(1/3)
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$\frac{x}{2\sqrt[3]{\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(b/x^3)^(-1/3),x]
```

```
[Out] x/(2*(b/x^3)^(1/3))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[3]{\frac{b}{x^3}}}\,dx &= \frac{\int x\,dx}{\sqrt[3]{\frac{b}{x^3}}\,x}\\ &= \frac{x}{2\sqrt[3]{\frac{b}{x^3}}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{x}{2\sqrt[3]{\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x^3)^(-1/3),x]
```

```
[Out] x/(2*(b/x^3)^(1/3))
```

fricas [A] time = 0.84, size = 15, normalized size = 1.07

$$\frac{x^4\left(\frac{b}{x^3}\right)^{\frac{2}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^3)^(1/3),x, algorithm="fricas")
[Out] 1/2*x^4*(b/x^3)^(2/3)/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{b}{x^3}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^3)^(1/3),x, algorithm="giac")
[Out] integrate((b/x^3)^(-1/3), x)
```

maple [A] time = 0.00, size = 11, normalized size = 0.79

$$\frac{x}{2\left(\frac{b}{x^3}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x^3)^(1/3),x)
[Out] 1/2*x/(b/x^3)^(1/3)
```

maxima [A] time = 1.31, size = 10, normalized size = 0.71

$$\frac{x}{2\left(\frac{b}{x^3}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^3)^(1/3),x, algorithm="maxima")
[Out] 1/2*x/(b/x^3)^(1/3)
```

mupad [B] time = 0.94, size = 15, normalized size = 1.07

$$\frac{x^4\left(\frac{b}{x^3}\right)^{2/3}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x^3)^(1/3),x)
[Out] (x^4*(b/x^3)^(2/3))/(2*b)
```

sympy [A] time = 0.45, size = 15, normalized size = 1.07

$$\frac{x}{2\sqrt[3]{b}\,\sqrt[3]{\frac{1}{x^3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x**3)**(1/3),x)
[Out] x/(2*b**(1/3)*(x**(-3))**(1/3))
```

3.124 $\int \frac{1}{(bx^n)^{2/3}}\,dx$

Optimal. Leaf size=19

$$\frac{3x}{(3-2n)\,(bx^n)^{2/3}}$$

```
[Out] 3*x/(3-2*n)/(b*x^n)^(2/3)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$\frac{3x}{(3-2n)\,(bx^n)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^n)^(-2/3),x]
```

```
[Out] (3*x)/((3 - 2*n)*(b*x^n)^(2/3))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(bx^n)^{2/3}}\,dx &= \frac{x^{2n/3}\int x^{-2n/3}\,dx}{(bx^n)^{2/3}}\\ &= \frac{3x}{(3-2n)\,(bx^n)^{2/3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.05

$$\frac{x}{\left(1-\frac{2n}{3}\right)(bx^n)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^n)^(-2/3),x]
```

```
[Out] x/((1 - (2*n)/3)*(b*x^n)^(2/3))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^n)^(2/3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^n)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^n)^(-2/3), x)
```

maple [**A**] time = 0.00, size = 18, normalized size = 0.95

$$-\frac{3x}{(2n-3)\,(b\,x^n)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^n)^(2/3),x)
```

```
[Out] -3*x/(2*n-3)/(b*x^n)^(2/3)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^n)^(2/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-(2*n)/3>0)', see `assume?` for mor
e details)Is -(2*n)/3 equal to -1?
```

mupad [**B**] time = 0.98, size = 26, normalized size = 1.37

$$-\frac{3\,x^{1-n}\,(b\,x^n)^{1/3}}{b\,(2\,n-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^n)^(2/3),x)
```

```
[Out] -(3*x^(1 - n)*(b*x^n)^(1/3))/(b*(2*n - 3))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**n)**(2/3),x)
```

```
[Out] Timed out
```

3.125 $\int \frac{1}{\left(bx^3\right)^{2/3}}\,dx$

Optimal. Leaf size=12

$$-\frac{x}{\left(bx^3\right)^{2/3}}$$

```
[Out] -x/(b*x^3)^(2/3)
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$-\frac{x}{\left(bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^3)^(-2/3),x]
```

```
[Out] -(x/(b*x^3)^(2/3))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(bx^3\right)^{2/3}}\,dx &= \frac{x^2 \int \frac{1}{x^2}\,dx}{\left(bx^3\right)^{2/3}} \\ &= -\frac{x}{\left(bx^3\right)^{2/3}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$-\frac{x}{\left(bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^3)^(-2/3),x]
```

```
[Out] -(x/(b*x^3)^(2/3))
```

fricas [A] time = 0.74, size = 15, normalized size = 1.25

$$-\frac{\left(bx^3\right)^{\frac{1}{3}}}{bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3)^(2/3),x, algorithm="fricas")
[Out] -(b*x^3)^(1/3)/(b*x^2)
```

giac [A] time = 0.15, size = 8, normalized size = 0.67

$$-\frac{1}{b^{\frac{2}{3}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3)^(2/3),x, algorithm="giac")
[Out] -1/(b^(2/3)*x)
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$-\frac{x}{\left(b\,x^3\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3)^(2/3),x)
[Out] -x/(b*x^3)^(2/3)
```

maxima [A] time = 1.29, size = 10, normalized size = 0.83

$$-\frac{x}{\left(bx^3\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3)^(2/3),x, algorithm="maxima")
[Out] -x/(b*x^3)^(2/3)
```

mupad [B] time = 0.93, size = 15, normalized size = 1.25

$$-\frac{\left(b\,x^3\right)^{1/3}}{b\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3)^(2/3),x)
[Out] -(b*x^3)^(1/3)/(b*x^2)
```

sympy [A] time = 0.54, size = 14, normalized size = 1.17

$$-\frac{x}{b^{\frac{2}{3}}\left(x^3\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3)**(2/3),x)
[Out] -x/(b**(2/3)*(x**3)**(2/3))
```

3.126 $\int \frac{1}{\left(bx^2\right)^{2/3}}\,dx$

Optimal. Leaf size=12

$$-\frac{3x}{\left(bx^2\right)^{2/3}}$$

```
[Out] -3*x/(b*x^2)^(2/3)
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$-\frac{3x}{\left(bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2)^(-2/3),x]
```

```
[Out] (-3*x)/(b*x^2)^(2/3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(bx^2\right)^{2/3}}\,dx &= \frac{x^{4/3}\int \frac{1}{x^{4/3}}\,dx}{\left(bx^2\right)^{2/3}}\\ &= -\frac{3x}{\left(bx^2\right)^{2/3}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$-\frac{3x}{\left(bx^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2)^(-2/3),x]
```

```
[Out] (-3*x)/(b*x^2)^(2/3)
```

fricas [A] time = 0.84, size = 15, normalized size = 1.25

$$-\frac{3\left(bx^2\right)^{\frac{1}{3}}}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2)^(2/3),x, algorithm="fricas")
[Out] -3*(b*x^2)^(1/3)/(b*x)
```

giac [A] time = 0.16, size = 12, normalized size = 1.00

$$-\frac{3\left(\frac{b}{x}\right)^{\frac{1}{3}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2)^(2/3),x, algorithm="giac")
[Out] -3*(b/x)^(1/3)/b
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$-\frac{3x}{\left(b\,x^2\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2)^(2/3),x)
[Out] -3*x/(b*x^2)^(2/3)
```

maxima [A] time = 1.34, size = 10, normalized size = 0.83

$$-\frac{3\,x}{\left(bx^2\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2)^(2/3),x, algorithm="maxima")
[Out] -3*x/(b*x^2)^(2/3)
```

mupad [B] time = 0.91, size = 13, normalized size = 1.08

$$-\frac{3\left(x^2\right)^{1/3}}{b^{2/3}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2)^(2/3),x)
[Out] -(3*(x^2)^(1/3))/(b^(2/3)*x)
```

sympy [A] time = 0.73, size = 15, normalized size = 1.25

$$-\frac{3x}{b^{\frac{2}{3}}\left(x^2\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2)**(2/3),x)
[Out] -3*x/(b**(2/3)*(x**2)**(2/3))
```

3.127 $\int \frac{1}{(bx)^{2/3}}\,dx$

Optimal. Leaf size=12

$$\frac{3\sqrt[3]{bx}}{b}$$

```
[Out] 3*(b*x)^(1/3)/b
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{3\sqrt[3]{bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x)^(-2/3),x]
```

```
[Out] (3*(b*x)^(1/3))/b
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(bx)^{2/3}}\,dx = \frac{3\sqrt[3]{bx}}{b}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 0.83

$$\frac{3x}{(bx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x)^(-2/3),x]
```

```
[Out] (3*x)/(b*x)^(2/3)
```

fricas [A] time = 0.87, size = 10, normalized size = 0.83

$$\frac{3\ (bx)^{\frac{1}{3}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(2/3),x, algorithm="fricas")
```

```
[Out] 3*(b*x)^(1/3)/b
```

giac [A] time = 0.18, size = 10, normalized size = 0.83

$$\frac{3\ (bx)^{\frac{1}{3}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(2/3),x, algorithm="giac")
[Out] 3*(b*x)^(1/3)/b
```

maple [A] time = 0.00, size = 9, normalized size = 0.75

$$\frac{3x}{(bx)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x)^(2/3),x)
[Out] 3*x/(b*x)^(2/3)
```

maxima [A] time = 1.30, size = 10, normalized size = 0.83

$$\frac{3\ (bx)^{\frac{1}{3}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(2/3),x, algorithm="maxima")
[Out] 3*(b*x)^(1/3)/b
```

mupad [B] time = 0.01, size = 10, normalized size = 0.83

$$\frac{3\,(b\,x)^{1/3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x)^(2/3),x)
[Out] (3*(b*x)^(1/3))/b
```

sympy [A] time = 0.10, size = 8, normalized size = 0.67

$$\frac{3\sqrt[3]{bx}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)**(2/3),x)
[Out] 3*(b*x)**(1/3)/b
```

3.128 $\int \frac{1}{\left(\frac{b}{x}\right)^{2/3}}\,dx$

Optimal. Leaf size=14

$$\frac{3x}{5\left(\frac{b}{x}\right)^{2/3}}$$

```
[Out] 3/5*x/(b/x)^(2/3)
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$\frac{3x}{5\left(\frac{b}{x}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b/x)^(-2/3),x]
```

```
[Out] (3*x)/(5*(b/x)^(2/3))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{b}{x}\right)^{2/3}}\,dx &= \frac{\int x^{2/3}\,dx}{\left(\frac{b}{x}\right)^{2/3} x^{2/3}}\\ &= \frac{3x}{5\left(\frac{b}{x}\right)^{2/3}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{3x}{5\left(\frac{b}{x}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x)^(-2/3),x]
```

```
[Out] (3*x)/(5*(b/x)^(2/3))
```

fricas [A] time = 0.72, size = 15, normalized size = 1.07

$$\frac{3\,x^2\left(\frac{b}{x}\right)^{\frac{1}{3}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x)^(2/3),x, algorithm="fricas")
[Out] 3/5*x^2*(b/x)^(1/3)/b
```

giac [A] time = 0.17, size = 10, normalized size = 0.71

$$\frac{3\,x}{5\left(\frac{b}{x}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x)^(2/3),x, algorithm="giac")
[Out] 3/5*x/(b/x)^(2/3)
```

maple [A] time = 0.00, size = 11, normalized size = 0.79

$$\frac{3x}{5\left(\frac{b}{x}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x)^(2/3),x)
[Out] 3/5*x/(b/x)^(2/3)
```

maxima [A] time = 1.29, size = 10, normalized size = 0.71

$$\frac{3\,x}{5\left(\frac{b}{x}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x)^(2/3),x, algorithm="maxima")
[Out] 3/5*x/(b/x)^(2/3)
```

mupad [B] time = 0.91, size = 15, normalized size = 1.07

$$\frac{3\,x^2\left(\frac{b}{x}\right)^{1/3}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x)^(2/3),x)
[Out] (3*x^2*(b/x)^(1/3))/(5*b)
```

sympy [A] time = 0.96, size = 15, normalized size = 1.07

$$\frac{3x}{5b^{\frac{2}{3}}\left(\frac{1}{x}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x)**(2/3),x)
[Out] 3*x/(5*b**(2/3)*(1/x)**(2/3))
```

3.129 $\int \frac{1}{\left(\frac{b}{x^2}\right)^{2/3}}\,dx$

Optimal. Leaf size=14

$$\frac{3x}{7\left(\frac{b}{x^2}\right)^{2/3}}$$

```
[Out] 3/7*x/(b/x^2)^(2/3)
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$\frac{3x}{7\left(\frac{b}{x^2}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b/x^2)^(-2/3),x]
```

```
[Out] (3*x)/(7*(b/x^2)^(2/3))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{b}{x^2}\right)^{2/3}}\,dx &= \frac{\int x^{4/3}\,dx}{\left(\frac{b}{x^2}\right)^{2/3} x^{4/3}}\\ &= \frac{3x}{7\left(\frac{b}{x^2}\right)^{2/3}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{3x}{7\left(\frac{b}{x^2}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x^2)^(-2/3),x]
```

```
[Out] (3*x)/(7*(b/x^2)^(2/3))
```

fricas [A] time = 0.77, size = 15, normalized size = 1.07

$$\frac{3\,x^3\left(\frac{b}{x^2}\right)^{\frac{1}{3}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^2)^(2/3),x, algorithm="fricas")
[Out] 3/7*x^3*(b/x^2)^(1/3)/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{b}{x^2}\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^2)^(2/3),x, algorithm="giac")
[Out] integrate((b/x^2)^(-2/3), x)
```

maple [A] time = 0.00, size = 11, normalized size = 0.79

$$\frac{3x}{7\left(\frac{b}{x^2}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x^2)^(2/3),x)
[Out] 3/7*x/(b/x^2)^(2/3)
```

maxima [A] time = 1.31, size = 10, normalized size = 0.71

$$\frac{3\,x}{7\left(\frac{b}{x^2}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^2)^(2/3),x, algorithm="maxima")
[Out] 3/7*x/(b/x^2)^(2/3)
```

mupad [B] time = 0.92, size = 13, normalized size = 0.93

$$\frac{3\,x^3\left(\frac{1}{x^2}\right)^{1/3}}{7\,b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x^2)^(2/3),x)
[Out] (3*x^3*(1/x^2)^(1/3))/(7*b^(2/3))
```

sympy [A] time = 0.68, size = 17, normalized size = 1.21

$$\frac{3x}{7b^{\frac{2}{3}}\left(\frac{1}{x^2}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x**2)**(2/3),x)
[Out] 3*x/(7*b**(2/3)*(x**(-2))**(2/3))
```

3.130 $\int \frac{1}{\left(\frac{b}{x^3}\right)^{2/3}}\,dx$

Optimal. Leaf size=14

$$\frac{x}{3\left(\frac{b}{x^3}\right)^{2/3}}$$

```
[Out] 1/3*x/(b/x^3)^(2/3)
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$\frac{x}{3\left(\frac{b}{x^3}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b/x^3)^(-2/3),x]
```

```
[Out] x/(3*(b/x^3)^(2/3))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{b}{x^3}\right)^{2/3}}\,dx &= \frac{\int x^2\,dx}{\left(\frac{b}{x^3}\right)^{2/3} x^2}\\ &= \frac{x}{3\left(\frac{b}{x^3}\right)^{2/3}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{x}{3\left(\frac{b}{x^3}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x^3)^(-2/3),x]
```

```
[Out] x/(3*(b/x^3)^(2/3))
```

fricas [A] time = 0.84, size = 15, normalized size = 1.07

$$\frac{x^4\left(\frac{b}{x^3}\right)^{\frac{1}{3}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^3)^(2/3),x, algorithm="fricas")
[Out] 1/3*x^4*(b/x^3)^(1/3)/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{b}{x^3}\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^3)^(2/3),x, algorithm="giac")
[Out] integrate((b/x^3)^(-2/3), x)
```

maple [A] time = 0.00, size = 11, normalized size = 0.79

$$\frac{x}{3\left(\frac{b}{x^3}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x^3)^(2/3),x)
[Out] 1/3*x/(b/x^3)^(2/3)
```

maxima [A] time = 1.32, size = 10, normalized size = 0.71

$$\frac{x}{3\left(\frac{b}{x^3}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^3)^(2/3),x, algorithm="maxima")
[Out] 1/3*x/(b/x^3)^(2/3)
```

mupad [B] time = 0.90, size = 15, normalized size = 1.07

$$\frac{x^4\left(\frac{b}{x^3}\right)^{1/3}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x^3)^(2/3),x)
[Out] (x^4*(b/x^3)^(1/3))/(3*b)
```

sympy [A] time = 0.73, size = 15, normalized size = 1.07

$$\frac{x}{3b^{\frac{2}{3}}\left(\frac{1}{x^3}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x**3)**(2/3),x)
[Out] x/(3*b**(2/3)*(x**(-3))**(2/3))
```

3.131 $\int x^2\sqrt{bx^n}\,dx$

Optimal. Leaf size=19

$$\frac{2x^3\sqrt{bx^n}}{n+6}$$

```
[Out] 2*x^3*(b*x^n)^(1/2)/(6+n)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$\frac{2x^3\sqrt{bx^n}}{n+6}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[b*x^n],x]
```

```
[Out] (2*x^3*Sqrt[b*x^n])/(6 + n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{bx^n}\,dx &= \left(x^{-n/2}\sqrt{bx^n}\right)\int x^{2+\frac{n}{2}}\,dx\\ &= \frac{2x^3\sqrt{bx^n}}{6+n}\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{2x^3\sqrt{bx^n}}{n+6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[b*x^n],x]
```

```
[Out] (2*x^3*Sqrt[b*x^n])/(6 + n)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^n}\, x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n)*x^2, x)
```

maple [A] time = 0.00, size = 18, normalized size = 0.95

$$\frac{2\sqrt{b\,x^n}\;x^3}{n+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^n)^(1/2),x)
```

```
[Out] 2*x^3*(b*x^n)^(1/2)/(6+n)
```

maxima [A] time = 1.24, size = 17, normalized size = 0.89

$$\frac{2\,\sqrt{bx^n}\,x^3}{n+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b*x^n)*x^3/(n + 6)
```

mupad [B] time = 1.00, size = 17, normalized size = 0.89

$$\frac{2\,x^3\,\sqrt{b\,x^n}}{n+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^n)^(1/2),x)
```

```
[Out] (2*x^3*(b*x^n)^(1/2))/(n + 6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{2\sqrt{b}\,x^3\sqrt{x^n}}{n+6} & \text{for } n \neq -6 \\ \int x^2\sqrt{\frac{b}{x^6}}\, dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**n)**(1/2),x)
```

```
[Out] Piecewise((2*sqrt(b)*x**3*sqrt(x**n)/(n + 6), Ne(n, -6)), (Integral(x**2*sq
rt(b/x**6), x), True))
```

3.132 $\int x\sqrt{bx^n}\,dx$

Optimal. Leaf size=19

$$\frac{2x^2\sqrt{bx^n}}{n+4}$$

```
[Out] 2*x^2*(b*x^n)^(1/2)/(4+n)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 30}

$$\frac{2x^2\sqrt{bx^n}}{n+4}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[b*x^n],x]
```

```
[Out] (2*x^2*Sqrt[b*x^n])/(4 + n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{bx^n}\,dx &= \left(x^{-n/2}\sqrt{bx^n}\right)\int x^{1+\frac{n}{2}}\,dx\\ &= \frac{2x^2\sqrt{bx^n}}{4+n}\end{aligned}$$

Mathematica [A] time = 0.02, size = 19, normalized size = 1.00

$$\frac{2x^2\sqrt{bx^n}}{n+4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[b*x^n],x]
```

```
[Out] (2*x^2*Sqrt[b*x^n])/(4 + n)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^n}\, x\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n)*x, x)
```

maple [A] time = 0.00, size = 18, normalized size = 0.95

$$\frac{2\sqrt{b\,x^n}\;x^2}{n+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^n)^(1/2),x)
```

```
[Out] 2*x^2*(b*x^n)^(1/2)/(4+n)
```

maxima [A] time = 1.28, size = 17, normalized size = 0.89

$$\frac{2\,\sqrt{bx^n}\,x^2}{n+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b*x^n)*x^2/(n + 4)
```

mupad [B] time = 0.95, size = 17, normalized size = 0.89

$$\frac{2\,x^2\,\sqrt{b\,x^n}}{n+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^n)^(1/2),x)
```

```
[Out] (2*x^2*(b*x^n)^(1/2))/(n + 4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{2\sqrt{b}\,x^2\sqrt{x^n}}{n+4} & \text{for } n \neq -4 \\ \int x\sqrt{\frac{b}{x^4}}\,dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**n)**(1/2),x)
```

```
[Out] Piecewise((2*sqrt(b)*x**2*sqrt(x**n)/(n + 4), Ne(n, -4)), (Integral(x*sqrt(
b/x**4), x), True))
```

3.133 $\int \sqrt{bx^n}\, dx$

Optimal. Leaf size=17

$$\frac{2x\sqrt{bx^n}}{n+2}$$

```
[Out] 2*x*(b*x^n)^(1/2)/(2+n)
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$\frac{2x\sqrt{bx^n}}{n+2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^n],x]
```

```
[Out] (2*x*Sqrt[b*x^n])/(2 + n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt{bx^n}\, dx &= \left(x^{-n/2}\sqrt{bx^n}\right)\int x^{n/2}\, dx \\ &= \frac{2x\sqrt{bx^n}}{2+n}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{2x\sqrt{bx^n}}{n+2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^n],x]
```

```
[Out] (2*x*Sqrt[b*x^n])/(2 + n)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^n), x)
```

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$\frac{2\sqrt{b\,x^n}\,x}{n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^(1/2),x)
[Out] 2*x*(b*x^n)^(1/2)/(2+n)
```

maxima [A] time = 1.31, size = 15, normalized size = 0.88

$$\frac{2\,\sqrt{bx^n}\,x}{n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(b*x^n)*x/(n + 2)
```

mupad [B] time = 1.48, size = 15, normalized size = 0.88

$$\frac{2\,x\,\sqrt{b\,x^n}}{n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^(1/2),x)
[Out] (2*x*(b*x^n)^(1/2))/(n + 2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{2\sqrt{b}\,x\sqrt{x^n}}{n+2} & \text{for } n \neq -2 \\ \int \sqrt{\frac{b}{x^2}}\, dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**n)**(1/2),x)
[Out] Piecewise((2*sqrt(b)*x*sqrt(x**n)/(n + 2), Ne(n, -2)), (Integral(sqrt(b/x**
2), x), True))
```

3.134 $\int \frac{\sqrt{bx^n}}{x}\,dx$

Optimal. Leaf size=14

$$\frac{2\sqrt{bx^n}}{n}$$

```
[Out] 2*(b*x^n)^(1/2)/n
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$\frac{2\sqrt{bx^n}}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^n]/x,x]
```

```
[Out] (2*Sqrt[b*x^n])/n
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{bx^n}}{x}\,dx &= \left(x^{-n/2}\sqrt{bx^n}\right)\int x^{-1+\frac{n}{2}}\,dx\\ &= \frac{2\sqrt{bx^n}}{n}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{2\sqrt{bx^n}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^n]/x,x]
```

```
[Out] (2*Sqrt[b*x^n])/n
```

fricas [A] time = 0.56, size = 12, normalized size = 0.86

$$\frac{2\,\sqrt{bx^n}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 2*sqrt(b*x^n)/n
```

giac [A] time = 0.18, size = 12, normalized size = 0.86

$$\frac{2\sqrt{bx^n}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^(1/2)/x,x, algorithm="giac")
[Out] 2*sqrt(b*x^n)/n
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$\frac{2\sqrt{b\,x^n}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^(1/2)/x,x)
[Out] 2*(b*x^n)^(1/2)/n
```

maxima [A] time = 1.32, size = 12, normalized size = 0.86

$$\frac{2\sqrt{bx^n}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^(1/2)/x,x, algorithm="maxima")
[Out] 2*sqrt(b*x^n)/n
```

mupad [B] time = 0.93, size = 12, normalized size = 0.86

$$\frac{2\sqrt{b\,x^n}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^(1/2)/x,x)
[Out] (2*(b*x^n)^(1/2))/n
```

sympy [A] time = 0.38, size = 22, normalized size = 1.57

$$\begin{cases} \frac{2\sqrt{b}\sqrt{x^n}}{n} & \text{for } n \neq 0 \\ \sqrt{b}\,\log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**n)**(1/2)/x,x)
[Out] Piecewise((2*sqrt(b)*sqrt(x**n)/n, Ne(n, 0)), (sqrt(b)*log(x), True))
```

3.135 $\int \frac{\sqrt{bx^n}}{x^2}\,dx$

Optimal. Leaf size=21

$$-\frac{2\sqrt{bx^n}}{(2-n)x}$$

```
[Out] -2*(b*x^n)^(1/2)/(2-n)/x
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$-\frac{2\sqrt{bx^n}}{(2-n)x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^n]/x^2,x]
```

```
[Out] (-2*Sqrt[b*x^n])/((2 - n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{bx^n}}{x^2}\,dx &= \left(x^{-n/2}\sqrt{bx^n}\right)\int x^{-2+\frac{n}{2}}\,dx \\ &= -\frac{2\sqrt{bx^n}}{(2-n)x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 0.90

$$\frac{2\sqrt{bx^n}}{(n-2)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^n]/x^2,x]
```

```
[Out] (2*Sqrt[b*x^n])/((-2 + n)*x)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^n}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n)/x^2, x)
```

maple [**A**] time = 0.00, size = 18, normalized size = 0.86

$$\frac{2\sqrt{b\,x^n}}{(n-2)\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^(1/2)/x^2,x)
```

```
[Out] 2/x/(-2+n)*(b*x^n)^(1/2)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(n/2-2>0)', see `assume?` for more d
etails)Is n/2-2 equal to -1?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sqrt{b\,x^n}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^(1/2)/x^2,x)
```

```
[Out] int((b*x^n)^(1/2)/x^2, x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{2\sqrt{b}\,\sqrt{x^n}}{nx-2x} & \text{for } n \neq 2 \\ \int \frac{\sqrt{bx^2}}{x^2}\,dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**n)**(1/2)/x**2,x)
```

```
[Out] Piecewise((2*sqrt(b)*sqrt(x**n)/(n*x - 2*x), Ne(n, 2)), (Integral(sqrt(b*x*
**2)/x**2, x), True))
```

3.136 $\int \frac{\sqrt{bx^n}}{x^3}\,dx$

Optimal. Leaf size=21

$$-\frac{2\sqrt{bx^n}}{(4-n)x^2}$$

```
[Out] -2*(b*x^n)^(1/2)/(4-n)/x^2
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$-\frac{2\sqrt{bx^n}}{(4-n)x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^n]/x^3,x]
```

```
[Out] (-2*Sqrt[b*x^n])/((4 - n)*x^2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{bx^n}}{x^3}\,dx &= \left(x^{-n/2}\sqrt{bx^n}\right)\int x^{-3+\frac{n}{2}}\,dx\\ &= -\frac{2\sqrt{bx^n}}{(4-n)x^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 0.90

$$\frac{2\sqrt{bx^n}}{(n-4)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^n]/x^3,x]
```

```
[Out] (2*Sqrt[b*x^n])/((-4 + n)*x^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^n}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n)/x^3, x)
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$\frac{2\sqrt{b\,x^n}}{(n-4)\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^(1/2)/x^3,x)
```

```
[Out] 2/x^2/(-4+n)*(b*x^n)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(n/2-3>0)', see `assume?` for more d
etails)Is n/2-3 equal to -1?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sqrt{b\,x^n}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^(1/2)/x^3,x)
```

```
[Out] int((b*x^n)^(1/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{2\sqrt{b}\,\sqrt{x^n}}{nx^2-4x^2} & \text{for } n \neq 4 \\ \int \frac{\sqrt{bx^4}}{x^3}\,dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**n)**(1/2)/x**3,x)
```

```
[Out] Piecewise((2*sqrt(b)*sqrt(x**n)/(n*x**2 - 4*x**2), Ne(n, 4)), (Integral(sqr
t(b*x**4)/x**3, x), True))
```

3.137 $\int x\,(bx^n)^{3/2}\,dx$

Optimal. Leaf size=24

$$\frac{2bx^{n+2}\sqrt{bx^n}}{3n+4}$$

```
[Out] 2*b*x^(2+n)*(b*x^n)^(1/2)/(4+3*n)
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 30}

$$\frac{2bx^{n+2}\sqrt{bx^n}}{3n+4}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b*x^n)^(3/2),x]
```

```
[Out] (2*b*x^(2 + n)*Sqrt[b*x^n])/(4 + 3*n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\,(bx^n)^{3/2}\,dx &= \left(bx^{-n/2}\sqrt{bx^n}\right)\int x^{1+\frac{3n}{2}}\,dx\\ &= \frac{2bx^{2+n}\sqrt{bx^n}}{4+3n}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.92

$$\frac{x^2\,(bx^n)^{3/2}}{\frac{3n}{2}+2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b*x^n)^(3/2),x]
```

```
[Out] (x^2*(b*x^n)^(3/2))/(2 + (3*n)/2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n)^{\frac{3}{2}}\, x\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^n)^(3/2)*x, x)
```

maple [A] time = 0.00, size = 20, normalized size = 0.83

$$\frac{2\,(b\,x^n)^{\frac{3}{2}}\,x^2}{3n+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^n)^(3/2),x)
```

```
[Out] 2*x^2/(4+3*n)*(b*x^n)^(3/2)
```

maxima [A] time = 1.31, size = 19, normalized size = 0.79

$$\frac{2\ (bx^n)^{\frac{3}{2}}\,x^2}{3\,n+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*(b*x^n)^(3/2)*x^2/(3*n + 4)
```

mupad [B] time = 0.98, size = 22, normalized size = 0.92

$$\frac{2\,b\,x^{n+2}\,\sqrt{b\,x^n}}{3\,n+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^n)^(3/2),x)
```

```
[Out] (2*b*x^(n + 2)*(b*x^n)^(1/2))/(3*n + 4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{2b^{\frac{3}{2}}x^2(x^n)^{\frac{3}{2}}}{3n+4} & \text{for } n \neq -\frac{4}{3} \\ \int x\left(\frac{b}{x^{\frac{4}{3}}}\right)^{\frac{3}{2}}\, dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**n)**(3/2),x)
```

```
[Out] Piecewise((2*b**(3/2)*x**2*(x**n)**(3/2)/(3*n + 4), Ne(n, -4/3)), (Integral
(x*(b/x**(4/3))**(3/2), x), True))
```

3.138 $\int (bx^n)^{3/2}\,dx$

Optimal. Leaf size=24

$$\frac{2bx^{n+1}\sqrt{bx^n}}{3n+2}$$

```
[Out] 2*b*x^(1+n)*(b*x^n)^(1/2)/(2+3*n)
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$\frac{2bx^{n+1}\sqrt{bx^n}}{3n+2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^n)^(3/2),x]
```

```
[Out] (2*b*x^(1 + n)*Sqrt[b*x^n])/(2 + 3*n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (bx^n)^{3/2}\,dx &= \left(bx^{-n/2}\sqrt{bx^n}\right)\int x^{3n/2}\,dx\\ &= \frac{2bx^{1+n}\sqrt{bx^n}}{2+3n}\end{aligned}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 0.83

$$\frac{x\,(bx^n)^{3/2}}{\frac{3n}{2}+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^n)^(3/2),x]
```

```
[Out] (x*(b*x^n)^(3/2))/(1 + (3*n)/2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^n)^(3/2), x)
```

maple [A] time = 0.00, size = 18, normalized size = 0.75

$$\frac{2\,(b\,x^n)^{\frac{3}{2}}\,x}{3n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^(3/2),x)
[Out] 2*x/(2+3*n)*(b*x^n)^(3/2)
```

maxima [A] time = 1.37, size = 17, normalized size = 0.71

$$\frac{2\,(bx^n)^{\frac{3}{2}}\,x}{3\,n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^(3/2),x, algorithm="maxima")
[Out] 2*(b*x^n)^(3/2)*x/(3*n + 2)
```

mupad [B] time = 0.97, size = 22, normalized size = 0.92

$$\frac{2\,b\,x^{n+1}\,\sqrt{b\,x^n}}{3\,n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^(3/2),x)
[Out] (2*b*x^(n + 1)*(b*x^n)^(1/2))/(3*n + 2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{2b^{\frac{3}{2}}x(x^n)^{\frac{3}{2}}}{3n+2} & \text{for } n \neq -\frac{2}{3} \\ \int \left(\frac{b}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}} dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**n)**(3/2),x)
[Out] Piecewise((2*b**(3/2)*x*(x**n)**(3/2)/(3*n + 2), Ne(n, -2/3)), (Integral((b
/x**(2/3))**(3/2), x), True))
```

3.139 $\int \frac{(bx^n)^{3/2}}{x}\,dx$

Optimal. Leaf size=20

$$\frac{2bx^n\sqrt{bx^n}}{3n}$$

```
[Out] 2/3*b*x^n*(b*x^n)^(1/2)/n
```

Rubi [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$\frac{2bx^n\sqrt{bx^n}}{3n}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^n)^(3/2)/x,x]
```

```
[Out] (2*b*x^n*Sqrt[b*x^n])/(3*n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^n)^{3/2}}{x}\,dx &= \left(bx^{-n/2}\sqrt{bx^n}\right)\int x^{-1+\frac{3n}{2}}\,dx\\ &= \frac{2bx^n\sqrt{bx^n}}{3n}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 0.80

$$\frac{2\,(bx^n)^{3/2}}{3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^n)^(3/2)/x,x]
```

```
[Out] (2*(b*x^n)^(3/2))/(3*n)
```

fricas [A] time = 0.86, size = 16, normalized size = 0.80

$$\frac{2\sqrt{bx^n}\,bx^n}{3\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*x^n)*b*x^n/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((b*x^n)^(3/2)/x, x)
```

maple [A] time = 0.00, size = 13, normalized size = 0.65

$$\frac{2\,(b\,x^n)^{\frac{3}{2}}}{3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^(3/2)/x,x)
```

```
[Out] 2/3/n*(b*x^n)^(3/2)
```

maxima [A] time = 1.32, size = 12, normalized size = 0.60

$$\frac{2\,(bx^n)^{\frac{3}{2}}}{3\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 2/3*(b*x^n)^(3/2)/n
```

mupad [B] time = 0.93, size = 16, normalized size = 0.80

$$\frac{2\,b\,x^n\,\sqrt{b\,x^n}}{3\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^(3/2)/x,x)
```

```
[Out] (2*b*x^n*(b*x^n)^(1/2))/(3*n)
```

sympy [A] time = 6.69, size = 24, normalized size = 1.20

$$\begin{cases} \frac{2b^{\frac{3}{2}}(x^n)^{\frac{3}{2}}}{3n} & \text{for } n \neq 0 \\ b^{\frac{3}{2}}\log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**n)**(3/2)/x,x)
```

```
[Out] Piecewise((2*b**(3/2)*(x**n)**(3/2)/(3*n), Ne(n, 0)), (b**(3/2)*log(x), Tru
e))
```

3.140 $\int \frac{(bx^n)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=24

$$-\frac{2bx^{n-1}\sqrt{bx^n}}{2-3n}$$

```
[Out] -2*b*x^(-1+n)*(b*x^n)^(1/2)/(2-3*n)
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$-\frac{2bx^{n-1}\sqrt{bx^n}}{2-3n}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^n)^(3/2)/x^2,x]
```

```
[Out] (-2*b*x^(-1 + n)*Sqrt[b*x^n])/(2 - 3*n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^n)^{3/2}}{x^2}\,dx &= \left(bx^{-n/2}\sqrt{bx^n}\right)\int x^{-2+\frac{3n}{2}}\,dx\\ &= -\frac{2bx^{-1+n}\sqrt{bx^n}}{2-3n}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.92

$$\frac{(bx^n)^{3/2}}{\left(\frac{3n}{2}-1\right)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^n)^(3/2)/x^2,x]
```

```
[Out] (b*x^n)^(3/2)/((-1 + (3*n)/2)*x)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^n)^(3/2)/x^2, x)
```

maple [A] time = 0.00, size = 20, normalized size = 0.83

$$\frac{2\,(b\,x^n)^{\frac{3}{2}}}{(3n-2)\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^(3/2)/x^2,x)
```

```
[Out] 2/x/(-2+3*n)*(b*x^n)^(3/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((3*n)/2-2>0)', see `assume?` for mo
re details)Is (3*n)/2-2 equal to -1?
```

mupad [B] time = 0.93, size = 22, normalized size = 0.92

$$\frac{2\,b\,x^{n-1}\,\sqrt{b\,x^n}}{3\,n-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^(3/2)/x^2,x)
```

```
[Out] (2*b*x^(n - 1)*(b*x^n)^(1/2))/(3*n - 2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{2b^{\frac{3}{2}}(x^n)^{\frac{3}{2}}}{3nx-2x} & \text{for } n \neq \frac{2}{3} \\ \int \frac{\left(bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^2}\,dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**n)**(3/2)/x**2,x)
```

```
[Out] Piecewise((2*b**(3/2)*(x**n)**(3/2)/(3*n*x - 2*x), Ne(n, 2/3)), (Integral((
b*x**(2/3))**(3/2)/x**2, x), True))
```

3.141 $\int \frac{(bx^n)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=24

$$-\frac{2bx^{n-2}\sqrt{bx^n}}{4-3n}$$

```
[Out] -2*b*x^(-2+n)*(b*x^n)^(1/2)/(4-3*n)
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$-\frac{2bx^{n-2}\sqrt{bx^n}}{4-3n}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^n)^(3/2)/x^3,x]
```

```
[Out] (-2*b*x^(-2 + n)*Sqrt[b*x^n])/(4 - 3*n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^n)^{3/2}}{x^3}\,dx &= \left(bx^{-n/2}\sqrt{bx^n}\right)\int x^{-3+\frac{3n}{2}}\,dx \\ &= -\frac{2bx^{-2+n}\sqrt{bx^n}}{4-3n}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.92

$$\frac{(bx^n)^{3/2}}{\left(\frac{3n}{2}-2\right)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^n)^(3/2)/x^3,x]
```

```
[Out] (b*x^n)^(3/2)/((-2 + (3*n)/2)*x^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^n)^(3/2)/x^3, x)
```

maple [A] time = 0.00, size = 20, normalized size = 0.83

$$\frac{2\,(b\,x^n)^{\frac{3}{2}}}{(3n-4)\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^(3/2)/x^3,x)
```

```
[Out] 2/x^2/(3*n-4)*(b*x^n)^(3/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((3*n)/2-3>0)', see `assume?` for mo
re details)Is (3*n)/2-3 equal to -1?
```

mupad [B] time = 0.96, size = 22, normalized size = 0.92

$$\frac{2\,b\,x^{n-2}\,\sqrt{b\,x^n}}{3\,n-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^(3/2)/x^3,x)
```

```
[Out] (2*b*x^(n - 2)*(b*x^n)^(1/2))/(3*n - 4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{2b^{\frac{3}{2}}(x^n)^{\frac{3}{2}}}{3nx^2-4x^2} & \text{for } n \neq \frac{4}{3} \\ \int \frac{\left(bx^{\frac{4}{3}}\right)^{\frac{3}{2}}}{x^3}\,dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**n)**(3/2)/x**3,x)
```

```
[Out] Piecewise((2*b**(3/2)*(x**n)**(3/2)/(3*n*x**2 - 4*x**2), Ne(n, 4/3)), (Inte
gral((b*x**(4/3))**(3/2)/x**3, x), True))
```

3.142 $\int \frac{(bx^n)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=26

$$-\frac{2bx^{n-3}\sqrt{bx^n}}{3(2-n)}$$

```
[Out] -2/3*b*x^(-3+n)*(b*x^n)^(1/2)/(2-n)
```

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$-\frac{2bx^{n-3}\sqrt{bx^n}}{3(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^n)^(3/2)/x^4,x]
```

```
[Out] (-2*b*x^(-3 + n)*Sqrt[b*x^n])/(3*(2 - n))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^n)^{3/2}}{x^4}\,dx &= \left(bx^{-n/2}\sqrt{bx^n}\right)\int x^{-4+\frac{3n}{2}}\,dx \\ &= -\frac{2bx^{-3+n}\sqrt{bx^n}}{3(2-n)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.85

$$\frac{(bx^n)^{3/2}}{\left(\frac{3n}{2}-3\right)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^n)^(3/2)/x^4,x]
```

```
[Out] (b*x^n)^(3/2)/((-3 + (3*n)/2)*x^3)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*x^n)^(3/2)/x^4, x)
```

maple [**A**] time = 0.00, size = 18, normalized size = 0.69

$$\frac{2\,(b\,x^n)^{\frac{3}{2}}}{3\,(n-2)\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^(3/2)/x^4,x)
```

```
[Out] 2/3/x^3/(n-2)*(b*x^n)^(3/2)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((3*n)/2-4>0)', see `assume?` for mo
re details)Is (3*n)/2-4 equal to -1?
```

mupad [**B**] time = 0.96, size = 22, normalized size = 0.85

$$\frac{2\,b\,x^{n-3}\,\sqrt{b\,x^n}}{3\,n-6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^(3/2)/x^4,x)
```

```
[Out] (2*b*x^(n - 3)*(b*x^n)^(1/2))/(3*n - 6)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{2b^{\frac{3}{2}}(x^n)^{\frac{3}{2}}}{3nx^3-6x^3} & \text{for } n \neq 2 \\ \int \frac{\left(bx^2\right)^{\frac{3}{2}}}{x^4}\,dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**n)**(3/2)/x**4,x)
```

```
[Out] Piecewise((2*b**(3/2)*(x**n)**(3/2)/(3*n*x**3 - 6*x**3), Ne(n, 2)), (Integr
al((b*x**2)**(3/2)/x**4, x), True))
```

3.143 $\int \frac{x^2}{\sqrt{bx^n}}\,dx$

Optimal. Leaf size=21

$$\frac{2x^3}{(6-n)\sqrt{bx^n}}$$

```
[Out] 2*x^3/(6-n)/(b*x^n)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$\frac{2x^3}{(6-n)\sqrt{bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[b*x^n],x]
```

```
[Out] (2*x^3)/((6 - n)*Sqrt[b*x^n])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{bx^n}}\,dx &= \frac{x^{n/2}\int x^{2-\frac{n}{2}}\,dx}{\sqrt{bx^n}}\\ &= \frac{2x^3}{(6-n)\sqrt{bx^n}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 0.90

$$-\frac{2x^3}{(n-6)\sqrt{bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[b*x^n],x]
```

```
[Out] (-2*x^3)/((-6 + n)*Sqrt[b*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(b*x^n), x)
```

maple [**A**] time = 0.00, size = 18, normalized size = 0.86

$$-\frac{2x^3}{(n-6)\sqrt{b\,x^n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^n)^(1/2),x)
```

```
[Out] -2*x^3/(-6+n)/(b*x^n)^(1/2)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(2-n/2>0)', see `assume?` for more d
etails)Is 2-n/2 equal to -1?
```

mupad [**B**] time = 1.04, size = 24, normalized size = 1.14

$$-\frac{2\,x^{3-n}\,\sqrt{b\,x^n}}{b\,(n-6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^n)^(1/2),x)
```

```
[Out] -(2*x^(3 - n)*(b*x^n)^(1/2))/(b*(n - 6))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} -\frac{2x^3}{\sqrt{b}\,n\sqrt{x^n}-6\sqrt{b}\,\sqrt{x^n}} & \text{for } n \neq 6 \\ \int \frac{x^2}{\sqrt{bx^6}}\,dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**n)**(1/2),x)
```

```
[Out] Piecewise((-2*x**3/(sqrt(b)*n*sqrt(x**n) - 6*sqrt(b)*sqrt(x**n)), Ne(n, 6))
, (Integral(x**2/sqrt(b*x**6), x), True))
```

3.144 $\int \frac{x}{\sqrt{bx^n}}\,dx$

Optimal. Leaf size=21

$$\frac{2x^2}{(4-n)\sqrt{bx^n}}$$

```
[Out] 2*x^2/(4-n)/(b*x^n)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 30}

$$\frac{2x^2}{(4-n)\sqrt{bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[b*x^n],x]
```

```
[Out] (2*x^2)/((4 - n)*Sqrt[b*x^n])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{bx^n}}\,dx &= \frac{x^{n/2}\int x^{1-\frac{n}{2}}\,dx}{\sqrt{bx^n}}\\ &= \frac{2x^2}{(4-n)\sqrt{bx^n}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 0.90

$$-\frac{2x^2}{(n-4)\sqrt{bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[b*x^n],x]
```

```
[Out] (-2*x^2)/((-4 + n)*Sqrt[b*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(b*x^n), x)
```

maple [**A**] time = 0.00, size = 18, normalized size = 0.86

$$-\frac{2x^2}{(n-4)\sqrt{b\,x^n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^n)^(1/2),x)
```

```
[Out] -2*x^2/(n-4)/(b*x^n)^(1/2)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(1-n/2>0)', see `assume?` for more d
etails)Is 1-n/2 equal to -1?
```

mupad [**B**] time = 1.04, size = 24, normalized size = 1.14

$$-\frac{2\,x^{2-n}\,\sqrt{b\,x^n}}{b\,(n-4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^n)^(1/2),x)
```

```
[Out] -(2*x^(2 - n)*(b*x^n)^(1/2))/(b*(n - 4))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} -\frac{2x^2}{\sqrt{b}\,n\sqrt{x^n}-4\sqrt{b}\,\sqrt{x^n}} & \text{for } n \neq 4 \\ \int \frac{x}{\sqrt{bx^4}}\,dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**n)**(1/2),x)
```

```
[Out] Piecewise((-2*x**2/(sqrt(b)*n*sqrt(x**n) - 4*sqrt(b)*sqrt(x**n)), Ne(n, 4))
, (Integral(x/sqrt(b*x**4), x), True))
```

3.145 $\int \frac{1}{\sqrt{bx^n}}\,dx$

Optimal. Leaf size=19

$$\frac{2x}{(2-n)\sqrt{bx^n}}$$

```
[Out] 2*x/(2-n)/(b*x^n)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$\frac{2x}{(2-n)\sqrt{bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*x^n],x]
```

```
[Out] (2*x)/((2 - n)*Sqrt[b*x^n])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{bx^n}}\,dx &= \frac{x^{n/2}\int x^{-n/2}\,dx}{\sqrt{bx^n}}\\ &= \frac{2x}{(2-n)\sqrt{bx^n}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 0.89

$$-\frac{2x}{(n-2)\sqrt{bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*x^n],x]
```

```
[Out] (-2*x)/((-2 + n)*Sqrt[b*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*x^n), x)
```

maple [A] time = 0.00, size = 16, normalized size = 0.84

$$-\frac{2x}{(n-2)\sqrt{b\,x^n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^n)^(1/2),x)
```

```
[Out] -2*x/(n-2)/(b*x^n)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-n/2>0)', see `assume?` for more de
tails)Is -n/2 equal to -1?
```

mupad [B] time = 0.97, size = 24, normalized size = 1.26

$$-\frac{2\,x^{1-n}\,\sqrt{b\,x^n}}{b\,(n-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^n)^(1/2),x)
```

```
[Out] -(2*x^(1 - n)*(b*x^n)^(1/2))/(b*(n - 2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} -\frac{2x}{\sqrt{b}\,n\sqrt{x^n}-2\sqrt{b}\,\sqrt{x^n}} & \text{for } n \neq 2 \\ \int \frac{1}{\sqrt{bx^2}}\,dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**n)**(1/2),x)
```

```
[Out] Piecewise((-2*x/(sqrt(b)*n*sqrt(x**n) - 2*sqrt(b)*sqrt(x**n)), Ne(n, 2)), (
Integral(1/sqrt(b*x**2), x), True))
```

3.146 $\int \frac{1}{x\sqrt{bx^n}}\,dx$

Optimal. Leaf size=14

$$-\frac{2}{n\sqrt{bx^n}}$$

```
[Out] -2/n/(b*x^n)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$-\frac{2}{n\sqrt{bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[b*x^n]),x]
```

```
[Out] -2/(n*Sqrt[b*x^n])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{bx^n}}\,dx &= \frac{x^{n/2}\int x^{-1-\frac{n}{2}}\,dx}{\sqrt{bx^n}}\\ &= -\frac{2}{n\sqrt{bx^n}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{2}{n\sqrt{bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[b*x^n]),x]
```

```
[Out] -2/(n*Sqrt[b*x^n])
```

fricas [A] time = 0.86, size = 20, normalized size = 1.43

$$-\frac{2\sqrt{bx^n}}{bnx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^n)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(b*x^n)/(b*n*x^n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^n}\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x^n)*x), x)
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$-\frac{2}{\sqrt{b\, x^n}\ n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^n)^(1/2),x)
[Out] -2/n/(b*x^n)^(1/2)
```

maxima [A] time = 1.29, size = 12, normalized size = 0.86

$$-\frac{2}{\sqrt{bx^n}\, n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^n)^(1/2),x, algorithm="maxima")
[Out] -2/(sqrt(b*x^n)*n)
```

mupad [B] time = 0.97, size = 20, normalized size = 1.43

$$-\frac{2\sqrt{b\, x^n}}{b\, n\, x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^n)^(1/2)),x)
[Out] -(2*(b*x^n)^(1/2))/(b*n*x^n)
```

sympy [A] time = 2.04, size = 24, normalized size = 1.71

$$\begin{cases} -\frac{2}{\sqrt{b}\, n \sqrt{x^n}} & \text{for } n \neq 0 \\ \frac{\log(x)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**n)**(1/2),x)
[Out] Piecewise((-2/(sqrt(b)*n*sqrt(x**n)), Ne(n, 0)), (log(x)/sqrt(b), True))
```

3.147 $\int \frac{1}{x^2\sqrt{bx^n}}\,dx$

Optimal. Leaf size=19

$$-\frac{2}{(n+2)x\sqrt{bx^n}}$$

```
[Out] -2/(2+n)/x/(b*x^n)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$-\frac{2}{(n+2)x\sqrt{bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[b*x^n]),x]
```

```
[Out] -2/((2 + n)*x*Sqrt[b*x^n])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{bx^n}}\,dx &= \frac{x^{n/2}\int x^{-2-\frac{n}{2}}\,dx}{\sqrt{bx^n}}\\ &= -\frac{2}{(2+n)x\sqrt{bx^n}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 19, normalized size = 1.00

$$-\frac{2}{(n+2)x\sqrt{bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[b*x^n]),x]
```

```
[Out] -2/((2 + n)*x*Sqrt[b*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^n}\, x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^n)*x^2), x)
```

maple [A] time = 0.00, size = 18, normalized size = 0.95

$$-\frac{2}{(n+2)\sqrt{b\,x^n}\, x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^n)^(1/2),x)
```

```
[Out] -2/(n+2)/x/(b*x^n)^(1/2)
```

maxima [A] time = 1.33, size = 17, normalized size = 0.89

$$-\frac{2}{\sqrt{bx^n}\,(n+2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/(sqrt(b*x^n)*(n + 2)*x)
```

mupad [B] time = 0.97, size = 24, normalized size = 1.26

$$-\frac{2\sqrt{b\,x^n}}{b\,x^{n+1}\,(n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x^n)^(1/2)),x)
```

```
[Out] -(2*(b*x^n)^(1/2))/(b*x^(n + 1)*(n + 2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} -\frac{2}{\sqrt{b}\,nx\sqrt{x^n}+2\sqrt{b}\,x\sqrt{x^n}} & \text{for } n \neq -2 \\ \int \frac{1}{x^2\sqrt{\frac{b}{x^2}}}\, dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**n)**(1/2),x)
```

```
[Out] Piecewise((-2/(sqrt(b)*n*x*sqrt(x**n) + 2*sqrt(b)*x*sqrt(x**n)), Ne(n, -2))
, (Integral(1/(x**2*sqrt(b/x**2)), x), True))
```

3.148 $\int \frac{1}{x^3\sqrt{bx^n}}\,dx$

Optimal. Leaf size=19

$$-\frac{2}{(n+4)x^2\sqrt{bx^n}}$$

```
[Out] -2/(4+n)/x^2/(b*x^n)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$-\frac{2}{(n+4)x^2\sqrt{bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[b*x^n]),x]
```

```
[Out] -2/((4 + n)*x^2*Sqrt[b*x^n])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{bx^n}}\,dx &= \frac{x^{n/2}\int x^{-3-\frac{n}{2}}\,dx}{\sqrt{bx^n}}\\ &= -\frac{2}{(4+n)x^2\sqrt{bx^n}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$-\frac{2}{(n+4)x^2\sqrt{bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[b*x^n]),x]
```

```
[Out] -2/((4 + n)*x^2*Sqrt[b*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^n}\, x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^n)*x^3), x)
```

maple [A] time = 0.00, size = 18, normalized size = 0.95

$$-\frac{2}{(n+4)\sqrt{b\,x^n}\;x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^n)^(1/2),x)
```

```
[Out] -2/(n+4)/x^2/(b*x^n)^(1/2)
```

maxima [A] time = 1.31, size = 17, normalized size = 0.89

$$-\frac{2}{\sqrt{bx^n}\,(n+4)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/(sqrt(b*x^n)*(n + 4)*x^2)
```

mupad [B] time = 0.98, size = 24, normalized size = 1.26

$$-\frac{2\sqrt{b\,x^n}}{b\,x^{n+2}\,(n+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b*x^n)^(1/2)),x)
```

```
[Out] -(2*(b*x^n)^(1/2))/(b*x^(n + 2)*(n + 4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} -\frac{2}{\sqrt{b}\,nx^2\sqrt{x^n}+4\sqrt{b}\,x^2\sqrt{x^n}} & \text{for } n \neq -4 \\ \int \frac{1}{x^3\sqrt{\frac{b}{x^4}}}\, dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**n)**(1/2),x)
```

```
[Out] Piecewise((-2/(sqrt(b)*n*x**2*sqrt(x**n) + 4*sqrt(b)*x**2*sqrt(x**n)), Ne(n
, -4)), (Integral(1/(x**3*sqrt(b/x**4)), x), True))
```

3.149 $\int \frac{x^2}{(bx^n)^{3/2}}\,dx$

Optimal. Leaf size=30

$$\frac{2x^{3-n}}{3b(2-n)\sqrt{bx^n}}$$

```
[Out] 2/3*x^(3-n)/b/(2-n)/(b*x^n)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$\frac{2x^{3-n}}{3b(2-n)\sqrt{bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(b*x^n)^(3/2),x]
```

```
[Out] (2*x^(3 - n))/(3*b*(2 - n)*Sqrt[b*x^n])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(bx^n)^{3/2}}\,dx &= \frac{x^{n/2}\int x^{2-\frac{3n}{2}}\,dx}{b\sqrt{bx^n}}\\ &= \frac{2x^{3-n}}{3b(2-n)\sqrt{bx^n}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.73

$$\frac{x^3}{\left(3-\frac{3n}{2}\right)(bx^n)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(b*x^n)^(3/2),x]
```

```
[Out] x^3/((3 - (3*n)/2)*(b*x^n)^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(bx^n)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*x^n)^(3/2), x)
```

maple [**A**] time = 0.00, size = 18, normalized size = 0.60

$$-\frac{2x^3}{3\,(n-2)\,(b\,x^n)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^n)^(3/2),x)
```

```
[Out] -2/3*x^3/(n-2)/(b*x^n)^(3/2)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(2-(3*n)/2>0)', see `assume?` for mo
re details)Is 2-(3*n)/2 equal to -1?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2}{(b\,x^n)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^n)^(3/2),x)
```

```
[Out] int(x^2/(b*x^n)^(3/2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} -\frac{2x^3}{3b^{\frac{3}{2}}n(x^n)^{\frac{3}{2}}-6b^{\frac{3}{2}}(x^n)^{\frac{3}{2}}} & \text{for } n \neq 2 \\ \int \frac{x^2}{\left(bx^2\right)^{\frac{3}{2}}}\,dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**n)**(3/2),x)
```

```
[Out] Piecewise((-2*x**3/(3*b**(3/2)*n*(x**n)**(3/2) - 6*b**(3/2)*(x**n)**(3/2)),
 Ne(n, 2)), (Integral(x**2/(b*x**2)**(3/2), x), True))
```

3.150 $\int \frac{x}{(bx^n)^{3/2}}\,dx$

Optimal. Leaf size=28

$$\frac{2x^{2-n}}{b(4-3n)\sqrt{bx^n}}$$

```
[Out] 2*x^(2-n)/b/(4-3*n)/(b*x^n)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 30}

$$\frac{2x^{2-n}}{b(4-3n)\sqrt{bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(b*x^n)^(3/2),x]
```

```
[Out] (2*x^(2 - n))/(b*(4 - 3*n)*Sqrt[b*x^n])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(bx^n)^{3/2}}\,dx &= \frac{x^{n/2}\int x^{1-\frac{3n}{2}}\,dx}{b\sqrt{bx^n}}\\ &= \frac{2x^{2-n}}{b(4-3n)\sqrt{bx^n}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.79

$$\frac{x^2}{\left(2-\frac{3n}{2}\right)(bx^n)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(b*x^n)^(3/2),x]
```

```
[Out] x^2/((2 - (3*n)/2)*(b*x^n)^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(bx^n)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/(b*x^n)^(3/2), x)
```

maple [**A**] time = 0.00, size = 20, normalized size = 0.71

$$-\frac{2x^2}{(3n-4)\,(b\,x^n)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^n)^(3/2),x)
```

```
[Out] -2*x^2/(3*n-4)/(b*x^n)^(3/2)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(1-(3*n)/2>0)', see `assume?` for mo
re details)Is 1-(3*n)/2 equal to -1?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x}{(b\,x^n)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^n)^(3/2),x)
```

```
[Out] int(x/(b*x^n)^(3/2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} -\frac{2x^2}{3b^{\frac{3}{2}}n(x^n)^{\frac{3}{2}} - 4b^{\frac{3}{2}}(x^n)^{\frac{3}{2}}} & \text{for } n \neq \frac{4}{3} \\ \int \frac{x}{\left(bx^{\frac{4}{3}}\right)^{\frac{3}{2}}}\,dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**n)**(3/2),x)
```

```
[Out] Piecewise((-2*x**2/(3*b**(3/2)*n*(x**n)**(3/2) - 4*b**(3/2)*(x**n)**(3/2)),
 Ne(n, 4/3)), (Integral(x/(b*x**(4/3))**(3/2), x), True))
```

3.151 $\int \frac{1}{(bx^n)^{3/2}}\,dx$

Optimal. Leaf size=28

$$\frac{2x^{1-n}}{b(2-3n)\sqrt{bx^n}}$$

```
[Out] 2*x^(1-n)/b/(2-3*n)/(b*x^n)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$\frac{2x^{1-n}}{b(2-3n)\sqrt{bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^n)^(-3/2),x]
```

```
[Out] (2*x^(1 - n))/(b*(2 - 3*n)*Sqrt[b*x^n])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(bx^n)^{3/2}}\,dx &= \frac{x^{n/2}\int x^{-3n/2}\,dx}{b\sqrt{bx^n}}\\ &= \frac{2x^{1-n}}{b(2-3n)\sqrt{bx^n}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.71

$$\frac{x}{\left(1-\frac{3n}{2}\right)(bx^n)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^n)^(-3/2),x]
```

```
[Out] x/((1 - (3*n)/2)*(b*x^n)^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^n)^(-3/2), x)
```

maple [A] time = 0.00, size = 18, normalized size = 0.64

$$-\frac{2x}{(3n-2)\,(b\,x^n)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^n)^(3/2),x)
```

```
[Out] -2*x/(3*n-2)/(b*x^n)^(3/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-(3*n)/2>0)', see `assume?` for mor
e details)Is -(3*n)/2 equal to -1?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(b\,x^n)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^n)^(3/2),x)
```

```
[Out] int(1/(b*x^n)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} -\frac{2x}{3b^{\frac{3}{2}}n(x^n)^{\frac{3}{2}}-2b^{\frac{3}{2}}(x^n)^{\frac{3}{2}}} & \text{for } n \neq \frac{2}{3} \\ \int \frac{1}{\left(bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**n)**(3/2),x)
```

```
[Out] Piecewise((-2*x/(3*b**(3/2)*n*(x**n)**(3/2) - 2*b**(3/2)*(x**n)**(3/2)), Ne
(n, 2/3)), (Integral((b*x**(2/3))**(-3/2), x), True))
```

3.152 $\int \frac{1}{x(bx^n)^{3/2}}\,dx$

Optimal. Leaf size=24

$$-\frac{2x^{-n}}{3bn\sqrt{bx^n}}$$

```
[Out] -2/3/b/n/(x^n)/(b*x^n)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$-\frac{2x^{-n}}{3bn\sqrt{bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(b*x^n)^(3/2)),x]
```

```
[Out] -2/(3*b*n*x^n*Sqrt[b*x^n])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\,(bx^n)^{3/2}}\,dx &= \frac{x^{n/2}\int x^{-1-\frac{3n}{2}}\,dx}{b\sqrt{bx^n}}\\ &= -\frac{2x^{-n}}{3bn\sqrt{bx^n}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 0.67

$$-\frac{2}{3n\,(bx^n)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(b*x^n)^(3/2)),x]
```

```
[Out] -2/(3*n*(b*x^n)^(3/2))
```

fricas [A] time = 0.89, size = 22, normalized size = 0.92

$$-\frac{2\sqrt{bx^n}}{3\,b^2nx^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^n)^(3/2),x, algorithm="fricas")
[Out] -2/3*sqrt(b*x^n)/(b^2*n*x^(2*n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n)^{\frac{3}{2}}\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^n)^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*x^n)^(3/2)*x), x)
```

maple [A] time = 0.00, size = 13, normalized size = 0.54

$$-\frac{2}{3\,(b\,x^n)^{\frac{3}{2}}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^n)^(3/2),x)
[Out] -2/3/n/(b*x^n)^(3/2)
```

maxima [A] time = 1.33, size = 12, normalized size = 0.50

$$-\frac{2}{3\ (bx^n)^{\frac{3}{2}}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^n)^(3/2),x, algorithm="maxima")
[Out] -2/3/((b*x^n)^(3/2)*n)
```

mupad [B] time = 0.93, size = 12, normalized size = 0.50

$$-\frac{2}{3\,n\,(b\,x^n)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^n)^(3/2)),x)
[Out] -2/(3*n*(b*x^n)^(3/2))
```

sympy [A] time = 3.31, size = 26, normalized size = 1.08

$$\begin{cases} -\frac{2}{3b^{\frac{3}{2}}n(x^n)^{\frac{3}{2}}} & \text{for } n \neq 0 \\ \frac{\log(x)}{b^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**n)**(3/2),x)
[Out] Piecewise((-2/(3*b**(3/2)*n*(x**n)**(3/2)), Ne(n, 0)), (log(x)/b**(3/2), Tr
ue))
```

3.153 $\int \frac{1}{x^2 (bx^n)^{3/2}}\, dx$

Optimal. Leaf size=28

$$-\frac{2x^{-n-1}}{b(3n+2)\sqrt{bx^n}}$$

```
[Out] -2*x^(-1-n)/b/(2+3*n)/(b*x^n)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$-\frac{2x^{-n-1}}{b(3n+2)\sqrt{bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(b*x^n)^(3/2)),x]
```

```
[Out] (-2*x^(-1 - n))/(b*(2 + 3*n)*Sqrt[b*x^n])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2 (bx^n)^{3/2}}\, dx &= \frac{x^{n/2} \int x^{-2-\frac{3n}{2}}\, dx}{b\sqrt{bx^n}} \\ &= -\frac{2x^{-1-n}}{b(2+3n)\sqrt{bx^n}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.79

$$\frac{1}{\left(-\frac{3n}{2}-1\right) x\, (bx^n)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(b*x^n)^(3/2)),x]
```

```
[Out] 1/((-1 - (3*n)/2)*x*(b*x^n)^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n)^{\frac{3}{2}}\, x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n)^(3/2)*x^2), x)
```

maple [A] time = 0.00, size = 20, normalized size = 0.71

$$-\frac{2}{(3n+2)\,(b\,x^n)^{\frac{3}{2}}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^n)^(3/2),x)
```

```
[Out] -2/x/(3*n+2)/(b*x^n)^(3/2)
```

maxima [A] time = 1.34, size = 19, normalized size = 0.68

$$-\frac{2}{(bx^n)^{\frac{3}{2}}\,(3\,n+2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/((b*x^n)^(3/2)*(3*n + 2)*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^2\,(b\,x^n)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x^n)^(3/2)),x)
```

```
[Out] int(1/(x^2*(b*x^n)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} -\frac{2}{3b^{\frac{3}{2}}nx(x^n)^{\frac{3}{2}}+2b^{\frac{3}{2}}x(x^n)^{\frac{3}{2}}} & \text{for } n \neq -\frac{2}{3} \\ \int \frac{1}{x^2\left(\frac{b}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}}}\, dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**n)**(3/2),x)
```

```
[Out] Piecewise((-2/(3*b**(3/2)*n*x*(x**n)**(3/2) + 2*b**(3/2)*x*(x**n)**(3/2)),
Ne(n, -2/3)), (Integral(1/(x**2*(b/x**(2/3))**(3/2)), x), True))
```

3.154 $\int \frac{1}{x^3 (bx^n)^{3/2}}\,dx$

Optimal. Leaf size=28

$$-\frac{2x^{-n-2}}{b(3n+4)\sqrt{bx^n}}$$

```
[Out] -2*x^(-2-n)/b/(4+3*n)/(b*x^n)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$-\frac{2x^{-n-2}}{b(3n+4)\sqrt{bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(b*x^n)^(3/2)),x]
```

```
[Out] (-2*x^(-2 - n))/(b*(4 + 3*n)*Sqrt[b*x^n])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3 (bx^n)^{3/2}}\,dx &= \frac{x^{n/2}\int x^{-3-\frac{3n}{2}}\,dx}{b\sqrt{bx^n}}\\ &= -\frac{2x^{-2-n}}{b(4+3n)\sqrt{bx^n}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.79

$$\frac{1}{\left(-\frac{3n}{2}-2\right)x^2 (bx^n)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(b*x^n)^(3/2)),x]
```

```
[Out] 1/((-2 - (3*n)/2)*x^2*(b*x^n)^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n)^{\frac{3}{2}}\, x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n)^(3/2)*x^3), x)
```

maple [A] time = 0.00, size = 20, normalized size = 0.71

$$-\frac{2}{(3n+4)\,(b\,x^n)^{\frac{3}{2}}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^n)^(3/2),x)
```

```
[Out] -2/x^2/(3*n+4)/(b*x^n)^(3/2)
```

maxima [A] time = 1.33, size = 19, normalized size = 0.68

$$-\frac{2}{(bx^n)^{\frac{3}{2}}\,(3\,n+4)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/((b*x^n)^(3/2)*(3*n + 4)*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^3\,(b\,x^n)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b*x^n)^(3/2)),x)
```

```
[Out] int(1/(x^3*(b*x^n)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} -\frac{2}{3b^{\frac{3}{2}}nx^2(x^n)^{\frac{3}{2}}+4b^{\frac{3}{2}}x^2(x^n)^{\frac{3}{2}}} & \text{for } n \neq -\frac{4}{3} \\ \int \frac{1}{x^3\left(\frac{b}{x^{\frac{4}{3}}}\right)^{\frac{3}{2}}}\, dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**n)**(3/2),x)
```

```
[Out] Piecewise((-2/(3*b**(3/2)*n*x**2*(x**n)**(3/2) + 4*b**(3/2)*x**2*(x**n)**(3
/2)), Ne(n, -4/3)), (Integral(1/(x**3*(b/x**(4/3))**(3/2)), x), True))
```

3.155 $\int \frac{1}{x^4 (bx^n)^{3/2}}\,dx$

Optimal. Leaf size=28

$$-\frac{2x^{-n-3}}{3b(n+2)\sqrt{bx^n}}$$

```
[Out] -2/3*x^(-3-n)/b/(2+n)/(b*x^n)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$-\frac{2x^{-n-3}}{3b(n+2)\sqrt{bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(b*x^n)^(3/2)),x]
```

```
[Out] (-2*x^(-3 - n))/(3*b*(2 + n)*Sqrt[b*x^n])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4 (bx^n)^{3/2}}\,dx &= \frac{x^{n/2}\int x^{-4-\frac{3n}{2}}\,dx}{b\sqrt{bx^n}}\\ &= -\frac{2x^{-3-n}}{3b(2+n)\sqrt{bx^n}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.79

$$\frac{1}{\left(-\frac{3n}{2}-3\right)x^3\,(bx^n)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(b*x^n)^(3/2)),x]
```

```
[Out] 1/((-3 - (3*n)/2)*x^3*(b*x^n)^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n)^{\frac{3}{2}}\, x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n)^(3/2)*x^4), x)
```

maple [A] time = 0.00, size = 18, normalized size = 0.64

$$-\frac{2}{3\,(n+2)\,(b\,x^n)^{\frac{3}{2}}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^n)^(3/2),x)
```

```
[Out] -2/3/x^3/(n+2)/(b*x^n)^(3/2)
```

maxima [A] time = 1.35, size = 17, normalized size = 0.61

$$-\frac{2}{3\ (bx^n)^{\frac{3}{2}}\,(n+2)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/3/((b*x^n)^(3/2)*(n + 2)*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^4\,(b\,x^n)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(b*x^n)^(3/2)),x)
```

```
[Out] int(1/(x^4*(b*x^n)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} -\frac{2}{3b^{\frac{3}{2}}nx^3(x^n)^{\frac{3}{2}}+6b^{\frac{3}{2}}x^3(x^n)^{\frac{3}{2}}} & \text{for } n \neq -2 \\ \int \frac{1}{x^4\left(\frac{b}{x^2}\right)^{\frac{3}{2}}}\, dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**n)**(3/2),x)
```

```
[Out] Piecewise((-2/(3*b**(3/2)*n*x**3*(x**n)**(3/2) + 6*b**(3/2)*x**3*(x**n)**(3
/2)), Ne(n, -2)), (Integral(1/(x**4*(b/x**2)**(3/2)), x), True))
```

3.156 $\int \frac{x^m}{(ax^n)^{3/2}}\,dx$

Optimal. Leaf size=32

$$\frac{2x^{m-n+1}}{a(2m-3n+2)\sqrt{ax^n}}$$

```
[Out] 2*x^(1+m-n)/a/(2+2*m-3*n)/(a*x^n)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$\frac{2x^{m-n+1}}{a(2m-3n+2)\sqrt{ax^n}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a*x^n)^(3/2),x]
```

```
[Out] (2*x^(1 + m - n))/(a*(2 + 2*m - 3*n)*Sqrt[a*x^n])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{(ax^n)^{3/2}}\,dx &= \frac{x^{n/2}\int x^{m-\frac{3n}{2}}\,dx}{a\sqrt{ax^n}}\\ &= \frac{2x^{1+m-n}}{a(2+2m-3n)\sqrt{ax^n}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.78

$$\frac{x^{m+1}}{\left(m-\frac{3n}{2}+1\right)(ax^n)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a*x^n)^(3/2),x]
```

```
[Out] x^(1 + m)/((1 + m - (3*n)/2)*(a*x^n)^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(ax^n)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/(a*x^n)^(3/2), x)
```

maple [A] time = 0.00, size = 25, normalized size = 0.78

$$\frac{2x^{m+1}}{(2m-3n+2)\,(a\,x^n)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a*x^n)^(3/2),x)
```

```
[Out] 2*x^(m+1)/(2+2*m-3*n)/(a*x^n)^(3/2)
```

maxima [A] time = 1.49, size = 24, normalized size = 0.75

$$\frac{2\,xx^m}{a^{\frac{3}{2}}(2\,m-3\,n+2)(x^n)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*x*x^m/(a^(3/2)*(2*m - 3*n + 2)*(x^n)^(3/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{(a\,x^n)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a*x^n)^(3/2),x)
```

```
[Out] int(x^m/(a*x^n)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{2xx^m}{2a^{\frac{3}{2}}m(x^n)^{\frac{3}{2}}-3a^{\frac{3}{2}}n(x^n)^{\frac{3}{2}}+2a^{\frac{3}{2}}(x^n)^{\frac{3}{2}}} & \text{for } m \neq \frac{3n}{2}-1 \\ \int \frac{x^{\frac{3n}{2}-1}}{(ax^n)^{\frac{3}{2}}}\,dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a*x**n)**(3/2),x)
```

```
[Out] Piecewise((2*x*x**m/(2*a**(3/2)*m*(x**n)**(3/2) - 3*a**(3/2)*n*(x**n)**(3/2
) + 2*a**(3/2)*(x**n)**(3/2)), Ne(m, 3*n/2 - 1)), (Integral(x**(3*n/2 - 1)/
(a*x**n)**(3/2), x), True))
```

3.157 $\int \frac{(cx)^m}{(ax^n)^{3/2}}\,dx$

Optimal. Leaf size=36

$$\frac{2x^{1-n}(cx)^m}{a(2m-3n+2)\sqrt{ax^n}}$$

```
[Out] 2*x^(1-n)*(c*x)^m/a/(2+2*m-3*n)/(a*x^n)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {15, 20, 30}

$$\frac{2x^{1-n}(cx)^m}{a(2m-3n+2)\sqrt{ax^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m/(a*x^n)^(3/2),x]
```

```
[Out] (2*x^(1 - n)*(c*x)^m)/(a*(2 + 2*m - 3*n)*Sqrt[a*x^n])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx)^m}{(ax^n)^{3/2}}\,dx &= \frac{x^{n/2}\int x^{-3n/2}(cx)^m\,dx}{a\sqrt{ax^n}}\\ &= \frac{\left(x^{-m+\frac{n}{2}}(cx)^m\right)\int x^{m-\frac{3n}{2}}\,dx}{a\sqrt{ax^n}}\\ &= \frac{2x^{1-n}(cx)^m}{a(2+2m-3n)\sqrt{ax^n}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.72

$$\frac{x(cx)^m}{\left(m-\frac{3n}{2}+1\right)(ax^n)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m/(a*x^n)^(3/2),x]
```

```
[Out] (x*(c*x)^m)/((1 + m - (3*n)/2)*(a*x^n)^(3/2))
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(a*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{(ax^n)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(a*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^m/(a*x^n)^(3/2), x)
```

maple [**A**] time = 0.00, size = 26, normalized size = 0.72

$$\frac{2x\,(cx)^m}{(2m-3n+2)\,(a\,x^n)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(a*x^n)^(3/2),x)
```

```
[Out] 2*x/(2*m-3*n+2)*(c*x)^m/(a*x^n)^(3/2)
```

maxima [**A**] time = 1.62, size = 27, normalized size = 0.75

$$\frac{2\,c^m x x^m}{a^{\frac{3}{2}}(2\,m-3\,n+2)(x^n)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(a*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*c^m*x*x^m/(a^(3/2)*(2*m - 3*n + 2)*(x^n)^(3/2))
```

mupad [**B**] time = 1.04, size = 34, normalized size = 0.94

$$\frac{2\,x^{1-2\,n}\,\sqrt{a\,x^n}\,(c\,x)^m}{a^2\,(2\,m-3\,n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(a*x^n)^(3/2),x)
```

```
[Out] (2*x^(1 - 2*n)*(a*x^n)^(1/2)*(c*x)^m)/(a^2*(2*m - 3*n + 2))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{2c^m x x^m}{2a^{\frac{3}{2}}m(x^n)^{\frac{3}{2}} - 3a^{\frac{3}{2}}n(x^n)^{\frac{3}{2}} + 2a^{\frac{3}{2}}(x^n)^{\frac{3}{2}}} & \text{for } m \neq \frac{3n}{2} - 1 \\ \int \frac{(cx)^{\frac{3n}{2}-1}}{(ax^n)^{\frac{3}{2}}}\,dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m/(a*x**n)**(3/2),x)
```

```
[Out] Piecewise((2*c**m*x*x**m/(2*a**(3/2)*m*(x**n)**(3/2) - 3*a**(3/2)*n*(x**n)*
*(3/2) + 2*a**(3/2)*(x**n)**(3/2)), Ne(m, 3*n/2 - 1)), (Integral((c*x)**(3*
n/2 - 1)/(a*x**n)**(3/2), x), True))
```

3.158 $\int x^m (bx^n)^{3/2}\, dx$

Optimal. Leaf size=28

$$\frac{2b\sqrt{bx^n}\, x^{m+n+1}}{2m+3n+2}$$

```
[Out] 2*b*x^(1+m+n)*(b*x^n)^(1/2)/(2+2*m+3*n)
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$\frac{2b\sqrt{bx^n}\, x^{m+n+1}}{2m+3n+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(b*x^n)^(3/2),x]
```

```
[Out] (2*b*x^(1 + m + n)*Sqrt[b*x^n])/(2 + 2*m + 3*n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^m (bx^n)^{3/2}\, dx &= \left(bx^{-n/2}\sqrt{bx^n}\right)\int x^{m+\frac{3n}{2}}\, dx \\ &= \frac{2bx^{1+m+n}\sqrt{bx^n}}{2+2m+3n}\end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 0.89

$$\frac{x^{m+1}(bx^n)^{3/2}}{m+\frac{3n}{2}+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(b*x^n)^(3/2),x]
```

```
[Out] (x^(1 + m)*(b*x^n)^(3/2))/(1 + m + (3*n)/2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n)^{\frac{3}{2}} x^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^n)^(3/2)*x^m, x)
```

maple [**A**] time = 0.00, size = 25, normalized size = 0.89

$$\frac{2\,(b\,x^n)^{\frac{3}{2}}\,x^{m+1}}{2m+3n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^n)^(3/2),x)
```

```
[Out] 2*x^(m+1)/(2+2*m+3*n)*(b*x^n)^(3/2)
```

maxima [**A**] time = 1.39, size = 24, normalized size = 0.86

$$\frac{2\,b^{\frac{3}{2}} x x^m (x^n)^{\frac{3}{2}}}{2\,m+3\,n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*b^(3/2)*x*x^m*(x^n)^(3/2)/(2*m + 3*n + 2)
```

mupad [**B**] time = 1.06, size = 26, normalized size = 0.93

$$\frac{2\,b\,x^{m+n+1}\,\sqrt{b\,x^n}}{2\,m+3\,n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^n)^(3/2),x)
```

```
[Out] (2*b*x^(m + n + 1)*(b*x^n)^(1/2))/(2*m + 3*n + 2)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**n)**(3/2),x)
```

```
[Out] Timed out
```

3.159 $\int x^m \sqrt{bx^n}\, dx$

Optimal. Leaf size=24

$$\frac{2x^{m+1}\sqrt{bx^n}}{2m+n+2}$$

```
[Out] 2*x^(1+m)*(b*x^n)^(1/2)/(2+2*m+n)
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$\frac{2x^{m+1}\sqrt{bx^n}}{2m+n+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sqrt[b*x^n],x]
```

```
[Out] (2*x^(1 + m)*Sqrt[b*x^n])/(2 + 2*m + n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^m \sqrt{bx^n}\, dx &= \left(x^{-n/2}\sqrt{bx^n}\right)\int x^{m+\frac{n}{2}}\, dx \\ &= \frac{2x^{1+m}\sqrt{bx^n}}{2+2m+n}\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.04

$$\frac{x^{m+1}\sqrt{bx^n}}{m+\frac{n}{2}+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sqrt[b*x^n],x]
```

```
[Out] (x^(1 + m)*Sqrt[b*x^n])/(1 + m + n/2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^n}\, x^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n)*x^m, x)
```

maple [A] time = 0.00, size = 23, normalized size = 0.96

$$\frac{2\sqrt{b\,x^n}\;x^{m+1}}{2m+n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^n)^(1/2),x)
```

```
[Out] 2*x^(m+1)*(b*x^n)^(1/2)/(2+2*m+n)
```

maxima [A] time = 1.52, size = 22, normalized size = 0.92

$$\frac{2\,\sqrt{b}\,x x^m \sqrt{x^n}}{2\,m+n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b)*x*x^m*sqrt(x^n)/(2*m + n + 2)
```

mupad [B] time = 1.04, size = 22, normalized size = 0.92

$$\frac{2\,x^{m+1}\,\sqrt{b\,x^n}}{2\,m+n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^n)^(1/2),x)
```

```
[Out] (2*x^(m + 1)*(b*x^n)^(1/2))/(2*m + n + 2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{2\sqrt{b}\,x x^m \sqrt{x^n}}{2m+n+2} & \text{for } m \neq -\frac{n}{2} - 1 \\ \int x^{-\frac{n}{2}-1}\sqrt{bx^n}\; dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**n)**(1/2),x)
```

```
[Out] Piecewise((2*sqrt(b)*x*x**m*sqrt(x**n)/(2*m + n + 2), Ne(m, -n/2 - 1)), (In
tegral(x**(-n/2 - 1)*sqrt(b*x**n), x), True))
```

3.160 $\int \frac{x^m}{\sqrt{bx^n}}\,dx$

Optimal. Leaf size=26

$$\frac{2x^{m+1}}{(2m-n+2)\sqrt{bx^n}}$$

```
[Out] 2*x^(1+m)/(2+2*m-n)/(b*x^n)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$\frac{2x^{m+1}}{(2m-n+2)\sqrt{bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/Sqrt[b*x^n],x]
```

```
[Out] (2*x^(1 + m))/((2 + 2*m - n)*Sqrt[b*x^n])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{\sqrt{bx^n}}\,dx &= \frac{x^{n/2}\int x^{m-\frac{n}{2}}\,dx}{\sqrt{bx^n}}\\ &= \frac{2x^{1+m}}{(2+2m-n)\sqrt{bx^n}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.96

$$\frac{x^{m+1}}{\left(m-\frac{n}{2}+1\right)\sqrt{bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/Sqrt[b*x^n],x]
```

```
[Out] x^(1 + m)/((1 + m - n/2)*Sqrt[b*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/sqrt(b*x^n), x)
```

maple [**A**] time = 0.00, size = 25, normalized size = 0.96

$$\frac{2x^{m+1}}{(2m-n+2)\sqrt{b\,x^n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^n)^(1/2),x)
```

```
[Out] 2*x^(m+1)/(2+2*m-n)/(b*x^n)^(1/2)
```

maxima [**A**] time = 1.57, size = 24, normalized size = 0.92

$$\frac{2\,xx^m}{\sqrt{b}\,(2\,m-n+2)\sqrt{x^n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*x*x^m/(sqrt(b)*(2*m - n + 2)*sqrt(x^n))
```

mupad [**B**] time = 1.09, size = 30, normalized size = 1.15

$$\frac{2\,x^{m-n+1}\,\sqrt{b\,x^n}}{b\,(2\,m-n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^n)^(1/2),x)
```

```
[Out] (2*x^(m - n + 1)*(b*x^n)^(1/2))/(b*(2*m - n + 2))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{2xx^m}{2\sqrt{b}\,m\sqrt{x^n}-\sqrt{b}\,n\sqrt{x^n}+2\sqrt{b}\,\sqrt{x^n}} & \text{for } m \neq \frac{n}{2}-1 \\ \int \frac{x^{\frac{n}{2}-1}}{\sqrt{bx^n}}\,dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x**n)**(1/2),x)
```

```
[Out] Piecewise((2*x*x**m/(2*sqrt(b)*m*sqrt(x**n) - sqrt(b)*n*sqrt(x**n) + 2*sqrt
(b)*sqrt(x**n)), Ne(m, n/2 - 1)), (Integral(x**(n/2 - 1)/sqrt(b*x**n), x),
True))
```

3.161 $\int \frac{x^m}{(bx^n)^{3/2}}\,dx$

Optimal. Leaf size=32

$$\frac{2x^{m-n+1}}{b(2m-3n+2)\sqrt{bx^n}}$$

```
[Out] 2*x^(1+m-n)/b/(2+2*m-3*n)/(b*x^n)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$\frac{2x^{m-n+1}}{b(2m-3n+2)\sqrt{bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(b*x^n)^(3/2),x]
```

```
[Out] (2*x^(1 + m - n))/(b*(2 + 2*m - 3*n)*Sqrt[b*x^n])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{(bx^n)^{3/2}}\,dx &= \frac{x^{n/2}\int x^{m-\frac{3n}{2}}\,dx}{b\sqrt{bx^n}}\\ &= \frac{2x^{1+m-n}}{b(2+2m-3n)\sqrt{bx^n}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.78

$$\frac{x^{m+1}}{\left(m-\frac{3n}{2}+1\right)(bx^n)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(b*x^n)^(3/2),x]
```

```
[Out] x^(1 + m)/((1 + m - (3*n)/2)*(b*x^n)^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(bx^n)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*x^n)^(3/2), x)
```

maple [A] time = 0.00, size = 25, normalized size = 0.78

$$\frac{2x^{m+1}}{(2m-3n+2)\,(b\,x^n)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^n)^(3/2),x)
```

```
[Out] 2*x^(m+1)/(2*m-3*n+2)/(b*x^n)^(3/2)
```

maxima [A] time = 1.52, size = 24, normalized size = 0.75

$$\frac{2\,x x^m}{b^{\frac{3}{2}}(2\,m-3\,n+2)(x^n)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*x*x^m/(b^(3/2)*(2*m - 3*n + 2)*(x^n)^(3/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{(b\,x^n)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^n)^(3/2),x)
```

```
[Out] int(x^m/(b*x^n)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{2xx^m}{2b^{\frac{3}{2}}m(x^n)^{\frac{3}{2}}-3b^{\frac{3}{2}}n(x^n)^{\frac{3}{2}}+2b^{\frac{3}{2}}(x^n)^{\frac{3}{2}}} & \text{for } m \neq \frac{3n}{2}-1 \\ \int \frac{x^{\frac{3n}{2}-1}}{(bx^n)^{\frac{3}{2}}}\,dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x**n)**(3/2),x)
```

```
[Out] Piecewise((2*x*x**m/(2*b**(3/2)*m*(x**n)**(3/2) - 3*b**(3/2)*n*(x**n)**(3/2
) + 2*b**(3/2)*(x**n)**(3/2)), Ne(m, 3*n/2 - 1)), (Integral(x**(3*n/2 - 1)/
(b*x**n)**(3/2), x), True))
```

3.162 $\int (cx)^m \left(bx^n\right)^{5/2}\, dx$

Optimal. Leaf size=31

$$\frac{2\left(bx^n\right)^{5/2}(cx)^{m+1}}{c(2m+5n+2)}$$

```
[Out] 2*(c*x)^(1+m)*(b*x^n)^(5/2)/c/(2+2*m+5*n)
```

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {15, 20, 30}

$$\frac{2b^2x^{2n+1}\sqrt{bx^n}\,(cx)^m}{2m+5n+2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*(b*x^n)^(5/2),x]
```

```
[Out] (2*b^2*x^(1 + 2*n)*(c*x)^m*Sqrt[b*x^n])/(2 + 2*m + 5*n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (cx)^m \left(bx^n\right)^{5/2}\, dx &= \left(b^2x^{-n/2}\sqrt{bx^n}\right)\int x^{5n/2}(cx)^m\, dx\\ &= \left(b^2x^{-m-\frac{n}{2}}(cx)^m\sqrt{bx^n}\right)\int x^{m+\frac{5n}{2}}\, dx\\ &= \frac{2b^2x^{1+2n}(cx)^m\sqrt{bx^n}}{2+2m+5n}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.84

$$\frac{x\left(bx^n\right)^{5/2}(cx)^m}{m+\frac{5n}{2}+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*(b*x^n)^(5/2),x]
```

```
[Out] (x*(c*x)^m*(b*x^n)^(5/2))/(1 + m + (5*n)/2)
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^n)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n)^{\frac{5}{2}} (cx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^n)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^n)^(5/2)*(c*x)^m, x)
```

maple [**A**] time = 0.00, size = 26, normalized size = 0.84

$$\frac{2\,(b\,x^n)^{\frac{5}{2}}\,x\,(cx)^m}{2m+5n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(b*x^n)^(5/2),x)
```

```
[Out] 2*x/(2+2*m+5*n)*(c*x)^m*(b*x^n)^(5/2)
```

maxima [**A**] time = 1.54, size = 27, normalized size = 0.87

$$\frac{2\,b^{\frac{5}{2}}c^m x x^m (x^n)^{\frac{5}{2}}}{2\,m+5\,n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^n)^(5/2),x, algorithm="maxima")
```

```
[Out] 2*b^(5/2)*c^m*x*x^m*(x^n)^(5/2)/(2*m + 5*n + 2)
```

mupad [**B**] time = 1.05, size = 34, normalized size = 1.10

$$\frac{2\,b^2\,x^{2\,n+1}\,\sqrt{b\,x^n}\,(c\,x)^m}{2\,m+5\,n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^(5/2)*(c*x)^m,x)
```

```
[Out] (2*b^2*x^(2*n + 1)*(b*x^n)^(1/2)*(c*x)^m)/(2*m + 5*n + 2)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(b*x**n)**(5/2),x)
```

```
[Out] Timed out
```

3.163 $\int (cx)^m \left(bx^n\right)^{3/2}\, dx$

Optimal. Leaf size=32

$$\frac{2bx^{n+1}\sqrt{bx^n}\,(cx)^m}{2m+3n+2}$$

```
[Out] 2*b*x^(1+n)*(c*x)^m*(b*x^n)^(1/2)/(2+2*m+3*n)
```

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {15, 20, 30}

$$\frac{2bx^{n+1}\sqrt{bx^n}\,(cx)^m}{2m+3n+2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*(b*x^n)^(3/2),x]
```

```
[Out] (2*b*x^(1 + n)*(c*x)^m*Sqrt[b*x^n])/(2 + 2*m + 3*n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (cx)^m \left(bx^n\right)^{3/2}\, dx &= \left(bx^{-n/2}\sqrt{bx^n}\right)\int x^{3n/2}(cx)^m\, dx \\ &= \left(bx^{-m-\frac{n}{2}}(cx)^m\sqrt{bx^n}\right)\int x^{m+\frac{3n}{2}}\, dx \\ &= \frac{2bx^{1+n}(cx)^m\sqrt{bx^n}}{2+2m+3n}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.81

$$\frac{x\left(bx^n\right)^{3/2}(cx)^m}{m+\frac{3n}{2}+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*(b*x^n)^(3/2),x]
```

```
[Out] (x*(c*x)^m*(b*x^n)^(3/2))/(1 + m + (3*n)/2)
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n)^{\frac{3}{2}} (cx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^n)^(3/2)*(c*x)^m, x)
```

maple [**A**] time = 0.00, size = 26, normalized size = 0.81

$$\frac{2\,(b\,x^n)^{\frac{3}{2}}\,x\,(cx)^m}{2m+3n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(b*x^n)^(3/2),x)
```

```
[Out] 2*x/(2*m+3*n+2)*(c*x)^m*(b*x^n)^(3/2)
```

maxima [**A**] time = 1.51, size = 27, normalized size = 0.84

$$\frac{2\,b^{\frac{3}{2}}c^m x x^m (x^n)^{\frac{3}{2}}}{2\,m+3\,n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*b^(3/2)*c^m*x*x^m*(x^n)^(3/2)/(2*m + 3*n + 2)
```

mupad [**B**] time = 1.00, size = 30, normalized size = 0.94

$$\frac{2\,b\,x^{n+1}\,\sqrt{b\,x^n}\,(c\,x)^m}{2\,m+3\,n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^(3/2)*(c*x)^m,x)
```

```
[Out] (2*b*x^(n + 1)*(b*x^n)^(1/2)*(c*x)^m)/(2*m + 3*n + 2)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(b*x**n)**(3/2),x)
```

```
[Out] Timed out
```

3.164 $\int (cx)^m \sqrt{bx^n}\, dx$

Optimal. Leaf size=29

$$\frac{2\sqrt{bx^n}\,(cx)^{m+1}}{c(2m+n+2)}$$

```
[Out] 2*(c*x)^(1+m)*(b*x^n)^(1/2)/c/(2+2*m+n)
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 0.86, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {15, 20, 30}

$$\frac{2x\sqrt{bx^n}\,(cx)^m}{2m+n+2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*Sqrt[b*x^n],x]
```

```
[Out] (2*x*(c*x)^m*Sqrt[b*x^n])/(2 + 2*m + n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (cx)^m \sqrt{bx^n}\, dx &= \left(x^{-n/2}\sqrt{bx^n}\right)\int x^{n/2}(cx)^m\, dx \\ &= \left(x^{-m-\frac{n}{2}}(cx)^m\sqrt{bx^n}\right)\int x^{m+\frac{n}{2}}\, dx \\ &= \frac{2x(cx)^m\sqrt{bx^n}}{2+2m+n}\end{aligned}$$

Mathematica [A] time = 0.00, size = 26, normalized size = 0.90

$$\frac{x\sqrt{bx^n}\,(cx)^m}{m+\frac{n}{2}+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*Sqrt[b*x^n],x]
```

```
[Out] (x*(c*x)^m*Sqrt[b*x^n])/(1 + m + n/2)
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^n}\ (cx)^m\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n)*(c*x)^m, x)
```

maple [**A**] time = 0.00, size = 24, normalized size = 0.83

$$\frac{2\sqrt{b\,x^n}\ x\,(cx)^m}{2m+n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(b*x^n)^(1/2),x)
```

```
[Out] 2*x*(c*x)^m*(b*x^n)^(1/2)/(2*m+n+2)
```

maxima [**A**] time = 1.47, size = 25, normalized size = 0.86

$$\frac{2\,\sqrt{b}\,c^m x x^m \sqrt{x^n}}{2\,m+n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b)*c^m*x*x^m*sqrt(x^n)/(2*m + n + 2)
```

mupad [**B**] time = 1.01, size = 23, normalized size = 0.79

$$\frac{2\,x\,\sqrt{b\,x^n}\ (c\,x)^m}{2\,m+n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^(1/2)*(c*x)^m,x)
```

```
[Out] (2*x*(b*x^n)^(1/2)*(c*x)^m)/(2*m + n + 2)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{2\sqrt{b}\,c^m x x^m \sqrt{x^n}}{2m+n+2} & \text{for } m \neq -\frac{n}{2}-1 \\ \int \sqrt{bx^n}\ (cx)^{-\frac{n}{2}-1}\ dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(b*x**n)**(1/2),x)
```

```
[Out] Piecewise((2*sqrt(b)*c**m*x*x**m*sqrt(x**n)/(2*m + n + 2), Ne(m, -n/2 - 1))
, (Integral(sqrt(b*x**n)*(c*x)**(-n/2 - 1), x), True))
```

3.165 $\int \frac{(cx)^m}{\sqrt{bx^n}}\,dx$

Optimal. Leaf size=27

$$\frac{2x(cx)^m}{(2m-n+2)\sqrt{bx^n}}$$

```
[Out] 2*x*(c*x)^m/(2+2*m-n)/(b*x^n)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {15, 20, 30}

$$\frac{2x(cx)^m}{(2m-n+2)\sqrt{bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m/Sqrt[b*x^n],x]
```

```
[Out] (2*x*(c*x)^m)/((2 + 2*m - n)*Sqrt[b*x^n])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx)^m}{\sqrt{bx^n}}\,dx &= \frac{x^{n/2}\int x^{-n/2}(cx)^m\,dx}{\sqrt{bx^n}}\\ &= \frac{\left(x^{-m+\frac{n}{2}}(cx)^m\right)\int x^{m-\frac{n}{2}}\,dx}{\sqrt{bx^n}}\\ &= \frac{2x(cx)^m}{(2+2m-n)\sqrt{bx^n}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.96

$$\frac{x(cx)^m}{\left(m-\frac{n}{2}+1\right)\sqrt{bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m/Sqrt[b*x^n],x]
```

```
[Out] (x*(c*x)^m)/((1 + m - n/2)*Sqrt[b*x^n])
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{\sqrt{bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^m/sqrt(b*x^n), x)
```

maple [**A**] time = 0.00, size = 26, normalized size = 0.96

$$\frac{2x\,(cx)^m}{(2m-n+2)\,\sqrt{b\,x^n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(b*x^n)^(1/2),x)
```

```
[Out] 2*x*(c*x)^m/(2*m-n+2)/(b*x^n)^(1/2)
```

maxima [**A**] time = 1.50, size = 27, normalized size = 1.00

$$\frac{2\,c^m xx^m}{\sqrt{b}\,(2\,m-n+2)\sqrt{x^n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*c^m*x*x^m/(sqrt(b)*(2*m - n + 2)*sqrt(x^n))
```

mupad [**B**] time = 1.03, size = 34, normalized size = 1.26

$$\frac{2\,x^{1-n}\,\sqrt{b\,x^n}\,(c\,x)^m}{b\,(2\,m-n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(b*x^n)^(1/2),x)
```

```
[Out] (2*x^(1 - n)*(b*x^n)^(1/2)*(c*x)^m)/(b*(2*m - n + 2))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{2c^m xx^m}{2\sqrt{b}\,m\sqrt{x^n}-\sqrt{b}\,n\sqrt{x^n}+2\sqrt{b}\,\sqrt{x^n}} & \text{for } m \neq \frac{n}{2}-1 \\ \int \frac{(cx)^{\frac{n}{2}-1}}{\sqrt{bx^n}}\,dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m/(b*x**n)**(1/2),x)
```

```
[Out] Piecewise((2*c**m*x*x**m/(2*sqrt(b)*m*sqrt(x**n) - sqrt(b)*n*sqrt(x**n) + 2
*sqrt(b)*sqrt(x**n)), Ne(m, n/2 - 1)), (Integral((c*x)**(n/2 - 1)/sqrt(b*x*
*n), x), True))
```

3.166 $\int \frac{(cx)^m}{(bx^n)^{3/2}}\,dx$

Optimal. Leaf size=36

$$\frac{2x^{1-n}(cx)^m}{b(2m-3n+2)\sqrt{bx^n}}$$

```
[Out] 2*x^(1-n)*(c*x)^m/b/(2+2*m-3*n)/(b*x^n)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {15, 20, 30}

$$\frac{2x^{1-n}(cx)^m}{b(2m-3n+2)\sqrt{bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m/(b*x^n)^(3/2),x]
```

```
[Out] (2*x^(1 - n)*(c*x)^m)/(b*(2 + 2*m - 3*n)*Sqrt[b*x^n])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx)^m}{(bx^n)^{3/2}}\,dx &= \frac{x^{n/2}\int x^{-3n/2}(cx)^m\,dx}{b\sqrt{bx^n}}\\ &= \frac{\left(x^{-m+\frac{n}{2}}(cx)^m\right)\int x^{m-\frac{3n}{2}}\,dx}{b\sqrt{bx^n}}\\ &= \frac{2x^{1-n}(cx)^m}{b(2+2m-3n)\sqrt{bx^n}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.72

$$\frac{x(cx)^m}{\left(m-\frac{3n}{2}+1\right)(bx^n)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m/(b*x^n)^(3/2),x]
```

```
[Out] (x*(c*x)^m)/((1 + m - (3*n)/2)*(b*x^n)^(3/2))
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{(bx^n)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^m/(b*x^n)^(3/2), x)
```

maple [**A**] time = 0.00, size = 26, normalized size = 0.72

$$\frac{2x\,(cx)^m}{(2m-3n+2)\,(b\,x^n)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(b*x^n)^(3/2),x)
```

```
[Out] 2*x/(2*m-3*n+2)*(c*x)^m/(b*x^n)^(3/2)
```

maxima [**A**] time = 1.52, size = 27, normalized size = 0.75

$$\frac{2\,c^m x x^m}{b^{\frac{3}{2}}(2\,m-3\,n+2)(x^n)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*c^m*x*x^m/(b^(3/2)*(2*m - 3*n + 2)*(x^n)^(3/2))
```

mupad [**B**] time = 1.03, size = 34, normalized size = 0.94

$$\frac{2\,x^{1-2\,n}\,\sqrt{b\,x^n}\,(c\,x)^m}{b^2\,(2\,m-3\,n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(b*x^n)^(3/2),x)
```

```
[Out] (2*x^(1 - 2*n)*(b*x^n)^(1/2)*(c*x)^m)/(b^2*(2*m - 3*n + 2))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{2c^m x x^m}{2b^{\frac{3}{2}}m(x^n)^{\frac{3}{2}}-3b^{\frac{3}{2}}n(x^n)^{\frac{3}{2}}+2b^{\frac{3}{2}}(x^n)^{\frac{3}{2}}} & \text{for } m \neq \frac{3n}{2}-1 \\ \int \frac{(cx)^{\frac{3n}{2}-1}}{(bx^n)^{\frac{3}{2}}}\,dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m/(b*x**n)**(3/2),x)
```

```
[Out] Piecewise((2*c**m*x*x**m/(2*b**(3/2)*m*(x**n)**(3/2) - 3*b**(3/2)*n*(x**n)*
**(3/2) + 2*b**(3/2)*(x**n)**(3/2)), Ne(m, 3*n/2 - 1)), (Integral((c*x)**(3*
n/2 - 1)/(b*x**n)**(3/2), x), True))
```

3.167 $\int \frac{(cx)^m}{(bx^n)^{5/2}}\,dx$

Optimal. Leaf size=36

$$\frac{2x^{1-2n}(cx)^m}{b^2(2m-5n+2)\sqrt{bx^n}}$$

```
[Out] 2*x^(1-2*n)*(c*x)^m/b^2/(2+2*m-5*n)/(b*x^n)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {15, 20, 30}

$$\frac{2x^{1-2n}(cx)^m}{b^2(2m-5n+2)\sqrt{bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m/(b*x^n)^(5/2),x]
```

```
[Out] (2*x^(1 - 2*n)*(c*x)^m)/(b^2*(2 + 2*m - 5*n)*Sqrt[b*x^n])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx)^m}{(bx^n)^{5/2}}\,dx &= \frac{x^{n/2}\int x^{-5n/2}(cx)^m\,dx}{b^2\sqrt{bx^n}}\\ &= \frac{\left(x^{-m+\frac{n}{2}}(cx)^m\right)\int x^{m-\frac{5n}{2}}\,dx}{b^2\sqrt{bx^n}}\\ &= \frac{2x^{1-2n}(cx)^m}{b^2(2+2m-5n)\sqrt{bx^n}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 0.72

$$\frac{x(cx)^m}{\left(m-\frac{5n}{2}+1\right)(bx^n)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m/(b*x^n)^(5/2),x]
```

```
[Out] (x*(c*x)^m)/((1 + m - (5*n)/2)*(b*x^n)^(5/2))
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(b*x^n)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{(bx^n)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(b*x^n)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^m/(b*x^n)^(5/2), x)
```

maple [**A**] time = 0.00, size = 26, normalized size = 0.72

$$\frac{2x\,(cx)^m}{(2m-5n+2)\,(b\,x^n)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(b*x^n)^(5/2),x)
```

```
[Out] 2*x/(2+2*m-5*n)*(c*x)^m/(b*x^n)^(5/2)
```

maxima [**A**] time = 1.57, size = 27, normalized size = 0.75

$$\frac{2\,c^m x x^m}{b^{\frac{5}{2}}(2\,m-5\,n+2)(x^n)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(b*x^n)^(5/2),x, algorithm="maxima")
```

```
[Out] 2*c^m*x*x^m/(b^(5/2)*(2*m - 5*n + 2)*(x^n)^(5/2))
```

mupad [**B**] time = 1.05, size = 34, normalized size = 0.94

$$\frac{2\,x^{1-3\,n}\,\sqrt{b\,x^n}\,(c\,x)^m}{b^3\,(2\,m-5\,n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(b*x^n)^(5/2),x)
```

```
[Out] (2*x^(1 - 3*n)*(b*x^n)^(1/2)*(c*x)^m)/(b^3*(2*m - 5*n + 2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m/(b*x**n)**(5/2),x)
```

```
[Out] Timed out
```

3.168 $\int x^{-1-\frac{3n}{2}} (bx^n)^{3/2}\, dx$

Optimal. Leaf size=20

$$bx^{-n/2}\log(x)\sqrt{bx^n}$$

```
[Out] b*ln(x)*(b*x^n)^(1/2)/(x^(1/2*n))
```

Rubi [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {15, 29}

$$bx^{-n/2}\log(x)\sqrt{bx^n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - (3*n)/2)*(b*x^n)^(3/2),x]
```

```
[Out] (b*Sqrt[b*x^n]*Log[x])/x^(n/2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int x^{-1-\frac{3n}{2}} (bx^n)^{3/2}\, dx &= \left(bx^{-n/2}\sqrt{bx^n}\right)\int \frac{1}{x}\, dx \\ &= bx^{-n/2}\sqrt{bx^n}\,\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 0.95

$$x^{-3n/2}\log(x)\,(bx^n)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - (3*n)/2)*(b*x^n)^(3/2),x]
```

```
[Out] ((b*x^n)^(3/2)*Log[x])/x^((3*n)/2)
```

fricas [B] time = 9.71, size = 110, normalized size = 5.50

$$\left[\frac{1}{2}\,b^{\frac{3}{2}}\log\left(\frac{bx^4+\left(x^4-1\right)\sqrt{b}\,x^{\frac{1}{3}}x^{-\frac{1}{2}n-\frac{1}{3}}\sqrt{\frac{b}{x^{\frac{2}{3}}x^{-n-\frac{2}{3}}}}+b}{x^2}\right),-\sqrt{-b}\,b\arctan\left(\frac{\left(x^2+1\right)\sqrt{-b}\,x^{\frac{1}{3}}x^{-\frac{1}{2}n-\frac{1}{3}}\sqrt{\frac{b}{x^{\frac{2}{3}}x^{-n-\frac{2}{3}}}}}{bx^2-b}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3/2*n)*(b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*b^(3/2)*log((b*x^4 + (x^4 - 1)*sqrt(b)*x^(1/3)*x^(-1/2*n - 1/3)*sqrt(b
/(x^(2/3)*x^(-n - 2/3))) + b)/x^2), -sqrt(-b)*b*arctan((x^2 + 1)*sqrt(-b)*x
^(1/3)*x^(-1/2*n - 1/3)*sqrt(b/(x^(2/3)*x^(-n - 2/3)))/(b*x^2 - b))]
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n)^{\frac{3}{2}}\, x^{-\frac{3}{2}n-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3/2*n)*(b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^n)^(3/2)*x^(-3/2*n - 1), x)
```

maple [**A**] time = 0.04, size = 23, normalized size = 1.15

$$\sqrt{b\,x^n}\; b\, x^{-\frac{n}{2}} \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-3/2*n)*(b*x^n)^(3/2),x)
```

```
[Out] b/(x^(1/2*n))*(b*(x^(1/2*n))^2)^(1/2)*ln(x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n)^{\frac{3}{2}}\, x^{-\frac{3}{2}n-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3/2*n)*(b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n)^(3/2)*x^(-3/2*n - 1), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(b\,x^n)^{3/2}}{x^{\frac{3n}{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^(3/2)/x^((3*n)/2 + 1),x)
```

```
[Out] int((b*x^n)^(3/2)/x^((3*n)/2 + 1), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-3/2*n)*(b*x**n)**(3/2),x)
```

```
[Out] Timed out
```

3.169 $\int x^{-1-\frac{n}{2}}\sqrt{bx^n}\,dx$

Optimal. Leaf size=19

$$x^{-n/2}\log(x)\sqrt{bx^n}$$

```
[Out] ln(x)*(b*x^n)^(1/2)/(x^(1/2*n))
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {15, 29}

$$x^{-n/2}\log(x)\sqrt{bx^n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n/2)*Sqrt[b*x^n],x]
```

```
[Out] (Sqrt[b*x^n]*Log[x])/x^(n/2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int x^{-1-\frac{n}{2}}\sqrt{bx^n}\,dx &= \left(x^{-n/2}\sqrt{bx^n}\right)\int \frac{1}{x}\,dx\\ &= x^{-n/2}\sqrt{bx^n}\,\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$x^{-n/2}\log(x)\sqrt{bx^n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n/2)*Sqrt[b*x^n],x]
```

```
[Out] (Sqrt[b*x^n]*Log[x])/x^(n/2)
```

fricas [A] time = 0.82, size = 27, normalized size = 1.42

$$xx^{-\frac{1}{2}n-1}\sqrt{\frac{b}{x^2x^{-n-2}}}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*n)*(b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] x*x^(-1/2*n - 1)*sqrt(b/(x^2*x^(-n - 2)))*log(x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^n}\,x^{-\frac{1}{2}n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*n)*(b*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^n)*x^(-1/2*n - 1), x)
```

maple [A] time = 0.03, size = 22, normalized size = 1.16

$$\sqrt{b\,x^n}\;x^{-\frac{n}{2}}\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-1/2*n)*(b*x^n)^(1/2),x)
[Out] (b*(x^(1/2*n))^2)^(1/2)/(x^(1/2*n))*ln(x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^n}\,x^{-\frac{1}{2}n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*n)*(b*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*x^n)*x^(-1/2*n - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sqrt{b\,x^n}}{x^{\frac{n}{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^(1/2)/x^(n/2 + 1),x)
[Out] int((b*x^n)^(1/2)/x^(n/2 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-\frac{n}{2}-1}\sqrt{bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-1/2*n)*(b*x**n)**(1/2),x)
[Out] Integral(x**(-n/2 - 1)*sqrt(b*x**n), x)
```

3.170 $\int \frac{x^{-1+\frac{n}{2}}}{\sqrt{bx^n}}\,dx$

Optimal. Leaf size=19

$$\frac{x^{n/2}\log(x)}{\sqrt{bx^n}}$$

```
[Out] x^(1/2*n)*ln(x)/(b*x^n)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {15, 29}

$$\frac{x^{n/2}\log(x)}{\sqrt{bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n/2)/Sqrt[b*x^n],x]
```

```
[Out] (x^(n/2)*Log[x])/Sqrt[b*x^n]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+\frac{n}{2}}}{\sqrt{bx^n}}\,dx &= \frac{x^{n/2}\int \frac{1}{x}\,dx}{\sqrt{bx^n}}\\ &= \frac{x^{n/2}\log(x)}{\sqrt{bx^n}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{x^{n/2}\log(x)}{\sqrt{bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n/2)/Sqrt[b*x^n],x]
```

```
[Out] (x^(n/2)*Log[x])/Sqrt[b*x^n]
```

fricas [A] time = 0.88, size = 30, normalized size = 1.58

$$\frac{\sqrt{bx^2x^{n-2}}\,\log(x)}{bxx^{\frac{1}{2}n-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*n)/(b*x^n)^(1/2),x, algorithm="fricas")
[Out] sqrt(b*x^2*x^(n - 2))*log(x)/(b*x*x^(1/2*n - 1))
```

giac [A] time = 0.21, size = 14, normalized size = 0.74

$$\frac{\log\left(x^n|b|\right)}{\sqrt{b}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*n)/(b*x^n)^(1/2),x, algorithm="giac")
[Out] log(x^n*abs(b))/(sqrt(b)*n)
```

maple [A] time = 0.03, size = 20, normalized size = 1.05

$$\frac{x^{\frac{n}{2}}\ln(x)}{\sqrt{b\,x^n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+1/2*n)/(b*x^n)^(1/2),x)
[Out] 1/(b*(x^(1/2*n))^2)^(1/2)*x^(1/2*n)*ln(x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{1}{2}n-1}}{\sqrt{bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*n)/(b*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(x^(1/2*n - 1)/sqrt(b*x^n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^{\frac{n}{2}-1}}{\sqrt{b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n/2 - 1)/(b*x^n)^(1/2),x)
[Out] int(x^(n/2 - 1)/(b*x^n)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{n}{2}-1}}{\sqrt{bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+1/2*n)/(b*x**n)**(1/2),x)
[Out] Integral(x**(n/2 - 1)/sqrt(b*x**n), x)
```

3.171 $\int \frac{x^{-1+\frac{3n}{2}}}{(bx^n)^{3/2}}\,dx$

Optimal. Leaf size=22

$$\frac{x^{n/2}\log(x)}{b\sqrt{bx^n}}$$

```
[Out] x^(1/2*n)*ln(x)/b/(b*x^n)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {15, 29}

$$\frac{x^{n/2}\log(x)}{b\sqrt{bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + (3*n)/2)/(b*x^n)^(3/2),x]
```

```
[Out] (x^(n/2)*Log[x])/(b*Sqrt[b*x^n])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+\frac{3n}{2}}}{(bx^n)^{3/2}}\,dx &= \frac{x^{n/2}\int \frac{1}{x}\,dx}{b\sqrt{bx^n}}\\ &= \frac{x^{n/2}\log(x)}{b\sqrt{bx^n}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 0.86

$$\frac{x^{3n/2}\log(x)}{(bx^n)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + (3*n)/2)/(b*x^n)^(3/2),x]
```

```
[Out] (x^((3*n)/2)*Log[x])/(b*x^n)^(3/2)
```

fricas [A] time = 0.54, size = 20, normalized size = 0.91

$$\frac{\sqrt{bx^n}\,\log(x)}{b^2 x^{\frac{1}{2}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3/2*n)/(b*x^n)^(3/2),x, algorithm="fricas")
[Out] sqrt(b*x^n)*log(x)/(b^2*x^(1/2*n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}n-1}}{(bx^n)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3/2*n)/(b*x^n)^(3/2),x, algorithm="giac")
[Out] integrate(x^(3/2*n - 1)/(b*x^n)^(3/2), x)
```

maple [A] time = 0.03, size = 23, normalized size = 1.05

$$\frac{x^{\frac{n}{2}}\ln(x)}{\sqrt{b\,x^n}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+3/2*n)/(b*x^n)^(3/2),x)
[Out] 1/b*x^(1/2*n)/(b*(x^(1/2*n))^2)^(1/2)*ln(x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}n-1}}{(bx^n)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3/2*n)/(b*x^n)^(3/2),x, algorithm="maxima")
[Out] integrate(x^(3/2*n - 1)/(b*x^n)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^{\frac{3n}{2}-1}}{(b\,x^n)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^((3*n)/2 - 1)/(b*x^n)^(3/2),x)
[Out] int(x^((3*n)/2 - 1)/(b*x^n)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3n}{2}-1}}{(bx^n)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3/2*n)/(b*x**n)**(3/2),x)
[Out] Integral(x**(3*n/2 - 1)/(b*x**n)**(3/2), x)
```

3.172 $\int x^m (bx^n)^p \, dx$

Optimal. Leaf size=21

$$\frac{x^{m+1} (bx^n)^p}{m+np+1}$$

```
[Out] x^(1+m)*(b*x^n)^p/(n*p+m+1)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 30}

$$\frac{x^{m+1} (bx^n)^p}{m+np+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(b*x^n)^p,x]
```

```
[Out] (x^(1 + m)*(b*x^n)^p)/(1 + m + n*p)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^m (bx^n)^p \, dx &= \left(x^{-np} (bx^n)^p\right) \int x^{m+np} \, dx \\ &= \frac{x^{1+m} (bx^n)^p}{1+m+np}\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{x^{m+1} (bx^n)^p}{m+np+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(b*x^n)^p,x]
```

```
[Out] (x^(1 + m)*(b*x^n)^p)/(1 + m + n*p)
```

fricas [A] time = 0.72, size = 24, normalized size = 1.14

$$\frac{x x^m e^{\left(np \log(x) + p \log(b)\right)}}{np+m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^n)^p,x, algorithm="fricas")
```

```
[Out] x*x^m*e^(n*p*log(x) + p*log(b))/(n*p + m + 1)
```

giac [A] time = 0.17, size = 24, normalized size = 1.14

$$\frac{x x^m e^{\left(np \log(x)+p \log(b)\right)}}{np+m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^n)^p,x, algorithm="giac")
[Out] x*x^m*e^(n*p*log(x) + p*log(b))/(n*p + m + 1)
```

maple [A] time = 0.00, size = 22, normalized size = 1.05

$$\frac{x^{m+1} (b\, x^n)^p}{np+m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^n)^p,x)
[Out] x^(m+1)*(b*x^n)^p/(n*p+m+1)
```

maxima [A] time = 1.48, size = 25, normalized size = 1.19

$$\frac{b^p x e^{\left(m \log(x)+p \log(x^n)\right)}}{np+m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^n)^p,x, algorithm="maxima")
[Out] b^p*x*e^(m*log(x) + p*log(x^n))/(n*p + m + 1)
```

mupad [B] time = 1.14, size = 21, normalized size = 1.00

$$\frac{x^{m+1} (b\, x^n)^p}{m+n\, p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^n)^p,x)
[Out] (x^(m + 1)*(b*x^n)^p)/(m + n*p + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{b^p x x^m (x^n)^p}{m+np+1} & \text{for } m \neq -np-1 \\ \int x^{-np-1} (bx^n)^p \, dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**n)**p,x)
[Out] Piecewise((b**p*x*x**m*(x**n)**p/(m + n*p + 1), Ne(m, -n*p - 1)), (Integral
(x**(-n*p - 1)*(b*x**n)**p, x), True))
```

3.173 $\int (cx)^m (bx^n)^p \, dx$

Optimal. Leaf size=26

$$\frac{(cx)^{m+1} (bx^n)^p}{c(m+np+1)}$$

```
[Out] (c*x)^(1+m)*(b*x^n)^p/c/(n*p+m+1)
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 0.85, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {15, 20, 30}

$$\frac{x(cx)^m (bx^n)^p}{m+np+1}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*(b*x^n)^p,x]
```

```
[Out] (x*(c*x)^m*(b*x^n)^p)/(1 + m + n*p)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (cx)^m (bx^n)^p \, dx &= \left(x^{-np} (bx^n)^p\right) \int x^{np} (cx)^m \, dx \\ &= \left(x^{-m-np} (cx)^m (bx^n)^p\right) \int x^{m+np} \, dx \\ &= \frac{x(cx)^m (bx^n)^p}{1+m+np}\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 0.85

$$\frac{x(cx)^m (bx^n)^p}{m+np+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*(b*x^n)^p,x]
```

```
[Out] (x*(c*x)^m*(b*x^n)^p)/(1 + m + n*p)
```

fricas [A] time = 0.70, size = 29, normalized size = 1.12

$$\frac{xe^{\left(np\log(x)+p\log(b)+m\log(c)+m\log(x)\right)}}{np+m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^n)^p,x, algorithm="fricas")
[Out] x*e^(n*p*log(x) + p*log(b) + m*log(c) + m*log(x))/(n*p + m + 1)
```

giac [A] time = 0.21, size = 29, normalized size = 1.12

$$\frac{xe^{\left(np\log(x)+p\log(b)+m\log(c)+m\log(x)\right)}}{np+m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^n)^p,x, algorithm="giac")
[Out] x*e^(n*p*log(x) + p*log(b) + m*log(c) + m*log(x))/(n*p + m + 1)
```

maple [A] time = 0.00, size = 23, normalized size = 0.88

$$\frac{x\left(b\,x^{n}\right)^{p}\left(cx\right)^{m}}{np+m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(b*x^n)^p,x)
[Out] x*(c*x)^m*(b*x^n)^p/(n*p+m+1)
```

maxima [A] time = 1.48, size = 28, normalized size = 1.08

$$\frac{b^{p}c^{m}xe^{\left(m\log(x)+p\log(x^{n})\right)}}{np+m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^n)^p,x, algorithm="maxima")
[Out] b^p*c^m*x*e^(m*log(x) + p*log(x^n))/(n*p + m + 1)
```

mupad [B] time = 1.01, size = 22, normalized size = 0.85

$$\frac{x\left(b\,x^{n}\right)^{p}\left(c\,x\right)^{m}}{m+n\,p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^p*(c*x)^m,x)
[Out] (x*(b*x^n)^p*(c*x)^m)/(m + n*p + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{b^{p}c^{m}xx^{m}(x^{n})^{p}}{m+np+1} & \text{for } m \neq -np-1 \\ \int \left(bx^{n}\right)^{p}\left(cx\right)^{-np-1}\,dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(b*x**n)**p,x)
```

```
[Out] Piecewise((b**p*c**m*x*x**m*(x**n)**p/(m + n*p + 1), Ne(m, -n*p - 1)), (Int
egral((b*x**n)**p*(c*x)**(-n*p - 1), x), True))
```

3.174 $\int x^2 (bx^n)^p \, dx$

Optimal. Leaf size=18

$$\frac{x^3 (bx^n)^p}{np+3}$$

```
[Out] x^3*(b*x^n)^p/(n*p+3)
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 30}

$$\frac{x^3 (bx^n)^p}{np+3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b*x^n)^p,x]
```

```
[Out] (x^3*(b*x^n)^p)/(3 + n*p)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2 (bx^n)^p \, dx &= \left(x^{-np} (bx^n)^p\right) \int x^{2+np} \, dx \\ &= \frac{x^3 (bx^n)^p}{3+np}\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{x^3 (bx^n)^p}{np+3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b*x^n)^p,x]
```

```
[Out] (x^3*(b*x^n)^p)/(3 + n*p)
```

fricas [A] time = 0.90, size = 22, normalized size = 1.22

$$\frac{x^3 e^{\left(np \log(x) + p \log(b)\right)}}{np+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^n)^p,x, algorithm="fricas")
```

```
[Out] x^3*e^(n*p*log(x) + p*log(b))/(n*p + 3)
```

giac [A] time = 0.19, size = 22, normalized size = 1.22

$$\frac{x^3 e^{\left(np\log(x)+p\log(b)\right)}}{np+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^n)^p,x, algorithm="giac")
```

```
[Out] x^3*e^(n*p*log(x) + p*log(b))/(n*p + 3)
```

maple [A] time = 0.00, size = 19, normalized size = 1.06

$$\frac{x^3 \left(b\, x^n\right)^p}{np+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^n)^p,x)
```

```
[Out] x^3*(b*x^n)^p/(n*p+3)
```

maxima [A] time = 1.48, size = 19, normalized size = 1.06

$$\frac{b^p x^3 (x^n)^p}{np+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^n)^p,x, algorithm="maxima")
```

```
[Out] b^p*x^3*(x^n)^p/(n*p + 3)
```

mupad [B] time = 1.08, size = 18, normalized size = 1.00

$$\frac{x^3 \left(b\, x^n\right)^p}{n\, p+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^n)^p,x)
```

```
[Out] (x^3*(b*x^n)^p)/(n*p + 3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{b^p x^3 (x^n)^p}{np+3} & \text{for } n \neq -\frac{3}{p} \\ \int x^2 \left(b x^{-\frac{3}{p}}\right)^p \, dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**n)**p,x)
```

```
[Out] Piecewise((b**p*x**3*(x**n)**p/(n*p + 3), Ne(n, -3/p)), (Integral(x**2*(b*x
**(-3/p))**p, x), True))
```

3.175 $\int x\,(bx^n)^p\,dx$

Optimal. Leaf size=18

$$\frac{x^2\,(bx^n)^p}{np+2}$$

```
[Out] x^2*(b*x^n)^p/(n*p+2)
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$\frac{x^2\,(bx^n)^p}{np+2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b*x^n)^p,x]
```

```
[Out] (x^2*(b*x^n)^p)/(2 + n*p)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\,(bx^n)^p\,dx &= \left(x^{-np}\,(bx^n)^p\right)\int x^{1+np}\,dx\\ &= \frac{x^2\,(bx^n)^p}{2+np}\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{x^2\,(bx^n)^p}{np+2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b*x^n)^p,x]
```

```
[Out] (x^2*(b*x^n)^p)/(2 + n*p)
```

fricas [A] time = 0.86, size = 22, normalized size = 1.22

$$\frac{x^2 e^{\left(np\log(x)+p\log(b)\right)}}{np+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^n)^p,x, algorithm="fricas")
```

```
[Out] x^2*e^(n*p*log(x) + p*log(b))/(n*p + 2)
```

giac [A] time = 0.17, size = 22, normalized size = 1.22

$$\frac{x^2 e^{\left(np\log(x)+p\log(b)\right)}}{np+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^n)^p,x, algorithm="giac")
[Out] x^2*e^(n*p*log(x) + p*log(b))/(n*p + 2)
```

maple [A] time = 0.00, size = 19, normalized size = 1.06

$$\frac{x^2 \left(b\, x^n\right)^p}{np+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^n)^p,x)
[Out] x^2*(b*x^n)^p/(n*p+2)
```

maxima [A] time = 1.42, size = 19, normalized size = 1.06

$$\frac{b^p x^2 (x^n)^p}{np+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^n)^p,x, algorithm="maxima")
[Out] b^p*x^2*(x^n)^p/(n*p + 2)
```

mupad [B] time = 1.06, size = 18, normalized size = 1.00

$$\frac{x^2 \left(b\, x^n\right)^p}{n\, p+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^n)^p,x)
[Out] (x^2*(b*x^n)^p)/(n*p + 2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{b^p x^2 (x^n)^p}{np+2} & \text{for } n \neq -\frac{2}{p} \\ \int x \left(b x^{-\frac{2}{p}}\right)^p \, dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**n)**p,x)
[Out] Piecewise((b**p*x**2*(x**n)**p/(n*p + 2), Ne(n, -2/p)), (Integral(x*(b*x**(
-2/p))**p, x), True))
```

3.176 $\int (bx^n)^p \, dx$

Optimal. Leaf size=16

$$\frac{x\,(bx^n)^p}{np+1}$$

```
[Out] x*(b*x^n)^p/(n*p+1)
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {15, 30}

$$\frac{x\,(bx^n)^p}{np+1}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^n)^p,x]
```

```
[Out] (x*(b*x^n)^p)/(1 + n*p)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (bx^n)^p \, dx &= \left(x^{-np}\,(bx^n)^p\right) \int x^{np}\, dx \\ &= \frac{x\,(bx^n)^p}{1+np}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{x\,(bx^n)^p}{np+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^n)^p,x]
```

```
[Out] (x*(b*x^n)^p)/(1 + n*p)
```

fricas [A] time = 0.93, size = 20, normalized size = 1.25

$$\frac{x e^{\left(np\log(x)+p\log(b)\right)}}{np+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^p,x, algorithm="fricas")
```

```
[Out] x*e^(n*p*log(x) + p*log(b))/(n*p + 1)
```

giac [**A**] time = 0.21, size = 20, normalized size = 1.25

$$\frac{xe^{\left(np\log(x)+p\log(b)\right)}}{np+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^p,x, algorithm="giac")
[Out] x*e^(n*p*log(x) + p*log(b))/(n*p + 1)
```

maple [**A**] time = 0.00, size = 17, normalized size = 1.06

$$\frac{x\left(b\,x^{n}\right)^{p}}{np+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^p,x)
[Out] x*(b*x^n)^p/(n*p+1)
```

maxima [**A**] time = 1.46, size = 17, normalized size = 1.06

$$\frac{b^{p}x(x^{n})^{p}}{np+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^p,x, algorithm="maxima")
[Out] b^p*x*(x^n)^p/(n*p + 1)
```

mupad [**B**] time = 1.19, size = 16, normalized size = 1.00

$$\frac{x\left(b\,x^{n}\right)^{p}}{n\,p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^p,x)
[Out] (x*(b*x^n)^p)/(n*p + 1)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{b^{p}x(x^{n})^{p}}{np+1} & \text{for } n \neq -\frac{1}{p} \\ \int \left(bx^{-\frac{1}{p}}\right)^{p} dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**n)**p,x)
[Out] Piecewise((b**p*x*(x**n)**p/(n*p + 1), Ne(n, -1/p)), (Integral((b*x**(-1/p)
)**p, x), True))
```

3.177 $\int \frac{(bx^n)^p}{x}\,dx$

Optimal. Leaf size=14

$$\frac{(bx^n)^p}{np}$$

```
[Out] (b*x^n)^p/n/p
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 30}

$$\frac{(bx^n)^p}{np}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^n)^p/x,x]
```

```
[Out] (b*x^n)^p/(n*p)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^n)^p}{x}\,dx &= \left(x^{-np}\,(bx^n)^p\right)\int x^{-1+np}\,dx\\ &= \frac{(bx^n)^p}{np}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{(bx^n)^p}{np}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^n)^p/x,x]
```

```
[Out] (b*x^n)^p/(n*p)
```

fricas [A] time = 0.65, size = 18, normalized size = 1.29

$$\frac{e^{\left(np\log(x)+p\log(b)\right)}}{np}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^p/x,x, algorithm="fricas")
```

```
[Out] e^(n*p*log(x) + p*log(b))/(n*p)
```

giac [A] time = 0.15, size = 14, normalized size = 1.00

$$\frac{(bx^n)^p}{np}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^p/x,x, algorithm="giac")
```

```
[Out] (b*x^n)^p/(n*p)
```

maple [A] time = 0.00, size = 15, normalized size = 1.07

$$\frac{(b\,x^n)^p}{np}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^p/x,x)
```

```
[Out] (b*x^n)^p/n/p
```

maxima [A] time = 1.42, size = 15, normalized size = 1.07

$$\frac{b^p(x^n)^p}{np}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^p/x,x, algorithm="maxima")
```

```
[Out] b^p*(x^n)^p/(n*p)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{(b\,x^n)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^p/x,x)
```

```
[Out] int((b*x^n)^p/x, x)
```

sympy [A] time = 0.32, size = 22, normalized size = 1.57

$$\begin{cases} \log(x) & \text{for } p = 0 \wedge \left(n = 0 \vee p = 0\right) \\ b^p \log(x) & \text{for } n = 0 \\ \frac{b^p (x^n)^p}{np} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**n)**p/x,x)
```

```
[Out] Piecewise((log(x), Eq(p, 0) & (Eq(n, 0) | Eq(p, 0))), (b**p*log(x), Eq(n, 0
)), (b**p*(x**n)**p/(n*p), True))
```

3.178 $\int \frac{(bx^n)^p}{x^2}\,dx$

Optimal. Leaf size=20

$$-\frac{(bx^n)^p}{x(1-np)}$$

```
[Out] -(b*x^n)^p/(-n*p+1)/x
```

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 30}

$$-\frac{(bx^n)^p}{x(1-np)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^n)^p/x^2,x]
```

```
[Out] -((b*x^n)^p/((1 - n*p)*x))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^n)^p}{x^2}\,dx &= \left(x^{-np}\,(bx^n)^p\right)\int x^{-2+np}\,dx \\ &= -\frac{(bx^n)^p}{(1-np)x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 0.90

$$\frac{(bx^n)^p}{x(np-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^n)^p/x^2,x]
```

```
[Out] (b*x^n)^p/((-1 + n*p)*x)
```

fricas [A] time = 0.93, size = 22, normalized size = 1.10

$$\frac{e^{\left(np\log(x)+p\log(b)\right)}}{\left(np-1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^p/x^2,x, algorithm="fricas")
```

```
[Out] e^(n*p*log(x) + p*log(b))/((n*p - 1)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^n)^p/x^2, x)
```

maple [A] time = 0.00, size = 19, normalized size = 0.95

$$\frac{(b\,x^n)^p}{\left(np-1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^p/x^2,x)
```

```
[Out] 1/x/(n*p-1)*(b*x^n)^p
```

maxima [A] time = 1.50, size = 19, normalized size = 0.95

$$\frac{b^p(x^n)^p}{\left(np-1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^p/x^2,x, algorithm="maxima")
```

```
[Out] b^p*(x^n)^p/((n*p - 1)*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(b\,x^n)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^p/x^2,x)
```

```
[Out] int((b*x^n)^p/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{b^p(x^n)^p}{npx-x} & \text{for } n \neq \frac{1}{p} \\ \int \frac{\left(bx^{\frac{1}{p}}\right)^p}{x^2}\,dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**n)**p/x**2,x)
```

```
[Out] Piecewise((b**p*(x**n)**p/(n*p*x - x), Ne(n, 1/p)), (Integral((b*x**(1/p))*
**p/x**2, x), True))
```

3.179 $\int \frac{(bx^n)^p}{x^3}\,dx$

Optimal. Leaf size=20

$$-\frac{(bx^n)^p}{x^2(2-np)}$$

```
[Out] -(b*x^n)^p/(-n*p+2)/x^2
```

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 30}

$$-\frac{(bx^n)^p}{x^2(2-np)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^n)^p/x^3,x]
```

```
[Out] -((b*x^n)^p/((2 - n*p)*x^2))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^n)^p}{x^3}\,dx &= \left(x^{-np}\,(bx^n)^p\right)\int x^{-3+np}\,dx\\ &= -\frac{(bx^n)^p}{(2-np)x^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 0.90

$$\frac{(bx^n)^p}{x^2(np-2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^n)^p/x^3,x]
```

```
[Out] (b*x^n)^p/((-2 + n*p)*x^2)
```

fricas [A] time = 0.87, size = 22, normalized size = 1.10

$$\frac{e^{\left(np\log(x)+p\log(b)\right)}}{\left(np-2\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^p/x^3,x, algorithm="fricas")
```

```
[Out] e^(n*p*log(x) + p*log(b))/((n*p - 2)*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^n)^p/x^3, x)
```

maple [A] time = 0.00, size = 19, normalized size = 0.95

$$\frac{(b\,x^n)^p}{\left(np-2\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^p/x^3,x)
```

```
[Out] 1/x^2/(n*p-2)*(b*x^n)^p
```

maxima [A] time = 1.46, size = 19, normalized size = 0.95

$$\frac{b^p(x^n)^p}{\left(np-2\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^p/x^3,x, algorithm="maxima")
```

```
[Out] b^p*(x^n)^p/((n*p - 2)*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(b\,x^n)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^p/x^3,x)
```

```
[Out] int((b*x^n)^p/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{b^p(x^n)^p}{npx^2-2x^2} & \text{for } n \neq \frac{2}{p} \\ \int \frac{\left(bx^{\frac{2}{p}}\right)^p}{x^3}\,dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**n)**p/x**3,x)
```

```
[Out] Piecewise((b**p*(x**n)**p/(n*p*x**2 - 2*x**2), Ne(n, 2/p)), (Integral((b*x*
**(2/p))**p/x**3, x), True))
```

3.180 $\int x^m \left(ax^n\right)^{-1/n}\, dx$

Optimal. Leaf size=20

$$\frac{x^{m+1}\left(ax^n\right)^{-1/n}}{m}$$

```
[Out] x^(1+m)/m/((a*x^n)^(1/n))
```

Rubi [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {15, 30}

$$\frac{x^{m+1}\left(ax^n\right)^{-1/n}}{m}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a*x^n)^n^(-1),x]
```

```
[Out] x^(1 + m)/(m*(a*x^n)^n^(-1))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^m \left(ax^n\right)^{-1/n}\, dx &= \left(x\left(ax^n\right)^{-1/n}\right)\int x^{-1+m}\, dx \\ &= \frac{x^{1+m}\left(ax^n\right)^{-1/n}}{m}\end{aligned}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$\frac{x^{m+1}\left(ax^n\right)^{-1/n}}{m}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a*x^n)^n^(-1),x]
```

```
[Out] x^(1 + m)/(m*(a*x^n)^n^(-1))
```

fricas [A] time = 0.81, size = 14, normalized size = 0.70

$$\frac{x^m}{a^{\left(\frac{1}{n}\right)}m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((a*x^n)^(1/n)),x, algorithm="fricas")
```

```
[Out] x^m/(a^(1/n)*m)
```

giac [A] time = 0.21, size = 14, normalized size = 0.70

$$\frac{x^m}{a^{\left(\frac{1}{n}\right)} m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((a*x^n)^(1/n)),x, algorithm="giac")
[Out] x^m/(a^(1/n)*m)
```

maple [A] time = 0.00, size = 21, normalized size = 1.05

$$\frac{x^{m+1} \left(a\, x^n\right)^{-\frac{1}{n}}}{m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a*x^n)^(1/n)),x)
[Out] x^(m+1)/m/((a*x^n)^(1/n))
```

maxima [A] time = 1.47, size = 27, normalized size = 1.35

$$\frac{x e^{\left(m \log(x) - \frac{\log\left(x^n\right)}{n}\right)}}{a^{\left(\frac{1}{n}\right)} m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((a*x^n)^(1/n)),x, algorithm="maxima")
[Out] x*e^(m*log(x) - log(x^n)/n)/(a^(1/n)*m)
```

mupad [B] time = 1.13, size = 20, normalized size = 1.00

$$\frac{x^{m+1}}{m \left(a\, x^n\right)^{1/n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a*x^n)^(1/n),x)
[Out] x^(m + 1)/(m*(a*x^n)^(1/n))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} -\frac{x}{0^n \tilde{\infty}^n (0^n)^{\frac{1}{n}} (x^n)^{\frac{1}{n}} - (0^n)^{\frac{1}{n}} (x^n)^{\frac{1}{n}}} & \text{for } a = 0^n \wedge m = 0 \\ \int (a x^n)^{-\frac{1}{n}}\, dx & \text{for } m = 0 \\ \frac{x x^m}{-0^n \tilde{\infty}^n (0^n)^{\frac{1}{n}} (x^n)^{\frac{1}{n}} + m (0^n)^{\frac{1}{n}} (x^n)^{\frac{1}{n}} + (0^n)^{\frac{1}{n}} (x^n)^{\frac{1}{n}}} & \text{for } a = 0^n \\ \frac{a^{-\frac{1}{n}} x x^m (x^n)^{-\frac{1}{n}}}{m} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/((a*x**n)**(1/n)),x)
```

```
[Out] Piecewise((-x/(0**n*zoo**n*(0**n)**(1/n)*(x**n)**(1/n) - (0**n)**(1/n)*(x**
n)**(1/n)), Eq(m, 0) & Eq(a, 0**n)), (Integral((a*x**n)**(-1/n), x), Eq(m,
0)), (x*x**m/(-0**n*zoo**n*(0**n)**(1/n)*(x**n)**(1/n) + m*(0**n)**(1/n)*(x
**n)**(1/n) + (0**n)**(1/n)*(x**n)**(1/n)), Eq(a, 0**n)), (a**(-1/n)*x*x**m
*(x**n)**(-1/n)/m, True))
```

3.181 $\int (cx)^m (ax^n)^{-1/n}\, dx$

Optimal. Leaf size=21

$$\frac{x (ax^n)^{-1/n} (cx)^m}{m}$$

```
[Out] x*(c*x)^m/m/((a*x^n)^(1/n))
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {15, 16, 32}

$$\frac{x (ax^n)^{-1/n} (cx)^m}{m}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m/(a*x^n)^n^(-1),x]
```

```
[Out] (x*(c*x)^m)/(m*(a*x^n)^n^(-1))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (cx)^m (ax^n)^{-1/n}\, dx &= \left(x (ax^n)^{-1/n}\right) \int \frac{(cx)^m}{x}\, dx \\ &= \left(cx (ax^n)^{-1/n}\right) \int (cx)^{-1+m}\, dx \\ &= \frac{x(cx)^m (ax^n)^{-1/n}}{m}\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{x (ax^n)^{-1/n} (cx)^m}{m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m/(a*x^n)^n^(-1),x]
```

```
[Out] (x*(c*x)^m)/(m*(a*x^n)^n^(-1))
```

fricas [A] time = 0.68, size = 21, normalized size = 1.00

$$\frac{e^{\left(m \log(c)+m \log(x)\right)}}{a^{\left(\frac{1}{n}\right)} m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/((a*x^n)^(1/n)),x, algorithm="fricas")
[Out] e^(m*log(c) + m*log(x))/(a^(1/n)*m)
```

giac [A] time = 0.22, size = 21, normalized size = 1.00

$$\frac{e^{\left(m \log(c)+m \log(x)\right)}}{a^{\left(\frac{1}{n}\right)} m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/((a*x^n)^(1/n)),x, algorithm="giac")
[Out] e^(m*log(c) + m*log(x))/(a^(1/n)*m)
```

maple [A] time = 0.00, size = 22, normalized size = 1.05

$$\frac{x \left(a x^{n}\right)^{-\frac{1}{n}} (cx)^{m}}{m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/((a*x^n)^(1/n)),x)
[Out] x*(c*x)^m/m/((a*x^n)^(1/n))
```

maxima [A] time = 1.52, size = 30, normalized size = 1.43

$$\frac{c^{m} x e^{\left(m \log(x)-\frac{\log\left(x^{n}\right)}{n}\right)}}{a^{\left(\frac{1}{n}\right)} m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/((a*x^n)^(1/n)),x, algorithm="maxima")
[Out] c^m*x*e^(m*log(x) - log(x^n)/n)/(a^(1/n)*m)
```

mupad [B] time = 0.99, size = 21, normalized size = 1.00

$$\frac{x \left(c\, x\right)^{m}}{m \left(a\, x^{n}\right)^{1/n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(a*x^n)^(1/n),x)
[Out] (x*(c*x)^m)/(m*(a*x^n)^(1/n))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} -\dfrac{x}{0^n \tilde{\infty}^n (0^n)^{\frac{1}{n}} (x^n)^{\frac{1}{n}} - (0^n)^{\frac{1}{n}} (x^n)^{\frac{1}{n}}} & \text{for } a = 0^n \wedge m = 0 \\ \int (ax^n)^{-\frac{1}{n}}\, dx & \text{for } m = 0 \\ \dfrac{c^m x x^m}{-0^n \tilde{\infty}^n (0^n)^{\frac{1}{n}} (x^n)^{\frac{1}{n}} + m (0^n)^{\frac{1}{n}} (x^n)^{\frac{1}{n}} + (0^n)^{\frac{1}{n}} (x^n)^{\frac{1}{n}}} & \text{for } a = 0^n \\ \dfrac{a^{-\frac{1}{n}} c^m x x^m (x^n)^{-\frac{1}{n}}}{m} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m/((a*x**n)**(1/n)),x)
```

```
[Out] Piecewise((-x/(0**n*zoo**n*(0**n)**(1/n)*(x**n)**(1/n) - (0**n)**(1/n)*(x**
n)**(1/n)), Eq(m, 0) & Eq(a, 0**n)), (Integral((a*x**n)**(-1/n), x), Eq(m,
0)), (c**m*x*x**m/(-0**n*zoo**n*(0**n)**(1/n)*(x**n)**(1/n) + m*(0**n)**(1/
n)*(x**n)**(1/n) + (0**n)**(1/n)*(x**n)**(1/n)), Eq(a, 0**n)), (a**(-1/n)*c
**m*x*x**m*(x**n)**(-1/n)/m, True))
```

3.182 $\int x^2 (ax^n)^{-1/n}\, dx$

Optimal. Leaf size=18

$$\frac{1}{2} x^3 (ax^n)^{-1/n}$$

```
[Out] 1/2*x^3/((a*x^n)^(1/n))
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {15, 30}

$$\frac{1}{2} x^3 (ax^n)^{-1/n}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x^n)^n^(-1),x]
```

```
[Out] x^3/(2*(a*x^n)^n^(-1))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2 (ax^n)^{-1/n}\, dx &= \left(x (ax^n)^{-1/n}\right) \int x\, dx \\ &= \frac{1}{2} x^3 (ax^n)^{-1/n}\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{1}{2} x^3 (ax^n)^{-1/n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x^n)^n^(-1),x]
```

```
[Out] x^3/(2*(a*x^n)^n^(-1))
```

fricas [A] time = 0.82, size = 12, normalized size = 0.67

$$\frac{x^2}{2\, a^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((a*x^n)^(1/n)),x, algorithm="fricas")
```

```
[Out] 1/2*x^2/a^(1/n)
```

giac [A] time = 0.19, size = 12, normalized size = 0.67

$$\frac{x^2}{2\,a^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((a*x^n)^(1/n)),x, algorithm="giac")
[Out] 1/2*x^2/a^(1/n)
```

maple [A] time = 0.00, size = 17, normalized size = 0.94

$$\frac{x^3\,(a\,x^n)^{-\frac{1}{n}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a*x^n)^(1/n)),x)
[Out] 1/2*x^3/((a*x^n)^(1/n))
```

maxima [A] time = 1.47, size = 21, normalized size = 1.17

$$\frac{x^3}{2\,a^{\left(\frac{1}{n}\right)}(x^n)^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((a*x^n)^(1/n)),x, algorithm="maxima")
[Out] 1/2*x^3/(a^(1/n)*(x^n)^(1/n))
```

mupad [B] time = 1.12, size = 16, normalized size = 0.89

$$\frac{x^3}{2\,(a\,x^n)^{1/n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^n)^(1/n),x)
[Out] x^3/(2*(a*x^n)^(1/n))
```

sympy [A] time = 3.56, size = 58, normalized size = 3.22

$$\begin{cases} \frac{a^{-\frac{1}{n}}x^3(x^n)^{-\frac{1}{n}}}{2} & \text{for } a \neq 0^n \\ -\frac{x^3}{0^n\tilde{\infty}^n(0^n)^{\frac{1}{n}}(x^n)^{\frac{1}{n}} - 3(0^n)^{\frac{1}{n}}(x^n)^{\frac{1}{n}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((a*x**n)**(1/n)),x)
[Out] Piecewise((a**(-1/n)*x**3*(x**n)**(-1/n)/2, Ne(a, 0**n)), (-x**3/(0**n*zoo*
**n*(0**n)**(1/n)*(x**n)**(1/n) - 3*(0**n)**(1/n)*(x**n)**(1/n)), True))
```

3.183 $\int x \left(ax^n\right)^{-1/n} \, dx$

Optimal. Leaf size=15

$$x^2 \left(ax^n\right)^{-1/n}$$

```
[Out] x^2/((a*x^n)^(1/n))
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 8}

$$x^2 \left(ax^n\right)^{-1/n}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x^n)^n^(-1),x]
```

```
[Out] x^2/(a*x^n)^n^(-1)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int x \left(ax^n\right)^{-1/n} \, dx &= \left(x \left(ax^n\right)^{-1/n}\right) \int 1 \, dx \\ &= x^2 \left(ax^n\right)^{-1/n}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$x^2 \left(ax^n\right)^{-1/n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x^n)^n^(-1),x]
```

```
[Out] x^2/(a*x^n)^n^(-1)
```

fricas [A] time = 0.79, size = 9, normalized size = 0.60

$$\frac{x}{a^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((a*x^n)^(1/n)),x, algorithm="fricas")
```

```
[Out] x/a^(1/n)
```

giac [A] time = 0.17, size = 9, normalized size = 0.60

$$\frac{x}{a^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((a*x^n)^(1/n)),x, algorithm="giac")
```

```
[Out] x/a^(1/n)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int x\,(a\,x^n)^{-\frac{1}{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a*x^n)^(1/n)),x)
```

```
[Out] int(x/((a*x^n)^(1/n)),x)
```

maxima [A] time = 1.46, size = 20, normalized size = 1.33

$$\frac{x^2}{a^{\left(\frac{1}{n}\right)}(x^n)^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((a*x^n)^(1/n)),x, algorithm="maxima")
```

```
[Out] x^2/(a^(1/n)*(x^n)^(1/n))
```

mupad [B] time = 1.06, size = 15, normalized size = 1.00

$$\frac{x^2}{(a\,x^n)^{1/n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^n)^(1/n),x)
```

```
[Out] x^2/(a*x^n)^(1/n)
```

sympy [A] time = 2.01, size = 56, normalized size = 3.73

$$\begin{cases} a^{-\frac{1}{n}}x^2\,(x^n)^{-\frac{1}{n}} & \text{for } a \neq 0^n \\ -\dfrac{x^2}{0^n\tilde{\infty}^n(0^n)^{\frac{1}{n}}(x^n)^{\frac{1}{n}} - 2(0^n)^{\frac{1}{n}}(x^n)^{\frac{1}{n}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((a*x**n)**(1/n)),x)
```

```
[Out] Piecewise((a**(-1/n)*x**2*(x**n)**(-1/n), Ne(a, 0**n)), (-x**2/(0**n*zoo**n
*(0**n)**(1/n)*(x**n)**(1/n) - 2*(0**n)**(1/n)*(x**n)**(1/n)), True))
```

3.184 $\int (ax^n)^{-1/n}\, dx$

Optimal. Leaf size=15

$$x \log(x) (ax^n)^{-1/n}$$

```
[Out] x*ln(x)/((a*x^n)^(1/n))
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 29}

$$x \log(x) (ax^n)^{-1/n}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^n)^(-n^(-1)),x]
```

```
[Out] (x*Log[x])/(a*x^n)^n^(-1)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int (ax^n)^{-1/n}\, dx &= \left(x (ax^n)^{-1/n}\right) \int \frac{1}{x}\, dx \\ &= x (ax^n)^{-1/n} \log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$x \log(x) (ax^n)^{-1/n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^n)^(-n^(-1)),x]
```

```
[Out] (x*Log[x])/(a*x^n)^n^(-1)
```

fricas [A] time = 0.85, size = 10, normalized size = 0.67

$$\frac{\log(x)}{a^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x^n)^(1/n)),x, algorithm="fricas")
```

```
[Out] log(x)/a^(1/n)
```

giac [A] time = 0.18, size = 10, normalized size = 0.67

$$\frac{\log(x)}{a^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x^n)^(1/n)),x, algorithm="giac")
[Out] log(x)/a^(1/n)
```

maple **[A]** time = 0.02, size = 29, normalized size = 1.93

$$\frac{x\,\mathrm{e}^{-\frac{\ln\left(a\,\mathrm{e}^{n\ln(x)}\right)}{n}}\ln\left(a\,\mathrm{e}^{n\ln(x)}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x^n)^(1/n)),x)
[Out] 1/n*x*ln(a*exp(n*ln(x)))/exp(1/n*ln(a*exp(n*ln(x))))
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax^n)^{\left(\frac{1}{n}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x^n)^(1/n)),x, algorithm="maxima")
[Out] integrate(1/((a*x^n)^(1/n)), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{1}{(a\,x^n)^{1/n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^n)^(1/n),x)
[Out] int(1/(a*x^n)^(1/n), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (ax^n)^{-\frac{1}{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x**n)**(1/n)),x)
[Out] Integral((a*x**n)**(-1/n), x)
```

3.185 $\int \frac{(ax^n)^{-1/n}}{x}\,dx$

Optimal. Leaf size=13

$$-(ax^n)^{-1/n}$$

```
[Out] -1/((a*x^n)^(1/n))
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {15, 30}

$$-(ax^n)^{-1/n}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*x^n)^n^(-1)),x]
```

```
[Out] -(a*x^n)^(-n^(-1))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(ax^n)^{-1/n}}{x}\,dx &= \left(x\,(ax^n)^{-1/n}\right)\int \frac{1}{x^2}\,dx \\ &= -(ax^n)^{-1/n}\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$-(ax^n)^{-1/n}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*x^n)^n^(-1)),x]
```

```
[Out] -(a*x^n)^(-n^(-1))
```

fricas [A] time = 0.76, size = 12, normalized size = 0.92

$$-\frac{1}{a^{\left(\frac{1}{n}\right)}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((a*x^n)^(1/n)),x, algorithm="fricas")
```

```
[Out] -1/(a^(1/n)*x)
```

giac [A] time = 0.16, size = 13, normalized size = 1.00

$$-\frac{1}{(ax^n)^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((a*x^n)^(1/n)),x, algorithm="giac")
[Out] -1/(a*x^n)^(1/n)
```

maple [A] time = 0.00, size = 14, normalized size = 1.08

$$-(a\,x^n)^{-\frac{1}{n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((a*x^n)^(1/n)),x)
[Out] -1/((a*x^n)^(1/n))
```

maxima [A] time = 1.50, size = 18, normalized size = 1.38

$$-\frac{1}{a^{\left(\frac{1}{n}\right)}(x^n)^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((a*x^n)^(1/n)),x, algorithm="maxima")
[Out] -1/(a^(1/n)*(x^n)^(1/n))
```

mupad [B] time = 1.02, size = 13, normalized size = 1.00

$$-\frac{1}{(a\,x^n)^{1/n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x^n)^(1/n)),x)
[Out] -1/(a*x^n)^(1/n)
```

sympy [A] time = 1.31, size = 32, normalized size = 2.46

$$\begin{cases} -a^{-\frac{1}{n}}\,(x^n)^{-\frac{1}{n}} & \text{for } a \neq 0^n \\ -(0^n)^{-\frac{1}{n}}\,(x^n)^{-\frac{1}{n}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((a*x**n)**(1/n)),x)
[Out] Piecewise((-a**(-1/n)*(x**n)**(-1/n), Ne(a, 0**n)), (-(0**n)**(-1/n)*(x**n)
**(-1/n), True))
```

3.186 $\int \frac{(ax^n)^{-1/n}}{x^2}\,dx$

Optimal. Leaf size=18

$$-\frac{(ax^n)^{-1/n}}{2x}$$

```
[Out] -1/2/x/((a*x^n)^(1/n))
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {15, 30}

$$-\frac{(ax^n)^{-1/n}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*x^n)^n^(-1)),x]
```

```
[Out] -1/(2*x*(a*x^n)^n^(-1))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(ax^n)^{-1/n}}{x^2}\,dx &= \left(x\,(ax^n)^{-1/n}\right)\int \frac{1}{x^3}\,dx \\ &= -\frac{(ax^n)^{-1/n}}{2x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$-\frac{(ax^n)^{-1/n}}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a*x^n)^n^(-1)),x]
```

```
[Out] -1/2*1/(x*(a*x^n)^n^(-1))
```

fricas [A] time = 0.60, size = 12, normalized size = 0.67

$$-\frac{1}{2\,a^{\left(\frac{1}{n}\right)}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((a*x^n)^(1/n)),x, algorithm="fricas")
```

```
[Out] -1/2/(a^(1/n)*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax^n)^{\left(\frac{1}{n}\right)} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((a*x^n)^(1/n)),x, algorithm="giac")
```

```
[Out] integrate(1/((a*x^n)^(1/n)*x^2), x)
```

maple [A] time = 0.00, size = 17, normalized size = 0.94

$$-\frac{(a\,x^n)^{-\frac{1}{n}}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/((a*x^n)^(1/n)),x)
```

```
[Out] -1/2/x/((a*x^n)^(1/n))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax^n)^{\left(\frac{1}{n}\right)} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((a*x^n)^(1/n)),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x^n)^(1/n)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{x^2\,(a\,x^n)^{1/n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x^n)^(1/n)),x)
```

```
[Out] int(1/(x^2*(a*x^n)^(1/n)), x)
```

sympy [A] time = 2.03, size = 60, normalized size = 3.33

$$\begin{cases} -\frac{a^{-\frac{1}{n}}(x^n)^{-\frac{1}{n}}}{2x} & \text{for } a \neq 0^n \\ -\frac{1}{0^n\tilde{\infty}^n x(0^n)^{\frac{1}{n}}(x^n)^{\frac{1}{n}} + x(0^n)^{\frac{1}{n}}(x^n)^{\frac{1}{n}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/((a*x**n)**(1/n)),x)
```

```
[Out] Piecewise((-a**(-1/n)*(x**n)**(-1/n)/(2*x), Ne(a, 0**n)), (-1/(0**n*zoo**n*
x*(0**n)**(1/n)*(x**n)**(1/n) + x*(0**n)**(1/n)*(x**n)**(1/n)), True))
```

3.187 $\int x^m \left(ax^n\right)^{-\frac{1+m}{n}}\, dx$

Optimal. Leaf size=22

$$x^{m+1} \log(x) \left(ax^n\right)^{-\frac{m+1}{n}}$$

```
[Out] x^(1+m)*ln(x)/((a*x^n)^((1+m)/n))
```

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 29}

$$x^{m+1} \log(x) \left(ax^n\right)^{-\frac{m+1}{n}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a*x^n)^((1 + m)/n),x]
```

```
[Out] (x^(1 + m)*Log[x])/(a*x^n)^((1 + m)/n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int x^m \left(ax^n\right)^{-\frac{1+m}{n}}\, dx &= \left(x^{1+m} \left(ax^n\right)^{-\frac{1+m}{n}}\right) \int \frac{1}{x}\, dx \\ &= x^{1+m} \left(ax^n\right)^{-\frac{1+m}{n}} \log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$x^{m+1} \log(x) \left(ax^n\right)^{-\frac{m+1}{n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a*x^n)^((1 + m)/n),x]
```

```
[Out] (x^(1 + m)*Log[x])/(a*x^n)^((1 + m)/n)
```

fricas [A] time = 0.99, size = 14, normalized size = 0.64

$$\frac{\log(x)}{a^{\frac{m+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((a*x^n)^((1+m)/n)),x, algorithm="fricas")
```

```
[Out] log(x)/a^((m + 1)/n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(ax^n)^{\frac{m+1}{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((a*x^n)^((1+m)/n)),x, algorithm="giac")
[Out] integrate(x^m/(a*x^n)^((m + 1)/n), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int x^m\,(a\,x^n)^{-\frac{m+1}{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a*x^n)^((m+1)/n)),x)
[Out] int(x^m/((a*x^n)^((m+1)/n)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(ax^n)^{\frac{m+1}{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/((a*x^n)^((1+m)/n)),x, algorithm="maxima")
[Out] integrate(x^m/(a*x^n)^((m + 1)/n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^m}{(a\,x^n)^{\frac{m+1}{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a*x^n)^((m + 1)/n),x)
[Out] int(x^m/(a*x^n)^((m + 1)/n), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m\,(ax^n)^{-\frac{m+1}{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/((a*x**n)**((1+m)/n)),x)
[Out] Integral(x**m*(a*x**n)**(-(m + 1)/n), x)
```

3.188 $\int x^{-1-np} (ax^n)^p \, dx$

Optimal. Leaf size=16

$$\log(x) x^{-np} (ax^n)^p$$

```
[Out] (a*x^n)^p*ln(x)/(x^(n*p))
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {15, 29}

$$\log(x) x^{-np} (ax^n)^p$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n*p)*(a*x^n)^p,x]
```

```
[Out] ((a*x^n)^p*Log[x])/x^(n*p)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int x^{-1-np} (ax^n)^p \, dx &= \left(x^{-np} (ax^n)^p\right) \int \frac{1}{x} \, dx \\ &= x^{-np} (ax^n)^p \log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\log(x) x^{-np} (ax^n)^p$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n*p)*(a*x^n)^p,x]
```

```
[Out] ((a*x^n)^p*Log[x])/x^(n*p)
```

fricas [A] time = 0.82, size = 6, normalized size = 0.38

$$a^p \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-n*p-1)*(a*x^n)^p,x, algorithm="fricas")
```

```
[Out] a^p*log(x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ax^n)^p x^{-np-1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-n*p-1)*(a*x^n)^p,x, algorithm="giac")
[Out] integrate((a*x^n)^p*x^(-n*p - 1), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int x^{-np-1}\,(a\,x^n)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-n*p-1)*(a*x^n)^p,x)
[Out] int(x^(-n*p-1)*(a*x^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ax^n)^p\,x^{-np-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-n*p-1)*(a*x^n)^p,x, algorithm="maxima")
[Out] integrate((a*x^n)^p*x^(-n*p - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{(a\,x^n)^p}{x^{n\,p+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^n)^p/x^(n*p + 1),x)
[Out] int((a*x^n)^p/x^(n*p + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-np-1}\,(ax^n)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-n*p-1)*(a*x**n)**p,x)
[Out] Integral(x**(-n*p - 1)*(a*x**n)**p, x)
```

3.189 $\int x^m \left(a\,(bx^n)^p\right)^q\,dx$

Optimal. Leaf size=26

$$\frac{x^{m+1}\left(a\,(bx^n)^p\right)^q}{m+npq+1}$$

```
[Out] x^(1+m)*(a*(b*x^n)^p)^q/(n*p*q+m+1)
```

Rubi [A] time = 0.05, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {6679, 30}

$$\frac{x^{m+1}\left(a\,(bx^n)^p\right)^q}{m+npq+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a*(b*x^n)^p)^q,x]
```

```
[Out] (x^(1 + m)*(a*(b*x^n)^p)^q)/(1 + m + n*p*q)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6679

```
Int[(u_.)*((c_.)*((d_.)*((a_.) + (b_.)*(x_))^(n_))^(p_))^(q_), x_Symbol] :>
 Dist[(c*(d*(a + b*x)^n)^p)^q/(a + b*x)^(n*p*q), Int[u*(a + b*x)^(n*p*q), x
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] &&  !IntegerQ[p] &&  !IntegerQ[q]
```

Rubi steps

$$\begin{aligned}\int x^m \left(a\,(bx^n)^p\right)^q\,dx &= \left(x^{-npq}\left(a\,(bx^n)^p\right)^q\right)\int x^{m+npq}\,dx\\ &= \frac{x^{1+m}\left(a\,(bx^n)^p\right)^q}{1+m+npq}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$\frac{x^{m+1}\left(a\,(bx^n)^p\right)^q}{m+npq+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a*(b*x^n)^p)^q,x]
```

```
[Out] (x^(1 + m)*(a*(b*x^n)^p)^q)/(1 + m + n*p*q)
```

fricas [A] time = 0.70, size = 31, normalized size = 1.19

$$\frac{x x^m e^{\left(npq\log(x)+pq\log(b)+q\log(a)\right)}}{npq+m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a*(b*x^n)^p)^q,x, algorithm="fricas")
[Out] x*x^m*e^(n*p*q*log(x) + p*q*log(b) + q*log(a))/(n*p*q + m + 1)
```

giac [A] time = 0.23, size = 31, normalized size = 1.19

$$\frac{x x^m e^{\left(npq \log(x)+pq \log(b)+q \log(a)\right)}}{npq+m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a*(b*x^n)^p)^q,x, algorithm="giac")
[Out] x*x^m*e^(n*p*q*log(x) + p*q*log(b) + q*log(a))/(n*p*q + m + 1)
```

maple [A] time = 0.00, size = 27, normalized size = 1.04

$$\frac{x^{m+1}\left(a\left(b\,x^n\right)^p\right)^q}{npq+m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a*(b*x^n)^p)^q,x)
[Out] x^(m+1)*(a*(b*x^n)^p)^q/(n*p*q+m+1)
```

maxima [A] time = 2.64, size = 33, normalized size = 1.27

$$\frac{a^q b^{pq} x e^{\left(m \log(x)+q \log\left(\left(x^n\right)^p\right)\right)}}{npq+m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a*(b*x^n)^p)^q,x, algorithm="maxima")
[Out] a^q*b^(p*q)*x*e^(m*log(x) + q*log((x^n)^p))/(n*p*q + m + 1)
```

mupad [B] time = 1.23, size = 26, normalized size = 1.00

$$\frac{x^{m+1}\left(a\left(b\,x^n\right)^p\right)^q}{m+n\,p\,q+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a*(b*x^n)^p)^q,x)
[Out] (x^(m + 1)*(a*(b*x^n)^p)^q)/(m + n*p*q + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \left(a\left(bx^n\right)^p\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a*(b*x**n)**p)**q,x)
[Out] Integral(x**m*(a*(b*x**n)**p)**q, x)
```

3.190 $\int x^2 \left(a\,(bx^n)^p\right)^q\,dx$

Optimal. Leaf size=23

$$\frac{x^3 \left(a\,(bx^n)^p\right)^q}{npq+3}$$

```
[Out] x^3*(a*(b*x^n)^p)^q/(n*p*q+3)
```

Rubi [A] time = 0.05, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {6679, 30}

$$\frac{x^3 \left(a\,(bx^n)^p\right)^q}{npq+3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a*(b*x^n)^p)^q,x]
```

```
[Out] (x^3*(a*(b*x^n)^p)^q)/(3 + n*p*q)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6679

```
Int[(u_.)*((c_.)*((d_.)*((a_.) + (b_.)*(x_))^(n_))^(p_))^(q_), x_Symbol] :>
 Dist[(c*(d*(a + b*x)^n)^p)^q/(a + b*x)^(n*p*q), Int[u*(a + b*x)^(n*p*q), x
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] &&  !IntegerQ[p] &&  !IntegerQ[q]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(a\,(bx^n)^p\right)^q\,dx &= \left(x^{-npq}\left(a\,(bx^n)^p\right)^q\right)\int x^{2+npq}\,dx\\ &= \frac{x^3 \left(a\,(bx^n)^p\right)^q}{3+npq}\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$\frac{x^3 \left(a\,(bx^n)^p\right)^q}{npq+3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a*(b*x^n)^p)^q,x]
```

```
[Out] (x^3*(a*(b*x^n)^p)^q)/(3 + n*p*q)
```

fricas [A] time = 0.71, size = 29, normalized size = 1.26

$$\frac{x^3 e^{\left(npq\log(x)+pq\log(b)+q\log(a)\right)}}{npq+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*(b*x^n)^p)^q,x, algorithm="fricas")
[Out] x^3*e^(n*p*q*log(x) + p*q*log(b) + q*log(a))/(n*p*q + 3)
```

giac [A] time = 0.26, size = 29, normalized size = 1.26

$$\frac{x^3 e^{\left(npq\log(x)+pq\log(b)+q\log(a)\right)}}{npq+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*(b*x^n)^p)^q,x, algorithm="giac")
[Out] x^3*e^(n*p*q*log(x) + p*q*log(b) + q*log(a))/(n*p*q + 3)
```

maple [A] time = 0.00, size = 24, normalized size = 1.04

$$\frac{x^3\left(a\left(b\,x^n\right)^p\right)^q}{npq+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*(b*x^n)^p)^q,x)
[Out] x^3*(a*(b*x^n)^p)^q/(n*p*q+3)
```

maxima [A] time = 2.68, size = 27, normalized size = 1.17

$$\frac{a^q b^{pq} x^3\left(\left(x^n\right)^p\right)^q}{npq+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*(b*x^n)^p)^q,x, algorithm="maxima")
[Out] a^q*b^(p*q)*x^3*((x^n)^p)^q/(n*p*q + 3)
```

mupad [B] time = 1.02, size = 23, normalized size = 1.00

$$\frac{x^3\left(a\left(b\,x^n\right)^p\right)^q}{n\,p\,q+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*(b*x^n)^p)^q,x)
[Out] (x^3*(a*(b*x^n)^p)^q)/(n*p*q + 3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a\left(bx^n\right)^p\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a*(b*x**n)**p)**q,x)
[Out] Integral(x**2*(a*(b*x**n)**p)**q, x)
```

3.191 $\int x \left(a \left(b x^n\right)^p\right)^q \, dx$

Optimal. Leaf size=23

$$\frac{x^2 \left(a \left(b x^n\right)^p\right)^q}{npq+2}$$

```
[Out] x^2*(a*(b*x^n)^p)^q/(n*p*q+2)
```

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {6679, 30}

$$\frac{x^2 \left(a \left(b x^n\right)^p\right)^q}{npq+2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a*(b*x^n)^p)^q,x]
```

```
[Out] (x^2*(a*(b*x^n)^p)^q)/(2 + n*p*q)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6679

```
Int[(u_.)*((c_.)*((d_.)*((a_.) + (b_.)*(x_))^(n_))^(p_))^(q_), x_Symbol] :>
 Dist[(c*(d*(a + b*x)^n)^p)^q/(a + b*x)^(n*p*q), Int[u*(a + b*x)^(n*p*q), x
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] &&  !IntegerQ[p] &&  !IntegerQ[q]
```

Rubi steps

$$\begin{aligned}\int x \left(a \left(b x^n\right)^p\right)^q \, dx &= \left(x^{-npq} \left(a \left(b x^n\right)^p\right)^q\right) \int x^{1+npq} \, dx \\ &= \frac{x^2 \left(a \left(b x^n\right)^p\right)^q}{2+npq}\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$\frac{x^2 \left(a \left(b x^n\right)^p\right)^q}{npq+2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a*(b*x^n)^p)^q,x]
```

```
[Out] (x^2*(a*(b*x^n)^p)^q)/(2 + n*p*q)
```

fricas [A] time = 0.60, size = 29, normalized size = 1.26

$$\frac{x^2 e^{\left(npq \log(x)+pq \log(b)+q \log(a)\right)}}{npq+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*(b*x^n)^p)^q,x, algorithm="fricas")
[Out] x^2*e^(n*p*q*log(x) + p*q*log(b) + q*log(a))/(n*p*q + 2)
```

giac [**A**] time = 0.24, size = 29, normalized size = 1.26

$$\frac{x^2 e^{\left(npq \log(x)+pq \log(b)+q \log(a)\right)}}{npq+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*(b*x^n)^p)^q,x, algorithm="giac")
[Out] x^2*e^(n*p*q*log(x) + p*q*log(b) + q*log(a))/(n*p*q + 2)
```

maple [**A**] time = 0.00, size = 24, normalized size = 1.04

$$\frac{x^2 \left(a \left(b\, x^n\right)^p\right)^q}{npq+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*(b*x^n)^p)^q,x)
[Out] x^2*(a*(b*x^n)^p)^q/(n*p*q+2)
```

maxima [**A**] time = 2.70, size = 27, normalized size = 1.17

$$\frac{a^q b^{pq} x^2 \left(\left(x^n\right)^p\right)^q}{npq+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*(b*x^n)^p)^q,x, algorithm="maxima")
[Out] a^q*b^(p*q)*x^2*((x^n)^p)^q/(n*p*q + 2)
```

mupad [**B**] time = 0.99, size = 23, normalized size = 1.00

$$\frac{x^2 \left(a \left(b\, x^n\right)^p\right)^q}{n\, p\, q+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*(b*x^n)^p)^q,x)
[Out] (x^2*(a*(b*x^n)^p)^q)/(n*p*q + 2)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int x \left(a \left(b x^n\right)^p\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*(b*x**n)**p)**q,x)
[Out] Integral(x*(a*(b*x**n)**p)**q, x)
```

3.192 $\int \left(a\,(bx^n)^p\right)^q\,dx$

Optimal. Leaf size=21

$$\frac{x\left(a\,(bx^n)^p\right)^q}{npq+1}$$

```
[Out] x*(a*(b*x^n)^p)^q/(n*p*q+1)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6679, 30}

$$\frac{x\left(a\,(bx^n)^p\right)^q}{npq+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a*(b*x^n)^p)^q,x]
```

```
[Out] (x*(a*(b*x^n)^p)^q)/(1 + n*p*q)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6679

```
Int[(u_.)*((c_.)*((d_.)*((a_.) + (b_.)*(x_))^(n_))^(p_))^(q_), x_Symbol] :>
 Dist[(c*(d*(a + b*x)^n)^p)^q/(a + b*x)^(n*p*q), Int[u*(a + b*x)^(n*p*q), x
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] &&  !IntegerQ[p] &&  !IntegerQ[q]
```

Rubi steps

$$\begin{aligned}\int \left(a\,(bx^n)^p\right)^q\,dx &= \left(x^{-npq}\left(a\,(bx^n)^p\right)^q\right)\int x^{npq}\,dx\\ &= \frac{x\left(a\,(bx^n)^p\right)^q}{1+npq}\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{x\left(a\,(bx^n)^p\right)^q}{npq+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*(b*x^n)^p)^q,x]
```

```
[Out] (x*(a*(b*x^n)^p)^q)/(1 + n*p*q)
```

fricas [A] time = 0.59, size = 27, normalized size = 1.29

$$\frac{xe^{\left(npq\log(x)+pq\log(b)+q\log(a)\right)}}{npq+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*x^n)^p)^q,x, algorithm="fricas")
[Out] x*e^(n*p*q*log(x) + p*q*log(b) + q*log(a))/(n*p*q + 1)
```

giac [**A**] time = 0.19, size = 27, normalized size = 1.29

$$\frac{xe^{\left(npq\log(x)+pq\log(b)+q\log(a)\right)}}{npq+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*x^n)^p)^q,x, algorithm="giac")
[Out] x*e^(n*p*q*log(x) + p*q*log(b) + q*log(a))/(n*p*q + 1)
```

maple [**A**] time = 0.00, size = 22, normalized size = 1.05

$$\frac{x\left(a\left(b\,x^{n}\right)^{p}\right)^{q}}{npq+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*(b*x^n)^p)^q,x)
[Out] x*(a*(b*x^n)^p)^q/(n*p*q+1)
```

maxima [**A**] time = 2.67, size = 25, normalized size = 1.19

$$\frac{a^{q}b^{pq}x\left(\left(x^{n}\right)^{p}\right)^{q}}{npq+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*x^n)^p)^q,x, algorithm="maxima")
[Out] a^q*b^(p*q)*x*((x^n)^p)^q/(n*p*q + 1)
```

mupad [**B**] time = 0.98, size = 21, normalized size = 1.00

$$\frac{x\left(a\left(b\,x^{n}\right)^{p}\right)^{q}}{n\,p\,q+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*(b*x^n)^p)^q,x)
[Out] (x*(a*(b*x^n)^p)^q)/(n*p*q + 1)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(bx^{n}\right)^{p}\right)^{q}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*x**n)**p)**q,x)
[Out] Integral((a*(b*x**n)**p)**q, x)
```

3.193 $\int \frac{\left(a(bx^n)^p\right)^q}{x}\,dx$

Optimal. Leaf size=21

$$\frac{\left(a\,(bx^n)^p\right)^q}{npq}$$

```
[Out] (a*(b*x^n)^p)^q/n/p/q
```

Rubi [A] time = 0.04, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {6679, 30}

$$\frac{\left(a\,(bx^n)^p\right)^q}{npq}$$

Antiderivative was successfully verified.

```
[In] Int[(a*(b*x^n)^p)^q/x,x]
```

```
[Out] (a*(b*x^n)^p)^q/(n*p*q)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6679

```
Int[(u_.)*((c_.)*((d_.)*((a_.) + (b_.)*(x_))^(n_))^(p_))^(q_), x_Symbol] :>
 Dist[(c*(d*(a + b*x)^n)^p)^q/(a + b*x)^(n*p*q), Int[u*(a + b*x)^(n*p*q), x
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] &&  !IntegerQ[p] &&  !IntegerQ[q]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a\,(bx^n)^p\right)^q}{x}\,dx &= \left(x^{-npq}\left(a\,(bx^n)^p\right)^q\right)\int x^{-1+npq}\,dx\\ &= \frac{\left(a\,(bx^n)^p\right)^q}{npq}\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{\left(a\,(bx^n)^p\right)^q}{npq}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*(b*x^n)^p)^q/x,x]
```

```
[Out] (a*(b*x^n)^p)^q/(n*p*q)
```

fricas [A] time = 0.66, size = 27, normalized size = 1.29

$$\frac{e^{\left(npq\log(x)+pq\log(b)+q\log(a)\right)}}{npq}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*x^n)^p)^q/x,x, algorithm="fricas")
[Out] e^(n*p*q*log(x) + p*q*log(b) + q*log(a))/(n*p*q)
```

giac [A] time = 0.16, size = 21, normalized size = 1.00

$$\frac{\left((bx^n)^p\, a\right)^q}{npq}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*x^n)^p)^q/x,x, algorithm="giac")
[Out] ((b*x^n)^p*a)^q/(n*p*q)
```

maple [A] time = 0.00, size = 22, normalized size = 1.05

$$\frac{\left(a\,(b\,x^n)^p\right)^q}{npq}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*(b*x^n)^p)^q/x,x)
[Out] (a*(b*x^n)^p)^q/n/p/q
```

maxima [A] time = 2.73, size = 25, normalized size = 1.19

$$\frac{a^q b^{pq}\left((x^n)^p\right)^q}{npq}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*x^n)^p)^q/x,x, algorithm="maxima")
[Out] a^q*b^(p*q)*((x^n)^p)^q/(n*p*q)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\left(a\,(b\,x^n)^p\right)^q}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*(b*x^n)^p)^q/x,x)
[Out] int((a*(b*x^n)^p)^q/x, x)
```

sympy [A] time = 0.72, size = 41, normalized size = 1.95

$$\begin{cases} \log(x) & \text{for } n = 0 \wedge p = 0 \wedge q = 0 \\ a^q \log(x) & \text{for } p = 0 \\ (ab^p)^q \log(x) & \text{for } n = 0 \\ \log(x) & \text{for } q = 0 \\ \frac{a^q (b^p)^q \left((x^n)^p\right)^q}{npq} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*x**n)**p)**q/x,x)
```

```
[Out] Piecewise((log(x), Eq(n, 0) & Eq(p, 0) & Eq(q, 0)), (a**q*log(x), Eq(p, 0))
, ((a*b**p)**q*log(x), Eq(n, 0)), (log(x), Eq(q, 0)), (a**q*(b**p)**q*((x**
n)**p)**q/(n*p*q), True))
```

3.194 $\int \frac{\left(a(bx^n)^p\right)^q}{x^2}\,dx$

Optimal. Leaf size=25

$$-\frac{\left(a\,(bx^n)^p\right)^q}{x(1-npq)}$$

```
[Out] -(a*(b*x^n)^p)^q/(-n*p*q+1)/x
```

Rubi [A] time = 0.05, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {6679, 30}

$$-\frac{\left(a\,(bx^n)^p\right)^q}{x(1-npq)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*(b*x^n)^p)^q/x^2,x]
```

```
[Out] -((a*(b*x^n)^p)^q/((1 - n*p*q)*x))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6679

```
Int[(u_.)*((c_.)*((d_.)*((a_.) + (b_.)*(x_))^(n_))^(p_))^(q_), x_Symbol] :>
 Dist[(c*(d*(a + b*x)^n)^p)^q/(a + b*x)^(n*p*q), Int[u*(a + b*x)^(n*p*q), x
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] &&  !IntegerQ[p] &&  !IntegerQ[q]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a\,(bx^n)^p\right)^q}{x^2}\,dx &= \left(x^{-npq}\left(a\,(bx^n)^p\right)^q\right)\int x^{-2+npq}\,dx\\ &= -\frac{\left(a\,(bx^n)^p\right)^q}{(1-npq)x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.92

$$\frac{\left(a\,(bx^n)^p\right)^q}{x(npq-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*(b*x^n)^p)^q/x^2,x]
```

```
[Out] (a*(b*x^n)^p)^q/((-1 + n*p*q)*x)
```

fricas [A] time = 0.56, size = 29, normalized size = 1.16

$$\frac{e^{\left(npq\log(x)+pq\log(b)+q\log(a)\right)}}{\left(npq-1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*x^n)^p)^q/x^2,x, algorithm="fricas")
[Out] e^(n*p*q*log(x) + p*q*log(b) + q*log(a))/((n*p*q - 1)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((bx^n)^p\, a\right)^q}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*x^n)^p)^q/x^2,x, algorithm="giac")
[Out] integrate(((b*x^n)^p*a)^q/x^2, x)
```

maple [A] time = 0.00, size = 24, normalized size = 0.96

$$\frac{\left(a\,(b\,x^n)^p\right)^q}{\left(npq-1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*(b*x^n)^p)^q/x^2,x)
[Out] 1/x/(n*p*q-1)*(a*(b*x^n)^p)^q
```

maxima [A] time = 2.80, size = 27, normalized size = 1.08

$$\frac{a^q b^{pq}\left((x^n)^p\right)^q}{\left(npq-1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*x^n)^p)^q/x^2,x, algorithm="maxima")
[Out] a^q*b^(p*q)*((x^n)^p)^q/((n*p*q - 1)*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a\,(b\,x^n)^p\right)^q}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*(b*x^n)^p)^q/x^2,x)
[Out] int((a*(b*x^n)^p)^q/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,(bx^n)^p\right)^q}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*x**n)**p)**q/x**2,x)
[Out] Integral((a*(b*x**n)**p)**q/x**2, x)
```

3.195 $\int \frac{\left(a(bx^n)^p\right)^q}{x^3}\,dx$

Optimal. Leaf size=25

$$-\frac{\left(a\,(bx^n)^p\right)^q}{x^2(2-npq)}$$

```
[Out] -(a*(b*x^n)^p)^q/(-n*p*q+2)/x^2
```

Rubi [A] time = 0.05, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {6679, 30}

$$-\frac{\left(a\,(bx^n)^p\right)^q}{x^2(2-npq)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*(b*x^n)^p)^q/x^3,x]
```

```
[Out] -((a*(b*x^n)^p)^q/((2 - n*p*q)*x^2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6679

```
Int[(u_.)*((c_.)*((d_.)*((a_.) + (b_.)*(x_))^(n_))^(p_))^(q_), x_Symbol] :>
 Dist[(c*(d*(a + b*x)^n)^p)^q/(a + b*x)^(n*p*q), Int[u*(a + b*x)^(n*p*q), x
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] &&  !IntegerQ[p] &&  !IntegerQ[q]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a\,(bx^n)^p\right)^q}{x^3}\,dx &= \left(x^{-npq}\left(a\,(bx^n)^p\right)^q\right)\int x^{-3+npq}\,dx\\ &= -\frac{\left(a\,(bx^n)^p\right)^q}{(2-npq)x^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 0.92

$$\frac{\left(a\,(bx^n)^p\right)^q}{x^2(npq-2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*(b*x^n)^p)^q/x^3,x]
```

```
[Out] (a*(b*x^n)^p)^q/((-2 + n*p*q)*x^2)
```

fricas [A] time = 0.71, size = 29, normalized size = 1.16

$$\frac{e^{\left(npq\log(x)+pq\log(b)+q\log(a)\right)}}{\left(npq-2\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*x^n)^p)^q/x^3,x, algorithm="fricas")
[Out] e^(n*p*q*log(x) + p*q*log(b) + q*log(a))/((n*p*q - 2)*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((bx^n)^p\, a\right)^q}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*x^n)^p)^q/x^3,x, algorithm="giac")
[Out] integrate(((b*x^n)^p*a)^q/x^3, x)
```

maple [A] time = 0.00, size = 24, normalized size = 0.96

$$\frac{\left(a\,(b\,x^n)^p\right)^q}{\left(npq-2\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*(b*x^n)^p)^q/x^3,x)
[Out] 1/x^2/(n*p*q-2)*(a*(b*x^n)^p)^q
```

maxima [A] time = 2.81, size = 27, normalized size = 1.08

$$\frac{a^q b^{pq}\left((x^n)^p\right)^q}{\left(npq-2\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*x^n)^p)^q/x^3,x, algorithm="maxima")
[Out] a^q*b^(p*q)*((x^n)^p)^q/((n*p*q - 2)*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a\,(b\,x^n)^p\right)^q}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*(b*x^n)^p)^q/x^3,x)
[Out] int((a*(b*x^n)^p)^q/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\,(bx^n)^p\right)^q}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*x**n)**p)**q/x**3,x)
[Out] Integral((a*(b*x**n)**p)**q/x**3, x)
```

3.196 $\int x^2 \left(a\left(bx^m\right)^n\right)^{-\frac{1}{mn}}\, dx$

Optimal. Leaf size=25

$$\frac{1}{2}x^3\left(a\left(bx^m\right)^n\right)^{-\frac{1}{mn}}$$

```
[Out] 1/2*x^3/((a*(b*x^m)^n)^(1/m/n))
```

Rubi [A] time = 0.05, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6679, 30}

$$\frac{1}{2}x^3\left(a\left(bx^m\right)^n\right)^{-\frac{1}{mn}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*(b*x^m)^n)^(1/(m*n)),x]
```

```
[Out] x^3/(2*(a*(b*x^m)^n)^(1/(m*n)))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6679

```
Int[(u_.)*((c_.)*((d_.)*((a_.) + (b_.)*(x_))^(n_))^(p_))^(q_), x_Symbol] :>
 Dist[(c*(d*(a + b*x)^n)^p)^q/(a + b*x)^(n*p*q), Int[u*(a + b*x)^(n*p*q), x
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] &&  !IntegerQ[p] &&  !IntegerQ[q]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a\left(bx^m\right)^n\right)^{-\frac{1}{mn}}\, dx &= \left(x\left(a\left(bx^m\right)^n\right)^{-\frac{1}{mn}}\right)\int x\, dx\\ &= \frac{1}{2}x^3\left(a\left(bx^m\right)^n\right)^{-\frac{1}{mn}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{1}{2}x^3\left(a\left(bx^m\right)^n\right)^{-\frac{1}{mn}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*(b*x^m)^n)^(1/(m*n)),x]
```

```
[Out] x^3/(2*(a*(b*x^m)^n)^(1/(m*n)))
```

fricas [A] time = 0.70, size = 21, normalized size = 0.84

$$\frac{1}{2}x^2 e^{\left(-\frac{n\log(b)+\log(a)}{mn}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((a*(b*x^m)^n)^(1/m/n)),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*e^(-(n*log(b) + log(a))/(m*n))
```

giac [A] time = 0.24, size = 21, normalized size = 0.84

$$\frac{1}{2}\,x^2 e^{\left(-\frac{n\log(b)+\log(a)}{mn}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((a*(b*x^m)^n)^(1/m/n)),x, algorithm="giac")
[Out] 1/2*x^2*e^(-(n*log(b) + log(a))/(m*n))
```

maple [A] time = 0.00, size = 25, normalized size = 1.00

$$\frac{x^3\left(a\,(b\,x^m)^n\right)^{-\frac{1}{mn}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a*(b*x^m)^n)^(1/m/n)),x)
[Out] 1/2*x^3/((a*(b*x^m)^n)^(1/m/n))
```

maxima [A] time = 3.05, size = 38, normalized size = 1.52

$$\frac{x^3}{2\,a^{\frac{1}{mn}}\,b^{\left(\frac{1}{m}\right)}\left((x^m)^n\right)^{\frac{1}{mn}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((a*(b*x^m)^n)^(1/m/n)),x, algorithm="maxima")
[Out] 1/2*x^3/(a^(1/(m*n))*b^(1/m)*((x^m)^n)^(1/(m*n)))
```

mupad [B] time = 1.01, size = 24, normalized size = 0.96

$$\frac{x^3}{2\left(a\,(b\,x^m)^n\right)^{\frac{1}{m\,n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*(b*x^m)^n)^(1/(m*n)),x)
[Out] x^3/(2*(a*(b*x^m)^n)^(1/(m*n)))
```

sympy [A] time = 20.01, size = 248, normalized size = 9.92

$$\begin{cases} -\dfrac{x^3}{0^{mn}\infty^{mn}(0^{mn})^{\frac{1}{mn}}\left((x^m)^n\right)^{\frac{1}{mn}}\left(\left((0^{mn})^{\frac{1}{n}}\right)^n\right)^{\frac{1}{mn}} - 3(0^{mn})^{\frac{1}{mn}}\left((x^m)^n\right)^{\frac{1}{mn}}\left(\left((0^{mn})^{\frac{1}{n}}\right)^n\right)^{\frac{1}{mn}}} & \text{for } a = 0^{mn} \wedge b = (0^{mn})^{\frac{1}{n}} \\ \dfrac{a^{-\frac{1}{mn}}x^3\left((x^m)^n\right)^{-\frac{1}{mn}}\left(\left((0^{mn})^{\frac{1}{n}}\right)^n\right)^{-\frac{1}{mn}}}{2} & \text{for } b = (0^{mn})^{\frac{1}{n}} \\ -\dfrac{x^3}{0^{mn}\infty^{mn}(0^{mn})^{\frac{1}{mn}}(b^n)^{\frac{1}{mn}}\left((x^m)^n\right)^{\frac{1}{mn}} - 3(0^{mn})^{\frac{1}{mn}}(b^n)^{\frac{1}{mn}}\left((x^m)^n\right)^{\frac{1}{mn}}} & \text{for } a = 0^{mn} \\ \dfrac{a^{-\frac{1}{mn}}x^3(b^n)^{-\frac{1}{mn}}\left((x^m)^n\right)^{-\frac{1}{mn}}}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((a*(b*x**m)**n)**(1/m/n)),x)
```

```
[Out] Piecewise((-x**3/(0**(m*n)*zoo**(m*n)*(0**(m*n))**(1/(m*n))*((x**m)**n)**(1
/(m*n))*(((0**(m*n))**(1/n))**n)**(1/(m*n)) - 3*(0**(m*n))**(1/(m*n))*((x**
m)**n)**(1/(m*n))*(((0**(m*n))**(1/n))**n)**(1/(m*n))), Eq(a, 0**(m*n)) & E
q(b, (0**(m*n))**(1/n))), (a**(-1/(m*n))*x**3*((x**m)**n)**(-1/(m*n))*(((0*
*(m*n))**(1/n))**n)**(-1/(m*n))/2, Eq(b, (0**(m*n))**(1/n))), (-x**3/(0**(m
*n)*zoo**(m*n)*(0**(m*n))**(1/(m*n))*(b**n)**(1/(m*n))*((x**m)**n)**(1/(m*n
)) - 3*(0**(m*n))**(1/(m*n))*(b**n)**(1/(m*n))*((x**m)**n)**(1/(m*n))), Eq(
a, 0**(m*n))), (a**(-1/(m*n))*x**3*(b**n)**(-1/(m*n))*((x**m)**n)**(-1/(m*n
))/2, True))
```

3.197 $\int x \left(a \left(b x^m\right)^n\right)^{-\frac{1}{mn}} \, dx$

Optimal. Leaf size=22

$$x^2 \left(a \left(b x^m\right)^n\right)^{-\frac{1}{mn}}$$

```
[Out] x^2/((a*(b*x^m)^n)^(1/m/n))
```

Rubi [A] time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6679, 8}

$$x^2 \left(a \left(b x^m\right)^n\right)^{-\frac{1}{mn}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*(b*x^m)^n)^(1/(m*n)),x]
```

```
[Out] x^2/(a*(b*x^m)^n)^(1/(m*n))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 6679

```
Int[(u_.)*((c_.)*((d_.)*((a_.) + (b_.)*(x_))^(n_))^(p_))^(q_), x_Symbol] :>
 Dist[(c*(d*(a + b*x)^n)^p)^q/(a + b*x)^(n*p*q), Int[u*(a + b*x)^(n*p*q), x
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] &&  !IntegerQ[p] &&  !IntegerQ[q]
```

Rubi steps

$$\begin{aligned}\int x \left(a \left(b x^m\right)^n\right)^{-\frac{1}{mn}} \, dx &= \left(x \left(a \left(b x^m\right)^n\right)^{-\frac{1}{mn}}\right) \int 1 \, dx \\ &= x^2 \left(a \left(b x^m\right)^n\right)^{-\frac{1}{mn}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$x^2 \left(a \left(b x^m\right)^n\right)^{-\frac{1}{mn}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*(b*x^m)^n)^(1/(m*n)),x]
```

```
[Out] x^2/(a*(b*x^m)^n)^(1/(m*n))
```

fricas [A] time = 0.67, size = 18, normalized size = 0.82

$$x e^{\left(-\frac{n \log(b) + \log(a)}{mn}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((a*(b*x^m)^n)^(1/m/n)),x, algorithm="fricas")
```

```
[Out] x*e^(-(n*log(b) + log(a))/(m*n))
```

giac [A] time = 0.23, size = 18, normalized size = 0.82

$$xe^{\left(-\frac{n\log(b)+\log(a)}{mn}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((a*(b*x^m)^n)^(1/m/n)),x, algorithm="giac")
[Out] x*e^(-(n*log(b) + log(a))/(m*n))
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int x\left(a\left(b\,x^m\right)^n\right)^{-\frac{1}{mn}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a*(b*x^m)^n)^(1/m/n)),x)
[Out] int(x/((a*(b*x^m)^n)^(1/m/n)),x)
```

maxima [A] time = 3.13, size = 37, normalized size = 1.68

$$\frac{x^2}{a^{\frac{1}{mn}}b^{\left(\frac{1}{m}\right)}\left(\left(x^m\right)^n\right)^{\frac{1}{mn}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((a*(b*x^m)^n)^(1/m/n)),x, algorithm="maxima")
[Out] x^2/(a^(1/(m*n))*b^(1/m)*((x^m)^n)^(1/(m*n)))
```

mupad [B] time = 0.98, size = 23, normalized size = 1.05

$$\frac{x^2}{\left(a\left(b\,x^m\right)^n\right)^{\frac{1}{m\,n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*(b*x^m)^n)^(1/(m*n)),x)
[Out] x^2/(a*(b*x^m)^n)^(1/(m*n))
```

sympy [A] time = 10.35, size = 245, normalized size = 11.14

$$\begin{cases} -\dfrac{x^2}{0^{mn}\infty^{mn}(0^{mn})^{\frac{1}{mn}}\left(\left(x^m\right)^n\right)^{\frac{1}{mn}}\left(\left((0^{mn})^{\frac{1}{n}}\right)^n\right)^{\frac{1}{mn}}-2(0^{mn})^{\frac{1}{mn}}\left(\left(x^m\right)^n\right)^{\frac{1}{mn}}\left(\left((0^{mn})^{\frac{1}{n}}\right)^n\right)^{\frac{1}{mn}}} & \text{for } a = 0^{mn} \wedge b = (0^{mn})^{\frac{1}{n}} \\ a^{-\frac{1}{mn}}x^2\left(\left(x^m\right)^n\right)^{-\frac{1}{mn}}\left(\left((0^{mn})^{\frac{1}{n}}\right)^n\right)^{-\frac{1}{mn}} & \text{for } b = (0^{mn})^{\frac{1}{n}} \\ -\dfrac{x^2}{0^{mn}\infty^{mn}(0^{mn})^{\frac{1}{mn}}(b^n)^{\frac{1}{mn}}\left(\left(x^m\right)^n\right)^{\frac{1}{mn}}-2(0^{mn})^{\frac{1}{mn}}(b^n)^{\frac{1}{mn}}\left(\left(x^m\right)^n\right)^{\frac{1}{mn}}} & \text{for } a = 0^{mn} \\ a^{-\frac{1}{mn}}x^2\left(b^n\right)^{-\frac{1}{mn}}\left(\left(x^m\right)^n\right)^{-\frac{1}{mn}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((a*(b*x**m)**n)**(1/m/n)),x)
```

```
[Out] Piecewise((-x**2/(0**(m*n)*zoo**(m*n)*(0**(m*n))**(1/(m*n))*((x**m)**n)**(1
/(m*n))*(((0**(m*n))**(1/n))**n)**(1/(m*n)) - 2*(0**(m*n))**(1/(m*n))*((x**
m)**n)**(1/(m*n))*(((0**(m*n))**(1/n))**n)**(1/(m*n))), Eq(a, 0**(m*n)) & E
q(b, (0**(m*n))**(1/n))), (a**(-1/(m*n))*x**2*((x**m)**n)**(-1/(m*n))*(((0*
*(m*n))**(1/n))**n)**(-1/(m*n)), Eq(b, (0**(m*n))**(1/n))), (-x**2/(0**(m*n
)*zoo**(m*n)*(0**(m*n))**(1/(m*n))*(b**n)**(1/(m*n))*((x**m)**n)**(1/(m*n))
 - 2*(0**(m*n))**(1/(m*n))*(b**n)**(1/(m*n))*((x**m)**n)**(1/(m*n))), Eq(a,
 0**(m*n))), (a**(-1/(m*n))*x**2*(b**n)**(-1/(m*n))*((x**m)**n)**(-1/(m*n))
, True))
```

3.198 $\int \left(a\,(bx^m)^n\right)^{-\frac{1}{mn}}\,dx$

Optimal. Leaf size=22

$$x\log(x)\left(a\,(bx^m)^n\right)^{-\frac{1}{mn}}$$

```
[Out] x*ln(x)/((a*(b*x^m)^n)^(1/m/n))
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6679, 29}

$$x\log(x)\left(a\,(bx^m)^n\right)^{-\frac{1}{mn}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*(b*x^m)^n)^(-(1/(m*n))),x]
```

```
[Out] (x*Log[x])/(a*(b*x^m)^n)^(1/(m*n))
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 6679

```
Int[(u_.)*((c_.)*((d_.)*((a_.) + (b_.)*(x_))^(n_))^(p_))^(q_), x_Symbol] :>
 Dist[(c*(d*(a + b*x)^n)^p)^q/(a + b*x)^(n*p*q), Int[u*(a + b*x)^(n*p*q), x
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] &&  !IntegerQ[p] &&  !IntegerQ[q]
```

Rubi steps

$$\begin{aligned}\int \left(a\,(bx^m)^n\right)^{-\frac{1}{mn}}\,dx &= \left(x\left(a\,(bx^m)^n\right)^{-\frac{1}{mn}}\right)\int \frac{1}{x}\,dx\\ &= x\left(a\,(bx^m)^n\right)^{-\frac{1}{mn}}\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$x\log(x)\left(a\,(bx^m)^n\right)^{-\frac{1}{mn}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*(b*x^m)^n)^(-(1/(m*n))),x]
```

```
[Out] (x*Log[x])/(a*(b*x^m)^n)^(1/(m*n))
```

fricas [A] time = 0.59, size = 19, normalized size = 0.86

$$e^{\left(-\frac{n\log(b)+\log(a)}{mn}\right)}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*(b*x^m)^n)^(1/m/n)),x, algorithm="fricas")
```

```
[Out] e^(-(n*log(b) + log(a))/(m*n))*log(x)
```

giac [**A**] time = 0.20, size = 19, normalized size = 0.86

$$e^{\left(-\frac{n\log(b)+\log(a)}{mn}\right)}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*(b*x^m)^n)^(1/m/n)),x, algorithm="giac")
[Out] e^(-(n*log(b) + log(a))/(m*n))*log(x)
```

maple [**F**] time = 0.05, size = 0, normalized size = 0.00

$$\int \left(a\,(b\,x^m)^n\right)^{-\frac{1}{mn}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*(b*x^m)^n)^(1/m/n)),x)
[Out] int(1/((a*(b*x^m)^n)^(1/m/n)),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((bx^m)^n\,a\right)^{\frac{1}{mn}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*(b*x^m)^n)^(1/m/n)),x, algorithm="maxima")
[Out] integrate(1/(((b*x^m)^n*a)^(1/(m*n))), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\left(a\,(b\,x^m)^n\right)^{\frac{1}{m\,n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*(b*x^m)^n)^(1/(m*n)),x)
[Out] int(1/(a*(b*x^m)^n)^(1/(m*n)), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\,(bx^m)^n\right)^{-\frac{1}{mn}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*(b*x**m)**n)**(1/m/n)),x)
[Out] Integral((a*(b*x**m)**n)**(-1/(m*n)), x)
```

3.199 $\int \frac{\left(a(bx^m)^n\right)^{-\frac{1}{mn}}}{x}\,dx$

Optimal. Leaf size=20

$$-\left(a\,(bx^m)^n\right)^{-\frac{1}{mn}}$$

```
[Out] -1/((a*(b*x^m)^n)^(1/m/n))
```

Rubi [A] time = 0.04, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6679, 30}

$$-\left(a\,(bx^m)^n\right)^{-\frac{1}{mn}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*(b*x^m)^n)^(1/(m*n))),x]
```

```
[Out] -(a*(b*x^m)^n)^(-(1/(m*n)))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6679

```
Int[(u_.)*((c_.)*((d_.)*((a_.) + (b_.)*(x_))^(n_))^(p_))^(q_), x_Symbol] :>
 Dist[(c*(d*(a + b*x)^n)^p)^q/(a + b*x)^(n*p*q), Int[u*(a + b*x)^(n*p*q), x
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] &&  !IntegerQ[p] &&  !IntegerQ[q]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a\,(bx^m)^n\right)^{-\frac{1}{mn}}}{x}\,dx &= \left(x\left(a\,(bx^m)^n\right)^{-\frac{1}{mn}}\right)\int \frac{1}{x^2}\,dx \\ &= -\left(a\,(bx^m)^n\right)^{-\frac{1}{mn}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$-\left(a\,(bx^m)^n\right)^{-\frac{1}{mn}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*(b*x^m)^n)^(1/(m*n))),x]
```

```
[Out] -(a*(b*x^m)^n)^(-(1/(m*n)))
```

fricas [A] time = 0.73, size = 21, normalized size = 1.05

$$-\frac{e^{\left(-\frac{n\log(b)+\log(a)}{mn}\right)}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((a*(b*x^m)^n)^(1/m/n)),x, algorithm="fricas")
[Out] -e^(-(n*log(b) + log(a))/(m*n))/x
```

giac [A] time = 0.17, size = 21, normalized size = 1.05

$$-\frac{1}{\left((bx^m)^n\, a\right)^{\frac{1}{mn}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((a*(b*x^m)^n)^(1/m/n)),x, algorithm="giac")
[Out] -1/((b*x^m)^n*a)^(1/(m*n))
```

maple [A] time = 0.00, size = 22, normalized size = 1.10

$$-\left(a\,(b\,x^m)^n\right)^{-\frac{1}{mn}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((a*(b*x^m)^n)^(1/m/n)),x)
[Out] -1/((a*(b*x^m)^n)^(1/m/n))
```

maxima [A] time = 3.11, size = 35, normalized size = 1.75

$$-\frac{1}{a^{\frac{1}{mn}} b^{\left(\frac{1}{m}\right)} \left((x^m)^n\right)^{\frac{1}{mn}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((a*(b*x^m)^n)^(1/m/n)),x, algorithm="maxima")
[Out] -1/(a^(1/(m*n))*b^(1/m)*((x^m)^n)^(1/(m*n)))
```

mupad [B] time = 0.97, size = 21, normalized size = 1.05

$$-\frac{1}{\left(a\,(b\,x^m)^n\right)^{\frac{1}{m\,n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*(b*x^m)^n)^(1/(m*n))),x)
[Out] -1/(a*(b*x^m)^n)^(1/(m*n))
```

sympy [A] time = 5.88, size = 153, normalized size = 7.65

$$\begin{cases} -(0^{mn})^{-\frac{1}{mn}} \left((x^m)^n\right)^{-\frac{1}{mn}} \left(\left((0^{mn})^{\frac{1}{n}}\right)^n\right)^{-\frac{1}{mn}} & \text{for } a = 0^{mn} \wedge b = (0^{mn})^{\frac{1}{n}} \\ -a^{-\frac{1}{mn}} \left((x^m)^n\right)^{-\frac{1}{mn}} \left(\left((0^{mn})^{\frac{1}{n}}\right)^n\right)^{-\frac{1}{mn}} & \text{for } b = (0^{mn})^{\frac{1}{n}} \\ -(0^{mn})^{-\frac{1}{mn}} (b^n)^{-\frac{1}{mn}} \left((x^m)^n\right)^{-\frac{1}{mn}} & \text{for } a = 0^{mn} \\ -a^{-\frac{1}{mn}} (b^n)^{-\frac{1}{mn}} \left((x^m)^n\right)^{-\frac{1}{mn}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((a*(b*x**m)**n)**(1/m/n)),x)
```

```
[Out] Piecewise((-(0**(m*n))**(-1/(m*n))*((x**m)**n)**(-1/(m*n))*(((0**(m*n))**(1
/n))**n)**(-1/(m*n)), Eq(a, 0**(m*n)) & Eq(b, (0**(m*n))**(1/n))), (-a**(-1
/(m*n))*((x**m)**n)**(-1/(m*n))*(((0**(m*n))**(1/n))**n)**(-1/(m*n)), Eq(b,
 (0**(m*n))**(1/n))), (-(0**(m*n))**(-1/(m*n))*(b**n)**(-1/(m*n))*((x**m)**
n)**(-1/(m*n)), Eq(a, 0**(m*n))), (-a**(-1/(m*n))*(b**n)**(-1/(m*n))*((x**m
)**n)**(-1/(m*n)), True))
```

3.200 $\int \frac{\left(a(bx^m)^n\right)^{-\frac{1}{mn}}}{x^2}\,dx$

Optimal. Leaf size=25

$$-\frac{\left(a\,(bx^m)^n\right)^{-\frac{1}{mn}}}{2x}$$

```
[Out] -1/2/x/((a*(b*x^m)^n)^(1/m/n))
```

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6679, 30}

$$-\frac{\left(a\,(bx^m)^n\right)^{-\frac{1}{mn}}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*(b*x^m)^n)^(1/(m*n))),x]
```

```
[Out] -1/(2*x*(a*(b*x^m)^n)^(1/(m*n)))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6679

```
Int[(u_.)*((c_.)*((d_.)*((a_.) + (b_.)*(x_))^(n_))^(p_))^(q_), x_Symbol] :>
 Dist[(c*(d*(a + b*x)^n)^p)^q/(a + b*x)^(n*p*q), Int[u*(a + b*x)^(n*p*q), x
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] &&  !IntegerQ[p] &&  !IntegerQ[q]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a\,(bx^m)^n\right)^{-\frac{1}{mn}}}{x^2}\,dx &= \left(x\left(a\,(bx^m)^n\right)^{-\frac{1}{mn}}\right)\int \frac{1}{x^3}\,dx \\ &= -\frac{\left(a\,(bx^m)^n\right)^{-\frac{1}{mn}}}{2x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$-\frac{\left(a\,(bx^m)^n\right)^{-\frac{1}{mn}}}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a*(b*x^m)^n)^(1/(m*n))),x]
```

```
[Out] -1/2*1/(x*(a*(b*x^m)^n)^(1/(m*n)))
```

fricas [A] time = 0.82, size = 21, normalized size = 0.84

$$-\frac{e^{\left(-\frac{n\log(b)+\log(a)}{mn}\right)}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((a*(b*x^m)^n)^(1/m/n)),x, algorithm="fricas")
[Out] -1/2*e^(-(n*log(b) + log(a))/(m*n))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((bx^m)^n a\right)^{\frac{1}{mn}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((a*(b*x^m)^n)^(1/m/n)),x, algorithm="giac")
[Out] integrate(1/(((b*x^m)^n*a)^(1/(m*n))*x^2), x)
```

maple [A] time = 0.00, size = 25, normalized size = 1.00

$$-\frac{\left(a\,(b\,x^m)^n\right)^{-\frac{1}{mn}}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/((a*(b*x^m)^n)^(1/m/n)),x)
[Out] -1/2/x/((a*(b*x^m)^n)^(1/m/n))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((bx^m)^n a\right)^{\frac{1}{mn}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((a*(b*x^m)^n)^(1/m/n)),x, algorithm="maxima")
[Out] integrate(1/(((b*x^m)^n*a)^(1/(m*n))*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^2\left(a\,(b\,x^m)^n\right)^{\frac{1}{m\,n}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*(b*x^m)^n)^(1/(m*n))),x)
[Out] int(1/(x^2*(a*(b*x^m)^n)^(1/(m*n))), x)
```

sympy [**A**] time = 11.06, size = 252, normalized size = 10.08

$$\begin{cases} -\dfrac{1}{0^{mn}\infty^{mn}x(0^{mn})^{\frac{1}{mn}}\left((x^m)^n\right)^{\frac{1}{mn}}\left(\left((0^{mn})^{\frac{1}{n}}\right)^n\right)^{\frac{1}{mn}}+x(0^{mn})^{\frac{1}{mn}}\left((x^m)^n\right)^{\frac{1}{mn}}\left(\left((0^{mn})^{\frac{1}{n}}\right)^n\right)^{\frac{1}{mn}}} & \text{for } a = 0^{mn} \wedge b = (0^{mn})^{\frac{1}{n}} \\ -\dfrac{a^{-\frac{1}{mn}}\left((x^m)^n\right)^{-\frac{1}{mn}}\left(\left((0^{mn})^{\frac{1}{n}}\right)^n\right)^{-\frac{1}{mn}}}{2x} & \text{for } b = (0^{mn})^{\frac{1}{n}} \\ -\dfrac{1}{0^{mn}\infty^{mn}x(0^{mn})^{\frac{1}{mn}}(b^n)^{\frac{1}{mn}}\left((x^m)^n\right)^{\frac{1}{mn}}+x(0^{mn})^{\frac{1}{mn}}(b^n)^{\frac{1}{mn}}\left((x^m)^n\right)^{\frac{1}{mn}}} & \text{for } a = 0^{mn} \\ -\dfrac{a^{-\frac{1}{mn}}(b^n)^{-\frac{1}{mn}}\left((x^m)^n\right)^{-\frac{1}{mn}}}{2x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/((a*(b*x**m)**n)**(1/m/n)),x)
```

```
[Out] Piecewise((-1/(0**(m*n)*zoo**(m*n)*x*(0**(m*n))**(1/(m*n))*((x**m)**n)**(1/
(m*n))*(((0**(m*n))**(1/n))**n)**(1/(m*n)) + x*(0**(m*n))**(1/(m*n))*((x**m
)**n)**(1/(m*n))*(((0**(m*n))**(1/n))**n)**(1/(m*n))), Eq(a, 0**(m*n)) & Eq
(b, (0**(m*n))**(1/n))), (-a**(-1/(m*n))*((x**m)**n)**(-1/(m*n))*(((0**(m*n
))**(1/n))**n)**(-1/(m*n))/(2*x), Eq(b, (0**(m*n))**(1/n))), (-1/(0**(m*n)*
zoo**(m*n)*x*(0**(m*n))**(1/(m*n))*(b**n)**(1/(m*n))*((x**m)**n)**(1/(m*n))
 + x*(0**(m*n))**(1/(m*n))*(b**n)**(1/(m*n))*((x**m)**n)**(1/(m*n))), Eq(a,
 0**(m*n))), (-a**(-1/(m*n))*(b**n)**(-1/(m*n))*((x**m)**n)**(-1/(m*n))/(2*
x), True))
```

3.201 $\int x^{2-npq}\left(a\left(bx^n\right)^p\right)^q\,dx$

Optimal. Leaf size=24

$$\frac{1}{3}x^{3-npq}\left(a\left(bx^n\right)^p\right)^q$$

```
[Out] 1/3*x^(-n*p*q+3)*(a*(b*x^n)^p)^q
```

Rubi [A] time = 0.04, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {6679, 30}

$$\frac{1}{3}x^{3-npq}\left(a\left(bx^n\right)^p\right)^q$$

Antiderivative was successfully verified.

```
[In] Int[x^(2 - n*p*q)*(a*(b*x^n)^p)^q,x]
```

```
[Out] (x^(3 - n*p*q)*(a*(b*x^n)^p)^q)/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6679

```
Int[(u_.)*((c_.)*((d_.)*((a_.) + (b_.)*(x_))^(n_))^(p_))^(q_), x_Symbol] :>
 Dist[(c*(d*(a + b*x)^n)^p)^q/(a + b*x)^(n*p*q), Int[u*(a + b*x)^(n*p*q), x
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] &&  !IntegerQ[p] &&  !IntegerQ[q]
```

Rubi steps

$$\begin{aligned}\int x^{2-npq}\left(a\left(bx^n\right)^p\right)^q\,dx &= \left(x^{-npq}\left(a\left(bx^n\right)^p\right)^q\right)\int x^2\,dx\\ &= \frac{1}{3}x^{3-npq}\left(a\left(bx^n\right)^p\right)^q\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$\frac{1}{3}x^{3-npq}\left(a\left(bx^n\right)^p\right)^q$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2 - n*p*q)*(a*(b*x^n)^p)^q,x]
```

```
[Out] (x^(3 - n*p*q)*(a*(b*x^n)^p)^q)/3
```

fricas [A] time = 0.68, size = 16, normalized size = 0.67

$$\frac{1}{3}x^3 e^{\left(pq\log(b)+q\log(a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-n*p*q+2)*(a*(b*x^n)^p)^q,x, algorithm="fricas")
```

```
[Out] 1/3*x^3*e^(p*q*log(b) + q*log(a))
```

giac [A] time = 0.28, size = 18, normalized size = 0.75

$$\frac{1}{3}\, x e^{\left(pq \log(b)+q \log(a)+2 \log(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-n*p*q+2)*(a*(b*x^n)^p)^q,x, algorithm="giac")
[Out] 1/3*x*e^(p*q*log(b) + q*log(a) + 2*log(x))
```

maple [A] time = 0.00, size = 23, normalized size = 0.96

$$\frac{x^{-npq+3} \left(a \left(b\, x^{n}\right)^{p}\right)^{q}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-n*p*q+2)*(a*(b*x^n)^p)^q,x)
[Out] 1/3*x^(-n*p*q+3)*(a*(b*x^n)^p)^q
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(b x^{n}\right)^{p} a\right)^{q} x^{-npq+2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-n*p*q+2)*(a*(b*x^n)^p)^q,x, algorithm="maxima")
[Out] integrate(((b*x^n)^p*a)^q*x^(-n*p*q + 2), x)
```

mupad [B] time = 1.02, size = 22, normalized size = 0.92

$$\frac{x^{3-n\,p\,q} \left(a \left(b\, x^{n}\right)^{p}\right)^{q}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2 - n*p*q)*(a*(b*x^n)^p)^q,x)
[Out] (x^(3 - n*p*q)*(a*(b*x^n)^p)^q)/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-npq+2} \left(a \left(b x^{n}\right)^{p}\right)^{q}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-n*p*q+2)*(a*(b*x**n)**p)**q,x)
[Out] Integral(x**(-n*p*q + 2)*(a*(b*x**n)**p)**q, x)
```

3.202 $\int x^{1-npq}\left(a\left(bx^n\right)^p\right)^q\,dx$

Optimal. Leaf size=24

$$\frac{1}{2}x^{2-npq}\left(a\left(bx^n\right)^p\right)^q$$

```
[Out] 1/2*x^(-n*p*q+2)*(a*(b*x^n)^p)^q
```

Rubi [A] time = 0.04, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {6679, 30}

$$\frac{1}{2}x^{2-npq}\left(a\left(bx^n\right)^p\right)^q$$

Antiderivative was successfully verified.

```
[In] Int[x^(1 - n*p*q)*(a*(b*x^n)^p)^q,x]
```

```
[Out] (x^(2 - n*p*q)*(a*(b*x^n)^p)^q)/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6679

```
Int[(u_.)*((c_.)*((d_.)*((a_.) + (b_.)*(x_))^(n_))^(p_))^(q_), x_Symbol] :>
 Dist[(c*(d*(a + b*x)^n)^p)^q/(a + b*x)^(n*p*q), Int[u*(a + b*x)^(n*p*q), x
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] &&  !IntegerQ[p] &&  !IntegerQ[q]
```

Rubi steps

$$\begin{aligned}\int x^{1-npq}\left(a\left(bx^n\right)^p\right)^q\,dx &= \left(x^{-npq}\left(a\left(bx^n\right)^p\right)^q\right)\int x\,dx\\ &= \frac{1}{2}x^{2-npq}\left(a\left(bx^n\right)^p\right)^q\end{aligned}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$\frac{1}{2}x^{2-npq}\left(a\left(bx^n\right)^p\right)^q$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1 - n*p*q)*(a*(b*x^n)^p)^q,x]
```

```
[Out] (x^(2 - n*p*q)*(a*(b*x^n)^p)^q)/2
```

fricas [A] time = 0.60, size = 16, normalized size = 0.67

$$\frac{1}{2}x^2e^{\left(pq\log(b)+q\log(a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-n*p*q+1)*(a*(b*x^n)^p)^q,x, algorithm="fricas")
```

```
[Out] 1/2*x^2*e^(p*q*log(b) + q*log(a))
```

giac [A] time = 0.28, size = 16, normalized size = 0.67

$$\frac{1}{2}\, xe^{\left(pq\log(b)+q\log(a)+\log(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-n*p*q+1)*(a*(b*x^n)^p)^q,x, algorithm="giac")
[Out] 1/2*x*e^(p*q*log(b) + q*log(a) + log(x))
```

maple [A] time = 0.00, size = 23, normalized size = 0.96

$$\frac{x^{-npq+2}\left(a\left(b\,x^{n}\right)^{p}\right)^{q}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-n*p*q+1)*(a*(b*x^n)^p)^q,x)
[Out] 1/2*x^(-n*p*q+2)*(a*(b*x^n)^p)^q
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(bx^{n}\right)^{p} a\right)^{q} x^{-npq+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-n*p*q+1)*(a*(b*x^n)^p)^q,x, algorithm="maxima")
[Out] integrate(((b*x^n)^p*a)^q*x^(-n*p*q + 1), x)
```

mupad [B] time = 1.00, size = 22, normalized size = 0.92

$$\frac{x^{2-n\,p\,q}\left(a\left(b\,x^{n}\right)^{p}\right)^{q}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1 - n*p*q)*(a*(b*x^n)^p)^q,x)
[Out] (x^(2 - n*p*q)*(a*(b*x^n)^p)^q)/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-npq+1}\left(a\left(bx^{n}\right)^{p}\right)^{q}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-n*p*q+1)*(a*(b*x**n)**p)**q,x)
[Out] Integral(x**(-n*p*q + 1)*(a*(b*x**n)**p)**q, x)
```

3.203 $\int x^{-npq}\left(a\left(bx^n\right)^p\right)^q\,dx$

Optimal. Leaf size=21

$$x^{1-npq}\left(a\left(bx^n\right)^p\right)^q$$

```
[Out] x^(-n*p*q+1)*(a*(b*x^n)^p)^q
```

Rubi [A] time = 0.04, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6679, 8}

$$x^{1-npq}\left(a\left(bx^n\right)^p\right)^q$$

Antiderivative was successfully verified.

```
[In] Int[(a*(b*x^n)^p)^q/x^(n*p*q),x]
```

```
[Out] x^(1 - n*p*q)*(a*(b*x^n)^p)^q
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 6679

```
Int[(u_.)*((c_.)*((d_.)*((a_.) + (b_.)*(x_))^(n_))^(p_))^(q_), x_Symbol] :>
 Dist[(c*(d*(a + b*x)^n)^p)^q/(a + b*x)^(n*p*q), Int[u*(a + b*x)^(n*p*q), x
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] &&  !IntegerQ[p] &&  !IntegerQ[q]
```

Rubi steps

$$\begin{aligned}\int x^{-npq}\left(a\left(bx^n\right)^p\right)^q\,dx &= \left(x^{-npq}\left(a\left(bx^n\right)^p\right)^q\right)\int 1\,dx\\ &= x^{1-npq}\left(a\left(bx^n\right)^p\right)^q\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$x^{1-npq}\left(a\left(bx^n\right)^p\right)^q$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*(b*x^n)^p)^q/x^(n*p*q),x]
```

```
[Out] x^(1 - n*p*q)*(a*(b*x^n)^p)^q
```

fricas [A] time = 0.74, size = 13, normalized size = 0.62

$$xe^{\left(pq\log(b)+q\log(a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*x^n)^p)^q/(x^(n*p*q)),x, algorithm="fricas")
```

```
[Out] x*e^(p*q*log(b) + q*log(a))
```

giac [A] time = 0.43, size = 27, normalized size = 1.29

$$\frac{xe^{\left(npq\log(x)+pq\log(b)+q\log(a)\right)}}{x^{npq}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*x^n)^p)^q/(x^(n*p*q)),x, algorithm="giac")
[Out] x*e^(n*p*q*log(x) + p*q*log(b) + q*log(a))/x^(n*p*q)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int x^{-npq} \left(a \left(b\, x^n\right)^p\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*(b*x^n)^p)^q/(x^(n*p*q)),x)
[Out] int((a*(b*x^n)^p)^q/(x^(n*p*q)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((b x^n)^p\, a\right)^q}{x^{npq}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*x^n)^p)^q/(x^(n*p*q)),x, algorithm="maxima")
[Out] integrate(((b*x^n)^p*a)^q/x^(n*p*q), x)
```

mupad [B] time = 1.00, size = 21, normalized size = 1.00

$$x^{1-n\,p\,q} \left(a \left(b\, x^n\right)^p\right)^q$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*(b*x^n)^p)^q/x^(n*p*q),x)
[Out] x^(1 - n*p*q)*(a*(b*x^n)^p)^q
```

sympy [A] time = 2.63, size = 22, normalized size = 1.05

$$a^q x x^{-npq} \left(b^p\right)^q \left(\left(x^n\right)^p\right)^q$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*x**n)**p)**q/(x**(n*p*q)),x)
[Out] a**q*x*x**(-n*p*q)*(b**p)**q*((x**n)**p)**q
```

3.204 $\int x^{-1-npq}\left(a\left(bx^n\right)^p\right)^q\,dx$

Optimal. Leaf size=21

$$\log(x)x^{-npq}\left(a\left(bx^n\right)^p\right)^q$$

```
[Out] (a*(b*x^n)^p)^q*ln(x)/(x^(n*p*q))
```

Rubi [A] time = 0.04, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {6679, 29}

$$\log(x)x^{-npq}\left(a\left(bx^n\right)^p\right)^q$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n*p*q)*(a*(b*x^n)^p)^q,x]
```

```
[Out] ((a*(b*x^n)^p)^q*Log[x])/x^(n*p*q)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 6679

```
Int[(u_.)*((c_.)*((d_.)*((a_.) + (b_.)*(x_))^(n_))^(p_))^(q_), x_Symbol] :>
 Dist[(c*(d*(a + b*x)^n)^p)^q/(a + b*x)^(n*p*q), Int[u*(a + b*x)^(n*p*q), x
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] &&  !IntegerQ[p] &&  !IntegerQ[q]
```

Rubi steps

$$\begin{aligned}\int x^{-1-npq}\left(a\left(bx^n\right)^p\right)^q\,dx &= \left(x^{-npq}\left(a\left(bx^n\right)^p\right)^q\right)\int\frac{1}{x}\,dx\\ &= x^{-npq}\left(a\left(bx^n\right)^p\right)^q\log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$\log(x)x^{-npq}\left(a\left(bx^n\right)^p\right)^q$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n*p*q)*(a*(b*x^n)^p)^q,x]
```

```
[Out] ((a*(b*x^n)^p)^q*Log[x])/x^(n*p*q)
```

fricas [A] time = 0.71, size = 14, normalized size = 0.67

$$e^{\left(pq\log(b)+q\log(a)\right)}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-n*p*q-1)*(a*(b*x^n)^p)^q,x, algorithm="fricas")
```

```
[Out] e^(p*q*log(b) + q*log(a))*log(x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\left(bx^n\right)^p a\right)^q x^{-npq-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-n*p*q-1)*(a*(b*x^n)^p)^q,x, algorithm="giac")
[Out] integrate(((b*x^n)^p*a)^q*x^(-n*p*q - 1), x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int x^{-npq-1} \left(a \left(b\, x^n\right)^p\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-n*p*q-1)*(a*(b*x^n)^p)^q,x)
[Out] int(x^(-n*p*q-1)*(a*(b*x^n)^p)^q,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(b x^n\right)^p a\right)^q x^{-npq-1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-n*p*q-1)*(a*(b*x^n)^p)^q,x, algorithm="maxima")
[Out] integrate(((b*x^n)^p*a)^q*x^(-n*p*q - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\left(a \left(b\, x^n\right)^p\right)^q}{x^{n\,p\,q+1}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*(b*x^n)^p)^q/x^(n*p*q + 1),x)
[Out] int((a*(b*x^n)^p)^q/x^(n*p*q + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-npq-1} \left(a \left(b x^n\right)^p\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-n*p*q-1)*(a*(b*x**n)**p)**q,x)
[Out] Integral(x**(-n*p*q - 1)*(a*(b*x**n)**p)**q, x)
```

3.205 $\int x^{-2-npq}\left(a\left(bx^n\right)^p\right)^q\,dx$

Optimal. Leaf size=22

$$-x^{-npq-1}\left(a\left(bx^n\right)^p\right)^q$$

```
[Out] -x^(-n*p*q-1)*(a*(b*x^n)^p)^q
```

Rubi [A] time = 0.04, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {6679, 30}

$$-x^{-npq-1}\left(a\left(bx^n\right)^p\right)^q$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2 - n*p*q)*(a*(b*x^n)^p)^q,x]
```

```
[Out] -(x^(-1 - n*p*q)*(a*(b*x^n)^p)^q)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6679

```
Int[(u_.)*((c_.)*((d_.)*((a_.) + (b_.)*(x_))^(n_))^(p_))^(q_), x_Symbol] :>
 Dist[(c*(d*(a + b*x)^n)^p)^q/(a + b*x)^(n*p*q), Int[u*(a + b*x)^(n*p*q), x
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] &&  !IntegerQ[p] &&  !IntegerQ[q]
```

Rubi steps

$$\begin{aligned}\int x^{-2-npq}\left(a\left(bx^n\right)^p\right)^q\,dx &= \left(x^{-npq}\left(a\left(bx^n\right)^p\right)^q\right)\int \frac{1}{x^2}\,dx\\ &= -x^{-1-npq}\left(a\left(bx^n\right)^p\right)^q\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$-x^{-npq-1}\left(a\left(bx^n\right)^p\right)^q$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2 - n*p*q)*(a*(b*x^n)^p)^q,x]
```

```
[Out] -(x^(-1 - n*p*q)*(a*(b*x^n)^p)^q)
```

fricas [A] time = 0.76, size = 16, normalized size = 0.73

$$-\frac{e^{\left(pq\log(b)+q\log(a)\right)}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-n*p*q-2)*(a*(b*x^n)^p)^q,x, algorithm="fricas")
```

```
[Out] -e^(p*q*log(b) + q*log(a))/x
```

giac [A] time = 0.28, size = 18, normalized size = 0.82

$$-xe^{\left(pq\log(b)+q\log(a)-2\log(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-n*p*q-2)*(a*(b*x^n)^p)^q,x, algorithm="giac")
[Out] -x*e^(p*q*log(b) + q*log(a) - 2*log(x))
```

maple [A] time = 0.00, size = 23, normalized size = 1.05

$$-x^{-npq-1}\left(a\left(b\,x^{n}\right)^{p}\right)^{q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-n*p*q-2)*(a*(b*x^n)^p)^q,x)
[Out] -x^(-n*p*q-1)*(a*(b*x^n)^p)^q
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(bx^{n}\right)^{p} a\right)^{q} x^{-npq-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-n*p*q-2)*(a*(b*x^n)^p)^q,x, algorithm="maxima")
[Out] integrate(((b*x^n)^p*a)^q*x^(-n*p*q - 2), x)
```

mupad [B] time = 1.01, size = 23, normalized size = 1.05

$$-\frac{\left(a\left(b\,x^{n}\right)^{p}\right)^{q}}{x^{n\,p\,q+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*(b*x^n)^p)^q/x^(n*p*q + 2),x)
[Out] -(a*(b*x^n)^p)^q/x^(n*p*q + 1)
```

sympy [A] time = 93.32, size = 24, normalized size = 1.09

$$-\frac{a^{q}x^{-npq}\left(b^{p}\right)^{q}\left(\left(x^{n}\right)^{p}\right)^{q}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-n*p*q-2)*(a*(b*x**n)**p)**q,x)
[Out] -a**q*x**(-n*p*q)*(b**p)**q*((x**n)**p)**q/x
```

3.206 $\int x^3 \left(a + bx^3\right) dx$

Optimal. Leaf size=17

$$\frac{ax^4}{4} + \frac{bx^7}{7}$$

```
[Out] 1/4*a*x^4+1/7*b*x^7
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$\frac{ax^4}{4} + \frac{bx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^3),x]
```

```
[Out] (a*x^4)/4 + (b*x^7)/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^3 \left(a + bx^3\right) dx &= \int \left(ax^3 + bx^6\right) dx \\ &= \frac{ax^4}{4} + \frac{bx^7}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^4}{4} + \frac{bx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^3),x]
```

```
[Out] (a*x^4)/4 + (b*x^7)/7
```

fricas [A] time = 0.47, size = 13, normalized size = 0.76

$$\frac{1}{7}x^7 b + \frac{1}{4}x^4 a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*b + 1/4*x^4*a
```

giac [A] time = 0.15, size = 13, normalized size = 0.76

$$\frac{1}{7}\, bx^7 + \frac{1}{4}\, ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a),x, algorithm="giac")
[Out] 1/7*b*x^7 + 1/4*a*x^4
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{7}b\,x^7+\frac{1}{4}a\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a),x)
[Out] 1/4*a*x^4+1/7*b*x^7
```

maxima [A] time = 1.31, size = 13, normalized size = 0.76

$$\frac{1}{7}\,bx^7+\frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a),x, algorithm="maxima")
[Out] 1/7*b*x^7 + 1/4*a*x^4
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{b\,x^7}{7}+\frac{a\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^3),x)
[Out] (a*x^4)/4 + (b*x^7)/7
```

sympy [A] time = 0.07, size = 12, normalized size = 0.71

$$\frac{ax^4}{4}+\frac{bx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a),x)
[Out] a*x**4/4 + b*x**7/7
```

3.207 $\int x^2 \left(a + bx^3\right) dx$

Optimal. Leaf size=17

$$\frac{ax^3}{3} + \frac{bx^6}{6}$$

```
[Out] 1/3*a*x^3+1/6*b*x^6
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$\frac{ax^3}{3} + \frac{bx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3),x]
```

```
[Out] (a*x^3)/3 + (b*x^6)/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(a + bx^3\right) dx &= \int \left(ax^2 + bx^5\right) dx \\ &= \frac{ax^3}{3} + \frac{bx^6}{6}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^3}{3} + \frac{bx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3),x]
```

```
[Out] (a*x^3)/3 + (b*x^6)/6
```

fricas [A] time = 0.50, size = 13, normalized size = 0.76

$$\frac{1}{6}x^6 b + \frac{1}{3}x^3 a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*x^6*b + 1/3*x^3*a
```

giac [A] time = 0.17, size = 13, normalized size = 0.76

$$\frac{1}{6}\, bx^6 + \frac{1}{3}\, ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a),x, algorithm="giac")
[Out] 1/6*b*x^6 + 1/3*a*x^3
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{6}b\,x^6+\frac{1}{3}a\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a),x)
[Out] 1/3*a*x^3+1/6*b*x^6
```

maxima [A] time = 1.36, size = 14, normalized size = 0.82

$$\frac{\left(bx^3+a\right)^2}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a),x, algorithm="maxima")
[Out] 1/6*(b*x^3 + a)^2/b
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{b\,x^6}{6}+\frac{a\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^3),x)
[Out] (a*x^3)/3 + (b*x^6)/6
```

sympy [A] time = 0.06, size = 12, normalized size = 0.71

$$\frac{ax^3}{3}+\frac{bx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a),x)
[Out] a*x**3/3 + b*x**6/6
```

3.208 $\int x\left(a+bx^3\right)\,dx$

Optimal. Leaf size=17

$$\frac{ax^2}{2}+\frac{bx^5}{5}$$

```
[Out] 1/2*a*x^2+1/5*b*x^5
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {14}

$$\frac{ax^2}{2}+\frac{bx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3),x]
```

```
[Out] (a*x^2)/2 + (b*x^5)/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^3\right)\,dx &= \int \left(ax+bx^4\right)\,dx\\ &= \frac{ax^2}{2}+\frac{bx^5}{5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^2}{2}+\frac{bx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3),x]
```

```
[Out] (a*x^2)/2 + (b*x^5)/5
```

fricas [A] time = 0.63, size = 13, normalized size = 0.76

$$\frac{1}{5}x^5b+\frac{1}{2}x^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/5*x^5*b + 1/2*x^2*a
```

giac [A] time = 0.15, size = 13, normalized size = 0.76

$$\frac{1}{5}\,bx^5+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a),x, algorithm="giac")
[Out] 1/5*b*x^5 + 1/2*a*x^2
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{5}b\,x^5+\frac{1}{2}a\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a),x)
[Out] 1/2*a*x^2+1/5*b*x^5
```

maxima [A] time = 1.30, size = 13, normalized size = 0.76

$$\frac{1}{5}\,bx^5+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a),x, algorithm="maxima")
[Out] 1/5*b*x^5 + 1/2*a*x^2
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{b\,x^5}{5}+\frac{a\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^3),x)
[Out] (a*x^2)/2 + (b*x^5)/5
```

sympy [A] time = 0.08, size = 12, normalized size = 0.71

$$\frac{ax^2}{2}+\frac{bx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a),x)
[Out] a*x**2/2 + b*x**5/5
```

3.209 $\int \left(a + bx^3\right) dx$

Optimal. Leaf size=12

$$ax + \frac{bx^4}{4}$$

```
[Out] a*x+1/4*b*x^4
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$ax + \frac{bx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*x^3,x]
```

```
[Out] a*x + (b*x^4)/4
```

Rubi steps

$$\int \left(a + bx^3\right) dx = ax + \frac{bx^4}{4}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$ax + \frac{bx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*x^3,x]
```

```
[Out] a*x + (b*x^4)/4
```

fricas [A] time = 0.50, size = 10, normalized size = 0.83

$$\frac{1}{4}x^4 b + xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^3+a,x, algorithm="fricas")
```

```
[Out] 1/4*x^4*b + x*a
```

giac [A] time = 0.15, size = 10, normalized size = 0.83

$$\frac{1}{4}\, bx^4 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^3+a,x, algorithm="giac")
```

```
[Out] 1/4*b*x^4 + a*x
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$\frac{1}{4} b\, x^4 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x^3+a,x)
[Out] a*x+1/4*b*x^4
```

maxima [A] time = 1.32, size = 10, normalized size = 0.83

$$\frac{1}{4}\,bx^4 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^3+a,x, algorithm="maxima")
[Out] 1/4*b*x^4 + a*x
```

mupad [B] time = 0.02, size = 10, normalized size = 0.83

$$\frac{b\,x^4}{4} + a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*x^3,x)
[Out] a*x + (b*x^4)/4
```

sympy [A] time = 0.09, size = 8, normalized size = 0.67

$$ax + \frac{bx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x**3+a,x)
[Out] a*x + b*x**4/4
```

3.210 $\int \frac{a+bx^3}{x}\,dx$

Optimal. Leaf size=13

$$a\log(x)+\frac{bx^3}{3}$$

```
[Out] 1/3*b*x^3+a*ln(x)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$a\log(x)+\frac{bx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)/x,x]
```

```
[Out] (b*x^3)/3 + a*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^3}{x}\,dx &= \int \left(\frac{a}{x}+bx^2\right)dx\\ &= \frac{bx^3}{3}+a\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$a\log(x)+\frac{bx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)/x,x]
```

```
[Out] (b*x^3)/3 + a*Log[x]
```

fricas [A] time = 0.51, size = 11, normalized size = 0.85

$$\frac{1}{3}bx^3+a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/x,x, algorithm="fricas")
```

```
[Out] 1/3*b*x^3 + a*log(x)
```

giac [A] time = 0.16, size = 12, normalized size = 0.92

$$\frac{1}{3}bx^3+a\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/x,x, algorithm="giac")
[Out] 1/3*b*x^3 + a*log(abs(x))
```

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$\frac{b\,x^3}{3} + a\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)/x,x)
[Out] 1/3*b*x^3+a*ln(x)
```

maxima [A] time = 1.25, size = 14, normalized size = 1.08

$$\frac{1}{3}\,bx^3 + \frac{1}{3}\,a\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/x,x, algorithm="maxima")
[Out] 1/3*b*x^3 + 1/3*a*log(x^3)
```

mupad [B] time = 0.02, size = 11, normalized size = 0.85

$$\frac{b\,x^3}{3} + a\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)/x,x)
[Out] (b*x^3)/3 + a*log(x)
```

sympy [A] time = 0.14, size = 10, normalized size = 0.77

$$a\log(x) + \frac{bx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)/x,x)
[Out] a*log(x) + b*x**3/3
```

3.211 $\int \frac{a+bx^3}{x^2}\,dx$

Optimal. Leaf size=15

$$\frac{bx^2}{2}-\frac{a}{x}$$

```
[Out] -a/x+1/2*b*x^2
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$\frac{bx^2}{2}-\frac{a}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)/x^2,x]
```

```
[Out] -(a/x) + (b*x^2)/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^3}{x^2}\,dx &= \int \left(\frac{a}{x^2}+bx\right)\,dx\\ &= -\frac{a}{x}+\frac{bx^2}{2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{bx^2}{2}-\frac{a}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)/x^2,x]
```

```
[Out] -(a/x) + (b*x^2)/2
```

fricas [A] time = 0.72, size = 14, normalized size = 0.93

$$\frac{bx^3-2\,a}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^3 - 2*a)/x
```

giac [A] time = 0.15, size = 13, normalized size = 0.87

$$\frac{1}{2}\,bx^2-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/x^2,x, algorithm="giac")
[Out] 1/2*b*x^2 - a/x
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{b\,x^2}{2} - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)/x^2,x)
[Out] -a/x+1/2*b*x^2
```

maxima [A] time = 1.28, size = 13, normalized size = 0.87

$$\frac{1}{2}\,bx^2 - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/x^2,x, algorithm="maxima")
[Out] 1/2*b*x^2 - a/x
```

mupad [B] time = 0.02, size = 13, normalized size = 0.87

$$\frac{b\,x^2}{2} - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)/x^2,x)
[Out] (b*x^2)/2 - a/x
```

sympy [A] time = 0.09, size = 8, normalized size = 0.53

$$-\frac{a}{x} + \frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)/x**2,x)
[Out] -a/x + b*x**2/2
```

3.212 $\int \frac{a+bx^3}{x^3}\,dx$

Optimal. Leaf size=12

$$bx - \frac{a}{2x^2}$$

```
[Out] -1/2*a/x^2+b*x
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$bx - \frac{a}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)/x^3,x]
```

```
[Out] -a/(2*x^2) + b*x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^3}{x^3}\,dx &= \int \left(b + \frac{a}{x^3}\right) dx \\ &= -\frac{a}{2x^2} + bx\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$bx - \frac{a}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)/x^3,x]
```

```
[Out] -1/2*a/x^2 + b*x
```

fricas [A] time = 0.58, size = 15, normalized size = 1.25

$$\frac{2\,bx^3 - a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x^3 - a)/x^2
```

giac [A] time = 0.16, size = 10, normalized size = 0.83

$$bx - \frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/x^3,x, algorithm="giac")
[Out] b*x - 1/2*a/x^2
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$bx - \frac{a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)/x^3,x)
[Out] -1/2*a/x^2+b*x
```

maxima [A] time = 1.33, size = 10, normalized size = 0.83

$$bx - \frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/x^3,x, algorithm="maxima")
[Out] b*x - 1/2*a/x^2
```

mupad [B] time = 0.02, size = 10, normalized size = 0.83

$$b\,x - \frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)/x^3,x)
[Out] b*x - a/(2*x^2)
```

sympy [A] time = 0.10, size = 8, normalized size = 0.67

$$-\frac{a}{2x^2} + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)/x**3,x)
[Out] -a/(2*x**2) + b*x
```

3.213 $\int \frac{a+bx^3}{x^4}\,dx$

Optimal. Leaf size=13

$$b\log(x)-\frac{a}{3x^3}$$

```
[Out] -1/3*a/x^3+b*ln(x)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$b\log(x)-\frac{a}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)/x^4,x]
```

```
[Out] -a/(3*x^3) + b*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^3}{x^4}\,dx &= \int \left(\frac{a}{x^4}+\frac{b}{x}\right)dx\\ &= -\frac{a}{3x^3}+b\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$b\log(x)-\frac{a}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)/x^4,x]
```

```
[Out] -1/3*a/x^3 + b*Log[x]
```

fricas [A] time = 0.71, size = 17, normalized size = 1.31

$$\frac{3\,bx^3\log(x)-a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*b*x^3*log(x) - a)/x^3
```

giac [A] time = 0.15, size = 18, normalized size = 1.38

$$b\log\left(|x|\right)-\frac{bx^3+a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/x^4,x, algorithm="giac")
[Out] b*log(abs(x)) - 1/3*(b*x^3 + a)/x^3
```

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$b \ln(x) - \frac{a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)/x^4,x)
[Out] -1/3*a/x^3+b*ln(x)
```

maxima [A] time = 1.31, size = 14, normalized size = 1.08

$$\frac{1}{3} b \log\left(x^3\right) - \frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/x^4,x, algorithm="maxima")
[Out] 1/3*b*log(x^3) - 1/3*a/x^3
```

mupad [B] time = 0.95, size = 11, normalized size = 0.85

$$b\,\ln(x) - \frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)/x^4,x)
[Out] b*log(x) - a/(3*x^3)
```

sympy [A] time = 0.14, size = 10, normalized size = 0.77

$$-\frac{a}{3x^3} + b \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)/x**4,x)
[Out] -a/(3*x**3) + b*log(x)
```

3.214 $\int \frac{a+bx^3}{x^5}\,dx$

Optimal. Leaf size=15

$$-\frac{a}{4x^4}-\frac{b}{x}$$

```
[Out] -1/4*a/x^4-b/x
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$-\frac{a}{4x^4}-\frac{b}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)/x^5,x]
```

```
[Out] -a/(4*x^4) - b/x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^3}{x^5}\,dx &= \int \left(\frac{a}{x^5}+\frac{b}{x^2}\right)dx\\ &= -\frac{a}{4x^4}-\frac{b}{x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$-\frac{a}{4x^4}-\frac{b}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)/x^5,x]
```

```
[Out] -1/4*a/x^4 - b/x
```

fricas [A] time = 0.72, size = 13, normalized size = 0.87

$$-\frac{4\,bx^3+a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/x^5,x, algorithm="fricas")
```

```
[Out] -1/4*(4*b*x^3 + a)/x^4
```

giac [A] time = 0.18, size = 13, normalized size = 0.87

$$-\frac{4\,bx^3+a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/x^5,x, algorithm="giac")
[Out] -1/4*(4*b*x^3 + a)/x^4
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$-\frac{b}{x}-\frac{a}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)/x^5,x)
[Out] -1/4*a/x^4-b/x
```

maxima [A] time = 1.24, size = 13, normalized size = 0.87

$$-\frac{4\,bx^3+a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/x^5,x, algorithm="maxima")
[Out] -1/4*(4*b*x^3 + a)/x^4
```

mupad [B] time = 0.02, size = 13, normalized size = 0.87

$$-\frac{4\,b\,x^3+a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)/x^5,x)
[Out] -(a + 4*b*x^3)/(4*x^4)
```

sympy [A] time = 0.16, size = 14, normalized size = 0.93

$$\frac{-a-4bx^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)/x**5,x)
[Out] (-a - 4*b*x**3)/(4*x**4)
```

3.215 $\int \frac{a+bx^3}{x^6}\,dx$

Optimal. Leaf size=17

$$-\frac{a}{5x^5}-\frac{b}{2x^2}$$

```
[Out] -1/5*a/x^5-1/2*b/x^2
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$-\frac{a}{5x^5}-\frac{b}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)/x^6,x]
```

```
[Out] -a/(5*x^5) - b/(2*x^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^3}{x^6}\,dx &= \int \left(\frac{a}{x^6}+\frac{b}{x^3}\right)dx\\ &= -\frac{a}{5x^5}-\frac{b}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$-\frac{a}{5x^5}-\frac{b}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)/x^6,x]
```

```
[Out] -1/5*a/x^5 - b/(2*x^2)
```

fricas [A] time = 0.59, size = 15, normalized size = 0.88

$$-\frac{5\,bx^3+2\,a}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/x^6,x, algorithm="fricas")
```

```
[Out] -1/10*(5*b*x^3 + 2*a)/x^5
```

giac [A] time = 0.15, size = 15, normalized size = 0.88

$$-\frac{5\,bx^3+2\,a}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/x^6,x, algorithm="giac")
[Out] -1/10*(5*b*x^3 + 2*a)/x^5
```

maple [A] time = 0.01, size = 14, normalized size = 0.82

$$-\frac{b}{2x^2}-\frac{a}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)/x^6,x)
[Out] -1/5*a/x^5-1/2*b/x^2
```

maxima [A] time = 1.29, size = 15, normalized size = 0.88

$$-\frac{5\,bx^3+2\,a}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/x^6,x, algorithm="maxima")
[Out] -1/10*(5*b*x^3 + 2*a)/x^5
```

mupad [B] time = 0.03, size = 15, normalized size = 0.88

$$-\frac{5\,b\,x^3+2\,a}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)/x^6,x)
[Out] -(2*a + 5*b*x^3)/(10*x^5)
```

sympy [A] time = 0.17, size = 15, normalized size = 0.88

$$\frac{-2a-5bx^3}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)/x**6,x)
[Out] (-2*a - 5*b*x**3)/(10*x**5)
```

3.216 $\int \frac{a+bx^3}{x^7}\,dx$

Optimal. Leaf size=17

$$-\frac{a}{6x^6}-\frac{b}{3x^3}$$

```
[Out] -1/6*a/x^6-1/3*b/x^3
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$-\frac{a}{6x^6}-\frac{b}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)/x^7,x]
```

```
[Out] -a/(6*x^6) - b/(3*x^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^3}{x^7}\,dx &= \int \left(\frac{a}{x^7}+\frac{b}{x^4}\right)dx\\ &= -\frac{a}{6x^6}-\frac{b}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$-\frac{a}{6x^6}-\frac{b}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)/x^7,x]
```

```
[Out] -1/6*a/x^6 - b/(3*x^3)
```

fricas [A] time = 0.83, size = 13, normalized size = 0.76

$$-\frac{2\,bx^3+a}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/x^7,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b*x^3 + a)/x^6
```

giac [A] time = 0.15, size = 13, normalized size = 0.76

$$-\frac{2\,bx^3+a}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/x^7,x, algorithm="giac")
[Out] -1/6*(2*b*x^3 + a)/x^6
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$-\frac{b}{3x^3}-\frac{a}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)/x^7,x)
[Out] -1/6*a/x^6-1/3*b/x^3
```

maxima [A] time = 1.32, size = 13, normalized size = 0.76

$$-\frac{2\,bx^3+a}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/x^7,x, algorithm="maxima")
[Out] -1/6*(2*b*x^3 + a)/x^6
```

mupad [B] time = 0.03, size = 13, normalized size = 0.76

$$-\frac{2\,b\,x^3+a}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)/x^7,x)
[Out] -(a + 2*b*x^3)/(6*x^6)
```

sympy [A] time = 0.16, size = 14, normalized size = 0.82

$$\frac{-a-2bx^3}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)/x**7,x)
[Out] (-a - 2*b*x**3)/(6*x**6)
```

3.217 $\int \frac{a+bx^3}{x^8}\,dx$

Optimal. Leaf size=17

$$-\frac{a}{7x^7}-\frac{b}{4x^4}$$

```
[Out] -1/7*a/x^7-1/4*b/x^4
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$-\frac{a}{7x^7}-\frac{b}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)/x^8,x]
```

```
[Out] -a/(7*x^7) - b/(4*x^4)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^3}{x^8}\,dx &= \int \left(\frac{a}{x^8}+\frac{b}{x^5}\right)dx\\ &= -\frac{a}{7x^7}-\frac{b}{4x^4}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$-\frac{a}{7x^7}-\frac{b}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)/x^8,x]
```

```
[Out] -1/7*a/x^7 - b/(4*x^4)
```

fricas [A] time = 0.94, size = 15, normalized size = 0.88

$$-\frac{7\,bx^3+4\,a}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/x^8,x, algorithm="fricas")
```

```
[Out] -1/28*(7*b*x^3 + 4*a)/x^7
```

giac [A] time = 0.18, size = 15, normalized size = 0.88

$$-\frac{7\,bx^3+4\,a}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/x^8,x, algorithm="giac")
[Out] -1/28*(7*b*x^3 + 4*a)/x^7
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$-\frac{b}{4x^4}-\frac{a}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)/x^8,x)
[Out] -1/7*a/x^7-1/4*b/x^4
```

maxima [A] time = 1.28, size = 15, normalized size = 0.88

$$-\frac{7\,bx^3+4\,a}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/x^8,x, algorithm="maxima")
[Out] -1/28*(7*b*x^3 + 4*a)/x^7
```

mupad [B] time = 0.03, size = 15, normalized size = 0.88

$$-\frac{7\,b\,x^3+4\,a}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)/x^8,x)
[Out] -(4*a + 7*b*x^3)/(28*x^7)
```

sympy [A] time = 0.16, size = 15, normalized size = 0.88

$$\frac{-4a-7bx^3}{28x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)/x**8,x)
[Out] (-4*a - 7*b*x**3)/(28*x**7)
```

3.218 $\int x^4 \left(a + bx^3\right)^2 \, dx$

Optimal. Leaf size=30

$$\frac{a^2 x^5}{5} + \frac{1}{4} abx^8 + \frac{b^2 x^{11}}{11}$$

```
[Out] 1/5*a^2*x^5+1/4*a*b*x^8+1/11*b^2*x^11
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{a^2 x^5}{5} + \frac{1}{4} abx^8 + \frac{b^2 x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^3)^2,x]
```

```
[Out] (a^2*x^5)/5 + (a*b*x^8)/4 + (b^2*x^11)/11
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^4 \left(a + bx^3\right)^2 \, dx &= \int \left(a^2 x^4 + 2abx^7 + b^2 x^{10}\right) \, dx \\ &= \frac{a^2 x^5}{5} + \frac{1}{4} abx^8 + \frac{b^2 x^{11}}{11}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2 x^5}{5} + \frac{1}{4} abx^8 + \frac{b^2 x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^3)^2,x]
```

```
[Out] (a^2*x^5)/5 + (a*b*x^8)/4 + (b^2*x^11)/11
```

fricas [A] time = 0.46, size = 24, normalized size = 0.80

$$\frac{1}{11} x^{11} b^2 + \frac{1}{4} x^8 ba + \frac{1}{5} x^5 a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*b^2 + 1/4*x^8*b*a + 1/5*x^5*a^2
```

giac [A] time = 0.15, size = 24, normalized size = 0.80

$$\frac{1}{11} b^2 x^{11} + \frac{1}{4} abx^8 + \frac{1}{5} a^2 x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^2,x, algorithm="giac")
[Out] 1/11*b^2*x^11 + 1/4*a*b*x^8 + 1/5*a^2*x^5
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$\frac{1}{11}b^2x^{11} + \frac{1}{4}ab\,x^8 + \frac{1}{5}a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)^2,x)
[Out] 1/5*a^2*x^5+1/4*a*b*x^8+1/11*b^2*x^11
```

maxima [A] time = 1.37, size = 24, normalized size = 0.80

$$\frac{1}{11}\,b^2x^{11} + \frac{1}{4}\,abx^8 + \frac{1}{5}\,a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^2,x, algorithm="maxima")
[Out] 1/11*b^2*x^11 + 1/4*a*b*x^8 + 1/5*a^2*x^5
```

mupad [B] time = 0.04, size = 24, normalized size = 0.80

$$\frac{a^2\,x^5}{5} + \frac{a\,b\,x^8}{4} + \frac{b^2\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^3)^2,x)
[Out] (a^2*x^5)/5 + (b^2*x^11)/11 + (a*b*x^8)/4
```

sympy [A] time = 0.07, size = 24, normalized size = 0.80

$$\frac{a^2x^5}{5} + \frac{abx^8}{4} + \frac{b^2x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)**2,x)
[Out] a**2*x**5/5 + a*b*x**8/4 + b**2*x**11/11
```

3.219 $\int x^3 \left(a + bx^3\right)^2 \, dx$

Optimal. Leaf size=30

$$\frac{a^2x^4}{4} + \frac{2}{7}abx^7 + \frac{b^2x^{10}}{10}$$

```
[Out] 1/4*a^2*x^4+2/7*a*b*x^7+1/10*b^2*x^10
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{a^2x^4}{4} + \frac{2}{7}abx^7 + \frac{b^2x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^3)^2,x]
```

```
[Out] (a^2*x^4)/4 + (2*a*b*x^7)/7 + (b^2*x^10)/10
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^3 \left(a + bx^3\right)^2 \, dx &= \int \left(a^2x^3 + 2abx^6 + b^2x^9\right) \, dx \\ &= \frac{a^2x^4}{4} + \frac{2}{7}abx^7 + \frac{b^2x^{10}}{10}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^4}{4} + \frac{2}{7}abx^7 + \frac{b^2x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^3)^2,x]
```

```
[Out] (a^2*x^4)/4 + (2*a*b*x^7)/7 + (b^2*x^10)/10
```

fricas [A] time = 0.39, size = 24, normalized size = 0.80

$$\frac{1}{10}x^{10}b^2 + \frac{2}{7}x^7ba + \frac{1}{4}x^4a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*b^2 + 2/7*x^7*b*a + 1/4*x^4*a^2
```

giac [A] time = 0.18, size = 24, normalized size = 0.80

$$\frac{1}{10}\,b^2x^{10} + \frac{2}{7}\,abx^7 + \frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^2,x, algorithm="giac")
[Out] 1/10*b^2*x^10 + 2/7*a*b*x^7 + 1/4*a^2*x^4
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$\frac{1}{10}b^2x^{10} + \frac{2}{7}ab\,x^7 + \frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)^2,x)
[Out] 1/4*x^4*a^2+2/7*a*b*x^7+1/10*b^2*x^10
```

maxima [A] time = 1.34, size = 24, normalized size = 0.80

$$\frac{1}{10}\,b^2x^{10} + \frac{2}{7}\,abx^7 + \frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^2,x, algorithm="maxima")
[Out] 1/10*b^2*x^10 + 2/7*a*b*x^7 + 1/4*a^2*x^4
```

mupad [B] time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^4}{4} + \frac{2\,a\,b\,x^7}{7} + \frac{b^2\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^3)^2,x)
[Out] (a^2*x^4)/4 + (b^2*x^10)/10 + (2*a*b*x^7)/7
```

sympy [A] time = 0.08, size = 26, normalized size = 0.87

$$\frac{a^2x^4}{4} + \frac{2abx^7}{7} + \frac{b^2x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**2,x)
[Out] a**2*x**4/4 + 2*a*b*x**7/7 + b**2*x**10/10
```

3.220 $\int x^2 \left(a + bx^3\right)^2 \, dx$

Optimal. Leaf size=16

$$\frac{\left(a + bx^3\right)^3}{9b}$$

```
[Out] 1/9*(b*x^3+a)^3/b
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$\frac{\left(a + bx^3\right)^3}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3)^2,x]
```

```
[Out] (a + b*x^3)^3/(9*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^2 \left(a + bx^3\right)^2 \, dx = \frac{\left(a + bx^3\right)^3}{9b}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.88

$$\frac{a^2 x^3}{3} + \frac{1}{3} abx^6 + \frac{b^2 x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)^2,x]
```

```
[Out] (a^2*x^3)/3 + (a*b*x^6)/3 + (b^2*x^9)/9
```

fricas [A] time = 0.66, size = 24, normalized size = 1.50

$$\frac{1}{9} x^9 b^2 + \frac{1}{3} x^6 ba + \frac{1}{3} x^3 a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*b^2 + 1/3*x^6*b*a + 1/3*x^3*a^2
```

giac [A] time = 0.16, size = 14, normalized size = 0.88

$$\frac{\left(bx^3 + a\right)^3}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^2,x, algorithm="giac")
[Out] 1/9*(b*x^3 + a)^3/b
```

maple [A] time = 0.00, size = 25, normalized size = 1.56

$$\frac{1}{9}b^2x^9 + \frac{1}{3}ab\,x^6 + \frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^2,x)
[Out] 1/9*b^2*x^9+1/3*a*b*x^6+1/3*a^2*x^3
```

maxima [A] time = 1.26, size = 14, normalized size = 0.88

$$\frac{\left(bx^3 + a\right)^3}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^2,x, algorithm="maxima")
[Out] 1/9*(b*x^3 + a)^3/b
```

mupad [B] time = 0.03, size = 24, normalized size = 1.50

$$\frac{a^2\,x^3}{3} + \frac{a\,b\,x^6}{3} + \frac{b^2\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^3)^2,x)
[Out] (a^2*x^3)/3 + (b^2*x^9)/9 + (a*b*x^6)/3
```

sympy [B] time = 0.08, size = 24, normalized size = 1.50

$$\frac{a^2x^3}{3} + \frac{abx^6}{3} + \frac{b^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**2,x)
[Out] a**2*x**3/3 + a*b*x**6/3 + b**2*x**9/9
```

3.221 $\int x\left(a+bx^3\right)^2\,dx$

Optimal. Leaf size=30

$$\frac{a^2x^2}{2}+\frac{2}{5}abx^5+\frac{b^2x^8}{8}$$

```
[Out] 1/2*a^2*x^2+2/5*a*b*x^5+1/8*b^2*x^8
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {270}

$$\frac{a^2x^2}{2}+\frac{2}{5}abx^5+\frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3)^2,x]
```

```
[Out] (a^2*x^2)/2 + (2*a*b*x^5)/5 + (b^2*x^8)/8
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^3\right)^2\,dx &= \int\left(a^2x+2abx^4+b^2x^7\right)\,dx\\ &= \frac{a^2x^2}{2}+\frac{2}{5}abx^5+\frac{b^2x^8}{8}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^2}{2}+\frac{2}{5}abx^5+\frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)^2,x]
```

```
[Out] (a^2*x^2)/2 + (2*a*b*x^5)/5 + (b^2*x^8)/8
```

fricas [A] time = 0.72, size = 24, normalized size = 0.80

$$\frac{1}{8}x^8b^2+\frac{2}{5}x^5ba+\frac{1}{2}x^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*b^2 + 2/5*x^5*b*a + 1/2*x^2*a^2
```

giac [A] time = 0.17, size = 24, normalized size = 0.80

$$\frac{1}{8}\,b^2x^8+\frac{2}{5}\,abx^5+\frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^2,x, algorithm="giac")
[Out] 1/8*b^2*x^8 + 2/5*a*b*x^5 + 1/2*a^2*x^2
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$\frac{1}{8}b^2x^8 + \frac{2}{5}ab\,x^5 + \frac{1}{2}a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^2,x)
[Out] 1/2*a^2*x^2+2/5*a*b*x^5+1/8*b^2*x^8
```

maxima [A] time = 1.27, size = 24, normalized size = 0.80

$$\frac{1}{8}\,b^2x^8 + \frac{2}{5}\,abx^5 + \frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^2,x, algorithm="maxima")
[Out] 1/8*b^2*x^8 + 2/5*a*b*x^5 + 1/2*a^2*x^2
```

mupad [B] time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^2}{2} + \frac{2\,a\,b\,x^5}{5} + \frac{b^2\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^3)^2,x)
[Out] (a^2*x^2)/2 + (b^2*x^8)/8 + (2*a*b*x^5)/5
```

sympy [A] time = 0.09, size = 26, normalized size = 0.87

$$\frac{a^2x^2}{2} + \frac{2abx^5}{5} + \frac{b^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**2,x)
[Out] a**2*x**2/2 + 2*a*b*x**5/5 + b**2*x**8/8
```

3.222 $\int \left(a + bx^3\right)^2 \, dx$

Optimal. Leaf size=25

$$a^2 x + \frac{1}{2} abx^4 + \frac{b^2 x^7}{7}$$

```
[Out] a^2*x+1/2*a*b*x^4+1/7*b^2*x^7
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {194}

$$a^2 x + \frac{1}{2} abx^4 + \frac{b^2 x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2,x]
```

```
[Out] a^2*x + (a*b*x^4)/2 + (b^2*x^7)/7
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a + bx^3\right)^2 \, dx &= \int \left(a^2 + 2abx^3 + b^2 x^6\right) \, dx \\ &= a^2 x + \frac{1}{2} abx^4 + \frac{b^2 x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$a^2 x + \frac{1}{2} abx^4 + \frac{b^2 x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2,x]
```

```
[Out] a^2*x + (a*b*x^4)/2 + (b^2*x^7)/7
```

fricas [A] time = 0.39, size = 21, normalized size = 0.84

$$\frac{1}{7} x^7 b^2 + \frac{1}{2} x^4 ba + xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*b^2 + 1/2*x^4*b*a + x*a^2
```

giac [A] time = 0.15, size = 21, normalized size = 0.84

$$\frac{1}{7} b^2 x^7 + \frac{1}{2} abx^4 + a^2 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2,x, algorithm="giac")
[Out] 1/7*b^2*x^7 + 1/2*a*b*x^4 + a^2*x
```

maple [A] time = 0.00, size = 22, normalized size = 0.88

$$\frac{1}{7}b^2x^7 + \frac{1}{2}ab\,x^4 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2,x)
[Out] a^2*x+1/2*a*b*x^4+1/7*b^2*x^7
```

maxima [A] time = 1.35, size = 21, normalized size = 0.84

$$\frac{1}{7}\,b^2x^7 + \frac{1}{2}\,abx^4 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2,x, algorithm="maxima")
[Out] 1/7*b^2*x^7 + 1/2*a*b*x^4 + a^2*x
```

mupad [B] time = 0.03, size = 21, normalized size = 0.84

$$a^2\,x + \frac{a\,b\,x^4}{2} + \frac{b^2\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^2,x)
[Out] a^2*x + (b^2*x^7)/7 + (a*b*x^4)/2
```

sympy [A] time = 0.08, size = 20, normalized size = 0.80

$$a^2x + \frac{abx^4}{2} + \frac{b^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2,x)
[Out] a**2*x + a*b*x**4/2 + b**2*x**7/7
```

3.223 $\int \frac{\left(a+bx^3\right)^2}{x}\,dx$

Optimal. Leaf size=26

$$a^2\log(x)+\frac{2}{3}abx^3+\frac{b^2x^6}{6}$$

```
[Out] 2/3*a*b*x^3+1/6*b^2*x^6+a^2*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$a^2\log(x)+\frac{2}{3}abx^3+\frac{b^2x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2/x,x]
```

```
[Out] (2*a*b*x^3)/3 + (b^2*x^6)/6 + a^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^2}{x}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^2}{x}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(2ab+\frac{a^2}{x}+b^2x\right)\,dx, x, x^3\right)\\ &= \frac{2}{3}abx^3+\frac{b^2x^6}{6}+a^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 26, normalized size = 1.00

$$a^2\log(x)+\frac{2}{3}abx^3+\frac{b^2x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2/x,x]
```

```
[Out] (2*a*b*x^3)/3 + (b^2*x^6)/6 + a^2*Log[x]
```

fricas [A] time = 0.81, size = 22, normalized size = 0.85

$$\frac{1}{6}\,b^2x^6+\frac{2}{3}\,abx^3+a^2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/x,x, algorithm="fricas")
[Out] 1/6*b^2*x^6 + 2/3*a*b*x^3 + a^2*log(x)
```

giac [A] time = 0.15, size = 23, normalized size = 0.88

$$\frac{1}{6}b^2x^6 + \frac{2}{3}abx^3 + a^2\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/x,x, algorithm="giac")
[Out] 1/6*b^2*x^6 + 2/3*a*b*x^3 + a^2*log(abs(x))
```

maple [A] time = 0.00, size = 23, normalized size = 0.88

$$\frac{b^2x^6}{6} + \frac{2ab\,x^3}{3} + a^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2/x,x)
[Out] 2/3*a*b*x^3+1/6*b^2*x^6+a^2*ln(x)
```

maxima [A] time = 1.27, size = 25, normalized size = 0.96

$$\frac{1}{6}b^2x^6 + \frac{2}{3}abx^3 + \frac{1}{3}a^2\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/x,x, algorithm="maxima")
[Out] 1/6*b^2*x^6 + 2/3*a*b*x^3 + 1/3*a^2*log(x^3)
```

mupad [B] time = 0.03, size = 22, normalized size = 0.85

$$a^2\,\ln(x) + \frac{b^2\,x^6}{6} + \frac{2\,a\,b\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^2/x,x)
[Out] a^2*log(x) + (b^2*x^6)/6 + (2*a*b*x^3)/3
```

sympy [A] time = 0.12, size = 24, normalized size = 0.92

$$a^2\log(x) + \frac{2abx^3}{3} + \frac{b^2x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2/x,x)
[Out] a**2*log(x) + 2*a*b*x**3/3 + b**2*x**6/6
```

3.224 $\int \frac{\left(a+bx^3\right)^2}{x^2}\,dx$

Optimal. Leaf size=25

$$-\frac{a^2}{x}+abx^2+\frac{b^2x^5}{5}$$

```
[Out] -a^2/x+a*b*x^2+1/5*b^2*x^5
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$-\frac{a^2}{x}+abx^2+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2/x^2,x]
```

```
[Out] -(a^2/x) + a*b*x^2 + (b^2*x^5)/5
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^2}{x^2}\,dx &= \int \left(\frac{a^2}{x^2}+2abx+b^2x^4\right)dx\\ &= -\frac{a^2}{x}+abx^2+\frac{b^2x^5}{5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$-\frac{a^2}{x}+abx^2+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2/x^2,x]
```

```
[Out] -(a^2/x) + a*b*x^2 + (b^2*x^5)/5
```

fricas [A] time = 0.84, size = 25, normalized size = 1.00

$$\frac{b^2x^6+5\,abx^3-5\,a^2}{5\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/5*(b^2*x^6 + 5*a*b*x^3 - 5*a^2)/x
```

giac [A] time = 0.15, size = 23, normalized size = 0.92

$$\frac{1}{5}\,b^2x^5+abx^2-\frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/x^2,x, algorithm="giac")
[Out] 1/5*b^2*x^5 + a*b*x^2 - a^2/x
```

maple [A] time = 0.00, size = 24, normalized size = 0.96

$$\frac{b^2x^5}{5} + ab\,x^2 - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2/x^2,x)
[Out] -a^2/x+a*b*x^2+1/5*b^2*x^5
```

maxima [A] time = 1.28, size = 23, normalized size = 0.92

$$\frac{1}{5}\,b^2x^5 + abx^2 - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/x^2,x, algorithm="maxima")
[Out] 1/5*b^2*x^5 + a*b*x^2 - a^2/x
```

mupad [B] time = 0.03, size = 24, normalized size = 0.96

$$\frac{-a^2 + a\,b\,x^3 + \frac{b^2\,x^6}{5}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^2/x^2,x)
[Out] ((b^2*x^6)/5 - a^2 + a*b*x^3)/x
```

sympy [A] time = 0.12, size = 19, normalized size = 0.76

$$-\frac{a^2}{x} + abx^2 + \frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2/x**2,x)
[Out] -a**2/x + a*b*x**2 + b**2*x**5/5
```

3.225 $\int \frac{\left(a+bx^3\right)^2}{x^3}\,dx$

Optimal. Leaf size=26

$$-\frac{a^2}{2x^2}+2abx+\frac{b^2x^4}{4}$$

```
[Out] -1/2*a^2/x^2+2*a*b*x+1/4*b^2*x^4
```

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$-\frac{a^2}{2x^2}+2abx+\frac{b^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2/x^3,x]
```

```
[Out] -a^2/(2*x^2) + 2*a*b*x + (b^2*x^4)/4
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^2}{x^3}\,dx &= \int \left(2ab+\frac{a^2}{x^3}+b^2x^3\right)dx\\ &= -\frac{a^2}{2x^2}+2abx+\frac{b^2x^4}{4}\end{aligned}$$

Mathematica [A] time = 0.00, size = 26, normalized size = 1.00

$$-\frac{a^2}{2x^2}+2abx+\frac{b^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2/x^3,x]
```

```
[Out] -1/2*a^2/x^2 + 2*a*b*x + (b^2*x^4)/4
```

fricas [A] time = 0.83, size = 25, normalized size = 0.96

$$\frac{b^2x^6+8\,abx^3-2\,a^2}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*x^6 + 8*a*b*x^3 - 2*a^2)/x^2
```

giac [A] time = 0.16, size = 22, normalized size = 0.85

$$\frac{1}{4}\,b^2x^4+2\,abx-\frac{a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/x^3,x, algorithm="giac")
[Out] 1/4*b^2*x^4 + 2*a*b*x - 1/2*a^2/x^2
```

maple [A] time = 0.00, size = 23, normalized size = 0.88

$$\frac{b^2x^4}{4} + 2abx - \frac{a^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2/x^3,x)
[Out] -1/2*a^2/x^2+2*a*b*x+1/4*b^2*x^4
```

maxima [A] time = 1.27, size = 22, normalized size = 0.85

$$\frac{1}{4}\,b^2x^4 + 2\,abx - \frac{a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/x^3,x, algorithm="maxima")
[Out] 1/4*b^2*x^4 + 2*a*b*x - 1/2*a^2/x^2
```

mupad [B] time = 0.03, size = 22, normalized size = 0.85

$$\frac{b^2\,x^4}{4} - \frac{a^2}{2\,x^2} + 2\,a\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^2/x^3,x)
[Out] (b^2*x^4)/4 - a^2/(2*x^2) + 2*a*b*x
```

sympy [A] time = 0.11, size = 22, normalized size = 0.85

$$-\frac{a^2}{2x^2} + 2abx + \frac{b^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2/x**3,x)
[Out] -a**2/(2*x**2) + 2*a*b*x + b**2*x**4/4
```

3.226 $\int \frac{\left(a+bx^3\right)^2}{x^4}\,dx$

Optimal. Leaf size=27

$$-\frac{a^2}{3x^3}+2ab\log(x)+\frac{b^2x^3}{3}$$

```
[Out] -1/3*a^2/x^3+1/3*b^2*x^3+2*a*b*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$-\frac{a^2}{3x^3}+2ab\log(x)+\frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2/x^4,x]
```

```
[Out] -a^2/(3*x^3) + (b^2*x^3)/3 + 2*a*b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^2}{x^4}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)^2}{x^2}\,dx, x, x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(b^2+\frac{a^2}{x^2}+\frac{2ab}{x}\right)\,dx, x, x^3\right)\\ &= -\frac{a^2}{3x^3}+\frac{b^2x^3}{3}+2ab\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.00

$$-\frac{a^2}{3x^3}+2ab\log(x)+\frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2/x^4,x]
```

```
[Out] -1/3*a^2/x^3 + (b^2*x^3)/3 + 2*a*b*Log[x]
```

fricas [A] time = 0.84, size = 27, normalized size = 1.00

$$\frac{b^2x^6+6\,abx^3\log(x)-a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/x^4,x, algorithm="fricas")
[Out] 1/3*(b^2*x^6 + 6*a*b*x^3*log(x) - a^2)/x^3
```

giac [A] time = 0.19, size = 32, normalized size = 1.19

$$\frac{1}{3}\,b^2x^3 + 2\,ab\log\left(|x|\right) - \frac{2\,abx^3 + a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/x^4,x, algorithm="giac")
[Out] 1/3*b^2*x^3 + 2*a*b*log(abs(x)) - 1/3*(2*a*b*x^3 + a^2)/x^3
```

maple [A] time = 0.01, size = 24, normalized size = 0.89

$$\frac{b^2x^3}{3} + 2ab\ln(x) - \frac{a^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2/x^4,x)
[Out] -1/3*a^2/x^3+1/3*b^2*x^3+2*a*b*ln(x)
```

maxima [A] time = 1.32, size = 25, normalized size = 0.93

$$\frac{1}{3}\,b^2x^3 + \frac{2}{3}\,ab\log\left(x^3\right) - \frac{a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/x^4,x, algorithm="maxima")
[Out] 1/3*b^2*x^3 + 2/3*a*b*log(x^3) - 1/3*a^2/x^3
```

mupad [B] time = 0.03, size = 23, normalized size = 0.85

$$\frac{b^2\,x^3}{3} - \frac{a^2}{3\,x^3} + 2\,a\,b\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^2/x^4,x)
[Out] (b^2*x^3)/3 - a^2/(3*x^3) + 2*a*b*log(x)
```

sympy [A] time = 0.21, size = 24, normalized size = 0.89

$$-\frac{a^2}{3x^3} + 2ab\log\left(x\right) + \frac{b^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2/x**4,x)
[Out] -a**2/(3*x**3) + 2*a*b*log(x) + b**2*x**3/3
```

3.227 $\int \frac{\left(a+bx^3\right)^2}{x^5}\,dx$

Optimal. Leaf size=28

$$-\frac{a^2}{4x^4}-\frac{2ab}{x}+\frac{b^2x^2}{2}$$

```
[Out] -1/4*a^2/x^4-2*a*b/x+1/2*b^2*x^2
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$-\frac{a^2}{4x^4}-\frac{2ab}{x}+\frac{b^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2/x^5,x]
```

```
[Out] -a^2/(4*x^4) - (2*a*b)/x + (b^2*x^2)/2
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^2}{x^5}\,dx &= \int \left(\frac{a^2}{x^5}+\frac{2ab}{x^2}+b^2x\right)dx\\ &= -\frac{a^2}{4x^4}-\frac{2ab}{x}+\frac{b^2x^2}{2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 28, normalized size = 1.00

$$-\frac{a^2}{4x^4}-\frac{2ab}{x}+\frac{b^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2/x^5,x]
```

```
[Out] -1/4*a^2/x^4 - (2*a*b)/x + (b^2*x^2)/2
```

fricas [A] time = 0.69, size = 26, normalized size = 0.93

$$\frac{2\,b^2x^6-8\,abx^3-a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*x^6 - 8*a*b*x^3 - a^2)/x^4
```

giac [A] time = 0.15, size = 25, normalized size = 0.89

$$\frac{1}{2}\,b^2x^2-\frac{8\,abx^3+a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/x^5,x, algorithm="giac")
[Out] 1/2*b^2*x^2 - 1/4*(8*a*b*x^3 + a^2)/x^4
```

maple [A] time = 0.00, size = 25, normalized size = 0.89

$$\frac{b^2x^2}{2} - \frac{2ab}{x} - \frac{a^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2/x^5,x)
[Out] -1/4*a^2/x^4-2*a*b/x+1/2*b^2*x^2
```

maxima [A] time = 1.37, size = 25, normalized size = 0.89

$$\frac{1}{2}\,b^2x^2 - \frac{8\,abx^3 + a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/x^5,x, algorithm="maxima")
[Out] 1/2*b^2*x^2 - 1/4*(8*a*b*x^3 + a^2)/x^4
```

mupad [B] time = 0.03, size = 27, normalized size = 0.96

$$\frac{b^2\,x^2}{2} - \frac{\frac{a^2}{4} + 2\,b\,a\,x^3}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^2/x^5,x)
[Out] (b^2*x^2)/2 - (a^2/4 + 2*a*b*x^3)/x^4
```

sympy [A] time = 0.18, size = 26, normalized size = 0.93

$$\frac{b^2x^2}{2} + \frac{-a^2 - 8abx^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2/x**5,x)
[Out] b**2*x**2/2 + (-a**2 - 8*a*b*x**3)/(4*x**4)
```

3.228 $\int \frac{\left(a+bx^3\right)^2}{x^6}\,dx$

Optimal. Leaf size=23

$$-\frac{a^2}{5x^5}-\frac{ab}{x^2}+b^2x$$

```
[Out] -1/5*a^2/x^5-a*b/x^2+b^2*x
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$-\frac{a^2}{5x^5}-\frac{ab}{x^2}+b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2/x^6,x]
```

```
[Out] -a^2/(5*x^5) - (a*b)/x^2 + b^2*x
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^2}{x^6}\,dx &= \int \left(b^2+\frac{a^2}{x^6}+\frac{2ab}{x^3}\right)dx\\ &= -\frac{a^2}{5x^5}-\frac{ab}{x^2}+b^2x\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$-\frac{a^2}{5x^5}-\frac{ab}{x^2}+b^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2/x^6,x]
```

```
[Out] -1/5*a^2/x^5 - (a*b)/x^2 + b^2*x
```

fricas [A] time = 0.83, size = 26, normalized size = 1.13

$$\frac{5\,b^2x^6-5\,abx^3-a^2}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/x^6,x, algorithm="fricas")
```

```
[Out] 1/5*(5*b^2*x^6 - 5*a*b*x^3 - a^2)/x^5
```

giac [A] time = 0.16, size = 22, normalized size = 0.96

$$b^2x-\frac{5\,abx^3+a^2}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/x^6,x, algorithm="giac")
[Out] b^2*x - 1/5*(5*a*b*x^3 + a^2)/x^5
```

maple [A] time = 0.01, size = 22, normalized size = 0.96

$$b^2x - \frac{ab}{x^2} - \frac{a^2}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2/x^6,x)
[Out] -1/5*a^2/x^5-a*b/x^2+b^2*x
```

maxima [A] time = 1.34, size = 22, normalized size = 0.96

$$b^2x - \frac{5\,abx^3 + a^2}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/x^6,x, algorithm="maxima")
[Out] b^2*x - 1/5*(5*a*b*x^3 + a^2)/x^5
```

mupad [B] time = 0.03, size = 23, normalized size = 1.00

$$b^2\,x - \frac{\frac{a^2}{5} + b\,a\,x^3}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^2/x^6,x)
[Out] b^2*x - (a^2/5 + a*b*x^3)/x^5
```

sympy [A] time = 0.18, size = 22, normalized size = 0.96

$$b^2x + \frac{-a^2 - 5abx^3}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2/x**6,x)
[Out] b**2*x + (-a**2 - 5*a*b*x**3)/(5*x**5)
```

3.229 $\int \frac{\left(a+bx^3\right)^2}{x^7}\,dx$

Optimal. Leaf size=26

$$-\frac{a^2}{6x^6}-\frac{2ab}{3x^3}+b^2\log(x)$$

```
[Out] -1/6*a^2/x^6-2/3*a*b/x^3+b^2*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$-\frac{a^2}{6x^6}-\frac{2ab}{3x^3}+b^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2/x^7,x]
```

```
[Out] -a^2/(6*x^6) - (2*a*b)/(3*x^3) + b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^2}{x^7}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^2}{x^3}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{a^2}{x^3}+\frac{2ab}{x^2}+\frac{b^2}{x}\right)dx, x, x^3\right)\\ &= -\frac{a^2}{6x^6}-\frac{2ab}{3x^3}+b^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 26, normalized size = 1.00

$$-\frac{a^2}{6x^6}-\frac{2ab}{3x^3}+b^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2/x^7,x]
```

```
[Out] -1/6*a^2/x^6 - (2*a*b)/(3*x^3) + b^2*Log[x]
```

fricas [A] time = 0.56, size = 28, normalized size = 1.08

$$\frac{6\,b^2x^6\log(x)-4\,abx^3-a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/x^7,x, algorithm="fricas")
[Out] 1/6*(6*b^2*x^6*log(x) - 4*a*b*x^3 - a^2)/x^6
```

giac [A] time = 0.15, size = 32, normalized size = 1.23

$$b^2 \log(|x|) - \frac{3\,b^2x^6 + 4\,abx^3 + a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/x^7,x, algorithm="giac")
[Out] b^2*log(abs(x)) - 1/6*(3*b^2*x^6 + 4*a*b*x^3 + a^2)/x^6
```

maple [A] time = 0.00, size = 23, normalized size = 0.88

$$b^2 \ln(x) - \frac{2ab}{3x^3} - \frac{a^2}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2/x^7,x)
[Out] -1/6*a^2/x^6-2/3*a*b/x^3+b^2*ln(x)
```

maxima [A] time = 1.33, size = 26, normalized size = 1.00

$$\frac{1}{3}\,b^2 \log\left(x^3\right) - \frac{4\,abx^3 + a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/x^7,x, algorithm="maxima")
[Out] 1/3*b^2*log(x^3) - 1/6*(4*a*b*x^3 + a^2)/x^6
```

mupad [B] time = 0.04, size = 25, normalized size = 0.96

$$b^2\,\ln(x) - \frac{\frac{a^2}{6} + \frac{2\,b\,a\,x^3}{3}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^2/x^7,x)
[Out] b^2*log(x) - (a^2/6 + (2*a*b*x^3)/3)/x^6
```

sympy [A] time = 0.25, size = 24, normalized size = 0.92

$$b^2 \log(x) + \frac{-a^2 - 4abx^3}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2/x**7,x)
[Out] b**2*log(x) + (-a**2 - 4*a*b*x**3)/(6*x**6)
```

3.230 $\int \frac{\left(a+bx^3\right)^2}{x^8}\,dx$

Optimal. Leaf size=28

$$-\frac{a^2}{7x^7}-\frac{ab}{2x^4}-\frac{b^2}{x}$$

```
[Out] -1/7*a^2/x^7-1/2*a*b/x^4-b^2/x
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$-\frac{a^2}{7x^7}-\frac{ab}{2x^4}-\frac{b^2}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2/x^8,x]
```

```
[Out] -a^2/(7*x^7) - (a*b)/(2*x^4) - b^2/x
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^2}{x^8}\,dx &= \int \left(\frac{a^2}{x^8}+\frac{2ab}{x^5}+\frac{b^2}{x^2}\right)dx\\ &= -\frac{a^2}{7x^7}-\frac{ab}{2x^4}-\frac{b^2}{x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 28, normalized size = 1.00

$$-\frac{a^2}{7x^7}-\frac{ab}{2x^4}-\frac{b^2}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2/x^8,x]
```

```
[Out] -1/7*a^2/x^7 - (a*b)/(2*x^4) - b^2/x
```

fricas [A] time = 0.74, size = 26, normalized size = 0.93

$$-\frac{14\,b^2x^6+7\,abx^3+2\,a^2}{14\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/x^8,x, algorithm="fricas")
```

```
[Out] -1/14*(14*b^2*x^6 + 7*a*b*x^3 + 2*a^2)/x^7
```

giac [A] time = 0.23, size = 26, normalized size = 0.93

$$-\frac{14\,b^2x^6+7\,abx^3+2\,a^2}{14\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/x^8,x, algorithm="giac")
[Out] -1/14*(14*b^2*x^6 + 7*a*b*x^3 + 2*a^2)/x^7
```

maple [A] time = 0.00, size = 25, normalized size = 0.89

$$-\frac{b^2}{x}-\frac{ab}{2x^4}-\frac{a^2}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2/x^8,x)
[Out] -1/7*a^2/x^7-1/2*a*b/x^4-b^2/x
```

maxima [A] time = 1.31, size = 26, normalized size = 0.93

$$-\frac{14\,b^2x^6+7\,abx^3+2\,a^2}{14\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/x^8,x, algorithm="maxima")
[Out] -1/14*(14*b^2*x^6 + 7*a*b*x^3 + 2*a^2)/x^7
```

mupad [B] time = 0.03, size = 25, normalized size = 0.89

$$-\frac{\frac{a^2}{7}+\frac{a\,b\,x^3}{2}+b^2\,x^6}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^2/x^8,x)
[Out] -(a^2/7 + b^2*x^6 + (a*b*x^3)/2)/x^7
```

sympy [A] time = 0.22, size = 27, normalized size = 0.96

$$\frac{-2a^2-7abx^3-14b^2x^6}{14x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2/x**8,x)
[Out] (-2*a**2 - 7*a*b*x**3 - 14*b**2*x**6)/(14*x**7)
```

3.231 $\int \frac{\left(a+bx^3\right)^2}{x^9}\,dx$

Optimal. Leaf size=30

$$-\frac{a^2}{8x^8}-\frac{2ab}{5x^5}-\frac{b^2}{2x^2}$$

```
[Out] -1/8*a^2/x^8-2/5*a*b/x^5-1/2*b^2/x^2
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$-\frac{a^2}{8x^8}-\frac{2ab}{5x^5}-\frac{b^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2/x^9,x]
```

```
[Out] -a^2/(8*x^8) - (2*a*b)/(5*x^5) - b^2/(2*x^2)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^2}{x^9}\,dx &= \int \left(\frac{a^2}{x^9}+\frac{2ab}{x^6}+\frac{b^2}{x^3}\right)dx\\ &= -\frac{a^2}{8x^8}-\frac{2ab}{5x^5}-\frac{b^2}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$-\frac{a^2}{8x^8}-\frac{2ab}{5x^5}-\frac{b^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2/x^9,x]
```

```
[Out] -1/8*a^2/x^8 - (2*a*b)/(5*x^5) - b^2/(2*x^2)
```

fricas [A] time = 0.49, size = 26, normalized size = 0.87

$$-\frac{20\,b^2x^6+16\,abx^3+5\,a^2}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/x^9,x, algorithm="fricas")
```

```
[Out] -1/40*(20*b^2*x^6 + 16*a*b*x^3 + 5*a^2)/x^8
```

giac [A] time = 0.17, size = 26, normalized size = 0.87

$$-\frac{20\,b^2x^6+16\,abx^3+5\,a^2}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/x^9,x, algorithm="giac")
[Out] -1/40*(20*b^2*x^6 + 16*a*b*x^3 + 5*a^2)/x^8
```

maple [A] time = 0.01, size = 25, normalized size = 0.83

$$-\frac{b^2}{2x^2}-\frac{2ab}{5x^5}-\frac{a^2}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2/x^9,x)
[Out] -1/8*a^2/x^8-2/5*a*b/x^5-1/2*b^2/x^2
```

maxima [A] time = 1.32, size = 26, normalized size = 0.87

$$-\frac{20\,b^2x^6+16\,abx^3+5\,a^2}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/x^9,x, algorithm="maxima")
[Out] -1/40*(20*b^2*x^6 + 16*a*b*x^3 + 5*a^2)/x^8
```

mupad [B] time = 0.03, size = 26, normalized size = 0.87

$$-\frac{\frac{a^2}{8}+\frac{2\,a\,b\,x^3}{5}+\frac{b^2\,x^6}{2}}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^2/x^9,x)
[Out] -(a^2/8 + (b^2*x^6)/2 + (2*a*b*x^3)/5)/x^8
```

sympy [A] time = 0.23, size = 27, normalized size = 0.90

$$\frac{-5a^2-16abx^3-20b^2x^6}{40x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2/x**9,x)
[Out] (-5*a**2 - 16*a*b*x**3 - 20*b**2*x**6)/(40*x**8)
```

3.232 $\int \frac{\left(a+bx^3\right)^2}{x^{10}}\,dx$

Optimal. Leaf size=19

$$-\frac{\left(a+bx^3\right)^3}{9ax^9}$$

```
[Out] -1/9*(b*x^3+a)^3/a/x^9
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {264}

$$-\frac{\left(a+bx^3\right)^3}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2/x^10,x]
```

```
[Out] -(a + b*x^3)^3/(9*a*x^9)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^2}{x^{10}}\,dx = -\frac{\left(a+bx^3\right)^3}{9ax^9}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.58

$$-\frac{a^2}{9x^9} - \frac{ab}{3x^6} - \frac{b^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2/x^10,x]
```

```
[Out] -1/9*a^2/x^9 - (a*b)/(3*x^6) - b^2/(3*x^3)
```

fricas [A] time = 0.80, size = 24, normalized size = 1.26

$$-\frac{3\,b^2x^6 + 3\,abx^3 + a^2}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/x^10,x, algorithm="fricas")
```

```
[Out] -1/9*(3*b^2*x^6 + 3*a*b*x^3 + a^2)/x^9
```

giac [A] time = 0.15, size = 24, normalized size = 1.26

$$-\frac{3\,b^2x^6 + 3\,abx^3 + a^2}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/x^10,x, algorithm="giac")
[Out] -1/9*(3*b^2*x^6 + 3*a*b*x^3 + a^2)/x^9
```

maple [A] time = 0.00, size = 25, normalized size = 1.32

$$-\frac{b^2}{3x^3}-\frac{ab}{3x^6}-\frac{a^2}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2/x^10,x)
[Out] -1/3*b^2/x^3-1/3*a*b/x^6-1/9*a^2/x^9
```

maxima [A] time = 1.40, size = 24, normalized size = 1.26

$$-\frac{3\,b^2x^6+3\,abx^3+a^2}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/x^10,x, algorithm="maxima")
[Out] -1/9*(3*b^2*x^6 + 3*a*b*x^3 + a^2)/x^9
```

mupad [B] time = 0.03, size = 26, normalized size = 1.37

$$-\frac{\frac{a^2}{9}+\frac{a\,b\,x^3}{3}+\frac{b^2\,x^6}{3}}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^2/x^10,x)
[Out] -(a^2/9 + (b^2*x^6)/3 + (a*b*x^3)/3)/x^9
```

sympy [A] time = 0.27, size = 26, normalized size = 1.37

$$\frac{-a^2-3abx^3-3b^2x^6}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2/x**10,x)
[Out] (-a**2 - 3*a*b*x**3 - 3*b**2*x**6)/(9*x**9)
```

3.233 $\int \frac{\left(a+bx^3\right)^2}{x^{11}}\,dx$

Optimal. Leaf size=30

$$-\frac{a^2}{10x^{10}}-\frac{2ab}{7x^7}-\frac{b^2}{4x^4}$$

```
[Out] -1/10*a^2/x^10-2/7*a*b/x^7-1/4*b^2/x^4
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$-\frac{a^2}{10x^{10}}-\frac{2ab}{7x^7}-\frac{b^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2/x^11,x]
```

```
[Out] -a^2/(10*x^10) - (2*a*b)/(7*x^7) - b^2/(4*x^4)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^2}{x^{11}}\,dx &= \int \left(\frac{a^2}{x^{11}}+\frac{2ab}{x^8}+\frac{b^2}{x^5}\right)dx\\ &= -\frac{a^2}{10x^{10}}-\frac{2ab}{7x^7}-\frac{b^2}{4x^4}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$-\frac{a^2}{10x^{10}}-\frac{2ab}{7x^7}-\frac{b^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2/x^11,x]
```

```
[Out] -1/10*a^2/x^10 - (2*a*b)/(7*x^7) - b^2/(4*x^4)
```

fricas [A] time = 0.85, size = 26, normalized size = 0.87

$$-\frac{35\,b^2x^6+40\,abx^3+14\,a^2}{140\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/x^11,x, algorithm="fricas")
```

```
[Out] -1/140*(35*b^2*x^6 + 40*a*b*x^3 + 14*a^2)/x^10
```

giac [A] time = 0.18, size = 26, normalized size = 0.87

$$-\frac{35\,b^2x^6+40\,abx^3+14\,a^2}{140\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/x^11,x, algorithm="giac")
[Out] -1/140*(35*b^2*x^6 + 40*a*b*x^3 + 14*a^2)/x^10
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$-\frac{b^2}{4x^4}-\frac{2ab}{7x^7}-\frac{a^2}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2/x^11,x)
[Out] -1/10*a^2/x^10-2/7*a*b/x^7-1/4*b^2/x^4
```

maxima [A] time = 1.32, size = 26, normalized size = 0.87

$$-\frac{35\,b^2x^6+40\,abx^3+14\,a^2}{140\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/x^11,x, algorithm="maxima")
[Out] -1/140*(35*b^2*x^6 + 40*a*b*x^3 + 14*a^2)/x^10
```

mupad [B] time = 0.95, size = 26, normalized size = 0.87

$$-\frac{\frac{a^2}{10}+\frac{2\,a\,b\,x^3}{7}+\frac{b^2\,x^6}{4}}{x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^2/x^11,x)
[Out] -(a^2/10 + (b^2*x^6)/4 + (2*a*b*x^3)/7)/x^10
```

sympy [A] time = 0.28, size = 27, normalized size = 0.90

$$\frac{-14a^2-40abx^3-35b^2x^6}{140x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2/x**11,x)
[Out] (-14*a**2 - 40*a*b*x**3 - 35*b**2*x**6)/(140*x**10)
```

3.234 $\int \frac{\left(a+bx^3\right)^2}{x^{12}}\,dx$

Optimal. Leaf size=30

$$-\frac{a^2}{11x^{11}}-\frac{ab}{4x^8}-\frac{b^2}{5x^5}$$

```
[Out] -1/11*a^2/x^11-1/4*a*b/x^8-1/5*b^2/x^5
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$-\frac{a^2}{11x^{11}}-\frac{ab}{4x^8}-\frac{b^2}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2/x^12,x]
```

```
[Out] -a^2/(11*x^11) - (a*b)/(4*x^8) - b^2/(5*x^5)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^2}{x^{12}}\,dx &= \int \left(\frac{a^2}{x^{12}}+\frac{2ab}{x^9}+\frac{b^2}{x^6}\right)dx \\ &= -\frac{a^2}{11x^{11}}-\frac{ab}{4x^8}-\frac{b^2}{5x^5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$-\frac{a^2}{11x^{11}}-\frac{ab}{4x^8}-\frac{b^2}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2/x^12,x]
```

```
[Out] -1/11*a^2/x^11 - (a*b)/(4*x^8) - b^2/(5*x^5)
```

fricas [A] time = 0.88, size = 26, normalized size = 0.87

$$-\frac{44\,b^2x^6+55\,abx^3+20\,a^2}{220\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/x^12,x, algorithm="fricas")
```

```
[Out] -1/220*(44*b^2*x^6 + 55*a*b*x^3 + 20*a^2)/x^11
```

giac [A] time = 0.15, size = 26, normalized size = 0.87

$$-\frac{44\,b^2x^6+55\,abx^3+20\,a^2}{220\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/x^12,x, algorithm="giac")
[Out] -1/220*(44*b^2*x^6 + 55*a*b*x^3 + 20*a^2)/x^11
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$-\frac{b^2}{5x^5}-\frac{ab}{4x^8}-\frac{a^2}{11x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2/x^12,x)
[Out] -1/11*a^2/x^11-1/4*a*b/x^8-1/5*b^2/x^5
```

maxima [A] time = 1.31, size = 26, normalized size = 0.87

$$-\frac{44\,b^2x^6+55\,abx^3+20\,a^2}{220\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/x^12,x, algorithm="maxima")
[Out] -1/220*(44*b^2*x^6 + 55*a*b*x^3 + 20*a^2)/x^11
```

mupad [B] time = 0.04, size = 26, normalized size = 0.87

$$-\frac{\frac{a^2}{11}+\frac{a\,b\,x^3}{4}+\frac{b^2\,x^6}{5}}{x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^2/x^12,x)
[Out] -(a^2/11 + (b^2*x^6)/5 + (a*b*x^3)/4)/x^11
```

sympy [A] time = 0.27, size = 27, normalized size = 0.90

$$\frac{-20a^2-55abx^3-44b^2x^6}{220x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2/x**12,x)
[Out] (-20*a**2 - 55*a*b*x**3 - 44*b**2*x**6)/(220*x**11)
```

3.235 $\int \frac{\left(a+bx^3\right)^2}{x^{13}}\,dx$

Optimal. Leaf size=30

$$-\frac{a^2}{12x^{12}}-\frac{2ab}{9x^9}-\frac{b^2}{6x^6}$$

```
[Out] -1/12*a^2/x^12-2/9*a*b/x^9-1/6*b^2/x^6
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$-\frac{a^2}{12x^{12}}-\frac{2ab}{9x^9}-\frac{b^2}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2/x^13,x]
```

```
[Out] -a^2/(12*x^12) - (2*a*b)/(9*x^9) - b^2/(6*x^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^2}{x^{13}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^2}{x^5}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{a^2}{x^5}+\frac{2ab}{x^4}+\frac{b^2}{x^3}\right)\,dx, x, x^3\right)\\ &= -\frac{a^2}{12x^{12}}-\frac{2ab}{9x^9}-\frac{b^2}{6x^6}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$-\frac{a^2}{12x^{12}}-\frac{2ab}{9x^9}-\frac{b^2}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2/x^13,x]
```

```
[Out] -1/12*a^2/x^12 - (2*a*b)/(9*x^9) - b^2/(6*x^6)
```

fricas [A] time = 0.65, size = 26, normalized size = 0.87

$$-\frac{6\,b^2x^6+8\,abx^3+3\,a^2}{36\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/x^13,x, algorithm="fricas")
[Out] -1/36*(6*b^2*x^6 + 8*a*b*x^3 + 3*a^2)/x^12
```

giac [A] time = 0.15, size = 26, normalized size = 0.87

$$-\frac{6\,b^2x^6+8\,abx^3+3\,a^2}{36\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/x^13,x, algorithm="giac")
[Out] -1/36*(6*b^2*x^6 + 8*a*b*x^3 + 3*a^2)/x^12
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$-\frac{b^2}{6x^6}-\frac{2ab}{9x^9}-\frac{a^2}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2/x^13,x)
[Out] -1/12*a^2/x^12-2/9*a*b/x^9-1/6*b^2/x^6
```

maxima [A] time = 1.26, size = 26, normalized size = 0.87

$$-\frac{6\,b^2x^6+8\,abx^3+3\,a^2}{36\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/x^13,x, algorithm="maxima")
[Out] -1/36*(6*b^2*x^6 + 8*a*b*x^3 + 3*a^2)/x^12
```

mupad [B] time = 0.04, size = 26, normalized size = 0.87

$$-\frac{\frac{a^2}{12}+\frac{2\,a\,b\,x^3}{9}+\frac{b^2\,x^6}{6}}{x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^2/x^13,x)
[Out] -(a^2/12 + (b^2*x^6)/6 + (2*a*b*x^3)/9)/x^12
```

sympy [A] time = 0.28, size = 27, normalized size = 0.90

$$\frac{-3a^2-8abx^3-6b^2x^6}{36x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2/x**13,x)
[Out] (-3*a**2 - 8*a*b*x**3 - 6*b**2*x**6)/(36*x**12)
```

3.236 $\int x^{14}\left(a+bx^3\right)^3\,dx$

Optimal. Leaf size=43

$$\frac{a^3x^{15}}{15}+\frac{1}{6}a^2bx^{18}+\frac{1}{7}ab^2x^{21}+\frac{b^3x^{24}}{24}$$

```
[Out] 1/15*a^3*x^15+1/6*a^2*b*x^18+1/7*a*b^2*x^21+1/24*b^3*x^24
```

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{1}{6}a^2bx^{18}+\frac{a^3x^{15}}{15}+\frac{1}{7}ab^2x^{21}+\frac{b^3x^{24}}{24}$$

Antiderivative was successfully verified.

```
[In] Int[x^14*(a + b*x^3)^3,x]
```

```
[Out] (a^3*x^15)/15 + (a^2*b*x^18)/6 + (a*b^2*x^21)/7 + (b^3*x^24)/24
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{14}\left(a+bx^3\right)^3\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int x^4(a+bx)^3\,dx,x,x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(a^3x^4+3a^2bx^5+3ab^2x^6+b^3x^7\right)\,dx,x,x^3\right)\\ &= \frac{a^3x^{15}}{15}+\frac{1}{6}a^2bx^{18}+\frac{1}{7}ab^2x^{21}+\frac{b^3x^{24}}{24}\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^{15}}{15}+\frac{1}{6}a^2bx^{18}+\frac{1}{7}ab^2x^{21}+\frac{b^3x^{24}}{24}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14*(a + b*x^3)^3,x]
```

```
[Out] (a^3*x^15)/15 + (a^2*b*x^18)/6 + (a*b^2*x^21)/7 + (b^3*x^24)/24
```

fricas [A] time = 0.72, size = 35, normalized size = 0.81

$$\frac{1}{24}x^{24}b^3+\frac{1}{7}x^{21}b^2a+\frac{1}{6}x^{18}ba^2+\frac{1}{15}x^{15}a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(b*x^3+a)^3,x, algorithm="fricas")
[Out] 1/24*x^24*b^3 + 1/7*x^21*b^2*a + 1/6*x^18*b*a^2 + 1/15*x^15*a^3
```

giac [A] time = 0.15, size = 35, normalized size = 0.81

$$\frac{1}{24}\,b^3x^{24} + \frac{1}{7}\,ab^2x^{21} + \frac{1}{6}\,a^2bx^{18} + \frac{1}{15}\,a^3x^{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(b*x^3+a)^3,x, algorithm="giac")
[Out] 1/24*b^3*x^24 + 1/7*a*b^2*x^21 + 1/6*a^2*b*x^18 + 1/15*a^3*x^15
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$\frac{1}{24}b^3x^{24} + \frac{1}{7}a\,b^2x^{21} + \frac{1}{6}a^2b\,x^{18} + \frac{1}{15}a^3x^{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(b*x^3+a)^3,x)
[Out] 1/15*a^3*x^15+1/6*a^2*b*x^18+1/7*a*b^2*x^21+1/24*b^3*x^24
```

maxima [A] time = 1.30, size = 35, normalized size = 0.81

$$\frac{1}{24}\,b^3x^{24} + \frac{1}{7}\,ab^2x^{21} + \frac{1}{6}\,a^2bx^{18} + \frac{1}{15}\,a^3x^{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(b*x^3+a)^3,x, algorithm="maxima")
[Out] 1/24*b^3*x^24 + 1/7*a*b^2*x^21 + 1/6*a^2*b*x^18 + 1/15*a^3*x^15
```

mupad [B] time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3\,x^{15}}{15} + \frac{a^2\,b\,x^{18}}{6} + \frac{a\,b^2\,x^{21}}{7} + \frac{b^3\,x^{24}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(a + b*x^3)^3,x)
[Out] (a^3*x^15)/15 + (b^3*x^24)/24 + (a^2*b*x^18)/6 + (a*b^2*x^21)/7
```

sympy [A] time = 0.07, size = 36, normalized size = 0.84

$$\frac{a^3x^{15}}{15} + \frac{a^2bx^{18}}{6} + \frac{ab^2x^{21}}{7} + \frac{b^3x^{24}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14*(b*x**3+a)**3,x)
[Out] a**3*x**15/15 + a**2*b*x**18/6 + a*b**2*x**21/7 + b**3*x**24/24
```

3.237 $\int x^{11}\left(a+bx^3\right)^3\,dx$

Optimal. Leaf size=43

$$\frac{a^3x^{12}}{12}+\frac{1}{5}a^2bx^{15}+\frac{1}{6}ab^2x^{18}+\frac{b^3x^{21}}{21}$$

```
[Out] 1/12*a^3*x^12+1/5*a^2*b*x^15+1/6*a*b^2*x^18+1/21*b^3*x^21
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{1}{5}a^2bx^{15}+\frac{a^3x^{12}}{12}+\frac{1}{6}ab^2x^{18}+\frac{b^3x^{21}}{21}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a + b*x^3)^3,x]
```

```
[Out] (a^3*x^12)/12 + (a^2*b*x^15)/5 + (a*b^2*x^18)/6 + (b^3*x^21)/21
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{11}\left(a+bx^3\right)^3\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int x^3(a+bx)^3\,dx,x,x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(a^3x^3+3a^2bx^4+3ab^2x^5+b^3x^6\right)\,dx,x,x^3\right)\\ &= \frac{a^3x^{12}}{12}+\frac{1}{5}a^2bx^{15}+\frac{1}{6}ab^2x^{18}+\frac{b^3x^{21}}{21}\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^{12}}{12}+\frac{1}{5}a^2bx^{15}+\frac{1}{6}ab^2x^{18}+\frac{b^3x^{21}}{21}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*x^3)^3,x]
```

```
[Out] (a^3*x^12)/12 + (a^2*b*x^15)/5 + (a*b^2*x^18)/6 + (b^3*x^21)/21
```

fricas [A] time = 0.72, size = 35, normalized size = 0.81

$$\frac{1}{21}x^{21}b^3+\frac{1}{6}x^{18}b^2a+\frac{1}{5}x^{15}ba^2+\frac{1}{12}x^{12}a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^3,x, algorithm="fricas")
[Out] 1/21*x^21*b^3 + 1/6*x^18*b^2*a + 1/5*x^15*b*a^2 + 1/12*x^12*a^3
```

giac [A] time = 0.26, size = 35, normalized size = 0.81

$$\frac{1}{21}\,b^3x^{21} + \frac{1}{6}\,ab^2x^{18} + \frac{1}{5}\,a^2bx^{15} + \frac{1}{12}\,a^3x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^3,x, algorithm="giac")
[Out] 1/21*b^3*x^21 + 1/6*a*b^2*x^18 + 1/5*a^2*b*x^15 + 1/12*a^3*x^12
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$\frac{1}{21}b^3x^{21} + \frac{1}{6}a\,b^2x^{18} + \frac{1}{5}a^2b\,x^{15} + \frac{1}{12}a^3x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x^3+a)^3,x)
[Out] 1/12*a^3*x^12+1/5*a^2*b*x^15+1/6*a*b^2*x^18+1/21*b^3*x^21
```

maxima [A] time = 1.29, size = 35, normalized size = 0.81

$$\frac{1}{21}\,b^3x^{21} + \frac{1}{6}\,ab^2x^{18} + \frac{1}{5}\,a^2bx^{15} + \frac{1}{12}\,a^3x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^3,x, algorithm="maxima")
[Out] 1/21*b^3*x^21 + 1/6*a*b^2*x^18 + 1/5*a^2*b*x^15 + 1/12*a^3*x^12
```

mupad [B] time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3\,x^{12}}{12} + \frac{a^2\,b\,x^{15}}{5} + \frac{a\,b^2\,x^{18}}{6} + \frac{b^3\,x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a + b*x^3)^3,x)
[Out] (a^3*x^12)/12 + (b^3*x^21)/21 + (a^2*b*x^15)/5 + (a*b^2*x^18)/6
```

sympy [A] time = 0.07, size = 36, normalized size = 0.84

$$\frac{a^3x^{12}}{12} + \frac{a^2bx^{15}}{5} + \frac{ab^2x^{18}}{6} + \frac{b^3x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x**3+a)**3,x)
[Out] a**3*x**12/12 + a**2*b*x**15/5 + a*b**2*x**18/6 + b**3*x**21/21
```

3.238 $\int x^8 \left(a+bx^3\right)^3 \, dx$

Optimal. Leaf size=43

$$\frac{a^3x^9}{9}+\frac{1}{4}a^2bx^{12}+\frac{1}{5}ab^2x^{15}+\frac{b^3x^{18}}{18}$$

```
[Out] 1/9*a^3*x^9+1/4*a^2*b*x^12+1/5*a*b^2*x^15+1/18*b^3*x^18
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{1}{4}a^2bx^{12}+\frac{a^3x^9}{9}+\frac{1}{5}ab^2x^{15}+\frac{b^3x^{18}}{18}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x^3)^3,x]
```

```
[Out] (a^3*x^9)/9 + (a^2*b*x^12)/4 + (a*b^2*x^15)/5 + (b^3*x^18)/18
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^8 \left(a+bx^3\right)^3 \, dx &= \frac{1}{3}\text{Subst}\left(\int x^2(a+bx)^3 \, dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(a^3x^2+3a^2bx^3+3ab^2x^4+b^3x^5\right) \, dx, x, x^3\right)\\ &= \frac{a^3x^9}{9}+\frac{1}{4}a^2bx^{12}+\frac{1}{5}ab^2x^{15}+\frac{b^3x^{18}}{18}\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^9}{9}+\frac{1}{4}a^2bx^{12}+\frac{1}{5}ab^2x^{15}+\frac{b^3x^{18}}{18}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x^3)^3,x]
```

```
[Out] (a^3*x^9)/9 + (a^2*b*x^12)/4 + (a*b^2*x^15)/5 + (b^3*x^18)/18
```

fricas [A] time = 0.41, size = 35, normalized size = 0.81

$$\frac{1}{18}x^{18}b^3+\frac{1}{5}x^{15}b^2a+\frac{1}{4}x^{12}ba^2+\frac{1}{9}x^9a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^3,x, algorithm="fricas")
[Out] 1/18*x^18*b^3 + 1/5*x^15*b^2*a + 1/4*x^12*b*a^2 + 1/9*x^9*a^3
```

giac [A] time = 0.16, size = 35, normalized size = 0.81

$$\frac{1}{18}\,b^3x^{18} + \frac{1}{5}\,ab^2x^{15} + \frac{1}{4}\,a^2bx^{12} + \frac{1}{9}\,a^3x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^3,x, algorithm="giac")
[Out] 1/18*b^3*x^18 + 1/5*a*b^2*x^15 + 1/4*a^2*b*x^12 + 1/9*a^3*x^9
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$\frac{1}{18}b^3x^{18} + \frac{1}{5}a\,b^2x^{15} + \frac{1}{4}a^2b\,x^{12} + \frac{1}{9}a^3x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^3+a)^3,x)
[Out] 1/9*a^3*x^9+1/4*a^2*b*x^12+1/5*a*b^2*x^15+1/18*b^3*x^18
```

maxima [A] time = 1.31, size = 35, normalized size = 0.81

$$\frac{1}{18}\,b^3x^{18} + \frac{1}{5}\,ab^2x^{15} + \frac{1}{4}\,a^2bx^{12} + \frac{1}{9}\,a^3x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^3,x, algorithm="maxima")
[Out] 1/18*b^3*x^18 + 1/5*a*b^2*x^15 + 1/4*a^2*b*x^12 + 1/9*a^3*x^9
```

mupad [B] time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3\,x^9}{9} + \frac{a^2\,b\,x^{12}}{4} + \frac{a\,b^2\,x^{15}}{5} + \frac{b^3\,x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*x^3)^3,x)
[Out] (a^3*x^9)/9 + (b^3*x^18)/18 + (a^2*b*x^12)/4 + (a*b^2*x^15)/5
```

sympy [A] time = 0.08, size = 36, normalized size = 0.84

$$\frac{a^3x^9}{9} + \frac{a^2bx^{12}}{4} + \frac{ab^2x^{15}}{5} + \frac{b^3x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**3+a)**3,x)
[Out] a**3*x**9/9 + a**2*b*x**12/4 + a*b**2*x**15/5 + b**3*x**18/18
```

3.239 $\int x^5 \left(a+bx^3\right)^3 \, dx$

Optimal. Leaf size=34

$$\frac{\left(a+bx^3\right)^5}{15b^2} - \frac{a\left(a+bx^3\right)^4}{12b^2}$$

```
[Out] -1/12*a*(b*x^3+a)^4/b^2+1/15*(b*x^3+a)^5/b^2
```

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{\left(a+bx^3\right)^5}{15b^2} - \frac{a\left(a+bx^3\right)^4}{12b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^3)^3,x]
```

```
[Out] -(a*(a + b*x^3)^4)/(12*b^2) + (a + b*x^3)^5/(15*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^5 \left(a+bx^3\right)^3 \, dx &= \frac{1}{3}\,\text{Subst}\left(\int x(a+bx)^3 \, dx, x, x^3\right) \\
&= \frac{1}{3}\,\text{Subst}\left(\int \left(-\frac{a(a+bx)^3}{b} + \frac{(a+bx)^4}{b}\right) dx, x, x^3\right) \\
&= -\frac{a\left(a+bx^3\right)^4}{12b^2} + \frac{\left(a+bx^3\right)^5}{15b^2}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.26

$$\frac{a^3x^6}{6} + \frac{1}{3}a^2bx^9 + \frac{1}{4}ab^2x^{12} + \frac{b^3x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^3)^3,x]
```

```
[Out] (a^3*x^6)/6 + (a^2*b*x^9)/3 + (a*b^2*x^12)/4 + (b^3*x^15)/15
```

fricas [A] time = 0.47, size = 35, normalized size = 1.03

$$\frac{1}{15}x^{15}b^3 + \frac{1}{4}x^{12}b^2a + \frac{1}{3}x^9ba^2 + \frac{1}{6}x^6a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^3,x, algorithm="fricas")
[Out] 1/15*x^15*b^3 + 1/4*x^12*b^2*a + 1/3*x^9*b*a^2 + 1/6*x^6*a^3
```

giac [A] time = 0.16, size = 35, normalized size = 1.03

$$\frac{1}{15}\,b^3x^{15} + \frac{1}{4}\,ab^2x^{12} + \frac{1}{3}\,a^2bx^9 + \frac{1}{6}\,a^3x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^3,x, algorithm="giac")
[Out] 1/15*b^3*x^15 + 1/4*a*b^2*x^12 + 1/3*a^2*b*x^9 + 1/6*a^3*x^6
```

maple [A] time = 0.00, size = 36, normalized size = 1.06

$$\frac{1}{15}b^3x^{15} + \frac{1}{4}a\,b^2x^{12} + \frac{1}{3}a^2b\,x^9 + \frac{1}{6}a^3x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^3+a)^3,x)
[Out] 1/15*b^3*x^15+1/4*a*b^2*x^12+1/3*a^2*b*x^9+1/6*a^3*x^6
```

maxima [A] time = 1.29, size = 35, normalized size = 1.03

$$\frac{1}{15}\,b^3x^{15} + \frac{1}{4}\,ab^2x^{12} + \frac{1}{3}\,a^2bx^9 + \frac{1}{6}\,a^3x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^3,x, algorithm="maxima")
[Out] 1/15*b^3*x^15 + 1/4*a*b^2*x^12 + 1/3*a^2*b*x^9 + 1/6*a^3*x^6
```

mupad [B] time = 0.04, size = 35, normalized size = 1.03

$$\frac{a^3\,x^6}{6} + \frac{a^2\,b\,x^9}{3} + \frac{a\,b^2\,x^{12}}{4} + \frac{b^3\,x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^3)^3,x)
[Out] (a^3*x^6)/6 + (b^3*x^15)/15 + (a^2*b*x^9)/3 + (a*b^2*x^12)/4
```

sympy [A] time = 0.09, size = 36, normalized size = 1.06

$$\frac{a^3x^6}{6} + \frac{a^2bx^9}{3} + \frac{ab^2x^{12}}{4} + \frac{b^3x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**3+a)**3,x)
[Out] a**3*x**6/6 + a**2*b*x**9/3 + a*b**2*x**12/4 + b**3*x**15/15
```

3.240 $\int x^2 \left(a + bx^3\right)^3 \, dx$

Optimal. Leaf size=16

$$\frac{\left(a + bx^3\right)^4}{12b}$$

```
[Out] 1/12*(b*x^3+a)^4/b
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$\frac{\left(a + bx^3\right)^4}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3)^3,x]
```

```
[Out] (a + b*x^3)^4/(12*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^2 \left(a + bx^3\right)^3 \, dx = \frac{\left(a + bx^3\right)^4}{12b}$$

Mathematica [B] time = 0.00, size = 43, normalized size = 2.69

$$\frac{a^3x^3}{3} + \frac{1}{2}a^2bx^6 + \frac{1}{3}ab^2x^9 + \frac{b^3x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)^3,x]
```

```
[Out] (a^3*x^3)/3 + (a^2*b*x^6)/2 + (a*b^2*x^9)/3 + (b^3*x^12)/12
```

fricas [B] time = 0.78, size = 35, normalized size = 2.19

$$\frac{1}{12}x^{12}b^3 + \frac{1}{3}x^9b^2a + \frac{1}{2}x^6ba^2 + \frac{1}{3}x^3a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*b^3 + 1/3*x^9*b^2*a + 1/2*x^6*b*a^2 + 1/3*x^3*a^3
```

giac [A] time = 0.15, size = 14, normalized size = 0.88

$$\frac{\left(bx^3 + a\right)^4}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^3,x, algorithm="giac")
[Out] 1/12*(b*x^3 + a)^4/b
```

maple [B] time = 0.00, size = 36, normalized size = 2.25

$$\frac{1}{12}b^3x^{12} + \frac{1}{3}a\,b^2x^9 + \frac{1}{2}a^2b\,x^6 + \frac{1}{3}a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^3,x)
[Out] 1/12*b^3*x^12+1/3*a*b^2*x^9+1/2*a^2*b*x^6+1/3*a^3*x^3
```

maxima [A] time = 1.27, size = 14, normalized size = 0.88

$$\frac{\left(bx^3 + a\right)^4}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^3,x, algorithm="maxima")
[Out] 1/12*(b*x^3 + a)^4/b
```

mupad [B] time = 0.04, size = 35, normalized size = 2.19

$$\frac{a^3\,x^3}{3} + \frac{a^2\,b\,x^6}{2} + \frac{a\,b^2\,x^9}{3} + \frac{b^3\,x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^3)^3,x)
[Out] (a^3*x^3)/3 + (b^3*x^12)/12 + (a^2*b*x^6)/2 + (a*b^2*x^9)/3
```

sympy [B] time = 0.08, size = 36, normalized size = 2.25

$$\frac{a^3x^3}{3} + \frac{a^2bx^6}{2} + \frac{ab^2x^9}{3} + \frac{b^3x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**3,x)
[Out] a**3*x**3/3 + a**2*b*x**6/2 + a*b**2*x**9/3 + b**3*x**12/12
```

3.241 $\int \frac{\left(a+bx^3\right)^3}{x}\,dx$

Optimal. Leaf size=36

$$a^3\log(x)+a^2bx^3+\frac{1}{2}ab^2x^6+\frac{b^3x^9}{9}$$

```
[Out] a^2*b*x^3+1/2*a*b^2*x^6+1/9*b^3*x^9+a^3*ln(x)
```

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$a^2bx^3+a^3\log(x)+\frac{1}{2}ab^2x^6+\frac{b^3x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^3/x,x]
```

```
[Out] a^2*b*x^3 + (a*b^2*x^6)/2 + (b^3*x^9)/9 + a^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^3}{x}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^3}{x}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(3a^2b+\frac{a^3}{x}+3ab^2x+b^3x^2\right)\,dx, x, x^3\right)\\ &= a^2bx^3+\frac{1}{2}ab^2x^6+\frac{b^3x^9}{9}+a^3\log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 1.00

$$a^3\log(x)+a^2bx^3+\frac{1}{2}ab^2x^6+\frac{b^3x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^3/x,x]
```

```
[Out] a^2*b*x^3 + (a*b^2*x^6)/2 + (b^3*x^9)/9 + a^3*Log[x]
```

fricas [A] time = 0.84, size = 32, normalized size = 0.89

$$\frac{1}{9}b^3x^9 + \frac{1}{2}ab^2x^6 + a^2bx^3 + a^3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/x,x, algorithm="fricas")
[Out] 1/9*b^3*x^9 + 1/2*a*b^2*x^6 + a^2*b*x^3 + a^3*log(x)
```

giac [A] time = 0.15, size = 33, normalized size = 0.92

$$\frac{1}{9}b^3x^9 + \frac{1}{2}ab^2x^6 + a^2bx^3 + a^3\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/x,x, algorithm="giac")
[Out] 1/9*b^3*x^9 + 1/2*a*b^2*x^6 + a^2*b*x^3 + a^3*log(abs(x))
```

maple [A] time = 0.00, size = 33, normalized size = 0.92

$$\frac{b^3x^9}{9} + \frac{a\,b^2x^6}{2} + a^2b\,x^3 + a^3\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3/x,x)
[Out] a^2*b*x^3+1/2*a*b^2*x^6+1/9*b^3*x^9+a^3*ln(x)
```

maxima [A] time = 1.37, size = 35, normalized size = 0.97

$$\frac{1}{9}b^3x^9 + \frac{1}{2}ab^2x^6 + a^2bx^3 + \frac{1}{3}a^3\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/x,x, algorithm="maxima")
[Out] 1/9*b^3*x^9 + 1/2*a*b^2*x^6 + a^2*b*x^3 + 1/3*a^3*log(x^3)
```

mupad [B] time = 0.03, size = 32, normalized size = 0.89

$$a^3\,\ln(x) + \frac{b^3\,x^9}{9} + a^2\,b\,x^3 + \frac{a\,b^2\,x^6}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^3/x,x)
[Out] a^3*log(x) + (b^3*x^9)/9 + a^2*b*x^3 + (a*b^2*x^6)/2
```

sympy [A] time = 0.13, size = 32, normalized size = 0.89

$$a^3\log(x) + a^2bx^3 + \frac{ab^2x^6}{2} + \frac{b^3x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3/x,x)
[Out] a**3*log(x) + a**2*b*x**3 + a*b**2*x**6/2 + b**3*x**9/9
```

3.242 $\int \frac{\left(a+bx^3\right)^3}{x^4}\,dx$

Optimal. Leaf size=37

$$-\frac{a^3}{3x^3}+3a^2b\log(x)+ab^2x^3+\frac{b^3x^6}{6}$$

```
[Out] -1/3*a^3/x^3+a*b^2*x^3+1/6*b^3*x^6+3*a^2*b*ln(x)
```

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$3a^2b\log(x)-\frac{a^3}{3x^3}+ab^2x^3+\frac{b^3x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^3/x^4,x]
```

```
[Out] -a^3/(3*x^3) + a*b^2*x^3 + (b^3*x^6)/6 + 3*a^2*b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^3}{x^4}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)^3}{x^2}\,dx, x, x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(3ab^2+\frac{a^3}{x^2}+\frac{3a^2b}{x}+b^3x\right)\,dx, x, x^3\right)\\ &= -\frac{a^3}{3x^3}+ab^2x^3+\frac{b^3x^6}{6}+3a^2b\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 37, normalized size = 1.00

$$-\frac{a^3}{3x^3}+3a^2b\log(x)+ab^2x^3+\frac{b^3x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^3/x^4,x]
```

```
[Out] -1/3*a^3/x^3 + a*b^2*x^3 + (b^3*x^6)/6 + 3*a^2*b*Log[x]
```

fricas [A] time = 0.54, size = 38, normalized size = 1.03

$$\frac{b^3x^9 + 6\,ab^2x^6 + 18\,a^2bx^3\log(x) - 2\,a^3}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/x^4,x, algorithm="fricas")
[Out] 1/6*(b^3*x^9 + 6*a*b^2*x^6 + 18*a^2*b*x^3*log(x) - 2*a^3)/x^3
```

giac [A] time = 0.17, size = 44, normalized size = 1.19

$$\frac{1}{6}\,b^3x^6 + ab^2x^3 + 3\,a^2b\log\left(|x|\right) - \frac{3\,a^2bx^3 + a^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/x^4,x, algorithm="giac")
[Out] 1/6*b^3*x^6 + a*b^2*x^3 + 3*a^2*b*log(abs(x)) - 1/3*(3*a^2*b*x^3 + a^3)/x^3
```

maple [A] time = 0.01, size = 34, normalized size = 0.92

$$\frac{b^3x^6}{6} + a\,b^2x^3 + 3a^2b\ln(x) - \frac{a^3}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3/x^4,x)
[Out] -1/3*a^3/x^3+a*b^2*x^3+1/6*b^3*x^6+3*a^2*b*ln(x)
```

maxima [A] time = 1.36, size = 34, normalized size = 0.92

$$\frac{1}{6}\,b^3x^6 + ab^2x^3 + a^2b\log\left(x^3\right) - \frac{a^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/x^4,x, algorithm="maxima")
[Out] 1/6*b^3*x^6 + a*b^2*x^3 + a^2*b*log(x^3) - 1/3*a^3/x^3
```

mupad [B] time = 0.03, size = 33, normalized size = 0.89

$$\frac{b^3\,x^6}{6} - \frac{a^3}{3\,x^3} + a\,b^2\,x^3 + 3\,a^2\,b\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^3/x^4,x)
[Out] (b^3*x^6)/6 - a^3/(3*x^3) + a*b^2*x^3 + 3*a^2*b*log(x)
```

sympy [A] time = 0.17, size = 34, normalized size = 0.92

$$-\frac{a^3}{3x^3} + 3a^2b\log(x) + ab^2x^3 + \frac{b^3x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3/x**4,x)
[Out] -a**3/(3*x**3) + 3*a**2*b*log(x) + a*b**2*x**3 + b**3*x**6/6
```

3.243 $\int \frac{\left(a+bx^3\right)^3}{x^7}\,dx$

Optimal. Leaf size=38

$$-\frac{a^3}{6x^6}-\frac{a^2b}{x^3}+3ab^2\log(x)+\frac{b^3x^3}{3}$$

```
[Out] -1/6*a^3/x^6-a^2*b/x^3+1/3*b^3*x^3+3*a*b^2*ln(x)
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$-\frac{a^2b}{x^3}-\frac{a^3}{6x^6}+3ab^2\log(x)+\frac{b^3x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^3/x^7,x]
```

```
[Out] -a^3/(6*x^6) - (a^2*b)/x^3 + (b^3*x^3)/3 + 3*a*b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^3}{x^7}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)^3}{x^3}\,dx, x, x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(b^3+\frac{a^3}{x^3}+\frac{3a^2b}{x^2}+\frac{3ab^2}{x}\right)\,dx, x, x^3\right)\\ &= -\frac{a^3}{6x^6}-\frac{a^2b}{x^3}+\frac{b^3x^3}{3}+3ab^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 38, normalized size = 1.00

$$-\frac{a^3}{6x^6}-\frac{a^2b}{x^3}+3ab^2\log(x)+\frac{b^3x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^3/x^7,x]
```

```
[Out] -1/6*a^3/x^6 - (a^2*b)/x^3 + (b^3*x^3)/3 + 3*a*b^2*Log[x]
```

fricas [A] time = 0.83, size = 39, normalized size = 1.03

$$\frac{2\,b^3x^9+18\,ab^2x^6\log(x)-6\,a^2bx^3-a^3}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/x^7,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^9 + 18*a*b^2*x^6*log(x) - 6*a^2*b*x^3 - a^3)/x^6
```

giac [A] time = 0.19, size = 45, normalized size = 1.18

$$\frac{1}{3}\,b^3x^3 + 3\,ab^2\log\left(|x|\right) - \frac{9\,ab^2x^6 + 6\,a^2bx^3 + a^3}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/x^7,x, algorithm="giac")
```

```
[Out] 1/3*b^3*x^3 + 3*a*b^2*log(abs(x)) - 1/6*(9*a*b^2*x^6 + 6*a^2*b*x^3 + a^3)/x
^6
```

maple [A] time = 0.01, size = 35, normalized size = 0.92

$$\frac{b^3x^3}{3} + 3a\,b^2\ln(x) - \frac{a^2b}{x^3} - \frac{a^3}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3/x^7,x)
```

```
[Out] -1/6*a^3/x^6-a^2*b/x^3+1/3*b^3*x^3+3*a*b^2*ln(x)
```

maxima [A] time = 1.35, size = 36, normalized size = 0.95

$$\frac{1}{3}\,b^3x^3 + ab^2\log\left(x^3\right) - \frac{6\,a^2bx^3 + a^3}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/x^7,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*x^3 + a*b^2*log(x^3) - 1/6*(6*a^2*b*x^3 + a^3)/x^6
```

mupad [B] time = 0.94, size = 36, normalized size = 0.95

$$\frac{b^3\,x^3}{3} - \frac{\frac{a^3}{6} + b\,a^2\,x^3}{x^6} + 3\,a\,b^2\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^3/x^7,x)
```

```
[Out] (b^3*x^3)/3 - (a^3/6 + a^2*b*x^3)/x^6 + 3*a*b^2*log(x)
```

sympy [A] time = 0.25, size = 37, normalized size = 0.97

$$3ab^2\log\left(x\right) + \frac{b^3x^3}{3} + \frac{-a^3 - 6a^2bx^3}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3/x**7,x)
```

```
[Out] 3*a*b**2*log(x) + b**3*x**3/3 + (-a**3 - 6*a**2*b*x**3)/(6*x**6)
```

3.244 $\int \frac{\left(a+bx^3\right)^3}{x^{10}}\,dx$

Optimal. Leaf size=37

$$-\frac{a^3}{9x^9}-\frac{a^2b}{2x^6}-\frac{ab^2}{x^3}+b^3\log(x)$$

```
[Out] -1/9*a^3/x^9-1/2*a^2*b/x^6-a*b^2/x^3+b^3*ln(x)
```

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$-\frac{a^2b}{2x^6}-\frac{a^3}{9x^9}-\frac{ab^2}{x^3}+b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^3/x^10,x]
```

```
[Out] -a^3/(9*x^9) - (a^2*b)/(2*x^6) - (a*b^2)/x^3 + b^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^3}{x^{10}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^3}{x^4}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{a^3}{x^4}+\frac{3a^2b}{x^3}+\frac{3ab^2}{x^2}+\frac{b^3}{x}\right)\,dx, x, x^3\right)\\ &= -\frac{a^3}{9x^9}-\frac{a^2b}{2x^6}-\frac{ab^2}{x^3}+b^3\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 37, normalized size = 1.00

$$-\frac{a^3}{9x^9}-\frac{a^2b}{2x^6}-\frac{ab^2}{x^3}+b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^3/x^10,x]
```

```
[Out] -1/9*a^3/x^9 - (a^2*b)/(2*x^6) - (a*b^2)/x^3 + b^3*Log[x]
```

fricas [A] time = 0.82, size = 39, normalized size = 1.05

$$\frac{18\,b^3x^9\log(x)-18\,ab^2x^6-9\,a^2bx^3-2\,a^3}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/x^10,x, algorithm="fricas")
```

```
[Out] 1/18*(18*b^3*x^9*log(x) - 18*a*b^2*x^6 - 9*a^2*b*x^3 - 2*a^3)/x^9
```

giac [A] time = 0.16, size = 45, normalized size = 1.22

$$b^3 \log(|x|) - \frac{11\,b^3x^9 + 18\,ab^2x^6 + 9\,a^2bx^3 + 2\,a^3}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/x^10,x, algorithm="giac")
```

```
[Out] b^3*log(abs(x)) - 1/18*(11*b^3*x^9 + 18*a*b^2*x^6 + 9*a^2*b*x^3 + 2*a^3)/x^
9
```

maple [A] time = 0.01, size = 34, normalized size = 0.92

$$b^3 \ln(x) - \frac{a\,b^2}{x^3} - \frac{a^2b}{2x^6} - \frac{a^3}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3/x^10,x)
```

```
[Out] -1/9*a^3/x^9-1/2*a^2*b/x^6-a*b^2/x^3+b^3*ln(x)
```

maxima [A] time = 1.33, size = 39, normalized size = 1.05

$$\frac{1}{3}\,b^3 \log\left(x^3\right) - \frac{18\,ab^2x^6 + 9\,a^2bx^3 + 2\,a^3}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/x^10,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*log(x^3) - 1/18*(18*a*b^2*x^6 + 9*a^2*b*x^3 + 2*a^3)/x^9
```

mupad [B] time = 0.95, size = 35, normalized size = 0.95

$$b^3\,\ln(x) - \frac{\frac{a^3}{9} + \frac{a^2\,b\,x^3}{2} + a\,b^2\,x^6}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^3/x^10,x)
```

```
[Out] b^3*log(x) - (a^3/9 + (a^2*b*x^3)/2 + a*b^2*x^6)/x^9
```

sympy [A] time = 0.32, size = 37, normalized size = 1.00

$$b^3 \log(x) + \frac{-2a^3 - 9a^2bx^3 - 18ab^2x^6}{18x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3/x**10,x)
```

```
[Out] b**3*log(x) + (-2*a**3 - 9*a**2*b*x**3 - 18*a*b**2*x**6)/(18*x**9)
```

3.245 $\int \frac{\left(a+bx^3\right)^3}{x^{13}}\,dx$

Optimal. Leaf size=19

$$-\frac{\left(a+bx^3\right)^4}{12ax^{12}}$$

```
[Out] -1/12*(b*x^3+a)^4/a/x^12
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {264}

$$-\frac{\left(a+bx^3\right)^4}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^3/x^13,x]
```

```
[Out] -(a + b*x^3)^4/(12*a*x^12)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^3}{x^{13}}\,dx = -\frac{\left(a+bx^3\right)^4}{12ax^{12}}$$

Mathematica [B] time = 0.01, size = 43, normalized size = 2.26

$$-\frac{a^3}{12x^{12}} - \frac{a^2b}{3x^9} - \frac{ab^2}{2x^6} - \frac{b^3}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^3/x^13,x]
```

```
[Out] -1/12*a^3/x^12 - (a^2*b)/(3*x^9) - (a*b^2)/(2*x^6) - b^3/(3*x^3)
```

fricas [B] time = 0.77, size = 35, normalized size = 1.84

$$-\frac{4\,b^3x^9 + 6\,ab^2x^6 + 4\,a^2bx^3 + a^3}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/x^13,x, algorithm="fricas")
```

```
[Out] -1/12*(4*b^3*x^9 + 6*a*b^2*x^6 + 4*a^2*b*x^3 + a^3)/x^12
```

giac [B] time = 0.15, size = 35, normalized size = 1.84

$$-\frac{4\,b^3x^9 + 6\,ab^2x^6 + 4\,a^2bx^3 + a^3}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/x^13,x, algorithm="giac")
[Out] -1/12*(4*b^3*x^9 + 6*a*b^2*x^6 + 4*a^2*b*x^3 + a^3)/x^12
```

maple [B] time = 0.00, size = 36, normalized size = 1.89

$$-\frac{b^3}{3x^3}-\frac{a\,b^2}{2x^6}-\frac{a^2b}{3x^9}-\frac{a^3}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3/x^13,x)
[Out] -1/3*b^3/x^3-1/12*a^3/x^12-1/2*a*b^2/x^6-1/3*a^2*b/x^9
```

maxima [B] time = 1.35, size = 35, normalized size = 1.84

$$-\frac{4\,b^3x^9+6\,ab^2x^6+4\,a^2bx^3+a^3}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/x^13,x, algorithm="maxima")
[Out] -1/12*(4*b^3*x^9 + 6*a*b^2*x^6 + 4*a^2*b*x^3 + a^3)/x^12
```

mupad [B] time = 0.03, size = 37, normalized size = 1.95

$$-\frac{\frac{a^3}{12}+\frac{a^2\,b\,x^3}{3}+\frac{a\,b^2\,x^6}{2}+\frac{b^3\,x^9}{3}}{x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^3/x^13,x)
[Out] -(a^3/12 + (b^3*x^9)/3 + (a^2*b*x^3)/3 + (a*b^2*x^6)/2)/x^12
```

sympy [B] time = 0.38, size = 37, normalized size = 1.95

$$\frac{-a^3-4a^2bx^3-6ab^2x^6-4b^3x^9}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3/x**13,x)
[Out] (-a**3 - 4*a**2*b*x**3 - 6*a*b**2*x**6 - 4*b**3*x**9)/(12*x**12)
```

3.246 $\int \frac{\left(a+bx^3\right)^3}{x^{16}}\,dx$

Optimal. Leaf size=40

$$\frac{b\left(a+bx^3\right)^4}{60a^2x^{12}}-\frac{\left(a+bx^3\right)^4}{15ax^{15}}$$

```
[Out] -1/15*(b*x^3+a)^4/a/x^15+1/60*b*(b*x^3+a)^4/a^2/x^12
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {266, 45, 37}

$$\frac{b\left(a+bx^3\right)^4}{60a^2x^{12}}-\frac{\left(a+bx^3\right)^4}{15ax^{15}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^3/x^16,x]
```

```
[Out] -(a + b*x^3)^4/(15*a*x^15) + (b*(a + b*x^3)^4)/(60*a^2*x^12)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^3}{x^{16}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^3}{x^6}\,dx, x, x^3\right)\\ &= -\frac{\left(a+bx^3\right)^4}{15ax^{15}}-\frac{b\,\text{Subst}\left(\int \frac{(a+bx)^3}{x^5}\,dx, x, x^3\right)}{15a}\\ &= -\frac{\left(a+bx^3\right)^4}{15ax^{15}}+\frac{b\left(a+bx^3\right)^4}{60a^2x^{12}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.08

$$-\frac{a^3}{15x^{15}}-\frac{a^2b}{4x^{12}}-\frac{ab^2}{3x^9}-\frac{b^3}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^3/x^16,x]
[Out] -1/15*a^3/x^15 - (a^2*b)/(4*x^12) - (a*b^2)/(3*x^9) - b^3/(6*x^6)
```

fricas [A] time = 0.78, size = 37, normalized size = 0.92

$$-\frac{10\,b^3x^9+20\,ab^2x^6+15\,a^2bx^3+4\,a^3}{60\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/x^16,x, algorithm="fricas")
[Out] -1/60*(10*b^3*x^9 + 20*a*b^2*x^6 + 15*a^2*b*x^3 + 4*a^3)/x^15
```

giac [A] time = 0.17, size = 37, normalized size = 0.92

$$-\frac{10\,b^3x^9+20\,ab^2x^6+15\,a^2bx^3+4\,a^3}{60\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/x^16,x, algorithm="giac")
[Out] -1/60*(10*b^3*x^9 + 20*a*b^2*x^6 + 15*a^2*b*x^3 + 4*a^3)/x^15
```

maple [A] time = 0.01, size = 36, normalized size = 0.90

$$-\frac{b^3}{6x^6}-\frac{a\,b^2}{3x^9}-\frac{a^2b}{4x^{12}}-\frac{a^3}{15x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3/x^16,x)
[Out] -1/15*a^3/x^15-1/6*b^3/x^6-1/4*a^2*b/x^12-1/3*a*b^2/x^9
```

maxima [A] time = 1.31, size = 37, normalized size = 0.92

$$-\frac{10\,b^3x^9+20\,ab^2x^6+15\,a^2bx^3+4\,a^3}{60\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/x^16,x, algorithm="maxima")
[Out] -1/60*(10*b^3*x^9 + 20*a*b^2*x^6 + 15*a^2*b*x^3 + 4*a^3)/x^15
```

mupad [B] time = 0.03, size = 37, normalized size = 0.92

$$-\frac{\frac{a^3}{15}+\frac{a^2\,b\,x^3}{4}+\frac{a\,b^2\,x^6}{3}+\frac{b^3\,x^9}{6}}{x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^3/x^16,x)
```

```
[Out] -(a^3/15 + (b^3*x^9)/6 + (a^2*b*x^3)/4 + (a*b^2*x^6)/3)/x^15
```

sympy [**A**] time = 0.51, size = 39, normalized size = 0.98

$$\frac{-4a^3 - 15a^2bx^3 - 20ab^2x^6 - 10b^3x^9}{60x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3/x**16,x)
[Out] (-4*a**3 - 15*a**2*b*x**3 - 20*a*b**2*x**6 - 10*b**3*x**9)/(60*x**15)
```

3.247 $\int \frac{\left(a+bx^3\right)^3}{x^{19}}\,dx$

Optimal. Leaf size=43

$$-\frac{a^3}{18x^{18}}-\frac{a^2b}{5x^{15}}-\frac{ab^2}{4x^{12}}-\frac{b^3}{9x^9}$$

```
[Out] -1/18*a^3/x^18-1/5*a^2*b/x^15-1/4*a*b^2/x^12-1/9*b^3/x^9
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$-\frac{a^2b}{5x^{15}}-\frac{a^3}{18x^{18}}-\frac{ab^2}{4x^{12}}-\frac{b^3}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^3/x^19,x]
```

```
[Out] -a^3/(18*x^18) - (a^2*b)/(5*x^15) - (a*b^2)/(4*x^12) - b^3/(9*x^9)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^3}{x^{19}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^3}{x^7}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{a^3}{x^7}+\frac{3a^2b}{x^6}+\frac{3ab^2}{x^5}+\frac{b^3}{x^4}\right)dx, x, x^3\right)\\ &= -\frac{a^3}{18x^{18}}-\frac{a^2b}{5x^{15}}-\frac{ab^2}{4x^{12}}-\frac{b^3}{9x^9}\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$-\frac{a^3}{18x^{18}}-\frac{a^2b}{5x^{15}}-\frac{ab^2}{4x^{12}}-\frac{b^3}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^3/x^19,x]
```

```
[Out] -1/18*a^3/x^18 - (a^2*b)/(5*x^15) - (a*b^2)/(4*x^12) - b^3/(9*x^9)
```

fricas [A] time = 0.55, size = 37, normalized size = 0.86

$$-\frac{20\,b^3x^9+45\,ab^2x^6+36\,a^2bx^3+10\,a^3}{180\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/x^19,x, algorithm="fricas")
[Out] -1/180*(20*b^3*x^9 + 45*a*b^2*x^6 + 36*a^2*b*x^3 + 10*a^3)/x^18
```

giac [A] time = 0.15, size = 37, normalized size = 0.86

$$-\frac{20\,b^3x^9+45\,ab^2x^6+36\,a^2bx^3+10\,a^3}{180\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/x^19,x, algorithm="giac")
[Out] -1/180*(20*b^3*x^9 + 45*a*b^2*x^6 + 36*a^2*b*x^3 + 10*a^3)/x^18
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$-\frac{b^3}{9x^9}-\frac{a\,b^2}{4x^{12}}-\frac{a^2b}{5x^{15}}-\frac{a^3}{18x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3/x^19,x)
[Out] -1/18*a^3/x^18-1/5*a^2*b/x^15-1/4*a*b^2/x^12-1/9*b^3/x^9
```

maxima [A] time = 1.35, size = 37, normalized size = 0.86

$$-\frac{20\,b^3x^9+45\,ab^2x^6+36\,a^2bx^3+10\,a^3}{180\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/x^19,x, algorithm="maxima")
[Out] -1/180*(20*b^3*x^9 + 45*a*b^2*x^6 + 36*a^2*b*x^3 + 10*a^3)/x^18
```

mupad [B] time = 0.03, size = 37, normalized size = 0.86

$$-\frac{\frac{a^3}{18}+\frac{a^2\,b\,x^3}{5}+\frac{a\,b^2\,x^6}{4}+\frac{b^3\,x^9}{9}}{x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^3/x^19,x)
[Out] -(a^3/18 + (b^3*x^9)/9 + (a^2*b*x^3)/5 + (a*b^2*x^6)/4)/x^18
```

sympy [A] time = 0.57, size = 39, normalized size = 0.91

$$\frac{-10a^3-36a^2bx^3-45ab^2x^6-20b^3x^9}{180x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3/x**19,x)
[Out] (-10*a**3 - 36*a**2*b*x**3 - 45*a*b**2*x**6 - 20*b**3*x**9)/(180*x**18)
```

3.248 $\int \frac{\left(a+bx^3\right)^3}{x^{22}}\,dx$

Optimal. Leaf size=43

$$-\frac{a^3}{21x^{21}}-\frac{a^2b}{6x^{18}}-\frac{ab^2}{5x^{15}}-\frac{b^3}{12x^{12}}$$

```
[Out] -1/21*a^3/x^21-1/6*a^2*b/x^18-1/5*a*b^2/x^15-1/12*b^3/x^12
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$-\frac{a^2b}{6x^{18}}-\frac{a^3}{21x^{21}}-\frac{ab^2}{5x^{15}}-\frac{b^3}{12x^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^3/x^22,x]
```

```
[Out] -a^3/(21*x^21) - (a^2*b)/(6*x^18) - (a*b^2)/(5*x^15) - b^3/(12*x^12)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^3}{x^{22}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^3}{x^8}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{a^3}{x^8}+\frac{3a^2b}{x^7}+\frac{3ab^2}{x^6}+\frac{b^3}{x^5}\right)dx, x, x^3\right)\\ &= -\frac{a^3}{21x^{21}}-\frac{a^2b}{6x^{18}}-\frac{ab^2}{5x^{15}}-\frac{b^3}{12x^{12}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$-\frac{a^3}{21x^{21}}-\frac{a^2b}{6x^{18}}-\frac{ab^2}{5x^{15}}-\frac{b^3}{12x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^3/x^22,x]
```

```
[Out] -1/21*a^3/x^21 - (a^2*b)/(6*x^18) - (a*b^2)/(5*x^15) - b^3/(12*x^12)
```

fricas [A] time = 0.82, size = 37, normalized size = 0.86

$$-\frac{35\,b^3x^9+84\,ab^2x^6+70\,a^2bx^3+20\,a^3}{420\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/x^22,x, algorithm="fricas")
[Out] -1/420*(35*b^3*x^9 + 84*a*b^2*x^6 + 70*a^2*b*x^3 + 20*a^3)/x^21
```

giac [A] time = 0.15, size = 37, normalized size = 0.86

$$-\frac{35\,b^3x^9+84\,ab^2x^6+70\,a^2bx^3+20\,a^3}{420\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/x^22,x, algorithm="giac")
[Out] -1/420*(35*b^3*x^9 + 84*a*b^2*x^6 + 70*a^2*b*x^3 + 20*a^3)/x^21
```

maple [A] time = 0.01, size = 36, normalized size = 0.84

$$-\frac{b^3}{12x^{12}}-\frac{a\,b^2}{5x^{15}}-\frac{a^2b}{6x^{18}}-\frac{a^3}{21x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3/x^22,x)
[Out] -1/21*a^3/x^21-1/6*a^2*b/x^18-1/5*a*b^2/x^15-1/12*b^3/x^12
```

maxima [A] time = 1.35, size = 37, normalized size = 0.86

$$-\frac{35\,b^3x^9+84\,ab^2x^6+70\,a^2bx^3+20\,a^3}{420\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/x^22,x, algorithm="maxima")
[Out] -1/420*(35*b^3*x^9 + 84*a*b^2*x^6 + 70*a^2*b*x^3 + 20*a^3)/x^21
```

mupad [B] time = 0.03, size = 37, normalized size = 0.86

$$-\frac{\frac{a^3}{21}+\frac{a^2\,b\,x^3}{6}+\frac{a\,b^2\,x^6}{5}+\frac{b^3\,x^9}{12}}{x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^3/x^22,x)
[Out] -(a^3/21 + (b^3*x^9)/12 + (a^2*b*x^3)/6 + (a*b^2*x^6)/5)/x^21
```

sympy [A] time = 0.49, size = 39, normalized size = 0.91

$$\frac{-20a^3-70a^2bx^3-84ab^2x^6-35b^3x^9}{420x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3/x**22,x)
[Out] (-20*a**3 - 70*a**2*b*x**3 - 84*a*b**2*x**6 - 35*b**3*x**9)/(420*x**21)
```

3.249 $\int x^4 \left(a + bx^3\right)^3 \, dx$

Optimal. Leaf size=43

$$\frac{a^3x^5}{5} + \frac{3}{8}a^2bx^8 + \frac{3}{11}ab^2x^{11} + \frac{b^3x^{14}}{14}$$

```
[Out] 1/5*a^3*x^5+3/8*a^2*b*x^8+3/11*a*b^2*x^11+1/14*b^3*x^14
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{3}{8}a^2bx^8 + \frac{a^3x^5}{5} + \frac{3}{11}ab^2x^{11} + \frac{b^3x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^3)^3,x]
```

```
[Out] (a^3*x^5)/5 + (3*a^2*b*x^8)/8 + (3*a*b^2*x^11)/11 + (b^3*x^14)/14
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^4 \left(a + bx^3\right)^3 \, dx &= \int \left(a^3x^4 + 3a^2bx^7 + 3ab^2x^{10} + b^3x^{13}\right) dx \\ &= \frac{a^3x^5}{5} + \frac{3}{8}a^2bx^8 + \frac{3}{11}ab^2x^{11} + \frac{b^3x^{14}}{14}\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^5}{5} + \frac{3}{8}a^2bx^8 + \frac{3}{11}ab^2x^{11} + \frac{b^3x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^3)^3,x]
```

```
[Out] (a^3*x^5)/5 + (3*a^2*b*x^8)/8 + (3*a*b^2*x^11)/11 + (b^3*x^14)/14
```

fricas [A] time = 0.58, size = 35, normalized size = 0.81

$$\frac{1}{14}x^{14}b^3 + \frac{3}{11}x^{11}b^2a + \frac{3}{8}x^8ba^2 + \frac{1}{5}x^5a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*b^3 + 3/11*x^11*b^2*a + 3/8*x^8*b*a^2 + 1/5*x^5*a^3
```

giac [A] time = 0.23, size = 35, normalized size = 0.81

$$\frac{1}{14}\,b^3x^{14} + \frac{3}{11}\,ab^2x^{11} + \frac{3}{8}\,a^2bx^8 + \frac{1}{5}\,a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^3,x, algorithm="giac")
[Out] 1/14*b^3*x^14 + 3/11*a*b^2*x^11 + 3/8*a^2*b*x^8 + 1/5*a^3*x^5
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$\frac{1}{14}b^3x^{14} + \frac{3}{11}a\,b^2x^{11} + \frac{3}{8}a^2b\,x^8 + \frac{1}{5}a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)^3,x)
[Out] 1/5*a^3*x^5+3/8*a^2*b*x^8+3/11*a*b^2*x^11+1/14*b^3*x^14
```

maxima [A] time = 1.33, size = 35, normalized size = 0.81

$$\frac{1}{14}\,b^3x^{14} + \frac{3}{11}\,ab^2x^{11} + \frac{3}{8}\,a^2bx^8 + \frac{1}{5}\,a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^3,x, algorithm="maxima")
[Out] 1/14*b^3*x^14 + 3/11*a*b^2*x^11 + 3/8*a^2*b*x^8 + 1/5*a^3*x^5
```

mupad [B] time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3\,x^5}{5} + \frac{3\,a^2\,b\,x^8}{8} + \frac{3\,a\,b^2\,x^{11}}{11} + \frac{b^3\,x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^3)^3,x)
[Out] (a^3*x^5)/5 + (b^3*x^14)/14 + (3*a^2*b*x^8)/8 + (3*a*b^2*x^11)/11
```

sympy [A] time = 0.13, size = 39, normalized size = 0.91

$$\frac{a^3x^5}{5} + \frac{3a^2bx^8}{8} + \frac{3ab^2x^{11}}{11} + \frac{b^3x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)**3,x)
[Out] a**3*x**5/5 + 3*a**2*b*x**8/8 + 3*a*b**2*x**11/11 + b**3*x**14/14
```

3.250 $\int x^3 \left(a + bx^3\right)^3 \, dx$

Optimal. Leaf size=43

$$\frac{a^3x^4}{4} + \frac{3}{7}a^2bx^7 + \frac{3}{10}ab^2x^{10} + \frac{b^3x^{13}}{13}$$

```
[Out] 1/4*a^3*x^4+3/7*a^2*b*x^7+3/10*a*b^2*x^10+1/13*b^3*x^13
```

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{3}{7}a^2bx^7 + \frac{a^3x^4}{4} + \frac{3}{10}ab^2x^{10} + \frac{b^3x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^3)^3,x]
```

```
[Out] (a^3*x^4)/4 + (3*a^2*b*x^7)/7 + (3*a*b^2*x^10)/10 + (b^3*x^13)/13
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^3 \left(a + bx^3\right)^3 \, dx &= \int \left(a^3x^3 + 3a^2bx^6 + 3ab^2x^9 + b^3x^{12}\right) dx \\ &= \frac{a^3x^4}{4} + \frac{3}{7}a^2bx^7 + \frac{3}{10}ab^2x^{10} + \frac{b^3x^{13}}{13}\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^4}{4} + \frac{3}{7}a^2bx^7 + \frac{3}{10}ab^2x^{10} + \frac{b^3x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^3)^3,x]
```

```
[Out] (a^3*x^4)/4 + (3*a^2*b*x^7)/7 + (3*a*b^2*x^10)/10 + (b^3*x^13)/13
```

fricas [A] time = 0.71, size = 35, normalized size = 0.81

$$\frac{1}{13}x^{13}b^3 + \frac{3}{10}x^{10}b^2a + \frac{3}{7}x^7ba^2 + \frac{1}{4}x^4a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*b^3 + 3/10*x^10*b^2*a + 3/7*x^7*b*a^2 + 1/4*x^4*a^3
```

giac [A] time = 0.18, size = 35, normalized size = 0.81

$$\frac{1}{13}\,b^3x^{13} + \frac{3}{10}\,ab^2x^{10} + \frac{3}{7}\,a^2bx^7 + \frac{1}{4}\,a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^3,x, algorithm="giac")
[Out] 1/13*b^3*x^13 + 3/10*a*b^2*x^10 + 3/7*a^2*b*x^7 + 1/4*a^3*x^4
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$\frac{1}{13}b^3x^{13} + \frac{3}{10}a\,b^2x^{10} + \frac{3}{7}a^2b\,x^7 + \frac{1}{4}a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)^3,x)
[Out] 1/4*a^3*x^4+3/7*a^2*b*x^7+3/10*a*b^2*x^10+1/13*b^3*x^13
```

maxima [A] time = 1.29, size = 35, normalized size = 0.81

$$\frac{1}{13}\,b^3x^{13} + \frac{3}{10}\,ab^2x^{10} + \frac{3}{7}\,a^2bx^7 + \frac{1}{4}\,a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^3,x, algorithm="maxima")
[Out] 1/13*b^3*x^13 + 3/10*a*b^2*x^10 + 3/7*a^2*b*x^7 + 1/4*a^3*x^4
```

mupad [B] time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3\,x^4}{4} + \frac{3\,a^2\,b\,x^7}{7} + \frac{3\,a\,b^2\,x^{10}}{10} + \frac{b^3\,x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^3)^3,x)
[Out] (a^3*x^4)/4 + (b^3*x^13)/13 + (3*a^2*b*x^7)/7 + (3*a*b^2*x^10)/10
```

sympy [A] time = 0.08, size = 39, normalized size = 0.91

$$\frac{a^3x^4}{4} + \frac{3a^2bx^7}{7} + \frac{3ab^2x^{10}}{10} + \frac{b^3x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**3,x)
[Out] a**3*x**4/4 + 3*a**2*b*x**7/7 + 3*a*b**2*x**10/10 + b**3*x**13/13
```

3.251 $\int x\left(a+bx^3\right)^3\,dx$

Optimal. Leaf size=43

$$\frac{a^3x^2}{2}+\frac{3}{5}a^2bx^5+\frac{3}{8}ab^2x^8+\frac{b^3x^{11}}{11}$$

```
[Out] 1/2*a^3*x^2+3/5*a^2*b*x^5+3/8*a*b^2*x^8+1/11*b^3*x^11
```

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {270}

$$\frac{3}{5}a^2bx^5+\frac{a^3x^2}{2}+\frac{3}{8}ab^2x^8+\frac{b^3x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3)^3,x]
```

```
[Out] (a^3*x^2)/2 + (3*a^2*b*x^5)/5 + (3*a*b^2*x^8)/8 + (b^3*x^11)/11
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^3\right)^3\,dx &= \int\left(a^3x+3a^2bx^4+3ab^2x^7+b^3x^{10}\right)dx\\ &= \frac{a^3x^2}{2}+\frac{3}{5}a^2bx^5+\frac{3}{8}ab^2x^8+\frac{b^3x^{11}}{11}\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^2}{2}+\frac{3}{5}a^2bx^5+\frac{3}{8}ab^2x^8+\frac{b^3x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)^3,x]
```

```
[Out] (a^3*x^2)/2 + (3*a^2*b*x^5)/5 + (3*a*b^2*x^8)/8 + (b^3*x^11)/11
```

fricas [A] time = 0.75, size = 35, normalized size = 0.81

$$\frac{1}{11}x^{11}b^3+\frac{3}{8}x^8b^2a+\frac{3}{5}x^5ba^2+\frac{1}{2}x^2a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*b^3 + 3/8*x^8*b^2*a + 3/5*x^5*b*a^2 + 1/2*x^2*a^3
```

giac [A] time = 0.15, size = 35, normalized size = 0.81

$$\frac{1}{11}\,b^3x^{11}+\frac{3}{8}\,ab^2x^8+\frac{3}{5}\,a^2bx^5+\frac{1}{2}\,a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^3,x, algorithm="giac")
[Out] 1/11*b^3*x^11 + 3/8*a*b^2*x^8 + 3/5*a^2*b*x^5 + 1/2*a^3*x^2
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$\frac{1}{11}b^3x^{11} + \frac{3}{8}a\,b^2x^8 + \frac{3}{5}a^2b\,x^5 + \frac{1}{2}a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^3,x)
[Out] 1/2*x^2*a^3+3/5*a^2*b*x^5+3/8*a*b^2*x^8+1/11*b^3*x^11
```

maxima [A] time = 1.37, size = 35, normalized size = 0.81

$$\frac{1}{11}\,b^3x^{11} + \frac{3}{8}\,ab^2x^8 + \frac{3}{5}\,a^2bx^5 + \frac{1}{2}\,a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^3,x, algorithm="maxima")
[Out] 1/11*b^3*x^11 + 3/8*a*b^2*x^8 + 3/5*a^2*b*x^5 + 1/2*a^3*x^2
```

mupad [B] time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3\,x^2}{2} + \frac{3\,a^2\,b\,x^5}{5} + \frac{3\,a\,b^2\,x^8}{8} + \frac{b^3\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^3)^3,x)
[Out] (a^3*x^2)/2 + (b^3*x^11)/11 + (3*a^2*b*x^5)/5 + (3*a*b^2*x^8)/8
```

sympy [A] time = 0.07, size = 39, normalized size = 0.91

$$\frac{a^3x^2}{2} + \frac{3a^2bx^5}{5} + \frac{3ab^2x^8}{8} + \frac{b^3x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**3,x)
[Out] a**3*x**2/2 + 3*a**2*b*x**5/5 + 3*a*b**2*x**8/8 + b**3*x**11/11
```

3.252 $\int \left(a + bx^3\right)^3 \, dx$

Optimal. Leaf size=38

$$a^3x + \frac{3}{4}a^2bx^4 + \frac{3}{7}ab^2x^7 + \frac{b^3x^{10}}{10}$$

```
[Out] a^3*x+3/4*a^2*b*x^4+3/7*a*b^2*x^7+1/10*b^3*x^10
```

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {194}

$$\frac{3}{4}a^2bx^4 + a^3x + \frac{3}{7}ab^2x^7 + \frac{b^3x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^3,x]
```

```
[Out] a^3*x + (3*a^2*b*x^4)/4 + (3*a*b^2*x^7)/7 + (b^3*x^10)/10
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a + bx^3\right)^3 \, dx &= \int \left(a^3 + 3a^2bx^3 + 3ab^2x^6 + b^3x^9\right) \, dx \\ &= a^3x + \frac{3}{4}a^2bx^4 + \frac{3}{7}ab^2x^7 + \frac{b^3x^{10}}{10}\end{aligned}$$

Mathematica [A] time = 0.00, size = 38, normalized size = 1.00

$$a^3x + \frac{3}{4}a^2bx^4 + \frac{3}{7}ab^2x^7 + \frac{b^3x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^3,x]
```

```
[Out] a^3*x + (3*a^2*b*x^4)/4 + (3*a*b^2*x^7)/7 + (b^3*x^10)/10
```

fricas [A] time = 0.74, size = 32, normalized size = 0.84

$$\frac{1}{10}x^{10}b^3 + \frac{3}{7}x^7b^2a + \frac{3}{4}x^4ba^2 + xa^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*b^3 + 3/7*x^7*b^2*a + 3/4*x^4*b*a^2 + x*a^3
```

giac [A] time = 0.16, size = 32, normalized size = 0.84

$$\frac{1}{10}\,b^3x^{10} + \frac{3}{7}\,ab^2x^7 + \frac{3}{4}\,a^2bx^4 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3,x, algorithm="giac")
[Out] 1/10*b^3*x^10 + 3/7*a*b^2*x^7 + 3/4*a^2*b*x^4 + a^3*x
```

maple [A] time = 0.00, size = 33, normalized size = 0.87

$$\frac{1}{10}b^3x^{10} + \frac{3}{7}a\,b^2x^7 + \frac{3}{4}a^2b\,x^4 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3,x)
[Out] a^3*x+3/4*a^2*b*x^4+3/7*a*b^2*x^7+1/10*b^3*x^10
```

maxima [A] time = 1.35, size = 32, normalized size = 0.84

$$\frac{1}{10}\,b^3x^{10} + \frac{3}{7}\,ab^2x^7 + \frac{3}{4}\,a^2bx^4 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3,x, algorithm="maxima")
[Out] 1/10*b^3*x^10 + 3/7*a*b^2*x^7 + 3/4*a^2*b*x^4 + a^3*x
```

mupad [B] time = 0.04, size = 32, normalized size = 0.84

$$a^3\,x + \frac{3\,a^2\,b\,x^4}{4} + \frac{3\,a\,b^2\,x^7}{7} + \frac{b^3\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^3,x)
[Out] a^3*x + (b^3*x^10)/10 + (3*a^2*b*x^4)/4 + (3*a*b^2*x^7)/7
```

sympy [A] time = 0.08, size = 36, normalized size = 0.95

$$a^3x + \frac{3a^2bx^4}{4} + \frac{3ab^2x^7}{7} + \frac{b^3x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3,x)
[Out] a**3*x + 3*a**2*b*x**4/4 + 3*a*b**2*x**7/7 + b**3*x**10/10
```

3.253 $\int \frac{\left(a+bx^3\right)^3}{x^2}\,dx$

Optimal. Leaf size=41

$$-\frac{a^3}{x}+\frac{3}{2}a^2bx^2+\frac{3}{5}ab^2x^5+\frac{b^3x^8}{8}$$

```
[Out] -a^3/x+3/2*a^2*b*x^2+3/5*a*b^2*x^5+1/8*b^3*x^8
```

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{3}{2}a^2bx^2-\frac{a^3}{x}+\frac{3}{5}ab^2x^5+\frac{b^3x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^3/x^2,x]
```

```
[Out] -(a^3/x) + (3*a^2*b*x^2)/2 + (3*a*b^2*x^5)/5 + (b^3*x^8)/8
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^3}{x^2}\,dx &= \int \left(\frac{a^3}{x^2}+3a^2bx+3ab^2x^4+b^3x^7\right)dx\\ &= -\frac{a^3}{x}+\frac{3}{2}a^2bx^2+\frac{3}{5}ab^2x^5+\frac{b^3x^8}{8}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.00

$$-\frac{a^3}{x}+\frac{3}{2}a^2bx^2+\frac{3}{5}ab^2x^5+\frac{b^3x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^3/x^2,x]
```

```
[Out] -(a^3/x) + (3*a^2*b*x^2)/2 + (3*a*b^2*x^5)/5 + (b^3*x^8)/8
```

fricas [A] time = 0.80, size = 37, normalized size = 0.90

$$\frac{5\,b^3x^9+24\,ab^2x^6+60\,a^2bx^3-40\,a^3}{40\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/40*(5*b^3*x^9 + 24*a*b^2*x^6 + 60*a^2*b*x^3 - 40*a^3)/x
```

giac [A] time = 0.19, size = 35, normalized size = 0.85

$$\frac{1}{8}\,b^3x^8+\frac{3}{5}\,ab^2x^5+\frac{3}{2}\,a^2bx^2-\frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/x^2,x, algorithm="giac")
[Out] 1/8*b^3*x^8 + 3/5*a*b^2*x^5 + 3/2*a^2*b*x^2 - a^3/x
```

maple [A] time = 0.00, size = 36, normalized size = 0.88

$$\frac{b^3x^8}{8} + \frac{3a\,b^2x^5}{5} + \frac{3a^2b\,x^2}{2} - \frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3/x^2,x)
[Out] -a^3/x+3/2*a^2*b*x^2+3/5*a*b^2*x^5+1/8*b^3*x^8
```

maxima [A] time = 1.32, size = 35, normalized size = 0.85

$$\frac{1}{8}\,b^3x^8 + \frac{3}{5}\,ab^2x^5 + \frac{3}{2}\,a^2bx^2 - \frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/x^2,x, algorithm="maxima")
[Out] 1/8*b^3*x^8 + 3/5*a*b^2*x^5 + 3/2*a^2*b*x^2 - a^3/x
```

mupad [B] time = 0.04, size = 35, normalized size = 0.85

$$\frac{b^3\,x^8}{8} - \frac{a^3}{x} + \frac{3\,a^2\,b\,x^2}{2} + \frac{3\,a\,b^2\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^3/x^2,x)
[Out] (b^3*x^8)/8 - a^3/x + (3*a^2*b*x^2)/2 + (3*a*b^2*x^5)/5
```

sympy [A] time = 0.12, size = 36, normalized size = 0.88

$$-\frac{a^3}{x} + \frac{3a^2bx^2}{2} + \frac{3ab^2x^5}{5} + \frac{b^3x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3/x**2,x)
[Out] -a**3/x + 3*a**2*b*x**2/2 + 3*a*b**2*x**5/5 + b**3*x**8/8
```

3.254 $\int \frac{\left(a+bx^3\right)^3}{x^3}\,dx$

Optimal. Leaf size=39

$$-\frac{a^3}{2x^2}+3a^2bx+\frac{3}{4}ab^2x^4+\frac{b^3x^7}{7}$$

```
[Out] -1/2*a^3/x^2+3*a^2*b*x+3/4*a*b^2*x^4+1/7*b^3*x^7
```

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$3a^2bx-\frac{a^3}{2x^2}+\frac{3}{4}ab^2x^4+\frac{b^3x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^3/x^3,x]
```

```
[Out] -a^3/(2*x^2) + 3*a^2*b*x + (3*a*b^2*x^4)/4 + (b^3*x^7)/7
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^3}{x^3}\,dx &= \int \left(3a^2b+\frac{a^3}{x^3}+3ab^2x^3+b^3x^6\right)dx\\ &= -\frac{a^3}{2x^2}+3a^2bx+\frac{3}{4}ab^2x^4+\frac{b^3x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 1.00

$$-\frac{a^3}{2x^2}+3a^2bx+\frac{3}{4}ab^2x^4+\frac{b^3x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^3/x^3,x]
```

```
[Out] -1/2*a^3/x^2 + 3*a^2*b*x + (3*a*b^2*x^4)/4 + (b^3*x^7)/7
```

fricas [A] time = 0.87, size = 37, normalized size = 0.95

$$\frac{4\,b^3x^9+21\,ab^2x^6+84\,a^2bx^3-14\,a^3}{28\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/28*(4*b^3*x^9 + 21*a*b^2*x^6 + 84*a^2*b*x^3 - 14*a^3)/x^2
```

giac [A] time = 0.15, size = 33, normalized size = 0.85

$$\frac{1}{7}\,b^3x^7+\frac{3}{4}\,ab^2x^4+3\,a^2bx-\frac{a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/x^3,x, algorithm="giac")
[Out] 1/7*b^3*x^7 + 3/4*a*b^2*x^4 + 3*a^2*b*x - 1/2*a^3/x^2
```

maple [A] time = 0.00, size = 34, normalized size = 0.87

$$\frac{b^3x^7}{7} + \frac{3a\,b^2x^4}{4} + 3a^2bx - \frac{a^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3/x^3,x)
[Out] -1/2*a^3/x^2+3*a^2*b*x+3/4*a*b^2*x^4+1/7*b^3*x^7
```

maxima [A] time = 1.32, size = 33, normalized size = 0.85

$$\frac{1}{7}\,b^3x^7 + \frac{3}{4}\,ab^2x^4 + 3\,a^2bx - \frac{a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/x^3,x, algorithm="maxima")
[Out] 1/7*b^3*x^7 + 3/4*a*b^2*x^4 + 3*a^2*b*x - 1/2*a^3/x^2
```

mupad [B] time = 0.04, size = 33, normalized size = 0.85

$$\frac{b^3\,x^7}{7} - \frac{a^3}{2\,x^2} + \frac{3\,a\,b^2\,x^4}{4} + 3\,a^2\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^3/x^3,x)
[Out] (b^3*x^7)/7 - a^3/(2*x^2) + (3*a*b^2*x^4)/4 + 3*a^2*b*x
```

sympy [A] time = 0.15, size = 36, normalized size = 0.92

$$-\frac{a^3}{2x^2} + 3a^2bx + \frac{3ab^2x^4}{4} + \frac{b^3x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3/x**3,x)
[Out] -a**3/(2*x**2) + 3*a**2*b*x + 3*a*b**2*x**4/4 + b**3*x**7/7
```

3.255 $\int \frac{\left(a+bx^3\right)^3}{x^5}\,dx$

Optimal. Leaf size=41

$$-\frac{a^3}{4x^4}-\frac{3a^2b}{x}+\frac{3}{2}ab^2x^2+\frac{b^3x^5}{5}$$

```
[Out] -1/4*a^3/x^4-3*a^2*b/x+3/2*a*b^2*x^2+1/5*b^3*x^5
```

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$-\frac{3a^2b}{x}-\frac{a^3}{4x^4}+\frac{3}{2}ab^2x^2+\frac{b^3x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^3/x^5,x]
```

```
[Out] -a^3/(4*x^4) - (3*a^2*b)/x + (3*a*b^2*x^2)/2 + (b^3*x^5)/5
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^3}{x^5}\,dx &= \int \left(\frac{a^3}{x^5}+\frac{3a^2b}{x^2}+3ab^2x+b^3x^4\right)dx\\ &= -\frac{a^3}{4x^4}-\frac{3a^2b}{x}+\frac{3}{2}ab^2x^2+\frac{b^3x^5}{5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 41, normalized size = 1.00

$$-\frac{a^3}{4x^4}-\frac{3a^2b}{x}+\frac{3}{2}ab^2x^2+\frac{b^3x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^3/x^5,x]
```

```
[Out] -1/4*a^3/x^4 - (3*a^2*b)/x + (3*a*b^2*x^2)/2 + (b^3*x^5)/5
```

fricas [A] time = 0.85, size = 37, normalized size = 0.90

$$\frac{4\,b^3x^9+30\,ab^2x^6-60\,a^2bx^3-5\,a^3}{20\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/x^5,x, algorithm="fricas")
```

```
[Out] 1/20*(4*b^3*x^9 + 30*a*b^2*x^6 - 60*a^2*b*x^3 - 5*a^3)/x^4
```

giac [A] time = 0.18, size = 36, normalized size = 0.88

$$\frac{1}{5}\,b^3x^5+\frac{3}{2}\,ab^2x^2-\frac{12\,a^2bx^3+a^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/x^5,x, algorithm="giac")
[Out] 1/5*b^3*x^5 + 3/2*a*b^2*x^2 - 1/4*(12*a^2*b*x^3 + a^3)/x^4
```

maple [A] time = 0.00, size = 36, normalized size = 0.88

$$\frac{b^3x^5}{5} + \frac{3a\,b^2x^2}{2} - \frac{3a^2b}{x} - \frac{a^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3/x^5,x)
[Out] -1/4*a^3/x^4-3*a^2*b/x+3/2*a*b^2*x^2+1/5*b^3*x^5
```

maxima [A] time = 1.31, size = 36, normalized size = 0.88

$$\frac{1}{5}\,b^3x^5 + \frac{3}{2}\,ab^2x^2 - \frac{12\,a^2bx^3 + a^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/x^5,x, algorithm="maxima")
[Out] 1/5*b^3*x^5 + 3/2*a*b^2*x^2 - 1/4*(12*a^2*b*x^3 + a^3)/x^4
```

mupad [B] time = 1.00, size = 38, normalized size = 0.93

$$\frac{b^3\,x^5}{5} - \frac{\frac{a^3}{4} + 3\,b\,a^2\,x^3}{x^4} + \frac{3\,a\,b^2\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^3/x^5,x)
[Out] (b^3*x^5)/5 - (a^3/4 + 3*a^2*b*x^3)/x^4 + (3*a*b^2*x^2)/2
```

sympy [A] time = 0.19, size = 39, normalized size = 0.95

$$\frac{3ab^2x^2}{2} + \frac{b^3x^5}{5} + \frac{-a^3 - 12a^2bx^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3/x**5,x)
[Out] 3*a*b**2*x**2/2 + b**3*x**5/5 + (-a**3 - 12*a**2*b*x**3)/(4*x**4)
```

3.256 $\int \frac{\left(a+bx^3\right)^3}{x^6}\,dx$

Optimal. Leaf size=39

$$-\frac{a^3}{5x^5}-\frac{3a^2b}{2x^2}+3ab^2x+\frac{b^3x^4}{4}$$

```
[Out] -1/5*a^3/x^5-3/2*a^2*b/x^2+3*a*b^2*x+1/4*b^3*x^4
```

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$-\frac{3a^2b}{2x^2}-\frac{a^3}{5x^5}+3ab^2x+\frac{b^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^3/x^6,x]
```

```
[Out] -a^3/(5*x^5) - (3*a^2*b)/(2*x^2) + 3*a*b^2*x + (b^3*x^4)/4
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^3}{x^6}\,dx &= \int \left(3ab^2+\frac{a^3}{x^6}+\frac{3a^2b}{x^3}+b^3x^3\right)dx\\ &= -\frac{a^3}{5x^5}-\frac{3a^2b}{2x^2}+3ab^2x+\frac{b^3x^4}{4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 1.00

$$-\frac{a^3}{5x^5}-\frac{3a^2b}{2x^2}+3ab^2x+\frac{b^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^3/x^6,x]
```

```
[Out] -1/5*a^3/x^5 - (3*a^2*b)/(2*x^2) + 3*a*b^2*x + (b^3*x^4)/4
```

fricas [A] time = 0.79, size = 37, normalized size = 0.95

$$\frac{5\,b^3x^9+60\,ab^2x^6-30\,a^2bx^3-4\,a^3}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/x^6,x, algorithm="fricas")
```

```
[Out] 1/20*(5*b^3*x^9 + 60*a*b^2*x^6 - 30*a^2*b*x^3 - 4*a^3)/x^5
```

giac [A] time = 0.15, size = 36, normalized size = 0.92

$$\frac{1}{4}\,b^3x^4+3\,ab^2x-\frac{15\,a^2bx^3+2\,a^3}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/x^6,x, algorithm="giac")
[Out] 1/4*b^3*x^4 + 3*a*b^2*x - 1/10*(15*a^2*b*x^3 + 2*a^3)/x^5
```

maple [A] time = 0.00, size = 34, normalized size = 0.87

$$\frac{b^3x^4}{4} + 3a\,b^2x - \frac{3a^2b}{2x^2} - \frac{a^3}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3/x^6,x)
[Out] -1/5*a^3/x^5-3/2*a^2*b/x^2+3*a*b^2*x+1/4*b^3*x^4
```

maxima [A] time = 1.32, size = 36, normalized size = 0.92

$$\frac{1}{4}\,b^3x^4 + 3\,ab^2x - \frac{15\,a^2bx^3 + 2\,a^3}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/x^6,x, algorithm="maxima")
[Out] 1/4*b^3*x^4 + 3*a*b^2*x - 1/10*(15*a^2*b*x^3 + 2*a^3)/x^5
```

mupad [B] time = 0.04, size = 36, normalized size = 0.92

$$\frac{b^3\,x^4}{4} - \frac{\frac{a^3}{5} + \frac{3\,b\,a^2\,x^3}{2}}{x^5} + 3\,a\,b^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^3/x^6,x)
[Out] (b^3*x^4)/4 - (a^3/5 + (3*a^2*b*x^3)/2)/x^5 + 3*a*b^2*x
```

sympy [A] time = 0.20, size = 37, normalized size = 0.95

$$3ab^2x + \frac{b^3x^4}{4} + \frac{-2a^3 - 15a^2bx^3}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3/x**6,x)
[Out] 3*a*b**2*x + b**3*x**4/4 + (-2*a**3 - 15*a**2*b*x**3)/(10*x**5)
```

3.257 $\int \frac{\left(a+bx^3\right)^3}{x^8}\,dx$

Optimal. Leaf size=41

$$-\frac{a^3}{7x^7}-\frac{3a^2b}{4x^4}-\frac{3ab^2}{x}+\frac{b^3x^2}{2}$$

```
[Out] -1/7*a^3/x^7-3/4*a^2*b/x^4-3*a*b^2/x+1/2*b^3*x^2
```

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$-\frac{3a^2b}{4x^4}-\frac{a^3}{7x^7}-\frac{3ab^2}{x}+\frac{b^3x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^3/x^8,x]
```

```
[Out] -a^3/(7*x^7) - (3*a^2*b)/(4*x^4) - (3*a*b^2)/x + (b^3*x^2)/2
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^3}{x^8}\,dx &= \int \left(\frac{a^3}{x^8}+\frac{3a^2b}{x^5}+\frac{3ab^2}{x^2}+b^3x\right)dx\\ &= -\frac{a^3}{7x^7}-\frac{3a^2b}{4x^4}-\frac{3ab^2}{x}+\frac{b^3x^2}{2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 41, normalized size = 1.00

$$-\frac{a^3}{7x^7}-\frac{3a^2b}{4x^4}-\frac{3ab^2}{x}+\frac{b^3x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^3/x^8,x]
```

```
[Out] -1/7*a^3/x^7 - (3*a^2*b)/(4*x^4) - (3*a*b^2)/x + (b^3*x^2)/2
```

fricas [A] time = 0.82, size = 37, normalized size = 0.90

$$\frac{14\,b^3x^9-84\,ab^2x^6-21\,a^2bx^3-4\,a^3}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/x^8,x, algorithm="fricas")
```

```
[Out] 1/28*(14*b^3*x^9 - 84*a*b^2*x^6 - 21*a^2*b*x^3 - 4*a^3)/x^7
```

giac [A] time = 0.16, size = 38, normalized size = 0.93

$$\frac{1}{2}\,b^3x^2-\frac{84\,ab^2x^6+21\,a^2bx^3+4\,a^3}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/x^8,x, algorithm="giac")
[Out] 1/2*b^3*x^2 - 1/28*(84*a*b^2*x^6 + 21*a^2*b*x^3 + 4*a^3)/x^7
```

maple [A] time = 0.01, size = 36, normalized size = 0.88

$$\frac{b^3x^2}{2} - \frac{3a\,b^2}{x} - \frac{3a^2b}{4x^4} - \frac{a^3}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3/x^8,x)
[Out] -1/7*a^3/x^7-3/4*a^2*b/x^4-3*a*b^2/x+1/2*b^3*x^2
```

maxima [A] time = 1.29, size = 38, normalized size = 0.93

$$\frac{1}{2}\,b^3x^2 - \frac{84\,ab^2x^6 + 21\,a^2bx^3 + 4\,a^3}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/x^8,x, algorithm="maxima")
[Out] 1/2*b^3*x^2 - 1/28*(84*a*b^2*x^6 + 21*a^2*b*x^3 + 4*a^3)/x^7
```

mupad [B] time = 0.98, size = 38, normalized size = 0.93

$$\frac{b^3\,x^2}{2} - \frac{\frac{a^3}{7} + \frac{3\,a^2\,b\,x^3}{4} + 3\,a\,b^2\,x^6}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^3/x^8,x)
[Out] (b^3*x^2)/2 - (a^3/7 + (3*a^2*b*x^3)/4 + 3*a*b^2*x^6)/x^7
```

sympy [A] time = 0.28, size = 39, normalized size = 0.95

$$\frac{b^3x^2}{2} + \frac{-4a^3 - 21a^2bx^3 - 84ab^2x^6}{28x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3/x**8,x)
[Out] b**3*x**2/2 + (-4*a**3 - 21*a**2*b*x**3 - 84*a*b**2*x**6)/(28*x**7)
```

3.258 $\int x^{17}\left(a+bx^3\right)^5\,dx$

Optimal. Leaf size=69

$$\frac{a^5x^{18}}{18}+\frac{5}{21}a^4bx^{21}+\frac{5}{12}a^3b^2x^{24}+\frac{10}{27}a^2b^3x^{27}+\frac{1}{6}ab^4x^{30}+\frac{b^5x^{33}}{33}$$

```
[Out] 1/18*a^5*x^18+5/21*a^4*b*x^21+5/12*a^3*b^2*x^24+10/27*a^2*b^3*x^27+1/6*a*b^
4*x^30+1/33*b^5*x^33
```

Rubi [A] time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{10}{27}a^2b^3x^{27}+\frac{5}{12}a^3b^2x^{24}+\frac{5}{21}a^4bx^{21}+\frac{a^5x^{18}}{18}+\frac{1}{6}ab^4x^{30}+\frac{b^5x^{33}}{33}$$

Antiderivative was successfully verified.

```
[In] Int[x^17*(a + b*x^3)^5,x]
```

```
[Out] (a^5*x^18)/18 + (5*a^4*b*x^21)/21 + (5*a^3*b^2*x^24)/12 + (10*a^2*b^3*x^27)
/27 + (a*b^4*x^30)/6 + (b^5*x^33)/33
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{17}\left(a+bx^3\right)^5\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int x^5(a+bx)^5\,dx,x,x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(a^5x^5+5a^4bx^6+10a^3b^2x^7+10a^2b^3x^8+5ab^4x^9+b^5x^{10}\right)\,dx,x,x^3\right)\\ &= \frac{a^5x^{18}}{18}+\frac{5}{21}a^4bx^{21}+\frac{5}{12}a^3b^2x^{24}+\frac{10}{27}a^2b^3x^{27}+\frac{1}{6}ab^4x^{30}+\frac{b^5x^{33}}{33}\end{aligned}$$

Mathematica [A] time = 0.00, size = 69, normalized size = 1.00

$$\frac{a^5x^{18}}{18}+\frac{5}{21}a^4bx^{21}+\frac{5}{12}a^3b^2x^{24}+\frac{10}{27}a^2b^3x^{27}+\frac{1}{6}ab^4x^{30}+\frac{b^5x^{33}}{33}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^17*(a + b*x^3)^5,x]
```

```
[Out] (a^5*x^18)/18 + (5*a^4*b*x^21)/21 + (5*a^3*b^2*x^24)/12 + (10*a^2*b^3*x^27)
/27 + (a*b^4*x^30)/6 + (b^5*x^33)/33
```

fricas [A] time = 0.77, size = 57, normalized size = 0.83

$$\frac{1}{33}x^{33}b^5 + \frac{1}{6}x^{30}b^4a + \frac{10}{27}x^{27}b^3a^2 + \frac{5}{12}x^{24}b^2a^3 + \frac{5}{21}x^{21}ba^4 + \frac{1}{18}x^{18}a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17*(b*x^3+a)^5,x, algorithm="fricas")
```

```
[Out] 1/33*x^33*b^5 + 1/6*x^30*b^4*a + 10/27*x^27*b^3*a^2 + 5/12*x^24*b^2*a^3 + 5
/21*x^21*b*a^4 + 1/18*x^18*a^5
```

giac [A] time = 0.21, size = 57, normalized size = 0.83

$$\frac{1}{33}\,b^5x^{33} + \frac{1}{6}\,ab^4x^{30} + \frac{10}{27}\,a^2b^3x^{27} + \frac{5}{12}\,a^3b^2x^{24} + \frac{5}{21}\,a^4bx^{21} + \frac{1}{18}\,a^5x^{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17*(b*x^3+a)^5,x, algorithm="giac")
```

```
[Out] 1/33*b^5*x^33 + 1/6*a*b^4*x^30 + 10/27*a^2*b^3*x^27 + 5/12*a^3*b^2*x^24 + 5
/21*a^4*b*x^21 + 1/18*a^5*x^18
```

maple [A] time = 0.00, size = 58, normalized size = 0.84

$$\frac{1}{33}b^5x^{33} + \frac{1}{6}a\,b^4x^{30} + \frac{10}{27}a^2b^3x^{27} + \frac{5}{12}a^3b^2x^{24} + \frac{5}{21}a^4b\,x^{21} + \frac{1}{18}a^5x^{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^17*(b*x^3+a)^5,x)
```

```
[Out] 1/18*a^5*x^18+5/21*a^4*b*x^21+5/12*a^3*b^2*x^24+10/27*a^2*b^3*x^27+1/6*a*b^
4*x^30+1/33*b^5*x^33
```

maxima [A] time = 1.35, size = 57, normalized size = 0.83

$$\frac{1}{33}\,b^5x^{33} + \frac{1}{6}\,ab^4x^{30} + \frac{10}{27}\,a^2b^3x^{27} + \frac{5}{12}\,a^3b^2x^{24} + \frac{5}{21}\,a^4bx^{21} + \frac{1}{18}\,a^5x^{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17*(b*x^3+a)^5,x, algorithm="maxima")
```

```
[Out] 1/33*b^5*x^33 + 1/6*a*b^4*x^30 + 10/27*a^2*b^3*x^27 + 5/12*a^3*b^2*x^24 + 5
/21*a^4*b*x^21 + 1/18*a^5*x^18
```

mupad [B] time = 0.02, size = 57, normalized size = 0.83

$$\frac{a^5\,x^{18}}{18} + \frac{5\,a^4\,b\,x^{21}}{21} + \frac{5\,a^3\,b^2\,x^{24}}{12} + \frac{10\,a^2\,b^3\,x^{27}}{27} + \frac{a\,b^4\,x^{30}}{6} + \frac{b^5\,x^{33}}{33}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^17*(a + b*x^3)^5,x)
```

```
[Out] (a^5*x^18)/18 + (b^5*x^33)/33 + (5*a^4*b*x^21)/21 + (a*b^4*x^30)/6 + (5*a^3
*b^2*x^24)/12 + (10*a^2*b^3*x^27)/27
```

sympy [A] time = 0.09, size = 65, normalized size = 0.94

$$\frac{a^5x^{18}}{18} + \frac{5a^4bx^{21}}{21} + \frac{5a^3b^2x^{24}}{12} + \frac{10a^2b^3x^{27}}{27} + \frac{ab^4x^{30}}{6} + \frac{b^5x^{33}}{33}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**17*(b*x**3+a)**5,x)
```

```
[Out] a**5*x**18/18 + 5*a**4*b*x**21/21 + 5*a**3*b**2*x**24/12 + 10*a**2*b**3*x**
27/27 + a*b**4*x**30/6 + b**5*x**33/33
```

3.259 $\int x^{14}\left(a+bx^3\right)^5\,dx$

Optimal. Leaf size=69

$$\frac{a^5x^{15}}{15}+\frac{5}{18}a^4bx^{18}+\frac{10}{21}a^3b^2x^{21}+\frac{5}{12}a^2b^3x^{24}+\frac{5}{27}ab^4x^{27}+\frac{b^5x^{30}}{30}$$

```
[Out] 1/15*a^5*x^15+5/18*a^4*b*x^18+10/21*a^3*b^2*x^21+5/12*a^2*b^3*x^24+5/27*a*b
^4*x^27+1/30*b^5*x^30
```

Rubi [A] time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{5}{12}a^2b^3x^{24}+\frac{10}{21}a^3b^2x^{21}+\frac{5}{18}a^4bx^{18}+\frac{a^5x^{15}}{15}+\frac{5}{27}ab^4x^{27}+\frac{b^5x^{30}}{30}$$

Antiderivative was successfully verified.

```
[In] Int[x^14*(a + b*x^3)^5,x]
```

```
[Out] (a^5*x^15)/15 + (5*a^4*b*x^18)/18 + (10*a^3*b^2*x^21)/21 + (5*a^2*b^3*x^24)
/12 + (5*a*b^4*x^27)/27 + (b^5*x^30)/30
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{14}\left(a+bx^3\right)^5\,dx &= \frac{1}{3}\text{Subst}\left(\int x^4(a+bx)^5\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(a^5x^4+5a^4bx^5+10a^3b^2x^6+10a^2b^3x^7+5ab^4x^8+b^5x^9\right)\,dx,x,x^3\right)\\ &= \frac{a^5x^{15}}{15}+\frac{5}{18}a^4bx^{18}+\frac{10}{21}a^3b^2x^{21}+\frac{5}{12}a^2b^3x^{24}+\frac{5}{27}ab^4x^{27}+\frac{b^5x^{30}}{30}\end{aligned}$$

Mathematica [A] time = 0.00, size = 69, normalized size = 1.00

$$\frac{a^5x^{15}}{15}+\frac{5}{18}a^4bx^{18}+\frac{10}{21}a^3b^2x^{21}+\frac{5}{12}a^2b^3x^{24}+\frac{5}{27}ab^4x^{27}+\frac{b^5x^{30}}{30}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14*(a + b*x^3)^5,x]
```

```
[Out] (a^5*x^15)/15 + (5*a^4*b*x^18)/18 + (10*a^3*b^2*x^21)/21 + (5*a^2*b^3*x^24)
/12 + (5*a*b^4*x^27)/27 + (b^5*x^30)/30
```

fricas [A] time = 0.75, size = 57, normalized size = 0.83

$$\frac{1}{30}x^{30}b^5 + \frac{5}{27}x^{27}b^4a + \frac{5}{12}x^{24}b^3a^2 + \frac{10}{21}x^{21}b^2a^3 + \frac{5}{18}x^{18}ba^4 + \frac{1}{15}x^{15}a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(b*x^3+a)^5,x, algorithm="fricas")
```

```
[Out] 1/30*x^30*b^5 + 5/27*x^27*b^4*a + 5/12*x^24*b^3*a^2 + 10/21*x^21*b^2*a^3 +
5/18*x^18*b*a^4 + 1/15*x^15*a^5
```

giac [A] time = 0.17, size = 57, normalized size = 0.83

$$\frac{1}{30}\,b^5x^{30} + \frac{5}{27}\,ab^4x^{27} + \frac{5}{12}\,a^2b^3x^{24} + \frac{10}{21}\,a^3b^2x^{21} + \frac{5}{18}\,a^4bx^{18} + \frac{1}{15}\,a^5x^{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(b*x^3+a)^5,x, algorithm="giac")
```

```
[Out] 1/30*b^5*x^30 + 5/27*a*b^4*x^27 + 5/12*a^2*b^3*x^24 + 10/21*a^3*b^2*x^21 +
5/18*a^4*b*x^18 + 1/15*a^5*x^15
```

maple [A] time = 0.00, size = 58, normalized size = 0.84

$$\frac{1}{30}b^5x^{30} + \frac{5}{27}a\,b^4x^{27} + \frac{5}{12}a^2b^3x^{24} + \frac{10}{21}a^3b^2x^{21} + \frac{5}{18}a^4b\,x^{18} + \frac{1}{15}a^5x^{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(b*x^3+a)^5,x)
```

```
[Out] 1/15*a^5*x^15+5/18*a^4*b*x^18+10/21*a^3*b^2*x^21+5/12*a^2*b^3*x^24+5/27*a*b
^4*x^27+1/30*b^5*x^30
```

maxima [A] time = 1.36, size = 57, normalized size = 0.83

$$\frac{1}{30}\,b^5x^{30} + \frac{5}{27}\,ab^4x^{27} + \frac{5}{12}\,a^2b^3x^{24} + \frac{10}{21}\,a^3b^2x^{21} + \frac{5}{18}\,a^4bx^{18} + \frac{1}{15}\,a^5x^{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(b*x^3+a)^5,x, algorithm="maxima")
```

```
[Out] 1/30*b^5*x^30 + 5/27*a*b^4*x^27 + 5/12*a^2*b^3*x^24 + 10/21*a^3*b^2*x^21 +
5/18*a^4*b*x^18 + 1/15*a^5*x^15
```

mupad [B] time = 0.02, size = 57, normalized size = 0.83

$$\frac{a^5\,x^{15}}{15} + \frac{5\,a^4\,b\,x^{18}}{18} + \frac{10\,a^3\,b^2\,x^{21}}{21} + \frac{5\,a^2\,b^3\,x^{24}}{12} + \frac{5\,a\,b^4\,x^{27}}{27} + \frac{b^5\,x^{30}}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(a + b*x^3)^5,x)
```

```
[Out] (a^5*x^15)/15 + (b^5*x^30)/30 + (5*a^4*b*x^18)/18 + (5*a*b^4*x^27)/27 + (10
*a^3*b^2*x^21)/21 + (5*a^2*b^3*x^24)/12
```

sympy [A] time = 0.10, size = 66, normalized size = 0.96

$$\frac{a^5x^{15}}{15} + \frac{5a^4bx^{18}}{18} + \frac{10a^3b^2x^{21}}{21} + \frac{5a^2b^3x^{24}}{12} + \frac{5ab^4x^{27}}{27} + \frac{b^5x^{30}}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14*(b*x**3+a)**5,x)
```

```
[Out] a**5*x**15/15 + 5*a**4*b*x**18/18 + 10*a**3*b**2*x**21/21 + 5*a**2*b**3*x**
24/12 + 5*a*b**4*x**27/27 + b**5*x**30/30
```

3.260 $\int x^{11}\left(a+bx^3\right)^5\,dx$

Optimal. Leaf size=72

$$-\frac{a^3\left(a+bx^3\right)^6}{18b^4}+\frac{a^2\left(a+bx^3\right)^7}{7b^4}+\frac{\left(a+bx^3\right)^9}{27b^4}-\frac{a\left(a+bx^3\right)^8}{8b^4}$$

```
[Out] -1/18*a^3*(b*x^3+a)^6/b^4+1/7*a^2*(b*x^3+a)^7/b^4-1/8*a*(b*x^3+a)^8/b^4+1/2
7*(b*x^3+a)^9/b^4
```

Rubi [A] time = 0.09, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{a^2\left(a+bx^3\right)^7}{7b^4}-\frac{a^3\left(a+bx^3\right)^6}{18b^4}+\frac{\left(a+bx^3\right)^9}{27b^4}-\frac{a\left(a+bx^3\right)^8}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a + b*x^3)^5,x]
```

```
[Out] -(a^3*(a + b*x^3)^6)/(18*b^4) + (a^2*(a + b*x^3)^7)/(7*b^4) - (a*(a + b*x^3
)^8)/(8*b^4) + (a + b*x^3)^9/(27*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{11}\left(a+bx^3\right)^5\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int x^3(a+bx)^5\,dx,x,x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int\left(-\frac{a^3(a+bx)^5}{b^3}+\frac{3a^2(a+bx)^6}{b^3}-\frac{3a(a+bx)^7}{b^3}+\frac{(a+bx)^8}{b^3}\right)dx,x,x^3\right)\\ &= -\frac{a^3\left(a+bx^3\right)^6}{18b^4}+\frac{a^2\left(a+bx^3\right)^7}{7b^4}-\frac{a\left(a+bx^3\right)^8}{8b^4}+\frac{\left(a+bx^3\right)^9}{27b^4}\end{aligned}$$

Mathematica [A] time = 0.00, size = 69, normalized size = 0.96

$$\frac{a^5x^{12}}{12}+\frac{1}{3}a^4bx^{15}+\frac{5}{9}a^3b^2x^{18}+\frac{10}{21}a^2b^3x^{21}+\frac{5}{24}ab^4x^{24}+\frac{b^5x^{27}}{27}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*x^3)^5,x]
```

```
[Out] (a^5*x^12)/12 + (a^4*b*x^15)/3 + (5*a^3*b^2*x^18)/9 + (10*a^2*b^3*x^21)/21
+ (5*a*b^4*x^24)/24 + (b^5*x^27)/27
```

fricas [A] time = 0.71, size = 57, normalized size = 0.79

$$\frac{1}{27}x^{27}b^5 + \frac{5}{24}x^{24}b^4a + \frac{10}{21}x^{21}b^3a^2 + \frac{5}{9}x^{18}b^2a^3 + \frac{1}{3}x^{15}ba^4 + \frac{1}{12}x^{12}a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^5,x, algorithm="fricas")
```

```
[Out] 1/27*x^27*b^5 + 5/24*x^24*b^4*a + 10/21*x^21*b^3*a^2 + 5/9*x^18*b^2*a^3 + 1
/3*x^15*b*a^4 + 1/12*x^12*a^5
```

giac [A] time = 0.15, size = 57, normalized size = 0.79

$$\frac{1}{27}\,b^5x^{27} + \frac{5}{24}\,ab^4x^{24} + \frac{10}{21}\,a^2b^3x^{21} + \frac{5}{9}\,a^3b^2x^{18} + \frac{1}{3}\,a^4bx^{15} + \frac{1}{12}\,a^5x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^5,x, algorithm="giac")
```

```
[Out] 1/27*b^5*x^27 + 5/24*a*b^4*x^24 + 10/21*a^2*b^3*x^21 + 5/9*a^3*b^2*x^18 + 1
/3*a^4*b*x^15 + 1/12*a^5*x^12
```

maple [A] time = 0.00, size = 58, normalized size = 0.81

$$\frac{1}{27}b^5x^{27} + \frac{5}{24}a\,b^4x^{24} + \frac{10}{21}a^2b^3x^{21} + \frac{5}{9}a^3b^2x^{18} + \frac{1}{3}a^4b\,x^{15} + \frac{1}{12}a^5x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x^3+a)^5,x)
```

```
[Out] 1/27*b^5*x^27+5/24*a*b^4*x^24+10/21*a^2*b^3*x^21+5/9*a^3*b^2*x^18+1/3*a^4*b
*x^15+1/12*a^5*x^12
```

maxima [A] time = 1.33, size = 57, normalized size = 0.79

$$\frac{1}{27}\,b^5x^{27} + \frac{5}{24}\,ab^4x^{24} + \frac{10}{21}\,a^2b^3x^{21} + \frac{5}{9}\,a^3b^2x^{18} + \frac{1}{3}\,a^4bx^{15} + \frac{1}{12}\,a^5x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^5,x, algorithm="maxima")
```

```
[Out] 1/27*b^5*x^27 + 5/24*a*b^4*x^24 + 10/21*a^2*b^3*x^21 + 5/9*a^3*b^2*x^18 + 1
/3*a^4*b*x^15 + 1/12*a^5*x^12
```

mupad [B] time = 0.02, size = 57, normalized size = 0.79

$$\frac{a^5\,x^{12}}{12} + \frac{a^4\,b\,x^{15}}{3} + \frac{5\,a^3\,b^2\,x^{18}}{9} + \frac{10\,a^2\,b^3\,x^{21}}{21} + \frac{5\,a\,b^4\,x^{24}}{24} + \frac{b^5\,x^{27}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a + b*x^3)^5,x)
```

```
[Out] (a^5*x^12)/12 + (b^5*x^27)/27 + (a^4*b*x^15)/3 + (5*a*b^4*x^24)/24 + (5*a^3
*b^2*x^18)/9 + (10*a^2*b^3*x^21)/21
```

sympy [A] time = 0.08, size = 65, normalized size = 0.90

$$\frac{a^5x^{12}}{12} + \frac{a^4bx^{15}}{3} + \frac{5a^3b^2x^{18}}{9} + \frac{10a^2b^3x^{21}}{21} + \frac{5ab^4x^{24}}{24} + \frac{b^5x^{27}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x**3+a)**5,x)
```

```
[Out] a**5*x**12/12 + a**4*b*x**15/3 + 5*a**3*b**2*x**18/9 + 10*a**2*b**3*x**21/2
1 + 5*a*b**4*x**24/24 + b**5*x**27/27
```

3.261 $\int x^8 \left(a+bx^3\right)^5 \, dx$

Optimal. Leaf size=53

$$\frac{a^2\left(a+bx^3\right)^6}{18b^3}+\frac{\left(a+bx^3\right)^8}{24b^3}-\frac{2a\left(a+bx^3\right)^7}{21b^3}$$

```
[Out] 1/18*a^2*(b*x^3+a)^6/b^3-2/21*a*(b*x^3+a)^7/b^3+1/24*(b*x^3+a)^8/b^3
```

Rubi [A] time = 0.06, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{a^2\left(a+bx^3\right)^6}{18b^3}+\frac{\left(a+bx^3\right)^8}{24b^3}-\frac{2a\left(a+bx^3\right)^7}{21b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x^3)^5,x]
```

```
[Out] (a^2*(a + b*x^3)^6)/(18*b^3) - (2*a*(a + b*x^3)^7)/(21*b^3) + (a + b*x^3)^8
/(24*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^8\left(a+bx^3\right)^5\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int x^2(a+bx)^5\,dx, x, x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int\left(\frac{a^2(a+bx)^5}{b^2}-\frac{2a(a+bx)^6}{b^2}+\frac{(a+bx)^7}{b^2}\right)dx, x, x^3\right)\\
&= \frac{a^2\left(a+bx^3\right)^6}{18b^3}-\frac{2a\left(a+bx^3\right)^7}{21b^3}+\frac{\left(a+bx^3\right)^8}{24b^3}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 69, normalized size = 1.30

$$\frac{a^5x^9}{9}+\frac{5}{12}a^4bx^{12}+\frac{2}{3}a^3b^2x^{15}+\frac{5}{9}a^2b^3x^{18}+\frac{5}{21}ab^4x^{21}+\frac{b^5x^{24}}{24}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x^3)^5,x]
```

```
[Out] (a^5*x^9)/9 + (5*a^4*b*x^12)/12 + (2*a^3*b^2*x^15)/3 + (5*a^2*b^3*x^18)/9 +
 (5*a*b^4*x^21)/21 + (b^5*x^24)/24
```

fricas [A] time = 0.71, size = 57, normalized size = 1.08

$$\frac{1}{24}x^{24}b^5 + \frac{5}{21}x^{21}b^4a + \frac{5}{9}x^{18}b^3a^2 + \frac{2}{3}x^{15}b^2a^3 + \frac{5}{12}x^{12}ba^4 + \frac{1}{9}x^9a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^5,x, algorithm="fricas")
```

```
[Out] 1/24*x^24*b^5 + 5/21*x^21*b^4*a + 5/9*x^18*b^3*a^2 + 2/3*x^15*b^2*a^3 + 5/1
2*x^12*b*a^4 + 1/9*x^9*a^5
```

giac [A] time = 0.15, size = 57, normalized size = 1.08

$$\frac{1}{24}\,b^5x^{24} + \frac{5}{21}\,ab^4x^{21} + \frac{5}{9}\,a^2b^3x^{18} + \frac{2}{3}\,a^3b^2x^{15} + \frac{5}{12}\,a^4bx^{12} + \frac{1}{9}\,a^5x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^5,x, algorithm="giac")
```

```
[Out] 1/24*b^5*x^24 + 5/21*a*b^4*x^21 + 5/9*a^2*b^3*x^18 + 2/3*a^3*b^2*x^15 + 5/1
2*a^4*b*x^12 + 1/9*a^5*x^9
```

maple [A] time = 0.00, size = 58, normalized size = 1.09

$$\frac{1}{24}b^5x^{24} + \frac{5}{21}a\,b^4x^{21} + \frac{5}{9}a^2b^3x^{18} + \frac{2}{3}a^3b^2x^{15} + \frac{5}{12}a^4b\,x^{12} + \frac{1}{9}a^5x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^3+a)^5,x)
```

```
[Out] 1/24*b^5*x^24+5/21*a*b^4*x^21+5/9*a^2*b^3*x^18+2/3*a^3*b^2*x^15+5/12*a^4*b*
x^12+1/9*a^5*x^9
```

maxima [A] time = 1.33, size = 57, normalized size = 1.08

$$\frac{1}{24}\,b^5x^{24} + \frac{5}{21}\,ab^4x^{21} + \frac{5}{9}\,a^2b^3x^{18} + \frac{2}{3}\,a^3b^2x^{15} + \frac{5}{12}\,a^4bx^{12} + \frac{1}{9}\,a^5x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^5,x, algorithm="maxima")
```

```
[Out] 1/24*b^5*x^24 + 5/21*a*b^4*x^21 + 5/9*a^2*b^3*x^18 + 2/3*a^3*b^2*x^15 + 5/1
2*a^4*b*x^12 + 1/9*a^5*x^9
```

mupad [B] time = 0.03, size = 57, normalized size = 1.08

$$\frac{a^5\,x^9}{9} + \frac{5\,a^4\,b\,x^{12}}{12} + \frac{2\,a^3\,b^2\,x^{15}}{3} + \frac{5\,a^2\,b^3\,x^{18}}{9} + \frac{5\,a\,b^4\,x^{21}}{21} + \frac{b^5\,x^{24}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*x^3)^5,x)
```

```
[Out] (a^5*x^9)/9 + (b^5*x^24)/24 + (5*a^4*b*x^12)/12 + (5*a*b^4*x^21)/21 + (2*a^
3*b^2*x^15)/3 + (5*a^2*b^3*x^18)/9
```

sympy [A] time = 0.08, size = 66, normalized size = 1.25

$$\frac{a^5x^9}{9} + \frac{5a^4bx^{12}}{12} + \frac{2a^3b^2x^{15}}{3} + \frac{5a^2b^3x^{18}}{9} + \frac{5ab^4x^{21}}{21} + \frac{b^5x^{24}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**3+a)**5,x)
```

```
[Out] a**5*x**9/9 + 5*a**4*b*x**12/12 + 2*a**3*b**2*x**15/3 + 5*a**2*b**3*x**18/9
 + 5*a*b**4*x**21/21 + b**5*x**24/24
```

3.262 $\int x^5 \left(a+bx^3\right)^5 dx$

Optimal. Leaf size=34

$$\frac{\left(a+bx^3\right)^7}{21b^2} - \frac{a\left(a+bx^3\right)^6}{18b^2}$$

```
[Out] -1/18*a*(b*x^3+a)^6/b^2+1/21*(b*x^3+a)^7/b^2
```

Rubi [A] time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{\left(a+bx^3\right)^7}{21b^2} - \frac{a\left(a+bx^3\right)^6}{18b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^3)^5,x]
```

```
[Out] -(a*(a + b*x^3)^6)/(18*b^2) + (a + b*x^3)^7/(21*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^5 \left(a+bx^3\right)^5 dx &= \frac{1}{3}\,\text{Subst}\left(\int x(a+bx)^5\,dx, x, x^3\right) \\
&= \frac{1}{3}\,\text{Subst}\left(\int \left(-\frac{a(a+bx)^5}{b} + \frac{(a+bx)^6}{b}\right) dx, x, x^3\right) \\
&= -\frac{a\left(a+bx^3\right)^6}{18b^2} + \frac{\left(a+bx^3\right)^7}{21b^2}
\end{aligned}$$

Mathematica [B] time = 0.00, size = 69, normalized size = 2.03

$$\frac{a^5x^6}{6} + \frac{5}{9}a^4bx^9 + \frac{5}{6}a^3b^2x^{12} + \frac{2}{3}a^2b^3x^{15} + \frac{5}{18}ab^4x^{18} + \frac{b^5x^{21}}{21}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^3)^5,x]
```

```
[Out] (a^5*x^6)/6 + (5*a^4*b*x^9)/9 + (5*a^3*b^2*x^12)/6 + (2*a^2*b^3*x^15)/3 + (
5*a*b^4*x^18)/18 + (b^5*x^21)/21
```

fricas [A] time = 0.70, size = 57, normalized size = 1.68

$$\frac{1}{21}x^{21}b^5 + \frac{5}{18}x^{18}b^4a + \frac{2}{3}x^{15}b^3a^2 + \frac{5}{6}x^{12}b^2a^3 + \frac{5}{9}x^9ba^4 + \frac{1}{6}x^6a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^5,x, algorithm="fricas")
```

```
[Out] 1/21*x^21*b^5 + 5/18*x^18*b^4*a + 2/3*x^15*b^3*a^2 + 5/6*x^12*b^2*a^3 + 5/9
*x^9*b*a^4 + 1/6*x^6*a^5
```

giac [A] time = 0.15, size = 57, normalized size = 1.68

$$\frac{1}{21}\,b^5x^{21} + \frac{5}{18}\,ab^4x^{18} + \frac{2}{3}\,a^2b^3x^{15} + \frac{5}{6}\,a^3b^2x^{12} + \frac{5}{9}\,a^4bx^9 + \frac{1}{6}\,a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^5,x, algorithm="giac")
```

```
[Out] 1/21*b^5*x^21 + 5/18*a*b^4*x^18 + 2/3*a^2*b^3*x^15 + 5/6*a^3*b^2*x^12 + 5/9
*a^4*b*x^9 + 1/6*a^5*x^6
```

maple [A] time = 0.00, size = 58, normalized size = 1.71

$$\frac{1}{21}b^5x^{21} + \frac{5}{18}a\,b^4x^{18} + \frac{2}{3}a^2b^3x^{15} + \frac{5}{6}a^3b^2x^{12} + \frac{5}{9}a^4b\,x^9 + \frac{1}{6}a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^3+a)^5,x)
```

```
[Out] 1/21*b^5*x^21+5/18*a*b^4*x^18+2/3*a^2*b^3*x^15+5/6*a^3*b^2*x^12+5/9*a^4*b*x
^9+1/6*a^5*x^6
```

maxima [A] time = 1.33, size = 57, normalized size = 1.68

$$\frac{1}{21}\,b^5x^{21} + \frac{5}{18}\,ab^4x^{18} + \frac{2}{3}\,a^2b^3x^{15} + \frac{5}{6}\,a^3b^2x^{12} + \frac{5}{9}\,a^4bx^9 + \frac{1}{6}\,a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^5,x, algorithm="maxima")
```

```
[Out] 1/21*b^5*x^21 + 5/18*a*b^4*x^18 + 2/3*a^2*b^3*x^15 + 5/6*a^3*b^2*x^12 + 5/9
*a^4*b*x^9 + 1/6*a^5*x^6
```

mupad [B] time = 0.02, size = 57, normalized size = 1.68

$$\frac{a^5\,x^6}{6} + \frac{5\,a^4\,b\,x^9}{9} + \frac{5\,a^3\,b^2\,x^{12}}{6} + \frac{2\,a^2\,b^3\,x^{15}}{3} + \frac{5\,a\,b^4\,x^{18}}{18} + \frac{b^5\,x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^3)^5,x)
```

```
[Out] (a^5*x^6)/6 + (b^5*x^21)/21 + (5*a^4*b*x^9)/9 + (5*a*b^4*x^18)/18 + (5*a^3*
b^2*x^12)/6 + (2*a^2*b^3*x^15)/3
```

sympy [B] time = 0.09, size = 66, normalized size = 1.94

$$\frac{a^5x^6}{6} + \frac{5a^4bx^9}{9} + \frac{5a^3b^2x^{12}}{6} + \frac{2a^2b^3x^{15}}{3} + \frac{5ab^4x^{18}}{18} + \frac{b^5x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**3+a)**5,x)
```

```
[Out] a**5*x**6/6 + 5*a**4*b*x**9/9 + 5*a**3*b**2*x**12/6 + 2*a**2*b**3*x**15/3 +
 5*a*b**4*x**18/18 + b**5*x**21/21
```

3.263 $\int x^2 \left(a + bx^3\right)^5 \, dx$

Optimal. Leaf size=16

$$\frac{\left(a + bx^3\right)^6}{18b}$$

```
[Out] 1/18*(b*x^3+a)^6/b
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$\frac{\left(a + bx^3\right)^6}{18b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3)^5,x]
```

```
[Out] (a + b*x^3)^6/(18*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^2 \left(a + bx^3\right)^5 \, dx = \frac{\left(a + bx^3\right)^6}{18b}$$

Mathematica [B] time = 0.00, size = 69, normalized size = 4.31

$$\frac{a^5 x^3}{3} + \frac{5}{6} a^4 b x^6 + \frac{10}{9} a^3 b^2 x^9 + \frac{5}{6} a^2 b^3 x^{12} + \frac{1}{3} a b^4 x^{15} + \frac{b^5 x^{18}}{18}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)^5,x]
```

```
[Out] (a^5*x^3)/3 + (5*a^4*b*x^6)/6 + (10*a^3*b^2*x^9)/9 + (5*a^2*b^3*x^12)/6 + (
a*b^4*x^15)/3 + (b^5*x^18)/18
```

fricas [B] time = 0.69, size = 57, normalized size = 3.56

$$\frac{1}{18} x^{18} b^5 + \frac{1}{3} x^{15} b^4 a + \frac{5}{6} x^{12} b^3 a^2 + \frac{10}{9} x^9 b^2 a^3 + \frac{5}{6} x^6 b a^4 + \frac{1}{3} x^3 a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^5,x, algorithm="fricas")
```

```
[Out] 1/18*x^18*b^5 + 1/3*x^15*b^4*a + 5/6*x^12*b^3*a^2 + 10/9*x^9*b^2*a^3 + 5/6*
x^6*b*a^4 + 1/3*x^3*a^5
```

giac [A] time = 0.16, size = 14, normalized size = 0.88

$$\frac{\left(bx^3 + a\right)^6}{18\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^5,x, algorithm="giac")
```

```
[Out] 1/18*(b*x^3 + a)^6/b
```

maple [B] time = 0.00, size = 58, normalized size = 3.62

$$\frac{1}{18}b^5x^{18} + \frac{1}{3}a\,b^4x^{15} + \frac{5}{6}a^2b^3x^{12} + \frac{10}{9}a^3b^2x^9 + \frac{5}{6}a^4b\,x^6 + \frac{1}{3}a^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^5,x)
```

```
[Out] 1/18*b^5*x^18+1/3*a*b^4*x^15+5/6*a^2*b^3*x^12+10/9*a^3*b^2*x^9+5/6*a^4*b*x^
6+1/3*a^5*x^3
```

maxima [A] time = 1.37, size = 14, normalized size = 0.88

$$\frac{\left(bx^3 + a\right)^6}{18\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^5,x, algorithm="maxima")
```

```
[Out] 1/18*(b*x^3 + a)^6/b
```

mupad [B] time = 0.03, size = 57, normalized size = 3.56

$$\frac{a^5\,x^3}{3} + \frac{5\,a^4\,b\,x^6}{6} + \frac{10\,a^3\,b^2\,x^9}{9} + \frac{5\,a^2\,b^3\,x^{12}}{6} + \frac{a\,b^4\,x^{15}}{3} + \frac{b^5\,x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^3)^5,x)
```

```
[Out] (a^5*x^3)/3 + (b^5*x^18)/18 + (5*a^4*b*x^6)/6 + (a*b^4*x^15)/3 + (10*a^3*b^
2*x^9)/9 + (5*a^2*b^3*x^12)/6
```

sympy [B] time = 0.09, size = 65, normalized size = 4.06

$$\frac{a^5x^3}{3} + \frac{5a^4bx^6}{6} + \frac{10a^3b^2x^9}{9} + \frac{5a^2b^3x^{12}}{6} + \frac{ab^4x^{15}}{3} + \frac{b^5x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**5,x)
```

```
[Out] a**5*x**3/3 + 5*a**4*b*x**6/6 + 10*a**3*b**2*x**9/9 + 5*a**2*b**3*x**12/6 +
 a*b**4*x**15/3 + b**5*x**18/18
```

3.264 $\int \frac{\left(a+bx^3\right)^5}{x}\,dx$

Optimal. Leaf size=65

$$a^5\log(x)+\frac{5}{3}a^4bx^3+\frac{5}{3}a^3b^2x^6+\frac{10}{9}a^2b^3x^9+\frac{5}{12}ab^4x^{12}+\frac{b^5x^{15}}{15}$$

```
[Out] 5/3*a^4*b*x^3+5/3*a^3*b^2*x^6+10/9*a^2*b^3*x^9+5/12*a*b^4*x^12+1/15*b^5*x^1
5+a^5*ln(x)
```

Rubi [A] time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{10}{9}a^2b^3x^9+\frac{5}{3}a^3b^2x^6+\frac{5}{3}a^4bx^3+a^5\log(x)+\frac{5}{12}ab^4x^{12}+\frac{b^5x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^5/x,x]
```

```
[Out] (5*a^4*b*x^3)/3 + (5*a^3*b^2*x^6)/3 + (10*a^2*b^3*x^9)/9 + (5*a*b^4*x^12)/1
2 + (b^5*x^15)/15 + a^5*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^5}{x}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)^5}{x}\,dx, x, x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(5a^4b+\frac{a^5}{x}+10a^3b^2x+10a^2b^3x^2+5ab^4x^3+b^5x^4\right)\,dx, x, x^3\right)\\ &= \frac{5}{3}a^4bx^3+\frac{5}{3}a^3b^2x^6+\frac{10}{9}a^2b^3x^9+\frac{5}{12}ab^4x^{12}+\frac{b^5x^{15}}{15}+a^5\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 65, normalized size = 1.00

$$a^5\log(x)+\frac{5}{3}a^4bx^3+\frac{5}{3}a^3b^2x^6+\frac{10}{9}a^2b^3x^9+\frac{5}{12}ab^4x^{12}+\frac{b^5x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^5/x,x]
```

```
[Out] (5*a^4*b*x^3)/3 + (5*a^3*b^2*x^6)/3 + (10*a^2*b^3*x^9)/9 + (5*a*b^4*x^12)/1
2 + (b^5*x^15)/15 + a^5*Log[x]
```

fricas [A] time = 0.82, size = 55, normalized size = 0.85

$$\frac{1}{15}b^5x^{15} + \frac{5}{12}ab^4x^{12} + \frac{10}{9}a^2b^3x^9 + \frac{5}{3}a^3b^2x^6 + \frac{5}{3}a^4bx^3 + a^5\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x,x, algorithm="fricas")
```

```
[Out] 1/15*b^5*x^15 + 5/12*a*b^4*x^12 + 10/9*a^2*b^3*x^9 + 5/3*a^3*b^2*x^6 + 5/3*
a^4*b*x^3 + a^5*log(x)
```

giac [A] time = 0.15, size = 56, normalized size = 0.86

$$\frac{1}{15}b^5x^{15} + \frac{5}{12}ab^4x^{12} + \frac{10}{9}a^2b^3x^9 + \frac{5}{3}a^3b^2x^6 + \frac{5}{3}a^4bx^3 + a^5\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x,x, algorithm="giac")
```

```
[Out] 1/15*b^5*x^15 + 5/12*a*b^4*x^12 + 10/9*a^2*b^3*x^9 + 5/3*a^3*b^2*x^6 + 5/3*
a^4*b*x^3 + a^5*log(abs(x))
```

maple [A] time = 0.00, size = 56, normalized size = 0.86

$$\frac{b^5x^{15}}{15} + \frac{5a\,b^4x^{12}}{12} + \frac{10a^2b^3x^9}{9} + \frac{5a^3b^2x^6}{3} + \frac{5a^4b\,x^3}{3} + a^5\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5/x,x)
```

```
[Out] 5/3*a^4*b*x^3+5/3*a^3*b^2*x^6+10/9*a^2*b^3*x^9+5/12*a*b^4*x^12+1/15*b^5*x^1
5+a^5*ln(x)
```

maxima [A] time = 1.28, size = 58, normalized size = 0.89

$$\frac{1}{15}b^5x^{15} + \frac{5}{12}ab^4x^{12} + \frac{10}{9}a^2b^3x^9 + \frac{5}{3}a^3b^2x^6 + \frac{5}{3}a^4bx^3 + \frac{1}{3}a^5\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x,x, algorithm="maxima")
```

```
[Out] 1/15*b^5*x^15 + 5/12*a*b^4*x^12 + 10/9*a^2*b^3*x^9 + 5/3*a^3*b^2*x^6 + 5/3*
a^4*b*x^3 + 1/3*a^5*log(x^3)
```

mupad [B] time = 0.03, size = 55, normalized size = 0.85

$$a^5\,\ln(x) + \frac{b^5\,x^{15}}{15} + \frac{5\,a^4\,b\,x^3}{3} + \frac{5\,a\,b^4\,x^{12}}{12} + \frac{5\,a^3\,b^2\,x^6}{3} + \frac{10\,a^2\,b^3\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^5/x,x)
```

```
[Out] a^5*log(x) + (b^5*x^15)/15 + (5*a^4*b*x^3)/3 + (5*a*b^4*x^12)/12 + (5*a^3*b
^2*x^6)/3 + (10*a^2*b^3*x^9)/9
```

sympy [A] time = 0.16, size = 65, normalized size = 1.00

$$a^5\log(x) + \frac{5a^4bx^3}{3} + \frac{5a^3b^2x^6}{3} + \frac{10a^2b^3x^9}{9} + \frac{5ab^4x^{12}}{12} + \frac{b^5x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5/x,x)
```

```
[Out] a**5*log(x) + 5*a**4*b*x**3/3 + 5*a**3*b**2*x**6/3 + 10*a**2*b**3*x**9/9 +
5*a*b**4*x**12/12 + b**5*x**15/15
```

3.265 $\int \frac{\left(a+bx^3\right)^5}{x^4}\,dx$

Optimal. Leaf size=66

$$-\frac{a^5}{3x^3}+5a^4b\log(x)+\frac{10}{3}a^3b^2x^3+\frac{5}{3}a^2b^3x^6+\frac{5}{9}ab^4x^9+\frac{b^5x^{12}}{12}$$

```
[Out] -1/3*a^5/x^3+10/3*a^3*b^2*x^3+5/3*a^2*b^3*x^6+5/9*a*b^4*x^9+1/12*b^5*x^12+5
*a^4*b*ln(x)
```

Rubi [A] time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{5}{3}a^2b^3x^6+\frac{10}{3}a^3b^2x^3+5a^4b\log(x)-\frac{a^5}{3x^3}+\frac{5}{9}ab^4x^9+\frac{b^5x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^5/x^4,x]
```

```
[Out] -a^5/(3*x^3) + (10*a^3*b^2*x^3)/3 + (5*a^2*b^3*x^6)/3 + (5*a*b^4*x^9)/9 + (
b^5*x^12)/12 + 5*a^4*b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^5}{x^4}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)^5}{x^2}\,dx, x, x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(10a^3b^2+\frac{a^5}{x^2}+\frac{5a^4b}{x}+10a^2b^3x+5ab^4x^2+b^5x^3\right)dx, x, x^3\right)\\ &= -\frac{a^5}{3x^3}+\frac{10}{3}a^3b^2x^3+\frac{5}{3}a^2b^3x^6+\frac{5}{9}ab^4x^9+\frac{b^5x^{12}}{12}+5a^4b\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 66, normalized size = 1.00

$$-\frac{a^5}{3x^3}+5a^4b\log(x)+\frac{10}{3}a^3b^2x^3+\frac{5}{3}a^2b^3x^6+\frac{5}{9}ab^4x^9+\frac{b^5x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^5/x^4,x]
```

```
[Out] -1/3*a^5/x^3 + (10*a^3*b^2*x^3)/3 + (5*a^2*b^3*x^6)/3 + (5*a*b^4*x^9)/9 + (
b^5*x^12)/12 + 5*a^4*b*Log[x]
```

fricas [A] time = 0.82, size = 61, normalized size = 0.92

$$\frac{3\,b^5x^{15} + 20\,ab^4x^{12} + 60\,a^2b^3x^9 + 120\,a^3b^2x^6 + 180\,a^4bx^3\log(x) - 12\,a^5}{36\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^4,x, algorithm="fricas")
```

```
[Out] 1/36*(3*b^5*x^15 + 20*a*b^4*x^12 + 60*a^2*b^3*x^9 + 120*a^3*b^2*x^6 + 180*a
^4*b*x^3*log(x) - 12*a^5)/x^3
```

giac [A] time = 0.15, size = 67, normalized size = 1.02

$$\frac{1}{12}\,b^5x^{12} + \frac{5}{9}\,ab^4x^9 + \frac{5}{3}\,a^2b^3x^6 + \frac{10}{3}\,a^3b^2x^3 + 5\,a^4b\log\left(|x|\right) - \frac{5\,a^4bx^3 + a^5}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^4,x, algorithm="giac")
```

```
[Out] 1/12*b^5*x^12 + 5/9*a*b^4*x^9 + 5/3*a^2*b^3*x^6 + 10/3*a^3*b^2*x^3 + 5*a^4*
b*log(abs(x)) - 1/3*(5*a^4*b*x^3 + a^5)/x^3
```

maple [A] time = 0.01, size = 57, normalized size = 0.86

$$\frac{b^5x^{12}}{12} + \frac{5a\,b^4x^9}{9} + \frac{5a^2b^3x^6}{3} + \frac{10a^3b^2x^3}{3} + 5a^4b\ln(x) - \frac{a^5}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5/x^4,x)
```

```
[Out] -1/3*a^5/x^3+10/3*a^3*b^2*x^3+5/3*a^2*b^3*x^6+5/9*a*b^4*x^9+1/12*b^5*x^12+5
*a^4*b*ln(x)
```

maxima [A] time = 1.37, size = 58, normalized size = 0.88

$$\frac{1}{12}\,b^5x^{12} + \frac{5}{9}\,ab^4x^9 + \frac{5}{3}\,a^2b^3x^6 + \frac{10}{3}\,a^3b^2x^3 + \frac{5}{3}\,a^4b\log\left(x^3\right) - \frac{a^5}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^4,x, algorithm="maxima")
```

```
[Out] 1/12*b^5*x^12 + 5/9*a*b^4*x^9 + 5/3*a^2*b^3*x^6 + 10/3*a^3*b^2*x^3 + 5/3*a^
4*b*log(x^3) - 1/3*a^5/x^3
```

mupad [B] time = 0.03, size = 56, normalized size = 0.85

$$\frac{b^5\,x^{12}}{12} - \frac{a^5}{3\,x^3} + \frac{5\,a\,b^4\,x^9}{9} + 5\,a^4\,b\,\ln(x) + \frac{10\,a^3\,b^2\,x^3}{3} + \frac{5\,a^2\,b^3\,x^6}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^5/x^4,x)
```

```
[Out] (b^5*x^12)/12 - a^5/(3*x^3) + (5*a*b^4*x^9)/9 + 5*a^4*b*log(x) + (10*a^3*b^
2*x^3)/3 + (5*a^2*b^3*x^6)/3
```

sympy [A] time = 0.22, size = 65, normalized size = 0.98

$$-\frac{a^5}{3x^3} + 5a^4b\log(x) + \frac{10a^3b^2x^3}{3} + \frac{5a^2b^3x^6}{3} + \frac{5ab^4x^9}{9} + \frac{b^5x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5/x**4,x)
```

```
[Out] -a**5/(3*x**3) + 5*a**4*b*log(x) + 10*a**3*b**2*x**3/3 + 5*a**2*b**3*x**6/3
 + 5*a*b**4*x**9/9 + b**5*x**12/12
```

3.266 $\int \frac{\left(a+bx^3\right)^5}{x^7}\,dx$

Optimal. Leaf size=66

$$-\frac{a^5}{6x^6}-\frac{5a^4b}{3x^3}+10a^3b^2\log(x)+\frac{10}{3}a^2b^3x^3+\frac{5}{6}ab^4x^6+\frac{b^5x^9}{9}$$

```
[Out] -1/6*a^5/x^6-5/3*a^4*b/x^3+10/3*a^2*b^3*x^3+5/6*a*b^4*x^6+1/9*b^5*x^9+10*a^
3*b^2*ln(x)
```

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{10}{3}a^2b^3x^3+10a^3b^2\log(x)-\frac{5a^4b}{3x^3}-\frac{a^5}{6x^6}+\frac{5}{6}ab^4x^6+\frac{b^5x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^5/x^7,x]
```

```
[Out] -a^5/(6*x^6) - (5*a^4*b)/(3*x^3) + (10*a^2*b^3*x^3)/3 + (5*a*b^4*x^6)/6 + (
b^5*x^9)/9 + 10*a^3*b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^5}{x^7}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)^5}{x^3}\,dx, x, x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(10a^2b^3+\frac{a^5}{x^3}+\frac{5a^4b}{x^2}+\frac{10a^3b^2}{x}+5ab^4x+b^5x^2\right)\,dx, x, x^3\right)\\ &= -\frac{a^5}{6x^6}-\frac{5a^4b}{3x^3}+\frac{10}{3}a^2b^3x^3+\frac{5}{6}ab^4x^6+\frac{b^5x^9}{9}+10a^3b^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 66, normalized size = 1.00

$$-\frac{a^5}{6x^6}-\frac{5a^4b}{3x^3}+10a^3b^2\log(x)+\frac{10}{3}a^2b^3x^3+\frac{5}{6}ab^4x^6+\frac{b^5x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^5/x^7,x]
```

```
[Out] -1/6*a^5/x^6 - (5*a^4*b)/(3*x^3) + (10*a^2*b^3*x^3)/3 + (5*a*b^4*x^6)/6 + (
b^5*x^9)/9 + 10*a^3*b^2*Log[x]
```

fricas [A] time = 0.79, size = 61, normalized size = 0.92

$$\frac{2\,b^5x^{15} + 15\,ab^4x^{12} + 60\,a^2b^3x^9 + 180\,a^3b^2x^6\log(x) - 30\,a^4bx^3 - 3\,a^5}{18\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^7,x, algorithm="fricas")
```

```
[Out] 1/18*(2*b^5*x^15 + 15*a*b^4*x^12 + 60*a^2*b^3*x^9 + 180*a^3*b^2*x^6*log(x)
- 30*a^4*b*x^3 - 3*a^5)/x^6
```

giac [A] time = 0.15, size = 69, normalized size = 1.05

$$\frac{1}{9}\,b^5x^9 + \frac{5}{6}\,ab^4x^6 + \frac{10}{3}\,a^2b^3x^3 + 10\,a^3b^2\log\left(|x|\right) - \frac{30\,a^3b^2x^6 + 10\,a^4bx^3 + a^5}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^7,x, algorithm="giac")
```

```
[Out] 1/9*b^5*x^9 + 5/6*a*b^4*x^6 + 10/3*a^2*b^3*x^3 + 10*a^3*b^2*log(abs(x)) - 1
/6*(30*a^3*b^2*x^6 + 10*a^4*b*x^3 + a^5)/x^6
```

maple [A] time = 0.01, size = 57, normalized size = 0.86

$$\frac{b^5x^9}{9} + \frac{5a\,b^4x^6}{6} + \frac{10a^2b^3x^3}{3} + 10a^3b^2\ln(x) - \frac{5a^4b}{3x^3} - \frac{a^5}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5/x^7,x)
```

```
[Out] -1/6*a^5/x^6-5/3*a^4*b/x^3+10/3*a^2*b^3*x^3+5/6*a*b^4*x^6+1/9*b^5*x^9+10*a^
3*b^2*ln(x)
```

maxima [A] time = 1.29, size = 59, normalized size = 0.89

$$\frac{1}{9}\,b^5x^9 + \frac{5}{6}\,ab^4x^6 + \frac{10}{3}\,a^2b^3x^3 + \frac{10}{3}\,a^3b^2\log\left(x^3\right) - \frac{10\,a^4bx^3 + a^5}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^7,x, algorithm="maxima")
```

```
[Out] 1/9*b^5*x^9 + 5/6*a*b^4*x^6 + 10/3*a^2*b^3*x^3 + 10/3*a^3*b^2*log(x^3) - 1/
6*(10*a^4*b*x^3 + a^5)/x^6
```

mupad [B] time = 0.03, size = 59, normalized size = 0.89

$$\frac{b^5\,x^9}{9} - \frac{\frac{a^5}{6} + \frac{5\,b\,a^4\,x^3}{3}}{x^6} + \frac{5\,a\,b^4\,x^6}{6} + \frac{10\,a^2\,b^3\,x^3}{3} + 10\,a^3\,b^2\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^5/x^7,x)
```

```
[Out] (b^5*x^9)/9 - (a^5/6 + (5*a^4*b*x^3)/3)/x^6 + (5*a*b^4*x^6)/6 + (10*a^2*b^3
*x^3)/3 + 10*a^3*b^2*log(x)
```

sympy [A] time = 0.33, size = 65, normalized size = 0.98

$$10a^3b^2\log(x) + \frac{10a^2b^3x^3}{3} + \frac{5ab^4x^6}{6} + \frac{b^5x^9}{9} + \frac{-a^5 - 10a^4bx^3}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5/x**7,x)
```

```
[Out] 10*a**3*b**2*log(x) + 10*a**2*b**3*x**3/3 + 5*a*b**4*x**6/6 + b**5*x**9/9 +
 (-a**5 - 10*a**4*b*x**3)/(6*x**6)
```

3.267 $\int \frac{\left(a+bx^3\right)^5}{x^{10}}\,dx$

Optimal. Leaf size=66

$$-\frac{a^5}{9x^9}-\frac{5a^4b}{6x^6}-\frac{10a^3b^2}{3x^3}+10a^2b^3\log(x)+\frac{5}{3}ab^4x^3+\frac{b^5x^6}{6}$$

```
[Out] -1/9*a^5/x^9-5/6*a^4*b/x^6-10/3*a^3*b^2/x^3+5/3*a*b^4*x^3+1/6*b^5*x^6+10*a^
2*b^3*ln(x)
```

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$-\frac{10a^3b^2}{3x^3}+10a^2b^3\log(x)-\frac{5a^4b}{6x^6}-\frac{a^5}{9x^9}+\frac{5}{3}ab^4x^3+\frac{b^5x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^5/x^10,x]
```

```
[Out] -a^5/(9*x^9) - (5*a^4*b)/(6*x^6) - (10*a^3*b^2)/(3*x^3) + (5*a*b^4*x^3)/3 +
 (b^5*x^6)/6 + 10*a^2*b^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^5}{x^{10}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^5}{x^4}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(5ab^4+\frac{a^5}{x^4}+\frac{5a^4b}{x^3}+\frac{10a^3b^2}{x^2}+\frac{10a^2b^3}{x}+b^5x\right)\,dx, x, x^3\right)\\ &= -\frac{a^5}{9x^9}-\frac{5a^4b}{6x^6}-\frac{10a^3b^2}{3x^3}+\frac{5}{3}ab^4x^3+\frac{b^5x^6}{6}+10a^2b^3\log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 66, normalized size = 1.00

$$-\frac{a^5}{9x^9}-\frac{5a^4b}{6x^6}-\frac{10a^3b^2}{3x^3}+10a^2b^3\log(x)+\frac{5}{3}ab^4x^3+\frac{b^5x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^5/x^10,x]
```

```
[Out] -1/9*a^5/x^9 - (5*a^4*b)/(6*x^6) - (10*a^3*b^2)/(3*x^3) + (5*a*b^4*x^3)/3 +
 (b^5*x^6)/6 + 10*a^2*b^3*Log[x]
```

fricas [A] time = 0.81, size = 61, normalized size = 0.92

$$\frac{3\,b^5x^{15} + 30\,ab^4x^{12} + 180\,a^2b^3x^9\log(x) - 60\,a^3b^2x^6 - 15\,a^4bx^3 - 2\,a^5}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^10,x, algorithm="fricas")
```

```
[Out] 1/18*(3*b^5*x^15 + 30*a*b^4*x^12 + 180*a^2*b^3*x^9*log(x) - 60*a^3*b^2*x^6
- 15*a^4*b*x^3 - 2*a^5)/x^9
```

giac [A] time = 0.19, size = 71, normalized size = 1.08

$$\frac{1}{6}\,b^5x^6 + \frac{5}{3}\,ab^4x^3 + 10\,a^2b^3\log\left(|x|\right) - \frac{110\,a^2b^3x^9 + 60\,a^3b^2x^6 + 15\,a^4bx^3 + 2\,a^5}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^10,x, algorithm="giac")
```

```
[Out] 1/6*b^5*x^6 + 5/3*a*b^4*x^3 + 10*a^2*b^3*log(abs(x)) - 1/18*(110*a^2*b^3*x^
9 + 60*a^3*b^2*x^6 + 15*a^4*b*x^3 + 2*a^5)/x^9
```

maple [A] time = 0.01, size = 57, normalized size = 0.86

$$\frac{b^5x^6}{6} + \frac{5a\,b^4x^3}{3} + 10a^2b^3\ln(x) - \frac{10a^3b^2}{3x^3} - \frac{5a^4b}{6x^6} - \frac{a^5}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5/x^10,x)
```

```
[Out] -1/9*a^5/x^9-5/6*a^4*b/x^6-10/3*a^3*b^2/x^3+5/3*a*b^4*x^3+1/6*b^5*x^6+10*a^
2*b^3*ln(x)
```

maxima [A] time = 1.32, size = 61, normalized size = 0.92

$$\frac{1}{6}\,b^5x^6 + \frac{5}{3}\,ab^4x^3 + \frac{10}{3}\,a^2b^3\log\left(x^3\right) - \frac{60\,a^3b^2x^6 + 15\,a^4bx^3 + 2\,a^5}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^10,x, algorithm="maxima")
```

```
[Out] 1/6*b^5*x^6 + 5/3*a*b^4*x^3 + 10/3*a^2*b^3*log(x^3) - 1/18*(60*a^3*b^2*x^6
+ 15*a^4*b*x^3 + 2*a^5)/x^9
```

mupad [B] time = 0.04, size = 59, normalized size = 0.89

$$\frac{b^5\,x^6}{6} - \frac{\frac{a^5}{9} + \frac{5\,a^4\,b\,x^3}{6} + \frac{10\,a^3\,b^2\,x^6}{3}}{x^9} + \frac{5\,a\,b^4\,x^3}{3} + 10\,a^2\,b^3\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^5/x^10,x)
```

```
[Out] (b^5*x^6)/6 - (a^5/9 + (5*a^4*b*x^3)/6 + (10*a^3*b^2*x^6)/3)/x^9 + (5*a*b^4
*x^3)/3 + 10*a^2*b^3*log(x)
```

sympy [A] time = 0.41, size = 65, normalized size = 0.98

$$10a^2b^3\log(x) + \frac{5ab^4x^3}{3} + \frac{b^5x^6}{6} + \frac{-2a^5 - 15a^4bx^3 - 60a^3b^2x^6}{18x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5/x**10,x)
```

```
[Out] 10*a**2*b**3*log(x) + 5*a*b**4*x**3/3 + b**5*x**6/6 + (-2*a**5 - 15*a**4*b*
x**3 - 60*a**3*b**2*x**6)/(18*x**9)
```

3.268 $\int \frac{\left(a+bx^3\right)^5}{x^{13}}\,dx$

Optimal. Leaf size=66

$$-\frac{a^5}{12x^{12}}-\frac{5a^4b}{9x^9}-\frac{5a^3b^2}{3x^6}-\frac{10a^2b^3}{3x^3}+5ab^4\log(x)+\frac{b^5x^3}{3}$$

```
[Out] -1/12*a^5/x^12-5/9*a^4*b/x^9-5/3*a^3*b^2/x^6-10/3*a^2*b^3/x^3+1/3*b^5*x^3+5
*a*b^4*ln(x)
```

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$-\frac{5a^3b^2}{3x^6}-\frac{10a^2b^3}{3x^3}-\frac{5a^4b}{9x^9}-\frac{a^5}{12x^{12}}+5ab^4\log(x)+\frac{b^5x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^5/x^13,x]
```

```
[Out] -a^5/(12*x^12) - (5*a^4*b)/(9*x^9) - (5*a^3*b^2)/(3*x^6) - (10*a^2*b^3)/(3*
x^3) + (b^5*x^3)/3 + 5*a*b^4*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^5}{x^{13}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^5}{x^5}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(b^5+\frac{a^5}{x^5}+\frac{5a^4b}{x^4}+\frac{10a^3b^2}{x^3}+\frac{10a^2b^3}{x^2}+\frac{5ab^4}{x}\right)dx, x, x^3\right)\\ &= -\frac{a^5}{12x^{12}}-\frac{5a^4b}{9x^9}-\frac{5a^3b^2}{3x^6}-\frac{10a^2b^3}{3x^3}+\frac{b^5x^3}{3}+5ab^4\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 66, normalized size = 1.00

$$-\frac{a^5}{12x^{12}}-\frac{5a^4b}{9x^9}-\frac{5a^3b^2}{3x^6}-\frac{10a^2b^3}{3x^3}+5ab^4\log(x)+\frac{b^5x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^5/x^13,x]
```

```
[Out] -1/12*a^5/x^12 - (5*a^4*b)/(9*x^9) - (5*a^3*b^2)/(3*x^6) - (10*a^2*b^3)/(3*
x^3) + (b^5*x^3)/3 + 5*a*b^4*Log[x]
```

fricas [A] time = 0.87, size = 61, normalized size = 0.92

$$\frac{12\,b^5x^{15} + 180\,ab^4x^{12}\log(x) - 120\,a^2b^3x^9 - 60\,a^3b^2x^6 - 20\,a^4bx^3 - 3\,a^5}{36\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^13,x, algorithm="fricas")
```

```
[Out] 1/36*(12*b^5*x^15 + 180*a*b^4*x^12*log(x) - 120*a^2*b^3*x^9 - 60*a^3*b^2*x^
6 - 20*a^4*b*x^3 - 3*a^5)/x^12
```

giac [A] time = 0.16, size = 69, normalized size = 1.05

$$\frac{1}{3}\,b^5x^3 + 5\,ab^4\log\left(|x|\right) - \frac{125\,ab^4x^{12} + 120\,a^2b^3x^9 + 60\,a^3b^2x^6 + 20\,a^4bx^3 + 3\,a^5}{36\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^13,x, algorithm="giac")
```

```
[Out] 1/3*b^5*x^3 + 5*a*b^4*log(abs(x)) - 1/36*(125*a*b^4*x^12 + 120*a^2*b^3*x^9
+ 60*a^3*b^2*x^6 + 20*a^4*b*x^3 + 3*a^5)/x^12
```

maple [A] time = 0.01, size = 57, normalized size = 0.86

$$\frac{b^5x^3}{3} + 5a\,b^4\ln(x) - \frac{10a^2b^3}{3x^3} - \frac{5a^3b^2}{3x^6} - \frac{5a^4b}{9x^9} - \frac{a^5}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5/x^13,x)
```

```
[Out] -1/12*a^5/x^12-5/9*a^4*b/x^9-5/3*a^3*b^2/x^6-10/3*a^2*b^3/x^3+1/3*b^5*x^3+5
*a*b^4*ln(x)
```

maxima [A] time = 1.31, size = 61, normalized size = 0.92

$$\frac{1}{3}\,b^5x^3 + \frac{5}{3}\,ab^4\log\left(x^3\right) - \frac{120\,a^2b^3x^9 + 60\,a^3b^2x^6 + 20\,a^4bx^3 + 3\,a^5}{36\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^13,x, algorithm="maxima")
```

```
[Out] 1/3*b^5*x^3 + 5/3*a*b^4*log(x^3) - 1/36*(120*a^2*b^3*x^9 + 60*a^3*b^2*x^6 +
 20*a^4*b*x^3 + 3*a^5)/x^12
```

mupad [B] time = 0.04, size = 59, normalized size = 0.89

$$\frac{b^5\,x^3}{3} - \frac{\frac{a^5}{12} + \frac{5\,a^4\,b\,x^3}{9} + \frac{5\,a^3\,b^2\,x^6}{3} + \frac{10\,a^2\,b^3\,x^9}{3}}{x^{12}} + 5\,a\,b^4\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^5/x^13,x)
```

```
[Out] (b^5*x^3)/3 - (a^5/12 + (5*a^4*b*x^3)/9 + (5*a^3*b^2*x^6)/3 + (10*a^2*b^3*x
^9)/3)/x^12 + 5*a*b^4*log(x)
```

sympy [A] time = 0.57, size = 63, normalized size = 0.95

$$5ab^4\log(x) + \frac{b^5x^3}{3} + \frac{-3a^5 - 20a^4bx^3 - 60a^3b^2x^6 - 120a^2b^3x^9}{36x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5/x**13,x)
```

```
[Out] 5*a*b**4*log(x) + b**5*x**3/3 + (-3*a**5 - 20*a**4*b*x**3 - 60*a**3*b**2*x**
**6 - 120*a**2*b**3*x**9)/(36*x**12)
```

3.269 $\int \frac{\left(a+bx^3\right)^5}{x^{16}}\,dx$

Optimal. Leaf size=65

$$-\frac{a^5}{15x^{15}}-\frac{5a^4b}{12x^{12}}-\frac{10a^3b^2}{9x^9}-\frac{5a^2b^3}{3x^6}-\frac{5ab^4}{3x^3}+b^5\log(x)$$

```
[Out] -1/15*a^5/x^15-5/12*a^4*b/x^12-10/9*a^3*b^2/x^9-5/3*a^2*b^3/x^6-5/3*a*b^4/x
^3+b^5*ln(x)
```

Rubi [A] time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$-\frac{10a^3b^2}{9x^9}-\frac{5a^2b^3}{3x^6}-\frac{5a^4b}{12x^{12}}-\frac{a^5}{15x^{15}}-\frac{5ab^4}{3x^3}+b^5\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^5/x^16,x]
```

```
[Out] -a^5/(15*x^15) - (5*a^4*b)/(12*x^12) - (10*a^3*b^2)/(9*x^9) - (5*a^2*b^3)/(
3*x^6) - (5*a*b^4)/(3*x^3) + b^5*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^5}{x^{16}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^5}{x^6}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{a^5}{x^6}+\frac{5a^4b}{x^5}+\frac{10a^3b^2}{x^4}+\frac{10a^2b^3}{x^3}+\frac{5ab^4}{x^2}+\frac{b^5}{x}\right)dx, x, x^3\right)\\ &= -\frac{a^5}{15x^{15}}-\frac{5a^4b}{12x^{12}}-\frac{10a^3b^2}{9x^9}-\frac{5a^2b^3}{3x^6}-\frac{5ab^4}{3x^3}+b^5\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 65, normalized size = 1.00

$$-\frac{a^5}{15x^{15}}-\frac{5a^4b}{12x^{12}}-\frac{10a^3b^2}{9x^9}-\frac{5a^2b^3}{3x^6}-\frac{5ab^4}{3x^3}+b^5\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^5/x^16,x]
```

```
[Out] -1/15*a^5/x^15 - (5*a^4*b)/(12*x^12) - (10*a^3*b^2)/(9*x^9) - (5*a^2*b^3)/(
3*x^6) - (5*a*b^4)/(3*x^3) + b^5*Log[x]
```

fricas [A] time = 0.70, size = 61, normalized size = 0.94

$$\frac{180\,b^5x^{15}\log(x) - 300\,ab^4x^{12} - 300\,a^2b^3x^9 - 200\,a^3b^2x^6 - 75\,a^4bx^3 - 12\,a^5}{180\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^16,x, algorithm="fricas")
```

```
[Out] 1/180*(180*b^5*x^15*log(x) - 300*a*b^4*x^12 - 300*a^2*b^3*x^9 - 200*a^3*b^2
*x^6 - 75*a^4*b*x^3 - 12*a^5)/x^15
```

giac [A] time = 0.16, size = 67, normalized size = 1.03

$$b^5\log(|x|) - \frac{137\,b^5x^{15} + 300\,ab^4x^{12} + 300\,a^2b^3x^9 + 200\,a^3b^2x^6 + 75\,a^4bx^3 + 12\,a^5}{180\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^16,x, algorithm="giac")
```

```
[Out] b^5*log(abs(x)) - 1/180*(137*b^5*x^15 + 300*a*b^4*x^12 + 300*a^2*b^3*x^9 +
200*a^3*b^2*x^6 + 75*a^4*b*x^3 + 12*a^5)/x^15
```

maple [A] time = 0.01, size = 56, normalized size = 0.86

$$b^5\ln(x) - \frac{5a\,b^4}{3x^3} - \frac{5a^2b^3}{3x^6} - \frac{10a^3b^2}{9x^9} - \frac{5a^4b}{12x^{12}} - \frac{a^5}{15x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5/x^16,x)
```

```
[Out] -1/15*a^5/x^15-5/12*a^4*b/x^12-10/9*a^3*b^2/x^9-5/3*a^2*b^3/x^6-5/3*a*b^4/x
^3+b^5*ln(x)
```

maxima [A] time = 1.35, size = 61, normalized size = 0.94

$$\frac{1}{3}\,b^5\log\left(x^3\right) - \frac{300\,ab^4x^{12} + 300\,a^2b^3x^9 + 200\,a^3b^2x^6 + 75\,a^4bx^3 + 12\,a^5}{180\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^16,x, algorithm="maxima")
```

```
[Out] 1/3*b^5*log(x^3) - 1/180*(300*a*b^4*x^12 + 300*a^2*b^3*x^9 + 200*a^3*b^2*x^
6 + 75*a^4*b*x^3 + 12*a^5)/x^15
```

mupad [B] time = 1.01, size = 58, normalized size = 0.89

$$b^5\,\ln(x) - \frac{\frac{a^5}{15} + \frac{5\,a^4\,b\,x^3}{12} + \frac{10\,a^3\,b^2\,x^6}{9} + \frac{5\,a^2\,b^3\,x^9}{3} + \frac{5\,a\,b^4\,x^{12}}{3}}{x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^5/x^16,x)
```

```
[Out] b^5*log(x) - (a^5/15 + (5*a^4*b*x^3)/12 + (5*a*b^4*x^12)/3 + (10*a^3*b^2*x^
6)/9 + (5*a^2*b^3*x^9)/3)/x^15
```

sympy [A] time = 0.68, size = 61, normalized size = 0.94

$$b^5\log(x) + \frac{-12a^5 - 75a^4bx^3 - 200a^3b^2x^6 - 300a^2b^3x^9 - 300ab^4x^{12}}{180x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5/x**16,x)
```

```
[Out] b**5*log(x) + (-12*a**5 - 75*a**4*b*x**3 - 200*a**3*b**2*x**6 - 300*a**2*b*
**3*x**9 - 300*a*b**4*x**12)/(180*x**15)
```

3.270 $\int \frac{\left(a+bx^3\right)^5}{x^{19}}\,dx$

Optimal. Leaf size=19

$$-\frac{\left(a+bx^3\right)^6}{18ax^{18}}$$

```
[Out] -1/18*(b*x^3+a)^6/a/x^18
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {264}

$$-\frac{\left(a+bx^3\right)^6}{18ax^{18}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^5/x^19,x]
```

```
[Out] -(a + b*x^3)^6/(18*a*x^18)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^5}{x^{19}}\,dx = -\frac{\left(a+bx^3\right)^6}{18ax^{18}}$$

Mathematica [B] time = 0.00, size = 69, normalized size = 3.63

$$-\frac{a^5}{18x^{18}} - \frac{a^4b}{3x^{15}} - \frac{5a^3b^2}{6x^{12}} - \frac{10a^2b^3}{9x^9} - \frac{5ab^4}{6x^6} - \frac{b^5}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^5/x^19,x]
```

```
[Out] -1/18*a^5/x^18 - (a^4*b)/(3*x^15) - (5*a^3*b^2)/(6*x^12) - (10*a^2*b^3)/(9*
x^9) - (5*a*b^4)/(6*x^6) - b^5/(3*x^3)
```

fricas [B] time = 1.06, size = 57, normalized size = 3.00

$$-\frac{6\,b^5x^{15} + 15\,ab^4x^{12} + 20\,a^2b^3x^9 + 15\,a^3b^2x^6 + 6\,a^4bx^3 + a^5}{18\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^19,x, algorithm="fricas")
```

```
[Out] -1/18*(6*b^5*x^15 + 15*a*b^4*x^12 + 20*a^2*b^3*x^9 + 15*a^3*b^2*x^6 + 6*a^4
*b*x^3 + a^5)/x^18
```

giac [B] time = 0.26, size = 57, normalized size = 3.00

$$-\frac{6\,b^5x^{15}+15\,ab^4x^{12}+20\,a^2b^3x^9+15\,a^3b^2x^6+6\,a^4bx^3+a^5}{18\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^19,x, algorithm="giac")
```

```
[Out] -1/18*(6*b^5*x^15 + 15*a*b^4*x^12 + 20*a^2*b^3*x^9 + 15*a^3*b^2*x^6 + 6*a^4
*b*x^3 + a^5)/x^18
```

maple [B] time = 0.01, size = 58, normalized size = 3.05

$$-\frac{b^5}{3x^3}-\frac{5a\,b^4}{6x^6}-\frac{10a^2b^3}{9x^9}-\frac{5a^3b^2}{6x^{12}}-\frac{a^4b}{3x^{15}}-\frac{a^5}{18x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5/x^19,x)
```

```
[Out] -1/3*b^5/x^3-1/18*a^5/x^18-5/6*a^3*b^2/x^12-5/6*a*b^4/x^6-10/9*a^2*b^3/x^9-
1/3*a^4*b/x^15
```

maxima [B] time = 1.30, size = 57, normalized size = 3.00

$$-\frac{6\,b^5x^{15}+15\,ab^4x^{12}+20\,a^2b^3x^9+15\,a^3b^2x^6+6\,a^4bx^3+a^5}{18\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^19,x, algorithm="maxima")
```

```
[Out] -1/18*(6*b^5*x^15 + 15*a*b^4*x^12 + 20*a^2*b^3*x^9 + 15*a^3*b^2*x^6 + 6*a^4
*b*x^3 + a^5)/x^18
```

mupad [B] time = 0.04, size = 59, normalized size = 3.11

$$-\frac{\frac{a^5}{18}+\frac{a^4\,b\,x^3}{3}+\frac{5\,a^3\,b^2\,x^6}{6}+\frac{10\,a^2\,b^3\,x^9}{9}+\frac{5\,a\,b^4\,x^{12}}{6}+\frac{b^5\,x^{15}}{3}}{x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^5/x^19,x)
```

```
[Out] -(a^5/18 + (b^5*x^15)/3 + (a^4*b*x^3)/3 + (5*a*b^4*x^12)/6 + (5*a^3*b^2*x^6
)/6 + (10*a^2*b^3*x^9)/9)/x^18
```

sympy [B] time = 0.76, size = 61, normalized size = 3.21

$$\frac{-a^5-6a^4bx^3-15a^3b^2x^6-20a^2b^3x^9-15ab^4x^{12}-6b^5x^{15}}{18x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5/x**19,x)
```

```
[Out] (-a**5 - 6*a**4*b*x**3 - 15*a**3*b**2*x**6 - 20*a**2*b**3*x**9 - 15*a*b**4*
x**12 - 6*b**5*x**15)/(18*x**18)
```

3.271 $\int \frac{\left(a+bx^3\right)^5}{x^{22}}\,dx$

Optimal. Leaf size=40

$$\frac{b\left(a+bx^3\right)^6}{126a^2x^{18}}-\frac{\left(a+bx^3\right)^6}{21ax^{21}}$$

```
[Out] -1/21*(b*x^3+a)^6/a/x^21+1/126*b*(b*x^3+a)^6/a^2/x^18
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {266, 45, 37}

$$\frac{b\left(a+bx^3\right)^6}{126a^2x^{18}}-\frac{\left(a+bx^3\right)^6}{21ax^{21}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^5/x^22,x]
```

```
[Out] -(a + b*x^3)^6/(21*a*x^21) + (b*(a + b*x^3)^6)/(126*a^2*x^18)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^5}{x^{22}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^5}{x^8}\,dx, x, x^3\right)\\ &= -\frac{\left(a+bx^3\right)^6}{21ax^{21}}-\frac{b\,\text{Subst}\left(\int \frac{(a+bx)^5}{x^7}\,dx, x, x^3\right)}{21a}\\ &= -\frac{\left(a+bx^3\right)^6}{21ax^{21}}+\frac{b\left(a+bx^3\right)^6}{126a^2x^{18}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 69, normalized size = 1.72

$$-\frac{a^5}{21x^{21}}-\frac{5a^4b}{18x^{18}}-\frac{2a^3b^2}{3x^{15}}-\frac{5a^2b^3}{6x^{12}}-\frac{5ab^4}{9x^9}-\frac{b^5}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^5/x^22,x]
```

```
[Out] -1/21*a^5/x^21 - (5*a^4*b)/(18*x^18) - (2*a^3*b^2)/(3*x^15) - (5*a^2*b^3)/(
6*x^12) - (5*a*b^4)/(9*x^9) - b^5/(6*x^6)
```

fricas [A] time = 1.03, size = 59, normalized size = 1.48

$$-\frac{21\,b^5x^{15}+70\,ab^4x^{12}+105\,a^2b^3x^9+84\,a^3b^2x^6+35\,a^4bx^3+6\,a^5}{126\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^22,x, algorithm="fricas")
```

```
[Out] -1/126*(21*b^5*x^15 + 70*a*b^4*x^12 + 105*a^2*b^3*x^9 + 84*a^3*b^2*x^6 + 35
*a^4*b*x^3 + 6*a^5)/x^21
```

giac [A] time = 0.17, size = 59, normalized size = 1.48

$$-\frac{21\,b^5x^{15}+70\,ab^4x^{12}+105\,a^2b^3x^9+84\,a^3b^2x^6+35\,a^4bx^3+6\,a^5}{126\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^22,x, algorithm="giac")
```

```
[Out] -1/126*(21*b^5*x^15 + 70*a*b^4*x^12 + 105*a^2*b^3*x^9 + 84*a^3*b^2*x^6 + 35
*a^4*b*x^3 + 6*a^5)/x^21
```

maple [A] time = 0.00, size = 58, normalized size = 1.45

$$-\frac{b^5}{6x^6}-\frac{5a\,b^4}{9x^9}-\frac{5a^2b^3}{6x^{12}}-\frac{2a^3b^2}{3x^{15}}-\frac{5a^4b}{18x^{18}}-\frac{a^5}{21x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5/x^22,x)
```

```
[Out] -2/3*a^3*b^2/x^15-5/6*a^2*b^3/x^12-5/18*a^4*b/x^18-1/6*b^5/x^6-1/21*a^5/x^2
1-5/9*a*b^4/x^9
```

maxima [A] time = 1.33, size = 59, normalized size = 1.48

$$-\frac{21\,b^5x^{15}+70\,ab^4x^{12}+105\,a^2b^3x^9+84\,a^3b^2x^6+35\,a^4bx^3+6\,a^5}{126\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^22,x, algorithm="maxima")
```

```
[Out] -1/126*(21*b^5*x^15 + 70*a*b^4*x^12 + 105*a^2*b^3*x^9 + 84*a^3*b^2*x^6 + 35
*a^4*b*x^3 + 6*a^5)/x^21
```

mupad [B] time = 0.95, size = 59, normalized size = 1.48

$$-\frac{\frac{a^5}{21}+\frac{5\,a^4\,b\,x^3}{18}+\frac{2\,a^3\,b^2\,x^6}{3}+\frac{5\,a^2\,b^3\,x^9}{6}+\frac{5\,a\,b^4\,x^{12}}{9}+\frac{b^5\,x^{15}}{6}}{x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^5/x^22,x)
```

```
[Out] -(a^5/21 + (b^5*x^15)/6 + (5*a^4*b*x^3)/18 + (5*a*b^4*x^12)/9 + (2*a^3*b^2*
x^6)/3 + (5*a^2*b^3*x^9)/6)/x^21
```

sympy [**A**] time = 0.73, size = 63, normalized size = 1.58

$$\frac{-6a^5 - 35a^4bx^3 - 84a^3b^2x^6 - 105a^2b^3x^9 - 70ab^4x^{12} - 21b^5x^{15}}{126x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5/x**22,x)
```

```
[Out] (-6*a**5 - 35*a**4*b*x**3 - 84*a**3*b**2*x**6 - 105*a**2*b**3*x**9 - 70*a*b
**4*x**12 - 21*b**5*x**15)/(126*x**21)
```

3.272 $\int \frac{\left(a+bx^3\right)^5}{x^{25}}\,dx$

Optimal. Leaf size=62

$$-\frac{b^2\left(a+bx^3\right)^6}{504a^3x^{18}}+\frac{b\left(a+bx^3\right)^6}{84a^2x^{21}}-\frac{\left(a+bx^3\right)^6}{24ax^{24}}$$

```
[Out] -1/24*(b*x^3+a)^6/a/x^24+1/84*b*(b*x^3+a)^6/a^2/x^21-1/504*b^2*(b*x^3+a)^6/
a^3/x^18
```

Rubi [A] time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {266, 45, 37}

$$-\frac{b^2\left(a+bx^3\right)^6}{504a^3x^{18}}+\frac{b\left(a+bx^3\right)^6}{84a^2x^{21}}-\frac{\left(a+bx^3\right)^6}{24ax^{24}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^5/x^25,x]
```

```
[Out] -(a + b*x^3)^6/(24*a*x^24) + (b*(a + b*x^3)^6)/(84*a^2*x^21) - (b^2*(a + b*
x^3)^6)/(504*a^3*x^18)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^5}{x^{25}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^5}{x^9}\,dx, x, x^3\right)\\ &= -\frac{\left(a+bx^3\right)^6}{24ax^{24}} - \frac{b\,\text{Subst}\left(\int \frac{(a+bx)^5}{x^8}\,dx, x, x^3\right)}{12a}\\ &= -\frac{\left(a+bx^3\right)^6}{24ax^{24}} + \frac{b\left(a+bx^3\right)^6}{84a^2x^{21}} + \frac{b^2\,\text{Subst}\left(\int \frac{(a+bx)^5}{x^7}\,dx, x, x^3\right)}{84a^2}\\ &= -\frac{\left(a+bx^3\right)^6}{24ax^{24}} + \frac{b\left(a+bx^3\right)^6}{84a^2x^{21}} - \frac{b^2\left(a+bx^3\right)^6}{504a^3x^{18}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 69, normalized size = 1.11

$$-\frac{a^5}{24x^{24}} - \frac{5a^4b}{21x^{21}} - \frac{5a^3b^2}{9x^{18}} - \frac{2a^2b^3}{3x^{15}} - \frac{5ab^4}{12x^{12}} - \frac{b^5}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^5/x^25,x]
```

```
[Out] -1/24*a^5/x^24 - (5*a^4*b)/(21*x^21) - (5*a^3*b^2)/(9*x^18) - (2*a^2*b^3)/(
3*x^15) - (5*a*b^4)/(12*x^12) - b^5/(9*x^9)
```

fricas [A] time = 0.84, size = 59, normalized size = 0.95

$$-\frac{56\,b^5x^{15} + 210\,ab^4x^{12} + 336\,a^2b^3x^9 + 280\,a^3b^2x^6 + 120\,a^4bx^3 + 21\,a^5}{504\,x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^25,x, algorithm="fricas")
```

```
[Out] -1/504*(56*b^5*x^15 + 210*a*b^4*x^12 + 336*a^2*b^3*x^9 + 280*a^3*b^2*x^6 +
120*a^4*b*x^3 + 21*a^5)/x^24
```

giac [A] time = 0.15, size = 59, normalized size = 0.95

$$-\frac{56\,b^5x^{15} + 210\,ab^4x^{12} + 336\,a^2b^3x^9 + 280\,a^3b^2x^6 + 120\,a^4bx^3 + 21\,a^5}{504\,x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^25,x, algorithm="giac")
```

```
[Out] -1/504*(56*b^5*x^15 + 210*a*b^4*x^12 + 336*a^2*b^3*x^9 + 280*a^3*b^2*x^6 +
120*a^4*b*x^3 + 21*a^5)/x^24
```

maple [A] time = 0.00, size = 58, normalized size = 0.94

$$-\frac{b^5}{9x^9} - \frac{5a\,b^4}{12x^{12}} - \frac{2a^2b^3}{3x^{15}} - \frac{5a^3b^2}{9x^{18}} - \frac{5a^4b}{21x^{21}} - \frac{a^5}{24x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5/x^25,x)
```

```
[Out] -2/3*a^2*b^3/x^15-5/21*a^4*b/x^21-1/24*a^5/x^24-5/12*a*b^4/x^12-5/9*a^3*b^2
/x^18-1/9*b^5/x^9
```

maxima [A] time = 1.32, size = 59, normalized size = 0.95

$$-\frac{56\,b^5x^{15}+210\,ab^4x^{12}+336\,a^2b^3x^9+280\,a^3b^2x^6+120\,a^4bx^3+21\,a^5}{504\,x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^25,x, algorithm="maxima")
```

```
[Out] -1/504*(56*b^5*x^15 + 210*a*b^4*x^12 + 336*a^2*b^3*x^9 + 280*a^3*b^2*x^6 +
120*a^4*b*x^3 + 21*a^5)/x^24
```

mupad [B] time = 0.95, size = 59, normalized size = 0.95

$$-\frac{\frac{a^5}{24}+\frac{5\,a^4\,b\,x^3}{21}+\frac{5\,a^3\,b^2\,x^6}{9}+\frac{2\,a^2\,b^3\,x^9}{3}+\frac{5\,a\,b^4\,x^{12}}{12}+\frac{b^5\,x^{15}}{9}}{x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^5/x^25,x)
```

```
[Out] -(a^5/24 + (b^5*x^15)/9 + (5*a^4*b*x^3)/21 + (5*a*b^4*x^12)/12 + (5*a^3*b^2
*x^6)/9 + (2*a^2*b^3*x^9)/3)/x^24
```

sympy [A] time = 0.88, size = 63, normalized size = 1.02

$$\frac{-21a^5-120a^4bx^3-280a^3b^2x^6-336a^2b^3x^9-210ab^4x^{12}-56b^5x^{15}}{504x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5/x**25,x)
```

```
[Out] (-21*a**5 - 120*a**4*b*x**3 - 280*a**3*b**2*x**6 - 336*a**2*b**3*x**9 - 210
*a*b**4*x**12 - 56*b**5*x**15)/(504*x**24)
```

3.273 $\int \frac{\left(a+bx^3\right)^5}{x^{28}}\,dx$

Optimal. Leaf size=69

$$-\frac{a^5}{27x^{27}}-\frac{5a^4b}{24x^{24}}-\frac{10a^3b^2}{21x^{21}}-\frac{5a^2b^3}{9x^{18}}-\frac{ab^4}{3x^{15}}-\frac{b^5}{12x^{12}}$$

```
[Out] -1/27*a^5/x^27-5/24*a^4*b/x^24-10/21*a^3*b^2/x^21-5/9*a^2*b^3/x^18-1/3*a*b^
4/x^15-1/12*b^5/x^12
```

Rubi [A] time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$-\frac{10a^3b^2}{21x^{21}}-\frac{5a^2b^3}{9x^{18}}-\frac{5a^4b}{24x^{24}}-\frac{a^5}{27x^{27}}-\frac{ab^4}{3x^{15}}-\frac{b^5}{12x^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^5/x^28,x]
```

```
[Out] -a^5/(27*x^27) - (5*a^4*b)/(24*x^24) - (10*a^3*b^2)/(21*x^21) - (5*a^2*b^3)
/(9*x^18) - (a*b^4)/(3*x^15) - b^5/(12*x^12)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^5}{x^{28}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^5}{x^{10}}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{a^5}{x^{10}}+\frac{5a^4b}{x^9}+\frac{10a^3b^2}{x^8}+\frac{10a^2b^3}{x^7}+\frac{5ab^4}{x^6}+\frac{b^5}{x^5}\right)dx, x, x^3\right)\\ &= -\frac{a^5}{27x^{27}}-\frac{5a^4b}{24x^{24}}-\frac{10a^3b^2}{21x^{21}}-\frac{5a^2b^3}{9x^{18}}-\frac{ab^4}{3x^{15}}-\frac{b^5}{12x^{12}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 69, normalized size = 1.00

$$-\frac{a^5}{27x^{27}}-\frac{5a^4b}{24x^{24}}-\frac{10a^3b^2}{21x^{21}}-\frac{5a^2b^3}{9x^{18}}-\frac{ab^4}{3x^{15}}-\frac{b^5}{12x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^5/x^28,x]
```

```
[Out] -1/27*a^5/x^27 - (5*a^4*b)/(24*x^24) - (10*a^3*b^2)/(21*x^21) - (5*a^2*b^3)
/(9*x^18) - (a*b^4)/(3*x^15) - b^5/(12*x^12)
```

fricas [A] time = 0.84, size = 59, normalized size = 0.86

$$-\frac{126\,b^5x^{15}+504\,ab^4x^{12}+840\,a^2b^3x^9+720\,a^3b^2x^6+315\,a^4bx^3+56\,a^5}{1512\,x^{27}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^28,x, algorithm="fricas")
```

```
[Out] -1/1512*(126*b^5*x^15 + 504*a*b^4*x^12 + 840*a^2*b^3*x^9 + 720*a^3*b^2*x^6
+ 315*a^4*b*x^3 + 56*a^5)/x^27
```

giac [A] time = 0.15, size = 59, normalized size = 0.86

$$-\frac{126\,b^5x^{15}+504\,ab^4x^{12}+840\,a^2b^3x^9+720\,a^3b^2x^6+315\,a^4bx^3+56\,a^5}{1512\,x^{27}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^28,x, algorithm="giac")
```

```
[Out] -1/1512*(126*b^5*x^15 + 504*a*b^4*x^12 + 840*a^2*b^3*x^9 + 720*a^3*b^2*x^6
+ 315*a^4*b*x^3 + 56*a^5)/x^27
```

maple [A] time = 0.00, size = 58, normalized size = 0.84

$$-\frac{b^5}{12x^{12}}-\frac{a\,b^4}{3x^{15}}-\frac{5a^2b^3}{9x^{18}}-\frac{10a^3b^2}{21x^{21}}-\frac{5a^4b}{24x^{24}}-\frac{a^5}{27x^{27}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5/x^28,x)
```

```
[Out] -1/27*a^5/x^27-5/24*a^4*b/x^24-10/21*a^3*b^2/x^21-5/9*a^2*b^3/x^18-1/3*a*b^
4/x^15-1/12*b^5/x^12
```

maxima [A] time = 1.33, size = 59, normalized size = 0.86

$$-\frac{126\,b^5x^{15}+504\,ab^4x^{12}+840\,a^2b^3x^9+720\,a^3b^2x^6+315\,a^4bx^3+56\,a^5}{1512\,x^{27}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^28,x, algorithm="maxima")
```

```
[Out] -1/1512*(126*b^5*x^15 + 504*a*b^4*x^12 + 840*a^2*b^3*x^9 + 720*a^3*b^2*x^6
+ 315*a^4*b*x^3 + 56*a^5)/x^27
```

mupad [B] time = 0.04, size = 59, normalized size = 0.86

$$-\frac{\frac{a^5}{27}+\frac{5\,a^4\,b\,x^3}{24}+\frac{10\,a^3\,b^2\,x^6}{21}+\frac{5\,a^2\,b^3\,x^9}{9}+\frac{a\,b^4\,x^{12}}{3}+\frac{b^5\,x^{15}}{12}}{x^{27}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^5/x^28,x)
```

```
[Out] -(a^5/27 + (b^5*x^15)/12 + (5*a^4*b*x^3)/24 + (a*b^4*x^12)/3 + (10*a^3*b^2*
x^6)/21 + (5*a^2*b^3*x^9)/9)/x^27
```

sympy [A] time = 0.79, size = 63, normalized size = 0.91

$$\frac{-56a^5-315a^4bx^3-720a^3b^2x^6-840a^2b^3x^9-504ab^4x^{12}-126b^5x^{15}}{1512x^{27}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5/x**28,x)
```

```
[Out] (-56*a**5 - 315*a**4*b*x**3 - 720*a**3*b**2*x**6 - 840*a**2*b**3*x**9 - 504
*a*b**4*x**12 - 126*b**5*x**15)/(1512*x**27)
```

3.274 $\int \frac{\left(a+bx^3\right)^5}{x^{31}}\,dx$

Optimal. Leaf size=69

$$-\frac{a^5}{30x^{30}}-\frac{5a^4b}{27x^{27}}-\frac{5a^3b^2}{12x^{24}}-\frac{10a^2b^3}{21x^{21}}-\frac{5ab^4}{18x^{18}}-\frac{b^5}{15x^{15}}$$

```
[Out] -1/30*a^5/x^30-5/27*a^4*b/x^27-5/12*a^3*b^2/x^24-10/21*a^2*b^3/x^21-5/18*a*
b^4/x^18-1/15*b^5/x^15
```

Rubi [A] time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$-\frac{5a^3b^2}{12x^{24}}-\frac{10a^2b^3}{21x^{21}}-\frac{5a^4b}{27x^{27}}-\frac{a^5}{30x^{30}}-\frac{5ab^4}{18x^{18}}-\frac{b^5}{15x^{15}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^5/x^31,x]
```

```
[Out] -a^5/(30*x^30) - (5*a^4*b)/(27*x^27) - (5*a^3*b^2)/(12*x^24) - (10*a^2*b^3)
/(21*x^21) - (5*a*b^4)/(18*x^18) - b^5/(15*x^15)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^5}{x^{31}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^5}{x^{11}}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{a^5}{x^{11}}+\frac{5a^4b}{x^{10}}+\frac{10a^3b^2}{x^9}+\frac{10a^2b^3}{x^8}+\frac{5ab^4}{x^7}+\frac{b^5}{x^6}\right)dx, x, x^3\right)\\ &= -\frac{a^5}{30x^{30}}-\frac{5a^4b}{27x^{27}}-\frac{5a^3b^2}{12x^{24}}-\frac{10a^2b^3}{21x^{21}}-\frac{5ab^4}{18x^{18}}-\frac{b^5}{15x^{15}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 69, normalized size = 1.00

$$-\frac{a^5}{30x^{30}}-\frac{5a^4b}{27x^{27}}-\frac{5a^3b^2}{12x^{24}}-\frac{10a^2b^3}{21x^{21}}-\frac{5ab^4}{18x^{18}}-\frac{b^5}{15x^{15}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^5/x^31,x]
```

```
[Out] -1/30*a^5/x^30 - (5*a^4*b)/(27*x^27) - (5*a^3*b^2)/(12*x^24) - (10*a^2*b^3)
/(21*x^21) - (5*a*b^4)/(18*x^18) - b^5/(15*x^15)
```

fricas [A] time = 0.84, size = 59, normalized size = 0.86

$$-\frac{252\,b^5x^{15}+1050\,ab^4x^{12}+1800\,a^2b^3x^9+1575\,a^3b^2x^6+700\,a^4bx^3+126\,a^5}{3780\,x^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^31,x, algorithm="fricas")
```

```
[Out] -1/3780*(252*b^5*x^15 + 1050*a*b^4*x^12 + 1800*a^2*b^3*x^9 + 1575*a^3*b^2*x
^6 + 700*a^4*b*x^3 + 126*a^5)/x^30
```

giac [A] time = 0.17, size = 59, normalized size = 0.86

$$-\frac{252\,b^5x^{15}+1050\,ab^4x^{12}+1800\,a^2b^3x^9+1575\,a^3b^2x^6+700\,a^4bx^3+126\,a^5}{3780\,x^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^31,x, algorithm="giac")
```

```
[Out] -1/3780*(252*b^5*x^15 + 1050*a*b^4*x^12 + 1800*a^2*b^3*x^9 + 1575*a^3*b^2*x
^6 + 700*a^4*b*x^3 + 126*a^5)/x^30
```

maple [A] time = 0.00, size = 58, normalized size = 0.84

$$-\frac{b^5}{15x^{15}}-\frac{5a\,b^4}{18x^{18}}-\frac{10a^2b^3}{21x^{21}}-\frac{5a^3b^2}{12x^{24}}-\frac{5a^4b}{27x^{27}}-\frac{a^5}{30x^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5/x^31,x)
```

```
[Out] -1/30*a^5/x^30-5/27*a^4*b/x^27-5/12*a^3*b^2/x^24-10/21*a^2*b^3/x^21-5/18*a*
b^4/x^18-1/15*b^5/x^15
```

maxima [A] time = 1.34, size = 59, normalized size = 0.86

$$-\frac{252\,b^5x^{15}+1050\,ab^4x^{12}+1800\,a^2b^3x^9+1575\,a^3b^2x^6+700\,a^4bx^3+126\,a^5}{3780\,x^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^31,x, algorithm="maxima")
```

```
[Out] -1/3780*(252*b^5*x^15 + 1050*a*b^4*x^12 + 1800*a^2*b^3*x^9 + 1575*a^3*b^2*x
^6 + 700*a^4*b*x^3 + 126*a^5)/x^30
```

mupad [B] time = 0.04, size = 59, normalized size = 0.86

$$-\frac{\frac{a^5}{30}+\frac{5\,a^4\,b\,x^3}{27}+\frac{5\,a^3\,b^2\,x^6}{12}+\frac{10\,a^2\,b^3\,x^9}{21}+\frac{5\,a\,b^4\,x^{12}}{18}+\frac{b^5\,x^{15}}{15}}{x^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^5/x^31,x)
```

```
[Out] -(a^5/30 + (b^5*x^15)/15 + (5*a^4*b*x^3)/27 + (5*a*b^4*x^12)/18 + (5*a^3*b^
2*x^6)/12 + (10*a^2*b^3*x^9)/21)/x^30
```

sympy [A] time = 1.00, size = 63, normalized size = 0.91

$$\frac{-126a^5-700a^4bx^3-1575a^3b^2x^6-1800a^2b^3x^9-1050ab^4x^{12}-252b^5x^{15}}{3780x^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5/x**31,x)
```

```
[Out] (-126*a**5 - 700*a**4*b*x**3 - 1575*a**3*b**2*x**6 - 1800*a**2*b**3*x**9 -
1050*a*b**4*x**12 - 252*b**5*x**15)/(3780*x**30)
```

3.275 $\int x^4 \left(a + bx^3\right)^5 \, dx$

Optimal. Leaf size=69

$$\frac{a^5x^5}{5} + \frac{5}{8}a^4bx^8 + \frac{10}{11}a^3b^2x^{11} + \frac{5}{7}a^2b^3x^{14} + \frac{5}{17}ab^4x^{17} + \frac{b^5x^{20}}{20}$$

```
[Out] 1/5*a^5*x^5+5/8*a^4*b*x^8+10/11*a^3*b^2*x^11+5/7*a^2*b^3*x^14+5/17*a*b^4*x^
17+1/20*b^5*x^20
```

Rubi [A] time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{5}{7}a^2b^3x^{14} + \frac{10}{11}a^3b^2x^{11} + \frac{5}{8}a^4bx^8 + \frac{a^5x^5}{5} + \frac{5}{17}ab^4x^{17} + \frac{b^5x^{20}}{20}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^3)^5,x]
```

```
[Out] (a^5*x^5)/5 + (5*a^4*b*x^8)/8 + (10*a^3*b^2*x^11)/11 + (5*a^2*b^3*x^14)/7 +
 (5*a*b^4*x^17)/17 + (b^5*x^20)/20
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^4 \left(a + bx^3\right)^5 \, dx &= \int \left(a^5x^4 + 5a^4bx^7 + 10a^3b^2x^{10} + 10a^2b^3x^{13} + 5ab^4x^{16} + b^5x^{19}\right) dx \\ &= \frac{a^5x^5}{5} + \frac{5}{8}a^4bx^8 + \frac{10}{11}a^3b^2x^{11} + \frac{5}{7}a^2b^3x^{14} + \frac{5}{17}ab^4x^{17} + \frac{b^5x^{20}}{20}\end{aligned}$$

Mathematica [A] time = 0.00, size = 69, normalized size = 1.00

$$\frac{a^5x^5}{5} + \frac{5}{8}a^4bx^8 + \frac{10}{11}a^3b^2x^{11} + \frac{5}{7}a^2b^3x^{14} + \frac{5}{17}ab^4x^{17} + \frac{b^5x^{20}}{20}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^3)^5,x]
```

```
[Out] (a^5*x^5)/5 + (5*a^4*b*x^8)/8 + (10*a^3*b^2*x^11)/11 + (5*a^2*b^3*x^14)/7 +
 (5*a*b^4*x^17)/17 + (b^5*x^20)/20
```

fricas [A] time = 0.77, size = 57, normalized size = 0.83

$$\frac{1}{20}x^{20}b^5 + \frac{5}{17}x^{17}b^4a + \frac{5}{7}x^{14}b^3a^2 + \frac{10}{11}x^{11}b^2a^3 + \frac{5}{8}x^8ba^4 + \frac{1}{5}x^5a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^5,x, algorithm="fricas")
```

```
[Out] 1/20*x^20*b^5 + 5/17*x^17*b^4*a + 5/7*x^14*b^3*a^2 + 10/11*x^11*b^2*a^3 + 5
/8*x^8*b*a^4 + 1/5*x^5*a^5
```

giac [A] time = 0.15, size = 57, normalized size = 0.83

$$\frac{1}{20}\,b^5x^{20} + \frac{5}{17}\,ab^4x^{17} + \frac{5}{7}\,a^2b^3x^{14} + \frac{10}{11}\,a^3b^2x^{11} + \frac{5}{8}\,a^4bx^8 + \frac{1}{5}\,a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^5,x, algorithm="giac")
```

```
[Out] 1/20*b^5*x^20 + 5/17*a*b^4*x^17 + 5/7*a^2*b^3*x^14 + 10/11*a^3*b^2*x^11 + 5
/8*a^4*b*x^8 + 1/5*a^5*x^5
```

maple [A] time = 0.00, size = 58, normalized size = 0.84

$$\frac{1}{20}b^5x^{20} + \frac{5}{17}a\,b^4x^{17} + \frac{5}{7}a^2b^3x^{14} + \frac{10}{11}a^3b^2x^{11} + \frac{5}{8}a^4b\,x^8 + \frac{1}{5}a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)^5,x)
```

```
[Out] 1/5*a^5*x^5+5/8*a^4*b*x^8+10/11*a^3*b^2*x^11+5/7*a^2*b^3*x^14+5/17*a*b^4*x^
17+1/20*b^5*x^20
```

maxima [A] time = 1.35, size = 57, normalized size = 0.83

$$\frac{1}{20}\,b^5x^{20} + \frac{5}{17}\,ab^4x^{17} + \frac{5}{7}\,a^2b^3x^{14} + \frac{10}{11}\,a^3b^2x^{11} + \frac{5}{8}\,a^4bx^8 + \frac{1}{5}\,a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^5,x, algorithm="maxima")
```

```
[Out] 1/20*b^5*x^20 + 5/17*a*b^4*x^17 + 5/7*a^2*b^3*x^14 + 10/11*a^3*b^2*x^11 + 5
/8*a^4*b*x^8 + 1/5*a^5*x^5
```

mupad [B] time = 0.02, size = 57, normalized size = 0.83

$$\frac{a^5\,x^5}{5} + \frac{5\,a^4\,b\,x^8}{8} + \frac{10\,a^3\,b^2\,x^{11}}{11} + \frac{5\,a^2\,b^3\,x^{14}}{7} + \frac{5\,a\,b^4\,x^{17}}{17} + \frac{b^5\,x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^3)^5,x)
```

```
[Out] (a^5*x^5)/5 + (b^5*x^20)/20 + (5*a^4*b*x^8)/8 + (5*a*b^4*x^17)/17 + (10*a^3
*b^2*x^11)/11 + (5*a^2*b^3*x^14)/7
```

sympy [A] time = 0.10, size = 66, normalized size = 0.96

$$\frac{a^5x^5}{5} + \frac{5a^4bx^8}{8} + \frac{10a^3b^2x^{11}}{11} + \frac{5a^2b^3x^{14}}{7} + \frac{5ab^4x^{17}}{17} + \frac{b^5x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)**5,x)
```

```
[Out] a**5*x**5/5 + 5*a**4*b*x**8/8 + 10*a**3*b**2*x**11/11 + 5*a**2*b**3*x**14/7
 + 5*a*b**4*x**17/17 + b**5*x**20/20
```

3.276 $\int x^3 \left(a + bx^3\right)^5 \, dx$

Optimal. Leaf size=66

$$\frac{a^5 x^4}{4} + \frac{5}{7} a^4 b x^7 + a^3 b^2 x^{10} + \frac{10}{13} a^2 b^3 x^{13} + \frac{5}{16} a b^4 x^{16} + \frac{b^5 x^{19}}{19}$$

```
[Out] 1/4*a^5*x^4+5/7*a^4*b*x^7+a^3*b^2*x^10+10/13*a^2*b^3*x^13+5/16*a*b^4*x^16+1
/19*b^5*x^19
```

Rubi [A] time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{10}{13} a^2 b^3 x^{13} + a^3 b^2 x^{10} + \frac{5}{7} a^4 b x^7 + \frac{a^5 x^4}{4} + \frac{5}{16} a b^4 x^{16} + \frac{b^5 x^{19}}{19}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^3)^5,x]
```

```
[Out] (a^5*x^4)/4 + (5*a^4*b*x^7)/7 + a^3*b^2*x^10 + (10*a^2*b^3*x^13)/13 + (5*a*
b^4*x^16)/16 + (b^5*x^19)/19
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^3 \left(a + bx^3\right)^5 \, dx &= \int \left(a^5 x^3 + 5a^4 b x^6 + 10a^3 b^2 x^9 + 10a^2 b^3 x^{12} + 5a b^4 x^{15} + b^5 x^{18}\right) dx \\ &= \frac{a^5 x^4}{4} + \frac{5}{7} a^4 b x^7 + a^3 b^2 x^{10} + \frac{10}{13} a^2 b^3 x^{13} + \frac{5}{16} a b^4 x^{16} + \frac{b^5 x^{19}}{19}\end{aligned}$$

Mathematica [A] time = 0.00, size = 66, normalized size = 1.00

$$\frac{a^5 x^4}{4} + \frac{5}{7} a^4 b x^7 + a^3 b^2 x^{10} + \frac{10}{13} a^2 b^3 x^{13} + \frac{5}{16} a b^4 x^{16} + \frac{b^5 x^{19}}{19}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^3)^5,x]
```

```
[Out] (a^5*x^4)/4 + (5*a^4*b*x^7)/7 + a^3*b^2*x^10 + (10*a^2*b^3*x^13)/13 + (5*a*
b^4*x^16)/16 + (b^5*x^19)/19
```

fricas [A] time = 0.71, size = 56, normalized size = 0.85

$$\frac{1}{19} x^{19} b^5 + \frac{5}{16} x^{16} b^4 a + \frac{10}{13} x^{13} b^3 a^2 + x^{10} b^2 a^3 + \frac{5}{7} x^7 b a^4 + \frac{1}{4} x^4 a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^5,x, algorithm="fricas")
```

```
[Out] 1/19*x^19*b^5 + 5/16*x^16*b^4*a + 10/13*x^13*b^3*a^2 + x^10*b^2*a^3 + 5/7*x
^7*b*a^4 + 1/4*x^4*a^5
```

giac [A] time = 0.18, size = 56, normalized size = 0.85

$$\frac{1}{19}b^5x^{19} + \frac{5}{16}ab^4x^{16} + \frac{10}{13}a^2b^3x^{13} + a^3b^2x^{10} + \frac{5}{7}a^4bx^7 + \frac{1}{4}a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^5,x, algorithm="giac")
```

```
[Out] 1/19*b^5*x^19 + 5/16*a*b^4*x^16 + 10/13*a^2*b^3*x^13 + a^3*b^2*x^10 + 5/7*a
^4*b*x^7 + 1/4*a^5*x^4
```

maple [A] time = 0.00, size = 57, normalized size = 0.86

$$\frac{1}{19}b^5x^{19} + \frac{5}{16}a\,b^4x^{16} + \frac{10}{13}a^2b^3x^{13} + a^3b^2x^{10} + \frac{5}{7}a^4b\,x^7 + \frac{1}{4}a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)^5,x)
```

```
[Out] 1/4*a^5*x^4+5/7*a^4*b*x^7+a^3*b^2*x^10+10/13*a^2*b^3*x^13+5/16*a*b^4*x^16+1
/19*b^5*x^19
```

maxima [A] time = 1.32, size = 56, normalized size = 0.85

$$\frac{1}{19}b^5x^{19} + \frac{5}{16}ab^4x^{16} + \frac{10}{13}a^2b^3x^{13} + a^3b^2x^{10} + \frac{5}{7}a^4bx^7 + \frac{1}{4}a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^5,x, algorithm="maxima")
```

```
[Out] 1/19*b^5*x^19 + 5/16*a*b^4*x^16 + 10/13*a^2*b^3*x^13 + a^3*b^2*x^10 + 5/7*a
^4*b*x^7 + 1/4*a^5*x^4
```

mupad [B] time = 0.02, size = 56, normalized size = 0.85

$$\frac{a^5\,x^4}{4} + \frac{5\,a^4\,b\,x^7}{7} + a^3\,b^2\,x^{10} + \frac{10\,a^2\,b^3\,x^{13}}{13} + \frac{5\,a\,b^4\,x^{16}}{16} + \frac{b^5\,x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^3)^5,x)
```

```
[Out] (a^5*x^4)/4 + (b^5*x^19)/19 + (5*a^4*b*x^7)/7 + (5*a*b^4*x^16)/16 + a^3*b^2
*x^10 + (10*a^2*b^3*x^13)/13
```

sympy [A] time = 0.09, size = 63, normalized size = 0.95

$$\frac{a^5x^4}{4} + \frac{5a^4bx^7}{7} + a^3b^2x^{10} + \frac{10a^2b^3x^{13}}{13} + \frac{5ab^4x^{16}}{16} + \frac{b^5x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**5,x)
```

```
[Out] a**5*x**4/4 + 5*a**4*b*x**7/7 + a**3*b**2*x**10 + 10*a**2*b**3*x**13/13 + 5
*a*b**4*x**16/16 + b**5*x**19/19
```

3.277 $\int x\left(a+bx^3\right)^5\,dx$

Optimal. Leaf size=66

$$\frac{a^5x^2}{2}+a^4bx^5+\frac{5}{4}a^3b^2x^8+\frac{10}{11}a^2b^3x^{11}+\frac{5}{14}ab^4x^{14}+\frac{b^5x^{17}}{17}$$

```
[Out] 1/2*a^5*x^2+a^4*b*x^5+5/4*a^3*b^2*x^8+10/11*a^2*b^3*x^11+5/14*a*b^4*x^14+1/
17*b^5*x^17
```

Rubi [A] time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {270}

$$\frac{10}{11}a^2b^3x^{11}+\frac{5}{4}a^3b^2x^8+a^4bx^5+\frac{a^5x^2}{2}+\frac{5}{14}ab^4x^{14}+\frac{b^5x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3)^5,x]
```

```
[Out] (a^5*x^2)/2 + a^4*b*x^5 + (5*a^3*b^2*x^8)/4 + (10*a^2*b^3*x^11)/11 + (5*a*b
^4*x^14)/14 + (b^5*x^17)/17
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^3\right)^5\,dx &= \int\left(a^5x+5a^4bx^4+10a^3b^2x^7+10a^2b^3x^{10}+5ab^4x^{13}+b^5x^{16}\right)dx\\ &= \frac{a^5x^2}{2}+a^4bx^5+\frac{5}{4}a^3b^2x^8+\frac{10}{11}a^2b^3x^{11}+\frac{5}{14}ab^4x^{14}+\frac{b^5x^{17}}{17}\end{aligned}$$

Mathematica [A] time = 0.00, size = 66, normalized size = 1.00

$$\frac{a^5x^2}{2}+a^4bx^5+\frac{5}{4}a^3b^2x^8+\frac{10}{11}a^2b^3x^{11}+\frac{5}{14}ab^4x^{14}+\frac{b^5x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)^5,x]
```

```
[Out] (a^5*x^2)/2 + a^4*b*x^5 + (5*a^3*b^2*x^8)/4 + (10*a^2*b^3*x^11)/11 + (5*a*b
^4*x^14)/14 + (b^5*x^17)/17
```

fricas [A] time = 0.56, size = 56, normalized size = 0.85

$$\frac{1}{17}x^{17}b^5+\frac{5}{14}x^{14}b^4a+\frac{10}{11}x^{11}b^3a^2+\frac{5}{4}x^8b^2a^3+x^5ba^4+\frac{1}{2}x^2a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^5,x, algorithm="fricas")
```

```
[Out] 1/17*x^17*b^5 + 5/14*x^14*b^4*a + 10/11*x^11*b^3*a^2 + 5/4*x^8*b^2*a^3 + x^
5*b*a^4 + 1/2*x^2*a^5
```

giac [A] time = 0.21, size = 56, normalized size = 0.85

$$\frac{1}{17}\,b^5x^{17} + \frac{5}{14}\,ab^4x^{14} + \frac{10}{11}\,a^2b^3x^{11} + \frac{5}{4}\,a^3b^2x^8 + a^4bx^5 + \frac{1}{2}\,a^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^5,x, algorithm="giac")
```

```
[Out] 1/17*b^5*x^17 + 5/14*a*b^4*x^14 + 10/11*a^2*b^3*x^11 + 5/4*a^3*b^2*x^8 + a^
4*b*x^5 + 1/2*a^5*x^2
```

maple [A] time = 0.00, size = 57, normalized size = 0.86

$$\frac{1}{17}b^5x^{17} + \frac{5}{14}a\,b^4x^{14} + \frac{10}{11}a^2b^3x^{11} + \frac{5}{4}a^3b^2x^8 + a^4b\,x^5 + \frac{1}{2}a^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^5,x)
```

```
[Out] 1/2*a^5*x^2+a^4*b*x^5+5/4*a^3*b^2*x^8+10/11*a^2*b^3*x^11+5/14*a*b^4*x^14+1/
17*b^5*x^17
```

maxima [A] time = 1.33, size = 56, normalized size = 0.85

$$\frac{1}{17}\,b^5x^{17} + \frac{5}{14}\,ab^4x^{14} + \frac{10}{11}\,a^2b^3x^{11} + \frac{5}{4}\,a^3b^2x^8 + a^4bx^5 + \frac{1}{2}\,a^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^5,x, algorithm="maxima")
```

```
[Out] 1/17*b^5*x^17 + 5/14*a*b^4*x^14 + 10/11*a^2*b^3*x^11 + 5/4*a^3*b^2*x^8 + a^
4*b*x^5 + 1/2*a^5*x^2
```

mupad [B] time = 0.02, size = 56, normalized size = 0.85

$$\frac{a^5\,x^2}{2} + a^4\,b\,x^5 + \frac{5\,a^3\,b^2\,x^8}{4} + \frac{10\,a^2\,b^3\,x^{11}}{11} + \frac{5\,a\,b^4\,x^{14}}{14} + \frac{b^5\,x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^3)^5,x)
```

```
[Out] (a^5*x^2)/2 + (b^5*x^17)/17 + a^4*b*x^5 + (5*a*b^4*x^14)/14 + (5*a^3*b^2*x^
8)/4 + (10*a^2*b^3*x^11)/11
```

sympy [A] time = 0.09, size = 63, normalized size = 0.95

$$\frac{a^5x^2}{2} + a^4bx^5 + \frac{5a^3b^2x^8}{4} + \frac{10a^2b^3x^{11}}{11} + \frac{5ab^4x^{14}}{14} + \frac{b^5x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**5,x)
```

```
[Out] a**5*x**2/2 + a**4*b*x**5 + 5*a**3*b**2*x**8/4 + 10*a**2*b**3*x**11/11 + 5*
a*b**4*x**14/14 + b**5*x**17/17
```

3.278 $\int \left(a + bx^3\right)^5 \, dx$

Optimal. Leaf size=61

$$a^5 x + \frac{5}{4} a^4 b x^4 + \frac{10}{7} a^3 b^2 x^7 + a^2 b^3 x^{10} + \frac{5}{13} a b^4 x^{13} + \frac{b^5 x^{16}}{16}$$

```
[Out] a^5*x+5/4*a^4*b*x^4+10/7*a^3*b^2*x^7+a^2*b^3*x^10+5/13*a*b^4*x^13+1/16*b^5*
x^16
```

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {194}

$$a^2 b^3 x^{10} + \frac{10}{7} a^3 b^2 x^7 + \frac{5}{4} a^4 b x^4 + a^5 x + \frac{5}{13} a b^4 x^{13} + \frac{b^5 x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^5,x]
```

```
[Out] a^5*x + (5*a^4*b*x^4)/4 + (10*a^3*b^2*x^7)/7 + a^2*b^3*x^10 + (5*a*b^4*x^13
)/13 + (b^5*x^16)/16
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a + bx^3\right)^5 \, dx &= \int \left(a^5 + 5a^4 b x^3 + 10 a^3 b^2 x^6 + 10 a^2 b^3 x^9 + 5 a b^4 x^{12} + b^5 x^{15}\right) \, dx \\ &= a^5 x + \frac{5}{4} a^4 b x^4 + \frac{10}{7} a^3 b^2 x^7 + a^2 b^3 x^{10} + \frac{5}{13} a b^4 x^{13} + \frac{b^5 x^{16}}{16}\end{aligned}$$

Mathematica [A] time = 0.00, size = 61, normalized size = 1.00

$$a^5 x + \frac{5}{4} a^4 b x^4 + \frac{10}{7} a^3 b^2 x^7 + a^2 b^3 x^{10} + \frac{5}{13} a b^4 x^{13} + \frac{b^5 x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^5,x]
```

```
[Out] a^5*x + (5*a^4*b*x^4)/4 + (10*a^3*b^2*x^7)/7 + a^2*b^3*x^10 + (5*a*b^4*x^13
)/13 + (b^5*x^16)/16
```

fricas [A] time = 0.73, size = 53, normalized size = 0.87

$$\frac{1}{16} x^{16} b^5 + \frac{5}{13} x^{13} b^4 a + x^{10} b^3 a^2 + \frac{10}{7} x^7 b^2 a^3 + \frac{5}{4} x^4 b a^4 + x a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*b^5 + 5/13*x^13*b^4*a + x^10*b^3*a^2 + 10/7*x^7*b^2*a^3 + 5/4*x^4
*b*a^4 + x*a^5
```

giac [**A**] time = 0.15, size = 53, normalized size = 0.87

$$\frac{1}{16}\,b^5x^{16} + \frac{5}{13}\,ab^4x^{13} + a^2b^3x^{10} + \frac{10}{7}\,a^3b^2x^7 + \frac{5}{4}\,a^4bx^4 + a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5,x, algorithm="giac")
```

```
[Out] 1/16*b^5*x^16 + 5/13*a*b^4*x^13 + a^2*b^3*x^10 + 10/7*a^3*b^2*x^7 + 5/4*a^4
*b*x^4 + a^5*x
```

maple [**A**] time = 0.00, size = 54, normalized size = 0.89

$$\frac{1}{16}b^5x^{16} + \frac{5}{13}a\,b^4x^{13} + a^2b^3x^{10} + \frac{10}{7}a^3b^2x^7 + \frac{5}{4}a^4b\,x^4 + a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5,x)
```

```
[Out] a^5*x+5/4*a^4*b*x^4+10/7*a^3*b^2*x^7+a^2*b^3*x^10+5/13*a*b^4*x^13+1/16*b^5*
x^16
```

maxima [**A**] time = 1.29, size = 53, normalized size = 0.87

$$\frac{1}{16}\,b^5x^{16} + \frac{5}{13}\,ab^4x^{13} + a^2b^3x^{10} + \frac{10}{7}\,a^3b^2x^7 + \frac{5}{4}\,a^4bx^4 + a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5,x, algorithm="maxima")
```

```
[Out] 1/16*b^5*x^16 + 5/13*a*b^4*x^13 + a^2*b^3*x^10 + 10/7*a^3*b^2*x^7 + 5/4*a^4
*b*x^4 + a^5*x
```

mupad [**B**] time = 0.02, size = 53, normalized size = 0.87

$$a^5\,x + \frac{5\,a^4\,b\,x^4}{4} + \frac{10\,a^3\,b^2\,x^7}{7} + a^2\,b^3\,x^{10} + \frac{5\,a\,b^4\,x^{13}}{13} + \frac{b^5\,x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^5,x)
```

```
[Out] a^5*x + (b^5*x^16)/16 + (5*a^4*b*x^4)/4 + (5*a*b^4*x^13)/13 + (10*a^3*b^2*x
^7)/7 + a^2*b^3*x^10
```

sympy [**A**] time = 0.11, size = 60, normalized size = 0.98

$$a^5x + \frac{5a^4bx^4}{4} + \frac{10a^3b^2x^7}{7} + a^2b^3x^{10} + \frac{5ab^4x^{13}}{13} + \frac{b^5x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5,x)
```

```
[Out] a**5*x + 5*a**4*b*x**4/4 + 10*a**3*b**2*x**7/7 + a**2*b**3*x**10 + 5*a*b**4
*x**13/13 + b**5*x**16/16
```

3.279 $\int \frac{\left(a+bx^3\right)^5}{x^2}\,dx$

Optimal. Leaf size=65

$$-\frac{a^5}{x}+\frac{5}{2}a^4bx^2+2a^3b^2x^5+\frac{5}{4}a^2b^3x^8+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{14}}{14}$$

```
[Out] -a^5/x+5/2*a^4*b*x^2+2*a^3*b^2*x^5+5/4*a^2*b^3*x^8+5/11*a*b^4*x^11+1/14*b^5
*x^14
```

Rubi [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{5}{4}a^2b^3x^8+2a^3b^2x^5+\frac{5}{2}a^4bx^2-\frac{a^5}{x}+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^5/x^2,x]
```

```
[Out] -(a^5/x) + (5*a^4*b*x^2)/2 + 2*a^3*b^2*x^5 + (5*a^2*b^3*x^8)/4 + (5*a*b^4*x
^11)/11 + (b^5*x^14)/14
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^5}{x^2}\,dx &= \int \left(\frac{a^5}{x^2}+5a^4bx+10a^3b^2x^4+10a^2b^3x^7+5ab^4x^{10}+b^5x^{13}\right)dx\\ &= -\frac{a^5}{x}+\frac{5}{2}a^4bx^2+2a^3b^2x^5+\frac{5}{4}a^2b^3x^8+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{14}}{14}\end{aligned}$$

Mathematica [A] time = 0.00, size = 65, normalized size = 1.00

$$-\frac{a^5}{x}+\frac{5}{2}a^4bx^2+2a^3b^2x^5+\frac{5}{4}a^2b^3x^8+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^5/x^2,x]
```

```
[Out] -(a^5/x) + (5*a^4*b*x^2)/2 + 2*a^3*b^2*x^5 + (5*a^2*b^3*x^8)/4 + (5*a*b^4*x
^11)/11 + (b^5*x^14)/14
```

fricas [A] time = 0.76, size = 59, normalized size = 0.91

$$\frac{22\,b^5x^{15}+140\,ab^4x^{12}+385\,a^2b^3x^9+616\,a^3b^2x^6+770\,a^4bx^3-308\,a^5}{308\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^2,x, algorithm="fricas")
```

```
[Out] 1/308*(22*b^5*x^15 + 140*a*b^4*x^12 + 385*a^2*b^3*x^9 + 616*a^3*b^2*x^6 + 7
70*a^4*b*x^3 - 308*a^5)/x
```

giac [A] time = 0.18, size = 57, normalized size = 0.88

$$\frac{1}{14}\,b^5x^{14} + \frac{5}{11}\,ab^4x^{11} + \frac{5}{4}\,a^2b^3x^8 + 2\,a^3b^2x^5 + \frac{5}{2}\,a^4bx^2 - \frac{a^5}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^2,x, algorithm="giac")
```

```
[Out] 1/14*b^5*x^14 + 5/11*a*b^4*x^11 + 5/4*a^2*b^3*x^8 + 2*a^3*b^2*x^5 + 5/2*a^4
*b*x^2 - a^5/x
```

maple [A] time = 0.00, size = 58, normalized size = 0.89

$$\frac{b^5x^{14}}{14} + \frac{5a\,b^4x^{11}}{11} + \frac{5a^2b^3x^8}{4} + 2a^3b^2x^5 + \frac{5a^4b\,x^2}{2} - \frac{a^5}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5/x^2,x)
```

```
[Out] -a^5/x+5/2*a^4*b*x^2+2*a^3*b^2*x^5+5/4*a^2*b^3*x^8+5/11*a*b^4*x^11+1/14*b^5
*x^14
```

maxima [A] time = 1.29, size = 57, normalized size = 0.88

$$\frac{1}{14}\,b^5x^{14} + \frac{5}{11}\,ab^4x^{11} + \frac{5}{4}\,a^2b^3x^8 + 2\,a^3b^2x^5 + \frac{5}{2}\,a^4bx^2 - \frac{a^5}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^2,x, algorithm="maxima")
```

```
[Out] 1/14*b^5*x^14 + 5/11*a*b^4*x^11 + 5/4*a^2*b^3*x^8 + 2*a^3*b^2*x^5 + 5/2*a^4
*b*x^2 - a^5/x
```

mupad [B] time = 0.02, size = 57, normalized size = 0.88

$$\frac{b^5\,x^{14}}{14} - \frac{a^5}{x} + \frac{5\,a^4\,b\,x^2}{2} + \frac{5\,a\,b^4\,x^{11}}{11} + 2\,a^3\,b^2\,x^5 + \frac{5\,a^2\,b^3\,x^8}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^5/x^2,x)
```

```
[Out] (b^5*x^14)/14 - a^5/x + (5*a^4*b*x^2)/2 + (5*a*b^4*x^11)/11 + 2*a^3*b^2*x^5
 + (5*a^2*b^3*x^8)/4
```

sympy [A] time = 0.15, size = 61, normalized size = 0.94

$$-\frac{a^5}{x} + \frac{5a^4bx^2}{2} + 2a^3b^2x^5 + \frac{5a^2b^3x^8}{4} + \frac{5ab^4x^{11}}{11} + \frac{b^5x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5/x**2,x)
```

```
[Out] -a**5/x + 5*a**4*b*x**2/2 + 2*a**3*b**2*x**5 + 5*a**2*b**3*x**8/4 + 5*a*b**
4*x**11/11 + b**5*x**14/14
```

3.280 $\int \frac{\left(a+bx^3\right)^5}{x^3}\,dx$

Optimal. Leaf size=65

$$-\frac{a^5}{2x^2}+5a^4bx+\frac{5}{2}a^3b^2x^4+\frac{10}{7}a^2b^3x^7+\frac{1}{2}ab^4x^{10}+\frac{b^5x^{13}}{13}$$

```
[Out] -1/2*a^5/x^2+5*a^4*b*x+5/2*a^3*b^2*x^4+10/7*a^2*b^3*x^7+1/2*a*b^4*x^10+1/13
*b^5*x^13
```

Rubi [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{10}{7}a^2b^3x^7+\frac{5}{2}a^3b^2x^4+5a^4bx-\frac{a^5}{2x^2}+\frac{1}{2}ab^4x^{10}+\frac{b^5x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^5/x^3,x]
```

```
[Out] -a^5/(2*x^2) + 5*a^4*b*x + (5*a^3*b^2*x^4)/2 + (10*a^2*b^3*x^7)/7 + (a*b^4*
x^10)/2 + (b^5*x^13)/13
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^5}{x^3}\,dx &= \int \left(5a^4b+\frac{a^5}{x^3}+10a^3b^2x^3+10a^2b^3x^6+5ab^4x^9+b^5x^{12}\right)dx\\ &= -\frac{a^5}{2x^2}+5a^4bx+\frac{5}{2}a^3b^2x^4+\frac{10}{7}a^2b^3x^7+\frac{1}{2}ab^4x^{10}+\frac{b^5x^{13}}{13}\end{aligned}$$

Mathematica [A] time = 0.00, size = 65, normalized size = 1.00

$$-\frac{a^5}{2x^2}+5a^4bx+\frac{5}{2}a^3b^2x^4+\frac{10}{7}a^2b^3x^7+\frac{1}{2}ab^4x^{10}+\frac{b^5x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^5/x^3,x]
```

```
[Out] -1/2*a^5/x^2 + 5*a^4*b*x + (5*a^3*b^2*x^4)/2 + (10*a^2*b^3*x^7)/7 + (a*b^4*
x^10)/2 + (b^5*x^13)/13
```

fricas [A] time = 0.49, size = 59, normalized size = 0.91

$$\frac{14\,b^5x^{15}+91\,ab^4x^{12}+260\,a^2b^3x^9+455\,a^3b^2x^6+910\,a^4bx^3-91\,a^5}{182\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^3,x, algorithm="fricas")
```

```
[Out] 1/182*(14*b^5*x^15 + 91*a*b^4*x^12 + 260*a^2*b^3*x^9 + 455*a^3*b^2*x^6 + 91
0*a^4*b*x^3 - 91*a^5)/x^2
```

giac [A] time = 0.37, size = 55, normalized size = 0.85

$$\frac{1}{13}\,b^5x^{13} + \frac{1}{2}\,ab^4x^{10} + \frac{10}{7}\,a^2b^3x^7 + \frac{5}{2}\,a^3b^2x^4 + 5\,a^4bx - \frac{a^5}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^3,x, algorithm="giac")
```

```
[Out] 1/13*b^5*x^13 + 1/2*a*b^4*x^10 + 10/7*a^2*b^3*x^7 + 5/2*a^3*b^2*x^4 + 5*a^4
*b*x - 1/2*a^5/x^2
```

maple [A] time = 0.00, size = 56, normalized size = 0.86

$$\frac{b^5x^{13}}{13} + \frac{a\,b^4x^{10}}{2} + \frac{10a^2b^3x^7}{7} + \frac{5a^3b^2x^4}{2} + 5a^4bx - \frac{a^5}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5/x^3,x)
```

```
[Out] -1/2*a^5/x^2+5*a^4*b*x+5/2*a^3*b^2*x^4+10/7*a^2*b^3*x^7+1/2*a*b^4*x^10+1/13
*b^5*x^13
```

maxima [A] time = 1.32, size = 55, normalized size = 0.85

$$\frac{1}{13}\,b^5x^{13} + \frac{1}{2}\,ab^4x^{10} + \frac{10}{7}\,a^2b^3x^7 + \frac{5}{2}\,a^3b^2x^4 + 5\,a^4bx - \frac{a^5}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^3,x, algorithm="maxima")
```

```
[Out] 1/13*b^5*x^13 + 1/2*a*b^4*x^10 + 10/7*a^2*b^3*x^7 + 5/2*a^3*b^2*x^4 + 5*a^4
*b*x - 1/2*a^5/x^2
```

mupad [B] time = 0.03, size = 55, normalized size = 0.85

$$\frac{b^5\,x^{13}}{13} - \frac{a^5}{2\,x^2} + \frac{a\,b^4\,x^{10}}{2} + \frac{5\,a^3\,b^2\,x^4}{2} + \frac{10\,a^2\,b^3\,x^7}{7} + 5\,a^4\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^5/x^3,x)
```

```
[Out] (b^5*x^13)/13 - a^5/(2*x^2) + (a*b^4*x^10)/2 + (5*a^3*b^2*x^4)/2 + (10*a^2*
b^3*x^7)/7 + 5*a^4*b*x
```

sympy [A] time = 0.16, size = 61, normalized size = 0.94

$$-\frac{a^5}{2x^2} + 5a^4bx + \frac{5a^3b^2x^4}{2} + \frac{10a^2b^3x^7}{7} + \frac{ab^4x^{10}}{2} + \frac{b^5x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5/x**3,x)
```

```
[Out] -a**5/(2*x**2) + 5*a**4*b*x + 5*a**3*b**2*x**4/2 + 10*a**2*b**3*x**7/7 + a*
b**4*x**10/2 + b**5*x**13/13
```

3.281 $\int \frac{\left(a+bx^3\right)^5}{x^5}\,dx$

Optimal. Leaf size=63

$$-\frac{a^5}{4x^4}-\frac{5a^4b}{x}+5a^3b^2x^2+2a^2b^3x^5+\frac{5}{8}ab^4x^8+\frac{b^5x^{11}}{11}$$

```
[Out] -1/4*a^5/x^4-5*a^4*b/x+5*a^3*b^2*x^2+2*a^2*b^3*x^5+5/8*a*b^4*x^8+1/11*b^5*x
^11
```

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$2a^2b^3x^5+5a^3b^2x^2-\frac{5a^4b}{x}-\frac{a^5}{4x^4}+\frac{5}{8}ab^4x^8+\frac{b^5x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^5/x^5,x]
```

```
[Out] -a^5/(4*x^4) - (5*a^4*b)/x + 5*a^3*b^2*x^2 + 2*a^2*b^3*x^5 + (5*a*b^4*x^8)/
8 + (b^5*x^11)/11
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^5}{x^5}\,dx &= \int \left(\frac{a^5}{x^5}+\frac{5a^4b}{x^2}+10a^3b^2x+10a^2b^3x^4+5ab^4x^7+b^5x^{10}\right)dx\\ &= -\frac{a^5}{4x^4}-\frac{5a^4b}{x}+5a^3b^2x^2+2a^2b^3x^5+\frac{5}{8}ab^4x^8+\frac{b^5x^{11}}{11}\end{aligned}$$

Mathematica [A] time = 0.01, size = 63, normalized size = 1.00

$$-\frac{a^5}{4x^4}-\frac{5a^4b}{x}+5a^3b^2x^2+2a^2b^3x^5+\frac{5}{8}ab^4x^8+\frac{b^5x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^5/x^5,x]
```

```
[Out] -1/4*a^5/x^4 - (5*a^4*b)/x + 5*a^3*b^2*x^2 + 2*a^2*b^3*x^5 + (5*a*b^4*x^8)/
8 + (b^5*x^11)/11
```

fricas [A] time = 0.85, size = 59, normalized size = 0.94

$$\frac{8\,b^5x^{15}+55\,ab^4x^{12}+176\,a^2b^3x^9+440\,a^3b^2x^6-440\,a^4bx^3-22\,a^5}{88\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^5,x, algorithm="fricas")
```

```
[Out] 1/88*(8*b^5*x^15 + 55*a*b^4*x^12 + 176*a^2*b^3*x^9 + 440*a^3*b^2*x^6 - 440*
a^4*b*x^3 - 22*a^5)/x^4
```

giac [A] time = 0.15, size = 58, normalized size = 0.92

$$\frac{1}{11}\,b^5x^{11} + \frac{5}{8}\,ab^4x^8 + 2\,a^2b^3x^5 + 5\,a^3b^2x^2 - \frac{20\,a^4bx^3 + a^5}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^5,x, algorithm="giac")
```

```
[Out] 1/11*b^5*x^11 + 5/8*a*b^4*x^8 + 2*a^2*b^3*x^5 + 5*a^3*b^2*x^2 - 1/4*(20*a^4
*b*x^3 + a^5)/x^4
```

maple [A] time = 0.00, size = 58, normalized size = 0.92

$$\frac{b^5x^{11}}{11} + \frac{5a\,b^4x^8}{8} + 2a^2b^3x^5 + 5a^3b^2x^2 - \frac{5a^4b}{x} - \frac{a^5}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5/x^5,x)
```

```
[Out] -1/4*a^5/x^4-5*a^4*b/x+5*a^3*b^2*x^2+2*a^2*b^3*x^5+5/8*a*b^4*x^8+1/11*b^5*x
^11
```

maxima [A] time = 1.35, size = 58, normalized size = 0.92

$$\frac{1}{11}\,b^5x^{11} + \frac{5}{8}\,ab^4x^8 + 2\,a^2b^3x^5 + 5\,a^3b^2x^2 - \frac{20\,a^4bx^3 + a^5}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^5,x, algorithm="maxima")
```

```
[Out] 1/11*b^5*x^11 + 5/8*a*b^4*x^8 + 2*a^2*b^3*x^5 + 5*a^3*b^2*x^2 - 1/4*(20*a^4
*b*x^3 + a^5)/x^4
```

mupad [B] time = 0.02, size = 60, normalized size = 0.95

$$\frac{b^5\,x^{11}}{11} - \frac{\frac{a^5}{4} + 5\,b\,a^4\,x^3}{x^4} + \frac{5\,a\,b^4\,x^8}{8} + 5\,a^3\,b^2\,x^2 + 2\,a^2\,b^3\,x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^5/x^5,x)
```

```
[Out] (b^5*x^11)/11 - (a^5/4 + 5*a^4*b*x^3)/x^4 + (5*a*b^4*x^8)/8 + 5*a^3*b^2*x^2
 + 2*a^2*b^3*x^5
```

sympy [A] time = 0.21, size = 63, normalized size = 1.00

$$5a^3b^2x^2 + 2a^2b^3x^5 + \frac{5ab^4x^8}{8} + \frac{b^5x^{11}}{11} + \frac{-a^5 - 20a^4bx^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5/x**5,x)
```

```
[Out] 5*a**3*b**2*x**2 + 2*a**2*b**3*x**5 + 5*a*b**4*x**8/8 + b**5*x**11/11 + (-a
**5 - 20*a**4*b*x**3)/(4*x**4)
```

3.282 $\int \frac{\left(a+bx^3\right)^5}{x^6}\,dx$

Optimal. Leaf size=65

$$-\frac{a^5}{5x^5}-\frac{5a^4b}{2x^2}+10a^3b^2x+\frac{5}{2}a^2b^3x^4+\frac{5}{7}ab^4x^7+\frac{b^5x^{10}}{10}$$

```
[Out] -1/5*a^5/x^5-5/2*a^4*b/x^2+10*a^3*b^2*x+5/2*a^2*b^3*x^4+5/7*a*b^4*x^7+1/10*
b^5*x^10
```

Rubi [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{5}{2}a^2b^3x^4+10a^3b^2x-\frac{5a^4b}{2x^2}-\frac{a^5}{5x^5}+\frac{5}{7}ab^4x^7+\frac{b^5x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^5/x^6,x]
```

```
[Out] -a^5/(5*x^5) - (5*a^4*b)/(2*x^2) + 10*a^3*b^2*x + (5*a^2*b^3*x^4)/2 + (5*a*
b^4*x^7)/7 + (b^5*x^10)/10
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^5}{x^6}\,dx &= \int \left(10a^3b^2+\frac{a^5}{x^6}+\frac{5a^4b}{x^3}+10a^2b^3x^3+5ab^4x^6+b^5x^9\right)dx\\ &= -\frac{a^5}{5x^5}-\frac{5a^4b}{2x^2}+10a^3b^2x+\frac{5}{2}a^2b^3x^4+\frac{5}{7}ab^4x^7+\frac{b^5x^{10}}{10}\end{aligned}$$

Mathematica [A] time = 0.00, size = 65, normalized size = 1.00

$$-\frac{a^5}{5x^5}-\frac{5a^4b}{2x^2}+10a^3b^2x+\frac{5}{2}a^2b^3x^4+\frac{5}{7}ab^4x^7+\frac{b^5x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^5/x^6,x]
```

```
[Out] -1/5*a^5/x^5 - (5*a^4*b)/(2*x^2) + 10*a^3*b^2*x + (5*a^2*b^3*x^4)/2 + (5*a*
b^4*x^7)/7 + (b^5*x^10)/10
```

fricas [A] time = 0.81, size = 59, normalized size = 0.91

$$\frac{7\,b^5x^{15}+50\,ab^4x^{12}+175\,a^2b^3x^9+700\,a^3b^2x^6-175\,a^4bx^3-14\,a^5}{70\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^6,x, algorithm="fricas")
```

```
[Out] 1/70*(7*b^5*x^15 + 50*a*b^4*x^12 + 175*a^2*b^3*x^9 + 700*a^3*b^2*x^6 - 175*
a^4*b*x^3 - 14*a^5)/x^5
```

giac [A] time = 0.18, size = 58, normalized size = 0.89

$$\frac{1}{10}\,b^5x^{10}+\frac{5}{7}\,ab^4x^7+\frac{5}{2}\,a^2b^3x^4+10\,a^3b^2x-\frac{25\,a^4bx^3+2\,a^5}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^6,x, algorithm="giac")
```

```
[Out] 1/10*b^5*x^10 + 5/7*a*b^4*x^7 + 5/2*a^2*b^3*x^4 + 10*a^3*b^2*x - 1/10*(25*a
^4*b*x^3 + 2*a^5)/x^5
```

maple [A] time = 0.00, size = 56, normalized size = 0.86

$$\frac{b^5x^{10}}{10}+\frac{5a\,b^4x^7}{7}+\frac{5a^2b^3x^4}{2}+10a^3b^2x-\frac{5a^4b}{2x^2}-\frac{a^5}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5/x^6,x)
```

```
[Out] -1/5*a^5/x^5-5/2*a^4*b/x^2+10*a^3*b^2*x+5/2*a^2*b^3*x^4+5/7*a*b^4*x^7+1/10*
b^5*x^10
```

maxima [A] time = 1.30, size = 58, normalized size = 0.89

$$\frac{1}{10}\,b^5x^{10}+\frac{5}{7}\,ab^4x^7+\frac{5}{2}\,a^2b^3x^4+10\,a^3b^2x-\frac{25\,a^4bx^3+2\,a^5}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^6,x, algorithm="maxima")
```

```
[Out] 1/10*b^5*x^10 + 5/7*a*b^4*x^7 + 5/2*a^2*b^3*x^4 + 10*a^3*b^2*x - 1/10*(25*a
^4*b*x^3 + 2*a^5)/x^5
```

mupad [B] time = 0.02, size = 58, normalized size = 0.89

$$\frac{b^5\,x^{10}}{10}-\frac{\frac{a^5}{5}+\frac{5\,b\,a^4\,x^3}{2}}{x^5}+10\,a^3\,b^2\,x+\frac{5\,a\,b^4\,x^7}{7}+\frac{5\,a^2\,b^3\,x^4}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^5/x^6,x)
```

```
[Out] (b^5*x^10)/10 - (a^5/5 + (5*a^4*b*x^3)/2)/x^5 + 10*a^3*b^2*x + (5*a*b^4*x^7
)/7 + (5*a^2*b^3*x^4)/2
```

sympy [A] time = 0.26, size = 65, normalized size = 1.00

$$10a^3b^2x+\frac{5a^2b^3x^4}{2}+\frac{5ab^4x^7}{7}+\frac{b^5x^{10}}{10}+\frac{-2a^5-25a^4bx^3}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5/x**6,x)
```

```
[Out] 10*a**3*b**2*x + 5*a**2*b**3*x**4/2 + 5*a*b**4*x**7/7 + b**5*x**10/10 + (-2
*a**5 - 25*a**4*b*x**3)/(10*x**5)
```

3.283 $\int \frac{\left(a+bx^3\right)^5}{x^8}\,dx$

Optimal. Leaf size=62

$$-\frac{a^5}{7x^7}-\frac{5a^4b}{4x^4}-\frac{10a^3b^2}{x}+5a^2b^3x^2+ab^4x^5+\frac{b^5x^8}{8}$$

```
[Out] -1/7*a^5/x^7-5/4*a^4*b/x^4-10*a^3*b^2/x+5*a^2*b^3*x^2+a*b^4*x^5+1/8*b^5*x^8
```

Rubi [A] time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$5a^2b^3x^2-\frac{10a^3b^2}{x}-\frac{5a^4b}{4x^4}-\frac{a^5}{7x^7}+ab^4x^5+\frac{b^5x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^5/x^8,x]
```

```
[Out] -a^5/(7*x^7) - (5*a^4*b)/(4*x^4) - (10*a^3*b^2)/x + 5*a^2*b^3*x^2 + a*b^4*x
^5 + (b^5*x^8)/8
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^5}{x^8}\,dx &= \int \left(\frac{a^5}{x^8}+\frac{5a^4b}{x^5}+\frac{10a^3b^2}{x^2}+10a^2b^3x+5ab^4x^4+b^5x^7\right)dx\\ &= -\frac{a^5}{7x^7}-\frac{5a^4b}{4x^4}-\frac{10a^3b^2}{x}+5a^2b^3x^2+ab^4x^5+\frac{b^5x^8}{8}\end{aligned}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 1.00

$$-\frac{a^5}{7x^7}-\frac{5a^4b}{4x^4}-\frac{10a^3b^2}{x}+5a^2b^3x^2+ab^4x^5+\frac{b^5x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^5/x^8,x]
```

```
[Out] -1/7*a^5/x^7 - (5*a^4*b)/(4*x^4) - (10*a^3*b^2)/x + 5*a^2*b^3*x^2 + a*b^4*x
^5 + (b^5*x^8)/8
```

fricas [A] time = 0.67, size = 59, normalized size = 0.95

$$\frac{7\,b^5x^{15}+56\,ab^4x^{12}+280\,a^2b^3x^9-560\,a^3b^2x^6-70\,a^4bx^3-8\,a^5}{56\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^8,x, algorithm="fricas")
```

```
[Out] 1/56*(7*b^5*x^15 + 56*a*b^4*x^12 + 280*a^2*b^3*x^9 - 560*a^3*b^2*x^6 - 70*a
^4*b*x^3 - 8*a^5)/x^7
```

giac [A] time = 0.15, size = 59, normalized size = 0.95

$$\frac{1}{8}b^5x^8 + ab^4x^5 + 5\,a^2b^3x^2 - \frac{280\,a^3b^2x^6 + 35\,a^4bx^3 + 4\,a^5}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^8,x, algorithm="giac")
```

```
[Out] 1/8*b^5*x^8 + a*b^4*x^5 + 5*a^2*b^3*x^2 - 1/28*(280*a^3*b^2*x^6 + 35*a^4*b*
x^3 + 4*a^5)/x^7
```

maple [A] time = 0.01, size = 57, normalized size = 0.92

$$\frac{b^5x^8}{8} + a\,b^4x^5 + 5a^2b^3x^2 - \frac{10a^3b^2}{x} - \frac{5a^4b}{4x^4} - \frac{a^5}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5/x^8,x)
```

```
[Out] -1/7*a^5/x^7-5/4*a^4*b/x^4-10*a^3*b^2/x+5*a^2*b^3*x^2+a*b^4*x^5+1/8*b^5*x^8
```

maxima [A] time = 1.30, size = 59, normalized size = 0.95

$$\frac{1}{8}b^5x^8 + ab^4x^5 + 5\,a^2b^3x^2 - \frac{280\,a^3b^2x^6 + 35\,a^4bx^3 + 4\,a^5}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^8,x, algorithm="maxima")
```

```
[Out] 1/8*b^5*x^8 + a*b^4*x^5 + 5*a^2*b^3*x^2 - 1/28*(280*a^3*b^2*x^6 + 35*a^4*b*
x^3 + 4*a^5)/x^7
```

mupad [B] time = 0.04, size = 59, normalized size = 0.95

$$\frac{b^5\,x^8}{8} - \frac{\frac{a^5}{7} + \frac{5\,a^4\,b\,x^3}{4} + 10\,a^3\,b^2\,x^6}{x^7} + a\,b^4\,x^5 + 5\,a^2\,b^3\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^5/x^8,x)
```

```
[Out] (b^5*x^8)/8 - (a^5/7 + (5*a^4*b*x^3)/4 + 10*a^3*b^2*x^6)/x^7 + a*b^4*x^5 +
5*a^2*b^3*x^2
```

sympy [A] time = 0.28, size = 61, normalized size = 0.98

$$5a^2b^3x^2 + ab^4x^5 + \frac{b^5x^8}{8} + \frac{-4a^5 - 35a^4bx^3 - 280a^3b^2x^6}{28x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5/x**8,x)
```

```
[Out] 5*a**2*b**3*x**2 + a*b**4*x**5 + b**5*x**8/8 + (-4*a**5 - 35*a**4*b*x**3 -
280*a**3*b**2*x**6)/(28*x**7)
```

3.284 $\int \frac{\left(a+bx^3\right)^5}{x^9}\,dx$

Optimal. Leaf size=61

$$-\frac{a^5}{8x^8}-\frac{a^4b}{x^5}-\frac{5a^3b^2}{x^2}+10a^2b^3x+\frac{5}{4}ab^4x^4+\frac{b^5x^7}{7}$$

```
[Out] -1/8*a^5/x^8-a^4*b/x^5-5*a^3*b^2/x^2+10*a^2*b^3*x+5/4*a*b^4*x^4+1/7*b^5*x^7
```

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$-\frac{5a^3b^2}{x^2}+10a^2b^3x-\frac{a^4b}{x^5}-\frac{a^5}{8x^8}+\frac{5}{4}ab^4x^4+\frac{b^5x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^5/x^9,x]
```

```
[Out] -a^5/(8*x^8) - (a^4*b)/x^5 - (5*a^3*b^2)/x^2 + 10*a^2*b^3*x + (5*a*b^4*x^4)
/4 + (b^5*x^7)/7
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^5}{x^9}\,dx &= \int \left(10a^2b^3+\frac{a^5}{x^9}+\frac{5a^4b}{x^6}+\frac{10a^3b^2}{x^3}+5ab^4x^3+b^5x^6\right)dx\\ &= -\frac{a^5}{8x^8}-\frac{a^4b}{x^5}-\frac{5a^3b^2}{x^2}+10a^2b^3x+\frac{5}{4}ab^4x^4+\frac{b^5x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 61, normalized size = 1.00

$$-\frac{a^5}{8x^8}-\frac{a^4b}{x^5}-\frac{5a^3b^2}{x^2}+10a^2b^3x+\frac{5}{4}ab^4x^4+\frac{b^5x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^5/x^9,x]
```

```
[Out] -1/8*a^5/x^8 - (a^4*b)/x^5 - (5*a^3*b^2)/x^2 + 10*a^2*b^3*x + (5*a*b^4*x^4)
/4 + (b^5*x^7)/7
```

fricas [A] time = 0.62, size = 59, normalized size = 0.97

$$\frac{8\,b^5x^{15}+70\,ab^4x^{12}+560\,a^2b^3x^9-280\,a^3b^2x^6-56\,a^4bx^3-7\,a^5}{56\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^9,x, algorithm="fricas")
```

```
[Out] 1/56*(8*b^5*x^15 + 70*a*b^4*x^12 + 560*a^2*b^3*x^9 - 280*a^3*b^2*x^6 - 56*a
^4*b*x^3 - 7*a^5)/x^8
```

giac [A] time = 0.15, size = 56, normalized size = 0.92

$$\frac{1}{7}b^5x^7+\frac{5}{4}ab^4x^4+10a^2b^3x-\frac{40a^3b^2x^6+8a^4bx^3+a^5}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^9,x, algorithm="giac")
```

```
[Out] 1/7*b^5*x^7 + 5/4*a*b^4*x^4 + 10*a^2*b^3*x - 1/8*(40*a^3*b^2*x^6 + 8*a^4*b*
x^3 + a^5)/x^8
```

maple [A] time = 0.00, size = 56, normalized size = 0.92

$$\frac{b^5x^7}{7}+\frac{5a\,b^4x^4}{4}+10a^2b^3x-\frac{5a^3b^2}{x^2}-\frac{a^4b}{x^5}-\frac{a^5}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5/x^9,x)
```

```
[Out] -1/8*a^5/x^8-a^4*b/x^5-5*a^3*b^2/x^2+10*a^2*b^3*x+5/4*a*b^4*x^4+1/7*b^5*x^7
```

maxima [A] time = 1.35, size = 56, normalized size = 0.92

$$\frac{1}{7}b^5x^7+\frac{5}{4}ab^4x^4+10a^2b^3x-\frac{40a^3b^2x^6+8a^4bx^3+a^5}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5/x^9,x, algorithm="maxima")
```

```
[Out] 1/7*b^5*x^7 + 5/4*a*b^4*x^4 + 10*a^2*b^3*x - 1/8*(40*a^3*b^2*x^6 + 8*a^4*b*
x^3 + a^5)/x^8
```

mupad [B] time = 0.05, size = 57, normalized size = 0.93

$$\frac{b^5\,x^7}{7}-\frac{\frac{a^5}{8}+a^4\,b\,x^3+5\,a^3\,b^2\,x^6}{x^8}+10\,a^2\,b^3\,x+\frac{5\,a\,b^4\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^5/x^9,x)
```

```
[Out] (b^5*x^7)/7 - (a^5/8 + a^4*b*x^3 + 5*a^3*b^2*x^6)/x^8 + 10*a^2*b^3*x + (5*a
*b^4*x^4)/4
```

sympy [A] time = 0.29, size = 61, normalized size = 1.00

$$10a^2b^3x+\frac{5ab^4x^4}{4}+\frac{b^5x^7}{7}+\frac{-a^5-8a^4bx^3-40a^3b^2x^6}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5/x**9,x)
```

```
[Out] 10*a**2*b**3*x + 5*a*b**4*x**4/4 + b**5*x**7/7 + (-a**5 - 8*a**4*b*x**3 - 4
0*a**3*b**2*x**6)/(8*x**8)
```

3.285 $\int x^{20}\left(a+bx^3\right)^8\,dx$

Optimal. Leaf size=129

$$\frac{a^6\left(a+bx^3\right)^9}{27b^7}-\frac{a^5\left(a+bx^3\right)^{10}}{5b^7}+\frac{5a^4\left(a+bx^3\right)^{11}}{11b^7}-\frac{5a^3\left(a+bx^3\right)^{12}}{9b^7}+\frac{5a^2\left(a+bx^3\right)^{13}}{13b^7}+\frac{\left(a+bx^3\right)^{15}}{45b^7}-\frac{a\left(a+bx^3\right)^{14}}{7b^7}$$

```
[Out] 1/27*a^6*(b*x^3+a)^9/b^7-1/5*a^5*(b*x^3+a)^10/b^7+5/11*a^4*(b*x^3+a)^11/b^7
-5/9*a^3*(b*x^3+a)^12/b^7+5/13*a^2*(b*x^3+a)^13/b^7-1/7*a*(b*x^3+a)^14/b^7+
1/45*(b*x^3+a)^15/b^7
```

Rubi [A] time = 0.22, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{5a^2\left(a+bx^3\right)^{13}}{13b^7}-\frac{5a^3\left(a+bx^3\right)^{12}}{9b^7}+\frac{5a^4\left(a+bx^3\right)^{11}}{11b^7}-\frac{a^5\left(a+bx^3\right)^{10}}{5b^7}+\frac{a^6\left(a+bx^3\right)^9}{27b^7}+\frac{\left(a+bx^3\right)^{15}}{45b^7}-\frac{a\left(a+bx^3\right)^{14}}{7b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^20*(a + b*x^3)^8,x]
```

```
[Out] (a^6*(a + b*x^3)^9)/(27*b^7) - (a^5*(a + b*x^3)^10)/(5*b^7) + (5*a^4*(a + b
*x^3)^11)/(11*b^7) - (5*a^3*(a + b*x^3)^12)/(9*b^7) + (5*a^2*(a + b*x^3)^13
)/(13*b^7) - (a*(a + b*x^3)^14)/(7*b^7) + (a + b*x^3)^15/(45*b^7)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{20}\left(a+bx^3\right)^8\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int x^6(a+bx)^8\,dx,x,x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int\left(\frac{a^6(a+bx)^8}{b^6}-\frac{6a^5(a+bx)^9}{b^6}+\frac{15a^4(a+bx)^{10}}{b^6}-\frac{20a^3(a+bx)^{11}}{b^6}+\frac{15a^2(a+b\right.\right.\\ &= \frac{a^6\left(a+bx^3\right)^9}{27b^7}-\frac{a^5\left(a+bx^3\right)^{10}}{5b^7}+\frac{5a^4\left(a+bx^3\right)^{11}}{11b^7}-\frac{5a^3\left(a+bx^3\right)^{12}}{9b^7}+\frac{5a^2\left(a+bx^3\right)^{13}}{13b^7}-\frac{a}{}\end{aligned}$$

Mathematica [A] time = 0.00, size = 108, normalized size = 0.84

$$\frac{a^8x^{21}}{21}+\frac{1}{3}a^7bx^{24}+\frac{28}{27}a^6b^2x^{27}+\frac{28}{15}a^5b^3x^{30}+\frac{70}{33}a^4b^4x^{33}+\frac{14}{9}a^3b^5x^{36}+\frac{28}{39}a^2b^6x^{39}+\frac{4}{21}ab^7x^{42}+\frac{b^8x^{45}}{45}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^20*(a + b*x^3)^8,x]
```

```
[Out] (a^8*x^21)/21 + (a^7*b*x^24)/3 + (28*a^6*b^2*x^27)/27 + (28*a^5*b^3*x^30)/1
5 + (70*a^4*b^4*x^33)/33 + (14*a^3*b^5*x^36)/9 + (28*a^2*b^6*x^39)/39 + (4*
a*b^7*x^42)/21 + (b^8*x^45)/45
```

fricas [A] time = 0.61, size = 90, normalized size = 0.70

$$\frac{1}{45}x^{45}b^8+\frac{4}{21}x^{42}b^7a+\frac{28}{39}x^{39}b^6a^2+\frac{14}{9}x^{36}b^5a^3+\frac{70}{33}x^{33}b^4a^4+\frac{28}{15}x^{30}b^3a^5+\frac{28}{27}x^{27}b^2a^6+\frac{1}{3}x^{24}ba^7+\frac{1}{21}x^{21}a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^20*(b*x^3+a)^8,x, algorithm="fricas")
```

```
[Out] 1/45*x^45*b^8 + 4/21*x^42*b^7*a + 28/39*x^39*b^6*a^2 + 14/9*x^36*b^5*a^3 +
70/33*x^33*b^4*a^4 + 28/15*x^30*b^3*a^5 + 28/27*x^27*b^2*a^6 + 1/3*x^24*b*a
^7 + 1/21*x^21*a^8
```

giac [A] time = 0.15, size = 90, normalized size = 0.70

$$\frac{1}{45}\,b^8x^{45}+\frac{4}{21}\,ab^7x^{42}+\frac{28}{39}\,a^2b^6x^{39}+\frac{14}{9}\,a^3b^5x^{36}+\frac{70}{33}\,a^4b^4x^{33}+\frac{28}{15}\,a^5b^3x^{30}+\frac{28}{27}\,a^6b^2x^{27}+\frac{1}{3}\,a^7bx^{24}+\frac{1}{21}\,a^8x^{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^20*(b*x^3+a)^8,x, algorithm="giac")
```

```
[Out] 1/45*b^8*x^45 + 4/21*a*b^7*x^42 + 28/39*a^2*b^6*x^39 + 14/9*a^3*b^5*x^36 +
70/33*a^4*b^4*x^33 + 28/15*a^5*b^3*x^30 + 28/27*a^6*b^2*x^27 + 1/3*a^7*b*x^
24 + 1/21*a^8*x^21
```

maple [A] time = 0.00, size = 91, normalized size = 0.71

$$\frac{1}{45}b^8x^{45}+\frac{4}{21}a\,b^7x^{42}+\frac{28}{39}a^2b^6x^{39}+\frac{14}{9}a^3b^5x^{36}+\frac{70}{33}a^4b^4x^{33}+\frac{28}{15}a^5b^3x^{30}+\frac{28}{27}a^6b^2x^{27}+\frac{1}{3}a^7b\,x^{24}+\frac{1}{21}a^8x^{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^20*(b*x^3+a)^8,x)
```

```
[Out] 1/45*b^8*x^45+4/21*a*b^7*x^42+28/39*a^2*b^6*x^39+14/9*a^3*b^5*x^36+70/33*a^
4*b^4*x^33+28/15*a^5*b^3*x^30+28/27*a^6*b^2*x^27+1/3*a^7*b*x^24+1/21*a^8*x^
21
```

maxima [A] time = 1.29, size = 90, normalized size = 0.70

$$\frac{1}{45}\,b^8x^{45}+\frac{4}{21}\,ab^7x^{42}+\frac{28}{39}\,a^2b^6x^{39}+\frac{14}{9}\,a^3b^5x^{36}+\frac{70}{33}\,a^4b^4x^{33}+\frac{28}{15}\,a^5b^3x^{30}+\frac{28}{27}\,a^6b^2x^{27}+\frac{1}{3}\,a^7bx^{24}+\frac{1}{21}\,a^8x^{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^20*(b*x^3+a)^8,x, algorithm="maxima")
```

```
[Out] 1/45*b^8*x^45 + 4/21*a*b^7*x^42 + 28/39*a^2*b^6*x^39 + 14/9*a^3*b^5*x^36 +
70/33*a^4*b^4*x^33 + 28/15*a^5*b^3*x^30 + 28/27*a^6*b^2*x^27 + 1/3*a^7*b*x^
24 + 1/21*a^8*x^21
```

mupad [B] time = 1.03, size = 90, normalized size = 0.70

$$\frac{a^8\,x^{21}}{21}+\frac{a^7\,b\,x^{24}}{3}+\frac{28\,a^6\,b^2\,x^{27}}{27}+\frac{28\,a^5\,b^3\,x^{30}}{15}+\frac{70\,a^4\,b^4\,x^{33}}{33}+\frac{14\,a^3\,b^5\,x^{36}}{9}+\frac{28\,a^2\,b^6\,x^{39}}{39}+\frac{4\,a\,b^7\,x^{42}}{21}+\frac{b^8\,x^{45}}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^20*(a + b*x^3)^8,x)
```

```
[Out] (a^8*x^21)/21 + (b^8*x^45)/45 + (a^7*b*x^24)/3 + (4*a*b^7*x^42)/21 + (28*a^
6*b^2*x^27)/27 + (28*a^5*b^3*x^30)/15 + (70*a^4*b^4*x^33)/33 + (14*a^3*b^5*
x^36)/9 + (28*a^2*b^6*x^39)/39
```

sympy [**A**] time = 0.11, size = 105, normalized size = 0.81

$$\frac{a^8x^{21}}{21}+\frac{a^7bx^{24}}{3}+\frac{28a^6b^2x^{27}}{27}+\frac{28a^5b^3x^{30}}{15}+\frac{70a^4b^4x^{33}}{33}+\frac{14a^3b^5x^{36}}{9}+\frac{28a^2b^6x^{39}}{39}+\frac{4ab^7x^{42}}{21}+\frac{b^8x^{45}}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**20*(b*x**3+a)**8,x)
```

```
[Out] a**8*x**21/21 + a**7*b*x**24/3 + 28*a**6*b**2*x**27/27 + 28*a**5*b**3*x**30
/15 + 70*a**4*b**4*x**33/33 + 14*a**3*b**5*x**36/9 + 28*a**2*b**6*x**39/39
+ 4*a*b**7*x**42/21 + b**8*x**45/45
```

3.286 $\int x^{17}\left(a+bx^3\right)^8\,dx$

Optimal. Leaf size=110

$$-\frac{a^5\left(a+bx^3\right)^9}{27b^6}+\frac{a^4\left(a+bx^3\right)^{10}}{6b^6}-\frac{10a^3\left(a+bx^3\right)^{11}}{33b^6}+\frac{5a^2\left(a+bx^3\right)^{12}}{18b^6}+\frac{\left(a+bx^3\right)^{14}}{42b^6}-\frac{5a\left(a+bx^3\right)^{13}}{39b^6}$$

```
[Out] -1/27*a^5*(b*x^3+a)^9/b^6+1/6*a^4*(b*x^3+a)^10/b^6-10/33*a^3*(b*x^3+a)^11/b
^6+5/18*a^2*(b*x^3+a)^12/b^6-5/39*a*(b*x^3+a)^13/b^6+1/42*(b*x^3+a)^14/b^6
```

Rubi [A] time = 0.17, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{5a^2\left(a+bx^3\right)^{12}}{18b^6}-\frac{10a^3\left(a+bx^3\right)^{11}}{33b^6}+\frac{a^4\left(a+bx^3\right)^{10}}{6b^6}-\frac{a^5\left(a+bx^3\right)^9}{27b^6}+\frac{\left(a+bx^3\right)^{14}}{42b^6}-\frac{5a\left(a+bx^3\right)^{13}}{39b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^17*(a + b*x^3)^8,x]
```

```
[Out] -(a^5*(a + b*x^3)^9)/(27*b^6) + (a^4*(a + b*x^3)^10)/(6*b^6) - (10*a^3*(a +
 b*x^3)^11)/(33*b^6) + (5*a^2*(a + b*x^3)^12)/(18*b^6) - (5*a*(a + b*x^3)^1
3)/(39*b^6) + (a + b*x^3)^14/(42*b^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{17}\left(a+bx^3\right)^8\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int x^5(a+bx)^8\,dx,x,x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int\left(-\frac{a^5(a+bx)^8}{b^5}+\frac{5a^4(a+bx)^9}{b^5}-\frac{10a^3(a+bx)^{10}}{b^5}+\frac{10a^2(a+bx)^{11}}{b^5}-\frac{5a(a+\right.\right.\\ &= -\frac{a^5\left(a+bx^3\right)^9}{27b^6}+\frac{a^4\left(a+bx^3\right)^{10}}{6b^6}-\frac{10a^3\left(a+bx^3\right)^{11}}{33b^6}+\frac{5a^2\left(a+bx^3\right)^{12}}{18b^6}-\frac{5a\left(a+bx^3\right)^{13}}{39b^6}\end{aligned}$$

Mathematica [A] time = 0.00, size = 108, normalized size = 0.98

$$\frac{a^8x^{18}}{18}+\frac{8}{21}a^7bx^{21}+\frac{7}{6}a^6b^2x^{24}+\frac{56}{27}a^5b^3x^{27}+\frac{7}{3}a^4b^4x^{30}+\frac{56}{33}a^3b^5x^{33}+\frac{7}{9}a^2b^6x^{36}+\frac{8}{39}ab^7x^{39}+\frac{b^8x^{42}}{42}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^17*(a + b*x^3)^8,x]
```

```
[Out] (a^8*x^18)/18 + (8*a^7*b*x^21)/21 + (7*a^6*b^2*x^24)/6 + (56*a^5*b^3*x^27)/
27 + (7*a^4*b^4*x^30)/3 + (56*a^3*b^5*x^33)/33 + (7*a^2*b^6*x^36)/9 + (8*a*
b^7*x^39)/39 + (b^8*x^42)/42
```

fricas [A] time = 0.74, size = 90, normalized size = 0.82

$$\frac{1}{42}x^{42}b^8+\frac{8}{39}x^{39}b^7a+\frac{7}{9}x^{36}b^6a^2+\frac{56}{33}x^{33}b^5a^3+\frac{7}{3}x^{30}b^4a^4+\frac{56}{27}x^{27}b^3a^5+\frac{7}{6}x^{24}b^2a^6+\frac{8}{21}x^{21}ba^7+\frac{1}{18}x^{18}a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17*(b*x^3+a)^8,x, algorithm="fricas")
```

```
[Out] 1/42*x^42*b^8 + 8/39*x^39*b^7*a + 7/9*x^36*b^6*a^2 + 56/33*x^33*b^5*a^3 + 7
/3*x^30*b^4*a^4 + 56/27*x^27*b^3*a^5 + 7/6*x^24*b^2*a^6 + 8/21*x^21*b*a^7 +
 1/18*x^18*a^8
```

giac [A] time = 0.15, size = 90, normalized size = 0.82

$$\frac{1}{42}\,b^8x^{42}+\frac{8}{39}\,ab^7x^{39}+\frac{7}{9}\,a^2b^6x^{36}+\frac{56}{33}\,a^3b^5x^{33}+\frac{7}{3}\,a^4b^4x^{30}+\frac{56}{27}\,a^5b^3x^{27}+\frac{7}{6}\,a^6b^2x^{24}+\frac{8}{21}\,a^7bx^{21}+\frac{1}{18}\,a^8x^{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17*(b*x^3+a)^8,x, algorithm="giac")
```

```
[Out] 1/42*b^8*x^42 + 8/39*a*b^7*x^39 + 7/9*a^2*b^6*x^36 + 56/33*a^3*b^5*x^33 + 7
/3*a^4*b^4*x^30 + 56/27*a^5*b^3*x^27 + 7/6*a^6*b^2*x^24 + 8/21*a^7*b*x^21 +
 1/18*a^8*x^18
```

maple [A] time = 0.00, size = 91, normalized size = 0.83

$$\frac{1}{42}b^8x^{42}+\frac{8}{39}a\,b^7x^{39}+\frac{7}{9}a^2b^6x^{36}+\frac{56}{33}a^3b^5x^{33}+\frac{7}{3}a^4b^4x^{30}+\frac{56}{27}a^5b^3x^{27}+\frac{7}{6}a^6b^2x^{24}+\frac{8}{21}a^7b\,x^{21}+\frac{1}{18}a^8x^{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^17*(b*x^3+a)^8,x)
```

```
[Out] 1/42*b^8*x^42+8/39*a*b^7*x^39+7/9*a^2*b^6*x^36+56/33*a^3*b^5*x^33+7/3*a^4*b
^4*x^30+56/27*a^5*b^3*x^27+7/6*a^6*b^2*x^24+8/21*a^7*b*x^21+1/18*a^8*x^18
```

maxima [A] time = 1.34, size = 90, normalized size = 0.82

$$\frac{1}{42}\,b^8x^{42}+\frac{8}{39}\,ab^7x^{39}+\frac{7}{9}\,a^2b^6x^{36}+\frac{56}{33}\,a^3b^5x^{33}+\frac{7}{3}\,a^4b^4x^{30}+\frac{56}{27}\,a^5b^3x^{27}+\frac{7}{6}\,a^6b^2x^{24}+\frac{8}{21}\,a^7bx^{21}+\frac{1}{18}\,a^8x^{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17*(b*x^3+a)^8,x, algorithm="maxima")
```

```
[Out] 1/42*b^8*x^42 + 8/39*a*b^7*x^39 + 7/9*a^2*b^6*x^36 + 56/33*a^3*b^5*x^33 + 7
/3*a^4*b^4*x^30 + 56/27*a^5*b^3*x^27 + 7/6*a^6*b^2*x^24 + 8/21*a^7*b*x^21 +
 1/18*a^8*x^18
```

mupad [B] time = 0.09, size = 90, normalized size = 0.82

$$\frac{a^8\,x^{18}}{18}+\frac{8\,a^7\,b\,x^{21}}{21}+\frac{7\,a^6\,b^2\,x^{24}}{6}+\frac{56\,a^5\,b^3\,x^{27}}{27}+\frac{7\,a^4\,b^4\,x^{30}}{3}+\frac{56\,a^3\,b^5\,x^{33}}{33}+\frac{7\,a^2\,b^6\,x^{36}}{9}+\frac{8\,a\,b^7\,x^{39}}{39}+\frac{b^8\,x^{42}}{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^17*(a + b*x^3)^8,x)
```

```
[Out] (a^8*x^18)/18 + (b^8*x^42)/42 + (8*a^7*b*x^21)/21 + (8*a*b^7*x^39)/39 + (7*
a^6*b^2*x^24)/6 + (56*a^5*b^3*x^27)/27 + (7*a^4*b^4*x^30)/3 + (56*a^3*b^5*x
^33)/33 + (7*a^2*b^6*x^36)/9
```

sympy [**A**] time = 0.11, size = 107, normalized size = 0.97

$$\frac{a^8x^{18}}{18}+\frac{8a^7bx^{21}}{21}+\frac{7a^6b^2x^{24}}{6}+\frac{56a^5b^3x^{27}}{27}+\frac{7a^4b^4x^{30}}{3}+\frac{56a^3b^5x^{33}}{33}+\frac{7a^2b^6x^{36}}{9}+\frac{8ab^7x^{39}}{39}+\frac{b^8x^{42}}{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**17*(b*x**3+a)**8,x)
```

```
[Out] a**8*x**18/18 + 8*a**7*b*x**21/21 + 7*a**6*b**2*x**24/6 + 56*a**5*b**3*x**2
7/27 + 7*a**4*b**4*x**30/3 + 56*a**3*b**5*x**33/33 + 7*a**2*b**6*x**36/9 +
8*a*b**7*x**39/39 + b**8*x**42/42
```

3.287 $\int x^{14}\left(a+bx^3\right)^8\,dx$

Optimal. Leaf size=91

$$\frac{a^4\left(a+bx^3\right)^9}{27b^5}-\frac{2a^3\left(a+bx^3\right)^{10}}{15b^5}+\frac{2a^2\left(a+bx^3\right)^{11}}{11b^5}+\frac{\left(a+bx^3\right)^{13}}{39b^5}-\frac{a\left(a+bx^3\right)^{12}}{9b^5}$$

```
[Out] 1/27*a^4*(b*x^3+a)^9/b^5-2/15*a^3*(b*x^3+a)^10/b^5+2/11*a^2*(b*x^3+a)^11/b^
5-1/9*a*(b*x^3+a)^12/b^5+1/39*(b*x^3+a)^13/b^5
```

Rubi [A] time = 0.14, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{2a^2\left(a+bx^3\right)^{11}}{11b^5}-\frac{2a^3\left(a+bx^3\right)^{10}}{15b^5}+\frac{a^4\left(a+bx^3\right)^9}{27b^5}+\frac{\left(a+bx^3\right)^{13}}{39b^5}-\frac{a\left(a+bx^3\right)^{12}}{9b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^14*(a + b*x^3)^8,x]
```

```
[Out] (a^4*(a + b*x^3)^9)/(27*b^5) - (2*a^3*(a + b*x^3)^10)/(15*b^5) + (2*a^2*(a
+ b*x^3)^11)/(11*b^5) - (a*(a + b*x^3)^12)/(9*b^5) + (a + b*x^3)^13/(39*b^5
)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{14}\left(a+bx^3\right)^8\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int x^4(a+bx)^8\,dx, x, x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int\left(\frac{a^4(a+bx)^8}{b^4}-\frac{4a^3(a+bx)^9}{b^4}+\frac{6a^2(a+bx)^{10}}{b^4}-\frac{4a(a+bx)^{11}}{b^4}+\frac{(a+bx)^{12}}{b^4}\right)d\\ &= \frac{a^4\left(a+bx^3\right)^9}{27b^5}-\frac{2a^3\left(a+bx^3\right)^{10}}{15b^5}+\frac{2a^2\left(a+bx^3\right)^{11}}{11b^5}-\frac{a\left(a+bx^3\right)^{12}}{9b^5}+\frac{\left(a+bx^3\right)^{13}}{39b^5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 108, normalized size = 1.19

$$\frac{a^8x^{15}}{15}+\frac{4}{9}a^7bx^{18}+\frac{4}{3}a^6b^2x^{21}+\frac{7}{3}a^5b^3x^{24}+\frac{70}{27}a^4b^4x^{27}+\frac{28}{15}a^3b^5x^{30}+\frac{28}{33}a^2b^6x^{33}+\frac{2}{9}ab^7x^{36}+\frac{b^8x^{39}}{39}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14*(a + b*x^3)^8,x]
```

```
[Out] (a^8*x^15)/15 + (4*a^7*b*x^18)/9 + (4*a^6*b^2*x^21)/3 + (7*a^5*b^3*x^24)/3
+ (70*a^4*b^4*x^27)/27 + (28*a^3*b^5*x^30)/15 + (28*a^2*b^6*x^33)/33 + (2*a
*b^7*x^36)/9 + (b^8*x^39)/39
```

fricas [A] time = 0.78, size = 90, normalized size = 0.99

$$\frac{1}{39}x^{39}b^8+\frac{2}{9}x^{36}b^7a+\frac{28}{33}x^{33}b^6a^2+\frac{28}{15}x^{30}b^5a^3+\frac{70}{27}x^{27}b^4a^4+\frac{7}{3}x^{24}b^3a^5+\frac{4}{3}x^{21}b^2a^6+\frac{4}{9}x^{18}ba^7+\frac{1}{15}x^{15}a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(b*x^3+a)^8,x, algorithm="fricas")
```

```
[Out] 1/39*x^39*b^8 + 2/9*x^36*b^7*a + 28/33*x^33*b^6*a^2 + 28/15*x^30*b^5*a^3 +
70/27*x^27*b^4*a^4 + 7/3*x^24*b^3*a^5 + 4/3*x^21*b^2*a^6 + 4/9*x^18*b*a^7 +
 1/15*x^15*a^8
```

giac [A] time = 0.21, size = 90, normalized size = 0.99

$$\frac{1}{39}\,b^8x^{39}+\frac{2}{9}\,ab^7x^{36}+\frac{28}{33}\,a^2b^6x^{33}+\frac{28}{15}\,a^3b^5x^{30}+\frac{70}{27}\,a^4b^4x^{27}+\frac{7}{3}\,a^5b^3x^{24}+\frac{4}{3}\,a^6b^2x^{21}+\frac{4}{9}\,a^7bx^{18}+\frac{1}{15}\,a^8x^{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(b*x^3+a)^8,x, algorithm="giac")
```

```
[Out] 1/39*b^8*x^39 + 2/9*a*b^7*x^36 + 28/33*a^2*b^6*x^33 + 28/15*a^3*b^5*x^30 +
70/27*a^4*b^4*x^27 + 7/3*a^5*b^3*x^24 + 4/3*a^6*b^2*x^21 + 4/9*a^7*b*x^18 +
 1/15*a^8*x^15
```

maple [A] time = 0.00, size = 91, normalized size = 1.00

$$\frac{1}{39}b^8x^{39}+\frac{2}{9}a\,b^7x^{36}+\frac{28}{33}a^2b^6x^{33}+\frac{28}{15}a^3b^5x^{30}+\frac{70}{27}a^4b^4x^{27}+\frac{7}{3}a^5b^3x^{24}+\frac{4}{3}a^6b^2x^{21}+\frac{4}{9}a^7b\,x^{18}+\frac{1}{15}a^8x^{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(b*x^3+a)^8,x)
```

```
[Out] 1/39*b^8*x^39+2/9*a*b^7*x^36+28/33*a^2*b^6*x^33+28/15*a^3*b^5*x^30+70/27*a^
4*b^4*x^27+7/3*a^5*b^3*x^24+4/3*a^6*b^2*x^21+4/9*a^7*b*x^18+1/15*a^8*x^15
```

maxima [A] time = 1.34, size = 90, normalized size = 0.99

$$\frac{1}{39}\,b^8x^{39}+\frac{2}{9}\,ab^7x^{36}+\frac{28}{33}\,a^2b^6x^{33}+\frac{28}{15}\,a^3b^5x^{30}+\frac{70}{27}\,a^4b^4x^{27}+\frac{7}{3}\,a^5b^3x^{24}+\frac{4}{3}\,a^6b^2x^{21}+\frac{4}{9}\,a^7bx^{18}+\frac{1}{15}\,a^8x^{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(b*x^3+a)^8,x, algorithm="maxima")
```

```
[Out] 1/39*b^8*x^39 + 2/9*a*b^7*x^36 + 28/33*a^2*b^6*x^33 + 28/15*a^3*b^5*x^30 +
70/27*a^4*b^4*x^27 + 7/3*a^5*b^3*x^24 + 4/3*a^6*b^2*x^21 + 4/9*a^7*b*x^18 +
 1/15*a^8*x^15
```

mupad [B] time = 0.09, size = 90, normalized size = 0.99

$$\frac{a^8\,x^{15}}{15}+\frac{4\,a^7\,b\,x^{18}}{9}+\frac{4\,a^6\,b^2\,x^{21}}{3}+\frac{7\,a^5\,b^3\,x^{24}}{3}+\frac{70\,a^4\,b^4\,x^{27}}{27}+\frac{28\,a^3\,b^5\,x^{30}}{15}+\frac{28\,a^2\,b^6\,x^{33}}{33}+\frac{2\,a\,b^7\,x^{36}}{9}+\frac{b^8\,x^{39}}{39}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(a + b*x^3)^8,x)
```

```
[Out] (a^8*x^15)/15 + (b^8*x^39)/39 + (4*a^7*b*x^18)/9 + (2*a*b^7*x^36)/9 + (4*a^
6*b^2*x^21)/3 + (7*a^5*b^3*x^24)/3 + (70*a^4*b^4*x^27)/27 + (28*a^3*b^5*x^3
0)/15 + (28*a^2*b^6*x^33)/33
```

sympy [**A**] time = 0.11, size = 107, normalized size = 1.18

$$\frac{a^8x^{15}}{15}+\frac{4a^7bx^{18}}{9}+\frac{4a^6b^2x^{21}}{3}+\frac{7a^5b^3x^{24}}{3}+\frac{70a^4b^4x^{27}}{27}+\frac{28a^3b^5x^{30}}{15}+\frac{28a^2b^6x^{33}}{33}+\frac{2ab^7x^{36}}{9}+\frac{b^8x^{39}}{39}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14*(b*x**3+a)**8,x)
```

```
[Out] a**8*x**15/15 + 4*a**7*b*x**18/9 + 4*a**6*b**2*x**21/3 + 7*a**5*b**3*x**24/
3 + 70*a**4*b**4*x**27/27 + 28*a**3*b**5*x**30/15 + 28*a**2*b**6*x**33/33 +
 2*a*b**7*x**36/9 + b**8*x**39/39
```

3.288 $\int x^{11}\left(a+bx^3\right)^8\,dx$

Optimal. Leaf size=72

$$-\frac{a^3\left(a+bx^3\right)^9}{27b^4}+\frac{a^2\left(a+bx^3\right)^{10}}{10b^4}+\frac{\left(a+bx^3\right)^{12}}{36b^4}-\frac{a\left(a+bx^3\right)^{11}}{11b^4}$$

```
[Out] -1/27*a^3*(b*x^3+a)^9/b^4+1/10*a^2*(b*x^3+a)^10/b^4-1/11*a*(b*x^3+a)^11/b^4
+1/36*(b*x^3+a)^12/b^4
```

Rubi [A] time = 0.11, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{a^2\left(a+bx^3\right)^{10}}{10b^4}-\frac{a^3\left(a+bx^3\right)^9}{27b^4}+\frac{\left(a+bx^3\right)^{12}}{36b^4}-\frac{a\left(a+bx^3\right)^{11}}{11b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a + b*x^3)^8,x]
```

```
[Out] -(a^3*(a + b*x^3)^9)/(27*b^4) + (a^2*(a + b*x^3)^10)/(10*b^4) - (a*(a + b*x
^3)^11)/(11*b^4) + (a + b*x^3)^12/(36*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{11}\left(a+bx^3\right)^8\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int x^3(a+bx)^8\,dx,x,x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int\left(-\frac{a^3(a+bx)^8}{b^3}+\frac{3a^2(a+bx)^9}{b^3}-\frac{3a(a+bx)^{10}}{b^3}+\frac{(a+bx)^{11}}{b^3}\right)dx,x,x^3\right)\\ &= -\frac{a^3\left(a+bx^3\right)^9}{27b^4}+\frac{a^2\left(a+bx^3\right)^{10}}{10b^4}-\frac{a\left(a+bx^3\right)^{11}}{11b^4}+\frac{\left(a+bx^3\right)^{12}}{36b^4}\end{aligned}$$

Mathematica [A] time = 0.02, size = 108, normalized size = 1.50

$$\frac{a^8x^{12}}{12}+\frac{8}{15}a^7bx^{15}+\frac{14}{9}a^6b^2x^{18}+\frac{8}{3}a^5b^3x^{21}+\frac{35}{12}a^4b^4x^{24}+\frac{56}{27}a^3b^5x^{27}+\frac{14}{15}a^2b^6x^{30}+\frac{8}{33}ab^7x^{33}+\frac{b^8x^{36}}{36}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*x^3)^8,x]
```

```
[Out] (a^8*x^12)/12 + (8*a^7*b*x^15)/15 + (14*a^6*b^2*x^18)/9 + (8*a^5*b^3*x^21)/
3 + (35*a^4*b^4*x^24)/12 + (56*a^3*b^5*x^27)/27 + (14*a^2*b^6*x^30)/15 + (8
*a*b^7*x^33)/33 + (b^8*x^36)/36
```

fricas [A] time = 0.68, size = 90, normalized size = 1.25

$$\frac{1}{36}x^{36}b^8+\frac{8}{33}x^{33}b^7a+\frac{14}{15}x^{30}b^6a^2+\frac{56}{27}x^{27}b^5a^3+\frac{35}{12}x^{24}b^4a^4+\frac{8}{3}x^{21}b^3a^5+\frac{14}{9}x^{18}b^2a^6+\frac{8}{15}x^{15}ba^7+\frac{1}{12}x^{12}a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^8,x, algorithm="fricas")
```

```
[Out] 1/36*x^36*b^8 + 8/33*x^33*b^7*a + 14/15*x^30*b^6*a^2 + 56/27*x^27*b^5*a^3 +
 35/12*x^24*b^4*a^4 + 8/3*x^21*b^3*a^5 + 14/9*x^18*b^2*a^6 + 8/15*x^15*b*a^
7 + 1/12*x^12*a^8
```

giac [A] time = 0.15, size = 90, normalized size = 1.25

$$\frac{1}{36}\,b^8x^{36}+\frac{8}{33}\,ab^7x^{33}+\frac{14}{15}\,a^2b^6x^{30}+\frac{56}{27}\,a^3b^5x^{27}+\frac{35}{12}\,a^4b^4x^{24}+\frac{8}{3}\,a^5b^3x^{21}+\frac{14}{9}\,a^6b^2x^{18}+\frac{8}{15}\,a^7bx^{15}+\frac{1}{12}\,a^8x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^8,x, algorithm="giac")
```

```
[Out] 1/36*b^8*x^36 + 8/33*a*b^7*x^33 + 14/15*a^2*b^6*x^30 + 56/27*a^3*b^5*x^27 +
 35/12*a^4*b^4*x^24 + 8/3*a^5*b^3*x^21 + 14/9*a^6*b^2*x^18 + 8/15*a^7*b*x^1
5 + 1/12*a^8*x^12
```

maple [A] time = 0.00, size = 91, normalized size = 1.26

$$\frac{1}{36}b^8x^{36}+\frac{8}{33}a\,b^7x^{33}+\frac{14}{15}a^2b^6x^{30}+\frac{56}{27}a^3b^5x^{27}+\frac{35}{12}a^4b^4x^{24}+\frac{8}{3}a^5b^3x^{21}+\frac{14}{9}a^6b^2x^{18}+\frac{8}{15}a^7b\,x^{15}+\frac{1}{12}a^8x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x^3+a)^8,x)
```

```
[Out] 1/36*b^8*x^36+8/33*a*b^7*x^33+14/15*a^2*b^6*x^30+56/27*a^3*b^5*x^27+35/12*a
^4*b^4*x^24+8/3*a^5*b^3*x^21+14/9*a^6*b^2*x^18+8/15*a^7*b*x^15+1/12*a^8*x^1
2
```

maxima [A] time = 1.35, size = 90, normalized size = 1.25

$$\frac{1}{36}\,b^8x^{36}+\frac{8}{33}\,ab^7x^{33}+\frac{14}{15}\,a^2b^6x^{30}+\frac{56}{27}\,a^3b^5x^{27}+\frac{35}{12}\,a^4b^4x^{24}+\frac{8}{3}\,a^5b^3x^{21}+\frac{14}{9}\,a^6b^2x^{18}+\frac{8}{15}\,a^7bx^{15}+\frac{1}{12}\,a^8x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^8,x, algorithm="maxima")
```

```
[Out] 1/36*b^8*x^36 + 8/33*a*b^7*x^33 + 14/15*a^2*b^6*x^30 + 56/27*a^3*b^5*x^27 +
 35/12*a^4*b^4*x^24 + 8/3*a^5*b^3*x^21 + 14/9*a^6*b^2*x^18 + 8/15*a^7*b*x^1
5 + 1/12*a^8*x^12
```

mupad [B] time = 1.00, size = 90, normalized size = 1.25

$$\frac{a^8\,x^{12}}{12}+\frac{8\,a^7\,b\,x^{15}}{15}+\frac{14\,a^6\,b^2\,x^{18}}{9}+\frac{8\,a^5\,b^3\,x^{21}}{3}+\frac{35\,a^4\,b^4\,x^{24}}{12}+\frac{56\,a^3\,b^5\,x^{27}}{27}+\frac{14\,a^2\,b^6\,x^{30}}{15}+\frac{8\,a\,b^7\,x^{33}}{33}+\frac{b^8\,x^{36}}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a + b*x^3)^8,x)
```

```
[Out] (a^8*x^12)/12 + (b^8*x^36)/36 + (8*a^7*b*x^15)/15 + (8*a*b^7*x^33)/33 + (14
*a^6*b^2*x^18)/9 + (8*a^5*b^3*x^21)/3 + (35*a^4*b^4*x^24)/12 + (56*a^3*b^5*
x^27)/27 + (14*a^2*b^6*x^30)/15
```

sympy [**A**] time = 0.16, size = 107, normalized size = 1.49

$$\frac{a^8x^{12}}{12}+\frac{8a^7bx^{15}}{15}+\frac{14a^6b^2x^{18}}{9}+\frac{8a^5b^3x^{21}}{3}+\frac{35a^4b^4x^{24}}{12}+\frac{56a^3b^5x^{27}}{27}+\frac{14a^2b^6x^{30}}{15}+\frac{8ab^7x^{33}}{33}+\frac{b^8x^{36}}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x**3+a)**8,x)
```

```
[Out] a**8*x**12/12 + 8*a**7*b*x**15/15 + 14*a**6*b**2*x**18/9 + 8*a**5*b**3*x**2
1/3 + 35*a**4*b**4*x**24/12 + 56*a**3*b**5*x**27/27 + 14*a**2*b**6*x**30/15
 + 8*a*b**7*x**33/33 + b**8*x**36/36
```

3.289 $\int x^8 \left(a+bx^3\right)^8 \, dx$

Optimal. Leaf size=53

$$\frac{a^2\left(a+bx^3\right)^9}{27b^3}+\frac{\left(a+bx^3\right)^{11}}{33b^3}-\frac{a\left(a+bx^3\right)^{10}}{15b^3}$$

```
[Out] 1/27*a^2*(b*x^3+a)^9/b^3-1/15*a*(b*x^3+a)^10/b^3+1/33*(b*x^3+a)^11/b^3
```

Rubi [A] time = 0.08, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{a^2\left(a+bx^3\right)^9}{27b^3}+\frac{\left(a+bx^3\right)^{11}}{33b^3}-\frac{a\left(a+bx^3\right)^{10}}{15b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x^3)^8,x]
```

```
[Out] (a^2*(a + b*x^3)^9)/(27*b^3) - (a*(a + b*x^3)^10)/(15*b^3) + (a + b*x^3)^11
/(33*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^8\left(a+bx^3\right)^8\,dx &= \frac{1}{3}\,\text{Subst}\left(\int x^2(a+bx)^8\,dx, x, x^3\right)\\
&= \frac{1}{3}\,\text{Subst}\left(\int\left(\frac{a^2(a+bx)^8}{b^2}-\frac{2a(a+bx)^9}{b^2}+\frac{(a+bx)^{10}}{b^2}\right)dx, x, x^3\right)\\
&= \frac{a^2\left(a+bx^3\right)^9}{27b^3}-\frac{a\left(a+bx^3\right)^{10}}{15b^3}+\frac{\left(a+bx^3\right)^{11}}{33b^3}
\end{aligned}$$

Mathematica [B] time = 0.00, size = 108, normalized size = 2.04

$$\frac{a^8x^9}{9}+\frac{2}{3}a^7bx^{12}+\frac{28}{15}a^6b^2x^{15}+\frac{28}{9}a^5b^3x^{18}+\frac{10}{3}a^4b^4x^{21}+\frac{7}{3}a^3b^5x^{24}+\frac{28}{27}a^2b^6x^{27}+\frac{4}{15}ab^7x^{30}+\frac{b^8x^{33}}{33}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x^3)^8,x]
```

```
[Out] (a^8*x^9)/9 + (2*a^7*b*x^12)/3 + (28*a^6*b^2*x^15)/15 + (28*a^5*b^3*x^18)/9
 + (10*a^4*b^4*x^21)/3 + (7*a^3*b^5*x^24)/3 + (28*a^2*b^6*x^27)/27 + (4*a*b
^7*x^30)/15 + (b^8*x^33)/33
```

fricas [A] time = 0.68, size = 90, normalized size = 1.70

$$\frac{1}{33}x^{33}b^8+\frac{4}{15}x^{30}b^7a+\frac{28}{27}x^{27}b^6a^2+\frac{7}{3}x^{24}b^5a^3+\frac{10}{3}x^{21}b^4a^4+\frac{28}{9}x^{18}b^3a^5+\frac{28}{15}x^{15}b^2a^6+\frac{2}{3}x^{12}ba^7+\frac{1}{9}x^9a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^8,x, algorithm="fricas")
```

```
[Out] 1/33*x^33*b^8 + 4/15*x^30*b^7*a + 28/27*x^27*b^6*a^2 + 7/3*x^24*b^5*a^3 + 1
0/3*x^21*b^4*a^4 + 28/9*x^18*b^3*a^5 + 28/15*x^15*b^2*a^6 + 2/3*x^12*b*a^7
+ 1/9*x^9*a^8
```

giac [A] time = 0.16, size = 90, normalized size = 1.70

$$\frac{1}{33}\,b^8x^{33}+\frac{4}{15}\,ab^7x^{30}+\frac{28}{27}\,a^2b^6x^{27}+\frac{7}{3}\,a^3b^5x^{24}+\frac{10}{3}\,a^4b^4x^{21}+\frac{28}{9}\,a^5b^3x^{18}+\frac{28}{15}\,a^6b^2x^{15}+\frac{2}{3}\,a^7bx^{12}+\frac{1}{9}\,a^8x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^8,x, algorithm="giac")
```

```
[Out] 1/33*b^8*x^33 + 4/15*a*b^7*x^30 + 28/27*a^2*b^6*x^27 + 7/3*a^3*b^5*x^24 + 1
0/3*a^4*b^4*x^21 + 28/9*a^5*b^3*x^18 + 28/15*a^6*b^2*x^15 + 2/3*a^7*b*x^12
+ 1/9*a^8*x^9
```

maple [A] time = 0.00, size = 91, normalized size = 1.72

$$\frac{1}{33}b^8x^{33}+\frac{4}{15}a\,b^7x^{30}+\frac{28}{27}a^2b^6x^{27}+\frac{7}{3}a^3b^5x^{24}+\frac{10}{3}a^4b^4x^{21}+\frac{28}{9}a^5b^3x^{18}+\frac{28}{15}a^6b^2x^{15}+\frac{2}{3}a^7b\,x^{12}+\frac{1}{9}a^8x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^3+a)^8,x)
```

```
[Out] 1/33*b^8*x^33+4/15*a*b^7*x^30+28/27*a^2*b^6*x^27+7/3*a^3*b^5*x^24+10/3*a^4*
b^4*x^21+28/9*a^5*b^3*x^18+28/15*a^6*b^2*x^15+2/3*a^7*b*x^12+1/9*a^8*x^9
```

maxima [A] time = 1.32, size = 90, normalized size = 1.70

$$\frac{1}{33}\,b^8x^{33}+\frac{4}{15}\,ab^7x^{30}+\frac{28}{27}\,a^2b^6x^{27}+\frac{7}{3}\,a^3b^5x^{24}+\frac{10}{3}\,a^4b^4x^{21}+\frac{28}{9}\,a^5b^3x^{18}+\frac{28}{15}\,a^6b^2x^{15}+\frac{2}{3}\,a^7bx^{12}+\frac{1}{9}\,a^8x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^8,x, algorithm="maxima")
```

```
[Out] 1/33*b^8*x^33 + 4/15*a*b^7*x^30 + 28/27*a^2*b^6*x^27 + 7/3*a^3*b^5*x^24 + 1
0/3*a^4*b^4*x^21 + 28/9*a^5*b^3*x^18 + 28/15*a^6*b^2*x^15 + 2/3*a^7*b*x^12
+ 1/9*a^8*x^9
```

mupad [B] time = 0.09, size = 90, normalized size = 1.70

$$\frac{a^8\,x^9}{9}+\frac{2\,a^7\,b\,x^{12}}{3}+\frac{28\,a^6\,b^2\,x^{15}}{15}+\frac{28\,a^5\,b^3\,x^{18}}{9}+\frac{10\,a^4\,b^4\,x^{21}}{3}+\frac{7\,a^3\,b^5\,x^{24}}{3}+\frac{28\,a^2\,b^6\,x^{27}}{27}+\frac{4\,a\,b^7\,x^{30}}{15}+\frac{b^8\,x^{33}}{33}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*x^3)^8,x)
```

```
[Out] (a^8*x^9)/9 + (b^8*x^33)/33 + (2*a^7*b*x^12)/3 + (4*a*b^7*x^30)/15 + (28*a^
6*b^2*x^15)/15 + (28*a^5*b^3*x^18)/9 + (10*a^4*b^4*x^21)/3 + (7*a^3*b^5*x^2
4)/3 + (28*a^2*b^6*x^27)/27
```

sympy [**B**] time = 0.11, size = 107, normalized size = 2.02

$$\frac{a^8x^9}{9}+\frac{2a^7bx^{12}}{3}+\frac{28a^6b^2x^{15}}{15}+\frac{28a^5b^3x^{18}}{9}+\frac{10a^4b^4x^{21}}{3}+\frac{7a^3b^5x^{24}}{3}+\frac{28a^2b^6x^{27}}{27}+\frac{4ab^7x^{30}}{15}+\frac{b^8x^{33}}{33}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**3+a)**8,x)
```

```
[Out] a**8*x**9/9 + 2*a**7*b*x**12/3 + 28*a**6*b**2*x**15/15 + 28*a**5*b**3*x**18
/9 + 10*a**4*b**4*x**21/3 + 7*a**3*b**5*x**24/3 + 28*a**2*b**6*x**27/27 + 4
*a*b**7*x**30/15 + b**8*x**33/33
```

3.290 $\int x^5 \left(a+bx^3\right)^8 dx$

Optimal. Leaf size=34

$$\frac{\left(a+bx^3\right)^{10}}{30b^2} - \frac{a\left(a+bx^3\right)^9}{27b^2}$$

```
[Out] -1/27*a*(b*x^3+a)^9/b^2+1/30*(b*x^3+a)^10/b^2
```

Rubi [A] time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{\left(a+bx^3\right)^{10}}{30b^2} - \frac{a\left(a+bx^3\right)^9}{27b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^3)^8,x]
```

```
[Out] -(a*(a + b*x^3)^9)/(27*b^2) + (a + b*x^3)^10/(30*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^5 \left(a+bx^3\right)^8 dx &= \frac{1}{3}\,\text{Subst}\left(\int x(a+bx)^8\,dx, x, x^3\right) \\
&= \frac{1}{3}\,\text{Subst}\left(\int \left(-\frac{a(a+bx)^8}{b} + \frac{(a+bx)^9}{b}\right) dx, x, x^3\right) \\
&= -\frac{a\left(a+bx^3\right)^9}{27b^2} + \frac{\left(a+bx^3\right)^{10}}{30b^2}
\end{aligned}$$

Mathematica [B] time = 0.00, size = 108, normalized size = 3.18

$$\frac{a^8x^6}{6} + \frac{8}{9}a^7bx^9 + \frac{7}{3}a^6b^2x^{12} + \frac{56}{15}a^5b^3x^{15} + \frac{35}{9}a^4b^4x^{18} + \frac{8}{3}a^3b^5x^{21} + \frac{7}{6}a^2b^6x^{24} + \frac{8}{27}ab^7x^{27} + \frac{b^8x^{30}}{30}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^3)^8,x]
```

```
[Out] (a^8*x^6)/6 + (8*a^7*b*x^9)/9 + (7*a^6*b^2*x^12)/3 + (56*a^5*b^3*x^15)/15 +
 (35*a^4*b^4*x^18)/9 + (8*a^3*b^5*x^21)/3 + (7*a^2*b^6*x^24)/6 + (8*a*b^7*x
^27)/27 + (b^8*x^30)/30
```

fricas [B] time = 0.72, size = 90, normalized size = 2.65

$$\frac{1}{30}x^{30}b^8+\frac{8}{27}x^{27}b^7a+\frac{7}{6}x^{24}b^6a^2+\frac{8}{3}x^{21}b^5a^3+\frac{35}{9}x^{18}b^4a^4+\frac{56}{15}x^{15}b^3a^5+\frac{7}{3}x^{12}b^2a^6+\frac{8}{9}x^9ba^7+\frac{1}{6}x^6a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^8,x, algorithm="fricas")
```

```
[Out] 1/30*x^30*b^8 + 8/27*x^27*b^7*a + 7/6*x^24*b^6*a^2 + 8/3*x^21*b^5*a^3 + 35/
9*x^18*b^4*a^4 + 56/15*x^15*b^3*a^5 + 7/3*x^12*b^2*a^6 + 8/9*x^9*b*a^7 + 1/
6*x^6*a^8
```

giac [B] time = 0.15, size = 90, normalized size = 2.65

$$\frac{1}{30}\,b^8x^{30}+\frac{8}{27}\,ab^7x^{27}+\frac{7}{6}\,a^2b^6x^{24}+\frac{8}{3}\,a^3b^5x^{21}+\frac{35}{9}\,a^4b^4x^{18}+\frac{56}{15}\,a^5b^3x^{15}+\frac{7}{3}\,a^6b^2x^{12}+\frac{8}{9}\,a^7bx^9+\frac{1}{6}\,a^8x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^8,x, algorithm="giac")
```

```
[Out] 1/30*b^8*x^30 + 8/27*a*b^7*x^27 + 7/6*a^2*b^6*x^24 + 8/3*a^3*b^5*x^21 + 35/
9*a^4*b^4*x^18 + 56/15*a^5*b^3*x^15 + 7/3*a^6*b^2*x^12 + 8/9*a^7*b*x^9 + 1/
6*a^8*x^6
```

maple [B] time = 0.00, size = 91, normalized size = 2.68

$$\frac{1}{30}b^8x^{30}+\frac{8}{27}a\,b^7x^{27}+\frac{7}{6}a^2b^6x^{24}+\frac{8}{3}a^3b^5x^{21}+\frac{35}{9}a^4b^4x^{18}+\frac{56}{15}a^5b^3x^{15}+\frac{7}{3}a^6b^2x^{12}+\frac{8}{9}a^7b\,x^9+\frac{1}{6}a^8x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^3+a)^8,x)
```

```
[Out] 1/30*b^8*x^30+8/27*a*b^7*x^27+7/6*a^2*b^6*x^24+8/3*a^3*b^5*x^21+35/9*a^4*b^
4*x^18+56/15*a^5*b^3*x^15+7/3*a^6*b^2*x^12+8/9*a^7*b*x^9+1/6*a^8*x^6
```

maxima [B] time = 1.33, size = 90, normalized size = 2.65

$$\frac{1}{30}\,b^8x^{30}+\frac{8}{27}\,ab^7x^{27}+\frac{7}{6}\,a^2b^6x^{24}+\frac{8}{3}\,a^3b^5x^{21}+\frac{35}{9}\,a^4b^4x^{18}+\frac{56}{15}\,a^5b^3x^{15}+\frac{7}{3}\,a^6b^2x^{12}+\frac{8}{9}\,a^7bx^9+\frac{1}{6}\,a^8x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^8,x, algorithm="maxima")
```

```
[Out] 1/30*b^8*x^30 + 8/27*a*b^7*x^27 + 7/6*a^2*b^6*x^24 + 8/3*a^3*b^5*x^21 + 35/
9*a^4*b^4*x^18 + 56/15*a^5*b^3*x^15 + 7/3*a^6*b^2*x^12 + 8/9*a^7*b*x^9 + 1/
6*a^8*x^6
```

mupad [B] time = 0.09, size = 90, normalized size = 2.65

$$\frac{a^8\,x^6}{6}+\frac{8\,a^7\,b\,x^9}{9}+\frac{7\,a^6\,b^2\,x^{12}}{3}+\frac{56\,a^5\,b^3\,x^{15}}{15}+\frac{35\,a^4\,b^4\,x^{18}}{9}+\frac{8\,a^3\,b^5\,x^{21}}{3}+\frac{7\,a^2\,b^6\,x^{24}}{6}+\frac{8\,a\,b^7\,x^{27}}{27}+\frac{b^8\,x^{30}}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^3)^8,x)
```

```
[Out] (a^8*x^6)/6 + (b^8*x^30)/30 + (8*a^7*b*x^9)/9 + (8*a*b^7*x^27)/27 + (7*a^6*
b^2*x^12)/3 + (56*a^5*b^3*x^15)/15 + (35*a^4*b^4*x^18)/9 + (8*a^3*b^5*x^21)
/3 + (7*a^2*b^6*x^24)/6
```

sympy [**B**] time = 0.11, size = 107, normalized size = 3.15

$$\frac{a^8x^6}{6}+\frac{8a^7bx^9}{9}+\frac{7a^6b^2x^{12}}{3}+\frac{56a^5b^3x^{15}}{15}+\frac{35a^4b^4x^{18}}{9}+\frac{8a^3b^5x^{21}}{3}+\frac{7a^2b^6x^{24}}{6}+\frac{8ab^7x^{27}}{27}+\frac{b^8x^{30}}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**3+a)**8,x)
```

```
[Out] a**8*x**6/6 + 8*a**7*b*x**9/9 + 7*a**6*b**2*x**12/3 + 56*a**5*b**3*x**15/15
 + 35*a**4*b**4*x**18/9 + 8*a**3*b**5*x**21/3 + 7*a**2*b**6*x**24/6 + 8*a*b
**7*x**27/27 + b**8*x**30/30
```

3.291 $\int x^2 \left(a + bx^3\right)^8 \, dx$

Optimal. Leaf size=16

$$\frac{\left(a + bx^3\right)^9}{27b}$$

```
[Out] 1/27*(b*x^3+a)^9/b
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$\frac{\left(a + bx^3\right)^9}{27b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3)^8,x]
```

```
[Out] (a + b*x^3)^9/(27*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^2 \left(a + bx^3\right)^8 \, dx = \frac{\left(a + bx^3\right)^9}{27b}$$

Mathematica [B] time = 0.00, size = 108, normalized size = 6.75

$$\frac{a^8 x^3}{3} + \frac{4}{3} a^7 b x^6 + \frac{28}{9} a^6 b^2 x^9 + \frac{14}{3} a^5 b^3 x^{12} + \frac{14}{3} a^4 b^4 x^{15} + \frac{28}{9} a^3 b^5 x^{18} + \frac{4}{3} a^2 b^6 x^{21} + \frac{1}{3} a b^7 x^{24} + \frac{b^8 x^{27}}{27}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)^8,x]
```

```
[Out] (a^8*x^3)/3 + (4*a^7*b*x^6)/3 + (28*a^6*b^2*x^9)/9 + (14*a^5*b^3*x^12)/3 +
(14*a^4*b^4*x^15)/3 + (28*a^3*b^5*x^18)/9 + (4*a^2*b^6*x^21)/3 + (a*b^7*x^2
4)/3 + (b^8*x^27)/27
```

fricas [B] time = 0.73, size = 90, normalized size = 5.62

$$\frac{1}{27} x^{27} b^8 + \frac{1}{3} x^{24} b^7 a + \frac{4}{3} x^{21} b^6 a^2 + \frac{28}{9} x^{18} b^5 a^3 + \frac{14}{3} x^{15} b^4 a^4 + \frac{14}{3} x^{12} b^3 a^5 + \frac{28}{9} x^9 b^2 a^6 + \frac{4}{3} x^6 b a^7 + \frac{1}{3} x^3 a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^8,x, algorithm="fricas")
```

```
[Out] 1/27*x^27*b^8 + 1/3*x^24*b^7*a + 4/3*x^21*b^6*a^2 + 28/9*x^18*b^5*a^3 + 14/
3*x^15*b^4*a^4 + 14/3*x^12*b^3*a^5 + 28/9*x^9*b^2*a^6 + 4/3*x^6*b*a^7 + 1/3
*x^3*a^8
```

giac [**A**] time = 0.18, size = 14, normalized size = 0.88

$$\frac{\left(bx^3+a\right)^9}{27\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^8,x, algorithm="giac")
```

```
[Out] 1/27*(b*x^3 + a)^9/b
```

maple [**B**] time = 0.00, size = 91, normalized size = 5.69

$$\frac{1}{27}b^8x^{27}+\frac{1}{3}a\,b^7x^{24}+\frac{4}{3}a^2b^6x^{21}+\frac{28}{9}a^3b^5x^{18}+\frac{14}{3}a^4b^4x^{15}+\frac{14}{3}a^5b^3x^{12}+\frac{28}{9}a^6b^2x^9+\frac{4}{3}a^7b\,x^6+\frac{1}{3}a^8x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^8,x)
```

```
[Out] 1/27*b^8*x^27+1/3*a*b^7*x^24+4/3*a^2*b^6*x^21+28/9*a^3*b^5*x^18+14/3*a^4*b^
4*x^15+14/3*a^5*b^3*x^12+28/9*a^6*b^2*x^9+4/3*a^7*b*x^6+1/3*a^8*x^3
```

maxima [**A**] time = 1.27, size = 14, normalized size = 0.88

$$\frac{\left(bx^3+a\right)^9}{27\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^8,x, algorithm="maxima")
```

```
[Out] 1/27*(b*x^3 + a)^9/b
```

mupad [**B**] time = 0.09, size = 14, normalized size = 0.88

$$\frac{\left(b\,x^3+a\right)^9}{27\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^3)^8,x)
```

```
[Out] (a + b*x^3)^9/(27*b)
```

sympy [**B**] time = 0.09, size = 105, normalized size = 6.56

$$\frac{a^8x^3}{3}+\frac{4a^7bx^6}{3}+\frac{28a^6b^2x^9}{9}+\frac{14a^5b^3x^{12}}{3}+\frac{14a^4b^4x^{15}}{3}+\frac{28a^3b^5x^{18}}{9}+\frac{4a^2b^6x^{21}}{3}+\frac{ab^7x^{24}}{3}+\frac{b^8x^{27}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**8,x)
```

```
[Out] a**8*x**3/3 + 4*a**7*b*x**6/3 + 28*a**6*b**2*x**9/9 + 14*a**5*b**3*x**12/3
+ 14*a**4*b**4*x**15/3 + 28*a**3*b**5*x**18/9 + 4*a**2*b**6*x**21/3 + a*b**
7*x**24/3 + b**8*x**27/27
```

3.292 $\int \frac{\left(a+bx^3\right)^8}{x}\,dx$

Optimal. Leaf size=104

$$a^8\log(x)+\frac{8}{3}a^7bx^3+\frac{14}{3}a^6b^2x^6+\frac{56}{9}a^5b^3x^9+\frac{35}{6}a^4b^4x^{12}+\frac{56}{15}a^3b^5x^{15}+\frac{14}{9}a^2b^6x^{18}+\frac{8}{21}ab^7x^{21}+\frac{b^8x^{24}}{24}$$

```
[Out] 8/3*a^7*b*x^3+14/3*a^6*b^2*x^6+56/9*a^5*b^3*x^9+35/6*a^4*b^4*x^12+56/15*a^3
*b^5*x^15+14/9*a^2*b^6*x^18+8/21*a*b^7*x^21+1/24*b^8*x^24+a^8*ln(x)
```

Rubi [A] time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{14}{9}a^2b^6x^{18}+\frac{56}{15}a^3b^5x^{15}+\frac{35}{6}a^4b^4x^{12}+\frac{56}{9}a^5b^3x^9+\frac{14}{3}a^6b^2x^6+\frac{8}{3}a^7bx^3+a^8\log(x)+\frac{8}{21}ab^7x^{21}+\frac{b^8x^{24}}{24}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^8/x,x]
```

```
[Out] (8*a^7*b*x^3)/3 + (14*a^6*b^2*x^6)/3 + (56*a^5*b^3*x^9)/9 + (35*a^4*b^4*x^1
2)/6 + (56*a^3*b^5*x^15)/15 + (14*a^2*b^6*x^18)/9 + (8*a*b^7*x^21)/21 + (b^
8*x^24)/24 + a^8*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^8}{x}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)^8}{x}\,dx, x, x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(8a^7b+\frac{a^8}{x}+28a^6b^2x+56a^5b^3x^2+70a^4b^4x^3+56a^3b^5x^4+28a^2b^6x^5+8ab^7x^6+\right.\right.\\ &= \frac{8}{3}a^7bx^3+\frac{14}{3}a^6b^2x^6+\frac{56}{9}a^5b^3x^9+\frac{35}{6}a^4b^4x^{12}+\frac{56}{15}a^3b^5x^{15}+\frac{14}{9}a^2b^6x^{18}+\frac{8}{21}ab^7x^{21}+\frac{b^8x^{24}}{24}\end{aligned}$$

Mathematica [A] time = 0.01, size = 104, normalized size = 1.00

$$a^8\log(x)+\frac{8}{3}a^7bx^3+\frac{14}{3}a^6b^2x^6+\frac{56}{9}a^5b^3x^9+\frac{35}{6}a^4b^4x^{12}+\frac{56}{15}a^3b^5x^{15}+\frac{14}{9}a^2b^6x^{18}+\frac{8}{21}ab^7x^{21}+\frac{b^8x^{24}}{24}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^8/x,x]
```

```
[Out] (8*a^7*b*x^3)/3 + (14*a^6*b^2*x^6)/3 + (56*a^5*b^3*x^9)/9 + (35*a^4*b^4*x^1
2)/6 + (56*a^3*b^5*x^15)/15 + (14*a^2*b^6*x^18)/9 + (8*a*b^7*x^21)/21 + (b^
8*x^24)/24 + a^8*Log[x]
```

fricas [A] time = 0.81, size = 88, normalized size = 0.85

$$\frac{1}{24}b^8x^{24}+\frac{8}{21}ab^7x^{21}+\frac{14}{9}a^2b^6x^{18}+\frac{56}{15}a^3b^5x^{15}+\frac{35}{6}a^4b^4x^{12}+\frac{56}{9}a^5b^3x^9+\frac{14}{3}a^6b^2x^6+\frac{8}{3}a^7bx^3+a^8\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x,x, algorithm="fricas")
```

```
[Out] 1/24*b^8*x^24 + 8/21*a*b^7*x^21 + 14/9*a^2*b^6*x^18 + 56/15*a^3*b^5*x^15 +
35/6*a^4*b^4*x^12 + 56/9*a^5*b^3*x^9 + 14/3*a^6*b^2*x^6 + 8/3*a^7*b*x^3 + a
^8*log(x)
```

giac [A] time = 0.16, size = 89, normalized size = 0.86

$$\frac{1}{24}b^8x^{24}+\frac{8}{21}ab^7x^{21}+\frac{14}{9}a^2b^6x^{18}+\frac{56}{15}a^3b^5x^{15}+\frac{35}{6}a^4b^4x^{12}+\frac{56}{9}a^5b^3x^9+\frac{14}{3}a^6b^2x^6+\frac{8}{3}a^7bx^3+a^8\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x,x, algorithm="giac")
```

```
[Out] 1/24*b^8*x^24 + 8/21*a*b^7*x^21 + 14/9*a^2*b^6*x^18 + 56/15*a^3*b^5*x^15 +
35/6*a^4*b^4*x^12 + 56/9*a^5*b^3*x^9 + 14/3*a^6*b^2*x^6 + 8/3*a^7*b*x^3 + a
^8*log(abs(x))
```

maple [A] time = 0.00, size = 89, normalized size = 0.86

$$\frac{b^8x^{24}}{24}+\frac{8a\,b^7x^{21}}{21}+\frac{14a^2b^6x^{18}}{9}+\frac{56a^3b^5x^{15}}{15}+\frac{35a^4b^4x^{12}}{6}+\frac{56a^5b^3x^9}{9}+\frac{14a^6b^2x^6}{3}+\frac{8a^7b\,x^3}{3}+a^8\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^8/x,x)
```

```
[Out] 8/3*a^7*b*x^3+14/3*a^6*b^2*x^6+56/9*a^5*b^3*x^9+35/6*a^4*b^4*x^12+56/15*a^3
*b^5*x^15+14/9*a^2*b^6*x^18+8/21*a*b^7*x^21+1/24*b^8*x^24+a^8*ln(x)
```

maxima [A] time = 1.28, size = 91, normalized size = 0.88

$$\frac{1}{24}b^8x^{24}+\frac{8}{21}ab^7x^{21}+\frac{14}{9}a^2b^6x^{18}+\frac{56}{15}a^3b^5x^{15}+\frac{35}{6}a^4b^4x^{12}+\frac{56}{9}a^5b^3x^9+\frac{14}{3}a^6b^2x^6+\frac{8}{3}a^7bx^3+\frac{1}{3}a^8\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x,x, algorithm="maxima")
```

```
[Out] 1/24*b^8*x^24 + 8/21*a*b^7*x^21 + 14/9*a^2*b^6*x^18 + 56/15*a^3*b^5*x^15 +
35/6*a^4*b^4*x^12 + 56/9*a^5*b^3*x^9 + 14/3*a^6*b^2*x^6 + 8/3*a^7*b*x^3 + 1
/3*a^8*log(x^3)
```

mupad [B] time = 0.05, size = 88, normalized size = 0.85

$$a^8\,\ln(x)+\frac{b^8\,x^{24}}{24}+\frac{8\,a^7\,b\,x^3}{3}+\frac{8\,a\,b^7\,x^{21}}{21}+\frac{14\,a^6\,b^2\,x^6}{3}+\frac{56\,a^5\,b^3\,x^9}{9}+\frac{35\,a^4\,b^4\,x^{12}}{6}+\frac{56\,a^3\,b^5\,x^{15}}{15}+\frac{14\,a^2\,b^6\,x^{18}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^8/x,x)
```

```
[Out] a^8*log(x) + (b^8*x^24)/24 + (8*a^7*b*x^3)/3 + (8*a*b^7*x^21)/21 + (14*a^6*
b^2*x^6)/3 + (56*a^5*b^3*x^9)/9 + (35*a^4*b^4*x^12)/6 + (56*a^3*b^5*x^15)/1
5 + (14*a^2*b^6*x^18)/9
```

sympy [**A**] time = 0.20, size = 105, normalized size = 1.01

$$a^8 \log(x)+\frac{8a^7bx^3}{3}+\frac{14a^6b^2x^6}{3}+\frac{56a^5b^3x^9}{9}+\frac{35a^4b^4x^{12}}{6}+\frac{56a^3b^5x^{15}}{15}+\frac{14a^2b^6x^{18}}{9}+\frac{8ab^7x^{21}}{21}+\frac{b^8x^{24}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**8/x,x)
```

```
[Out] a**8*log(x) + 8*a**7*b*x**3/3 + 14*a**6*b**2*x**6/3 + 56*a**5*b**3*x**9/9 +
 35*a**4*b**4*x**12/6 + 56*a**3*b**5*x**15/15 + 14*a**2*b**6*x**18/9 + 8*a*
b**7*x**21/21 + b**8*x**24/24
```

3.293 $\int \frac{\left(a+bx^3\right)^8}{x^4}\,dx$

Optimal. Leaf size=105

$$-\frac{a^8}{3x^3}+8a^7b\log(x)+\frac{28}{3}a^6b^2x^3+\frac{28}{3}a^5b^3x^6+\frac{70}{9}a^4b^4x^9+\frac{14}{3}a^3b^5x^{12}+\frac{28}{15}a^2b^6x^{15}+\frac{4}{9}ab^7x^{18}+\frac{b^8x^{21}}{21}$$

```
[Out] -1/3*a^8/x^3+28/3*a^6*b^2*x^3+28/3*a^5*b^3*x^6+70/9*a^4*b^4*x^9+14/3*a^3*b^
5*x^12+28/15*a^2*b^6*x^15+4/9*a*b^7*x^18+1/21*b^8*x^21+8*a^7*b*ln(x)
```

Rubi [A] time = 0.06, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{28}{15}a^2b^6x^{15}+\frac{14}{3}a^3b^5x^{12}+\frac{70}{9}a^4b^4x^9+\frac{28}{3}a^5b^3x^6+\frac{28}{3}a^6b^2x^3+8a^7b\log(x)-\frac{a^8}{3x^3}+\frac{4}{9}ab^7x^{18}+\frac{b^8x^{21}}{21}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^8/x^4,x]
```

```
[Out] -a^8/(3*x^3) + (28*a^6*b^2*x^3)/3 + (28*a^5*b^3*x^6)/3 + (70*a^4*b^4*x^9)/9
 + (14*a^3*b^5*x^12)/3 + (28*a^2*b^6*x^15)/15 + (4*a*b^7*x^18)/9 + (b^8*x^2
1)/21 + 8*a^7*b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^8}{x^4}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)^8}{x^2}\,dx, x, x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(28a^6b^2+\frac{a^8}{x^2}+\frac{8a^7b}{x}+56a^5b^3x+70a^4b^4x^2+56a^3b^5x^3+28a^2b^6x^4+8ab^7x^5\right.\right.\\ &= -\frac{a^8}{3x^3}+\frac{28}{3}a^6b^2x^3+\frac{28}{3}a^5b^3x^6+\frac{70}{9}a^4b^4x^9+\frac{14}{3}a^3b^5x^{12}+\frac{28}{15}a^2b^6x^{15}+\frac{4}{9}ab^7x^{18}+\frac{b^8x^{21}}{21}-\end{aligned}$$

Mathematica [A] time = 0.01, size = 105, normalized size = 1.00

$$-\frac{a^8}{3x^3}+8a^7b\log(x)+\frac{28}{3}a^6b^2x^3+\frac{28}{3}a^5b^3x^6+\frac{70}{9}a^4b^4x^9+\frac{14}{3}a^3b^5x^{12}+\frac{28}{15}a^2b^6x^{15}+\frac{4}{9}ab^7x^{18}+\frac{b^8x^{21}}{21}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^8/x^4,x]
```

```
[Out] -1/3*a^8/x^3 + (28*a^6*b^2*x^3)/3 + (28*a^5*b^3*x^6)/3 + (70*a^4*b^4*x^9)/9
 + (14*a^3*b^5*x^12)/3 + (28*a^2*b^6*x^15)/15 + (4*a*b^7*x^18)/9 + (b^8*x^2
1)/21 + 8*a^7*b*Log[x]
```

fricas [A] time = 0.62, size = 94, normalized size = 0.90

$$\frac{15\,b^8x^{24}+140\,ab^7x^{21}+588\,a^2b^6x^{18}+1470\,a^3b^5x^{15}+2450\,a^4b^4x^{12}+2940\,a^5b^3x^9+2940\,a^6b^2x^6+2520\,a^7bx^3\,\mathrm{lo}}{315\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^4,x, algorithm="fricas")
```

```
[Out] 1/315*(15*b^8*x^24 + 140*a*b^7*x^21 + 588*a^2*b^6*x^18 + 1470*a^3*b^5*x^15
+ 2450*a^4*b^4*x^12 + 2940*a^5*b^3*x^9 + 2940*a^6*b^2*x^6 + 2520*a^7*b*x^3*
log(x) - 105*a^8)/x^3
```

giac [A] time = 0.15, size = 100, normalized size = 0.95

$$\frac{1}{21}\,b^8x^{21}+\frac{4}{9}\,ab^7x^{18}+\frac{28}{15}\,a^2b^6x^{15}+\frac{14}{3}\,a^3b^5x^{12}+\frac{70}{9}\,a^4b^4x^9+\frac{28}{3}\,a^5b^3x^6+\frac{28}{3}\,a^6b^2x^3+8\,a^7b\log\left(|x|\right)-\frac{8\,a^7bx^3+a^8}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^4,x, algorithm="giac")
```

```
[Out] 1/21*b^8*x^21 + 4/9*a*b^7*x^18 + 28/15*a^2*b^6*x^15 + 14/3*a^3*b^5*x^12 + 7
0/9*a^4*b^4*x^9 + 28/3*a^5*b^3*x^6 + 28/3*a^6*b^2*x^3 + 8*a^7*b*log(abs(x))
 - 1/3*(8*a^7*b*x^3 + a^8)/x^3
```

maple [A] time = 0.01, size = 90, normalized size = 0.86

$$\frac{b^8x^{21}}{21}+\frac{4a\,b^7x^{18}}{9}+\frac{28a^2b^6x^{15}}{15}+\frac{14a^3b^5x^{12}}{3}+\frac{70a^4b^4x^9}{9}+\frac{28a^5b^3x^6}{3}+\frac{28a^6b^2x^3}{3}+8a^7b\ln(x)-\frac{a^8}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^8/x^4,x)
```

```
[Out] -1/3*a^8/x^3+28/3*a^6*b^2*x^3+28/3*a^5*b^3*x^6+70/9*a^4*b^4*x^9+14/3*a^3*b^
5*x^12+28/15*a^2*b^6*x^15+4/9*a*b^7*x^18+1/21*b^8*x^21+8*a^7*b*ln(x)
```

maxima [A] time = 1.30, size = 91, normalized size = 0.87

$$\frac{1}{21}\,b^8x^{21}+\frac{4}{9}\,ab^7x^{18}+\frac{28}{15}\,a^2b^6x^{15}+\frac{14}{3}\,a^3b^5x^{12}+\frac{70}{9}\,a^4b^4x^9+\frac{28}{3}\,a^5b^3x^6+\frac{28}{3}\,a^6b^2x^3+\frac{8}{3}\,a^7b\log\left(x^3\right)-\frac{a^8}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^4,x, algorithm="maxima")
```

```
[Out] 1/21*b^8*x^21 + 4/9*a*b^7*x^18 + 28/15*a^2*b^6*x^15 + 14/3*a^3*b^5*x^12 + 7
0/9*a^4*b^4*x^9 + 28/3*a^5*b^3*x^6 + 28/3*a^6*b^2*x^3 + 8/3*a^7*b*log(x^3)
- 1/3*a^8/x^3
```

mupad [B] time = 0.06, size = 89, normalized size = 0.85

$$\frac{b^8\,x^{21}}{21}-\frac{a^8}{3\,x^3}+\frac{4\,a\,b^7\,x^{18}}{9}+8\,a^7\,b\,\ln(x)+\frac{28\,a^6\,b^2\,x^3}{3}+\frac{28\,a^5\,b^3\,x^6}{3}+\frac{70\,a^4\,b^4\,x^9}{9}+\frac{14\,a^3\,b^5\,x^{12}}{3}+\frac{28\,a^2\,b^6\,x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^8/x^4,x)
```

```
[Out] (b^8*x^21)/21 - a^8/(3*x^3) + (4*a*b^7*x^18)/9 + 8*a^7*b*log(x) + (28*a^6*b
^2*x^3)/3 + (28*a^5*b^3*x^6)/3 + (70*a^4*b^4*x^9)/9 + (14*a^3*b^5*x^12)/3 +
 (28*a^2*b^6*x^15)/15
```

sympy [**A**] time = 0.30, size = 105, normalized size = 1.00

$$-\frac{a^8}{3x^3}+8a^7b\log(x)+\frac{28a^6b^2x^3}{3}+\frac{28a^5b^3x^6}{3}+\frac{70a^4b^4x^9}{9}+\frac{14a^3b^5x^{12}}{3}+\frac{28a^2b^6x^{15}}{15}+\frac{4ab^7x^{18}}{9}+\frac{b^8x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**8/x**4,x)
```

```
[Out] -a**8/(3*x**3) + 8*a**7*b*log(x) + 28*a**6*b**2*x**3/3 + 28*a**5*b**3*x**6/
3 + 70*a**4*b**4*x**9/9 + 14*a**3*b**5*x**12/3 + 28*a**2*b**6*x**15/15 + 4*
a*b**7*x**18/9 + b**8*x**21/21
```

3.294 $\int \frac{\left(a+bx^3\right)^8}{x^7}\,dx$

Optimal. Leaf size=105

$$-\frac{a^8}{6x^6}-\frac{8a^7b}{3x^3}+28a^6b^2\log(x)+\frac{56}{3}a^5b^3x^3+\frac{35}{3}a^4b^4x^6+\frac{56}{9}a^3b^5x^9+\frac{7}{3}a^2b^6x^{12}+\frac{8}{15}ab^7x^{15}+\frac{b^8x^{18}}{18}$$

```
[Out] -1/6*a^8/x^6-8/3*a^7*b/x^3+56/3*a^5*b^3*x^3+35/3*a^4*b^4*x^6+56/9*a^3*b^5*x
^9+7/3*a^2*b^6*x^12+8/15*a*b^7*x^15+1/18*b^8*x^18+28*a^6*b^2*ln(x)
```

Rubi [A] time = 0.06, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{7}{3}a^2b^6x^{12}+\frac{56}{9}a^3b^5x^9+\frac{35}{3}a^4b^4x^6+\frac{56}{3}a^5b^3x^3+28a^6b^2\log(x)-\frac{8a^7b}{3x^3}-\frac{a^8}{6x^6}+\frac{8}{15}ab^7x^{15}+\frac{b^8x^{18}}{18}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^8/x^7,x]
```

```
[Out] -a^8/(6*x^6) - (8*a^7*b)/(3*x^3) + (56*a^5*b^3*x^3)/3 + (35*a^4*b^4*x^6)/3
+ (56*a^3*b^5*x^9)/9 + (7*a^2*b^6*x^12)/3 + (8*a*b^7*x^15)/15 + (b^8*x^18)/
18 + 28*a^6*b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^8}{x^7}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^8}{x^3}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(56a^5b^3+\frac{a^8}{x^3}+\frac{8a^7b}{x^2}+\frac{28a^6b^2}{x}+70a^4b^4x+56a^3b^5x^2+28a^2b^6x^3+8ab^7x^4+b^8\right.\right.\\ &= -\frac{a^8}{6x^6}-\frac{8a^7b}{3x^3}+\frac{56}{3}a^5b^3x^3+\frac{35}{3}a^4b^4x^6+\frac{56}{9}a^3b^5x^9+\frac{7}{3}a^2b^6x^{12}+\frac{8}{15}ab^7x^{15}+\frac{b^8x^{18}}{18}+28a^6b^2\end{aligned}$$

Mathematica [A] time = 0.01, size = 105, normalized size = 1.00

$$-\frac{a^8}{6x^6}-\frac{8a^7b}{3x^3}+28a^6b^2\log(x)+\frac{56}{3}a^5b^3x^3+\frac{35}{3}a^4b^4x^6+\frac{56}{9}a^3b^5x^9+\frac{7}{3}a^2b^6x^{12}+\frac{8}{15}ab^7x^{15}+\frac{b^8x^{18}}{18}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^8/x^7,x]
```

```
[Out] -1/6*a^8/x^6 - (8*a^7*b)/(3*x^3) + (56*a^5*b^3*x^3)/3 + (35*a^4*b^4*x^6)/3
+ (56*a^3*b^5*x^9)/9 + (7*a^2*b^6*x^12)/3 + (8*a*b^7*x^15)/15 + (b^8*x^18)/
18 + 28*a^6*b^2*Log[x]
```

fricas [A] time = 0.56, size = 94, normalized size = 0.90

$$\frac{5\,b^8x^{24}+48\,ab^7x^{21}+210\,a^2b^6x^{18}+560\,a^3b^5x^{15}+1050\,a^4b^4x^{12}+1680\,a^5b^3x^9+2520\,a^6b^2x^6\log(x)-240\,a^7b}{90\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^7,x, algorithm="fricas")
```

```
[Out] 1/90*(5*b^8*x^24 + 48*a*b^7*x^21 + 210*a^2*b^6*x^18 + 560*a^3*b^5*x^15 + 10
50*a^4*b^4*x^12 + 1680*a^5*b^3*x^9 + 2520*a^6*b^2*x^6*log(x) - 240*a^7*b*x^
3 - 15*a^8)/x^6
```

giac [A] time = 0.19, size = 102, normalized size = 0.97

$$\frac{1}{18}\,b^8x^{18}+\frac{8}{15}\,ab^7x^{15}+\frac{7}{3}\,a^2b^6x^{12}+\frac{56}{9}\,a^3b^5x^9+\frac{35}{3}\,a^4b^4x^6+\frac{56}{3}\,a^5b^3x^3+28\,a^6b^2\log\left(|x|\right)-\frac{84\,a^6b^2x^6+16\,a^7bx^3+a}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^7,x, algorithm="giac")
```

```
[Out] 1/18*b^8*x^18 + 8/15*a*b^7*x^15 + 7/3*a^2*b^6*x^12 + 56/9*a^3*b^5*x^9 + 35/
3*a^4*b^4*x^6 + 56/3*a^5*b^3*x^3 + 28*a^6*b^2*log(abs(x)) - 1/6*(84*a^6*b^2
*x^6 + 16*a^7*b*x^3 + a^8)/x^6
```

maple [A] time = 0.01, size = 90, normalized size = 0.86

$$\frac{b^8x^{18}}{18}+\frac{8a\,b^7x^{15}}{15}+\frac{7a^2b^6x^{12}}{3}+\frac{56a^3b^5x^9}{9}+\frac{35a^4b^4x^6}{3}+\frac{56a^5b^3x^3}{3}+28a^6b^2\ln(x)-\frac{8a^7b}{3x^3}-\frac{a^8}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^8/x^7,x)
```

```
[Out] -1/6*a^8/x^6-8/3*a^7*b/x^3+56/3*a^5*b^3*x^3+35/3*a^4*b^4*x^6+56/9*a^3*b^5*x
^9+7/3*a^2*b^6*x^12+8/15*a*b^7*x^15+1/18*b^8*x^18+28*a^6*b^2*ln(x)
```

maxima [A] time = 1.35, size = 92, normalized size = 0.88

$$\frac{1}{18}\,b^8x^{18}+\frac{8}{15}\,ab^7x^{15}+\frac{7}{3}\,a^2b^6x^{12}+\frac{56}{9}\,a^3b^5x^9+\frac{35}{3}\,a^4b^4x^6+\frac{56}{3}\,a^5b^3x^3+\frac{28}{3}\,a^6b^2\log\left(x^3\right)-\frac{16\,a^7bx^3+a^8}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^7,x, algorithm="maxima")
```

```
[Out] 1/18*b^8*x^18 + 8/15*a*b^7*x^15 + 7/3*a^2*b^6*x^12 + 56/9*a^3*b^5*x^9 + 35/
3*a^4*b^4*x^6 + 56/3*a^5*b^3*x^3 + 28/3*a^6*b^2*log(x^3) - 1/6*(16*a^7*b*x^
3 + a^8)/x^6
```

mupad [B] time = 0.05, size = 92, normalized size = 0.88

$$\frac{b^8\,x^{18}}{18}-\frac{\frac{a^8}{6}+\frac{8\,b\,a^7\,x^3}{3}}{x^6}+\frac{8\,a\,b^7\,x^{15}}{15}+\frac{56\,a^5\,b^3\,x^3}{3}+\frac{35\,a^4\,b^4\,x^6}{3}+\frac{56\,a^3\,b^5\,x^9}{9}+\frac{7\,a^2\,b^6\,x^{12}}{3}+28\,a^6\,b^2\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^8/x^7,x)
```

```
[Out] (b^8*x^18)/18 - (a^8/6 + (8*a^7*b*x^3)/3)/x^6 + (8*a*b^7*x^15)/15 + (56*a^5
*b^3*x^3)/3 + (35*a^4*b^4*x^6)/3 + (56*a^3*b^5*x^9)/9 + (7*a^2*b^6*x^12)/3
+ 28*a^6*b^2*log(x)
```

sympy [**A**] time = 0.37, size = 105, normalized size = 1.00

$$28a^6b^2\log(x)+\frac{56a^5b^3x^3}{3}+\frac{35a^4b^4x^6}{3}+\frac{56a^3b^5x^9}{9}+\frac{7a^2b^6x^{12}}{3}+\frac{8ab^7x^{15}}{15}+\frac{b^8x^{18}}{18}+\frac{-a^8-16a^7bx^3}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**8/x**7,x)
```

```
[Out] 28*a**6*b**2*log(x) + 56*a**5*b**3*x**3/3 + 35*a**4*b**4*x**6/3 + 56*a**3*b
**5*x**9/9 + 7*a**2*b**6*x**12/3 + 8*a*b**7*x**15/15 + b**8*x**18/18 + (-a*
*8 - 16*a**7*b*x**3)/(6*x**6)
```

3.295 $\int \frac{\left(a+bx^3\right)^8}{x^{10}}\,dx$

Optimal. Leaf size=105

$$-\frac{a^8}{9x^9}-\frac{4a^7b}{3x^6}-\frac{28a^6b^2}{3x^3}+56a^5b^3\log(x)+\frac{70}{3}a^4b^4x^3+\frac{28}{3}a^3b^5x^6+\frac{28}{9}a^2b^6x^9+\frac{2}{3}ab^7x^{12}+\frac{b^8x^{15}}{15}$$

```
[Out] -1/9*a^8/x^9-4/3*a^7*b/x^6-28/3*a^6*b^2/x^3+70/3*a^4*b^4*x^3+28/3*a^3*b^5*x
^6+28/9*a^2*b^6*x^9+2/3*a*b^7*x^12+1/15*b^8*x^15+56*a^5*b^3*ln(x)
```

Rubi [A] time = 0.06, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{28}{9}a^2b^6x^9+\frac{28}{3}a^3b^5x^6+\frac{70}{3}a^4b^4x^3-\frac{28a^6b^2}{3x^3}+56a^5b^3\log(x)-\frac{4a^7b}{3x^6}-\frac{a^8}{9x^9}+\frac{2}{3}ab^7x^{12}+\frac{b^8x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^8/x^10,x]
```

```
[Out] -a^8/(9*x^9) - (4*a^7*b)/(3*x^6) - (28*a^6*b^2)/(3*x^3) + (70*a^4*b^4*x^3)/
3 + (28*a^3*b^5*x^6)/3 + (28*a^2*b^6*x^9)/9 + (2*a*b^7*x^12)/3 + (b^8*x^15)
/15 + 56*a^5*b^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^8}{x^{10}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^8}{x^4}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(70a^4b^4+\frac{a^8}{x^4}+\frac{8a^7b}{x^3}+\frac{28a^6b^2}{x^2}+\frac{56a^5b^3}{x}+56a^3b^5x+28a^2b^6x^2+8ab^7x^3+b\right.\right.\\ &= -\frac{a^8}{9x^9}-\frac{4a^7b}{3x^6}-\frac{28a^6b^2}{3x^3}+\frac{70}{3}a^4b^4x^3+\frac{28}{3}a^3b^5x^6+\frac{28}{9}a^2b^6x^9+\frac{2}{3}ab^7x^{12}+\frac{b^8x^{15}}{15}+56a^5b^3\end{aligned}$$

Mathematica [A] time = 0.01, size = 105, normalized size = 1.00

$$-\frac{a^8}{9x^9}-\frac{4a^7b}{3x^6}-\frac{28a^6b^2}{3x^3}+56a^5b^3\log(x)+\frac{70}{3}a^4b^4x^3+\frac{28}{3}a^3b^5x^6+\frac{28}{9}a^2b^6x^9+\frac{2}{3}ab^7x^{12}+\frac{b^8x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^8/x^10,x]
```

```
[Out] -1/9*a^8/x^9 - (4*a^7*b)/(3*x^6) - (28*a^6*b^2)/(3*x^3) + (70*a^4*b^4*x^3)/
3 + (28*a^3*b^5*x^6)/3 + (28*a^2*b^6*x^9)/9 + (2*a*b^7*x^12)/3 + (b^8*x^15)
/15 + 56*a^5*b^3*Log[x]
```

fricas [A] time = 0.77, size = 94, normalized size = 0.90

$$\frac{3\,b^8x^{24}+30\,ab^7x^{21}+140\,a^2b^6x^{18}+420\,a^3b^5x^{15}+1050\,a^4b^4x^{12}+2520\,a^5b^3x^9\log(x)-420\,a^6b^2x^6-60\,a^7bx^3-5}{45\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^10,x, algorithm="fricas")
```

```
[Out] 1/45*(3*b^8*x^24 + 30*a*b^7*x^21 + 140*a^2*b^6*x^18 + 420*a^3*b^5*x^15 + 10
50*a^4*b^4*x^12 + 2520*a^5*b^3*x^9*log(x) - 420*a^6*b^2*x^6 - 60*a^7*b*x^3
 - 5*a^8)/x^9
```

giac [A] time = 0.15, size = 102, normalized size = 0.97

$$\frac{1}{15}\,b^8x^{15}+\frac{2}{3}\,ab^7x^{12}+\frac{28}{9}\,a^2b^6x^9+\frac{28}{3}\,a^3b^5x^6+\frac{70}{3}\,a^4b^4x^3+56\,a^5b^3\log\left(|x|\right)-\frac{308\,a^5b^3x^9+84\,a^6b^2x^6+12\,a^7bx^3+a^8}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^10,x, algorithm="giac")
```

```
[Out] 1/15*b^8*x^15 + 2/3*a*b^7*x^12 + 28/9*a^2*b^6*x^9 + 28/3*a^3*b^5*x^6 + 70/3
*a^4*b^4*x^3 + 56*a^5*b^3*log(abs(x)) - 1/9*(308*a^5*b^3*x^9 + 84*a^6*b^2*x
^6 + 12*a^7*b*x^3 + a^8)/x^9
```

maple [A] time = 0.01, size = 90, normalized size = 0.86

$$\frac{b^8x^{15}}{15}+\frac{2a\,b^7x^{12}}{3}+\frac{28a^2b^6x^9}{9}+\frac{28a^3b^5x^6}{3}+\frac{70a^4b^4x^3}{3}+56a^5b^3\ln(x)-\frac{28a^6b^2}{3x^3}-\frac{4a^7b}{3x^6}-\frac{a^8}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^8/x^10,x)
```

```
[Out] -1/9*a^8/x^9-4/3*a^7*b/x^6-28/3*a^6*b^2/x^3+70/3*a^4*b^4*x^3+28/3*a^3*b^5*x
^6+28/9*a^2*b^6*x^9+2/3*a*b^7*x^12+1/15*b^8*x^15+56*a^5*b^3*ln(x)
```

maxima [A] time = 1.30, size = 92, normalized size = 0.88

$$\frac{1}{15}\,b^8x^{15}+\frac{2}{3}\,ab^7x^{12}+\frac{28}{9}\,a^2b^6x^9+\frac{28}{3}\,a^3b^5x^6+\frac{70}{3}\,a^4b^4x^3+\frac{56}{3}\,a^5b^3\log\left(x^3\right)-\frac{84\,a^6b^2x^6+12\,a^7bx^3+a^8}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^10,x, algorithm="maxima")
```

```
[Out] 1/15*b^8*x^15 + 2/3*a*b^7*x^12 + 28/9*a^2*b^6*x^9 + 28/3*a^3*b^5*x^6 + 70/3
*a^4*b^4*x^3 + 56/3*a^5*b^3*log(x^3) - 1/9*(84*a^6*b^2*x^6 + 12*a^7*b*x^3 +
 a^8)/x^9
```

mupad [B] time = 0.96, size = 92, normalized size = 0.88

$$\frac{b^8\,x^{15}}{15}-\frac{\frac{a^8}{9}+\frac{4\,a^7\,b\,x^3}{3}+\frac{28\,a^6\,b^2\,x^6}{3}}{x^9}+\frac{2\,a\,b^7\,x^{12}}{3}+\frac{70\,a^4\,b^4\,x^3}{3}+\frac{28\,a^3\,b^5\,x^6}{3}+\frac{28\,a^2\,b^6\,x^9}{9}+56\,a^5\,b^3\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^8/x^10,x)
```

```
[Out] (b^8*x^15)/15 - (a^8/9 + (4*a^7*b*x^3)/3 + (28*a^6*b^2*x^6)/3)/x^9 + (2*a*b
^7*x^12)/3 + (70*a^4*b^4*x^3)/3 + (28*a^3*b^5*x^6)/3 + (28*a^2*b^6*x^9)/9 +
 56*a^5*b^3*log(x)
```

sympy [**A**] time = 0.47, size = 104, normalized size = 0.99

$$56a^5b^3\log(x)+\frac{70a^4b^4x^3}{3}+\frac{28a^3b^5x^6}{3}+\frac{28a^2b^6x^9}{9}+\frac{2ab^7x^{12}}{3}+\frac{b^8x^{15}}{15}+\frac{-a^8-12a^7bx^3-84a^6b^2x^6}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**8/x**10,x)
```

```
[Out] 56*a**5*b**3*log(x) + 70*a**4*b**4*x**3/3 + 28*a**3*b**5*x**6/3 + 28*a**2*b
**6*x**9/9 + 2*a*b**7*x**12/3 + b**8*x**15/15 + (-a**8 - 12*a**7*b*x**3 - 8
4*a**6*b**2*x**6)/(9*x**9)
```

3.296 $\int \frac{\left(a+bx^3\right)^8}{x^{13}}\,dx$

Optimal. Leaf size=105

$$-\frac{a^8}{12x^{12}}-\frac{8a^7b}{9x^9}-\frac{14a^6b^2}{3x^6}-\frac{56a^5b^3}{3x^3}+70a^4b^4\log(x)+\frac{56}{3}a^3b^5x^3+\frac{14}{3}a^2b^6x^6+\frac{8}{9}ab^7x^9+\frac{b^8x^{12}}{12}$$

```
[Out] -1/12*a^8/x^12-8/9*a^7*b/x^9-14/3*a^6*b^2/x^6-56/3*a^5*b^3/x^3+56/3*a^3*b^5
*x^3+14/3*a^2*b^6*x^6+8/9*a*b^7*x^9+1/12*b^8*x^12+70*a^4*b^4*ln(x)
```

Rubi [A] time = 0.06, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{14}{3}a^2b^6x^6+\frac{56}{3}a^3b^5x^3-\frac{56a^5b^3}{3x^3}-\frac{14a^6b^2}{3x^6}+70a^4b^4\log(x)-\frac{8a^7b}{9x^9}-\frac{a^8}{12x^{12}}+\frac{8}{9}ab^7x^9+\frac{b^8x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^8/x^13,x]
```

```
[Out] -a^8/(12*x^12) - (8*a^7*b)/(9*x^9) - (14*a^6*b^2)/(3*x^6) - (56*a^5*b^3)/(3
*x^3) + (56*a^3*b^5*x^3)/3 + (14*a^2*b^6*x^6)/3 + (8*a*b^7*x^9)/9 + (b^8*x^
12)/12 + 70*a^4*b^4*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^8}{x^{13}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)^8}{x^5}\,dx, x, x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(56a^3b^5+\frac{a^8}{x^5}+\frac{8a^7b}{x^4}+\frac{28a^6b^2}{x^3}+\frac{56a^5b^3}{x^2}+\frac{70a^4b^4}{x}+28a^2b^6x+8ab^7x^2+b^8x^3\right)\\ &= -\frac{a^8}{12x^{12}}-\frac{8a^7b}{9x^9}-\frac{14a^6b^2}{3x^6}-\frac{56a^5b^3}{3x^3}+\frac{56}{3}a^3b^5x^3+\frac{14}{3}a^2b^6x^6+\frac{8}{9}ab^7x^9+\frac{b^8x^{12}}{12}+70a^4b^4\log(\end{aligned}$$

Mathematica [A] time = 0.01, size = 105, normalized size = 1.00

$$-\frac{a^8}{12x^{12}}-\frac{8a^7b}{9x^9}-\frac{14a^6b^2}{3x^6}-\frac{56a^5b^3}{3x^3}+70a^4b^4\log(x)+\frac{56}{3}a^3b^5x^3+\frac{14}{3}a^2b^6x^6+\frac{8}{9}ab^7x^9+\frac{b^8x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^8/x^13,x]
```

```
[Out] -1/12*a^8/x^12 - (8*a^7*b)/(9*x^9) - (14*a^6*b^2)/(3*x^6) - (56*a^5*b^3)/(3
*x^3) + (56*a^3*b^5*x^3)/3 + (14*a^2*b^6*x^6)/3 + (8*a*b^7*x^9)/9 + (b^8*x^
12)/12 + 70*a^4*b^4*Log[x]
```

fricas [A] time = 0.85, size = 94, normalized size = 0.90

$$\frac{3\,b^8x^{24}+32\,ab^7x^{21}+168\,a^2b^6x^{18}+672\,a^3b^5x^{15}+2520\,a^4b^4x^{12}\log(x)-672\,a^5b^3x^9-168\,a^6b^2x^6-32\,a^7bx^3-}{36\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^13,x, algorithm="fricas")
```

```
[Out] 1/36*(3*b^8*x^24 + 32*a*b^7*x^21 + 168*a^2*b^6*x^18 + 672*a^3*b^5*x^15 + 25
20*a^4*b^4*x^12*log(x) - 672*a^5*b^3*x^9 - 168*a^6*b^2*x^6 - 32*a^7*b*x^3 -
 3*a^8)/x^12
```

giac [A] time = 0.15, size = 104, normalized size = 0.99

$$\frac{1}{12}\,b^8x^{12}+\frac{8}{9}\,ab^7x^9+\frac{14}{3}\,a^2b^6x^6+\frac{56}{3}\,a^3b^5x^3+70\,a^4b^4\log\left(|x|\right)-\frac{1750\,a^4b^4x^{12}+672\,a^5b^3x^9+168\,a^6b^2x^6+32\,a^7bx}{36\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^13,x, algorithm="giac")
```

```
[Out] 1/12*b^8*x^12 + 8/9*a*b^7*x^9 + 14/3*a^2*b^6*x^6 + 56/3*a^3*b^5*x^3 + 70*a^
4*b^4*log(abs(x)) - 1/36*(1750*a^4*b^4*x^12 + 672*a^5*b^3*x^9 + 168*a^6*b^2
*x^6 + 32*a^7*b*x^3 + 3*a^8)/x^12
```

maple [A] time = 0.01, size = 90, normalized size = 0.86

$$\frac{b^8x^{12}}{12}+\frac{8a\,b^7x^9}{9}+\frac{14a^2b^6x^6}{3}+\frac{56a^3b^5x^3}{3}+70a^4b^4\ln(x)-\frac{56a^5b^3}{3x^3}-\frac{14a^6b^2}{3x^6}-\frac{8a^7b}{9x^9}-\frac{a^8}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^8/x^13,x)
```

```
[Out] -1/12*a^8/x^12-8/9*a^7*b/x^9-14/3*a^6*b^2/x^6-56/3*a^5*b^3/x^3+56/3*a^3*b^5
*x^3+14/3*a^2*b^6*x^6+8/9*a*b^7*x^9+1/12*b^8*x^12+70*a^4*b^4*ln(x)
```

maxima [A] time = 1.31, size = 94, normalized size = 0.90

$$\frac{1}{12}\,b^8x^{12}+\frac{8}{9}\,ab^7x^9+\frac{14}{3}\,a^2b^6x^6+\frac{56}{3}\,a^3b^5x^3+\frac{70}{3}\,a^4b^4\log\left(x^3\right)-\frac{672\,a^5b^3x^9+168\,a^6b^2x^6+32\,a^7bx^3+3\,a^8}{36\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^13,x, algorithm="maxima")
```

```
[Out] 1/12*b^8*x^12 + 8/9*a*b^7*x^9 + 14/3*a^2*b^6*x^6 + 56/3*a^3*b^5*x^3 + 70/3*
a^4*b^4*log(x^3) - 1/36*(672*a^5*b^3*x^9 + 168*a^6*b^2*x^6 + 32*a^7*b*x^3 +
 3*a^8)/x^12
```

mupad [B] time = 0.96, size = 92, normalized size = 0.88

$$\frac{b^8\,x^{12}}{12}-\frac{\frac{a^8}{12}+\frac{8\,a^7\,b\,x^3}{9}+\frac{14\,a^6\,b^2\,x^6}{3}+\frac{56\,a^5\,b^3\,x^9}{3}}{x^{12}}+\frac{8\,a\,b^7\,x^9}{9}+\frac{56\,a^3\,b^5\,x^3}{3}+\frac{14\,a^2\,b^6\,x^6}{3}+70\,a^4\,b^4\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^8/x^13,x)
```

```
[Out] (b^8*x^12)/12 - (a^8/12 + (8*a^7*b*x^3)/9 + (14*a^6*b^2*x^6)/3 + (56*a^5*b^
3*x^9)/3)/x^12 + (8*a*b^7*x^9)/9 + (56*a^3*b^5*x^3)/3 + (14*a^2*b^6*x^6)/3
+ 70*a^4*b^4*log(x)
```

sympy [**A**] time = 0.54, size = 104, normalized size = 0.99

$$70a^4b^4\log(x)+\frac{56a^3b^5x^3}{3}+\frac{14a^2b^6x^6}{3}+\frac{8ab^7x^9}{9}+\frac{b^8x^{12}}{12}+\frac{-3a^8-32a^7bx^3-168a^6b^2x^6-672a^5b^3x^9}{36x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**8/x**13,x)
```

```
[Out] 70*a**4*b**4*log(x) + 56*a**3*b**5*x**3/3 + 14*a**2*b**6*x**6/3 + 8*a*b**7*
x**9/9 + b**8*x**12/12 + (-3*a**8 - 32*a**7*b*x**3 - 168*a**6*b**2*x**6 - 6
72*a**5*b**3*x**9)/(36*x**12)
```

3.297 $\int \frac{\left(a+bx^3\right)^8}{x^{16}}\,dx$

Optimal. Leaf size=105

$$-\frac{a^8}{15x^{15}}-\frac{2a^7b}{3x^{12}}-\frac{28a^6b^2}{9x^9}-\frac{28a^5b^3}{3x^6}-\frac{70a^4b^4}{3x^3}+56a^3b^5\log(x)+\frac{28}{3}a^2b^6x^3+\frac{4}{3}ab^7x^6+\frac{b^8x^9}{9}$$

```
[Out] -1/15*a^8/x^15-2/3*a^7*b/x^12-28/9*a^6*b^2/x^9-28/3*a^5*b^3/x^6-70/3*a^4*b^
4/x^3+28/3*a^2*b^6*x^3+4/3*a*b^7*x^6+1/9*b^8*x^9+56*a^3*b^5*ln(x)
```

Rubi [A] time = 0.06, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{28}{3}a^2b^6x^3-\frac{70a^4b^4}{3x^3}-\frac{28a^5b^3}{3x^6}-\frac{28a^6b^2}{9x^9}+56a^3b^5\log(x)-\frac{2a^7b}{3x^{12}}-\frac{a^8}{15x^{15}}+\frac{4}{3}ab^7x^6+\frac{b^8x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^8/x^16,x]
```

```
[Out] -a^8/(15*x^15) - (2*a^7*b)/(3*x^12) - (28*a^6*b^2)/(9*x^9) - (28*a^5*b^3)/(
3*x^6) - (70*a^4*b^4)/(3*x^3) + (28*a^2*b^6*x^3)/3 + (4*a*b^7*x^6)/3 + (b^8
*x^9)/9 + 56*a^3*b^5*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^8}{x^{16}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)^8}{x^6}\,dx, x, x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(28a^2b^6+\frac{a^8}{x^6}+\frac{8a^7b}{x^5}+\frac{28a^6b^2}{x^4}+\frac{56a^5b^3}{x^3}+\frac{70a^4b^4}{x^2}+\frac{56a^3b^5}{x}+8ab^7x+b^8x^2\right.\right.\\ &= -\frac{a^8}{15x^{15}}-\frac{2a^7b}{3x^{12}}-\frac{28a^6b^2}{9x^9}-\frac{28a^5b^3}{3x^6}-\frac{70a^4b^4}{3x^3}+\frac{28}{3}a^2b^6x^3+\frac{4}{3}ab^7x^6+\frac{b^8x^9}{9}+56a^3b^5\log(\end{aligned}$$

Mathematica [A] time = 0.01, size = 105, normalized size = 1.00

$$-\frac{a^8}{15x^{15}}-\frac{2a^7b}{3x^{12}}-\frac{28a^6b^2}{9x^9}-\frac{28a^5b^3}{3x^6}-\frac{70a^4b^4}{3x^3}+56a^3b^5\log(x)+\frac{28}{3}a^2b^6x^3+\frac{4}{3}ab^7x^6+\frac{b^8x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^8/x^16,x]
```

```
[Out] -1/15*a^8/x^15 - (2*a^7*b)/(3*x^12) - (28*a^6*b^2)/(9*x^9) - (28*a^5*b^3)/(
3*x^6) - (70*a^4*b^4)/(3*x^3) + (28*a^2*b^6*x^3)/3 + (4*a*b^7*x^6)/3 + (b^8
*x^9)/9 + 56*a^3*b^5*Log[x]
```

fricas [A] time = 0.53, size = 94, normalized size = 0.90

$$\frac{5\,b^8x^{24}+60\,ab^7x^{21}+420\,a^2b^6x^{18}+2520\,a^3b^5x^{15}\log(x)-1050\,a^4b^4x^{12}-420\,a^5b^3x^9-140\,a^6b^2x^6-30\,a^7bx^3-3}{45\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^16,x, algorithm="fricas")
```

```
[Out] 1/45*(5*b^8*x^24 + 60*a*b^7*x^21 + 420*a^2*b^6*x^18 + 2520*a^3*b^5*x^15*log
(x) - 1050*a^4*b^4*x^12 - 420*a^5*b^3*x^9 - 140*a^6*b^2*x^6 - 30*a^7*b*x^3
- 3*a^8)/x^15
```

giac [A] time = 0.17, size = 104, normalized size = 0.99

$$\frac{1}{9}\,b^8x^9+\frac{4}{3}\,ab^7x^6+\frac{28}{3}\,a^2b^6x^3+56\,a^3b^5\log\left(|x|\right)-\frac{1918\,a^3b^5x^{15}+1050\,a^4b^4x^{12}+420\,a^5b^3x^9+140\,a^6b^2x^6+30\,a^7bx^3}{45\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^16,x, algorithm="giac")
```

```
[Out] 1/9*b^8*x^9 + 4/3*a*b^7*x^6 + 28/3*a^2*b^6*x^3 + 56*a^3*b^5*log(abs(x)) - 1
/45*(1918*a^3*b^5*x^15 + 1050*a^4*b^4*x^12 + 420*a^5*b^3*x^9 + 140*a^6*b^2*
x^6 + 30*a^7*b*x^3 + 3*a^8)/x^15
```

maple [A] time = 0.01, size = 90, normalized size = 0.86

$$\frac{b^8x^9}{9}+\frac{4a\,b^7x^6}{3}+\frac{28a^2b^6x^3}{3}+56a^3b^5\ln(x)-\frac{70a^4b^4}{3x^3}-\frac{28a^5b^3}{3x^6}-\frac{28a^6b^2}{9x^9}-\frac{2a^7b}{3x^{12}}-\frac{a^8}{15x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^8/x^16,x)
```

```
[Out] -1/15*a^8/x^15-2/3*a^7*b/x^12-28/9*a^6*b^2/x^9-28/3*a^5*b^3/x^6-70/3*a^4*b^
4/x^3+28/3*a^2*b^6*x^3+4/3*a*b^7*x^6+1/9*b^8*x^9+56*a^3*b^5*ln(x)
```

maxima [A] time = 1.32, size = 94, normalized size = 0.90

$$\frac{1}{9}\,b^8x^9+\frac{4}{3}\,ab^7x^6+\frac{28}{3}\,a^2b^6x^3+\frac{56}{3}\,a^3b^5\log\left(x^3\right)-\frac{1050\,a^4b^4x^{12}+420\,a^5b^3x^9+140\,a^6b^2x^6+30\,a^7bx^3+3\,a^8}{45\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^16,x, algorithm="maxima")
```

```
[Out] 1/9*b^8*x^9 + 4/3*a*b^7*x^6 + 28/3*a^2*b^6*x^3 + 56/3*a^3*b^5*log(x^3) - 1/
45*(1050*a^4*b^4*x^12 + 420*a^5*b^3*x^9 + 140*a^6*b^2*x^6 + 30*a^7*b*x^3 +
3*a^8)/x^15
```

mupad [B] time = 0.05, size = 92, normalized size = 0.88

$$\frac{b^8\,x^9}{9}-\frac{\frac{a^8}{15}+\frac{2\,a^7\,b\,x^3}{3}+\frac{28\,a^6\,b^2\,x^6}{9}+\frac{28\,a^5\,b^3\,x^9}{3}+\frac{70\,a^4\,b^4\,x^{12}}{3}}{x^{15}}+\frac{4\,a\,b^7\,x^6}{3}+\frac{28\,a^2\,b^6\,x^3}{3}+56\,a^3\,b^5\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^8/x^16,x)
```

```
[Out] (b^8*x^9)/9 - (a^8/15 + (2*a^7*b*x^3)/3 + (28*a^6*b^2*x^6)/9 + (28*a^5*b^3*
x^9)/3 + (70*a^4*b^4*x^12)/3)/x^15 + (4*a*b^7*x^6)/3 + (28*a^2*b^6*x^3)/3 +
 56*a^3*b^5*log(x)
```

sympy [**A**] time = 0.81, size = 102, normalized size = 0.97

$$56a^3b^5\log(x)+\frac{28a^2b^6x^3}{3}+\frac{4ab^7x^6}{3}+\frac{b^8x^9}{9}+\frac{-3a^8-30a^7bx^3-140a^6b^2x^6-420a^5b^3x^9-1050a^4b^4x^{12}}{45x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**8/x**16,x)
```

```
[Out] 56*a**3*b**5*log(x) + 28*a**2*b**6*x**3/3 + 4*a*b**7*x**6/3 + b**8*x**9/9 +
 (-3*a**8 - 30*a**7*b*x**3 - 140*a**6*b**2*x**6 - 420*a**5*b**3*x**9 - 1050
*a**4*b**4*x**12)/(45*x**15)
```

3.298 $\int \frac{\left(a+bx^3\right)^8}{x^{19}}\,dx$

Optimal. Leaf size=105

$$-\frac{a^8}{18x^{18}}-\frac{8a^7b}{15x^{15}}-\frac{7a^6b^2}{3x^{12}}-\frac{56a^5b^3}{9x^9}-\frac{35a^4b^4}{3x^6}-\frac{56a^3b^5}{3x^3}+28a^2b^6\log(x)+\frac{8}{3}ab^7x^3+\frac{b^8x^6}{6}$$

```
[Out] -1/18*a^8/x^18-8/15*a^7*b/x^15-7/3*a^6*b^2/x^12-56/9*a^5*b^3/x^9-35/3*a^4*b
^4/x^6-56/3*a^3*b^5/x^3+8/3*a*b^7*x^3+1/6*b^8*x^6+28*a^2*b^6*ln(x)
```

Rubi [A] time = 0.06, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$-\frac{7a^6b^2}{3x^{12}}-\frac{56a^5b^3}{9x^9}-\frac{35a^4b^4}{3x^6}-\frac{56a^3b^5}{3x^3}+28a^2b^6\log(x)-\frac{8a^7b}{15x^{15}}-\frac{a^8}{18x^{18}}+\frac{8}{3}ab^7x^3+\frac{b^8x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^8/x^19,x]
```

```
[Out] -a^8/(18*x^18) - (8*a^7*b)/(15*x^15) - (7*a^6*b^2)/(3*x^12) - (56*a^5*b^3)/
(9*x^9) - (35*a^4*b^4)/(3*x^6) - (56*a^3*b^5)/(3*x^3) + (8*a*b^7*x^3)/3 + (
b^8*x^6)/6 + 28*a^2*b^6*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^8}{x^{19}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^8}{x^7}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(8ab^7+\frac{a^8}{x^7}+\frac{8a^7b}{x^6}+\frac{28a^6b^2}{x^5}+\frac{56a^5b^3}{x^4}+\frac{70a^4b^4}{x^3}+\frac{56a^3b^5}{x^2}+\frac{28a^2b^6}{x}+b^8x\right)dx\right.\\ &= -\frac{a^8}{18x^{18}}-\frac{8a^7b}{15x^{15}}-\frac{7a^6b^2}{3x^{12}}-\frac{56a^5b^3}{9x^9}-\frac{35a^4b^4}{3x^6}-\frac{56a^3b^5}{3x^3}+\frac{8}{3}ab^7x^3+\frac{b^8x^6}{6}+28a^2b^6\log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 105, normalized size = 1.00

$$-\frac{a^8}{18x^{18}}-\frac{8a^7b}{15x^{15}}-\frac{7a^6b^2}{3x^{12}}-\frac{56a^5b^3}{9x^9}-\frac{35a^4b^4}{3x^6}-\frac{56a^3b^5}{3x^3}+28a^2b^6\log(x)+\frac{8}{3}ab^7x^3+\frac{b^8x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^8/x^19,x]
```

```
[Out] -1/18*a^8/x^18 - (8*a^7*b)/(15*x^15) - (7*a^6*b^2)/(3*x^12) - (56*a^5*b^3)/
(9*x^9) - (35*a^4*b^4)/(3*x^6) - (56*a^3*b^5)/(3*x^3) + (8*a*b^7*x^3)/3 + (
b^8*x^6)/6 + 28*a^2*b^6*Log[x]
```

fricas [A] time = 0.66, size = 94, normalized size = 0.90

$$\frac{15\,b^8x^{24}+240\,ab^7x^{21}+2520\,a^2b^6x^{18}\log(x)-1680\,a^3b^5x^{15}-1050\,a^4b^4x^{12}-560\,a^5b^3x^9-210\,a^6b^2x^6-48\,a^7}{90\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^19,x, algorithm="fricas")
```

```
[Out] 1/90*(15*b^8*x^24 + 240*a*b^7*x^21 + 2520*a^2*b^6*x^18*log(x) - 1680*a^3*b^
5*x^15 - 1050*a^4*b^4*x^12 - 560*a^5*b^3*x^9 - 210*a^6*b^2*x^6 - 48*a^7*b*x
^3 - 5*a^8)/x^18
```

giac [A] time = 0.16, size = 104, normalized size = 0.99

$$\frac{1}{6}\,b^8x^6+\frac{8}{3}\,ab^7x^3+28\,a^2b^6\log\left(|x|\right)-\frac{2058\,a^2b^6x^{18}+1680\,a^3b^5x^{15}+1050\,a^4b^4x^{12}+560\,a^5b^3x^9+210\,a^6b^2x^6+4}{90\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^19,x, algorithm="giac")
```

```
[Out] 1/6*b^8*x^6 + 8/3*a*b^7*x^3 + 28*a^2*b^6*log(abs(x)) - 1/90*(2058*a^2*b^6*x
^18 + 1680*a^3*b^5*x^15 + 1050*a^4*b^4*x^12 + 560*a^5*b^3*x^9 + 210*a^6*b^2
*x^6 + 48*a^7*b*x^3 + 5*a^8)/x^18
```

maple [A] time = 0.01, size = 90, normalized size = 0.86

$$\frac{b^8x^6}{6}+\frac{8a\,b^7x^3}{3}+28a^2b^6\ln(x)-\frac{56a^3b^5}{3x^3}-\frac{35a^4b^4}{3x^6}-\frac{56a^5b^3}{9x^9}-\frac{7a^6b^2}{3x^{12}}-\frac{8a^7b}{15x^{15}}-\frac{a^8}{18x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^8/x^19,x)
```

```
[Out] -1/18*a^8/x^18-8/15*a^7*b/x^15-7/3*a^6*b^2/x^12-56/9*a^5*b^3/x^9-35/3*a^4*b
^4/x^6-56/3*a^3*b^5/x^3+8/3*a*b^7*x^3+1/6*b^8*x^6+28*a^2*b^6*ln(x)
```

maxima [A] time = 1.36, size = 94, normalized size = 0.90

$$\frac{1}{6}\,b^8x^6+\frac{8}{3}\,ab^7x^3+\frac{28}{3}\,a^2b^6\log\left(x^3\right)-\frac{1680\,a^3b^5x^{15}+1050\,a^4b^4x^{12}+560\,a^5b^3x^9+210\,a^6b^2x^6+48\,a^7bx^3+5\,a^8}{90\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^19,x, algorithm="maxima")
```

```
[Out] 1/6*b^8*x^6 + 8/3*a*b^7*x^3 + 28/3*a^2*b^6*log(x^3) - 1/90*(1680*a^3*b^5*x^
15 + 1050*a^4*b^4*x^12 + 560*a^5*b^3*x^9 + 210*a^6*b^2*x^6 + 48*a^7*b*x^3 +
 5*a^8)/x^18
```

mupad [B] time = 0.98, size = 92, normalized size = 0.88

$$\frac{b^8\,x^6}{6}-\frac{\frac{a^8}{18}+\frac{8\,a^7\,b\,x^3}{15}+\frac{7\,a^6\,b^2\,x^6}{3}+\frac{56\,a^5\,b^3\,x^9}{9}+\frac{35\,a^4\,b^4\,x^{12}}{3}+\frac{56\,a^3\,b^5\,x^{15}}{3}}{x^{18}}+\frac{8\,a\,b^7\,x^3}{3}+28\,a^2\,b^6\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^8/x^19,x)
```

```
[Out] (b^8*x^6)/6 - (a^8/18 + (8*a^7*b*x^3)/15 + (7*a^6*b^2*x^6)/3 + (56*a^5*b^3*
x^9)/9 + (35*a^4*b^4*x^12)/3 + (56*a^3*b^5*x^15)/3)/x^18 + (8*a*b^7*x^3)/3
+ 28*a^2*b^6*log(x)
```

sympy [**A**] time = 0.83, size = 100, normalized size = 0.95

$$28a^2b^6\log(x)+\frac{8ab^7x^3}{3}+\frac{b^8x^6}{6}+\frac{-5a^8-48a^7bx^3-210a^6b^2x^6-560a^5b^3x^9-1050a^4b^4x^{12}-1680a^3b^5x^{15}}{90x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**8/x**19,x)
```

```
[Out] 28*a**2*b**6*log(x) + 8*a*b**7*x**3/3 + b**8*x**6/6 + (-5*a**8 - 48*a**7*b*
x**3 - 210*a**6*b**2*x**6 - 560*a**5*b**3*x**9 - 1050*a**4*b**4*x**12 - 168
0*a**3*b**5*x**15)/(90*x**18)
```

3.299 $\int \frac{\left(a+bx^3\right)^8}{x^{22}}\,dx$

Optimal. Leaf size=105

$$-\frac{a^8}{21x^{21}}-\frac{4a^7b}{9x^{18}}-\frac{28a^6b^2}{15x^{15}}-\frac{14a^5b^3}{3x^{12}}-\frac{70a^4b^4}{9x^9}-\frac{28a^3b^5}{3x^6}-\frac{28a^2b^6}{3x^3}+8ab^7\log(x)+\frac{b^8x^3}{3}$$

```
[Out] -1/21*a^8/x^21-4/9*a^7*b/x^18-28/15*a^6*b^2/x^15-14/3*a^5*b^3/x^12-70/9*a^4
*b^4/x^9-28/3*a^3*b^5/x^6-28/3*a^2*b^6/x^3+1/3*b^8*x^3+8*a*b^7*ln(x)
```

Rubi [A] time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$-\frac{28a^6b^2}{15x^{15}}-\frac{14a^5b^3}{3x^{12}}-\frac{70a^4b^4}{9x^9}-\frac{28a^3b^5}{3x^6}-\frac{28a^2b^6}{3x^3}-\frac{4a^7b}{9x^{18}}-\frac{a^8}{21x^{21}}+8ab^7\log(x)+\frac{b^8x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^8/x^22,x]
```

```
[Out] -a^8/(21*x^21) - (4*a^7*b)/(9*x^18) - (28*a^6*b^2)/(15*x^15) - (14*a^5*b^3)
/(3*x^12) - (70*a^4*b^4)/(9*x^9) - (28*a^3*b^5)/(3*x^6) - (28*a^2*b^6)/(3*x
^3) + (b^8*x^3)/3 + 8*a*b^7*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^8}{x^{22}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^8}{x^8}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(b^8+\frac{a^8}{x^8}+\frac{8a^7b}{x^7}+\frac{28a^6b^2}{x^6}+\frac{56a^5b^3}{x^5}+\frac{70a^4b^4}{x^4}+\frac{56a^3b^5}{x^3}+\frac{28a^2b^6}{x^2}+\frac{8ab^7}{x}\right)\right.\\ &= -\frac{a^8}{21x^{21}}-\frac{4a^7b}{9x^{18}}-\frac{28a^6b^2}{15x^{15}}-\frac{14a^5b^3}{3x^{12}}-\frac{70a^4b^4}{9x^9}-\frac{28a^3b^5}{3x^6}-\frac{28a^2b^6}{3x^3}+\frac{b^8x^3}{3}+8ab^7\log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 105, normalized size = 1.00

$$-\frac{a^8}{21x^{21}}-\frac{4a^7b}{9x^{18}}-\frac{28a^6b^2}{15x^{15}}-\frac{14a^5b^3}{3x^{12}}-\frac{70a^4b^4}{9x^9}-\frac{28a^3b^5}{3x^6}-\frac{28a^2b^6}{3x^3}+8ab^7\log(x)+\frac{b^8x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^8/x^22,x]
```

```
[Out] -1/21*a^8/x^21 - (4*a^7*b)/(9*x^18) - (28*a^6*b^2)/(15*x^15) - (14*a^5*b^3)
/(3*x^12) - (70*a^4*b^4)/(9*x^9) - (28*a^3*b^5)/(3*x^6) - (28*a^2*b^6)/(3*x
^3) + (b^8*x^3)/3 + 8*a*b^7*Log[x]
```

fricas [A] time = 0.68, size = 94, normalized size = 0.90

$$\frac{105\,b^8x^{24}+2520\,ab^7x^{21}\log(x)-2940\,a^2b^6x^{18}-2940\,a^3b^5x^{15}-2450\,a^4b^4x^{12}-1470\,a^5b^3x^9-588\,a^6b^2x^6-140\,a}{315\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^22,x, algorithm="fricas")
```

```
[Out] 1/315*(105*b^8*x^24 + 2520*a*b^7*x^21*log(x) - 2940*a^2*b^6*x^18 - 2940*a^3
*b^5*x^15 - 2450*a^4*b^4*x^12 - 1470*a^5*b^3*x^9 - 588*a^6*b^2*x^6 - 140*a^
7*b*x^3 - 15*a^8)/x^21
```

giac [A] time = 0.16, size = 102, normalized size = 0.97

$$\frac{1}{3}\,b^8x^3+8\,ab^7\log\left(|x|\right)-\frac{2178\,ab^7x^{21}+2940\,a^2b^6x^{18}+2940\,a^3b^5x^{15}+2450\,a^4b^4x^{12}+1470\,a^5b^3x^9+588\,a^6b^2x^6+}{315\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^22,x, algorithm="giac")
```

```
[Out] 1/3*b^8*x^3 + 8*a*b^7*log(abs(x)) - 1/315*(2178*a*b^7*x^21 + 2940*a^2*b^6*x
^18 + 2940*a^3*b^5*x^15 + 2450*a^4*b^4*x^12 + 1470*a^5*b^3*x^9 + 588*a^6*b^
2*x^6 + 140*a^7*b*x^3 + 15*a^8)/x^21
```

maple [A] time = 0.01, size = 90, normalized size = 0.86

$$\frac{b^8x^3}{3}+8a\,b^7\ln(x)-\frac{28a^2b^6}{3x^3}-\frac{28a^3b^5}{3x^6}-\frac{70a^4b^4}{9x^9}-\frac{14a^5b^3}{3x^{12}}-\frac{28a^6b^2}{15x^{15}}-\frac{4a^7b}{9x^{18}}-\frac{a^8}{21x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^8/x^22,x)
```

```
[Out] -1/21*a^8/x^21-4/9*a^7*b/x^18-28/15*a^6*b^2/x^15-14/3*a^5*b^3/x^12-70/9*a^4
*b^4/x^9-28/3*a^3*b^5/x^6-28/3*a^2*b^6/x^3+1/3*b^8*x^3+8*a*b^7*ln(x)
```

maxima [A] time = 1.39, size = 94, normalized size = 0.90

$$\frac{1}{3}\,b^8x^3+\frac{8}{3}\,ab^7\log\left(x^3\right)-\frac{2940\,a^2b^6x^{18}+2940\,a^3b^5x^{15}+2450\,a^4b^4x^{12}+1470\,a^5b^3x^9+588\,a^6b^2x^6+140\,a^7bx^3+1}{315\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^22,x, algorithm="maxima")
```

```
[Out] 1/3*b^8*x^3 + 8/3*a*b^7*log(x^3) - 1/315*(2940*a^2*b^6*x^18 + 2940*a^3*b^5*
x^15 + 2450*a^4*b^4*x^12 + 1470*a^5*b^3*x^9 + 588*a^6*b^2*x^6 + 140*a^7*b*x
^3 + 15*a^8)/x^21
```

mupad [B] time = 0.99, size = 94, normalized size = 0.90

$$-\frac{15\,a^8-105\,b^8\,x^{24}+140\,a^7\,b\,x^3+588\,a^6\,b^2\,x^6+1470\,a^5\,b^3\,x^9+2450\,a^4\,b^4\,x^{12}+2940\,a^3\,b^5\,x^{15}+2940\,a^2\,b^6\,x^{18}+}{315\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^8/x^22,x)
```

```
[Out] -(15*a^8 - 105*b^8*x^24 + 140*a^7*b*x^3 + 588*a^6*b^2*x^6 + 1470*a^5*b^3*x^
9 + 2450*a^4*b^4*x^12 + 2940*a^3*b^5*x^15 + 2940*a^2*b^6*x^18 - 2520*a*b^7*
x^21*log(x))/(315*x^21)
```

sympy [**A**] time = 1.18, size = 99, normalized size = 0.94

$$8ab^7\log(x)+\frac{b^8x^3}{3}+\frac{-15a^8-140a^7bx^3-588a^6b^2x^6-1470a^5b^3x^9-2450a^4b^4x^{12}-2940a^3b^5x^{15}-2940a^2b^6x^1}{315x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**8/x**22,x)
```

```
[Out] 8*a*b**7*log(x) + b**8*x**3/3 + (-15*a**8 - 140*a**7*b*x**3 - 588*a**6*b**2
*x**6 - 1470*a**5*b**3*x**9 - 2450*a**4*b**4*x**12 - 2940*a**3*b**5*x**15 -
 2940*a**2*b**6*x**18)/(315*x**21)
```

3.300 $\int \frac{\left(a+bx^3\right)^8}{x^{25}}\,dx$

Optimal. Leaf size=104

$$-\frac{a^8}{24x^{24}}-\frac{8a^7b}{21x^{21}}-\frac{14a^6b^2}{9x^{18}}-\frac{56a^5b^3}{15x^{15}}-\frac{35a^4b^4}{6x^{12}}-\frac{56a^3b^5}{9x^9}-\frac{14a^2b^6}{3x^6}-\frac{8ab^7}{3x^3}+b^8\log(x)$$

```
[Out] -1/24*a^8/x^24-8/21*a^7*b/x^21-14/9*a^6*b^2/x^18-56/15*a^5*b^3/x^15-35/6*a^
4*b^4/x^12-56/9*a^3*b^5/x^9-14/3*a^2*b^6/x^6-8/3*a*b^7/x^3+b^8*ln(x)
```

Rubi [A] time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$-\frac{14a^6b^2}{9x^{18}}-\frac{56a^5b^3}{15x^{15}}-\frac{35a^4b^4}{6x^{12}}-\frac{56a^3b^5}{9x^9}-\frac{14a^2b^6}{3x^6}-\frac{8a^7b}{21x^{21}}-\frac{a^8}{24x^{24}}-\frac{8ab^7}{3x^3}+b^8\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^8/x^25,x]
```

```
[Out] -a^8/(24*x^24) - (8*a^7*b)/(21*x^21) - (14*a^6*b^2)/(9*x^18) - (56*a^5*b^3)
/(15*x^15) - (35*a^4*b^4)/(6*x^12) - (56*a^3*b^5)/(9*x^9) - (14*a^2*b^6)/(3
*x^6) - (8*a*b^7)/(3*x^3) + b^8*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^8}{x^{25}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)^8}{x^9}\,dx, x, x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{a^8}{x^9}+\frac{8a^7b}{x^8}+\frac{28a^6b^2}{x^7}+\frac{56a^5b^3}{x^6}+\frac{70a^4b^4}{x^5}+\frac{56a^3b^5}{x^4}+\frac{28a^2b^6}{x^3}+\frac{8ab^7}{x^2}+\frac{b^8}{x}\right)dx\right.\\ &= -\frac{a^8}{24x^{24}}-\frac{8a^7b}{21x^{21}}-\frac{14a^6b^2}{9x^{18}}-\frac{56a^5b^3}{15x^{15}}-\frac{35a^4b^4}{6x^{12}}-\frac{56a^3b^5}{9x^9}-\frac{14a^2b^6}{3x^6}-\frac{8ab^7}{3x^3}+b^8\log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 104, normalized size = 1.00

$$-\frac{a^8}{24x^{24}}-\frac{8a^7b}{21x^{21}}-\frac{14a^6b^2}{9x^{18}}-\frac{56a^5b^3}{15x^{15}}-\frac{35a^4b^4}{6x^{12}}-\frac{56a^3b^5}{9x^9}-\frac{14a^2b^6}{3x^6}-\frac{8ab^7}{3x^3}+b^8\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^8/x^25,x]
```

```
[Out] -1/24*a^8/x^24 - (8*a^7*b)/(21*x^21) - (14*a^6*b^2)/(9*x^18) - (56*a^5*b^3)
/(15*x^15) - (35*a^4*b^4)/(6*x^12) - (56*a^3*b^5)/(9*x^9) - (14*a^2*b^6)/(3
*x^6) - (8*a*b^7)/(3*x^3) + b^8*Log[x]
```

fricas [A] time = 0.54, size = 94, normalized size = 0.90

$$\frac{2520\,b^8x^{24}\log(x) - 6720\,ab^7x^{21} - 11760\,a^2b^6x^{18} - 15680\,a^3b^5x^{15} - 14700\,a^4b^4x^{12} - 9408\,a^5b^3x^9 - 3920\,a^6b^2x}{2520\,x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^25,x, algorithm="fricas")
```

```
[Out] 1/2520*(2520*b^8*x^24*log(x) - 6720*a*b^7*x^21 - 11760*a^2*b^6*x^18 - 15680
*a^3*b^5*x^15 - 14700*a^4*b^4*x^12 - 9408*a^5*b^3*x^9 - 3920*a^6*b^2*x^6 -
960*a^7*b*x^3 - 105*a^8)/x^24
```

giac [A] time = 0.15, size = 100, normalized size = 0.96

$$b^8\log(|x|) - \frac{2283\,b^8x^{24} + 6720\,ab^7x^{21} + 11760\,a^2b^6x^{18} + 15680\,a^3b^5x^{15} + 14700\,a^4b^4x^{12} + 9408\,a^5b^3x^9 + 392}{2520\,x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^25,x, algorithm="giac")
```

```
[Out] b^8*log(abs(x)) - 1/2520*(2283*b^8*x^24 + 6720*a*b^7*x^21 + 11760*a^2*b^6*x
^18 + 15680*a^3*b^5*x^15 + 14700*a^4*b^4*x^12 + 9408*a^5*b^3*x^9 + 3920*a^6
*b^2*x^6 + 960*a^7*b*x^3 + 105*a^8)/x^24
```

maple [A] time = 0.01, size = 89, normalized size = 0.86

$$b^8\ln(x) - \frac{8a\,b^7}{3x^3} - \frac{14a^2b^6}{3x^6} - \frac{56a^3b^5}{9x^9} - \frac{35a^4b^4}{6x^{12}} - \frac{56a^5b^3}{15x^{15}} - \frac{14a^6b^2}{9x^{18}} - \frac{8a^7b}{21x^{21}} - \frac{a^8}{24x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^8/x^25,x)
```

```
[Out] -1/24*a^8/x^24-8/21*a^7*b/x^21-14/9*a^6*b^2/x^18-56/15*a^5*b^3/x^15-35/6*a^
4*b^4/x^12-56/9*a^3*b^5/x^9-14/3*a^2*b^6/x^6-8/3*a*b^7/x^3+b^8*ln(x)
```

maxima [A] time = 1.33, size = 94, normalized size = 0.90

$$\frac{1}{3}\,b^8\log\left(x^3\right) - \frac{6720\,ab^7x^{21} + 11760\,a^2b^6x^{18} + 15680\,a^3b^5x^{15} + 14700\,a^4b^4x^{12} + 9408\,a^5b^3x^9 + 3920\,a^6b^2x^6 + 9}{2520\,x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^25,x, algorithm="maxima")
```

```
[Out] 1/3*b^8*log(x^3) - 1/2520*(6720*a*b^7*x^21 + 11760*a^2*b^6*x^18 + 15680*a^3
*b^5*x^15 + 14700*a^4*b^4*x^12 + 9408*a^5*b^3*x^9 + 3920*a^6*b^2*x^6 + 960*
a^7*b*x^3 + 105*a^8)/x^24
```

mupad [B] time = 0.07, size = 91, normalized size = 0.88

$$b^8\,\ln(x) - \frac{\frac{a^8}{24} + \frac{8\,a^7\,b\,x^3}{21} + \frac{14\,a^6\,b^2\,x^6}{9} + \frac{56\,a^5\,b^3\,x^9}{15} + \frac{35\,a^4\,b^4\,x^{12}}{6} + \frac{56\,a^3\,b^5\,x^{15}}{9} + \frac{14\,a^2\,b^6\,x^{18}}{3} + \frac{8\,a\,b^7\,x^{21}}{3}}{x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^8/x^25,x)
```

```
[Out] b^8*log(x) - (a^8/24 + (8*a^7*b*x^3)/21 + (8*a*b^7*x^21)/3 + (14*a^6*b^2*x^
6)/9 + (56*a^5*b^3*x^9)/15 + (35*a^4*b^4*x^12)/6 + (56*a^3*b^5*x^15)/9 + (1
4*a^2*b^6*x^18)/3)/x^24
```

sympy [**A**] time = 1.20, size = 97, normalized size = 0.93

$$b^8 \log(x) + \frac{-105a^8 - 960a^7bx^3 - 3920a^6b^2x^6 - 9408a^5b^3x^9 - 14700a^4b^4x^{12} - 15680a^3b^5x^{15} - 11760a^2b^6x^{18} - 67}{2520x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**8/x**25,x)
```

```
[Out] b**8*log(x) + (-105*a**8 - 960*a**7*b*x**3 - 3920*a**6*b**2*x**6 - 9408*a**
5*b**3*x**9 - 14700*a**4*b**4*x**12 - 15680*a**3*b**5*x**15 - 11760*a**2*b*
*6*x**18 - 6720*a*b**7*x**21)/(2520*x**24)
```

3.301 $\int \frac{\left(a+bx^3\right)^8}{x^{28}}\,dx$

Optimal. Leaf size=19

$$-\frac{\left(a+bx^3\right)^9}{27ax^{27}}$$

```
[Out] -1/27*(b*x^3+a)^9/a/x^27
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {264}

$$-\frac{\left(a+bx^3\right)^9}{27ax^{27}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^8/x^28,x]
```

```
[Out] -(a + b*x^3)^9/(27*a*x^27)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^8}{x^{28}}\,dx = -\frac{\left(a+bx^3\right)^9}{27ax^{27}}$$

Mathematica [B] time = 0.01, size = 108, normalized size = 5.68

$$-\frac{a^8}{27x^{27}} - \frac{a^7 b}{3x^{24}} - \frac{4a^6b^2}{3x^{21}} - \frac{28a^5b^3}{9x^{18}} - \frac{14a^4b^4}{3x^{15}} - \frac{14a^3b^5}{3x^{12}} - \frac{28a^2b^6}{9x^9} - \frac{4ab^7}{3x^6} - \frac{b^8}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^8/x^28,x]
```

```
[Out] -1/27*a^8/x^27 - (a^7*b)/(3*x^24) - (4*a^6*b^2)/(3*x^21) - (28*a^5*b^3)/(9*
x^18) - (14*a^4*b^4)/(3*x^15) - (14*a^3*b^5)/(3*x^12) - (28*a^2*b^6)/(9*x^9
) - (4*a*b^7)/(3*x^6) - b^8/(3*x^3)
```

fricas [B] time = 0.69, size = 90, normalized size = 4.74

$$-\frac{9\,b^8x^{24} + 36\,ab^7x^{21} + 84\,a^2b^6x^{18} + 126\,a^3b^5x^{15} + 126\,a^4b^4x^{12} + 84\,a^5b^3x^9 + 36\,a^6b^2x^6 + 9\,a^7bx^3 + a^8}{27\,x^{27}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^28,x, algorithm="fricas")
```

```
[Out] -1/27*(9*b^8*x^24 + 36*a*b^7*x^21 + 84*a^2*b^6*x^18 + 126*a^3*b^5*x^15 + 12
6*a^4*b^4*x^12 + 84*a^5*b^3*x^9 + 36*a^6*b^2*x^6 + 9*a^7*b*x^3 + a^8)/x^27
```

giac [B] time = 0.15, size = 90, normalized size = 4.74

$$-\frac{9\,b^8x^{24}+36\,ab^7x^{21}+84\,a^2b^6x^{18}+126\,a^3b^5x^{15}+126\,a^4b^4x^{12}+84\,a^5b^3x^9+36\,a^6b^2x^6+9\,a^7bx^3+a^8}{27\,x^{27}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^28,x, algorithm="giac")
```

```
[Out] -1/27*(9*b^8*x^24 + 36*a*b^7*x^21 + 84*a^2*b^6*x^18 + 126*a^3*b^5*x^15 + 12
6*a^4*b^4*x^12 + 84*a^5*b^3*x^9 + 36*a^6*b^2*x^6 + 9*a^7*b*x^3 + a^8)/x^27
```

maple [B] time = 0.01, size = 91, normalized size = 4.79

$$-\frac{b^8}{3x^3}-\frac{4a\,b^7}{3x^6}-\frac{28a^2b^6}{9x^9}-\frac{14a^3b^5}{3x^{12}}-\frac{14a^4b^4}{3x^{15}}-\frac{28a^5b^3}{9x^{18}}-\frac{4a^6b^2}{3x^{21}}-\frac{a^7b}{3x^{24}}-\frac{a^8}{27x^{27}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^8/x^28,x)
```

```
[Out] -1/3*a^7*b/x^24-1/3*b^8/x^3-28/9*a^5*b^3/x^18-4/3*a^6*b^2/x^21-1/27*a^8/x^2
7-4/3*a*b^7/x^6-14/3*a^4*b^4/x^15-28/9*a^2*b^6/x^9-14/3*a^3*b^5/x^12
```

maxima [B] time = 1.35, size = 90, normalized size = 4.74

$$-\frac{9\,b^8x^{24}+36\,ab^7x^{21}+84\,a^2b^6x^{18}+126\,a^3b^5x^{15}+126\,a^4b^4x^{12}+84\,a^5b^3x^9+36\,a^6b^2x^6+9\,a^7bx^3+a^8}{27\,x^{27}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^28,x, algorithm="maxima")
```

```
[Out] -1/27*(9*b^8*x^24 + 36*a*b^7*x^21 + 84*a^2*b^6*x^18 + 126*a^3*b^5*x^15 + 12
6*a^4*b^4*x^12 + 84*a^5*b^3*x^9 + 36*a^6*b^2*x^6 + 9*a^7*b*x^3 + a^8)/x^27
```

mupad [B] time = 1.00, size = 92, normalized size = 4.84

$$-\frac{\frac{a^8}{27}+\frac{a^7\,b\,x^3}{3}+\frac{4\,a^6\,b^2\,x^6}{3}+\frac{28\,a^5\,b^3\,x^9}{9}+\frac{14\,a^4\,b^4\,x^{12}}{3}+\frac{14\,a^3\,b^5\,x^{15}}{3}+\frac{28\,a^2\,b^6\,x^{18}}{9}+\frac{4\,a\,b^7\,x^{21}}{3}+\frac{b^8\,x^{24}}{3}}{x^{27}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^8/x^28,x)
```

```
[Out] -(a^8/27 + (b^8*x^24)/3 + (a^7*b*x^3)/3 + (4*a*b^7*x^21)/3 + (4*a^6*b^2*x^6
)/3 + (28*a^5*b^3*x^9)/9 + (14*a^4*b^4*x^12)/3 + (14*a^3*b^5*x^15)/3 + (28*
a^2*b^6*x^18)/9)/x^27
```

sympy [B] time = 2.46, size = 97, normalized size = 5.11

$$\frac{-a^8-9a^7bx^3-36a^6b^2x^6-84a^5b^3x^9-126a^4b^4x^{12}-126a^3b^5x^{15}-84a^2b^6x^{18}-36ab^7x^{21}-9b^8x^{24}}{27x^{27}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**8/x**28,x)
```

```
[Out] (-a**8 - 9*a**7*b*x**3 - 36*a**6*b**2*x**6 - 84*a**5*b**3*x**9 - 126*a**4*b
**4*x**12 - 126*a**3*b**5*x**15 - 84*a**2*b**6*x**18 - 36*a*b**7*x**21 - 9*
b**8*x**24)/(27*x**27)
```

3.302 $\int \frac{\left(a+bx^3\right)^8}{x^{31}}\,dx$

Optimal. Leaf size=40

$$\frac{b\left(a+bx^3\right)^9}{270a^2x^{27}}-\frac{\left(a+bx^3\right)^9}{30ax^{30}}$$

```
[Out] -1/30*(b*x^3+a)^9/a/x^30+1/270*b*(b*x^3+a)^9/a^2/x^27
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {266, 45, 37}

$$\frac{b\left(a+bx^3\right)^9}{270a^2x^{27}}-\frac{\left(a+bx^3\right)^9}{30ax^{30}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^8/x^31,x]
```

```
[Out] -(a + b*x^3)^9/(30*a*x^30) + (b*(a + b*x^3)^9)/(270*a^2*x^27)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^8}{x^{31}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^8}{x^{11}}\,dx, x, x^3\right)\\ &= -\frac{\left(a+bx^3\right)^9}{30ax^{30}}-\frac{b\,\text{Subst}\left(\int \frac{(a+bx)^8}{x^{10}}\,dx, x, x^3\right)}{30a}\\ &= -\frac{\left(a+bx^3\right)^9}{30ax^{30}}+\frac{b\left(a+bx^3\right)^9}{270a^2x^{27}}\end{aligned}$$

Mathematica [B] time = 0.00, size = 108, normalized size = 2.70

$$-\frac{a^8}{30x^{30}}-\frac{8a^7b}{27x^{27}}-\frac{7a^6b^2}{6x^{24}}-\frac{8a^5b^3}{3x^{21}}-\frac{35a^4b^4}{9x^{18}}-\frac{56a^3b^5}{15x^{15}}-\frac{7a^2b^6}{3x^{12}}-\frac{8ab^7}{9x^9}-\frac{b^8}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^8/x^31,x]
```

```
[Out] -1/30*a^8/x^30 - (8*a^7*b)/(27*x^27) - (7*a^6*b^2)/(6*x^24) - (8*a^5*b^3)/(
3*x^21) - (35*a^4*b^4)/(9*x^18) - (56*a^3*b^5)/(15*x^15) - (7*a^2*b^6)/(3*x
^12) - (8*a*b^7)/(9*x^9) - b^8/(6*x^6)
```

fricas [B] time = 0.63, size = 92, normalized size = 2.30

$$-\frac{45\,b^8x^{24}+240\,ab^7x^{21}+630\,a^2b^6x^{18}+1008\,a^3b^5x^{15}+1050\,a^4b^4x^{12}+720\,a^5b^3x^9+315\,a^6b^2x^6+80\,a^7bx^3+9\,a}{270\,x^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^31,x, algorithm="fricas")
```

```
[Out] -1/270*(45*b^8*x^24 + 240*a*b^7*x^21 + 630*a^2*b^6*x^18 + 1008*a^3*b^5*x^15
 + 1050*a^4*b^4*x^12 + 720*a^5*b^3*x^9 + 315*a^6*b^2*x^6 + 80*a^7*b*x^3 + 9
*a^8)/x^30
```

giac [B] time = 0.15, size = 92, normalized size = 2.30

$$-\frac{45\,b^8x^{24}+240\,ab^7x^{21}+630\,a^2b^6x^{18}+1008\,a^3b^5x^{15}+1050\,a^4b^4x^{12}+720\,a^5b^3x^9+315\,a^6b^2x^6+80\,a^7bx^3+9\,a}{270\,x^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^31,x, algorithm="giac")
```

```
[Out] -1/270*(45*b^8*x^24 + 240*a*b^7*x^21 + 630*a^2*b^6*x^18 + 1008*a^3*b^5*x^15
 + 1050*a^4*b^4*x^12 + 720*a^5*b^3*x^9 + 315*a^6*b^2*x^6 + 80*a^7*b*x^3 + 9
*a^8)/x^30
```

maple [B] time = 0.01, size = 91, normalized size = 2.28

$$-\frac{b^8}{6x^6}-\frac{8a\,b^7}{9x^9}-\frac{7a^2b^6}{3x^{12}}-\frac{56a^3b^5}{15x^{15}}-\frac{35a^4b^4}{9x^{18}}-\frac{8a^5b^3}{3x^{21}}-\frac{7a^6b^2}{6x^{24}}-\frac{8a^7b}{27x^{27}}-\frac{a^8}{30x^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^8/x^31,x)
```

```
[Out] -7/6*a^6*b^2/x^24-56/15*a^3*b^5/x^15-8/3*a^5*b^3/x^21-35/9*a^4*b^4/x^18-1/3
0*a^8/x^30-8/27*a^7*b/x^27-1/6*b^8/x^6-8/9*a*b^7/x^9-7/3*a^2*b^6/x^12
```

maxima [B] time = 1.29, size = 92, normalized size = 2.30

$$-\frac{45\,b^8x^{24}+240\,ab^7x^{21}+630\,a^2b^6x^{18}+1008\,a^3b^5x^{15}+1050\,a^4b^4x^{12}+720\,a^5b^3x^9+315\,a^6b^2x^6+80\,a^7bx^3+9\,a}{270\,x^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^31,x, algorithm="maxima")
```

```
[Out] -1/270*(45*b^8*x^24 + 240*a*b^7*x^21 + 630*a^2*b^6*x^18 + 1008*a^3*b^5*x^15
 + 1050*a^4*b^4*x^12 + 720*a^5*b^3*x^9 + 315*a^6*b^2*x^6 + 80*a^7*b*x^3 + 9
*a^8)/x^30
```

mupad [B] time = 0.98, size = 92, normalized size = 2.30

$$-\frac{\frac{a^8}{30}+\frac{8a^7bx^3}{27}+\frac{7a^6b^2x^6}{6}+\frac{8a^5b^3x^9}{3}+\frac{35a^4b^4x^{12}}{9}+\frac{56a^3b^5x^{15}}{15}+\frac{7a^2b^6x^{18}}{3}+\frac{8ab^7x^{21}}{9}+\frac{b^8x^{24}}{6}}{x^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^8/x^31,x)
```

```
[Out] -(a^8/30 + (b^8*x^24)/6 + (8*a^7*b*x^3)/27 + (8*a*b^7*x^21)/9 + (7*a^6*b^2*
x^6)/6 + (8*a^5*b^3*x^9)/3 + (35*a^4*b^4*x^12)/9 + (56*a^3*b^5*x^15)/15 + (
7*a^2*b^6*x^18)/3)/x^30
```

sympy [B] time = 1.36, size = 99, normalized size = 2.48

$$\frac{-9a^8-80a^7bx^3-315a^6b^2x^6-720a^5b^3x^9-1050a^4b^4x^{12}-1008a^3b^5x^{15}-630a^2b^6x^{18}-240ab^7x^{21}-45b^8x^{24}}{270x^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**8/x**31,x)
```

```
[Out] (-9*a**8 - 80*a**7*b*x**3 - 315*a**6*b**2*x**6 - 720*a**5*b**3*x**9 - 1050*
a**4*b**4*x**12 - 1008*a**3*b**5*x**15 - 630*a**2*b**6*x**18 - 240*a*b**7*x
**21 - 45*b**8*x**24)/(270*x**30)
```

3.303 $\int \frac{\left(a+bx^3\right)^8}{x^{34}}\,dx$

Optimal. Leaf size=62

$$-\frac{b^2\left(a+bx^3\right)^9}{1485a^3x^{27}}+\frac{b\left(a+bx^3\right)^9}{165a^2x^{30}}-\frac{\left(a+bx^3\right)^9}{33ax^{33}}$$

```
[Out] -1/33*(b*x^3+a)^9/a/x^33+1/165*b*(b*x^3+a)^9/a^2/x^30-1/1485*b^2*(b*x^3+a)^
9/a^3/x^27
```

Rubi [A] time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {266, 45, 37}

$$-\frac{b^2\left(a+bx^3\right)^9}{1485a^3x^{27}}+\frac{b\left(a+bx^3\right)^9}{165a^2x^{30}}-\frac{\left(a+bx^3\right)^9}{33ax^{33}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^8/x^34,x]
```

```
[Out] -(a + b*x^3)^9/(33*a*x^33) + (b*(a + b*x^3)^9)/(165*a^2*x^30) - (b^2*(a + b
*x^3)^9)/(1485*a^3*x^27)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^8}{x^{34}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)^8}{x^{12}}\,dx, x, x^3\right)\\ &= -\frac{\left(a+bx^3\right)^9}{33ax^{33}} - \frac{(2b)\operatorname{Subst}\left(\int \frac{(a+bx)^8}{x^{11}}\,dx, x, x^3\right)}{33a}\\ &= -\frac{\left(a+bx^3\right)^9}{33ax^{33}} + \frac{b\left(a+bx^3\right)^9}{165a^2x^{30}} + \frac{b^2\operatorname{Subst}\left(\int \frac{(a+bx)^8}{x^{10}}\,dx, x, x^3\right)}{165a^2}\\ &= -\frac{\left(a+bx^3\right)^9}{33ax^{33}} + \frac{b\left(a+bx^3\right)^9}{165a^2x^{30}} - \frac{b^2\left(a+bx^3\right)^9}{1485a^3x^{27}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 108, normalized size = 1.74

$$-\frac{a^8}{33x^{33}} - \frac{4a^7b}{15x^{30}} - \frac{28a^6b^2}{27x^{27}} - \frac{7a^5b^3}{3x^{24}} - \frac{10a^4b^4}{3x^{21}} - \frac{28a^3b^5}{9x^{18}} - \frac{28a^2b^6}{15x^{15}} - \frac{2ab^7}{3x^{12}} - \frac{b^8}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^8/x^34,x]
```

```
[Out] -1/33*a^8/x^33 - (4*a^7*b)/(15*x^30) - (28*a^6*b^2)/(27*x^27) - (7*a^5*b^3)
/(3*x^24) - (10*a^4*b^4)/(3*x^21) - (28*a^3*b^5)/(9*x^18) - (28*a^2*b^6)/(1
5*x^15) - (2*a*b^7)/(3*x^12) - b^8/(9*x^9)
```

fricas [A] time = 0.60, size = 92, normalized size = 1.48

$$-\frac{165\,b^8x^{24} + 990\,ab^7x^{21} + 2772\,a^2b^6x^{18} + 4620\,a^3b^5x^{15} + 4950\,a^4b^4x^{12} + 3465\,a^5b^3x^9 + 1540\,a^6b^2x^6 + 396\,a^7}{1485\,x^{33}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^34,x, algorithm="fricas")
```

```
[Out] -1/1485*(165*b^8*x^24 + 990*a*b^7*x^21 + 2772*a^2*b^6*x^18 + 4620*a^3*b^5*x
^15 + 4950*a^4*b^4*x^12 + 3465*a^5*b^3*x^9 + 1540*a^6*b^2*x^6 + 396*a^7*b*x
^3 + 45*a^8)/x^33
```

giac [A] time = 0.16, size = 92, normalized size = 1.48

$$-\frac{165\,b^8x^{24} + 990\,ab^7x^{21} + 2772\,a^2b^6x^{18} + 4620\,a^3b^5x^{15} + 4950\,a^4b^4x^{12} + 3465\,a^5b^3x^9 + 1540\,a^6b^2x^6 + 396\,a^7}{1485\,x^{33}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^34,x, algorithm="giac")
```

```
[Out] -1/1485*(165*b^8*x^24 + 990*a*b^7*x^21 + 2772*a^2*b^6*x^18 + 4620*a^3*b^5*x
^15 + 4950*a^4*b^4*x^12 + 3465*a^5*b^3*x^9 + 1540*a^6*b^2*x^6 + 396*a^7*b*x
^3 + 45*a^8)/x^33
```

maple [A] time = 0.01, size = 91, normalized size = 1.47

$$-\frac{b^8}{9x^9} - \frac{2a\,b^7}{3x^{12}} - \frac{28a^2b^6}{15x^{15}} - \frac{28a^3b^5}{9x^{18}} - \frac{10a^4b^4}{3x^{21}} - \frac{7a^5b^3}{3x^{24}} - \frac{28a^6b^2}{27x^{27}} - \frac{4a^7b}{15x^{30}} - \frac{a^8}{33x^{33}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^8/x^34,x)
```

```
[Out] -28/9*a^3*b^5/x^18-10/3*a^4*b^4/x^21-1/33*a^8/x^33-7/3*a^5*b^3/x^24-4/15*a^
7*b/x^30-28/15*a^2*b^6/x^15-1/9*b^8/x^9-28/27*a^6*b^2/x^27-2/3*a*b^7/x^12
```

maxima [A] time = 1.37, size = 92, normalized size = 1.48

$$-\frac{165\,b^8x^{24}+990\,ab^7x^{21}+2772\,a^2b^6x^{18}+4620\,a^3b^5x^{15}+4950\,a^4b^4x^{12}+3465\,a^5b^3x^9+1540\,a^6b^2x^6+396\,a^7bx}{1485\,x^{33}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^34,x, algorithm="maxima")
```

```
[Out] -1/1485*(165*b^8*x^24 + 990*a*b^7*x^21 + 2772*a^2*b^6*x^18 + 4620*a^3*b^5*x
^15 + 4950*a^4*b^4*x^12 + 3465*a^5*b^3*x^9 + 1540*a^6*b^2*x^6 + 396*a^7*b*x
^3 + 45*a^8)/x^33
```

mupad [B] time = 0.08, size = 92, normalized size = 1.48

$$-\frac{\frac{a^8}{33}+\frac{4\,a^7\,b\,x^3}{15}+\frac{28\,a^6\,b^2\,x^6}{27}+\frac{7\,a^5\,b^3\,x^9}{3}+\frac{10\,a^4\,b^4\,x^{12}}{3}+\frac{28\,a^3\,b^5\,x^{15}}{9}+\frac{28\,a^2\,b^6\,x^{18}}{15}+\frac{2\,a\,b^7\,x^{21}}{3}+\frac{b^8\,x^{24}}{9}}{x^{33}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^8/x^34,x)
```

```
[Out] -(a^8/33 + (b^8*x^24)/9 + (4*a^7*b*x^3)/15 + (2*a*b^7*x^21)/3 + (28*a^6*b^2
*x^6)/27 + (7*a^5*b^3*x^9)/3 + (10*a^4*b^4*x^12)/3 + (28*a^3*b^5*x^15)/9 +
(28*a^2*b^6*x^18)/15)/x^33
```

sympy [A] time = 1.55, size = 99, normalized size = 1.60

$$\frac{-45a^8-396a^7bx^3-1540a^6b^2x^6-3465a^5b^3x^9-4950a^4b^4x^{12}-4620a^3b^5x^{15}-2772a^2b^6x^{18}-990ab^7x^{21}-165b^8}{1485x^{33}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**8/x**34,x)
```

```
[Out] (-45*a**8 - 396*a**7*b*x**3 - 1540*a**6*b**2*x**6 - 3465*a**5*b**3*x**9 - 4
950*a**4*b**4*x**12 - 4620*a**3*b**5*x**15 - 2772*a**2*b**6*x**18 - 990*a*b
**7*x**21 - 165*b**8*x**24)/(1485*x**33)
```

3.304 $\int \frac{\left(a+bx^3\right)^8}{x^{37}}\,dx$

Optimal. Leaf size=84

$$\frac{b^3\left(a+bx^3\right)^9}{5940a^4x^{27}}-\frac{b^2\left(a+bx^3\right)^9}{660a^3x^{30}}+\frac{b\left(a+bx^3\right)^9}{132a^2x^{33}}-\frac{\left(a+bx^3\right)^9}{36ax^{36}}$$

```
[Out] -1/36*(b*x^3+a)^9/a/x^36+1/132*b*(b*x^3+a)^9/a^2/x^33-1/660*b^2*(b*x^3+a)^9
/a^3/x^30+1/5940*b^3*(b*x^3+a)^9/a^4/x^27
```

Rubi [A] time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {266, 45, 37}

$$\frac{b^3\left(a+bx^3\right)^9}{5940a^4x^{27}}-\frac{b^2\left(a+bx^3\right)^9}{660a^3x^{30}}+\frac{b\left(a+bx^3\right)^9}{132a^2x^{33}}-\frac{\left(a+bx^3\right)^9}{36ax^{36}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^8/x^37,x]
```

```
[Out] -(a + b*x^3)^9/(36*a*x^36) + (b*(a + b*x^3)^9)/(132*a^2*x^33) - (b^2*(a + b
*x^3)^9)/(660*a^3*x^30) + (b^3*(a + b*x^3)^9)/(5940*a^4*x^27)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^8}{x^{37}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^8}{x^{13}}\,dx, x, x^3\right)$$
$$= -\frac{\left(a+bx^3\right)^9}{36ax^{36}} - \frac{b\,\text{Subst}\left(\int \frac{(a+bx)^8}{x^{12}}\,dx, x, x^3\right)}{12a}$$
$$= -\frac{\left(a+bx^3\right)^9}{36ax^{36}} + \frac{b\left(a+bx^3\right)^9}{132a^2x^{33}} + \frac{b^2\,\text{Subst}\left(\int \frac{(a+bx)^8}{x^{11}}\,dx, x, x^3\right)}{66a^2}$$
$$= -\frac{\left(a+bx^3\right)^9}{36ax^{36}} + \frac{b\left(a+bx^3\right)^9}{132a^2x^{33}} - \frac{b^2\left(a+bx^3\right)^9}{660a^3x^{30}} - \frac{b^3\,\text{Subst}\left(\int \frac{(a+bx)^8}{x^{10}}\,dx, x, x^3\right)}{660a^3}$$
$$= -\frac{\left(a+bx^3\right)^9}{36ax^{36}} + \frac{b\left(a+bx^3\right)^9}{132a^2x^{33}} - \frac{b^2\left(a+bx^3\right)^9}{660a^3x^{30}} + \frac{b^3\left(a+bx^3\right)^9}{5940a^4x^{27}}$$

Mathematica [A] time = 0.01, size = 108, normalized size = 1.29

$$-\frac{a^8}{36x^{36}} - \frac{8a^7b}{33x^{33}} - \frac{14a^6b^2}{15x^{30}} - \frac{56a^5b^3}{27x^{27}} - \frac{35a^4b^4}{12x^{24}} - \frac{8a^3b^5}{3x^{21}} - \frac{14a^2b^6}{9x^{18}} - \frac{8ab^7}{15x^{15}} - \frac{b^8}{12x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^8/x^37,x]
```

```
[Out] -1/36*a^8/x^36 - (8*a^7*b)/(33*x^33) - (14*a^6*b^2)/(15*x^30) - (56*a^5*b^3
)/(27*x^27) - (35*a^4*b^4)/(12*x^24) - (8*a^3*b^5)/(3*x^21) - (14*a^2*b^6)/
(9*x^18) - (8*a*b^7)/(15*x^15) - b^8/(12*x^12)
```

fricas [A] time = 0.79, size = 92, normalized size = 1.10

$$-\frac{495\,b^8x^{24} + 3168\,ab^7x^{21} + 9240\,a^2b^6x^{18} + 15840\,a^3b^5x^{15} + 17325\,a^4b^4x^{12} + 12320\,a^5b^3x^9 + 5544\,a^6b^2x^6 + 144(}{5940\,x^{36}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^37,x, algorithm="fricas")
```

```
[Out] -1/5940*(495*b^8*x^24 + 3168*a*b^7*x^21 + 9240*a^2*b^6*x^18 + 15840*a^3*b^5
*x^15 + 17325*a^4*b^4*x^12 + 12320*a^5*b^3*x^9 + 5544*a^6*b^2*x^6 + 1440*a^
7*b*x^3 + 165*a^8)/x^36
```

giac [A] time = 0.17, size = 92, normalized size = 1.10

$$-\frac{495\,b^8x^{24} + 3168\,ab^7x^{21} + 9240\,a^2b^6x^{18} + 15840\,a^3b^5x^{15} + 17325\,a^4b^4x^{12} + 12320\,a^5b^3x^9 + 5544\,a^6b^2x^6 + 144(}{5940\,x^{36}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^37,x, algorithm="giac")
```

```
[Out] -1/5940*(495*b^8*x^24 + 3168*a*b^7*x^21 + 9240*a^2*b^6*x^18 + 15840*a^3*b^5
*x^15 + 17325*a^4*b^4*x^12 + 12320*a^5*b^3*x^9 + 5544*a^6*b^2*x^6 + 1440*a^
7*b*x^3 + 165*a^8)/x^36
```

maple [A] time = 0.01, size = 91, normalized size = 1.08

$$-\frac{b^8}{12x^{12}} - \frac{8a\,b^7}{15x^{15}} - \frac{14a^2b^6}{9x^{18}} - \frac{8a^3b^5}{3x^{21}} - \frac{35a^4b^4}{12x^{24}} - \frac{56a^5b^3}{27x^{27}} - \frac{14a^6b^2}{15x^{30}} - \frac{8a^7b}{33x^{33}} - \frac{a^8}{36x^{36}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^8/x^37,x)
```

```
[Out] -8/3*a^3*b^5/x^21-8/15*a*b^7/x^15-1/36*a^8/x^36-56/27*a^5*b^3/x^27-35/12*a^
4*b^4/x^24-14/15*a^6*b^2/x^30-8/33*a^7*b/x^33-14/9*a^2*b^6/x^18-1/12*b^8/x^
12
```

maxima [A] time = 1.37, size = 92, normalized size = 1.10

$$-\frac{495\,b^8x^{24}+3168\,ab^7x^{21}+9240\,a^2b^6x^{18}+15840\,a^3b^5x^{15}+17325\,a^4b^4x^{12}+12320\,a^5b^3x^9+5544\,a^6b^2x^6+1}{5940\,x^{36}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^37,x, algorithm="maxima")
```

```
[Out] -1/5940*(495*b^8*x^24 + 3168*a*b^7*x^21 + 9240*a^2*b^6*x^18 + 15840*a^3*b^5
*x^15 + 17325*a^4*b^4*x^12 + 12320*a^5*b^3*x^9 + 5544*a^6*b^2*x^6 + 1440*a^
7*b*x^3 + 165*a^8)/x^36
```

mupad [B] time = 0.08, size = 92, normalized size = 1.10

$$-\frac{\frac{a^8}{36}+\frac{8\,a^7\,b\,x^3}{33}+\frac{14\,a^6\,b^2\,x^6}{15}+\frac{56\,a^5\,b^3\,x^9}{27}+\frac{35\,a^4\,b^4\,x^{12}}{12}+\frac{8\,a^3\,b^5\,x^{15}}{3}+\frac{14\,a^2\,b^6\,x^{18}}{9}+\frac{8\,a\,b^7\,x^{21}}{15}+\frac{b^8\,x^{24}}{12}}{x^{36}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^8/x^37,x)
```

```
[Out] -(a^8/36 + (b^8*x^24)/12 + (8*a^7*b*x^3)/33 + (8*a*b^7*x^21)/15 + (14*a^6*b
^2*x^6)/15 + (56*a^5*b^3*x^9)/27 + (35*a^4*b^4*x^12)/12 + (8*a^3*b^5*x^15)/
3 + (14*a^2*b^6*x^18)/9)/x^36
```

sympy [A] time = 1.49, size = 99, normalized size = 1.18

$$\frac{-165a^8-1440a^7bx^3-5544a^6b^2x^6-12320a^5b^3x^9-17325a^4b^4x^{12}-15840a^3b^5x^{15}-9240a^2b^6x^{18}-3168ab^7x}{5940x^{36}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**8/x**37,x)
```

```
[Out] (-165*a**8 - 1440*a**7*b*x**3 - 5544*a**6*b**2*x**6 - 12320*a**5*b**3*x**9
- 17325*a**4*b**4*x**12 - 15840*a**3*b**5*x**15 - 9240*a**2*b**6*x**18 - 31
68*a*b**7*x**21 - 495*b**8*x**24)/(5940*x**36)
```

3.305 $\int \frac{\left(a+bx^3\right)^8}{x^{40}}\,dx$

Optimal. Leaf size=106

$$-\frac{b^4\left(a+bx^3\right)^9}{19305a^5x^{27}}+\frac{b^3\left(a+bx^3\right)^9}{2145a^4x^{30}}-\frac{b^2\left(a+bx^3\right)^9}{429a^3x^{33}}+\frac{b\left(a+bx^3\right)^9}{117a^2x^{36}}-\frac{\left(a+bx^3\right)^9}{39ax^{39}}$$

```
[Out] -1/39*(b*x^3+a)^9/a/x^39+1/117*b*(b*x^3+a)^9/a^2/x^36-1/429*b^2*(b*x^3+a)^9
/a^3/x^33+1/2145*b^3*(b*x^3+a)^9/a^4/x^30-1/19305*b^4*(b*x^3+a)^9/a^5/x^27
```

Rubi [A] time = 0.05, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {266, 45, 37}

$$-\frac{b^4\left(a+bx^3\right)^9}{19305a^5x^{27}}+\frac{b^3\left(a+bx^3\right)^9}{2145a^4x^{30}}-\frac{b^2\left(a+bx^3\right)^9}{429a^3x^{33}}+\frac{b\left(a+bx^3\right)^9}{117a^2x^{36}}-\frac{\left(a+bx^3\right)^9}{39ax^{39}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^8/x^40,x]
```

```
[Out] -(a + b*x^3)^9/(39*a*x^39) + (b*(a + b*x^3)^9)/(117*a^2*x^36) - (b^2*(a + b
*x^3)^9)/(429*a^3*x^33) + (b^3*(a + b*x^3)^9)/(2145*a^4*x^30) - (b^4*(a + b
*x^3)^9)/(19305*a^5*x^27)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^8}{x^{40}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^8}{x^{14}}\,dx, x, x^3\right)$$
$$= -\frac{\left(a+bx^3\right)^9}{39ax^{39}} - \frac{(4b)\,\text{Subst}\left(\int \frac{(a+bx)^8}{x^{13}}\,dx, x, x^3\right)}{39a}$$
$$= -\frac{\left(a+bx^3\right)^9}{39ax^{39}} + \frac{b\left(a+bx^3\right)^9}{117a^2x^{36}} + \frac{b^2\,\text{Subst}\left(\int \frac{(a+bx)^8}{x^{12}}\,dx, x, x^3\right)}{39a^2}$$
$$= -\frac{\left(a+bx^3\right)^9}{39ax^{39}} + \frac{b\left(a+bx^3\right)^9}{117a^2x^{36}} - \frac{b^2\left(a+bx^3\right)^9}{429a^3x^{33}} - \frac{\left(2b^3\right)\text{Subst}\left(\int \frac{(a+bx)^8}{x^{11}}\,dx, x, x^3\right)}{429a^3}$$
$$= -\frac{\left(a+bx^3\right)^9}{39ax^{39}} + \frac{b\left(a+bx^3\right)^9}{117a^2x^{36}} - \frac{b^2\left(a+bx^3\right)^9}{429a^3x^{33}} + \frac{b^3\left(a+bx^3\right)^9}{2145a^4x^{30}} + \frac{b^4\,\text{Subst}\left(\int \frac{(a+bx)^8}{x^{10}}\,dx, x, x^3\right.}{2145a^4}$$
$$= -\frac{\left(a+bx^3\right)^9}{39ax^{39}} + \frac{b\left(a+bx^3\right)^9}{117a^2x^{36}} - \frac{b^2\left(a+bx^3\right)^9}{429a^3x^{33}} + \frac{b^3\left(a+bx^3\right)^9}{2145a^4x^{30}} - \frac{b^4\left(a+bx^3\right)^9}{19305a^5x^{27}}$$

Mathematica [A] time = 0.01, size = 108, normalized size = 1.02

$$-\frac{a^8}{39x^{39}} - \frac{2a^7b}{9x^{36}} - \frac{28a^6b^2}{33x^{33}} - \frac{28a^5b^3}{15x^{30}} - \frac{70a^4b^4}{27x^{27}} - \frac{7a^3b^5}{3x^{24}} - \frac{4a^2b^6}{3x^{21}} - \frac{4ab^7}{9x^{18}} - \frac{b^8}{15x^{15}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^8/x^40,x]
```

```
[Out] -1/39*a^8/x^39 - (2*a^7*b)/(9*x^36) - (28*a^6*b^2)/(33*x^33) - (28*a^5*b^3)
/(15*x^30) - (70*a^4*b^4)/(27*x^27) - (7*a^3*b^5)/(3*x^24) - (4*a^2*b^6)/(3
*x^21) - (4*a*b^7)/(9*x^18) - b^8/(15*x^15)
```

fricas [A] time = 0.70, size = 92, normalized size = 0.87

$$-\frac{1287\,b^8x^{24} + 8580\,ab^7x^{21} + 25740\,a^2b^6x^{18} + 45045\,a^3b^5x^{15} + 50050\,a^4b^4x^{12} + 36036\,a^5b^3x^9 + 16380\,a^6b^2x^6}{19305\,x^{39}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^40,x, algorithm="fricas")
```

```
[Out] -1/19305*(1287*b^8*x^24 + 8580*a*b^7*x^21 + 25740*a^2*b^6*x^18 + 45045*a^3*
b^5*x^15 + 50050*a^4*b^4*x^12 + 36036*a^5*b^3*x^9 + 16380*a^6*b^2*x^6 + 429
0*a^7*b*x^3 + 495*a^8)/x^39
```

giac [A] time = 0.15, size = 92, normalized size = 0.87

$$-\frac{1287\,b^8x^{24} + 8580\,ab^7x^{21} + 25740\,a^2b^6x^{18} + 45045\,a^3b^5x^{15} + 50050\,a^4b^4x^{12} + 36036\,a^5b^3x^9 + 16380\,a^6b^2x^6}{19305\,x^{39}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^40,x, algorithm="giac")
```

```
[Out] -1/19305*(1287*b^8*x^24 + 8580*a*b^7*x^21 + 25740*a^2*b^6*x^18 + 45045*a^3*
b^5*x^15 + 50050*a^4*b^4*x^12 + 36036*a^5*b^3*x^9 + 16380*a^6*b^2*x^6 + 429
0*a^7*b*x^3 + 495*a^8)/x^39
```

maple [A] time = 0.01, size = 91, normalized size = 0.86

$$-\frac{b^8}{15x^{15}} - \frac{4a\,b^7}{9x^{18}} - \frac{4a^2b^6}{3x^{21}} - \frac{7a^3b^5}{3x^{24}} - \frac{70a^4b^4}{27x^{27}} - \frac{28a^5b^3}{15x^{30}} - \frac{28a^6b^2}{33x^{33}} - \frac{2a^7b}{9x^{36}} - \frac{a^8}{39x^{39}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^8/x^40,x)
```

```
[Out] -2/9*a^7*b/x^36-28/33*a^6*b^2/x^33-4/3*a^2*b^6/x^21-7/3*a^3*b^5/x^24-28/15*
a^5*b^3/x^30-1/39*a^8/x^39-70/27*a^4*b^4/x^27-1/15*b^8/x^15-4/9*a*b^7/x^18
```

maxima [A] time = 1.36, size = 92, normalized size = 0.87

$$-\frac{1287\,b^8x^{24}+8580\,ab^7x^{21}+25740\,a^2b^6x^{18}+45045\,a^3b^5x^{15}+50050\,a^4b^4x^{12}+36036\,a^5b^3x^9+16380\,a^6b^2x^6+}{19305\,x^{39}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^40,x, algorithm="maxima")
```

```
[Out] -1/19305*(1287*b^8*x^24 + 8580*a*b^7*x^21 + 25740*a^2*b^6*x^18 + 45045*a^3*
b^5*x^15 + 50050*a^4*b^4*x^12 + 36036*a^5*b^3*x^9 + 16380*a^6*b^2*x^6 + 429
0*a^7*b*x^3 + 495*a^8)/x^39
```

mupad [B] time = 1.00, size = 92, normalized size = 0.87

$$-\frac{\frac{a^8}{39}+\frac{2\,a^7\,b\,x^3}{9}+\frac{28\,a^6\,b^2\,x^6}{33}+\frac{28\,a^5\,b^3\,x^9}{15}+\frac{70\,a^4\,b^4\,x^{12}}{27}+\frac{7\,a^3\,b^5\,x^{15}}{3}+\frac{4\,a^2\,b^6\,x^{18}}{3}+\frac{4\,a\,b^7\,x^{21}}{9}+\frac{b^8\,x^{24}}{15}}{x^{39}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^8/x^40,x)
```

```
[Out] -(a^8/39 + (b^8*x^24)/15 + (2*a^7*b*x^3)/9 + (4*a*b^7*x^21)/9 + (28*a^6*b^2
*x^6)/33 + (28*a^5*b^3*x^9)/15 + (70*a^4*b^4*x^12)/27 + (7*a^3*b^5*x^15)/3
+ (4*a^2*b^6*x^18)/3)/x^39
```

sympy [A] time = 2.96, size = 99, normalized size = 0.93

$$\frac{-495a^8-4290a^7bx^3-16380a^6b^2x^6-36036a^5b^3x^9-50050a^4b^4x^{12}-45045a^3b^5x^{15}-25740a^2b^6x^{18}-8580ab^7x}{19305x^{39}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**8/x**40,x)
```

```
[Out] (-495*a**8 - 4290*a**7*b*x**3 - 16380*a**6*b**2*x**6 - 36036*a**5*b**3*x**9
 - 50050*a**4*b**4*x**12 - 45045*a**3*b**5*x**15 - 25740*a**2*b**6*x**18 -
8580*a*b**7*x**21 - 1287*b**8*x**24)/(19305*x**39)
```

3.306 $\int \frac{\left(a+bx^3\right)^8}{x^{43}}\,dx$

Optimal. Leaf size=108

$$-\frac{a^8}{42x^{42}}-\frac{8a^7b}{39x^{39}}-\frac{7a^6b^2}{9x^{36}}-\frac{56a^5b^3}{33x^{33}}-\frac{7a^4b^4}{3x^{30}}-\frac{56a^3b^5}{27x^{27}}-\frac{7a^2b^6}{6x^{24}}-\frac{8ab^7}{21x^{21}}-\frac{b^8}{18x^{18}}$$

```
[Out] -1/42*a^8/x^42-8/39*a^7*b/x^39-7/9*a^6*b^2/x^36-56/33*a^5*b^3/x^33-7/3*a^4*
b^4/x^30-56/27*a^3*b^5/x^27-7/6*a^2*b^6/x^24-8/21*a*b^7/x^21-1/18*b^8/x^18
```

Rubi [A] time = 0.05, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$-\frac{7a^6b^2}{9x^{36}}-\frac{56a^5b^3}{33x^{33}}-\frac{7a^4b^4}{3x^{30}}-\frac{56a^3b^5}{27x^{27}}-\frac{7a^2b^6}{6x^{24}}-\frac{8a^7b}{39x^{39}}-\frac{a^8}{42x^{42}}-\frac{8ab^7}{21x^{21}}-\frac{b^8}{18x^{18}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^8/x^43,x]
```

```
[Out] -a^8/(42*x^42) - (8*a^7*b)/(39*x^39) - (7*a^6*b^2)/(9*x^36) - (56*a^5*b^3)/
(33*x^33) - (7*a^4*b^4)/(3*x^30) - (56*a^3*b^5)/(27*x^27) - (7*a^2*b^6)/(6*
x^24) - (8*a*b^7)/(21*x^21) - b^8/(18*x^18)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^8}{x^{43}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^8}{x^{15}}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{a^8}{x^{15}}+\frac{8a^7b}{x^{14}}+\frac{28a^6b^2}{x^{13}}+\frac{56a^5b^3}{x^{12}}+\frac{70a^4b^4}{x^{11}}+\frac{56a^3b^5}{x^{10}}+\frac{28a^2b^6}{x^9}+\frac{8ab^7}{x^8}+\frac{b^8}{x^7}\right)\right.\\ &= -\frac{a^8}{42x^{42}}-\frac{8a^7b}{39x^{39}}-\frac{7a^6b^2}{9x^{36}}-\frac{56a^5b^3}{33x^{33}}-\frac{7a^4b^4}{3x^{30}}-\frac{56a^3b^5}{27x^{27}}-\frac{7a^2b^6}{6x^{24}}-\frac{8ab^7}{21x^{21}}-\frac{b^8}{18x^{18}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 108, normalized size = 1.00

$$-\frac{a^8}{42x^{42}}-\frac{8a^7b}{39x^{39}}-\frac{7a^6b^2}{9x^{36}}-\frac{56a^5b^3}{33x^{33}}-\frac{7a^4b^4}{3x^{30}}-\frac{56a^3b^5}{27x^{27}}-\frac{7a^2b^6}{6x^{24}}-\frac{8ab^7}{21x^{21}}-\frac{b^8}{18x^{18}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^8/x^43,x]
```

```
[Out] -1/42*a^8/x^42 - (8*a^7*b)/(39*x^39) - (7*a^6*b^2)/(9*x^36) - (56*a^5*b^3)/
(33*x^33) - (7*a^4*b^4)/(3*x^30) - (56*a^3*b^5)/(27*x^27) - (7*a^2*b^6)/(6*
x^24) - (8*a*b^7)/(21*x^21) - b^8/(18*x^18)
```

fricas [A] time = 0.61, size = 92, normalized size = 0.85

$$-\frac{3003\,b^8x^{24}+20592\,ab^7x^{21}+63063\,a^2b^6x^{18}+112112\,a^3b^5x^{15}+126126\,a^4b^4x^{12}+91728\,a^5b^3x^9+42042\,a^6b^2x^6}{54054\,x^{42}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^43,x, algorithm="fricas")
```

```
[Out] -1/54054*(3003*b^8*x^24 + 20592*a*b^7*x^21 + 63063*a^2*b^6*x^18 + 112112*a^
3*b^5*x^15 + 126126*a^4*b^4*x^12 + 91728*a^5*b^3*x^9 + 42042*a^6*b^2*x^6 +
11088*a^7*b*x^3 + 1287*a^8)/x^42
```

giac [A] time = 0.16, size = 92, normalized size = 0.85

$$-\frac{3003\,b^8x^{24}+20592\,ab^7x^{21}+63063\,a^2b^6x^{18}+112112\,a^3b^5x^{15}+126126\,a^4b^4x^{12}+91728\,a^5b^3x^9+42042\,a^6b^2x^6}{54054\,x^{42}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^43,x, algorithm="giac")
```

```
[Out] -1/54054*(3003*b^8*x^24 + 20592*a*b^7*x^21 + 63063*a^2*b^6*x^18 + 112112*a^
3*b^5*x^15 + 126126*a^4*b^4*x^12 + 91728*a^5*b^3*x^9 + 42042*a^6*b^2*x^6 +
11088*a^7*b*x^3 + 1287*a^8)/x^42
```

maple [A] time = 0.01, size = 91, normalized size = 0.84

$$-\frac{b^8}{18x^{18}}-\frac{8a\,b^7}{21x^{21}}-\frac{7a^2b^6}{6x^{24}}-\frac{56a^3b^5}{27x^{27}}-\frac{7a^4b^4}{3x^{30}}-\frac{56a^5b^3}{33x^{33}}-\frac{7a^6b^2}{9x^{36}}-\frac{8a^7b}{39x^{39}}-\frac{a^8}{42x^{42}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^8/x^43,x)
```

```
[Out] -1/42*a^8/x^42-8/39*a^7*b/x^39-7/9*a^6*b^2/x^36-56/33*a^5*b^3/x^33-7/3*a^4*
b^4/x^30-56/27*a^3*b^5/x^27-7/6*a^2*b^6/x^24-8/21*a*b^7/x^21-1/18*b^8/x^18
```

maxima [A] time = 1.36, size = 92, normalized size = 0.85

$$-\frac{3003\,b^8x^{24}+20592\,ab^7x^{21}+63063\,a^2b^6x^{18}+112112\,a^3b^5x^{15}+126126\,a^4b^4x^{12}+91728\,a^5b^3x^9+42042\,a^6b^2x^6}{54054\,x^{42}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^43,x, algorithm="maxima")
```

```
[Out] -1/54054*(3003*b^8*x^24 + 20592*a*b^7*x^21 + 63063*a^2*b^6*x^18 + 112112*a^
3*b^5*x^15 + 126126*a^4*b^4*x^12 + 91728*a^5*b^3*x^9 + 42042*a^6*b^2*x^6 +
11088*a^7*b*x^3 + 1287*a^8)/x^42
```

mupad [B] time = 0.08, size = 92, normalized size = 0.85

$$-\frac{\frac{a^8}{42}+\frac{8\,a^7\,b\,x^3}{39}+\frac{7\,a^6\,b^2\,x^6}{9}+\frac{56\,a^5\,b^3\,x^9}{33}+\frac{7\,a^4\,b^4\,x^{12}}{3}+\frac{56\,a^3\,b^5\,x^{15}}{27}+\frac{7\,a^2\,b^6\,x^{18}}{6}+\frac{8\,a\,b^7\,x^{21}}{21}+\frac{b^8\,x^{24}}{18}}{x^{42}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^8/x^43,x)
```

```
[Out] -(a^8/42 + (b^8*x^24)/18 + (8*a^7*b*x^3)/39 + (8*a*b^7*x^21)/21 + (7*a^6*b^
2*x^6)/9 + (56*a^5*b^3*x^9)/33 + (7*a^4*b^4*x^12)/3 + (56*a^3*b^5*x^15)/27
+ (7*a^2*b^6*x^18)/6)/x^42
```

sympy [**A**] time = 3.02, size = 99, normalized size = 0.92

$$\frac{-1287a^8 - 11088a^7bx^3 - 42042a^6b^2x^6 - 91728a^5b^3x^9 - 126126a^4b^4x^{12} - 112112a^3b^5x^{15} - 63063a^2b^6x^{18} - 20}{54054x^{42}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**8/x**43,x)
```

```
[Out] (-1287*a**8 - 11088*a**7*b*x**3 - 42042*a**6*b**2*x**6 - 91728*a**5*b**3*x*
*9 - 126126*a**4*b**4*x**12 - 112112*a**3*b**5*x**15 - 63063*a**2*b**6*x**1
8 - 20592*a*b**7*x**21 - 3003*b**8*x**24)/(54054*x**42)
```

3.307 $\int \frac{\left(a+bx^3\right)^8}{x^{46}}\,dx$

Optimal. Leaf size=108

$$-\frac{a^8}{45x^{45}}-\frac{4a^7b}{21x^{42}}-\frac{28a^6b^2}{39x^{39}}-\frac{14a^5b^3}{9x^{36}}-\frac{70a^4b^4}{33x^{33}}-\frac{28a^3b^5}{15x^{30}}-\frac{28a^2b^6}{27x^{27}}-\frac{ab^7}{3x^{24}}-\frac{b^8}{21x^{21}}$$

```
[Out] -1/45*a^8/x^45-4/21*a^7*b/x^42-28/39*a^6*b^2/x^39-14/9*a^5*b^3/x^36-70/33*a
^4*b^4/x^33-28/15*a^3*b^5/x^30-28/27*a^2*b^6/x^27-1/3*a*b^7/x^24-1/21*b^8/x
^21
```

Rubi [A] time = 0.05, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$-\frac{28a^6b^2}{39x^{39}}-\frac{14a^5b^3}{9x^{36}}-\frac{70a^4b^4}{33x^{33}}-\frac{28a^3b^5}{15x^{30}}-\frac{28a^2b^6}{27x^{27}}-\frac{4a^7b}{21x^{42}}-\frac{a^8}{45x^{45}}-\frac{ab^7}{3x^{24}}-\frac{b^8}{21x^{21}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^8/x^46,x]
```

```
[Out] -a^8/(45*x^45) - (4*a^7*b)/(21*x^42) - (28*a^6*b^2)/(39*x^39) - (14*a^5*b^3
)/(9*x^36) - (70*a^4*b^4)/(33*x^33) - (28*a^3*b^5)/(15*x^30) - (28*a^2*b^6)
/(27*x^27) - (a*b^7)/(3*x^24) - b^8/(21*x^21)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^8}{x^{46}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)^8}{x^{16}}\,dx, x, x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{a^8}{x^{16}}+\frac{8a^7b}{x^{15}}+\frac{28a^6b^2}{x^{14}}+\frac{56a^5b^3}{x^{13}}+\frac{70a^4b^4}{x^{12}}+\frac{56a^3b^5}{x^{11}}+\frac{28a^2b^6}{x^{10}}+\frac{8ab^7}{x^9}+\frac{b^8}{x^8}\right)dx\right.\\ &= -\frac{a^8}{45x^{45}}-\frac{4a^7b}{21x^{42}}-\frac{28a^6b^2}{39x^{39}}-\frac{14a^5b^3}{9x^{36}}-\frac{70a^4b^4}{33x^{33}}-\frac{28a^3b^5}{15x^{30}}-\frac{28a^2b^6}{27x^{27}}-\frac{ab^7}{3x^{24}}-\frac{b^8}{21x^{21}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 108, normalized size = 1.00

$$-\frac{a^8}{45x^{45}}-\frac{4a^7b}{21x^{42}}-\frac{28a^6b^2}{39x^{39}}-\frac{14a^5b^3}{9x^{36}}-\frac{70a^4b^4}{33x^{33}}-\frac{28a^3b^5}{15x^{30}}-\frac{28a^2b^6}{27x^{27}}-\frac{ab^7}{3x^{24}}-\frac{b^8}{21x^{21}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^8/x^46,x]
```

```
[Out] -1/45*a^8/x^45 - (4*a^7*b)/(21*x^42) - (28*a^6*b^2)/(39*x^39) - (14*a^5*b^3
)/(9*x^36) - (70*a^4*b^4)/(33*x^33) - (28*a^3*b^5)/(15*x^30) - (28*a^2*b^6)
/(27*x^27) - (a*b^7)/(3*x^24) - b^8/(21*x^21)
```

fricas [A] time = 0.59, size = 92, normalized size = 0.85

$$-\frac{6435\,b^8x^{24}+45045\,ab^7x^{21}+140140\,a^2b^6x^{18}+252252\,a^3b^5x^{15}+286650\,a^4b^4x^{12}+210210\,a^5b^3x^9+97020\,a}{135135\,x^{45}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^46,x, algorithm="fricas")
```

```
[Out] -1/135135*(6435*b^8*x^24 + 45045*a*b^7*x^21 + 140140*a^2*b^6*x^18 + 252252*
a^3*b^5*x^15 + 286650*a^4*b^4*x^12 + 210210*a^5*b^3*x^9 + 97020*a^6*b^2*x^6
 + 25740*a^7*b*x^3 + 3003*a^8)/x^45
```

giac [A] time = 0.19, size = 92, normalized size = 0.85

$$-\frac{6435\,b^8x^{24}+45045\,ab^7x^{21}+140140\,a^2b^6x^{18}+252252\,a^3b^5x^{15}+286650\,a^4b^4x^{12}+210210\,a^5b^3x^9+97020\,a}{135135\,x^{45}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^46,x, algorithm="giac")
```

```
[Out] -1/135135*(6435*b^8*x^24 + 45045*a*b^7*x^21 + 140140*a^2*b^6*x^18 + 252252*
a^3*b^5*x^15 + 286650*a^4*b^4*x^12 + 210210*a^5*b^3*x^9 + 97020*a^6*b^2*x^6
 + 25740*a^7*b*x^3 + 3003*a^8)/x^45
```

maple [A] time = 0.01, size = 91, normalized size = 0.84

$$-\frac{b^8}{21x^{21}}-\frac{a\,b^7}{3x^{24}}-\frac{28a^2b^6}{27x^{27}}-\frac{28a^3b^5}{15x^{30}}-\frac{70a^4b^4}{33x^{33}}-\frac{14a^5b^3}{9x^{36}}-\frac{28a^6b^2}{39x^{39}}-\frac{4a^7b}{21x^{42}}-\frac{a^8}{45x^{45}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^8/x^46,x)
```

```
[Out] -1/45*a^8/x^45-4/21*a^7*b/x^42-28/39*a^6*b^2/x^39-14/9*a^5*b^3/x^36-70/33*a
^4*b^4/x^33-28/15*a^3*b^5/x^30-28/27*a^2*b^6/x^27-1/3*a*b^7/x^24-1/21*b^8/x
^21
```

maxima [A] time = 1.29, size = 92, normalized size = 0.85

$$-\frac{6435\,b^8x^{24}+45045\,ab^7x^{21}+140140\,a^2b^6x^{18}+252252\,a^3b^5x^{15}+286650\,a^4b^4x^{12}+210210\,a^5b^3x^9+97020\,a}{135135\,x^{45}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^46,x, algorithm="maxima")
```

```
[Out] -1/135135*(6435*b^8*x^24 + 45045*a*b^7*x^21 + 140140*a^2*b^6*x^18 + 252252*
a^3*b^5*x^15 + 286650*a^4*b^4*x^12 + 210210*a^5*b^3*x^9 + 97020*a^6*b^2*x^6
 + 25740*a^7*b*x^3 + 3003*a^8)/x^45
```

mupad [B] time = 1.00, size = 92, normalized size = 0.85

$$-\frac{\frac{a^8}{45}+\frac{4\,a^7\,b\,x^3}{21}+\frac{28\,a^6\,b^2\,x^6}{39}+\frac{14\,a^5\,b^3\,x^9}{9}+\frac{70\,a^4\,b^4\,x^{12}}{33}+\frac{28\,a^3\,b^5\,x^{15}}{15}+\frac{28\,a^2\,b^6\,x^{18}}{27}+\frac{a\,b^7\,x^{21}}{3}+\frac{b^8\,x^{24}}{21}}{x^{45}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^8/x^46,x)
```

```
[Out] -(a^8/45 + (b^8*x^24)/21 + (4*a^7*b*x^3)/21 + (a*b^7*x^21)/3 + (28*a^6*b^2*
x^6)/39 + (14*a^5*b^3*x^9)/9 + (70*a^4*b^4*x^12)/33 + (28*a^3*b^5*x^15)/15
+ (28*a^2*b^6*x^18)/27)/x^45
```

sympy [**A**] time = 1.92, size = 99, normalized size = 0.92

$$\frac{-3003a^8 - 25740a^7bx^3 - 97020a^6b^2x^6 - 210210a^5b^3x^9 - 286650a^4b^4x^{12} - 252252a^3b^5x^{15} - 140140a^2b^6x^{18} - 45}{135135x^{45}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**8/x**46,x)
```

```
[Out] (-3003*a**8 - 25740*a**7*b*x**3 - 97020*a**6*b**2*x**6 - 210210*a**5*b**3*x
**9 - 286650*a**4*b**4*x**12 - 252252*a**3*b**5*x**15 - 140140*a**2*b**6*x*
*18 - 45045*a*b**7*x**21 - 6435*b**8*x**24)/(135135*x**45)
```

3.308 $\int x^4 \left(a+bx^3\right)^8 dx$

Optimal. Leaf size=103

$$\frac{a^8x^5}{5}+a^7bx^8+\frac{28}{11}a^6b^2x^{11}+4a^5b^3x^{14}+\frac{70}{17}a^4b^4x^{17}+\frac{14}{5}a^3b^5x^{20}+\frac{28}{23}a^2b^6x^{23}+\frac{4}{13}ab^7x^{26}+\frac{b^8x^{29}}{29}$$

```
[Out] 1/5*a^8*x^5+a^7*b*x^8+28/11*a^6*b^2*x^11+4*a^5*b^3*x^14+70/17*a^4*b^4*x^17+
14/5*a^3*b^5*x^20+28/23*a^2*b^6*x^23+4/13*a*b^7*x^26+1/29*b^8*x^29
```

Rubi [A] time = 0.04, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{28}{23}a^2b^6x^{23}+\frac{14}{5}a^3b^5x^{20}+\frac{70}{17}a^4b^4x^{17}+4a^5b^3x^{14}+\frac{28}{11}a^6b^2x^{11}+a^7bx^8+\frac{a^8x^5}{5}+\frac{4}{13}ab^7x^{26}+\frac{b^8x^{29}}{29}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^3)^8,x]
```

```
[Out] (a^8*x^5)/5 + a^7*b*x^8 + (28*a^6*b^2*x^11)/11 + 4*a^5*b^3*x^14 + (70*a^4*b
^4*x^17)/17 + (14*a^3*b^5*x^20)/5 + (28*a^2*b^6*x^23)/23 + (4*a*b^7*x^26)/1
3 + (b^8*x^29)/29
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int x^4\left(a+bx^3\right)^8 dx = \int \left(a^8x^4+8a^7bx^7+28a^6b^2x^{10}+56a^5b^3x^{13}+70a^4b^4x^{16}+56a^3b^5x^{19}+28a^2b^6x^{22}+8a\right.$$
$$= \frac{a^8x^5}{5}+a^7bx^8+\frac{28}{11}a^6b^2x^{11}+4a^5b^3x^{14}+\frac{70}{17}a^4b^4x^{17}+\frac{14}{5}a^3b^5x^{20}+\frac{28}{23}a^2b^6x^{23}+\frac{4}{13}ab^7x$$

Mathematica [A] time = 0.00, size = 103, normalized size = 1.00

$$\frac{a^8x^5}{5}+a^7bx^8+\frac{28}{11}a^6b^2x^{11}+4a^5b^3x^{14}+\frac{70}{17}a^4b^4x^{17}+\frac{14}{5}a^3b^5x^{20}+\frac{28}{23}a^2b^6x^{23}+\frac{4}{13}ab^7x^{26}+\frac{b^8x^{29}}{29}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^3)^8,x]
```

```
[Out] (a^8*x^5)/5 + a^7*b*x^8 + (28*a^6*b^2*x^11)/11 + 4*a^5*b^3*x^14 + (70*a^4*b
^4*x^17)/17 + (14*a^3*b^5*x^20)/5 + (28*a^2*b^6*x^23)/23 + (4*a*b^7*x^26)/1
3 + (b^8*x^29)/29
```

fricas [A] time = 0.46, size = 89, normalized size = 0.86

$$\frac{1}{29}x^{29}b^8+\frac{4}{13}x^{26}b^7a+\frac{28}{23}x^{23}b^6a^2+\frac{14}{5}x^{20}b^5a^3+\frac{70}{17}x^{17}b^4a^4+4x^{14}b^3a^5+\frac{28}{11}x^{11}b^2a^6+x^8ba^7+\frac{1}{5}x^5a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^8,x, algorithm="fricas")
```

```
[Out] 1/29*x^29*b^8 + 4/13*x^26*b^7*a + 28/23*x^23*b^6*a^2 + 14/5*x^20*b^5*a^3 +
70/17*x^17*b^4*a^4 + 4*x^14*b^3*a^5 + 28/11*x^11*b^2*a^6 + x^8*b*a^7 + 1/5*
x^5*a^8
```

giac [A] time = 0.21, size = 89, normalized size = 0.86

$$\frac{1}{29}b^8x^{29}+\frac{4}{13}ab^7x^{26}+\frac{28}{23}a^2b^6x^{23}+\frac{14}{5}a^3b^5x^{20}+\frac{70}{17}a^4b^4x^{17}+4a^5b^3x^{14}+\frac{28}{11}a^6b^2x^{11}+a^7bx^8+\frac{1}{5}a^8x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^8,x, algorithm="giac")
```

```
[Out] 1/29*b^8*x^29 + 4/13*a*b^7*x^26 + 28/23*a^2*b^6*x^23 + 14/5*a^3*b^5*x^20 +
70/17*a^4*b^4*x^17 + 4*a^5*b^3*x^14 + 28/11*a^6*b^2*x^11 + a^7*b*x^8 + 1/5*
a^8*x^5
```

maple [A] time = 0.00, size = 90, normalized size = 0.87

$$\frac{1}{29}b^8x^{29}+\frac{4}{13}a\,b^7x^{26}+\frac{28}{23}a^2b^6x^{23}+\frac{14}{5}a^3b^5x^{20}+\frac{70}{17}a^4b^4x^{17}+4a^5b^3x^{14}+\frac{28}{11}a^6b^2x^{11}+a^7b\,x^8+\frac{1}{5}a^8x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)^8,x)
```

```
[Out] 1/5*a^8*x^5+a^7*b*x^8+28/11*a^6*b^2*x^11+4*a^5*b^3*x^14+70/17*a^4*b^4*x^17+
14/5*a^3*b^5*x^20+28/23*a^2*b^6*x^23+4/13*a*b^7*x^26+1/29*b^8*x^29
```

maxima [A] time = 1.30, size = 89, normalized size = 0.86

$$\frac{1}{29}b^8x^{29}+\frac{4}{13}ab^7x^{26}+\frac{28}{23}a^2b^6x^{23}+\frac{14}{5}a^3b^5x^{20}+\frac{70}{17}a^4b^4x^{17}+4a^5b^3x^{14}+\frac{28}{11}a^6b^2x^{11}+a^7bx^8+\frac{1}{5}a^8x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^8,x, algorithm="maxima")
```

```
[Out] 1/29*b^8*x^29 + 4/13*a*b^7*x^26 + 28/23*a^2*b^6*x^23 + 14/5*a^3*b^5*x^20 +
70/17*a^4*b^4*x^17 + 4*a^5*b^3*x^14 + 28/11*a^6*b^2*x^11 + a^7*b*x^8 + 1/5*
a^8*x^5
```

mupad [B] time = 1.01, size = 89, normalized size = 0.86

$$\frac{a^8\,x^5}{5}+a^7\,b\,x^8+\frac{28\,a^6\,b^2\,x^{11}}{11}+4\,a^5\,b^3\,x^{14}+\frac{70\,a^4\,b^4\,x^{17}}{17}+\frac{14\,a^3\,b^5\,x^{20}}{5}+\frac{28\,a^2\,b^6\,x^{23}}{23}+\frac{4\,a\,b^7\,x^{26}}{13}+\frac{b^8\,x^{29}}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^3)^8,x)
```

```
[Out] (a^8*x^5)/5 + (b^8*x^29)/29 + a^7*b*x^8 + (4*a*b^7*x^26)/13 + (28*a^6*b^2*x
^11)/11 + 4*a^5*b^3*x^14 + (70*a^4*b^4*x^17)/17 + (14*a^3*b^5*x^20)/5 + (28
*a^2*b^6*x^23)/23
```

sympy [A] time = 0.11, size = 102, normalized size = 0.99

$$\frac{a^8x^5}{5}+a^7bx^8+\frac{28a^6b^2x^{11}}{11}+4a^5b^3x^{14}+\frac{70a^4b^4x^{17}}{17}+\frac{14a^3b^5x^{20}}{5}+\frac{28a^2b^6x^{23}}{23}+\frac{4ab^7x^{26}}{13}+\frac{b^8x^{29}}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)**8,x)
```

```
[Out] a**8*x**5/5 + a**7*b*x**8 + 28*a**6*b**2*x**11/11 + 4*a**5*b**3*x**14 + 70*
a**4*b**4*x**17/17 + 14*a**3*b**5*x**20/5 + 28*a**2*b**6*x**23/23 + 4*a*b**
7*x**26/13 + b**8*x**29/29
```

3.309 $\int x^3 \left(a+bx^3\right)^8 dx$

Optimal. Leaf size=108

$$\frac{a^8x^4}{4}+\frac{8}{7}a^7bx^7+\frac{14}{5}a^6b^2x^{10}+\frac{56}{13}a^5b^3x^{13}+\frac{35}{8}a^4b^4x^{16}+\frac{56}{19}a^3b^5x^{19}+\frac{14}{11}a^2b^6x^{22}+\frac{8}{25}ab^7x^{25}+\frac{b^8x^{28}}{28}$$

```
[Out] 1/4*a^8*x^4+8/7*a^7*b*x^7+14/5*a^6*b^2*x^10+56/13*a^5*b^3*x^13+35/8*a^4*b^4
*x^16+56/19*a^3*b^5*x^19+14/11*a^2*b^6*x^22+8/25*a*b^7*x^25+1/28*b^8*x^28
```

Rubi [A] time = 0.04, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{14}{11}a^2b^6x^{22}+\frac{56}{19}a^3b^5x^{19}+\frac{35}{8}a^4b^4x^{16}+\frac{56}{13}a^5b^3x^{13}+\frac{14}{5}a^6b^2x^{10}+\frac{8}{7}a^7bx^7+\frac{a^8x^4}{4}+\frac{8}{25}ab^7x^{25}+\frac{b^8x^{28}}{28}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^3)^8,x]
```

```
[Out] (a^8*x^4)/4 + (8*a^7*b*x^7)/7 + (14*a^6*b^2*x^10)/5 + (56*a^5*b^3*x^13)/13
+ (35*a^4*b^4*x^16)/8 + (56*a^3*b^5*x^19)/19 + (14*a^2*b^6*x^22)/11 + (8*a*
b^7*x^25)/25 + (b^8*x^28)/28
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^3\right)^8 dx &= \int \left(a^8x^3+8a^7bx^6+28a^6b^2x^9+56a^5b^3x^{12}+70a^4b^4x^{15}+56a^3b^5x^{18}+28a^2b^6x^{21}+8ab\right.\\ &= \frac{a^8x^4}{4}+\frac{8}{7}a^7bx^7+\frac{14}{5}a^6b^2x^{10}+\frac{56}{13}a^5b^3x^{13}+\frac{35}{8}a^4b^4x^{16}+\frac{56}{19}a^3b^5x^{19}+\frac{14}{11}a^2b^6x^{22}+\frac{8}{25}\end{aligned}$$

Mathematica [A] time = 0.00, size = 108, normalized size = 1.00

$$\frac{a^8x^4}{4}+\frac{8}{7}a^7bx^7+\frac{14}{5}a^6b^2x^{10}+\frac{56}{13}a^5b^3x^{13}+\frac{35}{8}a^4b^4x^{16}+\frac{56}{19}a^3b^5x^{19}+\frac{14}{11}a^2b^6x^{22}+\frac{8}{25}ab^7x^{25}+\frac{b^8x^{28}}{28}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^3)^8,x]
```

```
[Out] (a^8*x^4)/4 + (8*a^7*b*x^7)/7 + (14*a^6*b^2*x^10)/5 + (56*a^5*b^3*x^13)/13
+ (35*a^4*b^4*x^16)/8 + (56*a^3*b^5*x^19)/19 + (14*a^2*b^6*x^22)/11 + (8*a*
b^7*x^25)/25 + (b^8*x^28)/28
```

fricas [A] time = 0.39, size = 90, normalized size = 0.83

$$\frac{1}{28}x^{28}b^8+\frac{8}{25}x^{25}b^7a+\frac{14}{11}x^{22}b^6a^2+\frac{56}{19}x^{19}b^5a^3+\frac{35}{8}x^{16}b^4a^4+\frac{56}{13}x^{13}b^3a^5+\frac{14}{5}x^{10}b^2a^6+\frac{8}{7}x^7ba^7+\frac{1}{4}x^4a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^8,x, algorithm="fricas")
```

```
[Out] 1/28*x^28*b^8 + 8/25*x^25*b^7*a + 14/11*x^22*b^6*a^2 + 56/19*x^19*b^5*a^3 +
 35/8*x^16*b^4*a^4 + 56/13*x^13*b^3*a^5 + 14/5*x^10*b^2*a^6 + 8/7*x^7*b*a^7
 + 1/4*x^4*a^8
```

giac [A] time = 0.15, size = 90, normalized size = 0.83

$$\frac{1}{28}b^8x^{28}+\frac{8}{25}ab^7x^{25}+\frac{14}{11}a^2b^6x^{22}+\frac{56}{19}a^3b^5x^{19}+\frac{35}{8}a^4b^4x^{16}+\frac{56}{13}a^5b^3x^{13}+\frac{14}{5}a^6b^2x^{10}+\frac{8}{7}a^7bx^7+\frac{1}{4}a^8x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^8,x, algorithm="giac")
```

```
[Out] 1/28*b^8*x^28 + 8/25*a*b^7*x^25 + 14/11*a^2*b^6*x^22 + 56/19*a^3*b^5*x^19 +
 35/8*a^4*b^4*x^16 + 56/13*a^5*b^3*x^13 + 14/5*a^6*b^2*x^10 + 8/7*a^7*b*x^7
 + 1/4*a^8*x^4
```

maple [A] time = 0.00, size = 91, normalized size = 0.84

$$\frac{1}{28}b^8x^{28}+\frac{8}{25}a\,b^7x^{25}+\frac{14}{11}a^2b^6x^{22}+\frac{56}{19}a^3b^5x^{19}+\frac{35}{8}a^4b^4x^{16}+\frac{56}{13}a^5b^3x^{13}+\frac{14}{5}a^6b^2x^{10}+\frac{8}{7}a^7b\,x^7+\frac{1}{4}a^8x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)^8,x)
```

```
[Out] 1/4*a^8*x^4+8/7*a^7*b*x^7+14/5*a^6*b^2*x^10+56/13*a^5*b^3*x^13+35/8*a^4*b^4
*x^16+56/19*a^3*b^5*x^19+14/11*a^2*b^6*x^22+8/25*a*b^7*x^25+1/28*b^8*x^28
```

maxima [A] time = 1.30, size = 90, normalized size = 0.83

$$\frac{1}{28}b^8x^{28}+\frac{8}{25}ab^7x^{25}+\frac{14}{11}a^2b^6x^{22}+\frac{56}{19}a^3b^5x^{19}+\frac{35}{8}a^4b^4x^{16}+\frac{56}{13}a^5b^3x^{13}+\frac{14}{5}a^6b^2x^{10}+\frac{8}{7}a^7bx^7+\frac{1}{4}a^8x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^8,x, algorithm="maxima")
```

```
[Out] 1/28*b^8*x^28 + 8/25*a*b^7*x^25 + 14/11*a^2*b^6*x^22 + 56/19*a^3*b^5*x^19 +
 35/8*a^4*b^4*x^16 + 56/13*a^5*b^3*x^13 + 14/5*a^6*b^2*x^10 + 8/7*a^7*b*x^7
 + 1/4*a^8*x^4
```

mupad [B] time = 0.09, size = 90, normalized size = 0.83

$$\frac{a^8\,x^4}{4}+\frac{8\,a^7\,b\,x^7}{7}+\frac{14\,a^6\,b^2\,x^{10}}{5}+\frac{56\,a^5\,b^3\,x^{13}}{13}+\frac{35\,a^4\,b^4\,x^{16}}{8}+\frac{56\,a^3\,b^5\,x^{19}}{19}+\frac{14\,a^2\,b^6\,x^{22}}{11}+\frac{8\,a\,b^7\,x^{25}}{25}+\frac{b^8\,x^{28}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^3)^8,x)
```

```
[Out] (a^8*x^4)/4 + (b^8*x^28)/28 + (8*a^7*b*x^7)/7 + (8*a*b^7*x^25)/25 + (14*a^6
*b^2*x^10)/5 + (56*a^5*b^3*x^13)/13 + (35*a^4*b^4*x^16)/8 + (56*a^3*b^5*x^1
9)/19 + (14*a^2*b^6*x^22)/11
```

sympy [A] time = 0.10, size = 107, normalized size = 0.99

$$\frac{a^8x^4}{4}+\frac{8a^7bx^7}{7}+\frac{14a^6b^2x^{10}}{5}+\frac{56a^5b^3x^{13}}{13}+\frac{35a^4b^4x^{16}}{8}+\frac{56a^3b^5x^{19}}{19}+\frac{14a^2b^6x^{22}}{11}+\frac{8ab^7x^{25}}{25}+\frac{b^8x^{28}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**8,x)
```

```
[Out] a**8*x**4/4 + 8*a**7*b*x**7/7 + 14*a**6*b**2*x**10/5 + 56*a**5*b**3*x**13/1
3 + 35*a**4*b**4*x**16/8 + 56*a**3*b**5*x**19/19 + 14*a**2*b**6*x**22/11 +
8*a*b**7*x**25/25 + b**8*x**28/28
```

3.310 $\int x\left(a+bx^3\right)^8\,dx$

Optimal. Leaf size=106

$$\frac{a^8x^2}{2}+\frac{8}{5}a^7bx^5+\frac{7}{2}a^6b^2x^8+\frac{56}{11}a^5b^3x^{11}+5a^4b^4x^{14}+\frac{56}{17}a^3b^5x^{17}+\frac{7}{5}a^2b^6x^{20}+\frac{8}{23}ab^7x^{23}+\frac{b^8x^{26}}{26}$$

```
[Out] 1/2*a^8*x^2+8/5*a^7*b*x^5+7/2*a^6*b^2*x^8+56/11*a^5*b^3*x^11+5*a^4*b^4*x^14
+56/17*a^3*b^5*x^17+7/5*a^2*b^6*x^20+8/23*a*b^7*x^23+1/26*b^8*x^26
```

Rubi [A] time = 0.04, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {270}

$$\frac{7}{5}a^2b^6x^{20}+\frac{56}{17}a^3b^5x^{17}+5a^4b^4x^{14}+\frac{56}{11}a^5b^3x^{11}+\frac{7}{2}a^6b^2x^8+\frac{8}{5}a^7bx^5+\frac{a^8x^2}{2}+\frac{8}{23}ab^7x^{23}+\frac{b^8x^{26}}{26}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3)^8,x]
```

```
[Out] (a^8*x^2)/2 + (8*a^7*b*x^5)/5 + (7*a^6*b^2*x^8)/2 + (56*a^5*b^3*x^11)/11 +
5*a^4*b^4*x^14 + (56*a^3*b^5*x^17)/17 + (7*a^2*b^6*x^20)/5 + (8*a*b^7*x^23)
/23 + (b^8*x^26)/26
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^3\right)^8\,dx &= \int\left(a^8x+8a^7bx^4+28a^6b^2x^7+56a^5b^3x^{10}+70a^4b^4x^{13}+56a^3b^5x^{16}+28a^2b^6x^{19}+8ab^7x\right.\\ &= \frac{a^8x^2}{2}+\frac{8}{5}a^7bx^5+\frac{7}{2}a^6b^2x^8+\frac{56}{11}a^5b^3x^{11}+5a^4b^4x^{14}+\frac{56}{17}a^3b^5x^{17}+\frac{7}{5}a^2b^6x^{20}+\frac{8}{23}ab^7x^{23}\end{aligned}$$

Mathematica [A] time = 0.00, size = 106, normalized size = 1.00

$$\frac{a^8x^2}{2}+\frac{8}{5}a^7bx^5+\frac{7}{2}a^6b^2x^8+\frac{56}{11}a^5b^3x^{11}+5a^4b^4x^{14}+\frac{56}{17}a^3b^5x^{17}+\frac{7}{5}a^2b^6x^{20}+\frac{8}{23}ab^7x^{23}+\frac{b^8x^{26}}{26}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)^8,x]
```

```
[Out] (a^8*x^2)/2 + (8*a^7*b*x^5)/5 + (7*a^6*b^2*x^8)/2 + (56*a^5*b^3*x^11)/11 +
5*a^4*b^4*x^14 + (56*a^3*b^5*x^17)/17 + (7*a^2*b^6*x^20)/5 + (8*a*b^7*x^23)
/23 + (b^8*x^26)/26
```

fricas [A] time = 0.48, size = 90, normalized size = 0.85

$$\frac{1}{26}x^{26}b^8+\frac{8}{23}x^{23}b^7a+\frac{7}{5}x^{20}b^6a^2+\frac{56}{17}x^{17}b^5a^3+5x^{14}b^4a^4+\frac{56}{11}x^{11}b^3a^5+\frac{7}{2}x^8b^2a^6+\frac{8}{5}x^5ba^7+\frac{1}{2}x^2a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^8,x, algorithm="fricas")
```

```
[Out] 1/26*x^26*b^8 + 8/23*x^23*b^7*a + 7/5*x^20*b^6*a^2 + 56/17*x^17*b^5*a^3 + 5
*x^14*b^4*a^4 + 56/11*x^11*b^3*a^5 + 7/2*x^8*b^2*a^6 + 8/5*x^5*b*a^7 + 1/2*
x^2*a^8
```

giac [A] time = 0.17, size = 90, normalized size = 0.85

$$\frac{1}{26}b^8x^{26}+\frac{8}{23}ab^7x^{23}+\frac{7}{5}a^2b^6x^{20}+\frac{56}{17}a^3b^5x^{17}+5a^4b^4x^{14}+\frac{56}{11}a^5b^3x^{11}+\frac{7}{2}a^6b^2x^8+\frac{8}{5}a^7bx^5+\frac{1}{2}a^8x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^8,x, algorithm="giac")
```

```
[Out] 1/26*b^8*x^26 + 8/23*a*b^7*x^23 + 7/5*a^2*b^6*x^20 + 56/17*a^3*b^5*x^17 + 5
*a^4*b^4*x^14 + 56/11*a^5*b^3*x^11 + 7/2*a^6*b^2*x^8 + 8/5*a^7*b*x^5 + 1/2*
a^8*x^2
```

maple [A] time = 0.00, size = 91, normalized size = 0.86

$$\frac{1}{26}b^8x^{26}+\frac{8}{23}a\,b^7x^{23}+\frac{7}{5}a^2b^6x^{20}+\frac{56}{17}a^3b^5x^{17}+5a^4b^4x^{14}+\frac{56}{11}a^5b^3x^{11}+\frac{7}{2}a^6b^2x^8+\frac{8}{5}a^7b\,x^5+\frac{1}{2}a^8x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^8,x)
```

```
[Out] 1/2*a^8*x^2+8/5*a^7*b*x^5+7/2*a^6*b^2*x^8+56/11*a^5*b^3*x^11+5*a^4*b^4*x^14
+56/17*a^3*b^5*x^17+7/5*a^2*b^6*x^20+8/23*a*b^7*x^23+1/26*b^8*x^26
```

maxima [A] time = 1.33, size = 90, normalized size = 0.85

$$\frac{1}{26}b^8x^{26}+\frac{8}{23}ab^7x^{23}+\frac{7}{5}a^2b^6x^{20}+\frac{56}{17}a^3b^5x^{17}+5a^4b^4x^{14}+\frac{56}{11}a^5b^3x^{11}+\frac{7}{2}a^6b^2x^8+\frac{8}{5}a^7bx^5+\frac{1}{2}a^8x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^8,x, algorithm="maxima")
```

```
[Out] 1/26*b^8*x^26 + 8/23*a*b^7*x^23 + 7/5*a^2*b^6*x^20 + 56/17*a^3*b^5*x^17 + 5
*a^4*b^4*x^14 + 56/11*a^5*b^3*x^11 + 7/2*a^6*b^2*x^8 + 8/5*a^7*b*x^5 + 1/2*
a^8*x^2
```

mupad [B] time = 0.98, size = 90, normalized size = 0.85

$$\frac{a^8\,x^2}{2}+\frac{8\,a^7\,b\,x^5}{5}+\frac{7\,a^6\,b^2\,x^8}{2}+\frac{56\,a^5\,b^3\,x^{11}}{11}+5\,a^4\,b^4\,x^{14}+\frac{56\,a^3\,b^5\,x^{17}}{17}+\frac{7\,a^2\,b^6\,x^{20}}{5}+\frac{8\,a\,b^7\,x^{23}}{23}+\frac{b^8\,x^{26}}{26}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^3)^8,x)
```

```
[Out] (a^8*x^2)/2 + (b^8*x^26)/26 + (8*a^7*b*x^5)/5 + (8*a*b^7*x^23)/23 + (7*a^6*
b^2*x^8)/2 + (56*a^5*b^3*x^11)/11 + 5*a^4*b^4*x^14 + (56*a^3*b^5*x^17)/17 +
 (7*a^2*b^6*x^20)/5
```

sympy [A] time = 0.09, size = 105, normalized size = 0.99

$$\frac{a^8x^2}{2}+\frac{8a^7bx^5}{5}+\frac{7a^6b^2x^8}{2}+\frac{56a^5b^3x^{11}}{11}+5a^4b^4x^{14}+\frac{56a^3b^5x^{17}}{17}+\frac{7a^2b^6x^{20}}{5}+\frac{8ab^7x^{23}}{23}+\frac{b^8x^{26}}{26}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**8,x)
```

```
[Out] a**8*x**2/2 + 8*a**7*b*x**5/5 + 7*a**6*b**2*x**8/2 + 56*a**5*b**3*x**11/11
+ 5*a**4*b**4*x**14 + 56*a**3*b**5*x**17/17 + 7*a**2*b**6*x**20/5 + 8*a*b**
7*x**23/23 + b**8*x**26/26
```

3.311 $\int \left(a+bx^3\right)^8 dx$

Optimal. Leaf size=99

$$a^8x+2a^7bx^4+4a^6b^2x^7+\frac{28}{5}a^5b^3x^{10}+\frac{70}{13}a^4b^4x^{13}+\frac{7}{2}a^3b^5x^{16}+\frac{28}{19}a^2b^6x^{19}+\frac{4}{11}ab^7x^{22}+\frac{b^8x^{25}}{25}$$

```
[Out] a^8*x+2*a^7*b*x^4+4*a^6*b^2*x^7+28/5*a^5*b^3*x^10+70/13*a^4*b^4*x^13+7/2*a^
3*b^5*x^16+28/19*a^2*b^6*x^19+4/11*a*b^7*x^22+1/25*b^8*x^25
```

Rubi [A] time = 0.04, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {194}

$$\frac{28}{19}a^2b^6x^{19}+\frac{7}{2}a^3b^5x^{16}+\frac{70}{13}a^4b^4x^{13}+\frac{28}{5}a^5b^3x^{10}+4a^6b^2x^7+2a^7bx^4+a^8x+\frac{4}{11}ab^7x^{22}+\frac{b^8x^{25}}{25}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^8,x]
```

```
[Out] a^8*x + 2*a^7*b*x^4 + 4*a^6*b^2*x^7 + (28*a^5*b^3*x^10)/5 + (70*a^4*b^4*x^1
3)/13 + (7*a^3*b^5*x^16)/2 + (28*a^2*b^6*x^19)/19 + (4*a*b^7*x^22)/11 + (b^
8*x^25)/25
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(a+bx^3\right)^8 dx = \int \left(a^8+8a^7bx^3+28a^6b^2x^6+56a^5b^3x^9+70a^4b^4x^{12}+56a^3b^5x^{15}+28a^2b^6x^{18}+8ab^7x^{21}+$$

$$= a^8x+2a^7bx^4+4a^6b^2x^7+\frac{28}{5}a^5b^3x^{10}+\frac{70}{13}a^4b^4x^{13}+\frac{7}{2}a^3b^5x^{16}+\frac{28}{19}a^2b^6x^{19}+\frac{4}{11}ab^7x^{22}+$$

Mathematica [A] time = 0.00, size = 99, normalized size = 1.00

$$a^8x+2a^7bx^4+4a^6b^2x^7+\frac{28}{5}a^5b^3x^{10}+\frac{70}{13}a^4b^4x^{13}+\frac{7}{2}a^3b^5x^{16}+\frac{28}{19}a^2b^6x^{19}+\frac{4}{11}ab^7x^{22}+\frac{b^8x^{25}}{25}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^8,x]
```

```
[Out] a^8*x + 2*a^7*b*x^4 + 4*a^6*b^2*x^7 + (28*a^5*b^3*x^10)/5 + (70*a^4*b^4*x^1
3)/13 + (7*a^3*b^5*x^16)/2 + (28*a^2*b^6*x^19)/19 + (4*a*b^7*x^22)/11 + (b^
8*x^25)/25
```

fricas [A] time = 0.43, size = 87, normalized size = 0.88

$$\frac{1}{25}x^{25}b^8+\frac{4}{11}x^{22}b^7a+\frac{28}{19}x^{19}b^6a^2+\frac{7}{2}x^{16}b^5a^3+\frac{70}{13}x^{13}b^4a^4+\frac{28}{5}x^{10}b^3a^5+4x^7b^2a^6+2x^4ba^7+xa^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8,x, algorithm="fricas")
```

```
[Out] 1/25*x^25*b^8 + 4/11*x^22*b^7*a + 28/19*x^19*b^6*a^2 + 7/2*x^16*b^5*a^3 + 7
0/13*x^13*b^4*a^4 + 28/5*x^10*b^3*a^5 + 4*x^7*b^2*a^6 + 2*x^4*b*a^7 + x*a^8
```

giac [A] time = 0.15, size = 87, normalized size = 0.88

$$\frac{1}{25}b^8x^{25}+\frac{4}{11}ab^7x^{22}+\frac{28}{19}a^2b^6x^{19}+\frac{7}{2}a^3b^5x^{16}+\frac{70}{13}a^4b^4x^{13}+\frac{28}{5}a^5b^3x^{10}+4a^6b^2x^7+2a^7bx^4+a^8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8,x, algorithm="giac")
```

```
[Out] 1/25*b^8*x^25 + 4/11*a*b^7*x^22 + 28/19*a^2*b^6*x^19 + 7/2*a^3*b^5*x^16 + 7
0/13*a^4*b^4*x^13 + 28/5*a^5*b^3*x^10 + 4*a^6*b^2*x^7 + 2*a^7*b*x^4 + a^8*x
```

maple [A] time = 0.00, size = 88, normalized size = 0.89

$$\frac{1}{25}b^8x^{25}+\frac{4}{11}a\,b^7x^{22}+\frac{28}{19}a^2b^6x^{19}+\frac{7}{2}a^3b^5x^{16}+\frac{70}{13}a^4b^4x^{13}+\frac{28}{5}a^5b^3x^{10}+4a^6b^2x^7+2a^7b\,x^4+a^8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^8,x)
```

```
[Out] a^8*x+2*a^7*b*x^4+4*a^6*b^2*x^7+28/5*a^5*b^3*x^10+70/13*a^4*b^4*x^13+7/2*a^
3*b^5*x^16+28/19*a^2*b^6*x^19+4/11*a*b^7*x^22+1/25*b^8*x^25
```

maxima [A] time = 1.26, size = 87, normalized size = 0.88

$$\frac{1}{25}b^8x^{25}+\frac{4}{11}ab^7x^{22}+\frac{28}{19}a^2b^6x^{19}+\frac{7}{2}a^3b^5x^{16}+\frac{70}{13}a^4b^4x^{13}+\frac{28}{5}a^5b^3x^{10}+4a^6b^2x^7+2a^7bx^4+a^8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8,x, algorithm="maxima")
```

```
[Out] 1/25*b^8*x^25 + 4/11*a*b^7*x^22 + 28/19*a^2*b^6*x^19 + 7/2*a^3*b^5*x^16 + 7
0/13*a^4*b^4*x^13 + 28/5*a^5*b^3*x^10 + 4*a^6*b^2*x^7 + 2*a^7*b*x^4 + a^8*x
```

mupad [B] time = 0.04, size = 87, normalized size = 0.88

$$a^8\,x+2\,a^7\,b\,x^4+4\,a^6\,b^2\,x^7+\frac{28\,a^5\,b^3\,x^{10}}{5}+\frac{70\,a^4\,b^4\,x^{13}}{13}+\frac{7\,a^3\,b^5\,x^{16}}{2}+\frac{28\,a^2\,b^6\,x^{19}}{19}+\frac{4\,a\,b^7\,x^{22}}{11}+\frac{b^8\,x^{25}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^8,x)
```

```
[Out] a^8*x + (b^8*x^25)/25 + 2*a^7*b*x^4 + (4*a*b^7*x^22)/11 + 4*a^6*b^2*x^7 + (
28*a^5*b^3*x^10)/5 + (70*a^4*b^4*x^13)/13 + (7*a^3*b^5*x^16)/2 + (28*a^2*b^
6*x^19)/19
```

sympy [A] time = 0.09, size = 100, normalized size = 1.01

$$a^8x + 2a^7bx^4 + 4a^6b^2x^7 + \frac{28a^5b^3x^{10}}{5} + \frac{70a^4b^4x^{13}}{13} + \frac{7a^3b^5x^{16}}{2} + \frac{28a^2b^6x^{19}}{19} + \frac{4ab^7x^{22}}{11} + \frac{b^8x^{25}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**8,x)
```

```
[Out] a**8*x + 2*a**7*b*x**4 + 4*a**6*b**2*x**7 + 28*a**5*b**3*x**10/5 + 70*a**4*
b**4*x**13/13 + 7*a**3*b**5*x**16/2 + 28*a**2*b**6*x**19/19 + 4*a*b**7*x**2
2/11 + b**8*x**25/25
```

3.312 $\int \frac{\left(a+bx^3\right)^8}{x^2}\,dx$

Optimal. Leaf size=100

$$-\frac{a^8}{x}+4a^7bx^2+\frac{28}{5}a^6b^2x^5+7a^5b^3x^8+\frac{70}{11}a^4b^4x^{11}+4a^3b^5x^{14}+\frac{28}{17}a^2b^6x^{17}+\frac{2}{5}ab^7x^{20}+\frac{b^8x^{23}}{23}$$

```
[Out] -a^8/x+4*a^7*b*x^2+28/5*a^6*b^2*x^5+7*a^5*b^3*x^8+70/11*a^4*b^4*x^11+4*a^3*
b^5*x^14+28/17*a^2*b^6*x^17+2/5*a*b^7*x^20+1/23*b^8*x^23
```

Rubi [A] time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{28}{17}a^2b^6x^{17}+4a^3b^5x^{14}+\frac{70}{11}a^4b^4x^{11}+7a^5b^3x^8+\frac{28}{5}a^6b^2x^5+4a^7bx^2-\frac{a^8}{x}+\frac{2}{5}ab^7x^{20}+\frac{b^8x^{23}}{23}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^8/x^2,x]
```

```
[Out] -(a^8/x) + 4*a^7*b*x^2 + (28*a^6*b^2*x^5)/5 + 7*a^5*b^3*x^8 + (70*a^4*b^4*x
^11)/11 + 4*a^3*b^5*x^14 + (28*a^2*b^6*x^17)/17 + (2*a*b^7*x^20)/5 + (b^8*x
^23)/23
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^8}{x^2}\,dx = \int\left(\frac{a^8}{x^2}+8a^7bx+28a^6b^2x^4+56a^5b^3x^7+70a^4b^4x^{10}+56a^3b^5x^{13}+28a^2b^6x^{16}+8ab^7x^{19}+$$
$$= -\frac{a^8}{x}+4a^7bx^2+\frac{28}{5}a^6b^2x^5+7a^5b^3x^8+\frac{70}{11}a^4b^4x^{11}+4a^3b^5x^{14}+\frac{28}{17}a^2b^6x^{17}+\frac{2}{5}ab^7x^{20}+\frac{b}{}$$

Mathematica [A] time = 0.01, size = 100, normalized size = 1.00

$$-\frac{a^8}{x}+4a^7bx^2+\frac{28}{5}a^6b^2x^5+7a^5b^3x^8+\frac{70}{11}a^4b^4x^{11}+4a^3b^5x^{14}+\frac{28}{17}a^2b^6x^{17}+\frac{2}{5}ab^7x^{20}+\frac{b^8x^{23}}{23}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^8/x^2,x]
```

```
[Out] -(a^8/x) + 4*a^7*b*x^2 + (28*a^6*b^2*x^5)/5 + 7*a^5*b^3*x^8 + (70*a^4*b^4*x
^11)/11 + 4*a^3*b^5*x^14 + (28*a^2*b^6*x^17)/17 + (2*a*b^7*x^20)/5 + (b^8*x
^23)/23
```

fricas [A] time = 0.56, size = 92, normalized size = 0.92

$$\frac{935\,b^8x^{24}+8602\,ab^7x^{21}+35420\,a^2b^6x^{18}+86020\,a^3b^5x^{15}+136850\,a^4b^4x^{12}+150535\,a^5b^3x^9+120428\,a^6b^2x}{21505\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^2,x, algorithm="fricas")
```

```
[Out] 1/21505*(935*b^8*x^24 + 8602*a*b^7*x^21 + 35420*a^2*b^6*x^18 + 86020*a^3*b^
5*x^15 + 136850*a^4*b^4*x^12 + 150535*a^5*b^3*x^9 + 120428*a^6*b^2*x^6 + 86
020*a^7*b*x^3 - 21505*a^8)/x
```

giac [A] time = 0.16, size = 90, normalized size = 0.90

$$\frac{1}{23}b^8x^{23}+\frac{2}{5}ab^7x^{20}+\frac{28}{17}a^2b^6x^{17}+4a^3b^5x^{14}+\frac{70}{11}a^4b^4x^{11}+7a^5b^3x^8+\frac{28}{5}a^6b^2x^5+4a^7bx^2-\frac{a^8}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^2,x, algorithm="giac")
```

```
[Out] 1/23*b^8*x^23 + 2/5*a*b^7*x^20 + 28/17*a^2*b^6*x^17 + 4*a^3*b^5*x^14 + 70/1
1*a^4*b^4*x^11 + 7*a^5*b^3*x^8 + 28/5*a^6*b^2*x^5 + 4*a^7*b*x^2 - a^8/x
```

maple [A] time = 0.00, size = 91, normalized size = 0.91

$$\frac{b^8x^{23}}{23}+\frac{2a\,b^7x^{20}}{5}+\frac{28a^2b^6x^{17}}{17}+4a^3b^5x^{14}+\frac{70a^4b^4x^{11}}{11}+7a^5b^3x^8+\frac{28a^6b^2x^5}{5}+4a^7b\,x^2-\frac{a^8}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^8/x^2,x)
```

```
[Out] -a^8/x+4*a^7*b*x^2+28/5*a^6*b^2*x^5+7*a^5*b^3*x^8+70/11*a^4*b^4*x^11+4*a^3*
b^5*x^14+28/17*a^2*b^6*x^17+2/5*a*b^7*x^20+1/23*b^8*x^23
```

maxima [A] time = 1.33, size = 90, normalized size = 0.90

$$\frac{1}{23}b^8x^{23}+\frac{2}{5}ab^7x^{20}+\frac{28}{17}a^2b^6x^{17}+4a^3b^5x^{14}+\frac{70}{11}a^4b^4x^{11}+7a^5b^3x^8+\frac{28}{5}a^6b^2x^5+4a^7bx^2-\frac{a^8}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^2,x, algorithm="maxima")
```

```
[Out] 1/23*b^8*x^23 + 2/5*a*b^7*x^20 + 28/17*a^2*b^6*x^17 + 4*a^3*b^5*x^14 + 70/1
1*a^4*b^4*x^11 + 7*a^5*b^3*x^8 + 28/5*a^6*b^2*x^5 + 4*a^7*b*x^2 - a^8/x
```

mupad [B] time = 0.05, size = 90, normalized size = 0.90

$$\frac{b^8\,x^{23}}{23}-\frac{a^8}{x}+4\,a^7\,b\,x^2+\frac{2\,a\,b^7\,x^{20}}{5}+\frac{28\,a^6\,b^2\,x^5}{5}+7\,a^5\,b^3\,x^8+\frac{70\,a^4\,b^4\,x^{11}}{11}+4\,a^3\,b^5\,x^{14}+\frac{28\,a^2\,b^6\,x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^8/x^2,x)
```

```
[Out] (b^8*x^23)/23 - a^8/x + 4*a^7*b*x^2 + (2*a*b^7*x^20)/5 + (28*a^6*b^2*x^5)/5
 + 7*a^5*b^3*x^8 + (70*a^4*b^4*x^11)/11 + 4*a^3*b^5*x^14 + (28*a^2*b^6*x^17
)/17
```

sympy [A] time = 0.21, size = 99, normalized size = 0.99

$$-\frac{a^8}{x}+4a^7bx^2+\frac{28a^6b^2x^5}{5}+7a^5b^3x^8+\frac{70a^4b^4x^{11}}{11}+4a^3b^5x^{14}+\frac{28a^2b^6x^{17}}{17}+\frac{2ab^7x^{20}}{5}+\frac{b^8x^{23}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**8/x**2,x)
```

```
[Out] -a**8/x + 4*a**7*b*x**2 + 28*a**6*b**2*x**5/5 + 7*a**5*b**3*x**8 + 70*a**4*
b**4*x**11/11 + 4*a**3*b**5*x**14 + 28*a**2*b**6*x**17/17 + 2*a*b**7*x**20/
5 + b**8*x**23/23
```

3.313 $\int \frac{\left(a+bx^3\right)^8}{x^3}\,dx$

Optimal. Leaf size=98

$$-\frac{a^8}{2x^2}+8a^7bx+7a^6b^2x^4+8a^5b^3x^7+7a^4b^4x^{10}+\frac{56}{13}a^3b^5x^{13}+\frac{7}{4}a^2b^6x^{16}+\frac{8}{19}ab^7x^{19}+\frac{b^8x^{22}}{22}$$

```
[Out] -1/2*a^8/x^2+8*a^7*b*x+7*a^6*b^2*x^4+8*a^5*b^3*x^7+7*a^4*b^4*x^10+56/13*a^3
*b^5*x^13+7/4*a^2*b^6*x^16+8/19*a*b^7*x^19+1/22*b^8*x^22
```

Rubi [A] time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{7}{4}a^2b^6x^{16}+\frac{56}{13}a^3b^5x^{13}+7a^4b^4x^{10}+8a^5b^3x^7+7a^6b^2x^4+8a^7bx-\frac{a^8}{2x^2}+\frac{8}{19}ab^7x^{19}+\frac{b^8x^{22}}{22}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^8/x^3,x]
```

```
[Out] -a^8/(2*x^2) + 8*a^7*b*x + 7*a^6*b^2*x^4 + 8*a^5*b^3*x^7 + 7*a^4*b^4*x^10 +
 (56*a^3*b^5*x^13)/13 + (7*a^2*b^6*x^16)/4 + (8*a*b^7*x^19)/19 + (b^8*x^22)
/22
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^8}{x^3}\,dx = \int \left(8a^7b+\frac{a^8}{x^3}+28a^6b^2x^3+56a^5b^3x^6+70a^4b^4x^9+56a^3b^5x^{12}+28a^2b^6x^{15}+8ab^7x^{18}+b^8\right.$$
$$= -\frac{a^8}{2x^2}+8a^7bx+7a^6b^2x^4+8a^5b^3x^7+7a^4b^4x^{10}+\frac{56}{13}a^3b^5x^{13}+\frac{7}{4}a^2b^6x^{16}+\frac{8}{19}ab^7x^{19}+\frac{b^8}{2}$$

Mathematica [A] time = 0.01, size = 98, normalized size = 1.00

$$-\frac{a^8}{2x^2}+8a^7bx+7a^6b^2x^4+8a^5b^3x^7+7a^4b^4x^{10}+\frac{56}{13}a^3b^5x^{13}+\frac{7}{4}a^2b^6x^{16}+\frac{8}{19}ab^7x^{19}+\frac{b^8x^{22}}{22}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^8/x^3,x]
```

```
[Out] -1/2*a^8/x^2 + 8*a^7*b*x + 7*a^6*b^2*x^4 + 8*a^5*b^3*x^7 + 7*a^4*b^4*x^10 +
 (56*a^3*b^5*x^13)/13 + (7*a^2*b^6*x^16)/4 + (8*a*b^7*x^19)/19 + (b^8*x^22)
/22
```

fricas [A] time = 0.53, size = 92, normalized size = 0.94

$$\frac{494\,b^8x^{24}+4576\,ab^7x^{21}+19019\,a^2b^6x^{18}+46816\,a^3b^5x^{15}+76076\,a^4b^4x^{12}+86944\,a^5b^3x^9+76076\,a^6b^2x^6+}{10868\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^3,x, algorithm="fricas")
```

```
[Out] 1/10868*(494*b^8*x^24 + 4576*a*b^7*x^21 + 19019*a^2*b^6*x^18 + 46816*a^3*b^
5*x^15 + 76076*a^4*b^4*x^12 + 86944*a^5*b^3*x^9 + 76076*a^6*b^2*x^6 + 86944
*a^7*b*x^3 - 5434*a^8)/x^2
```

giac [A] time = 0.15, size = 88, normalized size = 0.90

$$\frac{1}{22}b^8x^{22}+\frac{8}{19}ab^7x^{19}+\frac{7}{4}a^2b^6x^{16}+\frac{56}{13}a^3b^5x^{13}+7a^4b^4x^{10}+8a^5b^3x^7+7a^6b^2x^4+8a^7bx-\frac{a^8}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^3,x, algorithm="giac")
```

```
[Out] 1/22*b^8*x^22 + 8/19*a*b^7*x^19 + 7/4*a^2*b^6*x^16 + 56/13*a^3*b^5*x^13 + 7
*a^4*b^4*x^10 + 8*a^5*b^3*x^7 + 7*a^6*b^2*x^4 + 8*a^7*b*x - 1/2*a^8/x^2
```

maple [A] time = 0.00, size = 89, normalized size = 0.91

$$\frac{b^8x^{22}}{22}+\frac{8ab^7x^{19}}{19}+\frac{7a^2b^6x^{16}}{4}+\frac{56a^3b^5x^{13}}{13}+7a^4b^4x^{10}+8a^5b^3x^7+7a^6b^2x^4+8a^7bx-\frac{a^8}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^8/x^3,x)
```

```
[Out] -1/2*a^8/x^2+8*a^7*b*x+7*a^6*b^2*x^4+8*a^5*b^3*x^7+7*a^4*b^4*x^10+56/13*a^3
*b^5*x^13+7/4*a^2*b^6*x^16+8/19*a*b^7*x^19+1/22*b^8*x^22
```

maxima [A] time = 1.34, size = 88, normalized size = 0.90

$$\frac{1}{22}b^8x^{22}+\frac{8}{19}ab^7x^{19}+\frac{7}{4}a^2b^6x^{16}+\frac{56}{13}a^3b^5x^{13}+7a^4b^4x^{10}+8a^5b^3x^7+7a^6b^2x^4+8a^7bx-\frac{a^8}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^3,x, algorithm="maxima")
```

```
[Out] 1/22*b^8*x^22 + 8/19*a*b^7*x^19 + 7/4*a^2*b^6*x^16 + 56/13*a^3*b^5*x^13 + 7
*a^4*b^4*x^10 + 8*a^5*b^3*x^7 + 7*a^6*b^2*x^4 + 8*a^7*b*x - 1/2*a^8/x^2
```

mupad [B] time = 0.05, size = 88, normalized size = 0.90

$$\frac{b^8\,x^{22}}{22}-\frac{a^8}{2\,x^2}+\frac{8\,a\,b^7\,x^{19}}{19}+7\,a^6\,b^2\,x^4+8\,a^5\,b^3\,x^7+7\,a^4\,b^4\,x^{10}+\frac{56\,a^3\,b^5\,x^{13}}{13}+\frac{7\,a^2\,b^6\,x^{16}}{4}+8\,a^7\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^8/x^3,x)
```

```
[Out] (b^8*x^22)/22 - a^8/(2*x^2) + (8*a*b^7*x^19)/19 + 7*a^6*b^2*x^4 + 8*a^5*b^3
*x^7 + 7*a^4*b^4*x^10 + (56*a^3*b^5*x^13)/13 + (7*a^2*b^6*x^16)/4 + 8*a^7*b
*x
```

sympy [A] time = 0.21, size = 99, normalized size = 1.01

$$-\frac{a^8}{2x^2}+8a^7bx+7a^6b^2x^4+8a^5b^3x^7+7a^4b^4x^{10}+\frac{56a^3b^5x^{13}}{13}+\frac{7a^2b^6x^{16}}{4}+\frac{8ab^7x^{19}}{19}+\frac{b^8x^{22}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**8/x**3,x)
```

```
[Out] -a**8/(2*x**2) + 8*a**7*b*x + 7*a**6*b**2*x**4 + 8*a**5*b**3*x**7 + 7*a**4*
b**4*x**10 + 56*a**3*b**5*x**13/13 + 7*a**2*b**6*x**16/4 + 8*a*b**7*x**19/1
9 + b**8*x**22/22
```

3.314 $\int \frac{\left(a+bx^3\right)^8}{x^5}\,dx$

Optimal. Leaf size=102

$$-\frac{a^8}{4x^4}-\frac{8a^7b}{x}+14a^6b^2x^2+\frac{56}{5}a^5b^3x^5+\frac{35}{4}a^4b^4x^8+\frac{56}{11}a^3b^5x^{11}+2a^2b^6x^{14}+\frac{8}{17}ab^7x^{17}+\frac{b^8x^{20}}{20}$$

```
[Out] -1/4*a^8/x^4-8*a^7*b/x+14*a^6*b^2*x^2+56/5*a^5*b^3*x^5+35/4*a^4*b^4*x^8+56/
11*a^3*b^5*x^11+2*a^2*b^6*x^14+8/17*a*b^7*x^17+1/20*b^8*x^20
```

Rubi [A] time = 0.04, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$2a^2b^6x^{14}+\frac{56}{11}a^3b^5x^{11}+\frac{35}{4}a^4b^4x^8+\frac{56}{5}a^5b^3x^5+14a^6b^2x^2-\frac{8a^7b}{x}-\frac{a^8}{4x^4}+\frac{8}{17}ab^7x^{17}+\frac{b^8x^{20}}{20}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^8/x^5,x]
```

```
[Out] -a^8/(4*x^4) - (8*a^7*b)/x + 14*a^6*b^2*x^2 + (56*a^5*b^3*x^5)/5 + (35*a^4*
b^4*x^8)/4 + (56*a^3*b^5*x^11)/11 + 2*a^2*b^6*x^14 + (8*a*b^7*x^17)/17 + (b
^8*x^20)/20
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^8}{x^5}\,dx &= \int\left(\frac{a^8}{x^5}+\frac{8a^7b}{x^2}+28a^6b^2x+56a^5b^3x^4+70a^4b^4x^7+56a^3b^5x^{10}+28a^2b^6x^{13}+8ab^7x^{16}+b^8\right.\\ &= -\frac{a^8}{4x^4}-\frac{8a^7b}{x}+14a^6b^2x^2+\frac{56}{5}a^5b^3x^5+\frac{35}{4}a^4b^4x^8+\frac{56}{11}a^3b^5x^{11}+2a^2b^6x^{14}+\frac{8}{17}ab^7x^{17}+\end{aligned}$$

Mathematica [A] time = 0.01, size = 102, normalized size = 1.00

$$-\frac{a^8}{4x^4}-\frac{8a^7b}{x}+14a^6b^2x^2+\frac{56}{5}a^5b^3x^5+\frac{35}{4}a^4b^4x^8+\frac{56}{11}a^3b^5x^{11}+2a^2b^6x^{14}+\frac{8}{17}ab^7x^{17}+\frac{b^8x^{20}}{20}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^8/x^5,x]
```

```
[Out] -1/4*a^8/x^4 - (8*a^7*b)/x + 14*a^6*b^2*x^2 + (56*a^5*b^3*x^5)/5 + (35*a^4*
b^4*x^8)/4 + (56*a^3*b^5*x^11)/11 + 2*a^2*b^6*x^14 + (8*a*b^7*x^17)/17 + (b
^8*x^20)/20
```

fricas [A] time = 0.61, size = 92, normalized size = 0.90

$$\frac{187\,b^8x^{24}+1760\,ab^7x^{21}+7480\,a^2b^6x^{18}+19040\,a^3b^5x^{15}+32725\,a^4b^4x^{12}+41888\,a^5b^3x^9+52360\,a^6b^2x^6-2}{3740\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^5,x, algorithm="fricas")
```

```
[Out] 1/3740*(187*b^8*x^24 + 1760*a*b^7*x^21 + 7480*a^2*b^6*x^18 + 19040*a^3*b^5*
x^15 + 32725*a^4*b^4*x^12 + 41888*a^5*b^3*x^9 + 52360*a^6*b^2*x^6 - 29920*a
^7*b*x^3 - 935*a^8)/x^4
```

giac [A] time = 0.16, size = 91, normalized size = 0.89

$$\frac{1}{20}\,b^8x^{20}+\frac{8}{17}\,ab^7x^{17}+2\,a^2b^6x^{14}+\frac{56}{11}\,a^3b^5x^{11}+\frac{35}{4}\,a^4b^4x^8+\frac{56}{5}\,a^5b^3x^5+14\,a^6b^2x^2-\frac{32\,a^7bx^3+a^8}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^5,x, algorithm="giac")
```

```
[Out] 1/20*b^8*x^20 + 8/17*a*b^7*x^17 + 2*a^2*b^6*x^14 + 56/11*a^3*b^5*x^11 + 35/
4*a^4*b^4*x^8 + 56/5*a^5*b^3*x^5 + 14*a^6*b^2*x^2 - 1/4*(32*a^7*b*x^3 + a^8
)/x^4
```

maple [A] time = 0.01, size = 91, normalized size = 0.89

$$\frac{b^8x^{20}}{20}+\frac{8a\,b^7x^{17}}{17}+2a^2b^6x^{14}+\frac{56a^3b^5x^{11}}{11}+\frac{35a^4b^4x^8}{4}+\frac{56a^5b^3x^5}{5}+14a^6b^2x^2-\frac{8a^7b}{x}-\frac{a^8}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^8/x^5,x)
```

```
[Out] -1/4*a^8/x^4-8*a^7*b/x+14*a^6*b^2*x^2+56/5*a^5*b^3*x^5+35/4*a^4*b^4*x^8+56/
11*a^3*b^5*x^11+2*a^2*b^6*x^14+8/17*a*b^7*x^17+1/20*b^8*x^20
```

maxima [A] time = 1.33, size = 91, normalized size = 0.89

$$\frac{1}{20}\,b^8x^{20}+\frac{8}{17}\,ab^7x^{17}+2\,a^2b^6x^{14}+\frac{56}{11}\,a^3b^5x^{11}+\frac{35}{4}\,a^4b^4x^8+\frac{56}{5}\,a^5b^3x^5+14\,a^6b^2x^2-\frac{32\,a^7bx^3+a^8}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^5,x, algorithm="maxima")
```

```
[Out] 1/20*b^8*x^20 + 8/17*a*b^7*x^17 + 2*a^2*b^6*x^14 + 56/11*a^3*b^5*x^11 + 35/
4*a^4*b^4*x^8 + 56/5*a^5*b^3*x^5 + 14*a^6*b^2*x^2 - 1/4*(32*a^7*b*x^3 + a^8
)/x^4
```

mupad [B] time = 0.05, size = 93, normalized size = 0.91

$$\frac{b^8\,x^{20}}{20}-\frac{\frac{a^8}{4}+8\,b\,a^7\,x^3}{x^4}+\frac{8\,a\,b^7\,x^{17}}{17}+14\,a^6\,b^2\,x^2+\frac{56\,a^5\,b^3\,x^5}{5}+\frac{35\,a^4\,b^4\,x^8}{4}+\frac{56\,a^3\,b^5\,x^{11}}{11}+2\,a^2\,b^6\,x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^8/x^5,x)
```

```
[Out] (b^8*x^20)/20 - (a^8/4 + 8*a^7*b*x^3)/x^4 + (8*a*b^7*x^17)/17 + 14*a^6*b^2*
x^2 + (56*a^5*b^3*x^5)/5 + (35*a^4*b^4*x^8)/4 + (56*a^3*b^5*x^11)/11 + 2*a^
2*b^6*x^14
```

sympy [A] time = 0.30, size = 104, normalized size = 1.02

$$14a^6b^2x^2+\frac{56a^5b^3x^5}{5}+\frac{35a^4b^4x^8}{4}+\frac{56a^3b^5x^{11}}{11}+2a^2b^6x^{14}+\frac{8ab^7x^{17}}{17}+\frac{b^8x^{20}}{20}+\frac{-a^8-32a^7bx^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**8/x**5,x)

[Out] 14*a**6*b**2*x**2 + 56*a**5*b**3*x**5/5 + 35*a**4*b**4*x**8/4 + 56*a**3*b**
5*x**11/11 + 2*a**2*b**6*x**14 + 8*a*b**7*x**17/17 + b**8*x**20/20 + (-a**8
 - 32*a**7*b*x**3)/(4*x**4)
```

3.315 $\int \frac{\left(a+bx^3\right)^8}{x^6}\,dx$

Optimal. Leaf size=98

$$-\frac{a^8}{5x^5}-\frac{4a^7b}{x^2}+28a^6b^2x+14a^5b^3x^4+10a^4b^4x^7+\frac{28}{5}a^3b^5x^{10}+\frac{28}{13}a^2b^6x^{13}+\frac{1}{2}ab^7x^{16}+\frac{b^8x^{19}}{19}$$

```
[Out] -1/5*a^8/x^5-4*a^7*b/x^2+28*a^6*b^2*x+14*a^5*b^3*x^4+10*a^4*b^4*x^7+28/5*a^
3*b^5*x^10+28/13*a^2*b^6*x^13+1/2*a*b^7*x^16+1/19*b^8*x^19
```

Rubi [A] time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{28}{13}a^2b^6x^{13}+\frac{28}{5}a^3b^5x^{10}+10a^4b^4x^7+14a^5b^3x^4+28a^6b^2x-\frac{4a^7b}{x^2}-\frac{a^8}{5x^5}+\frac{1}{2}ab^7x^{16}+\frac{b^8x^{19}}{19}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^8/x^6,x]
```

```
[Out] -a^8/(5*x^5) - (4*a^7*b)/x^2 + 28*a^6*b^2*x + 14*a^5*b^3*x^4 + 10*a^4*b^4*x
^7 + (28*a^3*b^5*x^10)/5 + (28*a^2*b^6*x^13)/13 + (a*b^7*x^16)/2 + (b^8*x^1
9)/19
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^8}{x^6}\,dx &= \int \left(28a^6b^2+\frac{a^8}{x^6}+\frac{8a^7b}{x^3}+56a^5b^3x^3+70a^4b^4x^6+56a^3b^5x^9+28a^2b^6x^{12}+8ab^7x^{15}+b^8x^{18}\right)\\ &= -\frac{a^8}{5x^5}-\frac{4a^7b}{x^2}+28a^6b^2x+14a^5b^3x^4+10a^4b^4x^7+\frac{28}{5}a^3b^5x^{10}+\frac{28}{13}a^2b^6x^{13}+\frac{1}{2}ab^7x^{16}+\frac{b^8x^{19}}{19}\end{aligned}$$

Mathematica [A] time = 0.01, size = 98, normalized size = 1.00

$$-\frac{a^8}{5x^5}-\frac{4a^7b}{x^2}+28a^6b^2x+14a^5b^3x^4+10a^4b^4x^7+\frac{28}{5}a^3b^5x^{10}+\frac{28}{13}a^2b^6x^{13}+\frac{1}{2}ab^7x^{16}+\frac{b^8x^{19}}{19}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^8/x^6,x]
```

```
[Out] -1/5*a^8/x^5 - (4*a^7*b)/x^2 + 28*a^6*b^2*x + 14*a^5*b^3*x^4 + 10*a^4*b^4*x
^7 + (28*a^3*b^5*x^10)/5 + (28*a^2*b^6*x^13)/13 + (a*b^7*x^16)/2 + (b^8*x^1
9)/19
```

fricas [A] time = 0.54, size = 92, normalized size = 0.94

$$\frac{130\,b^8x^{24}+1235\,ab^7x^{21}+5320\,a^2b^6x^{18}+13832\,a^3b^5x^{15}+24700\,a^4b^4x^{12}+34580\,a^5b^3x^9+69160\,a^6b^2x^6-9880}{2470\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^6,x, algorithm="fricas")
```

```
[Out] 1/2470*(130*b^8*x^24 + 1235*a*b^7*x^21 + 5320*a^2*b^6*x^18 + 13832*a^3*b^5*
x^15 + 24700*a^4*b^4*x^12 + 34580*a^5*b^3*x^9 + 69160*a^6*b^2*x^6 - 9880*a^
7*b*x^3 - 494*a^8)/x^5
```

giac [A] time = 0.15, size = 89, normalized size = 0.91

$$\frac{1}{19}b^8x^{19}+\frac{1}{2}ab^7x^{16}+\frac{28}{13}a^2b^6x^{13}+\frac{28}{5}a^3b^5x^{10}+10a^4b^4x^7+14a^5b^3x^4+28a^6b^2x-\frac{20a^7bx^3+a^8}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^6,x, algorithm="giac")
```

```
[Out] 1/19*b^8*x^19 + 1/2*a*b^7*x^16 + 28/13*a^2*b^6*x^13 + 28/5*a^3*b^5*x^10 + 1
0*a^4*b^4*x^7 + 14*a^5*b^3*x^4 + 28*a^6*b^2*x - 1/5*(20*a^7*b*x^3 + a^8)/x^
5
```

maple [A] time = 0.00, size = 89, normalized size = 0.91

$$\frac{b^8x^{19}}{19}+\frac{ab^7x^{16}}{2}+\frac{28a^2b^6x^{13}}{13}+\frac{28a^3b^5x^{10}}{5}+10a^4b^4x^7+14a^5b^3x^4+28a^6b^2x-\frac{4a^7b}{x^2}-\frac{a^8}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^8/x^6,x)
```

```
[Out] -1/5*a^8/x^5-4*a^7*b/x^2+28*a^6*b^2*x+14*a^5*b^3*x^4+10*a^4*b^4*x^7+28/5*a^
3*b^5*x^10+28/13*a^2*b^6*x^13+1/2*a*b^7*x^16+1/19*b^8*x^19
```

maxima [A] time = 1.36, size = 89, normalized size = 0.91

$$\frac{1}{19}b^8x^{19}+\frac{1}{2}ab^7x^{16}+\frac{28}{13}a^2b^6x^{13}+\frac{28}{5}a^3b^5x^{10}+10a^4b^4x^7+14a^5b^3x^4+28a^6b^2x-\frac{20a^7bx^3+a^8}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^6,x, algorithm="maxima")
```

```
[Out] 1/19*b^8*x^19 + 1/2*a*b^7*x^16 + 28/13*a^2*b^6*x^13 + 28/5*a^3*b^5*x^10 + 1
0*a^4*b^4*x^7 + 14*a^5*b^3*x^4 + 28*a^6*b^2*x - 1/5*(20*a^7*b*x^3 + a^8)/x^
5
```

mupad [B] time = 0.05, size = 91, normalized size = 0.93

$$\frac{b^8x^{19}}{19}-\frac{\frac{a^8}{5}+4ba^7x^3}{x^5}+28a^6b^2x+\frac{ab^7x^{16}}{2}+14a^5b^3x^4+10a^4b^4x^7+\frac{28a^3b^5x^{10}}{5}+\frac{28a^2b^6x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^8/x^6,x)
```

```
[Out] (b^8*x^19)/19 - (a^8/5 + 4*a^7*b*x^3)/x^5 + 28*a^6*b^2*x + (a*b^7*x^16)/2 +
 14*a^5*b^3*x^4 + 10*a^4*b^4*x^7 + (28*a^3*b^5*x^10)/5 + (28*a^2*b^6*x^13)/
13
```

sympy [A] time = 0.28, size = 99, normalized size = 1.01

$$28a^6b^2x+14a^5b^3x^4+10a^4b^4x^7+\frac{28a^3b^5x^{10}}{5}+\frac{28a^2b^6x^{13}}{13}+\frac{ab^7x^{16}}{2}+\frac{b^8x^{19}}{19}+\frac{-a^8-20a^7bx^3}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**8/x**6,x)
```

```
[Out] 28*a**6*b**2*x + 14*a**5*b**3*x**4 + 10*a**4*b**4*x**7 + 28*a**3*b**5*x**10
/5 + 28*a**2*b**6*x**13/13 + a*b**7*x**16/2 + b**8*x**19/19 + (-a**8 - 20*a
**7*b*x**3)/(5*x**5)
```

3.316 $\int \frac{\left(a+bx^3\right)^8}{x^8}\,dx$

Optimal. Leaf size=98

$$-\frac{a^8}{7x^7}-\frac{2a^7b}{x^4}-\frac{28a^6b^2}{x}+28a^5b^3x^2+14a^4b^4x^5+7a^3b^5x^8+\frac{28}{11}a^2b^6x^{11}+\frac{4}{7}ab^7x^{14}+\frac{b^8x^{17}}{17}$$

```
[Out] -1/7*a^8/x^7-2*a^7*b/x^4-28*a^6*b^2/x+28*a^5*b^3*x^2+14*a^4*b^4*x^5+7*a^3*b
^5*x^8+28/11*a^2*b^6*x^11+4/7*a*b^7*x^14+1/17*b^8*x^17
```

Rubi [A] time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{28}{11}a^2b^6x^{11}+7a^3b^5x^8+14a^4b^4x^5+28a^5b^3x^2-\frac{28a^6b^2}{x}-\frac{2a^7b}{x^4}-\frac{a^8}{7x^7}+\frac{4}{7}ab^7x^{14}+\frac{b^8x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^8/x^8,x]
```

```
[Out] -a^8/(7*x^7) - (2*a^7*b)/x^4 - (28*a^6*b^2)/x + 28*a^5*b^3*x^2 + 14*a^4*b^4
*x^5 + 7*a^3*b^5*x^8 + (28*a^2*b^6*x^11)/11 + (4*a*b^7*x^14)/7 + (b^8*x^17)
/17
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^8}{x^8}\,dx &= \int\left(\frac{a^8}{x^8}+\frac{8a^7b}{x^5}+\frac{28a^6b^2}{x^2}+56a^5b^3x+70a^4b^4x^4+56a^3b^5x^7+28a^2b^6x^{10}+8ab^7x^{13}+b^8x^{1}\right.\\ &= -\frac{a^8}{7x^7}-\frac{2a^7b}{x^4}-\frac{28a^6b^2}{x}+28a^5b^3x^2+14a^4b^4x^5+7a^3b^5x^8+\frac{28}{11}a^2b^6x^{11}+\frac{4}{7}ab^7x^{14}+\frac{b^8x^{17}}{17}\end{aligned}$$

Mathematica [A] time = 0.01, size = 98, normalized size = 1.00

$$-\frac{a^8}{7x^7}-\frac{2a^7b}{x^4}-\frac{28a^6b^2}{x}+28a^5b^3x^2+14a^4b^4x^5+7a^3b^5x^8+\frac{28}{11}a^2b^6x^{11}+\frac{4}{7}ab^7x^{14}+\frac{b^8x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^8/x^8,x]
```

```
[Out] -1/7*a^8/x^7 - (2*a^7*b)/x^4 - (28*a^6*b^2)/x + 28*a^5*b^3*x^2 + 14*a^4*b^4
*x^5 + 7*a^3*b^5*x^8 + (28*a^2*b^6*x^11)/11 + (4*a*b^7*x^14)/7 + (b^8*x^17)
/17
```

fricas [A] time = 0.59, size = 92, normalized size = 0.94

$$\frac{77\,b^8x^{24}+748\,ab^7x^{21}+3332\,a^2b^6x^{18}+9163\,a^3b^5x^{15}+18326\,a^4b^4x^{12}+36652\,a^5b^3x^9-36652\,a^6b^2x^6-2618}{1309\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^8,x, algorithm="fricas")
```

```
[Out] 1/1309*(77*b^8*x^24 + 748*a*b^7*x^21 + 3332*a^2*b^6*x^18 + 9163*a^3*b^5*x^1
5 + 18326*a^4*b^4*x^12 + 36652*a^5*b^3*x^9 - 36652*a^6*b^2*x^6 - 2618*a^7*b
*x^3 - 187*a^8)/x^7
```

giac [A] time = 0.15, size = 91, normalized size = 0.93

$$\frac{1}{17}b^8x^{17}+\frac{4}{7}ab^7x^{14}+\frac{28}{11}a^2b^6x^{11}+7a^3b^5x^8+14a^4b^4x^5+28a^5b^3x^2-\frac{196a^6b^2x^6+14a^7bx^3+a^8}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^8,x, algorithm="giac")
```

```
[Out] 1/17*b^8*x^17 + 4/7*a*b^7*x^14 + 28/11*a^2*b^6*x^11 + 7*a^3*b^5*x^8 + 14*a^
4*b^4*x^5 + 28*a^5*b^3*x^2 - 1/7*(196*a^6*b^2*x^6 + 14*a^7*b*x^3 + a^8)/x^7
```

maple [A] time = 0.01, size = 91, normalized size = 0.93

$$\frac{b^8x^{17}}{17}+\frac{4ab^7x^{14}}{7}+\frac{28a^2b^6x^{11}}{11}+7a^3b^5x^8+14a^4b^4x^5+28a^5b^3x^2-\frac{28a^6b^2}{x}-\frac{2a^7b}{x^4}-\frac{a^8}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^8/x^8,x)
```

```
[Out] -1/7*a^8/x^7-2*a^7*b/x^4-28*a^6*b^2/x+28*a^5*b^3*x^2+14*a^4*b^4*x^5+7*a^3*b
^5*x^8+28/11*a^2*b^6*x^11+4/7*a*b^7*x^14+1/17*b^8*x^17
```

maxima [A] time = 1.28, size = 91, normalized size = 0.93

$$\frac{1}{17}b^8x^{17}+\frac{4}{7}ab^7x^{14}+\frac{28}{11}a^2b^6x^{11}+7a^3b^5x^8+14a^4b^4x^5+28a^5b^3x^2-\frac{196a^6b^2x^6+14a^7bx^3+a^8}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^8,x, algorithm="maxima")
```

```
[Out] 1/17*b^8*x^17 + 4/7*a*b^7*x^14 + 28/11*a^2*b^6*x^11 + 7*a^3*b^5*x^8 + 14*a^
4*b^4*x^5 + 28*a^5*b^3*x^2 - 1/7*(196*a^6*b^2*x^6 + 14*a^7*b*x^3 + a^8)/x^7
```

mupad [B] time = 0.95, size = 93, normalized size = 0.95

$$\frac{b^8\,x^{17}}{17}-\frac{\frac{a^8}{7}+2\,a^7\,b\,x^3+28\,a^6\,b^2\,x^6}{x^7}+\frac{4\,a\,b^7\,x^{14}}{7}+28\,a^5\,b^3\,x^2+14\,a^4\,b^4\,x^5+7\,a^3\,b^5\,x^8+\frac{28\,a^2\,b^6\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^8/x^8,x)
```

```
[Out] (b^8*x^17)/17 - (a^8/7 + 2*a^7*b*x^3 + 28*a^6*b^2*x^6)/x^7 + (4*a*b^7*x^14)
/7 + 28*a^5*b^3*x^2 + 14*a^4*b^4*x^5 + 7*a^3*b^5*x^8 + (28*a^2*b^6*x^11)/11
```

sympy [A] time = 0.34, size = 100, normalized size = 1.02

$$28a^5b^3x^2+14a^4b^4x^5+7a^3b^5x^8+\frac{28a^2b^6x^{11}}{11}+\frac{4ab^7x^{14}}{7}+\frac{b^8x^{17}}{17}+\frac{-a^8-14a^7bx^3-196a^6b^2x^6}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**8/x**8,x)
```

```
[Out] 28*a**5*b**3*x**2 + 14*a**4*b**4*x**5 + 7*a**3*b**5*x**8 + 28*a**2*b**6*x**
11/11 + 4*a*b**7*x**14/7 + b**8*x**17/17 + (-a**8 - 14*a**7*b*x**3 - 196*a*
*6*b**2*x**6)/(7*x**7)
```

3.317 $\int \frac{\left(a+bx^3\right)^8}{x^9}\,dx$

Optimal. Leaf size=100

$$-\frac{a^8}{8x^8}-\frac{8a^7b}{5x^5}-\frac{14a^6b^2}{x^2}+56a^5b^3x+\frac{35}{2}a^4b^4x^4+8a^3b^5x^7+\frac{14}{5}a^2b^6x^{10}+\frac{8}{13}ab^7x^{13}+\frac{b^8x^{16}}{16}$$

```
[Out] -1/8*a^8/x^8-8/5*a^7*b/x^5-14*a^6*b^2/x^2+56*a^5*b^3*x+35/2*a^4*b^4*x^4+8*a
^3*b^5*x^7+14/5*a^2*b^6*x^10+8/13*a*b^7*x^13+1/16*b^8*x^16
```

Rubi [A] time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{14}{5}a^2b^6x^{10}+8a^3b^5x^7+\frac{35}{2}a^4b^4x^4-\frac{14a^6b^2}{x^2}+56a^5b^3x-\frac{8a^7b}{5x^5}-\frac{a^8}{8x^8}+\frac{8}{13}ab^7x^{13}+\frac{b^8x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^8/x^9,x]
```

```
[Out] -a^8/(8*x^8) - (8*a^7*b)/(5*x^5) - (14*a^6*b^2)/x^2 + 56*a^5*b^3*x + (35*a^
4*b^4*x^4)/2 + 8*a^3*b^5*x^7 + (14*a^2*b^6*x^10)/5 + (8*a*b^7*x^13)/13 + (b
^8*x^16)/16
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^8}{x^9}\,dx &= \int \left(56a^5b^3+\frac{a^8}{x^9}+\frac{8a^7b}{x^6}+\frac{28a^6b^2}{x^3}+70a^4b^4x^3+56a^3b^5x^6+28a^2b^6x^9+8ab^7x^{12}+b^8x^{15}\right)\\ &= -\frac{a^8}{8x^8}-\frac{8a^7b}{5x^5}-\frac{14a^6b^2}{x^2}+56a^5b^3x+\frac{35}{2}a^4b^4x^4+8a^3b^5x^7+\frac{14}{5}a^2b^6x^{10}+\frac{8}{13}ab^7x^{13}+\frac{b^8x^{16}}{16}\end{aligned}$$

Mathematica [A] time = 0.00, size = 100, normalized size = 1.00

$$-\frac{a^8}{8x^8}-\frac{8a^7b}{5x^5}-\frac{14a^6b^2}{x^2}+56a^5b^3x+\frac{35}{2}a^4b^4x^4+8a^3b^5x^7+\frac{14}{5}a^2b^6x^{10}+\frac{8}{13}ab^7x^{13}+\frac{b^8x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^8/x^9,x]
```

```
[Out] -1/8*a^8/x^8 - (8*a^7*b)/(5*x^5) - (14*a^6*b^2)/x^2 + 56*a^5*b^3*x + (35*a^
4*b^4*x^4)/2 + 8*a^3*b^5*x^7 + (14*a^2*b^6*x^10)/5 + (8*a*b^7*x^13)/13 + (b
^8*x^16)/16
```

fricas [A] time = 0.68, size = 92, normalized size = 0.92

$$\frac{65\,b^8x^{24}+640\,ab^7x^{21}+2912\,a^2b^6x^{18}+8320\,a^3b^5x^{15}+18200\,a^4b^4x^{12}+58240\,a^5b^3x^9-14560\,a^6b^2x^6-1664}{1040\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^9,x, algorithm="fricas")
```

```
[Out] 1/1040*(65*b^8*x^24 + 640*a*b^7*x^21 + 2912*a^2*b^6*x^18 + 8320*a^3*b^5*x^1
5 + 18200*a^4*b^4*x^12 + 58240*a^5*b^3*x^9 - 14560*a^6*b^2*x^6 - 1664*a^7*b
*x^3 - 130*a^8)/x^8
```

giac [A] time = 0.21, size = 91, normalized size = 0.91

$$\frac{1}{16}\,b^8x^{16}+\frac{8}{13}\,ab^7x^{13}+\frac{14}{5}\,a^2b^6x^{10}+8\,a^3b^5x^7+\frac{35}{2}\,a^4b^4x^4+56\,a^5b^3x-\frac{560\,a^6b^2x^6+64\,a^7bx^3+5\,a^8}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^9,x, algorithm="giac")
```

```
[Out] 1/16*b^8*x^16 + 8/13*a*b^7*x^13 + 14/5*a^2*b^6*x^10 + 8*a^3*b^5*x^7 + 35/2*
a^4*b^4*x^4 + 56*a^5*b^3*x - 1/40*(560*a^6*b^2*x^6 + 64*a^7*b*x^3 + 5*a^8)/
x^8
```

maple [A] time = 0.01, size = 89, normalized size = 0.89

$$\frac{b^8x^{16}}{16}+\frac{8a\,b^7x^{13}}{13}+\frac{14a^2b^6x^{10}}{5}+8a^3b^5x^7+\frac{35a^4b^4x^4}{2}+56a^5b^3x-\frac{14a^6b^2}{x^2}-\frac{8a^7b}{5x^5}-\frac{a^8}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^8/x^9,x)
```

```
[Out] -1/8*a^8/x^8-8/5*a^7*b/x^5-14*a^6*b^2/x^2+56*a^5*b^3*x+35/2*a^4*b^4*x^4+8*a
^3*b^5*x^7+14/5*a^2*b^6*x^10+8/13*a*b^7*x^13+1/16*b^8*x^16
```

maxima [A] time = 1.32, size = 91, normalized size = 0.91

$$\frac{1}{16}\,b^8x^{16}+\frac{8}{13}\,ab^7x^{13}+\frac{14}{5}\,a^2b^6x^{10}+8\,a^3b^5x^7+\frac{35}{2}\,a^4b^4x^4+56\,a^5b^3x-\frac{560\,a^6b^2x^6+64\,a^7bx^3+5\,a^8}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^9,x, algorithm="maxima")
```

```
[Out] 1/16*b^8*x^16 + 8/13*a*b^7*x^13 + 14/5*a^2*b^6*x^10 + 8*a^3*b^5*x^7 + 35/2*
a^4*b^4*x^4 + 56*a^5*b^3*x - 1/40*(560*a^6*b^2*x^6 + 64*a^7*b*x^3 + 5*a^8)/
x^8
```

mupad [B] time = 0.05, size = 91, normalized size = 0.91

$$\frac{b^8\,x^{16}}{16}-\frac{\frac{a^8}{8}+\frac{8\,a^7\,b\,x^3}{5}+14\,a^6\,b^2\,x^6}{x^8}+56\,a^5\,b^3\,x+\frac{8\,a\,b^7\,x^{13}}{13}+\frac{35\,a^4\,b^4\,x^4}{2}+8\,a^3\,b^5\,x^7+\frac{14\,a^2\,b^6\,x^{10}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^8/x^9,x)
```

```
[Out] (b^8*x^16)/16 - (a^8/8 + (8*a^7*b*x^3)/5 + 14*a^6*b^2*x^6)/x^8 + 56*a^5*b^3
*x + (8*a*b^7*x^13)/13 + (35*a^4*b^4*x^4)/2 + 8*a^3*b^5*x^7 + (14*a^2*b^6*x
^10)/5
```

sympy [A] time = 0.36, size = 102, normalized size = 1.02

$$56a^5b^3x+\frac{35a^4b^4x^4}{2}+8a^3b^5x^7+\frac{14a^2b^6x^{10}}{5}+\frac{8ab^7x^{13}}{13}+\frac{b^8x^{16}}{16}+\frac{-5a^8-64a^7bx^3-560a^6b^2x^6}{40x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**8/x**9,x)
```

```
[Out] 56*a**5*b**3*x + 35*a**4*b**4*x**4/2 + 8*a**3*b**5*x**7 + 14*a**2*b**6*x**1
0/5 + 8*a*b**7*x**13/13 + b**8*x**16/16 + (-5*a**8 - 64*a**7*b*x**3 - 560*a
**6*b**2*x**6)/(40*x**8)
```

3.318 $\int \frac{x^8}{a+bx^3}\,dx$

Optimal. Leaf size=40

$$\frac{a^2\log\left(a+bx^3\right)}{3b^3}-\frac{ax^3}{3b^2}+\frac{x^6}{6b}$$

```
[Out] -1/3*a*x^3/b^2+1/6*x^6/b+1/3*a^2*ln(b*x^3+a)/b^3
```

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{a^2\log\left(a+bx^3\right)}{3b^3}-\frac{ax^3}{3b^2}+\frac{x^6}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^3),x]
```

```
[Out] -(a*x^3)/(3*b^2) + x^6/(6*b) + (a^2*Log[a + b*x^3])/(3*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{a+bx^3}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{a+bx}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)\,dx, x, x^3\right)\\ &= -\frac{ax^3}{3b^2}+\frac{x^6}{6b}+\frac{a^2\log\left(a+bx^3\right)}{3b^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.00

$$\frac{a^2\log\left(a+bx^3\right)}{3b^3}-\frac{ax^3}{3b^2}+\frac{x^6}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^3),x]
```

```
[Out] -1/3*(a*x^3)/b^2 + x^6/(6*b) + (a^2*Log[a + b*x^3])/(3*b^3)
```

fricas [A] time = 0.72, size = 33, normalized size = 0.82

$$\frac{b^2x^6-2\,abx^3+2\,a^2\log\left(bx^3+a\right)}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a),x, algorithm="fricas")
[Out] 1/6*(b^2*x^6 - 2*a*b*x^3 + 2*a^2*log(b*x^3 + a))/b^3
```

giac [A] time = 0.18, size = 35, normalized size = 0.88

$$\frac{a^2 \log\left(\left|bx^3+a\right|\right)}{3\,b^3}+\frac{bx^6-2\,ax^3}{6\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a),x, algorithm="giac")
[Out] 1/3*a^2*log(abs(b*x^3 + a))/b^3 + 1/6*(b*x^6 - 2*a*x^3)/b^2
```

maple [A] time = 0.00, size = 35, normalized size = 0.88

$$\frac{x^6}{6b}-\frac{a\,x^3}{3b^2}+\frac{a^2\ln\left(b\,x^3+a\right)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a),x)
[Out] -1/3*a*x^3/b^2+1/6*x^6/b+1/3*a^2*ln(b*x^3+a)/b^3
```

maxima [A] time = 1.32, size = 34, normalized size = 0.85

$$\frac{a^2 \log\left(bx^3+a\right)}{3\,b^3}+\frac{bx^6-2\,ax^3}{6\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a),x, algorithm="maxima")
[Out] 1/3*a^2*log(b*x^3 + a)/b^3 + 1/6*(b*x^6 - 2*a*x^3)/b^2
```

mupad [B] time = 0.05, size = 33, normalized size = 0.82

$$\frac{2\,a^2\,\ln\left(b\,x^3+a\right)+b^2\,x^6-2\,a\,b\,x^3}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^3),x)
[Out] (2*a^2*log(a + b*x^3) + b^2*x^6 - 2*a*b*x^3)/(6*b^3)
```

sympy [A] time = 0.37, size = 32, normalized size = 0.80

$$\frac{a^2 \log\left(a+bx^3\right)}{3b^3}-\frac{ax^3}{3b^2}+\frac{x^6}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a),x)
[Out] a**2*log(a + b*x**3)/(3*b**3) - a*x**3/(3*b**2) + x**6/(6*b)
```

3.319 $\int \frac{x^5}{a+bx^3}\,dx$

Optimal. Leaf size=27

$$\frac{x^3}{3b} - \frac{a\log\left(a+bx^3\right)}{3b^2}$$

```
[Out] 1/3*x^3/b-1/3*a*ln(b*x^3+a)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{x^3}{3b} - \frac{a\log\left(a+bx^3\right)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^3),x]
```

```
[Out] x^3/(3*b) - (a*Log[a + b*x^3])/(3*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{a+bx^3}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{a+bx}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{1}{b} - \frac{a}{b(a+bx)}\right)\,dx, x, x^3\right)\\ &= \frac{x^3}{3b} - \frac{a\log\left(a+bx^3\right)}{3b^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.00

$$\frac{x^3}{3b} - \frac{a\log\left(a+bx^3\right)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^3),x]
```

```
[Out] x^3/(3*b) - (a*Log[a + b*x^3])/(3*b^2)
```

fricas [A] time = 0.61, size = 22, normalized size = 0.81

$$\frac{bx^3 - a\log\left(bx^3+a\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a),x, algorithm="fricas")
[Out] 1/3*(b*x^3 - a*log(b*x^3 + a))/b^2
```

giac [A] time = 0.18, size = 24, normalized size = 0.89

$$\frac{x^3}{3\,b} - \frac{a\log\left(\left|bx^3+a\right|\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a),x, algorithm="giac")
[Out] 1/3*x^3/b - 1/3*a*log(abs(b*x^3 + a))/b^2
```

maple [A] time = 0.00, size = 24, normalized size = 0.89

$$\frac{x^3}{3b} - \frac{a\ln\left(b\,x^3+a\right)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a),x)
[Out] 1/3*x^3/b-1/3*a*ln(b*x^3+a)/b^2
```

maxima [A] time = 1.35, size = 23, normalized size = 0.85

$$\frac{x^3}{3\,b} - \frac{a\log\left(bx^3+a\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a),x, algorithm="maxima")
[Out] 1/3*x^3/b - 1/3*a*log(b*x^3 + a)/b^2
```

mupad [B] time = 0.95, size = 22, normalized size = 0.81

$$-\frac{a\,\ln\left(b\,x^3+a\right)-b\,x^3}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^3),x)
[Out] -(a*log(a + b*x^3) - b*x^3)/(3*b^2)
```

sympy [A] time = 0.20, size = 20, normalized size = 0.74

$$-\frac{a\log\left(a+bx^3\right)}{3b^2} + \frac{x^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a),x)
[Out] -a*log(a + b*x**3)/(3*b**2) + x**3/(3*b)
```

3.320 $\int \frac{x^2}{a+bx^3}\,dx$

Optimal. Leaf size=15

$$\frac{\log\left(a+bx^3\right)}{3b}$$

```
[Out] 1/3*ln(b*x^3+a)/b
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {260}

$$\frac{\log\left(a+bx^3\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^3),x]
```

```
[Out] Log[a + b*x^3]/(3*b)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^2}{a+bx^3}\,dx = \frac{\log\left(a+bx^3\right)}{3b}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log\left(a+bx^3\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^3),x]
```

```
[Out] Log[a + b*x^3]/(3*b)
```

fricas [A] time = 0.64, size = 13, normalized size = 0.87

$$\frac{\log\left(bx^3+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/3*log(b*x^3 + a)/b
```

giac [A] time = 0.16, size = 14, normalized size = 0.93

$$\frac{\log\left(\left|bx^3+a\right|\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a),x, algorithm="giac")
[Out] 1/3*log(abs(b*x^3 + a))/b
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{\ln\left(b\,x^3+a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a),x)
[Out] 1/3*ln(b*x^3+a)/b
```

maxima [A] time = 1.32, size = 13, normalized size = 0.87

$$\frac{\log\left(bx^3+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a),x, algorithm="maxima")
[Out] 1/3*log(b*x^3 + a)/b
```

mupad [B] time = 0.03, size = 13, normalized size = 0.87

$$\frac{\ln\left(b\,x^3+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^3),x)
[Out] log(a + b*x^3)/(3*b)
```

sympy [A] time = 0.17, size = 10, normalized size = 0.67

$$\frac{\log\left(a+bx^3\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a),x)
[Out] log(a + b*x**3)/(3*b)
```

3.321 $\int \frac{1}{x\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=22

$$\frac{\log(x)}{a}-\frac{\log\left(a+bx^3\right)}{3a}$$

```
[Out] ln(x)/a-1/3*ln(b*x^3+a)/a
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {266, 36, 29, 31}

$$\frac{\log(x)}{a}-\frac{\log\left(a+bx^3\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3)),x]
```

```
[Out] Log[x]/a - Log[a + b*x^3]/(3*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a+bx^3\right)}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx, x, x^3\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^3\right)}{3a}-\frac{b\,\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, x^3\right)}{3a}\\ &= \frac{\log(x)}{a}-\frac{\log\left(a+bx^3\right)}{3a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{\log(x)}{a}-\frac{\log\left(a+bx^3\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3)),x]
[Out] Log[x]/a - Log[a + b*x^3]/(3*a)
```

fricas [A] time = 0.61, size = 18, normalized size = 0.82

$$-\frac{\log\left(bx^3+a\right)-3\,\log(x)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a),x, algorithm="fricas")
[Out] -1/3*(log(b*x^3 + a) - 3*log(x))/a
```

giac [A] time = 0.15, size = 22, normalized size = 1.00

$$-\frac{\log\left(\left|bx^3+a\right|\right)}{3\,a}+\frac{\log\left(|x|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a),x, algorithm="giac")
[Out] -1/3*log(abs(b*x^3 + a))/a + log(abs(x))/a
```

maple [A] time = 0.00, size = 21, normalized size = 0.95

$$\frac{\ln(x)}{a}-\frac{\ln\left(b\,x^3+a\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a),x)
[Out] ln(x)/a-1/3*ln(b*x^3+a)/a
```

maxima [A] time = 1.32, size = 23, normalized size = 1.05

$$-\frac{\log\left(bx^3+a\right)}{3\,a}+\frac{\log\left(x^3\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a),x, algorithm="maxima")
[Out] -1/3*log(b*x^3 + a)/a + 1/3*log(x^3)/a
```

mupad [B] time = 1.00, size = 18, normalized size = 0.82

$$-\frac{\ln\left(b\,x^3+a\right)-3\,\ln(x)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3)),x)
[Out] -(log(a + b*x^3) - 3*log(x))/(3*a)
```

sympy [A] time = 0.29, size = 15, normalized size = 0.68

$$\frac{\log(x)}{a}-\frac{\log\left(\frac{a}{b}+x^3\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a),x)
[Out] log(x)/a - log(a/b + x**3)/(3*a)
```

3.322 $\int \frac{1}{x^4\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=35

$$\frac{b\log\left(a+bx^3\right)}{3a^2}-\frac{b\log(x)}{a^2}-\frac{1}{3ax^3}$$

```
[Out] -1/3/a/x^3-b*ln(x)/a^2+1/3*b*ln(b*x^3+a)/a^2
```

Rubi [A] time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 44}

$$\frac{b\log\left(a+bx^3\right)}{3a^2}-\frac{b\log(x)}{a^2}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3)),x]
```

```
[Out] -1/(3*a*x^3) - (b*Log[x])/a^2 + (b*Log[a + b*x^3])/(3*a^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(a+bx^3\right)}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx, x, x^3\right)\\ &= -\frac{1}{3ax^3}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^3\right)}{3a^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.00

$$\frac{b\log\left(a+bx^3\right)}{3a^2}-\frac{b\log(x)}{a^2}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3)),x]
```

```
[Out] -1/3*1/(a*x^3) - (b*Log[x])/a^2 + (b*Log[a + b*x^3])/(3*a^2)
```

fricas [A] time = 0.58, size = 33, normalized size = 0.94

$$\frac{bx^3\log\left(bx^3+a\right)-3\,bx^3\log(x)-a}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/3*(b*x^3*log(b*x^3 + a) - 3*b*x^3*log(x) - a)/(a^2*x^3)
```

giac [A] time = 0.16, size = 42, normalized size = 1.20

$$\frac{b\log\left(\left|bx^3+a\right|\right)}{3\,a^2}-\frac{b\log\left(|x|\right)}{a^2}+\frac{bx^3-a}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*b*log(abs(b*x^3 + a))/a^2 - b*log(abs(x))/a^2 + 1/3*(b*x^3 - a)/(a^2*x^
3)
```

maple [A] time = 0.01, size = 32, normalized size = 0.91

$$-\frac{b\ln(x)}{a^2}+\frac{b\ln\left(b\,x^3+a\right)}{3a^2}-\frac{1}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a),x)
```

```
[Out] -1/3/a/x^3-b*ln(x)/a^2+1/3*b*ln(b*x^3+a)/a^2
```

maxima [A] time = 1.29, size = 33, normalized size = 0.94

$$\frac{b\log\left(bx^3+a\right)}{3\,a^2}-\frac{b\log\left(x^3\right)}{3\,a^2}-\frac{1}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*b*log(b*x^3 + a)/a^2 - 1/3*b*log(x^3)/a^2 - 1/3/(a*x^3)
```

mupad [B] time = 0.08, size = 31, normalized size = 0.89

$$\frac{b\,\ln\left(b\,x^3+a\right)}{3\,a^2}-\frac{1}{3\,a\,x^3}-\frac{b\,\ln(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3)),x)
```

```
[Out] (b*log(a + b*x^3))/(3*a^2) - 1/(3*a*x^3) - (b*log(x))/a^2
```

sympy [A] time = 0.69, size = 31, normalized size = 0.89

$$-\frac{1}{3ax^3}-\frac{b\log(x)}{a^2}+\frac{b\log\left(\frac{a}{b}+x^3\right)}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a),x)
```

```
[Out] -1/(3*a*x**3) - b*log(x)/a**2 + b*log(a/b + x**3)/(3*a**2)
```

3.323 $\int \frac{x^4}{a+bx^3}\,dx$

Optimal. Leaf size=124

$$-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{5/3}}+\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{5/3}}+\frac{a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{5/3}}+\frac{x^2}{2b}$$

```
[Out] 1/2*x^2/b+1/3*a^(2/3)*ln(a^(1/3)+b^(1/3)*x)/b^(5/3)-1/6*a^(2/3)*ln(a^(2/3)-
a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(5/3)+1/3*a^(2/3)*arctan(1/3*(a^(1/3)-2*b^
(1/3)*x)/a^(1/3)*3^(1/2))/b^(5/3)*3^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {321, 292, 31, 634, 617, 204, 628}

$$-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{5/3}}+\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{5/3}}+\frac{a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{5/3}}+\frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^3),x]
```

```
[Out] x^2/(2*b) + (a^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sq
rt[3]*b^(5/3)) + (a^(2/3)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(5/3)) - (a^(2/3)*
Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{a+bx^3}\,dx &= \frac{x^2}{2b} - \frac{a\int \frac{x}{a+bx^3}\,dx}{b} \\
&= \frac{x^2}{2b} + \frac{a^{2/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3b^{4/3}} - \frac{a^{2/3}\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3b^{4/3}} \\
&= \frac{x^2}{2b} + \frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{5/3}} - \frac{a^{2/3}\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{6b^{5/3}} - \frac{a\int \frac{1}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{2b^{4/3}} \\
&= \frac{x^2}{2b} + \frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{5/3}} - \frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{5/3}} - \frac{a^{2/3}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2}{\sqrt[3]{a}}\right)}{b^{5/3}} \\
&= \frac{x^2}{2b} + \frac{a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{5/3}} + \frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{5/3}} - \frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{5/3}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 111, normalized size = 0.90

$$\frac{-a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)+2a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+2\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+3b^{2/3}x^2}{6b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^3),x]
```

```
[Out] (3*b^(2/3)*x^2 + 2*Sqrt[3]*a^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[
3]] + 2*a^(2/3)*Log[a^(1/3) + b^(1/3)*x] - a^(2/3)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/(6*b^(5/3))
```

fricas [A] time = 0.57, size = 123, normalized size = 0.99

$$\frac{3\,x^2-2\sqrt{3}\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,bx\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}-\sqrt{3}\,a}{3\,a}\right)-\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax^2-bx\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}+a\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right)+2\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax+b\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(3*x^2 - 2*sqrt(3)*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^
(1/3) - sqrt(3)*a)/a) - (a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)^(2/3) + a
*(a^2/b^2)^(1/3)) + 2*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^(2/3)))/b
```

giac [A] time = 0.19, size = 114, normalized size = 0.92

$$\frac{x^2}{2\,b}+\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,b}+\frac{\sqrt{3}\,\left(-a b^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^3}-\frac{\left(-a b^2\right)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/2*x^2/b + 1/3*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/b + 1/3*sqrt(3)*(-a
*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^3 - 1/6
*(-a*b^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^3
```

maple [A] time = 0.00, size = 102, normalized size = 0.82

$$\frac{x^2}{2b}-\frac{\sqrt{3}\;a\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}+\frac{a\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}-\frac{a\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a),x)
```

```
[Out] 1/2*x^2/b+1/3*a/b^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-1/6*a/b^2/(a/b)^(1/3)*ln(
x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-1/3*a/b^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(
1/2)*(2/(a/b)^(1/3)*x-1))
```

maxima [A] time = 2.88, size = 109, normalized size = 0.88

$$\frac{x^2}{2\,b}-\frac{\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{a\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{a\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/b - 1/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3
))/(b^2*(a/b)^(1/3)) - 1/6*a*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a
/b)^(1/3)) + 1/3*a*log(x + (a/b)^(1/3))/(b^2*(a/b)^(1/3))
```

mupad [B] time = 0.21, size = 120, normalized size = 0.97

$$\frac{x^2}{2\,b}+\frac{a^{2/3}\,\ln\left(\frac{a^{7/3}}{b^{4/3}}+\frac{a^2\,x}{b}\right)}{3\,b^{5/3}}-\frac{a^{2/3}\,\ln\left(\frac{a^2\,x}{b}+\frac{a^{7/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{b^{4/3}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,b^{5/3}}+\frac{a^{2/3}\,\ln\left(\frac{a^2\,x}{b}+\frac{9\,a^{7/3}\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2}{b^{4/3}}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}}{6}\right.}{b^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^3),x)
```

```
[Out] x^2/(2*b) + (a^(2/3)*log(a^(7/3)/b^(4/3) + (a^2*x)/b))/(3*b^(5/3)) - (a^(2/
3)*log((a^2*x)/b + (a^(7/3)*((3^(1/2)*1i)/2 + 1/2)^2)/b^(4/3))*((3^(1/2)*1i
)/2 + 1/2))/(3*b^(5/3)) + (a^(2/3)*log((a^2*x)/b + (9*a^(7/3)*((3^(1/2)*1i)
/6 - 1/6)^2)/b^(4/3))*((3^(1/2)*1i)/6 - 1/6))/b^(5/3)
```

sympy [**A**] time = 0.21, size = 32, normalized size = 0.26

$$\operatorname{RootSum}\left(27t^3b^5 - a^2, \left(t \mapsto t\log\left(\frac{9t^2b^3}{a} + x\right)\right)\right) + \frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*b**5 - a**2, Lambda(_t, _t*log(9*_t**2*b**3/a + x))) + x**
2/(2*b)
```

3.324 $\int \frac{x^3}{a+bx^3}\,dx$

Optimal. Leaf size=119

$$\frac{\sqrt[3]{a}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{4/3}}-\frac{\sqrt[3]{a}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{4/3}}+\frac{\sqrt[3]{a}\,\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{4/3}}+\frac{x}{b}$$

```
[Out] x/b-1/3*a^(1/3)*ln(a^(1/3)+b^(1/3)*x)/b^(4/3)+1/6*a^(1/3)*ln(a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/b^(4/3)+1/3*a^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*
x)/a^(1/3)*3^(1/2))/b^(4/3)*3^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {321, 200, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{a}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{4/3}}-\frac{\sqrt[3]{a}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{4/3}}+\frac{\sqrt[3]{a}\,\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{4/3}}+\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^3),x]
```

```
[Out] x/b + (a^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*
b^(4/3)) - (a^(1/3)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(4/3)) + (a^(1/3)*Log[a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{a+bx^3}\,dx &= \frac{x}{b} - \frac{a\int \frac{1}{a+bx^3}\,dx}{b} \\
&= \frac{x}{b} - \frac{\sqrt[3]{a}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3b} - \frac{\sqrt[3]{a}\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3b} \\
&= \frac{x}{b} - \frac{\sqrt[3]{a}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{4/3}} + \frac{\sqrt[3]{a}\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{6b^{4/3}} - \frac{a^{2/3}\int \frac{1}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{2b} \\
&= \frac{x}{b} - \frac{\sqrt[3]{a}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{4/3}} + \frac{\sqrt[3]{a}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{4/3}} - \frac{\sqrt[3]{a}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{}}{\sqrt[3]{}}\right.}{b^{4/3}} \\
&= \frac{x}{b} + \frac{\sqrt[3]{a}\,\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{4/3}} - \frac{\sqrt[3]{a}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{4/3}} + \frac{\sqrt[3]{a}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{4/3}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 108, normalized size = 0.91

$$\frac{\sqrt[3]{a}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)-2\sqrt[3]{a}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+2\sqrt{3}\,\sqrt[3]{a}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+6\sqrt[3]{b}\,x}{6b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^3),x]
```

```
[Out] (6*b^(1/3)*x + 2*Sqrt[3]*a^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]
] - 2*a^(1/3)*Log[a^(1/3) + b^(1/3)*x] + a^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/(6*b^(4/3))
```

fricas [A] time = 0.65, size = 106, normalized size = 0.89

$$\frac{2\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,a}{3a}\right)-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)+2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)+6x}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3
)*a)/a) - (-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)) + 2*(-a/b)^
(1/3)*log(x - (-a/b)^(1/3)) + 6*x)/b
```

giac [A] time = 0.18, size = 111, normalized size = 0.93

$$\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,b}+\frac{x}{b}-\frac{\sqrt{3}\,\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^2}-\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/b + x/b - 1/3*sqrt(3)*(-a*b^2)^
(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^2 - 1/6*(-a*b
^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^2
```

maple [A] time = 0.00, size = 99, normalized size = 0.83

$$-\frac{\sqrt{3}\;a\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}-\frac{a\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}+\frac{a\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a),x)
```

```
[Out] x/b-1/3*a/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+1/6*a/b^2/(a/b)^(2/3)*ln(x^2-(a
/b)^(1/3)*x+(a/b)^(2/3))-1/3*a/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(
2/(a/b)^(1/3)*x-1))
```

maxima [A] time = 2.92, size = 106, normalized size = 0.89

$$\frac{x}{b}-\frac{\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{a\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{a\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] x/b - 1/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^
2*(a/b)^(2/3)) + 1/6*a*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a/b)^(2
/3)) - 1/3*a*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))
```

mupad [B] time = 1.16, size = 114, normalized size = 0.96

$$\frac{x}{b}+\frac{(-a)^{1/3}\,\ln\left((-a)^{4/3}+a\,b^{1/3}\,x\right)}{3\,b^{4/3}}-\frac{(-a)^{1/3}\,\ln\left(3\,(-a)^{4/3}\,b^{2/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-3\,a\,b\,x\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,b^{4/3}}+\frac{(-a)^{1/3}\,\ln\left(9\,(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^3),x)
```

```
[Out] x/b + ((-a)^(1/3)*log((-a)^(4/3) + a*b^(1/3)*x))/(3*b^(4/3)) - ((-a)^(1/3)*
log(3*(-a)^(4/3)*b^(2/3)*((3^(1/2)*1i)/2 + 1/2) - 3*a*b*x)*((3^(1/2)*1i)/2
+ 1/2))/(3*b^(4/3)) + ((-a)^(1/3)*log(9*(-a)^(4/3)*b^(2/3)*((3^(1/2)*1i)/6
- 1/6) + 3*a*b*x)*((3^(1/2)*1i)/6 - 1/6))/b^(4/3)
```

sympy [**A**] time = 0.23, size = 22, normalized size = 0.18

$$\operatorname{RootSum}\left(27t^3b^4 + a, \left(t \mapsto t\log\left(-3tb + x\right)\right)\right) + \frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*b**4 + a, Lambda(_t, _t*log(-3*_t*b + x))) + x/b
```

3.325 $\int \frac{x}{a+bx^3}\,dx$

Optimal. Leaf size=115

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6\sqrt[3]{a}\,b^{2/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{2/3}}$$

```
[Out] -1/3*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(2/3)+1/6*ln(a^(2/3)-a^(1/3)*b^(1/3)*x
+b^(2/3)*x^2)/a^(1/3)/b^(2/3)-1/3*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*
3^(1/2))/a^(1/3)/b^(2/3)*3^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {292, 31, 634, 617, 204, 628}

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6\sqrt[3]{a}\,b^{2/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^3),x]
```

```
[Out] -(ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(1/3)*b^(2/3
))) - Log[a^(1/3) + b^(1/3)*x]/(3*a^(1/3)*b^(2/3)) + Log[a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2]/(6*a^(1/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{a+bx^3}\,dx &= -\frac{\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3\sqrt[3]{a}\,\sqrt[3]{b}} + \frac{\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3\sqrt[3]{a}\,\sqrt[3]{b}} \\ &= -\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}} + \frac{\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{6\sqrt[3]{a}\,b^{2/3}} + \frac{\int \frac{1}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{2\sqrt[3]{b}} \\ &= -\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6\sqrt[3]{a}\,b^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}\,b^{2/3}} \\ &= -\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{2/3}} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6\sqrt[3]{a}\,b^{2/3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 89, normalized size = 0.77

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-2\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6\sqrt[3]{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^3),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3) + b
^(1/3)*x] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3)*b^(2
/3))
```

fricas [A] time = 0.81, size = 304, normalized size = 2.64

$$\left[\frac{3\sqrt{\frac{1}{3}}\,ab\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\,\log\left(\frac{2b^2x^3-ab+3\sqrt{\frac{1}{3}}\left(abx+2\left(-ab^2\right)^{\frac{2}{3}}x^2+\left(-ab^2\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}-3\left(-ab^2\right)^{\frac{2}{3}}x}{bx^3+a}\right)+\left(-ab^2\right)^{\frac{2}{3}}\log\left(b^2x^2+\left(-ab^2\right)^{\frac{1}{3}}b\right.}{6\,ab^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(
1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/
```

```
a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + (-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^
2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3))
)/(a*b^2), 1/6*(6*sqrt(1/3)*a*b*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2
*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (-a*b^2)^(2/3)*log(b^2*
x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*log(b*x - (-a
*b^2)^(1/3)))/(a*b^2)]
```

giac [A] time = 0.20, size = 112, normalized size = 0.97

$$-\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a}-\frac{\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^2}+\frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/a - 1/3*sqrt(3)*(-a*b^2)^(2/3)
*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2) + 1/6*(-a*b^
2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2)
```

maple [A] time = 0.00, size = 91, normalized size = 0.79

$$\frac{\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b}-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a),x)
```

```
[Out] -1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x
+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x
-1))
```

maxima [A] time = 3.03, size = 98, normalized size = 0.85

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^(1
/3)) + 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(1/3)) - 1/3*log
(x + (a/b)^(1/3))/(b*(a/b)^(1/3))
```

mupad [B] time = 1.17, size = 111, normalized size = 0.97

$$\frac{\ln\left(b^{1/3}x-(-a)^{1/3}\right)}{3(-a)^{1/3}b^{2/3}}+\frac{\ln\left(bx-\frac{(-a)^{1/3}b^{2/3}\left(-1+\sqrt{3}\ 1\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\ 1\mathrm{i}\right)}{6(-a)^{1/3}b^{2/3}}-\frac{\ln\left(bx-\frac{(-a)^{1/3}b^{2/3}\left(1+\sqrt{3}\ 1\mathrm{i}\right)^2}{4}\right)\left(1+\sqrt{3}\ 1\mathrm{i}\right)}{6(-a)^{1/3}b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^3),x)
```

```
[Out] log(b^(1/3)*x - (-a)^(1/3))/(3*(-a)^(1/3)*b^(2/3)) + (log(b*x - ((-a)^(1/3)
*b^(2/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(6*(-a)^(1/3)*b^(2/3)) -
(log(b*x - ((-a)^(1/3)*b^(2/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/(6*
(-a)^(1/3)*b^(2/3))
```

sympy [**A**] time = 0.15, size = 24, normalized size = 0.21

$$\operatorname{RootSum}\left(27t^3ab^2+1,\left(t\mapsto t\log\left(9t^2ab+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a*b**2 + 1, Lambda(_t, _t*log(9*_t**2*a*b + x)))
```

3.326 $\int \frac{1}{a+bx^3}\,dx$

Optimal. Leaf size=115

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}}$$

```
[Out] 1/3*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(1/3)-1/6*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+
b^(2/3)*x^2)/a^(2/3)/b^(1/3)-1/3*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3
^(1/2))/a^(2/3)/b^(1/3)*3^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(-1),x]
```

```
[Out] -(ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3)*b^(1/3
))) + Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) - Log[a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2]/(6*a^(2/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+bx^3}\,dx &= \frac{\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a^{2/3}} + \frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{2/3}} \\ &= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}\sqrt[3]{b}} + \frac{\int \frac{1}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{2\sqrt[3]{a}} - \frac{\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{6a^{2/3}\sqrt[3]{b}} \\ &= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}\sqrt[3]{b}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{b}} \\ &= -\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}\sqrt[3]{b}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 89, normalized size = 0.77

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+2\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6a^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(-1),x]
```

```
[Out] -1/6*(2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3)
 + b^(1/3)*x] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(a^(2/3)*b^
(1/3))
```

fricas [A] time = 0.52, size = 299, normalized size = 2.60

$$\left[\frac{3\sqrt{\frac{1}{3}}\,ab\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\,\log\left(\frac{2\,abx^3-3\left(a^2b\right)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2\,abx^2+\left(a^2b\right)^{\frac{2}{3}}x-\left(a^2b\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}}{bx^3+a}\right)-\left(a^2b\right)^{\frac{2}{3}}\log\left(abx^2-\left(a^2b\right)^{\frac{2}{3}}x+\left(\right.\right.\right.}{6\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/
3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*
```

```
sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - (a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^
(2/3)*x + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a
^2*b), 1/6*(6*sqrt(1/3)*a*b*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*
b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - (a^2*b)^(2/3)*lo
g(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*x
+ (a^2*b)^(2/3)))/(a^2*b)]
```

giac [A] time = 0.16, size = 112, normalized size = 0.97

$$-\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a}+\frac{\sqrt{3}\,\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab}+\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a + 1/3*sqrt(3)*(-a*b^2)^(1/3)
*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b) + 1/6*(-a*b^2)
^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b)
```

maple [A] time = 0.00, size = 91, normalized size = 0.79

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}+\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a),x)
```

```
[Out] 1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+
(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-
1))
```

maxima [A] time = 2.93, size = 98, normalized size = 0.85

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^(2
/3)) - 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)) + 1/3*log
(x + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

mupad [B] time = 1.14, size = 99, normalized size = 0.86

$$\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)}{3\,a^{2/3}\,b^{1/3}}+\frac{\ln\left(3\,b^2\,x+\frac{3\,a^{1/3}\,b^{5/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}}-\frac{\ln\left(3\,b^2\,x-\frac{3\,a^{1/3}\,b^{5/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^3),x)
```

```
[Out] log(b^(1/3)*x + a^(1/3))/(3*a^(2/3)*b^(1/3)) + (log(3*b^2*x + (3*a^(1/3)*b^
(5/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(6*a^(2/3)*b^(1/3)) - (log(3*b
^2*x - (3*a^(1/3)*b^(5/3)*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(6*a^(2/3)
*b^(1/3))
```

sympy [**A**] time = 0.17, size = 20, normalized size = 0.17

$$\operatorname{RootSum}\left(27t^3a^2b - 1, \left(t \mapsto t\log\left(3ta + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(3*_t*a + x)))
```

3.327 $\int \frac{1}{x^2\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=122

$$-\frac{\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{4/3}}+\frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{4/3}}+\frac{\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}}-\frac{1}{ax}$$

```
[Out] -1/a/x+1/3*b^(1/3)*ln(a^(1/3)+b^(1/3)*x)/a^(4/3)-1/6*b^(1/3)*ln(a^(2/3)-a^(
1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)+1/3*b^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/
3)*x)/a^(1/3)*3^(1/2))/a^(4/3)*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {325, 292, 31, 634, 617, 204, 628}

$$-\frac{\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{4/3}}+\frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{4/3}}+\frac{\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^3)),x]
```

```
[Out] -(1/(a*x)) + (b^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(S
qrt[3]*a^(4/3)) + (b^(1/3)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(4/3)) - (b^(1/3)
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\left(a+bx^3\right)}\,dx &= -\frac{1}{ax} - \frac{b\int \frac{x}{a+bx^3}\,dx}{a} \\
&= -\frac{1}{ax} + \frac{b^{2/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a^{4/3}} - \frac{b^{2/3}\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{4/3}} \\
&= -\frac{1}{ax} + \frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{4/3}} - \frac{\sqrt[3]{b}\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{6a^{4/3}} - \frac{b^{2/3}\int \frac{1}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{2a} \\
&= -\frac{1}{ax} + \frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{4/3}} - \frac{\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{4/3}} - \frac{\sqrt[3]{b}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x\right.}{a^{4/3}} \\
&= -\frac{1}{ax} + \frac{\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}} + \frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{4/3}} - \frac{\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{4/3}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 114, normalized size = 0.93

$$\frac{-\sqrt[3]{b}\,x\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)+2\sqrt[3]{b}\,x\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+2\sqrt{3}\,\sqrt[3]{b}\,x\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-6\sqrt[3]{a}}{6a^{4/3}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^3)),x]
```

```
[Out] (-6*a^(1/3) + 2*Sqrt[3]*b^(1/3)*x*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3
]] + 2*b^(1/3)*x*Log[a^(1/3) + b^(1/3)*x] - b^(1/3)*x*Log[a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)*x)
```

fricas [A] time = 0.48, size = 103, normalized size = 0.84

$$-\frac{2\sqrt{3}\,x\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\,x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+x\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(\frac{b}{a}\right)^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-2\,x\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)+}{6\,ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*x*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3
)) + x*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) - 2*x*(b/a)
^(1/3)*log(b*x + a*(b/a)^(2/3)) + 6)/(a*x)
```

giac [A] time = 0.18, size = 121, normalized size = 0.99

$$\frac{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^2}+\frac{\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^2b}-\frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^2b}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*b*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 1/3*sqrt(3)*(-a*b^2)^(2
/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b) - 1/6*(-a
*b^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b) - 1/(a*x)
```

maple [A] time = 0.00, size = 99, normalized size = 0.81

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a}+\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}a}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a),x)
```

```
[Out] 1/3/a/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-1/6/a/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+
(a/b)^(2/3))-1/3/a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-
1))-1/a/x
```

maxima [A] time = 3.00, size = 106, normalized size = 0.87

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*(a/b)^(
1/3)) - 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(1/3)) + 1/3*lo
g(x + (a/b)^(1/3))/(a*(a/b)^(1/3)) - 1/(a*x)
```

mupad [B] time = 1.16, size = 102, normalized size = 0.84

$$\frac{b^{1/3}\ln\left(b^{1/3}x+a^{1/3}\right)}{3a^{4/3}}-\frac{1}{ax}-\frac{b^{1/3}\ln\left(4b^{1/3}x-2a^{1/3}+\sqrt{3}\,a^{1/3}2\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3a^{4/3}}+\frac{b^{1/3}\ln\left(4b^{1/3}x-2a^{1/3}-\sqrt{3}\,a^{1}\right.}{a^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^3)),x)
```

```
[Out] (b^(1/3)*log(b^(1/3)*x + a^(1/3)))/(3*a^(4/3)) - 1/(a*x) - (b^(1/3)*log(3^(
1/2)*a^(1/3)*2i + 4*b^(1/3)*x - 2*a^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(3*a^(4/
3)) + (b^(1/3)*log(4*b^(1/3)*x - 3^(1/2)*a^(1/3)*2i - 2*a^(1/3))*((3^(1/2)*
1i)/6 - 1/6))/a^(4/3)
```

sympy [**A**] time = 0.20, size = 29, normalized size = 0.24

$$\operatorname{RootSum}\left(27t^3a^4 - b, \left(t \mapsto t\log\left(\frac{9t^2a^3}{b} + x\right)\right)\right) - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**4 - b, Lambda(_t, _t*log(9*_t**2*a**3/b + x))) - 1/(a*x
)
```

3.328 $\int \frac{1}{x^3\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=124

$$\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{5/3}}-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{5/3}}+\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}}-\frac{1}{2ax^2}$$

```
[Out] -1/2/a/x^2-1/3*b^(2/3)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)+1/6*b^(2/3)*ln(a^(2/3)
-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)+1/3*b^(2/3)*arctan(1/3*(a^(1/3)-2*b
^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)*3^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {325, 200, 31, 634, 617, 204, 628}

$$\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{5/3}}-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{5/3}}+\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^3)),x]
```

```
[Out] -1/(2*a*x^2) + (b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/
(Sqrt[3]*a^(5/3)) - (b^(2/3)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(5/3)) + (b^(2/
3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^3\right)}\,dx = -\frac{1}{2ax^2} - \frac{b\int \frac{1}{a+bx^3}\,dx}{a}$$

$$= -\frac{1}{2ax^2} - \frac{b\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a^{5/3}} - \frac{b\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{5/3}}$$

$$= -\frac{1}{2ax^2} - \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{5/3}} + \frac{b^{2/3}\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{6a^{5/3}} - \frac{b\int \frac{1}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{2a^{4/3}}$$

$$= -\frac{1}{2ax^2} - \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{5/3}} + \frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{5/3}} - \frac{b^{2/3}\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx\right.}{a^{5/3}}$$

$$= -\frac{1}{2ax^2} + \frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}} - \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{5/3}} + \frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{5/3}}$$

Mathematica [A] time = 0.02, size = 119, normalized size = 0.96

$$\frac{b^{2/3}x^2\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right) - 3a^{2/3} - 2b^{2/3}x^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right) + 2\sqrt{3}\,b^{2/3}x^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6a^{5/3}x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^3)),x]
```

```
[Out] (-3*a^(2/3) + 2*Sqrt[3]*b^(2/3)*x^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt
[3]] - 2*b^(2/3)*x^2*Log[a^(1/3) + b^(1/3)*x] + b^(2/3)*x^2*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(5/3)*x^2)
```

fricas [A] time = 0.59, size = 143, normalized size = 1.15

$$\frac{2\sqrt{3}\,x^2\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3\,b}\right) - x^2\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2+abx\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right) + 2\,x^2\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b\right.}{6\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*x^2*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2
/3) - sqrt(3)*b)/b) - x^2*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(
1/3) + a^2*(-b^2/a^2)^(2/3)) + 2*x^2*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2
)^(1/3)) - 3)/(a*x^2)
```

giac [A] time = 0.17, size = 115, normalized size = 0.93

$$\frac{b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^2}-\frac{\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^2}-\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^2}-\frac{1}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*b*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 - 1/3*sqrt(3)*(-a*b^2)^(1
/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^2 - 1/6*(-a*b^2
)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^2 - 1/2/(a*x^2)
```

maple [A] time = 0.00, size = 99, normalized size = 0.80

$$-\frac{\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a}-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}a}-\frac{1}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a),x)
```

```
[Out] -1/3/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+1/6/a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x
+(a/b)^(2/3))-1/3/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x
-1))-1/2/a/x^2
```

maxima [A] time = 3.05, size = 106, normalized size = 0.85

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{1}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*(a/b)^(
2/3)) + 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(2/3)) - 1/3*lo
g(x + (a/b)^(1/3))/(a*(a/b)^(2/3)) - 1/2/(a*x^2)
```

mupad [B] time = 0.22, size = 128, normalized size = 1.03

$$\frac{b^{2/3}\ln\left((-a)^{7/3}-a^2b^{1/3}x\right)}{3(-a)^{5/3}}-\frac{1}{2ax^2}-\frac{b^{2/3}\ln\left(3a^2b^3x+3(-a)^{7/3}b^{8/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3(-a)^{5/3}}+\frac{b^{2/3}\ln\left(3a^2b^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^3)),x)
```

```
[Out] (b^(2/3)*log((-a)^(7/3) - a^2*b^(1/3)*x))/(3*(-a)^(5/3)) - 1/(2*a*x^2) - (b
^(2/3)*log(3*a^2*b^3*x + 3*(-a)^(7/3)*b^(8/3)*((3^(1/2)*1i)/2 + 1/2))*((3^(
1/2)*1i)/2 + 1/2))/(3*(-a)^(5/3)) + (b^(2/3)*log(3*a^2*b^3*x - 9*(-a)^(7/3)
*b^(8/3)*((3^(1/2)*1i)/6 - 1/6))*((3^(1/2)*1i)/6 - 1/6))/(-a)^(5/3)
```

sympy [**A**] time = 0.32, size = 32, normalized size = 0.26

$$\operatorname{RootSum}\left(27t^3a^5 + b^2, \left(t \mapsto t\log\left(-\frac{3ta^2}{b} + x\right)\right)\right) - \frac{1}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**5 + b**2, Lambda(_t, _t*log(-3*_t*a**2/b + x))) - 1/(2*
a*x**2)
```

3.329 $\int \frac{x^8}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=46

$$-\frac{a^2}{3b^3\left(a+bx^3\right)}-\frac{2a\log\left(a+bx^3\right)}{3b^3}+\frac{x^3}{3b^2}$$

```
[Out] 1/3*x^3/b^2-1/3*a^2/b^3/(b*x^3+a)-2/3*a*ln(b*x^3+a)/b^3
```

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$-\frac{a^2}{3b^3\left(a+bx^3\right)}-\frac{2a\log\left(a+bx^3\right)}{3b^3}+\frac{x^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^3)^2,x]
```

```
[Out] x^3/(3*b^2) - a^2/(3*b^3*(a + b*x^3)) - (2*a*Log[a + b*x^3])/(3*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{\left(a+bx^3\right)^2}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{(a+bx)^2}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{1}{b^2}+\frac{a^2}{b^2(a+bx)^2}-\frac{2a}{b^2(a+bx)}\right)\,dx, x, x^3\right)\\ &= \frac{x^3}{3b^2}-\frac{a^2}{3b^3\left(a+bx^3\right)}-\frac{2a\log\left(a+bx^3\right)}{3b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 0.83

$$\frac{-\frac{a^2}{a+bx^3}-2a\log\left(a+bx^3\right)+bx^3}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^3)^2,x]
```

```
[Out] (b*x^3 - a^2/(a + b*x^3) - 2*a*Log[a + b*x^3])/(3*b^3)
```

fricas [A] time = 0.61, size = 56, normalized size = 1.22

$$\frac{b^2x^6 + abx^3 - a^2 - 2\left(abx^3 + a^2\right)\log\left(bx^3 + a\right)}{3\left(b^4x^3 + ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*x^6 + a*b*x^3 - a^2 - 2*(a*b*x^3 + a^2)*log(b*x^3 + a))/(b^4*x^3 +
 a*b^3)
```

giac [A] time = 0.17, size = 49, normalized size = 1.07

$$\frac{x^3}{3\,b^2} - \frac{2\,a\log\left(\left|bx^3 + a\right|\right)}{3\,b^3} + \frac{2\,abx^3 + a^2}{3\left(bx^3 + a\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3/b^2 - 2/3*a*log(abs(b*x^3 + a))/b^3 + 1/3*(2*a*b*x^3 + a^2)/((b*x^3
 + a)*b^3)
```

maple [A] time = 0.01, size = 41, normalized size = 0.89

$$\frac{x^3}{3b^2} - \frac{a^2}{3\left(b\,x^3 + a\right)b^3} - \frac{2a\ln\left(b\,x^3 + a\right)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a)^2,x)
```

```
[Out] 1/3*x^3/b^2-1/3*a^2/b^3/(b*x^3+a)-2/3*a*ln(b*x^3+a)/b^3
```

maxima [A] time = 1.37, size = 43, normalized size = 0.93

$$-\frac{a^2}{3\left(b^4x^3 + ab^3\right)} + \frac{x^3}{3\,b^2} - \frac{2\,a\log\left(bx^3 + a\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*a^2/(b^4*x^3 + a*b^3) + 1/3*x^3/b^2 - 2/3*a*log(b*x^3 + a)/b^3
```

mupad [B] time = 0.05, size = 45, normalized size = 0.98

$$\frac{x^3}{3\,b^2} - \frac{a^2}{3\,\left(b^4\,x^3 + a\,b^3\right)} - \frac{2\,a\,\ln\left(b\,x^3 + a\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^3)^2,x)
```

```
[Out] x^3/(3*b^2) - a^2/(3*(a*b^3 + b^4*x^3)) - (2*a*log(a + b*x^3))/(3*b^3)
```

sympy [**A**] time = 0.37, size = 42, normalized size = 0.91

$$-\frac{a^2}{3ab^3+3b^4x^3}-\frac{2a\log\left(a+bx^3\right)}{3b^3}+\frac{x^3}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a)**2,x)
```

```
[Out] -a**2/(3*a*b**3 + 3*b**4*x**3) - 2*a*log(a + b*x**3)/(3*b**3) + x**3/(3*b**
2)
```

3.330 $\int \frac{x^5}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=33

$$\frac{a}{3b^2\left(a+bx^3\right)}+\frac{\log\left(a+bx^3\right)}{3b^2}$$

```
[Out] 1/3*a/b^2/(b*x^3+a)+1/3*ln(b*x^3+a)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{a}{3b^2\left(a+bx^3\right)}+\frac{\log\left(a+bx^3\right)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^3)^2,x]
```

```
[Out] a/(3*b^2*(a + b*x^3)) + Log[a + b*x^3]/(3*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\left(a+bx^3\right)^2}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{(a+bx)^2}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^2}+\frac{1}{b(a+bx)}\right)\,dx, x, x^3\right)\\ &= \frac{a}{3b^2\left(a+bx^3\right)}+\frac{\log\left(a+bx^3\right)}{3b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.82

$$\frac{\frac{a}{a+bx^3}+\log\left(a+bx^3\right)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^3)^2,x]
```

```
[Out] (a/(a + b*x^3) + Log[a + b*x^3])/(3*b^2)
```

fricas [A] time = 0.59, size = 35, normalized size = 1.06

$$\frac{\left(bx^3+a\right)\log\left(bx^3+a\right)+a}{3\left(b^3x^3+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^2,x, algorithm="fricas")
[Out] 1/3*((b*x^3 + a)*log(b*x^3 + a) + a)/(b^3*x^3 + a*b^2)
```

giac [A] time = 0.16, size = 48, normalized size = 1.45

$$-\frac{\frac{\log\left(\frac{\left|bx^3+a\right|}{\left(bx^3+a\right)^2|b|}\right)}{b}-\frac{a}{\left(bx^3+a\right)b}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^2,x, algorithm="giac")
[Out] -1/3*(log(abs(b*x^3 + a)/((b*x^3 + a)^2*abs(b)))/b - a/((b*x^3 + a)*b))/b
```

maple [A] time = 0.01, size = 30, normalized size = 0.91

$$\frac{a}{3\left(b\,x^3+a\right)b^2}+\frac{\ln\left(b\,x^3+a\right)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a)^2,x)
[Out] 1/3*a/b^2/(b*x^3+a)+1/3*ln(b*x^3+a)/b^2
```

maxima [A] time = 1.34, size = 32, normalized size = 0.97

$$\frac{a}{3\left(b^3x^3+ab^2\right)}+\frac{\log\left(bx^3+a\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^2,x, algorithm="maxima")
[Out] 1/3*a/(b^3*x^3 + a*b^2) + 1/3*log(b*x^3 + a)/b^2
```

mupad [B] time = 0.04, size = 29, normalized size = 0.88

$$\frac{\ln\left(b\,x^3+a\right)}{3\,b^2}+\frac{a}{3\,b^2\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^3)^2,x)
[Out] log(a + b*x^3)/(3*b^2) + a/(3*b^2*(a + b*x^3))
```

sympy [A] time = 0.33, size = 29, normalized size = 0.88

$$\frac{a}{3ab^2+3b^3x^3}+\frac{\log\left(a+bx^3\right)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)**2,x)
[Out] a/(3*a*b**2 + 3*b**3*x**3) + log(a + b*x**3)/(3*b**2)
```

3.331 $\int \frac{x^2}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{3b\left(a+bx^3\right)}$$

```
[Out] -1/3/b/(b*x^3+a)
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$-\frac{1}{3b\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^3)^2,x]
```

```
[Out] -1/(3*b*(a + b*x^3))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^3\right)^2}\,dx = -\frac{1}{3b\left(a+bx^3\right)}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{3b\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^3)^2,x]
```

```
[Out] -1/3*1/(b*(a + b*x^3))
```

fricas [A] time = 0.59, size = 15, normalized size = 0.94

$$-\frac{1}{3\left(b^2x^3+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/3/(b^2*x^3 + a*b)
```

giac [A] time = 0.17, size = 14, normalized size = 0.88

$$-\frac{1}{3\left(bx^3+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^2,x, algorithm="giac")
[Out] -1/3/((b*x^3 + a)*b)
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$-\frac{1}{3\left(b\,x^3+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)^2,x)
[Out] -1/3/b/(b*x^3+a)
```

maxima [A] time = 1.30, size = 14, normalized size = 0.88

$$-\frac{1}{3\left(bx^3+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^2,x, algorithm="maxima")
[Out] -1/3/((b*x^3 + a)*b)
```

mupad [B] time = 0.97, size = 14, normalized size = 0.88

$$-\frac{1}{3\,b\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^3)^2,x)
[Out] -1/(3*b*(a + b*x^3))
```

sympy [A] time = 0.25, size = 15, normalized size = 0.94

$$-\frac{1}{3ab+3b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)**2,x)
[Out] -1/(3*a*b + 3*b**2*x**3)
```

3.332 $\int \frac{1}{x\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=38

$$-\frac{\log\left(a+bx^3\right)}{3a^2}+\frac{\log(x)}{a^2}+\frac{1}{3a\left(a+bx^3\right)}$$

```
[Out] 1/3/a/(b*x^3+a)+ln(x)/a^2-1/3*ln(b*x^3+a)/a^2
```

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 44}

$$-\frac{\log\left(a+bx^3\right)}{3a^2}+\frac{\log(x)}{a^2}+\frac{1}{3a\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3)^2),x]
```

```
[Out] 1/(3*a*(a + b*x^3)) + Log[x]/a^2 - Log[a + b*x^3]/(3*a^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a+bx^3\right)^2}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^2}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right)\,dx, x, x^3\right)\\ &= \frac{1}{3a\left(a+bx^3\right)}+\frac{\log(x)}{a^2}-\frac{\log\left(a+bx^3\right)}{3a^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.87

$$\frac{\frac{a}{a+bx^3}-\log\left(a+bx^3\right)+3\log(x)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3)^2),x]
```

```
[Out] (a/(a + b*x^3) + 3*Log[x] - Log[a + b*x^3])/(3*a^2)
```

fricas [A] time = 0.77, size = 47, normalized size = 1.24

$$-\frac{\left(bx^3+a\right)\log\left(bx^3+a\right)-3\left(bx^3+a\right)\log(x)-a}{3\left(a^2bx^3+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/3*((b*x^3 + a)*log(b*x^3 + a) - 3*(b*x^3 + a)*log(x) - a)/(a^2*b*x^3 + a
^3)
```

giac [A] time = 0.15, size = 45, normalized size = 1.18

$$-\frac{\log\left(\left|bx^3+a\right|\right)}{3\,a^2}+\frac{\log\left(|x|\right)}{a^2}+\frac{bx^3+2\,a}{3\left(bx^3+a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*log(abs(b*x^3 + a))/a^2 + log(abs(x))/a^2 + 1/3*(b*x^3 + 2*a)/((b*x^3
+ a)*a^2)
```

maple [A] time = 0.01, size = 35, normalized size = 0.92

$$\frac{1}{3\left(b\,x^3+a\right)a}+\frac{\ln(x)}{a^2}-\frac{\ln\left(b\,x^3+a\right)}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)^2,x)
```

```
[Out] 1/3/a/(b*x^3+a)+ln(x)/a^2-1/3*ln(b*x^3+a)/a^2
```

maxima [A] time = 1.32, size = 37, normalized size = 0.97

$$\frac{1}{3\left(abx^3+a^2\right)}-\frac{\log\left(bx^3+a\right)}{3\,a^2}+\frac{\log\left(x^3\right)}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3/(a*b*x^3 + a^2) - 1/3*log(b*x^3 + a)/a^2 + 1/3*log(x^3)/a^2
```

mupad [B] time = 1.00, size = 34, normalized size = 0.89

$$\frac{\ln(x)}{a^2}+\frac{1}{3\,a\,\left(b\,x^3+a\right)}-\frac{\ln\left(b\,x^3+a\right)}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3)^2),x)
```

```
[Out] log(x)/a^2 + 1/(3*a*(a + b*x^3)) - log(a + b*x^3)/(3*a^2)
```

sympy [A] time = 0.43, size = 34, normalized size = 0.89

$$\frac{1}{3a^2+3abx^3}+\frac{\log(x)}{a^2}-\frac{\log\left(\frac{a}{b}+x^3\right)}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)**2,x)
[Out] 1/(3*a**2 + 3*a*b*x**3) + log(x)/a**2 - log(a/b + x**3)/(3*a**2)
```

3.333 $\int \frac{1}{x^4\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=52

$$\frac{2b\log\left(a+bx^3\right)}{3a^3}-\frac{2b\log(x)}{a^3}-\frac{b}{3a^2\left(a+bx^3\right)}-\frac{1}{3a^2x^3}$$

```
[Out] -1/3/a^2/x^3-1/3*b/a^2/(b*x^3+a)-2*b*ln(x)/a^3+2/3*b*ln(b*x^3+a)/a^3
```

Rubi [A] time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 44}

$$-\frac{b}{3a^2\left(a+bx^3\right)}+\frac{2b\log\left(a+bx^3\right)}{3a^3}-\frac{2b\log(x)}{a^3}-\frac{1}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3)^2),x]
```

```
[Out] -1/(3*a^2*x^3) - b/(3*a^2*(a + b*x^3)) - (2*b*Log[x])/a^3 + (2*b*Log[a + b*
x^3])/(3*a^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(a+bx^3\right)^2}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)^2}\,dx,x,x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int\left(\frac{1}{a^2x^2}-\frac{2b}{a^3x}+\frac{b^2}{a^2(a+bx)^2}+\frac{2b^2}{a^3(a+bx)}\right)dx,x,x^3\right)\\ &= -\frac{1}{3a^2x^3}-\frac{b}{3a^2\left(a+bx^3\right)}-\frac{2b\log(x)}{a^3}+\frac{2b\log\left(a+bx^3\right)}{3a^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 41, normalized size = 0.79

$$-\frac{a\left(\frac{b}{a+bx^3}+\frac{1}{x^3}\right)-2b\log\left(a+bx^3\right)+6b\log(x)}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3)^2),x]
```

```
[Out] -1/3*(a*(x^(-3) + b/(a + b*x^3)) + 6*b*Log[x] - 2*b*Log[a + b*x^3])/a^3
```

fricas [A] time = 0.80, size = 73, normalized size = 1.40

$$-\frac{2\,abx^3+a^2-2\left(b^2x^6+abx^3\right)\log\left(bx^3+a\right)+6\left(b^2x^6+abx^3\right)\log(x)}{3\left(a^3bx^6+a^4x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(2*a*b*x^3 + a^2 - 2*(b^2*x^6 + a*b*x^3)*log(b*x^3 + a) + 6*(b^2*x^6 +
 a*b*x^3)*log(x))/(a^3*b*x^6 + a^4*x^3)
```

giac [A] time = 0.18, size = 51, normalized size = 0.98

$$\frac{2\,b\log\left(\left|bx^3+a\right|\right)}{3\,a^3}-\frac{2\,b\log\left(|x|\right)}{a^3}-\frac{2\,bx^3+a}{3\left(bx^6+ax^3\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 2/3*b*log(abs(b*x^3 + a))/a^3 - 2*b*log(abs(x))/a^3 - 1/3*(2*b*x^3 + a)/((b
*x^6 + a*x^3)*a^2)
```

maple [A] time = 0.01, size = 47, normalized size = 0.90

$$-\frac{b}{3\left(b\,x^3+a\right)a^2}-\frac{2b\ln(x)}{a^3}+\frac{2b\ln\left(b\,x^3+a\right)}{3a^3}-\frac{1}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)^2,x)
```

```
[Out] -1/3/a^2/x^3-1/3*b/a^2/(b*x^3+a)-2*b*ln(x)/a^3+2/3*b*ln(b*x^3+a)/a^3
```

maxima [A] time = 1.33, size = 53, normalized size = 1.02

$$-\frac{2\,bx^3+a}{3\left(a^2bx^6+a^3x^3\right)}+\frac{2\,b\log\left(bx^3+a\right)}{3\,a^3}-\frac{2\,b\log\left(x^3\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(2*b*x^3 + a)/(a^2*b*x^6 + a^3*x^3) + 2/3*b*log(b*x^3 + a)/a^3 - 2/3*b
*log(x^3)/a^3
```

mupad [B] time = 0.07, size = 53, normalized size = 1.02

$$\frac{2\,b\,\ln\left(b\,x^3+a\right)}{3\,a^3}-\frac{\frac{1}{3\,a}+\frac{2\,b\,x^3}{3\,a^2}}{b\,x^6+a\,x^3}-\frac{2\,b\,\ln(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3)^2),x)
```

```
[Out] (2*b*log(a + b*x^3))/(3*a^3) - (1/(3*a) + (2*b*x^3)/(3*a^2))/(a*x^3 + b*x^6
) - (2*b*log(x))/a^3
```

sympy [**A**] time = 0.56, size = 54, normalized size = 1.04

$$\frac{-a-2bx^3}{3a^3x^3+3a^2bx^6}-\frac{2b\log(x)}{a^3}+\frac{2b\log\left(\frac{a}{b}+x^3\right)}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)**2,x)
```

```
[Out] (-a - 2*b*x**3)/(3*a**3*x**3 + 3*a**2*b*x**6) - 2*b*log(x)/a**3 + 2*b*log(a
/b + x**3)/(3*a**3)
```

3.334 $\int \frac{x^4}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=136

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{9\sqrt[3]{a}\,b^{5/3}}-\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9\sqrt[3]{a}\,b^{5/3}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,\sqrt[3]{a}\,b^{5/3}}-\frac{x^2}{3b\left(a+bx^3\right)}$$

```
[Out] -1/3*x^2/b/(b*x^3+a)-2/9*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(5/3)+1/9*ln(a^(2/
3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1/3)/b^(5/3)-2/9*arctan(1/3*(a^(1/3)-2
*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(1/3)/b^(5/3)*3^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {288, 292, 31, 634, 617, 204, 628}

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{9\sqrt[3]{a}\,b^{5/3}}-\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9\sqrt[3]{a}\,b^{5/3}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,\sqrt[3]{a}\,b^{5/3}}-\frac{x^2}{3b\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^3)^2,x]
```

```
[Out] -x^2/(3*b*(a + b*x^3)) - (2*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3)
)])/(3*Sqrt[3]*a^(1/3)*b^(5/3)) - (2*Log[a^(1/3) + b^(1/3)*x])/(9*a^(1/3)*b
^(5/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(9*a^(1/3)*b^(5/3)
)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^3\right)^2}\,dx = -\frac{x^2}{3b\left(a+bx^3\right)} + \frac{2\int \frac{x}{a+bx^3}\,dx}{3b}$$

$$= -\frac{x^2}{3b\left(a+bx^3\right)} - \frac{2\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{9\sqrt[3]{a}\,b^{4/3}} + \frac{2\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{9\sqrt[3]{a}\,b^{4/3}}$$

$$= -\frac{x^2}{3b\left(a+bx^3\right)} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9\sqrt[3]{a}\,b^{5/3}} + \frac{\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{9\sqrt[3]{a}\,b^{5/3}} + \frac{\int \frac{1}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3b^{4/3}}$$

$$= -\frac{x^2}{3b\left(a+bx^3\right)} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9\sqrt[3]{a}\,b^{5/3}} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{9\sqrt[3]{a}\,b^{5/3}} + \frac{2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x\right.}{3\sqrt[3]{a}\,b^{5/3}}$$

$$= -\frac{x^2}{3b\left(a+bx^3\right)} - \frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,\sqrt[3]{a}\,b^{5/3}} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9\sqrt[3]{a}\,b^{5/3}} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{9\sqrt[3]{a}\,b^{5/3}}$$

Mathematica [A] time = 0.08, size = 119, normalized size = 0.88

$$\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{\sqrt[3]{a}} - \frac{3b^{2/3}x^2}{a+bx^3} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a}} - \frac{2\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{9b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^3)^2,x]
```

```
[Out] ((-3*b^(2/3)*x^2)/(a + b*x^3) - (2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3
))/Sqrt[3]])/a^(1/3) - (2*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) + Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/a^(1/3))/(9*b^(5/3))
```

fricas [A] time = 0.59, size = 400, normalized size = 2.94

$$\left[-\frac{3\,ab^2x^2-3\sqrt{\frac{1}{3}}\left(ab^2x^3+a^2b\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\,\log\left(\frac{2\,b^2x^3-ab+3\,\sqrt{\frac{1}{3}}\left(abx+2\left(-ab^2\right)^{\frac{2}{3}}x^2+\left(-ab^2\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}-3\left(-ab^2\right)^{\frac{2}{3}}x}{bx^3+a}\right)-\left(bx^3\right.}{9\left(ab^4x^3+a^2b^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/9*(3*a*b^2*x^2 - 3*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*sqrt((-a*b^2)^(1/3)/a)
*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2
)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - (b*x
^3 + a)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) +
 2*(b*x^3 + a)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a*b^4*x^3 + a^2*b
^3), -1/9*(3*a*b^2*x^2 - 6*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*sqrt(-(-a*b^2)^(1/
3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b)
 - (b*x^3 + a)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2
/3)) + 2*(b*x^3 + a)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a*b^4*x^3 +
 a^2*b^3)]
```

giac [A] time = 0.16, size = 132, normalized size = 0.97

$$-\frac{x^2}{3\left(bx^3+a\right)b}-\frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab}-\frac{2\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^3}+\frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right.}{9\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*x^2/((b*x^3 + a)*b) - 2/9*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a*b
) - 2/9*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/
b)^(1/3))/(a*b^3) + 1/9*(-a*b^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2
/3))/(a*b^3)
```

maple [A] time = 0.01, size = 108, normalized size = 0.79

$$-\frac{x^2}{3\left(b\,x^3+a\right)b}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}-\frac{2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a)^2,x)
```

```
[Out] -1/3*x^2/b/(b*x^3+a)-2/9/b^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/9/b^2/(a/b)^(1
/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+2/9/b^2*3^(1/2)/(a/b)^(1/3)*arctan(1/
3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

maxima [A] time = 2.99, size = 116, normalized size = 0.85

$$-\frac{x^2}{3\left(b^2x^3+ab\right)}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{2\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*x^2/(b^2*x^3 + a*b) + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3
))/(a/b)^(1/3))/(b^2*(a/b)^(1/3)) + 1/9*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/
3))/(b^2*(a/b)^(1/3)) - 2/9*log(x + (a/b)^(1/3))/(b^2*(a/b)^(1/3))
```

mupad [B] time = 0.22, size = 138, normalized size = 1.01

$$\frac{2\ln\left(\frac{4x}{9b}-\frac{4(-a)^{1/3}}{9b^{4/3}}\right)}{9(-a)^{1/3}b^{5/3}}-\frac{x^2}{3b\left(bx^3+a\right)}+\frac{\ln\left(\frac{4x}{9b}-\frac{(-a)^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{9b^{4/3}}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{9(-a)^{1/3}b^{5/3}}-\frac{\ln\left(\frac{4x}{9b}-\frac{(-a)^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{9b^{4/3}}\right)\left(1+\sqrt{3}\right.}{9(-a)^{1/3}b^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^3)^2,x)
```

```
[Out] (2*log((4*x)/(9*b) - (4*(-a)^(1/3))/(9*b^(4/3))))/(9*(-a)^(1/3)*b^(5/3)) -
x^2/(3*b*(a + b*x^3)) + (log((4*x)/(9*b) - ((-a)^(1/3)*(3^(1/2)*1i - 1)^2)/
(9*b^(4/3)))*(3^(1/2)*1i - 1))/(9*(-a)^(1/3)*b^(5/3)) - (log((4*x)/(9*b) -
((-a)^(1/3)*(3^(1/2)*1i + 1)^2)/(9*b^(4/3)))*(3^(1/2)*1i + 1))/(9*(-a)^(1/3
)*b^(5/3))
```

sympy [A] time = 0.28, size = 44, normalized size = 0.32

$$-\frac{x^2}{3ab+3b^2x^3}+\operatorname{RootSum}\left(729t^3ab^5+8,\left(t\mapsto t\log\left(\frac{81t^2ab^3}{4}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a)**2,x)
```

```
[Out] -x**2/(3*a*b + 3*b**2*x**3) + RootSum(729*_t**3*a*b**5 + 8, Lambda(_t, _t*l
og(81*_t**2*a*b**3/4 + x)))
```

3.335 $\int \frac{x^3}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=134

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{2/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{2/3}b^{4/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}b^{4/3}}-\frac{x}{3b\left(a+bx^3\right)}$$

```
[Out] -1/3*x/b/(b*x^3+a)+1/9*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(4/3)-1/18*ln(a^(2/3
)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(4/3)-1/9*arctan(1/3*(a^(1/3)-2*
b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(4/3)*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {288, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{2/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{2/3}b^{4/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}b^{4/3}}-\frac{x}{3b\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^3)^2,x]
```

```
[Out] -x/(3*b*(a + b*x^3)) - ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(3
*Sqrt[3]*a^(2/3)*b^(4/3)) + Log[a^(1/3) + b^(1/3)*x]/(9*a^(2/3)*b^(4/3)) -
Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(18*a^(2/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^3\right)^2}\,dx = -\frac{x}{3b\left(a+bx^3\right)} + \frac{\int \frac{1}{a+bx^3}\,dx}{3b}$$

$$= -\frac{x}{3b\left(a+bx^3\right)} + \frac{\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{9a^{2/3}b} + \frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{9a^{2/3}b}$$

$$= -\frac{x}{3b\left(a+bx^3\right)} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{2/3}b^{4/3}} - \frac{\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{18a^{2/3}b^{4/3}} + \frac{\int \frac{1}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{6\sqrt[3]{a}\,b}$$

$$= -\frac{x}{3b\left(a+bx^3\right)} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{2/3}b^{4/3}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{2/3}b^{4/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\right.}{3a^{2/3}b^{4/3}}$$

$$= -\frac{x}{3b\left(a+bx^3\right)} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}b^{4/3}} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{2/3}b^{4/3}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{2/3}b^{4/3}}$$

Mathematica [A] time = 0.06, size = 118, normalized size = 0.88

$$\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{2/3}} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{2/3}} - \frac{2\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} - \frac{6\sqrt[3]{b}\,x}{a+bx^3}}{18b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^3)^2,x]
```

```
[Out] ((-6*b^(1/3)*x)/(a + b*x^3) - (2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))
/Sqrt[3]])/a^(2/3) + (2*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) - Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/a^(2/3))/(18*b^(4/3))
```

fricas [A] time = 0.61, size = 391, normalized size = 2.92

$$\left[-\frac{6\,a^2bx-3\sqrt{\frac{1}{3}}\left(ab^2x^3+a^2b\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\,\log\left(\frac{2\,abx^3-3\left(a^2b\right)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2\,abx^2+\left(a^2b\right)^{\frac{2}{3}}x-\left(a^2b\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}}{bx^3+a}\right)+\left(bx^3+\right.}{18\left(a^2b^3x^3+a^3b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/18*(6*a^2*b*x - 3*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*sqrt(-(a^2*b)^(1/3)/b)*
log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*
b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) + (b*x^3
 + a)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 2*(b
*x^3 + a)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^2*b^3*x^3 + a^3*b^2)
, -1/18*(6*a^2*b*x - 6*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*sqrt((a^2*b)^(1/3)/b)*
arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b
)/a^2) + (b*x^3 + a)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^
(1/3)*a) - 2*(b*x^3 + a)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^2*b^3
*x^3 + a^3*b^2)]
```

giac [A] time = 0.24, size = 130, normalized size = 0.97

$$-\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab}-\frac{x}{3\left(bx^3+a\right)b}+\frac{\sqrt{3}\,\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^2}+\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\right.}{18\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b) - 1/3*x/((b*x^3 + a)*b)
+ 1/9*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)
^(1/3))/(a*b^2) + 1/18*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/
3))/(a*b^2)
```

maple [A] time = 0.01, size = 106, normalized size = 0.79

$$-\frac{x}{3\left(b\,x^3+a\right)b}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}+\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a)^2,x)
```

```
[Out] -1/3*x/b/(b*x^3+a)+1/9/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/18/b^2/(a/b)^(2/
3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/9/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3
*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

maxima [A] time = 2.84, size = 114, normalized size = 0.85

$$-\frac{x}{3\left(b^2x^3+ab\right)}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*x/(b^2*x^3 + a*b) + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))
/(a/b)^(1/3))/(b^2*(a/b)^(2/3)) - 1/18*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3
))/(b^2*(a/b)^(2/3)) + 1/9*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))
```

mupad [B] time = 1.15, size = 108, normalized size = 0.81

$$\frac{\ln\left(b^{1/3}x+a^{1/3}\right)}{9a^{2/3}b^{4/3}}-\frac{x}{3b\left(bx^3+a\right)}+\frac{\ln\left(bx+\frac{a^{1/3}b^{2/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{18a^{2/3}b^{4/3}}-\frac{\ln\left(bx-\frac{a^{1/3}b^{2/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right.}{18a^{2/3}b^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^3)^2,x)
```

```
[Out] log(b^(1/3)*x + a^(1/3))/(9*a^(2/3)*b^(4/3)) - x/(3*b*(a + b*x^3)) + (log(b
*x + (a^(1/3)*b^(2/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(18*a^(2/3)*b^
(4/3)) - (log(b*x - (a^(1/3)*b^(2/3)*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))
/(18*a^(2/3)*b^(4/3))
```

sympy [A] time = 0.28, size = 39, normalized size = 0.29

$$-\frac{x}{3ab+3b^2x^3}+\operatorname{RootSum}\left(729t^3a^2b^4-1,\left(t\mapsto t\log\left(9tab+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a)**2,x)
```

```
[Out] -x/(3*a*b + 3*b**2*x**3) + RootSum(729*_t**3*a**2*b**4 - 1, Lambda(_t, _t*l
og(9*_t*a*b + x)))
```

3.336 $\int \frac{x}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=136

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{4/3}b^{2/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{4/3}b^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}b^{2/3}}+\frac{x^2}{3a\left(a+bx^3\right)}$$

```
[Out] 1/3*x^2/a/(b*x^3+a)-1/9*ln(a^(1/3)+b^(1/3)*x)/a^(4/3)/b^(2/3)+1/18*ln(a^(2/
3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/b^(2/3)-1/9*arctan(1/3*(a^(1/3)-2
*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(4/3)/b^(2/3)*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {290, 292, 31, 634, 617, 204, 628}

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{4/3}b^{2/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{4/3}b^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}b^{2/3}}+\frac{x^2}{3a\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^3)^2,x]
```

```
[Out] x^2/(3*a*(a + b*x^3)) - ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(
3*Sqrt[3]*a^(4/3)*b^(2/3)) - Log[a^(1/3) + b^(1/3)*x]/(9*a^(4/3)*b^(2/3)) +
 Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(18*a^(4/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^3\right)^2}\,dx = \frac{x^2}{3a\left(a+bx^3\right)} + \frac{\int \frac{x}{a+bx^3}\,dx}{3a}$$

$$= \frac{x^2}{3a\left(a+bx^3\right)} - \frac{\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{9a^{4/3}\sqrt[3]{b}} + \frac{\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{9a^{4/3}\sqrt[3]{b}}$$

$$= \frac{x^2}{3a\left(a+bx^3\right)} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{4/3}b^{2/3}} + \frac{\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{18a^{4/3}b^{2/3}} + \frac{\int \frac{1}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{6a\sqrt[3]{b}}$$

$$= \frac{x^2}{3a\left(a+bx^3\right)} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{4/3}b^{2/3}} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{4/3}b^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\right.}{3a^{4/3}b^{2/3}}$$

$$= \frac{x^2}{3a\left(a+bx^3\right)} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}b^{2/3}} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{4/3}b^{2/3}} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{4/3}b^{2/3}}$$

Mathematica [A] time = 0.06, size = 119, normalized size = 0.88

$$\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{b^{2/3}} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{b^{2/3}} - \frac{2\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}} + \frac{6\sqrt[3]{a}\,x^2}{a+bx^3}}{18a^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^3)^2,x]
```

```
[Out] ((6*a^(1/3)*x^2)/(a + b*x^3) - (2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3)
)/Sqrt[3]])/b^(2/3) - (2*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/b^(2/3))/(18*a^(4/3))
```

fricas [A] time = 0.55, size = 402, normalized size = 2.96

$$\left[\frac{6\,ab^2x^2+3\sqrt{\frac{1}{3}}\left(ab^2x^3+a^2b\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\,\log\left(\frac{2b^2x^3-ab+3\sqrt{\frac{1}{3}}\left(abx+2\left(-ab^2\right)^{\frac{2}{3}}x^2+\left(-ab^2\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}-3\left(-ab^2\right)^{\frac{2}{3}}x}{bx^3+a}\right)+\left(bx^3+\right.}{18\left(a^2b^3x^3+a^3b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/18*(6*a*b^2*x^2 + 3*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*sqrt((-a*b^2)^(1/3)/a)
*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2
)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + (b*x
^3 + a)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) -
 2*(b*x^3 + a)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^3*x^3 + a^3
*b^2), 1/18*(6*a*b^2*x^2 + 6*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*sqrt(-(-a*b^2)^(
1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b
) + (b*x^3 + a)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^
(2/3)) - 2*(b*x^3 + a)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^3*x
^3 + a^3*b^2)]
```

giac [A] time = 0.20, size = 129, normalized size = 0.95

$$\frac{x^2}{3\left(bx^3+a\right)a}-\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2}-\frac{\sqrt{3}\,\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b^2}+\frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\right.\right.}{18\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*x^2/((b*x^3 + a)*a) - 1/9*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/a^2 -
 1/9*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^
(1/3))/(a^2*b^2) + 1/18*(-a*b^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2
/3))/(a^2*b^2)
```

maple [A] time = 0.00, size = 117, normalized size = 0.86

$$\frac{x^2}{3\left(b\,x^3+a\right)a}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a)^2,x)
```

```
[Out] 1/3*x^2/a/(b*x^3+a)-1/9/a/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/18/a/b/(a/b)^(1
/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/9/a*3^(1/2)/b/(a/b)^(1/3)*arctan(1/
3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

maxima [A] time = 3.00, size = 124, normalized size = 0.91

$$\frac{x^2}{3\left(abx^3+a^2\right)}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^2/(a*b*x^3 + a^2) + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3)
)/(a/b)^(1/3))/(a*b*(a/b)^(1/3)) + 1/18*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/
3))/(a*b*(a/b)^(1/3)) - 1/9*log(x + (a/b)^(1/3))/(a*b*(a/b)^(1/3))
```

mupad [B] time = 1.20, size = 138, normalized size = 1.01

$$\frac{x^2}{3\,a\,\left(b\,x^3+a\right)}+\frac{(-1)^{1/3}\,\ln\left(\frac{(-1)^{2/3}\,b^{2/3}}{9\,a^{5/3}}+\frac{b\,x}{9\,a^2}\right)}{9\,a^{4/3}\,b^{2/3}}-\frac{(-1)^{1/3}\,\ln\left((-1)^{2/3}\,a^{1/3}-2\,b^{1/3}\,x+(-1)^{1/6}\,\sqrt{3}\,a^{1/3}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,a^{4/3}\,b^{2/3}}+\frac{(-1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^3)^2,x)
```

```
[Out] x^2/(3*a*(a + b*x^3)) + ((-1)^(1/3)*log(((-1)^(2/3)*b^(2/3))/(9*a^(5/3)) +
(b*x)/(9*a^2)))/(9*a^(4/3)*b^(2/3)) - ((-1)^(1/3)*log((-1)^(2/3)*a^(1/3) -
2*b^(1/3)*x + (-1)^(1/6)*3^(1/2)*a^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(9*a^(4/3
)*b^(2/3)) + ((-1)^(1/3)*log(2*b^(1/3)*x - (-1)^(2/3)*a^(1/3) + (-1)^(1/6)*
3^(1/2)*a^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(9*a^(4/3)*b^(2/3))
```

sympy [A] time = 0.29, size = 44, normalized size = 0.32

$$\frac{x^2}{3a^2+3abx^3}+\operatorname{RootSum}\left(729t^3a^4b^2+1,\left(t\mapsto t\log\left(81t^2a^3b+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a)**2,x)
```

```
[Out] x**2/(3*a**2 + 3*a*b*x**3) + RootSum(729*_t**3*a**4*b**2 + 1, Lambda(_t, _t
*log(81*_t**2*a**3*b + x)))
```

3.337 $\int \frac{1}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=134

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{9a^{5/3}\sqrt[3]{b}}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}\sqrt[3]{b}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}\sqrt[3]{b}}+\frac{x}{3a\left(a+bx^3\right)}$$

```
[Out] 1/3*x/a/(b*x^3+a)+2/9*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(1/3)-1/9*ln(a^(2/3)-
a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(1/3)-2/9*arctan(1/3*(a^(1/3)-2*b^
(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(1/3)*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {199, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{9a^{5/3}\sqrt[3]{b}}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}\sqrt[3]{b}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}\sqrt[3]{b}}+\frac{x}{3a\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(-2),x]
```

```
[Out] x/(3*a*(a + b*x^3)) - (2*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])
/(3*Sqrt[3]*a^(5/3)*b^(1/3)) + (2*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(1
/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(9*a^(5/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^2}\,dx = \frac{x}{3a\left(a+bx^3\right)} + \frac{2\int \frac{1}{a+bx^3}\,dx}{3a}$$

$$= \frac{x}{3a\left(a+bx^3\right)} + \frac{2\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{9a^{5/3}} + \frac{2\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{9a^{5/3}}$$

$$= \frac{x}{3a\left(a+bx^3\right)} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}\sqrt[3]{b}} + \frac{\int \frac{1}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{4/3}} - \frac{\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{9a^{5/3}\sqrt[3]{b}}$$

$$= \frac{x}{3a\left(a+bx^3\right)} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}\sqrt[3]{b}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{9a^{5/3}\sqrt[3]{b}} + \frac{2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, \right.}{3a^{5/3}\sqrt[3]{b}}$$

$$= \frac{x}{3a\left(a+bx^3\right)} - \frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}\sqrt[3]{b}} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}\sqrt[3]{b}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{9a^{5/3}\sqrt[3]{b}}$$

Mathematica [A] time = 0.06, size = 118, normalized size = 0.88

$$\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{\sqrt[3]{b}} + \frac{3a^{2/3}x}{a+bx^3} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}} - \frac{2\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{9a^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(-2),x]
```

```
[Out] ((3*a^(2/3)*x)/(a + b*x^3) - (2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/
Sqrt[3]])/b^(1/3) + (2*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) - Log[a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2]/b^(1/3))/(9*a^(5/3))
```

fricas [A] time = 0.82, size = 389, normalized size = 2.90

$$\left[\frac{3\,a^2bx+3\sqrt{\frac{1}{3}}\left(ab^2x^3+a^2b\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\,\log\left(\frac{2\,abx^3-3\left(a^2b\right)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2\,abx^2+\left(a^2b\right)^{\frac{2}{3}}x-\left(a^2b\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}}{bx^3+a}\right)-\left(bx^3+a\right.}{9\left(a^3b^2x^3+a^4b\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/9*(3*a^2*b*x + 3*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*sqrt(-(a^2*b)^(1/3)/b)*lo
g((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)
^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - (b*x^3 +
 a)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(b*x
^3 + a)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^3*b^2*x^3 + a^4*b), 1/
9*(3*a^2*b*x + 6*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*sqrt((a^2*b)^(1/3)/b)*arctan
(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2)
 - (b*x^3 + a)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*
a) + 2*(b*x^3 + a)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^3*b^2*x^3 +
 a^4*b)]
```

giac [A] time = 0.17, size = 127, normalized size = 0.95

$$-\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2}+\frac{x}{3\left(bx^3+a\right)a}+\frac{2\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b}+\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right.}{9\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -2/9*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 1/3*x/((b*x^3 + a)*a) +
2/9*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(
1/3))/(a^2*b) + 1/9*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))
/(a^2*b)
```

maple [A] time = 0.00, size = 115, normalized size = 0.86

$$\frac{x}{3\left(b\,x^3+a\right)a}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}+\frac{2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^2,x)
```

```
[Out] 1/3*x/a/(b*x^3+a)+2/9/a/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/9/a/b/(a/b)^(2/3)
*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+2/9/a/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3
^(1/2)*(2/(a/b)^(1/3)*x-1))
```

maxima [A] time = 2.96, size = 122, normalized size = 0.91

$$\frac{x}{3\left(abx^3+a^2\right)}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*x/(a*b*x^3 + a^2) + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/
(a/b)^(1/3))/(a*b*(a/b)^(2/3)) - 1/9*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))
/(a*b*(a/b)^(2/3)) + 2/9*log(x + (a/b)^(1/3))/(a*b*(a/b)^(2/3))
```

mupad [B] time = 1.11, size = 128, normalized size = 0.96

$$\frac{x}{3a\left(bx^3+a\right)}+\frac{2\ln\left(\frac{2b^{5/3}}{a^{2/3}}+\frac{2b^2x}{a}\right)}{9a^{5/3}b^{1/3}}+\frac{\ln\left(\frac{2b^2x}{a}+\frac{b^{5/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{a^{2/3}}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{9a^{5/3}b^{1/3}}-\frac{\ln\left(\frac{2b^2x}{a}-\frac{b^{5/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{a^{2/3}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right.}{9a^{5/3}b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^3)^2,x)
```

```
[Out] x/(3*a*(a + b*x^3)) + (2*log((2*b^(5/3))/a^(2/3) + (2*b^2*x)/a))/(9*a^(5/3)
*b^(1/3)) + (log((2*b^2*x)/a + (b^(5/3)*(3^(1/2)*1i - 1))/a^(2/3))*(3^(1/2)
*1i - 1))/(9*a^(5/3)*b^(1/3)) - (log((2*b^2*x)/a - (b^(5/3)*(3^(1/2)*1i + 1
))/a^(2/3))*(3^(1/2)*1i + 1))/(9*a^(5/3)*b^(1/3))
```

sympy [A] time = 0.31, size = 39, normalized size = 0.29

$$\frac{x}{3a^2+3abx^3}+\operatorname{RootSum}\left(729t^3a^5b-8,\left(t\mapsto t\log\left(\frac{9ta^2}{2}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**2,x)
```

```
[Out] x/(3*a**2 + 3*a*b*x**3) + RootSum(729*_t**3*a**5*b - 8, Lambda(_t, _t*log(9
*_t*a**2/2 + x)))
```

3.338 $\int \frac{1}{x^2\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=146

$$-\frac{2\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{9a^{7/3}}+\frac{4\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{7/3}}+\frac{4\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}}-\frac{4}{3a^2x}+\frac{1}{3ax\left(a+bx^3\right)}$$

```
[Out] -4/3/a^2/x+1/3/a/x/(b*x^3+a)+4/9*b^(1/3)*ln(a^(1/3)+b^(1/3)*x)/a^(7/3)-2/9*
b^(1/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(7/3)+4/9*b^(1/3)*arcta
n(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(7/3)*3^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {290, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{2\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{9a^{7/3}}+\frac{4\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{7/3}}+\frac{4\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}}-\frac{4}{3a^2x}+\frac{1}{3ax\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^3)^2),x]
```

```
[Out] -4/(3*a^2*x) + 1/(3*a*x*(a + b*x^3)) + (4*b^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/
3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)) + (4*b^(1/3)*Log[a^(1/3) + b^
(1/3)*x])/(9*a^(7/3)) - (2*b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2])/(9*a^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\left(a+bx^3\right)^2}\,dx &= \frac{1}{3ax\left(a+bx^3\right)} + \frac{4\int \frac{1}{x^2\left(a+bx^3\right)}\,dx}{3a} \\
&= -\frac{4}{3a^2x} + \frac{1}{3ax\left(a+bx^3\right)} - \frac{(4b)\int \frac{x}{a+bx^3}\,dx}{3a^2} \\
&= -\frac{4}{3a^2x} + \frac{1}{3ax\left(a+bx^3\right)} + \frac{\left(4b^{2/3}\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{9a^{7/3}} - \frac{\left(4b^{2/3}\right)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{9a^{7/3}} \\
&= -\frac{4}{3a^2x} + \frac{1}{3ax\left(a+bx^3\right)} + \frac{4\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{7/3}} - \frac{\left(2\sqrt[3]{b}\right)\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{9a^{7/3}} - \frac{\left(2b^{2/3}\right.}{} \\
&= -\frac{4}{3a^2x} + \frac{1}{3ax\left(a+bx^3\right)} + \frac{4\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{7/3}} - \frac{2\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{9a^{7/3}} - \frac{\left(\right.}{} \\
&= -\frac{4}{3a^2x} + \frac{1}{3ax\left(a+bx^3\right)} + \frac{4\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}} + \frac{4\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{7/3}} - \frac{2\sqrt[3]{b}\,\log\left(a^{2/}\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 131, normalized size = 0.90

$$\frac{-2\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right) - \frac{3\sqrt[3]{a}\,bx^2}{a+bx^3} + 4\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right) + 4\sqrt{3}\,\sqrt[3]{b}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right) - \frac{9\sqrt[3]{a}}{x}}{9a^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^3)^2),x]
```

```
[Out] ((-9*a^(1/3))/x - (3*a^(1/3)*b*x^2)/(a + b*x^3) + 4*Sqrt[3]*b^(1/3)*ArcTan[
(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 4*b^(1/3)*Log[a^(1/3) + b^(1/3)*x] -
 2*b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(9*a^(7/3))
```

fricas [A] time = 0.58, size = 146, normalized size = 1.00

$$-\frac{12\,bx^3+4\sqrt{3}\left(bx^4+ax\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\,x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+2\left(bx^4+ax\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(\frac{b}{a}\right)^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right.}{9\left(a^2bx^4+a^3x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/9*(12*b*x^3 + 4*sqrt(3)*(b*x^4 + a*x)*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(
b/a)^(1/3) - 1/3*sqrt(3)) + 2*(b*x^4 + a*x)*(b/a)^(1/3)*log(b*x^2 - a*x*(b/
a)^(2/3) + a*(b/a)^(1/3)) - 4*(b*x^4 + a*x)*(b/a)^(1/3)*log(b*x + a*(b/a)^(
2/3)) + 9*a)/(a^2*b*x^4 + a^3*x)
```

giac [A] time = 0.17, size = 139, normalized size = 0.95

$$\frac{4\,b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3}+\frac{4\sqrt{3}\,\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3b}-\frac{4\,bx^3+3\,a}{3\left(bx^4+ax\right)a^2}-\frac{2\,\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^2+x\left(-\right.\right.}{9\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 4/9*b*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/a^3 + 4/9*sqrt(3)*(-a*b^2)^(2
/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b) - 1/3*(4*
b*x^3 + 3*a)/((b*x^4 + a*x)*a^2) - 2/9*(-a*b^2)^(2/3)*log(x^2 + x*(-a/b)^(1
/3) + (-a/b)^(2/3))/(a^3*b)
```

maple [A] time = 0.01, size = 117, normalized size = 0.80

$$-\frac{b\,x^2}{3\left(b\,x^3+a\right)a^2}-\frac{4\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}+\frac{4\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}-\frac{2\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}-\frac{1}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a)^2,x)
```

```
[Out] -1/3/a^2*b*x^2/(b*x^3+a)+4/9/a^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-2/9/a^2/(a/b
)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-4/9/a^2*3^(1/2)/(a/b)^(1/3)*arcta
n(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/a^2/x
```

maxima [A] time = 2.98, size = 126, normalized size = 0.86

$$-\frac{4\,bx^3+3\,a}{3\left(a^2bx^4+a^3x\right)}-\frac{4\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{2\,\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{4\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(4*b*x^3 + 3*a)/(a^2*b*x^4 + a^3*x) - 4/9*sqrt(3)*arctan(1/3*sqrt(3)*(
2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*(a/b)^(1/3)) - 2/9*log(x^2 - x*(a/b)^(
1/3) + (a/b)^(2/3))/(a^2*(a/b)^(1/3)) + 4/9*log(x + (a/b)^(1/3))/(a^2*(a/b)
^(1/3))
```

mupad **[B]** time = 0.24, size = 120, normalized size = 0.82

$$\frac{4\,b^{1/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)}{9\,a^{7/3}}-\frac{\frac{1}{a}+\frac{4\,b\,x^3}{3\,a^2}}{b\,x^4+a\,x}-\frac{4\,b^{1/3}\,\ln\left(4\,b^{1/3}\,x-2\,a^{1/3}+\sqrt{3}\,a^{1/3}\,2\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,a^{7/3}}+\frac{b^{1/3}\,\ln\left(4\,b^{1/3}\,x-2\,a^{1/3}-\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^3)^2),x)
```

```
[Out] (4*b^(1/3)*log(b^(1/3)*x + a^(1/3)))/(9*a^(7/3)) - (1/a + (4*b*x^3)/(3*a^2)
)/(a*x + b*x^4) - (4*b^(1/3)*log(3^(1/2)*a^(1/3)*2i + 4*b^(1/3)*x - 2*a^(1/
3))*((3^(1/2)*1i)/2 + 1/2))/(9*a^(7/3)) + (b^(1/3)*log(4*b^(1/3)*x - 3^(1/2
)*a^(1/3)*2i - 2*a^(1/3))*((3^(1/2)*2i)/9 - 2/9))/a^(7/3)
```

sympy **[A]** time = 0.38, size = 56, normalized size = 0.38

$$\frac{-3a-4bx^3}{3a^3x+3a^2bx^4}+\operatorname{RootSum}\left(729t^3a^7-64b,\left(t\mapsto t\log\left(\frac{81t^2a^5}{16b}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a)**2,x)
```

```
[Out] (-3*a - 4*b*x**3)/(3*a**3*x + 3*a**2*b*x**4) + RootSum(729*_t**3*a**7 - 64*
b, Lambda(_t, _t*log(81*_t**2*a**5/(16*b) + x)))
```

3.339 $\int \frac{1}{x^3\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=146

$$\frac{5b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{8/3}}-\frac{5b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{8/3}}+\frac{5b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{8/3}}-\frac{5}{6a^2x^2}+\frac{1}{3ax^2\left(a+bx^3\right)}$$

```
[Out] -5/6/a^2/x^2+1/3/a/x^2/(b*x^3+a)-5/9*b^(2/3)*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)+
5/18*b^(2/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(8/3)+5/9*b^(2/3)*
arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)*3^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {290, 325, 200, 31, 634, 617, 204, 628}

$$\frac{5b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{8/3}}-\frac{5b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{8/3}}+\frac{5b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{8/3}}-\frac{5}{6a^2x^2}+\frac{1}{3ax^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^3)^2),x]
```

```
[Out] -5/(6*a^2*x^2) + 1/(3*a*x^2*(a + b*x^3)) + (5*b^(2/3)*ArcTan[(a^(1/3) - 2*b
^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)) - (5*b^(2/3)*Log[a^(1/3)
+ b^(1/3)*x])/(9*a^(8/3)) + (5*b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2])/(18*a^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(a+bx^3\right)^2}\,dx &= \frac{1}{3ax^2\left(a+bx^3\right)} + \frac{5\int \frac{1}{x^3\left(a+bx^3\right)}\,dx}{3a} \\
&= -\frac{5}{6a^2x^2} + \frac{1}{3ax^2\left(a+bx^3\right)} - \frac{(5b)\int \frac{1}{a+bx^3}\,dx}{3a^2} \\
&= -\frac{5}{6a^2x^2} + \frac{1}{3ax^2\left(a+bx^3\right)} - \frac{(5b)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{9a^{8/3}} - \frac{(5b)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{9a^{8/3}} \\
&= -\frac{5}{6a^2x^2} + \frac{1}{3ax^2\left(a+bx^3\right)} - \frac{5b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{8/3}} + \frac{\left(5b^{2/3}\right)\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{18a^{8/3}} - \frac{(5b)}{} \\
&= -\frac{5}{6a^2x^2} + \frac{1}{3ax^2\left(a+bx^3\right)} - \frac{5b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{8/3}} + \frac{5b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{8/3}} - \\
&= -\frac{5}{6a^2x^2} + \frac{1}{3ax^2\left(a+bx^3\right)} + \frac{5b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{8/3}} - \frac{5b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{8/3}} + \frac{5b^{2/3}\log\left(}{}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 129, normalized size = 0.88

$$\frac{5b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right) - \frac{6a^{2/3}bx}{a+bx^3} - \frac{9a^{2/3}}{x^2} - 10b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right) + 10\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{18a^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^3)^2),x]
```

```
[Out] ((-9*a^(2/3))/x^2 - (6*a^(2/3)*b*x)/(a + b*x^3) + 10*Sqrt[3]*b^(2/3)*ArcTan
[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 10*b^(2/3)*Log[a^(1/3) + b^(1/3)*x]
 + 5*b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(8/3))
```

fricas [A] time = 0.79, size = 187, normalized size = 1.28

$$-\frac{15\,bx^3-10\sqrt{3}\left(bx^5+ax^2\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3\,b}\right)+5\left(bx^5+ax^2\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2+abx\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}$$
$$18\left(a^2bx^5+a^3x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/18*(15*b*x^3 - 10*sqrt(3)*(b*x^5 + a*x^2)*(-b^2/a^2)^(1/3)*arctan(1/3*(2
*sqrt(3)*a*x*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) + 5*(b*x^5 + a*x^2)*(-b^2/a^2
)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) - 10*(
b*x^5 + a*x^2)*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)) + 9*a)/(a^2*b
*x^5 + a^3*x^2)
```

giac [A] time = 0.19, size = 131, normalized size = 0.90

$$\frac{5\,b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3}-\frac{bx}{3\left(bx^3+a\right)a^2}-\frac{5\sqrt{3}\,\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3}-\frac{5\,\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right.\right.}{18\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 5/9*b*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^3 - 1/3*b*x/((b*x^3 + a)*a^
2) - 5/9*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a
/b)^(1/3))/a^3 - 5/18*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3
))/a^3 - 1/2/(a^2*x^2)
```

maple [A] time = 0.01, size = 115, normalized size = 0.79

$$-\frac{bx}{3\left(b\,x^3+a\right)a^2}-\frac{5\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2}-\frac{5\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2}+\frac{5\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2}-\frac{1}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a)^2,x)
```

```
[Out] -1/3/a^2*b*x/(b*x^3+a)-5/9/a^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+5/18/a^2/(a/b)
^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-5/9/a^2/(a/b)^(2/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/2/a^2/x^2
```

maxima [A] time = 3.01, size = 128, normalized size = 0.88

$$-\frac{5bx^3+3a}{6\left(a^2bx^5+a^3x^2\right)}-\frac{5\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{5\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{5\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(5*b*x^3 + 3*a)/(a^2*b*x^5 + a^3*x^2) - 5/9*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*(a/b)^(2/3)) + 5/18*log(x^2 - x*(a/b
)^(1/3) + (a/b)^(2/3))/(a^2*(a/b)^(2/3)) - 5/9*log(x + (a/b)^(1/3))/(a^2*(a
/b)^(2/3))
```

mupad [B] time = 1.34, size = 146, normalized size = 1.00

$$\frac{5(-1)^{1/3}b^{2/3}\ln\left((-1)^{1/3}a^{1/3}-b^{1/3}x\right)}{9a^{8/3}}-\frac{\frac{1}{2a}+\frac{5bx^3}{6a^2}}{bx^5+ax^2}-\frac{5(-1)^{1/3}b^{2/3}\ln\left((-1)^{1/3}a^{1/3}+2b^{1/3}x+(-1)^{5/6}\sqrt{3}\,a^{1/3}\right)\left(\frac{1}{2}+\right.}{9a^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^3)^2),x)
```

```
[Out] (5*(-1)^(1/3)*b^(2/3)*log((-1)^(1/3)*a^(1/3) - b^(1/3)*x))/(9*a^(8/3)) - (1
/(2*a) + (5*b*x^3)/(6*a^2))/(a*x^2 + b*x^5) - (5*(-1)^(1/3)*b^(2/3)*log((-1
)^(1/3)*a^(1/3) + 2*b^(1/3)*x + (-1)^(5/6)*3^(1/2)*a^(1/3))*((3^(1/2)*1i)/2
 + 1/2))/(9*a^(8/3)) + (5*(-1)^(1/3)*b^(2/3)*log((-1)^(1/3)*a^(1/3) + 2*b^(
1/3)*x - (-1)^(5/6)*3^(1/2)*a^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(9*a^(8/3))
```

sympy [A] time = 0.46, size = 58, normalized size = 0.40

$$\frac{-3a-5bx^3}{6a^3x^2+6a^2bx^5}+\operatorname{RootSum}\left(729t^3a^8+125b^2,\left(t\mapsto t\log\left(-\frac{9ta^3}{5b}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a)**2,x)
```

```
[Out] (-3*a - 5*b*x**3)/(6*a**3*x**2 + 6*a**2*b*x**5) + RootSum(729*_t**3*a**8 +
125*b**2, Lambda(_t, _t*log(-9*_t*a**3/(5*b) + x)))
```

3.340 $\int \frac{x^{11}}{\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=61

$$\frac{a^3}{6b^4\left(a+bx^3\right)^2}-\frac{a^2}{b^4\left(a+bx^3\right)}-\frac{a\log\left(a+bx^3\right)}{b^4}+\frac{x^3}{3b^3}$$

```
[Out] 1/3*x^3/b^3+1/6*a^3/b^4/(b*x^3+a)^2-a^2/b^4/(b*x^3+a)-a*ln(b*x^3+a)/b^4
```

Rubi [A] time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{a^3}{6b^4\left(a+bx^3\right)^2}-\frac{a^2}{b^4\left(a+bx^3\right)}-\frac{a\log\left(a+bx^3\right)}{b^4}+\frac{x^3}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + b*x^3)^3,x]
```

```
[Out] x^3/(3*b^3) + a^3/(6*b^4*(a + b*x^3)^2) - a^2/(b^4*(a + b*x^3)) - (a*Log[a
+ b*x^3])/b^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{\left(a+bx^3\right)^3}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^3}{(a+bx)^3}\,dx, x, x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{1}{b^3}-\frac{a^3}{b^3(a+bx)^3}+\frac{3a^2}{b^3(a+bx)^2}-\frac{3a}{b^3(a+bx)}\right)dx, x, x^3\right)\\ &= \frac{x^3}{3b^3}+\frac{a^3}{6b^4\left(a+bx^3\right)^2}-\frac{a^2}{b^4\left(a+bx^3\right)}-\frac{a\log\left(a+bx^3\right)}{b^4}\end{aligned}$$

Mathematica [A] time = 0.06, size = 48, normalized size = 0.79

$$-\frac{\frac{a^2\left(5a+6bx^3\right)}{\left(a+bx^3\right)^2}+6a\log\left(a+bx^3\right)-2bx^3}{6b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + b*x^3)^3,x]
```

```
[Out] -1/6*(-2*b*x^3 + (a^2*(5*a + 6*b*x^3))/(a + b*x^3)^2 + 6*a*Log[a + b*x^3])/
b^4
```

fricas [A] time = 0.65, size = 91, normalized size = 1.49

$$\frac{2\,b^3x^9 + 4\,ab^2x^6 - 4\,a^2bx^3 - 5\,a^3 - 6\left(ab^2x^6 + 2\,a^2bx^3 + a^3\right)\log\left(bx^3 + a\right)}{6\left(b^6x^6 + 2\,ab^5x^3 + a^2b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^9 + 4*a*b^2*x^6 - 4*a^2*b*x^3 - 5*a^3 - 6*(a*b^2*x^6 + 2*a^2*b
*x^3 + a^3)*log(b*x^3 + a))/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4)
```

giac [A] time = 0.17, size = 62, normalized size = 1.02

$$\frac{x^3}{3\,b^3} - \frac{a\log\left(\left|bx^3 + a\right|\right)}{b^4} + \frac{9\,ab^2x^6 + 12\,a^2bx^3 + 4\,a^3}{6\left(bx^3 + a\right)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/3*x^3/b^3 - a*log(abs(b*x^3 + a))/b^4 + 1/6*(9*a*b^2*x^6 + 12*a^2*b*x^3 +
 4*a^3)/((b*x^3 + a)^2*b^4)
```

maple [A] time = 0.01, size = 58, normalized size = 0.95

$$\frac{x^3}{3b^3} + \frac{a^3}{6\left(b\,x^3 + a\right)^2b^4} - \frac{a^2}{\left(b\,x^3 + a\right)b^4} - \frac{a\ln\left(b\,x^3 + a\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^3+a)^3,x)
```

```
[Out] 1/3*x^3/b^3+1/6*a^3/b^4/(b*x^3+a)^2-a^2/b^4/(b*x^3+a)-a*ln(b*x^3+a)/b^4
```

maxima [A] time = 1.33, size = 66, normalized size = 1.08

$$-\frac{6\,a^2bx^3 + 5\,a^3}{6\left(b^6x^6 + 2\,ab^5x^3 + a^2b^4\right)} + \frac{x^3}{3\,b^3} - \frac{a\log\left(bx^3 + a\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(6*a^2*b*x^3 + 5*a^3)/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4) + 1/3*x^3/b^3
 - a*log(b*x^3 + a)/b^4
```

mupad [B] time = 1.02, size = 67, normalized size = 1.10

$$\frac{x^3}{3\,b^3} - \frac{\frac{5\,a^3}{6\,b} + a^2\,x^3}{a^2\,b^3 + 2\,a\,b^4\,x^3 + b^5\,x^6} - \frac{a\,\ln\left(b\,x^3 + a\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a + b*x^3)^3,x)
```

```
[Out] x^3/(3*b^3) - ((5*a^3)/(6*b) + a^2*x^3)/(a^2*b^3 + b^5*x^6 + 2*a*b^4*x^3) -
 (a*log(a + b*x^3))/b^4
```

sympy [**A**] time = 0.53, size = 65, normalized size = 1.07

$$-\frac{a\log\left(a+bx^3\right)}{b^4}+\frac{-5a^3-6a^2bx^3}{6a^2b^4+12ab^5x^3+6b^6x^6}+\frac{x^3}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**3+a)**3,x)
```

```
[Out] -a*log(a + b*x**3)/b**4 + (-5*a**3 - 6*a**2*b*x**3)/(6*a**2*b**4 + 12*a*b**
5*x**3 + 6*b**6*x**6) + x**3/(3*b**3)
```

3.341 $\int \frac{x^8}{\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=52

$$-\frac{a^2}{6b^3\left(a+bx^3\right)^2}+\frac{2a}{3b^3\left(a+bx^3\right)}+\frac{\log\left(a+bx^3\right)}{3b^3}$$

```
[Out] -1/6*a^2/b^3/(b*x^3+a)^2+2/3*a/b^3/(b*x^3+a)+1/3*ln(b*x^3+a)/b^3
```

Rubi [A] time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$-\frac{a^2}{6b^3\left(a+bx^3\right)^2}+\frac{2a}{3b^3\left(a+bx^3\right)}+\frac{\log\left(a+bx^3\right)}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^3)^3,x]
```

```
[Out] -a^2/(6*b^3*(a + b*x^3)^2) + (2*a)/(3*b^3*(a + b*x^3)) + Log[a + b*x^3]/(3*
b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{\left(a+bx^3\right)^3}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{(a+bx)^3}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{a^2}{b^2(a+bx)^3}-\frac{2a}{b^2(a+bx)^2}+\frac{1}{b^2(a+bx)}\right)dx, x, x^3\right)\\ &= -\frac{a^2}{6b^3\left(a+bx^3\right)^2}+\frac{2a}{3b^3\left(a+bx^3\right)}+\frac{\log\left(a+bx^3\right)}{3b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.75

$$\frac{\frac{a\left(3a+4bx^3\right)}{\left(a+bx^3\right)^2}+2\log\left(a+bx^3\right)}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^3)^3,x]
```

```
[Out] ((a*(3*a + 4*b*x^3))/(a + b*x^3)^2 + 2*Log[a + b*x^3])/(6*b^3)
```

fricas [A] time = 0.65, size = 69, normalized size = 1.33

$$\frac{4\,abx^3 + 3\,a^2 + 2\left(b^2x^6 + 2\,abx^3 + a^2\right)\log\left(bx^3 + a\right)}{6\left(b^5x^6 + 2\,ab^4x^3 + a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(4*a*b*x^3 + 3*a^2 + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*log(b*x^3 + a))/(b^5
*x^6 + 2*a*b^4*x^3 + a^2*b^3)
```

giac [A] time = 0.20, size = 42, normalized size = 0.81

$$\frac{\log\left(\left|bx^3 + a\right|\right)}{3\,b^3} - \frac{3\,bx^6 + 2\,ax^3}{6\left(bx^3 + a\right)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/3*log(abs(b*x^3 + a))/b^3 - 1/6*(3*b*x^6 + 2*a*x^3)/((b*x^3 + a)^2*b^2)
```

maple [A] time = 0.01, size = 47, normalized size = 0.90

$$-\frac{a^2}{6\left(b\,x^3 + a\right)^2b^3} + \frac{2a}{3\left(b\,x^3 + a\right)b^3} + \frac{\ln\left(b\,x^3 + a\right)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a)^3,x)
```

```
[Out] -1/6*a^2/b^3/(b*x^3+a)^2+2/3*a/b^3/(b*x^3+a)+1/3*ln(b*x^3+a)/b^3
```

maxima [A] time = 1.35, size = 55, normalized size = 1.06

$$\frac{4\,abx^3 + 3\,a^2}{6\left(b^5x^6 + 2\,ab^4x^3 + a^2b^3\right)} + \frac{\log\left(bx^3 + a\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(4*a*b*x^3 + 3*a^2)/(b^5*x^6 + 2*a*b^4*x^3 + a^2*b^3) + 1/3*log(b*x^3 +
 a)/b^3
```

mupad [B] time = 0.06, size = 53, normalized size = 1.02

$$\frac{\frac{a^2}{2\,b^3} + \frac{2\,a\,x^3}{3\,b^2}}{a^2 + 2\,a\,b\,x^3 + b^2\,x^6} + \frac{\ln\left(b\,x^3 + a\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^3)^3,x)
```

```
[Out] (a^2/(2*b^3) + (2*a*x^3)/(3*b^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) + log(a + b*x
^3)/(3*b^3)
```

sympy [**A**] time = 0.46, size = 53, normalized size = 1.02

$$\frac{3a^2 + 4abx^3}{6a^2b^3 + 12ab^4x^3 + 6b^5x^6} + \frac{\log\left(a + bx^3\right)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a)**3,x)
```

```
[Out] (3*a**2 + 4*a*b*x**3)/(6*a**2*b**3 + 12*a*b**4*x**3 + 6*b**5*x**6) + log(a
+ b*x**3)/(3*b**3)
```

3.342 $\int \frac{x^5}{\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=19

$$\frac{x^6}{6a\left(a+bx^3\right)^2}$$

```
[Out] 1/6*x^6/a/(b*x^3+a)^2
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {264}

$$\frac{x^6}{6a\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^3)^3,x]
```

```
[Out] x^6/(6*a*(a + b*x^3)^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^5}{\left(a+bx^3\right)^3}\,dx = \frac{x^6}{6a\left(a+bx^3\right)^2}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.26

$$-\frac{a+2bx^3}{6b^2\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^3)^3,x]
```

```
[Out] -1/6*(a + 2*b*x^3)/(b^2*(a + b*x^3)^2)
```

fricas [B] time = 0.51, size = 36, normalized size = 1.89

$$-\frac{2\,bx^3+a}{6\left(b^4x^6+2\,ab^3x^3+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b*x^3 + a)/(b^4*x^6 + 2*a*b^3*x^3 + a^2*b^2)
```

giac [A] time = 0.17, size = 22, normalized size = 1.16

$$-\frac{2\,bx^3+a}{6\left(bx^3+a\right)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^3,x, algorithm="giac")
[Out] -1/6*(2*b*x^3 + a)/((b*x^3 + a)^2*b^2)
```

maple [A] time = 0.01, size = 31, normalized size = 1.63

$$\frac{a}{6\left(b\,x^3+a\right)^2b^2}-\frac{1}{3\left(b\,x^3+a\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a)^3,x)
[Out] -1/3/b^2/(b*x^3+a)+1/6*a/b^2/(b*x^3+a)^2
```

maxima [B] time = 1.33, size = 36, normalized size = 1.89

$$-\frac{2\,bx^3+a}{6\left(b^4x^6+2\,ab^3x^3+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^3,x, algorithm="maxima")
[Out] -1/6*(2*b*x^3 + a)/(b^4*x^6 + 2*a*b^3*x^3 + a^2*b^2)
```

mupad [B] time = 0.03, size = 37, normalized size = 1.95

$$-\frac{\frac{a}{6\,b^2}+\frac{x^3}{3\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^3)^3,x)
[Out] -(a/(6*b^2) + x^3/(3*b))/(a^2 + b^2*x^6 + 2*a*b*x^3)
```

sympy [B] time = 0.39, size = 36, normalized size = 1.89

$$\frac{-a-2bx^3}{6a^2b^2+12ab^3x^3+6b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)**3,x)
[Out] (-a - 2*b*x**3)/(6*a**2*b**2 + 12*a*b**3*x**3 + 6*b**4*x**6)
```

3.343 $\int \frac{x^2}{\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{6b\left(a+bx^3\right)^2}$$

```
[Out] -1/6/b/(b*x^3+a)^2
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$-\frac{1}{6b\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^3)^3,x]
```

```
[Out] -1/(6*b*(a + b*x^3)^2)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^3\right)^3}\,dx = -\frac{1}{6b\left(a+bx^3\right)^2}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{6b\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^3)^3,x]
```

```
[Out] -1/6*1/(b*(a + b*x^3)^2)
```

fricas [A] time = 0.45, size = 26, normalized size = 1.62

$$-\frac{1}{6\left(b^3x^6+2\,ab^2x^3+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/6/(b^3*x^6 + 2*a*b^2*x^3 + a^2*b)
```

giac [A] time = 0.19, size = 14, normalized size = 0.88

$$-\frac{1}{6\left(bx^3+a\right)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^3,x, algorithm="giac")
[Out] -1/6/((b*x^3 + a)^2*b)
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$-\frac{1}{6\left(b\,x^3+a\right)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)^3,x)
[Out] -1/6/b/(b*x^3+a)^2
```

maxima [A] time = 1.35, size = 14, normalized size = 0.88

$$-\frac{1}{6\left(bx^3+a\right)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^3,x, algorithm="maxima")
[Out] -1/6/((b*x^3 + a)^2*b)
```

mupad [B] time = 0.94, size = 28, normalized size = 1.75

$$-\frac{1}{6\,a^2\,b+12\,a\,b^2\,x^3+6\,b^3\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^3)^3,x)
[Out] -1/(6*a^2*b + 6*b^3*x^6 + 12*a*b^2*x^3)
```

sympy [A] time = 0.32, size = 27, normalized size = 1.69

$$-\frac{1}{6a^2b+12ab^2x^3+6b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)**3,x)
[Out] -1/(6*a**2*b + 12*a*b**2*x**3 + 6*b**3*x**6)
```

3.344 $\int \frac{1}{x\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=54

$$-\frac{\log\left(a+bx^3\right)}{3a^3}+\frac{\log(x)}{a^3}+\frac{1}{3a^2\left(a+bx^3\right)}+\frac{1}{6a\left(a+bx^3\right)^2}$$

```
[Out] 1/6/a/(b*x^3+a)^2+1/3/a^2/(b*x^3+a)+ln(x)/a^3-1/3*ln(b*x^3+a)/a^3
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 44}

$$\frac{1}{3a^2\left(a+bx^3\right)}-\frac{\log\left(a+bx^3\right)}{3a^3}+\frac{\log(x)}{a^3}+\frac{1}{6a\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3)^3),x]
```

```
[Out] 1/(6*a*(a + b*x^3)^2) + 1/(3*a^2*(a + b*x^3)) + Log[x]/a^3 - Log[a + b*x^3]
/(3*a^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a+bx^3\right)^3}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^3}\,dx, x, x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int\left(\frac{1}{a^3x}-\frac{b}{a(a+bx)^3}-\frac{b}{a^2(a+bx)^2}-\frac{b}{a^3(a+bx)}\right)dx, x, x^3\right)\\ &= \frac{1}{6a\left(a+bx^3\right)^2}+\frac{1}{3a^2\left(a+bx^3\right)}+\frac{\log(x)}{a^3}-\frac{\log\left(a+bx^3\right)}{3a^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 43, normalized size = 0.80

$$\frac{\frac{a\left(3a+2bx^3\right)}{\left(a+bx^3\right)^2}-2\log\left(a+bx^3\right)+6\log(x)}{6a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3)^3),x]
```

```
[Out] ((a*(3*a + 2*b*x^3))/(a + b*x^3)^2 + 6*Log[x] - 2*Log[a + b*x^3])/(6*a^3)
```

fricas [A] time = 0.67, size = 90, normalized size = 1.67

$$\frac{2\,abx^3 + 3\,a^2 - 2\left(b^2x^6 + 2\,abx^3 + a^2\right)\log\left(bx^3 + a\right) + 6\left(b^2x^6 + 2\,abx^3 + a^2\right)\log(x)}{6\left(a^3b^2x^6 + 2\,a^4bx^3 + a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*b*x^3 + 3*a^2 - 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*log(b*x^3 + a) + 6*(
b^2*x^6 + 2*a*b*x^3 + a^2)*log(x))/(a^3*b^2*x^6 + 2*a^4*b*x^3 + a^5)
```

giac [A] time = 0.16, size = 57, normalized size = 1.06

$$-\frac{\log\left(\left|bx^3 + a\right|\right)}{3\,a^3} + \frac{\log\left(|x|\right)}{a^3} + \frac{3\,b^2x^6 + 8\,abx^3 + 6\,a^2}{6\left(bx^3 + a\right)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/3*log(abs(b*x^3 + a))/a^3 + log(abs(x))/a^3 + 1/6*(3*b^2*x^6 + 8*a*b*x^3
 + 6*a^2)/((b*x^3 + a)^2*a^3)
```

maple [A] time = 0.01, size = 49, normalized size = 0.91

$$\frac{1}{6\left(b\,x^3 + a\right)^2a} + \frac{1}{3\left(b\,x^3 + a\right)a^2} + \frac{\ln(x)}{a^3} - \frac{\ln\left(b\,x^3 + a\right)}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)^3,x)
```

```
[Out] 1/6/a/(b*x^3+a)^2+1/3/a^2/(b*x^3+a)+ln(x)/a^3-1/3*ln(b*x^3+a)/a^3
```

maxima [A] time = 1.35, size = 60, normalized size = 1.11

$$\frac{2\,bx^3 + 3\,a}{6\left(a^2b^2x^6 + 2\,a^3bx^3 + a^4\right)} - \frac{\log\left(bx^3 + a\right)}{3\,a^3} + \frac{\log\left(x^3\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b*x^3 + 3*a)/(a^2*b^2*x^6 + 2*a^3*b*x^3 + a^4) - 1/3*log(b*x^3 + a)/
a^3 + 1/3*log(x^3)/a^3
```

mupad [B] time = 0.07, size = 56, normalized size = 1.04

$$\frac{\ln(x)}{a^3} + \frac{\frac{1}{2\,a} + \frac{b\,x^3}{3\,a^2}}{a^2 + 2\,a\,b\,x^3 + b^2\,x^6} - \frac{\ln\left(b\,x^3 + a\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3)^3),x)
```

```
[Out] log(x)/a^3 + (1/(2*a) + (b*x^3)/(3*a^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) - log(
a + b*x^3)/(3*a^3)
```

sympy [A] time = 0.54, size = 56, normalized size = 1.04

$$\frac{3a + 2bx^3}{6a^4 + 12a^3bx^3 + 6a^2b^2x^6} + \frac{\log(x)}{a^3} - \frac{\log\left(\frac{a}{b} + x^3\right)}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)**3,x)
```

```
[Out] (3*a + 2*b*x**3)/(6*a**4 + 12*a**3*b*x**3 + 6*a**2*b**2*x**6) + log(x)/a**3
 - log(a/b + x**3)/(3*a**3)
```

3.345 $\int \frac{1}{x^4\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=66

$$\frac{b\log\left(a+bx^3\right)}{a^4}-\frac{3b\log(x)}{a^4}-\frac{2b}{3a^3\left(a+bx^3\right)}-\frac{1}{3a^3x^3}-\frac{b}{6a^2\left(a+bx^3\right)^2}$$

```
[Out] -1/3/a^3/x^3-1/6*b/a^2/(b*x^3+a)^2-2/3*b/a^3/(b*x^3+a)-3*b*ln(x)/a^4+b*ln(b
*x^3+a)/a^4
```

Rubi [A] time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 44}

$$-\frac{2b}{3a^3\left(a+bx^3\right)}-\frac{b}{6a^2\left(a+bx^3\right)^2}+\frac{b\log\left(a+bx^3\right)}{a^4}-\frac{3b\log(x)}{a^4}-\frac{1}{3a^3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3)^3),x]
```

```
[Out] -1/(3*a^3*x^3) - b/(6*a^2*(a + b*x^3)^2) - (2*b)/(3*a^3*(a + b*x^3)) - (3*b
*Log[x])/a^4 + (b*Log[a + b*x^3])/a^4
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(a+bx^3\right)^3}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)^3}\,dx, x, x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int\left(\frac{1}{a^3x^2}-\frac{3b}{a^4x}+\frac{b^2}{a^2(a+bx)^3}+\frac{2b^2}{a^3(a+bx)^2}+\frac{3b^2}{a^4(a+bx)}\right)dx, x, x^3\right)\\ &= -\frac{1}{3a^3x^3}-\frac{b}{6a^2\left(a+bx^3\right)^2}-\frac{2b}{3a^3\left(a+bx^3\right)}-\frac{3b\log(x)}{a^4}+\frac{b\log\left(a+bx^3\right)}{a^4}\end{aligned}$$

Mathematica [A] time = 0.06, size = 59, normalized size = 0.89

$$-\frac{\frac{a\left(2a^2+9abx^3+6b^2x^6\right)}{x^3\left(a+bx^3\right)^2}-6b\log\left(a+bx^3\right)+18b\log(x)}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3)^3),x]
```

```
[Out] -1/6*((a*(2*a^2 + 9*a*b*x^3 + 6*b^2*x^6))/(x^3*(a + b*x^3)^2) + 18*b*Log[x]
 - 6*b*Log[a + b*x^3])/a^4
```

fricas [A] time = 0.57, size = 119, normalized size = 1.80

$$-\frac{6\,ab^2x^6+9\,a^2bx^3+2\,a^3-6\left(b^3x^9+2\,ab^2x^6+a^2bx^3\right)\log\left(bx^3+a\right)+18\left(b^3x^9+2\,ab^2x^6+a^2bx^3\right)\log(x)}{6\left(a^4b^2x^9+2\,a^5bx^6+a^6x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/6*(6*a*b^2*x^6 + 9*a^2*b*x^3 + 2*a^3 - 6*(b^3*x^9 + 2*a*b^2*x^6 + a^2*b*
x^3)*log(b*x^3 + a) + 18*(b^3*x^9 + 2*a*b^2*x^6 + a^2*b*x^3)*log(x))/(a^4*b
^2*x^9 + 2*a^5*b*x^6 + a^6*x^3)
```

giac [A] time = 0.19, size = 80, normalized size = 1.21

$$\frac{b\log\left(\left|bx^3+a\right|\right)}{a^4}-\frac{3\,b\log\left(|x|\right)}{a^4}-\frac{9\,b^3x^6+22\,ab^2x^3+14\,a^2b}{6\left(bx^3+a\right)^2a^4}+\frac{3\,bx^3-a}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] b*log(abs(b*x^3 + a))/a^4 - 3*b*log(abs(x))/a^4 - 1/6*(9*b^3*x^6 + 22*a*b^2
*x^3 + 14*a^2*b)/((b*x^3 + a)^2*a^4) + 1/3*(3*b*x^3 - a)/(a^4*x^3)
```

maple [A] time = 0.01, size = 61, normalized size = 0.92

$$-\frac{b}{6\left(b\,x^3+a\right)^2a^2}-\frac{2b}{3\left(b\,x^3+a\right)a^3}-\frac{3b\ln(x)}{a^4}+\frac{b\ln\left(b\,x^3+a\right)}{a^4}-\frac{1}{3a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)^3,x)
```

```
[Out] -1/3/a^3/x^3-1/6*b/a^2/(b*x^3+a)^2-2/3*b/a^3/(b*x^3+a)-3*b*ln(x)/a^4+b*ln(b
*x^3+a)/a^4
```

maxima [A] time = 1.36, size = 76, normalized size = 1.15

$$-\frac{6\,b^2x^6+9\,abx^3+2\,a^2}{6\left(a^3b^2x^9+2\,a^4bx^6+a^5x^3\right)}+\frac{b\log\left(bx^3+a\right)}{a^4}-\frac{b\log\left(x^3\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(6*b^2*x^6 + 9*a*b*x^3 + 2*a^2)/(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3)
 + b*log(b*x^3 + a)/a^4 - b*log(x^3)/a^4
```

mupad [B] time = 0.09, size = 73, normalized size = 1.11

$$\frac{b\,\ln\left(b\,x^3+a\right)}{a^4}-\frac{\frac{1}{3\,a}+\frac{3\,b\,x^3}{2\,a^2}+\frac{b^2\,x^6}{a^3}}{a^2\,x^3+2\,a\,b\,x^6+b^2\,x^9}-\frac{3\,b\,\ln(x)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3)^3),x)
```

```
[Out] (b*log(a + b*x^3))/a^4 - (1/(3*a) + (3*b*x^3)/(2*a^2) + (b^2*x^6)/a^3)/(a^2
*x^3 + b^2*x^9 + 2*a*b*x^6) - (3*b*log(x))/a^4
```

sympy [**A**] time = 0.68, size = 76, normalized size = 1.15

$$\frac{-2a^2 - 9abx^3 - 6b^2x^6}{6a^5x^3 + 12a^4bx^6 + 6a^3b^2x^9} - \frac{3b\log(x)}{a^4} + \frac{b\log\left(\frac{a}{b} + x^3\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)**3,x)
```

```
[Out] (-2*a**2 - 9*a*b*x**3 - 6*b**2*x**6)/(6*a**5*x**3 + 12*a**4*b*x**6 + 6*a**3
*b**2*x**9) - 3*b*log(x)/a**4 + b*log(a/b + x**3)/a**4
```

3.346 $\int \frac{x^7}{\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=155

$$\frac{5\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54\sqrt[3]{a}\,b^{8/3}}-\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27\sqrt[3]{a}\,b^{8/3}}-\frac{5\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,\sqrt[3]{a}\,b^{8/3}}-\frac{5x^2}{18b^2\left(a+bx^3\right)}-\frac{x^5}{6b\left(a+bx^3\right)^2}$$

```
[Out] -1/6*x^5/b/(b*x^3+a)^2-5/18*x^2/b^2/(b*x^3+a)-5/27*ln(a^(1/3)+b^(1/3)*x)/a^
(1/3)/b^(8/3)+5/54*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1/3)/b^(8/3
)-5/27*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(1/3)/b^(8/3)*3^
(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {288, 292, 31, 634, 617, 204, 628}

$$\frac{5\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54\sqrt[3]{a}\,b^{8/3}}-\frac{5x^2}{18b^2\left(a+bx^3\right)}-\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27\sqrt[3]{a}\,b^{8/3}}-\frac{5\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,\sqrt[3]{a}\,b^{8/3}}-\frac{x^5}{6b\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^3)^3,x]
```

```
[Out] -x^5/(6*b*(a + b*x^3)^2) - (5*x^2)/(18*b^2*(a + b*x^3)) - (5*ArcTan[(a^(1/3
) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(1/3)*b^(8/3)) - (5*Log[a
^(1/3) + b^(1/3)*x])/(27*a^(1/3)*b^(8/3)) + (5*Log[a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2])/(54*a^(1/3)*b^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^7}{\left(a+bx^3\right)^3}\,dx = -\frac{x^5}{6b\left(a+bx^3\right)^2} + \frac{5\int \frac{x^4}{\left(a+bx^3\right)^2}\,dx}{6b}$$

$$= -\frac{x^5}{6b\left(a+bx^3\right)^2} - \frac{5x^2}{18b^2\left(a+bx^3\right)} + \frac{5\int \frac{x}{a+bx^3}\,dx}{9b^2}$$

$$= -\frac{x^5}{6b\left(a+bx^3\right)^2} - \frac{5x^2}{18b^2\left(a+bx^3\right)} - \frac{5\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{27\sqrt[3]{a}\,b^{7/3}} + \frac{5\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{27\sqrt[3]{a}\,b^{7/3}}$$

$$= -\frac{x^5}{6b\left(a+bx^3\right)^2} - \frac{5x^2}{18b^2\left(a+bx^3\right)} - \frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27\sqrt[3]{a}\,b^{8/3}} + \frac{5\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{54\sqrt[3]{a}\,b^{8/3}} + \frac{5\int \frac{}{a^{2/3}-}}{}$$

$$= -\frac{x^5}{6b\left(a+bx^3\right)^2} - \frac{5x^2}{18b^2\left(a+bx^3\right)} - \frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27\sqrt[3]{a}\,b^{8/3}} + \frac{5\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54\sqrt[3]{a}\,b^{8/3}} + \frac{5}{}$$

$$= -\frac{x^5}{6b\left(a+bx^3\right)^2} - \frac{5x^2}{18b^2\left(a+bx^3\right)} - \frac{5\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,\sqrt[3]{a}\,b^{8/3}} - \frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27\sqrt[3]{a}\,b^{8/3}} + \frac{5\log\left(a^{2/3}-}{54}$$

Mathematica [A] time = 0.07, size = 140, normalized size = 0.90

$$\frac{\frac{5\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{\sqrt[3]{a}} - \frac{24b^{2/3}x^2}{a+bx^3} + \frac{9ab^{2/3}x^2}{\left(a+bx^3\right)^2} - \frac{10\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a}} - \frac{10\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{54b^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^3)^3,x]
```

```
[Out] ((9*a*b^(2/3)*x^2)/(a + b*x^3)^2 - (24*b^(2/3)*x^2)/(a + b*x^3) - (10*Sqrt[
3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) - (10*Log[a^(1/3) +
 b^(1/3)*x])/a^(1/3) + (5*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a
^(1/3))/(54*b^(8/3))
```

fricas [B] time = 0.64, size = 510, normalized size = 3.29

$$\left[-\frac{24\,ab^3x^5+15\,a^2b^2x^2-15\sqrt{\frac{1}{3}}\left(ab^3x^6+2\,a^2b^2x^3+a^3b\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\,\log\left(\frac{2\,b^2x^3-ab+3\sqrt{\frac{1}{3}}\left(abx+2\left(-ab^2\right)^{\frac{2}{3}}x^2+\left(-ab^2\right)^{\frac{1}{3}}a\right)\right.}{bx^3+a}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/54*(24*a*b^3*x^5 + 15*a^2*b^2*x^2 - 15*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2
*x^3 + a^3*b)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*
b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(
-a*b^2)^(2/3)*x)/(b*x^3 + a)) - 5*(b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^(2/3
)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 10*(b^2*x^6 + 2*a*b*
x^3 + a^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a*b^6*x^6 + 2*a^2*b^5
*x^3 + a^3*b^4), -1/54*(24*a*b^3*x^5 + 15*a^2*b^2*x^2 - 30*sqrt(1/3)*(a*b^3
*x^6 + 2*a^2*b^2*x^3 + a^3*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b
*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - 5*(b^2*x^6 + 2*a*b*x^3 +
a^2)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 10
*(b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a*b
^6*x^6 + 2*a^2*b^5*x^3 + a^3*b^4)]
```

giac [A] time = 0.18, size = 142, normalized size = 0.92

$$-\frac{5\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^2}-\frac{5\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^4}-\frac{8\,bx^5+5\,ax^2}{18\left(bx^3+a\right)^2b^2}+\frac{5\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^2+x\left(\right.\right.}{54\,ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -5/27*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2) - 5/27*sqrt(3)*(-a*b^
2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^4) - 1/
18*(8*b*x^5 + 5*a*x^2)/((b*x^3 + a)^2*b^2) + 5/54*(-a*b^2)^(2/3)*log(x^2 +
x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^4)
```

maple [A] time = 0.01, size = 119, normalized size = 0.77

$$\frac{5\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3}-\frac{5\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3}+\frac{5\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3}+\frac{-\frac{4x^5}{9b}-\frac{5a\,x^2}{18b^2}}{\left(b\,x^3+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^3+a)^3,x)
```

```
[Out] (-4/9*x^5/b-5/18*a*x^2/b^2)/(b*x^3+a)^2-5/27/b^3/(a/b)^(1/3)*ln(x+(a/b)^(1/
3))+5/54/b^3/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+5/27/b^3*3^(1/2)
/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

maxima [A] time = 3.06, size = 139, normalized size = 0.90

$$-\frac{8bx^5+5ax^2}{18\left(b^4x^6+2ab^3x^3+a^2b^2\right)}+\frac{5\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{5\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*(8*b*x^5 + 5*a*x^2)/(b^4*x^6 + 2*a*b^3*x^3 + a^2*b^2) + 5/27*sqrt(3)*
arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(1/3)) + 5/5
4*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(1/3)) - 5/27*log(x + (
a/b)^(1/3))/(b^3*(a/b)^(1/3))
```

mupad [B] time = 1.15, size = 161, normalized size = 1.04

$$\frac{5\ln\left(\frac{25x}{81b^3}-\frac{25(-a)^{1/3}}{81b^{10/3}}\right)}{27(-a)^{1/3}b^{8/3}}-\frac{\frac{4x^5}{9b}+\frac{5ax^2}{18b^2}}{a^2+2abx^3+b^2x^6}+\frac{\ln\left(\frac{25x}{81b^3}-\frac{(-a)^{1/3}\left(-5+\sqrt{3}\,5\mathrm{i}\right)^2}{324b^{10/3}}\right)\left(-5+\sqrt{3}\,5\mathrm{i}\right)}{54(-a)^{1/3}b^{8/3}}-\frac{\ln\left(\frac{25x}{81b^3}-\frac{(-a)^{1/3}\left(5+\sqrt{3}\,5\mathrm{i}\right.}{324b^{10/3}}\right.}{54(-a)^{1/3}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^3)^3,x)
```

```
[Out] (5*log((25*x)/(81*b^3) - (25*(-a)^(1/3))/(81*b^(10/3))))/(27*(-a)^(1/3)*b^(
8/3)) - ((4*x^5)/(9*b) + (5*a*x^2)/(18*b^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) +
(log((25*x)/(81*b^3) - ((-a)^(1/3)*(3^(1/2)*5i - 5)^2)/(324*b^(10/3)))*(3^(
1/2)*5i - 5))/(54*(-a)^(1/3)*b^(8/3)) - (log((25*x)/(81*b^3) - ((-a)^(1/3)*
(3^(1/2)*5i + 5)^2)/(324*b^(10/3)))*(3^(1/2)*5i + 5))/(54*(-a)^(1/3)*b^(8/3
))
```

sympy [A] time = 0.51, size = 70, normalized size = 0.45

$$\frac{-5ax^2-8bx^5}{18a^2b^2+36ab^3x^3+18b^4x^6}+\operatorname{RootSum}\left(19683t^3ab^8+125,\left(t\mapsto t\log\left(\frac{729t^2ab^5}{25}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**3+a)**3,x)
```

```
[Out] (-5*a*x**2 - 8*b*x**5)/(18*a**2*b**2 + 36*a*b**3*x**3 + 18*b**4*x**6) + Roo
tSum(19683*_t**3*a*b**8 + 125, Lambda(_t, _t*log(729*_t**2*a*b**5/25 + x)))
```

3.347 $\int \frac{x^6}{\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=153

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{27a^{2/3}b^{7/3}}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{2/3}b^{7/3}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{2/3}b^{7/3}}-\frac{2x}{9b^2\left(a+bx^3\right)}-\frac{x^4}{6b\left(a+bx^3\right)^2}$$

```
[Out] -1/6*x^4/b/(b*x^3+a)^2-2/9*x/b^2/(b*x^3+a)+2/27*ln(a^(1/3)+b^(1/3)*x)/a^(2/
3)/b^(7/3)-1/27*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(7/3)-2
/27*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(7/3)*3^(1/
2)
```

Rubi [A] time = 0.08, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {288, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{27a^{2/3}b^{7/3}}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{2/3}b^{7/3}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{2/3}b^{7/3}}-\frac{2x}{9b^2\left(a+bx^3\right)}-\frac{x^4}{6b\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^3)^3,x]
```

```
[Out] -x^4/(6*b*(a + b*x^3)^2) - (2*x)/(9*b^2*(a + b*x^3)) - (2*ArcTan[(a^(1/3) -
 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(2/3)*b^(7/3)) + (2*Log[a^(1
/3) + b^(1/3)*x])/(27*a^(2/3)*b^(7/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2]/(27*a^(2/3)*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^6}{\left(a+bx^3\right)^3}\,dx = -\frac{x^4}{6b\left(a+bx^3\right)^2} + \frac{2\int \frac{x^3}{\left(a+bx^3\right)^2}\,dx}{3b}$$

$$= -\frac{x^4}{6b\left(a+bx^3\right)^2} - \frac{2x}{9b^2\left(a+bx^3\right)} + \frac{2\int \frac{1}{a+bx^3}\,dx}{9b^2}$$

$$= -\frac{x^4}{6b\left(a+bx^3\right)^2} - \frac{2x}{9b^2\left(a+bx^3\right)} + \frac{2\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{27a^{2/3}b^2} + \frac{2\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{27a^{2/3}b^2}$$

$$= -\frac{x^4}{6b\left(a+bx^3\right)^2} - \frac{2x}{9b^2\left(a+bx^3\right)} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{2/3}b^{7/3}} - \frac{\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{27a^{2/3}b^{7/3}} + \frac{\int \frac{1}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}}}{9\sqrt[3]{a}}$$

$$= -\frac{x^4}{6b\left(a+bx^3\right)^2} - \frac{2x}{9b^2\left(a+bx^3\right)} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{2/3}b^{7/3}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{27a^{2/3}b^{7/3}} + \frac{2\,\mathrm{Su}}{}$$

$$= -\frac{x^4}{6b\left(a+bx^3\right)^2} - \frac{2x}{9b^2\left(a+bx^3\right)} - \frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{2/3}b^{7/3}} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{2/3}b^{7/3}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\right.}{27a^{2/}}$$

Mathematica [A] time = 0.07, size = 136, normalized size = 0.89

$$\frac{-\frac{2\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{2/3}} + \frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{2/3}} - \frac{4\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} - \frac{21\sqrt[3]{b}\,x}{a+bx^3} + \frac{9a\sqrt[3]{b}\,x}{\left(a+bx^3\right)^2}}{54b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^3)^3,x]
```

```
[Out] ((9*a*b^(1/3)*x)/(a + b*x^3)^2 - (21*b^(1/3)*x)/(a + b*x^3) - (4*Sqrt[3]*Ar
cTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (4*Log[a^(1/3) + b^(1/
3)*x])/a^(2/3) - (2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3)
)/(54*b^(7/3))
```

fricas [B] time = 0.66, size = 503, normalized size = 3.29

$$\left[-\frac{21\,a^2b^2x^4+12\,a^3bx-6\sqrt{\frac{1}{3}}\left(ab^3x^6+2\,a^2b^2x^3+a^3b\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\,\log\left(\frac{2\,abx^3-3\left(a^2b\right)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2\,abx^2+\left(a^2b\right)^{\frac{2}{3}}x-\left(a^2\right.\right.\right.}{bx^3+a}\right.\right.}{54}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/54*(21*a^2*b^2*x^4 + 12*a^3*b*x - 6*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x^
3 + a^3*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^
2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*
b)^(1/3)/b))/(b*x^3 + a)) + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log
(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 4*(b^2*x^6 + 2*a*b*x^3 + a^
2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^2*b^5*x^6 + 2*a^3*b^4*x^3 +
 a^4*b^3), -1/54*(21*a^2*b^2*x^4 + 12*a^3*b*x - 12*sqrt(1/3)*(a*b^3*x^6 + 2
*a^2*b^2*x^3 + a^3*b)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/
3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) + 2*(b^2*x^6 + 2*a*b*x^3
 + a^2)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 4*
(b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^2*
b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3)]
```

giac [A] time = 0.18, size = 140, normalized size = 0.92

$$-\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^2}+\frac{2\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^3}+\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,ab^3}-\frac{}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -2/27*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2) + 2/27*sqrt(3)*(-a*b^
2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^3) + 1/
27*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^3) - 1/18*(
7*b*x^4 + 4*a*x)/((b*x^3 + a)^2*b^2)
```

maple [A] time = 0.01, size = 117, normalized size = 0.76

$$\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}+\frac{2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}+\frac{-\frac{7x^4}{18b}-\frac{2ax}{9b^2}}{\left(b\,x^3+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^3+a)^3,x)
```

```
[Out] (-7/18/b*x^4-2/9*a*x/b^2)/(b*x^3+a)^2+2/27/b^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3)
)-1/27/b^3/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+2/27/b^3/(a/b)^(2/
3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

maxima [A] time = 2.99, size = 137, normalized size = 0.90

$$-\frac{7bx^4+4ax}{18\left(b^4x^6+2ab^3x^3+a^2b^2\right)}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*(7*b*x^4 + 4*a*x)/(b^4*x^6 + 2*a*b^3*x^3 + a^2*b^2) + 2/27*sqrt(3)*ar
ctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(2/3)) - 1/27*
log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3)) + 2/27*log(x + (a/
b)^(1/3))/(b^3*(a/b)^(2/3))
```

mupad [B] time = 0.31, size = 127, normalized size = 0.83

$$\frac{2\ln\left(x+\frac{a^{1/3}}{b^{1/3}}\right)}{27a^{2/3}b^{7/3}}-\frac{\frac{7x^4}{18b}+\frac{2ax}{9b^2}}{a^2+2abx^3+b^2x^6}+\frac{\ln\left(x+\frac{a^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2b^{1/3}}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{27a^{2/3}b^{7/3}}-\frac{\ln\left(x-\frac{a^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2b^{1/3}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{27a^{2/3}b^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^3)^3,x)
```

```
[Out] (2*log(x + a^(1/3)/b^(1/3)))/(27*a^(2/3)*b^(7/3)) - ((7*x^4)/(18*b) + (2*a*
x)/(9*b^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) + (log(x + (a^(1/3)*(3^(1/2)*1i - 1
))/(2*b^(1/3)))*(3^(1/2)*1i - 1))/(27*a^(2/3)*b^(7/3)) - (log(x - (a^(1/3)*
(3^(1/2)*1i + 1))/(2*b^(1/3)))*(3^(1/2)*1i + 1))/(27*a^(2/3)*b^(7/3))
```

sympy [A] time = 0.47, size = 68, normalized size = 0.44

$$\frac{-4ax-7bx^4}{18a^2b^2+36ab^3x^3+18b^4x^6}+\operatorname{RootSum}\left(19683t^3a^2b^7-8,\left(t\mapsto t\log\left(\frac{27tab^2}{2}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**3+a)**3,x)
```

```
[Out] (-4*a*x - 7*b*x**4)/(18*a**2*b**2 + 36*a*b**3*x**3 + 18*b**4*x**6) + RootSu
m(19683*_t**3*a**2*b**7 - 8, Lambda(_t, _t*log(27*_t*a*b**2/2 + x)))
```

3.348 $\int \frac{x^4}{\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=158

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{4/3}b^{5/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{4/3}b^{5/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{4/3}b^{5/3}}+\frac{x^2}{9ab\left(a+bx^3\right)}-\frac{x^2}{6b\left(a+bx^3\right)^2}$$

```
[Out] -1/6*x^2/b/(b*x^3+a)^2+1/9*x^2/a/b/(b*x^3+a)-1/27*ln(a^(1/3)+b^(1/3)*x)/a^(
4/3)/b^(5/3)+1/54*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/b^(5/3)
-1/27*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(4/3)/b^(5/3)*3^(
1/2)
```

Rubi [A] time = 0.08, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {288, 290, 292, 31, 634, 617, 204, 628}

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{4/3}b^{5/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{4/3}b^{5/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{4/3}b^{5/3}}+\frac{x^2}{9ab\left(a+bx^3\right)}-\frac{x^2}{6b\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^3)^3,x]
```

```
[Out] -x^2/(6*b*(a + b*x^3)^2) + x^2/(9*a*b*(a + b*x^3)) - ArcTan[(a^(1/3) - 2*b^
(1/3)*x)/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(4/3)*b^(5/3)) - Log[a^(1/3) + b^(
1/3)*x]/(27*a^(4/3)*b^(5/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2]/(54*a^(4/3)*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^3\right)^3}\,dx = -\frac{x^2}{6b\left(a+bx^3\right)^2} + \frac{\int \frac{x}{\left(a+bx^3\right)^2}\,dx}{3b}$$

$$= -\frac{x^2}{6b\left(a+bx^3\right)^2} + \frac{x^2}{9ab\left(a+bx^3\right)} + \frac{\int \frac{x}{a+bx^3}\,dx}{9ab}$$

$$= -\frac{x^2}{6b\left(a+bx^3\right)^2} + \frac{x^2}{9ab\left(a+bx^3\right)} - \frac{\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{27a^{4/3}b^{4/3}} + \frac{\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{27a^{4/3}b^{4/3}}$$

$$= -\frac{x^2}{6b\left(a+bx^3\right)^2} + \frac{x^2}{9ab\left(a+bx^3\right)} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{4/3}b^{5/3}} + \frac{\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{54a^{4/3}b^{5/3}} + \frac{\int \frac{1}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x}}{18ab^4}$$

$$= -\frac{x^2}{6b\left(a+bx^3\right)^2} + \frac{x^2}{9ab\left(a+bx^3\right)} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{4/3}b^{5/3}} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{4/3}b^{5/3}} + \frac{\text{Subst}}{}$$

$$= -\frac{x^2}{6b\left(a+bx^3\right)^2} + \frac{x^2}{9ab\left(a+bx^3\right)} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{4/3}b^{5/3}} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{4/3}b^{5/3}} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\right.}{54a^{4/3}b^{5}}$$

Mathematica [A] time = 0.11, size = 141, normalized size = 0.89

$$\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{4/3}}-\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{4/3}}-\frac{2\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}}+\frac{6b^{2/3}x^2}{a^2+abx^3}-\frac{9b^{2/3}x^2}{\left(a+bx^3\right)^2}}{54b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^3)^3,x]
```

```
[Out] ((-9*b^(2/3)*x^2)/(a + b*x^3)^2 + (6*b^(2/3)*x^2)/(a^2 + a*b*x^3) - (2*Sqrt
[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(4/3) - (2*Log[a^(1/3) +
 b^(1/3)*x])/a^(4/3) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/a^(4/
3))/(54*b^(5/3))
```

fricas [A] time = 0.66, size = 512, normalized size = 3.24

$$\left[\frac{6\,ab^3x^5-3\,a^2b^2x^2+3\sqrt{\frac{1}{3}}\left(ab^3x^6+2\,a^2b^2x^3+a^3b\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\,\log\left(\frac{2\,b^2x^3-ab+3\sqrt{\frac{1}{3}}\left(abx+2\left(-ab^2\right)^{\frac{2}{3}}x^2+\left(-ab^2\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-ab^2\right.}{a}}}{bx^3+a}\right.}{54}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(6*a*b^3*x^5 - 3*a^2*b^2*x^2 + 3*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x^3
 + a^3*b)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x
+ 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b
^2)^(2/3)*x)/(b*x^3 + a)) + (b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^(2/3)*log(
b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(b^2*x^6 + 2*a*b*x^3 + a
^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^5*x^6 + 2*a^3*b^4*x^3
+ a^4*b^3), 1/54*(6*a*b^3*x^5 - 3*a^2*b^2*x^2 + 6*sqrt(1/3)*(a*b^3*x^6 + 2*
a^2*b^2*x^3 + a^3*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*
b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^
2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(b^2*x^6 +
2*a*b*x^3 + a^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^5*x^6 + 2
*a^3*b^4*x^3 + a^4*b^3)]
```

giac [A] time = 0.17, size = 145, normalized size = 0.92

$$-\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^2b}-\frac{\sqrt{3}\,\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^3}+\frac{2\,bx^5-ax^2}{18\left(bx^3+a\right)^2ab}+\frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right.}{54\,a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b) - 1/27*sqrt(3)*(-a*b^
2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^3) +
```

```
1/18*(2*b*x^5 - a*x^2)/((b*x^3 + a)^2*a*b) + 1/54*(-a*b^2)^(2/3)*log(x^2 +
x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^3)
```

maple [A] time = 0.01, size = 127, normalized size = 0.80

$$\frac{\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a\,b^2}-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a\,b^2}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{1}{3}}a\,b^2}+\frac{\frac{x^5}{9a}-\frac{x^2}{18b}}{\left(b\,x^3+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a)^3,x)
```

```
[Out] (1/9*x^5/a-1/18/b*x^2)/(b*x^3+a)^2-1/27/b^2/a/(a/b)^(1/3)*ln(x+(a/b)^(1/3))
+1/54/b^2/a/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/27/b^2/a*3^(1/2
)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

maxima [A] time = 2.97, size = 149, normalized size = 0.94

$$\frac{2\,bx^5-ax^2}{18\left(ab^3x^6+2\,a^2b^2x^3+a^3b\right)}+\frac{\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(2*b*x^5 - a*x^2)/(a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b) + 1/27*sqrt(3)*a
rctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(1/3)) + 1/
54*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(1/3)) - 1/27*log(x
+ (a/b)^(1/3))/(a*b^2*(a/b)^(1/3))
```

mupad [B] time = 1.17, size = 168, normalized size = 1.06

$$\frac{\frac{x^5}{9\,a}-\frac{x^2}{18\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6}+\frac{\ln\left(\frac{1}{81\,a^{5/3}\,(-b)^{4/3}}+\frac{x}{81\,a^2\,b}\right)}{27\,a^{4/3}\,(-b)^{5/3}}+\frac{\ln\left(\frac{x}{81\,a^2\,b}+\frac{\left(-1+\sqrt{3}\ 1\mathrm{i}\right)^2}{324\,a^{5/3}\,(-b)^{4/3}}\right)\left(-1+\sqrt{3}\ 1\mathrm{i}\right)}{54\,a^{4/3}\,(-b)^{5/3}}-\frac{\ln\left(\frac{x}{81\,a^2\,b}+\frac{\left(1+\sqrt{3}\ 1\mathrm{i}\right)^2}{324\,a^{5/3}\,(-b)^{4/3}}\right)}{54\,a^{4/3}\,(-b)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^3)^3,x)
```

```
[Out] (x^5/(9*a) - x^2/(18*b))/(a^2 + b^2*x^6 + 2*a*b*x^3) + log(1/(81*a^(5/3)*(-
b)^(4/3)) + x/(81*a^2*b))/(27*a^(4/3)*(-b)^(5/3)) + (log(x/(81*a^2*b) + (3^
(1/2)*1i - 1)^2/(324*a^(5/3)*(-b)^(4/3)))*(3^(1/2)*1i - 1))/(54*a^(4/3)*(-b
)^(5/3)) - (log(x/(81*a^2*b) + (3^(1/2)*1i + 1)^2/(324*a^(5/3)*(-b)^(4/3)))
*(3^(1/2)*1i + 1))/(54*a^(4/3)*(-b)^(5/3))
```

sympy [A] time = 0.47, size = 70, normalized size = 0.44

$$\frac{-ax^2+2bx^5}{18a^3b+36a^2b^2x^3+18ab^3x^6}+\operatorname{RootSum}\left(19683t^3a^4b^5+1,\left(t\mapsto t\log\left(729t^2a^3b^3+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a)**3,x)

[Out] (-a*x**2 + 2*b*x**5)/(18*a**3*b + 36*a**2*b**2*x**3 + 18*a*b**3*x**6) + Roo
tSum(19683*_t**3*a**4*b**5 + 1, Lambda(_t, _t*log(729*_t**2*a**3*b**3 + x))
)
```

3.349 $\int \frac{x^3}{\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=154

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{5/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{5/3}b^{4/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{5/3}b^{4/3}}+\frac{x}{18ab\left(a+bx^3\right)}-\frac{x}{6b\left(a+bx^3\right)^2}$$

```
[Out] -1/6*x/b/(b*x^3+a)^2+1/18*x/a/b/(b*x^3+a)+1/27*ln(a^(1/3)+b^(1/3)*x)/a^(5/3
)/b^(4/3)-1/54*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(4/3)-1/
27*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(4/3)*3^(1/2
)
```

Rubi [A] time = 0.08, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {288, 199, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{5/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{5/3}b^{4/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{5/3}b^{4/3}}+\frac{x}{18ab\left(a+bx^3\right)}-\frac{x}{6b\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^3)^3,x]
```

```
[Out] -x/(6*b*(a + b*x^3)^2) + x/(18*a*b*(a + b*x^3)) - ArcTan[(a^(1/3) - 2*b^(1/
3)*x)/(Sqrt[3]*a^(1/3))]/(9*Sqrt[3]*a^(5/3)*b^(4/3)) + Log[a^(1/3) + b^(1/3
)*x]/(27*a^(5/3)*b^(4/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/
(54*a^(5/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^3\right)^3}\,dx = -\frac{x}{6b\left(a+bx^3\right)^2} + \frac{\int \frac{1}{\left(a+bx^3\right)^2}\,dx}{6b}$$

$$= -\frac{x}{6b\left(a+bx^3\right)^2} + \frac{x}{18ab\left(a+bx^3\right)} + \frac{\int \frac{1}{a+bx^3}\,dx}{9ab}$$

$$= -\frac{x}{6b\left(a+bx^3\right)^2} + \frac{x}{18ab\left(a+bx^3\right)} + \frac{\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{27a^{5/3}b} + \frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{27a^{5/3}b}$$

$$= -\frac{x}{6b\left(a+bx^3\right)^2} + \frac{x}{18ab\left(a+bx^3\right)} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{5/3}b^{4/3}} - \frac{\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{54a^{5/3}b^{4/3}} + \frac{\int}{1}$$

$$= -\frac{x}{6b\left(a+bx^3\right)^2} + \frac{x}{18ab\left(a+bx^3\right)} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{5/3}b^{4/3}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{5/3}b^{4/3}} + \frac{S}{-}$$

$$= -\frac{x}{6b\left(a+bx^3\right)^2} + \frac{x}{18ab\left(a+bx^3\right)} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{5/3}b^{4/3}} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{5/3}b^{4/3}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\right.}{54a^{5}}$$

Mathematica [A] time = 0.08, size = 138, normalized size = 0.90

$$\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{5/3}}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{5/3}}-\frac{2\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}}+\frac{3\sqrt[3]{b}\,x}{a^2+abx^3}-\frac{9\sqrt[3]{b}\,x}{\left(a+bx^3\right)^2}}{54b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^3)^3,x]
```

```
[Out] ((-9*b^(1/3)*x)/(a + b*x^3)^2 + (3*b^(1/3)*x)/(a^2 + a*b*x^3) - (2*Sqrt[3]*
ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(5/3) + (2*Log[a^(1/3) + b^(
1/3)*x])/a^(5/3) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/a^(5/3))/
(54*b^(4/3))
```

fricas [B] time = 0.61, size = 503, normalized size = 3.27

$$\left[\frac{3\,a^2b^2x^4-6\,a^3bx+3\sqrt{\frac{1}{3}}\left(ab^3x^6+2\,a^2b^2x^3+a^3b\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\,\log\left(\frac{2\,abx^3-3\left(a^2b\right)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2\,abx^2+\left(a^2b\right)^{\frac{2}{3}}x-\left(a^2b\right)^{\frac{1}{3}}a\right)\sqrt{}}{bx^3+a}\right.}{54\left(a^3b^4x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(3*a^2*b^2*x^4 - 6*a^3*b*x + 3*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x^3 +
 a^3*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 +
 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^
(1/3)/b))/(b*x^3 + a)) - (b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(a*b*
x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*(a
^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^3*b^4*x^6 + 2*a^4*b^3*x^3 + a^5*
b^2), 1/54*(3*a^2*b^2*x^4 - 6*a^3*b*x + 6*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*
x^3 + a^3*b)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a
^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - (b^2*x^6 + 2*a*b*x^3 + a^2)*(a^
2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(b^2*x^6 +
2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^3*b^4*x^6 + 2
*a^4*b^3*x^3 + a^5*b^2)]
```

giac [A] time = 0.18, size = 142, normalized size = 0.92

$$-\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^2b}+\frac{\sqrt{3}\,\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^2}+\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b^2}+\frac{bx^4-}{18\left(bx^3-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b) + 1/27*sqrt(3)*(-a*b^
2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^2) +
```

```
1/54*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^2) + 1/
18*(b*x^4 - 2*a*x)/((b*x^3 + a)^2*a*b)
```

maple [A] time = 0.01, size = 125, normalized size = 0.81

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,b^2}+\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,b^2}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,b^2}+\frac{\frac{x^4}{18a}-\frac{x}{9b}}{\left(b\,x^3+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a)^3,x)
```

```
[Out] (1/18*x^4/a-1/9/b*x)/(b*x^3+a)^2+1/27/b^2/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1
/54/b^2/a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/27/b^2/a/(a/b)^(2
/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

maxima [A] time = 2.95, size = 146, normalized size = 0.95

$$\frac{bx^4-2\,ax}{18\left(ab^3x^6+2\,a^2b^2x^3+a^3b\right)}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(b*x^4 - 2*a*x)/(a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b) + 1/27*sqrt(3)*arc
tan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(2/3)) - 1/54
*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(2/3)) + 1/27*log(x +
(a/b)^(1/3))/(a*b^2*(a/b)^(2/3))
```

mupad [B] time = 1.13, size = 145, normalized size = 0.94

$$\frac{\ln\left(\frac{b^{2/3}}{3\,a^{2/3}}+\frac{b\,x}{3\,a}\right)}{27\,a^{5/3}\,b^{4/3}}-\frac{\frac{x}{9\,b}-\frac{x^4}{18\,a}}{a^2+2\,a\,b\,x^3+b^2\,x^6}+\frac{\ln\left(\frac{b\,x}{3\,a}+\frac{b^{2/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{54\,a^{5/3}\,b^{4/3}}-\frac{\ln\left(\frac{b\,x}{3\,a}-\frac{b^{2/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}}\right)\left(1+\sqrt{3}\,1\right.}{54\,a^{5/3}\,b^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^3)^3,x)
```

```
[Out] log(b^(2/3)/(3*a^(2/3)) + (b*x)/(3*a))/(27*a^(5/3)*b^(4/3)) - (x/(9*b) - x^
4/(18*a))/(a^2 + b^2*x^6 + 2*a*b*x^3) + (log((b*x)/(3*a) + (b^(2/3)*(3^(1/2
)*1i - 1))/(6*a^(2/3)))*(3^(1/2)*1i - 1))/(54*a^(5/3)*b^(4/3)) - (log((b*x)
/(3*a) - (b^(2/3)*(3^(1/2)*1i + 1))/(6*a^(2/3)))*(3^(1/2)*1i + 1))/(54*a^(5
/3)*b^(4/3))
```

sympy [A] time = 0.45, size = 65, normalized size = 0.42

$$\frac{-2ax+bx^4}{18a^3b+36a^2b^2x^3+18ab^3x^6}+\operatorname{RootSum}\left(19683t^3a^5b^4-1,\left(t\mapsto t\log\left(27ta^2b+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a)**3,x)

[Out] (-2*a*x + b*x**4)/(18*a**3*b + 36*a**2*b**2*x**3 + 18*a*b**3*x**6) + RootSu
m(19683*_t**3*a**5*b**4 - 1, Lambda(_t, _t*log(27*_t*a**2*b + x)))
```

3.350 $\int \frac{x}{\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=155

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{27a^{7/3}b^{2/3}}-\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{7/3}b^{2/3}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{7/3}b^{2/3}}+\frac{2x^2}{9a^2\left(a+bx^3\right)}+\frac{x^2}{6a\left(a+bx^3\right)^2}$$

```
[Out] 1/6*x^2/a/(b*x^3+a)^2+2/9*x^2/a^2/(b*x^3+a)-2/27*ln(a^(1/3)+b^(1/3)*x)/a^(7
/3)/b^(2/3)+1/27*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(7/3)/b^(2/3)-
2/27*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(7/3)/b^(2/3)*3^(1
/2)
```

Rubi [A] time = 0.08, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {290, 292, 31, 634, 617, 204, 628}

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{27a^{7/3}b^{2/3}}-\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{7/3}b^{2/3}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{7/3}b^{2/3}}+\frac{2x^2}{9a^2\left(a+bx^3\right)}+\frac{x^2}{6a\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^3)^3,x]
```

```
[Out] x^2/(6*a*(a + b*x^3)^2) + (2*x^2)/(9*a^2*(a + b*x^3)) - (2*ArcTan[(a^(1/3)
- 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(7/3)*b^(2/3)) - (2*Log[a^(
1/3) + b^(1/3)*x])/(27*a^(7/3)*b^(2/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2]/(27*a^(7/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(a+bx^3\right)^3}\,dx &= \frac{x^2}{6a\left(a+bx^3\right)^2} + \frac{2\int \frac{x}{\left(a+bx^3\right)^2}\,dx}{3a} \\
&= \frac{x^2}{6a\left(a+bx^3\right)^2} + \frac{2x^2}{9a^2\left(a+bx^3\right)} + \frac{2\int \frac{x}{a+bx^3}\,dx}{9a^2} \\
&= \frac{x^2}{6a\left(a+bx^3\right)^2} + \frac{2x^2}{9a^2\left(a+bx^3\right)} - \frac{2\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{27a^{7/3}\sqrt[3]{b}} + \frac{2\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{27a^{7/3}\sqrt[3]{b}} \\
&= \frac{x^2}{6a\left(a+bx^3\right)^2} + \frac{2x^2}{9a^2\left(a+bx^3\right)} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{7/3}b^{2/3}} + \frac{\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{27a^{7/3}b^{2/3}} + \frac{\int \frac{1}{}}{9a^2\sqrt[3]{} \\
&= \frac{x^2}{6a\left(a+bx^3\right)^2} + \frac{2x^2}{9a^2\left(a+bx^3\right)} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{7/3}b^{2/3}} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{27a^{7/3}b^{2/3}} + \frac{2\,\mathrm{Sub}}{} \\
&= \frac{x^2}{6a\left(a+bx^3\right)^2} + \frac{2x^2}{9a^2\left(a+bx^3\right)} - \frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{7/3}b^{2/3}} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{7/3}b^{2/3}} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{}\right.}{27a^{7/3}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 139, normalized size = 0.90

$$\frac{\frac{2\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{b^{2/3}} + \frac{9a^{4/3}x^2}{\left(a+bx^3\right)^2} - \frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{b^{2/3}} - \frac{4\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}} + \frac{12\sqrt[3]{a}\,x^2}{a+bx^3}}{54a^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^3)^3,x]
```

```
[Out] ((9*a^(4/3)*x^2)/(a + b*x^3)^2 + (12*a^(1/3)*x^2)/(a + b*x^3) - (4*Sqrt[3]*
ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) - (4*Log[a^(1/3) + b^(
```

```
1/3)*x])/b^(2/3) + (2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/
3))/(54*a^(7/3))
```

fricas [B] time = 0.79, size = 514, normalized size = 3.32

$$\left[\frac{12\,ab^3x^5+21\,a^2b^2x^2+6\sqrt{\frac{1}{3}}\left(ab^3x^6+2\,a^2b^2x^3+a^3b\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\,\log\left(\frac{2\,b^2x^3-ab+3\sqrt{\frac{1}{3}}\left(abx+2\left(-ab^2\right)^{\frac{2}{3}}x^2+\left(-ab^2\right)^{\frac{1}{3}}a\right)\sqrt{\left(-\right.}}{bx^3+a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(12*a*b^3*x^5 + 21*a^2*b^2*x^2 + 6*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x
^3 + a^3*b)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*
x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a
*b^2)^(2/3)*x)/(b*x^3 + a)) + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^(2/3)*
log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 4*(b^2*x^6 + 2*a*b*x^3
 + a^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^3*b^4*x^6 + 2*a^4*b^3*
x^3 + a^5*b^2), 1/54*(12*a*b^3*x^5 + 21*a^2*b^2*x^2 + 12*sqrt(1/3)*(a*b^3*x
^6 + 2*a^2*b^2*x^3 + a^3*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x
 + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 2*(b^2*x^6 + 2*a*b*x^3 + a^
2)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 4*(b
^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^3*b^
4*x^6 + 2*a^4*b^3*x^3 + a^5*b^2)]
```

giac [A] time = 0.17, size = 139, normalized size = 0.90

$$-\frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3}-\frac{2\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b^2}+\frac{4\,bx^5+7\,ax^2}{18\left(bx^3+a\right)^2a^2}+\frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^2+x\left(-\right.\right.}{27\,a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -2/27*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/a^3 - 2/27*sqrt(3)*(-a*b^2)^(
2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b^2) + 1/18
*(4*b*x^5 + 7*a*x^2)/((b*x^3 + a)^2*a^2) + 1/27*(-a*b^2)^(2/3)*log(x^2 + x*
(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b^2)
```

maple [A] time = 0.00, size = 134, normalized size = 0.86

$$\frac{x^2}{6\left(b\,x^3+a\right)^2a}+\frac{2x^2}{9\left(b\,x^3+a\right)a^2}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2b}-\frac{2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2b}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a)^3,x)
```

```
[Out] 1/6*x^2/a/(b*x^3+a)^2+2/9*x^2/a^2/(b*x^3+a)-2/27/a^2/b/(a/b)^(1/3)*ln(x+(a/
b)^(1/3))+1/27/a^2/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+2/27/a^2
*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

maxima [A] time = 2.98, size = 147, normalized size = 0.95

$$\frac{4bx^5+7ax^2}{18\left(a^2b^2x^6+2a^3bx^3+a^4\right)}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{2\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(4*b*x^5 + 7*a*x^2)/(a^2*b^2*x^6 + 2*a^3*b*x^3 + a^4) + 2/27*sqrt(3)*a
rctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)^(1/3)) + 1/
27*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(1/3)) - 2/27*log(x
+ (a/b)^(1/3))/(a^2*b*(a/b)^(1/3))
```

mupad [B] time = 0.23, size = 163, normalized size = 1.05

$$\frac{\frac{7x^2}{18a}+\frac{2bx^5}{9a^2}}{a^2+2abx^3+b^2x^6}+\frac{2\ln\left(\frac{4bx}{81a^4}-\frac{4b^{2/3}}{81(-a)^{11/3}}\right)}{27(-a)^{7/3}b^{2/3}}+\frac{\ln\left(\frac{4bx}{81a^4}-\frac{b^{2/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{81(-a)^{11/3}}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{27(-a)^{7/3}b^{2/3}}-\frac{\ln\left(\frac{4bx}{81a^4}-\frac{b^{2/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{81(-a)^{11/3}}\right)}{27(-a)^{7/3}b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^3)^3,x)
```

```
[Out] ((7*x^2)/(18*a) + (2*b*x^5)/(9*a^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) + (2*log((
4*b*x)/(81*a^4) - (4*b^(2/3))/(81*(-a)^(11/3))))/(27*(-a)^(7/3)*b^(2/3)) +
(log((4*b*x)/(81*a^4) - (b^(2/3)*(3^(1/2)*1i - 1)^2)/(81*(-a)^(11/3)))*(3^(
1/2)*1i - 1))/(27*(-a)^(7/3)*b^(2/3)) - (log((4*b*x)/(81*a^4) - (b^(2/3)*(3
^(1/2)*1i + 1)^2)/(81*(-a)^(11/3)))*(3^(1/2)*1i + 1))/(27*(-a)^(7/3)*b^(2/3
))
```

sympy [A] time = 0.40, size = 70, normalized size = 0.45

$$\frac{7ax^2+4bx^5}{18a^4+36a^3bx^3+18a^2b^2x^6}+\operatorname{RootSum}\left(19683t^3a^7b^2+8,\left(t\mapsto t\log\left(\frac{729t^2a^5b}{4}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a)**3,x)
```

```
[Out] (7*a*x**2 + 4*b*x**5)/(18*a**4 + 36*a**3*b*x**3 + 18*a**2*b**2*x**6) + Root
Sum(19683*_t**3*a**7*b**2 + 8, Lambda(_t, _t*log(729*_t**2*a**5*b/4 + x)))
```

3.351 $\int \frac{1}{\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=151

$$-\frac{5\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{8/3}\sqrt[3]{b}}+\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{8/3}\sqrt[3]{b}}-\frac{5\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{8/3}\sqrt[3]{b}}+\frac{5x}{18a^2\left(a+bx^3\right)}+\frac{x}{6a\left(a+bx^3\right)^2}$$

```
[Out] 1/6*x/a/(b*x^3+a)^2+5/18*x/a^2/(b*x^3+a)+5/27*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)
/b^(1/3)-5/54*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(8/3)/b^(1/3)-5/2
7*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)/b^(1/3)*3^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {199, 200, 31, 634, 617, 204, 628}

$$-\frac{5\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{8/3}\sqrt[3]{b}}+\frac{5x}{18a^2\left(a+bx^3\right)}+\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{8/3}\sqrt[3]{b}}-\frac{5\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{8/3}\sqrt[3]{b}}+\frac{x}{6a\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(-3),x]
```

```
[Out] x/(6*a*(a + b*x^3)^2) + (5*x)/(18*a^2*(a + b*x^3)) - (5*ArcTan[(a^(1/3) - 2
*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8/3)*b^(1/3)) + (5*Log[a^(1/3
) + b^(1/3)*x])/(27*a^(8/3)*b^(1/3)) - (5*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2])/(54*a^(8/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^3}\,dx = \frac{x}{6a\left(a+bx^3\right)^2} + \frac{5\int \frac{1}{\left(a+bx^3\right)^2}\,dx}{6a}$$

$$= \frac{x}{6a\left(a+bx^3\right)^2} + \frac{5x}{18a^2\left(a+bx^3\right)} + \frac{5\int \frac{1}{a+bx^3}\,dx}{9a^2}$$

$$= \frac{x}{6a\left(a+bx^3\right)^2} + \frac{5x}{18a^2\left(a+bx^3\right)} + \frac{5\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{27a^{8/3}} + \frac{5\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{27a^{8/3}}$$

$$= \frac{x}{6a\left(a+bx^3\right)^2} + \frac{5x}{18a^2\left(a+bx^3\right)} + \frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{8/3}\sqrt[3]{b}} + \frac{5\int \frac{1}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{18a^{7/3}} - \frac{5\int \frac{-\sqrt[3]{a}}{a^{2/3}-\sqrt[3]{a}}}{54a}$$

$$= \frac{x}{6a\left(a+bx^3\right)^2} + \frac{5x}{18a^2\left(a+bx^3\right)} + \frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{8/3}\sqrt[3]{b}} - \frac{5\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{8/3}\sqrt[3]{b}} + \frac{5S}{}$$

$$= \frac{x}{6a\left(a+bx^3\right)^2} + \frac{5x}{18a^2\left(a+bx^3\right)} - \frac{5\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{8/3}\sqrt[3]{b}} + \frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{8/3}\sqrt[3]{b}} - \frac{5\log\left(a^{2/3}-\sqrt[3]{}\right.}{54a^{8}}$$

Mathematica [A] time = 0.07, size = 135, normalized size = 0.89

$$\frac{-\frac{5\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{\sqrt[3]{b}} + \frac{9a^{5/3}x}{\left(a+bx^3\right)^2} + \frac{15a^{2/3}x}{a+bx^3} + \frac{10\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}} - \frac{10\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{54a^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(-3),x]
```

```
[Out] ((9*a^(5/3)*x)/(a + b*x^3)^2 + (15*a^(2/3)*x)/(a + b*x^3) - (10*Sqrt[3]*Arc
Tan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + (10*Log[a^(1/3) + b^(1/
```

```
3)*x])/b^(1/3) - (5*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3)
)/(54*a^(8/3))
```

fricas [B] time = 0.77, size = 499, normalized size = 3.30

$$\left[\frac{15\,a^2b^2x^4+24\,a^3bx+15\sqrt{\frac{1}{3}}\left(ab^3x^6+2\,a^2b^2x^3+a^3b\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\,\log\left(\frac{2\,abx^3-3\left(a^2b\right)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2\,abx^2+\left(a^2b\right)^{\frac{2}{3}}x-\left(a^2\right.\right.\right.}{bx^3+a}\right.\right.}{54}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(15*a^2*b^2*x^4 + 24*a^3*b*x + 15*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x^
3 + a^3*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^
2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*
b)^(1/3)/b))/(b*x^3 + a)) - 5*(b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log
(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 10*(b^2*x^6 + 2*a*b*x^3 + a
^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^4*b^3*x^6 + 2*a^5*b^2*x^3
+ a^6*b), 1/54*(15*a^2*b^2*x^4 + 24*a^3*b*x + 30*sqrt(1/3)*(a*b^3*x^6 + 2*a
^2*b^2*x^3 + a^3*b)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)
*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 5*(b^2*x^6 + 2*a*b*x^3 +
 a^2)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 10*(
b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^4*b
^3*x^6 + 2*a^5*b^2*x^3 + a^6*b)]
```

giac [A] time = 0.16, size = 137, normalized size = 0.91

$$-\frac{5\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3}+\frac{5\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b}+\frac{5\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -5/27*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^3 + 5/27*sqrt(3)*(-a*b^2)^(
1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b) + 5/54*(
-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b) + 1/18*(5*b*
x^4 + 8*a*x)/((b*x^3 + a)^2*a^2)
```

maple [A] time = 0.00, size = 130, normalized size = 0.86

$$\frac{x}{6\left(b\,x^3+a\right)^2a}+\frac{5x}{18\left(b\,x^3+a\right)a^2}+\frac{5\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2b}+\frac{5\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2b}-\frac{5\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^3,x)
```

```
[Out] 1/6*x/a/(b*x^3+a)^2+5/18*x/a^2/(b*x^3+a)+5/27/a^2/b/(a/b)^(2/3)*ln(x+(a/b)^
(1/3))-5/54/a^2/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+5/27/a^2/b/
(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

maxima [A] time = 2.90, size = 145, normalized size = 0.96

$$\frac{5bx^4+8ax}{18\left(a^2b^2x^6+2a^3bx^3+a^4\right)}+\frac{5\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{5\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{5\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(5*b*x^4 + 8*a*x)/(a^2*b^2*x^6 + 2*a^3*b*x^3 + a^4) + 5/27*sqrt(3)*arc
tan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)^(2/3)) - 5/54
*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(2/3)) + 5/27*log(x +
(a/b)^(1/3))/(a^2*b*(a/b)^(2/3))
```

mupad [B] time = 1.11, size = 142, normalized size = 0.94

$$\frac{\frac{4x}{9a}+\frac{5bx^4}{18a^2}}{a^2+2abx^3+b^2x^6}+\frac{5\ln\left(b^{1/3}x+a^{1/3}\right)}{27a^{8/3}b^{1/3}}+\frac{\ln\left(\frac{5b^2x}{3a^2}+\frac{b^{5/3}\left(-5+\sqrt{3}\,5\mathrm{i}\right)}{6a^{5/3}}\right)\left(-5+\sqrt{3}\,5\mathrm{i}\right)}{54a^{8/3}b^{1/3}}-\frac{\ln\left(\frac{5b^2x}{3a^2}-\frac{b^{5/3}\left(5+\sqrt{3}\,5\mathrm{i}\right)}{6a^{5/3}}\right)\left(5+\right.}{54a^{8/3}b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^3)^3,x)
```

```
[Out] ((4*x)/(9*a) + (5*b*x^4)/(18*a^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) + (5*log(b^(
1/3)*x + a^(1/3)))/(27*a^(8/3)*b^(1/3)) + (log((5*b^2*x)/(3*a^2) + (b^(5/3)
*(3^(1/2)*5i - 5))/(6*a^(5/3)))*(3^(1/2)*5i - 5))/(54*a^(8/3)*b^(1/3)) - (l
og((5*b^2*x)/(3*a^2) - (b^(5/3)*(3^(1/2)*5i + 5))/(6*a^(5/3)))*(3^(1/2)*5i
+ 5))/(54*a^(8/3)*b^(1/3))
```

sympy [A] time = 0.45, size = 63, normalized size = 0.42

$$\frac{8ax+5bx^4}{18a^4+36a^3bx^3+18a^2b^2x^6}+\operatorname{RootSum}\left(19683t^3a^8b-125,\left(t\mapsto t\log\left(\frac{27ta^3}{5}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**3,x)
```

```
[Out] (8*a*x + 5*b*x**4)/(18*a**4 + 36*a**3*b*x**3 + 18*a**2*b**2*x**6) + RootSum
(19683*_t**3*a**8*b - 125, Lambda(_t, _t*log(27*_t*a**3/5 + x)))
```

3.352 $\int \frac{x^8}{a-bx^3}\,dx$

Optimal. Leaf size=41

$$-\frac{a^2\log\left(a-bx^3\right)}{3b^3}-\frac{ax^3}{3b^2}-\frac{x^6}{6b}$$

```
[Out] -1/3*a*x^3/b^2-1/6*x^6/b-1/3*a^2*ln(-b*x^3+a)/b^3
```

Rubi [A] time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {266, 43}

$$-\frac{a^2\log\left(a-bx^3\right)}{3b^3}-\frac{ax^3}{3b^2}-\frac{x^6}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a - b*x^3),x]
```

```
[Out] -(a*x^3)/(3*b^2) - x^6/(6*b) - (a^2*Log[a - b*x^3])/(3*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{a-bx^3}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{a-bx}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(-\frac{a}{b^2}-\frac{x}{b}-\frac{a^2}{b^2(-a+bx)}\right)dx, x, x^3\right)\\ &= -\frac{ax^3}{3b^2}-\frac{x^6}{6b}-\frac{a^2\log\left(a-bx^3\right)}{3b^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.00

$$-\frac{a^2\log\left(a-bx^3\right)}{3b^3}-\frac{ax^3}{3b^2}-\frac{x^6}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a - b*x^3),x]
```

```
[Out] -1/3*(a*x^3)/b^2 - x^6/(6*b) - (a^2*Log[a - b*x^3])/(3*b^3)
```

fricas [A] time = 0.59, size = 35, normalized size = 0.85

$$-\frac{b^2x^6+2\,abx^3+2\,a^2\log\left(bx^3-a\right)}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-b*x^3+a),x, algorithm="fricas")
[Out] -1/6*(b^2*x^6 + 2*a*b*x^3 + 2*a^2*log(b*x^3 - a))/b^3
```

giac [A] time = 0.16, size = 37, normalized size = 0.90

$$-\frac{a^2 \log\left(\left|bx^3-a\right|\right)}{3\,b^3}-\frac{bx^6+2\,ax^3}{6\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-b*x^3+a),x, algorithm="giac")
[Out] -1/3*a^2*log(abs(b*x^3 - a))/b^3 - 1/6*(b*x^6 + 2*a*x^3)/b^2
```

maple [A] time = 0.00, size = 37, normalized size = 0.90

$$-\frac{x^6}{6b}-\frac{a\,x^3}{3b^2}-\frac{a^2\ln\left(b\,x^3-a\right)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-b*x^3+a),x)
[Out] -1/6/b*x^6-1/3*a/b^2*x^3-1/3*a^2/b^3*ln(b*x^3-a)
```

maxima [A] time = 1.34, size = 36, normalized size = 0.88

$$-\frac{a^2 \log\left(bx^3-a\right)}{3\,b^3}-\frac{bx^6+2\,ax^3}{6\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-b*x^3+a),x, algorithm="maxima")
[Out] -1/3*a^2*log(b*x^3 - a)/b^3 - 1/6*(b*x^6 + 2*a*x^3)/b^2
```

mupad [B] time = 0.06, size = 35, normalized size = 0.85

$$-\frac{2\,a^2\,\ln\left(b\,x^3-a\right)+b^2\,x^6+2\,a\,b\,x^3}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a - b*x^3),x)
[Out] -(2*a^2*log(b*x^3 - a) + b^2*x^6 + 2*a*b*x^3)/(6*b^3)
```

sympy [A] time = 0.23, size = 34, normalized size = 0.83

$$-\frac{a^2 \log\left(-a+bx^3\right)}{3b^3}-\frac{ax^3}{3b^2}-\frac{x^6}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-b*x**3+a),x)
[Out] -a**2*log(-a + b*x**3)/(3*b**3) - a*x**3/(3*b**2) - x**6/(6*b)
```

3.353 $\int \frac{x^5}{a-bx^3}\,dx$

Optimal. Leaf size=28

$$-\frac{a\log\left(a-bx^3\right)}{3b^2}-\frac{x^3}{3b}$$

```
[Out] -1/3*x^3/b-1/3*a*ln(-b*x^3+a)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {266, 43}

$$-\frac{a\log\left(a-bx^3\right)}{3b^2}-\frac{x^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a - b*x^3),x]
```

```
[Out] -x^3/(3*b) - (a*Log[a - b*x^3])/(3*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{a-bx^3}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{a-bx}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(-\frac{1}{b}-\frac{a}{b(-a+bx)}\right)dx, x, x^3\right)\\ &= -\frac{x^3}{3b}-\frac{a\log\left(a-bx^3\right)}{3b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$-\frac{a\log\left(a-bx^3\right)}{3b^2}-\frac{x^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a - b*x^3),x]
```

```
[Out] -1/3*x^3/b - (a*Log[a - b*x^3])/(3*b^2)
```

fricas [A] time = 0.53, size = 23, normalized size = 0.82

$$-\frac{bx^3+a\log\left(bx^3-a\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-b*x^3+a),x, algorithm="fricas")
[Out] -1/3*(b*x^3 + a*log(b*x^3 - a))/b^2
```

giac [A] time = 0.19, size = 26, normalized size = 0.93

$$-\frac{x^3}{3\,b}-\frac{a\log\left(\left|bx^3-a\right|\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-b*x^3+a),x, algorithm="giac")
[Out] -1/3*x^3/b - 1/3*a*log(abs(b*x^3 - a))/b^2
```

maple [A] time = 0.00, size = 26, normalized size = 0.93

$$-\frac{x^3}{3b}-\frac{a\ln\left(b\,x^3-a\right)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-b*x^3+a),x)
[Out] -1/3/b*x^3-1/3*a/b^2*ln(b*x^3-a)
```

maxima [A] time = 1.30, size = 25, normalized size = 0.89

$$-\frac{x^3}{3\,b}-\frac{a\log\left(bx^3-a\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-b*x^3+a),x, algorithm="maxima")
[Out] -1/3*x^3/b - 1/3*a*log(b*x^3 - a)/b^2
```

mupad [B] time = 0.04, size = 23, normalized size = 0.82

$$-\frac{b\,x^3+a\,\ln\left(b\,x^3-a\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a - b*x^3),x)
[Out] -(b*x^3 + a*log(b*x^3 - a))/(3*b^2)
```

sympy [A] time = 0.22, size = 22, normalized size = 0.79

$$-\frac{a\log\left(-a+bx^3\right)}{3b^2}-\frac{x^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-b*x**3+a),x)
[Out] -a*log(-a + b*x**3)/(3*b**2) - x**3/(3*b)
```

3.354 $\int \frac{x^2}{a-bx^3}\,dx$

Optimal. Leaf size=16

$$-\frac{\log\left(a-bx^3\right)}{3b}$$

```
[Out] -1/3*ln(-b*x^3+a)/b
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {260}

$$-\frac{\log\left(a-bx^3\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a - b*x^3),x]
```

```
[Out] -Log[a - b*x^3]/(3*b)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^2}{a-bx^3}\,dx = -\frac{\log\left(a-bx^3\right)}{3b}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{\log\left(a-bx^3\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a - b*x^3),x]
```

```
[Out] -1/3*Log[a - b*x^3]/b
```

fricas [A] time = 0.64, size = 15, normalized size = 0.94

$$-\frac{\log\left(bx^3-a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/3*log(b*x^3 - a)/b
```

giac [A] time = 0.16, size = 16, normalized size = 1.00

$$-\frac{\log\left(\left|bx^3-a\right|\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^3+a),x, algorithm="giac")
[Out] -1/3*log(abs(b*x^3 - a))/b
```

maple [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{\ln\left(b\,x^3-a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^3+a),x)
[Out] -1/3/b*ln(b*x^3-a)
```

maxima [A] time = 1.34, size = 15, normalized size = 0.94

$$-\frac{\log\left(bx^3-a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^3+a),x, algorithm="maxima")
[Out] -1/3*log(b*x^3 - a)/b
```

mupad [B] time = 0.04, size = 15, normalized size = 0.94

$$-\frac{\ln\left(b\,x^3-a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a - b*x^3),x)
[Out] -log(b*x^3 - a)/(3*b)
```

sympy [A] time = 0.16, size = 12, normalized size = 0.75

$$-\frac{\log\left(-a+bx^3\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**3+a),x)
[Out] -log(-a + b*x**3)/(3*b)
```

3.355 $\int \frac{1}{x\left(a-bx^3\right)}\,dx$

Optimal. Leaf size=23

$$\frac{\log(x)}{a}-\frac{\log\left(a-bx^3\right)}{3a}$$

```
[Out] ln(x)/a-1/3*ln(-b*x^3+a)/a
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {266, 36, 29, 31}

$$\frac{\log(x)}{a}-\frac{\log\left(a-bx^3\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a - b*x^3)),x]
```

```
[Out] Log[x]/a - Log[a - b*x^3]/(3*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a-bx^3\right)}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x(a-bx)}\,dx, x, x^3\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^3\right)}{3a}+\frac{b\,\text{Subst}\left(\int \frac{1}{a-bx}\,dx, x, x^3\right)}{3a}\\ &= \frac{\log(x)}{a}-\frac{\log\left(a-bx^3\right)}{3a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{\log(x)}{a}-\frac{\log\left(a-bx^3\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a - b*x^3)),x]
[Out] Log[x]/a - Log[a - b*x^3]/(3*a)
```

fricas [A] time = 0.59, size = 20, normalized size = 0.87

$$-\frac{\log\left(bx^3-a\right)-3\,\log(x)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^3+a),x, algorithm="fricas")
[Out] -1/3*(log(b*x^3 - a) - 3*log(x))/a
```

giac [A] time = 0.16, size = 24, normalized size = 1.04

$$-\frac{\log\left(\left|bx^3-a\right|\right)}{3\,a}+\frac{\log\left(|x|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^3+a),x, algorithm="giac")
[Out] -1/3*log(abs(b*x^3 - a))/a + log(abs(x))/a
```

maple [A] time = 0.00, size = 23, normalized size = 1.00

$$\frac{\ln(x)}{a}-\frac{\ln\left(b\,x^3-a\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-b*x^3+a),x)
[Out] -1/3/a*ln(b*x^3-a)+1/a*ln(x)
```

maxima [A] time = 1.34, size = 25, normalized size = 1.09

$$-\frac{\log\left(bx^3-a\right)}{3\,a}+\frac{\log\left(x^3\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^3+a),x, algorithm="maxima")
[Out] -1/3*log(b*x^3 - a)/a + 1/3*log(x^3)/a
```

mupad [B] time = 0.10, size = 21, normalized size = 0.91

$$\frac{\ln(x)}{a}-\frac{\ln\left(a-b\,x^3\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a - b*x^3)),x)
[Out] log(x)/a - log(a - b*x^3)/(3*a)
```

sympy [A] time = 0.27, size = 15, normalized size = 0.65

$$\frac{\log\left(x\right)}{a}-\frac{\log\left(-\frac{a}{b}+x^3\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x**3+a),x)
[Out] log(x)/a - log(-a/b + x**3)/(3*a)
```

3.356 $\int \frac{1}{x^4\left(a-bx^3\right)}\,dx$

Optimal. Leaf size=35

$$-\frac{b\log\left(a-bx^3\right)}{3a^2}+\frac{b\log(x)}{a^2}-\frac{1}{3ax^3}$$

```
[Out] -1/3/a/x^3+b*ln(x)/a^2-1/3*b*ln(-b*x^3+a)/a^2
```

Rubi [A] time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {266, 44}

$$-\frac{b\log\left(a-bx^3\right)}{3a^2}+\frac{b\log(x)}{a^2}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a - b*x^3)),x]
```

```
[Out] -1/(3*a*x^3) + (b*Log[x])/a^2 - (b*Log[a - b*x^3])/(3*a^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(a-bx^3\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x^2(a-bx)}\,dx, x, x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{1}{ax^2}+\frac{b}{a^2x}+\frac{b^2}{a^2(a-bx)}\right)dx, x, x^3\right)\\ &= -\frac{1}{3ax^3}+\frac{b\log(x)}{a^2}-\frac{b\log\left(a-bx^3\right)}{3a^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.00

$$-\frac{b\log\left(a-bx^3\right)}{3a^2}+\frac{b\log(x)}{a^2}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a - b*x^3)),x]
```

```
[Out] -1/3*1/(a*x^3) + (b*Log[x])/a^2 - (b*Log[a - b*x^3])/(3*a^2)
```

fricas [A] time = 0.85, size = 33, normalized size = 0.94

$$-\frac{bx^3\log\left(bx^3-a\right)-3\,bx^3\log(x)+a}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/3*(b*x^3*log(b*x^3 - a) - 3*b*x^3*log(x) + a)/(a^2*x^3)
```

giac [A] time = 0.21, size = 41, normalized size = 1.17

$$-\frac{b\log\left(\left|bx^3-a\right|\right)}{3\,a^2}+\frac{b\log\left(|x|\right)}{a^2}-\frac{bx^3+a}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*b*log(abs(b*x^3 - a))/a^2 + b*log(abs(x))/a^2 - 1/3*(b*x^3 + a)/(a^2*x
^3)
```

maple [A] time = 0.01, size = 33, normalized size = 0.94

$$\frac{b\ln(x)}{a^2}-\frac{b\ln\left(b\,x^3-a\right)}{3a^2}-\frac{1}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-b*x^3+a),x)
```

```
[Out] -1/3/a^2*b*ln(b*x^3-a)-1/3/a/x^3+1/a^2*b*ln(x)
```

maxima [A] time = 1.32, size = 35, normalized size = 1.00

$$-\frac{b\log\left(bx^3-a\right)}{3\,a^2}+\frac{b\log\left(x^3\right)}{3\,a^2}-\frac{1}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*b*log(b*x^3 - a)/a^2 + 1/3*b*log(x^3)/a^2 - 1/3/(a*x^3)
```

mupad [B] time = 0.07, size = 31, normalized size = 0.89

$$\frac{b\,\ln(x)}{a^2}-\frac{b\,\ln\left(a-b\,x^3\right)}{3\,a^2}-\frac{1}{3\,a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a - b*x^3)),x)
```

```
[Out] (b*log(x))/a^2 - (b*log(a - b*x^3))/(3*a^2) - 1/(3*a*x^3)
```

sympy [A] time = 0.43, size = 31, normalized size = 0.89

$$-\frac{1}{3ax^3}+\frac{b\log(x)}{a^2}-\frac{b\log\left(-\frac{a}{b}+x^3\right)}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-b*x**3+a),x)
```

```
[Out] -1/(3*a*x**3) + b*log(x)/a**2 - b*log(-a/b + x**3)/(3*a**2)
```

3.357 $\int \frac{x^4}{a-bx^3}\,dx$

Optimal. Leaf size=125

$$\frac{a^{2/3}\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{5/3}}-\frac{a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{3b^{5/3}}-\frac{a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{5/3}}-\frac{x^2}{2b}$$

```
[Out] -1/2*x^2/b-1/3*a^(2/3)*ln(a^(1/3)-b^(1/3)*x)/b^(5/3)+1/6*a^(2/3)*ln(a^(2/3)
+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(5/3)-1/3*a^(2/3)*arctan(1/3*(a^(1/3)+2*b
^(1/3)*x)/a^(1/3)*3^(1/2))/b^(5/3)*3^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {321, 292, 31, 634, 617, 204, 628}

$$\frac{a^{2/3}\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{5/3}}-\frac{a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{3b^{5/3}}-\frac{a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{5/3}}-\frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a - b*x^3),x]
```

```
[Out] -x^2/(2*b) - (a^(2/3)*ArcTan[(a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(S
qrt[3]*b^(5/3)) - (a^(2/3)*Log[a^(1/3) - b^(1/3)*x])/(3*b^(5/3)) + (a^(2/3)
*Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{a-bx^3}\,dx &= -\frac{x^2}{2b} + \frac{a\int \frac{x}{a-bx^3}\,dx}{b} \\
&= -\frac{x^2}{2b} + \frac{a^{2/3}\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{b}\,x}\,dx}{3b^{4/3}} - \frac{a^{2/3}\int \frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3b^{4/3}} \\
&= -\frac{x^2}{2b} - \frac{a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{3b^{5/3}} + \frac{a^{2/3}\int \frac{\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{6b^{5/3}} - \frac{a\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{2b^{4/3}} \\
&= -\frac{x^2}{2b} - \frac{a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{3b^{5/3}} + \frac{a^{2/3}\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{5/3}} + \frac{a^{2/3}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x,\right.}{b^{5/3}} \\
&= -\frac{x^2}{2b} - \frac{a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{5/3}} - \frac{a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{3b^{5/3}} + \frac{a^{2/3}\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{5/3}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 111, normalized size = 0.89

$$-\frac{-a^{2/3}\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)+2a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)+2\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)+3b^{2/3}x^2}{6b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a - b*x^3),x]
```

```
[Out] -1/6*(3*b^(2/3)*x^2 + 2*Sqrt[3]*a^(2/3)*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3))/
Sqrt[3]] + 2*a^(2/3)*Log[a^(1/3) - b^(1/3)*x] - a^(2/3)*Log[a^(2/3) + a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/b^(5/3)
```

fricas [A] time = 0.76, size = 130, normalized size = 1.04

$$-\frac{3\,x^2+2\sqrt{3}\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,bx\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}-\sqrt{3}\,a}{3\,a}\right)+\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax^2+bx\left(-\frac{a^2}{b^2}\right)^{\frac{2}{3}}-a\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right)-2\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(\right.}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/6*(3*x^2 + 2*sqrt(3)*(-a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a^2/b^
2)^(1/3) - sqrt(3)*a)/a) + (-a^2/b^2)^(1/3)*log(a*x^2 + b*x*(-a^2/b^2)^(2/3
) - a*(-a^2/b^2)^(1/3)) - 2*(-a^2/b^2)^(1/3)*log(a*x - b*(-a^2/b^2)^(2/3)))
/b
```

giac [A] time = 0.18, size = 106, normalized size = 0.85

$$-\frac{x^2}{2\,b}-\frac{\left(\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,b}-\frac{\sqrt{3}\,\left(a b^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^3}+\frac{\left(a b^2\right)^{\frac{2}{3}}\log\left(x^2+x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/2*x^2/b - 1/3*(a/b)^(2/3)*log(abs(x - (a/b)^(1/3)))/b - 1/3*sqrt(3)*(a*b
^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/b^3 + 1/6*(a*
b^2)^(2/3)*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/b^3
```

maple [A] time = 0.00, size = 103, normalized size = 0.82

$$-\frac{x^2}{2b}-\frac{\sqrt{3}\;a\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}-\frac{a\ln\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}+\frac{a\ln\left(x^2+\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-b*x^3+a),x)
```

```
[Out] -1/2/b*x^2-1/3*a/b^2/(a/b)^(1/3)*ln(x-(a/b)^(1/3))+1/6*a/b^2/(a/b)^(1/3)*ln
(x^2+(a/b)^(1/3)*x+(a/b)^(2/3))-1/3*a/b^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^
(1/2)*(2/(a/b)^(1/3)*x+1))
```

maxima [A] time = 3.00, size = 108, normalized size = 0.86

$$-\frac{x^2}{2\,b}-\frac{\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{a\log\left(x^2+x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{a\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/2*x^2/b - 1/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/
3))/(b^2*(a/b)^(1/3)) + 1/6*a*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(
a/b)^(1/3)) - 1/3*a*log(x - (a/b)^(1/3))/(b^2*(a/b)^(1/3))
```

mupad [B] time = 1.16, size = 134, normalized size = 1.07

$$\frac{a^{2/3}\,\ln\left(\frac{a^2\,x}{b}-\frac{a^{7/3}}{(-b)^{4/3}}\right)}{3\,(-b)^{5/3}}-\frac{x^2}{2\,b}-\frac{a^{2/3}\,\ln\left(\frac{a^2\,x}{b}-\frac{a^{7/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{(-b)^{4/3}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,(-b)^{5/3}}+\frac{a^{2/3}\,\ln\left(\frac{a^2\,x}{b}-\frac{9\,a^{7/3}\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2}{(-b)^{4/3}}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right.}{(-b)^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a - b*x^3),x)
```

```
[Out] (a^(2/3)*log((a^2*x)/b - a^(7/3)/(-b)^(4/3)))/(3*(-b)^(5/3)) - x^2/(2*b) -
(a^(2/3)*log((a^2*x)/b - (a^(7/3)*((3^(1/2)*1i)/2 + 1/2)^2)/(-b)^(4/3))*((3
^(1/2)*1i)/2 + 1/2))/(3*(-b)^(5/3)) + (a^(2/3)*log((a^2*x)/b - (9*a^(7/3)*(
(3^(1/2)*1i)/6 - 1/6)^2)/(-b)^(4/3))*((3^(1/2)*1i)/6 - 1/6))/(-b)^(5/3)
```

sympy [**A**] time = 0.20, size = 34, normalized size = 0.27

$$-\operatorname{RootSum}\left(27t^3b^5 - a^2, \left(t \mapsto t\log\left(-\frac{9t^2b^3}{a} + x\right)\right)\right) - \frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-b*x**3+a),x)
```

```
[Out] -RootSum(27*_t**3*b**5 - a**2, Lambda(_t, _t*log(-9*_t**2*b**3/a + x))) - x
**2/(2*b)
```

3.358 $\int \frac{x^3}{a-bx^3}\,dx$

Optimal. Leaf size=120

$$\frac{\sqrt[3]{a}\,\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{4/3}}-\frac{\sqrt[3]{a}\,\log\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{3b^{4/3}}+\frac{\sqrt[3]{a}\,\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{4/3}}-\frac{x}{b}$$

```
[Out] -x/b-1/3*a^(1/3)*ln(a^(1/3)-b^(1/3)*x)/b^(4/3)+1/6*a^(1/3)*ln(a^(2/3)+a^(1/
3)*b^(1/3)*x+b^(2/3)*x^2)/b^(4/3)+1/3*a^(1/3)*arctan(1/3*(a^(1/3)+2*b^(1/3)
*x)/a^(1/3)*3^(1/2))/b^(4/3)*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {321, 200, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{a}\,\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{4/3}}-\frac{\sqrt[3]{a}\,\log\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{3b^{4/3}}+\frac{\sqrt[3]{a}\,\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{4/3}}-\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a - b*x^3),x]
```

```
[Out] -(x/b) + (a^(1/3)*ArcTan[(a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[
3]*b^(4/3)) - (a^(1/3)*Log[a^(1/3) - b^(1/3)*x])/(3*b^(4/3)) + (a^(1/3)*Log
[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{a-bx^3}\,dx &= -\frac{x}{b} + \frac{a\int \frac{1}{a-bx^3}\,dx}{b} \\
&= -\frac{x}{b} + \frac{\sqrt[3]{a}\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{b}\,x}\,dx}{3b} + \frac{\sqrt[3]{a}\int \frac{2\sqrt[3]{a}+\sqrt[3]{b}\,x}{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3b} \\
&= -\frac{x}{b} - \frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{3b^{4/3}} + \frac{\sqrt[3]{a}\int \frac{\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{6b^{4/3}} + \frac{a^{2/3}\int \frac{1}{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{2b} \\
&= -\frac{x}{b} - \frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{3b^{4/3}} + \frac{\sqrt[3]{a}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{4/3}} - \frac{\sqrt[3]{a}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1\right.}{b^{4/3}} \\
&= -\frac{x}{b} + \frac{\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{4/3}} - \frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{3b^{4/3}} + \frac{\sqrt[3]{a}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{4/3}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 108, normalized size = 0.90

$$\frac{\sqrt[3]{a}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right) - 2\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right) + 2\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}+1}{\sqrt{3}}\right) - 6\sqrt[3]{b}\,x}{6b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a - b*x^3),x]
```

```
[Out] (-6*b^(1/3)*x + 2*Sqrt[3]*a^(1/3)*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3))/Sqrt[3
]] - 2*a^(1/3)*Log[a^(1/3) - b^(1/3)*x] + a^(1/3)*Log[a^(2/3) + a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2])/(6*b^(4/3))
```

fricas [A] time = 0.70, size = 103, normalized size = 0.86

$$-\frac{2\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}+\sqrt{3}\,a}{3a}\right) + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2 - x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right) - 2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right) + 6x}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) + sqrt(
3)*a)/a) + (-a/b)^(1/3)*log(x^2 - x*(-a/b)^(1/3) + (-a/b)^(2/3)) - 2*(-a/b)
^(1/3)*log(x + (-a/b)^(1/3)) + 6*x)/b
```

giac [A] time = 0.17, size = 104, normalized size = 0.87

$$-\frac{\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,b}-\frac{x}{b}+\frac{\sqrt{3}\,\left(ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^2}+\frac{\left(ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*(a/b)^(1/3)*log(abs(x - (a/b)^(1/3)))/b - x/b + 1/3*sqrt(3)*(a*b^2)^(1
/3)*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/b^2 + 1/6*(a*b^2)^(
1/3)*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/b^2
```

maple [A] time = 0.00, size = 101, normalized size = 0.84

$$\frac{\sqrt{3}\;a\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}-\frac{a\ln\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}+\frac{a\ln\left(x^2+\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}-\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-b*x^3+a),x)
```

```
[Out] -1/b*x-1/3*a/b^2/(a/b)^(2/3)*ln(x-(a/b)^(1/3))+1/6*a/b^2/(a/b)^(2/3)*ln(x^2
+(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*a/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2
)*(2/(a/b)^(1/3)*x+1))
```

maxima [A] time = 3.02, size = 106, normalized size = 0.88

$$-\frac{x}{b}+\frac{\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{a\log\left(x^2+x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{a\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^3+a),x, algorithm="maxima")
```

```
[Out] -x/b + 1/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/(b
^2*(a/b)^(2/3)) + 1/6*a*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a/b)^(
2/3)) - 1/3*a*log(x - (a/b)^(1/3))/(b^2*(a/b)^(2/3))
```

mupad [B] time = 1.12, size = 116, normalized size = 0.97

$$\frac{(-a)^{1/3}\,\ln\left((-a)^{4/3}-a\,b^{1/3}\,x\right)}{3\,b^{4/3}}-\frac{x}{b}-\frac{(-a)^{1/3}\,\ln\left(3\,(-a)^{4/3}\,b^{2/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)+3\,a\,b\,x\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,b^{4/3}}+\frac{(-a)^{1/3}\,\ln\left(9\,(-a)^{4/}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a - b*x^3),x)
```

```
[Out] ((-a)^(1/3)*log((-a)^(4/3) - a*b^(1/3)*x))/(3*b^(4/3)) - x/b - ((-a)^(1/3)*
log(3*(-a)^(4/3)*b^(2/3)*((3^(1/2)*1i)/2 + 1/2) + 3*a*b*x)*((3^(1/2)*1i)/2
+ 1/2))/(3*b^(4/3)) + ((-a)^(1/3)*log(9*(-a)^(4/3)*b^(2/3)*((3^(1/2)*1i)/6
- 1/6) - 3*a*b*x)*((3^(1/2)*1i)/6 - 1/6))/b^(4/3)
```

sympy [A] time = 0.24, size = 24, normalized size = 0.20

$$-\operatorname{RootSum}\left(27t^3b^4 - a, \left(t \mapsto t\log\left(-3tb + x\right)\right)\right) - \frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-b*x**3+a),x)
```

```
[Out] -RootSum(27*_t**3*b**4 - a, Lambda(_t, _t*log(-3*_t*b + x))) - x/b
```

3.359 $\int \frac{x}{a-bx^3}\,dx$

Optimal. Leaf size=115

$$\frac{\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6\sqrt[3]{a}\,b^{2/3}}-\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{2/3}}$$

```
[Out] -1/3*ln(a^(1/3)-b^(1/3)*x)/a^(1/3)/b^(2/3)+1/6*ln(a^(2/3)+a^(1/3)*b^(1/3)*x
+b^(2/3)*x^2)/a^(1/3)/b^(2/3)-1/3*arctan(1/3*(a^(1/3)+2*b^(1/3)*x)/a^(1/3)*
3^(1/2))/a^(1/3)/b^(2/3)*3^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {292, 31, 634, 617, 204, 628}

$$\frac{\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6\sqrt[3]{a}\,b^{2/3}}-\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a - b*x^3),x]
```

```
[Out] -(ArcTan[(a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(1/3)*b^(2/3
))) - Log[a^(1/3) - b^(1/3)*x]/(3*a^(1/3)*b^(2/3)) + Log[a^(2/3) + a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2]/(6*a^(1/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{a-bx^3}\,dx &= \frac{\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{b}\,x}\,dx}{3\sqrt[3]{a}\,\sqrt[3]{b}} - \frac{\int \frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3\sqrt[3]{a}\,\sqrt[3]{b}} \\ &= -\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}} + \frac{\int \frac{\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{6\sqrt[3]{a}\,b^{2/3}} - \frac{\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{2\sqrt[3]{b}} \\ &= -\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}} + \frac{\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6\sqrt[3]{a}\,b^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}\,b^{2/3}} \\ &= -\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{2/3}} - \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}} + \frac{\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6\sqrt[3]{a}\,b^{2/3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 89, normalized size = 0.77

$$\frac{\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)-2\log\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)-2\sqrt{3}\,\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{6\sqrt[3]{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a - b*x^3),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3) - b
^(1/3)*x] + Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3)*b^(2
/3))
```

fricas [A] time = 0.83, size = 307, normalized size = 2.67

$$\left[\frac{3\sqrt{\frac{1}{3}}\,ab\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\,\log\left(\frac{2b^2x^3+ab+3\sqrt{\frac{1}{3}}\left(abx-2\left(-ab^2\right)^{\frac{2}{3}}x^2-\left(-ab^2\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}-3\left(-ab^2\right)^{\frac{2}{3}}x}{bx^3-a}\right)+\left(-ab^2\right)^{\frac{2}{3}}\log\left(b^2x^2-\left(-ab^2\right)^{\frac{2}{3}}\right.}{6\,ab^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 + a*b + 3*sqrt(
1/3)*(a*b*x - 2*(-a*b^2)^(2/3)*x^2 - (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/
```

```
a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 - a)) + (-a*b^2)^(2/3)*log(b^2*x^2 - (-a*b^
2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*log(b*x + (-a*b^2)^(1/3))
)/(a*b^2), 1/6*(6*sqrt(1/3)*a*b*sqrt(-(-a*b^2)^(1/3)/a)*arctan(-sqrt(1/3)*(
2*b*x - (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (-a*b^2)^(2/3)*log(b^2
*x^2 - (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*log(b*x + (-
a*b^2)^(1/3)))/(a*b^2)]
```

giac [A] time = 0.19, size = 104, normalized size = 0.90

$$-\frac{\left(\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a}-\frac{\sqrt{3}\,\left(ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^2}+\frac{\left(ab^2\right)^{\frac{2}{3}}\log\left(x^2+x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*(a/b)^(2/3)*log(abs(x - (a/b)^(1/3)))/a - 1/3*sqrt(3)*(a*b^2)^(2/3)*ar
ctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2) + 1/6*(a*b^2)^(2/
3)*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2)
```

maple [A] time = 0.00, size = 92, normalized size = 0.80

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b}-\frac{\ln\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b}+\frac{\ln\left(x^2+\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^3+a),x)
```

```
[Out] -1/3/b/(a/b)^(1/3)*ln(x-(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2+(a/b)^(1/3)*x
+(a/b)^(2/3))-1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x
+1))
```

maxima [A] time = 2.89, size = 97, normalized size = 0.84

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^2+x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^(
1/3)) + 1/6*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(1/3)) - 1/3*lo
g(x - (a/b)^(1/3))/(b*(a/b)^(1/3))
```

mupad [B] time = 1.11, size = 109, normalized size = 0.95

$$\frac{\ln\left((-a)^{1/3}+b^{1/3}\,x\right)}{3\,(-a)^{1/3}\,b^{2/3}}+\frac{\ln\left(b\,x+\frac{(-a)^{1/3}\,b^{2/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,(-a)^{1/3}\,b^{2/3}}-\frac{\ln\left(b\,x+\frac{(-a)^{1/3}\,b^{2/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,(-a)^{1/3}\,b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a - b*x^3),x)
```

```
[Out] log((-a)^(1/3) + b^(1/3)*x)/(3*(-a)^(1/3)*b^(2/3)) + (log(b*x + ((-a)^(1/3)
*b^(2/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(6*(-a)^(1/3)*b^(2/3)) -
(log(b*x + ((-a)^(1/3)*b^(2/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/(6*
(-a)^(1/3)*b^(2/3))
```

sympy [**A**] time = 0.17, size = 26, normalized size = 0.23

$$-\operatorname{RootSum}\left(27t^3ab^2-1,\left(t\mapsto t\log\left(-9t^2ab+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**3+a),x)
```

```
[Out] -RootSum(27*_t**3*a*b**2 - 1, Lambda(_t, _t*log(-9*_t**2*a*b + x)))
```

3.360 $\int \frac{1}{a-bx^3}\,dx$

Optimal. Leaf size=114

$$\frac{\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}}$$

```
[Out] -1/3*ln(a^(1/3)-b^(1/3)*x)/a^(2/3)/b^(1/3)+1/6*ln(a^(2/3)+a^(1/3)*b^(1/3)*x
+b^(2/3)*x^2)/a^(2/3)/b^(1/3)+1/3*arctan(1/3*(a^(1/3)+2*b^(1/3)*x)/a^(1/3)*
3^(1/2))/a^(2/3)/b^(1/3)*3^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {200, 31, 634, 617, 204, 628}

$$\frac{\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^(-1),x]
```

```
[Out] ArcTan[(a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3)*b^(1/3))
 - Log[a^(1/3) - b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + Log[a^(2/3) + a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2]/(6*a^(2/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a-bx^3}\,dx &= \frac{\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{b}\,x}\,dx}{3a^{2/3}} + \frac{\int \frac{2\sqrt[3]{a}+\sqrt[3]{b}\,x}{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{2/3}} \\ &= -\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{3a^{2/3}\sqrt[3]{b}} + \frac{\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{2\sqrt[3]{a}} + \frac{\int \frac{\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{6a^{2/3}\sqrt[3]{b}} \\ &= -\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{3a^{2/3}\sqrt[3]{b}} + \frac{\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{b}} \\ &= \frac{\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}} - \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{3a^{2/3}\sqrt[3]{b}} + \frac{\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 89, normalized size = 0.78

$$\frac{\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)-2\log\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)+2\sqrt{3}\,\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{6a^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^(-1),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3) - b^
(1/3)*x] + Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/
3))
```

fricas [A] time = 0.95, size = 320, normalized size = 2.81

$$\left[\frac{3\sqrt{\frac{1}{3}}\,ab\sqrt{\frac{\left(-a^2b\right)^{\frac{1}{3}}}{b}}\,\log\left(\frac{2\,abx^3+3\left(-a^2b\right)^{\frac{1}{3}}ax+a^2+3\sqrt{\frac{1}{3}}\left(2\,abx^2-\left(-a^2b\right)^{\frac{2}{3}}x+\left(-a^2b\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-a^2b\right)^{\frac{1}{3}}}{b}}}{bx^3-a}\right)+\left(-a^2b\right)^{\frac{2}{3}}\log\left(abx^2+\left(-a^2\right.\right.\right.}{6\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt((-a^2*b)^(1/3)/b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1
/3)*a*x + a^2 + 3*sqrt(1/3)*(2*a*b*x^2 - (-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*
```

```
a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 - a)) + (-a^2*b)^(2/3)*log(a*b*x^2 + (-a^
2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) - 2*(-a^2*b)^(2/3)*log(a*b*x - (-a^2*b)^(2
/3)))/(a^2*b), 1/6*(6*sqrt(1/3)*a*b*sqrt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3
)*(2*(-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) + (-
a^2*b)^(2/3)*log(a*b*x^2 + (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) - 2*(-a^2*b
)^(2/3)*log(a*b*x - (-a^2*b)^(2/3)))/(a^2*b)]
```

giac [A] time = 0.16, size = 104, normalized size = 0.91

$$-\frac{\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a}+\frac{\sqrt{3}\,\left(ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab}+\frac{\left(ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*(a/b)^(1/3)*log(abs(x - (a/b)^(1/3)))/a + 1/3*sqrt(3)*(a*b^2)^(1/3)*ar
ctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/(a*b) + 1/6*(a*b^2)^(1/3)
*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b)
```

maple [A] time = 0.00, size = 92, normalized size = 0.81

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}-\frac{\ln\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}+\frac{\ln\left(x^2+\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^3+a),x)
```

```
[Out] -1/3/b/(a/b)^(2/3)*ln(x-(a/b)^(1/3))+1/6/b/(a/b)^(2/3)*ln(x^2+(a/b)^(1/3)*x
+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x
+1))
```

maxima [A] time = 2.97, size = 97, normalized size = 0.85

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\log\left(x^2+x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^(2
/3)) + 1/6*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)) - 1/3*log
(x - (a/b)^(1/3))/(b*(a/b)^(2/3))
```

mupad [B] time = 1.23, size = 115, normalized size = 1.01

$$\frac{\ln\left(a^{1/3}\,(-b)^{5/3}+b^2\,x\right)}{3\,a^{2/3}\,(-b)^{1/3}}+\frac{\ln\left(3\,b^2\,x+\frac{3\,a^{1/3}\,(-b)^{5/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,(-b)^{1/3}}-\frac{\ln\left(3\,b^2\,x-\frac{3\,a^{1/3}\,(-b)^{5/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(1+\sqrt{3}\right.}{6\,a^{2/3}\,(-b)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*x^3),x)
```

```
[Out] log(a^(1/3)*(-b)^(5/3) + b^2*x)/(3*a^(2/3)*(-b)^(1/3)) + (log(3*b^2*x + (3*
a^(1/3)*(-b)^(5/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(6*a^(2/3)*(-b)^(
1/3)) - (log(3*b^2*x - (3*a^(1/3)*(-b)^(5/3)*(3^(1/2)*1i + 1))/2)*(3^(1/2)*
1i + 1))/(6*a^(2/3)*(-b)^(1/3))
```

sympy [**A**] time = 0.25, size = 22, normalized size = 0.19

$$-\operatorname{RootSum}\left(27t^3a^2b-1,\left(t\mapsto t\log\left(-3ta+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**3+a),x)
```

```
[Out] -RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(-3*_t*a + x)))
```

3.361 $\int \frac{1}{x^2\left(a-bx^3\right)}\,dx$

Optimal. Leaf size=123

$$\frac{\sqrt[3]{b}\,\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{4/3}}-\frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{3a^{4/3}}-\frac{\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}}-\frac{1}{ax}$$

```
[Out] -1/a/x-1/3*b^(1/3)*ln(a^(1/3)-b^(1/3)*x)/a^(4/3)+1/6*b^(1/3)*ln(a^(2/3)+a^(
1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)-1/3*b^(1/3)*arctan(1/3*(a^(1/3)+2*b^(1/
3)*x)/a^(1/3)*3^(1/2))/a^(4/3)*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {325, 292, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{b}\,\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{4/3}}-\frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{3a^{4/3}}-\frac{\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a - b*x^3)),x]
```

```
[Out] -(1/(a*x)) - (b^(1/3)*ArcTan[(a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(S
qrt[3]*a^(4/3)) - (b^(1/3)*Log[a^(1/3) - b^(1/3)*x])/(3*a^(4/3)) + (b^(1/3)
*Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^2\left(a-bx^3\right)}\,dx = -\frac{1}{ax} + \frac{b\int \frac{x}{a-bx^3}\,dx}{a}$$

$$= -\frac{1}{ax} + \frac{b^{2/3}\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{b}\,x}\,dx}{3a^{4/3}} - \frac{b^{2/3}\int \frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{4/3}}$$

$$= -\frac{1}{ax} - \frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{3a^{4/3}} + \frac{\sqrt[3]{b}\int \frac{\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{6a^{4/3}} - \frac{b^{2/3}\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{2a}$$

$$= -\frac{1}{ax} - \frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{3a^{4/3}} + \frac{\sqrt[3]{b}\,\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{4/3}} + \frac{\sqrt[3]{b}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,d\right.}{a^{4/3}}$$

$$= -\frac{1}{ax} - \frac{\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}} - \frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{3a^{4/3}} + \frac{\sqrt[3]{b}\,\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{4/3}}$$

Mathematica [A] time = 0.04, size = 114, normalized size = 0.93

$$-\frac{-\sqrt[3]{b}\,x\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)+2\sqrt[3]{b}\,x\log\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)+2\sqrt{3}\,\sqrt[3]{b}\,x\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)+6\sqrt[3]{a}}{6a^{4/3}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a - b*x^3)),x]
```

```
[Out] -1/6*(6*a^(1/3) + 2*Sqrt[3]*b^(1/3)*x*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3))/Sq
rt[3]] + 2*b^(1/3)*x*Log[a^(1/3) - b^(1/3)*x] - b^(1/3)*x*Log[a^(2/3) + a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/(a^(4/3)*x)
```

fricas [A] time = 0.77, size = 111, normalized size = 0.90

$$-\frac{2\sqrt{3}\,x\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\,x\left(-\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+x\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2+ax\left(-\frac{b}{a}\right)^{\frac{2}{3}}-a\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)-2\,x\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx\right.}{6\,ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*x*(-b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^(1/3) - 1/3*sqrt
(3)) + x*(-b/a)^(1/3)*log(b*x^2 + a*x*(-b/a)^(2/3) - a*(-b/a)^(1/3)) - 2*x*
(-b/a)^(1/3)*log(b*x - a*(-b/a)^(2/3)) + 6)/(a*x)
```

giac [A] time = 0.19, size = 113, normalized size = 0.92

$$-\frac{b\left(\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^2}-\frac{\sqrt{3}\left(ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^2b}+\frac{\left(ab^2\right)^{\frac{2}{3}}\log\left(x^2+x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^2b}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*b*(a/b)^(2/3)*log(abs(x - (a/b)^(1/3)))/a^2 - 1/3*sqrt(3)*(a*b^2)^(2/3
)*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b) + 1/6*(a*b^2)
^(2/3)*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b) - 1/(a*x)
```

maple [A] time = 0.01, size = 100, normalized size = 0.81

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a}-\frac{\ln\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a}+\frac{\ln\left(x^2+\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}a}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-b*x^3+a),x)
```

```
[Out] -1/a/x-1/3/a/(a/b)^(1/3)*ln(x-(a/b)^(1/3))+1/6/a/(a/b)^(1/3)*ln(x^2+(a/b)^(
1/3)*x+(a/b)^(2/3))-1/3/a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(
1/3)*x+1))
```

maxima [A] time = 2.85, size = 105, normalized size = 0.85

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^2+x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/(a*(a/b)^(
1/3)) + 1/6*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(1/3)) - 1/3*lo
g(x - (a/b)^(1/3))/(a*(a/b)^(1/3)) - 1/(a*x)
```

mupad [B] time = 1.15, size = 127, normalized size = 1.03

$$\frac{(-b)^{1/3}\ln\left(b^3x-a^{1/3}(-b)^{8/3}\right)}{3a^{4/3}}-\frac{1}{ax}-\frac{(-b)^{1/3}\ln\left(ab^3x-a^{4/3}(-b)^{8/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3a^{4/3}}+\frac{(-b)^{1/3}\ln\left(ab^3x-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a - b*x^3)),x)
```

```
[Out] ((-b)^(1/3)*log(b^3*x - a^(1/3)*(-b)^(8/3)))/(3*a^(4/3)) - 1/(a*x) - ((-b)^
(1/3)*log(a*b^3*x - a^(4/3)*(-b)^(8/3)*((3^(1/2)*1i)/2 + 1/2)^2)*((3^(1/2)*
1i)/2 + 1/2))/(3*a^(4/3)) + ((-b)^(1/3)*log(a*b^3*x - 9*a^(4/3)*(-b)^(8/3)*
((3^(1/2)*1i)/6 - 1/6)^2)*((3^(1/2)*1i)/6 - 1/6))/a^(4/3)
```

sympy [**A**] time = 0.25, size = 31, normalized size = 0.25

$$-\operatorname{RootSum}\left(27t^3a^4 - b, \left(t \mapsto t\log\left(-\frac{9t^2a^3}{b} + x\right)\right)\right) - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-b*x**3+a),x)
```

```
[Out] -RootSum(27*_t**3*a**4 - b, Lambda(_t, _t*log(-9*_t**2*a**3/b + x))) - 1/(a
*x)
```

3.362 $\int \frac{1}{x^3\left(a-bx^3\right)}\,dx$

Optimal. Leaf size=124

$$\frac{b^{2/3}\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{5/3}}-\frac{b^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{3a^{5/3}}+\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}}-\frac{1}{2ax^2}$$

```
[Out] -1/2/a/x^2-1/3*b^(2/3)*ln(a^(1/3)-b^(1/3)*x)/a^(5/3)+1/6*b^(2/3)*ln(a^(2/3)
+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)+1/3*b^(2/3)*arctan(1/3*(a^(1/3)+2*b
^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {325, 200, 31, 634, 617, 204, 628}

$$\frac{b^{2/3}\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{5/3}}-\frac{b^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{3a^{5/3}}+\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a - b*x^3)),x]
```

```
[Out] -1/(2*a*x^2) + (b^(2/3)*ArcTan[(a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/
(Sqrt[3]*a^(5/3)) - (b^(2/3)*Log[a^(1/3) - b^(1/3)*x])/(3*a^(5/3)) + (b^(2/
3)*Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(a-bx^3\right)}\,dx &= -\frac{1}{2ax^2} + \frac{b\int \frac{1}{a-bx^3}\,dx}{a} \\
&= -\frac{1}{2ax^2} + \frac{b\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{b}\,x}\,dx}{3a^{5/3}} + \frac{b\int \frac{2\sqrt[3]{a}+\sqrt[3]{b}\,x}{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{5/3}} \\
&= -\frac{1}{2ax^2} - \frac{b^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{3a^{5/3}} + \frac{b^{2/3}\int \frac{\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{6a^{5/3}} + \frac{b\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{2a^{4/3}} \\
&= -\frac{1}{2ax^2} - \frac{b^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{3a^{5/3}} + \frac{b^{2/3}\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{5/3}} - \frac{b^{2/3}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\right.}{a^{5/} \\
&= -\frac{1}{2ax^2} + \frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}} - \frac{b^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{3a^{5/3}} + \frac{b^{2/3}\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right.}{6a^{5/3}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 119, normalized size = 0.96

$$\frac{b^{2/3}x^2\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)-3a^{2/3}-2b^{2/3}x^2\log\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)+2\sqrt{3}\,b^{2/3}x^2\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{6a^{5/3}x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a - b*x^3)),x]
```

```
[Out] (-3*a^(2/3) + 2*Sqrt[3]*b^(2/3)*x^2*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3))/Sqrt
[3]] - 2*b^(2/3)*x^2*Log[a^(1/3) - b^(1/3)*x] + b^(2/3)*x^2*Log[a^(2/3) + a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(5/3)*x^2)
```

fricas [A] time = 0.92, size = 141, normalized size = 1.14

$$-\frac{2\sqrt{3}\,x^2\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}+\sqrt{3}\,b}{3\,b}\right)+x^2\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2-abx\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)-2\,x^2\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\text{lo}}{6\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*x^2*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(
2/3) + sqrt(3)*b)/b) + x^2*(-b^2/a^2)^(1/3)*log(b^2*x^2 - a*b*x*(-b^2/a^2)^
(1/3) + a^2*(-b^2/a^2)^(2/3)) - 2*x^2*(-b^2/a^2)^(1/3)*log(b*x + a*(-b^2/a^
2)^(1/3)) + 3)/(a*x^2)
```

giac [A] time = 0.16, size = 107, normalized size = 0.86

$$-\frac{b\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^2}+\frac{\sqrt{3}\left(ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^2}+\frac{\left(ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^2}-\frac{1}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*b*(a/b)^(1/3)*log(abs(x - (a/b)^(1/3)))/a^2 + 1/3*sqrt(3)*(a*b^2)^(1/3
)*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/a^2 + 1/6*(a*b^2)^(1/
3)*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/a^2 - 1/2/(a*x^2)
```

maple [A] time = 0.00, size = 100, normalized size = 0.81

$$\frac{\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a}-\frac{\ln\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a}+\frac{\ln\left(x^2+\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}a}-\frac{1}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-b*x^3+a),x)
```

```
[Out] -1/2/a/x^2-1/3/a/(a/b)^(2/3)*ln(x-(a/b)^(1/3))+1/6/a/(a/b)^(2/3)*ln(x^2+(a/
b)^(1/3)*x+(a/b)^(2/3))+1/3/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/
b)^(1/3)*x+1))
```

maxima [A] time = 2.95, size = 105, normalized size = 0.85

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\log\left(x^2+x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{1}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/(a*(a/b)^(2
/3)) + 1/6*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(2/3)) - 1/3*log
(x - (a/b)^(1/3))/(a*(a/b)^(2/3)) - 1/2/(a*x^2)
```

mupad [B] time = 1.11, size = 127, normalized size = 1.02

$$\frac{b^{2/3}\ln\left((-a)^{7/3}+a^2b^{1/3}x\right)}{3(-a)^{5/3}}-\frac{1}{2ax^2}-\frac{b^{2/3}\ln\left(3a^2b^3x-3(-a)^{7/3}b^{8/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3(-a)^{5/3}}+\frac{b^{2/3}\ln\left(3a^2b^3x+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a - b*x^3)),x)
```

```
[Out] (b^(2/3)*log((-a)^(7/3) + a^2*b^(1/3)*x))/(3*(-a)^(5/3)) - 1/(2*a*x^2) - (b
^(2/3)*log(3*a^2*b^3*x - 3*(-a)^(7/3)*b^(8/3)*((3^(1/2)*1i)/2 + 1/2))*((3^(
1/2)*1i)/2 + 1/2))/(3*(-a)^(5/3)) + (b^(2/3)*log(3*a^2*b^3*x + 9*(-a)^(7/3)
*b^(8/3)*((3^(1/2)*1i)/6 - 1/6))*((3^(1/2)*1i)/6 - 1/6))/(-a)^(5/3)
```

sympy [**A**] time = 0.59, size = 34, normalized size = 0.27

$$-\operatorname{RootSum}\left(27t^3a^5 - b^2, \left(t \mapsto t\log\left(-\frac{3ta^2}{b} + x\right)\right)\right) - \frac{1}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-b*x**3+a),x)
```

```
[Out] -RootSum(27*_t**3*a**5 - b**2, Lambda(_t, _t*log(-3*_t*a**2/b + x))) - 1/(2
*a*x**2)
```

3.363 $\int \frac{1}{1+a+bx^3}\,dx$

Optimal. Leaf size=125

$$-\frac{\log\left(-\sqrt[3]{a+1}\,\sqrt[3]{b}\,x+(a+1)^{2/3}+b^{2/3}x^2\right)}{6(a+1)^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a+1}+\sqrt[3]{b}\,x\right)}{3(a+1)^{2/3}\sqrt[3]{b}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+1}}}{\sqrt{3}}\right)}{\sqrt{3}\,(a+1)^{2/3}\sqrt[3]{b}}$$

```
[Out] 1/3*ln((1+a)^(1/3)+b^(1/3)*x)/(1+a)^(2/3)/b^(1/3)-1/6*ln((1+a)^(2/3)-(1+a)^
(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(1+a)^(2/3)/b^(1/3)-1/3*arctan(1/3*(1-2*b^(1/3
)*x/(1+a)^(1/3))*3^(1/2))/(1+a)^(2/3)/b^(1/3)*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(-\sqrt[3]{a+1}\,\sqrt[3]{b}\,x+(a+1)^{2/3}+b^{2/3}x^2\right)}{6(a+1)^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a+1}+\sqrt[3]{b}\,x\right)}{3(a+1)^{2/3}\sqrt[3]{b}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+1}}}{\sqrt{3}}\right)}{\sqrt{3}\,(a+1)^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a + b*x^3)^(-1),x]
```

```
[Out] -(ArcTan[(1 - (2*b^(1/3)*x)/(1 + a)^(1/3))/Sqrt[3]]/(Sqrt[3]*(1 + a)^(2/3)*
b^(1/3))) + Log[(1 + a)^(1/3) + b^(1/3)*x]/(3*(1 + a)^(2/3)*b^(1/3)) - Log[
(1 + a)^(2/3) - (1 + a)^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(6*(1 + a)^(2/3)*b^(
1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{1+a+bx^3}\,dx = \frac{\int \frac{1}{\sqrt[3]{1+a}+\sqrt[3]{b}\,x}\,dx}{3(1+a)^{2/3}} + \frac{\int \frac{2\sqrt[3]{1+a}-\sqrt[3]{b}\,x}{(1+a)^{2/3}-\sqrt[3]{1+a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3(1+a)^{2/3}}$$

$$= \frac{\log\left(\sqrt[3]{1+a}+\sqrt[3]{b}\,x\right)}{3(1+a)^{2/3}\sqrt[3]{b}} + \frac{\int \frac{1}{(1+a)^{2/3}-\sqrt[3]{1+a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{2\sqrt[3]{1+a}} - \frac{\int \frac{-\sqrt[3]{1+a}\,\sqrt[3]{b}+2b^{2/3}x}{(1+a)^{2/3}-\sqrt[3]{1+a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{6(1+a)^{2/3}\sqrt[3]{b}}$$

$$= \frac{\log\left(\sqrt[3]{1+a}+\sqrt[3]{b}\,x\right)}{3(1+a)^{2/3}\sqrt[3]{b}} - \frac{\log\left((1+a)^{2/3}-\sqrt[3]{1+a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6(1+a)^{2/3}\sqrt[3]{b}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\right.}{(1+a)^{2/3}\sqrt[3]{b}}$$

$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{1+a}}}{\sqrt{3}}\right)}{\sqrt{3}\,(1+a)^{2/3}\sqrt[3]{b}} + \frac{\log\left(\sqrt[3]{1+a}+\sqrt[3]{b}\,x\right)}{3(1+a)^{2/3}\sqrt[3]{b}} - \frac{\log\left((1+a)^{2/3}-\sqrt[3]{1+a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6(1+a)^{2/3}\sqrt[3]{b}}$$

Mathematica [A] time = 0.05, size = 101, normalized size = 0.81

$$\frac{-\log\left(-\sqrt[3]{a+1}\,\sqrt[3]{b}\,x+(a+1)^{2/3}+b^{2/3}x^2\right)+2\log\left(\sqrt[3]{a+1}+\sqrt[3]{b}\,x\right)+2\sqrt{3}\,\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+1}}-1}{\sqrt{3}}\right)}{6(a+1)^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a + b*x^3)^(-1),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-1 + (2*b^(1/3)*x)/(1 + a)^(1/3))/Sqrt[3]] + 2*Log[(1 +
a)^(1/3) + b^(1/3)*x] - Log[(1 + a)^(2/3) - (1 + a)^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/(6*(1 + a)^(2/3)*b^(1/3))
```

fricas [B] time = 0.81, size = 446, normalized size = 3.57

$$\left[\frac{3\sqrt{\frac{1}{3}}\,(a+1)b\sqrt{-\frac{\left(\left(a^2+2a+1\right)b\right)^{\frac{1}{3}}}{b}}\,\log\left(\frac{2(a+1)bx^3-3\left(\left(a^2+2a+1\right)b\right)^{\frac{1}{3}}(a+1)x-a^2+3\sqrt{\frac{1}{3}}\left(2(a+1)bx^2+\left(\left(a^2+2a+1\right)b\right)^{\frac{2}{3}}x-\left(\left(a^2+2a+1\right)b\right)^{\frac{1}{3}}\right.\right.}{bx^3+a+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a+1),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*(a + 1)*b*sqrt(-((a^2 + 2*a + 1)*b)^(1/3)/b)*log((2*(a +
1)*b*x^3 - 3*((a^2 + 2*a + 1)*b)^(1/3)*(a + 1)*x - a^2 + 3*sqrt(1/3)*(2*(a
+ 1)*b*x^2 + ((a^2 + 2*a + 1)*b)^(2/3)*x - ((a^2 + 2*a + 1)*b)^(1/3)*(a + 1
))*sqrt(-((a^2 + 2*a + 1)*b)^(1/3)/b) - 2*a - 1)/(b*x^3 + a + 1)) - ((a^2 +
 2*a + 1)*b)^(2/3)*log((a + 1)*b*x^2 - ((a^2 + 2*a + 1)*b)^(2/3)*x + ((a^2
+ 2*a + 1)*b)^(1/3)*(a + 1)) + 2*((a^2 + 2*a + 1)*b)^(2/3)*log((a + 1)*b*x
+ ((a^2 + 2*a + 1)*b)^(2/3)))/((a^2 + 2*a + 1)*b), 1/6*(6*sqrt(1/3)*(a + 1)
*b*sqrt(((a^2 + 2*a + 1)*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*((a^2 + 2*a + 1)*b
)^(2/3)*x - ((a^2 + 2*a + 1)*b)^(1/3)*(a + 1))*sqrt(((a^2 + 2*a + 1)*b)^(1/
3)/b)/(a^2 + 2*a + 1)) - ((a^2 + 2*a + 1)*b)^(2/3)*log((a + 1)*b*x^2 - ((a^
2 + 2*a + 1)*b)^(2/3)*x + ((a^2 + 2*a + 1)*b)^(1/3)*(a + 1)) + 2*((a^2 + 2*
a + 1)*b)^(2/3)*log((a + 1)*b*x + ((a^2 + 2*a + 1)*b)^(2/3)))/((a^2 + 2*a +
 1)*b)]
```

giac [A] time = 0.16, size = 143, normalized size = 1.14

$$\frac{\left(-ab^2-b^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a+1}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a+1}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,ab+\sqrt{3}\,b}+\frac{\left(-ab^2-b^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a+1}{b}\right)^{\frac{1}{3}}+\left(-\frac{a+1}{b}\right)^{\frac{2}{3}}\right)}{6\,(ab+b)}-\frac{\left(-\frac{a+1}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a+1}{b}\right.\right.\right.}{3\,(a+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a+1),x, algorithm="giac")
```

```
[Out] (-a*b^2 - b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-(a + 1)/b)^(1/3))/(-(a + 1
)/b)^(1/3))/(sqrt(3)*a*b + sqrt(3)*b) + 1/6*(-a*b^2 - b^2)^(1/3)*log(x^2 +
x*(-(a + 1)/b)^(1/3) + (-(a + 1)/b)^(2/3))/(a*b + b) - 1/3*(-(a + 1)/b)^(1/
3)*log(abs(x - (-(a + 1)/b)^(1/3)))/(a + 1)
```

maple [A] time = 0.00, size = 105, normalized size = 0.84

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a+1}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a+1}{b}\right)^{\frac{2}{3}}b}+\frac{\ln\left(x+\left(\frac{a+1}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a+1}{b}\right)^{\frac{2}{3}}b}-\frac{\ln\left(x^2-\left(\frac{a+1}{b}\right)^{\frac{1}{3}}x+\left(\frac{a+1}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a+1}{b}\right)^{\frac{2}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a+1),x)
```

```
[Out] 1/3/b/((1+a)/b)^(2/3)*ln(x+((1+a)/b)^(1/3))-1/6/b/((1+a)/b)^(2/3)*ln(x^2-((
1+a)/b)^(1/3)*x+((1+a)/b)^(2/3))+1/3/b/((1+a)/b)^(2/3)*3^(1/2)*arctan(1/3*3
^(1/2)*(2/((1+a)/b)^(1/3)*x-1))
```

maxima [A] time = 2.92, size = 114, normalized size = 0.91

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a+1}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a+1}{b}\right)^{\frac{1}{3}}}\right)}{3b\left(\frac{a+1}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^2-x\left(\frac{a+1}{b}\right)^{\frac{1}{3}}+\left(\frac{a+1}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a+1}{b}\right)^{\frac{2}{3}}}+\frac{\log\left(x+\left(\frac{a+1}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a+1}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a+1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - ((a + 1)/b)^(1/3))/((a + 1)/b)^(1/3))
/(b*((a + 1)/b)^(2/3)) - 1/6*log(x^2 - x*((a + 1)/b)^(1/3) + ((a + 1)/b)^(2
/3))/(b*((a + 1)/b)^(2/3)) + 1/3*log(x + ((a + 1)/b)^(1/3))/(b*((a + 1)/b)^
(2/3))
```

mupad **[B]** time = 1.21, size = 137, normalized size = 1.10

$$\frac{\ln\left(a+b^{1/3}\,x\,(a+1)^{2/3}+1\right)}{3\,b^{1/3}\,(a+1)^{2/3}}+\frac{\ln\left(3\,b^2\,x+\frac{\left(9\,a\,b^2+9\,b^2\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,b^{1/3}\,(a+1)^{2/3}}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,b^{1/3}\,(a+1)^{2/3}}-\frac{\ln\left(3\,b^2\,x-\frac{\left(9\,a\,b^2+9\,b^2\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,b^{1/3}\,(a+1)^{2/3}}\right)}{6\,b^{1/3}\,(a+1)^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^3 + 1),x)
```

```
[Out] log(a + b^(1/3)*x*(a + 1)^(2/3) + 1)/(3*b^(1/3)*(a + 1)^(2/3)) + (log(3*b^2
*x + ((9*a*b^2 + 9*b^2)*(3^(1/2)*1i - 1))/(6*b^(1/3)*(a + 1)^(2/3)))*(3^(1/
2)*1i - 1))/(6*b^(1/3)*(a + 1)^(2/3)) - (log(3*b^2*x - ((9*a*b^2 + 9*b^2)*(
3^(1/2)*1i + 1))/(6*b^(1/3)*(a + 1)^(2/3)))*(3^(1/2)*1i + 1))/(6*b^(1/3)*(a
 + 1)^(2/3))
```

sympy **[A]** time = 0.32, size = 32, normalized size = 0.26

$$\operatorname{RootSum}\left(t^3\left(27a^2b+54ab+27b\right)-1,\left(t\mapsto t\log\left(3ta+3t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a+1),x)
```

```
[Out] RootSum(_t**3*(27*a**2*b + 54*a*b + 27*b) - 1, Lambda(_t, _t*log(3*_t*a + 3
*_t + x)))
```

3.364 $\int \frac{1}{1+a-bx^3}\,dx$

Optimal. Leaf size=124

$$\frac{\log\left(\sqrt[3]{a+1}\,\sqrt[3]{b}\,x+(a+1)^{2/3}+b^{2/3}x^2\right)}{6(a+1)^{2/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+1}-\sqrt[3]{b}\,x\right)}{3(a+1)^{2/3}\sqrt[3]{b}}+\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+1}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,(a+1)^{2/3}\sqrt[3]{b}}$$

```
[Out] -1/3*ln((1+a)^(1/3)-b^(1/3)*x)/(1+a)^(2/3)/b^(1/3)+1/6*ln((1+a)^(2/3)+(1+a)
^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(1+a)^(2/3)/b^(1/3)+1/3*arctan(1/3*(1+2*b^(1/
3)*x/(1+a)^(1/3))*3^(1/2))/(1+a)^(2/3)/b^(1/3)*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {200, 31, 634, 617, 204, 628}

$$\frac{\log\left(\sqrt[3]{a+1}\,\sqrt[3]{b}\,x+(a+1)^{2/3}+b^{2/3}x^2\right)}{6(a+1)^{2/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+1}-\sqrt[3]{b}\,x\right)}{3(a+1)^{2/3}\sqrt[3]{b}}+\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+1}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,(a+1)^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a - b*x^3)^(-1),x]
```

```
[Out] ArcTan[(1 + (2*b^(1/3)*x)/(1 + a)^(1/3))/Sqrt[3]]/(Sqrt[3]*(1 + a)^(2/3)*b^
(1/3)) - Log[(1 + a)^(1/3) - b^(1/3)*x]/(3*(1 + a)^(2/3)*b^(1/3)) + Log[(1
+ a)^(2/3) + (1 + a)^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(6*(1 + a)^(2/3)*b^(1/3
))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{1+a-bx^3}\,dx = \frac{\int \frac{1}{\sqrt[3]{1+a}-\sqrt[3]{b}\,x}\,dx}{3(1+a)^{2/3}} + \frac{\int \frac{2\sqrt[3]{1+a}+\sqrt[3]{b}\,x}{(1+a)^{2/3}+\sqrt[3]{1+a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3(1+a)^{2/3}}$$

$$= -\frac{\log\left(\sqrt[3]{1+a}-\sqrt[3]{b}\,x\right)}{3(1+a)^{2/3}\sqrt[3]{b}} + \frac{\int \frac{1}{(1+a)^{2/3}+\sqrt[3]{1+a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{2\sqrt[3]{1+a}} + \frac{\int \frac{\sqrt[3]{1+a}\,\sqrt[3]{b}+2b^{2/3}x}{(1+a)^{2/3}+\sqrt[3]{1+a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{6(1+a)^{2/3}\sqrt[3]{b}}$$

$$= -\frac{\log\left(\sqrt[3]{1+a}-\sqrt[3]{b}\,x\right)}{3(1+a)^{2/3}\sqrt[3]{b}} + \frac{\log\left((1+a)^{2/3}+\sqrt[3]{1+a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6(1+a)^{2/3}\sqrt[3]{b}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1\right.}{(1+a)^{2/3}\sqrt[3]{b}}$$

$$= \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{1+a}}}{\sqrt{3}}\right)}{\sqrt{3}\,(1+a)^{2/3}\sqrt[3]{b}} - \frac{\log\left(\sqrt[3]{1+a}-\sqrt[3]{b}\,x\right)}{3(1+a)^{2/3}\sqrt[3]{b}} + \frac{\log\left((1+a)^{2/3}+\sqrt[3]{1+a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6(1+a)^{2/3}\sqrt[3]{b}}$$

Mathematica [A] time = 0.09, size = 124, normalized size = 1.00

$$\frac{(-1)^{2/3}\left(\log\left(-\sqrt[3]{-1}\,\sqrt[3]{a+1}\,\sqrt[3]{b}\,x+(a+1)^{2/3}+(-1)^{2/3}b^{2/3}x^2\right)-2\log\left(\sqrt[3]{a+1}+\sqrt[3]{-1}\,\sqrt[3]{b}\,x\right)-2\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{-1}}{\sqrt[3]{a}}\right.\right.}{6(a+1)^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a - b*x^3)^(-1),x]
```

```
[Out] ((-1)^(2/3)*(-2*Sqrt[3]*ArcTan[(-1 + (2*(-1)^(1/3)*b^(1/3)*x)/(1 + a)^(1/3)
)/Sqrt[3]] - 2*Log[(1 + a)^(1/3) + (-1)^(1/3)*b^(1/3)*x] + Log[(1 + a)^(2/3
) - (-1)^(1/3)*(1 + a)^(1/3)*b^(1/3)*x + (-1)^(2/3)*b^(2/3)*x^2]))/(6*(1 +
a)^(2/3)*b^(1/3))
```

fricas [B] time = 0.70, size = 467, normalized size = 3.77

$$\left[\frac{3\sqrt{\frac{1}{3}}(a+1)b\sqrt{\frac{\left(-\left(a^2+2a+1\right)b\right)^{\frac{1}{3}}}{b}}\,\log\left(\frac{2(a+1)bx^3+3\left(-\left(a^2+2a+1\right)b\right)^{\frac{1}{3}}(a+1)x+a^2+3\sqrt{\frac{1}{3}}\left(2(a+1)bx^2-\left(-\left(a^2+2a+1\right)b\right)^{\frac{2}{3}}x+\left(-\left(a^2+2a+1\right)\right.\right.\right.}{bx^3-a-1}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^3+a+1),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*(a + 1)*b*sqrt((-(a^2 + 2*a + 1)*b)^(1/3)/b)*log((2*(a +
1)*b*x^3 + 3*(-(a^2 + 2*a + 1)*b)^(1/3)*(a + 1)*x + a^2 + 3*sqrt(1/3)*(2*(a
 + 1)*b*x^2 - (-(a^2 + 2*a + 1)*b)^(2/3)*x + (-(a^2 + 2*a + 1)*b)^(1/3)*(a
+ 1))*sqrt((-(a^2 + 2*a + 1)*b)^(1/3)/b) + 2*a + 1)/(b*x^3 - a - 1)) + (-(a
^2 + 2*a + 1)*b)^(2/3)*log((a + 1)*b*x^2 + (-(a^2 + 2*a + 1)*b)^(2/3)*x - (
-(a^2 + 2*a + 1)*b)^(1/3)*(a + 1)) - 2*(-(a^2 + 2*a + 1)*b)^(2/3)*log((a +
1)*b*x - (-(a^2 + 2*a + 1)*b)^(2/3)))/((a^2 + 2*a + 1)*b), 1/6*(6*sqrt(1/3)
*(a + 1)*b*sqrt(-(-(a^2 + 2*a + 1)*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-(a^2 +
 2*a + 1)*b)^(2/3)*x - (-(a^2 + 2*a + 1)*b)^(1/3)*(a + 1))*sqrt(-(-(a^2 + 2
*a + 1)*b)^(1/3)/b)/(a^2 + 2*a + 1)) + (-(a^2 + 2*a + 1)*b)^(2/3)*log((a +
1)*b*x^2 + (-(a^2 + 2*a + 1)*b)^(2/3)*x - (-(a^2 + 2*a + 1)*b)^(1/3)*(a + 1
)) - 2*(-(a^2 + 2*a + 1)*b)^(2/3)*log((a + 1)*b*x - (-(a^2 + 2*a + 1)*b)^(2
/3)))/((a^2 + 2*a + 1)*b)]
```

giac [A] time = 0.16, size = 131, normalized size = 1.06

$$\frac{\left(ab^2+b^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a+1}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a+1}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,ab+\sqrt{3}\,b}+\frac{\left(ab^2+b^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(\frac{a+1}{b}\right)^{\frac{1}{3}}+\left(\frac{a+1}{b}\right)^{\frac{2}{3}}\right)}{6\,(ab+b)}-\frac{\left(\frac{a+1}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{a+1}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,(a+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^3+a+1),x, algorithm="giac")
```

```
[Out] (a*b^2 + b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + ((a + 1)/b)^(1/3))/((a + 1)/b
)^(1/3))/(sqrt(3)*a*b + sqrt(3)*b) + 1/6*(a*b^2 + b^2)^(1/3)*log(x^2 + x*((
a + 1)/b)^(1/3) + ((a + 1)/b)^(2/3))/(a*b + b) - 1/3*((a + 1)/b)^(1/3)*log(
abs(x - ((a + 1)/b)^(1/3)))/(a + 1)
```

maple [A] time = 0.00, size = 106, normalized size = 0.85

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a+1}{b}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{3\left(\frac{a+1}{b}\right)^{\frac{2}{3}}b}-\frac{\ln\left(x-\left(\frac{a+1}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a+1}{b}\right)^{\frac{2}{3}}b}+\frac{\ln\left(x^2+\left(\frac{a+1}{b}\right)^{\frac{1}{3}}x+\left(\frac{a+1}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a+1}{b}\right)^{\frac{2}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^3+a+1),x)
```

```
[Out] -1/3/b/((a+1)/b)^(2/3)*ln(x-((a+1)/b)^(1/3))+1/6/b/((a+1)/b)^(2/3)*ln(x^2+(
(a+1)/b)^(1/3)*x+((a+1)/b)^(2/3))+1/3/b/((a+1)/b)^(2/3)*3^(1/2)*arctan(1/3*
3^(1/2)*(2/((a+1)/b)^(1/3)*x+1))
```

maxima [A] time = 3.00, size = 113, normalized size = 0.91

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a+1}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a+1}{b}\right)^{\frac{1}{3}}}\right)}{3\,b\left(\frac{a+1}{b}\right)^{\frac{2}{3}}}+\frac{\log\left(x^2+x\left(\frac{a+1}{b}\right)^{\frac{1}{3}}+\left(\frac{a+1}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a+1}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x-\left(\frac{a+1}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a+1}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^3+a+1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + ((a + 1)/b)^(1/3))/((a + 1)/b)^(1/3))
/(b*((a + 1)/b)^(2/3)) + 1/6*log(x^2 + x*((a + 1)/b)^(1/3) + ((a + 1)/b)^(2
/3))/(b*((a + 1)/b)^(2/3)) - 1/3*log(x - ((a + 1)/b)^(1/3))/(b*((a + 1)/b)^
(2/3))
```

mupad **[B]** time = 1.32, size = 165, normalized size = 1.33

$$\frac{\ln\left(3\,b^2\,x+\frac{9\,a\,b^2+9\,b^2}{3\,(-b)^{1/3}\,(a+1)^{2/3}}\right)}{3\,(-b)^{1/3}\,(a+1)^{2/3}}+\frac{\ln\left(3\,b^2\,x+\frac{\left(9\,a\,b^2+9\,b^2\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,(-b)^{1/3}\,(a+1)^{2/3}}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,(-b)^{1/3}\,(a+1)^{2/3}}-\frac{\ln\left(3\,b^2\,x-\frac{\left(9\,a\,b^2+9\,b^2\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,(-b)^{1/3}\,(a+1)^{2/3}}\right)}{6\,(-b)^{1/3}\,(a+1)^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*x^3 + 1),x)
```

```
[Out] log(3*b^2*x + (9*a*b^2 + 9*b^2)/(3*(-b)^(1/3)*(a + 1)^(2/3)))/(3*(-b)^(1/3)
*(a + 1)^(2/3)) + (log(3*b^2*x + ((9*a*b^2 + 9*b^2)*(3^(1/2)*1i - 1))/(6*(-
b)^(1/3)*(a + 1)^(2/3)))*(3^(1/2)*1i - 1))/(6*(-b)^(1/3)*(a + 1)^(2/3)) - (
log(3*b^2*x - ((9*a*b^2 + 9*b^2)*(3^(1/2)*1i + 1))/(6*(-b)^(1/3)*(a + 1)^(2
/3)))*(3^(1/2)*1i + 1))/(6*(-b)^(1/3)*(a + 1)^(2/3))
```

sympy **[A]** time = 0.42, size = 34, normalized size = 0.27

$$-\operatorname{RootSum}\left(t^3\left(27a^2b+54ab+27b\right)-1,\left(t\mapsto t\log\left(-3ta-3t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**3+a+1),x)
```

```
[Out] -RootSum(_t**3*(27*a**2*b + 54*a*b + 27*b) - 1, Lambda(_t, _t*log(-3*_t*a -
 3*_t + x)))
```

3.365 $\int \frac{1}{-1+a+bx^3}\,dx$

Optimal. Leaf size=139

$$-\frac{\log\left(\sqrt[3]{1-a}\,\sqrt[3]{b}\,x+(1-a)^{2/3}+b^{2/3}x^2\right)}{6(1-a)^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{1-a}-\sqrt[3]{b}\,x\right)}{3(1-a)^{2/3}\sqrt[3]{b}}-\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{1-a}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,(1-a)^{2/3}\sqrt[3]{b}}$$

```
[Out] 1/3*ln((1-a)^(1/3)-b^(1/3)*x)/(1-a)^(2/3)/b^(1/3)-1/6*ln((1-a)^(2/3)+(1-a)^
(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(1-a)^(2/3)/b^(1/3)-1/3*arctan(1/3*(1+2*b^(1/3
)*x/(1-a)^(1/3))*3^(1/2))/(1-a)^(2/3)/b^(1/3)*3^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(\sqrt[3]{1-a}\,\sqrt[3]{b}\,x+(1-a)^{2/3}+b^{2/3}x^2\right)}{6(1-a)^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{1-a}-\sqrt[3]{b}\,x\right)}{3(1-a)^{2/3}\sqrt[3]{b}}-\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{1-a}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,(1-a)^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + a + b*x^3)^(-1),x]
```

```
[Out] -(ArcTan[(1 + (2*b^(1/3)*x)/(1 - a)^(1/3))/Sqrt[3]]/(Sqrt[3]*(1 - a)^(2/3)*
b^(1/3))) + Log[(1 - a)^(1/3) - b^(1/3)*x]/(3*(1 - a)^(2/3)*b^(1/3)) - Log[
(1 - a)^(2/3) + (1 - a)^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(6*(1 - a)^(2/3)*b^(
1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{-1+a+bx^3}\,dx &= \frac{\int \frac{1}{-\sqrt[3]{1-a}+\sqrt[3]{b}\,x}\,dx}{3(1-a)^{2/3}} + \frac{\int \frac{-2\sqrt[3]{1-a}-\sqrt[3]{b}\,x}{(1-a)^{2/3}+\sqrt[3]{1-a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3(1-a)^{2/3}} \\
&= \frac{\log\left(\sqrt[3]{1-a}-\sqrt[3]{b}\,x\right)}{3(1-a)^{2/3}\sqrt[3]{b}} - \frac{\int \frac{1}{(1-a)^{2/3}+\sqrt[3]{1-a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{2\sqrt[3]{1-a}} - \frac{\int \frac{\sqrt[3]{1-a}\,\sqrt[3]{b}+2b^{2/3}x}{(1-a)^{2/3}+\sqrt[3]{1-a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{6(1-a)^{2/3}\sqrt[3]{b}} \\
&= \frac{\log\left(\sqrt[3]{1-a}-\sqrt[3]{b}\,x\right)}{3(1-a)^{2/3}\sqrt[3]{b}} - \frac{\log\left((1-a)^{2/3}+\sqrt[3]{1-a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6(1-a)^{2/3}\sqrt[3]{b}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1}{(1-a)^{2/3}\sqrt[3]{b}} \\
&= -\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{1-a}}}{\sqrt{3}}\right)}{\sqrt{3}\,(1-a)^{2/3}\sqrt[3]{b}} + \frac{\log\left(\sqrt[3]{1-a}-\sqrt[3]{b}\,x\right)}{3(1-a)^{2/3}\sqrt[3]{b}} - \frac{\log\left((1-a)^{2/3}+\sqrt[3]{1-a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6(1-a)^{2/3}\sqrt[3]{b}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 101, normalized size = 0.73

$$\frac{-\log\left(-\sqrt[3]{a-1}\,\sqrt[3]{b}\,x+(a-1)^{2/3}+b^{2/3}x^2\right)+2\log\left(\sqrt[3]{a-1}+\sqrt[3]{b}\,x\right)+2\sqrt{3}\,\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a-1}}-1}{\sqrt{3}}\right)}{6(a-1)^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + a + b*x^3)^(-1),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-1 + (2*b^(1/3)*x)/(-1 + a)^(1/3))/Sqrt[3]] + 2*Log[(-1
+ a)^(1/3) + b^(1/3)*x] - Log[(-1 + a)^(2/3) - (-1 + a)^(1/3)*b^(1/3)*x + b
^(2/3)*x^2])/(6*(-1 + a)^(2/3)*b^(1/3))
```

fricas [A] time = 0.94, size = 446, normalized size = 3.21

$$\left[\frac{3\sqrt{\frac{1}{3}}\,(a-1)b\sqrt{-\frac{\left(\left(a^2-2a+1\right)b\right)^{\frac{1}{3}}}{b}}\,\log\left(\frac{2(a-1)bx^3-3\left(\left(a^2-2a+1\right)b\right)^{\frac{1}{3}}(a-1)x-a^2+3\sqrt{\frac{1}{3}}\left(2(a-1)bx^2+\left(\left(a^2-2a+1\right)b\right)^{\frac{2}{3}}x-\left(\left(a^2-2a+1\right)b\right)^{\frac{1}{3}}(a\right.}{bx^3+a-1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a-1),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*(a - 1)*b*sqrt(-((a^2 - 2*a + 1)*b)^(1/3)/b)*log((2*(a -
1)*b*x^3 - 3*((a^2 - 2*a + 1)*b)^(1/3)*(a - 1)*x - a^2 + 3*sqrt(1/3)*(2*(a
- 1)*b*x^2 + ((a^2 - 2*a + 1)*b)^(2/3)*x - ((a^2 - 2*a + 1)*b)^(1/3)*(a - 1
))*sqrt(-((a^2 - 2*a + 1)*b)^(1/3)/b) + 2*a - 1)/(b*x^3 + a - 1)) - ((a^2 -
 2*a + 1)*b)^(2/3)*log((a - 1)*b*x^2 - ((a^2 - 2*a + 1)*b)^(2/3)*x + ((a^2
- 2*a + 1)*b)^(1/3)*(a - 1)) + 2*((a^2 - 2*a + 1)*b)^(2/3)*log((a - 1)*b*x
+ ((a^2 - 2*a + 1)*b)^(2/3)))/((a^2 - 2*a + 1)*b), 1/6*(6*sqrt(1/3)*(a - 1)
*b*sqrt(((a^2 - 2*a + 1)*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*((a^2 - 2*a + 1)*b
)^(2/3)*x - ((a^2 - 2*a + 1)*b)^(1/3)*(a - 1))*sqrt(((a^2 - 2*a + 1)*b)^(1/
3)/b)/(a^2 - 2*a + 1)) - ((a^2 - 2*a + 1)*b)^(2/3)*log((a - 1)*b*x^2 - ((a^
2 - 2*a + 1)*b)^(2/3)*x + ((a^2 - 2*a + 1)*b)^(1/3)*(a - 1)) + 2*((a^2 - 2*
a + 1)*b)^(2/3)*log((a - 1)*b*x + ((a^2 - 2*a + 1)*b)^(2/3)))/((a^2 - 2*a +
 1)*b)]
```

giac [A] time = 0.16, size = 142, normalized size = 1.02

$$\frac{\left(-ab^2+b^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a-1}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a-1}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,ab-\sqrt{3}\,b}+\frac{\left(-ab^2+b^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a-1}{b}\right)^{\frac{1}{3}}+\left(-\frac{a-1}{b}\right)^{\frac{2}{3}}\right)}{6\,(ab-b)}-\frac{\left(-\frac{a-1}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a-1}{b}\right.\right.\right.}{3\,(a-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a-1),x, algorithm="giac")
```

```
[Out] (-a*b^2 + b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-(a - 1)/b)^(1/3))/(-(a - 1
)/b)^(1/3))/(sqrt(3)*a*b - sqrt(3)*b) + 1/6*(-a*b^2 + b^2)^(1/3)*log(x^2 +
x*(-(a - 1)/b)^(1/3) + (-(a - 1)/b)^(2/3))/(a*b - b) - 1/3*(-(a - 1)/b)^(1/
3)*log(abs(x - (-(a - 1)/b)^(1/3)))/(a - 1)
```

maple [A] time = 0.00, size = 105, normalized size = 0.76

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a-1}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a-1}{b}\right)^{\frac{2}{3}}b}+\frac{\ln\left(x+\left(\frac{a-1}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a-1}{b}\right)^{\frac{2}{3}}b}-\frac{\ln\left(x^2-\left(\frac{a-1}{b}\right)^{\frac{1}{3}}x+\left(\frac{a-1}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a-1}{b}\right)^{\frac{2}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a-1),x)
```

```
[Out] 1/3/b/((-1+a)/b)^(2/3)*ln(x+((-1+a)/b)^(1/3))-1/6/b/((-1+a)/b)^(2/3)*ln(x^2
-((-1+a)/b)^(1/3)*x+((-1+a)/b)^(2/3))+1/3/b/((-1+a)/b)^(2/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2/((-1+a)/b)^(1/3)*x-1))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a-1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a-1.0>0)', see `assume?` for more d
etails)Is a-1.0 positive or negative?
```

mupad [B] time = 1.23, size = 137, normalized size = 0.99

$$\frac{\ln\left(a+b^{1/3}\,x\,(a-1)^{2/3}-1\right)}{3\,b^{1/3}\,(a-1)^{2/3}}+\frac{\ln\left(3\,b^2\,x+\frac{\left(9\,a\,b^2-9\,b^2\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,b^{1/3}\,(a-1)^{2/3}}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,b^{1/3}\,(a-1)^{2/3}}-\frac{\ln\left(3\,b^2\,x-\frac{\left(9\,a\,b^2-9\,b^2\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,b^{1/3}\,(a-1)^{2/3}}\right)}{6\,b^{1/3}\,(a-1)^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^3 - 1),x)
```

```
[Out] log(a + b^(1/3)*x*(a - 1)^(2/3) - 1)/(3*b^(1/3)*(a - 1)^(2/3)) + (log(3*b^2
*x + ((9*a*b^2 - 9*b^2)*(3^(1/2)*1i - 1))/(6*b^(1/3)*(a - 1)^(2/3)))*(3^(1/
2)*1i - 1))/(6*b^(1/3)*(a - 1)^(2/3)) - (log(3*b^2*x - ((9*a*b^2 - 9*b^2)*(
3^(1/2)*1i + 1))/(6*b^(1/3)*(a - 1)^(2/3)))*(3^(1/2)*1i + 1))/(6*b^(1/3)*(a
 - 1)^(2/3))
```

sympy [A] time = 0.37, size = 32, normalized size = 0.23

$$\mathrm{RootSum}\left(t^3\left(27a^2b-54ab+27b\right)-1,\left(t\mapsto t\log\left(3ta-3t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a-1),x)
```

```
[Out] RootSum(_t**3*(27*a**2*b - 54*a*b + 27*b) - 1, Lambda(_t, _t*log(3*_t*a - 3
*_t + x)))
```

3.366 $\int \frac{1}{-1+a-bx^3}\,dx$

Optimal. Leaf size=138

$$\frac{\log\left(-\sqrt[3]{1-a}\,\sqrt[3]{b}\,x+(1-a)^{2/3}+b^{2/3}x^2\right)}{6(1-a)^{2/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{1-a}+\sqrt[3]{b}\,x\right)}{3(1-a)^{2/3}\sqrt[3]{b}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{1-a}}}{\sqrt{3}}\right)}{\sqrt{3}\,(1-a)^{2/3}\sqrt[3]{b}}$$

```
[Out] -1/3*ln((1-a)^(1/3)+b^(1/3)*x)/(1-a)^(2/3)/b^(1/3)+1/6*ln((1-a)^(2/3)-(1-a)
^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(1-a)^(2/3)/b^(1/3)+1/3*arctan(1/3*(1-2*b^(1/
3)*x/(1-a)^(1/3))*3^(1/2))/(1-a)^(2/3)/b^(1/3)*3^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {200, 31, 634, 617, 204, 628}

$$\frac{\log\left(-\sqrt[3]{1-a}\,\sqrt[3]{b}\,x+(1-a)^{2/3}+b^{2/3}x^2\right)}{6(1-a)^{2/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{1-a}+\sqrt[3]{b}\,x\right)}{3(1-a)^{2/3}\sqrt[3]{b}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{1-a}}}{\sqrt{3}}\right)}{\sqrt{3}\,(1-a)^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + a - b*x^3)^(-1),x]
```

```
[Out] ArcTan[(1 - (2*b^(1/3)*x)/(1 - a)^(1/3))/Sqrt[3]]/(Sqrt[3]*(1 - a)^(2/3)*b^
(1/3)) - Log[(1 - a)^(1/3) + b^(1/3)*x]/(3*(1 - a)^(2/3)*b^(1/3)) + Log[(1
 - a)^(2/3) - (1 - a)^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(6*(1 - a)^(2/3)*b^(1/3
))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{-1+a-bx^3}\,dx = \frac{\int \frac{1}{-\sqrt[3]{1-a}-\sqrt[3]{b}\,x}\,dx}{3(1-a)^{2/3}} + \frac{\int \frac{-2\sqrt[3]{1-a}+\sqrt[3]{b}\,x}{(1-a)^{2/3}-\sqrt[3]{1-a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3(1-a)^{2/3}}$$

$$= -\frac{\log\left(\sqrt[3]{1-a}+\sqrt[3]{b}\,x\right)}{3(1-a)^{2/3}\sqrt[3]{b}} - \frac{\int \frac{1}{(1-a)^{2/3}-\sqrt[3]{1-a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{2\sqrt[3]{1-a}} + \frac{\int \frac{-\sqrt[3]{1-a}\,\sqrt[3]{b}+2b^{2/3}x}{(1-a)^{2/3}-\sqrt[3]{1-a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{6(1-a)^{2/3}\sqrt[3]{b}}$$

$$= -\frac{\log\left(\sqrt[3]{1-a}+\sqrt[3]{b}\,x\right)}{3(1-a)^{2/3}\sqrt[3]{b}} + \frac{\log\left((1-a)^{2/3}-\sqrt[3]{1-a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6(1-a)^{2/3}\sqrt[3]{b}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x,\right.}{(1-a)^{2/3}\sqrt[3]{}}$$

$$= \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{1-a}}}{\sqrt{3}}\right)}{\sqrt{3}\,(1-a)^{2/3}\sqrt[3]{b}} - \frac{\log\left(\sqrt[3]{1-a}+\sqrt[3]{b}\,x\right)}{3(1-a)^{2/3}\sqrt[3]{b}} + \frac{\log\left((1-a)^{2/3}-\sqrt[3]{1-a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6(1-a)^{2/3}\sqrt[3]{b}}$$

Mathematica [A] time = 0.09, size = 124, normalized size = 0.90

$$\frac{(-1)^{2/3}\left(\log\left(-\sqrt[3]{-1}\,\sqrt[3]{a-1}\,\sqrt[3]{b}\,x+(a-1)^{2/3}+(-1)^{2/3}b^{2/3}x^2\right)-2\log\left(\sqrt[3]{a-1}+\sqrt[3]{-1}\,\sqrt[3]{b}\,x\right)-2\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{-1}}{\sqrt[3]{a-}}\right.\right.}{6(a-1)^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + a - b*x^3)^(-1),x]
```

```
[Out] ((-1)^(2/3)*(-2*Sqrt[3]*ArcTan[(-1 + (2*(-1)^(1/3)*b^(1/3)*x)/(-1 + a)^(1/3
))/Sqrt[3]] - 2*Log[(-1 + a)^(1/3) + (-1)^(1/3)*b^(1/3)*x] + Log[(-1 + a)^(
2/3) - (-1)^(1/3)*(-1 + a)^(1/3)*b^(1/3)*x + (-1)^(2/3)*b^(2/3)*x^2]))/(6*(
-1 + a)^(2/3)*b^(1/3))
```

fricas [B] time = 0.92, size = 467, normalized size = 3.38

$$\left[\frac{3\sqrt{\frac{1}{3}}(a-1)b\sqrt{\frac{\left(-\left(a^2-2a+1\right)b\right)^{\frac{1}{3}}}{b}}\,\log\left(\frac{2(a-1)bx^3+3\left(-\left(a^2-2a+1\right)b\right)^{\frac{1}{3}}(a-1)x+a^2+3\sqrt{\frac{1}{3}}\left(2(a-1)bx^2-\left(-\left(a^2-2a+1\right)b\right)^{\frac{2}{3}}x+\left(-\left(a^2-2a+1\right)b\right)\right.\right.}{bx^3-a+1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^3+a-1),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*(a - 1)*b*sqrt((-(a^2 - 2*a + 1)*b)^(1/3)/b)*log((2*(a -
1)*b*x^3 + 3*(-(a^2 - 2*a + 1)*b)^(1/3)*(a - 1)*x + a^2 + 3*sqrt(1/3)*(2*(a
 - 1)*b*x^2 - (-(a^2 - 2*a + 1)*b)^(2/3)*x + (-(a^2 - 2*a + 1)*b)^(1/3)*(a
- 1))*sqrt((-(a^2 - 2*a + 1)*b)^(1/3)/b) - 2*a + 1)/(b*x^3 - a + 1)) + (-(a
^2 - 2*a + 1)*b)^(2/3)*log((a - 1)*b*x^2 + (-(a^2 - 2*a + 1)*b)^(2/3)*x - (
-(a^2 - 2*a + 1)*b)^(1/3)*(a - 1)) - 2*(-(a^2 - 2*a + 1)*b)^(2/3)*log((a -
1)*b*x - (-(a^2 - 2*a + 1)*b)^(2/3)))/((a^2 - 2*a + 1)*b), 1/6*(6*sqrt(1/3)
*(a - 1)*b*sqrt(-(-(a^2 - 2*a + 1)*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-(a^2 -
 2*a + 1)*b)^(2/3)*x - (-(a^2 - 2*a + 1)*b)^(1/3)*(a - 1))*sqrt(-(-(a^2 - 2
*a + 1)*b)^(1/3)/b)/(a^2 - 2*a + 1)) + (-(a^2 - 2*a + 1)*b)^(2/3)*log((a -
1)*b*x^2 + (-(a^2 - 2*a + 1)*b)^(2/3)*x - (-(a^2 - 2*a + 1)*b)^(1/3)*(a - 1
)) - 2*(-(a^2 - 2*a + 1)*b)^(2/3)*log((a - 1)*b*x - (-(a^2 - 2*a + 1)*b)^(2
/3)))/((a^2 - 2*a + 1)*b)]
```

giac [A] time = 0.20, size = 138, normalized size = 1.00

$$\frac{\left(ab^2-b^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a-1}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a-1}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,ab-\sqrt{3}\,b}+\frac{\left(ab^2-b^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(\frac{a-1}{b}\right)^{\frac{1}{3}}+\left(\frac{a-1}{b}\right)^{\frac{2}{3}}\right)}{6\,(ab-b)}-\frac{\left(\frac{a-1}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{a-1}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,(a-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^3+a-1),x, algorithm="giac")
```

```
[Out] (a*b^2 - b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + ((a - 1)/b)^(1/3))/((a - 1)/b
)^(1/3))/(sqrt(3)*a*b - sqrt(3)*b) + 1/6*(a*b^2 - b^2)^(1/3)*log(x^2 + x*((
a - 1)/b)^(1/3) + ((a - 1)/b)^(2/3))/(a*b - b) - 1/3*((a - 1)/b)^(1/3)*log(
abs(x - ((a - 1)/b)^(1/3)))/(a - 1)
```

maple [A] time = 0.00, size = 106, normalized size = 0.77

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a-1}{b}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{3\left(\frac{a-1}{b}\right)^{\frac{2}{3}}b}-\frac{\ln\left(x-\left(\frac{a-1}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a-1}{b}\right)^{\frac{2}{3}}b}+\frac{\ln\left(x^2+\left(\frac{a-1}{b}\right)^{\frac{1}{3}}x+\left(\frac{a-1}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a-1}{b}\right)^{\frac{2}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^3+a-1),x)
```

```
[Out] -1/3/b/((a-1)/b)^(2/3)*ln(x-((a-1)/b)^(1/3))+1/6/b/((a-1)/b)^(2/3)*ln(x^2+(
(a-1)/b)^(1/3)*x+((a-1)/b)^(2/3))+1/3/b/((a-1)/b)^(2/3)*3^(1/2)*arctan(1/3*
3^(1/2)*(2/((a-1)/b)^(1/3)*x+1))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^3+a-1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a-1.0>0)', see `assume?` for more d
etails)Is a-1.0 positive or negative?
```

mupad [B] time = 1.34, size = 165, normalized size = 1.20

$$\frac{\ln\left(3b^2x+\frac{9ab^2-9b^2}{3(-b)^{1/3}(a-1)^{2/3}}\right)}{3(-b)^{1/3}(a-1)^{2/3}}+\frac{\ln\left(3b^2x+\frac{\left(9ab^2-9b^2\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6(-b)^{1/3}(a-1)^{2/3}}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6(-b)^{1/3}(a-1)^{2/3}}-\frac{\ln\left(3b^2x-\frac{\left(9ab^2-9b^2\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6(-b)^{1/3}(a-1)^{2/3}}\right)\left($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(b*x^3 - a + 1),x)
```

```
[Out] log(3*b^2*x + (9*a*b^2 - 9*b^2)/(3*(-b)^(1/3)*(a - 1)^(2/3)))/(3*(-b)^(1/3)
*(a - 1)^(2/3)) + (log(3*b^2*x + ((9*a*b^2 - 9*b^2)*(3^(1/2)*1i - 1))/(6*(-
b)^(1/3)*(a - 1)^(2/3)))*(3^(1/2)*1i - 1))/(6*(-b)^(1/3)*(a - 1)^(2/3)) - (
log(3*b^2*x - ((9*a*b^2 - 9*b^2)*(3^(1/2)*1i + 1))/(6*(-b)^(1/3)*(a - 1)^(2
/3)))*(3^(1/2)*1i + 1))/(6*(-b)^(1/3)*(a - 1)^(2/3))
```

sympy [A] time = 0.28, size = 34, normalized size = 0.25

$$-\operatorname{RootSum}\left(t^3\left(27a^2b-54ab+27b\right)-1,\left(t\mapsto t\log\left(-3ta+3t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**3+a-1),x)
```

```
[Out] -RootSum(_t**3*(27*a**2*b - 54*a*b + 27*b) - 1, Lambda(_t, _t*log(-3*_t*a +
 3*_t + x)))
```

3.367 $\int \frac{\sqrt{x}}{1+x^3}\,dx$

Optimal. Leaf size=10

$$\frac{2}{3}\tan^{-1}\left(x^{3/2}\right)$$

```
[Out] 2/3*arctan(x^(3/2))
```

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {329, 275, 203}

$$\frac{2}{3}\tan^{-1}\left(x^{3/2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(1 + x^3),x]
```

```
[Out] (2*ArcTan[x^(3/2)])/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{1+x^3}\,dx &= 2\,\text{Subst}\left(\int \frac{x^2}{1+x^6}\,dx, x, \sqrt{x}\right)\\ &= \frac{2}{3}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, x^{3/2}\right)\\ &= \frac{2}{3}\tan^{-1}\left(x^{3/2}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{2}{3}\tan^{-1}\left(x^{3/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(1 + x^3),x]
```

```
[Out] (2*ArcTan[x^(3/2)])/3
```

fricas [A] time = 0.58, size = 6, normalized size = 0.60

$$\frac{2}{3}\arctan\left(x^{\frac{3}{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 2/3*arctan(x^(3/2))
```

giac [A] time = 0.15, size = 6, normalized size = 0.60

$$\frac{2}{3}\arctan\left(x^{\frac{3}{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^3+1),x, algorithm="giac")
```

```
[Out] 2/3*arctan(x^(3/2))
```

maple [A] time = 0.01, size = 7, normalized size = 0.70

$$\frac{2\arctan\left(x^{\frac{3}{2}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^3+1),x)
```

```
[Out] 2/3*arctan(x^(3/2))
```

maxima [A] time = 2.99, size = 6, normalized size = 0.60

$$\frac{2}{3}\arctan\left(x^{\frac{3}{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 2/3*arctan(x^(3/2))
```

mupad [B] time = 0.15, size = 6, normalized size = 0.60

$$\frac{2\operatorname{atan}\left(x^{3/2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^3 + 1),x)
```

```
[Out] (2*atan(x^(3/2)))/3
```

sympy [B] time = 1.22, size = 42, normalized size = 4.20

$$-\frac{2\operatorname{atan}\left(\sqrt{x}\right)}{3}+\frac{2\operatorname{atan}\left(2\sqrt{x}-\sqrt{3}\right)}{3}+\frac{2\operatorname{atan}\left(2\sqrt{x}+\sqrt{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(x**3+1),x)
```

```
[Out] -2*atan(sqrt(x))/3 + 2*atan(2*sqrt(x) - sqrt(3))/3 + 2*atan(2*sqrt(x) + sqr
t(3))/3
```

3.368 $\int x^{11}\sqrt{a+bx^3}\,dx$

Optimal. Leaf size=80

$$-\frac{2a^3\left(a+bx^3\right)^{3/2}}{9b^4}+\frac{2a^2\left(a+bx^3\right)^{5/2}}{5b^4}+\frac{2\left(a+bx^3\right)^{9/2}}{27b^4}-\frac{2a\left(a+bx^3\right)^{7/2}}{7b^4}$$

```
[Out] -2/9*a^3*(b*x^3+a)^(3/2)/b^4+2/5*a^2*(b*x^3+a)^(5/2)/b^4-2/7*a*(b*x^3+a)^(7
/2)/b^4+2/27*(b*x^3+a)^(9/2)/b^4
```

Rubi [A] time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{2a^2\left(a+bx^3\right)^{5/2}}{5b^4}-\frac{2a^3\left(a+bx^3\right)^{3/2}}{9b^4}+\frac{2\left(a+bx^3\right)^{9/2}}{27b^4}-\frac{2a\left(a+bx^3\right)^{7/2}}{7b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*Sqrt[a + b*x^3],x]
```

```
[Out] (-2*a^3*(a + b*x^3)^(3/2))/(9*b^4) + (2*a^2*(a + b*x^3)^(5/2))/(5*b^4) - (2
*a*(a + b*x^3)^(7/2))/(7*b^4) + (2*(a + b*x^3)^(9/2))/(27*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{11}\sqrt{a+bx^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int x^3\sqrt{a+bx}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int\left(-\frac{a^3\sqrt{a+bx}}{b^3}+\frac{3a^2(a+bx)^{3/2}}{b^3}-\frac{3a(a+bx)^{5/2}}{b^3}+\frac{(a+bx)^{7/2}}{b^3}\right)dx, x, x^3\right)\\ &= -\frac{2a^3\left(a+bx^3\right)^{3/2}}{9b^4}+\frac{2a^2\left(a+bx^3\right)^{5/2}}{5b^4}-\frac{2a\left(a+bx^3\right)^{7/2}}{7b^4}+\frac{2\left(a+bx^3\right)^{9/2}}{27b^4}\end{aligned}$$

Mathematica [A] time = 0.05, size = 50, normalized size = 0.62

$$\frac{2\left(a+bx^3\right)^{3/2}\left(-16a^3+24a^2bx^3-30ab^2x^6+35b^3x^9\right)}{945b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*Sqrt[a + b*x^3],x]
```

```
[Out] (2*(a + b*x^3)^(3/2)*(-16*a^3 + 24*a^2*b*x^3 - 30*a*b^2*x^6 + 35*b^3*x^9))/
(945*b^4)
```

fricas [A] time = 0.76, size = 57, normalized size = 0.71

$$\frac{2\left(35\,b^4x^{12}+5\,ab^3x^9-6\,a^2b^2x^6+8\,a^3bx^3-16\,a^4\right)\sqrt{bx^3+a}}{945\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/945*(35*b^4*x^12 + 5*a*b^3*x^9 - 6*a^2*b^2*x^6 + 8*a^3*b*x^3 - 16*a^4)*sq
rt(b*x^3 + a)/b^4
```

giac [A] time = 0.16, size = 57, normalized size = 0.71

$$\frac{2\left(35\left(bx^3+a\right)^{\frac{9}{2}}-135\left(bx^3+a\right)^{\frac{7}{2}}a+189\left(bx^3+a\right)^{\frac{5}{2}}a^2-105\left(bx^3+a\right)^{\frac{3}{2}}a^3\right)}{945\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/945*(35*(b*x^3 + a)^(9/2) - 135*(b*x^3 + a)^(7/2)*a + 189*(b*x^3 + a)^(5/
2)*a^2 - 105*(b*x^3 + a)^(3/2)*a^3)/b^4
```

maple [A] time = 0.01, size = 47, normalized size = 0.59

$$-\frac{2\left(b\,x^3+a\right)^{\frac{3}{2}}\left(-35b^3x^9+30a\,b^2x^6-24a^2b\,x^3+16a^3\right)}{945b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x^3+a)^(1/2),x)
```

```
[Out] -2/945*(b*x^3+a)^(3/2)*(-35*b^3*x^9+30*a*b^2*x^6-24*a^2*b*x^3+16*a^3)/b^4
```

maxima [A] time = 1.28, size = 64, normalized size = 0.80

$$\frac{2\left(bx^3+a\right)^{\frac{9}{2}}}{27\,b^4}-\frac{2\left(bx^3+a\right)^{\frac{7}{2}}a}{7\,b^4}+\frac{2\left(bx^3+a\right)^{\frac{5}{2}}a^2}{5\,b^4}-\frac{2\left(bx^3+a\right)^{\frac{3}{2}}a^3}{9\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/27*(b*x^3 + a)^(9/2)/b^4 - 2/7*(b*x^3 + a)^(7/2)*a/b^4 + 2/5*(b*x^3 + a)^
(5/2)*a^2/b^4 - 2/9*(b*x^3 + a)^(3/2)*a^3/b^4
```

mupad [B] time = 1.12, size = 90, normalized size = 1.12

$$\frac{2\,x^{12}\,\sqrt{b\,x^3+a}}{27}-\frac{32\,a^4\,\sqrt{b\,x^3+a}}{945\,b^4}+\frac{2\,a\,x^9\,\sqrt{b\,x^3+a}}{189\,b}+\frac{16\,a^3\,x^3\,\sqrt{b\,x^3+a}}{945\,b^3}-\frac{4\,a^2\,x^6\,\sqrt{b\,x^3+a}}{315\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a + b*x^3)^(1/2),x)
```

```
[Out] (2*x^12*(a + b*x^3)^(1/2))/27 - (32*a^4*(a + b*x^3)^(1/2))/(945*b^4) + (2*a
*x^9*(a + b*x^3)^(1/2))/(189*b) + (16*a^3*x^3*(a + b*x^3)^(1/2))/(945*b^3)
- (4*a^2*x^6*(a + b*x^3)^(1/2))/(315*b^2)
```

sympy [**A**] time = 4.98, size = 114, normalized size = 1.42

$$\begin{cases} -\frac{32a^4\sqrt{a+bx^3}}{945b^4} + \frac{16a^3x^3\sqrt{a+bx^3}}{945b^3} - \frac{4a^2x^6\sqrt{a+bx^3}}{315b^2} + \frac{2ax^9\sqrt{a+bx^3}}{189b} + \frac{2x^{12}\sqrt{a+bx^3}}{27} & \text{for } b \neq 0 \\ \frac{\sqrt{a}\,x^{12}}{12} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x**3+a)**(1/2),x)
```

```
[Out] Piecewise((-32*a**4*sqrt(a + b*x**3)/(945*b**4) + 16*a**3*x**3*sqrt(a + b*x
**3)/(945*b**3) - 4*a**2*x**6*sqrt(a + b*x**3)/(315*b**2) + 2*a*x**9*sqrt(a
 + b*x**3)/(189*b) + 2*x**12*sqrt(a + b*x**3)/27, Ne(b, 0)), (sqrt(a)*x**12
/12, True))
```

3.369 $\int x^8\sqrt{a+bx^3}\,dx$

Optimal. Leaf size=59

$$\frac{2a^2\left(a+bx^3\right)^{3/2}}{9b^3}+\frac{2\left(a+bx^3\right)^{7/2}}{21b^3}-\frac{4a\left(a+bx^3\right)^{5/2}}{15b^3}$$

```
[Out] 2/9*a^2*(b*x^3+a)^(3/2)/b^3-4/15*a*(b*x^3+a)^(5/2)/b^3+2/21*(b*x^3+a)^(7/2)
/b^3
```

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{2a^2\left(a+bx^3\right)^{3/2}}{9b^3}+\frac{2\left(a+bx^3\right)^{7/2}}{21b^3}-\frac{4a\left(a+bx^3\right)^{5/2}}{15b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*Sqrt[a + b*x^3],x]
```

```
[Out] (2*a^2*(a + b*x^3)^(3/2))/(9*b^3) - (4*a*(a + b*x^3)^(5/2))/(15*b^3) + (2*(
a + b*x^3)^(7/2))/(21*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^8\sqrt{a+bx^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int x^2\sqrt{a+bx}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int\left(\frac{a^2\sqrt{a+bx}}{b^2}-\frac{2a(a+bx)^{3/2}}{b^2}+\frac{(a+bx)^{5/2}}{b^2}\right)dx, x, x^3\right)\\ &= \frac{2a^2\left(a+bx^3\right)^{3/2}}{9b^3}-\frac{4a\left(a+bx^3\right)^{5/2}}{15b^3}+\frac{2\left(a+bx^3\right)^{7/2}}{21b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.66

$$\frac{2\left(a+bx^3\right)^{3/2}\left(8a^2-12abx^3+15b^2x^6\right)}{315b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*Sqrt[a + b*x^3],x]
```

```
[Out] (2*(a + b*x^3)^(3/2)*(8*a^2 - 12*a*b*x^3 + 15*b^2*x^6))/(315*b^3)
```

fricas [A] time = 0.75, size = 46, normalized size = 0.78

$$\frac{2\left(15\,b^3x^9+3\,ab^2x^6-4\,a^2bx^3+8\,a^3\right)\sqrt{bx^3+a}}{315\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(15*b^3*x^9 + 3*a*b^2*x^6 - 4*a^2*b*x^3 + 8*a^3)*sqrt(b*x^3 + a)/b^3
```

giac [A] time = 0.19, size = 43, normalized size = 0.73

$$\frac{2\left(15\left(bx^3+a\right)^{\frac{7}{2}}-42\left(bx^3+a\right)^{\frac{5}{2}}a+35\left(bx^3+a\right)^{\frac{3}{2}}a^2\right)}{315\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(15*(b*x^3 + a)^(7/2) - 42*(b*x^3 + a)^(5/2)*a + 35*(b*x^3 + a)^(3/2)
*a^2)/b^3
```

maple [A] time = 0.01, size = 36, normalized size = 0.61

$$\frac{2\left(b\,x^3+a\right)^{\frac{3}{2}}\left(15b^2x^6-12ab\,x^3+8a^2\right)}{315b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^3+a)^(1/2),x)
```

```
[Out] 2/315*(b*x^3+a)^(3/2)*(15*b^2*x^6-12*a*b*x^3+8*a^2)/b^3
```

maxima [A] time = 1.29, size = 47, normalized size = 0.80

$$\frac{2\left(bx^3+a\right)^{\frac{7}{2}}}{21\,b^3}-\frac{4\left(bx^3+a\right)^{\frac{5}{2}}a}{15\,b^3}+\frac{2\left(bx^3+a\right)^{\frac{3}{2}}a^2}{9\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/21*(b*x^3 + a)^(7/2)/b^3 - 4/15*(b*x^3 + a)^(5/2)*a/b^3 + 2/9*(b*x^3 + a)
^(3/2)*a^2/b^3
```

mupad [B] time = 1.09, size = 70, normalized size = 1.19

$$\frac{2\,x^9\,\sqrt{b\,x^3+a}}{21}+\frac{16\,a^3\,\sqrt{b\,x^3+a}}{315\,b^3}+\frac{2\,a\,x^6\,\sqrt{b\,x^3+a}}{105\,b}-\frac{8\,a^2\,x^3\,\sqrt{b\,x^3+a}}{315\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*x^3)^(1/2),x)
```

```
[Out] (2*x^9*(a + b*x^3)^(1/2))/21 + (16*a^3*(a + b*x^3)^(1/2))/(315*b^3) + (2*a*
x^6*(a + b*x^3)^(1/2))/(105*b) - (8*a^2*x^3*(a + b*x^3)^(1/2))/(315*b^2)
```

sympy [**A**] time = 2.39, size = 90, normalized size = 1.53

$$\begin{cases} \frac{16a^3\sqrt{a+bx^3}}{315b^3} - \frac{8a^2x^3\sqrt{a+bx^3}}{315b^2} + \frac{2ax^6\sqrt{a+bx^3}}{105b} + \frac{2x^9\sqrt{a+bx^3}}{21} & \text{for } b \neq 0 \\ \frac{\sqrt{a}\,x^9}{9} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**3+a)**(1/2),x)
```

```
[Out] Piecewise((16*a**3*sqrt(a + b*x**3)/(315*b**3) - 8*a**2*x**3*sqrt(a + b*x**
3)/(315*b**2) + 2*a*x**6*sqrt(a + b*x**3)/(105*b) + 2*x**9*sqrt(a + b*x**3)
/21, Ne(b, 0)), (sqrt(a)*x**9/9, True))
```

3.370 $\int x^5\sqrt{a+bx^3}\,dx$

Optimal. Leaf size=38

$$\frac{2\left(a+bx^3\right)^{5/2}}{15b^2}-\frac{2a\left(a+bx^3\right)^{3/2}}{9b^2}$$

```
[Out] -2/9*a*(b*x^3+a)^(3/2)/b^2+2/15*(b*x^3+a)^(5/2)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{2\left(a+bx^3\right)^{5/2}}{15b^2}-\frac{2a\left(a+bx^3\right)^{3/2}}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[a + b*x^3],x]
```

```
[Out] (-2*a*(a + b*x^3)^(3/2))/(9*b^2) + (2*(a + b*x^3)^(5/2))/(15*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\sqrt{a+bx^3}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int x\sqrt{a+bx}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int\left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx, x, x^3\right)\\ &= -\frac{2a\left(a+bx^3\right)^{3/2}}{9b^2}+\frac{2\left(a+bx^3\right)^{5/2}}{15b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.74

$$\frac{2\left(a+bx^3\right)^{3/2}\left(3bx^3-2a\right)}{45b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[a + b*x^3],x]
```

```
[Out] (2*(a + b*x^3)^(3/2)*(-2*a + 3*b*x^3))/(45*b^2)
```

fricas [A] time = 0.80, size = 34, normalized size = 0.89

$$\frac{2\left(3\,b^2x^6+abx^3-2\,a^2\right)\sqrt{bx^3+a}}{45\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] 2/45*(3*b^2*x^6 + a*b*x^3 - 2*a^2)*sqrt(b*x^3 + a)/b^2
```

giac [A] time = 0.15, size = 29, normalized size = 0.76

$$\frac{2\left(3\left(bx^3+a\right)^{\frac{5}{2}}-5\left(bx^3+a\right)^{\frac{3}{2}}a\right)}{45\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] 2/45*(3*(b*x^3 + a)^(5/2) - 5*(b*x^3 + a)^(3/2)*a)/b^2
```

maple [A] time = 0.00, size = 25, normalized size = 0.66

$$-\frac{2\left(b\,x^3+a\right)^{\frac{3}{2}}\left(-3b\,x^3+2a\right)}{45b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^3+a)^(1/2),x)
[Out] -2/45*(b*x^3+a)^(3/2)*(-3*b*x^3+2*a)/b^2
```

maxima [A] time = 1.29, size = 30, normalized size = 0.79

$$\frac{2\left(bx^3+a\right)^{\frac{5}{2}}}{15\,b^2}-\frac{2\left(bx^3+a\right)^{\frac{3}{2}}a}{9\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] 2/15*(b*x^3 + a)^(5/2)/b^2 - 2/9*(b*x^3 + a)^(3/2)*a/b^2
```

mupad [B] time = 1.09, size = 29, normalized size = 0.76

$$-\frac{10\,a\left(b\,x^3+a\right)^{3/2}-6\left(b\,x^3+a\right)^{5/2}}{45\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^3)^(1/2),x)
[Out] -(10*a*(a + b*x^3)^(3/2) - 6*(a + b*x^3)^(5/2))/(45*b^2)
```

sympy [A] time = 0.83, size = 66, normalized size = 1.74

$$\begin{cases} -\frac{4a^2\sqrt{a+bx^3}}{45b^2}+\frac{2ax^3\sqrt{a+bx^3}}{45b}+\frac{2x^6\sqrt{a+bx^3}}{15} & \text{for } b\neq 0 \\ \frac{\sqrt{a}\,x^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**3+a)**(1/2),x)
[Out] Piecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/(
45*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True))
```

3.371 $\int x^2\sqrt{a+bx^3}\,dx$

Optimal. Leaf size=18

$$\frac{2\left(a+bx^3\right)^{3/2}}{9b}$$

```
[Out] 2/9*(b*x^3+a)^(3/2)/b
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$\frac{2\left(a+bx^3\right)^{3/2}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*x^3],x]
```

```
[Out] (2*(a + b*x^3)^(3/2))/(9*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^2\sqrt{a+bx^3}\,dx = \frac{2\left(a+bx^3\right)^{3/2}}{9b}$$

Mathematica [A] time = 0.02, size = 18, normalized size = 1.00

$$\frac{2\left(a+bx^3\right)^{3/2}}{9b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b*x^3],x]
```

```
[Out] (2*(a + b*x^3)^(3/2))/(9*b)
```

fricas [A] time = 1.00, size = 14, normalized size = 0.78

$$\frac{2\left(bx^3+a\right)^{\frac{3}{2}}}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9*(b*x^3 + a)^(3/2)/b
```

giac [A] time = 0.15, size = 14, normalized size = 0.78

$$\frac{2\left(bx^3+a\right)^{\frac{3}{2}}}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] 2/9*(b*x^3 + a)^(3/2)/b
```

maple [A] time = 0.01, size = 15, normalized size = 0.83

$$\frac{2\left(b\,x^3+a\right)^{\frac{3}{2}}}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^(1/2),x)
[Out] 2/9*(b*x^3+a)^(3/2)/b
```

maxima [A] time = 1.28, size = 14, normalized size = 0.78

$$\frac{2\left(bx^3+a\right)^{\frac{3}{2}}}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] 2/9*(b*x^3 + a)^(3/2)/b
```

mupad [B] time = 1.09, size = 14, normalized size = 0.78

$$\frac{2\left(b\,x^3+a\right)^{3/2}}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^3)^(1/2),x)
[Out] (2*(a + b*x^3)^(3/2))/(9*b)
```

sympy [A] time = 0.22, size = 42, normalized size = 2.33

$$\begin{cases} \frac{2a\sqrt{a+bx^3}}{9b}+\frac{2x^3\sqrt{a+bx^3}}{9} & \text{for } b\neq 0 \\ \frac{\sqrt{a}\,x^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**(1/2),x)
[Out] Piecewise((2*a*sqrt(a + b*x**3)/(9*b) + 2*x**3*sqrt(a + b*x**3)/9, Ne(b, 0)
), (sqrt(a)*x**3/3, True))
```

3.372 $\int \frac{\sqrt{a+bx^3}}{x}\,dx$

Optimal. Leaf size=43

$$\frac{2}{3}\sqrt{a+bx^3} - \frac{2}{3}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$

```
[Out] -2/3*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+2/3*(b*x^3+a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 50, 63, 208}

$$\frac{2}{3}\sqrt{a+bx^3} - \frac{2}{3}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3]/x,x]
```

```
[Out] (2*Sqrt[a + b*x^3])/3 - (2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx^3}}{x}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^3\right)\\ &= \frac{2}{3}\sqrt{a+bx^3} + \frac{1}{3}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right)\\ &= \frac{2}{3}\sqrt{a+bx^3} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)}{3b}\\ &= \frac{2}{3}\sqrt{a+bx^3} - \frac{2}{3}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 1.00

$$\frac{2}{3}\sqrt{a+bx^3} - \frac{2}{3}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3]/x,x]
```

```
[Out] (2*Sqrt[a + b*x^3])/3 - (2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3
```

fricas [A] time = 0.91, size = 84, normalized size = 1.95

$$\left[\frac{1}{3}\sqrt{a}\,\log\left(\frac{bx^3 - 2\sqrt{bx^3+a}\sqrt{a} + 2a}{x^3}\right) + \frac{2}{3}\sqrt{bx^3+a}, \frac{2}{3}\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx^3+a}\sqrt{-a}}{a}\right) + \frac{2}{3}\sqrt{bx^3+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/3*sqrt(a)*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2/3*sqrt(
b*x^3 + a), 2/3*sqrt(-a)*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + 2/3*sqrt(b*x^
3 + a)]
```

giac [A] time = 0.16, size = 36, normalized size = 0.84

$$\frac{2\,a\,\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\sqrt{-a}} + \frac{2}{3}\sqrt{bx^3+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2/3*a*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a) + 2/3*sqrt(b*x^3 + a)
```

maple [A] time = 0.28, size = 32, normalized size = 0.74

$$-\frac{2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3} + \frac{2\sqrt{b\,x^3+a}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/2)/x,x)
```

```
[Out] -2/3*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+2/3*(b*x^3+a)^(1/2)
```

maxima [A] time = 2.95, size = 49, normalized size = 1.14

$$\frac{1}{3}\sqrt{a}\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)+\frac{2}{3}\sqrt{bx^3+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(a)*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a))) +
2/3*sqrt(b*x^3 + a)
```

mupad [B] time = 1.21, size = 52, normalized size = 1.21

$$\frac{\sqrt{a}\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3}+\frac{2\sqrt{b\,x^3+a}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/2)/x,x)
```

```
[Out] (a^(1/2)*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2))
)/x^6))/3 + (2*(a + b*x^3)^(1/2))/3
```

sympy [B] time = 2.07, size = 76, normalized size = 1.77

$$-\frac{2\sqrt{a}\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{3}+\frac{2a}{3\sqrt{b}\,x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{2\sqrt{b}\,x^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/2)/x,x)
```

```
[Out] -2*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + 2*a/(3*sqrt(b)*x**(3/2)*sq
rt(a/(b*x**3) + 1)) + 2*sqrt(b)*x**(3/2)/(3*sqrt(a/(b*x**3) + 1))
```

3.373 $\int \frac{\sqrt{a+bx^3}}{x^4}\,dx$

Optimal. Leaf size=47

$$-\frac{\sqrt{a+bx^3}}{3x^3}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

```
[Out] -1/3*b*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)-1/3*(b*x^3+a)^(1/2)/x^3
```

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 47, 63, 208}

$$-\frac{\sqrt{a+bx^3}}{3x^3}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3]/x^4,x]
```

```
[Out] -Sqrt[a + b*x^3]/(3*x^3) - (b*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx^3}}{x^4}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx, x, x^3\right)\\ &= -\frac{\sqrt{a+bx^3}}{3x^3} + \frac{1}{6}b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right)\\ &= -\frac{\sqrt{a+bx^3}}{3x^3} + \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)\\ &= -\frac{\sqrt{a+bx^3}}{3x^3} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 59, normalized size = 1.26

$$-\frac{bx^3\sqrt{\frac{bx^3}{a}+1}\,\tanh^{-1}\left(\sqrt{\frac{bx^3}{a}+1}\right)+a+bx^3}{3x^3\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3]/x^4,x]
```

```
[Out] -1/3*(a + b*x^3 + b*x^3*Sqrt[1 + (b*x^3)/a]*ArcTanh[Sqrt[1 + (b*x^3)/a]])/(
x^3*Sqrt[a + b*x^3])
```

fricas [A] time = 0.95, size = 108, normalized size = 2.30

$$\left[\frac{\sqrt{a}\,bx^3\log\left(\frac{bx^3-2\sqrt{bx^3+a}\sqrt{a}+2a}{x^3}\right)-2\sqrt{bx^3+a}\,a}{6ax^3}, \frac{\sqrt{-a}\,bx^3\arctan\left(\frac{\sqrt{bx^3+a}\sqrt{-a}}{a}\right)-\sqrt{bx^3+a}\,a}{3ax^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/6*(sqrt(a)*b*x^3*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) - 2*
sqrt(b*x^3 + a)*a)/(a*x^3), 1/3*(sqrt(-a)*b*x^3*arctan(sqrt(b*x^3 + a)*sqrt
(-a)/a) - sqrt(b*x^3 + a)*a)/(a*x^3)]
```

giac [A] time = 0.16, size = 46, normalized size = 0.98

$$\frac{\frac{b^2\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\sqrt{bx^3+a}\,b}{x^3}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/3*(b^2*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a) - sqrt(b*x^3 + a)*b/x^3)
/b
```

maple [A] time = 0.02, size = 36, normalized size = 0.77

$$-\frac{b\,\text{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3\sqrt{a}}-\frac{\sqrt{b\,x^3+a}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/2)/x^4,x)
```

```
[Out] -1/3*b*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)-1/3*(b*x^3+a)^(1/2)/x^3
```

maxima [A] time = 2.94, size = 53, normalized size = 1.13

$$\frac{b \log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{6\sqrt{a}} - \frac{\sqrt{bx^3+a}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*b*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/sqrt(a)
- 1/3*sqrt(b*x^3 + a)/x^3
```

mupad [B] time = 1.26, size = 56, normalized size = 1.19

$$\frac{b \ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{6\sqrt{a}} - \frac{\sqrt{b\,x^3+a}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/2)/x^4,x)
```

```
[Out] (b*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6)
)/(6*a^(1/2)) - (a + b*x^3)^(1/2)/(3*x^3)
```

sympy [A] time = 2.84, size = 49, normalized size = 1.04

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} - \frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/2)/x**4,x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - b*asinh(sqrt(a)/(sqrt(b)*x**(3
/2)))/(3*sqrt(a))
```

3.374 $\int \frac{\sqrt{a+bx^3}}{x^7}\,dx$

Optimal. Leaf size=71

$$\frac{b^2 \tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}} - \frac{b\sqrt{a+bx^3}}{12ax^3} - \frac{\sqrt{a+bx^3}}{6x^6}$$

```
[Out] 1/12*b^2*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)-1/6*(b*x^3+a)^(1/2)/x^6-1
/12*b*(b*x^3+a)^(1/2)/a/x^3
```

Rubi [A] time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {266, 47, 51, 63, 208}

$$\frac{b^2 \tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}} - \frac{b\sqrt{a+bx^3}}{12ax^3} - \frac{\sqrt{a+bx^3}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3]/x^7,x]
```

```
[Out] -Sqrt[a + b*x^3]/(6*x^6) - (b*Sqrt[a + b*x^3])/(12*a*x^3) + (b^2*ArcTanh[Sq
rt[a + b*x^3]/Sqrt[a]])/(12*a^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^3}}{x^7}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^3}\,dx, x, x^3\right) \\
&= -\frac{\sqrt{a+bx^3}}{6x^6} + \frac{1}{12}b\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^3\right) \\
&= -\frac{\sqrt{a+bx^3}}{6x^6} - \frac{b\sqrt{a+bx^3}}{12ax^3} - \frac{b^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right)}{24a} \\
&= -\frac{\sqrt{a+bx^3}}{6x^6} - \frac{b\sqrt{a+bx^3}}{12ax^3} - \frac{b\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)}{12a} \\
&= -\frac{\sqrt{a+bx^3}}{6x^6} - \frac{b\sqrt{a+bx^3}}{12ax^3} + \frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.55

$$-\frac{2b^2\left(a+bx^3\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{bx^3}{a}+1\right)}{9a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3]/x^7,x]
```

```
[Out] (-2*b^2*(a + b*x^3)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + (b*x^3)/a])/(9
*a^3)
```

fricas [A] time = 0.94, size = 133, normalized size = 1.87

$$\left[\frac{\sqrt{a}\,b^2x^6\log\left(\frac{bx^3+2\sqrt{bx^3+a}\sqrt{a}+2a}{x^3}\right)-2\left(abx^3+2a^2\right)\sqrt{bx^3+a}}{24a^2x^6}, -\frac{\sqrt{-a}\,b^2x^6\arctan\left(\frac{\sqrt{bx^3+a}\sqrt{-a}}{a}\right)+\left(abx^3+2a^2\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/24*(sqrt(a)*b^2*x^6*log((b*x^3 + 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) -
 2*(a*b*x^3 + 2*a^2)*sqrt(b*x^3 + a))/(a^2*x^6), -1/12*(sqrt(-a)*b^2*x^6*ar
ctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (a*b*x^3 + 2*a^2)*sqrt(b*x^3 + a))/(a^2*
x^6)]
```

giac [A] time = 0.16, size = 72, normalized size = 1.01

$$-\frac{\frac{b^3\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{\left(bx^3+a\right)^{\frac{3}{2}}b^3+\sqrt{bx^3+a}\,ab^3}{ab^2x^6}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/12*(b^3*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a) + ((b*x^3 + a)^(3/
2)*b^3 + sqrt(b*x^3 + a)*a*b^3)/(a*b^2*x^6))/b
```

maple [A] time = 0.03, size = 56, normalized size = 0.79

$$\frac{b^2 \operatorname{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{12a^{\frac{3}{2}}} - \frac{\sqrt{b\,x^3+a}\;b}{12a\,x^3} - \frac{\sqrt{b\,x^3+a}}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/2)/x^7,x)
```

```
[Out] 1/12*b^2*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)-1/6*(b*x^3+a)^(1/2)/x^6-1
/12*b*(b*x^3+a)^(1/2)/a/x^3
```

maxima [A] time = 2.90, size = 100, normalized size = 1.41

$$-\frac{b^2 \log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{24\,a^{\frac{3}{2}}} - \frac{\left(bx^3+a\right)^{\frac{3}{2}}b^2 + \sqrt{bx^3+a}\,ab^2}{12\left(\left(bx^3+a\right)^2 a - 2\left(bx^3+a\right)a^2 + a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/24*b^2*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(3
/2) - 1/12*((b*x^3 + a)^(3/2)*b^2 + sqrt(b*x^3 + a)*a*b^2)/((b*x^3 + a)^2*a
 - 2*(b*x^3 + a)*a^2 + a^3)
```

mupad [B] time = 1.35, size = 76, normalized size = 1.07

$$\frac{b^2 \ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)^3}{x^6}\right)}{24\,a^{3/2}} - \frac{\sqrt{b\,x^3+a}}{6\,x^6} - \frac{b\sqrt{b\,x^3+a}}{12\,a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/2)/x^7,x)
```

```
[Out] (b^2*log((((a + b*x^3)^(1/2) - a^(1/2))*((a + b*x^3)^(1/2) + a^(1/2))^3)/x^
6))/(24*a^(3/2)) - (a + b*x^3)^(1/2)/(6*x^6) - (b*(a + b*x^3)^(1/2))/(12*a*
x^3)
```

sympy [A] time = 6.01, size = 100, normalized size = 1.41

$$-\frac{a}{6\sqrt{b}\,x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{\sqrt{b}}{4x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{b^{\frac{3}{2}}}{12ax^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{b^2 \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{12a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/2)/x**7,x)
```

```
[Out] -a/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - sqrt(b)/(4*x**(9/2)*sqrt(a/
(b*x**3) + 1)) - b**(3/2)/(12*a*x**(3/2)*sqrt(a/(b*x**3) + 1)) + b**2*asinh
(sqrt(a)/(sqrt(b)*x**(3/2)))/(12*a**(3/2))
```

3.375 $\int \frac{\sqrt{a+bx^3}}{x^{10}}\,dx$

Optimal. Leaf size=95

$$-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{24a^{5/2}}+\frac{b^2\sqrt{a+bx^3}}{24a^2x^3}-\frac{\sqrt{a+bx^3}}{9x^9}-\frac{b\sqrt{a+bx^3}}{36ax^6}$$

```
[Out] -1/24*b^3*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(5/2)-1/9*(b*x^3+a)^(1/2)/x^9-
1/36*b*(b*x^3+a)^(1/2)/a/x^6+1/24*b^2*(b*x^3+a)^(1/2)/a^2/x^3
```

Rubi [A] time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {266, 47, 51, 63, 208}

$$\frac{b^2\sqrt{a+bx^3}}{24a^2x^3}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{24a^{5/2}}-\frac{b\sqrt{a+bx^3}}{36ax^6}-\frac{\sqrt{a+bx^3}}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3]/x^10,x]
```

```
[Out] -Sqrt[a + b*x^3]/(9*x^9) - (b*Sqrt[a + b*x^3])/(36*a*x^6) + (b^2*Sqrt[a + b
*x^3])/(24*a^2*x^3) - (b^3*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(24*a^(5/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^3}}{x^{10}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^4}\,dx, x, x^3\right)\\
&= -\frac{\sqrt{a+bx^3}}{9x^9} + \frac{1}{18}b\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx, x, x^3\right)\\
&= -\frac{\sqrt{a+bx^3}}{9x^9} - \frac{b\sqrt{a+bx^3}}{36ax^6} - \frac{b^2\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^3\right)}{24a}\\
&= -\frac{\sqrt{a+bx^3}}{9x^9} - \frac{b\sqrt{a+bx^3}}{36ax^6} + \frac{b^2\sqrt{a+bx^3}}{24a^2x^3} + \frac{b^3\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right)}{48a^2}\\
&= -\frac{\sqrt{a+bx^3}}{9x^9} - \frac{b\sqrt{a+bx^3}}{36ax^6} + \frac{b^2\sqrt{a+bx^3}}{24a^2x^3} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)}{24a^2}\\
&= -\frac{\sqrt{a+bx^3}}{9x^9} - \frac{b\sqrt{a+bx^3}}{36ax^6} + \frac{b^2\sqrt{a+bx^3}}{24a^2x^3} - \frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{24a^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 39, normalized size = 0.41

$$\frac{2b^3\left(a+bx^3\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},4;\frac{5}{2};\frac{bx^3}{a}+1\right)}{9a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3]/x^10,x]
```

```
[Out] (2*b^3*(a + b*x^3)^(3/2)*Hypergeometric2F1[3/2, 4, 5/2, 1 + (b*x^3)/a])/(9*
a^4)
```

fricas [A] time = 0.89, size = 159, normalized size = 1.67

$$\left[\frac{3\sqrt{a}\,b^3x^9\log\left(\frac{bx^3-2\sqrt{bx^3+a}\sqrt{a}+2a}{x^3}\right)+2\left(3ab^2x^6-2a^2bx^3-8a^3\right)\sqrt{bx^3+a}}{144a^3x^9}, \frac{3\sqrt{-a}\,b^3x^9\arctan\left(\frac{\sqrt{bx^3+a}\sqrt{-a}}{a}\right)+\left(3\right.}{72a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/2)/x^10,x, algorithm="fricas")
```

```
[Out] [1/144*(3*sqrt(a)*b^3*x^9*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3
) + 2*(3*a*b^2*x^6 - 2*a^2*b*x^3 - 8*a^3)*sqrt(b*x^3 + a))/(a^3*x^9), 1/72*
(3*sqrt(-a)*b^3*x^9*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (3*a*b^2*x^6 - 2*a
^2*b*x^3 - 8*a^3)*sqrt(b*x^3 + a))/(a^3*x^9)]
```

giac [A] time = 0.18, size = 92, normalized size = 0.97

$$\frac{\frac{3\,b^4\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{3\left(bx^3+a\right)^{\frac{5}{2}}b^4-8\left(bx^3+a\right)^{\frac{3}{2}}ab^4-3\sqrt{bx^3+a}\,a^2b^4}{a^2b^3x^9}}{72\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/2)/x^10,x, algorithm="giac")
```

```
[Out] 1/72*(3*b^4*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x^3 + a
)^(5/2)*b^4 - 8*(b*x^3 + a)^(3/2)*a*b^4 - 3*sqrt(b*x^3 + a)*a^2*b^4)/(a^2*b
^3*x^9))/b
```

maple [A] time = 0.03, size = 76, normalized size = 0.80

$$-\frac{b^3 \operatorname{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{24a^{\frac{5}{2}}}+\frac{\sqrt{b\,x^3+a}\;b^2}{24a^2x^3}-\frac{\sqrt{b\,x^3+a}\;b}{36a\,x^6}-\frac{\sqrt{b\,x^3+a}}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/2)/x^10,x)
```

```
[Out] -1/24*b^3*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(5/2)-1/9*(b*x^3+a)^(1/2)/x^9-
1/36*b*(b*x^3+a)^(1/2)/x^6/a+1/24*b^2*(b*x^3+a)^(1/2)/a^2/x^3
```

maxima [A] time = 2.94, size = 137, normalized size = 1.44

$$\frac{b^3 \log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{48\,a^{\frac{5}{2}}}+\frac{3\left(bx^3+a\right)^{\frac{5}{2}}b^3-8\left(bx^3+a\right)^{\frac{3}{2}}ab^3-3\sqrt{bx^3+a}\,a^2b^3}{72\left(\left(bx^3+a\right)^3a^2-3\left(bx^3+a\right)^2a^3+3\left(bx^3+a\right)a^4-a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/2)/x^10,x, algorithm="maxima")
```

```
[Out] 1/48*b^3*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(5/
2) + 1/72*(3*(b*x^3 + a)^(5/2)*b^3 - 8*(b*x^3 + a)^(3/2)*a*b^3 - 3*sqrt(b*x
^3 + a)*a^2*b^3)/((b*x^3 + a)^3*a^2 - 3*(b*x^3 + a)^2*a^3 + 3*(b*x^3 + a)*a
^4 - a^5)
```

mupad [B] time = 1.50, size = 96, normalized size = 1.01

$$\frac{b^3\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{48\,a^{5/2}}-\frac{\sqrt{b\,x^3+a}}{9\,x^9}-\frac{b\,\sqrt{b\,x^3+a}}{36\,a\,x^6}+\frac{b^2\,\sqrt{b\,x^3+a}}{24\,a^2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/2)/x^10,x)
```

```
[Out] (b^3*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^
6))/(48*a^(5/2)) - (a + b*x^3)^(1/2)/(9*x^9) - (b*(a + b*x^3)^(1/2))/(36*a*
x^6) + (b^2*(a + b*x^3)^(1/2))/(24*a^2*x^3)
```

sympy [A] time = 7.15, size = 129, normalized size = 1.36

$$-\frac{a}{9\sqrt{b}\,x^{\frac{21}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{5\sqrt{b}}{36x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{b^{\frac{3}{2}}}{72ax^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{b^{\frac{5}{2}}}{24a^2x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{b^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{24a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/2)/x**10,x)
```

```
[Out] -a/(9*sqrt(b)*x**(21/2)*sqrt(a/(b*x**3) + 1)) - 5*sqrt(b)/(36*x**(15/2)*sqr
t(a/(b*x**3) + 1)) + b**(3/2)/(72*a*x**(9/2)*sqrt(a/(b*x**3) + 1)) + b**(5/
2)/(24*a**2*x**(3/2)*sqrt(a/(b*x**3) + 1)) - b**3*asinh(sqrt(a)/(sqrt(b)*x*
*(3/2)))/(24*a**(5/2))
```

3.376 $\int x^6\sqrt{a+bx^3}\,dx$

Optimal. Leaf size=275

$$-\frac{48a^2x\sqrt{a+bx^3}}{935b^2}+\frac{32\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^3\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right.}{935b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}$$

```
[Out] -48/935*a^2*x*(b*x^3+a)^(1/2)/b^2+6/187*a*x^4*(b*x^3+a)^(1/2)/b+2/17*x^7*(b
*x^3+a)^(1/2)+32/935*3^(3/4)*a^3*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a
^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(
1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2)))^2)^(1/2)/b^(7/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*
x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {279, 321, 218}

$$-\frac{48a^2x\sqrt{a+bx^3}}{935b^2}+\frac{32\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^3\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right.}{935b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*Sqrt[a + b*x^3],x]
```

```
[Out] (-48*a^2*x*Sqrt[a + b*x^3])/(935*b^2) + (6*a*x^4*Sqrt[a + b*x^3])/(187*b) +
 (2*x^7*Sqrt[a + b*x^3])/17 + (32*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^3*(a^(1/3) +
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(935*b^(7/3)*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt
[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^6\sqrt{a+bx^3}\,dx = \frac{2}{17}x^7\sqrt{a+bx^3} + \frac{1}{17}(3a)\int \frac{x^6}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{6ax^4\sqrt{a+bx^3}}{187b} + \frac{2}{17}x^7\sqrt{a+bx^3} - \frac{\left(24a^2\right)\int \frac{x^3}{\sqrt{a+bx^3}}\,dx}{187b}$$

$$= -\frac{48a^2x\sqrt{a+bx^3}}{935b^2} + \frac{6ax^4\sqrt{a+bx^3}}{187b} + \frac{2}{17}x^7\sqrt{a+bx^3} + \frac{\left(48a^3\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{935b^2}$$

$$= -\frac{48a^2x\sqrt{a+bx^3}}{935b^2} + \frac{6ax^4\sqrt{a+bx^3}}{187b} + \frac{2}{17}x^7\sqrt{a+bx^3} + \frac{32\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^3\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{}}{935b^{7/3}}$$

Mathematica [C] time = 0.07, size = 94, normalized size = 0.34

$$\frac{2x\sqrt{a+bx^3}\left(\sqrt{\frac{bx^3}{a}+1}\left(-8a^2+3abx^3+11b^2x^6\right)+8a^2\,{}_2F_1\left(-\frac{1}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)\right)}{187b^2\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*Sqrt[a + b*x^3],x]
```

```
[Out] (2*x*Sqrt[a + b*x^3]*(Sqrt[1 + (b*x^3)/a]*(-8*a^2 + 3*a*b*x^3 + 11*b^2*x^6)
 + 8*a^2*Hypergeometric2F1[-1/2, 1/3, 4/3, -((b*x^3)/a)]))/(187*b^2*Sqrt[1
+ (b*x^3)/a])
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{bx^3+a}\,x^6, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)*x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^3+a}\,x^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^3 + a)*x^6, x)
```

maple [A] time = 0.02, size = 337, normalized size = 1.23

$$32i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+$$

$$\frac{2\sqrt{b\,x^3+a}\,x^7}{17}+\frac{6\sqrt{b\,x^3+a}\,a\,x^4}{187b}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^3+a)^(1/2),x)
```

```
[Out] 2/17*x^7*(b*x^3+a)^(1/2)+6/187*a*x^4*(b*x^3+a)^(1/2)/b-48/935*a^2*x*(b*x^3+
a)^(1/2)/b^2-32/935*I*a^3/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/
b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^3+a}\,x^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a)*x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^6\,\sqrt{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b*x^3)^(1/2),x)
```

```
[Out] int(x^6*(a + b*x^3)^(1/2), x)
```

sympy [A] time = 1.07, size = 39, normalized size = 0.14

$$\frac{\sqrt{a}\,x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**3+a)**(1/2),x)
```

```
[Out] sqrt(a)*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(10/3))
```

3.377 $\int x^3\sqrt{a+bx^3}\,dx$

Optimal. Leaf size=251

$$-\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^2\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{55b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\frac{6ax\sqrt{a+bx^3}}{55b}+\frac{2}{11}x$$

```
[Out] 6/55*a*x*(b*x^3+a)^(1/2)/b+2/11*x^4*(b*x^3+a)^(1/2)-4/55*3^(3/4)*a^2*(a^(1/
3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*
(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(4/3)/(b*x^3
+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(
1/2)
```

Rubi [A] time = 0.07, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {279, 321, 218}

$$-\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^2\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{55b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\frac{2}{11}x^4\sqrt{a+bx^3}+\frac{6a}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + b*x^3],x]
```

```
[Out] (6*a*x*Sqrt[a + b*x^3])/(55*b) + (2*x^4*Sqrt[a + b*x^3])/11 - (4*3^(3/4)*Sq
rt[2 + Sqrt[3]]*a^2*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
 4*Sqrt[3]])/(55*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^3\sqrt{a+bx^3}\,dx = \frac{2}{11}x^4\sqrt{a+bx^3} + \frac{1}{11}(3a)\int \frac{x^3}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{6ax\sqrt{a+bx^3}}{55b} + \frac{2}{11}x^4\sqrt{a+bx^3} - \frac{\left(6a^2\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{55b}$$

$$= \frac{6ax\sqrt{a+bx^3}}{55b} + \frac{2}{11}x^4\sqrt{a+bx^3} - \frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}\;a^2\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left($$

$$55b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{}$$

Mathematica [C] time = 0.06, size = 62, normalized size = 0.25

$$\frac{2x\sqrt{a+bx^3}\left(-\frac{a\,{}_2F_1\left(-\frac{1}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}}+a+bx^3\right)}{11b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + b*x^3],x]
```

```
[Out] (2*x*Sqrt[a + b*x^3]*(a + b*x^3 - (a*Hypergeometric2F1[-1/2, 1/3, 4/3, -((b
*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(11*b)
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{bx^3+a}\,x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)*x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^3+a}\,x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^3 + a)*x^3, x)
```

maple [A] time = 0.02, size = 317, normalized size = 1.26

$$4i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\,\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}$$

$$\frac{2\sqrt{b\,x^3+a}\;x^4}{11}+\frac{}{55\sqrt{b\,x^3+a}\;b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)^(1/2),x)
```

```
[Out] 2/11*x^4*(b*x^3+a)^(1/2)+6/55*a*x*(b*x^3+a)^(1/2)/b+4/55*I*a^2/b^2*3^(1/2)*
(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3
^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b
+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3
^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)
/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2
)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^3+a}\,x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a)*x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\sqrt{b\,x^3+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^3)^(1/2),x)
```

```
[Out] int(x^3*(a + b*x^3)^(1/2), x)
```

sympy [A] time = 0.94, size = 39, normalized size = 0.16

$$\frac{\sqrt{a}\,x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**(1/2),x)
```

```
[Out] sqrt(a)*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(7/3))
```

3.378 $\int \sqrt{a+bx^3}\,dx$

Optimal. Leaf size=227

$$\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{5\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{2}{5}x\sqrt{a+bx^3}$$

```
[Out] 2/5*x*(b*x^3+a)^(1/2)+2/5*3^(3/4)*a*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*
x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*
6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(1/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/
3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {195, 218}

$$\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{5\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{2}{5}x\sqrt{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3],x]
```

```
[Out] (2*x*Sqrt[a + b*x^3])/5 + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(a^(1/3) + b^(1/3)
*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*b^(1/3)*Sqrt[(a^(1/3)*
(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^
3])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
 + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \sqrt{a+bx^3}\,dx = \frac{2}{5}x\sqrt{a+bx^3} + \frac{1}{5}(3a)\int \frac{1}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{2}{5}x\sqrt{a+bx^3} + \frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\;a\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}\right)|-\right.}{5\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] time = 0.00, size = 46, normalized size = 0.20

$$\frac{x\sqrt{a+bx^3}\;{}_2F_1\left(-\frac{1}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3],x]
```

```
[Out] (x*Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, 1/3, 4/3, -((b*x^3)/a)])/Sqrt[1
+ (b*x^3)/a]
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{bx^3+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^3 + a), x)
```

maple [A] time = 0.02, size = 297, normalized size = 1.31

$$-\frac{2i\sqrt{3}\;\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\;b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\;b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\;a\,\text{EllipticF}\left(\sqrt{\right.}}{5\sqrt{b\,x^3+a}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/2),x)
```

```
[Out] 2/5*x*(b*x^3+a)^(1/2)-2/5*I*a*3^(1/2)*(-a*b^2)^(1/3)/b*(I*(x+1/2*(-a*b^2)^(
1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-
(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(
1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-
a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b
^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)
,(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1
/3)/b)/b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a), x)
```

mupad [B] time = 1.02, size = 37, normalized size = 0.16

$$\frac{x\sqrt{b\,x^3+a}\;{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{1}{3};\;\frac{4}{3};\;-\frac{b\,x^3}{a}\right)}{\sqrt{\frac{b\,x^3}{a}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/2),x)
```

```
[Out] (x*(a + b*x^3)^(1/2)*hypergeom([-1/2, 1/3], 4/3, -(b*x^3)/a))/((b*x^3)/a +
1)^(1/2)
```

sympy [A] time = 0.87, size = 37, normalized size = 0.16

$$\frac{\sqrt{a}\,x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\,\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/2),x)
```

```
[Out] sqrt(a)*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(
3*gamma(4/3))
```

3.379 $\int \frac{\sqrt{a+bx^3}}{x^3}\,dx$

Optimal. Leaf size=228

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}-\frac{\sqrt{a+bx^3}}{2x^2}$$

```
[Out] -1/2*(b*x^3+a)^(1/2)/x^2+1/2*3^(3/4)*b^(2/3)*(a^(1/3)+b^(1/3)*x)*EllipticF(
(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2
*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(
1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1
/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {277, 218}

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}-\frac{\sqrt{a+bx^3}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3]/x^3,x]
```

```
[Out] -Sqrt[a + b*x^3]/(2*x^2) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(2/3)*(a^(1/3) + b^
(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*Sqrt[(a^(1/3)*(a^
(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}}{x^3}\,dx = -\frac{\sqrt{a+bx^3}}{2x^2} + \frac{1}{4}(3b)\int \frac{1}{\sqrt{a+bx^3}}\,dx$$

$$= -\frac{\sqrt{a+bx^3}}{2x^2} + \frac{3^{3/4}\sqrt{2+\sqrt{3}}\; b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}\right.\right.}{2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.22

$$-\frac{\sqrt{a+bx^3}\;{}_2F_1\left(-\frac{2}{3},-\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a}\right)}{2x^2\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3]/x^3,x]
```

```
[Out] -1/2*(Sqrt[a + b*x^3]*Hypergeometric2F1[-2/3, -1/2, 1/3, -((b*x^3)/a)])/(x^
2*Sqrt[1 + (b*x^3)/a])
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+a}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^3+a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^3 + a)/x^3, x)
```

maple [A] time = 0.02, size = 295, normalized size = 1.29

$$-\frac{i\sqrt{3}\;\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\;b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\;b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\;\text{EllipticF}\left(\sqrt{-}\right.}{2\sqrt{b\,x^3+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/2)/x^3,x)
```

```
[Out] -1/2*(b*x^3+a)^(1/2)/x^2-1/2*I*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1
/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(
-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1
/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a
*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^
2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),
(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/
3)/b)/b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^3+a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,x^3+a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/2)/x^3,x)
```

```
[Out] int((a + b*x^3)^(1/2)/x^3, x)
```

sympy [A] time = 1.54, size = 42, normalized size = 0.18

$$\frac{\sqrt{a}\,\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3}, -\frac{1}{2}\\ \frac{1}{3}\end{matrix}\right| \frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/2)/x**3,x)
```

```
[Out] sqrt(a)*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(
3*x**2*gamma(1/3))
```

3.380 $\int \frac{\sqrt{a+bx^3}}{x^6}\,dx$

Optimal. Leaf size=253

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{5/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{20a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}-\frac{\sqrt{a+bx^3}}{5x^5}-\frac{3b\sqrt{}}{2}$$

```
[Out] -1/5*(b*x^3+a)^(1/2)/x^5-3/20*b*(b*x^3+a)^(1/2)/a/x^2-1/20*3^(3/4)*b^(5/3)*
(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/
3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a/(b*x^3
+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(
1/2)
```

Rubi [A] time = 0.07, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {277, 325, 218}

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{5/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{20a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}-\frac{3b\sqrt{a+bx^3}}{20ax^2}-\frac{\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3]/x^6,x]
```

```
[Out] -Sqrt[a + b*x^3]/(5*x^5) - (3*b*Sqrt[a + b*x^3])/(20*a*x^2) - (3^(3/4)*Sqrt
[2 + Sqrt[3]]*b^(5/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[(
(1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
 - 4*Sqrt[3]])/(20*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}}{x^6}\,dx = -\frac{\sqrt{a+bx^3}}{5x^5} + \frac{1}{10}(3b)\int \frac{1}{x^3\sqrt{a+bx^3}}\,dx$$

$$= -\frac{\sqrt{a+bx^3}}{5x^5} - \frac{3b\sqrt{a+bx^3}}{20ax^2} - \frac{\left(3b^2\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{40a}$$

$$= -\frac{\sqrt{a+bx^3}}{5x^5} - \frac{3b\sqrt{a+bx^3}}{20ax^2} - \frac{3^{3/4}\sqrt{2+\sqrt{3}}\; b^{5/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{(1-}{(1+}\right.\right.}{20a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.20

$$-\frac{\sqrt{a+bx^3}\;{}_2F_1\left(-\frac{5}{3},-\frac{1}{2};-\frac{2}{3};-\frac{bx^3}{a}\right)}{5x^5\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3]/x^6,x]
```

```
[Out] -1/5*(Sqrt[a + b*x^3]*Hypergeometric2F1[-5/3, -1/2, -2/3, -((b*x^3)/a)])/(x
^5*Sqrt[1 + (b*x^3)/a])
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+a}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^3+a}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^3 + a)/x^6, x)
```

maple [A] time = 0.02, size = 317, normalized size = 1.25

$$\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\,b\,\mathrm{EllipticF}\left(\sqrt{\right.}}{20\sqrt{b\,x^3+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/2)/x^6,x)
```

```
[Out] -1/5*(b*x^3+a)^(1/2)/x^5-3/20*b*(b*x^3+a)^(1/2)/a/x^2+1/20*I*b/a*3^(1/2)*(-
a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(
1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1
/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(
1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b
)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^
(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^3+a}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a)/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,x^3+a}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/2)/x^6,x)
```

```
[Out] int((a + b*x^3)^(1/2)/x^6, x)
```

sympy [A] time = 1.19, size = 46, normalized size = 0.18

$$\frac{\sqrt{a}\,\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{5}{3}, -\frac{1}{2}\\ -\frac{2}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/2)/x**6,x)
```

```
[Out] sqrt(a)*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/
(3*x**5*gamma(-2/3))
```

3.381 $\int \frac{\sqrt{a+bx^3}}{x^9}\,dx$

Optimal. Leaf size=277

$$\frac{21b^2\sqrt{a+bx^3}}{320a^2x^2}+\frac{7\,3^{3/4}\sqrt{2+\sqrt{3}}\,b^{8/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{320a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}-\frac{\sqrt{a}}{}$$

```
[Out] -1/8*(b*x^3+a)^(1/2)/x^8-3/80*b*(b*x^3+a)^(1/2)/a/x^5+21/320*b^2*(b*x^3+a)^
(1/2)/a^2/x^2+7/320*3^(3/4)*b^(8/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*
x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*
6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2)))^2)^(1/2)/a^2/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x
)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {277, 325, 218}

$$\frac{21b^2\sqrt{a+bx^3}}{320a^2x^2}+\frac{7\,3^{3/4}\sqrt{2+\sqrt{3}}\,b^{8/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{320a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}-\frac{3b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3]/x^9,x]
```

```
[Out] -Sqrt[a + b*x^3]/(8*x^8) - (3*b*Sqrt[a + b*x^3])/(80*a*x^5) + (21*b^2*Sqrt[
a + b*x^3])/(320*a^2*x^2) + (7*3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(8/3)*(a^(1/3) +
 b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*
x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(320*a^2*Sqrt[(a^
(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
+ b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^3}}{x^9}\,dx &= -\frac{\sqrt{a+bx^3}}{8x^8} + \frac{1}{16}(3b)\int \frac{1}{x^6\sqrt{a+bx^3}}\,dx \\
&= -\frac{\sqrt{a+bx^3}}{8x^8} - \frac{3b\sqrt{a+bx^3}}{80ax^5} - \frac{\left(21b^2\right)\int \frac{1}{x^3\sqrt{a+bx^3}}\,dx}{160a} \\
&= -\frac{\sqrt{a+bx^3}}{8x^8} - \frac{3b\sqrt{a+bx^3}}{80ax^5} + \frac{21b^2\sqrt{a+bx^3}}{320a^2x^2} + \frac{\left(21b^3\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{640a^2} \\
&= -\frac{\sqrt{a+bx^3}}{8x^8} - \frac{3b\sqrt{a+bx^3}}{80ax^5} + \frac{21b^2\sqrt{a+bx^3}}{320a^2x^2} + \frac{7\,3^{3/4}\sqrt{2+\sqrt{3}}\,b^{8/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}}{\left((1+\sqrt{3}\right)}}}{320a^2\sqrt{\frac{\sqrt[3]{a}\left(}{\left((1+\sqrt{3}\right)}}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.18

$$-\frac{\sqrt{a+bx^3}\;{}_2F_1\left(-\frac{8}{3},-\frac{1}{2};-\frac{5}{3};-\frac{bx^3}{a}\right)}{8x^8\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3]/x^9,x]
```

```
[Out] -1/8*(Sqrt[a + b*x^3]*Hypergeometric2F1[-8/3, -1/2, -5/3, -((b*x^3)/a)])/(x
^8*Sqrt[1 + (b*x^3)/a])
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+a}}{x^9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/2)/x^9,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)/x^9, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^3+a}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^3 + a)/x^9, x)
```

maple [A] time = 0.03, size = 339, normalized size = 1.22

$$-\frac{7i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\,b^2\,\mathrm{EllipticF}\left(\right.}{320\sqrt{b\,x^3+a}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/2)/x^9,x)
```

```
[Out] -1/8*(b*x^3+a)^(1/2)/x^8-3/80*b*(b*x^3+a)^(1/2)/a/x^5+21/320*b^2*(b*x^3+a)^
(1/2)/a^2/x^2-7/320*I*b^2/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/
3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-
a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/
2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*
b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2
)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(
I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3
)/b)/b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^3+a}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/2)/x^9,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a)/x^9, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,x^3+a}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/2)/x^9,x)
```

```
[Out] int((a + b*x^3)^(1/2)/x^9, x)
```

sympy [A] time = 1.39, size = 46, normalized size = 0.17

$$\frac{\sqrt{a}\,\Gamma\left(-\frac{8}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{8}{3}, -\frac{1}{2}\\ -\frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3x^8\Gamma\left(-\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/2)/x**9,x)
```

```
[Out] sqrt(a)*gamma(-8/3)*hyper((-8/3, -1/2), (-5/3,), b*x**3*exp_polar(I*pi)/a)/
(3*x**8*gamma(-5/3))
```

3.382 $\int x^7 \sqrt{a+bx^3}\, dx$

Optimal. Leaf size=535

$$\frac{80\sqrt{2}\,3^{3/4}a^{10/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{1729b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{120\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{10/}}{}$$

```
[Out] -60/1729*a^2*x^2*(b*x^3+a)^(1/2)/b^2+6/247*a*x^5*(b*x^3+a)^(1/2)/b+2/19*x^8
*(b*x^3+a)^(1/2)+240/1729*a^3*(b*x^3+a)^(1/2)/b^(8/3)/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2)))+80/1729*3^(3/4)*a^(10/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*
x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/
2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
^2)^(1/2)/b^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a
^(1/3)*(1+3^(1/2)))^2)^(1/2)-120/1729*3^(1/4)*a^(10/3)*(a^(1/3)+b^(1/3)*x)*
EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I
*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)
*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(8/3)/(b*x^3+a)^(1/2)/(a^(
1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 535, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {279, 321, 303, 218, 1877}

$$-\frac{60a^2x^2\sqrt{a+bx^3}}{1729b^2}+\frac{240a^3\sqrt{a+bx^3}}{1729b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{80\sqrt{2}\,3^{3/4}a^{10/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-}\right.}{1729b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*Sqrt[a + b*x^3],x]
```

```
[Out] (-60*a^2*x^2*Sqrt[a + b*x^3])/(1729*b^2) + (6*a*x^5*Sqrt[a + b*x^3])/(247*b
) + (2*x^8*Sqrt[a + b*x^3])/19 + (240*a^3*Sqrt[a + b*x^3])/(1729*b^(8/3)*((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (120*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(10/3
)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3
) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729
*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*Sqrt[a + b*x^3]) + (80*Sqrt[2]*3^(3/4)*a^(10/3)*(a^(1/3) + b^(1/3
)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(8/3)*Sqrt[(a^(1
/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int x^7\sqrt{a+bx^3}\,dx = \frac{2}{19}x^8\sqrt{a+bx^3} + \frac{1}{19}(3a)\int \frac{x^7}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{6ax^5\sqrt{a+bx^3}}{247b} + \frac{2}{19}x^8\sqrt{a+bx^3} - \frac{\left(30a^2\right)\int \frac{x^4}{\sqrt{a+bx^3}}\,dx}{247b}$$

$$= -\frac{60a^2x^2\sqrt{a+bx^3}}{1729b^2} + \frac{6ax^5\sqrt{a+bx^3}}{247b} + \frac{2}{19}x^8\sqrt{a+bx^3} + \frac{\left(120a^3\right)\int \frac{x}{\sqrt{a+bx^3}}\,dx}{1729b^2}$$

$$= -\frac{60a^2x^2\sqrt{a+bx^3}}{1729b^2} + \frac{6ax^5\sqrt{a+bx^3}}{247b} + \frac{2}{19}x^8\sqrt{a+bx^3} + \frac{\left(120a^3\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{1729b^{7/3}} + \frac{\left(120\right.}{}$$

$$= -\frac{60a^2x^2\sqrt{a+bx^3}}{1729b^2} + \frac{6ax^5\sqrt{a+bx^3}}{247b} + \frac{2}{19}x^8\sqrt{a+bx^3} + \frac{240a^3\sqrt{a+bx^3}}{1729b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} -$$

Mathematica [C] time = 0.08, size = 96, normalized size = 0.18

$$\frac{2x^2\sqrt{a+bx^3}\left(\sqrt{\frac{bx^3}{a}+1}\left(-10a^2+3abx^3+13b^2x^6\right)+10a^2\,{}_2F_1\left(-\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{247b^2\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*Sqrt[a + b*x^3],x]
```

```
[Out] (2*x^2*Sqrt[a + b*x^3]*(Sqrt[1 + (b*x^3)/a]*(-10*a^2 + 3*a*b*x^3 + 13*b^2*x
^6) + 10*a^2*Hypergeometric2F1[-1/2, 2/3, 5/3, -((b*x^3)/a)]))/(247*b^2*Sqr
t[1 + (b*x^3)/a])
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{bx^3+a}\,x^7, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)*x^7, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^3+a}\,x^7\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^3 + a)*x^7, x)
```

maple [A] time = 0.02, size = 491, normalized size = 0.92

$$80i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(}{}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}$$

$$\frac{2\sqrt{b\,x^3+a}\,x^8}{19}+\frac{6\sqrt{b\,x^3+a}\,a\,x^5}{247b}-\frac{60\sqrt{b\,x^3+a}\,a^2x^2}{1729b^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/2)*x^7,x)
```

```
[Out] 2/19*x^8*(b*x^3+a)^(1/2)+6/247*a*x^5*(b*x^3+a)^(1/2)/b-60/1729*a^2*x^2*(b*x
^3+a)^(1/2)/b^2-80/1729*I*a^3/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)
```

```
^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((
x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))
^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/
(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/
2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I
*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b
^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+
(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(
1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^
(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^3 + a}\, x^7\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a)*x^7, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^7 \sqrt{b\,x^3 + a}\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^3)^(1/2),x)
```

```
[Out] int(x^7*(a + b*x^3)^(1/2), x)
```

sympy [A] time = 2.38, size = 39, normalized size = 0.07

$$\frac{\sqrt{a}\, x^8 \Gamma\left(\frac{8}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{8}{3} \\ \frac{11}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**3+a)**(1/2),x)
```

```
[Out] sqrt(a)*x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(11/3))
```

3.383 $\int x^4\sqrt{a+bx^3}\,dx$

Optimal. Leaf size=511

$$-\frac{8\sqrt{2}\,3^{3/4}a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{91b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{7/3}\left($$

```
[Out] 6/91*a*x^2*(b*x^3+a)^(1/2)/b+2/13*x^5*(b*x^3+a)^(1/2)-24/91*a^2*(b*x^3+a)^(
1/2)/b^(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-8/91*3^(3/4)*a^(7/3)*(a^(1/3)+
b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(
b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a
^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+12/91*3^(1/4)*a^
(7/3)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3
)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)
-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^
(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 511, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {279, 321, 303, 218, 1877}

$$-\frac{24a^2\sqrt{a+bx^3}}{91b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{8\sqrt{2}\,3^{3/4}a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7\right.}{91b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[a + b*x^3],x]
```

```
[Out] (6*a*x^2*Sqrt[a + b*x^3])/(91*b) + (2*x^5*Sqrt[a + b*x^3])/13 - (24*a^2*Sqr
t[a + b*x^3])/(91*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (12*3^(1/4
)*Sqrt[2 - Sqrt[3]]*a^(7/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[Ar
cSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)], -7 - 4*Sqrt[3]])/(91*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (8*Sqrt[2]*3^(3/4)*a^(
7/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(
1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(9
1*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int x^4\sqrt{a+bx^3}\,dx = \frac{2}{13}x^5\sqrt{a+bx^3} + \frac{1}{13}(3a)\int \frac{x^4}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{6ax^2\sqrt{a+bx^3}}{91b} + \frac{2}{13}x^5\sqrt{a+bx^3} - \frac{\left(12a^2\right)\int \frac{x}{\sqrt{a+bx^3}}\,dx}{91b}$$

$$= \frac{6ax^2\sqrt{a+bx^3}}{91b} + \frac{2}{13}x^5\sqrt{a+bx^3} - \frac{\left(12a^2\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{91b^{4/3}} - \frac{\left(12\sqrt{2\left(2-\sqrt{3}\right)}\,a^{7/3}\right)\int$$

$$= \frac{6ax^2\sqrt{a+bx^3}}{91b} + \frac{2}{13}x^5\sqrt{a+bx^3} - \frac{24a^2\sqrt{a+bx^3}}{91b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} + \frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{7/3}\left(\sqrt[3]{a}\right.}{}$$

Mathematica [C] time = 0.08, size = 64, normalized size = 0.13

$$\frac{2x^2\sqrt{a+bx^3}\left(-\frac{a\,{}_2F_1\left(-\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}}+a+bx^3\right)}{13b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[a + b*x^3],x]
```

```
[Out] (2*x^2*Sqrt[a + b*x^3]*(a + b*x^3 - (a*Hypergeometric2F1[-1/2, 2/3, 5/3, -(
(b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(13*b)
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{bx^3+a}\,x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)*x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^3+a}\,x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^3 + a)*x^4, x)
```

maple [A] time = 0.02, size = 471, normalized size = 0.92

$$8i\sqrt{3}\,\left(-ab^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-ab^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-ab^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-ab^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(}{}\right.}}$$

$$\frac{2\sqrt{b\,x^3+a}\,x^5}{13}+\frac{6\sqrt{b\,x^3+a}\,a\,x^2}{91b}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/2)*x^4,x)
```

```
[Out] 2/13*x^5*(b*x^3+a)^(1/2)+6/91*a*x^2*(b*x^3+a)^(1/2)/b+8/91*I*a^2/b^2*3^(1/2
)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)
*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)
/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I
*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*
((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^(1/
2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b
^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3
^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(
I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^
```

```
(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/
2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^3 + a}\, x^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a)*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 \sqrt{b\,x^3 + a}\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^3)^(1/2),x)
```

```
[Out] int(x^4*(a + b*x^3)^(1/2), x)
```

sympy [A] time = 1.24, size = 39, normalized size = 0.08

$$\frac{\sqrt{a}\, x^5 \Gamma\left(\frac{5}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{5}{3} \\ \frac{8}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)**(1/2),x)
```

```
[Out] sqrt(a)*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(8/3))
```

3.384 $\int x\sqrt{a+bx^3}\,dx$

Optimal. Leaf size=487

$$\frac{2\sqrt{2}\,3^{3/4}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}\right.}{}$$

```
[Out] 2/7*x^2*(b*x^3+a)^(1/2)+6/7*a*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2)))+2/7*3^(3/4)*a^(4/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(
1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((
a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(
1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3
)*(1+3^(1/2)))^2)^(1/2)-3/7*3^(1/4)*a^(4/3)*(a^(1/3)+b^(1/3)*x)*EllipticE((
b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*
I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1
/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/
3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 487, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {279, 303, 218, 1877}

$$\frac{2\sqrt{2}\,3^{3/4}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*x^3],x]
```

```
[Out] (2*x^2*Sqrt[a + b*x^3])/7 + (6*a*Sqrt[a + b*x^3])/(7*b^(2/3)*((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(a^(1/3) + b^
(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*b^(2/3)*Sqrt[(a^(
1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
 b*x^3]) + (2*Sqrt[2]*3^(3/4)*a^(4/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*El
lipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*
x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int x\sqrt{a+bx^3}\,dx = \frac{2}{7}x^2\sqrt{a+bx^3} + \frac{1}{7}(3a)\int \frac{x}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{2}{7}x^2\sqrt{a+bx^3} + \frac{(3a)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{7\sqrt[3]{b}} + \frac{\left(3\sqrt{2\left(2-\sqrt{3}\right)}\,a^{4/3}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{7\sqrt[3]{b}}$$

$$= \frac{2}{7}x^2\sqrt{a+bx^3} + \frac{6a\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\right.}}}{7b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\right.}}}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.10

$$\frac{x^2\sqrt{a+bx^3}\;{}_2F_1\left(-\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + b*x^3],x]
```

```
[Out] (x^2*Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, 2/3, 5/3, -((b*x^3)/a)])/(2*Sq
rt[1 + (b*x^3)/a])
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{bx^3+a}\,x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^3 + a}\, x\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^3 + a)*x, x)
```

maple [A] time = 0.02, size = 451, normalized size = 0.93

$$2i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{}}{\left(-a\,b^2\right)^{\frac{1}{3}}}}$$

$$\frac{2\sqrt{b\,x^3+a}\;x^2}{7}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/2)*x,x)
```

```
[Out] 2/7*x^2*(b*x^3+a)^(1/2)-2/7*I*a*3^(1/2)*(-a*b^2)^(1/3)/b*(I*(x+1/2*(-a*b^2)
^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((
x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))
^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/
(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/
2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I
*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b
^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+
(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(
1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^
(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^3 + a}\, x\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\sqrt{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^3)^(1/2),x)
```

```
[Out] int(x*(a + b*x^3)^(1/2), x)
```

sympy [A] time = 1.24, size = 39, normalized size = 0.08

$$\frac{\sqrt{a}\,x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**(1/2),x)
```

```
[Out] sqrt(a)*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(5/3))
```

3.385 $\int \frac{\sqrt{a+bx^3}}{x^2}\,dx$

Optimal. Leaf size=479

$$\frac{\sqrt{2}\,3^{3/4}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}} - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,\sqrt[3]{b}}{\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}$$

```
[Out] -(b*x^3+a)^(1/2)/x+3*b^(1/3)*(b*x^3+a)^(1/2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))
)+3^(3/4)*a^(1/3)*b^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*
(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/
3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/
(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))
)^2)^(1/2)-3/2*3^(1/4)*a^(1/3)*b^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/
3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1
/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)
/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 479, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {277, 303, 218, 1877}

$$\frac{\sqrt{2}\,3^{3/4}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}} - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,\sqrt[3]{b}}{\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3]/x^2,x]
```

```
[Out] -(Sqrt[a + b*x^3]/x) + (3*b^(1/3)*Sqrt[a + b*x^3])/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*b^(1/3)*(a^(1/3) + b^(1/
3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*Sqrt[(a^(1/3)*(a^(1/
3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) +
(Sqrt[2]*3^(3/4)*a^(1/3)*b^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)], -7 - 4*Sqrt[3]])/(Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}}{x^2}\,dx = -\frac{\sqrt{a+bx^3}}{x} + \frac{1}{2}(3b)\int \frac{x}{\sqrt{a+bx^3}}\,dx$$

$$= -\frac{\sqrt{a+bx^3}}{x} + \frac{1}{2}\left(3b^{2/3}\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a} + \sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx + \frac{\left(3\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,b^{2/3}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{\sqrt{2}}$$

$$= -\frac{\sqrt{a+bx^3}}{x} + \frac{3\sqrt[3]{b}\,\sqrt{a+bx^3}}{\left(1+\sqrt{3}\right)\sqrt[3]{a} + \sqrt[3]{b}\,x} - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,\sqrt[3]{b}\left(\sqrt[3]{a} + \sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3} - \sqrt[3]{a}\,\sqrt[3]{b}\,x + b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a} + \sqrt[3]{b}\,x\right)^2}}}{2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a} + \sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a} + \sqrt[3]{b}\,x\right)^2}}\sqrt{}}$$

Mathematica [C] time = 0.01, size = 49, normalized size = 0.10

$$-\frac{\sqrt{a+bx^3}\;{}_2F_1\left(-\frac{1}{2}, -\frac{1}{3}; \frac{2}{3}; -\frac{bx^3}{a}\right)}{x\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3]/x^2,x]
```

```
[Out] -((Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, -1/3, 2/3, -((b*x^3)/a)])/(x*Sqr
t[1 + (b*x^3)/a]))
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+a}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^3+a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^3 + a)/x^2, x)
```

maple [A] time = 0.02, size = 447, normalized size = 0.93

$$i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}$$

$$-\frac{\sqrt{b\,x^3+a}}{x}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/2)/x^2,x)
```

```
[Out] -(b*x^3+a)^(1/2)/x-I*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*
I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1
/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x
+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3
)*b)^(1/2)/(b*x^3+a)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(
1/3)/b)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*
b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3
/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3
)/b*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)
^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(
-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^3+a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a)/x^2, x)
```

mupad [B] time = 1.27, size = 40, normalized size = 0.08

$$\frac{2\sqrt{b\,x^3+a}\;{}_2\mathrm{F}_1\left(-\frac{1}{2},-\frac{1}{6};\;\frac{5}{6};\;-\frac{a}{b\,x^3}\right)}{x\sqrt{\frac{a}{b\,x^3}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/2)/x^2,x)
```

```
[Out] (2*(a + b*x^3)^(1/2)*hypergeom([-1/2, -1/6], 5/6, -a/(b*x^3)))/(x*(a/(b*x^3
) + 1)^(1/2))
```

sympy [A] time = 1.70, size = 41, normalized size = 0.09

$$\frac{\sqrt{a}\,\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\,\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/2)/x**2,x)
```

```
[Out] sqrt(a)*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(
3*x*gamma(2/3))
```

3.386 $\int \frac{\sqrt{a+bx^3}}{x^5}\,dx$

Optimal. Leaf size=511

$$\frac{3^{3/4}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\,a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}} - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{}\right.}{}$$

```
[Out] -1/4*(b*x^3+a)^(1/2)/x^4-3/8*b*(b*x^3+a)^(1/2)/a/x+3/8*b^(4/3)*(b*x^3+a)^(1
/2)/a/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+1/8*3^(3/4)*b^(4/3)*(a^(1/3)+b^(1/3)*
x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))
),I*3^(1/2)+2*I)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3
)*(1+3^(1/2)))^2)^(1/2)/a^(2/3)*2^(1/2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b
^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-3/16*3^(1/4)*b^(4/3)*(a^
(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*
b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*
x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2
)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 511, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {277, 325, 303, 218, 1877}

$$\frac{3^{3/4}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\,a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}} - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3]/x^5,x]
```

```
[Out] -Sqrt[a + b*x^3]/(4*x^4) - (3*b*Sqrt[a + b*x^3])/(8*a*x) + (3*b^(4/3)*Sqrt[
a + b*x^3])/(8*a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3*3^(1/4)*Sqrt[2 -
 Sqrt[3]]*b^(4/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 -
 Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4
*Sqrt[3]])/(16*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (3^(3/4)*b^(4/3)*(a^(1/3) + b^(1
/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(4*Sqrt[2]*a^(2/3)*Sqr
t[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sq
rt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
 + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
  b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
  x]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}}{x^5}\,dx = -\frac{\sqrt{a+bx^3}}{4x^4} + \frac{1}{8}(3b)\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx$$

$$= -\frac{\sqrt{a+bx^3}}{4x^4} - \frac{3b\sqrt{a+bx^3}}{8ax} + \frac{\left(3b^2\right)\int \frac{x}{\sqrt{a+bx^3}}\,dx}{16a}$$

$$= -\frac{\sqrt{a+bx^3}}{4x^4} - \frac{3b\sqrt{a+bx^3}}{8ax} + \frac{\left(3b^{5/3}\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{16a} + \frac{\left(3\sqrt{\frac{1}{2}\left(2-\sqrt{3}\right)}\,b^{5/3}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{8a^{2/3}}$$

$$= -\frac{\sqrt{a+bx^3}}{4x^4} - \frac{3b\sqrt{a+bx^3}}{8ax} + \frac{3b^{4/3}\sqrt{a+bx^3}}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{}}{16a^{2/3}}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.10

$$-\frac{\sqrt{a+bx^3}\;{}_2F_1\left(-\frac{4}{3},-\frac{1}{2};-\frac{1}{3};-\frac{bx^3}{a}\right)}{4x^4\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3]/x^5,x]
```

```
[Out] -1/4*(Sqrt[a + b*x^3]*Hypergeometric2F1[-4/3, -1/2, -1/3, -((b*x^3)/a)])/(x
^4*Sqrt[1 + (b*x^3)/a])
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+a}}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)/x^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^3+a}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^3 + a)/x^5, x)
```

maple [A] time = 0.02, size = 469, normalized size = 0.92

$$-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\left(\left(-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/2)/x^5,x)
```

```
[Out] -1/4*(b*x^3+a)^(1/2)/x^4-3/8*b*(b*x^3+a)^(1/2)/a/x-1/8*I*b/a*3^(1/2)*(-a*b^
2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)
/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I
*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)
*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*((-3/2*(
-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^(1/2)*(I*(x
+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3
)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(
```

```
-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(I*(x+1/2
*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)
^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b
^2)^(1/3)/b)/b)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^3+a}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a)/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,x^3+a}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/2)/x^5,x)
```

```
[Out] int((a + b*x^3)^(1/2)/x^5, x)
```

sympy [A] time = 3.42, size = 46, normalized size = 0.09

$$\frac{\sqrt{a}\,\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{4}{3}, -\frac{1}{2}\\ -\frac{1}{3}\end{matrix}\right| \frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/2)/x**5,x)
```

```
[Out] sqrt(a)*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/
(3*x**4*gamma(-1/3))
```

3.387 $\int x^{11}\left(a+bx^3\right)^{3/2}\,dx$

Optimal. Leaf size=80

$$-\frac{2a^3\left(a+bx^3\right)^{5/2}}{15b^4}+\frac{2a^2\left(a+bx^3\right)^{7/2}}{7b^4}+\frac{2\left(a+bx^3\right)^{11/2}}{33b^4}-\frac{2a\left(a+bx^3\right)^{9/2}}{9b^4}$$

```
[Out] -2/15*a^3*(b*x^3+a)^(5/2)/b^4+2/7*a^2*(b*x^3+a)^(7/2)/b^4-2/9*a*(b*x^3+a)^(
9/2)/b^4+2/33*(b*x^3+a)^(11/2)/b^4
```

Rubi [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{2a^2\left(a+bx^3\right)^{7/2}}{7b^4}-\frac{2a^3\left(a+bx^3\right)^{5/2}}{15b^4}+\frac{2\left(a+bx^3\right)^{11/2}}{33b^4}-\frac{2a\left(a+bx^3\right)^{9/2}}{9b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*a^3*(a + b*x^3)^(5/2))/(15*b^4) + (2*a^2*(a + b*x^3)^(7/2))/(7*b^4) - (
2*a*(a + b*x^3)^(9/2))/(9*b^4) + (2*(a + b*x^3)^(11/2))/(33*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{11}\left(a+bx^3\right)^{3/2}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int x^3(a+bx)^{3/2}\,dx, x, x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int\left(-\frac{a^3(a+bx)^{3/2}}{b^3}+\frac{3a^2(a+bx)^{5/2}}{b^3}-\frac{3a(a+bx)^{7/2}}{b^3}+\frac{(a+bx)^{9/2}}{b^3}\right)dx, x, x^3\right)\\ &= -\frac{2a^3\left(a+bx^3\right)^{5/2}}{15b^4}+\frac{2a^2\left(a+bx^3\right)^{7/2}}{7b^4}-\frac{2a\left(a+bx^3\right)^{9/2}}{9b^4}+\frac{2\left(a+bx^3\right)^{11/2}}{33b^4}\end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.62

$$\frac{2\left(a+bx^3\right)^{5/2}\left(-16a^3+40a^2bx^3-70ab^2x^6+105b^3x^9\right)}{3465b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(a + b*x^3)^(5/2)*(-16*a^3 + 40*a^2*b*x^3 - 70*a*b^2*x^6 + 105*b^3*x^9))
/(3465*b^4)
```

fricas [A] time = 0.78, size = 68, normalized size = 0.85

$$\frac{2\left(105\,b^5x^{15}+140\,ab^4x^{12}+5\,a^2b^3x^9-6\,a^3b^2x^6+8\,a^4bx^3-16\,a^5\right)\sqrt{bx^3+a}}{3465\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3465*(105*b^5*x^15 + 140*a*b^4*x^12 + 5*a^2*b^3*x^9 - 6*a^3*b^2*x^6 + 8*a
^4*b*x^3 - 16*a^5)*sqrt(b*x^3 + a)/b^4
```

giac [A] time = 0.16, size = 57, normalized size = 0.71

$$\frac{2\left(105\left(bx^3+a\right)^{\frac{11}{2}}-385\left(bx^3+a\right)^{\frac{9}{2}}a+495\left(bx^3+a\right)^{\frac{7}{2}}a^2-231\left(bx^3+a\right)^{\frac{5}{2}}a^3\right)}{3465\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3465*(105*(b*x^3 + a)^(11/2) - 385*(b*x^3 + a)^(9/2)*a + 495*(b*x^3 + a)^
(7/2)*a^2 - 231*(b*x^3 + a)^(5/2)*a^3)/b^4
```

maple [A] time = 0.01, size = 47, normalized size = 0.59

$$-\frac{2\left(b\,x^3+a\right)^{\frac{5}{2}}\left(-105b^3x^9+70a\,b^2x^6-40a^2b\,x^3+16a^3\right)}{3465b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x^3+a)^(3/2),x)
```

```
[Out] -2/3465*(b*x^3+a)^(5/2)*(-105*b^3*x^9+70*a*b^2*x^6-40*a^2*b*x^3+16*a^3)/b^4
```

maxima [A] time = 1.26, size = 64, normalized size = 0.80

$$\frac{2\left(bx^3+a\right)^{\frac{11}{2}}}{33\,b^4}-\frac{2\left(bx^3+a\right)^{\frac{9}{2}}a}{9\,b^4}+\frac{2\left(bx^3+a\right)^{\frac{7}{2}}a^2}{7\,b^4}-\frac{2\left(bx^3+a\right)^{\frac{5}{2}}a^3}{15\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/33*(b*x^3 + a)^(11/2)/b^4 - 2/9*(b*x^3 + a)^(9/2)*a/b^4 + 2/7*(b*x^3 + a)
^(7/2)*a^2/b^4 - 2/15*(b*x^3 + a)^(5/2)*a^3/b^4
```

mupad [B] time = 1.12, size = 108, normalized size = 1.35

$$\frac{8\,a\,x^{12}\sqrt{b\,x^3+a}}{99}+\frac{2\,b\,x^{15}\sqrt{b\,x^3+a}}{33}-\frac{32\,a^5\sqrt{b\,x^3+a}}{3465\,b^4}+\frac{16\,a^4\,x^3\sqrt{b\,x^3+a}}{3465\,b^3}-\frac{4\,a^3\,x^6\sqrt{b\,x^3+a}}{1155\,b^2}+\frac{2\,a^2\,x^9\sqrt{b\,x^3+a}}{693\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a + b*x^3)^(3/2),x)
```

```
[Out] (8*a*x^12*(a + b*x^3)^(1/2))/99 + (2*b*x^15*(a + b*x^3)^(1/2))/33 - (32*a^5
*(a + b*x^3)^(1/2))/(3465*b^4) + (16*a^4*x^3*(a + b*x^3)^(1/2))/(3465*b^3)
```

```
- (4*a^3*x^6*(a + b*x^3)^(1/2))/(1155*b^2) + (2*a^2*x^9*(a + b*x^3)^(1/2))/
(693*b)
```

sympy [A] time = 9.73, size = 136, normalized size = 1.70

$$\begin{cases} -\frac{32a^5\sqrt{a+bx^3}}{3465b^4} + \frac{16a^4x^3\sqrt{a+bx^3}}{3465b^3} - \frac{4a^3x^6\sqrt{a+bx^3}}{1155b^2} + \frac{2a^2x^9\sqrt{a+bx^3}}{693b} + \frac{8ax^{12}\sqrt{a+bx^3}}{99} + \frac{2bx^{15}\sqrt{a+bx^3}}{33} & \text{for } b \neq 0 \\ \frac{a^{\frac{3}{2}}x^{12}}{12} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x**3+a)**(3/2),x)
```

```
[Out] Piecewise((-32*a**5*sqrt(a + b*x**3)/(3465*b**4) + 16*a**4*x**3*sqrt(a + b*
x**3)/(3465*b**3) - 4*a**3*x**6*sqrt(a + b*x**3)/(1155*b**2) + 2*a**2*x**9*
sqrt(a + b*x**3)/(693*b) + 8*a*x**12*sqrt(a + b*x**3)/99 + 2*b*x**15*sqrt(a
 + b*x**3)/33, Ne(b, 0)), (a**(3/2)*x**12/12, True))
```

3.388 $\int x^8 \left(a+bx^3\right)^{3/2}\,dx$

Optimal. Leaf size=59

$$\frac{2a^2\left(a+bx^3\right)^{5/2}}{15b^3}+\frac{2\left(a+bx^3\right)^{9/2}}{27b^3}-\frac{4a\left(a+bx^3\right)^{7/2}}{21b^3}$$

```
[Out] 2/15*a^2*(b*x^3+a)^(5/2)/b^3-4/21*a*(b*x^3+a)^(7/2)/b^3+2/27*(b*x^3+a)^(9/2
)/b^3
```

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{2a^2\left(a+bx^3\right)^{5/2}}{15b^3}+\frac{2\left(a+bx^3\right)^{9/2}}{27b^3}-\frac{4a\left(a+bx^3\right)^{7/2}}{21b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x^3)^(3/2),x]
```

```
[Out] (2*a^2*(a + b*x^3)^(5/2))/(15*b^3) - (4*a*(a + b*x^3)^(7/2))/(21*b^3) + (2*
(a + b*x^3)^(9/2))/(27*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^8\left(a+bx^3\right)^{3/2}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int x^2(a+bx)^{3/2}\,dx,x,x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int\left(\frac{a^2(a+bx)^{3/2}}{b^2}-\frac{2a(a+bx)^{5/2}}{b^2}+\frac{(a+bx)^{7/2}}{b^2}\right)dx,x,x^3\right)\\ &= \frac{2a^2\left(a+bx^3\right)^{5/2}}{15b^3}-\frac{4a\left(a+bx^3\right)^{7/2}}{21b^3}+\frac{2\left(a+bx^3\right)^{9/2}}{27b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.66

$$\frac{2\left(a+bx^3\right)^{5/2}\left(8a^2-20abx^3+35b^2x^6\right)}{945b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(a + b*x^3)^(5/2)*(8*a^2 - 20*a*b*x^3 + 35*b^2*x^6))/(945*b^3)
```

fricas [A] time = 0.87, size = 57, normalized size = 0.97

$$\frac{2\left(35\,b^4x^{12}+50\,ab^3x^9+3\,a^2b^2x^6-4\,a^3bx^3+8\,a^4\right)\sqrt{bx^3+a}}{945\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/945*(35*b^4*x^12 + 50*a*b^3*x^9 + 3*a^2*b^2*x^6 - 4*a^3*b*x^3 + 8*a^4)*sq
rt(b*x^3 + a)/b^3
```

giac [A] time = 0.16, size = 43, normalized size = 0.73

$$\frac{2\left(35\left(bx^3+a\right)^{\frac{9}{2}}-90\left(bx^3+a\right)^{\frac{7}{2}}a+63\left(bx^3+a\right)^{\frac{5}{2}}a^2\right)}{945\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/945*(35*(b*x^3 + a)^(9/2) - 90*(b*x^3 + a)^(7/2)*a + 63*(b*x^3 + a)^(5/2)
*a^2)/b^3
```

maple [A] time = 0.01, size = 36, normalized size = 0.61

$$\frac{2\left(b\,x^3+a\right)^{\frac{5}{2}}\left(35b^2x^6-20ab\,x^3+8a^2\right)}{945b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^3+a)^(3/2),x)
```

```
[Out] 2/945*(b*x^3+a)^(5/2)*(35*b^2*x^6-20*a*b*x^3+8*a^2)/b^3
```

maxima [A] time = 1.38, size = 47, normalized size = 0.80

$$\frac{2\left(bx^3+a\right)^{\frac{9}{2}}}{27\,b^3}-\frac{4\left(bx^3+a\right)^{\frac{7}{2}}a}{21\,b^3}+\frac{2\left(bx^3+a\right)^{\frac{5}{2}}a^2}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/27*(b*x^3 + a)^(9/2)/b^3 - 4/21*(b*x^3 + a)^(7/2)*a/b^3 + 2/15*(b*x^3 + a
)^(5/2)*a^2/b^3
```

mupad [B] time = 1.07, size = 88, normalized size = 1.49

$$\frac{20\,a\,x^9\,\sqrt{b\,x^3+a}}{189}+\frac{2\,b\,x^{12}\,\sqrt{b\,x^3+a}}{27}+\frac{16\,a^4\,\sqrt{b\,x^3+a}}{945\,b^3}-\frac{8\,a^3\,x^3\,\sqrt{b\,x^3+a}}{945\,b^2}+\frac{2\,a^2\,x^6\,\sqrt{b\,x^3+a}}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*x^3)^(3/2),x)
```

```
[Out] (20*a*x^9*(a + b*x^3)^(1/2))/189 + (2*b*x^12*(a + b*x^3)^(1/2))/27 + (16*a^
4*(a + b*x^3)^(1/2))/(945*b^3) - (8*a^3*x^3*(a + b*x^3)^(1/2))/(945*b^2) +
(2*a^2*x^6*(a + b*x^3)^(1/2))/(315*b)
```

sympy [**A**] time = 4.81, size = 112, normalized size = 1.90

$$\begin{cases} \frac{16a^4\sqrt{a+bx^3}}{945b^3} - \frac{8a^3x^3\sqrt{a+bx^3}}{945b^2} + \frac{2a^2x^6\sqrt{a+bx^3}}{315b} + \frac{20ax^9\sqrt{a+bx^3}}{189} + \frac{2bx^{12}\sqrt{a+bx^3}}{27} & \text{for } b \neq 0 \\ \frac{a^{\frac{3}{2}}x^9}{9} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**3+a)**(3/2),x)
```

```
[Out] Piecewise((16*a**4*sqrt(a + b*x**3)/(945*b**3) - 8*a**3*x**3*sqrt(a + b*x**
3)/(945*b**2) + 2*a**2*x**6*sqrt(a + b*x**3)/(315*b) + 20*a*x**9*sqrt(a + b
*x**3)/189 + 2*b*x**12*sqrt(a + b*x**3)/27, Ne(b, 0)), (a**(3/2)*x**9/9, Tr
ue))
```

3.389 $\int x^5 \left(a+bx^3\right)^{3/2}\,dx$

Optimal. Leaf size=38

$$\frac{2\left(a+bx^3\right)^{7/2}}{21b^2}-\frac{2a\left(a+bx^3\right)^{5/2}}{15b^2}$$

```
[Out] -2/15*a*(b*x^3+a)^(5/2)/b^2+2/21*(b*x^3+a)^(7/2)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{2\left(a+bx^3\right)^{7/2}}{21b^2}-\frac{2a\left(a+bx^3\right)^{5/2}}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*a*(a + b*x^3)^(5/2))/(15*b^2) + (2*(a + b*x^3)^(7/2))/(21*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+bx^3\right)^{3/2}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int x(a+bx)^{3/2}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int\left(-\frac{a(a+bx)^{3/2}}{b}+\frac{(a+bx)^{5/2}}{b}\right)dx, x, x^3\right)\\ &= -\frac{2a\left(a+bx^3\right)^{5/2}}{15b^2}+\frac{2\left(a+bx^3\right)^{7/2}}{21b^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 0.74

$$\frac{2\left(a+bx^3\right)^{5/2}\left(5bx^3-2a\right)}{105b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(a + b*x^3)^(5/2)*(-2*a + 5*b*x^3))/(105*b^2)
```

fricas [A] time = 0.82, size = 45, normalized size = 1.18

$$\frac{2\left(5\,b^3x^9+8\,ab^2x^6+a^2bx^3-2\,a^3\right)\sqrt{bx^3+a}}{105\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/105*(5*b^3*x^9 + 8*a*b^2*x^6 + a^2*b*x^3 - 2*a^3)*sqrt(b*x^3 + a)/b^2
```

giac [A] time = 0.17, size = 29, normalized size = 0.76

$$\frac{2\left(5\left(bx^3+a\right)^{\frac{7}{2}}-7\left(bx^3+a\right)^{\frac{5}{2}}a\right)}{105\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/105*(5*(b*x^3 + a)^(7/2) - 7*(b*x^3 + a)^(5/2)*a)/b^2
```

maple [A] time = 0.00, size = 25, normalized size = 0.66

$$-\frac{2\left(b\,x^3+a\right)^{\frac{5}{2}}\left(-5b\,x^3+2a\right)}{105b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^3+a)^(3/2),x)
```

```
[Out] -2/105*(b*x^3+a)^(5/2)*(-5*b*x^3+2*a)/b^2
```

maxima [A] time = 1.34, size = 30, normalized size = 0.79

$$\frac{2\left(bx^3+a\right)^{\frac{7}{2}}}{21\,b^2}-\frac{2\left(bx^3+a\right)^{\frac{5}{2}}a}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/21*(b*x^3 + a)^(7/2)/b^2 - 2/15*(b*x^3 + a)^(5/2)*a/b^2
```

mupad [B] time = 1.10, size = 68, normalized size = 1.79

$$\frac{16\,a\,x^6\,\sqrt{b\,x^3+a}}{105}+\frac{2\,b\,x^9\,\sqrt{b\,x^3+a}}{21}-\frac{4\,a^3\,\sqrt{b\,x^3+a}}{105\,b^2}+\frac{2\,a^2\,x^3\,\sqrt{b\,x^3+a}}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^3)^(3/2),x)
```

```
[Out] (16*a*x^6*(a + b*x^3)^(1/2))/105 + (2*b*x^9*(a + b*x^3)^(1/2))/21 - (4*a^3*
(a + b*x^3)^(1/2))/(105*b^2) + (2*a^2*x^3*(a + b*x^3)^(1/2))/(105*b)
```

sympy [A] time = 4.02, size = 88, normalized size = 2.32

$$\begin{cases} -\frac{4a^3\sqrt{a+bx^3}}{105b^2}+\frac{2a^2x^3\sqrt{a+bx^3}}{105b}+\frac{16ax^6\sqrt{a+bx^3}}{105}+\frac{2bx^9\sqrt{a+bx^3}}{21} & \text{for } b\neq 0 \\ \frac{a^{\frac{3}{2}}x^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**3+a)**(3/2),x)
```

```
[Out] Piecewise((-4*a**3*sqrt(a + b*x**3)/(105*b**2) + 2*a**2*x**3*sqrt(a + b*x**
3)/(105*b) + 16*a*x**6*sqrt(a + b*x**3)/105 + 2*b*x**9*sqrt(a + b*x**3)/21,
 Ne(b, 0)), (a**(3/2)*x**6/6, True))
```

3.390 $\int x^2 \left(a + bx^3\right)^{3/2} dx$

Optimal. Leaf size=18

$$\frac{2\left(a + bx^3\right)^{5/2}}{15b}$$

```
[Out] 2/15*(b*x^3+a)^(5/2)/b
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$\frac{2\left(a + bx^3\right)^{5/2}}{15b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(a + b*x^3)^(5/2))/(15*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^2 \left(a + bx^3\right)^{3/2} dx = \frac{2\left(a + bx^3\right)^{5/2}}{15b}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{2\left(a + bx^3\right)^{5/2}}{15b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(a + b*x^3)^(5/2))/(15*b)
```

fricas [B] time = 0.87, size = 32, normalized size = 1.78

$$\frac{2\left(b^2x^6 + 2\,abx^3 + a^2\right)\sqrt{bx^3 + a}}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15*(b^2*x^6 + 2*a*b*x^3 + a^2)*sqrt(b*x^3 + a)/b
```

giac [A] time = 0.15, size = 14, normalized size = 0.78

$$\frac{2\left(bx^3 + a\right)^{\frac{5}{2}}}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(3/2),x, algorithm="giac")
[Out] 2/15*(b*x^3 + a)^(5/2)/b
```

maple [A] time = 0.01, size = 15, normalized size = 0.83

$$\frac{2\left(b\,x^3+a\right)^{\frac{5}{2}}}{15b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^(3/2),x)
[Out] 2/15*(b*x^3+a)^(5/2)/b
```

maxima [A] time = 1.35, size = 14, normalized size = 0.78

$$\frac{2\left(bx^3+a\right)^{\frac{5}{2}}}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(3/2),x, algorithm="maxima")
[Out] 2/15*(b*x^3 + a)^(5/2)/b
```

mupad [B] time = 1.09, size = 14, normalized size = 0.78

$$\frac{2\left(b\,x^3+a\right)^{5/2}}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^3)^(3/2),x)
[Out] (2*(a + b*x^3)^(5/2))/(15*b)
```

sympy [A] time = 1.97, size = 65, normalized size = 3.61

$$\begin{cases} \frac{2a^2\sqrt{a+bx^3}}{15b}+\frac{4ax^3\sqrt{a+bx^3}}{15}+\frac{2bx^6\sqrt{a+bx^3}}{15} & \text{for } b\neq 0 \\ \frac{a^{\frac{3}{2}}x^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**(3/2),x)
[Out] Piecewise((2*a**2*sqrt(a + b*x**3)/(15*b) + 4*a*x**3*sqrt(a + b*x**3)/15 +
2*b*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (a**(3/2)*x**3/3, True))
```

3.391 $\int \frac{\left(a+bx^3\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=59

$$-\frac{2}{3}a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{2}{3}a\sqrt{a+bx^3}+\frac{2}{9}\left(a+bx^3\right)^{3/2}$$

```
[Out] 2/9*(b*x^3+a)^(3/2)-2/3*a^(3/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2))+2/3*a*(b*x
^3+a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 50, 63, 208}

$$-\frac{2}{3}a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{2}{3}a\sqrt{a+bx^3}+\frac{2}{9}\left(a+bx^3\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(3/2)/x,x]
```

```
[Out] (2*a*Sqrt[a + b*x^3])/3 + (2*(a + b*x^3)^(3/2))/9 - (2*a^(3/2)*ArcTanh[Sqrt
[a + b*x^3]/Sqrt[a]])/3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^{3/2}}{x}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, x^3\right)\\ &= \frac{2}{9}\left(a+bx^3\right)^{3/2} + \frac{1}{3}a\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^3\right)\\ &= \frac{2}{3}a\sqrt{a+bx^3} + \frac{2}{9}\left(a+bx^3\right)^{3/2} + \frac{1}{3}a^2\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right)\\ &= \frac{2}{3}a\sqrt{a+bx^3} + \frac{2}{9}\left(a+bx^3\right)^{3/2} + \frac{\left(2a^2\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)}{3b}\\ &= \frac{2}{3}a\sqrt{a+bx^3} + \frac{2}{9}\left(a+bx^3\right)^{3/2} - \frac{2}{3}a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.86

$$\frac{2}{9}\left(\sqrt{a+bx^3}\left(4a+bx^3\right) - 3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(3/2)/x,x]
```

```
[Out] (2*(Sqrt[a + b*x^3]*(4*a + b*x^3) - 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[
a]]))/9
```

fricas [A] time = 0.96, size = 103, normalized size = 1.75

$$\left[\frac{1}{3}a^{\frac{3}{2}}\log\left(\frac{bx^3 - 2\sqrt{bx^3+a}\sqrt{a} + 2a}{x^3}\right) + \frac{2}{9}\left(bx^3+4a\right)\sqrt{bx^3+a}, \frac{2}{3}\sqrt{-a}a\arctan\left(\frac{\sqrt{bx^3+a}\sqrt{-a}}{a}\right) + \frac{2}{9}\left(bx^3+4\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/3*a^(3/2)*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2/9*(b*x^
3 + 4*a)*sqrt(b*x^3 + a), 2/3*sqrt(-a)*a*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a)
 + 2/9*(b*x^3 + 4*a)*sqrt(b*x^3 + a)]
```

giac [A] time = 0.17, size = 50, normalized size = 0.85

$$\frac{2a^2\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\sqrt{-a}} + \frac{2}{9}\left(bx^3+a\right)^{\frac{3}{2}} + \frac{2}{3}\sqrt{bx^3+a}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 2/3*a^2*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a) + 2/9*(b*x^3 + a)^(3/2) +
 2/3*sqrt(b*x^3 + a)*a
```

maple [A] time = 0.02, size = 48, normalized size = 0.81

$$\frac{2\sqrt{b\,x^3+a}\;b\,x^3}{9} - \frac{2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3} + \frac{8\sqrt{b\,x^3+a}\;a}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)/x,x)
```

```
[Out] 2/9*b*x^3*(b*x^3+a)^(1/2)+8/9*a*(b*x^3+a)^(1/2)-2/3*a^(3/2)*arctanh((b*x^3+
a)^(1/2)/a^(1/2))
```

maxima [A] time = 2.89, size = 61, normalized size = 1.03

$$\frac{1}{3}\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)+\frac{2}{9}\left(bx^3+a\right)^{\frac{3}{2}}+\frac{2}{3}\sqrt{bx^3+a}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 1/3*a^(3/2)*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a))) +
2/9*(b*x^3 + a)^(3/2) + 2/3*sqrt(b*x^3 + a)*a
```

mupad [B] time = 1.18, size = 68, normalized size = 1.15

$$\frac{a^{3/2}\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3}+\frac{8\,a\,\sqrt{b\,x^3+a}}{9}+\frac{2\,b\,x^3\,\sqrt{b\,x^3+a}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(3/2)/x,x)
```

```
[Out] (a^(3/2)*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2))
)/x^6))/3 + (8*a*(a + b*x^3)^(1/2))/9 + (2*b*x^3*(a + b*x^3)^(1/2))/9
```

sympy [A] time = 6.24, size = 83, normalized size = 1.41

$$\frac{8a^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{9}+\frac{a^{\frac{3}{2}}\log\left(\frac{bx^3}{a}\right)}{3}-\frac{2a^{\frac{3}{2}}\log\left(\sqrt{1+\frac{bx^3}{a}}+1\right)}{3}+\frac{2\sqrt{a}\,bx^3\sqrt{1+\frac{bx^3}{a}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)/x,x)
```

```
[Out] 8*a**(3/2)*sqrt(1 + b*x**3/a)/9 + a**(3/2)*log(b*x**3/a)/3 - 2*a**(3/2)*log
(sqrt(1 + b*x**3/a) + 1)/3 + 2*sqrt(a)*b*x**3*sqrt(1 + b*x**3/a)/9
```

3.392 $\int \frac{\left(a+bx^3\right)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=58

$$-\frac{\left(a+bx^3\right)^{3/2}}{3x^3}+b\sqrt{a+bx^3}-\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$

```
[Out] -1/3*(b*x^3+a)^(3/2)/x^3-b*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+b*(b*x^
3+a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {266, 47, 50, 63, 208}

$$-\frac{\left(a+bx^3\right)^{3/2}}{3x^3}+b\sqrt{a+bx^3}-\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(3/2)/x^4,x]
```

```
[Out] b*Sqrt[a + b*x^3] - (a + b*x^3)^(3/2)/(3*x^3) - Sqrt[a]*b*ArcTanh[Sqrt[a +
b*x^3]/Sqrt[a]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^3\right)^{3/2}}{x^4}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2}\,dx, x, x^3\right) \\
&= -\frac{\left(a+bx^3\right)^{3/2}}{3x^3} + \frac{1}{2}b\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^3\right) \\
&= b\sqrt{a+bx^3} - \frac{\left(a+bx^3\right)^{3/2}}{3x^3} + \frac{1}{2}(ab)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right) \\
&= b\sqrt{a+bx^3} - \frac{\left(a+bx^3\right)^{3/2}}{3x^3} + a\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right) \\
&= b\sqrt{a+bx^3} - \frac{\left(a+bx^3\right)^{3/2}}{3x^3} - \sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.64

$$\frac{2b\left(a+bx^3\right)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{bx^3}{a}+1\right)}{15a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(3/2)/x^4,x]
```

```
[Out] (2*b*(a + b*x^3)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^3)/a])/(15*a
^2)
```

fricas [A] time = 0.82, size = 121, normalized size = 2.09

$$\left[\frac{3\sqrt{a}\,bx^3\log\left(\frac{bx^3-2\sqrt{bx^3+a}\,\sqrt{a}+2a}{x^3}\right)+2\left(2bx^3-a\right)\sqrt{bx^3+a}}{6x^3},\ \frac{3\sqrt{-a}\,bx^3\arctan\left(\frac{\sqrt{bx^3+a}\,\sqrt{-a}}{a}\right)+\left(2bx^3-a\right)\sqrt{bx^3}}{3x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(a)*b*x^3*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) +
2*(2*b*x^3 - a)*sqrt(b*x^3 + a))/x^3, 1/3*(3*sqrt(-a)*b*x^3*arctan(sqrt(b*x
^3 + a)*sqrt(-a)/a) + (2*b*x^3 - a)*sqrt(b*x^3 + a))/x^3]
```

giac [A] time = 0.18, size = 63, normalized size = 1.09

$$\frac{\frac{3ab^2\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{bx^3+a}\,b^2-\frac{\sqrt{bx^3+a}\,ab}{x^3}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*a*b^2*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a) + 2*sqrt(b*x^3 + a)*
b^2 - sqrt(b*x^3 + a)*a*b/x^3)/b
```

maple [A] time = 0.03, size = 49, normalized size = 0.84

$$-\sqrt{a}\, b \operatorname{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)+\frac{2\sqrt{b\,x^3+a}\, b}{3}-\frac{\sqrt{b\,x^3+a}\, a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)/x^4,x)
```

```
[Out] -1/3*a*(b*x^3+a)^(1/2)/x^3+2/3*b*(b*x^3+a)^(1/2)-b*arctanh((b*x^3+a)^(1/2)/
a^(1/2))*a^(1/2)
```

maxima [A] time = 2.99, size = 66, normalized size = 1.14

$$\frac{1}{2}\sqrt{a}\, b \log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)+\frac{2}{3}\sqrt{bx^3+a}\, b-\frac{\sqrt{bx^3+a}\, a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a)*b*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))
+ 2/3*sqrt(b*x^3 + a)*b - 1/3*sqrt(b*x^3 + a)*a/x^3
```

mupad [B] time = 1.31, size = 69, normalized size = 1.19

$$\frac{2\,b\sqrt{b\,x^3+a}}{3}-\frac{a\sqrt{b\,x^3+a}}{3\,x^3}+\frac{\sqrt{a}\; b\;\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(3/2)/x^4,x)
```

```
[Out] (2*b*(a + b*x^3)^(1/2))/3 - (a*(a + b*x^3)^(1/2))/(3*x^3) + (a^(1/2)*b*log(
(((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6))/2
```

sympy [B] time = 5.25, size = 100, normalized size = 1.72

$$-\sqrt{a}\, b \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\, x^{\frac{3}{2}}}\right)-\frac{a^2}{3\sqrt{b}\, x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{a\sqrt{b}}{3x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{2b^{\frac{3}{2}}x^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)/x**4,x)
```

```
[Out] -sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*x**(3/2))) - a**2/(3*sqrt(b)*x**(9/2)*sqr
t(a/(b*x**3) + 1)) + a*sqrt(b)/(3*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*b**(3/
2)*x**(3/2)/(3*sqrt(a/(b*x**3) + 1))
```

3.393 $\int \frac{\left(a+bx^3\right)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=68

$$-\frac{b^2 \tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4\sqrt{a}} - \frac{b\sqrt{a+bx^3}}{4x^3} - \frac{\left(a+bx^3\right)^{3/2}}{6x^6}$$

```
[Out] -1/6*(b*x^3+a)^(3/2)/x^6-1/4*b^2*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)-1
/4*b*(b*x^3+a)^(1/2)/x^3
```

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 47, 63, 208}

$$-\frac{b^2 \tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4\sqrt{a}} - \frac{b\sqrt{a+bx^3}}{4x^3} - \frac{\left(a+bx^3\right)^{3/2}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(3/2)/x^7,x]
```

```
[Out] -(b*Sqrt[a + b*x^3])/(4*x^3) - (a + b*x^3)^(3/2)/(6*x^6) - (b^2*ArcTanh[Sqr
t[a + b*x^3]/Sqrt[a]])/(4*Sqrt[a])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^{3/2}}{x^7}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^3}\,dx, x, x^3\right)\\ &= -\frac{\left(a+bx^3\right)^{3/2}}{6x^6} + \frac{1}{4}b\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx, x, x^3\right)\\ &= -\frac{b\sqrt{a+bx^3}}{4x^3} - \frac{\left(a+bx^3\right)^{3/2}}{6x^6} + \frac{1}{8}b^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right)\\ &= -\frac{b\sqrt{a+bx^3}}{4x^3} - \frac{\left(a+bx^3\right)^{3/2}}{6x^6} + \frac{1}{4}b\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)\\ &= -\frac{b\sqrt{a+bx^3}}{4x^3} - \frac{\left(a+bx^3\right)^{3/2}}{6x^6} - \frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 76, normalized size = 1.12

$$-\frac{2a^2+3b^2x^6\sqrt{\frac{bx^3}{a}+1}\,\tanh^{-1}\left(\sqrt{\frac{bx^3}{a}+1}\right)+7abx^3+5b^2x^6}{12x^6\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(3/2)/x^7,x]
```

```
[Out] -1/12*(2*a^2 + 7*a*b*x^3 + 5*b^2*x^6 + 3*b^2*x^6*Sqrt[1 + (b*x^3)/a]*ArcTan
h[Sqrt[1 + (b*x^3)/a]])/(x^6*Sqrt[a + b*x^3])
```

fricas [A] time = 0.77, size = 138, normalized size = 2.03

$$\left[\frac{3\sqrt{a}\,b^2x^6\log\left(\frac{bx^3-2\sqrt{bx^3+a}\sqrt{a}+2a}{x^3}\right)-2\left(5abx^3+2a^2\right)\sqrt{bx^3+a}}{24ax^6}, \frac{3\sqrt{-a}\,b^2x^6\arctan\left(\frac{\sqrt{bx^3+a}\sqrt{-a}}{a}\right)-\left(5abx^3+2a\right.}{12ax^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/24*(3*sqrt(a)*b^2*x^6*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3)
 - 2*(5*a*b*x^3 + 2*a^2)*sqrt(b*x^3 + a))/(a*x^6), 1/12*(3*sqrt(-a)*b^2*x^6
*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) - (5*a*b*x^3 + 2*a^2)*sqrt(b*x^3 + a))/
(a*x^6)]
```

giac [A] time = 0.16, size = 70, normalized size = 1.03

$$\frac{\frac{3b^3\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{5\left(bx^3+a\right)^{\frac{3}{2}}b^3-3\sqrt{bx^3+a}\,ab^3}{b^2x^6}}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/12*(3*b^3*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a) - (5*(b*x^3 + a)^(3/2
)*b^3 - 3*sqrt(b*x^3 + a)*a*b^3)/(b^2*x^6))/b
```

maple [A] time = 0.03, size = 54, normalized size = 0.79

$$-\frac{b^2 \operatorname{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{4\sqrt{a}} - \frac{5\sqrt{b\,x^3+a}\;b}{12x^3} - \frac{\sqrt{b\,x^3+a}\;a}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)/x^7,x)
```

```
[Out] -1/6*a*(b*x^3+a)^(1/2)/x^6-5/12*b*(b*x^3+a)^(1/2)/x^3-1/4*b^2*arctanh((b*x^
3+a)^(1/2)/a^(1/2))/a^(1/2)
```

maxima [A] time = 2.90, size = 98, normalized size = 1.44

$$\frac{b^2 \log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{8\sqrt{a}} - \frac{5\left(bx^3+a\right)^{\frac{3}{2}}b^2 - 3\sqrt{bx^3+a}\,ab^2}{12\left(\left(bx^3+a\right)^2 - 2\left(bx^3+a\right)a + a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/sqrt(a
) - 1/12*(5*(b*x^3 + a)^(3/2)*b^2 - 3*sqrt(b*x^3 + a)*a*b^2)/((b*x^3 + a)^2
 - 2*(b*x^3 + a)*a + a^2)
```

mupad [B] time = 1.44, size = 74, normalized size = 1.09

$$\frac{b^2 \ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{8\sqrt{a}} - \frac{5\,b\sqrt{b\,x^3+a}}{12\,x^3} - \frac{a\sqrt{b\,x^3+a}}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(3/2)/x^7,x)
```

```
[Out] (b^2*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^
6))/(8*a^(1/2)) - (5*b*(a + b*x^3)^(1/2))/(12*x^3) - (a*(a + b*x^3)^(1/2))/
(6*x^6)
```

sympy [A] time = 7.54, size = 78, normalized size = 1.15

$$-\frac{a\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{6x^{\frac{9}{2}}} - \frac{5b^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}{12x^{\frac{3}{2}}} - \frac{b^2 \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)/x**7,x)
```

```
[Out] -a*sqrt(b)*sqrt(a/(b*x**3) + 1)/(6*x**(9/2)) - 5*b**(3/2)*sqrt(a/(b*x**3) +
 1)/(12*x**(3/2)) - b**2*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(4*sqrt(a))
```

3.394 $\int x^6 \left(a+bx^3\right)^{3/2}\,dx$

Optimal. Leaf size=296

$$-\frac{432a^3x\sqrt{a+bx^3}}{21505b^2}+\frac{54a^2x^4\sqrt{a+bx^3}}{4301b}+\frac{288\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^4\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{}\right.\right.\right.}{\sqrt[3]{b}\,x+\left(1+\sqrt{}\right.}\right.$$

$$21505b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}$$

```
[Out] 2/23*x^7*(b*x^3+a)^(3/2)-432/21505*a^3*x*(b*x^3+a)^(1/2)/b^2+54/4301*a^2*x^
4*(b*x^3+a)^(1/2)/b+18/391*a*x^7*(b*x^3+a)^(1/2)+288/21505*3^(3/4)*a^4*(a^(
1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3
)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(7/3)/(b*x
^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)
^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {279, 321, 218}

$$-\frac{432a^3x\sqrt{a+bx^3}}{21505b^2}+\frac{288\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^4\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right.}{21505b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x^3)^(3/2),x]
```

```
[Out] (-432*a^3*x*Sqrt[a + b*x^3])/(21505*b^2) + (54*a^2*x^4*Sqrt[a + b*x^3])/(43
01*b) + (18*a*x^7*Sqrt[a + b*x^3])/391 + (2*x^7*(a + b*x^3)^(3/2))/23 + (28
8*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^4*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Elliptic
F[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)], -7 - 4*Sqrt[3]])/(21505*b^(7/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x)
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int x^6\left(a+bx^3\right)^{3/2}\,dx &= \frac{2}{23}x^7\left(a+bx^3\right)^{3/2} + \frac{1}{23}(9a)\int x^6\sqrt{a+bx^3}\,dx \\
&= \frac{18}{391}ax^7\sqrt{a+bx^3} + \frac{2}{23}x^7\left(a+bx^3\right)^{3/2} + \frac{1}{391}\left(27a^2\right)\int \frac{x^6}{\sqrt{a+bx^3}}\,dx \\
&= \frac{54a^2x^4\sqrt{a+bx^3}}{4301b} + \frac{18}{391}ax^7\sqrt{a+bx^3} + \frac{2}{23}x^7\left(a+bx^3\right)^{3/2} - \frac{\left(216a^3\right)\int \frac{x^3}{\sqrt{a+bx^3}}\,dx}{4301b} \\
&= -\frac{432a^3x\sqrt{a+bx^3}}{21505b^2} + \frac{54a^2x^4\sqrt{a+bx^3}}{4301b} + \frac{18}{391}ax^7\sqrt{a+bx^3} + \frac{2}{23}x^7\left(a+bx^3\right)^{3/2} + \frac{\left(43\right.}{} \\
&= -\frac{432a^3x\sqrt{a+bx^3}}{21505b^2} + \frac{54a^2x^4\sqrt{a+bx^3}}{4301b} + \frac{18}{391}ax^7\sqrt{a+bx^3} + \frac{2}{23}x^7\left(a+bx^3\right)^{3/2} + \frac{288}{}
\end{aligned}$$

Mathematica [C] time = 0.08, size = 79, normalized size = 0.27

$$\frac{2x\sqrt{a+bx^3}\left(\frac{8a^3\,{}_2F_1\left(-\frac{3}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}} - \left(8a-17bx^3\right)\left(a+bx^3\right)^2\right)}{391b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x^3)^(3/2),x]
```

```
[Out] (2*x*Sqrt[a + b*x^3]*(-((8*a - 17*b*x^3)*(a + b*x^3)^2) + (8*a^3*Hypergeome
tric2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(391*b^2)
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^9+ax^6\right)\sqrt{bx^3+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^9 + a*x^6)*sqrt(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{3}{2}}x^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(3/2)*x^6, x)
```

maple [A] time = 0.02, size = 355, normalized size = 1.20

$$288i\sqrt{3}\ \left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x}{-\frac{3\left(-a\,b^2\right.}{2b}}}$$

$$\frac{2\sqrt{b\,x^3+a}\ b\,x^{10}}{23}+\frac{52\sqrt{b\,x^3+a}\ a\,x^7}{391}+\frac{54\sqrt{b\,x^3+a}\ a^2x^4}{4301b}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^3+a)^(3/2),x)
```

```
[Out] 2/23*b*x^10*(b*x^3+a)^(1/2)+52/391*a*x^7*(b*x^3+a)^(1/2)+54/4301*a^2*x^4*(b
*x^3+a)^(1/2)/b-432/21505*a^3*x*(b*x^3+a)^(1/2)/b^2-288/21505*I*a^4/b^3*3^(
1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)
/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1
/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/
2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/
2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^
(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-
a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{3}{2}}x^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(3/2)*x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^6\left(b\,x^3+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b*x^3)^(3/2),x)
```

```
[Out] int(x^6*(a + b*x^3)^(3/2), x)
```

sympy [A] time = 1.84, size = 39, normalized size = 0.13

$$\frac{a^{\frac{3}{2}}x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**3+a)**(3/2),x)
```

```
[Out] a**(3/2)*x**7*gamma(7/3)*hyper((-3/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(10/3))
```

3.395 $\int x^3 \left(a+bx^3\right)^{3/2}\,dx$

Optimal. Leaf size=272

$$\frac{54a^2x\sqrt{a+bx^3}}{935b}-\frac{36\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^3\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{935b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}$$

```
[Out] 2/17*x^4*(b*x^3+a)^(3/2)+54/935*a^2*x*(b*x^3+a)^(1/2)/b+18/187*a*x^4*(b*x^3
+a)^(1/2)-36/935*3^(3/4)*a^3*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/
3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)
+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(
1+3^(1/2)))^2)^(1/2)/b^(4/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(
b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {279, 321, 218}

$$-\frac{36\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^3\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{935b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\frac{54a^2x\sqrt{a+bx}}{935b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^3)^(3/2),x]
```

```
[Out] (54*a^2*x*Sqrt[a + b*x^3])/(935*b) + (18*a*x^4*Sqrt[a + b*x^3])/187 + (2*x^
4*(a + b*x^3)^(3/2))/17 - (36*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^3*(a^(1/3) + b^(1
/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(935*b^(4/3)*Sqrt[(a^(
1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
 b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int x^3 \left(a+bx^3\right)^{3/2}\,dx &= \frac{2}{17}x^4\left(a+bx^3\right)^{3/2} + \frac{1}{17}(9a)\int x^3\sqrt{a+bx^3}\,dx \\
&= \frac{18}{187}ax^4\sqrt{a+bx^3} + \frac{2}{17}x^4\left(a+bx^3\right)^{3/2} + \frac{1}{187}\left(27a^2\right)\int \frac{x^3}{\sqrt{a+bx^3}}\,dx \\
&= \frac{54a^2x\sqrt{a+bx^3}}{935b} + \frac{18}{187}ax^4\sqrt{a+bx^3} + \frac{2}{17}x^4\left(a+bx^3\right)^{3/2} - \frac{\left(54a^3\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{935b} \\
&= \frac{54a^2x\sqrt{a+bx^3}}{935b} + \frac{18}{187}ax^4\sqrt{a+bx^3} + \frac{2}{17}x^4\left(a+bx^3\right)^{3/2} - \frac{36\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^3\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{93}
\end{aligned}$$

Mathematica [C] time = 0.06, size = 67, normalized size = 0.25

$$\frac{2x\sqrt{a+bx^3}\left(\left(a+bx^3\right)^2 - \frac{a^2\,{}_2F_1\left(-\frac{3}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}}\right)}{17b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^3)^(3/2),x]
```

```
[Out] (2*x*Sqrt[a + b*x^3]*((a + b*x^3)^2 - (a^2*Hypergeometric2F1[-3/2, 1/3, 4/3
, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(17*b)
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^6+ax^3\right)\sqrt{bx^3+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^6 + a*x^3)*sqrt(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{3}{2}}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(3/2)*x^3, x)
```

maple [A] time = 0.02, size = 335, normalized size = 1.23

$$36i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-$$

$$\frac{2\sqrt{b\,x^3+a}\,b\,x^7}{17}+\frac{40\sqrt{b\,x^3+a}\,a\,x^4}{187}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)^(3/2),x)
```

```
[Out] 2/17*b*x^7*(b*x^3+a)^(1/2)+40/187*a*x^4*(b*x^3+a)^(1/2)+54/935*a^2*x*(b*x^3
+a)^(1/2)/b+36/935*I*a^3/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3
)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a
*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2
)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b
^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)
^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I
*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)
/b)/b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{3}{2}}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(3/2)*x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\left(b\,x^3+a\right)^{3/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^3)^(3/2),x)
```

```
[Out] int(x^3*(a + b*x^3)^(3/2), x)
```

sympy [A] time = 2.57, size = 39, normalized size = 0.14

$$\frac{a^{\frac{3}{2}}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**(3/2),x)
```

```
[Out] a**(3/2)*x**4*gamma(4/3)*hyper((-3/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(7/3))
```

3.396 $\int \left(a+bx^3\right)^{3/2}\,dx$

Optimal. Leaf size=246

$$\frac{18\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^2\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{55\sqrt[3]{b}\,\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\frac{18}{55}ax\sqrt{a+bx^3}+\frac{2}{11}$$

```
[Out] 2/11*x*(b*x^3+a)^(3/2)+18/55*a*x*(b*x^3+a)^(1/2)+18/55*3^(3/4)*a^2*(a^(1/3)
+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/
3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(1/3)/(b*x^3+a
)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/
2)
```

Rubi [A] time = 0.06, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {195, 218}

$$\frac{18\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^2\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{55\sqrt[3]{b}\,\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\frac{18}{55}ax\sqrt{a+bx^3}+\frac{2}{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(3/2),x]
```

```
[Out] (18*a*x*Sqrt[a + b*x^3])/55 + (2*x*(a + b*x^3)^(3/2))/11 + (18*3^(3/4)*Sqrt
[2 + Sqrt[3]]*a^2*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
 Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4
*Sqrt[3]])/(55*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \left(a+bx^3\right)^{3/2}\,dx = \frac{2}{11}x\left(a+bx^3\right)^{3/2} + \frac{1}{11}(9a)\int \sqrt{a+bx^3}\,dx$$
$$= \frac{18}{55}ax\sqrt{a+bx^3} + \frac{2}{11}x\left(a+bx^3\right)^{3/2} + \frac{1}{55}\left(27a^2\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx$$
$$= \frac{18}{55}ax\sqrt{a+bx^3} + \frac{2}{11}x\left(a+bx^3\right)^{3/2} + \frac{18\,3^{3/4}\sqrt{2+\sqrt{3}}\;a^2\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}}{55\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right.}}}$$

Mathematica [C] time = 0.01, size = 47, normalized size = 0.19

$$\frac{ax\sqrt{a+bx^3}\;{}_2F_1\left(-\frac{3}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(3/2),x]
```

```
[Out] (a*x*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)])/Sqrt[
1 + (b*x^3)/a]
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^3+a\right)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(3/2), x)
```

maple [A] time = 0.02, size = 315, normalized size = 1.28

$$18i\sqrt{3}\;\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right.}{\left(-a\,b^2\right)^{\frac{1}{3}}}}$$

$$\frac{2\sqrt{b\,x^3+a}\;b\,x^4}{11} - \frac{}{55\sqrt{b\,x^3+a}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2),x)
```

```
[Out] 2/11*b*x^4*(b*x^3+a)^(1/2)+28/55*a*x*(b*x^3+a)^(1/2)-18/55*I*a^2*3^(1/2)*(-
a*b^2)^(1/3)/b*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3
^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b
+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3
^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)
/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2
)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3 + a\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(3/2), x)
```

mupad [B] time = 1.02, size = 37, normalized size = 0.15

$$\frac{x\left(b\,x^3 + a\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2}, \frac{1}{3};\ \frac{4}{3};\ -\frac{b\,x^3}{a}\right)}{\left(\frac{b\,x^3}{a} + 1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(3/2),x)
```

```
[Out] (x*(a + b*x^3)^(3/2)*hypergeom([-3/2, 1/3], 4/3, -(b*x^3)/a))/((b*x^3)/a +
1)^(3/2)
```

sympy [A] time = 1.33, size = 37, normalized size = 0.15

$$\frac{a^{\frac{3}{2}} x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\left.\begin{matrix} -\frac{3}{2}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\right| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2),x)
```

```
[Out] a**(3/2)*x*gamma(1/3)*hyper((-3/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(4/3))
```

3.397 $\int \frac{\left(a+bx^3\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=246

$$\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,ab^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{10\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\frac{9}{10}bx\sqrt{a+bx^3}$$

```
[Out] -1/2*(b*x^3+a)^(3/2)/x^2+9/10*b*x*(b*x^3+a)^(1/2)+9/10*3^(3/4)*a*b^(2/3)*(a
^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1
/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)
*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/(b*x^3+a)^
(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {277, 195, 218}

$$\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,ab^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{10\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}-\frac{\left(a+bx^3\right)^{3/2}}{2x^2}+\frac{1}{1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(3/2)/x^3,x]
```

```
[Out] (9*b*x*Sqrt[a + b*x^3])/10 - (a + b*x^3)^(3/2)/(2*x^2) + (9*3^(3/4)*Sqrt[2
+ Sqrt[3]]*a*b^(2/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((
1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
- 4*Sqrt[3]])/(10*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
```

```
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}}{x^3}\,dx = -\frac{\left(a+bx^3\right)^{3/2}}{2x^2} + \frac{1}{4}(9b)\int \sqrt{a+bx^3}\,dx$$

$$= \frac{9}{10}bx\sqrt{a+bx^3} - \frac{\left(a+bx^3\right)^{3/2}}{2x^2} + \frac{1}{20}(27ab)\int \frac{1}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{9}{10}bx\sqrt{a+bx^3} - \frac{\left(a+bx^3\right)^{3/2}}{2x^2} + \frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\;ab^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{si}\right.}{10\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+}}$$

Mathematica [C] time = 0.02, size = 52, normalized size = 0.21

$$-\frac{a\sqrt{a+bx^3}\;{}_2F_1\left(-\frac{3}{2},-\frac{2}{3};\frac{1}{3};-\frac{bx^3}{a}\right)}{2x^2\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(3/2)/x^3,x]
```

```
[Out] -1/2*(a*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, -2/3, 1/3, -((b*x^3)/a)])/(
x^2*Sqrt[1 + (b*x^3)/a])
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^3+a\right)^{\frac{3}{2}}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(3/2)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(3/2)/x^3, x)
```

maple [A] time = 0.02, size = 310, normalized size = 1.26

$$9i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\,a\,\mathrm{EllipticF}$$

$$-\frac{}{10\sqrt{b\,x^3+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)/x^3,x)
```

```
[Out] -1/2*a*(b*x^3+a)^(1/2)/x^2+2/5*b*x*(b*x^3+a)^(1/2)-9/10*I*a*3^(1/2)*(-a*b^2
)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/
(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*
3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*
(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*EllipticF
(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(
1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)
/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(3/2)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(3/2)/x^3,x)
```

```
[Out] int((a + b*x^3)^(3/2)/x^3, x)
```

sympy [A] time = 1.28, size = 42, normalized size = 0.17

$$\frac{a^{\frac{3}{2}}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{2},-\frac{2}{3}\\ \frac{1}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)/x**3,x)
```

```
[Out] a**(3/2)*gamma(-2/3)*hyper((-3/2, -2/3), (1/3,), b*x**3*exp_polar(I*pi)/a)/
(3*x**2*gamma(1/3))
```

3.398 $\int \frac{\left(a+bx^3\right)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=247

$$\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,b^{5/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{20\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}-\frac{\left(a+bx^3\right)^{3/2}}{5x^5}-\frac{9b\sqrt{a}}{2}$$

```
[Out] -1/5*(b*x^3+a)^(3/2)/x^5-9/20*b*(b*x^3+a)^(1/2)/x^2+9/20*3^(3/4)*b^(5/3)*(a
^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1
/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)
*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/(b*x^3+a)^
(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {277, 218}

$$\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,b^{5/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{20\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}-\frac{9b\sqrt{a+bx^3}}{20x^2}-\frac{\left(a+\right.}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(3/2)/x^6,x]
```

```
[Out] (-9*b*Sqrt[a + b*x^3])/(20*x^2) - (a + b*x^3)^(3/2)/(5*x^5) + (9*3^(3/4)*Sq
rt[2 + Sqrt[3]]*b^(5/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin
[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)],
-7 - 4*Sqrt[3]])/(20*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}}{x^6}\,dx = -\frac{\left(a+bx^3\right)^{3/2}}{5x^5} + \frac{1}{10}(9b)\int \frac{\sqrt{a+bx^3}}{x^3}\,dx$$

$$= -\frac{9b\sqrt{a+bx^3}}{20x^2} - \frac{\left(a+bx^3\right)^{3/2}}{5x^5} + \frac{1}{40}\left(27b^2\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx$$

$$= -\frac{9b\sqrt{a+bx^3}}{20x^2} - \frac{\left(a+bx^3\right)^{3/2}}{5x^5} + \frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,b^{5/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{s}\right.}{20\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+}}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.21

$$-\frac{a\sqrt{a+bx^3}\;{}_2F_1\left(-\frac{5}{3},-\frac{3}{2};-\frac{2}{3};-\frac{bx^3}{a}\right)}{5x^5\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(3/2)/x^6,x]
```

```
[Out] -1/5*(a*Sqrt[a + b*x^3]*Hypergeometric2F1[-5/3, -3/2, -2/3, -((b*x^3)/a)])/
(x^5*Sqrt[1 + (b*x^3)/a])
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^3+a\right)^{\frac{3}{2}}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(3/2)/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(3/2)/x^6, x)
```

maple [A] time = 0.03, size = 312, normalized size = 1.26

$$9i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\,b\,\text{EllipticF}$$

$$-\frac{}{20\sqrt{b\,x^3+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)/x^6,x)
```

```
[Out] -1/5*a*(b*x^3+a)^(1/2)/x^5-13/20*b*(b*x^3+a)^(1/2)/x^2-9/20*I*b*3^(1/2)*(-a
*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1
/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/
2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1
/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)
*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(
1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(3/2)/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{3/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(3/2)/x^6,x)
```

```
[Out] int((a + b*x^3)^(3/2)/x^6, x)
```

sympy [A] time = 1.57, size = 46, normalized size = 0.19

$$\frac{a^{\frac{3}{2}}\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3}, -\frac{3}{2} \\ -\frac{2}{3}\end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)/x**6,x)
```

```
[Out] a**(3/2)*gamma(-5/3)*hyper((-5/3, -3/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)
/(3*x**5*gamma(-2/3))
```

3.399 $\int x^7 \left(a+bx^3\right)^{3/2}\,dx$

Optimal. Leaf size=556

$$\frac{144\sqrt{2}\,3^{3/4}a^{13/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\quad 216\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{13}}{8645b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}-$$

```
[Out] 2/25*x^8*(b*x^3+a)^(3/2)-108/8645*a^3*x^2*(b*x^3+a)^(1/2)/b^2+54/6175*a^2*x
^5*(b*x^3+a)^(1/2)/b+18/475*a*x^8*(b*x^3+a)^(1/2)+432/8645*a^4*(b*x^3+a)^(1
/2)/b^(8/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+144/8645*3^(3/4)*a^(13/3)*(a^(1
/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)
*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^
2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3
)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-216/8645*3^(
1/4)*a^(13/3)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*(
(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^
(1/2)/b^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 556, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {279, 321, 303, 218, 1877}

$$-\frac{108a^3x^2\sqrt{a+bx^3}}{8645b^2}+\frac{432a^4\sqrt{a+bx^3}}{8645b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{144\sqrt{2}\,3^{3/4}a^{13/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{si}\right.}{8645b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^3)^(3/2),x]
```

```
[Out] (-108*a^3*x^2*Sqrt[a + b*x^3])/(8645*b^2) + (54*a^2*x^5*Sqrt[a + b*x^3])/(6
175*b) + (18*a*x^8*Sqrt[a + b*x^3])/475 + (432*a^4*Sqrt[a + b*x^3])/(8645*b
^(8/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*x^8*(a + b*x^3)^(3/2))/25
- (216*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(13/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(8645*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b
^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (144*S
qrt[2]*3^(3/4)*a^(13/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin
[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)],
-7 - 4*Sqrt[3]])/(8645*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int x^7 \left(a+bx^3\right)^{3/2}\,dx = \frac{2}{25}x^8\left(a+bx^3\right)^{3/2} + \frac{1}{25}(9a)\int x^7\sqrt{a+bx^3}\,dx$$

$$= \frac{18}{475}ax^8\sqrt{a+bx^3} + \frac{2}{25}x^8\left(a+bx^3\right)^{3/2} + \frac{1}{475}\left(27a^2\right)\int \frac{x^7}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{54a^2x^5\sqrt{a+bx^3}}{6175b} + \frac{18}{475}ax^8\sqrt{a+bx^3} + \frac{2}{25}x^8\left(a+bx^3\right)^{3/2} - \frac{\left(54a^3\right)\int \frac{x^4}{\sqrt{a+bx^3}}\,dx}{1235b}$$

$$= -\frac{108a^3x^2\sqrt{a+bx^3}}{8645b^2} + \frac{54a^2x^5\sqrt{a+bx^3}}{6175b} + \frac{18}{475}ax^8\sqrt{a+bx^3} + \frac{2}{25}x^8\left(a+bx^3\right)^{3/2} + \frac{\left(216a^{\prime}\right.}{}$$

$$= -\frac{108a^3x^2\sqrt{a+bx^3}}{8645b^2} + \frac{54a^2x^5\sqrt{a+bx^3}}{6175b} + \frac{18}{475}ax^8\sqrt{a+bx^3} + \frac{2}{25}x^8\left(a+bx^3\right)^{3/2} + \frac{\left(216a^{\prime}\right.}{}$$

$$= -\frac{108a^3x^2\sqrt{a+bx^3}}{8645b^2} + \frac{54a^2x^5\sqrt{a+bx^3}}{6175b} + \frac{18}{475}ax^8\sqrt{a+bx^3} + \frac{432a^4\sqrt{a+bx^3}}{8645b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\right.}$$

Mathematica [C] time = 0.11, size = 81, normalized size = 0.15

$$\frac{2x^2\sqrt{a+bx^3}\left(\frac{10a^3\,{}_2F_1\left(-\frac{3}{2},\frac{2}{3},\frac{5}{3};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}}-\left(10a-19bx^3\right)\left(a+bx^3\right)^2\right)}{475b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^3)^(3/2),x]
```

```
[Out] (2*x^2*Sqrt[a + b*x^3]*(-((10*a - 19*b*x^3)*(a + b*x^3)^2) + (10*a^3*Hyperg
eometric2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(475*b^2)
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^{10}+ax^7\right)\sqrt{bx^3+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^10 + a*x^7)*sqrt(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{3}{2}}x^7\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(3/2)*x^7, x)
```

maple [A] time = 0.02, size = 509, normalized size = 0.92

$$144i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-}{\left(-\right.}}$$

$$\frac{2\sqrt{b\,x^3+a}\,b\,x^{11}}{25}+\frac{56\sqrt{b\,x^3+a}\,a\,x^8}{475}+\frac{54\sqrt{b\,x^3+a}\,a^2x^5}{6175b}-\frac{108\sqrt{b\,x^3+a}\,a^3x^2}{8645b^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^3+a)^(3/2),x)
```

```
[Out] 2/25*b*x^11*(b*x^3+a)^(1/2)+56/475*a*x^8*(b*x^3+a)^(1/2)+54/6175*a^2*x^5*(b
*x^3+a)^(1/2)/b-108/8645*a^3*x^2*(b*x^3+a)^(1/2)/b^2-144/8645*I*a^4/b^3*3^(
```

```
1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)
/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1
/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/
2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/
2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^
(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-
a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*
I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2
)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^
2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^
(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3 + a\right)^{\frac{3}{2}} x^7 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(3/2)*x^7, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^7 \left(b\,x^3 + a\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^3)^(3/2),x)
```

```
[Out] int(x^7*(a + b*x^3)^(3/2), x)
```

sympy [A] time = 3.75, size = 39, normalized size = 0.07

$$\frac{a^{\frac{3}{2}} x^8 \Gamma\left(\frac{8}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{8}{3} \\ \frac{11}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**3+a)**(3/2),x)
```

```
[Out] a**(3/2)*x**8*gamma(8/3)*hyper((-3/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(11/3))
```

3.400 $\int x^4 \left(a+bx^3\right)^{3/2}\,dx$

Optimal. Leaf size=532

$$-\frac{72\sqrt{2}\,3^{3/4}a^{10/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{1729b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\frac{108\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^1}{}$$

```
[Out] 2/19*x^5*(b*x^3+a)^(3/2)+54/1729*a^2*x^2*(b*x^3+a)^(1/2)/b+18/247*a*x^5*(b*
x^3+a)^(1/2)-216/1729*a^3*(b*x^3+a)^(1/2)/b^(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(
1/2)))-72/1729*3^(3/4)*a^(10/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^
(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*(
(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^
(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2)))^2)^(1/2)+108/1729*3^(1/4)*a^(10/3)*(a^(1/3)+b^(1/3)*x)*Elli
pticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(
1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2
)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)
*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 532, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {279, 321, 303, 218, 1877}

$$-\frac{216a^3\sqrt{a+bx^3}}{1729b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{72\sqrt{2}\,3^{3/4}a^{10/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|\right.}{1729b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^3)^(3/2),x]
```

```
[Out] (54*a^2*x^2*Sqrt[a + b*x^3])/(1729*b) + (18*a*x^5*Sqrt[a + b*x^3])/247 - (2
16*a^3*Sqrt[a + b*x^3])/(1729*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x))
+ (2*x^5*(a + b*x^3)^(3/2))/19 + (108*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(10/3)*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(
5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*Sqrt[a + b*x^3]) - (72*Sqrt[2]*3^(3/4)*a^(10/3)*(a^(1/3) + b^(1/3)*x)
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(5/3)*Sqrt[(a^(1/3)*
(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^
3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int x^4 \left(a+bx^3\right)^{3/2}\,dx = \frac{2}{19}x^5\left(a+bx^3\right)^{3/2} + \frac{1}{19}(9a)\int x^4\sqrt{a+bx^3}\,dx$$

$$= \frac{18}{247}ax^5\sqrt{a+bx^3} + \frac{2}{19}x^5\left(a+bx^3\right)^{3/2} + \frac{1}{247}\left(27a^2\right)\int \frac{x^4}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{54a^2x^2\sqrt{a+bx^3}}{1729b} + \frac{18}{247}ax^5\sqrt{a+bx^3} + \frac{2}{19}x^5\left(a+bx^3\right)^{3/2} - \frac{\left(108a^3\right)\int \frac{x}{\sqrt{a+bx^3}}\,dx}{1729b}$$

$$= \frac{54a^2x^2\sqrt{a+bx^3}}{1729b} + \frac{18}{247}ax^5\sqrt{a+bx^3} + \frac{2}{19}x^5\left(a+bx^3\right)^{3/2} - \frac{\left(108a^3\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{1729b^{4/3}}$$

$$= \frac{54a^2x^2\sqrt{a+bx^3}}{1729b} + \frac{18}{247}ax^5\sqrt{a+bx^3} - \frac{216a^3\sqrt{a+bx^3}}{1729b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} + \frac{2}{19}x^5\left(a+bx^3\right.$$

Mathematica [C] time = 0.08, size = 69, normalized size = 0.13

$$\frac{2x^2\sqrt{a+bx^3}\left(\left(a+bx^3\right)^2-\frac{a^2\,{}_2F_1\left(-\frac{3}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}}\right)}{19b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^3)^(3/2),x]
```

```
[Out] (2*x^2*Sqrt[a + b*x^3]*((a + b*x^3)^2 - (a^2*Hypergeometric2F1[-3/2, 2/3, 5
/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(19*b)
```

fricas [F] time = 1.08, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^7+ax^4\right)\sqrt{bx^3+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^7 + a*x^4)*sqrt(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{3}{2}}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(3/2)*x^4, x)
```

maple [A] time = 0.02, size = 489, normalized size = 0.92

$$72i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{}{-\frac{3\left(-a\,b\right.}{2b}}}$$

$$\frac{2\sqrt{b\,x^3+a}\,b\,x^8}{19}+\frac{44\sqrt{b\,x^3+a}\,a\,x^5}{247}+\frac{54\sqrt{b\,x^3+a}\,a^2x^2}{1729b}+\frac{}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)^(3/2),x)
```

```
[Out] 2/19*b*x^8*(b*x^3+a)^(1/2)+44/247*a*x^5*(b*x^3+a)^(1/2)+54/1729*a^2*x^2*(b*
x^3+a)^(1/2)/b+72/1729*I*a^3/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^
```

```
(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x
-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^
(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(
-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2
)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*
3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^
2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(
-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1
/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(
1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3 + a\right)^{\frac{3}{2}} x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(3/2)*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 \left(b\,x^3 + a\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^3)^(3/2),x)
```

```
[Out] int(x^4*(a + b*x^3)^(3/2), x)
```

sympy [A] time = 1.94, size = 39, normalized size = 0.07

$$\frac{a^{\frac{3}{2}} x^5 \Gamma\left(\frac{5}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{5}{3} \\ \frac{8}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)**(3/2),x)
```

```
[Out] a**(3/2)*x**5*gamma(5/3)*hyper((-3/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(8/3))
```

3.401 $\int x\left(a+bx^3\right)^{3/2}\,dx$

Optimal. Leaf size=508

$$\frac{18\sqrt{2}\,3^{3/4}a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{91b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,E\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{91b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{54a^2\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}$$

```
[Out] 2/13*x^2*(b*x^3+a)^(3/2)+18/91*a*x^2*(b*x^3+a)^(1/2)+54/91*a^2*(b*x^3+a)^(1
/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+18/91*3^(3/4)*a^(7/3)*(a^(1/3)+
b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(
b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a
^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-27/91*3^(1/4)*a^
(7/3)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3
)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)
-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^
(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 508, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {279, 303, 218, 1877}

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3)^(3/2),x]
```

```
[Out] (18*a*x^2*Sqrt[a + b*x^3])/91 + (54*a^2*Sqrt[a + b*x^3])/(91*b^(2/3)*((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*x^2*(a + b*x^3)^(3/2))/13 - (27*3^(1/4)
*Sqrt[2 - Sqrt[3]]*a^(7/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[Arc
Sin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
], -7 - 4*Sqrt[3]])/(91*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (18*Sqrt[2]*3^(3/4)*a^(
7/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(
1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(9
1*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int x\left(a+bx^3\right)^{3/2}\,dx = \frac{2}{13}x^2\left(a+bx^3\right)^{3/2} + \frac{1}{13}(9a)\int x\sqrt{a+bx^3}\,dx$$

$$= \frac{18}{91}ax^2\sqrt{a+bx^3} + \frac{2}{13}x^2\left(a+bx^3\right)^{3/2} + \frac{1}{91}\left(27a^2\right)\int \frac{x}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{18}{91}ax^2\sqrt{a+bx^3} + \frac{2}{13}x^2\left(a+bx^3\right)^{3/2} + \frac{\left(27a^2\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{91\sqrt[3]{b}} + \frac{\left(27\sqrt{2\left(2-\sqrt{3}\right)}\,a^{7/3}\right.}{91\sqrt[3]{b}}$$

$$= \frac{18}{91}ax^2\sqrt{a+bx^3} + \frac{54a^2\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} + \frac{2}{13}x^2\left(a+bx^3\right)^{3/2} - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{7/}}{}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.10

$$\frac{ax^2\sqrt{a+bx^3}\;{}_2F_1\left(-\frac{3}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)^(3/2),x]
```

```
[Out] (a*x^2*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)])/(2*
Sqrt[1 + (b*x^3)/a])
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^4+ax\right)\sqrt{bx^3+a}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a*x)*sqrt(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{3}{2}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(3/2)*x, x)
```

maple [A] time = 0.02, size = 469, normalized size = 0.92

$$18i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-$$

$$\frac{2\sqrt{b\,x^3+a}\,b\,x^5}{13}+\frac{32\sqrt{b\,x^3+a}\,a\,x^2}{91}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^(3/2),x)
```

```
[Out] 2/13*b*x^5*(b*x^3+a)^(1/2)+32/91*a*x^2*(b*x^3+a)^(1/2)-18/91*I*a^2*3^(1/2)*
(-a*b^2)^(1/3)/b*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)
*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)
/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I
*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*
((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^(1/
2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b
^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3
^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(
I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^
(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/
2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{3}{2}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(3/2)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x \left(b\,x^3 + a\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^3)^(3/2),x)
```

```
[Out] int(x*(a + b*x^3)^(3/2), x)
```

sympy [A] time = 2.35, size = 39, normalized size = 0.08

$$\frac{a^{\frac{3}{2}} x^2 \Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{2}{3} \\ \frac{5}{3} \end{matrix} \middle| \frac{b x^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**(3/2),x)
```

```
[Out] a**(3/2)*x**2*gamma(2/3)*hyper((-3/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(5/3))
```

3.402 $\int \frac{\left(a+bx^3\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=504

$$\frac{9\sqrt{2}\,3^{3/4}a^{4/3}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}} - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}}{}$$

```
[Out] -(b*x^3+a)^(3/2)/x+9/7*b*x^2*(b*x^3+a)^(1/2)+27/7*a*b^(1/3)*(b*x^3+a)^(1/2)
/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+9/7*3^(3/4)*a^(4/3)*b^(1/3)*(a^(1/3)+b^(1/
3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/
3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3
)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-27/14*3^(1/4)*a^(4/3)*b^(1/3)
*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a
^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1
/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/(b*x^3+
a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1
/2)
```

Rubi [A] time = 0.19, antiderivative size = 504, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {277, 279, 303, 218, 1877}

$$\frac{9\sqrt{2}\,3^{3/4}a^{4/3}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}} - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(3/2)/x^2,x]
```

```
[Out] (9*b*x^2*Sqrt[a + b*x^3])/7 + (27*a*b^(1/3)*Sqrt[a + b*x^3])/(7*((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)) - (a + b*x^3)^(3/2)/x - (27*3^(1/4)*Sqrt[2 - Sqrt
[3]]*a^(4/3)*b^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((
1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
- 4*Sqrt[3]])/(14*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*Sqrt[2]*3^(3/4)*a^(4/3)*b^(1/3)*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*Sqrt[(
a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[
a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}}{x^2}\,dx = -\frac{\left(a+bx^3\right)^{3/2}}{x} + \frac{1}{2}(9b)\int x\sqrt{a+bx^3}\,dx$$

$$= \frac{9}{7}bx^2\sqrt{a+bx^3} - \frac{\left(a+bx^3\right)^{3/2}}{x} + \frac{1}{14}(27ab)\int \frac{x}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{9}{7}bx^2\sqrt{a+bx^3} - \frac{\left(a+bx^3\right)^{3/2}}{x} + \frac{1}{14}\left(27ab^{2/3}\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a} + \sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx + \frac{1}{7}\left(27\sqrt{\frac{1}{2}\left(2-\sqrt{}\right.\right.$$

$$= \frac{9}{7}bx^2\sqrt{a+bx^3} + \frac{27a\sqrt[3]{b}\,\sqrt{a+bx^3}}{7\left(\left(1+\sqrt{3}\right)\sqrt[3]{a} + \sqrt[3]{b}\,x\right)} - \frac{\left(a+bx^3\right)^{3/2}}{x} - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}\sqrt[3]{b}\left(\sqrt[3]{a}+\right.}{}$$

Mathematica [C] time = 0.01, size = 50, normalized size = 0.10

$$-\frac{a\sqrt{a+bx^3}\;{}_2F_1\left(-\frac{3}{2},-\frac{1}{3};\frac{2}{3};-\frac{bx^3}{a}\right)}{x\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(3/2)/x^2,x]
```

```
[Out] -((a*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, -1/3, 2/3, -((b*x^3)/a)])/(x*S
qrt[1 + (b*x^3)/a]))
```

fricas **[F]** time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^3+a\right)^{\frac{3}{2}}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(3/2)/x^2, x)
```

giac **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(3/2)/x^2, x)
```

maple **[A]** time = 0.02, size = 464, normalized size = 0.92

$$9i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)}{\left(-a\,b^2\right)^{\frac{1}{3}}}}$$

$$\frac{2\sqrt{b\,x^3+a}\,b\,x^2}{7}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)/x^2,x)
```

```
[Out] -a*(b*x^3+a)^(1/2)/x+2/7*b*x^2*(b*x^3+a)^(1/2)-9/7*I*a*3^(1/2)*(-a*b^2)^(1/
3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b
^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/
2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b
^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*((-3/2*(-a*b^2
)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(
-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(
```

```
1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2
)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b
^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)
,(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1
/3)/b)/b)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(3/2)/x^2, x)
```

mupad [B] time = 1.58, size = 40, normalized size = 0.08

$$\frac{2\left(b\,x^3+a\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{7}{6};\,-\frac{1}{6};\,-\frac{a}{b\,x^3}\right)}{7\,x\left(\frac{a}{b\,x^3}+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(3/2)/x^2,x)
```

```
[Out] (2*(a + b*x^3)^(3/2)*hypergeom([-3/2, -7/6], -1/6, -a/(b*x^3)))/(7*x*(a/(b*
x^3) + 1)^(3/2))
```

sympy [A] time = 1.37, size = 41, normalized size = 0.08

$$\frac{a^{\frac{3}{2}}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{3}{2},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)/x**2,x)
```

```
[Out] a**(3/2)*gamma(-1/3)*hyper((-3/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/
(3*x*gamma(2/3))
```

3.403 $\int \frac{\left(a+bx^3\right)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=505

$$\frac{9\,3^{3/4}\sqrt[3]{a}\,b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\quad 27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\;\sqrt[3]{a}\,b^{4/3}}{4\sqrt{2}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -1/4*(b*x^3+a)^(3/2)/x^4-9/8*b*(b*x^3+a)^(1/2)/x+27/8*b^(4/3)*(b*x^3+a)^(1/
2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+9/8*3^(3/4)*a^(1/3)*b^(4/3)*(a^(1/3)+b^(
1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(
1/2))),I*3^(1/2)+2*I)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a
^(1/3)*(1+3^(1/2)))^2)^(1/2)*2^(1/2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1
/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-27/16*3^(1/4)*a^(1/3)*b^(4/
3)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^
(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/(b*x^
3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^
(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 505, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {277, 303, 218, 1877}

$$\frac{9\,3^{3/4}\sqrt[3]{a}\,b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\quad 27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\;\sqrt[3]{a}\,b^{4/3}}{4\sqrt{2}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(3/2)/x^5,x]
```

```
[Out] (-9*b*Sqrt[a + b*x^3])/(8*x) + (27*b^(4/3)*Sqrt[a + b*x^3])/(8*((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)) - (a + b*x^3)^(3/2)/(4*x^4) - (27*3^(1/4)*Sqrt[2 -
 Sqrt[3]]*a^(1/3)*b^(4/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcS
in[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(16*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9*3^(3/4)*a^(1/3)*b^(4/3)*(a^(1
/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(4*Sqrt[2]*S
qrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}}{x^5}\,dx = -\frac{\left(a+bx^3\right)^{3/2}}{4x^4} + \frac{1}{8}(9b)\int \frac{\sqrt{a+bx^3}}{x^2}\,dx$$

$$= -\frac{9b\sqrt{a+bx^3}}{8x} - \frac{\left(a+bx^3\right)^{3/2}}{4x^4} + \frac{1}{16}\left(27b^2\right)\int \frac{x}{\sqrt{a+bx^3}}\,dx$$

$$= -\frac{9b\sqrt{a+bx^3}}{8x} - \frac{\left(a+bx^3\right)^{3/2}}{4x^4} + \frac{1}{16}\left(27b^{5/3}\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx + \frac{\left(27\sqrt{2-\sqrt{3}}\ \sqrt[3]{a}\,b\right.}{8\sqrt{}}$$

$$= -\frac{9b\sqrt{a+bx^3}}{8x} + \frac{27b^{4/3}\sqrt{a+bx^3}}{8\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} - \frac{\left(a+bx^3\right)^{3/2}}{4x^4} - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\ \sqrt[3]{a}\,b^{4/3}\left(\sqrt[3]{a}+\right.}{}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.10

$$-\frac{a\sqrt{a+bx^3}\ {}_2F_1\left(-\frac{3}{2},-\frac{4}{3};-\frac{1}{3};-\frac{bx^3}{a}\right)}{4x^4\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(3/2)/x^5,x]
```

```
[Out] -1/4*(a*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, -4/3, -1/3, -((b*x^3)/a)])/
(x^4*Sqrt[1 + (b*x^3)/a])
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^3+a\right)^{\frac{3}{2}}}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)/x^5,x, algorithm="fricas")
[Out] integral((b*x^3 + a)^(3/2)/x^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)/x^5,x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(3/2)/x^5, x)
```

maple [A] time = 0.03, size = 464, normalized size = 0.92

$$9i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\left(\left(-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right.\right.$$

$$-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)/x^5,x)
[Out] -1/4*a*(b*x^3+a)^(1/2)/x^4-11/8*b*(b*x^3+a)^(1/2)/x-9/8*I*b*3^(1/2)*(-a*b^2
)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/
(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*
3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*
(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*((-3/2*(-
a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^(1/2)*(I*(x+
1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)
*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-
a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(I*(x+1/2*
(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^
(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^
2)^(1/3)/b)/b)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(3/2)/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(3/2)/x^5,x)
```

```
[Out] int((a + b*x^3)^(3/2)/x^5, x)
```

sympy [A] time = 2.64, size = 46, normalized size = 0.09

$$\frac{a^{\frac{3}{2}}\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{3}{2}, -\frac{4}{3} \\ -\frac{1}{3}\end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)/x**5,x)
```

```
[Out] a**(3/2)*gamma(-4/3)*hyper((-3/2, -4/3), (-1/3,), b*x**3*exp_polar(I*pi)/a)
/(3*x**4*gamma(-1/3))
```

3.404 $\int \frac{x^{11}}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=80

$$-\frac{2a^3\sqrt{a+bx^3}}{3b^4}+\frac{2a^2\left(a+bx^3\right)^{3/2}}{3b^4}+\frac{2\left(a+bx^3\right)^{7/2}}{21b^4}-\frac{2a\left(a+bx^3\right)^{5/2}}{5b^4}$$

```
[Out] 2/3*a^2*(b*x^3+a)^(3/2)/b^4-2/5*a*(b*x^3+a)^(5/2)/b^4+2/21*(b*x^3+a)^(7/2)/
b^4-2/3*a^3*(b*x^3+a)^(1/2)/b^4
```

Rubi [A] time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{2a^2\left(a+bx^3\right)^{3/2}}{3b^4}-\frac{2a^3\sqrt{a+bx^3}}{3b^4}+\frac{2\left(a+bx^3\right)^{7/2}}{21b^4}-\frac{2a\left(a+bx^3\right)^{5/2}}{5b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/Sqrt[a + b*x^3],x]
```

```
[Out] (-2*a^3*Sqrt[a + b*x^3])/(3*b^4) + (2*a^2*(a + b*x^3)^(3/2))/(3*b^4) - (2*a
*(a + b*x^3)^(5/2))/(5*b^4) + (2*(a + b*x^3)^(7/2))/(21*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{\sqrt{a+bx^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^3}{\sqrt{a+bx}}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int\left(-\frac{a^3}{b^3\sqrt{a+bx}}+\frac{3a^2\sqrt{a+bx}}{b^3}-\frac{3a(a+bx)^{3/2}}{b^3}+\frac{(a+bx)^{5/2}}{b^3}\right)dx, x, x^3\right)\\ &= -\frac{2a^3\sqrt{a+bx^3}}{3b^4}+\frac{2a^2\left(a+bx^3\right)^{3/2}}{3b^4}-\frac{2a\left(a+bx^3\right)^{5/2}}{5b^4}+\frac{2\left(a+bx^3\right)^{7/2}}{21b^4}\end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.62

$$\frac{2\sqrt{a+bx^3}\left(-16a^3+8a^2bx^3-6ab^2x^6+5b^3x^9\right)}{105b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/Sqrt[a + b*x^3],x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(-16*a^3 + 8*a^2*b*x^3 - 6*a*b^2*x^6 + 5*b^3*x^9))/(105*
b^4)
```

fricas [A] time = 0.86, size = 46, normalized size = 0.58

$$\frac{2\left(5\,b^3x^9-6\,ab^2x^6+8\,a^2bx^3-16\,a^3\right)\sqrt{bx^3+a}}{105\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(5*b^3*x^9 - 6*a*b^2*x^6 + 8*a^2*b*x^3 - 16*a^3)*sqrt(b*x^3 + a)/b^4
```

giac [A] time = 0.19, size = 61, normalized size = 0.76

$$-\frac{2\sqrt{bx^3+a}\,a^3}{3\,b^4}+\frac{2\left(5\left(bx^3+a\right)^{\frac{7}{2}}-21\left(bx^3+a\right)^{\frac{5}{2}}a+35\left(bx^3+a\right)^{\frac{3}{2}}a^2\right)}{105\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(b*x^3 + a)*a^3/b^4 + 2/105*(5*(b*x^3 + a)^(7/2) - 21*(b*x^3 + a)^
(5/2)*a + 35*(b*x^3 + a)^(3/2)*a^2)/b^4
```

maple [A] time = 0.01, size = 47, normalized size = 0.59

$$-\frac{2\sqrt{b\,x^3+a}\,\left(-5b^3x^9+6a\,b^2x^6-8a^2b\,x^3+16a^3\right)}{105b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^3+a)^(1/2),x)
```

```
[Out] -2/105*(b*x^3+a)^(1/2)*(-5*b^3*x^9+6*a*b^2*x^6-8*a^2*b*x^3+16*a^3)/b^4
```

maxima [A] time = 1.34, size = 64, normalized size = 0.80

$$\frac{2\left(bx^3+a\right)^{\frac{7}{2}}}{21\,b^4}-\frac{2\left(bx^3+a\right)^{\frac{5}{2}}a}{5\,b^4}+\frac{2\left(bx^3+a\right)^{\frac{3}{2}}a^2}{3\,b^4}-\frac{2\sqrt{bx^3+a}\,a^3}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/21*(b*x^3 + a)^(7/2)/b^4 - 2/5*(b*x^3 + a)^(5/2)*a/b^4 + 2/3*(b*x^3 + a)^
(3/2)*a^2/b^4 - 2/3*sqrt(b*x^3 + a)*a^3/b^4
```

mupad [B] time = 1.13, size = 73, normalized size = 0.91

$$\frac{2\,x^9\,\sqrt{b\,x^3+a}}{21\,b}-\frac{32\,a^3\,\sqrt{b\,x^3+a}}{105\,b^4}-\frac{4\,a\,x^6\,\sqrt{b\,x^3+a}}{35\,b^2}+\frac{16\,a^2\,x^3\,\sqrt{b\,x^3+a}}{105\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a + b*x^3)^(1/2),x)
```

```
[Out] (2*x^9*(a + b*x^3)^(1/2))/(21*b) - (32*a^3*(a + b*x^3)^(1/2))/(105*b^4) - (
4*a*x^6*(a + b*x^3)^(1/2))/(35*b^2) + (16*a^2*x^3*(a + b*x^3)^(1/2))/(105*b
^3)
```

sympy [**A**] time = 3.62, size = 94, normalized size = 1.18

$$\begin{cases} -\frac{32a^3\sqrt{a+bx^3}}{105b^4} + \frac{16a^2x^3\sqrt{a+bx^3}}{105b^3} - \frac{4ax^6\sqrt{a+bx^3}}{35b^2} + \frac{2x^9\sqrt{a+bx^3}}{21b} & \text{for } b \neq 0 \\ \frac{x^{12}}{12\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**3+a)**(1/2),x)
```

```
[Out] Piecewise((-32*a**3*sqrt(a + b*x**3)/(105*b**4) + 16*a**2*x**3*sqrt(a + b*x
**3)/(105*b**3) - 4*a*x**6*sqrt(a + b*x**3)/(35*b**2) + 2*x**9*sqrt(a + b*x
**3)/(21*b), Ne(b, 0)), (x**12/(12*sqrt(a)), True))
```

3.405 $\int \frac{x^8}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=59

$$\frac{2a^2\sqrt{a+bx^3}}{3b^3}+\frac{2\left(a+bx^3\right)^{5/2}}{15b^3}-\frac{4a\left(a+bx^3\right)^{3/2}}{9b^3}$$

```
[Out] -4/9*a*(b*x^3+a)^(3/2)/b^3+2/15*(b*x^3+a)^(5/2)/b^3+2/3*a^2*(b*x^3+a)^(1/2)
/b^3
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{2a^2\sqrt{a+bx^3}}{3b^3}+\frac{2\left(a+bx^3\right)^{5/2}}{15b^3}-\frac{4a\left(a+bx^3\right)^{3/2}}{9b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/Sqrt[a + b*x^3],x]
```

```
[Out] (2*a^2*Sqrt[a + b*x^3])/(3*b^3) - (4*a*(a + b*x^3)^(3/2))/(9*b^3) + (2*(a +
 b*x^3)^(5/2))/(15*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{\sqrt{a+bx^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx}}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int\left(\frac{a^2}{b^2\sqrt{a+bx}}-\frac{2a\sqrt{a+bx}}{b^2}+\frac{(a+bx)^{3/2}}{b^2}\right)dx, x, x^3\right)\\ &= \frac{2a^2\sqrt{a+bx^3}}{3b^3}-\frac{4a\left(a+bx^3\right)^{3/2}}{9b^3}+\frac{2\left(a+bx^3\right)^{5/2}}{15b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.66

$$\frac{2\sqrt{a+bx^3}\left(8a^2-4abx^3+3b^2x^6\right)}{45b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/Sqrt[a + b*x^3],x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(8*a^2 - 4*a*b*x^3 + 3*b^2*x^6))/(45*b^3)
```

fricas [A] time = 0.83, size = 35, normalized size = 0.59

$$\frac{2\left(3\,b^2x^6-4\,abx^3+8\,a^2\right)\sqrt{bx^3+a}}{45\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*(3*b^2*x^6 - 4*a*b*x^3 + 8*a^2)*sqrt(b*x^3 + a)/b^3
```

giac [A] time = 0.16, size = 47, normalized size = 0.80

$$\frac{2\sqrt{bx^3+a}\,a^2}{3\,b^3}+\frac{2\left(3\left(bx^3+a\right)^{\frac{5}{2}}-10\left(bx^3+a\right)^{\frac{3}{2}}a\right)}{45\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(b*x^3 + a)*a^2/b^3 + 2/45*(3*(b*x^3 + a)^(5/2) - 10*(b*x^3 + a)^(3
/2)*a)/b^3
```

maple [A] time = 0.01, size = 36, normalized size = 0.61

$$\frac{2\sqrt{b\,x^3+a}\,\left(3b^2x^6-4ab\,x^3+8a^2\right)}{45b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a)^(1/2),x)
```

```
[Out] 2/45*(b*x^3+a)^(1/2)*(3*b^2*x^6-4*a*b*x^3+8*a^2)/b^3
```

maxima [A] time = 1.36, size = 47, normalized size = 0.80

$$\frac{2\left(bx^3+a\right)^{\frac{5}{2}}}{15\,b^3}-\frac{4\left(bx^3+a\right)^{\frac{3}{2}}a}{9\,b^3}+\frac{2\sqrt{bx^3+a}\,a^2}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(b*x^3 + a)^(5/2)/b^3 - 4/9*(b*x^3 + a)^(3/2)*a/b^3 + 2/3*sqrt(b*x^3 +
 a)*a^2/b^3
```

mupad [B] time = 1.11, size = 35, normalized size = 0.59

$$\frac{2\sqrt{b\,x^3+a}\,\left(8\,a^2-4\,a\,b\,x^3+3\,b^2\,x^6\right)}{45\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^3)^(1/2),x)
```

```
[Out] (2*(a + b*x^3)^(1/2)*(8*a^2 + 3*b^2*x^6 - 4*a*b*x^3))/(45*b^3)
```

sympy [**A**] time = 2.85, size = 70, normalized size = 1.19

$$\begin{cases} \frac{16a^2\sqrt{a+bx^3}}{45b^3} - \frac{8ax^3\sqrt{a+bx^3}}{45b^2} + \frac{2x^6\sqrt{a+bx^3}}{15b} & \text{for } b \neq 0 \\ \frac{x^9}{9\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a)**(1/2),x)
```

```
[Out] Piecewise((16*a**2*sqrt(a + b*x**3)/(45*b**3) - 8*a*x**3*sqrt(a + b*x**3)/(
45*b**2) + 2*x**6*sqrt(a + b*x**3)/(15*b), Ne(b, 0)), (x**9/(9*sqrt(a)), Tr
ue))
```

3.406 $\int \frac{x^5}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=38

$$\frac{2\left(a+bx^3\right)^{3/2}}{9b^2}-\frac{2a\sqrt{a+bx^3}}{3b^2}$$

```
[Out] 2/9*(b*x^3+a)^(3/2)/b^2-2/3*a*(b*x^3+a)^(1/2)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{2\left(a+bx^3\right)^{3/2}}{9b^2}-\frac{2a\sqrt{a+bx^3}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[a + b*x^3],x]
```

```
[Out] (-2*a*Sqrt[a + b*x^3])/(3*b^2) + (2*(a + b*x^3)^(3/2))/(9*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\sqrt{a+bx^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(-\frac{a}{b\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b}\right)dx, x, x^3\right)\\ &= -\frac{2a\sqrt{a+bx^3}}{3b^2}+\frac{2\left(a+bx^3\right)^{3/2}}{9b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.71

$$\frac{2\left(bx^3-2a\right)\sqrt{a+bx^3}}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[a + b*x^3],x]
```

```
[Out] (2*(-2*a + b*x^3)*Sqrt[a + b*x^3])/(9*b^2)
```

fricas [A] time = 0.81, size = 23, normalized size = 0.61

$$\frac{2\sqrt{bx^3+a}\left(bx^3-2a\right)}{9b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] 2/9*sqrt(b*x^3 + a)*(b*x^3 - 2*a)/b^2
```

giac [A] time = 0.15, size = 30, normalized size = 0.79

$$\frac{2\left(bx^3+a\right)^{\frac{3}{2}}}{9b^2}-\frac{2\sqrt{bx^3+a}\,a}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] 2/9*(b*x^3 + a)^(3/2)/b^2 - 2/3*sqrt(b*x^3 + a)*a/b^2
```

maple [A] time = 0.00, size = 25, normalized size = 0.66

$$-\frac{2\sqrt{b\,x^3+a}\,\left(-b\,x^3+2a\right)}{9b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a)^(1/2),x)
[Out] -2/9*(b*x^3+a)^(1/2)*(-b*x^3+2*a)/b^2
```

maxima [A] time = 1.32, size = 30, normalized size = 0.79

$$\frac{2\left(bx^3+a\right)^{\frac{3}{2}}}{9b^2}-\frac{2\sqrt{bx^3+a}\,a}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] 2/9*(b*x^3 + a)^(3/2)/b^2 - 2/3*sqrt(b*x^3 + a)*a/b^2
```

mupad [B] time = 1.09, size = 24, normalized size = 0.63

$$-\frac{2\sqrt{b\,x^3+a}\,\left(2\,a-b\,x^3\right)}{9\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^3)^(1/2),x)
[Out] -(2*(a + b*x^3)^(1/2)*(2*a - b*x^3))/(9*b^2)
```

sympy [A] time = 1.73, size = 46, normalized size = 1.21

$$\begin{cases} -\frac{4a\sqrt{a+bx^3}}{9b^2}+\frac{2x^3\sqrt{a+bx^3}}{9b} & \text{for } b\neq 0 \\ \frac{x^6}{6\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)**(1/2),x)
```

```
[Out] Piecewise((-4*a*sqrt(a + b*x**3)/(9*b**2) + 2*x**3*sqrt(a + b*x**3)/(9*b),
Ne(b, 0)), (x**6/(6*sqrt(a)), True))
```

3.407 $\int \frac{x^2}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=18

$$\frac{2\sqrt{a+bx^3}}{3b}$$

```
[Out] 2/3*(b*x^3+a)^(1/2)/b
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$\frac{2\sqrt{a+bx^3}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*x^3],x]
```

```
[Out] (2*Sqrt[a + b*x^3])/(3*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{a+bx^3}}{3b}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{2\sqrt{a+bx^3}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b*x^3],x]
```

```
[Out] (2*Sqrt[a + b*x^3])/(3*b)
```

fricas [A] time = 0.81, size = 14, normalized size = 0.78

$$\frac{2\sqrt{bx^3+a}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*x^3 + a)/b
```

giac [A] time = 0.16, size = 14, normalized size = 0.78

$$\frac{2\sqrt{bx^3+a}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] 2/3*sqrt(b*x^3 + a)/b
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$\frac{2\sqrt{b\,x^3+a}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)^(1/2),x)
[Out] 2/3*(b*x^3+a)^(1/2)/b
```

maxima [A] time = 1.30, size = 14, normalized size = 0.78

$$\frac{2\,\sqrt{bx^3+a}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] 2/3*sqrt(b*x^3 + a)/b
```

mupad [B] time = 1.10, size = 14, normalized size = 0.78

$$\frac{2\,\sqrt{b\,x^3+a}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^3)^(1/2),x)
[Out] (2*(a + b*x^3)^(1/2))/(3*b)
```

sympy [A] time = 0.84, size = 24, normalized size = 1.33

$$\begin{cases} \frac{2\sqrt{a+bx^3}}{3b} & \text{for } b \neq 0 \\ \frac{x^3}{3\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)**(1/2),x)
[Out] Piecewise((2*sqrt(a + b*x**3)/(3*b), Ne(b, 0)), (x**3/(3*sqrt(a)), True))
```

3.408 $\int \frac{1}{x\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=27

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

```
[Out] -2/3*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {266, 63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{a+bx^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right)\\ &= \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)}{3b}\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a])
```

fricas [A] time = 0.93, size = 63, normalized size = 2.33

$$\left[\frac{\log\left(\frac{bx^3-2\sqrt{bx^3+a}\sqrt{a}+2a}{x^3}\right)}{3\sqrt{a}}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{bx^3+a}\sqrt{-a}}{a}\right)}{3a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3)/sqrt(a), 2/3*sqrt(-
a)*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a)/a]
```

giac [A] time = 0.15, size = 23, normalized size = 0.85

$$\frac{2\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a)
```

maple [A] time = 0.02, size = 20, normalized size = 0.74

$$-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)^(1/2),x)
```

```
[Out] -2/3*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)
```

maxima [A] time = 2.88, size = 37, normalized size = 1.37

$$\frac{\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/sqrt(a)
```

mupad [B] time = 1.23, size = 40, normalized size = 1.48

$$\frac{\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3)^(1/2)),x)
```

```
[Out] log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6)/(3
*a^(1/2))
```

sympy [A] time = 3.24, size = 26, normalized size = 0.96

$$-\frac{2\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)**(1/2),x)
```

```
[Out] -2*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*sqrt(a))
```

3.409 $\int \frac{1}{x^4\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=50

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}} - \frac{\sqrt{a+bx^3}}{3ax^3}$$

```
[Out] 1/3*b*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)-1/3*(b*x^3+a)^(1/2)/a/x^3
```

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 51, 63, 208}

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}} - \frac{\sqrt{a+bx^3}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a + b*x^3]),x]
```

```
[Out] -Sqrt[a + b*x^3]/(3*a*x^3) + (b*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(3/2
))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{a+bx^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^3\right)\\ &= -\frac{\sqrt{a+bx^3}}{3ax^3} - \frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right)}{6a}\\ &= -\frac{\sqrt{a+bx^3}}{3ax^3} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)}{3a}\\ &= -\frac{\sqrt{a+bx^3}}{3ax^3} + \frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 64, normalized size = 1.28

$$\frac{2b\sqrt{a+bx^3}\left(\frac{\tanh^{-1}\left(\sqrt{\frac{bx^3}{a}+1}\right)}{2\sqrt{\frac{bx^3}{a}+1}} - \frac{a}{2bx^3}\right)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*b*Sqrt[a + b*x^3]*(-1/2*a/(b*x^3) + ArcTanh[Sqrt[1 + (b*x^3)/a]]/(2*Sqrt
[1 + (b*x^3)/a])))/(3*a^2)
```

fricas [A] time = 0.85, size = 107, normalized size = 2.14

$$\left[\frac{\sqrt{a}\,bx^3\log\left(\frac{bx^3+2\sqrt{bx^3+a}\sqrt{a}+2a}{x^3}\right) - 2\sqrt{bx^3+a}\,a}{6a^2x^3}, -\frac{\sqrt{-a}\,bx^3\arctan\left(\frac{\sqrt{bx^3+a}\sqrt{-a}}{a}\right) + \sqrt{bx^3+a}\,a}{3a^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(sqrt(a)*b*x^3*log((b*x^3 + 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) - 2*
sqrt(b*x^3 + a)*a)/(a^2*x^3), -1/3*(sqrt(-a)*b*x^3*arctan(sqrt(b*x^3 + a)*s
qrt(-a)/a) + sqrt(b*x^3 + a)*a)/(a^2*x^3)]
```

giac [A] time = 0.20, size = 51, normalized size = 1.02

$$-\frac{\frac{b^2\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a} + \frac{\sqrt{bx^3+a}\,b}{ax^3}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*(b^2*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(b*x^3 + a)*b
/(a*x^3))/b
```

maple [A] time = 0.02, size = 39, normalized size = 0.78

$$\frac{b\,\text{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}} - \frac{\sqrt{b\,x^3+a}}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)^(1/2),x)
```

```
[Out] 1/3*b*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)-1/3*(b*x^3+a)^(1/2)/a/x^3
```

maxima [A] time = 2.88, size = 68, normalized size = 1.36

$$-\frac{\sqrt{bx^3+a}\,b}{3\left(\left(bx^3+a\right)a-a^2\right)}-\frac{b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{6\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(b*x^3 + a)*b/((b*x^3 + a)*a - a^2) - 1/6*b*log((sqrt(b*x^3 + a) -
 sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(3/2)
```

mupad [B] time = 1.28, size = 59, normalized size = 1.18

$$\frac{b\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)^3}{x^6}\right)}{6\,a^{3/2}}-\frac{\sqrt{b\,x^3+a}}{3\,a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3)^(1/2)),x)
```

```
[Out] (b*log((((a + b*x^3)^(1/2) - a^(1/2))*((a + b*x^3)^(1/2) + a^(1/2))^3)/x^6)
)/(6*a^(3/2)) - (a + b*x^3)^(1/2)/(3*a*x^3)
```

sympy [A] time = 5.73, size = 49, normalized size = 0.98

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3ax^{\frac{3}{2}}}+\frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{3a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)**(1/2),x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x**3) + 1)/(3*a*x**(3/2)) + b*asinh(sqrt(a)/(sqrt(b)*x**
(3/2)))/(3*a**(3/2))
```

3.410 $\int \frac{1}{x^7\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=74

$$-\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4a^{5/2}}+\frac{b\sqrt{a+bx^3}}{4a^2x^3}-\frac{\sqrt{a+bx^3}}{6ax^6}$$

```
[Out] -1/4*b^2*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(5/2)-1/6*(b*x^3+a)^(1/2)/a/x^6
+1/4*b*(b*x^3+a)^(1/2)/a^2/x^3
```

Rubi [A] time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 51, 63, 208}

$$-\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4a^{5/2}}+\frac{b\sqrt{a+bx^3}}{4a^2x^3}-\frac{\sqrt{a+bx^3}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*Sqrt[a + b*x^3]),x]
```

```
[Out] -Sqrt[a + b*x^3]/(6*a*x^6) + (b*Sqrt[a + b*x^3])/(4*a^2*x^3) - (b^2*ArcTanh
[Sqrt[a + b*x^3]/Sqrt[a]])/(4*a^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7\sqrt{a+bx^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx, x, x^3\right)\\ &= -\frac{\sqrt{a+bx^3}}{6ax^6} - \frac{b\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^3\right)}{4a}\\ &= -\frac{\sqrt{a+bx^3}}{6ax^6} + \frac{b\sqrt{a+bx^3}}{4a^2x^3} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right)}{8a^2}\\ &= -\frac{\sqrt{a+bx^3}}{6ax^6} + \frac{b\sqrt{a+bx^3}}{4a^2x^3} + \frac{b\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)}{4a^2}\\ &= -\frac{\sqrt{a+bx^3}}{6ax^6} + \frac{b\sqrt{a+bx^3}}{4a^2x^3} - \frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4a^{5/2}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.53

$$-\frac{2b^2\sqrt{a+bx^3}\;{}_2F_1\left(\frac{1}{2}, 3; \frac{3}{2}; \frac{bx^3}{a}+1\right)}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*b^2*Sqrt[a + b*x^3]*Hypergeometric2F1[1/2, 3, 3/2, 1 + (b*x^3)/a])/(3*a
^3)
```

fricas [A] time = 0.85, size = 137, normalized size = 1.85

$$\left[\frac{3\sqrt{a}\,b^2x^6\log\left(\frac{bx^3-2\sqrt{bx^3+a}\sqrt{a}+2a}{x^3}\right)+2\left(3abx^3-2a^2\right)\sqrt{bx^3+a}}{24a^3x^6}, \frac{3\sqrt{-a}\,b^2x^6\arctan\left(\frac{\sqrt{bx^3+a}\sqrt{-a}}{a}\right)+\left(3abx^3-\right.}{12a^3x^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*sqrt(a)*b^2*x^6*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3)
 + 2*(3*a*b*x^3 - 2*a^2)*sqrt(b*x^3 + a))/(a^3*x^6), 1/12*(3*sqrt(-a)*b^2*x
^6*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (3*a*b*x^3 - 2*a^2)*sqrt(b*x^3 + a)
)/(a^3*x^6)]
```

giac [A] time = 0.17, size = 75, normalized size = 1.01

$$\frac{\frac{3b^3\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{3\left(bx^3+a\right)^{\frac{3}{2}}b^3-5\sqrt{bx^3+a}\,ab^3}{a^2b^2x^6}}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*(3*b^3*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x^3 + a
)^(3/2)*b^3 - 5*sqrt(b*x^3 + a)*a*b^3)/(a^2*b^2*x^6))/b
```

maple [A] time = 0.02, size = 59, normalized size = 0.80

$$-\frac{b^2 \operatorname{arctanh}\left(\frac{\sqrt{b x^3+a}}{\sqrt{a}}\right)}{4a^{\frac{5}{2}}}+\frac{\sqrt{b x^3+a}\, b}{4a^2x^3}-\frac{\sqrt{b x^3+a}}{6a x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^3+a)^(1/2),x)
```

```
[Out] -1/4*b^2*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(5/2)-1/6*(b*x^3+a)^(1/2)/x^6/a
+1/4*b*(b*x^3+a)^(1/2)/a^2/x^3
```

maxima [A] time = 3.01, size = 104, normalized size = 1.41

$$\frac{b^2 \log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{8\,a^{\frac{5}{2}}}+\frac{3\left(bx^3+a\right)^{\frac{3}{2}}b^2-5\sqrt{bx^3+a}\,ab^2}{12\left(\left(bx^3+a\right)^2a^2-2\left(bx^3+a\right)a^3+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*b^2*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(5/2
) + 1/12*(3*(b*x^3 + a)^(3/2)*b^2 - 5*sqrt(b*x^3 + a)*a*b^2)/((b*x^3 + a)^2
*a^2 - 2*(b*x^3 + a)*a^3 + a^4)
```

mupad [B] time = 1.32, size = 79, normalized size = 1.07

$$\frac{b^2\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{8\,a^{5/2}}-\frac{\sqrt{b\,x^3+a}}{6\,a\,x^6}+\frac{b\,\sqrt{b\,x^3+a}}{4\,a^2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^3)^(1/2)),x)
```

```
[Out] (b^2*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^
6))/(8*a^(5/2)) - (a + b*x^3)^(1/2)/(6*a*x^6) + (b*(a + b*x^3)^(1/2))/(4*a^
2*x^3)
```

sympy [A] time = 10.87, size = 104, normalized size = 1.41

$$-\frac{1}{6\sqrt{b}\,x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{\sqrt{b}}{12ax^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{b^{\frac{3}{2}}}{4a^2x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{4a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**3+a)**(1/2),x)
```

```
[Out] -1/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) + sqrt(b)/(12*a*x**(9/2)*sqrt
(a/(b*x**3) + 1)) + b**(3/2)/(4*a**2*x**(3/2)*sqrt(a/(b*x**3) + 1)) - b**2*
asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(4*a**(5/2))
```

3.411 $\int \frac{x^6}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=254

$$\frac{32\sqrt{2+\sqrt{3}}\,a^2\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{55\sqrt[4]{3}\,b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}-\frac{16ax\sqrt{a+bx^3}}{55b^2}+\frac{2x^4}{}$$

```
[Out] -16/55*a*x*(b*x^3+a)^(1/2)/b^2+2/11*x^4*(b*x^3+a)^(1/2)/b+32/165*a^2*(a^(1/
3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*
(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(7/3
)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2
)))^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {321, 218}

$$\frac{32\sqrt{2+\sqrt{3}}\,a^2\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{55\sqrt[4]{3}\,b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}-\frac{16ax\sqrt{a+bx^3}}{55b^2}+\frac{2x^4}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/Sqrt[a + b*x^3],x]
```

```
[Out] (-16*a*x*Sqrt[a + b*x^3])/(55*b^2) + (2*x^4*Sqrt[a + b*x^3])/(11*b) + (32*S
qrt[2 + Sqrt[3]]*a^2*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((
1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
- 4*Sqrt[3]])/(55*3^(1/4)*b^(7/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^6}{\sqrt{a+bx^3}}\,dx = \frac{2x^4\sqrt{a+bx^3}}{11b} - \frac{(8a)\int \frac{x^3}{\sqrt{a+bx^3}}\,dx}{11b}$$

$$= -\frac{16ax\sqrt{a+bx^3}}{55b^2} + \frac{2x^4\sqrt{a+bx^3}}{11b} + \frac{\left(16a^2\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{55b^2}$$

$$= -\frac{16ax\sqrt{a+bx^3}}{55b^2} + \frac{2x^4\sqrt{a+bx^3}}{11b} + \frac{32\sqrt{2+\sqrt{3}}\;a^2\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\right.\right.}{55\sqrt[4]{3}\,b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a-}}$$

Mathematica [C] time = 0.04, size = 79, normalized size = 0.31

$$\frac{2\left(8a^2x\sqrt{\frac{bx^3}{a}+1}\;{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)-8a^2x-3abx^4+5b^2x^7\right)}{55b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/Sqrt[a + b*x^3],x]
```

```
[Out] (2*(-8*a^2*x - 3*a*b*x^4 + 5*b^2*x^7 + 8*a^2*x*Sqrt[1 + (b*x^3)/a]*Hypergeo
metric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]))/(55*b^2*Sqrt[a + b*x^3])
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^6}{\sqrt{bx^3+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^6/sqrt(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/sqrt(b*x^3 + a), x)
```

maple [A] time = 0.02, size = 320, normalized size = 1.26

$$32i\sqrt{3}\;\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\;b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\;i}{\left(-a\,b^2\right)^{\frac{1}{3}}}}$$

$$\frac{2\sqrt{b\,x^3+a}\;x^4}{11b} - \frac{}{165\sqrt{b\,x^3+a}\;b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^3+a)^(1/2),x)
```

```
[Out] 2/11*x^4*(b*x^3+a)^(1/2)/b-16/55*a*x*(b*x^3+a)^(1/2)/b^2-32/165*I*a^2/b^3*3
^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/
3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^
(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+
1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(
1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2
)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*
(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/sqrt(b*x^3 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^3)^(1/2),x)
```

```
[Out] int(x^6/(a + b*x^3)^(1/2), x)
```

sympy [A] time = 2.01, size = 37, normalized size = 0.15

$$\frac{x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**3+a)**(1/2),x)
```

```
[Out] x**7*gamma(7/3)*hyper((1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqr
t(a)*gamma(10/3))
```

3.412 $\int \frac{x^3}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=230

$$\frac{2x\sqrt{a+bx^3}}{5b}-\frac{4\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\,b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}$$

```
[Out] 2/5*x*(b*x^3+a)^(1/2)/b-4/15*a*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(
1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/
2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)
*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(4/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^
(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {321, 218}

$$\frac{2x\sqrt{a+bx^3}}{5b}-\frac{4\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\,b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b*x^3],x]
```

```
[Out] (2*x*Sqrt[a + b*x^3])/(5*b) - (4*Sqrt[2 + Sqrt[3]]*a*(a^(1/3) + b^(1/3)*x)*
Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*b^(4/3)*Sqrt[(a^(1
/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+bx^3}}\,dx = \frac{2x\sqrt{a+bx^3}}{5b} - \frac{(2a)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{5b}$$

$$= \frac{2x\sqrt{a+bx^3}}{5b} - \frac{4\sqrt{2+\sqrt{3}}\; a\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}\right)|-\right.}{5\sqrt[4]{3}\,b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}$$

Mathematica [C] time = 0.02, size = 62, normalized size = 0.27

$$\frac{2x\left(-a\sqrt{\frac{bx^3}{a}+1}\;{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+a+bx^3\right)}{5b\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + b*x^3],x]
```

```
[Out] (2*x*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -(
(b*x^3)/a)]))/(5*b*Sqrt[a + b*x^3])
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3}{\sqrt{bx^3+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^3/sqrt(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(b*x^3 + a), x)
```

maple [A] time = 0.02, size = 300, normalized size = 1.30

$$4i\sqrt{3}\;\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\;b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\;b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\;a\,\text{EllipticF}\left(\right.$$

$$15\sqrt{b\,x^3+a}\;b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a)^(1/2),x)
```

```
[Out] 2/5*x*(b*x^3+a)^(1/2)/b+4/15*I*a/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b
^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)
*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/
b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/
2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*
(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^
(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^
2)^(1/3)/b)/b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{bx^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(b*x^3 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\sqrt{b\,x^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^3)^(1/2),x)
```

```
[Out] int(x^3/(a + b*x^3)^(1/2), x)
```

sympy [A] time = 1.45, size = 37, normalized size = 0.16

$$\frac{x^4 \Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{4}{3} \\ \frac{7}{3}\end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a)**(1/2),x)
```

```
[Out] x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt
(a)*gamma(7/3))
```

3.413 $\int \frac{1}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=207

$$\frac{2\sqrt{2+\sqrt{3}}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,\sqrt[3]{b}\,\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}$$

```
[Out] 2/3*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*
x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a
^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3
/4)/b^(1/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)
*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {218}

$$\frac{2\sqrt{2+\sqrt{3}}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,\sqrt[3]{b}\,\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*x^3],x]
```

```
[Out] (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((
1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
- 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{2+\sqrt{3}}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,\sqrt[3]{b}\,\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}$$

Mathematica [C] time = 0.01, size = 46, normalized size = 0.22

$$\frac{x\sqrt{\frac{bx^3}{a}+1}\ {}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)}{\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*x^3],x]
```

```
[Out] (x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)])/Sqrt
[a + b*x^3]
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{bx^3+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*x^3 + a), x)
```

maple [A] time = 0.02, size = 283, normalized size = 1.37

$$-\frac{2i\sqrt{3}\ \left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\ \text{EllipticF}\left(\frac{\sqrt{3}}{}\right.}{3\sqrt{b\,x^3+a}\ b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(-a*b^2)^(1/3)/b*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-
a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2
*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2
)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(
b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/
2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1
/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*x^3 + a), x)
```

mupad **[B]** time = 1.02, size = 37, normalized size = 0.18

$$\frac{x\sqrt{\frac{b\,x^3}{a}+1}\;{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\;\frac{4}{3};\;-\frac{b\,x^3}{a}\right)}{\sqrt{b\,x^3+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^3)^(1/2),x)
```

```
[Out] (x*((b*x^3)/a + 1)^(1/2)*hypergeom([1/3, 1/2], 4/3, -(b*x^3)/a))/(a + b*x^3
)^(1/2)
```

sympy **[A]** time = 1.64, size = 36, normalized size = 0.17

$$\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\,\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(1/2),x)
```

```
[Out] x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)
*gamma(4/3))
```

3.414 $\int \frac{1}{x^3\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=234

$$-\frac{\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\,a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt{a+bx^3}}{2ax^2}$$

```
[Out] -1/2*(b*x^3+a)^(1/2)/a/x^2-1/6*b^(2/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/
3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1
/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+
b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {325, 218}

$$-\frac{\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\,a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt{a+bx^3}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*x^3]),x]
```

```
[Out] -Sqrt[a + b*x^3]/(2*a*x^2) - (Sqrt[2 + Sqrt[3]]*b^(2/3)*(a^(1/3) + b^(1/3)*
x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*a*Sqrt[(a^(1/3)
*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x
^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{a+bx^3}}\,dx = -\frac{\sqrt{a+bx^3}}{2ax^2} - \frac{b\int \frac{1}{\sqrt{a+bx^3}}\,dx}{4a}$$

$$= -\frac{\sqrt{a+bx^3}}{2ax^2} - \frac{\sqrt{2+\sqrt{3}}\; b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}\right)\right|$$

$$2\sqrt[4]{3}\,a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.22

$$-\frac{\sqrt{\frac{bx^3}{a}+1}\;{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a}\right)}{2x^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + b*x^3]),x]
```

```
[Out] -1/2*(Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-2/3, 1/2, 1/3, -((b*x^3)/a)])/
(x^2*Sqrt[a + b*x^3])
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+a}}{bx^6+ax^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)/(b*x^6 + a*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^3+a}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a)*x^3), x)
```

maple [A] time = 0.02, size = 301, normalized size = 1.29

$$i\sqrt{3}\;\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\;b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\;b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\;\text{EllipticF}\left(\begin{array}{l}\sqrt{3}\\ \\ \\ \\ \end{array}\right.$$

$$6\sqrt{b\,x^3+a}\;a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a)^(1/2),x)
```

```
[Out] -1/2*(b*x^3+a)^(1/2)/a/x^2+1/6*I/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2
)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*(
(x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)
)^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)
/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-
a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1
/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)
^(1/3)/b)/b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^3 + a}\, x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3 \sqrt{b\, x^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^3)^(1/2)),x)
```

```
[Out] int(1/(x^3*(a + b*x^3)^(1/2)), x)
```

sympy [A] time = 1.59, size = 41, normalized size = 0.18

$$\frac{\Gamma\left(-\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{1}{2} \\ \frac{1}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}\, x^2 \Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a)**(1/2),x)
```

```
[Out] gamma(-2/3)*hyper((-2/3, 1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)
*x**2*gamma(1/3))
```

3.415 $\int \frac{1}{x^6\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=256

$$\frac{7b\sqrt{a+bx^3}}{20a^2x^2}+\frac{7\sqrt{2+\sqrt{3}}\,b^{5/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{20\sqrt[4]{3}\,a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}-\frac{\sqrt{a+}}{5ax}$$

```
[Out] -1/5*(b*x^3+a)^(1/2)/a/x^5+7/20*b*(b*x^3+a)^(1/2)/a^2/x^2+7/60*b^(5/3)*(a^(
1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3
)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^2/
(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))
)^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {325, 218}

$$\frac{7\sqrt{2+\sqrt{3}}\,b^{5/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{20\sqrt[4]{3}\,a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\frac{7b\sqrt{a+bx^3}}{20a^2x^2}-\frac{\sqrt{a+}}{5ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*Sqrt[a + b*x^3]),x]
```

```
[Out] -Sqrt[a + b*x^3]/(5*a*x^5) + (7*b*Sqrt[a + b*x^3])/(20*a^2*x^2) + (7*Sqrt[2
 + Sqrt[3]]*b^(5/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
 4*Sqrt[3]])/(20*3^(1/4)*a^2*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^6\sqrt{a+bx^3}}\,dx = -\frac{\sqrt{a+bx^3}}{5ax^5} - \frac{(7b)\int \frac{1}{x^3\sqrt{a+bx^3}}\,dx}{10a}$$

$$= -\frac{\sqrt{a+bx^3}}{5ax^5} + \frac{7b\sqrt{a+bx^3}}{20a^2x^2} + \frac{\left(7b^2\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{40a^2}$$

$$= -\frac{\sqrt{a+bx^3}}{5ax^5} + \frac{7b\sqrt{a+bx^3}}{20a^2x^2} + \frac{7\sqrt{2+\sqrt{3}}\,b^{5/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{(1-}{(1+}\right.\right.}{20\sqrt[4]{3}\,a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^}}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.20

$$-\frac{\sqrt{\frac{bx^3}{a}+1}\;{}_2F_1\left(-\frac{5}{3},\frac{1}{2};-\frac{2}{3};-\frac{bx^3}{a}\right)}{5x^5\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*Sqrt[a + b*x^3]),x]
```

```
[Out] -1/5*(Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-5/3, 1/2, -2/3, -((b*x^3)/a)])
/(x^5*Sqrt[a + b*x^3])
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+a}}{bx^9+ax^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)/(b*x^9 + a*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^3+a}\,x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a)*x^6), x)
```

maple [A] time = 0.02, size = 320, normalized size = 1.25

$$-\frac{7i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\,b\,\text{EllipticF}\left(\sqrt{}\right.}{60\sqrt{b\,x^3+a}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^3+a)^(1/2),x)
```

```
[Out] -1/5*(b*x^3+a)^(1/2)/a/x^5+7/20*b*(b*x^3+a)^(1/2)/a^2/x^2-7/60*I/a^2*b*3^(1
/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/
b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/
3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2
*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2
)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(
1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a
*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^3 + a}\, x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^6 \sqrt{b\, x^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^3)^(1/2)),x)
```

```
[Out] int(1/(x^6*(a + b*x^3)^(1/2)), x)
```

sympy [A] time = 2.59, size = 44, normalized size = 0.17

$$\frac{\Gamma\left(-\frac{5}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{3}, \frac{1}{2} \\ -\frac{2}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}\, x^5 \Gamma\left(-\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**3+a)**(1/2),x)
```

```
[Out] gamma(-5/3)*hyper((-5/3, 1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a
)*x**5*gamma(-2/3))
```

3.416 $\int \frac{x^7}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=514

$$\frac{80\sqrt{2}\,a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{91\sqrt[4]{3}\,b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{40\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{}\right.}{}$$

```
[Out] -20/91*a*x^2*(b*x^3+a)^(1/2)/b^2+2/13*x^5*(b*x^3+a)^(1/2)/b+80/91*a^2*(b*x^
3+a)^(1/2)/b^(8/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+80/273*a^(7/3)*(a^(1/3)+
b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(
b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(8/3)/(b*x^3+a)^(1/2)/(a^
(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-40/91*3^
(1/4)*a^(7/3)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*(
(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^
(1/2)/b^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 514, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {321, 303, 218, 1877}

$$\frac{80a^2\sqrt{a+bx^3}}{91b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{80\sqrt{2}\,a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right.}{91\sqrt[4]{3}\,b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/Sqrt[a + b*x^3],x]
```

```
[Out] (-20*a*x^2*Sqrt[a + b*x^3])/(91*b^2) + (2*x^5*Sqrt[a + b*x^3])/(13*b) + (80
*a^2*Sqrt[a + b*x^3])/(91*b^(8/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (4
0*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Elli
pticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x
))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (80*Sqrt[2]*a^
(7/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^
(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(
91*3^(1/4)*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^7}{\sqrt{a+bx^3}}\,dx = \frac{2x^5\sqrt{a+bx^3}}{13b} - \frac{(10a)\int \frac{x^4}{\sqrt{a+bx^3}}\,dx}{13b}$$

$$= -\frac{20ax^2\sqrt{a+bx^3}}{91b^2} + \frac{2x^5\sqrt{a+bx^3}}{13b} + \frac{\left(40a^2\right)\int \frac{x}{\sqrt{a+bx^3}}\,dx}{91b^2}$$

$$= -\frac{20ax^2\sqrt{a+bx^3}}{91b^2} + \frac{2x^5\sqrt{a+bx^3}}{13b} + \frac{\left(40a^2\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{91b^{7/3}} + \frac{\left(40\sqrt{2\left(2-\sqrt{3}\right)}\,a^{7/3}\right)\int$$

$$= -\frac{20ax^2\sqrt{a+bx^3}}{91b^2} + \frac{2x^5\sqrt{a+bx^3}}{13b} + \frac{80a^2\sqrt{a+bx^3}}{91b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} - \frac{40\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{7/3}\left($$

Mathematica [C] time = 0.03, size = 80, normalized size = 0.16

$$\frac{2x^2\left(10a^2\sqrt{\frac{bx^3}{a}+1}\;{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)-10a^2-3abx^3+7b^2x^6\right)}{91b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/Sqrt[a + b*x^3],x]
```

```
[Out] (2*x^2*(-10*a^2 - 3*a*b*x^3 + 7*b^2*x^6 + 10*a^2*Sqrt[1 + (b*x^3)/a]*Hyperg
eometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)]))/(91*b^2*Sqrt[a + b*x^3])
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^7}{\sqrt{bx^3+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^7/sqrt(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^7/sqrt(b*x^3 + a), x)
```

maple [A] time = 0.02, size = 474, normalized size = 0.92

$$80i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}$$

$$\frac{2\sqrt{b\,x^3+a}\,x^5}{13b}-\frac{20\sqrt{b\,x^3+a}\,a\,x^2}{91b^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^3+a)^(1/2),x)
```

```
[Out] 2/13*x^5*(b*x^3+a)^(1/2)/b-20/91*a*x^2*(b*x^3+a)^(1/2)/b^2-80/273*I*a^2/b^3
*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(
1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2
)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/
b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)
^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/
3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2
)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+
1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^
(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-
a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*
I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^7/sqrt(b*x^3 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7}{\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^3)^(1/2),x)
```

```
[Out] int(x^7/(a + b*x^3)^(1/2), x)
```

sympy [A] time = 2.31, size = 37, normalized size = 0.07

$$\frac{x^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**3+a)**(1/2),x)
```

```
[Out] x**8*gamma(8/3)*hyper((1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqr
t(a)*gamma(11/3))
```

3.417 $\int \frac{x^4}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=490

$$-\frac{8\sqrt{2}\,a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7\sqrt[4]{3}\,b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{}\right.}{}$$

```
[Out] 2/7*x^2*(b*x^3+a)^(1/2)/b-8/7*a*(b*x^3+a)^(1/2)/b^(5/3)/(b^(1/3)*x+a^(1/3)*
(1+3^(1/2)))-8/21*a^(4/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*
(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/
3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*
3^(3/4)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2)))^2)^(1/2)+4/7*3^(1/4)*a^(4/3)*(a^(1/3)+b^(1/3)*x)*Elliptic
E((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)
+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b
^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^
(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 490, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {321, 303, 218, 1877}

$$-\frac{8\sqrt{2}\,a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7\sqrt[4]{3}\,b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a + b*x^3],x]
```

```
[Out] (2*x^2*Sqrt[a + b*x^3])/(7*b) - (8*a*Sqrt[a + b*x^3])/(7*b^(5/3)*((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(a^(1/3)
+ b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)
*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*b^(5/3)*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt
[a + b*x^3]) - (8*Sqrt[2]*a^(4/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)], -7 - 4*Sqrt[3]])/(7*3^(1/4)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1
/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{a+bx^3}}\,dx = \frac{2x^2\sqrt{a+bx^3}}{7b} - \frac{(4a)\int \frac{x}{\sqrt{a+bx^3}}\,dx}{7b}$$

$$= \frac{2x^2\sqrt{a+bx^3}}{7b} - \frac{(4a)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{7b^{4/3}} - \frac{\left(4\sqrt{2\left(2-\sqrt{3}\right)}\,a^{4/3}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{7b^{4/3}}$$

$$= \frac{2x^2\sqrt{a+bx^3}}{7b} - \frac{8a\sqrt{a+bx^3}}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} + \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{}}{\left(\left(1+\sqrt{3}\right)\right.}}}{7b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{}\right.}{\left(\left(1+\sqrt{3}\right)\right.}}}$$

Mathematica [C] time = 0.02, size = 64, normalized size = 0.13

$$\frac{2x^2\left(-a\sqrt{\frac{bx^3}{a}+1}\;{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)+a+bx^3\right)}{7b\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a + b*x^3],x]
```

```
[Out] (2*x^2*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2/3, 5/3,
-((b*x^3)/a)]))/(7*b*Sqrt[a + b*x^3])
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^4}{\sqrt{bx^3+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^4/sqrt(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(b*x^3 + a), x)
```

maple [A] time = 0.02, size = 454, normalized size = 0.93

$$8i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}$$

$$\frac{2\sqrt{b\,x^3+a}\,x^2}{7b}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a)^(1/2),x)
```

```
[Out] 2/7*x^2*(b*x^3+a)^(1/2)/b+8/21*I*a/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a
*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/
2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3
)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(
1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*
3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-
1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*
(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1
/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*
I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*
b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(b*x^3 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^3)^(1/2),x)
```

```
[Out] int(x^4/(a + b*x^3)^(1/2), x)
```

sympy [A] time = 0.96, size = 37, normalized size = 0.08

$$\frac{x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a)**(1/2),x)
```

```
[Out] x**5*gamma(5/3)*hyper((1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt
(a)*gamma(8/3))
```

3.418 $\int \frac{x}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=462

$$\frac{2\sqrt{2}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right.}{b^{}}$$

```
[Out] 2*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+2/3*a^(1/3)*(a^(1
/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)
*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^
2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(b*x^3+a)^(1/2)
/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-3^(1
/4)*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(
b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a
^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1
/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)
*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 462, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {303, 218, 1877}

$$\frac{2\sqrt{2}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right.}{b^{}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*x^3],x]
```

```
[Out] (2*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4
)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[Ar
cSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2]*a^(1/3)*(a^(1/
3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2
/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+bx^3}}\,dx = \frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{\sqrt[3]{b}} + \frac{\left(\sqrt{2\left(2-\sqrt{3}\right)}\,\sqrt[3]{a}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{\sqrt[3]{b}}$$

$$= \frac{2\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,E\left(\sin^{-1}\left(\right.\right.}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.11

$$\frac{x^2\sqrt{\frac{bx^3}{a}+1}\;{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b*x^3],x]
```

```
[Out] (x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)])/(2
*Sqrt[a + b*x^3])
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{\sqrt{bx^3+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x/sqrt(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(b*x^3 + a), x)
```

maple [A] time = 0.02, size = 435, normalized size = 0.94

$$-\frac{2i\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-ab^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-ab^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-ab^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-ab^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-ab^2\right)^{\frac{1}{3}}}}\left(\left(-\frac{3\left(-ab^2\right)^{\frac{1}{3}}}{2b}+\right.\right.}{\left.\left.\left.

sympy [**A**] time = 0.97, size = 37, normalized size = 0.08

$$\frac{x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a)**(1/2),x)
```

```
[Out] x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt
(a)*gamma(5/3))
```

3.419 $\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=484

$$\frac{\sqrt{2}\,\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}} - \frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{2a^2}$$

```
[Out] -(b*x^3+a)^(1/2)/a/x+b^(1/3)*(b*x^3+a)^(1/2)/a/(b^(1/3)*x+a^(1/3)*(1+3^(1/2
))))+1/3*b^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)
))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)
*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^
(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2)))^2)^(1/2)-1/2*3^(1/4)*b^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)
*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2
*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a
^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1
/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 484, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {325, 303, 218, 1877}

$$\frac{\sqrt{2}\,\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}} - \frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*x^3]),x]
```

```
[Out] -(Sqrt[a + b*x^3]/(a*x)) + (b^(1/3)*Sqrt[a + b*x^3])/(a*((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*(a^(1/3) + b^(1/3)*x
)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*a^(2/3)*Sqrt[(a^(1/3)*(a
^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]
) + (Sqrt[2]*b^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((
1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
 - 4*Sqrt[3]])/(3^(1/4)*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
 + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
 + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx = -\frac{\sqrt{a+bx^3}}{ax} + \frac{b\int \frac{x}{\sqrt{a+bx^3}}\,dx}{2a}$$

$$= -\frac{\sqrt{a+bx^3}}{ax} + \frac{b^{2/3}\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{2a} + \frac{\left(\sqrt{\frac{1}{2}\left(2-\sqrt{3}\right)}\,b^{2/3}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{a^{2/3}}$$

$$= -\frac{\sqrt{a+bx^3}}{ax} + \frac{\sqrt[3]{b}\,\sqrt{a+bx^3}}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\;\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b}}}{2a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{}\right.}}$$

Mathematica [C] time = 0.01, size = 49, normalized size = 0.10

$$-\frac{\sqrt{\frac{bx^3}{a}+1}\;{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a}\right)}{x\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x^3]),x]
```

```
[Out] -((Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-1/3, 1/2, 2/3, -((b*x^3)/a)])/(x*
Sqrt[a + b*x^3]))
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+a}}{bx^5+ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)/(b*x^5 + a*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^3+a}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a)*x^2), x)
```

maple [A] time = 0.02, size = 453, normalized size = 0.94

$$-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\left(-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\right.}{\left.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a)^(1/2),x)
```

```
[Out] -(b*x^3+a)^(1/2)/a/x-1/3*I/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3
)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a
*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2
)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b
^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-
a*b^2)^(1/3)/b)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1
/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(
1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b
^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*
(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)
/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^3+a}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a)*x^2), x)
```

mupad [B] time = 1.28, size = 40, normalized size = 0.08

$$-\frac{2\sqrt{\frac{a}{b\,x^3}+1}\;{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{5}{6};\;\frac{11}{6};\;-\frac{a}{b\,x^3}\right)}{5\,x\,\sqrt{b\,x^3+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^3)^(1/2)),x)
```

```
[Out] -(2*(a/(b*x^3) + 1)^(1/2)*hypergeom([1/2, 5/6], 11/6, -a/(b*x^3)))/(5*x*(a
+ b*x^3)^(1/2))
```

sympy [A] time = 1.78, size = 39, normalized size = 0.08

$$\frac{\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},\frac{1}{2}\\ \frac{2}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,x\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a)**(1/2),x)
```

```
[Out] gamma(-1/3)*hyper((-1/3, 1/2), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)
*x*gamma(2/3))
```

3.420 $\int \frac{1}{x^5\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=514

$$-\frac{5b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\,\sqrt[4]{3}\,a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\frac{5\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{16a^{5}}$$

```
[Out] -1/4*(b*x^3+a)^(1/2)/a/x^4+5/8*b*(b*x^3+a)^(1/2)/a^2/x-5/8*b^(4/3)*(b*x^3+a
)^(1/2)/a^2/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-5/24*b^(4/3)*(a^(1/3)+b^(1/3)*x
)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
,I*3^(1/2)+2*I)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)
*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(5/3)*2^(1/2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^
(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+5/16*3^(1/4)*b^(4
/3)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*
x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a
^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(5
/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1
/2)))^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 514, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {325, 303, 218, 1877}

$$-\frac{5b^{4/3}\sqrt{a+bx^3}}{8a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{5b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\,\sqrt[4]{3}\,a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\frac{5\sqrt[4]{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[a + b*x^3]),x]
```

```
[Out] -Sqrt[a + b*x^3]/(4*a*x^4) + (5*b*Sqrt[a + b*x^3])/(8*a^2*x) - (5*b^(4/3)*S
qrt[a + b*x^3])/(8*a^2*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (5*3^(1/4)*Sq
rt[2 - Sqrt[3]]*b^(4/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin
[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)],
-7 - 4*Sqrt[3]])/(16*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (5*b^(4/3)*(a^(1/3) + b^(1
/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(4*Sqrt[2]*3^(1/4)*a^(
5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
 + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^5\sqrt{a+bx^3}}\,dx = -\frac{\sqrt{a+bx^3}}{4ax^4} - \frac{(5b)\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx}{8a}$$

$$= -\frac{\sqrt{a+bx^3}}{4ax^4} + \frac{5b\sqrt{a+bx^3}}{8a^2x} - \frac{\left(5b^2\right)\int \frac{x}{\sqrt{a+bx^3}}\,dx}{16a^2}$$

$$= -\frac{\sqrt{a+bx^3}}{4ax^4} + \frac{5b\sqrt{a+bx^3}}{8a^2x} - \frac{\left(5b^{5/3}\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{16a^2} - \frac{\left(5\sqrt{\frac{1}{2}\left(2-\sqrt{3}\right)}\,b^{5/3}\right)\int \frac{1}{\sqrt{a+bx}}}{8a^{5/3}}$$

$$= -\frac{\sqrt{a+bx^3}}{4ax^4} + \frac{5b\sqrt{a+bx^3}}{8a^2x} - \frac{5b^{4/3}\sqrt{a+bx^3}}{8a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} + \frac{5\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{}\right.}{}$$

1

Mathematica [C] time = 0.01, size = 51, normalized size = 0.10

$$-\frac{\sqrt{\frac{bx^3}{a}+1}\;{}_2F_1\left(-\frac{4}{3},\frac{1}{2};-\frac{1}{3};-\frac{bx^3}{a}\right)}{4x^4\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[a + b*x^3]),x]
```

```
[Out] -1/4*(Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-4/3, 1/2, -1/3, -((b*x^3)/a)])
/(x^4*Sqrt[a + b*x^3])
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+a}}{bx^8+ax^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)/(b*x^8 + a*x^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^3+a}\,x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a)*x^5), x)
```

maple [A] time = 0.02, size = 472, normalized size = 0.92

$$\frac{5i\sqrt{3}\,\left(-ab^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-ab^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-ab^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-ab^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-ab^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-ab^2\right)^{\frac{1}{3}}}}\left(\left(-\frac{3\left(-ab^2\right)^{\frac{1}{3}}}{2b}+\frac{i}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^3+a)^(1/2),x)
```

```
[Out] -1/4*(b*x^3+a)^(1/2)/a/x^4+5/8*b*(b*x^3+a)^(1/2)/a^2/x+5/24*I/a^2*b*3^(1/2)
*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*
3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/
b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*
3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*(
(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^(1/2
)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^
2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^
(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(I
*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(
1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2
)*(-a*b^2)^(1/3)/b)/b)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^3+a}\,x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^5 \sqrt{b\,x^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^3)^(1/2)),x)
```

```
[Out] int(1/(x^5*(a + b*x^3)^(1/2)), x)
```

sympy [A] time = 1.43, size = 44, normalized size = 0.09

$$\frac{\Gamma\left(-\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{4}{3}, \frac{1}{2} \\ -\frac{1}{3} \end{matrix} \middle| \frac{b x^3 e^{i\pi}}{a}\right)}{3\sqrt{a}\, x^4 \Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**3+a)**(1/2),x)
```

```
[Out] gamma(-4/3)*hyper((-4/3, 1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a
)*x**4*gamma(-1/3))
```

3.421 $\int \frac{x^{11}}{\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=78

$$\frac{2a^3}{3b^4\sqrt{a+bx^3}}+\frac{2a^2\sqrt{a+bx^3}}{b^4}-\frac{2a\left(a+bx^3\right)^{3/2}}{3b^4}+\frac{2\left(a+bx^3\right)^{5/2}}{15b^4}$$

```
[Out] -2/3*a*(b*x^3+a)^(3/2)/b^4+2/15*(b*x^3+a)^(5/2)/b^4+2/3*a^3/b^4/(b*x^3+a)^(
1/2)+2*a^2*(b*x^3+a)^(1/2)/b^4
```

Rubi [A] time = 0.04, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{2a^3}{3b^4\sqrt{a+bx^3}}+\frac{2a^2\sqrt{a+bx^3}}{b^4}-\frac{2a\left(a+bx^3\right)^{3/2}}{3b^4}+\frac{2\left(a+bx^3\right)^{5/2}}{15b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*a^3)/(3*b^4*Sqrt[a + b*x^3]) + (2*a^2*Sqrt[a + b*x^3])/b^4 - (2*a*(a + b
*x^3)^(3/2))/(3*b^4) + (2*(a + b*x^3)^(5/2))/(15*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{\left(a+bx^3\right)^{3/2}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^3}{(a+bx)^{3/2}}\,dx, x, x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int\left(-\frac{a^3}{b^3(a+bx)^{3/2}}+\frac{3a^2}{b^3\sqrt{a+bx}}-\frac{3a\sqrt{a+bx}}{b^3}+\frac{(a+bx)^{3/2}}{b^3}\right)dx, x, x^3\right)\\ &= \frac{2a^3}{3b^4\sqrt{a+bx^3}}+\frac{2a^2\sqrt{a+bx^3}}{b^4}-\frac{2a\left(a+bx^3\right)^{3/2}}{3b^4}+\frac{2\left(a+bx^3\right)^{5/2}}{15b^4}\end{aligned}$$

Mathematica [A] time = 0.03, size = 49, normalized size = 0.63

$$\frac{2\left(16a^3+8a^2bx^3-2ab^2x^6+b^3x^9\right)}{15b^4\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(16*a^3 + 8*a^2*b*x^3 - 2*a*b^2*x^6 + b^3*x^9))/(15*b^4*Sqrt[a + b*x^3])
```

fricas [A] time = 0.90, size = 57, normalized size = 0.73

$$\frac{2\left(b^3x^9 - 2\,ab^2x^6 + 8\,a^2bx^3 + 16\,a^3\right)\sqrt{bx^3+a}}{15\left(b^5x^3 + ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15*(b^3*x^9 - 2*a*b^2*x^6 + 8*a^2*b*x^3 + 16*a^3)*sqrt(b*x^3 + a)/(b^5*x^
3 + a*b^4)
```

giac [A] time = 0.15, size = 69, normalized size = 0.88

$$\frac{2\,a^3}{3\sqrt{bx^3+a}\,b^4} + \frac{2\left(\left(bx^3+a\right)^{\frac{5}{2}}b^{16} - 5\left(bx^3+a\right)^{\frac{3}{2}}ab^{16} + 15\sqrt{bx^3+a}\,a^2b^{16}\right)}{15\,b^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*a^3/(sqrt(b*x^3 + a)*b^4) + 2/15*((b*x^3 + a)^(5/2)*b^16 - 5*(b*x^3 + a
)^(3/2)*a*b^16 + 15*sqrt(b*x^3 + a)*a^2*b^16)/b^20
```

maple [A] time = 0.01, size = 46, normalized size = 0.59

$$\frac{\frac{2}{15}b^3x^9 - \frac{4}{15}a\,b^2x^6 + \frac{16}{15}a^2b\,x^3 + \frac{32}{15}a^3}{\sqrt{b\,x^3+a}\;b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^3+a)^(3/2),x)
```

```
[Out] 2/15/(b*x^3+a)^(1/2)*(b^3*x^9-2*a*b^2*x^6+8*a^2*b*x^3+16*a^3)/b^4
```

maxima [A] time = 1.33, size = 64, normalized size = 0.82

$$\frac{2\left(bx^3+a\right)^{\frac{5}{2}}}{15\,b^4} - \frac{2\left(bx^3+a\right)^{\frac{3}{2}}a}{3\,b^4} + \frac{2\sqrt{bx^3+a}\,a^2}{b^4} + \frac{2\,a^3}{3\sqrt{bx^3+a}\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*(b*x^3 + a)^(5/2)/b^4 - 2/3*(b*x^3 + a)^(3/2)*a/b^4 + 2*sqrt(b*x^3 + a
)*a^2/b^4 + 2/3*a^3/(sqrt(b*x^3 + a)*b^4)
```

mupad [B] time = 1.34, size = 70, normalized size = 0.90

$$\frac{22\,a^2\sqrt{b\,x^3+a}}{15\,b^4} + \frac{2\,a^3}{3\,b^4\sqrt{b\,x^3+a}} + \frac{2\,x^6\sqrt{b\,x^3+a}}{15\,b^2} - \frac{2\,a\,x^3\sqrt{b\,x^3+a}}{5\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a + b*x^3)^(3/2),x)
```

```
[Out] (22*a^2*(a + b*x^3)^(1/2))/(15*b^4) + (2*a^3)/(3*b^4*(a + b*x^3)^(1/2)) + (
2*x^6*(a + b*x^3)^(1/2))/(15*b^2) - (2*a*x^3*(a + b*x^3)^(1/2))/(5*b^3)
```

sympy [**A**] time = 4.62, size = 94, normalized size = 1.21

$$\begin{cases} \frac{32a^3}{15b^4\sqrt{a+bx^3}} + \frac{16a^2x^3}{15b^3\sqrt{a+bx^3}} - \frac{4ax^6}{15b^2\sqrt{a+bx^3}} + \frac{2x^9}{15b\sqrt{a+bx^3}} & \text{for } b \neq 0 \\ \frac{x^{12}}{12a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**3+a)**(3/2),x)
```

```
[Out] Piecewise((32*a**3/(15*b**4*sqrt(a + b*x**3)) + 16*a**2*x**3/(15*b**3*sqrt(
a + b*x**3)) - 4*a*x**6/(15*b**2*sqrt(a + b*x**3)) + 2*x**9/(15*b*sqrt(a +
b*x**3)), Ne(b, 0)), (x**12/(12*a**(3/2)), True))
```

3.422 $\int \frac{x^8}{\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=59

$$-\frac{2a^2}{3b^3\sqrt{a+bx^3}}-\frac{4a\sqrt{a+bx^3}}{3b^3}+\frac{2\left(a+bx^3\right)^{3/2}}{9b^3}$$

```
[Out] 2/9*(b*x^3+a)^(3/2)/b^3-2/3*a^2/b^3/(b*x^3+a)^(1/2)-4/3*a*(b*x^3+a)^(1/2)/b
^3
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{2a^2}{3b^3\sqrt{a+bx^3}}-\frac{4a\sqrt{a+bx^3}}{3b^3}+\frac{2\left(a+bx^3\right)^{3/2}}{9b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*a^2)/(3*b^3*Sqrt[a + b*x^3]) - (4*a*Sqrt[a + b*x^3])/(3*b^3) + (2*(a +
b*x^3)^(3/2))/(9*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{\left(a+bx^3\right)^{3/2}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{(a+bx)^{3/2}}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int\left(\frac{a^2}{b^2(a+bx)^{3/2}}-\frac{2a}{b^2\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b^2}\right)dx, x, x^3\right)\\ &= -\frac{2a^2}{3b^3\sqrt{a+bx^3}}-\frac{4a\sqrt{a+bx^3}}{3b^3}+\frac{2\left(a+bx^3\right)^{3/2}}{9b^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 38, normalized size = 0.64

$$\frac{2\left(-8a^2-4abx^3+b^2x^6\right)}{9b^3\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(-8*a^2 - 4*a*b*x^3 + b^2*x^6))/(9*b^3*Sqrt[a + b*x^3])
```

fricas [A] time = 0.85, size = 46, normalized size = 0.78

$$\frac{2\left(b^2x^6 - 4\,abx^3 - 8\,a^2\right)\sqrt{bx^3+a}}{9\left(b^4x^3 + ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/9*(b^2*x^6 - 4*a*b*x^3 - 8*a^2)*sqrt(b*x^3 + a)/(b^4*x^3 + a*b^3)
```

giac [A] time = 0.16, size = 52, normalized size = 0.88

$$-\frac{2\,a^2}{3\sqrt{bx^3+a}\,b^3} + \frac{2\left(\left(bx^3+a\right)^{\frac{3}{2}}b^6 - 6\sqrt{bx^3+a}\,ab^6\right)}{9\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*a^2/(sqrt(b*x^3 + a)*b^3) + 2/9*((b*x^3 + a)^(3/2)*b^6 - 6*sqrt(b*x^3
+ a)*a*b^6)/b^9
```

maple [A] time = 0.01, size = 36, normalized size = 0.61

$$-\frac{2\left(-b^2x^6 + 4ab\,x^3 + 8a^2\right)}{9\sqrt{b\,x^3+a}\;b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a)^(3/2),x)
```

```
[Out] -2/9/(b*x^3+a)^(1/2)*(-b^2*x^6+4*a*b*x^3+8*a^2)/b^3
```

maxima [A] time = 1.32, size = 47, normalized size = 0.80

$$\frac{2\left(bx^3+a\right)^{\frac{3}{2}}}{9\,b^3} - \frac{4\sqrt{bx^3+a}\,a}{3\,b^3} - \frac{2\,a^2}{3\sqrt{bx^3+a}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/9*(b*x^3 + a)^(3/2)/b^3 - 4/3*sqrt(b*x^3 + a)*a/b^3 - 2/3*a^2/(sqrt(b*x^3
 + a)*b^3)
```

mupad [B] time = 1.18, size = 41, normalized size = 0.69

$$-\frac{12\,a\,\left(b\,x^3+a\right) - 2\left(b\,x^3+a\right)^2 + 6\,a^2}{9\,b^3\sqrt{b\,x^3+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^3)^(3/2),x)
```

```
[Out] -(12*a*(a + b*x^3) - 2*(a + b*x^3)^2 + 6*a^2)/(9*b^3*(a + b*x^3)^(1/2))
```

sympy [**A**] time = 4.92, size = 70, normalized size = 1.19

$$\begin{cases} -\frac{16a^2}{9b^3\sqrt{a+bx^3}} - \frac{8ax^3}{9b^2\sqrt{a+bx^3}} + \frac{2x^6}{9b\sqrt{a+bx^3}} & \text{for } b \neq 0 \\ \frac{x^9}{9a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a)**(3/2),x)
```

```
[Out] Piecewise((-16*a**2/(9*b**3*sqrt(a + b*x**3)) - 8*a*x**3/(9*b**2*sqrt(a + b
*x**3)) + 2*x**6/(9*b*sqrt(a + b*x**3)), Ne(b, 0)), (x**9/(9*a**(3/2)), Tru
e))
```

3.423 $\int \frac{x^5}{\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=38

$$\frac{2a}{3b^2\sqrt{a+bx^3}}+\frac{2\sqrt{a+bx^3}}{3b^2}$$

```
[Out] 2/3*a/b^2/(b*x^3+a)^(1/2)+2/3*(b*x^3+a)^(1/2)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{2a}{3b^2\sqrt{a+bx^3}}+\frac{2\sqrt{a+bx^3}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*a)/(3*b^2*Sqrt[a + b*x^3]) + (2*Sqrt[a + b*x^3])/(3*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\left(a+bx^3\right)^{3/2}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{(a+bx)^{3/2}}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^{3/2}}+\frac{1}{b\sqrt{a+bx}}\right)dx, x, x^3\right)\\ &= \frac{2a}{3b^2\sqrt{a+bx^3}}+\frac{2\sqrt{a+bx^3}}{3b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.71

$$\frac{2\left(2a+bx^3\right)}{3b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(2*a + b*x^3))/(3*b^2*Sqrt[a + b*x^3])
```

fricas [A] time = 0.67, size = 35, normalized size = 0.92

$$\frac{2\left(bx^3+2a\right)\sqrt{bx^3+a}}{3\left(b^3x^3+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(3/2),x, algorithm="fricas")
[Out] 2/3*(b*x^3 + 2*a)*sqrt(b*x^3 + a)/(b^3*x^3 + a*b^2)
```

giac [A] time = 0.16, size = 33, normalized size = 0.87

$$\frac{2\left(\frac{\sqrt{bx^3+a}}{b}+\frac{a}{\sqrt{bx^3+a}\,b}\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(3/2),x, algorithm="giac")
[Out] 2/3*(sqrt(b*x^3 + a)/b + a/(sqrt(b*x^3 + a)*b))/b
```

maple [A] time = 0.00, size = 24, normalized size = 0.63

$$\frac{\frac{2b\,x^3}{3}+\frac{4a}{3}}{\sqrt{b\,x^3+a}\;b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a)^(3/2),x)
[Out] 2/3/(b*x^3+a)^(1/2)*(b*x^3+2*a)/b^2
```

maxima [A] time = 1.33, size = 30, normalized size = 0.79

$$\frac{2\sqrt{bx^3+a}}{3\,b^2}+\frac{2\,a}{3\sqrt{bx^3+a}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(3/2),x, algorithm="maxima")
[Out] 2/3*sqrt(b*x^3 + a)/b^2 + 2/3*a/(sqrt(b*x^3 + a)*b^2)
```

mupad [B] time = 1.14, size = 24, normalized size = 0.63

$$\frac{2\,b\,x^3+4\,a}{3\,b^2\,\sqrt{b\,x^3+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^3)^(3/2),x)
[Out] (4*a + 2*b*x^3)/(3*b^2*(a + b*x^3)^(1/2))
```

sympy [A] time = 1.16, size = 46, normalized size = 1.21

$$\begin{cases} \frac{4a}{3b^2\sqrt{a+bx^3}}+\frac{2x^3}{3b\sqrt{a+bx^3}} & \text{for } b\neq 0 \\ \frac{x^6}{6a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)**(3/2),x)
```

```
[Out] Piecewise((4*a/(3*b**2*sqrt(a + b*x**3)) + 2*x**3/(3*b*sqrt(a + b*x**3)), N
e(b, 0)), (x**6/(6*a**(3/2)), True))
```

3.424 $\int \frac{x^2}{\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=18

$$-\frac{2}{3b\sqrt{a+bx^3}}$$

```
[Out] -2/3/b/(b*x^3+a)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$-\frac{2}{3b\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^3)^(3/2),x]
```

```
[Out] -2/(3*b*Sqrt[a + b*x^3])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^3\right)^{3/2}}\,dx = -\frac{2}{3b\sqrt{a+bx^3}}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$-\frac{2}{3b\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^3)^(3/2),x]
```

```
[Out] -2/(3*b*Sqrt[a + b*x^3])
```

fricas [A] time = 0.82, size = 24, normalized size = 1.33

$$-\frac{2\sqrt{bx^3+a}}{3\left(b^2x^3+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(b*x^3 + a)/(b^2*x^3 + a*b)
```

giac [A] time = 0.16, size = 14, normalized size = 0.78

$$-\frac{2}{3\sqrt{bx^3+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(3/2),x, algorithm="giac")
[Out] -2/3/(sqrt(b*x^3 + a)*b)
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$-\frac{2}{3\sqrt{b\,x^3+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)^(3/2),x)
[Out] -2/3/b/(b*x^3+a)^(1/2)
```

maxima [A] time = 1.36, size = 14, normalized size = 0.78

$$-\frac{2}{3\,\sqrt{b x^3+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(3/2),x, algorithm="maxima")
[Out] -2/3/(sqrt(b*x^3 + a)*b)
```

mupad [B] time = 1.10, size = 14, normalized size = 0.78

$$-\frac{2}{3\,b\,\sqrt{b\,x^3+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^3)^(3/2),x)
[Out] -2/(3*b*(a + b*x^3)^(1/2))
```

sympy [A] time = 0.67, size = 26, normalized size = 1.44

$$\begin{cases} -\frac{2}{3b\sqrt{a+bx^3}} & \text{for } b \neq 0 \\ \frac{x^3}{3a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)**(3/2),x)
[Out] Piecewise((-2/(3*b*sqrt(a + b*x**3)), Ne(b, 0)), (x**3/(3*a**(3/2)), True))
```

3.425 $\int \frac{1}{x\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=46

$$\frac{2}{3a\sqrt{a+bx^3}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}$$

```
[Out] -2/3*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)+2/3/a/(b*x^3+a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 51, 63, 208}

$$\frac{2}{3a\sqrt{a+bx^3}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3)^(3/2)),x]
```

```
[Out] 2/(3*a*Sqrt[a + b*x^3]) - (2*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(3/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^3\right)^{3/2}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, x^3\right)$$
$$= \frac{2}{3a\sqrt{a+bx^3}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right)}{3a}$$
$$= \frac{2}{3a\sqrt{a+bx^3}} + \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)}{3ab}$$
$$= \frac{2}{3a\sqrt{a+bx^3}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}$$

Mathematica [C] time = 0.01, size = 36, normalized size = 0.78

$$\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx^3}{a}+1\right)}{3a\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3)^(3/2)),x]
```

```
[Out] (2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^3)/a])/(3*a*Sqrt[a + b*x^3])
```

fricas [A] time = 0.87, size = 129, normalized size = 2.80

$$\left[\frac{\left(bx^3+a\right)\sqrt{a}\,\log\left(\frac{bx^3-2\sqrt{bx^3+a}\sqrt{a}+2a}{x^3}\right)+2\sqrt{bx^3+a}\,a}{3\left(a^2bx^3+a^3\right)}, \frac{2\left(\left(bx^3+a\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx^3+a}\sqrt{-a}}{a}\right)+\sqrt{bx^3+a}\,a\right)}{3\left(a^2bx^3+a^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/3*((b*x^3 + a)*sqrt(a)*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3
) + 2*sqrt(b*x^3 + a)*a)/(a^2*b*x^3 + a^3), 2/3*((b*x^3 + a)*sqrt(-a)*arcta
n(sqrt(b*x^3 + a)*sqrt(-a)/a) + sqrt(b*x^3 + a)*a)/(a^2*b*x^3 + a^3)]
```

giac [A] time = 0.16, size = 41, normalized size = 0.89

$$\frac{2\,\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\sqrt{-a}\,a} + \frac{2}{3\sqrt{bx^3+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a) + 2/3/(sqrt(b*x^3 + a)*a)
```

maple [A] time = 0.03, size = 39, normalized size = 0.85

$$-\frac{2\,\text{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}} + \frac{2}{3\sqrt{\left(x^3+\frac{a}{b}\right)b}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)^(3/2),x)
```

```
[Out] 2/3/a/((x^3+a/b)*b)^(1/2)-2/3*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)
```

maxima [A] time = 2.92, size = 52, normalized size = 1.13

$$\frac{\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{3\,a^{\frac{3}{2}}}+\frac{2}{3\,\sqrt{bx^3+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(3/2) +
2/3/(sqrt(b*x^3 + a)*a)
```

mupad [B] time = 1.23, size = 55, normalized size = 1.20

$$\frac{\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3\,a^{3/2}}+\frac{2}{3\,a\,\sqrt{b\,x^3+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3)^(3/2)),x)
```

```
[Out] log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6)/(3
*a^(3/2)) + 2/(3*a*(a + b*x^3)^(1/2))
```

sympy [B] time = 2.53, size = 184, normalized size = 4.00

$$\frac{2a^3\sqrt{1+\frac{bx^3}{a}}}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}+\frac{a^3\log\left(\frac{bx^3}{a}\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^3}{a}}+1\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}+\frac{a^2bx^3\log\left(\frac{bx^3}{a}\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}-\frac{2a^2bx^3\log\left(\sqrt{1+\frac{bx^3}{a}}+1\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)**(3/2),x)
```

```
[Out] 2*a**3*sqrt(1 + b*x**3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a**3*log(b*x**
3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) - 2*a**3*log(sqrt(1 + b*x**3/a) + 1)/
(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a**2*b*x**3*log(b*x**3/a)/(3*a**(9/2) +
3*a**(7/2)*b*x**3) - 2*a**2*b*x**3*log(sqrt(1 + b*x**3/a) + 1)/(3*a**(9/2)
+ 3*a**(7/2)*b*x**3)
```

3.426 $\int \frac{1}{x^4\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=64

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{b}{a^2\sqrt{a+bx^3}}-\frac{1}{3ax^3\sqrt{a+bx^3}}$$

```
[Out] b*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(5/2)-b/a^2/(b*x^3+a)^(1/2)-1/3/a/x^3/
(b*x^3+a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 51, 63, 208}

$$-\frac{\sqrt{a+bx^3}}{a^2x^3}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2}{3ax^3\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3)^(3/2)),x]
```

```
[Out] 2/(3*a*x^3*Sqrt[a + b*x^3]) - Sqrt[a + b*x^3]/(a^2*x^3) + (b*ArcTanh[Sqrt[a
 + b*x^3]/Sqrt[a]])/a^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(a+bx^3\right)^{3/2}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx, x, x^3\right)\\ &= \frac{2}{3ax^3\sqrt{a+bx^3}} + \frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^3\right)}{a}\\ &= \frac{2}{3ax^3\sqrt{a+bx^3}} - \frac{\sqrt{a+bx^3}}{a^2x^3} - \frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right)}{2a^2}\\ &= \frac{2}{3ax^3\sqrt{a+bx^3}} - \frac{\sqrt{a+bx^3}}{a^2x^3} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)}{a^2}\\ &= \frac{2}{3ax^3\sqrt{a+bx^3}} - \frac{\sqrt{a+bx^3}}{a^2x^3} + \frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{a^{5/2}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.58

$$-\frac{2b\,{}_2F_1\left(-\frac{1}{2}, 2; \frac{1}{2}; \frac{bx^3}{a}+1\right)}{3a^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-2*b*Hypergeometric2F1[-1/2, 2, 1/2, 1 + (b*x^3)/a])/(3*a^2*Sqrt[a + b*x^3
])
```

fricas [A] time = 0.79, size = 173, normalized size = 2.70

$$\left[\frac{3\left(b^2x^6+abx^3\right)\sqrt{a}\,\log\left(\frac{bx^3+2\sqrt{bx^3+a}\sqrt{a}+2a}{x^3}\right) - 2\left(3abx^3+a^2\right)\sqrt{bx^3+a}}{6\left(a^3bx^6+a^4x^3\right)}, -\frac{3\left(b^2x^6+abx^3\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx^3+}}{a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^2*x^6 + a*b*x^3)*sqrt(a)*log((b*x^3 + 2*sqrt(b*x^3 + a)*sqrt(a)
+ 2*a)/x^3) - 2*(3*a*b*x^3 + a^2)*sqrt(b*x^3 + a))/(a^3*b*x^6 + a^4*x^3), -
1/3*(3*(b^2*x^6 + a*b*x^3)*sqrt(-a)*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (3
*a*b*x^3 + a^2)*sqrt(b*x^3 + a))/(a^3*b*x^6 + a^4*x^3)]
```

giac [A] time = 0.18, size = 72, normalized size = 1.12

$$-\frac{b\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2} - \frac{3\left(bx^3+a\right)b-2ab}{3\left(\left(bx^3+a\right)^{\frac{3}{2}}-\sqrt{bx^3+a}\,a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] -b*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^2) - 1/3*(3*(b*x^3 + a)*b -
 2*a*b)/(((b*x^3 + a)^(3/2) - sqrt(b*x^3 + a)*a)*a^2)
```

maple [A] time = 0.03, size = 57, normalized size = 0.89

$$\frac{b \operatorname{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}} - \frac{2b}{3\sqrt{\left(x^3+\frac{a}{b}\right)b}\;a^2} - \frac{\sqrt{b\,x^3+a}}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)^(3/2),x)
```

```
[Out] -1/3/a^2*(b*x^3+a)^(1/2)/x^3-2/3*b/a^2/((x^3+a/b)*b)^(1/2)+b*arctanh((b*x^3
+a)^(1/2)/a^(1/2))/a^(5/2)
```

maxima [A] time = 2.98, size = 86, normalized size = 1.34

$$-\frac{3\left(bx^3+a\right)b-2\,ab}{3\left(\left(bx^3+a\right)^{\frac{3}{2}}a^2-\sqrt{bx^3+a}\,a^3\right)} - \frac{b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{2\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*(b*x^3 + a)*b - 2*a*b)/((b*x^3 + a)^(3/2)*a^2 - sqrt(b*x^3 + a)*a^3
) - 1/2*b*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(5
/2)
```

mupad [B] time = 1.34, size = 74, normalized size = 1.16

$$\frac{b\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)^3}{x^6}\right)}{2\,a^{5/2}} - \frac{2\,b}{3\,a^2\,\sqrt{b\,x^3+a}} - \frac{\sqrt{b\,x^3+a}}{3\,a^2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3)^(3/2)),x)
```

```
[Out] (b*log((((a + b*x^3)^(1/2) - a^(1/2))*((a + b*x^3)^(1/2) + a^(1/2))^3)/x^6)
)/(2*a^(5/2)) - (2*b)/(3*a^2*(a + b*x^3)^(1/2)) - (a + b*x^3)^(1/2)/(3*a^2*
x^3)
```

sympy [A] time = 4.18, size = 75, normalized size = 1.17

$$-\frac{1}{3a\sqrt{b}\,x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{\sqrt{b}}{a^2x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)**(3/2),x)
```

```
[Out] -1/(3*a*sqrt(b)*x**(9/2)*sqrt(a/(b*x**3) + 1)) - sqrt(b)/(a**2*x**(3/2)*sqr
t(a/(b*x**3) + 1)) + b*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/a**(5/2)
```

3.427 $\int \frac{1}{x^7\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=95

$$-\frac{5b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4a^{7/2}}+\frac{5b^2}{4a^3\sqrt{a+bx^3}}+\frac{5b}{12a^2x^3\sqrt{a+bx^3}}-\frac{1}{6ax^6\sqrt{a+bx^3}}$$

```
[Out] -5/4*b^2*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(7/2)+5/4*b^2/a^3/(b*x^3+a)^(1/
2)-1/6/a/x^6/(b*x^3+a)^(1/2)+5/12*b/a^2/x^3/(b*x^3+a)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 51, 63, 208}

$$-\frac{5b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4a^{7/2}}+\frac{5b\sqrt{a+bx^3}}{4a^3x^3}-\frac{5\sqrt{a+bx^3}}{6a^2x^6}+\frac{2}{3ax^6\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^3)^(3/2)),x]
```

```
[Out] 2/(3*a*x^6*Sqrt[a + b*x^3]) - (5*Sqrt[a + b*x^3])/(6*a^2*x^6) + (5*b*Sqrt[a
 + b*x^3])/(4*a^3*x^3) - (5*b^2*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(4*a^(7/2
))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^7\left(a+bx^3\right)^{3/2}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x^3(a+bx)^{3/2}}\,dx,x,x^3\right)$$

$$= \frac{2}{3ax^6\sqrt{a+bx^3}} + \frac{5\operatorname{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx,x,x^3\right)}{3a}$$

$$= \frac{2}{3ax^6\sqrt{a+bx^3}} - \frac{5\sqrt{a+bx^3}}{6a^2x^6} - \frac{(5b)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx,x,x^3\right)}{4a^2}$$

$$= \frac{2}{3ax^6\sqrt{a+bx^3}} - \frac{5\sqrt{a+bx^3}}{6a^2x^6} + \frac{5b\sqrt{a+bx^3}}{4a^3x^3} + \frac{\left(5b^2\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^3\right)}{8a^3}$$

$$= \frac{2}{3ax^6\sqrt{a+bx^3}} - \frac{5\sqrt{a+bx^3}}{6a^2x^6} + \frac{5b\sqrt{a+bx^3}}{4a^3x^3} + \frac{(5b)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^3}\right)}{4a^3}$$

$$= \frac{2}{3ax^6\sqrt{a+bx^3}} - \frac{5\sqrt{a+bx^3}}{6a^2x^6} + \frac{5b\sqrt{a+bx^3}}{4a^3x^3} - \frac{5b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4a^{7/2}}$$

Mathematica [C] time = 0.03, size = 39, normalized size = 0.41

$$\frac{2b^2\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\frac{bx^3}{a}+1\right)}{3a^3\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^3)^(3/2)),x]
```

```
[Out] (2*b^2*Hypergeometric2F1[-1/2, 3, 1/2, 1 + (b*x^3)/a])/(3*a^3*Sqrt[a + b*x^
3])
```

fricas [A] time = 0.78, size = 203, normalized size = 2.14

$$\left[\frac{15\left(b^3x^9+ab^2x^6\right)\sqrt{a}\,\log\left(\frac{bx^3-2\sqrt{bx^3+a}\sqrt{a}+2a}{x^3}\right)+2\left(15ab^2x^6+5a^2bx^3-2a^3\right)\sqrt{bx^3+a}}{24\left(a^4bx^9+a^5x^6\right)}, \frac{15\left(b^3x^9+ab^2x^6\right)\sqrt{-a}\,a}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/24*(15*(b^3*x^9 + a*b^2*x^6)*sqrt(a)*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt
(a) + 2*a)/x^3) + 2*(15*a*b^2*x^6 + 5*a^2*b*x^3 - 2*a^3)*sqrt(b*x^3 + a))/(
a^4*b*x^9 + a^5*x^6), 1/12*(15*(b^3*x^9 + a*b^2*x^6)*sqrt(-a)*arctan(sqrt(b
*x^3 + a)*sqrt(-a)/a) + (15*a*b^2*x^6 + 5*a^2*b*x^3 - 2*a^3)*sqrt(b*x^3 + a
))/(a^4*b*x^9 + a^5*x^6)]
```

giac [A] time = 0.16, size = 88, normalized size = 0.93

$$\frac{5\,b^2\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{4\sqrt{-a}\,a^3} + \frac{2\,b^2}{3\sqrt{bx^3+a}\,a^3} + \frac{7\left(bx^3+a\right)^{\frac{3}{2}}b^2-9\sqrt{bx^3+a}\,ab^2}{12\,a^3b^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] 5/4*b^2*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^3) + 2/3*b^2/(sqrt(b*x
^3 + a)*a^3) + 1/12*(7*(b*x^3 + a)^(3/2)*b^2 - 9*sqrt(b*x^3 + a)*a*b^2)/(a^
3*b^2*x^6)
```

maple [A] time = 0.03, size = 80, normalized size = 0.84

$$-\frac{5b^2\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{4a^{\frac{7}{2}}}+\frac{2b^2}{3\sqrt{\left(x^3+\frac{a}{b}\right)b}\;a^3}+\frac{7\sqrt{b\,x^3+a}\;b}{12a^3x^3}-\frac{\sqrt{b\,x^3+a}}{6a^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^3+a)^(3/2),x)
```

```
[Out] -1/6/a^2*(b*x^3+a)^(1/2)/x^6+7/12/a^3*b*(b*x^3+a)^(1/2)/x^3+2/3*b^2/a^3/((x
^3+a/b)*b)^(1/2)-5/4*b^2*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(7/2)
```

maxima [A] time = 2.99, size = 122, normalized size = 1.28

$$\frac{15\left(bx^3+a\right)^2b^2-25\left(bx^3+a\right)ab^2+8\,a^2b^2}{12\left(\left(bx^3+a\right)^{\frac{5}{2}}a^3-2\left(bx^3+a\right)^{\frac{3}{2}}a^4+\sqrt{bx^3+a}\,a^5\right)}+\frac{5\,b^2\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{8\,a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(15*(b*x^3 + a)^2*b^2 - 25*(b*x^3 + a)*a*b^2 + 8*a^2*b^2)/((b*x^3 + a)
^(5/2)*a^3 - 2*(b*x^3 + a)^(3/2)*a^4 + sqrt(b*x^3 + a)*a^5) + 5/8*b^2*log((
sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(7/2)
```

mupad [B] time = 1.47, size = 96, normalized size = 1.01

$$\frac{2\,b^2}{3\,a^3\,\sqrt{b\,x^3+a}}-\frac{\sqrt{b\,x^3+a}}{6\,a^2\,x^6}+\frac{5\,b^2\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{8\,a^{7/2}}+\frac{7\,b\,\sqrt{b\,x^3+a}}{12\,a^3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^3)^(3/2)),x)
```

```
[Out] (2*b^2)/(3*a^3*(a + b*x^3)^(1/2)) - (a + b*x^3)^(1/2)/(6*a^2*x^6) + (5*b^2*
log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6))/(
8*a^(7/2)) + (7*b*(a + b*x^3)^(1/2))/(12*a^3*x^3)
```

sympy [A] time = 7.24, size = 112, normalized size = 1.18

$$-\frac{1}{6a\sqrt{b}\,x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{5\sqrt{b}}{12a^2x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{5b^{\frac{3}{2}}}{4a^3x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{5b^2\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{4a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**3+a)**(3/2),x)
```

```
[Out] -1/(6*a*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) + 5*sqrt(b)/(12*a**2*x**(9/
2)*sqrt(a/(b*x**3) + 1)) + 5*b**(3/2)/(4*a**3*x**(3/2)*sqrt(a/(b*x**3) + 1)
) - 5*b**2*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(4*a**(7/2))
```

3.428 $\int \frac{x^6}{\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=251

$$-\frac{32\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{15\sqrt[4]{3}\,b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\frac{16x\sqrt{a+bx^3}}{15b^2}-\frac{2x^4}{3b\sqrt{a+}}$$

```
[Out] -2/3*x^4/b/(b*x^3+a)^(1/2)+16/15*x*(b*x^3+a)^(1/2)/b^2-32/45*a*(a^(1/3)+b^(
1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(
1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x
+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(7/3)/(b*x
^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)
^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {288, 321, 218}

$$-\frac{32\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{15\sqrt[4]{3}\,b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\frac{16x\sqrt{a+bx^3}}{15b^2}-\frac{2x^4}{3b\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*x^4)/(3*b*Sqrt[a + b*x^3]) + (16*x*Sqrt[a + b*x^3])/(15*b^2) - (32*Sqrt
[2 + Sqrt[3]]*a*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - S
qrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*S
qrt[3]])/(15*3^(1/4)*b^(7/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^6}{\left(a+bx^3\right)^{3/2}}\,dx = -\frac{2x^4}{3b\sqrt{a+bx^3}} + \frac{8\int \frac{x^3}{\sqrt{a+bx^3}}\,dx}{3b}$$

$$= -\frac{2x^4}{3b\sqrt{a+bx^3}} + \frac{16x\sqrt{a+bx^3}}{15b^2} - \frac{(16a)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{15b^2}$$

$$= -\frac{2x^4}{3b\sqrt{a+bx^3}} + \frac{16x\sqrt{a+bx^3}}{15b^2} - \frac{32\sqrt{2+\sqrt{3}}\;a\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\right.}{15\sqrt[4]{3}\,b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{}}$$

Mathematica [C] time = 0.05, size = 65, normalized size = 0.26

$$\frac{2x\left(-8a\sqrt{\frac{bx^3}{a}+1}\;{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+8a+3bx^3\right)}{15b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*x*(8*a + 3*b*x^3 - 8*a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4
/3, -((b*x^3)/a)]))/(15*b^2*Sqrt[a + b*x^3])
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+a}\,x^6}{b^2x^6+2\,abx^3+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)*x^6/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/(b*x^3 + a)^(3/2), x)
```

maple [A] time = 0.02, size = 320, normalized size = 1.27

$$32i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}$$

$$\frac{2ax}{3\sqrt{\left(x^3+\frac{a}{b}\right)b}\,b^2}+\frac{}{45\sqrt{b\,x^3+a}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^3+a)^(3/2),x)
```

```
[Out] 2/3/b^2*a*x/((x^3+a/b)*b)^(1/2)+2/5*x*(b*x^3+a)^(1/2)/b^2+32/45*I*a/b^3*3^(
1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)
/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1
/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/
2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/
2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^
(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-
a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(b*x^3 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{\left(b\,x^3+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^3)^(3/2),x)
```

```
[Out] int(x^6/(a + b*x^3)^(3/2), x)
```

sympy [A] time = 2.74, size = 37, normalized size = 0.15

$$\frac{x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**3+a)**(3/2),x)
```

```
[Out] x**7*gamma(7/3)*hyper((3/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**
(3/2)*gamma(10/3))
```

3.429 $\int \frac{x^3}{\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=229

$$\frac{4\sqrt{2+\sqrt{3}}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}-\frac{2x}{3b\sqrt{a+bx^3}}$$

```
[Out] -2/3*x/b/(b*x^3+a)^(1/2)+4/9*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/
3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)
+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(
1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(4/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1
/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {288, 218}

$$\frac{4\sqrt{2+\sqrt{3}}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}-\frac{2x}{3b\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*x)/(3*b*Sqrt[a + b*x^3]) + (4*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1/3)*x)*S
qrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(4/3)*Sqrt[(a^(1/
3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b
*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^3\right)^{3/2}}\,dx = -\frac{2x}{3b\sqrt{a+bx^3}} + \frac{2\int \frac{1}{\sqrt{a+bx^3}}\,dx}{3b}$$

$$= -\frac{2x}{3b\sqrt{a+bx^3}} + \frac{4\sqrt{2+\sqrt{3}}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}\right)|-7\right.}{3\sqrt[4]{3}\,b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}$$

Mathematica [C] time = 0.02, size = 55, normalized size = 0.24

$$\frac{2x\left(\sqrt{\frac{bx^3}{a}+1}\;{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)-1\right)}{3b\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*x*(-1 + Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a
)]))/(3*b*Sqrt[a + b*x^3])
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+a}\,x^3}{b^2x^6+2\,abx^3+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)*x^3/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(b*x^3 + a)^(3/2), x)
```

maple [A] time = 0.02, size = 303, normalized size = 1.32

$$4i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{}}{\left(-a\,b^2\right)^{\frac{1}{3}}}}$$

$$-\frac{2x}{3\sqrt{\left(x^3+\frac{a}{b}\right)b}\;b}-\frac{}{9\sqrt{b\,x^3+a}\;b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a)^(3/2),x)
```

```
[Out] -2/3/b*x/((x^3+a/b)*b)^(1/2)-4/9*I/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a
*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/
2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3
)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(
1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/
2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b
)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*
b^2)^(1/3)/b)/b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(b*x^3 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\left(b\,x^3+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^3)^(3/2),x)
```

```
[Out] int(x^3/(a + b*x^3)^(3/2), x)
```

sympy [A] time = 2.00, size = 37, normalized size = 0.16

$$\frac{x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{4}{3}, \frac{3}{2}\\ \frac{7}{3}\end{matrix}\right| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a)**(3/2),x)
```

```
[Out] x**4*gamma(4/3)*hyper((4/3, 3/2), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
3/2)*gamma(7/3))
```

3.430 $\int \frac{1}{\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=232

$$\frac{2\sqrt{2+\sqrt{3}}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,a\sqrt[3]{b}\,\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\frac{2x}{3a\sqrt{a+bx^3}}$$

```
[Out] 2/3*x/a/(b*x^3+a)^(1/2)+2/9*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3
)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+
1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2)))^2)^(1/2)*3^(3/4)/a/b^(1/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(
1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {199, 218}

$$\frac{2\sqrt{2+\sqrt{3}}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,a\sqrt[3]{b}\,\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\frac{2x}{3a\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(-3/2),x]
```

```
[Out] (2*x)/(3*a*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1/3)*x)*Sq
rt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a*b^(1/3)*Sqrt[(a^(1
/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
b*x^3])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^{3/2}}\,dx = \frac{2x}{3a\sqrt{a+bx^3}} + \frac{\int \frac{1}{\sqrt{a+bx^3}}\,dx}{3a}$$

$$= \frac{2x}{3a\sqrt{a+bx^3}} + \frac{2\sqrt{2+\sqrt{3}}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}\right)|\,-\right.}{3\sqrt[4]{3}\,a\sqrt[3]{b}\,\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}$$

Mathematica [C] time = 0.01, size = 55, normalized size = 0.24

$$\frac{x\left(\sqrt{\frac{bx^3}{a}+1}\;{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+2\right)}{3a\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(-3/2),x]
```

```
[Out] (x*(2 + Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)])
)/(3*a*Sqrt[a + b*x^3])
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+a}}{b^2x^6+2\,abx^3+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(-3/2), x)
```

maple [A] time = 0.02, size = 306, normalized size = 1.32

$$2i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)}{\left(-a\,b^2\right)^{\frac{1}{3}}}}$$

$$\frac{2x}{3\sqrt{\left(x^3+\frac{a}{b}\right)b}\;a} - \frac{}{9\sqrt{b\,x^3+a}\;ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(3/2),x)
```

```
[Out] 2/3/a*x/((x^3+a/b)*b)^(1/2)-2/9*I/a*3^(1/2)*(-a*b^2)^(1/3)/b*(I*(x+1/2*(-a*
b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2
)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)
/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1
/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2
*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)
^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b
^2)^(1/3)/b)/b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(-3/2), x)
```

mupad [B] time = 1.13, size = 37, normalized size = 0.16

$$\frac{x\left(\frac{b\,x^3}{a}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{3}{2};\ \frac{4}{3};\ -\frac{b\,x^3}{a}\right)}{\left(b\,x^3+a\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^3)^(3/2),x)
```

```
[Out] (x*((b*x^3)/a + 1)^(3/2)*hypergeom([1/3, 3/2], 4/3, -(b*x^3)/a))/(a + b*x^3
)^(3/2)
```

sympy [A] time = 1.92, size = 36, normalized size = 0.16

$$\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(3/2),x)
```

```
[Out] x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2
)*gamma(4/3))
```

3.431 $\int \frac{1}{x^3\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=255

$$-\frac{7\sqrt{a+bx^3}}{6a^2x^2}-\frac{7\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{6\sqrt[4]{3}\,a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{}{3ax^2\sqrt{}}$$

```
[Out] 2/3/a/x^2/(b*x^3+a)^(1/2)-7/6*(b*x^3+a)^(1/2)/a^2/x^2-7/18*b^(2/3)*(a^(1/3)
+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/
3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^2/(b*x
^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)
^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {290, 325, 218}

$$-\frac{7\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{6\sqrt[4]{3}\,a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{7\sqrt{a+bx^3}}{6a^2x^2}+\frac{}{3ax^2\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^3)^(3/2)),x]
```

```
[Out] 2/(3*a*x^2*Sqrt[a + b*x^3]) - (7*Sqrt[a + b*x^3])/(6*a^2*x^2) - (7*Sqrt[2 +
 Sqrt[3]]*b^(2/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
 Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4
*Sqrt[3]])/(6*3^(1/4)*a^2*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^3\right)^{3/2}}\,dx = \frac{2}{3ax^2\sqrt{a+bx^3}} + \frac{7\int \frac{1}{x^3\sqrt{a+bx^3}}\,dx}{3a}$$

$$= \frac{2}{3ax^2\sqrt{a+bx^3}} - \frac{7\sqrt{a+bx^3}}{6a^2x^2} - \frac{(7b)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{12a^2}$$

$$= \frac{2}{3ax^2\sqrt{a+bx^3}} - \frac{7\sqrt{a+bx^3}}{6a^2x^2} - \frac{7\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\right.}{6\sqrt[4]{3}\,a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+}}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.21

$$-\frac{\sqrt{\frac{bx^3}{a}+1}\;{}_2F_1\left(-\frac{2}{3},\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a}\right)}{2ax^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^3)^(3/2)),x]
```

```
[Out] -1/2*(Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-2/3, 3/2, 1/3, -((b*x^3)/a)])/
(a*x^2*Sqrt[a + b*x^3])
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+a}}{b^2x^9+2\,abx^6+a^2x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)/(b^2*x^9 + 2*a*b*x^6 + a^2*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(3/2)*x^3), x)
```

maple [A] time = 0.03, size = 321, normalized size = 1.26

$$7i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right.}{\left(-a\,b^2\right)^{\frac{1}{3}}}}$$

$$-\frac{2bx}{3\sqrt{\left(x^3+\frac{a}{b}\right)b}\,a^2}+\frac{}{18\sqrt{b\,x^3+a}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a)^(3/2),x)
```

```
[Out] -2/3*b/a^2*x/((x^3+a/b)*b)^(1/2)-1/2*(b*x^3+a)^(1/2)/a^2/x^2+7/18*I/a^2*3^(
1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)
/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1
/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/
2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/
2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^
(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-
a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(3/2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\left(b\,x^3+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^3)^(3/2)),x)
```

```
[Out] int(1/(x^3*(a + b*x^3)^(3/2)), x)
```

sympy [A] time = 1.29, size = 41, normalized size = 0.16

$$\frac{\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{3}{2}\\ \frac{1}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x^2\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a)**(3/2),x)
```

```
[Out] gamma(-2/3)*hyper((-2/3, 3/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2
)*x**2*gamma(1/3))
```

3.432 $\int \frac{1}{x^6\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=277

$$\frac{91b\sqrt{a+bx^3}}{60a^3x^2}-\frac{13\sqrt{a+bx^3}}{15a^2x^5}+\frac{91\sqrt{2+\sqrt{3}}\,b^{5/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4$$
$$60\sqrt[4]{3}\,a^3\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}$$

```
[Out] 2/3/a/x^5/(b*x^3+a)^(1/2)-13/15*(b*x^3+a)^(1/2)/a^2/x^5+91/60*b*(b*x^3+a)^(
1/2)/a^3/x^2+91/180*b^(5/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3
)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+
1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2)))^2)^(1/2)*3^(3/4)/a^3/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x
)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {290, 325, 218}

$$\frac{91\sqrt{2+\sqrt{3}}\,b^{5/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{60\sqrt[4]{3}\,a^3\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{91b\sqrt{a+bx^3}}{60a^3x^2}-\frac{13\sqrt{a+}}{15a^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^3)^(3/2)),x]
```

```
[Out] 2/(3*a*x^5*Sqrt[a + b*x^3]) - (13*Sqrt[a + b*x^3])/(15*a^2*x^5) + (91*b*Sqr
t[a + b*x^3])/(60*a^3*x^2) + (91*Sqrt[2 + Sqrt[3]]*b^(5/3)*(a^(1/3) + b^(1/
3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(60*3^(1/4)*a^3*Sqrt[(a
^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
 + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
 + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^6\left(a+bx^3\right)^{3/2}}\,dx &= \frac{2}{3ax^5\sqrt{a+bx^3}} + \frac{13\int \frac{1}{x^6\sqrt{a+bx^3}}\,dx}{3a} \\
&= \frac{2}{3ax^5\sqrt{a+bx^3}} - \frac{13\sqrt{a+bx^3}}{15a^2x^5} - \frac{(91b)\int \frac{1}{x^3\sqrt{a+bx^3}}\,dx}{30a^2} \\
&= \frac{2}{3ax^5\sqrt{a+bx^3}} - \frac{13\sqrt{a+bx^3}}{15a^2x^5} + \frac{91b\sqrt{a+bx^3}}{60a^3x^2} + \frac{\left(91b^2\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{120a^3} \\
&= \frac{2}{3ax^5\sqrt{a+bx^3}} - \frac{13\sqrt{a+bx^3}}{15a^2x^5} + \frac{91b\sqrt{a+bx^3}}{60a^3x^2} + \frac{91\sqrt{2+\sqrt{3}}\,b^{5/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-}{\left((1+}}}{60\sqrt[4]{3}\,a^3\sqrt{}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.19

$$-\frac{\sqrt{\frac{bx^3}{a}+1}\;{}_2F_1\left(-\frac{5}{3},\frac{3}{2};-\frac{2}{3};-\frac{bx^3}{a}\right)}{5ax^5\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^3)^(3/2)),x]
```

```
[Out] -1/5*(Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-5/3, 3/2, -2/3, -((b*x^3)/a)])
/(a*x^5*Sqrt[a + b*x^3])
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+a}}{b^2x^{12}+2\,abx^9+a^2x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)/(b^2*x^12 + 2*a*b*x^9 + a^2*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{3}{2}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(3/2)*x^6), x)
```

maple [A] time = 0.03, size = 342, normalized size = 1.23

$$91i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{}}{\left(-a\,b^2\right)^{\frac{1}{3}}}}$$

$$\frac{2b^2x}{3\sqrt{\left(x^3+\frac{a}{b}\right)b}\;a^3}-\frac{}{180\sqrt{b\,x^3+a}\;a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^3+a)^(3/2),x)
```

```
[Out] -1/5*(b*x^3+a)^(1/2)/a^2/x^5+17/20*b*(b*x^3+a)^(1/2)/x^2/a^3+2/3*b^2/a^3*x/
((x^3+a/b)*b)^(1/2)-91/180*I/a^3*b*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2
)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*(
(x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)
)^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)
/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-
a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1
/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)
^(1/3)/b)/b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{3}{2}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(3/2)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^6\left(b\,x^3+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^3)^(3/2)),x)
```

```
[Out] int(1/(x^6*(a + b*x^3)^(3/2)), x)
```

sympy [A] time = 2.99, size = 44, normalized size = 0.16

$$\frac{\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{5}{3},\frac{3}{2}\\-\frac{2}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x^5\Gamma\left(-\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**3+a)**(3/2),x)
```

```
[Out] gamma(-5/3)*hyper((-5/3, 3/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/
2)*x**5*gamma(-2/3))
```

3.433 $\int \frac{x^7}{\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=511

$$-\frac{80\sqrt{2}\,a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{21\sqrt[4]{3}\,b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\frac{40\sqrt{2-\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{7}$$

```
[Out] -2/3*x^5/b/(b*x^3+a)^(1/2)+20/21*x^2*(b*x^3+a)^(1/2)/b^2-80/21*a*(b*x^3+a)^
(1/2)/b^(8/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-80/63*a^(4/3)*(a^(1/3)+b^(1/3
)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2
))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3
)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*
(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+40/21*a^(4/3)*
(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/
3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/
b^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 511, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {288, 321, 303, 218, 1877}

$$-\frac{80\sqrt{2}\,a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{21\sqrt[4]{3}\,b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\frac{40\sqrt{2-\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*x^5)/(3*b*Sqrt[a + b*x^3]) + (20*x^2*Sqrt[a + b*x^3])/(21*b^2) - (80*a*
Sqrt[a + b*x^3])/(21*b^(8/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (40*Sqr
t[2 - Sqrt[3]]*a^(4/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[
((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -
7 - 4*Sqrt[3]])/(7*3^(3/4)*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (80*Sqrt[2]*a^(4/3)*
(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3)
+ b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(21*3^(
1/4)*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^7}{\left(a+bx^3\right)^{3/2}}\,dx = -\frac{2x^5}{3b\sqrt{a+bx^3}} + \frac{10\int \frac{x^4}{\sqrt{a+bx^3}}\,dx}{3b}$$

$$= -\frac{2x^5}{3b\sqrt{a+bx^3}} + \frac{20x^2\sqrt{a+bx^3}}{21b^2} - \frac{(40a)\int \frac{x}{\sqrt{a+bx^3}}\,dx}{21b^2}$$

$$= -\frac{2x^5}{3b\sqrt{a+bx^3}} + \frac{20x^2\sqrt{a+bx^3}}{21b^2} - \frac{(40a)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{21b^{7/3}} - \frac{\left(40\sqrt{2\left(2-\sqrt{3}\right)}\,a^{4/3}\right)\int$$

$$= -\frac{2x^5}{3b\sqrt{a+bx^3}} + \frac{20x^2\sqrt{a+bx^3}}{21b^2} - \frac{80a\sqrt{a+bx^3}}{21b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} + \frac{40\sqrt{2-\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}\right.}{}$$

Mathematica [C] time = 0.03, size = 66, normalized size = 0.13

$$\frac{2x^2\left(10a\sqrt{\frac{bx^3}{a}+1}\;{}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a}\right)-10a+bx^3\right)}{7b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*x^2*(-10*a + b*x^3 + 10*a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3/2
, 5/3, -((b*x^3)/a)]))/(7*b^2*Sqrt[a + b*x^3])
```

fricas **[F]** time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+a}\,x^7}{b^2x^6+2\,abx^3+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)*x^7/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

giac **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^7/(b*x^3 + a)^(3/2), x)
```

maple **[A]** time = 0.03, size = 476, normalized size = 0.93

$$80i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(-a}{}\right.}$$

$$\frac{2a\,x^2}{3\sqrt{\left(x^3+\frac{a}{b}\right)b}\,b^2}+\frac{2\sqrt{b\,x^3+a}\,x^2}{7b^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^3+a)^(3/2),x)
```

```
[Out] 2/3/b^2*a*x^2/((x^3+a/b)*b)^(1/2)+2/7*x^2*(b*x^3+a)^(1/2)/b^2+80/63*I*a/b^3
*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(
1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2
)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/
b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)
^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/
3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2
```

```
)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+
1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^
(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-
a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*
I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^7/(b*x^3 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7}{\left(b\,x^3+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^3)^(3/2),x)
```

```
[Out] int(x^7/(a + b*x^3)^(3/2), x)
```

sympy [A] time = 2.43, size = 37, normalized size = 0.07

$$\frac{x^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**3+a)**(3/2),x)
```

```
[Out] x**8*gamma(8/3)*hyper((3/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**
(3/2)*gamma(11/3))
```

3.434 $\int \frac{x^4}{\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=487

$$\frac{8\sqrt{2}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}-\frac{4\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{}}{3^{3/4}b}$$

```
[Out] -2/3*x^2/b/(b*x^3+a)^(1/2)+8/3*(b*x^3+a)^(1/2)/b^(5/3)/(b^(1/3)*x+a^(1/3)*(
1+3^(1/2)))+8/9*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1
-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)
-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^
(3/4)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2)))^2)^(1/2)-4/3*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)
*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2
*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a
^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1
/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 487, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {288, 303, 218, 1877}

$$\frac{8\sqrt{2}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}-\frac{4\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{}}{3^{3/4}b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*x^2)/(3*b*Sqrt[a + b*x^3]) + (8*Sqrt[a + b*x^3])/(3*b^(5/3)*((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)) - (4*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)
*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*b^(5/3)*Sqrt[(a^
(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
 + b*x^3]) + (8*Sqrt[2]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF
[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)
*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
 + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^3\right)^{3/2}}\,dx = -\frac{2x^2}{3b\sqrt{a+bx^3}} + \frac{4\int \frac{x}{\sqrt{a+bx^3}}\,dx}{3b}$$

$$= -\frac{2x^2}{3b\sqrt{a+bx^3}} + \frac{4\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{3b^{4/3}} + \frac{\left(4\sqrt{2\left(2-\sqrt{3}\right)}\,\sqrt[3]{a}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{3b^{4/3}}$$

$$= -\frac{2x^2}{3b\sqrt{a+bx^3}} + \frac{8\sqrt{a+bx^3}}{3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} - \frac{4\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{}\right.}}}{3^{3/4}b^{5/3}\sqrt{\frac{\sqrt[3]{a}}{\left(\left(1+\sqrt{}\right.\right.}}}$$

Mathematica [C] time = 0.02, size = 55, normalized size = 0.11

$$-\frac{2x^2\left(\sqrt{\frac{bx^3}{a}+1}\;{}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a}\right)-1\right)}{b\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*x^2*(-1 + Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3/2, 5/3, -((b*x^3
)/a)]))/(b*Sqrt[a + b*x^3])
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+a}\,x^4}{b^2x^6+2\,abx^3+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)*x^4/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/(b*x^3 + a)^(3/2), x)
```

maple [A] time = 0.02, size = 457, normalized size = 0.94

$$8i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{}}{\left(-a\,b^2\right)^{\frac{1}{3}}}}$$

$$-\frac{2x^2}{3\sqrt{\left(x^3+\frac{a}{b}\right)b}\,b}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a)^(3/2),x)
```

```
[Out] -2/3/b*x^2/((x^3+a/b)*b)^(1/2)-8/9*I/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(
-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(
1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1
/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3
^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*
I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/
b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2
)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^
(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/
2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-
a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2
)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(b*x^3 + a)^(3/2), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(b\,x^3+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^3)^(3/2),x)
```

```
[Out] int(x^4/(a + b*x^3)^(3/2), x)
```

sympy **[A]** time = 2.10, size = 37, normalized size = 0.08

$$\frac{x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2}, \frac{5}{3} \\ \frac{8}{3}\end{matrix}\right| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a)**(3/2),x)
```

```
[Out] x**5*gamma(5/3)*hyper((3/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
3/2)*gamma(8/3))
```

3.435 $\int \frac{x}{\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=489

$$-\frac{2\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/3*x^2/a/(b*x^3+a)^(1/2)-2/3*(b*x^3+a)^(1/2)/a/b^(2/3)/(b^(1/3)*x+a^(1/3)*
(1+3^(1/2)))-2/9*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2
)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a
^(2/3)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1
/3)*(1+3^(1/2)))^2)^(1/2)+1/3*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1
/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2
)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*
(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(2/3)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(
1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 489, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {290, 303, 218, 1877}

$$-\frac{2\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*x^2)/(3*a*Sqrt[a + b*x^3]) - (2*Sqrt[a + b*x^3])/(3*a*b^(2/3)*((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)) + (Sqrt[2 - Sqrt[3]]*(a^(1/3) + b^(1/3)*x)*Sqrt[
(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*a^(2/3)*b^(2/3)*Sqrt[(a^(
1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
 b*x^3]) - (2*Sqrt[2]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[(
(1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
 - 4*Sqrt[3]])/(3*3^(1/4)*a^(2/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*
x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^3\right)^{3/2}}\,dx = \frac{2x^2}{3a\sqrt{a+bx^3}} - \frac{\int \frac{x}{\sqrt{a+bx^3}}\,dx}{3a}$$

$$= \frac{2x^2}{3a\sqrt{a+bx^3}} - \frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{3a\sqrt[3]{b}} - \frac{\sqrt{2\left(2-\sqrt{3}\right)}\int \frac{1}{\sqrt{a+bx^3}}\,dx}{3a^{2/3}\sqrt[3]{b}}$$

$$= \frac{2x^2}{3a\sqrt{a+bx^3}} - \frac{2\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} + \frac{\sqrt{2-\sqrt{3}}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\right.}}}{3^{3/4}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(}{\left((1+\sqrt{3}\right.}}}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.11

$$\frac{x^2\sqrt{\frac{bx^3}{a}+1}\;{}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a}\right)}{2a\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^3)^(3/2),x]
```

```
[Out] (x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3/2, 5/3, -((b*x^3)/a)])/(2
*a*Sqrt[a + b*x^3])
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+a}\,x}{b^2x^6+2\,abx^3+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)*x/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/(b*x^3 + a)^(3/2), x)
```

maple [A] time = 0.02, size = 460, normalized size = 0.94

$$2i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}}{\left(-a\,b^2\right)^{\frac{1}{3}}}}$$

$$\frac{2x^2}{3\sqrt{\left(x^3+\frac{a}{b}\right)b}\,a}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a)^(3/2),x)
```

```
[Out] 2/3/a*x^2/((x^3+a/b)*b)^(1/2)+2/9*I/a*3^(1/2)*(-a*b^2)^(1/3)/b*(I*(x+1/2*(-
a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1
/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/
3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^
(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I
*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b
-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)
*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(
1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2
*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a
*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)
))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*x^3 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(b\,x^3+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^3)^(3/2),x)
```

```
[Out] int(x/(a + b*x^3)^(3/2), x)
```

sympy [A] time = 1.93, size = 37, normalized size = 0.08

$$\frac{x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{2}{3},\frac{3}{2}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a)**(3/2),x)
```

```
[Out] x**2*gamma(2/3)*hyper((2/3, 3/2), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
3/2)*gamma(5/3))
```

3.436 $\int \frac{1}{x^2\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=513

$$\frac{5\sqrt{2}\,\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}-\frac{5\sqrt{2-\sqrt{3}}\,\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{}}{2\,3^{3/4}}$$

```
[Out] 2/3/a/x/(b*x^3+a)^(1/2)-5/3*(b*x^3+a)^(1/2)/a^2/x+5/3*b^(1/3)*(b*x^3+a)^(1/
2)/a^2/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+5/9*b^(1/3)*(a^(1/3)+b^(1/3)*x)*Elli
pticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(
1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+
b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-5/6*b^(1/3)*(a^(1/3)+b^
(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*
x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(5/3)/(b*
x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2
)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 513, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {290, 325, 303, 218, 1877}

$$\frac{5\sqrt{2}\,\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}-\frac{5\sqrt{2-\sqrt{3}}\,\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{}}{2\,3^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^3)^(3/2)),x]
```

```
[Out] 2/(3*a*x*Sqrt[a + b*x^3]) - (5*Sqrt[a + b*x^3])/(3*a^2*x) + (5*b^(1/3)*Sqrt
[a + b*x^3])/(3*a^2*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (5*Sqrt[2 - Sqrt
[3]]*b^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt
[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt
[3]])/(2*3^(3/4)*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (5*Sqrt[2]*b^(1/3)*(a^(1/3) +
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(5/3)
*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
 + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^3\right)^{3/2}}\,dx = \frac{2}{3ax\sqrt{a+bx^3}} + \frac{5\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx}{3a}$$

$$= \frac{2}{3ax\sqrt{a+bx^3}} - \frac{5\sqrt{a+bx^3}}{3a^2x} + \frac{(5b)\int \frac{x}{\sqrt{a+bx^3}}\,dx}{6a^2}$$

$$= \frac{2}{3ax\sqrt{a+bx^3}} - \frac{5\sqrt{a+bx^3}}{3a^2x} + \frac{\left(5b^{2/3}\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{6a^2} + \frac{\left(5\sqrt{\frac{1}{2}\left(2-\sqrt{3}\right)}\,b^{2/3}\right)\int \frac{}{\sqrt{}}}{3a^{5/3}}$$

$$= \frac{2}{3ax\sqrt{a+bx^3}} - \frac{5\sqrt{a+bx^3}}{3a^2x} + \frac{5\sqrt[3]{b}\,\sqrt{a+bx^3}}{3a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} - \frac{5\sqrt{2-\sqrt{3}}\,\sqrt[3]{b}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{2}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.10

$$-\frac{\sqrt{\frac{bx^3}{a}+1}\;{}_2F_1\left(-\frac{1}{3},\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a}\right)}{ax\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^3)^(3/2)),x]
```

```
[Out] -((Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-1/3, 3/2, 2/3, -((b*x^3)/a)])/(a*
x*Sqrt[a + b*x^3]))
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+a}}{b^2x^8+2\,abx^5+a^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)/(b^2*x^8 + 2*a*b*x^5 + a^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(3/2)*x^2), x)
```

maple [A] time = 0.02, size = 475, normalized size = 0.93

$$5i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)}{\left(-a\,b^2\right)^{\frac{1}{3}}}}$$

$$-\frac{2b\,x^2}{3\sqrt{\left(x^3+\frac{a}{b}\right)b}\,a^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a)^(3/2),x)
```

```
[Out] -2/3*b/a^2*x^2/((x^3+a/b)*b)^(1/2)-(b*x^3+a)^(1/2)/a^2/x-5/9*I/a^2*3^(1/2)*
(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3
^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b
+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3
^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*((
-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^(1/2)
*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2
```

```
)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(
1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(I*
(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1
/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)
*(-a*b^2)^(1/3)/b)/b)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(3/2)*x^2), x)
```

mupad [B] time = 1.37, size = 40, normalized size = 0.08

$$-\frac{2\left(\frac{a}{b\,x^3}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{11}{6};\ \frac{17}{6};\ -\frac{a}{b\,x^3}\right)}{11\,x\left(b\,x^3+a\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^3)^(3/2)),x)
```

```
[Out] -(2*(a/(b*x^3) + 1)^(3/2)*hypergeom([3/2, 11/6], 17/6, -a/(b*x^3)))/(11*x*(
a + b*x^3)^(3/2))
```

sympy [A] time = 1.99, size = 39, normalized size = 0.08

$$\frac{\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},\frac{3}{2}\\ \frac{2}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a)**(3/2),x)
```

```
[Out] gamma(-1/3)*hyper((-1/3, 3/2), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2
)*x*gamma(2/3))
```

3.437 $\int \frac{1}{x^5\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=535

$$-\frac{55b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{12\sqrt{2}\,\sqrt[4]{3}\,a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\frac{55\sqrt{2-\sqrt{3}}\,b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{16\,3^{3/4}}$$

```
[Out] 2/3/a/x^4/(b*x^3+a)^(1/2)-11/12*(b*x^3+a)^(1/2)/a^2/x^4+55/24*b*(b*x^3+a)^(
1/2)/a^3/x-55/24*b^(4/3)*(b*x^3+a)^(1/2)/a^3/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))
)-55/72*b^(4/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)
))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((a^(2/3)-a^(1/3)*b^(1/3)
*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(8/3)*2^
(1/2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2)))^2)^(1/2)+55/48*b^(4/3)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(
1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/
2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)
*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^
(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 535, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {290, 325, 303, 218, 1877}

$$-\frac{55b^{4/3}\sqrt{a+bx^3}}{24a^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{55b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{12\sqrt{2}\,\sqrt[4]{3}\,a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^3)^(3/2)),x]
```

```
[Out] 2/(3*a*x^4*Sqrt[a + b*x^3]) - (11*Sqrt[a + b*x^3])/(12*a^2*x^4) + (55*b*Sqr
t[a + b*x^3])/(24*a^3*x) - (55*b^(4/3)*Sqrt[a + b*x^3])/(24*a^3*((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)) + (55*Sqrt[2 - Sqrt[3]]*b^(4/3)*(a^(1/3) + b^(1/3
)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*3^(3/4)*a^(8/3)*Sqrt
[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqr
t[a + b*x^3]) - (55*b^(4/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[Ar
cSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)], -7 - 4*Sqrt[3]])/(12*Sqrt[2]*3^(1/4)*a^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b
^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^5\left(a+bx^3\right)^{3/2}}\,dx = \frac{2}{3ax^4\sqrt{a+bx^3}} + \frac{11\int \frac{1}{x^5\sqrt{a+bx^3}}\,dx}{3a}$$

$$= \frac{2}{3ax^4\sqrt{a+bx^3}} - \frac{11\sqrt{a+bx^3}}{12a^2x^4} - \frac{(55b)\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx}{24a^2}$$

$$= \frac{2}{3ax^4\sqrt{a+bx^3}} - \frac{11\sqrt{a+bx^3}}{12a^2x^4} + \frac{55b\sqrt{a+bx^3}}{24a^3x} - \frac{\left(55b^2\right)\int \frac{x}{\sqrt{a+bx^3}}\,dx}{48a^3}$$

$$= \frac{2}{3ax^4\sqrt{a+bx^3}} - \frac{11\sqrt{a+bx^3}}{12a^2x^4} + \frac{55b\sqrt{a+bx^3}}{24a^3x} - \frac{\left(55b^{5/3}\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{48a^3} - \frac{\left(55\sqrt{}\right.}{}$$

$$= \frac{2}{3ax^4\sqrt{a+bx^3}} - \frac{11\sqrt{a+bx^3}}{12a^2x^4} + \frac{55b\sqrt{a+bx^3}}{24a^3x} - \frac{55b^{4/3}\sqrt{a+bx^3}}{24a^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} + \frac{55\sqrt{2}}{}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.10

$$-\frac{\sqrt{\frac{bx^3}{a}+1}\ {}_2F_1\left(-\frac{4}{3},\frac{3}{2};-\frac{1}{3};-\frac{bx^3}{a}\right)}{4ax^4\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^3)^(3/2)),x]
```

```
[Out] -1/4*(Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-4/3, 3/2, -1/3, -((b*x^3)/a)])
/(a*x^4*Sqrt[a + b*x^3])
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+a}}{b^2x^{11}+2\,abx^8+a^2x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)/(b^2*x^11 + 2*a*b*x^8 + a^2*x^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{3}{2}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(3/2)*x^5), x)
```

maple [A] time = 0.03, size = 496, normalized size = 0.93

$$55i\sqrt{3}\ \left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\ b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)}{\left(-a\,b^2\right)^{\frac{1}{3}}}}$$

$$\frac{2b^2x^2}{3\sqrt{\left(x^3+\frac{a}{b}\right)b}\ a^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^3+a)^(3/2),x)
```

```
[Out] 2/3*b^2/a^3*x^2/((x^3+a/b)*b)^(1/2)-1/4*(b*x^3+a)^(1/2)/x^4/a^2+13/8*b*(b*x
^3+a)^(1/2)/a^3/x+55/72*I/a^3*b*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(
```

```
1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-
(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(
1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-
a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)
*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3
^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2
)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-
a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/
2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1
/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{3}{2}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(3/2)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^5\left(b\,x^3+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^3)^(3/2)),x)
```

```
[Out] int(1/(x^5*(a + b*x^3)^(3/2)), x)
```

sympy [A] time = 2.30, size = 44, normalized size = 0.08

$$\frac{\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{4}{3}, \frac{3}{2} \\ -\frac{1}{3}\end{matrix}\right| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x^4\Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**3+a)**(3/2),x)
```

```
[Out] gamma(-4/3)*hyper((-4/3, 3/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/
2)*x**4*gamma(-1/3))
```

3.438 $\int \frac{x^{11}}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=53

$$\frac{2}{21}\left(x^3+1\right)^{7/2}-\frac{2}{5}\left(x^3+1\right)^{5/2}+\frac{2}{3}\left(x^3+1\right)^{3/2}-\frac{2\sqrt{x^3+1}}{3}$$

```
[Out] 2/3*(x^3+1)^(3/2)-2/5*(x^3+1)^(5/2)+2/21*(x^3+1)^(7/2)-2/3*(x^3+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{2}{21}\left(x^3+1\right)^{7/2}-\frac{2}{5}\left(x^3+1\right)^{5/2}+\frac{2}{3}\left(x^3+1\right)^{3/2}-\frac{2\sqrt{x^3+1}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/Sqrt[1 + x^3],x]
```

```
[Out] (-2*Sqrt[1 + x^3])/3 + (2*(1 + x^3)^(3/2))/3 - (2*(1 + x^3)^(5/2))/5 + (2*(
1 + x^3)^(7/2))/21
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{\sqrt{1+x^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^3}{\sqrt{1+x}}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(-\frac{1}{\sqrt{1+x}}+3\sqrt{1+x}-3(1+x)^{3/2}+(1+x)^{5/2}\right)dx, x, x^3\right)\\ &= -\frac{2}{3}\sqrt{1+x^3}+\frac{2}{3}\left(1+x^3\right)^{3/2}-\frac{2}{5}\left(1+x^3\right)^{5/2}+\frac{2}{21}\left(1+x^3\right)^{7/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.57

$$\frac{2}{105}\sqrt{x^3+1}\left(5x^9-6x^6+8x^3-16\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/Sqrt[1 + x^3],x]
```

```
[Out] (2*Sqrt[1 + x^3]*(-16 + 8*x^3 - 6*x^6 + 5*x^9))/105
```

fricas [A] time = 0.72, size = 26, normalized size = 0.49

$$\frac{2}{105}\left(5x^9-6x^6+8x^3-16\right)\sqrt{x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(5*x^9 - 6*x^6 + 8*x^3 - 16)*sqrt(x^3 + 1)
```

giac [A] time = 0.16, size = 37, normalized size = 0.70

$$\frac{2}{21}\left(x^3+1\right)^{\frac{7}{2}}-\frac{2}{5}\left(x^3+1\right)^{\frac{5}{2}}+\frac{2}{3}\left(x^3+1\right)^{\frac{3}{2}}-\frac{2}{3}\sqrt{x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] 2/21*(x^3 + 1)^(7/2) - 2/5*(x^3 + 1)^(5/2) + 2/3*(x^3 + 1)^(3/2) - 2/3*sqrt
(x^3 + 1)
```

maple [A] time = 0.00, size = 38, normalized size = 0.72

$$\frac{2\left(x+1\right)\left(x^2-x+1\right)\left(5x^9-6x^6+8x^3-16\right)}{105\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^3+1)^(1/2),x)
```

```
[Out] 2/105*(x+1)*(x^2-x+1)*(5*x^9-6*x^6+8*x^3-16)/(x^3+1)^(1/2)
```

maxima [A] time = 1.36, size = 37, normalized size = 0.70

$$\frac{2}{21}\left(x^3+1\right)^{\frac{7}{2}}-\frac{2}{5}\left(x^3+1\right)^{\frac{5}{2}}+\frac{2}{3}\left(x^3+1\right)^{\frac{3}{2}}-\frac{2}{3}\sqrt{x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/21*(x^3 + 1)^(7/2) - 2/5*(x^3 + 1)^(5/2) + 2/3*(x^3 + 1)^(3/2) - 2/3*sqrt
(x^3 + 1)
```

mupad [B] time = 1.00, size = 46, normalized size = 0.87

$$\frac{16\,x^3\,\sqrt{x^3+1}}{105}-\frac{32\,\sqrt{x^3+1}}{105}-\frac{4\,x^6\,\sqrt{x^3+1}}{35}+\frac{2\,x^9\,\sqrt{x^3+1}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^3 + 1)^(1/2),x)
```

```
[Out] (16*x^3*(x^3 + 1)^(1/2))/105 - (32*(x^3 + 1)^(1/2))/105 - (4*x^6*(x^3 + 1)^
(1/2))/35 + (2*x^9*(x^3 + 1)^(1/2))/21
```

sympy [A] time = 2.91, size = 56, normalized size = 1.06

$$\frac{2x^9\sqrt{x^3+1}}{21}-\frac{4x^6\sqrt{x^3+1}}{35}+\frac{16x^3\sqrt{x^3+1}}{105}-\frac{32\sqrt{x^3+1}}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(x**3+1)**(1/2),x)
```

```
[Out] 2*x**9*sqrt(x**3 + 1)/21 - 4*x**6*sqrt(x**3 + 1)/35 + 16*x**3*sqrt(x**3 + 1
)/105 - 32*sqrt(x**3 + 1)/105
```

3.439 $\int \frac{x^8}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=40

$$\frac{2}{15}\left(x^3+1\right)^{5/2}-\frac{4}{9}\left(x^3+1\right)^{3/2}+\frac{2\sqrt{x^3+1}}{3}$$

```
[Out] -4/9*(x^3+1)^(3/2)+2/15*(x^3+1)^(5/2)+2/3*(x^3+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{2}{15}\left(x^3+1\right)^{5/2}-\frac{4}{9}\left(x^3+1\right)^{3/2}+\frac{2\sqrt{x^3+1}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/Sqrt[1 + x^3],x]
```

```
[Out] (2*Sqrt[1 + x^3])/3 - (4*(1 + x^3)^(3/2))/9 + (2*(1 + x^3)^(5/2))/15
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{\sqrt{1+x^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x}}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{1}{\sqrt{1+x}}-2\sqrt{1+x}+(1+x)^{3/2}\right)dx, x, x^3\right)\\ &= \frac{2\sqrt{1+x^3}}{3}-\frac{4}{9}\left(1+x^3\right)^{3/2}+\frac{2}{15}\left(1+x^3\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.62

$$\frac{2}{45}\sqrt{x^3+1}\left(3x^6-4x^3+8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/Sqrt[1 + x^3],x]
```

```
[Out] (2*Sqrt[1 + x^3]*(8 - 4*x^3 + 3*x^6))/45
```

fricas [A] time = 0.83, size = 21, normalized size = 0.52

$$\frac{2}{45}\left(3\,x^6-4\,x^3+8\right)\sqrt{x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] 2/45*(3*x^6 - 4*x^3 + 8)*sqrt(x^3 + 1)
```

giac [A] time = 0.16, size = 28, normalized size = 0.70

$$\frac{2}{15}\left(x^3+1\right)^{\frac{5}{2}}-\frac{4}{9}\left(x^3+1\right)^{\frac{3}{2}}+\frac{2}{3}\sqrt{x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^3+1)^(1/2),x, algorithm="giac")
[Out] 2/15*(x^3 + 1)^(5/2) - 4/9*(x^3 + 1)^(3/2) + 2/3*sqrt(x^3 + 1)
```

maple [A] time = 0.00, size = 33, normalized size = 0.82

$$\frac{2\left(x+1\right)\left(x^2-x+1\right)\left(3x^6-4x^3+8\right)}{45\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^3+1)^(1/2),x)
[Out] 2/45*(x+1)*(x^2-x+1)*(3*x^6-4*x^3+8)/(x^3+1)^(1/2)
```

maxima [A] time = 1.32, size = 28, normalized size = 0.70

$$\frac{2}{15}\left(x^3+1\right)^{\frac{5}{2}}-\frac{4}{9}\left(x^3+1\right)^{\frac{3}{2}}+\frac{2}{3}\sqrt{x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] 2/15*(x^3 + 1)^(5/2) - 4/9*(x^3 + 1)^(3/2) + 2/3*sqrt(x^3 + 1)
```

mupad [B] time = 0.03, size = 21, normalized size = 0.52

$$\frac{2\sqrt{x^3+1}\,\left(3\,x^6-4\,x^3+8\right)}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^3 + 1)^(1/2),x)
[Out] (2*(x^3 + 1)^(1/2)*(3*x^6 - 4*x^3 + 8))/45
```

sympy [A] time = 1.47, size = 41, normalized size = 1.02

$$\frac{2x^6\sqrt{x^3+1}}{15}-\frac{8x^3\sqrt{x^3+1}}{45}+\frac{16\sqrt{x^3+1}}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**3+1)**(1/2),x)
[Out] 2*x**6*sqrt(x**3 + 1)/15 - 8*x**3*sqrt(x**3 + 1)/45 + 16*sqrt(x**3 + 1)/45
```

3.440 $\int \frac{x^5}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=27

$$\frac{2}{9}\left(x^3+1\right)^{3/2}-\frac{2\sqrt{x^3+1}}{3}$$

```
[Out] 2/9*(x^3+1)^(3/2)-2/3*(x^3+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{2}{9}\left(x^3+1\right)^{3/2}-\frac{2\sqrt{x^3+1}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[1 + x^3],x]
```

```
[Out] (-2*Sqrt[1 + x^3])/3 + (2*(1 + x^3)^(3/2))/9
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\sqrt{1+x^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{\sqrt{1+x}}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(-\frac{1}{\sqrt{1+x}}+\sqrt{1+x}\right)\,dx, x, x^3\right)\\ &= -\frac{2}{3}\sqrt{1+x^3}+\frac{2}{9}\left(1+x^3\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 0.67

$$\frac{2}{9}\left(x^3-2\right)\sqrt{x^3+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[1 + x^3],x]
```

```
[Out] (2*(-2 + x^3)*Sqrt[1 + x^3])/9
```

fricas [A] time = 0.67, size = 14, normalized size = 0.52

$$\frac{2}{9}\sqrt{x^3+1}\left(x^3-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] 2/9*sqrt(x^3 + 1)*(x^3 - 2)
```

giac [A] time = 0.17, size = 19, normalized size = 0.70

$$\frac{2}{9}\left(x^3+1\right)^{\frac{3}{2}}-\frac{2}{3}\sqrt{x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^3+1)^(1/2),x, algorithm="giac")
[Out] 2/9*(x^3 + 1)^(3/2) - 2/3*sqrt(x^3 + 1)
```

maple [A] time = 0.00, size = 26, normalized size = 0.96

$$\frac{2\left(x+1\right)\left(x^2-x+1\right)\left(x^3-2\right)}{9\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^3+1)^(1/2),x)
[Out] 2/9*(x+1)*(x^2-x+1)*(x^3-2)/(x^3+1)^(1/2)
```

maxima [A] time = 1.32, size = 19, normalized size = 0.70

$$\frac{2}{9}\left(x^3+1\right)^{\frac{3}{2}}-\frac{2}{3}\sqrt{x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] 2/9*(x^3 + 1)^(3/2) - 2/3*sqrt(x^3 + 1)
```

mupad [B] time = 0.02, size = 14, normalized size = 0.52

$$\frac{2\sqrt{x^3+1}\,\left(x^3-2\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^3 + 1)^(1/2),x)
[Out] (2*(x^3 + 1)^(1/2)*(x^3 - 2))/9
```

sympy [A] time = 0.39, size = 26, normalized size = 0.96

$$\frac{2x^3\sqrt{x^3+1}}{9}-\frac{4\sqrt{x^3+1}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**3+1)**(1/2),x)
[Out] 2*x**3*sqrt(x**3 + 1)/9 - 4*sqrt(x**3 + 1)/9
```

3.441 $\int \frac{x^2}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=13

$$\frac{2\sqrt{x^3+1}}{3}$$

```
[Out] 2/3*(x^3+1)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$\frac{2\sqrt{x^3+1}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[1 + x^3],x]
```

```
[Out] (2*Sqrt[1 + x^3])/3
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{1+x^3}}\,dx = \frac{2\sqrt{1+x^3}}{3}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{2\sqrt{x^3+1}}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[1 + x^3],x]
```

```
[Out] (2*Sqrt[1 + x^3])/3
```

fricas [A] time = 0.87, size = 9, normalized size = 0.69

$$\frac{2}{3}\sqrt{x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(x^3 + 1)
```

giac [A] time = 0.15, size = 9, normalized size = 0.69

$$\frac{2}{3}\sqrt{x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^3+1)^(1/2),x, algorithm="giac")
[Out] 2/3*sqrt(x^3 + 1)
```

maple [B] time = 0.00, size = 21, normalized size = 1.62

$$\frac{2\,(x+1)\left(x^2-x+1\right)}{3\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^3+1)^(1/2),x)
[Out] 2/3*(x+1)*(x^2-x+1)/(x^3+1)^(1/2)
```

maxima [A] time = 1.40, size = 9, normalized size = 0.69

$$\frac{2}{3}\sqrt{x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] 2/3*sqrt(x^3 + 1)
```

mupad [B] time = 0.02, size = 9, normalized size = 0.69

$$\frac{2\sqrt{x^3+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^3 + 1)^(1/2),x)
[Out] (2*(x^3 + 1)^(1/2))/3
```

sympy [A] time = 0.35, size = 10, normalized size = 0.77

$$\frac{2\sqrt{x^3+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**3+1)**(1/2),x)
[Out] 2*sqrt(x**3 + 1)/3
```

3.442 $\int \frac{1}{x\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=14

$$-\frac{2}{3}\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

```
[Out] -2/3*arctanh((x^3+1)^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {266, 63, 207}

$$-\frac{2}{3}\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[1 + x^3]])/3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{1+x^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^3\right)\\ &= \frac{2}{3}\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^3}\right)\\ &= -\frac{2}{3}\tanh^{-1}\left(\sqrt{1+x^3}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{2}{3}\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[1 + x^3]),x]
[Out] (-2*ArcTanh[Sqrt[1 + x^3]])/3
```

fricas [B] time = 0.61, size = 25, normalized size = 1.79

$$-\frac{1}{3}\log\left(\sqrt{x^3+1}+1\right)+\frac{1}{3}\log\left(\sqrt{x^3+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] -1/3*log(sqrt(x^3 + 1) + 1) + 1/3*log(sqrt(x^3 + 1) - 1)
```

giac [B] time = 0.15, size = 26, normalized size = 1.86

$$-\frac{1}{3}\log\left(\sqrt{x^3+1}+1\right)+\frac{1}{3}\log\left(\left|\sqrt{x^3+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^3+1)^(1/2),x, algorithm="giac")
[Out] -1/3*log(sqrt(x^3 + 1) + 1) + 1/3*log(abs(sqrt(x^3 + 1) - 1))
```

maple [A] time = 0.16, size = 11, normalized size = 0.79

$$-\frac{2\,\mathrm{arctanh}\left(\sqrt{x^3+1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^3+1)^(1/2),x)
[Out] -2/3*arctanh((x^3+1)^(1/2))
```

maxima [B] time = 1.32, size = 25, normalized size = 1.79

$$-\frac{1}{3}\log\left(\sqrt{x^3+1}+1\right)+\frac{1}{3}\log\left(\sqrt{x^3+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] -1/3*log(sqrt(x^3 + 1) + 1) + 1/3*log(sqrt(x^3 + 1) - 1)
```

mupad [B] time = 1.14, size = 164, normalized size = 11.71

$$-\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^3 + 1)^(1/2)),x)
[Out] -((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1
/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(((x + 1)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))
```

```
)/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*
1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

sympy [**A**] time = 1.27, size = 12, normalized size = 0.86

$$-\frac{2\,\mathrm{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**3+1)**(1/2),x)
```

```
[Out] -2*asinh(x**(-3/2))/3
```

3.443 $\int \frac{1}{x^4\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=31

$$\frac{1}{3}\tanh^{-1}\left(\sqrt{x^3+1}\right)-\frac{\sqrt{x^3+1}}{3x^3}$$

```
[Out] 1/3*arctanh((x^3+1)^(1/2))-1/3*(x^3+1)^(1/2)/x^3
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {266, 51, 63, 207}

$$\frac{1}{3}\tanh^{-1}\left(\sqrt{x^3+1}\right)-\frac{\sqrt{x^3+1}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[1 + x^3]),x]
```

```
[Out] -Sqrt[1 + x^3]/(3*x^3) + ArcTanh[Sqrt[1 + x^3]]/3
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{1+x^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x}}\,dx, x, x^3\right)\\ &= -\frac{\sqrt{1+x^3}}{3x^3} - \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^3\right)\\ &= -\frac{\sqrt{1+x^3}}{3x^3} - \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^3}\right)\\ &= -\frac{\sqrt{1+x^3}}{3x^3} + \frac{1}{3}\tanh^{-1}\left(\sqrt{1+x^3}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{1}{3}\tanh^{-1}\left(\sqrt{x^3+1}\right) - \frac{\sqrt{x^3+1}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[1 + x^3]),x]
```

```
[Out] -1/3*Sqrt[1 + x^3]/x^3 + ArcTanh[Sqrt[1 + x^3]]/3
```

fricas [A] time = 0.74, size = 44, normalized size = 1.42

$$\frac{x^3\log\left(\sqrt{x^3+1}+1\right) - x^3\log\left(\sqrt{x^3+1}-1\right) - 2\sqrt{x^3+1}}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(x^3*log(sqrt(x^3 + 1) + 1) - x^3*log(sqrt(x^3 + 1) - 1) - 2*sqrt(x^3 +
 1))/x^3
```

giac [A] time = 0.23, size = 38, normalized size = 1.23

$$-\frac{\sqrt{x^3+1}}{3\,x^3} + \frac{1}{6}\log\left(\sqrt{x^3+1}+1\right) - \frac{1}{6}\log\left(\left|\sqrt{x^3+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(x^3 + 1)/x^3 + 1/6*log(sqrt(x^3 + 1) + 1) - 1/6*log(abs(sqrt(x^3
+ 1) - 1))
```

maple [A] time = 0.02, size = 24, normalized size = 0.77

$$\frac{\operatorname{arctanh}\left(\sqrt{x^3+1}\right)}{3} - \frac{\sqrt{x^3+1}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^3+1)^(1/2),x)
```

```
[Out] 1/3*arctanh((x^3+1)^(1/2))-1/3*(x^3+1)^(1/2)/x^3
```

maxima [A] time = 1.30, size = 37, normalized size = 1.19

$$-\frac{\sqrt{x^3+1}}{3x^3}+\frac{1}{6}\log\left(\sqrt{x^3+1}+1\right)-\frac{1}{6}\log\left(\sqrt{x^3+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(x^3 + 1)/x^3 + 1/6*log(sqrt(x^3 + 1) + 1) - 1/6*log(sqrt(x^3 + 1)
 - 1)
```

mupad [B] time = 0.99, size = 176, normalized size = 5.68

$$-\frac{\sqrt{x^3+1}}{3x^3}+\frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x^3 + 1)^(1/2)),x)
```

```
[Out] (((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))
^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/(
(3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(((x + 1)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 -
3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(
1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2) - (x^3 + 1)^(1/2)/(3*x^3)
```

sympy [A] time = 2.49, size = 26, normalized size = 0.84

$$\frac{\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}-\frac{\sqrt{1+\frac{1}{x^3}}}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**3+1)**(1/2),x)
```

```
[Out] asinh(x**(-3/2))/3 - sqrt(1 + x**(-3))/(3*x**(3/2))
```

3.444 $\int \frac{1}{x^7\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=47

$$\frac{\sqrt{x^3+1}}{4x^3}-\frac{1}{4}\tanh^{-1}\left(\sqrt{x^3+1}\right)-\frac{\sqrt{x^3+1}}{6x^6}$$

```
[Out] -1/4*arctanh((x^3+1)^(1/2))-1/6*(x^3+1)^(1/2)/x^6+1/4*(x^3+1)^(1/2)/x^3
```

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {266, 51, 63, 207}

$$\frac{\sqrt{x^3+1}}{4x^3}-\frac{\sqrt{x^3+1}}{6x^6}-\frac{1}{4}\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*Sqrt[1 + x^3]),x]
```

```
[Out] -Sqrt[1 + x^3]/(6*x^6) + Sqrt[1 + x^3]/(4*x^3) - ArcTanh[Sqrt[1 + x^3]]/4
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7\sqrt{1+x^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{1+x}}\,dx, x, x^3\right)\\ &= -\frac{\sqrt{1+x^3}}{6x^6} - \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x}}\,dx, x, x^3\right)\\ &= -\frac{\sqrt{1+x^3}}{6x^6} + \frac{\sqrt{1+x^3}}{4x^3} + \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^3\right)\\ &= -\frac{\sqrt{1+x^3}}{6x^6} + \frac{\sqrt{1+x^3}}{4x^3} + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^3}\right)\\ &= -\frac{\sqrt{1+x^3}}{6x^6} + \frac{\sqrt{1+x^3}}{4x^3} - \frac{1}{4}\tanh^{-1}\left(\sqrt{1+x^3}\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 26, normalized size = 0.55

$$-\frac{2}{3}\sqrt{x^3+1}\;{}_2F_1\left(\frac{1}{2}, 3; \frac{3}{2}; x^3+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*Sqrt[1 + x^3]*Hypergeometric2F1[1/2, 3, 3/2, 1 + x^3])/3
```

fricas [A] time = 0.80, size = 52, normalized size = 1.11

$$-\frac{3\,x^6\log\left(\sqrt{x^3+1}+1\right) - 3\,x^6\log\left(\sqrt{x^3+1}-1\right) - 2\left(3\,x^3-2\right)\sqrt{x^3+1}}{24\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(3*x^6*log(sqrt(x^3 + 1) + 1) - 3*x^6*log(sqrt(x^3 + 1) - 1) - 2*(3*x
^3 - 2)*sqrt(x^3 + 1))/x^6
```

giac [A] time = 0.15, size = 50, normalized size = 1.06

$$\frac{3\left(x^3+1\right)^{\frac{3}{2}} - 5\sqrt{x^3+1}}{12\,x^6} - \frac{1}{8}\log\left(\sqrt{x^3+1}+1\right) + \frac{1}{8}\log\left(\left|\sqrt{x^3+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*(3*(x^3 + 1)^(3/2) - 5*sqrt(x^3 + 1))/x^6 - 1/8*log(sqrt(x^3 + 1) + 1)
 + 1/8*log(abs(sqrt(x^3 + 1) - 1))
```

maple [A] time = 0.02, size = 36, normalized size = 0.77

$$-\frac{\operatorname{arctanh}\left(\sqrt{x^3+1}\right)}{4} + \frac{\sqrt{x^3+1}}{4x^3} - \frac{\sqrt{x^3+1}}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^3+1)^(1/2),x)
```

```
[Out] -1/4*arctanh((x^3+1)^(1/2))-1/6*(x^3+1)^(1/2)/x^6+1/4*(x^3+1)^(1/2)/x^3
```

maxima [A] time = 1.36, size = 64, normalized size = 1.36

$$-\frac{3\left(x^3+1\right)^{\frac{3}{2}}-5\sqrt{x^3+1}}{12\left(2x^3-\left(x^3+1\right)^2+1\right)}-\frac{1}{8}\log\left(\sqrt{x^3+1}+1\right)+\frac{1}{8}\log\left(\sqrt{x^3+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/12*(3*(x^3 + 1)^(3/2) - 5*sqrt(x^3 + 1))/(2*x^3 - (x^3 + 1)^2 + 1) - 1/8
*log(sqrt(x^3 + 1) + 1) + 1/8*log(sqrt(x^3 + 1) - 1)
```

mupad [B] time = 0.06, size = 189, normalized size = 4.02

$$\frac{\sqrt{x^3+1}}{4x^3}-\frac{\sqrt{x^3+1}}{6x^6}-\frac{3\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{4\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(x^3 + 1)^(1/2)),x)
```

```
[Out] (x^3 + 1)^(1/2)/(4*x^3) - (x^3 + 1)^(1/2)/(6*x^6) - (3*((3^(1/2)*1i)/2 + 3/
2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/
2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(4*(x^3 - x*((
(3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*
((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

sympy [A] time = 6.95, size = 65, normalized size = 1.38

$$-\frac{\mathrm{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{4}+\frac{1}{4x^{\frac{3}{2}}\sqrt{1+\frac{1}{x^3}}}+\frac{1}{12x^{\frac{9}{2}}\sqrt{1+\frac{1}{x^3}}}-\frac{1}{6x^{\frac{15}{2}}\sqrt{1+\frac{1}{x^3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**3+1)**(1/2),x)
```

```
[Out] -asinh(x**(-3/2))/4 + 1/(4*x**(3/2)*sqrt(1 + x**(-3))) + 1/(12*x**(9/2)*sqr
t(1 + x**(-3))) - 1/(6*x**(15/2)*sqrt(1 + x**(-3)))
```

3.445 $\int \frac{1}{x^{10}\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=63

$$-\frac{5\sqrt{x^3+1}}{24x^3}+\frac{5}{24}\tanh^{-1}\left(\sqrt{x^3+1}\right)-\frac{\sqrt{x^3+1}}{9x^9}+\frac{5\sqrt{x^3+1}}{36x^6}$$

```
[Out] 5/24*arctanh((x^3+1)^(1/2))-1/9*(x^3+1)^(1/2)/x^9+5/36*(x^3+1)^(1/2)/x^6-5/
24*(x^3+1)^(1/2)/x^3
```

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {266, 51, 63, 207}

$$-\frac{5\sqrt{x^3+1}}{24x^3}+\frac{5\sqrt{x^3+1}}{36x^6}-\frac{\sqrt{x^3+1}}{9x^9}+\frac{5}{24}\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*Sqrt[1 + x^3]),x]
```

```
[Out] -Sqrt[1 + x^3]/(9*x^9) + (5*Sqrt[1 + x^3])/(36*x^6) - (5*Sqrt[1 + x^3])/(24
*x^3) + (5*ArcTanh[Sqrt[1 + x^3]])/24
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{10}\sqrt{1+x^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^4\sqrt{1+x}}\,dx, x, x^3\right)\\ &= -\frac{\sqrt{1+x^3}}{9x^9} - \frac{5}{18}\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{1+x}}\,dx, x, x^3\right)\\ &= -\frac{\sqrt{1+x^3}}{9x^9} + \frac{5\sqrt{1+x^3}}{36x^6} + \frac{5}{24}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x}}\,dx, x, x^3\right)\\ &= -\frac{\sqrt{1+x^3}}{9x^9} + \frac{5\sqrt{1+x^3}}{36x^6} - \frac{5\sqrt{1+x^3}}{24x^3} - \frac{5}{48}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^3\right)\\ &= -\frac{\sqrt{1+x^3}}{9x^9} + \frac{5\sqrt{1+x^3}}{36x^6} - \frac{5\sqrt{1+x^3}}{24x^3} - \frac{5}{24}\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^3}\right)\\ &= -\frac{\sqrt{1+x^3}}{9x^9} + \frac{5\sqrt{1+x^3}}{36x^6} - \frac{5\sqrt{1+x^3}}{24x^3} + \frac{5}{24}\tanh^{-1}\left(\sqrt{1+x^3}\right)\end{aligned}$$

Mathematica [C] time = 0.00, size = 26, normalized size = 0.41

$$\frac{2}{3}\sqrt{x^3+1}\;{}_2F_1\left(\frac{1}{2}, 4; \frac{3}{2}; x^3+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^10*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[1 + x^3]*Hypergeometric2F1[1/2, 4, 3/2, 1 + x^3])/3
```

fricas [A] time = 0.91, size = 57, normalized size = 0.90

$$\frac{15\,x^9\log\left(\sqrt{x^3+1}+1\right) - 15\,x^9\log\left(\sqrt{x^3+1}-1\right) - 2\left(15\,x^6 - 10\,x^3 + 8\right)\sqrt{x^3+1}}{144\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/144*(15*x^9*log(sqrt(x^3 + 1) + 1) - 15*x^9*log(sqrt(x^3 + 1) - 1) - 2*(1
5*x^6 - 10*x^3 + 8)*sqrt(x^3 + 1))/x^9
```

giac [A] time = 0.17, size = 59, normalized size = 0.94

$$-\frac{15\left(x^3+1\right)^{\frac{5}{2}} - 40\left(x^3+1\right)^{\frac{3}{2}} + 33\sqrt{x^3+1}}{72\,x^9} + \frac{5}{48}\log\left(\sqrt{x^3+1}+1\right) - \frac{5}{48}\log\left(\left|\sqrt{x^3+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/72*(15*(x^3 + 1)^(5/2) - 40*(x^3 + 1)^(3/2) + 33*sqrt(x^3 + 1))/x^9 + 5/
48*log(sqrt(x^3 + 1) + 1) - 5/48*log(abs(sqrt(x^3 + 1) - 1))
```

maple [A] time = 0.02, size = 48, normalized size = 0.76

$$\frac{5\,\text{arctanh}\left(\sqrt{x^3+1}\right)}{24} - \frac{5\sqrt{x^3+1}}{24x^3} + \frac{5\sqrt{x^3+1}}{36x^6} - \frac{\sqrt{x^3+1}}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(x^3+1)^(1/2),x)
```

```
[Out] 5/24*arctanh((x^3+1)^(1/2))-1/9*(x^3+1)^(1/2)/x^9+5/36*(x^3+1)^(1/2)/x^6-5/
24*(x^3+1)^(1/2)/x^3
```

maxima [A] time = 1.33, size = 80, normalized size = 1.27

$$-\frac{15\left(x^3+1\right)^{\frac{5}{2}}-40\left(x^3+1\right)^{\frac{3}{2}}+33\sqrt{x^3+1}}{72\left(\left(x^3+1\right)^3+3x^3-3\left(x^3+1\right)^2+2\right)}+\frac{5}{48}\log\left(\sqrt{x^3+1}+1\right)-\frac{5}{48}\log\left(\sqrt{x^3+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/72*(15*(x^3 + 1)^(5/2) - 40*(x^3 + 1)^(3/2) + 33*sqrt(x^3 + 1))/((x^3 +
1)^3 + 3*x^3 - 3*(x^3 + 1)^2 + 2) + 5/48*log(sqrt(x^3 + 1) + 1) - 5/48*log(
sqrt(x^3 + 1) - 1)
```

mupad [B] time = 0.04, size = 201, normalized size = 3.19

$$\frac{5\sqrt{x^3+1}}{36x^6}-\frac{5\sqrt{x^3+1}}{24x^3}-\frac{\sqrt{x^3+1}}{9x^9}+\frac{5\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{\frac{x}{\frac{3}{2}+}}\right.\right.}{8\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^10*(x^3 + 1)^(1/2)),x)
```

```
[Out] (5*(x^3 + 1)^(1/2))/(36*x^6) - (5*(x^3 + 1)^(1/2))/(24*x^3) - (x^3 + 1)^(1/
2)/(9*x^9) + (5*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2
)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)
/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2
, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3
^(1/2)*1i)/2 - 3/2)))/(8*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

sympy [A] time = 5.98, size = 85, normalized size = 1.35

$$\frac{5\,\mathrm{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{24}-\frac{5}{24x^{\frac{3}{2}}\sqrt{1+\frac{1}{x^3}}}-\frac{5}{72x^{\frac{9}{2}}\sqrt{1+\frac{1}{x^3}}}+\frac{1}{36x^{\frac{15}{2}}\sqrt{1+\frac{1}{x^3}}}-\frac{1}{9x^{\frac{21}{2}}\sqrt{1+\frac{1}{x^3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(x**3+1)**(1/2),x)
```

```
[Out] 5*asinh(x**(-3/2))/24 - 5/(24*x**(3/2)*sqrt(1 + x**(-3))) - 5/(72*x**(9/2)*
sqrt(1 + x**(-3))) + 1/(36*x**(15/2)*sqrt(1 + x**(-3))) - 1/(9*x**(21/2)*sq
rt(1 + x**(-3)))
```

3.446 $\int \frac{x^6}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=136

$$-\frac{16}{55}\sqrt{x^3+1}\,x+\frac{2}{11}\sqrt{x^3+1}\,x^4+\frac{32\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{55\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

```
[Out] -16/55*x*(x^3+1)^(1/2)+2/11*x^4*(x^3+1)^(1/2)+32/165*(1+x)*EllipticF((1+x-3
^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(
1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {321, 218}

$$\frac{2}{11}\sqrt{x^3+1}\,x^4-\frac{16}{55}\sqrt{x^3+1}\,x+\frac{32\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{55\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/Sqrt[1 + x^3],x]
```

```
[Out] (-16*x*Sqrt[1 + x^3])/55 + (2*x^4*Sqrt[1 + x^3])/11 + (32*Sqrt[2 + Sqrt[3]]
*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt
[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(55*3^(1/4)*Sqrt[(1 + x)/(1 +
 Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^6}{\sqrt{1+x^3}}\,dx = \frac{2}{11}x^4\sqrt{1+x^3} - \frac{8}{11}\int \frac{x^3}{\sqrt{1+x^3}}\,dx$$
$$= -\frac{16}{55}x\sqrt{1+x^3} + \frac{2}{11}x^4\sqrt{1+x^3} + \frac{16}{55}\int \frac{1}{\sqrt{1+x^3}}\,dx$$
$$= -\frac{16}{55}x\sqrt{1+x^3} + \frac{2}{11}x^4\sqrt{1+x^3} + \frac{32\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\right.}{55\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.29

$$\frac{2}{55}x\left(8\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)+\sqrt{x^3+1}\left(5x^3-8\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/Sqrt[1 + x^3],x]
```

```
[Out] (2*x*(Sqrt[1 + x^3]*(-8 + 5*x^3) + 8*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3]
))/55
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^6}{\sqrt{x^3+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^6/sqrt(x^3 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/sqrt(x^3 + 1), x)
```

maple [A] time = 0.02, size = 139, normalized size = 1.02

$$\frac{2\sqrt{x^3+1}\,x^4}{11} - \frac{16\sqrt{x^3+1}\,x}{55} + \frac{32\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\text{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{55\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^3+1)^(1/2),x)
```

```
[Out] 2/11*x^4*(x^3+1)^(1/2)-16/55*x*(x^3+1)^(1/2)+32/55*(3/2-1/2*I*3^(1/2))*((x+
1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^
(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*Elli
```

```
pticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3
^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/sqrt(x^3 + 1), x)
```

mupad [B] time = 0.98, size = 179, normalized size = 1.32

$$\frac{2x^4\sqrt{x^3+1}}{11}-\frac{16x\sqrt{x^3+1}}{55}+\frac{32\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{55\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^3 + 1)^(1/2),x)
```

```
[Out] (2*x^4*(x^3 + 1)^(1/2))/11 - (16*x*(x^3 + 1)^(1/2))/55 + (32*((3^(1/2)*1i)/
2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2
+ 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^
(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(55*(x^3 - x*(((3^(1/2)*1i)/2 -
 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2
+ 1/2))^(1/2))
```

sympy [A] time = 2.11, size = 29, normalized size = 0.21

$$\frac{x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**3+1)**(1/2),x)
```

```
[Out] x**7*gamma(7/3)*hyper((1/2, 7/3), (10/3,), x**3*exp_polar(I*pi))/(3*gamma(1
0/3))
```

3.447 $\int \frac{x^3}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=120

$$\frac{2}{5}x\sqrt{x^3+1}-\frac{4\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^3+1}}$$

```
[Out] 2/5*x*(x^3+1)^(1/2)-4/15*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1
/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4
)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {321, 218}

$$\frac{2}{5}x\sqrt{x^3+1}-\frac{4\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[1 + x^3],x]
```

```
[Out] (2*x*Sqrt[1 + x^3])/5 - (4*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1
+ Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7
 - 4*Sqrt[3]])/(5*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{1+x^3}}\,dx = \frac{2}{5}x\sqrt{1+x^3}-\frac{2}{5}\int \frac{1}{\sqrt{1+x^3}}\,dx$$

$$= \frac{2}{5}x\sqrt{1+x^3}-\frac{4\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{1+x^3}}$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.27

$$\frac{2}{5}x\left(\sqrt{x^3+1}-{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[1 + x^3],x]
[Out] (2*x*(Sqrt[1 + x^3] - Hypergeometric2F1[1/3, 1/2, 4/3, -x^3]))/5
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3}{\sqrt{x^3+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] integral(x^3/sqrt(x^3 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^3/sqrt(x^3 + 1), x)
```

maple [A] time = 0.02, size = 127, normalized size = 1.06

$$\frac{2\sqrt{x^3+1}\,x}{5}-\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\text{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{5\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^3+1)^(1/2),x)
[Out] 2/5*(x^3+1)^(1/2)*x-4/5*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/
2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2)
)/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(
1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/sqrt(x^3 + 1), x)
```

mupad [B] time = 0.04, size = 167, normalized size = 1.39

$$\frac{2x\sqrt{x^3+1}}{5}-\frac{4\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{5\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^3 + 1)^(1/2),x)
```

```
[Out] (2*x*(x^3 + 1)^(1/2))/5 - (4*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 -
1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(
((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x
 + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)
/2 - 3/2)))/(5*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1)
 - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

sympy [A] time = 1.65, size = 29, normalized size = 0.24

$$\frac{x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**3+1)**(1/2),x)
```

```
[Out] x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), x**3*exp_polar(I*pi))/(3*gamma(7/
3))
```

3.448 $\int \frac{1}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=103

$$\frac{2\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

```
[Out] 2/3*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)
+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x
)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {218}

$$\frac{2\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + x^3],x]
```

```
[Out] (2*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ellipt
icF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*
Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+x^3}}\,dx = \frac{2\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] time = 0.00, size = 17, normalized size = 0.17

$$x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + x^3],x]
```

```
[Out] x*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3]
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{x^3+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(x^3 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(x^3 + 1), x)
```

maple [A] time = 0.02, size = 116, normalized size = 1.13

$$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\text{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(x^3 + 1), x)
```

mupad [B] time = 0.04, size = 155, normalized size = 1.50

$$\frac{\left(3+\sqrt{3}\,1\text{i}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\text{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\text{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\text{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\text{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\text{i}}{2}}}\,\text{F}\left(\text{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\text{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\text{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\text{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\text{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\text{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\text{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\text{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3 + 1)^(1/2),x)
```

```
[Out] ((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)
*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/
2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i
)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1
i)/2 + 1/2))^(1/2)
```

sympy [**A**] time = 1.50, size = 27, normalized size = 0.26

$$\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3}, \frac{1}{2} \\ \frac{4}{3}\end{matrix}\right| x^3 e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3+1)**(1/2),x)
```

```
[Out] x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/3))
```

3.449 $\int \frac{1}{x^3\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=122

$$-\frac{\sqrt{x^3+1}}{2x^2}-\frac{\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\,\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^3+1}}$$

```
[Out] -1/2*(x^3+1)^(1/2)/x^2-1/6*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^
(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3
/4)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {325, 218}

$$-\frac{\sqrt{x^3+1}}{2x^2}-\frac{\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\,\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[1 + x^3]),x]
```

```
[Out] -Sqrt[1 + x^3]/(2*x^2) - (Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 +
 Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7
- 4*Sqrt[3]])/(2*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{1+x^3}}\,dx = -\frac{\sqrt{1+x^3}}{2x^2}-\frac{1}{4}\int \frac{1}{\sqrt{1+x^3}}\,dx$$

$$= -\frac{\sqrt{1+x^3}}{2x^2}-\frac{\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\,\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{1+x^3}}$$

Mathematica [C] time = 0.00, size = 22, normalized size = 0.18

$$-\frac{{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};-x^3\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[1 + x^3]),x]
[Out] -1/2*Hypergeometric2F1[-2/3, 1/2, 1/3, -x^3]/x^2
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^3+1}}{x^6+x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x^3 + 1)/(x^6 + x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^3+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^3 + 1)*x^3), x)
```

maple [A] time = 0.02, size = 129, normalized size = 1.06

$$-\frac{\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\text{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{2\sqrt{x^3+1}}-\frac{\sqrt{x^3+1}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^3+1)^(1/2),x)
[Out] -1/2*(x^3+1)^(1/2)/x^2-1/2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^
(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1
/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*
3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^3+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^3 + 1)*x^3), x)
```

mupad [B] time = 0.99, size = 169, normalized size = 1.39

$$-\frac{\sqrt{x^3+1}}{2x^2}-\frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{2\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x^3 + 1)^(1/2)),x)
```

```
[Out] - (x^3 + 1)^(1/2)/(2*x^2) - (((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 -
1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(
((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x
 + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)
/2 - 3/2)))/(2*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1)
 - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

sympy [A] time = 1.70, size = 32, normalized size = 0.26

$$\frac{\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{1}{2}\\ \frac{1}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**3+1)**(1/2),x)
```

```
[Out] gamma(-2/3)*hyper((-2/3, 1/2), (1/3,), x**3*exp_polar(I*pi))/(3*x**2*gamma(
1/3))
```

3.450 $\int \frac{1}{x^6\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=138

$$-\frac{\sqrt{x^3+1}}{5x^5}+\frac{7\sqrt{x^3+1}}{20x^2}+\frac{7\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{20\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

```
[Out] -1/5*(x^3+1)^(1/2)/x^5+7/20*(x^3+1)^(1/2)/x^2+7/60*(1+x)*EllipticF((1+x-3^(
1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+
x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {325, 218}

$$\frac{7\sqrt{x^3+1}}{20x^2}-\frac{\sqrt{x^3+1}}{5x^5}+\frac{7\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{20\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*Sqrt[1 + x^3]),x]
```

```
[Out] -Sqrt[1 + x^3]/(5*x^5) + (7*Sqrt[1 + x^3])/(20*x^2) + (7*Sqrt[2 + Sqrt[3]]*
(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[
3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(20*3^(1/4)*Sqrt[(1 + x)/(1 +
Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^6\sqrt{1+x^3}}\,dx = -\frac{\sqrt{1+x^3}}{5x^5} - \frac{7}{10}\int \frac{1}{x^3\sqrt{1+x^3}}\,dx$$
$$= -\frac{\sqrt{1+x^3}}{5x^5} + \frac{7\sqrt{1+x^3}}{20x^2} + \frac{7}{40}\int \frac{1}{\sqrt{1+x^3}}\,dx$$
$$= -\frac{\sqrt{1+x^3}}{5x^5} + \frac{7\sqrt{1+x^3}}{20x^2} + \frac{7\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{20\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] time = 0.00, size = 22, normalized size = 0.16

$$-\frac{{}_2F_1\left(-\frac{5}{3},\frac{1}{2};-\frac{2}{3};-x^3\right)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*Sqrt[1 + x^3]),x]
[Out] -1/5*Hypergeometric2F1[-5/3, 1/2, -2/3, -x^3]/x^5
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^3+1}}{x^9+x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x^3 + 1)/(x^9 + x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^3+1}\,x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^3 + 1)*x^6), x)
```

maple [A] time = 0.02, size = 141, normalized size = 1.02

$$\frac{7\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\text{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{20\sqrt{x^3+1}} + \frac{7\sqrt{x^3+1}}{20x^2} - \frac{\sqrt{x^3+1}}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^3+1)^(1/2),x)
[Out] -1/5*(x^3+1)^(1/2)/x^5+7/20*(x^3+1)^(1/2)/x^2+7/20*(3/2-1/2*I*3^(1/2))*((x+
1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^
(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*Elli
```

```
pticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3
^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^3+1}\,x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^3 + 1)*x^6), x)
```

mupad [B] time = 1.00, size = 181, normalized size = 1.31

$$\frac{7\sqrt{x^3+1}}{20\,x^2}-\frac{\sqrt{x^3+1}}{5\,x^5}+\frac{7\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{20\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(x^3 + 1)^(1/2)),x)
```

```
[Out] (7*(x^3 + 1)^(1/2))/(20*x^2) - (x^3 + 1)^(1/2)/(5*x^5) + (7*((3^(1/2)*1i)/2
 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/
((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(
1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(20*(x^3 - x*(((3^(1/2)*1i)/2 -
1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
 1/2))^(1/2))
```

sympy [A] time = 1.97, size = 36, normalized size = 0.26

$$\frac{\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{5}{3},\frac{1}{2}\\-\frac{2}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**3+1)**(1/2),x)
```

```
[Out] gamma(-5/3)*hyper((-5/3, 1/2), (-2/3,), x**3*exp_polar(I*pi))/(3*x**5*gamma
(-2/3))
```

3.451 $\int \frac{x^7}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=262

$$\frac{80\sqrt{x^3+1}}{91\left(x+\sqrt{3}+1\right)}+\frac{2}{13}\sqrt{x^3+1}\,x^5-\frac{20}{91}\sqrt{x^3+1}\,x^2+\frac{80\sqrt{2}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)\quad 40}{91\sqrt[4]{3}\,\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^3+1}}--$$

```
[Out] -20/91*x^2*(x^3+1)^(1/2)+2/13*x^5*(x^3+1)^(1/2)+80/91*(x^3+1)^(1/2)/(1+x+3^
(1/2))+80/273*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^
(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(1+x+3
^(1/2))^2)^(1/2)-40/91*3^(1/4)*(1+x)*EllipticE((1+x-3^(1/2))/(1+x+3^(1/2)),
I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/
(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {321, 303, 218, 1877}

$$\frac{2}{13}\sqrt{x^3+1}\,x^5-\frac{20}{91}\sqrt{x^3+1}\,x^2+\frac{80\sqrt{x^3+1}}{91\left(x+\sqrt{3}+1\right)}+\frac{80\sqrt{2}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)\quad 40}{91\sqrt[4]{3}\,\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^3+1}}--$$

Antiderivative was successfully verified.

```
[In] Int[x^7/Sqrt[1 + x^3],x]
```

```
[Out] (-20*x^2*Sqrt[1 + x^3])/91 + (2*x^5*Sqrt[1 + x^3])/13 + (80*Sqrt[1 + x^3])/
(91*(1 + Sqrt[3] + x)) - (40*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x
+ x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3]
 + x)], -7 - 4*Sqrt[3]])/(91*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3
]) + (80*Sqrt[2]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[
ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(91*3^(1/4)*S
qrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{\sqrt{1+x^3}}\,dx &= \frac{2}{13}x^5\sqrt{1+x^3} - \frac{10}{13}\int \frac{x^4}{\sqrt{1+x^3}}\,dx \\
&= -\frac{20}{91}x^2\sqrt{1+x^3} + \frac{2}{13}x^5\sqrt{1+x^3} + \frac{40}{91}\int \frac{x}{\sqrt{1+x^3}}\,dx \\
&= -\frac{20}{91}x^2\sqrt{1+x^3} + \frac{2}{13}x^5\sqrt{1+x^3} + \frac{40}{91}\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx + \frac{1}{91}\left(40\sqrt{2\left(2-\sqrt{3}\right)}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx \\
&= -\frac{20}{91}x^2\sqrt{1+x^3} + \frac{2}{13}x^5\sqrt{1+x^3} + \frac{80\sqrt{1+x^3}}{91\left(1+\sqrt{3}+x\right)} - \frac{40\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,E\left(\text{sin}\right.}{91\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1-}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 42, normalized size = 0.16

$$\frac{2}{91}x^2\left(10\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^3\right)+\sqrt{x^3+1}\,\left(7x^3-10\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/Sqrt[1 + x^3],x]
```

```
[Out] (2*x^2*(Sqrt[1 + x^3]*(-10 + 7*x^3) + 10*Hypergeometric2F1[1/2, 2/3, 5/3, -
x^3]))/91
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^7}{\sqrt{x^3+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^7/sqrt(x^3 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^7/sqrt(x^3 + 1), x)
```

maple [A] time = 0.02, size = 198, normalized size = 0.76

$$\frac{2\sqrt{x^3+1}\,x^5}{13}-\frac{20\sqrt{x^3+1}\,x^2}{91}+\frac{80\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\text{EllipticE}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\right.\right.}{91\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^3+1)^(1/2),x)
```

```
[Out] 2/13*x^5*(x^3+1)^(1/2)-20/91*x^2*(x^3+1)^(1/2)+80/91*(3/2-1/2*I*3^(1/2))*((
x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2))
)^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*((
-3/2-1/2*I*3^(1/2))*EllipticE(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*
I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2+1/2*I*3^(1/2))*EllipticF(((x+1
)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1
/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^7/sqrt(x^3 + 1), x)
```

mupad [B] time = 0.99, size = 239, normalized size = 0.91

$$\frac{2\,x^5\,\sqrt{x^3+1}}{13}-\frac{20\,x^2\,\sqrt{x^3+1}}{91}-\frac{80\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{3}{2}}\right.\right.}{91\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^3 + 1)^(1/2),x)
```

```
[Out] (2*x^5*(x^3 + 1)^(1/2))/13 - (20*x^2*(x^3 + 1)^(1/2))/91 - (80*(((3^(1/2)*1
i)/2 - 1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1
/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1i)/2 - 3/2)*ellipticE
(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^
(1/2)*1i)/2 - 3/2)))*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3
^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2
)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(91*(x^3 - x*(((3^(1/2)*1
i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*
1i)/2 + 1/2))^(1/2))
```

sympy [A] time = 1.44, size = 29, normalized size = 0.11

$$\frac{x^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**3+1)**(1/2),x)
```

```
[Out] x**8*gamma(8/3)*hyper((1/2, 8/3), (11/3,), x**3*exp_polar(I*pi))/(3*gamma(1
1/3))
```

3.452 $\int \frac{x^4}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=246

$$-\frac{8\sqrt{x^3+1}}{7\left(x+\sqrt{3}+1\right)}+\frac{2}{7}\sqrt{x^3+1}\,x^2-\frac{8\sqrt{2}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{7\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(x+$$

```
[Out] 2/7*x^2*(x^3+1)^(1/2)-8/7*(x^3+1)^(1/2)/(1+x+3^(1/2))-8/21*(1+x)*EllipticF(
(1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*((x^2-x+1)/(1+x+3^(1/2))
^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)+4/7*3^(1/4)*(
1+x)*EllipticE((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*
2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2
))^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {321, 303, 218, 1877}

$$\frac{2}{7}\sqrt{x^3+1}\,x^2-\frac{8\sqrt{x^3+1}}{7\left(x+\sqrt{3}+1\right)}-\frac{8\sqrt{2}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{7\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(x+1$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[1 + x^3],x]
```

```
[Out] (2*x^2*Sqrt[1 + x^3])/7 - (8*Sqrt[1 + x^3])/(7*(1 + Sqrt[3] + x)) + (4*3^(1
/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ellip
ticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(7*Sqrt[
(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) - (8*Sqrt[2]*(1 + x)*Sqrt[(1 -
x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[
3] + x)], -7 - 4*Sqrt[3]])/(7*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqr
t[1 + x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{1+x^3}}\,dx = \frac{2}{7}x^2\sqrt{1+x^3} - \frac{4}{7}\int \frac{x}{\sqrt{1+x^3}}\,dx$$

$$= \frac{2}{7}x^2\sqrt{1+x^3} - \frac{4}{7}\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx - \frac{1}{7}\left(4\sqrt{2\left(2-\sqrt{3}\right)}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx$$

$$= \frac{2}{7}x^2\sqrt{1+x^3} - \frac{8\sqrt{1+x^3}}{7\left(1+\sqrt{3}+x\right)} + \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\right.}{7\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.14

$$\frac{2}{7}x^2\left(\sqrt{x^3+1} - {}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[1 + x^3],x]
[Out] (2*x^2*(Sqrt[1 + x^3] - Hypergeometric2F1[1/2, 2/3, 5/3, -x^3]))/7
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^4}{\sqrt{x^3+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] integral(x^4/sqrt(x^3 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(x^3 + 1), x)
```

maple [A] time = 0.02, size = 186, normalized size = 0.76

$$\frac{2\sqrt{x^3+1}\,x^2}{7}-\frac{8\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\text{EllipticE}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\frac{1}{2}\right.\right.}{7\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^3+1)^(1/2),x)
```

```
[Out] 2/7*(x^3+1)^(1/2)*x^2-8/7*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(
1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/
2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*((-3/2-1/2*I*3^(1/2))*Ellipti
cE(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1
/2)))^(1/2))+(1/2+1/2*I*3^(1/2))*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2
),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(x^3 + 1), x)
```

mupad [B] time = 1.00, size = 227, normalized size = 0.92

$$\frac{2x^2\sqrt{x^3+1}}{7}+\frac{8\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right.\right.}{7\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^3 + 1)^(1/2),x)
```

```
[Out] (2*x^2*(x^3 + 1)^(1/2))/7 + (8*(((3^(1/2)*1i)/2 - 1/2)*ellipticF(asin(((x +
 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2
 - 3/2)) - ((3^(1/2)*1i)/2 - 3/2)*ellipticE(asin(((x + 1)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*((3^(1/2)*1
i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x +
 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)
/2 + 3/2))^(1/2))/(7*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2
) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

sympy [A] time = 0.80, size = 29, normalized size = 0.12

$$\frac{x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**3+1)**(1/2),x)

[Out] x**5*gamma(5/3)*hyper((1/2, 5/3), (8/3,), x**3*exp_polar(I*pi))/(3*gamma(8/
3))
```

3.453 $\int \frac{x}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=224

$$\frac{2\sqrt{x^3+1}}{x+\sqrt{3}+1}+\frac{2\sqrt{2}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,E\left(\sin$$

$$\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3}$$

```
[Out] 2*(x^3+1)^(1/2)/(1+x+3^(1/2))+2/3*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2
))),I*3^(1/2)+2*I)*2^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)
^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)-3^(1/4)*(1+x)*EllipticE((1+x-3^(1/2))/
(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1
/2))^2)^(1/2)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {303, 218, 1877}

$$\frac{2\sqrt{x^3+1}}{x+\sqrt{3}+1}+\frac{2\sqrt{2}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,E\left(\sin$$

$$\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[1 + x^3],x]
```

```
[Out] (2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sq
rt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1
 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[
1 + x^3]) + (2*Sqrt[2]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Elli
pticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4
)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
```

```
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{1+x^3}}\,dx = \sqrt{2\left(2-\sqrt{3}\right)}\int \frac{1}{\sqrt{1+x^3}}\,dx + \int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx$$

$$= \frac{2\sqrt{1+x^3}}{1+\sqrt{3}+x} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{2\sqrt{2}\,(1+x)}{}$$

Mathematica [C] time = 0.00, size = 22, normalized size = 0.10

$$\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[1 + x^3],x]
[Out] (x^2*Hypergeometric2F1[1/2, 2/3, 5/3, -x^3])/2
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{\sqrt{x^3+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] integral(x/sqrt(x^3 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(x/sqrt(x^3 + 1), x)
```

maple [A] time = 0.02, size = 173, normalized size = 0.77

$$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\text{EllipticE}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\text{EllipticF}\right.}{\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^3+1)^(1/2),x)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*((-3/2-1/2*I*3^(1/2))*EllipticE(((x+1)/(3/2-1/2*I*3^(
1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2+1/2*I*
3^(1/2))*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/
(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(x^3 + 1), x)
```

mupad [B] time = 0.05, size = 214, normalized size = 0.96

$$-\frac{2\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right.\right.}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^3 + 1)^(1/2),x)
```

```
[Out] -(2*(((3^(1/2)*1i)/2 - 1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))
^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1i)/2
- 3/2)*ellipticE(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1
i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*
1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))
^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(x^3 - x*
(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

sympy [A] time = 1.51, size = 29, normalized size = 0.13

$$\frac{x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**3+1)**(1/2),x)
```

```
[Out] x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamma(5/
3))
```

3.454 $\int \frac{1}{x^2\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=238

$$-\frac{\sqrt{x^3+1}}{x}+\frac{\sqrt{x^3+1}}{x+\sqrt{3}+1}+\frac{\sqrt{2}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right.}}}{2\sqrt{\frac{x+}{\left(x+\sqrt{3}\right.}}}$$

```
[Out] -(x^3+1)^(1/2)/x+(x^3+1)^(1/2)/(1+x+3^(1/2))+1/3*(1+x)*EllipticF((1+x-3^(1/
2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*
3^(3/4)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)-1/2*3^(1/4)*(1+x)*Ellip
ticE((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*(
(x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2
)
```

Rubi [A] time = 0.05, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {325, 303, 218, 1877}

$$-\frac{\sqrt{x^3+1}}{x}+\frac{\sqrt{x^3+1}}{x+\sqrt{3}+1}+\frac{\sqrt{2}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right.}}}{2\sqrt{\frac{x+}{\left(x+\sqrt{3}\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[1 + x^3]),x]
```

```
[Out] -(Sqrt[1 + x^3]/x) + Sqrt[1 + x^3]/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sq
rt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1
 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(2*Sqrt[(1 + x)/(1 + Sq
rt[3] + x)^2]*Sqrt[1 + x^3]) + (Sqrt[2]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqr
t[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*
Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
 + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
 + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{1+x^3}}\,dx &= -\frac{\sqrt{1+x^3}}{x} + \frac{1}{2}\int \frac{x}{\sqrt{1+x^3}}\,dx \\ &= -\frac{\sqrt{1+x^3}}{x} + \frac{1}{2}\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx + \sqrt{\frac{1}{2}\left(2-\sqrt{3}\right)}\int \frac{1}{\sqrt{1+x^3}}\,dx \\ &= -\frac{\sqrt{1+x^3}}{x} + \frac{\sqrt{1+x^3}}{1+\sqrt{3}+x} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{}\right.}{2\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{1+x^3}}\end{aligned}$$

Mathematica [C] time = 0.00, size = 20, normalized size = 0.08

$$-\frac{{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};-x^3\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[1 + x^3]),x]
```

```
[Out] -(Hypergeometric2F1[-1/3, 1/2, 2/3, -x^3]/x)
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^3+1}}{x^5+x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^3 + 1)/(x^5 + x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^3+1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^3 + 1)*x^2), x)
```

maple [A] time = 0.02, size = 185, normalized size = 0.78

$$-\frac{\sqrt{x^3+1}}{x}+\frac{\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\text{EllipticE}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\right.}{\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^3+1)^(1/2),x)
```

```
[Out] -(x^3+1)^(1/2)/x+(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-
1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2
+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*((-3/2-1/2*I*3^(1/2))*EllipticE(((x+1)
/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/
2))+(1/2+1/2*I*3^(1/2))*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+
1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^3+1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^3 + 1)*x^2), x)
```

mupad [B] time = 0.07, size = 227, normalized size = 0.95

$$-\frac{\sqrt{x^3+1}}{x}-\frac{\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)\left(\frac{3}{2}+\right.}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^3 + 1)^(1/2)),x)
```

```
[Out] - (x^3 + 1)^(1/2)/x - ((((3^(1/2)*1i)/2 - 1/2)*ellipticF(asin(((x + 1)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))
 - ((3^(1/2)*1i)/2 - 3/2)*ellipticE(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(
1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*((3^(1/2)*1i)/2 + 3
/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2
))^(1/2))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((
3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

sympy [A] time = 1.02, size = 31, normalized size = 0.13

$$\frac{\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},\frac{1}{2}\\ \frac{2}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{3x\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**3+1)**(1/2),x)

[Out] gamma(-1/3)*hyper((-1/3, 1/2), (2/3,), x**3*exp_polar(I*pi))/(3*x*gamma(2/3
))
```

3.455 $\int \frac{1}{x^5\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=262

$$\frac{5\sqrt{x^3+1}}{8x}-\frac{5\sqrt{x^3+1}}{8\left(x+\sqrt{3}+1\right)}-\frac{\sqrt{x^3+1}}{4x^4}-\frac{5(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\,\sqrt[4]{3}\,\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^3+1}}+\frac{5\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,(x+$$

```
[Out] -1/4*(x^3+1)^(1/2)/x^4+5/8*(x^3+1)^(1/2)/x-5/8*(x^3+1)^(1/2)/(1+x+3^(1/2))-
5/24*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*((x
^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2
)^(1/2)+5/16*3^(1/4)*(1+x)*EllipticE((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+
2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1
/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {325, 303, 218, 1877}

$$\frac{5\sqrt{x^3+1}}{8x}-\frac{5\sqrt{x^3+1}}{8\left(x+\sqrt{3}+1\right)}-\frac{\sqrt{x^3+1}}{4x^4}-\frac{5(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\,\sqrt[4]{3}\,\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^3+1}}+\frac{5\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,(x+$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[1 + x^3]),x]
```

```
[Out] -Sqrt[1 + x^3]/(4*x^4) + (5*Sqrt[1 + x^3])/(8*x) - (5*Sqrt[1 + x^3])/(8*(1
+ Sqrt[3] + x)) + (5*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(
1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)],
-7 - 4*Sqrt[3]])/(16*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) - (5*
(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[
3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(4*Sqrt[2]*3^(1/4)*Sqrt[(1 + x
)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\sqrt{1+x^3}}\,dx &= -\frac{\sqrt{1+x^3}}{4x^4} - \frac{5}{8}\int \frac{1}{x^2\sqrt{1+x^3}}\,dx \\
&= -\frac{\sqrt{1+x^3}}{4x^4} + \frac{5\sqrt{1+x^3}}{8x} - \frac{5}{16}\int \frac{x}{\sqrt{1+x^3}}\,dx \\
&= -\frac{\sqrt{1+x^3}}{4x^4} + \frac{5\sqrt{1+x^3}}{8x} - \frac{5}{16}\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx - \frac{1}{8}\left(5\sqrt{\frac{1}{2}\left(2-\sqrt{3}\right)}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx \\
&= -\frac{\sqrt{1+x^3}}{4x^4} + \frac{5\sqrt{1+x^3}}{8x} - \frac{5\sqrt{1+x^3}}{8\left(1+\sqrt{3}+x\right)} + \frac{5\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1}{1}\right.\right.}{16\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{1+x^3}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 22, normalized size = 0.08

$$-\frac{{}_2F_1\left(-\frac{4}{3},\frac{1}{2};-\frac{1}{3};-x^3\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[1 + x^3]),x]
```

```
[Out] -1/4*Hypergeometric2F1[-4/3, 1/2, -1/3, -x^3]/x^4
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^3+1}}{x^8+x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^3 + 1)/(x^8 + x^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^3+1}\,x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^3 + 1)*x^5), x)
```

maple [A] time = 0.02, size = 198, normalized size = 0.76

$$\frac{5\sqrt{x^3+1}}{8x}-\frac{\sqrt{x^3+1}}{4x^4}-\frac{5\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\text{EllipticE}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)\right.}{8\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^3+1)^(1/2),x)
```

```
[Out] -1/4*(x^3+1)^(1/2)/x^4+5/8*(x^3+1)^(1/2)/x-5/8*(3/2-1/2*I*3^(1/2))*((x+1)/(
3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2
)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*((-3/2-1
/2*I*3^(1/2))*EllipticE(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+(1/2+1/2*I*3^(1/2))*EllipticF(((x+1)/(3/2
-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^3+1}\,x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^3 + 1)*x^5), x)
```

mupad [B] time = 1.01, size = 239, normalized size = 0.91

$$\frac{5\sqrt{x^3+1}}{8x}-\frac{\sqrt{x^3+1}}{4x^4}+\frac{5\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\middle|-\frac{\frac{3}{2}+}{-\frac{3}{2}}\right.\right.}{8\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(x^3 + 1)^(1/2)),x)
```

```
[Out] (5*(x^3 + 1)^(1/2))/(8*x) - (x^3 + 1)^(1/2)/(4*x^4) + (5*(((3^(1/2)*1i)/2 -
 1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i
)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1i)/2 - 3/2)*ellipticE(asin(
((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*
1i)/2 - 3/2)))*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)
*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/
2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(8*(x^3 - x*(((3^(1/2)*1i)/2 -
1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
 1/2))^(1/2))
```

sympy [A] time = 1.42, size = 36, normalized size = 0.14

$$\frac{\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{4}{3},\frac{1}{2}\\-\frac{1}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**3+1)**(1/2),x)
```

```
[Out] gamma(-4/3)*hyper((-4/3, 1/2), (-1/3,), x**3*exp_polar(I*pi))/(3*x**4*gamma
(-1/3))
```

3.456 $\int \frac{x^{11}}{\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=61

$$\frac{2}{21}\left(1-x^3\right)^{7/2}-\frac{2}{5}\left(1-x^3\right)^{5/2}+\frac{2}{3}\left(1-x^3\right)^{3/2}-\frac{2\sqrt{1-x^3}}{3}$$

```
[Out] 2/3*(-x^3+1)^(3/2)-2/5*(-x^3+1)^(5/2)+2/21*(-x^3+1)^(7/2)-2/3*(-x^3+1)^(1/2
)
```

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{2}{21}\left(1-x^3\right)^{7/2}-\frac{2}{5}\left(1-x^3\right)^{5/2}+\frac{2}{3}\left(1-x^3\right)^{3/2}-\frac{2\sqrt{1-x^3}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/Sqrt[1 - x^3],x]
```

```
[Out] (-2*Sqrt[1 - x^3])/3 + (2*(1 - x^3)^(3/2))/3 - (2*(1 - x^3)^(5/2))/5 + (2*(
1 - x^3)^(7/2))/21
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{\sqrt{1-x^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^3}{\sqrt{1-x}}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{1}{\sqrt{1-x}}-3\sqrt{1-x}+3(1-x)^{3/2}-(1-x)^{5/2}\right)dx, x, x^3\right)\\ &= -\frac{2}{3}\sqrt{1-x^3}+\frac{2}{3}\left(1-x^3\right)^{3/2}-\frac{2}{5}\left(1-x^3\right)^{5/2}+\frac{2}{21}\left(1-x^3\right)^{7/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 0.52

$$-\frac{2}{105}\sqrt{1-x^3}\left(5x^9+6x^6+8x^3+16\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/Sqrt[1 - x^3],x]
```

```
[Out] (-2*Sqrt[1 - x^3]*(16 + 8*x^3 + 6*x^6 + 5*x^9))/105
```

fricas [A] time = 0.88, size = 28, normalized size = 0.46

$$-\frac{2}{105}\left(5x^9+6x^6+8x^3+16\right)\sqrt{-x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/105*(5*x^9 + 6*x^6 + 8*x^3 + 16)*sqrt(-x^3 + 1)
```

giac [A] time = 0.18, size = 59, normalized size = 0.97

$$-\frac{2}{21}\left(x^3-1\right)^3\sqrt{-x^3+1}-\frac{2}{5}\left(x^3-1\right)^2\sqrt{-x^3+1}+\frac{2}{3}\left(-x^3+1\right)^{\frac{3}{2}}-\frac{2}{3}\sqrt{-x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] -2/21*(x^3 - 1)^3*sqrt(-x^3 + 1) - 2/5*(x^3 - 1)^2*sqrt(-x^3 + 1) + 2/3*(-x
^3 + 1)^(3/2) - 2/3*sqrt(-x^3 + 1)
```

maple [A] time = 0.01, size = 38, normalized size = 0.62

$$\frac{2(x-1)\left(x^2+x+1\right)\left(5x^9+6x^6+8x^3+16\right)}{105\sqrt{-x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(-x^3+1)^(1/2),x)
```

```
[Out] 2/105*(x-1)*(x^2+x+1)*(5*x^9+6*x^6+8*x^3+16)/(-x^3+1)^(1/2)
```

maxima [A] time = 1.33, size = 45, normalized size = 0.74

$$\frac{2}{21}\left(-x^3+1\right)^{\frac{7}{2}}-\frac{2}{5}\left(-x^3+1\right)^{\frac{5}{2}}+\frac{2}{3}\left(-x^3+1\right)^{\frac{3}{2}}-\frac{2}{3}\sqrt{-x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/21*(-x^3 + 1)^(7/2) - 2/5*(-x^3 + 1)^(5/2) + 2/3*(-x^3 + 1)^(3/2) - 2/3*s
qrt(-x^3 + 1)
```

mupad [B] time = 0.04, size = 54, normalized size = 0.89

$$-\frac{16x^3\sqrt{1-x^3}}{105}-\frac{4x^6\sqrt{1-x^3}}{35}-\frac{2x^9\sqrt{1-x^3}}{21}-\frac{32\sqrt{1-x^3}}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(1 - x^3)^(1/2),x)
```

```
[Out] - (16*x^3*(1 - x^3)^(1/2))/105 - (4*x^6*(1 - x^3)^(1/2))/35 - (2*x^9*(1 - x
^3)^(1/2))/21 - (32*(1 - x^3)^(1/2))/105
```

sympy [A] time = 1.91, size = 58, normalized size = 0.95

$$-\frac{2x^9\sqrt{1-x^3}}{21}-\frac{4x^6\sqrt{1-x^3}}{35}-\frac{16x^3\sqrt{1-x^3}}{105}-\frac{32\sqrt{1-x^3}}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(-x**3+1)**(1/2),x)
```

```
[Out] -2*x**9*sqrt(1 - x**3)/21 - 4*x**6*sqrt(1 - x**3)/35 - 16*x**3*sqrt(1 - x**
3)/105 - 32*sqrt(1 - x**3)/105
```

3.457 $\int \frac{x^8}{\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=46

$$-\frac{2}{15}\left(1-x^3\right)^{5/2}+\frac{4}{9}\left(1-x^3\right)^{3/2}-\frac{2\sqrt{1-x^3}}{3}$$

```
[Out] 4/9*(-x^3+1)^(3/2)-2/15*(-x^3+1)^(5/2)-2/3*(-x^3+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{2}{15}\left(1-x^3\right)^{5/2}+\frac{4}{9}\left(1-x^3\right)^{3/2}-\frac{2\sqrt{1-x^3}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/Sqrt[1 - x^3],x]
```

```
[Out] (-2*Sqrt[1 - x^3])/3 + (4*(1 - x^3)^(3/2))/9 - (2*(1 - x^3)^(5/2))/15
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{\sqrt{1-x^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1-x}}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{1}{\sqrt{1-x}}-2\sqrt{1-x}+(1-x)^{3/2}\right)dx, x, x^3\right)\\ &= -\frac{2}{3}\sqrt{1-x^3}+\frac{4}{9}\left(1-x^3\right)^{3/2}-\frac{2}{15}\left(1-x^3\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.59

$$-\frac{2}{45}\sqrt{1-x^3}\left(3x^6+4x^3+8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/Sqrt[1 - x^3],x]
```

```
[Out] (-2*Sqrt[1 - x^3]*(8 + 4*x^3 + 3*x^6))/45
```

fricas [A] time = 0.54, size = 23, normalized size = 0.50

$$-\frac{2}{45}\left(3\,x^6+4\,x^3+8\right)\sqrt{-x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/45*(3*x^6 + 4*x^3 + 8)*sqrt(-x^3 + 1)
```

giac [A] time = 0.15, size = 41, normalized size = 0.89

$$-\frac{2}{15}\left(x^3-1\right)^2\sqrt{-x^3+1}+\frac{4}{9}\left(-x^3+1\right)^{\frac{3}{2}}-\frac{2}{3}\sqrt{-x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] -2/15*(x^3 - 1)^2*sqrt(-x^3 + 1) + 4/9*(-x^3 + 1)^(3/2) - 2/3*sqrt(-x^3 + 1
)
```

maple [A] time = 0.01, size = 33, normalized size = 0.72

$$\frac{2\left(x-1\right)\left(x^2+x+1\right)\left(3x^6+4x^3+8\right)}{45\sqrt{-x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-x^3+1)^(1/2),x)
```

```
[Out] 2/45*(x-1)*(x^2+x+1)*(3*x^6+4*x^3+8)/(-x^3+1)^(1/2)
```

maxima [A] time = 1.32, size = 34, normalized size = 0.74

$$-\frac{2}{15}\left(-x^3+1\right)^{\frac{5}{2}}+\frac{4}{9}\left(-x^3+1\right)^{\frac{3}{2}}-\frac{2}{3}\sqrt{-x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/15*(-x^3 + 1)^(5/2) + 4/9*(-x^3 + 1)^(3/2) - 2/3*sqrt(-x^3 + 1)
```

mupad [B] time = 0.03, size = 23, normalized size = 0.50

$$-\frac{2\sqrt{1-x^3}\,\left(3\,x^6+4\,x^3+8\right)}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(1 - x^3)^(1/2),x)
```

```
[Out] -(2*(1 - x^3)^(1/2)*(4*x^3 + 3*x^6 + 8))/45
```

sympy [A] time = 0.78, size = 42, normalized size = 0.91

$$-\frac{2x^6\sqrt{1-x^3}}{15}-\frac{8x^3\sqrt{1-x^3}}{45}-\frac{16\sqrt{1-x^3}}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-x**3+1)**(1/2),x)
```

```
[Out] -2*x**6*sqrt(1 - x**3)/15 - 8*x**3*sqrt(1 - x**3)/45 - 16*sqrt(1 - x**3)/45
```

3.458 $\int \frac{x^5}{\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=31

$$\frac{2}{9}\left(1-x^3\right)^{3/2}-\frac{2\sqrt{1-x^3}}{3}$$

```
[Out] 2/9*(-x^3+1)^(3/2)-2/3*(-x^3+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{2}{9}\left(1-x^3\right)^{3/2}-\frac{2\sqrt{1-x^3}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[1 - x^3],x]
```

```
[Out] (-2*Sqrt[1 - x^3])/3 + (2*(1 - x^3)^(3/2))/9
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\sqrt{1-x^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{\sqrt{1-x}}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{1}{\sqrt{1-x}}-\sqrt{1-x}\right)dx, x, x^3\right)\\ &= -\frac{2}{3}\sqrt{1-x^3}+\frac{2}{9}\left(1-x^3\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.65

$$-\frac{2}{9}\sqrt{1-x^3}\left(x^3+2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[1 - x^3],x]
```

```
[Out] (-2*Sqrt[1 - x^3]*(2 + x^3))/9
```

fricas [A] time = 0.84, size = 16, normalized size = 0.52

$$-\frac{2}{9}\left(x^3+2\right)\sqrt{-x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+1)^(1/2),x, algorithm="fricas")
[Out] -2/9*(x^3 + 2)*sqrt(-x^3 + 1)
```

giac [A] time = 0.21, size = 23, normalized size = 0.74

$$\frac{2}{9}\left(-x^3+1\right)^{\frac{3}{2}}-\frac{2}{3}\sqrt{-x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+1)^(1/2),x, algorithm="giac")
[Out] 2/9*(-x^3 + 1)^(3/2) - 2/3*sqrt(-x^3 + 1)
```

maple [A] time = 0.00, size = 26, normalized size = 0.84

$$\frac{2\left(x-1\right)\left(x^2+x+1\right)\left(x^3+2\right)}{9\sqrt{-x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-x^3+1)^(1/2),x)
[Out] 2/9*(x-1)*(x^2+x+1)*(x^3+2)/(-x^3+1)^(1/2)
```

maxima [A] time = 1.29, size = 23, normalized size = 0.74

$$\frac{2}{9}\left(-x^3+1\right)^{\frac{3}{2}}-\frac{2}{3}\sqrt{-x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+1)^(1/2),x, algorithm="maxima")
[Out] 2/9*(-x^3 + 1)^(3/2) - 2/3*sqrt(-x^3 + 1)
```

mupad [B] time = 0.03, size = 16, normalized size = 0.52

$$-\frac{2\sqrt{1-x^3}\,\left(x^3+2\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(1 - x^3)^(1/2),x)
[Out] -(2*(1 - x^3)^(1/2)*(x^3 + 2))/9
```

sympy [A] time = 0.69, size = 27, normalized size = 0.87

$$-\frac{2x^3\sqrt{1-x^3}}{9}-\frac{4\sqrt{1-x^3}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-x**3+1)**(1/2),x)
[Out] -2*x**3*sqrt(1 - x**3)/9 - 4*sqrt(1 - x**3)/9
```

3.459 $\int \frac{x^2}{\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=15

$$-\frac{2}{3}\sqrt{1-x^3}$$

```
[Out] -2/3*(-x^3+1)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$-\frac{2}{3}\sqrt{1-x^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[1 - x^3],x]
```

```
[Out] (-2*Sqrt[1 - x^3])/3
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{1-x^3}}\,dx = -\frac{2}{3}\sqrt{1-x^3}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$-\frac{2}{3}\sqrt{1-x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[1 - x^3],x]
```

```
[Out] (-2*Sqrt[1 - x^3])/3
```

fricas [A] time = 0.86, size = 11, normalized size = 0.73

$$-\frac{2}{3}\sqrt{-x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(-x^3 + 1)
```

giac [A] time = 0.15, size = 11, normalized size = 0.73

$$-\frac{2}{3}\sqrt{-x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(1/2),x, algorithm="giac")
[Out] -2/3*sqrt(-x^3 + 1)
```

maple [A] time = 0.01, size = 21, normalized size = 1.40

$$\frac{2\,(x-1)\left(x^2+x+1\right)}{3\sqrt{-x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^3+1)^(1/2),x)
[Out] 2/3*(x-1)*(x^2+x+1)/(-x^3+1)^(1/2)
```

maxima [A] time = 1.34, size = 11, normalized size = 0.73

$$-\frac{2}{3}\sqrt{-x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(1/2),x, algorithm="maxima")
[Out] -2/3*sqrt(-x^3 + 1)
```

mupad [B] time = 0.02, size = 11, normalized size = 0.73

$$-\frac{2\sqrt{1-x^3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1 - x^3)^(1/2),x)
[Out] -(2*(1 - x^3)^(1/2))/3
```

sympy [A] time = 0.34, size = 12, normalized size = 0.80

$$-\frac{2\sqrt{1-x^3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**3+1)**(1/2),x)
[Out] -2*sqrt(1 - x**3)/3
```

3.460 $\int \frac{1}{x\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=16

$$-\frac{2}{3}\tanh^{-1}\left(\sqrt{1-x^3}\right)$$

```
[Out] -2/3*arctanh((-x^3+1)^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {266, 63, 206}

$$-\frac{2}{3}\tanh^{-1}\left(\sqrt{1-x^3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[1 - x^3]])/3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{1-x^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, x^3\right)\\ &= -\left(\frac{2}{3}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-x^3}\right)\right)\\ &= -\frac{2}{3}\tanh^{-1}\left(\sqrt{1-x^3}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{2}{3}\tanh^{-1}\left(\sqrt{1-x^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[1 - x^3]),x]
[Out] (-2*ArcTanh[Sqrt[1 - x^3]])/3
```

fricas [B] time = 0.86, size = 29, normalized size = 1.81

$$-\frac{1}{3}\log\left(\sqrt{-x^3+1}+1\right)+\frac{1}{3}\log\left(\sqrt{-x^3+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(1/2),x, algorithm="fricas")
[Out] -1/3*log(sqrt(-x^3 + 1) + 1) + 1/3*log(sqrt(-x^3 + 1) - 1)
```

giac [B] time = 0.15, size = 30, normalized size = 1.88

$$-\frac{1}{3}\log\left(\sqrt{-x^3+1}+1\right)+\frac{1}{3}\log\left(\left|\sqrt{-x^3+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(1/2),x, algorithm="giac")
[Out] -1/3*log(sqrt(-x^3 + 1) + 1) + 1/3*log(abs(sqrt(-x^3 + 1) - 1))
```

maple [A] time = 0.06, size = 13, normalized size = 0.81

$$-\frac{2\operatorname{arctanh}\left(\sqrt{-x^3+1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^3+1)^(1/2),x)
[Out] -2/3*arctanh((-x^3+1)^(1/2))
```

maxima [B] time = 1.30, size = 29, normalized size = 1.81

$$-\frac{1}{3}\log\left(\sqrt{-x^3+1}+1\right)+\frac{1}{3}\log\left(\sqrt{-x^3+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(1/2),x, algorithm="maxima")
[Out] -1/3*log(sqrt(-x^3 + 1) + 1) + 1/3*log(sqrt(-x^3 + 1) - 1)
```

mupad [B] time = 0.19, size = 180, normalized size = 11.25

$$-\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\operatorname{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1 - x^3)^(1/2)),x)
[Out] -(2*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3
^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/
2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2
 + 3/2, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3
```

```
/2)/((3^(1/2)*1i)/2 - 3/2)))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(
1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) +
 x^3)^(1/2))
```

sympy [**A**] time = 1.95, size = 31, normalized size = 1.94

$$\begin{cases} -\frac{2\,\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \frac{1}{|x^3|} > 1 \\ \frac{2i\,\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**3+1)**(1/2),x)
```

```
[Out] Piecewise((-2*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (2*I*asin(x**(-3/2))/3,
 True))
```

3.461 $\int \frac{1}{x^4\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=35

$$-\frac{\sqrt{1-x^3}}{3x^3}-\frac{1}{3}\tanh^{-1}\left(\sqrt{1-x^3}\right)$$

```
[Out] -1/3*arctanh((-x^3+1)^(1/2))-1/3*(-x^3+1)^(1/2)/x^3
```

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 51, 63, 206}

$$-\frac{\sqrt{1-x^3}}{3x^3}-\frac{1}{3}\tanh^{-1}\left(\sqrt{1-x^3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[1 - x^3]),x]
```

```
[Out] -Sqrt[1 - x^3]/(3*x^3) - ArcTanh[Sqrt[1 - x^3]]/3
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{1-x^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x^2}\,dx, x, x^3\right)\\ &= -\frac{\sqrt{1-x^3}}{3x^3} + \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, x^3\right)\\ &= -\frac{\sqrt{1-x^3}}{3x^3} - \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-x^3}\right)\\ &= -\frac{\sqrt{1-x^3}}{3x^3} - \frac{1}{3}\tanh^{-1}\left(\sqrt{1-x^3}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.00

$$-\frac{\sqrt{1-x^3}}{3x^3} - \frac{1}{3}\tanh^{-1}\left(\sqrt{1-x^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[1 - x^3]),x]
```

```
[Out] -1/3*Sqrt[1 - x^3]/x^3 - ArcTanh[Sqrt[1 - x^3]]/3
```

fricas [A] time = 0.88, size = 50, normalized size = 1.43

$$-\frac{x^3\log\left(\sqrt{-x^3+1}+1\right) - x^3\log\left(\sqrt{-x^3+1}-1\right) + 2\sqrt{-x^3+1}}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(x^3*log(sqrt(-x^3 + 1) + 1) - x^3*log(sqrt(-x^3 + 1) - 1) + 2*sqrt(-x
^3 + 1))/x^3
```

giac [A] time = 0.16, size = 44, normalized size = 1.26

$$-\frac{\sqrt{-x^3+1}}{3\,x^3} - \frac{1}{6}\log\left(\sqrt{-x^3+1}+1\right) + \frac{1}{6}\log\left(\left|\sqrt{-x^3+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(-x^3 + 1)/x^3 - 1/6*log(sqrt(-x^3 + 1) + 1) + 1/6*log(abs(sqrt(-x
^3 + 1) - 1))
```

maple [A] time = 0.03, size = 28, normalized size = 0.80

$$-\frac{\operatorname{arctanh}\left(\sqrt{-x^3+1}\right)}{3} - \frac{\sqrt{-x^3+1}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-x^3+1)^(1/2),x)
```

```
[Out] -1/3*arctanh((-x^3+1)^(1/2))-1/3*(-x^3+1)^(1/2)/x^3
```

maxima [A] time = 1.34, size = 43, normalized size = 1.23

$$-\frac{\sqrt{-x^3+1}}{3x^3}-\frac{1}{6}\log\left(\sqrt{-x^3+1}+1\right)+\frac{1}{6}\log\left(\sqrt{-x^3+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-x^3 + 1)/x^3 - 1/6*log(sqrt(-x^3 + 1) + 1) + 1/6*log(sqrt(-x^3 +
 1) - 1)
```

mupad [B] time = 0.07, size = 195, normalized size = 5.57

$$-\frac{\sqrt{1-x^3}}{3x^3}-\frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\operatorname{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+}{-\frac{3}{2}+}\right.}{\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(1 - x^3)^(1/2)),x)
```

```
[Out] - (1 - x^3)^(1/2)/(3*x^3) - (((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x -
(3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 +
1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*
ellipticPi((3^(1/2)*1i)/2 + 3/2, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/
2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((1 - x^3)^(1/2)*(((3
^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^
(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

sympy [A] time = 3.79, size = 82, normalized size = 2.34

$$\begin{cases} -\frac{\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}-\frac{\sqrt{-1+\frac{1}{x^3}}}{3x^{\frac{3}{2}}} & \text{for } \frac{1}{|x^3|}>1 \\ \frac{i\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}-\frac{i}{3x^{\frac{3}{2}}\sqrt{1-\frac{1}{x^3}}}+\frac{i}{3x^{\frac{9}{2}}\sqrt{1-\frac{1}{x^3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-x**3+1)**(1/2),x)
```

```
[Out] Piecewise((-acosh(x**(-3/2))/3 - sqrt(-1 + x**(-3))/(3*x**(3/2)), 1/Abs(x**
3) > 1), (I*asin(x**(-3/2))/3 - I/(3*x**(3/2)*sqrt(1 - 1/x**3)) + I/(3*x**(
9/2)*sqrt(1 - 1/x**3)), True))
```

3.462 $\int \frac{1}{x^7\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=53

$$-\frac{\sqrt{1-x^3}}{4x^3}-\frac{1}{4}\tanh^{-1}\left(\sqrt{1-x^3}\right)-\frac{\sqrt{1-x^3}}{6x^6}$$

```
[Out] -1/4*arctanh((-x^3+1)^(1/2))-1/6*(-x^3+1)^(1/2)/x^6-1/4*(-x^3+1)^(1/2)/x^3
```

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 51, 63, 206}

$$-\frac{\sqrt{1-x^3}}{4x^3}-\frac{\sqrt{1-x^3}}{6x^6}-\frac{1}{4}\tanh^{-1}\left(\sqrt{1-x^3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*Sqrt[1 - x^3]),x]
```

```
[Out] -Sqrt[1 - x^3]/(6*x^6) - Sqrt[1 - x^3]/(4*x^3) - ArcTanh[Sqrt[1 - x^3]]/4
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7\sqrt{1-x^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x^3}\,dx, x, x^3\right)\\ &= -\frac{\sqrt{1-x^3}}{6x^6} + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x^2}\,dx, x, x^3\right)\\ &= -\frac{\sqrt{1-x^3}}{6x^6} - \frac{\sqrt{1-x^3}}{4x^3} + \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, x^3\right)\\ &= -\frac{\sqrt{1-x^3}}{6x^6} - \frac{\sqrt{1-x^3}}{4x^3} - \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-x^3}\right)\\ &= -\frac{\sqrt{1-x^3}}{6x^6} - \frac{\sqrt{1-x^3}}{4x^3} - \frac{1}{4}\tanh^{-1}\left(\sqrt{1-x^3}\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 30, normalized size = 0.57

$$-\frac{2}{3}\sqrt{1-x^3}\;{}_2F_1\left(\frac{1}{2}, 3; \frac{3}{2}; 1-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*Sqrt[1 - x^3]*Hypergeometric2F1[1/2, 3, 3/2, 1 - x^3])/3
```

fricas [A] time = 0.54, size = 58, normalized size = 1.09

$$-\frac{3\,x^6\log\left(\sqrt{-x^3+1}+1\right) - 3\,x^6\log\left(\sqrt{-x^3+1}-1\right) + 2\left(3\,x^3+2\right)\sqrt{-x^3+1}}{24\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(3*x^6*log(sqrt(-x^3 + 1) + 1) - 3*x^6*log(sqrt(-x^3 + 1) - 1) + 2*(3
*x^3 + 2)*sqrt(-x^3 + 1))/x^6
```

giac [A] time = 0.15, size = 58, normalized size = 1.09

$$\frac{3\left(-x^3+1\right)^{\frac{3}{2}} - 5\sqrt{-x^3+1}}{12\,x^6} - \frac{1}{8}\log\left(\sqrt{-x^3+1}+1\right) + \frac{1}{8}\log\left(\left|\sqrt{-x^3+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*(3*(-x^3 + 1)^(3/2) - 5*sqrt(-x^3 + 1))/x^6 - 1/8*log(sqrt(-x^3 + 1) +
 1) + 1/8*log(abs(sqrt(-x^3 + 1) - 1))
```

maple [A] time = 0.03, size = 42, normalized size = 0.79

$$-\frac{\operatorname{arctanh}\left(\sqrt{-x^3+1}\right)}{4} - \frac{\sqrt{-x^3+1}}{4x^3} - \frac{\sqrt{-x^3+1}}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(-x^3+1)^(1/2),x)
```

```
[Out] -1/4*arctanh((-x^3+1)^(1/2))-1/6*(-x^3+1)^(1/2)/x^6-1/4*(-x^3+1)^(1/2)/x^3
```

maxima [A] time = 1.34, size = 70, normalized size = 1.32

$$\frac{3\left(-x^3+1\right)^{\frac{3}{2}}-5\sqrt{-x^3+1}}{12\left(2x^3+\left(x^3-1\right)^2-1\right)}-\frac{1}{8}\log\left(\sqrt{-x^3+1}+1\right)+\frac{1}{8}\log\left(\sqrt{-x^3+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(3*(-x^3 + 1)^(3/2) - 5*sqrt(-x^3 + 1))/(2*x^3 + (x^3 - 1)^2 - 1) - 1/
8*log(sqrt(-x^3 + 1) + 1) + 1/8*log(sqrt(-x^3 + 1) - 1)
```

mupad [B] time = 0.04, size = 209, normalized size = 3.94

$$-\frac{\sqrt{1-x^3}}{4x^3}-\frac{\sqrt{1-x^3}}{6x^6}-\frac{3\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}}{2}}}\right.\right.}{4\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1}{2}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(1 - x^3)^(1/2)),x)
```

```
[Out] - (1 - x^3)^(1/2)/(4*x^3) - (1 - x^3)^(1/2)/(6*x^6) - (3*((3^(1/2)*1i)/2 +
3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(
1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((
3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin((-(x - 1)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 -
3/2)))/(4*(1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) -
x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

sympy [A] time = 5.11, size = 138, normalized size = 2.60

$$\begin{cases} -\frac{\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{4}+\frac{1}{4x^{\frac{3}{2}}\sqrt{-1+\frac{1}{x^3}}}-\frac{1}{12x^{\frac{9}{2}}\sqrt{-1+\frac{1}{x^3}}}-\frac{1}{6x^{\frac{15}{2}}\sqrt{-1+\frac{1}{x^3}}} & \text{for } \frac{1}{|x^3|}>1 \\ \frac{i\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{4}-\frac{i}{4x^{\frac{3}{2}}\sqrt{1-\frac{1}{x^3}}}+\frac{i}{12x^{\frac{9}{2}}\sqrt{1-\frac{1}{x^3}}}+\frac{i}{6x^{\frac{15}{2}}\sqrt{1-\frac{1}{x^3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(-x**3+1)**(1/2),x)
```

```
[Out] Piecewise((-acosh(x**(-3/2))/4 + 1/(4*x**(3/2)*sqrt(-1 + x**(-3))) - 1/(12*
x**(9/2)*sqrt(-1 + x**(-3))) - 1/(6*x**(15/2)*sqrt(-1 + x**(-3))), 1/Abs(x*
*3) > 1), (I*asin(x**(-3/2))/4 - I/(4*x**(3/2)*sqrt(1 - 1/x**3)) + I/(12*x*
*(9/2)*sqrt(1 - 1/x**3)) + I/(6*x**(15/2)*sqrt(1 - 1/x**3)), True))
```

3.463 $\int \frac{1}{x^{10}\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=71

$$-\frac{5\sqrt{1-x^3}}{24x^3}-\frac{5}{24}\tanh^{-1}\left(\sqrt{1-x^3}\right)-\frac{\sqrt{1-x^3}}{9x^9}-\frac{5\sqrt{1-x^3}}{36x^6}$$

```
[Out] -5/24*arctanh((-x^3+1)^(1/2))-1/9*(-x^3+1)^(1/2)/x^9-5/36*(-x^3+1)^(1/2)/x^
6-5/24*(-x^3+1)^(1/2)/x^3
```

Rubi [A] time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 51, 63, 206}

$$-\frac{5\sqrt{1-x^3}}{24x^3}-\frac{5\sqrt{1-x^3}}{36x^6}-\frac{\sqrt{1-x^3}}{9x^9}-\frac{5}{24}\tanh^{-1}\left(\sqrt{1-x^3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*Sqrt[1 - x^3]),x]
```

```
[Out] -Sqrt[1 - x^3]/(9*x^9) - (5*Sqrt[1 - x^3])/(36*x^6) - (5*Sqrt[1 - x^3])/(24
*x^3) - (5*ArcTanh[Sqrt[1 - x^3]])/24
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{10}\sqrt{1-x^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x^4}\,dx, x, x^3\right)\\ &= -\frac{\sqrt{1-x^3}}{9x^9} + \frac{5}{18}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x^3}\,dx, x, x^3\right)\\ &= -\frac{\sqrt{1-x^3}}{9x^9} - \frac{5\sqrt{1-x^3}}{36x^6} + \frac{5}{24}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x^2}\,dx, x, x^3\right)\\ &= -\frac{\sqrt{1-x^3}}{9x^9} - \frac{5\sqrt{1-x^3}}{36x^6} - \frac{5\sqrt{1-x^3}}{24x^3} + \frac{5}{48}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, x^3\right)\\ &= -\frac{\sqrt{1-x^3}}{9x^9} - \frac{5\sqrt{1-x^3}}{36x^6} - \frac{5\sqrt{1-x^3}}{24x^3} - \frac{5}{24}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-x^3}\right)\\ &= -\frac{\sqrt{1-x^3}}{9x^9} - \frac{5\sqrt{1-x^3}}{36x^6} - \frac{5\sqrt{1-x^3}}{24x^3} - \frac{5}{24}\tanh^{-1}\left(\sqrt{1-x^3}\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 30, normalized size = 0.42

$$-\frac{2}{3}\sqrt{1-x^3}\;{}_2F_1\left(\frac{1}{2}, 4; \frac{3}{2}; 1-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^10*Sqrt[1 - x^3]),x]
[Out] (-2*Sqrt[1 - x^3]*Hypergeometric2F1[1/2, 4, 3/2, 1 - x^3])/3
```

fricas [A] time = 0.82, size = 63, normalized size = 0.89

$$-\frac{15\,x^9\log\left(\sqrt{-x^3+1}+1\right) - 15\,x^9\log\left(\sqrt{-x^3+1}-1\right) + 2\left(15\,x^6+10\,x^3+8\right)\sqrt{-x^3+1}}{144\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(-x^3+1)^(1/2),x, algorithm="fricas")
[Out] -1/144*(15*x^9*log(sqrt(-x^3 + 1) + 1) - 15*x^9*log(sqrt(-x^3 + 1) - 1) + 2
*(15*x^6 + 10*x^3 + 8)*sqrt(-x^3 + 1))/x^9
```

giac [A] time = 0.17, size = 76, normalized size = 1.07

$$-\frac{15\left(x^3-1\right)^2\sqrt{-x^3+1} - 40\left(-x^3+1\right)^{\frac{3}{2}} + 33\sqrt{-x^3+1}}{72\,x^9} - \frac{5}{48}\log\left(\sqrt{-x^3+1}+1\right) + \frac{5}{48}\log\left(\left|\sqrt{-x^3+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(-x^3+1)^(1/2),x, algorithm="giac")
[Out] -1/72*(15*(x^3 - 1)^2*sqrt(-x^3 + 1) - 40*(-x^3 + 1)^(3/2) + 33*sqrt(-x^3 +
 1))/x^9 - 5/48*log(sqrt(-x^3 + 1) + 1) + 5/48*log(abs(sqrt(-x^3 + 1) - 1))
```

maple [A] time = 0.03, size = 56, normalized size = 0.79

$$-\frac{5\,\text{arctanh}\left(\sqrt{-x^3+1}\right)}{24} - \frac{5\sqrt{-x^3+1}}{24x^3} - \frac{5\sqrt{-x^3+1}}{36x^6} - \frac{\sqrt{-x^3+1}}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(-x^3+1)^(1/2),x)
```

```
[Out] -5/24*arctanh((-x^3+1)^(1/2))-1/9*(-x^3+1)^(1/2)/x^9-5/36*(-x^3+1)^(1/2)/x^
6-5/24*(-x^3+1)^(1/2)/x^3
```

maxima [A] time = 1.40, size = 90, normalized size = 1.27

$$-\frac{15\left(-x^3+1\right)^{\frac{5}{2}}-40\left(-x^3+1\right)^{\frac{3}{2}}+33\sqrt{-x^3+1}}{72\left(\left(x^3-1\right)^3+3x^3+3\left(x^3-1\right)^2-2\right)}-\frac{5}{48}\log\left(\sqrt{-x^3+1}+1\right)+\frac{5}{48}\log\left(\sqrt{-x^3+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/72*(15*(-x^3 + 1)^(5/2) - 40*(-x^3 + 1)^(3/2) + 33*sqrt(-x^3 + 1))/((x^3
 - 1)^3 + 3*x^3 + 3*(x^3 - 1)^2 - 2) - 5/48*log(sqrt(-x^3 + 1) + 1) + 5/48*
log(sqrt(-x^3 + 1) - 1)
```

mupad [B] time = 0.03, size = 223, normalized size = 3.14

$$-\frac{5\sqrt{1-x^3}}{24x^3}-\frac{5\sqrt{1-x^3}}{36x^6}-\frac{\sqrt{1-x^3}}{9x^9}-\frac{5\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right.}{8\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^10*(1 - x^3)^(1/2)),x)
```

```
[Out] - (5*(1 - x^3)^(1/2))/(24*x^3) - (5*(1 - x^3)^(1/2))/(36*x^6) - (1 - x^3)^(
1/2)/(9*x^9) - (5*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i
)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1
/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(
(3^(1/2)*1i)/2 + 3/2, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(
1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(8*(1 - x^3)^(1/2)*(((3^(1/2)*1i
)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)
/2 + 1/2) + 1) + x^3)^(1/2))
```

sympy [A] time = 6.90, size = 182, normalized size = 2.56

$$\begin{cases} -\frac{5\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{24}+\frac{5}{24x^{\frac{3}{2}}\sqrt{-1+\frac{1}{x^3}}}-\frac{5}{72x^{\frac{9}{2}}\sqrt{-1+\frac{1}{x^3}}}-\frac{1}{36x^{\frac{15}{2}}\sqrt{-1+\frac{1}{x^3}}}-\frac{1}{9x^{\frac{21}{2}}\sqrt{-1+\frac{1}{x^3}}} & \text{for } \frac{1}{|x^3|}>1 \\ \frac{5i\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{24}-\frac{5i}{24x^{\frac{3}{2}}\sqrt{1-\frac{1}{x^3}}}+\frac{5i}{72x^{\frac{9}{2}}\sqrt{1-\frac{1}{x^3}}}+\frac{i}{36x^{\frac{15}{2}}\sqrt{1-\frac{1}{x^3}}}+\frac{i}{9x^{\frac{21}{2}}\sqrt{1-\frac{1}{x^3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(-x**3+1)**(1/2),x)
```

```
[Out] Piecewise((-5*acosh(x**(-3/2))/24 + 5/(24*x**(3/2)*sqrt(-1 + x**(-3))) - 5/
(72*x**(9/2)*sqrt(-1 + x**(-3))) - 1/(36*x**(15/2)*sqrt(-1 + x**(-3))) - 1/
(9*x**(21/2)*sqrt(-1 + x**(-3))), 1/Abs(x**3) > 1), (5*I*asin(x**(-3/2))/24
 - 5*I/(24*x**(3/2)*sqrt(1 - 1/x**3)) + 5*I/(72*x**(9/2)*sqrt(1 - 1/x**3))
+ I/(36*x**(15/2)*sqrt(1 - 1/x**3)) + I/(9*x**(21/2)*sqrt(1 - 1/x**3)), Tru
e))
```

3.464 $\int \frac{x^6}{\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=152

$$-\frac{16}{55}\sqrt{1-x^3}\,x-\frac{2}{11}\sqrt{1-x^3}\,x^4-\frac{32\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{55\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -16/55*x*(-x^3+1)^(1/2)-2/11*x^4*(-x^3+1)^(1/2)-32/165*(1-x)*EllipticF((1-x
-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)
/(1-x+3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2
)
```

Rubi [A] time = 0.04, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {321, 218}

$$-\frac{2}{11}\sqrt{1-x^3}\,x^4-\frac{16}{55}\sqrt{1-x^3}\,x-\frac{32\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{55\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/Sqrt[1 - x^3],x]
```

```
[Out] (-16*x*Sqrt[1 - x^3])/55 - (2*x^4*Sqrt[1 - x^3])/11 - (32*Sqrt[2 + Sqrt[3]]
*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt
[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(55*3^(1/4)*Sqrt[(1 - x)/(1 +
 Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^6}{\sqrt{1-x^3}}\,dx = -\frac{2}{11}x^4\sqrt{1-x^3} + \frac{8}{11}\int \frac{x^3}{\sqrt{1-x^3}}\,dx$$
$$= -\frac{16}{55}x\sqrt{1-x^3} - \frac{2}{11}x^4\sqrt{1-x^3} + \frac{16}{55}\int \frac{1}{\sqrt{1-x^3}}\,dx$$
$$= -\frac{16}{55}x\sqrt{1-x^3} - \frac{2}{11}x^4\sqrt{1-x^3} - \frac{32\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{}\right.}{55\sqrt[4]{3}\,\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\,\sqrt{1-x^3}}$$

Mathematica [C] time = 0.03, size = 40, normalized size = 0.26

$$-\frac{2}{55}x\left(\sqrt{1-x^3}\left(5x^3+8\right) - 8\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/Sqrt[1 - x^3],x]
```

```
[Out] (-2*x*(Sqrt[1 - x^3]*(8 + 5*x^3) - 8*Hypergeometric2F1[1/3, 1/2, 4/3, x^3])
)/55
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^3+1}\,x^6}{x^3-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 + 1)*x^6/(x^3 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/sqrt(-x^3 + 1), x)
```

maple [A] time = 0.02, size = 134, normalized size = 0.88

$$-\frac{2\sqrt{-x^3+1}\,x^4}{11} - \frac{16\sqrt{-x^3+1}\,x}{55} - \frac{32i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\text{EllipticF}\left(\right.}{165\sqrt{-x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-x^3+1)^(1/2),x)
```

```
[Out] -2/11*x^4*(-x^3+1)^(1/2)-16/55*x*(-x^3+1)^(1/2)-32/165*I*3^(1/2)*(I*(x+1/2-
1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2
```

```
+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/sqrt(-x^3 + 1), x)
```

mupad [B] time = 0.04, size = 199, normalized size = 1.31

$$-\frac{2x^4\sqrt{1-x^3}}{11}-\frac{16x\sqrt{1-x^3}}{55}-\frac{32\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1}{2}}}\right.\right.}{55\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(1 - x^3)^(1/2),x)
```

```
[Out] - (2*x^4*(1 - x^3)^(1/2))/11 - (16*x*(1 - x^3)^(1/2))/55 - (32*((3^(1/2)*1i
)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3
/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x -
 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2
+ 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(55*(1 - x
^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)
/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

sympy [A] time = 1.69, size = 31, normalized size = 0.20

$$\frac{x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\right|x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-x**3+1)**(1/2),x)
```

```
[Out] x**7*gamma(7/3)*hyper((1/2, 7/3), (10/3,), x**3*exp_polar(2*I*pi))/(3*gamma
(10/3))
```

3.465 $\int \frac{x^3}{\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=134

$$-\frac{2}{5}\sqrt{1-x^3}\,x-\frac{4\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -2/5*x*(-x^3+1)^(1/2)-4/15*(1-x)*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^
(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(3
/4)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {321, 218}

$$-\frac{2}{5}\sqrt{1-x^3}\,x-\frac{4\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[1 - x^3],x]
```

```
[Out] (-2*x*Sqrt[1 - x^3])/5 - (4*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1
 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -
7 - 4*Sqrt[3]])/(5*3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{1-x^3}}\,dx = -\frac{2}{5}x\sqrt{1-x^3}+\frac{2}{5}\int \frac{1}{\sqrt{1-x^3}}\,dx$$

$$= -\frac{2}{5}x\sqrt{1-x^3}-\frac{4\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.24

$$\frac{2}{5}x\left({}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)-\sqrt{1-x^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[1 - x^3],x]
[Out] (2*x*(-Sqrt[1 - x^3] + Hypergeometric2F1[1/3, 1/2, 4/3, x^3]))/5
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^3+1}\,x^3}{x^3-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-x^3 + 1)*x^3/(x^3 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^3/sqrt(-x^3 + 1), x)
```

maple [A] time = 0.02, size = 120, normalized size = 0.90

$$-\frac{2\sqrt{-x^3+1}\,x}{5}-\frac{4i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\ \text{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\right.}}{15\sqrt{-x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-x^3+1)^(1/2),x)
[Out] -2/5*(-x^3+1)^(1/2)*x-4/15*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2
)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/
2)/(-x^3+1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(
1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/sqrt(-x^3 + 1), x)
```

mupad [B] time = 0.99, size = 185, normalized size = 1.38

$$-\frac{2x\sqrt{1-x^3}}{5}-\frac{4\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{5\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(1 - x^3)^(1/2),x)
```

```
[Out] - (2*x*(1 - x^3)^(1/2))/5 - (4*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x
- (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2
+ 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2
 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(5*(1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)
*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
+ 1) + x^3)^(1/2))
```

sympy [A] time = 2.08, size = 31, normalized size = 0.23

$$\frac{x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\right|x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-x**3+1)**(1/2),x)
```

```
[Out] x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), x**3*exp_polar(2*I*pi))/(3*gamma(
7/3))
```

3.466 $\int \frac{1}{\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=115

$$-\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -2/3*(1-x)*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2
)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3+1)^(1/2)/((1
-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {218}

$$-\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 - x^3],x]
```

```
[Out] (-2*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*Ellip
ticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)
*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-x^3}}\,dx = -\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] time = 0.00, size = 15, normalized size = 0.13

$$x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 - x^3],x]
```

```
[Out] x*Hypergeometric2F1[1/3, 1/2, 4/3, x^3]
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^3+1}}{x^3-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 + 1)/(x^3 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-x^3 + 1), x)
```

maple [A] time = 0.02, size = 107, normalized size = 0.93

$$-\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\;\text{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}, \sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^3+1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+
1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-x^3 + 1), x)
```

mupad [B] time = 0.03, size = 172, normalized size = 1.50

$$-\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\;\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1 - x^3)^(1/2),x)
```

```
[Out] -(2*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3
^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/
2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/
((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3
/2)))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(
((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

sympy [**A**] time = 1.24, size = 29, normalized size = 0.25

$$\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3}, \frac{1}{2} \\ \frac{4}{3}\end{matrix}\right| x^3 e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**3+1)**(1/2),x)
```

```
[Out] x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(4/3
))
```

3.467 $\int \frac{1}{x^3\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=136

$$-\frac{\sqrt{1-x^3}}{2x^2}-\frac{\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -1/2*(-x^3+1)^(1/2)/x^2-1/6*(1-x)*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3
^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(
3/4)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {325, 218}

$$-\frac{\sqrt{1-x^3}}{2x^2}-\frac{\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[1 - x^3]),x]
```

```
[Out] -Sqrt[1 - x^3]/(2*x^2) - (Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 +
 Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7
- 4*Sqrt[3]])/(2*3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{1-x^3}}\,dx = -\frac{\sqrt{1-x^3}}{2x^2}+\frac{1}{4}\int \frac{1}{\sqrt{1-x^3}}\,dx$$

$$= -\frac{\sqrt{1-x^3}}{2x^2}-\frac{\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] time = 0.00, size = 20, normalized size = 0.15

$$-\frac{{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};x^3\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[1 - x^3]),x]
[Out] -1/2*Hypergeometric2F1[-2/3, 1/2, 1/3, x^3]/x^2
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^3+1}}{x^6-x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-x^3 + 1)/(x^6 - x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^3+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-x^3 + 1)*x^3), x)
```

maple [A] time = 0.02, size = 122, normalized size = 0.90

$$-\frac{i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\text{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{6\sqrt{-x^3+1}}-\frac{\sqrt{-x^3}}{2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-x^3+1)^(1/2),x)
[Out] -1/2*(-x^3+1)^(1/2)/x^2-1/6*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/
2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1
/2)/(-x^3+1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^
(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^3+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-x^3 + 1)*x^3), x)
```

mupad [B] time = 0.06, size = 187, normalized size = 1.38

$$-\frac{\sqrt{1-x^3}}{2x^2}-\frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{2\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(1 - x^3)^(1/2)),x)
```

```
[Out] - (1 - x^3)^(1/2)/(2*x^2) - (((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x -
(3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 +
1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*
ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 +
 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(2*(1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*(
(3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) +
1) + x^3)^(1/2))
```

sympy [A] time = 1.56, size = 34, normalized size = 0.25

$$\frac{\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{1}{2}\\ \frac{1}{3}\end{matrix}\right|x^3e^{2i\pi}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-x**3+1)**(1/2),x)
```

```
[Out] gamma(-2/3)*hyper((-2/3, 1/2), (1/3,), x**3*exp_polar(2*I*pi))/(3*x**2*gamm
a(1/3))
```

3.468 $\int \frac{1}{x^6\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=154

$$-\frac{\sqrt{1-x^3}}{5x^5}-\frac{7\sqrt{1-x^3}}{20x^2}-\frac{7\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{20\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -1/5*(-x^3+1)^(1/2)/x^5-7/20*(-x^3+1)^(1/2)/x^2-7/60*(1-x)*EllipticF((1-x-3
^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(
1-x+3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {325, 218}

$$-\frac{7\sqrt{1-x^3}}{20x^2}-\frac{\sqrt{1-x^3}}{5x^5}-\frac{7\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{20\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*Sqrt[1 - x^3]),x]
```

```
[Out] -Sqrt[1 - x^3]/(5*x^5) - (7*Sqrt[1 - x^3])/(20*x^2) - (7*Sqrt[2 + Sqrt[3]]*
(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[
3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(20*3^(1/4)*Sqrt[(1 - x)/(1 +
Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^6\sqrt{1-x^3}}\,dx = -\frac{\sqrt{1-x^3}}{5x^5} + \frac{7}{10}\int \frac{1}{x^3\sqrt{1-x^3}}\,dx$$
$$= -\frac{\sqrt{1-x^3}}{5x^5} - \frac{7\sqrt{1-x^3}}{20x^2} + \frac{7}{40}\int \frac{1}{\sqrt{1-x^3}}\,dx$$
$$= -\frac{\sqrt{1-x^3}}{5x^5} - \frac{7\sqrt{1-x^3}}{20x^2} - \frac{7\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{20\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] time = 0.00, size = 20, normalized size = 0.13

$$-\frac{{}_2F_1\left(-\frac{5}{3},\frac{1}{2};-\frac{2}{3};x^3\right)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*Sqrt[1 - x^3]),x]
[Out] -1/5*Hypergeometric2F1[-5/3, 1/2, -2/3, x^3]/x^5
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^3+1}}{x^9-x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^3+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-x^3 + 1)/(x^9 - x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^3+1}\,x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-x^3 + 1)*x^6), x)
```

maple [A] time = 0.03, size = 136, normalized size = 0.88

$$-\frac{7i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\text{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}, \sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{60\sqrt{-x^3+1}} - \frac{7}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-x^3+1)^(1/2),x)
[Out] -1/5*(-x^3+1)^(1/2)/x^5-7/20*(-x^3+1)^(1/2)/x^2-7/60*I*3^(1/2)*(I*(x+1/2-1/
2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1
```

```
/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2
-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^3+1}\,x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^3 + 1)*x^6), x)
```

mupad [B] time = 0.04, size = 201, normalized size = 1.31

$$-\frac{7\sqrt{1-x^3}}{20\,x^2}-\frac{\sqrt{1-x^3}}{5\,x^5}-\frac{7\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+}{-\frac{3}{2}+}\right.}{20\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(1 - x^3)^(1/2)),x)
```

```
[Out] - (7*(1 - x^3)^(1/2))/(20*x^2) - (1 - x^3)^(1/2)/(5*x^5) - (7*((3^(1/2)*1i)
/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/
2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x -
1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(20*(1 - x^
3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/
2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

sympy [A] time = 1.64, size = 37, normalized size = 0.24

$$\frac{\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{5}{3}, \frac{1}{2} \\ -\frac{2}{3}\end{matrix}\right| x^3 e^{2i\pi}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-x**3+1)**(1/2),x)
```

```
[Out] gamma(-5/3)*hyper((-5/3, 1/2), (-2/3,), x**3*exp_polar(2*I*pi))/(3*x**5*gam
ma(-2/3))
```

3.469 $\int \frac{x^7}{\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=294

$$\frac{80\sqrt{1-x^3}}{91\left(-x+\sqrt{3}+1\right)}-\frac{2}{13}\sqrt{1-x^3}\,x^5-\frac{20}{91}\sqrt{1-x^3}\,x^2+\frac{80\sqrt{2}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)\middle|-7-4\sqrt{3}\right)}{91\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -20/91*x^2*(-x^3+1)^(1/2)-2/13*x^5*(-x^3+1)^(1/2)+80/91*(-x^3+1)^(1/2)/(1-x
+3^(1/2))+80/273*(1-x)*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)
*2^(1/2)*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3+1)^(1/2)/((1-x)/(1
-x+3^(1/2))^2)^(1/2)-40/91*3^(1/4)*(1-x)*EllipticE((1-x-3^(1/2))/(1-x+3^(1/
2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1
/2)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {321, 303, 218, 1877}

$$-\frac{2}{13}\sqrt{1-x^3}\,x^5-\frac{20}{91}\sqrt{1-x^3}\,x^2+\frac{80\sqrt{1-x^3}}{91\left(-x+\sqrt{3}+1\right)}+\frac{80\sqrt{2}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)\middle|-7-4\sqrt{3}\right)}{91\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/Sqrt[1 - x^3],x]
```

```
[Out] (80*Sqrt[1 - x^3])/(91*(1 + Sqrt[3] - x)) - (20*x^2*Sqrt[1 - x^3])/91 - (2*
x^5*Sqrt[1 - x^3])/13 - (40*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x +
 x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3]
- x)], -7 - 4*Sqrt[3]])/(91*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]
) + (80*Sqrt[2]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[A
rcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(91*3^(1/4)*Sq
rt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{\sqrt{1-x^3}}\,dx &= -\frac{2}{13}x^5\sqrt{1-x^3} + \frac{10}{13}\int \frac{x^4}{\sqrt{1-x^3}}\,dx \\
&= -\frac{20}{91}x^2\sqrt{1-x^3} - \frac{2}{13}x^5\sqrt{1-x^3} + \frac{40}{91}\int \frac{x}{\sqrt{1-x^3}}\,dx \\
&= -\frac{20}{91}x^2\sqrt{1-x^3} - \frac{2}{13}x^5\sqrt{1-x^3} - \frac{40}{91}\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx - \frac{1}{91}\left(40\sqrt{2\left(2-\sqrt{3}\right)}\right)\int \frac{1}{\sqrt{1-x^3}}\,dx \\
&= \frac{80\sqrt{1-x^3}}{91\left(1+\sqrt{3}-x\right)} - \frac{20}{91}x^2\sqrt{1-x^3} - \frac{2}{13}x^5\sqrt{1-x^3} - \frac{40\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,E\left(\sin^{-1}\right.}{91\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 42, normalized size = 0.14

$$-\frac{2}{91}x^2\left(\sqrt{1-x^3}\left(7x^3+10\right) - 10\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};x^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/Sqrt[1 - x^3],x]
```

```
[Out] (-2*x^2*(Sqrt[1 - x^3]*(10 + 7*x^3) - 10*Hypergeometric2F1[1/2, 2/3, 5/3, x
^3]))/91
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^3+1}\,x^7}{x^3-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 + 1)*x^7/(x^3 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^7/sqrt(-x^3 + 1), x)
```

maple [A] time = 0.02, size = 187, normalized size = 0.64

$$-\frac{2\sqrt{-x^3+1}\,x^5}{13}-\frac{20\sqrt{-x^3+1}\,x^2}{91}-\frac{80i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\left(\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-x^3+1)^(1/2),x)
```

```
[Out] -2/13*x^5*(-x^3+1)^(1/2)-20/91*x^2*(-x^3+1)^(1/2)-80/273*I*3^(1/2)*(I*(x+1/
2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1
/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*((-3/2+1/2*I*3^(1/2))*Ellip
ticE(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1
/2*I*3^(1/2)))^(1/2))+EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2
))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^7/sqrt(-x^3 + 1), x)
```

mupad [B] time = 0.05, size = 260, normalized size = 0.88

$$-\frac{20\,x^2\,\sqrt{1-x^3}}{91}-\frac{2\,x^5\,\sqrt{1-x^3}}{13}-\frac{80\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{E}\left(\mathrm{asin}\left(\sqrt{}\right.\right.}{91\,\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(1 - x^3)^(1/2),x)
```

```
[Out] - (20*x^2*(1 - x^3)^(1/2))/91 - (2*x^5*(1 - x^3)^(1/2))/13 - (80*(((3^(1/2)
*1i)/2 - 1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3
^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1i)/2 - 3/2)*ellipt
icE(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/
((3^(1/2)*1i)/2 - 3/2)))*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(
1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(91
*(1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1
/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

sympy [A] time = 1.80, size = 31, normalized size = 0.11

$$\frac{x^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\right|x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-x**3+1)**(1/2),x)
```

```
[Out] x**8*gamma(8/3)*hyper((1/2, 8/3), (11/3,), x**3*exp_polar(2*I*pi))/(3*gamma
(11/3))
```

3.470 $\int \frac{x^4}{\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=276

$$\frac{8\sqrt{1-x^3}}{7\left(-x+\sqrt{3}+1\right)}-\frac{2}{7}\sqrt{1-x^3}\,x^2+\frac{8\sqrt{2}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{7\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1$$

```
[Out] -2/7*x^2*(-x^3+1)^(1/2)+8/7*(-x^3+1)^(1/2)/(1-x+3^(1/2))+8/21*(1-x)*Ellipti
cF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*((x^2+x+1)/(1-x+3^(1/
2))^2)^(1/2)*3^(3/4)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)-4/7*3^(1/
4)*(1-x)*EllipticE((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-
1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/(-x^3+1)^(1/2)/((1-x)/(1-x+3
^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {321, 303, 218, 1877}

$$-\frac{2}{7}\sqrt{1-x^3}\,x^2+\frac{8\sqrt{1-x^3}}{7\left(-x+\sqrt{3}+1\right)}+\frac{8\sqrt{2}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{7\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[1 - x^3],x]
```

```
[Out] (8*Sqrt[1 - x^3])/(7*(1 + Sqrt[3] - x)) - (2*x^2*Sqrt[1 - x^3])/7 - (4*3^(1
/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*Ellip
ticE[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(7*Sqrt[
(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]) + (8*Sqrt[2]*(1 - x)*Sqrt[(1 +
x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[
3] - x)], -7 - 4*Sqrt[3]])/(7*3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqr
t[1 - x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt{1-x^3}}\,dx &= -\frac{2}{7}x^2\sqrt{1-x^3} + \frac{4}{7}\int \frac{x}{\sqrt{1-x^3}}\,dx \\ &= -\frac{2}{7}x^2\sqrt{1-x^3} - \frac{4}{7}\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx - \frac{1}{7}\left(4\sqrt{2\left(2-\sqrt{3}\right)}\right)\int \frac{1}{\sqrt{1-x^3}}\,dx \\ &= \frac{8\sqrt{1-x^3}}{7\left(1+\sqrt{3}-x\right)} - \frac{2}{7}x^2\sqrt{1-x^3} - \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{}\right.}{7\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.12

$$\frac{2}{7}x^2\left({}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};x^3\right)-\sqrt{1-x^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[1 - x^3],x]
[Out] (2*x^2*(-Sqrt[1 - x^3] + Hypergeometric2F1[1/2, 2/3, 5/3, x^3]))/7
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^3+1}\,x^4}{x^3-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-x^3 + 1)*x^4/(x^3 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(-x^3 + 1), x)
```

maple [A] time = 0.02, size = 173, normalized size = 0.63

$$-\frac{2\sqrt{-x^3+1}\,x^2}{7}-\frac{8i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\left(\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\text{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{}\right.\right.}{21\sqrt{-x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^3+1)^(1/2),x)
```

```
[Out] -2/7*(-x^3+1)^(1/2)*x^2-8/21*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1
/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(
1/2)/(-x^3+1)^(1/2)*((-3/2+1/2*I*3^(1/2))*EllipticE(1/3*3^(1/2)*(I*(x+1/2-1
/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+Ellipt
icF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/
2*I*3^(1/2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(-x^3 + 1), x)
```

mupad [B] time = 0.06, size = 246, normalized size = 0.89

$$-\frac{2x^2\sqrt{1-x^3}}{7}-\frac{8\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\text{i}}{2}\right)\text{F}\left(\text{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\text{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\text{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\text{i}}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1\text{i}}{2}\right)\text{E}\left(\text{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\text{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+}{-\frac{3}{2}+}\right.\right.}{7\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\text{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\text{i}}{2}\right)-1\right)x+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(1 - x^3)^(1/2),x)
```

```
[Out] - (2*x^2*(1 - x^3)^(1/2))/7 - (8*(((3^(1/2)*1i)/2 - 1/2)*ellipticF(asin((-(
x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i
)/2 - 3/2)) - ((3^(1/2)*1i)/2 - 3/2)*ellipticE(asin((-(x - 1)/((3^(1/2)*1i)
/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*((3^(1/
2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/
2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(
-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(7*(1 - x^3)^(1/2)*(((3^(1/2)*1i)/2
 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2
+ 1/2) + 1) + x^3)^(1/2))
```

sympy [A] time = 1.02, size = 31, normalized size = 0.11

$$\frac{x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\right|x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**3+1)**(1/2),x)
```

```
[Out] x**5*gamma(5/3)*hyper((1/2, 5/3), (8/3,), x**3*exp_polar(2*I*pi))/(3*gamma(
8/3))
```

3.471 $\int \frac{x}{\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=252

$$\frac{2\sqrt{1-x^3}}{-x+\sqrt{3}+1}+\frac{2\sqrt{2}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\,\sqrt{1-x^3}}-\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,E}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}}$$

```
[Out] 2*(-x^3+1)^(1/2)/(1-x+3^(1/2))+2/3*(1-x)*EllipticF((1-x-3^(1/2))/(1-x+3^(1/
2)),I*3^(1/2)+2*I)*2^(1/2)*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3+
1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)-3^(1/4)*(1-x)*EllipticE((1-x-3^(1/2)
)/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x+3^
(1/2))^2)^(1/2)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {303, 218, 1877}

$$\frac{2\sqrt{1-x^3}}{-x+\sqrt{3}+1}+\frac{2\sqrt{2}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\,\sqrt{1-x^3}}-\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,E}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[1 - x^3],x]
```

```
[Out] (2*Sqrt[1 - x^3])/(1 + Sqrt[3] - x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sq
rt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] - x)/(1
 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[
1 - x^3]) + (2*Sqrt[2]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*Elli
pticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4
)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
```

```
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{1-x^3}}\,dx = -\left(\sqrt{2\left(2-\sqrt{3}\right)}\int \frac{1}{\sqrt{1-x^3}}\,dx\right) - \int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx$$

$$= \frac{2\sqrt{1-x^3}}{1+\sqrt{3}-x} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} + \frac{2\sqrt{2}\,(1-x)}{\sqrt{}}$$

Mathematica [C] time = 0.00, size = 20, normalized size = 0.08

$$\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[1 - x^3],x]
[Out] (x^2*Hypergeometric2F1[1/2, 2/3, 5/3, x^3])/2
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^3+1}\,x}{x^3-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-x^3 + 1)*x/(x^3 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(x/sqrt(-x^3 + 1), x)
```

maple [A] time = 0.02, size = 158, normalized size = 0.63

$$-\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\left(\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\text{EllipticE}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}, \sqrt{\frac{i}{-\frac{3}{2}}}\right.\right.}{3\sqrt{-x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^3+1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*((-3
/2+1/2*I*3^(1/2))*EllipticE(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(
1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+EllipticF(1/3*3^(1/2)*(I*(x+1/
2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(-x^3 + 1), x)
```

mupad [B] time = 0.04, size = 231, normalized size = 0.92

$$-\frac{2\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)\left(\frac{3}{2}+\sqrt{}\right.}{\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1 - x^3)^(1/2),x)
```

```
[Out] -(2*(((3^(1/2)*1i)/2 - 1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1i)/2
 - 3/2)*ellipticE(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)
*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/
2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2
)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/
2))^(1/2))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
- x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

sympy [A] time = 0.83, size = 31, normalized size = 0.12

$$\frac{x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**3+1)**(1/2),x)
```

```
[Out] x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(2*I*pi))/(3*gamma(
5/3))
```

3.472 $\int \frac{1}{x^2\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=270

$$-\frac{\sqrt{1-x^3}}{-x+\sqrt{3}+1}-\frac{\sqrt{1-x^3}}{x}-\frac{\sqrt{2}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2}{(-x+}}}{2\sqrt{\frac{}{(-}}}$$

```
[Out] -(-x^3+1)^(1/2)/x-(-x^3+1)^(1/2)/(1-x+3^(1/2))-1/3*(1-x)*EllipticF((1-x-3^(
1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2
)*3^(3/4)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)+1/2*3^(1/4)*(1-x)*El
lipticE((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2)
)*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^
(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {325, 303, 218, 1877}

$$-\frac{\sqrt{1-x^3}}{-x+\sqrt{3}+1}-\frac{\sqrt{1-x^3}}{x}-\frac{\sqrt{2}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2}{(-x+}}}{2\sqrt{\frac{}{(-}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[1 - x^3]),x]
```

```
[Out] -(Sqrt[1 - x^3]/(1 + Sqrt[3] - x)) - Sqrt[1 - x^3]/x + (3^(1/4)*Sqrt[2 - Sq
rt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[ArcSin[(1
 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(2*Sqrt[(1 - x)/(1 + Sq
rt[3] - x)^2]*Sqrt[1 - x^3]) - (Sqrt[2]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqr
t[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*
Sqrt[3]])/(3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
 + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
 + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{1-x^3}}\,dx &= -\frac{\sqrt{1-x^3}}{x} - \frac{1}{2}\int \frac{x}{\sqrt{1-x^3}}\,dx \\ &= -\frac{\sqrt{1-x^3}}{x} + \frac{1}{2}\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx + \sqrt{\frac{1}{2}\left(2-\sqrt{3}\right)}\int \frac{1}{\sqrt{1-x^3}}\,dx \\ &= -\frac{\sqrt{1-x^3}}{1+\sqrt{3}-x} - \frac{\sqrt{1-x^3}}{x} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right.}{2\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\,\sqrt{1-x^3}}\end{aligned}$$

Mathematica [C] time = 0.00, size = 18, normalized size = 0.07

$$-\frac{{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};x^3\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[1 - x^3]),x]
```

```
[Out] -(Hypergeometric2F1[-1/3, 1/2, 2/3, x^3]/x)
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^3+1}}{x^5-x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 + 1)/(x^5 - x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^3+1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^3 + 1)*x^2), x)
```

maple [A] time = 0.02, size = 173, normalized size = 0.64

$$-\frac{\sqrt{-x^3+1}}{x}+\frac{i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\left(\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\text{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right.}\right.\right.}{3}}{3\sqrt{-x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-x^3+1)^(1/2),x)
```

```
[Out] -(-x^3+1)^(1/2)/x+1/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x
-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-
x^3+1)^(1/2)*((-3/2+1/2*I*3^(1/2))*EllipticE(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^
(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+EllipticF(1/3
*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(
1/2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^3+1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^3 + 1)*x^2), x)
```

mupad [B] time = 1.02, size = 245, normalized size = 0.91

$$-\frac{\sqrt{1-x^3}}{x}+\frac{\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)\left(}{\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(1 - x^3)^(1/2)),x)
```

```
[Out] ((((3^(1/2)*1i)/2 - 1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(
1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1i)/2 -
3/2)*ellipticE(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i
)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*
(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1
i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))
^(1/2))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x
*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) - (1 - x
^3)^(1/2)/x
```

sympy [A] time = 1.36, size = 32, normalized size = 0.12

$$\frac{\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},\frac{1}{2}\\ \frac{2}{3}\end{matrix}\right|x^3e^{2i\pi}\right)}{3x\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-x**3+1)**(1/2),x)
```

```
[Out] gamma(-1/3)*hyper((-1/3, 1/2), (2/3,), x**3*exp_polar(2*I*pi))/(3*x*gamma(2
/3))
```

3.473 $\int \frac{1}{x^5\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=294

$$-\frac{5\sqrt{1-x^3}}{8\left(-x+\sqrt{3}+1\right)}-\frac{5\sqrt{1-x^3}}{8x}-\frac{\sqrt{1-x^3}}{4x^4}-\frac{5(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\,\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}+\frac{5\sqrt[4]{3}\sqrt{2-\sqrt{3}}$$

```
[Out] -1/4*(-x^3+1)^(1/2)/x^4-5/8*(-x^3+1)^(1/2)/x-5/8*(-x^3+1)^(1/2)/(1-x+3^(1/2
))-5/24*(1-x)*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*
((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2
))^2)^(1/2)+5/16*3^(1/4)*(1-x)*EllipticE((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1
/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/(-x^3+
1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {325, 303, 218, 1877}

$$-\frac{5\sqrt{1-x^3}}{8\left(-x+\sqrt{3}+1\right)}-\frac{5\sqrt{1-x^3}}{8x}-\frac{\sqrt{1-x^3}}{4x^4}-\frac{5(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\,\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}+\frac{5\sqrt[4]{3}\sqrt{2-\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[1 - x^3]),x]
```

```
[Out] (-5*Sqrt[1 - x^3])/(8*(1 + Sqrt[3] - x)) - Sqrt[1 - x^3]/(4*x^4) - (5*Sqrt[
1 - x^3])/(8*x) + (5*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(
1 + Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)],
-7 - 4*Sqrt[3]])/(16*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]) - (5*
(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[
3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(4*Sqrt[2]*3^(1/4)*Sqrt[(1 - x
)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\sqrt{1-x^3}}\,dx &= -\frac{\sqrt{1-x^3}}{4x^4} + \frac{5}{8}\int \frac{1}{x^2\sqrt{1-x^3}}\,dx \\
&= -\frac{\sqrt{1-x^3}}{4x^4} - \frac{5\sqrt{1-x^3}}{8x} - \frac{5}{16}\int \frac{x}{\sqrt{1-x^3}}\,dx \\
&= -\frac{\sqrt{1-x^3}}{4x^4} - \frac{5\sqrt{1-x^3}}{8x} + \frac{5}{16}\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx + \frac{1}{8}\left(5\sqrt{\frac{1}{2}\left(2-\sqrt{3}\right)}\right)\int \frac{1}{\sqrt{1-x^3}}\,dx \\
&= -\frac{5\sqrt{1-x^3}}{8\left(1+\sqrt{3}-x\right)} - \frac{\sqrt{1-x^3}}{4x^4} - \frac{5\sqrt{1-x^3}}{8x} + \frac{5\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1-}{1+}\right.\right.}{16\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}
\end{aligned}$$

Mathematica [C] time = 0.00, size = 20, normalized size = 0.07

$$-\frac{{}_2F_1\left(-\frac{4}{3},\frac{1}{2};-\frac{1}{3};x^3\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[1 - x^3]),x]
```

```
[Out] -1/4*Hypergeometric2F1[-4/3, 1/2, -1/3, x^3]/x^4
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^3+1}}{x^8-x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 + 1)/(x^8 - x^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^3+1}\,x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^3 + 1)*x^5), x)
```

maple [A] time = 0.03, size = 187, normalized size = 0.64

$$-\frac{5\sqrt{-x^3+1}}{8x}-\frac{\sqrt{-x^3+1}}{4x^4}+\frac{5i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\left(\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\text{EllipticE}\right.}{24\sqrt{-x^3+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-x^3+1)^(1/2),x)
```

```
[Out] -1/4*(-x^3+1)^(1/2)/x^4-5/8*(-x^3+1)^(1/2)/x+5/24*I*3^(1/2)*(I*(x+1/2-1/2*I
*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*
I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*((-3/2+1/2*I*3^(1/2))*EllipticE(1/
3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^
(1/2)))^(1/2))+EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2
),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^3+1}\,x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^3 + 1)*x^5), x)
```

mupad [B] time = 0.05, size = 260, normalized size = 0.88

$$-\frac{5\sqrt{1-x^3}}{8\,x}-\frac{\sqrt{1-x^3}}{4\,x^4}+\frac{5\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\right|$$

$$8\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(1 - x^3)^(1/2)),x)
```

```
[Out] (5*(((3^(1/2)*1i)/2 - 1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))
^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1i)/2
- 3/2)*ellipticE(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*
1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2
)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)
*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2
))^(1/2))/(8*(1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) - (1
 - x^3)^(1/2)/(4*x^4) - (5*(1 - x^3)^(1/2))/(8*x)
```

sympy [**A**] time = 1.58, size = 37, normalized size = 0.13

$$\frac{\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{4}{3}, \frac{1}{2} \\ -\frac{1}{3}\end{matrix}\right| x^3 e^{2i\pi}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-x**3+1)**(1/2),x)
```

```
[Out] gamma(-4/3)*hyper((-4/3, 1/2), (-1/3,), x**3*exp_polar(2*I*pi))/(3*x**4*gam
ma(-1/3))
```

3.474 $\int \frac{x^{11}}{\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=53

$$\frac{2}{21}\left(x^3-1\right)^{7/2}+\frac{2}{5}\left(x^3-1\right)^{5/2}+\frac{2}{3}\left(x^3-1\right)^{3/2}+\frac{2\sqrt{x^3-1}}{3}$$

```
[Out] 2/3*(x^3-1)^(3/2)+2/5*(x^3-1)^(5/2)+2/21*(x^3-1)^(7/2)+2/3*(x^3-1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{2}{21}\left(x^3-1\right)^{7/2}+\frac{2}{5}\left(x^3-1\right)^{5/2}+\frac{2}{3}\left(x^3-1\right)^{3/2}+\frac{2\sqrt{x^3-1}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/Sqrt[-1 + x^3],x]
```

```
[Out] (2*Sqrt[-1 + x^3])/3 + (2*(-1 + x^3)^(3/2))/3 + (2*(-1 + x^3)^(5/2))/5 + (2
*(-1 + x^3)^(7/2))/21
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{\sqrt{-1+x^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^3}{\sqrt{-1+x}}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{1}{\sqrt{-1+x}}+3\sqrt{-1+x}+3(-1+x)^{3/2}+(-1+x)^{5/2}\right)dx, x, x^3\right)\\ &= \frac{2}{3}\sqrt{-1+x^3}+\frac{2}{3}\left(-1+x^3\right)^{3/2}+\frac{2}{5}\left(-1+x^3\right)^{5/2}+\frac{2}{21}\left(-1+x^3\right)^{7/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.57

$$\frac{2}{105}\sqrt{x^3-1}\left(5x^9+6x^6+8x^3+16\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/Sqrt[-1 + x^3],x]
```

```
[Out] (2*Sqrt[-1 + x^3]*(16 + 8*x^3 + 6*x^6 + 5*x^9))/105
```

fricas [A] time = 0.89, size = 26, normalized size = 0.49

$$\frac{2}{105}\left(5x^9+6x^6+8x^3+16\right)\sqrt{x^3-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(5*x^9 + 6*x^6 + 8*x^3 + 16)*sqrt(x^3 - 1)
```

giac [A] time = 0.17, size = 37, normalized size = 0.70

$$\frac{2}{21}\left(x^3-1\right)^{\frac{7}{2}}+\frac{2}{5}\left(x^3-1\right)^{\frac{5}{2}}+\frac{2}{3}\left(x^3-1\right)^{\frac{3}{2}}+\frac{2}{3}\sqrt{x^3-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] 2/21*(x^3 - 1)^(7/2) + 2/5*(x^3 - 1)^(5/2) + 2/3*(x^3 - 1)^(3/2) + 2/3*sqrt
(x^3 - 1)
```

maple [A] time = 0.00, size = 36, normalized size = 0.68

$$\frac{2\left(x-1\right)\left(x^2+x+1\right)\left(5x^9+6x^6+8x^3+16\right)}{105\sqrt{x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^3-1)^(1/2),x)
```

```
[Out] 2/105*(x-1)*(x^2+x+1)*(5*x^9+6*x^6+8*x^3+16)/(x^3-1)^(1/2)
```

maxima [A] time = 1.29, size = 37, normalized size = 0.70

$$\frac{2}{21}\left(x^3-1\right)^{\frac{7}{2}}+\frac{2}{5}\left(x^3-1\right)^{\frac{5}{2}}+\frac{2}{3}\left(x^3-1\right)^{\frac{3}{2}}+\frac{2}{3}\sqrt{x^3-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/21*(x^3 - 1)^(7/2) + 2/5*(x^3 - 1)^(5/2) + 2/3*(x^3 - 1)^(3/2) + 2/3*sqrt
(x^3 - 1)
```

mupad [B] time = 0.04, size = 46, normalized size = 0.87

$$\frac{32\sqrt{x^3-1}}{105}+\frac{16x^3\sqrt{x^3-1}}{105}+\frac{4x^6\sqrt{x^3-1}}{35}+\frac{2x^9\sqrt{x^3-1}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^3 - 1)^(1/2),x)
```

```
[Out] (32*(x^3 - 1)^(1/2))/105 + (16*x^3*(x^3 - 1)^(1/2))/105 + (4*x^6*(x^3 - 1)^
(1/2))/35 + (2*x^9*(x^3 - 1)^(1/2))/21
```

sympy [A] time = 3.08, size = 56, normalized size = 1.06

$$\frac{2x^9\sqrt{x^3-1}}{21}+\frac{4x^6\sqrt{x^3-1}}{35}+\frac{16x^3\sqrt{x^3-1}}{105}+\frac{32\sqrt{x^3-1}}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(x**3-1)**(1/2),x)
```

```
[Out] 2*x**9*sqrt(x**3 - 1)/21 + 4*x**6*sqrt(x**3 - 1)/35 + 16*x**3*sqrt(x**3 - 1
)/105 + 32*sqrt(x**3 - 1)/105
```

3.475 $\int \frac{x^8}{\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=40

$$\frac{2}{15}\left(x^3-1\right)^{5/2}+\frac{4}{9}\left(x^3-1\right)^{3/2}+\frac{2\sqrt{x^3-1}}{3}$$

```
[Out] 4/9*(x^3-1)^(3/2)+2/15*(x^3-1)^(5/2)+2/3*(x^3-1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{2}{15}\left(x^3-1\right)^{5/2}+\frac{4}{9}\left(x^3-1\right)^{3/2}+\frac{2\sqrt{x^3-1}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/Sqrt[-1 + x^3],x]
```

```
[Out] (2*Sqrt[-1 + x^3])/3 + (4*(-1 + x^3)^(3/2))/9 + (2*(-1 + x^3)^(5/2))/15
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{\sqrt{-1+x^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{-1+x}}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{1}{\sqrt{-1+x}}+2\sqrt{-1+x}+(-1+x)^{3/2}\right)dx, x, x^3\right)\\ &= \frac{2}{3}\sqrt{-1+x^3}+\frac{4}{9}\left(-1+x^3\right)^{3/2}+\frac{2}{15}\left(-1+x^3\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.62

$$\frac{2}{45}\sqrt{x^3-1}\left(3x^6+4x^3+8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/Sqrt[-1 + x^3],x]
```

```
[Out] (2*Sqrt[-1 + x^3]*(8 + 4*x^3 + 3*x^6))/45
```

fricas [A] time = 0.83, size = 21, normalized size = 0.52

$$\frac{2}{45}\left(3\,x^6+4\,x^3+8\right)\sqrt{x^3-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^3-1)^(1/2),x, algorithm="fricas")
[Out] 2/45*(3*x^6 + 4*x^3 + 8)*sqrt(x^3 - 1)
```

giac [A] time = 0.15, size = 28, normalized size = 0.70

$$\frac{2}{15}\left(x^3-1\right)^{\frac{5}{2}}+\frac{4}{9}\left(x^3-1\right)^{\frac{3}{2}}+\frac{2}{3}\sqrt{x^3-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^3-1)^(1/2),x, algorithm="giac")
[Out] 2/15*(x^3 - 1)^(5/2) + 4/9*(x^3 - 1)^(3/2) + 2/3*sqrt(x^3 - 1)
```

maple [A] time = 0.00, size = 31, normalized size = 0.78

$$\frac{2\left(x-1\right)\left(x^2+x+1\right)\left(3x^6+4x^3+8\right)}{45\sqrt{x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^3-1)^(1/2),x)
[Out] 2/45*(x-1)*(x^2+x+1)*(3*x^6+4*x^3+8)/(x^3-1)^(1/2)
```

maxima [A] time = 1.40, size = 28, normalized size = 0.70

$$\frac{2}{15}\left(x^3-1\right)^{\frac{5}{2}}+\frac{4}{9}\left(x^3-1\right)^{\frac{3}{2}}+\frac{2}{3}\sqrt{x^3-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^3-1)^(1/2),x, algorithm="maxima")
[Out] 2/15*(x^3 - 1)^(5/2) + 4/9*(x^3 - 1)^(3/2) + 2/3*sqrt(x^3 - 1)
```

mupad [B] time = 0.03, size = 21, normalized size = 0.52

$$\frac{2\sqrt{x^3-1}\,\left(3\,x^6+4\,x^3+8\right)}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^3 - 1)^(1/2),x)
[Out] (2*(x^3 - 1)^(1/2)*(4*x^3 + 3*x^6 + 8))/45
```

sympy [A] time = 1.38, size = 41, normalized size = 1.02

$$\frac{2x^6\sqrt{x^3-1}}{15}+\frac{8x^3\sqrt{x^3-1}}{45}+\frac{16\sqrt{x^3-1}}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**3-1)**(1/2),x)
[Out] 2*x**6*sqrt(x**3 - 1)/15 + 8*x**3*sqrt(x**3 - 1)/45 + 16*sqrt(x**3 - 1)/45
```

3.476 $\int \frac{x^5}{\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=27

$$\frac{2}{9}\left(x^3-1\right)^{3/2}+\frac{2\sqrt{x^3-1}}{3}$$

```
[Out] 2/9*(x^3-1)^(3/2)+2/3*(x^3-1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{2}{9}\left(x^3-1\right)^{3/2}+\frac{2\sqrt{x^3-1}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[-1 + x^3],x]
```

```
[Out] (2*Sqrt[-1 + x^3])/3 + (2*(-1 + x^3)^(3/2))/9
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\sqrt{-1+x^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{\sqrt{-1+x}}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{1}{\sqrt{-1+x}}+\sqrt{-1+x}\right)dx, x, x^3\right)\\ &= \frac{2}{3}\sqrt{-1+x^3}+\frac{2}{9}\left(-1+x^3\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 0.67

$$\frac{2}{9}\sqrt{x^3-1}\,\left(x^3+2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[-1 + x^3],x]
```

```
[Out] (2*Sqrt[-1 + x^3]*(2 + x^3))/9
```

fricas [A] time = 0.57, size = 14, normalized size = 0.52

$$\frac{2}{9}\left(x^3+2\right)\sqrt{x^3-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^3-1)^(1/2),x, algorithm="fricas")
[Out] 2/9*(x^3 + 2)*sqrt(x^3 - 1)
```

giac [**A**] time = 0.16, size = 19, normalized size = 0.70

$$\frac{2}{9}\left(x^3-1\right)^{\frac{3}{2}}+\frac{2}{3}\sqrt{x^3-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^3-1)^(1/2),x, algorithm="giac")
[Out] 2/9*(x^3 - 1)^(3/2) + 2/3*sqrt(x^3 - 1)
```

maple [**A**] time = 0.01, size = 24, normalized size = 0.89

$$\frac{2\left(x-1\right)\left(x^2+x+1\right)\left(x^3+2\right)}{9\sqrt{x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^3-1)^(1/2),x)
[Out] 2/9*(x-1)*(x^2+x+1)*(x^3+2)/(x^3-1)^(1/2)
```

maxima [**A**] time = 1.27, size = 19, normalized size = 0.70

$$\frac{2}{9}\left(x^3-1\right)^{\frac{3}{2}}+\frac{2}{3}\sqrt{x^3-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^3-1)^(1/2),x, algorithm="maxima")
[Out] 2/9*(x^3 - 1)^(3/2) + 2/3*sqrt(x^3 - 1)
```

mupad [**B**] time = 0.02, size = 14, normalized size = 0.52

$$\frac{2\sqrt{x^3-1}\,\left(x^3+2\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^3 - 1)^(1/2),x)
[Out] (2*(x^3 - 1)^(1/2)*(x^3 + 2))/9
```

sympy [**A**] time = 0.51, size = 26, normalized size = 0.96

$$\frac{2x^3\sqrt{x^3-1}}{9}+\frac{4\sqrt{x^3-1}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**3-1)**(1/2),x)
[Out] 2*x**3*sqrt(x**3 - 1)/9 + 4*sqrt(x**3 - 1)/9
```

3.477 $\int \frac{x^2}{\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=13

$$\frac{2\sqrt{x^3-1}}{3}$$

```
[Out] 2/3*(x^3-1)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$\frac{2\sqrt{x^3-1}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[-1 + x^3],x]
```

```
[Out] (2*Sqrt[-1 + x^3])/3
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{-1+x^3}}\,dx = \frac{2}{3}\sqrt{-1+x^3}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{2\sqrt{x^3-1}}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[-1 + x^3],x]
```

```
[Out] (2*Sqrt[-1 + x^3])/3
```

fricas [A] time = 0.94, size = 9, normalized size = 0.69

$$\frac{2}{3}\sqrt{x^3-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(x^3 - 1)
```

giac [A] time = 0.15, size = 9, normalized size = 0.69

$$\frac{2}{3}\sqrt{x^3-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^3-1)^(1/2),x, algorithm="giac")
[Out] 2/3*sqrt(x^3 - 1)
```

maple [A] time = 0.00, size = 19, normalized size = 1.46

$$\frac{2\,(x-1)\left(x^2+x+1\right)}{3\sqrt{x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^3-1)^(1/2),x)
[Out] 2/3*(x-1)*(x^2+x+1)/(x^3-1)^(1/2)
```

maxima [A] time = 1.27, size = 9, normalized size = 0.69

$$\frac{2}{3}\sqrt{x^3-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^3-1)^(1/2),x, algorithm="maxima")
[Out] 2/3*sqrt(x^3 - 1)
```

mupad [B] time = 0.02, size = 9, normalized size = 0.69

$$\frac{2\sqrt{x^3-1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^3 - 1)^(1/2),x)
[Out] (2*(x^3 - 1)^(1/2))/3
```

sympy [A] time = 0.22, size = 10, normalized size = 0.77

$$\frac{2\sqrt{x^3-1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**3-1)**(1/2),x)
[Out] 2*sqrt(x**3 - 1)/3
```

3.478 $\int \frac{1}{x\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=14

$$\frac{2}{3}\tan^{-1}\left(\sqrt{x^3-1}\right)$$

```
[Out] 2/3*arctan((x^3-1)^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {266, 63, 203}

$$\frac{2}{3}\tan^{-1}\left(\sqrt{x^3-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*ArcTan[Sqrt[-1 + x^3]])/3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{-1+x^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx, x, x^3\right)\\ &= \frac{2}{3}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x^3}\right)\\ &= \frac{2}{3}\tan^{-1}\left(\sqrt{-1+x^3}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{2}{3}\tan^{-1}\left(\sqrt{x^3-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-1 + x^3]),x]
[Out] (2*ArcTan[Sqrt[-1 + x^3]])/3
```

fricas [A] time = 0.75, size = 10, normalized size = 0.71

$$\frac{2}{3}\arctan\left(\sqrt{x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^3-1)^(1/2),x, algorithm="fricas")
[Out] 2/3*arctan(sqrt(x^3 - 1))
```

giac [A] time = 0.15, size = 10, normalized size = 0.71

$$\frac{2}{3}\arctan\left(\sqrt{x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^3-1)^(1/2),x, algorithm="giac")
[Out] 2/3*arctan(sqrt(x^3 - 1))
```

maple [A] time = 0.10, size = 11, normalized size = 0.79

$$\frac{2\arctan\left(\sqrt{x^3-1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^3-1)^(1/2),x)
[Out] 2/3*arctan((x^3-1)^(1/2))
```

maxima [A] time = 2.90, size = 10, normalized size = 0.71

$$\frac{2}{3}\arctan\left(\sqrt{x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^3-1)^(1/2),x, algorithm="maxima")
[Out] 2/3*arctan(sqrt(x^3 - 1))
```

mupad [B] time = 0.04, size = 164, normalized size = 11.71

$$-\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\operatorname{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^3 - 1)^(1/2)),x)
[Out] -((3^(1/2)*1i + 3)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/
2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin((-(x - 1)/(
(3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/
```

```
2)))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 -
1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)
```

sympy [**A**] time = 1.35, size = 31, normalized size = 2.21

$$\begin{cases} \dfrac{2i\,\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \dfrac{1}{|x^3|} > 1 \\ -\dfrac{2\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**3-1)**(1/2),x)
```

```
[Out] Piecewise((2*I*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (-2*asin(x**(-3/2))/3,
 True))
```

3.479 $\int \frac{1}{x^4\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=31

$$\frac{\sqrt{x^3-1}}{3x^3}+\frac{1}{3}\tan^{-1}\left(\sqrt{x^3-1}\right)$$

```
[Out] 1/3*arctan((x^3-1)^(1/2))+1/3*(x^3-1)^(1/2)/x^3
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {266, 51, 63, 203}

$$\frac{\sqrt{x^3-1}}{3x^3}+\frac{1}{3}\tan^{-1}\left(\sqrt{x^3-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[-1 + x^3]),x]
```

```
[Out] Sqrt[-1 + x^3]/(3*x^3) + ArcTan[Sqrt[-1 + x^3]]/3
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{-1+x^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x^2}\,dx, x, x^3\right)\\ &= \frac{\sqrt{-1+x^3}}{3x^3} + \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx, x, x^3\right)\\ &= \frac{\sqrt{-1+x^3}}{3x^3} + \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x^3}\right)\\ &= \frac{\sqrt{-1+x^3}}{3x^3} + \frac{1}{3}\tan^{-1}\left(\sqrt{-1+x^3}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 41, normalized size = 1.32

$$\frac{1}{3}\sqrt{x^3-1}\left(\frac{1}{x^3} + \frac{\tanh^{-1}\left(\sqrt{1-x^3}\right)}{\sqrt{1-x^3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[-1 + x^3]),x]
[Out] (Sqrt[-1 + x^3]*(x^(-3) + ArcTanh[Sqrt[1 - x^3]]/Sqrt[1 - x^3]))/3
```

fricas [A] time = 0.79, size = 25, normalized size = 0.81

$$\frac{x^3\arctan\left(\sqrt{x^3-1}\right) + \sqrt{x^3-1}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^3-1)^(1/2),x, algorithm="fricas")
[Out] 1/3*(x^3*arctan(sqrt(x^3 - 1)) + sqrt(x^3 - 1))/x^3
```

giac [A] time = 0.15, size = 23, normalized size = 0.74

$$\frac{\sqrt{x^3-1}}{3\,x^3} + \frac{1}{3}\arctan\left(\sqrt{x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^3-1)^(1/2),x, algorithm="giac")
[Out] 1/3*sqrt(x^3 - 1)/x^3 + 1/3*arctan(sqrt(x^3 - 1))
```

maple [A] time = 0.02, size = 24, normalized size = 0.77

$$\frac{\arctan\left(\sqrt{x^3-1}\right)}{3} + \frac{\sqrt{x^3-1}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^3-1)^(1/2),x)
[Out] 1/3*arctan((x^3-1)^(1/2))+1/3*(x^3-1)^(1/2)/x^3
```

maxima [A] time = 3.00, size = 23, normalized size = 0.74

$$\frac{\sqrt{x^3-1}}{3\,x^3} + \frac{1}{3}\arctan\left(\sqrt{x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(x^3 - 1)/x^3 + 1/3*arctan(sqrt(x^3 - 1))
```

mupad [B] time = 1.00, size = 177, normalized size = 5.71

$$\frac{\sqrt{x^3-1}}{3x^3}-\frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\operatorname{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x^3 - 1)^(1/2)),x)
```

```
[Out] (x^3 - 1)^(1/2)/(3*x^3) - (((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1
/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)
/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2
)*1i)/2 + 3/2, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i
)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2
 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2
)
```

sympy [A] time = 3.64, size = 82, normalized size = 2.65

$$\begin{cases} \dfrac{i\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}+\dfrac{i\sqrt{-1+\frac{1}{x^3}}}{3x^{\frac{3}{2}}} & \text{for } \dfrac{1}{|x^3|}>1 \\ -\dfrac{\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}+\dfrac{1}{3x^{\frac{3}{2}}\sqrt{1-\frac{1}{x^3}}}-\dfrac{1}{3x^{\frac{9}{2}}\sqrt{1-\frac{1}{x^3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**3-1)**(1/2),x)
```

```
[Out] Piecewise((I*acosh(x**(-3/2))/3 + I*sqrt(-1 + x**(-3))/(3*x**(3/2)), 1/Abs(
x**3) > 1), (-asin(x**(-3/2))/3 + 1/(3*x**(3/2)*sqrt(1 - 1/x**3)) - 1/(3*x*
*(9/2)*sqrt(1 - 1/x**3)), True))
```

3.480 $\int \frac{1}{x^7 \sqrt{-1+x^3}}\, dx$

Optimal. Leaf size=47

$$\frac{\sqrt{x^3-1}}{4x^3} + \frac{1}{4}\tan^{-1}\left(\sqrt{x^3-1}\right) + \frac{\sqrt{x^3-1}}{6x^6}$$

```
[Out] 1/4*arctan((x^3-1)^(1/2))+1/6*(x^3-1)^(1/2)/x^6+1/4*(x^3-1)^(1/2)/x^3
```

Rubi [A] time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {266, 51, 63, 203}

$$\frac{\sqrt{x^3-1}}{4x^3} + \frac{\sqrt{x^3-1}}{6x^6} + \frac{1}{4}\tan^{-1}\left(\sqrt{x^3-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*Sqrt[-1 + x^3]),x]
```

```
[Out] Sqrt[-1 + x^3]/(6*x^6) + Sqrt[-1 + x^3]/(4*x^3) + ArcTan[Sqrt[-1 + x^3]]/4
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7\sqrt{-1+x^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x^3}\,dx, x, x^3\right)\\ &= \frac{\sqrt{-1+x^3}}{6x^6} + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x^2}\,dx, x, x^3\right)\\ &= \frac{\sqrt{-1+x^3}}{6x^6} + \frac{\sqrt{-1+x^3}}{4x^3} + \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx, x, x^3\right)\\ &= \frac{\sqrt{-1+x^3}}{6x^6} + \frac{\sqrt{-1+x^3}}{4x^3} + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x^3}\right)\\ &= \frac{\sqrt{-1+x^3}}{6x^6} + \frac{\sqrt{-1+x^3}}{4x^3} + \frac{1}{4}\tan^{-1}\left(\sqrt{-1+x^3}\right)\end{aligned}$$

Mathematica [C] time = 0.00, size = 28, normalized size = 0.60

$$\frac{2}{3}\sqrt{x^3-1}\;{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*Sqrt[-1 + x^3]),x]
[Out] (2*Sqrt[-1 + x^3]*Hypergeometric2F1[1/2, 3, 3/2, 1 - x^3])/3
```

fricas [A] time = 0.54, size = 34, normalized size = 0.72

$$\frac{3\,x^6\arctan\left(\sqrt{x^3-1}\right)+\left(3\,x^3+2\right)\sqrt{x^3-1}}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^3-1)^(1/2),x, algorithm="fricas")
[Out] 1/12*(3*x^6*arctan(sqrt(x^3 - 1)) + (3*x^3 + 2)*sqrt(x^3 - 1))/x^6
```

giac [A] time = 0.15, size = 35, normalized size = 0.74

$$\frac{3\left(x^3-1\right)^{\frac{3}{2}}+5\sqrt{x^3-1}}{12\,x^6}+\frac{1}{4}\arctan\left(\sqrt{x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^3-1)^(1/2),x, algorithm="giac")
[Out] 1/12*(3*(x^3 - 1)^(3/2) + 5*sqrt(x^3 - 1))/x^6 + 1/4*arctan(sqrt(x^3 - 1))
```

maple [A] time = 0.02, size = 36, normalized size = 0.77

$$\frac{\arctan\left(\sqrt{x^3-1}\right)}{4}+\frac{\sqrt{x^3-1}}{4x^3}+\frac{\sqrt{x^3-1}}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^3-1)^(1/2),x)
[Out] 1/4*arctan((x^3-1)^(1/2))+1/6*(x^3-1)^(1/2)/x^6+1/4*(x^3-1)^(1/2)/x^3
```

maxima [A] time = 2.91, size = 48, normalized size = 1.02

$$\frac{3\left(x^3-1\right)^{\frac{3}{2}}+5\sqrt{x^3-1}}{12\left(2x^3+\left(x^3-1\right)^2-1\right)}+\frac{1}{4}\arctan\left(\sqrt{x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(3*(x^3 - 1)^(3/2) + 5*sqrt(x^3 - 1))/(2*x^3 + (x^3 - 1)^2 - 1) + 1/4*
arctan(sqrt(x^3 - 1))
```

mupad [B] time = 1.00, size = 189, normalized size = 4.02

$$\frac{\sqrt{x^3-1}}{4x^3}+\frac{\sqrt{x^3-1}}{6x^6}-\frac{3\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\operatorname{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\sqrt{}}{-\frac{3}{2}+}\right.}{4\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(x^3 - 1)^(1/2)),x)
```

```
[Out] (x^3 - 1)^(1/2)/(4*x^3) + (x^3 - 1)^(1/2)/(6*x^6) - (3*((3^(1/2)*1i)/2 + 3/
2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2
)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/
2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin((-(x - 1)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(4*(((3^(1/
2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2
)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

sympy [A] time = 3.74, size = 138, normalized size = 2.94

$$\begin{cases}\frac{i\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{4}-\frac{i}{4x^{\frac{3}{2}}\sqrt{-1+\frac{1}{x^3}}}+\frac{i}{12x^{\frac{9}{2}}\sqrt{-1+\frac{1}{x^3}}}+\frac{i}{6x^{\frac{15}{2}}\sqrt{-1+\frac{1}{x^3}}} & \text{for } \frac{1}{|x^3|}>1\\ -\frac{\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{4}+\frac{1}{4x^{\frac{3}{2}}\sqrt{1-\frac{1}{x^3}}}-\frac{1}{12x^{\frac{9}{2}}\sqrt{1-\frac{1}{x^3}}}-\frac{1}{6x^{\frac{15}{2}}\sqrt{1-\frac{1}{x^3}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**3-1)**(1/2),x)
```

```
[Out] Piecewise((I*acosh(x**(-3/2))/4 - I/(4*x**(3/2)*sqrt(-1 + x**(-3))) + I/(12
*x**(9/2)*sqrt(-1 + x**(-3))) + I/(6*x**(15/2)*sqrt(-1 + x**(-3))), 1/Abs(x
**3) > 1), (-asin(x**(-3/2))/4 + 1/(4*x**(3/2)*sqrt(1 - 1/x**3)) - 1/(12*x*
*(9/2)*sqrt(1 - 1/x**3)) - 1/(6*x**(15/2)*sqrt(1 - 1/x**3)), True))
```

3.481 $\int \frac{1}{x^{10}\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=63

$$\frac{5\sqrt{x^3-1}}{24x^3}+\frac{5}{24}\tan^{-1}\left(\sqrt{x^3-1}\right)+\frac{\sqrt{x^3-1}}{9x^9}+\frac{5\sqrt{x^3-1}}{36x^6}$$

```
[Out] 5/24*arctan((x^3-1)^(1/2))+1/9*(x^3-1)^(1/2)/x^9+5/36*(x^3-1)^(1/2)/x^6+5/2
4*(x^3-1)^(1/2)/x^3
```

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {266, 51, 63, 203}

$$\frac{5\sqrt{x^3-1}}{24x^3}+\frac{5\sqrt{x^3-1}}{36x^6}+\frac{\sqrt{x^3-1}}{9x^9}+\frac{5}{24}\tan^{-1}\left(\sqrt{x^3-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*Sqrt[-1 + x^3]),x]
```

```
[Out] Sqrt[-1 + x^3]/(9*x^9) + (5*Sqrt[-1 + x^3])/(36*x^6) + (5*Sqrt[-1 + x^3])/(
24*x^3) + (5*ArcTan[Sqrt[-1 + x^3]])/24
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{10}\sqrt{-1+x^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x^4}\,dx, x, x^3\right)\\ &= \frac{\sqrt{-1+x^3}}{9x^9} + \frac{5}{18}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x^3}\,dx, x, x^3\right)\\ &= \frac{\sqrt{-1+x^3}}{9x^9} + \frac{5\sqrt{-1+x^3}}{36x^6} + \frac{5}{24}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x^2}\,dx, x, x^3\right)\\ &= \frac{\sqrt{-1+x^3}}{9x^9} + \frac{5\sqrt{-1+x^3}}{36x^6} + \frac{5\sqrt{-1+x^3}}{24x^3} + \frac{5}{48}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx, x, x^3\right)\\ &= \frac{\sqrt{-1+x^3}}{9x^9} + \frac{5\sqrt{-1+x^3}}{36x^6} + \frac{5\sqrt{-1+x^3}}{24x^3} + \frac{5}{24}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x^3}\right)\\ &= \frac{\sqrt{-1+x^3}}{9x^9} + \frac{5\sqrt{-1+x^3}}{36x^6} + \frac{5\sqrt{-1+x^3}}{24x^3} + \frac{5}{24}\tan^{-1}\left(\sqrt{-1+x^3}\right)\end{aligned}$$

Mathematica [C] time = 0.02, size = 28, normalized size = 0.44

$$\frac{2}{3}\sqrt{x^3-1}\;{}_2F_1\left(\frac{1}{2}, 4; \frac{3}{2}; 1-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^10*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[-1 + x^3]*Hypergeometric2F1[1/2, 4, 3/2, 1 - x^3])/3
```

fricas [A] time = 0.83, size = 39, normalized size = 0.62

$$\frac{15\,x^9\arctan\left(\sqrt{x^3-1}\right) + \left(15\,x^6+10\,x^3+8\right)\sqrt{x^3-1}}{72\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/72*(15*x^9*arctan(sqrt(x^3 - 1)) + (15*x^6 + 10*x^3 + 8)*sqrt(x^3 - 1))/x
^9
```

giac [A] time = 0.15, size = 44, normalized size = 0.70

$$\frac{15\left(x^3-1\right)^{\frac{5}{2}} + 40\left(x^3-1\right)^{\frac{3}{2}} + 33\sqrt{x^3-1}}{72\,x^9} + \frac{5}{24}\arctan\left(\sqrt{x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/72*(15*(x^3 - 1)^(5/2) + 40*(x^3 - 1)^(3/2) + 33*sqrt(x^3 - 1))/x^9 + 5/2
4*arctan(sqrt(x^3 - 1))
```

maple [A] time = 0.02, size = 48, normalized size = 0.76

$$\frac{5\arctan\left(\sqrt{x^3-1}\right)}{24} + \frac{5\sqrt{x^3-1}}{24x^3} + \frac{5\sqrt{x^3-1}}{36x^6} + \frac{\sqrt{x^3-1}}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(x^3-1)^(1/2),x)
```

```
[Out] 5/24*arctan((x^3-1)^(1/2))+1/9*(x^3-1)^(1/2)/x^9+5/36*(x^3-1)^(1/2)/x^6+5/2
4*(x^3-1)^(1/2)/x^3
```

maxima [A] time = 3.03, size = 66, normalized size = 1.05

$$\frac{15\left(x^3-1\right)^{\frac{5}{2}}+40\left(x^3-1\right)^{\frac{3}{2}}+33\sqrt{x^3-1}}{72\left(\left(x^3-1\right)^3+3x^3+3\left(x^3-1\right)^2-2\right)}+\frac{5}{24}\arctan\left(\sqrt{x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/72*(15*(x^3 - 1)^(5/2) + 40*(x^3 - 1)^(3/2) + 33*sqrt(x^3 - 1))/((x^3 - 1
)^3 + 3*x^3 + 3*(x^3 - 1)^2 - 2) + 5/24*arctan(sqrt(x^3 - 1))
```

mupad [B] time = 0.04, size = 201, normalized size = 3.19

$$\frac{5\sqrt{x^3-1}}{24x^3}+\frac{5\sqrt{x^3-1}}{36x^6}+\frac{\sqrt{x^3-1}}{9x^9}-\frac{5\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\operatorname{asin}\left(\sqrt{\right.\right.}\right.}{8\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^10*(x^3 - 1)^(1/2)),x)
```

```
[Out] (5*(x^3 - 1)^(1/2))/(24*x^3) + (5*(x^3 - 1)^(1/2))/(36*x^6) + (x^3 - 1)^(1/
2)/(9*x^9) - (5*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/
2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3
/2, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/
((3^(1/2)*1i)/2 - 3/2)))/(8*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
- x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

sympy [A] time = 5.57, size = 182, normalized size = 2.89

$$\begin{cases}\frac{5i\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{24}-\frac{5i}{24x^{\frac{3}{2}}\sqrt{-1+\frac{1}{x^3}}}+\frac{5i}{72x^{\frac{9}{2}}\sqrt{-1+\frac{1}{x^3}}}+\frac{i}{36x^{\frac{15}{2}}\sqrt{-1+\frac{1}{x^3}}}+\frac{i}{9x^{\frac{21}{2}}\sqrt{-1+\frac{1}{x^3}}} & \text{for } \frac{1}{|x^3|}>1\\ -\frac{5\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{24}+\frac{5}{24x^{\frac{3}{2}}\sqrt{1-\frac{1}{x^3}}}-\frac{5}{72x^{\frac{9}{2}}\sqrt{1-\frac{1}{x^3}}}-\frac{1}{36x^{\frac{15}{2}}\sqrt{1-\frac{1}{x^3}}}-\frac{1}{9x^{\frac{21}{2}}\sqrt{1-\frac{1}{x^3}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(x**3-1)**(1/2),x)
```

```
[Out] Piecewise((5*I*acosh(x**(-3/2))/24 - 5*I/(24*x**(3/2)*sqrt(-1 + x**(-3))) +
 5*I/(72*x**(9/2)*sqrt(-1 + x**(-3))) + I/(36*x**(15/2)*sqrt(-1 + x**(-3)))
 + I/(9*x**(21/2)*sqrt(-1 + x**(-3))), 1/Abs(x**3) > 1), (-5*asin(x**(-3/2)
)/24 + 5/(24*x**(3/2)*sqrt(1 - 1/x**3)) - 5/(72*x**(9/2)*sqrt(1 - 1/x**3))
- 1/(36*x**(15/2)*sqrt(1 - 1/x**3)) - 1/(9*x**(21/2)*sqrt(1 - 1/x**3)), Tru
e))
```

3.482 $\int \frac{x^6}{\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=153

$$\frac{16}{55}\sqrt{x^3-1}\,x+\frac{2}{11}\sqrt{x^3-1}\,x^4-\frac{32\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{55\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[Out] 16/55*x*(x^3-1)^(1/2)+2/11*x^4*(x^3-1)^(1/2)-32/165*(1-x)*EllipticF((1-x+3^
(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1
-x-3^(1/2))^2)^(1/2)*3^(3/4)/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {321, 219}

$$\frac{2}{11}\sqrt{x^3-1}\,x^4+\frac{16}{55}\sqrt{x^3-1}\,x-\frac{32\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{55\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/Sqrt[-1 + x^3],x]
```

```
[Out] (16*x*Sqrt[-1 + x^3])/55 + (2*x^4*Sqrt[-1 + x^3])/11 - (32*Sqrt[2 - Sqrt[3]
]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqr
t[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(55*3^(1/4)*Sqrt[-((1 - x)/(
1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^6}{\sqrt{-1+x^3}}\,dx = \frac{2}{11}x^4\sqrt{-1+x^3} + \frac{8}{11}\int \frac{x^3}{\sqrt{-1+x^3}}\,dx$$
$$= \frac{16}{55}x\sqrt{-1+x^3} + \frac{2}{11}x^4\sqrt{-1+x^3} + \frac{16}{55}\int \frac{1}{\sqrt{-1+x^3}}\,dx$$
$$= \frac{16}{55}x\sqrt{-1+x^3} + \frac{2}{11}x^4\sqrt{-1+x^3} - \frac{32\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7-\right.}{55\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.34

$$\frac{2x\left(8\sqrt{1-x^3}\;{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)+5x^6+3x^3-8\right)}{55\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/Sqrt[-1 + x^3],x]
```

```
[Out] (2*x*(-8 + 3*x^3 + 5*x^6 + 8*Sqrt[1 - x^3]*Hypergeometric2F1[1/3, 1/2, 4/3,
 x^3]))/(55*Sqrt[-1 + x^3])
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^6}{\sqrt{x^3-1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^6/sqrt(x^3 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/sqrt(x^3 - 1), x)
```

maple [A] time = 0.02, size = 139, normalized size = 0.91

$$\frac{2\sqrt{x^3-1}\;x^4}{11}+\frac{16\sqrt{x^3-1}\;x}{55}+\frac{32\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\;\text{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{55\sqrt{x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^3-1)^(1/2),x)
```

```
[Out] 2/11*x^4*(x^3-1)^(1/2)+16/55*x*(x^3-1)^(1/2)+32/55*(-3/2-1/2*I*3^(1/2))*((x
-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))
```

```
^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*Elli
pticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^
(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/sqrt(x^3 - 1), x)
```

mupad [B] time = 0.05, size = 179, normalized size = 1.17

$$\frac{16\,x\,\sqrt{x^3-1}}{55}+\frac{2\,x^4\,\sqrt{x^3-1}}{11}-\frac{32\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\sqrt{}}{-\frac{3}{2}+\sqrt{}}\right.}{55\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^3 - 1)^(1/2),x)
```

```
[Out] (16*x*(x^3 - 1)^(1/2))/55 + (2*x^4*(x^3 - 1)^(1/2))/11 - (32*((3^(1/2)*1i)/
2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (
3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/
2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -(
(3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(55*(((3^(1/2)*1i)/2 - 1/2)*
((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) +
 1) + x^3)^(1/2))
```

sympy [A] time = 0.99, size = 27, normalized size = 0.18

$$-\frac{ix^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\right|x^3\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**3-1)**(1/2),x)
```

```
[Out] -I*x**7*gamma(7/3)*hyper((1/2, 7/3), (10/3,), x**3)/(3*gamma(10/3))
```

3.483 $\int \frac{x^3}{\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=137

$$\frac{2}{5}x\sqrt{x^3-1}-\frac{4\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\,\sqrt{x^3-1}}$$

```
[Out] 2/5*x*(x^3-1)^(1/2)-4/15*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*
3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/4
)/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {321, 219}

$$\frac{2}{5}x\sqrt{x^3-1}-\frac{4\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\,\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[-1 + x^3],x]
```

```
[Out] (2*x*Sqrt[-1 + x^3])/5 - (4*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1
 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -
7 + 4*Sqrt[3]])/(5*3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x
^3])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{-1+x^3}}\,dx = \frac{2}{5}x\sqrt{-1+x^3} + \frac{2}{5}\int \frac{1}{\sqrt{-1+x^3}}\,dx$$

$$= \frac{2}{5}x\sqrt{-1+x^3} - \frac{4\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)\middle|-7+4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] time = 0.01, size = 44, normalized size = 0.32

$$\frac{2x\left(\sqrt{1-x^3}\;{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)+x^3-1\right)}{5\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[-1 + x^3],x]
```

```
[Out] (2*x*(-1 + x^3 + Sqrt[1 - x^3]*Hypergeometric2F1[1/3, 1/2, 4/3, x^3]))/(5*S
qrt[-1 + x^3])
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3}{\sqrt{x^3-1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^3/sqrt(x^3 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(x^3 - 1), x)
```

maple [A] time = 0.02, size = 127, normalized size = 0.93

$$\frac{2\sqrt{x^3-1}\,x}{5} + \frac{4\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\;\text{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{5\sqrt{x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^3-1)^(1/2),x)
```

```
[Out] 2/5*(x^3-1)^(1/2)*x+4/5*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(
1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2
))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^
(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(x^3 - 1), x)
```

mupad [B] time = 0.04, size = 167, normalized size = 1.22

$$\frac{2x\sqrt{x^3-1}}{5}-\frac{4\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{5\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^3 - 1)^(1/2),x)
```

```
[Out] (2*x*(x^3 - 1)^(1/2))/5 - (4*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 +
 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1
i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((
-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*
1i)/2 - 3/2)))/(5*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(
1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

sympy [A] time = 1.75, size = 27, normalized size = 0.20

$$-\frac{ix^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\right|x^3\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**3-1)**(1/2),x)
```

```
[Out] -I*x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), x**3)/(3*gamma(7/3))
```

3.484 $\int \frac{1}{\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=120

$$-\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\,\sqrt{x^3-1}}$$

```
[Out] -2/3*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2
)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(x^3-1)^(1/2)/((-1
+x)/(1-x-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {219}

$$-\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\,\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-1 + x^3],x]
```

```
[Out] (-2*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*Ellip
ticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)
*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+x^3}}\,dx = -\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}}$$

Mathematica [C] time = 0.00, size = 35, normalized size = 0.29

$$\frac{x\sqrt{1-x^3}\;{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)}{\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-1 + x^3],x]
```

```
[Out] (x*Sqrt[1 - x^3]*Hypergeometric2F1[1/3, 1/2, 4/3, x^3])/Sqrt[-1 + x^3]
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{x^3-1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(x^3 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(x^3 - 1), x)
```

maple [A] time = 0.02, size = 116, normalized size = 0.97

$$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\text{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3-1)^(1/2),x)
```

```
[Out] 2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(
1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/
2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(x^3 - 1), x)
```

mupad [B] time = 0.04, size = 156, normalized size = 1.30

$$-\frac{\left(3+\sqrt{3}\,1\text{i}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\text{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\text{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\text{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\text{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\text{i}}{2}}}\,\text{F}\left(\text{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\text{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\text{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\text{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\text{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\text{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\text{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\text{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3 - 1)^(1/2),x)
```

```
[Out] -((3^(1/2)*1i + 3)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/
2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1/2)*1i)/2 -
 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
1/2) + 1) + x^3)^(1/2)
```

sympy [**A**] time = 0.99, size = 26, normalized size = 0.22

$$-\frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3}, \frac{1}{2} \\ \frac{4}{3}\end{matrix}\right| x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3-1)**(1/2),x)
```

```
[Out] -I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3))
```

3.485 $\int \frac{1}{x^3\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=139

$$\frac{\sqrt{x^3-1}}{2x^2}-\frac{\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[Out] 1/2*(x^3-1)^(1/2)/x^2-1/6*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I
*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/
4)/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {325, 219}

$$\frac{\sqrt{x^3-1}}{2x^2}-\frac{\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[-1 + x^3]),x]
```

```
[Out] Sqrt[-1 + x^3]/(2*x^2) - (Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 -
 Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7
+ 4*Sqrt[3]])/(2*3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3
])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{-1+x^3}}\,dx = \frac{\sqrt{-1+x^3}}{2x^2} + \frac{1}{4}\int \frac{1}{\sqrt{-1+x^3}}\,dx$$
$$= \frac{\sqrt{-1+x^3}}{2x^2} - \frac{\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.29

$$-\frac{\sqrt{1-x^3}\;{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};x^3\right)}{2x^2\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[-1 + x^3]),x]
```

```
[Out] -1/2*(Sqrt[1 - x^3]*Hypergeometric2F1[-2/3, 1/2, 1/3, x^3])/(x^2*Sqrt[-1 +
x^3])
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^3-1}}{x^6-x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^3 - 1)/(x^6 - x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^3-1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^3 - 1)*x^3), x)
```

maple [A] time = 0.02, size = 129, normalized size = 0.93

$$\frac{\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\;\text{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{2\sqrt{x^3-1}} + \frac{\sqrt{x^3-1}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^3-1)^(1/2),x)
```

```
[Out] 1/2*(x^3-1)^(1/2)/x^2+1/2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))
^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1
/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*
3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^3-1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^3 - 1)*x^3), x)
```

mupad [B] time = 0.05, size = 169, normalized size = 1.22

$$\frac{\sqrt{x^3-1}}{2x^2}-\frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{2\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x^3 - 1)^(1/2)),x)
```

```
[Out] (x^3 - 1)^(1/2)/(2*x^2) - (((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1
/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)
/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(
x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i
)/2 - 3/2)))/(2*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/
2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

sympy [A] time = 1.58, size = 31, normalized size = 0.22

$$-\frac{i\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{1}{2}\\ \frac{1}{3}\end{matrix}\right|x^3\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**3-1)**(1/2),x)
```

```
[Out] -I*gamma(-2/3)*hyper((-2/3, 1/2), (1/3,), x**3)/(3*x**2*gamma(1/3))
```

3.486 $\int \frac{1}{x^6\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=155

$$\frac{\sqrt{x^3-1}}{5x^5}+\frac{7\sqrt{x^3-1}}{20x^2}-\frac{7\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{20\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[Out] 1/5*(x^3-1)^(1/2)/x^5+7/20*(x^3-1)^(1/2)/x^2-7/60*(1-x)*EllipticF((1-x+3^(1
/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x
-3^(1/2))^2)^(1/2)*3^(3/4)/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {325, 219}

$$\frac{7\sqrt{x^3-1}}{20x^2}+\frac{\sqrt{x^3-1}}{5x^5}-\frac{7\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{20\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*Sqrt[-1 + x^3]),x]
```

```
[Out] Sqrt[-1 + x^3]/(5*x^5) + (7*Sqrt[-1 + x^3])/(20*x^2) - (7*Sqrt[2 - Sqrt[3]]
*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt
[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(20*3^(1/4)*Sqrt[-((1 - x)/(1
 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^6\sqrt{-1+x^3}}\,dx = \frac{\sqrt{-1+x^3}}{5x^5} + \frac{7}{10}\int \frac{1}{x^3\sqrt{-1+x^3}}\,dx$$
$$= \frac{\sqrt{-1+x^3}}{5x^5} + \frac{7\sqrt{-1+x^3}}{20x^2} + \frac{7}{40}\int \frac{1}{\sqrt{-1+x^3}}\,dx$$
$$= \frac{\sqrt{-1+x^3}}{5x^5} + \frac{7\sqrt{-1+x^3}}{20x^2} - \frac{7\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right.}{20\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.26

$$-\frac{\sqrt{1-x^3}\;{}_2F_1\left(-\frac{5}{3},\frac{1}{2};-\frac{2}{3};x^3\right)}{5x^5\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*Sqrt[-1 + x^3]),x]
```

```
[Out] -1/5*(Sqrt[1 - x^3]*Hypergeometric2F1[-5/3, 1/2, -2/3, x^3])/(x^5*Sqrt[-1 +
 x^3])
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^3-1}}{x^9-x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^3 - 1)/(x^9 - x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^3-1}\,x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^3 - 1)*x^6), x)
```

maple [A] time = 0.02, size = 141, normalized size = 0.91

$$\frac{7\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\;\text{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{20\sqrt{x^3-1}} + \frac{7\sqrt{x^3-1}}{20x^2} + \frac{\sqrt{x^3-1}}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^3-1)^(1/2),x)
```

```
[Out] 1/5*(x^3-1)^(1/2)/x^5+7/20*(x^3-1)^(1/2)/x^2+7/20*(-3/2-1/2*I*3^(1/2))*((x-
1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^
```

```
(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*Ellip
ticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(
1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^3-1}\,x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^3 - 1)*x^6), x)
```

mupad [B] time = 0.99, size = 181, normalized size = 1.17

$$\frac{7\sqrt{x^3-1}}{20\,x^2}+\frac{\sqrt{x^3-1}}{5\,x^5}-\frac{7\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{20\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(x^3 - 1)^(1/2)),x)
```

```
[Out] (7*(x^3 - 1)^(1/2))/(20*x^2) + (x^3 - 1)^(1/2)/(5*x^5) - (7*((3^(1/2)*1i)/2
 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3
^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2
 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((
3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(20*(((3^(1/2)*1i)/2 - 1/2)*(
(3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) +
1) + x^3)^(1/2))
```

sympy [A] time = 2.20, size = 34, normalized size = 0.22

$$-\frac{i\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{5}{3},\frac{1}{2}\\-\frac{2}{3}\end{matrix}\right|x^3\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**3-1)**(1/2),x)
```

```
[Out] -I*gamma(-5/3)*hyper((-5/3, 1/2), (-2/3,), x**3)/(3*x**5*gamma(-2/3))
```

3.487 $\int \frac{x^7}{\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=294

$$-\frac{80\sqrt{x^3-1}}{91\left(-x-\sqrt{3}+1\right)}+\frac{2}{13}\sqrt{x^3-1}\,x^5+\frac{20}{91}\sqrt{x^3-1}\,x^2-\frac{80\sqrt{2}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right.}{91\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

```
[Out] 20/91*x^2*(x^3-1)^(1/2)+2/13*x^5*(x^3-1)^(1/2)-80/91*(x^3-1)^(1/2)/(1-x-3^(
1/2))-80/273*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*2^(
1/2)*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(x^3-1)^(1/2)/((-1+x)/(1-x-3
^(1/2))^2)^(1/2)+40/91*3^(1/4)*(1-x)*EllipticE((1-x+3^(1/2))/(1-x-3^(1/2)),
2*I-I*3^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/
(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {321, 304, 219, 1879}

$$\frac{2}{13}\sqrt{x^3-1}\,x^5+\frac{20}{91}\sqrt{x^3-1}\,x^2-\frac{80\sqrt{x^3-1}}{91\left(-x-\sqrt{3}+1\right)}-\frac{80\sqrt{2}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{91\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/Sqrt[-1 + x^3],x]
```

```
[Out] (-80*Sqrt[-1 + x^3])/(91*(1 - Sqrt[3] - x)) + (20*x^2*Sqrt[-1 + x^3])/91 +
(2*x^5*Sqrt[-1 + x^3])/13 + (40*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 +
 x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt
[3] - x)], -7 + 4*Sqrt[3]])/(91*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-
1 + x^3]) - (80*Sqrt[2]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*Ell
ipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(91*3^
(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{\sqrt{-1+x^3}}\,dx &= \frac{2}{13}x^5\sqrt{-1+x^3} + \frac{10}{13}\int \frac{x^4}{\sqrt{-1+x^3}}\,dx \\
&= \frac{20}{91}x^2\sqrt{-1+x^3} + \frac{2}{13}x^5\sqrt{-1+x^3} + \frac{40}{91}\int \frac{x}{\sqrt{-1+x^3}}\,dx \\
&= \frac{20}{91}x^2\sqrt{-1+x^3} + \frac{2}{13}x^5\sqrt{-1+x^3} - \frac{40}{91}\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx + \frac{1}{91}\left(40\sqrt{2\left(2+\sqrt{3}\right)}\right)\int \frac{1}{\sqrt{-1+x}} \\
&= -\frac{80\sqrt{-1+x^3}}{91\left(1-\sqrt{3}-x\right)} + \frac{20}{91}x^2\sqrt{-1+x^3} + \frac{2}{13}x^5\sqrt{-1+x^3} + \frac{40\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,E}{91\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.18

$$\frac{2x^2\left(10\sqrt{1-x^3}\;{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};x^3\right)+7x^6+3x^3-10\right)}{91\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/Sqrt[-1 + x^3],x]
```

```
[Out] (2*x^2*(-10 + 3*x^3 + 7*x^6 + 10*Sqrt[1 - x^3]*Hypergeometric2F1[1/2, 2/3,
5/3, x^3]))/(91*Sqrt[-1 + x^3])
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^7}{\sqrt{x^3-1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^7/sqrt(x^3 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^7/sqrt(x^3 - 1), x)
```

maple [A] time = 0.02, size = 198, normalized size = 0.67

$$\frac{2\sqrt{x^3-1}\,x^5}{13}+\frac{20\sqrt{x^3-1}\,x^2}{91}+\frac{80\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\text{EllipticE}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right.\right.}{91\sqrt{x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^3-1)^(1/2),x)
```

```
[Out] 2/13*x^5*(x^3-1)^(1/2)+20/91*x^2*(x^3-1)^(1/2)+80/91*(-3/2-1/2*I*3^(1/2))*(
(x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)
))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*((
3/2-1/2*I*3^(1/2))*EllipticE(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I
*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+(-1/2+1/2*I*3^(1/2))*EllipticF(((x-1)
/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2
)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^7/sqrt(x^3 - 1), x)
```

mupad [B] time = 0.97, size = 240, normalized size = 0.82

$$\frac{20\,x^2\,\sqrt{x^3-1}}{91}+\frac{2\,x^5\,\sqrt{x^3-1}}{13}-\frac{80\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\right.\right.\right.}}{91\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^3 - 1)^(1/2),x)
```

```
[Out] (20*x^2*(x^3 - 1)^(1/2))/91 + (2*x^5*(x^3 - 1)^(1/2))/13 - (80*(((3^(1/2)*1
i)/2 - 1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(
1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1i)/2 - 3/2)*elliptic
E(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((
3^(1/2)*1i)/2 - 3/2)))*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/
((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(91*(((3^(1/2)*1i)/2
- 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
 1/2) + 1) + x^3)^(1/2))
```

sympy [A] time = 1.64, size = 27, normalized size = 0.09

$$-\frac{ix^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\right|x^3\right)}{3\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**3-1)**(1/2),x)
[Out] -I*x**8*gamma(8/3)*hyper((1/2, 8/3), (11/3,), x**3)/(3*gamma(11/3))
```

3.488 $\int \frac{x^4}{\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=278

$$-\frac{8\sqrt{x^3-1}}{7\left(-x-\sqrt{3}+1\right)}+\frac{2}{7}\sqrt{x^3-1}\,x^2-\frac{8\sqrt{2}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{7\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}+\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(\cdots}{\cdots}$$

```
[Out] 2/7*x^2*(x^3-1)^(1/2)-8/7*(x^3-1)^(1/2)/(1-x-3^(1/2))-8/21*(1-x)*EllipticF(
(1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*2^(1/2)*((x^2+x+1)/(1-x-3^(1/2))
^2)^(1/2)*3^(3/4)/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)+4/7*3^(1/4)*
(1-x)*EllipticE((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*((x^2+x+1)/(1-x-
3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1
/2))^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {321, 304, 219, 1879}

$$\frac{2}{7}\sqrt{x^3-1}\,x^2-\frac{8\sqrt{x^3-1}}{7\left(-x-\sqrt{3}+1\right)}-\frac{8\sqrt{2}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{7\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}+\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[-1 + x^3],x]
```

```
[Out] (-8*Sqrt[-1 + x^3])/(7*(1 - Sqrt[3] - x)) + (2*x^2*Sqrt[-1 + x^3])/7 + (4*3
^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*El
lipticE[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(7*Sq
rt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]) - (8*Sqrt[2]*(1 - x)*Sqr
t[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1
 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(7*3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] -
x)^2)]*Sqrt[-1 + x^3])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
 + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{-1+x^3}}\,dx = \frac{2}{7}x^2\sqrt{-1+x^3} + \frac{4}{7}\int \frac{x}{\sqrt{-1+x^3}}\,dx$$

$$= \frac{2}{7}x^2\sqrt{-1+x^3} - \frac{4}{7}\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx + \frac{1}{7}\left(4\sqrt{2\left(2+\sqrt{3}\right)}\right)\int \frac{1}{\sqrt{-1+x^3}}\,dx$$

$$= -\frac{8\sqrt{-1+x^3}}{7\left(1-\sqrt{3}-x\right)} + \frac{2}{7}x^2\sqrt{-1+x^3} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7\right.}{7\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}}$$

Mathematica [C] time = 0.01, size = 46, normalized size = 0.17

$$\frac{2x^2\left(\sqrt{1-x^3}\;{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};x^3\right)+x^3-1\right)}{7\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[-1 + x^3],x]
```

```
[Out] (2*x^2*(-1 + x^3 + Sqrt[1 - x^3]*Hypergeometric2F1[1/2, 2/3, 5/3, x^3]))/(7
*Sqrt[-1 + x^3])
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^4}{\sqrt{x^3-1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^4/sqrt(x^3 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(x^3 - 1), x)
```

maple [A] time = 0.02, size = 186, normalized size = 0.67

$$\frac{2\sqrt{x^3-1}\,x^2}{7}+\frac{8\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\text{EllipticE}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\right.\right.}{7\sqrt{x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^3-1)^(1/2),x)
```

```
[Out] 2/7*(x^3-1)^(1/2)*x^2+8/7*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))
^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1
/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*((3/2-1/2*I*3^(1/2))*Elliptic
E(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2
)))^(1/2))+(-1/2+1/2*I*3^(1/2))*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2
),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(x^3 - 1), x)
```

mupad [B] time = 0.97, size = 228, normalized size = 0.82

$$\frac{2x^2\sqrt{x^3-1}}{7}-\frac{8\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\sqrt{}}{-\frac{3}{2}+}\right.\right.}{7\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{}{}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^3 - 1)^(1/2),x)
```

```
[Out] (2*x^2*(x^3 - 1)^(1/2))/7 - (8*(((3^(1/2)*1i)/2 - 1/2)*ellipticF(asin((-(x
- 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/
2 - 3/2)) - ((3^(1/2)*1i)/2 - 3/2)*ellipticE(asin((-(x - 1)/((3^(1/2)*1i)/2
 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*((3^(1/2)
*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((
x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)
*1i)/2 + 3/2))^(1/2))/(7*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x
*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

sympy [A] time = 1.28, size = 27, normalized size = 0.10

$$-\frac{ix^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\right|x^3\right)}{3\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**3-1)**(1/2),x)
[Out] -I*x**5*gamma(5/3)*hyper((1/2, 5/3), (8/3,), x**3)/(3*gamma(8/3))
```

3.489 $\int \frac{x}{\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=255

$$-\frac{2\sqrt{x^3-1}}{-x-\sqrt{3}+1}-\frac{2\sqrt{2}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+\right.}}}$$

```
[Out] -2*(x^3-1)^(1/2)/(1-x-3^(1/2))-2/3*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/
2)),2*I-I*3^(1/2))*2^(1/2)*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(x^3-1
)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)+3^(1/4)*(1-x)*EllipticE((1-x+3^(1/2)
)/(1-x-3^(1/2)),2*I-I*3^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*(1/2*6^(1/
2)+1/2*2^(1/2))/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {304, 219, 1879}

$$-\frac{2\sqrt{x^3-1}}{-x-\sqrt{3}+1}-\frac{2\sqrt{2}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[-1 + x^3],x]
```

```
[Out] (-2*Sqrt[-1 + x^3])/(1 - Sqrt[3] - x) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*
Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] - x)/
(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*
Sqrt[-1 + x^3]) - (2*Sqrt[2]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2
]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(
3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
```

```
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{-1+x^3}}\,dx = \sqrt{2\left(2+\sqrt{3}\right)}\int \frac{1}{\sqrt{-1+x^3}}\,dx - \int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx$$

$$= -\frac{2\sqrt{-1+x^3}}{1-\sqrt{3}-x} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}} - \frac{2\sqrt{2}\,(1-x$$

Mathematica [C] time = 0.00, size = 40, normalized size = 0.16

$$\frac{x^2\sqrt{1-x^3}\;{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};x^3\right)}{2\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[-1 + x^3],x]
```

```
[Out] (x^2*Sqrt[1 - x^3]*Hypergeometric2F1[1/2, 2/3, 5/3, x^3])/(2*Sqrt[-1 + x^3]
)
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{\sqrt{x^3-1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x/sqrt(x^3 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(x^3 - 1), x)
```

maple [A] time = 0.02, size = 173, normalized size = 0.68

$$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\text{EllipticE}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\text{Ellipti}\right.}{\sqrt{x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^3-1)^(1/2),x)
```

```
[Out] 2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(
1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3-1)^(1/2)*((3/2-1/2*I*3^(1/2))*EllipticE(((x-1)/(-3/2-1/2*I*3^(
1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+(-1/2+1/2*I*3
^(1/2))*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(
3/2-1/2*I*3^(1/2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(x^3 - 1), x)
```

mupad [B] time = 0.04, size = 215, normalized size = 0.84

$$-\frac{2\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)\right)\left(\frac{3}{2}+\frac{\sqrt{}}{}\right.}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}}{2}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^3 - 1)^(1/2),x)
```

```
[Out] -(2*(((3^(1/2)*1i)/2 - 1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1i)/2
 - 3/2)*ellipticE(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)
*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/
2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/(
(3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(((3^
(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(
1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)
```

sympy [A] time = 1.12, size = 27, normalized size = 0.11

$$-\frac{ix^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|x^3\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**3-1)**(1/2),x)
```

```
[Out] -I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3)/(3*gamma(5/3))
```

3.490 $\int \frac{1}{x^2\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=269

$$\frac{\sqrt{x^3-1}}{-x-\sqrt{3}+1}+\frac{\sqrt{x^3-1}}{x}+\frac{\sqrt{2}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\,\sqrt{x^3-1}}-\frac{\sqrt[4]{3}\,\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x}{(-x-\sqrt{3}}}}{2\sqrt{-\frac{}{(-}}}$$

```
[Out] (x^3-1)^(1/2)/x+(x^3-1)^(1/2)/(1-x-3^(1/2))+1/3*(1-x)*EllipticF((1-x+3^(1/2
))/(1-x-3^(1/2)),2*I-I*3^(1/2))*2^(1/2)*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3
^(3/4)/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)-1/2*3^(1/4)*(1-x)*Ellip
ticE((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)
^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/
2)
```

Rubi [A] time = 0.06, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {325, 304, 219, 1879}

$$\frac{\sqrt{x^3-1}}{-x-\sqrt{3}+1}+\frac{\sqrt{x^3-1}}{x}+\frac{\sqrt{2}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\,\sqrt{x^3-1}}-\frac{\sqrt[4]{3}\,\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x}{(-x-\sqrt{3}}}}{2\sqrt{-\frac{}{(-}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[-1 + x^3]),x]
```

```
[Out] Sqrt[-1 + x^3]/(1 - Sqrt[3] - x) + Sqrt[-1 + x^3]/x - (3^(1/4)*Sqrt[2 + Sqr
t[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 +
 Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(2*Sqrt[-((1 - x)/(1 - S
qrt[3] - x)^2)]*Sqrt[-1 + x^3]) + (Sqrt[2]*(1 - x)*Sqrt[(1 + x + x^2)/(1 -
Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 +
 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{-1+x^3}}\,dx &= \frac{\sqrt{-1+x^3}}{x} - \frac{1}{2}\int \frac{x}{\sqrt{-1+x^3}}\,dx \\ &= \frac{\sqrt{-1+x^3}}{x} + \frac{1}{2}\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx - \sqrt{\frac{1}{2}\left(2+\sqrt{3}\right)}\int \frac{1}{\sqrt{-1+x^3}}\,dx \\ &= \frac{\sqrt{-1+x^3}}{1-\sqrt{3}-x} + \frac{\sqrt{-1+x^3}}{x} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\right.}{2\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 38, normalized size = 0.14

$$-\frac{\sqrt{1-x^3}\;{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};x^3\right)}{x\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[-1 + x^3]),x]
```

```
[Out] -((Sqrt[1 - x^3]*Hypergeometric2F1[-1/3, 1/2, 2/3, x^3])/(x*Sqrt[-1 + x^3])
)
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^3-1}}{x^5-x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^3 - 1)/(x^5 - x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^3-1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^3 - 1)*x^2), x)
```

maple [A] time = 0.02, size = 185, normalized size = 0.69

$$\frac{\sqrt{x^3-1}}{x}-\frac{\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\text{EllipticE}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right.\right.}{\sqrt{x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^3-1)^(1/2),x)
```

```
[Out] (x^3-1)^(1/2)/x-(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x
+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+
1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*((3/2-1/2*I*3^(1/2))*EllipticE(((x-1)/(
-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
+(-1/2+1/2*I*3^(1/2))*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/
2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^3-1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^3 - 1)*x^2), x)
```

mupad [B] time = 1.00, size = 226, normalized size = 0.84

$$\frac{\sqrt{x^3-1}}{x}+\frac{\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)\left(\frac{3}{2}\right.\right.}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}-\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^3 - 1)^(1/2)),x)
```

```
[Out] (x^3 - 1)^(1/2)/x + ((((3^(1/2)*1i)/2 - 1/2)*ellipticF(asin((-(x - 1)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))
- ((3^(1/2)*1i)/2 - 3/2)*ellipticE(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(
1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*((3^(1/2)*1i)/2 + 3
/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/
2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3
/2))^(1/2))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1
i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)
```

sympy [A] time = 0.94, size = 29, normalized size = 0.11

$$-\frac{i\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},\frac{1}{2}\\ \frac{2}{3}\end{matrix}\right|x^3\right)}{3x\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**3-1)**(1/2),x)
```

```
[Out] -I*gamma(-1/3)*hyper((-1/3, 1/2), (2/3,), x**3)/(3*x*gamma(2/3))
```

3.491 $\int \frac{1}{x^5\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=294

$$\frac{5\sqrt{x^3-1}}{8\left(-x-\sqrt{3}+1\right)}+\frac{5\sqrt{x^3-1}}{8x}+\frac{\sqrt{x^3-1}}{4x^4}+\frac{5(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{4\sqrt{2}\,\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\,\sqrt{x^3-1}}-\frac{5\sqrt[4]{3}\sqrt{2+\sqrt{}}}{}$$

```
[Out] 1/4*(x^3-1)^(1/2)/x^4+5/8*(x^3-1)^(1/2)/x+5/8*(x^3-1)^(1/2)/(1-x-3^(1/2))+5
/24*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*2^(1/2)*((x^
2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2
)^(1/2)-5/16*3^(1/4)*(1-x)*EllipticE((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1
/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/(x^3-1)^(1
/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {325, 304, 219, 1879}

$$\frac{5\sqrt{x^3-1}}{8\left(-x-\sqrt{3}+1\right)}+\frac{5\sqrt{x^3-1}}{8x}+\frac{\sqrt{x^3-1}}{4x^4}+\frac{5(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{4\sqrt{2}\,\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\,\sqrt{x^3-1}}-\frac{5\sqrt[4]{3}\sqrt{2+\sqrt{}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[-1 + x^3]),x]
```

```
[Out] (5*Sqrt[-1 + x^3])/(8*(1 - Sqrt[3] - x)) + Sqrt[-1 + x^3]/(4*x^4) + (5*Sqrt
[-1 + x^3])/(8*x) - (5*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)
/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)]
, -7 + 4*Sqrt[3]])/(16*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
 + (5*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 +
 Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(4*Sqrt[2]*3^(1/4)*Sqrt[
-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\sqrt{-1+x^3}}\,dx &= \frac{\sqrt{-1+x^3}}{4x^4} + \frac{5}{8}\int \frac{1}{x^2\sqrt{-1+x^3}}\,dx \\
&= \frac{\sqrt{-1+x^3}}{4x^4} + \frac{5\sqrt{-1+x^3}}{8x} - \frac{5}{16}\int \frac{x}{\sqrt{-1+x^3}}\,dx \\
&= \frac{\sqrt{-1+x^3}}{4x^4} + \frac{5\sqrt{-1+x^3}}{8x} + \frac{5}{16}\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx - \frac{1}{8}\left(5\sqrt{\frac{1}{2}\left(2+\sqrt{3}\right)}\right)\int \frac{1}{\sqrt{-1+x^3}}\,dx \\
&= \frac{5\sqrt{-1+x^3}}{8\left(1-\sqrt{3}-x\right)} + \frac{\sqrt{-1+x^3}}{4x^4} + \frac{5\sqrt{-1+x^3}}{8x} - \frac{5\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1}{1}\right.\right.}{16\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.14

$$-\frac{\sqrt{1-x^3}\;{}_2F_1\left(-\frac{4}{3},\frac{1}{2};-\frac{1}{3};x^3\right)}{4x^4\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[-1 + x^3]),x]
```

```
[Out] -1/4*(Sqrt[1 - x^3]*Hypergeometric2F1[-4/3, 1/2, -1/3, x^3])/(x^4*Sqrt[-1 +
 x^3])
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^3-1}}{x^8-x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^3 - 1)/(x^8 - x^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^3-1}\,x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^3 - 1)*x^5), x)
```

maple [A] time = 0.02, size = 198, normalized size = 0.67

$$\frac{5\sqrt{x^3-1}}{8x}+\frac{\sqrt{x^3-1}}{4x^4}-\frac{5\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\text{EllipticE}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i}{2}}{\frac{3}{2}-\frac{i}{2}}}\right.\right.}{8\sqrt{x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^3-1)^(1/2),x)
```

```
[Out] 1/4*(x^3-1)^(1/2)/x^4+5/8*(x^3-1)^(1/2)/x-5/8*(-3/2-1/2*I*3^(1/2))*((x-1)/(
-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2
)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*((3/2-1/2
*I*3^(1/2))*EllipticE(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2
))/(3/2-1/2*I*3^(1/2)))^(1/2))+(-1/2+1/2*I*3^(1/2))*EllipticF(((x-1)/(-3/2-
1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^3-1}\,x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^3 - 1)*x^5), x)
```

mupad [B] time = 1.00, size = 240, normalized size = 0.82

$$\frac{5\sqrt{x^3-1}}{8x}+\frac{\sqrt{x^3-1}}{4x^4}+\frac{5\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\text{i}}{2}\right)\text{F}\left(\text{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\text{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\text{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\text{i}}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1\text{i}}{2}\right)\text{E}\left(\text{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\text{i}}{2}}}\right.\right.\right.}{8\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\text{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\text{i}}{2}\right)-1\right)x+(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(x^3 - 1)^(1/2)),x)
```

```
[Out] (5*(x^3 - 1)^(1/2))/(8*x) + (x^3 - 1)^(1/2)/(4*x^4) + (5*(((3^(1/2)*1i)/2 -
 1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1
i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1i)/2 - 3/2)*ellipticE(asin
((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2
)*1i)/2 - 3/2)))*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1
/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))
^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(8*(((3^(1/2)*1i)/2 - 1/2)*
((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) +
 1) + x^3)^(1/2))
```

sympy [A] time = 1.00, size = 34, normalized size = 0.12

$$-\frac{i\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{4}{3},\frac{1}{2}\\-\frac{1}{3}\end{matrix}\right|x^3\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**3-1)**(1/2),x)
[Out] -I*gamma(-4/3)*hyper((-4/3, 1/2), (-1/3,), x**3)/(3*x**4*gamma(-1/3))
```

3.492 $\int \frac{x^{11}}{\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=61

$$\frac{2}{21}\left(-x^3-1\right)^{7/2}+\frac{2}{5}\left(-x^3-1\right)^{5/2}+\frac{2}{3}\left(-x^3-1\right)^{3/2}+\frac{2}{3}\sqrt{-x^3-1}$$

```
[Out] 2/3*(-x^3-1)^(3/2)+2/5*(-x^3-1)^(5/2)+2/21*(-x^3-1)^(7/2)+2/3*(-x^3-1)^(1/2
)
```

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{2}{21}\left(-x^3-1\right)^{7/2}+\frac{2}{5}\left(-x^3-1\right)^{5/2}+\frac{2}{3}\left(-x^3-1\right)^{3/2}+\frac{2}{3}\sqrt{-x^3-1}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/Sqrt[-1 - x^3],x]
```

```
[Out] (2*Sqrt[-1 - x^3])/3 + (2*(-1 - x^3)^(3/2))/3 + (2*(-1 - x^3)^(5/2))/5 + (2
*(-1 - x^3)^(7/2))/21
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{\sqrt{-1-x^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^3}{\sqrt{-1-x}}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{1}{\sqrt{-1-x}}-3\sqrt{-1-x}-3(-1-x)^{3/2}-(-1-x)^{5/2}\right)dx, x, x^3\right)\\ &= \frac{2}{3}\sqrt{-1-x^3}+\frac{2}{3}\left(-1-x^3\right)^{3/2}+\frac{2}{5}\left(-1-x^3\right)^{5/2}+\frac{2}{21}\left(-1-x^3\right)^{7/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 0.52

$$-\frac{2}{105}\sqrt{-x^3-1}\left(5x^9-6x^6+8x^3-16\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/Sqrt[-1 - x^3],x]
```

```
[Out] (-2*Sqrt[-1 - x^3]*(-16 + 8*x^3 - 6*x^6 + 5*x^9))/105
```

fricas [A] time = 0.81, size = 28, normalized size = 0.46

$$-\frac{2}{105}\left(5x^9-6x^6+8x^3-16\right)\sqrt{-x^3-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/105*(5*x^9 - 6*x^6 + 8*x^3 - 16)*sqrt(-x^3 - 1)
```

giac [A] time = 0.17, size = 59, normalized size = 0.97

$$-\frac{2}{21}\left(x^3+1\right)^3\sqrt{-x^3-1}+\frac{2}{5}\left(x^3+1\right)^2\sqrt{-x^3-1}+\frac{2}{3}\left(-x^3-1\right)^{\frac{3}{2}}+\frac{2}{3}\sqrt{-x^3-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] -2/21*(x^3 + 1)^3*sqrt(-x^3 - 1) + 2/5*(x^3 + 1)^2*sqrt(-x^3 - 1) + 2/3*(-x
^3 - 1)^(3/2) + 2/3*sqrt(-x^3 - 1)
```

maple [A] time = 0.01, size = 40, normalized size = 0.66

$$\frac{2\left(x+1\right)\left(x^2-x+1\right)\left(5x^9-6x^6+8x^3-16\right)}{105\sqrt{-x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(-x^3-1)^(1/2),x)
```

```
[Out] 2/105*(x+1)*(x^2-x+1)*(5*x^9-6*x^6+8*x^3-16)/(-x^3-1)^(1/2)
```

maxima [A] time = 1.32, size = 45, normalized size = 0.74

$$\frac{2}{21}\left(-x^3-1\right)^{\frac{7}{2}}+\frac{2}{5}\left(-x^3-1\right)^{\frac{5}{2}}+\frac{2}{3}\left(-x^3-1\right)^{\frac{3}{2}}+\frac{2}{3}\sqrt{-x^3-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/21*(-x^3 - 1)^(7/2) + 2/5*(-x^3 - 1)^(5/2) + 2/3*(-x^3 - 1)^(3/2) + 2/3*s
qrt(-x^3 - 1)
```

mupad [B] time = 0.03, size = 54, normalized size = 0.89

$$\frac{4x^6\sqrt{-x^3-1}}{35}-\frac{16x^3\sqrt{-x^3-1}}{105}-\frac{2x^9\sqrt{-x^3-1}}{21}+\frac{32\sqrt{-x^3-1}}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(- x^3 - 1)^(1/2),x)
```

```
[Out] (4*x^6*(- x^3 - 1)^(1/2))/35 - (16*x^3*(- x^3 - 1)^(1/2))/105 - (2*x^9*(- x
^3 - 1)^(1/2))/21 + (32*(- x^3 - 1)^(1/2))/105
```

sympy [A] time = 1.90, size = 63, normalized size = 1.03

$$-\frac{2x^9\sqrt{-x^3-1}}{21}+\frac{4x^6\sqrt{-x^3-1}}{35}-\frac{16x^3\sqrt{-x^3-1}}{105}+\frac{32\sqrt{-x^3-1}}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(-x**3-1)**(1/2),x)
```

```
[Out] -2*x**9*sqrt(-x**3 - 1)/21 + 4*x**6*sqrt(-x**3 - 1)/35 - 16*x**3*sqrt(-x**3
 - 1)/105 + 32*sqrt(-x**3 - 1)/105
```

3.493 $\int \frac{x^8}{\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=46

$$-\frac{2}{15}\left(-x^3-1\right)^{5/2}-\frac{4}{9}\left(-x^3-1\right)^{3/2}-\frac{2}{3}\sqrt{-x^3-1}$$

```
[Out] -4/9*(-x^3-1)^(3/2)-2/15*(-x^3-1)^(5/2)-2/3*(-x^3-1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{2}{15}\left(-x^3-1\right)^{5/2}-\frac{4}{9}\left(-x^3-1\right)^{3/2}-\frac{2}{3}\sqrt{-x^3-1}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/Sqrt[-1 - x^3],x]
```

```
[Out] (-2*Sqrt[-1 - x^3])/3 - (4*(-1 - x^3)^(3/2))/9 - (2*(-1 - x^3)^(5/2))/15
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{\sqrt{-1-x^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{-1-x}}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{1}{\sqrt{-1-x}}+2\sqrt{-1-x}+(-1-x)^{3/2}\right)dx, x, x^3\right)\\ &= -\frac{2}{3}\sqrt{-1-x^3}-\frac{4}{9}\left(-1-x^3\right)^{3/2}-\frac{2}{15}\left(-1-x^3\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.59

$$-\frac{2}{45}\sqrt{-x^3-1}\left(3x^6-4x^3+8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/Sqrt[-1 - x^3],x]
```

```
[Out] (-2*Sqrt[-1 - x^3]*(8 - 4*x^3 + 3*x^6))/45
```

fricas [A] time = 0.61, size = 23, normalized size = 0.50

$$-\frac{2}{45}\left(3\,x^6-4\,x^3+8\right)\sqrt{-x^3-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/45*(3*x^6 - 4*x^3 + 8)*sqrt(-x^3 - 1)
```

giac [A] time = 0.15, size = 41, normalized size = 0.89

$$-\frac{2}{15}\left(x^3+1\right)^2\sqrt{-x^3-1}-\frac{4}{9}\left(-x^3-1\right)^{\frac{3}{2}}-\frac{2}{3}\sqrt{-x^3-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] -2/15*(x^3 + 1)^2*sqrt(-x^3 - 1) - 4/9*(-x^3 - 1)^(3/2) - 2/3*sqrt(-x^3 - 1
)
```

maple [A] time = 0.00, size = 35, normalized size = 0.76

$$\frac{2\left(x+1\right)\left(x^2-x+1\right)\left(3x^6-4x^3+8\right)}{45\sqrt{-x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-x^3-1)^(1/2),x)
```

```
[Out] 2/45*(x+1)*(x^2-x+1)*(3*x^6-4*x^3+8)/(-x^3-1)^(1/2)
```

maxima [A] time = 1.34, size = 34, normalized size = 0.74

$$-\frac{2}{15}\left(-x^3-1\right)^{\frac{5}{2}}-\frac{4}{9}\left(-x^3-1\right)^{\frac{3}{2}}-\frac{2}{3}\sqrt{-x^3-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/15*(-x^3 - 1)^(5/2) - 4/9*(-x^3 - 1)^(3/2) - 2/3*sqrt(-x^3 - 1)
```

mupad [B] time = 0.03, size = 23, normalized size = 0.50

$$-\frac{2\sqrt{-x^3-1}\,\left(3\,x^6-4\,x^3+8\right)}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(- x^3 - 1)^(1/2),x)
```

```
[Out] -(2*(- x^3 - 1)^(1/2)*(3*x^6 - 4*x^3 + 8))/45
```

sympy [A] time = 0.81, size = 46, normalized size = 1.00

$$-\frac{2x^6\sqrt{-x^3-1}}{15}+\frac{8x^3\sqrt{-x^3-1}}{45}-\frac{16\sqrt{-x^3-1}}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-x**3-1)**(1/2),x)
```

```
[Out] -2*x**6*sqrt(-x**3 - 1)/15 + 8*x**3*sqrt(-x**3 - 1)/45 - 16*sqrt(-x**3 - 1)
/45
```

3.494 $\int \frac{x^5}{\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=31

$$\frac{2}{9}\left(-x^3-1\right)^{3/2}+\frac{2}{3}\sqrt{-x^3-1}$$

```
[Out] 2/9*(-x^3-1)^(3/2)+2/3*(-x^3-1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{2}{9}\left(-x^3-1\right)^{3/2}+\frac{2}{3}\sqrt{-x^3-1}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[-1 - x^3],x]
```

```
[Out] (2*Sqrt[-1 - x^3])/3 + (2*(-1 - x^3)^(3/2))/9
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\sqrt{-1-x^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{\sqrt{-1-x}}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(-\frac{1}{\sqrt{-1-x}}-\sqrt{-1-x}\right)\,dx, x, x^3\right)\\ &= \frac{2}{3}\sqrt{-1-x^3}+\frac{2}{9}\left(-1-x^3\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.65

$$-\frac{2}{9}\sqrt{-x^3-1}\left(x^3-2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[-1 - x^3],x]
```

```
[Out] (-2*Sqrt[-1 - x^3]*(-2 + x^3))/9
```

fricas [A] time = 0.73, size = 16, normalized size = 0.52

$$-\frac{2}{9}\left(x^3-2\right)\sqrt{-x^3-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3-1)^(1/2),x, algorithm="fricas")
[Out] -2/9*(x^3 - 2)*sqrt(-x^3 - 1)
```

giac [A] time = 0.15, size = 23, normalized size = 0.74

$$\frac{2}{9}\left(-x^3-1\right)^{\frac{3}{2}}+\frac{2}{3}\sqrt{-x^3-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3-1)^(1/2),x, algorithm="giac")
[Out] 2/9*(-x^3 - 1)^(3/2) + 2/3*sqrt(-x^3 - 1)
```

maple [A] time = 0.01, size = 28, normalized size = 0.90

$$\frac{2\left(x+1\right)\left(x^2-x+1\right)\left(x^3-2\right)}{9\sqrt{-x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-x^3-1)^(1/2),x)
[Out] 2/9*(x+1)*(x^2-x+1)*(x^3-2)/(-x^3-1)^(1/2)
```

maxima [A] time = 1.32, size = 23, normalized size = 0.74

$$\frac{2}{9}\left(-x^3-1\right)^{\frac{3}{2}}+\frac{2}{3}\sqrt{-x^3-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3-1)^(1/2),x, algorithm="maxima")
[Out] 2/9*(-x^3 - 1)^(3/2) + 2/3*sqrt(-x^3 - 1)
```

mupad [B] time = 0.03, size = 16, normalized size = 0.52

$$-\frac{2\sqrt{-x^3-1}\ \left(x^3-2\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(- x^3 - 1)^(1/2),x)
[Out] -(2*(- x^3 - 1)^(1/2)*(x^3 - 2))/9
```

sympy [A] time = 0.66, size = 29, normalized size = 0.94

$$-\frac{2x^3\sqrt{-x^3-1}}{9}+\frac{4\sqrt{-x^3-1}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-x**3-1)**(1/2),x)
[Out] -2*x**3*sqrt(-x**3 - 1)/9 + 4*sqrt(-x**3 - 1)/9
```

3.495 $\int \frac{x^2}{\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=15

$$-\frac{2}{3}\sqrt{-x^3-1}$$

```
[Out] -2/3*(-x^3-1)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$-\frac{2}{3}\sqrt{-x^3-1}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[-1 - x^3],x]
```

```
[Out] (-2*Sqrt[-1 - x^3])/3
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{-1-x^3}}\,dx = -\frac{2}{3}\sqrt{-1-x^3}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$-\frac{2}{3}\sqrt{-x^3-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[-1 - x^3],x]
```

```
[Out] (-2*Sqrt[-1 - x^3])/3
```

fricas [A] time = 0.84, size = 11, normalized size = 0.73

$$-\frac{2}{3}\sqrt{-x^3-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(-x^3 - 1)
```

giac [A] time = 0.15, size = 11, normalized size = 0.73

$$-\frac{2}{3}\sqrt{-x^3-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3-1)^(1/2),x, algorithm="giac")
[Out] -2/3*sqrt(-x^3 - 1)
```

maple [A] time = 0.01, size = 23, normalized size = 1.53

$$\frac{2\,(x+1)\left(x^2-x+1\right)}{3\sqrt{-x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^3-1)^(1/2),x)
[Out] 2/3*(x+1)*(x^2-x+1)/(-x^3-1)^(1/2)
```

maxima [A] time = 1.24, size = 11, normalized size = 0.73

$$-\frac{2}{3}\sqrt{-x^3-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3-1)^(1/2),x, algorithm="maxima")
[Out] -2/3*sqrt(-x^3 - 1)
```

mupad [B] time = 0.02, size = 11, normalized size = 0.73

$$-\frac{2\sqrt{-x^3-1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(- x^3 - 1)^(1/2),x)
[Out] -(2*(- x^3 - 1)^(1/2))/3
```

sympy [A] time = 0.21, size = 14, normalized size = 0.93

$$-\frac{2\sqrt{-x^3-1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**3-1)**(1/2),x)
[Out] -2*sqrt(-x**3 - 1)/3
```

3.496 $\int \frac{1}{x\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=16

$$\frac{2}{3}\tan^{-1}\left(\sqrt{-x^3-1}\right)$$

```
[Out] 2/3*arctan((-x^3-1)^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {266, 63, 204}

$$\frac{2}{3}\tan^{-1}\left(\sqrt{-x^3-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*ArcTan[Sqrt[-1 - x^3]])/3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{-1-x^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{-1-x}\,x}\,dx, x, x^3\right)\\ &= -\left(\frac{2}{3}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{-1-x^3}\right)\right)\\ &= \frac{2}{3}\tan^{-1}\left(\sqrt{-1-x^3}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{2}{3}\tan^{-1}\left(\sqrt{-x^3-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-1 - x^3]),x]
[Out] (2*ArcTan[Sqrt[-1 - x^3]])/3
```

fricas [A] time = 0.76, size = 12, normalized size = 0.75

$$\frac{2}{3}\arctan\left(\sqrt{-x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3-1)^(1/2),x, algorithm="fricas")
[Out] 2/3*arctan(sqrt(-x^3 - 1))
```

giac [A] time = 0.16, size = 12, normalized size = 0.75

$$\frac{2}{3}\arctan\left(\sqrt{-x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3-1)^(1/2),x, algorithm="giac")
[Out] 2/3*arctan(sqrt(-x^3 - 1))
```

maple [A] time = 0.03, size = 13, normalized size = 0.81

$$\frac{2\arctan\left(\sqrt{-x^3-1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^3-1)^(1/2),x)
[Out] 2/3*arctan((-x^3-1)^(1/2))
```

maxima [A] time = 2.93, size = 12, normalized size = 0.75

$$\frac{2}{3}\arctan\left(\sqrt{-x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3-1)^(1/2),x, algorithm="maxima")
[Out] 2/3*arctan(sqrt(-x^3 - 1))
```

mupad [B] time = 0.03, size = 180, normalized size = 11.25

$$-\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(- x^3 - 1)^(1/2)),x)
[Out] -(2*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^
(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)
*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 +
 3/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)
```

```
/((3^(1/2)*1i)/2 - 3/2)))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/
2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1
/2))^(1/2))
```

sympy [**C**] time = 1.59, size = 12, normalized size = 0.75

$$\frac{2i\,\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**3-1)**(1/2),x)
```

```
[Out] 2*I*asinh(x**(-3/2))/3
```

3.497 $\int \frac{1}{x^4\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=35

$$\frac{\sqrt{-x^3-1}}{3x^3}-\frac{1}{3}\tan^{-1}\left(\sqrt{-x^3-1}\right)$$

```
[Out] -1/3*arctan((-x^3-1)^(1/2))+1/3*(-x^3-1)^(1/2)/x^3
```

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 51, 63, 204}

$$\frac{\sqrt{-x^3-1}}{3x^3}-\frac{1}{3}\tan^{-1}\left(\sqrt{-x^3-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[-1 - x^3]),x]
```

```
[Out] Sqrt[-1 - x^3]/(3*x^3) - ArcTan[Sqrt[-1 - x^3]]/3
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{-1-x^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1-x}\,x^2}\,dx, x, x^3\right)\\ &= \frac{\sqrt{-1-x^3}}{3x^3} - \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1-x}\,x}\,dx, x, x^3\right)\\ &= \frac{\sqrt{-1-x^3}}{3x^3} + \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{-1-x^3}\right)\\ &= \frac{\sqrt{-1-x^3}}{3x^3} - \frac{1}{3}\tan^{-1}\left(\sqrt{-1-x^3}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 1.14

$$\frac{1}{3}\sqrt{-x^3-1}\left(\frac{1}{x^3} - \frac{\tanh^{-1}\left(\sqrt{x^3+1}\right)}{\sqrt{x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[-1 - x^3]),x]
[Out] (Sqrt[-1 - x^3]*(x^(-3) - ArcTanh[Sqrt[1 + x^3]]/Sqrt[1 + x^3]))/3
```

fricas [A] time = 0.93, size = 31, normalized size = 0.89

$$-\frac{x^3\arctan\left(\sqrt{-x^3-1}\right) - \sqrt{-x^3-1}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3-1)^(1/2),x, algorithm="fricas")
[Out] -1/3*(x^3*arctan(sqrt(-x^3 - 1)) - sqrt(-x^3 - 1))/x^3
```

giac [A] time = 0.24, size = 27, normalized size = 0.77

$$\frac{\sqrt{-x^3-1}}{3\,x^3} - \frac{1}{3}\,\arctan\left(\sqrt{-x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3-1)^(1/2),x, algorithm="giac")
[Out] 1/3*sqrt(-x^3 - 1)/x^3 - 1/3*arctan(sqrt(-x^3 - 1))
```

maple [A] time = 0.03, size = 28, normalized size = 0.80

$$-\frac{\arctan\left(\sqrt{-x^3-1}\right)}{3} + \frac{\sqrt{-x^3-1}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-x^3-1)^(1/2),x)
[Out] -1/3*arctan((-x^3-1)^(1/2))+1/3*(-x^3-1)^(1/2)/x^3
```

maxima [A] time = 2.98, size = 27, normalized size = 0.77

$$\frac{\sqrt{-x^3-1}}{3\,x^3} - \frac{1}{3}\,\arctan\left(\sqrt{-x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(-x^3 - 1)/x^3 - 1/3*arctan(sqrt(-x^3 - 1))
```

mupad [B] time = 0.98, size = 194, normalized size = 5.54

$$\frac{\sqrt{-x^3-1}}{3\,x^3}+\frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(- x^3 - 1)^(1/2)),x)
```

```
[Out] (- x^3 - 1)^(1/2)/(3*x^3) + (((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (
3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2
 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*el
lipticPi((3^(1/2)*1i)/2 + 3/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2))
, -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((- x^3 - 1)^(1/2)*(x^3
- x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 -
 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

sympy [C] time = 2.15, size = 29, normalized size = 0.83

$$-\frac{i\,\mathrm{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}+\frac{i\sqrt{1+\frac{1}{x^3}}}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-x**3-1)**(1/2),x)
```

```
[Out] -I*asinh(x**(-3/2))/3 + I*sqrt(1 + x**(-3))/(3*x**(3/2))
```

3.498 $\int \frac{1}{x^7\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=53

$$-\frac{\sqrt{-x^3-1}}{4x^3}+\frac{1}{4}\tan^{-1}\left(\sqrt{-x^3-1}\right)+\frac{\sqrt{-x^3-1}}{6x^6}$$

```
[Out] 1/4*arctan((-x^3-1)^(1/2))+1/6*(-x^3-1)^(1/2)/x^6-1/4*(-x^3-1)^(1/2)/x^3
```

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 51, 63, 204}

$$-\frac{\sqrt{-x^3-1}}{4x^3}+\frac{\sqrt{-x^3-1}}{6x^6}+\frac{1}{4}\tan^{-1}\left(\sqrt{-x^3-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*Sqrt[-1 - x^3]),x]
```

```
[Out] Sqrt[-1 - x^3]/(6*x^6) - Sqrt[-1 - x^3]/(4*x^3) + ArcTan[Sqrt[-1 - x^3]]/4
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7\sqrt{-1-x^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1-x}\,x^3}\,dx, x, x^3\right)\\ &= \frac{\sqrt{-1-x^3}}{6x^6} - \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1-x}\,x^2}\,dx, x, x^3\right)\\ &= \frac{\sqrt{-1-x^3}}{6x^6} - \frac{\sqrt{-1-x^3}}{4x^3} + \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1-x}\,x}\,dx, x, x^3\right)\\ &= \frac{\sqrt{-1-x^3}}{6x^6} - \frac{\sqrt{-1-x^3}}{4x^3} - \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{-1-x^3}\right)\\ &= \frac{\sqrt{-1-x^3}}{6x^6} - \frac{\sqrt{-1-x^3}}{4x^3} + \frac{1}{4}\tan^{-1}\left(\sqrt{-1-x^3}\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 28, normalized size = 0.53

$$\frac{2}{3}\sqrt{-x^3-1}\;{}_2F_1\left(\frac{1}{2}, 3; \frac{3}{2}; x^3+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[-1 - x^3]*Hypergeometric2F1[1/2, 3, 3/2, 1 + x^3])/3
```

fricas [A] time = 0.85, size = 39, normalized size = 0.74

$$\frac{3\,x^6\arctan\left(\sqrt{-x^3-1}\right) - \left(3\,x^3-2\right)\sqrt{-x^3-1}}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(3*x^6*arctan(sqrt(-x^3 - 1)) - (3*x^3 - 2)*sqrt(-x^3 - 1))/x^6
```

giac [A] time = 0.15, size = 41, normalized size = 0.77

$$\frac{3\left(-x^3-1\right)^{\frac{3}{2}} + 5\sqrt{-x^3-1}}{12\,x^6} + \frac{1}{4}\arctan\left(\sqrt{-x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*(3*(-x^3 - 1)^(3/2) + 5*sqrt(-x^3 - 1))/x^6 + 1/4*arctan(sqrt(-x^3 - 1
))
```

maple [A] time = 0.03, size = 42, normalized size = 0.79

$$\frac{\arctan\left(\sqrt{-x^3-1}\right)}{4} - \frac{\sqrt{-x^3-1}}{4x^3} + \frac{\sqrt{-x^3-1}}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(-x^3-1)^(1/2),x)
```

```
[Out] 1/4*arctan((-x^3-1)^(1/2))+1/6*(-x^3-1)^(1/2)/x^6-1/4*(-x^3-1)^(1/2)/x^3
```

maxima [A] time = 3.02, size = 56, normalized size = 1.06

$$-\frac{3\left(-x^3-1\right)^{\frac{3}{2}}+5\sqrt{-x^3-1}}{12\left(2x^3-\left(x^3+1\right)^2+1\right)}+\frac{1}{4}\arctan\left(\sqrt{-x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/12*(3*(-x^3 - 1)^(3/2) + 5*sqrt(-x^3 - 1))/(2*x^3 - (x^3 + 1)^2 + 1) + 1
/4*arctan(sqrt(-x^3 - 1))
```

mupad [B] time = 0.04, size = 209, normalized size = 3.94

$$\frac{\sqrt{-x^3-1}}{6x^6}-\frac{\sqrt{-x^3-1}}{4x^3}-\frac{3\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}}{2}}}\right.\right.}{4\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(- x^3 - 1)^(1/2)),x)
```

```
[Out] (- x^3 - 1)^(1/2)/(6*x^6) - (- x^3 - 1)^(1/2)/(4*x^3) - (3*((3^(1/2)*1i)/2
+ 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^
(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((
3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(((x + 1)/
((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3
/2)))/(4*(- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2
 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

sympy [C] time = 4.74, size = 66, normalized size = 1.25

$$\frac{i\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{4}-\frac{i}{4x^{\frac{3}{2}}\sqrt{1+\frac{1}{x^3}}}-\frac{i}{12x^{\frac{9}{2}}\sqrt{1+\frac{1}{x^3}}}+\frac{i}{6x^{\frac{15}{2}}\sqrt{1+\frac{1}{x^3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(-x**3-1)**(1/2),x)
```

```
[Out] I*asinh(x**(-3/2))/4 - I/(4*x**(3/2)*sqrt(1 + x**(-3))) - I/(12*x**(9/2)*sq
rt(1 + x**(-3))) + I/(6*x**(15/2)*sqrt(1 + x**(-3)))
```

3.499 $\int \frac{1}{x^{10}\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=71

$$\frac{5\sqrt{-x^3-1}}{24x^3} - \frac{5}{24}\tan^{-1}\left(\sqrt{-x^3-1}\right) + \frac{\sqrt{-x^3-1}}{9x^9} - \frac{5\sqrt{-x^3-1}}{36x^6}$$

```
[Out] -5/24*arctan((-x^3-1)^(1/2))+1/9*(-x^3-1)^(1/2)/x^9-5/36*(-x^3-1)^(1/2)/x^6
+5/24*(-x^3-1)^(1/2)/x^3
```

Rubi [A] time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 51, 63, 204}

$$\frac{5\sqrt{-x^3-1}}{24x^3} - \frac{5\sqrt{-x^3-1}}{36x^6} + \frac{\sqrt{-x^3-1}}{9x^9} - \frac{5}{24}\tan^{-1}\left(\sqrt{-x^3-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*Sqrt[-1 - x^3]),x]
```

```
[Out] Sqrt[-1 - x^3]/(9*x^9) - (5*Sqrt[-1 - x^3])/(36*x^6) + (5*Sqrt[-1 - x^3])/(
24*x^3) - (5*ArcTan[Sqrt[-1 - x^3]])/24
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{10}\sqrt{-1-x^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1-x}\,x^4}\,dx, x, x^3\right) \\
&= \frac{\sqrt{-1-x^3}}{9x^9} - \frac{5}{18}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1-x}\,x^3}\,dx, x, x^3\right) \\
&= \frac{\sqrt{-1-x^3}}{9x^9} - \frac{5\sqrt{-1-x^3}}{36x^6} + \frac{5}{24}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1-x}\,x^2}\,dx, x, x^3\right) \\
&= \frac{\sqrt{-1-x^3}}{9x^9} - \frac{5\sqrt{-1-x^3}}{36x^6} + \frac{5\sqrt{-1-x^3}}{24x^3} - \frac{5}{48}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1-x}\,x}\,dx, x, x^3\right) \\
&= \frac{\sqrt{-1-x^3}}{9x^9} - \frac{5\sqrt{-1-x^3}}{36x^6} + \frac{5\sqrt{-1-x^3}}{24x^3} + \frac{5}{24}\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{-1-x^3}\right) \\
&= \frac{\sqrt{-1-x^3}}{9x^9} - \frac{5\sqrt{-1-x^3}}{36x^6} + \frac{5\sqrt{-1-x^3}}{24x^3} - \frac{5}{24}\tan^{-1}\left(\sqrt{-1-x^3}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 28, normalized size = 0.39

$$-\frac{2}{3}\sqrt{-x^3-1}\;{}_2F_1\left(\frac{1}{2}, 4; \frac{3}{2}; x^3+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^10*Sqrt[-1 - x^3]),x]
```

```
[Out] (-2*Sqrt[-1 - x^3]*Hypergeometric2F1[1/2, 4, 3/2, 1 + x^3])/3
```

fricas [A] time = 0.85, size = 44, normalized size = 0.62

$$-\frac{15\,x^9\arctan\left(\sqrt{-x^3-1}\right) - \left(15\,x^6 - 10\,x^3 + 8\right)\sqrt{-x^3-1}}{72\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/72*(15*x^9*arctan(sqrt(-x^3 - 1)) - (15*x^6 - 10*x^3 + 8)*sqrt(-x^3 - 1)
)/x^9
```

giac [A] time = 0.17, size = 59, normalized size = 0.83

$$\frac{15\left(x^3+1\right)^2\sqrt{-x^3-1} + 40\left(-x^3-1\right)^{\frac{3}{2}} + 33\sqrt{-x^3-1}}{72\,x^9} - \frac{5}{24}\arctan\left(\sqrt{-x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/72*(15*(x^3 + 1)^2*sqrt(-x^3 - 1) + 40*(-x^3 - 1)^(3/2) + 33*sqrt(-x^3 -
1))/x^9 - 5/24*arctan(sqrt(-x^3 - 1))
```

maple [A] time = 0.03, size = 56, normalized size = 0.79

$$-\frac{5\arctan\left(\sqrt{-x^3-1}\right)}{24} + \frac{5\sqrt{-x^3-1}}{24x^3} - \frac{5\sqrt{-x^3-1}}{36x^6} + \frac{\sqrt{-x^3-1}}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(-x^3-1)^(1/2),x)
```

```
[Out] -5/24*arctan((-x^3-1)^(1/2))+1/9*(-x^3-1)^(1/2)/x^9-5/36*(-x^3-1)^(1/2)/x^6
+5/24*(-x^3-1)^(1/2)/x^3
```

maxima [A] time = 3.00, size = 74, normalized size = 1.04

$$\frac{15\left(-x^3-1\right)^{\frac{5}{2}}+40\left(-x^3-1\right)^{\frac{3}{2}}+33\sqrt{-x^3-1}}{72\left(\left(x^3+1\right)^3+3x^3-3\left(x^3+1\right)^2+2\right)}-\frac{5}{24}\arctan\left(\sqrt{-x^3-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/72*(15*(-x^3 - 1)^(5/2) + 40*(-x^3 - 1)^(3/2) + 33*sqrt(-x^3 - 1))/((x^3
+ 1)^3 + 3*x^3 - 3*(x^3 + 1)^2 + 2) - 5/24*arctan(sqrt(-x^3 - 1))
```

mupad [B] time = 0.03, size = 223, normalized size = 3.14

$$\frac{5\sqrt{-x^3-1}}{24x^3}-\frac{5\sqrt{-x^3-1}}{36x^6}+\frac{\sqrt{-x^3-1}}{9x^9}+\frac{5\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3+1}\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{a}\right.}{8\sqrt{-x^3-1}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^10*(- x^3 - 1)^(1/2)),x)
```

```
[Out] (5*(- x^3 - 1)^(1/2))/(24*x^3) - (5*(- x^3 - 1)^(1/2))/(36*x^6) + (- x^3 -
1)^(1/2)/(9*x^9) + (5*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)
*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticP
i((3^(1/2)*1i)/2 + 3/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^
(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(8*(- x^3 - 1)^(1/2)*(x^3 - x*(
((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)
*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

sympy [C] time = 8.48, size = 90, normalized size = 1.27

$$-\frac{5i\,\mathrm{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{24}+\frac{5i}{24x^{\frac{3}{2}}\sqrt{1+\frac{1}{x^3}}}+\frac{5i}{72x^{\frac{9}{2}}\sqrt{1+\frac{1}{x^3}}}-\frac{i}{36x^{\frac{15}{2}}\sqrt{1+\frac{1}{x^3}}}+\frac{i}{9x^{\frac{21}{2}}\sqrt{1+\frac{1}{x^3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(-x**3-1)**(1/2),x)
```

```
[Out] -5*I*asinh(x**(-3/2))/24 + 5*I/(24*x**(3/2)*sqrt(1 + x**(-3))) + 5*I/(72*x*
*(9/2)*sqrt(1 + x**(-3))) - I/(36*x**(15/2)*sqrt(1 + x**(-3))) + I/(9*x**(2
1/2)*sqrt(1 + x**(-3)))
```

3.500 $\int \frac{x^6}{\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=149

$$\frac{16}{55}\sqrt{-x^3-1}\,x-\frac{2}{11}\sqrt{-x^3-1}\,x^4+\frac{32\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{55\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[Out] 16/55*x*(-x^3-1)^(1/2)-2/11*x^4*(-x^3-1)^(1/2)+32/165*(1+x)*EllipticF((1+x+
3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/
(1+x-3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2
)
```

Rubi [A] time = 0.04, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {321, 219}

$$-\frac{2}{11}\sqrt{-x^3-1}\,x^4+\frac{16}{55}\sqrt{-x^3-1}\,x+\frac{32\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{55\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/Sqrt[-1 - x^3],x]
```

```
[Out] (16*x*Sqrt[-1 - x^3])/55 - (2*x^4*Sqrt[-1 - x^3])/11 + (32*Sqrt[2 - Sqrt[3]
]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqr
t[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(55*3^(1/4)*Sqrt[-((1 + x)/(
1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^6}{\sqrt{-1-x^3}}\,dx = -\frac{2}{11}x^4\sqrt{-1-x^3} - \frac{8}{11}\int \frac{x^3}{\sqrt{-1-x^3}}\,dx$$
$$= \frac{16}{55}x\sqrt{-1-x^3} - \frac{2}{11}x^4\sqrt{-1-x^3} + \frac{16}{55}\int \frac{1}{\sqrt{-1-x^3}}\,dx$$
$$= \frac{16}{55}x\sqrt{-1-x^3} - \frac{2}{11}x^4\sqrt{-1-x^3} + \frac{32\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\right.}{55\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.36

$$\frac{2x\left(8\sqrt{x^3+1}\;{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)+5x^6-3x^3-8\right)}{55\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/Sqrt[-1 - x^3],x]
```

```
[Out] (2*x*(-8 - 3*x^3 + 5*x^6 + 8*Sqrt[1 + x^3]*Hypergeometric2F1[1/3, 1/2, 4/3,
 -x^3]))/(55*Sqrt[-1 - x^3])
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^3-1}\,x^6}{x^3+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 - 1)*x^6/(x^3 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/sqrt(-x^3 - 1), x)
```

maple [A] time = 0.02, size = 134, normalized size = 0.90

$$-\frac{2\sqrt{-x^3-1}\;x^4}{11}+\frac{16\sqrt{-x^3-1}\;x}{55}-\frac{32i\sqrt{3}\;\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\;\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\;\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\;\text{EllipticF}\left(\frac{\sqrt{3}\;\sqrt{i}\right.}{165\sqrt{-x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-x^3-1)^(1/2),x)
```

```
[Out] -2/11*x^4*(-x^3-1)^(1/2)+16/55*x*(-x^3-1)^(1/2)-32/165*I*3^(1/2)*(I*(x-1/2-
1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+
1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x-1/
2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/sqrt(-x^3 - 1), x)
```

mupad [B] time = 0.04, size = 199, normalized size = 1.34

$$\frac{16\,x\,\sqrt{-x^3-1}}{55}-\frac{2\,x^4\,\sqrt{-x^3-1}}{11}+\frac{32\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right.\right.}{55\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(- x^3 - 1)^(1/2),x)
```

```
[Out] (16*x*(- x^3 - 1)^(1/2))/55 - (2*x^4*(- x^3 - 1)^(1/2))/11 + (32*((3^(1/2)*
1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 -
3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1
/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(55*(- x^3
- 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - (
(3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

sympy [A] time = 1.20, size = 32, normalized size = 0.21

$$-\frac{ix^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-x**3-1)**(1/2),x)
```

```
[Out] -I*x**7*gamma(7/3)*hyper((1/2, 7/3), (10/3,), x**3*exp_polar(I*pi))/(3*gamm
a(10/3))
```

3.501 $\int \frac{x^3}{\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=131

$$-\frac{2}{5}\sqrt{-x^3-1}\,x-\frac{4\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[Out] -2/5*x*(-x^3-1)^(1/2)-4/15*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-
I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*3^(3
/4)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {321, 219}

$$-\frac{2}{5}\sqrt{-x^3-1}\,x-\frac{4\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[-1 - x^3],x]
```

```
[Out] (-2*x*Sqrt[-1 - x^3])/5 - (4*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(
1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)],
-7 + 4*Sqrt[3]])/(5*3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 -
x^3])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{-1-x^3}}\,dx = -\frac{2}{5}x\sqrt{-1-x^3} - \frac{2}{5}\int \frac{1}{\sqrt{-1-x^3}}\,dx$$
$$= -\frac{2}{5}x\sqrt{-1-x^3} - \frac{4\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] time = 0.02, size = 47, normalized size = 0.36

$$\frac{2\left(-\sqrt{x^3+1}\,x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)+x^4+x\right)}{5\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[-1 - x^3],x]
```

```
[Out] (2*(x + x^4 - x*Sqrt[1 + x^3]*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3]))/(5*S
qrt[-1 - x^3])
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^3-1}\,x^3}{x^3+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 - 1)*x^3/(x^3 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(-x^3 - 1), x)
```

maple [A] time = 0.02, size = 120, normalized size = 0.92

$$-\frac{2\sqrt{-x^3-1}\,x}{5} + \frac{4i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\ \text{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}, \sqrt{}\right.}{15\sqrt{-x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-x^3-1)^(1/2),x)
```

```
[Out] -2/5*(-x^3-1)^(1/2)*x+4/15*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2
)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2
```

```
)/(-x^3-1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1
/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(-x^3 - 1), x)
```

mupad [B] time = 0.05, size = 185, normalized size = 1.41

$$-\frac{2x\sqrt{-x^3-1}}{5}-\frac{4\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{5\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(- x^3 - 1)^(1/2),x)
```

```
[Out] - (2*x*(- x^3 - 1)^(1/2))/5 - (4*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x
 + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1
i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2
 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(5*(- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i
)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1
i)/2 + 1/2))^(1/2))
```

sympy [A] time = 1.51, size = 32, normalized size = 0.24

$$-\frac{ix^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-x**3-1)**(1/2),x)
```

```
[Out] -I*x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), x**3*exp_polar(I*pi))/(3*gamma
(7/3))
```

3.502 $\int \frac{1}{\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=112

$$\frac{2\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[Out] 2/3*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)
-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3-1)^(1/2)/((-1
-x)/(1+x-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {219}

$$\frac{2\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-1 - x^3],x]
```

```
[Out] (2*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*Ellipt
icF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*
Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1-x^3}}\,dx = \frac{2\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] time = 0.00, size = 37, normalized size = 0.33

$$\frac{x\sqrt{x^3+1}\;{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)}{\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-1 - x^3],x]
```

```
[Out] (x*Sqrt[1 + x^3]*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3])/Sqrt[-1 - x^3]
```

fricas **[F]** time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^3-1}}{x^3+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 - 1)/(x^3 + 1), x)
```

giac **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-x^3 - 1), x)
```

maple **[A]** time = 0.02, size = 107, normalized size = 0.96

$$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\text{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}, \sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/
2*I*3^(1/2)))^(1/2))
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-x^3 - 1), x)
```

mupad **[B]** time = 0.03, size = 172, normalized size = 1.54

$$\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\text{i}}{2}\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\text{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\text{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\text{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\text{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\text{i}}{2}}}\,\text{F}\left(\text{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\text{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\text{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\text{i}}{2}}\right)}{\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\text{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\text{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\text{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\text{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- x^3 - 1)^(1/2),x)
```

```
[Out] (2*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(
1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*
1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))
)/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2
) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

sympy [A] time = 1.20, size = 31, normalized size = 0.28

$$-\frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3}, \frac{1}{2} \\ \frac{4}{3}\end{matrix}\right| x^3 e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**3-1)**(1/2),x)
```

```
[Out] -I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/
3))
```

3.503 $\int \frac{1}{x^3\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=133

$$\frac{\sqrt{-x^3-1}}{2x^2}-\frac{\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[Out] 1/2*(-x^3-1)^(1/2)/x^2-1/6*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-
I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*3^(3
/4)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {325, 219}

$$\frac{\sqrt{-x^3-1}}{2x^2}-\frac{\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[-1 - x^3]),x]
```

```
[Out] Sqrt[-1 - x^3]/(2*x^2) - (Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 -
 Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7
+ 4*Sqrt[3]])/(2*3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3
])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{-1-x^3}}\,dx = \frac{\sqrt{-1-x^3}}{2x^2} - \frac{1}{4}\int \frac{1}{\sqrt{-1-x^3}}\,dx$$
$$= \frac{\sqrt{-1-x^3}}{2x^2} - \frac{\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)\middle|-7+4\sqrt{3}\right)}{2\sqrt[4]{3}\,\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}$$

Mathematica [C] time = 0.01, size = 42, normalized size = 0.32

$$-\frac{\sqrt{x^3+1}\;{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};-x^3\right)}{2x^2\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[-1 - x^3]),x]
```

```
[Out] -1/2*(Sqrt[1 + x^3]*Hypergeometric2F1[-2/3, 1/2, 1/3, -x^3])/(x^2*Sqrt[-1 -
 x^3])
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^3-1}}{x^6+x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 - 1)/(x^6 + x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^3-1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^3 - 1)*x^3), x)
```

maple [A] time = 0.02, size = 122, normalized size = 0.92

$$\frac{i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\;\text{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}, \sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{6\sqrt{-x^3-1}} + \frac{\sqrt{-x^3-}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-x^3-1)^(1/2),x)
```

```
[Out] 1/2*(-x^3-1)^(1/2)/x^2+1/6*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2
)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2
```

```
)/(-x^3-1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1
/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^3-1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^3 - 1)*x^3), x)
```

mupad [B] time = 0.99, size = 187, normalized size = 1.41

$$\frac{\sqrt{-x^3-1}}{2x^2}-\frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{2\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(- x^3 - 1)^(1/2)),x)
```

```
[Out] (- x^3 - 1)^(1/2)/(2*x^2) - (((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (
3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2
 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*el
lipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/
2)/((3^(1/2)*1i)/2 - 3/2)))/(2*(- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2
 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2
 + 1/2))^(1/2))
```

sympy [A] time = 0.96, size = 36, normalized size = 0.27

$$-\frac{i\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{1}{2}\\ \frac{1}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-x**3-1)**(1/2),x)
```

```
[Out] -I*gamma(-2/3)*hyper((-2/3, 1/2), (1/3,), x**3*exp_polar(I*pi))/(3*x**2*gam
ma(1/3))
```

3.504 $\int \frac{1}{x^6\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=151

$$\frac{\sqrt{-x^3-1}}{5x^5}-\frac{7\sqrt{-x^3-1}}{20x^2}+\frac{7\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{20\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

```
[Out] 1/5*(-x^3-1)^(1/2)/x^5-7/20*(-x^3-1)^(1/2)/x^2+7/60*(1+x)*EllipticF((1+x+3^
(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1
+x-3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {325, 219}

$$-\frac{7\sqrt{-x^3-1}}{20x^2}+\frac{\sqrt{-x^3-1}}{5x^5}+\frac{7\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{20\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*Sqrt[-1 - x^3]),x]
```

```
[Out] Sqrt[-1 - x^3]/(5*x^5) - (7*Sqrt[-1 - x^3])/(20*x^2) + (7*Sqrt[2 - Sqrt[3]]
*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt
[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(20*3^(1/4)*Sqrt[-((1 + x)/(1
 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^6\sqrt{-1-x^3}}\,dx = \frac{\sqrt{-1-x^3}}{5x^5} - \frac{7}{10}\int \frac{1}{x^3\sqrt{-1-x^3}}\,dx$$
$$= \frac{\sqrt{-1-x^3}}{5x^5} - \frac{7\sqrt{-1-x^3}}{20x^2} + \frac{7}{40}\int \frac{1}{\sqrt{-1-x^3}}\,dx$$
$$= \frac{\sqrt{-1-x^3}}{5x^5} - \frac{7\sqrt{-1-x^3}}{20x^2} + \frac{7\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)\middle|-7+4\sqrt{3}\right)}{20\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] time = 0.03, size = 42, normalized size = 0.28

$$-\frac{\sqrt{x^3+1}\;{}_2F_1\left(-\frac{5}{3},\frac{1}{2};-\frac{2}{3};-x^3\right)}{5x^5\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*Sqrt[-1 - x^3]),x]
```

```
[Out] -1/5*(Sqrt[1 + x^3]*Hypergeometric2F1[-5/3, 1/2, -2/3, -x^3])/(x^5*Sqrt[-1
- x^3])
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^3-1}}{x^9+x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 - 1)/(x^9 + x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^3-1}\,x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^3 - 1)*x^6), x)
```

maple [A] time = 0.03, size = 136, normalized size = 0.90

$$-\frac{7i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\;\text{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}, \sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{60\sqrt{-x^3-1}} - \frac{7\sqrt{-x^3}}{20x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-x^3-1)^(1/2),x)
```

```
[Out] 1/5*(-x^3-1)^(1/2)/x^5-7/20*(-x^3-1)^(1/2)/x^2-7/60*I*3^(1/2)*(I*(x-1/2-1/2
*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2
*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x-1/2-1
/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^3-1}\,x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^3 - 1)*x^6), x)
```

mupad [B] time = 0.04, size = 201, normalized size = 1.33

$$\frac{\sqrt{-x^3-1}}{5x^5}-\frac{7\sqrt{-x^3-1}}{20x^2}+\frac{7\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+}{-\frac{3}{2}}\right.}{20\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1}{2}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(- x^3 - 1)^(1/2)),x)
```

```
[Out] (- x^3 - 1)^(1/2)/(5*x^5) - (7*(- x^3 - 1)^(1/2))/(20*x^2) + (7*((3^(1/2)*1
i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3
/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/
2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 +
3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(20*(- x^3 -
 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((
3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

sympy [A] time = 2.00, size = 39, normalized size = 0.26

$$-\frac{i\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{5}{3},\frac{1}{2}\\-\frac{2}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-x**3-1)**(1/2),x)
```

```
[Out] -I*gamma(-5/3)*hyper((-5/3, 1/2), (-2/3,), x**3*exp_polar(I*pi))/(3*x**5*ga
mma(-2/3))
```

3.505 $\int \frac{x^7}{\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=282

$$-\frac{80\sqrt{-x^3-1}}{91\left(x-\sqrt{3}+1\right)}-\frac{2}{13}\sqrt{-x^3-1}\,x^5+\frac{20}{91}\sqrt{-x^3-1}\,x^2-\frac{80\sqrt{2}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{91\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+$$

```
[Out] 20/91*x^2*(-x^3-1)^(1/2)-2/13*x^5*(-x^3-1)^(1/2)-80/91*(-x^3-1)^(1/2)/(1+x-
3^(1/2))-80/273*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*
2^(1/2)*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/(1
+x-3^(1/2))^2)^(1/2)+40/91*3^(1/4)*(1+x)*EllipticE((1+x+3^(1/2))/(1+x-3^(1/
2)),2*I-I*3^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/
2))/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {321, 304, 219, 1879}

$$-\frac{2}{13}\sqrt{-x^3-1}\,x^5+\frac{20}{91}\sqrt{-x^3-1}\,x^2-\frac{80\sqrt{-x^3-1}}{91\left(x-\sqrt{3}+1\right)}-\frac{80\sqrt{2}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{91\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+$$

Antiderivative was successfully verified.

```
[In] Int[x^7/Sqrt[-1 - x^3],x]
```

```
[Out] (20*x^2*Sqrt[-1 - x^3])/91 - (2*x^5*Sqrt[-1 - x^3])/13 - (80*Sqrt[-1 - x^3]
)/(91*(1 - Sqrt[3] + x)) + (40*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 -
x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[
3] + x)], -7 + 4*Sqrt[3]])/(91*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1
 - x^3]) - (80*Sqrt[2]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*Elli
pticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(91*3^(
1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{\sqrt{-1-x^3}}\,dx &= -\frac{2}{13}x^5\sqrt{-1-x^3} - \frac{10}{13}\int \frac{x^4}{\sqrt{-1-x^3}}\,dx \\
&= \frac{20}{91}x^2\sqrt{-1-x^3} - \frac{2}{13}x^5\sqrt{-1-x^3} + \frac{40}{91}\int \frac{x}{\sqrt{-1-x^3}}\,dx \\
&= \frac{20}{91}x^2\sqrt{-1-x^3} - \frac{2}{13}x^5\sqrt{-1-x^3} + \frac{40}{91}\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx - \frac{1}{91}\left(40\sqrt{2\left(2+\sqrt{3}\right)}\right)\int \frac{1}{\sqrt{-1-} \\
&= \frac{20}{91}x^2\sqrt{-1-x^3} - \frac{2}{13}x^5\sqrt{-1-x^3} - \frac{80\sqrt{-1-x^3}}{91\left(1-\sqrt{3}+x\right)} + \frac{40\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}}{91\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 56, normalized size = 0.20

$$\frac{2x^2\left(10\sqrt{x^3+1}\;{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^3\right)+7x^6-3x^3-10\right)}{91\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/Sqrt[-1 - x^3],x]
```

```
[Out] (2*x^2*(-10 - 3*x^3 + 7*x^6 + 10*Sqrt[1 + x^3]*Hypergeometric2F1[1/2, 2/3,
5/3, -x^3]))/(91*Sqrt[-1 - x^3])
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^3-1}\,x^7}{x^3+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 - 1)*x^7/(x^3 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^7/sqrt(-x^3 - 1), x)
```

maple [A] time = 0.02, size = 189, normalized size = 0.67

$$-\frac{2\sqrt{-x^3-1}\,x^5}{13}+\frac{20\sqrt{-x^3-1}\,x^2}{91}-\frac{80i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\left(\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\text{Elli}\right.}{273\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-x^3-1)^(1/2),x)
```

```
[Out] -2/13*x^5*(-x^3-1)^(1/2)+20/91*x^2*(-x^3-1)^(1/2)-80/273*I*3^(1/2)*(I*(x-1/
2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/
2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*((3/2+1/2*I*3^(1/2))*Ellipti
cE(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*
I*3^(1/2)))^(1/2))-EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^
(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^7/sqrt(-x^3 - 1), x)
```

mupad [B] time = 0.04, size = 259, normalized size = 0.92

$$\frac{20\,x^2\,\sqrt{-x^3-1}}{91}-\frac{2\,x^5\,\sqrt{-x^3-1}}{13}-\frac{80\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\text{i}}{2}\right)\text{F}\left(\text{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\text{i}}{2}}}\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\text{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\text{i}}{2}}\right)\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1\text{i}}{2}\right)\text{E}\left(\text{asin}\left(\sqrt{\frac{3}{2}}\right.\right.\right.}{91\,\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\text{i}}{2}\right)\left(\frac{1}{2}+\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(- x^3 - 1)^(1/2),x)
```

```
[Out] (20*x^2*(- x^3 - 1)^(1/2))/91 - (2*x^5*(- x^3 - 1)^(1/2))/13 - (80*(((3^(1/
2)*1i)/2 - 1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((
3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1i)/2 - 3/2)*ellip
ticE(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/
((3^(1/2)*1i)/2 - 3/2)))*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1
/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3
/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(91*(
- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) +
1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

sympy [**A**] time = 1.27, size = 32, normalized size = 0.11

$$-\frac{ix^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{8}{3} \\ \frac{11}{3}\end{matrix}\right| x^3 e^{i\pi}\right)}{3\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-x**3-1)**(1/2),x)
```

```
[Out] -I*x**8*gamma(8/3)*hyper((1/2, 8/3), (11/3,), x**3*exp_polar(I*pi))/(3*gamm
a(11/3))
```

3.506 $\int \frac{x^4}{\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=264

$$\frac{8\sqrt{-x^3-1}}{7\left(x-\sqrt{3}+1\right)}-\frac{2}{7}\sqrt{-x^3-1}\,x^2+\frac{8\sqrt{2}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{7\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(x+1)}{7}$$

```
[Out] -2/7*x^2*(-x^3-1)^(1/2)+8/7*(-x^3-1)^(1/2)/(1+x-3^(1/2))+8/21*(1+x)*Ellipti
cF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*2^(1/2)*((x^2-x+1)/(1+x-3^(1/
2))^2)^(1/2)*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)-4/7*3^(1
/4)*(1+x)*EllipticE((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*((x^2-x+1)/(
1+x-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/(-x^3-1)^(1/2)/((-1-x)/(1+x
-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {321, 304, 219, 1879}

$$-\frac{2}{7}\sqrt{-x^3-1}\,x^2+\frac{8\sqrt{-x^3-1}}{7\left(x-\sqrt{3}+1\right)}+\frac{8\sqrt{2}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{7\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(x+1)}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[-1 - x^3],x]
```

```
[Out] (-2*x^2*Sqrt[-1 - x^3])/7 + (8*Sqrt[-1 - x^3])/(7*(1 - Sqrt[3] + x)) - (4*3
^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*El
lipticE[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(7*Sq
rt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3]) + (8*Sqrt[2]*(1 + x)*Sqr
t[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1
 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(7*3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] +
 x)^2)]*Sqrt[-1 - x^3])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
 + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt{-1-x^3}}\,dx &= -\frac{2}{7}x^2\sqrt{-1-x^3} - \frac{4}{7}\int \frac{x}{\sqrt{-1-x^3}}\,dx \\ &= -\frac{2}{7}x^2\sqrt{-1-x^3} - \frac{4}{7}\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx + \frac{1}{7}\left(4\sqrt{2\left(2+\sqrt{3}\right)}\right)\int \frac{1}{\sqrt{-1-x^3}}\,dx \\ &= -\frac{2}{7}x^2\sqrt{-1-x^3} + \frac{8\sqrt{-1-x^3}}{7\left(1-\sqrt{3}+x\right)} - \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)\right|}{7\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 49, normalized size = 0.19

$$\frac{2x^2\left(-\sqrt{x^3+1}\;{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^3\right)+x^3+1\right)}{7\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[-1 - x^3],x]
```

```
[Out] (2*x^2*(1 + x^3 - Sqrt[1 + x^3]*Hypergeometric2F1[1/2, 2/3, 5/3, -x^3]))/(7
*Sqrt[-1 - x^3])
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^3-1}\,x^4}{x^3+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 - 1)*x^4/(x^3 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(-x^3 - 1), x)
```

maple [A] time = 0.02, size = 175, normalized size = 0.66

$$-\frac{2\sqrt{-x^3-1}\,x^2}{7}+\frac{8i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\left(\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\text{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\right.}}{3}\right.\right.}{21\sqrt{-x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^3-1)^(1/2),x)
```

```
[Out] -2/7*(-x^3-1)^(1/2)*x^2+8/21*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1
/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1
/2)/(-x^3-1)^(1/2)*((3/2+1/2*I*3^(1/2))*EllipticE(1/3*3^(1/2)*(I*(x-1/2-1/2
*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))-EllipticF
(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*
3^(1/2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(-x^3 - 1), x)
```

mupad [B] time = 0.06, size = 245, normalized size = 0.93

$$-\frac{2\,x^2\,\sqrt{-x^3-1}}{7}+\frac{8\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right.\right.}{7\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(- x^3 - 1)^(1/2),x)
```

```
[Out] (8*(((3^(1/2)*1i)/2 - 1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1i)/2 -
 3/2)*ellipticE(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i
)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*
((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2
)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(
1/2))/(7*(- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2
 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (2*x
^2*(- x^3 - 1)^(1/2))/7
```

sympy [A] time = 1.32, size = 32, normalized size = 0.12

$$-\frac{ix^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**3-1)**(1/2),x)
```

```
[Out] -I*x**5*gamma(5/3)*hyper((1/2, 5/3), (8/3,), x**3*exp_polar(I*pi))/(3*gamma
(8/3))
```

3.507 $\int \frac{x}{\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=239

$$-\frac{2\sqrt{-x^3-1}}{x-\sqrt{3}+1}-\frac{2\sqrt{2}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,E\left(\sin^{-1}\right.}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3}}$$

```
[Out] -2*(-x^3-1)^(1/2)/(1+x-3^(1/2))-2/3*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1
/2)),2*I-I*3^(1/2))*2^(1/2)*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3
-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)+3^(1/4)*(1+x)*EllipticE((1+x+3^(1/
2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*(1/2*6^(
1/2)+1/2*2^(1/2))/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {304, 219, 1879}

$$-\frac{2\sqrt{-x^3-1}}{x-\sqrt{3}+1}-\frac{2\sqrt{2}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,E\left(\sin^{-1}\right.}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[-1 - x^3],x]
```

```
[Out] (-2*Sqrt[-1 - x^3])/(1 - Sqrt[3] + x) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*
Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] + x)/
(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*
Sqrt[-1 - x^3]) - (2*Sqrt[2]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2
]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(
3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
```

```
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{-1-x^3}}\,dx = -\left(\sqrt{2\left(2+\sqrt{3}\right)}\int \frac{1}{\sqrt{-1-x^3}}\,dx\right) + \int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx$$

$$= -\frac{2\sqrt{-1-x^3}}{1-\sqrt{3}+x} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{2\sqrt{2}\,(1}{}$$

Mathematica [C] time = 0.00, size = 42, normalized size = 0.18

$$\frac{x^2\sqrt{x^3+1}\;{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^3\right)}{2\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[-1 - x^3],x]
```

```
[Out] (x^2*Sqrt[1 + x^3]*Hypergeometric2F1[1/2, 2/3, 5/3, -x^3])/(2*Sqrt[-1 - x^3
])
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^3-1}\,x}{x^3+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 - 1)*x/(x^3 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(-x^3 - 1), x)
```

maple [A] time = 0.02, size = 160, normalized size = 0.67

$$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\left(\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\text{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}, \sqrt{\frac{i}{\frac{3}{2}+}\right.\right.}{3\sqrt{-x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^3-1)^(1/2),x)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*((3/2
+1/2*I*3^(1/2))*EllipticE(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/
2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))-EllipticF(1/3*3^(1/2)*(I*(x-1/2-1
/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(-x^3 - 1), x)
```

mupad [B] time = 0.04, size = 230, normalized size = 0.96

$$-\frac{2\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(- x^3 - 1)^(1/2),x)
```

```
[Out] -(2*(((3^(1/2)*1i)/2 - 1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))
^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1i)/2
- 3/2)*ellipticE(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1
i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)
*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/
2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^
(1/2))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2
+ 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

sympy [A] time = 0.84, size = 32, normalized size = 0.13

$$-\frac{ix^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**3-1)**(1/2),x)
```

```
[Out] -I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamma
(5/3))
```

3.508 $\int \frac{1}{x^2\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=257

$$\frac{\sqrt{-x^3-1}}{x}-\frac{\sqrt{-x^3-1}}{x-\sqrt{3}+1}-\frac{\sqrt{2}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-}{(x-\sqrt{3}}}}{2\sqrt{-\frac{}{(x-}}}$$

```
[Out] (-x^3-1)^(1/2)/x-(-x^3-1)^(1/2)/(1+x-3^(1/2))-1/3*(1+x)*EllipticF((1+x+3^(1
/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*2^(1/2)*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)
*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)+1/2*3^(1/4)*(1+x)*El
lipticE((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*((x^2-x+1)/(1+x-3^(1/2))
^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)
^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {325, 304, 219, 1879}

$$\frac{\sqrt{-x^3-1}}{x}-\frac{\sqrt{-x^3-1}}{x-\sqrt{3}+1}-\frac{\sqrt{2}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-}{(x-\sqrt{3}}}}{2\sqrt{-\frac{}{(x-}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[-1 - x^3]),x]
```

```
[Out] Sqrt[-1 - x^3]/x - Sqrt[-1 - x^3]/(1 - Sqrt[3] + x) + (3^(1/4)*Sqrt[2 + Sqr
t[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 +
 Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(2*Sqrt[-((1 + x)/(1 - S
qrt[3] + x)^2)]*Sqrt[-1 - x^3]) - (Sqrt[2]*(1 + x)*Sqrt[(1 - x + x^2)/(1 -
Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 +
 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{-1-x^3}}\,dx &= \frac{\sqrt{-1-x^3}}{x} + \frac{1}{2}\int \frac{x}{\sqrt{-1-x^3}}\,dx \\ &= \frac{\sqrt{-1-x^3}}{x} + \frac{1}{2}\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx - \sqrt{\frac{1}{2}\left(2+\sqrt{3}\right)}\int \frac{1}{\sqrt{-1-x^3}}\,dx \\ &= \frac{\sqrt{-1-x^3}}{x} - \frac{\sqrt{-1-x^3}}{1-\sqrt{3}+x} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right.}{2\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.16

$$-\frac{\sqrt{x^3+1}\;{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};-x^3\right)}{x\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[-1 - x^3]),x]
```

```
[Out] -((Sqrt[1 + x^3]*Hypergeometric2F1[-1/3, 1/2, 2/3, -x^3])/(x*Sqrt[-1 - x^3]
))
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^3-1}}{x^5+x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 - 1)/(x^5 + x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^3-1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^3 - 1)*x^2), x)
```

maple [A] time = 0.02, size = 174, normalized size = 0.68

$$\frac{\sqrt{-x^3-1}}{x}-\frac{i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\left(\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\text{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}}{3}\right.\right.}{3\sqrt{-x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-x^3-1)^(1/2),x)
```

```
[Out] (-x^3-1)^(1/2)/x-1/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+
1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^
3-1)^(1/2)*((3/2+1/2*I*3^(1/2))*EllipticE(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/
2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))-EllipticF(1/3*3^(
1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2))
)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^3-1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^3 - 1)*x^2), x)
```

mupad [B] time = 1.03, size = 244, normalized size = 0.95

$$\frac{\sqrt{-x^3-1}}{x}-\frac{\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{F}\left(\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{E}\left(\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)\left(\right.}{\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(- x^3 - 1)^(1/2)),x)
```

```
[Out] (- x^3 - 1)^(1/2)/x - ((((3^(1/2)*1i)/2 - 1/2)*ellipticF(asin(((x + 1)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))
 - ((3^(1/2)*1i)/2 - 3/2)*ellipticE(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(
1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*((3^(1/2)*1i)/2 + 3
/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/
2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(
1/2)*1i)/2 + 3/2))^(1/2))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/
2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1
/2))^(1/2))
```

sympy [A] time = 1.72, size = 34, normalized size = 0.13

$$-\frac{i\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},\frac{1}{2}\\ \frac{2}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{3x\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-x**3-1)**(1/2),x)
```

```
[Out] -I*gamma(-1/3)*hyper((-1/3, 1/2), (2/3,), x**3*exp_polar(I*pi))/(3*x*gamma(
2/3))
```

3.509 $\int \frac{1}{x^5\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=282

$$-\frac{5\sqrt{-x^3-1}}{8x}+\frac{5\sqrt{-x^3-1}}{8\left(x-\sqrt{3}+1\right)}+\frac{\sqrt{-x^3-1}}{4x^4}+\frac{5(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{4\sqrt{2}\,\sqrt[4]{3}\,\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\sqrt{-x^3-1}}-\frac{5\sqrt[4]{3}\sqrt{2+}}{}$$

```
[Out] 1/4*(-x^3-1)^(1/2)/x^4-5/8*(-x^3-1)^(1/2)/x+5/8*(-x^3-1)^(1/2)/(1+x-3^(1/2)
)+5/24*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*2^(1/2)*(
(x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2
))^2)^(1/2)-5/16*3^(1/4)*(1+x)*EllipticE((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*
3^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/(-x^3-
1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {325, 304, 219, 1879}

$$-\frac{5\sqrt{-x^3-1}}{8x}+\frac{5\sqrt{-x^3-1}}{8\left(x-\sqrt{3}+1\right)}+\frac{\sqrt{-x^3-1}}{4x^4}+\frac{5(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{4\sqrt{2}\,\sqrt[4]{3}\,\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\sqrt{-x^3-1}}-\frac{5\sqrt[4]{3}\sqrt{2+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[-1 - x^3]),x]
```

```
[Out] Sqrt[-1 - x^3]/(4*x^4) - (5*Sqrt[-1 - x^3])/(8*x) + (5*Sqrt[-1 - x^3])/(8*(
1 - Sqrt[3] + x)) - (5*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)
/(1 - Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)]
, -7 + 4*Sqrt[3]])/(16*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
 + (5*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 +
 Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(4*Sqrt[2]*3^(1/4)*Sqrt[
-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\sqrt{-1-x^3}}\,dx &= \frac{\sqrt{-1-x^3}}{4x^4} - \frac{5}{8}\int \frac{1}{x^2\sqrt{-1-x^3}}\,dx \\
&= \frac{\sqrt{-1-x^3}}{4x^4} - \frac{5\sqrt{-1-x^3}}{8x} - \frac{5}{16}\int \frac{x}{\sqrt{-1-x^3}}\,dx \\
&= \frac{\sqrt{-1-x^3}}{4x^4} - \frac{5\sqrt{-1-x^3}}{8x} - \frac{5}{16}\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx + \frac{1}{8}\left(5\sqrt{\frac{1}{2}\left(2+\sqrt{3}\right)}\right)\int \frac{1}{\sqrt{-1-x^3}}\,dx \\
&= \frac{\sqrt{-1-x^3}}{4x^4} - \frac{5\sqrt{-1-x^3}}{8x} + \frac{5\sqrt{-1-x^3}}{8\left(1-\sqrt{3}+x\right)} - \frac{5\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1-}{1-}\right.\right.}{16\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 42, normalized size = 0.15

$$-\frac{\sqrt{x^3+1}\;{}_2F_1\left(-\frac{4}{3},\frac{1}{2};-\frac{1}{3};-x^3\right)}{4x^4\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[-1 - x^3]),x]
```

```
[Out] -1/4*(Sqrt[1 + x^3]*Hypergeometric2F1[-4/3, 1/2, -1/3, -x^3])/(x^4*Sqrt[-1
- x^3])
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^3-1}}{x^8+x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 - 1)/(x^8 + x^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^3-1}\,x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^3 - 1)*x^5), x)
```

maple [A] time = 0.03, size = 189, normalized size = 0.67

$$-\frac{5\sqrt{-x^3-1}}{8x}+\frac{\sqrt{-x^3-1}}{4x^4}+\frac{5i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\left(\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\text{EllipticE}\right.}{24\sqrt{-x^3-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-x^3-1)^(1/2),x)
```

```
[Out] 1/4*(-x^3-1)^(1/2)/x^4-5/8*(-x^3-1)^(1/2)/x+5/24*I*3^(1/2)*(I*(x-1/2-1/2*I*
3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*
3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*((3/2+1/2*I*3^(1/2))*EllipticE(1/3*3
^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2
)))^(1/2))-EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I
*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^3-1}\,x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^3 - 1)*x^5), x)
```

mupad [B] time = 0.05, size = 259, normalized size = 0.92

$$\frac{\sqrt{-x^3-1}}{4\,x^4}-\frac{5\,\sqrt{-x^3-1}}{8\,x}+\frac{5\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right.\right.\right.}{8\,\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(- x^3 - 1)^(1/2)),x)
```

```
[Out] (- x^3 - 1)^(1/2)/(4*x^4) - (5*(- x^3 - 1)^(1/2))/(8*x) + (5*(((3^(1/2)*1i)
/2 - 1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2
)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1i)/2 - 3/2)*ellipticE(a
sin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1
/2)*1i)/2 - 3/2)))*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i
)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(
1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(8*(- x^3 -
 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((
3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

sympy [**A**] time = 1.69, size = 39, normalized size = 0.14

$$-\frac{i\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3}, \frac{1}{2} \\ -\frac{1}{3}\end{matrix}\middle| x^3 e^{i\pi}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-x**3-1)**(1/2),x)
```

```
[Out] -I*gamma(-4/3)*hyper((-4/3, 1/2), (-1/3,), x**3*exp_polar(I*pi))/(3*x**4*ga
mma(-1/3))
```

3.510 $\int x^{11}\sqrt[3]{a+bx^3}\,dx$

Optimal. Leaf size=80

$$-\frac{a^3\left(a+bx^3\right)^{4/3}}{4b^4}+\frac{3a^2\left(a+bx^3\right)^{7/3}}{7b^4}+\frac{\left(a+bx^3\right)^{13/3}}{13b^4}-\frac{3a\left(a+bx^3\right)^{10/3}}{10b^4}$$

```
[Out] -1/4*a^3*(b*x^3+a)^(4/3)/b^4+3/7*a^2*(b*x^3+a)^(7/3)/b^4-3/10*a*(b*x^3+a)^(
10/3)/b^4+1/13*(b*x^3+a)^(13/3)/b^4
```

Rubi [A] time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{3a^2\left(a+bx^3\right)^{7/3}}{7b^4}-\frac{a^3\left(a+bx^3\right)^{4/3}}{4b^4}+\frac{\left(a+bx^3\right)^{13/3}}{13b^4}-\frac{3a\left(a+bx^3\right)^{10/3}}{10b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a + b*x^3)^(1/3),x]
```

```
[Out] -(a^3*(a + b*x^3)^(4/3))/(4*b^4) + (3*a^2*(a + b*x^3)^(7/3))/(7*b^4) - (3*a
*(a + b*x^3)^(10/3))/(10*b^4) + (a + b*x^3)^(13/3)/(13*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{11}\sqrt[3]{a+bx^3}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int x^3\sqrt[3]{a+bx}\,dx, x, x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int\left(-\frac{a^3\sqrt[3]{a+bx}}{b^3}+\frac{3a^2(a+bx)^{4/3}}{b^3}-\frac{3a(a+bx)^{7/3}}{b^3}+\frac{(a+bx)^{10/3}}{b^3}\right)dx, x, x^3\right)\\ &= -\frac{a^3\left(a+bx^3\right)^{4/3}}{4b^4}+\frac{3a^2\left(a+bx^3\right)^{7/3}}{7b^4}-\frac{3a\left(a+bx^3\right)^{10/3}}{10b^4}+\frac{\left(a+bx^3\right)^{13/3}}{13b^4}\end{aligned}$$

Mathematica [A] time = 0.04, size = 50, normalized size = 0.62

$$\frac{\left(a+bx^3\right)^{4/3}\left(-81a^3+108a^2bx^3-126ab^2x^6+140b^3x^9\right)}{1820b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*x^3)^(1/3),x]
```

```
[Out] ((a + b*x^3)^(4/3)*(-81*a^3 + 108*a^2*b*x^3 - 126*a*b^2*x^6 + 140*b^3*x^9))
/(1820*b^4)
```

fricas [A] time = 0.83, size = 57, normalized size = 0.71

$$\frac{\left(140\,b^4x^{12}+14\,ab^3x^9-18\,a^2b^2x^6+27\,a^3bx^3-81\,a^4\right)\left(bx^3+a\right)^{\frac{1}{3}}}{1820\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/1820*(140*b^4*x^12 + 14*a*b^3*x^9 - 18*a^2*b^2*x^6 + 27*a^3*b*x^3 - 81*a^
4)*(b*x^3 + a)^(1/3)/b^4
```

giac [A] time = 0.19, size = 57, normalized size = 0.71

$$\frac{140\left(bx^3+a\right)^{\frac{13}{3}}-546\left(bx^3+a\right)^{\frac{10}{3}}a+780\left(bx^3+a\right)^{\frac{7}{3}}a^2-455\left(bx^3+a\right)^{\frac{4}{3}}a^3}{1820\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] 1/1820*(140*(b*x^3 + a)^(13/3) - 546*(b*x^3 + a)^(10/3)*a + 780*(b*x^3 + a)
^(7/3)*a^2 - 455*(b*x^3 + a)^(4/3)*a^3)/b^4
```

maple [A] time = 0.01, size = 47, normalized size = 0.59

$$-\frac{\left(b\,x^3+a\right)^{\frac{4}{3}}\left(-140b^3x^9+126a\,b^2x^6-108a^2b\,x^3+81a^3\right)}{1820b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x^3+a)^(1/3),x)
```

```
[Out] -1/1820*(b*x^3+a)^(4/3)*(-140*b^3*x^9+126*a*b^2*x^6-108*a^2*b*x^3+81*a^3)/b
^4
```

maxima [A] time = 1.34, size = 64, normalized size = 0.80

$$\frac{\left(bx^3+a\right)^{\frac{13}{3}}}{13\,b^4}-\frac{3\left(bx^3+a\right)^{\frac{10}{3}}a}{10\,b^4}+\frac{3\left(bx^3+a\right)^{\frac{7}{3}}a^2}{7\,b^4}-\frac{\left(bx^3+a\right)^{\frac{4}{3}}a^3}{4\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/13*(b*x^3 + a)^(13/3)/b^4 - 3/10*(b*x^3 + a)^(10/3)*a/b^4 + 3/7*(b*x^3 +
a)^(7/3)*a^2/b^4 - 1/4*(b*x^3 + a)^(4/3)*a^3/b^4
```

mupad [B] time = 1.11, size = 55, normalized size = 0.69

$$\left(b\,x^3+a\right)^{1/3}\left(\frac{x^{12}}{13}-\frac{81\,a^4}{1820\,b^4}+\frac{a\,x^9}{130\,b}+\frac{27\,a^3\,x^3}{1820\,b^3}-\frac{9\,a^2\,x^6}{910\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a + b*x^3)^(1/3),x)
```

```
[Out] (a + b*x^3)^(1/3)*(x^12/13 - (81*a^4)/(1820*b^4) + (a*x^9)/(130*b) + (27*a^
3*x^3)/(1820*b^3) - (9*a^2*x^6)/(910*b^2))
```

sympy [**A**] time = 5.55, size = 110, normalized size = 1.38

$$\begin{cases} -\frac{81a^4\sqrt[3]{a+bx^3}}{1820b^4} + \frac{27a^3x^3\sqrt[3]{a+bx^3}}{1820b^3} - \frac{9a^2x^6\sqrt[3]{a+bx^3}}{910b^2} + \frac{ax^9\sqrt[3]{a+bx^3}}{130b} + \frac{x^{12}\sqrt[3]{a+bx^3}}{13} & \text{for } b \neq 0 \\ \frac{\sqrt[3]{a}\,x^{12}}{12} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x**3+a)**(1/3),x)
```

```
[Out] Piecewise((-81*a**4*(a + b*x**3)**(1/3)/(1820*b**4) + 27*a**3*x**3*(a + b*x
**3)**(1/3)/(1820*b**3) - 9*a**2*x**6*(a + b*x**3)**(1/3)/(910*b**2) + a*x*
*9*(a + b*x**3)**(1/3)/(130*b) + x**12*(a + b*x**3)**(1/3)/13, Ne(b, 0)), (
a**(1/3)*x**12/12, True))
```

3.511 $\int x^8 \sqrt[3]{a+bx^3}\,dx$

Optimal. Leaf size=59

$$\frac{a^2\left(a+bx^3\right)^{4/3}}{4b^3}+\frac{\left(a+bx^3\right)^{10/3}}{10b^3}-\frac{2a\left(a+bx^3\right)^{7/3}}{7b^3}$$

```
[Out] 1/4*a^2*(b*x^3+a)^(4/3)/b^3-2/7*a*(b*x^3+a)^(7/3)/b^3+1/10*(b*x^3+a)^(10/3)
/b^3
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{a^2\left(a+bx^3\right)^{4/3}}{4b^3}+\frac{\left(a+bx^3\right)^{10/3}}{10b^3}-\frac{2a\left(a+bx^3\right)^{7/3}}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x^3)^(1/3),x]
```

```
[Out] (a^2*(a + b*x^3)^(4/3))/(4*b^3) - (2*a*(a + b*x^3)^(7/3))/(7*b^3) + (a + b*
x^3)^(10/3)/(10*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^8\sqrt[3]{a+bx^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int x^2\sqrt[3]{a+bx}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int\left(\frac{a^2\sqrt[3]{a+bx}}{b^2}-\frac{2a(a+bx)^{4/3}}{b^2}+\frac{(a+bx)^{7/3}}{b^2}\right)dx, x, x^3\right)\\ &= \frac{a^2\left(a+bx^3\right)^{4/3}}{4b^3}-\frac{2a\left(a+bx^3\right)^{7/3}}{7b^3}+\frac{\left(a+bx^3\right)^{10/3}}{10b^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 39, normalized size = 0.66

$$\frac{\left(a+bx^3\right)^{4/3}\left(9a^2-12abx^3+14b^2x^6\right)}{140b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x^3)^(1/3),x]
```

```
[Out] ((a + b*x^3)^(4/3)*(9*a^2 - 12*a*b*x^3 + 14*b^2*x^6))/(140*b^3)
```

fricas [A] time = 0.83, size = 46, normalized size = 0.78

$$\frac{\left(14\,b^3x^9+2\,ab^2x^6-3\,a^2bx^3+9\,a^3\right)\left(bx^3+a\right)^{\frac{1}{3}}}{140\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/140*(14*b^3*x^9 + 2*a*b^2*x^6 - 3*a^2*b*x^3 + 9*a^3)*(b*x^3 + a)^(1/3)/b^
3
```

giac [A] time = 0.16, size = 43, normalized size = 0.73

$$\frac{14\left(bx^3+a\right)^{\frac{10}{3}}-40\left(bx^3+a\right)^{\frac{7}{3}}a+35\left(bx^3+a\right)^{\frac{4}{3}}a^2}{140\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] 1/140*(14*(b*x^3 + a)^(10/3) - 40*(b*x^3 + a)^(7/3)*a + 35*(b*x^3 + a)^(4/3
)*a^2)/b^3
```

maple [A] time = 0.01, size = 36, normalized size = 0.61

$$\frac{\left(b\,x^3+a\right)^{\frac{4}{3}}\left(14b^2x^6-12ab\,x^3+9a^2\right)}{140b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^3+a)^(1/3),x)
```

```
[Out] 1/140*(b*x^3+a)^(4/3)*(14*b^2*x^6-12*a*b*x^3+9*a^2)/b^3
```

maxima [A] time = 1.32, size = 47, normalized size = 0.80

$$\frac{\left(bx^3+a\right)^{\frac{10}{3}}}{10\,b^3}-\frac{2\left(bx^3+a\right)^{\frac{7}{3}}a}{7\,b^3}+\frac{\left(bx^3+a\right)^{\frac{4}{3}}a^2}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/10*(b*x^3 + a)^(10/3)/b^3 - 2/7*(b*x^3 + a)^(7/3)*a/b^3 + 1/4*(b*x^3 + a)
^(4/3)*a^2/b^3
```

mupad [B] time = 1.05, size = 44, normalized size = 0.75

$$\left(b\,x^3+a\right)^{1/3}\left(\frac{x^9}{10}+\frac{9\,a^3}{140\,b^3}+\frac{a\,x^6}{70\,b}-\frac{3\,a^2\,x^3}{140\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*x^3)^(1/3),x)
```

```
[Out] (a + b*x^3)^(1/3)*(x^9/10 + (9*a^3)/(140*b^3) + (a*x^6)/(70*b) - (3*a^2*x^3
)/(140*b^2))
```

sympy [**A**] time = 4.47, size = 87, normalized size = 1.47

$$\begin{cases} \frac{9a^3\sqrt[3]{a+bx^3}}{140b^3} - \frac{3a^2x^3\sqrt[3]{a+bx^3}}{140b^2} + \frac{ax^6\sqrt[3]{a+bx^3}}{70b} + \frac{x^9\sqrt[3]{a+bx^3}}{10} & \text{for } b \neq 0 \\ \frac{\sqrt[3]{a}\,x^9}{9} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**3+a)**(1/3),x)
```

```
[Out] Piecewise((9*a**3*(a + b*x**3)**(1/3)/(140*b**3) - 3*a**2*x**3*(a + b*x**3)
**(1/3)/(140*b**2) + a*x**6*(a + b*x**3)**(1/3)/(70*b) + x**9*(a + b*x**3)*
*(1/3)/10, Ne(b, 0)), (a**(1/3)*x**9/9, True))
```

3.512 $\int x^5 \sqrt[3]{a+bx^3}\, dx$

Optimal. Leaf size=38

$$\frac{\left(a+bx^3\right)^{7/3}}{7b^2} - \frac{a\left(a+bx^3\right)^{4/3}}{4b^2}$$

```
[Out] -1/4*a*(b*x^3+a)^(4/3)/b^2+1/7*(b*x^3+a)^(7/3)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{\left(a+bx^3\right)^{7/3}}{7b^2} - \frac{a\left(a+bx^3\right)^{4/3}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^3)^(1/3),x]
```

```
[Out] -(a*(a + b*x^3)^(4/3))/(4*b^2) + (a + b*x^3)^(7/3)/(7*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^5 \sqrt[3]{a+bx^3}\, dx &= \frac{1}{3}\operatorname{Subst}\left(\int x\sqrt[3]{a+bx}\, dx, x, x^3\right) \\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(-\frac{a\sqrt[3]{a+bx}}{b} + \frac{(a+bx)^{4/3}}{b}\right) dx, x, x^3\right) \\
&= -\frac{a\left(a+bx^3\right)^{4/3}}{4b^2} + \frac{\left(a+bx^3\right)^{7/3}}{7b^2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 0.74

$$\frac{\left(a+bx^3\right)^{4/3}\left(4bx^3-3a\right)}{28b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^3)^(1/3),x]
```

```
[Out] ((a + b*x^3)^(4/3)*(-3*a + 4*b*x^3))/(28*b^2)
```

fricas [A] time = 0.58, size = 34, normalized size = 0.89

$$\frac{\left(4\,b^2x^6+abx^3-3\,a^2\right)\left(bx^3+a\right)^{\frac{1}{3}}}{28\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(1/3),x, algorithm="fricas")
[Out] 1/28*(4*b^2*x^6 + a*b*x^3 - 3*a^2)*(b*x^3 + a)^(1/3)/b^2
```

giac [A] time = 0.16, size = 29, normalized size = 0.76

$$\frac{4\left(bx^3+a\right)^{\frac{7}{3}}-7\left(bx^3+a\right)^{\frac{4}{3}}a}{28\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(1/3),x, algorithm="giac")
[Out] 1/28*(4*(b*x^3 + a)^(7/3) - 7*(b*x^3 + a)^(4/3)*a)/b^2
```

maple [A] time = 0.01, size = 25, normalized size = 0.66

$$-\frac{\left(b\,x^3+a\right)^{\frac{4}{3}}\left(-4b\,x^3+3a\right)}{28b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^3+a)^(1/3),x)
[Out] -1/28*(b*x^3+a)^(4/3)*(-4*b*x^3+3*a)/b^2
```

maxima [A] time = 1.37, size = 30, normalized size = 0.79

$$\frac{\left(bx^3+a\right)^{\frac{7}{3}}}{7\,b^2}-\frac{\left(bx^3+a\right)^{\frac{4}{3}}a}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(1/3),x, algorithm="maxima")
[Out] 1/7*(b*x^3 + a)^(7/3)/b^2 - 1/4*(b*x^3 + a)^(4/3)*a/b^2
```

mupad [B] time = 1.03, size = 33, normalized size = 0.87

$$\left(b\,x^3+a\right)^{1/3}\left(\frac{x^6}{7}-\frac{3\,a^2}{28\,b^2}+\frac{a\,x^3}{28\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^3)^(1/3),x)
[Out] (a + b*x^3)^(1/3)*(x^6/7 - (3*a^2)/(28*b^2) + (a*x^3)/(28*b))
```

sympy [A] time = 1.82, size = 63, normalized size = 1.66

$$\begin{cases} -\frac{3a^2\sqrt[3]{a+bx^3}}{28b^2}+\frac{ax^3\sqrt[3]{a+bx^3}}{28b}+\frac{x^6\sqrt[3]{a+bx^3}}{7} & \text{for } b\neq 0 \\ \frac{\sqrt[3]{a}\,x^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**3+a)**(1/3),x)
```

```
[Out] Piecewise((-3*a**2*(a + b*x**3)**(1/3)/(28*b**2) + a*x**3*(a + b*x**3)**(1/
3)/(28*b) + x**6*(a + b*x**3)**(1/3)/7, Ne(b, 0)), (a**(1/3)*x**6/6, True))
```

3.513 $\int x^2 \sqrt[3]{a+bx^3}\, dx$

Optimal. Leaf size=18

$$\frac{\left(a+bx^3\right)^{4/3}}{4b}$$

```
[Out] 1/4*(b*x^3+a)^(4/3)/b
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$\frac{\left(a+bx^3\right)^{4/3}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3)^(1/3),x]
```

```
[Out] (a + b*x^3)^(4/3)/(4*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^2 \sqrt[3]{a+bx^3}\, dx = \frac{\left(a+bx^3\right)^{4/3}}{4b}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{\left(a+bx^3\right)^{4/3}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)^(1/3),x]
```

```
[Out] (a + b*x^3)^(4/3)/(4*b)
```

fricas [A] time = 0.83, size = 14, normalized size = 0.78

$$\frac{\left(bx^3+a\right)^{\frac{4}{3}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/4*(b*x^3 + a)^(4/3)/b
```

giac [A] time = 0.15, size = 14, normalized size = 0.78

$$\frac{\left(bx^3+a\right)^{\frac{4}{3}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] 1/4*(b*x^3 + a)^(4/3)/b
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$\frac{\left(b\,x^3+a\right)^{\frac{4}{3}}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^(1/3),x)
```

```
[Out] 1/4*(b*x^3+a)^(4/3)/b
```

maxima [A] time = 1.30, size = 14, normalized size = 0.78

$$\frac{\left(bx^3+a\right)^{\frac{4}{3}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^3 + a)^(4/3)/b
```

mupad [B] time = 1.03, size = 14, normalized size = 0.78

$$\frac{\left(b\,x^3+a\right)^{4/3}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^3)^(1/3),x)
```

```
[Out] (a + b*x^3)^(4/3)/(4*b)
```

sympy [A] time = 0.49, size = 39, normalized size = 2.17

$$\begin{cases} \frac{a\sqrt[3]{a+bx^3}}{4b}+\frac{x^3\sqrt[3]{a+bx^3}}{4} & \text{for } b\neq 0 \\ \frac{\sqrt[3]{a}\,x^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**(1/3),x)
```

```
[Out] Piecewise((a*(a + b*x**3)**(1/3)/(4*b) + x**3*(a + b*x**3)**(1/3)/4, Ne(b,
0)), (a**(1/3)*x**3/3, True))
```

3.514 $\int \frac{\sqrt[3]{a+bx^3}}{x}\,dx$

Optimal. Leaf size=95

$$\sqrt[3]{a+bx^3}+\frac{1}{2}\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)-\frac{\sqrt[3]{a}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}}-\frac{1}{2}\sqrt[3]{a}\log(x)$$

```
[Out] (b*x^3+a)^(1/3)-1/2*a^(1/3)*ln(x)+1/2*a^(1/3)*ln(a^(1/3)-(b*x^3+a)^(1/3))-1
/3*a^(1/3)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {266, 50, 57, 617, 204, 31}

$$\sqrt[3]{a+bx^3}+\frac{1}{2}\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)-\frac{\sqrt[3]{a}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}}-\frac{1}{2}\sqrt[3]{a}\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/x,x]
```

```
[Out] (a + b*x^3)^(1/3) - (a^(1/3)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3
]*a^(1/3))])/Sqrt[3] - (a^(1/3)*Log[x])/2 + (a^(1/3)*Log[a^(1/3) - (a + b*x
^3)^(1/3)])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{a+bx^3}}{x}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x}\,dx, x, x^3\right) \\
&= \sqrt[3]{a+bx^3} + \frac{1}{3}a\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^3\right) \\
&= \sqrt[3]{a+bx^3} - \frac{1}{2}\sqrt[3]{a}\,\log(x) - \frac{1}{2}\sqrt[3]{a}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right) - \frac{1}{2}a^{2/3}\,\text{Subst}\left(\int \frac{}{a^{2/3}}\right. \\
&= \sqrt[3]{a+bx^3} - \frac{1}{2}\sqrt[3]{a}\,\log(x) + \frac{1}{2}\sqrt[3]{a}\,\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right) + \sqrt[3]{a}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\right. \\
&= \sqrt[3]{a+bx^3} - \frac{\sqrt[3]{a}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{2}\sqrt[3]{a}\,\log(x) + \frac{1}{2}\sqrt[3]{a}\,\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)
\end{aligned}$$

Mathematica [A] time = 0.07, size = 124, normalized size = 1.31

$$-\frac{1}{6}\sqrt[3]{a}\,\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)+\sqrt[3]{a+bx^3}+\frac{1}{3}\sqrt[3]{a}\,\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)-\frac{\sqrt[3]{a}\,\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+}{\sqrt{3}}\right.}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/x,x]
```

```
[Out] (a + b*x^3)^(1/3) - (a^(1/3)*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqr
t[3]])/Sqrt[3] + (a^(1/3)*Log[a^(1/3) - (a + b*x^3)^(1/3)])/3 - (a^(1/3)*Lo
g[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/6
```

fricas [A] time = 0.78, size = 100, normalized size = 1.05

$$-\frac{1}{3}\sqrt{3}\,a^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}\,a}{3\,a}\right)-\frac{1}{6}a^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\frac{1}{3}a^{\frac{1}{3}}\log\left(\left(bx^3+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x,x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*a^(1/3)*arctan(1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*a^(2/3) + sqrt
(3)*a)/a) - 1/6*a^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) +
 a^(2/3)) + 1/3*a^(1/3)*log((b*x^3 + a)^(1/3) - a^(1/3)) + (b*x^3 + a)^(1/3
)
```

giac [A] time = 0.67, size = 96, normalized size = 1.01

$$-\frac{1}{3}\sqrt{3}\,a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-\frac{1}{6}\,a^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\frac{1}{3}\,a^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x,x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*a^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(
1/3)) - 1/6*a^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(
2/3)) + 1/3*a^(1/3)*log(abs((b*x^3 + a)^(1/3) - a^(1/3))) + (b*x^3 + a)^(1/
3)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x,x)
```

```
[Out] int((b*x^3+a)^(1/3)/x,x)
```

maxima [A] time = 2.88, size = 95, normalized size = 1.00

$$-\frac{1}{3}\sqrt{3}\,a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-\frac{1}{6}\,a^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\frac{1}{3}\,a^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{1}{3}}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x,x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*a^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(
1/3)) - 1/6*a^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(
2/3)) + 1/3*a^(1/3)*log((b*x^3 + a)^(1/3) - a^(1/3)) + (b*x^3 + a)^(1/3)
```

mupad [B] time = 1.08, size = 112, normalized size = 1.18

$$\frac{a^{1/3}\,\ln\left(a\left(b\,x^3+a\right)^{1/3}-a^{4/3}\right)}{3}+\left(b\,x^3+a\right)^{1/3}-\frac{a^{1/3}\,\ln\left(3\,a^{4/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)+3\,a\left(b\,x^3+a\right)^{1/3}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3}+a^{1/3}\,\ln\left($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/x,x)
```

```
[Out] (a^(1/3)*log(a*(a + b*x^3)^(1/3) - a^(4/3)))/3 + (a + b*x^3)^(1/3) - (a^(1/
3)*log(3*a^(4/3)*((3^(1/2)*1i)/2 + 1/2) + 3*a*(a + b*x^3)^(1/3))*((3^(1/2)*
1i)/2 + 1/2))/3 + a^(1/3)*log(3*a*(a + b*x^3)^(1/3) - 9*a^(4/3)*((3^(1/2)*1
i)/6 - 1/6))*((3^(1/2)*1i)/6 - 1/6)
```

sympy [C] time = 2.31, size = 42, normalized size = 0.44

$$-\frac{\sqrt[3]{b}\,x\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\right|\frac{ae^{i\pi}}{bx^3}\right)}{3\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x,x)
```

```
[Out] -b**(1/3)*x*gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), a*exp_polar(I*pi)/(b*x*
**3))/(3*gamma(2/3))
```

3.515 $\int \frac{\sqrt[3]{a+bx^3}}{x^4}\,dx$

Optimal. Leaf size=107

$$\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{2/3}}-\frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}}-\frac{b\log(x)}{6a^{2/3}}-\frac{\sqrt[3]{a+bx^3}}{3x^3}$$

```
[Out] -1/3*(b*x^3+a)^(1/3)/x^3-1/6*b*ln(x)/a^(2/3)+1/6*b*ln(a^(1/3)-(b*x^3+a)^(1/
3))/a^(2/3)-1/9*b*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/a
^(2/3)*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {266, 47, 57, 617, 204, 31}

$$\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{2/3}}-\frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}}-\frac{b\log(x)}{6a^{2/3}}-\frac{\sqrt[3]{a+bx^3}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/x^4,x]
```

```
[Out] -(a + b*x^3)^(1/3)/(3*x^3) - (b*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqr
t[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)) - (b*Log[x])/(6*a^(2/3)) + (b*Log[a^(1/
3) - (a + b*x^3)^(1/3)])/(6*a^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{a+bx^3}}{x^4}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x^2}\,dx, x, x^3\right) \\
&= -\frac{\sqrt[3]{a+bx^3}}{3x^3} + \frac{1}{9}b\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^3\right) \\
&= -\frac{\sqrt[3]{a+bx^3}}{3x^3} - \frac{b\log(x)}{6a^{2/3}} - \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{6a^{2/3}} - \frac{b\,\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx, x,\right.}{6\sqrt[3]{a}} \\
&= -\frac{\sqrt[3]{a+bx^3}}{3x^3} - \frac{b\log(x)}{6a^{2/3}} + \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{2/3}} + \frac{b\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{3a^{2/3}} \\
&= -\frac{\sqrt[3]{a+bx^3}}{3x^3} - \frac{b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{2/3}} - \frac{b\log(x)}{6a^{2/3}} + \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{2/3}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.35

$$\frac{b\left(a+bx^3\right)^{4/3}\,{}_2F_1\left(\frac{4}{3},2;\frac{7}{3};\frac{bx^3}{a}+1\right)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/x^4,x]
```

```
[Out] (b*(a + b*x^3)^(4/3)*Hypergeometric2F1[4/3, 2, 7/3, 1 + (b*x^3)/a])/(4*a^2)
```

fricas [A] time = 0.81, size = 155, normalized size = 1.45

$$-\frac{2\sqrt{3}\left(a^2\right)^{\frac{1}{6}}abx^3\arctan\left(\frac{\left(a^2\right)^{\frac{1}{6}}\left(\sqrt{3}\left(a^2\right)^{\frac{1}{3}}a+2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{2}{3}}\right)}{3a^2}\right)+\left(a^2\right)^{\frac{2}{3}}bx^3\log\left(\left(bx^3+a\right)^{\frac{2}{3}}a+\left(a^2\right)^{\frac{1}{3}}a+\left(bx^3+a\right)^{\frac{1}{3}}\right.}{18\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^4,x, algorithm="fricas")
```

```
[Out] -1/18*(2*sqrt(3)*(a^2)^(1/6)*a*b*x^3*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2)^
(1/3)*a + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(a^2)^(2/3))/a^2) + (a^2)^(2/3)*b*x^3
*log((b*x^3 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(a^2)^(2/3)) -
 2*(a^2)^(2/3)*b*x^3*log((b*x^3 + a)^(1/3)*a - (a^2)^(2/3)) + 6*(b*x^3 + a)
^(1/3)*a^2)/(a^2*x^3)
```

giac [A] time = 0.69, size = 115, normalized size = 1.07

$$-\frac{\frac{2\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}}+\frac{b^2\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{2}{3}}}-\frac{2b^2\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{2}{3}}}+\frac{6\left(bx^3+a\right)^{\frac{1}{3}}b}{x^3}}{18\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^4,x, algorithm="giac")
```

```
[Out] -1/18*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(
1/3))/a^(2/3) + b^2*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(
2/3))/a^(2/3) - 2*b^2*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/a^(2/3) + 6*(b*
x^3 + a)^(1/3)*b/x^3)/b
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^4,x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^4,x)
```

maxima [A] time = 2.97, size = 103, normalized size = 0.96

$$-\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{9\,a^{\frac{2}{3}}}-\frac{b\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{\frac{2}{3}}}+\frac{b\log\left(\left(bx^3+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9\,a^{\frac{2}{3}}}-\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^4,x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(2/3) - 1/18*b*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3
))/a^(2/3) + 1/9*b*log((b*x^3 + a)^(1/3) - a^(1/3))/a^(2/3) - 1/3*(b*x^3 +
a)^(1/3)/x^3
```

mupad [B] time = 1.21, size = 122, normalized size = 1.14

$$\frac{b\,\ln\left(b\left(b\,x^3+a\right)^{1/3}-a^{1/3}\,b\right)}{9\,a^{2/3}}-\frac{\left(b\,x^3+a\right)^{1/3}}{3\,x^3}-\frac{\ln\left(\frac{a^{1/3}\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{2}+b\left(b\,x^3+a\right)^{1/3}\right)\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{18\,a^{2/3}}-\frac{\ln\left(\frac{a^{1/3}\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{2}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/x^4,x)
```

```
[Out] (b*log(b*(a + b*x^3)^(1/3) - a^(1/3)*b))/(9*a^(2/3)) - (a + b*x^3)^(1/3)/(3
*x^3) - (log((a^(1/3)*(b - 3^(1/2)*b*1i))/2 + b*(a + b*x^3)^(1/3))*(b - 3^(
1/2)*b*1i))/(18*a^(2/3)) - (log((a^(1/3)*(b + 3^(1/2)*b*1i))/2 + b*(a + b*x
^3)^(1/3))*(b + 3^(1/2)*b*1i))/(18*a^(2/3))
```

sympy [C] time = 2.62, size = 41, normalized size = 0.38

$$-\frac{\sqrt[3]{b}\,\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3}, \frac{2}{3} \\ \frac{5}{3}\end{matrix}\right| \frac{ae^{i\pi}}{bx^3}\right)}{3x^2\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**4,x)
```

```
[Out] -b**(1/3)*gamma(2/3)*hyper((-1/3, 2/3), (5/3,), a*exp_polar(I*pi)/(b*x**3))
/(3*x**2*gamma(5/3))
```

3.516 $\int x^4 \sqrt[3]{a+bx^3}\, dx$

Optimal. Leaf size=120

$$\frac{a^2 \log\left(\sqrt[3]{b}\, x - \sqrt[3]{a+bx^3}\right)}{18b^{5/3}} + \frac{a^2 \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\, x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\, b^{5/3}} + \frac{1}{6}x^5 \sqrt[3]{a+bx^3} + \frac{ax^2 \sqrt[3]{a+bx^3}}{18b}$$

```
[Out] 1/18*a*x^2*(b*x^3+a)^(1/3)/b+1/6*x^5*(b*x^3+a)^(1/3)+1/18*a^2*ln(b^(1/3)*x-
(b*x^3+a)^(1/3))/b^(5/3)+1/27*a^2*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3)
)*3^(1/2))/b^(5/3)*3^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 173, normalized size of antiderivative = 1.44, number of steps used = 9, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {279, 321, 331, 292, 31, 634, 617, 204, 628}

$$\frac{a^2 \log\left(1 - \frac{\sqrt[3]{b}\, x}{\sqrt[3]{a+bx^3}}\right)}{27b^{5/3}} - \frac{a^2 \log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}} + \frac{\sqrt[3]{b}\, x}{\sqrt[3]{a+bx^3}} + 1\right)}{54b^{5/3}} + \frac{a^2 \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\, x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\, b^{5/3}} + \frac{1}{6}x^5 \sqrt[3]{a+bx^3} + \frac{ax^2 \sqrt[3]{a+bx^3}}{18b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^3)^(1/3),x]
```

```
[Out] (a*x^2*(a + b*x^3)^(1/3))/(18*b) + (x^5*(a + b*x^3)^(1/3))/6 + (a^2*ArcTan[
(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(5/3)) + (a^2*
Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(27*b^(5/3)) - (a^2*Log[1 + (b^(2/3
)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(54*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int x^4 \sqrt[3]{a+bx^3}\,dx = \frac{1}{6}x^5\sqrt[3]{a+bx^3} + \frac{1}{6}a\int \frac{x^4}{\left(a+bx^3\right)^{2/3}}\,dx$$

$$= \frac{ax^2\sqrt[3]{a+bx^3}}{18b} + \frac{1}{6}x^5\sqrt[3]{a+bx^3} - \frac{a^2\int \frac{x}{\left(a+bx^3\right)^{2/3}}\,dx}{9b}$$

$$= \frac{ax^2\sqrt[3]{a+bx^3}}{18b} + \frac{1}{6}x^5\sqrt[3]{a+bx^3} - \frac{a^2\,\text{Subst}\left(\int \frac{x}{1-bx^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{9b}$$

$$= \frac{ax^2\sqrt[3]{a+bx^3}}{18b} + \frac{1}{6}x^5\sqrt[3]{a+bx^3} - \frac{a^2\,\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{b}x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{27b^{4/3}} + \frac{a^2\,\text{Subst}\left(\int \frac{1-\sqrt[3]{b}x}{1+\sqrt[3]{b}x+b^{2/3}}\right.}{27b^{4/3}}$$

$$= \frac{ax^2\sqrt[3]{a+bx^3}}{18b} + \frac{1}{6}x^5\sqrt[3]{a+bx^3} + \frac{a^2\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{27b^{5/3}} - \frac{a^2\,\text{Subst}\left(\int \frac{\sqrt[3]{b}+2b^{2/3}x}{1+\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+bx}}\right.}{54b^{5/3}}$$

$$= \frac{ax^2\sqrt[3]{a+bx^3}}{18b} + \frac{1}{6}x^5\sqrt[3]{a+bx^3} + \frac{a^2\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{27b^{5/3}} - \frac{a^2\log\left(1+\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{54b^{5/3}} - \frac{a^2}{}$$

$$= \frac{ax^2\sqrt[3]{a+bx^3}}{18b} + \frac{1}{6}x^5\sqrt[3]{a+bx^3} + \frac{a^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}\,b^{5/3}} + \frac{a^2\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{27b^{5/3}} - \frac{a^2\log\left(1+\frac{}{(a}\right.}{5}$$

Mathematica [C] time = 0.09, size = 64, normalized size = 0.53

$$\frac{x^2\sqrt[3]{a+bx^3}\left(-\frac{a\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{\sqrt[3]{\frac{bx^3}{a}+1}} + a + bx^3\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^3)^(1/3),x]
```

```
[Out] (x^2*(a + b*x^3)^(1/3)*(a + b*x^3 - (a*Hypergeometric2F1[-1/3, 2/3, 5/3, -(
(b*x^3)/a)])/(1 + (b*x^3)/a)^(1/3)))/(6*b)
```

fricas [A] time = 0.86, size = 181, normalized size = 1.51

$$-\frac{2\sqrt{3}\,a^2\left(b^2\right)^{\frac{1}{6}}b\arctan\left(\frac{\left(\sqrt{3}\left(b^2\right)^{\frac{1}{3}}bx+2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(b^2\right)^{\frac{2}{3}}\right)\left(b^2\right)^{\frac{1}{6}}}{3\,b^2x}\right) - 2\,a^2\left(b^2\right)^{\frac{2}{3}}\log\left(-\frac{\left(b^2\right)^{\frac{2}{3}}x-\left(bx^3+a\right)^{\frac{1}{3}}b}{x}\right) + a^2\left(b^2\right)^{\frac{2}{3}}\log\left(\frac{\left(b^2\right)^{}}{}\right.}{54\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/54*(2*sqrt(3)*a^2*(b^2)^(1/6)*b*arctan(1/3*(sqrt(3)*(b^2)^(1/3)*b*x + 2*
sqrt(3)*(b*x^3 + a)^(1/3)*(b^2)^(2/3))*(b^2)^(1/6)/(b^2*x)) - 2*a^2*(b^2)^(
2/3)*log(-((b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + a^2*(b^2)^(2/3)*log(((
b^2)^(1/3)*b*x^2 + (b*x^3 + a)^(1/3)*(b^2)^(2/3)*x + (b*x^3 + a)^(2/3)*b)/x
^2) - 3*(3*b^3*x^5 + a*b^2*x^2)*(b*x^3 + a)^(1/3))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{1}{3}}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*x^4, x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \left(b\,x^3+a\right)^{\frac{1}{3}} x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)^(1/3),x)
```

```
[Out] int(x^4*(b*x^3+a)^(1/3),x)
```

maxima [A] time = 2.99, size = 181, normalized size = 1.51

$$-\frac{\sqrt{3}\,a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{27\,b^{\frac{5}{3}}}-\frac{a^2\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{54\,b^{\frac{5}{3}}}+\frac{a^2\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{27\,b^{\frac{5}{3}}}+\frac{\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}}{18\left(b^3-\frac{2\left(b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/27*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1
/3))/b^(5/3) - 1/54*a^2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3
+ a)^(2/3)/x^2)/b^(5/3) + 1/27*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(5
/3) + 1/18*(2*(b*x^3 + a)^(1/3)*a^2*b/x + (b*x^3 + a)^(4/3)*a^2/x^4)/(b^3 -
 2*(b*x^3 + a)*b^2/x^3 + (b*x^3 + a)^2*b/x^6)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\left(b\,x^3+a\right)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^3)^(1/3),x)
```

```
[Out] int(x^4*(a + b*x^3)^(1/3), x)
```

sympy [C] time = 1.62, size = 39, normalized size = 0.32

$$\frac{\sqrt[3]{a}\,x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)**(1/3),x)
```

```
[Out] a**(1/3)*x**5*gamma(5/3)*hyper((-1/3, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(8/3))
```

3.517 $\int x\sqrt[3]{a+bx^3}\,dx$

Optimal. Leaf size=94

$$-\frac{a\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{6b^{2/3}}-\frac{a\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{2/3}}+\frac{1}{3}x^2\sqrt[3]{a+bx^3}$$

```
[Out] 1/3*x^2*(b*x^3+a)^(1/3)-1/6*a*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)-1/9*a*a
rctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)*3^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 145, normalized size of antiderivative = 1.54, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {279, 331, 292, 31, 634, 617, 204, 628}

$$-\frac{a\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{9b^{2/3}}+\frac{a\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1\right)}{18b^{2/3}}-\frac{a\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{2/3}}+\frac{1}{3}x^2\sqrt[3]{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3)^(1/3),x]
```

```
[Out] (x^2*(a + b*x^3)^(1/3))/3 - (a*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))
/Sqrt[3]])/(3*Sqrt[3]*b^(2/3)) - (a*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)])
/(9*b^(2/3)) + (a*Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a
+ b*x^3)^(1/3)])/(18*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
```

```
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int x\sqrt[3]{a+bx^3}\,dx &= \frac{1}{3}x^2\sqrt[3]{a+bx^3} + \frac{1}{3}a\int \frac{x}{\left(a+bx^3\right)^{2/3}}\,dx \\
&= \frac{1}{3}x^2\sqrt[3]{a+bx^3} + \frac{1}{3}a\,\text{Subst}\left(\int \frac{x}{1-bx^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right) \\
&= \frac{1}{3}x^2\sqrt[3]{a+bx^3} + \frac{a\,\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{b}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{9\sqrt[3]{b}} - \frac{a\,\text{Subst}\left(\int \frac{1-\sqrt[3]{b}\,x}{1+\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{9\sqrt[3]{b}} \\
&= \frac{1}{3}x^2\sqrt[3]{a+bx^3} - \frac{a\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{9b^{2/3}} + \frac{a\,\text{Subst}\left(\int \frac{\sqrt[3]{b}+2b^{2/3}x}{1+\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{18b^{2/3}} - \frac{a\,\text{Subst}\left(\int \right.}{} \\
&= \frac{1}{3}x^2\sqrt[3]{a+bx^3} - \frac{a\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{9b^{2/3}} + \frac{a\log\left(1+\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{18b^{2/3}} + \frac{a\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,d\right.}{3b^2} \\
&= \frac{1}{3}x^2\sqrt[3]{a+bx^3} - \frac{a\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{2/3}} - \frac{a\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{9b^{2/3}} + \frac{a\log\left(1+\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{18b^{2/3}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.54

$$\frac{x^2\sqrt[3]{a+bx^3}\;{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)^(1/3),x]
```

```
[Out] (x^2*(a + b*x^3)^(1/3)*Hypergeometric2F1[-1/3, 2/3, 5/3, -((b*x^3)/a)])/(2*
(1 + (b*x^3)/a)^(1/3))
```

fricas [B] time = 0.86, size = 193, normalized size = 2.05

$$\frac{6\left(bx^3+a\right)^{\frac{1}{3}}b^2x^2+2\sqrt{3}\,ab\sqrt{-\left(-b^2\right)^{\frac{1}{3}}}\arctan\left(-\frac{\left(\sqrt{3}\left(-b^2\right)^{\frac{1}{3}}bx-2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(-b^2\right)^{\frac{2}{3}}\right)\sqrt{-\left(-b^2\right)^{\frac{1}{3}}}}{3b^2x}\right)-2\left(-b^2\right)^{\frac{2}{3}}a\log\left(-\frac{(-b^2}{}\right.}{18b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/18*(6*(b*x^3 + a)^(1/3)*b^2*x^2 + 2*sqrt(3)*a*b*sqrt(-(-b^2)^(1/3))*arcta
n(-1/3*(sqrt(3)*(-b^2)^(1/3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(2/3)
)*sqrt(-(-b^2)^(1/3))/(b^2*x)) - 2*(-b^2)^(2/3)*a*log(-((-b^2)^(2/3)*x - (b
*x^3 + a)^(1/3)*b)/x) + (-b^2)^(2/3)*a*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 +
a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{1}{3}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*x, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \left(b\,x^3+a\right)^{\frac{1}{3}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^(1/3),x)
```

```
[Out] int(x*(b*x^3+a)^(1/3),x)
```

maxima [A] time = 2.98, size = 134, normalized size = 1.43

$$\frac{\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{9b^{\frac{2}{3}}}+\frac{a\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{18b^{\frac{2}{3}}}-\frac{a\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{9b^{\frac{2}{3}}}-\frac{\left(bx^3+a\right)^{\frac{1}{3}}a}{3\left(b-\frac{bx^3+a}{x^3}\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))
/b^(2/3) + 1/18*a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(
2/3)/x^2)/b^(2/3) - 1/9*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(2/3) - 1/3
*(b*x^3 + a)^(1/3)*a/((b - (b*x^3 + a)/x^3)*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \left(b\,x^3 + a\right)^{1/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^3)^(1/3),x)
```

```
[Out] int(x*(a + b*x^3)^(1/3), x)
```

sympy [C] time = 1.21, size = 39, normalized size = 0.41

$$\frac{\sqrt[3]{a}\,x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3}, \frac{2}{3} \\ \frac{5}{3}\end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**(1/3),x)
```

```
[Out] a**(1/3)*x**2*gamma(2/3)*hyper((-1/3, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(5/3))
```

3.518 $\int \frac{\sqrt[3]{a+bx^3}}{x^2}\,dx$

Optimal. Leaf size=88

$$-\frac{\sqrt[3]{a+bx^3}}{x}-\frac{1}{2}\sqrt[3]{b}\,\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)-\frac{\sqrt[3]{b}\,\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -(b*x^3+a)^(1/3)/x-1/2*b^(1/3)*ln(b^(1/3)*x-(b*x^3+a)^(1/3))-1/3*b^(1/3)*ar
ctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 138, normalized size of antiderivative = 1.57, number of steps used = 8, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {277, 331, 292, 31, 634, 617, 204, 628}

$$\frac{1}{6}\sqrt[3]{b}\,\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1\right)-\frac{\sqrt[3]{a+bx^3}}{x}-\frac{1}{3}\sqrt[3]{b}\,\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)-\frac{\sqrt[3]{b}\,\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/x^2,x]
```

```
[Out] -((a + b*x^3)^(1/3)/x) - (b^(1/3)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/
3))/Sqrt[3]])/Sqrt[3] - (b^(1/3)*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)])/3
+ (b^(1/3)*Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3
)^(1/3)])/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{a+bx^3}}{x^2}\,dx &= -\frac{\sqrt[3]{a+bx^3}}{x} + b\int \frac{x}{\left(a+bx^3\right)^{2/3}}\,dx \\
&= -\frac{\sqrt[3]{a+bx^3}}{x} + b\,\text{Subst}\left(\int \frac{x}{1-bx^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right) \\
&= -\frac{\sqrt[3]{a+bx^3}}{x} + \frac{1}{3}b^{2/3}\,\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{b}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right) - \frac{1}{3}b^{2/3}\,\text{Subst}\left(\int \frac{1-\sqrt[3]{b}\,x}{1+\sqrt[3]{b}\,x+b^{2/3}} \right. \\
&= -\frac{\sqrt[3]{a+bx^3}}{x} - \frac{1}{3}\sqrt[3]{b}\,\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right) + \frac{1}{6}\sqrt[3]{b}\,\text{Subst}\left(\int \frac{\sqrt[3]{b}+2b^{2/3}x}{1+\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+bx}} \right. \\
&= -\frac{\sqrt[3]{a+bx^3}}{x} - \frac{1}{3}\sqrt[3]{b}\,\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right) + \frac{1}{6}\sqrt[3]{b}\,\log\left(1+\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right) + \sqrt[3]{b}\,\text{S} \\
&= -\frac{\sqrt[3]{a+bx^3}}{x} - \frac{\sqrt[3]{b}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{3}\sqrt[3]{b}\,\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right) + \frac{1}{6}\sqrt[3]{b}\,\log\left(1+\frac{b^{2/3}x^2}{\left(a+bx^3\right)}\right.
\end{aligned}$$

Mathematica [C] time = 0.01, size = 49, normalized size = 0.56

$$-\frac{\sqrt[3]{a+bx^3}\;{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\frac{2}{3};-\frac{bx^3}{a}\right)}{x\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/x^2,x]
```

```
[Out] -(((a + b*x^3)^(1/3)*Hypergeometric2F1[-1/3, -1/3, 2/3, -((b*x^3)/a)])/(x*(
1 + (b*x^3)/a)^(1/3)))
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/x^2, x)
```

maple [**F**] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^2,x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^2,x)
```

maxima [**A**] time = 2.87, size = 114, normalized size = 1.30

$$\frac{1}{3}\sqrt{3}\,b^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)+\frac{1}{6}\,b^{\frac{1}{3}}\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)-\frac{1}{3}\,b^{\frac{1}{3}}\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*b^(1/3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^
(1/3)) + 1/6*b^(1/3)*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a
)^(2/3)/x^2) - 1/3*b^(1/3)*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x) - (b*x^3 + a
)^(1/3)/x
```

mupad [**B**] time = 1.21, size = 40, normalized size = 0.45

$$-\frac{\left(b\,x^3+a\right)^{1/3}\,{}_2\mathrm{F}_1\left(-\frac{1}{3},-\frac{1}{3};\ \frac{2}{3};\ -\frac{b\,x^3}{a}\right)}{x\left(\frac{b\,x^3}{a}+1\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/x^2,x)
```

```
[Out] -((a + b*x^3)^(1/3)*hypergeom([-1/3, -1/3], 2/3, -(b*x^3)/a))/(x*((b*x^3)/a
 + 1)^(1/3))
```

sympy [C] time = 2.21, size = 41, normalized size = 0.47

$$\frac{\sqrt[3]{a}\,\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3}, -\frac{1}{3} \\ \frac{2}{3}\end{matrix}\right| \frac{bx^3 e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**2,x)
```

```
[Out] a**(1/3)*gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/
(3*x*gamma(2/3))
```

3.519 $\int \frac{\sqrt[3]{a+bx^3}}{x^5}\,dx$

Optimal. Leaf size=21

$$-\frac{\left(a+bx^3\right)^{4/3}}{4ax^4}$$

```
[Out] -1/4*(b*x^3+a)^(4/3)/a/x^4
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {264}

$$-\frac{\left(a+bx^3\right)^{4/3}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/x^5,x]
```

```
[Out] -(a + b*x^3)^(4/3)/(4*a*x^4)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5}\,dx = -\frac{\left(a+bx^3\right)^{4/3}}{4ax^4}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$-\frac{\left(a+bx^3\right)^{4/3}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/x^5,x]
```

```
[Out] -1/4*(a + b*x^3)^(4/3)/(a*x^4)
```

fricas [A] time = 0.88, size = 17, normalized size = 0.81

$$-\frac{\left(bx^3+a\right)^{\frac{4}{3}}}{4\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^5,x, algorithm="fricas")
```

```
[Out] -1/4*(b*x^3 + a)^(4/3)/(a*x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/x^5, x)
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$-\frac{\left(b\,x^3+a\right)^{\frac{4}{3}}}{4a\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^5,x)
```

```
[Out] -1/4*(b*x^3+a)^(4/3)/a/x^4
```

maxima [A] time = 1.32, size = 17, normalized size = 0.81

$$-\frac{\left(bx^3+a\right)^{\frac{4}{3}}}{4\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*(b*x^3 + a)^(4/3)/(a*x^4)
```

mupad [B] time = 1.11, size = 17, normalized size = 0.81

$$-\frac{\left(b\,x^3+a\right)^{4/3}}{4\,a\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/x^5,x)
```

```
[Out] -(a + b*x^3)^(4/3)/(4*a*x^4)
```

sympy [B] time = 1.47, size = 68, normalized size = 3.24

$$\frac{\sqrt[3]{b}\,\sqrt[3]{\frac{a}{bx^3}+1}\,\Gamma\left(-\frac{4}{3}\right)}{3x^3\Gamma\left(-\frac{1}{3}\right)}+\frac{b^{\frac{4}{3}}\sqrt[3]{\frac{a}{bx^3}+1}\,\Gamma\left(-\frac{4}{3}\right)}{3a\Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**5,x)
```

```
[Out] b**(1/3)*(a/(b*x**3) + 1)**(1/3)*gamma(-4/3)/(3*x**3*gamma(-1/3)) + b**(4/3
)*(a/(b*x**3) + 1)**(1/3)*gamma(-4/3)/(3*a*gamma(-1/3))
```

3.520 $\int \frac{\sqrt[3]{a+bx^3}}{x^8}\,dx$

Optimal. Leaf size=44

$$\frac{3b\left(a+bx^3\right)^{4/3}}{28a^2x^4}-\frac{\left(a+bx^3\right)^{4/3}}{7ax^7}$$

```
[Out] -1/7*(b*x^3+a)^(4/3)/a/x^7+3/28*b*(b*x^3+a)^(4/3)/a^2/x^4
```

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$\frac{3b\left(a+bx^3\right)^{4/3}}{28a^2x^4}-\frac{\left(a+bx^3\right)^{4/3}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/x^8,x]
```

```
[Out] -(a + b*x^3)^(4/3)/(7*a*x^7) + (3*b*(a + b*x^3)^(4/3))/(28*a^2*x^4)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^3}}{x^8}\,dx &= -\frac{\left(a+bx^3\right)^{4/3}}{7ax^7}-\frac{(3b)\int \frac{\sqrt[3]{a+bx^3}}{x^5}\,dx}{7a}\\ &= -\frac{\left(a+bx^3\right)^{4/3}}{7ax^7}+\frac{3b\left(a+bx^3\right)^{4/3}}{28a^2x^4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.70

$$\frac{\left(a+bx^3\right)^{4/3}\left(3bx^3-4a\right)}{28a^2x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/x^8,x]
```

```
[Out] ((a + b*x^3)^(4/3)*(-4*a + 3*b*x^3))/(28*a^2*x^7)
```

fricas [A] time = 1.09, size = 38, normalized size = 0.86

$$\frac{\left(3\,b^2x^6-abx^3-4\,a^2\right)\left(bx^3+a\right)^{\frac{1}{3}}}{28\,a^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^8,x, algorithm="fricas")
```

```
[Out] 1/28*(3*b^2*x^6 - a*b*x^3 - 4*a^2)*(b*x^3 + a)^(1/3)/(a^2*x^7)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^8,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/x^8, x)
```

maple [A] time = 0.01, size = 28, normalized size = 0.64

$$-\frac{\left(b\,x^3+a\right)^{\frac{4}{3}}\left(-3b\,x^3+4a\right)}{28a^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^8,x)
```

```
[Out] -1/28*(b*x^3+a)^(4/3)*(-3*b*x^3+4*a)/a^2/x^7
```

maxima [A] time = 1.34, size = 35, normalized size = 0.80

$$\frac{\frac{7\left(bx^3+a\right)^{\frac{4}{3}}b}{x^4}-\frac{4\left(bx^3+a\right)^{\frac{7}{3}}}{x^7}}{28\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^8,x, algorithm="maxima")
```

```
[Out] 1/28*(7*(b*x^3 + a)^(4/3)*b/x^4 - 4*(b*x^3 + a)^(7/3)/x^7)/a^2
```

mupad [B] time = 1.23, size = 32, normalized size = 0.73

$$-\frac{7\,a\left(b\,x^3+a\right)^{4/3}-3\left(b\,x^3+a\right)^{7/3}}{28\,a^2\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/x^8,x)
```

```
[Out] -(7*a*(a + b*x^3)^(4/3) - 3*(a + b*x^3)^(7/3))/(28*a^2*x^7)
```

sympy [B] time = 1.23, size = 109, normalized size = 2.48

$$-\frac{4\sqrt[3]{b}\,\sqrt[3]{\frac{a}{bx^3}+1}\,\Gamma\left(-\frac{7}{3}\right)}{9x^6\Gamma\left(-\frac{1}{3}\right)}-\frac{b^{\frac{4}{3}}\sqrt[3]{\frac{a}{bx^3}+1}\,\Gamma\left(-\frac{7}{3}\right)}{9ax^3\Gamma\left(-\frac{1}{3}\right)}+\frac{b^{\frac{7}{3}}\sqrt[3]{\frac{a}{bx^3}+1}\,\Gamma\left(-\frac{7}{3}\right)}{3a^2\Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**8,x)
```

```
[Out] -4*b**(1/3)*(a/(b*x**3) + 1)**(1/3)*gamma(-7/3)/(9*x**6*gamma(-1/3)) - b**(
4/3)*(a/(b*x**3) + 1)**(1/3)*gamma(-7/3)/(9*a*x**3*gamma(-1/3)) + b**(7/3)*
(a/(b*x**3) + 1)**(1/3)*gamma(-7/3)/(3*a**2*gamma(-1/3))
```

3.521 $\int \frac{\sqrt[3]{a+bx^3}}{x^{11}}\,dx$

Optimal. Leaf size=68

$$-\frac{9b^2\left(a+bx^3\right)^{4/3}}{140a^3x^4}+\frac{3b\left(a+bx^3\right)^{4/3}}{35a^2x^7}-\frac{\left(a+bx^3\right)^{4/3}}{10ax^{10}}$$

```
[Out] -1/10*(b*x^3+a)^(4/3)/a/x^10+3/35*b*(b*x^3+a)^(4/3)/a^2/x^7-9/140*b^2*(b*x^
3+a)^(4/3)/a^3/x^4
```

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$-\frac{9b^2\left(a+bx^3\right)^{4/3}}{140a^3x^4}+\frac{3b\left(a+bx^3\right)^{4/3}}{35a^2x^7}-\frac{\left(a+bx^3\right)^{4/3}}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/x^11,x]
```

```
[Out] -(a + b*x^3)^(4/3)/(10*a*x^10) + (3*b*(a + b*x^3)^(4/3))/(35*a^2*x^7) - (9*
b^2*(a + b*x^3)^(4/3))/(140*a^3*x^4)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^3}}{x^{11}}\,dx &= -\frac{\left(a+bx^3\right)^{4/3}}{10ax^{10}}-\frac{(3b)\int \frac{\sqrt[3]{a+bx^3}}{x^8}\,dx}{5a}\\ &= -\frac{\left(a+bx^3\right)^{4/3}}{10ax^{10}}+\frac{3b\left(a+bx^3\right)^{4/3}}{35a^2x^7}+\frac{\left(9b^2\right)\int \frac{\sqrt[3]{a+bx^3}}{x^5}\,dx}{35a^2}\\ &= -\frac{\left(a+bx^3\right)^{4/3}}{10ax^{10}}+\frac{3b\left(a+bx^3\right)^{4/3}}{35a^2x^7}-\frac{9b^2\left(a+bx^3\right)^{4/3}}{140a^3x^4}\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.62

$$-\frac{\left(a+bx^3\right)^{4/3}\left(14a^2-12abx^3+9b^2x^6\right)}{140a^3x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/x^11,x]
```

```
[Out] -1/140*((a + b*x^3)^(4/3)*(14*a^2 - 12*a*b*x^3 + 9*b^2*x^6))/(a^3*x^10)
```

fricas [A] time = 0.88, size = 49, normalized size = 0.72

$$-\frac{\left(9\,b^3x^9-3\,ab^2x^6+2\,a^2bx^3+14\,a^3\right)\left(bx^3+a\right)^{\frac{1}{3}}}{140\,a^3x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^11,x, algorithm="fricas")
```

```
[Out] -1/140*(9*b^3*x^9 - 3*a*b^2*x^6 + 2*a^2*b*x^3 + 14*a^3)*(b*x^3 + a)^(1/3)/(
a^3*x^10)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^11,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/x^11, x)
```

maple [A] time = 0.01, size = 39, normalized size = 0.57

$$-\frac{\left(b\,x^3+a\right)^{\frac{4}{3}}\left(9b^2x^6-12ab\,x^3+14a^2\right)}{140a^3x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^11,x)
```

```
[Out] -1/140*(b*x^3+a)^(4/3)*(9*b^2*x^6-12*a*b*x^3+14*a^2)/x^10/a^3
```

maxima [A] time = 1.34, size = 52, normalized size = 0.76

$$-\frac{\frac{35\left(bx^3+a\right)^{\frac{4}{3}}b^2}{x^4}-\frac{40\left(bx^3+a\right)^{\frac{7}{3}}b}{x^7}+\frac{14\left(bx^3+a\right)^{\frac{10}{3}}}{x^{10}}}{140\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^11,x, algorithm="maxima")
```

```
[Out] -1/140*(35*(b*x^3 + a)^(4/3)*b^2/x^4 - 40*(b*x^3 + a)^(7/3)*b/x^7 + 14*(b*x
^3 + a)^(10/3)/x^10)/a^3
```

mupad [B] time = 1.35, size = 73, normalized size = 1.07

$$\frac{3\,b^2\left(b\,x^3+a\right)^{1/3}}{140\,a^2\,x^4}-\frac{b\left(b\,x^3+a\right)^{1/3}}{70\,a\,x^7}-\frac{9\,b^3\left(b\,x^3+a\right)^{1/3}}{140\,a^3\,x}-\frac{\left(b\,x^3+a\right)^{1/3}}{10\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/x^11,x)
```

```
[Out] (3*b^2*(a + b*x^3)^(1/3))/(140*a^2*x^4) - (b*(a + b*x^3)^(1/3))/(70*a*x^7)
- (9*b^3*(a + b*x^3)^(1/3))/(140*a^3*x) - (a + b*x^3)^(1/3)/(10*x^10)
```

sympy [**B**] time = 3.23, size = 520, normalized size = 7.65

$$\frac{28a^5b^{\frac{13}{3}}\sqrt[3]{\frac{a}{bx^3}+1}\,\Gamma\left(-\frac{10}{3}\right)}{27a^5b^4x^9\Gamma\left(-\frac{1}{3}\right)+54a^4b^5x^{12}\Gamma\left(-\frac{1}{3}\right)+27a^3b^6x^{15}\Gamma\left(-\frac{1}{3}\right)}+\frac{60a^4b^{\frac{16}{3}}x^3\sqrt[3]{\frac{a}{bx^3}+1}\,\Gamma\left(-\frac{10}{3}\right)}{27a^5b^4x^9\Gamma\left(-\frac{1}{3}\right)+54a^4b^5x^{12}\Gamma\left(-\frac{1}{3}\right)+27a^3b^6x^{15}\Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**11,x)
```

```
[Out] 28*a**5*b**(13/3)*(a/(b*x**3) + 1)**(1/3)*gamma(-10/3)/(27*a**5*b**4*x**9*g
amma(-1/3) + 54*a**4*b**5*x**12*gamma(-1/3) + 27*a**3*b**6*x**15*gamma(-1/3
)) + 60*a**4*b**(16/3)*x**3*(a/(b*x**3) + 1)**(1/3)*gamma(-10/3)/(27*a**5*b
**4*x**9*gamma(-1/3) + 54*a**4*b**5*x**12*gamma(-1/3) + 27*a**3*b**6*x**15*
gamma(-1/3)) + 30*a**3*b**(19/3)*x**6*(a/(b*x**3) + 1)**(1/3)*gamma(-10/3)/
(27*a**5*b**4*x**9*gamma(-1/3) + 54*a**4*b**5*x**12*gamma(-1/3) + 27*a**3*b
**6*x**15*gamma(-1/3)) + 10*a**2*b**(22/3)*x**9*(a/(b*x**3) + 1)**(1/3)*gam
ma(-10/3)/(27*a**5*b**4*x**9*gamma(-1/3) + 54*a**4*b**5*x**12*gamma(-1/3) +
 27*a**3*b**6*x**15*gamma(-1/3)) + 30*a*b**(25/3)*x**12*(a/(b*x**3) + 1)**(
1/3)*gamma(-10/3)/(27*a**5*b**4*x**9*gamma(-1/3) + 54*a**4*b**5*x**12*gamma
(-1/3) + 27*a**3*b**6*x**15*gamma(-1/3)) + 18*b**(28/3)*x**15*(a/(b*x**3) +
 1)**(1/3)*gamma(-10/3)/(27*a**5*b**4*x**9*gamma(-1/3) + 54*a**4*b**5*x**12
*gamma(-1/3) + 27*a**3*b**6*x**15*gamma(-1/3))
```

3.522 $\int \frac{\sqrt[3]{a+bx^3}}{x^{14}}\,dx$

Optimal. Leaf size=92

$$\frac{81b^3\left(a+bx^3\right)^{4/3}}{1820a^4x^4}-\frac{27b^2\left(a+bx^3\right)^{4/3}}{455a^3x^7}+\frac{9b\left(a+bx^3\right)^{4/3}}{130a^2x^{10}}-\frac{\left(a+bx^3\right)^{4/3}}{13ax^{13}}$$

```
[Out] -1/13*(b*x^3+a)^(4/3)/a/x^13+9/130*b*(b*x^3+a)^(4/3)/a^2/x^10-27/455*b^2*(b
*x^3+a)^(4/3)/a^3/x^7+81/1820*b^3*(b*x^3+a)^(4/3)/a^4/x^4
```

Rubi [A] time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$\frac{81b^3\left(a+bx^3\right)^{4/3}}{1820a^4x^4}-\frac{27b^2\left(a+bx^3\right)^{4/3}}{455a^3x^7}+\frac{9b\left(a+bx^3\right)^{4/3}}{130a^2x^{10}}-\frac{\left(a+bx^3\right)^{4/3}}{13ax^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/x^14,x]
```

```
[Out] -(a + b*x^3)^(4/3)/(13*a*x^13) + (9*b*(a + b*x^3)^(4/3))/(130*a^2*x^10) - (
27*b^2*(a + b*x^3)^(4/3))/(455*a^3*x^7) + (81*b^3*(a + b*x^3)^(4/3))/(1820*
a^4*x^4)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{a+bx^3}}{x^{14}}\,dx &= -\frac{\left(a+bx^3\right)^{4/3}}{13ax^{13}}-\frac{(9b)\int \frac{\sqrt[3]{a+bx^3}}{x^{11}}\,dx}{13a}\\
&= -\frac{\left(a+bx^3\right)^{4/3}}{13ax^{13}}+\frac{9b\left(a+bx^3\right)^{4/3}}{130a^2x^{10}}+\frac{\left(27b^2\right)\int \frac{\sqrt[3]{a+bx^3}}{x^8}\,dx}{65a^2}\\
&= -\frac{\left(a+bx^3\right)^{4/3}}{13ax^{13}}+\frac{9b\left(a+bx^3\right)^{4/3}}{130a^2x^{10}}-\frac{27b^2\left(a+bx^3\right)^{4/3}}{455a^3x^7}-\frac{\left(81b^3\right)\int \frac{\sqrt[3]{a+bx^3}}{x^5}\,dx}{455a^3}\\
&= -\frac{\left(a+bx^3\right)^{4/3}}{13ax^{13}}+\frac{9b\left(a+bx^3\right)^{4/3}}{130a^2x^{10}}-\frac{27b^2\left(a+bx^3\right)^{4/3}}{455a^3x^7}+\frac{81b^3\left(a+bx^3\right)^{4/3}}{1820a^4x^4}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 53, normalized size = 0.58

$$\frac{\left(a+bx^3\right)^{4/3}\left(-140a^3+126a^2bx^3-108ab^2x^6+81b^3x^9\right)}{1820a^4x^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/x^14,x]
```

```
[Out] ((a + b*x^3)^(4/3)*(-140*a^3 + 126*a^2*b*x^3 - 108*a*b^2*x^6 + 81*b^3*x^9))
/(1820*a^4*x^13)
```

fricas [A] time = 0.70, size = 60, normalized size = 0.65

$$\frac{\left(81\,b^4x^{12}-27\,ab^3x^9+18\,a^2b^2x^6-14\,a^3bx^3-140\,a^4\right)\left(bx^3+a\right)^{\frac{1}{3}}}{1820\,a^4x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^14,x, algorithm="fricas")
```

```
[Out] 1/1820*(81*b^4*x^12 - 27*a*b^3*x^9 + 18*a^2*b^2*x^6 - 14*a^3*b*x^3 - 140*a^
4)*(b*x^3 + a)^(1/3)/(a^4*x^13)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^14,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/x^14, x)
```

maple [A] time = 0.01, size = 50, normalized size = 0.54

$$-\frac{\left(b\,x^3+a\right)^{\frac{4}{3}}\left(-81b^3x^9+108a\,b^2x^6-126a^2b\,x^3+140a^3\right)}{1820a^4x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^14,x)
```

```
[Out] -1/1820*(b*x^3+a)^(4/3)*(-81*b^3*x^9+108*a*b^2*x^6-126*a^2*b*x^3+140*a^3)/x
^13/a^4
```

maxima [A] time = 1.37, size = 69, normalized size = 0.75

$$\frac{\frac{455\left(bx^3+a\right)^{\frac{4}{3}}b^3}{x^4}-\frac{780\left(bx^3+a\right)^{\frac{7}{3}}b^2}{x^7}+\frac{546\left(bx^3+a\right)^{\frac{10}{3}}b}{x^{10}}-\frac{140\left(bx^3+a\right)^{\frac{13}{3}}}{x^{13}}}{1820\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^14,x, algorithm="maxima")
```

```
[Out] 1/1820*(455*(b*x^3 + a)^(4/3)*b^3/x^4 - 780*(b*x^3 + a)^(7/3)*b^2/x^7 + 546
*(b*x^3 + a)^(10/3)*b/x^10 - 140*(b*x^3 + a)^(13/3)/x^13)/a^4
```

mupad [B] time = 1.50, size = 93, normalized size = 1.01

$$\frac{81\,b^4\left(b\,x^3+a\right)^{1/3}}{1820\,a^4\,x}-\frac{b\left(b\,x^3+a\right)^{1/3}}{130\,a\,x^{10}}-\frac{\left(b\,x^3+a\right)^{1/3}}{13\,x^{13}}-\frac{27\,b^3\left(b\,x^3+a\right)^{1/3}}{1820\,a^3\,x^4}+\frac{9\,b^2\left(b\,x^3+a\right)^{1/3}}{910\,a^2\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/x^14,x)
```

```
[Out] (81*b^4*(a + b*x^3)^(1/3))/(1820*a^4*x) - (b*(a + b*x^3)^(1/3))/(130*a*x^10
) - (a + b*x^3)^(1/3)/(13*x^13) - (27*b^3*(a + b*x^3)^(1/3))/(1820*a^3*x^4)
 + (9*b^2*(a + b*x^3)^(1/3))/(910*a^2*x^7)
```

sympy [B] time = 2.81, size = 847, normalized size = 9.21

$$-\frac{280a^7b^{\frac{28}{3}}\sqrt[3]{\frac{a}{bx^3}+1}\,\Gamma\left(-\frac{13}{3}\right)}{81a^7b^9x^{12}\Gamma\left(-\frac{1}{3}\right)+243a^6b^{10}x^{15}\Gamma\left(-\frac{1}{3}\right)+243a^5b^{11}x^{18}\Gamma\left(-\frac{1}{3}\right)+81a^4b^{12}x^{21}\Gamma\left(-\frac{1}{3}\right)}-\frac{}{81a^7b^9x^{12}\Gamma\left(-\frac{1}{3}\right)+243a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**14,x)
```

```
[Out] -280*a**7*b**(28/3)*(a/(b*x**3) + 1)**(1/3)*gamma(-13/3)/(81*a**7*b**9*x**1
2*gamma(-1/3) + 243*a**6*b**10*x**15*gamma(-1/3) + 243*a**5*b**11*x**18*gam
ma(-1/3) + 81*a**4*b**12*x**21*gamma(-1/3)) - 868*a**6*b**(31/3)*x**3*(a/(b
*x**3) + 1)**(1/3)*gamma(-13/3)/(81*a**7*b**9*x**12*gamma(-1/3) + 243*a**6*
b**10*x**15*gamma(-1/3) + 243*a**5*b**11*x**18*gamma(-1/3) + 81*a**4*b**12*
x**21*gamma(-1/3)) - 888*a**5*b**(34/3)*x**6*(a/(b*x**3) + 1)**(1/3)*gamma(
-13/3)/(81*a**7*b**9*x**12*gamma(-1/3) + 243*a**6*b**10*x**15*gamma(-1/3) +
 243*a**5*b**11*x**18*gamma(-1/3) + 81*a**4*b**12*x**21*gamma(-1/3)) - 310*
a**4*b**(37/3)*x**9*(a/(b*x**3) + 1)**(1/3)*gamma(-13/3)/(81*a**7*b**9*x**1
2*gamma(-1/3) + 243*a**6*b**10*x**15*gamma(-1/3) + 243*a**5*b**11*x**18*gam
ma(-1/3) + 81*a**4*b**12*x**21*gamma(-1/3)) + 80*a**3*b**(40/3)*x**12*(a/(b
*x**3) + 1)**(1/3)*gamma(-13/3)/(81*a**7*b**9*x**12*gamma(-1/3) + 243*a**6*
b**10*x**15*gamma(-1/3) + 243*a**5*b**11*x**18*gamma(-1/3) + 81*a**4*b**12*
x**21*gamma(-1/3)) + 360*a**2*b**(43/3)*x**15*(a/(b*x**3) + 1)**(1/3)*gamma
(-13/3)/(81*a**7*b**9*x**12*gamma(-1/3) + 243*a**6*b**10*x**15*gamma(-1/3)
+ 243*a**5*b**11*x**18*gamma(-1/3) + 81*a**4*b**12*x**21*gamma(-1/3)) + 432
*a*b**(46/3)*x**18*(a/(b*x**3) + 1)**(1/3)*gamma(-13/3)/(81*a**7*b**9*x**12
*gamma(-1/3) + 243*a**6*b**10*x**15*gamma(-1/3) + 243*a**5*b**11*x**18*gamm
a(-1/3) + 81*a**4*b**12*x**21*gamma(-1/3)) + 162*b**(49/3)*x**21*(a/(b*x**3
) + 1)**(1/3)*gamma(-13/3)/(81*a**7*b**9*x**12*gamma(-1/3) + 243*a**6*b**10
*x**15*gamma(-1/3) + 243*a**5*b**11*x**18*gamma(-1/3) + 81*a**4*b**12*x**21
*gamma(-1/3))
```

3.523 $\int x^3 \sqrt[3]{a+bx^3}\, dx$

Optimal. Leaf size=38

$$\frac{x^4 \left(a+bx^3\right)^{4/3}\, {}_2F_1\left(1, \frac{8}{3}; \frac{7}{3}; -\frac{bx^3}{a}\right)}{4a}$$

```
[Out] 1/4*x^4*(b*x^3+a)^(4/3)*hypergeom([1, 8/3],[7/3],-b*x^3/a)/a
```

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.34, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {365, 364}

$$\frac{x^4 \sqrt[3]{a+bx^3}\, {}_2F_1\left(-\frac{1}{3}, \frac{4}{3}; \frac{7}{3}; -\frac{bx^3}{a}\right)}{4\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^3)^(1/3),x]
```

```
[Out] (x^4*(a + b*x^3)^(1/3)*Hypergeometric2F1[-1/3, 4/3, 7/3, -((b*x^3)/a)])/(4*
(1 + (b*x^3)/a)^(1/3))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^3 \sqrt[3]{a+bx^3}\, dx &= \frac{\sqrt[3]{a+bx^3} \int x^3 \sqrt[3]{1+\frac{bx^3}{a}}\, dx}{\sqrt[3]{1+\frac{bx^3}{a}}} \\ &= \frac{x^4 \sqrt[3]{a+bx^3}\, {}_2F_1\left(-\frac{1}{3}, \frac{4}{3}; \frac{7}{3}; -\frac{bx^3}{a}\right)}{4\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 1.34

$$\frac{x^4 \sqrt[3]{a+bx^3}\, {}_2F_1\left(-\frac{1}{3}, \frac{4}{3}; \frac{7}{3}; -\frac{bx^3}{a}\right)}{4\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^3)^(1/3),x]
```

```
[Out] (x^4*(a + b*x^3)^(1/3)*Hypergeometric2F1[-1/3, 4/3, 7/3, -((b*x^3)/a)])/(4*
(1 + (b*x^3)/a)^(1/3))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^3+a\right)^{\frac{1}{3}}x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(1/3)*x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{1}{3}}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*x^3, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \left(b\,x^3+a\right)^{\frac{1}{3}}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)^(1/3),x)
```

```
[Out] int(x^3*(b*x^3+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{1}{3}}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x^3\left(b\,x^3+a\right)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^3)^(1/3),x)
```

```
[Out] int(x^3*(a + b*x^3)^(1/3), x)
```

sympy [C] time = 1.83, size = 39, normalized size = 1.03

$$\frac{\sqrt[3]{a}\,x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3}, \frac{4}{3}\\ \frac{7}{3}\end{matrix}\right| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**(1/3),x)
```

```
[Out] a**(1/3)*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(7/3))
```

3.524 $\int \sqrt[3]{a+bx^3}\,dx$

Optimal. Leaf size=33

$$\frac{x\left(a+bx^3\right)^{4/3}\,{}_2F_1\left(1,\frac{5}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{a}$$

```
[Out] x*(b*x^3+a)^(4/3)*hypergeom([1, 5/3],[4/3],-b*x^3/a)/a
```

Rubi [A] time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.39, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {246, 245}

$$\frac{x\sqrt[3]{a+bx^3}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3),x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a)])/(1 +
(b*x^3)/a)^(1/3)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{a+bx^3}\,dx &= \frac{\sqrt[3]{a+bx^3}\int \sqrt[3]{1+\frac{bx^3}{a}}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= \frac{x\sqrt[3]{a+bx^3}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

Mathematica [C] time = 0.25, size = 196, normalized size = 5.94

$$\frac{3\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt[3]{a+bx^3}\,F_1\left(\frac{4}{3};-\frac{1}{3},-\frac{1}{3};\frac{7}{3};-\frac{i\left(\sqrt[3]{b}\,x+(-1)^{2/3}\sqrt[3]{a}\right)}{\sqrt{3}\,\sqrt[3]{a}},\frac{-\frac{2i\sqrt[3]{b}\,x}{\sqrt[3]{a}}+\sqrt{3}+i}{3i+\sqrt{3}}\right)}{4\sqrt[3]{2}\,\sqrt[3]{b}\,\sqrt[3]{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\,\sqrt[3]{\frac{i\left(\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}+1\right)}{\sqrt{3}+3i}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3),x]
```

```
[Out] (3*((-1)^(2/3)*a^(1/3) + b^(1/3)*x)*(a + b*x^3)^(1/3)*AppellF1[4/3, -1/3, -
1/3, 7/3, ((-I)*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(Sqrt[3]*a^(1/3)), (I + S
qrt[3] - ((2*I)*b^(1/3)*x)/a^(1/3))/(3*I + Sqrt[3])])/(4*2^(1/3)*b^(1/3)*((
a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3)))^(1/3)*((I*(1 +
(b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3]))^(1/3))
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^3+a\right)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3), x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \left(b\,x^3+a\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3),x)
```

```
[Out] int((b*x^3+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3), x)
```

mupad [B] time = 1.01, size = 37, normalized size = 1.12

$$\frac{x\left(b\,x^3+a\right)^{1/3}\,{}_2\mathrm{F}_1\left(-\frac{1}{3},\frac{1}{3};\ \frac{4}{3};\ -\frac{b\,x^3}{a}\right)}{\left(\frac{b\,x^3}{a}+1\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3),x)
```

```
[Out] (x*(a + b*x^3)^(1/3)*hypergeom([-1/3, 1/3], 4/3, -(b*x^3)/a))/((b*x^3)/a +
1)^(1/3)
```

sympy [C] time = 0.98, size = 37, normalized size = 1.12

$$\frac{\sqrt[3]{a}\, x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3),x)
```

```
[Out] a**(1/3)*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(4/3))
```

3.525 $\int \frac{\sqrt[3]{a+bx^3}}{x^3}\,dx$

Optimal. Leaf size=38

$$-\frac{\left(a+bx^3\right)^{4/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{1}{3};-\frac{bx^3}{a}\right)}{2ax^2}$$

```
[Out] -1/2*(b*x^3+a)^(4/3)*hypergeom([2/3, 1],[1/3],-b*x^3/a)/a/x^2
```

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.34, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {365, 364}

$$-\frac{\sqrt[3]{a+bx^3}\,{}_2F_1\left(-\frac{2}{3},-\frac{1}{3};\frac{1}{3};-\frac{bx^3}{a}\right)}{2x^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/x^3,x]
```

```
[Out] -((a + b*x^3)^(1/3)*Hypergeometric2F1[-2/3, -1/3, 1/3, -((b*x^3)/a)])/(2*x^
2*(1 + (b*x^3)/a)^(1/3))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^3}}{x^3}\,dx &= \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= -\frac{\sqrt[3]{a+bx^3}\,{}_2F_1\left(-\frac{2}{3},-\frac{1}{3};\frac{1}{3};-\frac{bx^3}{a}\right)}{2x^2\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 1.34

$$-\frac{\sqrt[3]{a+bx^3}\,{}_2F_1\left(-\frac{2}{3},-\frac{1}{3};\frac{1}{3};-\frac{bx^3}{a}\right)}{2x^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/x^3,x]
```

```
[Out] -1/2*((a + b*x^3)^(1/3)*Hypergeometric2F1[-2/3, -1/3, 1/3, -((b*x^3)/a)])/(
x^2*(1 + (b*x^3)/a)^(1/3))
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(1/3)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/x^3, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^3,x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(b\,x^3+a\right)^{1/3}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/x^3,x)
```

```
[Out] int((a + b*x^3)^(1/3)/x^3, x)
```

sympy [C] time = 1.52, size = 42, normalized size = 1.11

$$\frac{\sqrt[3]{a}\,\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3}, -\frac{1}{3} \\ \frac{1}{3}\end{matrix}\right| \frac{bx^3 e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**3,x)
```

```
[Out] a**(1/3)*gamma(-2/3)*hyper((-2/3, -1/3), (1/3,), b*x**3*exp_polar(I*pi)/a)/
(3*x**2*gamma(1/3))
```

3.526 $\int \frac{\sqrt[3]{a+bx^3}}{x^6}\,dx$

Optimal. Leaf size=38

$$-\frac{\left(a+bx^3\right)^{4/3}\,{}_2F_1\left(-\frac{1}{3},1;-\frac{2}{3};-\frac{bx^3}{a}\right)}{5ax^5}$$

```
[Out] -1/5*(b*x^3+a)^(4/3)*hypergeom([-1/3, 1],[-2/3],-b*x^3/a)/a/x^5
```

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.34, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {365, 364}

$$-\frac{\sqrt[3]{a+bx^3}\,{}_2F_1\left(-\frac{5}{3},-\frac{1}{3};-\frac{2}{3};-\frac{bx^3}{a}\right)}{5x^5\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/x^6,x]
```

```
[Out] -((a + b*x^3)^(1/3)*Hypergeometric2F1[-5/3, -1/3, -2/3, -((b*x^3)/a)])/(5*x
^5*(1 + (b*x^3)/a)^(1/3))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^3}}{x^6}\,dx &= \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^6}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= -\frac{\sqrt[3]{a+bx^3}\,{}_2F_1\left(-\frac{5}{3},-\frac{1}{3};-\frac{2}{3};-\frac{bx^3}{a}\right)}{5x^5\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 1.34

$$-\frac{\sqrt[3]{a+bx^3}\,{}_2F_1\left(-\frac{5}{3},-\frac{1}{3};-\frac{2}{3};-\frac{bx^3}{a}\right)}{5x^5\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/x^6,x]
```

```
[Out] -1/5*((a + b*x^3)^(1/3)*Hypergeometric2F1[-5/3, -1/3, -2/3, -((b*x^3)/a)])/
(x^5*(1 + (b*x^3)/a)^(1/3))
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^6,x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(1/3)/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/x^6, x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^6,x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^6,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(b\,x^3+a\right)^{1/3}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/x^6,x)
```

```
[Out] int((a + b*x^3)^(1/3)/x^6, x)
```

sympy [C] time = 1.17, size = 42, normalized size = 1.11

$$\frac{\sqrt[3]{b}\,\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3}, \frac{4}{3} \\ \frac{7}{3}\end{matrix}\right| \frac{ae^{i\pi}}{bx^3}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**6,x)
```

```
[Out] b**(1/3)*gamma(-4/3)*hyper((-1/3, 4/3), (7/3,), a*exp_polar(I*pi)/(b*x**3))
/(3*x**4*gamma(-1/3))
```

3.527 $\int x^{11}\left(a+bx^3\right)^{2/3}\,dx$

Optimal. Leaf size=80

$$-\frac{a^3\left(a+bx^3\right)^{5/3}}{5b^4}+\frac{3a^2\left(a+bx^3\right)^{8/3}}{8b^4}+\frac{\left(a+bx^3\right)^{14/3}}{14b^4}-\frac{3a\left(a+bx^3\right)^{11/3}}{11b^4}$$

```
[Out] -1/5*a^3*(b*x^3+a)^(5/3)/b^4+3/8*a^2*(b*x^3+a)^(8/3)/b^4-3/11*a*(b*x^3+a)^(
11/3)/b^4+1/14*(b*x^3+a)^(14/3)/b^4
```

Rubi [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{3a^2\left(a+bx^3\right)^{8/3}}{8b^4}-\frac{a^3\left(a+bx^3\right)^{5/3}}{5b^4}+\frac{\left(a+bx^3\right)^{14/3}}{14b^4}-\frac{3a\left(a+bx^3\right)^{11/3}}{11b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a + b*x^3)^(2/3),x]
```

```
[Out] -(a^3*(a + b*x^3)^(5/3))/(5*b^4) + (3*a^2*(a + b*x^3)^(8/3))/(8*b^4) - (3*a
*(a + b*x^3)^(11/3))/(11*b^4) + (a + b*x^3)^(14/3)/(14*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{11}\left(a+bx^3\right)^{2/3}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int x^3(a+bx)^{2/3}\,dx,x,x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int\left(-\frac{a^3(a+bx)^{2/3}}{b^3}+\frac{3a^2(a+bx)^{5/3}}{b^3}-\frac{3a(a+bx)^{8/3}}{b^3}+\frac{(a+bx)^{11/3}}{b^3}\right)dx,x,x^3\right)\\ &= -\frac{a^3\left(a+bx^3\right)^{5/3}}{5b^4}+\frac{3a^2\left(a+bx^3\right)^{8/3}}{8b^4}-\frac{3a\left(a+bx^3\right)^{11/3}}{11b^4}+\frac{\left(a+bx^3\right)^{14/3}}{14b^4}\end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.62

$$\frac{\left(a+bx^3\right)^{5/3}\left(-81a^3+135a^2bx^3-180ab^2x^6+220b^3x^9\right)}{3080b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*x^3)^(2/3),x]
```

```
[Out] ((a + b*x^3)^(5/3)*(-81*a^3 + 135*a^2*b*x^3 - 180*a*b^2*x^6 + 220*b^3*x^9))
/(3080*b^4)
```

fricas [A] time = 0.59, size = 57, normalized size = 0.71

$$\frac{\left(220\,b^4x^{12}+40\,ab^3x^9-45\,a^2b^2x^6+54\,a^3bx^3-81\,a^4\right)\left(bx^3+a\right)^{\frac{2}{3}}}{3080\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/3080*(220*b^4*x^12 + 40*a*b^3*x^9 - 45*a^2*b^2*x^6 + 54*a^3*b*x^3 - 81*a^
4)*(b*x^3 + a)^(2/3)/b^4
```

giac [A] time = 0.19, size = 57, normalized size = 0.71

$$\frac{220\left(bx^3+a\right)^{\frac{14}{3}}-840\left(bx^3+a\right)^{\frac{11}{3}}a+1155\left(bx^3+a\right)^{\frac{8}{3}}a^2-616\left(bx^3+a\right)^{\frac{5}{3}}a^3}{3080\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] 1/3080*(220*(b*x^3 + a)^(14/3) - 840*(b*x^3 + a)^(11/3)*a + 1155*(b*x^3 + a
)^(8/3)*a^2 - 616*(b*x^3 + a)^(5/3)*a^3)/b^4
```

maple [A] time = 0.01, size = 47, normalized size = 0.59

$$-\frac{\left(b\,x^3+a\right)^{\frac{5}{3}}\left(-220b^3x^9+180a\,b^2x^6-135a^2b\,x^3+81a^3\right)}{3080b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x^3+a)^(2/3),x)
```

```
[Out] -1/3080*(b*x^3+a)^(5/3)*(-220*b^3*x^9+180*a*b^2*x^6-135*a^2*b*x^3+81*a^3)/b
^4
```

maxima [A] time = 1.35, size = 64, normalized size = 0.80

$$\frac{\left(bx^3+a\right)^{\frac{14}{3}}}{14\,b^4}-\frac{3\left(bx^3+a\right)^{\frac{11}{3}}a}{11\,b^4}+\frac{3\left(bx^3+a\right)^{\frac{8}{3}}a^2}{8\,b^4}-\frac{\left(bx^3+a\right)^{\frac{5}{3}}a^3}{5\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 1/14*(b*x^3 + a)^(14/3)/b^4 - 3/11*(b*x^3 + a)^(11/3)*a/b^4 + 3/8*(b*x^3 +
a)^(8/3)*a^2/b^4 - 1/5*(b*x^3 + a)^(5/3)*a^3/b^4
```

mupad [B] time = 1.11, size = 55, normalized size = 0.69

$$\left(b\,x^3+a\right)^{2/3}\left(\frac{x^{12}}{14}-\frac{81\,a^4}{3080\,b^4}+\frac{a\,x^9}{77\,b}+\frac{27\,a^3\,x^3}{1540\,b^3}-\frac{9\,a^2\,x^6}{616\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a + b*x^3)^(2/3),x)
```

```
[Out] (a + b*x^3)^(2/3)*(x^12/14 - (81*a^4)/(3080*b^4) + (a*x^9)/(77*b) + (27*a^3
*x^3)/(1540*b^3) - (9*a^2*x^6)/(616*b^2))
```

sympy [A] time = 12.62, size = 110, normalized size = 1.38

$$\begin{cases} -\frac{81a^4\left(a+bx^3\right)^{\frac{2}{3}}}{3080b^4}+\frac{27a^3x^3\left(a+bx^3\right)^{\frac{2}{3}}}{1540b^3}-\frac{9a^2x^6\left(a+bx^3\right)^{\frac{2}{3}}}{616b^2}+\frac{ax^9\left(a+bx^3\right)^{\frac{2}{3}}}{77b}+\frac{x^{12}\left(a+bx^3\right)^{\frac{2}{3}}}{14} & \text{for } b\neq 0 \\ \frac{a^{\frac{2}{3}}x^{12}}{12} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x**3+a)**(2/3),x)
```

```
[Out] Piecewise((-81*a**4*(a + b*x**3)**(2/3)/(3080*b**4) + 27*a**3*x**3*(a + b*x
**3)**(2/3)/(1540*b**3) - 9*a**2*x**6*(a + b*x**3)**(2/3)/(616*b**2) + a*x*
*9*(a + b*x**3)**(2/3)/(77*b) + x**12*(a + b*x**3)**(2/3)/14, Ne(b, 0)), (a
**(2/3)*x**12/12, True))
```

3.528 $\int x^8 \left(a+bx^3\right)^{2/3}\,dx$

Optimal. Leaf size=59

$$\frac{a^2\left(a+bx^3\right)^{5/3}}{5b^3}+\frac{\left(a+bx^3\right)^{11/3}}{11b^3}-\frac{a\left(a+bx^3\right)^{8/3}}{4b^3}$$

```
[Out] 1/5*a^2*(b*x^3+a)^(5/3)/b^3-1/4*a*(b*x^3+a)^(8/3)/b^3+1/11*(b*x^3+a)^(11/3)
/b^3
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{a^2\left(a+bx^3\right)^{5/3}}{5b^3}+\frac{\left(a+bx^3\right)^{11/3}}{11b^3}-\frac{a\left(a+bx^3\right)^{8/3}}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x^3)^(2/3),x]
```

```
[Out] (a^2*(a + b*x^3)^(5/3))/(5*b^3) - (a*(a + b*x^3)^(8/3))/(4*b^3) + (a + b*x^
3)^(11/3)/(11*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^8\left(a+bx^3\right)^{2/3}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int x^2(a+bx)^{2/3}\,dx,x,x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int\left(\frac{a^2(a+bx)^{2/3}}{b^2}-\frac{2a(a+bx)^{5/3}}{b^2}+\frac{(a+bx)^{8/3}}{b^2}\right)dx,x,x^3\right)\\ &= \frac{a^2\left(a+bx^3\right)^{5/3}}{5b^3}-\frac{a\left(a+bx^3\right)^{8/3}}{4b^3}+\frac{\left(a+bx^3\right)^{11/3}}{11b^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 39, normalized size = 0.66

$$\frac{\left(a+bx^3\right)^{5/3}\left(9a^2-15abx^3+20b^2x^6\right)}{220b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x^3)^(2/3),x]
```

```
[Out] ((a + b*x^3)^(5/3)*(9*a^2 - 15*a*b*x^3 + 20*b^2*x^6))/(220*b^3)
```

fricas [A] time = 0.56, size = 46, normalized size = 0.78

$$\frac{\left(20\,b^3x^9+5\,ab^2x^6-6\,a^2bx^3+9\,a^3\right)\left(bx^3+a\right)^{\frac{2}{3}}}{220\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/220*(20*b^3*x^9 + 5*a*b^2*x^6 - 6*a^2*b*x^3 + 9*a^3)*(b*x^3 + a)^(2/3)/b^
3
```

giac [A] time = 0.16, size = 43, normalized size = 0.73

$$\frac{20\left(bx^3+a\right)^{\frac{11}{3}}-55\left(bx^3+a\right)^{\frac{8}{3}}a+44\left(bx^3+a\right)^{\frac{5}{3}}a^2}{220\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] 1/220*(20*(b*x^3 + a)^(11/3) - 55*(b*x^3 + a)^(8/3)*a + 44*(b*x^3 + a)^(5/3
)*a^2)/b^3
```

maple [A] time = 0.01, size = 36, normalized size = 0.61

$$\frac{\left(b\,x^3+a\right)^{\frac{5}{3}}\left(20b^2x^6-15ab\,x^3+9a^2\right)}{220b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^3+a)^(2/3),x)
```

```
[Out] 1/220*(b*x^3+a)^(5/3)*(20*b^2*x^6-15*a*b*x^3+9*a^2)/b^3
```

maxima [A] time = 1.31, size = 47, normalized size = 0.80

$$\frac{\left(bx^3+a\right)^{\frac{11}{3}}}{11\,b^3}-\frac{\left(bx^3+a\right)^{\frac{8}{3}}a}{4\,b^3}+\frac{\left(bx^3+a\right)^{\frac{5}{3}}a^2}{5\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 1/11*(b*x^3 + a)^(11/3)/b^3 - 1/4*(b*x^3 + a)^(8/3)*a/b^3 + 1/5*(b*x^3 + a)
^(5/3)*a^2/b^3
```

mupad [B] time = 1.07, size = 44, normalized size = 0.75

$$\left(b\,x^3+a\right)^{2/3}\left(\frac{x^9}{11}+\frac{9\,a^3}{220\,b^3}+\frac{a\,x^6}{44\,b}-\frac{3\,a^2\,x^3}{110\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*x^3)^(2/3),x)
```

```
[Out] (a + b*x^3)^(2/3)*(x^9/11 + (9*a^3)/(220*b^3) + (a*x^6)/(44*b) - (3*a^2*x^3
)/(110*b^2))
```

sympy [**A**] time = 4.21, size = 87, normalized size = 1.47

$$\begin{cases} \frac{9a^3\left(a+bx^3\right)^{\frac{2}{3}}}{220b^3} - \frac{3a^2x^3\left(a+bx^3\right)^{\frac{2}{3}}}{110b^2} + \frac{ax^6\left(a+bx^3\right)^{\frac{2}{3}}}{44b} + \frac{x^9\left(a+bx^3\right)^{\frac{2}{3}}}{11} & \text{for } b \neq 0 \\ \frac{a^{\frac{2}{3}}x^9}{9} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**3+a)**(2/3),x)
```

```
[Out] Piecewise((9*a**3*(a + b*x**3)**(2/3)/(220*b**3) - 3*a**2*x**3*(a + b*x**3)
**(2/3)/(110*b**2) + a*x**6*(a + b*x**3)**(2/3)/(44*b) + x**9*(a + b*x**3)*
*(2/3)/11, Ne(b, 0)), (a**(2/3)*x**9/9, True))
```

3.529 $\int x^5 \left(a+bx^3\right)^{2/3}\,dx$

Optimal. Leaf size=38

$$\frac{\left(a+bx^3\right)^{8/3}}{8b^2}-\frac{a\left(a+bx^3\right)^{5/3}}{5b^2}$$

```
[Out] -1/5*a*(b*x^3+a)^(5/3)/b^2+1/8*(b*x^3+a)^(8/3)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{\left(a+bx^3\right)^{8/3}}{8b^2}-\frac{a\left(a+bx^3\right)^{5/3}}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^3)^(2/3),x]
```

```
[Out] -(a*(a + b*x^3)^(5/3))/(5*b^2) + (a + b*x^3)^(8/3)/(8*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+bx^3\right)^{2/3}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int x(a+bx)^{2/3}\,dx, x, x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int\left(-\frac{a(a+bx)^{2/3}}{b}+\frac{(a+bx)^{5/3}}{b}\right)dx, x, x^3\right)\\ &= -\frac{a\left(a+bx^3\right)^{5/3}}{5b^2}+\frac{\left(a+bx^3\right)^{8/3}}{8b^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 0.74

$$\frac{\left(a+bx^3\right)^{5/3}\left(5bx^3-3a\right)}{40b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^3)^(2/3),x]
```

```
[Out] ((a + b*x^3)^(5/3)*(-3*a + 5*b*x^3))/(40*b^2)
```

fricas [A] time = 0.73, size = 35, normalized size = 0.92

$$\frac{\left(5\,b^2x^6+2\,abx^3-3\,a^2\right)\left(bx^3+a\right)^{\frac{2}{3}}}{40\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(2/3),x, algorithm="fricas")
[Out] 1/40*(5*b^2*x^6 + 2*a*b*x^3 - 3*a^2)*(b*x^3 + a)^(2/3)/b^2
```

giac [A] time = 0.16, size = 29, normalized size = 0.76

$$\frac{5\left(bx^3+a\right)^{\frac{8}{3}}-8\left(bx^3+a\right)^{\frac{5}{3}}a}{40\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(2/3),x, algorithm="giac")
[Out] 1/40*(5*(b*x^3 + a)^(8/3) - 8*(b*x^3 + a)^(5/3)*a)/b^2
```

maple [A] time = 0.00, size = 25, normalized size = 0.66

$$-\frac{\left(b\,x^3+a\right)^{\frac{5}{3}}\left(-5b\,x^3+3a\right)}{40b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^3+a)^(2/3),x)
[Out] -1/40*(b*x^3+a)^(5/3)*(-5*b*x^3+3*a)/b^2
```

maxima [A] time = 1.32, size = 30, normalized size = 0.79

$$\frac{\left(bx^3+a\right)^{\frac{8}{3}}}{8\,b^2}-\frac{\left(bx^3+a\right)^{\frac{5}{3}}a}{5\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(2/3),x, algorithm="maxima")
[Out] 1/8*(b*x^3 + a)^(8/3)/b^2 - 1/5*(b*x^3 + a)^(5/3)*a/b^2
```

mupad [B] time = 1.11, size = 33, normalized size = 0.87

$$\left(b\,x^3+a\right)^{2/3}\left(\frac{x^6}{8}-\frac{3\,a^2}{40\,b^2}+\frac{a\,x^3}{20\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^3)^(2/3),x)
[Out] (a + b*x^3)^(2/3)*(x^6/8 - (3*a^2)/(40*b^2) + (a*x^3)/(20*b))
```

sympy [A] time = 1.94, size = 63, normalized size = 1.66

$$\begin{cases} -\frac{3a^2\left(a+bx^3\right)^{\frac{2}{3}}}{40b^2}+\frac{ax^3\left(a+bx^3\right)^{\frac{2}{3}}}{20b}+\frac{x^6\left(a+bx^3\right)^{\frac{2}{3}}}{8} & \text{for } b\neq 0 \\ \frac{a^{\frac{2}{3}}x^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**3+a)**(2/3),x)
```

```
[Out] Piecewise((-3*a**2*(a + b*x**3)**(2/3)/(40*b**2) + a*x**3*(a + b*x**3)**(2/
3)/(20*b) + x**6*(a + b*x**3)**(2/3)/8, Ne(b, 0)), (a**(2/3)*x**6/6, True))
```

3.530 $\int x^2 \left(a + bx^3\right)^{2/3} \, dx$

Optimal. Leaf size=18

$$\frac{\left(a + bx^3\right)^{5/3}}{5b}$$

```
[Out] 1/5*(b*x^3+a)^(5/3)/b
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$\frac{\left(a + bx^3\right)^{5/3}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3)^(2/3),x]
```

```
[Out] (a + b*x^3)^(5/3)/(5*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^2 \left(a + bx^3\right)^{2/3} \, dx = \frac{\left(a + bx^3\right)^{5/3}}{5b}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{\left(a + bx^3\right)^{5/3}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)^(2/3),x]
```

```
[Out] (a + b*x^3)^(5/3)/(5*b)
```

fricas [A] time = 0.54, size = 14, normalized size = 0.78

$$\frac{\left(bx^3 + a\right)^{\frac{5}{3}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/5*(b*x^3 + a)^(5/3)/b
```

giac [A] time = 0.17, size = 14, normalized size = 0.78

$$\frac{\left(bx^3 + a\right)^{\frac{5}{3}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] 1/5*(b*x^3 + a)^(5/3)/b
```

maple [A] time = 0.01, size = 15, normalized size = 0.83

$$\frac{\left(b\,x^3+a\right)^{\frac{5}{3}}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^(2/3),x)
```

```
[Out] 1/5*(b*x^3+a)^(5/3)/b
```

maxima [A] time = 1.33, size = 14, normalized size = 0.78

$$\frac{\left(bx^3+a\right)^{\frac{5}{3}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 1/5*(b*x^3 + a)^(5/3)/b
```

mupad [B] time = 1.05, size = 14, normalized size = 0.78

$$\frac{\left(b\,x^3+a\right)^{5/3}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^3)^(2/3),x)
```

```
[Out] (a + b*x^3)^(5/3)/(5*b)
```

sympy [A] time = 0.93, size = 39, normalized size = 2.17

$$\begin{cases} \frac{a\left(a+bx^3\right)^{\frac{2}{3}}}{5b}+\frac{x^3\left(a+bx^3\right)^{\frac{2}{3}}}{5} & \text{for } b\neq 0 \\ \frac{a^{\frac{2}{3}}x^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**(2/3),x)
```

```
[Out] Piecewise((a*(a + b*x**3)**(2/3)/(5*b) + x**3*(a + b*x**3)**(2/3)/5, Ne(b,
0)), (a**(2/3)*x**3/3, True))
```

3.531 $\int \frac{\left(a+bx^3\right)^{2/3}}{x}\,dx$

Optimal. Leaf size=98

$$\frac{1}{2}a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+\frac{a^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}}-\frac{1}{2}a^{2/3}\log(x)+\frac{1}{2}\left(a+bx^3\right)^{2/3}$$

```
[Out] 1/2*(b*x^3+a)^(2/3)-1/2*a^(2/3)*ln(x)+1/2*a^(2/3)*ln(a^(1/3)-(b*x^3+a)^(1/3
))+1/3*a^(2/3)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1
/2)
```

Rubi [A] time = 0.07, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 50, 55, 617, 204, 31}

$$\frac{1}{2}a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+\frac{a^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}}-\frac{1}{2}a^{2/3}\log(x)+\frac{1}{2}\left(a+bx^3\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/x,x]
```

```
[Out] (a + b*x^3)^(2/3)/2 + (a^(2/3)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt
[3]*a^(1/3))])/Sqrt[3] - (a^(2/3)*Log[x])/2 + (a^(2/3)*Log[a^(1/3) - (a + b
*x^3)^(1/3)])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^3\right)^{2/3}}{x}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x}\,dx, x, x^3\right)\\
&= \frac{1}{2}\left(a+bx^3\right)^{2/3} + \frac{1}{3}a\,\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^3\right)\\
&= \frac{1}{2}\left(a+bx^3\right)^{2/3} - \frac{1}{2}a^{2/3}\log(x) - \frac{1}{2}a^{2/3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right) + \frac{1}{2}a\,\text{Subst}\left(\int \frac{}{a^2}\right.\\
&= \frac{1}{2}\left(a+bx^3\right)^{2/3} - \frac{1}{2}a^{2/3}\log(x) + \frac{1}{2}a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right) - a^{2/3}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1\right.\\
&= \frac{1}{2}\left(a+bx^3\right)^{2/3} + \frac{a^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{2}a^{2/3}\log(x) + \frac{1}{2}a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)
\end{aligned}$$

Mathematica [A] time = 0.08, size = 90, normalized size = 0.92

$$\frac{1}{2}\left(a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right) - a^{2/3}\log(x) + \left(a+bx^3\right)^{2/3}\right) + \frac{a^{2/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/x,x]
```

```
[Out] (a^(2/3)*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/Sqrt[3] + ((a
 + b*x^3)^(2/3) - a^(2/3)*Log[x] + a^(2/3)*Log[a^(1/3) - (a + b*x^3)^(1/3)]
)/2
```

fricas [A] time = 0.78, size = 122, normalized size = 1.24

$$\frac{1}{3}\sqrt{3}\left(a^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,a + 2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{1}{3}}}{3\,a}\right) - \frac{1}{6}\left(a^2\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}a + \left(a^2\right)^{\frac{1}{3}}a + \left(bx^3+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{2}{3}}\right) + \frac{1}{3}\left(a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x,x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*(a^2)^(1/3)*arctan(1/3*(sqrt(3)*a + 2*sqrt(3)*(b*x^3 + a)^(1/3)
*(a^2)^(1/3))/a) - 1/6*(a^2)^(1/3)*log((b*x^3 + a)^(2/3)*a + (a^2)^(1/3)*a
```

```
+ (b*x^3 + a)^(1/3)*(a^2)^(2/3)) + 1/3*(a^2)^(1/3)*log((b*x^3 + a)^(1/3)*a
- (a^2)^(2/3)) + 1/2*(b*x^3 + a)^(2/3)
```

giac [A] time = 0.69, size = 98, normalized size = 1.00

$$\frac{1}{3}\sqrt{3}\,a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-\frac{1}{6}\,a^{\frac{2}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\frac{1}{3}\,a^{\frac{2}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x,x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*a^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1
/3)) - 1/6*a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2
/3)) + 1/3*a^(2/3)*log(abs((b*x^3 + a)^(1/3) - a^(1/3))) + 1/2*(b*x^3 + a)^
(2/3)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x,x)
```

```
[Out] int((b*x^3+a)^(2/3)/x,x)
```

maxima [A] time = 2.95, size = 97, normalized size = 0.99

$$\frac{1}{3}\sqrt{3}\,a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-\frac{1}{6}\,a^{\frac{2}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\frac{1}{3}\,a^{\frac{2}{3}}\log\left(\left(bx^3+a\right)^{\frac{1}{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*a^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1
/3)) - 1/6*a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2
/3)) + 1/3*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)) + 1/2*(b*x^3 + a)^(2/3)
```

mupad [B] time = 1.08, size = 122, normalized size = 1.24

$$\frac{\left(b\,x^3+a\right)^{2/3}}{2}+\frac{a^{2/3}\,\ln\left(a^2\left(b\,x^3+a\right)^{1/3}-a^{7/3}\right)}{3}-\frac{a^{2/3}\,\ln\left(a^2\left(b\,x^3+a\right)^{1/3}-a^{7/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3}+a^{2/3}\,\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/x,x)
```

```
[Out] (a + b*x^3)^(2/3)/2 + (a^(2/3)*log(a^2*(a + b*x^3)^(1/3) - a^(7/3)))/3 - (a
^(2/3)*log(a^2*(a + b*x^3)^(1/3) - a^(7/3)*((3^(1/2)*1i)/2 + 1/2)^2)*((3^(1
/2)*1i)/2 + 1/2))/3 + a^(2/3)*log(a^2*(a + b*x^3)^(1/3) - 9*a^(7/3)*((3^(1/
2)*1i)/6 - 1/6)^2)*((3^(1/2)*1i)/6 - 1/6)
```

sympy [C] time = 2.19, size = 44, normalized size = 0.45

$$-\frac{b^{\frac{2}{3}}x^2\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},-\frac{2}{3}\\ \frac{1}{3}\end{matrix}\right|\frac{ae^{i\pi}}{bx^3}\right)}{3\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x,x)
```

```
[Out] -b**(2/3)*x**2*gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), a*exp_polar(I*pi)/(b
*x**3))/(3*gamma(1/3))
```

3.532 $\int \frac{\left(a+bx^3\right)^{2/3}}{x^4}\,dx$

Optimal. Leaf size=107

$$-\frac{\left(a+bx^3\right)^{2/3}}{3x^3}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{3\sqrt[3]{a}}+\frac{2b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,\sqrt[3]{a}}-\frac{b\log(x)}{3\sqrt[3]{a}}$$

```
[Out] -1/3*(b*x^3+a)^(2/3)/x^3-1/3*b*ln(x)/a^(1/3)+1/3*b*ln(a^(1/3)-(b*x^3+a)^(1/
3))/a^(1/3)+2/9*b*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/a
^(1/3)*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {266, 47, 55, 617, 204, 31}

$$-\frac{\left(a+bx^3\right)^{2/3}}{3x^3}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{3\sqrt[3]{a}}+\frac{2b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,\sqrt[3]{a}}-\frac{b\log(x)}{3\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/x^4,x]
```

```
[Out] -(a + b*x^3)^(2/3)/(3*x^3) + (2*b*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(S
qrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(1/3)) - (b*Log[x])/(3*a^(1/3)) + (b*Log[a^(
1/3) - (a + b*x^3)^(1/3)])/(3*a^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^3\right)^{2/3}}{x^4}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x^2}\,dx, x, x^3\right) \\
&= -\frac{\left(a+bx^3\right)^{2/3}}{3x^3} + \frac{1}{9}(2b)\,\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^3\right) \\
&= -\frac{\left(a+bx^3\right)^{2/3}}{3x^3} - \frac{b\log(x)}{3\sqrt[3]{a}} + \frac{1}{3}b\,\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right) - \frac{b\,\text{Subst}\left(\int \frac{}{\sqrt[3]{a}}\right.}{} \\
&= -\frac{\left(a+bx^3\right)^{2/3}}{3x^3} - \frac{b\log(x)}{3\sqrt[3]{a}} + \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{3\sqrt[3]{a}} - \frac{(2b)\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right.}{3\sqrt[3]{a}} \\
&= -\frac{\left(a+bx^3\right)^{2/3}}{3x^3} + \frac{2b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{a}} - \frac{b\log(x)}{3\sqrt[3]{a}} + \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{3\sqrt[3]{a}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.35

$$\frac{b\left(a+bx^3\right)^{5/3}\,{}_2F_1\left(\frac{5}{3},2;\frac{8}{3};\frac{bx^3}{a}+1\right)}{5a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/x^4,x]
```

```
[Out] (b*(a + b*x^3)^(5/3)*Hypergeometric2F1[5/3, 2, 8/3, 1 + (b*x^3)/a])/(5*a^2)
```

fricas [A] time = 0.67, size = 290, normalized size = 2.71

$$\left[\frac{3\sqrt{\frac{1}{3}}abx^3\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2bx^3+3\sqrt{\frac{1}{3}}\left(2\left(bx^3+a\right)^{\frac{2}{3}}a^{\frac{2}{3}}-\left(bx^3+a\right)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{2}{3}}+3a}{x^3}\right)-a^{\frac{2}{3}}bx^3\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3\right.\right.\right.}{9ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^4,x, algorithm="fricas")
```

```
[Out] [1/9*(3*sqrt(1/3)*a*b*x^3*sqrt(-1/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b
*x^3 + a)^(2/3)*a^(2/3) - (b*x^3 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) -
 3*(b*x^3 + a)^(1/3)*a^(2/3) + 3*a)/x^3) - a^(2/3)*b*x^3*log((b*x^3 + a)^(2
/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*a^(2/3)*b*x^3*log((b*x^3 + a
)^(1/3) - a^(1/3)) - 3*(b*x^3 + a)^(2/3)*a)/(a*x^3), 1/9*(6*sqrt(1/3)*a^(2/
3)*b*x^3*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3)) - a^(2/3
)*b*x^3*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*a^
(2/3)*b*x^3*log((b*x^3 + a)^(1/3) - a^(1/3)) - 3*(b*x^3 + a)^(2/3)*a)/(a*x^
3)]
```

giac [A] time = 0.71, size = 116, normalized size = 1.08

$$\frac{2\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}-\frac{b^2\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{1}{3}}}+\frac{2\,b^2\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{1}{3}}}-\frac{3\left(bx^3+a\right)^{\frac{2}{3}}b}{x^3}}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^4,x, algorithm="giac")
```

```
[Out] 1/9*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/
3))/a^(1/3) - b^2*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/
3))/a^(1/3) + 2*b^2*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/a^(1/3) - 3*(b*x^
3 + a)^(2/3)*b/x^3)/b
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^4,x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^4,x)
```

maxima [A] time = 2.94, size = 103, normalized size = 0.96

$$\frac{2\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{9\,a^{\frac{1}{3}}}-\frac{b\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9\,a^{\frac{1}{3}}}+\frac{2\,b\log\left(\left(bx^3+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9\,a^{\frac{1}{3}}}-\frac{\left(bx^3+\right.}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^4,x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/a
^(1/3) - 1/9*b*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))
/a^(1/3) + 2/9*b*log((b*x^3 + a)^(1/3) - a^(1/3))/a^(1/3) - 1/3*(b*x^3 + a)
^(2/3)/x^3
```

mupad [B] time = 1.19, size = 137, normalized size = 1.28

$$\frac{2\,b\,\ln\left(\frac{4\,a^{1/3}\,b^2}{9}-\frac{4\,b^2\left(b\,x^3+a\right)^{1/3}}{9}\right)}{9\,a^{1/3}}-\frac{\left(b\,x^3+a\right)^{2/3}}{3\,x^3}-\frac{\ln\left(\frac{a^{1/3}\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)^2}{9}-\frac{4\,b^2\left(b\,x^3+a\right)^{1/3}}{9}\right)\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{9\,a^{1/3}}-\frac{\ln\left(\frac{a^{1/3}\left(b+\sqrt{3}\,b\,1\right.\right.}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/x^4,x)
```

```
[Out] (2*b*log((4*a^(1/3)*b^2)/9 - (4*b^2*(a + b*x^3)^(1/3))/9))/(9*a^(1/3)) - (a
 + b*x^3)^(2/3)/(3*x^3) - (log((a^(1/3)*(b - 3^(1/2)*b*1i)^2)/9 - (4*b^2*(a
 + b*x^3)^(1/3))/9)*(b - 3^(1/2)*b*1i))/(9*a^(1/3)) - (log((a^(1/3)*(b + 3^
(1/2)*b*1i)^2)/9 - (4*b^2*(a + b*x^3)^(1/3))/9)*(b + 3^(1/2)*b*1i))/(9*a^(1
/3))
```

sympy [C] time = 1.35, size = 39, normalized size = 0.36

$$-\frac{b^{\frac{2}{3}}\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3}, \frac{1}{3} \\ \frac{4}{3}\end{matrix}\middle| \frac{ae^{i\pi}}{bx^3}\right)}{3x\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**4,x)
```

```
[Out] -b**(2/3)*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), a*exp_polar(I*pi)/(b*x**3))
/(3*x*gamma(4/3))
```

3.533 $\int x^4 \left(a+bx^3\right)^{2/3}\,dx$

Optimal. Leaf size=38

$$\frac{x^5 \left(a+bx^3\right)^{5/3}\ {}_2F_1\left(1,\frac{10}{3};\frac{8}{3};-\frac{bx^3}{a}\right)}{5a}$$

```
[Out] 1/5*x^5*(b*x^3+a)^(5/3)*hypergeom([1, 10/3],[8/3],-b*x^3/a)/a
```

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.34, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {365, 364}

$$\frac{x^5 \left(a+bx^3\right)^{2/3}\ {}_2F_1\left(-\frac{2}{3},\frac{5}{3};\frac{8}{3};-\frac{bx^3}{a}\right)}{5\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^3)^(2/3),x]
```

```
[Out] (x^5*(a + b*x^3)^(2/3)*Hypergeometric2F1[-2/3, 5/3, 8/3, -((b*x^3)/a)])/(5*
(1 + (b*x^3)/a)^(2/3))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^4 \left(a+bx^3\right)^{2/3}\,dx &= \frac{\left(a+bx^3\right)^{2/3}\int x^4\left(1+\frac{bx^3}{a}\right)^{2/3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\ &= \frac{x^5\left(a+bx^3\right)^{2/3}\ {}_2F_1\left(-\frac{2}{3},\frac{5}{3};\frac{8}{3};-\frac{bx^3}{a}\right)}{5\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 1.34

$$\frac{x^5 \left(a+bx^3\right)^{2/3}\ {}_2F_1\left(-\frac{2}{3},\frac{5}{3};\frac{8}{3};-\frac{bx^3}{a}\right)}{5\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^3)^(2/3),x]
```

```
[Out] (x^5*(a + b*x^3)^(2/3)*Hypergeometric2F1[-2/3, 5/3, 8/3, -((b*x^3)/a)])/(5*
(1 + (b*x^3)/a)^(2/3))
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^3+a\right)^{\frac{2}{3}}x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(2/3)*x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{2}{3}}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)*x^4, x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \left(b\,x^3+a\right)^{\frac{2}{3}}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)^(2/3),x)
```

```
[Out] int(x^4*(b*x^3+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{2}{3}}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x^4\left(b\,x^3+a\right)^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^3)^(2/3),x)
```

```
[Out] int(x^4*(a + b*x^3)^(2/3), x)
```

sympy [C] time = 2.05, size = 39, normalized size = 1.03

$$\frac{a^{\frac{2}{3}}x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3}, \frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\,\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)**(2/3),x)
```

```
[Out] a**(2/3)*x**5*gamma(5/3)*hyper((-2/3, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(8/3))
```

3.534 $\int x\left(a+bx^3\right)^{2/3}\,dx$

Optimal. Leaf size=38

$$\frac{x^2\left(a+bx^3\right)^{5/3}\,{}_2F_1\left(1,\frac{7}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2a}$$

```
[Out] 1/2*x^2*(b*x^3+a)^(5/3)*hypergeom([1, 7/3],[5/3],-b*x^3/a)/a
```

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.34, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {365, 364}

$$\frac{x^2\left(a+bx^3\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3)^(2/3),x]
```

```
[Out] (x^2*(a + b*x^3)^(2/3)*Hypergeometric2F1[-2/3, 2/3, 5/3, -((b*x^3)/a)])/(2*
(1 + (b*x^3)/a)^(2/3))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^3\right)^{2/3}\,dx &= \frac{\left(a+bx^3\right)^{2/3}\int x\left(1+\frac{bx^3}{a}\right)^{2/3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\ &= \frac{x^2\left(a+bx^3\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 1.34

$$\frac{x^2\left(a+bx^3\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)^(2/3),x]
```

```
[Out] (x^2*(a + b*x^3)^(2/3)*Hypergeometric2F1[-2/3, 2/3, 5/3, -((b*x^3)/a)])/(2*
(1 + (b*x^3)/a)^(2/3))
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^3+a\right)^{\frac{2}{3}}x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(2/3)*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{2}{3}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)*x, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \left(b\,x^3+a\right)^{\frac{2}{3}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^(2/3),x)
```

```
[Out] int(x*(b*x^3+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{2}{3}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x\left(b\,x^3+a\right)^{2/3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^3)^(2/3),x)
```

```
[Out] int(x*(a + b*x^3)^(2/3), x)
```

sympy [C] time = 1.37, size = 39, normalized size = 1.03

$$\frac{a^{\frac{2}{3}}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3}, \frac{2}{3} \\ \frac{5}{3}\end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**(2/3),x)
```

```
[Out] a**(2/3)*x**2*gamma(2/3)*hyper((-2/3, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(5/3))
```

3.535 $\int \frac{\left(a+bx^3\right)^{2/3}}{x^2}\,dx$

Optimal. Leaf size=36

$$-\frac{\left(a+bx^3\right)^{5/3}\,{}_2F_1\left(1,\frac{4}{3};\frac{2}{3};-\frac{bx^3}{a}\right)}{ax}$$

```
[Out] -(b*x^3+a)^(5/3)*hypergeom([1, 4/3],[2/3],-b*x^3/a)/a/x
```

Rubi [A] time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.36, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {365, 364}

$$-\frac{\left(a+bx^3\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},-\frac{1}{3};\frac{2}{3};-\frac{bx^3}{a}\right)}{x\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/x^2,x]
```

```
[Out] -(((a + b*x^3)^(2/3)*Hypergeometric2F1[-2/3, -1/3, 2/3, -((b*x^3)/a)])/(x*(
1 + (b*x^3)/a)^(2/3)))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^{2/3}}{x^2}\,dx &= \frac{\left(a+bx^3\right)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^2}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\ &= -\frac{\left(a+bx^3\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},-\frac{1}{3};\frac{2}{3};-\frac{bx^3}{a}\right)}{x\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 1.36

$$-\frac{\left(a+bx^3\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},-\frac{1}{3};\frac{2}{3};-\frac{bx^3}{a}\right)}{x\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/x^2,x]
```

```
[Out] -(((a + b*x^3)^(2/3)*Hypergeometric2F1[-2/3, -1/3, 2/3, -((b*x^3)/a)])/(x*(
1 + (b*x^3)/a)^(2/3)))
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(2/3)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/x^2, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^2,x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/x^2, x)
```

mupad [B] time = 1.26, size = 39, normalized size = 1.08

$$\frac{\left(b\,x^3+a\right)^{2/3}\,{}_2\mathrm{F}_1\left(-\frac{2}{3},-\frac{1}{3};\ \frac{2}{3};\ -\frac{a}{b\,x^3}\right)}{x\left(\frac{a}{b\,x^3}+1\right)^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/x^2,x)
```

```
[Out] ((a + b*x^3)^(2/3)*hypergeom([-2/3, -1/3], 2/3, -a/(b*x^3)))/(x*(a/(b*x^3)
+ 1)^(2/3))
```

sympy [C] time = 1.56, size = 41, normalized size = 1.14

$$\frac{a^{\frac{2}{3}}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3}, -\frac{1}{3} \\ \frac{2}{3}\end{matrix}\right| \frac{bx^3 e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**2,x)
```

```
[Out] a**(2/3)*gamma(-1/3)*hyper((-2/3, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/
(3*x*gamma(2/3))
```

3.536 $\int \frac{\left(a+bx^3\right)^{2/3}}{x^5}\,dx$

Optimal. Leaf size=38

$$-\frac{\left(a+bx^3\right)^{5/3}\,{}_2F_1\left(\frac{1}{3},1;-\frac{1}{3};-\frac{bx^3}{a}\right)}{4ax^4}$$

```
[Out] -1/4*(b*x^3+a)^(5/3)*hypergeom([1/3, 1],[-1/3],-b*x^3/a)/a/x^4
```

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.34, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {365, 364}

$$-\frac{\left(a+bx^3\right)^{2/3}\,{}_2F_1\left(-\frac{4}{3},-\frac{2}{3};-\frac{1}{3};-\frac{bx^3}{a}\right)}{4x^4\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/x^5,x]
```

```
[Out] -((a + b*x^3)^(2/3)*Hypergeometric2F1[-4/3, -2/3, -1/3, -((b*x^3)/a)])/(4*x
^4*(1 + (b*x^3)/a)^(2/3))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^{2/3}}{x^5}\,dx &= \frac{\left(a+bx^3\right)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^5}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\ &= -\frac{\left(a+bx^3\right)^{2/3}\,{}_2F_1\left(-\frac{4}{3},-\frac{2}{3};-\frac{1}{3};-\frac{bx^3}{a}\right)}{4x^4\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 1.34

$$-\frac{\left(a+bx^3\right)^{2/3}\,{}_2F_1\left(-\frac{4}{3},-\frac{2}{3};-\frac{1}{3};-\frac{bx^3}{a}\right)}{4x^4\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/x^5,x]
```

```
[Out] -1/4*((a + b*x^3)^(2/3)*Hypergeometric2F1[-4/3, -2/3, -1/3, -((b*x^3)/a)])/
(x^4*(1 + (b*x^3)/a)^(2/3))
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^5,x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(2/3)/x^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/x^5, x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^5,x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^5,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(b\,x^3+a\right)^{2/3}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/x^5,x)
```

```
[Out] int((a + b*x^3)^(2/3)/x^5, x)
```

sympy [C] time = 1.32, size = 46, normalized size = 1.21

$$\frac{a^{\frac{2}{3}}\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3}, -\frac{2}{3} \\ -\frac{1}{3}\end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**5,x)
```

```
[Out] a**(2/3)*gamma(-4/3)*hyper((-4/3, -2/3), (-1/3,), b*x**3*exp_polar(I*pi)/a)
/(3*x**4*gamma(-1/3))
```

3.537 $\int x^3 \left(a+bx^3\right)^{2/3}\,dx$

Optimal. Leaf size=117

$$\frac{a^2 \log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{18b^{4/3}}-\frac{a^2\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,b^{4/3}}+\frac{ax\left(a+bx^3\right)^{2/3}}{9b}+\frac{1}{6}x^4\left(a+bx^3\right)^{2/3}$$

```
[Out] 1/9*a*x*(b*x^3+a)^(2/3)/b+1/6*x^4*(b*x^3+a)^(2/3)+1/18*a^2*ln(-b^(1/3)*x+(b
*x^3+a)^(1/3))/b^(4/3)-1/27*a^2*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*
3^(1/2))/b^(4/3)*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {279, 321, 239}

$$\frac{a^2 \log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{18b^{4/3}}-\frac{a^2\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,b^{4/3}}+\frac{1}{6}x^4\left(a+bx^3\right)^{2/3}+\frac{ax\left(a+bx^3\right)^{2/3}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^3)^(2/3),x]
```

```
[Out] (a*x*(a + b*x^3)^(2/3))/(9*b) + (x^4*(a + b*x^3)^(2/3))/6 - (a^2*ArcTan[(1
+ (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(4/3)) + (a^2*Log
[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(18*b^(4/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^3 \left(a+bx^3\right)^{2/3}\,dx = \frac{1}{6}x^4\left(a+bx^3\right)^{2/3} + \frac{1}{3}a\int \frac{x^3}{\sqrt[3]{a+bx^3}}\,dx$$

$$= \frac{ax\left(a+bx^3\right)^{2/3}}{9b} + \frac{1}{6}x^4\left(a+bx^3\right)^{2/3} - \frac{a^2\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{9b}$$

$$= \frac{ax\left(a+bx^3\right)^{2/3}}{9b} + \frac{1}{6}x^4\left(a+bx^3\right)^{2/3} - \frac{a^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}\,b^{4/3}} + \frac{a^2\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)}{18b^{4/3}}$$

Mathematica [C] time = 0.06, size = 62, normalized size = 0.53

$$\frac{x\left(a+bx^3\right)^{2/3}\left(-\frac{a\,{}_2F_1\left(-\frac{2}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(\frac{bx^3}{a}+1\right)^{2/3}}+a+bx^3\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^3)^(2/3),x]
```

```
[Out] (x*(a + b*x^3)^(2/3)*(a + b*x^3 - (a*Hypergeometric2F1[-2/3, 1/3, 4/3, -((b
*x^3)/a)])/(1 + (b*x^3)/a)^(2/3)))/(6*b)
```

fricas [A] time = 0.67, size = 345, normalized size = 2.95

$$\left[\frac{3\sqrt{\frac{1}{3}}\,a^2b\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3bx^3-3\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3+\left(bx^3+a\right)^{\frac{1}{3}}bx^2-2\left(bx^3+a\right)^{\frac{2}{3}}b^{\frac{2}{3}}x\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}+2a\right)}{54\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] [1/54*(3*sqrt(1/3)*a^2*b*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)
*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^
3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 2*a^2*b^(2/3)*log(-(b^(1/
3)*x - (b*x^3 + a)^(1/3))/x) - a^2*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(
1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*(3*b^2*x^4 + 2*a*b*x)*(b*x^3 +
 a)^(2/3))/b^2, 1/54*(6*sqrt(1/3)*a^2*b^(2/3)*arctan(sqrt(1/3)*(b^(1/3)*x +
 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x)) + 2*a^2*b^(2/3)*log(-(b^(1/3)*x - (b*x^3
 + a)^(1/3))/x) - a^2*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*
x + (b*x^3 + a)^(2/3))/x^2) + 3*(3*b^2*x^4 + 2*a*b*x)*(b*x^3 + a)^(2/3))/b^
2]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{2}{3}}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)*x^3, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \left(b\,x^3+a\right)^{\frac{2}{3}} x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)^(2/3),x)
```

```
[Out] int(x^3*(b*x^3+a)^(2/3),x)
```

maxima [B] time = 2.90, size = 181, normalized size = 1.55

$$\frac{\sqrt{3}\,a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{27\,b^{\frac{4}{3}}}-\frac{a^2\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{54\,b^{\frac{4}{3}}}+\frac{a^2\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{27\,b^{\frac{4}{3}}}+\frac{\frac{\left(bx^3+a\right)^{\frac{2}{3}}a^2b}{x^2}}{18\left(b^3-\frac{2\left(bx^3\right.}{x}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 1/27*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/
3))/b^(4/3) - 1/54*a^2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 +
 a)^(2/3)/x^2)/b^(4/3) + 1/27*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/
3) + 1/18*((b*x^3 + a)^(2/3)*a^2*b/x^2 + 2*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^3
 - 2*(b*x^3 + a)*b^2/x^3 + (b*x^3 + a)^2*b/x^6)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\left(b\,x^3+a\right)^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^3)^(2/3),x)
```

```
[Out] int(x^3*(a + b*x^3)^(2/3), x)
```

sympy [C] time = 2.50, size = 39, normalized size = 0.33

$$\frac{a^{\frac{2}{3}}x^4\,\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\,\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**(2/3),x)
```

```
[Out] a**(2/3)*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(7/3))
```

3.538 $\int \left(a+bx^3\right)^{2/3}\,dx$

Optimal. Leaf size=91

$$\frac{1}{3}x\left(a+bx^3\right)^{2/3}-\frac{a\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{3\sqrt[3]{b}}+\frac{2a\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,\sqrt[3]{b}}$$

```
[Out] 1/3*x*(b*x^3+a)^(2/3)-1/3*a*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)+2/9*a*ar
ctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/3)*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {195, 239}

$$\frac{1}{3}x\left(a+bx^3\right)^{2/3}-\frac{a\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{3\sqrt[3]{b}}+\frac{2a\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3),x]
```

```
[Out] (x*(a + b*x^3)^(2/3))/3 + (2*a*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))
/Sqrt[3]])/(3*Sqrt[3]*b^(1/3)) - (a*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/
(3*b^(1/3))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^3\right)^{2/3}\,dx &= \frac{1}{3}x\left(a+bx^3\right)^{2/3}+\frac{1}{3}(2a)\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx\\ &= \frac{1}{3}x\left(a+bx^3\right)^{2/3}+\frac{2a\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,\sqrt[3]{b}}-\frac{a\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)}{3\sqrt[3]{b}}\end{aligned}$$

Mathematica [C] time = 0.22, size = 196, normalized size = 2.15

$$\frac{3\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(a+bx^3\right)^{2/3}F_1\left(\frac{5}{3};-\frac{2}{3},-\frac{2}{3};\frac{8}{3};-\frac{i\left(\sqrt[3]{b}\,x+(-1)^{2/3}\sqrt[3]{a}\right)}{\sqrt{3}\,\sqrt[3]{a}},\frac{-\frac{2i\sqrt[3]{b}\,x}{\sqrt[3]{a}}+\sqrt{3}+i}{3i+\sqrt{3}}\right)}{5\,2^{2/3}\sqrt[3]{b}\left(\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}\right)^{2/3}\left(\frac{i\left(\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}+1\right)}{\sqrt{3}+3i}\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3),x]
```

```
[Out] (3*((-1)^(2/3)*a^(1/3) + b^(1/3)*x)*(a + b*x^3)^(2/3)*AppellF1[5/3, -2/3, -
2/3, 8/3, ((-I)*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(Sqrt[3]*a^(1/3)), (I + S
qrt[3] - ((2*I)*b^(1/3)*x)/a^(1/3))/(3*I + Sqrt[3])])/(5*2^(2/3)*b^(1/3)*((
a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3)))^(2/3)*((I*(1 +
(b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3]))^(2/3))
```

fricas [B] time = 0.66, size = 362, normalized size = 3.98

$$\left[\frac{3\sqrt{\frac{1}{3}}\,ab\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3bx^3-3\left(bx^3+a\right)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}bx^3-\left(bx^3+a\right)^{\frac{1}{3}}bx^2+2\left(bx^3+a\right)^{\frac{2}{3}}(-b)^{\frac{2}{3}}x\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] [1/9*(3*sqrt(1/3)*a*b*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*
(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 +
2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) + 3*(b*x^3 + a)
^(2/3)*b*x - 2*a*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + a*(
-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a
)^(2/3))/x^2))/b, -1/9*(6*sqrt(1/3)*a*b*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/
3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt(-(-b)^(1/3)/b)/x) - 3*(b*x^3 +
 a)^(2/3)*b*x + 2*a*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) -
a*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3
+ a)^(2/3))/x^2))/b]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3), x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \left(b\,x^3+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3),x)
```

```
[Out] int((b*x^3+a)^(2/3),x)
```

maxima [A] time = 3.01, size = 134, normalized size = 1.47

$$-\frac{2\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{9b^{\frac{1}{3}}}+\frac{a\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{9b^{\frac{1}{3}}}-\frac{2a\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{9b^{\frac{1}{3}}}-\frac{\left(bx^3+a\right)^{\frac{2}{3}}a}{3\left(b-\frac{bx^3+a}{x^3}\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3)
)/b^(1/3) + 1/9*a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(
2/3)/x^2)/b^(1/3) - 2/9*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3) - 1/3
*(b*x^3 + a)^(2/3)*a/((b - (b*x^3 + a)/x^3)*x^2)
```

mupad [B] time = 1.02, size = 37, normalized size = 0.41

$$\frac{x\left(b\,x^3+a\right)^{2/3}\,{}_2\mathrm{F}_1\left(-\frac{2}{3},\frac{1}{3};\ \frac{4}{3};\ -\frac{b\,x^3}{a}\right)}{\left(\frac{b\,x^3}{a}+1\right)^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3),x)
```

```
[Out] (x*(a + b*x^3)^(2/3)*hypergeom([-2/3, 1/3], 4/3, -(b*x^3)/a))/((b*x^3)/a +
1)^(2/3)
```

sympy [C] time = 2.19, size = 37, normalized size = 0.41

$$\frac{a^{\frac{2}{3}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3),x)
```

```
[Out] a**(2/3)*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(4/3))
```

3.539 $\int \frac{\left(a+bx^3\right)^{2/3}}{x^3}\,dx$

Optimal. Leaf size=88

$$-\frac{1}{2}b^{2/3}\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)+\frac{b^{2/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\left(a+bx^3\right)^{2/3}}{2x^2}$$

```
[Out] -1/2*(b*x^3+a)^(2/3)/x^2-1/2*b^(2/3)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))+1/3*b^(
2/3)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {277, 239}

$$-\frac{1}{2}b^{2/3}\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)+\frac{b^{2/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\left(a+bx^3\right)^{2/3}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/x^3,x]
```

```
[Out] -(a + b*x^3)^(2/3)/(2*x^2) + (b^(2/3)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)
^(1/3))/Sqrt[3]])/Sqrt[3] - (b^(2/3)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])
/2
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^3}\,dx = -\frac{\left(a+bx^3\right)^{2/3}}{2x^2}+b\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx$$

$$= -\frac{\left(a+bx^3\right)^{2/3}}{2x^2}+\frac{b^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}b^{2/3}\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.58

$$-\frac{\left(a+bx^3\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},-\frac{2}{3};\frac{1}{3};-\frac{bx^3}{a}\right)}{2x^2\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/x^3,x]
```

```
[Out] -1/2*((a + b*x^3)^(2/3)*Hypergeometric2F1[-2/3, -2/3, 1/3, -((b*x^3)/a)])/(
x^2*(1 + (b*x^3)/a)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/x^3, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^3,x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^3,x)
```

maxima [A] time = 3.02, size = 114, normalized size = 1.30

$$-\frac{1}{3}\sqrt{3}\,b^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)+\frac{1}{6}\,b^{\frac{2}{3}}\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)-\frac{1}{3}\,b^{\frac{2}{3}}\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right.}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^3,x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*b^(2/3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b
^(1/3)) + 1/6*b^(2/3)*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 +
a)^(2/3)/x^2) - 1/3*b^(2/3)*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x) - 1/2*(b*x^
3 + a)^(2/3)/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^3+a\right)^{2/3}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/x^3,x)
```

```
[Out] int((a + b*x^3)^(2/3)/x^3, x)
```

sympy [C] time = 2.10, size = 42, normalized size = 0.48

$$\frac{a^{\frac{2}{3}}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3}, -\frac{2}{3} \\ \frac{1}{3}\end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**3,x)
```

```
[Out] a**(2/3)*gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), b*x**3*exp_polar(I*pi)/a)/
(3*x**2*gamma(1/3))
```

3.540 $\int \frac{\left(a+bx^3\right)^{2/3}}{x^6}\,dx$

Optimal. Leaf size=21

$$-\frac{\left(a+bx^3\right)^{5/3}}{5ax^5}$$

```
[Out] -1/5*(b*x^3+a)^(5/3)/a/x^5
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {264}

$$-\frac{\left(a+bx^3\right)^{5/3}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/x^6,x]
```

```
[Out] -(a + b*x^3)^(5/3)/(5*a*x^5)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^6}\,dx = -\frac{\left(a+bx^3\right)^{5/3}}{5ax^5}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$-\frac{\left(a+bx^3\right)^{5/3}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/x^6,x]
```

```
[Out] -1/5*(a + b*x^3)^(5/3)/(a*x^5)
```

fricas [A] time = 0.88, size = 17, normalized size = 0.81

$$-\frac{\left(bx^3+a\right)^{\frac{5}{3}}}{5\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^6,x, algorithm="fricas")
```

```
[Out] -1/5*(b*x^3 + a)^(5/3)/(a*x^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^6,x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(2/3)/x^6, x)
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$-\frac{\left(b\,x^3+a\right)^{\frac{5}{3}}}{5a\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^6,x)
[Out] -1/5*(b*x^3+a)^(5/3)/a/x^5
```

maxima [A] time = 1.34, size = 17, normalized size = 0.81

$$-\frac{\left(bx^3+a\right)^{\frac{5}{3}}}{5\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^6,x, algorithm="maxima")
[Out] -1/5*(b*x^3 + a)^(5/3)/(a*x^5)
```

mupad [B] time = 1.12, size = 17, normalized size = 0.81

$$-\frac{\left(b\,x^3+a\right)^{5/3}}{5\,a\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/x^6,x)
[Out] -(a + b*x^3)^(5/3)/(5*a*x^5)
```

sympy [B] time = 1.27, size = 68, normalized size = 3.24

$$\frac{b^{\frac{2}{3}}\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{3x^3\Gamma\left(-\frac{2}{3}\right)}+\frac{b^{\frac{5}{3}}\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{3a\Gamma\left(-\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**6,x)
[Out] b**(2/3)*(a/(b*x**3) + 1)**(2/3)*gamma(-5/3)/(3*x**3*gamma(-2/3)) + b**(5/3
)*(a/(b*x**3) + 1)**(2/3)*gamma(-5/3)/(3*a*gamma(-2/3))
```

3.541 $\int \frac{\left(a+bx^3\right)^{2/3}}{x^9}\,dx$

Optimal. Leaf size=44

$$\frac{3b\left(a+bx^3\right)^{5/3}}{40a^2x^5}-\frac{\left(a+bx^3\right)^{5/3}}{8ax^8}$$

```
[Out] -1/8*(b*x^3+a)^(5/3)/a/x^8+3/40*b*(b*x^3+a)^(5/3)/a^2/x^5
```

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$\frac{3b\left(a+bx^3\right)^{5/3}}{40a^2x^5}-\frac{\left(a+bx^3\right)^{5/3}}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/x^9,x]
```

```
[Out] -(a + b*x^3)^(5/3)/(8*a*x^8) + (3*b*(a + b*x^3)^(5/3))/(40*a^2*x^5)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^{2/3}}{x^9}\,dx &= -\frac{\left(a+bx^3\right)^{5/3}}{8ax^8}-\frac{(3b)\int \frac{\left(a+bx^3\right)^{2/3}}{x^6}\,dx}{8a}\\ &= -\frac{\left(a+bx^3\right)^{5/3}}{8ax^8}+\frac{3b\left(a+bx^3\right)^{5/3}}{40a^2x^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.70

$$\frac{\left(a+bx^3\right)^{5/3}\left(3bx^3-5a\right)}{40a^2x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/x^9,x]
```

```
[Out] ((a + b*x^3)^(5/3)*(-5*a + 3*b*x^3))/(40*a^2*x^8)
```

fricas [A] time = 0.87, size = 38, normalized size = 0.86

$$\frac{\left(3\,b^2x^6-2\,abx^3-5\,a^2\right)\left(bx^3+a\right)^{\frac{2}{3}}}{40\,a^2x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^9,x, algorithm="fricas")
```

```
[Out] 1/40*(3*b^2*x^6 - 2*a*b*x^3 - 5*a^2)*(b*x^3 + a)^(2/3)/(a^2*x^8)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^9,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/x^9, x)
```

maple [A] time = 0.00, size = 28, normalized size = 0.64

$$-\frac{\left(b\,x^3+a\right)^{\frac{5}{3}}\left(-3b\,x^3+5a\right)}{40a^2x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^9,x)
```

```
[Out] -1/40*(b*x^3+a)^(5/3)*(-3*b*x^3+5*a)/x^8/a^2
```

maxima [A] time = 1.24, size = 35, normalized size = 0.80

$$\frac{\frac{8\left(bx^3+a\right)^{\frac{5}{3}}b}{x^5}-\frac{5\left(bx^3+a\right)^{\frac{8}{3}}}{x^8}}{40\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^9,x, algorithm="maxima")
```

```
[Out] 1/40*(8*(b*x^3 + a)^(5/3)*b/x^5 - 5*(b*x^3 + a)^(8/3)/x^8)/a^2
```

mupad [B] time = 1.21, size = 38, normalized size = 0.86

$$-\frac{\left(b\,x^3+a\right)^{2/3}\left(5\,a^2+2\,a\,b\,x^3-3\,b^2\,x^6\right)}{40\,a^2\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/x^9,x)
```

```
[Out] -((a + b*x^3)^(2/3)*(5*a^2 - 3*b^2*x^6 + 2*a*b*x^3))/(40*a^2*x^8)
```

sympy [B] time = 1.99, size = 110, normalized size = 2.50

$$-\frac{5b^{\frac{2}{3}}\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{9x^6\Gamma\left(-\frac{2}{3}\right)}-\frac{2b^{\frac{5}{3}}\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{9ax^3\Gamma\left(-\frac{2}{3}\right)}+\frac{b^{\frac{8}{3}}\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{3a^2\Gamma\left(-\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**9,x)
```

```
[Out] -5*b**(2/3)*(a/(b*x**3) + 1)**(2/3)*gamma(-8/3)/(9*x**6*gamma(-2/3)) - 2*b*
*(5/3)*(a/(b*x**3) + 1)**(2/3)*gamma(-8/3)/(9*a*x**3*gamma(-2/3)) + b**(8/3
)*(a/(b*x**3) + 1)**(2/3)*gamma(-8/3)/(3*a**2*gamma(-2/3))
```

3.542 $\int \frac{\left(a+bx^3\right)^{2/3}}{x^{12}}\,dx$

Optimal. Leaf size=68

$$-\frac{9b^2\left(a+bx^3\right)^{5/3}}{220a^3x^5}+\frac{3b\left(a+bx^3\right)^{5/3}}{44a^2x^8}-\frac{\left(a+bx^3\right)^{5/3}}{11ax^{11}}$$

```
[Out] -1/11*(b*x^3+a)^(5/3)/a/x^11+3/44*b*(b*x^3+a)^(5/3)/a^2/x^8-9/220*b^2*(b*x^
3+a)^(5/3)/a^3/x^5
```

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$-\frac{9b^2\left(a+bx^3\right)^{5/3}}{220a^3x^5}+\frac{3b\left(a+bx^3\right)^{5/3}}{44a^2x^8}-\frac{\left(a+bx^3\right)^{5/3}}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/x^12,x]
```

```
[Out] -(a + b*x^3)^(5/3)/(11*a*x^11) + (3*b*(a + b*x^3)^(5/3))/(44*a^2*x^8) - (9*
b^2*(a + b*x^3)^(5/3))/(220*a^3*x^5)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^{2/3}}{x^{12}}\,dx &= -\frac{\left(a+bx^3\right)^{5/3}}{11ax^{11}}-\frac{(6b)\int \frac{\left(a+bx^3\right)^{2/3}}{x^9}\,dx}{11a}\\ &= -\frac{\left(a+bx^3\right)^{5/3}}{11ax^{11}}+\frac{3b\left(a+bx^3\right)^{5/3}}{44a^2x^8}+\frac{\left(9b^2\right)\int \frac{\left(a+bx^3\right)^{2/3}}{x^6}\,dx}{44a^2}\\ &= -\frac{\left(a+bx^3\right)^{5/3}}{11ax^{11}}+\frac{3b\left(a+bx^3\right)^{5/3}}{44a^2x^8}-\frac{9b^2\left(a+bx^3\right)^{5/3}}{220a^3x^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.62

$$-\frac{\left(a+bx^3\right)^{5/3}\left(20a^2-15abx^3+9b^2x^6\right)}{220a^3x^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/x^12,x]
```

```
[Out] -1/220*((a + b*x^3)^(5/3)*(20*a^2 - 15*a*b*x^3 + 9*b^2*x^6))/(a^3*x^11)
```

fricas [A] time = 0.98, size = 49, normalized size = 0.72

$$-\frac{\left(9\,b^3x^9-6\,ab^2x^6+5\,a^2bx^3+20\,a^3\right)\left(bx^3+a\right)^{\frac{2}{3}}}{220\,a^3x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^12,x, algorithm="fricas")
```

```
[Out] -1/220*(9*b^3*x^9 - 6*a*b^2*x^6 + 5*a^2*b*x^3 + 20*a^3)*(b*x^3 + a)^(2/3)/(
a^3*x^11)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^12,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/x^12, x)
```

maple [A] time = 0.01, size = 39, normalized size = 0.57

$$-\frac{\left(b\,x^3+a\right)^{\frac{5}{3}}\left(9b^2x^6-15ab\,x^3+20a^2\right)}{220a^3x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^12,x)
```

```
[Out] -1/220*(b*x^3+a)^(5/3)*(9*b^2*x^6-15*a*b*x^3+20*a^2)/x^11/a^3
```

maxima [A] time = 1.31, size = 52, normalized size = 0.76

$$-\frac{\frac{44\left(bx^3+a\right)^{\frac{5}{3}}b^2}{x^5}-\frac{55\left(bx^3+a\right)^{\frac{8}{3}}b}{x^8}+\frac{20\left(bx^3+a\right)^{\frac{11}{3}}}{x^{11}}}{220\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^12,x, algorithm="maxima")
```

```
[Out] -1/220*(44*(b*x^3 + a)^(5/3)*b^2/x^5 - 55*(b*x^3 + a)^(8/3)*b/x^8 + 20*(b*x
^3 + a)^(11/3)/x^11)/a^3
```

mupad [B] time = 1.35, size = 73, normalized size = 1.07

$$\frac{3\,b^2\left(b\,x^3+a\right)^{2/3}}{110\,a^2\,x^5}-\frac{b\left(b\,x^3+a\right)^{2/3}}{44\,a\,x^8}-\frac{9\,b^3\left(b\,x^3+a\right)^{2/3}}{220\,a^3\,x^2}-\frac{\left(b\,x^3+a\right)^{2/3}}{11\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/x^12,x)
```

```
[Out] (3*b^2*(a + b*x^3)^(2/3))/(110*a^2*x^5) - (b*(a + b*x^3)^(2/3))/(44*a*x^8)
- (9*b^3*(a + b*x^3)^(2/3))/(220*a^3*x^2) - (a + b*x^3)^(2/3)/(11*x^11)
```

sympy [**B**] time = 2.23, size = 520, normalized size = 7.65

$$\frac{40a^5b^{\frac{14}{3}}\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{11}{3}\right)}{27a^5b^4x^9\Gamma\left(-\frac{2}{3}\right)+54a^4b^5x^{12}\Gamma\left(-\frac{2}{3}\right)+27a^3b^6x^{15}\Gamma\left(-\frac{2}{3}\right)}+\frac{90a^4b^{\frac{17}{3}}x^3\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{11}{3}\right)}{27a^5b^4x^9\Gamma\left(-\frac{2}{3}\right)+54a^4b^5x^{12}\Gamma\left(-\frac{2}{3}\right)+27a^3b^6x^{15}\Gamma\left(-\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**12,x)
```

```
[Out] 40*a**5*b**(14/3)*(a/(b*x**3) + 1)**(2/3)*gamma(-11/3)/(27*a**5*b**4*x**9*g
amma(-2/3) + 54*a**4*b**5*x**12*gamma(-2/3) + 27*a**3*b**6*x**15*gamma(-2/3
)) + 90*a**4*b**(17/3)*x**3*(a/(b*x**3) + 1)**(2/3)*gamma(-11/3)/(27*a**5*b
**4*x**9*gamma(-2/3) + 54*a**4*b**5*x**12*gamma(-2/3) + 27*a**3*b**6*x**15*
gamma(-2/3)) + 48*a**3*b**(20/3)*x**6*(a/(b*x**3) + 1)**(2/3)*gamma(-11/3)/
(27*a**5*b**4*x**9*gamma(-2/3) + 54*a**4*b**5*x**12*gamma(-2/3) + 27*a**3*b
**6*x**15*gamma(-2/3)) + 4*a**2*b**(23/3)*x**9*(a/(b*x**3) + 1)**(2/3)*gamm
a(-11/3)/(27*a**5*b**4*x**9*gamma(-2/3) + 54*a**4*b**5*x**12*gamma(-2/3) +
27*a**3*b**6*x**15*gamma(-2/3)) + 24*a*b**(26/3)*x**12*(a/(b*x**3) + 1)**(2
/3)*gamma(-11/3)/(27*a**5*b**4*x**9*gamma(-2/3) + 54*a**4*b**5*x**12*gamma(
-2/3) + 27*a**3*b**6*x**15*gamma(-2/3)) + 18*b**(29/3)*x**15*(a/(b*x**3) +
1)**(2/3)*gamma(-11/3)/(27*a**5*b**4*x**9*gamma(-2/3) + 54*a**4*b**5*x**12*
gamma(-2/3) + 27*a**3*b**6*x**15*gamma(-2/3))
```

3.543 $\int x^8 \left(1-x^3\right)^{6/5}\, dx$

Optimal. Leaf size=46

$$-\frac{5}{63}\left(1-x^3\right)^{21/5}+\frac{5}{24}\left(1-x^3\right)^{16/5}-\frac{5}{33}\left(1-x^3\right)^{11/5}$$

```
[Out] -5/33*(-x^3+1)^(11/5)+5/24*(-x^3+1)^(16/5)-5/63*(-x^3+1)^(21/5)
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{5}{63}\left(1-x^3\right)^{21/5}+\frac{5}{24}\left(1-x^3\right)^{16/5}-\frac{5}{33}\left(1-x^3\right)^{11/5}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(1 - x^3)^(6/5),x]
```

```
[Out] (-5*(1 - x^3)^(11/5))/33 + (5*(1 - x^3)^(16/5))/24 - (5*(1 - x^3)^(21/5))/6
3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^8\left(1-x^3\right)^{6/5}\, dx &= \frac{1}{3}\text{Subst}\left(\int (1-x)^{6/5}x^2\, dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left((1-x)^{6/5}-2(1-x)^{11/5}+(1-x)^{16/5}\right)\, dx, x, x^3\right)\\ &= -\frac{5}{33}\left(1-x^3\right)^{11/5}+\frac{5}{24}\left(1-x^3\right)^{16/5}-\frac{5}{63}\left(1-x^3\right)^{21/5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.59

$$-\frac{5\left(1-x^3\right)^{11/5}\left(88x^6+55x^3+25\right)}{5544}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(1 - x^3)^(6/5),x]
```

```
[Out] (-5*(1 - x^3)^(11/5)*(25 + 55*x^3 + 88*x^6))/5544
```

fricas [A] time = 0.80, size = 33, normalized size = 0.72

$$-\frac{5}{5544}\left(88\,x^{12}-121\,x^9+3\,x^6+5\,x^3+25\right)\left(-x^3+1\right)^{\frac{1}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(-x^3+1)^(6/5),x, algorithm="fricas")
```

```
[Out] -5/5544*(88*x^12 - 121*x^9 + 3*x^6 + 5*x^3 + 25)*(-x^3 + 1)^(1/5)
```

giac [A] time = 0.16, size = 55, normalized size = 1.20

$$-\frac{5}{63}\left(x^3-1\right)^4\left(-x^3+1\right)^{\frac{1}{5}}-\frac{5}{24}\left(x^3-1\right)^3\left(-x^3+1\right)^{\frac{1}{5}}-\frac{5}{33}\left(x^3-1\right)^2\left(-x^3+1\right)^{\frac{1}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(-x^3+1)^(6/5),x, algorithm="giac")
```

```
[Out] -5/63*(x^3 - 1)^4*(-x^3 + 1)^(1/5) - 5/24*(x^3 - 1)^3*(-x^3 + 1)^(1/5) - 5/
33*(x^3 - 1)^2*(-x^3 + 1)^(1/5)
```

maple [A] time = 0.01, size = 33, normalized size = 0.72

$$\frac{5\left(x-1\right)\left(x^2+x+1\right)\left(88x^6+55x^3+25\right)\left(-x^3+1\right)^{\frac{6}{5}}}{5544}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(-x^3+1)^(6/5),x)
```

```
[Out] 5/5544*(x-1)*(x^2+x+1)*(88*x^6+55*x^3+25)*(-x^3+1)^(6/5)
```

maxima [A] time = 1.32, size = 34, normalized size = 0.74

$$-\frac{5}{63}\left(-x^3+1\right)^{\frac{21}{5}}+\frac{5}{24}\left(-x^3+1\right)^{\frac{16}{5}}-\frac{5}{33}\left(-x^3+1\right)^{\frac{11}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(-x^3+1)^(6/5),x, algorithm="maxima")
```

```
[Out] -5/63*(-x^3 + 1)^(21/5) + 5/24*(-x^3 + 1)^(16/5) - 5/33*(-x^3 + 1)^(11/5)
```

mupad [B] time = 1.08, size = 33, normalized size = 0.72

$$-\left(1-x^3\right)^{1/5}\left(\frac{5\,x^{12}}{63}-\frac{55\,x^9}{504}+\frac{5\,x^6}{1848}+\frac{25\,x^3}{5544}+\frac{125}{5544}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(1 - x^3)^(6/5),x)
```

```
[Out] -(1 - x^3)^(1/5)*((25*x^3)/5544 + (5*x^6)/1848 - (55*x^9)/504 + (5*x^12)/63
 + 125/5544)
```

sympy [B] time = 17.00, size = 71, normalized size = 1.54

$$-\frac{5x^{12}\sqrt[5]{1-x^3}}{63}+\frac{55x^9\sqrt[5]{1-x^3}}{504}-\frac{5x^6\sqrt[5]{1-x^3}}{1848}-\frac{25x^3\sqrt[5]{1-x^3}}{5544}-\frac{125\sqrt[5]{1-x^3}}{5544}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(-x**3+1)**(6/5),x)
```

```
[Out] -5*x**12*(1 - x**3)**(1/5)/63 + 55*x**9*(1 - x**3)**(1/5)/504 - 5*x**6*(1 -
 x**3)**(1/5)/1848 - 25*x**3*(1 - x**3)**(1/5)/5544 - 125*(1 - x**3)**(1/5)
/5544
```

3.544 $\int \frac{x^{11}}{\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=80

$$-\frac{a^3\left(a+bx^3\right)^{2/3}}{2b^4}+\frac{3a^2\left(a+bx^3\right)^{5/3}}{5b^4}+\frac{\left(a+bx^3\right)^{11/3}}{11b^4}-\frac{3a\left(a+bx^3\right)^{8/3}}{8b^4}$$

```
[Out] -1/2*a^3*(b*x^3+a)^(2/3)/b^4+3/5*a^2*(b*x^3+a)^(5/3)/b^4-3/8*a*(b*x^3+a)^(8
/3)/b^4+1/11*(b*x^3+a)^(11/3)/b^4
```

Rubi [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{3a^2\left(a+bx^3\right)^{5/3}}{5b^4}-\frac{a^3\left(a+bx^3\right)^{2/3}}{2b^4}+\frac{\left(a+bx^3\right)^{11/3}}{11b^4}-\frac{3a\left(a+bx^3\right)^{8/3}}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + b*x^3)^(1/3),x]
```

```
[Out] -(a^3*(a + b*x^3)^(2/3))/(2*b^4) + (3*a^2*(a + b*x^3)^(5/3))/(5*b^4) - (3*a
*(a + b*x^3)^(8/3))/(8*b^4) + (a + b*x^3)^(11/3)/(11*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{\sqrt[3]{a+bx^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^3}{\sqrt[3]{a+bx}}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(-\frac{a^3}{b^3\sqrt[3]{a+bx}}+\frac{3a^2(a+bx)^{2/3}}{b^3}-\frac{3a(a+bx)^{5/3}}{b^3}+\frac{(a+bx)^{8/3}}{b^3}\right)dx, x, x^3\right)\\ &= -\frac{a^3\left(a+bx^3\right)^{2/3}}{2b^4}+\frac{3a^2\left(a+bx^3\right)^{5/3}}{5b^4}-\frac{3a\left(a+bx^3\right)^{8/3}}{8b^4}+\frac{\left(a+bx^3\right)^{11/3}}{11b^4}\end{aligned}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 0.62

$$\frac{\left(a+bx^3\right)^{2/3}\left(-81a^3+54a^2bx^3-45ab^2x^6+40b^3x^9\right)}{440b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + b*x^3)^(1/3),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(-81*a^3 + 54*a^2*b*x^3 - 45*a*b^2*x^6 + 40*b^3*x^9))/(4
40*b^4)
```

fricas [A] time = 0.73, size = 46, normalized size = 0.58

$$\frac{\left(40\,b^3x^9 - 45\,ab^2x^6 + 54\,a^2bx^3 - 81\,a^3\right)\left(bx^3+a\right)^{\frac{2}{3}}}{440\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/440*(40*b^3*x^9 - 45*a*b^2*x^6 + 54*a^2*b*x^3 - 81*a^3)*(b*x^3 + a)^(2/3)
/b^4
```

giac [A] time = 0.18, size = 61, normalized size = 0.76

$$-\frac{\left(bx^3+a\right)^{\frac{2}{3}}a^3}{2\,b^4} + \frac{40\left(bx^3+a\right)^{\frac{11}{3}} - 165\left(bx^3+a\right)^{\frac{8}{3}}a + 264\left(bx^3+a\right)^{\frac{5}{3}}a^2}{440\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] -1/2*(b*x^3 + a)^(2/3)*a^3/b^4 + 1/440*(40*(b*x^3 + a)^(11/3) - 165*(b*x^3
+ a)^(8/3)*a + 264*(b*x^3 + a)^(5/3)*a^2)/b^4
```

maple [A] time = 0.01, size = 47, normalized size = 0.59

$$-\frac{\left(b\,x^3+a\right)^{\frac{2}{3}}\left(-40b^3x^9 + 45a\,b^2x^6 - 54a^2b\,x^3 + 81a^3\right)}{440b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^3+a)^(1/3),x)
```

```
[Out] -1/440*(b*x^3+a)^(2/3)*(-40*b^3*x^9+45*a*b^2*x^6-54*a^2*b*x^3+81*a^3)/b^4
```

maxima [A] time = 1.38, size = 64, normalized size = 0.80

$$\frac{\left(bx^3+a\right)^{\frac{11}{3}}}{11\,b^4} - \frac{3\left(bx^3+a\right)^{\frac{8}{3}}a}{8\,b^4} + \frac{3\left(bx^3+a\right)^{\frac{5}{3}}a^2}{5\,b^4} - \frac{\left(bx^3+a\right)^{\frac{2}{3}}a^3}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/11*(b*x^3 + a)^(11/3)/b^4 - 3/8*(b*x^3 + a)^(8/3)*a/b^4 + 3/5*(b*x^3 + a)
^(5/3)*a^2/b^4 - 1/2*(b*x^3 + a)^(2/3)*a^3/b^4
```

mupad [B] time = 1.11, size = 48, normalized size = 0.60

$$-\left(b\,x^3+a\right)^{2/3}\left(\frac{81\,a^3}{440\,b^4} - \frac{x^9}{11\,b} + \frac{9\,a\,x^6}{88\,b^2} - \frac{27\,a^2\,x^3}{220\,b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a + b*x^3)^(1/3),x)
```

```
[Out] -(a + b*x^3)^(2/3)*((81*a^3)/(440*b^4) - x^9/(11*b) + (9*a*x^6)/(88*b^2) -
(27*a^2*x^3)/(220*b^3))
```

sympy [**A**] time = 3.31, size = 92, normalized size = 1.15

$$\begin{cases} -\frac{81a^3\left(a+bx^3\right)^{\frac{2}{3}}}{440b^4}+\frac{27a^2x^3\left(a+bx^3\right)^{\frac{2}{3}}}{220b^3}-\frac{9ax^6\left(a+bx^3\right)^{\frac{2}{3}}}{88b^2}+\frac{x^9\left(a+bx^3\right)^{\frac{2}{3}}}{11b} & \text{for } b\neq 0 \\ \frac{x^{12}}{12\sqrt[3]{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**3+a)**(1/3),x)
```

```
[Out] Piecewise((-81*a**3*(a + b*x**3)**(2/3)/(440*b**4) + 27*a**2*x**3*(a + b*x*
*3)**(2/3)/(220*b**3) - 9*a*x**6*(a + b*x**3)**(2/3)/(88*b**2) + x**9*(a +
b*x**3)**(2/3)/(11*b), Ne(b, 0)), (x**12/(12*a**(1/3)), True))
```

3.545 $\int \frac{x^8}{\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=59

$$\frac{a^2\left(a+bx^3\right)^{2/3}}{2b^3}+\frac{\left(a+bx^3\right)^{8/3}}{8b^3}-\frac{2a\left(a+bx^3\right)^{5/3}}{5b^3}$$

```
[Out] 1/2*a^2*(b*x^3+a)^(2/3)/b^3-2/5*a*(b*x^3+a)^(5/3)/b^3+1/8*(b*x^3+a)^(8/3)/b
^3
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{a^2\left(a+bx^3\right)^{2/3}}{2b^3}+\frac{\left(a+bx^3\right)^{8/3}}{8b^3}-\frac{2a\left(a+bx^3\right)^{5/3}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^3)^(1/3),x]
```

```
[Out] (a^2*(a + b*x^3)^(2/3))/(2*b^3) - (2*a*(a + b*x^3)^(5/3))/(5*b^3) + (a + b*
x^3)^(8/3)/(8*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{\sqrt[3]{a+bx^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{\sqrt[3]{a+bx}}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{a^2}{b^2\sqrt[3]{a+bx}}-\frac{2a(a+bx)^{2/3}}{b^2}+\frac{(a+bx)^{5/3}}{b^2}\right)dx, x, x^3\right)\\ &= \frac{a^2\left(a+bx^3\right)^{2/3}}{2b^3}-\frac{2a\left(a+bx^3\right)^{5/3}}{5b^3}+\frac{\left(a+bx^3\right)^{8/3}}{8b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.66

$$\frac{\left(a+bx^3\right)^{2/3}\left(9a^2-6abx^3+5b^2x^6\right)}{40b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^3)^(1/3),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(9*a^2 - 6*a*b*x^3 + 5*b^2*x^6))/(40*b^3)
```

fricas [A] time = 0.84, size = 35, normalized size = 0.59

$$\frac{\left(5\,b^2x^6 - 6\,abx^3 + 9\,a^2\right)\left(bx^3 + a\right)^{\frac{2}{3}}}{40\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/40*(5*b^2*x^6 - 6*a*b*x^3 + 9*a^2)*(b*x^3 + a)^(2/3)/b^3
```

giac [A] time = 0.16, size = 47, normalized size = 0.80

$$\frac{\left(bx^3 + a\right)^{\frac{2}{3}}a^2}{2\,b^3} + \frac{5\left(bx^3 + a\right)^{\frac{8}{3}} - 16\left(bx^3 + a\right)^{\frac{5}{3}}a}{40\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] 1/2*(b*x^3 + a)^(2/3)*a^2/b^3 + 1/40*(5*(b*x^3 + a)^(8/3) - 16*(b*x^3 + a)^
(5/3)*a)/b^3
```

maple [A] time = 0.01, size = 36, normalized size = 0.61

$$\frac{\left(b\,x^3 + a\right)^{\frac{2}{3}}\left(5b^2x^6 - 6ab\,x^3 + 9a^2\right)}{40b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a)^(1/3),x)
```

```
[Out] 1/40*(b*x^3+a)^(2/3)*(5*b^2*x^6-6*a*b*x^3+9*a^2)/b^3
```

maxima [A] time = 1.33, size = 47, normalized size = 0.80

$$\frac{\left(bx^3 + a\right)^{\frac{8}{3}}}{8\,b^3} - \frac{2\left(bx^3 + a\right)^{\frac{5}{3}}a}{5\,b^3} + \frac{\left(bx^3 + a\right)^{\frac{2}{3}}a^2}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/8*(b*x^3 + a)^(8/3)/b^3 - 2/5*(b*x^3 + a)^(5/3)*a/b^3 + 1/2*(b*x^3 + a)^(
2/3)*a^2/b^3
```

mupad [B] time = 1.08, size = 36, normalized size = 0.61

$$\left(b\,x^3 + a\right)^{2/3}\left(\frac{9\,a^2}{40\,b^3} + \frac{x^6}{8\,b} - \frac{3\,a\,x^3}{20\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^3)^(1/3),x)
```

```
[Out] (a + b*x^3)^(2/3)*((9*a^2)/(40*b^3) + x^6/(8*b) - (3*a*x^3)/(20*b^2))
```

sympy [**A**] time = 2.73, size = 68, normalized size = 1.15

$$\begin{cases} \frac{9a^2\left(a+bx^3\right)^{\frac{2}{3}}}{40b^3} - \frac{3ax^3\left(a+bx^3\right)^{\frac{2}{3}}}{20b^2} + \frac{x^6\left(a+bx^3\right)^{\frac{2}{3}}}{8b} & \text{for } b \neq 0 \\ \frac{x^9}{9\sqrt[3]{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a)**(1/3),x)
```

```
[Out] Piecewise((9*a**2*(a + b*x**3)**(2/3)/(40*b**3) - 3*a*x**3*(a + b*x**3)**(2
/3)/(20*b**2) + x**6*(a + b*x**3)**(2/3)/(8*b), Ne(b, 0)), (x**9/(9*a**(1/3
)), True))
```

3.546 $\int \frac{x^5}{\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=38

$$\frac{\left(a+bx^3\right)^{5/3}}{5b^2}-\frac{a\left(a+bx^3\right)^{2/3}}{2b^2}$$

```
[Out] -1/2*a*(b*x^3+a)^(2/3)/b^2+1/5*(b*x^3+a)^(5/3)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{\left(a+bx^3\right)^{5/3}}{5b^2}-\frac{a\left(a+bx^3\right)^{2/3}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^3)^(1/3),x]
```

```
[Out] -(a*(a + b*x^3)^(2/3))/(2*b^2) + (a + b*x^3)^(5/3)/(5*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\sqrt[3]{a+bx^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{\sqrt[3]{a+bx}}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(-\frac{a}{b\sqrt[3]{a+bx}}+\frac{(a+bx)^{2/3}}{b}\right)dx, x, x^3\right)\\ &= -\frac{a\left(a+bx^3\right)^{2/3}}{2b^2}+\frac{\left(a+bx^3\right)^{5/3}}{5b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.74

$$\frac{\left(a+bx^3\right)^{2/3}\left(2bx^3-3a\right)}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^3)^(1/3),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(-3*a + 2*b*x^3))/(10*b^2)
```

fricas [A] time = 0.69, size = 24, normalized size = 0.63

$$\frac{\left(2\,bx^3-3\,a\right)\left(bx^3+a\right)^{\frac{2}{3}}}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(1/3),x, algorithm="fricas")
[Out] 1/10*(2*b*x^3 - 3*a)*(b*x^3 + a)^(2/3)/b^2
```

giac [A] time = 0.17, size = 30, normalized size = 0.79

$$\frac{\left(bx^3+a\right)^{\frac{5}{3}}}{5\,b^2}-\frac{\left(bx^3+a\right)^{\frac{2}{3}}a}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(1/3),x, algorithm="giac")
[Out] 1/5*(b*x^3 + a)^(5/3)/b^2 - 1/2*(b*x^3 + a)^(2/3)*a/b^2
```

maple [A] time = 0.01, size = 25, normalized size = 0.66

$$-\frac{\left(b\,x^3+a\right)^{\frac{2}{3}}\left(-2b\,x^3+3a\right)}{10b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a)^(1/3),x)
[Out] -1/10*(b*x^3+a)^(2/3)*(-2*b*x^3+3*a)/b^2
```

maxima [A] time = 1.32, size = 30, normalized size = 0.79

$$\frac{\left(bx^3+a\right)^{\frac{5}{3}}}{5\,b^2}-\frac{\left(bx^3+a\right)^{\frac{2}{3}}a}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(1/3),x, algorithm="maxima")
[Out] 1/5*(b*x^3 + a)^(5/3)/b^2 - 1/2*(b*x^3 + a)^(2/3)*a/b^2
```

mupad [B] time = 1.07, size = 24, normalized size = 0.63

$$-\frac{\left(b\,x^3+a\right)^{2/3}\left(3\,a-2\,b\,x^3\right)}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^3)^(1/3),x)
[Out] -((a + b*x^3)^(2/3)*(3*a - 2*b*x^3))/(10*b^2)
```

sympy [A] time = 1.40, size = 44, normalized size = 1.16

$$\begin{cases} -\frac{3a\left(a+bx^3\right)^{\frac{2}{3}}}{10b^2}+\frac{x^3\left(a+bx^3\right)^{\frac{2}{3}}}{5b} & \text{for } b\neq 0 \\ \frac{x^6}{6\sqrt[3]{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)**(1/3),x)
```

```
[Out] Piecewise((-3*a*(a + b*x**3)**(2/3)/(10*b**2) + x**3*(a + b*x**3)**(2/3)/(5
*b), Ne(b, 0)), (x**6/(6*a**(1/3)), True))
```

3.547 $\int \frac{x^2}{\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=18

$$\frac{\left(a+bx^3\right)^{2/3}}{2b}$$

```
[Out] 1/2*(b*x^3+a)^(2/3)/b
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$\frac{\left(a+bx^3\right)^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^3)^(1/3),x]
```

```
[Out] (a + b*x^3)^(2/3)/(2*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^2}{\sqrt[3]{a+bx^3}}\,dx = \frac{\left(a+bx^3\right)^{2/3}}{2b}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{\left(a+bx^3\right)^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^3)^(1/3),x]
```

```
[Out] (a + b*x^3)^(2/3)/(2*b)
```

fricas [A] time = 0.46, size = 14, normalized size = 0.78

$$\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^3 + a)^(2/3)/b
```

giac [A] time = 0.17, size = 14, normalized size = 0.78

$$\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(1/3),x, algorithm="giac")
[Out] 1/2*(b*x^3 + a)^(2/3)/b
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$\frac{\left(b\,x^3+a\right)^{\frac{2}{3}}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)^(1/3),x)
[Out] 1/2*(b*x^3+a)^(2/3)/b
```

maxima [A] time = 1.31, size = 14, normalized size = 0.78

$$\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(1/3),x, algorithm="maxima")
[Out] 1/2*(b*x^3 + a)^(2/3)/b
```

mupad [B] time = 1.08, size = 14, normalized size = 0.78

$$\frac{\left(b\,x^3+a\right)^{2/3}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^3)^(1/3),x)
[Out] (a + b*x^3)^(2/3)/(2*b)
```

sympy [A] time = 0.49, size = 22, normalized size = 1.22

$$\begin{cases} \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{2b} & \text{for } b \neq 0 \\ \frac{x^3}{3\sqrt[3]{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)**(1/3),x)
[Out] Piecewise(((a + b*x**3)**(2/3)/(2*b), Ne(b, 0)), (x**3/(3*a**(1/3)), True))
```

3.548 $\int \frac{1}{x\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=83

$$\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}$$

```
[Out] -1/2*ln(x)/a^(1/3)+1/2*ln(a^(1/3)-(b*x^3+a)^(1/3))/a^(1/3)+1/3*arctan(1/3*(
a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/a^(1/3)*3^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {266, 55, 617, 204, 31}

$$\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3)^(1/3)),x]
```

```
[Out] ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(1/3))
 - Log[x]/(2*a^(1/3)) + Log[a^(1/3) - (a + b*x^3)^(1/3)]/(2*a^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{a+bx^3}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^3\right)$$

$$= -\frac{\log(x)}{2\sqrt[3]{a}} + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{a^{2/3} + \sqrt[3]{a}\,x + x^2}\,dx, x, \sqrt[3]{a+bx^3}\right) - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx}\right.}{2\sqrt[3]{a}}$$

$$= -\frac{\log(x)}{2\sqrt[3]{a}} + \frac{\log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 + \frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}}$$

$$= \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{a}} - \frac{\log(x)}{2\sqrt[3]{a}} + \frac{\log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}$$

Mathematica [A] time = 0.02, size = 70, normalized size = 0.84

$$\frac{3\log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^3}\right) + 2\sqrt{3}\,\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right) - 3\log(x)}{6\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 3*Log[x] +
 3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(6*a^(1/3))
```

fricas [A] time = 0.64, size = 236, normalized size = 2.84

$$\left[\frac{3\sqrt{\frac{1}{3}}\,a\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\,\log\left(\frac{2\,bx^3+3\sqrt{\frac{1}{3}}\left(2\left(bx^3+a\right)^{\frac{2}{3}}a^{\frac{2}{3}}-\left(bx^3+a\right)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{2}{3}}+3\,a}{x^3}\right) - a^{\frac{2}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}} + \left(bx^3+a\right.\right.\right.}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*sqrt(-1/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 +
 a)^(2/3)*a^(2/3) - (b*x^3 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*
x^3 + a)^(1/3)*a^(2/3) + 3*a)/x^3) - a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3
 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3))
)/a, 1/6*(6*sqrt(1/3)*a^(2/3)*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3) + a^(1/
3))/a^(1/3)) - a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) +
a^(2/3)) + 2*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)))/a]
```

giac [A] time = 0.72, size = 87, normalized size = 1.05

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{1}{3}}} - \frac{\log\left(\left(bx^3+a\right)^{\frac{2}{3}} + \left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{2}{3}}\right)}{6\,a^{\frac{1}{3}}} + \frac{\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}} - a^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(
1/3) - 1/6*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/a^(
1/3) + 1/3*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/a^(1/3)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{1}{3}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)^(1/3),x)
```

```
[Out] int(1/x/(b*x^3+a)^(1/3),x)
```

maxima [A] time = 3.02, size = 86, normalized size = 1.04

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{1}{3}}}-\frac{\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{1}{3}}}+\frac{\log\left(\left(bx^3+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(
1/3) - 1/6*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/a^(
1/3) + 1/3*log((b*x^3 + a)^(1/3) - a^(1/3))/a^(1/3)
```

mupad [B] time = 1.20, size = 100, normalized size = 1.20

$$\frac{\ln\left(\left(b\,x^3+a\right)^{1/3}-a^{1/3}\right)}{3\,a^{1/3}}+\frac{\ln\left(\left(b\,x^3+a\right)^{1/3}-\frac{a^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{1/3}}-\frac{\ln\left(\left(b\,x^3+a\right)^{1/3}-\frac{a^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(1+\right.}{6\,a^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3)^(1/3)),x)
```

```
[Out] log((a + b*x^3)^(1/3) - a^(1/3))/(3*a^(1/3)) + (log((a + b*x^3)^(1/3) - (a^
(1/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(6*a^(1/3)) - (log((a + b*x^
3)^(1/3) - (a^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/(6*a^(1/3))
```

sympy [C] time = 1.11, size = 37, normalized size = 0.45

$$-\frac{\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\right|\frac{ae^{i\pi}}{bx^3}\right)}{3\sqrt[3]{b}\,x\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)**(1/3),x)
```

```
[Out] -gamma(1/3)*hyper((1/3, 1/3), (4/3,), a*exp_polar(I*pi)/(b*x**3))/(3*b**(1/
3)*x*gamma(4/3))
```

3.549 $\int \frac{1}{x^4 \sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=110

$$-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{4/3}}-\frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}}+\frac{b\log(x)}{6a^{4/3}}-\frac{\left(a+bx^3\right)^{2/3}}{3ax^3}$$

```
[Out] -1/3*(b*x^3+a)^(2/3)/a/x^3+1/6*b*ln(x)/a^(4/3)-1/6*b*ln(a^(1/3)-(b*x^3+a)^(
1/3))/a^(4/3)-1/9*b*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))
/a^(4/3)*3^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {266, 51, 55, 617, 204, 31}

$$-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{4/3}}-\frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}}+\frac{b\log(x)}{6a^{4/3}}-\frac{\left(a+bx^3\right)^{2/3}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3)^(1/3)),x]
```

```
[Out] -(a + b*x^3)^(2/3)/(3*a*x^3) - (b*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(S
qrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)) + (b*Log[x])/(6*a^(4/3)) - (b*Log[a^(
1/3) - (a + b*x^3)^(1/3)])/(6*a^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4 \sqrt[3]{a+bx^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^2 \sqrt[3]{a+bx}}\,dx, x, x^3\right) \\
&= -\frac{\left(a+bx^3\right)^{2/3}}{3ax^3} - \frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^3\right)}{9a} \\
&= -\frac{\left(a+bx^3\right)^{2/3}}{3ax^3} + \frac{b\log(x)}{6a^{4/3}} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{6a^{4/3}} - \frac{b\,\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx,\right.}{6a} \\
&= -\frac{\left(a+bx^3\right)^{2/3}}{3ax^3} + \frac{b\log(x)}{6a^{4/3}} - \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{4/3}} + \frac{b\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{3a^{4/3}} \\
&= -\frac{\left(a+bx^3\right)^{2/3}}{3ax^3} - \frac{b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{4/3}} + \frac{b\log(x)}{6a^{4/3}} - \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{4/3}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.34

$$\frac{b\left(a+bx^3\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{bx^3}{a}+1\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3)^(1/3)),x]
```

```
[Out] (b*(a + b*x^3)^(2/3)*Hypergeometric2F1[2/3, 2, 5/3, 1 + (b*x^3)/a])/(2*a^2)
```

fricas [A] time = 0.71, size = 344, normalized size = 3.13

$$\left[\frac{3\sqrt{\frac{1}{3}}\,abx^3\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\,\log\left(\frac{2bx^3-3\sqrt{\frac{1}{3}}\left(2\left(bx^3+a\right)^{\frac{2}{3}}(-a)^{\frac{2}{3}}-\left(bx^3+a\right)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3\left(bx^3+a\right)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+3a}{x^3}\right)+(-a)^{\frac{2}{3}}\,bx^3\log\left(\left(bx^3\right.\right.}{18\,a^2x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*a*b*x^3*sqrt((-a)^(1/3)/a)*log((2*b*x^3 - 3*sqrt(1/3)*(2
*(b*x^3 + a)^(2/3)*(-a)^(2/3) - (b*x^3 + a)^(1/3)*a + (-a)^(1/3)*a)*sqrt((-
```

```
a)^(1/3)/a) - 3*(b*x^3 + a)^(1/3)*(-a)^(2/3) + 3*a)/x^3) + (-a)^(2/3)*b*x^3
*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(-a
)^(2/3)*b*x^3*log((b*x^3 + a)^(1/3) + (-a)^(1/3)) - 6*(b*x^3 + a)^(2/3)*a)/
(a^2*x^3), -1/18*(6*sqrt(1/3)*a*b*x^3*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*
(2*(b*x^3 + a)^(1/3) - (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) - (-a)^(2/3)*b*x^3*
log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) + 2*(-a)
^(2/3)*b*x^3*log((b*x^3 + a)^(1/3) + (-a)^(1/3)) + 6*(b*x^3 + a)^(2/3)*a)/(
a^2*x^3)]
```

giac [A] time = 0.71, size = 119, normalized size = 1.08

$$-\frac{\frac{2\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}}-\frac{b^2\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{4}{3}}}+\frac{2\,b^2\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{4}{3}}}+\frac{6\left(bx^3+a\right)^{\frac{2}{3}}b}{ax^3}}{18\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] -1/18*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(
1/3))/a^(4/3) - b^2*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(
2/3))/a^(4/3) + 2*b^2*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/a^(4/3) + 6*(b*
x^3 + a)^(2/3)*b/(a*x^3))/b
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{1}{3}}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)^(1/3),x)
```

```
[Out] int(1/x^4/(b*x^3+a)^(1/3),x)
```

maxima [A] time = 3.03, size = 118, normalized size = 1.07

$$-\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{4}{3}}}-\frac{\left(bx^3+a\right)^{\frac{2}{3}}b}{3\left(\left(bx^3+a\right)a-a^2\right)}+\frac{b\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{\frac{4}{3}}}-\frac{b\log\left(\left(bx^3\right.\right.}{9\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(4/3) - 1/3*(b*x^3 + a)^(2/3)*b/((b*x^3 + a)*a - a^2) + 1/18*b*log((b*x^3
 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/a^(4/3) - 1/9*b*log((b*x
^3 + a)^(1/3) - a^(1/3))/a^(4/3)
```

mupad [B] time = 1.34, size = 138, normalized size = 1.25

$$-\frac{b\,\ln\left(\left(b\,x^3+a\right)^{1/3}-a^{1/3}\right)}{9\,a^{4/3}}-\frac{\left(b\,x^3+a\right)^{2/3}}{3\,a\,x^3}+\frac{\ln\left(\frac{\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)^2}{36\,a^{5/3}}-\frac{b^2\left(b\,x^3+a\right)^{1/3}}{9\,a^2}\right)\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{18\,a^{4/3}}+\frac{\ln\left(\frac{\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)^2}{36\,a^{5/3}}-\frac{b^2\left(b\,x\right.}{9}\right.}{18\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3)^(1/3)),x)
```

```
[Out] (log((b - 3^(1/2)*b*1i)^2/(36*a^(5/3)) - (b^2*(a + b*x^3)^(1/3))/(9*a^2))*(
b - 3^(1/2)*b*1i))/(18*a^(4/3)) - (a + b*x^3)^(2/3)/(3*a*x^3) - (b*log((a +
 b*x^3)^(1/3) - a^(1/3)))/(9*a^(4/3)) + (log((b + 3^(1/2)*b*1i)^2/(36*a^(5/
3)) - (b^2*(a + b*x^3)^(1/3))/(9*a^2))*(b + 3^(1/2)*b*1i))/(18*a^(4/3))
```

sympy [C] time = 1.81, size = 39, normalized size = 0.35

$$-\frac{\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3}, \frac{4}{3} \\ \frac{7}{3}\end{matrix}\right| \frac{ae^{i\pi}}{bx^3}\right)}{3\sqrt[3]{b}\,x^4\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)**(1/3),x)
```

```
[Out] -gamma(4/3)*hyper((1/3, 4/3), (7/3,), a*exp_polar(I*pi)/(b*x**3))/(3*b**(1/
3)*x**4*gamma(7/3))
```

3.550 $\int \frac{x^7}{\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=38

$$\frac{x^8\left(a+bx^3\right)^{2/3}\,{}_2F_1\left(1,\frac{10}{3};\frac{11}{3};-\frac{bx^3}{a}\right)}{8a}$$

```
[Out] 1/8*x^8*(b*x^3+a)^(2/3)*hypergeom([1, 10/3],[11/3],-b*x^3/a)/a
```

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.34, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {365, 364}

$$\frac{x^8\sqrt[3]{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{8}{3};\frac{11}{3};-\frac{bx^3}{a}\right)}{8\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^3)^(1/3),x]
```

```
[Out] (x^8*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 8/3, 11/3, -((b*x^3)/a)])
/(8*(a + b*x^3)^(1/3))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{\sqrt[3]{a+bx^3}}\,dx &= \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x^7}{\sqrt[3]{1+\frac{bx^3}{a}}}\,dx}{\sqrt[3]{a+bx^3}}\\ &= \frac{x^8\sqrt[3]{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{8}{3};\frac{11}{3};-\frac{bx^3}{a}\right)}{8\sqrt[3]{a+bx^3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 1.34

$$\frac{x^8\sqrt[3]{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{8}{3};\frac{11}{3};-\frac{bx^3}{a}\right)}{8\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^3)^(1/3),x]
```

```
[Out] (x^8*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 8/3, 11/3, -((b*x^3)/a)])
/(8*(a + b*x^3)^(1/3))
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^7}{\left(bx^3+a\right)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(x^7/(b*x^3 + a)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(bx^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(x^7/(b*x^3 + a)^(1/3), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(b\,x^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^3+a)^(1/3),x)
```

```
[Out] int(x^7/(b*x^3+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(bx^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(x^7/(b*x^3 + a)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^7}{\left(b\,x^3+a\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^3)^(1/3),x)
```

```
[Out] int(x^7/(a + b*x^3)^(1/3), x)
```

sympy [C] time = 1.85, size = 37, normalized size = 0.97

$$\frac{x^8 \Gamma\left(\frac{8}{3}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{3}, \frac{8}{3} \\ \frac{11}{3}\end{matrix}\right| \frac{b x^3 e^{i\pi}}{a}\right)}{3 \sqrt[3]{a} \, \Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**3+a)**(1/3),x)
```

```
[Out] x**8*gamma(8/3)*hyper((1/3, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**
(1/3)*gamma(11/3))
```

3.551 $\int \frac{x^4}{\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=38

$$\frac{x^5\left(a+bx^3\right)^{2/3}\,{}_2F_1\left(1,\frac{7}{3};\frac{8}{3};-\frac{bx^3}{a}\right)}{5a}$$

```
[Out] 1/5*x^5*(b*x^3+a)^(2/3)*hypergeom([1, 7/3],[8/3],-b*x^3/a)/a
```

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.34, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {365, 364}

$$\frac{x^5\sqrt[3]{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{5}{3};\frac{8}{3};-\frac{bx^3}{a}\right)}{5\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^3)^(1/3),x]
```

```
[Out] (x^5*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 5/3, 8/3, -((b*x^3)/a)])/
(5*(a + b*x^3)^(1/3))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt[3]{a+bx^3}}\,dx &= \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x^4}{\sqrt[3]{1+\frac{bx^3}{a}}}\,dx}{\sqrt[3]{a+bx^3}}\\ &= \frac{x^5\sqrt[3]{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{5}{3};\frac{8}{3};-\frac{bx^3}{a}\right)}{5\sqrt[3]{a+bx^3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 1.34

$$\frac{x^5\sqrt[3]{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{5}{3};\frac{8}{3};-\frac{bx^3}{a}\right)}{5\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^3)^(1/3),x]
```

```
[Out] (x^5*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 5/3, 8/3, -((b*x^3)/a)])/
(5*(a + b*x^3)^(1/3))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^4}{\left(bx^3+a\right)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(x^4/(b*x^3 + a)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(x^4/(b*x^3 + a)^(1/3), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(b\,x^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a)^(1/3),x)
```

```
[Out] int(x^4/(b*x^3+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(b*x^3 + a)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^4}{\left(b\,x^3+a\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^3)^(1/3),x)
```

```
[Out] int(x^4/(a + b*x^3)^(1/3), x)
```

sympy [C] time = 1.42, size = 37, normalized size = 0.97

$$\frac{x^5 \Gamma\left(\frac{5}{3}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{3}, \frac{5}{3} \\ \frac{8}{3}\end{matrix}\right| \frac{b x^3 e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a)**(1/3),x)
```

```
[Out] x**5*gamma(5/3)*hyper((1/3, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
1/3)*gamma(8/3))
```

3.552 $\int \frac{x}{\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=38

$$\frac{x^2\left(a+bx^3\right)^{2/3}\,{}_2F_1\left(1,\frac{4}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2a}$$

```
[Out] 1/2*x^2*(b*x^3+a)^(2/3)*hypergeom([1, 4/3],[5/3],-b*x^3/a)/a
```

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.34, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {365, 364}

$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^3)^(1/3),x]
```

```
[Out] (x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*x^3)/a)])/
(2*(a + b*x^3)^(1/3))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt[3]{a+bx^3}}\,dx &= \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x}{\sqrt[3]{1+\frac{bx^3}{a}}}\,dx}{\sqrt[3]{a+bx^3}}\\ &= \frac{x^2\sqrt[3]{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2\sqrt[3]{a+bx^3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 1.34

$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^3)^(1/3),x]
```

```
[Out] (x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*x^3)/a)])/
(2*(a + b*x^3)^(1/3))
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{\left(bx^3+a\right)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(x/(b*x^3 + a)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(x/(b*x^3 + a)^(1/3), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(b\,x^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a)^(1/3),x)
```

```
[Out] int(x/(b*x^3+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*x^3 + a)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{\left(b\,x^3+a\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^3)^(1/3),x)
```

```
[Out] int(x/(a + b*x^3)^(1/3), x)
```

sympy [C] time = 1.42, size = 37, normalized size = 0.97

$$\frac{x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3}, \frac{2}{3} \\ \frac{5}{3}\end{matrix}\right| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a)**(1/3),x)
```

```
[Out] x**2*gamma(2/3)*hyper((1/3, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
1/3)*gamma(5/3))
```

3.553 $\int \frac{1}{x^2 \sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=36

$$-\frac{\left(a+bx^3\right)^{2/3}\ {}_2F_1\left(\frac{1}{3},1;\frac{2}{3};-\frac{bx^3}{a}\right)}{ax}$$

```
[Out] -(b*x^3+a)^(2/3)*hypergeom([1/3, 1],[2/3],-b*x^3/a)/a/x
```

Rubi [A] time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.36, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {365, 364}

$$-\frac{\sqrt[3]{\frac{bx^3}{a}+1}\ {}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{2}{3};-\frac{bx^3}{a}\right)}{x\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^3)^(1/3)),x]
```

```
[Out] -(((1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[-1/3, 1/3, 2/3, -((b*x^3)/a)])/(
x*(a + b*x^3)^(1/3)))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt[3]{a+bx^3}}\,dx &= \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{x^2\sqrt[3]{1+\frac{bx^3}{a}}}\,dx}{\sqrt[3]{a+bx^3}}\\ &= -\frac{\sqrt[3]{1+\frac{bx^3}{a}}\ {}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{2}{3};-\frac{bx^3}{a}\right)}{x\sqrt[3]{a+bx^3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.36

$$-\frac{\sqrt[3]{\frac{bx^3}{a}+1}\ {}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{2}{3};-\frac{bx^3}{a}\right)}{x\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^3)^(1/3)),x]
```

```
[Out] -(((1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[-1/3, 1/3, 2/3, -((b*x^3)/a)])/(
x*(a + b*x^3)^(1/3)))
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{bx^5+ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(2/3)/(b*x^5 + a*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*x^2), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{1}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a)^(1/3),x)
```

```
[Out] int(1/x^2/(b*x^3+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*x^2), x)
```

mupad [B] time = 1.22, size = 40, normalized size = 1.11

$$-\frac{\left(\frac{a}{b\,x^3}+1\right)^{1/3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{2}{3};\ \frac{5}{3};\ -\frac{a}{b\,x^3}\right)}{2\,x\left(b\,x^3+a\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^3)^(1/3)),x)
```

```
[Out] -((a/(b*x^3) + 1)^(1/3)*hypergeom([1/3, 2/3], 5/3, -a/(b*x^3)))/(2*x*(a + b
*x^3)^(1/3))
```

sympy [C] time = 1.54, size = 39, normalized size = 1.08

$$\frac{\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3}, \frac{1}{3} \\ \frac{2}{3}\end{matrix}\right| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\, x\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a)**(1/3),x)
```

```
[Out] gamma(-1/3)*hyper((-1/3, 1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3
)*x*gamma(2/3))
```

3.554 $\int \frac{1}{x^5 \sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=38

$$-\frac{\left(a+bx^3\right)^{2/3}\ {}_2F_1\left(-\frac{2}{3},1;-\frac{1}{3};-\frac{bx^3}{a}\right)}{4ax^4}$$

```
[Out] -1/4*(b*x^3+a)^(2/3)*hypergeom([-2/3, 1],[-1/3],-b*x^3/a)/a/x^4
```

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.34, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {365, 364}

$$-\frac{\sqrt[3]{\frac{bx^3}{a}+1}\ {}_2F_1\left(-\frac{4}{3},\frac{1}{3};-\frac{1}{3};-\frac{bx^3}{a}\right)}{4x^4\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^3)^(1/3)),x]
```

```
[Out] -((1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[-4/3, 1/3, -1/3, -((b*x^3)/a)])/(
4*x^4*(a + b*x^3)^(1/3))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5 \sqrt[3]{a+bx^3}}\,dx &= \frac{\sqrt[3]{1+\frac{bx^3}{a}} \int \frac{1}{x^5 \sqrt[3]{1+\frac{bx^3}{a}}}\,dx}{\sqrt[3]{a+bx^3}} \\ &= -\frac{\sqrt[3]{1+\frac{bx^3}{a}}\ {}_2F_1\left(-\frac{4}{3},\frac{1}{3};-\frac{1}{3};-\frac{bx^3}{a}\right)}{4x^4\sqrt[3]{a+bx^3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 1.34

$$-\frac{\sqrt[3]{\frac{bx^3}{a}+1}\ {}_2F_1\left(-\frac{4}{3},\frac{1}{3};-\frac{1}{3};-\frac{bx^3}{a}\right)}{4x^4\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^3)^(1/3)),x]
```

```
[Out] -1/4*((1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[-4/3, 1/3, -1/3, -((b*x^3)/a)
])/(x^4*(a + b*x^3)^(1/3))
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{bx^8+ax^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(2/3)/(b*x^8 + a*x^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*x^5), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{1}{3}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^3+a)^(1/3),x)
```

```
[Out] int(1/x^5/(b*x^3+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x^5\left(b\,x^3+a\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^3)^(1/3)),x)
```

```
[Out] int(1/(x^5*(a + b*x^3)^(1/3)), x)
```

sympy [C] time = 1.23, size = 44, normalized size = 1.16

$$\frac{\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{4}{3}, \frac{1}{3} \\ -\frac{1}{3}\end{matrix}\right| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\, x^4 \Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**3+a)**(1/3),x)
```

```
[Out] gamma(-4/3)*hyper((-4/3, 1/3), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/
3)*x**4*gamma(-1/3))
```

3.555 $\int \frac{x^3}{\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=94

$$\frac{a\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{6b^{4/3}}-\frac{a\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{4/3}}+\frac{x\left(a+bx^3\right)^{2/3}}{3b}$$

```
[Out] 1/3*x*(b*x^3+a)^(2/3)/b+1/6*a*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(4/3)-1/9*a*
arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(4/3)*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {321, 239}

$$\frac{a\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{6b^{4/3}}-\frac{a\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{4/3}}+\frac{x\left(a+bx^3\right)^{2/3}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^3)^(1/3),x]
```

```
[Out] (x*(a + b*x^3)^(2/3))/(3*b) - (a*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3
))/Sqrt[3]])/(3*Sqrt[3]*b^(4/3)) + (a*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]
)/(6*b^(4/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt[3]{a+bx^3}}\,dx &= \frac{x\left(a+bx^3\right)^{2/3}}{3b}-\frac{a\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{3b}\\ &= \frac{x\left(a+bx^3\right)^{2/3}}{3b}-\frac{a\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{4/3}}+\frac{a\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)}{6b^{4/3}}\end{aligned}$$

Mathematica [A] time = 0.11, size = 134, normalized size = 1.43

$$\frac{-a\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1\right)+6\sqrt[3]{b}\,x\left(a+bx^3\right)^{2/3}+2a\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)-2\sqrt{3}\,a\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{18b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^3)^(1/3),x]
```

```
[Out] (6*b^(1/3)*x*(a + b*x^3)^(2/3) - 2*Sqrt[3]*a*ArcTan[(1 + (2*b^(1/3)*x)/(a +
 b*x^3)^(1/3))/Sqrt[3]] + 2*a*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)] - a*Lo
g[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(18
*b^(4/3))
```

fricas [A] time = 0.67, size = 309, normalized size = 3.29

$$\left[\frac{3\sqrt{\frac{1}{3}}\,ab\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3bx^3-3\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3+\left(bx^3+a\right)^{\frac{1}{3}}bx^2-2\left(bx^3+a\right)^{\frac{2}{3}}b^{\frac{2}{3}}x\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}+2a\right.}{18\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*a*b*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b
^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3
 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 6*(b*x^3 + a)^(2/3)*b*x + 2
*a*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - a*b^(2/3)*log((b^(2/3)
*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/b^2, 1/18*(6*
sqrt(1/3)*a*b^(2/3)*arctan(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(
1/3)*x)) + 6*(b*x^3 + a)^(2/3)*b*x + 2*a*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 +
 a)^(1/3))/x) - a*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x +
(b*x^3 + a)^(2/3))/x^2))/b^2]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(bx^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(x^3/(b*x^3 + a)^(1/3), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(b\,x^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a)^(1/3),x)
```

```
[Out] int(x^3/(b*x^3+a)^(1/3),x)
```

maxima [A] time = 2.96, size = 137, normalized size = 1.46

$$\frac{\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{9\,b^{\frac{4}{3}}}-\frac{a\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{18\,b^{\frac{4}{3}}}+\frac{a\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{9\,b^{\frac{4}{3}}}-\frac{\left(bx^3+a\right)^{\frac{2}{3}}a}{3\left(b^2-\frac{\left(bx^3+a\right)b}{x^3}\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))
/b^(4/3) - 1/18*a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(
2/3)/x^2)/b^(4/3) + 1/9*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) - 1/3
*(b*x^3 + a)^(2/3)*a/((b^2 - (b*x^3 + a)*b/x^3)*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\left(b\,x^3+a\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^3)^(1/3),x)
```

```
[Out] int(x^3/(a + b*x^3)^(1/3), x)
```

sympy [C] time = 1.33, size = 37, normalized size = 0.39

$$\frac{x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\,\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a)**(1/3),x)
```

```
[Out] x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
1/3)*gamma(7/3))
```

3.556 $\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=70

$$\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2\sqrt[3]{b}}$$

```
[Out] -1/2*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)+1/3*arctan(1/3*(1+2*b^(1/3)*x/(
b*x^3+a)^(1/3))*3^(1/2))/b^(1/3)*3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {239}

$$\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(-1/3),x]
```

```
[Out] ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - L
og[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx=\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}-\frac{\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}$$

Mathematica [A] time = 0.02, size = 110, normalized size = 1.57

$$\frac{\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1\right)-2\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\,\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{6\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(-1/3),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]] - 2*Log[1
- (b^(1/3)*x)/(a + b*x^3)^(1/3)] + Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3)
+ (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*b^(1/3))
```

fricas [B] time = 0.68, size = 330, normalized size = 4.71

$$\left[\frac{3\sqrt{\frac{1}{3}}\,b\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\,\log\left(3\,bx^3-3\left(bx^3+a\right)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}bx^3-\left(bx^3+a\right)^{\frac{1}{3}}bx^2+2\left(bx^3+a\right)^{\frac{2}{3}}(-b)^{\frac{2}{3}}x\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\right)}{6\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*b*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-
b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*
(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 2*(-b)^(2/3)*lo
g(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + (-b)^(2/3)*log(((-b)^(2/3)*x^2 -
(b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/b, -1/6*(6*sqrt(1
/3)*b*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(
1/3))*sqrt(-(-b)^(1/3)/b)/x) + 2*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)
^(1/3))/x) - (-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*
x + (b*x^3 + a)^(2/3))/x^2))/b]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(-1/3), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(1/3),x)
```

```
[Out] int(1/(b*x^3+a)^(1/3),x)
```

maxima [A] time = 3.03, size = 100, normalized size = 1.43

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{3\,b^{\frac{1}{3}}}+\frac{\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{6\,b^{\frac{1}{3}}}-\frac{\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/
b^(1/3) + 1/6*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)
/x^2)/b^(1/3) - 1/3*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3)
```

mupad [B] time = 1.03, size = 37, normalized size = 0.53

$$\frac{x\left(\frac{b\,x^3}{a}+1\right)^{1/3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{3};\ \frac{4}{3};\ -\frac{b\,x^3}{a}\right)}{\left(b\,x^3+a\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^3)^(1/3),x)
```

```
[Out] (x*((b*x^3)/a + 1)^(1/3)*hypergeom([1/3, 1/3], 4/3, -(b*x^3)/a))/(a + b*x^3
)^(1/3)
```

sympy [C] time = 1.80, size = 36, normalized size = 0.51

$$\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(1/3),x)
```

```
[Out] x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3
)*gamma(4/3))
```

3.557 $\int \frac{1}{x^3 \sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=21

$$-\frac{\left(a+bx^3\right)^{2/3}}{2ax^2}$$

```
[Out] -1/2*(b*x^3+a)^(2/3)/a/x^2
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {264}

$$-\frac{\left(a+bx^3\right)^{2/3}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^3)^(1/3)),x]
```

```
[Out] -(a + b*x^3)^(2/3)/(2*a*x^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^3 \sqrt[3]{a+bx^3}}\,dx = -\frac{\left(a+bx^3\right)^{2/3}}{2ax^2}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$-\frac{\left(a+bx^3\right)^{2/3}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^3)^(1/3)),x]
```

```
[Out] -1/2*(a + b*x^3)^(2/3)/(a*x^2)
```

fricas [A] time = 0.66, size = 17, normalized size = 0.81

$$-\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/2*(b*x^3 + a)^(2/3)/(a*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(1/3),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(1/3)*x^3), x)
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$-\frac{\left(b\,x^3+a\right)^{\frac{2}{3}}}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a)^(1/3),x)
[Out] -1/2*(b*x^3+a)^(2/3)/a/x^2
```

maxima [A] time = 1.34, size = 17, normalized size = 0.81

$$-\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(1/3),x, algorithm="maxima")
[Out] -1/2*(b*x^3 + a)^(2/3)/(a*x^2)
```

mupad [B] time = 1.03, size = 17, normalized size = 0.81

$$-\frac{\left(b\,x^3+a\right)^{2/3}}{2\,a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^3)^(1/3)),x)
[Out] -(a + b*x^3)^(2/3)/(2*a*x^2)
```

sympy [A] time = 1.29, size = 31, normalized size = 1.48

$$\frac{b^{\frac{2}{3}}\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{2}{3}\right)}{3a\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a)**(1/3),x)
[Out] b**(2/3)*(a/(b*x**3) + 1)**(2/3)*gamma(-2/3)/(3*a*gamma(1/3))
```

3.558 $\int \frac{1}{x^6 \sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=44

$$\frac{3b\left(a+bx^3\right)^{2/3}}{10a^2x^2}-\frac{\left(a+bx^3\right)^{2/3}}{5ax^5}$$

```
[Out] -1/5*(b*x^3+a)^(2/3)/a/x^5+3/10*b*(b*x^3+a)^(2/3)/a^2/x^2
```

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$\frac{3b\left(a+bx^3\right)^{2/3}}{10a^2x^2}-\frac{\left(a+bx^3\right)^{2/3}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^3)^(1/3)),x]
```

```
[Out] -(a + b*x^3)^(2/3)/(5*a*x^5) + (3*b*(a + b*x^3)^(2/3))/(10*a^2*x^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^6 \sqrt[3]{a+bx^3}}\,dx &= -\frac{\left(a+bx^3\right)^{2/3}}{5ax^5}-\frac{(3b)\int \frac{1}{x^3 \sqrt[3]{a+bx^3}}\,dx}{5a}\\ &= -\frac{\left(a+bx^3\right)^{2/3}}{5ax^5}+\frac{3b\left(a+bx^3\right)^{2/3}}{10a^2x^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.70

$$\frac{\left(a+bx^3\right)^{2/3}\left(3bx^3-2a\right)}{10a^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^3)^(1/3)),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(-2*a + 3*b*x^3))/(10*a^2*x^5)
```

fricas [A] time = 0.63, size = 27, normalized size = 0.61

$$\frac{\left(3\,bx^3-2\,a\right)\left(bx^3+a\right)^{\frac{2}{3}}}{10\,a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/10*(3*b*x^3 - 2*a)*(b*x^3 + a)^(2/3)/(a^2*x^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}} x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*x^6), x)
```

maple [A] time = 0.00, size = 28, normalized size = 0.64

$$-\frac{\left(b\,x^3+a\right)^{\frac{2}{3}}\left(-3b\,x^3+2a\right)}{10a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^3+a)^(1/3),x)
```

```
[Out] -1/10*(b*x^3+a)^(2/3)*(-3*b*x^3+2*a)/a^2/x^5
```

maxima [A] time = 1.35, size = 35, normalized size = 0.80

$$\frac{\frac{5\left(bx^3+a\right)^{\frac{2}{3}}b}{x^2}-\frac{2\left(bx^3+a\right)^{\frac{5}{3}}}{x^5}}{10\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/10*(5*(b*x^3 + a)^(2/3)*b/x^2 - 2*(b*x^3 + a)^(5/3)/x^5)/a^2
```

mupad [B] time = 1.08, size = 27, normalized size = 0.61

$$-\frac{\left(b\,x^3+a\right)^{2/3}\left(2\,a-3\,b\,x^3\right)}{10\,a^2\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^3)^(1/3)),x)
```

```
[Out] -((a + b*x^3)^(2/3)*(2*a - 3*b*x^3))/(10*a^2*x^5)
```

sympy [A] time = 2.03, size = 70, normalized size = 1.59

$$-\frac{2b^{\frac{2}{3}}\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{9ax^3\Gamma\left(\frac{1}{3}\right)}+\frac{b^{\frac{5}{3}}\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{3a^2\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**3+a)**(1/3),x)
```

```
[Out] -2*b**(2/3)*(a/(b*x**3) + 1)**(2/3)*gamma(-5/3)/(9*a*x**3*gamma(1/3)) + b**
(5/3)*(a/(b*x**3) + 1)**(2/3)*gamma(-5/3)/(3*a**2*gamma(1/3))
```

3.559 $\int \frac{1}{x^9 \sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=68

$$-\frac{9b^2\left(a+bx^3\right)^{2/3}}{40a^3x^2}+\frac{3b\left(a+bx^3\right)^{2/3}}{20a^2x^5}-\frac{\left(a+bx^3\right)^{2/3}}{8ax^8}$$

```
[Out] -1/8*(b*x^3+a)^(2/3)/a/x^8+3/20*b*(b*x^3+a)^(2/3)/a^2/x^5-9/40*b^2*(b*x^3+a
)^(2/3)/a^3/x^2
```

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$-\frac{9b^2\left(a+bx^3\right)^{2/3}}{40a^3x^2}+\frac{3b\left(a+bx^3\right)^{2/3}}{20a^2x^5}-\frac{\left(a+bx^3\right)^{2/3}}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(a + b*x^3)^(1/3)),x]
```

```
[Out] -(a + b*x^3)^(2/3)/(8*a*x^8) + (3*b*(a + b*x^3)^(2/3))/(20*a^2*x^5) - (9*b^
2*(a + b*x^3)^(2/3))/(40*a^3*x^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^9 \sqrt[3]{a+bx^3}}\,dx &= -\frac{\left(a+bx^3\right)^{2/3}}{8ax^8}-\frac{(3b)\int \frac{1}{x^6 \sqrt[3]{a+bx^3}}\,dx}{4a}\\ &= -\frac{\left(a+bx^3\right)^{2/3}}{8ax^8}+\frac{3b\left(a+bx^3\right)^{2/3}}{20a^2x^5}+\frac{\left(9b^2\right)\int \frac{1}{x^3 \sqrt[3]{a+bx^3}}\,dx}{20a^2}\\ &= -\frac{\left(a+bx^3\right)^{2/3}}{8ax^8}+\frac{3b\left(a+bx^3\right)^{2/3}}{20a^2x^5}-\frac{9b^2\left(a+bx^3\right)^{2/3}}{40a^3x^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 42, normalized size = 0.62

$$-\frac{\left(a+bx^3\right)^{2/3}\left(5a^2-6abx^3+9b^2x^6\right)}{40a^3x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^9*(a + b*x^3)^(1/3)),x]
```

```
[Out] -1/40*((a + b*x^3)^(2/3)*(5*a^2 - 6*a*b*x^3 + 9*b^2*x^6))/(a^3*x^8)
```

fricas [A] time = 0.76, size = 38, normalized size = 0.56

$$-\frac{\left(9\,b^2x^6-6\,abx^3+5\,a^2\right)\left(bx^3+a\right)^{\frac{2}{3}}}{40\,a^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/40*(9*b^2*x^6 - 6*a*b*x^3 + 5*a^2)*(b*x^3 + a)^(2/3)/(a^3*x^8)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*x^9), x)
```

maple [A] time = 0.01, size = 39, normalized size = 0.57

$$-\frac{\left(b\,x^3+a\right)^{\frac{2}{3}}\left(9b^2x^6-6ab\,x^3+5a^2\right)}{40a^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(b*x^3+a)^(1/3),x)
```

```
[Out] -1/40*(b*x^3+a)^(2/3)*(9*b^2*x^6-6*a*b*x^3+5*a^2)/x^8/a^3
```

maxima [A] time = 1.31, size = 52, normalized size = 0.76

$$-\frac{\frac{20\left(bx^3+a\right)^{\frac{2}{3}}b^2}{x^2}-\frac{16\left(bx^3+a\right)^{\frac{5}{3}}b}{x^5}+\frac{5\left(bx^3+a\right)^{\frac{8}{3}}}{x^8}}{40\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/40*(20*(b*x^3 + a)^(2/3)*b^2/x^2 - 16*(b*x^3 + a)^(5/3)*b/x^5 + 5*(b*x^3
 + a)^(8/3)/x^8)/a^3
```

mupad [B] time = 1.14, size = 38, normalized size = 0.56

$$-\frac{\left(b\,x^3+a\right)^{2/3}\left(5\,a^2-6\,a\,b\,x^3+9\,b^2\,x^6\right)}{40\,a^3\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(a + b*x^3)^(1/3)),x)
```

```
[Out] -((a + b*x^3)^(2/3)*(5*a^2 + 9*b^2*x^6 - 6*a*b*x^3))/(40*a^3*x^8)
```

sympy [**B**] time = 2.58, size = 406, normalized size = 5.97

$$\frac{10a^4b^{\frac{14}{3}}\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{27a^5b^4x^6\Gamma\left(\frac{1}{3}\right)+54a^4b^5x^9\Gamma\left(\frac{1}{3}\right)+27a^3b^6x^{12}\Gamma\left(\frac{1}{3}\right)}+\frac{8a^3b^{\frac{17}{3}}x^3\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{27a^5b^4x^6\Gamma\left(\frac{1}{3}\right)+54a^4b^5x^9\Gamma\left(\frac{1}{3}\right)+27a^3b^6x^{12}\Gamma\left(\frac{1}{3}\right)}+\frac{}{27a^5b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(b*x**3+a)**(1/3),x)
```

```
[Out] 10*a**4*b**(14/3)*(a/(b*x**3) + 1)**(2/3)*gamma(-8/3)/(27*a**5*b**4*x**6*ga
mma(1/3) + 54*a**4*b**5*x**9*gamma(1/3) + 27*a**3*b**6*x**12*gamma(1/3)) +
8*a**3*b**(17/3)*x**3*(a/(b*x**3) + 1)**(2/3)*gamma(-8/3)/(27*a**5*b**4*x**
6*gamma(1/3) + 54*a**4*b**5*x**9*gamma(1/3) + 27*a**3*b**6*x**12*gamma(1/3)
) + 4*a**2*b**(20/3)*x**6*(a/(b*x**3) + 1)**(2/3)*gamma(-8/3)/(27*a**5*b**4
*x**6*gamma(1/3) + 54*a**4*b**5*x**9*gamma(1/3) + 27*a**3*b**6*x**12*gamma(
1/3)) + 24*a*b**(23/3)*x**9*(a/(b*x**3) + 1)**(2/3)*gamma(-8/3)/(27*a**5*b*
*4*x**6*gamma(1/3) + 54*a**4*b**5*x**9*gamma(1/3) + 27*a**3*b**6*x**12*gamm
a(1/3)) + 18*b**(26/3)*x**12*(a/(b*x**3) + 1)**(2/3)*gamma(-8/3)/(27*a**5*b
**4*x**6*gamma(1/3) + 54*a**4*b**5*x**9*gamma(1/3) + 27*a**3*b**6*x**12*gam
ma(1/3))
```

3.560 $\int \frac{1}{x^{12}\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=92

$$\frac{81b^3\left(a+bx^3\right)^{2/3}}{440a^4x^2}-\frac{27b^2\left(a+bx^3\right)^{2/3}}{220a^3x^5}+\frac{9b\left(a+bx^3\right)^{2/3}}{88a^2x^8}-\frac{\left(a+bx^3\right)^{2/3}}{11ax^{11}}$$

```
[Out] -1/11*(b*x^3+a)^(2/3)/a/x^11+9/88*b*(b*x^3+a)^(2/3)/a^2/x^8-27/220*b^2*(b*x
^3+a)^(2/3)/a^3/x^5+81/440*b^3*(b*x^3+a)^(2/3)/a^4/x^2
```

Rubi [A] time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$\frac{81b^3\left(a+bx^3\right)^{2/3}}{440a^4x^2}-\frac{27b^2\left(a+bx^3\right)^{2/3}}{220a^3x^5}+\frac{9b\left(a+bx^3\right)^{2/3}}{88a^2x^8}-\frac{\left(a+bx^3\right)^{2/3}}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^12*(a + b*x^3)^(1/3)),x]
```

```
[Out] -(a + b*x^3)^(2/3)/(11*a*x^11) + (9*b*(a + b*x^3)^(2/3))/(88*a^2*x^8) - (27
*b^2*(a + b*x^3)^(2/3))/(220*a^3*x^5) + (81*b^3*(a + b*x^3)^(2/3))/(440*a^4
*x^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{12}\sqrt[3]{a+bx^3}}\,dx &= -\frac{\left(a+bx^3\right)^{2/3}}{11ax^{11}}-\frac{(9b)\int \frac{1}{x^9\sqrt[3]{a+bx^3}}\,dx}{11a}\\
&= -\frac{\left(a+bx^3\right)^{2/3}}{11ax^{11}}+\frac{9b\left(a+bx^3\right)^{2/3}}{88a^2x^8}+\frac{\left(27b^2\right)\int \frac{1}{x^6\sqrt[3]{a+bx^3}}\,dx}{44a^2}\\
&= -\frac{\left(a+bx^3\right)^{2/3}}{11ax^{11}}+\frac{9b\left(a+bx^3\right)^{2/3}}{88a^2x^8}-\frac{27b^2\left(a+bx^3\right)^{2/3}}{220a^3x^5}-\frac{\left(81b^3\right)\int \frac{1}{x^3\sqrt[3]{a+bx^3}}\,dx}{220a^3}\\
&= -\frac{\left(a+bx^3\right)^{2/3}}{11ax^{11}}+\frac{9b\left(a+bx^3\right)^{2/3}}{88a^2x^8}-\frac{27b^2\left(a+bx^3\right)^{2/3}}{220a^3x^5}+\frac{81b^3\left(a+bx^3\right)^{2/3}}{440a^4x^2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.58

$$\frac{\left(a+bx^3\right)^{2/3}\left(-40a^3+45a^2bx^3-54ab^2x^6+81b^3x^9\right)}{440a^4x^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^12*(a + b*x^3)^(1/3)),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(-40*a^3 + 45*a^2*b*x^3 - 54*a*b^2*x^6 + 81*b^3*x^9))/(4
40*a^4*x^11)
```

fricas [A] time = 0.61, size = 49, normalized size = 0.53

$$\frac{\left(81\,b^3x^9 - 54\,ab^2x^6 + 45\,a^2bx^3 - 40\,a^3\right)\left(bx^3 + a\right)^{\frac{2}{3}}}{440\,a^4x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^12/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/440*(81*b^3*x^9 - 54*a*b^2*x^6 + 45*a^2*b*x^3 - 40*a^3)*(b*x^3 + a)^(2/3)
/(a^4*x^11)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3 + a\right)^{\frac{1}{3}}x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^12/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*x^12), x)
```

maple [A] time = 0.01, size = 50, normalized size = 0.54

$$-\frac{\left(b\,x^3 + a\right)^{\frac{2}{3}}\left(-81b^3x^9 + 54a\,b^2x^6 - 45a^2b\,x^3 + 40a^3\right)}{440a^4x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^12/(b*x^3+a)^(1/3),x)
```

```
[Out] -1/440*(b*x^3+a)^(2/3)*(-81*b^3*x^9+54*a*b^2*x^6-45*a^2*b*x^3+40*a^3)/x^11/
a^4
```

maxima [A] time = 1.29, size = 69, normalized size = 0.75

$$\frac{\frac{220\left(bx^3+a\right)^{\frac{2}{3}}b^3}{x^2} - \frac{264\left(bx^3+a\right)^{\frac{5}{3}}b^2}{x^5} + \frac{165\left(bx^3+a\right)^{\frac{8}{3}}b}{x^8} - \frac{40\left(bx^3+a\right)^{\frac{11}{3}}}{x^{11}}}{440\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^12/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/440*(220*(b*x^3 + a)^(2/3)*b^3/x^2 - 264*(b*x^3 + a)^(5/3)*b^2/x^5 + 165*
(b*x^3 + a)^(8/3)*b/x^8 - 40*(b*x^3 + a)^(11/3)/x^11)/a^4
```

mupad [B] time = 1.20, size = 76, normalized size = 0.83

$$\frac{9\,b\left(b\,x^3 + a\right)^{2/3}}{88\,a^2\,x^8} - \frac{\left(b\,x^3 + a\right)^{2/3}}{11\,a\,x^{11}} + \frac{81\,b^3\left(b\,x^3 + a\right)^{2/3}}{440\,a^4\,x^2} - \frac{27\,b^2\left(b\,x^3 + a\right)^{2/3}}{220\,a^3\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^12*(a + b*x^3)^(1/3)),x)
```

```
[Out] (9*b*(a + b*x^3)^(2/3))/(88*a^2*x^8) - (a + b*x^3)^(2/3)/(11*a*x^11) + (81*
b^3*(a + b*x^3)^(2/3))/(440*a^4*x^2) - (27*b^2*(a + b*x^3)^(2/3))/(220*a^3*
x^5)
```

sympy [**B**] time = 3.44, size = 692, normalized size = 7.52

$$-\frac{80a^6b^{\frac{29}{3}}\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{11}{3}\right)}{81a^7b^9x^9\Gamma\left(\frac{1}{3}\right)+243a^6b^{10}x^{12}\Gamma\left(\frac{1}{3}\right)+243a^5b^{11}x^{15}\Gamma\left(\frac{1}{3}\right)+81a^4b^{12}x^{18}\Gamma\left(\frac{1}{3}\right)}-\frac{150a^5b^{\frac{32}{3}}x}{81a^7b^9x^9\Gamma\left(\frac{1}{3}\right)+243a^6b^{10}x^{12}\Gamma\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**12/(b*x**3+a)**(1/3),x)
```

```
[Out] -80*a**6*b**(29/3)*(a/(b*x**3) + 1)**(2/3)*gamma(-11/3)/(81*a**7*b**9*x**9*
gamma(1/3) + 243*a**6*b**10*x**12*gamma(1/3) + 243*a**5*b**11*x**15*gamma(1
/3) + 81*a**4*b**12*x**18*gamma(1/3)) - 150*a**5*b**(32/3)*x**3*(a/(b*x**3)
 + 1)**(2/3)*gamma(-11/3)/(81*a**7*b**9*x**9*gamma(1/3) + 243*a**6*b**10*x*
*12*gamma(1/3) + 243*a**5*b**11*x**15*gamma(1/3) + 81*a**4*b**12*x**18*gamm
a(1/3)) - 78*a**4*b**(35/3)*x**6*(a/(b*x**3) + 1)**(2/3)*gamma(-11/3)/(81*a
**7*b**9*x**9*gamma(1/3) + 243*a**6*b**10*x**12*gamma(1/3) + 243*a**5*b**11
*x**15*gamma(1/3) + 81*a**4*b**12*x**18*gamma(1/3)) + 28*a**3*b**(38/3)*x**
9*(a/(b*x**3) + 1)**(2/3)*gamma(-11/3)/(81*a**7*b**9*x**9*gamma(1/3) + 243*
a**6*b**10*x**12*gamma(1/3) + 243*a**5*b**11*x**15*gamma(1/3) + 81*a**4*b**
12*x**18*gamma(1/3)) + 252*a**2*b**(41/3)*x**12*(a/(b*x**3) + 1)**(2/3)*gam
ma(-11/3)/(81*a**7*b**9*x**9*gamma(1/3) + 243*a**6*b**10*x**12*gamma(1/3) +
 243*a**5*b**11*x**15*gamma(1/3) + 81*a**4*b**12*x**18*gamma(1/3)) + 378*a*
b**(44/3)*x**15*(a/(b*x**3) + 1)**(2/3)*gamma(-11/3)/(81*a**7*b**9*x**9*gam
ma(1/3) + 243*a**6*b**10*x**12*gamma(1/3) + 243*a**5*b**11*x**15*gamma(1/3)
 + 81*a**4*b**12*x**18*gamma(1/3)) + 162*b**(47/3)*x**18*(a/(b*x**3) + 1)**
(2/3)*gamma(-11/3)/(81*a**7*b**9*x**9*gamma(1/3) + 243*a**6*b**10*x**12*gam
ma(1/3) + 243*a**5*b**11*x**15*gamma(1/3) + 81*a**4*b**12*x**18*gamma(1/3))
```

3.561 $\int \frac{x^{11}}{\left(a+bx^3\right)^{2/3}}\,dx$

Optimal. Leaf size=78

$$-\frac{a^3\sqrt[3]{a+bx^3}}{b^4}+\frac{3a^2\left(a+bx^3\right)^{4/3}}{4b^4}+\frac{\left(a+bx^3\right)^{10/3}}{10b^4}-\frac{3a\left(a+bx^3\right)^{7/3}}{7b^4}$$

```
[Out] -a^3*(b*x^3+a)^(1/3)/b^4+3/4*a^2*(b*x^3+a)^(4/3)/b^4-3/7*a*(b*x^3+a)^(7/3)/
b^4+1/10*(b*x^3+a)^(10/3)/b^4
```

Rubi [A] time = 0.04, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{3a^2\left(a+bx^3\right)^{4/3}}{4b^4}-\frac{a^3\sqrt[3]{a+bx^3}}{b^4}+\frac{\left(a+bx^3\right)^{10/3}}{10b^4}-\frac{3a\left(a+bx^3\right)^{7/3}}{7b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + b*x^3)^(2/3),x]
```

```
[Out] -((a^3*(a + b*x^3)^(1/3))/b^4) + (3*a^2*(a + b*x^3)^(4/3))/(4*b^4) - (3*a*(
a + b*x^3)^(7/3))/(7*b^4) + (a + b*x^3)^(10/3)/(10*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{\left(a+bx^3\right)^{2/3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^3}{(a+bx)^{2/3}}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int\left(-\frac{a^3}{b^3(a+bx)^{2/3}}+\frac{3a^2\sqrt[3]{a+bx}}{b^3}-\frac{3a(a+bx)^{4/3}}{b^3}+\frac{(a+bx)^{7/3}}{b^3}\right)dx, x, x^3\right)\\ &= -\frac{a^3\sqrt[3]{a+bx^3}}{b^4}+\frac{3a^2\left(a+bx^3\right)^{4/3}}{4b^4}-\frac{3a\left(a+bx^3\right)^{7/3}}{7b^4}+\frac{\left(a+bx^3\right)^{10/3}}{10b^4}\end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.64

$$\frac{\sqrt[3]{a+bx^3}\left(-81a^3+27a^2bx^3-18ab^2x^6+14b^3x^9\right)}{140b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + b*x^3)^(2/3),x]
```

```
[Out] ((a + b*x^3)^(1/3)*(-81*a^3 + 27*a^2*b*x^3 - 18*a*b^2*x^6 + 14*b^3*x^9))/(1
40*b^4)
```

fricas [A] time = 0.62, size = 46, normalized size = 0.59

$$\frac{\left(14\,b^3x^9 - 18\,ab^2x^6 + 27\,a^2bx^3 - 81\,a^3\right)\left(bx^3+a\right)^{\frac{1}{3}}}{140\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/140*(14*b^3*x^9 - 18*a*b^2*x^6 + 27*a^2*b*x^3 - 81*a^3)*(b*x^3 + a)^(1/3)
/b^4
```

giac [A] time = 0.16, size = 61, normalized size = 0.78

$$-\frac{\left(bx^3+a\right)^{\frac{1}{3}}a^3}{b^4} + \frac{14\left(bx^3+a\right)^{\frac{10}{3}} - 60\left(bx^3+a\right)^{\frac{7}{3}}a + 105\left(bx^3+a\right)^{\frac{4}{3}}a^2}{140\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] -(b*x^3 + a)^(1/3)*a^3/b^4 + 1/140*(14*(b*x^3 + a)^(10/3) - 60*(b*x^3 + a)^
(7/3)*a + 105*(b*x^3 + a)^(4/3)*a^2)/b^4
```

maple [A] time = 0.01, size = 47, normalized size = 0.60

$$-\frac{\left(b\,x^3+a\right)^{\frac{1}{3}}\left(-14b^3x^9 + 18a\,b^2x^6 - 27a^2b\,x^3 + 81a^3\right)}{140b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^3+a)^(2/3),x)
```

```
[Out] -1/140*(b*x^3+a)^(1/3)*(-14*b^3*x^9+18*a*b^2*x^6-27*a^2*b*x^3+81*a^3)/b^4
```

maxima [A] time = 1.35, size = 64, normalized size = 0.82

$$\frac{\left(bx^3+a\right)^{\frac{10}{3}}}{10\,b^4} - \frac{3\left(bx^3+a\right)^{\frac{7}{3}}a}{7\,b^4} + \frac{3\left(bx^3+a\right)^{\frac{4}{3}}a^2}{4\,b^4} - \frac{\left(bx^3+a\right)^{\frac{1}{3}}a^3}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 1/10*(b*x^3 + a)^(10/3)/b^4 - 3/7*(b*x^3 + a)^(7/3)*a/b^4 + 3/4*(b*x^3 + a)
^(4/3)*a^2/b^4 - (b*x^3 + a)^(1/3)*a^3/b^4
```

mupad [B] time = 1.10, size = 48, normalized size = 0.62

$$-\left(b\,x^3+a\right)^{1/3}\left(\frac{81\,a^3}{140\,b^4} - \frac{x^9}{10\,b} + \frac{9\,a\,x^6}{70\,b^2} - \frac{27\,a^2\,x^3}{140\,b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a + b*x^3)^(2/3),x)
```

```
[Out] -(a + b*x^3)^(1/3)*((81*a^3)/(140*b^4) - x^9/(10*b) + (9*a*x^6)/(70*b^2) -
(27*a^2*x^3)/(140*b^3))
```

sympy [**A**] time = 3.36, size = 92, normalized size = 1.18

$$\begin{cases} -\frac{81a^3\sqrt[3]{a+bx^3}}{140b^4} + \frac{27a^2x^3\sqrt[3]{a+bx^3}}{140b^3} - \frac{9ax^6\sqrt[3]{a+bx^3}}{70b^2} + \frac{x^9\sqrt[3]{a+bx^3}}{10b} & \text{for } b \neq 0 \\ \frac{x^{12}}{12a^{\frac{2}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**3+a)**(2/3),x)
```

```
[Out] Piecewise((-81*a**3*(a + b*x**3)**(1/3)/(140*b**4) + 27*a**2*x**3*(a + b*x*
*3)**(1/3)/(140*b**3) - 9*a*x**6*(a + b*x**3)**(1/3)/(70*b**2) + x**9*(a +
b*x**3)**(1/3)/(10*b), Ne(b, 0)), (x**12/(12*a**(2/3)), True))
```

3.562 $\int \frac{x^8}{\left(a+bx^3\right)^{2/3}}\,dx$

Optimal. Leaf size=56

$$\frac{a^2\sqrt[3]{a+bx^3}}{b^3}+\frac{\left(a+bx^3\right)^{7/3}}{7b^3}-\frac{a\left(a+bx^3\right)^{4/3}}{2b^3}$$

```
[Out] a^2*(b*x^3+a)^(1/3)/b^3-1/2*a*(b*x^3+a)^(4/3)/b^3+1/7*(b*x^3+a)^(7/3)/b^3
```

Rubi [A] time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{a^2\sqrt[3]{a+bx^3}}{b^3}+\frac{\left(a+bx^3\right)^{7/3}}{7b^3}-\frac{a\left(a+bx^3\right)^{4/3}}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^3)^(2/3),x]
```

```
[Out] (a^2*(a + b*x^3)^(1/3))/b^3 - (a*(a + b*x^3)^(4/3))/(2*b^3) + (a + b*x^3)^(
7/3)/(7*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{\left(a+bx^3\right)^{2/3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(a+bx)^{2/3}}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int\left(\frac{a^2}{b^2(a+bx)^{2/3}}-\frac{2a\sqrt[3]{a+bx}}{b^2}+\frac{(a+bx)^{4/3}}{b^2}\right)dx, x, x^3\right)\\ &= \frac{a^2\sqrt[3]{a+bx^3}}{b^3}-\frac{a\left(a+bx^3\right)^{4/3}}{2b^3}+\frac{\left(a+bx^3\right)^{7/3}}{7b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.70

$$\frac{\sqrt[3]{a+bx^3}\left(9a^2-3abx^3+2b^2x^6\right)}{14b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^3)^(2/3),x]
```

```
[Out] ((a + b*x^3)^(1/3)*(9*a^2 - 3*a*b*x^3 + 2*b^2*x^6))/(14*b^3)
```

fricas [A] time = 0.56, size = 35, normalized size = 0.62

$$\frac{\left(2\,b^2x^6 - 3\,abx^3 + 9\,a^2\right)\left(bx^3 + a\right)^{\frac{1}{3}}}{14\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/14*(2*b^2*x^6 - 3*a*b*x^3 + 9*a^2)*(b*x^3 + a)^(1/3)/b^3
```

giac [A] time = 0.15, size = 46, normalized size = 0.82

$$\frac{\left(bx^3 + a\right)^{\frac{1}{3}}a^2}{b^3} + \frac{2\left(bx^3 + a\right)^{\frac{7}{3}} - 7\left(bx^3 + a\right)^{\frac{4}{3}}a}{14\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] (b*x^3 + a)^(1/3)*a^2/b^3 + 1/14*(2*(b*x^3 + a)^(7/3) - 7*(b*x^3 + a)^(4/3)
*a)/b^3
```

maple [A] time = 0.01, size = 36, normalized size = 0.64

$$\frac{\left(b\,x^3 + a\right)^{\frac{1}{3}}\left(2b^2x^6 - 3ab\,x^3 + 9a^2\right)}{14b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a)^(2/3),x)
```

```
[Out] 1/14*(b*x^3+a)^(1/3)*(2*b^2*x^6-3*a*b*x^3+9*a^2)/b^3
```

maxima [A] time = 1.28, size = 46, normalized size = 0.82

$$\frac{\left(bx^3 + a\right)^{\frac{7}{3}}}{7\,b^3} - \frac{\left(bx^3 + a\right)^{\frac{4}{3}}a}{2\,b^3} + \frac{\left(bx^3 + a\right)^{\frac{1}{3}}a^2}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 1/7*(b*x^3 + a)^(7/3)/b^3 - 1/2*(b*x^3 + a)^(4/3)*a/b^3 + (b*x^3 + a)^(1/3)
*a^2/b^3
```

mupad [B] time = 1.08, size = 36, normalized size = 0.64

$$\left(b\,x^3 + a\right)^{1/3}\left(\frac{9\,a^2}{14\,b^3} + \frac{x^6}{7\,b} - \frac{3\,a\,x^3}{14\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^3)^(2/3),x)
```

```
[Out] (a + b*x^3)^(1/3)*((9*a^2)/(14*b^3) + x^6/(7*b) - (3*a*x^3)/(14*b^2))
```

sympy [**A**] time = 2.12, size = 68, normalized size = 1.21

$$\begin{cases} \frac{9a^2\sqrt[3]{a+bx^3}}{14b^3} - \frac{3ax^3\sqrt[3]{a+bx^3}}{14b^2} + \frac{x^6\sqrt[3]{a+bx^3}}{7b} & \text{for } b \neq 0 \\ \frac{x^9}{9a^{\frac{2}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a)**(2/3),x)
```

```
[Out] Piecewise((9*a**2*(a + b*x**3)**(1/3)/(14*b**3) - 3*a*x**3*(a + b*x**3)**(1
/3)/(14*b**2) + x**6*(a + b*x**3)**(1/3)/(7*b), Ne(b, 0)), (x**9/(9*a**(2/3
)), True))
```

3.563 $\int \frac{x^5}{\left(a+bx^3\right)^{2/3}}\,dx$

Optimal. Leaf size=36

$$\frac{\left(a+bx^3\right)^{4/3}}{4b^2}-\frac{a\sqrt[3]{a+bx^3}}{b^2}$$

```
[Out] -a*(b*x^3+a)^(1/3)/b^2+1/4*(b*x^3+a)^(4/3)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{\left(a+bx^3\right)^{4/3}}{4b^2}-\frac{a\sqrt[3]{a+bx^3}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^3)^(2/3),x]
```

```
[Out] -((a*(a + b*x^3)^(1/3))/b^2) + (a + b*x^3)^(4/3)/(4*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\left(a+bx^3\right)^{2/3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{(a+bx)^{2/3}}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int\left(-\frac{a}{b(a+bx)^{2/3}}+\frac{\sqrt[3]{a+bx}}{b}\right)dx, x, x^3\right)\\ &= -\frac{a\sqrt[3]{a+bx^3}}{b^2}+\frac{\left(a+bx^3\right)^{4/3}}{4b^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 27, normalized size = 0.75

$$\frac{\left(bx^3-3a\right)\sqrt[3]{a+bx^3}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^3)^(2/3),x]
```

```
[Out] ((-3*a + b*x^3)*(a + b*x^3)^(1/3))/(4*b^2)
```

fricas [A] time = 0.77, size = 23, normalized size = 0.64

$$\frac{\left(bx^3+a\right)^{\frac{1}{3}}\left(bx^3-3\,a\right)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(2/3),x, algorithm="fricas")
[Out] 1/4*(b*x^3 + a)^(1/3)*(b*x^3 - 3*a)/b^2
```

giac [A] time = 0.18, size = 30, normalized size = 0.83

$$\frac{\left(bx^3+a\right)^{\frac{4}{3}}}{4\,b^2}-\frac{\left(bx^3+a\right)^{\frac{1}{3}}a}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(2/3),x, algorithm="giac")
[Out] 1/4*(b*x^3 + a)^(4/3)/b^2 - (b*x^3 + a)^(1/3)*a/b^2
```

maple [A] time = 0.00, size = 25, normalized size = 0.69

$$-\frac{\left(b\,x^3+a\right)^{\frac{1}{3}}\left(-b\,x^3+3a\right)}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a)^(2/3),x)
[Out] -1/4*(b*x^3+a)^(1/3)*(-b*x^3+3*a)/b^2
```

maxima [A] time = 1.39, size = 30, normalized size = 0.83

$$\frac{\left(bx^3+a\right)^{\frac{4}{3}}}{4\,b^2}-\frac{\left(bx^3+a\right)^{\frac{1}{3}}a}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(2/3),x, algorithm="maxima")
[Out] 1/4*(b*x^3 + a)^(4/3)/b^2 - (b*x^3 + a)^(1/3)*a/b^2
```

mupad [B] time = 1.07, size = 24, normalized size = 0.67

$$-\frac{\left(b\,x^3+a\right)^{1/3}\left(3\,a-b\,x^3\right)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^3)^(2/3),x)
[Out] -((a + b*x^3)^(1/3)*(3*a - b*x^3))/(4*b^2)
```

sympy [A] time = 0.90, size = 44, normalized size = 1.22

$$\begin{cases} -\frac{3a\sqrt[3]{a+bx^3}}{4b^2}+\frac{x^3\sqrt[3]{a+bx^3}}{4b} & \text{for } b\neq 0 \\ \frac{x^6}{6a^{\frac{2}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)**(2/3),x)
```

```
[Out] Piecewise((-3*a*(a + b*x**3)**(1/3)/(4*b**2) + x**3*(a + b*x**3)**(1/3)/(4*
b), Ne(b, 0)), (x**6/(6*a**(2/3)), True))
```

3.564 $\int \frac{x^2}{\left(a+bx^3\right)^{2/3}}\,dx$

Optimal. Leaf size=15

$$\frac{\sqrt[3]{a+bx^3}}{b}$$

```
[Out] (b*x^3+a)^(1/3)/b
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$\frac{\sqrt[3]{a+bx^3}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^3)^(2/3),x]
```

```
[Out] (a + b*x^3)^(1/3)/b
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^3\right)^{2/3}}\,dx = \frac{\sqrt[3]{a+bx^3}}{b}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\sqrt[3]{a+bx^3}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^3)^(2/3),x]
```

```
[Out] (a + b*x^3)^(1/3)/b
```

fricas [A] time = 0.68, size = 13, normalized size = 0.87

$$\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] (b*x^3 + a)^(1/3)/b
```

giac [A] time = 0.16, size = 13, normalized size = 0.87

$$\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(2/3),x, algorithm="giac")
[Out] (b*x^3 + a)^(1/3)/b
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)^(2/3),x)
[Out] (b*x^3+a)^(1/3)/b
```

maxima [A] time = 1.27, size = 13, normalized size = 0.87

$$\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(2/3),x, algorithm="maxima")
[Out] (b*x^3 + a)^(1/3)/b
```

mupad [B] time = 1.05, size = 13, normalized size = 0.87

$$\frac{\left(b\,x^3+a\right)^{1/3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^3)^(2/3),x)
[Out] (a + b*x^3)^(1/3)/b
```

sympy [A] time = 0.76, size = 20, normalized size = 1.33

$$\begin{cases} \frac{\sqrt[3]{a+bx^3}}{b} & \text{for } b \neq 0 \\ \frac{x^3}{3a^{\frac{2}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)**(2/3),x)
[Out] Piecewise(((a + b*x**3)**(1/3)/b, Ne(b, 0)), (x**3/(3*a**(2/3)), True))
```

3.565 $\int \frac{1}{x\left(a+bx^3\right)^{2/3}}\,dx$

Optimal. Leaf size=84

$$\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}}-\frac{\log(x)}{2a^{2/3}}$$

```
[Out] -1/2*ln(x)/a^(2/3)+1/2*ln(a^(1/3)-(b*x^3+a)^(1/3))/a^(2/3)-1/3*arctan(1/3*(
a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/a^(2/3)*3^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {266, 57, 617, 204, 31}

$$\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}}-\frac{\log(x)}{2a^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3)^(2/3)),x]
```

```
[Out] -(ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3
))) - Log[x]/(2*a^(2/3)) + Log[a^(1/3) - (a + b*x^3)^(1/3)]/(2*a^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^3\right)^{2/3}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^3\right)$$

$$= -\frac{\log(x)}{2a^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2a^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}$$

$$= -\frac{\log(x)}{2a^{2/3}} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{a^{2/3}}$$

$$= -\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}} - \frac{\log(x)}{2a^{2/3}} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}$$

Mathematica [A] time = 0.05, size = 101, normalized size = 1.20

$$-\frac{\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right) - 2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right) + 2\sqrt{3}\,\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{6a^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3)^(2/3)),x]
```

```
[Out] -1/6*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 2*Log
[a^(1/3) - (a + b*x^3)^(1/3)] + Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (
a + b*x^3)^(2/3)])/a^(2/3)
```

fricas [B] time = 0.46, size = 126, normalized size = 1.50

$$-\frac{2\sqrt{3}\left(a^2\right)^{\frac{1}{6}} a \arctan\left(\frac{\left(a^2\right)^{\frac{1}{6}}\left(\sqrt{3}\left(a^2\right)^{\frac{1}{3}}a+2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{2}{3}}\right)}{3a^2}\right) + \left(a^2\right)^{\frac{2}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}a+\left(a^2\right)^{\frac{1}{3}}a+\left(bx^3+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{2}{3}}\right) -}{6a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(a^2)^(1/6)*a*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2)^(1/3)*a
 + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(a^2)^(2/3))/a^2) + (a^2)^(2/3)*log((b*x^3 +
 a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(a^2)^(2/3)) - 2*(a^2)^(2/3
)*log((b*x^3 + a)^(1/3)*a - (a^2)^(2/3)))/a^2
```

giac [A] time = 0.70, size = 87, normalized size = 1.04

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{3a^{\frac{2}{3}}} - \frac{\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{2}{3}}} + \frac{\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/a^
(2/3) - 1/6*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/a^
(2/3) + 1/3*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/a^(2/3)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{2}{3}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)^(2/3),x)
```

```
[Out] int(1/x/(b*x^3+a)^(2/3),x)
```

maxima [A] time = 2.98, size = 86, normalized size = 1.02

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{2}{3}}}-\frac{\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{2}{3}}}+\frac{\log\left(\left(bx^3+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/a^
(2/3) - 1/6*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/a^
(2/3) + 1/3*log((b*x^3 + a)^(1/3) - a^(1/3))/a^(2/3)
```

mupad [B] time = 1.23, size = 102, normalized size = 1.21

$$\frac{\ln\left(3\left(b\,x^3+a\right)^{1/3}-3\,a^{1/3}\right)}{3\,a^{2/3}}+\frac{\ln\left(\frac{3\,a^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}-3\left(b\,x^3+a\right)^{1/3}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}}-\frac{\ln\left(\frac{3\,a^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}+3\left(b\,x^3+a\right.\right.}{6\,a^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3)^(2/3)),x)
```

```
[Out] log(3*(a + b*x^3)^(1/3) - 3*a^(1/3))/(3*a^(2/3)) + (log((3*a^(1/3)*(3^(1/2)
*1i - 1))/2 - 3*(a + b*x^3)^(1/3))*(3^(1/2)*1i - 1))/(6*a^(2/3)) - (log((3*
a^(1/3)*(3^(1/2)*1i + 1))/2 + 3*(a + b*x^3)^(1/3))*(3^(1/2)*1i + 1))/(6*a^(
2/3))
```

sympy [C] time = 1.76, size = 39, normalized size = 0.46

$$-\frac{\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{ae^{i\pi}}{bx^3}\right)}{3b^{\frac{2}{3}}x^2\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)**(2/3),x)
```

```
[Out] -gamma(2/3)*hyper((2/3, 2/3), (5/3,), a*exp_polar(I*pi)/(b*x**3))/(3*b**(2/
3)*x**2*gamma(5/3))
```

3.566 $\int \frac{1}{x^4\left(a+bx^3\right)^{2/3}}\,dx$

Optimal. Leaf size=110

$$-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{3a^{5/3}}+\frac{2b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}}+\frac{b\log(x)}{3a^{5/3}}-\frac{\sqrt[3]{a+bx^3}}{3ax^3}$$

```
[Out] -1/3*(b*x^3+a)^(1/3)/a/x^3+1/3*b*ln(x)/a^(5/3)-1/3*b*ln(a^(1/3)-(b*x^3+a)^(
1/3))/a^(5/3)+2/9*b*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))
/a^(5/3)*3^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {266, 51, 57, 617, 204, 31}

$$-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{3a^{5/3}}+\frac{2b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}}+\frac{b\log(x)}{3a^{5/3}}-\frac{\sqrt[3]{a+bx^3}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3)^(2/3)),x]
```

```
[Out] -(a + b*x^3)^(1/3)/(3*a*x^3) + (2*b*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/
(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)) + (b*Log[x])/(3*a^(5/3)) - (b*Log[a
^(1/3) - (a + b*x^3)^(1/3)])/(3*a^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^3\right)^{2/3}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{2/3}}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt[3]{a+bx^3}}{3ax^3} - \frac{(2b)\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^3\right)}{9a}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{3ax^3} + \frac{b\log(x)}{3a^{5/3}} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{3a^{5/3}} + \frac{b\,\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\right.}{3a^{4/3}}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{3ax^3} + \frac{b\log(x)}{3a^{5/3}} - \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{3a^{5/3}} - \frac{(2b)\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a}}{\sqrt[3]{}}\right.}{3a^{5/3}}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{3ax^3} + \frac{2b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{5/3}} + \frac{b\log(x)}{3a^{5/3}} - \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{3a^{5/3}}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.31

$$\frac{b\sqrt[3]{a+bx^3}\;{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};\frac{bx^3}{a}+1\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3)^(2/3)),x]
```

```
[Out] (b*(a + b*x^3)^(1/3)*Hypergeometric2F1[1/3, 2, 4/3, 1 + (b*x^3)/a])/a^2
```

fricas [B] time = 0.61, size = 182, normalized size = 1.65

$$\frac{2\sqrt{3}\,abx^3\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}\,\arctan\left(-\frac{\left(\sqrt{3}\left(-a^2\right)^{\frac{1}{3}}a-2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(-a^2\right)^{\frac{2}{3}}\right)\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}}{3a^2}\right)+\left(-a^2\right)^{\frac{2}{3}}bx^3\log\left(\left(bx^3+a\right)^{\frac{2}{3}}a-\left(-a^2\right.\right.}{9\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/9*(2*sqrt(3)*a*b*x^3*sqrt(-(-a^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-a^2)^(1/3
)*a - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-a^2)^(2/3))*sqrt(-(-a^2)^(1/3))/a^2) +
(-a^2)^(2/3)*b*x^3*log((b*x^3 + a)^(2/3)*a - (-a^2)^(1/3)*a + (b*x^3 + a)^(
```

```
1/3)*(-a^2)^(2/3)) - 2*(-a^2)^(2/3)*b*x^3*log((b*x^3 + a)^(1/3)*a - (-a^2)^
(2/3)) - 3*(b*x^3 + a)^(1/3)*a^2)/(a^3*x^3)
```

giac [A] time = 0.70, size = 118, normalized size = 1.07

$$\frac{\dfrac{2\sqrt{3}\,b^2\arctan\left(\dfrac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{5}{3}}}+\dfrac{b^2\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{5}{3}}}-\dfrac{2b^2\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{5}{3}}}-\dfrac{3\left(bx^3+a\right)^{\frac{1}{3}}b}{ax^3}}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] 1/9*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/
3))/a^(5/3) + b^2*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/
3))/a^(5/3) - 2*b^2*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/a^(5/3) - 3*(b*x^
3 + a)^(1/3)*b/(a*x^3))/b
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{2}{3}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)^(2/3),x)
```

```
[Out] int(1/x^4/(b*x^3+a)^(2/3),x)
```

maxima [A] time = 2.96, size = 118, normalized size = 1.07

$$\frac{2\sqrt{3}\,b\arctan\left(\dfrac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{9a^{\frac{5}{3}}}-\frac{\left(bx^3+a\right)^{\frac{1}{3}}b}{3\left(\left(bx^3+a\right)a-a^2\right)}+\frac{b\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{5}{3}}}-\frac{2b\log\left(\left(bx^3+\right.\right.}{9a^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/a
^(5/3) - 1/3*(b*x^3 + a)^(1/3)*b/((b*x^3 + a)*a - a^2) + 1/9*b*log((b*x^3 +
 a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/a^(5/3) - 2/9*b*log((b*x^3
 + a)^(1/3) - a^(1/3))/a^(5/3)
```

mupad [B] time = 1.32, size = 128, normalized size = 1.16

$$\frac{\ln\left(\frac{b-\sqrt{3}\,b\,1\mathrm{i}}{a^{2/3}}+\frac{2\,b\left(b\,x^3+a\right)^{1/3}}{a}\right)\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{9\,a^{5/3}}+\frac{\ln\left(\frac{b+\sqrt{3}\,b\,1\mathrm{i}}{a^{2/3}}+\frac{2\,b\left(b\,x^3+a\right)^{1/3}}{a}\right)\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{9\,a^{5/3}}-\frac{2\,b\,\ln\left(\left(b\,x^3+a\right)^{1/3}-a^{1/3}\right)}{9\,a^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3)^(2/3)),x)
```

```
[Out] (log((b - 3^(1/2)*b*1i)/a^(2/3) + (2*b*(a + b*x^3)^(1/3))/a)*(b - 3^(1/2)*b
*1i))/(9*a^(5/3)) + (log((b + 3^(1/2)*b*1i)/a^(2/3) + (2*b*(a + b*x^3)^(1/3
```

```
))/a)*(b + 3^(1/2)*b*1i))/(9*a^(5/3)) - (2*b*log((a + b*x^3)^(1/3) - a^(1/3
)))/(9*a^(5/3)) - (a + b*x^3)^(1/3)/(3*a*x^3)
```

sympy [C] time = 2.26, size = 39, normalized size = 0.35

$$-\frac{\Gamma\left(\frac{5}{3}\right) {}_2F_1\left(\begin{matrix}\frac{2}{3}, \frac{5}{3} \\ \frac{8}{3}\end{matrix}\middle| \frac{ae^{i\pi}}{bx^3}\right)}{3b^{\frac{2}{3}}x^5\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)**(2/3),x)
```

```
[Out] -gamma(5/3)*hyper((2/3, 5/3), (8/3,), a*exp_polar(I*pi)/(b*x**3))/(3*b**(2/
3)*x**5*gamma(8/3))
```

3.567 $\int \frac{x^7}{\left(a+bx^3\right)^{2/3}}\,dx$

Optimal. Leaf size=123

$$-\frac{5a^2\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{18b^{8/3}}-\frac{5a^2\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,b^{8/3}}-\frac{5ax^2\sqrt[3]{a+bx^3}}{18b^2}+\frac{x^5\sqrt[3]{a+bx^3}}{6b}$$

```
[Out] -5/18*a*x^2*(b*x^3+a)^(1/3)/b^2+1/6*x^5*(b*x^3+a)^(1/3)/b-5/18*a^2*ln(b^(1/
3)*x-(b*x^3+a)^(1/3))/b^(8/3)-5/27*a^2*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^
(1/3))*3^(1/2))/b^(8/3)*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 176, normalized size of antiderivative = 1.43, number of steps used = 9, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {321, 331, 292, 31, 634, 617, 204, 628}

$$-\frac{5a^2\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{27b^{8/3}}+\frac{5a^2\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1\right)}{54b^{8/3}}-\frac{5a^2\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,b^{8/3}}-\frac{5ax^2\sqrt[3]{a+bx^3}}{18b^2}+\frac{x^5\sqrt[3]{a+bx^3}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^3)^(2/3),x]
```

```
[Out] (-5*a*x^2*(a + b*x^3)^(1/3))/(18*b^2) + (x^5*(a + b*x^3)^(1/3))/(6*b) - (5*
a^2*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(8/
3)) - (5*a^2*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(27*b^(8/3)) + (5*a^2*
Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(
54*b^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^7}{\left(a+bx^3\right)^{2/3}}\,dx = \frac{x^5\sqrt[3]{a+bx^3}}{6b} - \frac{(5a)\int \frac{x^4}{\left(a+bx^3\right)^{2/3}}\,dx}{6b}$$

$$= -\frac{5ax^2\sqrt[3]{a+bx^3}}{18b^2} + \frac{x^5\sqrt[3]{a+bx^3}}{6b} + \frac{\left(5a^2\right)\int \frac{x}{\left(a+bx^3\right)^{2/3}}\,dx}{9b^2}$$

$$= -\frac{5ax^2\sqrt[3]{a+bx^3}}{18b^2} + \frac{x^5\sqrt[3]{a+bx^3}}{6b} + \frac{\left(5a^2\right)\text{Subst}\left(\int \frac{x}{1-bx^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{9b^2}$$

$$= -\frac{5ax^2\sqrt[3]{a+bx^3}}{18b^2} + \frac{x^5\sqrt[3]{a+bx^3}}{6b} + \frac{\left(5a^2\right)\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{b}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{27b^{7/3}} - \frac{\left(5a^2\right)\text{Subst}\left(\int$$

$$= -\frac{5ax^2\sqrt[3]{a+bx^3}}{18b^2} + \frac{x^5\sqrt[3]{a+bx^3}}{6b} - \frac{5a^2\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{27b^{8/3}} + \frac{\left(5a^2\right)\text{Subst}\left(\int \frac{\sqrt[3]{b}+2b^{2/3}x}{1+\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx\right.}{54b^{8/3}}$$

$$= -\frac{5ax^2\sqrt[3]{a+bx^3}}{18b^2} + \frac{x^5\sqrt[3]{a+bx^3}}{6b} - \frac{5a^2\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{27b^{8/3}} + \frac{5a^2\log\left(1+\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right.}{54b^{8/3}}$$

$$= -\frac{5ax^2\sqrt[3]{a+bx^3}}{18b^2} + \frac{x^5\sqrt[3]{a+bx^3}}{6b} - \frac{5a^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}\,b^{8/3}} - \frac{5a^2\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{27b^{8/3}} + \frac{5a^2\log}{}$$

Mathematica [C] time = 0.02, size = 69, normalized size = 0.56

$$\frac{x^2\left(5a^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{bx^3}{bx^3+a}\right)-5a^2-2abx^3+3b^2x^6\right)}{18b^2\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^3)^(2/3),x]
```

```
[Out] (x^2*(-5*a^2 - 2*a*b*x^3 + 3*b^2*x^6 + 5*a^2*Hypergeometric2F1[2/3, 1, 5/3,
 (b*x^3)/(a + b*x^3)]))/(18*b^2*(a + b*x^3)^(2/3))
```

fricas [B] time = 0.66, size = 212, normalized size = 1.72

$$\frac{10\sqrt{3}\,a^2b\sqrt{-\left(-b^2\right)^{\frac{1}{3}}}\arctan\left(-\frac{\left(\sqrt{3}\left(-b^2\right)^{\frac{1}{3}}bx-2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(-b^2\right)^{\frac{2}{3}}\right)\sqrt{-\left(-b^2\right)^{\frac{1}{3}}}}{3b^2x}\right)-10\left(-b^2\right)^{\frac{2}{3}}a^2\log\left(-\frac{\left(-b^2\right)^{\frac{2}{3}}x-\left(bx^3+a\right)^{\frac{1}{3}}b}{x}\right)-}{54\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/54*(10*sqrt(3)*a^2*b*sqrt(-(-b^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-b^2)^(1/3
)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(2/3))*sqrt(-(-b^2)^(1/3))/(b^2*
x)) - 10*(-b^2)^(2/3)*a^2*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) +
5*(-b^2)^(2/3)*a^2*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3
)*x - (b*x^3 + a)^(2/3)*b)/x^2) + 3*(3*b^3*x^5 - 5*a*b^2*x^2)*(b*x^3 + a)^(
1/3))/b^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(bx^3+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(x^7/(b*x^3 + a)^(2/3), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(b\,x^3+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^3+a)^(2/3),x)
```

```
[Out] int(x^7/(b*x^3+a)^(2/3),x)
```

maxima [A] time = 3.00, size = 184, normalized size = 1.50

$$\frac{5\sqrt{3}\,a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{27\,b^{\frac{8}{3}}}+\frac{5\,a^2\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{54\,b^{\frac{8}{3}}}-\frac{5\,a^2\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{27\,b^{\frac{8}{3}}}+\frac{\frac{8\left(bx^3+a\right)^{\frac{1}{3}}}{x}}{18\left(b^4-\frac{2\left(b\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 5/27*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/
3))/b^(8/3) + 5/54*a^2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 +
 a)^(2/3)/x^2)/b^(8/3) - 5/27*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(8/
3) + 1/18*(8*(b*x^3 + a)^(1/3)*a^2*b/x - 5*(b*x^3 + a)^(4/3)*a^2/x^4)/(b^4
- 2*(b*x^3 + a)*b^3/x^3 + (b*x^3 + a)^2*b^2/x^6)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7}{\left(b\,x^3+a\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^3)^(2/3),x)
```

```
[Out] int(x^7/(a + b*x^3)^(2/3), x)
```

sympy **[C]** time = 2.84, size = 37, normalized size = 0.30

$$\frac{x^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\,\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**3+a)**(2/3),x)
```

```
[Out] x**8*gamma(8/3)*hyper((2/3, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**
(2/3)*gamma(11/3))
```

3.568 $\int \frac{x^4}{\left(a+bx^3\right)^{2/3}}\,dx$

Optimal. Leaf size=97

$$\frac{a\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{3b^{5/3}}+\frac{2a\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{5/3}}+\frac{x^2\sqrt[3]{a+bx^3}}{3b}$$

```
[Out] 1/3*x^2*(b*x^3+a)^(1/3)/b+1/3*a*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(5/3)+2/9*a
*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(5/3)*3^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 148, normalized size of antiderivative = 1.53, number of steps used = 8, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {321, 331, 292, 31, 634, 617, 204, 628}

$$\frac{2a\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{9b^{5/3}}-\frac{a\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1\right)}{9b^{5/3}}+\frac{2a\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{5/3}}+\frac{x^2\sqrt[3]{a+bx^3}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^3)^(2/3),x]
```

```
[Out] (x^2*(a + b*x^3)^(1/3))/(3*b) + (2*a*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^
(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(5/3)) + (2*a*Log[1 - (b^(1/3)*x)/(a + b*x^3)
^(1/3)])/(9*b^(5/3)) - (a*Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3
)*x)/(a + b*x^3)^(1/3)])/(9*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^3\right)^{2/3}}\,dx = \frac{x^2\sqrt[3]{a+bx^3}}{3b} - \frac{(2a)\int \frac{x}{\left(a+bx^3\right)^{2/3}}\,dx}{3b}$$

$$= \frac{x^2\sqrt[3]{a+bx^3}}{3b} - \frac{(2a)\,\text{Subst}\left(\int \frac{x}{1-bx^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3b}$$

$$= \frac{x^2\sqrt[3]{a+bx^3}}{3b} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{b}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{9b^{4/3}} + \frac{(2a)\,\text{Subst}\left(\int \frac{1-\sqrt[3]{b}\,x}{1+\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, \frac{}{\sqrt[3]{a}}\right.}{9b^{4/3}}$$

$$= \frac{x^2\sqrt[3]{a+bx^3}}{3b} + \frac{2a\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{9b^{5/3}} - \frac{a\,\text{Subst}\left(\int \frac{\sqrt[3]{b}+2b^{2/3}x}{1+\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{9b^{5/3}} + \frac{a\,\text{Subst}\left(}{}$$

$$= \frac{x^2\sqrt[3]{a+bx^3}}{3b} + \frac{2a\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{9b^{5/3}} - \frac{a\log\left(1+\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{9b^{5/3}} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{-3-}\right.}{3}$$

$$= \frac{x^2\sqrt[3]{a+bx^3}}{3b} + \frac{2a\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{5/3}} + \frac{2a\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{9b^{5/3}} - \frac{a\log\left(1+\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx}}\right.}{9b^{5/3}}$$

Mathematica [C] time = 0.01, size = 53, normalized size = 0.55

$$\frac{x^2\left(-a\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{bx^3}{bx^3+a}\right)+a+bx^3\right)}{3b\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^3)^(2/3),x]
```

```
[Out] (x^2*(a + b*x^3 - a*Hypergeometric2F1[2/3, 1, 5/3, (b*x^3)/(a + b*x^3)]))/(
3*b*(a + b*x^3)^(2/3))
```

fricas [B] time = 0.56, size = 165, normalized size = 1.70

$$\frac{3\left(bx^3+a\right)^{\frac{1}{3}}b^2x^2-2\sqrt{3}\,a\left(b^2\right)^{\frac{1}{6}}b\arctan\left(\frac{\left(\sqrt{3}\left(b^2\right)^{\frac{1}{3}}bx+2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(b^2\right)^{\frac{2}{3}}\right)\left(b^2\right)^{\frac{1}{6}}}{3b^2x}\right)+2a\left(b^2\right)^{\frac{2}{3}}\log\left(-\frac{\left(b^2\right)^{\frac{2}{3}}x-\left(bx^3+a\right)^{\frac{1}{3}}b}{x}\right)-a}{9b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/9*(3*(b*x^3 + a)^(1/3)*b^2*x^2 - 2*sqrt(3)*a*(b^2)^(1/6)*b*arctan(1/3*(sq
rt(3)*(b^2)^(1/3)*b*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(b^2)^(2/3))*(b^2)^(1/6
)/(b^2*x)) + 2*a*(b^2)^(2/3)*log(-((b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x)
- a*(b^2)^(2/3)*log(((b^2)^(1/3)*b*x^2 + (b*x^3 + a)^(1/3)*(b^2)^(2/3)*x +
(b*x^3 + a)^(2/3)*b)/x^2))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^3+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(x^4/(b*x^3 + a)^(2/3), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(b\,x^3+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a)^(2/3),x)
```

```
[Out] int(x^4/(b*x^3+a)^(2/3),x)
```

maxima [A] time = 2.90, size = 137, normalized size = 1.41

$$-\frac{2\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{9b^{\frac{5}{3}}}-\frac{a\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{9b^{\frac{5}{3}}}+\frac{2a\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{9b^{\frac{5}{3}}}-\frac{\left(bx^3+a\right)^{\frac{1}{3}}a}{3\left(b^2-\frac{\left(bx^3+a\right)b}{x^3}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3)
)/b^(5/3) - 1/9*a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(
```

```
2/3)/x^2)/b^(5/3) + 2/9*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(5/3) - 1/3
*(b*x^3 + a)^(1/3)*a/((b^2 - (b*x^3 + a)*b/x^3)*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\left(b\,x^3+a\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^3)^(2/3),x)
```

```
[Out] int(x^4/(a + b*x^3)^(2/3), x)
```

sympy [C] time = 2.76, size = 37, normalized size = 0.38

$$\frac{x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3}, \frac{5}{3} \\ \frac{8}{3}\end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a)**(2/3),x)
```

```
[Out] x**5*gamma(5/3)*hyper((2/3, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
2/3)*gamma(8/3))
```

3.569 $\int \frac{x}{\left(a+bx^3\right)^{2/3}}\,dx$

Optimal. Leaf size=72

$$-\frac{\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}-\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}}$$

```
[Out] -1/2*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)-1/3*arctan(1/3*(1+2*b^(1/3)*x/(b
*x^3+a)^(1/3))*3^(1/2))/b^(2/3)*3^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 122, normalized size of antiderivative = 1.69, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {331, 292, 31, 634, 617, 204, 628}

$$-\frac{\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}}+\frac{\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1\right)}{6b^{2/3}}-\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^3)^(2/3),x]
```

```
[Out] -(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(2/3)))
- Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)]/(3*b^(2/3)) + Log[1 + (b^(2/3)*x^2
)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)]/(6*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^3\right)^{2/3}}\,dx = \text{Subst}\left(\int \frac{x}{1-bx^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{b}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}} - \frac{\text{Subst}\left(\int \frac{1-\sqrt[3]{b}\,x}{1+\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}$$

$$= -\frac{\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}} + \frac{\text{Subst}\left(\int \frac{\sqrt[3]{b}+2b^{2/3}x}{1+\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{6b^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x,\right.}{2\sqrt[3]{b}}$$

$$= -\frac{\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}} + \frac{\log\left(1+\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{6b^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{b^{2/3}}$$

$$= -\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}} - \frac{\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}} + \frac{\log\left(1+\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{6b^{2/3}}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.56

$$\frac{x^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{bx^3}{bx^3+a}\right)}{2\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^3)^(2/3),x]
```

```
[Out] (x^2*Hypergeometric2F1[2/3, 1, 5/3, (b*x^3)/(a + b*x^3)])/(2*(a + b*x^3)^(2
/3))
```

fricas [B] time = 0.69, size = 173, normalized size = 2.40

$$\frac{2\sqrt{3}\,b\sqrt{-\left(-b^2\right)^{\frac{1}{3}}}\,\arctan\left(-\frac{\left(\sqrt{3}\left(-b^2\right)^{\frac{1}{3}}bx-2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(-b^2\right)^{\frac{2}{3}}\right)\sqrt{-\left(-b^2\right)^{\frac{1}{3}}}}{3\,b^2x}\right) - 2\left(-b^2\right)^{\frac{2}{3}}\log\left(-\frac{\left(-b^2\right)^{\frac{2}{3}}x-\left(bx^3+a\right)^{\frac{1}{3}}b}{x}\right)+\left(-}{6\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*b*sqrt(-(-b^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-b^2)^(1/3)*b*x
- 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(2/3))*sqrt(-(-b^2)^(1/3))/(b^2*x)) -
2*(-b^2)^(2/3)*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (-b^2)^(2/3
)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)
^(2/3)*b)/x^2))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^3+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(x/(b*x^3 + a)^(2/3), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(b\,x^3+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a)^(2/3),x)
```

```
[Out] int(x/(b*x^3+a)^(2/3),x)
```

maxima [A] time = 2.88, size = 100, normalized size = 1.39

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{3\,b^{\frac{2}{3}}}+\frac{\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{6\,b^{\frac{2}{3}}}-\frac{\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b
^(2/3) + 1/6*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/
x^2)/b^(2/3) - 1/3*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(2/3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\left(b\,x^3+a\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^3)^(2/3),x)
```

```
[Out] int(x/(a + b*x^3)^(2/3), x)
```

sympy [C] time = 2.52, size = 37, normalized size = 0.51

$$\frac{x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{2}{3}, \frac{2}{3} \\ \frac{5}{3}\end{matrix}\right| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a)**(2/3),x)
```

```
[Out] x**2*gamma(2/3)*hyper((2/3, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
2/3)*gamma(5/3))
```

3.570 $\int \frac{1}{x^2\left(a+bx^3\right)^{2/3}}\,dx$

Optimal. Leaf size=19

$$-\frac{\sqrt[3]{a+bx^3}}{ax}$$

```
[Out] -(b*x^3+a)^(1/3)/a/x
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {264}

$$-\frac{\sqrt[3]{a+bx^3}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^3)^(2/3)),x]
```

```
[Out] -((a + b*x^3)^(1/3)/(a*x))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^3\right)^{2/3}}\,dx = -\frac{\sqrt[3]{a+bx^3}}{ax}$$

Mathematica [A] time = 0.02, size = 19, normalized size = 1.00

$$-\frac{\sqrt[3]{a+bx^3}}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^3)^(2/3)),x]
```

```
[Out] -((a + b*x^3)^(1/3)/(a*x))
```

fricas [A] time = 0.57, size = 17, normalized size = 0.89

$$-\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] -(b*x^3 + a)^(1/3)/(a*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(2/3),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(2/3)*x^2), x)
```

maple [A] time = 0.00, size = 18, normalized size = 0.95

$$-\frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a)^(2/3),x)
[Out] -(b*x^3+a)^(1/3)/a/x
```

maxima [A] time = 1.34, size = 17, normalized size = 0.89

$$-\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(2/3),x, algorithm="maxima")
[Out] -(b*x^3 + a)^(1/3)/(a*x)
```

mupad [B] time = 1.03, size = 17, normalized size = 0.89

$$-\frac{\left(b\,x^3+a\right)^{1/3}}{a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^3)^(2/3)),x)
[Out] -(a + b*x^3)^(1/3)/(a*x)
```

sympy [B] time = 1.15, size = 31, normalized size = 1.63

$$\frac{\sqrt[3]{b}\,\sqrt[3]{\frac{a}{bx^3}+1}\,\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a)**(2/3),x)
[Out] b**(1/3)*(a/(b*x**3) + 1)**(1/3)*gamma(-1/3)/(3*a*gamma(2/3))
```

3.571 $\int \frac{1}{x^5\left(a+bx^3\right)^{2/3}}\,dx$

Optimal. Leaf size=44

$$\frac{3b\sqrt[3]{a+bx^3}}{4a^2x}-\frac{\sqrt[3]{a+bx^3}}{4ax^4}$$

```
[Out] -1/4*(b*x^3+a)^(1/3)/a/x^4+3/4*b*(b*x^3+a)^(1/3)/a^2/x
```

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$\frac{3b\sqrt[3]{a+bx^3}}{4a^2x}-\frac{\sqrt[3]{a+bx^3}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^3)^(2/3)),x]
```

```
[Out] -(a + b*x^3)^(1/3)/(4*a*x^4) + (3*b*(a + b*x^3)^(1/3))/(4*a^2*x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\left(a+bx^3\right)^{2/3}}\,dx &= -\frac{\sqrt[3]{a+bx^3}}{4ax^4}-\frac{(3b)\int \frac{1}{x^2\left(a+bx^3\right)^{2/3}}\,dx}{4a}\\ &= -\frac{\sqrt[3]{a+bx^3}}{4ax^4}+\frac{3b\sqrt[3]{a+bx^3}}{4a^2x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.66

$$-\frac{\left(a-3bx^3\right)\sqrt[3]{a+bx^3}}{4a^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^3)^(2/3)),x]
```

```
[Out] -1/4*((a - 3*b*x^3)*(a + b*x^3)^(1/3))/(a^2*x^4)
```

fricas [A] time = 0.51, size = 27, normalized size = 0.61

$$\frac{\left(3\,bx^3-a\right)\left(bx^3+a\right)^{\frac{1}{3}}}{4\,a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)^(2/3),x, algorithm="fricas")
[Out] 1/4*(3*b*x^3 - a)*(b*x^3 + a)^(1/3)/(a^2*x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)^(2/3),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(2/3)*x^5), x)
```

maple [A] time = 0.00, size = 26, normalized size = 0.59

$$-\frac{\left(b\,x^3+a\right)^{\frac{1}{3}}\left(-3b\,x^3+a\right)}{4a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^3+a)^(2/3),x)
[Out] -1/4*(b*x^3+a)^(1/3)*(-3*b*x^3+a)/a^2/x^4
```

maxima [A] time = 1.32, size = 35, normalized size = 0.80

$$\frac{\frac{4\left(bx^3+a\right)^{\frac{1}{3}}b}{x}-\frac{\left(bx^3+a\right)^{\frac{4}{3}}}{x^4}}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)^(2/3),x, algorithm="maxima")
[Out] 1/4*(4*(b*x^3 + a)^(1/3)*b/x - (b*x^3 + a)^(4/3)/x^4)/a^2
```

mupad [B] time = 1.08, size = 25, normalized size = 0.57

$$-\frac{\left(b\,x^3+a\right)^{1/3}\left(a-3\,b\,x^3\right)}{4\,a^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^3)^(2/3)),x)
[Out] -((a + b*x^3)^(1/3)*(a - 3*b*x^3))/(4*a^2*x^4)
```

sympy [A] time = 1.54, size = 68, normalized size = 1.55

$$-\frac{\sqrt[3]{b}\,\sqrt[3]{\frac{a}{bx^3}+1}\,\Gamma\left(-\frac{4}{3}\right)}{9ax^3\Gamma\left(\frac{2}{3}\right)}+\frac{b^{\frac{4}{3}}\sqrt[3]{\frac{a}{bx^3}+1}\,\Gamma\left(-\frac{4}{3}\right)}{3a^2\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**3+a)**(2/3),x)
[Out] -b**(1/3)*(a/(b*x**3) + 1)**(1/3)*gamma(-4/3)/(9*a*x**3*gamma(2/3)) + b**(4
/3)*(a/(b*x**3) + 1)**(1/3)*gamma(-4/3)/(3*a**2*gamma(2/3))
```

3.572 $\int \frac{1}{x^8\left(a+bx^3\right)^{2/3}}\,dx$

Optimal. Leaf size=68

$$-\frac{9b^2\sqrt[3]{a+bx^3}}{14a^3x}+\frac{3b\sqrt[3]{a+bx^3}}{14a^2x^4}-\frac{\sqrt[3]{a+bx^3}}{7ax^7}$$

```
[Out] -1/7*(b*x^3+a)^(1/3)/a/x^7+3/14*b*(b*x^3+a)^(1/3)/a^2/x^4-9/14*b^2*(b*x^3+a
)^(1/3)/a^3/x
```

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$-\frac{9b^2\sqrt[3]{a+bx^3}}{14a^3x}+\frac{3b\sqrt[3]{a+bx^3}}{14a^2x^4}-\frac{\sqrt[3]{a+bx^3}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(a + b*x^3)^(2/3)),x]
```

```
[Out] -(a + b*x^3)^(1/3)/(7*a*x^7) + (3*b*(a + b*x^3)^(1/3))/(14*a^2*x^4) - (9*b^
2*(a + b*x^3)^(1/3))/(14*a^3*x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^8\left(a+bx^3\right)^{2/3}}\,dx &= -\frac{\sqrt[3]{a+bx^3}}{7ax^7}-\frac{(6b)\int \frac{1}{x^5\left(a+bx^3\right)^{2/3}}\,dx}{7a}\\ &= -\frac{\sqrt[3]{a+bx^3}}{7ax^7}+\frac{3b\sqrt[3]{a+bx^3}}{14a^2x^4}+\frac{\left(9b^2\right)\int \frac{1}{x^2\left(a+bx^3\right)^{2/3}}\,dx}{14a^2}\\ &= -\frac{\sqrt[3]{a+bx^3}}{7ax^7}+\frac{3b\sqrt[3]{a+bx^3}}{14a^2x^4}-\frac{9b^2\sqrt[3]{a+bx^3}}{14a^3x}\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.62

$$-\frac{\sqrt[3]{a+bx^3}\left(2a^2-3abx^3+9b^2x^6\right)}{14a^3x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(a + b*x^3)^(2/3)),x]
```

```
[Out] -1/14*((a + b*x^3)^(1/3)*(2*a^2 - 3*a*b*x^3 + 9*b^2*x^6))/(a^3*x^7)
```

fricas [A] time = 0.59, size = 38, normalized size = 0.56

$$-\frac{\left(9\,b^2x^6-3\,abx^3+2\,a^2\right)\left(bx^3+a\right)^{\frac{1}{3}}}{14\,a^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/14*(9*b^2*x^6 - 3*a*b*x^3 + 2*a^2)*(b*x^3 + a)^(1/3)/(a^3*x^7)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*x^8), x)
```

maple [A] time = 0.00, size = 39, normalized size = 0.57

$$-\frac{\left(b\,x^3+a\right)^{\frac{1}{3}}\left(9b^2x^6-3ab\,x^3+2a^2\right)}{14a^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(b*x^3+a)^(2/3),x)
```

```
[Out] -1/14*(b*x^3+a)^(1/3)*(9*b^2*x^6-3*a*b*x^3+2*a^2)/a^3/x^7
```

maxima [A] time = 1.28, size = 52, normalized size = 0.76

$$-\frac{\frac{14\left(bx^3+a\right)^{\frac{1}{3}}b^2}{x}-\frac{7\left(bx^3+a\right)^{\frac{4}{3}}b}{x^4}+\frac{2\left(bx^3+a\right)^{\frac{7}{3}}}{x^7}}{14\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/14*(14*(b*x^3 + a)^(1/3)*b^2/x - 7*(b*x^3 + a)^(4/3)*b/x^4 + 2*(b*x^3 +
a)^(7/3)/x^7)/a^3
```

mupad [B] time = 1.15, size = 38, normalized size = 0.56

$$-\frac{\left(b\,x^3+a\right)^{1/3}\left(2\,a^2-3\,a\,b\,x^3+9\,b^2\,x^6\right)}{14\,a^3\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(a + b*x^3)^(2/3)),x)
```

```
[Out] -((a + b*x^3)^(1/3)*(2*a^2 + 9*b^2*x^6 - 3*a*b*x^3))/(14*a^3*x^7)
```

sympy [**B**] time = 3.05, size = 406, normalized size = 5.97

$$\frac{4a^4b^{\frac{13}{3}}\sqrt[3]{\frac{a}{bx^3}+1}\,\Gamma\left(-\frac{7}{3}\right)}{27a^5b^4x^6\Gamma\left(\frac{2}{3}\right)+54a^4b^5x^9\Gamma\left(\frac{2}{3}\right)+27a^3b^6x^{12}\Gamma\left(\frac{2}{3}\right)}+\frac{2a^3b^{\frac{16}{3}}x^3\sqrt[3]{\frac{a}{bx^3}+1}\,\Gamma\left(-\frac{7}{3}\right)}{27a^5b^4x^6\Gamma\left(\frac{2}{3}\right)+54a^4b^5x^9\Gamma\left(\frac{2}{3}\right)+27a^3b^6x^{12}\Gamma\left(\frac{2}{3}\right)}+\frac{}{27a^5b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(b*x**3+a)**(2/3),x)
```

```
[Out] 4*a**4*b**(13/3)*(a/(b*x**3) + 1)**(1/3)*gamma(-7/3)/(27*a**5*b**4*x**6*gam
ma(2/3) + 54*a**4*b**5*x**9*gamma(2/3) + 27*a**3*b**6*x**12*gamma(2/3)) + 2
*a**3*b**(16/3)*x**3*(a/(b*x**3) + 1)**(1/3)*gamma(-7/3)/(27*a**5*b**4*x**6
*gamma(2/3) + 54*a**4*b**5*x**9*gamma(2/3) + 27*a**3*b**6*x**12*gamma(2/3))
 + 10*a**2*b**(19/3)*x**6*(a/(b*x**3) + 1)**(1/3)*gamma(-7/3)/(27*a**5*b**4
*x**6*gamma(2/3) + 54*a**4*b**5*x**9*gamma(2/3) + 27*a**3*b**6*x**12*gamma(
2/3)) + 30*a*b**(22/3)*x**9*(a/(b*x**3) + 1)**(1/3)*gamma(-7/3)/(27*a**5*b*
*4*x**6*gamma(2/3) + 54*a**4*b**5*x**9*gamma(2/3) + 27*a**3*b**6*x**12*gamm
a(2/3)) + 18*b**(25/3)*x**12*(a/(b*x**3) + 1)**(1/3)*gamma(-7/3)/(27*a**5*b
**4*x**6*gamma(2/3) + 54*a**4*b**5*x**9*gamma(2/3) + 27*a**3*b**6*x**12*gam
ma(2/3))
```

3.573 $\int \frac{1}{x^{11}\left(a+bx^3\right)^{2/3}}\,dx$

Optimal. Leaf size=92

$$\frac{81b^3\sqrt[3]{a+bx^3}}{140a^4x}-\frac{27b^2\sqrt[3]{a+bx^3}}{140a^3x^4}+\frac{9b\sqrt[3]{a+bx^3}}{70a^2x^7}-\frac{\sqrt[3]{a+bx^3}}{10ax^{10}}$$

```
[Out] -1/10*(b*x^3+a)^(1/3)/a/x^10+9/70*b*(b*x^3+a)^(1/3)/a^2/x^7-27/140*b^2*(b*x
^3+a)^(1/3)/a^3/x^4+81/140*b^3*(b*x^3+a)^(1/3)/a^4/x
```

Rubi [A] time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$\frac{81b^3\sqrt[3]{a+bx^3}}{140a^4x}-\frac{27b^2\sqrt[3]{a+bx^3}}{140a^3x^4}+\frac{9b\sqrt[3]{a+bx^3}}{70a^2x^7}-\frac{\sqrt[3]{a+bx^3}}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^11*(a + b*x^3)^(2/3)),x]
```

```
[Out] -(a + b*x^3)^(1/3)/(10*a*x^10) + (9*b*(a + b*x^3)^(1/3))/(70*a^2*x^7) - (27
*b^2*(a + b*x^3)^(1/3))/(140*a^3*x^4) + (81*b^3*(a + b*x^3)^(1/3))/(140*a^4
*x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{11}\left(a+bx^3\right)^{2/3}}\,dx &= -\frac{\sqrt[3]{a+bx^3}}{10ax^{10}}-\frac{(9b)\int \frac{1}{x^8\left(a+bx^3\right)^{2/3}}\,dx}{10a}\\ &= -\frac{\sqrt[3]{a+bx^3}}{10ax^{10}}+\frac{9b\sqrt[3]{a+bx^3}}{70a^2x^7}+\frac{\left(27b^2\right)\int \frac{1}{x^5\left(a+bx^3\right)^{2/3}}\,dx}{35a^2}\\ &= -\frac{\sqrt[3]{a+bx^3}}{10ax^{10}}+\frac{9b\sqrt[3]{a+bx^3}}{70a^2x^7}-\frac{27b^2\sqrt[3]{a+bx^3}}{140a^3x^4}-\frac{\left(81b^3\right)\int \frac{1}{x^2\left(a+bx^3\right)^{2/3}}\,dx}{140a^3}\\ &= -\frac{\sqrt[3]{a+bx^3}}{10ax^{10}}+\frac{9b\sqrt[3]{a+bx^3}}{70a^2x^7}-\frac{27b^2\sqrt[3]{a+bx^3}}{140a^3x^4}+\frac{81b^3\sqrt[3]{a+bx^3}}{140a^4x}\end{aligned}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.58

$$\frac{\sqrt[3]{a+bx^3}\left(-14a^3+18a^2bx^3-27ab^2x^6+81b^3x^9\right)}{140a^4x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^11*(a + b*x^3)^(2/3)),x]
```

```
[Out] ((a + b*x^3)^(1/3)*(-14*a^3 + 18*a^2*b*x^3 - 27*a*b^2*x^6 + 81*b^3*x^9))/(140*a^4*x^10)
```

fricas [A] time = 0.59, size = 49, normalized size = 0.53

$$\frac{\left(81\,b^3x^9 - 27\,ab^2x^6 + 18\,a^2bx^3 - 14\,a^3\right)\left(bx^3 + a\right)^{\frac{1}{3}}}{140\,a^4x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/140*(81*b^3*x^9 - 27*a*b^2*x^6 + 18*a^2*b*x^3 - 14*a^3)*(b*x^3 + a)^(1/3)/(a^4*x^10)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3 + a\right)^{\frac{2}{3}} x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*x^11), x)
```

maple [A] time = 0.01, size = 50, normalized size = 0.54

$$-\frac{\left(b\,x^3 + a\right)^{\frac{1}{3}}\left(-81b^3x^9 + 27a\,b^2x^6 - 18a^2b\,x^3 + 14a^3\right)}{140a^4x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^11/(b*x^3+a)^(2/3),x)
```

```
[Out] -1/140*(b*x^3+a)^(1/3)*(-81*b^3*x^9+27*a*b^2*x^6-18*a^2*b*x^3+14*a^3)/x^10/a^4
```

maxima [A] time = 1.27, size = 69, normalized size = 0.75

$$\frac{\frac{140\left(bx^3+a\right)^{\frac{1}{3}}b^3}{x} - \frac{105\left(bx^3+a\right)^{\frac{4}{3}}b^2}{x^4} + \frac{60\left(bx^3+a\right)^{\frac{7}{3}}b}{x^7} - \frac{14\left(bx^3+a\right)^{\frac{10}{3}}}{x^{10}}}{140\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 1/140*(140*(b*x^3 + a)^(1/3)*b^3/x - 105*(b*x^3 + a)^(4/3)*b^2/x^4 + 60*(b*x^3 + a)^(7/3)*b/x^7 - 14*(b*x^3 + a)^(10/3)/x^10)/a^4
```

mupad [B] time = 1.20, size = 76, normalized size = 0.83

$$\frac{9\,b\left(b\,x^3 + a\right)^{1/3}}{70\,a^2\,x^7} - \frac{\left(b\,x^3 + a\right)^{1/3}}{10\,a\,x^{10}} + \frac{81\,b^3\left(b\,x^3 + a\right)^{1/3}}{140\,a^4\,x} - \frac{27\,b^2\left(b\,x^3 + a\right)^{1/3}}{140\,a^3\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^11*(a + b*x^3)^(2/3)),x)
```

```
[Out] (9*b*(a + b*x^3)^(1/3))/(70*a^2*x^7) - (a + b*x^3)^(1/3)/(10*a*x^10) + (81*
b^3*(a + b*x^3)^(1/3))/(140*a^4*x) - (27*b^2*(a + b*x^3)^(1/3))/(140*a^3*x^
4)
```

sympy [B] time = 3.18, size = 692, normalized size = 7.52

$$-\frac{28a^6b^{\frac{28}{3}}\sqrt[3]{\frac{a}{bx^3}+1}\,\Gamma\left(-\frac{10}{3}\right)}{81a^7b^9x^9\Gamma\left(\frac{2}{3}\right)+243a^6b^{10}x^{12}\Gamma\left(\frac{2}{3}\right)+243a^5b^{11}x^{15}\Gamma\left(\frac{2}{3}\right)+81a^4b^{12}x^{18}\Gamma\left(\frac{2}{3}\right)}-\frac{48a^5b^{\frac{31}{3}}x}{81a^7b^9x^9\Gamma\left(\frac{2}{3}\right)+243a^6b^{10}x^{12}\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**11/(b*x**3+a)**(2/3),x)
```

```
[Out] -28*a**6*b**(28/3)*(a/(b*x**3) + 1)**(1/3)*gamma(-10/3)/(81*a**7*b**9*x**9*
gamma(2/3) + 243*a**6*b**10*x**12*gamma(2/3) + 243*a**5*b**11*x**15*gamma(2
/3) + 81*a**4*b**12*x**18*gamma(2/3)) - 48*a**5*b**(31/3)*x**3*(a/(b*x**3)
+ 1)**(1/3)*gamma(-10/3)/(81*a**7*b**9*x**9*gamma(2/3) + 243*a**6*b**10*x**
12*gamma(2/3) + 243*a**5*b**11*x**15*gamma(2/3) + 81*a**4*b**12*x**18*gamma
(2/3)) - 30*a**4*b**(34/3)*x**6*(a/(b*x**3) + 1)**(1/3)*gamma(-10/3)/(81*a*
*7*b**9*x**9*gamma(2/3) + 243*a**6*b**10*x**12*gamma(2/3) + 243*a**5*b**11*
x**15*gamma(2/3) + 81*a**4*b**12*x**18*gamma(2/3)) + 80*a**3*b**(37/3)*x**9
*(a/(b*x**3) + 1)**(1/3)*gamma(-10/3)/(81*a**7*b**9*x**9*gamma(2/3) + 243*a
**6*b**10*x**12*gamma(2/3) + 243*a**5*b**11*x**15*gamma(2/3) + 81*a**4*b**1
2*x**18*gamma(2/3)) + 360*a**2*b**(40/3)*x**12*(a/(b*x**3) + 1)**(1/3)*gamm
a(-10/3)/(81*a**7*b**9*x**9*gamma(2/3) + 243*a**6*b**10*x**12*gamma(2/3) +
243*a**5*b**11*x**15*gamma(2/3) + 81*a**4*b**12*x**18*gamma(2/3)) + 432*a*b
**(43/3)*x**15*(a/(b*x**3) + 1)**(1/3)*gamma(-10/3)/(81*a**7*b**9*x**9*gamm
a(2/3) + 243*a**6*b**10*x**12*gamma(2/3) + 243*a**5*b**11*x**15*gamma(2/3)
+ 81*a**4*b**12*x**18*gamma(2/3)) + 162*b**(46/3)*x**18*(a/(b*x**3) + 1)**(
1/3)*gamma(-10/3)/(81*a**7*b**9*x**9*gamma(2/3) + 243*a**6*b**10*x**12*gamm
a(2/3) + 243*a**5*b**11*x**15*gamma(2/3) + 81*a**4*b**12*x**18*gamma(2/3))
```

3.574 $\int \frac{x^6}{\left(a+bx^3\right)^{2/3}}\,dx$

Optimal. Leaf size=38

$$\frac{x^7\sqrt[3]{a+bx^3}\;{}_2F_1\left(1,\frac{8}{3};\frac{10}{3};-\frac{bx^3}{a}\right)}{7a}$$

```
[Out] 1/7*x^7*(b*x^3+a)^(1/3)*hypergeom([1, 8/3],[10/3],-b*x^3/a)/a
```

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.34, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {365, 364}

$$\frac{x^7\left(\frac{bx^3}{a}+1\right)^{2/3}\;{}_2F_1\left(\frac{2}{3},\frac{7}{3};\frac{10}{3};-\frac{bx^3}{a}\right)}{7\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^3)^(2/3),x]
```

```
[Out] (x^7*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 7/3, 10/3, -((b*x^3)/a)])
/(7*(a + b*x^3)^(2/3))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{\left(a+bx^3\right)^{2/3}}\,dx &= \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{x^6}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{\left(a+bx^3\right)^{2/3}}\\ &= \frac{x^7\left(1+\frac{bx^3}{a}\right)^{2/3}\;{}_2F_1\left(\frac{2}{3},\frac{7}{3};\frac{10}{3};-\frac{bx^3}{a}\right)}{7\left(a+bx^3\right)^{2/3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 1.34

$$\frac{x^7\left(\frac{bx^3}{a}+1\right)^{2/3}\;{}_2F_1\left(\frac{2}{3},\frac{7}{3};\frac{10}{3};-\frac{bx^3}{a}\right)}{7\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^3)^(2/3),x]
```

```
[Out] (x^7*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 7/3, 10/3, -((b*x^3)/a)])
/(7*(a + b*x^3)^(2/3))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^6}{\left(bx^3+a\right)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral(x^6/(b*x^3 + a)^(2/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(bx^3+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(x^6/(b*x^3 + a)^(2/3), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(b\,x^3+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^3+a)^(2/3),x)
```

```
[Out] int(x^6/(b*x^3+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(bx^3+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(b*x^3 + a)^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^6}{\left(b\,x^3+a\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^3)^(2/3),x)
```

```
[Out] int(x^6/(a + b*x^3)^(2/3), x)
```

sympy [C] time = 1.48, size = 37, normalized size = 0.97

$$\frac{x^7 \Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\begin{matrix}\frac{2}{3}, \frac{7}{3} \\ \frac{10}{3}\end{matrix} \middle| \frac{b x^3 e^{i\pi}}{a}\right)}{3 a^{\frac{2}{3}} \Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**3+a)**(2/3),x)
```

```
[Out] x**7*gamma(7/3)*hyper((2/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**
(2/3)*gamma(10/3))
```

3.575 $\int \frac{x^3}{\left(a+bx^3\right)^{2/3}}\,dx$

Optimal. Leaf size=38

$$\frac{x^4\sqrt[3]{a+bx^3}\;{}_2F_1\left(1,\frac{5}{3};\frac{7}{3};-\frac{bx^3}{a}\right)}{4a}$$

```
[Out] 1/4*x^4*(b*x^3+a)^(1/3)*hypergeom([1, 5/3],[7/3],-b*x^3/a)/a
```

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.34, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {365, 364}

$$\frac{x^4\left(\frac{bx^3}{a}+1\right)^{2/3}\;{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{7}{3};-\frac{bx^3}{a}\right)}{4\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^3)^(2/3),x]
```

```
[Out] (x^4*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 4/3, 7/3, -((b*x^3)/a)])/
(4*(a + b*x^3)^(2/3))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(a+bx^3\right)^{2/3}}\,dx &= \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{x^3}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{\left(a+bx^3\right)^{2/3}}\\ &= \frac{x^4\left(1+\frac{bx^3}{a}\right)^{2/3}\;{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{7}{3};-\frac{bx^3}{a}\right)}{4\left(a+bx^3\right)^{2/3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 1.34

$$\frac{x^4\left(\frac{bx^3}{a}+1\right)^{2/3}\;{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{7}{3};-\frac{bx^3}{a}\right)}{4\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^3)^(2/3),x]
```

```
[Out] (x^4*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 4/3, 7/3, -((b*x^3)/a)])/
(4*(a + b*x^3)^(2/3))
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3}{\left(bx^3+a\right)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral(x^3/(b*x^3 + a)^(2/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(bx^3+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(x^3/(b*x^3 + a)^(2/3), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(b\,x^3+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a)^(2/3),x)
```

```
[Out] int(x^3/(b*x^3+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(bx^3+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(b*x^3 + a)^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^3}{\left(b\,x^3+a\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^3)^(2/3),x)
```

```
[Out] int(x^3/(a + b*x^3)^(2/3), x)
```

sympy [C] time = 1.38, size = 37, normalized size = 0.97

$$\frac{x^4 \Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix}\frac{2}{3}, \frac{4}{3} \\ \frac{7}{3}\end{matrix} \middle| \frac{b x^3 e^{i\pi}}{a}\right)}{3 a^{\frac{2}{3}} \Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a)**(2/3),x)
```

```
[Out] x**4*gamma(4/3)*hyper((2/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
2/3)*gamma(7/3))
```

3.576 $\int \frac{1}{\left(a+bx^3\right)^{2/3}}\,dx$

Optimal. Leaf size=33

$$\frac{x\sqrt[3]{a+bx^3}\;{}_2F_1\left(\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a}\right)}{a}$$

```
[Out] x*(b*x^3+a)^(1/3)*hypergeom([2/3, 1],[4/3],-b*x^3/a)/a
```

Rubi [A] time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.39, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {246, 245}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\;{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(-2/3),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(a
 + b*x^3)^(2/3)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^{2/3}}\,dx = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{\left(a+bx^3\right)^{2/3}} = \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\;{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}}$$

Mathematica [C] time = 0.22, size = 177, normalized size = 5.36

$$\frac{3\sqrt[3]{2}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}\right)^{2/3}\sqrt{\frac{i\left(\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}+1\right)}{\sqrt{3}+3i}}\;{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};\frac{\left(1-i\sqrt{3}\right)\sqrt[3]{b}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{a}}{2\left(\sqrt[3]{b}\,x+\sqrt[3]{a}\right)}\right)}{\sqrt[3]{b}\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(-2/3),x]
```

```
[Out] (3*2^(1/3)*((-1)^(2/3)*a^(1/3) + b^(1/3)*x)*((a^(1/3) + (-1)^(2/3)*b^(1/3)*
x)/((1 + (-1)^(1/3))*a^(1/3)))^(2/3)*((I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I +
Sqrt[3]))^(1/3)*Hypergeometric2F1[1/3, 2/3, 4/3, ((1 + I*Sqrt[3])*a^(1/3) +
 (1 - I*Sqrt[3])*b^(1/3)*x)/(2*(a^(1/3) + b^(1/3)*x))])/(b^(1/3)*(a + b*x^3
)^(2/3))
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(-2/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(-2/3), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(2/3),x)
```

```
[Out] int(1/(b*x^3+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(-2/3), x)
```

mupad [B] time = 1.02, size = 37, normalized size = 1.12

$$\frac{x\left(\frac{b\,x^3}{a}+1\right)^{2/3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{2}{3};\ \frac{4}{3};\ -\frac{b\,x^3}{a}\right)}{\left(b\,x^3+a\right)^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^3)^(2/3),x)
```

```
[Out] (x*((b*x^3)/a + 1)^(2/3)*hypergeom([1/3, 2/3], 4/3, -(b*x^3)/a))/(a + b*x^3
)^(2/3)
```

sympy [C] time = 0.90, size = 36, normalized size = 1.09

$$\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3}, \frac{2}{3} \\ \frac{4}{3}\end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(2/3),x)
```

```
[Out] x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2/3
)*gamma(4/3))
```

3.577 $\int \frac{1}{x^3\left(a+bx^3\right)^{2/3}}\,dx$

Optimal. Leaf size=38

$$-\frac{\sqrt[3]{a+bx^3}\;{}_2F_1\left(-\frac{1}{3},1;\frac{1}{3};-\frac{bx^3}{a}\right)}{2ax^2}$$

```
[Out] -1/2*(b*x^3+a)^(1/3)*hypergeom([-1/3, 1],[1/3],-b*x^3/a)/a/x^2
```

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.34, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {365, 364}

$$-\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\;{}_2F_1\left(-\frac{2}{3},\frac{2}{3};\frac{1}{3};-\frac{bx^3}{a}\right)}{2x^2\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^3)^(2/3)),x]
```

```
[Out] -((1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[-2/3, 2/3, 1/3, -((b*x^3)/a)])/(2
*x^2*(a + b*x^3)^(2/3))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^3\right)^{2/3}}\,dx = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{x^3\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{\left(a+bx^3\right)^{2/3}}$$
$$= -\frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\;{}_2F_1\left(-\frac{2}{3},\frac{2}{3};\frac{1}{3};-\frac{bx^3}{a}\right)}{2x^2\left(a+bx^3\right)^{2/3}}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 1.34

$$-\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\;{}_2F_1\left(-\frac{2}{3},\frac{2}{3};\frac{1}{3};-\frac{bx^3}{a}\right)}{2x^2\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^3)^(2/3)),x]
```

```
[Out] -1/2*((1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[-2/3, 2/3, 1/3, -((b*x^3)/a)]
)/(x^2*(a + b*x^3)^(2/3))
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{bx^6+ax^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(1/3)/(b*x^6 + a*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*x^3), x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{2}{3}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a)^(2/3),x)
```

```
[Out] int(1/x^3/(b*x^3+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x^3\left(b\,x^3+a\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^3)^(2/3)),x)
```

```
[Out] int(1/(x^3*(a + b*x^3)^(2/3)), x)
```

sympy [C] time = 1.18, size = 41, normalized size = 1.08

$$\frac{\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3}, \frac{2}{3} \\ \frac{1}{3}\end{matrix}\right| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}x^2\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a)**(2/3),x)
```

```
[Out] gamma(-2/3)*hyper((-2/3, 2/3), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2/3
)*x**2*gamma(1/3))
```

3.578 $\int \frac{1}{x^6\left(a+bx^3\right)^{2/3}}\,dx$

Optimal. Leaf size=38

$$-\frac{\sqrt[3]{a+bx^3}\ {}_2F_1\left(-\frac{4}{3},1;-\frac{2}{3};-\frac{bx^3}{a}\right)}{5ax^5}$$

```
[Out] -1/5*(b*x^3+a)^(1/3)*hypergeom([-4/3, 1],[-2/3],-b*x^3/a)/a/x^5
```

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.34, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {365, 364}

$$-\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\ {}_2F_1\left(-\frac{5}{3},\frac{2}{3};-\frac{2}{3};-\frac{bx^3}{a}\right)}{5x^5\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^3)^(2/3)),x]
```

```
[Out] -((1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[-5/3, 2/3, -2/3, -((b*x^3)/a)])/(
5*x^5*(a + b*x^3)^(2/3))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^6\left(a+bx^3\right)^{2/3}}\,dx &= \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{x^6\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{\left(a+bx^3\right)^{2/3}}\\ &= -\frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\ {}_2F_1\left(-\frac{5}{3},\frac{2}{3};-\frac{2}{3};-\frac{bx^3}{a}\right)}{5x^5\left(a+bx^3\right)^{2/3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 1.34

$$-\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\ {}_2F_1\left(-\frac{5}{3},\frac{2}{3};-\frac{2}{3};-\frac{bx^3}{a}\right)}{5x^5\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^3)^(2/3)),x]
```

```
[Out] -1/5*((1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[-5/3, 2/3, -2/3, -((b*x^3)/a)
])/(x^5*(a + b*x^3)^(2/3))
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{bx^9+ax^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(1/3)/(b*x^9 + a*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*x^6), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{2}{3}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^3+a)^(2/3),x)
```

```
[Out] int(1/x^6/(b*x^3+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x^6\left(b\,x^3+a\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^3)^(2/3)),x)
```

```
[Out] int(1/(x^6*(a + b*x^3)^(2/3)), x)
```

sympy [C] time = 1.59, size = 44, normalized size = 1.16

$$\frac{\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{5}{3}, \frac{2}{3} \\ -\frac{2}{3}\end{matrix}\right| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}x^5\Gamma\left(-\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**3+a)**(2/3),x)
```

```
[Out] gamma(-5/3)*hyper((-5/3, 2/3), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2/
3)*x**5*gamma(-2/3))
```

3.579 $\int \frac{1}{\sqrt[3]{a-bx^3}}\,dx$

Optimal. Leaf size=72

$$\frac{\log\left(\sqrt[3]{a-bx^3}+\sqrt[3]{b}\,x\right)}{2\sqrt[3]{b}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a-bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}$$

```
[Out] 1/2*ln(b^(1/3)*x+(-b*x^3+a)^(1/3))/b^(1/3)-1/3*arctan(1/3*(1-2*b^(1/3)*x/(-
b*x^3+a)^(1/3))*3^(1/2))/b^(1/3)*3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {239}

$$\frac{\log\left(\sqrt[3]{a-bx^3}+\sqrt[3]{b}\,x\right)}{2\sqrt[3]{b}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a-bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^(-1/3),x]
```

```
[Out] -(ArcTan[(1 - (2*b^(1/3)*x)/(a - b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)))
+ Log[b^(1/3)*x + (a - b*x^3)^(1/3)]/(2*b^(1/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a-bx^3}}\,dx=-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a-bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{b}\,x+\sqrt[3]{a-bx^3}\right)}{2\sqrt[3]{b}}$$

Mathematica [A] time = 0.11, size = 116, normalized size = 1.61

$$\frac{-\log\left(\frac{b^{2/3}x^2}{\left(a-bx^3\right)^{2/3}}-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a-bx^3}}+1\right)+2\log\left(\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a-bx^3}}+1\right)+2\sqrt{3}\,\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a-bx^3}}-1}{\sqrt{3}}\right)}{6\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^(-1/3),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-1 + (2*b^(1/3)*x)/(a - b*x^3)^(1/3))/Sqrt[3]] - Log[1 +
 (b^(2/3)*x^2)/(a - b*x^3)^(2/3) - (b^(1/3)*x)/(a - b*x^3)^(1/3)] + 2*Log[1
 + (b^(1/3)*x)/(a - b*x^3)^(1/3)])/(6*b^(1/3))
```

fricas [B] time = 0.59, size = 284, normalized size = 3.94

$$\left[\frac{3\sqrt{\frac{1}{3}}b\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(-3bx^3-3\left(-bx^3+a\right)^{\frac{1}{3}}b^{\frac{2}{3}}x^2+3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3-\left(-bx^3+a\right)^{\frac{1}{3}}bx^2-2\left(-bx^3+a\right)^{\frac{2}{3}}b^{\frac{2}{3}}x\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}+2\right.}{6b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*b*sqrt(-1/b^(2/3))*log(-3*b*x^3 - 3*(-b*x^3 + a)^(1/3)*b^
(2/3)*x^2 + 3*sqrt(1/3)*(b^(4/3)*x^3 - (-b*x^3 + a)^(1/3)*b*x^2 - 2*(-b*x^3
 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 2*b^(2/3)*log((b^(1/3)*x +
 (-b*x^3 + a)^(1/3))/x) - b^(2/3)*log((b^(2/3)*x^2 - (-b*x^3 + a)^(1/3)*b^(
1/3)*x + (-b*x^3 + a)^(2/3))/x^2))/b, 1/6*(6*sqrt(1/3)*b^(2/3)*arctan(sqrt(
1/3)*(b^(1/3)*x - 2*(-b*x^3 + a)^(1/3))/(b^(1/3)*x)) + 2*b^(2/3)*log((b^(1/
3)*x + (-b*x^3 + a)^(1/3))/x) - b^(2/3)*log((b^(2/3)*x^2 - (-b*x^3 + a)^(1/
3)*b^(1/3)*x + (-b*x^3 + a)^(2/3))/x^2))/b]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((-b*x^3 + a)^(-1/3), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^3+a)^(1/3),x)
```

```
[Out] int(1/(-b*x^3+a)^(1/3),x)
```

maxima [A] time = 2.95, size = 103, normalized size = 1.43

$$-\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(b^{\frac{1}{3}}-\frac{2\left(-bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{3b^{\frac{1}{3}}}-\frac{\log\left(b^{\frac{2}{3}}-\frac{\left(-bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(-bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{6b^{\frac{1}{3}}}+\frac{\log\left(b^{\frac{1}{3}}+\frac{\left(-bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(b^(1/3) - 2*(-b*x^3 + a)^(1/3)/x)/b^(1/3)
)/b^(1/3) - 1/6*log(b^(2/3) - (-b*x^3 + a)^(1/3)*b^(1/3)/x + (-b*x^3 + a)^(
2/3)/x^2)/b^(1/3) + 1/3*log(b^(1/3) + (-b*x^3 + a)^(1/3)/x)/b^(1/3)
```

mupad [B] time = 1.07, size = 38, normalized size = 0.53

$$\frac{x\left(1-\frac{b\,x^3}{a}\right)^{1/3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{3};\ \frac{4}{3};\ \frac{b\,x^3}{a}\right)}{\left(a-b\,x^3\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*x^3)^(1/3),x)
```

```
[Out] (x*(1 - (b*x^3)/a)^(1/3)*hypergeom([1/3, 1/3], 4/3, (b*x^3)/a))/(a - b*x^3)
^(1/3)
```

sympy [C] time = 1.73, size = 37, normalized size = 0.51

$$\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\,\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**3+a)**(1/3),x)
```

```
[Out] x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*a**(1
/3)*gamma(4/3))
```

3.580 $\int \frac{1}{\sqrt[3]{2+x^3}}\,dx$

Optimal. Leaf size=46

$$\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(\sqrt[3]{x^3+2}-x\right)$$

```
[Out] -1/2*ln(-x+(x^3+2)^(1/3))+1/3*arctan(1/3*(1+2*x/(x^3+2)^(1/3))*3^(1/2))*3^(
1/2)
```

Rubi [A] time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {239}

$$\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(\sqrt[3]{x^3+2}-x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^3)^(-1/3),x]
```

```
[Out] ArcTan[(1 + (2*x)/(2 + x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[-x + (2 + x^3)^(1
/3)]/2
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{2+x^3}}\,dx=\frac{\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(-x+\sqrt[3]{2+x^3}\right)$$

Mathematica [A] time = 0.05, size = 78, normalized size = 1.70

$$-\frac{1}{3}\log\left(1-\frac{x}{\sqrt[3]{x^3+2}}\right)+\frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{6}\log\left(\frac{x}{\sqrt[3]{x^3+2}}+\frac{x^2}{\left(x^3+2\right)^{2/3}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^3)^(-1/3),x]
```

```
[Out] ArcTan[(1 + (2*x)/(2 + x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[1 - x/(2 + x^3)^(
1/3)]/3 + Log[1 + x^2/(2 + x^3)^(2/3) + x/(2 + x^3)^(1/3)]/6
```

fricas [A] time = 0.62, size = 76, normalized size = 1.65

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}\left(x^3+2\right)^{\frac{1}{3}}}{3x}\right)-\frac{1}{3}\log\left(-\frac{x-\left(x^3+2\right)^{\frac{1}{3}}}{x}\right)+\frac{1}{6}\log\left(\frac{x^2+\left(x^3+2\right)^{\frac{1}{3}}x+\left(x^3+2\right)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+2)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^3 + 2)^(1/3))/x) - 1/3*lo
g(-(x - (x^3 + 2)^(1/3))/x) + 1/6*log((x^2 + (x^3 + 2)^(1/3)*x + (x^3 + 2)^
(2/3))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^3 + 2)^(-1/3), x)
```

maple [C] time = 0.16, size = 18, normalized size = 0.39

$$\frac{2^{\frac{2}{3}}x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],-\frac{x^3}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3+2)^(1/3),x)
```

```
[Out] 1/2*2^(2/3)*x*hypergeom([1/3,1/3],[4/3],-1/2*x^3)
```

maxima [A] time = 2.89, size = 69, normalized size = 1.50

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^3+2\right)^{\frac{1}{3}}}{x}+1\right)\right)+\frac{1}{6}\log\left(\frac{\left(x^3+2\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3+2\right)^{\frac{2}{3}}}{x^2}+1\right)-\frac{1}{3}\log\left(\frac{\left(x^3+2\right)^{\frac{1}{3}}}{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+2)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + 2)^(1/3)/x + 1)) + 1/6*log((x^3 +
2)^(1/3)/x + (x^3 + 2)^(2/3)/x^2 + 1) - 1/3*log((x^3 + 2)^(1/3)/x - 1)
```

mupad [B] time = 1.04, size = 16, normalized size = 0.35

$$\frac{2^{2/3}x\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{3};\ \frac{4}{3};\ -\frac{x^3}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3 + 2)^(1/3),x)
```

```
[Out] (2^(2/3)*x*hypergeom([1/3, 1/3], 4/3, -x^3/2))/2
```

sympy [C] time = 1.81, size = 34, normalized size = 0.74

$$\frac{2^{\frac{2}{3}} x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{3}, \frac{1}{3} \\ \frac{4}{3}\end{matrix}\right| \frac{x^3 e^{i\pi}}{2}\right)}{6\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3+2)**(1/3),x)
```

```
[Out] 2**(2/3)*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), x**3*exp_polar(I*pi)/2)/(6*
gamma(4/3))
```

3.581 $\int \frac{x}{\left(1-x^3\right)^{2/3}}\,dx$

Optimal. Leaf size=53

$$-\frac{1}{2}\log\left(-\sqrt[3]{1-x^3}-x\right)-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -1/2*ln(-x-(-x^3+1)^(1/3))-1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3
^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.64, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {331, 292, 31, 634, 618, 204, 628}

$$\frac{1}{6}\log\left(\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}+1\right)-\frac{1}{3}\log\left(\frac{x}{\sqrt[3]{1-x^3}}+1\right)-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 - x^3)^(2/3),x]
```

```
[Out] -(ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + Log[1 + x^2/(1 - x
^3)^(2/3) - x/(1 - x^3)^(1/3)]/6 - Log[1 + x/(1 - x^3)^(1/3)]/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(1-x^3\right)^{2/3}}\,dx &= \text{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) \\
&= -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)\right) + \frac{1}{3}\,\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) \\
&= -\frac{1}{3}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{6}\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx\right. \\
&= \frac{1}{6}\log\left(1+\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{1}{3}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) - \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+\frac{\sqrt[3]{}}{}\right. \\
&= \frac{\tan^{-1}\left(\frac{-1+\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{6}\log\left(1+\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{1}{3}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.70

$$\frac{x^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{x^3}{x^3-1}\right)}{2\left(1-x^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 - x^3)^(2/3),x]
```

```
[Out] (x^2*Hypergeometric2F1[2/3, 1, 5/3, x^3/(-1 + x^3)])/(2*(1 - x^3)^(2/3))
```

fricas [A] time = 0.71, size = 82, normalized size = 1.55

$$-\frac{1}{3}\sqrt{3}\,\arctan\left(-\frac{\sqrt{3}\,x-2\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}}{3\,x}\right)-\frac{1}{3}\,\log\left(\frac{x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)+\frac{1}{6}\,\log\left(\frac{x^2-\left(-x^3+1\right)^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x) - 1/3*
log((x + (-x^3 + 1)^(1/3))/x) + 1/6*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 +
 1)^(2/3))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-x^3+1\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(x/(-x^3 + 1)^(2/3), x)
```

maple [C] time = 0.11, size = 15, normalized size = 0.28

$$\frac{x^2\,\mathrm{hypergeom}\left(\left[\frac{2}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^3+1)^(2/3),x)
```

```
[Out] 1/2*x^2*hypergeom([2/3,2/3],[5/3],x^3)
```

maxima [A] time = 2.89, size = 78, normalized size = 1.47

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(-x^3+1\right)^{\frac{1}{3}}}{x}-1\right)\right)-\frac{1}{3}\log\left(\frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x}+1\right)+\frac{1}{6}\log\left(-\frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x}+\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3)/x - 1)) - 1/3*log((-x^3
 + 1)^(1/3)/x + 1) + 1/6*log(-(-x^3 + 1)^(1/3)/x + (-x^3 + 1)^(2/3)/x^2 + 1
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\left(1-x^3\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1 - x^3)^(2/3),x)
```

```
[Out] int(x/(1 - x^3)^(2/3), x)
```

sympy [C] time = 2.01, size = 31, normalized size = 0.58

$$\frac{x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**3+1)**(2/3),x)
```

```
[Out] x**2*gamma(2/3)*hyper((2/3, 2/3), (5/3,), x**3*exp_polar(2*I*pi))/(3*gamma(
5/3))
```

3.582 $\int \frac{x^2}{\sqrt[4]{2+x^3}}\,dx$

Optimal. Leaf size=13

$$\frac{4}{9}\left(x^3+2\right)^{3/4}$$

```
[Out] 4/9*(x^3+2)^(3/4)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$\frac{4}{9}\left(x^3+2\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 + x^3)^(1/4),x]
```

```
[Out] (4*(2 + x^3)^(3/4))/9
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^2}{\sqrt[4]{2+x^3}}\,dx = \frac{4}{9}\left(2+x^3\right)^{3/4}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{4}{9}\left(x^3+2\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 + x^3)^(1/4),x]
```

```
[Out] (4*(2 + x^3)^(3/4))/9
```

fricas [A] time = 0.74, size = 9, normalized size = 0.69

$$\frac{4}{9}\left(x^3+2\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^3+2)^(1/4),x, algorithm="fricas")
```

```
[Out] 4/9*(x^3 + 2)^(3/4)
```

giac [A] time = 0.15, size = 9, normalized size = 0.69

$$\frac{4}{9}\left(x^3+2\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^3+2)^(1/4),x, algorithm="giac")
[Out] 4/9*(x^3 + 2)^(3/4)
```

maple [A] time = 0.00, size = 10, normalized size = 0.77

$$\frac{4\left(x^3+2\right)^{\frac{3}{4}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^3+2)^(1/4),x)
[Out] 4/9*(x^3+2)^(3/4)
```

maxima [A] time = 1.27, size = 9, normalized size = 0.69

$$\frac{4}{9}\left(x^3+2\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^3+2)^(1/4),x, algorithm="maxima")
[Out] 4/9*(x^3 + 2)^(3/4)
```

mupad [B] time = 1.13, size = 9, normalized size = 0.69

$$\frac{4\left(x^3+2\right)^{3/4}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^3 + 2)^(1/4),x)
[Out] (4*(x^3 + 2)^(3/4))/9
```

sympy [A] time = 0.72, size = 10, normalized size = 0.77

$$\frac{4\left(x^3+2\right)^{\frac{3}{4}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**3+2)**(1/4),x)
[Out] 4*(x**3 + 2)**(3/4)/9
```

3.583 $\int x^m \left(a+bx^3\right)^8 dx$

Optimal. Leaf size=151

$$\frac{a^8 x^{m+1}}{m+1}+\frac{8a^7 b x^{m+4}}{m+4}+\frac{28a^6 b^2 x^{m+7}}{m+7}+\frac{56a^5 b^3 x^{m+10}}{m+10}+\frac{70a^4 b^4 x^{m+13}}{m+13}+\frac{56a^3 b^5 x^{m+16}}{m+16}+\frac{28a^2 b^6 x^{m+19}}{m+19}+\frac{8ab^7 x^{m+22}}{m+22}+\frac{b^8 x^{m+}}{m+2}$$

```
[Out] a^8*x^(1+m)/(1+m)+8*a^7*b*x^(4+m)/(4+m)+28*a^6*b^2*x^(7+m)/(7+m)+56*a^5*b^3
*x^(10+m)/(10+m)+70*a^4*b^4*x^(13+m)/(13+m)+56*a^3*b^5*x^(16+m)/(16+m)+28*a
^2*b^6*x^(19+m)/(19+m)+8*a*b^7*x^(22+m)/(22+m)+b^8*x^(25+m)/(25+m)
```

Rubi [A] time = 0.08, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{28a^6 b^2 x^{m+7}}{m+7}+\frac{56a^5 b^3 x^{m+10}}{m+10}+\frac{70a^4 b^4 x^{m+13}}{m+13}+\frac{56a^3 b^5 x^{m+16}}{m+16}+\frac{28a^2 b^6 x^{m+19}}{m+19}+\frac{8a^7 b x^{m+4}}{m+4}+\frac{a^8 x^{m+1}}{m+1}+\frac{8ab^7 x^{m+22}}{m+22}+\frac{b^8 x^{m+}}{m+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^3)^8,x]
```

```
[Out] (a^8*x^(1 + m))/(1 + m) + (8*a^7*b*x^(4 + m))/(4 + m) + (28*a^6*b^2*x^(7 +
m))/(7 + m) + (56*a^5*b^3*x^(10 + m))/(10 + m) + (70*a^4*b^4*x^(13 + m))/(1
3 + m) + (56*a^3*b^5*x^(16 + m))/(16 + m) + (28*a^2*b^6*x^(19 + m))/(19 + m
) + (8*a*b^7*x^(22 + m))/(22 + m) + (b^8*x^(25 + m))/(25 + m)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int x^m \left(a+bx^3\right)^8 dx = \int \left(a^8 x^m + 8a^7 b x^{3+m} + 28a^6 b^2 x^{6+m} + 56a^5 b^3 x^{9+m} + 70a^4 b^4 x^{12+m} + 56a^3 b^5 x^{15+m} + 28a^2 b^{}$$
$$= \frac{a^8 x^{1+m}}{1+m} + \frac{8a^7 b x^{4+m}}{4+m} + \frac{28a^6 b^2 x^{7+m}}{7+m} + \frac{56a^5 b^3 x^{10+m}}{10+m} + \frac{70a^4 b^4 x^{13+m}}{13+m} + \frac{56a^3 b^5 x^{16+m}}{16+m} + \frac{28a}{1}$$

Mathematica [A] time = 0.11, size = 136, normalized size = 0.90

$$x^{m+1}\left(\frac{a^8}{m+1}+\frac{8a^7 b x^3}{m+4}+\frac{28a^6 b^2 x^6}{m+7}+\frac{56a^5 b^3 x^9}{m+10}+\frac{70a^4 b^4 x^{12}}{m+13}+\frac{56a^3 b^5 x^{15}}{m+16}+\frac{28a^2 b^6 x^{18}}{m+19}+\frac{8ab^7 x^{21}}{m+22}+\frac{b^8 x^{24}}{m+25}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^3)^8,x]
```

```
[Out] x^(1 + m)*(a^8/(1 + m) + (8*a^7*b*x^3)/(4 + m) + (28*a^6*b^2*x^6)/(7 + m) +
 (56*a^5*b^3*x^9)/(10 + m) + (70*a^4*b^4*x^12)/(13 + m) + (56*a^3*b^5*x^15)
/(16 + m) + (28*a^2*b^6*x^18)/(19 + m) + (8*a*b^7*x^21)/(22 + m) + (b^8*x^2
4)/(25 + m))
```

fricas [B] time = 0.73, size = 847, normalized size = 5.61

$$\left(\left(b^8 m^8 + 92\,b^8 m^7 + 3514\,b^8 m^6 + 72128\,b^8 m^5 + 859369\,b^8 m^4 + 5974388\,b^8 m^3 + 22963996\,b^8 m^2 + 42124592\,b^8 m\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)^8,x, algorithm="fricas")
```

```
[Out] ((b^8*m^8 + 92*b^8*m^7 + 3514*b^8*m^6 + 72128*b^8*m^5 + 859369*b^8*m^4 + 59
74388*b^8*m^3 + 22963996*b^8*m^2 + 42124592*b^8*m + 24344320*b^8)*x^25 + 8*
(a*b^7*m^8 + 95*a*b^7*m^7 + 3724*a*b^7*m^6 + 78050*a*b^7*m^5 + 945469*a*b^7
*m^4 + 6658295*a*b^7*m^3 + 25841206*a*b^7*m^2 + 47717960*a*b^7*m + 27664000
*a*b^7)*x^22 + 28*(a^2*b^6*m^8 + 98*a^2*b^6*m^7 + 3952*a^2*b^6*m^6 + 84890*
a^2*b^6*m^5 + 1049659*a^2*b^6*m^4 + 7515092*a^2*b^6*m^3 + 29536948*a^2*b^6*
m^2 + 55022480*a^2*b^6*m + 32032000*a^2*b^6)*x^19 + 56*(a^3*b^5*m^8 + 101*a
^3*b^5*m^7 + 4198*a^3*b^5*m^6 + 92810*a^3*b^5*m^5 + 1177609*a^3*b^5*m^4 + 8
616869*a^3*b^5*m^3 + 34453792*a^3*b^5*m^2 + 64963820*a^3*b^5*m + 38038000*a
^3*b^5)*x^16 + 70*(a^4*b^4*m^8 + 104*a^4*b^4*m^7 + 4462*a^4*b^4*m^6 + 10197
2*a^4*b^4*m^5 + 1336933*a^4*b^4*m^4 + 10078484*a^4*b^4*m^3 + 41303404*a^4*b
^4*m^2 + 79280240*a^4*b^4*m + 46816000*a^4*b^4)*x^13 + 56*(a^5*b^3*m^8 + 10
7*a^5*b^3*m^7 + 4744*a^5*b^3*m^6 + 112538*a^5*b^3*m^5 + 1537189*a^5*b^3*m^4
 + 12086723*a^5*b^3*m^3 + 51456466*a^5*b^3*m^2 + 101659832*a^5*b^3*m + 6086
0800*a^5*b^3)*x^10 + 28*(a^6*b^2*m^8 + 110*a^6*b^2*m^7 + 5044*a^6*b^2*m^6 +
 124670*a^6*b^2*m^5 + 1789879*a^6*b^2*m^4 + 14929460*a^6*b^2*m^3 + 67817476
*a^6*b^2*m^2 + 141502160*a^6*b^2*m + 86944000*a^6*b^2)*x^7 + 8*(a^7*b*m^8 +
 113*a^7*b*m^7 + 5362*a^7*b*m^6 + 138530*a^7*b*m^5 + 2108449*a^7*b*m^4 + 19
024817*a^7*b*m^3 + 96224428*a^7*b*m^2 + 231326780*a^7*b*m + 152152000*a^7*b
)*x^4 + (a^8*m^8 + 116*a^8*m^7 + 5698*a^8*m^6 + 154280*a^8*m^5 + 2508289*a^
8*m^4 + 24950324*a^8*m^3 + 147373372*a^8*m^2 + 468851120*a^8*m + 608608000*
a^8)*x)*x^m/(m^9 + 117*m^8 + 5814*m^7 + 159978*m^6 + 2662569*m^5 + 27458613
*m^4 + 172323696*m^3 + 616224492*m^2 + 1077459120*m + 608608000)
```

giac [**B**] time = 0.32, size = 1269, normalized size = 8.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)^8,x, algorithm="giac")
```

```
[Out] (b^8*m^8*x^25*x^m + 92*b^8*m^7*x^25*x^m + 3514*b^8*m^6*x^25*x^m + 8*a*b^7*m
^8*x^22*x^m + 72128*b^8*m^5*x^25*x^m + 760*a*b^7*m^7*x^22*x^m + 859369*b^8*
m^4*x^25*x^m + 29792*a*b^7*m^6*x^22*x^m + 5974388*b^8*m^3*x^25*x^m + 28*a^2
*b^6*m^8*x^19*x^m + 624400*a*b^7*m^5*x^22*x^m + 22963996*b^8*m^2*x^25*x^m +
 2744*a^2*b^6*m^7*x^19*x^m + 7563752*a*b^7*m^4*x^22*x^m + 42124592*b^8*m*x^
25*x^m + 110656*a^2*b^6*m^6*x^19*x^m + 53266360*a*b^7*m^3*x^22*x^m + 243443
20*b^8*x^25*x^m + 56*a^3*b^5*m^8*x^16*x^m + 2376920*a^2*b^6*m^5*x^19*x^m +
206729648*a*b^7*m^2*x^22*x^m + 5656*a^3*b^5*m^7*x^16*x^m + 29390452*a^2*b^6
*m^4*x^19*x^m + 381743680*a*b^7*m*x^22*x^m + 235088*a^3*b^5*m^6*x^16*x^m +
210422576*a^2*b^6*m^3*x^19*x^m + 221312000*a*b^7*x^22*x^m + 70*a^4*b^4*m^8*
x^13*x^m + 5197360*a^3*b^5*m^5*x^16*x^m + 827034544*a^2*b^6*m^2*x^19*x^m +
7280*a^4*b^4*m^7*x^13*x^m + 65946104*a^3*b^5*m^4*x^16*x^m + 1540629440*a^2*
b^6*m*x^19*x^m + 312340*a^4*b^4*m^6*x^13*x^m + 482544664*a^3*b^5*m^3*x^16*x
^m + 896896000*a^2*b^6*x^19*x^m + 56*a^5*b^3*m^8*x^10*x^m + 7138040*a^4*b^4
*m^5*x^13*x^m + 1929412352*a^3*b^5*m^2*x^16*x^m + 5992*a^5*b^3*m^7*x^10*x^m
 + 93585310*a^4*b^4*m^4*x^13*x^m + 3637973920*a^3*b^5*m*x^16*x^m + 265664*a
^5*b^3*m^6*x^10*x^m + 705493880*a^4*b^4*m^3*x^13*x^m + 2130128000*a^3*b^5*x
^16*x^m + 28*a^6*b^2*m^8*x^7*x^m + 6302128*a^5*b^3*m^5*x^10*x^m + 289123828
0*a^4*b^4*m^2*x^13*x^m + 3080*a^6*b^2*m^7*x^7*x^m + 86082584*a^5*b^3*m^4*x^
10*x^m + 5549616800*a^4*b^4*m*x^13*x^m + 141232*a^6*b^2*m^6*x^7*x^m + 67685
6488*a^5*b^3*m^3*x^10*x^m + 3277120000*a^4*b^4*x^13*x^m + 8*a^7*b*m^8*x^4*x
^m + 3490760*a^6*b^2*m^5*x^7*x^m + 2881562096*a^5*b^3*m^2*x^10*x^m + 904*a^
7*b*m^7*x^4*x^m + 50116612*a^6*b^2*m^4*x^7*x^m + 5692950592*a^5*b^3*m*x^10*
x^m + 42896*a^7*b*m^6*x^4*x^m + 418024880*a^6*b^2*m^3*x^7*x^m + 3408204800*
a^5*b^3*x^10*x^m + a^8*m^8*x*x^m + 1108240*a^7*b*m^5*x^4*x^m + 1898889328*a
```

```
^6*b^2*m^2*x^7*x^m + 116*a^8*m^7*x*x^m + 16867592*a^7*b*m^4*x^4*x^m + 39620
60480*a^6*b^2*m*x^7*x^m + 5698*a^8*m^6*x*x^m + 152198536*a^7*b*m^3*x^4*x^m
+ 2434432000*a^6*b^2*x^7*x^m + 154280*a^8*m^5*x*x^m + 769795424*a^7*b*m^2*x
^4*x^m + 2508289*a^8*m^4*x*x^m + 1850614240*a^7*b*m*x^4*x^m + 24950324*a^8*
m^3*x*x^m + 1217216000*a^7*b*x^4*x^m + 147373372*a^8*m^2*x*x^m + 468851120*
a^8*m*x*x^m + 608608000*a^8*x*x^m)/(m^9 + 117*m^8 + 5814*m^7 + 159978*m^6 +
 2662569*m^5 + 27458613*m^4 + 172323696*m^3 + 616224492*m^2 + 1077459120*m
+ 608608000)
```

maple [B] time = 0.01, size = 1023, normalized size = 6.77

$$\frac{\left(b^8m^8x^{24}+92b^8m^7x^{24}+3514b^8m^6x^{24}+8a\,b^7m^8x^{21}+72128b^8m^5x^{24}+760a\,b^7m^7x^{21}+859369b^8m^4x^{24}+2979\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^3+a)^8,x)
```

```
[Out] x^(m+1)*(b^8*m^8*x^24+92*b^8*m^7*x^24+3514*b^8*m^6*x^24+8*a*b^7*m^8*x^21+72
128*b^8*m^5*x^24+760*a*b^7*m^7*x^21+859369*b^8*m^4*x^24+29792*a*b^7*m^6*x^2
1+5974388*b^8*m^3*x^24+28*a^2*b^6*m^8*x^18+624400*a*b^7*m^5*x^21+22963996*b
^8*m^2*x^24+2744*a^2*b^6*m^7*x^18+7563752*a*b^7*m^4*x^21+42124592*b^8*m*x^2
4+110656*a^2*b^6*m^6*x^18+53266360*a*b^7*m^3*x^21+24344320*b^8*x^24+56*a^3*
b^5*m^8*x^15+2376920*a^2*b^6*m^5*x^18+206729648*a*b^7*m^2*x^21+5656*a^3*b^5
*m^7*x^15+29390452*a^2*b^6*m^4*x^18+381743680*a*b^7*m*x^21+235088*a^3*b^5*m
^6*x^15+210422576*a^2*b^6*m^3*x^18+221312000*a*b^7*x^21+70*a^4*b^4*m^8*x^12
+5197360*a^3*b^5*m^5*x^15+827034544*a^2*b^6*m^2*x^18+7280*a^4*b^4*m^7*x^12+
65946104*a^3*b^5*m^4*x^15+1540629440*a^2*b^6*m*x^18+312340*a^4*b^4*m^6*x^12
+482544664*a^3*b^5*m^3*x^15+896896000*a^2*b^6*x^18+56*a^5*b^3*m^8*x^9+71380
40*a^4*b^4*m^5*x^12+1929412352*a^3*b^5*m^2*x^15+5992*a^5*b^3*m^7*x^9+935853
10*a^4*b^4*m^4*x^12+3637973920*a^3*b^5*m*x^15+265664*a^5*b^3*m^6*x^9+705493
880*a^4*b^4*m^3*x^12+2130128000*a^3*b^5*x^15+28*a^6*b^2*m^8*x^6+6302128*a^5
*b^3*m^5*x^9+2891238280*a^4*b^4*m^2*x^12+3080*a^6*b^2*m^7*x^6+86082584*a^5*
b^3*m^4*x^9+5549616800*a^4*b^4*m*x^12+141232*a^6*b^2*m^6*x^6+676856488*a^5*
b^3*m^3*x^9+3277120000*a^4*b^4*x^12+8*a^7*b*m^8*x^3+3490760*a^6*b^2*m^5*x^6
+2881562096*a^5*b^3*m^2*x^9+904*a^7*b*m^7*x^3+50116612*a^6*b^2*m^4*x^6+5692
950592*a^5*b^3*m*x^9+42896*a^7*b*m^6*x^3+418024880*a^6*b^2*m^3*x^6+34082048
00*a^5*b^3*x^9+a^8*m^8+1108240*a^7*b*m^5*x^3+1898889328*a^6*b^2*m^2*x^6+116
*a^8*m^7+16867592*a^7*b*m^4*x^3+3962060480*a^6*b^2*m*x^6+5698*a^8*m^6+15219
8536*a^7*b*m^3*x^3+2434432000*a^6*b^2*x^6+154280*a^8*m^5+769795424*a^7*b*m^
2*x^3+2508289*a^8*m^4+1850614240*a^7*b*m*x^3+24950324*a^8*m^3+1217216000*a^
7*b*x^3+147373372*a^8*m^2+468851120*a^8*m+608608000*a^8)/(m+1)/(10+m)/(m+13
)/(16+m)/(19+m)/(22+m)/(25+m)/(m+4)/(m+7)
```

maxima [A] time = 1.36, size = 151, normalized size = 1.00

$$\frac{b^8x^{m+25}}{m+25}+\frac{8\,ab^7x^{m+22}}{m+22}+\frac{28\,a^2b^6x^{m+19}}{m+19}+\frac{56\,a^3b^5x^{m+16}}{m+16}+\frac{70\,a^4b^4x^{m+13}}{m+13}+\frac{56\,a^5b^3x^{m+10}}{m+10}+\frac{28\,a^6b^2x^{m+7}}{m+7}+\frac{8\,a^7bx^{m+4}}{m+4}+\frac{a^8}{m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)^8,x, algorithm="maxima")
```

```
[Out] b^8*x^(m + 25)/(m + 25) + 8*a*b^7*x^(m + 22)/(m + 22) + 28*a^2*b^6*x^(m + 1
9)/(m + 19) + 56*a^3*b^5*x^(m + 16)/(m + 16) + 70*a^4*b^4*x^(m + 13)/(m + 1
3) + 56*a^5*b^3*x^(m + 10)/(m + 10) + 28*a^6*b^2*x^(m + 7)/(m + 7) + 8*a^7*
b*x^(m + 4)/(m + 4) + a^8*x^(m + 1)/(m + 1)
```

mupad [B] time = 2.18, size = 860, normalized size = 5.70

$$\frac{a^8\,x\,x^m\left(m^8+116\,m^7+5698\,m^6+154280\,m^5+2508289\,m^4+24950324\,m^3+147373372\,m^2+468851120\right.}{m^9+117\,m^8+5814\,m^7+159978\,m^6+2662569\,m^5+27458613\,m^4+172323696\,m^3+616224492\,m^2+107745}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^3)^8,x)
```

```
[Out] (a^8*x*x^m*(468851120*m + 147373372*m^2 + 24950324*m^3 + 2508289*m^4 + 1542
80*m^5 + 5698*m^6 + 116*m^7 + m^8 + 608608000))/(1077459120*m + 616224492*m
^2 + 172323696*m^3 + 27458613*m^4 + 2662569*m^5 + 159978*m^6 + 5814*m^7 + 1
17*m^8 + m^9 + 608608000) + (b^8*x^m*x^25*(42124592*m + 22963996*m^2 + 5974
388*m^3 + 859369*m^4 + 72128*m^5 + 3514*m^6 + 92*m^7 + m^8 + 24344320))/(10
77459120*m + 616224492*m^2 + 172323696*m^3 + 27458613*m^4 + 2662569*m^5 + 1
59978*m^6 + 5814*m^7 + 117*m^8 + m^9 + 608608000) + (28*a^2*b^6*x^m*x^19*(5
5022480*m + 29536948*m^2 + 7515092*m^3 + 1049659*m^4 + 84890*m^5 + 3952*m^6
 + 98*m^7 + m^8 + 32032000))/(1077459120*m + 616224492*m^2 + 172323696*m^3
+ 27458613*m^4 + 2662569*m^5 + 159978*m^6 + 5814*m^7 + 117*m^8 + m^9 + 6086
08000) + (56*a^3*b^5*x^m*x^16*(64963820*m + 34453792*m^2 + 8616869*m^3 + 11
77609*m^4 + 92810*m^5 + 4198*m^6 + 101*m^7 + m^8 + 38038000))/(1077459120*m
 + 616224492*m^2 + 172323696*m^3 + 27458613*m^4 + 2662569*m^5 + 159978*m^6
+ 5814*m^7 + 117*m^8 + m^9 + 608608000) + (70*a^4*b^4*x^m*x^13*(79280240*m
+ 41303404*m^2 + 10078484*m^3 + 1336933*m^4 + 101972*m^5 + 4462*m^6 + 104*m
^7 + m^8 + 46816000))/(1077459120*m + 616224492*m^2 + 172323696*m^3 + 27458
613*m^4 + 2662569*m^5 + 159978*m^6 + 5814*m^7 + 117*m^8 + m^9 + 608608000)
+ (56*a^5*b^3*x^m*x^10*(101659832*m + 51456466*m^2 + 12086723*m^3 + 1537189
*m^4 + 112538*m^5 + 4744*m^6 + 107*m^7 + m^8 + 60860800))/(1077459120*m + 6
16224492*m^2 + 172323696*m^3 + 27458613*m^4 + 2662569*m^5 + 159978*m^6 + 58
14*m^7 + 117*m^8 + m^9 + 608608000) + (28*a^6*b^2*x^m*x^7*(141502160*m + 67
817476*m^2 + 14929460*m^3 + 1789879*m^4 + 124670*m^5 + 5044*m^6 + 110*m^7 +
 m^8 + 86944000))/(1077459120*m + 616224492*m^2 + 172323696*m^3 + 27458613*
m^4 + 2662569*m^5 + 159978*m^6 + 5814*m^7 + 117*m^8 + m^9 + 608608000) + (8
*a*b^7*x^m*x^22*(47717960*m + 25841206*m^2 + 6658295*m^3 + 945469*m^4 + 780
50*m^5 + 3724*m^6 + 95*m^7 + m^8 + 27664000))/(1077459120*m + 616224492*m^2
 + 172323696*m^3 + 27458613*m^4 + 2662569*m^5 + 159978*m^6 + 5814*m^7 + 117
*m^8 + m^9 + 608608000) + (8*a^7*b*x^m*x^4*(231326780*m + 96224428*m^2 + 19
024817*m^3 + 2108449*m^4 + 138530*m^5 + 5362*m^6 + 113*m^7 + m^8 + 15215200
0))/(1077459120*m + 616224492*m^2 + 172323696*m^3 + 27458613*m^4 + 2662569*
m^5 + 159978*m^6 + 5814*m^7 + 117*m^8 + m^9 + 608608000)
```

sympy [A] time = 47.37, size = 5902, normalized size = 39.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**3+a)**8,x)
```

```
[Out] Piecewise((-a**8/(24*x**24) - 8*a**7*b/(21*x**21) - 14*a**6*b**2/(9*x**18)
- 56*a**5*b**3/(15*x**15) - 35*a**4*b**4/(6*x**12) - 56*a**3*b**5/(9*x**9)
- 14*a**2*b**6/(3*x**6) - 8*a*b**7/(3*x**3) + b**8*log(x), Eq(m, -25)), (-a
**8/(21*x**21) - 4*a**7*b/(9*x**18) - 28*a**6*b**2/(15*x**15) - 14*a**5*b**
3/(3*x**12) - 70*a**4*b**4/(9*x**9) - 28*a**3*b**5/(3*x**6) - 28*a**2*b**6/
(3*x**3) + 8*a*b**7*log(x) + b**8*x**3/3, Eq(m, -22)), (-a**8/(18*x**18) -
8*a**7*b/(15*x**15) - 7*a**6*b**2/(3*x**12) - 56*a**5*b**3/(9*x**9) - 35*a*
*4*b**4/(3*x**6) - 56*a**3*b**5/(3*x**3) + 28*a**2*b**6*log(x) + 8*a*b**7*x
**3/3 + b**8*x**6/6, Eq(m, -19)), (-a**8/(15*x**15) - 2*a**7*b/(3*x**12) -
28*a**6*b**2/(9*x**9) - 28*a**5*b**3/(3*x**6) - 70*a**4*b**4/(3*x**3) + 56*
a**3*b**5*log(x) + 28*a**2*b**6*x**3/3 + 4*a*b**7*x**6/3 + b**8*x**9/9, Eq(
m, -16)), (-a**8/(12*x**12) - 8*a**7*b/(9*x**9) - 14*a**6*b**2/(3*x**6) - 5
6*a**5*b**3/(3*x**3) + 70*a**4*b**4*log(x) + 56*a**3*b**5*x**3/3 + 14*a**2*
b**6*x**6/3 + 8*a*b**7*x**9/9 + b**8*x**12/12, Eq(m, -13)), (-a**8/(9*x**9)
 - 4*a**7*b/(3*x**6) - 28*a**6*b**2/(3*x**3) + 56*a**5*b**3*log(x) + 70*a**
4*b**4*x**3/3 + 28*a**3*b**5*x**6/3 + 28*a**2*b**6*x**9/9 + 2*a*b**7*x**12/
3 + b**8*x**15/15, Eq(m, -10)), (-a**8/(6*x**6) - 8*a**7*b/(3*x**3) + 28*a*
```

```
*6*b**2*log(x) + 56*a**5*b**3*x**3/3 + 35*a**4*b**4*x**6/3 + 56*a**3*b**5*x
**9/9 + 7*a**2*b**6*x**12/3 + 8*a*b**7*x**15/15 + b**8*x**18/18, Eq(m, -7))
, (-a**8/(3*x**3) + 8*a**7*b*log(x) + 28*a**6*b**2*x**3/3 + 28*a**5*b**3*x*
*6/3 + 70*a**4*b**4*x**9/9 + 14*a**3*b**5*x**12/3 + 28*a**2*b**6*x**15/15 +
 4*a*b**7*x**18/9 + b**8*x**21/21, Eq(m, -4)), (a**8*log(x) + 8*a**7*b*x**3
/3 + 14*a**6*b**2*x**6/3 + 56*a**5*b**3*x**9/9 + 35*a**4*b**4*x**12/6 + 56*
a**3*b**5*x**15/15 + 14*a**2*b**6*x**18/9 + 8*a*b**7*x**21/21 + b**8*x**24/
24, Eq(m, -1)), (a**8*m**8*x*x**m/(m**9 + 117*m**8 + 5814*m**7 + 159978*m**
6 + 2662569*m**5 + 27458613*m**4 + 172323696*m**3 + 616224492*m**2 + 107745
9120*m + 608608000) + 116*a**8*m**7*x*x**m/(m**9 + 117*m**8 + 5814*m**7 + 1
59978*m**6 + 2662569*m**5 + 27458613*m**4 + 172323696*m**3 + 616224492*m**2
 + 1077459120*m + 608608000) + 5698*a**8*m**6*x*x**m/(m**9 + 117*m**8 + 581
4*m**7 + 159978*m**6 + 2662569*m**5 + 27458613*m**4 + 172323696*m**3 + 6162
24492*m**2 + 1077459120*m + 608608000) + 154280*a**8*m**5*x*x**m/(m**9 + 11
7*m**8 + 5814*m**7 + 159978*m**6 + 2662569*m**5 + 27458613*m**4 + 172323696
*m**3 + 616224492*m**2 + 1077459120*m + 608608000) + 2508289*a**8*m**4*x*x*
*m/(m**9 + 117*m**8 + 5814*m**7 + 159978*m**6 + 2662569*m**5 + 27458613*m**
4 + 172323696*m**3 + 616224492*m**2 + 1077459120*m + 608608000) + 24950324*
a**8*m**3*x*x**m/(m**9 + 117*m**8 + 5814*m**7 + 159978*m**6 + 2662569*m**5
+ 27458613*m**4 + 172323696*m**3 + 616224492*m**2 + 1077459120*m + 60860800
0) + 147373372*a**8*m**2*x*x**m/(m**9 + 117*m**8 + 5814*m**7 + 159978*m**6
+ 2662569*m**5 + 27458613*m**4 + 172323696*m**3 + 616224492*m**2 + 10774591
20*m + 608608000) + 468851120*a**8*m*x*x**m/(m**9 + 117*m**8 + 5814*m**7 +
159978*m**6 + 2662569*m**5 + 27458613*m**4 + 172323696*m**3 + 616224492*m**
2 + 1077459120*m + 608608000) + 608608000*a**8*x*x**m/(m**9 + 117*m**8 + 58
14*m**7 + 159978*m**6 + 2662569*m**5 + 27458613*m**4 + 172323696*m**3 + 616
224492*m**2 + 1077459120*m + 608608000) + 8*a**7*b*m**8*x**4*x**m/(m**9 + 1
17*m**8 + 5814*m**7 + 159978*m**6 + 2662569*m**5 + 27458613*m**4 + 17232369
6*m**3 + 616224492*m**2 + 1077459120*m + 608608000) + 904*a**7*b*m**7*x**4*
x**m/(m**9 + 117*m**8 + 5814*m**7 + 159978*m**6 + 2662569*m**5 + 27458613*m
**4 + 172323696*m**3 + 616224492*m**2 + 1077459120*m + 608608000) + 42896*a
**7*b*m**6*x**4*x**m/(m**9 + 117*m**8 + 5814*m**7 + 159978*m**6 + 2662569*m
**5 + 27458613*m**4 + 172323696*m**3 + 616224492*m**2 + 1077459120*m + 6086
08000) + 1108240*a**7*b*m**5*x**4*x**m/(m**9 + 117*m**8 + 5814*m**7 + 15997
8*m**6 + 2662569*m**5 + 27458613*m**4 + 172323696*m**3 + 616224492*m**2 + 1
077459120*m + 608608000) + 16867592*a**7*b*m**4*x**4*x**m/(m**9 + 117*m**8
+ 5814*m**7 + 159978*m**6 + 2662569*m**5 + 27458613*m**4 + 172323696*m**3 +
 616224492*m**2 + 1077459120*m + 608608000) + 152198536*a**7*b*m**3*x**4*x*
*m/(m**9 + 117*m**8 + 5814*m**7 + 159978*m**6 + 2662569*m**5 + 27458613*m**
4 + 172323696*m**3 + 616224492*m**2 + 1077459120*m + 608608000) + 769795424
*a**7*b*m**2*x**4*x**m/(m**9 + 117*m**8 + 5814*m**7 + 159978*m**6 + 2662569
*m**5 + 27458613*m**4 + 172323696*m**3 + 616224492*m**2 + 1077459120*m + 60
8608000) + 1850614240*a**7*b*m*x**4*x**m/(m**9 + 117*m**8 + 5814*m**7 + 159
978*m**6 + 2662569*m**5 + 27458613*m**4 + 172323696*m**3 + 616224492*m**2 +
 1077459120*m + 608608000) + 1217216000*a**7*b*x**4*x**m/(m**9 + 117*m**8 +
 5814*m**7 + 159978*m**6 + 2662569*m**5 + 27458613*m**4 + 172323696*m**3 +
616224492*m**2 + 1077459120*m + 608608000) + 28*a**6*b**2*m**8*x**7*x**m/(m
**9 + 117*m**8 + 5814*m**7 + 159978*m**6 + 2662569*m**5 + 27458613*m**4 + 1
72323696*m**3 + 616224492*m**2 + 1077459120*m + 608608000) + 3080*a**6*b**2
*m**7*x**7*x**m/(m**9 + 117*m**8 + 5814*m**7 + 159978*m**6 + 2662569*m**5 +
 27458613*m**4 + 172323696*m**3 + 616224492*m**2 + 1077459120*m + 608608000
) + 141232*a**6*b**2*m**6*x**7*x**m/(m**9 + 117*m**8 + 5814*m**7 + 159978*m
**6 + 2662569*m**5 + 27458613*m**4 + 172323696*m**3 + 616224492*m**2 + 1077
459120*m + 608608000) + 3490760*a**6*b**2*m**5*x**7*x**m/(m**9 + 117*m**8 +
 5814*m**7 + 159978*m**6 + 2662569*m**5 + 27458613*m**4 + 172323696*m**3 +
616224492*m**2 + 1077459120*m + 608608000) + 50116612*a**6*b**2*m**4*x**7*x
**m/(m**9 + 117*m**8 + 5814*m**7 + 159978*m**6 + 2662569*m**5 + 27458613*m*
*4 + 172323696*m**3 + 616224492*m**2 + 1077459120*m + 608608000) + 41802488
0*a**6*b**2*m**3*x**7*x**m/(m**9 + 117*m**8 + 5814*m**7 + 159978*m**6 + 266
```

```
2569*m**5 + 27458613*m**4 + 172323696*m**3 + 616224492*m**2 + 1077459120*m
 + 608608000) + 1898889328*a**6*b**2*m**2*x**7*x**m/(m**9 + 117*m**8 + 5814*
m**7 + 159978*m**6 + 2662569*m**5 + 27458613*m**4 + 172323696*m**3 + 616224
492*m**2 + 1077459120*m + 608608000) + 3962060480*a**6*b**2*m*x**7*x**m/(m*
*9 + 117*m**8 + 5814*m**7 + 159978*m**6 + 2662569*m**5 + 27458613*m**4 + 17
2323696*m**3 + 616224492*m**2 + 1077459120*m + 608608000) + 2434432000*a**6
*b**2*x**7*x**m/(m**9 + 117*m**8 + 5814*m**7 + 159978*m**6 + 2662569*m**5 +
 27458613*m**4 + 172323696*m**3 + 616224492*m**2 + 1077459120*m + 608608000
) + 56*a**5*b**3*m**8*x**10*x**m/(m**9 + 117*m**8 + 5814*m**7 + 159978*m**6
 + 2662569*m**5 + 27458613*m**4 + 172323696*m**3 + 616224492*m**2 + 1077459
120*m + 608608000) + 5992*a**5*b**3*m**7*x**10*x**m/(m**9 + 117*m**8 + 5814
*m**7 + 159978*m**6 + 2662569*m**5 + 27458613*m**4 + 172323696*m**3 + 61622
4492*m**2 + 1077459120*m + 608608000) + 265664*a**5*b**3*m**6*x**10*x**m/(m
**9 + 117*m**8 + 5814*m**7 + 159978*m**6 + 2662569*m**5 + 27458613*m**4 + 1
72323696*m**3 + 616224492*m**2 + 1077459120*m + 608608000) + 6302128*a**5*b
**3*m**5*x**10*x**m/(m**9 + 117*m**8 + 5814*m**7 + 159978*m**6 + 2662569*m*
*5 + 27458613*m**4 + 172323696*m**3 + 616224492*m**2 + 1077459120*m + 60860
8000) + 86082584*a**5*b**3*m**4*x**10*x**m/(m**9 + 117*m**8 + 5814*m**7 + 1
59978*m**6 + 2662569*m**5 + 27458613*m**4 + 172323696*m**3 + 616224492*m**2
 + 1077459120*m + 608608000) + 676856488*a**5*b**3*m**3*x**10*x**m/(m**9 +
117*m**8 + 5814*m**7 + 159978*m**6 + 2662569*m**5 + 27458613*m**4 + 1723236
96*m**3 + 616224492*m**2 + 1077459120*m + 608608000) + 2881562096*a**5*b**3
*m**2*x**10*x**m/(m**9 + 117*m**8 + 5814*m**7 + 159978*m**6 + 2662569*m**5
 + 27458613*m**4 + 172323696*m**3 + 616224492*m**2 + 1077459120*m + 60860800
0) + 5692950592*a**5*b**3*m*x**10*x**m/(m**9 + 117*m**8 + 5814*m**7 + 15997
8*m**6 + 2662569*m**5 + 27458613*m**4 + 172323696*m**3 + 616224492*m**2 + 1
077459120*m + 608608000) + 3408204800*a**5*b**3*x**10*x**m/(m**9 + 117*m**8
 + 5814*m**7 + 159978*m**6 + 2662569*m**5 + 27458613*m**4 + 172323696*m**3
 + 616224492*m**2 + 1077459120*m + 608608000) + 70*a**4*b**4*m**8*x**13*x**m
/(m**9 + 117*m**8 + 5814*m**7 + 159978*m**6 + 2662569*m**5 + 27458613*m**4
 + 172323696*m**3 + 616224492*m**2 + 1077459120*m + 608608000) + 7280*a**4*b
**4*m**7*x**13*x**m/(m**9 + 117*m**8 + 5814*m**7 + 159978*m**6 + 2662569*m*
*5 + 27458613*m**4 + 172323696*m**3 + 616224492*m**2 + 1077459120*m + 60860
8000) + 312340*a**4*b**4*m**6*x**13*x**m/(m**9 + 117*m**8 + 5814*m**7 + 159
978*m**6 + 2662569*m**5 + 27458613*m**4 + 172323696*m**3 + 616224492*m**2 +
 1077459120*m + 608608000) + 7138040*a**4*b**4*m**5*x**13*x**m/(m**9 + 117*
m**8 + 5814*m**7 + 159978*m**6 + 2662569*m**5 + 27458613*m**4 + 172323696*m
**3 + 616224492*m**2 + 1077459120*m + 608608000) + 93585310*a**4*b**4*m**4*
x**13*x**m/(m**9 + 117*m**8 + 5814*m**7 + 159978*m**6 + 2662569*m**5 + 2745
8613*m**4 + 172323696*m**3 + 616224492*m**2 + 1077459120*m + 608608000) + 7
05493880*a**4*b**4*m**3*x**13*x**m/(m**9 + 117*m**8 + 5814*m**7 + 159978*m*
*6 + 2662569*m**5 + 27458613*m**4 + 172323696*m**3 + 616224492*m**2 + 10774
59120*m + 608608000) + 2891238280*a**4*b**4*m**2*x**13*x**m/(m**9 + 117*m**
8 + 5814*m**7 + 159978*m**6 + 2662569*m**5 + 27458613*m**4 + 172323696*m**3
 + 616224492*m**2 + 1077459120*m + 608608000) + 5549616800*a**4*b**4*m*x**1
3*x**m/(m**9 + 117*m**8 + 5814*m**7 + 159978*m**6 + 2662569*m**5 + 27458613
*m**4 + 172323696*m**3 + 616224492*m**2 + 1077459120*m + 608608000) + 32771
20000*a**4*b**4*x**13*x**m/(m**9 + 117*m**8 + 5814*m**7 + 159978*m**6 + 266
2569*m**5 + 27458613*m**4 + 172323696*m**3 + 616224492*m**2 + 1077459120*m
 + 608608000) + 56*a**3*b**5*m**8*x**16*x**m/(m**9 + 117*m**8 + 5814*m**7 +
159978*m**6 + 2662569*m**5 + 27458613*m**4 + 172323696*m**3 + 616224492*m**
2 + 1077459120*m + 608608000) + 5656*a**3*b**5*m**7*x**16*x**m/(m**9 + 117*
m**8 + 5814*m**7 + 159978*m**6 + 2662569*m**5 + 27458613*m**4 + 172323696*m
**3 + 616224492*m**2 + 1077459120*m + 608608000) + 235088*a**3*b**5*m**6*x*
*16*x**m/(m**9 + 117*m**8 + 5814*m**7 + 159978*m**6 + 2662569*m**5 + 274586
13*m**4 + 172323696*m**3 + 616224492*m**2 + 1077459120*m + 608608000) + 519
7360*a**3*b**5*m**5*x**16*x**m/(m**9 + 117*m**8 + 5814*m**7 + 159978*m**6 +
 2662569*m**5 + 27458613*m**4 + 172323696*m**3 + 616224492*m**2 + 107745912
0*m + 608608000) + 65946104*a**3*b**5*m**4*x**16*x**m/(m**9 + 117*m**8 + 58
```

```
14*m**7 + 159978*m**6 + 2662569*m**5 + 27458613*m**4 + 172323696*m**3 + 616
224492*m**2 + 1077459120*m + 608608000) + 482544664*a**3*b**5*m**3*x**16*x*
*m/(m**9 + 117*m**8 + 5814*m**7 + 159978*m**6 + 2662569*m**5 + 27458613*m**
4 + 172323696*m**3 + 616224492*m**2 + 1077459120*m + 608608000) + 192941235
2*a**3*b**5*m**2*x**16*x**m/(m**9 + 117*m**8 + 5814*m**7 + 159978*m**6 + 26
62569*m**5 + 27458613*m**4 + 172323696*m**3 + 616224492*m**2 + 1077459120*m
 + 608608000) + 3637973920*a**3*b**5*m*x**16*x**m/(m**9 + 117*m**8 + 5814*m
**7 + 159978*m**6 + 2662569*m**5 + 27458613*m**4 + 172323696*m**3 + 6162244
92*m**2 + 1077459120*m + 608608000) + 2130128000*a**3*b**5*x**16*x**m/(m**9
 + 117*m**8 + 5814*m**7 + 159978*m**6 + 2662569*m**5 + 27458613*m**4 + 1723
23696*m**3 + 616224492*m**2 + 1077459120*m + 608608000) + 28*a**2*b**6*m**8
*x**19*x**m/(m**9 + 117*m**8 + 5814*m**7 + 159978*m**6 + 2662569*m**5 + 274
58613*m**4 + 172323696*m**3 + 616224492*m**2 + 1077459120*m + 608608000) +
2744*a**2*b**6*m**7*x**19*x**m/(m**9 + 117*m**8 + 5814*m**7 + 159978*m**6 +
 2662569*m**5 + 27458613*m**4 + 172323696*m**3 + 616224492*m**2 + 107745912
0*m + 608608000) + 110656*a**2*b**6*m**6*x**19*x**m/(m**9 + 117*m**8 + 5814
*m**7 + 159978*m**6 + 2662569*m**5 + 27458613*m**4 + 172323696*m**3 + 61622
4492*m**2 + 1077459120*m + 608608000) + 2376920*a**2*b**6*m**5*x**19*x**m/(
m**9 + 117*m**8 + 5814*m**7 + 159978*m**6 + 2662569*m**5 + 27458613*m**4 +
172323696*m**3 + 616224492*m**2 + 1077459120*m + 608608000) + 29390452*a**2
*b**6*m**4*x**19*x**m/(m**9 + 117*m**8 + 5814*m**7 + 159978*m**6 + 2662569*
m**5 + 27458613*m**4 + 172323696*m**3 + 616224492*m**2 + 1077459120*m + 608
608000) + 210422576*a**2*b**6*m**3*x**19*x**m/(m**9 + 117*m**8 + 5814*m**7
+ 159978*m**6 + 2662569*m**5 + 27458613*m**4 + 172323696*m**3 + 616224492*m
**2 + 1077459120*m + 608608000) + 827034544*a**2*b**6*m**2*x**19*x**m/(m**9
 + 117*m**8 + 5814*m**7 + 159978*m**6 + 2662569*m**5 + 27458613*m**4 + 1723
23696*m**3 + 616224492*m**2 + 1077459120*m + 608608000) + 1540629440*a**2*b
**6*m*x**19*x**m/(m**9 + 117*m**8 + 5814*m**7 + 159978*m**6 + 2662569*m**5
+ 27458613*m**4 + 172323696*m**3 + 616224492*m**2 + 1077459120*m + 60860800
0) + 896896000*a**2*b**6*x**19*x**m/(m**9 + 117*m**8 + 5814*m**7 + 159978*m
**6 + 2662569*m**5 + 27458613*m**4 + 172323696*m**3 + 616224492*m**2 + 1077
459120*m + 608608000) + 8*a*b**7*m**8*x**22*x**m/(m**9 + 117*m**8 + 5814*m*
*7 + 159978*m**6 + 2662569*m**5 + 27458613*m**4 + 172323696*m**3 + 61622449
2*m**2 + 1077459120*m + 608608000) + 760*a*b**7*m**7*x**22*x**m/(m**9 + 117
*m**8 + 5814*m**7 + 159978*m**6 + 2662569*m**5 + 27458613*m**4 + 172323696*
m**3 + 616224492*m**2 + 1077459120*m + 608608000) + 29792*a*b**7*m**6*x**22
*x**m/(m**9 + 117*m**8 + 5814*m**7 + 159978*m**6 + 2662569*m**5 + 27458613*
m**4 + 172323696*m**3 + 616224492*m**2 + 1077459120*m + 608608000) + 624400
*a*b**7*m**5*x**22*x**m/(m**9 + 117*m**8 + 5814*m**7 + 159978*m**6 + 266256
9*m**5 + 27458613*m**4 + 172323696*m**3 + 616224492*m**2 + 1077459120*m + 6
08608000) + 7563752*a*b**7*m**4*x**22*x**m/(m**9 + 117*m**8 + 5814*m**7 + 1
59978*m**6 + 2662569*m**5 + 27458613*m**4 + 172323696*m**3 + 616224492*m**2
 + 1077459120*m + 608608000) + 53266360*a*b**7*m**3*x**22*x**m/(m**9 + 117*
m**8 + 5814*m**7 + 159978*m**6 + 2662569*m**5 + 27458613*m**4 + 172323696*m
**3 + 616224492*m**2 + 1077459120*m + 608608000) + 206729648*a*b**7*m**2*x*
*22*x**m/(m**9 + 117*m**8 + 5814*m**7 + 159978*m**6 + 2662569*m**5 + 274586
13*m**4 + 172323696*m**3 + 616224492*m**2 + 1077459120*m + 608608000) + 381
743680*a*b**7*m*x**22*x**m/(m**9 + 117*m**8 + 5814*m**7 + 159978*m**6 + 266
2569*m**5 + 27458613*m**4 + 172323696*m**3 + 616224492*m**2 + 1077459120*m
+ 608608000) + 221312000*a*b**7*x**22*x**m/(m**9 + 117*m**8 + 5814*m**7 + 1
59978*m**6 + 2662569*m**5 + 27458613*m**4 + 172323696*m**3 + 616224492*m**2
 + 1077459120*m + 608608000) + b**8*m**8*x**25*x**m/(m**9 + 117*m**8 + 5814
*m**7 + 159978*m**6 + 2662569*m**5 + 27458613*m**4 + 172323696*m**3 + 61622
4492*m**2 + 1077459120*m + 608608000) + 92*b**8*m**7*x**25*x**m/(m**9 + 117
*m**8 + 5814*m**7 + 159978*m**6 + 2662569*m**5 + 27458613*m**4 + 172323696*
m**3 + 616224492*m**2 + 1077459120*m + 608608000) + 3514*b**8*m**6*x**25*x*
*m/(m**9 + 117*m**8 + 5814*m**7 + 159978*m**6 + 2662569*m**5 + 27458613*m**
4 + 172323696*m**3 + 616224492*m**2 + 1077459120*m + 608608000) + 72128*b**
8*m**5*x**25*x**m/(m**9 + 117*m**8 + 5814*m**7 + 159978*m**6 + 2662569*m**5
```

```
 + 27458613*m**4 + 172323696*m**3 + 616224492*m**2 + 1077459120*m + 6086080
00) + 859369*b**8*m**4*x**25*x**m/(m**9 + 117*m**8 + 5814*m**7 + 159978*m**
6 + 2662569*m**5 + 27458613*m**4 + 172323696*m**3 + 616224492*m**2 + 107745
9120*m + 608608000) + 5974388*b**8*m**3*x**25*x**m/(m**9 + 117*m**8 + 5814*
m**7 + 159978*m**6 + 2662569*m**5 + 27458613*m**4 + 172323696*m**3 + 616224
492*m**2 + 1077459120*m + 608608000) + 22963996*b**8*m**2*x**25*x**m/(m**9
+ 117*m**8 + 5814*m**7 + 159978*m**6 + 2662569*m**5 + 27458613*m**4 + 17232
3696*m**3 + 616224492*m**2 + 1077459120*m + 608608000) + 42124592*b**8*m*x*
*25*x**m/(m**9 + 117*m**8 + 5814*m**7 + 159978*m**6 + 2662569*m**5 + 274586
13*m**4 + 172323696*m**3 + 616224492*m**2 + 1077459120*m + 608608000) + 243
44320*b**8*x**25*x**m/(m**9 + 117*m**8 + 5814*m**7 + 159978*m**6 + 2662569*
m**5 + 27458613*m**4 + 172323696*m**3 + 616224492*m**2 + 1077459120*m + 608
608000), True))
```

3.584 $\int x^m \left(a+bx^3\right)^5 dx$

Optimal. Leaf size=97

$$\frac{a^5x^{m+1}}{m+1}+\frac{5a^4bx^{m+4}}{m+4}+\frac{10a^3b^2x^{m+7}}{m+7}+\frac{10a^2b^3x^{m+10}}{m+10}+\frac{5ab^4x^{m+13}}{m+13}+\frac{b^5x^{m+16}}{m+16}$$

```
[Out] a^5*x^(1+m)/(1+m)+5*a^4*b*x^(4+m)/(4+m)+10*a^3*b^2*x^(7+m)/(7+m)+10*a^2*b^3
*x^(10+m)/(10+m)+5*a*b^4*x^(13+m)/(13+m)+b^5*x^(16+m)/(16+m)
```

Rubi [A] time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{10a^3b^2x^{m+7}}{m+7}+\frac{10a^2b^3x^{m+10}}{m+10}+\frac{5a^4bx^{m+4}}{m+4}+\frac{a^5x^{m+1}}{m+1}+\frac{5ab^4x^{m+13}}{m+13}+\frac{b^5x^{m+16}}{m+16}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^3)^5,x]
```

```
[Out] (a^5*x^(1 + m))/(1 + m) + (5*a^4*b*x^(4 + m))/(4 + m) + (10*a^3*b^2*x^(7 +
m))/(7 + m) + (10*a^2*b^3*x^(10 + m))/(10 + m) + (5*a*b^4*x^(13 + m))/(13 +
 m) + (b^5*x^(16 + m))/(16 + m)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^m\left(a+bx^3\right)^5 dx &= \int \left(a^5x^m+5a^4bx^{3+m}+10a^3b^2x^{6+m}+10a^2b^3x^{9+m}+5ab^4x^{12+m}+b^5x^{15+m}\right) dx\\ &= \frac{a^5x^{1+m}}{1+m}+\frac{5a^4bx^{4+m}}{4+m}+\frac{10a^3b^2x^{7+m}}{7+m}+\frac{10a^2b^3x^{10+m}}{10+m}+\frac{5ab^4x^{13+m}}{13+m}+\frac{b^5x^{16+m}}{16+m}\end{aligned}$$

Mathematica [A] time = 0.05, size = 88, normalized size = 0.91

$$x^{m+1}\left(\frac{a^5}{m+1}+\frac{5a^4bx^3}{m+4}+\frac{10a^3b^2x^6}{m+7}+\frac{10a^2b^3x^9}{m+10}+\frac{5ab^4x^{12}}{m+13}+\frac{b^5x^{15}}{m+16}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^3)^5,x]
```

```
[Out] x^(1 + m)*(a^5/(1 + m) + (5*a^4*b*x^3)/(4 + m) + (10*a^3*b^2*x^6)/(7 + m) +
 (10*a^2*b^3*x^9)/(10 + m) + (5*a*b^4*x^12)/(13 + m) + (b^5*x^15)/(16 + m))
```

fricas [B] time = 0.57, size = 367, normalized size = 3.78

$$\frac{\left(\left(b^5m^5+35\,b^5m^4+445\,b^5m^3+2485\,b^5m^2+5714\,b^5m+3640\,b^5\right)x^{16}+5\left(ab^4m^5+38\,ab^4m^4+511\,ab^4m^3+296\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)^5,x, algorithm="fricas")
```

```
[Out] ((b^5*m^5 + 35*b^5*m^4 + 445*b^5*m^3 + 2485*b^5*m^2 + 5714*b^5*m + 3640*b^5
)*x^16 + 5*(a*b^4*m^5 + 38*a*b^4*m^4 + 511*a*b^4*m^3 + 2962*a*b^4*m^2 + 696
8*a*b^4*m + 4480*a*b^4)*x^13 + 10*(a^2*b^3*m^5 + 41*a^2*b^3*m^4 + 595*a^2*b
^3*m^3 + 3655*a^2*b^3*m^2 + 8924*a^2*b^3*m + 5824*a^2*b^3)*x^10 + 10*(a^3*b
^2*m^5 + 44*a^3*b^2*m^4 + 697*a^3*b^2*m^3 + 4726*a^3*b^2*m^2 + 12392*a^3*b^
2*m + 8320*a^3*b^2)*x^7 + 5*(a^4*b*m^5 + 47*a^4*b*m^4 + 817*a^4*b*m^3 + 633
7*a^4*b*m^2 + 20126*a^4*b*m + 14560*a^4*b)*x^4 + (a^5*m^5 + 50*a^5*m^4 + 95
5*a^5*m^3 + 8650*a^5*m^2 + 36824*a^5*m + 58240*a^5)*x)*x^m/(m^6 + 51*m^5 +
1005*m^4 + 9605*m^3 + 45474*m^2 + 95064*m + 58240)
```

giac [B] time = 0.22, size = 540, normalized size = 5.57

$$b^5m^5x^{16}x^m + 35\,b^5m^4x^{16}x^m + 445\,b^5m^3x^{16}x^m + 5\,ab^4m^5x^{13}x^m + 2485\,b^5m^2x^{16}x^m + 190\,ab^4m^4x^{13}x^m + 5714\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)^5,x, algorithm="giac")
```

```
[Out] (b^5*m^5*x^16*x^m + 35*b^5*m^4*x^16*x^m + 445*b^5*m^3*x^16*x^m + 5*a*b^4*m^
5*x^13*x^m + 2485*b^5*m^2*x^16*x^m + 190*a*b^4*m^4*x^13*x^m + 5714*b^5*m*x^
16*x^m + 2555*a*b^4*m^3*x^13*x^m + 3640*b^5*x^16*x^m + 10*a^2*b^3*m^5*x^10*
x^m + 14810*a*b^4*m^2*x^13*x^m + 410*a^2*b^3*m^4*x^10*x^m + 34840*a*b^4*m*x
^13*x^m + 5950*a^2*b^3*m^3*x^10*x^m + 22400*a*b^4*x^13*x^m + 10*a^3*b^2*m^5
*x^7*x^m + 36550*a^2*b^3*m^2*x^10*x^m + 440*a^3*b^2*m^4*x^7*x^m + 89240*a^2
*b^3*m*x^10*x^m + 6970*a^3*b^2*m^3*x^7*x^m + 58240*a^2*b^3*x^10*x^m + 5*a^4
*b*m^5*x^4*x^m + 47260*a^3*b^2*m^2*x^7*x^m + 235*a^4*b*m^4*x^4*x^m + 123920
*a^3*b^2*m*x^7*x^m + 4085*a^4*b*m^3*x^4*x^m + 83200*a^3*b^2*x^7*x^m + a^5*m
^5*x*x^m + 31685*a^4*b*m^2*x^4*x^m + 50*a^5*m^4*x*x^m + 100630*a^4*b*m*x^4*
x^m + 955*a^5*m^3*x*x^m + 72800*a^4*b*x^4*x^m + 8650*a^5*m^2*x*x^m + 36824*
a^5*m*x*x^m + 58240*a^5*x*x^m)/(m^6 + 51*m^5 + 1005*m^4 + 9605*m^3 + 45474*
m^2 + 95064*m + 58240)
```

maple [B] time = 0.01, size = 432, normalized size = 4.45

$$\left(b^5m^5x^{15} + 35b^5m^4x^{15} + 445b^5m^3x^{15} + 5a\,b^4m^5x^{12} + 2485b^5m^2x^{15} + 190a\,b^4m^4x^{12} + 5714b^5m\,x^{15} + 2555a\,b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^3+a)^5,x)
```

```
[Out] x^(m+1)*(b^5*m^5*x^15+35*b^5*m^4*x^15+445*b^5*m^3*x^15+5*a*b^4*m^5*x^12+248
5*b^5*m^2*x^15+190*a*b^4*m^4*x^12+5714*b^5*m*x^15+2555*a*b^4*m^3*x^12+3640*
b^5*x^15+10*a^2*b^3*m^5*x^9+14810*a*b^4*m^2*x^12+410*a^2*b^3*m^4*x^9+34840*
a*b^4*m*x^12+5950*a^2*b^3*m^3*x^9+22400*a*b^4*x^12+10*a^3*b^2*m^5*x^6+36550
*a^2*b^3*m^2*x^9+440*a^3*b^2*m^4*x^6+89240*a^2*b^3*m*x^9+6970*a^3*b^2*m^3*x
^6+58240*a^2*b^3*x^9+5*a^4*b*m^5*x^3+47260*a^3*b^2*m^2*x^6+235*a^4*b*m^4*x^
3+123920*a^3*b^2*m*x^6+4085*a^4*b*m^3*x^3+83200*a^3*b^2*x^6+a^5*m^5+31685*a
^4*b*m^2*x^3+50*a^5*m^4+100630*a^4*b*m*x^3+955*a^5*m^3+72800*a^4*b*x^3+8650
*a^5*m^2+36824*a^5*m+58240*a^5)/(m+1)/(m+4)/(m+7)/(m+10)/(m+13)/(m+16)
```

maxima [A] time = 1.34, size = 97, normalized size = 1.00

$$\frac{b^5x^{m+16}}{m+16} + \frac{5\,ab^4x^{m+13}}{m+13} + \frac{10\,a^2b^3x^{m+10}}{m+10} + \frac{10\,a^3b^2x^{m+7}}{m+7} + \frac{5\,a^4bx^{m+4}}{m+4} + \frac{a^5x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)^5,x, algorithm="maxima")
```

```
[Out] b^5*x^(m + 16)/(m + 16) + 5*a*b^4*x^(m + 13)/(m + 13) + 10*a^2*b^3*x^(m + 1
0)/(m + 10) + 10*a^3*b^2*x^(m + 7)/(m + 7) + 5*a^4*b*x^(m + 4)/(m + 4) + a^
5*x^(m + 1)/(m + 1)
```

mupad [**B**] time = 1.55, size = 389, normalized size = 4.01

$$\frac{a^5\,x\,x^m\left(m^5+50\,m^4+955\,m^3+8650\,m^2+36824\,m+58240\right)}{m^6+51\,m^5+1005\,m^4+9605\,m^3+45474\,m^2+95064\,m+58240}+\frac{b^5\,x^m\,x^{16}\left(m^5+35\,m^4+445\,m^3+2485\,m\right.}{m^6+51\,m^5+1005\,m^4+9605\,m^3+45474\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^3)^5,x)
```

```
[Out] (a^5*x*x^m*(36824*m + 8650*m^2 + 955*m^3 + 50*m^4 + m^5 + 58240))/(95064*m
+ 45474*m^2 + 9605*m^3 + 1005*m^4 + 51*m^5 + m^6 + 58240) + (b^5*x^m*x^16*(
5714*m + 2485*m^2 + 445*m^3 + 35*m^4 + m^5 + 3640))/(95064*m + 45474*m^2 +
9605*m^3 + 1005*m^4 + 51*m^5 + m^6 + 58240) + (5*a*b^4*x^m*x^13*(6968*m + 2
962*m^2 + 511*m^3 + 38*m^4 + m^5 + 4480))/(95064*m + 45474*m^2 + 9605*m^3 +
 1005*m^4 + 51*m^5 + m^6 + 58240) + (5*a^4*b*x^m*x^4*(20126*m + 6337*m^2 +
817*m^3 + 47*m^4 + m^5 + 14560))/(95064*m + 45474*m^2 + 9605*m^3 + 1005*m^4
 + 51*m^5 + m^6 + 58240) + (10*a^2*b^3*x^m*x^10*(8924*m + 3655*m^2 + 595*m^
3 + 41*m^4 + m^5 + 5824))/(95064*m + 45474*m^2 + 9605*m^3 + 1005*m^4 + 51*m
^5 + m^6 + 58240) + (10*a^3*b^2*x^m*x^7*(12392*m + 4726*m^2 + 697*m^3 + 44*
m^4 + m^5 + 8320))/(95064*m + 45474*m^2 + 9605*m^3 + 1005*m^4 + 51*m^5 + m^
6 + 58240)
```

sympy [**A**] time = 14.93, size = 2006, normalized size = 20.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**3+a)**5,x)
```

```
[Out] Piecewise((-a**5/(15*x**15) - 5*a**4*b/(12*x**12) - 10*a**3*b**2/(9*x**9) -
 5*a**2*b**3/(3*x**6) - 5*a*b**4/(3*x**3) + b**5*log(x), Eq(m, -16)), (-a**
5/(12*x**12) - 5*a**4*b/(9*x**9) - 5*a**3*b**2/(3*x**6) - 10*a**2*b**3/(3*x
**3) + 5*a*b**4*log(x) + b**5*x**3/3, Eq(m, -13)), (-a**5/(9*x**9) - 5*a**4
*b/(6*x**6) - 10*a**3*b**2/(3*x**3) + 10*a**2*b**3*log(x) + 5*a*b**4*x**3/3
 + b**5*x**6/6, Eq(m, -10)), (-a**5/(6*x**6) - 5*a**4*b/(3*x**3) + 10*a**3*
b**2*log(x) + 10*a**2*b**3*x**3/3 + 5*a*b**4*x**6/6 + b**5*x**9/9, Eq(m, -7
)), (-a**5/(3*x**3) + 5*a**4*b*log(x) + 10*a**3*b**2*x**3/3 + 5*a**2*b**3*x
**6/3 + 5*a*b**4*x**9/9 + b**5*x**12/12, Eq(m, -4)), (a**5*log(x) + 5*a**4*
b*x**3/3 + 5*a**3*b**2*x**6/3 + 10*a**2*b**3*x**9/9 + 5*a*b**4*x**12/12 + b
**5*x**15/15, Eq(m, -1)), (a**5*m**5*x*x**m/(m**6 + 51*m**5 + 1005*m**4 + 9
605*m**3 + 45474*m**2 + 95064*m + 58240) + 50*a**5*m**4*x*x**m/(m**6 + 51*m
**5 + 1005*m**4 + 9605*m**3 + 45474*m**2 + 95064*m + 58240) + 955*a**5*m**3
*x*x**m/(m**6 + 51*m**5 + 1005*m**4 + 9605*m**3 + 45474*m**2 + 95064*m + 58
240) + 8650*a**5*m**2*x*x**m/(m**6 + 51*m**5 + 1005*m**4 + 9605*m**3 + 4547
4*m**2 + 95064*m + 58240) + 36824*a**5*m*x*x**m/(m**6 + 51*m**5 + 1005*m**4
 + 9605*m**3 + 45474*m**2 + 95064*m + 58240) + 58240*a**5*x*x**m/(m**6 + 51
*m**5 + 1005*m**4 + 9605*m**3 + 45474*m**2 + 95064*m + 58240) + 5*a**4*b*m*
*5*x**4*x**m/(m**6 + 51*m**5 + 1005*m**4 + 9605*m**3 + 45474*m**2 + 95064*m
 + 58240) + 235*a**4*b*m**4*x**4*x**m/(m**6 + 51*m**5 + 1005*m**4 + 9605*m*
*3 + 45474*m**2 + 95064*m + 58240) + 4085*a**4*b*m**3*x**4*x**m/(m**6 + 51*
m**5 + 1005*m**4 + 9605*m**3 + 45474*m**2 + 95064*m + 58240) + 31685*a**4*b
*m**2*x**4*x**m/(m**6 + 51*m**5 + 1005*m**4 + 9605*m**3 + 45474*m**2 + 9506
4*m + 58240) + 100630*a**4*b*m*x**4*x**m/(m**6 + 51*m**5 + 1005*m**4 + 9605
*m**3 + 45474*m**2 + 95064*m + 58240) + 72800*a**4*b*x**4*x**m/(m**6 + 51*m
**5 + 1005*m**4 + 9605*m**3 + 45474*m**2 + 95064*m + 58240) + 10*a**3*b**2*
m**5*x**7*x**m/(m**6 + 51*m**5 + 1005*m**4 + 9605*m**3 + 45474*m**2 + 95064
```

```
*m + 58240) + 440*a**3*b**2*m**4*x**7*x**m/(m**6 + 51*m**5 + 1005*m**4 + 96
05*m**3 + 45474*m**2 + 95064*m + 58240) + 6970*a**3*b**2*m**3*x**7*x**m/(m*
*6 + 51*m**5 + 1005*m**4 + 9605*m**3 + 45474*m**2 + 95064*m + 58240) + 4726
0*a**3*b**2*m**2*x**7*x**m/(m**6 + 51*m**5 + 1005*m**4 + 9605*m**3 + 45474*
m**2 + 95064*m + 58240) + 123920*a**3*b**2*m*x**7*x**m/(m**6 + 51*m**5 + 10
05*m**4 + 9605*m**3 + 45474*m**2 + 95064*m + 58240) + 83200*a**3*b**2*x**7*
x**m/(m**6 + 51*m**5 + 1005*m**4 + 9605*m**3 + 45474*m**2 + 95064*m + 58240
) + 10*a**2*b**3*m**5*x**10*x**m/(m**6 + 51*m**5 + 1005*m**4 + 9605*m**3 +
45474*m**2 + 95064*m + 58240) + 410*a**2*b**3*m**4*x**10*x**m/(m**6 + 51*m*
*5 + 1005*m**4 + 9605*m**3 + 45474*m**2 + 95064*m + 58240) + 5950*a**2*b**3
*m**3*x**10*x**m/(m**6 + 51*m**5 + 1005*m**4 + 9605*m**3 + 45474*m**2 + 950
64*m + 58240) + 36550*a**2*b**3*m**2*x**10*x**m/(m**6 + 51*m**5 + 1005*m**4
 + 9605*m**3 + 45474*m**2 + 95064*m + 58240) + 89240*a**2*b**3*m*x**10*x**m
/(m**6 + 51*m**5 + 1005*m**4 + 9605*m**3 + 45474*m**2 + 95064*m + 58240) +
58240*a**2*b**3*x**10*x**m/(m**6 + 51*m**5 + 1005*m**4 + 9605*m**3 + 45474*
m**2 + 95064*m + 58240) + 5*a*b**4*m**5*x**13*x**m/(m**6 + 51*m**5 + 1005*m
**4 + 9605*m**3 + 45474*m**2 + 95064*m + 58240) + 190*a*b**4*m**4*x**13*x**
m/(m**6 + 51*m**5 + 1005*m**4 + 9605*m**3 + 45474*m**2 + 95064*m + 58240) +
 2555*a*b**4*m**3*x**13*x**m/(m**6 + 51*m**5 + 1005*m**4 + 9605*m**3 + 4547
4*m**2 + 95064*m + 58240) + 14810*a*b**4*m**2*x**13*x**m/(m**6 + 51*m**5 +
1005*m**4 + 9605*m**3 + 45474*m**2 + 95064*m + 58240) + 34840*a*b**4*m*x**1
3*x**m/(m**6 + 51*m**5 + 1005*m**4 + 9605*m**3 + 45474*m**2 + 95064*m + 582
40) + 22400*a*b**4*x**13*x**m/(m**6 + 51*m**5 + 1005*m**4 + 9605*m**3 + 454
74*m**2 + 95064*m + 58240) + b**5*m**5*x**16*x**m/(m**6 + 51*m**5 + 1005*m*
*4 + 9605*m**3 + 45474*m**2 + 95064*m + 58240) + 35*b**5*m**4*x**16*x**m/(m
**6 + 51*m**5 + 1005*m**4 + 9605*m**3 + 45474*m**2 + 95064*m + 58240) + 445
*b**5*m**3*x**16*x**m/(m**6 + 51*m**5 + 1005*m**4 + 9605*m**3 + 45474*m**2
+ 95064*m + 58240) + 2485*b**5*m**2*x**16*x**m/(m**6 + 51*m**5 + 1005*m**4
+ 9605*m**3 + 45474*m**2 + 95064*m + 58240) + 5714*b**5*m*x**16*x**m/(m**6
+ 51*m**5 + 1005*m**4 + 9605*m**3 + 45474*m**2 + 95064*m + 58240) + 3640*b*
*5*x**16*x**m/(m**6 + 51*m**5 + 1005*m**4 + 9605*m**3 + 45474*m**2 + 95064*
m + 58240), True))
```

3.585 $\int x^m \left(a+bx^3\right)^3 \, dx$

Optimal. Leaf size=61

$$\frac{a^3 x^{m+1}}{m+1}+\frac{3a^2 b x^{m+4}}{m+4}+\frac{3ab^2 x^{m+7}}{m+7}+\frac{b^3 x^{m+10}}{m+10}$$

```
[Out] a^3*x^(1+m)/(1+m)+3*a^2*b*x^(4+m)/(4+m)+3*a*b^2*x^(7+m)/(7+m)+b^3*x^(10+m)/
(10+m)
```

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{3a^2 b x^{m+4}}{m+4}+\frac{a^3 x^{m+1}}{m+1}+\frac{3ab^2 x^{m+7}}{m+7}+\frac{b^3 x^{m+10}}{m+10}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^3)^3,x]
```

```
[Out] (a^3*x^(1 + m))/(1 + m) + (3*a^2*b*x^(4 + m))/(4 + m) + (3*a*b^2*x^(7 + m))
/(7 + m) + (b^3*x^(10 + m))/(10 + m)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^m \left(a+bx^3\right)^3 \, dx &= \int \left(a^3 x^m+3a^2 b x^{3+m}+3ab^2 x^{6+m}+b^3 x^{9+m}\right) \, dx \\ &= \frac{a^3 x^{1+m}}{1+m}+\frac{3a^2 b x^{4+m}}{4+m}+\frac{3ab^2 x^{7+m}}{7+m}+\frac{b^3 x^{10+m}}{10+m}\end{aligned}$$

Mathematica [A] time = 0.03, size = 56, normalized size = 0.92

$$x^{m+1}\left(\frac{a^3}{m+1}+\frac{3a^2 b x^3}{m+4}+\frac{3ab^2 x^6}{m+7}+\frac{b^3 x^9}{m+10}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^3)^3,x]
```

```
[Out] x^(1 + m)*(a^3/(1 + m) + (3*a^2*b*x^3)/(4 + m) + (3*a*b^2*x^6)/(7 + m) + (b
^3*x^9)/(10 + m))
```

fricas [B] time = 0.68, size = 157, normalized size = 2.57

$$\frac{\left(\left(b^3 m^3+12\,b^3 m^2+39\,b^3 m+28\,b^3\right)x^{10}+3\left(ab^2 m^3+15\,ab^2 m^2+54\,ab^2 m+40\,ab^2\right)x^7+3\left(a^2 bm^3+18\,a^2 bm^2+\right.\right.}{m^4+22\,m^3+159\,m^2+418\,m+280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] ((b^3*m^3 + 12*b^3*m^2 + 39*b^3*m + 28*b^3)*x^10 + 3*(a*b^2*m^3 + 15*a*b^2*
m^2 + 54*a*b^2*m + 40*a*b^2)*x^7 + 3*(a^2*b*m^3 + 18*a^2*b*m^2 + 87*a^2*b*m
 + 70*a^2*b)*x^4 + (a^3*m^3 + 21*a^3*m^2 + 138*a^3*m + 280*a^3)*x)*x^m/(m^4
 + 22*m^3 + 159*m^2 + 418*m + 280)
```

giac [B] time = 0.25, size = 224, normalized size = 3.67

$$\frac{b^3m^3x^{10}x^m + 12\,b^3m^2x^{10}x^m + 39\,b^3mx^{10}x^m + 3\,ab^2m^3x^7x^m + 28\,b^3x^{10}x^m + 45\,ab^2m^2x^7x^m + 162\,ab^2mx^7x^m +}{m^4 + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] (b^3*m^3*x^10*x^m + 12*b^3*m^2*x^10*x^m + 39*b^3*m*x^10*x^m + 3*a*b^2*m^3*x
^7*x^m + 28*b^3*x^10*x^m + 45*a*b^2*m^2*x^7*x^m + 162*a*b^2*m*x^7*x^m + 3*a
^2*b*m^3*x^4*x^m + 120*a*b^2*x^7*x^m + 54*a^2*b*m^2*x^4*x^m + 261*a^2*b*m*x
^4*x^m + a^3*m^3*x*x^m + 210*a^2*b*x^4*x^m + 21*a^3*m^2*x*x^m + 138*a^3*m*x
*x^m + 280*a^3*x*x^m)/(m^4 + 22*m^3 + 159*m^2 + 418*m + 280)
```

maple [B] time = 0.00, size = 178, normalized size = 2.92

$$\frac{\left(b^3m^3x^9 + 12b^3m^2x^9 + 39b^3m\,x^9 + 3a\,b^2m^3x^6 + 28b^3x^9 + 45a\,b^2m^2x^6 + 162a\,b^2m\,x^6 + 3a^2b\,m^3x^3 + 120a\,b^2x^6\right.}{(m+10)\,(m+7)\,(m+4)\,(m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^3+a)^3,x)
```

```
[Out] x^(m+1)*(b^3*m^3*x^9+12*b^3*m^2*x^9+39*b^3*m*x^9+3*a*b^2*m^3*x^6+28*b^3*x^9
+45*a*b^2*m^2*x^6+162*a*b^2*m*x^6+3*a^2*b*m^3*x^3+120*a*b^2*x^6+54*a^2*b*m^
2*x^3+261*a^2*b*m*x^3+a^3*m^3+210*a^2*b*x^3+21*a^3*m^2+138*a^3*m+280*a^3)/(
m+10)/(m+7)/(m+4)/(m+1)
```

maxima [A] time = 1.36, size = 61, normalized size = 1.00

$$\frac{b^3x^{m+10}}{m+10} + \frac{3\,ab^2x^{m+7}}{m+7} + \frac{3\,a^2bx^{m+4}}{m+4} + \frac{a^3x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] b^3*x^(m + 10)/(m + 10) + 3*a*b^2*x^(m + 7)/(m + 7) + 3*a^2*b*x^(m + 4)/(m
+ 4) + a^3*x^(m + 1)/(m + 1)
```

mupad [B] time = 1.35, size = 167, normalized size = 2.74

$$x^m\left(\frac{a^3\,x\,\left(m^3+21\,m^2+138\,m+280\right)}{m^4+22\,m^3+159\,m^2+418\,m+280} + \frac{b^3\,x^{10}\,\left(m^3+12\,m^2+39\,m+28\right)}{m^4+22\,m^3+159\,m^2+418\,m+280} + \frac{3\,a\,b^2\,x^7\,\left(m^3+15\,m^2+5\right.}{m^4+22\,m^3+159\,m^2+4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^3)^3,x)
```

```
[Out] x^m*((a^3*x*(138*m + 21*m^2 + m^3 + 280))/(418*m + 159*m^2 + 22*m^3 + m^4 +
 280) + (b^3*x^10*(39*m + 12*m^2 + m^3 + 28))/(418*m + 159*m^2 + 22*m^3 + m
^4 + 280) + (3*a*b^2*x^7*(54*m + 15*m^2 + m^3 + 40))/(418*m + 159*m^2 + 22*
m^3 + m^4 + 280) + (3*a^2*b*x^4*(87*m + 18*m^2 + m^3 + 70))/(418*m + 159*m^
2 + 22*m^3 + m^4 + 280))
```

sympy [A] time = 4.19, size = 666, normalized size = 10.92

$$\begin{cases} -\frac{a^3}{9x^9} - \frac{a^2b}{2x^6} - \frac{ab^2}{x^3} + b^3\log(x) \\ -\frac{a^3}{6x^6} - \frac{a^2b}{x^3} + 3ab^2\log(x) + \frac{b^3x^3}{3} \\ -\frac{a^3}{3x^3} + 3a^2b\log(x) + ab^2x^3 + \frac{b^3x^6}{6} \\ a^3\log(x) + a^2bx^3 + \frac{ab^2x^6}{2} + \frac{b^3x^9}{9} \\ \frac{a^3m^3xx^m}{m^4+22m^3+159m^2+418m+280} + \frac{21a^3m^2xx^m}{m^4+22m^3+159m^2+418m+280} + \frac{138a^3mxx^m}{m^4+22m^3+159m^2+418m+280} + \frac{280a^3xx^m}{m^4+22m^3+159m^2+418m+280} + \frac{}{m^4+22n} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**3+a)**3,x)
```

```
[Out] Piecewise((-a**3/(9*x**9) - a**2*b/(2*x**6) - a*b**2/x**3 + b**3*log(x), Eq
(m, -10)), (-a**3/(6*x**6) - a**2*b/x**3 + 3*a*b**2*log(x) + b**3*x**3/3, E
q(m, -7)), (-a**3/(3*x**3) + 3*a**2*b*log(x) + a*b**2*x**3 + b**3*x**6/6, E
q(m, -4)), (a**3*log(x) + a**2*b*x**3 + a*b**2*x**6/2 + b**3*x**9/9, Eq(m,
-1)), (a**3*m**3*x*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 21*a**3
*m**2*x*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 138*a**3*m*x*x**m/
(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 280*a**3*x*x**m/(m**4 + 22*m**3
 + 159*m**2 + 418*m + 280) + 3*a**2*b*m**3*x**4*x**m/(m**4 + 22*m**3 + 159*
m**2 + 418*m + 280) + 54*a**2*b*m**2*x**4*x**m/(m**4 + 22*m**3 + 159*m**2 +
 418*m + 280) + 261*a**2*b*m*x**4*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m +
 280) + 210*a**2*b*x**4*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 3*
a*b**2*m**3*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 45*a*b**2
*m**2*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 162*a*b**2*m*x*
*7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 120*a*b**2*x**7*x**m/(m
**4 + 22*m**3 + 159*m**2 + 418*m + 280) + b**3*m**3*x**10*x**m/(m**4 + 22*m
**3 + 159*m**2 + 418*m + 280) + 12*b**3*m**2*x**10*x**m/(m**4 + 22*m**3 + 1
59*m**2 + 418*m + 280) + 39*b**3*m*x**10*x**m/(m**4 + 22*m**3 + 159*m**2 +
418*m + 280) + 28*b**3*x**10*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280)
, True))
```

3.586 $\int x^m \left(a + bx^3\right)^2 dx$

Optimal. Leaf size=43

$$\frac{a^2 x^{m+1}}{m+1} + \frac{2abx^{m+4}}{m+4} + \frac{b^2 x^{m+7}}{m+7}$$

```
[Out] a^2*x^(1+m)/(1+m)+2*a*b*x^(4+m)/(4+m)+b^2*x^(7+m)/(7+m)
```

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{a^2 x^{m+1}}{m+1} + \frac{2abx^{m+4}}{m+4} + \frac{b^2 x^{m+7}}{m+7}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^3)^2,x]
```

```
[Out] (a^2*x^(1 + m))/(1 + m) + (2*a*b*x^(4 + m))/(4 + m) + (b^2*x^(7 + m))/(7 + m)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^m \left(a + bx^3\right)^2 dx &= \int \left(a^2 x^m + 2abx^{3+m} + b^2 x^{6+m}\right) dx \\ &= \frac{a^2 x^{1+m}}{1+m} + \frac{2abx^{4+m}}{4+m} + \frac{b^2 x^{7+m}}{7+m}\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 0.93

$$x^{m+1}\left(\frac{a^2}{m+1} + \frac{2abx^3}{m+4} + \frac{b^2 x^6}{m+7}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^3)^2,x]
```

```
[Out] x^(1 + m)*(a^2/(1 + m) + (2*a*b*x^3)/(4 + m) + (b^2*x^6)/(7 + m))
```

fricas [A] time = 0.80, size = 85, normalized size = 1.98

$$\frac{\left(\left(b^2 m^2 + 5\,b^2 m + 4\,b^2\right)x^7 + 2\left(abm^2 + 8\,abm + 7\,ab\right)x^4 + \left(a^2 m^2 + 11\,a^2 m + 28\,a^2\right)x\right)x^m}{m^3 + 12\,m^2 + 39\,m + 28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*m^2 + 5*b^2*m + 4*b^2)*x^7 + 2*(a*b*m^2 + 8*a*b*m + 7*a*b)*x^4 + (a^2*m^2 + 11*a^2*m + 28*a^2)*x)*x^m/(m^3 + 12*m^2 + 39*m + 28)
```

giac [B] time = 0.17, size = 117, normalized size = 2.72

$$\frac{b^2m^2x^7x^m + 5\,b^2mx^7x^m + 4\,b^2x^7x^m + 2\,abm^2x^4x^m + 16\,abmx^4x^m + 14\,abx^4x^m + a^2m^2xx^m + 11\,a^2mxx^m + 28\,a^2x}{m^3 + 12\,m^2 + 39\,m + 28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] (b^2*m^2*x^7*x^m + 5*b^2*m*x^7*x^m + 4*b^2*x^7*x^m + 2*a*b*m^2*x^4*x^m + 16
*a*b*m*x^4*x^m + 14*a*b*x^4*x^m + a^2*m^2*x*x^m + 11*a^2*m*x*x^m + 28*a^2*x
*x^m)/(m^3 + 12*m^2 + 39*m + 28)
```

maple [B] time = 0.01, size = 93, normalized size = 2.16

$$\frac{\left(b^2m^2x^6 + 5b^2m\,x^6 + 4b^2x^6 + 2ab\,m^2x^3 + 16abm\,x^3 + 14ab\,x^3 + a^2m^2 + 11a^2m + 28a^2\right)x^{m+1}}{(m+7)\,(m+4)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^3+a)^2,x)
```

```
[Out] x^(m+1)*(b^2*m^2*x^6+5*b^2*m*x^6+4*b^2*x^6+2*a*b*m^2*x^3+16*a*b*m*x^3+14*a*
b*x^3+a^2*m^2+11*a^2*m+28*a^2)/(m+7)/(m+4)/(m+1)
```

maxima [A] time = 1.33, size = 43, normalized size = 1.00

$$\frac{b^2x^{m+7}}{m+7} + \frac{2\,abx^{m+4}}{m+4} + \frac{a^2x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] b^2*x^(m + 7)/(m + 7) + 2*a*b*x^(m + 4)/(m + 4) + a^2*x^(m + 1)/(m + 1)
```

mupad [B] time = 1.31, size = 93, normalized size = 2.16

$$x^m\left(\frac{a^2\,x\,\left(m^2+11\,m+28\right)}{m^3+12\,m^2+39\,m+28} + \frac{b^2\,x^7\,\left(m^2+5\,m+4\right)}{m^3+12\,m^2+39\,m+28} + \frac{2\,a\,b\,x^4\,\left(m^2+8\,m+7\right)}{m^3+12\,m^2+39\,m+28}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^3)^2,x)
```

```
[Out] x^m*((a^2*x*(11*m + m^2 + 28))/(39*m + 12*m^2 + m^3 + 28) + (b^2*x^7*(5*m +
 m^2 + 4))/(39*m + 12*m^2 + m^3 + 28) + (2*a*b*x^4*(8*m + m^2 + 7))/(39*m +
 12*m^2 + m^3 + 28))
```

sympy [A] time = 2.72, size = 313, normalized size = 7.28

$$\begin{cases} -\frac{a^2}{6x^6} - \frac{2ab}{3x^3} + b^2\log(x) \\ -\frac{a^2}{3x^3} + 2ab\log(x) + \frac{b^2x^3}{3} \\ a^2\log(x) + \frac{2abx^3}{3} + \frac{b^2x^6}{6} \\ \frac{a^2m^2xx^m}{m^3+12m^2+39m+28} + \frac{11a^2mxx^m}{m^3+12m^2+39m+28} + \frac{28a^2xx^m}{m^3+12m^2+39m+28} + \frac{2abm^2x^4x^m}{m^3+12m^2+39m+28} + \frac{16abmx^4x^m}{m^3+12m^2+39m+28} + \frac{14abx^4x^m}{m^3+12m^2+39m+28} + \frac{}{m} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**3+a)**2,x)
```

```
[Out] Piecewise((-a**2/(6*x**6) - 2*a*b/(3*x**3) + b**2*log(x), Eq(m, -7)), (-a**
2/(3*x**3) + 2*a*b*log(x) + b**2*x**3/3, Eq(m, -4)), (a**2*log(x) + 2*a*b*x
**3/3 + b**2*x**6/6, Eq(m, -1)), (a**2*m**2*x*x**m/(m**3 + 12*m**2 + 39*m +
 28) + 11*a**2*m*x*x**m/(m**3 + 12*m**2 + 39*m + 28) + 28*a**2*x*x**m/(m**3
 + 12*m**2 + 39*m + 28) + 2*a*b*m**2*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28)
 + 16*a*b*m*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) + 14*a*b*x**4*x**m/(m**3
 + 12*m**2 + 39*m + 28) + b**2*m**2*x**7*x**m/(m**3 + 12*m**2 + 39*m + 28)
+ 5*b**2*m*x**7*x**m/(m**3 + 12*m**2 + 39*m + 28) + 4*b**2*x**7*x**m/(m**3
+ 12*m**2 + 39*m + 28), True))
```

3.587 $\int x^m \left(a + bx^3\right) dx$

Optimal. Leaf size=25

$$\frac{ax^{m+1}}{m+1} + \frac{bx^{m+4}}{m+4}$$

```
[Out] a*x^(1+m)/(1+m)+b*x^(4+m)/(4+m)
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$\frac{ax^{m+1}}{m+1} + \frac{bx^{m+4}}{m+4}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^3),x]
```

```
[Out] (a*x^(1 + m))/(1 + m) + (b*x^(4 + m))/(4 + m)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^m \left(a + bx^3\right) dx &= \int \left(ax^m + bx^{3+m}\right) dx \\ &= \frac{ax^{1+m}}{1+m} + \frac{bx^{4+m}}{4+m}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{ax^{m+1}}{m+1} + \frac{bx^{m+4}}{m+4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^3),x]
```

```
[Out] (a*x^(1 + m))/(1 + m) + (b*x^(4 + m))/(4 + m)
```

fricas [A] time = 0.57, size = 33, normalized size = 1.32

$$\frac{\left((bm+b)x^4 + (am+4\,a)x\right)x^m}{m^2+5\,m+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a),x, algorithm="fricas")
```

```
[Out] ((b*m + b)*x^4 + (a*m + 4*a)*x)*x^m/(m^2 + 5*m + 4)
```

giac [A] time = 0.16, size = 43, normalized size = 1.72

$$\frac{bmx^4x^m + bx^4x^m + amxx^m + 4\,axx^m}{m^2+5\,m+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a),x, algorithm="giac")
```

```
[Out] (b*m*x^4*x^m + b*x^4*x^m + a*m*x*x^m + 4*a*x*x^m)/(m^2 + 5*m + 4)
```

maple [A] time = 0.00, size = 35, normalized size = 1.40

$$\frac{\left(bm\,x^3 + b\,x^3 + am + 4a\right)x^{m+1}}{(m+4)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^3+a),x)
```

```
[Out] x^(m+1)*(b*m*x^3+b*x^3+a*m+4*a)/(m+4)/(m+1)
```

maxima [A] time = 1.30, size = 25, normalized size = 1.00

$$\frac{bx^{m+4}}{m+4} + \frac{ax^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a),x, algorithm="maxima")
```

```
[Out] b*x^(m + 4)/(m + 4) + a*x^(m + 1)/(m + 1)
```

mupad [B] time = 1.21, size = 34, normalized size = 1.36

$$\frac{x^{m+1}\left(4\,a + a\,m + b\,x^3 + b\,m\,x^3\right)}{m^2 + 5\,m + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^3),x)
```

```
[Out] (x^(m + 1)*(4*a + a*m + b*x^3 + b*m*x^3))/(5*m + m^2 + 4)
```

sympy [A] time = 1.09, size = 94, normalized size = 3.76

$$\begin{cases} -\frac{a}{3x^3} + b\log(x) & \text{for } m = -4 \\ a\log(x) + \frac{bx^3}{3} & \text{for } m = -1 \\ \frac{amxx^m}{m^2+5m+4} + \frac{4axx^m}{m^2+5m+4} + \frac{bmx^4x^m}{m^2+5m+4} + \frac{bx^4x^m}{m^2+5m+4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**3+a),x)
```

```
[Out] Piecewise((-a/(3*x**3) + b*log(x), Eq(m, -4)), (a*log(x) + b*x**3/3, Eq(m,
-1)), (a*m*x*x**m/(m**2 + 5*m + 4) + 4*a*x*x**m/(m**2 + 5*m + 4) + b*m*x**4
*x**m/(m**2 + 5*m + 4) + b*x**4*x**m/(m**2 + 5*m + 4), True))
```

3.588 $\int \frac{x^m}{a+bx^3}\,dx$

Optimal. Leaf size=39

$$\frac{x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{a(m+1)}$$

```
[Out] x^(1+m)*hypergeom([1, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/a/(1+m)
```

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {364}

$$\frac{x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{a(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x^3),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(a*(1
+ m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^m}{a+bx^3}\,dx = \frac{x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{a(1+m)}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.05

$$\frac{x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+1}{3}+1;-\frac{bx^3}{a}\right)}{a(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x^3),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, 1 + (1 + m)/3, -((b*x^3)/a)])/(a
*(1 + m))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m}{bx^3+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral(x^m/(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{bx^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*x^3 + a), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{x^m}{b\,x^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^3+a),x)
```

```
[Out] int(x^m/(b*x^3+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{bx^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(b*x^3 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{b\,x^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^3),x)
```

```
[Out] int(x^m/(a + b*x^3), x)
```

sympy [C] time = 8.75, size = 88, normalized size = 2.26

$$\frac{mxx^m\Phi\left(\frac{bx^3e^{i\pi}}{a}, 1, \frac{m}{3} + \frac{1}{3}\right)\Gamma\left(\frac{m}{3} + \frac{1}{3}\right)}{9a\Gamma\left(\frac{m}{3} + \frac{4}{3}\right)} + \frac{xx^m\Phi\left(\frac{bx^3e^{i\pi}}{a}, 1, \frac{m}{3} + \frac{1}{3}\right)\Gamma\left(\frac{m}{3} + \frac{1}{3}\right)}{9a\Gamma\left(\frac{m}{3} + \frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x**3+a),x)
```

```
[Out] m*x*x**m*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 1/3)*gamma(m/3 + 1/3)/
(9*a*gamma(m/3 + 4/3)) + x*x**m*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 +
 1/3)*gamma(m/3 + 1/3)/(9*a*gamma(m/3 + 4/3))
```

3.589 $\int \frac{x^m}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=39

$$\frac{x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{a^2(m+1)}$$

```
[Out] x^(1+m)*hypergeom([2, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/a^2/(1+m)
```

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {364}

$$\frac{x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{a^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x^3)^2,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(a^2*(
1 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^m}{\left(a+bx^3\right)^2}\,dx = \frac{x^{1+m}\,{}_2F_1\left(2,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{a^2(1+m)}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.05

$$\frac{x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{3};\frac{m+1}{3}+1;-\frac{bx^3}{a}\right)}{a^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x^3)^2,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/3, 1 + (1 + m)/3, -((b*x^3)/a)])/(a
^2*(1 + m))
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m}{b^2x^6+2\,abx^3+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(bx^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^3+a)^2,x, algorithm="giac")
[Out] integrate(x^m/(b*x^3 + a)^2, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(b\,x^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^3+a)^2,x)
[Out] int(x^m/(b*x^3+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(bx^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^3+a)^2,x, algorithm="maxima")
[Out] integrate(x^m/(b*x^3 + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{\left(b\,x^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^3)^2,x)
[Out] int(x^m/(a + b*x^3)^2, x)
```

sympy [C] time = 88.02, size = 515, normalized size = 13.21

$$-\frac{am^2xx^m\Phi\left(\frac{bx^3e^{i\pi}}{a},1,\frac{m}{3}+\frac{1}{3}\right)\Gamma\left(\frac{m}{3}+\frac{1}{3}\right)}{27a^3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)+27a^2bx^3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}+\frac{amxx^m\Phi\left(\frac{bx^3e^{i\pi}}{a},1,\frac{m}{3}+\frac{1}{3}\right)\Gamma\left(\frac{m}{3}+\frac{1}{3}\right)}{27a^3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)+27a^2bx^3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}+\frac{3amxx^m\Gamma\left(\frac{m}{3}+\frac{1}{3}\right)}{27a^3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)+27a^2bx^3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x**3+a)**2,x)
[Out] -a*m**2*x*x**m*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 1/3)*gamma(m/3 +
 1/3)/(27*a**3*gamma(m/3 + 4/3) + 27*a**2*b*x**3*gamma(m/3 + 4/3)) + a*m*x*
x**m*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 1/3)*gamma(m/3 + 1/3)/(27*
a**3*gamma(m/3 + 4/3) + 27*a**2*b*x**3*gamma(m/3 + 4/3)) + 3*a*m*x*x**m*gam
ma(m/3 + 1/3)/(27*a**3*gamma(m/3 + 4/3) + 27*a**2*b*x**3*gamma(m/3 + 4/3))
```

```
+ 2*a*x*x**m*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 1/3)*gamma(m/3 + 1
/3)/(27*a**3*gamma(m/3 + 4/3) + 27*a**2*b*x**3*gamma(m/3 + 4/3)) + 3*a*x*x*
*m*gamma(m/3 + 1/3)/(27*a**3*gamma(m/3 + 4/3) + 27*a**2*b*x**3*gamma(m/3 +
4/3)) - b*m**2*x**4*x**m*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 1/3)*g
amma(m/3 + 1/3)/(27*a**3*gamma(m/3 + 4/3) + 27*a**2*b*x**3*gamma(m/3 + 4/3)
) + b*m*x**4*x**m*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 1/3)*gamma(m/
3 + 1/3)/(27*a**3*gamma(m/3 + 4/3) + 27*a**2*b*x**3*gamma(m/3 + 4/3)) + 2*b
*x**4*x**m*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 1/3)*gamma(m/3 + 1/3
)/(27*a**3*gamma(m/3 + 4/3) + 27*a**2*b*x**3*gamma(m/3 + 4/3))
```

3.590 $\int \frac{x^m}{\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=39

$$\frac{x^{m+1}\,{}_2F_1\left(3,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{a^3(m+1)}$$

```
[Out] x^(1+m)*hypergeom([3, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/a^3/(1+m)
```

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {364}

$$\frac{x^{m+1}\,{}_2F_1\left(3,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{a^3(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x^3)^3,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[3, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(a^3*(
1 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^m}{\left(a+bx^3\right)^3}\,dx = \frac{x^{1+m}\,{}_2F_1\left(3,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{a^3(1+m)}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.05

$$\frac{x^{m+1}\,{}_2F_1\left(3,\frac{m+1}{3};\frac{m+1}{3}+1;-\frac{bx^3}{a}\right)}{a^3(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x^3)^3,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[3, (1 + m)/3, 1 + (1 + m)/3, -((b*x^3)/a)])/(a
^3*(1 + m))
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m}{b^3x^9+3\,ab^2x^6+3\,a^2bx^3+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m/(b^3*x^9 + 3*a*b^2*x^6 + 3*a^2*b*x^3 + a^3), x)
```

giac **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(bx^3+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^3+a)^3,x, algorithm="giac")
[Out] integrate(x^m/(b*x^3 + a)^3, x)
```

maple **[F]** time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(b\,x^3+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^3+a)^3,x)
[Out] int(x^m/(b*x^3+a)^3,x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(bx^3+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^3+a)^3,x, algorithm="maxima")
[Out] integrate(x^m/(b*x^3 + a)^3, x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{\left(b\,x^3+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^3)^3,x)
[Out] int(x^m/(a + b*x^3)^3, x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x**3+a)**3,x)
[Out] Timed out
```

3.591 $\int x^m \left(a+bx^3\right)^{3/2}\,dx$

Optimal. Leaf size=64

$$\frac{ax^{m+1}\sqrt{a+bx^3}\;{}_2F_1\left(-\frac{3}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{(m+1)\sqrt{\frac{bx^3}{a}+1}}$$

```
[Out] a*x^(1+m)*hypergeom([-3/2, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)*(b*x^3+a)^(1/2)
/(1+m)/(1+b*x^3/a)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {365, 364}

$$\frac{ax^{m+1}\sqrt{a+bx^3}\;{}_2F_1\left(-\frac{3}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{(m+1)\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^3)^(3/2),x]
```

```
[Out] (a*x^(1 + m)*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, (1 + m)/3, (4 + m)/3,
-((b*x^3)/a)])/((1 + m)*Sqrt[1 + (b*x^3)/a])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^m\left(a+bx^3\right)^{3/2}\,dx &= \frac{\left(a\sqrt{a+bx^3}\right)\int x^m\left(1+\frac{bx^3}{a}\right)^{3/2}\,dx}{\sqrt{1+\frac{bx^3}{a}}}\\ &= \frac{ax^{1+m}\sqrt{a+bx^3}\;{}_2F_1\left(-\frac{3}{2},\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{(1+m)\sqrt{1+\frac{bx^3}{a}}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 1.03

$$\frac{ax^{m+1}\sqrt{a+bx^3}\;{}_2F_1\left(-\frac{3}{2},\frac{m+1}{3};\frac{m+1}{3}+1;-\frac{bx^3}{a}\right)}{(m+1)\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^3)^(3/2),x]
```

```
[Out] (a*x^(1 + m)*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, (1 + m)/3, 1 + (1 + m)
/3, -((b*x^3)/a)])/((1 + m)*Sqrt[1 + (b*x^3)/a])
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^3+a\right)^{\frac{3}{2}}x^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(3/2)*x^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{3}{2}}x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(3/2)*x^m, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \left(b\,x^3+a\right)^{\frac{3}{2}}x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^3+a)^(3/2),x)
```

```
[Out] int(x^m*(b*x^3+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{3}{2}}x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(3/2)*x^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^m\left(b\,x^3+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^3)^(3/2),x)
```

```
[Out] int(x^m*(a + b*x^3)^(3/2), x)
```

sympy [C] time = 4.35, size = 54, normalized size = 0.84

$$\frac{a^{\frac{3}{2}} x x^{m} \Gamma\left(\frac{m}{3}+\frac{1}{3}\right) {}_2F_1\left(\begin{matrix}-\frac{3}{2}, \frac{m}{3}+\frac{1}{3} \\ \frac{m}{3}+\frac{4}{3}\end{matrix}\middle| \frac{b x^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**3+a)**(3/2),x)
```

```
[Out] a**(3/2)*x*x**m*gamma(m/3 + 1/3)*hyper((-3/2, m/3 + 1/3), (m/3 + 4/3,), b*x
**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 4/3))
```

3.592 $\int x^m \sqrt{a+bx^3}\, dx$

Optimal. Leaf size=63

$$\frac{x^{m+1}\sqrt{a+bx^3}\ {}_2F_1\left(-\frac{1}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{(m+1)\sqrt{\frac{bx^3}{a}+1}}$$

```
[Out] x^(1+m)*hypergeom([-1/2, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)*(b*x^3+a)^(1/2)/(
1+m)/(1+b*x^3/a)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {365, 364}

$$\frac{x^{m+1}\sqrt{a+bx^3}\ {}_2F_1\left(-\frac{1}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{(m+1)\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sqrt[a + b*x^3],x]
```

```
[Out] (x^(1 + m)*Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, (1 + m)/3, (4 + m)/3, -(
(b*x^3)/a)])/((1 + m)*Sqrt[1 + (b*x^3)/a])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^m\sqrt{a+bx^3}\, dx &= \frac{\sqrt{a+bx^3}\int x^m\sqrt{1+\frac{bx^3}{a}}\, dx}{\sqrt{1+\frac{bx^3}{a}}}\\ &= \frac{x^{1+m}\sqrt{a+bx^3}\ {}_2F_1\left(-\frac{1}{2},\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{(1+m)\sqrt{1+\frac{bx^3}{a}}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 65, normalized size = 1.03

$$\frac{x^{m+1}\sqrt{a+bx^3}\ {}_2F_1\left(-\frac{1}{2},\frac{m+1}{3};\frac{m+1}{3}+1;-\frac{bx^3}{a}\right)}{(m+1)\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sqrt[a + b*x^3],x]
```

```
[Out] (x^(1 + m)*Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, (1 + m)/3, 1 + (1 + m)/3
, -((b*x^3)/a)])/((1 + m)*Sqrt[1 + (b*x^3)/a])
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{bx^3 + a}\, x^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)*x^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^3 + a}\, x^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^3 + a)*x^m, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \sqrt{b\,x^3 + a}\ x^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^3+a)^(1/2),x)
```

```
[Out] int(x^m*(b*x^3+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^3 + a}\, x^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a)*x^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^m\, \sqrt{b\,x^3 + a}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^3)^(1/2),x)
```

```
[Out] int(x^m*(a + b*x^3)^(1/2), x)
```

sympy [C] time = 1.84, size = 54, normalized size = 0.86

$$\frac{\sqrt{a}\, x x^m \Gamma\left(\frac{m}{3} + \frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{3} + \frac{1}{3} \\ \frac{m}{3} + \frac{4}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3} + \frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**3+a)**(1/2),x)
```

```
[Out] sqrt(a)*x*x**m*gamma(m/3 + 1/3)*hyper((-1/2, m/3 + 1/3), (m/3 + 4/3,), b*x*
**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 4/3))
```

3.593 $\int \frac{x^m}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=52

$$\frac{x^{m+1}\sqrt{a+bx^3}\;{}_2F_1\left(1,\frac{1}{6}(2m+5);\frac{m+4}{3};-\frac{bx^3}{a}\right)}{a(m+1)}$$

```
[Out] x^(1+m)*hypergeom([1, 5/6+1/3*m],[4/3+1/3*m],-b*x^3/a)*(b*x^3+a)^(1/2)/a/(1
+m)
```

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {365, 364}

$$\frac{x^{m+1}\sqrt{\frac{bx^3}{a}+1}\;{}_2F_1\left(\frac{1}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{(m+1)\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/Sqrt[a + b*x^3],x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, (1 + m)/3, (4 + m)/3,
 -((b*x^3)/a)])/((1 + m)*Sqrt[a + b*x^3])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{\sqrt{a+bx^3}}\,dx &= \frac{\sqrt{1+\frac{bx^3}{a}}\int \frac{x^m}{\sqrt{1+\frac{bx^3}{a}}}\,dx}{\sqrt{a+bx^3}}\\ &= \frac{x^{1+m}\sqrt{1+\frac{bx^3}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{(1+m)\sqrt{a+bx^3}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 65, normalized size = 1.25

$$\frac{x^{m+1}\sqrt{\frac{bx^3}{a}+1}\;{}_2F_1\left(\frac{1}{2},\frac{m+1}{3};\frac{m+1}{3}+1;-\frac{bx^3}{a}\right)}{(m+1)\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/Sqrt[a + b*x^3],x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, (1 + m)/3, 1 + (1 + m
)/3, -((b*x^3)/a)])/((1 + m)*Sqrt[a + b*x^3])
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m}{\sqrt{bx^3+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/sqrt(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/sqrt(b*x^3 + a), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^3+a)^(1/2),x)
```

```
[Out] int(x^m/(b*x^3+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/sqrt(b*x^3 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^3)^(1/2),x)
```

```
[Out] int(x^m/(a + b*x^3)^(1/2), x)
```

sympy [C] time = 2.03, size = 53, normalized size = 1.02

$$\frac{x x^m \Gamma\left(\frac{m}{3}+\frac{1}{3}\right) {}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{m}{3}+\frac{1}{3} \\ \frac{m}{3}+\frac{4}{3}\end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x**3+a)**(1/2),x)
```

```
[Out] x*x**m*gamma(m/3 + 1/3)*hyper((1/2, m/3 + 1/3), (m/3 + 4/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*sqrt(a)*gamma(m/3 + 4/3))
```

3.594 $\int \frac{x^m}{\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=66

$$\frac{x^{m+1}\sqrt{\frac{bx^3}{a}+1}\;{}_2F_1\left(\frac{3}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{a(m+1)\sqrt{a+bx^3}}$$

```
[Out] x^(1+m)*hypergeom([3/2, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)*(1+b*x^3/a)^(1/2)/
a/(1+m)/(b*x^3+a)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {365, 364}

$$\frac{x^{m+1}\sqrt{\frac{bx^3}{a}+1}\;{}_2F_1\left(\frac{3}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{a(m+1)\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x^3)^(3/2),x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[3/2, (1 + m)/3, (4 + m)/3,
 -((b*x^3)/a)])/(a*(1 + m)*Sqrt[a + b*x^3])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^m}{\left(a+bx^3\right)^{3/2}}\,dx = \frac{\sqrt{1+\frac{bx^3}{a}}\int \frac{x^m}{\left(1+\frac{bx^3}{a}\right)^{3/2}}\,dx}{a\sqrt{a+bx^3}}$$

$$= \frac{x^{1+m}\sqrt{1+\frac{bx^3}{a}}\;{}_2F_1\left(\frac{3}{2},\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{a(1+m)\sqrt{a+bx^3}}$$

Mathematica [A] time = 0.02, size = 68, normalized size = 1.03

$$\frac{x^{m+1}\sqrt{\frac{bx^3}{a}+1}\;{}_2F_1\left(\frac{3}{2},\frac{m+1}{3};\frac{m+1}{3}+1;-\frac{bx^3}{a}\right)}{a(m+1)\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x^3)^(3/2),x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[3/2, (1 + m)/3, 1 + (1 + m
)/3, -((b*x^3)/a)])/(a*(1 + m)*Sqrt[a + b*x^3])
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+a}\,x^m}{b^2x^6+2\,abx^3+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)*x^m/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*x^3 + a)^(3/2), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(b\,x^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^3+a)^(3/2),x)
```

```
[Out] int(x^m/(b*x^3+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(b*x^3 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{\left(b\,x^3+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^3)^(3/2),x)
```

```
[Out] int(x^m/(a + b*x^3)^(3/2), x)
```

sympy [C] time = 2.06, size = 53, normalized size = 0.80

$$\frac{xx^m\Gamma\left(\frac{m}{3}+\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2}, \frac{m}{3}+\frac{1}{3} \\ \frac{m}{3}+\frac{4}{3}\end{matrix}\right| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x**3+a)**(3/2),x)
```

```
[Out] x*x**m*gamma(m/3 + 1/3)*hyper((3/2, m/3 + 1/3), (m/3 + 4/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(3/2)*gamma(m/3 + 4/3))
```

3.595 $\int (cx)^m \left(a+bx^3\right)^{4/3}\,dx$

Optimal. Leaf size=69

$$\frac{a\sqrt[3]{a+bx^3}\,(cx)^{m+1}\,{}_2F_1\left(-\frac{4}{3},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{c(m+1)\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[Out] a*(c*x)^(1+m)*(b*x^3+a)^(1/3)*hypergeom([-4/3, 1/3+1/3*m],[4/3+1/3*m],-b*x^
3/a)/c/(1+m)/(1+b*x^3/a)^(1/3)
```

Rubi [A] time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {365, 364}

$$\frac{a\sqrt[3]{a+bx^3}\,(cx)^{m+1}\,{}_2F_1\left(-\frac{4}{3},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{c(m+1)\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*(a + b*x^3)^(4/3),x]
```

```
[Out] (a*(c*x)^(1 + m)*(a + b*x^3)^(1/3)*Hypergeometric2F1[-4/3, (1 + m)/3, (4 +
m)/3, -((b*x^3)/a)])/(c*(1 + m)*(1 + (b*x^3)/a)^(1/3))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (cx)^m\left(a+bx^3\right)^{4/3}\,dx &= \frac{\left(a\sqrt[3]{a+bx^3}\right)\int (cx)^m\left(1+\frac{bx^3}{a}\right)^{4/3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= \frac{a(cx)^{1+m}\sqrt[3]{a+bx^3}\;{}_2F_1\left(-\frac{4}{3},\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{c(1+m)\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 67, normalized size = 0.97

$$\frac{ax\sqrt[3]{a+bx^3}\,(cx)^m\,{}_2F_1\left(-\frac{4}{3},\frac{m+1}{3};\frac{m+1}{3}+1;-\frac{bx^3}{a}\right)}{(m+1)\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*(a + b*x^3)^(4/3),x]
```

```
[Out] (a*x*(c*x)^m*(a + b*x^3)^(1/3)*Hypergeometric2F1[-4/3, (1 + m)/3, 1 + (1 +
m)/3, -((b*x^3)/a)])/((1 + m)*(1 + (b*x^3)/a)^(1/3))
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^3+a\right)^{\frac{4}{3}}(cx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^3+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(4/3)*(c*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{4}{3}}(cx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^3+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)*(c*x)^m, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \left(b\,x^3+a\right)^{\frac{4}{3}}(cx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(b*x^3+a)^(4/3),x)
```

```
[Out] int((c*x)^m*(b*x^3+a)^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{4}{3}}(cx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^3+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)*(c*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x)^m\left(b\,x^3+a\right)^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(a + b*x^3)^(4/3),x)
```

```
[Out] int((c*x)^m*(a + b*x^3)^(4/3), x)
```

sympy [C] time = 12.60, size = 58, normalized size = 0.84

$$\frac{a^{\frac{4}{3}} c^m x x^m \Gamma\left(\frac{m}{3}+\frac{1}{3}\right) {}_2F_1\left(\begin{matrix}-\frac{4}{3}, \frac{m}{3}+\frac{1}{3} \\ \frac{m}{3}+\frac{4}{3}\end{matrix}\middle| \frac{b x^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(b*x**3+a)**(4/3),x)
```

```
[Out] a**(4/3)*c**m*x*x**m*gamma(m/3 + 1/3)*hyper((-4/3, m/3 + 1/3), (m/3 + 4/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 4/3))
```

3.596 $\int (cx)^m \left(a+bx^3\right)^{2/3}\,dx$

Optimal. Leaf size=61

$$\frac{(cx)^{m+1}\left(a+bc^3x^3\right)^{5/3}\,{}_2F_1\left(1,\frac{m+6}{3};\frac{m+4}{3};-\frac{bc^3x^3}{a}\right)}{ac(m+1)}$$

```
[Out] (c*x)^(1+m)*(b*c^3*x^3+a)^(5/3)*hypergeom([1, 2+1/3*m],[4/3+1/3*m],-b*c^3*x
^3/a)/a/c/(1+m)
```

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {365, 364}

$$\frac{\left(a+bx^3\right)^{2/3}(cx)^{m+1}\,{}_2F_1\left(-\frac{2}{3},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{c(m+1)\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*(a + b*x^3)^(2/3),x]
```

```
[Out] ((c*x)^(1 + m)*(a + b*x^3)^(2/3)*Hypergeometric2F1[-2/3, (1 + m)/3, (4 + m)
/3, -((b*x^3)/a)])/(c*(1 + m)*(1 + (b*x^3)/a)^(2/3))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (cx)^m\left(a+bx^3\right)^{2/3}\,dx &= \frac{\left(a+bx^3\right)^{2/3}\int (cx)^m\left(1+\frac{bx^3}{a}\right)^{2/3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\ &= \frac{(cx)^{1+m}\left(a+bx^3\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{c(1+m)\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 66, normalized size = 1.08

$$\frac{x\left(a+bx^3\right)^{2/3}(cx)^m\,{}_2F_1\left(-\frac{2}{3},\frac{m+1}{3};\frac{m+1}{3}+1;-\frac{bx^3}{a}\right)}{(m+1)\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*(a + b*x^3)^(2/3),x]
```

```
[Out] (x*(c*x)^m*(a + b*x^3)^(2/3)*Hypergeometric2F1[-2/3, (1 + m)/3, 1 + (1 + m)
/3, -((b*x^3)/a)])/((1 + m)*(1 + (b*x^3)/a)^(2/3))
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^3+a\right)^{\frac{2}{3}}(cx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(2/3)*(c*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{2}{3}}(cx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)*(c*x)^m, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \left(b\,x^3+a\right)^{\frac{2}{3}}(cx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(b*x^3+a)^(2/3),x)
```

```
[Out] int((c*x)^m*(b*x^3+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{2}{3}}(cx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)*(c*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (c\,x)^m\left(b\,x^3+a\right)^{2/3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(a + b*x^3)^(2/3),x)
```

```
[Out] int((c*x)^m*(a + b*x^3)^(2/3), x)
```

sympy [C] time = 2.79, size = 58, normalized size = 0.95

$$\frac{a^{\frac{2}{3}} c^m x x^m \Gamma\left(\frac{m}{3}+\frac{1}{3}\right) {}_2F_1\left(\left.\begin{matrix}-\frac{2}{3}, \frac{m}{3}+\frac{1}{3} \\ \frac{m}{3}+\frac{4}{3}\end{matrix}\right| \frac{b x^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(b*x**3+a)**(2/3),x)
```

```
[Out] a**(2/3)*c**m*x*x**m*gamma(m/3 + 1/3)*hyper((-2/3, m/3 + 1/3), (m/3 + 4/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 4/3))
```

3.597 $\int (cx)^m \sqrt[3]{a+bx^3}\,dx$

Optimal. Leaf size=61

$$\frac{(cx)^{m+1}\left(a+bc^3x^3\right)^{4/3}\,{}_2F_1\left(1,\frac{m+5}{3};\frac{m+4}{3};-\frac{bc^3x^3}{a}\right)}{ac(m+1)}$$

```
[Out] (c*x)^(1+m)*(b*c^3*x^3+a)^(4/3)*hypergeom([1, 5/3+1/3*m],[4/3+1/3*m],-b*c^3
*x^3/a)/a/c/(1+m)
```

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {365, 364}

$$\frac{\sqrt[3]{a+bx^3}\,(cx)^{m+1}\,{}_2F_1\left(-\frac{1}{3},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{c(m+1)\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*(a + b*x^3)^(1/3),x]
```

```
[Out] ((c*x)^(1 + m)*(a + b*x^3)^(1/3)*Hypergeometric2F1[-1/3, (1 + m)/3, (4 + m)
/3, -((b*x^3)/a)])/(c*(1 + m)*(1 + (b*x^3)/a)^(1/3))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (cx)^m \sqrt[3]{a+bx^3}\,dx &= \frac{\sqrt[3]{a+bx^3}\int (cx)^m \sqrt[3]{1+\frac{bx^3}{a}}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= \frac{(cx)^{1+m}\sqrt[3]{a+bx^3}\,{}_2F_1\left(-\frac{1}{3},\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{c(1+m)\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 1.08

$$\frac{x\sqrt[3]{a+bx^3}\,(cx)^m\,{}_2F_1\left(-\frac{1}{3},\frac{m+1}{3};\frac{m+1}{3}+1;-\frac{bx^3}{a}\right)}{(m+1)\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*(a + b*x^3)^(1/3),x]
```

```
[Out] (x*(c*x)^m*(a + b*x^3)^(1/3)*Hypergeometric2F1[-1/3, (1 + m)/3, 1 + (1 + m)
/3, -((b*x^3)/a)])/((1 + m)*(1 + (b*x^3)/a)^(1/3))
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^3+a\right)^{\frac{1}{3}}(cx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(1/3)*(c*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{1}{3}}(cx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*(c*x)^m, x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \left(b\,x^3+a\right)^{\frac{1}{3}}(cx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(b*x^3+a)^(1/3),x)
```

```
[Out] int((c*x)^m*(b*x^3+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{1}{3}}(cx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*(c*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (c\,x)^m\left(b\,x^3+a\right)^{1/3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(a + b*x^3)^(1/3),x)
```

```
[Out] int((c*x)^m*(a + b*x^3)^(1/3), x)
```

sympy [C] time = 2.55, size = 58, normalized size = 0.95

$$\frac{\sqrt[3]{a}\,c^m x x^m \Gamma\left(\frac{m}{3}+\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3}, \frac{m}{3}+\frac{1}{3} \\ \frac{m}{3}+\frac{4}{3}\end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(b*x**3+a)**(1/3),x)
```

```
[Out] a**(1/3)*c**m*x*x**m*gamma(m/3 + 1/3)*hyper((-1/3, m/3 + 1/3), (m/3 + 4/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 4/3))
```

3.598 $\int (cx)^m \left(a+bx^3\right)^p \, dx$

Optimal. Leaf size=66

$$\frac{(cx)^{m+1}\left(a+bx^3\right)^p \left(\frac{bx^3}{a}+1\right)^{-p} {}_2F_1\left(\frac{m+1}{3},-p;\frac{m+4}{3};-\frac{bx^3}{a}\right)}{c(m+1)}$$

```
[Out] (c*x)^(1+m)*(b*x^3+a)^p*hypergeom([-p, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/c/(
1+m)/((1+b*x^3/a)^p)
```

Rubi [A] time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {365, 364}

$$\frac{(cx)^{m+1}\left(a+bx^3\right)^p \left(\frac{bx^3}{a}+1\right)^{-p} {}_2F_1\left(\frac{m+1}{3},-p;\frac{m+4}{3};-\frac{bx^3}{a}\right)}{c(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*(a + b*x^3)^p,x]
```

```
[Out] ((c*x)^(1 + m)*(a + b*x^3)^p*Hypergeometric2F1[(1 + m)/3, -p, (4 + m)/3, -(
(b*x^3)/a)])/(c*(1 + m)*(1 + (b*x^3)/a)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (cx)^m \left(a+bx^3\right)^p \, dx &= \left(\left(a+bx^3\right)^p \left(1+\frac{bx^3}{a}\right)^{-p}\right)\int (cx)^m \left(1+\frac{bx^3}{a}\right)^p \, dx \\ &= \frac{(cx)^{1+m}\left(a+bx^3\right)^p \left(1+\frac{bx^3}{a}\right)^{-p} {}_2F_1\left(\frac{1+m}{3},-p;\frac{4+m}{3};-\frac{bx^3}{a}\right)}{c(1+m)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 64, normalized size = 0.97

$$\frac{x(cx)^m \left(a+bx^3\right)^p \left(\frac{bx^3}{a}+1\right)^{-p} {}_2F_1\left(\frac{m+1}{3},-p;\frac{m+1}{3}+1;-\frac{bx^3}{a}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*(a + b*x^3)^p,x]
```

```
[Out] (x*(c*x)^m*(a + b*x^3)^p*Hypergeometric2F1[(1 + m)/3, -p, 1 + (1 + m)/3, -(
(b*x^3)/a)])/((1 + m)*(1 + (b*x^3)/a)^p)
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^3+a\right)^p (cx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^3+a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^p*(c*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^p (cx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^3+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^p*(c*x)^m, x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int (cx)^m \left(b\,x^3+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(b*x^3+a)^p,x)
```

```
[Out] int((c*x)^m*(b*x^3+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^p (cx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^3+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^p*(c*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (c\,x)^m \left(b\,x^3+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(a + b*x^3)^p,x)
```

```
[Out] int((c*x)^m*(a + b*x^3)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(b*x**3+a)**p,x)
```

```
[Out] Timed out
```

3.599 $\int x^2 \left(a + bx^3\right)^p \, dx$

Optimal. Leaf size=23

$$\frac{\left(a + bx^3\right)^{p+1}}{3b(p+1)}$$

```
[Out] 1/3*(b*x^3+a)^(1+p)/b/(1+p)
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$\frac{\left(a + bx^3\right)^{p+1}}{3b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3)^p,x]
```

```
[Out] (a + b*x^3)^(1 + p)/(3*b*(1 + p))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^2 \left(a + bx^3\right)^p \, dx = \frac{\left(a + bx^3\right)^{1+p}}{3b(1+p)}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{\left(a + bx^3\right)^{p+1}}{3b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)^p,x]
```

```
[Out] (a + b*x^3)^(1 + p)/(3*b*(1 + p))
```

fricas [A] time = 0.63, size = 25, normalized size = 1.09

$$\frac{\left(bx^3 + a\right)\left(bx^3 + a\right)^p}{3\left(bp + b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^p,x, algorithm="fricas")
```

```
[Out] 1/3*(b*x^3 + a)*(b*x^3 + a)^p/(b*p + b)
```

giac [A] time = 0.15, size = 21, normalized size = 0.91

$$\frac{\left(bx^3+a\right)^{p+1}}{3\,b\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^p,x, algorithm="giac")
[Out] 1/3*(b*x^3 + a)^(p + 1)/(b*(p + 1))
```

maple [A] time = 0.00, size = 22, normalized size = 0.96

$$\frac{\left(b\,x^3+a\right)^{p+1}}{3\left(p+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^p,x)
[Out] 1/3*(b*x^3+a)^(p+1)/b/(p+1)
```

maxima [A] time = 1.36, size = 21, normalized size = 0.91

$$\frac{\left(bx^3+a\right)^{p+1}}{3\,b\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^p,x, algorithm="maxima")
[Out] 1/3*(b*x^3 + a)^(p + 1)/(b*(p + 1))
```

mupad [B] time = 1.09, size = 21, normalized size = 0.91

$$\frac{\left(b\,x^3+a\right)^{p+1}}{3\,b\,\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^3)^p,x)
[Out] (a + b*x^3)^(p + 1)/(3*b*(p + 1))
```

sympy [A] time = 2.13, size = 134, normalized size = 5.83

$$\begin{cases} \frac{x^3}{3a} & \text{for } b = 0 \wedge p = -1 \\ \frac{a^p x^3}{3} & \text{for } b = 0 \\ \frac{\log\left(-\sqrt[3]{-1}\,\sqrt[3]{a}\,\sqrt[3]{\frac{1}{b}}+x\right)}{3b} + \frac{\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-1}\,\sqrt[3]{a}\,x\sqrt[3]{\frac{1}{b}}+4x^2\right)}{3b} & \text{for } p = -1 \\ \frac{a\left(a+bx^3\right)^p}{3bp+3b} + \frac{bx^3\left(a+bx^3\right)^p}{3bp+3b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**p,x)
```

```
[Out] Piecewise((x**3/(3*a), Eq(b, 0) & Eq(p, -1)), (a**p*x**3/3, Eq(b, 0)), (log
(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x)/(3*b) + log(4*(-1)**(2/3)*a**(2/3)
*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x*(1/b)**(1/3) + 4*x**2)/(3*b), Eq(p
, -1)), (a*(a + b*x**3)**p/(3*b*p + 3*b) + b*x**3*(a + b*x**3)**p/(3*b*p +
3*b), True))
```

3.600 $\int x^5 \left(a+bx^3\right)^p \, dx$

Optimal. Leaf size=48

$$\frac{\left(a+bx^3\right)^{p+2}}{3b^2(p+2)} - \frac{a\left(a+bx^3\right)^{p+1}}{3b^2(p+1)}$$

```
[Out] -1/3*a*(b*x^3+a)^(1+p)/b^2/(1+p)+1/3*(b*x^3+a)^(2+p)/b^2/(2+p)
```

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{\left(a+bx^3\right)^{p+2}}{3b^2(p+2)} - \frac{a\left(a+bx^3\right)^{p+1}}{3b^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^3)^p,x]
```

```
[Out] -(a*(a + b*x^3)^(1 + p))/(3*b^2*(1 + p)) + (a + b*x^3)^(2 + p)/(3*b^2*(2 + p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^5 \left(a+bx^3\right)^p \, dx &= \frac{1}{3}\operatorname{Subst}\left(\int x(a+bx)^p \, dx, x, x^3\right) \\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(-\frac{a(a+bx)^p}{b} + \frac{(a+bx)^{1+p}}{b}\right) dx, x, x^3\right) \\
&= -\frac{a\left(a+bx^3\right)^{1+p}}{3b^2(1+p)} + \frac{\left(a+bx^3\right)^{2+p}}{3b^2(2+p)}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 40, normalized size = 0.83

$$\frac{\left(a+bx^3\right)^{p+1}\left(b(p+1)x^3 - a\right)}{3b^2(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^3)^p,x]
```

```
[Out] ((a + b*x^3)^(1 + p)*(-a + b*(1 + p)*x^3))/(3*b^2*(1 + p)*(2 + p))
```

fricas [A] time = 0.89, size = 58, normalized size = 1.21

$$\frac{\left(\left(b^2p+b^2\right)x^6+abpx^3-a^2\right)\left(bx^3+a\right)^p}{3\left(b^2p^2+3\,b^2p+2\,b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^p,x, algorithm="fricas")
```

```
[Out] 1/3*((b^2*p + b^2)*x^6 + a*b*p*x^3 - a^2)*(b*x^3 + a)^p/(b^2*p^2 + 3*b^2*p
+ 2*b^2)
```

giac [B] time = 0.17, size = 94, normalized size = 1.96

$$\frac{\left(bx^3+a\right)^2\left(bx^3+a\right)^p p-\left(bx^3+a\right)\left(bx^3+a\right)^p ap+\left(bx^3+a\right)^2\left(bx^3+a\right)^p-2\left(bx^3+a\right)\left(bx^3+a\right)^p a}{3\left(p^2+3\,p+2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^p,x, algorithm="giac")
```

```
[Out] 1/3*((b*x^3 + a)^2*(b*x^3 + a)^p*p - (b*x^3 + a)*(b*x^3 + a)^p*a*p + (b*x^3
 + a)^2*(b*x^3 + a)^p - 2*(b*x^3 + a)*(b*x^3 + a)^p*a)/((p^2 + 3*p + 2)*b^2
)
```

maple [A] time = 0.00, size = 42, normalized size = 0.88

$$-\frac{\left(-pb\,x^3-b\,x^3+a\right)\left(b\,x^3+a\right)^{p+1}}{3\left(p^2+3p+2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^3+a)^p,x)
```

```
[Out] -1/3*(b*x^3+a)^(p+1)*(-b*p*x^3-b*x^3+a)/b^2/(p^2+3*p+2)
```

maxima [A] time = 1.33, size = 47, normalized size = 0.98

$$\frac{\left(b^2\left(p+1\right)x^6+abpx^3-a^2\right)\left(bx^3+a\right)^p}{3\left(p^2+3\,p+2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^p,x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*(p + 1)*x^6 + a*b*p*x^3 - a^2)*(b*x^3 + a)^p/((p^2 + 3*p + 2)*b^2)
```

mupad [B] time = 1.08, size = 68, normalized size = 1.42

$$\left(b\,x^3+a\right)^p\left(\frac{x^6\,\left(p+1\right)}{3\,\left(p^2+3\,p+2\right)}-\frac{a^2}{3\,b^2\,\left(p^2+3\,p+2\right)}+\frac{a\,p\,x^3}{3\,b\,\left(p^2+3\,p+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^3)^p,x)
```

```
[Out] (a + b*x^3)^p*((x^6*(p + 1))/(3*(3*p + p^2 + 2)) - a^2/(3*b^2*(3*p + p^2 +
2)) + (a*p*x^3)/(3*b*(3*p + p^2 + 2)))
```

sympy [A] time = 8.07, size = 524, normalized size = 10.92

$$\begin{cases} \frac{a^p x^6}{6} \\ \frac{a \log\left(-\sqrt[3]{-1}\,\sqrt[3]{a}\,\sqrt[3]{\frac{1}{b}}+x\right)}{3ab^2+3b^3x^3} + \frac{a \log\left(4(-1)^{\frac{2}{3}} a^{\frac{2}{3}} \left(\frac{1}{b}\right)^{\frac{2}{3}} + 4\sqrt[3]{-1}\,\sqrt[3]{a}\,x\sqrt[3]{\frac{1}{b}}+4x^2\right)}{3ab^2+3b^3x^3} - \frac{2a\log(2)}{3ab^2+3b^3x^3} + \frac{a}{3ab^2+3b^3x^3} + \frac{bx^3 \log\left(-\sqrt[3]{-1}\,\sqrt[3]{a}\,\sqrt[3]{\frac{1}{b}}+x\right)}{3ab^2+3b^3x^3} + \\ -\frac{a \log\left(-\sqrt[3]{-1}\,\sqrt[3]{a}\,\sqrt[3]{\frac{1}{b}}+x\right)}{3b^2} - \frac{a \log\left(4(-1)^{\frac{2}{3}} a^{\frac{2}{3}} \left(\frac{1}{b}\right)^{\frac{2}{3}} + 4\sqrt[3]{-1}\,\sqrt[3]{a}\,x\sqrt[3]{\frac{1}{b}}+4x^2\right)}{3b^2} + \frac{x^3}{3b} \\ -\frac{a^2\left(a+bx^3\right)^p}{3b^2p^2+9b^2p+6b^2} + \frac{abpx^3\left(a+bx^3\right)^p}{3b^2p^2+9b^2p+6b^2} + \frac{b^2px^6\left(a+bx^3\right)^p}{3b^2p^2+9b^2p+6b^2} + \frac{b^2x^6\left(a+bx^3\right)^p}{3b^2p^2+9b^2p+6b^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**3+a)**p,x)
```

```
[Out] Piecewise((a**p*x**6/6, Eq(b, 0)), (a*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3
) + x)/(3*a*b**2 + 3*b**3*x**3) + a*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3)
 + 4*(-1)**(1/3)*a**(1/3)*x*(1/b)**(1/3) + 4*x**2)/(3*a*b**2 + 3*b**3*x**3)
 - 2*a*log(2)/(3*a*b**2 + 3*b**3*x**3) + a/(3*a*b**2 + 3*b**3*x**3) + b*x**
3*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x)/(3*a*b**2 + 3*b**3*x**3) + b*
x**3*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x*(1/
b)**(1/3) + 4*x**2)/(3*a*b**2 + 3*b**3*x**3) - 2*b*x**3*log(2)/(3*a*b**2 +
3*b**3*x**3), Eq(p, -2)), (-a*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x)/(
3*b**2) - a*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3
)*x*(1/b)**(1/3) + 4*x**2)/(3*b**2) + x**3/(3*b), Eq(p, -1)), (-a**2*(a + b
*x**3)**p/(3*b**2*p**2 + 9*b**2*p + 6*b**2) + a*b*p*x**3*(a + b*x**3)**p/(3
*b**2*p**2 + 9*b**2*p + 6*b**2) + b**2*p*x**6*(a + b*x**3)**p/(3*b**2*p**2
+ 9*b**2*p + 6*b**2) + b**2*x**6*(a + b*x**3)**p/(3*b**2*p**2 + 9*b**2*p +
6*b**2), True))
```

3.601 $\int x^8 \left(a+bx^3\right)^p \, dx$

Optimal. Leaf size=74

$$\frac{a^2 \left(a+bx^3\right)^{p+1}}{3b^3(p+1)} - \frac{2a \left(a+bx^3\right)^{p+2}}{3b^3(p+2)} + \frac{\left(a+bx^3\right)^{p+3}}{3b^3(p+3)}$$

```
[Out] 1/3*a^2*(b*x^3+a)^(1+p)/b^3/(1+p)-2/3*a*(b*x^3+a)^(2+p)/b^3/(2+p)+1/3*(b*x^
3+a)^(3+p)/b^3/(3+p)
```

Rubi [A] time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{a^2 \left(a+bx^3\right)^{p+1}}{3b^3(p+1)} - \frac{2a \left(a+bx^3\right)^{p+2}}{3b^3(p+2)} + \frac{\left(a+bx^3\right)^{p+3}}{3b^3(p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x^3)^p,x]
```

```
[Out] (a^2*(a + b*x^3)^(1 + p))/(3*b^3*(1 + p)) - (2*a*(a + b*x^3)^(2 + p))/(3*b^
3*(2 + p)) + (a + b*x^3)^(3 + p)/(3*b^3*(3 + p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^8 \left(a+bx^3\right)^p \, dx &= \frac{1}{3} \text{Subst}\left(\int x^2 (a+bx)^p \, dx, x, x^3\right) \\
&= \frac{1}{3} \text{Subst}\left(\int \left(\frac{a^2(a+bx)^p}{b^2} - \frac{2a(a+bx)^{1+p}}{b^2} + \frac{(a+bx)^{2+p}}{b^2}\right) dx, x, x^3\right) \\
&= \frac{a^2 \left(a+bx^3\right)^{1+p}}{3b^3(1+p)} - \frac{2a \left(a+bx^3\right)^{2+p}}{3b^3(2+p)} + \frac{\left(a+bx^3\right)^{3+p}}{3b^3(3+p)}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 64, normalized size = 0.86

$$\frac{\left(a+bx^3\right)^{p+1} \left(2a^2 - 2ab(p+1)x^3 + b^2\left(p^2+3p+2\right)x^6\right)}{3b^3(p+1)(p+2)(p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x^3)^p,x]
```

```
[Out] ((a + b*x^3)^(1 + p)*(2*a^2 - 2*a*b*(1 + p)*x^3 + b^2*(2 + 3*p + p^2)*x^6))
/(3*b^3*(1 + p)*(2 + p)*(3 + p))
```

fricas [A] time = 0.84, size = 98, normalized size = 1.32

$$\frac{\left(\left(b^3p^2+3\,b^3p+2\,b^3\right)x^9-2\,a^2bpx^3+\left(ab^2p^2+ab^2p\right)x^6+2\,a^3\right)\left(bx^3+a\right)^p}{3\left(b^3p^3+6\,b^3p^2+11\,b^3p+6\,b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^p,x, algorithm="fricas")
```

```
[Out] 1/3*((b^3*p^2 + 3*b^3*p + 2*b^3)*x^9 - 2*a^2*b*p*x^3 + (a*b^2*p^2 + a*b^2*p
)*x^6 + 2*a^3)*(b*x^3 + a)^p/(b^3*p^3 + 6*b^3*p^2 + 11*b^3*p + 6*b^3)
```

giac [B] time = 0.16, size = 231, normalized size = 3.12

$$\frac{\left(bx^3+a\right)^3\left(bx^3+a\right)^p p^2-2\left(bx^3+a\right)^2\left(bx^3+a\right)^p ap^2+\left(bx^3+a\right)\left(bx^3+a\right)^p a^2p^2+3\left(bx^3+a\right)^3\left(bx^3+a\right)^p p-8($$
$$3\left(b^2p^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^p,x, algorithm="giac")
```

```
[Out] 1/3*((b*x^3 + a)^3*(b*x^3 + a)^p*p^2 - 2*(b*x^3 + a)^2*(b*x^3 + a)^p*a*p^2
+ (b*x^3 + a)*(b*x^3 + a)^p*a^2*p^2 + 3*(b*x^3 + a)^3*(b*x^3 + a)^p*p - 8*(
b*x^3 + a)^2*(b*x^3 + a)^p*a*p + 5*(b*x^3 + a)*(b*x^3 + a)^p*a^2*p + 2*(b*x
^3 + a)^3*(b*x^3 + a)^p - 6*(b*x^3 + a)^2*(b*x^3 + a)^p*a + 6*(b*x^3 + a)*(
b*x^3 + a)^p*a^2)/((b^2*p^3 + 6*b^2*p^2 + 11*b^2*p + 6*b^2)*b)
```

maple [A] time = 0.01, size = 80, normalized size = 1.08

$$\frac{\left(b^2p^2x^6+3b^2p\,x^6+2b^2x^6-2abp\,x^3-2ab\,x^3+2a^2\right)\left(b\,x^3+a\right)^{p+1}}{3\left(p^3+6p^2+11p+6\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^3+a)^p,x)
```

```
[Out] 1/3*(b*x^3+a)^(p+1)*(b^2*p^2*x^6+3*b^2*p*x^6+2*b^2*x^6-2*a*b*p*x^3-2*a*b*x^
3+2*a^2)/b^3/(p^3+6*p^2+11*p+6)
```

maxima [A] time = 1.44, size = 73, normalized size = 0.99

$$\frac{\left(\left(p^2+3\,p+2\right)b^3x^9+\left(p^2+p\right)ab^2x^6-2\,a^2bpx^3+2\,a^3\right)\left(bx^3+a\right)^p}{3\left(p^3+6\,p^2+11\,p+6\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^p,x, algorithm="maxima")
```

```
[Out] 1/3*((p^2 + 3*p + 2)*b^3*x^9 + (p^2 + p)*a*b^2*x^6 - 2*a^2*b*p*x^3 + 2*a^3)
*(b*x^3 + a)^p/((p^3 + 6*p^2 + 11*p + 6)*b^3)
```

mupad [B] time = 1.09, size = 118, normalized size = 1.59

$$\left(b\,x^3+a\right)^p\left(\frac{2\,a^3}{3\,b^3\left(p^3+6\,p^2+11\,p+6\right)}+\frac{x^9\left(p^2+3\,p+2\right)}{3\left(p^3+6\,p^2+11\,p+6\right)}-\frac{2\,a^2\,p\,x^3}{3\,b^2\left(p^3+6\,p^2+11\,p+6\right)}+\frac{a\,p\,x^6}{3\,b\left(p^3+6\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*x^3)^p,x)
```

```
[Out] (a + b*x^3)^p*((2*a^3)/(3*b^3*(11*p + 6*p^2 + p^3 + 6)) + (x^9*(3*p + p^2 +
 2))/(3*(11*p + 6*p^2 + p^3 + 6)) - (2*a^2*p*x^3)/(3*b^2*(11*p + 6*p^2 + p^
3 + 6)) + (a*p*x^6*(p + 1))/(3*b*(11*p + 6*p^2 + p^3 + 6)))
```

sympy [**A**] time = 32.72, size = 1370, normalized size = 18.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**3+a)**p,x)
```

```
[Out] Piecewise((a**p*x**9/9, Eq(b, 0)), (2*a**2*log(-(-1)**(1/3)*a**(1/3)*(1/b)*
*(1/3) + x)/(6*a**2*b**3 + 12*a*b**4*x**3 + 6*b**5*x**6) + 2*a**2*log(4*(-1
)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x*(1/b)**(1/3) + 4*
x**2)/(6*a**2*b**3 + 12*a*b**4*x**3 + 6*b**5*x**6) - 4*a**2*log(2)/(6*a**2*
b**3 + 12*a*b**4*x**3 + 6*b**5*x**6) + 3*a**2/(6*a**2*b**3 + 12*a*b**4*x**3
 + 6*b**5*x**6) + 4*a*b*x**3*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x)/(6
*a**2*b**3 + 12*a*b**4*x**3 + 6*b**5*x**6) + 4*a*b*x**3*log(4*(-1)**(2/3)*a
**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x*(1/b)**(1/3) + 4*x**2)/(6*a
**2*b**3 + 12*a*b**4*x**3 + 6*b**5*x**6) - 8*a*b*x**3*log(2)/(6*a**2*b**3 +
 12*a*b**4*x**3 + 6*b**5*x**6) + 4*a*b*x**3/(6*a**2*b**3 + 12*a*b**4*x**3 +
 6*b**5*x**6) + 2*b**2*x**6*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x)/(6*
a**2*b**3 + 12*a*b**4*x**3 + 6*b**5*x**6) + 2*b**2*x**6*log(4*(-1)**(2/3)*a
**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x*(1/b)**(1/3) + 4*x**2)/(6*a
**2*b**3 + 12*a*b**4*x**3 + 6*b**5*x**6) - 4*b**2*x**6*log(2)/(6*a**2*b**3
+ 12*a*b**4*x**3 + 6*b**5*x**6), Eq(p, -3)), (-2*a**2*log(-(-1)**(1/3)*a**(
1/3)*(1/b)**(1/3) + x)/(3*a*b**3 + 3*b**4*x**3) - 2*a**2*log(4*(-1)**(2/3)*
a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x*(1/b)**(1/3) + 4*x**2)/(3*
a*b**3 + 3*b**4*x**3) - 2*a**2/(3*a*b**3 + 3*b**4*x**3) + 4*a**2*log(2)/(3*
a*b**3 + 3*b**4*x**3) - 2*a*b*x**3*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) +
 x)/(3*a*b**3 + 3*b**4*x**3) - 2*a*b*x**3*log(4*(-1)**(2/3)*a**(2/3)*(1/b)*
*(2/3) + 4*(-1)**(1/3)*a**(1/3)*x*(1/b)**(1/3) + 4*x**2)/(3*a*b**3 + 3*b**4
*x**3) + 4*a*b*x**3*log(2)/(3*a*b**3 + 3*b**4*x**3) + b**2*x**6/(3*a*b**3 +
 3*b**4*x**3), Eq(p, -2)), (a**2*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x
)/(3*b**3) + a**2*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a
**(1/3)*x*(1/b)**(1/3) + 4*x**2)/(3*b**3) - a*x**3/(3*b**2) + x**6/(6*b), E
q(p, -1)), (2*a**3*(a + b*x**3)**p/(3*b**3*p**3 + 18*b**3*p**2 + 33*b**3*p
+ 18*b**3) - 2*a**2*b*p*x**3*(a + b*x**3)**p/(3*b**3*p**3 + 18*b**3*p**2 +
33*b**3*p + 18*b**3) + a*b**2*p**2*x**6*(a + b*x**3)**p/(3*b**3*p**3 + 18*b
**3*p**2 + 33*b**3*p + 18*b**3) + a*b**2*p*x**6*(a + b*x**3)**p/(3*b**3*p**
3 + 18*b**3*p**2 + 33*b**3*p + 18*b**3) + b**3*p**2*x**9*(a + b*x**3)**p/(3
*b**3*p**3 + 18*b**3*p**2 + 33*b**3*p + 18*b**3) + 3*b**3*p*x**9*(a + b*x**
3)**p/(3*b**3*p**3 + 18*b**3*p**2 + 33*b**3*p + 18*b**3) + 2*b**3*x**9*(a +
 b*x**3)**p/(3*b**3*p**3 + 18*b**3*p**2 + 33*b**3*p + 18*b**3), True))
```

3.602 $\int x^{11}\left(a+bx^3\right)^p\,dx$

Optimal. Leaf size=95

$$-\frac{a^3\left(a+bx^3\right)^{p+1}}{3b^4(p+1)}+\frac{a^2\left(a+bx^3\right)^{p+2}}{b^4(p+2)}-\frac{a\left(a+bx^3\right)^{p+3}}{b^4(p+3)}+\frac{\left(a+bx^3\right)^{p+4}}{3b^4(p+4)}$$

```
[Out] -1/3*a^3*(b*x^3+a)^(1+p)/b^4/(1+p)+a^2*(b*x^3+a)^(2+p)/b^4/(2+p)-a*(b*x^3+a
)^(3+p)/b^4/(3+p)+1/3*(b*x^3+a)^(4+p)/b^4/(4+p)
```

Rubi [A] time = 0.06, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$-\frac{a^3\left(a+bx^3\right)^{p+1}}{3b^4(p+1)}+\frac{a^2\left(a+bx^3\right)^{p+2}}{b^4(p+2)}-\frac{a\left(a+bx^3\right)^{p+3}}{b^4(p+3)}+\frac{\left(a+bx^3\right)^{p+4}}{3b^4(p+4)}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a + b*x^3)^p,x]
```

```
[Out] -(a^3*(a + b*x^3)^(1 + p))/(3*b^4*(1 + p)) + (a^2*(a + b*x^3)^(2 + p))/(b^4
*(2 + p)) - (a*(a + b*x^3)^(3 + p))/(b^4*(3 + p)) + (a + b*x^3)^(4 + p)/(3*
b^4*(4 + p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{11}\left(a+bx^3\right)^p\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int x^3(a+bx)^p\,dx,x,x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int\left(-\frac{a^3(a+bx)^p}{b^3}+\frac{3a^2(a+bx)^{1+p}}{b^3}-\frac{3a(a+bx)^{2+p}}{b^3}+\frac{(a+bx)^{3+p}}{b^3}\right)dx,x,x^3\right)\\ &= -\frac{a^3\left(a+bx^3\right)^{1+p}}{3b^4(1+p)}+\frac{a^2\left(a+bx^3\right)^{2+p}}{b^4(2+p)}-\frac{a\left(a+bx^3\right)^{3+p}}{b^4(3+p)}+\frac{\left(a+bx^3\right)^{4+p}}{3b^4(4+p)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 95, normalized size = 1.00

$$-\frac{a^3\left(a+bx^3\right)^{p+1}}{3b^4(p+1)}+\frac{a^2\left(a+bx^3\right)^{p+2}}{b^4(p+2)}-\frac{a\left(a+bx^3\right)^{p+3}}{b^4(p+3)}+\frac{\left(a+bx^3\right)^{p+4}}{3b^4(p+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*x^3)^p,x]
```

```
[Out] -1/3*(a^3*(a + b*x^3)^(1 + p))/(b^4*(1 + p)) + (a^2*(a + b*x^3)^(2 + p))/(b
^4*(2 + p)) - (a*(a + b*x^3)^(3 + p))/(b^4*(3 + p)) + (a + b*x^3)^(4 + p)/(
3*b^4*(4 + p))
```

fricas [A] time = 0.71, size = 148, normalized size = 1.56

$$\frac{\left(\left(b^4p^3+6\,b^4p^2+11\,b^4p+6\,b^4\right)x^{12}+\left(ab^3p^3+3\,ab^3p^2+2\,ab^3p\right)x^9+6\,a^3bpx^3-3\left(a^2b^2p^2+a^2b^2p\right)x^6-6\,a^4\right)\left(bx^3\right.}{3\left(b^4p^4+10\,b^4p^3+35\,b^4p^2+50\,b^4p+24\,b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^p,x, algorithm="fricas")
```

```
[Out] 1/3*((b^4*p^3 + 6*b^4*p^2 + 11*b^4*p + 6*b^4)*x^12 + (a*b^3*p^3 + 3*a*b^3*p
^2 + 2*a*b^3*p)*x^9 + 6*a^3*b*p*x^3 - 3*(a^2*b^2*p^2 + a^2*b^2*p)*x^6 - 6*a
^4)*(b*x^3 + a)^p/(b^4*p^4 + 10*b^4*p^3 + 35*b^4*p^2 + 50*b^4*p + 24*b^4)
```

giac [B] time = 0.19, size = 410, normalized size = 4.32

$$\frac{\left(bx^3+a\right)^4\left(bx^3+a\right)^p p^3-3\left(bx^3+a\right)^3\left(bx^3+a\right)^p ap^3+3\left(bx^3+a\right)^2\left(bx^3+a\right)^p a^2p^3-\left(bx^3+a\right)\left(bx^3+a\right)^p a^3p^3+6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^p,x, algorithm="giac")
```

```
[Out] 1/3*((b*x^3 + a)^4*(b*x^3 + a)^p*p^3 - 3*(b*x^3 + a)^3*(b*x^3 + a)^p*a*p^3
+ 3*(b*x^3 + a)^2*(b*x^3 + a)^p*a^2*p^3 - (b*x^3 + a)*(b*x^3 + a)^p*a^3*p^3
 + 6*(b*x^3 + a)^4*(b*x^3 + a)^p*p^2 - 21*(b*x^3 + a)^3*(b*x^3 + a)^p*a*p^2
 + 24*(b*x^3 + a)^2*(b*x^3 + a)^p*a^2*p^2 - 9*(b*x^3 + a)*(b*x^3 + a)^p*a^3
*p^2 + 11*(b*x^3 + a)^4*(b*x^3 + a)^p*p - 42*(b*x^3 + a)^3*(b*x^3 + a)^p*a*
p + 57*(b*x^3 + a)^2*(b*x^3 + a)^p*a^2*p - 26*(b*x^3 + a)*(b*x^3 + a)^p*a^3
*p + 6*(b*x^3 + a)^4*(b*x^3 + a)^p - 24*(b*x^3 + a)^3*(b*x^3 + a)^p*a + 36*
(b*x^3 + a)^2*(b*x^3 + a)^p*a^2 - 24*(b*x^3 + a)*(b*x^3 + a)^p*a^3)/((b^3*p
^4 + 10*b^3*p^3 + 35*b^3*p^2 + 50*b^3*p + 24*b^3)*b)
```

maple [A] time = 0.01, size = 132, normalized size = 1.39

$$-\frac{\left(-b^3p^3x^9-6b^3p^2x^9-11b^3p\,x^9-6b^3x^9+3a\,b^2p^2x^6+9a\,b^2p\,x^6+6a\,b^2x^6-6a^2bp\,x^3-6a^2b\,x^3+6a^3\right)\left(b\,x^3+a\right.}{3\left(p^4+10p^3+35p^2+50p+24\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x^3+a)^p,x)
```

```
[Out] -1/3*(b*x^3+a)^(p+1)*(-b^3*p^3*x^9-6*b^3*p^2*x^9-11*b^3*p*x^9-6*b^3*x^9+3*a
*b^2*p^2*x^6+9*a*b^2*p*x^6+6*a*b^2*x^6-6*a^2*b*p*x^3-6*a^2*b*x^3+6*a^3)/b^4
/(p^4+10*p^3+35*p^2+50*p+24)
```

maxima [A] time = 1.43, size = 106, normalized size = 1.12

$$\frac{\left(\left(p^3+6\,p^2+11\,p+6\right)b^4x^{12}+\left(p^3+3\,p^2+2\,p\right)ab^3x^9-3\left(p^2+p\right)a^2b^2x^6+6\,a^3bpx^3-6\,a^4\right)\left(bx^3+a\right)^p}{3\left(p^4+10\,p^3+35\,p^2+50\,p+24\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^p,x, algorithm="maxima")
```

```
[Out] 1/3*((p^3 + 6*p^2 + 11*p + 6)*b^4*x^12 + (p^3 + 3*p^2 + 2*p)*a*b^3*x^9 - 3*
(p^2 + p)*a^2*b^2*x^6 + 6*a^3*b*p*x^3 - 6*a^4)*(b*x^3 + a)^p/((p^4 + 10*p^3
 + 35*p^2 + 50*p + 24)*b^4)
```

mupad [B] time = 1.12, size = 183, normalized size = 1.93

$$\left(b\,x^3+a\right)^p\left(\frac{x^{12}\left(p^3+6\,p^2+11\,p+6\right)}{3\left(p^4+10\,p^3+35\,p^2+50\,p+24\right)}-\frac{2\,a^4}{b^4\left(p^4+10\,p^3+35\,p^2+50\,p+24\right)}+\frac{2\,a^3\,p\,x^3}{b^3\left(p^4+10\,p^3+35\,p\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a + b*x^3)^p,x)
```

```
[Out] (a + b*x^3)^p*((x^12*(11*p + 6*p^2 + p^3 + 6))/(3*(50*p + 35*p^2 + 10*p^3 +
 p^4 + 24)) - (2*a^4)/(b^4*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) + (2*a^3*p*
x^3)/(b^3*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) + (a*p*x^9*(3*p + p^2 + 2))/
(3*b*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) - (a^2*p*x^6*(p + 1))/(b^2*(50*p
+ 35*p^2 + 10*p^3 + p^4 + 24)))
```

sympy [A] time = 80.69, size = 2773, normalized size = 29.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x**3+a)**p,x)
```

```
[Out] Piecewise((a**p*x**12/12, Eq(b, 0)), (6*a**3*log(-(-1)**(1/3)*a**(1/3)*(1/b
)**(1/3) + x)/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*
x**9) + 6*a**3*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(
1/3)*x*(1/b)**(1/3) + 4*x**2)/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6
*x**6 + 18*b**7*x**9) - 12*a**3*log(2)/(18*a**3*b**4 + 54*a**2*b**5*x**3 +
54*a*b**6*x**6 + 18*b**7*x**9) + 11*a**3/(18*a**3*b**4 + 54*a**2*b**5*x**3
+ 54*a*b**6*x**6 + 18*b**7*x**9) + 18*a**2*b*x**3*log(-(-1)**(1/3)*a**(1/3)
*(1/b)**(1/3) + x)/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*
b**7*x**9) + 18*a**2*b*x**3*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1
)**(1/3)*a**(1/3)*x*(1/b)**(1/3) + 4*x**2)/(18*a**3*b**4 + 54*a**2*b**5*x**
3 + 54*a*b**6*x**6 + 18*b**7*x**9) - 36*a**2*b*x**3*log(2)/(18*a**3*b**4 +
54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 27*a**2*b*x**3/(18*a**
3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 18*a*b**2*x**
6*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x)/(18*a**3*b**4 + 54*a**2*b**5*
x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 18*a*b**2*x**6*log(4*(-1)**(2/3)*a*
*(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x*(1/b)**(1/3) + 4*x**2)/(18*a
**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) - 36*a*b**2*x
**6*log(2)/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**
9) + 18*a*b**2*x**6/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18
*b**7*x**9) + 6*b**3*x**9*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x)/(18*a
**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 6*b**3*x**9
*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x*(1/b)**
(1/3) + 4*x**2)/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**
7*x**9) - 12*b**3*x**9*log(2)/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6
*x**6 + 18*b**7*x**9), Eq(p, -4)), (-6*a**3*log(-(-1)**(1/3)*a**(1/3)*(1/b)
**(1/3) + x)/(6*a**2*b**4 + 12*a*b**5*x**3 + 6*b**6*x**6) - 6*a**3*log(4*(-
1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x*(1/b)**(1/3) + 4
*x**2)/(6*a**2*b**4 + 12*a*b**5*x**3 + 6*b**6*x**6) - 9*a**3/(6*a**2*b**4 +
 12*a*b**5*x**3 + 6*b**6*x**6) + 12*a**3*log(2)/(6*a**2*b**4 + 12*a*b**5*x*
*3 + 6*b**6*x**6) - 12*a**2*b*x**3*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) +
 x)/(6*a**2*b**4 + 12*a*b**5*x**3 + 6*b**6*x**6) - 12*a**2*b*x**3*log(4*(-1
)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x*(1/b)**(1/3) + 4*
x**2)/(6*a**2*b**4 + 12*a*b**5*x**3 + 6*b**6*x**6) - 12*a**2*b*x**3/(6*a**2
*b**4 + 12*a*b**5*x**3 + 6*b**6*x**6) + 24*a**2*b*x**3*log(2)/(6*a**2*b**4
+ 12*a*b**5*x**3 + 6*b**6*x**6) - 6*a*b**2*x**6*log(-(-1)**(1/3)*a**(1/3)*(
1/b)**(1/3) + x)/(6*a**2*b**4 + 12*a*b**5*x**3 + 6*b**6*x**6) - 6*a*b**2*x*
*6*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x*(1/b)
```

```
**(1/3) + 4*x**2)/(6*a**2*b**4 + 12*a*b**5*x**3 + 6*b**6*x**6) + 12*a*b**2*
x**6*log(2)/(6*a**2*b**4 + 12*a*b**5*x**3 + 6*b**6*x**6) + 2*b**3*x**9/(6*a
**2*b**4 + 12*a*b**5*x**3 + 6*b**6*x**6), Eq(p, -3)), (6*a**3*log(-(-1)**(1
/3)*a**(1/3)*(1/b)**(1/3) + x)/(6*a*b**4 + 6*b**5*x**3) + 6*a**3*log(4*(-1)
**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x*(1/b)**(1/3) + 4*x
**2)/(6*a*b**4 + 6*b**5*x**3) - 12*a**3*log(2)/(6*a*b**4 + 6*b**5*x**3) + 6
*a**3/(6*a*b**4 + 6*b**5*x**3) + 6*a**2*b*x**3*log(-(-1)**(1/3)*a**(1/3)*(1
/b)**(1/3) + x)/(6*a*b**4 + 6*b**5*x**3) + 6*a**2*b*x**3*log(4*(-1)**(2/3)*
a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x*(1/b)**(1/3) + 4*x**2)/(6*
a*b**4 + 6*b**5*x**3) - 12*a**2*b*x**3*log(2)/(6*a*b**4 + 6*b**5*x**3) - 3*
a*b**2*x**6/(6*a*b**4 + 6*b**5*x**3) + b**3*x**9/(6*a*b**4 + 6*b**5*x**3),
Eq(p, -2)), (-a**3*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x)/(3*b**4) - a
**3*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x*(1/b
)**(1/3) + 4*x**2)/(3*b**4) + a**2*x**3/(3*b**3) - a*x**6/(6*b**2) + x**9/(
9*b), Eq(p, -1)), (-6*a**4*(a + b*x**3)**p/(3*b**4*p**4 + 30*b**4*p**3 + 10
5*b**4*p**2 + 150*b**4*p + 72*b**4) + 6*a**3*b*p*x**3*(a + b*x**3)**p/(3*b*
*4*p**4 + 30*b**4*p**3 + 105*b**4*p**2 + 150*b**4*p + 72*b**4) - 3*a**2*b**
2*p**2*x**6*(a + b*x**3)**p/(3*b**4*p**4 + 30*b**4*p**3 + 105*b**4*p**2 + 1
50*b**4*p + 72*b**4) - 3*a**2*b**2*p*x**6*(a + b*x**3)**p/(3*b**4*p**4 + 30
*b**4*p**3 + 105*b**4*p**2 + 150*b**4*p + 72*b**4) + a*b**3*p**3*x**9*(a +
b*x**3)**p/(3*b**4*p**4 + 30*b**4*p**3 + 105*b**4*p**2 + 150*b**4*p + 72*b*
*4) + 3*a*b**3*p**2*x**9*(a + b*x**3)**p/(3*b**4*p**4 + 30*b**4*p**3 + 105*
b**4*p**2 + 150*b**4*p + 72*b**4) + 2*a*b**3*p*x**9*(a + b*x**3)**p/(3*b**4
*p**4 + 30*b**4*p**3 + 105*b**4*p**2 + 150*b**4*p + 72*b**4) + b**4*p**3*x*
*12*(a + b*x**3)**p/(3*b**4*p**4 + 30*b**4*p**3 + 105*b**4*p**2 + 150*b**4*
p + 72*b**4) + 6*b**4*p**2*x**12*(a + b*x**3)**p/(3*b**4*p**4 + 30*b**4*p**
3 + 105*b**4*p**2 + 150*b**4*p + 72*b**4) + 11*b**4*p*x**12*(a + b*x**3)**p
/(3*b**4*p**4 + 30*b**4*p**3 + 105*b**4*p**2 + 150*b**4*p + 72*b**4) + 6*b*
*4*x**12*(a + b*x**3)**p/(3*b**4*p**4 + 30*b**4*p**3 + 105*b**4*p**2 + 150*
b**4*p + 72*b**4), True))
```

3.603 $\int x^m \left(a + bx^4\right) dx$

Optimal. Leaf size=25

$$\frac{ax^{m+1}}{m+1} + \frac{bx^{m+5}}{m+5}$$

```
[Out] a*x^(1+m)/(1+m)+b*x^(5+m)/(5+m)
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$\frac{ax^{m+1}}{m+1} + \frac{bx^{m+5}}{m+5}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^4),x]
```

```
[Out] (a*x^(1 + m))/(1 + m) + (b*x^(5 + m))/(5 + m)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^m \left(a + bx^4\right) dx &= \int \left(ax^m + bx^{4+m}\right) dx \\ &= \frac{ax^{1+m}}{1+m} + \frac{bx^{5+m}}{5+m}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{ax^{m+1}}{m+1} + \frac{bx^{m+5}}{m+5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^4),x]
```

```
[Out] (a*x^(1 + m))/(1 + m) + (b*x^(5 + m))/(5 + m)
```

fricas [A] time = 0.86, size = 33, normalized size = 1.32

$$\frac{\left((bm+b)x^5 + (am+5\,a)x\right)x^m}{m^2+6\,m+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^4+a),x, algorithm="fricas")
```

```
[Out] ((b*m + b)*x^5 + (a*m + 5*a)*x)*x^m/(m^2 + 6*m + 5)
```

giac [A] time = 0.17, size = 43, normalized size = 1.72

$$\frac{bmx^5x^m + bx^5x^m + amxx^m + 5\,axx^m}{m^2+6\,m+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^4+a),x, algorithm="giac")
```

```
[Out] (b*m*x^5*x^m + b*x^5*x^m + a*m*x*x^m + 5*a*x*x^m)/(m^2 + 6*m + 5)
```

maple [A] time = 0.00, size = 35, normalized size = 1.40

$$\frac{\left(bm\,x^4 + b\,x^4 + am + 5a\right)x^{m+1}}{(m+5)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^4+a),x)
```

```
[Out] x^(m+1)*(b*m*x^4+b*x^4+a*m+5*a)/(m+5)/(m+1)
```

maxima [A] time = 1.29, size = 25, normalized size = 1.00

$$\frac{bx^{m+5}}{m+5} + \frac{ax^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^4+a),x, algorithm="maxima")
```

```
[Out] b*x^(m + 5)/(m + 5) + a*x^(m + 1)/(m + 1)
```

mupad [B] time = 1.24, size = 34, normalized size = 1.36

$$\frac{x^{m+1}\left(5\,a + a\,m + b\,x^4 + b\,m\,x^4\right)}{m^2 + 6\,m + 5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^4),x)
```

```
[Out] (x^(m + 1)*(5*a + a*m + b*x^4 + b*m*x^4))/(6*m + m^2 + 5)
```

sympy [A] time = 1.30, size = 94, normalized size = 3.76

$$\begin{cases} -\frac{a}{4x^4} + b\log(x) & \text{for } m = -5 \\ a\log(x) + \frac{bx^4}{4} & \text{for } m = -1 \\ \frac{amxx^m}{m^2+6m+5} + \frac{5axx^m}{m^2+6m+5} + \frac{bmx^5x^m}{m^2+6m+5} + \frac{bx^5x^m}{m^2+6m+5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**4+a),x)
```

```
[Out] Piecewise((-a/(4*x**4) + b*log(x), Eq(m, -5)), (a*log(x) + b*x**4/4, Eq(m,
-1)), (a*m*x*x**m/(m**2 + 6*m + 5) + 5*a*x*x**m/(m**2 + 6*m + 5) + b*m*x**5
*x**m/(m**2 + 6*m + 5) + b*x**5*x**m/(m**2 + 6*m + 5), True))
```

3.604 $\int x^5 \left(a + bx^4\right) dx$

Optimal. Leaf size=17

$$\frac{ax^6}{6} + \frac{bx^{10}}{10}$$

```
[Out] 1/6*a*x^6+1/10*b*x^10
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$\frac{ax^6}{6} + \frac{bx^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^4),x]
```

```
[Out] (a*x^6)/6 + (b*x^10)/10
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^5 \left(a + bx^4\right) dx &= \int \left(ax^5 + bx^9\right) dx \\ &= \frac{ax^6}{6} + \frac{bx^{10}}{10}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^6}{6} + \frac{bx^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^4),x]
```

```
[Out] (a*x^6)/6 + (b*x^10)/10
```

fricas [A] time = 0.46, size = 13, normalized size = 0.76

$$\frac{1}{10}x^{10}b + \frac{1}{6}x^6 a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/10*x^10*b + 1/6*x^6*a
```

giac [A] time = 0.15, size = 13, normalized size = 0.76

$$\frac{1}{10}\, bx^{10} + \frac{1}{6}\, ax^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^4+a),x, algorithm="giac")
[Out] 1/10*b*x^10 + 1/6*a*x^6
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{10} b\, x^{10} + \frac{1}{6} a\, x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^4+a),x)
[Out] 1/6*x^6*a+1/10*b*x^10
```

maxima [A] time = 1.29, size = 13, normalized size = 0.76

$$\frac{1}{10}\, bx^{10} + \frac{1}{6}\, ax^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^4+a),x, algorithm="maxima")
[Out] 1/10*b*x^10 + 1/6*a*x^6
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{b\, x^{10}}{10} + \frac{a\, x^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^4),x)
[Out] (a*x^6)/6 + (b*x^10)/10
```

sympy [A] time = 0.08, size = 12, normalized size = 0.71

$$\frac{ax^{6}}{6} + \frac{bx^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**4+a),x)
[Out] a*x**6/6 + b*x**10/10
```

3.605 $\int x^4 \left(a + bx^4\right) dx$

Optimal. Leaf size=17

$$\frac{ax^5}{5} + \frac{bx^9}{9}$$

```
[Out] 1/5*a*x^5+1/9*b*x^9
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$\frac{ax^5}{5} + \frac{bx^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^4),x]
```

```
[Out] (a*x^5)/5 + (b*x^9)/9
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^4 \left(a + bx^4\right) dx &= \int \left(ax^4 + bx^8\right) dx \\ &= \frac{ax^5}{5} + \frac{bx^9}{9}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^5}{5} + \frac{bx^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^4),x]
```

```
[Out] (a*x^5)/5 + (b*x^9)/9
```

fricas [A] time = 0.47, size = 13, normalized size = 0.76

$$\frac{1}{9}x^9 b + \frac{1}{5}x^5 a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/9*x^9*b + 1/5*x^5*a
```

giac [A] time = 0.15, size = 13, normalized size = 0.76

$$\frac{1}{9}\, bx^9 + \frac{1}{5}\, ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^4+a),x, algorithm="giac")
[Out] 1/9*b*x^9 + 1/5*a*x^5
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{9}b\,x^9+\frac{1}{5}a\,x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^4+a),x)
[Out] 1/5*a*x^5+1/9*b*x^9
```

maxima [A] time = 1.32, size = 13, normalized size = 0.76

$$\frac{1}{9}\,bx^9+\frac{1}{5}\,ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^4+a),x, algorithm="maxima")
[Out] 1/9*b*x^9 + 1/5*a*x^5
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{b\,x^9}{9}+\frac{a\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^4),x)
[Out] (a*x^5)/5 + (b*x^9)/9
```

sympy [A] time = 0.08, size = 12, normalized size = 0.71

$$\frac{ax^5}{5}+\frac{bx^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**4+a),x)
[Out] a*x**5/5 + b*x**9/9
```

3.606 $\int x^3 \left(a + bx^4\right) dx$

Optimal. Leaf size=17

$$\frac{ax^4}{4} + \frac{bx^8}{8}$$

```
[Out] 1/4*a*x^4+1/8*b*x^8
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$\frac{ax^4}{4} + \frac{bx^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^4),x]
```

```
[Out] (a*x^4)/4 + (b*x^8)/8
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^3 \left(a + bx^4\right) dx &= \int \left(ax^3 + bx^7\right) dx \\ &= \frac{ax^4}{4} + \frac{bx^8}{8}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^4}{4} + \frac{bx^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^4),x]
```

```
[Out] (a*x^4)/4 + (b*x^8)/8
```

fricas [A] time = 0.63, size = 13, normalized size = 0.76

$$\frac{1}{8}x^8 b + \frac{1}{4}x^4 a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/8*x^8*b + 1/4*x^4*a
```

giac [A] time = 0.15, size = 13, normalized size = 0.76

$$\frac{1}{8}\, bx^8 + \frac{1}{4}\, ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^4+a),x, algorithm="giac")
[Out] 1/8*b*x^8 + 1/4*a*x^4
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{8}b\,x^8+\frac{1}{4}a\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^4+a),x)
[Out] 1/4*a*x^4+1/8*b*x^8
```

maxima [A] time = 1.29, size = 14, normalized size = 0.82

$$\frac{\left(bx^4+a\right)^2}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^4+a),x, algorithm="maxima")
[Out] 1/8*(b*x^4 + a)^2/b
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{b\,x^8}{8}+\frac{a\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^4),x)
[Out] (a*x^4)/4 + (b*x^8)/8
```

sympy [A] time = 0.18, size = 12, normalized size = 0.71

$$\frac{ax^4}{4}+\frac{bx^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**4+a),x)
[Out] a*x**4/4 + b*x**8/8
```

3.607 $\int x^2 \left(a + bx^4\right) dx$

Optimal. Leaf size=17

$$\frac{ax^3}{3} + \frac{bx^7}{7}$$

```
[Out] 1/3*a*x^3+1/7*b*x^7
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$\frac{ax^3}{3} + \frac{bx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^4),x]
```

```
[Out] (a*x^3)/3 + (b*x^7)/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(a + bx^4\right) dx &= \int \left(ax^2 + bx^6\right) dx \\ &= \frac{ax^3}{3} + \frac{bx^7}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^3}{3} + \frac{bx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^4),x]
```

```
[Out] (a*x^3)/3 + (b*x^7)/7
```

fricas [A] time = 0.86, size = 13, normalized size = 0.76

$$\frac{1}{7}x^7 b + \frac{1}{3}x^3 a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*b + 1/3*x^3*a
```

giac [A] time = 0.15, size = 13, normalized size = 0.76

$$\frac{1}{7}\, bx^7 + \frac{1}{3}\, ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^4+a),x, algorithm="giac")
[Out] 1/7*b*x^7 + 1/3*a*x^3
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{7}b\,x^7 + \frac{1}{3}a\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^4+a),x)
[Out] 1/3*a*x^3+1/7*b*x^7
```

maxima [A] time = 1.32, size = 13, normalized size = 0.76

$$\frac{1}{7}\,bx^7 + \frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^4+a),x, algorithm="maxima")
[Out] 1/7*b*x^7 + 1/3*a*x^3
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{b\,x^7}{7} + \frac{a\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^4),x)
[Out] (a*x^3)/3 + (b*x^7)/7
```

sympy [A] time = 0.07, size = 12, normalized size = 0.71

$$\frac{ax^3}{3} + \frac{bx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**4+a),x)
[Out] a*x**3/3 + b*x**7/7
```

3.608 $\int x\left(a+bx^4\right)\,dx$

Optimal. Leaf size=17

$$\frac{ax^2}{2}+\frac{bx^6}{6}$$

```
[Out] 1/2*a*x^2+1/6*b*x^6
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {14}

$$\frac{ax^2}{2}+\frac{bx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^4),x]
```

```
[Out] (a*x^2)/2 + (b*x^6)/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^4\right)\,dx &= \int \left(ax+bx^5\right)\,dx\\ &= \frac{ax^2}{2}+\frac{bx^6}{6}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^2}{2}+\frac{bx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^4),x]
```

```
[Out] (a*x^2)/2 + (b*x^6)/6
```

fricas [A] time = 0.48, size = 13, normalized size = 0.76

$$\frac{1}{6}x^6b+\frac{1}{2}x^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/6*x^6*b + 1/2*x^2*a
```

giac [A] time = 0.15, size = 13, normalized size = 0.76

$$\frac{1}{6}\,bx^6+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^4+a),x, algorithm="giac")
[Out] 1/6*b*x^6 + 1/2*a*x^2
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{6}b\,x^6+\frac{1}{2}a\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^4+a),x)
[Out] 1/2*a*x^2+1/6*b*x^6
```

maxima [A] time = 1.33, size = 13, normalized size = 0.76

$$\frac{1}{6}\,bx^6+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^4+a),x, algorithm="maxima")
[Out] 1/6*b*x^6 + 1/2*a*x^2
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{b\,x^6}{6}+\frac{a\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^4),x)
[Out] (a*x^2)/2 + (b*x^6)/6
```

sympy [A] time = 0.08, size = 12, normalized size = 0.71

$$\frac{ax^2}{2}+\frac{bx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**4+a),x)
[Out] a*x**2/2 + b*x**6/6
```

3.609 $\int \left(a + bx^4\right)\, dx$

Optimal. Leaf size=12

$$ax + \frac{bx^5}{5}$$

```
[Out] a*x+1/5*b*x^5
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$ax + \frac{bx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*x^4,x]
```

```
[Out] a*x + (b*x^5)/5
```

Rubi steps

$$\int \left(a + bx^4\right)\, dx = ax + \frac{bx^5}{5}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$ax + \frac{bx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*x^4,x]
```

```
[Out] a*x + (b*x^5)/5
```

fricas [A] time = 0.53, size = 10, normalized size = 0.83

$$\frac{1}{5}x^5 b + xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^4+a,x, algorithm="fricas")
```

```
[Out] 1/5*x^5*b + x*a
```

giac [A] time = 0.15, size = 10, normalized size = 0.83

$$\frac{1}{5}\, bx^5 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^4+a,x, algorithm="giac")
```

```
[Out] 1/5*b*x^5 + a*x
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$\frac{1}{5} b\, x^5 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x^4+a,x)
[Out] a*x+1/5*b*x^5
```

maxima [**A**] time = 1.28, size = 10, normalized size = 0.83

$$\frac{1}{5}\,bx^5 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^4+a,x, algorithm="maxima")
[Out] 1/5*b*x^5 + a*x
```

mupad [**B**] time = 0.02, size = 10, normalized size = 0.83

$$\frac{b\,x^5}{5} + a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*x^4,x)
[Out] a*x + (b*x^5)/5
```

sympy [**A**] time = 0.10, size = 8, normalized size = 0.67

$$ax + \frac{bx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x**4+a,x)
[Out] a*x + b*x**5/5
```

3.610 $\int \frac{a+bx^4}{x}\,dx$

Optimal. Leaf size=13

$$a\log(x)+\frac{bx^4}{4}$$

```
[Out] 1/4*b*x^4+a*ln(x)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$a\log(x)+\frac{bx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)/x,x]
```

```
[Out] (b*x^4)/4 + a*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^4}{x}\,dx &= \int \left(\frac{a}{x}+bx^3\right)\,dx\\ &= \frac{bx^4}{4}+a\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$a\log(x)+\frac{bx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)/x,x]
```

```
[Out] (b*x^4)/4 + a*Log[x]
```

fricas [A] time = 0.65, size = 11, normalized size = 0.85

$$\frac{1}{4}\,bx^4+a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/x,x, algorithm="fricas")
```

```
[Out] 1/4*b*x^4 + a*log(x)
```

giac [A] time = 0.19, size = 14, normalized size = 1.08

$$\frac{1}{4}\,bx^4+\frac{1}{4}\,a\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/x,x, algorithm="giac")
[Out] 1/4*b*x^4 + 1/4*a*log(x^4)
```

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$\frac{b\,x^4}{4} + a\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)/x,x)
[Out] 1/4*b*x^4+a*ln(x)
```

maxima [A] time = 1.32, size = 14, normalized size = 1.08

$$\frac{1}{4}\,bx^4 + \frac{1}{4}\,a\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/x,x, algorithm="maxima")
[Out] 1/4*b*x^4 + 1/4*a*log(x^4)
```

mupad [B] time = 0.02, size = 11, normalized size = 0.85

$$\frac{b\,x^4}{4} + a\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)/x,x)
[Out] (b*x^4)/4 + a*log(x)
```

sympy [A] time = 0.10, size = 10, normalized size = 0.77

$$a\log(x) + \frac{bx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)/x,x)
[Out] a*log(x) + b*x**4/4
```

3.611 $\int \frac{a+bx^4}{x^2}\,dx$

Optimal. Leaf size=15

$$\frac{bx^3}{3} - \frac{a}{x}$$

```
[Out] -a/x+1/3*b*x^3
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$\frac{bx^3}{3} - \frac{a}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)/x^2,x]
```

```
[Out] -(a/x) + (b*x^3)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^4}{x^2}\,dx &= \int \left(\frac{a}{x^2} + bx^2\right) dx \\ &= -\frac{a}{x} + \frac{bx^3}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{bx^3}{3} - \frac{a}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)/x^2,x]
```

```
[Out] -(a/x) + (b*x^3)/3
```

fricas [A] time = 0.65, size = 14, normalized size = 0.93

$$\frac{bx^4 - 3\,a}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/x^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b*x^4 - 3*a)/x
```

giac [A] time = 0.15, size = 13, normalized size = 0.87

$$\frac{1}{3}\,bx^3 - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/x^2,x, algorithm="giac")
[Out] 1/3*b*x^3 - a/x
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{b\,x^3}{3} - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)/x^2,x)
[Out] -a/x+1/3*b*x^3
```

maxima [A] time = 1.37, size = 13, normalized size = 0.87

$$\frac{1}{3}\,bx^3 - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/x^2,x, algorithm="maxima")
[Out] 1/3*b*x^3 - a/x
```

mupad [B] time = 0.02, size = 13, normalized size = 0.87

$$\frac{b\,x^3}{3} - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)/x^2,x)
[Out] (b*x^3)/3 - a/x
```

sympy [A] time = 0.14, size = 8, normalized size = 0.53

$$-\frac{a}{x} + \frac{bx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)/x**2,x)
[Out] -a/x + b*x**3/3
```

3.612 $\int \frac{a+bx^4}{x^3}\,dx$

Optimal. Leaf size=17

$$\frac{bx^2}{2}-\frac{a}{2x^2}$$

```
[Out] -1/2*a/x^2+1/2*b*x^2
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$\frac{bx^2}{2}-\frac{a}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)/x^3,x]
```

```
[Out] -a/(2*x^2) + (b*x^2)/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^4}{x^3}\,dx &= \int \left(\frac{a}{x^3}+bx\right)\,dx\\ &= -\frac{a}{2x^2}+\frac{bx^2}{2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{bx^2}{2}-\frac{a}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)/x^3,x]
```

```
[Out] -1/2*a/x^2 + (b*x^2)/2
```

fricas [A] time = 0.75, size = 14, normalized size = 0.82

$$\frac{bx^4-a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^4 - a)/x^2
```

giac [A] time = 0.17, size = 13, normalized size = 0.76

$$\frac{1}{2}\,bx^2-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/x^3,x, algorithm="giac")
[Out] 1/2*b*x^2 - 1/2*a/x^2
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{b\,x^2}{2}-\frac{a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)/x^3,x)
[Out] -1/2*a/x^2+1/2*b*x^2
```

maxima [A] time = 1.33, size = 13, normalized size = 0.76

$$\frac{1}{2}\,bx^2-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/x^3,x, algorithm="maxima")
[Out] 1/2*b*x^2 - 1/2*a/x^2
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$-\frac{a-b\,x^4}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)/x^3,x)
[Out] -(a - b*x^4)/(2*x^2)
```

sympy [A] time = 0.17, size = 12, normalized size = 0.71

$$-\frac{a}{2x^2}+\frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)/x**3,x)
[Out] -a/(2*x**2) + b*x**2/2
```

3.613 $\int \frac{a+bx^4}{x^4}\,dx$

Optimal. Leaf size=12

$$bx - \frac{a}{3x^3}$$

```
[Out] -1/3*a/x^3+b*x
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$bx - \frac{a}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)/x^4,x]
```

```
[Out] -a/(3*x^3) + b*x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^4}{x^4}\,dx &= \int \left(b + \frac{a}{x^4}\right) dx \\ &= -\frac{a}{3x^3} + bx\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$bx - \frac{a}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)/x^4,x]
```

```
[Out] -1/3*a/x^3 + b*x
```

fricas [A] time = 0.75, size = 15, normalized size = 1.25

$$\frac{3\,bx^4 - a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*b*x^4 - a)/x^3
```

giac [A] time = 0.15, size = 10, normalized size = 0.83

$$bx - \frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/x^4,x, algorithm="giac")
[Out] b*x - 1/3*a/x^3
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$bx - \frac{a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)/x^4,x)
[Out] -1/3*a/x^3+b*x
```

maxima [A] time = 1.34, size = 10, normalized size = 0.83

$$bx - \frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/x^4,x, algorithm="maxima")
[Out] b*x - 1/3*a/x^3
```

mupad [B] time = 0.02, size = 10, normalized size = 0.83

$$b\,x - \frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)/x^4,x)
[Out] b*x - a/(3*x^3)
```

sympy [A] time = 0.22, size = 8, normalized size = 0.67

$$-\frac{a}{3x^3} + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)/x**4,x)
[Out] -a/(3*x**3) + b*x
```

3.614 $\int \frac{a+bx^4}{x^5}\,dx$

Optimal. Leaf size=13

$$b\log(x)-\frac{a}{4x^4}$$

```
[Out] -1/4*a/x^4+b*ln(x)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$b\log(x)-\frac{a}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)/x^5,x]
```

```
[Out] -a/(4*x^4) + b*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^4}{x^5}\,dx &= \int \left(\frac{a}{x^5}+\frac{b}{x}\right)dx\\ &= -\frac{a}{4x^4}+b\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$b\log(x)-\frac{a}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)/x^5,x]
```

```
[Out] -1/4*a/x^4 + b*Log[x]
```

fricas [A] time = 0.59, size = 17, normalized size = 1.31

$$\frac{4\,bx^4\log(x)-a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b*x^4*log(x) - a)/x^4
```

giac [A] time = 0.15, size = 20, normalized size = 1.54

$$\frac{1}{4}\,b\log\left(x^4\right)-\frac{bx^4+a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/x^5,x, algorithm="giac")
[Out] 1/4*b*log(x^4) - 1/4*(b*x^4 + a)/x^4
```

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$b\ln(x) - \frac{a}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)/x^5,x)
[Out] -1/4*a/x^4+b*ln(x)
```

maxima [A] time = 1.32, size = 14, normalized size = 1.08

$$\frac{1}{4}\,b\log\left(x^4\right) - \frac{a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/x^5,x, algorithm="maxima")
[Out] 1/4*b*log(x^4) - 1/4*a/x^4
```

mupad [B] time = 0.04, size = 11, normalized size = 0.85

$$b\,\ln(x) - \frac{a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)/x^5,x)
[Out] b*log(x) - a/(4*x^4)
```

sympy [A] time = 0.26, size = 10, normalized size = 0.77

$$-\frac{a}{4x^4} + b\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)/x**5,x)
[Out] -a/(4*x**4) + b*log(x)
```

3.615 $\int \frac{a+bx^4}{x^6}\,dx$

Optimal. Leaf size=15

$$-\frac{a}{5x^5}-\frac{b}{x}$$

```
[Out] -1/5*a/x^5-b/x
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$-\frac{a}{5x^5}-\frac{b}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)/x^6,x]
```

```
[Out] -a/(5*x^5) - b/x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^4}{x^6}\,dx &= \int \left(\frac{a}{x^6}+\frac{b}{x^2}\right)dx \\ &= -\frac{a}{5x^5}-\frac{b}{x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$-\frac{a}{5x^5}-\frac{b}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)/x^6,x]
```

```
[Out] -1/5*a/x^5 - b/x
```

fricas [A] time = 0.61, size = 13, normalized size = 0.87

$$-\frac{5\,bx^4+a}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/x^6,x, algorithm="fricas")
```

```
[Out] -1/5*(5*b*x^4 + a)/x^5
```

giac [A] time = 0.15, size = 13, normalized size = 0.87

$$-\frac{5\,bx^4+a}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/x^6,x, algorithm="giac")
[Out] -1/5*(5*b*x^4 + a)/x^5
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$-\frac{b}{x}-\frac{a}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)/x^6,x)
[Out] -1/5*a/x^5-b/x
```

maxima [A] time = 1.34, size = 13, normalized size = 0.87

$$-\frac{5\,bx^4+a}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/x^6,x, algorithm="maxima")
[Out] -1/5*(5*b*x^4 + a)/x^5
```

mupad [B] time = 0.02, size = 13, normalized size = 0.87

$$-\frac{5\,b\,x^4+a}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)/x^6,x)
[Out] -(a + 5*b*x^4)/(5*x^5)
```

sympy [A] time = 0.39, size = 14, normalized size = 0.93

$$\frac{-a-5bx^4}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)/x**6,x)
[Out] (-a - 5*b*x**4)/(5*x**5)
```

3.616 $\int \frac{a+bx^4}{x^7}\,dx$

Optimal. Leaf size=17

$$-\frac{a}{6x^6}-\frac{b}{2x^2}$$

```
[Out] -1/6*a/x^6-1/2*b/x^2
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$-\frac{a}{6x^6}-\frac{b}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)/x^7,x]
```

```
[Out] -a/(6*x^6) - b/(2*x^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^4}{x^7}\,dx &= \int \left(\frac{a}{x^7}+\frac{b}{x^3}\right)dx \\ &= -\frac{a}{6x^6}-\frac{b}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$-\frac{a}{6x^6}-\frac{b}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)/x^7,x]
```

```
[Out] -1/6*a/x^6 - b/(2*x^2)
```

fricas [A] time = 0.55, size = 13, normalized size = 0.76

$$-\frac{3\,bx^4+a}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/x^7,x, algorithm="fricas")
```

```
[Out] -1/6*(3*b*x^4 + a)/x^6
```

giac [A] time = 0.14, size = 13, normalized size = 0.76

$$-\frac{3\,bx^4+a}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/x^7,x, algorithm="giac")
[Out] -1/6*(3*b*x^4 + a)/x^6
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$-\frac{b}{2x^2}-\frac{a}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)/x^7,x)
[Out] -1/6*a/x^6-1/2*b/x^2
```

maxima [A] time = 1.26, size = 13, normalized size = 0.76

$$-\frac{3\,bx^4+a}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/x^7,x, algorithm="maxima")
[Out] -1/6*(3*b*x^4 + a)/x^6
```

mupad [B] time = 0.03, size = 13, normalized size = 0.76

$$-\frac{3\,b\,x^4+a}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)/x^7,x)
[Out] -(a + 3*b*x^4)/(6*x^6)
```

sympy [A] time = 0.37, size = 14, normalized size = 0.82

$$\frac{-a-3bx^4}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)/x**7,x)
[Out] (-a - 3*b*x**4)/(6*x**6)
```

3.617 $\int \frac{a+bx^4}{x^8}\,dx$

Optimal. Leaf size=17

$$-\frac{a}{7x^7}-\frac{b}{3x^3}$$

```
[Out] -1/7*a/x^7-1/3*b/x^3
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$-\frac{a}{7x^7}-\frac{b}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)/x^8,x]
```

```
[Out] -a/(7*x^7) - b/(3*x^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^4}{x^8}\,dx &= \int \left(\frac{a}{x^8}+\frac{b}{x^4}\right)dx \\ &= -\frac{a}{7x^7}-\frac{b}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$-\frac{a}{7x^7}-\frac{b}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)/x^8,x]
```

```
[Out] -1/7*a/x^7 - b/(3*x^3)
```

fricas [A] time = 0.65, size = 15, normalized size = 0.88

$$-\frac{7\,bx^4+3\,a}{21\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/x^8,x, algorithm="fricas")
```

```
[Out] -1/21*(7*b*x^4 + 3*a)/x^7
```

giac [A] time = 0.15, size = 15, normalized size = 0.88

$$-\frac{7\,bx^4+3\,a}{21\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/x^8,x, algorithm="giac")
[Out] -1/21*(7*b*x^4 + 3*a)/x^7
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$-\frac{b}{3x^3}-\frac{a}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)/x^8,x)
[Out] -1/7*a/x^7-1/3*b/x^3
```

maxima [A] time = 1.34, size = 15, normalized size = 0.88

$$-\frac{7\,bx^4+3\,a}{21\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/x^8,x, algorithm="maxima")
[Out] -1/21*(7*b*x^4 + 3*a)/x^7
```

mupad [B] time = 0.03, size = 15, normalized size = 0.88

$$-\frac{7\,b\,x^4+3\,a}{21\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)/x^8,x)
[Out] -(3*a + 7*b*x^4)/(21*x^7)
```

sympy [A] time = 0.32, size = 15, normalized size = 0.88

$$\frac{-3a-7bx^4}{21x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)/x**8,x)
[Out] (-3*a - 7*b*x**4)/(21*x**7)
```

3.618 $\int \frac{a+bx^4}{x^9}\,dx$

Optimal. Leaf size=17

$$-\frac{a}{8x^8}-\frac{b}{4x^4}$$

```
[Out] -1/8*a/x^8-1/4*b/x^4
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$-\frac{a}{8x^8}-\frac{b}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)/x^9,x]
```

```
[Out] -a/(8*x^8) - b/(4*x^4)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^4}{x^9}\,dx &= \int \left(\frac{a}{x^9}+\frac{b}{x^5}\right)dx\\ &= -\frac{a}{8x^8}-\frac{b}{4x^4}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$-\frac{a}{8x^8}-\frac{b}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)/x^9,x]
```

```
[Out] -1/8*a/x^8 - b/(4*x^4)
```

fricas [A] time = 0.53, size = 13, normalized size = 0.76

$$-\frac{2\,bx^4+a}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/x^9,x, algorithm="fricas")
```

```
[Out] -1/8*(2*b*x^4 + a)/x^8
```

giac [A] time = 0.18, size = 13, normalized size = 0.76

$$-\frac{2\,bx^4+a}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/x^9,x, algorithm="giac")
[Out] -1/8*(2*b*x^4 + a)/x^8
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$-\frac{b}{4x^4}-\frac{a}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)/x^9,x)
[Out] -1/8*a/x^8-1/4*b/x^4
```

maxima [A] time = 1.29, size = 13, normalized size = 0.76

$$-\frac{2\,bx^4+a}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/x^9,x, algorithm="maxima")
[Out] -1/8*(2*b*x^4 + a)/x^8
```

mupad [B] time = 0.03, size = 13, normalized size = 0.76

$$-\frac{2\,b\,x^4+a}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)/x^9,x)
[Out] -(a + 2*b*x^4)/(8*x^8)
```

sympy [A] time = 0.18, size = 14, normalized size = 0.82

$$\frac{-a-2bx^4}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)/x**9,x)
[Out] (-a - 2*b*x**4)/(8*x**8)
```

3.619 $\int \frac{a+bx^4}{x^{10}}\,dx$

Optimal. Leaf size=17

$$-\frac{a}{9x^9}-\frac{b}{5x^5}$$

```
[Out] -1/9*a/x^9-1/5*b/x^5
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$-\frac{a}{9x^9}-\frac{b}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)/x^10,x]
```

```
[Out] -a/(9*x^9) - b/(5*x^5)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^4}{x^{10}}\,dx &= \int \left(\frac{a}{x^{10}}+\frac{b}{x^6}\right)dx\\ &= -\frac{a}{9x^9}-\frac{b}{5x^5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$-\frac{a}{9x^9}-\frac{b}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)/x^10,x]
```

```
[Out] -1/9*a/x^9 - b/(5*x^5)
```

fricas [A] time = 0.79, size = 15, normalized size = 0.88

$$-\frac{9\,bx^4+5\,a}{45\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/x^10,x, algorithm="fricas")
```

```
[Out] -1/45*(9*b*x^4 + 5*a)/x^9
```

giac [A] time = 0.15, size = 15, normalized size = 0.88

$$-\frac{9\,bx^4+5\,a}{45\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/x^10,x, algorithm="giac")
[Out] -1/45*(9*b*x^4 + 5*a)/x^9
```

maple [A] time = 0.01, size = 14, normalized size = 0.82

$$-\frac{b}{5x^5}-\frac{a}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)/x^10,x)
[Out] -1/9*a/x^9-1/5*b/x^5
```

maxima [A] time = 1.28, size = 15, normalized size = 0.88

$$-\frac{9\,bx^4+5\,a}{45\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/x^10,x, algorithm="maxima")
[Out] -1/45*(9*b*x^4 + 5*a)/x^9
```

mupad [B] time = 0.03, size = 15, normalized size = 0.88

$$-\frac{9\,b\,x^4+5\,a}{45\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)/x^10,x)
[Out] -(5*a + 9*b*x^4)/(45*x^9)
```

sympy [A] time = 0.21, size = 15, normalized size = 0.88

$$\frac{-5a-9bx^4}{45x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)/x**10,x)
[Out] (-5*a - 9*b*x**4)/(45*x**9)
```

3.620 $\int x^m \left(a + bx^4\right)^2 dx$

Optimal. Leaf size=43

$$\frac{a^2 x^{m+1}}{m+1} + \frac{2abx^{m+5}}{m+5} + \frac{b^2 x^{m+9}}{m+9}$$

```
[Out] a^2*x^(1+m)/(1+m)+2*a*b*x^(5+m)/(5+m)+b^2*x^(9+m)/(9+m)
```

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{a^2 x^{m+1}}{m+1} + \frac{2abx^{m+5}}{m+5} + \frac{b^2 x^{m+9}}{m+9}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^4)^2,x]
```

```
[Out] (a^2*x^(1 + m))/(1 + m) + (2*a*b*x^(5 + m))/(5 + m) + (b^2*x^(9 + m))/(9 +
m)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^m \left(a + bx^4\right)^2 dx &= \int \left(a^2 x^m + 2abx^{4+m} + b^2 x^{8+m}\right) dx \\ &= \frac{a^2 x^{1+m}}{1+m} + \frac{2abx^{5+m}}{5+m} + \frac{b^2 x^{9+m}}{9+m}\end{aligned}$$

Mathematica [A] time = 0.03, size = 40, normalized size = 0.93

$$x^{m+1}\left(\frac{a^2}{m+1} + \frac{2abx^4}{m+5} + \frac{b^2 x^8}{m+9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^4)^2,x]
```

```
[Out] x^(1 + m)*(a^2/(1 + m) + (2*a*b*x^4)/(5 + m) + (b^2*x^8)/(9 + m))
```

fricas [A] time = 0.64, size = 85, normalized size = 1.98

$$\frac{\left(\left(b^2 m^2 + 6\,b^2 m + 5\,b^2\right)x^9 + 2\left(abm^2 + 10\,abm + 9\,ab\right)x^5 + \left(a^2 m^2 + 14\,a^2 m + 45\,a^2\right)x\right)x^m}{m^3 + 15\,m^2 + 59\,m + 45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*m^2 + 6*b^2*m + 5*b^2)*x^9 + 2*(a*b*m^2 + 10*a*b*m + 9*a*b)*x^5 + (a^
2*m^2 + 14*a^2*m + 45*a^2)*x)*x^m/(m^3 + 15*m^2 + 59*m + 45)
```

giac [B] time = 0.16, size = 117, normalized size = 2.72

$$\frac{b^2m^2x^9x^m + 6\,b^2mx^9x^m + 5\,b^2x^9x^m + 2\,abm^2x^5x^m + 20\,abmx^5x^m + 18\,abx^5x^m + a^2m^2xx^m + 14\,a^2mxx^m + 45\,a^2x}{m^3 + 15\,m^2 + 59\,m + 45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] (b^2*m^2*x^9*x^m + 6*b^2*m*x^9*x^m + 5*b^2*x^9*x^m + 2*a*b*m^2*x^5*x^m + 20
*a*b*m*x^5*x^m + 18*a*b*x^5*x^m + a^2*m^2*x*x^m + 14*a^2*m*x*x^m + 45*a^2*x
*x^m)/(m^3 + 15*m^2 + 59*m + 45)
```

maple [B] time = 0.01, size = 93, normalized size = 2.16

$$\frac{\left(b^2m^2x^8 + 6b^2m\,x^8 + 5b^2x^8 + 2ab\,m^2x^4 + 20abm\,x^4 + 18ab\,x^4 + a^2m^2 + 14a^2m + 45a^2\right)x^{m+1}}{(m+9)\,(m+5)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^4+a)^2,x)
```

```
[Out] x^(m+1)*(b^2*m^2*x^8+6*b^2*m*x^8+5*b^2*x^8+2*a*b*m^2*x^4+20*a*b*m*x^4+18*a*
b*x^4+a^2*m^2+14*a^2*m+45*a^2)/(m+9)/(m+5)/(m+1)
```

maxima [A] time = 1.32, size = 43, normalized size = 1.00

$$\frac{b^2x^{m+9}}{m+9} + \frac{2\,abx^{m+5}}{m+5} + \frac{a^2x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] b^2*x^(m + 9)/(m + 9) + 2*a*b*x^(m + 5)/(m + 5) + a^2*x^(m + 1)/(m + 1)
```

mupad [B] time = 1.32, size = 93, normalized size = 2.16

$$x^m\left(\frac{a^2\,x\,\left(m^2+14\,m+45\right)}{m^3+15\,m^2+59\,m+45} + \frac{b^2\,x^9\,\left(m^2+6\,m+5\right)}{m^3+15\,m^2+59\,m+45} + \frac{2\,a\,b\,x^5\,\left(m^2+10\,m+9\right)}{m^3+15\,m^2+59\,m+45}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^4)^2,x)
```

```
[Out] x^m*((a^2*x*(14*m + m^2 + 45))/(59*m + 15*m^2 + m^3 + 45) + (b^2*x^9*(6*m +
 m^2 + 5))/(59*m + 15*m^2 + m^3 + 45) + (2*a*b*x^5*(10*m + m^2 + 9))/(59*m
+ 15*m^2 + m^3 + 45))
```

sympy [A] time = 3.94, size = 309, normalized size = 7.19

$$\begin{cases} -\frac{a^2}{8x^8} - \frac{ab}{2x^4} + b^2\log(x) \\ -\frac{a^2}{4x^4} + 2ab\log(x) + \frac{b^2x^4}{4} \\ a^2\log(x) + \frac{abx^4}{2} + \frac{b^2x^8}{8} \\ \frac{a^2m^2xx^m}{m^3+15m^2+59m+45} + \frac{14a^2mxx^m}{m^3+15m^2+59m+45} + \frac{45a^2xx^m}{m^3+15m^2+59m+45} + \frac{2abm^2x^5x^m}{m^3+15m^2+59m+45} + \frac{20abmx^5x^m}{m^3+15m^2+59m+45} + \frac{18abx^5x^m}{m^3+15m^2+59m+45} + \frac{}{m} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**4+a)**2,x)
```

```
[Out] Piecewise((-a**2/(8*x**8) - a*b/(2*x**4) + b**2*log(x), Eq(m, -9)), (-a**2/
(4*x**4) + 2*a*b*log(x) + b**2*x**4/4, Eq(m, -5)), (a**2*log(x) + a*b*x**4/
2 + b**2*x**8/8, Eq(m, -1)), (a**2*m**2*x*x**m/(m**3 + 15*m**2 + 59*m + 45)
 + 14*a**2*m*x*x**m/(m**3 + 15*m**2 + 59*m + 45) + 45*a**2*x*x**m/(m**3 + 1
5*m**2 + 59*m + 45) + 2*a*b*m**2*x**5*x**m/(m**3 + 15*m**2 + 59*m + 45) + 2
0*a*b*m*x**5*x**m/(m**3 + 15*m**2 + 59*m + 45) + 18*a*b*x**5*x**m/(m**3 + 1
5*m**2 + 59*m + 45) + b**2*m**2*x**9*x**m/(m**3 + 15*m**2 + 59*m + 45) + 6*
b**2*m*x**9*x**m/(m**3 + 15*m**2 + 59*m + 45) + 5*b**2*x**9*x**m/(m**3 + 15
*m**2 + 59*m + 45), True))
```

3.621 $\int x^5 \left(a + bx^4\right)^2 \, dx$

Optimal. Leaf size=30

$$\frac{a^2x^6}{6} + \frac{1}{5}abx^{10} + \frac{b^2x^{14}}{14}$$

```
[Out] 1/6*a^2*x^6+1/5*a*b*x^10+1/14*b^2*x^14
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{a^2x^6}{6} + \frac{1}{5}abx^{10} + \frac{b^2x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^4)^2,x]
```

```
[Out] (a^2*x^6)/6 + (a*b*x^10)/5 + (b^2*x^14)/14
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^5 \left(a + bx^4\right)^2 \, dx &= \int \left(a^2x^5 + 2abx^9 + b^2x^{13}\right) \, dx \\ &= \frac{a^2x^6}{6} + \frac{1}{5}abx^{10} + \frac{b^2x^{14}}{14}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^6}{6} + \frac{1}{5}abx^{10} + \frac{b^2x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^4)^2,x]
```

```
[Out] (a^2*x^6)/6 + (a*b*x^10)/5 + (b^2*x^14)/14
```

fricas [A] time = 0.53, size = 24, normalized size = 0.80

$$\frac{1}{14}x^{14}b^2 + \frac{1}{5}x^{10}ba + \frac{1}{6}x^6a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*b^2 + 1/5*x^10*b*a + 1/6*x^6*a^2
```

giac [A] time = 0.15, size = 24, normalized size = 0.80

$$\frac{1}{14}\,b^2x^{14} + \frac{1}{5}\,abx^{10} + \frac{1}{6}\,a^2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^4+a)^2,x, algorithm="giac")
[Out] 1/14*b^2*x^14 + 1/5*a*b*x^10 + 1/6*a^2*x^6
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$\frac{1}{14}b^2x^{14} + \frac{1}{5}ab\,x^{10} + \frac{1}{6}a^2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^4+a)^2,x)
[Out] 1/6*a^2*x^6+1/5*a*b*x^10+1/14*b^2*x^14
```

maxima [A] time = 1.35, size = 24, normalized size = 0.80

$$\frac{1}{14}\,b^2x^{14} + \frac{1}{5}\,abx^{10} + \frac{1}{6}\,a^2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^4+a)^2,x, algorithm="maxima")
[Out] 1/14*b^2*x^14 + 1/5*a*b*x^10 + 1/6*a^2*x^6
```

mupad [B] time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^6}{6} + \frac{a\,b\,x^{10}}{5} + \frac{b^2\,x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^4)^2,x)
[Out] (a^2*x^6)/6 + (b^2*x^14)/14 + (a*b*x^10)/5
```

sympy [A] time = 0.10, size = 24, normalized size = 0.80

$$\frac{a^2x^6}{6} + \frac{abx^{10}}{5} + \frac{b^2x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**4+a)**2,x)
[Out] a**2*x**6/6 + a*b*x**10/5 + b**2*x**14/14
```

3.622 $\int x^4 \left(a+bx^4\right)^2 dx$

Optimal. Leaf size=30

$$\frac{a^2x^5}{5}+\frac{2}{9}abx^9+\frac{b^2x^{13}}{13}$$

```
[Out] 1/5*a^2*x^5+2/9*a*b*x^9+1/13*b^2*x^13
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{a^2x^5}{5}+\frac{2}{9}abx^9+\frac{b^2x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^4)^2,x]
```

```
[Out] (a^2*x^5)/5 + (2*a*b*x^9)/9 + (b^2*x^13)/13
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^4\left(a+bx^4\right)^2 dx &= \int \left(a^2x^4+2abx^8+b^2x^{12}\right) dx \\ &= \frac{a^2x^5}{5}+\frac{2}{9}abx^9+\frac{b^2x^{13}}{13}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^5}{5}+\frac{2}{9}abx^9+\frac{b^2x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^4)^2,x]
```

```
[Out] (a^2*x^5)/5 + (2*a*b*x^9)/9 + (b^2*x^13)/13
```

fricas [A] time = 0.62, size = 24, normalized size = 0.80

$$\frac{1}{13}x^{13}b^2+\frac{2}{9}x^9ba+\frac{1}{5}x^5a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*b^2 + 2/9*x^9*b*a + 1/5*x^5*a^2
```

giac [A] time = 0.15, size = 24, normalized size = 0.80

$$\frac{1}{13}\,b^2x^{13}+\frac{2}{9}\,abx^9+\frac{1}{5}\,a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^4+a)^2,x, algorithm="giac")
[Out] 1/13*b^2*x^13 + 2/9*a*b*x^9 + 1/5*a^2*x^5
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$\frac{1}{13}b^2x^{13} + \frac{2}{9}ab\,x^9 + \frac{1}{5}a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^4+a)^2,x)
[Out] 1/5*a^2*x^5+2/9*a*b*x^9+1/13*b^2*x^13
```

maxima [A] time = 1.33, size = 24, normalized size = 0.80

$$\frac{1}{13}\,b^2x^{13} + \frac{2}{9}\,abx^9 + \frac{1}{5}\,a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^4+a)^2,x, algorithm="maxima")
[Out] 1/13*b^2*x^13 + 2/9*a*b*x^9 + 1/5*a^2*x^5
```

mupad [B] time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^5}{5} + \frac{2\,a\,b\,x^9}{9} + \frac{b^2\,x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^4)^2,x)
[Out] (a^2*x^5)/5 + (b^2*x^13)/13 + (2*a*b*x^9)/9
```

sympy [A] time = 0.07, size = 26, normalized size = 0.87

$$\frac{a^2x^5}{5} + \frac{2abx^9}{9} + \frac{b^2x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**4+a)**2,x)
[Out] a**2*x**5/5 + 2*a*b*x**9/9 + b**2*x**13/13
```

3.623 $\int x^3 \left(a + bx^4\right)^2 \, dx$

Optimal. Leaf size=16

$$\frac{\left(a + bx^4\right)^3}{12b}$$

```
[Out] 1/12*(b*x^4+a)^3/b
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$\frac{\left(a + bx^4\right)^3}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^4)^2,x]
```

```
[Out] (a + b*x^4)^3/(12*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^3 \left(a + bx^4\right)^2 \, dx = \frac{\left(a + bx^4\right)^3}{12b}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.88

$$\frac{a^2 x^4}{4} + \frac{1}{4} abx^8 + \frac{b^2 x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^4)^2,x]
```

```
[Out] (a^2*x^4)/4 + (a*b*x^8)/4 + (b^2*x^12)/12
```

fricas [A] time = 0.64, size = 24, normalized size = 1.50

$$\frac{1}{12} x^{12} b^2 + \frac{1}{4} x^8 ba + \frac{1}{4} x^4 a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*b^2 + 1/4*x^8*b*a + 1/4*x^4*a^2
```

giac [A] time = 0.15, size = 14, normalized size = 0.88

$$\frac{\left(bx^4 + a\right)^3}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^4+a)^2,x, algorithm="giac")
[Out] 1/12*(b*x^4 + a)^3/b
```

maple [A] time = 0.00, size = 25, normalized size = 1.56

$$\frac{1}{12}b^2x^{12} + \frac{1}{4}ab\,x^8 + \frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^4+a)^2,x)
[Out] 1/12*b^2*x^12+1/4*a*b*x^8+1/4*a^2*x^4
```

maxima [A] time = 1.35, size = 14, normalized size = 0.88

$$\frac{\left(bx^4 + a\right)^3}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^4+a)^2,x, algorithm="maxima")
[Out] 1/12*(b*x^4 + a)^3/b
```

mupad [B] time = 0.03, size = 24, normalized size = 1.50

$$\frac{a^2\,x^4}{4} + \frac{a\,b\,x^8}{4} + \frac{b^2\,x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^4)^2,x)
[Out] (a^2*x^4)/4 + (b^2*x^12)/12 + (a*b*x^8)/4
```

sympy [B] time = 0.20, size = 24, normalized size = 1.50

$$\frac{a^2x^4}{4} + \frac{abx^8}{4} + \frac{b^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**4+a)**2,x)
[Out] a**2*x**4/4 + a*b*x**8/4 + b**2*x**12/12
```

3.624 $\int x^2 \left(a+bx^4\right)^2 \, dx$

Optimal. Leaf size=30

$$\frac{a^2x^3}{3}+\frac{2}{7}abx^7+\frac{b^2x^{11}}{11}$$

```
[Out] 1/3*a^2*x^3+2/7*a*b*x^7+1/11*b^2*x^11
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{a^2x^3}{3}+\frac{2}{7}abx^7+\frac{b^2x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^4)^2,x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^7)/7 + (b^2*x^11)/11
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(a+bx^4\right)^2 \, dx &= \int \left(a^2x^2+2abx^6+b^2x^{10}\right) \, dx \\ &= \frac{a^2x^3}{3}+\frac{2}{7}abx^7+\frac{b^2x^{11}}{11}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^3}{3}+\frac{2}{7}abx^7+\frac{b^2x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^4)^2,x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^7)/7 + (b^2*x^11)/11
```

fricas [A] time = 0.50, size = 24, normalized size = 0.80

$$\frac{1}{11}x^{11}b^2+\frac{2}{7}x^7ba+\frac{1}{3}x^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*b^2 + 2/7*x^7*b*a + 1/3*x^3*a^2
```

giac [A] time = 0.15, size = 24, normalized size = 0.80

$$\frac{1}{11}\,b^2x^{11}+\frac{2}{7}\,abx^7+\frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^4+a)^2,x, algorithm="giac")
[Out] 1/11*b^2*x^11 + 2/7*a*b*x^7 + 1/3*a^2*x^3
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$\frac{1}{11}b^2x^{11} + \frac{2}{7}ab\,x^7 + \frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^4+a)^2,x)
[Out] 1/3*a^2*x^3+2/7*a*b*x^7+1/11*b^2*x^11
```

maxima [A] time = 1.34, size = 24, normalized size = 0.80

$$\frac{1}{11}\,b^2x^{11} + \frac{2}{7}\,abx^7 + \frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^4+a)^2,x, algorithm="maxima")
[Out] 1/11*b^2*x^11 + 2/7*a*b*x^7 + 1/3*a^2*x^3
```

mupad [B] time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^3}{3} + \frac{2\,a\,b\,x^7}{7} + \frac{b^2\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^4)^2,x)
[Out] (a^2*x^3)/3 + (b^2*x^11)/11 + (2*a*b*x^7)/7
```

sympy [A] time = 0.08, size = 26, normalized size = 0.87

$$\frac{a^2x^3}{3} + \frac{2abx^7}{7} + \frac{b^2x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**4+a)**2,x)
[Out] a**2*x**3/3 + 2*a*b*x**7/7 + b**2*x**11/11
```

3.625 $\int x\left(a+bx^4\right)^2\,dx$

Optimal. Leaf size=30

$$\frac{a^2x^2}{2}+\frac{1}{3}abx^6+\frac{b^2x^{10}}{10}$$

```
[Out] 1/2*a^2*x^2+1/3*a*b*x^6+1/10*b^2*x^10
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {270}

$$\frac{a^2x^2}{2}+\frac{1}{3}abx^6+\frac{b^2x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^4)^2,x]
```

```
[Out] (a^2*x^2)/2 + (a*b*x^6)/3 + (b^2*x^10)/10
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^4\right)^2\,dx &= \int\left(a^2x+2abx^5+b^2x^9\right)\,dx\\ &= \frac{a^2x^2}{2}+\frac{1}{3}abx^6+\frac{b^2x^{10}}{10}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^2}{2}+\frac{1}{3}abx^6+\frac{b^2x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^4)^2,x]
```

```
[Out] (a^2*x^2)/2 + (a*b*x^6)/3 + (b^2*x^10)/10
```

fricas [A] time = 0.48, size = 24, normalized size = 0.80

$$\frac{1}{10}x^{10}b^2+\frac{1}{3}x^6ba+\frac{1}{2}x^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*b^2 + 1/3*x^6*b*a + 1/2*x^2*a^2
```

giac [A] time = 0.15, size = 24, normalized size = 0.80

$$\frac{1}{10}\,b^2x^{10}+\frac{1}{3}\,abx^6+\frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^4+a)^2,x, algorithm="giac")
[Out] 1/10*b^2*x^10 + 1/3*a*b*x^6 + 1/2*a^2*x^2
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$\frac{1}{10}b^2x^{10} + \frac{1}{3}ab\,x^6 + \frac{1}{2}a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^4+a)^2,x)
[Out] 1/2*a^2*x^2+1/3*a*b*x^6+1/10*b^2*x^10
```

maxima [A] time = 1.32, size = 24, normalized size = 0.80

$$\frac{1}{10}\,b^2x^{10} + \frac{1}{3}\,abx^6 + \frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^4+a)^2,x, algorithm="maxima")
[Out] 1/10*b^2*x^10 + 1/3*a*b*x^6 + 1/2*a^2*x^2
```

mupad [B] time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^2}{2} + \frac{a\,b\,x^6}{3} + \frac{b^2\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^4)^2,x)
[Out] (a^2*x^2)/2 + (b^2*x^10)/10 + (a*b*x^6)/3
```

sympy [A] time = 0.09, size = 24, normalized size = 0.80

$$\frac{a^2x^2}{2} + \frac{abx^6}{3} + \frac{b^2x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**4+a)**2,x)
[Out] a**2*x**2/2 + a*b*x**6/3 + b**2*x**10/10
```

3.626 $\int \left(a + bx^4\right)^2 \, dx$

Optimal. Leaf size=25

$$a^2x + \frac{2}{5}abx^5 + \frac{b^2x^9}{9}$$

```
[Out] a^2*x+2/5*a*b*x^5+1/9*b^2*x^9
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {194}

$$a^2x + \frac{2}{5}abx^5 + \frac{b^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^2,x]
```

```
[Out] a^2*x + (2*a*b*x^5)/5 + (b^2*x^9)/9
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a + bx^4\right)^2 \, dx &= \int \left(a^2 + 2abx^4 + b^2x^8\right) \, dx \\ &= a^2x + \frac{2}{5}abx^5 + \frac{b^2x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$a^2x + \frac{2}{5}abx^5 + \frac{b^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^2,x]
```

```
[Out] a^2*x + (2*a*b*x^5)/5 + (b^2*x^9)/9
```

fricas [A] time = 0.42, size = 21, normalized size = 0.84

$$\frac{1}{9}x^9b^2 + \frac{2}{5}x^5ba + xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*b^2 + 2/5*x^5*b*a + x*a^2
```

giac [A] time = 0.15, size = 21, normalized size = 0.84

$$\frac{1}{9}\,b^2x^9 + \frac{2}{5}\,abx^5 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2,x, algorithm="giac")
[Out] 1/9*b^2*x^9 + 2/5*a*b*x^5 + a^2*x
```

maple [A] time = 0.00, size = 22, normalized size = 0.88

$$\frac{1}{9}b^2x^9 + \frac{2}{5}ab\,x^5 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^2,x)
[Out] a^2*x+2/5*a*b*x^5+1/9*b^2*x^9
```

maxima [A] time = 1.33, size = 21, normalized size = 0.84

$$\frac{1}{9}\,b^2x^9 + \frac{2}{5}\,abx^5 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2,x, algorithm="maxima")
[Out] 1/9*b^2*x^9 + 2/5*a*b*x^5 + a^2*x
```

mupad [B] time = 0.03, size = 21, normalized size = 0.84

$$a^2\,x + \frac{2\,a\,b\,x^5}{5} + \frac{b^2\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^2,x)
[Out] a^2*x + (b^2*x^9)/9 + (2*a*b*x^5)/5
```

sympy [A] time = 0.09, size = 22, normalized size = 0.88

$$a^2x + \frac{2abx^5}{5} + \frac{b^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**2,x)
[Out] a**2*x + 2*a*b*x**5/5 + b**2*x**9/9
```

3.627 $\int \frac{\left(a+bx^4\right)^2}{x}\,dx$

Optimal. Leaf size=26

$$a^2\log(x)+\frac{1}{2}abx^4+\frac{b^2x^8}{8}$$

```
[Out] 1/2*a*b*x^4+1/8*b^2*x^8+a^2*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$a^2\log(x)+\frac{1}{2}abx^4+\frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^2/x,x]
```

```
[Out] (a*b*x^4)/2 + (b^2*x^8)/8 + a^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^4\right)^2}{x}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{(a+bx)^2}{x}\,dx, x, x^4\right)\\ &= \frac{1}{4}\operatorname{Subst}\left(\int \left(2ab+\frac{a^2}{x}+b^2x\right)\,dx, x, x^4\right)\\ &= \frac{1}{2}abx^4+\frac{b^2x^8}{8}+a^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 26, normalized size = 1.00

$$a^2\log(x)+\frac{1}{2}abx^4+\frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^2/x,x]
```

```
[Out] (a*b*x^4)/2 + (b^2*x^8)/8 + a^2*Log[x]
```

fricas [A] time = 0.55, size = 22, normalized size = 0.85

$$\frac{1}{8}b^2x^8+\frac{1}{2}abx^4+a^2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/x,x, algorithm="fricas")
[Out] 1/8*b^2*x^8 + 1/2*a*b*x^4 + a^2*log(x)
```

giac [A] time = 0.15, size = 25, normalized size = 0.96

$$\frac{1}{8}b^2x^8 + \frac{1}{2}abx^4 + \frac{1}{4}a^2\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/x,x, algorithm="giac")
[Out] 1/8*b^2*x^8 + 1/2*a*b*x^4 + 1/4*a^2*log(x^4)
```

maple [A] time = 0.00, size = 23, normalized size = 0.88

$$\frac{b^2x^8}{8} + \frac{ab\,x^4}{2} + a^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^2/x,x)
[Out] 1/2*a*b*x^4+1/8*b^2*x^8+a^2*ln(x)
```

maxima [A] time = 1.32, size = 25, normalized size = 0.96

$$\frac{1}{8}b^2x^8 + \frac{1}{2}abx^4 + \frac{1}{4}a^2\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/x,x, algorithm="maxima")
[Out] 1/8*b^2*x^8 + 1/2*a*b*x^4 + 1/4*a^2*log(x^4)
```

mupad [B] time = 0.96, size = 22, normalized size = 0.85

$$a^2\,\ln(x) + \frac{b^2\,x^8}{8} + \frac{a\,b\,x^4}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^2/x,x)
[Out] a^2*log(x) + (b^2*x^8)/8 + (a*b*x^4)/2
```

sympy [A] time = 0.18, size = 22, normalized size = 0.85

$$a^2\log(x) + \frac{abx^4}{2} + \frac{b^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**2/x,x)
[Out] a**2*log(x) + a*b*x**4/2 + b**2*x**8/8
```

3.628 $\int \frac{\left(a+bx^4\right)^2}{x^2}\,dx$

Optimal. Leaf size=28

$$-\frac{a^2}{x}+\frac{2}{3}abx^3+\frac{b^2x^7}{7}$$

```
[Out] -a^2/x+2/3*a*b*x^3+1/7*b^2*x^7
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$-\frac{a^2}{x}+\frac{2}{3}abx^3+\frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^2/x^2,x]
```

```
[Out] -(a^2/x) + (2*a*b*x^3)/3 + (b^2*x^7)/7
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^4\right)^2}{x^2}\,dx &= \int \left(\frac{a^2}{x^2}+2abx^2+b^2x^6\right)dx\\ &= -\frac{a^2}{x}+\frac{2}{3}abx^3+\frac{b^2x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 28, normalized size = 1.00

$$-\frac{a^2}{x}+\frac{2}{3}abx^3+\frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^2/x^2,x]
```

```
[Out] -(a^2/x) + (2*a*b*x^3)/3 + (b^2*x^7)/7
```

fricas [A] time = 0.76, size = 26, normalized size = 0.93

$$\frac{3\,b^2x^8+14\,abx^4-21\,a^2}{21\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/21*(3*b^2*x^8 + 14*a*b*x^4 - 21*a^2)/x
```

giac [A] time = 0.15, size = 24, normalized size = 0.86

$$\frac{1}{7}\,b^2x^7+\frac{2}{3}\,abx^3-\frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/x^2,x, algorithm="giac")
[Out] 1/7*b^2*x^7 + 2/3*a*b*x^3 - a^2/x
```

maple [A] time = 0.00, size = 25, normalized size = 0.89

$$\frac{b^2x^7}{7} + \frac{2ab\,x^3}{3} - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^2/x^2,x)
[Out] -a^2/x+2/3*a*b*x^3+1/7*b^2*x^7
```

maxima [A] time = 1.30, size = 24, normalized size = 0.86

$$\frac{1}{7}\,b^2x^7 + \frac{2}{3}\,abx^3 - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/x^2,x, algorithm="maxima")
[Out] 1/7*b^2*x^7 + 2/3*a*b*x^3 - a^2/x
```

mupad [B] time = 0.03, size = 25, normalized size = 0.89

$$\frac{-a^2 + \frac{2\,a\,b\,x^4}{3} + \frac{b^2\,x^8}{7}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^2/x^2,x)
[Out] ((b^2*x^8)/7 - a^2 + (2*a*b*x^4)/3)/x
```

sympy [A] time = 0.20, size = 22, normalized size = 0.79

$$-\frac{a^2}{x} + \frac{2abx^3}{3} + \frac{b^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**2/x**2,x)
[Out] -a**2/x + 2*a*b*x**3/3 + b**2*x**7/7
```

3.629 $\int \frac{\left(a+bx^4\right)^2}{x^3}\,dx$

Optimal. Leaf size=27

$$-\frac{a^2}{2x^2}+abx^2+\frac{b^2x^6}{6}$$

```
[Out] -1/2*a^2/x^2+a*b*x^2+1/6*b^2*x^6
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$-\frac{a^2}{2x^2}+abx^2+\frac{b^2x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^2/x^3,x]
```

```
[Out] -a^2/(2*x^2) + a*b*x^2 + (b^2*x^6)/6
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^4\right)^2}{x^3}\,dx &= \int \left(\frac{a^2}{x^3}+2abx+b^2x^5\right)dx\\ &= -\frac{a^2}{2x^2}+abx^2+\frac{b^2x^6}{6}\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.00

$$-\frac{a^2}{2x^2}+abx^2+\frac{b^2x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^2/x^3,x]
```

```
[Out] -1/2*a^2/x^2 + a*b*x^2 + (b^2*x^6)/6
```

fricas [A] time = 0.81, size = 25, normalized size = 0.93

$$\frac{b^2x^8+6\,abx^4-3\,a^2}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*(b^2*x^8 + 6*a*b*x^4 - 3*a^2)/x^2
```

giac [A] time = 0.15, size = 23, normalized size = 0.85

$$\frac{1}{6}\,b^2x^6+abx^2-\frac{a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/x^3,x, algorithm="giac")
[Out] 1/6*b^2*x^6 + a*b*x^2 - 1/2*a^2/x^2
```

maple [A] time = 0.00, size = 24, normalized size = 0.89

$$\frac{b^2x^6}{6} + ab\,x^2 - \frac{a^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^2/x^3,x)
[Out] -1/2*a^2/x^2+a*b*x^2+1/6*b^2*x^6
```

maxima [A] time = 1.32, size = 23, normalized size = 0.85

$$\frac{1}{6}\,b^2x^6 + abx^2 - \frac{a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/x^3,x, algorithm="maxima")
[Out] 1/6*b^2*x^6 + a*b*x^2 - 1/2*a^2/x^2
```

mupad [B] time = 0.03, size = 25, normalized size = 0.93

$$\frac{-3\,a^2 + 6\,a\,b\,x^4 + b^2\,x^8}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^2/x^3,x)
[Out] (b^2*x^8 - 3*a^2 + 6*a*b*x^4)/(6*x^2)
```

sympy [A] time = 0.16, size = 22, normalized size = 0.81

$$-\frac{a^2}{2x^2} + abx^2 + \frac{b^2x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**2/x**3,x)
[Out] -a**2/(2*x**2) + a*b*x**2 + b**2*x**6/6
```

3.630 $\int \frac{\left(a+bx^4\right)^2}{x^4}\,dx$

Optimal. Leaf size=26

$$-\frac{a^2}{3x^3}+2abx+\frac{b^2x^5}{5}$$

```
[Out] -1/3*a^2/x^3+2*a*b*x+1/5*b^2*x^5
```

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$-\frac{a^2}{3x^3}+2abx+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^2/x^4,x]
```

```
[Out] -a^2/(3*x^3) + 2*a*b*x + (b^2*x^5)/5
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^4\right)^2}{x^4}\,dx &= \int \left(2ab+\frac{a^2}{x^4}+b^2x^4\right)dx\\ &= -\frac{a^2}{3x^3}+2abx+\frac{b^2x^5}{5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 26, normalized size = 1.00

$$-\frac{a^2}{3x^3}+2abx+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^2/x^4,x]
```

```
[Out] -1/3*a^2/x^3 + 2*a*b*x + (b^2*x^5)/5
```

fricas [A] time = 0.72, size = 26, normalized size = 1.00

$$\frac{3\,b^2x^8+30\,abx^4-5\,a^2}{15\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^2*x^8 + 30*a*b*x^4 - 5*a^2)/x^3
```

giac [A] time = 0.15, size = 22, normalized size = 0.85

$$\frac{1}{5}\,b^2x^5+2\,abx-\frac{a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/x^4,x, algorithm="giac")
[Out] 1/5*b^2*x^5 + 2*a*b*x - 1/3*a^2/x^3
```

maple [A] time = 0.00, size = 23, normalized size = 0.88

$$\frac{b^2x^5}{5} + 2abx - \frac{a^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^2/x^4,x)
[Out] -1/3*a^2/x^3+2*a*b*x+1/5*b^2*x^5
```

maxima [A] time = 1.35, size = 22, normalized size = 0.85

$$\frac{1}{5}\,b^2x^5 + 2\,abx - \frac{a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/x^4,x, algorithm="maxima")
[Out] 1/5*b^2*x^5 + 2*a*b*x - 1/3*a^2/x^3
```

mupad [B] time = 0.03, size = 22, normalized size = 0.85

$$\frac{b^2\,x^5}{5} - \frac{a^2}{3\,x^3} + 2\,a\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^2/x^4,x)
[Out] (b^2*x^5)/5 - a^2/(3*x^3) + 2*a*b*x
```

sympy [A] time = 0.19, size = 22, normalized size = 0.85

$$-\frac{a^2}{3x^3} + 2abx + \frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**2/x**4,x)
[Out] -a**2/(3*x**3) + 2*a*b*x + b**2*x**5/5
```

3.631 $\int \frac{\left(a+bx^4\right)^2}{x^5}\,dx$

Optimal. Leaf size=27

$$-\frac{a^2}{4x^4}+2ab\log(x)+\frac{b^2x^4}{4}$$

```
[Out] -1/4*a^2/x^4+1/4*b^2*x^4+2*a*b*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$-\frac{a^2}{4x^4}+2ab\log(x)+\frac{b^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^2/x^5,x]
```

```
[Out] -a^2/(4*x^4) + (b^2*x^4)/4 + 2*a*b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^4\right)^2}{x^5}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{(a+bx)^2}{x^2}\,dx, x, x^4\right)\\ &= \frac{1}{4}\operatorname{Subst}\left(\int \left(b^2+\frac{a^2}{x^2}+\frac{2ab}{x}\right)\,dx, x, x^4\right)\\ &= -\frac{a^2}{4x^4}+\frac{b^2x^4}{4}+2ab\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.00

$$-\frac{a^2}{4x^4}+2ab\log(x)+\frac{b^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^2/x^5,x]
```

```
[Out] -1/4*a^2/x^4 + (b^2*x^4)/4 + 2*a*b*Log[x]
```

fricas [A] time = 0.59, size = 27, normalized size = 1.00

$$\frac{b^2x^8+8\,abx^4\log(x)-a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/x^5,x, algorithm="fricas")
[Out] 1/4*(b^2*x^8 + 8*a*b*x^4*log(x) - a^2)/x^4
```

giac [A] time = 0.15, size = 33, normalized size = 1.22

$$\frac{1}{4}\,b^2x^4+\frac{1}{2}\,ab\log\left(x^4\right)-\frac{2\,abx^4+a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/x^5,x, algorithm="giac")
[Out] 1/4*b^2*x^4 + 1/2*a*b*log(x^4) - 1/4*(2*a*b*x^4 + a^2)/x^4
```

maple [A] time = 0.01, size = 24, normalized size = 0.89

$$\frac{b^2x^4}{4}+2ab\ln(x)-\frac{a^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^2/x^5,x)
[Out] -1/4*a^2/x^4+1/4*b^2*x^4+2*a*b*ln(x)
```

maxima [A] time = 1.28, size = 25, normalized size = 0.93

$$\frac{1}{4}\,b^2x^4+\frac{1}{2}\,ab\log\left(x^4\right)-\frac{a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/x^5,x, algorithm="maxima")
[Out] 1/4*b^2*x^4 + 1/2*a*b*log(x^4) - 1/4*a^2/x^4
```

mupad [B] time = 0.03, size = 23, normalized size = 0.85

$$\frac{b^2\,x^4}{4}-\frac{a^2}{4\,x^4}+2\,a\,b\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^2/x^5,x)
[Out] (b^2*x^4)/4 - a^2/(4*x^4) + 2*a*b*log(x)
```

sympy [A] time = 0.20, size = 24, normalized size = 0.89

$$-\frac{a^2}{4x^4}+2ab\log(x)+\frac{b^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**2/x**5,x)
[Out] -a**2/(4*x**4) + 2*a*b*log(x) + b**2*x**4/4
```

3.632 $\int x^m \left(a + bx^4\right)^3 \, dx$

Optimal. Leaf size=61

$$\frac{a^3 x^{m+1}}{m+1} + \frac{3a^2 b x^{m+5}}{m+5} + \frac{3ab^2 x^{m+9}}{m+9} + \frac{b^3 x^{m+13}}{m+13}$$

```
[Out] a^3*x^(1+m)/(1+m)+3*a^2*b*x^(5+m)/(5+m)+3*a*b^2*x^(9+m)/(9+m)+b^3*x^(13+m)/
(13+m)
```

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{3a^2 b x^{m+5}}{m+5} + \frac{a^3 x^{m+1}}{m+1} + \frac{3ab^2 x^{m+9}}{m+9} + \frac{b^3 x^{m+13}}{m+13}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^4)^3,x]
```

```
[Out] (a^3*x^(1 + m))/(1 + m) + (3*a^2*b*x^(5 + m))/(5 + m) + (3*a*b^2*x^(9 + m))
/(9 + m) + (b^3*x^(13 + m))/(13 + m)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^m \left(a + bx^4\right)^3 \, dx &= \int \left(a^3 x^m + 3a^2 b x^{4+m} + 3ab^2 x^{8+m} + b^3 x^{12+m}\right) dx \\ &= \frac{a^3 x^{1+m}}{1+m} + \frac{3a^2 b x^{5+m}}{5+m} + \frac{3ab^2 x^{9+m}}{9+m} + \frac{b^3 x^{13+m}}{13+m}\end{aligned}$$

Mathematica [A] time = 0.03, size = 56, normalized size = 0.92

$$x^{m+1}\left(\frac{a^3}{m+1} + \frac{3a^2 b x^4}{m+5} + \frac{3ab^2 x^8}{m+9} + \frac{b^3 x^{12}}{m+13}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^4)^3,x]
```

```
[Out] x^(1 + m)*(a^3/(1 + m) + (3*a^2*b*x^4)/(5 + m) + (3*a*b^2*x^8)/(9 + m) + (b
^3*x^12)/(13 + m))
```

fricas [B] time = 0.58, size = 157, normalized size = 2.57

$$\frac{\left(\left(b^3 m^3 + 15\,b^3 m^2 + 59\,b^3 m + 45\,b^3\right)x^{13} + 3\left(ab^2 m^3 + 19\,ab^2 m^2 + 83\,ab^2 m + 65\,ab^2\right)x^9 + 3\left(a^2 bm^3 + 23\,a^2 bm^2 +\right.\right.}{m^4 + 28\,m^3 + 254\,m^2 + 812\,m + 585}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] ((b^3*m^3 + 15*b^3*m^2 + 59*b^3*m + 45*b^3)*x^13 + 3*(a*b^2*m^3 + 19*a*b^2*
m^2 + 83*a*b^2*m + 65*a*b^2)*x^9 + 3*(a^2*b*m^3 + 23*a^2*b*m^2 + 139*a^2*b*
m + 117*a^2*b)*x^5 + (a^3*m^3 + 27*a^3*m^2 + 227*a^3*m + 585*a^3)*x)*x^m/(m
^4 + 28*m^3 + 254*m^2 + 812*m + 585)
```

giac [B] time = 0.18, size = 224, normalized size = 3.67

$$\frac{b^3m^3x^{13}x^m + 15\,b^3m^2x^{13}x^m + 59\,b^3mx^{13}x^m + 45\,b^3x^{13}x^m + 3\,ab^2m^3x^9x^m + 57\,ab^2m^2x^9x^m + 249\,ab^2mx^9x^m +}{m^4 + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^4+a)^3,x, algorithm="giac")
```

```
[Out] (b^3*m^3*x^13*x^m + 15*b^3*m^2*x^13*x^m + 59*b^3*m*x^13*x^m + 45*b^3*x^13*x
^m + 3*a*b^2*m^3*x^9*x^m + 57*a*b^2*m^2*x^9*x^m + 249*a*b^2*m*x^9*x^m + 195
*a*b^2*x^9*x^m + 3*a^2*b*m^3*x^5*x^m + 69*a^2*b*m^2*x^5*x^m + 417*a^2*b*m*x
^5*x^m + 351*a^2*b*x^5*x^m + a^3*m^3*x*x^m + 27*a^3*m^2*x*x^m + 227*a^3*m*x
*x^m + 585*a^3*x*x^m)/(m^4 + 28*m^3 + 254*m^2 + 812*m + 585)
```

maple [B] time = 0.01, size = 178, normalized size = 2.92

$$\frac{\left(b^3m^3x^{12} + 15b^3m^2x^{12} + 59b^3m\,x^{12} + 45b^3x^{12} + 3a\,b^2m^3x^8 + 57a\,b^2m^2x^8 + 249a\,b^2m\,x^8 + 195a\,b^2x^8 + 3a^2b\,m\right.}{(m+13)\,(m+9)\,(m+5)\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^4+a)^3,x)
```

```
[Out] x^(m+1)*(b^3*m^3*x^12+15*b^3*m^2*x^12+59*b^3*m*x^12+45*b^3*x^12+3*a*b^2*m^3
*x^8+57*a*b^2*m^2*x^8+249*a*b^2*m*x^8+195*a*b^2*x^8+3*a^2*b*m^3*x^4+69*a^2*
b*m^2*x^4+417*a^2*b*m*x^4+351*a^2*b*x^4+a^3*m^3+27*a^3*m^2+227*a^3*m+585*a^
3)/(m+13)/(m+9)/(m+5)/(m+1)
```

maxima [A] time = 1.34, size = 61, normalized size = 1.00

$$\frac{b^3x^{m+13}}{m+13} + \frac{3\,ab^2x^{m+9}}{m+9} + \frac{3\,a^2bx^{m+5}}{m+5} + \frac{a^3x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] b^3*x^(m + 13)/(m + 13) + 3*a*b^2*x^(m + 9)/(m + 9) + 3*a^2*b*x^(m + 5)/(m
+ 5) + a^3*x^(m + 1)/(m + 1)
```

mupad [B] time = 1.33, size = 167, normalized size = 2.74

$$x^m\left(\frac{a^3\,x\,\left(m^3+27\,m^2+227\,m+585\right)}{m^4+28\,m^3+254\,m^2+812\,m+585} + \frac{b^3\,x^{13}\,\left(m^3+15\,m^2+59\,m+45\right)}{m^4+28\,m^3+254\,m^2+812\,m+585} + \frac{3\,a\,b^2\,x^9\,\left(m^3+19\,m^2+\right.}{m^4+28\,m^3+254\,m^2+8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^4)^3,x)
```

```
[Out] x^m*((a^3*x*(227*m + 27*m^2 + m^3 + 585))/(812*m + 254*m^2 + 28*m^3 + m^4 +
 585) + (b^3*x^13*(59*m + 15*m^2 + m^3 + 45))/(812*m + 254*m^2 + 28*m^3 + m
^4 + 585) + (3*a*b^2*x^9*(83*m + 19*m^2 + m^3 + 65))/(812*m + 254*m^2 + 28*
m^3 + m^4 + 585) + (3*a^2*b*x^5*(139*m + 23*m^2 + m^3 + 117))/(812*m + 254*
m^2 + 28*m^3 + m^4 + 585))
```

sympy [A] time = 7.36, size = 683, normalized size = 11.20

$$\begin{cases} -\frac{a^3}{12x^{12}} - \frac{3a^2b}{8x^8} - \frac{3ab^2}{4x^4} + b^3\log(x) \\ -\frac{a^3}{8x^8} - \frac{3a^2b}{4x^4} + 3ab^2\log(x) + \frac{b^3x^4}{4} \\ -\frac{a^3}{4x^4} + 3a^2b\log(x) + \frac{3ab^2x^4}{4} + \frac{b^3x^8}{8} \\ a^3\log(x) + \frac{3a^2bx^4}{4} + \frac{3ab^2x^8}{8} + \frac{b^3x^{12}}{12} \\ \frac{a^3m^3xx^m}{m^4+28m^3+254m^2+812m+585} + \frac{27a^3m^2xx^m}{m^4+28m^3+254m^2+812m+585} + \frac{227a^3mxx^m}{m^4+28m^3+254m^2+812m+585} + \frac{585a^3xx^m}{m^4+28m^3+254m^2+812m+585} + \frac{}{m^4+28m} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**4+a)**3,x)
```

```
[Out] Piecewise((-a**3/(12*x**12) - 3*a**2*b/(8*x**8) - 3*a*b**2/(4*x**4) + b**3*
log(x), Eq(m, -13)), (-a**3/(8*x**8) - 3*a**2*b/(4*x**4) + 3*a*b**2*log(x)
+ b**3*x**4/4, Eq(m, -9)), (-a**3/(4*x**4) + 3*a**2*b*log(x) + 3*a*b**2*x**
4/4 + b**3*x**8/8, Eq(m, -5)), (a**3*log(x) + 3*a**2*b*x**4/4 + 3*a*b**2*x*
*8/8 + b**3*x**12/12, Eq(m, -1)), (a**3*m**3*x*x**m/(m**4 + 28*m**3 + 254*m
**2 + 812*m + 585) + 27*a**3*m**2*x*x**m/(m**4 + 28*m**3 + 254*m**2 + 812*m
 + 585) + 227*a**3*m*x*x**m/(m**4 + 28*m**3 + 254*m**2 + 812*m + 585) + 585
*a**3*x*x**m/(m**4 + 28*m**3 + 254*m**2 + 812*m + 585) + 3*a**2*b*m**3*x**5
*x**m/(m**4 + 28*m**3 + 254*m**2 + 812*m + 585) + 69*a**2*b*m**2*x**5*x**m/
(m**4 + 28*m**3 + 254*m**2 + 812*m + 585) + 417*a**2*b*m*x**5*x**m/(m**4 +
28*m**3 + 254*m**2 + 812*m + 585) + 351*a**2*b*x**5*x**m/(m**4 + 28*m**3 +
254*m**2 + 812*m + 585) + 3*a*b**2*m**3*x**9*x**m/(m**4 + 28*m**3 + 254*m**
2 + 812*m + 585) + 57*a*b**2*m**2*x**9*x**m/(m**4 + 28*m**3 + 254*m**2 + 81
2*m + 585) + 249*a*b**2*m*x**9*x**m/(m**4 + 28*m**3 + 254*m**2 + 812*m + 58
5) + 195*a*b**2*x**9*x**m/(m**4 + 28*m**3 + 254*m**2 + 812*m + 585) + b**3*
m**3*x**13*x**m/(m**4 + 28*m**3 + 254*m**2 + 812*m + 585) + 15*b**3*m**2*x*
*13*x**m/(m**4 + 28*m**3 + 254*m**2 + 812*m + 585) + 59*b**3*m*x**13*x**m/(
m**4 + 28*m**3 + 254*m**2 + 812*m + 585) + 45*b**3*x**13*x**m/(m**4 + 28*m*
*3 + 254*m**2 + 812*m + 585), True))
```

3.633 $\int x^5 \left(a + bx^4\right)^3 \, dx$

Optimal. Leaf size=43

$$\frac{a^3x^6}{6} + \frac{3}{10}a^2bx^{10} + \frac{3}{14}ab^2x^{14} + \frac{b^3x^{18}}{18}$$

```
[Out] 1/6*a^3*x^6+3/10*a^2*b*x^10+3/14*a*b^2*x^14+1/18*b^3*x^18
```

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{3}{10}a^2bx^{10} + \frac{a^3x^6}{6} + \frac{3}{14}ab^2x^{14} + \frac{b^3x^{18}}{18}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^4)^3,x]
```

```
[Out] (a^3*x^6)/6 + (3*a^2*b*x^10)/10 + (3*a*b^2*x^14)/14 + (b^3*x^18)/18
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^5 \left(a + bx^4\right)^3 \, dx &= \int \left(a^3x^5 + 3a^2bx^9 + 3ab^2x^{13} + b^3x^{17}\right) dx \\ &= \frac{a^3x^6}{6} + \frac{3}{10}a^2bx^{10} + \frac{3}{14}ab^2x^{14} + \frac{b^3x^{18}}{18}\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^6}{6} + \frac{3}{10}a^2bx^{10} + \frac{3}{14}ab^2x^{14} + \frac{b^3x^{18}}{18}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^4)^3,x]
```

```
[Out] (a^3*x^6)/6 + (3*a^2*b*x^10)/10 + (3*a*b^2*x^14)/14 + (b^3*x^18)/18
```

fricas [A] time = 0.52, size = 35, normalized size = 0.81

$$\frac{1}{18}x^{18}b^3 + \frac{3}{14}x^{14}b^2a + \frac{3}{10}x^{10}ba^2 + \frac{1}{6}x^6a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 1/18*x^18*b^3 + 3/14*x^14*b^2*a + 3/10*x^10*b*a^2 + 1/6*x^6*a^3
```

giac [A] time = 0.18, size = 35, normalized size = 0.81

$$\frac{1}{18}\,b^3x^{18} + \frac{3}{14}\,ab^2x^{14} + \frac{3}{10}\,a^2bx^{10} + \frac{1}{6}\,a^3x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^4+a)^3,x, algorithm="giac")
[Out] 1/18*b^3*x^18 + 3/14*a*b^2*x^14 + 3/10*a^2*b*x^10 + 1/6*a^3*x^6
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$\frac{1}{18}b^3x^{18} + \frac{3}{14}a\,b^2x^{14} + \frac{3}{10}a^2b\,x^{10} + \frac{1}{6}a^3x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^4+a)^3,x)
[Out] 1/6*a^3*x^6+3/10*a^2*b*x^10+3/14*a*b^2*x^14+1/18*b^3*x^18
```

maxima [A] time = 1.34, size = 35, normalized size = 0.81

$$\frac{1}{18}\,b^3x^{18} + \frac{3}{14}\,ab^2x^{14} + \frac{3}{10}\,a^2bx^{10} + \frac{1}{6}\,a^3x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^4+a)^3,x, algorithm="maxima")
[Out] 1/18*b^3*x^18 + 3/14*a*b^2*x^14 + 3/10*a^2*b*x^10 + 1/6*a^3*x^6
```

mupad [B] time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3\,x^6}{6} + \frac{3\,a^2\,b\,x^{10}}{10} + \frac{3\,a\,b^2\,x^{14}}{14} + \frac{b^3\,x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^4)^3,x)
[Out] (a^3*x^6)/6 + (b^3*x^18)/18 + (3*a^2*b*x^10)/10 + (3*a*b^2*x^14)/14
```

sympy [A] time = 0.32, size = 39, normalized size = 0.91

$$\frac{a^3x^6}{6} + \frac{3a^2bx^{10}}{10} + \frac{3ab^2x^{14}}{14} + \frac{b^3x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**4+a)**3,x)
[Out] a**3*x**6/6 + 3*a**2*b*x**10/10 + 3*a*b**2*x**14/14 + b**3*x**18/18
```

3.634 $\int x^4 \left(a + bx^4\right)^3 \, dx$

Optimal. Leaf size=43

$$\frac{a^3x^5}{5} + \frac{1}{3}a^2bx^9 + \frac{3}{13}ab^2x^{13} + \frac{b^3x^{17}}{17}$$

```
[Out] 1/5*a^3*x^5+1/3*a^2*b*x^9+3/13*a*b^2*x^13+1/17*b^3*x^17
```

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{1}{3}a^2bx^9 + \frac{a^3x^5}{5} + \frac{3}{13}ab^2x^{13} + \frac{b^3x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^4)^3,x]
```

```
[Out] (a^3*x^5)/5 + (a^2*b*x^9)/3 + (3*a*b^2*x^13)/13 + (b^3*x^17)/17
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^4 \left(a + bx^4\right)^3 \, dx &= \int \left(a^3x^4 + 3a^2bx^8 + 3ab^2x^{12} + b^3x^{16}\right) dx \\ &= \frac{a^3x^5}{5} + \frac{1}{3}a^2bx^9 + \frac{3}{13}ab^2x^{13} + \frac{b^3x^{17}}{17}\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^5}{5} + \frac{1}{3}a^2bx^9 + \frac{3}{13}ab^2x^{13} + \frac{b^3x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^4)^3,x]
```

```
[Out] (a^3*x^5)/5 + (a^2*b*x^9)/3 + (3*a*b^2*x^13)/13 + (b^3*x^17)/17
```

fricas [A] time = 0.56, size = 35, normalized size = 0.81

$$\frac{1}{17}x^{17}b^3 + \frac{3}{13}x^{13}b^2a + \frac{1}{3}x^9ba^2 + \frac{1}{5}x^5a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 1/17*x^17*b^3 + 3/13*x^13*b^2*a + 1/3*x^9*b*a^2 + 1/5*x^5*a^3
```

giac [A] time = 0.15, size = 35, normalized size = 0.81

$$\frac{1}{17}\,b^3x^{17} + \frac{3}{13}\,ab^2x^{13} + \frac{1}{3}\,a^2bx^9 + \frac{1}{5}\,a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^4+a)^3,x, algorithm="giac")
[Out] 1/17*b^3*x^17 + 3/13*a*b^2*x^13 + 1/3*a^2*b*x^9 + 1/5*a^3*x^5
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$\frac{1}{17}b^3x^{17} + \frac{3}{13}a\,b^2x^{13} + \frac{1}{3}a^2b\,x^9 + \frac{1}{5}a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^4+a)^3,x)
[Out] 1/5*a^3*x^5+1/3*a^2*b*x^9+3/13*a*b^2*x^13+1/17*b^3*x^17
```

maxima [A] time = 1.34, size = 35, normalized size = 0.81

$$\frac{1}{17}\,b^3x^{17} + \frac{3}{13}\,ab^2x^{13} + \frac{1}{3}\,a^2bx^9 + \frac{1}{5}\,a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^4+a)^3,x, algorithm="maxima")
[Out] 1/17*b^3*x^17 + 3/13*a*b^2*x^13 + 1/3*a^2*b*x^9 + 1/5*a^3*x^5
```

mupad [B] time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3\,x^5}{5} + \frac{a^2\,b\,x^9}{3} + \frac{3\,a\,b^2\,x^{13}}{13} + \frac{b^3\,x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^4)^3,x)
[Out] (a^3*x^5)/5 + (b^3*x^17)/17 + (a^2*b*x^9)/3 + (3*a*b^2*x^13)/13
```

sympy [A] time = 0.13, size = 37, normalized size = 0.86

$$\frac{a^3x^5}{5} + \frac{a^2bx^9}{3} + \frac{3ab^2x^{13}}{13} + \frac{b^3x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**4+a)**3,x)
[Out] a**3*x**5/5 + a**2*b*x**9/3 + 3*a*b**2*x**13/13 + b**3*x**17/17
```

3.635 $\int x^3 \left(a + bx^4\right)^3 \, dx$

Optimal. Leaf size=16

$$\frac{\left(a + bx^4\right)^4}{16b}$$

```
[Out] 1/16*(b*x^4+a)^4/b
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$\frac{\left(a + bx^4\right)^4}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^4)^3,x]
```

```
[Out] (a + b*x^4)^4/(16*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^3 \left(a + bx^4\right)^3 \, dx = \frac{\left(a + bx^4\right)^4}{16b}$$

Mathematica [B] time = 0.00, size = 43, normalized size = 2.69

$$\frac{a^3 x^4}{4} + \frac{3}{8} a^2 b x^8 + \frac{1}{4} a b^2 x^{12} + \frac{b^3 x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^4)^3,x]
```

```
[Out] (a^3*x^4)/4 + (3*a^2*b*x^8)/8 + (a*b^2*x^12)/4 + (b^3*x^16)/16
```

fricas [B] time = 0.49, size = 35, normalized size = 2.19

$$\frac{1}{16} x^{16} b^3 + \frac{1}{4} x^{12} b^2 a + \frac{3}{8} x^8 b a^2 + \frac{1}{4} x^4 a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*b^3 + 1/4*x^12*b^2*a + 3/8*x^8*b*a^2 + 1/4*x^4*a^3
```

giac [A] time = 0.15, size = 14, normalized size = 0.88

$$\frac{\left(bx^4 + a\right)^4}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^4+a)^3,x, algorithm="giac")
[Out] 1/16*(b*x^4 + a)^4/b
```

maple [B] time = 0.00, size = 36, normalized size = 2.25

$$\frac{1}{16}b^3x^{16} + \frac{1}{4}a\,b^2x^{12} + \frac{3}{8}a^2b\,x^8 + \frac{1}{4}a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^4+a)^3,x)
[Out] 1/16*b^3*x^16+1/4*a*b^2*x^12+3/8*a^2*b*x^8+1/4*a^3*x^4
```

maxima [A] time = 1.32, size = 14, normalized size = 0.88

$$\frac{\left(bx^4 + a\right)^4}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^4+a)^3,x, algorithm="maxima")
[Out] 1/16*(b*x^4 + a)^4/b
```

mupad [B] time = 0.04, size = 35, normalized size = 2.19

$$\frac{a^3\,x^4}{4} + \frac{3\,a^2\,b\,x^8}{8} + \frac{a\,b^2\,x^{12}}{4} + \frac{b^3\,x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^4)^3,x)
[Out] (a^3*x^4)/4 + (b^3*x^16)/16 + (3*a^2*b*x^8)/8 + (a*b^2*x^12)/4
```

sympy [B] time = 0.09, size = 37, normalized size = 2.31

$$\frac{a^3x^4}{4} + \frac{3a^2bx^8}{8} + \frac{ab^2x^{12}}{4} + \frac{b^3x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**4+a)**3,x)
[Out] a**3*x**4/4 + 3*a**2*b*x**8/8 + a*b**2*x**12/4 + b**3*x**16/16
```

3.636 $\int x^2 \left(a + bx^4\right)^3 \, dx$

Optimal. Leaf size=43

$$\frac{a^3x^3}{3} + \frac{3}{7}a^2bx^7 + \frac{3}{11}ab^2x^{11} + \frac{b^3x^{15}}{15}$$

```
[Out] 1/3*a^3*x^3+3/7*a^2*b*x^7+3/11*a*b^2*x^11+1/15*b^3*x^15
```

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{3}{7}a^2bx^7 + \frac{a^3x^3}{3} + \frac{3}{11}ab^2x^{11} + \frac{b^3x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^4)^3,x]
```

```
[Out] (a^3*x^3)/3 + (3*a^2*b*x^7)/7 + (3*a*b^2*x^11)/11 + (b^3*x^15)/15
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(a + bx^4\right)^3 \, dx &= \int \left(a^3x^2 + 3a^2bx^6 + 3ab^2x^{10} + b^3x^{14}\right) dx \\ &= \frac{a^3x^3}{3} + \frac{3}{7}a^2bx^7 + \frac{3}{11}ab^2x^{11} + \frac{b^3x^{15}}{15}\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^3}{3} + \frac{3}{7}a^2bx^7 + \frac{3}{11}ab^2x^{11} + \frac{b^3x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^4)^3,x]
```

```
[Out] (a^3*x^3)/3 + (3*a^2*b*x^7)/7 + (3*a*b^2*x^11)/11 + (b^3*x^15)/15
```

fricas [A] time = 0.65, size = 35, normalized size = 0.81

$$\frac{1}{15}x^{15}b^3 + \frac{3}{11}x^{11}b^2a + \frac{3}{7}x^7ba^2 + \frac{1}{3}x^3a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*b^3 + 3/11*x^11*b^2*a + 3/7*x^7*b*a^2 + 1/3*x^3*a^3
```

giac [A] time = 0.15, size = 35, normalized size = 0.81

$$\frac{1}{15}\,b^3x^{15} + \frac{3}{11}\,ab^2x^{11} + \frac{3}{7}\,a^2bx^7 + \frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^4+a)^3,x, algorithm="giac")
[Out] 1/15*b^3*x^15 + 3/11*a*b^2*x^11 + 3/7*a^2*b*x^7 + 1/3*a^3*x^3
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$\frac{1}{15}b^3x^{15} + \frac{3}{11}a\,b^2x^{11} + \frac{3}{7}a^2b\,x^7 + \frac{1}{3}a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^4+a)^3,x)
[Out] 1/3*a^3*x^3+3/7*a^2*b*x^7+3/11*a*b^2*x^11+1/15*b^3*x^15
```

maxima [A] time = 1.24, size = 35, normalized size = 0.81

$$\frac{1}{15}\,b^3x^{15} + \frac{3}{11}\,ab^2x^{11} + \frac{3}{7}\,a^2bx^7 + \frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^4+a)^3,x, algorithm="maxima")
[Out] 1/15*b^3*x^15 + 3/11*a*b^2*x^11 + 3/7*a^2*b*x^7 + 1/3*a^3*x^3
```

mupad [B] time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3\,x^3}{3} + \frac{3\,a^2\,b\,x^7}{7} + \frac{3\,a\,b^2\,x^{11}}{11} + \frac{b^3\,x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^4)^3,x)
[Out] (a^3*x^3)/3 + (b^3*x^15)/15 + (3*a^2*b*x^7)/7 + (3*a*b^2*x^11)/11
```

sympy [A] time = 0.08, size = 39, normalized size = 0.91

$$\frac{a^3x^3}{3} + \frac{3a^2bx^7}{7} + \frac{3ab^2x^{11}}{11} + \frac{b^3x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**4+a)**3,x)
[Out] a**3*x**3/3 + 3*a**2*b*x**7/7 + 3*a*b**2*x**11/11 + b**3*x**15/15
```

3.637 $\int x\left(a+bx^4\right)^3\,dx$

Optimal. Leaf size=43

$$\frac{a^3x^2}{2}+\frac{1}{2}a^2bx^6+\frac{3}{10}ab^2x^{10}+\frac{b^3x^{14}}{14}$$

```
[Out] 1/2*a^3*x^2+1/2*a^2*b*x^6+3/10*a*b^2*x^10+1/14*b^3*x^14
```

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {270}

$$\frac{1}{2}a^2bx^6+\frac{a^3x^2}{2}+\frac{3}{10}ab^2x^{10}+\frac{b^3x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^4)^3,x]
```

```
[Out] (a^3*x^2)/2 + (a^2*b*x^6)/2 + (3*a*b^2*x^10)/10 + (b^3*x^14)/14
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^4\right)^3\,dx &= \int\left(a^3x+3a^2bx^5+3ab^2x^9+b^3x^{13}\right)dx\\ &= \frac{a^3x^2}{2}+\frac{1}{2}a^2bx^6+\frac{3}{10}ab^2x^{10}+\frac{b^3x^{14}}{14}\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^2}{2}+\frac{1}{2}a^2bx^6+\frac{3}{10}ab^2x^{10}+\frac{b^3x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^4)^3,x]
```

```
[Out] (a^3*x^2)/2 + (a^2*b*x^6)/2 + (3*a*b^2*x^10)/10 + (b^3*x^14)/14
```

fricas [A] time = 0.45, size = 35, normalized size = 0.81

$$\frac{1}{14}x^{14}b^3+\frac{3}{10}x^{10}b^2a+\frac{1}{2}x^6ba^2+\frac{1}{2}x^2a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*b^3 + 3/10*x^10*b^2*a + 1/2*x^6*b*a^2 + 1/2*x^2*a^3
```

giac [A] time = 0.16, size = 35, normalized size = 0.81

$$\frac{1}{14}\,b^3x^{14}+\frac{3}{10}\,ab^2x^{10}+\frac{1}{2}\,a^2bx^6+\frac{1}{2}\,a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^4+a)^3,x, algorithm="giac")
[Out] 1/14*b^3*x^14 + 3/10*a*b^2*x^10 + 1/2*a^2*b*x^6 + 1/2*a^3*x^2
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$\frac{1}{14}b^3x^{14} + \frac{3}{10}a\,b^2x^{10} + \frac{1}{2}a^2b\,x^6 + \frac{1}{2}a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^4+a)^3,x)
[Out] 1/2*a^3*x^2+1/2*a^2*b*x^6+3/10*a*b^2*x^10+1/14*b^3*x^14
```

maxima [A] time = 1.34, size = 35, normalized size = 0.81

$$\frac{1}{14}\,b^3x^{14} + \frac{3}{10}\,ab^2x^{10} + \frac{1}{2}\,a^2bx^6 + \frac{1}{2}\,a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^4+a)^3,x, algorithm="maxima")
[Out] 1/14*b^3*x^14 + 3/10*a*b^2*x^10 + 1/2*a^2*b*x^6 + 1/2*a^3*x^2
```

mupad [B] time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3\,x^2}{2} + \frac{a^2\,b\,x^6}{2} + \frac{3\,a\,b^2\,x^{10}}{10} + \frac{b^3\,x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^4)^3,x)
[Out] (a^3*x^2)/2 + (b^3*x^14)/14 + (a^2*b*x^6)/2 + (3*a*b^2*x^10)/10
```

sympy [A] time = 0.09, size = 37, normalized size = 0.86

$$\frac{a^3x^2}{2} + \frac{a^2bx^6}{2} + \frac{3ab^2x^{10}}{10} + \frac{b^3x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**4+a)**3,x)
[Out] a**3*x**2/2 + a**2*b*x**6/2 + 3*a*b**2*x**10/10 + b**3*x**14/14
```

3.638 $\int \left(a + bx^4\right)^3 \, dx$

Optimal. Leaf size=38

$$a^3x + \frac{3}{5}a^2bx^5 + \frac{1}{3}ab^2x^9 + \frac{b^3x^{13}}{13}$$

```
[Out] a^3*x+3/5*a^2*b*x^5+1/3*a*b^2*x^9+1/13*b^3*x^13
```

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {194}

$$\frac{3}{5}a^2bx^5 + a^3x + \frac{1}{3}ab^2x^9 + \frac{b^3x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^3,x]
```

```
[Out] a^3*x + (3*a^2*b*x^5)/5 + (a*b^2*x^9)/3 + (b^3*x^13)/13
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a + bx^4\right)^3 \, dx &= \int \left(a^3 + 3a^2bx^4 + 3ab^2x^8 + b^3x^{12}\right) \, dx \\ &= a^3x + \frac{3}{5}a^2bx^5 + \frac{1}{3}ab^2x^9 + \frac{b^3x^{13}}{13}\end{aligned}$$

Mathematica [A] time = 0.00, size = 38, normalized size = 1.00

$$a^3x + \frac{3}{5}a^2bx^5 + \frac{1}{3}ab^2x^9 + \frac{b^3x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^3,x]
```

```
[Out] a^3*x + (3*a^2*b*x^5)/5 + (a*b^2*x^9)/3 + (b^3*x^13)/13
```

fricas [A] time = 0.39, size = 32, normalized size = 0.84

$$\frac{1}{13}x^{13}b^3 + \frac{1}{3}x^9b^2a + \frac{3}{5}x^5ba^2 + xa^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*b^3 + 1/3*x^9*b^2*a + 3/5*x^5*b*a^2 + x*a^3
```

giac [A] time = 0.17, size = 32, normalized size = 0.84

$$\frac{1}{13}\,b^3x^{13} + \frac{1}{3}\,ab^2x^9 + \frac{3}{5}\,a^2bx^5 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^3,x, algorithm="giac")
[Out] 1/13*b^3*x^13 + 1/3*a*b^2*x^9 + 3/5*a^2*b*x^5 + a^3*x
```

maple [A] time = 0.00, size = 33, normalized size = 0.87

$$\frac{1}{13}b^3x^{13} + \frac{1}{3}a\,b^2x^9 + \frac{3}{5}a^2b\,x^5 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^3,x)
[Out] a^3*x+3/5*a^2*b*x^5+1/3*a*b^2*x^9+1/13*b^3*x^13
```

maxima [A] time = 1.35, size = 32, normalized size = 0.84

$$\frac{1}{13}\,b^3x^{13} + \frac{1}{3}\,ab^2x^9 + \frac{3}{5}\,a^2bx^5 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^3,x, algorithm="maxima")
[Out] 1/13*b^3*x^13 + 1/3*a*b^2*x^9 + 3/5*a^2*b*x^5 + a^3*x
```

mupad [B] time = 0.04, size = 32, normalized size = 0.84

$$a^3\,x + \frac{3\,a^2\,b\,x^5}{5} + \frac{a\,b^2\,x^9}{3} + \frac{b^3\,x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^3,x)
[Out] a^3*x + (b^3*x^13)/13 + (3*a^2*b*x^5)/5 + (a*b^2*x^9)/3
```

sympy [A] time = 0.08, size = 34, normalized size = 0.89

$$a^3x + \frac{3a^2bx^5}{5} + \frac{ab^2x^9}{3} + \frac{b^3x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**3,x)
[Out] a**3*x + 3*a**2*b*x**5/5 + a*b**2*x**9/3 + b**3*x**13/13
```

3.639 $\int \frac{\left(a+bx^4\right)^3}{x}\,dx$

Optimal. Leaf size=39

$$a^3\log(x)+\frac{3}{4}a^2bx^4+\frac{3}{8}ab^2x^8+\frac{b^3x^{12}}{12}$$

```
[Out] 3/4*a^2*b*x^4+3/8*a*b^2*x^8+1/12*b^3*x^12+a^3*ln(x)
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{3}{4}a^2bx^4+a^3\log(x)+\frac{3}{8}ab^2x^8+\frac{b^3x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^3/x,x]
```

```
[Out] (3*a^2*b*x^4)/4 + (3*a*b^2*x^8)/8 + (b^3*x^12)/12 + a^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^4\right)^3}{x}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{(a+bx)^3}{x}\,dx, x, x^4\right)\\ &= \frac{1}{4}\operatorname{Subst}\left(\int \left(3a^2b+\frac{a^3}{x}+3ab^2x+b^3x^2\right)\,dx, x, x^4\right)\\ &= \frac{3}{4}a^2bx^4+\frac{3}{8}ab^2x^8+\frac{b^3x^{12}}{12}+a^3\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 39, normalized size = 1.00

$$a^3\log(x)+\frac{3}{4}a^2bx^4+\frac{3}{8}ab^2x^8+\frac{b^3x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^3/x,x]
```

```
[Out] (3*a^2*b*x^4)/4 + (3*a*b^2*x^8)/8 + (b^3*x^12)/12 + a^3*Log[x]
```

fricas [A] time = 0.63, size = 33, normalized size = 0.85

$$\frac{1}{12}b^3x^{12} + \frac{3}{8}ab^2x^8 + \frac{3}{4}a^2bx^4 + a^3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^3/x,x, algorithm="fricas")
[Out] 1/12*b^3*x^12 + 3/8*a*b^2*x^8 + 3/4*a^2*b*x^4 + a^3*log(x)
```

giac [A] time = 0.15, size = 36, normalized size = 0.92

$$\frac{1}{12}b^3x^{12} + \frac{3}{8}ab^2x^8 + \frac{3}{4}a^2bx^4 + \frac{1}{4}a^3\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^3/x,x, algorithm="giac")
[Out] 1/12*b^3*x^12 + 3/8*a*b^2*x^8 + 3/4*a^2*b*x^4 + 1/4*a^3*log(x^4)
```

maple [A] time = 0.00, size = 34, normalized size = 0.87

$$\frac{b^3x^{12}}{12} + \frac{3a\,b^2x^8}{8} + \frac{3a^2b\,x^4}{4} + a^3\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^3/x,x)
[Out] 3/4*a^2*b*x^4+3/8*a*b^2*x^8+1/12*b^3*x^12+a^3*ln(x)
```

maxima [A] time = 1.34, size = 36, normalized size = 0.92

$$\frac{1}{12}b^3x^{12} + \frac{3}{8}ab^2x^8 + \frac{3}{4}a^2bx^4 + \frac{1}{4}a^3\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^3/x,x, algorithm="maxima")
[Out] 1/12*b^3*x^12 + 3/8*a*b^2*x^8 + 3/4*a^2*b*x^4 + 1/4*a^3*log(x^4)
```

mupad [B] time = 0.04, size = 33, normalized size = 0.85

$$a^3\,\ln(x) + \frac{b^3\,x^{12}}{12} + \frac{3\,a^2\,b\,x^4}{4} + \frac{3\,a\,b^2\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^3/x,x)
[Out] a^3*log(x) + (b^3*x^12)/12 + (3*a^2*b*x^4)/4 + (3*a*b^2*x^8)/8
```

sympy [A] time = 0.14, size = 37, normalized size = 0.95

$$a^3\log(x) + \frac{3a^2bx^4}{4} + \frac{3ab^2x^8}{8} + \frac{b^3x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**3/x,x)
[Out] a**3*log(x) + 3*a**2*b*x**4/4 + 3*a*b**2*x**8/8 + b**3*x**12/12
```

3.640 $\int \frac{\left(a+bx^4\right)^3}{x^2}\,dx$

Optimal. Leaf size=38

$$-\frac{a^3}{x}+a^2bx^3+\frac{3}{7}ab^2x^7+\frac{b^3x^{11}}{11}$$

```
[Out] -a^3/x+a^2*b*x^3+3/7*a*b^2*x^7+1/11*b^3*x^11
```

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$a^2bx^3-\frac{a^3}{x}+\frac{3}{7}ab^2x^7+\frac{b^3x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^3/x^2,x]
```

```
[Out] -(a^3/x) + a^2*b*x^3 + (3*a*b^2*x^7)/7 + (b^3*x^11)/11
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^4\right)^3}{x^2}\,dx &= \int \left(\frac{a^3}{x^2}+3a^2bx^2+3ab^2x^6+b^3x^{10}\right)dx\\ &= -\frac{a^3}{x}+a^2bx^3+\frac{3}{7}ab^2x^7+\frac{b^3x^{11}}{11}\end{aligned}$$

Mathematica [A] time = 0.00, size = 38, normalized size = 1.00

$$-\frac{a^3}{x}+a^2bx^3+\frac{3}{7}ab^2x^7+\frac{b^3x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^3/x^2,x]
```

```
[Out] -(a^3/x) + a^2*b*x^3 + (3*a*b^2*x^7)/7 + (b^3*x^11)/11
```

fricas [A] time = 0.57, size = 37, normalized size = 0.97

$$\frac{7\,b^3x^{12}+33\,ab^2x^8+77\,a^2bx^4-77\,a^3}{77\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/77*(7*b^3*x^12 + 33*a*b^2*x^8 + 77*a^2*b*x^4 - 77*a^3)/x
```

giac [A] time = 0.23, size = 34, normalized size = 0.89

$$\frac{1}{11}\,b^3x^{11}+\frac{3}{7}\,ab^2x^7+a^2bx^3-\frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^3/x^2,x, algorithm="giac")
[Out] 1/11*b^3*x^11 + 3/7*a*b^2*x^7 + a^2*b*x^3 - a^3/x
```

maple [A] time = 0.00, size = 35, normalized size = 0.92

$$\frac{b^3x^{11}}{11} + \frac{3a\,b^2x^7}{7} + a^2b\,x^3 - \frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^3/x^2,x)
[Out] -a^3/x+a^2*b*x^3+3/7*a*b^2*x^7+1/11*b^3*x^11
```

maxima [A] time = 1.27, size = 34, normalized size = 0.89

$$\frac{1}{11}\,b^3x^{11} + \frac{3}{7}\,ab^2x^7 + a^2bx^3 - \frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^3/x^2,x, algorithm="maxima")
[Out] 1/11*b^3*x^11 + 3/7*a*b^2*x^7 + a^2*b*x^3 - a^3/x
```

mupad [B] time = 0.04, size = 34, normalized size = 0.89

$$\frac{b^3\,x^{11}}{11} - \frac{a^3}{x} + a^2\,b\,x^3 + \frac{3\,a\,b^2\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^3/x^2,x)
[Out] (b^3*x^11)/11 - a^3/x + a^2*b*x^3 + (3*a*b^2*x^7)/7
```

sympy [A] time = 0.15, size = 32, normalized size = 0.84

$$-\frac{a^3}{x} + a^2bx^3 + \frac{3ab^2x^7}{7} + \frac{b^3x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**3/x**2,x)
[Out] -a**3/x + a**2*b*x**3 + 3*a*b**2*x**7/7 + b**3*x**11/11
```

3.641 $\int \frac{\left(a+bx^4\right)^3}{x^3}\,dx$

Optimal. Leaf size=43

$$-\frac{a^3}{2x^2}+\frac{3}{2}a^2bx^2+\frac{1}{2}ab^2x^6+\frac{b^3x^{10}}{10}$$

```
[Out] -1/2*a^3/x^2+3/2*a^2*b*x^2+1/2*a*b^2*x^6+1/10*b^3*x^10
```

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{3}{2}a^2bx^2-\frac{a^3}{2x^2}+\frac{1}{2}ab^2x^6+\frac{b^3x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^3/x^3,x]
```

```
[Out] -a^3/(2*x^2) + (3*a^2*b*x^2)/2 + (a*b^2*x^6)/2 + (b^3*x^10)/10
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^4\right)^3}{x^3}\,dx &= \int \left(\frac{a^3}{x^3}+3a^2bx+3ab^2x^5+b^3x^9\right)dx\\ &= -\frac{a^3}{2x^2}+\frac{3}{2}a^2bx^2+\frac{1}{2}ab^2x^6+\frac{b^3x^{10}}{10}\end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 1.00

$$-\frac{a^3}{2x^2}+\frac{3}{2}a^2bx^2+\frac{1}{2}ab^2x^6+\frac{b^3x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^3/x^3,x]
```

```
[Out] -1/2*a^3/x^2 + (3*a^2*b*x^2)/2 + (a*b^2*x^6)/2 + (b^3*x^10)/10
```

fricas [A] time = 0.68, size = 36, normalized size = 0.84

$$\frac{b^3x^{12}+5\,ab^2x^8+15\,a^2bx^4-5\,a^3}{10\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/10*(b^3*x^12 + 5*a*b^2*x^8 + 15*a^2*b*x^4 - 5*a^3)/x^2
```

giac [A] time = 0.15, size = 35, normalized size = 0.81

$$\frac{1}{10}\,b^3x^{10}+\frac{1}{2}\,ab^2x^6+\frac{3}{2}\,a^2bx^2-\frac{a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^3/x^3,x, algorithm="giac")
[Out] 1/10*b^3*x^10 + 1/2*a*b^2*x^6 + 3/2*a^2*b*x^2 - 1/2*a^3/x^2
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$\frac{b^3x^{10}}{10} + \frac{a\,b^2x^6}{2} + \frac{3a^2b\,x^2}{2} - \frac{a^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^3/x^3,x)
[Out] -1/2*a^3/x^2+3/2*a^2*b*x^2+1/2*a*b^2*x^6+1/10*b^3*x^10
```

maxima [A] time = 1.31, size = 35, normalized size = 0.81

$$\frac{1}{10}\,b^3x^{10} + \frac{1}{2}\,ab^2x^6 + \frac{3}{2}\,a^2bx^2 - \frac{a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^3/x^3,x, algorithm="maxima")
[Out] 1/10*b^3*x^10 + 1/2*a*b^2*x^6 + 3/2*a^2*b*x^2 - 1/2*a^3/x^2
```

mupad [B] time = 0.04, size = 35, normalized size = 0.81

$$\frac{b^3\,x^{10}}{10} - \frac{a^3}{2\,x^2} + \frac{3\,a^2\,b\,x^2}{2} + \frac{a\,b^2\,x^6}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^3/x^3,x)
[Out] (b^3*x^10)/10 - a^3/(2*x^2) + (3*a^2*b*x^2)/2 + (a*b^2*x^6)/2
```

sympy [A] time = 0.31, size = 37, normalized size = 0.86

$$-\frac{a^3}{2x^2} + \frac{3a^2bx^2}{2} + \frac{ab^2x^6}{2} + \frac{b^3x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**3/x**3,x)
[Out] -a**3/(2*x**2) + 3*a**2*b*x**2/2 + a*b**2*x**6/2 + b**3*x**10/10
```

3.642 $\int \frac{\left(a+bx^4\right)^3}{x^4}\,dx$

Optimal. Leaf size=39

$$-\frac{a^3}{3x^3}+3a^2bx+\frac{3}{5}ab^2x^5+\frac{b^3x^9}{9}$$

```
[Out] -1/3*a^3/x^3+3*a^2*b*x+3/5*a*b^2*x^5+1/9*b^3*x^9
```

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$3a^2bx-\frac{a^3}{3x^3}+\frac{3}{5}ab^2x^5+\frac{b^3x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^3/x^4,x]
```

```
[Out] -a^3/(3*x^3) + 3*a^2*b*x + (3*a*b^2*x^5)/5 + (b^3*x^9)/9
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^4\right)^3}{x^4}\,dx &= \int \left(3a^2b+\frac{a^3}{x^4}+3ab^2x^4+b^3x^8\right)dx\\ &= -\frac{a^3}{3x^3}+3a^2bx+\frac{3}{5}ab^2x^5+\frac{b^3x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.00, size = 39, normalized size = 1.00

$$-\frac{a^3}{3x^3}+3a^2bx+\frac{3}{5}ab^2x^5+\frac{b^3x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^3/x^4,x]
```

```
[Out] -1/3*a^3/x^3 + 3*a^2*b*x + (3*a*b^2*x^5)/5 + (b^3*x^9)/9
```

fricas [A] time = 0.70, size = 37, normalized size = 0.95

$$\frac{5\,b^3x^{12}+27\,ab^2x^8+135\,a^2bx^4-15\,a^3}{45\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/45*(5*b^3*x^12 + 27*a*b^2*x^8 + 135*a^2*b*x^4 - 15*a^3)/x^3
```

giac [A] time = 0.15, size = 33, normalized size = 0.85

$$\frac{1}{9}\,b^3x^9+\frac{3}{5}\,ab^2x^5+3\,a^2bx-\frac{a^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^3/x^4,x, algorithm="giac")
[Out] 1/9*b^3*x^9 + 3/5*a*b^2*x^5 + 3*a^2*b*x - 1/3*a^3/x^3
```

maple [A] time = 0.00, size = 34, normalized size = 0.87

$$\frac{b^3x^9}{9} + \frac{3a\,b^2x^5}{5} + 3a^2bx - \frac{a^3}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^3/x^4,x)
[Out] -1/3*a^3/x^3+3*a^2*b*x+3/5*a*b^2*x^5+1/9*b^3*x^9
```

maxima [A] time = 1.34, size = 33, normalized size = 0.85

$$\frac{1}{9}\,b^3x^9 + \frac{3}{5}\,ab^2x^5 + 3\,a^2bx - \frac{a^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^3/x^4,x, algorithm="maxima")
[Out] 1/9*b^3*x^9 + 3/5*a*b^2*x^5 + 3*a^2*b*x - 1/3*a^3/x^3
```

mupad [B] time = 0.04, size = 33, normalized size = 0.85

$$\frac{b^3\,x^9}{9} - \frac{a^3}{3\,x^3} + \frac{3\,a\,b^2\,x^5}{5} + 3\,a^2\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^3/x^4,x)
[Out] (b^3*x^9)/9 - a^3/(3*x^3) + (3*a*b^2*x^5)/5 + 3*a^2*b*x
```

sympy [A] time = 0.30, size = 36, normalized size = 0.92

$$-\frac{a^3}{3x^3} + 3a^2bx + \frac{3ab^2x^5}{5} + \frac{b^3x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**3/x**4,x)
[Out] -a**3/(3*x**3) + 3*a**2*b*x + 3*a*b**2*x**5/5 + b**3*x**9/9
```

3.643 $\int \frac{\left(a+bx^4\right)^3}{x^5}\,dx$

Optimal. Leaf size=40

$$-\frac{a^3}{4x^4}+3a^2b\log(x)+\frac{3}{4}ab^2x^4+\frac{b^3x^8}{8}$$

```
[Out] -1/4*a^3/x^4+3/4*a*b^2*x^4+1/8*b^3*x^8+3*a^2*b*ln(x)
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$3a^2b\log(x)-\frac{a^3}{4x^4}+\frac{3}{4}ab^2x^4+\frac{b^3x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^3/x^5,x]
```

```
[Out] -a^3/(4*x^4) + (3*a*b^2*x^4)/4 + (b^3*x^8)/8 + 3*a^2*b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^4\right)^3}{x^5}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{(a+bx)^3}{x^2}\,dx, x, x^4\right)\\ &= \frac{1}{4}\operatorname{Subst}\left(\int \left(3ab^2+\frac{a^3}{x^2}+\frac{3a^2b}{x}+b^3x\right)\,dx, x, x^4\right)\\ &= -\frac{a^3}{4x^4}+\frac{3}{4}ab^2x^4+\frac{b^3x^8}{8}+3a^2b\log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.00

$$-\frac{a^3}{4x^4}+3a^2b\log(x)+\frac{3}{4}ab^2x^4+\frac{b^3x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^3/x^5,x]
```

```
[Out] -1/4*a^3/x^4 + (3*a*b^2*x^4)/4 + (b^3*x^8)/8 + 3*a^2*b*Log[x]
```

fricas [A] time = 0.59, size = 38, normalized size = 0.95

$$\frac{b^3x^{12} + 6\,ab^2x^8 + 24\,a^2bx^4\log(x) - 2\,a^3}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^3/x^5,x, algorithm="fricas")
```

```
[Out] 1/8*(b^3*x^12 + 6*a*b^2*x^8 + 24*a^2*b*x^4*log(x) - 2*a^3)/x^4
```

giac [A] time = 0.15, size = 46, normalized size = 1.15

$$\frac{1}{8}\,b^3x^8 + \frac{3}{4}\,ab^2x^4 + \frac{3}{4}\,a^2b\log\left(x^4\right) - \frac{3\,a^2bx^4 + a^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^3/x^5,x, algorithm="giac")
```

```
[Out] 1/8*b^3*x^8 + 3/4*a*b^2*x^4 + 3/4*a^2*b*log(x^4) - 1/4*(3*a^2*b*x^4 + a^3)/
x^4
```

maple [A] time = 0.01, size = 35, normalized size = 0.88

$$\frac{b^3x^8}{8} + \frac{3a\,b^2x^4}{4} + 3a^2b\ln(x) - \frac{a^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^3/x^5,x)
```

```
[Out] -1/4*a^3/x^4+3/4*a*b^2*x^4+1/8*b^3*x^8+3*a^2*b*ln(x)
```

maxima [A] time = 1.31, size = 36, normalized size = 0.90

$$\frac{1}{8}\,b^3x^8 + \frac{3}{4}\,ab^2x^4 + \frac{3}{4}\,a^2b\log\left(x^4\right) - \frac{a^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^3/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*b^3*x^8 + 3/4*a*b^2*x^4 + 3/4*a^2*b*log(x^4) - 1/4*a^3/x^4
```

mupad [B] time = 0.03, size = 34, normalized size = 0.85

$$\frac{b^3\,x^8}{8} - \frac{a^3}{4\,x^4} + \frac{3\,a\,b^2\,x^4}{4} + 3\,a^2\,b\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^3/x^5,x)
```

```
[Out] (b^3*x^8)/8 - a^3/(4*x^4) + (3*a*b^2*x^4)/4 + 3*a^2*b*log(x)
```

sympy [A] time = 0.32, size = 37, normalized size = 0.92

$$-\frac{a^3}{4x^4} + 3a^2b\log(x) + \frac{3ab^2x^4}{4} + \frac{b^3x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**3/x**5,x)
```

```
[Out] -a**3/(4*x**4) + 3*a**2*b*log(x) + 3*a*b**2*x**4/4 + b**3*x**8/8
```

3.644 $\int \frac{x^9}{a+cx^4}\,dx$

Optimal. Leaf size=51

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2c^{5/2}}-\frac{ax^2}{2c^2}+\frac{x^6}{6c}$$

```
[Out] -1/2*a*x^2/c^2+1/6*x^6/c+1/2*a^(3/2)*arctan(x^2*c^(1/2)/a^(1/2))/c^(5/2)
```

Rubi [A] time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {275, 302, 205}

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2c^{5/2}}-\frac{ax^2}{2c^2}+\frac{x^6}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + c*x^4),x]
```

```
[Out] -(a*x^2)/(2*c^2) + x^6/(6*c) + (a^(3/2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*c
^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{a+cx^4}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{a+cx^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{a}{c^2}+\frac{x^2}{c}+\frac{a^2}{c^2\left(a+cx^2\right)}\right)dx, x, x^2\right)\\ &= -\frac{ax^2}{2c^2}+\frac{x^6}{6c}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{2c^2}\\ &= -\frac{ax^2}{2c^2}+\frac{x^6}{6c}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2c^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 48, normalized size = 0.94

$$\frac{1}{6}\left(\frac{3a^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{c^{5/2}}+\frac{cx^6-3ax^2}{c^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + c*x^4),x]
```

```
[Out] ((-3*a*x^2 + c*x^6)/c^2 + (3*a^(3/2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/c^(5/2)
)/6
```

fricas [A] time = 0.69, size = 107, normalized size = 2.10

$$\left[\frac{2\,cx^6 - 6\,ax^2 + 3\,a\sqrt{-\frac{a}{c}}\,\log\left(\frac{cx^4+2\,cx^2\sqrt{-\frac{a}{c}}-a}{cx^4+a}\right)}{12\,c^2},\ \frac{cx^6 - 3\,ax^2 + 3\,a\sqrt{\frac{a}{c}}\,\arctan\left(\frac{cx^2\sqrt{\frac{a}{c}}}{a}\right)}{6\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/12*(2*c*x^6 - 6*a*x^2 + 3*a*sqrt(-a/c)*log((c*x^4 + 2*c*x^2*sqrt(-a/c) -
 a)/(c*x^4 + a)))/c^2, 1/6*(c*x^6 - 3*a*x^2 + 3*a*sqrt(a/c)*arctan(c*x^2*sq
rt(a/c)/a))/c^2]
```

giac [A] time = 0.16, size = 45, normalized size = 0.88

$$\frac{a^2\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\sqrt{ac}\,c^2} + \frac{c^2x^6 - 3\,acx^2}{6\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*a^2*arctan(c*x^2/sqrt(a*c))/(sqrt(a*c)*c^2) + 1/6*(c^2*x^6 - 3*a*c*x^2)
/c^3
```

maple [A] time = 0.00, size = 43, normalized size = 0.84

$$\frac{x^6}{6c} + \frac{a^2\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{2\sqrt{ac}\;c^2} - \frac{a\,x^2}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(c*x^4+a),x)
```

```
[Out] 1/6*x^6/c-1/2*a*x^2/c^2+1/2*a^2/c^2/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2))
```

maxima [A] time = 2.94, size = 42, normalized size = 0.82

$$\frac{a^2\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\sqrt{ac}\,c^2} + \frac{cx^6 - 3\,ax^2}{6\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*arctan(c*x^2/sqrt(a*c))/(sqrt(a*c)*c^2) + 1/6*(c*x^6 - 3*a*x^2)/c^2
```

mupad [B] time = 0.09, size = 37, normalized size = 0.73

$$\frac{x^6}{6\,c} - \frac{a\,x^2}{2\,c^2} + \frac{a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\;x^2}{\sqrt{a}}\right)}{2\,c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + c*x^4),x)
```

```
[Out] x^6/(6*c) - (a*x^2)/(2*c^2) + (a^(3/2)*atan((c^(1/2)*x^2)/a^(1/2)))/(2*c^(5
/2))
```

sympy [B] time = 0.28, size = 87, normalized size = 1.71

$$-\frac{ax^2}{2c^2} - \frac{\sqrt{-\frac{a^3}{c^5}}\log\left(x^2 - \frac{c^2\sqrt{-\frac{a^3}{c^5}}}{a}\right)}{4} + \frac{\sqrt{-\frac{a^3}{c^5}}\log\left(x^2 + \frac{c^2\sqrt{-\frac{a^3}{c^5}}}{a}\right)}{4} + \frac{x^6}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(c*x**4+a),x)
```

```
[Out] -a*x**2/(2*c**2) - sqrt(-a**3/c**5)*log(x**2 - c**2*sqrt(-a**3/c**5)/a)/4 +
 sqrt(-a**3/c**5)*log(x**2 + c**2*sqrt(-a**3/c**5)/a)/4 + x**6/(6*c)
```

3.645 $\int \frac{x^7}{a+cx^4}\,dx$

Optimal. Leaf size=27

$$\frac{x^4}{4c} - \frac{a\log\left(a+cx^4\right)}{4c^2}$$

```
[Out] 1/4*x^4/c-1/4*a*ln(c*x^4+a)/c^2
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{x^4}{4c} - \frac{a\log\left(a+cx^4\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + c*x^4),x]
```

```
[Out] x^4/(4*c) - (a*Log[a + c*x^4])/(4*c^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{a+cx^4}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x}{a+cx}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int \left(\frac{1}{c} - \frac{a}{c(a+cx)}\right)dx, x, x^4\right)\\ &= \frac{x^4}{4c} - \frac{a\log\left(a+cx^4\right)}{4c^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{x^4}{4c} - \frac{a\log\left(a+cx^4\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + c*x^4),x]
```

```
[Out] x^4/(4*c) - (a*Log[a + c*x^4])/(4*c^2)
```

fricas [A] time = 0.50, size = 22, normalized size = 0.81

$$\frac{cx^4 - a\log\left(cx^4+a\right)}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+a),x, algorithm="fricas")
[Out] 1/4*(c*x^4 - a*log(c*x^4 + a))/c^2
```

giac [A] time = 0.16, size = 24, normalized size = 0.89

$$\frac{x^4}{4\,c} - \frac{a\log\left(\left|cx^4+a\right|\right)}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+a),x, algorithm="giac")
[Out] 1/4*x^4/c - 1/4*a*log(abs(c*x^4 + a))/c^2
```

maple [A] time = 0.00, size = 24, normalized size = 0.89

$$\frac{x^4}{4c} - \frac{a\ln\left(c\,x^4+a\right)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+a),x)
[Out] 1/4*x^4/c-1/4*a*ln(c*x^4+a)/c^2
```

maxima [A] time = 1.27, size = 23, normalized size = 0.85

$$\frac{x^4}{4\,c} - \frac{a\log\left(cx^4+a\right)}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+a),x, algorithm="maxima")
[Out] 1/4*x^4/c - 1/4*a*log(c*x^4 + a)/c^2
```

mupad [B] time = 0.04, size = 22, normalized size = 0.81

$$-\frac{a\,\ln\left(c\,x^4+a\right)-c\,x^4}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + c*x^4),x)
[Out] -(a*log(a + c*x^4) - c*x^4)/(4*c^2)
```

sympy [A] time = 0.24, size = 20, normalized size = 0.74

$$-\frac{a\log\left(a+cx^4\right)}{4c^2} + \frac{x^4}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+a),x)
[Out] -a*log(a + c*x**4)/(4*c**2) + x**4/(4*c)
```

3.646 $\int \frac{x^5}{a+cx^4}\,dx$

Optimal. Leaf size=40

$$\frac{x^2}{2c} - \frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2c^{3/2}}$$

```
[Out] 1/2*x^2/c-1/2*arctan(x^2*c^(1/2)/a^(1/2))*a^(1/2)/c^(3/2)
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {275, 321, 205}

$$\frac{x^2}{2c} - \frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + c*x^4),x]
```

```
[Out] x^2/(2*c) - (Sqrt[a]*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*c^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{a+cx^4}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{a+cx^2}\,dx, x, x^2\right)\\ &= \frac{x^2}{2c} - \frac{a\,\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{2c}\\ &= \frac{x^2}{2c} - \frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2c^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.00

$$\frac{x^2}{2c} - \frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + c*x^4),x]
```

```
[Out] x^2/(2*c) - (Sqrt[a]*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*c^(3/2))
```

fricas [A] time = 0.63, size = 89, normalized size = 2.22

$$\left[\frac{2\,x^2+\sqrt{-\frac{a}{c}}\,\log\left(\frac{cx^4-2\,cx^2\sqrt{-\frac{a}{c}}-a}{cx^4+a}\right)}{4\,c},\ \frac{x^2-\sqrt{\frac{a}{c}}\,\arctan\left(\frac{cx^2\sqrt{\frac{a}{c}}}{a}\right)}{2\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*x^2 + sqrt(-a/c)*log((c*x^4 - 2*c*x^2*sqrt(-a/c) - a)/(c*x^4 + a)))
/c, 1/2*(x^2 - sqrt(a/c)*arctan(c*x^2*sqrt(a/c)/a))/c]
```

giac [A] time = 0.15, size = 31, normalized size = 0.78

$$\frac{x^2}{2\,c}-\frac{a\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\,\sqrt{ac}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*x^2/c - 1/2*a*arctan(c*x^2/sqrt(a*c))/(sqrt(a*c)*c)
```

maple [A] time = 0.00, size = 32, normalized size = 0.80

$$-\frac{a\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{2\sqrt{ac}\ c}+\frac{x^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+a),x)
```

```
[Out] 1/2*x^2/c-1/2*a/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x^2)
```

maxima [A] time = 2.97, size = 31, normalized size = 0.78

$$\frac{x^2}{2\,c}-\frac{a\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\,\sqrt{ac}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/c - 1/2*a*arctan(c*x^2/sqrt(a*c))/(sqrt(a*c)*c)
```

mupad [B] time = 0.04, size = 28, normalized size = 0.70

$$\frac{x^2}{2\,c}-\frac{\sqrt{a}\ \mathrm{atan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + c*x^4),x)
```

```
[Out] x^2/(2*c) - (a^(1/2)*atan((c^(1/2)*x^2)/a^(1/2)))/(2*c^(3/2))
```

sympy [A] time = 0.32, size = 63, normalized size = 1.58

$$\frac{\sqrt{-\frac{a}{c^3}}\,\log\left(-c\sqrt{-\frac{a}{c^3}}+x^2\right)}{4}-\frac{\sqrt{-\frac{a}{c^3}}\,\log\left(c\sqrt{-\frac{a}{c^3}}+x^2\right)}{4}+\frac{x^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+a),x)
```

```
[Out] sqrt(-a/c**3)*log(-c*sqrt(-a/c**3) + x**2)/4 - sqrt(-a/c**3)*log(c*sqrt(-a/
c**3) + x**2)/4 + x**2/(2*c)
```

3.647 $\int \frac{x^3}{a+cx^4}\,dx$

Optimal. Leaf size=15

$$\frac{\log\left(a+cx^4\right)}{4c}$$

```
[Out] 1/4*ln(c*x^4+a)/c
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {260}

$$\frac{\log\left(a+cx^4\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + c*x^4),x]
```

```
[Out] Log[a + c*x^4]/(4*c)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^3}{a+cx^4}\,dx = \frac{\log\left(a+cx^4\right)}{4c}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log\left(a+cx^4\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + c*x^4),x]
```

```
[Out] Log[a + c*x^4]/(4*c)
```

fricas [A] time = 0.49, size = 13, normalized size = 0.87

$$\frac{\log\left(cx^4+a\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*log(c*x^4 + a)/c
```

giac [A] time = 0.17, size = 14, normalized size = 0.93

$$\frac{\log\left(\left|cx^4+a\right|\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+a),x, algorithm="giac")
[Out] 1/4*log(abs(c*x^4 + a))/c
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{\ln\left(c\,x^4+a\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+a),x)
[Out] 1/4*ln(c*x^4+a)/c
```

maxima [A] time = 1.36, size = 13, normalized size = 0.87

$$\frac{\log\left(cx^4+a\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+a),x, algorithm="maxima")
[Out] 1/4*log(c*x^4 + a)/c
```

mupad [B] time = 0.03, size = 13, normalized size = 0.87

$$\frac{\ln\left(c\,x^4+a\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + c*x^4),x)
[Out] log(a + c*x^4)/(4*c)
```

sympy [A] time = 0.19, size = 10, normalized size = 0.67

$$\frac{\log\left(a+cx^4\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+a),x)
[Out] log(a + c*x**4)/(4*c)
```

3.648 $\int \frac{x}{a+cx^4}\,dx$

Optimal. Leaf size=29

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{c}}$$

```
[Out] 1/2*arctan(x^2*c^(1/2)/a^(1/2))/a^(1/2)/c^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {275, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + c*x^4),x]
```

```
[Out] ArcTan[(Sqrt[c]*x^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[c])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{a+cx^4}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + c*x^4),x]
```

```
[Out] ArcTan[(Sqrt[c]*x^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[c])
```

fricas [A] time = 0.54, size = 72, normalized size = 2.48

$$\left[-\frac{\sqrt{-ac}\,\log\left(\frac{cx^4-2\sqrt{-ac}\,x^2-a}{cx^4+a}\right)}{4\,ac}, -\frac{\sqrt{ac}\,\arctan\left(\frac{\sqrt{ac}}{cx^2}\right)}{2\,ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-a*c)*log((c*x^4 - 2*sqrt(-a*c)*x^2 - a)/(c*x^4 + a))/(a*c), -1/
2*sqrt(a*c)*arctan(sqrt(a*c)/(c*x^2))/(a*c)]
```

giac [A] time = 0.18, size = 18, normalized size = 0.62

$$\frac{\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*arctan(c*x^2/sqrt(a*c))/sqrt(a*c)
```

maple [A] time = 0.00, size = 19, normalized size = 0.66

$$\frac{\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{2\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+a),x)
```

```
[Out] 1/2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x^2)
```

maxima [A] time = 2.94, size = 18, normalized size = 0.62

$$\frac{\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(c*x^2/sqrt(a*c))/sqrt(a*c)
```

mupad [B] time = 0.05, size = 19, normalized size = 0.66

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + c*x^4),x)
```

```
[Out] atan((c^(1/2)*x^2)/a^(1/2))/(2*a^(1/2)*c^(1/2))
```

sympy [B] time = 0.22, size = 56, normalized size = 1.93

$$-\frac{\sqrt{-\frac{1}{ac}}\log\left(-a\sqrt{-\frac{1}{ac}}+x^2\right)}{4}+\frac{\sqrt{-\frac{1}{ac}}\log\left(a\sqrt{-\frac{1}{ac}}+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+a),x)
```

```
[Out] -sqrt(-1/(a*c))*log(-a*sqrt(-1/(a*c)) + x**2)/4 + sqrt(-1/(a*c))*log(a*sqrt
(-1/(a*c)) + x**2)/4
```

3.649 $\int \frac{1}{x\left(a+cx^4\right)}\,dx$

Optimal. Leaf size=22

$$\frac{\log(x)}{a}-\frac{\log\left(a+cx^4\right)}{4a}$$

```
[Out] ln(x)/a-1/4*ln(c*x^4+a)/a
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {266, 36, 29, 31}

$$\frac{\log(x)}{a}-\frac{\log\left(a+cx^4\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + c*x^4)),x]
```

```
[Out] Log[x]/a - Log[a + c*x^4]/(4*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a+cx^4\right)}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x(a+cx)}\,dx, x, x^4\right)\\ &= \frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, x^4\right)}{4a}-\frac{c\operatorname{Subst}\left(\int \frac{1}{a+cx}\,dx, x, x^4\right)}{4a}\\ &= \frac{\log(x)}{a}-\frac{\log\left(a+cx^4\right)}{4a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{\log(x)}{a}-\frac{\log\left(a+cx^4\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + c*x^4)),x]
[Out] Log[x]/a - Log[a + c*x^4]/(4*a)
```

fricas [A] time = 0.52, size = 18, normalized size = 0.82

$$-\frac{\log\left(cx^4+a\right)-4\,\log(x)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+a),x, algorithm="fricas")
[Out] -1/4*(log(c*x^4 + a) - 4*log(x))/a
```

giac [A] time = 0.16, size = 24, normalized size = 1.09

$$\frac{\log\left(x^4\right)}{4\,a}-\frac{\log\left(\left|cx^4+a\right|\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+a),x, algorithm="giac")
[Out] 1/4*log(x^4)/a - 1/4*log(abs(c*x^4 + a))/a
```

maple [A] time = 0.01, size = 21, normalized size = 0.95

$$\frac{\ln(x)}{a}-\frac{\ln\left(c\,x^4+a\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+a),x)
[Out] 1/a*ln(x)-1/4*ln(c*x^4+a)/a
```

maxima [A] time = 1.29, size = 23, normalized size = 1.05

$$-\frac{\log\left(cx^4+a\right)}{4\,a}+\frac{\log\left(x^4\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+a),x, algorithm="maxima")
[Out] -1/4*log(c*x^4 + a)/a + 1/4*log(x^4)/a
```

mupad [B] time = 1.00, size = 18, normalized size = 0.82

$$-\frac{\ln\left(c\,x^4+a\right)-4\,\ln(x)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + c*x^4)),x)
[Out] -(log(a + c*x^4) - 4*log(x))/(4*a)
```

sympy [A] time = 0.61, size = 15, normalized size = 0.68

$$\frac{\log(x)}{a}-\frac{\log\left(\frac{a}{c}+x^4\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+a),x)
[Out] log(x)/a - log(a/c + x**4)/(4*a)
```

3.650 $\int \frac{1}{x^3\left(a+cx^4\right)}\,dx$

Optimal. Leaf size=40

$$-\frac{\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{1}{2ax^2}$$

```
[Out] -1/2/a/x^2-1/2*arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/a^(3/2)
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {275, 325, 205}

$$-\frac{\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + c*x^4)),x]
```

```
[Out] -1/(2*a*x^2) - (Sqrt[c]*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*a^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a+cx^4\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\left(a+cx^2\right)}\,dx, x, x^2\right)\\ &= -\frac{1}{2ax^2}-\frac{c\,\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{2a}\\ &= -\frac{1}{2ax^2}-\frac{\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2a^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 79, normalized size = 1.98

$$\frac{\sqrt{c}\,x^2\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+\sqrt{c}\,x^2\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)-\sqrt{a}}{2a^{3/2}x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + c*x^4)),x]
```

```
[Out] (-Sqrt[a] + Sqrt[c]*x^2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + Sqrt[c]*x
^2*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*a^(3/2)*x^2)
```

fricas [A] time = 0.53, size = 94, normalized size = 2.35

$$\left[\frac{x^2\sqrt{-\frac{c}{a}}\log\left(\frac{cx^4-2\,ax^2\sqrt{-\frac{c}{a}}-a}{cx^4+a}\right)-2}{4\,ax^2},\ \frac{x^2\sqrt{\frac{c}{a}}\arctan\left(\frac{a\sqrt{\frac{c}{a}}}{cx^2}\right)-1}{2\,ax^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(x^2*sqrt(-c/a)*log((c*x^4 - 2*a*x^2*sqrt(-c/a) - a)/(c*x^4 + a)) - 2)
/(a*x^2), 1/2*(x^2*sqrt(c/a)*arctan(a*sqrt(c/a)/(c*x^2)) - 1)/(a*x^2)]
```

giac [A] time = 0.16, size = 31, normalized size = 0.78

$$-\frac{c\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\sqrt{ac}\,a}-\frac{1}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -1/2*c*arctan(c*x^2/sqrt(a*c))/(sqrt(a*c)*a) - 1/2/(a*x^2)
```

maple [A] time = 0.00, size = 32, normalized size = 0.80

$$-\frac{c\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{2\sqrt{ac}\;a}-\frac{1}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+a),x)
```

```
[Out] -1/2*c/a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x^2)-1/2/a/x^2
```

maxima [A] time = 2.98, size = 31, normalized size = 0.78

$$-\frac{c\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\sqrt{ac}\,a}-\frac{1}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] -1/2*c*arctan(c*x^2/sqrt(a*c))/(sqrt(a*c)*a) - 1/2/(a*x^2)
```

mupad [B] time = 0.05, size = 28, normalized size = 0.70

$$-\frac{1}{2\,a\,x^2}-\frac{\sqrt{c}\;\mathrm{atan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + c*x^4)),x)
```

```
[Out] - 1/(2*a*x^2) - (c^(1/2)*atan((c^(1/2)*x^2)/a^(1/2)))/(2*a^(3/2))
```

sympy [A] time = 0.47, size = 71, normalized size = 1.78

$$\frac{\sqrt{-\frac{c}{a^3}}\log\left(-\frac{a^2\sqrt{-\frac{c}{a^3}}}{c}+x^2\right)}{4}-\frac{\sqrt{-\frac{c}{a^3}}\log\left(\frac{a^2\sqrt{-\frac{c}{a^3}}}{c}+x^2\right)}{4}-\frac{1}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+a),x)
```

```
[Out] sqrt(-c/a**3)*log(-a**2*sqrt(-c/a**3)/c + x**2)/4 - sqrt(-c/a**3)*log(a**2*
sqrt(-c/a**3)/c + x**2)/4 - 1/(2*a*x**2)
```

3.651 $\int \frac{1}{x^5\left(a+cx^4\right)}\,dx$

Optimal. Leaf size=35

$$\frac{c\log\left(a+cx^4\right)}{4a^2}-\frac{c\log(x)}{a^2}-\frac{1}{4ax^4}$$

```
[Out] -1/4/a/x^4-c*ln(x)/a^2+1/4*c*ln(c*x^4+a)/a^2
```

Rubi [A] time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 44}

$$\frac{c\log\left(a+cx^4\right)}{4a^2}-\frac{c\log(x)}{a^2}-\frac{1}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + c*x^4)),x]
```

```
[Out] -1/(4*a*x^4) - (c*Log[x])/a^2 + (c*Log[a + c*x^4])/(4*a^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\left(a+cx^4\right)}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x^2(a+cx)}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int\left(\frac{1}{ax^2}-\frac{c}{a^2x}+\frac{c^2}{a^2(a+cx)}\right)dx, x, x^4\right)\\ &= -\frac{1}{4ax^4}-\frac{c\log(x)}{a^2}+\frac{c\log\left(a+cx^4\right)}{4a^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.00

$$\frac{c\log\left(a+cx^4\right)}{4a^2}-\frac{c\log(x)}{a^2}-\frac{1}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + c*x^4)),x]
```

```
[Out] -1/4*1/(a*x^4) - (c*Log[x])/a^2 + (c*Log[a + c*x^4])/(4*a^2)
```

fricas [A] time = 0.54, size = 33, normalized size = 0.94

$$\frac{cx^4\log\left(cx^4+a\right)-4\,cx^4\log(x)-a}{4\,a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*(c*x^4*log(c*x^4 + a) - 4*c*x^4*log(x) - a)/(a^2*x^4)
```

giac [A] time = 0.15, size = 43, normalized size = 1.23

$$-\frac{c\log\left(x^4\right)}{4\,a^2}+\frac{c\log\left(\left|cx^4+a\right|\right)}{4\,a^2}+\frac{cx^4-a}{4\,a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*c*log(x^4)/a^2 + 1/4*c*log(abs(c*x^4 + a))/a^2 + 1/4*(c*x^4 - a)/(a^2*
x^4)
```

maple [A] time = 0.01, size = 32, normalized size = 0.91

$$-\frac{c\ln(x)}{a^2}+\frac{c\ln\left(c\,x^4+a\right)}{4a^2}-\frac{1}{4a\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(c*x^4+a),x)
```

```
[Out] -1/4/a/x^4-1/a^2*c*ln(x)+1/4*c*ln(c*x^4+a)/a^2
```

maxima [A] time = 1.29, size = 33, normalized size = 0.94

$$\frac{c\log\left(cx^4+a\right)}{4\,a^2}-\frac{c\log\left(x^4\right)}{4\,a^2}-\frac{1}{4\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/4*c*log(c*x^4 + a)/a^2 - 1/4*c*log(x^4)/a^2 - 1/4/(a*x^4)
```

mupad [B] time = 0.07, size = 31, normalized size = 0.89

$$\frac{c\,\ln\left(c\,x^4+a\right)}{4\,a^2}-\frac{1}{4\,a\,x^4}-\frac{c\,\ln(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + c*x^4)),x)
```

```
[Out] (c*log(a + c*x^4))/(4*a^2) - 1/(4*a*x^4) - (c*log(x))/a^2
```

sympy [A] time = 0.83, size = 31, normalized size = 0.89

$$-\frac{1}{4ax^4}-\frac{c\log(x)}{a^2}+\frac{c\log\left(\frac{a}{c}+x^4\right)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(c*x**4+a),x)
```

```
[Out] -1/(4*a*x**4) - c*log(x)/a**2 + c*log(a/c + x**4)/(4*a**2)
```

3.652 $\int \frac{1}{x^7\left(a+cx^4\right)}\,dx$

Optimal. Leaf size=51

$$\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2a^{5/2}}+\frac{c}{2a^2x^2}-\frac{1}{6ax^6}$$

```
[Out] -1/6/a/x^6+1/2*c/a^2/x^2+1/2*c^(3/2)*arctan(x^2*c^(1/2)/a^(1/2))/a^(5/2)
```

Rubi [A] time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {275, 325, 205}

$$\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2a^{5/2}}+\frac{c}{2a^2x^2}-\frac{1}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + c*x^4)),x]
```

```
[Out] -1/(6*a*x^6) + c/(2*a^2*x^2) + (c^(3/2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*a
^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^7\left(a+cx^4\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^4\left(a+cx^2\right)}\,dx, x, x^2\right)\\
&= -\frac{1}{6ax^6}-\frac{c\,\text{Subst}\left(\int \frac{1}{x^2\left(a+cx^2\right)}\,dx, x, x^2\right)}{2a}\\
&= -\frac{1}{6ax^6}+\frac{c}{2a^2x^2}+\frac{c^2\,\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{2a^2}\\
&= -\frac{1}{6ax^6}+\frac{c}{2a^2x^2}+\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2a^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 88, normalized size = 1.73

$$-\frac{3c^{3/2}x^6\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+3c^{3/2}x^6\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)+\sqrt{a}\left(a-3cx^4\right)}{6a^{5/2}x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + c*x^4)),x]
```

```
[Out] -1/6*(Sqrt[a]*(a - 3*c*x^4) + 3*c^(3/2)*x^6*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/
a^(1/4)] + 3*c^(3/2)*x^6*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(5/2)*
x^6)
```

fricas [A] time = 0.66, size = 112, normalized size = 2.20

$$\left[\frac{3\,cx^6\sqrt{-\frac{c}{a}}\log\left(\frac{cx^4+2\,ax^2\sqrt{-\frac{c}{a}}-a}{cx^4+a}\right)+6\,cx^4-2\,a}{12\,a^2x^6},-\frac{3\,cx^6\sqrt{\frac{c}{a}}\arctan\left(\frac{a\sqrt{\frac{c}{a}}}{cx^2}\right)-3\,cx^4+a}{6\,a^2x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/12*(3*c*x^6*sqrt(-c/a)*log((c*x^4 + 2*a*x^2*sqrt(-c/a) - a)/(c*x^4 + a))
 + 6*c*x^4 - 2*a)/(a^2*x^6), -1/6*(3*c*x^6*sqrt(c/a)*arctan(a*sqrt(c/a)/(c*
x^2)) - 3*c*x^4 + a)/(a^2*x^6)]
```

giac [A] time = 0.18, size = 43, normalized size = 0.84

$$\frac{c^2\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\sqrt{ac}\,a^2}+\frac{3\,cx^4-a}{6\,a^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*c^2*arctan(c*x^2/sqrt(a*c))/(sqrt(a*c)*a^2) + 1/6*(3*c*x^4 - a)/(a^2*x^
6)
```

maple [A] time = 0.01, size = 43, normalized size = 0.84

$$\frac{c^2\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{2\sqrt{ac}\;a^2}+\frac{c}{2a^2x^2}-\frac{1}{6a\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(c*x^4+a),x)
```

```
[Out] 1/2*c^2/a^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x^2)-1/6/x^6/a+1/2*c/a^2/x^2
```

maxima [A] time = 3.09, size = 43, normalized size = 0.84

$$\frac{c^2\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\,\sqrt{ac}\,a^2}+\frac{3\,cx^4-a}{6\,a^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*c^2*arctan(c*x^2/sqrt(a*c))/(sqrt(a*c)*a^2) + 1/6*(3*c*x^4 - a)/(a^2*x^
6)
```

mupad [B] time = 0.06, size = 40, normalized size = 0.78

$$\frac{c^{3/2}\,\operatorname{atan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\,a^{5/2}}-\frac{\frac{1}{6\,a}-\frac{c\,x^4}{2\,a^2}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + c*x^4)),x)
```

```
[Out] (c^(3/2)*atan((c^(1/2)*x^2)/a^(1/2)))/(2*a^(5/2)) - (1/(6*a) - (c*x^4)/(2*a
^2))/x^6
```

sympy [B] time = 0.53, size = 90, normalized size = 1.76

$$-\frac{\sqrt{-\frac{c^3}{a^5}}\,\log\left(-\frac{a^3\sqrt{-\frac{c^3}{a^5}}}{c^2}+x^2\right)}{4}+\frac{\sqrt{-\frac{c^3}{a^5}}\,\log\left(\frac{a^3\sqrt{-\frac{c^3}{a^5}}}{c^2}+x^2\right)}{4}+\frac{-a+3cx^4}{6a^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(c*x**4+a),x)
```

```
[Out] -sqrt(-c**3/a**5)*log(-a**3*sqrt(-c**3/a**5)/c**2 + x**2)/4 + sqrt(-c**3/a*
*5)*log(a**3*sqrt(-c**3/a**5)/c**2 + x**2)/4 + (-a + 3*c*x**4)/(6*a**2*x**6
)
```

3.653 $\int \frac{x^4}{a+cx^4}\,dx$

Optimal. Leaf size=190

$$\frac{\sqrt[4]{a}\,\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{5/4}}-\frac{\sqrt[4]{a}\,\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{5/4}}+\frac{\sqrt[4]{a}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,c^{5/4}}-\frac{\sqrt[4]{a}\,\tan^{-1}}{2\sqrt{}}$$

```
[Out] x/c-1/4*a^(1/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/c^(5/4)*2^(1/2)-1/4*a^
(1/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))/c^(5/4)*2^(1/2)+1/8*a^(1/4)*ln(-a
^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/c^(5/4)*2^(1/2)-1/8*a^(1/4)*l
n(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/c^(5/4)*2^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {321, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt[4]{a}\,\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{5/4}}-\frac{\sqrt[4]{a}\,\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{5/4}}+\frac{\sqrt[4]{a}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,c^{5/4}}-\frac{\sqrt[4]{a}\,\tan^{-1}}{2\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + c*x^4),x]
```

```
[Out] x/c + (a^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(5/4))
 - (a^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(5/4)) +
(a^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]
*c^(5/4)) - (a^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]
)/(4*Sqrt[2]*c^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^4}{a+cx^4}\,dx = \frac{x}{c} - \frac{a\int \frac{1}{a+cx^4}\,dx}{c}$$

$$= \frac{x}{c} - \frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{c}\,x^2}{a+cx^4}\,dx}{2c} - \frac{\sqrt{a}\int \frac{\sqrt{a}+\sqrt{c}\,x^2}{a+cx^4}\,dx}{2c}$$

$$= \frac{x}{c} - \frac{\sqrt{a}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4c^{3/2}} - \frac{\sqrt{a}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4c^{3/2}} + \frac{\sqrt[4]{a}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}\,c^{5/4}} + \frac{\sqrt[4]{a}\int \frac{\sqrt{}}{-\frac{\sqrt{a}}{\sqrt{c}}}}{4\sqrt{}}$$

$$= \frac{x}{c} + \frac{\sqrt[4]{a}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{5/4}} - \frac{\sqrt[4]{a}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{5/4}} - \frac{\sqrt[4]{a}\,\text{Subst}}{}$$

$$= \frac{x}{c} + \frac{\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,c^{5/4}} - \frac{\sqrt[4]{a}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,c^{5/4}} + \frac{\sqrt[4]{a}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{5/4}}$$

Mathematica [A] time = 0.03, size = 173, normalized size = 0.91

$$\frac{\sqrt{2}\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)-\sqrt{2}\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)+2\sqrt{2}\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}}{\sqrt[4]{}}\right.}{8c^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + c*x^4),x]
```

```
[Out] (8*c^(1/4)*x + 2*Sqrt[2]*a^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] -
2*Sqrt[2]*a^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + Sqrt[2]*a^(1/4)
*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Sqrt[2]*a^(1/4)*L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(8*c^(5/4))
```

fricas [A] time = 0.66, size = 119, normalized size = 0.63

$$-\frac{4c\left(-\frac{a}{c^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{c^4x\left(-\frac{a}{c^5}\right)^{\frac{3}{4}}-\sqrt{c^2\sqrt{-\frac{a}{c^5}}+x^2}\,c^4\left(-\frac{a}{c^5}\right)^{\frac{3}{4}}}{a}\right)+c\left(-\frac{a}{c^5}\right)^{\frac{1}{4}}\log\left(c\left(-\frac{a}{c^5}\right)^{\frac{1}{4}}+x\right)-c\left(-\frac{a}{c^5}\right)^{\frac{1}{4}}\log\left(-c\left(-\frac{a}{c^5}\right)^{\frac{1}{4}}+x\right.}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*(4*c*(-a/c^5)^(1/4)*arctan(-(c^4*x*(-a/c^5)^(3/4) - sqrt(c^2*sqrt(-a/c
^5) + x^2)*c^4*(-a/c^5)^(3/4))/a) + c*(-a/c^5)^(1/4)*log(c*(-a/c^5)^(1/4) +
 x) - c*(-a/c^5)^(1/4)*log(-c*(-a/c^5)^(1/4) + x) - 4*x)/c
```

giac [A] time = 0.18, size = 172, normalized size = 0.91

$$\frac{x}{c}-\frac{\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4c^2}-\frac{\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4c^2}-\frac{\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{}\right.}{8c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+a),x, algorithm="giac")
```

```
[Out] x/c - 1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/
4))/(a/c)^(1/4))/c^2 - 1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x -
sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/c^2 - 1/8*sqrt(2)*(a*c^3)^(1/4)*log(x^2 +
 sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/c^2 + 1/8*sqrt(2)*(a*c^3)^(1/4)*log(x^2
 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/c^2
```

maple [A] time = 0.01, size = 133, normalized size = 0.70

$$\frac{x}{c}-\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4c}-\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4c}-\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\ \ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+a),x)
```

```
[Out] x/c-1/8/c*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x
^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))-1/4/c*(a/c)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(a/c)^(1/4)*x+1)-1/4/c*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*
x-1)
```

maxima [A] time = 2.92, size = 179, normalized size = 0.94

$$-\frac{\frac{2\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}}+\frac{2\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}}+\frac{\sqrt{2}a^{\frac{1}{4}}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{c^{\frac{1}{4}}}-\frac{\sqrt{2}a^{\frac{1}{4}}\log\left(\sqrt{c}x^2-\sqrt{2}a\right.}{c^{\frac{1}{4}}}}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] -1/8*(2*sqrt(2)*sqrt(a)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c
^(1/4))/sqrt(sqrt(a)*sqrt(c)))/sqrt(sqrt(a)*sqrt(c)) + 2*sqrt(2)*sqrt(a)*ar
ctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(
c)))/sqrt(sqrt(a)*sqrt(c)) + sqrt(2)*a^(1/4)*log(sqrt(c)*x^2 + sqrt(2)*a^(1
/4)*c^(1/4)*x + sqrt(a))/c^(1/4) - sqrt(2)*a^(1/4)*log(sqrt(c)*x^2 - sqrt(2
)*a^(1/4)*c^(1/4)*x + sqrt(a))/c^(1/4))/c + x/c
```

mupad **[B]** time = 1.04, size = 48, normalized size = 0.25

$$\frac{x}{c} - \frac{(-a)^{1/4}\,\operatorname{atan}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{2\,c^{5/4}} - \frac{(-a)^{1/4}\,\operatorname{atanh}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{2\,c^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + c*x^4),x)
```

```
[Out] x/c - ((-a)^(1/4)*atan((c^(1/4)*x)/(-a)^(1/4)))/(2*c^(5/4)) - ((-a)^(1/4)*a
tanh((c^(1/4)*x)/(-a)^(1/4)))/(2*c^(5/4))
```

sympy **[A]** time = 0.23, size = 22, normalized size = 0.12

$$\operatorname{RootSum}\left(256t^4c^5 + a, \left(t \mapsto t\log\left(-4tc + x\right)\right)\right) + \frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*c**5 + a, Lambda(_t, _t*log(-4*_t*c + x))) + x/c
```

3.654 $\int \frac{x^2}{a+cx^4}\,dx$

Optimal. Leaf size=185

$$\frac{\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,\sqrt[4]{a}\,c^{3/4}}-\frac{\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,\sqrt[4]{a}\,c^{3/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,\sqrt[4]{a}\,c^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,\sqrt[4]{a}\,c^{3/4}}$$

```
[Out] 1/4*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(1/4)/c^(3/4)*2^(1/2)+1/4*arctan
(1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(1/4)/c^(3/4)*2^(1/2)+1/8*ln(-a^(1/4)*c^(1/
4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(1/4)/c^(3/4)*2^(1/2)-1/8*ln(a^(1/4)*c^
(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(1/4)/c^(3/4)*2^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {297, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,\sqrt[4]{a}\,c^{3/4}}-\frac{\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,\sqrt[4]{a}\,c^{3/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,\sqrt[4]{a}\,c^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,\sqrt[4]{a}\,c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + c*x^4),x]
```

```
[Out] -ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(1/4)*c^(3/4)) + ArcT
an[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(1/4)*c^(3/4)) + Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(4*Sqrt[2]*a^(1/4)*c^(3/4)) -
 Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(4*Sqrt[2]*a^(1/4)*
c^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^2}{a+cx^4}\,dx = -\frac{\int \frac{\sqrt{a}-\sqrt{c}\,x^2}{a+cx^4}\,dx}{2\sqrt{c}} + \frac{\int \frac{\sqrt{a}+\sqrt{c}\,x^2}{a+cx^4}\,dx}{2\sqrt{c}}$$

$$= \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}} + x^2}\,dx}{4c} + \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}} + x^2}\,dx}{4c} + \frac{\int \frac{\frac{\sqrt{2}\,\sqrt[4]{a}}{\sqrt[4]{c}} + 2x}{-\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}} - x^2}\,dx}{4\sqrt{2}\,\sqrt[4]{a}\,c^{3/4}} + \frac{\int \frac{\frac{\sqrt{2}\,\sqrt[4]{a}}{\sqrt[4]{c}} - 2x}{-\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}} - x^2}\,dx}{4\sqrt{2}\,\sqrt[4]{a}\,c^{3/4}}$$

$$= \frac{\log\left(\sqrt{a} - \sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x + \sqrt{c}\,x^2\right)}{4\sqrt{2}\,\sqrt[4]{a}\,c^{3/4}} - \frac{\log\left(\sqrt{a} + \sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x + \sqrt{c}\,x^2\right)}{4\sqrt{2}\,\sqrt[4]{a}\,c^{3/4}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1\right.}{2\sqrt{2}\,\sqrt[4]{a}\,c^{3/4}}$$

$$= -\frac{\tan^{-1}\left(1 - \frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,\sqrt[4]{a}\,c^{3/4}} + \frac{\tan^{-1}\left(1 + \frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,\sqrt[4]{a}\,c^{3/4}} + \frac{\log\left(\sqrt{a} - \sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x + \sqrt{c}\,x^2\right)}{4\sqrt{2}\,\sqrt[4]{a}\,c^{3/4}} - \frac{\log\left(\sqrt{a} + \sqrt{}\right.}{4}$$

Mathematica [A] time = 0.02, size = 134, normalized size = 0.72

$$\frac{\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x + \sqrt{a} + \sqrt{c}\,x^2\right) - \log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x + \sqrt{a} + \sqrt{c}\,x^2\right) - 2\tan^{-1}\left(1 - \frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right) + 2\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right.}{4\sqrt{2}\,\sqrt[4]{a}\,c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + c*x^4),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*c^(1/4)
*x)/a^(1/4)] + Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(1/4)*c^(
3/4))
```

fricas [A] time = 0.80, size = 122, normalized size = 0.66

$$-\left(-\frac{1}{ac^3}\right)^{\frac{1}{4}} \arctan\left(-cx\left(-\frac{1}{ac^3}\right)^{\frac{1}{4}} + \sqrt{-ac\sqrt{-\frac{1}{ac^3}} + x^2}\,c\left(-\frac{1}{ac^3}\right)^{\frac{1}{4}}\right) + \frac{1}{4}\left(-\frac{1}{ac^3}\right)^{\frac{1}{4}} \log\left(ac^2\left(-\frac{1}{ac^3}\right)^{\frac{3}{4}} + x\right) - \frac{1}{4}\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] -(-1/(a*c^3))^(1/4)*arctan(-c*x*(-1/(a*c^3))^(1/4) + sqrt(-a*c*sqrt(-1/(a*c
^3)) + x^2)*c*(-1/(a*c^3))^(1/4)) + 1/4*(-1/(a*c^3))^(1/4)*log(a*c^2*(-1/(a
*c^3))^(3/4) + x) - 1/4*(-1/(a*c^3))^(1/4)*log(-a*c^2*(-1/(a*c^3))^(3/4) +
x)
```

giac [A] time = 0.16, size = 179, normalized size = 0.97

$$\frac{\sqrt{2}\left(ac^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac^3}+\frac{\sqrt{2}\left(ac^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac^3}-\frac{\sqrt{2}\left(ac^3\right)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(a*c^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a
/c)^(1/4))/(a*c^3) + 1/4*sqrt(2)*(a*c^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sq
rt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3) - 1/8*sqrt(2)*(a*c^3)^(3/4)*log(x^2
 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3) + 1/8*sqrt(2)*(a*c^3)^(3/4)*l
og(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3)
```

maple [A] time = 0.00, size = 128, normalized size = 0.69

$$\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4\left(\frac{a}{c}\right)^{\frac{1}{4}}c}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4\left(\frac{a}{c}\right)^{\frac{1}{4}}c}+\frac{\sqrt{2}\,\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{8\left(\frac{a}{c}\right)^{\frac{1}{4}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+a),x)
```

```
[Out] 1/8/c/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(
a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+1/4/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)
/(a/c)^(1/4)*x+1)+1/4/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)
```

maxima [A] time = 3.02, size = 169, normalized size = 0.91

$$\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\,\sqrt{c}}}\right)}{4\sqrt{\sqrt{a}\,\sqrt{c}}\,\sqrt{c}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\,\sqrt{c}}}\right)}{4\sqrt{\sqrt{a}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\,\log\left(\sqrt{c}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8a^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(\sqrt{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt
(sqrt(a)*sqrt(c)))/(sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 1/4*sqrt(2)*arctan(1/2
*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sq
rt(sqrt(a)*sqrt(c))*sqrt(c)) - 1/8*sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4
)*c^(1/4)*x + sqrt(a))/(a^(1/4)*c^(3/4)) + 1/8*sqrt(2)*log(sqrt(c)*x^2 - sq
rt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(1/4)*c^(3/4))
```

mupad [B] time = 1.04, size = 35, normalized size = 0.19

$$\frac{\operatorname{atan}\left(\frac{c^{1/4}x}{(-a)^{1/4}}\right)-\operatorname{atanh}\left(\frac{c^{1/4}x}{(-a)^{1/4}}\right)}{2\,(-a)^{1/4}\,c^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + c*x^4),x)
```

```
[Out] (atan((c^(1/4)*x)/(-a)^(1/4)) - atanh((c^(1/4)*x)/(-a)^(1/4)))/(2*(-a)^(1/4
)*c^(3/4))
```

sympy [A] time = 0.22, size = 26, normalized size = 0.14

$$\text{RootSum}\left(256t^4ac^3+1,\left(t\mapsto t\log\left(64t^3ac^2+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a*c**3 + 1, Lambda(_t, _t*log(64*_t**3*a*c**2 + x)))
```

3.655 $\int \frac{1}{a+cx^4}\,dx$

Optimal. Leaf size=185

$$-\frac{\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}}+\frac{\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}}$$

```
[Out] 1/4*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/c^(1/4)*2^(1/2)+1/4*arctan
(1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/c^(1/4)*2^(1/2)-1/8*ln(-a^(1/4)*c^(1/
4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(3/4)/c^(1/4)*2^(1/2)+1/8*ln(a^(1/4)*c^
(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(3/4)/c^(1/4)*2^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}}+\frac{\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^(-1),x]
```

```
[Out] -ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(1/4)) + ArcT
an[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(1/4)) - Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(4*Sqrt[2]*a^(3/4)*c^(1/4)) +
 Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(4*Sqrt[2]*a^(3/4)*
c^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{a+cx^4}\,dx = \frac{\int \frac{\sqrt{a}-\sqrt{c}\,x^2}{a+cx^4}\,dx}{2\sqrt{a}} + \frac{\int \frac{\sqrt{a}+\sqrt{c}\,x^2}{a+cx^4}\,dx}{2\sqrt{a}}$$

$$= \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}} + x^2}\,dx}{4\sqrt{a}\,\sqrt{c}} + \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}} + x^2}\,dx}{4\sqrt{a}\,\sqrt{c}} - \frac{\int \frac{\frac{\sqrt{2}\,\sqrt[4]{a}}{\sqrt[4]{c}} + 2x}{-\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}} - x^2}\,dx}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}} - \frac{\int \frac{\frac{\sqrt{2}\,\sqrt[4]{a}}{\sqrt[4]{c}} - 2x}{-\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}} - x^2}\,dx}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}}$$

$$= -\frac{\log\left(\sqrt{a} - \sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x + \sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}} + \frac{\log\left(\sqrt{a} + \sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x + \sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x,\right.}{2\sqrt{2}\,a^{3/4}\sqrt[4]{}}$$

$$= -\frac{\tan^{-1}\left(1 - \frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}} + \frac{\tan^{-1}\left(1 + \frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}} - \frac{\log\left(\sqrt{a} - \sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x + \sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}} + \frac{\log\left(\sqrt{a} + \sqrt{}\right.}{4\sqrt{}}$$

Mathematica [A] time = 0.02, size = 134, normalized size = 0.72

$$\frac{-\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x + \sqrt{a} + \sqrt{c}\,x^2\right) + \log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x + \sqrt{a} + \sqrt{c}\,x^2\right) - 2\tan^{-1}\left(1 - \frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right) + 2\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{}}{\sqrt[4]{a}}\right.}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^(-1),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*c^(1/4)
*x)/a^(1/4)] - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(
1/4))
```

fricas [A] time = 0.52, size = 121, normalized size = 0.65

$$\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}} \arctan\left(-a^2cx\left(-\frac{1}{a^3c}\right)^{\frac{3}{4}} + \sqrt{a^2\sqrt{-\frac{1}{a^3c}} + x^2}\,a^2c\left(-\frac{1}{a^3c}\right)^{\frac{3}{4}}\right) + \frac{1}{4}\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}} \log\left(a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}} + x\right) - \frac{1}{4}\left(-\frac{1}{a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] (-1/(a^3*c))^(1/4)*arctan(-a^2*c*x*(-1/(a^3*c))^(3/4) + sqrt(a^2*sqrt(-1/(a
^3*c)) + x^2)*a^2*c*(-1/(a^3*c))^(3/4)) + 1/4*(-1/(a^3*c))^(1/4)*log(a*(-1/
(a^3*c))^(1/4) + x) - 1/4*(-1/(a^3*c))^(1/4)*log(-a*(-1/(a^3*c))^(1/4) + x)
```

giac [A] time = 0.17, size = 179, normalized size = 0.97

$$\frac{\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac}+\frac{\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac}+\frac{\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right.}{8ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a
/c)^(1/4))/(a*c) + 1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x - sqrt
(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c) + 1/8*sqrt(2)*(a*c^3)^(1/4)*log(x^2 + s
qrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c) - 1/8*sqrt(2)*(a*c^3)^(1/4)*log(x^2
 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c)
```

maple [A] time = 0.00, size = 128, normalized size = 0.69

$$\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+a),x)
```

```
[Out] 1/8*(a/c)^(1/4)/a*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(
a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+1/4*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)
/(a/c)^(1/4)*x+1)+1/4*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)
```

maxima [A] time = 2.93, size = 169, normalized size = 0.91

$$\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\,\sqrt{c}}}\right)}{4\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{c}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\,\sqrt{c}}}\right)}{4\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{c}}}+\frac{\sqrt{2}\,\log\left(\sqrt{c}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\,\log\left(\sqrt{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt
(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + 1/4*sqrt(2)*arctan(1/2
*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sq
rt(a)*sqrt(sqrt(a)*sqrt(c))) + 1/8*sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4
)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - 1/8*sqrt(2)*log(sqrt(c)*x^2 - sq
rt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4))
```

mupad [B] time = 1.01, size = 33, normalized size = 0.18

$$-\frac{\operatorname{atan}\left(\frac{c^{1/4}x}{(-a)^{1/4}}\right)+\operatorname{atanh}\left(\frac{c^{1/4}x}{(-a)^{1/4}}\right)}{2\,(-a)^{3/4}\,c^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x^4),x)
```

```
[Out] -(atan((c^(1/4)*x)/(-a)^(1/4)) + atanh((c^(1/4)*x)/(-a)^(1/4)))/(2*(-a)^(3/
4)*c^(1/4))
```

sympy [A] time = 0.24, size = 20, normalized size = 0.11

$$\text{RootSum}\left(256t^4a^3c+1,\left(t\mapsto t\log\left(4ta+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c + 1, Lambda(_t, _t*log(4*_t*a + x)))
```

3.656 $\int \frac{1}{x^2\left(a+cx^4\right)}\,dx$

Optimal. Leaf size=193

$$-\frac{\sqrt[4]{c}\,\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{5/4}}+\frac{\sqrt[4]{c}\,\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{5/4}}+\frac{\sqrt[4]{c}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{5/4}}-\frac{\sqrt[4]{c}\,\tan^{-}}{2}$$

```
[Out] -1/a/x-1/4*c^(1/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(5/4)*2^(1/2)-1/4
*c^(1/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(5/4)*2^(1/2)-1/8*c^(1/4)*ln
(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(5/4)*2^(1/2)+1/8*c^(1/4
)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(5/4)*2^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {325, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{c}\,\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{5/4}}+\frac{\sqrt[4]{c}\,\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{5/4}}+\frac{\sqrt[4]{c}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{5/4}}-\frac{\sqrt[4]{c}\,\tan^{-}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + c*x^4)),x]
```

```
[Out] -(1/(a*x)) + (c^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a
^(5/4)) - (c^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(5
/4)) - (c^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*
Sqrt[2]*a^(5/4)) + (c^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[
c]*x^2])/(4*Sqrt[2]*a^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+cx^4\right)}\,dx = -\frac{1}{ax} - \frac{c\int \frac{x^2}{a+cx^4}\,dx}{a}$$

$$= -\frac{1}{ax} + \frac{\sqrt{c}\int \frac{\sqrt{a}-\sqrt{c}\,x^2}{a+cx^4}\,dx}{2a} - \frac{\sqrt{c}\int \frac{\sqrt{a}+\sqrt{c}\,x^2}{a+cx^4}\,dx}{2a}$$

$$= -\frac{1}{ax} - \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2}\,dx}{4a} - \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2}\,dx}{4a} - \frac{\sqrt[4]{c}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} - x^2}\,dx}{4\sqrt{2}\,a^{5/4}} - \frac{\sqrt[4]{c}\int \frac{\frac{\sqrt{2}}{\sqrt[4]{}}}{-\frac{\sqrt{a}}{\sqrt{c}}+}}{4\sqrt{2}}$$

$$= -\frac{1}{ax} - \frac{\sqrt[4]{c}\,\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x + \sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{5/4}} + \frac{\sqrt[4]{c}\,\log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x + \sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{5/4}} - \frac{\sqrt[4]{c}}{}$$

$$= -\frac{1}{ax} + \frac{\sqrt[4]{c}\,\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{5/4}} - \frac{\sqrt[4]{c}\,\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{5/4}} - \frac{\sqrt[4]{c}\,\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x + \right.}{4\sqrt{2}\,a^{5/4}}$$

Mathematica [A] time = 0.03, size = 179, normalized size = 0.93

$$\frac{-\sqrt{2}\sqrt[4]{c}\,x\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x + \sqrt{a} + \sqrt{c}\,x^2\right) + \sqrt{2}\sqrt[4]{c}\,x\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x + \sqrt{a} + \sqrt{c}\,x^2\right) + 2\sqrt{2}\sqrt[4]{c}\,x\tan^{-1}\left(1 - \right.}{8a^{5/4}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + c*x^4)),x]
```

```
[Out] (-8*a^(1/4) + 2*Sqrt[2]*c^(1/4)*x*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] -
 2*Sqrt[2]*c^(1/4)*x*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - Sqrt[2]*c^(1
/4)*x*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sqrt[2]*c^(1
/4)*x*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(8*a^(5/4)*x)
```

fricas [A] time = 0.65, size = 130, normalized size = 0.67

$$\frac{4\,ax\left(-\frac{c}{a^5}\right)^{\frac{1}{4}}\arctan\left(-ax\left(-\frac{c}{a^5}\right)^{\frac{1}{4}}+a\sqrt{-\frac{a^3\sqrt{-\frac{c}{a^5}}-cx^2}{c}}\left(-\frac{c}{a^5}\right)^{\frac{1}{4}}\right)-ax\left(-\frac{c}{a^5}\right)^{\frac{1}{4}}\log\left(a^4\left(-\frac{c}{a^5}\right)^{\frac{3}{4}}+cx\right)+ax\left(-\frac{c}{a^5}\right)^{\frac{1}{4}}\log\left(-\right.}{4\,ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*(4*a*x*(-c/a^5)^(1/4)*arctan(-a*x*(-c/a^5)^(1/4) + a*sqrt(-(a^3*sqrt(-c
/a^5) - c*x^2)/c)*(-c/a^5)^(1/4)) - a*x*(-c/a^5)^(1/4)*log(a^4*(-c/a^5)^(3/
4) + c*x) + a*x*(-c/a^5)^(1/4)*log(-a^4*(-c/a^5)^(3/4) + c*x) - 4)/(a*x)
```

giac [A] time = 0.19, size = 187, normalized size = 0.97

$$-\frac{\sqrt{2}\left(ac^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,a^2c^2}-\frac{\sqrt{2}\left(ac^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,a^2c^2}+\frac{\sqrt{2}\left(ac^3\right)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right.}{8\,a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(a*c^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(
a/c)^(1/4))/(a^2*c^2) - 1/4*sqrt(2)*(a*c^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x -
 sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^2) + 1/8*sqrt(2)*(a*c^3)^(3/4)*lo
g(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^2) - 1/8*sqrt(2)*(a*c^3)^
(3/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^2) - 1/(a*x)
```

maple [A] time = 0.01, size = 136, normalized size = 0.70

$$-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4\left(\frac{a}{c}\right)^{\frac{1}{4}}a}-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4\left(\frac{a}{c}\right)^{\frac{1}{4}}a}-\frac{\sqrt{2}\,\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{8\left(\frac{a}{c}\right)^{\frac{1}{4}}a}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+a),x)
```

```
[Out] -1/8/a/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+
(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))-1/4/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2
)/(a/c)^(1/4)*x+1)-1/4/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1
)-1/a/x
```

maxima [A] time = 2.97, size = 183, normalized size = 0.95

$$-\frac{c\left(\frac{2\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{c}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{c}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{\right.}}{a^{\frac{1}{4}}c^{\frac{3}{4}}}\right.}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] -1/8*c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4)
)/sqrt(sqrt(a)*sqrt(c)))/(sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan
(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))
/(sqrt(sqrt(a)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4
)*c^(1/4)*x + sqrt(a))/(a^(1/4)*c^(3/4)) + sqrt(2)*log(sqrt(c)*x^2 - sqrt(2
)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(1/4)*c^(3/4)))/a - 1/(a*x)
```

mupad **[B]** time = 0.10, size = 51, normalized size = 0.26

$$\frac{(-c)^{1/4}\,\operatorname{atanh}\left(\frac{(-c)^{1/4}\,x}{a^{1/4}}\right)}{2\,a^{5/4}}-\frac{(-c)^{1/4}\,\operatorname{atan}\left(\frac{(-c)^{1/4}\,x}{a^{1/4}}\right)}{2\,a^{5/4}}-\frac{1}{a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + c*x^4)),x)
```

```
[Out] ((-c)^(1/4)*atanh(((-c)^(1/4)*x)/a^(1/4)))/(2*a^(5/4)) - ((-c)^(1/4)*atan((
(-c)^(1/4)*x)/a^(1/4)))/(2*a^(5/4)) - 1/(a*x)
```

sympy **[A]** time = 0.27, size = 29, normalized size = 0.15

$$\operatorname{RootSum}\left(256t^4a^5+c,\left(t\mapsto t\log\left(-\frac{64t^3a^4}{c}+x\right)\right)\right)-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**5 + c, Lambda(_t, _t*log(-64*_t**3*a**4/c + x))) - 1/(
a*x)
```

3.657 $\int \frac{1}{x^4\left(a+cx^4\right)}\,dx$

Optimal. Leaf size=195

$$\frac{c^{3/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{7/4}}-\frac{c^{3/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{7/4}}+\frac{c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{7/4}}-\frac{c^{3/4}\tan^{-1}}{2\sqrt{}}$$

```
[Out] -1/3/a/x^3-1/4*c^(3/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)*2^(1/2)
-1/4*c^(3/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)*2^(1/2)+1/8*c^(3/4
)*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(7/4)*2^(1/2)-1/8*c^
(3/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(7/4)*2^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {325, 211, 1165, 628, 1162, 617, 204}

$$\frac{c^{3/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{7/4}}-\frac{c^{3/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{7/4}}+\frac{c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{7/4}}-\frac{c^{3/4}\tan^{-1}}{2\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + c*x^4)),x]
```

```
[Out] -1/(3*a*x^3) + (c^(3/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]
*a^(7/4)) - (c^(3/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^
(7/4)) + (c^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(
4*Sqrt[2]*a^(7/4)) - (c^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqr
t[c]*x^2])/(4*Sqrt[2]*a^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+cx^4\right)}\,dx = -\frac{1}{3ax^3} - \frac{c\int \frac{1}{a+cx^4}\,dx}{a}$$

$$= -\frac{1}{3ax^3} - \frac{c\int \frac{\sqrt{a}-\sqrt{c}\,x^2}{a+cx^4}\,dx}{2a^{3/2}} - \frac{c\int \frac{\sqrt{a}+\sqrt{c}\,x^2}{a+cx^4}\,dx}{2a^{3/2}}$$

$$= -\frac{1}{3ax^3} - \frac{\sqrt{c}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{4a^{3/2}} - \frac{\sqrt{c}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{4a^{3/2}} + \frac{c^{3/4}\int \frac{\frac{\sqrt{2}\,\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}\,a^{7/4}} + \frac{c^{3}}{}$$

$$= -\frac{1}{3ax^3} + \frac{c^{3/4}\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{7/4}} - \frac{c^{3/4}\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{7/4}} - \frac{c}{}$$

$$= -\frac{1}{3ax^3} + \frac{c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{7/4}} - \frac{c^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{7/4}} + \frac{c^{3/4}\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x\right.}{4\sqrt{2}\,a^{7/4}}$$

Mathematica [A] time = 0.03, size = 188, normalized size = 0.96

$$\frac{-8a^{3/4}+6\sqrt{2}\,c^{3/4}x^3\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)-6\sqrt{2}\,c^{3/4}x^3\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)+3\sqrt{2}\,c^{3/4}x^3\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}\right.}{24a^{7/4}x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + c*x^4)),x]
```

```
[Out] (-8*a^(3/4) + 6*Sqrt[2]*c^(3/4)*x^3*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]
 - 6*Sqrt[2]*c^(3/4)*x^3*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 3*Sqrt[2
]*c^(3/4)*x^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - 3*Sq
rt[2]*c^(3/4)*x^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(
24*a^(7/4)*x^3)
```

fricas [A] time = 0.69, size = 162, normalized size = 0.83

$$-\frac{12\,ax^3\left(-\frac{c^3}{a^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{a^5x\left(-\frac{c^3}{a^7}\right)^{\frac{3}{4}}-a^5\sqrt{\frac{a^4\sqrt{-\frac{c^3}{a^7}}+c^2x^2}{c^2}}\left(-\frac{c^3}{a^7}\right)^{\frac{3}{4}}}{c^2}\right)+3\,ax^3\left(-\frac{c^3}{a^7}\right)^{\frac{1}{4}}\log\left(a^2\left(-\frac{c^3}{a^7}\right)^{\frac{1}{4}}+cx\right)-3\,ax^3\left(-\frac{c^3}{a^7}\right)^{\frac{1}{4}}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/12*(12*a*x^3*(-c^3/a^7)^(1/4)*arctan(-(a^5*x*(-c^3/a^7)^(3/4) - a^5*sqrt
((a^4*sqrt(-c^3/a^7) + c^2*x^2)/c^2)*(-c^3/a^7)^(3/4))/c^2) + 3*a*x^3*(-c^3
/a^7)^(1/4)*log(a^2*(-c^3/a^7)^(1/4) + c*x) - 3*a*x^3*(-c^3/a^7)^(1/4)*log(
-a^2*(-c^3/a^7)^(1/4) + c*x) + 4)/(a*x^3)
```

giac [A] time = 0.16, size = 175, normalized size = 0.90

$$-\frac{\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,a^2}-\frac{\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,a^2}-\frac{\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right.}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(
a/c)^(1/4))/a^2 - 1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(
2)*(a/c)^(1/4))/(a/c)^(1/4))/a^2 - 1/8*sqrt(2)*(a*c^3)^(1/4)*log(x^2 + sqrt
(2)*x*(a/c)^(1/4) + sqrt(a/c))/a^2 + 1/8*sqrt(2)*(a*c^3)^(1/4)*log(x^2 - sq
rt(2)*x*(a/c)^(1/4) + sqrt(a/c))/a^2 - 1/3/(a*x^3)
```

maple [A] time = 0.01, size = 139, normalized size = 0.71

$$-\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4a^2}-\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4a^2}-\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{8a^2}-\frac{1}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^4+a),x)
```

```
[Out] -1/3/a/x^3-1/8*c/a^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c
)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))-1/4*c/a^2*(a/c)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)-1/4*c/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(a/c)^(1/4)*x-1)
```

maxima [A] time = 3.00, size = 184, normalized size = 0.94

$$-\frac{\frac{2\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{2\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{\sqrt{2}\,c^{\frac{3}{4}}\log\left(\sqrt{c}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}\,c^{\frac{3}{4}}\log\left(\sqrt{c}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right.}{a^{\frac{3}{4}}}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] -1/8*(2*sqrt(2)*c*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4)
)/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + 2*sqrt(2)*c*arct
an(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)
))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + sqrt(2)*c^(3/4)*log(sqrt(c)*x^2 + sqrt
(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/a^(3/4) - sqrt(2)*c^(3/4)*log(sqrt(c)*x^2
- sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/a^(3/4))/a - 1/3/(a*x^3)
```

mupad **[B]** time = 0.10, size = 51, normalized size = 0.26

$$\frac{(-c)^{3/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\,x}{a^{1/4}}\right)}{2\,a^{7/4}}-\frac{1}{3\,a\,x^3}+\frac{(-c)^{3/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\,x}{a^{1/4}}\right)}{2\,a^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + c*x^4)),x)
```

```
[Out] ((-c)^(3/4)*atan(((-c)^(1/4)*x)/a^(1/4)))/(2*a^(7/4)) - 1/(3*a*x^3) + ((-c)
^(3/4)*atanh(((-c)^(1/4)*x)/a^(1/4)))/(2*a^(7/4))
```

sympy **[A]** time = 0.34, size = 32, normalized size = 0.16

$$\mathrm{RootSum}\left(256t^4a^7+c^3,\left(t\mapsto t\log\left(-\frac{4ta^2}{c}+x\right)\right)\right)-\frac{1}{3ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**7 + c**3, Lambda(_t, _t*log(-4*_t*a**2/c + x))) - 1/(3
*a*x**3)
```

3.658 $\int \frac{x^{11}}{\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=46

$$-\frac{a^2}{4c^3\left(a+cx^4\right)}-\frac{a\log\left(a+cx^4\right)}{2c^3}+\frac{x^4}{4c^2}$$

```
[Out] 1/4*x^4/c^2-1/4*a^2/c^3/(c*x^4+a)-1/2*a*ln(c*x^4+a)/c^3
```

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$-\frac{a^2}{4c^3\left(a+cx^4\right)}-\frac{a\log\left(a+cx^4\right)}{2c^3}+\frac{x^4}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + c*x^4)^2,x]
```

```
[Out] x^4/(4*c^2) - a^2/(4*c^3*(a + c*x^4)) - (a*Log[a + c*x^4])/(2*c^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{\left(a+cx^4\right)^2}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x^2}{(a+cx)^2}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int \left(\frac{1}{c^2}+\frac{a^2}{c^2(a+cx)^2}-\frac{2a}{c^2(a+cx)}\right)dx, x, x^4\right)\\ &= \frac{x^4}{4c^2}-\frac{a^2}{4c^3\left(a+cx^4\right)}-\frac{a\log\left(a+cx^4\right)}{2c^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 0.83

$$\frac{-\frac{a^2}{a+cx^4}-2a\log\left(a+cx^4\right)+cx^4}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + c*x^4)^2,x]
```

```
[Out] (c*x^4 - a^2/(a + c*x^4) - 2*a*Log[a + c*x^4])/(4*c^3)
```

fricas [A] time = 0.63, size = 56, normalized size = 1.22

$$\frac{c^2x^8 + acx^4 - a^2 - 2\left(acx^4 + a^2\right)\log\left(cx^4 + a\right)}{4\left(c^4x^4 + ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(c^2*x^8 + a*c*x^4 - a^2 - 2*(a*c*x^4 + a^2)*log(c*x^4 + a))/(c^4*x^4 +
 a*c^3)
```

giac [A] time = 0.19, size = 49, normalized size = 1.07

$$\frac{x^4}{4\,c^2} - \frac{a\log\left(\left|cx^4 + a\right|\right)}{2\,c^3} + \frac{2\,acx^4 + a^2}{4\left(cx^4 + a\right)c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*x^4/c^2 - 1/2*a*log(abs(c*x^4 + a))/c^3 + 1/4*(2*a*c*x^4 + a^2)/((c*x^4
 + a)*c^3)
```

maple [A] time = 0.01, size = 41, normalized size = 0.89

$$\frac{x^4}{4c^2} - \frac{a^2}{4\left(c\,x^4 + a\right)c^3} - \frac{a\ln\left(c\,x^4 + a\right)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(c*x^4+a)^2,x)
```

```
[Out] 1/4*x^4/c^2-1/4*a^2/c^3/(c*x^4+a)-1/2*a*ln(c*x^4+a)/c^3
```

maxima [A] time = 1.34, size = 43, normalized size = 0.93

$$\frac{x^4}{4\,c^2} - \frac{a^2}{4\left(c^4x^4 + ac^3\right)} - \frac{a\log\left(cx^4 + a\right)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^4/c^2 - 1/4*a^2/(c^4*x^4 + a*c^3) - 1/2*a*log(c*x^4 + a)/c^3
```

mupad [B] time = 0.99, size = 45, normalized size = 0.98

$$\frac{x^4}{4\,c^2} - \frac{a^2}{4\,\left(c^4\,x^4 + a\,c^3\right)} - \frac{a\,\ln\left(c\,x^4 + a\right)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a + c*x^4)^2,x)
```

```
[Out] x^4/(4*c^2) - a^2/(4*(a*c^3 + c^4*x^4)) - (a*log(a + c*x^4))/(2*c^3)
```

sympy [**A**] time = 0.52, size = 41, normalized size = 0.89

$$-\frac{a^2}{4ac^3+4c^4x^4}-\frac{a\log\left(a+cx^4\right)}{2c^3}+\frac{x^4}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(c*x**4+a)**2,x)
[Out] -a**2/(4*a*c**3 + 4*c**4*x**4) - a*log(a + c*x**4)/(2*c**3) + x**4/(4*c**2)
```

3.659 $\int \frac{x^9}{\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=59

$$-\frac{3\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4c^{5/2}}-\frac{x^6}{4c\left(a+cx^4\right)}+\frac{3x^2}{4c^2}$$

```
[Out] 3/4*x^2/c^2-1/4*x^6/c/(c*x^4+a)-3/4*arctan(x^2*c^(1/2)/a^(1/2))*a^(1/2)/c^(
5/2)
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {275, 288, 321, 205}

$$-\frac{3\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4c^{5/2}}-\frac{x^6}{4c\left(a+cx^4\right)}+\frac{3x^2}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + c*x^4)^2,x]
```

```
[Out] (3*x^2)/(4*c^2) - x^6/(4*c*(a + c*x^4)) - (3*Sqrt[a]*ArcTan[(Sqrt[c]*x^2)/S
qrt[a]])/(4*c^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^9}{\left(a+cx^4\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{\left(a+cx^2\right)^2}\,dx, x, x^2\right)$$
$$= -\frac{x^6}{4c\left(a+cx^4\right)} + \frac{3\,\text{Subst}\left(\int \frac{x^2}{a+cx^2}\,dx, x, x^2\right)}{4c}$$
$$= \frac{3x^2}{4c^2} - \frac{x^6}{4c\left(a+cx^4\right)} - \frac{(3a)\,\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{4c^2}$$
$$= \frac{3x^2}{4c^2} - \frac{x^6}{4c\left(a+cx^4\right)} - \frac{3\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4c^{5/2}}$$

Mathematica [A] time = 0.05, size = 60, normalized size = 1.02

$$-\frac{3\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4c^{5/2}} + \frac{ax^2}{4c^2\left(a+cx^4\right)} + \frac{x^2}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + c*x^4)^2,x]
```

```
[Out] x^2/(2*c^2) + (a*x^2)/(4*c^2*(a + c*x^4)) - (3*Sqrt[a]*ArcTan[(Sqrt[c]*x^2)
/Sqrt[a]])/(4*c^(5/2))
```

fricas [A] time = 0.59, size = 144, normalized size = 2.44

$$\left[\frac{4\,cx^6 + 6\,ax^2 + 3\left(cx^4+a\right)\sqrt{-\frac{a}{c}}\,\log\left(\frac{cx^4 - 2\,cx^2\sqrt{-\frac{a}{c}} - a}{cx^4+a}\right)}{8\left(c^3x^4+ac^2\right)}, \frac{2\,cx^6 + 3\,ax^2 - 3\left(cx^4+a\right)\sqrt{\frac{a}{c}}\,\arctan\left(\frac{cx^2\sqrt{\frac{a}{c}}}{a}\right)}{4\left(c^3x^4+ac^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*c*x^6 + 6*a*x^2 + 3*(c*x^4 + a)*sqrt(-a/c)*log((c*x^4 - 2*c*x^2*sqr
t(-a/c) - a)/(c*x^4 + a)))/(c^3*x^4 + a*c^2), 1/4*(2*c*x^6 + 3*a*x^2 - 3*(c
*x^4 + a)*sqrt(a/c)*arctan(c*x^2*sqrt(a/c)/a))/(c^3*x^4 + a*c^2)]
```

giac [A] time = 0.16, size = 49, normalized size = 0.83

$$\frac{ax^2}{4\left(cx^4+a\right)c^2} + \frac{x^2}{2\,c^2} - \frac{3\,a\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\sqrt{ac}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*a*x^2/((c*x^4 + a)*c^2) + 1/2*x^2/c^2 - 3/4*a*arctan(c*x^2/sqrt(a*c))/(
sqrt(a*c)*c^2)
```

maple [A] time = 0.01, size = 50, normalized size = 0.85

$$\frac{a\,x^2}{4\left(c\,x^4+a\right)c^2} - \frac{3a\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{4\sqrt{ac}\;c^2} + \frac{x^2}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(c*x^4+a)^2,x)
```

```
[Out] 1/2*x^2/c^2+1/4*a/c^2*x^2/(c*x^4+a)-3/4*a/c^2/(a*c)^(1/2)*arctan(1/(a*c)^(1
/2)*c*x^2)
```

maxima [A] time = 2.95, size = 52, normalized size = 0.88

$$\frac{ax^2}{4\left(c^3x^4+ac^2\right)}+\frac{x^2}{2\,c^2}-\frac{3\,a\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\,\sqrt{ac}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*a*x^2/(c^3*x^4 + a*c^2) + 1/2*x^2/c^2 - 3/4*a*arctan(c*x^2/sqrt(a*c))/(
sqrt(a*c)*c^2)
```

mupad [B] time = 1.07, size = 50, normalized size = 0.85

$$\frac{x^2}{2\,c^2}+\frac{a\,x^2}{4\,\left(c^3\,x^4+a\,c^2\right)}-\frac{3\,\sqrt{a}\,\operatorname{atan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4\,c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + c*x^4)^2,x)
```

```
[Out] x^2/(2*c^2) + (a*x^2)/(4*(a*c^2 + c^3*x^4)) - (3*a^(1/2)*atan((c^(1/2)*x^2)
/a^(1/2)))/(4*c^(5/2))
```

sympy [A] time = 0.85, size = 92, normalized size = 1.56

$$\frac{ax^2}{4ac^2+4c^3x^4}+\frac{3\sqrt{-\frac{a}{c^5}}\,\log\left(-c^2\sqrt{-\frac{a}{c^5}}+x^2\right)}{8}-\frac{3\sqrt{-\frac{a}{c^5}}\,\log\left(c^2\sqrt{-\frac{a}{c^5}}+x^2\right)}{8}+\frac{x^2}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(c*x**4+a)**2,x)
```

```
[Out] a*x**2/(4*a*c**2 + 4*c**3*x**4) + 3*sqrt(-a/c**5)*log(-c**2*sqrt(-a/c**5) +
 x**2)/8 - 3*sqrt(-a/c**5)*log(c**2*sqrt(-a/c**5) + x**2)/8 + x**2/(2*c**2)
```

3.660 $\int \frac{x^7}{\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=33

$$\frac{a}{4c^2\left(a+cx^4\right)}+\frac{\log\left(a+cx^4\right)}{4c^2}$$

```
[Out] 1/4*a/c^2/(c*x^4+a)+1/4*ln(c*x^4+a)/c^2
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{a}{4c^2\left(a+cx^4\right)}+\frac{\log\left(a+cx^4\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + c*x^4)^2,x]
```

```
[Out] a/(4*c^2*(a + c*x^4)) + Log[a + c*x^4]/(4*c^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{\left(a+cx^4\right)^2}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x}{(a+cx)^2}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int \left(-\frac{a}{c(a+cx)^2}+\frac{1}{c(a+cx)}\right)\,dx, x, x^4\right)\\ &= \frac{a}{4c^2\left(a+cx^4\right)}+\frac{\log\left(a+cx^4\right)}{4c^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.82

$$\frac{\frac{a}{a+cx^4}+\log\left(a+cx^4\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + c*x^4)^2,x]
```

```
[Out] (a/(a + c*x^4) + Log[a + c*x^4])/(4*c^2)
```

fricas [A] time = 0.64, size = 35, normalized size = 1.06

$$\frac{\left(cx^4+a\right)\log\left(cx^4+a\right)+a}{4\left(c^3x^4+ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+a)^2,x, algorithm="fricas")
[Out] 1/4*((c*x^4 + a)*log(c*x^4 + a) + a)/(c^3*x^4 + a*c^2)
```

giac [A] time = 0.20, size = 48, normalized size = 1.45

$$-\frac{\frac{\log\left(\frac{\left|cx^4+a\right|}{\left(cx^4+a\right)^2|c|}\right)}{c}-\frac{a}{\left(cx^4+a\right)c}}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+a)^2,x, algorithm="giac")
[Out] -1/4*(log(abs(c*x^4 + a)/((c*x^4 + a)^2*abs(c)))/c - a/((c*x^4 + a)*c))/c
```

maple [A] time = 0.01, size = 30, normalized size = 0.91

$$\frac{a}{4\left(c\,x^4+a\right)c^2}+\frac{\ln\left(c\,x^4+a\right)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+a)^2,x)
[Out] 1/4*a/c^2/(c*x^4+a)+1/4*ln(c*x^4+a)/c^2
```

maxima [A] time = 1.28, size = 32, normalized size = 0.97

$$\frac{a}{4\left(c^3x^4+ac^2\right)}+\frac{\log\left(cx^4+a\right)}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+a)^2,x, algorithm="maxima")
[Out] 1/4*a/(c^3*x^4 + a*c^2) + 1/4*log(c*x^4 + a)/c^2
```

mupad [B] time = 0.05, size = 29, normalized size = 0.88

$$\frac{\ln\left(c\,x^4+a\right)}{4\,c^2}+\frac{a}{4\,c^2\left(c\,x^4+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + c*x^4)^2,x)
[Out] log(a + c*x^4)/(4*c^2) + a/(4*c^2*(a + c*x^4))
```

sympy [A] time = 0.34, size = 29, normalized size = 0.88

$$\frac{a}{4ac^2+4c^3x^4}+\frac{\log\left(a+cx^4\right)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+a)**2,x)
[Out] a/(4*a*c**2 + 4*c**3*x**4) + log(a + c*x**4)/(4*c**2)
```

3.661 $\int \frac{x^5}{\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=49

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4\sqrt{a}\,c^{3/2}}-\frac{x^2}{4c\left(a+cx^4\right)}$$

```
[Out] -1/4*x^2/c/(c*x^4+a)+1/4*arctan(x^2*c^(1/2)/a^(1/2))/c^(3/2)/a^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {275, 288, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4\sqrt{a}\,c^{3/2}}-\frac{x^2}{4c\left(a+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + c*x^4)^2,x]
```

```
[Out] -x^2/(4*c*(a + c*x^4)) + ArcTan[(Sqrt[c]*x^2)/Sqrt[a]]/(4*Sqrt[a]*c^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\left(a+cx^4\right)^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\left(a+cx^2\right)^2}\,dx, x, x^2\right)\\ &= -\frac{x^2}{4c\left(a+cx^4\right)}+\frac{\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{4c}\\ &= -\frac{x^2}{4c\left(a+cx^4\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4\sqrt{a}\,c^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4\sqrt{a}\,c^{3/2}} - \frac{x^2}{4c\left(a+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + c*x^4)^2,x]
```

```
[Out] -1/4*x^2/(c*(a + c*x^4)) + ArcTan[(Sqrt[c]*x^2)/Sqrt[a]]/(4*Sqrt[a]*c^(3/2)
)
```

fricas [A] time = 0.61, size = 127, normalized size = 2.59

$$\left[-\frac{2\,acx^2+\left(cx^4+a\right)\sqrt{-ac}\,\log\left(\frac{cx^4-2\,\sqrt{-ac}\,x^2-a}{cx^4+a}\right)}{8\left(ac^3x^4+a^2c^2\right)}, -\frac{acx^2+\left(cx^4+a\right)\sqrt{ac}\,\arctan\left(\frac{\sqrt{ac}}{cx^2}\right)}{4\left(ac^3x^4+a^2c^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(2*a*c*x^2 + (c*x^4 + a)*sqrt(-a*c)*log((c*x^4 - 2*sqrt(-a*c)*x^2 - a
)/(c*x^4 + a)))/(a*c^3*x^4 + a^2*c^2), -1/4*(a*c*x^2 + (c*x^4 + a)*sqrt(a*c
)*arctan(sqrt(a*c)/(c*x^2)))/(a*c^3*x^4 + a^2*c^2)]
```

giac [A] time = 0.16, size = 39, normalized size = 0.80

$$-\frac{x^2}{4\left(cx^4+a\right)c} + \frac{\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\sqrt{ac}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*x^2/((c*x^4 + a)*c) + 1/4*arctan(c*x^2/sqrt(a*c))/(sqrt(a*c)*c)
```

maple [A] time = 0.01, size = 40, normalized size = 0.82

$$-\frac{x^2}{4\left(c\,x^4+a\right)c} + \frac{\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{4\sqrt{ac}\;c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+a)^2,x)
```

```
[Out] -1/4*x^2/c/(c*x^4+a)+1/4/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x^2)
```

maxima [A] time = 2.97, size = 40, normalized size = 0.82

$$-\frac{x^2}{4\left(c^2x^4+ac\right)} + \frac{\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\sqrt{ac}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*x^2/(c^2*x^4 + a*c) + 1/4*arctan(c*x^2/sqrt(a*c))/(sqrt(a*c)*c)
```

mupad [B] time = 0.07, size = 37, normalized size = 0.76

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4\sqrt{a}\,c^{3/2}}-\frac{x^2}{4c\left(cx^4+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + c*x^4)^2,x)
```

```
[Out] atan((c^(1/2)*x^2)/a^(1/2))/(4*a^(1/2)*c^(3/2)) - x^2/(4*c*(a + c*x^4))
```

sympy [B] time = 0.48, size = 83, normalized size = 1.69

$$-\frac{x^2}{4ac+4c^2x^4}-\frac{\sqrt{-\frac{1}{ac^3}}\log\left(-ac\sqrt{-\frac{1}{ac^3}}+x^2\right)}{8}+\frac{\sqrt{-\frac{1}{ac^3}}\log\left(ac\sqrt{-\frac{1}{ac^3}}+x^2\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+a)**2,x)
```

```
[Out] -x**2/(4*a*c + 4*c**2*x**4) - sqrt(-1/(a*c**3))*log(-a*c*sqrt(-1/(a*c**3))
+ x**2)/8 + sqrt(-1/(a*c**3))*log(a*c*sqrt(-1/(a*c**3)) + x**2)/8
```

3.662 $\int \frac{x^3}{\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{4c\left(a+cx^4\right)}$$

```
[Out] -1/4/c/(c*x^4+a)
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$-\frac{1}{4c\left(a+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + c*x^4)^2,x]
```

```
[Out] -1/(4*c*(a + c*x^4))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^3}{\left(a+cx^4\right)^2}\,dx = -\frac{1}{4c\left(a+cx^4\right)}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{4c\left(a+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + c*x^4)^2,x]
```

```
[Out] -1/4*1/(c*(a + c*x^4))
```

fricas [A] time = 0.73, size = 15, normalized size = 0.94

$$-\frac{1}{4\left(c^2x^4+ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4/(c^2*x^4 + a*c)
```

giac [A] time = 0.16, size = 14, normalized size = 0.88

$$-\frac{1}{4\left(cx^4+a\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+a)^2,x, algorithm="giac")
[Out] -1/4/((c*x^4 + a)*c)
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$-\frac{1}{4\left(c\,x^4+a\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+a)^2,x)
[Out] -1/4/c/(c*x^4+a)
```

maxima [A] time = 1.33, size = 14, normalized size = 0.88

$$-\frac{1}{4\left(cx^4+a\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+a)^2,x, algorithm="maxima")
[Out] -1/4/((c*x^4 + a)*c)
```

mupad [B] time = 0.95, size = 14, normalized size = 0.88

$$-\frac{1}{4\,c\,\left(c\,x^4+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + c*x^4)^2,x)
[Out] -1/(4*c*(a + c*x^4))
```

sympy [A] time = 0.65, size = 15, normalized size = 0.94

$$-\frac{1}{4ac+4c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+a)**2,x)
[Out] -1/(4*a*c + 4*c**2*x**4)
```

3.663 $\int \frac{x}{\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=49

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}}+\frac{x^2}{4a\left(a+cx^4\right)}$$

```
[Out] 1/4*x^2/a/(c*x^4+a)+1/4*arctan(x^2*c^(1/2)/a^(1/2))/a^(3/2)/c^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {275, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}}+\frac{x^2}{4a\left(a+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + c*x^4)^2,x]
```

```
[Out] x^2/(4*a*(a + c*x^4)) + ArcTan[(Sqrt[c]*x^2)/Sqrt[a]]/(4*a^(3/2)*Sqrt[c])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a+cx^4\right)^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\left(a+cx^2\right)^2}\,dx, x, x^2\right)\\ &= \frac{x^2}{4a\left(a+cx^4\right)}+\frac{\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{4a}\\ &= \frac{x^2}{4a\left(a+cx^4\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 49, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}} + \frac{x^2}{4a\left(a+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + c*x^4)^2,x]
```

```
[Out] x^2/(4*a*(a + c*x^4)) + ArcTan[(Sqrt[c]*x^2)/Sqrt[a]]/(4*a^(3/2)*Sqrt[c])
```

fricas [A] time = 0.63, size = 129, normalized size = 2.63

$$\left[\frac{2\,acx^2 - \left(cx^4+a\right)\sqrt{-ac}\,\log\left(\frac{cx^4-2\sqrt{-ac}\,x^2-a}{cx^4+a}\right)}{8\left(a^2c^2x^4+a^3c\right)}, \frac{acx^2 - \left(cx^4+a\right)\sqrt{ac}\,\arctan\left(\frac{\sqrt{ac}}{cx^2}\right)}{4\left(a^2c^2x^4+a^3c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(2*a*c*x^2 - (c*x^4 + a)*sqrt(-a*c)*log((c*x^4 - 2*sqrt(-a*c)*x^2 - a)
/(c*x^4 + a)))/(a^2*c^2*x^4 + a^3*c), 1/4*(a*c*x^2 - (c*x^4 + a)*sqrt(a*c)*
arctan(sqrt(a*c)/(c*x^2)))/(a^2*c^2*x^4 + a^3*c)]
```

giac [A] time = 0.16, size = 39, normalized size = 0.80

$$\frac{x^2}{4\left(cx^4+a\right)a} + \frac{\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\sqrt{ac}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*x^2/((c*x^4 + a)*a) + 1/4*arctan(c*x^2/sqrt(a*c))/(sqrt(a*c)*a)
```

maple [A] time = 0.01, size = 40, normalized size = 0.82

$$\frac{x^2}{4\left(c\,x^4+a\right)a} + \frac{\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{4\sqrt{ac}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+a)^2,x)
```

```
[Out] 1/4*x^2/a/(c*x^4+a)+1/4/a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x^2)
```

maxima [A] time = 2.89, size = 39, normalized size = 0.80

$$\frac{x^2}{4\left(acx^4+a^2\right)} + \frac{\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\sqrt{ac}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^2/(a*c*x^4 + a^2) + 1/4*arctan(c*x^2/sqrt(a*c))/(sqrt(a*c)*a)
```

mupad [B] time = 0.04, size = 37, normalized size = 0.76

$$\frac{x^2}{4\,a\,\left(c\,x^4+a\right)}+\frac{\operatorname{atan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4\,a^{3/2}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + c*x^4)^2,x)
```

```
[Out] x^2/(4*a*(a + c*x^4)) + atan((c^(1/2)*x^2)/a^(1/2))/(4*a^(3/2)*c^(1/2))
```

sympy [B] time = 0.53, size = 83, normalized size = 1.69

$$\frac{x^2}{4a^2+4acx^4}-\frac{\sqrt{-\frac{1}{a^3c}}\,\log\left(-a^2\sqrt{-\frac{1}{a^3c}}+x^2\right)}{8}+\frac{\sqrt{-\frac{1}{a^3c}}\,\log\left(a^2\sqrt{-\frac{1}{a^3c}}+x^2\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+a)**2,x)
```

```
[Out] x**2/(4*a**2 + 4*a*c*x**4) - sqrt(-1/(a**3*c))*log(-a**2*sqrt(-1/(a**3*c))
+ x**2)/8 + sqrt(-1/(a**3*c))*log(a**2*sqrt(-1/(a**3*c)) + x**2)/8
```

3.664 $\int \frac{1}{x\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=38

$$-\frac{\log\left(a+cx^4\right)}{4a^2}+\frac{\log(x)}{a^2}+\frac{1}{4a\left(a+cx^4\right)}$$

```
[Out] 1/4/a/(c*x^4+a)+ln(x)/a^2-1/4*ln(c*x^4+a)/a^2
```

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 44}

$$-\frac{\log\left(a+cx^4\right)}{4a^2}+\frac{\log(x)}{a^2}+\frac{1}{4a\left(a+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + c*x^4)^2),x]
```

```
[Out] 1/(4*a*(a + c*x^4)) + Log[x]/a^2 - Log[a + c*x^4]/(4*a^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a+cx^4\right)^2}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x(a+cx)^2}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int \left(\frac{1}{a^2x}-\frac{c}{a(a+cx)^2}-\frac{c}{a^2(a+cx)}\right)dx, x, x^4\right)\\ &= \frac{1}{4a\left(a+cx^4\right)}+\frac{\log(x)}{a^2}-\frac{\log\left(a+cx^4\right)}{4a^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.87

$$\frac{\frac{a}{a+cx^4}-\log\left(a+cx^4\right)+4\log(x)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + c*x^4)^2),x]
```

```
[Out] (a/(a + c*x^4) + 4*Log[x] - Log[a + c*x^4])/(4*a^2)
```

fricas [A] time = 0.68, size = 47, normalized size = 1.24

$$-\frac{\left(cx^4+a\right)\log\left(cx^4+a\right)-4\left(cx^4+a\right)\log(x)-a}{4\left(a^2cx^4+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*((c*x^4 + a)*log(c*x^4 + a) - 4*(c*x^4 + a)*log(x) - a)/(a^2*c*x^4 + a
^3)
```

giac [A] time = 0.15, size = 47, normalized size = 1.24

$$\frac{\log\left(x^4\right)}{4\,a^2}-\frac{\log\left(\left|cx^4+a\right|\right)}{4\,a^2}+\frac{cx^4+2\,a}{4\left(cx^4+a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*log(x^4)/a^2 - 1/4*log(abs(c*x^4 + a))/a^2 + 1/4*(c*x^4 + 2*a)/((c*x^4
+ a)*a^2)
```

maple [A] time = 0.02, size = 35, normalized size = 0.92

$$\frac{1}{4\left(c\,x^4+a\right)a}+\frac{\ln(x)}{a^2}-\frac{\ln\left(c\,x^4+a\right)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+a)^2,x)
```

```
[Out] 1/4/a/(c*x^4+a)+1/a^2*ln(x)-1/4*ln(c*x^4+a)/a^2
```

maxima [A] time = 1.27, size = 37, normalized size = 0.97

$$\frac{1}{4\left(acx^4+a^2\right)}-\frac{\log\left(cx^4+a\right)}{4\,a^2}+\frac{\log\left(x^4\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4/(a*c*x^4 + a^2) - 1/4*log(c*x^4 + a)/a^2 + 1/4*log(x^4)/a^2
```

mupad [B] time = 0.07, size = 34, normalized size = 0.89

$$\frac{\ln(x)}{a^2}+\frac{1}{4\,a\,\left(c\,x^4+a\right)}-\frac{\ln\left(c\,x^4+a\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + c*x^4)^2),x)
```

```
[Out] log(x)/a^2 + 1/(4*a*(a + c*x^4)) - log(a + c*x^4)/(4*a^2)
```

sympy [A] time = 0.57, size = 34, normalized size = 0.89

$$\frac{1}{4a^2+4acx^4}+\frac{\log(x)}{a^2}-\frac{\log\left(\frac{a}{c}+x^4\right)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+a)**2,x)
[Out] 1/(4*a**2 + 4*a*c*x**4) + log(x)/a**2 - log(a/c + x**4)/(4*a**2)
```

3.665 $\int \frac{1}{x^3\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=59

$$-\frac{3\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{5/2}}-\frac{3}{4a^2x^2}+\frac{1}{4ax^2\left(a+cx^4\right)}$$

```
[Out] -3/4/a^2/x^2+1/4/a/x^2/(c*x^4+a)-3/4*arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/a^
(5/2)
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {275, 290, 325, 205}

$$-\frac{3\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{5/2}}-\frac{3}{4a^2x^2}+\frac{1}{4ax^2\left(a+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + c*x^4)^2),x]
```

```
[Out] -3/(4*a^2*x^2) + 1/(4*a*x^2*(a + c*x^4)) - (3*Sqrt[c]*ArcTan[(Sqrt[c]*x^2)/
Sqrt[a]])/(4*a^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+cx^4\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\left(a+cx^2\right)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{4ax^2\left(a+cx^4\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{x^2\left(a+cx^2\right)}\,dx, x, x^2\right)}{4a}$$
$$= -\frac{3}{4a^2x^2} + \frac{1}{4ax^2\left(a+cx^4\right)} - \frac{(3c)\,\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{4a^2}$$
$$= -\frac{3}{4a^2x^2} + \frac{1}{4ax^2\left(a+cx^4\right)} - \frac{3\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{5/2}}$$

Mathematica [A] time = 0.05, size = 94, normalized size = 1.59

$$\frac{-\frac{\sqrt{a}\left(2a+3cx^4\right)}{x^2\left(a+cx^4\right)} + 3\sqrt{c}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right) + 3\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{4a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + c*x^4)^2),x]
```

```
[Out] (-((Sqrt[a]*(2*a + 3*c*x^4))/(x^2*(a + c*x^4))) + 3*Sqrt[c]*ArcTan[1 - (Sqr
t[2]*c^(1/4)*x)/a^(1/4)] + 3*Sqrt[c]*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)
])/(4*a^(5/2))
```

fricas [A] time = 0.72, size = 152, normalized size = 2.58

$$\left[-\frac{6\,cx^4 - 3\left(cx^6+ax^2\right)\sqrt{-\frac{c}{a}}\,\log\left(\frac{cx^4-2\,ax^2\sqrt{-\frac{c}{a}}-a}{cx^4+a}\right) + 4\,a}{8\left(a^2cx^6+a^3x^2\right)}, -\frac{3\,cx^4 - 3\left(cx^6+ax^2\right)\sqrt{\frac{c}{a}}\,\arctan\left(\frac{a\sqrt{\frac{c}{a}}}{cx^2}\right) + 2\,a}{4\left(a^2cx^6+a^3x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(6*c*x^4 - 3*(c*x^6 + a*x^2)*sqrt(-c/a)*log((c*x^4 - 2*a*x^2*sqrt(-c/
a) - a)/(c*x^4 + a)) + 4*a)/(a^2*c*x^6 + a^3*x^2), -1/4*(3*c*x^4 - 3*(c*x^6
 + a*x^2)*sqrt(c/a)*arctan(a*sqrt(c/a)/(c*x^2)) + 2*a)/(a^2*c*x^6 + a^3*x^2
)]
```

giac [A] time = 0.19, size = 51, normalized size = 0.86

$$-\frac{3\,c\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\sqrt{ac}\,a^2} - \frac{3\,cx^4+2\,a}{4\left(cx^6+ax^2\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -3/4*c*arctan(c*x^2/sqrt(a*c))/(sqrt(a*c)*a^2) - 1/4*(3*c*x^4 + 2*a)/((c*x^
6 + a*x^2)*a^2)
```

maple [A] time = 0.02, size = 50, normalized size = 0.85

$$-\frac{c\,x^2}{4\left(c\,x^4+a\right)a^2}-\frac{3c\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{4\sqrt{ac}\;a^2}-\frac{1}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+a)^2,x)
```

```
[Out] -1/4*c/a^2*x^2/(c*x^4+a)-3/4*c/a^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x^2)-
1/2/a^2/x^2
```

maxima [A] time = 2.99, size = 53, normalized size = 0.90

$$-\frac{3\,cx^4+2\,a}{4\left(a^2cx^6+a^3x^2\right)}-\frac{3\,c\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\,\sqrt{ac}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(3*c*x^4 + 2*a)/(a^2*c*x^6 + a^3*x^2) - 3/4*c*arctan(c*x^2/sqrt(a*c))/
(sqrt(a*c)*a^2)
```

mupad [B] time = 1.02, size = 50, normalized size = 0.85

$$-\frac{\frac{1}{2\,a}+\frac{3\,c\,x^4}{4\,a^2}}{c\,x^6+a\,x^2}-\frac{3\,\sqrt{c}\;\mathrm{atan}\left(\frac{\sqrt{c}\;x^2}{\sqrt{a}}\right)}{4\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + c*x^4)^2),x)
```

```
[Out] - (1/(2*a) + (3*c*x^4)/(4*a^2))/(a*x^2 + c*x^6) - (3*c^(1/2)*atan((c^(1/2)*
x^2)/a^(1/2)))/(4*a^(5/2))
```

sympy [A] time = 0.87, size = 97, normalized size = 1.64

$$\frac{3\sqrt{-\frac{c}{a^5}}\,\log\left(-\frac{a^3\sqrt{-\frac{c}{a^5}}}{c}+x^2\right)}{8}-\frac{3\sqrt{-\frac{c}{a^5}}\,\log\left(\frac{a^3\sqrt{-\frac{c}{a^5}}}{c}+x^2\right)}{8}+\frac{-2a-3cx^4}{4a^3x^2+4a^2cx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+a)**2,x)
```

```
[Out] 3*sqrt(-c/a**5)*log(-a**3*sqrt(-c/a**5)/c + x**2)/8 - 3*sqrt(-c/a**5)*log(a
**3*sqrt(-c/a**5)/c + x**2)/8 + (-2*a - 3*c*x**4)/(4*a**3*x**2 + 4*a**2*c*x
**6)
```

3.666 $\int \frac{x^6}{\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=204

$$\frac{3\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,\sqrt[4]{a}\,c^{7/4}}-\frac{3\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,\sqrt[4]{a}\,c^{7/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,\sqrt[4]{a}\,c^{7/4}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}}{\sqrt[4]{a}}\right.}{8\sqrt{2}\,\sqrt[4]{a}\,c}$$

```
[Out] -1/4*x^3/c/(c*x^4+a)+3/16*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(1/4)/c^(7
/4)*2^(1/2)+3/16*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(1/4)/c^(7/4)*2^(1/2
)+3/32*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(1/4)/c^(7/4)*2
^(1/2)-3/32*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(1/4)/c^(7/
4)*2^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {288, 297, 1162, 617, 204, 1165, 628}

$$\frac{3\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,\sqrt[4]{a}\,c^{7/4}}-\frac{3\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,\sqrt[4]{a}\,c^{7/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,\sqrt[4]{a}\,c^{7/4}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}}{\sqrt[4]{a}}\right.}{8\sqrt{2}\,\sqrt[4]{a}\,c}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + c*x^4)^2,x]
```

```
[Out] -x^3/(4*c*(a + c*x^4)) - (3*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqr
t[2]*a^(1/4)*c^(7/4)) + (3*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt
[2]*a^(1/4)*c^(7/4)) + (3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]
*x^2])/(16*Sqrt[2]*a^(1/4)*c^(7/4)) - (3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(1/4)*c^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^6}{\left(a+cx^4\right)^2}\,dx = -\frac{x^3}{4c\left(a+cx^4\right)} + \frac{3\int \frac{x^2}{a+cx^4}\,dx}{4c}$$

$$= -\frac{x^3}{4c\left(a+cx^4\right)} - \frac{3\int \frac{\sqrt{a}-\sqrt{c}\,x^2}{a+cx^4}\,dx}{8c^{3/2}} + \frac{3\int \frac{\sqrt{a}+\sqrt{c}\,x^2}{a+cx^4}\,dx}{8c^{3/2}}$$

$$= -\frac{x^3}{4c\left(a+cx^4\right)} + \frac{3\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{16c^2} + \frac{3\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{16c^2} + \frac{3\int \frac{\frac{\sqrt{2}\,\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}\,\sqrt[4]{a}\,c^{7/4}} + \frac{3\int -$$

$$= -\frac{x^3}{4c\left(a+cx^4\right)} + \frac{3\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,\sqrt[4]{a}\,c^{7/4}} - \frac{3\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,\sqrt[4]{a}\,c^{7/4}} + \frac{3\,9}{-}$$

$$= -\frac{x^3}{4c\left(a+cx^4\right)} - \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,\sqrt[4]{a}\,c^{7/4}} + \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,\sqrt[4]{a}\,c^{7/4}} + \frac{3\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{}\right.}{16\sqrt{2}\,\sqrt[4]{a}\,c^{7/4}}$$

Mathematica [A] time = 0.12, size = 185, normalized size = 0.91

$$\frac{-\frac{8c^{3/4}x^3}{a+cx^4} + \frac{3\sqrt{2}\,\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{\sqrt[4]{a}} - \frac{3\sqrt{2}\,\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{\sqrt[4]{a}} - \frac{6\sqrt{2}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}} + \frac{6\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{\sqrt[4]{a}}}{32c^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + c*x^4)^2,x]
```

```
[Out] ((-8*c^(3/4)*x^3)/(a + c*x^4) - (6*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a
^(1/4)])/a^(1/4) + (6*Sqrt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(1
/4) + (3*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^
```

```
(1/4) - (3*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/
a^(1/4))/(32*c^(7/4))
```

fricas [A] time = 0.73, size = 181, normalized size = 0.89

$$-\frac{4\,x^3+12\left(c^2x^4+ac\right)\left(-\frac{1}{ac^7}\right)^{\frac{1}{4}}\arctan\left(-c^2x\left(-\frac{1}{ac^7}\right)^{\frac{1}{4}}+\sqrt{-ac^3\sqrt{-\frac{1}{ac^7}}+x^2}\,c^2\left(-\frac{1}{ac^7}\right)^{\frac{1}{4}}\right)-3\left(c^2x^4+ac\right)\left(-\frac{1}{ac^7}\right)^{\frac{1}{4}}}{16\left(c^2x^4+ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*x^3 + 12*(c^2*x^4 + a*c)*(-1/(a*c^7))^(1/4)*arctan(-c^2*x*(-1/(a*c
^7))^(1/4) + sqrt(-a*c^3*sqrt(-1/(a*c^7)) + x^2)*c^2*(-1/(a*c^7))^(1/4)) -
3*(c^2*x^4 + a*c)*(-1/(a*c^7))^(1/4)*log(a*c^5*(-1/(a*c^7))^(3/4) + x) + 3*
(c^2*x^4 + a*c)*(-1/(a*c^7))^(1/4)*log(-a*c^5*(-1/(a*c^7))^(3/4) + x))/(c^2
*x^4 + a*c)
```

giac [A] time = 0.18, size = 196, normalized size = 0.96

$$-\frac{x^3}{4\left(cx^4+a\right)c}+\frac{3\sqrt{2}\left(ac^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,ac^4}+\frac{3\sqrt{2}\left(ac^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,ac^4}-\frac{3\sqrt{2}\left(ac^3\right)^{\frac{3}{4}}\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*x^3/((c*x^4 + a)*c) + 3/16*sqrt(2)*(a*c^3)^(3/4)*arctan(1/2*sqrt(2)*(2
*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^4) + 3/16*sqrt(2)*(a*c^3)^(3/4)
*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^4) - 3/32
*sqrt(2)*(a*c^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^4)
 + 3/32*sqrt(2)*(a*c^3)^(3/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/
(a*c^4)
```

maple [A] time = 0.01, size = 145, normalized size = 0.71

$$-\frac{x^3}{4\left(c\,x^4+a\right)c}+\frac{3\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{16\left(\frac{a}{c}\right)^{\frac{1}{4}}c^2}+\frac{3\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{16\left(\frac{a}{c}\right)^{\frac{1}{4}}c^2}+\frac{3\sqrt{2}\ \ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{32\left(\frac{a}{c}\right)^{\frac{1}{4}}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+a)^2,x)
```

```
[Out] -1/4*x^3/c/(c*x^4+a)+3/32/c^2/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/
2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+3/16/c^2/(a/c)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+3/16/c^2/(a/c)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(a/c)^(1/4)*x-1)
```

maxima [A] time = 3.01, size = 192, normalized size = 0.94

$$-\frac{x^3}{4\left(c^2x^4+ac\right)}+\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left($$

$$32\,c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*x^3/(c^2*x^4 + a*c) + 3/32*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x
+ sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(sqrt(a)*sqrt(c))*sq
rt(c)) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4
))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(sqrt(a)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqr
t(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(1/4)*c^(3/4)) + sqrt(2)
*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(1/4)*c^(3/4)))/
c
```

mupad [B] time = 1.08, size = 60, normalized size = 0.29

$$\frac{3\,\mathrm{atan}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{8\,(-a)^{1/4}\,c^{7/4}}-\frac{3\,\mathrm{atanh}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{8\,(-a)^{1/4}\,c^{7/4}}-\frac{x^3}{4\,c\,\left(c\,x^4+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + c*x^4)^2,x)
```

```
[Out] (3*atan((c^(1/4)*x)/(-a)^(1/4)))/(8*(-a)^(1/4)*c^(7/4)) - (3*atanh((c^(1/4)
*x)/(-a)^(1/4)))/(8*(-a)^(1/4)*c^(7/4)) - x^3/(4*c*(a + c*x^4))
```

sympy [A] time = 0.37, size = 44, normalized size = 0.22

$$-\frac{x^3}{4ac+4c^2x^4}+\mathrm{RootSum}\left(65536t^4ac^7+81,\left(t\mapsto t\log\left(\frac{4096t^3ac^5}{27}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+a)**2,x)
```

```
[Out] -x**3/(4*a*c + 4*c**2*x**4) + RootSum(65536*_t**4*a*c**7 + 81, Lambda(_t, _
t*log(4096*_t**3*a*c**5/27 + x)))
```

3.667 $\int \frac{x^4}{\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=202

$$-\frac{\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{3/4}c^{5/4}}+\frac{\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{3/4}c^{5/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{3/4}c^{5/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right.}{8\sqrt{2}\,a^{3/4}c^{5/4}}$$

```
[Out] -1/4*x/c/(c*x^4+a)+1/16*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/c^(5/4
)*2^(1/2)+1/16*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/c^(5/4)*2^(1/2)-
1/32*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(3/4)/c^(5/4)*2^(
1/2)+1/32*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(3/4)/c^(5/4)
*2^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {288, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{3/4}c^{5/4}}+\frac{\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{3/4}c^{5/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{3/4}c^{5/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right.}{8\sqrt{2}\,a^{3/4}c^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + c*x^4)^2,x]
```

```
[Out] -x/(4*c*(a + c*x^4)) - ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(8*Sqrt[2]*a
^(3/4)*c^(5/4)) + ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(8*Sqrt[2]*a^(3/4
)*c^(5/4)) - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(16*Sqr
t[2]*a^(3/4)*c^(5/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x
^2]/(16*Sqrt[2]*a^(3/4)*c^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^4}{\left(a+cx^4\right)^2}\,dx = -\frac{x}{4c\left(a+cx^4\right)} + \frac{\int \frac{1}{a+cx^4}\,dx}{4c}$$

$$= -\frac{x}{4c\left(a+cx^4\right)} + \frac{\int \frac{\sqrt{a}-\sqrt{c}\,x^2}{a+cx^4}\,dx}{8\sqrt{a}\,c} + \frac{\int \frac{\sqrt{a}+\sqrt{c}\,x^2}{a+cx^4}\,dx}{8\sqrt{a}\,c}$$

$$= -\frac{x}{4c\left(a+cx^4\right)} + \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{16\sqrt{a}\,c^{3/2}} + \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{16\sqrt{a}\,c^{3/2}} - \frac{\int \frac{\frac{\sqrt{2}\,\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}\,a^{3/4}c^{5/4}} - \frac{\int \frac{\frac{\sqrt{2}\,\sqrt[4]{a}}{\sqrt[4]{c}}}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}}{}}}{16\sqrt{2}\,a}$$

$$= -\frac{x}{4c\left(a+cx^4\right)} - \frac{\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{3/4}c^{5/4}} + \frac{\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{3/4}c^{5/4}} + \frac{\text{Subs}}{}$$

$$= -\frac{x}{4c\left(a+cx^4\right)} - \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{3/4}c^{5/4}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{3/4}c^{5/4}} - \frac{\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{3/4}c^{5/4}}$$

Mathematica [A] time = 0.12, size = 182, normalized size = 0.90

$$\frac{-\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{a^{3/4}} + \frac{\sqrt{2}\,\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{a^{3/4}} - \frac{2\sqrt{2}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{a^{3/4}} + \frac{2\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{a^{3/4}} - \frac{8\sqrt[4]{c}\,x}{a+cx^4}}{32c^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + c*x^4)^2,x]
```

```
[Out] ((-8*c^(1/4)*x)/(a + c*x^4) - (2*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(
1/4)])/a^(3/4) + (2*Sqrt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(3/4
) - (Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(3/4
```

```
) + (Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(3/4
))/(32*c^(5/4))
```

fricas [A] time = 0.80, size = 181, normalized size = 0.90

$$\frac{4\left(c^2x^4+ac\right)\left(-\frac{1}{a^3c^5}\right)^{\frac{1}{4}}\arctan\left(-a^2c^4x\left(-\frac{1}{a^3c^5}\right)^{\frac{3}{4}}+\sqrt{a^2c^2\sqrt{-\frac{1}{a^3c^5}}+x^2}\,a^2c^4\left(-\frac{1}{a^3c^5}\right)^{\frac{3}{4}}\right)+\left(c^2x^4+ac\right)\left(-\frac{1}{a^3c^5}\right)^{\frac{1}{4}}\mathrm{lo}}{16\left(c^2x^4+ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*(c^2*x^4 + a*c)*(-1/(a^3*c^5))^(1/4)*arctan(-a^2*c^4*x*(-1/(a^3*c^5
))^(3/4) + sqrt(a^2*c^2*sqrt(-1/(a^3*c^5)) + x^2)*a^2*c^4*(-1/(a^3*c^5))^(3
/4)) + (c^2*x^4 + a*c)*(-1/(a^3*c^5))^(1/4)*log(a*c*(-1/(a^3*c^5))^(1/4) +
x) - (c^2*x^4 + a*c)*(-1/(a^3*c^5))^(1/4)*log(-a*c*(-1/(a^3*c^5))^(1/4) + x
) - 4*x)/(c^2*x^4 + a*c)
```

giac [A] time = 0.16, size = 194, normalized size = 0.96

$$-\frac{x}{4\left(cx^4+a\right)c}+\frac{\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,ac^2}+\frac{\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,ac^2}+\frac{\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\log\left(x^2-\right.}{32\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*x/((c*x^4 + a)*c) + 1/16*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x
 + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^2) + 1/16*sqrt(2)*(a*c^3)^(1/4)*a
rctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^2) + 1/32*s
qrt(2)*(a*c^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^2) -
 1/32*sqrt(2)*(a*c^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a
*c^2)
```

maple [A] time = 0.01, size = 152, normalized size = 0.75

$$-\frac{x}{4\left(c\,x^4+a\right)c}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{16ac}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{16ac}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{32ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+a)^2,x)
```

```
[Out] -1/4*x/c/(c*x^4+a)+1/32/c*(a/c)^(1/4)/a*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)
*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+1/16/c*(a/c)^(1/4)
/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+1/16/c*(a/c)^(1/4)/a*2^(1/2)*arc
tan(2^(1/2)/(a/c)^(1/4)*x-1)
```

maxima [A] time = 2.96, size = 190, normalized size = 0.94

$$-\frac{x}{4\left(c^2x^4+ac\right)}+\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\right.}{}}{32\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*x/(c^2*x^4 + a*c) + 1/32*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x +
sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(
c))) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))
/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + sqrt(2)*log(sqrt(
c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - sqrt(2)*l
og(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)))/c
```

mupad [**B**] time = 1.06, size = 58, normalized size = 0.29

$$-\frac{x}{4\,c\,\left(c\,x^4+a\right)}-\frac{\mathrm{atan}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{8\,(-a)^{3/4}\,c^{5/4}}-\frac{\mathrm{atanh}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{8\,(-a)^{3/4}\,c^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + c*x^4)^2,x)
```

```
[Out] - x/(4*c*(a + c*x^4)) - atan((c^(1/4)*x)/(-a)^(1/4))/(8*(-a)^(3/4)*c^(5/4))
 - atanh((c^(1/4)*x)/(-a)^(1/4))/(8*(-a)^(3/4)*c^(5/4))
```

sympy [**A**] time = 0.54, size = 39, normalized size = 0.19

$$-\frac{x}{4ac+4c^2x^4}+\mathrm{RootSum}\left(65536t^4a^3c^5+1,\left(t\mapsto t\log\left(16tac+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+a)**2,x)
```

```
[Out] -x/(4*a*c + 4*c**2*x**4) + RootSum(65536*_t**4*a**3*c**5 + 1, Lambda(_t, _t
*log(16*_t*a*c + x)))
```

3.668 $\int \frac{x^2}{\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=204

$$\frac{\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{5/4}c^{3/4}}-\frac{\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{5/4}c^{3/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{5/4}c^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}\,a^{5/4}c^{3/4}}$$

```
[Out] 1/4*x^3/a/(c*x^4+a)+1/16*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(5/4)/c^(3/
4)*2^(1/2)+1/16*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(5/4)/c^(3/4)*2^(1/2)
+1/32*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(5/4)/c^(3/4)*2^
(1/2)-1/32*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(5/4)/c^(3/4
)*2^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {290, 297, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{5/4}c^{3/4}}-\frac{\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{5/4}c^{3/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{5/4}c^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}\,a^{5/4}c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + c*x^4)^2,x]
```

```
[Out] x^3/(4*a*(a + c*x^4)) - ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(8*Sqrt[2]*
a^(5/4)*c^(3/4)) + ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(8*Sqrt[2]*a^(5/
4)*c^(3/4)) + Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(16*Sq
rt[2]*a^(5/4)*c^(3/4)) - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*
x^2]/(16*Sqrt[2]*a^(5/4)*c^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^2}{\left(a+cx^4\right)^2}\,dx = \frac{x^3}{4a\left(a+cx^4\right)} + \frac{\int \frac{x^2}{a+cx^4}\,dx}{4a}$$

$$= \frac{x^3}{4a\left(a+cx^4\right)} - \frac{\int \frac{\sqrt{a}-\sqrt{c}\,x^2}{a+cx^4}\,dx}{8a\sqrt{c}} + \frac{\int \frac{\sqrt{a}+\sqrt{c}\,x^2}{a+cx^4}\,dx}{8a\sqrt{c}}$$

$$= \frac{x^3}{4a\left(a+cx^4\right)} + \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}} + x^2}\,dx}{16ac} + \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}} + x^2}\,dx}{16ac} + \frac{\int \frac{\frac{\sqrt{2}\,\sqrt[4]{a}}{\sqrt[4]{c}} + 2x}{-\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}} - x^2}\,dx}{16\sqrt{2}\,a^{5/4}c^{3/4}} + \frac{\int \frac{\frac{\sqrt{2}\,\sqrt[4]{a}}{\sqrt[4]{c}}}{-\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\,\sqrt[4]{a}}{\sqrt[4]{c}}}}{16\sqrt{2}\,a^{5}}$$

$$= \frac{x^3}{4a\left(a+cx^4\right)} + \frac{\log\left(\sqrt{a} - \sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x + \sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{5/4}c^{3/4}} - \frac{\log\left(\sqrt{a} + \sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x + \sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{5/4}c^{3/4}} + \frac{\text{Subst}}{}$$

$$= \frac{x^3}{4a\left(a+cx^4\right)} - \frac{\tan^{-1}\left(1 - \frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{5/4}c^{3/4}} + \frac{\tan^{-1}\left(1 + \frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{5/4}c^{3/4}} + \frac{\log\left(\sqrt{a} - \sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x + \sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{5/4}c^{3/4}} -$$

Mathematica [A] time = 0.11, size = 184, normalized size = 0.90

$$\frac{\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{c^{3/4}} - \frac{\sqrt{2}\,\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{c^{3/4}} - \frac{2\sqrt{2}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{c^{3/4}} + \frac{2\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{c^{3/4}} + \frac{8\sqrt[4]{a}\,x^3}{a+cx^4}}{32a^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + c*x^4)^2,x]
```

```
[Out] ((8*a^(1/4)*x^3)/(a + c*x^4) - (2*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^
(1/4)])/c^(3/4) + (2*Sqrt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/
4) + (Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/
```

```
4) - (Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/
4))/(32*a^(5/4))
```

fricas [A] time = 0.66, size = 178, normalized size = 0.87

$$\frac{4\,x^3 - 4\left(acx^4+a^2\right)\left(-\frac{1}{a^5c^3}\right)^{\frac{1}{4}}\arctan\left(-acx\left(-\frac{1}{a^5c^3}\right)^{\frac{1}{4}} + \sqrt{-a^3c\sqrt{-\frac{1}{a^5c^3}}+x^2}\,ac\left(-\frac{1}{a^5c^3}\right)^{\frac{1}{4}}\right) + \left(acx^4+a^2\right)\left(-\frac{1}{a^5c^3}\right)}{16\left(acx^4+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*x^3 - 4*(a*c*x^4 + a^2)*(-1/(a^5*c^3))^(1/4)*arctan(-a*c*x*(-1/(a^5
*c^3))^(1/4) + sqrt(-a^3*c*sqrt(-1/(a^5*c^3)) + x^2)*a*c*(-1/(a^5*c^3))^(1/
4)) + (a*c*x^4 + a^2)*(-1/(a^5*c^3))^(1/4)*log(a^4*c^2*(-1/(a^5*c^3))^(3/4)
 + x) - (a*c*x^4 + a^2)*(-1/(a^5*c^3))^(1/4)*log(-a^4*c^2*(-1/(a^5*c^3))^(3
/4) + x))/(a*c*x^4 + a^2)
```

giac [A] time = 0.18, size = 196, normalized size = 0.96

$$\frac{x^3}{4\left(cx^4+a\right)a} + \frac{\sqrt{2}\,\left(ac^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3} + \frac{\sqrt{2}\,\left(ac^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3} - \frac{\sqrt{2}\,\left(ac^3\right)^{\frac{3}{4}}\log\left(x^2+\right.}{32\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*x^3/((c*x^4 + a)*a) + 1/16*sqrt(2)*(a*c^3)^(3/4)*arctan(1/2*sqrt(2)*(2*
x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) + 1/16*sqrt(2)*(a*c^3)^(3/4
)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) - 1
/32*sqrt(2)*(a*c^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2
*c^3) + 1/32*sqrt(2)*(a*c^3)^(3/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a
/c))/(a^2*c^3)
```

maple [A] time = 0.00, size = 154, normalized size = 0.75

$$\frac{x^3}{4\left(c\,x^4+a\right)a} + \frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{16\left(\frac{a}{c}\right)^{\frac{1}{4}}ac} + \frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{16\left(\frac{a}{c}\right)^{\frac{1}{4}}ac} + \frac{\sqrt{2}\,\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{32\left(\frac{a}{c}\right)^{\frac{1}{4}}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+a)^2,x)
```

```
[Out] 1/4*x^3/a/(c*x^4+a)+1/32/a/c/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2
)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+1/16/a/c/(a/c)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+1/16/a/c/(a/c)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(a/c)^(1/4)*x-1)
```

maxima [A] time = 3.08, size = 191, normalized size = 0.94

$$\frac{x^3}{4\left(acx^4+a^2\right)} + \frac{\frac{2\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{2\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} - \frac{\sqrt{2}\log\left(\sqrt{c}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}c^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(\sqrt{\right.}\right.}{32\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^3/(a*c*x^4 + a^2) + 1/32*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x +
 sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(sqrt(a)*sqrt(c))*sqr
t(c)) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4)
)/sqrt(sqrt(a)*sqrt(c)))/(sqrt(sqrt(a)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt
(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(1/4)*c^(3/4)) + sqrt(2)*
log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(1/4)*c^(3/4)))/a
```

mupad **[B]** time = 0.11, size = 60, normalized size = 0.29

$$\frac{\operatorname{atanh}\left(\frac{c^{1/4}x}{(-a)^{1/4}}\right)}{8\,(-a)^{5/4}\,c^{3/4}}-\frac{\operatorname{atan}\left(\frac{c^{1/4}x}{(-a)^{1/4}}\right)}{8\,(-a)^{5/4}\,c^{3/4}}+\frac{x^3}{4\,a\,\left(c\,x^4+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + c*x^4)^2,x)
```

```
[Out] atanh((c^(1/4)*x)/(-a)^(1/4))/(8*(-a)^(5/4)*c^(3/4)) - atan((c^(1/4)*x)/(-a
)^(1/4))/(8*(-a)^(5/4)*c^(3/4)) + x^3/(4*a*(a + c*x^4))
```

sympy **[A]** time = 0.64, size = 46, normalized size = 0.23

$$\frac{x^3}{4a^2+4acx^4}+\operatorname{RootSum}\left(65536t^4a^5c^3+1,\left(t\mapsto t\log\left(4096t^3a^4c^2+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+a)**2,x)
```

```
[Out] x**3/(4*a**2 + 4*a*c*x**4) + RootSum(65536*_t**4*a**5*c**3 + 1, Lambda(_t,
_t*log(4096*_t**3*a**4*c**2 + x)))
```

3.669 $\int \frac{1}{\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=202

$$-\frac{3\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}}+\frac{3\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{c}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}}{\sqrt[4]}\right.}{8\sqrt{2}\,a^{7/}}$$

```
[Out] 1/4*x/a/(c*x^4+a)+3/16*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/c^(1/4)
*2^(1/2)+3/16*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/c^(1/4)*2^(1/2)-3
/32*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(7/4)/c^(1/4)*2^(1
/2)+3/32*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(7/4)/c^(1/4)*
2^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {199, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}}+\frac{3\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{c}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}}{\sqrt[4]}\right.}{8\sqrt{2}\,a^{7/}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^(-2),x]
```

```
[Out] x/(4*a*(a + c*x^4)) - (3*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2
]*a^(7/4)*c^(1/4)) + (3*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]
*a^(7/4)*c^(1/4)) - (3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(16*Sqrt[2]*a^(7/4)*c^(1/4)) + (3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(1/4))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\left(a+cx^4\right)^2}\,dx = \frac{x}{4a\left(a+cx^4\right)} + \frac{3\int \frac{1}{a+cx^4}\,dx}{4a}$$

$$= \frac{x}{4a\left(a+cx^4\right)} + \frac{3\int \frac{\sqrt{a}-\sqrt{c}\,x^2}{a+cx^4}\,dx}{8a^{3/2}} + \frac{3\int \frac{\sqrt{a}+\sqrt{c}\,x^2}{a+cx^4}\,dx}{8a^{3/2}}$$

$$= \frac{x}{4a\left(a+cx^4\right)} + \frac{3\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{16a^{3/2}\sqrt{c}} + \frac{3\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{16a^{3/2}\sqrt{c}} - \frac{3\int \frac{\frac{\sqrt{2}\,\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}} - \frac{3\int$$

$$= \frac{x}{4a\left(a+cx^4\right)} - \frac{3\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}} + \frac{3\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}} + \frac{3\,\mathrm{Su}}{}$$

$$= \frac{x}{4a\left(a+cx^4\right)} - \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{c}} + \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{c}} - \frac{3\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\right.}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}}$$

Mathematica [A] time = 0.10, size = 183, normalized size = 0.91

$$\frac{\frac{8a^{3/4}x}{a+cx^4} - \frac{3\sqrt{2}\,\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{\sqrt[4]{c}} + \frac{3\sqrt{2}\,\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{\sqrt[4]{c}} - \frac{6\sqrt{2}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}} + \frac{6\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{\sqrt[4]{c}}}{32a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^(-2),x]
```

```
[Out] ((8*a^(3/4)*x)/(a + c*x^4) - (6*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1
/4)])/c^(1/4) + (6*Sqrt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(1/4)
 - (3*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/
```

```
4) + (3*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(
1/4))/(32*a^(7/4))
```

fricas [A] time = 0.69, size = 173, normalized size = 0.86

$$\frac{12\left(acx^4+a^2\right)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}\arctan\left(-a^5cx\left(-\frac{1}{a^7c}\right)^{\frac{3}{4}}+\sqrt{a^4\sqrt{-\frac{1}{a^7c}}+x^2}\,a^5c\left(-\frac{1}{a^7c}\right)^{\frac{3}{4}}\right)+3\left(acx^4+a^2\right)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}\log\left(a^2\right.}{16\left(acx^4+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(12*(a*c*x^4 + a^2)*(-1/(a^7*c))^(1/4)*arctan(-a^5*c*x*(-1/(a^7*c))^(3
/4) + sqrt(a^4*sqrt(-1/(a^7*c)) + x^2)*a^5*c*(-1/(a^7*c))^(3/4)) + 3*(a*c*x
^4 + a^2)*(-1/(a^7*c))^(1/4)*log(a^2*(-1/(a^7*c))^(1/4) + x) - 3*(a*c*x^4 +
 a^2)*(-1/(a^7*c))^(1/4)*log(-a^2*(-1/(a^7*c))^(1/4) + x) + 4*x)/(a*c*x^4 +
 a^2)
```

giac [A] time = 0.18, size = 194, normalized size = 0.96

$$\frac{x}{4\left(cx^4+a\right)a}+\frac{3\sqrt{2}\,\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c}+\frac{3\sqrt{2}\,\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c}+\frac{3\sqrt{2}\,\left(ac^3\right)^{\frac{1}{4}}\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*x/((c*x^4 + a)*a) + 3/16*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x
+ sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c) + 3/16*sqrt(2)*(a*c^3)^(1/4)*ar
ctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c) + 3/32*sq
rt(2)*(a*c^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c) -
3/32*sqrt(2)*(a*c^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^
2*c)
```

maple [A] time = 0.00, size = 143, normalized size = 0.71

$$\frac{x}{4\left(c\,x^4+a\right)a}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{16a^2}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{16a^2}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{32a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+a)^2,x)
```

```
[Out] 1/4*x/a/(c*x^4+a)+3/32/a^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*
x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+3/16/a^2*(a/c)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+3/16/a^2*(a/c)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(a/c)^(1/4)*x-1)
```

maxima [A] time = 2.97, size = 189, normalized size = 0.94

$$\frac{x}{4\left(acx^4+a^2\right)}+\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{c}\right.}{}\right.}{32\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x/(a*c*x^4 + a^2) + 3/32*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + s
qrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c
))) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/
sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + sqrt(2)*log(sqrt(c
)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - sqrt(2)*lo
g(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)))/a
```

mupad [B] time = 1.05, size = 58, normalized size = 0.29

$$\frac{x}{4\,a\,\left(c\,x^4+a\right)}+\frac{3\,\mathrm{atan}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{8\,(-a)^{7/4}\,c^{1/4}}+\frac{3\,\mathrm{atanh}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{8\,(-a)^{7/4}\,c^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x^4)^2,x)
```

```
[Out] x/(4*a*(a + c*x^4)) + (3*atan((c^(1/4)*x)/(-a)^(1/4)))/(8*(-a)^(7/4)*c^(1/4
)) + (3*atanh((c^(1/4)*x)/(-a)^(1/4)))/(8*(-a)^(7/4)*c^(1/4))
```

sympy [A] time = 0.40, size = 39, normalized size = 0.19

$$\frac{x}{4a^2+4acx^4}+\mathrm{RootSum}\left(65536t^4a^7c+81,\left(t\mapsto t\log\left(\frac{16ta^2}{3}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+a)**2,x)
```

```
[Out] x/(4*a**2 + 4*a*c*x**4) + RootSum(65536*_t**4*a**7*c + 81, Lambda(_t, _t*lo
g(16*_t*a**2/3 + x)))
```

3.670 $\int \frac{1}{x^2\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=214

$$-\frac{5\sqrt[4]{c}\,\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{9/4}}+\frac{5\sqrt[4]{c}\,\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{9/4}}+\frac{5\sqrt[4]{c}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{9/4}}-\frac{5}{}$$

```
[Out] -5/4/a^2/x+1/4/a/x/(c*x^4+a)-5/16*c^(1/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/
4))/a^(9/4)*2^(1/2)-5/16*c^(1/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(9/4
)*2^(1/2)-5/32*c^(1/4)*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a
^(9/4)*2^(1/2)+5/32*c^(1/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2
))/a^(9/4)*2^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {290, 325, 297, 1162, 617, 204, 1165, 628}

$$-\frac{5\sqrt[4]{c}\,\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{9/4}}+\frac{5\sqrt[4]{c}\,\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{9/4}}+\frac{5\sqrt[4]{c}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{9/4}}-\frac{5}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + c*x^4)^2),x]
```

```
[Out] -5/(4*a^2*x) + 1/(4*a*x*(a + c*x^4)) + (5*c^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(9/4)) - (5*c^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4
)*x)/a^(1/4)])/(8*Sqrt[2]*a^(9/4)) - (5*c^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(9/4)) + (5*c^(1/4)*Log[Sqrt[a]
+ Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+cx^4\right)^2}\,dx = \frac{1}{4ax\left(a+cx^4\right)} + \frac{5\int \frac{1}{x^2\left(a+cx^4\right)}\,dx}{4a}$$

$$= -\frac{5}{4a^2x} + \frac{1}{4ax\left(a+cx^4\right)} - \frac{(5c)\int \frac{x^2}{a+cx^4}\,dx}{4a^2}$$

$$= -\frac{5}{4a^2x} + \frac{1}{4ax\left(a+cx^4\right)} + \frac{\left(5\sqrt{c}\right)\int \frac{\sqrt{a}-\sqrt{c}\,x^2}{a+cx^4}\,dx}{8a^2} - \frac{\left(5\sqrt{c}\right)\int \frac{\sqrt{a}+\sqrt{c}\,x^2}{a+cx^4}\,dx}{8a^2}$$

$$= -\frac{5}{4a^2x} + \frac{1}{4ax\left(a+cx^4\right)} - \frac{5\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}} + x^2}\,dx}{16a^2} - \frac{5\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}} + x^2}\,dx}{16a^2} - \frac{\left(5\sqrt[4]{c}\right)\int \frac{\frac{\sqrt{2}\,\sqrt[4]{a}}{\sqrt[4]{c}}}{-\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}} - x^2}\,dx}{16\sqrt{2}\,a^{9/4}}$$

$$= -\frac{5}{4a^2x} + \frac{1}{4ax\left(a+cx^4\right)} - \frac{5\sqrt[4]{c}\,\log\left(\sqrt{a} - \sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x + \sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{9/4}} + \frac{5\sqrt[4]{c}\,\log\left(\sqrt{a} + \sqrt{2}\,\sqrt[4]{a}\,\right.}{16\sqrt{2}\,a^{9/4}}$$

$$= -\frac{5}{4a^2x} + \frac{1}{4ax\left(a+cx^4\right)} + \frac{5\sqrt[4]{c}\,\tan^{-1}\left(1 - \frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{9/4}} - \frac{5\sqrt[4]{c}\,\tan^{-1}\left(1 + \frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{9/4}} - \frac{5\sqrt[4]{c}\,\log\left(\right.}{}$$

Mathematica [A] time = 0.13, size = 196, normalized size = 0.92

$$\frac{-5\sqrt{2}\,\sqrt[4]{c}\,\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)+5\sqrt{2}\,\sqrt[4]{c}\,\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)-\frac{8\sqrt[4]{a}\,cx^3}{a+cx^4}+10\sqrt{2}\,\sqrt[4]{c}}{32a^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + c*x^4)^2),x]
```

```
[Out] ((-32*a^(1/4))/x - (8*a^(1/4)*c*x^3)/(a + c*x^4) + 10*Sqrt[2]*c^(1/4)*ArcTa
n[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - 10*Sqrt[2]*c^(1/4)*ArcTan[1 + (Sqrt[2]
*c^(1/4)*x)/a^(1/4)] - 5*Sqrt[2]*c^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2] + 5*Sqrt[2]*c^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2])/(32*a^(9/4))
```

fricas [A] time = 0.70, size = 192, normalized size = 0.90

$$-\frac{20\,cx^4-20\left(a^2cx^5+a^3x\right)\left(-\frac{c}{a^9}\right)^{\frac{1}{4}}\arctan\left(-a^2x\left(-\frac{c}{a^9}\right)^{\frac{1}{4}}+a^2\sqrt{-\frac{a^5\sqrt{-\frac{c}{a^9}}-cx^2}{c}}\left(-\frac{c}{a^9}\right)^{\frac{1}{4}}\right)+5\left(a^2cx^5+a^3x\right)\left(-\frac{c}{a^9}\right)^{\frac{1}{4}}}{16\left(a^2cx^5+a^3x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(20*c*x^4 - 20*(a^2*c*x^5 + a^3*x)*(-c/a^9)^(1/4)*arctan(-a^2*x*(-c/a
^9)^(1/4) + a^2*sqrt(-(a^5*sqrt(-c/a^9) - c*x^2)/c)*(-c/a^9)^(1/4)) + 5*(a^
2*c*x^5 + a^3*x)*(-c/a^9)^(1/4)*log(125*a^7*(-c/a^9)^(3/4) + 125*c*x) - 5*(
a^2*c*x^5 + a^3*x)*(-c/a^9)^(1/4)*log(-125*a^7*(-c/a^9)^(3/4) + 125*c*x) +
16*a)/(a^2*c*x^5 + a^3*x)
```

giac [A] time = 0.20, size = 205, normalized size = 0.96

$$-\frac{5\,cx^4+4\,a}{4\left(cx^5+ax\right)a^2}-\frac{5\sqrt{2}\,\left(ac^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^3c^2}-\frac{5\sqrt{2}\,\left(ac^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^3c^2}+\frac{5\sqrt{2}\,\left(ac^3\right)^{\frac{3}{4}}\,\mathrm{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*(5*c*x^4 + 4*a)/((c*x^5 + a*x)*a^2) - 5/16*sqrt(2)*(a*c^3)^(3/4)*arcta
n(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^2) - 5/16*sqr
t(2)*(a*c^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/
4))/(a^3*c^2) + 5/32*sqrt(2)*(a*c^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/c)^(1/4)
+ sqrt(a/c))/(a^3*c^2) - 5/32*sqrt(2)*(a*c^3)^(3/4)*log(x^2 - sqrt(2)*x*(a/
c)^(1/4) + sqrt(a/c))/(a^3*c^2)
```

maple [A] time = 0.01, size = 154, normalized size = 0.72

$$-\frac{c\,x^3}{4\left(c\,x^4+a\right)a^2}-\frac{5\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{16\left(\frac{a}{c}\right)^{\frac{1}{4}}a^2}-\frac{5\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{16\left(\frac{a}{c}\right)^{\frac{1}{4}}a^2}-\frac{5\sqrt{2}\,\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{32\left(\frac{a}{c}\right)^{\frac{1}{4}}a^2}-\frac{1}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+a)^2,x)
```

```
[Out] -1/4*c/a^2*x^3/(c*x^4+a)-5/32/a^2/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2
^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))-5/16/a^2/(a/
c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)-5/16/a^2/(a/c)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)-1/a^2/x
```

maxima [A] time = 2.95, size = 203, normalized size = 0.95

$$-\frac{5cx^4+4a}{4\left(a^2cx^5+a^3x\right)}-\frac{5c\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\,l}{}\right.}{32a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(5*c*x^4 + 4*a)/(a^2*c*x^5 + a^3*x) - 5/32*c*(2*sqrt(2)*arctan(1/2*sqr
t(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(s
qrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt
(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(sqrt(a)*sqrt(c))*sqrt(c))
 - sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(1/4)*
c^(3/4)) + sqrt(2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(
a^(1/4)*c^(3/4)))/a^2
```

mupad [B] time = 1.03, size = 69, normalized size = 0.32

$$\frac{5\,(-c)^{1/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\,x}{a^{1/4}}\right)}{8\,a^{9/4}}-\frac{5\,(-c)^{1/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\,x}{a^{1/4}}\right)}{8\,a^{9/4}}-\frac{\frac{1}{a}+\frac{5\,c\,x^4}{4\,a^2}}{c\,x^5+a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + c*x^4)^2),x)
```

```
[Out] (5*(-c)^(1/4)*atanh(((-c)^(1/4)*x)/a^(1/4)))/(8*a^(9/4)) - (5*(-c)^(1/4)*at
an(((-c)^(1/4)*x)/a^(1/4)))/(8*a^(9/4)) - (1/a + (5*c*x^4)/(4*a^2))/(a*x +
c*x^5)
```

sympy [A] time = 0.71, size = 56, normalized size = 0.26

$$\frac{-4a-5cx^4}{4a^3x+4a^2cx^5}+\mathrm{RootSum}\left(65536t^4a^9+625c,\left(t\mapsto t\log\left(-\frac{4096t^3a^7}{125c}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+a)**2,x)
```

```
[Out] (-4*a - 5*c*x**4)/(4*a**3*x + 4*a**2*c*x**5) + RootSum(65536*_t**4*a**9 + 6
25*c, Lambda(_t, _t*log(-4096*_t**3*a**7/(125*c) + x)))
```

3.671 $\int \frac{1}{x^4\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=214

$$\frac{7c^{3/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{11/4}}-\frac{7c^{3/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{11/4}}+\frac{7c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{11/4}}-\frac{7c^{3}}{}$$

```
[Out] -7/12/a^2/x^3+1/4/a/x^3/(c*x^4+a)-7/16*c^(3/4)*arctan(-1+c^(1/4)*x*2^(1/2)/
a^(1/4))/a^(11/4)*2^(1/2)-7/16*c^(3/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))/
a^(11/4)*2^(1/2)+7/32*c^(3/4)*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(
1/2))/a^(11/4)*2^(1/2)-7/32*c^(3/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^
2*c^(1/2))/a^(11/4)*2^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {290, 325, 211, 1165, 628, 1162, 617, 204}

$$\frac{7c^{3/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{11/4}}-\frac{7c^{3/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{11/4}}+\frac{7c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{11/4}}-\frac{7c^{3}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + c*x^4)^2),x]
```

```
[Out] -7/(12*a^2*x^3) + 1/(4*a*x^3*(a + c*x^4)) + (7*c^(3/4)*ArcTan[1 - (Sqrt[2]*
c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(11/4)) - (7*c^(3/4)*ArcTan[1 + (Sqrt[2]*
c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(11/4)) + (7*c^(3/4)*Log[Sqrt[a] - Sqrt[2
]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(11/4)) - (7*c^(3/4)*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(11/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+cx^4\right)^2}\,dx = \frac{1}{4ax^3\left(a+cx^4\right)} + \frac{7\int \frac{1}{x^4\left(a+cx^4\right)}\,dx}{4a}$$

$$= -\frac{7}{12a^2x^3} + \frac{1}{4ax^3\left(a+cx^4\right)} - \frac{(7c)\int \frac{1}{a+cx^4}\,dx}{4a^2}$$

$$= -\frac{7}{12a^2x^3} + \frac{1}{4ax^3\left(a+cx^4\right)} - \frac{(7c)\int \frac{\sqrt{a}-\sqrt{c}\,x^2}{a+cx^4}\,dx}{8a^{5/2}} - \frac{(7c)\int \frac{\sqrt{a}+\sqrt{c}\,x^2}{a+cx^4}\,dx}{8a^{5/2}}$$

$$= -\frac{7}{12a^2x^3} + \frac{1}{4ax^3\left(a+cx^4\right)} - \frac{\left(7\sqrt{c}\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}} + x^2}\,dx}{16a^{5/2}} - \frac{\left(7\sqrt{c}\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}} + x^2}\,dx}{16a^{5/2}} + \frac{\left(7c^{3}\right.}{}$$

$$= -\frac{7}{12a^2x^3} + \frac{1}{4ax^3\left(a+cx^4\right)} + \frac{7c^{3/4}\log\left(\sqrt{a} - \sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x + \sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{11/4}} - \frac{7c^{3/4}\log\left(\sqrt{a} + \sqrt{2}\,\right.}{16\sqrt{2}\,a}$$

$$= -\frac{7}{12a^2x^3} + \frac{1}{4ax^3\left(a+cx^4\right)} + \frac{7c^{3/4}\tan^{-1}\left(1 - \frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{11/4}} - \frac{7c^{3/4}\tan^{-1}\left(1 + \frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{11/4}} + \frac{7c^{3/4}\,\mathrm{lo}}{}$$

Mathematica [A] time = 0.14, size = 194, normalized size = 0.91

$$\frac{-\frac{24a^{3/4}cx}{a+cx^4}-\frac{32a^{3/4}}{x^3}+21\sqrt{2}\,c^{3/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)-21\sqrt{2}\,c^{3/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)+}{96a^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + c*x^4)^2),x]
```

```
[Out] ((-32*a^(3/4))/x^3 - (24*a^(3/4)*c*x)/(a + c*x^4) + 42*Sqrt[2]*c^(3/4)*ArcT
an[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - 42*Sqrt[2]*c^(3/4)*ArcTan[1 + (Sqrt[2
]*c^(1/4)*x)/a^(1/4)] + 21*Sqrt[2]*c^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^
(1/4)*x + Sqrt[c]*x^2] - 21*Sqrt[2]*c^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c
^(1/4)*x + Sqrt[c]*x^2])/(96*a^(11/4))
```

fricas [A] time = 0.62, size = 221, normalized size = 1.03

$$-\frac{28\,cx^4+84\left(a^2cx^7+a^3x^3\right)\left(-\frac{c^3}{a^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{a^8x\left(-\frac{c^3}{a^{11}}\right)^{\frac{3}{4}}-a^8\sqrt{\frac{a^6\sqrt{-\frac{c^3}{a^{11}}}+c^2x^2}{c^2}}\left(-\frac{c^3}{a^{11}}\right)^{\frac{3}{4}}}{c^2}\right)+21\left(a^2cx^7+a^3x^3\right)\left(-\frac{c^3}{a^{11}}\right)^{\frac{1}{4}}}{48\left(a^2cx^7+a^3x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] -1/48*(28*c*x^4 + 84*(a^2*c*x^7 + a^3*x^3)*(-c^3/a^11)^(1/4)*arctan(-(a^8*x
*(-c^3/a^11)^(3/4) - a^8*sqrt((a^6*sqrt(-c^3/a^11) + c^2*x^2)/c^2)*(-c^3/a^
11)^(3/4))/c^2) + 21*(a^2*c*x^7 + a^3*x^3)*(-c^3/a^11)^(1/4)*log(7*a^3*(-c^
3/a^11)^(1/4) + 7*c*x) - 21*(a^2*c*x^7 + a^3*x^3)*(-c^3/a^11)^(1/4)*log(-7*
a^3*(-c^3/a^11)^(1/4) + 7*c*x) + 16*a)/(a^2*c*x^7 + a^3*x^3)
```

giac [A] time = 0.17, size = 191, normalized size = 0.89

$$-\frac{cx}{4\left(cx^4+a\right)a^2}-\frac{7\sqrt{2}\,\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^3}-\frac{7\sqrt{2}\,\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^3}-\frac{7\sqrt{2}\,\left(ac^3\right)^{\frac{1}{4}}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*c*x/((c*x^4 + a)*a^2) - 7/16*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*
(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/a^3 - 7/16*sqrt(2)*(a*c^3)^(1/4)*a
rctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/a^3 - 7/32*sqrt(
2)*(a*c^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/a^3 + 7/32*sq
rt(2)*(a*c^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/a^3 - 1/3/
(a^2*x^3)
```

maple [A] time = 0.01, size = 155, normalized size = 0.72

$$-\frac{cx}{4\left(c\,x^4+a\right)a^2}-\frac{7\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{16a^3}-\frac{7\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{16a^3}-\frac{7\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+}\right.}{32a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^4+a)^2,x)
```

```
[Out] -1/4/a^2*c*x/(c*x^4+a)-7/32/a^3*c*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2
^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))-7/16/a^3*c*(
a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)-7/16/a^3*c*(a/c)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)-1/3/a^2/x^3
```

maxima [A] time = 3.03, size = 206, normalized size = 0.96

$$-\frac{7cx^4+4a}{12\left(a^2cx^7+a^3x^3\right)}-\frac{7\left(\frac{2\sqrt{2}c\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{2\sqrt{2}c\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{\sqrt{2}c^{\frac{3}{4}}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\right.}{32a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -1/12*(7*c*x^4 + 4*a)/(a^2*c*x^7 + a^3*x^3) - 7/32*(2*sqrt(2)*c*arctan(1/2*
sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqr
t(a)*sqrt(sqrt(a)*sqrt(c))) + 2*sqrt(2)*c*arctan(1/2*sqrt(2)*(2*sqrt(c)*x -
 sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt
(c))) + sqrt(2)*c^(3/4)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(
a))/a^(3/4) - sqrt(2)*c^(3/4)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x +
 sqrt(a))/a^(3/4))/a^2
```

mupad [B] time = 0.11, size = 73, normalized size = 0.34

$$\frac{7(-c)^{3/4}\operatorname{atan}\left(\frac{(-c)^{1/4}x}{a^{1/4}}\right)}{8a^{11/4}}-\frac{\frac{1}{3a}+\frac{7cx^4}{12a^2}}{cx^7+ax^3}+\frac{7(-c)^{3/4}\operatorname{atanh}\left(\frac{(-c)^{1/4}x}{a^{1/4}}\right)}{8a^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + c*x^4)^2),x)
```

```
[Out] (7*(-c)^(3/4)*atan(((-c)^(1/4)*x)/a^(1/4)))/(8*a^(11/4)) - (1/(3*a) + (7*c*
x^4)/(12*a^2))/(a*x^3 + c*x^7) + (7*(-c)^(3/4)*atanh(((-c)^(1/4)*x)/a^(1/4)
))/(8*a^(11/4))
```

sympy [A] time = 0.88, size = 58, normalized size = 0.27

$$\frac{-4a-7cx^4}{12a^3x^3+12a^2cx^7}+\operatorname{RootSum}\left(65536t^4a^{11}+2401c^3,\left(t\mapsto t\log\left(-\frac{16ta^3}{7c}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**4+a)**2,x)
```

```
[Out] (-4*a - 7*c*x**4)/(12*a**3*x**3 + 12*a**2*c*x**7) + RootSum(65536*_t**4*a**
11 + 2401*c**3, Lambda(_t, _t*log(-16*_t*a**3/(7*c) + x)))
```

3.672 $\int \frac{x^{11}}{\left(a+cx^4\right)^3}\,dx$

Optimal. Leaf size=52

$$-\frac{a^2}{8c^3\left(a+cx^4\right)^2}+\frac{a}{2c^3\left(a+cx^4\right)}+\frac{\log\left(a+cx^4\right)}{4c^3}$$

```
[Out] -1/8*a^2/c^3/(c*x^4+a)^2+1/2*a/c^3/(c*x^4+a)+1/4*ln(c*x^4+a)/c^3
```

Rubi [A] time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$-\frac{a^2}{8c^3\left(a+cx^4\right)^2}+\frac{a}{2c^3\left(a+cx^4\right)}+\frac{\log\left(a+cx^4\right)}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + c*x^4)^3,x]
```

```
[Out] -a^2/(8*c^3*(a + c*x^4)^2) + a/(2*c^3*(a + c*x^4)) + Log[a + c*x^4]/(4*c^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{\left(a+cx^4\right)^3}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x^2}{(a+cx)^3}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int \left(\frac{a^2}{c^2(a+cx)^3}-\frac{2a}{c^2(a+cx)^2}+\frac{1}{c^2(a+cx)}\right)dx, x, x^4\right)\\ &= -\frac{a^2}{8c^3\left(a+cx^4\right)^2}+\frac{a}{2c^3\left(a+cx^4\right)}+\frac{\log\left(a+cx^4\right)}{4c^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.75

$$\frac{\frac{a\left(3a+4cx^4\right)}{\left(a+cx^4\right)^2}+2\log\left(a+cx^4\right)}{8c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + c*x^4)^3,x]
```

```
[Out] ((a*(3*a + 4*c*x^4))/(a + c*x^4)^2 + 2*Log[a + c*x^4])/(8*c^3)
```

fricas [A] time = 0.56, size = 69, normalized size = 1.33

$$\frac{4\,acx^4 + 3\,a^2 + 2\left(c^2x^8 + 2\,acx^4 + a^2\right)\log\left(cx^4 + a\right)}{8\left(c^5x^8 + 2\,ac^4x^4 + a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(4*a*c*x^4 + 3*a^2 + 2*(c^2*x^8 + 2*a*c*x^4 + a^2)*log(c*x^4 + a))/(c^5
*x^8 + 2*a*c^4*x^4 + a^2*c^3)
```

giac [A] time = 0.27, size = 42, normalized size = 0.81

$$\frac{\log\left(\left|cx^4 + a\right|\right)}{4\,c^3} - \frac{3\,cx^8 + 2\,ax^4}{8\left(cx^4 + a\right)^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*log(abs(c*x^4 + a))/c^3 - 1/8*(3*c*x^8 + 2*a*x^4)/((c*x^4 + a)^2*c^2)
```

maple [A] time = 0.01, size = 47, normalized size = 0.90

$$-\frac{a^2}{8\left(c\,x^4 + a\right)^2c^3} + \frac{a}{2\left(c\,x^4 + a\right)c^3} + \frac{\ln\left(c\,x^4 + a\right)}{4c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(c*x^4+a)^3,x)
```

```
[Out] -1/8*a^2/c^3/(c*x^4+a)^2+1/2*a/c^3/(c*x^4+a)+1/4*ln(c*x^4+a)/c^3
```

maxima [A] time = 1.34, size = 55, normalized size = 1.06

$$\frac{4\,acx^4 + 3\,a^2}{8\left(c^5x^8 + 2\,ac^4x^4 + a^2c^3\right)} + \frac{\log\left(cx^4 + a\right)}{4\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(4*a*c*x^4 + 3*a^2)/(c^5*x^8 + 2*a*c^4*x^4 + a^2*c^3) + 1/4*log(c*x^4 +
 a)/c^3
```

mupad [B] time = 0.07, size = 53, normalized size = 1.02

$$\frac{\frac{3\,a^2}{8\,c^3} + \frac{a\,x^4}{2\,c^2}}{a^2 + 2\,a\,c\,x^4 + c^2\,x^8} + \frac{\ln\left(c\,x^4 + a\right)}{4\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a + c*x^4)^3,x)
```

```
[Out] ((3*a^2)/(8*c^3) + (a*x^4)/(2*c^2))/(a^2 + c^2*x^8 + 2*a*c*x^4) + log(a + c
*x^4)/(4*c^3)
```

sympy [**A**] time = 0.74, size = 53, normalized size = 1.02

$$\frac{3a^2 + 4acx^4}{8a^2c^3 + 16ac^4x^4 + 8c^5x^8} + \frac{\log\left(a + cx^4\right)}{4c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(c*x**4+a)**3,x)
```

```
[Out] (3*a**2 + 4*a*c*x**4)/(8*a**2*c**3 + 16*a*c**4*x**4 + 8*c**5*x**8) + log(a
+ c*x**4)/(4*c**3)
```

3.673 $\int \frac{x^9}{\left(a+cx^4\right)^3}\,dx$

Optimal. Leaf size=68

$$\frac{3\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{16\sqrt{a}\,c^{5/2}} - \frac{3x^2}{16c^2\left(a+cx^4\right)} - \frac{x^6}{8c\left(a+cx^4\right)^2}$$

```
[Out] -1/8*x^6/c/(c*x^4+a)^2-3/16*x^2/c^2/(c*x^4+a)+3/16*arctan(x^2*c^(1/2)/a^(1/
2))/c^(5/2)/a^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {275, 288, 205}

$$-\frac{3x^2}{16c^2\left(a+cx^4\right)} + \frac{3\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{16\sqrt{a}\,c^{5/2}} - \frac{x^6}{8c\left(a+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + c*x^4)^3,x]
```

```
[Out] -x^6/(8*c*(a + c*x^4)^2) - (3*x^2)/(16*c^2*(a + c*x^4)) + (3*ArcTan[(Sqrt[c
]*x^2)/Sqrt[a]])/(16*Sqrt[a]*c^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^9}{\left(a+cx^4\right)^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{\left(a+cx^2\right)^3}\,dx, x, x^2\right)$$

$$= -\frac{x^6}{8c\left(a+cx^4\right)^2} + \frac{3\,\text{Subst}\left(\int \frac{x^2}{\left(a+cx^2\right)^2}\,dx, x, x^2\right)}{8c}$$

$$= -\frac{x^6}{8c\left(a+cx^4\right)^2} - \frac{3x^2}{16c^2\left(a+cx^4\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{16c^2}$$

$$= -\frac{x^6}{8c\left(a+cx^4\right)^2} - \frac{3x^2}{16c^2\left(a+cx^4\right)} + \frac{3\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{16\sqrt{a}\,c^{5/2}}$$

Mathematica [A] time = 0.05, size = 58, normalized size = 0.85

$$\frac{1}{16}\left(\frac{3\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{\sqrt{a}\,c^{5/2}} + \frac{-3ax^2-5cx^6}{c^2\left(a+cx^4\right)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + c*x^4)^3,x]
```

```
[Out] ((-3*a*x^2 - 5*c*x^6)/(c^2*(a + c*x^4)^2) + (3*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]
])/(Sqrt[a]*c^(5/2)))/16
```

fricas [A] time = 0.55, size = 196, normalized size = 2.88

$$\left[-\frac{10\,ac^2x^6+6\,a^2cx^2+3\left(c^2x^8+2\,acx^4+a^2\right)\sqrt{-ac}\,\log\left(\frac{cx^4-2\sqrt{-ac}\,x^2-a}{cx^4+a}\right)}{32\left(ac^5x^8+2\,a^2c^4x^4+a^3c^3\right)}, -\frac{5\,ac^2x^6+3\,a^2cx^2+3\left(c^2x^8+2\,acx^4\right.}{16\left(ac^5x^8+2\,a^2c^4x^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(10*a*c^2*x^6 + 6*a^2*c*x^2 + 3*(c^2*x^8 + 2*a*c*x^4 + a^2)*sqrt(-a*
c)*log((c*x^4 - 2*sqrt(-a*c)*x^2 - a)/(c*x^4 + a)))/(a*c^5*x^8 + 2*a^2*c^4*
x^4 + a^3*c^3), -1/16*(5*a*c^2*x^6 + 3*a^2*c*x^2 + 3*(c^2*x^8 + 2*a*c*x^4 +
 a^2)*sqrt(a*c)*arctan(sqrt(a*c)/(c*x^2)))/(a*c^5*x^8 + 2*a^2*c^4*x^4 + a^3
*c^3)]
```

giac [A] time = 0.16, size = 49, normalized size = 0.72

$$\frac{3\,\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{16\sqrt{ac}\,c^2} - \frac{5\,cx^6+3\,ax^2}{16\left(cx^4+a\right)^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 3/16*arctan(c*x^2/sqrt(a*c))/(sqrt(a*c)*c^2) - 1/16*(5*c*x^6 + 3*a*x^2)/((c
*x^4 + a)^2*c^2)
```

maple [A] time = 0.01, size = 52, normalized size = 0.76

$$\frac{3\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{16\sqrt{ac}\,c^2}+\frac{-\frac{5x^6}{8c}-\frac{3a\,x^2}{8c^2}}{2\left(c\,x^4+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(c*x^4+a)^3,x)
```

```
[Out] 1/2*(-5/8/c*x^6-3/8*a/c^2*x^2)/(c*x^4+a)^2+3/16/c^2/(a*c)^(1/2)*arctan(1/(a
*c)^(1/2)*c*x^2)
```

maxima [A] time = 3.03, size = 63, normalized size = 0.93

$$-\frac{5\,cx^6+3\,ax^2}{16\left(c^4x^8+2\,ac^3x^4+a^2c^2\right)}+\frac{3\,\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{16\,\sqrt{ac}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(5*c*x^6 + 3*a*x^2)/(c^4*x^8 + 2*a*c^3*x^4 + a^2*c^2) + 3/16*arctan(c
*x^2/sqrt(a*c))/(sqrt(a*c)*c^2)
```

mupad [B] time = 1.04, size = 60, normalized size = 0.88

$$\frac{3\,\mathrm{atan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{16\,\sqrt{a}\,c^{5/2}}-\frac{\frac{5\,x^6}{16\,c}+\frac{3\,a\,x^2}{16\,c^2}}{a^2+2\,a\,c\,x^4+c^2\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + c*x^4)^3,x)
```

```
[Out] (3*atan((c^(1/2)*x^2)/a^(1/2)))/(16*a^(1/2)*c^(5/2)) - ((5*x^6)/(16*c) + (3
*a*x^2)/(16*c^2))/(a^2 + c^2*x^8 + 2*a*c*x^4)
```

sympy [A] time = 1.04, size = 116, normalized size = 1.71

$$-\frac{3\sqrt{-\frac{1}{ac^5}}\,\log\left(-ac^2\sqrt{-\frac{1}{ac^5}}+x^2\right)}{32}+\frac{3\sqrt{-\frac{1}{ac^5}}\,\log\left(ac^2\sqrt{-\frac{1}{ac^5}}+x^2\right)}{32}+\frac{-3ax^2-5cx^6}{16a^2c^2+32ac^3x^4+16c^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(c*x**4+a)**3,x)
```

```
[Out] -3*sqrt(-1/(a*c**5))*log(-a*c**2*sqrt(-1/(a*c**5)) + x**2)/32 + 3*sqrt(-1/(
a*c**5))*log(a*c**2*sqrt(-1/(a*c**5)) + x**2)/32 + (-3*a*x**2 - 5*c*x**6)/(
16*a**2*c**2 + 32*a*c**3*x**4 + 16*c**4*x**8)
```

3.674 $\int \frac{x^7}{\left(a+cx^4\right)^3}\,dx$

Optimal. Leaf size=19

$$\frac{x^8}{8a\left(a+cx^4\right)^2}$$

```
[Out] 1/8*x^8/a/(c*x^4+a)^2
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {264}

$$\frac{x^8}{8a\left(a+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + c*x^4)^3,x]
```

```
[Out] x^8/(8*a*(a + c*x^4)^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^7}{\left(a+cx^4\right)^3}\,dx = \frac{x^8}{8a\left(a+cx^4\right)^2}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.26

$$-\frac{a+2cx^4}{8c^2\left(a+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + c*x^4)^3,x]
```

```
[Out] -1/8*(a + 2*c*x^4)/(c^2*(a + c*x^4)^2)
```

fricas [B] time = 0.55, size = 36, normalized size = 1.89

$$-\frac{2\,cx^4+a}{8\left(c^4x^8+2\,ac^3x^4+a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*c*x^4 + a)/(c^4*x^8 + 2*a*c^3*x^4 + a^2*c^2)
```

giac [A] time = 0.16, size = 22, normalized size = 1.16

$$-\frac{2\,cx^4+a}{8\left(cx^4+a\right)^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+a)^3,x, algorithm="giac")
[Out] -1/8*(2*c*x^4 + a)/((c*x^4 + a)^2*c^2)
```

maple [A] time = 0.01, size = 31, normalized size = 1.63

$$\frac{a}{8\left(c\,x^4+a\right)^2c^2}-\frac{1}{4\left(c\,x^4+a\right)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+a)^3,x)
[Out] -1/4/c^2/(c*x^4+a)+1/8*a/c^2/(c*x^4+a)^2
```

maxima [B] time = 1.32, size = 36, normalized size = 1.89

$$-\frac{2\,cx^4+a}{8\left(c^4x^8+2\,ac^3x^4+a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+a)^3,x, algorithm="maxima")
[Out] -1/8*(2*c*x^4 + a)/(c^4*x^8 + 2*a*c^3*x^4 + a^2*c^2)
```

mupad [B] time = 0.03, size = 37, normalized size = 1.95

$$-\frac{\frac{a}{8\,c^2}+\frac{x^4}{4\,c}}{a^2+2\,a\,c\,x^4+c^2\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + c*x^4)^3,x)
[Out] -(a/(8*c^2) + x^4/(4*c))/(a^2 + c^2*x^8 + 2*a*c*x^4)
```

sympy [B] time = 0.50, size = 36, normalized size = 1.89

$$\frac{-a-2cx^4}{8a^2c^2+16ac^3x^4+8c^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+a)**3,x)
[Out] (-a - 2*c*x**4)/(8*a**2*c**2 + 16*a*c**3*x**4 + 8*c**4*x**8)
```

3.675 $\int \frac{x^5}{\left(a+cx^4\right)^3}\,dx$

Optimal. Leaf size=71

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{16a^{3/2}c^{3/2}}+\frac{x^2}{16ac\left(a+cx^4\right)}-\frac{x^2}{8c\left(a+cx^4\right)^2}$$

```
[Out] -1/8*x^2/c/(c*x^4+a)^2+1/16*x^2/a/c/(c*x^4+a)+1/16*arctan(x^2*c^(1/2)/a^(1/
2))/a^(3/2)/c^(3/2)
```

Rubi [A] time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {275, 288, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{16a^{3/2}c^{3/2}}+\frac{x^2}{16ac\left(a+cx^4\right)}-\frac{x^2}{8c\left(a+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + c*x^4)^3,x]
```

```
[Out] -x^2/(8*c*(a + c*x^4)^2) + x^2/(16*a*c*(a + c*x^4)) + ArcTan[(Sqrt[c]*x^2)/
Sqrt[a]]/(16*a^(3/2)*c^(3/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^5}{\left(a+cx^4\right)^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\left(a+cx^2\right)^3}\,dx, x, x^2\right)$$
$$= -\frac{x^2}{8c\left(a+cx^4\right)^2} + \frac{\text{Subst}\left(\int \frac{1}{\left(a+cx^2\right)^2}\,dx, x, x^2\right)}{8c}$$
$$= -\frac{x^2}{8c\left(a+cx^4\right)^2} + \frac{x^2}{16ac\left(a+cx^4\right)} + \frac{\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{16ac}$$
$$= -\frac{x^2}{8c\left(a+cx^4\right)^2} + \frac{x^2}{16ac\left(a+cx^4\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{16a^{3/2}c^{3/2}}$$

Mathematica [A] time = 0.03, size = 62, normalized size = 0.87

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right) + \frac{\sqrt{a}\,\sqrt{c}\,x^2\left(cx^4-a\right)}{\left(a+cx^4\right)^2}}{16a^{3/2}c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + c*x^4)^3,x]
```

```
[Out] ((Sqrt[a]*Sqrt[c]*x^2*(-a + c*x^4))/(a + c*x^4)^2 + ArcTan[(Sqrt[c]*x^2)/Sq
rt[a]])/(16*a^(3/2)*c^(3/2))
```

fricas [A] time = 0.81, size = 199, normalized size = 2.80

$$\left[\frac{2\,ac^2x^6 - 2\,a^2cx^2 - \left(c^2x^8 + 2\,acx^4 + a^2\right)\sqrt{-ac}\,\log\left(\frac{cx^4 - 2\sqrt{-ac}\,x^2 - a}{cx^4+a}\right)}{32\left(a^2c^4x^8 + 2\,a^3c^3x^4 + a^4c^2\right)}, \frac{ac^2x^6 - a^2cx^2 - \left(c^2x^8 + 2\,acx^4 + a^2\right)\sqrt{ac}\,\text{arc}}{16\left(a^2c^4x^8 + 2\,a^3c^3x^4 + a^4c^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(2*a*c^2*x^6 - 2*a^2*c*x^2 - (c^2*x^8 + 2*a*c*x^4 + a^2)*sqrt(-a*c)*l
og((c*x^4 - 2*sqrt(-a*c)*x^2 - a)/(c*x^4 + a)))/(a^2*c^4*x^8 + 2*a^3*c^3*x^
4 + a^4*c^2), 1/16*(a*c^2*x^6 - a^2*c*x^2 - (c^2*x^8 + 2*a*c*x^4 + a^2)*sqr
t(a*c)*arctan(sqrt(a*c)/(c*x^2)))/(a^2*c^4*x^8 + 2*a^3*c^3*x^4 + a^4*c^2)]
```

giac [A] time = 0.17, size = 54, normalized size = 0.76

$$\frac{\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{16\sqrt{ac}\,ac} + \frac{cx^6 - ax^2}{16\left(cx^4+a\right)^2 ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/16*arctan(c*x^2/sqrt(a*c))/(sqrt(a*c)*a*c) + 1/16*(c*x^6 - a*x^2)/((c*x^4
 + a)^2*a*c)
```

maple [A] time = 0.01, size = 54, normalized size = 0.76

$$\frac{\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{16\sqrt{ac}\,ac}+\frac{\frac{x^6}{8a}-\frac{x^2}{8c}}{2\left(c\,x^4+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+a)^3,x)
```

```
[Out] 1/2*(1/8*x^6/a-1/8/c*x^2)/(c*x^4+a)^2+1/16/c/a/(a*c)^(1/2)*arctan(1/(a*c)^(
1/2)*c*x^2)
```

maxima [A] time = 2.92, size = 66, normalized size = 0.93

$$\frac{cx^6-ax^2}{16\left(ac^3x^8+2\,a^2c^2x^4+a^3c\right)}+\frac{\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{16\sqrt{ac}\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(c*x^6 - a*x^2)/(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c) + 1/16*arctan(c*x^
2/sqrt(a*c))/(sqrt(a*c)*a*c)
```

mupad [B] time = 1.04, size = 58, normalized size = 0.82

$$\frac{\frac{x^6}{16\,a}-\frac{x^2}{16\,c}}{a^2+2\,a\,c\,x^4+c^2\,x^8}+\frac{\operatorname{atan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{16\,a^{3/2}\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + c*x^4)^3,x)
```

```
[Out] (x^6/(16*a) - x^2/(16*c))/(a^2 + c^2*x^8 + 2*a*c*x^4) + atan((c^(1/2)*x^2)/
a^(1/2))/(16*a^(3/2)*c^(3/2))
```

sympy [B] time = 0.81, size = 116, normalized size = 1.63

$$-\frac{\sqrt{-\frac{1}{a^3c^3}}\,\log\left(-a^2c\sqrt{-\frac{1}{a^3c^3}}+x^2\right)}{32}+\frac{\sqrt{-\frac{1}{a^3c^3}}\,\log\left(a^2c\sqrt{-\frac{1}{a^3c^3}}+x^2\right)}{32}+\frac{-ax^2+cx^6}{16a^3c+32a^2c^2x^4+16ac^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+a)**3,x)
```

```
[Out] -sqrt(-1/(a**3*c**3))*log(-a**2*c*sqrt(-1/(a**3*c**3)) + x**2)/32 + sqrt(-1
/(a**3*c**3))*log(a**2*c*sqrt(-1/(a**3*c**3)) + x**2)/32 + (-a*x**2 + c*x**
6)/(16*a**3*c + 32*a**2*c**2*x**4 + 16*a*c**3*x**8)
```

3.676 $\int \frac{x^3}{\left(a+cx^4\right)^3}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{8c\left(a+cx^4\right)^2}$$

```
[Out] -1/8/c/(c*x^4+a)^2
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$-\frac{1}{8c\left(a+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + c*x^4)^3,x]
```

```
[Out] -1/(8*c*(a + c*x^4)^2)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^3}{\left(a+cx^4\right)^3}\,dx = -\frac{1}{8c\left(a+cx^4\right)^2}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{8c\left(a+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + c*x^4)^3,x]
```

```
[Out] -1/8*1/(c*(a + c*x^4)^2)
```

fricas [A] time = 0.56, size = 26, normalized size = 1.62

$$-\frac{1}{8\left(c^3x^8+2\,ac^2x^4+a^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] -1/8/(c^3*x^8 + 2*a*c^2*x^4 + a^2*c)
```

giac [A] time = 0.18, size = 14, normalized size = 0.88

$$-\frac{1}{8\left(cx^4+a\right)^2 c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+a)^3,x, algorithm="giac")
[Out] -1/8/((c*x^4 + a)^2*c)
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$-\frac{1}{8\left(c\,x^4+a\right)^2 c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+a)^3,x)
[Out] -1/8/c/(c*x^4+a)^2
```

maxima [A] time = 1.33, size = 14, normalized size = 0.88

$$-\frac{1}{8\left(cx^4+a\right)^2 c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+a)^3,x, algorithm="maxima")
[Out] -1/8/((c*x^4 + a)^2*c)
```

mupad [B] time = 0.03, size = 25, normalized size = 1.56

$$-\frac{1}{8\,c\,\left(a^2+2\,a\,c\,x^4+c^2\,x^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + c*x^4)^3,x)
[Out] -1/(8*c*(a^2 + c^2*x^8 + 2*a*c*x^4))
```

sympy [A] time = 0.48, size = 27, normalized size = 1.69

$$-\frac{1}{8a^2c+16ac^2x^4+8c^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+a)**3,x)
[Out] -1/(8*a**2*c + 16*a*c**2*x**4 + 8*c**3*x**8)
```

3.677 $\int \frac{x}{\left(a+cx^4\right)^3}\,dx$

Optimal. Leaf size=68

$$\frac{3\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}}+\frac{3x^2}{16a^2\left(a+cx^4\right)}+\frac{x^2}{8a\left(a+cx^4\right)^2}$$

```
[Out] 1/8*x^2/a/(c*x^4+a)^2+3/16*x^2/a^2/(c*x^4+a)+3/16*arctan(x^2*c^(1/2)/a^(1/2
))/a^(5/2)/c^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {275, 199, 205}

$$\frac{3x^2}{16a^2\left(a+cx^4\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}}+\frac{x^2}{8a\left(a+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + c*x^4)^3,x]
```

```
[Out] x^2/(8*a*(a + c*x^4)^2) + (3*x^2)/(16*a^2*(a + c*x^4)) + (3*ArcTan[(Sqrt[c]
*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[c])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x}{\left(a+cx^4\right)^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\left(a+cx^2\right)^3}\,dx, x, x^2\right)$$
$$= \frac{x^2}{8a\left(a+cx^4\right)^2} + \frac{3\,\text{Subst}\left(\int \frac{1}{\left(a+cx^2\right)^2}\,dx, x, x^2\right)}{8a}$$
$$= \frac{x^2}{8a\left(a+cx^4\right)^2} + \frac{3x^2}{16a^2\left(a+cx^4\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{16a^2}$$
$$= \frac{x^2}{8a\left(a+cx^4\right)^2} + \frac{3x^2}{16a^2\left(a+cx^4\right)} + \frac{3\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}}$$

Mathematica [A] time = 0.05, size = 58, normalized size = 0.85

$$\frac{1}{16}\left(\frac{3\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{a^{5/2}\sqrt{c}} + \frac{5ax^2+3cx^6}{a^2\left(a+cx^4\right)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + c*x^4)^3,x]
```

```
[Out] ((5*a*x^2 + 3*c*x^6)/(a^2*(a + c*x^4)^2) + (3*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]]
)/(a^(5/2)*Sqrt[c]))/16
```

fricas [A] time = 0.65, size = 196, normalized size = 2.88

$$\left[\frac{6\,ac^2x^6+10\,a^2cx^2-3\left(c^2x^8+2\,acx^4+a^2\right)\sqrt{-ac}\,\log\left(\frac{cx^4-2\sqrt{-ac}\,x^2-a}{cx^4+a}\right)}{32\left(a^3c^3x^8+2\,a^4c^2x^4+a^5c\right)}, \frac{3\,ac^2x^6+5\,a^2cx^2-3\left(c^2x^8+2\,acx^4+\right.}{16\left(a^3c^3x^8+2\,a^4c^2x^4+\right.}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(6*a*c^2*x^6 + 10*a^2*c*x^2 - 3*(c^2*x^8 + 2*a*c*x^4 + a^2)*sqrt(-a*c
)*log((c*x^4 - 2*sqrt(-a*c)*x^2 - a)/(c*x^4 + a)))/(a^3*c^3*x^8 + 2*a^4*c^2
*x^4 + a^5*c), 1/16*(3*a*c^2*x^6 + 5*a^2*c*x^2 - 3*(c^2*x^8 + 2*a*c*x^4 + a
^2)*sqrt(a*c)*arctan(sqrt(a*c)/(c*x^2)))/(a^3*c^3*x^8 + 2*a^4*c^2*x^4 + a^5
*c)]
```

giac [A] time = 0.16, size = 49, normalized size = 0.72

$$\frac{3\,\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{16\sqrt{ac}\,a^2} + \frac{3\,cx^6+5\,ax^2}{16\left(cx^4+a\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 3/16*arctan(c*x^2/sqrt(a*c))/(sqrt(a*c)*a^2) + 1/16*(3*c*x^6 + 5*a*x^2)/((c
*x^4 + a)^2*a^2)
```

maple [A] time = 0.01, size = 57, normalized size = 0.84

$$\frac{x^2}{8\left(c\,x^4+a\right)^2 a}+\frac{3x^2}{16\left(c\,x^4+a\right)a^2}+\frac{3\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{16\sqrt{ac}\;a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+a)^3,x)
```

```
[Out] 1/8*x^2/a/(c*x^4+a)^2+3/16*x^2/a^2/(c*x^4+a)+3/16/a^2/(a*c)^(1/2)*arctan(1/
(a*c)^(1/2)*c*x^2)
```

maxima [A] time = 2.98, size = 62, normalized size = 0.91

$$\frac{3\,cx^6+5\,ax^2}{16\left(a^2c^2x^8+2\,a^3cx^4+a^4\right)}+\frac{3\,\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{16\,\sqrt{ac}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(3*c*x^6 + 5*a*x^2)/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4) + 3/16*arctan(c*
x^2/sqrt(a*c))/(sqrt(a*c)*a^2)
```

mupad [B] time = 1.00, size = 59, normalized size = 0.87

$$\frac{\frac{5\,x^2}{16\,a}+\frac{3\,c\,x^6}{16\,a^2}}{a^2+2\,a\,c\,x^4+c^2\,x^8}+\frac{3\,\mathrm{atan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{16\,a^{5/2}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + c*x^4)^3,x)
```

```
[Out] ((5*x^2)/(16*a) + (3*c*x^6)/(16*a^2))/(a^2 + c^2*x^8 + 2*a*c*x^4) + (3*atan
((c^(1/2)*x^2)/a^(1/2)))/(16*a^(5/2)*c^(1/2))
```

sympy [A] time = 0.58, size = 110, normalized size = 1.62

$$-\frac{3\sqrt{-\frac{1}{a^5c}}\,\log\left(-a^3\sqrt{-\frac{1}{a^5c}}+x^2\right)}{32}+\frac{3\sqrt{-\frac{1}{a^5c}}\,\log\left(a^3\sqrt{-\frac{1}{a^5c}}+x^2\right)}{32}+\frac{5ax^2+3cx^6}{16a^4+32a^3cx^4+16a^2c^2x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+a)**3,x)
```

```
[Out] -3*sqrt(-1/(a**5*c))*log(-a**3*sqrt(-1/(a**5*c)) + x**2)/32 + 3*sqrt(-1/(a*
*5*c))*log(a**3*sqrt(-1/(a**5*c)) + x**2)/32 + (5*a*x**2 + 3*c*x**6)/(16*a*
*4 + 32*a**3*c*x**4 + 16*a**2*c**2*x**8)
```

3.678 $\int \frac{1}{x\left(a+cx^4\right)^3}\,dx$

Optimal. Leaf size=54

$$-\frac{\log\left(a+cx^4\right)}{4a^3}+\frac{\log(x)}{a^3}+\frac{1}{4a^2\left(a+cx^4\right)}+\frac{1}{8a\left(a+cx^4\right)^2}$$

```
[Out] 1/8/a/(c*x^4+a)^2+1/4/a^2/(c*x^4+a)+ln(x)/a^3-1/4*ln(c*x^4+a)/a^3
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 44}

$$\frac{1}{4a^2\left(a+cx^4\right)}-\frac{\log\left(a+cx^4\right)}{4a^3}+\frac{\log(x)}{a^3}+\frac{1}{8a\left(a+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + c*x^4)^3),x]
```

```
[Out] 1/(8*a*(a + c*x^4)^2) + 1/(4*a^2*(a + c*x^4)) + Log[x]/a^3 - Log[a + c*x^4]
/(4*a^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a+cx^4\right)^3}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x(a+cx)^3}\,dx, x, x^4\right)\\ &= \frac{1}{4}\operatorname{Subst}\left(\int \left(\frac{1}{a^3x}-\frac{c}{a(a+cx)^3}-\frac{c}{a^2(a+cx)^2}-\frac{c}{a^3(a+cx)}\right)dx, x, x^4\right)\\ &= \frac{1}{8a\left(a+cx^4\right)^2}+\frac{1}{4a^2\left(a+cx^4\right)}+\frac{\log(x)}{a^3}-\frac{\log\left(a+cx^4\right)}{4a^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 43, normalized size = 0.80

$$\frac{\frac{a\left(3a+2cx^4\right)}{\left(a+cx^4\right)^2}-2\log\left(a+cx^4\right)+8\log(x)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + c*x^4)^3),x]
```

```
[Out] ((a*(3*a + 2*c*x^4))/(a + c*x^4)^2 + 8*Log[x] - 2*Log[a + c*x^4])/(8*a^3)
```

fricas [A] time = 0.51, size = 90, normalized size = 1.67

$$\frac{2\,acx^4 + 3\,a^2 - 2\left(c^2x^8 + 2\,acx^4 + a^2\right)\log\left(cx^4 + a\right) + 8\left(c^2x^8 + 2\,acx^4 + a^2\right)\log(x)}{8\left(a^3c^2x^8 + 2\,a^4cx^4 + a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(2*a*c*x^4 + 3*a^2 - 2*(c^2*x^8 + 2*a*c*x^4 + a^2)*log(c*x^4 + a) + 8*(
c^2*x^8 + 2*a*c*x^4 + a^2)*log(x))/(a^3*c^2*x^8 + 2*a^4*c*x^4 + a^5)
```

giac [A] time = 0.16, size = 59, normalized size = 1.09

$$\frac{\log\left(x^4\right)}{4\,a^3} - \frac{\log\left(\left|cx^4 + a\right|\right)}{4\,a^3} + \frac{3\,c^2x^8 + 8\,acx^4 + 6\,a^2}{8\left(cx^4 + a\right)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*log(x^4)/a^3 - 1/4*log(abs(c*x^4 + a))/a^3 + 1/8*(3*c^2*x^8 + 8*a*c*x^4
 + 6*a^2)/((c*x^4 + a)^2*a^3)
```

maple [A] time = 0.02, size = 49, normalized size = 0.91

$$\frac{1}{8\left(c\,x^4 + a\right)^2a} + \frac{1}{4\left(c\,x^4 + a\right)a^2} + \frac{\ln(x)}{a^3} - \frac{\ln\left(c\,x^4 + a\right)}{4a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+a)^3,x)
```

```
[Out] 1/8/a/(c*x^4+a)^2+1/4/a^2/(c*x^4+a)+1/a^3*ln(x)-1/4*ln(c*x^4+a)/a^3
```

maxima [A] time = 1.36, size = 60, normalized size = 1.11

$$\frac{2\,cx^4 + 3\,a}{8\left(a^2c^2x^8 + 2\,a^3cx^4 + a^4\right)} - \frac{\log\left(cx^4 + a\right)}{4\,a^3} + \frac{\log\left(x^4\right)}{4\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(2*c*x^4 + 3*a)/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4) - 1/4*log(c*x^4 + a)/
a^3 + 1/4*log(x^4)/a^3
```

mupad [B] time = 1.04, size = 56, normalized size = 1.04

$$\frac{\ln(x)}{a^3} + \frac{\frac{3}{8\,a} + \frac{c\,x^4}{4\,a^2}}{a^2 + 2\,a\,c\,x^4 + c^2\,x^8} - \frac{\ln\left(c\,x^4 + a\right)}{4\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + c*x^4)^3),x)
```

```
[Out] log(x)/a^3 + (3/(8*a) + (c*x^4)/(4*a^2))/(a^2 + c^2*x^8 + 2*a*c*x^4) - log(
a + c*x^4)/(4*a^3)
```

sympy [A] time = 0.78, size = 56, normalized size = 1.04

$$\frac{3a + 2cx^4}{8a^4 + 16a^3cx^4 + 8a^2c^2x^8} + \frac{\log(x)}{a^3} - \frac{\log\left(\frac{a}{c} + x^4\right)}{4a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+a)**3,x)
```

```
[Out] (3*a + 2*c*x**4)/(8*a**4 + 16*a**3*c*x**4 + 8*a**2*c**2*x**8) + log(x)/a**3
 - log(a/c + x**4)/(4*a**3)
```

3.679 $\int \frac{1}{x^3\left(a+cx^4\right)^3}\,dx$

Optimal. Leaf size=78

$$-\frac{15\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{16a^{7/2}}-\frac{15}{16a^3x^2}+\frac{5}{16a^2x^2\left(a+cx^4\right)}+\frac{1}{8ax^2\left(a+cx^4\right)^2}$$

```
[Out] -15/16/a^3/x^2+1/8/a/x^2/(c*x^4+a)^2+5/16/a^2/x^2/(c*x^4+a)-15/16*arctan(x^
2*c^(1/2)/a^(1/2))*c^(1/2)/a^(7/2)
```

Rubi [A] time = 0.04, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {275, 290, 325, 205}

$$\frac{5}{16a^2x^2\left(a+cx^4\right)}-\frac{15\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{16a^{7/2}}-\frac{15}{16a^3x^2}+\frac{1}{8ax^2\left(a+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + c*x^4)^3),x]
```

```
[Out] -15/(16*a^3*x^2) + 1/(8*a*x^2*(a + c*x^4)^2) + 5/(16*a^2*x^2*(a + c*x^4)) -
 (15*Sqrt[c]*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(16*a^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+cx^4\right)^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\left(a+cx^2\right)^3}\,dx, x, x^2\right)$$

$$= \frac{1}{8ax^2\left(a+cx^4\right)^2} + \frac{5\,\text{Subst}\left(\int \frac{1}{x^2\left(a+cx^2\right)^2}\,dx, x, x^2\right)}{8a}$$

$$= \frac{1}{8ax^2\left(a+cx^4\right)^2} + \frac{5}{16a^2x^2\left(a+cx^4\right)} + \frac{15\,\text{Subst}\left(\int \frac{1}{x^2\left(a+cx^2\right)}\,dx, x, x^2\right)}{16a^2}$$

$$= -\frac{15}{16a^3x^2} + \frac{1}{8ax^2\left(a+cx^4\right)^2} + \frac{5}{16a^2x^2\left(a+cx^4\right)} - \frac{(15c)\,\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{16a^3}$$

$$= -\frac{15}{16a^3x^2} + \frac{1}{8ax^2\left(a+cx^4\right)^2} + \frac{5}{16a^2x^2\left(a+cx^4\right)} - \frac{15\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{16a^{7/2}}$$

Mathematica [A] time = 0.09, size = 105, normalized size = 1.35

$$\frac{-\frac{\sqrt{a}\left(8a^2+25acx^4+15c^2x^8\right)}{x^2\left(a+cx^4\right)^2} + 15\sqrt{c}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right) + 15\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{16a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + c*x^4)^3),x]
```

```
[Out] (-((Sqrt[a]*(8*a^2 + 25*a*c*x^4 + 15*c^2*x^8))/(x^2*(a + c*x^4)^2)) + 15*Sq
rt[c]*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 15*Sqrt[c]*ArcTan[1 + (Sqrt
[2]*c^(1/4)*x)/a^(1/4)])/(16*a^(7/2))
```

fricas [A] time = 0.57, size = 218, normalized size = 2.79

$$\left[-\frac{30\,c^2x^8 + 50\,acx^4 - 15\left(c^2x^{10} + 2\,acx^6 + a^2x^2\right)\sqrt{-\frac{c}{a}}\,\log\left(\frac{cx^4 - 2\,ax^2\sqrt{-\frac{c}{a}} - a}{cx^4 + a}\right) + 16\,a^2}{32\left(a^3c^2x^{10} + 2\,a^4cx^6 + a^5x^2\right)}, -\frac{15\,c^2x^8 + 25\,acx^4 - 15\left(c\right.}{16\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(30*c^2*x^8 + 50*a*c*x^4 - 15*(c^2*x^10 + 2*a*c*x^6 + a^2*x^2)*sqrt(
-c/a)*log((c*x^4 - 2*a*x^2*sqrt(-c/a) - a)/(c*x^4 + a)) + 16*a^2)/(a^3*c^2*
x^10 + 2*a^4*c*x^6 + a^5*x^2), -1/16*(15*c^2*x^8 + 25*a*c*x^4 - 15*(c^2*x^1
0 + 2*a*c*x^6 + a^2*x^2)*sqrt(c/a)*arctan(a*sqrt(c/a)/(c*x^2)) + 8*a^2)/(a^
3*c^2*x^10 + 2*a^4*c*x^6 + a^5*x^2)]
```

giac [A] time = 0.16, size = 61, normalized size = 0.78

$$-\frac{15\,c\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{16\sqrt{ac}\,a^3} - \frac{7\,c^2x^6 + 9\,acx^2}{16\left(cx^4+a\right)^2a^3} - \frac{1}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] -15/16*c*arctan(c*x^2/sqrt(a*c))/(sqrt(a*c)*a^3) - 1/16*(7*c^2*x^6 + 9*a*c*
x^2)/((c*x^4 + a)^2*a^3) - 1/2/(a^3*x^2)
```

maple [A] time = 0.02, size = 70, normalized size = 0.90

$$-\frac{7c^2x^6}{16\left(c\,x^4+a\right)^2a^3}-\frac{9c\,x^2}{16\left(c\,x^4+a\right)^2a^2}-\frac{15c\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{16\sqrt{ac}\,a^3}-\frac{1}{2a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+a)^3,x)
```

```
[Out] -7/16*c^2/a^3/(c*x^4+a)^2*x^6-9/16*c/a^2/(c*x^4+a)^2*x^2-15/16*c/a^3/(a*c)^
(1/2)*arctan(1/(a*c)^(1/2)*c*x^2)-1/2/x^2/a^3
```

maxima [A] time = 2.95, size = 75, normalized size = 0.96

$$-\frac{15\,c^2x^8+25\,acx^4+8\,a^2}{16\left(a^3c^2x^{10}+2\,a^4cx^6+a^5x^2\right)}-\frac{15\,c\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{16\,\sqrt{ac}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(15*c^2*x^8 + 25*a*c*x^4 + 8*a^2)/(a^3*c^2*x^10 + 2*a^4*c*x^6 + a^5*x
^2) - 15/16*c*arctan(c*x^2/sqrt(a*c))/(sqrt(a*c)*a^3)
```

mupad [B] time = 1.05, size = 72, normalized size = 0.92

$$-\frac{\frac{1}{2\,a}+\frac{25\,c\,x^4}{16\,a^2}+\frac{15\,c^2\,x^8}{16\,a^3}}{a^2\,x^2+2\,a\,c\,x^6+c^2\,x^{10}}-\frac{15\,\sqrt{c}\,\operatorname{atan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{16\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + c*x^4)^3),x)
```

```
[Out] - (1/(2*a) + (25*c*x^4)/(16*a^2) + (15*c^2*x^8)/(16*a^3))/(a^2*x^2 + c^2*x^
10 + 2*a*c*x^6) - (15*c^(1/2)*atan((c^(1/2)*x^2)/a^(1/2)))/(16*a^(7/2))
```

sympy [A] time = 0.82, size = 121, normalized size = 1.55

$$\frac{15\sqrt{-\frac{c}{a^7}}\,\log\left(-\frac{a^4\sqrt{-\frac{c}{a^7}}}{c}+x^2\right)}{32}-\frac{15\sqrt{-\frac{c}{a^7}}\,\log\left(\frac{a^4\sqrt{-\frac{c}{a^7}}}{c}+x^2\right)}{32}+\frac{-8a^2-25acx^4-15c^2x^8}{16a^5x^2+32a^4cx^6+16a^3c^2x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+a)**3,x)
```

```
[Out] 15*sqrt(-c/a**7)*log(-a**4*sqrt(-c/a**7)/c + x**2)/32 - 15*sqrt(-c/a**7)*lo
g(a**4*sqrt(-c/a**7)/c + x**2)/32 + (-8*a**2 - 25*a*c*x**4 - 15*c**2*x**8)/
(16*a**5*x**2 + 32*a**4*c*x**6 + 16*a**3*c**2*x**10)
```

3.680 $\int \frac{x^{10}}{\left(a+cx^4\right)^3}\,dx$

Optimal. Leaf size=223

$$\frac{21\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,\sqrt[4]{a}\,c^{11/4}}-\frac{21\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,\sqrt[4]{a}\,c^{11/4}}-\frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,\sqrt[4]{a}\,c^{11/4}}+\frac{21\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,\sqrt[4]{a}\,c^{11/4}}$$

```
[Out] -1/8*x^7/c/(c*x^4+a)^2-7/32*x^3/c^2/(c*x^4+a)+21/128*arctan(-1+c^(1/4)*x*2^
(1/2)/a^(1/4))/a^(1/4)/c^(11/4)*2^(1/2)+21/128*arctan(1+c^(1/4)*x*2^(1/2)/a
^(1/4))/a^(1/4)/c^(11/4)*2^(1/2)+21/256*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/
2)+x^2*c^(1/2))/a^(1/4)/c^(11/4)*2^(1/2)-21/256*ln(a^(1/4)*c^(1/4)*x*2^(1/2
)+a^(1/2)+x^2*c^(1/2))/a^(1/4)/c^(11/4)*2^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {288, 297, 1162, 617, 204, 1165, 628}

$$-\frac{7x^3}{32c^2\left(a+cx^4\right)}+\frac{21\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,\sqrt[4]{a}\,c^{11/4}}-\frac{21\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,\sqrt[4]{a}\,c^{11/4}}-\frac{21\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{64\sqrt{2}\,\sqrt[4]{a}\,c^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + c*x^4)^3,x]
```

```
[Out] -x^7/(8*c*(a + c*x^4)^2) - (7*x^3)/(32*c^2*(a + c*x^4)) - (21*ArcTan[1 - (S
qrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(1/4)*c^(11/4)) + (21*ArcTan[1 +
(Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(1/4)*c^(11/4)) + (21*Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(1/4)*c^(11/4
)) - (21*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[
2]*a^(1/4)*c^(11/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{10}}{\left(a+cx^4\right)^3}\,dx = -\frac{x^7}{8c\left(a+cx^4\right)^2} + \frac{7\int \frac{x^6}{\left(a+cx^4\right)^2}\,dx}{8c}$$

$$= -\frac{x^7}{8c\left(a+cx^4\right)^2} - \frac{7x^3}{32c^2\left(a+cx^4\right)} + \frac{21\int \frac{x^2}{a+cx^4}\,dx}{32c^2}$$

$$= -\frac{x^7}{8c\left(a+cx^4\right)^2} - \frac{7x^3}{32c^2\left(a+cx^4\right)} - \frac{21\int \frac{\sqrt{a}-\sqrt{c}\,x^2}{a+cx^4}\,dx}{64c^{5/2}} + \frac{21\int \frac{\sqrt{a}+\sqrt{c}\,x^2}{a+cx^4}\,dx}{64c^{5/2}}$$

$$= -\frac{x^7}{8c\left(a+cx^4\right)^2} - \frac{7x^3}{32c^2\left(a+cx^4\right)} + \frac{21\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{128c^3} + \frac{21\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{128c^3} + \frac{21\int \frac{1}{-\frac{\sqrt{a}}{\sqrt{c}}}}{128\sqrt{}}$$

$$= -\frac{x^7}{8c\left(a+cx^4\right)^2} - \frac{7x^3}{32c^2\left(a+cx^4\right)} + \frac{21\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{128\sqrt{2}\sqrt[4]{a}\,c^{11/4}} - \frac{21\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{}\right.}{128\sqrt{2}\sqrt[4]{a}}$$

$$= -\frac{x^7}{8c\left(a+cx^4\right)^2} - \frac{7x^3}{32c^2\left(a+cx^4\right)} - \frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\sqrt[4]{a}\,c^{11/4}} + \frac{21\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\sqrt[4]{a}\,c^{11/4}} + \frac{21\log\left(}{}$$

Mathematica [A] time = 0.10, size = 205, normalized size = 0.92

$$\frac{-\frac{88c^{3/4}x^3}{a+cx^4} + \frac{32ac^{3/4}x^3}{\left(a+cx^4\right)^2} + \frac{21\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{\sqrt[4]{a}} - \frac{21\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{\sqrt[4]{a}} - \frac{42\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}} + \frac{42\sqrt{2}\tan}{}}{256c^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + c*x^4)^3,x]
```

```
[Out] ((32*a*c^(3/4)*x^3)/(a + c*x^4)^2 - (88*c^(3/4)*x^3)/(a + c*x^4) - (42*Sqrt
[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(1/4) + (42*Sqrt[2]*ArcTan[1
 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(1/4) + (21*Sqrt[2]*Log[Sqrt[a] - Sqrt[2
]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(1/4) - (21*Sqrt[2]*Log[Sqrt[a] + Sqr
t[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(1/4))/(256*c^(11/4))
```

fricas [A] time = 0.66, size = 240, normalized size = 1.08

$$-\frac{44\,cx^7+28\,ax^3+84\left(c^4x^8+2\,ac^3x^4+a^2c^2\right)\left(-\frac{1}{ac^{11}}\right)^{\frac{1}{4}}\arctan\left(-c^3x\left(-\frac{1}{ac^{11}}\right)^{\frac{1}{4}}+\sqrt{-ac^5\sqrt{-\frac{1}{ac^{11}}}+x^2}\,c^3\left(-\frac{1}{ac^{11}}\right.\right.}{128\left(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(44*c*x^7 + 28*a*x^3 + 84*(c^4*x^8 + 2*a*c^3*x^4 + a^2*c^2)*(-1/(a*c
^11))^(1/4)*arctan(-c^3*x*(-1/(a*c^11))^(1/4) + sqrt(-a*c^5*sqrt(-1/(a*c^11
)) + x^2)*c^3*(-1/(a*c^11))^(1/4)) - 21*(c^4*x^8 + 2*a*c^3*x^4 + a^2*c^2)*(
-1/(a*c^11))^(1/4)*log(a*c^8*(-1/(a*c^11))^(3/4) + x) + 21*(c^4*x^8 + 2*a*c
^3*x^4 + a^2*c^2)*(-1/(a*c^11))^(1/4)*log(-a*c^8*(-1/(a*c^11))^(3/4) + x))/
(c^4*x^8 + 2*a*c^3*x^4 + a^2*c^2)
```

giac [A] time = 0.17, size = 206, normalized size = 0.92

$$-\frac{11\,cx^7+7\,ax^3}{32\left(cx^4+a\right)^2c^2}+\frac{21\sqrt{2}\,\left(ac^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,ac^5}+\frac{21\sqrt{2}\,\left(ac^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,ac^5}-\frac{21\sqrt{2}\,\left(ac^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] -1/32*(11*c*x^7 + 7*a*x^3)/((c*x^4 + a)^2*c^2) + 21/128*sqrt(2)*(a*c^3)^(3/
4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^5) + 21
/128*sqrt(2)*(a*c^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(
a/c)^(1/4))/(a*c^5) - 21/256*sqrt(2)*(a*c^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/c
)^(1/4) + sqrt(a/c))/(a*c^5) + 21/256*sqrt(2)*(a*c^3)^(3/4)*log(x^2 - sqrt(
2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^5)
```

maple [A] time = 0.01, size = 156, normalized size = 0.70

$$\frac{21\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{128\left(\frac{a}{c}\right)^{\frac{1}{4}}c^3}+\frac{21\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{128\left(\frac{a}{c}\right)^{\frac{1}{4}}c^3}+\frac{21\sqrt{2}\,\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{256\left(\frac{a}{c}\right)^{\frac{1}{4}}c^3}+\frac{-\frac{11x^7}{32c}-\frac{7ax^3}{32c^2}}{\left(c\,x^4+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(c*x^4+a)^3,x)
```

```
[Out] (-11/32/c*x^7-7/32*a/c^2*x^3)/(c*x^4+a)^2+21/256/c^3/(a/c)^(1/4)*2^(1/2)*ln
((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(
1/2)))+21/128/c^3/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+21/12
8/c^3/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)
```

maxima [A] time = 3.03, size = 215, normalized size = 0.96

$$-\frac{11cx^7+7ax^3}{32\left(c^4x^8+2ac^3x^4+a^2c^2\right)}+\frac{21\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x-\right.}{a^{\frac{1}{4}}c^{\frac{3}{4}}}\right.}{256c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] -1/32*(11*c*x^7 + 7*a*x^3)/(c^4*x^8 + 2*a*c^3*x^4 + a^2*c^2) + 21/256*(2*sq
rt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(
a)*sqrt(c)))/(sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(1/2*sqrt(2)
*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(sqrt(
a)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x
+ sqrt(a))/(a^(1/4)*c^(3/4)) + sqrt(2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^
(1/4)*x + sqrt(a))/(a^(1/4)*c^(3/4)))/c^2
```

mupad [B] time = 1.07, size = 83, normalized size = 0.37

$$\frac{21\operatorname{atan}\left(\frac{c^{1/4}x}{(-a)^{1/4}}\right)}{64(-a)^{1/4}c^{11/4}}-\frac{\frac{11x^7}{32c}+\frac{7ax^3}{32c^2}}{a^2+2acx^4+c^2x^8}-\frac{21\operatorname{atanh}\left(\frac{c^{1/4}x}{(-a)^{1/4}}\right)}{64(-a)^{1/4}c^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a + c*x^4)^3,x)
```

```
[Out] (21*atan((c^(1/4)*x)/(-a)^(1/4)))/(64*(-a)^(1/4)*c^(11/4)) - ((11*x^7)/(32*
c) + (7*a*x^3)/(32*c^2))/(a^2 + c^2*x^8 + 2*a*c*x^4) - (21*atanh((c^(1/4)*x
)/(-a)^(1/4)))/(64*(-a)^(1/4)*c^(11/4))
```

sympy [A] time = 0.68, size = 70, normalized size = 0.31

$$\frac{-7ax^3-11cx^7}{32a^2c^2+64ac^3x^4+32c^4x^8}+\operatorname{RootSum}\left(268435456t^4ac^{11}+194481,\left(t\mapsto t\log\left(\frac{2097152t^3ac^8}{9261}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(c*x**4+a)**3,x)
```

```
[Out] (-7*a*x**3 - 11*c*x**7)/(32*a**2*c**2 + 64*a*c**3*x**4 + 32*c**4*x**8) + Ro
otSum(268435456*_t**4*a*c**11 + 194481, Lambda(_t, _t*log(2097152*_t**3*a*c
**8/9261 + x)))
```

3.681 $\int \frac{x^8}{\left(a+cx^4\right)^3}\,dx$

Optimal. Leaf size=221

$$-\frac{5\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{3/4}c^{9/4}}+\frac{5\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{3/4}c^{9/4}}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{3/4}c^{9/4}}+\frac{5\tan^{-1}\left(\frac{\sqrt{2}}{\sqrt[4]{}}\right.}{64\sqrt{2}\,a^{3/}}$$

```
[Out] -1/8*x^5/c/(c*x^4+a)^2-5/32*x/c^2/(c*x^4+a)+5/128*arctan(-1+c^(1/4)*x*2^(1/
2)/a^(1/4))/a^(3/4)/c^(9/4)*2^(1/2)+5/128*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4
))/a^(3/4)/c^(9/4)*2^(1/2)-5/256*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*
c^(1/2))/a^(3/4)/c^(9/4)*2^(1/2)+5/256*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)
+x^2*c^(1/2))/a^(3/4)/c^(9/4)*2^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {288, 211, 1165, 628, 1162, 617, 204}

$$-\frac{5\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{3/4}c^{9/4}}+\frac{5\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{3/4}c^{9/4}}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{3/4}c^{9/4}}+\frac{5\tan^{-1}\left(\frac{\sqrt{2}}{\sqrt[4]{}}\right.}{64\sqrt{2}\,a^{3/}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + c*x^4)^3,x]
```

```
[Out] -x^5/(8*c*(a + c*x^4)^2) - (5*x)/(32*c^2*(a + c*x^4)) - (5*ArcTan[1 - (Sqrt
[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(3/4)*c^(9/4)) + (5*ArcTan[1 + (Sqrt
[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(3/4)*c^(9/4)) - (5*Log[Sqrt[a] - Sq
rt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(3/4)*c^(9/4)) + (5*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(3/4
)*c^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^8}{\left(a+cx^4\right)^3}\,dx = -\frac{x^5}{8c\left(a+cx^4\right)^2} + \frac{5\int \frac{x^4}{\left(a+cx^4\right)^2}\,dx}{8c}$$

$$= -\frac{x^5}{8c\left(a+cx^4\right)^2} - \frac{5x}{32c^2\left(a+cx^4\right)} + \frac{5\int \frac{1}{a+cx^4}\,dx}{32c^2}$$

$$= -\frac{x^5}{8c\left(a+cx^4\right)^2} - \frac{5x}{32c^2\left(a+cx^4\right)} + \frac{5\int \frac{\sqrt{a}-\sqrt{c}\,x^2}{a+cx^4}\,dx}{64\sqrt{a}\,c^2} + \frac{5\int \frac{\sqrt{a}+\sqrt{c}\,x^2}{a+cx^4}\,dx}{64\sqrt{a}\,c^2}$$

$$= -\frac{x^5}{8c\left(a+cx^4\right)^2} - \frac{5x}{32c^2\left(a+cx^4\right)} + \frac{5\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}} + x^2}\,dx}{128\sqrt{a}\,c^{5/2}} + \frac{5\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}} + x^2}\,dx}{128\sqrt{a}\,c^{5/2}} - \frac{5\int \frac{\frac{\sqrt{2}\,\sqrt[4]{}}{\sqrt[4]{c}}}{-\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}}{}}}{128\sqrt{2}\,}$$

$$= -\frac{x^5}{8c\left(a+cx^4\right)^2} - \frac{5x}{32c^2\left(a+cx^4\right)} - \frac{5\log\left(\sqrt{a} - \sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x + \sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{3/4}c^{9/4}} + \frac{5\log\left(\sqrt{a} + \sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{}\right.}{128\sqrt{2}\,a^{3/4}}$$

$$= -\frac{x^5}{8c\left(a+cx^4\right)^2} - \frac{5x}{32c^2\left(a+cx^4\right)} - \frac{5\tan^{-1}\left(1 - \frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{3/4}c^{9/4}} + \frac{5\tan^{-1}\left(1 + \frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{3/4}c^{9/4}} - \frac{5\log\left(\sqrt{a}\right.}{1}$$

Mathematica [A] time = 0.10, size = 201, normalized size = 0.91

$$\frac{-\frac{5\sqrt{2}\,\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{a^{3/4}} + \frac{5\sqrt{2}\,\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{a^{3/4}} - \frac{10\sqrt{2}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{a^{3/4}} + \frac{10\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{a^{3/4}} - \frac{72\sqrt[4]{c}\,x}{a+cx^4}}{256c^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + c*x^4)^3,x]
```

```
[Out] ((32*a*c^(1/4)*x)/(a + c*x^4)^2 - (72*c^(1/4)*x)/(a + c*x^4) - (10*Sqrt[2]*
ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(3/4) + (10*Sqrt[2]*ArcTan[1 + (
Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(3/4) - (5*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(
1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(3/4) + (5*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a
^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(3/4))/(256*c^(9/4))
```

fricas [A] time = 0.66, size = 245, normalized size = 1.11

$$-\frac{36\,cx^5-20\left(c^4x^8+2\,ac^3x^4+a^2c^2\right)\left(-\frac{1}{a^3c^9}\right)^{\frac{1}{4}}\arctan\left(-a^2c^7x\left(-\frac{1}{a^3c^9}\right)^{\frac{3}{4}}+\sqrt{a^2c^4\sqrt{-\frac{1}{a^3c^9}}+x^2}\,a^2c^7\left(-\frac{1}{a^3c^9}\right)^{\frac{3}{4}}\right)-}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(36*c*x^5 - 20*(c^4*x^8 + 2*a*c^3*x^4 + a^2*c^2)*(-1/(a^3*c^9))^(1/4
)*arctan(-a^2*c^7*x*(-1/(a^3*c^9))^(3/4) + sqrt(a^2*c^4*sqrt(-1/(a^3*c^9))
+ x^2)*a^2*c^7*(-1/(a^3*c^9))^(3/4)) - 5*(c^4*x^8 + 2*a*c^3*x^4 + a^2*c^2)*
(-1/(a^3*c^9))^(1/4)*log(a*c^2*(-1/(a^3*c^9))^(1/4) + x) + 5*(c^4*x^8 + 2*a
*c^3*x^4 + a^2*c^2)*(-1/(a^3*c^9))^(1/4)*log(-a*c^2*(-1/(a^3*c^9))^(1/4) +
x) + 20*a*x)/(c^4*x^8 + 2*a*c^3*x^4 + a^2*c^2)
```

giac [A] time = 0.17, size = 204, normalized size = 0.92

$$\frac{5\sqrt{2}\,\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,ac^3}+\frac{5\sqrt{2}\,\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,ac^3}+\frac{5\sqrt{2}\,\left(ac^3\right)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}\right.}{256\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 5/128*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/
(a/c)^(1/4))/(a*c^3) + 5/128*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x
- sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3) + 5/256*sqrt(2)*(a*c^3)^(1/4)*l
og(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3) - 5/256*sqrt(2)*(a*c^3)
^(1/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3) - 1/32*(9*c*x^5
 + 5*a*x)/((c*x^4 + a)^2*c^2)
```

maple [A] time = 0.01, size = 163, normalized size = 0.74

$$\frac{5\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{128a\,c^2}+\frac{5\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{128a\,c^2}+\frac{5\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{256a\,c^2}+\frac{-\frac{9x^5}{32c}-\frac{5ax}{32c^2}}{\left(c\,x^4+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^4+a)^3,x)
```

```
[Out] (-9/32/c*x^5-5/32*a/c^2*x)/(c*x^4+a)^2+5/256/c^2*(a/c)^(1/4)/a*2^(1/2)*ln((
x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/
2)))+5/128/c^2*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+5/128/
c^2*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)
```

maxima [A] time = 3.03, size = 213, normalized size = 0.96

$$-\frac{9cx^5+5ax}{32\left(c^4x^8+2ac^3x^4+a^2c^2\right)}+\frac{5\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\right.}{a^{\frac{3}{4}}c^{\frac{1}{4}}}\right.}{256c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] -1/32*(9*c*x^5 + 5*a*x)/(c^4*x^8 + 2*a*c^3*x^4 + a^2*c^2) + 5/256*(2*sqrt(2
)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*s
qrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*
sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(s
qrt(a)*sqrt(c))) + sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sq
rt(a))/(a^(3/4)*c^(1/4)) - sqrt(2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4
)*x + sqrt(a))/(a^(3/4)*c^(1/4)))/c^2
```

mupad [B] time = 1.08, size = 81, normalized size = 0.37

$$-\frac{\frac{9x^5}{32c}+\frac{5ax}{32c^2}}{a^2+2acx^4+c^2x^8}-\frac{5\,\mathrm{atan}\left(\frac{c^{1/4}x}{(-a)^{1/4}}\right)}{64(-a)^{3/4}c^{9/4}}-\frac{5\,\mathrm{atanh}\left(\frac{c^{1/4}x}{(-a)^{1/4}}\right)}{64(-a)^{3/4}c^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + c*x^4)^3,x)
```

```
[Out] - ((9*x^5)/(32*c) + (5*a*x)/(32*c^2))/(a^2 + c^2*x^8 + 2*a*c*x^4) - (5*atan
((c^(1/4)*x)/(-a)^(1/4)))/(64*(-a)^(3/4)*c^(9/4)) - (5*atanh((c^(1/4)*x)/(-
a)^(1/4)))/(64*(-a)^(3/4)*c^(9/4))
```

sympy [A] time = 0.96, size = 68, normalized size = 0.31

$$\frac{-5ax-9cx^5}{32a^2c^2+64ac^3x^4+32c^4x^8}+\mathrm{RootSum}\left(268435456t^4a^3c^9+625,\left(t\mapsto t\log\left(\frac{128tac^2}{5}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**4+a)**3,x)
```

```
[Out] (-5*a*x - 9*c*x**5)/(32*a**2*c**2 + 64*a*c**3*x**4 + 32*c**4*x**8) + RootSu
m(268435456*_t**4*a**3*c**9 + 625, Lambda(_t, _t*log(128*_t*a*c**2/5 + x)))
```

3.682 $\int \frac{x^6}{\left(a+cx^4\right)^3}\,dx$

Optimal. Leaf size=226

$$\frac{3\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{5/4}c^{7/4}}-\frac{3\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{5/4}c^{7/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{5/4}c^{7/4}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}}{\sqrt[4]{a}}\right.}{64\sqrt{2}\,a^{5/4}}$$

```
[Out] -1/8*x^3/c/(c*x^4+a)^2+3/32*x^3/a/c/(c*x^4+a)+3/128*arctan(-1+c^(1/4)*x*2^(
1/2)/a^(1/4))/a^(5/4)/c^(7/4)*2^(1/2)+3/128*arctan(1+c^(1/4)*x*2^(1/2)/a^(1
/4))/a^(5/4)/c^(7/4)*2^(1/2)+3/256*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^
2*c^(1/2))/a^(5/4)/c^(7/4)*2^(1/2)-3/256*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/
2)+x^2*c^(1/2))/a^(5/4)/c^(7/4)*2^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {288, 290, 297, 1162, 617, 204, 1165, 628}

$$\frac{3\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{5/4}c^{7/4}}-\frac{3\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{5/4}c^{7/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{5/4}c^{7/4}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}}{\sqrt[4]{a}}\right.}{64\sqrt{2}\,a^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + c*x^4)^3,x]
```

```
[Out] -x^3/(8*c*(a + c*x^4)^2) + (3*x^3)/(32*a*c*(a + c*x^4)) - (3*ArcTan[1 - (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(5/4)*c^(7/4)) + (3*ArcTan[1 + (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(5/4)*c^(7/4)) + (3*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(5/4)*c^(7/4)) - (
3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(5
/4)*c^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^6}{\left(a+cx^4\right)^3}\,dx = -\frac{x^3}{8c\left(a+cx^4\right)^2} + \frac{3\int \frac{x^2}{\left(a+cx^4\right)^2}\,dx}{8c}$$

$$= -\frac{x^3}{8c\left(a+cx^4\right)^2} + \frac{3x^3}{32ac\left(a+cx^4\right)} + \frac{3\int \frac{x^2}{a+cx^4}\,dx}{32ac}$$

$$= -\frac{x^3}{8c\left(a+cx^4\right)^2} + \frac{3x^3}{32ac\left(a+cx^4\right)} - \frac{3\int \frac{\sqrt{a}-\sqrt{c}\,x^2}{a+cx^4}\,dx}{64ac^{3/2}} + \frac{3\int \frac{\sqrt{a}+\sqrt{c}\,x^2}{a+cx^4}\,dx}{64ac^{3/2}}$$

$$= -\frac{x^3}{8c\left(a+cx^4\right)^2} + \frac{3x^3}{32ac\left(a+cx^4\right)} + \frac{3\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{128ac^2} + \frac{3\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{128ac^2} + \frac{3\int \frac{\frac{\sqrt{2}\,\sqrt[4]{}}{\sqrt[4]{c}}}{-\frac{\sqrt{a}}{\sqrt{c}}-}}{128\sqrt{2}}$$

$$= -\frac{x^3}{8c\left(a+cx^4\right)^2} + \frac{3x^3}{32ac\left(a+cx^4\right)} + \frac{3\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{5/4}c^{7/4}} - \frac{3\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\right.}{128\sqrt{2}\,a^{5/4}}$$

$$= -\frac{x^3}{8c\left(a+cx^4\right)^2} + \frac{3x^3}{32ac\left(a+cx^4\right)} - \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{5/4}c^{7/4}} + \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{5/4}c^{7/4}} + \frac{3\log\left(\sqrt{a}\right.}{1}$$

Mathematica [A] time = 0.11, size = 207, normalized size = 0.92

$$\frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{5/4}}-\frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{5/4}}-\frac{6\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{5/4}}+\frac{6\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{a^{5/4}}+\frac{24c^{3/4}x^3}{a^2+acx^4}$$
$$256c^{7/4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + c*x^4)^3,x]
```

```
[Out] ((-32*c^(3/4)*x^3)/(a + c*x^4)^2 + (24*c^(3/4)*x^3)/(a^2 + a*c*x^4) - (6*Sq
rt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(5/4) + (6*Sqrt[2]*ArcTan[
1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(5/4) + (3*Sqrt[2]*Log[Sqrt[a] - Sqrt[2
]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(5/4) - (3*Sqrt[2]*Log[Sqrt[a] + Sqrt
[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(5/4))/(256*c^(7/4))
```

fricas [A] time = 0.64, size = 252, normalized size = 1.12

$$\frac{12\,cx^7-4\,ax^3-12\left(ac^3x^8+2\,a^2c^2x^4+a^3c\right)\left(-\frac{1}{a^5c^7}\right)^{\frac{1}{4}}\arctan\left(-ac^2x\left(-\frac{1}{a^5c^7}\right)^{\frac{1}{4}}+\sqrt{-a^3c^3\sqrt{-\frac{1}{a^5c^7}}+x^2}\,ac^2\left(-\frac{1}{a^5c}\right.\right.}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(12*c*x^7 - 4*a*x^3 - 12*(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c)*(-1/(a^5
*c^7))^(1/4)*arctan(-a*c^2*x*(-1/(a^5*c^7))^(1/4) + sqrt(-a^3*c^3*sqrt(-1/(
a^5*c^7)) + x^2)*a*c^2*(-1/(a^5*c^7))^(1/4)) + 3*(a*c^3*x^8 + 2*a^2*c^2*x^4
 + a^3*c)*(-1/(a^5*c^7))^(1/4)*log(a^4*c^5*(-1/(a^5*c^7))^(3/4) + x) - 3*(a
*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c)*(-1/(a^5*c^7))^(1/4)*log(-a^4*c^5*(-1/(a^
5*c^7))^(3/4) + x))/(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c)
```

giac [A] time = 0.17, size = 209, normalized size = 0.92

$$\frac{3\,cx^7-ax^3}{32\left(cx^4+a\right)^2ac}+\frac{3\sqrt{2}\left(ac^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^2c^4}+\frac{3\sqrt{2}\left(ac^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^2c^4}-\frac{3\sqrt{2}\left(ac^3\right)^{\frac{3}{4}}\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/32*(3*c*x^7 - a*x^3)/((c*x^4 + a)^2*a*c) + 3/128*sqrt(2)*(a*c^3)^(3/4)*ar
ctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^4) + 3/128
*sqrt(2)*(a*c^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)
^(1/4))/(a^2*c^4) - 3/256*sqrt(2)*(a*c^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/c)^(
1/4) + sqrt(a/c))/(a^2*c^4) + 3/256*sqrt(2)*(a*c^3)^(3/4)*log(x^2 - sqrt(2)
*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^4)
```

maple [A] time = 0.01, size = 164, normalized size = 0.73

$$\frac{3\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{128\left(\frac{a}{c}\right)^{\frac{1}{4}}a\,c^2}+\frac{3\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{128\left(\frac{a}{c}\right)^{\frac{1}{4}}a\,c^2}+\frac{3\sqrt{2}\,\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{256\left(\frac{a}{c}\right)^{\frac{1}{4}}a\,c^2}+\frac{\frac{3x^7}{32a}-\frac{x^3}{32c}}{\left(c\,x^4+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+a)^3,x)
```

```
[Out] (3/32*x^7/a-1/32/c*x^3)/(c*x^4+a)^2+3/256/c^2/a/(a/c)^(1/4)*2^(1/2)*ln((x^2
-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))
)+3/128/c^2/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+3/128/c^2
/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)
```

maxima [A] time = 3.03, size = 219, normalized size = 0.97

$$\frac{3cx^7 - ax^3}{32\left(ac^3x^8 + 2a^2c^2x^4 + a^3c\right)} + \frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x - \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} - \frac{\sqrt{2}\log\left(\sqrt{c}x^2 + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \right.}{a^{\frac{1}{4}}c^{\frac{3}{4}}}\right.}{256\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(3*c*x^7 - a*x^3)/(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c) + 3/256*(2*sqrt(
2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*
sqrt(c)))/(sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2
*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(sqrt(a)*
sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + s
qrt(a))/(a^(1/4)*c^(3/4)) + sqrt(2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/
4)*x + sqrt(a))/(a^(1/4)*c^(3/4)))/(a*c)
```

mupad [B] time = 1.08, size = 81, normalized size = 0.36

$$\frac{\frac{3x^7}{32a} - \frac{x^3}{32c}}{a^2 + 2acx^4 + c^2x^8} - \frac{3\operatorname{atan}\left(\frac{c^{1/4}x}{(-a)^{1/4}}\right)}{64(-a)^{5/4}c^{7/4}} + \frac{3\operatorname{atanh}\left(\frac{c^{1/4}x}{(-a)^{1/4}}\right)}{64(-a)^{5/4}c^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + c*x^4)^3,x)
```

```
[Out] ((3*x^7)/(32*a) - x^3/(32*c))/(a^2 + c^2*x^8 + 2*a*c*x^4) - (3*atan((c^(1/4
)*x)/(-a)^(1/4)))/(64*(-a)^(5/4)*c^(7/4)) + (3*atanh((c^(1/4)*x)/(-a)^(1/4)
))/(64*(-a)^(5/4)*c^(7/4))
```

sympy [A] time = 0.98, size = 71, normalized size = 0.31

$$\frac{-ax^3 + 3cx^7}{32a^3c + 64a^2c^2x^4 + 32ac^3x^8} + \operatorname{RootSum}\left(268435456t^4a^5c^7 + 81, \left(t \mapsto t\log\left(\frac{2097152t^3a^4c^5}{27} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+a)**3,x)
```

```
[Out] (-a*x**3 + 3*c*x**7)/(32*a**3*c + 64*a**2*c**2*x**4 + 32*a*c**3*x**8) + Roo
tSum(268435456*_t**4*a**5*c**7 + 81, Lambda(_t, _t*log(2097152*_t**3*a**4*c
**5/27 + x)))
```

3.683 $\int \frac{x^4}{\left(a+cx^4\right)^3}\,dx$

Optimal. Leaf size=222

$$-\frac{3\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{7/4}c^{5/4}}+\frac{3\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{7/4}c^{5/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{7/4}c^{5/4}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}}{\sqrt[4]{}}\right.}{64\sqrt{2}\,a^{7/}}$$

```
[Out] -1/8*x/c/(c*x^4+a)^2+1/32*x/a/c/(c*x^4+a)+3/128*arctan(-1+c^(1/4)*x*2^(1/2)
/a^(1/4))/a^(7/4)/c^(5/4)*2^(1/2)+3/128*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))
/a^(7/4)/c^(5/4)*2^(1/2)-3/256*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^
(1/2))/a^(7/4)/c^(5/4)*2^(1/2)+3/256*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x
^2*c^(1/2))/a^(7/4)/c^(5/4)*2^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {288, 199, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{7/4}c^{5/4}}+\frac{3\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{7/4}c^{5/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{7/4}c^{5/4}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}}{\sqrt[4]{}}\right.}{64\sqrt{2}\,a^{7/}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + c*x^4)^3,x]
```

```
[Out] -x/(8*c*(a + c*x^4)^2) + x/(32*a*c*(a + c*x^4)) - (3*ArcTan[1 - (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(7/4)*c^(5/4)) + (3*ArcTan[1 + (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(7/4)*c^(5/4)) - (3*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(7/4)*c^(5/4)) + (3*Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(7/4)*c^(5
/4))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
```

```
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^4}{\left(a+cx^4\right)^3}\,dx = -\frac{x}{8c\left(a+cx^4\right)^2} + \frac{\int \frac{1}{\left(a+cx^4\right)^2}\,dx}{8c}$$

$$= -\frac{x}{8c\left(a+cx^4\right)^2} + \frac{x}{32ac\left(a+cx^4\right)} + \frac{3\int \frac{1}{a+cx^4}\,dx}{32ac}$$

$$= -\frac{x}{8c\left(a+cx^4\right)^2} + \frac{x}{32ac\left(a+cx^4\right)} + \frac{3\int \frac{\sqrt{a}-\sqrt{c}\,x^2}{a+cx^4}\,dx}{64a^{3/2}c} + \frac{3\int \frac{\sqrt{a}+\sqrt{c}\,x^2}{a+cx^4}\,dx}{64a^{3/2}c}$$

$$= -\frac{x}{8c\left(a+cx^4\right)^2} + \frac{x}{32ac\left(a+cx^4\right)} + \frac{3\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{128a^{3/2}c^{3/2}} + \frac{3\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{128a^{3/2}c^{3/2}} - \frac{3\int \frac{\frac{\sqrt{2}\,\sqrt[4]{}}{\sqrt[4]{c}}}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{}}{}}}{128\sqrt{2}\,a}$$

$$= -\frac{x}{8c\left(a+cx^4\right)^2} + \frac{x}{32ac\left(a+cx^4\right)} - \frac{3\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{7/4}c^{5/4}} + \frac{3\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\right.}{128\sqrt{2}\,a^{7/4}}$$

$$= -\frac{x}{8c\left(a+cx^4\right)^2} + \frac{x}{32ac\left(a+cx^4\right)} - \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{7/4}c^{5/4}} + \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{7/4}c^{5/4}} - \frac{3\log\left(\sqrt{a}\right.}{1}$$

Mathematica [A] time = 0.10, size = 203, normalized size = 0.91

$$\frac{-\frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{7/4}}+\frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{7/4}}-\frac{6\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{7/4}}+\frac{6\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{a^{7/4}}+\frac{8\sqrt[4]{c}x}{a^2+acx}}{256c^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + c*x^4)^3,x]
```

```
[Out] ((-32*c^(1/4)*x)/(a + c*x^4)^2 + (8*c^(1/4)*x)/(a^2 + a*c*x^4) - (6*Sqrt[2]
*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) + (6*Sqrt[2]*ArcTan[1 + (
Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) - (3*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(
1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4) + (3*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a
^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4))/(256*c^(5/4))
```

fricas [A] time = 0.81, size = 249, normalized size = 1.12

$$\frac{4cx^5+12\left(ac^3x^8+2a^2c^2x^4+a^3c\right)\left(-\frac{1}{a^7c^5}\right)^{\frac{1}{4}}\arctan\left(-a^5c^4x\left(-\frac{1}{a^7c^5}\right)^{\frac{3}{4}}+\sqrt{a^4c^2\sqrt{-\frac{1}{a^7c^5}}+x^2}\,a^5c^4\left(-\frac{1}{a^7c^5}\right)^{\frac{3}{4}}\right)+3}{128\left(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(4*c*x^5 + 12*(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c)*(-1/(a^7*c^5))^(1/4
)*arctan(-a^5*c^4*x*(-1/(a^7*c^5))^(3/4) + sqrt(a^4*c^2*sqrt(-1/(a^7*c^5))
+ x^2)*a^5*c^4*(-1/(a^7*c^5))^(3/4)) + 3*(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c
)*(-1/(a^7*c^5))^(1/4)*log(a^2*c*(-1/(a^7*c^5))^(1/4) + x) - 3*(a*c^3*x^8 +
 2*a^2*c^2*x^4 + a^3*c)*(-1/(a^7*c^5))^(1/4)*log(-a^2*c*(-1/(a^7*c^5))^(1/4
) + x) - 12*a*x)/(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c)
```

giac [A] time = 0.17, size = 206, normalized size = 0.93

$$\frac{3\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128a^2c^2}+\frac{3\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128a^2c^2}+\frac{3\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}\right.}{256a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 3/128*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/
(a/c)^(1/4))/(a^2*c^2) + 3/128*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*
x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^2) + 3/256*sqrt(2)*(a*c^3)^(1/
4)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^2) - 3/256*sqrt(2)*(
a*c^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^2) + 1/32*
(c*x^5 - 3*a*x)/((c*x^4 + a)^2*a*c)
```

maple [A] time = 0.01, size = 162, normalized size = 0.73

$$\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{128a^2c}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{128a^2c}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\ \ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{256a^2c}+\frac{\frac{x^5}{32a}-\frac{3x}{32c}}{\left(cx^4+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+a)^3,x)
```

```
[Out] (1/32/a*x^5-3/32/c*x)/(c*x^4+a)^2+3/256/c/a^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(
a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+
3/128/c/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+3/128/c/a^2
*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)
```

maxima [A] time = 3.01, size = 216, normalized size = 0.97

$$\frac{cx^5 - 3ax}{32\left(ac^3x^8 + 2a^2c^2x^4 + a^3c\right)} + \frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x - \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} + \frac{\sqrt{2}\log\left(\sqrt{c}x^2 + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \right.}{a^{\frac{3}{4}}c^{\frac{1}{4}}}\right.}{256\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(c*x^5 - 3*a*x)/(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c) + 3/256*(2*sqrt(2)
*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sq
rt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*s
qrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sq
rt(a)*sqrt(c))) + sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqr
t(a))/(a^(3/4)*c^(1/4)) - sqrt(2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)
*x + sqrt(a))/(a^(3/4)*c^(1/4)))/(a*c)
```

mupad [B] time = 0.12, size = 80, normalized size = 0.36

$$\frac{3\,\mathrm{atan}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{64\,(-a)^{7/4}\,c^{5/4}} - \frac{\frac{3\,x}{32\,c} - \frac{x^5}{32\,a}}{a^2 + 2\,a\,c\,x^4 + c^2\,x^8} + \frac{3\,\mathrm{atanh}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{64\,(-a)^{7/4}\,c^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + c*x^4)^3,x)
```

```
[Out] (3*atan((c^(1/4)*x)/(-a)^(1/4)))/(64*(-a)^(7/4)*c^(5/4)) - ((3*x)/(32*c) -
x^5/(32*a))/(a^2 + c^2*x^8 + 2*a*c*x^4) + (3*atanh((c^(1/4)*x)/(-a)^(1/4)))
/(64*(-a)^(7/4)*c^(5/4))
```

sympy [A] time = 0.85, size = 66, normalized size = 0.30

$$\frac{-3ax + cx^5}{32a^3c + 64a^2c^2x^4 + 32ac^3x^8} + \mathrm{RootSum}\left(268435456t^4a^7c^5 + 81, \left(t \mapsto t\log\left(\frac{128ta^2c}{3} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+a)**3,x)
```

```
[Out] (-3*a*x + c*x**5)/(32*a**3*c + 64*a**2*c**2*x**4 + 32*a*c**3*x**8) + RootSu
m(268435456*_t**4*a**7*c**5 + 81, Lambda(_t, _t*log(128*_t*a**2*c/3 + x)))
```

3.684 $\int \frac{x^2}{\left(a+cx^4\right)^3}\,dx$

Optimal. Leaf size=223

$$\frac{5\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{9/4}c^{3/4}}-\frac{5\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{9/4}c^{3/4}}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{9/4}c^{3/4}}+\frac{5\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}}{\sqrt[4]{a}}\right.}{64\sqrt{2}\,a^{9/4}}$$

```
[Out] 1/8*x^3/a/(c*x^4+a)^2+5/32*x^3/a^2/(c*x^4+a)+5/128*arctan(-1+c^(1/4)*x*2^(1
/2)/a^(1/4))/a^(9/4)/c^(3/4)*2^(1/2)+5/128*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/
4))/a^(9/4)/c^(3/4)*2^(1/2)+5/256*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2
*c^(1/2))/a^(9/4)/c^(3/4)*2^(1/2)-5/256*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2
)+x^2*c^(1/2))/a^(9/4)/c^(3/4)*2^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {290, 297, 1162, 617, 204, 1165, 628}

$$\frac{5\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{9/4}c^{3/4}}-\frac{5\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{9/4}c^{3/4}}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{9/4}c^{3/4}}+\frac{5\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}}{\sqrt[4]{a}}\right.}{64\sqrt{2}\,a^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + c*x^4)^3,x]
```

```
[Out] x^3/(8*a*(a + c*x^4)^2) + (5*x^3)/(32*a^2*(a + c*x^4)) - (5*ArcTan[1 - (Sqr
t[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(9/4)*c^(3/4)) + (5*ArcTan[1 + (Sqr
t[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(9/4)*c^(3/4)) + (5*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(9/4)*c^(3/4)) - (5
*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(9/
4)*c^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^2}{\left(a+cx^4\right)^3}\,dx = \frac{x^3}{8a\left(a+cx^4\right)^2} + \frac{5\int \frac{x^2}{\left(a+cx^4\right)^2}\,dx}{8a}$$

$$= \frac{x^3}{8a\left(a+cx^4\right)^2} + \frac{5x^3}{32a^2\left(a+cx^4\right)} + \frac{5\int \frac{x^2}{a+cx^4}\,dx}{32a^2}$$

$$= \frac{x^3}{8a\left(a+cx^4\right)^2} + \frac{5x^3}{32a^2\left(a+cx^4\right)} - \frac{5\int \frac{\sqrt{a}-\sqrt{c}\,x^2}{a+cx^4}\,dx}{64a^2\sqrt{c}} + \frac{5\int \frac{\sqrt{a}+\sqrt{c}\,x^2}{a+cx^4}\,dx}{64a^2\sqrt{c}}$$

$$= \frac{x^3}{8a\left(a+cx^4\right)^2} + \frac{5x^3}{32a^2\left(a+cx^4\right)} + \frac{5\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}} + x^2}\,dx}{128a^2c} + \frac{5\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}} + x^2}\,dx}{128a^2c} + \frac{5\int \frac{\frac{\sqrt{2}\,\sqrt[4]{a}}{\sqrt[4]{c}}}{-\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}}{\sqrt[4]{}}}}$$

$$= \frac{x^3}{8a\left(a+cx^4\right)^2} + \frac{5x^3}{32a^2\left(a+cx^4\right)} + \frac{5\log\left(\sqrt{a} - \sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x + \sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{9/4}c^{3/4}} - \frac{5\log\left(\sqrt{a} + \sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{}\right.}{128\sqrt{2}\,a^{9/4}c^{}}$$

$$= \frac{x^3}{8a\left(a+cx^4\right)^2} + \frac{5x^3}{32a^2\left(a+cx^4\right)} - \frac{5\tan^{-1}\left(1 - \frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{9/4}c^{3/4}} + \frac{5\tan^{-1}\left(1 + \frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{9/4}c^{3/4}} + \frac{5\log\left(\sqrt{a} -\right.}{12}$$

Mathematica [A] time = 0.09, size = 204, normalized size = 0.91

$$\frac{\frac{32a^{5/4}x^3}{\left(a+cx^4\right)^2} + \frac{5\sqrt{2}\,\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{c^{3/4}} - \frac{5\sqrt{2}\,\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{c^{3/4}} - \frac{10\sqrt{2}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{c^{3/4}} + \frac{10\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{c^{3/4}} -}{256a^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + c*x^4)^3,x]
```

```
[Out] ((32*a^(5/4)*x^3)/(a + c*x^4)^2 + (40*a^(1/4)*x^3)/(a + c*x^4) - (10*Sqrt[2
]*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4) + (10*Sqrt[2]*ArcTan[1 +
 (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4) + (5*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4) - (5*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]
*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4))/(256*a^(9/4))
```

fricas [A] time = 0.71, size = 242, normalized size = 1.09

$$\frac{20\,cx^7+36\,ax^3-20\left(a^2c^2x^8+2\,a^3cx^4+a^4\right)\left(-\frac{1}{a^9c^3}\right)^{\frac{1}{4}}\arctan\left(-a^2cx\left(-\frac{1}{a^9c^3}\right)^{\frac{1}{4}}+\sqrt{-a^5c\sqrt{-\frac{1}{a^9c^3}}+x^2}\,a^2c\left(-\frac{1}{a^9c}\right.\right.}{128\left(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(20*c*x^7 + 36*a*x^3 - 20*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^9*
c^3))^(1/4)*arctan(-a^2*c*x*(-1/(a^9*c^3))^(1/4) + sqrt(-a^5*c*sqrt(-1/(a^9
*c^3)) + x^2)*a^2*c*(-1/(a^9*c^3))^(1/4)) + 5*(a^2*c^2*x^8 + 2*a^3*c*x^4 +
a^4)*(-1/(a^9*c^3))^(1/4)*log(a^7*c^2*(-1/(a^9*c^3))^(3/4) + x) - 5*(a^2*c^
2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^9*c^3))^(1/4)*log(-a^7*c^2*(-1/(a^9*c^3))
^(3/4) + x))/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)
```

giac [A] time = 0.19, size = 206, normalized size = 0.92

$$\frac{5\,cx^7+9\,ax^3}{32\left(cx^4+a\right)^2a^2}+\frac{5\sqrt{2}\,\left(ac^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c^3}+\frac{5\sqrt{2}\,\left(ac^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c^3}-\frac{5\sqrt{2}\,\left(ac^3\right)^{\frac{3}{4}}\,\mathrm{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/32*(5*c*x^7 + 9*a*x^3)/((c*x^4 + a)^2*a^2) + 5/128*sqrt(2)*(a*c^3)^(3/4)*
arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^3) + 5/1
28*sqrt(2)*(a*c^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/
c)^(1/4))/(a^3*c^3) - 5/256*sqrt(2)*(a*c^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/c)
^(1/4) + sqrt(a/c))/(a^3*c^3) + 5/256*sqrt(2)*(a*c^3)^(3/4)*log(x^2 - sqrt(
2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^3)
```

maple [A] time = 0.00, size = 171, normalized size = 0.77

$$\frac{x^3}{8\left(c\,x^4+a\right)^2a}+\frac{5x^3}{32\left(c\,x^4+a\right)a^2}+\frac{5\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{128\left(\frac{a}{c}\right)^{\frac{1}{4}}a^2c}+\frac{5\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{128\left(\frac{a}{c}\right)^{\frac{1}{4}}a^2c}+\frac{5\sqrt{2}\,\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right.}{256\left(\frac{a}{c}\right)^{\frac{1}{4}}a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+a)^3,x)
```

```
[Out] 1/8*x^3/a/(c*x^4+a)^2+5/32*x^3/a^2/(c*x^4+a)+5/256/a^2/c/(a/c)^(1/4)*2^(1/2
)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/
c)^(1/2)))+5/128/a^2/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+
5/128/a^2/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)
```

maxima [A] time = 3.04, size = 214, normalized size = 0.96

$$\frac{5\,cx^7+9\,ax^3}{32\left(a^2c^2x^8+2\,a^3cx^4+a^4\right)}+\frac{5\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{}\right.}{a^{\frac{1}{4}}c^{\frac{3}{4}}}\right.}{256\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(5*c*x^7 + 9*a*x^3)/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4) + 5/256*(2*sqrt(
2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*
sqrt(c)))/(sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2
*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(sqrt(a)*
sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + s
qrt(a))/(a^(1/4)*c^(3/4)) + sqrt(2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/
4)*x + sqrt(a))/(a^(1/4)*c^(3/4)))/a^2
```

mupad [B] time = 1.07, size = 82, normalized size = 0.37

$$\frac{\frac{9\,x^3}{32\,a}+\frac{5\,c\,x^7}{32\,a^2}}{a^2+2\,a\,c\,x^4+c^2\,x^8}+\frac{5\,\mathrm{atan}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{64\,(-a)^{9/4}\,c^{3/4}}-\frac{5\,\mathrm{atanh}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{64\,(-a)^{9/4}\,c^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + c*x^4)^3,x)
```

```
[Out] ((9*x^3)/(32*a) + (5*c*x^7)/(32*a^2))/(a^2 + c^2*x^8 + 2*a*c*x^4) + (5*atan
((c^(1/4)*x)/(-a)^(1/4)))/(64*(-a)^(9/4)*c^(3/4)) - (5*atanh((c^(1/4)*x)/(-
a)^(1/4)))/(64*(-a)^(9/4)*c^(3/4))
```

sympy [A] time = 0.58, size = 71, normalized size = 0.32

$$\frac{9ax^3+5cx^7}{32a^4+64a^3cx^4+32a^2c^2x^8}+\mathrm{RootSum}\left(268435456t^4a^9c^3+625,\left(t\mapsto t\log\left(\frac{2097152t^3a^7c^2}{125}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+a)**3,x)
```

```
[Out] (9*a*x**3 + 5*c*x**7)/(32*a**4 + 64*a**3*c*x**4 + 32*a**2*c**2*x**8) + Root
Sum(268435456*_t**4*a**9*c**3 + 625, Lambda(_t, _t*log(2097152*_t**3*a**7*c
**2/125 + x)))
```

3.685 $\int \frac{1}{\left(a+cx^4\right)^3}\,dx$

Optimal. Leaf size=219

$$-\frac{21\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}\sqrt[4]{c}}+\frac{21\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}\sqrt[4]{c}}-\frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{c}}+\frac{21\tan^{-1}}{64\sqrt{2}}$$

```
[Out] 1/8*x/a/(c*x^4+a)^2+7/32*x/a^2/(c*x^4+a)+21/128*arctan(-1+c^(1/4)*x*2^(1/2)
/a^(1/4))/a^(11/4)/c^(1/4)*2^(1/2)+21/128*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4
))/a^(11/4)/c^(1/4)*2^(1/2)-21/256*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^
2*c^(1/2))/a^(11/4)/c^(1/4)*2^(1/2)+21/256*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(
1/2)+x^2*c^(1/2))/a^(11/4)/c^(1/4)*2^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {199, 211, 1165, 628, 1162, 617, 204}

$$\frac{7x}{32a^2\left(a+cx^4\right)}-\frac{21\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}\sqrt[4]{c}}+\frac{21\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}\sqrt[4]{c}}-\frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{}}{\sqrt[4]{}}\right.}{64\sqrt{2}\,a^{11/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^(-3),x]
```

```
[Out] x/(8*a*(a + c*x^4)^2) + (7*x)/(32*a^2*(a + c*x^4)) - (21*ArcTan[1 - (Sqrt[2
]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(1/4)) + (21*ArcTan[1 + (Sqrt
[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(1/4)) - (21*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*c^(1/4)) +
(21*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^
(11/4)*c^(1/4))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+cx^4\right)^3}\,dx &= \frac{x}{8a\left(a+cx^4\right)^2} + \frac{7\int \frac{1}{\left(a+cx^4\right)^2}\,dx}{8a} \\
&= \frac{x}{8a\left(a+cx^4\right)^2} + \frac{7x}{32a^2\left(a+cx^4\right)} + \frac{21\int \frac{1}{a+cx^4}\,dx}{32a^2} \\
&= \frac{x}{8a\left(a+cx^4\right)^2} + \frac{7x}{32a^2\left(a+cx^4\right)} + \frac{21\int \frac{\sqrt{a}-\sqrt{c}\,x^2}{a+cx^4}\,dx}{64a^{5/2}} + \frac{21\int \frac{\sqrt{a}+\sqrt{c}\,x^2}{a+cx^4}\,dx}{64a^{5/2}} \\
&= \frac{x}{8a\left(a+cx^4\right)^2} + \frac{7x}{32a^2\left(a+cx^4\right)} + \frac{21\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}} + x^2}\,dx}{128a^{5/2}\sqrt{c}} + \frac{21\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}} + x^2}\,dx}{128a^{5/2}\sqrt{c}} - \frac{21\int \frac{\frac{\sqrt{2}}{\sqrt{c}}}{-\frac{\sqrt{a}}{\sqrt{c}}}}{128\sqrt{2} \\
&= \frac{x}{8a\left(a+cx^4\right)^2} + \frac{7x}{32a^2\left(a+cx^4\right)} - \frac{21\log\left(\sqrt{a} - \sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x + \sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}\sqrt[4]{c}} + \frac{21\log\left(\sqrt{a} + \sqrt{2}\,\sqrt[4]{a}\right.}{128\sqrt{2}\,a^{11/} \\
&= \frac{x}{8a\left(a+cx^4\right)^2} + \frac{7x}{32a^2\left(a+cx^4\right)} - \frac{21\tan^{-1}\left(1 - \frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{c}} + \frac{21\tan^{-1}\left(1 + \frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{c}} - \frac{21\log\left(\sqrt{}\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 200, normalized size = 0.91

$$\frac{\frac{32a^{7/4}x}{\left(a+cx^4\right)^2} + \frac{56a^{3/4}x}{a+cx^4} - \frac{21\sqrt{2}\,\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{\sqrt[4]{c}} + \frac{21\sqrt{2}\,\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{\sqrt[4]{c}} - \frac{42\sqrt{2}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}} + \frac{42\sqrt{2}\,\tan^{-1}\left(}{\sqrt[4]{}}}{256a^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^(-3),x]
```

```
[Out] ((32*a^(7/4)*x)/(a + c*x^4)^2 + (56*a^(3/4)*x)/(a + c*x^4) - (42*Sqrt[2]*Ar
cTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(1/4) + (42*Sqrt[2]*ArcTan[1 + (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/c^(1/4) - (21*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4) + (21*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a
^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4))/(256*a^(11/4))
```

fricas [A] time = 0.67, size = 232, normalized size = 1.06

$$\frac{28\,cx^5 + 84\left(a^2c^2x^8 + 2\,a^3cx^4 + a^4\right)\left(-\frac{1}{a^{11}c}\right)^{\frac{1}{4}}\arctan\left(-a^8cx\left(-\frac{1}{a^{11}c}\right)^{\frac{3}{4}} + \sqrt{a^6\sqrt{-\frac{1}{a^{11}c}} + x^2}\,a^8c\left(-\frac{1}{a^{11}c}\right)^{\frac{3}{4}}\right) + 21\,(a}{128\left(a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(28*c*x^5 + 84*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^11*c))^(1/4)*
arctan(-a^8*c*x*(-1/(a^11*c))^(3/4) + sqrt(a^6*sqrt(-1/(a^11*c)) + x^2)*a^8
*c*(-1/(a^11*c))^(3/4)) + 21*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^11*c)
)^(1/4)*log(a^3*(-1/(a^11*c))^(1/4) + x) - 21*(a^2*c^2*x^8 + 2*a^3*c*x^4 +
a^4)*(-1/(a^11*c))^(1/4)*log(-a^3*(-1/(a^11*c))^(1/4) + x) + 44*a*x)/(a^2*c
^2*x^8 + 2*a^3*c*x^4 + a^4)
```

giac [A] time = 0.16, size = 204, normalized size = 0.93

$$\frac{21\sqrt{2}\,\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c} + \frac{21\sqrt{2}\,\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c} + \frac{21\sqrt{2}\,\left(ac^3\right)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}\,x\right.}{256\,a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 21/128*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))
/(a/c)^(1/4))/(a^3*c) + 21/128*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*
x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c) + 21/256*sqrt(2)*(a*c^3)^(1/4
)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c) - 21/256*sqrt(2)*(a*
c^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c) + 1/32*(7*c
*x^5 + 11*a*x)/((c*x^4 + a)^2*a^2)
```

maple [A] time = 0.01, size = 158, normalized size = 0.72

$$\frac{x}{8\left(c\,x^4+a\right)^2a} + \frac{7x}{32\left(c\,x^4+a\right)a^2} + \frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{128a^3} + \frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{128a^3} + \frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+a)^3,x)
```

```
[Out] 1/8*x/a/(c*x^4+a)^2+7/32*x/a^2/(c*x^4+a)+21/256/a^3*(a/c)^(1/4)*2^(1/2)*ln(
(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1
/2)))+21/128/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+21/128
/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)
```

maxima [A] time = 2.95, size = 212, normalized size = 0.97

$$\frac{7cx^5+11ax}{32\left(a^2c^2x^8+2a^3cx^4+a^4\right)}+\frac{21\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\right.}{a^{\frac{3}{4}}c^{\frac{1}{4}}}\right.}{256a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(7*c*x^5 + 11*a*x)/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4) + 21/256*(2*sqrt(
2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*
sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2
*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(
sqrt(a)*sqrt(c))) + sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + s
qrt(a))/(a^(3/4)*c^(1/4)) - sqrt(2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/
4)*x + sqrt(a))/(a^(3/4)*c^(1/4)))/a^2
```

mupad [B] time = 1.08, size = 80, normalized size = 0.37

$$\frac{\frac{11x}{32a}+\frac{7cx^5}{32a^2}}{a^2+2acx^4+c^2x^8}-\frac{21\operatorname{atan}\left(\frac{c^{1/4}x}{(-a)^{1/4}}\right)}{64(-a)^{11/4}c^{1/4}}-\frac{21\operatorname{atanh}\left(\frac{c^{1/4}x}{(-a)^{1/4}}\right)}{64(-a)^{11/4}c^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x^4)^3,x)
```

```
[Out] ((11*x)/(32*a) + (7*c*x^5)/(32*a^2))/(a^2 + c^2*x^8 + 2*a*c*x^4) - (21*atan
((c^(1/4)*x)/(-a)^(1/4)))/(64*(-a)^(11/4)*c^(1/4)) - (21*atanh((c^(1/4)*x)/
(-a)^(1/4)))/(64*(-a)^(11/4)*c^(1/4))
```

sympy [A] time = 0.58, size = 63, normalized size = 0.29

$$\frac{11ax+7cx^5}{32a^4+64a^3cx^4+32a^2c^2x^8}+\operatorname{RootSum}\left(268435456t^4a^{11}c+194481,\left(t\mapsto t\log\left(\frac{128ta^3}{21}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+a)**3,x)
```

```
[Out] (11*a*x + 7*c*x**5)/(32*a**4 + 64*a**3*c*x**4 + 32*a**2*c**2*x**8) + RootSu
m(268435456*_t**4*a**11*c + 194481, Lambda(_t, _t*log(128*_t*a**3/21 + x)))
```

3.686 $\int \frac{1}{x^2\left(a+cx^4\right)^3}\,dx$

Optimal. Leaf size=233

$$-\frac{45\sqrt[4]{c}\,\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{13/4}}+\frac{45\sqrt[4]{c}\,\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{13/4}}+\frac{45\sqrt[4]{c}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{13/4}}$$

```
[Out] -45/32/a^3/x+1/8/a/x/(c*x^4+a)^2+9/32/a^2/x/(c*x^4+a)-45/128*c^(1/4)*arctan
(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(13/4)*2^(1/2)-45/128*c^(1/4)*arctan(1+c^(
1/4)*x*2^(1/2)/a^(1/4))/a^(13/4)*2^(1/2)-45/256*c^(1/4)*ln(-a^(1/4)*c^(1/4)
*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(13/4)*2^(1/2)+45/256*c^(1/4)*ln(a^(1/4)*
c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(13/4)*2^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {290, 325, 297, 1162, 617, 204, 1165, 628}

$$\frac{9}{32a^2x\left(a+cx^4\right)}-\frac{45\sqrt[4]{c}\,\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{13/4}}+\frac{45\sqrt[4]{c}\,\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{13/4}}+\frac{45\sqrt[4]{c}\,\tan}{64}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + c*x^4)^3),x]
```

```
[Out] -45/(32*a^3*x) + 1/(8*a*x*(a + c*x^4)^2) + 9/(32*a^2*x*(a + c*x^4)) + (45*c
^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(13/4)) - (45
*c^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(13/4)) - (
45*c^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqr
t[2]*a^(13/4)) + (45*c^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/(128*Sqrt[2]*a^(13/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+cx^4\right)^3}\,dx = \frac{1}{8ax\left(a+cx^4\right)^2} + \frac{9\int \frac{1}{x^2\left(a+cx^4\right)^2}\,dx}{8a}$$

$$= \frac{1}{8ax\left(a+cx^4\right)^2} + \frac{9}{32a^2x\left(a+cx^4\right)} + \frac{45\int \frac{1}{x^2\left(a+cx^4\right)}\,dx}{32a^2}$$

$$= -\frac{45}{32a^3x} + \frac{1}{8ax\left(a+cx^4\right)^2} + \frac{9}{32a^2x\left(a+cx^4\right)} - \frac{(45c)\int \frac{x^2}{a+cx^4}\,dx}{32a^3}$$

$$= -\frac{45}{32a^3x} + \frac{1}{8ax\left(a+cx^4\right)^2} + \frac{9}{32a^2x\left(a+cx^4\right)} + \frac{\left(45\sqrt{c}\right)\int \frac{\sqrt{a}-\sqrt{c}\,x^2}{a+cx^4}\,dx}{64a^3} - \frac{\left(45\sqrt{c}\right)\int \frac{\sqrt{a}+}{a+c}}{64a^3}$$

$$= -\frac{45}{32a^3x} + \frac{1}{8ax\left(a+cx^4\right)^2} + \frac{9}{32a^2x\left(a+cx^4\right)} - \frac{45\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{128a^3} - \frac{45\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}}}{128a^3}$$

$$= -\frac{45}{32a^3x} + \frac{1}{8ax\left(a+cx^4\right)^2} + \frac{9}{32a^2x\left(a+cx^4\right)} - \frac{45\sqrt[4]{c}\,\log\left(\sqrt{a}\,-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{13/4}} +$$

$$= -\frac{45}{32a^3x} + \frac{1}{8ax\left(a+cx^4\right)^2} + \frac{9}{32a^2x\left(a+cx^4\right)} + \frac{45\sqrt[4]{c}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{13/4}} - \frac{45\sqrt[4]{c}\,\tan^{-1}}{64\sqrt{}}$$

Mathematica [A] time = 0.10, size = 216, normalized size = 0.93

$$\frac{-\frac{32a^{5/4}cx^3}{\left(a+cx^4\right)^2} - 45\sqrt{2}\,\sqrt[4]{c}\,\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}\,+\sqrt{c}\,x^2\right) + 45\sqrt{2}\,\sqrt[4]{c}\,\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}\,+\sqrt{c}\,x^2\right) - \frac{104\sqrt[4]{a}\,c}{a+cx^4}}{256a^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + c*x^4)^3),x]
```

```
[Out] ((-256*a^(1/4))/x - (32*a^(5/4)*c*x^3)/(a + c*x^4)^2 - (104*a^(1/4)*c*x^3)/
(a + c*x^4) + 90*Sqrt[2]*c^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] -
90*Sqrt[2]*c^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - 45*Sqrt[2]*c^(
1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + 45*Sqrt[2]*c^
(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(256*a^(13/4)
)
```

fricas [A] time = 0.56, size = 247, normalized size = 1.06

$$-\frac{180\,c^2x^8 + 324\,acx^4 - 180\left(a^3c^2x^9 + 2\,a^4cx^5 + a^5x\right)\left(-\frac{c}{a^{13}}\right)^{\frac{1}{4}}\arctan\left(-a^3x\left(-\frac{c}{a^{13}}\right)^{\frac{1}{4}} + a^3\sqrt{-\frac{a^7\sqrt{-\frac{c}{a^{13}}}-cx^2}{c}}\left(-\frac{c}{a^{13}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(180*c^2*x^8 + 324*a*c*x^4 - 180*(a^3*c^2*x^9 + 2*a^4*c*x^5 + a^5*x)
*(-c/a^13)^(1/4)*arctan(-a^3*x*(-c/a^13)^(1/4) + a^3*sqrt(-(a^7*sqrt(-c/a^1
```

```
3) - c*x^2)/c)*(-c/a^13)^(1/4)) + 45*(a^3*c^2*x^9 + 2*a^4*c*x^5 + a^5*x)*(-
c/a^13)^(1/4)*log(91125*a^10*(-c/a^13)^(3/4) + 91125*c*x) - 45*(a^3*c^2*x^9
 + 2*a^4*c*x^5 + a^5*x)*(-c/a^13)^(1/4)*log(-91125*a^10*(-c/a^13)^(3/4) + 9
1125*c*x) + 128*a^2)/(a^3*c^2*x^9 + 2*a^4*c*x^5 + a^5*x)
```

giac [A] time = 0.25, size = 217, normalized size = 0.93

$$-\frac{45\sqrt{2}\left(ac^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128a^4c^2}-\frac{45\sqrt{2}\left(ac^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128a^4c^2}+\frac{45\sqrt{2}\left(ac^3\right)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right.\right.}{256a^4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] -45/128*sqrt(2)*(a*c^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4)
)/(a/c)^(1/4))/(a^4*c^2) - 45/128*sqrt(2)*(a*c^3)^(3/4)*arctan(1/2*sqrt(2)*
(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^4*c^2) + 45/256*sqrt(2)*(a*c^3)
^(3/4)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^4*c^2) - 45/256*sqrt
(2)*(a*c^3)^(3/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^4*c^2) -
1/32*(13*c^2*x^7 + 17*a*c*x^3)/((c*x^4 + a)^2*a^3) - 1/(a^3*x)
```

maple [A] time = 0.02, size = 174, normalized size = 0.75

$$-\frac{13c^2x^7}{32\left(cx^4+a\right)^2a^3}-\frac{17cx^3}{32\left(cx^4+a\right)^2a^2}-\frac{45\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{128\left(\frac{a}{c}\right)^{\frac{1}{4}}a^3}-\frac{45\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{128\left(\frac{a}{c}\right)^{\frac{1}{4}}a^3}-\frac{45\sqrt{2}\ \ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x}\right.}{256\left(\frac{a}{c}\right)^{\frac{1}{4}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+a)^3,x)
```

```
[Out] -13/32*c^2/a^3/(c*x^4+a)^2*x^7-17/32*c/a^2/(c*x^4+a)^2*x^3-45/256/a^3/(a/c)
^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*
2^(1/2)*x+(a/c)^(1/2)))-45/128/a^3/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)
^(1/4)*x+1)-45/128/a^3/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)-
1/a^3/x
```

maxima [A] time = 3.04, size = 225, normalized size = 0.97

$$-\frac{45c^2x^8+81acx^4+32a^2}{32\left(a^3c^2x^9+2a^4cx^5+a^5x\right)}-\frac{45c\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c\right.}{a^{\frac{1}{4}}c^{\frac{3}{4}}}\right.}{256a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] -1/32*(45*c^2*x^8 + 81*a*c*x^4 + 32*a^2)/(a^3*c^2*x^9 + 2*a^4*c*x^5 + a^5*x
) - 45/256*c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c
^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*
arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqr
```

```
t(c)))/(sqrt(sqrt(a)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*
a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(1/4)*c^(3/4)) + sqrt(2)*log(sqrt(c)*x^2 -
sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(1/4)*c^(3/4)))/a^3
```

mupad **[B]** time = 0.11, size = 91, normalized size = 0.39

$$\frac{45\,(-c)^{1/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\,x}{a^{1/4}}\right)}{64\,a^{13/4}}-\frac{45\,(-c)^{1/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\,x}{a^{1/4}}\right)}{64\,a^{13/4}}-\frac{\frac{1}{a}+\frac{81\,c\,x^4}{32\,a^2}+\frac{45\,c^2\,x^8}{32\,a^3}}{a^2\,x+2\,a\,c\,x^5+c^2\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + c*x^4)^3),x)
```

```
[Out] (45*(-c)^(1/4)*atanh(((-c)^(1/4)*x)/a^(1/4)))/(64*a^(13/4)) - (45*(-c)^(1/4
)*atan(((-c)^(1/4)*x)/a^(1/4)))/(64*a^(13/4)) - (1/a + (81*c*x^4)/(32*a^2)
+ (45*c^2*x^8)/(32*a^3))/(a^2*x + c^2*x^9 + 2*a*c*x^5)
```

sympy **[A]** time = 1.04, size = 80, normalized size = 0.34

$$\frac{-32a^2-81acx^4-45c^2x^8}{32a^5x+64a^4cx^5+32a^3c^2x^9}+\mathrm{RootSum}\left(268435456t^4a^{13}+4100625c,\left(t\mapsto t\log\left(-\frac{2097152t^3a^{10}}{91125c}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+a)**3,x)
```

```
[Out] (-32*a**2 - 81*a*c*x**4 - 45*c**2*x**8)/(32*a**5*x + 64*a**4*c*x**5 + 32*a*
*3*c**2*x**9) + RootSum(268435456*_t**4*a**13 + 4100625*c, Lambda(_t, _t*lo
g(-2097152*_t**3*a**10/(91125*c) + x)))
```

3.687 $\int \frac{x^9}{2+3x^4}\,dx$

Optimal. Leaf size=38

$$\frac{x^6}{18}-\frac{x^2}{9}+\frac{1}{9}\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)$$

```
[Out] -1/9*x^2+1/18*x^6+1/27*arctan(1/2*x^2*6^(1/2))*6^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {275, 302, 203}

$$\frac{x^6}{18}-\frac{x^2}{9}+\frac{1}{9}\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(2 + 3*x^4),x]
```

```
[Out] -x^2/9 + x^6/18 + (Sqrt[2/3]*ArcTan[Sqrt[3/2]*x^2])/9
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9}{2+3x^4}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{2+3x^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{2}{9}+\frac{x^2}{3}+\frac{4}{9\left(2+3x^2\right)}\right)dx, x, x^2\right)\\
&= -\frac{x^2}{9}+\frac{x^6}{18}+\frac{2}{9}\,\text{Subst}\left(\int \frac{1}{2+3x^2}\,dx, x, x^2\right)\\
&= -\frac{x^2}{9}+\frac{x^6}{18}+\frac{1}{9}\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 0.89

$$\frac{1}{54}\left(3x^6-6x^2+2\sqrt{6}\tan^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(2 + 3*x^4),x]
[Out] (-6*x^2 + 3*x^6 + 2*Sqrt[6]*ArcTan[Sqrt[3/2]*x^2])/54
```

fricas [A] time = 0.77, size = 31, normalized size = 0.82

$$\frac{1}{18}x^6 - \frac{1}{9}x^2 + \frac{1}{27}\sqrt{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{3}\sqrt{2}x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(3*x^4+2),x, algorithm="fricas")
[Out] 1/18*x^6 - 1/9*x^2 + 1/27*sqrt(3)*sqrt(2)*arctan(1/2*sqrt(3)*sqrt(2)*x^2)
```

giac [A] time = 0.17, size = 25, normalized size = 0.66

$$\frac{1}{18}x^6 - \frac{1}{9}x^2 + \frac{1}{27}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(3*x^4+2),x, algorithm="giac")
[Out] 1/18*x^6 - 1/9*x^2 + 1/27*sqrt(6)*arctan(1/2*sqrt(6)*x^2)
```

maple [A] time = 0.00, size = 26, normalized size = 0.68

$$\frac{x^6}{18} - \frac{x^2}{9} + \frac{\sqrt{6}\ \arctan\left(\frac{\sqrt{6}\,x^2}{2}\right)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(3*x^4+2),x)
[Out] -1/9*x^2+1/18*x^6+1/27*arctan(1/2*x^2*6^(1/2))*6^(1/2)
```

maxima [A] time = 2.89, size = 25, normalized size = 0.66

$$\frac{1}{18}x^6 - \frac{1}{9}x^2 + \frac{1}{27}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(3*x^4+2),x, algorithm="maxima")
[Out] 1/18*x^6 - 1/9*x^2 + 1/27*sqrt(6)*arctan(1/2*sqrt(6)*x^2)
```

mupad [B] time = 0.04, size = 25, normalized size = 0.66

$$\frac{\sqrt{6}\ \operatorname{atan}\left(\frac{\sqrt{6}\,x^2}{2}\right)}{27} - \frac{x^2}{9} + \frac{x^6}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(3*x^4 + 2),x)
[Out] (6^(1/2)*atan((6^(1/2)*x^2)/2))/27 - x^2/9 + x^6/18
```

sympy [**A**] time = 0.24, size = 27, normalized size = 0.71

$$\frac{x^6}{18} - \frac{x^2}{9} + \frac{\sqrt{6}\,\operatorname{atan}\left(\frac{\sqrt{6}\,x^2}{2}\right)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(3*x**4+2),x)
[Out] x**6/18 - x**2/9 + sqrt(6)*atan(sqrt(6)*x**2/2)/27
```

3.688 $\int \frac{x^7}{2+3x^4}\,dx$

Optimal. Leaf size=20

$$\frac{x^4}{12}-\frac{1}{18}\log\left(3x^4+2\right)$$

```
[Out] 1/12*x^4-1/18*ln(3*x^4+2)
```

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{x^4}{12}-\frac{1}{18}\log\left(3x^4+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(2 + 3*x^4),x]
```

```
[Out] x^4/12 - Log[2 + 3*x^4]/18
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{2+3x^4}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x}{2+3x}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int \left(\frac{1}{3}-\frac{2}{3(2+3x)}\right)\,dx, x, x^4\right)\\ &= \frac{x^4}{12}-\frac{1}{18}\log\left(2+3x^4\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.05

$$\frac{1}{36}\left(3x^4-2\log\left(3x^4+2\right)+2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(2 + 3*x^4),x]
```

```
[Out] (2 + 3*x^4 - 2*Log[2 + 3*x^4])/36
```

fricas [A] time = 0.53, size = 16, normalized size = 0.80

$$\frac{1}{12}\,x^4-\frac{1}{18}\,\log\left(3\,x^4+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(3*x^4+2),x, algorithm="fricas")
[Out] 1/12*x^4 - 1/18*log(3*x^4 + 2)
```

giac [A] time = 0.17, size = 16, normalized size = 0.80

$$\frac{1}{12}\,x^4 - \frac{1}{18}\,\log\left(3\,x^4+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(3*x^4+2),x, algorithm="giac")
[Out] 1/12*x^4 - 1/18*log(3*x^4 + 2)
```

maple [A] time = 0.00, size = 17, normalized size = 0.85

$$\frac{x^4}{12} - \frac{\ln\left(3x^4+2\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(3*x^4+2),x)
[Out] 1/12*x^4-1/18*ln(3*x^4+2)
```

maxima [A] time = 1.30, size = 16, normalized size = 0.80

$$\frac{1}{12}\,x^4 - \frac{1}{18}\,\log\left(3\,x^4+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(3*x^4+2),x, algorithm="maxima")
[Out] 1/12*x^4 - 1/18*log(3*x^4 + 2)
```

mupad [B] time = 0.03, size = 14, normalized size = 0.70

$$\frac{x^4}{12} - \frac{\ln\left(x^4+\frac{2}{3}\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(3*x^4 + 2),x)
[Out] x^4/12 - log(x^4 + 2/3)/18
```

sympy [A] time = 0.25, size = 14, normalized size = 0.70

$$\frac{x^4}{12} - \frac{\log\left(3x^4+2\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(3*x**4+2),x)
[Out] x**4/12 - log(3*x**4 + 2)/18
```

3.689 $\int \frac{x^5}{2+3x^4}\,dx$

Optimal. Leaf size=29

$$\frac{x^2}{6}-\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{3\sqrt{6}}$$

```
[Out] 1/6*x^2-1/18*arctan(1/2*x^2*6^(1/2))*6^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {275, 321, 203}

$$\frac{x^2}{6}-\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{3\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(2 + 3*x^4),x]
```

```
[Out] x^2/6 - ArcTan[Sqrt[3/2]*x^2]/(3*Sqrt[6])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{2+3x^4}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{2+3x^2}\,dx, x, x^2\right)\\ &= \frac{x^2}{6}-\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{2+3x^2}\,dx, x, x^2\right)\\ &= \frac{x^2}{6}-\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{3\sqrt{6}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{x^2}{6}-\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{3\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(2 + 3*x^4),x]
[Out] x^2/6 - ArcTan[Sqrt[3/2]*x^2]/(3*Sqrt[6])
```

fricas [A] time = 0.62, size = 20, normalized size = 0.69

$$\frac{1}{6}x^2 - \frac{1}{18}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(3*x^4+2),x, algorithm="fricas")
[Out] 1/6*x^2 - 1/18*sqrt(6)*arctan(1/2*sqrt(6)*x^2)
```

giac [A] time = 0.19, size = 20, normalized size = 0.69

$$\frac{1}{6}x^2 - \frac{1}{18}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(3*x^4+2),x, algorithm="giac")
[Out] 1/6*x^2 - 1/18*sqrt(6)*arctan(1/2*sqrt(6)*x^2)
```

maple [A] time = 0.00, size = 21, normalized size = 0.72

$$\frac{x^2}{6} - \frac{\sqrt{6}\ \arctan\left(\frac{\sqrt{6}\,x^2}{2}\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(3*x^4+2),x)
[Out] 1/6*x^2-1/18*6^(1/2)*arctan(1/2*6^(1/2)*x^2)
```

maxima [A] time = 2.93, size = 20, normalized size = 0.69

$$\frac{1}{6}x^2 - \frac{1}{18}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(3*x^4+2),x, algorithm="maxima")
[Out] 1/6*x^2 - 1/18*sqrt(6)*arctan(1/2*sqrt(6)*x^2)
```

mupad [B] time = 1.01, size = 20, normalized size = 0.69

$$\frac{x^2}{6} - \frac{\sqrt{6}\ \operatorname{atan}\left(\frac{\sqrt{6}\,x^2}{2}\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(3*x^4 + 2),x)
[Out] x^2/6 - (6^(1/2)*atan((6^(1/2)*x^2)/2))/18
```

sympy [**A**] time = 0.16, size = 22, normalized size = 0.76

$$\frac{x^2}{6} - \frac{\sqrt{6}\,\operatorname{atan}\left(\frac{\sqrt{6}\,x^2}{2}\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(3*x**4+2),x)
[Out] x**2/6 - sqrt(6)*atan(sqrt(6)*x**2/2)/18
```

3.690 $\int \frac{x^3}{2+3x^4}\,dx$

Optimal. Leaf size=12

$$\frac{1}{12}\log\left(3x^4+2\right)$$

```
[Out] 1/12*ln(3*x^4+2)
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {260}

$$\frac{1}{12}\log\left(3x^4+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(2 + 3*x^4),x]
```

```
[Out] Log[2 + 3*x^4]/12
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^3}{2+3x^4}\,dx = \frac{1}{12}\log\left(2+3x^4\right)$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\frac{1}{12}\log\left(3x^4+2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(2 + 3*x^4),x]
```

```
[Out] Log[2 + 3*x^4]/12
```

fricas [A] time = 0.48, size = 10, normalized size = 0.83

$$\frac{1}{12}\log\left(3\,x^4+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] 1/12*log(3*x^4 + 2)
```

giac [A] time = 0.17, size = 10, normalized size = 0.83

$$\frac{1}{12}\log\left(3\,x^4+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(3*x^4+2),x, algorithm="giac")
[Out] 1/12*log(3*x^4 + 2)
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$\frac{\ln\left(3x^4+2\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(3*x^4+2),x)
[Out] 1/12*ln(3*x^4+2)
```

maxima [A] time = 1.30, size = 10, normalized size = 0.83

$$\frac{1}{12}\log\left(3\,x^4+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(3*x^4+2),x, algorithm="maxima")
[Out] 1/12*log(3*x^4 + 2)
```

mupad [B] time = 0.98, size = 8, normalized size = 0.67

$$\frac{\ln\left(x^4+\frac{2}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(3*x^4 + 2),x)
[Out] log(x^4 + 2/3)/12
```

sympy [A] time = 0.10, size = 8, normalized size = 0.67

$$\frac{\log\left(3x^4+2\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(3*x**4+2),x)
[Out] log(3*x**4 + 2)/12
```

3.691 $\int \frac{x}{2+3x^4}\,dx$

Optimal. Leaf size=21

$$\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{2\sqrt{6}}$$

```
[Out] 1/12*arctan(1/2*x^2*6^(1/2))*6^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {275, 203}

$$\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(2 + 3*x^4),x]
```

```
[Out] ArcTan[Sqrt[3/2]*x^2]/(2*Sqrt[6])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{2+3x^4}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{2+3x^2}\,dx, x, x^2\right)\\ &= \frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{2\sqrt{6}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(2 + 3*x^4),x]
```

```
[Out] ArcTan[Sqrt[3/2]*x^2]/(2*Sqrt[6])
```

fricas [A] time = 0.57, size = 14, normalized size = 0.67

$$\frac{1}{12}\sqrt{6}\,\arctan\left(\frac{1}{2}\sqrt{6}\,x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x^4+2),x, algorithm="fricas")
[Out] 1/12*sqrt(6)*arctan(1/2*sqrt(6)*x^2)
```

giac [**A**] time = 0.17, size = 14, normalized size = 0.67

$$\frac{1}{12}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x^4+2),x, algorithm="giac")
[Out] 1/12*sqrt(6)*arctan(1/2*sqrt(6)*x^2)
```

maple [**A**] time = 0.00, size = 15, normalized size = 0.71

$$\frac{\sqrt{6}\ \arctan\left(\frac{\sqrt{6}\,x^2}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(3*x^4+2),x)
[Out] 1/12*6^(1/2)*arctan(1/2*6^(1/2)*x^2)
```

maxima [**A**] time = 2.95, size = 14, normalized size = 0.67

$$\frac{1}{12}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x^4+2),x, algorithm="maxima")
[Out] 1/12*sqrt(6)*arctan(1/2*sqrt(6)*x^2)
```

mupad [**B**] time = 0.03, size = 14, normalized size = 0.67

$$\frac{\sqrt{6}\ \operatorname{atan}\left(\frac{\sqrt{6}\,x^2}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(3*x^4 + 2),x)
[Out] (6^(1/2)*atan((6^(1/2)*x^2)/2))/12
```

sympy [**A**] time = 0.12, size = 17, normalized size = 0.81

$$\frac{\sqrt{6}\ \operatorname{atan}\left(\frac{\sqrt{6}x^2}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x**4+2),x)
[Out] sqrt(6)*atan(sqrt(6)*x**2/2)/12
```

3.692 $\int \frac{1}{x\left(2+3x^4\right)}\,dx$

Optimal. Leaf size=19

$$\frac{\log(x)}{2}-\frac{1}{8}\log\left(3x^4+2\right)$$

```
[Out] 1/2*ln(x)-1/8*ln(3*x^4+2)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {266, 36, 29, 31}

$$\frac{\log(x)}{2}-\frac{1}{8}\log\left(3x^4+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(2 + 3*x^4)),x]
```

```
[Out] Log[x]/2 - Log[2 + 3*x^4]/8
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(2+3x^4\right)}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x(2+3x)}\,dx, x, x^4\right)\\ &= \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^4\right)-\frac{3}{8}\,\text{Subst}\left(\int \frac{1}{2+3x}\,dx, x, x^4\right)\\ &= \frac{\log(x)}{2}-\frac{1}{8}\log\left(2+3x^4\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{\log(x)}{2}-\frac{1}{8}\log\left(3x^4+2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(2 + 3*x^4)),x]
[Out] Log[x]/2 - Log[2 + 3*x^4]/8
```

fricas [A] time = 0.61, size = 15, normalized size = 0.79

$$-\frac{1}{8}\log\left(3x^4+2\right)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^4+2),x, algorithm="fricas")
[Out] -1/8*log(3*x^4 + 2) + 1/2*log(x)
```

giac [A] time = 0.18, size = 17, normalized size = 0.89

$$-\frac{1}{8}\log\left(3x^4+2\right)+\frac{1}{8}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^4+2),x, algorithm="giac")
[Out] -1/8*log(3*x^4 + 2) + 1/8*log(x^4)
```

maple [A] time = 0.00, size = 16, normalized size = 0.84

$$\frac{\ln(x)}{2}-\frac{\ln\left(3x^4+2\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(3*x^4+2),x)
[Out] 1/2*ln(x)-1/8*ln(3*x^4+2)
```

maxima [A] time = 1.34, size = 17, normalized size = 0.89

$$-\frac{1}{8}\log\left(3x^4+2\right)+\frac{1}{8}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^4+2),x, algorithm="maxima")
[Out] -1/8*log(3*x^4 + 2) + 1/8*log(x^4)
```

mupad [B] time = 0.07, size = 13, normalized size = 0.68

$$\frac{\ln(x)}{2}-\frac{\ln\left(x^4+\frac{2}{3}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(3*x^4 + 2)),x)
[Out] log(x)/2 - log(x^4 + 2/3)/8
```

sympy [A] time = 0.12, size = 14, normalized size = 0.74

$$\frac{\log(x)}{2}-\frac{\log\left(3x^4+2\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x**4+2),x)
[Out] log(x)/2 - log(3*x**4 + 2)/8
```

3.693 $\int \frac{1}{x^3\left(2+3x^4\right)}\,dx$

Optimal. Leaf size=31

$$-\frac{1}{4x^2}-\frac{1}{4}\sqrt{\frac{3}{2}}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)$$

```
[Out] -1/4/x^2-1/8*arctan(1/2*x^2*6^(1/2))*6^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {275, 325, 203}

$$-\frac{1}{4x^2}-\frac{1}{4}\sqrt{\frac{3}{2}}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(2 + 3*x^4)),x]
```

```
[Out] -1/(4*x^2) - (Sqrt[3/2]*ArcTan[Sqrt[3/2]*x^2])/4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(2+3x^4\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\left(2+3x^2\right)}\,dx, x, x^2\right)\\ &= -\frac{1}{4x^2}-\frac{3}{4}\,\text{Subst}\left(\int \frac{1}{2+3x^2}\,dx, x, x^2\right)\\ &= -\frac{1}{4x^2}-\frac{1}{4}\sqrt{\frac{3}{2}}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 48, normalized size = 1.55

$$\frac{\sqrt{6}\,x^2\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)+\sqrt{6}\,x^2\tan^{-1}\left(\sqrt[4]{6}\,x+1\right)-2}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(2 + 3*x^4)),x]
```

```
[Out] (-2 + Sqrt[6]*x^2*ArcTan[1 - 6^(1/4)*x] + Sqrt[6]*x^2*ArcTan[1 + 6^(1/4)*x]
)/(8*x^2)
```

fricas [A] time = 0.55, size = 29, normalized size = 0.94

$$-\frac{\sqrt{3}\,\sqrt{2}\,x^2 \arctan\left(\frac{1}{2}\,\sqrt{3}\,\sqrt{2}\,x^2\right) + 2}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] -1/8*(sqrt(3)*sqrt(2)*x^2*arctan(1/2*sqrt(3)*sqrt(2)*x^2) + 2)/x^2
```

giac [A] time = 0.18, size = 20, normalized size = 0.65

$$-\frac{1}{8}\,\sqrt{6}\,\arctan\left(\frac{1}{2}\,\sqrt{6}\,x^2\right) - \frac{1}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(3*x^4+2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(6)*arctan(1/2*sqrt(6)*x^2) - 1/4/x^2
```

maple [A] time = 0.00, size = 21, normalized size = 0.68

$$-\frac{\sqrt{6}\,\arctan\left(\frac{\sqrt{6}\,x^2}{2}\right)}{8} - \frac{1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(3*x^4+2),x)
```

```
[Out] -1/4/x^2-1/8*6^(1/2)*arctan(1/2*6^(1/2)*x^2)
```

maxima [A] time = 2.89, size = 20, normalized size = 0.65

$$-\frac{1}{8}\,\sqrt{6}\,\arctan\left(\frac{1}{2}\,\sqrt{6}\,x^2\right) - \frac{1}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(6)*arctan(1/2*sqrt(6)*x^2) - 1/4/x^2
```

mupad [B] time = 1.00, size = 20, normalized size = 0.65

$$-\frac{\sqrt{6}\,\operatorname{atan}\left(\frac{\sqrt{6}\,x^2}{2}\right)}{8} - \frac{1}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(3*x^4 + 2)),x)
```

```
[Out] - (6^(1/2)*atan((6^(1/2)*x^2)/2))/8 - 1/(4*x^2)
```

sympy [**A**] time = 0.15, size = 26, normalized size = 0.84

$$-\frac{\sqrt{6}\,\operatorname{atan}\left(\frac{\sqrt{6}\,x^2}{2}\right)}{8}-\frac{1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(3*x**4+2),x)
[Out] -sqrt(6)*atan(sqrt(6)*x**2/2)/8 - 1/(4*x**2)
```

3.694 $\int \frac{x^6}{2+3x^4}\,dx$

Optimal. Leaf size=104

$$\frac{x^3}{9}-\frac{\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{6\,6^{3/4}}+\frac{\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{6\,6^{3/4}}+\frac{\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{3\,6^{3/4}}-\frac{\tan^{-1}\left(\sqrt[4]{6}\,x+1\right)}{3\,6^{3/4}}$$

```
[Out] 1/9*x^3-1/18*arctan(-1+6^(1/4)*x)*6^(1/4)-1/18*arctan(1+6^(1/4)*x)*6^(1/4)-
1/36*ln(-6^(3/4)*x+3*x^2+6^(1/2))*6^(1/4)+1/36*ln(6^(3/4)*x+3*x^2+6^(1/2))*
6^(1/4)
```

Rubi [A] time = 0.10, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {321, 297, 1162, 617, 204, 1165, 628}

$$\frac{x^3}{9}-\frac{\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{6\,6^{3/4}}+\frac{\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{6\,6^{3/4}}+\frac{\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{3\,6^{3/4}}-\frac{\tan^{-1}\left(\sqrt[4]{6}\,x+1\right)}{3\,6^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(2 + 3*x^4),x]
```

```
[Out] x^3/9 + ArcTan[1 - 6^(1/4)*x]/(3*6^(3/4)) - ArcTan[1 + 6^(1/4)*x]/(3*6^(3/4
)) - Log[Sqrt[6] - 6^(3/4)*x + 3*x^2]/(6*6^(3/4)) + Log[Sqrt[6] + 6^(3/4)*x
 + 3*x^2]/(6*6^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^6}{2+3x^4}\,dx = \frac{x^3}{9} - \frac{2}{3}\int \frac{x^2}{2+3x^4}\,dx$$

$$= \frac{x^3}{9} + \frac{\int \frac{\sqrt{2}-\sqrt{3}\,x^2}{2+3x^4}\,dx}{3\sqrt{3}} - \frac{\int \frac{\sqrt{2}+\sqrt{3}\,x^2}{2+3x^4}\,dx}{3\sqrt{3}}$$

$$= \frac{x^3}{9} - \frac{1}{18}\int \frac{1}{\sqrt{\frac{2}{3}} - \frac{2^{3/4}x}{\sqrt[4]{3}} + x^2}\,dx - \frac{1}{18}\int \frac{1}{\sqrt{\frac{2}{3}} + \frac{2^{3/4}x}{\sqrt[4]{3}} + x^2}\,dx - \frac{\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{6\,6^{3/4}} - \frac{\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}-2x}{-\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}-}}{6\,6^{3/4}}$$

$$= \frac{x^3}{9} - \frac{\log\left(\sqrt{6} - 6^{3/4}x + 3x^2\right)}{6\,6^{3/4}} + \frac{\log\left(\sqrt{6} + 6^{3/4}x + 3x^2\right)}{6\,6^{3/4}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt[4]{6}\,x\right)}{3\,6^{3/4}} + \frac{\text{Su}}{}$$

$$= \frac{x^3}{9} + \frac{\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{3\,6^{3/4}} - \frac{\tan^{-1}\left(1+\sqrt[4]{6}\,x\right)}{3\,6^{3/4}} - \frac{\log\left(\sqrt{6} - 6^{3/4}x + 3x^2\right)}{6\,6^{3/4}} + \frac{\log\left(\sqrt{6} + 6^{3/4}x + 3x^2\right)}{6\,6^{3/4}}$$

Mathematica [A] time = 0.03, size = 98, normalized size = 0.94

$$\frac{1}{36}\left(4x^3 - \sqrt[4]{6}\log\left(\sqrt{6}\,x^2 - 2\sqrt[4]{6}\,x + 2\right) + \sqrt[4]{6}\log\left(\sqrt{6}\,x^2 + 2\sqrt[4]{6}\,x + 2\right) + 2\sqrt[4]{6}\tan^{-1}\left(1 - \sqrt[4]{6}\,x\right) - 2\sqrt[4]{6}\tan^{-1}\left(\sqrt[4]{6}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(2 + 3*x^4),x]
```

```
[Out] (4*x^3 + 2*6^(1/4)*ArcTan[1 - 6^(1/4)*x] - 2*6^(1/4)*ArcTan[1 + 6^(1/4)*x]
- 6^(1/4)*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] + 6^(1/4)*Log[2 + 2*6^(1/4)*x
+ Sqrt[6]*x^2])/36
```

fricas [B] time = 0.65, size = 164, normalized size = 1.58

$$\frac{1}{9}x^3 + \frac{1}{162}\cdot 54^{\frac{3}{4}}\sqrt{2}\arctan\left(-\frac{1}{18}\cdot 54^{\frac{3}{4}}\sqrt{2}\,x + \frac{1}{54}\cdot 54^{\frac{3}{4}}\sqrt{2}\sqrt{9x^2 + 3\cdot 54^{\frac{1}{4}}\sqrt{2}\,x + 3\sqrt{6}} - 1\right) + \frac{1}{162}\cdot 54^{\frac{3}{4}}\sqrt{2}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] 1/9*x^3 + 1/162*54^(3/4)*sqrt(2)*arctan(-1/18*54^(3/4)*sqrt(2)*x + 1/54*54^
(3/4)*sqrt(2)*sqrt(9*x^2 + 3*54^(1/4)*sqrt(2)*x + 3*sqrt(6)) - 1) + 1/162*5
```

```
4^(3/4)*sqrt(2)*arctan(-1/18*54^(3/4)*sqrt(2)*x + 1/54*54^(3/4)*sqrt(2)*sqr
t(9*x^2 - 3*54^(1/4)*sqrt(2)*x + 3*sqrt(6)) + 1) + 1/648*54^(3/4)*sqrt(2)*l
og(9*x^2 + 3*54^(1/4)*sqrt(2)*x + 3*sqrt(6)) - 1/648*54^(3/4)*sqrt(2)*log(9
*x^2 - 3*54^(1/4)*sqrt(2)*x + 3*sqrt(6))
```

giac [A] time = 0.19, size = 100, normalized size = 0.96

$$\frac{1}{9}x^3-\frac{1}{18}\cdot 6^{\frac{1}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)-\frac{1}{18}\cdot 6^{\frac{1}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{36}\cdot 6^{\frac{1}{4}}\log\left($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/9*x^3 - 1/18*6^(1/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^
(1/4))) - 1/18*6^(1/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^
(1/4))) + 1/36*6^(1/4)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/36*
6^(1/4)*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

maple [A] time = 0.01, size = 116, normalized size = 1.12

$$\frac{x^3}{9}-\frac{\sqrt{3}\ 6^{\frac{3}{4}}\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\ \sqrt{3}\ 6^{\frac{3}{4}}x}{6}-1\right)}{108}-\frac{\sqrt{3}\ 6^{\frac{3}{4}}\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\ \sqrt{3}\ 6^{\frac{3}{4}}x}{6}+1\right)}{108}-\frac{\sqrt{3}\ 6^{\frac{3}{4}}\sqrt{2}\ \ln\left(\frac{x^2-\frac{\sqrt{3}\ 6^{\frac{1}{4}}\sqrt{2}\ x}{3}+\frac{\sqrt{6}}{3}}{x^2+\frac{\sqrt{3}\ 6^{\frac{1}{4}}\sqrt{2}\ x}{3}+\frac{\sqrt{6}}{3}}\right)}{216}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(3*x^4+2),x)
```

```
[Out] 1/9*x^3-1/216*3^(1/2)*6^(3/4)*2^(1/2)*ln((x^2-1/3*3^(1/2)*6^(1/4)*x*2^(1/2)
+1/3*6^(1/2))/(x^2+1/3*3^(1/2)*6^(1/4)*x*2^(1/2)+1/3*6^(1/2)))-1/108*3^(1/2
)*6^(3/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)-1/108*3^(1/2)*6^(
3/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)
```

maxima [A] time = 2.94, size = 126, normalized size = 1.21

$$\frac{1}{9}x^3-\frac{1}{18}\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)-\frac{1}{18}\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{36}\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/9*x^3 - 1/18*3^(1/4)*2^(1/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^
(1/4)*2^(3/4))) - 1/18*3^(1/4)*2^(1/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3
)*x - 3^(1/4)*2^(3/4))) + 1/36*3^(1/4)*2^(1/4)*log(sqrt(3)*x^2 + 3^(1/4)*2^
(3/4)*x + sqrt(2)) - 1/36*3^(1/4)*2^(1/4)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)
*x + sqrt(2))
```

mupad [B] time = 0.12, size = 38, normalized size = 0.37

$$\frac{x^3}{9}+6^{1/4}\,\mathrm{atan}\left(6^{1/4}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{18}+\frac{1}{18}\mathrm{i}\right)+6^{1/4}\,\mathrm{atan}\left(6^{1/4}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{18}-\frac{1}{18}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(3*x^4 + 2),x)
```

```
[Out] x^3/9 - 6^(1/4)*atan(6^(1/4)*x*(1/2 + 1i/2))*(1/18 + 1i/18) - 6^(1/4)*atan(
6^(1/4)*x*(1/2 - 1i/2))*(1/18 - 1i/18)
```

sympy [**A**] time = 0.47, size = 92, normalized size = 0.88

$$\frac{x^3}{9}-\frac{\sqrt[4]{6}\,\log\left(x^2-\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{36}+\frac{\sqrt[4]{6}\,\log\left(x^2+\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{36}-\frac{\sqrt[4]{6}\,\operatorname{atan}\left(\sqrt[4]{6}\,x-1\right)}{18}-\frac{\sqrt[4]{6}\,\operatorname{atan}\left(\sqrt[4]{6}\,x+1\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(3*x**4+2),x)
```

```
[Out] x**3/9 - 6**(1/4)*log(x**2 - 6**(3/4)*x/3 + sqrt(6)/3)/36 + 6**(1/4)*log(x*
*2 + 6**(3/4)*x/3 + sqrt(6)/3)/36 - 6**(1/4)*atan(6**(1/4)*x - 1)/18 - 6**(
1/4)*atan(6**(1/4)*x + 1)/18
```

3.695 $\int \frac{x^4}{2+3x^4}\,dx$

Optimal. Leaf size=102

$$\frac{\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{12\sqrt[4]{6}}-\frac{\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{12\sqrt[4]{6}}+\frac{x}{3}+\frac{\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{6\sqrt[4]{6}}-\frac{\tan^{-1}\left(\sqrt[4]{6}\,x+1\right)}{6\sqrt[4]{6}}$$

```
[Out] 1/3*x-1/36*arctan(-1+6^(1/4)*x)*6^(3/4)-1/36*arctan(1+6^(1/4)*x)*6^(3/4)+1/
72*ln(-6^(3/4)*x+3*x^2+6^(1/2))*6^(3/4)-1/72*ln(6^(3/4)*x+3*x^2+6^(1/2))*6^
(3/4)
```

Rubi [A] time = 0.08, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {321, 211, 1165, 628, 1162, 617, 204}

$$\frac{\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{12\sqrt[4]{6}}-\frac{\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{12\sqrt[4]{6}}+\frac{x}{3}+\frac{\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{6\sqrt[4]{6}}-\frac{\tan^{-1}\left(\sqrt[4]{6}\,x+1\right)}{6\sqrt[4]{6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(2 + 3*x^4),x]
```

```
[Out] x/3 + ArcTan[1 - 6^(1/4)*x]/(6*6^(1/4)) - ArcTan[1 + 6^(1/4)*x]/(6*6^(1/4))
 + Log[Sqrt[6] - 6^(3/4)*x + 3*x^2]/(12*6^(1/4)) - Log[Sqrt[6] + 6^(3/4)*x
+ 3*x^2]/(12*6^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{2+3x^4}\,dx &= \frac{x}{3} - \frac{2}{3}\int \frac{1}{2+3x^4}\,dx \\
&= \frac{x}{3} - \frac{\int \frac{\sqrt{2}-\sqrt{3}\,x^2}{2+3x^4}\,dx}{3\sqrt{2}} - \frac{\int \frac{\sqrt{2}+\sqrt{3}\,x^2}{2+3x^4}\,dx}{3\sqrt{2}} \\
&= \frac{x}{3} - \frac{\int \frac{1}{\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx}{6\sqrt{6}} - \frac{\int \frac{1}{\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx}{6\sqrt{6}} + \frac{\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{12\sqrt[4]{6}} + \frac{\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}-2x}{-\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{12\sqrt[4]{6}} \\
&= \frac{x}{3} + \frac{\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{12\sqrt[4]{6}} - \frac{\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{12\sqrt[4]{6}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt[4]{6}\,x\right)}{6\sqrt[4]{6}} + \frac{\text{Sub}}{} \\
&= \frac{x}{3} + \frac{\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{6\sqrt[4]{6}} - \frac{\tan^{-1}\left(1+\sqrt[4]{6}\,x\right)}{6\sqrt[4]{6}} + \frac{\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{12\sqrt[4]{6}} - \frac{\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{12\sqrt[4]{6}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 96, normalized size = 0.94

$$\frac{1}{72}\left(6^{3/4}\log\left(\sqrt{6}\,x^2-2\sqrt[4]{6}\,x+2\right)-6^{3/4}\log\left(\sqrt{6}\,x^2+2\sqrt[4]{6}\,x+2\right)+24x+2\,6^{3/4}\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)-2\,6^{3/4}\tan^{-1}\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(2 + 3*x^4),x]
```

```
[Out] (24*x + 2*6^(3/4)*ArcTan[1 - 6^(1/4)*x] - 2*6^(3/4)*ArcTan[1 + 6^(1/4)*x] +
 6^(3/4)*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] - 6^(3/4)*Log[2 + 2*6^(1/4)*x +
 Sqrt[6]*x^2])/72
```

fricas [B] time = 0.70, size = 166, normalized size = 1.63

$$\frac{1}{72}\cdot 24^{\frac{3}{4}}\sqrt{2}\,\arctan\left(\frac{1}{12}\cdot 24^{\frac{1}{4}}\sqrt{3}\,\sqrt{2}\,\sqrt{24^{\frac{3}{4}}\sqrt{2}\,x+12\,x^2+4\,\sqrt{6}}\;-\frac{1}{2}\cdot 24^{\frac{1}{4}}\sqrt{2}\,x-1\right)+\frac{1}{72}\cdot 24^{\frac{3}{4}}\sqrt{2}\,\arctan\left(\frac{1}{12}\cdot 24^{\frac{1}{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] 1/72*24^(3/4)*sqrt(2)*arctan(1/12*24^(1/4)*sqrt(3)*sqrt(2)*sqrt(24^(3/4)*sq
rt(2)*x + 12*x^2 + 4*sqrt(6)) - 1/2*24^(1/4)*sqrt(2)*x - 1) + 1/72*24^(3/4)
```

```
*sqrt(2)*arctan(1/12*24^(1/4)*sqrt(3)*sqrt(2)*sqrt(-24^(3/4)*sqrt(2)*x + 12
*x^2 + 4*sqrt(6)) - 1/2*24^(1/4)*sqrt(2)*x + 1) - 1/288*24^(3/4)*sqrt(2)*lo
g(24^(3/4)*sqrt(2)*x + 12*x^2 + 4*sqrt(6)) + 1/288*24^(3/4)*sqrt(2)*log(-24
^(3/4)*sqrt(2)*x + 12*x^2 + 4*sqrt(6)) + 1/3*x
```

giac [A] time = 0.19, size = 98, normalized size = 0.96

$$-\frac{1}{36}\cdot 6^{\frac{3}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)-\frac{1}{36}\cdot 6^{\frac{3}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)-\frac{1}{72}\cdot 6^{\frac{3}{4}}\log\left(x^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^4+2),x, algorithm="giac")
```

```
[Out] -1/36*6^(3/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4))) -
 1/36*6^(3/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) -
 1/72*6^(3/4)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) + 1/72*6^(3/4)*l
og(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) + 1/3*x
```

maple [A] time = 0.00, size = 114, normalized size = 1.12

$$\frac{x}{3}-\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)}{36}-\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)}{36}-\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,x}{3}+\frac{\sqrt{6}}{3}}{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,x}{3}+\frac{\sqrt{6}}{3}}\right)}{72}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(3*x^4+2),x)
```

```
[Out] 1/3*x-1/36*3^(1/2)*6^(1/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)-
1/72*3^(1/2)*6^(1/4)*2^(1/2)*ln((x^2+1/3*3^(1/2)*6^(1/4)*2^(1/2)*x+1/3*6^(1
/2))/(x^2-1/3*3^(1/2)*6^(1/4)*2^(1/2)*x+1/3*6^(1/2)))-1/36*3^(1/2)*6^(1/4)*
2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)
```

maxima [A] time = 3.00, size = 124, normalized size = 1.22

$$-\frac{1}{36}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)-\frac{1}{36}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)-\frac{1}{72}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\log\left(\sqrt{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] -1/36*3^(3/4)*2^(3/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^(
3/4))) - 1/36*3^(3/4)*2^(3/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(
1/4)*2^(3/4))) - 1/72*3^(3/4)*2^(3/4)*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x +
 sqrt(2)) + 1/72*3^(3/4)*2^(3/4)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt
(2)) + 1/3*x
```

mupad [B] time = 1.09, size = 36, normalized size = 0.35

$$\frac{x}{3}+6^{3/4}\operatorname{atan}\left(6^{1/4}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{36}-\frac{1}{36}\mathrm{i}\right)+6^{3/4}\operatorname{atan}\left(6^{1/4}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{36}+\frac{1}{36}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(3*x^4 + 2),x)
```

```
[Out] x/3 - 6^(3/4)*atan(6^(1/4)*x*(1/2 - 1i/2))*(1/36 + 1i/36) - 6^(3/4)*atan(6^
(1/4)*x*(1/2 + 1i/2))*(1/36 - 1i/36)
```

sympy [**A**] time = 0.49, size = 90, normalized size = 0.88

$$\frac{x}{3}+\frac{6^{\frac{3}{4}}\log\left(x^2-\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{72}-\frac{6^{\frac{3}{4}}\log\left(x^2+\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{72}-\frac{6^{\frac{3}{4}}\operatorname{atan}\left(\sqrt[4]{6}\,x-1\right)}{36}-\frac{6^{\frac{3}{4}}\operatorname{atan}\left(\sqrt[4]{6}\,x+1\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(3*x**4+2),x)
```

```
[Out] x/3 + 6**(3/4)*log(x**2 - 6**(3/4)*x/3 + sqrt(6)/3)/72 - 6**(3/4)*log(x**2
+ 6**(3/4)*x/3 + sqrt(6)/3)/72 - 6**(3/4)*atan(6**(1/4)*x - 1)/36 - 6**(3/4
)*atan(6**(1/4)*x + 1)/36
```

3.696 $\int \frac{x^2}{2+3x^4}\,dx$

Optimal. Leaf size=97

$$\frac{\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{4\,6^{3/4}}-\frac{\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{4\,6^{3/4}}-\frac{\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{2\,6^{3/4}}+\frac{\tan^{-1}\left(\sqrt[4]{6}\,x+1\right)}{2\,6^{3/4}}$$

```
[Out] 1/12*arctan(-1+6^(1/4)*x)*6^(1/4)+1/12*arctan(1+6^(1/4)*x)*6^(1/4)+1/24*ln(
-6^(3/4)*x+3*x^2+6^(1/2))*6^(1/4)-1/24*ln(6^(3/4)*x+3*x^2+6^(1/2))*6^(1/4)
```

Rubi [A] time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {297, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{4\,6^{3/4}}-\frac{\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{4\,6^{3/4}}-\frac{\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{2\,6^{3/4}}+\frac{\tan^{-1}\left(\sqrt[4]{6}\,x+1\right)}{2\,6^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 + 3*x^4),x]
```

```
[Out] -ArcTan[1 - 6^(1/4)*x]/(2*6^(3/4)) + ArcTan[1 + 6^(1/4)*x]/(2*6^(3/4)) + Lo
g[Sqrt[6] - 6^(3/4)*x + 3*x^2]/(4*6^(3/4)) - Log[Sqrt[6] + 6^(3/4)*x + 3*x^
2]/(4*6^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^2}{2+3x^4}\,dx = -\frac{\int \frac{\sqrt{2}-\sqrt{3}\,x^2}{2+3x^4}\,dx}{2\sqrt{3}} + \frac{\int \frac{\sqrt{2}+\sqrt{3}\,x^2}{2+3x^4}\,dx}{2\sqrt{3}}$$

$$= \frac{1}{12}\int \frac{1}{\sqrt{\frac{2}{3}} - \frac{2^{3/4}x}{\sqrt[4]{3}} + x^2}\,dx + \frac{1}{12}\int \frac{1}{\sqrt{\frac{2}{3}} + \frac{2^{3/4}x}{\sqrt[4]{3}} + x^2}\,dx + \frac{\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{4\,6^{3/4}} + \frac{\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}-2x}{-\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{4\,6^{3/4}}$$

$$= \frac{\log\left(\sqrt{6} - 6^{3/4}x + 3x^2\right)}{4\,6^{3/4}} - \frac{\log\left(\sqrt{6} + 6^{3/4}x + 3x^2\right)}{4\,6^{3/4}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt[4]{6}\,x\right)}{2\,6^{3/4}} - \frac{\text{Subst}\left(\int$$

$$= -\frac{\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{2\,6^{3/4}} + \frac{\tan^{-1}\left(1+\sqrt[4]{6}\,x\right)}{2\,6^{3/4}} + \frac{\log\left(\sqrt{6} - 6^{3/4}x + 3x^2\right)}{4\,6^{3/4}} - \frac{\log\left(\sqrt{6} + 6^{3/4}x + 3x^2\right)}{4\,6^{3/4}}$$

Mathematica [A] time = 0.01, size = 77, normalized size = 0.79

$$\frac{\log\left(\sqrt{6}\,x^2 - 2\sqrt[4]{6}\,x + 2\right) - \log\left(\sqrt{6}\,x^2 + 2\sqrt[4]{6}\,x + 2\right) - 2\tan^{-1}\left(1-\sqrt[4]{6}\,x\right) + 2\tan^{-1}\left(\sqrt[4]{6}\,x + 1\right)}{4\,6^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 + 3*x^4),x]
```

```
[Out] (-2*ArcTan[1 - 6^(1/4)*x] + 2*ArcTan[1 + 6^(1/4)*x] + Log[2 - 2*6^(1/4)*x +
 Sqrt[6]*x^2] - Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2])/(4*6^(3/4))
```

fricas [B] time = 0.78, size = 159, normalized size = 1.64

$$-\frac{1}{108}\cdot 54^{\frac{3}{4}}\sqrt{2}\,\arctan\left(-\frac{1}{18}\cdot 54^{\frac{3}{4}}\sqrt{2}\,x + \frac{1}{54}\cdot 54^{\frac{3}{4}}\sqrt{2}\,\sqrt{9\,x^2 + 3\cdot 54^{\frac{1}{4}}\sqrt{2}\,x + 3\,\sqrt{6}}\ - 1\right) - \frac{1}{108}\cdot 54^{\frac{3}{4}}\sqrt{2}\,\arctan\left(-\frac{1}{18}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] -1/108*54^(3/4)*sqrt(2)*arctan(-1/18*54^(3/4)*sqrt(2)*x + 1/54*54^(3/4)*sqr
t(2)*sqrt(9*x^2 + 3*54^(1/4)*sqrt(2)*x + 3*sqrt(6)) - 1) - 1/108*54^(3/4)*s
qrt(2)*arctan(-1/18*54^(3/4)*sqrt(2)*x + 1/54*54^(3/4)*sqrt(2)*sqrt(9*x^2 -
 3*54^(1/4)*sqrt(2)*x + 3*sqrt(6)) + 1) - 1/432*54^(3/4)*sqrt(2)*log(9*x^2
+ 3*54^(1/4)*sqrt(2)*x + 3*sqrt(6)) + 1/432*54^(3/4)*sqrt(2)*log(9*x^2 - 3*
54^(1/4)*sqrt(2)*x + 3*sqrt(6))
```

giac [A] time = 0.35, size = 95, normalized size = 0.98

$$\frac{1}{12}\cdot 6^{\frac{1}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{12}\cdot 6^{\frac{1}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) - \frac{1}{24}\cdot 6^{\frac{1}{4}}\log\left(x^2 + \sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/12*6^(1/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4))) +
1/12*6^(1/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) -
1/24*6^(1/4)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) + 1/24*6^(1/4)*lo
g(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

maple [A] time = 0.00, size = 111, normalized size = 1.14

$$\frac{\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)}{72}+\frac{\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)}{72}+\frac{\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\,\ln\left(\frac{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,x}{3}+\frac{\sqrt{6}}{3}}{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,x}{3}+\frac{\sqrt{6}}{3}}\right)}{144}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^4+2),x)
```

```
[Out] 1/72*3^(1/2)*6^(3/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)+1/72*3
^(1/2)*6^(3/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)+1/144*3^(1/2
)*6^(3/4)*2^(1/2)*ln((x^2-1/3*3^(1/2)*6^(1/4)*2^(1/2)*x+1/3*6^(1/2))/(x^2+1
/3*3^(1/2)*6^(1/4)*2^(1/2)*x+1/3*6^(1/2)))
```

maxima [A] time = 3.06, size = 121, normalized size = 1.25

$$\frac{1}{12}\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}\,x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{12}\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}\,x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)-\frac{1}{24}\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}\log\left(\sqrt{3}\,x^{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/12*3^(1/4)*2^(1/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^(3
/4))) + 1/12*3^(1/4)*2^(1/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1
/4)*2^(3/4))) - 1/24*3^(1/4)*2^(1/4)*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x +
sqrt(2)) + 1/24*3^(1/4)*2^(1/4)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(
2))
```

mupad [B] time = 0.11, size = 33, normalized size = 0.34

$$6^{1/4}\,\mathrm{atan}\left(6^{1/4}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{12}-\frac{1}{12}\mathrm{i}\right)+6^{1/4}\,\mathrm{atan}\left(6^{1/4}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{12}+\frac{1}{12}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^4 + 2),x)
```

```
[Out] 6^(1/4)*atan(6^(1/4)*x*(1/2 - 1i/2))*(1/12 - 1i/12) + 6^(1/4)*atan(6^(1/4)*
x*(1/2 + 1i/2))*(1/12 + 1i/12)
```

sympy [A] time = 0.48, size = 87, normalized size = 0.90

$$\frac{\sqrt[4]{6}\,\log\left(x^2-\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{24}-\frac{\sqrt[4]{6}\,\log\left(x^2+\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{24}+\frac{\sqrt[4]{6}\,\mathrm{atan}\left(\sqrt[4]{6}\,x-1\right)}{12}+\frac{\sqrt[4]{6}\,\mathrm{atan}\left(\sqrt[4]{6}\,x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(3*x**4+2),x)
```

```
[Out] 6**(1/4)*log(x**2 - 6**(3/4)*x/3 + sqrt(6)/3)/24 - 6**(1/4)*log(x**2 + 6**(
3/4)*x/3 + sqrt(6)/3)/24 + 6**(1/4)*atan(6**(1/4)*x - 1)/12 + 6**(1/4)*atan
(6**(1/4)*x + 1)/12
```

3.697 $\int \frac{1}{2+3x^4}\,dx$

Optimal. Leaf size=97

$$-\frac{\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}+\frac{\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}-\frac{\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{4\sqrt[4]{6}}+\frac{\tan^{-1}\left(\sqrt[4]{6}\,x+1\right)}{4\sqrt[4]{6}}$$

```
[Out] 1/24*arctan(-1+6^(1/4)*x)*6^(3/4)+1/24*arctan(1+6^(1/4)*x)*6^(3/4)-1/48*ln(
-6^(3/4)*x+3*x^2+6^(1/2))*6^(3/4)+1/48*ln(6^(3/4)*x+3*x^2+6^(1/2))*6^(3/4)
```

Rubi [A] time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}+\frac{\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}-\frac{\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{4\sqrt[4]{6}}+\frac{\tan^{-1}\left(\sqrt[4]{6}\,x+1\right)}{4\sqrt[4]{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^4)^(-1),x]
```

```
[Out] -ArcTan[1 - 6^(1/4)*x]/(4*6^(1/4)) + ArcTan[1 + 6^(1/4)*x]/(4*6^(1/4)) - Lo
g[Sqrt[6] - 6^(3/4)*x + 3*x^2]/(8*6^(1/4)) + Log[Sqrt[6] + 6^(3/4)*x + 3*x^
2]/(8*6^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{2+3x^4}\,dx = \frac{\int \frac{\sqrt{2}-\sqrt{3}\,x^2}{2+3x^4}\,dx}{2\sqrt{2}} + \frac{\int \frac{\sqrt{2}+\sqrt{3}\,x^2}{2+3x^4}\,dx}{2\sqrt{2}}$$

$$= \frac{\int \frac{1}{\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx}{4\sqrt{6}} + \frac{\int \frac{1}{\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx}{4\sqrt{6}} - \frac{\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\sqrt[4]{6}} - \frac{\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}-2x}{-\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\sqrt[4]{6}}$$

$$= -\frac{\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt[4]{6}\,x\right)}{4\sqrt[4]{6}} - \frac{\text{Sub}}{}$$

$$= -\frac{\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{4\sqrt[4]{6}} + \frac{\tan^{-1}\left(1+\sqrt[4]{6}\,x\right)}{4\sqrt[4]{6}} - \frac{\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}} + \frac{\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\sqrt[4]{6}}$$

Mathematica [A] time = 0.02, size = 77, normalized size = 0.79

$$\frac{-\log\left(\sqrt{6}\,x^2-2\sqrt[4]{6}\,x+2\right)+\log\left(\sqrt{6}\,x^2+2\sqrt[4]{6}\,x+2\right)-2\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)+2\tan^{-1}\left(\sqrt[4]{6}\,x+1\right)}{8\sqrt[4]{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^4)^(-1),x]
```

```
[Out] (-2*ArcTan[1 - 6^(1/4)*x] + 2*ArcTan[1 + 6^(1/4)*x] - Log[2 - 2*6^(1/4)*x +
 Sqrt[6]*x^2] + Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2])/(8*6^(1/4))
```

fricas [B] time = 0.75, size = 163, normalized size = 1.68

$$-\frac{1}{48}\cdot 24^{\frac{3}{4}}\sqrt{2}\,\arctan\left(\frac{1}{12}\cdot 24^{\frac{1}{4}}\sqrt{3}\,\sqrt{2}\,\sqrt{24^{\frac{3}{4}}\sqrt{2}\,x+12\,x^2+4\,\sqrt{6}}\;-\frac{1}{2}\cdot 24^{\frac{1}{4}}\sqrt{2}\,x-1\right)-\frac{1}{48}\cdot 24^{\frac{3}{4}}\sqrt{2}\,\arctan\left(\frac{1}{12}\cdot 2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] -1/48*24^(3/4)*sqrt(2)*arctan(1/12*24^(1/4)*sqrt(3)*sqrt(2)*sqrt(24^(3/4)*s
qrt(2)*x + 12*x^2 + 4*sqrt(6)) - 1/2*24^(1/4)*sqrt(2)*x - 1) - 1/48*24^(3/4
)*sqrt(2)*arctan(1/12*24^(1/4)*sqrt(3)*sqrt(2)*sqrt(-24^(3/4)*sqrt(2)*x + 1
2*x^2 + 4*sqrt(6)) - 1/2*24^(1/4)*sqrt(2)*x + 1) + 1/192*24^(3/4)*sqrt(2)*l
og(24^(3/4)*sqrt(2)*x + 12*x^2 + 4*sqrt(6)) - 1/192*24^(3/4)*sqrt(2)*log(-2
4^(3/4)*sqrt(2)*x + 12*x^2 + 4*sqrt(6))
```

giac [A] time = 0.20, size = 95, normalized size = 0.98

$$\frac{1}{24}\cdot 6^{\frac{3}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{24}\cdot 6^{\frac{3}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{48}\cdot 6^{\frac{3}{4}}\log\left(x^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/24*6^(3/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4))) +
1/24*6^(3/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) +
1/48*6^(3/4)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/48*6^(3/4)*lo
g(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

maple [A] time = 0.00, size = 111, normalized size = 1.14

$$\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)}{24}+\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)}{24}+\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,x}{3}+\frac{\sqrt{6}}{3}}{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,x}{3}+\frac{\sqrt{6}}{3}}\right)}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+2),x)
```

```
[Out] 1/48*3^(1/2)*6^(1/4)*2^(1/2)*ln((x^2+1/3*3^(1/2)*6^(1/4)*2^(1/2)*x+1/3*6^(1
/2))/(x^2-1/3*3^(1/2)*6^(1/4)*2^(1/2)*x+1/3*6^(1/2)))+1/24*3^(1/2)*6^(1/4)*
2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)+1/24*3^(1/2)*6^(1/4)*2^(1/2
)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)
```

maxima [A] time = 2.94, size = 121, normalized size = 1.25

$$\frac{1}{24}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}\,x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{24}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}\,x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{48}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\log\left(\sqrt{3}\,x^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/24*3^(3/4)*2^(3/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^(3
/4))) + 1/24*3^(3/4)*2^(3/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1
/4)*2^(3/4))) + 1/48*3^(3/4)*2^(3/4)*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x +
sqrt(2)) - 1/48*3^(3/4)*2^(3/4)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(
2))
```

mupad [B] time = 1.06, size = 33, normalized size = 0.34

$$6^{3/4}\,\mathrm{atan}\left(6^{1/4}\,x\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{24}+\frac{1}{24}\mathrm{i}\right)+6^{3/4}\,\mathrm{atan}\left(6^{1/4}\,x\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{24}-\frac{1}{24}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4 + 2),x)
```

```
[Out] 6^(3/4)*atan(6^(1/4)*x*(1/2 - 1i/2))*(1/24 + 1i/24) + 6^(3/4)*atan(6^(1/4)*
x*(1/2 + 1i/2))*(1/24 - 1i/24)
```

sympy [A] time = 0.47, size = 87, normalized size = 0.90

$$-\frac{6^{\frac{3}{4}}\log\left(x^2-\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{48}+\frac{6^{\frac{3}{4}}\log\left(x^2+\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{48}+\frac{6^{\frac{3}{4}}\,\mathrm{atan}\left(\sqrt[4]{6}\,x-1\right)}{24}+\frac{6^{\frac{3}{4}}\,\mathrm{atan}\left(\sqrt[4]{6}\,x+1\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+2),x)
```

```
[Out] -6**(3/4)*log(x**2 - 6**(3/4)*x/3 + sqrt(6)/3)/48 + 6**(3/4)*log(x**2 + 6**
(3/4)*x/3 + sqrt(6)/3)/48 + 6**(3/4)*atan(6**(1/4)*x - 1)/24 + 6**(3/4)*ata
n(6**(1/4)*x + 1)/24
```

3.698 $\int \frac{1}{x^2\left(2+3x^4\right)}\,dx$

Optimal. Leaf size=124

$$-\frac{\sqrt[4]{3}\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\,2^{3/4}}+\frac{\sqrt[4]{3}\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\,2^{3/4}}-\frac{1}{2x}+\frac{\sqrt[4]{3}\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{4\,2^{3/4}}-\frac{\sqrt[4]{3}\tan^{-1}\left(\sqrt[4]{6}\,x+1\right)}{4\,2^{3/4}}$$

```
[Out] -1/2/x-1/8*3^(1/4)*arctan(-1+6^(1/4)*x)*2^(1/4)-1/8*3^(1/4)*arctan(1+6^(1/4
)*x)*2^(1/4)-1/16*3^(1/4)*ln(-6^(3/4)*x+3*x^2+6^(1/2))*2^(1/4)+1/16*3^(1/4)
*ln(6^(3/4)*x+3*x^2+6^(1/2))*2^(1/4)
```

Rubi [A] time = 0.08, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {325, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{3}\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\,2^{3/4}}+\frac{\sqrt[4]{3}\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\,2^{3/4}}-\frac{1}{2x}+\frac{\sqrt[4]{3}\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{4\,2^{3/4}}-\frac{\sqrt[4]{3}\tan^{-1}\left(\sqrt[4]{6}\,x+1\right)}{4\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(2 + 3*x^4)),x]
```

```
[Out] -1/(2*x) + (3^(1/4)*ArcTan[1 - 6^(1/4)*x])/(4*2^(3/4)) - (3^(1/4)*ArcTan[1
+ 6^(1/4)*x])/(4*2^(3/4)) - (3^(1/4)*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*2
^(3/4)) + (3^(1/4)*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*2^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^2\left(2+3x^4\right)}\,dx = -\frac{1}{2x} - \frac{3}{2}\int \frac{x^2}{2+3x^4}\,dx$$

$$= -\frac{1}{2x} + \frac{1}{4}\sqrt{3}\int \frac{\sqrt{2}-\sqrt{3}\,x^2}{2+3x^4}\,dx - \frac{1}{4}\sqrt{3}\int \frac{\sqrt{2}+\sqrt{3}\,x^2}{2+3x^4}\,dx$$

$$= -\frac{1}{2x} - \frac{1}{8}\int \frac{1}{\sqrt{\frac{2}{3}} - \frac{2^{3/4}x}{\sqrt[4]{3}} + x^2}\,dx - \frac{1}{8}\int \frac{1}{\sqrt{\frac{2}{3}} + \frac{2^{3/4}x}{\sqrt[4]{3}} + x^2}\,dx - \frac{\sqrt[4]{3}\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{8\,2^{3/4}} - \frac{\sqrt[4]{3}\int$$

$$= -\frac{1}{2x} - \frac{\sqrt[4]{3}\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\,2^{3/4}} + \frac{\sqrt[4]{3}\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{8\,2^{3/4}} - \frac{\sqrt[4]{3}\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,\right.}{4\,2^{3/4}}$$

$$= -\frac{1}{2x} + \frac{\sqrt[4]{3}\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{4\,2^{3/4}} - \frac{\sqrt[4]{3}\tan^{-1}\left(1+\sqrt[4]{6}\,x\right)}{4\,2^{3/4}} - \frac{\sqrt[4]{3}\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{8\,2^{3/4}} + \frac{\sqrt[4]{3}\,\log}{}$$

Mathematica [A] time = 0.03, size = 101, normalized size = 0.81

$$-\frac{\sqrt[4]{6}\,x\log\left(\sqrt{6}\,x^2-2\sqrt[4]{6}\,x+2\right)-\sqrt[4]{6}\,x\log\left(\sqrt{6}\,x^2+2\sqrt[4]{6}\,x+2\right)-2\sqrt[4]{6}\,x\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)+2\sqrt[4]{6}\,x\tan^{-1}\left(\sqrt[4]{6}\,x+\right.}{16x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(2 + 3*x^4)),x]
```

```
[Out] -1/16*(8 - 2*6^(1/4)*x*ArcTan[1 - 6^(1/4)*x] + 2*6^(1/4)*x*ArcTan[1 + 6^(1/
4)*x] + 6^(1/4)*x*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] - 6^(1/4)*x*Log[2 + 2*
6^(1/4)*x + Sqrt[6]*x^2])/x
```

fricas [B] time = 0.80, size = 174, normalized size = 1.40

$$\frac{4\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}x\arctan\left(\frac{1}{3}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\sqrt{3^{\frac{3}{4}}2^{\frac{3}{4}}x+3\,x^2+\sqrt{3}\,\sqrt{2}}\;-3^{\frac{1}{4}}2^{\frac{1}{4}}x-1\right)+4\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}x\arctan\left(\frac{1}{3}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\sqrt{-3^{\frac{3}{4}}2^{\frac{3}{4}}x+3\,x^2}\right.}{16\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^4+2),x, algorithm="fricas")
```

```
[Out] 1/16*(4*3^(1/4)*2^(1/4)*x*arctan(1/3*3^(3/4)*2^(1/4)*sqrt(3^(3/4)*2^(3/4)*x
 + 3*x^2 + sqrt(3)*sqrt(2)) - 3^(1/4)*2^(1/4)*x - 1) + 4*3^(1/4)*2^(1/4)*x*
arctan(1/3*3^(3/4)*2^(1/4)*sqrt(-3^(3/4)*2^(3/4)*x + 3*x^2 + sqrt(3)*sqrt(2
)) - 3^(1/4)*2^(1/4)*x + 1) + 3^(1/4)*2^(1/4)*x*log(3^(3/4)*2^(3/4)*x + 3*x
^2 + sqrt(3)*sqrt(2)) - 3^(1/4)*2^(1/4)*x*log(-3^(3/4)*2^(3/4)*x + 3*x^2 +
sqrt(3)*sqrt(2)) - 8)/x
```

giac [A] time = 0.19, size = 100, normalized size = 0.81

$$-\frac{1}{8}\cdot 6^{\frac{1}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)-\frac{1}{8}\cdot 6^{\frac{1}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{16}\cdot 6^{\frac{1}{4}}\log\left(x^2+\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^4+2),x, algorithm="giac")
```

```
[Out] -1/8*6^(1/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4))) -
1/8*6^(1/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) + 1
/16*6^(1/4)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/16*6^(1/4)*log
(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/2/x
```

maple [A] time = 0.01, size = 116, normalized size = 0.94

$$-\frac{\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)}{48}-\frac{\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)}{48}-\frac{\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\,\ln\left(\frac{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,x}{3}+\frac{\sqrt{6}}{3}}{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\,x}{3}+\frac{\sqrt{6}}{3}}\right)}{96}-\frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(3*x^4+2),x)
```

```
[Out] -1/48*3^(1/2)*6^(3/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)-1/96*
3^(1/2)*6^(3/4)*2^(1/2)*ln((x^2-1/3*3^(1/2)*6^(1/4)*2^(1/2)*x+1/3*6^(1/2))/
(x^2+1/3*3^(1/2)*6^(1/4)*2^(1/2)*x+1/3*6^(1/2)))-1/48*3^(1/2)*6^(3/4)*2^(1/
2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)-1/2/x
```

maxima [A] time = 3.01, size = 126, normalized size = 1.02

$$-\frac{1}{8}\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}\,x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)-\frac{1}{8}\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}\,x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{16}\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}\log\left(\sqrt{3}\,x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^4+2),x, algorithm="maxima")
```

```
[Out] -1/8*3^(1/4)*2^(1/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^(3
/4))) - 1/8*3^(1/4)*2^(1/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/
4)*2^(3/4))) + 1/16*3^(1/4)*2^(1/4)*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + s
qrt(2)) - 1/16*3^(1/4)*2^(1/4)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2
)) - 1/2/x
```

mupad [B] time = 0.11, size = 38, normalized size = 0.31

$$-\frac{1}{2x}+6^{1/4}\,\mathrm{atan}\left(6^{1/4}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)+6^{1/4}\,\mathrm{atan}\left(6^{1/4}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(3*x^4 + 2)),x)
```

```
[Out] - 6^(1/4)*atan(6^(1/4)*x*(1/2 - 1i/2))*(1/8 - 1i/8) - 6^(1/4)*atan(6^(1/4)*
x*(1/2 + 1i/2))*(1/8 + 1i/8) - 1/(2*x)
```

sympy [**A**] time = 0.47, size = 92, normalized size = 0.74

$$-\frac{\sqrt[4]{6}\log\left(x^2-\frac{\sqrt[4]{6}^3x}{3}+\frac{\sqrt{6}}{3}\right)}{16}+\frac{\sqrt[4]{6}\log\left(x^2+\frac{\sqrt[4]{6}^3x}{3}+\frac{\sqrt{6}}{3}\right)}{16}-\frac{\sqrt[4]{6}\operatorname{atan}\left(\sqrt[4]{6}x-1\right)}{8}-\frac{\sqrt[4]{6}\operatorname{atan}\left(\sqrt[4]{6}x+1\right)}{8}-\frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(3*x**4+2),x)
```

```
[Out] -6**(1/4)*log(x**2 - 6**(3/4)*x/3 + sqrt(6)/3)/16 + 6**(1/4)*log(x**2 + 6**
(3/4)*x/3 + sqrt(6)/3)/16 - 6**(1/4)*atan(6**(1/4)*x - 1)/8 - 6**(1/4)*atan
(6**(1/4)*x + 1)/8 - 1/(2*x)
```

3.699 $\int \frac{x^3}{\left(2+3x^4\right)^2}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{12\left(3x^4+2\right)}$$

```
[Out] -1/12/(3*x^4+2)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$-\frac{1}{12\left(3x^4+2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(2 + 3*x^4)^2,x]
```

```
[Out] -1/(12*(2 + 3*x^4))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^3}{\left(2+3x^4\right)^2}\,dx = -\frac{1}{12\left(2+3x^4\right)}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$-\frac{1}{12\left(3x^4+2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(2 + 3*x^4)^2,x]
```

```
[Out] -1/12*1/(2 + 3*x^4)
```

fricas [A] time = 0.57, size = 11, normalized size = 0.85

$$-\frac{1}{12\left(3\,x^4+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(3*x^4+2)^2,x, algorithm="fricas")
```

```
[Out] -1/12/(3*x^4 + 2)
```

giac [A] time = 0.21, size = 11, normalized size = 0.85

$$-\frac{1}{12\left(3\,x^4+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(3*x^4+2)^2,x, algorithm="giac")
[Out] -1/12/(3*x^4 + 2)
```

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$-\frac{1}{12\left(3x^4+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(3*x^4+2)^2,x)
[Out] -1/12/(3*x^4+2)
```

maxima [A] time = 1.37, size = 11, normalized size = 0.85

$$-\frac{1}{12\left(3\,x^4+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(3*x^4+2)^2,x, algorithm="maxima")
[Out] -1/12/(3*x^4 + 2)
```

mupad [B] time = 1.00, size = 11, normalized size = 0.85

$$-\frac{1}{36\,x^4+24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(3*x^4 + 2)^2,x)
[Out] -1/(36*x^4 + 24)
```

sympy [A] time = 0.18, size = 8, normalized size = 0.62

$$-\frac{1}{36x^4+24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(3*x**4+2)**2,x)
[Out] -1/(36*x**4 + 24)
```

3.700 $\int \frac{x}{\left(2+3x^4\right)^2}\,dx$

Optimal. Leaf size=38

$$\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{8\sqrt{6}}+\frac{x^2}{8\left(3x^4+2\right)}$$

```
[Out] 1/8*x^2/(3*x^4+2)+1/48*arctan(1/2*x^2*6^(1/2))*6^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {275, 199, 203}

$$\frac{x^2}{8\left(3x^4+2\right)}+\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{8\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(2 + 3*x^4)^2,x]
```

```
[Out] x^2/(8*(2 + 3*x^4)) + ArcTan[Sqrt[3/2]*x^2]/(8*Sqrt[6])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(2+3x^4\right)^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\left(2+3x^2\right)^2}\,dx, x, x^2\right)\\ &= \frac{x^2}{8\left(2+3x^4\right)}+\frac{1}{8}\,\text{Subst}\left(\int \frac{1}{2+3x^2}\,dx, x, x^2\right)\\ &= \frac{x^2}{8\left(2+3x^4\right)}+\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{8\sqrt{6}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 1.00

$$\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)}{8\sqrt{6}} + \frac{x^2}{8\left(3x^4+2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(2 + 3*x^4)^2,x]
[Out] x^2/(8*(2 + 3*x^4)) + ArcTan[Sqrt[3/2]*x^2]/(8*Sqrt[6])
```

fricas [A] time = 0.68, size = 37, normalized size = 0.97

$$\frac{\sqrt{6}\left(3\,x^4+2\right)\arctan\left(\frac{1}{2}\,\sqrt{6}\,x^2\right)+6\,x^2}{48\left(3\,x^4+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x^4+2)^2,x, algorithm="fricas")
[Out] 1/48*(sqrt(6)*(3*x^4 + 2)*arctan(1/2*sqrt(6)*x^2) + 6*x^2)/(3*x^4 + 2)
```

giac [A] time = 0.16, size = 29, normalized size = 0.76

$$\frac{1}{48}\,\sqrt{6}\,\arctan\left(\frac{1}{2}\,\sqrt{6}\,x^2\right)+\frac{x^2}{8\left(3\,x^4+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x^4+2)^2,x, algorithm="giac")
[Out] 1/48*sqrt(6)*arctan(1/2*sqrt(6)*x^2) + 1/8*x^2/(3*x^4 + 2)
```

maple [A] time = 0.01, size = 30, normalized size = 0.79

$$\frac{x^2}{24x^4+16}+\frac{\sqrt{6}\,\arctan\left(\frac{\sqrt{6}\,x^2}{2}\right)}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(3*x^4+2)^2,x)
[Out] 1/8*x^2/(3*x^4+2)+1/48*6^(1/2)*arctan(1/2*6^(1/2)*x^2)
```

maxima [A] time = 2.91, size = 29, normalized size = 0.76

$$\frac{1}{48}\,\sqrt{6}\,\arctan\left(\frac{1}{2}\,\sqrt{6}\,x^2\right)+\frac{x^2}{8\left(3\,x^4+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x^4+2)^2,x, algorithm="maxima")
[Out] 1/48*sqrt(6)*arctan(1/2*sqrt(6)*x^2) + 1/8*x^2/(3*x^4 + 2)
```

mupad [B] time = 0.04, size = 28, normalized size = 0.74

$$\frac{\sqrt{6}\,\operatorname{atan}\left(\frac{\sqrt{6}\,x^2}{2}\right)}{48}+\frac{x^2}{24\left(x^4+\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(3*x^4 + 2)^2,x)
[Out] (6^(1/2)*atan((6^(1/2)*x^2)/2))/48 + x^2/(24*(x^4 + 2/3))
```

sympy [**A**] time = 0.17, size = 27, normalized size = 0.71

$$\frac{x^2}{24x^4+16}+\frac{\sqrt{6}\,\operatorname{atan}\left(\frac{\sqrt{6}\,x^2}{2}\right)}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x**4+2)**2,x)
[Out] x**2/(24*x**4 + 16) + sqrt(6)*atan(sqrt(6)*x**2/2)/48
```

3.701 $\int \frac{1}{x\left(2+3x^4\right)^2}\,dx$

Optimal. Leaf size=32

$$\frac{1}{8\left(3x^4+2\right)}-\frac{1}{16}\log\left(3x^4+2\right)+\frac{\log(x)}{4}$$

```
[Out] 1/8/(3*x^4+2)+1/4*ln(x)-1/16*ln(3*x^4+2)
```

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 44}

$$\frac{1}{8\left(3x^4+2\right)}-\frac{1}{16}\log\left(3x^4+2\right)+\frac{\log(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(2 + 3*x^4)^2),x]
```

```
[Out] 1/(8*(2 + 3*x^4)) + Log[x]/4 - Log[2 + 3*x^4]/16
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(2+3x^4\right)^2}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x(2+3x)^2}\,dx, x, x^4\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{4x}-\frac{3}{2(2+3x)^2}-\frac{3}{4(2+3x)}\right)dx, x, x^4\right)\\ &= \frac{1}{8\left(2+3x^4\right)}+\frac{\log(x)}{4}-\frac{1}{16}\log\left(2+3x^4\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 1.00

$$\frac{1}{8\left(3x^4+2\right)}-\frac{1}{16}\log\left(3x^4+2\right)+\frac{\log(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(2 + 3*x^4)^2),x]
```

```
[Out] 1/(8*(2 + 3*x^4)) + Log[x]/4 - Log[2 + 3*x^4]/16
```

fricas [A] time = 0.51, size = 40, normalized size = 1.25

$$-\frac{\left(3x^4+2\right)\log\left(3x^4+2\right)-4\left(3x^4+2\right)\log(x)-2}{16\left(3x^4+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^4+2)^2,x, algorithm="fricas")
[Out] -1/16*((3*x^4 + 2)*log(3*x^4 + 2) - 4*(3*x^4 + 2)*log(x) - 2)/(3*x^4 + 2)
```

giac [A] time = 0.17, size = 35, normalized size = 1.09

$$\frac{3x^4+4}{16\left(3x^4+2\right)}-\frac{1}{16}\log\left(3x^4+2\right)+\frac{1}{16}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^4+2)^2,x, algorithm="giac")
[Out] 1/16*(3*x^4 + 4)/(3*x^4 + 2) - 1/16*log(3*x^4 + 2) + 1/16*log(x^4)
```

maple [A] time = 0.01, size = 27, normalized size = 0.84

$$\frac{\ln(x)}{4}-\frac{\ln\left(3x^4+2\right)}{16}+\frac{1}{24x^4+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(3*x^4+2)^2,x)
[Out] 1/8/(3*x^4+2)+1/4*ln(x)-1/16*ln(3*x^4+2)
```

maxima [A] time = 1.31, size = 28, normalized size = 0.88

$$\frac{1}{8\left(3x^4+2\right)}-\frac{1}{16}\log\left(3x^4+2\right)+\frac{1}{16}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^4+2)^2,x, algorithm="maxima")
[Out] 1/8/(3*x^4 + 2) - 1/16*log(3*x^4 + 2) + 1/16*log(x^4)
```

mupad [B] time = 0.07, size = 22, normalized size = 0.69

$$\frac{\ln(x)}{4}-\frac{\ln\left(x^4+\frac{2}{3}\right)}{16}+\frac{1}{24\left(x^4+\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(3*x^4 + 2)^2),x)
[Out] log(x)/4 - log(x^4 + 2/3)/16 + 1/(24*(x^4 + 2/3))
```

sympy [A] time = 0.17, size = 22, normalized size = 0.69

$$\frac{\log(x)}{4}-\frac{\log\left(3x^4+2\right)}{16}+\frac{1}{24x^4+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x**4+2)**2,x)
[Out] log(x)/4 - log(3*x**4 + 2)/16 + 1/(24*x**4 + 16)
```

3.702 $\int \frac{1}{x^3\left(2+3x^4\right)^2}\,dx$

Optimal. Leaf size=47

$$-\frac{3}{16x^2}-\frac{3}{16}\sqrt{\frac{3}{2}}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)+\frac{1}{8x^2\left(3x^4+2\right)}$$

```
[Out] -3/16/x^2+1/8/x^2/(3*x^4+2)-3/32*arctan(1/2*x^2*6^(1/2))*6^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {275, 290, 325, 203}

$$\frac{1}{8x^2\left(3x^4+2\right)}-\frac{3}{16x^2}-\frac{3}{16}\sqrt{\frac{3}{2}}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(2 + 3*x^4)^2),x]
```

```
[Out] -3/(16*x^2) + 1/(8*x^2*(2 + 3*x^4)) - (3*Sqrt[3/2]*ArcTan[Sqrt[3/2]*x^2])/1
6
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(2+3x^4\right)^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\left(2+3x^2\right)^2}\,dx, x, x^2\right)\\ &= \frac{1}{8x^2\left(2+3x^4\right)} + \frac{3}{8}\,\text{Subst}\left(\int \frac{1}{x^2\left(2+3x^2\right)}\,dx, x, x^2\right)\\ &= -\frac{3}{16x^2} + \frac{1}{8x^2\left(2+3x^4\right)} - \frac{9}{16}\,\text{Subst}\left(\int \frac{1}{2+3x^2}\,dx, x, x^2\right)\\ &= -\frac{3}{16x^2} + \frac{1}{8x^2\left(2+3x^4\right)} - \frac{3}{16}\sqrt{\frac{3}{2}}\,\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x^2\right)\end{aligned}$$

Mathematica [A] time = 0.05, size = 59, normalized size = 1.26

$$\frac{1}{32}\left(-\frac{4}{x^2} - \frac{6x^2}{3x^4+2} + 3\sqrt{6}\,\tan^{-1}\left(1-\sqrt[4]{6}\,x\right) + 3\sqrt{6}\,\tan^{-1}\left(\sqrt[4]{6}\,x+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(2 + 3*x^4)^2),x]
```

```
[Out] (-4/x^2 - (6*x^2)/(2 + 3*x^4) + 3*Sqrt[6]*ArcTan[1 - 6^(1/4)*x] + 3*Sqrt[6]
*ArcTan[1 + 6^(1/4)*x])/32
```

fricas [A] time = 0.65, size = 53, normalized size = 1.13

$$-\frac{18\,x^4 + 3\,\sqrt{3}\,\sqrt{2}\left(3\,x^6+2\,x^2\right)\arctan\left(\frac{1}{2}\,\sqrt{3}\,\sqrt{2}\,x^2\right) + 8}{32\left(3\,x^6+2\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(3*x^4+2)^2,x, algorithm="fricas")
```

```
[Out] -1/32*(18*x^4 + 3*sqrt(3)*sqrt(2)*(3*x^6 + 2*x^2)*arctan(1/2*sqrt(3)*sqrt(2
)*x^2) + 8)/(3*x^6 + 2*x^2)
```

giac [A] time = 0.17, size = 37, normalized size = 0.79

$$-\frac{3}{32}\,\sqrt{6}\,\arctan\left(\frac{1}{2}\,\sqrt{6}\,x^2\right) - \frac{9\,x^4+4}{16\left(3\,x^6+2\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(3*x^4+2)^2,x, algorithm="giac")
```

```
[Out] -3/32*sqrt(6)*arctan(1/2*sqrt(6)*x^2) - 1/16*(9*x^4 + 4)/(3*x^6 + 2*x^2)
```

maple [A] time = 0.01, size = 33, normalized size = 0.70

$$-\frac{x^2}{16\left(x^4+\frac{2}{3}\right)} - \frac{3\sqrt{6}\,\arctan\left(\frac{\sqrt{6}\,x^2}{2}\right)}{32} - \frac{1}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(3*x^4+2)^2,x)
```

```
[Out] -1/16*x^2/(x^4+2/3)-3/32*6^(1/2)*arctan(1/2*6^(1/2)*x^2)-1/8/x^2
```

maxima [A] time = 2.88, size = 37, normalized size = 0.79

$$-\frac{3}{32}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}\,x^2\right)-\frac{9\,x^4+4}{16\left(3\,x^6+2\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(3*x^4+2)^2,x, algorithm="maxima")
```

```
[Out] -3/32*sqrt(6)*arctan(1/2*sqrt(6)*x^2) - 1/16*(9*x^4 + 4)/(3*x^6 + 2*x^2)
```

mupad [B] time = 1.00, size = 41, normalized size = 0.87

$$-\frac{\frac{3x^4}{16}+\frac{1}{12}}{x^6+\frac{2x^2}{3}}-\frac{3\sqrt{2}\,\sqrt{3}\,\operatorname{atan}\left(\frac{\sqrt{2}\,\sqrt{3}\,x^2}{2}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(3*x^4 + 2)^2),x)
```

```
[Out] - ((3*x^4)/16 + 1/12)/((2*x^2)/3 + x^6) - (3*2^(1/2)*3^(1/2)*atan((2^(1/2)*
3^(1/2)*x^2)/2))/32
```

sympy [A] time = 0.30, size = 37, normalized size = 0.79

$$\frac{-9x^4-4}{48x^6+32x^2}-\frac{3\sqrt{6}\,\operatorname{atan}\left(\frac{\sqrt{6}\,x^2}{2}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(3*x**4+2)**2,x)
```

```
[Out] (-9*x**4 - 4)/(48*x**6 + 32*x**2) - 3*sqrt(6)*atan(sqrt(6)*x**2/2)/32
```

3.703 $\int \frac{x^4}{\left(2+3x^4\right)^2}\,dx$

Optimal. Leaf size=111

$$-\frac{x}{12\left(3x^4+2\right)}-\frac{\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{96\sqrt[4]{6}}+\frac{\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{96\sqrt[4]{6}}-\frac{\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{48\sqrt[4]{6}}+\frac{\tan^{-1}\left(\sqrt[4]{6}\,x+1\right)}{48\sqrt[4]{6}}$$

```
[Out] -1/12*x/(3*x^4+2)+1/288*arctan(-1+6^(1/4)*x)*6^(3/4)+1/288*arctan(1+6^(1/4)
*x)*6^(3/4)-1/576*ln(-6^(3/4)*x+3*x^2+6^(1/2))*6^(3/4)+1/576*ln(6^(3/4)*x+3
*x^2+6^(1/2))*6^(3/4)
```

Rubi [A] time = 0.08, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {288, 211, 1165, 628, 1162, 617, 204}

$$-\frac{x}{12\left(3x^4+2\right)}-\frac{\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{96\sqrt[4]{6}}+\frac{\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{96\sqrt[4]{6}}-\frac{\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{48\sqrt[4]{6}}+\frac{\tan^{-1}\left(\sqrt[4]{6}\,x+1\right)}{48\sqrt[4]{6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(2 + 3*x^4)^2,x]
```

```
[Out] -x/(12*(2 + 3*x^4)) - ArcTan[1 - 6^(1/4)*x]/(48*6^(1/4)) + ArcTan[1 + 6^(1/
4)*x]/(48*6^(1/4)) - Log[Sqrt[6] - 6^(3/4)*x + 3*x^2]/(96*6^(1/4)) + Log[Sq
rt[6] + 6^(3/4)*x + 3*x^2]/(96*6^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^4}{\left(2+3x^4\right)^2}\,dx = -\frac{x}{12\left(2+3x^4\right)} + \frac{1}{12}\int \frac{1}{2+3x^4}\,dx$$

$$= -\frac{x}{12\left(2+3x^4\right)} + \frac{\int \frac{\sqrt{2}-\sqrt{3}\,x^2}{2+3x^4}\,dx}{24\sqrt{2}} + \frac{\int \frac{\sqrt{2}+\sqrt{3}\,x^2}{2+3x^4}\,dx}{24\sqrt{2}}$$

$$= -\frac{x}{12\left(2+3x^4\right)} + \frac{\int \frac{1}{\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx}{48\sqrt{6}} + \frac{\int \frac{1}{\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}+x^2}\,dx}{48\sqrt{6}} - \frac{\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{96\sqrt[4]{6}} - \frac{\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}-2x}{-\sqrt{\frac{2}{3}}+\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}}{96\sqrt[4]{6}}$$

$$= -\frac{x}{12\left(2+3x^4\right)} - \frac{\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{96\sqrt[4]{6}} + \frac{\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{96\sqrt[4]{6}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1\right.}{48\sqrt[4]{6}}$$

$$= -\frac{x}{12\left(2+3x^4\right)} - \frac{\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{48\sqrt[4]{6}} + \frac{\tan^{-1}\left(1+\sqrt[4]{6}\,x\right)}{48\sqrt[4]{6}} - \frac{\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{96\sqrt[4]{6}} + \frac{\log\left(\sqrt{6}\right.}{9}$$

Mathematica [A] time = 0.12, size = 105, normalized size = 0.95

$$\frac{1}{576}\left(-\frac{48x}{3x^4+2} - 6^{3/4}\log\left(\sqrt{6}\,x^2-2\sqrt[4]{6}\,x+2\right) + 6^{3/4}\log\left(\sqrt{6}\,x^2+2\sqrt[4]{6}\,x+2\right) - 2\,6^{3/4}\tan^{-1}\left(1-\sqrt[4]{6}\,x\right) + 2\,6^{3/4}\,t\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(2 + 3*x^4)^2,x]
```

```
[Out] ((-48*x)/(2 + 3*x^4) - 2*6^(3/4)*ArcTan[1 - 6^(1/4)*x] + 2*6^(3/4)*ArcTan[1
 + 6^(1/4)*x] - 6^(3/4)*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] + 6^(3/4)*Log[2
+ 2*6^(1/4)*x + Sqrt[6]*x^2])/576
```

fricas [B] time = 0.58, size = 204, normalized size = 1.84

$$-\frac{4\cdot 24^{\frac{3}{4}}\sqrt{2}\left(3x^4+2\right)\arctan\left(\frac{1}{12}\cdot 24^{\frac{1}{4}}\sqrt{3}\sqrt{2}\sqrt{24^{\frac{3}{4}}\sqrt{2}\,x+12x^2+4\sqrt{6}}\;-\frac{1}{2}\cdot 24^{\frac{1}{4}}\sqrt{2}\,x-1\right)+4\cdot 24^{\frac{3}{4}}\sqrt{2}\left(3x^4+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^4+2)^2,x, algorithm="fricas")
```

```
[Out] -1/2304*(4*24^(3/4)*sqrt(2)*(3*x^4 + 2)*arctan(1/12*24^(1/4)*sqrt(3)*sqrt(2
)*sqrt(24^(3/4)*sqrt(2)*x + 12*x^2 + 4*sqrt(6)) - 1/2*24^(1/4)*sqrt(2)*x -
1) + 4*24^(3/4)*sqrt(2)*(3*x^4 + 2)*arctan(1/12*24^(1/4)*sqrt(3)*sqrt(2)*sq
rt(-24^(3/4)*sqrt(2)*x + 12*x^2 + 4*sqrt(6)) - 1/2*24^(1/4)*sqrt(2)*x + 1)
- 24^(3/4)*sqrt(2)*(3*x^4 + 2)*log(24^(3/4)*sqrt(2)*x + 12*x^2 + 4*sqrt(6))
 + 24^(3/4)*sqrt(2)*(3*x^4 + 2)*log(-24^(3/4)*sqrt(2)*x + 12*x^2 + 4*sqrt(6
)) + 192*x)/(3*x^4 + 2)
```

giac [A] time = 0.20, size = 107, normalized size = 0.96

$$\frac{1}{288}\cdot 6^{\frac{3}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{288}\cdot 6^{\frac{3}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{576}\cdot 6^{\frac{3}{4}}\log\left(x^{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^4+2)^2,x, algorithm="giac")
```

```
[Out] 1/288*6^(3/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4))) +
 1/288*6^(3/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4)))
+ 1/576*6^(3/4)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/576*6^(3/4
)*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/12*x/(3*x^4 + 2)
```

maple [A] time = 0.01, size = 121, normalized size = 1.09

$$-\frac{x}{36\left(x^{4}+\frac{2}{3}\right)}+\frac{\sqrt{3}\ 6^{\frac{1}{4}}\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\ \sqrt{3}\ 6^{\frac{3}{4}}x}{6}-1\right)}{288}+\frac{\sqrt{3}\ 6^{\frac{1}{4}}\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\ \sqrt{3}\ 6^{\frac{3}{4}}x}{6}+1\right)}{288}+\frac{\sqrt{3}\ 6^{\frac{1}{4}}\sqrt{2}\ \ln\left(\frac{x^{2}+\frac{\sqrt{3}\,6^{\frac{1}{4}}}{3}}{x^{2}-\frac{\sqrt{3}\,6^{\frac{1}{4}}}{3}}\right.}{576}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(3*x^4+2)^2,x)
```

```
[Out] -1/36*x/(x^4+2/3)+1/288*3^(1/2)*6^(1/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*
6^(3/4)*x+1)+1/288*3^(1/2)*6^(1/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/
4)*x-1)+1/576*3^(1/2)*6^(1/4)*2^(1/2)*ln((x^2+1/3*3^(1/2)*6^(1/4)*2^(1/2)*x
+1/3*6^(1/2))/(x^2-1/3*3^(1/2)*6^(1/4)*2^(1/2)*x+1/3*6^(1/2)))
```

maxima [A] time = 3.00, size = 133, normalized size = 1.20

$$\frac{1}{288}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{288}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{576}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\log\left(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^4+2)^2,x, algorithm="maxima")
```

```
[Out] 1/288*3^(3/4)*2^(3/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^(
3/4))) + 1/288*3^(3/4)*2^(3/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^
(1/4)*2^(3/4))) + 1/576*3^(3/4)*2^(3/4)*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x
 + sqrt(2)) - 1/576*3^(3/4)*2^(3/4)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + s
qrt(2)) - 1/12*x/(3*x^4 + 2)
```

mupad [B] time = 1.16, size = 45, normalized size = 0.41

$$-\frac{x}{36\left(x^{4}+\frac{2}{3}\right)}+6^{3/4}\operatorname{atan}\left(6^{1/4}x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{288}+\frac{1}{288}\mathrm{i}\right)+6^{3/4}\operatorname{atan}\left(6^{1/4}x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{288}-\frac{1}{288}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(3*x^4 + 2)^2,x)
```

```
[Out] 6^(3/4)*atan(6^(1/4)*x*(1/2 - 1i/2))*(1/288 + 1i/288) + 6^(3/4)*atan(6^(1/4
)*x*(1/2 + 1i/2))*(1/288 - 1i/288) - x/(36*(x^4 + 2/3))
```

sympy [**A**] time = 0.79, size = 95, normalized size = 0.86

$$-\frac{x}{36x^4+24}-\frac{6^{\frac{3}{4}}\log\left(x^2-\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{576}+\frac{6^{\frac{3}{4}}\log\left(x^2+\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{576}+\frac{6^{\frac{3}{4}}\operatorname{atan}\left(\sqrt[4]{6}\,x-1\right)}{288}+\frac{6^{\frac{3}{4}}\operatorname{atan}\left(\sqrt[4]{6}\,x+1\right)}{288}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(3*x**4+2)**2,x)
```

```
[Out] -x/(36*x**4 + 24) - 6**(3/4)*log(x**2 - 6**(3/4)*x/3 + sqrt(6)/3)/576 + 6**
(3/4)*log(x**2 + 6**(3/4)*x/3 + sqrt(6)/3)/576 + 6**(3/4)*atan(6**(1/4)*x -
 1)/288 + 6**(3/4)*atan(6**(1/4)*x + 1)/288
```

3.704 $\int \frac{x^2}{\left(2+3x^4\right)^2}\,dx$

Optimal. Leaf size=113

$$\frac{\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{32\,6^{3/4}}-\frac{\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{32\,6^{3/4}}+\frac{x^3}{8\left(3x^4+2\right)}-\frac{\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{16\,6^{3/4}}+\frac{\tan^{-1}\left(\sqrt[4]{6}\,x+1\right)}{16\,6^{3/4}}$$

```
[Out] 1/8*x^3/(3*x^4+2)+1/96*arctan(-1+6^(1/4)*x)*6^(1/4)+1/96*arctan(1+6^(1/4)*x
)*6^(1/4)+1/192*ln(-6^(3/4)*x+3*x^2+6^(1/2))*6^(1/4)-1/192*ln(6^(3/4)*x+3*x
^2+6^(1/2))*6^(1/4)
```

Rubi [A] time = 0.08, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {290, 297, 1162, 617, 204, 1165, 628}

$$\frac{x^3}{8\left(3x^4+2\right)}+\frac{\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{32\,6^{3/4}}-\frac{\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{32\,6^{3/4}}-\frac{\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{16\,6^{3/4}}+\frac{\tan^{-1}\left(\sqrt[4]{6}\,x+1\right)}{16\,6^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 + 3*x^4)^2,x]
```

```
[Out] x^3/(8*(2 + 3*x^4)) - ArcTan[1 - 6^(1/4)*x]/(16*6^(3/4)) + ArcTan[1 + 6^(1/
4)*x]/(16*6^(3/4)) + Log[Sqrt[6] - 6^(3/4)*x + 3*x^2]/(32*6^(3/4)) - Log[Sq
rt[6] + 6^(3/4)*x + 3*x^2]/(32*6^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^2}{\left(2+3x^4\right)^2}\,dx = \frac{x^3}{8\left(2+3x^4\right)} + \frac{1}{8}\int \frac{x^2}{2+3x^4}\,dx$$

$$= \frac{x^3}{8\left(2+3x^4\right)} - \frac{\int \frac{\sqrt{2}-\sqrt{3}\,x^2}{2+3x^4}\,dx}{16\sqrt{3}} + \frac{\int \frac{\sqrt{2}+\sqrt{3}\,x^2}{2+3x^4}\,dx}{16\sqrt{3}}$$

$$= \frac{x^3}{8\left(2+3x^4\right)} + \frac{1}{96}\int \frac{1}{\sqrt{\frac{2}{3}} - \frac{2^{3/4}x}{\sqrt[4]{3}} + x^2}\,dx + \frac{1}{96}\int \frac{1}{\sqrt{\frac{2}{3}} + \frac{2^{3/4}x}{\sqrt[4]{3}} + x^2}\,dx + \frac{\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}+2x}{-\sqrt{\frac{2}{3}}-\frac{2^{3/4}x}{\sqrt[4]{3}}-x^2}\,dx}{32\,6^{3/4}} +$$

$$= \frac{x^3}{8\left(2+3x^4\right)} + \frac{\log\left(\sqrt{6} - 6^{3/4}x + 3x^2\right)}{32\,6^{3/4}} - \frac{\log\left(\sqrt{6} + 6^{3/4}x + 3x^2\right)}{32\,6^{3/4}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1 -}{16\,6^{3/4}}$$

$$= \frac{x^3}{8\left(2+3x^4\right)} - \frac{\tan^{-1}\left(1 - \sqrt[4]{6}\,x\right)}{16\,6^{3/4}} + \frac{\tan^{-1}\left(1 + \sqrt[4]{6}\,x\right)}{16\,6^{3/4}} + \frac{\log\left(\sqrt{6} - 6^{3/4}x + 3x^2\right)}{32\,6^{3/4}} - \frac{\log\left(\sqrt{6} + 6}{32\,6}$$

Mathematica [A] time = 0.09, size = 107, normalized size = 0.95

$$\frac{1}{192}\left(\sqrt[4]{6}\log\left(\sqrt{6}\,x^2 - 2\sqrt[4]{6}\,x + 2\right) - \sqrt[4]{6}\log\left(\sqrt{6}\,x^2 + 2\sqrt[4]{6}\,x + 2\right) + \frac{24x^3}{3x^4+2} - 2\sqrt[4]{6}\tan^{-1}\left(1 - \sqrt[4]{6}\,x\right) + 2\sqrt[4]{6}\tan^{-1}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 + 3*x^4)^2,x]
```

```
[Out] ((24*x^3)/(2 + 3*x^4) - 2*6^(1/4)*ArcTan[1 - 6^(1/4)*x] + 2*6^(1/4)*ArcTan[
1 + 6^(1/4)*x] + 6^(1/4)*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] - 6^(1/4)*Log[2
 + 2*6^(1/4)*x + Sqrt[6]*x^2])/192
```

fricas [B] time = 0.63, size = 202, normalized size = 1.79

$$-\frac{4\cdot 54^{\frac{3}{4}}\sqrt{2}\left(3x^4+2\right)\arctan\left(-\frac{1}{18}\cdot 54^{\frac{3}{4}}\sqrt{2}\,x + \frac{1}{54}\cdot 54^{\frac{3}{4}}\sqrt{2}\sqrt{9x^2 + 3\cdot 54^{\frac{1}{4}}\sqrt{2}\,x + 3\sqrt{6}} - 1\right) + 4\cdot 54^{\frac{3}{4}}\sqrt{2}\left(3x^4 +}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^4+2)^2,x, algorithm="fricas")
```

```
[Out] -1/3456*(4*54^(3/4)*sqrt(2)*(3*x^4 + 2)*arctan(-1/18*54^(3/4)*sqrt(2)*x + 1
/54*54^(3/4)*sqrt(2)*sqrt(9*x^2 + 3*54^(1/4)*sqrt(2)*x + 3*sqrt(6)) - 1) +
4*54^(3/4)*sqrt(2)*(3*x^4 + 2)*arctan(-1/18*54^(3/4)*sqrt(2)*x + 1/54*54^(3
/4)*sqrt(2)*sqrt(9*x^2 - 3*54^(1/4)*sqrt(2)*x + 3*sqrt(6)) + 1) + 54^(3/4)*
sqrt(2)*(3*x^4 + 2)*log(9*x^2 + 3*54^(1/4)*sqrt(2)*x + 3*sqrt(6)) - 54^(3/4
)*sqrt(2)*(3*x^4 + 2)*log(9*x^2 - 3*54^(1/4)*sqrt(2)*x + 3*sqrt(6)) - 432*x
^3)/(3*x^4 + 2)
```

giac [A] time = 0.19, size = 109, normalized size = 0.96

$$\frac{x^3}{8\left(3x^4+2\right)}+\frac{1}{96}\cdot 6^{\frac{1}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{96}\cdot 6^{\frac{1}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)-\frac{1}{192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^4+2)^2,x, algorithm="giac")
```

```
[Out] 1/8*x^3/(3*x^4 + 2) + 1/96*6^(1/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sq
rt(2)*(2/3)^(1/4))) + 1/96*6^(1/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sq
rt(2)*(2/3)^(1/4))) - 1/192*6^(1/4)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(
2/3)) + 1/192*6^(1/4)*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

maple [A] time = 0.00, size = 125, normalized size = 1.11

$$\frac{x^3}{24x^4+16}+\frac{\sqrt{3}\ 6^{\frac{3}{4}}\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\ \sqrt{3}\ 6^{\frac{3}{4}}x}{6}-1\right)}{576}+\frac{\sqrt{3}\ 6^{\frac{3}{4}}\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\ \sqrt{3}\ 6^{\frac{3}{4}}x}{6}+1\right)}{576}+\frac{\sqrt{3}\ 6^{\frac{3}{4}}\sqrt{2}\ \ln\left(\frac{x^2-\frac{\sqrt{3}\ 6^{\frac{1}{4}}\sqrt{2}\ x}{3}}{x^2+\frac{\sqrt{3}\ 6^{\frac{1}{4}}\sqrt{2}\ x}{3}}\right.}{1152}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^4+2)^2,x)
```

```
[Out] 1/8*x^3/(3*x^4+2)+1/576*3^(1/2)*6^(3/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*
6^(3/4)*x-1)+1/1152*3^(1/2)*6^(3/4)*2^(1/2)*ln((x^2-1/3*3^(1/2)*6^(1/4)*2^(
1/2)*x+1/3*6^(1/2))/(x^2+1/3*3^(1/2)*6^(1/4)*2^(1/2)*x+1/3*6^(1/2)))+1/576*
3^(1/2)*6^(3/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)
```

maxima [A] time = 3.17, size = 135, normalized size = 1.19

$$\frac{x^3}{8\left(3x^4+2\right)}+\frac{1}{96}\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{96}\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)-\frac{1}{192}\cdot 3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^4+2)^2,x, algorithm="maxima")
```

```
[Out] 1/8*x^3/(3*x^4 + 2) + 1/96*3^(1/4)*2^(1/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sq
rt(3)*x + 3^(1/4)*2^(3/4))) + 1/96*3^(1/4)*2^(1/4)*arctan(1/6*3^(3/4)*2^(1/
4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4))) - 1/192*3^(1/4)*2^(1/4)*log(sqrt(3)*x^2
 + 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/192*3^(1/4)*2^(1/4)*log(sqrt(3)*x^2 - 3
^(1/4)*2^(3/4)*x + sqrt(2))
```

mupad [B] time = 0.12, size = 46, normalized size = 0.41

$$\frac{x^3}{24\left(x^4+\frac{2}{3}\right)}+6^{1/4}\operatorname{atan}\left(6^{1/4}x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{96}-\frac{1}{96}\mathrm{i}\right)+6^{1/4}\operatorname{atan}\left(6^{1/4}x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{96}+\frac{1}{96}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^4 + 2)^2,x)
```

```
[Out] 6^(1/4)*atan(6^(1/4)*x*(1/2 - 1i/2))*(1/96 - 1i/96) + 6^(1/4)*atan(6^(1/4)*
x*(1/2 + 1i/2))*(1/96 + 1i/96) + x^3/(24*(x^4 + 2/3))
```

sympy [**A**] time = 0.48, size = 97, normalized size = 0.86

$$\frac{x^3}{24x^4+16}+\frac{\sqrt[4]{6}\,\log\left(x^2-\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{192}-\frac{\sqrt[4]{6}\,\log\left(x^2+\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{192}+\frac{\sqrt[4]{6}\,\operatorname{atan}\left(\sqrt[4]{6}\,x-1\right)}{96}+\frac{\sqrt[4]{6}\,\operatorname{atan}\left(\sqrt[4]{6}\,x+1\right)}{96}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(3*x**4+2)**2,x)
```

```
[Out] x**3/(24*x**4 + 16) + 6**(1/4)*log(x**2 - 6**(3/4)*x/3 + sqrt(6)/3)/192 - 6
**(1/4)*log(x**2 + 6**(3/4)*x/3 + sqrt(6)/3)/192 + 6**(1/4)*atan(6**(1/4)*x
 - 1)/96 + 6**(1/4)*atan(6**(1/4)*x + 1)/96
```

3.705 $\int \frac{1}{\left(2+3x^4\right)^2}\,dx$

Optimal. Leaf size=131

$$\frac{x}{8\left(3x^4+2\right)}-\frac{3^{3/4}\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{64\sqrt[4]{2}}+\frac{3^{3/4}\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{64\sqrt[4]{2}}-\frac{3^{3/4}\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{32\sqrt[4]{2}}+\frac{3^{3/4}\tan^{-1}\left(\sqrt[4]{6}\right.}{32\sqrt[4]{2}}$$

```
[Out] 1/8*x/(3*x^4+2)+1/64*3^(3/4)*arctan(-1+6^(1/4)*x)*2^(3/4)+1/64*3^(3/4)*arct
an(1+6^(1/4)*x)*2^(3/4)-1/128*3^(3/4)*ln(-6^(3/4)*x+3*x^2+6^(1/2))*2^(3/4)+
1/128*3^(3/4)*ln(6^(3/4)*x+3*x^2+6^(1/2))*2^(3/4)
```

Rubi [A] time = 0.07, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {199, 211, 1165, 628, 1162, 617, 204}

$$\frac{x}{8\left(3x^4+2\right)}-\frac{3^{3/4}\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{64\sqrt[4]{2}}+\frac{3^{3/4}\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{64\sqrt[4]{2}}-\frac{3^{3/4}\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{32\sqrt[4]{2}}+\frac{3^{3/4}\tan^{-1}\left(\sqrt[4]{6}\right.}{32\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^4)^(-2),x]
```

```
[Out] x/(8*(2 + 3*x^4)) - (3^(3/4)*ArcTan[1 - 6^(1/4)*x])/(32*2^(1/4)) + (3^(3/4)
*ArcTan[1 + 6^(1/4)*x])/(32*2^(1/4)) - (3^(3/4)*Log[Sqrt[6] - 6^(3/4)*x + 3
*x^2])/(64*2^(1/4)) + (3^(3/4)*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(64*2^(1/4
))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\left(2+3x^4\right)^2}\,dx = \frac{x}{8\left(2+3x^4\right)} + \frac{3}{8}\int \frac{1}{2+3x^4}\,dx$$

$$= \frac{x}{8\left(2+3x^4\right)} + \frac{3\int \frac{\sqrt{2}-\sqrt{3}\,x^2}{2+3x^4}\,dx}{16\sqrt{2}} + \frac{3\int \frac{\sqrt{2}+\sqrt{3}\,x^2}{2+3x^4}\,dx}{16\sqrt{2}}$$

$$= \frac{x}{8\left(2+3x^4\right)} + \frac{1}{32}\sqrt{\frac{3}{2}}\int \frac{1}{\sqrt{\frac{2}{3}} - \frac{2^{3/4}x}{\sqrt[4]{3}} + x^2}\,dx + \frac{1}{32}\sqrt{\frac{3}{2}}\int \frac{1}{\sqrt{\frac{2}{3}} + \frac{2^{3/4}x}{\sqrt[4]{3}} + x^2}\,dx - \frac{3^{3/4}\int \frac{\frac{2^{3/4}}{\sqrt[4]{3}}}{-\sqrt{\frac{2}{3}}-}}{64\sqrt[4]{}}$$

$$= \frac{x}{8\left(2+3x^4\right)} - \frac{3^{3/4}\log\left(\sqrt{6} - 6^{3/4}x + 3x^2\right)}{64\sqrt[4]{2}} + \frac{3^{3/4}\log\left(\sqrt{6} + 6^{3/4}x + 3x^2\right)}{64\sqrt[4]{2}} + \frac{3^{3/4}\,\text{Subst}\left(\int \frac{1}{-1-}\right.}{32}$$

$$= \frac{x}{8\left(2+3x^4\right)} - \frac{3^{3/4}\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{32\sqrt[4]{2}} + \frac{3^{3/4}\tan^{-1}\left(1+\sqrt[4]{6}\,x\right)}{32\sqrt[4]{2}} - \frac{3^{3/4}\log\left(\sqrt{6} - 6^{3/4}x + 3x^2\right)}{64\sqrt[4]{2}} +$$

Mathematica [A] time = 0.08, size = 105, normalized size = 0.80

$$\frac{1}{128}\left(\frac{16x}{3x^4+2} - 6^{3/4}\log\left(\sqrt{6}\,x^2 - 2\sqrt[4]{6}\,x + 2\right) + 6^{3/4}\log\left(\sqrt{6}\,x^2 + 2\sqrt[4]{6}\,x + 2\right) - 2\,6^{3/4}\tan^{-1}\left(1-\sqrt[4]{6}\,x\right) + 2\,6^{3/4}\,\text{tan}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^4)^(-2),x]
```

```
[Out] ((16*x)/(2 + 3*x^4) - 2*6^(3/4)*ArcTan[1 - 6^(1/4)*x] + 2*6^(3/4)*ArcTan[1
+ 6^(1/4)*x] - 6^(3/4)*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] + 6^(3/4)*Log[2 +
 2*6^(1/4)*x + Sqrt[6]*x^2])/128
```

fricas [B] time = 0.71, size = 248, normalized size = 1.89

$$\frac{4\cdot 27^{\frac{1}{4}}8^{\frac{3}{4}}\sqrt{2}\left(3\,x^4+2\right)\arctan\left(-\frac{1}{18}\cdot 27^{\frac{3}{4}}8^{\frac{1}{4}}\sqrt{2}\,x + \frac{1}{108}\cdot 27^{\frac{3}{4}}8^{\frac{1}{4}}\sqrt{2}\sqrt{3\cdot 27^{\frac{1}{4}}8^{\frac{3}{4}}\sqrt{2}\,x + 36\,x^2 + 12\sqrt{3}\sqrt{2}}\ -1\right) +}{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+2)^2,x, algorithm="fricas")
```

```
[Out] -1/512*(4*27^(1/4)*8^(3/4)*sqrt(2)*(3*x^4 + 2)*arctan(-1/18*27^(3/4)*8^(1/4
)*sqrt(2)*x + 1/108*27^(3/4)*8^(1/4)*sqrt(2)*sqrt(3*27^(1/4)*8^(3/4)*sqrt(2
)*x + 36*x^2 + 12*sqrt(3)*sqrt(2)) - 1) + 4*27^(1/4)*8^(3/4)*sqrt(2)*(3*x^4
 + 2)*arctan(-1/18*27^(3/4)*8^(1/4)*sqrt(2)*x + 1/108*27^(3/4)*8^(1/4)*sqrt
(2)*sqrt(-3*27^(1/4)*8^(3/4)*sqrt(2)*x + 36*x^2 + 12*sqrt(3)*sqrt(2)) + 1)
- 27^(1/4)*8^(3/4)*sqrt(2)*(3*x^4 + 2)*log(3*27^(1/4)*8^(3/4)*sqrt(2)*x + 3
6*x^2 + 12*sqrt(3)*sqrt(2)) + 27^(1/4)*8^(3/4)*sqrt(2)*(3*x^4 + 2)*log(-3*2
7^(1/4)*8^(3/4)*sqrt(2)*x + 36*x^2 + 12*sqrt(3)*sqrt(2)) - 64*x)/(3*x^4 + 2
)
```

giac [A] time = 0.20, size = 107, normalized size = 0.82

$$\frac{1}{64}\cdot 6^{\frac{3}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{64}\cdot 6^{\frac{3}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{128}\cdot 6^{\frac{3}{4}}\log\left(x^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+2)^2,x, algorithm="giac")
```

```
[Out] 1/64*6^(3/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4))) +
1/64*6^(3/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) +
1/128*6^(3/4)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/128*6^(3/4)*
log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) + 1/8*x/(3*x^4 + 2)
```

maple [A] time = 0.00, size = 123, normalized size = 0.94

$$\frac{x}{24x^4+16}+\frac{\sqrt{3}\ 6^{\frac{1}{4}}\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\ \sqrt{3}\ 6^{\frac{3}{4}}x}{6}-1\right)}{64}+\frac{\sqrt{3}\ 6^{\frac{1}{4}}\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\ \sqrt{3}\ 6^{\frac{3}{4}}x}{6}+1\right)}{64}+\frac{\sqrt{3}\ 6^{\frac{1}{4}}\sqrt{2}\ \ln\left(\frac{x^2+\frac{\sqrt{3}\ 6^{\frac{1}{4}}\sqrt{2}\ }{3}}{x^2-\frac{\sqrt{3}\ 6^{\frac{1}{4}}\sqrt{2}\ }{3}}\right.}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+2)^2,x)
```

```
[Out] 1/8*x/(3*x^4+2)+1/64*3^(1/2)*6^(1/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(
3/4)*x-1)+1/128*3^(1/2)*6^(1/4)*2^(1/2)*ln((x^2+1/3*3^(1/2)*6^(1/4)*2^(1/2)
*x+1/3*6^(1/2))/(x^2-1/3*3^(1/2)*6^(1/4)*2^(1/2)*x+1/3*6^(1/2)))+1/64*3^(1/
2)*6^(1/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)
```

maxima [A] time = 3.08, size = 133, normalized size = 1.02

$$\frac{1}{64}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{64}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{128}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\log\left(\sqrt{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+2)^2,x, algorithm="maxima")
```

```
[Out] 1/64*3^(3/4)*2^(3/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^(3
/4))) + 1/64*3^(3/4)*2^(3/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1
/4)*2^(3/4))) + 1/128*3^(3/4)*2^(3/4)*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x +
 sqrt(2)) - 1/128*3^(3/4)*2^(3/4)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqr
t(2)) + 1/8*x/(3*x^4 + 2)
```

mupad [B] time = 1.06, size = 44, normalized size = 0.34

$$\frac{x}{24\left(x^4+\frac{2}{3}\right)}+6^{3/4}\operatorname{atan}\left(6^{1/4}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{64}+\frac{1}{64}\mathrm{i}\right)+6^{3/4}\operatorname{atan}\left(6^{1/4}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{64}-\frac{1}{64}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4 + 2)^2,x)
```

```
[Out] 6^(3/4)*atan(6^(1/4)*x*(1/2 - 1i/2))*(1/64 + 1i/64) + 6^(3/4)*atan(6^(1/4)*
x*(1/2 + 1i/2))*(1/64 - 1i/64) + x/(24*(x^4 + 2/3))
```

sympy [**A**] time = 0.60, size = 95, normalized size = 0.73

$$\frac{x}{24x^4+16}-\frac{6^{\frac{3}{4}}\log\left(x^2-\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{128}+\frac{6^{\frac{3}{4}}\log\left(x^2+\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{128}+\frac{6^{\frac{3}{4}}\operatorname{atan}\left(\sqrt[4]{6}\,x-1\right)}{64}+\frac{6^{\frac{3}{4}}\operatorname{atan}\left(\sqrt[4]{6}\,x+1\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+2)**2,x)
```

```
[Out] x/(24*x**4 + 16) - 6**(3/4)*log(x**2 - 6**(3/4)*x/3 + sqrt(6)/3)/128 + 6**(
3/4)*log(x**2 + 6**(3/4)*x/3 + sqrt(6)/3)/128 + 6**(3/4)*atan(6**(1/4)*x -
1)/64 + 6**(3/4)*atan(6**(1/4)*x + 1)/64
```

3.706 $\int \frac{1}{x^2\left(2+3x^4\right)^2}\,dx$

Optimal. Leaf size=140

$$\frac{1}{8x\left(3x^4+2\right)}-\frac{5\sqrt[4]{3}\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{64\,2^{3/4}}+\frac{5\sqrt[4]{3}\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{64\,2^{3/4}}-\frac{5}{16x}+\frac{5\sqrt[4]{3}\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{32\,2^{3/4}}-\frac{5\sqrt[4]{3}}{}$$

```
[Out] -5/16/x+1/8/x/(3*x^4+2)-5/64*3^(1/4)*arctan(-1+6^(1/4)*x)*2^(1/4)-5/64*3^(1
/4)*arctan(1+6^(1/4)*x)*2^(1/4)-5/128*3^(1/4)*ln(-6^(3/4)*x+3*x^2+6^(1/2))*
2^(1/4)+5/128*3^(1/4)*ln(6^(3/4)*x+3*x^2+6^(1/2))*2^(1/4)
```

Rubi [A] time = 0.08, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {290, 325, 297, 1162, 617, 204, 1165, 628}

$$\frac{1}{8x\left(3x^4+2\right)}-\frac{5\sqrt[4]{3}\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{64\,2^{3/4}}+\frac{5\sqrt[4]{3}\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{64\,2^{3/4}}-\frac{5}{16x}+\frac{5\sqrt[4]{3}\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{32\,2^{3/4}}-\frac{5\sqrt[4]{3}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(2 + 3*x^4)^2),x]
```

```
[Out] -5/(16*x) + 1/(8*x*(2 + 3*x^4)) + (5*3^(1/4)*ArcTan[1 - 6^(1/4)*x])/(32*2^(
3/4)) - (5*3^(1/4)*ArcTan[1 + 6^(1/4)*x])/(32*2^(3/4)) - (5*3^(1/4)*Log[Sqr
t[6] - 6^(3/4)*x + 3*x^2])/(64*2^(3/4)) + (5*3^(1/4)*Log[Sqrt[6] + 6^(3/4)*
x + 3*x^2])/(64*2^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
 + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\left(2+3x^4\right)^2}\,dx &= \frac{1}{8x\left(2+3x^4\right)} + \frac{5}{8}\int \frac{1}{x^2\left(2+3x^4\right)}\,dx \\
&= -\frac{5}{16x} + \frac{1}{8x\left(2+3x^4\right)} - \frac{15}{16}\int \frac{x^2}{2+3x^4}\,dx \\
&= -\frac{5}{16x} + \frac{1}{8x\left(2+3x^4\right)} + \frac{1}{32}\left(5\sqrt{3}\right)\int \frac{\sqrt{2}-\sqrt{3}\,x^2}{2+3x^4}\,dx - \frac{1}{32}\left(5\sqrt{3}\right)\int \frac{\sqrt{2}+\sqrt{3}\,x^2}{2+3x^4}\,dx \\
&= -\frac{5}{16x} + \frac{1}{8x\left(2+3x^4\right)} - \frac{5}{64}\int \frac{1}{\sqrt{\frac{2}{3}} - \frac{2^{3/4}x}{\sqrt[4]{3}} + x^2}\,dx - \frac{5}{64}\int \frac{1}{\sqrt{\frac{2}{3}} + \frac{2^{3/4}x}{\sqrt[4]{3}} + x^2}\,dx - \frac{\left(5\sqrt[4]{3}\right)\int}{} \\
&= -\frac{5}{16x} + \frac{1}{8x\left(2+3x^4\right)} - \frac{5\sqrt[4]{3}\,\log\left(\sqrt{6}-6^{3/4}x+3x^2\right)}{64\,2^{3/4}} + \frac{5\sqrt[4]{3}\,\log\left(\sqrt{6}+6^{3/4}x+3x^2\right)}{64\,2^{3/4}} - \frac{\left(5\right.}{} \\
&= -\frac{5}{16x} + \frac{1}{8x\left(2+3x^4\right)} + \frac{5\sqrt[4]{3}\,\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)}{32\,2^{3/4}} - \frac{5\sqrt[4]{3}\,\tan^{-1}\left(1+\sqrt[4]{6}\,x\right)}{32\,2^{3/4}} - \frac{5\sqrt[4]{3}\,\log\left(\sqrt{6}-\right.}{64\,2^{3}}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 113, normalized size = 0.81

$$\frac{1}{128}\left(-5\sqrt[4]{6}\,\log\left(\sqrt{6}\,x^2-2\sqrt[4]{6}\,x+2\right)+5\sqrt[4]{6}\,\log\left(\sqrt{6}\,x^2+2\sqrt[4]{6}\,x+2\right)-\frac{24x^3}{3x^4+2}-\frac{32}{x}+10\sqrt[4]{6}\,\tan^{-1}\left(1-\sqrt[4]{6}\,x\right)-\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(2 + 3*x^4)^2),x]
```

```
[Out] (-32/x - (24*x^3)/(2 + 3*x^4) + 10*6^(1/4)*ArcTan[1 - 6^(1/4)*x] - 10*6^(1/
4)*ArcTan[1 + 6^(1/4)*x] - 5*6^(1/4)*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] + 5
*6^(1/4)*Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2])/128
```

fricas [B] time = 0.72, size = 220, normalized size = 1.57

$$-\frac{120\,x^4 - 20\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}\left(3\,x^5+2\,x\right)\arctan\left(\frac{1}{3}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\sqrt{3^{\frac{3}{4}}2^{\frac{3}{4}}x+3\,x^2+\sqrt{3}\,\sqrt{2}}\;-3^{\frac{1}{4}}2^{\frac{1}{4}}x-1\right)-20\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}\left(3\,x^5+2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^4+2)^2,x, algorithm="fricas")
```

```
[Out] -1/128*(120*x^4 - 20*3^(1/4)*2^(1/4)*(3*x^5 + 2*x)*arctan(1/3*3^(3/4)*2^(1/
4)*sqrt(3^(3/4)*2^(3/4)*x + 3*x^2 + sqrt(3)*sqrt(2)) - 3^(1/4)*2^(1/4)*x -
1) - 20*3^(1/4)*2^(1/4)*(3*x^5 + 2*x)*arctan(1/3*3^(3/4)*2^(1/4)*sqrt(-3^(3
/4)*2^(3/4)*x + 3*x^2 + sqrt(3)*sqrt(2)) - 3^(1/4)*2^(1/4)*x + 1) - 5*3^(1/
4)*2^(1/4)*(3*x^5 + 2*x)*log(3^(3/4)*2^(3/4)*x + 3*x^2 + sqrt(3)*sqrt(2)) +
 5*3^(1/4)*2^(1/4)*(3*x^5 + 2*x)*log(-3^(3/4)*2^(3/4)*x + 3*x^2 + sqrt(3)*s
qrt(2)) + 64)/(3*x^5 + 2*x)
```

giac [A] time = 0.19, size = 115, normalized size = 0.82

$$-\frac{5}{64}\cdot 6^{\frac{1}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)-\frac{5}{64}\cdot 6^{\frac{1}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+\frac{5}{128}\cdot 6^{\frac{1}{4}}\log\left(x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^4+2)^2,x, algorithm="giac")
```

```
[Out] -5/64*6^(1/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4))) -
 5/64*6^(1/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) +
 5/128*6^(1/4)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 5/128*6^(1/4)
*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/16*(15*x^4 + 8)/(3*x^5 +
2*x)
```

maple [A] time = 0.01, size = 128, normalized size = 0.91

$$-\frac{x^3}{16\left(x^4+\frac{2}{3}\right)}-\frac{5\sqrt{3}\;6^{\frac{3}{4}}\sqrt{2}\;\arctan\left(\frac{\sqrt{2}\;\sqrt{3}\;6^{\frac{3}{4}}x}{6}-1\right)}{384}-\frac{5\sqrt{3}\;6^{\frac{3}{4}}\sqrt{2}\;\arctan\left(\frac{\sqrt{2}\;\sqrt{3}\;6^{\frac{3}{4}}x}{6}+1\right)}{384}-\frac{5\sqrt{3}\;6^{\frac{3}{4}}\sqrt{2}\;\ln\left(\frac{x^2-}{x^2+}\right.}{768}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(3*x^4+2)^2,x)
```

```
[Out] -1/16*x^3/(x^4+2/3)-5/768*3^(1/2)*6^(3/4)*2^(1/2)*ln((x^2-1/3*3^(1/2)*6^(1/
4)*2^(1/2)*x+1/3*6^(1/2))/(x^2+1/3*3^(1/2)*6^(1/4)*2^(1/2)*x+1/3*6^(1/2)))-
5/384*3^(1/2)*6^(3/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x+1)-5/384
*3^(1/2)*6^(3/4)*2^(1/2)*arctan(1/6*2^(1/2)*3^(1/2)*6^(3/4)*x-1)-1/4/x
```

maxima [A] time = 3.06, size = 141, normalized size = 1.01

$$-\frac{5}{64}\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}\,x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)-\frac{5}{64}\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}\,x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{5}{128}\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}\log\left(\sqrt{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(3*x^4+2)^2,x, algorithm="maxima")
```

```
[Out] -5/64*3^(1/4)*2^(1/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^(
3/4))) - 5/64*3^(1/4)*2^(1/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(
1/4)*2^(3/4))) + 5/128*3^(1/4)*2^(1/4)*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x
+ sqrt(2)) - 5/128*3^(1/4)*2^(1/4)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sq
rt(2)) - 1/16*(15*x^4 + 8)/(3*x^5 + 2*x)
```

mupad [B] time = 1.10, size = 51, normalized size = 0.36

$$-\frac{\frac{5x^4}{16}+\frac{1}{6}}{x^5+\frac{2x}{3}}+6^{1/4}\operatorname{atan}\left(6^{1/4}x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{5}{64}+\frac{5}{64}\mathrm{i}\right)+6^{1/4}\operatorname{atan}\left(6^{1/4}x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{5}{64}-\frac{5}{64}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(3*x^4 + 2)^2),x)
```

```
[Out] - 6^(1/4)*atan(6^(1/4)*x*(1/2 - 1i/2))*(5/64 - 5i/64) - 6^(1/4)*atan(6^(1/4
)*x*(1/2 + 1i/2))*(5/64 + 5i/64) - ((5*x^4)/16 + 1/6)/((2*x)/3 + x^5)
```

sympy [A] time = 0.67, size = 110, normalized size = 0.79

$$\frac{-15x^4-8}{48x^5+32x}-\frac{5\sqrt[4]{6}\log\left(x^2-\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{128}+\frac{5\sqrt[4]{6}\log\left(x^2+\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{128}-\frac{5\sqrt[4]{6}\operatorname{atan}\left(\sqrt[4]{6}x-1\right)}{64}-\frac{5\sqrt[4]{6}\operatorname{atan}\left(\sqrt[4]{6}x+1\right.}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(3*x**4+2)**2,x)
```

```
[Out] (-15*x**4 - 8)/(48*x**5 + 32*x) - 5*6**(1/4)*log(x**2 - 6**(3/4)*x/3 + sqrt
(6)/3)/128 + 5*6**(1/4)*log(x**2 + 6**(3/4)*x/3 + sqrt(6)/3)/128 - 5*6**(1/
4)*atan(6**(1/4)*x - 1)/64 - 5*6**(1/4)*atan(6**(1/4)*x + 1)/64
```

3.707 $\int \frac{x^2}{3+x^4}\,dx$

Optimal. Leaf size=133

$$\frac{\log\left(x^2-\sqrt{2}\,\sqrt[4]{3}\,x+\sqrt{3}\right)}{4\sqrt{2}\,\sqrt[4]{3}}-\frac{\log\left(x^2+\sqrt{2}\,\sqrt[4]{3}\,x+\sqrt{3}\right)}{4\sqrt{2}\,\sqrt[4]{3}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,x}{\sqrt[4]{3}}\right)}{2\sqrt{2}\,\sqrt[4]{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt[4]{3}}+1\right)}{2\sqrt{2}\,\sqrt[4]{3}}$$

```
[Out] 1/12*arctan(-1+1/3*x*2^(1/2)*3^(3/4))*3^(3/4)*2^(1/2)+1/12*arctan(1+1/3*x*2
^(1/2)*3^(3/4))*3^(3/4)*2^(1/2)+1/24*ln(x^2-3^(1/4)*x*2^(1/2)+3^(1/2))*3^(3
/4)*2^(1/2)-1/24*ln(x^2+3^(1/4)*x*2^(1/2)+3^(1/2))*3^(3/4)*2^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {297, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(x^2-\sqrt{2}\,\sqrt[4]{3}\,x+\sqrt{3}\right)}{4\sqrt{2}\,\sqrt[4]{3}}-\frac{\log\left(x^2+\sqrt{2}\,\sqrt[4]{3}\,x+\sqrt{3}\right)}{4\sqrt{2}\,\sqrt[4]{3}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,x}{\sqrt[4]{3}}\right)}{2\sqrt{2}\,\sqrt[4]{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt[4]{3}}+1\right)}{2\sqrt{2}\,\sqrt[4]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(3 + x^4),x]
```

```
[Out] -ArcTan[1 - (Sqrt[2]*x)/3^(1/4)]/(2*Sqrt[2]*3^(1/4)) + ArcTan[1 + (Sqrt[2]*
x)/3^(1/4)]/(2*Sqrt[2]*3^(1/4)) + Log[Sqrt[3] - Sqrt[2]*3^(1/4)*x + x^2]/(4
*Sqrt[2]*3^(1/4)) - Log[Sqrt[3] + Sqrt[2]*3^(1/4)*x + x^2]/(4*Sqrt[2]*3^(1/
4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^2}{3+x^4}\,dx = -\left(\frac{1}{2}\int \frac{\sqrt{3}-x^2}{3+x^4}\,dx\right)+\frac{1}{2}\int \frac{\sqrt{3}+x^2}{3+x^4}\,dx$$

$$= \frac{1}{4}\int \frac{1}{\sqrt{3}-\sqrt{2}\sqrt[4]{3}\,x+x^2}\,dx+\frac{1}{4}\int \frac{1}{\sqrt{3}+\sqrt{2}\sqrt[4]{3}\,x+x^2}\,dx+\frac{\int \frac{\sqrt{2}\sqrt[4]{3}+2x}{-\sqrt{3}-\sqrt{2}\sqrt[4]{3}\,x-x^2}\,dx}{4\sqrt{2}\sqrt[4]{3}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{3}-}{-\sqrt{3}+\sqrt{2}\sqrt[4]{}}}{4\sqrt{2}\sqrt[4]{}}$$

$$= \frac{\log\left(\sqrt{3}-\sqrt{2}\sqrt[4]{3}\,x+x^2\right)}{4\sqrt{2}\sqrt[4]{3}}-\frac{\log\left(\sqrt{3}+\sqrt{2}\sqrt[4]{3}\,x+x^2\right)}{4\sqrt{2}\sqrt[4]{3}}+\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\,x}{\sqrt[4]{3}}\right)}{2\sqrt{2}\sqrt[4]{3}}-\frac{\text{Subst}}{}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,x}{\sqrt[4]{3}}\right)}{2\sqrt{2}\sqrt[4]{3}}+\frac{\tan^{-1}\left(1+\frac{\sqrt{2}\,x}{\sqrt[4]{3}}\right)}{2\sqrt{2}\sqrt[4]{3}}+\frac{\log\left(\sqrt{3}-\sqrt{2}\sqrt[4]{3}\,x+x^2\right)}{4\sqrt{2}\sqrt[4]{3}}-\frac{\log\left(\sqrt{3}+\sqrt{2}\sqrt[4]{3}\,x+x^2\right)}{4\sqrt{2}\sqrt[4]{3}}$$

Mathematica [A] time = 0.05, size = 101, normalized size = 0.76

$$\frac{\log\left(\sqrt{3}\,x^2-\sqrt{2}\,3^{3/4}x+3\right)-\log\left(\sqrt{3}\,x^2+\sqrt{2}\,3^{3/4}x+3\right)-2\tan^{-1}\left(1-\frac{\sqrt{2}\,x}{\sqrt[4]{3}}\right)+2\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt[4]{3}}+1\right)}{4\sqrt{2}\sqrt[4]{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(3 + x^4),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*x)/3^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*x)/3^(1/4)] + L
og[3 - Sqrt[2]*3^(3/4)*x + Sqrt[3]*x^2] - Log[3 + Sqrt[2]*3^(3/4)*x + Sqrt[
3]*x^2])/(4*Sqrt[2]*3^(1/4))
```

fricas [A] time = 0.81, size = 141, normalized size = 1.06

$$-\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\,\arctan\left(-\frac{1}{3}\cdot 3^{\frac{3}{4}}\sqrt{2}\,x+\frac{1}{3}\cdot 3^{\frac{3}{4}}\sqrt{2}\,\sqrt{x^2+3^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{3}}\;-1\right)-\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\,\arctan\left(-\frac{1}{3}\cdot 3^{\frac{3}{4}}\sqrt{2}\,x+\frac{1}{3}\cdot 3^{\frac{3}{4}}\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+3),x, algorithm="fricas")
```

```
[Out] -1/6*3^(3/4)*sqrt(2)*arctan(-1/3*3^(3/4)*sqrt(2)*x + 1/3*3^(3/4)*sqrt(2)*sq
rt(x^2 + 3^(1/4)*sqrt(2)*x + sqrt(3)) - 1) - 1/6*3^(3/4)*sqrt(2)*arctan(-1/
3*3^(3/4)*sqrt(2)*x + 1/3*3^(3/4)*sqrt(2)*sqrt(x^2 - 3^(1/4)*sqrt(2)*x + sq
rt(3)) + 1) - 1/24*3^(3/4)*sqrt(2)*log(x^2 + 3^(1/4)*sqrt(2)*x + sqrt(3)) +
 1/24*3^(3/4)*sqrt(2)*log(x^2 - 3^(1/4)*sqrt(2)*x + sqrt(3))
```

giac [A] time = 0.18, size = 95, normalized size = 0.71

$$\frac{1}{12}\cdot 108^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\,x+3^{\frac{1}{4}}\sqrt{2}\right)\right)+\frac{1}{12}\cdot 108^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\,x-3^{\frac{1}{4}}\sqrt{2}\right)\right)-\frac{1}{24}\cdot 108^{\frac{1}{4}}\log\left(x^2+3^{\frac{1}{4}}\sqrt{2}\,x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+3),x, algorithm="giac")
```

```
[Out] 1/12*108^(1/4)*arctan(1/6*3^(3/4)*sqrt(2)*(2*x + 3^(1/4)*sqrt(2))) + 1/12*1
08^(1/4)*arctan(1/6*3^(3/4)*sqrt(2)*(2*x - 3^(1/4)*sqrt(2))) - 1/24*108^(1/
4)*log(x^2 + 3^(1/4)*sqrt(2)*x + sqrt(3)) + 1/24*108^(1/4)*log(x^2 - 3^(1/4
)*sqrt(2)*x + sqrt(3))
```

maple [A] time = 0.00, size = 85, normalized size = 0.64

$$\frac{3^{\frac{3}{4}}\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\,3^{\frac{3}{4}}x}{3}-1\right)}{12}+\frac{3^{\frac{3}{4}}\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\,3^{\frac{3}{4}}x}{3}+1\right)}{12}+\frac{3^{\frac{3}{4}}\sqrt{2}\ \ln\left(\frac{x^2-3^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{3}}{x^2+3^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{3}}\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4+3),x)
```

```
[Out] 1/12*arctan(-1+1/3*x*2^(1/2)*3^(3/4))*3^(3/4)*2^(1/2)+1/24*3^(3/4)*2^(1/2)*
ln((x^2-3^(1/4)*x*2^(1/2)+3^(1/2))/(x^2+3^(1/4)*x*2^(1/2)+3^(1/2)))+1/12*ar
ctan(1+1/3*x*2^(1/2)*3^(3/4))*3^(3/4)*2^(1/2)
```

maxima [A] time = 3.01, size = 107, normalized size = 0.80

$$\frac{1}{12}\cdot 3^{\frac{3}{4}}\sqrt{2}\,\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\,x+3^{\frac{1}{4}}\sqrt{2}\right)\right)+\frac{1}{12}\cdot 3^{\frac{3}{4}}\sqrt{2}\,\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\,x-3^{\frac{1}{4}}\sqrt{2}\right)\right)-\frac{1}{24}\cdot 3^{\frac{3}{4}}\sqrt{2}\,\log\left(x^2+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+3),x, algorithm="maxima")
```

```
[Out] 1/12*3^(3/4)*sqrt(2)*arctan(1/6*3^(3/4)*sqrt(2)*(2*x + 3^(1/4)*sqrt(2))) +
1/12*3^(3/4)*sqrt(2)*arctan(1/6*3^(3/4)*sqrt(2)*(2*x - 3^(1/4)*sqrt(2))) -
1/24*3^(3/4)*sqrt(2)*log(x^2 + 3^(1/4)*sqrt(2)*x + sqrt(3)) + 1/24*3^(3/4)*
sqrt(2)*log(x^2 - 3^(1/4)*sqrt(2)*x + sqrt(3))
```

mupad [B] time = 0.11, size = 45, normalized size = 0.34

$$\sqrt{2}\ 3^{3/4}\,\mathrm{atan}\left(\sqrt{2}\ 3^{3/4}\,x\left(\frac{1}{6}-\frac{1}{6}\mathrm{i}\right)\right)\left(\frac{1}{12}-\frac{1}{12}\mathrm{i}\right)+\sqrt{2}\ 3^{3/4}\,\mathrm{atan}\left(\sqrt{2}\ 3^{3/4}\,x\left(\frac{1}{6}+\frac{1}{6}\mathrm{i}\right)\right)\left(\frac{1}{12}+\frac{1}{12}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4 + 3),x)
```

```
[Out] 2^(1/2)*3^(3/4)*atan(2^(1/2)*3^(3/4)*x*(1/6 - 1i/6))*(1/12 - 1i/12) + 2^(1/
2)*3^(3/4)*atan(2^(1/2)*3^(3/4)*x*(1/6 + 1i/6))*(1/12 + 1i/12)
```

sympy [A] time = 0.47, size = 124, normalized size = 0.93

$$\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\log\left(x^2-\sqrt{2}\sqrt[4]{3}\,x+\sqrt{3}\right)}{24}-\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\log\left(x^2+\sqrt{2}\sqrt[4]{3}\,x+\sqrt{3}\right)}{24}+\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\,\mathrm{atan}\left(\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}x}{3}-1\right)}{12}+\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**4+3),x)
```

```
[Out] sqrt(2)*3**(3/4)*log(x**2 - sqrt(2)*3**(1/4)*x + sqrt(3))/24 - sqrt(2)*3**(
3/4)*log(x**2 + sqrt(2)*3**(1/4)*x + sqrt(3))/24 + sqrt(2)*3**(3/4)*atan(sq
rt(2)*3**(3/4)*x/3 - 1)/12 + sqrt(2)*3**(3/4)*atan(sqrt(2)*3**(3/4)*x/3 + 1
)/12
```

3.708 $\int \frac{1}{1+a+(-1+a)x^4}\,dx$

Optimal. Leaf size=83

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{1-a}\,x}{\sqrt[4]{a+1}}\right)}{2\sqrt{a+1}\,\sqrt[4]{1-a^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{1-a}\,x}{\sqrt[4]{a+1}}\right)}{2\sqrt{a+1}\,\sqrt[4]{1-a^2}}$$

```
[Out] 1/2*arctan((1-a)^(1/4)*x/(1+a)^(1/4))/(-a^2+1)^(1/4)/(1+a)^(1/2)+1/2*arctan
h((1-a)^(1/4)*x/(1+a)^(1/4))/(-a^2+1)^(1/4)/(1+a)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {212, 208, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{1-a}\,x}{\sqrt[4]{a+1}}\right)}{2\sqrt{a+1}\,\sqrt[4]{1-a^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{1-a}\,x}{\sqrt[4]{a+1}}\right)}{2\sqrt{a+1}\,\sqrt[4]{1-a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a + (-1 + a)*x^4)^(-1),x]
```

```
[Out] ArcTan[((1 - a)^(1/4)*x)/(1 + a)^(1/4)]/(2*Sqrt[1 + a]*(1 - a^2)^(1/4)) + A
rcTanh[((1 - a)^(1/4)*x)/(1 + a)^(1/4)]/(2*Sqrt[1 + a]*(1 - a^2)^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+a+(-1+a)x^4}\,dx &= \frac{\int \frac{1}{\sqrt{1+a}-\sqrt{1-a}\,x^2}\,dx}{2\sqrt{1+a}}+\frac{\int \frac{1}{\sqrt{1+a}+\sqrt{1-a}\,x^2}\,dx}{2\sqrt{1+a}}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt[4]{1-a}\,x}{\sqrt[4]{1+a}}\right)}{2\sqrt{1+a}\,\sqrt[4]{1-a^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{1-a}\,x}{\sqrt[4]{1+a}}\right)}{2\sqrt{1+a}\,\sqrt[4]{1-a^2}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 160, normalized size = 1.93

$$\frac{-\log\left(\sqrt{a-1}\,x^2-\sqrt{2}\,\sqrt[4]{a-1}\,\sqrt[4]{a+1}\,x+\sqrt{a+1}\right)+\log\left(\sqrt{a-1}\,x^2+\sqrt{2}\,\sqrt[4]{a-1}\,\sqrt[4]{a+1}\,x+\sqrt{a+1}\right)-2\tan^{-1}\left(1\right.}{4\sqrt{2}\,\sqrt[4]{a-1}\,(a+1)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + a + (-1 + a)*x^4)^(-1),x]
```

```
[Out] (-2*ArcTan[1 - Sqrt[2]*((-1 + a)/(1 + a))^(1/4)*x] + 2*ArcTan[1 + Sqrt[2]*(
(-1 + a)/(1 + a))^(1/4)*x] - Log[Sqrt[1 + a] - Sqrt[2]*(-1 + a)^(1/4)*(1 +
a)^(1/4)*x + Sqrt[-1 + a]*x^2] + Log[Sqrt[1 + a] + Sqrt[2]*(-1 + a)^(1/4)*(
1 + a)^(1/4)*x + Sqrt[-1 + a]*x^2])/(4*Sqrt[2]*(-1 + a)^(1/4)*(1 + a)^(3/4)
)
```

fricas [B] time = 0.73, size = 216, normalized size = 2.60

$$\left(-\frac{1}{a^4+2\,a^3-2\,a-1}\right)^{\frac{1}{4}}\arctan\left(-\left(a^3+a^2-a-1\right)x\left(-\frac{1}{a^4+2\,a^3-2\,a-1}\right)^{\frac{3}{4}}+\left(a^3+a^2-a-1\right)\sqrt{x^2+\left(a^2+2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+a+(-1+a)*x^4),x, algorithm="fricas")
```

```
[Out] (-1/(a^4 + 2*a^3 - 2*a - 1))^(1/4)*arctan(-(a^3 + a^2 - a - 1)*x*(-1/(a^4 +
 2*a^3 - 2*a - 1))^(3/4) + (a^3 + a^2 - a - 1)*sqrt(x^2 + (a^2 + 2*a + 1)*s
qrt(-1/(a^4 + 2*a^3 - 2*a - 1)))*(-1/(a^4 + 2*a^3 - 2*a - 1))^(3/4)) + 1/4*
(-1/(a^4 + 2*a^3 - 2*a - 1))^(1/4)*log((a + 1)*(-1/(a^4 + 2*a^3 - 2*a - 1))
^(1/4) + x) - 1/4*(-1/(a^4 + 2*a^3 - 2*a - 1))^(1/4)*log(-(a + 1)*(-1/(a^4
+ 2*a^3 - 2*a - 1))^(1/4) + x)
```

giac [B] time = 0.17, size = 267, normalized size = 3.22

$$\frac{\left(a^4-2\,a^3+2\,a-1\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a+1}{a-1}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a+1}{a-1}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,a^2-\sqrt{2}\right)}+\frac{\left(a^4-2\,a^3+2\,a-1\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a+1}{a-1}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a+1}{a-1}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,a^2-\sqrt{2}\right)}+\frac{\left(a^4-2\,a^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+a+(-1+a)*x^4),x, algorithm="giac")
```

```
[Out] 1/2*(a^4 - 2*a^3 + 2*a - 1)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*((a + 1
)/(a - 1))^(1/4))/((a + 1)/(a - 1))^(1/4))/(sqrt(2)*a^2 - sqrt(2)) + 1/2*(a
^4 - 2*a^3 + 2*a - 1)^(1/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*((a + 1)/(a -
 1))^(1/4))/((a + 1)/(a - 1))^(1/4))/(sqrt(2)*a^2 - sqrt(2)) + 1/4*(a^4 - 2
*a^3 + 2*a - 1)^(1/4)*log(x^2 + sqrt(2)*x*((a + 1)/(a - 1))^(1/4) + sqrt((a
 + 1)/(a - 1)))/(sqrt(2)*a^2 - sqrt(2)) - 1/4*(a^4 - 2*a^3 + 2*a - 1)^(1/4)
*log(x^2 - sqrt(2)*x*((a + 1)/(a - 1))^(1/4) + sqrt((a + 1)/(a - 1)))/(sqrt
(2)*a^2 - sqrt(2))
```

maple [B] time = 0.01, size = 170, normalized size = 2.05

$$\frac{\left(\frac{a+1}{a-1}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a+1}{a-1}\right)^{\frac{1}{4}}}-1\right)}{4a+4}+\frac{\left(\frac{a+1}{a-1}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a+1}{a-1}\right)^{\frac{1}{4}}}+1\right)}{4a+4}+\frac{\left(\frac{a+1}{a-1}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x^2+\left(\frac{a+1}{a-1}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a+1}{a-1}}}{x^2-\left(\frac{a+1}{a-1}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a+1}{a-1}}}\right)}{8a+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+a+(a-1)*x^4),x)
```

```
[Out] 1/8*((a+1)/(a-1))^(1/4)/(a+1)*2^(1/2)*ln((x^2+((a+1)/(a-1))^(1/4)*x*2^(1/2)
+((a+1)/(a-1))^(1/2))/(x^2-((a+1)/(a-1))^(1/4)*x*2^(1/2)+((a+1)/(a-1))^(1/2
```

```
)))+1/4*((a+1)/(a-1))^(1/4)/(a+1)*2^(1/2)*arctan(2^(1/2)/((a+1)/(a-1))^(1/4
)*x+1)+1/4*((a+1)/(a-1))^(1/4)/(a+1)*2^(1/2)*arctan(2^(1/2)/((a+1)/(a-1))^(
1/4)*x-1)
```

maxima [B] time = 3.16, size = 318, normalized size = 3.83

$$\frac{\sqrt{2}\log\left(\sqrt{a-1}\,x^2+\sqrt{2}\,(a+1)^{\frac{1}{4}}(a-1)^{\frac{1}{4}}x+\sqrt{a+1}\right)}{8\,(a+1)^{\frac{3}{4}}(a-1)^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{a-1}\,x^2-\sqrt{2}\,(a+1)^{\frac{1}{4}}(a-1)^{\frac{1}{4}}x+\sqrt{a+1}\right)}{8\,(a+1)^{\frac{3}{4}}(a-1)^{\frac{1}{4}}}+\frac{\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+a+(-1+a)*x^4),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*log(sqrt(a - 1)*x^2 + sqrt(2)*(a + 1)^(1/4)*(a - 1)^(1/4)*x + s
qrt(a + 1))/((a + 1)^(3/4)*(a - 1)^(1/4)) - 1/8*sqrt(2)*log(sqrt(a - 1)*x^2
 - sqrt(2)*(a + 1)^(1/4)*(a - 1)^(1/4)*x + sqrt(a + 1))/((a + 1)^(3/4)*(a -
 1)^(1/4)) + 1/8*sqrt(2)*log((2*sqrt(a - 1)*x - sqrt(2)*sqrt(-sqrt(a + 1)*s
qrt(a - 1)) + sqrt(2)*(a + 1)^(1/4)*(a - 1)^(1/4))/(2*sqrt(a - 1)*x + sqrt(
2)*sqrt(-sqrt(a + 1)*sqrt(a - 1)) + sqrt(2)*(a + 1)^(1/4)*(a - 1)^(1/4)))/(
sqrt(-sqrt(a + 1)*sqrt(a - 1))*sqrt(a + 1)) + 1/8*sqrt(2)*log((2*sqrt(a - 1
)*x - sqrt(2)*sqrt(-sqrt(a + 1)*sqrt(a - 1)) - sqrt(2)*(a + 1)^(1/4)*(a - 1
)^(1/4))/(2*sqrt(a - 1)*x + sqrt(2)*sqrt(-sqrt(a + 1)*sqrt(a - 1)) - sqrt(2
)*(a + 1)^(1/4)*(a - 1)^(1/4)))/(sqrt(-sqrt(a + 1)*sqrt(a - 1))*sqrt(a + 1)
)
```

mupad [B] time = 1.18, size = 543, normalized size = 6.54

$$\operatorname{atan}\left(\frac{\left(\frac{x\left(4a^3-12a^2+12a-4\right)}{4}-\frac{4a^4-8a^3+8a-4}{4(1-a)^{1/4}(a+1)^{3/4}}\right)1\mathrm{i}}{(1-a)^{1/4}(a+1)^{3/4}}+\frac{\left(\frac{x\left(4a^3-12a^2+12a-4\right)}{4}+\frac{4a^4-8a^3+8a-4}{4(1-a)^{1/4}(a+1)^{3/4}}\right)1\mathrm{i}}{(1-a)^{1/4}(a+1)^{3/4}}}{\frac{x\left(4a^3-12a^2+12a-4\right)}{4}-\frac{4a^4-8a^3+8a-4}{4(1-a)^{1/4}(a+1)^{3/4}}}{(1-a)^{1/4}(a+1)^{3/4}}-\frac{\frac{x\left(4a^3-12a^2+12a-4\right)}{4}+\frac{4a^4-8a^3+8a-4}{4(1-a)^{1/4}(a+1)^{3/4}}}{(1-a)^{1/4}(a+1)^{3/4}}}\right)1\mathrm{i}}{2\,(1-a)^{1/4}\,(a+1)^{3/4}}+\frac{\operatorname{atan}\left(\frac{\frac{x\left(4a^3-12a^2+12a-4\right)}{4}-\frac{\left(4a^4-8a^3+8a-4\right)}{4(1-a)^{1/4}(a+1)^{3/4}}}{(1-a)^{1/4}(a+1)^{3/4}}}{\left(\frac{x\left(4a^3-12a^2+12a-4\right)}{4}-\frac{\left(4a^4-8a^3+8a-4\right)1\mathrm{i}}{4(1-a)^{1/4}(a+1)^{3/4}}\right.}\right.}{(1-a)^{1/4}(a+1)^{3/4}}}{2\,(1-a)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + x^4*(a - 1) + 1),x)
```

```
[Out] (atan(((((x*(12*a - 12*a^2 + 4*a^3 - 4))/4 - (8*a - 8*a^3 + 4*a^4 - 4)/(4*(
1 - a)^(1/4)*(a + 1)^(3/4)))*1i)/((1 - a)^(1/4)*(a + 1)^(3/4)) + (((x*(12*a
 - 12*a^2 + 4*a^3 - 4))/4 + (8*a - 8*a^3 + 4*a^4 - 4)/(4*(1 - a)^(1/4)*(a +
 1)^(3/4)))*1i)/((1 - a)^(1/4)*(a + 1)^(3/4)))/(((x*(12*a - 12*a^2 + 4*a^3
 - 4))/4 - (8*a - 8*a^3 + 4*a^4 - 4)/(4*(1 - a)^(1/4)*(a + 1)^(3/4)))/((1 -
a)^(1/4)*(a + 1)^(3/4)) - ((x*(12*a - 12*a^2 + 4*a^3 - 4))/4 + (8*a - 8*a^3
 + 4*a^4 - 4)/(4*(1 - a)^(1/4)*(a + 1)^(3/4)))/((1 - a)^(1/4)*(a + 1)^(3/4)
)))*1i)/(2*(1 - a)^(1/4)*(a + 1)^(3/4)) + atan((((x*(12*a - 12*a^2 + 4*a^3
 - 4))/4 - ((8*a - 8*a^3 + 4*a^4 - 4)*1i)/(4*(1 - a)^(1/4)*(a + 1)^(3/4)))/(
(1 - a)^(1/4)*(a + 1)^(3/4)) + ((x*(12*a - 12*a^2 + 4*a^3 - 4))/4 + ((8*a -
 8*a^3 + 4*a^4 - 4)*1i)/(4*(1 - a)^(1/4)*(a + 1)^(3/4)))/((1 - a)^(1/4)*(a
 + 1)^(3/4)))/((((x*(12*a - 12*a^2 + 4*a^3 - 4))/4 - ((8*a - 8*a^3 + 4*a^4 -
 4)*1i)/(4*(1 - a)^(1/4)*(a + 1)^(3/4)))*1i)/((1 - a)^(1/4)*(a + 1)^(3/4))
 - (((x*(12*a - 12*a^2 + 4*a^3 - 4))/4 + ((8*a - 8*a^3 + 4*a^4 - 4)*1i)/(4*(
1 - a)^(1/4)*(a + 1)^(3/4)))*1i)/((1 - a)^(1/4)*(a + 1)^(3/4))))/(2*(1 - a)
^(1/4)*(a + 1)^(3/4))
```

sympy [A] time = 0.49, size = 32, normalized size = 0.39

$$\operatorname{RootSum}\left(t^4\left(256a^4+512a^3-512a-256\right)+1,\left(t\mapsto t\log\left(4ta+4t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+a+(-1+a)*x**4),x)
```

```
[Out] RootSum(_t**4*(256*a**4 + 512*a**3 - 512*a - 256) + 1, Lambda(_t, _t*log(4*
_t*a + 4*_t + x)))
```

3.709 $\int \frac{1}{2a+2b+x^4}\,dx$

Optimal. Leaf size=79

$$-\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{-a-b}}\right)}{2\,2^{3/4}(-a-b)^{3/4}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{-a-b}}\right)}{2\,2^{3/4}(-a-b)^{3/4}}$$

```
[Out] -1/4*arctan(1/2*x*2^(3/4)/(-a-b)^(1/4))*2^(1/4)/(-a-b)^(3/4)-1/4*arctanh(1/
2*x*2^(3/4)/(-a-b)^(1/4))*2^(1/4)/(-a-b)^(3/4)
```

Rubi [A] time = 0.06, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {212, 206, 203}

$$-\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{-a-b}}\right)}{2\,2^{3/4}(-a-b)^{3/4}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{-a-b}}\right)}{2\,2^{3/4}(-a-b)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*b + x^4)^(-1),x]
```

```
[Out] -ArcTan[x/(2^(1/4)*(-a - b)^(1/4))]/(2*2^(3/4)*(-a - b)^(3/4)) - ArcTanh[x/
(2^(1/4)*(-a - b)^(1/4))]/(2*2^(3/4)*(-a - b)^(3/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2a+2b+x^4}\,dx &= -\frac{\int \frac{1}{\sqrt{2}\,\sqrt{-a-b}-x^2}\,dx}{2\sqrt{2}\,\sqrt{-a-b}}-\frac{\int \frac{1}{\sqrt{2}\,\sqrt{-a-b}+x^2}\,dx}{2\sqrt{2}\,\sqrt{-a-b}}\\ &= -\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{-a-b}}\right)}{2\,2^{3/4}(-a-b)^{3/4}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{-a-b}}\right)}{2\,2^{3/4}(-a-b)^{3/4}}\end{aligned}$$

Mathematica [A] time = 0.10, size = 128, normalized size = 1.62

$$\frac{-\log\left(-2\sqrt[4]{2}\,x\sqrt[4]{a+b}+2\sqrt{a+b}+\sqrt{2}\,x^2\right)+\log\left(2\sqrt[4]{2}\,x\sqrt[4]{a+b}+2\sqrt{a+b}+\sqrt{2}\,x^2\right)-2\tan^{-1}\left(1-\frac{\sqrt[4]{2}\,x}{\sqrt[4]{a+b}}\right)+2\,\text{ta}}{8\sqrt[4]{2}\,(a+b)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*b + x^4)^(-1),x]
```

```
[Out] (-2*ArcTan[1 - (2^(1/4)*x)/(a + b)^(1/4)] + 2*ArcTan[1 + (2^(1/4)*x)/(a + b
)^(1/4)] - Log[2*Sqrt[a + b] - 2*2^(1/4)*(a + b)^(1/4)*x + Sqrt[2]*x^2] + L
og[2*Sqrt[a + b] + 2*2^(1/4)*(a + b)^(1/4)*x + Sqrt[2]*x^2])/(8*2^(1/4)*(a
+ b)^(3/4))
```

fricas [B] time = 0.72, size = 294, normalized size = 3.72

$$\left(\frac{1}{8}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3+3\,a^2b+3\,ab^2+b^3}\right)^{\frac{1}{4}}\arctan\left(-4\left(\frac{1}{8}\right)^{\frac{3}{4}}\left(a^2+2\,ab+b^2\right)x\left(-\frac{1}{a^3+3\,a^2b+3\,ab^2+b^3}\right)^{\frac{3}{4}}+4\left(\frac{1}{8}\right)^{\frac{3}{4}}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a+2*b),x, algorithm="fricas")
```

```
[Out] (1/8)^(1/4)*(-1/(a^3 + 3*a^2*b + 3*a*b^2 + b^3))^(1/4)*arctan(-4*(1/8)^(3/4
)*(a^2 + 2*a*b + b^2)*x*(-1/(a^3 + 3*a^2*b + 3*a*b^2 + b^3))^(3/4) + 4*(1/8
)^(3/4)*(a^2 + 2*a*b + b^2)*sqrt(x^2 + 2*sqrt(1/2)*(a^2 + 2*a*b + b^2)*sqrt
(-1/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)))*(-1/(a^3 + 3*a^2*b + 3*a*b^2 + b^3))^
(3/4)) + 1/4*(1/8)^(1/4)*(-1/(a^3 + 3*a^2*b + 3*a*b^2 + b^3))^(1/4)*log(2*(
1/8)^(1/4)*(a + b)*(-1/(a^3 + 3*a^2*b + 3*a*b^2 + b^3))^(1/4) + x) - 1/4*(1
/8)^(1/4)*(-1/(a^3 + 3*a^2*b + 3*a*b^2 + b^3))^(1/4)*log(-2*(1/8)^(1/4)*(a
+ b)*(-1/(a^3 + 3*a^2*b + 3*a*b^2 + b^3))^(1/4) + x)
```

giac [B] time = 0.17, size = 219, normalized size = 2.77

$$\frac{(2\,a+2\,b)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\,(2\,a+2\,b)^{\frac{1}{4}}\right)}{2\,(2\,a+2\,b)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}\,a+\sqrt{2}\,b\right)}+\frac{(2\,a+2\,b)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\,(2\,a+2\,b)^{\frac{1}{4}}\right)}{2\,(2\,a+2\,b)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}\,a+\sqrt{2}\,b\right)}+\frac{(2\,a+2\,b)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}\right.}{8\left(\sqrt{2}\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a+2*b),x, algorithm="giac")
```

```
[Out] 1/4*(2*a + 2*b)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(2*a + 2*b)^(1/4))/
(2*a + 2*b)^(1/4))/(sqrt(2)*a + sqrt(2)*b) + 1/4*(2*a + 2*b)^(1/4)*arctan(1
/2*sqrt(2)*(2*x - sqrt(2)*(2*a + 2*b)^(1/4))/(2*a + 2*b)^(1/4))/(sqrt(2)*a
+ sqrt(2)*b) + 1/8*(2*a + 2*b)^(1/4)*log(x^2 + sqrt(2)*(2*a + 2*b)^(1/4)*x
+ sqrt(2*a + 2*b))/(sqrt(2)*a + sqrt(2)*b) - 1/8*(2*a + 2*b)^(1/4)*log(x^2
- sqrt(2)*(2*a + 2*b)^(1/4)*x + sqrt(2*a + 2*b))/(sqrt(2)*a + sqrt(2)*b)
```

maple [B] time = 0.01, size = 137, normalized size = 1.73

$$\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{(2a+2b)^{\frac{1}{4}}}-1\right)}{4\,(2a+2b)^{\frac{3}{4}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{(2a+2b)^{\frac{1}{4}}}+1\right)}{4\,(2a+2b)^{\frac{3}{4}}}+\frac{\sqrt{2}\,\ln\left(\frac{x^2+(2a+2b)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{2a+2b}}{x^2-(2a+2b)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{2a+2b}}\right)}{8\,(2a+2b)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+2*a+2*b),x)
```

```
[Out] 1/8/(2*a+2*b)^(3/4)*2^(1/2)*ln((x^2+(2*a+2*b)^(1/4)*x*2^(1/2)+(2*a+2*b)^(1/
2))/(x^2-(2*a+2*b)^(1/4)*x*2^(1/2)+(2*a+2*b)^(1/2)))+1/4/(2*a+2*b)^(3/4)*2^
(1/2)*arctan(2^(1/2)/(2*a+2*b)^(1/4)*x+1)+1/4/(2*a+2*b)^(3/4)*2^(1/2)*arcta
n(2^(1/2)/(2*a+2*b)^(1/4)*x-1)
```

maxima [B] time = 3.01, size = 179, normalized size = 2.27

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}(2a+2b)^{\frac{1}{4}}\right)}{2(2a+2b)^{\frac{1}{4}}}\right)}{4(2a+2b)^{\frac{3}{4}}}+\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}(2a+2b)^{\frac{1}{4}}\right)}{2(2a+2b)^{\frac{1}{4}}}\right)}{4(2a+2b)^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(x^2+\sqrt{2}(2a+2b)^{\frac{1}{4}}x+\sqrt{2a+2b}\right.}{8(2a+2b)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a+2*b),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(2*a + 2*b)^(1/4))/(2*a + 2*b
)^(1/4))/(2*a + 2*b)^(3/4) + 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*
(2*a + 2*b)^(1/4))/(2*a + 2*b)^(1/4))/(2*a + 2*b)^(3/4) + 1/8*sqrt(2)*log(x
^2 + sqrt(2)*(2*a + 2*b)^(1/4)*x + sqrt(2*a + 2*b))/(2*a + 2*b)^(3/4) - 1/8
*sqrt(2)*log(x^2 - sqrt(2)*(2*a + 2*b)^(1/4)*x + sqrt(2*a + 2*b))/(2*a + 2*
b)^(3/4)
```

mupad [B] time = 0.12, size = 121, normalized size = 1.53

$$\frac{2^{1/4}\operatorname{atan}\left(\frac{2^{1/4}x}{\left(\frac{\sqrt{2}\,a}{(-a-b)^{3/2}}+\frac{\sqrt{2}\,b}{(-a-b)^{3/2}}\right)(-a-b)^{3/4}}\right)}{4(-a-b)^{3/4}}+\frac{2^{1/4}\operatorname{atanh}\left(\frac{2^{1/4}x}{\left(\frac{\sqrt{2}\,a}{(-a-b)^{3/2}}+\frac{\sqrt{2}\,b}{(-a-b)^{3/2}}\right)(-a-b)^{3/4}}\right)}{4(-a-b)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a + 2*b + x^4),x)
```

```
[Out] (2^(1/4)*atan((2^(1/4)*x)/(((2^(1/2)*a)/(- a - b)^(3/2) + (2^(1/2)*b)/(- a
- b)^(3/2))*(- a - b)^(3/4))))/(4*(- a - b)^(3/4)) + (2^(1/4)*atanh((2^(1/4
)*x)/(((2^(1/2)*a)/(- a - b)^(3/2) + (2^(1/2)*b)/(- a - b)^(3/2))*(- a - b)
^(3/4))))/(4*(- a - b)^(3/4))
```

sympy [A] time = 0.43, size = 42, normalized size = 0.53

$$\operatorname{RootSum}\left(t^4\left(2048a^3+6144a^2b+6144ab^2+2048b^3\right)+1,\left(t\mapsto t\log\left(8ta+8tb+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+2*a+2*b),x)
```

```
[Out] RootSum(_t**4*(2048*a**3 + 6144*a**2*b + 6144*a*b**2 + 2048*b**3) + 1, Lamb
da(_t, _t*log(8*_t*a + 8*_t*b + x)))
```

3.710 $\int \frac{1}{2(a+b)+x^4}\,dx$

Optimal. Leaf size=79

$$-\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{-a-b}}\right)}{2\,2^{3/4}(-a-b)^{3/4}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{-a-b}}\right)}{2\,2^{3/4}(-a-b)^{3/4}}$$

```
[Out] -1/4*arctan(1/2*x*2^(3/4)/(-a-b)^(1/4))*2^(1/4)/(-a-b)^(3/4)-1/4*arctanh(1/
2*x*2^(3/4)/(-a-b)^(1/4))*2^(1/4)/(-a-b)^(3/4)
```

Rubi [A] time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {212, 206, 203}

$$-\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{-a-b}}\right)}{2\,2^{3/4}(-a-b)^{3/4}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{-a-b}}\right)}{2\,2^{3/4}(-a-b)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*(a + b) + x^4)^(-1),x]
```

```
[Out] -ArcTan[x/(2^(1/4)*(-a - b)^(1/4))]/(2*2^(3/4)*(-a - b)^(3/4)) - ArcTanh[x/
(2^(1/4)*(-a - b)^(1/4))]/(2*2^(3/4)*(-a - b)^(3/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2(a+b)+x^4}\,dx &= -\frac{\int \frac{1}{\sqrt{2}\sqrt{-a-b}-x^2}\,dx}{2\sqrt{2}\sqrt{-a-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{-a-b}+x^2}\,dx}{2\sqrt{2}\sqrt{-a-b}}\\ &= -\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{-a-b}}\right)}{2\,2^{3/4}(-a-b)^{3/4}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{-a-b}}\right)}{2\,2^{3/4}(-a-b)^{3/4}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 128, normalized size = 1.62

$$\frac{-\log\left(-2\sqrt[4]{2}\,x\sqrt[4]{a+b}+2\sqrt{a+b}+\sqrt{2}\,x^2\right)+\log\left(2\sqrt[4]{2}\,x\sqrt[4]{a+b}+2\sqrt{a+b}+\sqrt{2}\,x^2\right)-2\tan^{-1}\left(1-\frac{\sqrt[4]{2}\,x}{\sqrt[4]{a+b}}\right)+}{8\sqrt[4]{2}\,(a+b)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*(a + b) + x^4)^(-1),x]
```

```
[Out] (-2*ArcTan[1 - (2^(1/4)*x)/(a + b)^(1/4)] + 2*ArcTan[1 + (2^(1/4)*x)/(a + b
)^(1/4)] - Log[2*Sqrt[a + b] - 2*2^(1/4)*(a + b)^(1/4)*x + Sqrt[2]*x^2] + L
og[2*Sqrt[a + b] + 2*2^(1/4)*(a + b)^(1/4)*x + Sqrt[2]*x^2])/(8*2^(1/4)*(a
+ b)^(3/4))
```

fricas [B] time = 0.66, size = 294, normalized size = 3.72

$$\left(\frac{1}{8}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3+3a^2b+3ab^2+b^3}\right)^{\frac{1}{4}}\arctan\left(-4\left(\frac{1}{8}\right)^{\frac{3}{4}}\left(a^2+2ab+b^2\right)x\left(-\frac{1}{a^3+3a^2b+3ab^2+b^3}\right)^{\frac{3}{4}}+4\left(\frac{1}{8}\right)^{\frac{3}{4}}\left(a^2+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a+2*b),x, algorithm="fricas")
```

```
[Out] (1/8)^(1/4)*(-1/(a^3 + 3*a^2*b + 3*a*b^2 + b^3))^(1/4)*arctan(-4*(1/8)^(3/4
)*(a^2 + 2*a*b + b^2)*x*(-1/(a^3 + 3*a^2*b + 3*a*b^2 + b^3))^(3/4) + 4*(1/8
)^(3/4)*(a^2 + 2*a*b + b^2)*sqrt(x^2 + 2*sqrt(1/2)*(a^2 + 2*a*b + b^2)*sqrt
(-1/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)))*(-1/(a^3 + 3*a^2*b + 3*a*b^2 + b^3))^
(3/4)) + 1/4*(1/8)^(1/4)*(-1/(a^3 + 3*a^2*b + 3*a*b^2 + b^3))^(1/4)*log(2*(
1/8)^(1/4)*(a + b)*(-1/(a^3 + 3*a^2*b + 3*a*b^2 + b^3))^(1/4) + x) - 1/4*(1
/8)^(1/4)*(-1/(a^3 + 3*a^2*b + 3*a*b^2 + b^3))^(1/4)*log(-2*(1/8)^(1/4)*(a
+ b)*(-1/(a^3 + 3*a^2*b + 3*a*b^2 + b^3))^(1/4) + x)
```

giac [B] time = 0.21, size = 219, normalized size = 2.77

$$\frac{(2a+2b)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}(2a+2b)^{\frac{1}{4}}\right)}{2(2a+2b)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}a+\sqrt{2}b\right)}+\frac{(2a+2b)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}(2a+2b)^{\frac{1}{4}}\right)}{2(2a+2b)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}a+\sqrt{2}b\right)}+\frac{(2a+2b)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}(2a\right.}{8\left(\sqrt{2}a+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a+2*b),x, algorithm="giac")
```

```
[Out] 1/4*(2*a + 2*b)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(2*a + 2*b)^(1/4))/
(2*a + 2*b)^(1/4))/(sqrt(2)*a + sqrt(2)*b) + 1/4*(2*a + 2*b)^(1/4)*arctan(1
/2*sqrt(2)*(2*x - sqrt(2)*(2*a + 2*b)^(1/4))/(2*a + 2*b)^(1/4))/(sqrt(2)*a
+ sqrt(2)*b) + 1/8*(2*a + 2*b)^(1/4)*log(x^2 + sqrt(2)*(2*a + 2*b)^(1/4)*x
+ sqrt(2*a + 2*b))/(sqrt(2)*a + sqrt(2)*b) - 1/8*(2*a + 2*b)^(1/4)*log(x^2
- sqrt(2)*(2*a + 2*b)^(1/4)*x + sqrt(2*a + 2*b))/(sqrt(2)*a + sqrt(2)*b)
```

maple [B] time = 0.00, size = 137, normalized size = 1.73

$$\frac{\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\,x}{(2a+2b)^{\frac{1}{4}}}-1\right)}{4(2a+2b)^{\frac{3}{4}}}+\frac{\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\,x}{(2a+2b)^{\frac{1}{4}}}+1\right)}{4(2a+2b)^{\frac{3}{4}}}+\frac{\sqrt{2}\ \ln\left(\frac{x^2+(2a+2b)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{2a+2b}}{x^2-(2a+2b)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{2a+2b}}\right)}{8(2a+2b)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+2*a+2*b),x)
```

```
[Out] 1/4/(2*a+2*b)^(3/4)*2^(1/2)*arctan(2^(1/2)/(2*a+2*b)^(1/4)*x-1)+1/4/(2*a+2*
b)^(3/4)*2^(1/2)*arctan(2^(1/2)/(2*a+2*b)^(1/4)*x+1)+1/8/(2*a+2*b)^(3/4)*2^
(1/2)*ln((x^2+(2*a+2*b)^(1/4)*2^(1/2)*x+(2*a+2*b)^(1/2))/(x^2-(2*a+2*b)^(1/
4)*2^(1/2)*x+(2*a+2*b)^(1/2)))
```

maxima [B] time = 3.02, size = 179, normalized size = 2.27

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}(2a+2b)^{\frac{1}{4}}\right)}{2(2a+2b)^{\frac{1}{4}}}\right)}{4(2a+2b)^{\frac{3}{4}}}+\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}(2a+2b)^{\frac{1}{4}}\right)}{2(2a+2b)^{\frac{1}{4}}}\right)}{4(2a+2b)^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(x^2+\sqrt{2}(2a+2b)^{\frac{1}{4}}x+\sqrt{2a+2}\right.}{8(2a+2b)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a+2*b),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(2*a + 2*b)^(1/4))/(2*a + 2*b
)^(1/4))/(2*a + 2*b)^(3/4) + 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*
(2*a + 2*b)^(1/4))/(2*a + 2*b)^(1/4))/(2*a + 2*b)^(3/4) + 1/8*sqrt(2)*log(x
^2 + sqrt(2)*(2*a + 2*b)^(1/4)*x + sqrt(2*a + 2*b))/(2*a + 2*b)^(3/4) - 1/8
*sqrt(2)*log(x^2 - sqrt(2)*(2*a + 2*b)^(1/4)*x + sqrt(2*a + 2*b))/(2*a + 2*
b)^(3/4)
```

mupad [B] time = 0.00, size = 121, normalized size = 1.53

$$\frac{2^{1/4}\operatorname{atan}\left(\frac{2^{1/4}x}{\left(\frac{\sqrt{2}\,a}{(-a-b)^{3/2}}+\frac{\sqrt{2}\,b}{(-a-b)^{3/2}}\right)(-a-b)^{3/4}}\right)}{4(-a-b)^{3/4}}+\frac{2^{1/4}\operatorname{atanh}\left(\frac{2^{1/4}x}{\left(\frac{\sqrt{2}\,a}{(-a-b)^{3/2}}+\frac{\sqrt{2}\,b}{(-a-b)^{3/2}}\right)(-a-b)^{3/4}}\right)}{4(-a-b)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a + 2*b + x^4),x)
```

```
[Out] (2^(1/4)*atan((2^(1/4)*x)/(((2^(1/2)*a)/(- a - b)^(3/2) + (2^(1/2)*b)/(- a
- b)^(3/2))*(- a - b)^(3/4))))/(4*(- a - b)^(3/4)) + (2^(1/4)*atanh((2^(1/4
)*x)/(((2^(1/2)*a)/(- a - b)^(3/2) + (2^(1/2)*b)/(- a - b)^(3/2))*(- a - b)
^(3/4))))/(4*(- a - b)^(3/4))
```

sympy [A] time = 0.55, size = 42, normalized size = 0.53

$$\operatorname{RootSum}\left(t^4\left(2048a^3+6144a^2b+6144ab^2+2048b^3\right)+1,\left(t\mapsto t\log\left(8ta+8tb+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+2*a+2*b),x)
```

```
[Out] RootSum(_t**4*(2048*a**3 + 6144*a**2*b + 6144*a*b**2 + 2048*b**3) + 1, Lamb
da(_t, _t*log(8*_t*a + 8*_t*b + x)))
```

3.711 $\int \frac{x}{2a+2b+x^4}\,dx$

Optimal. Leaf size=33

$$\frac{\tan^{-1}\left(\frac{x^2}{\sqrt{2}\sqrt{a+b}}\right)}{2\sqrt{2}\sqrt{a+b}}$$

```
[Out] 1/4*arctan(1/2*x^2*2^(1/2)/(a+b)^(1/2))*2^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {275, 203}

$$\frac{\tan^{-1}\left(\frac{x^2}{\sqrt{2}\sqrt{a+b}}\right)}{2\sqrt{2}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(2*a + 2*b + x^4),x]
```

```
[Out] ArcTan[x^2/(Sqrt[2]*Sqrt[a + b])]/(2*Sqrt[2]*Sqrt[a + b])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{2a+2b+x^4}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{2a+2b+x^2}\,dx, x, x^2\right)\\ &= \frac{\tan^{-1}\left(\frac{x^2}{\sqrt{2}\sqrt{a+b}}\right)}{2\sqrt{2}\sqrt{a+b}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{x^2}{\sqrt{2}\sqrt{a+b}}\right)}{2\sqrt{2}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(2*a + 2*b + x^4),x]
```

```
[Out] ArcTan[x^2/(Sqrt[2]*Sqrt[a + b])]/(2*Sqrt[2]*Sqrt[a + b])
```

fricas [A] time = 0.64, size = 91, normalized size = 2.76

$$\left[-\frac{\sqrt{-2a-2b}\,\log\left(\frac{x^4-2\sqrt{-2a-2b}\,x^2-2a-2b}{x^4+2a+2b}\right)}{8(a+b)}, \frac{\sqrt{2a+2b}\,\arctan\left(\frac{\sqrt{2a+2b}\,x^2}{2(a+b)}\right)}{4(a+b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*a+2*b),x, algorithm="fricas")
```

```
[Out] [-1/8*sqrt(-2*a - 2*b)*log((x^4 - 2*sqrt(-2*a - 2*b)*x^2 - 2*a - 2*b)/(x^4
+ 2*a + 2*b))/(a + b), 1/4*sqrt(2*a + 2*b)*arctan(1/2*sqrt(2*a + 2*b)*x^2/(
a + b))/(a + b)]
```

giac [A] time = 0.16, size = 24, normalized size = 0.73

$$\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x^2}{2\,\sqrt{a+b}}\right)}{4\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*a+2*b),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*x^2/sqrt(a + b))/sqrt(a + b)
```

maple [A] time = 0.00, size = 26, normalized size = 0.79

$$\frac{\arctan\left(\frac{x^2}{\sqrt{2a+2b}}\right)}{2\sqrt{2a+2b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4+2*a+2*b),x)
```

```
[Out] 1/2/(2*a+2*b)^(1/2)*arctan(x^2/(2*a+2*b)^(1/2))
```

maxima [A] time = 3.03, size = 25, normalized size = 0.76

$$\frac{\arctan\left(\frac{x^2}{\sqrt{2\,a+2\,b}}\right)}{2\,\sqrt{2\,a+2\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*a+2*b),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(x^2/sqrt(2*a + 2*b))/sqrt(2*a + 2*b)
```

mupad [B] time = 1.04, size = 32, normalized size = 0.97

$$\frac{\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,x^2\,\sqrt{a+b}}{2\,a+2\,b}\right)}{4\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2*a + 2*b + x^4),x)
```

```
[Out] (2^(1/2)*atan((2^(1/2)*x^2*(a + b)^(1/2))/(2*a + 2*b)))/(4*(a + b)^(1/2))
```

sympy [B] time = 0.49, size = 110, normalized size = 3.33

$$-\frac{\sqrt{2}\,\sqrt{-\frac{1}{a+b}}\,\log\left(-\sqrt{2}\,a\sqrt{-\frac{1}{a+b}}-\sqrt{2}\,b\sqrt{-\frac{1}{a+b}}+x^2\right)}{8}+\frac{\sqrt{2}\,\sqrt{-\frac{1}{a+b}}\,\log\left(\sqrt{2}\,a\sqrt{-\frac{1}{a+b}}+\sqrt{2}\,b\sqrt{-\frac{1}{a+b}}+x^2\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4+2*a+2*b),x)
```

```
[Out] -sqrt(2)*sqrt(-1/(a + b))*log(-sqrt(2)*a*sqrt(-1/(a + b)) - sqrt(2)*b*sqrt(
-1/(a + b)) + x**2)/8 + sqrt(2)*sqrt(-1/(a + b))*log(sqrt(2)*a*sqrt(-1/(a +
 b)) + sqrt(2)*b*sqrt(-1/(a + b)) + x**2)/8
```

3.712 $\int \frac{x}{2(a+b)+x^4}\,dx$

Optimal. Leaf size=33

$$\frac{\tan^{-1}\left(\frac{x^2}{\sqrt{2}\sqrt{a+b}}\right)}{2\sqrt{2}\sqrt{a+b}}$$

```
[Out] 1/4*arctan(1/2*x^2*2^(1/2)/(a+b)^(1/2))*2^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {275, 203}

$$\frac{\tan^{-1}\left(\frac{x^2}{\sqrt{2}\sqrt{a+b}}\right)}{2\sqrt{2}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(2*(a + b) + x^4),x]
```

```
[Out] ArcTan[x^2/(Sqrt[2]*Sqrt[a + b])]/(2*Sqrt[2]*Sqrt[a + b])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{2(a+b)+x^4}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{2(a+b)+x^2}\,dx, x, x^2\right)\\ &= \frac{\tan^{-1}\left(\frac{x^2}{\sqrt{2}\sqrt{a+b}}\right)}{2\sqrt{2}\sqrt{a+b}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{x^2}{\sqrt{2}\sqrt{a+b}}\right)}{2\sqrt{2}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(2*(a + b) + x^4),x]
```

```
[Out] ArcTan[x^2/(Sqrt[2]*Sqrt[a + b])]/(2*Sqrt[2]*Sqrt[a + b])
```

fricas [A] time = 0.60, size = 91, normalized size = 2.76

$$\left[-\frac{\sqrt{-2a-2b}\,\log\left(\frac{x^4-2\sqrt{-2a-2b}\,x^2-2a-2b}{x^4+2a+2b}\right)}{8(a+b)}, \frac{\sqrt{2a+2b}\,\arctan\left(\frac{\sqrt{2a+2b}\,x^2}{2(a+b)}\right)}{4(a+b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*a+2*b),x, algorithm="fricas")
```

```
[Out] [-1/8*sqrt(-2*a - 2*b)*log((x^4 - 2*sqrt(-2*a - 2*b)*x^2 - 2*a - 2*b)/(x^4
+ 2*a + 2*b))/(a + b), 1/4*sqrt(2*a + 2*b)*arctan(1/2*sqrt(2*a + 2*b)*x^2/(
a + b))/(a + b)]
```

giac [A] time = 0.15, size = 24, normalized size = 0.73

$$\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x^2}{2\,\sqrt{a+b}}\right)}{4\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*a+2*b),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*x^2/sqrt(a + b))/sqrt(a + b)
```

maple [A] time = 0.00, size = 26, normalized size = 0.79

$$\frac{\arctan\left(\frac{x^2}{\sqrt{2a+2b}}\right)}{2\sqrt{2a+2b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4+2*a+2*b),x)
```

```
[Out] 1/2/(2*a+2*b)^(1/2)*arctan(1/(2*a+2*b)^(1/2)*x^2)
```

maxima [A] time = 2.94, size = 25, normalized size = 0.76

$$\frac{\arctan\left(\frac{x^2}{\sqrt{2\,a+2\,b}}\right)}{2\,\sqrt{2\,a+2\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*a+2*b),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(x^2/sqrt(2*a + 2*b))/sqrt(2*a + 2*b)
```

mupad [B] time = 0.00, size = 32, normalized size = 0.97

$$\frac{\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,x^2\,\sqrt{a+b}}{2\,a+2\,b}\right)}{4\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2*a + 2*b + x^4),x)
```

```
[Out] (2^(1/2)*atan((2^(1/2)*x^2*(a + b)^(1/2))/(2*a + 2*b)))/(4*(a + b)^(1/2))
```

sympy [B] time = 0.29, size = 110, normalized size = 3.33

$$-\frac{\sqrt{2}\,\sqrt{-\frac{1}{a+b}}\,\log\left(-\sqrt{2}\,a\sqrt{-\frac{1}{a+b}}-\sqrt{2}\,b\sqrt{-\frac{1}{a+b}}+x^2\right)}{8}+\frac{\sqrt{2}\,\sqrt{-\frac{1}{a+b}}\,\log\left(\sqrt{2}\,a\sqrt{-\frac{1}{a+b}}+\sqrt{2}\,b\sqrt{-\frac{1}{a+b}}+x^2\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4+2*a+2*b),x)
```

```
[Out] -sqrt(2)*sqrt(-1/(a + b))*log(-sqrt(2)*a*sqrt(-1/(a + b)) - sqrt(2)*b*sqrt(
-1/(a + b)) + x**2)/8 + sqrt(2)*sqrt(-1/(a + b))*log(sqrt(2)*a*sqrt(-1/(a +
 b)) + sqrt(2)*b*sqrt(-1/(a + b)) + x**2)/8
```

3.713 $\int \frac{x^2}{2a+2b+x^4}\,dx$

Optimal. Leaf size=79

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{-a-b}}\right)}{2\sqrt[4]{2}\,\sqrt[4]{-a-b}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{-a-b}}\right)}{2\sqrt[4]{2}\,\sqrt[4]{-a-b}}$$

```
[Out] 1/4*arctan(1/2*x*2^(3/4)/(-a-b)^(1/4))*2^(3/4)/(-a-b)^(1/4)-1/4*arctanh(1/2
*x*2^(3/4)/(-a-b)^(1/4))*2^(3/4)/(-a-b)^(1/4)
```

Rubi [A] time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {298, 203, 206}

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{-a-b}}\right)}{2\sqrt[4]{2}\,\sqrt[4]{-a-b}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{-a-b}}\right)}{2\sqrt[4]{2}\,\sqrt[4]{-a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2*a + 2*b + x^4),x]
```

```
[Out] ArcTan[x/(2^(1/4)*(-a - b)^(1/4))]/(2*2^(1/4)*(-a - b)^(1/4)) - ArcTanh[x/(
2^(1/4)*(-a - b)^(1/4))]/(2*2^(1/4)*(-a - b)^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{2a+2b+x^4}\,dx &= -\left(\frac{1}{2}\int \frac{1}{\sqrt{2}\,\sqrt{-a-b}\,-x^2}\,dx\right)+\frac{1}{2}\int \frac{1}{\sqrt{2}\,\sqrt{-a-b}\,+x^2}\,dx\\ &= \frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{-a-b}}\right)}{2\sqrt[4]{2}\,\sqrt[4]{-a-b}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{-a-b}}\right)}{2\sqrt[4]{2}\,\sqrt[4]{-a-b}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 128, normalized size = 1.62

$$\frac{\log\left(-2\sqrt[4]{2}\,x\sqrt[4]{a+b}\,+2\sqrt{a+b}\,+\sqrt{2}\,x^2\right)-\log\left(2\sqrt[4]{2}\,x\sqrt[4]{a+b}\,+2\sqrt{a+b}\,+\sqrt{2}\,x^2\right)-2\tan^{-1}\left(1-\frac{\sqrt[4]{2}\,x}{\sqrt[4]{a+b}}\right)+2\tan}{4\,2^{3/4}\sqrt[4]{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2*a + 2*b + x^4),x]
```

```
[Out] (-2*ArcTan[1 - (2^(1/4)*x)/(a + b)^(1/4)] + 2*ArcTan[1 + (2^(1/4)*x)/(a + b
)^(1/4)] + Log[2*Sqrt[a + b] - 2*2^(1/4)*(a + b)^(1/4)*x + Sqrt[2]*x^2] - L
og[2*Sqrt[a + b] + 2*2^(1/4)*(a + b)^(1/4)*x + Sqrt[2]*x^2])/(4*2^(3/4)*(a
+ b)^(1/4))
```

fricas [B] time = 0.69, size = 136, normalized size = 1.72

$$-\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(-\frac{1}{a+b}\right)^{\frac{1}{4}}\arctan\left(-\left(\frac{1}{2}\right)^{\frac{1}{4}}x\left(-\frac{1}{a+b}\right)^{\frac{1}{4}}+\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{x^2-2\sqrt{\frac{1}{2}}(a+b)\sqrt{-\frac{1}{a+b}}}\left(-\frac{1}{a+b}\right)^{\frac{1}{4}}\right)+\frac{1}{4}\left(\frac{1}{2}\right)^{\frac{1}{4}}\left($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+2*a+2*b),x, algorithm="fricas")
```

```
[Out] -(1/2)^(1/4)*(-1/(a + b))^(1/4)*arctan(-(1/2)^(1/4)*x*(-1/(a + b))^(1/4) +
(1/2)^(1/4)*sqrt(x^2 - 2*sqrt(1/2)*(a + b)*sqrt(-1/(a + b)))*(-1/(a + b))^(
1/4)) + 1/4*(1/2)^(1/4)*(-1/(a + b))^(1/4)*log(2*(1/2)^(3/4)*(a + b)*(-1/(a
 + b))^(3/4) + x) - 1/4*(1/2)^(1/4)*(-1/(a + b))^(1/4)*log(-2*(1/2)^(3/4)*(
a + b)*(-1/(a + b))^(3/4) + x)
```

giac [B] time = 0.17, size = 219, normalized size = 2.77

$$\frac{(2a+2b)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}(2a+2b)^{\frac{1}{4}}\right)}{2(2a+2b)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}a+\sqrt{2}b\right)}+\frac{(2a+2b)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}(2a+2b)^{\frac{1}{4}}\right)}{2(2a+2b)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}a+\sqrt{2}b\right)}-\frac{(2a+2b)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}\right.}{8\left(\sqrt{2}a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+2*a+2*b),x, algorithm="giac")
```

```
[Out] 1/4*(2*a + 2*b)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(2*a + 2*b)^(1/4))/
(2*a + 2*b)^(1/4))/(sqrt(2)*a + sqrt(2)*b) + 1/4*(2*a + 2*b)^(3/4)*arctan(1
/2*sqrt(2)*(2*x - sqrt(2)*(2*a + 2*b)^(1/4))/(2*a + 2*b)^(1/4))/(sqrt(2)*a
+ sqrt(2)*b) - 1/8*(2*a + 2*b)^(3/4)*log(x^2 + sqrt(2)*(2*a + 2*b)^(1/4)*x
+ sqrt(2*a + 2*b))/(sqrt(2)*a + sqrt(2)*b) + 1/8*(2*a + 2*b)^(3/4)*log(x^2
- sqrt(2)*(2*a + 2*b)^(1/4)*x + sqrt(2*a + 2*b))/(sqrt(2)*a + sqrt(2)*b)
```

maple [B] time = 0.00, size = 137, normalized size = 1.73

$$\frac{\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\,x}{(2a+2b)^{\frac{1}{4}}}-1\right)}{4(2a+2b)^{\frac{1}{4}}}+\frac{\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\,x}{(2a+2b)^{\frac{1}{4}}}+1\right)}{4(2a+2b)^{\frac{1}{4}}}+\frac{\sqrt{2}\ \ln\left(\frac{x^2-(2a+2b)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{2a+2b}}{x^2+(2a+2b)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{2a+2b}}\right)}{8(2a+2b)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4+2*a+2*b),x)
```

```
[Out] 1/8/(2*a+2*b)^(1/4)*2^(1/2)*ln((x^2-(2*a+2*b)^(1/4)*2^(1/2)*x+(2*a+2*b)^(1/
2))/(x^2+(2*a+2*b)^(1/4)*2^(1/2)*x+(2*a+2*b)^(1/2)))+1/4/(2*a+2*b)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(2*a+2*b)^(1/4)*x+1)+1/4/(2*a+2*b)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(2*a+2*b)^(1/4)*x-1)
```

maxima [B] time = 3.04, size = 179, normalized size = 2.27

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}(2a+2b)^{\frac{1}{4}}\right)}{2(2a+2b)^{\frac{1}{4}}}\right)}{4(2a+2b)^{\frac{1}{4}}}+\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}(2a+2b)^{\frac{1}{4}}\right)}{2(2a+2b)^{\frac{1}{4}}}\right)}{4(2a+2b)^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(x^2+\sqrt{2}(2a+2b)^{\frac{1}{4}}x+\sqrt{2a+2b}\right.}{8(2a+2b)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+2*a+2*b),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(2*a + 2*b)^(1/4))/(2*a + 2*b
)^(1/4))/(2*a + 2*b)^(1/4) + 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*
(2*a + 2*b)^(1/4))/(2*a + 2*b)^(1/4))/(2*a + 2*b)^(1/4) - 1/8*sqrt(2)*log(x
^2 + sqrt(2)*(2*a + 2*b)^(1/4)*x + sqrt(2*a + 2*b))/(2*a + 2*b)^(1/4) + 1/8
*sqrt(2)*log(x^2 - sqrt(2)*(2*a + 2*b)^(1/4)*x + sqrt(2*a + 2*b))/(2*a + 2*
b)^(1/4)
```

mupad [B] time = 0.13, size = 53, normalized size = 0.67

$$\frac{8^{1/4}\operatorname{atan}\left(\frac{8^{1/4}x}{2(-a-b)^{1/4}}\right)-8^{1/4}\operatorname{atanh}\left(\frac{8^{1/4}x}{2(-a-b)^{1/4}}\right)}{4(-a-b)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2*a + 2*b + x^4),x)
```

```
[Out] (8^(1/4)*atan((8^(1/4)*x)/(2*(- a - b)^(1/4))) - 8^(1/4)*atanh((8^(1/4)*x)/
(2*(- a - b)^(1/4))))/(4*(- a - b)^(1/4))
```

sympy [A] time = 0.28, size = 29, normalized size = 0.37

$$\operatorname{RootSum}\left(t^4(512a+512b)+1,\left(t\mapsto t\log\left(128t^3a+128t^3b+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**4+2*a+2*b),x)
```

```
[Out] RootSum(_t**4*(512*a + 512*b) + 1, Lambda(_t, _t*log(128*_t**3*a + 128*_t**
3*b + x)))
```

3.714 $\int \frac{x^2}{2(a+b)+x^4}\,dx$

Optimal. Leaf size=79

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{-a-b}}\right)}{2\sqrt[4]{2}\,\sqrt[4]{-a-b}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{-a-b}}\right)}{2\sqrt[4]{2}\,\sqrt[4]{-a-b}}$$

```
[Out] 1/4*arctan(1/2*x*2^(3/4)/(-a-b)^(1/4))*2^(3/4)/(-a-b)^(1/4)-1/4*arctanh(1/2
*x*2^(3/4)/(-a-b)^(1/4))*2^(3/4)/(-a-b)^(1/4)
```

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {298, 203, 206}

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{-a-b}}\right)}{2\sqrt[4]{2}\,\sqrt[4]{-a-b}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{-a-b}}\right)}{2\sqrt[4]{2}\,\sqrt[4]{-a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2*(a + b) + x^4),x]
```

```
[Out] ArcTan[x/(2^(1/4)*(-a - b)^(1/4))]/(2*2^(1/4)*(-a - b)^(1/4)) - ArcTanh[x/(
2^(1/4)*(-a - b)^(1/4))]/(2*2^(1/4)*(-a - b)^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{2(a+b)+x^4}\,dx &= -\left(\frac{1}{2}\int \frac{1}{\sqrt{2}\,\sqrt{-a-b}-x^2}\,dx\right)+\frac{1}{2}\int \frac{1}{\sqrt{2}\,\sqrt{-a-b}+x^2}\,dx\\ &= \frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{-a-b}}\right)}{2\sqrt[4]{2}\,\sqrt[4]{-a-b}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{-a-b}}\right)}{2\sqrt[4]{2}\,\sqrt[4]{-a-b}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 128, normalized size = 1.62

$$\frac{\log\left(-2\sqrt[4]{2}\,x\sqrt[4]{a+b}+2\sqrt{a+b}+\sqrt{2}\,x^2\right)-\log\left(2\sqrt[4]{2}\,x\sqrt[4]{a+b}+2\sqrt{a+b}+\sqrt{2}\,x^2\right)-2\tan^{-1}\left(1-\frac{\sqrt[4]{2}\,x}{\sqrt[4]{a+b}}\right)+2\,t}{4\,2^{3/4}\sqrt[4]{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2*(a + b) + x^4),x]
```

```
[Out] (-2*ArcTan[1 - (2^(1/4)*x)/(a + b)^(1/4)] + 2*ArcTan[1 + (2^(1/4)*x)/(a + b
)^(1/4)] + Log[2*Sqrt[a + b] - 2*2^(1/4)*(a + b)^(1/4)*x + Sqrt[2]*x^2] - L
og[2*Sqrt[a + b] + 2*2^(1/4)*(a + b)^(1/4)*x + Sqrt[2]*x^2])/(4*2^(3/4)*(a
+ b)^(1/4))
```

fricas [B] time = 0.66, size = 136, normalized size = 1.72

$$-\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(-\frac{1}{a+b}\right)^{\frac{1}{4}}\arctan\left(-\left(\frac{1}{2}\right)^{\frac{1}{4}}x\left(-\frac{1}{a+b}\right)^{\frac{1}{4}}+\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{x^2-2\sqrt{\frac{1}{2}}(a+b)\sqrt{-\frac{1}{a+b}}}\left(-\frac{1}{a+b}\right)^{\frac{1}{4}}\right)+\frac{1}{4}\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(-\frac{}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+2*a+2*b),x, algorithm="fricas")
```

```
[Out] -(1/2)^(1/4)*(-1/(a + b))^(1/4)*arctan(-(1/2)^(1/4)*x*(-1/(a + b))^(1/4) +
(1/2)^(1/4)*sqrt(x^2 - 2*sqrt(1/2)*(a + b)*sqrt(-1/(a + b)))*(-1/(a + b))^(
1/4)) + 1/4*(1/2)^(1/4)*(-1/(a + b))^(1/4)*log(2*(1/2)^(3/4)*(a + b)*(-1/(a
 + b))^(3/4) + x) - 1/4*(1/2)^(1/4)*(-1/(a + b))^(1/4)*log(-2*(1/2)^(3/4)*(
a + b)*(-1/(a + b))^(3/4) + x)
```

giac [B] time = 0.19, size = 219, normalized size = 2.77

$$\frac{(2a+2b)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}(2a+2b)^{\frac{1}{4}}\right)}{2(2a+2b)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}a+\sqrt{2}b\right)}+\frac{(2a+2b)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}(2a+2b)^{\frac{1}{4}}\right)}{2(2a+2b)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}a+\sqrt{2}b\right)}-\frac{(2a+2b)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}(2a\right.}{8\left(\sqrt{2}a+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+2*a+2*b),x, algorithm="giac")
```

```
[Out] 1/4*(2*a + 2*b)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(2*a + 2*b)^(1/4))/
(2*a + 2*b)^(1/4))/(sqrt(2)*a + sqrt(2)*b) + 1/4*(2*a + 2*b)^(3/4)*arctan(1
/2*sqrt(2)*(2*x - sqrt(2)*(2*a + 2*b)^(1/4))/(2*a + 2*b)^(1/4))/(sqrt(2)*a
+ sqrt(2)*b) - 1/8*(2*a + 2*b)^(3/4)*log(x^2 + sqrt(2)*(2*a + 2*b)^(1/4)*x
+ sqrt(2*a + 2*b))/(sqrt(2)*a + sqrt(2)*b) + 1/8*(2*a + 2*b)^(3/4)*log(x^2
- sqrt(2)*(2*a + 2*b)^(1/4)*x + sqrt(2*a + 2*b))/(sqrt(2)*a + sqrt(2)*b)
```

maple [B] time = 0.00, size = 137, normalized size = 1.73

$$\frac{\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\,x}{(2a+2b)^{\frac{1}{4}}}-1\right)}{4(2a+2b)^{\frac{1}{4}}}+\frac{\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\,x}{(2a+2b)^{\frac{1}{4}}}+1\right)}{4(2a+2b)^{\frac{1}{4}}}+\frac{\sqrt{2}\ \ln\left(\frac{x^2-(2a+2b)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{2a+2b}}{x^2+(2a+2b)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{2a+2b}}\right)}{8(2a+2b)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4+2*a+2*b),x)
```

```
[Out] 1/4/(2*a+2*b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(2*a+2*b)^(1/4)*x-1)+1/4/(2*a+2*
b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(2*a+2*b)^(1/4)*x+1)+1/8/(2*a+2*b)^(1/4)*2^
(1/2)*ln((x^2-(2*a+2*b)^(1/4)*2^(1/2)*x+(2*a+2*b)^(1/2))/(x^2+(2*a+2*b)^(1/
4)*2^(1/2)*x+(2*a+2*b)^(1/2)))
```

maxima [B] time = 3.11, size = 179, normalized size = 2.27

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}(2a+2b)^{\frac{1}{4}}\right)}{2(2a+2b)^{\frac{1}{4}}}\right)}{4(2a+2b)^{\frac{1}{4}}}+\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}(2a+2b)^{\frac{1}{4}}\right)}{2(2a+2b)^{\frac{1}{4}}}\right)}{4(2a+2b)^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(x^2+\sqrt{2}(2a+2b)^{\frac{1}{4}}x+\sqrt{2a+2}\right.}{8(2a+2b)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+2*a+2*b),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(2*a + 2*b)^(1/4))/(2*a + 2*b
)^(1/4))/(2*a + 2*b)^(1/4) + 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*
(2*a + 2*b)^(1/4))/(2*a + 2*b)^(1/4))/(2*a + 2*b)^(1/4) - 1/8*sqrt(2)*log(x
^2 + sqrt(2)*(2*a + 2*b)^(1/4)*x + sqrt(2*a + 2*b))/(2*a + 2*b)^(1/4) + 1/8
*sqrt(2)*log(x^2 - sqrt(2)*(2*a + 2*b)^(1/4)*x + sqrt(2*a + 2*b))/(2*a + 2*
b)^(1/4)
```

mupad [B] time = 0.00, size = 53, normalized size = 0.67

$$\frac{8^{1/4}\operatorname{atan}\left(\frac{8^{1/4}x}{2(-a-b)^{1/4}}\right)-8^{1/4}\operatorname{atanh}\left(\frac{8^{1/4}x}{2(-a-b)^{1/4}}\right)}{4(-a-b)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2*a + 2*b + x^4),x)
```

```
[Out] (8^(1/4)*atan((8^(1/4)*x)/(2*(- a - b)^(1/4))) - 8^(1/4)*atanh((8^(1/4)*x)/
(2*(- a - b)^(1/4))))/(4*(- a - b)^(1/4))
```

sympy [A] time = 0.49, size = 29, normalized size = 0.37

$$\operatorname{RootSum}\left(t^4(512a+512b)+1,\left(t\mapsto t\log\left(128t^3a+128t^3b+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**4+2*a+2*b),x)
```

```
[Out] RootSum(_t**4*(512*a + 512*b) + 1, Lambda(_t, _t*log(128*_t**3*a + 128*_t**
3*b + x)))
```

3.715 $\int \frac{x^3}{2a+2b+x^4}\,dx$

Optimal. Leaf size=14

$$\frac{1}{4}\log\left(2(a+b)+x^4\right)$$

```
[Out] 1/4*ln(x^4+2*a+2*b)
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {260}

$$\frac{1}{4}\log\left(2(a+b)+x^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(2*a + 2*b + x^4),x]
```

```
[Out] Log[2*(a + b) + x^4]/4
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^3}{2a+2b+x^4}\,dx = \frac{1}{4}\log\left(2(a+b)+x^4\right)$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.07

$$\frac{1}{4}\log\left(2a+2b+x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(2*a + 2*b + x^4),x]
```

```
[Out] Log[2*a + 2*b + x^4]/4
```

fricas [A] time = 0.53, size = 13, normalized size = 0.93

$$\frac{1}{4}\log\left(x^4+2\,a+2\,b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4+2*a+2*b),x, algorithm="fricas")
```

```
[Out] 1/4*log(x^4 + 2*a + 2*b)
```

giac [A] time = 0.15, size = 14, normalized size = 1.00

$$\frac{1}{4}\log\left(\left|x^4+2\,a+2\,b\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4+2*a+2*b),x, algorithm="giac")
[Out] 1/4*log(abs(x^4 + 2*a + 2*b))
```

maple [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{\ln\left(x^4+2a+2b\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^4+2*a+2*b),x)
[Out] 1/4*ln(x^4+2*a+2*b)
```

maxima [A] time = 1.40, size = 13, normalized size = 0.93

$$\frac{1}{4}\log\left(x^4+2\,a+2\,b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4+2*a+2*b),x, algorithm="maxima")
[Out] 1/4*log(x^4 + 2*a + 2*b)
```

mupad [B] time = 0.04, size = 13, normalized size = 0.93

$$\frac{\ln\left(x^4+2\,a+2\,b\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(2*a + 2*b + x^4),x)
[Out] log(2*a + 2*b + x^4)/4
```

sympy [A] time = 0.46, size = 12, normalized size = 0.86

$$\frac{\log\left(2a+2b+x^4\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**4+2*a+2*b),x)
[Out] log(2*a + 2*b + x**4)/4
```

3.716 $\int \frac{x^3}{2(a+b)+x^4}\,dx$

Optimal. Leaf size=14

$$\frac{1}{4}\log\left(2(a+b)+x^4\right)$$

```
[Out] 1/4*ln(x^4+2*a+2*b)
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {260}

$$\frac{1}{4}\log\left(2(a+b)+x^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(2*(a + b) + x^4),x]
```

```
[Out] Log[2*(a + b) + x^4]/4
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^3}{2(a+b)+x^4}\,dx = \frac{1}{4}\log\left(2(a+b)+x^4\right)$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.07

$$\frac{1}{4}\log\left(2a+2b+x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(2*(a + b) + x^4),x]
```

```
[Out] Log[2*a + 2*b + x^4]/4
```

fricas [A] time = 0.74, size = 13, normalized size = 0.93

$$\frac{1}{4}\log\left(x^4+2\,a+2\,b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4+2*a+2*b),x, algorithm="fricas")
```

```
[Out] 1/4*log(x^4 + 2*a + 2*b)
```

giac [A] time = 0.17, size = 14, normalized size = 1.00

$$\frac{1}{4}\log\left(\left|x^4+2\,a+2\,b\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4+2*a+2*b),x, algorithm="giac")
[Out] 1/4*log(abs(x^4 + 2*a + 2*b))
```

maple [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{\ln\left(x^4+2a+2b\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^4+2*a+2*b),x)
[Out] 1/4*ln(x^4+2*a+2*b)
```

maxima [A] time = 1.34, size = 13, normalized size = 0.93

$$\frac{1}{4}\log\left(x^4+2\,a+2\,b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4+2*a+2*b),x, algorithm="maxima")
[Out] 1/4*log(x^4 + 2*a + 2*b)
```

mupad [B] time = 0.00, size = 13, normalized size = 0.93

$$\frac{\ln\left(x^4+2\,a+2\,b\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(2*a + 2*b + x^4),x)
[Out] log(2*a + 2*b + x^4)/4
```

sympy [A] time = 0.21, size = 12, normalized size = 0.86

$$\frac{\log\left(2a+2b+x^4\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**4+2*a+2*b),x)
[Out] log(2*a + 2*b + x**4)/4
```

3.717 $\int x^{5/2}\left(a+cx^4\right)\,dx$

Optimal. Leaf size=21

$$\frac{2}{7}ax^{7/2}+\frac{2}{15}cx^{15/2}$$

```
[Out] 2/7*a*x^(7/2)+2/15*c*x^(15/2)
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$\frac{2}{7}ax^{7/2}+\frac{2}{15}cx^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + c*x^4),x]
```

```
[Out] (2*a*x^(7/2))/7 + (2*c*x^(15/2))/15
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+cx^4\right)\,dx &= \int \left(ax^{5/2}+cx^{13/2}\right)\,dx\\ &= \frac{2}{7}ax^{7/2}+\frac{2}{15}cx^{15/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{2}{7}ax^{7/2}+\frac{2}{15}cx^{15/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + c*x^4),x]
```

```
[Out] (2*a*x^(7/2))/7 + (2*c*x^(15/2))/15
```

fricas [A] time = 0.48, size = 18, normalized size = 0.86

$$\frac{2}{105}\left(7\,cx^7+15\,ax^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+a),x, algorithm="fricas")
```

```
[Out] 2/105*(7*c*x^7 + 15*a*x^3)*sqrt(x)
```

giac [A] time = 0.17, size = 13, normalized size = 0.62

$$\frac{2}{15}cx^{\frac{15}{2}}+\frac{2}{7}ax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+a),x, algorithm="giac")
[Out] 2/15*c*x^(15/2) + 2/7*a*x^(7/2)
```

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$\frac{2\left(7c\,x^4+15a\right)x^{\frac{7}{2}}}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(c*x^4+a),x)
[Out] 2/105*x^(7/2)*(7*c*x^4+15*a)
```

maxima [A] time = 1.36, size = 13, normalized size = 0.62

$$\frac{2}{15}\,cx^{\frac{15}{2}}+\frac{2}{7}\,ax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+a),x, algorithm="maxima")
[Out] 2/15*c*x^(15/2) + 2/7*a*x^(7/2)
```

mupad [B] time = 1.05, size = 15, normalized size = 0.71

$$\frac{2\,x^{7/2}\,\left(7\,c\,x^4+15\,a\right)}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + c*x^4),x)
[Out] (2*x^(7/2)*(15*a + 7*c*x^4))/105
```

sympy [A] time = 7.94, size = 19, normalized size = 0.90

$$\frac{2ax^{\frac{7}{2}}}{7}+\frac{2cx^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(c*x**4+a),x)
[Out] 2*a*x**(7/2)/7 + 2*c*x**(15/2)/15
```

3.718 $\int x^{3/2}\left(a+cx^4\right)\,dx$

Optimal. Leaf size=21

$$\frac{2}{5}ax^{5/2}+\frac{2}{13}cx^{13/2}$$

```
[Out] 2/5*a*x^(5/2)+2/13*c*x^(13/2)
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$\frac{2}{5}ax^{5/2}+\frac{2}{13}cx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + c*x^4),x]
```

```
[Out] (2*a*x^(5/2))/5 + (2*c*x^(13/2))/13
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+cx^4\right)\,dx &= \int \left(ax^{3/2}+cx^{11/2}\right)\,dx\\ &= \frac{2}{5}ax^{5/2}+\frac{2}{13}cx^{13/2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{2}{5}ax^{5/2}+\frac{2}{13}cx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + c*x^4),x]
```

```
[Out] (2*a*x^(5/2))/5 + (2*c*x^(13/2))/13
```

fricas [A] time = 0.68, size = 18, normalized size = 0.86

$$\frac{2}{65}\left(5\,cx^6+13\,ax^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+a),x, algorithm="fricas")
```

```
[Out] 2/65*(5*c*x^6 + 13*a*x^2)*sqrt(x)
```

giac [A] time = 0.15, size = 13, normalized size = 0.62

$$\frac{2}{13}\,cx^{\frac{13}{2}}+\frac{2}{5}\,ax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+a),x, algorithm="giac")
[Out] 2/13*c*x^(13/2) + 2/5*a*x^(5/2)
```

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$\frac{2\left(5c\,x^4+13a\right)x^{\frac{5}{2}}}{65}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(c*x^4+a),x)
[Out] 2/65*x^(5/2)*(5*c*x^4+13*a)
```

maxima [A] time = 1.37, size = 13, normalized size = 0.62

$$\frac{2}{13}\,cx^{\frac{13}{2}}+\frac{2}{5}\,ax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+a),x, algorithm="maxima")
[Out] 2/13*c*x^(13/2) + 2/5*a*x^(5/2)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.71

$$\frac{2\,x^{5/2}\,\left(5\,c\,x^4+13\,a\right)}{65}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + c*x^4),x)
[Out] (2*x^(5/2)*(13*a + 5*c*x^4))/65
```

sympy [A] time = 4.01, size = 19, normalized size = 0.90

$$\frac{2ax^{\frac{5}{2}}}{5}+\frac{2cx^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(c*x**4+a),x)
[Out] 2*a*x**(5/2)/5 + 2*c*x**(13/2)/13
```

3.719 $\int \sqrt{x}\left(a+cx^4\right)\,dx$

Optimal. Leaf size=21

$$\frac{2}{3}ax^{3/2}+\frac{2}{11}cx^{11/2}$$

```
[Out] 2/3*a*x^(3/2)+2/11*c*x^(11/2)
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$\frac{2}{3}ax^{3/2}+\frac{2}{11}cx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + c*x^4),x]
```

```
[Out] (2*a*x^(3/2))/3 + (2*c*x^(11/2))/11
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+cx^4\right)\,dx &= \int \left(a\sqrt{x}+cx^{9/2}\right)\,dx\\ &= \frac{2}{3}ax^{3/2}+\frac{2}{11}cx^{11/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{2}{3}ax^{3/2}+\frac{2}{11}cx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + c*x^4),x]
```

```
[Out] (2*a*x^(3/2))/3 + (2*c*x^(11/2))/11
```

fricas [A] time = 0.72, size = 16, normalized size = 0.76

$$\frac{2}{33}\left(3\,cx^5+11\,ax\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/33*(3*c*x^5 + 11*a*x)*sqrt(x)
```

giac [A] time = 0.15, size = 13, normalized size = 0.62

$$\frac{2}{11}cx^{\frac{11}{2}}+\frac{2}{3}ax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)*x^(1/2),x, algorithm="giac")
[Out] 2/11*c*x^(11/2) + 2/3*a*x^(3/2)
```

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$\frac{2\left(3c\,x^4+11a\right)x^{\frac{3}{2}}}{33}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)*x^(1/2),x)
[Out] 2/33*x^(3/2)*(3*c*x^4+11*a)
```

maxima [A] time = 1.34, size = 13, normalized size = 0.62

$$\frac{2}{11}\,cx^{\frac{11}{2}}+\frac{2}{3}\,ax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)*x^(1/2),x, algorithm="maxima")
[Out] 2/11*c*x^(11/2) + 2/3*a*x^(3/2)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.71

$$\frac{2\,x^{3/2}\,\left(3\,c\,x^4+11\,a\right)}{33}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + c*x^4),x)
[Out] (2*x^(3/2)*(11*a + 3*c*x^4))/33
```

sympy [A] time = 2.78, size = 19, normalized size = 0.90

$$\frac{2ax^{\frac{3}{2}}}{3}+\frac{2cx^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)*x**(1/2),x)
[Out] 2*a*x**(3/2)/3 + 2*c*x**(11/2)/11
```

3.720 $\int \frac{a+cx^4}{\sqrt{x}}\,dx$

Optimal. Leaf size=19

$$2a\sqrt{x}+\frac{2}{9}cx^{9/2}$$

```
[Out] 2/9*c*x^(9/2)+2*a*x^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$2a\sqrt{x}+\frac{2}{9}cx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)/Sqrt[x],x]
```

```
[Out] 2*a*Sqrt[x] + (2*c*x^(9/2))/9
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+cx^4}{\sqrt{x}}\,dx &= \int \left(\frac{a}{\sqrt{x}}+cx^{7/2}\right)dx\\ &= 2a\sqrt{x}+\frac{2}{9}cx^{9/2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$2a\sqrt{x}+\frac{2}{9}cx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)/Sqrt[x],x]
```

```
[Out] 2*a*Sqrt[x] + (2*c*x^(9/2))/9
```

fricas [A] time = 0.63, size = 14, normalized size = 0.74

$$\frac{2}{9}\left(cx^4+9\,a\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9*(c*x^4 + 9*a)*sqrt(x)
```

giac [A] time = 0.15, size = 13, normalized size = 0.68

$$\frac{2}{9}\,cx^{\frac{9}{2}}+2\,a\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/x^(1/2),x, algorithm="giac")
[Out] 2/9*c*x^(9/2) + 2*a*sqrt(x)
```

maple [A] time = 0.00, size = 15, normalized size = 0.79

$$\frac{2\left(c\,x^4+9a\right)\sqrt{x}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)/x^(1/2),x)
[Out] 2/9*x^(1/2)*(c*x^4+9*a)
```

maxima [A] time = 1.33, size = 13, normalized size = 0.68

$$\frac{2}{9}cx^{\frac{9}{2}}+2\,a\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/x^(1/2),x, algorithm="maxima")
[Out] 2/9*c*x^(9/2) + 2*a*sqrt(x)
```

mupad [B] time = 0.03, size = 14, normalized size = 0.74

$$\frac{2\sqrt{x}\,\left(c\,x^4+9\,a\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)/x^(1/2),x)
[Out] (2*x^(1/2)*(9*a + c*x^4))/9
```

sympy [A] time = 1.69, size = 17, normalized size = 0.89

$$2a\sqrt{x}+\frac{2cx^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)/x**(1/2),x)
[Out] 2*a*sqrt(x) + 2*c*x**(9/2)/9
```

3.721 $\int \frac{a+cx^4}{x^{3/2}}\,dx$

Optimal. Leaf size=19

$$\frac{2}{7}cx^{7/2}-\frac{2a}{\sqrt{x}}$$

```
[Out] 2/7*c*x^(7/2)-2*a/x^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$\frac{2}{7}cx^{7/2}-\frac{2a}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)/x^(3/2),x]
```

```
[Out] (-2*a)/Sqrt[x] + (2*c*x^(7/2))/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+cx^4}{x^{3/2}}\,dx &= \int \left(\frac{a}{x^{3/2}}+cx^{5/2}\right)dx\\ &= -\frac{2a}{\sqrt{x}}+\frac{2}{7}cx^{7/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{2}{7}cx^{7/2}-\frac{2a}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)/x^(3/2),x]
```

```
[Out] (-2*a)/Sqrt[x] + (2*c*x^(7/2))/7
```

fricas [A] time = 0.76, size = 14, normalized size = 0.74

$$\frac{2\left(cx^4-7\,a\right)}{7\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/7*(c*x^4 - 7*a)/sqrt(x)
```

giac [A] time = 0.15, size = 13, normalized size = 0.68

$$\frac{2}{7}\,cx^{\frac{7}{2}}-\frac{2\,a}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/x^(3/2),x, algorithm="giac")
[Out] 2/7*c*x^(7/2) - 2*a/sqrt(x)
```

maple [A] time = 0.00, size = 16, normalized size = 0.84

$$-\frac{2\left(-c\,x^4+7a\right)}{7\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)/x^(3/2),x)
[Out] -2/7*(-c*x^4+7*a)/x^(1/2)
```

maxima [A] time = 1.38, size = 13, normalized size = 0.68

$$\frac{2}{7}\,cx^{\frac{7}{2}}-\frac{2\,a}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/x^(3/2),x, algorithm="maxima")
[Out] 2/7*c*x^(7/2) - 2*a/sqrt(x)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.79

$$-\frac{14\,a-2\,c\,x^4}{7\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)/x^(3/2),x)
[Out] -(14*a - 2*c*x^4)/(7*x^(1/2))
```

sympy [A] time = 1.66, size = 17, normalized size = 0.89

$$-\frac{2a}{\sqrt{x}}+\frac{2cx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)/x**(3/2),x)
[Out] -2*a/sqrt(x) + 2*c*x**(7/2)/7
```

3.722 $\int \frac{a+cx^4}{x^{5/2}}\,dx$

Optimal. Leaf size=21

$$\frac{2}{5}cx^{5/2}-\frac{2a}{3x^{3/2}}$$

```
[Out] -2/3*a/x^(3/2)+2/5*c*x^(5/2)
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$\frac{2}{5}cx^{5/2}-\frac{2a}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)/x^(5/2),x]
```

```
[Out] (-2*a)/(3*x^(3/2)) + (2*c*x^(5/2))/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+cx^4}{x^{5/2}}\,dx &= \int \left(\frac{a}{x^{5/2}}+cx^{3/2}\right)dx\\ &= -\frac{2a}{3x^{3/2}}+\frac{2}{5}cx^{5/2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 21, normalized size = 1.00

$$\frac{2}{5}cx^{5/2}-\frac{2a}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)/x^(5/2),x]
```

```
[Out] (-2*a)/(3*x^(3/2)) + (2*c*x^(5/2))/5
```

fricas [A] time = 0.62, size = 15, normalized size = 0.71

$$\frac{2\left(3\,cx^4-5\,a\right)}{15\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*c*x^4 - 5*a)/x^(3/2)
```

giac [A] time = 0.19, size = 13, normalized size = 0.62

$$\frac{2}{5}\,cx^{\frac{5}{2}}-\frac{2\,a}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/x^(5/2),x, algorithm="giac")
[Out] 2/5*c*x^(5/2) - 2/3*a/x^(3/2)
```

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$-\frac{2\left(-3c\,x^4+5a\right)}{15x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)/x^(5/2),x)
[Out] -2/15*(-3*c*x^4+5*a)/x^(3/2)
```

maxima [A] time = 1.35, size = 13, normalized size = 0.62

$$\frac{2}{5}cx^{\frac{5}{2}}-\frac{2\,a}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/x^(5/2),x, algorithm="maxima")
[Out] 2/5*c*x^(5/2) - 2/3*a/x^(3/2)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.71

$$-\frac{10\,a-6\,c\,x^4}{15\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)/x^(5/2),x)
[Out] -(10*a - 6*c*x^4)/(15*x^(3/2))
```

sympy [A] time = 1.56, size = 19, normalized size = 0.90

$$-\frac{2a}{3x^{\frac{3}{2}}}+\frac{2cx^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)/x**(5/2),x)
[Out] -2*a/(3*x**(3/2)) + 2*c*x**(5/2)/5
```

3.723 $\int \frac{a+cx^4}{x^{7/2}}\,dx$

Optimal. Leaf size=21

$$\frac{2}{3}cx^{3/2}-\frac{2a}{5x^{5/2}}$$

```
[Out] -2/5*a/x^(5/2)+2/3*c*x^(3/2)
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$\frac{2}{3}cx^{3/2}-\frac{2a}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)/x^(7/2),x]
```

```
[Out] (-2*a)/(5*x^(5/2)) + (2*c*x^(3/2))/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+cx^4}{x^{7/2}}\,dx &= \int \left(\frac{a}{x^{7/2}}+c\sqrt{x}\right)dx\\ &= -\frac{2a}{5x^{5/2}}+\frac{2}{3}cx^{3/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{2}{3}cx^{3/2}-\frac{2a}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)/x^(7/2),x]
```

```
[Out] (-2*a)/(5*x^(5/2)) + (2*c*x^(3/2))/3
```

fricas [A] time = 0.45, size = 15, normalized size = 0.71

$$\frac{2\left(5\,cx^4-3\,a\right)}{15\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(5*c*x^4 - 3*a)/x^(5/2)
```

giac [A] time = 0.16, size = 13, normalized size = 0.62

$$\frac{2}{3}\,cx^{\frac{3}{2}}-\frac{2\,a}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/x^(7/2),x, algorithm="giac")
[Out] 2/3*c*x^(3/2) - 2/5*a/x^(5/2)
```

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$-\frac{2\left(-5c\,x^4+3a\right)}{15x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)/x^(7/2),x)
[Out] -2/15*(-5*c*x^4+3*a)/x^(5/2)
```

maxima [A] time = 1.37, size = 13, normalized size = 0.62

$$\frac{2}{3}cx^{\frac{3}{2}}-\frac{2\,a}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/x^(7/2),x, algorithm="maxima")
[Out] 2/3*c*x^(3/2) - 2/5*a/x^(5/2)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.71

$$-\frac{6\,a-10\,c\,x^4}{15\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)/x^(7/2),x)
[Out] -(6*a - 10*c*x^4)/(15*x^(5/2))
```

sympy [A] time = 3.38, size = 19, normalized size = 0.90

$$-\frac{2a}{5x^{\frac{5}{2}}}+\frac{2cx^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)/x**(7/2),x)
[Out] -2*a/(5*x**(5/2)) + 2*c*x**(3/2)/3
```

3.724 $\int x^{5/2}\left(a+cx^4\right)^2\,dx$

Optimal. Leaf size=36

$$\frac{2}{7}a^2x^{7/2}+\frac{4}{15}acx^{15/2}+\frac{2}{23}c^2x^{23/2}$$

```
[Out] 2/7*a^2*x^(7/2)+4/15*a*c*x^(15/2)+2/23*c^2*x^(23/2)
```

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {270}

$$\frac{2}{7}a^2x^{7/2}+\frac{4}{15}acx^{15/2}+\frac{2}{23}c^2x^{23/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + c*x^4)^2,x]
```

```
[Out] (2*a^2*x^(7/2))/7 + (4*a*c*x^(15/2))/15 + (2*c^2*x^(23/2))/23
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+cx^4\right)^2\,dx &= \int\left(a^2x^{5/2}+2acx^{13/2}+c^2x^{21/2}\right)dx\\ &= \frac{2}{7}a^2x^{7/2}+\frac{4}{15}acx^{15/2}+\frac{2}{23}c^2x^{23/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.83

$$\frac{2x^{7/2}\left(345a^2+322acx^4+105c^2x^8\right)}{2415}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + c*x^4)^2,x]
```

```
[Out] (2*x^(7/2)*(345*a^2 + 322*a*c*x^4 + 105*c^2*x^8))/2415
```

fricas [A] time = 0.61, size = 29, normalized size = 0.81

$$\frac{2}{2415}\left(105\,c^2x^{11}+322\,acx^7+345\,a^2x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 2/2415*(105*c^2*x^11 + 322*a*c*x^7 + 345*a^2*x^3)*sqrt(x)
```

giac [A] time = 0.15, size = 24, normalized size = 0.67

$$\frac{2}{23}\,c^2x^{\frac{23}{2}}+\frac{4}{15}\,acx^{\frac{15}{2}}+\frac{2}{7}\,a^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+a)^2,x, algorithm="giac")
[Out] 2/23*c^2*x^(23/2) + 4/15*a*c*x^(15/2) + 2/7*a^2*x^(7/2)
```

maple [A] time = 0.01, size = 27, normalized size = 0.75

$$\frac{2\left(105c^2x^8+322acx^4+345a^2\right)x^{\frac{7}{2}}}{2415}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(c*x^4+a)^2,x)
[Out] 2/2415*x^(7/2)*(105*c^2*x^8+322*a*c*x^4+345*a^2)
```

maxima [A] time = 1.30, size = 24, normalized size = 0.67

$$\frac{2}{23}c^2x^{\frac{23}{2}}+\frac{4}{15}acx^{\frac{15}{2}}+\frac{2}{7}a^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+a)^2,x, algorithm="maxima")
[Out] 2/23*c^2*x^(23/2) + 4/15*a*c*x^(15/2) + 2/7*a^2*x^(7/2)
```

mupad [B] time = 1.02, size = 25, normalized size = 0.69

$$x^{7/2}\left(\frac{2\,a^2}{7}+\frac{4\,a\,c\,x^4}{15}+\frac{2\,c^2\,x^8}{23}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + c*x^4)^2,x)
[Out] x^(7/2)*((2*a^2)/7 + (2*c^2*x^8)/23 + (4*a*c*x^4)/15)
```

sympy [A] time = 24.25, size = 34, normalized size = 0.94

$$\frac{2a^2x^{\frac{7}{2}}}{7}+\frac{4acx^{\frac{15}{2}}}{15}+\frac{2c^2x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(c*x**4+a)**2,x)
[Out] 2*a**2*x**(7/2)/7 + 4*a*c*x**(15/2)/15 + 2*c**2*x**(23/2)/23
```

3.725 $\int x^{3/2}\left(a+cx^4\right)^2\,dx$

Optimal. Leaf size=36

$$\frac{2}{5}a^2x^{5/2}+\frac{4}{13}acx^{13/2}+\frac{2}{21}c^2x^{21/2}$$

```
[Out] 2/5*a^2*x^(5/2)+4/13*a*c*x^(13/2)+2/21*c^2*x^(21/2)
```

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {270}

$$\frac{2}{5}a^2x^{5/2}+\frac{4}{13}acx^{13/2}+\frac{2}{21}c^2x^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + c*x^4)^2,x]
```

```
[Out] (2*a^2*x^(5/2))/5 + (4*a*c*x^(13/2))/13 + (2*c^2*x^(21/2))/21
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+cx^4\right)^2\,dx &= \int\left(a^2x^{3/2}+2acx^{11/2}+c^2x^{19/2}\right)dx\\ &= \frac{2}{5}a^2x^{5/2}+\frac{4}{13}acx^{13/2}+\frac{2}{21}c^2x^{21/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.83

$$\frac{2x^{5/2}\left(273a^2+210acx^4+65c^2x^8\right)}{1365}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + c*x^4)^2,x]
```

```
[Out] (2*x^(5/2)*(273*a^2 + 210*a*c*x^4 + 65*c^2*x^8))/1365
```

fricas [A] time = 0.68, size = 29, normalized size = 0.81

$$\frac{2}{1365}\left(65\,c^2x^{10}+210\,acx^6+273\,a^2x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 2/1365*(65*c^2*x^10 + 210*a*c*x^6 + 273*a^2*x^2)*sqrt(x)
```

giac [A] time = 0.15, size = 24, normalized size = 0.67

$$\frac{2}{21}\,c^2x^{\frac{21}{2}}+\frac{4}{13}\,acx^{\frac{13}{2}}+\frac{2}{5}\,a^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+a)^2,x, algorithm="giac")
[Out] 2/21*c^2*x^(21/2) + 4/13*a*c*x^(13/2) + 2/5*a^2*x^(5/2)
```

maple [A] time = 0.00, size = 27, normalized size = 0.75

$$\frac{2\left(65c^2x^8+210acx^4+273a^2\right)x^{\frac{5}{2}}}{1365}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(c*x^4+a)^2,x)
[Out] 2/1365*x^(5/2)*(65*c^2*x^8+210*a*c*x^4+273*a^2)
```

maxima [A] time = 1.31, size = 24, normalized size = 0.67

$$\frac{2}{21}c^2x^{\frac{21}{2}}+\frac{4}{13}acx^{\frac{13}{2}}+\frac{2}{5}a^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+a)^2,x, algorithm="maxima")
[Out] 2/21*c^2*x^(21/2) + 4/13*a*c*x^(13/2) + 2/5*a^2*x^(5/2)
```

mupad [B] time = 0.04, size = 26, normalized size = 0.72

$$\frac{2\,x^{5/2}\left(273\,a^2+210\,a\,c\,x^4+65\,c^2\,x^8\right)}{1365}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + c*x^4)^2,x)
[Out] (2*x^(5/2)*(273*a^2 + 65*c^2*x^8 + 210*a*c*x^4))/1365
```

sympy [A] time = 17.04, size = 34, normalized size = 0.94

$$\frac{2a^2x^{\frac{5}{2}}}{5}+\frac{4acx^{\frac{13}{2}}}{13}+\frac{2c^2x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(c*x**4+a)**2,x)
[Out] 2*a**2*x**(5/2)/5 + 4*a*c*x**(13/2)/13 + 2*c**2*x**(21/2)/21
```

3.726 $\int \sqrt{x}\left(a+cx^4\right)^2\,dx$

Optimal. Leaf size=36

$$\frac{2}{3}a^2x^{3/2}+\frac{4}{11}acx^{11/2}+\frac{2}{19}c^2x^{19/2}$$

```
[Out] 2/3*a^2*x^(3/2)+4/11*a*c*x^(11/2)+2/19*c^2*x^(19/2)
```

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {270}

$$\frac{2}{3}a^2x^{3/2}+\frac{4}{11}acx^{11/2}+\frac{2}{19}c^2x^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + c*x^4)^2,x]
```

```
[Out] (2*a^2*x^(3/2))/3 + (4*a*c*x^(11/2))/11 + (2*c^2*x^(19/2))/19
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+cx^4\right)^2\,dx &= \int \left(a^2\sqrt{x}+2acx^{9/2}+c^2x^{17/2}\right)dx\\ &= \frac{2}{3}a^2x^{3/2}+\frac{4}{11}acx^{11/2}+\frac{2}{19}c^2x^{19/2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 0.83

$$\frac{2}{627}x^{3/2}\left(209a^2+114acx^4+33c^2x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + c*x^4)^2,x]
```

```
[Out] (2*x^(3/2)*(209*a^2 + 114*a*c*x^4 + 33*c^2*x^8))/627
```

fricas [A] time = 0.56, size = 27, normalized size = 0.75

$$\frac{2}{627}\left(33\,c^2x^9+114\,acx^5+209\,a^2x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/627*(33*c^2*x^9 + 114*a*c*x^5 + 209*a^2*x)*sqrt(x)
```

giac [A] time = 0.18, size = 24, normalized size = 0.67

$$\frac{2}{19}\,c^2x^{\frac{19}{2}}+\frac{4}{11}\,acx^{\frac{11}{2}}+\frac{2}{3}\,a^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2*x^(1/2),x, algorithm="giac")
[Out] 2/19*c^2*x^(19/2) + 4/11*a*c*x^(11/2) + 2/3*a^2*x^(3/2)
```

maple [A] time = 0.01, size = 27, normalized size = 0.75

$$\frac{2\left(33c^2x^8+114acx^4+209a^2\right)x^{\frac{3}{2}}}{627}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^2*x^(1/2),x)
[Out] 2/627*x^(3/2)*(33*c^2*x^8+114*a*c*x^4+209*a^2)
```

maxima [A] time = 1.34, size = 24, normalized size = 0.67

$$\frac{2}{19}c^2x^{\frac{19}{2}}+\frac{4}{11}acx^{\frac{11}{2}}+\frac{2}{3}a^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2*x^(1/2),x, algorithm="maxima")
[Out] 2/19*c^2*x^(19/2) + 4/11*a*c*x^(11/2) + 2/3*a^2*x^(3/2)
```

mupad [B] time = 1.06, size = 26, normalized size = 0.72

$$\frac{2\,x^{3/2}\left(209\,a^2+114\,a\,c\,x^4+33\,c^2\,x^8\right)}{627}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + c*x^4)^2,x)
[Out] (2*x^(3/2)*(209*a^2 + 33*c^2*x^8 + 114*a*c*x^4))/627
```

sympy [A] time = 2.96, size = 34, normalized size = 0.94

$$\frac{2a^2x^{\frac{3}{2}}}{3}+\frac{4acx^{\frac{11}{2}}}{11}+\frac{2c^2x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**2*x**(1/2),x)
[Out] 2*a**2*x**(3/2)/3 + 4*a*c*x**(11/2)/11 + 2*c**2*x**(19/2)/19
```

3.727 $\int \frac{\left(a+cx^4\right)^2}{\sqrt{x}}\,dx$

Optimal. Leaf size=34

$$2a^2\sqrt{x}+\frac{4}{9}acx^{9/2}+\frac{2}{17}c^2x^{17/2}$$

```
[Out] 4/9*a*c*x^(9/2)+2/17*c^2*x^(17/2)+2*a^2*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {270}

$$2a^2\sqrt{x}+\frac{4}{9}acx^{9/2}+\frac{2}{17}c^2x^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^2/Sqrt[x],x]
```

```
[Out] 2*a^2*Sqrt[x] + (4*a*c*x^(9/2))/9 + (2*c^2*x^(17/2))/17
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+cx^4\right)^2}{\sqrt{x}}\,dx &= \int \left(\frac{a^2}{\sqrt{x}}+2acx^{7/2}+c^2x^{15/2}\right)dx\\ &= 2a^2\sqrt{x}+\frac{4}{9}acx^{9/2}+\frac{2}{17}c^2x^{17/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.88

$$\frac{2}{153}\sqrt{x}\left(153a^2+34acx^4+9c^2x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^2/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(153*a^2 + 34*a*c*x^4 + 9*c^2*x^8))/153
```

fricas [A] time = 0.70, size = 26, normalized size = 0.76

$$\frac{2}{153}\left(9\,c^2x^8+34\,acx^4+153\,a^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/153*(9*c^2*x^8 + 34*a*c*x^4 + 153*a^2)*sqrt(x)
```

giac [A] time = 0.15, size = 24, normalized size = 0.71

$$\frac{2}{17}c^2x^{\frac{17}{2}}+\frac{4}{9}acx^{\frac{9}{2}}+2\,a^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/x^(1/2),x, algorithm="giac")
[Out] 2/17*c^2*x^(17/2) + 4/9*a*c*x^(9/2) + 2*a^2*sqrt(x)
```

maple [A] time = 0.01, size = 27, normalized size = 0.79

$$\frac{2\left(9c^2x^8+34ac\,x^4+153a^2\right)\sqrt{x}}{153}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^2/x^(1/2),x)
[Out] 2/153*x^(1/2)*(9*c^2*x^8+34*a*c*x^4+153*a^2)
```

maxima [A] time = 1.46, size = 24, normalized size = 0.71

$$\frac{2}{17}\,c^2x^{\frac{17}{2}}+\frac{4}{9}\,acx^{\frac{9}{2}}+2\,a^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/x^(1/2),x, algorithm="maxima")
[Out] 2/17*c^2*x^(17/2) + 4/9*a*c*x^(9/2) + 2*a^2*sqrt(x)
```

mupad [B] time = 0.04, size = 26, normalized size = 0.76

$$\frac{2\sqrt{x}\,\left(153\,a^2+34\,a\,c\,x^4+9\,c^2\,x^8\right)}{153}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^2/x^(1/2),x)
[Out] (2*x^(1/2)*(153*a^2 + 9*c^2*x^8 + 34*a*c*x^4))/153
```

sympy [A] time = 7.34, size = 32, normalized size = 0.94

$$2a^2\sqrt{x}+\frac{4acx^{\frac{9}{2}}}{9}+\frac{2c^2x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**2/x**(1/2),x)
[Out] 2*a**2*sqrt(x) + 4*a*c*x**(9/2)/9 + 2*c**2*x**(17/2)/17
```

3.728 $\int \frac{\left(a+cx^4\right)^2}{x^{3/2}}\,dx$

Optimal. Leaf size=34

$$-\frac{2a^2}{\sqrt{x}}+\frac{4}{7}acx^{7/2}+\frac{2}{15}c^2x^{15/2}$$

```
[Out] 4/7*a*c*x^(7/2)+2/15*c^2*x^(15/2)-2*a^2/x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {270}

$$-\frac{2a^2}{\sqrt{x}}+\frac{4}{7}acx^{7/2}+\frac{2}{15}c^2x^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^2/x^(3/2),x]
```

```
[Out] (-2*a^2)/Sqrt[x] + (4*a*c*x^(7/2))/7 + (2*c^2*x^(15/2))/15
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+cx^4\right)^2}{x^{3/2}}\,dx &= \int \left(\frac{a^2}{x^{3/2}}+2acx^{5/2}+c^2x^{13/2}\right)dx\\ &= -\frac{2a^2}{\sqrt{x}}+\frac{4}{7}acx^{7/2}+\frac{2}{15}c^2x^{15/2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 30, normalized size = 0.88

$$\frac{2\left(-105a^2+30acx^4+7c^2x^8\right)}{105\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^2/x^(3/2),x]
```

```
[Out] (2*(-105*a^2 + 30*a*c*x^4 + 7*c^2*x^8))/(105*Sqrt[x])
```

fricas [A] time = 0.80, size = 26, normalized size = 0.76

$$\frac{2\left(7\,c^2x^8+30\,acx^4-105\,a^2\right)}{105\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/105*(7*c^2*x^8 + 30*a*c*x^4 - 105*a^2)/sqrt(x)
```

giac [A] time = 0.17, size = 24, normalized size = 0.71

$$\frac{2}{15}c^2x^{\frac{15}{2}} + \frac{4}{7}acx^{\frac{7}{2}} - \frac{2a^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/x^(3/2),x, algorithm="giac")
[Out] 2/15*c^2*x^(15/2) + 4/7*a*c*x^(7/2) - 2*a^2/sqrt(x)
```

maple [A] time = 0.01, size = 27, normalized size = 0.79

$$-\frac{2\left(-7c^2x^8 - 30acx^4 + 105a^2\right)}{105\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^2/x^(3/2),x)
[Out] -2/105*(-7*c^2*x^8-30*a*c*x^4+105*a^2)/x^(1/2)
```

maxima [A] time = 1.35, size = 24, normalized size = 0.71

$$\frac{2}{15}c^2x^{\frac{15}{2}} + \frac{4}{7}acx^{\frac{7}{2}} - \frac{2a^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/x^(3/2),x, algorithm="maxima")
[Out] 2/15*c^2*x^(15/2) + 4/7*a*c*x^(7/2) - 2*a^2/sqrt(x)
```

mupad [B] time = 0.04, size = 25, normalized size = 0.74

$$\frac{-2a^2 + \frac{4acx^4}{7} + \frac{2c^2x^8}{15}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^2/x^(3/2),x)
[Out] ((2*c^2*x^8)/15 - 2*a^2 + (4*a*c*x^4)/7)/x^(1/2)
```

sympy [A] time = 5.89, size = 32, normalized size = 0.94

$$-\frac{2a^2}{\sqrt{x}} + \frac{4acx^{\frac{7}{2}}}{7} + \frac{2c^2x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**2/x**(3/2),x)
[Out] -2*a**2/sqrt(x) + 4*a*c*x**(7/2)/7 + 2*c**2*x**(15/2)/15
```

3.729 $\int \frac{\left(a+cx^4\right)^2}{x^{5/2}}\,dx$

Optimal. Leaf size=36

$$-\frac{2a^2}{3x^{3/2}}+\frac{4}{5}acx^{5/2}+\frac{2}{13}c^2x^{13/2}$$

```
[Out] -2/3*a^2/x^(3/2)+4/5*a*c*x^(5/2)+2/13*c^2*x^(13/2)
```

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {270}

$$-\frac{2a^2}{3x^{3/2}}+\frac{4}{5}acx^{5/2}+\frac{2}{13}c^2x^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^2/x^(5/2),x]
```

```
[Out] (-2*a^2)/(3*x^(3/2)) + (4*a*c*x^(5/2))/5 + (2*c^2*x^(13/2))/13
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+cx^4\right)^2}{x^{5/2}}\,dx &= \int \left(\frac{a^2}{x^{5/2}}+2acx^{3/2}+c^2x^{11/2}\right)dx\\ &= -\frac{2a^2}{3x^{3/2}}+\frac{4}{5}acx^{5/2}+\frac{2}{13}c^2x^{13/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.83

$$\frac{2\left(-65a^2+78acx^4+15c^2x^8\right)}{195x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^2/x^(5/2),x]
```

```
[Out] (2*(-65*a^2 + 78*a*c*x^4 + 15*c^2*x^8))/(195*x^(3/2))
```

fricas [A] time = 0.64, size = 26, normalized size = 0.72

$$\frac{2\left(15\,c^2x^8+78\,acx^4-65\,a^2\right)}{195\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/195*(15*c^2*x^8 + 78*a*c*x^4 - 65*a^2)/x^(3/2)
```

giac [A] time = 0.15, size = 24, normalized size = 0.67

$$\frac{2}{13}c^2x^{\frac{13}{2}} + \frac{4}{5}acx^{\frac{5}{2}} - \frac{2a^2}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/x^(5/2),x, algorithm="giac")
[Out] 2/13*c^2*x^(13/2) + 4/5*a*c*x^(5/2) - 2/3*a^2/x^(3/2)
```

maple [A] time = 0.00, size = 27, normalized size = 0.75

$$-\frac{2\left(-15c^2x^8 - 78acx^4 + 65a^2\right)}{195x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^2/x^(5/2),x)
[Out] -2/195*(-15*c^2*x^8-78*a*c*x^4+65*a^2)/x^(3/2)
```

maxima [A] time = 1.37, size = 24, normalized size = 0.67

$$\frac{2}{13}c^2x^{\frac{13}{2}} + \frac{4}{5}acx^{\frac{5}{2}} - \frac{2a^2}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/x^(5/2),x, algorithm="maxima")
[Out] 2/13*c^2*x^(13/2) + 4/5*a*c*x^(5/2) - 2/3*a^2/x^(3/2)
```

mupad [B] time = 0.04, size = 26, normalized size = 0.72

$$\frac{-130a^2 + 156acx^4 + 30c^2x^8}{195x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^2/x^(5/2),x)
[Out] (30*c^2*x^8 - 130*a^2 + 156*a*c*x^4)/(195*x^(3/2))
```

sympy [A] time = 7.43, size = 34, normalized size = 0.94

$$-\frac{2a^2}{3x^{\frac{3}{2}}} + \frac{4acx^{\frac{5}{2}}}{5} + \frac{2c^2x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**2/x**(5/2),x)
[Out] -2*a**2/(3*x**(3/2)) + 4*a*c*x**(5/2)/5 + 2*c**2*x**(13/2)/13
```

3.730 $\int \frac{\left(a+cx^4\right)^2}{x^{7/2}}\,dx$

Optimal. Leaf size=36

$$-\frac{2a^2}{5x^{5/2}}+\frac{4}{3}acx^{3/2}+\frac{2}{11}c^2x^{11/2}$$

```
[Out] -2/5*a^2/x^(5/2)+4/3*a*c*x^(3/2)+2/11*c^2*x^(11/2)
```

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {270}

$$-\frac{2a^2}{5x^{5/2}}+\frac{4}{3}acx^{3/2}+\frac{2}{11}c^2x^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^2/x^(7/2),x]
```

```
[Out] (-2*a^2)/(5*x^(5/2)) + (4*a*c*x^(3/2))/3 + (2*c^2*x^(11/2))/11
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+cx^4\right)^2}{x^{7/2}}\,dx &= \int \left(\frac{a^2}{x^{7/2}}+2ac\sqrt{x}+c^2x^{9/2}\right)dx\\ &= -\frac{2a^2}{5x^{5/2}}+\frac{4}{3}acx^{3/2}+\frac{2}{11}c^2x^{11/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.83

$$\frac{2\left(-33a^2+110acx^4+15c^2x^8\right)}{165x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^2/x^(7/2),x]
```

```
[Out] (2*(-33*a^2 + 110*a*c*x^4 + 15*c^2*x^8))/(165*x^(5/2))
```

fricas [A] time = 0.63, size = 26, normalized size = 0.72

$$\frac{2\left(15\,c^2x^8+110\,acx^4-33\,a^2\right)}{165\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/165*(15*c^2*x^8 + 110*a*c*x^4 - 33*a^2)/x^(5/2)
```

giac [A] time = 0.19, size = 24, normalized size = 0.67

$$\frac{2}{11}c^2x^{\frac{11}{2}} + \frac{4}{3}acx^{\frac{3}{2}} - \frac{2a^2}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/x^(7/2),x, algorithm="giac")
[Out] 2/11*c^2*x^(11/2) + 4/3*a*c*x^(3/2) - 2/5*a^2/x^(5/2)
```

maple [A] time = 0.01, size = 27, normalized size = 0.75

$$-\frac{2\left(-15c^2x^8 - 110acx^4 + 33a^2\right)}{165x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^2/x^(7/2),x)
[Out] -2/165*(-15*c^2*x^8-110*a*c*x^4+33*a^2)/x^(5/2)
```

maxima [A] time = 1.35, size = 24, normalized size = 0.67

$$\frac{2}{11}c^2x^{\frac{11}{2}} + \frac{4}{3}acx^{\frac{3}{2}} - \frac{2a^2}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/x^(7/2),x, algorithm="maxima")
[Out] 2/11*c^2*x^(11/2) + 4/3*a*c*x^(3/2) - 2/5*a^2/x^(5/2)
```

mupad [B] time = 0.04, size = 26, normalized size = 0.72

$$\frac{-66\,a^2 + 220\,a\,c\,x^4 + 30\,c^2\,x^8}{165\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^2/x^(7/2),x)
[Out] (30*c^2*x^8 - 66*a^2 + 220*a*c*x^4)/(165*x^(5/2))
```

sympy [A] time = 8.96, size = 34, normalized size = 0.94

$$-\frac{2a^2}{5x^{\frac{5}{2}}} + \frac{4acx^{\frac{3}{2}}}{3} + \frac{2c^2x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**2/x**(7/2),x)
[Out] -2*a**2/(5*x**(5/2)) + 4*a*c*x**(3/2)/3 + 2*c**2*x**(11/2)/11
```

3.731 $\int x^{5/2}\left(a+cx^4\right)^3\,dx$

Optimal. Leaf size=51

$$\frac{2}{7}a^3x^{7/2}+\frac{2}{5}a^2cx^{15/2}+\frac{6}{23}ac^2x^{23/2}+\frac{2}{31}c^3x^{31/2}$$

```
[Out] 2/7*a^3*x^(7/2)+2/5*a^2*c*x^(15/2)+6/23*a*c^2*x^(23/2)+2/31*c^3*x^(31/2)
```

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {270}

$$\frac{2}{5}a^2cx^{15/2}+\frac{2}{7}a^3x^{7/2}+\frac{6}{23}ac^2x^{23/2}+\frac{2}{31}c^3x^{31/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + c*x^4)^3,x]
```

```
[Out] (2*a^3*x^(7/2))/7 + (2*a^2*c*x^(15/2))/5 + (6*a*c^2*x^(23/2))/23 + (2*c^3*x
^(31/2))/31
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+cx^4\right)^3\,dx &= \int\left(a^3x^{5/2}+3a^2cx^{13/2}+3ac^2x^{21/2}+c^3x^{29/2}\right)dx\\ &= \frac{2}{7}a^3x^{7/2}+\frac{2}{5}a^2cx^{15/2}+\frac{6}{23}ac^2x^{23/2}+\frac{2}{31}c^3x^{31/2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 1.00

$$\frac{2}{7}a^3x^{7/2}+\frac{2}{5}a^2cx^{15/2}+\frac{6}{23}ac^2x^{23/2}+\frac{2}{31}c^3x^{31/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + c*x^4)^3,x]
```

```
[Out] (2*a^3*x^(7/2))/7 + (2*a^2*c*x^(15/2))/5 + (6*a*c^2*x^(23/2))/23 + (2*c^3*x
^(31/2))/31
```

fricas [A] time = 0.65, size = 40, normalized size = 0.78

$$\frac{2}{24955}\left(805\,c^3x^{15}+3255\,ac^2x^{11}+4991\,a^2cx^7+3565\,a^3x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 2/24955*(805*c^3*x^15 + 3255*a*c^2*x^11 + 4991*a^2*c*x^7 + 3565*a^3*x^3)*sq
rt(x)
```

giac [A] time = 0.15, size = 35, normalized size = 0.69

$$\frac{2}{31}c^3x^{\frac{31}{2}} + \frac{6}{23}ac^2x^{\frac{23}{2}} + \frac{2}{5}a^2cx^{\frac{15}{2}} + \frac{2}{7}a^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 2/31*c^3*x^(31/2) + 6/23*a*c^2*x^(23/2) + 2/5*a^2*c*x^(15/2) + 2/7*a^3*x^(7
/2)
```

maple [A] time = 0.01, size = 38, normalized size = 0.75

$$\frac{2\left(805c^3x^{12} + 3255a\,c^2x^8 + 4991a^2c\,x^4 + 3565a^3\right)x^{\frac{7}{2}}}{24955}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(c*x^4+a)^3,x)
```

```
[Out] 2/24955*x^(7/2)*(805*c^3*x^12+3255*a*c^2*x^8+4991*a^2*c*x^4+3565*a^3)
```

maxima [A] time = 1.35, size = 35, normalized size = 0.69

$$\frac{2}{31}c^3x^{\frac{31}{2}} + \frac{6}{23}ac^2x^{\frac{23}{2}} + \frac{2}{5}a^2cx^{\frac{15}{2}} + \frac{2}{7}a^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 2/31*c^3*x^(31/2) + 6/23*a*c^2*x^(23/2) + 2/5*a^2*c*x^(15/2) + 2/7*a^3*x^(7
/2)
```

mupad [B] time = 0.04, size = 35, normalized size = 0.69

$$\frac{2\,a^3\,x^{7/2}}{7} + \frac{2\,c^3\,x^{31/2}}{31} + \frac{2\,a^2\,c\,x^{15/2}}{5} + \frac{6\,a\,c^2\,x^{23/2}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + c*x^4)^3,x)
```

```
[Out] (2*a^3*x^(7/2))/7 + (2*c^3*x^(31/2))/31 + (2*a^2*c*x^(15/2))/5 + (6*a*c^2*x
^(23/2))/23
```

sympy [A] time = 70.18, size = 49, normalized size = 0.96

$$\frac{2a^3x^{\frac{7}{2}}}{7} + \frac{2a^2cx^{\frac{15}{2}}}{5} + \frac{6ac^2x^{\frac{23}{2}}}{23} + \frac{2c^3x^{\frac{31}{2}}}{31}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(c*x**4+a)**3,x)
```

```
[Out] 2*a**3*x**(7/2)/7 + 2*a**2*c*x**(15/2)/5 + 6*a*c**2*x**(23/2)/23 + 2*c**3*x
**(31/2)/31
```

3.732 $\int x^{3/2}\left(a+cx^4\right)^3\,dx$

Optimal. Leaf size=51

$$\frac{2}{5}a^3x^{5/2}+\frac{6}{13}a^2cx^{13/2}+\frac{2}{7}ac^2x^{21/2}+\frac{2}{29}c^3x^{29/2}$$

```
[Out] 2/5*a^3*x^(5/2)+6/13*a^2*c*x^(13/2)+2/7*a*c^2*x^(21/2)+2/29*c^3*x^(29/2)
```

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {270}

$$\frac{6}{13}a^2cx^{13/2}+\frac{2}{5}a^3x^{5/2}+\frac{2}{7}ac^2x^{21/2}+\frac{2}{29}c^3x^{29/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + c*x^4)^3,x]
```

```
[Out] (2*a^3*x^(5/2))/5 + (6*a^2*c*x^(13/2))/13 + (2*a*c^2*x^(21/2))/7 + (2*c^3*x
^(29/2))/29
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+cx^4\right)^3\,dx &= \int\left(a^3x^{3/2}+3a^2cx^{11/2}+3ac^2x^{19/2}+c^3x^{27/2}\right)dx\\ &= \frac{2}{5}a^3x^{5/2}+\frac{6}{13}a^2cx^{13/2}+\frac{2}{7}ac^2x^{21/2}+\frac{2}{29}c^3x^{29/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 1.00

$$\frac{2}{5}a^3x^{5/2}+\frac{6}{13}a^2cx^{13/2}+\frac{2}{7}ac^2x^{21/2}+\frac{2}{29}c^3x^{29/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + c*x^4)^3,x]
```

```
[Out] (2*a^3*x^(5/2))/5 + (6*a^2*c*x^(13/2))/13 + (2*a*c^2*x^(21/2))/7 + (2*c^3*x
^(29/2))/29
```

fricas [A] time = 0.55, size = 40, normalized size = 0.78

$$\frac{2}{13195}\left(455\,c^3x^{14}+1885\,ac^2x^{10}+3045\,a^2cx^6+2639\,a^3x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 2/13195*(455*c^3*x^14 + 1885*a*c^2*x^10 + 3045*a^2*c*x^6 + 2639*a^3*x^2)*sq
rt(x)
```

giac [A] time = 0.15, size = 35, normalized size = 0.69

$$\frac{2}{29}c^3x^{\frac{29}{2}} + \frac{2}{7}ac^2x^{\frac{21}{2}} + \frac{6}{13}a^2cx^{\frac{13}{2}} + \frac{2}{5}a^3x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 2/29*c^3*x^(29/2) + 2/7*a*c^2*x^(21/2) + 6/13*a^2*c*x^(13/2) + 2/5*a^3*x^(5
/2)
```

maple [A] time = 0.01, size = 38, normalized size = 0.75

$$\frac{2\left(455c^3x^{12} + 1885a\,c^2x^8 + 3045a^2c\,x^4 + 2639a^3\right)x^{\frac{5}{2}}}{13195}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(c*x^4+a)^3,x)
```

```
[Out] 2/13195*x^(5/2)*(455*c^3*x^12+1885*a*c^2*x^8+3045*a^2*c*x^4+2639*a^3)
```

maxima [A] time = 1.38, size = 35, normalized size = 0.69

$$\frac{2}{29}c^3x^{\frac{29}{2}} + \frac{2}{7}ac^2x^{\frac{21}{2}} + \frac{6}{13}a^2cx^{\frac{13}{2}} + \frac{2}{5}a^3x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 2/29*c^3*x^(29/2) + 2/7*a*c^2*x^(21/2) + 6/13*a^2*c*x^(13/2) + 2/5*a^3*x^(5
/2)
```

mupad [B] time = 0.04, size = 35, normalized size = 0.69

$$\frac{2\,a^3\,x^{5/2}}{5} + \frac{2\,c^3\,x^{29/2}}{29} + \frac{6\,a^2\,c\,x^{13/2}}{13} + \frac{2\,a\,c^2\,x^{21/2}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + c*x^4)^3,x)
```

```
[Out] (2*a^3*x^(5/2))/5 + (2*c^3*x^(29/2))/29 + (6*a^2*c*x^(13/2))/13 + (2*a*c^2*
x^(21/2))/7
```

sympy [A] time = 48.12, size = 49, normalized size = 0.96

$$\frac{2a^3x^{\frac{5}{2}}}{5} + \frac{6a^2cx^{\frac{13}{2}}}{13} + \frac{2ac^2x^{\frac{21}{2}}}{7} + \frac{2c^3x^{\frac{29}{2}}}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(c*x**4+a)**3,x)
```

```
[Out] 2*a**3*x**(5/2)/5 + 6*a**2*c*x**(13/2)/13 + 2*a*c**2*x**(21/2)/7 + 2*c**3*x
**(29/2)/29
```

3.733 $\int \sqrt{x}\left(a+cx^4\right)^3\,dx$

Optimal. Leaf size=51

$$\frac{2}{3}a^3x^{3/2}+\frac{6}{11}a^2cx^{11/2}+\frac{6}{19}ac^2x^{19/2}+\frac{2}{27}c^3x^{27/2}$$

```
[Out] 2/3*a^3*x^(3/2)+6/11*a^2*c*x^(11/2)+6/19*a*c^2*x^(19/2)+2/27*c^3*x^(27/2)
```

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {270}

$$\frac{6}{11}a^2cx^{11/2}+\frac{2}{3}a^3x^{3/2}+\frac{6}{19}ac^2x^{19/2}+\frac{2}{27}c^3x^{27/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + c*x^4)^3,x]
```

```
[Out] (2*a^3*x^(3/2))/3 + (6*a^2*c*x^(11/2))/11 + (6*a*c^2*x^(19/2))/19 + (2*c^3*
x^(27/2))/27
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+cx^4\right)^3\,dx &= \int \left(a^3\sqrt{x}+3a^2cx^{9/2}+3ac^2x^{17/2}+c^3x^{25/2}\right)dx\\ &= \frac{2}{3}a^3x^{3/2}+\frac{6}{11}a^2cx^{11/2}+\frac{6}{19}ac^2x^{19/2}+\frac{2}{27}c^3x^{27/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.80

$$\frac{2x^{3/2}\left(1881a^3+1539a^2cx^4+891ac^2x^8+209c^3x^{12}\right)}{5643}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + c*x^4)^3,x]
```

```
[Out] (2*x^(3/2)*(1881*a^3 + 1539*a^2*c*x^4 + 891*a*c^2*x^8 + 209*c^3*x^12))/5643
```

fricas [A] time = 0.69, size = 38, normalized size = 0.75

$$\frac{2}{5643}\left(209\,c^3x^{13}+891\,ac^2x^9+1539\,a^2cx^5+1881\,a^3x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^3*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5643*(209*c^3*x^13 + 891*a*c^2*x^9 + 1539*a^2*c*x^5 + 1881*a^3*x)*sqrt(x)
```

giac [A] time = 0.17, size = 35, normalized size = 0.69

$$\frac{2}{27}c^3x^{\frac{27}{2}}+\frac{6}{19}ac^2x^{\frac{19}{2}}+\frac{6}{11}a^2cx^{\frac{11}{2}}+\frac{2}{3}a^3x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^3*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/27*c^3*x^(27/2) + 6/19*a*c^2*x^(19/2) + 6/11*a^2*c*x^(11/2) + 2/3*a^3*x^(
3/2)
```

maple [A] time = 0.01, size = 38, normalized size = 0.75

$$\frac{2\left(209c^3x^{12}+891a\,c^2x^8+1539a^2c\,x^4+1881a^3\right)x^{\frac{3}{2}}}{5643}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^3*x^(1/2),x)
```

```
[Out] 2/5643*x^(3/2)*(209*c^3*x^12+891*a*c^2*x^8+1539*a^2*c*x^4+1881*a^3)
```

maxima [A] time = 1.31, size = 35, normalized size = 0.69

$$\frac{2}{27}c^3x^{\frac{27}{2}}+\frac{6}{19}ac^2x^{\frac{19}{2}}+\frac{6}{11}a^2cx^{\frac{11}{2}}+\frac{2}{3}a^3x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^3*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/27*c^3*x^(27/2) + 6/19*a*c^2*x^(19/2) + 6/11*a^2*c*x^(11/2) + 2/3*a^3*x^(
3/2)
```

mupad [B] time = 0.04, size = 35, normalized size = 0.69

$$\frac{2\,a^3\,x^{3/2}}{3}+\frac{2\,c^3\,x^{27/2}}{27}+\frac{6\,a^2\,c\,x^{11/2}}{11}+\frac{6\,a\,c^2\,x^{19/2}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + c*x^4)^3,x)
```

```
[Out] (2*a^3*x^(3/2))/3 + (2*c^3*x^(27/2))/27 + (6*a^2*c*x^(11/2))/11 + (6*a*c^2*
x^(19/2))/19
```

sympy [A] time = 5.95, size = 49, normalized size = 0.96

$$\frac{2a^3x^{\frac{3}{2}}}{3}+\frac{6a^2cx^{\frac{11}{2}}}{11}+\frac{6ac^2x^{\frac{19}{2}}}{19}+\frac{2c^3x^{\frac{27}{2}}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**3*x**(1/2),x)
```

```
[Out] 2*a**3*x**(3/2)/3 + 6*a**2*c*x**(11/2)/11 + 6*a*c**2*x**(19/2)/19 + 2*c**3*
x**(27/2)/27
```

3.734 $\int \frac{\left(a+cx^4\right)^3}{\sqrt{x}}\,dx$

Optimal. Leaf size=49

$$2a^3\sqrt{x}+\frac{2}{3}a^2cx^{9/2}+\frac{6}{17}ac^2x^{17/2}+\frac{2}{25}c^3x^{25/2}$$

```
[Out] 2/3*a^2*c*x^(9/2)+6/17*a*c^2*x^(17/2)+2/25*c^3*x^(25/2)+2*a^3*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {270}

$$\frac{2}{3}a^2cx^{9/2}+2a^3\sqrt{x}+\frac{6}{17}ac^2x^{17/2}+\frac{2}{25}c^3x^{25/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^3/Sqrt[x],x]
```

```
[Out] 2*a^3*Sqrt[x] + (2*a^2*c*x^(9/2))/3 + (6*a*c^2*x^(17/2))/17 + (2*c^3*x^(25/
2))/25
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+cx^4\right)^3}{\sqrt{x}}\,dx &= \int\left(\frac{a^3}{\sqrt{x}}+3a^2cx^{7/2}+3ac^2x^{15/2}+c^3x^{23/2}\right)dx\\ &= 2a^3\sqrt{x}+\frac{2}{3}a^2cx^{9/2}+\frac{6}{17}ac^2x^{17/2}+\frac{2}{25}c^3x^{25/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.84

$$\frac{2\sqrt{x}\left(1275a^3+425a^2cx^4+225ac^2x^8+51c^3x^{12}\right)}{1275}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^3/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(1275*a^3 + 425*a^2*c*x^4 + 225*a*c^2*x^8 + 51*c^3*x^12))/1275
```

fricas [A] time = 0.56, size = 37, normalized size = 0.76

$$\frac{2}{1275}\left(51\,c^3x^{12}+225\,ac^2x^8+425\,a^2cx^4+1275\,a^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/1275*(51*c^3*x^12 + 225*a*c^2*x^8 + 425*a^2*c*x^4 + 1275*a^3)*sqrt(x)
```

giac [A] time = 0.15, size = 35, normalized size = 0.71

$$\frac{2}{25}c^3x^{\frac{25}{2}} + \frac{6}{17}ac^2x^{\frac{17}{2}} + \frac{2}{3}a^2cx^{\frac{9}{2}} + 2a^3\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/25*c^3*x^(25/2) + 6/17*a*c^2*x^(17/2) + 2/3*a^2*c*x^(9/2) + 2*a^3*sqrt(x)
```

maple [A] time = 0.01, size = 38, normalized size = 0.78

$$\frac{2\left(51c^3x^{12} + 225a\,c^2x^8 + 425a^2c\,x^4 + 1275a^3\right)\sqrt{x}}{1275}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^3/x^(1/2),x)
```

```
[Out] 2/1275*x^(1/2)*(51*c^3*x^12+225*a*c^2*x^8+425*a^2*c*x^4+1275*a^3)
```

maxima [A] time = 1.35, size = 35, normalized size = 0.71

$$\frac{2}{25}c^3x^{\frac{25}{2}} + \frac{6}{17}ac^2x^{\frac{17}{2}} + \frac{2}{3}a^2cx^{\frac{9}{2}} + 2a^3\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/25*c^3*x^(25/2) + 6/17*a*c^2*x^(17/2) + 2/3*a^2*c*x^(9/2) + 2*a^3*sqrt(x)
```

mupad [B] time = 0.04, size = 35, normalized size = 0.71

$$2\,a^3\,\sqrt{x} + \frac{2\,c^3\,x^{25/2}}{25} + \frac{2\,a^2\,c\,x^{9/2}}{3} + \frac{6\,a\,c^2\,x^{17/2}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^3/x^(1/2),x)
```

```
[Out] 2*a^3*x^(1/2) + (2*c^3*x^(25/2))/25 + (2*a^2*c*x^(9/2))/3 + (6*a*c^2*x^(17/
2))/17
```

sympy [A] time = 24.75, size = 48, normalized size = 0.98

$$2a^3\sqrt{x} + \frac{2a^2cx^{\frac{9}{2}}}{3} + \frac{6ac^2x^{\frac{17}{2}}}{17} + \frac{2c^3x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**3/x**(1/2),x)
```

```
[Out] 2*a**3*sqrt(x) + 2*a**2*c*x**(9/2)/3 + 6*a*c**2*x**(17/2)/17 + 2*c**3*x**(2
5/2)/25
```

3.735 $\int \frac{\left(a+cx^4\right)^3}{x^{3/2}}\,dx$

Optimal. Leaf size=49

$$-\frac{2a^3}{\sqrt{x}}+\frac{6}{7}a^2cx^{7/2}+\frac{2}{5}ac^2x^{15/2}+\frac{2}{23}c^3x^{23/2}$$

```
[Out] 6/7*a^2*c*x^(7/2)+2/5*a*c^2*x^(15/2)+2/23*c^3*x^(23/2)-2*a^3/x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {270}

$$\frac{6}{7}a^2cx^{7/2}-\frac{2a^3}{\sqrt{x}}+\frac{2}{5}ac^2x^{15/2}+\frac{2}{23}c^3x^{23/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^3/x^(3/2),x]
```

```
[Out] (-2*a^3)/Sqrt[x] + (6*a^2*c*x^(7/2))/7 + (2*a*c^2*x^(15/2))/5 + (2*c^3*x^(2
3/2))/23
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+cx^4\right)^3}{x^{3/2}}\,dx &= \int \left(\frac{a^3}{x^{3/2}}+3a^2cx^{5/2}+3ac^2x^{13/2}+c^3x^{21/2}\right)dx\\ &= -\frac{2a^3}{\sqrt{x}}+\frac{6}{7}a^2cx^{7/2}+\frac{2}{5}ac^2x^{15/2}+\frac{2}{23}c^3x^{23/2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 0.84

$$\frac{2\left(-805a^3+345a^2cx^4+161ac^2x^8+35c^3x^{12}\right)}{805\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^3/x^(3/2),x]
```

```
[Out] (2*(-805*a^3 + 345*a^2*c*x^4 + 161*a*c^2*x^8 + 35*c^3*x^12))/(805*Sqrt[x])
```

fricas [A] time = 0.62, size = 37, normalized size = 0.76

$$\frac{2\left(35\,c^3x^{12}+161\,ac^2x^8+345\,a^2cx^4-805\,a^3\right)}{805\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^3/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/805*(35*c^3*x^12 + 161*a*c^2*x^8 + 345*a^2*c*x^4 - 805*a^3)/sqrt(x)
```

giac [A] time = 0.21, size = 35, normalized size = 0.71

$$\frac{2}{23}c^3x^{\frac{23}{2}} + \frac{2}{5}ac^2x^{\frac{15}{2}} + \frac{6}{7}a^2cx^{\frac{7}{2}} - \frac{2a^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^3/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/23*c^3*x^(23/2) + 2/5*a*c^2*x^(15/2) + 6/7*a^2*c*x^(7/2) - 2*a^3/sqrt(x)
```

maple [A] time = 0.01, size = 38, normalized size = 0.78

$$-\frac{2\left(-35c^3x^{12} - 161ac^2x^8 - 345a^2cx^4 + 805a^3\right)}{805\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^3/x^(3/2),x)
```

```
[Out] -2/805*(-35*c^3*x^12-161*a*c^2*x^8-345*a^2*c*x^4+805*a^3)/x^(1/2)
```

maxima [A] time = 1.39, size = 35, normalized size = 0.71

$$\frac{2}{23}c^3x^{\frac{23}{2}} + \frac{2}{5}ac^2x^{\frac{15}{2}} + \frac{6}{7}a^2cx^{\frac{7}{2}} - \frac{2a^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^3/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/23*c^3*x^(23/2) + 2/5*a*c^2*x^(15/2) + 6/7*a^2*c*x^(7/2) - 2*a^3/sqrt(x)
```

mupad [B] time = 0.05, size = 35, normalized size = 0.71

$$\frac{2c^3x^{23/2}}{23} - \frac{2a^3}{\sqrt{x}} + \frac{6a^2cx^{7/2}}{7} + \frac{2ac^2x^{15/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^3/x^(3/2),x)
```

```
[Out] (2*c^3*x^(23/2))/23 - (2*a^3)/x^(1/2) + (6*a^2*c*x^(7/2))/7 + (2*a*c^2*x^(1
5/2))/5
```

sympy [A] time = 27.82, size = 48, normalized size = 0.98

$$-\frac{2a^3}{\sqrt{x}} + \frac{6a^2cx^{\frac{7}{2}}}{7} + \frac{2ac^2x^{\frac{15}{2}}}{5} + \frac{2c^3x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**3/x**(3/2),x)
```

```
[Out] -2*a**3/sqrt(x) + 6*a**2*c*x**(7/2)/7 + 2*a*c**2*x**(15/2)/5 + 2*c**3*x**(2
3/2)/23
```

3.736 $\int \frac{\left(a+cx^4\right)^3}{x^{5/2}}\,dx$

Optimal. Leaf size=51

$$-\frac{2a^3}{3x^{3/2}}+\frac{6}{5}a^2cx^{5/2}+\frac{6}{13}ac^2x^{13/2}+\frac{2}{21}c^3x^{21/2}$$

```
[Out] -2/3*a^3/x^(3/2)+6/5*a^2*c*x^(5/2)+6/13*a*c^2*x^(13/2)+2/21*c^3*x^(21/2)
```

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {270}

$$\frac{6}{5}a^2cx^{5/2}-\frac{2a^3}{3x^{3/2}}+\frac{6}{13}ac^2x^{13/2}+\frac{2}{21}c^3x^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^3/x^(5/2),x]
```

```
[Out] (-2*a^3)/(3*x^(3/2)) + (6*a^2*c*x^(5/2))/5 + (6*a*c^2*x^(13/2))/13 + (2*c^3
*x^(21/2))/21
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+cx^4\right)^3}{x^{5/2}}\,dx &= \int \left(\frac{a^3}{x^{5/2}}+3a^2cx^{3/2}+3ac^2x^{11/2}+c^3x^{19/2}\right)dx\\ &= -\frac{2a^3}{3x^{3/2}}+\frac{6}{5}a^2cx^{5/2}+\frac{6}{13}ac^2x^{13/2}+\frac{2}{21}c^3x^{21/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.80

$$\frac{2\left(-455a^3+819a^2cx^4+315ac^2x^8+65c^3x^{12}\right)}{1365x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^3/x^(5/2),x]
```

```
[Out] (2*(-455*a^3 + 819*a^2*c*x^4 + 315*a*c^2*x^8 + 65*c^3*x^12))/(1365*x^(3/2))
```

fricas [A] time = 0.61, size = 37, normalized size = 0.73

$$\frac{2\left(65\,c^3x^{12}+315\,ac^2x^8+819\,a^2cx^4-455\,a^3\right)}{1365\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^3/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/1365*(65*c^3*x^12 + 315*a*c^2*x^8 + 819*a^2*c*x^4 - 455*a^3)/x^(3/2)
```

giac [A] time = 0.21, size = 35, normalized size = 0.69

$$\frac{2}{21}c^3x^{\frac{21}{2}} + \frac{6}{13}ac^2x^{\frac{13}{2}} + \frac{6}{5}a^2cx^{\frac{5}{2}} - \frac{2a^3}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^3/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/21*c^3*x^(21/2) + 6/13*a*c^2*x^(13/2) + 6/5*a^2*c*x^(5/2) - 2/3*a^3/x^(3/
2)
```

maple [A] time = 0.01, size = 38, normalized size = 0.75

$$-\frac{2\left(-65c^3x^{12} - 315a\,c^2x^8 - 819a^2c\,x^4 + 455a^3\right)}{1365x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^3/x^(5/2),x)
```

```
[Out] -2/1365*(-65*c^3*x^12-315*a*c^2*x^8-819*a^2*c*x^4+455*a^3)/x^(3/2)
```

maxima [A] time = 1.16, size = 35, normalized size = 0.69

$$\frac{2}{21}c^3x^{\frac{21}{2}} + \frac{6}{13}ac^2x^{\frac{13}{2}} + \frac{6}{5}a^2cx^{\frac{5}{2}} - \frac{2a^3}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^3/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/21*c^3*x^(21/2) + 6/13*a*c^2*x^(13/2) + 6/5*a^2*c*x^(5/2) - 2/3*a^3/x^(3/
2)
```

mupad [B] time = 0.04, size = 35, normalized size = 0.69

$$\frac{2\,c^3\,x^{21/2}}{21} - \frac{2\,a^3}{3\,x^{3/2}} + \frac{6\,a^2\,c\,x^{5/2}}{5} + \frac{6\,a\,c^2\,x^{13/2}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^3/x^(5/2),x)
```

```
[Out] (2*c^3*x^(21/2))/21 - (2*a^3)/(3*x^(3/2)) + (6*a^2*c*x^(5/2))/5 + (6*a*c^2*
x^(13/2))/13
```

sympy [A] time = 37.07, size = 49, normalized size = 0.96

$$-\frac{2a^3}{3x^{\frac{3}{2}}} + \frac{6a^2cx^{\frac{5}{2}}}{5} + \frac{6ac^2x^{\frac{13}{2}}}{13} + \frac{2c^3x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**3/x**(5/2),x)
```

```
[Out] -2*a**3/(3*x**(3/2)) + 6*a**2*c*x**(5/2)/5 + 6*a*c**2*x**(13/2)/13 + 2*c**3
*x**(21/2)/21
```

3.737 $\int \frac{\left(a+cx^4\right)^3}{x^{7/2}}\,dx$

Optimal. Leaf size=49

$$-\frac{2a^3}{5x^{5/2}}+2a^2cx^{3/2}+\frac{6}{11}ac^2x^{11/2}+\frac{2}{19}c^3x^{19/2}$$

```
[Out] -2/5*a^3/x^(5/2)+2*a^2*c*x^(3/2)+6/11*a*c^2*x^(11/2)+2/19*c^3*x^(19/2)
```

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {270}

$$2a^2cx^{3/2}-\frac{2a^3}{5x^{5/2}}+\frac{6}{11}ac^2x^{11/2}+\frac{2}{19}c^3x^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^3/x^(7/2),x]
```

```
[Out] (-2*a^3)/(5*x^(5/2)) + 2*a^2*c*x^(3/2) + (6*a*c^2*x^(11/2))/11 + (2*c^3*x^(
19/2))/19
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+cx^4\right)^3}{x^{7/2}}\,dx &= \int \left(\frac{a^3}{x^{7/2}}+3a^2c\sqrt{x}+3ac^2x^{9/2}+c^3x^{17/2}\right)dx\\ &= -\frac{2a^3}{5x^{5/2}}+2a^2cx^{3/2}+\frac{6}{11}ac^2x^{11/2}+\frac{2}{19}c^3x^{19/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.84

$$\frac{2\left(-209a^3+1045a^2cx^4+285ac^2x^8+55c^3x^{12}\right)}{1045x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^3/x^(7/2),x]
```

```
[Out] (2*(-209*a^3 + 1045*a^2*c*x^4 + 285*a*c^2*x^8 + 55*c^3*x^12))/(1045*x^(5/2)
)
```

fricas [A] time = 0.52, size = 37, normalized size = 0.76

$$\frac{2\left(55\,c^3x^{12}+285\,ac^2x^8+1045\,a^2cx^4-209\,a^3\right)}{1045\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^3/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/1045*(55*c^3*x^12 + 285*a*c^2*x^8 + 1045*a^2*c*x^4 - 209*a^3)/x^(5/2)
```

giac [A] time = 0.19, size = 35, normalized size = 0.71

$$\frac{2}{19}c^3x^{\frac{19}{2}} + \frac{6}{11}ac^2x^{\frac{11}{2}} + 2a^2cx^{\frac{3}{2}} - \frac{2a^3}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^3/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/19*c^3*x^(19/2) + 6/11*a*c^2*x^(11/2) + 2*a^2*c*x^(3/2) - 2/5*a^3/x^(5/2)
```

maple [A] time = 0.01, size = 38, normalized size = 0.78

$$-\frac{2\left(-55c^3x^{12} - 285a\,c^2x^8 - 1045a^2c\,x^4 + 209a^3\right)}{1045x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^3/x^(7/2),x)
```

```
[Out] -2/1045*(-55*c^3*x^12-285*a*c^2*x^8-1045*a^2*c*x^4+209*a^3)/x^(5/2)
```

maxima [A] time = 1.27, size = 35, normalized size = 0.71

$$\frac{2}{19}c^3x^{\frac{19}{2}} + \frac{6}{11}ac^2x^{\frac{11}{2}} + 2a^2cx^{\frac{3}{2}} - \frac{2a^3}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^3/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/19*c^3*x^(19/2) + 6/11*a*c^2*x^(11/2) + 2*a^2*c*x^(3/2) - 2/5*a^3/x^(5/2)
```

mupad [B] time = 0.04, size = 35, normalized size = 0.71

$$\frac{2\,c^3\,x^{19/2}}{19} - \frac{2\,a^3}{5\,x^{5/2}} + 2\,a^2\,c\,x^{3/2} + \frac{6\,a\,c^2\,x^{11/2}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^3/x^(7/2),x)
```

```
[Out] (2*c^3*x^(19/2))/19 - (2*a^3)/(5*x^(5/2)) + 2*a^2*c*x^(3/2) + (6*a*c^2*x^(1
1/2))/11
```

sympy [A] time = 37.21, size = 48, normalized size = 0.98

$$-\frac{2a^3}{5x^{\frac{5}{2}}} + 2a^2cx^{\frac{3}{2}} + \frac{6ac^2x^{\frac{11}{2}}}{11} + \frac{2c^3x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**3/x**(7/2),x)
```

```
[Out] -2*a**3/(5*x**(5/2)) + 2*a**2*c*x**(3/2) + 6*a*c**2*x**(11/2)/11 + 2*c**3*x
**(19/2)/19
```

3.738 $\int \frac{x^{9/2}}{a+cx^4}\,dx$

Optimal. Leaf size=299

$$-\frac{(-a)^{3/8}\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{4\sqrt{2}\,c^{11/8}}+\frac{(-a)^{3/8}\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{4\sqrt{2}\,c^{11/8}}+\frac{(-a)^{3/8}\tan^{-1}\left(1-\right.}{2\sqrt{2}\,c^{11/}}$$

```
[Out] 2/3*x^(3/2)/c+1/2*(-a)^(3/8)*arctan(c^(1/8)*x^(1/2)/(-a)^(1/8))/c^(11/8)-1/
2*(-a)^(3/8)*arctanh(c^(1/8)*x^(1/2)/(-a)^(1/8))/c^(11/8)-1/4*(-a)^(3/8)*ar
ctan(-1+c^(1/8)*2^(1/2)*x^(1/2)/(-a)^(1/8))/c^(11/8)*2^(1/2)-1/4*(-a)^(3/8)
*arctan(1+c^(1/8)*2^(1/2)*x^(1/2)/(-a)^(1/8))/c^(11/8)*2^(1/2)-1/8*(-a)^(3/
8)*ln((-a)^(1/4)+c^(1/4)*x-(-a)^(1/8)*c^(1/8)*2^(1/2)*x^(1/2))/c^(11/8)*2^(
1/2)+1/8*(-a)^(3/8)*ln((-a)^(1/4)+c^(1/4)*x+(-a)^(1/8)*c^(1/8)*2^(1/2)*x^(1
/2))/c^(11/8)*2^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {321, 329, 300, 297, 1162, 617, 204, 1165, 628, 298, 205, 208}

$$-\frac{(-a)^{3/8}\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{4\sqrt{2}\,c^{11/8}}+\frac{(-a)^{3/8}\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{4\sqrt{2}\,c^{11/8}}+\frac{(-a)^{3/8}\tan^{-1}\left(1-\right.}{2\sqrt{2}\,c^{11/}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/(a + c*x^4),x]
```

```
[Out] (2*x^(3/2))/(3*c) + ((-a)^(3/8)*ArcTan[1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(
1/8)])/(2*Sqrt[2]*c^(11/8)) - ((-a)^(3/8)*ArcTan[1 + (Sqrt[2]*c^(1/8)*Sqrt[
x])/(-a)^(1/8)])/(2*Sqrt[2]*c^(11/8)) + ((-a)^(3/8)*ArcTan[(c^(1/8)*Sqrt[x]
)/(-a)^(1/8)])/(2*c^(11/8)) - ((-a)^(3/8)*ArcTanh[(c^(1/8)*Sqrt[x])/(-a)^(1
/8)])/(2*c^(11/8)) - ((-a)^(3/8)*Log[(-a)^(1/4) - Sqrt[2]*(-a)^(1/8)*c^(1/8
)*Sqrt[x] + c^(1/4)*x])/(4*Sqrt[2]*c^(11/8)) + ((-a)^(3/8)*Log[(-a)^(1/4) +
 Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x])/(4*Sqrt[2]*c^(11/8))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 300

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[x^m/(r + s
*x^(n/2)), x], x] + Dist[r/(2*a), Int[x^m/(r - s*x^(n/2)), x], x]] /; FreeQ
[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n/2] &&  !GtQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{9/2}}{a+cx^4}\,dx &= \frac{2x^{3/2}}{3c} - \frac{a\int \frac{\sqrt{x}}{a+cx^4}\,dx}{c} \\
&= \frac{2x^{3/2}}{3c} - \frac{(2a)\,\text{Subst}\left(\int \frac{x^2}{a+cx^8}\,dx, x, \sqrt{x}\right)}{c} \\
&= \frac{2x^{3/2}}{3c} - \frac{\sqrt{-a}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{-a}-\sqrt{c}\,x^4}\,dx, x, \sqrt{x}\right)}{c} - \frac{\sqrt{-a}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{-a}+\sqrt{c}\,x^4}\,dx, x, \sqrt{x}\right)}{c} \\
&= \frac{2x^{3/2}}{3c} - \frac{\sqrt{-a}\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}\,x^2}\,dx, x, \sqrt{x}\right)}{2c^{5/4}} + \frac{\sqrt{-a}\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}+\sqrt[4]{c}\,x^2}\,dx, x, \sqrt{x}\right)}{2c^{5/4}} + \frac{\sqrt{-a}\,\text{Sub}}{} \\
&= \frac{2x^{3/2}}{3c} + \frac{(-a)^{3/8}\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2c^{11/8}} - \frac{(-a)^{3/8}\tanh^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2c^{11/8}} - \frac{\sqrt{-a}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[4]{-a}}{\sqrt[4]{c}} - \frac{\sqrt{2}\,\sqrt[8]{-a}\,x}{\sqrt[8]{c}} + x^2}\,dx, x, \right.}{4c^{3/2}} \\
&= \frac{2x^{3/2}}{3c} + \frac{(-a)^{3/8}\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2c^{11/8}} - \frac{(-a)^{3/8}\tanh^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2c^{11/8}} - \frac{(-a)^{3/8}\log\left(\sqrt[4]{-a} - \sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x} + \right.}{4\sqrt{2}\,c^{11/8}} \\
&= \frac{2x^{3/2}}{3c} + \frac{(-a)^{3/8}\tan^{-1}\left(1 - \frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}\,c^{11/8}} - \frac{(-a)^{3/8}\tan^{-1}\left(1 + \frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}\,c^{11/8}} + \frac{(-a)^{3/8}\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2c^{11/8}} - \frac{(}{}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 31, normalized size = 0.10

$$-\frac{2x^{3/2}\left({}_2F_1\left(\frac{3}{8},1;\frac{11}{8};-\frac{cx^4}{a}\right)-1\right)}{3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/(a + c*x^4),x]
```

```
[Out] (-2*x^(3/2)*(-1 + Hypergeometric2F1[3/8, 1, 11/8, -((c*x^4)/a)]))/(3*c)
```

fricas [B] time = 0.63, size = 491, normalized size = 1.64

$$12\sqrt{2}\,c\left(-\frac{a^3}{c^{11}}\right)^{\frac{1}{8}}\arctan\left(-\frac{\sqrt{2}\,ac^7\sqrt{x}\left(-\frac{a^3}{c^{11}}\right)^{\frac{5}{8}} - \sqrt{2}\,\sqrt{c^8\left(-\frac{a^3}{c^{11}}\right)^{\frac{3}{4}} + \sqrt{2}\,ac^4\sqrt{x}\left(-\frac{a^3}{c^{11}}\right)^{\frac{3}{8}} + a^2x\,c^7\left(-\frac{a^3}{c^{11}}\right)^{\frac{5}{8}} - a^3}}{a^3}\right) + 12\sqrt{2}\,c\left(-\frac{a^3}{c^{11}}\right)^{\frac{1}{8}}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/24*(12*sqrt(2)*c*(-a^3/c^11)^(1/8)*arctan(-(sqrt(2)*a*c^7*sqrt(x)*(-a^3/c
^11)^(5/8) - sqrt(2)*sqrt(c^8*(-a^3/c^11)^(3/4) + sqrt(2)*a*c^4*sqrt(x)*(-a
^3/c^11)^(3/8) + a^2*x)*c^7*(-a^3/c^11)^(5/8) - a^3)/a^3) + 12*sqrt(2)*c*(-
a^3/c^11)^(1/8)*arctan(-(sqrt(2)*a*c^7*sqrt(x)*(-a^3/c^11)^(5/8) - sqrt(2)*
sqrt(c^8*(-a^3/c^11)^(3/4) - sqrt(2)*a*c^4*sqrt(x)*(-a^3/c^11)^(3/8) + a^2*
x)*c^7*(-a^3/c^11)^(5/8) + a^3)/a^3) - 3*sqrt(2)*c*(-a^3/c^11)^(1/8)*log(c^
8*(-a^3/c^11)^(3/4) + sqrt(2)*a*c^4*sqrt(x)*(-a^3/c^11)^(3/8) + a^2*x) + 3*
sqrt(2)*c*(-a^3/c^11)^(1/8)*log(c^8*(-a^3/c^11)^(3/4) - sqrt(2)*a*c^4*sqrt(
x)*(-a^3/c^11)^(3/8) + a^2*x) - 24*c*(-a^3/c^11)^(1/8)*arctan(-(a*c^7*sqrt(
x)*(-a^3/c^11)^(5/8) - sqrt(c^8*(-a^3/c^11)^(3/4) + a^2*x)*c^7*(-a^3/c^11)^
```

```
(5/8))/a^3) + 6*c*(-a^3/c^11)^(1/8)*log(c^4*(-a^3/c^11)^(3/8) + a*sqrt(x))
- 6*c*(-a^3/c^11)^(1/8)*log(-c^4*(-a^3/c^11)^(3/8) + a*sqrt(x)) + 16*x^(3/2
))/c
```

giac [B] time = 0.64, size = 453, normalized size = 1.52

$$\frac{2x^{\frac{3}{2}}}{3c}+\frac{\left(\frac{a}{c}\right)^{\frac{3}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2c\sqrt{2\sqrt{2}+4}}+\frac{\left(\frac{a}{c}\right)^{\frac{3}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2c\sqrt{2\sqrt{2}+4}}-\frac{\left(\frac{a}{c}\right)^{\frac{3}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2c\sqrt{-2\sqrt{2}+4}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 2/3*x^(3/2)/c + 1/2*(a/c)^(3/8)*arctan((sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + 2*
sqrt(x))/(sqrt(sqrt(2) + 2)*(a/c)^(1/8)))/(c*sqrt(2*sqrt(2) + 4)) + 1/2*(a/
c)^(3/8)*arctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(sqrt(2)
 + 2)*(a/c)^(1/8)))/(c*sqrt(2*sqrt(2) + 4)) - 1/2*(a/c)^(3/8)*arctan((sqrt(
sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(c*
sqrt(-2*sqrt(2) + 4)) - 1/2*(a/c)^(3/8)*arctan(-(sqrt(sqrt(2) + 2)*(a/c)^(1
/8) - 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(c*sqrt(-2*sqrt(2) + 4))
 - 1/4*(a/c)^(3/8)*log(sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1
/4))/(c*sqrt(2*sqrt(2) + 4)) + 1/4*(a/c)^(3/8)*log(-sqrt(x)*sqrt(sqrt(2) +
2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(c*sqrt(2*sqrt(2) + 4)) + 1/4*(a/c)^(3/8)
*log(sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(c*sqrt(-2*s
qrt(2) + 4)) - 1/4*(a/c)^(3/8)*log(-sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8)
+ x + (a/c)^(1/4))/(c*sqrt(-2*sqrt(2) + 4))
```

maple [C] time = 0.03, size = 39, normalized size = 0.13

$$\frac{2x^{\frac{3}{2}}}{3c}-\frac{a\ln\left(-\operatorname{RootOf}\left(c_Z^8+a\right)+\sqrt{x}\right)}{4c^2\operatorname{RootOf}\left(c_Z^8+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(c*x^4+a),x)
```

```
[Out] 2/3*x^(3/2)/c-1/4*a/c^2*sum(1/_R^5*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-a\int\frac{\sqrt{x}}{c^2x^4+ac}\,dx+\frac{2x^{\frac{3}{2}}}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] -a*integrate(sqrt(x)/(c^2*x^4 + a*c), x) + 2/3*x^(3/2)/c
```

mupad [B] time = 1.11, size = 126, normalized size = 0.42

$$\frac{2x^{3/2}}{3c}+\frac{(-a)^{3/8}\operatorname{atan}\left(\frac{c^{1/8}\sqrt{x}}{(-a)^{1/8}}\right)}{2c^{11/8}}+\frac{(-a)^{3/8}\operatorname{atan}\left(\frac{c^{1/8}\sqrt{x}\,1\mathrm{i}}{(-a)^{1/8}}\right)1\mathrm{i}}{2c^{11/8}}+\frac{\sqrt{2}\,(-a)^{3/8}\operatorname{atan}\left(\frac{\sqrt{2}\,c^{1/8}\sqrt{x}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\left(-\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)}{c^{11/8}}+\frac{\sqrt{2}\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(a + c*x^4),x)
```

```
[Out] (2*x^(3/2))/(3*c) + ((-a)^(3/8)*atan((c^(1/8)*x^(1/2))/(-a)^(1/8)))/(2*c^(1
1/8)) + ((-a)^(3/8)*atan((c^(1/8)*x^(1/2)*1i)/(-a)^(1/8))*1i)/(2*c^(11/8))
- (2^(1/2)*(-a)^(3/8)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^(1/8
))*(1/4 - 1i/4))/c^(11/8) - (2^(1/2)*(-a)^(3/8)*atan((2^(1/2)*c^(1/8)*x^(1/
2)*(1/2 + 1i/2))/(-a)^(1/8))*(1/4 + 1i/4))/c^(11/8)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(c*x**4+a),x)
```

```
[Out] Timed out
```

3.739 $\int \frac{x^{7/2}}{a+cx^4}\,dx$

Optimal. Leaf size=297

$$\frac{\sqrt[8]{-a}\,\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{4\sqrt{2}\,c^{9/8}}-\frac{\sqrt[8]{-a}\,\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{4\sqrt{2}\,c^{9/8}}+\frac{\sqrt[8]{-a}\,\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{2\sqrt{2}\,c^{9/8}}$$

```
[Out] -1/2*(-a)^(1/8)*arctan(c^(1/8)*x^(1/2)/(-a)^(1/8))/c^(9/8)-1/2*(-a)^(1/8)*a
rctanh(c^(1/8)*x^(1/2)/(-a)^(1/8))/c^(9/8)-1/4*(-a)^(1/8)*arctan(-1+c^(1/8)
*2^(1/2)*x^(1/2)/(-a)^(1/8))/c^(9/8)*2^(1/2)-1/4*(-a)^(1/8)*arctan(1+c^(1/8
)*2^(1/2)*x^(1/2)/(-a)^(1/8))/c^(9/8)*2^(1/2)+1/8*(-a)^(1/8)*ln((-a)^(1/4)+
c^(1/4)*x-(-a)^(1/8)*c^(1/8)*2^(1/2)*x^(1/2))/c^(9/8)*2^(1/2)-1/8*(-a)^(1/8
)*ln((-a)^(1/4)+c^(1/4)*x+(-a)^(1/8)*c^(1/8)*2^(1/2)*x^(1/2))/c^(9/8)*2^(1/
2)+2*x^(1/2)/c
```

Rubi [A] time = 0.29, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {321, 329, 214, 212, 208, 205, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt[8]{-a}\,\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{4\sqrt{2}\,c^{9/8}}-\frac{\sqrt[8]{-a}\,\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{4\sqrt{2}\,c^{9/8}}+\frac{\sqrt[8]{-a}\,\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{2\sqrt{2}\,c^{9/8}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(a + c*x^4),x]
```

```
[Out] (2*Sqrt[x])/c + ((-a)^(1/8)*ArcTan[1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)
])/(2*Sqrt[2]*c^(9/8)) - ((-a)^(1/8)*ArcTan[1 + (Sqrt[2]*c^(1/8)*Sqrt[x])/(
-a)^(1/8)])/(2*Sqrt[2]*c^(9/8)) - ((-a)^(1/8)*ArcTan[(c^(1/8)*Sqrt[x])/(-a)
^(1/8)])/(2*c^(9/8)) - ((-a)^(1/8)*ArcTanh[(c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(
2*c^(9/8)) + ((-a)^(1/8)*Log[(-a)^(1/4) - Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x
] + c^(1/4)*x])/(4*Sqrt[2]*c^(9/8)) - ((-a)^(1/8)*Log[(-a)^(1/4) + Sqrt[2]*
(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x])/(4*Sqrt[2]*c^(9/8))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{7/2}}{a+cx^4}\,dx = \frac{2\sqrt{x}}{c} - \frac{a\int \frac{1}{\sqrt{x}\left(a+cx^4\right)}\,dx}{c}$$

$$= \frac{2\sqrt{x}}{c} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{a+cx^8}\,dx, x, \sqrt{x}\right)}{c}$$

$$= \frac{2\sqrt{x}}{c} - \frac{\sqrt{-a}\,\text{Subst}\left(\int \frac{1}{\sqrt{-a}-\sqrt{c}\,x^4}\,dx, x, \sqrt{x}\right)}{c} - \frac{\sqrt{-a}\,\text{Subst}\left(\int \frac{1}{\sqrt{-a}+\sqrt{c}\,x^4}\,dx, x, \sqrt{x}\right)}{c}$$

$$= \frac{2\sqrt{x}}{c} - \frac{\sqrt[4]{-a}\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}\,x^2}\,dx, x, \sqrt{x}\right)}{2c} - \frac{\sqrt[4]{-a}\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}+\sqrt[4]{c}\,x^2}\,dx, x, \sqrt{x}\right)}{2c} - \frac{\sqrt[4]{-a}\,\text{S}}{}$$

$$= \frac{2\sqrt{x}}{c} - \frac{\sqrt[8]{-a}\,\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2c^{9/8}} - \frac{\sqrt[8]{-a}\,\tanh^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2c^{9/8}} - \frac{\sqrt[4]{-a}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[4]{-a}}{\sqrt{c}} - \frac{\sqrt{2}\,\sqrt[8]{-a}\,x}{\sqrt[8]{c}} + x^2}\,dx, x, \sqrt{}\right.}{4c^{5/4}}$$

$$= \frac{2\sqrt{x}}{c} - \frac{\sqrt[8]{-a}\,\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2c^{9/8}} - \frac{\sqrt[8]{-a}\,\tanh^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2c^{9/8}} + \frac{\sqrt[8]{-a}\,\log\left(\sqrt[4]{-a} - \sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x} + \sqrt[4]{}\right.}{4\sqrt{2}\,c^{9/8}}$$

$$= \frac{2\sqrt{x}}{c} + \frac{\sqrt[8]{-a}\,\tan^{-1}\left(1 - \frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}\,c^{9/8}} - \frac{\sqrt[8]{-a}\,\tan^{-1}\left(1 + \frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}\,c^{9/8}} - \frac{\sqrt[8]{-a}\,\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2c^{9/8}} - \frac{\sqrt[8]{-}}{}$$

Mathematica [C] time = 0.02, size = 29, normalized size = 0.10

$$-\frac{2\sqrt{x}\left({}_2F_1\left(\frac{1}{8}, 1; \frac{9}{8}; -\frac{cx^4}{a}\right) - 1\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(a + c*x^4),x]
```

```
[Out] (-2*Sqrt[x]*(-1 + Hypergeometric2F1[1/8, 1, 9/8, -((c*x^4)/a)]))/c
```

fricas [A] time = 0.71, size = 392, normalized size = 1.32

$$-\frac{4\sqrt{2}\,c\left(-\frac{a}{c^9}\right)^{\frac{1}{8}}\arctan\left(\frac{\sqrt{2}\,\sqrt{c^2\left(-\frac{a}{c^9}\right)^{\frac{1}{4}} + \sqrt{2}\,c\sqrt{x}\left(-\frac{a}{c^9}\right)^{\frac{1}{8}} + x\,c^8\left(-\frac{a}{c^9}\right)^{\frac{7}{8}} - \sqrt{2}\,c^8\sqrt{x}\left(-\frac{a}{c^9}\right)^{\frac{7}{8}} + a}}{a}\right) + 4\sqrt{2}\,c\left(-\frac{a}{c^9}\right)^{\frac{1}{8}}\arctan\left(\frac{\sqrt{2}\,\sqrt{c^2\left(-\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/8*(4*sqrt(2)*c*(-a/c^9)^(1/8)*arctan((sqrt(2)*sqrt(c^2*(-a/c^9)^(1/4) +
sqrt(2)*c*sqrt(x)*(-a/c^9)^(1/8) + x)*c^8*(-a/c^9)^(7/8) - sqrt(2)*c^8*sqrt
(x)*(-a/c^9)^(7/8) + a)/a) + 4*sqrt(2)*c*(-a/c^9)^(1/8)*arctan((sqrt(2)*sqr
t(c^2*(-a/c^9)^(1/4) - sqrt(2)*c*sqrt(x)*(-a/c^9)^(1/8) + x)*c^8*(-a/c^9)^(
7/8) - sqrt(2)*c^8*sqrt(x)*(-a/c^9)^(7/8) - a)/a) + sqrt(2)*c*(-a/c^9)^(1/8
)*log(c^2*(-a/c^9)^(1/4) + sqrt(2)*c*sqrt(x)*(-a/c^9)^(1/8) + x) - sqrt(2)*
c*(-a/c^9)^(1/8)*log(c^2*(-a/c^9)^(1/4) - sqrt(2)*c*sqrt(x)*(-a/c^9)^(1/8)
+ x) + 8*c*(-a/c^9)^(1/8)*arctan((sqrt(c^2*(-a/c^9)^(1/4) + x)*c^8*(-a/c^9)
^(7/8) - c^8*sqrt(x)*(-a/c^9)^(7/8))/a) + 2*c*(-a/c^9)^(1/8)*log(c*(-a/c^9)
^(1/8) + sqrt(x)) - 2*c*(-a/c^9)^(1/8)*log(-c*(-a/c^9)^(1/8) + sqrt(x)) - 1
6*sqrt(x))/c
```

giac [B] time = 0.45, size = 453, normalized size = 1.53

$$-\frac{\left(\frac{a}{c}\right)^{\frac{1}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2c\sqrt{-2\sqrt{2}+4}}-\frac{\left(\frac{a}{c}\right)^{\frac{1}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2c\sqrt{-2\sqrt{2}+4}}-\frac{\left(\frac{a}{c}\right)^{\frac{1}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2c\sqrt{2\sqrt{2}+4}}-\frac{\left(\frac{a}{c}\right)^{\frac{1}{8}}\text{ arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -1/2*(a/c)^(1/8)*arctan((sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(
sqrt(2) + 2)*(a/c)^(1/8)))/(c*sqrt(-2*sqrt(2) + 4)) - 1/2*(a/c)^(1/8)*arcta
n(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(sqrt(2) + 2)*(a/c)^(1
/8)))/(c*sqrt(-2*sqrt(2) + 4)) - 1/2*(a/c)^(1/8)*arctan((sqrt(sqrt(2) + 2)*
(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(c*sqrt(2*sqrt(2
) + 4)) - 1/2*(a/c)^(1/8)*arctan(-(sqrt(sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x
))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(c*sqrt(2*sqrt(2) + 4)) - 1/4*(a/c)^(1
/8)*log(sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(c*sqrt(-2
*sqrt(2) + 4)) + 1/4*(a/c)^(1/8)*log(-sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8)
 + x + (a/c)^(1/4))/(c*sqrt(-2*sqrt(2) + 4)) - 1/4*(a/c)^(1/8)*log(sqrt(x)*
sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(c*sqrt(2*sqrt(2) + 4)) +
 1/4*(a/c)^(1/8)*log(-sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1
/4))/(c*sqrt(2*sqrt(2) + 4)) + 2*sqrt(x)/c
```

maple [C] time = 0.02, size = 39, normalized size = 0.13

$$-\frac{a\ln\left(-\text{RootOf}\left(c_Z^8+a\right)+\sqrt{x}\right)}{4c^2\,\text{RootOf}\left(c_Z^8+a\right)^7}+\frac{2\sqrt{x}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(c*x^4+a),x)
```

```
[Out] 2*x^(1/2)/c-1/4*a/c^2*sum(1/_R^7*ln(-_R+x^(1/2)),_R=RootOf(_Z^8*c+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{7}{2}}}{cx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(7/2)/(c*x^4 + a), x)
```

mupad [B] time = 1.10, size = 126, normalized size = 0.42

$$\frac{2\sqrt{x}}{c}-\frac{(-a)^{1/8}\,\text{atan}\left(\frac{c^{1/8}\sqrt{x}}{(-a)^{1/8}}\right)}{2c^{9/8}}+\frac{(-a)^{1/8}\,\text{atan}\left(\frac{c^{1/8}\sqrt{x}\,1\text{i}}{(-a)^{1/8}}\right)1\text{i}}{2c^{9/8}}+\frac{\sqrt{2}\,(-a)^{1/8}\,\text{atan}\left(\frac{\sqrt{2}\,c^{1/8}\sqrt{x}\left(\frac{1}{2}-\frac{1}{2}\text{i}\right)}{(-a)^{1/8}}\right)\left(-\frac{1}{4}-\frac{1}{4}\text{i}\right)}{c^{9/8}}+\frac{\sqrt{2}\,(-a)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(a + c*x^4),x)
```

```
[Out] (2*x^(1/2))/c - ((-a)^(1/8)*atan((c^(1/8)*x^(1/2))/(-a)^(1/8)))/(2*c^(9/8))
 + ((-a)^(1/8)*atan((c^(1/8)*x^(1/2)*1i)/(-a)^(1/8))*1i)/(2*c^(9/8)) - (2^(
1/2)*(-a)^(1/8)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^(1/8))*(1/
```

```
4 + 1i/4))/c^(9/8) - (2^(1/2)*(-a)^(1/8)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2
 + 1i/2))/(-a)^(1/8))*(1/4 - 1i/4))/c^(9/8)
```

sympy [A] time = 168.07, size = 459, normalized size = 1.55

$$\begin{cases} \tilde{\infty}\sqrt{x} \\ \frac{2x^{\frac{9}{2}}}{9a} \\ \frac{2\sqrt{x}}{c} \\ \frac{\sqrt[8]{-1}\sqrt[8]{a}\sqrt[8]{\frac{1}{c}}\log\left(-\sqrt[8]{-1}\sqrt[8]{a}\sqrt[8]{\frac{1}{c}}+\sqrt{x}\right)}{4c} - \frac{\sqrt[8]{-1}\sqrt[8]{a}\sqrt[8]{\frac{1}{c}}\log\left(\sqrt[8]{-1}\sqrt[8]{a}\sqrt[8]{\frac{1}{c}}+\sqrt{x}\right)}{4c} + \frac{\sqrt[8]{-1}\sqrt{2}\sqrt[8]{a}\sqrt[8]{\frac{1}{c}}\log\left(-4\sqrt[8]{-1}\sqrt{2}\sqrt[8]{a}\sqrt{x}\sqrt[8]{\frac{1}{c}}+4\sqrt[4]{-1}\sqrt[4]{a}\right.}{8c} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(c*x**4+a),x)
```

```
[Out] Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(c, 0)), (2*x**(9/2)/(9*a), Eq(c, 0)),
 (2*sqrt(x)/c, Eq(a, 0)), ((-1)**(1/8)*a**(1/8)*(1/c)**(1/8)*log(-(-1)**(1/
8)*a**(1/8)*(1/c)**(1/8) + sqrt(x))/(4*c) - (-1)**(1/8)*a**(1/8)*(1/c)**(1/
8)*log((-1)**(1/8)*a**(1/8)*(1/c)**(1/8) + sqrt(x))/(4*c) + (-1)**(1/8)*sqr
t(2)*a**(1/8)*(1/c)**(1/8)*log(-4*(-1)**(1/8)*sqrt(2)*a**(1/8)*sqrt(x)*(1/c
)**(1/8) + 4*(-1)**(1/4)*a**(1/4)*(1/c)**(1/4) + 4*x)/(8*c) - (-1)**(1/8)*s
qrt(2)*a**(1/8)*(1/c)**(1/8)*log(4*(-1)**(1/8)*sqrt(2)*a**(1/8)*sqrt(x)*(1/
c)**(1/8) + 4*(-1)**(1/4)*a**(1/4)*(1/c)**(1/4) + 4*x)/(8*c) + (-1)**(1/8)*
a**(1/8)*(1/c)**(1/8)*atan((-1)**(7/8)*sqrt(x)/(a**(1/8)*(1/c)**(1/8)))/(2*
c) - (-1)**(1/8)*sqrt(2)*a**(1/8)*(1/c)**(1/8)*atan(1 - (-1)**(7/8)*sqrt(2)
*sqrt(x)/(a**(1/8)*(1/c)**(1/8)))/(4*c) + (-1)**(1/8)*sqrt(2)*a**(1/8)*(1/c
)**(1/8)*atan(1 + (-1)**(7/8)*sqrt(2)*sqrt(x)/(a**(1/8)*(1/c)**(1/8)))/(4*c
) + 2*sqrt(x)/c, True))
```

3.740 $\int \frac{x^{5/2}}{a+cx^4}\,dx$

Optimal. Leaf size=287

$$\frac{\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{4\sqrt{2}\,\sqrt[8]{-a}\,c^{7/8}}-\frac{\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{4\sqrt{2}\,\sqrt[8]{-a}\,c^{7/8}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}\,\sqrt[8]{-a}\,c^{7/8}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[8]{}}{}\right.}{2\sqrt{2}\,\sqrt[8]{}}$$

```
[Out] 1/2*arctan(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(1/8)/c^(7/8)-1/2*arctanh(c^(1/
8)*x^(1/2)/(-a)^(1/8))/(-a)^(1/8)/c^(7/8)+1/4*arctan(-1+c^(1/8)*2^(1/2)*x^(
1/2)/(-a)^(1/8))/(-a)^(1/8)/c^(7/8)*2^(1/2)+1/4*arctan(1+c^(1/8)*2^(1/2)*x^
(1/2)/(-a)^(1/8))/(-a)^(1/8)/c^(7/8)*2^(1/2)+1/8*ln((-a)^(1/4)+c^(1/4)*x-(-
a)^(1/8)*c^(1/8)*2^(1/2)*x^(1/2))/(-a)^(1/8)/c^(7/8)*2^(1/2)-1/8*ln((-a)^(1
/4)+c^(1/4)*x+(-a)^(1/8)*c^(1/8)*2^(1/2)*x^(1/2))/(-a)^(1/8)/c^(7/8)*2^(1/2
)
```

Rubi [A] time = 0.23, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$, Rules used = {329, 301, 297, 1162, 617, 204, 1165, 628, 298, 205, 208}

$$\frac{\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{4\sqrt{2}\,\sqrt[8]{-a}\,c^{7/8}}-\frac{\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{4\sqrt{2}\,\sqrt[8]{-a}\,c^{7/8}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}\,\sqrt[8]{-a}\,c^{7/8}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[8]{}}{}\right.}{2\sqrt{2}\,\sqrt[8]{}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + c*x^4),x]
```

```
[Out] -ArcTan[1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*Sqrt[2]*(-a)^(1/8)*c^(
7/8)) + ArcTan[1 + (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*Sqrt[2]*(-a)^(1
/8)*c^(7/8)) + ArcTan[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*(-a)^(1/8)*c^(7/8))
- ArcTanh[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*(-a)^(1/8)*c^(7/8)) + Log[(-a)^(
1/4) - Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x]/(4*Sqrt[2]*(-a)^(1/8
)*c^(7/8)) - Log[(-a)^(1/4) + Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*
x]/(4*Sqrt[2]*(-a)^(1/8)*c^(7/8))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 301

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[x^(m - n/2
)/(r + s*x^(n/2)), x], x] - Dist[s/(2*b), Int[x^(m - n/2)/(r - s*x^(n/2)),
x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] &&
 LtQ[m, n] &&  !GtQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{5/2}}{a+cx^4}\,dx = 2\,\text{Subst}\left(\int \frac{x^6}{a+cx^8}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{x^2}{\sqrt{-a}-\sqrt{c}\,x^4}\,dx, x, \sqrt{x}\right)}{\sqrt{c}} + \frac{\text{Subst}\left(\int \frac{x^2}{\sqrt{-a}+\sqrt{c}\,x^4}\,dx, x, \sqrt{x}\right)}{\sqrt{c}}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}\,x^2}\,dx, x, \sqrt{x}\right)}{2c^{3/4}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}+\sqrt[4]{c}\,x^2}\,dx, x, \sqrt{x}\right)}{2c^{3/4}} - \frac{\text{Subst}\left(\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}\,x^2}{\sqrt{-a}+\sqrt{c}\,x^4}\,dx, x, \sqrt{}\right.}{2c^{3/4}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}\,c^{7/8}} - \frac{\tanh^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}\,c^{7/8}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}-\frac{\sqrt{2}\,\sqrt[8]{-a}\,x}{\sqrt[8]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{4c} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}+\frac{\sqrt{2}\,\sqrt[8]{-a}}{\sqrt[8]{c}}}\right.}{4c}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}\,c^{7/8}} - \frac{\tanh^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}\,c^{7/8}} + \frac{\log\left(\sqrt[4]{-a}-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{c}\,x\right)}{4\sqrt{2}\,\sqrt[8]{-a}\,c^{7/8}} - \frac{\log\left(\sqrt[4]{-a}+\sqrt{2}\,\sqrt[8]{-a}\right.}{4\sqrt{2}\,\sqrt[8]{-a}}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}\,\sqrt[8]{-a}\,c^{7/8}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}\,\sqrt[8]{-a}\,c^{7/8}} + \frac{\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}\,c^{7/8}} - \frac{\tanh^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}\,c^{7/8}} + \frac{\log\left(\sqrt[4]{-a}\right.}{}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.10

$$\frac{2x^{7/2}\,{}_2F_1\left(\frac{7}{8},1;\frac{15}{8};-\frac{cx^4}{a}\right)}{7a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + c*x^4),x]
```

```
[Out] (2*x^(7/2)*Hypergeometric2F1[7/8, 1, 15/8, -((c*x^4)/a)])/(7*a)
```

fricas [B] time = 0.62, size = 426, normalized size = 1.48

$$-\frac{1}{2}\sqrt{2}\left(-\frac{1}{ac^7}\right)^{\frac{1}{8}}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\,ac^6\sqrt{x}\left(-\frac{1}{ac^7}\right)^{\frac{7}{8}} - ac^5\left(-\frac{1}{ac^7}\right)^{\frac{3}{4}} + x}\,c\left(-\frac{1}{ac^7}\right)^{\frac{1}{8}} - \sqrt{2}\,c\sqrt{x}\left(-\frac{1}{ac^7}\right)^{\frac{1}{8}} + 1\right) - \frac{1}{2}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*(-1/(a*c^7))^(1/8)*arctan(sqrt(2)*sqrt(sqrt(2)*a*c^6*sqrt(x)*(
-1/(a*c^7))^(7/8) - a*c^5*(-1/(a*c^7))^(3/4) + x)*c*(-1/(a*c^7))^(1/8) - sq
rt(2)*c*sqrt(x)*(-1/(a*c^7))^(1/8) + 1) - 1/2*sqrt(2)*(-1/(a*c^7))^(1/8)*ar
ctan(sqrt(2)*sqrt(-sqrt(2)*a*c^6*sqrt(x)*(-1/(a*c^7))^(7/8) - a*c^5*(-1/(a*
c^7))^(3/4) + x)*c*(-1/(a*c^7))^(1/8) - sqrt(2)*c*sqrt(x)*(-1/(a*c^7))^(1/8
) - 1) + 1/8*sqrt(2)*(-1/(a*c^7))^(1/8)*log(sqrt(2)*a*c^6*sqrt(x)*(-1/(a*c^
7))^(7/8) - a*c^5*(-1/(a*c^7))^(3/4) + x) - 1/8*sqrt(2)*(-1/(a*c^7))^(1/8)*
log(-sqrt(2)*a*c^6*sqrt(x)*(-1/(a*c^7))^(7/8) - a*c^5*(-1/(a*c^7))^(3/4) +
x) - (-1/(a*c^7))^(1/8)*arctan(sqrt(-a*c^5*(-1/(a*c^7))^(3/4) + x)*c*(-1/(a
*c^7))^(1/8) - c*sqrt(x)*(-1/(a*c^7))^(1/8)) + 1/4*(-1/(a*c^7))^(1/8)*log(a
*c^6*(-1/(a*c^7))^(7/8) + sqrt(x)) - 1/4*(-1/(a*c^7))^(1/8)*log(-a*c^6*(-1/
(a*c^7))^(7/8) + sqrt(x))
```

giac [B] time = 0.53, size = 445, normalized size = 1.55

$$\frac{\left(\frac{a}{c}\right)^{\frac{7}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2a\sqrt{-2\sqrt{2}+4}}+\frac{\left(\frac{a}{c}\right)^{\frac{7}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2a\sqrt{-2\sqrt{2}+4}}+\frac{\left(\frac{a}{c}\right)^{\frac{7}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2a\sqrt{2\sqrt{2}+4}}+\frac{\left(\frac{a}{c}\right)^{\frac{7}{8}}\text{a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*(a/c)^(7/8)*arctan((sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(s
qrt(2) + 2)*(a/c)^(1/8)))/(a*sqrt(-2*sqrt(2) + 4)) + 1/2*(a/c)^(7/8)*arctan
(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(sqrt(2) + 2)*(a/c)^(1/
8)))/(a*sqrt(-2*sqrt(2) + 4)) + 1/2*(a/c)^(7/8)*arctan((sqrt(sqrt(2) + 2)*(
a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a*sqrt(2*sqrt(2)
 + 4)) + 1/2*(a/c)^(7/8)*arctan(-(sqrt(sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x)
)/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a*sqrt(2*sqrt(2) + 4)) - 1/4*(a/c)^(7/
8)*log(sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a*sqrt(-2*
sqrt(2) + 4)) + 1/4*(a/c)^(7/8)*log(-sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8)
+ x + (a/c)^(1/4))/(a*sqrt(-2*sqrt(2) + 4)) - 1/4*(a/c)^(7/8)*log(sqrt(x)*s
qrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a*sqrt(2*sqrt(2) + 4)) +
1/4*(a/c)^(7/8)*log(-sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/
4))/(a*sqrt(2*sqrt(2) + 4))
```

maple [C] time = 0.02, size = 29, normalized size = 0.10

$$\frac{\ln\left(-\text{RootOf}\left(c_Z^8+a\right)+\sqrt{x}\right)}{4c\,\text{RootOf}\left(c_Z^8+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(c*x^4+a),x)
```

```
[Out] 1/4/c*sum(1/_R*ln(-_R+x^(1/2)),_R=RootOf(_Z^8*c+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{cx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(5/2)/(c*x^4 + a), x)
```

mupad [B] time = 1.11, size = 118, normalized size = 0.41

$$\frac{\text{atan}\left(\frac{c^{1/8}\sqrt{x}}{(-a)^{1/8}}\right)}{2\,(-a)^{1/8}\,c^{7/8}}+\frac{\text{atan}\left(\frac{c^{1/8}\sqrt{x}\,1\text{i}}{(-a)^{1/8}}\right)1\text{i}}{2\,(-a)^{1/8}\,c^{7/8}}+\frac{\sqrt{2}\,\text{atan}\left(\frac{\sqrt{2}\,c^{1/8}\sqrt{x}\left(\frac{1}{2}-\frac{1}{2}\text{i}\right)}{(-a)^{1/8}}\right)\left(\frac{1}{4}-\frac{1}{4}\text{i}\right)}{(-a)^{1/8}\,c^{7/8}}+\frac{\sqrt{2}\,\text{atan}\left(\frac{\sqrt{2}\,c^{1/8}\sqrt{x}\left(\frac{1}{2}+\frac{1}{2}\text{i}\right)}{(-a)^{1/8}}\right)\left(\frac{1}{4}+\frac{1}{4}\text{i}\right)}{(-a)^{1/8}\,c^{7/8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + c*x^4),x)
```

```
[Out] atan((c^(1/8)*x^(1/2))/(-a)^(1/8))/(2*(-a)^(1/8)*c^(7/8)) + (atan((c^(1/8)*
x^(1/2)*1i)/(-a)^(1/8))*1i)/(2*(-a)^(1/8)*c^(7/8)) + (2^(1/2)*atan((2^(1/2)
*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^(1/8))*(1/4 - 1i/4))/((-a)^(1/8)*c^(7/8
```

```
)) + (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 + 1i/2))/(-a)^(1/8))*(1/4
+ 1i/4))/((-a)^(1/8)*c^(7/8))
```

sympy [**A**] time = 94.13, size = 452, normalized size = 1.57

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} \\ \frac{2x^{\frac{7}{2}}}{7a} \\ -\frac{2}{c\sqrt{x}} \\ -\frac{(-1)^{\frac{7}{8}}\log\left(-\sqrt[8]{-1}\,\sqrt[8]{a}\,\sqrt[8]{\frac{1}{c}}+\sqrt{x}\right)}{4\sqrt[8]{a}\,c\sqrt[8]{\frac{1}{c}}}+\frac{(-1)^{\frac{7}{8}}\log\left(\sqrt[8]{-1}\,\sqrt[8]{a}\,\sqrt[8]{\frac{1}{c}}+\sqrt{x}\right)}{4\sqrt[8]{a}\,c\sqrt[8]{\frac{1}{c}}}-\frac{(-1)^{\frac{7}{8}}\sqrt{2}\log\left(-4\sqrt[8]{-1}\,\sqrt{2}\,\sqrt[8]{a}\,\sqrt{x}\,\sqrt[8]{\frac{1}{c}}+4\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{c}}+4x\right)}{8\sqrt[8]{a}\,c\sqrt[8]{\frac{1}{c}}}+\frac{(-1)^{\frac{7}{8}}\sqrt{2}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(c*x**4+a),x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(c, 0)), (2*x**(7/2)/(7*a), Eq(c, 0)),
 (-2/(c*sqrt(x)), Eq(a, 0)), (-(-1)**(7/8)*log(-(-1)**(1/8)*a**(1/8)*(1/c)*
*(1/8) + sqrt(x))/(4*a**(1/8)*c*(1/c)**(1/8)) + (-1)**(7/8)*log((-1)**(1/8)
*a**(1/8)*(1/c)**(1/8) + sqrt(x))/(4*a**(1/8)*c*(1/c)**(1/8)) - (-1)**(7/8)
*sqrt(2)*log(-4*(-1)**(1/8)*sqrt(2)*a**(1/8)*sqrt(x)*(1/c)**(1/8) + 4*(-1)*
*(1/4)*a**(1/4)*(1/c)**(1/4) + 4*x)/(8*a**(1/8)*c*(1/c)**(1/8)) + (-1)**(7/
8)*sqrt(2)*log(4*(-1)**(1/8)*sqrt(2)*a**(1/8)*sqrt(x)*(1/c)**(1/8) + 4*(-1)
**(1/4)*a**(1/4)*(1/c)**(1/4) + 4*x)/(8*a**(1/8)*c*(1/c)**(1/8)) + (-1)**(7
/8)*atan((-1)**(7/8)*sqrt(x)/(a**(1/8)*(1/c)**(1/8)))/(2*a**(1/8)*c*(1/c)**
(1/8)) - (-1)**(7/8)*sqrt(2)*atan(1 - (-1)**(7/8)*sqrt(2)*sqrt(x)/(a**(1/8)
*(1/c)**(1/8)))/(4*a**(1/8)*c*(1/c)**(1/8)) + (-1)**(7/8)*sqrt(2)*atan(1 +
(-1)**(7/8)*sqrt(2)*sqrt(x)/(a**(1/8)*(1/c)**(1/8)))/(4*a**(1/8)*c*(1/c)**(
1/8)), True))
```

3.741 $\int \frac{x^{3/2}}{a+cx^4}\,dx$

Optimal. Leaf size=287

$$-\frac{\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{4\sqrt{2}\,(-a)^{3/8}c^{5/8}}+\frac{\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{4\sqrt{2}\,(-a)^{3/8}c^{5/8}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}\,(-a)^{3/8}c^{5/8}}+\frac{\tan^{-1}}{2\sqrt{}}$$

```
[Out] -1/2*arctan(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(3/8)/c^(5/8)-1/2*arctanh(c^(1
/8)*x^(1/2)/(-a)^(1/8))/(-a)^(3/8)/c^(5/8)+1/4*arctan(-1+c^(1/8)*2^(1/2)*x^
(1/2)/(-a)^(1/8))/(-a)^(3/8)/c^(5/8)*2^(1/2)+1/4*arctan(1+c^(1/8)*2^(1/2)*x
^(1/2)/(-a)^(1/8))/(-a)^(3/8)/c^(5/8)*2^(1/2)-1/8*ln((-a)^(1/4)+c^(1/4)*x-(
-a)^(1/8)*c^(1/8)*2^(1/2)*x^(1/2))/(-a)^(3/8)/c^(5/8)*2^(1/2)+1/8*ln((-a)^(
1/4)+c^(1/4)*x+(-a)^(1/8)*c^(1/8)*2^(1/2)*x^(1/2))/(-a)^(3/8)/c^(5/8)*2^(1/
2)
```

Rubi [A] time = 0.23, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$, Rules used = {329, 301, 211, 1165, 628, 1162, 617, 204, 212, 208, 205}

$$-\frac{\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{4\sqrt{2}\,(-a)^{3/8}c^{5/8}}+\frac{\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{4\sqrt{2}\,(-a)^{3/8}c^{5/8}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}\,(-a)^{3/8}c^{5/8}}+\frac{\tan^{-1}}{2\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + c*x^4),x]
```

```
[Out] -ArcTan[1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*Sqrt[2]*(-a)^(3/8)*c^(
5/8)) + ArcTan[1 + (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*Sqrt[2]*(-a)^(3
/8)*c^(5/8)) - ArcTan[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*(-a)^(3/8)*c^(5/8))
- ArcTanh[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*(-a)^(3/8)*c^(5/8)) - Log[(-a)^(
1/4) - Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x]/(4*Sqrt[2]*(-a)^(3/8
)*c^(5/8)) + Log[(-a)^(1/4) + Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*
x]/(4*Sqrt[2]*(-a)^(3/8)*c^(5/8))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 301

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[x^(m - n/2
)/(r + s*x^(n/2)), x], x] - Dist[s/(2*b), Int[x^(m - n/2)/(r - s*x^(n/2)),
x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] &&
 LtQ[m, n] &&  !GtQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{3/2}}{a+cx^4}\,dx = 2\,\text{Subst}\left(\int \frac{x^4}{a+cx^8}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{-a}-\sqrt{c}\,x^4}\,dx, x, \sqrt{x}\right)}{\sqrt{c}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{-a}+\sqrt{c}\,x^4}\,dx, x, \sqrt{x}\right)}{\sqrt{c}}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}\,x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt[4]{-a}\,\sqrt{c}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}+\sqrt[4]{c}\,x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt[4]{-a}\,\sqrt{c}} + \frac{\text{Subst}\left(\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}\,x^2}{\sqrt{-a}+\sqrt{c}\,x^4}\,dx, x\right.}{2\sqrt[4]{-a}\,\sqrt{c}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}c^{5/8}} - \frac{\tanh^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}c^{5/8}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[4]{-a}}{\sqrt{c}} - \frac{\sqrt{2}\,\sqrt[8]{-a}\,x}{\sqrt[8]{c}} + x^2}\,dx, x, \sqrt{x}\right)}{4\sqrt[4]{-a}\,c^{3/4}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[4]{-a}}{\sqrt{c}} + \sqrt{}\right.}{4\sqrt[4]{}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}c^{5/8}} - \frac{\tanh^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}c^{5/8}} - \frac{\log\left(\sqrt[4]{-a} - \sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x} + \sqrt[4]{c}\,x\right)}{4\sqrt{2}\,(-a)^{3/8}c^{5/8}} + \frac{\log\left(\sqrt[4]{-a} + \sqrt{2}\right.}{4\sqrt{2}}$$

$$= -\frac{\tan^{-1}\left(1 - \frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}\,(-a)^{3/8}c^{5/8}} + \frac{\tan^{-1}\left(1 + \frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}\,(-a)^{3/8}c^{5/8}} - \frac{\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}c^{5/8}} - \frac{\tanh^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}c^{5/8}} - \frac{\log\left(\sqrt[4]{-}\right.}{}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.10

$$\frac{2x^{5/2}\,{}_2F_1\left(\frac{5}{8}, 1; \frac{13}{8}; -\frac{cx^4}{a}\right)}{5a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + c*x^4),x]
[Out] (2*x^(5/2)*Hypergeometric2F1[5/8, 1, 13/8, -((c*x^4)/a)])/(5*a)
```

fricas [B] time = 0.72, size = 446, normalized size = 1.55

$$\frac{1}{2}\sqrt{2}\left(-\frac{1}{a^3c^5}\right)^{\frac{1}{8}}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\,a^2c^3\sqrt{x}\left(-\frac{1}{a^3c^5}\right)^{\frac{5}{8}} - ac\left(-\frac{1}{a^3c^5}\right)^{\frac{1}{4}} + x}\,ac^2\left(-\frac{1}{a^3c^5}\right)^{\frac{3}{8}} - \sqrt{2}\,ac^2\sqrt{x}\left(-\frac{1}{a^3c^5}\right)^{\frac{3}{8}} +\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+a),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*(-1/(a^3*c^5))^(1/8)*arctan(sqrt(2)*sqrt(sqrt(2)*a^2*c^3*sqrt(x
)*(-1/(a^3*c^5))^(5/8) - a*c*(-1/(a^3*c^5))^(1/4) + x)*a*c^2*(-1/(a^3*c^5))
^(3/8) - sqrt(2)*a*c^2*sqrt(x)*(-1/(a^3*c^5))^(3/8) + 1) + 1/2*sqrt(2)*(-1/
(a^3*c^5))^(1/8)*arctan(sqrt(2)*sqrt(-sqrt(2)*a^2*c^3*sqrt(x)*(-1/(a^3*c^5)
)^(5/8) - a*c*(-1/(a^3*c^5))^(1/4) + x)*a*c^2*(-1/(a^3*c^5))^(3/8) - sqrt(2
)*a*c^2*sqrt(x)*(-1/(a^3*c^5))^(3/8) - 1) + 1/8*sqrt(2)*(-1/(a^3*c^5))^(1/8
)*log(sqrt(2)*a^2*c^3*sqrt(x)*(-1/(a^3*c^5))^(5/8) - a*c*(-1/(a^3*c^5))^(1/
4) + x) - 1/8*sqrt(2)*(-1/(a^3*c^5))^(1/8)*log(-sqrt(2)*a^2*c^3*sqrt(x)*(-1
/(a^3*c^5))^(5/8) - a*c*(-1/(a^3*c^5))^(1/4) + x) - (-1/(a^3*c^5))^(1/8)*ar
ctan(sqrt(-a*c*(-1/(a^3*c^5))^(1/4) + x)*a*c^2*(-1/(a^3*c^5))^(3/8) - a*c^2
*sqrt(x)*(-1/(a^3*c^5))^(3/8)) - 1/4*(-1/(a^3*c^5))^(1/8)*log(a^2*c^3*(-1/(
a^3*c^5))^(5/8) + sqrt(x)) + 1/4*(-1/(a^3*c^5))^(1/8)*log(-a^2*c^3*(-1/(a^3
*c^5))^(5/8) + sqrt(x))
```

giac [B] time = 0.55, size = 445, normalized size = 1.55

$$-\frac{\left(\frac{a}{c}\right)^{\frac{5}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2a\sqrt{2\sqrt{2}+4}}-\frac{\left(\frac{a}{c}\right)^{\frac{5}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2a\sqrt{2\sqrt{2}+4}}+\frac{\left(\frac{a}{c}\right)^{\frac{5}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2a\sqrt{-2\sqrt{2}+4}}+\frac{\left(\frac{a}{c}\right)^{\frac{5}{8}}\text{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -1/2*(a/c)^(5/8)*arctan((sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(
sqrt(2) + 2)*(a/c)^(1/8)))/(a*sqrt(2*sqrt(2) + 4)) - 1/2*(a/c)^(5/8)*arctan
(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(sqrt(2) + 2)*(a/c)^(1/
8)))/(a*sqrt(2*sqrt(2) + 4)) + 1/2*(a/c)^(5/8)*arctan((sqrt(sqrt(2) + 2)*(a
/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a*sqrt(-2*sqrt(2)
 + 4)) + 1/2*(a/c)^(5/8)*arctan(-(sqrt(sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x)
)/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a*sqrt(-2*sqrt(2) + 4)) - 1/4*(a/c)^(5
/8)*log(sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a*sqrt(2*
sqrt(2) + 4)) + 1/4*(a/c)^(5/8)*log(-sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8)
+ x + (a/c)^(1/4))/(a*sqrt(2*sqrt(2) + 4)) + 1/4*(a/c)^(5/8)*log(sqrt(x)*sq
rt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a*sqrt(-2*sqrt(2) + 4)) -
1/4*(a/c)^(5/8)*log(-sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/
4))/(a*sqrt(-2*sqrt(2) + 4))
```

maple [C] time = 0.02, size = 29, normalized size = 0.10

$$\frac{\ln\left(-\text{RootOf}\left(c_Z^8+a\right)+\sqrt{x}\right)}{4c\,\text{RootOf}\left(c_Z^8+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(c*x^4+a),x)
```

```
[Out] 1/4/c*sum(1/_R^3*ln(-_R+x^(1/2)),_R=RootOf(_Z^8*c+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{cx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/(c*x^4 + a), x)
```

mupad [B] time = 1.11, size = 118, normalized size = 0.41

$$-\frac{\text{atan}\left(\frac{c^{1/8}\sqrt{x}}{(-a)^{1/8}}\right)}{2(-a)^{3/8}c^{5/8}}+\frac{\text{atan}\left(\frac{c^{1/8}\sqrt{x}\,1\text{i}}{(-a)^{1/8}}\right)1\text{i}}{2(-a)^{3/8}c^{5/8}}+\frac{\sqrt{2}\,\text{atan}\left(\frac{\sqrt{2}\,c^{1/8}\sqrt{x}\left(\frac{1}{2}-\frac{1}{2}\text{i}\right)}{(-a)^{1/8}}\right)\left(\frac{1}{4}+\frac{1}{4}\text{i}\right)}{(-a)^{3/8}c^{5/8}}+\frac{\sqrt{2}\,\text{atan}\left(\frac{\sqrt{2}\,c^{1/8}\sqrt{x}\left(\frac{1}{2}+\frac{1}{2}\text{i}\right)}{(-a)^{1/8}}\right)\left(\frac{1}{4}-\frac{1}{4}\text{i}\right)}{(-a)^{3/8}c^{5/8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + c*x^4),x)
```

```
[Out] (atan((c^(1/8)*x^(1/2)*1i)/(-a)^(1/8))*1i)/(2*(-a)^(3/8)*c^(5/8)) - atan((c
^(1/8)*x^(1/2))/(-a)^(1/8))/(2*(-a)^(3/8)*c^(5/8)) + (2^(1/2)*atan((2^(1/2)
```

```
*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^(1/8))*(1/4 + 1i/4))/((-a)^(3/8)*c^(5/8
)) + (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 + 1i/2))/(-a)^(1/8))*(1/4
- 1i/4))/((-a)^(3/8)*c^(5/8))
```

sympy [A] time = 74.19, size = 454, normalized size = 1.58

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \\ \frac{2x^{\frac{5}{2}}}{5a} & \\ -\frac{2}{3cx^{\frac{3}{2}}} & \\ -\frac{(-1)^{\frac{5}{8}}\log\left(-\sqrt[8]{-1}\sqrt[8]{a}\sqrt[8]{\frac{1}{c}}+\sqrt{x}\right)}{4a^{\frac{3}{8}}c\left(\frac{1}{c}\right)^{\frac{3}{8}}}+\frac{(-1)^{\frac{5}{8}}\log\left(\sqrt[8]{-1}\sqrt[8]{a}\sqrt[8]{\frac{1}{c}}+\sqrt{x}\right)}{4a^{\frac{3}{8}}c\left(\frac{1}{c}\right)^{\frac{3}{8}}}+\frac{(-1)^{\frac{5}{8}}\sqrt{2}\log\left(-4\sqrt[8]{-1}\sqrt{2}\sqrt[8]{a}\sqrt{x}\sqrt[8]{\frac{1}{c}}+4\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{c}}+4x\right)}{8a^{\frac{3}{8}}c\left(\frac{1}{c}\right)^{\frac{3}{8}}}-\frac{(-1)^{\frac{5}{8}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(c*x**4+a),x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(c, 0)), (2*x**(5/2)/(5*a), Eq(c, 0))
, (-2/(3*c*x**(3/2)), Eq(a, 0)), (-(-1)**(5/8)*log(-(-1)**(1/8)*a**(1/8)*(1
/c)**(1/8) + sqrt(x))/(4*a**(3/8)*c*(1/c)**(3/8)) + (-1)**(5/8)*log((-1)**(
1/8)*a**(1/8)*(1/c)**(1/8) + sqrt(x))/(4*a**(3/8)*c*(1/c)**(3/8)) + (-1)**(
5/8)*sqrt(2)*log(-4*(-1)**(1/8)*sqrt(2)*a**(1/8)*sqrt(x)*(1/c)**(1/8) + 4*(
-1)**(1/4)*a**(1/4)*(1/c)**(1/4) + 4*x)/(8*a**(3/8)*c*(1/c)**(3/8)) - (-1)*
*(5/8)*sqrt(2)*log(4*(-1)**(1/8)*sqrt(2)*a**(1/8)*sqrt(x)*(1/c)**(1/8) + 4*
(-1)**(1/4)*a**(1/4)*(1/c)**(1/4) + 4*x)/(8*a**(3/8)*c*(1/c)**(3/8)) - (-1)
**(5/8)*atan((-1)**(7/8)*sqrt(x)/(a**(1/8)*(1/c)**(1/8)))/(2*a**(3/8)*c*(1/
c)**(3/8)) - (-1)**(5/8)*sqrt(2)*atan(1 - (-1)**(7/8)*sqrt(2)*sqrt(x)/(a**(
1/8)*(1/c)**(1/8)))/(4*a**(3/8)*c*(1/c)**(3/8)) + (-1)**(5/8)*sqrt(2)*atan(
1 + (-1)**(7/8)*sqrt(2)*sqrt(x)/(a**(1/8)*(1/c)**(1/8)))/(4*a**(3/8)*c*(1/c
)**(3/8)), True))
```

3.742 $\int \frac{\sqrt{x}}{a+cx^4}\,dx$

Optimal. Leaf size=287

$$-\frac{\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{4\sqrt{2}\,(-a)^{5/8}c^{3/8}}+\frac{\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{4\sqrt{2}\,(-a)^{5/8}c^{3/8}}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}\,(-a)^{5/8}c^{3/8}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}}{2\sqrt{2}\,(-}\right.}{}$$

```
[Out] 1/2*arctan(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(5/8)/c^(3/8)-1/2*arctanh(c^(1/
8)*x^(1/2)/(-a)^(1/8))/(-a)^(5/8)/c^(3/8)-1/4*arctan(-1+c^(1/8)*2^(1/2)*x^(
1/2)/(-a)^(1/8))/(-a)^(5/8)/c^(3/8)*2^(1/2)-1/4*arctan(1+c^(1/8)*2^(1/2)*x^
(1/2)/(-a)^(1/8))/(-a)^(5/8)/c^(3/8)*2^(1/2)-1/8*ln((-a)^(1/4)+c^(1/4)*x-(-
a)^(1/8)*c^(1/8)*2^(1/2)*x^(1/2))/(-a)^(5/8)/c^(3/8)*2^(1/2)+1/8*ln((-a)^(1
/4)+c^(1/4)*x+(-a)^(1/8)*c^(1/8)*2^(1/2)*x^(1/2))/(-a)^(5/8)/c^(3/8)*2^(1/2
)
```

Rubi [A] time = 0.24, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$, Rules used = {329, 300, 297, 1162, 617, 204, 1165, 628, 298, 205, 208}

$$-\frac{\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{4\sqrt{2}\,(-a)^{5/8}c^{3/8}}+\frac{\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{4\sqrt{2}\,(-a)^{5/8}c^{3/8}}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}\,(-a)^{5/8}c^{3/8}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}}{2\sqrt{2}\,(-}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + c*x^4),x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*Sqrt[2]*(-a)^(5/8)*c^(3
/8)) - ArcTan[1 + (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*Sqrt[2]*(-a)^(5/
8)*c^(3/8)) + ArcTan[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*(-a)^(5/8)*c^(3/8)) -
 ArcTanh[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*(-a)^(5/8)*c^(3/8)) - Log[(-a)^(1
/4) - Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x]/(4*Sqrt[2]*(-a)^(5/8)
*c^(3/8)) + Log[(-a)^(1/4) + Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x
]/(4*Sqrt[2]*(-a)^(5/8)*c^(3/8))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 300

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[x^m/(r + s
*x^(n/2)), x], x] + Dist[r/(2*a), Int[x^m/(r - s*x^(n/2)), x], x]] /; FreeQ
[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n/2] &&  !GtQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{x}}{a+cx^4}\,dx = 2\,\text{Subst}\left(\int \frac{x^2}{a+cx^8}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{x^2}{\sqrt{-a}-\sqrt{c}\,x^4}\,dx, x, \sqrt{x}\right)}{\sqrt{-a}} - \frac{\text{Subst}\left(\int \frac{x^2}{\sqrt{-a}+\sqrt{c}\,x^4}\,dx, x, \sqrt{x}\right)}{\sqrt{-a}}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}\,x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{-a}\,\sqrt[4]{c}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}+\sqrt[4]{c}\,x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{-a}\,\sqrt[4]{c}} + \frac{\text{Subst}\left(\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}\,x^2}{\sqrt{-a}+\sqrt{c}\,x^4}\,dx, x, \sqrt{x}\right)}{2\sqrt{-a}\,\sqrt[4]{c}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{5/8}c^{3/8}} - \frac{\tanh^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{5/8}c^{3/8}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[4]{-a}}{\sqrt[4]{c}} - \frac{\sqrt{2}\,\sqrt[8]{-a}\,x}{\sqrt[8]{c}} + x^2}\,dx, x, \sqrt{x}\right)}{4\sqrt{-a}\,\sqrt{c}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[4]{-a}}{\sqrt[4]{c}} + \frac{\sqrt{2}\,\sqrt[8]{-a}\,x}{\sqrt[8]{c}}}\right.}{4\sqrt{-a}\,\sqrt{}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{5/8}c^{3/8}} - \frac{\tanh^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{5/8}c^{3/8}} - \frac{\log\left(\sqrt[4]{-a} - \sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x} + \sqrt[4]{c}\,x\right)}{4\sqrt{2}\,(-a)^{5/8}c^{3/8}} + \frac{\log\left(\sqrt[4]{-a} + \sqrt{2}\,\sqrt[8]{-a}\right.}{4\sqrt{2}\,(-a)^{5}}$$

$$= \frac{\tan^{-1}\left(1 - \frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}\,(-a)^{5/8}c^{3/8}} - \frac{\tan^{-1}\left(1 + \frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}\,(-a)^{5/8}c^{3/8}} + \frac{\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{5/8}c^{3/8}} - \frac{\tanh^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{5/8}c^{3/8}} - \frac{\log\left(\sqrt[4]{-a} -\right.}{4}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.10

$$\frac{2x^{3/2}\,{}_2F_1\left(\frac{3}{8}, 1; \frac{11}{8}; -\frac{cx^4}{a}\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + c*x^4),x]
```

```
[Out] (2*x^(3/2)*Hypergeometric2F1[3/8, 1, 11/8, -((c*x^4)/a)])/(3*a)
```

fricas [B] time = 0.79, size = 460, normalized size = 1.60

$$-\frac{1}{2}\sqrt{2}\left(-\frac{1}{a^5c^3}\right)^{\frac{1}{8}}\arctan\left(\sqrt{2}\sqrt{a^4c^2\left(-\frac{1}{a^5c^3}\right)^{\frac{3}{4}} + \sqrt{2}\,a^2c\sqrt{x}\left(-\frac{1}{a^5c^3}\right)^{\frac{3}{8}} + x\,a^3c^2\left(-\frac{1}{a^5c^3}\right)^{\frac{5}{8}} - \sqrt{2}\,a^3c^2\sqrt{x}\left(-\frac{1}{a^5c^3}\right)^{\frac{5}{8}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*(-1/(a^5*c^3))^(1/8)*arctan(sqrt(2)*sqrt(a^4*c^2*(-1/(a^5*c^3)
)^(3/4) + sqrt(2)*a^2*c*sqrt(x)*(-1/(a^5*c^3))^(3/8) + x)*a^3*c^2*(-1/(a^5*
c^3))^(5/8) - sqrt(2)*a^3*c^2*sqrt(x)*(-1/(a^5*c^3))^(5/8) + 1) - 1/2*sqrt(
2)*(-1/(a^5*c^3))^(1/8)*arctan(sqrt(2)*sqrt(a^4*c^2*(-1/(a^5*c^3))^(3/4) -
sqrt(2)*a^2*c*sqrt(x)*(-1/(a^5*c^3))^(3/8) + x)*a^3*c^2*(-1/(a^5*c^3))^(5/8
) - sqrt(2)*a^3*c^2*sqrt(x)*(-1/(a^5*c^3))^(5/8) - 1) + 1/8*sqrt(2)*(-1/(a^
5*c^3))^(1/8)*log(a^4*c^2*(-1/(a^5*c^3))^(3/4) + sqrt(2)*a^2*c*sqrt(x)*(-1/
(a^5*c^3))^(3/8) + x) - 1/8*sqrt(2)*(-1/(a^5*c^3))^(1/8)*log(a^4*c^2*(-1/(a
^5*c^3))^(3/4) - sqrt(2)*a^2*c*sqrt(x)*(-1/(a^5*c^3))^(3/8) + x) + (-1/(a^5
*c^3))^(1/8)*arctan(sqrt(a^4*c^2*(-1/(a^5*c^3))^(3/4) + x)*a^3*c^2*(-1/(a^5
*c^3))^(5/8) - a^3*c^2*sqrt(x)*(-1/(a^5*c^3))^(5/8)) - 1/4*(-1/(a^5*c^3))^(
1/8)*log(a^2*c*(-1/(a^5*c^3))^(3/8) + sqrt(x)) + 1/4*(-1/(a^5*c^3))^(1/8)*l
og(-a^2*c*(-1/(a^5*c^3))^(3/8) + sqrt(x))
```

giac [B] time = 0.51, size = 445, normalized size = 1.55

$$-\frac{\left(\frac{a}{c}\right)^{\frac{3}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2a\sqrt{2\sqrt{2}+4}}-\frac{\left(\frac{a}{c}\right)^{\frac{3}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2a\sqrt{2\sqrt{2}+4}}+\frac{\left(\frac{a}{c}\right)^{\frac{3}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2a\sqrt{-2\sqrt{2}+4}}+\frac{\left(\frac{a}{c}\right)^{\frac{3}{8}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -1/2*(a/c)^(3/8)*arctan((sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(
sqrt(2) + 2)*(a/c)^(1/8)))/(a*sqrt(2*sqrt(2) + 4)) - 1/2*(a/c)^(3/8)*arctan
(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(sqrt(2) + 2)*(a/c)^(1/
8)))/(a*sqrt(2*sqrt(2) + 4)) + 1/2*(a/c)^(3/8)*arctan((sqrt(sqrt(2) + 2)*(a
/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a*sqrt(-2*sqrt(2)
 + 4)) + 1/2*(a/c)^(3/8)*arctan(-(sqrt(sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x)
)/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a*sqrt(-2*sqrt(2) + 4)) + 1/4*(a/c)^(3
/8)*log(sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a*sqrt(2*
sqrt(2) + 4)) - 1/4*(a/c)^(3/8)*log(-sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8)
+ x + (a/c)^(1/4))/(a*sqrt(2*sqrt(2) + 4)) - 1/4*(a/c)^(3/8)*log(sqrt(x)*sq
rt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a*sqrt(-2*sqrt(2) + 4)) +
1/4*(a/c)^(3/8)*log(-sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/
4))/(a*sqrt(-2*sqrt(2) + 4))
```

maple [C] time = 0.01, size = 29, normalized size = 0.10

$$\frac{\ln\left(-\operatorname{RootOf}\left(c_Z^8+a\right)+\sqrt{x}\right)}{4c\operatorname{RootOf}\left(c_Z^8+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(c*x^4+a),x)
```

```
[Out] 1/4/c*sum(1/_R^5*ln(-_R+x^(1/2)),_R=RootOf(_Z^8*c+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{cx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/(c*x^4 + a), x)
```

mupad [B] time = 0.13, size = 118, normalized size = 0.41

$$\frac{\operatorname{atan}\left(\frac{c^{1/8}\sqrt{x}}{(-a)^{1/8}}\right)}{2(-a)^{5/8}c^{3/8}}+\frac{\operatorname{atan}\left(\frac{c^{1/8}\sqrt{x}\,1\mathrm{i}}{(-a)^{1/8}}\right)1\mathrm{i}}{2(-a)^{5/8}c^{3/8}}+\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\,c^{1/8}\sqrt{x}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\left(-\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)}{(-a)^{5/8}c^{3/8}}+\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\,c^{1/8}\sqrt{x}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\left(-\frac{1}{4}-\right.}{(-a)^{5/8}c^{3/8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + c*x^4),x)
```

```
[Out] atan((c^(1/8)*x^(1/2))/(-a)^(1/8))/(2*(-a)^(5/8)*c^(3/8)) + (atan((c^(1/8)*
x^(1/2)*1i)/(-a)^(1/8))*1i)/(2*(-a)^(5/8)*c^(3/8)) - (2^(1/2)*atan((2^(1/2)
*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^(1/8))*(1/4 - 1i/4))/((-a)^(5/8)*c^(3/8
```

```
)) - (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 + 1i/2))/(-a)^(1/8))*(1/4
+ 1i/4))/((-a)^(5/8)*c^(3/8))
```

sympy [A] time = 41.00, size = 442, normalized size = 1.54

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} \\ \frac{2x^{\frac{3}{2}}}{3a} \\ -\frac{2}{5cx^{\frac{5}{2}}} \\ -\frac{(-1)^{\frac{3}{8}}\left(\frac{1}{c}\right)^{\frac{3}{8}}\log\left(-\sqrt[8]{-1}\,\sqrt[8]{a}\,\sqrt[8]{\frac{1}{c}}+\sqrt{x}\right)}{4a^{\frac{5}{8}}}+\frac{(-1)^{\frac{3}{8}}\left(\frac{1}{c}\right)^{\frac{3}{8}}\log\left(\sqrt[8]{-1}\,\sqrt[8]{a}\,\sqrt[8]{\frac{1}{c}}+\sqrt{x}\right)}{4a^{\frac{5}{8}}}+\frac{(-1)^{\frac{3}{8}}\sqrt{2}\left(\frac{1}{c}\right)^{\frac{3}{8}}\log\left(-4\sqrt[8]{-1}\,\sqrt{2}\,\sqrt[8]{a}\,\sqrt{x}\,\sqrt[8]{\frac{1}{c}}+4\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{c}}+4\right.}{8a^{\frac{5}{8}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**4+a),x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(c, 0)), (2*x**(3/2)/(3*a), Eq(c, 0))
, (-2/(5*c*x**(5/2)), Eq(a, 0)), (-(-1)**(3/8)*(1/c)**(3/8)*log(-(-1)**(1/8
)*a**(1/8)*(1/c)**(1/8) + sqrt(x))/(4*a**(5/8)) + (-1)**(3/8)*(1/c)**(3/8)*
log((-1)**(1/8)*a**(1/8)*(1/c)**(1/8) + sqrt(x))/(4*a**(5/8)) + (-1)**(3/8)
*sqrt(2)*(1/c)**(3/8)*log(-4*(-1)**(1/8)*sqrt(2)*a**(1/8)*sqrt(x)*(1/c)**(1
/8) + 4*(-1)**(1/4)*a**(1/4)*(1/c)**(1/4) + 4*x)/(8*a**(5/8)) - (-1)**(3/8)
*sqrt(2)*(1/c)**(3/8)*log(4*(-1)**(1/8)*sqrt(2)*a**(1/8)*sqrt(x)*(1/c)**(1/
8) + 4*(-1)**(1/4)*a**(1/4)*(1/c)**(1/4) + 4*x)/(8*a**(5/8)) + (-1)**(3/8)*
(1/c)**(3/8)*atan((-1)**(7/8)*sqrt(x)/(a**(1/8)*(1/c)**(1/8)))/(2*a**(5/8))
 + (-1)**(3/8)*sqrt(2)*(1/c)**(3/8)*atan(1 - (-1)**(7/8)*sqrt(2)*sqrt(x)/(a
**(1/8)*(1/c)**(1/8)))/(4*a**(5/8)) - (-1)**(3/8)*sqrt(2)*(1/c)**(3/8)*atan
(1 + (-1)**(7/8)*sqrt(2)*sqrt(x)/(a**(1/8)*(1/c)**(1/8)))/(4*a**(5/8)), Tru
e))
```

3.743 $\int \frac{1}{\sqrt{x}\left(a+cx^4\right)}\,dx$

Optimal. Leaf size=287

$$\frac{\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{4\sqrt{2}\,(-a)^{7/8}\sqrt[8]{c}}-\frac{\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{4\sqrt{2}\,(-a)^{7/8}\sqrt[8]{c}}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}\,(-a)^{7/8}\sqrt[8]{c}}-\frac{\tan^{-1}\left(1\right.}{2\sqrt{2}}$$

```
[Out] -1/2*arctan(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(7/8)/c^(1/8)-1/2*arctanh(c^(1
/8)*x^(1/2)/(-a)^(1/8))/(-a)^(7/8)/c^(1/8)-1/4*arctan(-1+c^(1/8)*2^(1/2)*x^
(1/2)/(-a)^(1/8))/(-a)^(7/8)/c^(1/8)*2^(1/2)-1/4*arctan(1+c^(1/8)*2^(1/2)*x
^(1/2)/(-a)^(1/8))/(-a)^(7/8)/c^(1/8)*2^(1/2)+1/8*ln((-a)^(1/4)+c^(1/4)*x-(
-a)^(1/8)*c^(1/8)*2^(1/2)*x^(1/2))/(-a)^(7/8)/c^(1/8)*2^(1/2)-1/8*ln((-a)^(
1/4)+c^(1/4)*x+(-a)^(1/8)*c^(1/8)*2^(1/2)*x^(1/2))/(-a)^(7/8)/c^(1/8)*2^(1/
2)
```

Rubi [A] time = 0.22, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$, Rules used = {329, 214, 212, 208, 205, 211, 1165, 628, 1162, 617, 204}

$$\frac{\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{4\sqrt{2}\,(-a)^{7/8}\sqrt[8]{c}}-\frac{\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{4\sqrt{2}\,(-a)^{7/8}\sqrt[8]{c}}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}\,(-a)^{7/8}\sqrt[8]{c}}-\frac{\tan^{-1}\left(1\right.}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + c*x^4)),x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*Sqrt[2]*(-a)^(7/8)*c^(1
/8)) - ArcTan[1 + (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*Sqrt[2]*(-a)^(7/
8)*c^(1/8)) - ArcTan[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*(-a)^(7/8)*c^(1/8)) -
 ArcTanh[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*(-a)^(7/8)*c^(1/8)) + Log[(-a)^(1
/4) - Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x]/(4*Sqrt[2]*(-a)^(7/8)
*c^(1/8)) - Log[(-a)^(1/4) + Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x
]/(4*Sqrt[2]*(-a)^(7/8)*c^(1/8))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(a+cx^4\right)}\,dx = 2\,\text{Subst}\left(\int \frac{1}{a+cx^8}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{-a}-\sqrt{c}\,x^4}\,dx, x, \sqrt{x}\right)}{\sqrt{-a}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{-a}+\sqrt{c}\,x^4}\,dx, x, \sqrt{x}\right)}{\sqrt{-a}}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}\,x^2}\,dx, x, \sqrt{x}\right)}{2(-a)^{3/4}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}+\sqrt[4]{c}\,x^2}\,dx, x, \sqrt{x}\right)}{2(-a)^{3/4}} - \frac{\text{Subst}\left(\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}\,x}{\sqrt{-a}+\sqrt{c}\,x}\right.}{2(-a)^{3/}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{7/8}\sqrt[8]{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{7/8}\sqrt[8]{c}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[4]{-a}}{\sqrt[4]{c}} - \frac{\sqrt{2}\,\sqrt[8]{-a}\,x}{\sqrt[8]{c}} + x^2}\,dx, x, \sqrt{x}\right)}{4(-a)^{3/4}\sqrt[4]{c}} - \frac{\text{Subst}\left(\int -\right.}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{7/8}\sqrt[8]{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{7/8}\sqrt[8]{c}} + \frac{\log\left(\sqrt[4]{-a} - \sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x} + \sqrt[4]{c}\,x\right)}{4\sqrt{2}\,(-a)^{7/8}\sqrt[8]{c}} - \frac{\log\left(\sqrt[4]{-a}\right.}{}$$

$$= \frac{\tan^{-1}\left(1 - \frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}\,(-a)^{7/8}\sqrt[8]{c}} - \frac{\tan^{-1}\left(1 + \frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}\,(-a)^{7/8}\sqrt[8]{c}} - \frac{\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{7/8}\sqrt[8]{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{7/8}\sqrt[8]{c}} + \frac{\log}{}$$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.09

$$\frac{2\sqrt{x}\;{}_2F_1\left(\frac{1}{8}, 1; \frac{9}{8}; -\frac{cx^4}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + c*x^4)),x]
```

```
[Out] (2*Sqrt[x]*Hypergeometric2F1[1/8, 1, 9/8, -((c*x^4)/a)])/a
```

fricas [B] time = 0.76, size = 415, normalized size = 1.45

$$\frac{1}{2}\sqrt{2}\left(-\frac{1}{a^7c}\right)^{\frac{1}{8}}\arctan\left(\sqrt{2}\sqrt{a^2\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}} + \sqrt{2}\,a\sqrt{x}\left(-\frac{1}{a^7c}\right)^{\frac{1}{8}} + x}\,a^6c\left(-\frac{1}{a^7c}\right)^{\frac{7}{8}} - \sqrt{2}\,a^6c\sqrt{x}\left(-\frac{1}{a^7c}\right)^{\frac{7}{8}} + 1\right) + \frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*(-1/(a^7*c))^(1/8)*arctan(sqrt(2)*sqrt(a^2*(-1/(a^7*c))^(1/4) +
 sqrt(2)*a*sqrt(x)*(-1/(a^7*c))^(1/8) + x)*a^6*c*(-1/(a^7*c))^(7/8) - sqrt(
2)*a^6*c*sqrt(x)*(-1/(a^7*c))^(7/8) + 1) + 1/2*sqrt(2)*(-1/(a^7*c))^(1/8)*a
rctan(sqrt(2)*sqrt(a^2*(-1/(a^7*c))^(1/4) - sqrt(2)*a*sqrt(x)*(-1/(a^7*c))^
(1/8) + x)*a^6*c*(-1/(a^7*c))^(7/8) - sqrt(2)*a^6*c*sqrt(x)*(-1/(a^7*c))^(7
/8) - 1) + 1/8*sqrt(2)*(-1/(a^7*c))^(1/8)*log(a^2*(-1/(a^7*c))^(1/4) + sqrt
(2)*a*sqrt(x)*(-1/(a^7*c))^(1/8) + x) - 1/8*sqrt(2)*(-1/(a^7*c))^(1/8)*log(
a^2*(-1/(a^7*c))^(1/4) - sqrt(2)*a*sqrt(x)*(-1/(a^7*c))^(1/8) + x) + (-1/(a
^7*c))^(1/8)*arctan(sqrt(a^2*(-1/(a^7*c))^(1/4) + x)*a^6*c*(-1/(a^7*c))^(7/
8) - a^6*c*sqrt(x)*(-1/(a^7*c))^(7/8)) + 1/4*(-1/(a^7*c))^(1/8)*log(a*(-1/(
a^7*c))^(1/8) + sqrt(x)) - 1/4*(-1/(a^7*c))^(1/8)*log(-a*(-1/(a^7*c))^(1/8)
 + sqrt(x))
```

giac [B] time = 0.36, size = 445, normalized size = 1.55

$$\frac{\left(\frac{a}{c}\right)^{\frac{1}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2a\sqrt{-2\sqrt{2}+4}}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2a\sqrt{-2\sqrt{2}+4}}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2a\sqrt{2\sqrt{2}+4}}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{8}}\text{arct}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)/x^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(a/c)^(1/8)*arctan((sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(s
qrt(2) + 2)*(a/c)^(1/8)))/(a*sqrt(-2*sqrt(2) + 4)) + 1/2*(a/c)^(1/8)*arctan
(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(sqrt(2) + 2)*(a/c)^(1/
8)))/(a*sqrt(-2*sqrt(2) + 4)) + 1/2*(a/c)^(1/8)*arctan((sqrt(sqrt(2) + 2)*(
a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a*sqrt(2*sqrt(2)
 + 4)) + 1/2*(a/c)^(1/8)*arctan(-(sqrt(sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x)
)/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a*sqrt(2*sqrt(2) + 4)) + 1/4*(a/c)^(1/
8)*log(sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a*sqrt(-2*
sqrt(2) + 4)) - 1/4*(a/c)^(1/8)*log(-sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8)
+ x + (a/c)^(1/4))/(a*sqrt(-2*sqrt(2) + 4)) + 1/4*(a/c)^(1/8)*log(sqrt(x)*s
qrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a*sqrt(2*sqrt(2) + 4)) -
1/4*(a/c)^(1/8)*log(-sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/
4))/(a*sqrt(2*sqrt(2) + 4))
```

maple [C] time = 0.00, size = 29, normalized size = 0.10

$$\frac{\ln\left(-\text{RootOf}\left(c_Z^8+a\right)+\sqrt{x}\right)}{4c\,\text{RootOf}\left(c_Z^8+a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+a)/x^(1/2),x)
```

```
[Out] 1/4/c*sum(1/_R^7*ln(-_R+x^(1/2)),_R=RootOf(_Z^8*c+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-c\int\frac{x^{\frac{7}{2}}}{acx^4+a^2}\,dx+\frac{2\sqrt{x}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -c*integrate(x^(7/2)/(a*c*x^4 + a^2), x) + 2*sqrt(x)/a
```

mupad [B] time = 1.10, size = 118, normalized size = 0.41

$$-\frac{\text{atan}\left(\frac{c^{1/8}\sqrt{x}}{(-a)^{1/8}}\right)}{2(-a)^{7/8}c^{1/8}}+\frac{\text{atan}\left(\frac{c^{1/8}\sqrt{x}\,1\text{i}}{(-a)^{1/8}}\right)1\text{i}}{2(-a)^{7/8}c^{1/8}}+\frac{\sqrt{2}\,\text{atan}\left(\frac{\sqrt{2}\,c^{1/8}\sqrt{x}\left(\frac{1}{2}-\frac{1}{2}\text{i}\right)}{(-a)^{1/8}}\right)\left(-\frac{1}{4}-\frac{1}{4}\text{i}\right)}{(-a)^{7/8}c^{1/8}}+\frac{\sqrt{2}\,\text{atan}\left(\frac{\sqrt{2}\,c^{1/8}\sqrt{x}\left(\frac{1}{2}+\frac{1}{2}\text{i}\right)}{(-a)^{1/8}}\right)\left(-\frac{1}{4}+\frac{1}{4}\right.}{(-a)^{7/8}c^{1/8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + c*x^4)),x)
```

```
[Out] (atan((c^(1/8)*x^(1/2)*1i)/(-a)^(1/8))*1i)/(2*(-a)^(7/8)*c^(1/8)) - atan((c
^(1/8)*x^(1/2))/(-a)^(1/8))/(2*(-a)^(7/8)*c^(1/8)) - (2^(1/2)*atan((2^(1/2)
```

```
*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^(1/8))*(1/4 + 1i/4))/((-a)^(7/8)*c^(1/8
)) - (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 + 1i/2))/(-a)^(1/8))*(1/4
- 1i/4))/((-a)^(7/8)*c^(1/8))
```

sympy [A] time = 58.92, size = 440, normalized size = 1.53

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \\ \frac{2\sqrt{x}}{a} & \\ -\frac{2}{7cx^{\frac{7}{2}}} & \\ -\frac{\sqrt[8]{-1}\sqrt[8]{\frac{1}{c}}\log\left(-\sqrt[8]{-1}\sqrt[8]{a}\sqrt[8]{\frac{1}{c}}+\sqrt{x}\right)}{4a^{\frac{7}{8}}}+\frac{\sqrt[8]{-1}\sqrt[8]{\frac{1}{c}}\log\left(\sqrt[8]{-1}\sqrt[8]{a}\sqrt[8]{\frac{1}{c}}+\sqrt{x}\right)}{4a^{\frac{7}{8}}}-\frac{\sqrt[8]{-1}\sqrt{2}\sqrt[8]{\frac{1}{c}}\log\left(-4\sqrt[8]{-1}\sqrt{2}\sqrt[8]{a}\sqrt{x}\sqrt[8]{\frac{1}{c}}+4\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{c}}+4x\right.}{8a^{\frac{7}{8}}} & \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+a)/x**(1/2),x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(c, 0)), (2*sqrt(x)/a, Eq(c, 0)), (-2
/(7*c*x**(7/2)), Eq(a, 0)), (-(-1)**(1/8)*(1/c)**(1/8)*log(-(-1)**(1/8)*a**
(1/8)*(1/c)**(1/8) + sqrt(x))/(4*a**(7/8)) + (-1)**(1/8)*(1/c)**(1/8)*log((
-1)**(1/8)*a**(1/8)*(1/c)**(1/8) + sqrt(x))/(4*a**(7/8)) - (-1)**(1/8)*sqrt
(2)*(1/c)**(1/8)*log(-4*(-1)**(1/8)*sqrt(2)*a**(1/8)*sqrt(x)*(1/c)**(1/8) +
 4*(-1)**(1/4)*a**(1/4)*(1/c)**(1/4) + 4*x)/(8*a**(7/8)) + (-1)**(1/8)*sqrt
(2)*(1/c)**(1/8)*log(4*(-1)**(1/8)*sqrt(2)*a**(1/8)*sqrt(x)*(1/c)**(1/8) +
4*(-1)**(1/4)*a**(1/4)*(1/c)**(1/4) + 4*x)/(8*a**(7/8)) - (-1)**(1/8)*(1/c)
**(1/8)*atan((-1)**(7/8)*sqrt(x)/(a**(1/8)*(1/c)**(1/8)))/(2*a**(7/8)) + (-
1)**(1/8)*sqrt(2)*(1/c)**(1/8)*atan(1 - (-1)**(7/8)*sqrt(2)*sqrt(x)/(a**(1/
8)*(1/c)**(1/8)))/(4*a**(7/8)) - (-1)**(1/8)*sqrt(2)*(1/c)**(1/8)*atan(1 +
(-1)**(7/8)*sqrt(2)*sqrt(x)/(a**(1/8)*(1/c)**(1/8)))/(4*a**(7/8)), True))
```

3.744 $\int \frac{1}{x^{3/2}\left(a+cx^4\right)}\,dx$

Optimal. Leaf size=297

$$\frac{\sqrt[8]{c}\,\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{4\sqrt{2}\,(-a)^{9/8}}-\frac{\sqrt[8]{c}\,\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{4\sqrt{2}\,(-a)^{9/8}}-\frac{\sqrt[8]{c}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}\,(-a)^{9/8}}+$$

```
[Out] 1/2*c^(1/8)*arctan(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(9/8)-1/2*c^(1/8)*arcta
nh(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(9/8)+1/4*c^(1/8)*arctan(-1+c^(1/8)*2^(
1/2)*x^(1/2)/(-a)^(1/8))/(-a)^(9/8)*2^(1/2)+1/4*c^(1/8)*arctan(1+c^(1/8)*2^
(1/2)*x^(1/2)/(-a)^(1/8))/(-a)^(9/8)*2^(1/2)+1/8*c^(1/8)*ln((-a)^(1/4)+c^(1
/4)*x-(-a)^(1/8)*c^(1/8)*2^(1/2)*x^(1/2))/(-a)^(9/8)*2^(1/2)-1/8*c^(1/8)*ln
((-a)^(1/4)+c^(1/4)*x+(-a)^(1/8)*c^(1/8)*2^(1/2)*x^(1/2))/(-a)^(9/8)*2^(1/2
)-2/a/x^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {325, 329, 301, 297, 1162, 617, 204, 1165, 628, 298, 205, 208}

$$\frac{\sqrt[8]{c}\,\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{4\sqrt{2}\,(-a)^{9/8}}-\frac{\sqrt[8]{c}\,\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{4\sqrt{2}\,(-a)^{9/8}}-\frac{\sqrt[8]{c}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}\,(-a)^{9/8}}+$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + c*x^4)),x]
```

```
[Out] -2/(a*Sqrt[x]) - (c^(1/8)*ArcTan[1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)])
/(2*Sqrt[2]*(-a)^(9/8)) + (c^(1/8)*ArcTan[1 + (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a
)^(1/8)])/(2*Sqrt[2]*(-a)^(9/8)) + (c^(1/8)*ArcTan[(c^(1/8)*Sqrt[x])/(-a)^(
1/8)])/(2*(-a)^(9/8)) - (c^(1/8)*ArcTanh[(c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(2*
(-a)^(9/8)) + (c^(1/8)*Log[(-a)^(1/4) - Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x]
+ c^(1/4)*x])/(4*Sqrt[2]*(-a)^(9/8)) - (c^(1/8)*Log[(-a)^(1/4) + Sqrt[2]*(-
a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x])/(4*Sqrt[2]*(-a)^(9/8))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 301

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[x^(m - n/2
)/(r + s*x^(n/2)), x], x] - Dist[s/(2*b), Int[x^(m - n/2)/(r - s*x^(n/2)),
x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] &&
 LtQ[m, n] &&  !GtQ[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(a+cx^4\right)}\,dx = -\frac{2}{a\sqrt{x}} - \frac{c\int \frac{x^{5/2}}{a+cx^4}\,dx}{a}$$

$$= -\frac{2}{a\sqrt{x}} - \frac{(2c)\,\text{Subst}\left(\int \frac{x^6}{a+cx^8}\,dx, x, \sqrt{x}\right)}{a}$$

$$= -\frac{2}{a\sqrt{x}} + \frac{\sqrt{c}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{-a}-\sqrt{c}x^4}\,dx, x, \sqrt{x}\right)}{a} - \frac{\sqrt{c}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{-a}+\sqrt{c}x^4}\,dx, x, \sqrt{x}\right)}{a}$$

$$= -\frac{2}{a\sqrt{x}} + \frac{\sqrt[4]{c}\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x^2}\,dx, x, \sqrt{x}\right)}{2a} - \frac{\sqrt[4]{c}\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}+\sqrt[4]{c}x^2}\,dx, x, \sqrt{x}\right)}{2a} + \frac{\sqrt[4]{c}\,\text{S}}{}$$

$$= -\frac{2}{a\sqrt{x}} + \frac{\sqrt[8]{c}\,\tan^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{9/8}} - \frac{\sqrt[8]{c}\,\tanh^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{9/8}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[4]{-a}}{\sqrt[4]{c}} - \frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{c}} + x^2}\,dx, x, \sqrt{x}\right)}{4a}$$

$$= -\frac{2}{a\sqrt{x}} + \frac{\sqrt[8]{c}\,\tan^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{9/8}} - \frac{\sqrt[8]{c}\,\tanh^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{9/8}} + \frac{\sqrt[8]{c}\,\log\left(\sqrt[4]{-a} - \sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x} + \sqrt[4]{\right.}}{4\sqrt{2}\,(-a)^{9/8}}$$

$$= -\frac{2}{a\sqrt{x}} - \frac{\sqrt[8]{c}\,\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}\,(-a)^{9/8}} + \frac{\sqrt[8]{c}\,\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}\,(-a)^{9/8}} + \frac{\sqrt[8]{c}\,\tan^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{9/8}} - \frac{\sqrt[8]{c}\,t}{}$$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.09

$$-\frac{2\,{}_2F_1\left(-\frac{1}{8}, 1; \frac{7}{8}; -\frac{cx^4}{a}\right)}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + c*x^4)),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/8, 1, 7/8, -((c*x^4)/a)])/(a*Sqrt[x])
```

fricas [B] time = 0.69, size = 446, normalized size = 1.50

$$\frac{4\sqrt{2}\,ax\left(-\frac{c}{a^9}\right)^{\frac{1}{8}}\arctan\left(-\frac{\sqrt{2}\,ac\sqrt{x}\left(-\frac{c}{a^9}\right)^{\frac{1}{8}} - \sqrt{2}\sqrt{\sqrt{2}\,a^8c\sqrt{x}\left(-\frac{c}{a^9}\right)^{\frac{7}{8}} - a^7c\left(-\frac{c}{a^9}\right)^{\frac{3}{4}} + c^2x}\,a\left(-\frac{c}{a^9}\right)^{\frac{1}{8}} - c}{c}\right) + 4\sqrt{2}\,ax\left(-\frac{c}{a^9}\right)^{\frac{1}{8}}\arctan\left(-\frac{\sqrt{2}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/8*(4*sqrt(2)*a*x*(-c/a^9)^(1/8)*arctan(-(sqrt(2)*a*c*sqrt(x)*(-c/a^9)^(1/
8) - sqrt(2)*sqrt(sqrt(2)*a^8*c*sqrt(x)*(-c/a^9)^(7/8) - a^7*c*(-c/a^9)^(3/
4) + c^2*x)*a*(-c/a^9)^(1/8) - c)/c) + 4*sqrt(2)*a*x*(-c/a^9)^(1/8)*arctan(
-(sqrt(2)*a*c*sqrt(x)*(-c/a^9)^(1/8) - sqrt(2)*sqrt(-sqrt(2)*a^8*c*sqrt(x)*
(-c/a^9)^(7/8) - a^7*c*(-c/a^9)^(3/4) + c^2*x)*a*(-c/a^9)^(1/8) + c)/c) - s
qrt(2)*a*x*(-c/a^9)^(1/8)*log(sqrt(2)*a^8*c*sqrt(x)*(-c/a^9)^(7/8) - a^7*c*
(-c/a^9)^(3/4) + c^2*x) + sqrt(2)*a*x*(-c/a^9)^(1/8)*log(-sqrt(2)*a^8*c*sqr
```

```
t(x)*(-c/a^9)^(7/8) - a^7*c*(-c/a^9)^(3/4) + c^2*x) + 8*a*x*(-c/a^9)^(1/8)*
arctan(-(a*c*sqrt(x)*(-c/a^9)^(1/8) - sqrt(-a^7*c*(-c/a^9)^(3/4) + c^2*x)*a
*(-c/a^9)^(1/8))/c) - 2*a*x*(-c/a^9)^(1/8)*log(a^8*(-c/a^9)^(7/8) + c*sqrt(
x)) + 2*a*x*(-c/a^9)^(1/8)*log(-a^8*(-c/a^9)^(7/8) + c*sqrt(x)) - 16*sqrt(x
))/(a*x)
```

giac [B] time = 0.55, size = 461, normalized size = 1.55

$$-\frac{c\left(\frac{a}{c}\right)^{\frac{7}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2a^2\sqrt{-2\sqrt{2}+4}}-\frac{c\left(\frac{a}{c}\right)^{\frac{7}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2a^2\sqrt{-2\sqrt{2}+4}}-\frac{c\left(\frac{a}{c}\right)^{\frac{7}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2a^2\sqrt{2\sqrt{2}+4}}-c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -1/2*c*(a/c)^(7/8)*arctan((sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqr
t(sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*sqrt(-2*sqrt(2) + 4)) - 1/2*c*(a/c)^(7/8)
*arctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(sqrt(2) + 2)*(a
/c)^(1/8)))/(a^2*sqrt(-2*sqrt(2) + 4)) - 1/2*c*(a/c)^(7/8)*arctan((sqrt(sqr
t(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*s
qrt(2*sqrt(2) + 4)) - 1/2*c*(a/c)^(7/8)*arctan(-(sqrt(sqrt(2) + 2)*(a/c)^(1
/8) - 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*sqrt(2*sqrt(2) + 4)
) + 1/4*c*(a/c)^(7/8)*log(sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)
^(1/4))/(a^2*sqrt(-2*sqrt(2) + 4)) - 1/4*c*(a/c)^(7/8)*log(-sqrt(x)*sqrt(sq
rt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*sqrt(-2*sqrt(2) + 4)) + 1/4*
c*(a/c)^(7/8)*log(sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))
/(a^2*sqrt(2*sqrt(2) + 4)) - 1/4*c*(a/c)^(7/8)*log(-sqrt(x)*sqrt(-sqrt(2) +
 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*sqrt(2*sqrt(2) + 4)) - 2/(a*sqrt(x)
)
```

maple [C] time = 0.01, size = 38, normalized size = 0.13

$$-\frac{\ln\left(-\operatorname{RootOf}\left(c_Z^8+a\right)+\sqrt{x}\right)}{4a\operatorname{RootOf}\left(c_Z^8+a\right)}-\frac{2}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(c*x^4+a),x)
```

```
[Out] -1/4/a*sum(1/_R*ln(-_R+x^(1/2)),_R=RootOf(_Z^8*c+a))-2/a/x^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-c\int\frac{x^{\frac{5}{2}}}{acx^4+a^2}\,dx-\frac{2}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] -c*integrate(x^(5/2)/(a*c*x^4 + a^2), x) - 2/(a*sqrt(x))
```

mupad [B] time = 1.09, size = 126, normalized size = 0.42

$$-\frac{2}{a\sqrt{x}}-\frac{(-c)^{1/8}\operatorname{atan}\left(\frac{(-c)^{1/8}\sqrt{x}}{a^{1/8}}\right)}{2a^{9/8}}-\frac{(-c)^{1/8}\operatorname{atan}\left(\frac{(-c)^{1/8}\sqrt{x}\,1\mathrm{i}}{a^{1/8}}\right)1\mathrm{i}}{2a^{9/8}}+\frac{\sqrt{2}\,(-c)^{1/8}\operatorname{atan}\left(\frac{\sqrt{2}\,(-c)^{1/8}\sqrt{x}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{a^{1/8}}\right)\left(-\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)}{a^{9/8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + c*x^4)),x)
```

```
[Out] - 2/(a*x^(1/2)) - ((-c)^(1/8)*atan(((-c)^(1/8)*x^(1/2))/a^(1/8)))/(2*a^(9/8
)) - ((-c)^(1/8)*atan(((-c)^(1/8)*x^(1/2)*1i)/a^(1/8))*1i)/(2*a^(9/8)) - (2
^(1/2)*(-c)^(1/8)*atan((2^(1/2)*(-c)^(1/8)*x^(1/2)*(1/2 - 1i/2))/a^(1/8))*(
1/4 - 1i/4))/a^(9/8) - (2^(1/2)*(-c)^(1/8)*atan((2^(1/2)*(-c)^(1/8)*x^(1/2)
*(1/2 + 1i/2))/a^(1/8))*(1/4 + 1i/4))/a^(9/8)
```

sympy [A] time = 148.29, size = 450, normalized size = 1.52

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{9}{2}}} \\ -\frac{2}{9cx^{\frac{9}{2}}} \\ -\frac{2}{a\sqrt{x}} \\ -\frac{2}{a\sqrt{x}} + \frac{(-1)^{\frac{7}{8}}\log\left(-\sqrt[8]{-1}\sqrt[8]{a}\sqrt[8]{\frac{1}{c}}+\sqrt{x}\right)}{4a^{\frac{9}{8}}\sqrt[8]{\frac{1}{c}}} - \frac{(-1)^{\frac{7}{8}}\log\left(\sqrt[8]{-1}\sqrt[8]{a}\sqrt[8]{\frac{1}{c}}+\sqrt{x}\right)}{4a^{\frac{9}{8}}\sqrt[8]{\frac{1}{c}}} + \frac{(-1)^{\frac{7}{8}}\sqrt{2}\log\left(-4\sqrt[8]{-1}\sqrt{2}\sqrt[8]{a}\sqrt{x}\sqrt[8]{\frac{1}{c}}+4\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{c}}+4x\right)}{8a^{\frac{9}{8}}\sqrt[8]{\frac{1}{c}}} - \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(c*x**4+a),x)
```

```
[Out] Piecewise((zoo/x**(9/2), Eq(a, 0) & Eq(c, 0)), (-2/(9*c*x**(9/2)), Eq(a, 0)
), (-2/(a*sqrt(x)), Eq(c, 0)), (-2/(a*sqrt(x)) + (-1)**(7/8)*log(-(-1)**(1/
8)*a**(1/8)*(1/c)**(1/8) + sqrt(x))/(4*a**(9/8)*(1/c)**(1/8)) - (-1)**(7/8)
*log((-1)**(1/8)*a**(1/8)*(1/c)**(1/8) + sqrt(x))/(4*a**(9/8)*(1/c)**(1/8))
 + (-1)**(7/8)*sqrt(2)*log(-4*(-1)**(1/8)*sqrt(2)*a**(1/8)*sqrt(x)*(1/c)**(
1/8) + 4*(-1)**(1/4)*a**(1/4)*(1/c)**(1/4) + 4*x)/(8*a**(9/8)*(1/c)**(1/8))
 - (-1)**(7/8)*sqrt(2)*log(4*(-1)**(1/8)*sqrt(2)*a**(1/8)*sqrt(x)*(1/c)**(1
/8) + 4*(-1)**(1/4)*a**(1/4)*(1/c)**(1/4) + 4*x)/(8*a**(9/8)*(1/c)**(1/8))
- (-1)**(7/8)*atan((-1)**(7/8)*sqrt(x)/(a**(1/8)*(1/c)**(1/8)))/(2*a**(9/8)
*(1/c)**(1/8)) + (-1)**(7/8)*sqrt(2)*atan(1 - (-1)**(7/8)*sqrt(2)*sqrt(x)/(
a**(1/8)*(1/c)**(1/8)))/(4*a**(9/8)*(1/c)**(1/8)) - (-1)**(7/8)*sqrt(2)*ata
n(1 + (-1)**(7/8)*sqrt(2)*sqrt(x)/(a**(1/8)*(1/c)**(1/8)))/(4*a**(9/8)*(1/c
)**(1/8)), True))
```

3.745 $\int \frac{1}{x^{5/2}\left(a+cx^4\right)}\,dx$

Optimal. Leaf size=299

$$-\frac{c^{3/8}\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{4\sqrt{2}\,(-a)^{11/8}}+\frac{c^{3/8}\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{4\sqrt{2}\,(-a)^{11/8}}-\frac{c^{3/8}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{}}{\sqrt[8]{-a}}\right.}{2\sqrt{2}\,(-a)^{11/8}}$$

```
[Out] -2/3/a/x^(3/2)-1/2*c^(3/8)*arctan(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(11/8)-1
/2*c^(3/8)*arctanh(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(11/8)+1/4*c^(3/8)*arct
an(-1+c^(1/8)*2^(1/2)*x^(1/2)/(-a)^(1/8))/(-a)^(11/8)*2^(1/2)+1/4*c^(3/8)*a
rctan(1+c^(1/8)*2^(1/2)*x^(1/2)/(-a)^(1/8))/(-a)^(11/8)*2^(1/2)-1/8*c^(3/8)
*ln((-a)^(1/4)+c^(1/4)*x-(-a)^(1/8)*c^(1/8)*2^(1/2)*x^(1/2))/(-a)^(11/8)*2^
(1/2)+1/8*c^(3/8)*ln((-a)^(1/4)+c^(1/4)*x+(-a)^(1/8)*c^(1/8)*2^(1/2)*x^(1/2
))/(-a)^(11/8)*2^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {325, 329, 301, 211, 1165, 628, 1162, 617, 204, 212, 208, 205}

$$-\frac{c^{3/8}\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{4\sqrt{2}\,(-a)^{11/8}}+\frac{c^{3/8}\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{4\sqrt{2}\,(-a)^{11/8}}-\frac{c^{3/8}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{}}{\sqrt[8]{-a}}\right.}{2\sqrt{2}\,(-a)^{11/8}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + c*x^4)),x]
```

```
[Out] -2/(3*a*x^(3/2)) - (c^(3/8)*ArcTan[1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)
])/(2*Sqrt[2]*(-a)^(11/8)) + (c^(3/8)*ArcTan[1 + (Sqrt[2]*c^(1/8)*Sqrt[x])/
(-a)^(1/8)])/(2*Sqrt[2]*(-a)^(11/8)) - (c^(3/8)*ArcTan[(c^(1/8)*Sqrt[x])/(-
a)^(1/8)])/(2*(-a)^(11/8)) - (c^(3/8)*ArcTanh[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]
)/(2*(-a)^(11/8)) - (c^(3/8)*Log[(-a)^(1/4) - Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sq
rt[x] + c^(1/4)*x])/(4*Sqrt[2]*(-a)^(11/8)) + (c^(3/8)*Log[(-a)^(1/4) + Sqr
t[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x])/(4*Sqrt[2]*(-a)^(11/8))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 301

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[x^(m - n/2
)/(r + s*x^(n/2)), x], x] - Dist[s/(2*b), Int[x^(m - n/2)/(r - s*x^(n/2)),
x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] &&
 LtQ[m, n] &&  !GtQ[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{5/2}\left(a+cx^4\right)}\,dx = -\frac{2}{3ax^{3/2}} - \frac{c\int \frac{x^{3/2}}{a+cx^4}\,dx}{a}$$

$$= -\frac{2}{3ax^{3/2}} - \frac{(2c)\,\text{Subst}\left(\int \frac{x^4}{a+cx^8}\,dx, x, \sqrt{x}\right)}{a}$$

$$= -\frac{2}{3ax^{3/2}} + \frac{\sqrt{c}\,\text{Subst}\left(\int \frac{1}{\sqrt{-a}-\sqrt{c}\,x^4}\,dx, x, \sqrt{x}\right)}{a} - \frac{\sqrt{c}\,\text{Subst}\left(\int \frac{1}{\sqrt{-a}+\sqrt{c}\,x^4}\,dx, x, \sqrt{x}\right)}{a}$$

$$= -\frac{2}{3ax^{3/2}} - \frac{\sqrt{c}\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}\,x^2}\,dx, x, \sqrt{x}\right)}{2(-a)^{5/4}} - \frac{\sqrt{c}\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}+\sqrt[4]{c}\,x^2}\,dx, x, \sqrt{x}\right)}{2(-a)^{5/4}} + -$$

$$= -\frac{2}{3ax^{3/2}} - \frac{c^{3/8}\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{11/8}} - \frac{c^{3/8}\tanh^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{11/8}} + \frac{\sqrt[4]{c}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[4]{-a}}{\sqrt[4]{c}} - \frac{\sqrt{2}\,\sqrt[8]{-a}\,x}{\sqrt[8]{c}} + x^2}\,dx,\right.}{4(-a)^{5/4}}$$

$$= -\frac{2}{3ax^{3/2}} - \frac{c^{3/8}\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{11/8}} - \frac{c^{3/8}\tanh^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{11/8}} - \frac{c^{3/8}\log\left(\sqrt[4]{-a} - \sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}\right.}{4\sqrt{2}\,(-a)^{11/8}}$$

$$= -\frac{2}{3ax^{3/2}} - \frac{c^{3/8}\tan^{-1}\left(1 - \frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}\,(-a)^{11/8}} + \frac{c^{3/8}\tan^{-1}\left(1 + \frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}\,(-a)^{11/8}} - \frac{c^{3/8}\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{11/8}} -$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.10

$$-\frac{2\,{}_2F_1\left(-\frac{3}{8}, 1; \frac{5}{8}; -\frac{cx^4}{a}\right)}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + c*x^4)),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/8, 1, 5/8, -((c*x^4)/a)])/(3*a*x^(3/2))
```

fricas [B] time = 0.85, size = 553, normalized size = 1.85

$$12\sqrt{2}\,ax^2\left(-\frac{c^3}{a^{11}}\right)^{\frac{1}{8}}\arctan\left(-\frac{\sqrt{2}\,a^4c^2\sqrt{x}\left(-\frac{c^3}{a^{11}}\right)^{\frac{3}{8}} - \sqrt{2}\,\sqrt{\sqrt{2}\,a^7c^2\sqrt{x}\left(-\frac{c^3}{a^{11}}\right)^{\frac{5}{8}} - a^3c^3\left(-\frac{c^3}{a^{11}}\right)^{\frac{1}{4}} + c^4x\,a^4\left(-\frac{c^3}{a^{11}}\right)^{\frac{3}{8}} - c^3}}{c^3}\right) + 12\sqrt{2}\,ax^2\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/24*(12*sqrt(2)*a*x^2*(-c^3/a^11)^(1/8)*arctan(-(sqrt(2)*a^4*c^2*sqrt(x)*
(-c^3/a^11)^(3/8) - sqrt(2)*sqrt(sqrt(2)*a^7*c^2*sqrt(x)*(-c^3/a^11)^(5/8)
- a^3*c^3*(-c^3/a^11)^(1/4) + c^4*x)*a^4*(-c^3/a^11)^(3/8) - c^3)/c^3) + 12
*sqrt(2)*a*x^2*(-c^3/a^11)^(1/8)*arctan(-(sqrt(2)*a^4*c^2*sqrt(x)*(-c^3/a^1
1)^(3/8) - sqrt(2)*sqrt(-sqrt(2)*a^7*c^2*sqrt(x)*(-c^3/a^11)^(5/8) - a^3*c^
3*(-c^3/a^11)^(1/4) + c^4*x)*a^4*(-c^3/a^11)^(3/8) + c^3)/c^3) + 3*sqrt(2)*
a*x^2*(-c^3/a^11)^(1/8)*log(sqrt(2)*a^7*c^2*sqrt(x)*(-c^3/a^11)^(5/8) - a^3
*c^3*(-c^3/a^11)^(1/4) + c^4*x) - 3*sqrt(2)*a*x^2*(-c^3/a^11)^(1/8)*log(-sq
```

```
rt(2)*a^7*c^2*sqrt(x)*(-c^3/a^11)^(5/8) - a^3*c^3*(-c^3/a^11)^(1/4) + c^4*x
) - 24*a*x^2*(-c^3/a^11)^(1/8)*arctan(-(a^4*c^2*sqrt(x)*(-c^3/a^11)^(3/8) -
 sqrt(-a^3*c^3*(-c^3/a^11)^(1/4) + c^4*x)*a^4*(-c^3/a^11)^(3/8))/c^3) - 6*a
*x^2*(-c^3/a^11)^(1/8)*log(a^7*(-c^3/a^11)^(5/8) + c^2*sqrt(x)) + 6*a*x^2*(
-c^3/a^11)^(1/8)*log(-a^7*(-c^3/a^11)^(5/8) + c^2*sqrt(x)) + 16*sqrt(x))/(a
*x^2)
```

giac [B] time = 0.55, size = 461, normalized size = 1.54

$$\frac{c\left(\frac{a}{c}\right)^{\frac{5}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2a^2\sqrt{2\sqrt{2}+4}}+\frac{c\left(\frac{a}{c}\right)^{\frac{5}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2a^2\sqrt{2\sqrt{2}+4}}-\frac{c\left(\frac{a}{c}\right)^{\frac{5}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2a^2\sqrt{-2\sqrt{2}+4}}-\frac{c\left(\frac{a}{c}\right)^{\frac{5}{8}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*c*(a/c)^(5/8)*arctan((sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt
(sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*sqrt(2*sqrt(2) + 4)) + 1/2*c*(a/c)^(5/8)*a
rctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(sqrt(2) + 2)*(a/c
)^(1/8)))/(a^2*sqrt(2*sqrt(2) + 4)) - 1/2*c*(a/c)^(5/8)*arctan((sqrt(sqrt(2
) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*sqrt
(-2*sqrt(2) + 4)) - 1/2*c*(a/c)^(5/8)*arctan(-(sqrt(sqrt(2) + 2)*(a/c)^(1/8
) - 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*sqrt(-2*sqrt(2) + 4))
 + 1/4*c*(a/c)^(5/8)*log(sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^
(1/4))/(a^2*sqrt(2*sqrt(2) + 4)) - 1/4*c*(a/c)^(5/8)*log(-sqrt(x)*sqrt(sqrt
(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*sqrt(2*sqrt(2) + 4)) - 1/4*c*(
a/c)^(5/8)*log(sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a
^2*sqrt(-2*sqrt(2) + 4)) + 1/4*c*(a/c)^(5/8)*log(-sqrt(x)*sqrt(-sqrt(2) + 2
)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*sqrt(-2*sqrt(2) + 4)) - 2/3/(a*x^(3/2
))
```

maple [C] time = 0.01, size = 38, normalized size = 0.13

$$-\frac{\ln\left(-\operatorname{RootOf}\left(c_Z^8+a\right)+\sqrt{x}\right)}{4a\operatorname{RootOf}\left(c_Z^8+a\right)^3}-\frac{2}{3a\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(c*x^4+a),x)
```

```
[Out] -2/3/a/x^(3/2)-1/4/a*sum(1/_R^3*ln(-_R+x^(1/2)),_R=RootOf(_Z^8*c+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-c\int\frac{x^{\frac{3}{2}}}{acx^4+a^2}\,dx-\frac{2}{3\,ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] -c*integrate(x^(3/2)/(a*c*x^4 + a^2), x) - 2/3/(a*x^(3/2))
```

mupad [B] time = 1.08, size = 126, normalized size = 0.42

$$\frac{(-c)^{3/8}\operatorname{atan}\left(\frac{(-c)^{1/8}\sqrt{x}}{a^{1/8}}\right)}{2a^{11/8}}-\frac{2}{3\,a\,x^{3/2}}-\frac{(-c)^{3/8}\operatorname{atan}\left(\frac{(-c)^{1/8}\sqrt{x}\,1\mathrm{i}}{a^{1/8}}\right)1\mathrm{i}}{2a^{11/8}}+\frac{\sqrt{2}\,(-c)^{3/8}\operatorname{atan}\left(\frac{\sqrt{2}\,(-c)^{1/8}\sqrt{x}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{a^{1/8}}\right)\left(-\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)}{a^{11/8}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + c*x^4)),x)
```

```
[Out] ((-c)^(3/8)*atan(((-c)^(1/8)*x^(1/2))/a^(1/8)))/(2*a^(11/8)) - 2/(3*a*x^(3/
2)) - ((-c)^(3/8)*atan(((-c)^(1/8)*x^(1/2)*1i)/a^(1/8))*1i)/(2*a^(11/8)) -
(2^(1/2)*(-c)^(3/8)*atan((2^(1/2)*(-c)^(1/8)*x^(1/2)*(1/2 - 1i/2))/a^(1/8))
*(1/4 + 1i/4))/a^(11/8) - (2^(1/2)*(-c)^(3/8)*atan((2^(1/2)*(-c)^(1/8)*x^(1
/2)*(1/2 + 1i/2))/a^(1/8))*(1/4 - 1i/4))/a^(11/8)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(c*x**4+a),x)
```

```
[Out] Timed out
```

3.746 $\int \frac{x^{13/2}}{\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=308

$$\frac{7\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{32\sqrt{2}\,\sqrt[8]{-a}\,c^{15/8}}-\frac{7\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{32\sqrt{2}\,\sqrt[8]{-a}\,c^{15/8}}-\frac{7\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}\,\sqrt[8]{-a}\,c^{15/8}}+\frac{7\tan^{-}}{16}$$

```
[Out] -1/4*x^(7/2)/c/(c*x^4+a)+7/16*arctan(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(1/8)
/c^(15/8)-7/16*arctanh(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(1/8)/c^(15/8)+7/32
*arctan(-1+c^(1/8)*2^(1/2)*x^(1/2)/(-a)^(1/8))/(-a)^(1/8)/c^(15/8)*2^(1/2)+
7/32*arctan(1+c^(1/8)*2^(1/2)*x^(1/2)/(-a)^(1/8))/(-a)^(1/8)/c^(15/8)*2^(1/
2)+7/64*ln((-a)^(1/4)+c^(1/4)*x-(-a)^(1/8)*c^(1/8)*2^(1/2)*x^(1/2))/(-a)^(1
/8)/c^(15/8)*2^(1/2)-7/64*ln((-a)^(1/4)+c^(1/4)*x+(-a)^(1/8)*c^(1/8)*2^(1/2
)*x^(1/2))/(-a)^(1/8)/c^(15/8)*2^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {288, 329, 301, 297, 1162, 617, 204, 1165, 628, 298, 205, 208}

$$\frac{7\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{32\sqrt{2}\,\sqrt[8]{-a}\,c^{15/8}}-\frac{7\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{32\sqrt{2}\,\sqrt[8]{-a}\,c^{15/8}}-\frac{7\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}\,\sqrt[8]{-a}\,c^{15/8}}+\frac{7\tan^{-}}{16}$$

Antiderivative was successfully verified.

```
[In] Int[x^(13/2)/(a + c*x^4)^2,x]
```

```
[Out] -x^(7/2)/(4*c*(a + c*x^4)) - (7*ArcTan[1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(
1/8)])/(16*Sqrt[2]*(-a)^(1/8)*c^(15/8)) + (7*ArcTan[1 + (Sqrt[2]*c^(1/8)*Sq
rt[x])/(-a)^(1/8)])/(16*Sqrt[2]*(-a)^(1/8)*c^(15/8)) + (7*ArcTan[(c^(1/8)*S
qrt[x])/(-a)^(1/8)])/(16*(-a)^(1/8)*c^(15/8)) - (7*ArcTanh[(c^(1/8)*Sqrt[x]
)/(-a)^(1/8)])/(16*(-a)^(1/8)*c^(15/8)) + (7*Log[(-a)^(1/4) - Sqrt[2]*(-a)^
(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x])/(32*Sqrt[2]*(-a)^(1/8)*c^(15/8)) - (7*L
og[(-a)^(1/4) + Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x])/(32*Sqrt[2
]*(-a)^(1/8)*c^(15/8))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 301

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[x^(m - n/2
)/(r + s*x^(n/2)), x], x] - Dist[s/(2*b), Int[x^(m - n/2)/(r - s*x^(n/2)),
x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] &&
 LtQ[m, n] &&  !GtQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{13/2}}{\left(a+cx^4\right)^2}\,dx = -\frac{x^{7/2}}{4c\left(a+cx^4\right)} + \frac{7\int \frac{x^{5/2}}{a+cx^4}\,dx}{8c}$$

$$= -\frac{x^{7/2}}{4c\left(a+cx^4\right)} + \frac{7\,\text{Subst}\left(\int \frac{x^6}{a+cx^8}\,dx, x, \sqrt{x}\right)}{4c}$$

$$= -\frac{x^{7/2}}{4c\left(a+cx^4\right)} - \frac{7\,\text{Subst}\left(\int \frac{x^2}{\sqrt{-a}-\sqrt{c}\,x^4}\,dx, x, \sqrt{x}\right)}{8c^{3/2}} + \frac{7\,\text{Subst}\left(\int \frac{x^2}{\sqrt{-a}+\sqrt{c}\,x^4}\,dx, x, \sqrt{x}\right)}{8c^{3/2}}$$

$$= -\frac{x^{7/2}}{4c\left(a+cx^4\right)} - \frac{7\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}\,x^2}\,dx, x, \sqrt{x}\right)}{16c^{7/4}} + \frac{7\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}+\sqrt[4]{c}\,x^2}\,dx, x, \sqrt{x}\right)}{16c^{7/4}} - \frac{7\,\text{Sub}}{}$$

$$= -\frac{x^{7/2}}{4c\left(a+cx^4\right)} + \frac{7\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt[8]{-a}\,c^{15/8}} - \frac{7\tanh^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt[8]{-a}\,c^{15/8}} + \frac{7\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}-\frac{\sqrt{2}\,\sqrt[8]{-a}\,x}{\sqrt[8]{c}}+x^2}\,dx, x, \sqrt{x}\right.}{32c^2}$$

$$= -\frac{x^{7/2}}{4c\left(a+cx^4\right)} + \frac{7\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt[8]{-a}\,c^{15/8}} - \frac{7\tanh^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt[8]{-a}\,c^{15/8}} + \frac{7\log\left(\sqrt[4]{-a}-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{c}\,x\right.}{32\sqrt{2}\,\sqrt[8]{-a}\,c^{15/8}}$$

$$= -\frac{x^{7/2}}{4c\left(a+cx^4\right)} - \frac{7\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}\,\sqrt[8]{-a}\,c^{15/8}} + \frac{7\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}\,\sqrt[8]{-a}\,c^{15/8}} + \frac{7\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt[8]{-a}\,c^{15/8}} - \frac{7\tanh}{16\sqrt[8]{}}$$

Mathematica [C] time = 0.04, size = 43, normalized size = 0.14

$$\frac{2x^{7/2}\left(\frac{{}_2F_1\left(\frac{7}{8},2;\frac{15}{8};-\frac{cx^4}{a}\right)}{a} - \frac{1}{a+cx^4}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(13/2)/(a + c*x^4)^2,x]
```

```
[Out] (2*x^(7/2)*(-(a + c*x^4)^(-1) + Hypergeometric2F1[7/8, 2, 15/8, -((c*x^4)/a
)]/a))/c
```

fricas [B] time = 0.65, size = 535, normalized size = 1.74

$$-\frac{16\,x^{\frac{7}{2}} + 28\sqrt{2}\left(c^2x^4+ac\right)\left(-\frac{1}{ac^{15}}\right)^{\frac{1}{8}}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\,ac^{13}\sqrt{x}\left(-\frac{1}{ac^{15}}\right)^{\frac{7}{8}} - ac^{11}\left(-\frac{1}{ac^{15}}\right)^{\frac{3}{4}} + x\,c^2\left(-\frac{1}{ac^{15}}\right)^{\frac{1}{8}} - \sqrt{2}\,c^2\sqrt{}\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] -1/64*(16*x^(7/2) + 28*sqrt(2)*(c^2*x^4 + a*c)*(-1/(a*c^15))^(1/8)*arctan(s
qrt(2)*sqrt(sqrt(2)*a*c^13*sqrt(x)*(-1/(a*c^15))^(7/8) - a*c^11*(-1/(a*c^15
```

```
))^(3/4) + x)*c^2*(-1/(a*c^15))^(1/8) - sqrt(2)*c^2*sqrt(x)*(-1/(a*c^15))^(
1/8) + 1) + 28*sqrt(2)*(c^2*x^4 + a*c)*(-1/(a*c^15))^(1/8)*arctan(sqrt(2)*s
qrt(-sqrt(2)*a*c^13*sqrt(x)*(-1/(a*c^15))^(7/8) - a*c^11*(-1/(a*c^15))^(3/4
) + x)*c^2*(-1/(a*c^15))^(1/8) - sqrt(2)*c^2*sqrt(x)*(-1/(a*c^15))^(1/8) -
1) - 7*sqrt(2)*(c^2*x^4 + a*c)*(-1/(a*c^15))^(1/8)*log(sqrt(2)*a*c^13*sqrt(
x)*(-1/(a*c^15))^(7/8) - a*c^11*(-1/(a*c^15))^(3/4) + x) + 7*sqrt(2)*(c^2*x
^4 + a*c)*(-1/(a*c^15))^(1/8)*log(-sqrt(2)*a*c^13*sqrt(x)*(-1/(a*c^15))^(7/
8) - a*c^11*(-1/(a*c^15))^(3/4) + x) + 56*(c^2*x^4 + a*c)*(-1/(a*c^15))^(1/
8)*arctan(sqrt(-a*c^11*(-1/(a*c^15))^(3/4) + x)*c^2*(-1/(a*c^15))^(1/8) - c
^2*sqrt(x)*(-1/(a*c^15))^(1/8)) - 14*(c^2*x^4 + a*c)*(-1/(a*c^15))^(1/8)*lo
g(a*c^13*(-1/(a*c^15))^(7/8) + sqrt(x)) + 14*(c^2*x^4 + a*c)*(-1/(a*c^15))^
(1/8)*log(-a*c^13*(-1/(a*c^15))^(7/8) + sqrt(x)))/(c^2*x^4 + a*c)
```

giac [B] time = 0.74, size = 486, normalized size = 1.58

$$-\frac{x^{\frac{7}{2}}}{4\left(cx^4+a\right)c}+\frac{7\left(\frac{a}{c}\right)^{\frac{7}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16\,ac\sqrt{-2\sqrt{2}+4}}+\frac{7\left(\frac{a}{c}\right)^{\frac{7}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16\,ac\sqrt{-2\sqrt{2}+4}}+\frac{7\left(\frac{a}{c}\right)^{\frac{7}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}}}{\sqrt{}}\right.}{16\,ac\sqrt{2\sqrt{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*x^(7/2)/((c*x^4 + a)*c) + 7/16*(a/c)^(7/8)*arctan((sqrt(-sqrt(2) + 2)*
(a/c)^(1/8) + 2*sqrt(x))/(sqrt(sqrt(2) + 2)*(a/c)^(1/8)))/(a*c*sqrt(-2*sqrt
(2) + 4)) + 7/16*(a/c)^(7/8)*arctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sq
rt(x))/(sqrt(sqrt(2) + 2)*(a/c)^(1/8)))/(a*c*sqrt(-2*sqrt(2) + 4)) + 7/16*(
a/c)^(7/8)*arctan((sqrt(sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2
) + 2)*(a/c)^(1/8)))/(a*c*sqrt(2*sqrt(2) + 4)) + 7/16*(a/c)^(7/8)*arctan(-(
sqrt(sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8))
)/(a*c*sqrt(2*sqrt(2) + 4)) - 7/32*(a/c)^(7/8)*log(sqrt(x)*sqrt(sqrt(2) + 2
)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a*c*sqrt(-2*sqrt(2) + 4)) + 7/32*(a/c)^(7
/8)*log(-sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a*c*sqrt
(-2*sqrt(2) + 4)) - 7/32*(a/c)^(7/8)*log(sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(
1/8) + x + (a/c)^(1/4))/(a*c*sqrt(2*sqrt(2) + 4)) + 7/32*(a/c)^(7/8)*log(-s
qrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a*c*sqrt(2*sqrt(2
) + 4))
```

maple [C] time = 0.02, size = 47, normalized size = 0.15

$$-\frac{x^{\frac{7}{2}}}{4\left(c\,x^4+a\right)c}+\frac{7\ln\left(-\operatorname{RootOf}\left(c_Z^8+a\right)+\sqrt{x}\right)}{32c^2\operatorname{RootOf}\left(c_Z^8+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(c*x^4+a)^2,x)
```

```
[Out] -1/4*x^(7/2)/c/(c*x^4+a)+7/32/c^2*sum(1/_R*ln(-_R+x^(1/2)),_R=RootOf(_Z^8*c
+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{x^{\frac{7}{2}}}{4\left(c^2x^4+ac\right)}+7\int\frac{x^{\frac{5}{2}}}{8\left(c^2x^4+ac\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*x^(7/2)/(c^2*x^4 + a*c) + 7*integrate(1/8*x^(5/2)/(c^2*x^4 + a*c), x)
```

mupad [B] time = 0.15, size = 135, normalized size = 0.44

$$\frac{7\,\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}}{(-a)^{1/8}}\right)}{16\,(-a)^{1/8}\,c^{15/8}}-\frac{x^{7/2}}{4\,c\,\left(c\,x^4+a\right)}+\frac{\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}\,1\mathrm{i}}{(-a)^{1/8}}\right)\,7\mathrm{i}}{16\,(-a)^{1/8}\,c^{15/8}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\left(\frac{7}{32}-\frac{7}{32}\mathrm{i}\right)}{(-a)^{1/8}\,c^{15/8}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\right.\right.}{(-a)^{1/8}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(a + c*x^4)^2,x)
```

```
[Out] (7*atan((c^(1/8)*x^(1/2))/(-a)^(1/8)))/(16*(-a)^(1/8)*c^(15/8)) - x^(7/2)/(
4*c*(a + c*x^4)) + (atan((c^(1/8)*x^(1/2)*1i)/(-a)^(1/8))*7i)/(16*(-a)^(1/8
)*c^(15/8)) + (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^(1/
8))*(7/32 - 7i/32))/((-a)^(1/8)*c^(15/8)) + (2^(1/2)*atan((2^(1/2)*c^(1/8)*
x^(1/2)*(1/2 + 1i/2))/(-a)^(1/8))*(7/32 + 7i/32))/((-a)^(1/8)*c^(15/8))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.747 $\int \frac{x^{11/2}}{\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=308

$$-\frac{5\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{32\sqrt{2}\,(-a)^{3/8}c^{13/8}}+\frac{5\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{32\sqrt{2}\,(-a)^{3/8}c^{13/8}}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}\,(-a)^{3/8}c^{13/8}}+\frac{5}{-}$$

```
[Out] -1/4*x^(5/2)/c/(c*x^4+a)-5/16*arctan(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(3/8)
/c^(13/8)-5/16*arctanh(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(3/8)/c^(13/8)+5/32
*arctan(-1+c^(1/8)*2^(1/2)*x^(1/2)/(-a)^(1/8))/(-a)^(3/8)/c^(13/8)*2^(1/2)+
5/32*arctan(1+c^(1/8)*2^(1/2)*x^(1/2)/(-a)^(1/8))/(-a)^(3/8)/c^(13/8)*2^(1/
2)-5/64*ln((-a)^(1/4)+c^(1/4)*x-(-a)^(1/8)*c^(1/8)*2^(1/2)*x^(1/2))/(-a)^(3
/8)/c^(13/8)*2^(1/2)+5/64*ln((-a)^(1/4)+c^(1/4)*x+(-a)^(1/8)*c^(1/8)*2^(1/2
)*x^(1/2))/(-a)^(3/8)/c^(13/8)*2^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {288, 329, 301, 211, 1165, 628, 1162, 617, 204, 212, 208, 205}

$$-\frac{5\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{32\sqrt{2}\,(-a)^{3/8}c^{13/8}}+\frac{5\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{32\sqrt{2}\,(-a)^{3/8}c^{13/8}}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}\,(-a)^{3/8}c^{13/8}}+\frac{5}{-}$$

Antiderivative was successfully verified.

```
[In] Int[x^(11/2)/(a + c*x^4)^2,x]
```

```
[Out] -x^(5/2)/(4*c*(a + c*x^4)) - (5*ArcTan[1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(
1/8)])/(16*Sqrt[2]*(-a)^(3/8)*c^(13/8)) + (5*ArcTan[1 + (Sqrt[2]*c^(1/8)*Sq
rt[x])/(-a)^(1/8)])/(16*Sqrt[2]*(-a)^(3/8)*c^(13/8)) - (5*ArcTan[(c^(1/8)*S
qrt[x])/(-a)^(1/8)])/(16*(-a)^(3/8)*c^(13/8)) - (5*ArcTanh[(c^(1/8)*Sqrt[x]
)/(-a)^(1/8)])/(16*(-a)^(3/8)*c^(13/8)) - (5*Log[(-a)^(1/4) - Sqrt[2]*(-a)^
(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x])/(32*Sqrt[2]*(-a)^(3/8)*c^(13/8)) + (5*L
og[(-a)^(1/4) + Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x])/(32*Sqrt[2
]*(-a)^(3/8)*c^(13/8))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 301

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[x^(m - n/2
)/(r + s*x^(n/2)), x], x] - Dist[s/(2*b), Int[x^(m - n/2)/(r - s*x^(n/2)),
x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] &&
 LtQ[m, n] &&  !GtQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{11/2}}{\left(a+cx^4\right)^2}\,dx = -\frac{x^{5/2}}{4c\left(a+cx^4\right)} + \frac{5\int \frac{x^{3/2}}{a+cx^4}\,dx}{8c}$$

$$= -\frac{x^{5/2}}{4c\left(a+cx^4\right)} + \frac{5\,\text{Subst}\left(\int \frac{x^4}{a+cx^8}\,dx, x, \sqrt{x}\right)}{4c}$$

$$= -\frac{x^{5/2}}{4c\left(a+cx^4\right)} - \frac{5\,\text{Subst}\left(\int \frac{1}{\sqrt{-a}-\sqrt{c}\,x^4}\,dx, x, \sqrt{x}\right)}{8c^{3/2}} + \frac{5\,\text{Subst}\left(\int \frac{1}{\sqrt{-a}+\sqrt{c}\,x^4}\,dx, x, \sqrt{x}\right)}{8c^{3/2}}$$

$$= -\frac{x^{5/2}}{4c\left(a+cx^4\right)} - \frac{5\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}\,x^2}\,dx, x, \sqrt{x}\right)}{16\sqrt[4]{-a}\,c^{3/2}} - \frac{5\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}+\sqrt[4]{c}\,x^2}\,dx, x, \sqrt{x}\right)}{16\sqrt[4]{-a}\,c^{3/2}} + \frac{5\,S}{}$$

$$= -\frac{x^{5/2}}{4c\left(a+cx^4\right)} - \frac{5\tan^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{3/8}c^{13/8}} - \frac{5\tanh^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{3/8}c^{13/8}} + \frac{5\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[4]{-a}}{\sqrt[4]{c}} - \frac{\sqrt{2}\sqrt[8]{-a}\,x}{\sqrt[8]{c}} + x^2}\,dx, x, \sqrt{}\right.}{32\sqrt[4]{-a}\,c^{7/4}}$$

$$= -\frac{x^{5/2}}{4c\left(a+cx^4\right)} - \frac{5\tan^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{3/8}c^{13/8}} - \frac{5\tanh^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{3/8}c^{13/8}} - \frac{5\log\left(\sqrt[4]{-a} - \sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x} + \sqrt[4]{}\right.}{32\sqrt{2}\,(-a)^{3/8}c^{13/8}}$$

$$= -\frac{x^{5/2}}{4c\left(a+cx^4\right)} - \frac{5\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}\,(-a)^{3/8}c^{13/8}} + \frac{5\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}\,(-a)^{3/8}c^{13/8}} - \frac{5\tan^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{3/8}c^{13/8}} - \frac{5\,ta}{16}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.18

$$\frac{2x^{5/2}\,{}_2F_1\left(\frac{5}{8}, 2; \frac{13}{8}; -\frac{cx^4}{a}\right)}{3ac} - \frac{2x^{5/2}}{3c\left(a+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(11/2)/(a + c*x^4)^2,x]
```

```
[Out] (-2*x^(5/2))/(3*c*(a + c*x^4)) + (2*x^(5/2)*Hypergeometric2F1[5/8, 2, 13/8,
 -((c*x^4)/a)])/(3*a*c)
```

fricas [B] time = 0.61, size = 553, normalized size = 1.80

$$\frac{20\sqrt{2}\left(c^2x^4+ac\right)\left(-\frac{1}{a^3c^{13}}\right)^{\frac{1}{8}}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\,a^2c^8\sqrt{x}\left(-\frac{1}{a^3c^{13}}\right)^{\frac{5}{8}} - ac^3\left(-\frac{1}{a^3c^{13}}\right)^{\frac{1}{4}} + x}\,ac^5\left(-\frac{1}{a^3c^{13}}\right)^{\frac{3}{8}} - \sqrt{2}\,ac^5\sqrt{x}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/64*(20*sqrt(2)*(c^2*x^4 + a*c)*(-1/(a^3*c^13))^(1/8)*arctan(sqrt(2)*sqrt(
sqrt(2)*a^2*c^8*sqrt(x)*(-1/(a^3*c^13))^(5/8) - a*c^3*(-1/(a^3*c^13))^(1/4)
```

```
 + x)*a*c^5*(-1/(a^3*c^13))^(3/8) - sqrt(2)*a*c^5*sqrt(x)*(-1/(a^3*c^13))^(
3/8) + 1) + 20*sqrt(2)*(c^2*x^4 + a*c)*(-1/(a^3*c^13))^(1/8)*arctan(sqrt(2)
*sqrt(-sqrt(2)*a^2*c^8*sqrt(x)*(-1/(a^3*c^13))^(5/8) - a*c^3*(-1/(a^3*c^13)
)^(1/4) + x)*a*c^5*(-1/(a^3*c^13))^(3/8) - sqrt(2)*a*c^5*sqrt(x)*(-1/(a^3*c
^13))^(3/8) - 1) + 5*sqrt(2)*(c^2*x^4 + a*c)*(-1/(a^3*c^13))^(1/8)*log(sqrt
(2)*a^2*c^8*sqrt(x)*(-1/(a^3*c^13))^(5/8) - a*c^3*(-1/(a^3*c^13))^(1/4) + x
) - 5*sqrt(2)*(c^2*x^4 + a*c)*(-1/(a^3*c^13))^(1/8)*log(-sqrt(2)*a^2*c^8*sq
rt(x)*(-1/(a^3*c^13))^(5/8) - a*c^3*(-1/(a^3*c^13))^(1/4) + x) - 16*x^(5/2)
 - 40*(c^2*x^4 + a*c)*(-1/(a^3*c^13))^(1/8)*arctan(sqrt(-a*c^3*(-1/(a^3*c^1
3))^(1/4) + x)*a*c^5*(-1/(a^3*c^13))^(3/8) - a*c^5*sqrt(x)*(-1/(a^3*c^13))^
(3/8)) - 10*(c^2*x^4 + a*c)*(-1/(a^3*c^13))^(1/8)*log(a^2*c^8*(-1/(a^3*c^13
))^(5/8) + sqrt(x)) + 10*(c^2*x^4 + a*c)*(-1/(a^3*c^13))^(1/8)*log(-a^2*c^8
*(-1/(a^3*c^13))^(5/8) + sqrt(x)))/(c^2*x^4 + a*c)
```

giac [B] time = 0.75, size = 486, normalized size = 1.58

$$-\frac{x^{\frac{5}{2}}}{4\left(cx^4+a\right)c}-\frac{5\left(\frac{a}{c}\right)^{\frac{5}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16ac\sqrt{2\sqrt{2}+4}}-\frac{5\left(\frac{a}{c}\right)^{\frac{5}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16ac\sqrt{2\sqrt{2}+4}}+\frac{5\left(\frac{a}{c}\right)^{\frac{5}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}+2}\left(\right.\right.}{\sqrt{-\sqrt{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*x^(5/2)/((c*x^4 + a)*c) - 5/16*(a/c)^(5/8)*arctan((sqrt(-sqrt(2) + 2)*
(a/c)^(1/8) + 2*sqrt(x))/(sqrt(sqrt(2) + 2)*(a/c)^(1/8)))/(a*c*sqrt(2*sqrt(
2) + 4)) - 5/16*(a/c)^(5/8)*arctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqr
t(x))/(sqrt(sqrt(2) + 2)*(a/c)^(1/8)))/(a*c*sqrt(2*sqrt(2) + 4)) + 5/16*(a/
c)^(5/8)*arctan((sqrt(sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2)
 + 2)*(a/c)^(1/8)))/(a*c*sqrt(-2*sqrt(2) + 4)) + 5/16*(a/c)^(5/8)*arctan(-(s
qrt(sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))
/(a*c*sqrt(-2*sqrt(2) + 4)) - 5/32*(a/c)^(5/8)*log(sqrt(x)*sqrt(sqrt(2) + 2
)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a*c*sqrt(2*sqrt(2) + 4)) + 5/32*(a/c)^(5/
8)*log(-sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a*c*sqrt(
2*sqrt(2) + 4)) + 5/32*(a/c)^(5/8)*log(sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/
8) + x + (a/c)^(1/4))/(a*c*sqrt(-2*sqrt(2) + 4)) - 5/32*(a/c)^(5/8)*log(-sq
rt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a*c*sqrt(-2*sqrt(2
) + 4))
```

maple [C] time = 0.02, size = 47, normalized size = 0.15

$$-\frac{x^{\frac{5}{2}}}{4\left(c\,x^4+a\right)c}+\frac{5\ln\left(-\operatorname{RootOf}\left(c_Z^8+a\right)+\sqrt{x}\right)}{32c^2\operatorname{RootOf}\left(c_Z^8+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(c*x^4+a)^2,x)
```

```
[Out] -1/4*x^(5/2)/c/(c*x^4+a)+5/32/c^2*sum(1/_R^3*ln(-_R+x^(1/2)),_R=RootOf(_Z^8
*c+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{x^{\frac{5}{2}}}{4\left(c^2x^4+ac\right)}+5\int\frac{x^{\frac{3}{2}}}{8\left(c^2x^4+ac\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*x^(5/2)/(c^2*x^4 + a*c) + 5*integrate(1/8*x^(3/2)/(c^2*x^4 + a*c), x)
```

mupad [B] time = 1.09, size = 135, normalized size = 0.44

$$-\frac{x^{5/2}}{4\,c\,\left(c\,x^4+a\right)}-\frac{5\,\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}}{(-a)^{1/8}}\right)}{16\,(-a)^{3/8}\,c^{13/8}}+\frac{\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}\,1\mathrm{i}}{(-a)^{1/8}}\right)\,5\mathrm{i}}{16\,(-a)^{3/8}\,c^{13/8}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\left(\frac{5}{32}+\frac{5}{32}\mathrm{i}\right)}{(-a)^{3/8}\,c^{13/8}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/}}{(-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(a + c*x^4)^2,x)
```

```
[Out] (atan((c^(1/8)*x^(1/2)*1i)/(-a)^(1/8))*5i)/(16*(-a)^(3/8)*c^(13/8)) - (5*at
an((c^(1/8)*x^(1/2))/(-a)^(1/8)))/(16*(-a)^(3/8)*c^(13/8)) - x^(5/2)/(4*c*(
a + c*x^4)) + (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^(1/
8))*(5/32 + 5i/32))/((-a)^(3/8)*c^(13/8)) + (2^(1/2)*atan((2^(1/2)*c^(1/8)*
x^(1/2)*(1/2 + 1i/2))/(-a)^(1/8))*(5/32 - 5i/32))/((-a)^(3/8)*c^(13/8))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.748 $\int \frac{x^{9/2}}{\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=308

$$-\frac{3\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{32\sqrt{2}\,(-a)^{5/8}c^{11/8}}+\frac{3\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{32\sqrt{2}\,(-a)^{5/8}c^{11/8}}+\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}\,(-a)^{5/8}c^{11/8}}-\frac{3\tan}{16}$$

```
[Out] -1/4*x^(3/2)/c/(c*x^4+a)+3/16*arctan(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(5/8)
/c^(11/8)-3/16*arctanh(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(5/8)/c^(11/8)-3/32
*arctan(-1+c^(1/8)*2^(1/2)*x^(1/2)/(-a)^(1/8))/(-a)^(5/8)/c^(11/8)*2^(1/2)-
3/32*arctan(1+c^(1/8)*2^(1/2)*x^(1/2)/(-a)^(1/8))/(-a)^(5/8)/c^(11/8)*2^(1/
2)-3/64*ln((-a)^(1/4)+c^(1/4)*x-(-a)^(1/8)*c^(1/8)*2^(1/2)*x^(1/2))/(-a)^(5
/8)/c^(11/8)*2^(1/2)+3/64*ln((-a)^(1/4)+c^(1/4)*x+(-a)^(1/8)*c^(1/8)*2^(1/2
)*x^(1/2))/(-a)^(5/8)/c^(11/8)*2^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {288, 329, 300, 297, 1162, 617, 204, 1165, 628, 298, 205, 208}

$$-\frac{3\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{32\sqrt{2}\,(-a)^{5/8}c^{11/8}}+\frac{3\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{32\sqrt{2}\,(-a)^{5/8}c^{11/8}}+\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}\,(-a)^{5/8}c^{11/8}}-\frac{3\tan}{16}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/(a + c*x^4)^2,x]
```

```
[Out] -x^(3/2)/(4*c*(a + c*x^4)) + (3*ArcTan[1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(
1/8)])/(16*Sqrt[2]*(-a)^(5/8)*c^(11/8)) - (3*ArcTan[1 + (Sqrt[2]*c^(1/8)*Sq
rt[x])/(-a)^(1/8)])/(16*Sqrt[2]*(-a)^(5/8)*c^(11/8)) + (3*ArcTan[(c^(1/8)*S
qrt[x])/(-a)^(1/8)])/(16*(-a)^(5/8)*c^(11/8)) - (3*ArcTanh[(c^(1/8)*Sqrt[x]
)/(-a)^(1/8)])/(16*(-a)^(5/8)*c^(11/8)) - (3*Log[(-a)^(1/4) - Sqrt[2]*(-a)^
(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x])/(32*Sqrt[2]*(-a)^(5/8)*c^(11/8)) + (3*L
og[(-a)^(1/4) + Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x])/(32*Sqrt[2
]*(-a)^(5/8)*c^(11/8))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 300

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[x^m/(r + s
*x^(n/2)), x], x] + Dist[r/(2*a), Int[x^m/(r - s*x^(n/2)), x], x]] /; FreeQ
[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n/2] &&  !GtQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{9/2}}{\left(a+cx^4\right)^2}\,dx = -\frac{x^{3/2}}{4c\left(a+cx^4\right)} + \frac{3\int \frac{\sqrt{x}}{a+cx^4}\,dx}{8c}$$

$$= -\frac{x^{3/2}}{4c\left(a+cx^4\right)} + \frac{3\,\text{Subst}\left(\int \frac{x^2}{a+cx^8}\,dx, x, \sqrt{x}\right)}{4c}$$

$$= -\frac{x^{3/2}}{4c\left(a+cx^4\right)} - \frac{3\,\text{Subst}\left(\int \frac{x^2}{\sqrt{-a}-\sqrt{c}\,x^4}\,dx, x, \sqrt{x}\right)}{8\sqrt{-a}\,c} - \frac{3\,\text{Subst}\left(\int \frac{x^2}{\sqrt{-a}+\sqrt{c}\,x^4}\,dx, x, \sqrt{x}\right)}{8\sqrt{-a}\,c}$$

$$= -\frac{x^{3/2}}{4c\left(a+cx^4\right)} - \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}\,x^2}\,dx, x, \sqrt{x}\right)}{16\sqrt{-a}\,c^{5/4}} + \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}+\sqrt[4]{c}\,x^2}\,dx, x, \sqrt{x}\right)}{16\sqrt{-a}\,c^{5/4}} + \frac{3\,\text{Sub}}{}$$

$$= -\frac{x^{3/2}}{4c\left(a+cx^4\right)} + \frac{3\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{5/8}c^{11/8}} - \frac{3\tanh^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{5/8}c^{11/8}} - \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[4]{-a}}{\sqrt[4]{c}} - \frac{\sqrt{2}\,\sqrt[8]{-a}\,x}{\sqrt[8]{c}} + x^2}\,dx, x, \sqrt{x}\right.}{32\sqrt{-a}\,c^{3/2}}$$

$$= -\frac{x^{3/2}}{4c\left(a+cx^4\right)} + \frac{3\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{5/8}c^{11/8}} - \frac{3\tanh^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{5/8}c^{11/8}} - \frac{3\log\left(\sqrt[4]{-a} - \sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x} + \sqrt[4]{c}\,x\right.}{32\sqrt{2}\,(-a)^{5/8}c^{11/8}}$$

$$= -\frac{x^{3/2}}{4c\left(a+cx^4\right)} + \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}\,(-a)^{5/8}c^{11/8}} - \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}\,(-a)^{5/8}c^{11/8}} + \frac{3\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{5/8}c^{11/8}} - \frac{3\tanh}{16(-}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.18

$$\frac{2x^{3/2}\,{}_2F_1\left(\frac{3}{8}, 2; \frac{11}{8}; -\frac{cx^4}{a}\right)}{5ac} - \frac{2x^{3/2}}{5c\left(a+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/(a + c*x^4)^2,x]
```

```
[Out] (-2*x^(3/2))/(5*c*(a + c*x^4)) + (2*x^(3/2)*Hypergeometric2F1[3/8, 2, 11/8,
 -((c*x^4)/a)])/(5*a*c)
```

fricas [B] time = 0.78, size = 570, normalized size = 1.85

$$-\frac{12\sqrt{2}\left(c^2x^4+ac\right)\left(-\frac{1}{a^5c^{11}}\right)^{\frac{1}{8}}\arctan\left(\sqrt{2}\sqrt{a^4c^8\left(-\frac{1}{a^5c^{11}}\right)^{\frac{3}{4}} + \sqrt{2}\,a^2c^4\sqrt{x}\left(-\frac{1}{a^5c^{11}}\right)^{\frac{3}{8}} + x}\,a^3c^7\left(-\frac{1}{a^5c^{11}}\right)^{\frac{5}{8}} - \sqrt{2}\,a^3c^7\sqrt{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] -1/64*(12*sqrt(2)*(c^2*x^4 + a*c)*(-1/(a^5*c^11))^(1/8)*arctan(sqrt(2)*sqrt
(a^4*c^8*(-1/(a^5*c^11))^(3/4) + sqrt(2)*a^2*c^4*sqrt(x)*(-1/(a^5*c^11))^(3
/8) + x)*a^3*c^7*(-1/(a^5*c^11))^(5/8) - sqrt(2)*a^3*c^7*sqrt(x)*(-1/(a^5*c
^11))^(5/8) + 1) + 12*sqrt(2)*(c^2*x^4 + a*c)*(-1/(a^5*c^11))^(1/8)*arctan(
sqrt(2)*sqrt(a^4*c^8*(-1/(a^5*c^11))^(3/4) - sqrt(2)*a^2*c^4*sqrt(x)*(-1/(a
```

```
^5*c^11))^(3/8) + x)*a^3*c^7*(-1/(a^5*c^11))^(5/8) - sqrt(2)*a^3*c^7*sqrt(x
)*(-1/(a^5*c^11))^(5/8) - 1) - 3*sqrt(2)*(c^2*x^4 + a*c)*(-1/(a^5*c^11))^(1
/8)*log(a^4*c^8*(-1/(a^5*c^11))^(3/4) + sqrt(2)*a^2*c^4*sqrt(x)*(-1/(a^5*c^
11))^(3/8) + x) + 3*sqrt(2)*(c^2*x^4 + a*c)*(-1/(a^5*c^11))^(1/8)*log(a^4*c
^8*(-1/(a^5*c^11))^(3/4) - sqrt(2)*a^2*c^4*sqrt(x)*(-1/(a^5*c^11))^(3/8) +
x) - 24*(c^2*x^4 + a*c)*(-1/(a^5*c^11))^(1/8)*arctan(sqrt(a^4*c^8*(-1/(a^5*
c^11))^(3/4) + x)*a^3*c^7*(-1/(a^5*c^11))^(5/8) - a^3*c^7*sqrt(x)*(-1/(a^5*
c^11))^(5/8)) + 6*(c^2*x^4 + a*c)*(-1/(a^5*c^11))^(1/8)*log(a^2*c^4*(-1/(a^
5*c^11))^(3/8) + sqrt(x)) - 6*(c^2*x^4 + a*c)*(-1/(a^5*c^11))^(1/8)*log(-a^
2*c^4*(-1/(a^5*c^11))^(3/8) + sqrt(x)) + 16*x^(3/2))/(c^2*x^4 + a*c)
```

giac [B] time = 0.73, size = 486, normalized size = 1.58

$$-\frac{x^{\frac{3}{2}}}{4\left(cx^4+a\right)c}-\frac{3\left(\frac{a}{c}\right)^{\frac{3}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16\,ac\sqrt{2\sqrt{2}+4}}-\frac{3\left(\frac{a}{c}\right)^{\frac{3}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16\,ac\sqrt{2\sqrt{2}+4}}+\frac{3\left(\frac{a}{c}\right)^{\frac{3}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}}}{\sqrt{-}}\right.}{16\,ac\sqrt{-2\sqrt{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*x^(3/2)/((c*x^4 + a)*c) - 3/16*(a/c)^(3/8)*arctan((sqrt(-sqrt(2) + 2)*
(a/c)^(1/8) + 2*sqrt(x))/(sqrt(sqrt(2) + 2)*(a/c)^(1/8)))/(a*c*sqrt(2*sqrt(
2) + 4)) - 3/16*(a/c)^(3/8)*arctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqr
t(x))/(sqrt(sqrt(2) + 2)*(a/c)^(1/8)))/(a*c*sqrt(2*sqrt(2) + 4)) + 3/16*(a/
c)^(3/8)*arctan((sqrt(sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2)
+ 2)*(a/c)^(1/8)))/(a*c*sqrt(-2*sqrt(2) + 4)) + 3/16*(a/c)^(3/8)*arctan(-(s
qrt(sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))
/(a*c*sqrt(-2*sqrt(2) + 4)) + 3/32*(a/c)^(3/8)*log(sqrt(x)*sqrt(sqrt(2) + 2
)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a*c*sqrt(2*sqrt(2) + 4)) - 3/32*(a/c)^(3/
8)*log(-sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a*c*sqrt(
2*sqrt(2) + 4)) - 3/32*(a/c)^(3/8)*log(sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/
8) + x + (a/c)^(1/4))/(a*c*sqrt(-2*sqrt(2) + 4)) + 3/32*(a/c)^(3/8)*log(-sq
rt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a*c*sqrt(-2*sqrt(2
) + 4))
```

maple [C] time = 0.02, size = 47, normalized size = 0.15

$$-\frac{x^{\frac{3}{2}}}{4\left(c\,x^4+a\right)c}+\frac{3\ln\left(-\operatorname{RootOf}\left(c_Z^8+a\right)+\sqrt{x}\right)}{32c^2\operatorname{RootOf}\left(c_Z^8+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(c*x^4+a)^2,x)
```

```
[Out] -1/4*x^(3/2)/c/(c*x^4+a)+3/32/c^2*sum(1/_R^5*ln(-_R+x^(1/2)),_R=RootOf(_Z^8
*c+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{x^{\frac{3}{2}}}{4\left(c^2x^4+ac\right)}+3\int\frac{\sqrt{x}}{8\left(c^2x^4+ac\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*x^(3/2)/(c^2*x^4 + a*c) + 3*integrate(1/8*sqrt(x)/(c^2*x^4 + a*c), x)
```

mupad [B] time = 1.06, size = 135, normalized size = 0.44

$$\frac{3\,\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}}{(-a)^{1/8}}\right)}{16\,(-a)^{5/8}\,c^{11/8}}-\frac{x^{3/2}}{4\,c\,\left(c\,x^4+a\right)}+\frac{\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}\,1\mathrm{i}}{(-a)^{1/8}}\right)\,3\mathrm{i}}{16\,(-a)^{5/8}\,c^{11/8}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\left(-\frac{3}{32}+\frac{3}{32}\mathrm{i}\right)}{(-a)^{5/8}\,c^{11/8}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{}}{(-a)}\right.}{(-a)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(a + c*x^4)^2,x)
```

```
[Out] (3*atan((c^(1/8)*x^(1/2))/(-a)^(1/8)))/(16*(-a)^(5/8)*c^(11/8)) - x^(3/2)/(
4*c*(a + c*x^4)) + (atan((c^(1/8)*x^(1/2)*1i)/(-a)^(1/8))*3i)/(16*(-a)^(5/8
)*c^(11/8)) - (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^(1/
8))*(3/32 - 3i/32))/((-a)^(5/8)*c^(11/8)) - (2^(1/2)*atan((2^(1/2)*c^(1/8)*
x^(1/2)*(1/2 + 1i/2))/(-a)^(1/8))*(3/32 + 3i/32))/((-a)^(5/8)*c^(11/8))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.749 $\int \frac{x^{7/2}}{\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=308

$$\frac{\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{32\sqrt{2}\,(-a)^{7/8}c^{9/8}}-\frac{\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{32\sqrt{2}\,(-a)^{7/8}c^{9/8}}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}\,(-a)^{7/8}c^{9/8}}-\frac{\tan^{-1}\left(1\right.}{16\sqrt{2}}$$

```
[Out] -1/16*arctan(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(7/8)/c^(9/8)-1/16*arctanh(c^
(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(7/8)/c^(9/8)-1/32*arctan(-1+c^(1/8)*2^(1/2)
*x^(1/2)/(-a)^(1/8))/(-a)^(7/8)/c^(9/8)*2^(1/2)-1/32*arctan(1+c^(1/8)*2^(1/
2)*x^(1/2)/(-a)^(1/8))/(-a)^(7/8)/c^(9/8)*2^(1/2)+1/64*ln((-a)^(1/4)+c^(1/4
)*x-(-a)^(1/8)*c^(1/8)*2^(1/2)*x^(1/2))/(-a)^(7/8)/c^(9/8)*2^(1/2)-1/64*ln(
(-a)^(1/4)+c^(1/4)*x+(-a)^(1/8)*c^(1/8)*2^(1/2)*x^(1/2))/(-a)^(7/8)/c^(9/8)
*2^(1/2)-1/4*x^(1/2)/c/(c*x^4+a)
```

Rubi [A] time = 0.25, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {288, 329, 214, 212, 208, 205, 211, 1165, 628, 1162, 617, 204}

$$\frac{\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{32\sqrt{2}\,(-a)^{7/8}c^{9/8}}-\frac{\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{32\sqrt{2}\,(-a)^{7/8}c^{9/8}}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}\,(-a)^{7/8}c^{9/8}}-\frac{\tan^{-1}\left(1\right.}{16\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(a + c*x^4)^2,x]
```

```
[Out] -Sqrt[x]/(4*c*(a + c*x^4)) + ArcTan[1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8
)]/(16*Sqrt[2]*(-a)^(7/8)*c^(9/8)) - ArcTan[1 + (Sqrt[2]*c^(1/8)*Sqrt[x])/(
-a)^(1/8)]/(16*Sqrt[2]*(-a)^(7/8)*c^(9/8)) - ArcTan[(c^(1/8)*Sqrt[x])/(-a)^
(1/8)]/(16*(-a)^(7/8)*c^(9/8)) - ArcTanh[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(16*
(-a)^(7/8)*c^(9/8)) + Log[(-a)^(1/4) - Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] +
 c^(1/4)*x]/(32*Sqrt[2]*(-a)^(7/8)*c^(9/8)) - Log[(-a)^(1/4) + Sqrt[2]*(-a)
^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x]/(32*Sqrt[2]*(-a)^(7/8)*c^(9/8))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{7/2}}{\left(a+cx^4\right)^2}\,dx = -\frac{\sqrt{x}}{4c\left(a+cx^4\right)} + \frac{\int \frac{1}{\sqrt{x}\left(a+cx^4\right)}\,dx}{8c}$$

$$= -\frac{\sqrt{x}}{4c\left(a+cx^4\right)} + \frac{\text{Subst}\left(\int \frac{1}{a+cx^8}\,dx, x, \sqrt{x}\right)}{4c}$$

$$= -\frac{\sqrt{x}}{4c\left(a+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{-a}-\sqrt{c}\,x^4}\,dx, x, \sqrt{x}\right)}{8\sqrt{-a}\,c} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{-a}+\sqrt{c}\,x^4}\,dx, x, \sqrt{x}\right)}{8\sqrt{-a}\,c}$$

$$= -\frac{\sqrt{x}}{4c\left(a+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}\,x^2}\,dx, x, \sqrt{x}\right)}{16(-a)^{3/4}c} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}+\sqrt[4]{c}\,x^2}\,dx, x, \sqrt{x}\right)}{16(-a)^{3/4}c} - \frac{\text{Subst}}{}$$

$$= -\frac{\sqrt{x}}{4c\left(a+cx^4\right)} - \frac{\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{7/8}c^{9/8}} - \frac{\tanh^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{7/8}c^{9/8}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[4]{-a}}{\sqrt[4]{c}} - \frac{\sqrt{2}\,\sqrt[8]{-a}\,x}{\sqrt[8]{c}} + x^2}\,dx, x, \sqrt{x}\right)}{32(-a)^{3/4}c^{5/4}} -$$

$$= -\frac{\sqrt{x}}{4c\left(a+cx^4\right)} - \frac{\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{7/8}c^{9/8}} - \frac{\tanh^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{7/8}c^{9/8}} + \frac{\log\left(\sqrt[4]{-a} - \sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x} + \sqrt[4]{c}\,x\right)}{32\sqrt{2}\,(-a)^{7/8}c^{9/8}}$$

$$= -\frac{\sqrt{x}}{4c\left(a+cx^4\right)} + \frac{\tan^{-1}\left(1 - \frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}\,(-a)^{7/8}c^{9/8}} - \frac{\tan^{-1}\left(1 + \frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}\,(-a)^{7/8}c^{9/8}} - \frac{\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{7/8}c^{9/8}} - \frac{\tanh^{-1}\left(}{16(-a)^{7}}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.18

$$\frac{\sqrt{x}\;{}_2F_1\left(\frac{1}{8}, 1; \frac{9}{8}; -\frac{cx^4}{a}\right)}{4ac} - \frac{\sqrt{x}}{4c\left(a+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(a + c*x^4)^2,x]
```

```
[Out] -1/4*Sqrt[x]/(c*(a + c*x^4)) + (Sqrt[x]*Hypergeometric2F1[1/8, 1, 9/8, -((c
*x^4)/a)])/(4*a*c)
```

fricas [B] time = 0.73, size = 545, normalized size = 1.77

$$\frac{4\sqrt{2}\left(c^2x^4+ac\right)\left(-\frac{1}{a^7c^9}\right)^{\frac{1}{8}}\arctan\left(\sqrt{2}\sqrt{a^2c^2\left(-\frac{1}{a^7c^9}\right)^{\frac{1}{4}} + \sqrt{2}\,ac\sqrt{x}\left(-\frac{1}{a^7c^9}\right)^{\frac{1}{8}} + x}\,a^6c^8\left(-\frac{1}{a^7c^9}\right)^{\frac{7}{8}} - \sqrt{2}\,a^6c^8\sqrt{x}\left(-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/64*(4*sqrt(2)*(c^2*x^4 + a*c)*(-1/(a^7*c^9))^(1/8)*arctan(sqrt(2)*sqrt(a^
2*c^2*(-1/(a^7*c^9))^(1/4) + sqrt(2)*a*c*sqrt(x)*(-1/(a^7*c^9))^(1/8) + x)*
a^6*c^8*(-1/(a^7*c^9))^(7/8) - sqrt(2)*a^6*c^8*sqrt(x)*(-1/(a^7*c^9))^(7/8)
 + 1) + 4*sqrt(2)*(c^2*x^4 + a*c)*(-1/(a^7*c^9))^(1/8)*arctan(sqrt(2)*sqrt(
a^2*c^2*(-1/(a^7*c^9))^(1/4) - sqrt(2)*a*c*sqrt(x)*(-1/(a^7*c^9))^(1/8) + x
)*a^6*c^8*(-1/(a^7*c^9))^(7/8) - sqrt(2)*a^6*c^8*sqrt(x)*(-1/(a^7*c^9))^(7/
```

```
8) - 1) + sqrt(2)*(c^2*x^4 + a*c)*(-1/(a^7*c^9))^(1/8)*log(a^2*c^2*(-1/(a^7
*c^9))^(1/4) + sqrt(2)*a*c*sqrt(x)*(-1/(a^7*c^9))^(1/8) + x) - sqrt(2)*(c^2
*x^4 + a*c)*(-1/(a^7*c^9))^(1/8)*log(a^2*c^2*(-1/(a^7*c^9))^(1/4) - sqrt(2)
*a*c*sqrt(x)*(-1/(a^7*c^9))^(1/8) + x) + 8*(c^2*x^4 + a*c)*(-1/(a^7*c^9))^(
1/8)*arctan(sqrt(a^2*c^2*(-1/(a^7*c^9))^(1/4) + x)*a^6*c^8*(-1/(a^7*c^9))^(
7/8) - a^6*c^8*sqrt(x)*(-1/(a^7*c^9))^(7/8)) + 2*(c^2*x^4 + a*c)*(-1/(a^7*c
^9))^(1/8)*log(a*c*(-1/(a^7*c^9))^(1/8) + sqrt(x)) - 2*(c^2*x^4 + a*c)*(-1/
(a^7*c^9))^(1/8)*log(-a*c*(-1/(a^7*c^9))^(1/8) + sqrt(x)) - 16*sqrt(x))/(c^
2*x^4 + a*c)
```

giac [B] time = 0.58, size = 486, normalized size = 1.58

$$\frac{\left(\frac{a}{c}\right)^{\frac{1}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16\,ac\sqrt{-2\sqrt{2}+4}}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16\,ac\sqrt{-2\sqrt{2}+4}}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16\,ac\sqrt{2\sqrt{2}+4}}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{8}}\arct}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/16*(a/c)^(1/8)*arctan((sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(
sqrt(2) + 2)*(a/c)^(1/8)))/(a*c*sqrt(-2*sqrt(2) + 4)) + 1/16*(a/c)^(1/8)*ar
ctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(sqrt(2) + 2)*(a/c)
^(1/8)))/(a*c*sqrt(-2*sqrt(2) + 4)) + 1/16*(a/c)^(1/8)*arctan((sqrt(sqrt(2)
 + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a*c*sqrt(
2*sqrt(2) + 4)) + 1/16*(a/c)^(1/8)*arctan(-(sqrt(sqrt(2) + 2)*(a/c)^(1/8) -
 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a*c*sqrt(2*sqrt(2) + 4)) + 1
/32*(a/c)^(1/8)*log(sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4)
)/(a*c*sqrt(-2*sqrt(2) + 4)) - 1/32*(a/c)^(1/8)*log(-sqrt(x)*sqrt(sqrt(2) +
 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a*c*sqrt(-2*sqrt(2) + 4)) + 1/32*(a/c)^
(1/8)*log(sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a*c*sq
rt(2*sqrt(2) + 4)) - 1/32*(a/c)^(1/8)*log(-sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)
^(1/8) + x + (a/c)^(1/4))/(a*c*sqrt(2*sqrt(2) + 4)) - 1/4*sqrt(x)/((c*x^4 +
 a)*c)
```

maple [C] time = 0.02, size = 47, normalized size = 0.15

$$-\frac{\sqrt{x}}{4\left(c\,x^4+a\right)c}+\frac{\ln\left(-\operatorname{RootOf}\left(c_Z^8+a\right)+\sqrt{x}\right)}{32c^2\operatorname{RootOf}\left(c_Z^8+a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(c*x^4+a)^2,x)
```

```
[Out] -1/4*x^(1/2)/c/(c*x^4+a)+1/32/c^2*sum(1/_R^7*ln(-_R+x^(1/2)),_R=RootOf(_Z^8
*c+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x^{\frac{9}{2}}}{4\left(acx^4+a^2\right)}-\int\frac{x^{\frac{7}{2}}}{8\left(acx^4+a^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^(9/2)/(a*c*x^4 + a^2) - integrate(1/8*x^(7/2)/(a*c*x^4 + a^2), x)
```

mupad [B] time = 1.07, size = 135, normalized size = 0.44

$$-\frac{\sqrt{x}}{4\,c\left(c\,x^4+a\right)}-\frac{\operatorname{atan}\left(\frac{c^{1/8}\,\sqrt{x}}{(-a)^{1/8}}\right)}{16\,(-a)^{7/8}\,c^{9/8}}+\frac{\operatorname{atan}\left(\frac{c^{1/8}\,\sqrt{x}\,1\mathrm{i}}{(-a)^{1/8}}\right)1\mathrm{i}}{16\,(-a)^{7/8}\,c^{9/8}}+\frac{\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\left(-\frac{1}{32}-\frac{1}{32}\mathrm{i}\right)}{(-a)^{7/8}\,c^{9/8}}+\frac{\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,c^{1/8}}{(}\right.}{(-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(a + c*x^4)^2,x)
```

```
[Out] (atan((c^(1/8)*x^(1/2)*1i)/(-a)^(1/8))*1i)/(16*(-a)^(7/8)*c^(9/8)) - atan((
c^(1/8)*x^(1/2))/(-a)^(1/8))/(16*(-a)^(7/8)*c^(9/8)) - x^(1/2)/(4*c*(a + c*
x^4)) - (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^(1/8))*(1
/32 + 1i/32))/((-a)^(7/8)*c^(9/8)) - (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)
*(1/2 + 1i/2))/(-a)^(1/8))*(1/32 - 1i/32))/((-a)^(7/8)*c^(9/8))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.750 $\int \frac{x^{5/2}}{\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=308

$$-\frac{\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{32\sqrt{2}\,(-a)^{9/8}c^{7/8}}+\frac{\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{32\sqrt{2}\,(-a)^{9/8}c^{7/8}}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}\,(-a)^{9/8}c^{7/8}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}}{16\sqrt{2}\,(}\right.}$$

```
[Out] 1/4*x^(7/2)/a/(c*x^4+a)-1/16*arctan(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(9/8)/
c^(7/8)+1/16*arctanh(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(9/8)/c^(7/8)-1/32*ar
ctan(-1+c^(1/8)*2^(1/2)*x^(1/2)/(-a)^(1/8))/(-a)^(9/8)/c^(7/8)*2^(1/2)-1/32
*arctan(1+c^(1/8)*2^(1/2)*x^(1/2)/(-a)^(1/8))/(-a)^(9/8)/c^(7/8)*2^(1/2)-1/
64*ln((-a)^(1/4)+c^(1/4)*x-(-a)^(1/8)*c^(1/8)*2^(1/2)*x^(1/2))/(-a)^(9/8)/c
^(7/8)*2^(1/2)+1/64*ln((-a)^(1/4)+c^(1/4)*x+(-a)^(1/8)*c^(1/8)*2^(1/2)*x^(1
/2))/(-a)^(9/8)/c^(7/8)*2^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {290, 329, 301, 297, 1162, 617, 204, 1165, 628, 298, 205, 208}

$$-\frac{\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{32\sqrt{2}\,(-a)^{9/8}c^{7/8}}+\frac{\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{32\sqrt{2}\,(-a)^{9/8}c^{7/8}}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}\,(-a)^{9/8}c^{7/8}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}}{16\sqrt{2}\,(}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + c*x^4)^2,x]
```

```
[Out] x^(7/2)/(4*a*(a + c*x^4)) + ArcTan[1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)
]/(16*Sqrt[2]*(-a)^(9/8)*c^(7/8)) - ArcTan[1 + (Sqrt[2]*c^(1/8)*Sqrt[x])/(-
a)^(1/8)]/(16*Sqrt[2]*(-a)^(9/8)*c^(7/8)) - ArcTan[(c^(1/8)*Sqrt[x])/(-a)^(
1/8)]/(16*(-a)^(9/8)*c^(7/8)) + ArcTanh[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(16*(
-a)^(9/8)*c^(7/8)) - Log[(-a)^(1/4) - Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] +
c^(1/4)*x]/(32*Sqrt[2]*(-a)^(9/8)*c^(7/8)) + Log[(-a)^(1/4) + Sqrt[2]*(-a)^
(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x]/(32*Sqrt[2]*(-a)^(9/8)*c^(7/8))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
```

```
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 301

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[x^(m - n/2
)/(r + s*x^(n/2)), x], x] - Dist[s/(2*b), Int[x^(m - n/2)/(r - s*x^(n/2)),
x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] &&
 LtQ[m, n] &&  !GtQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{5/2}}{\left(a+cx^4\right)^2}\,dx = \frac{x^{7/2}}{4a\left(a+cx^4\right)} + \frac{\int \frac{x^{5/2}}{a+cx^4}\,dx}{8a}$$

$$= \frac{x^{7/2}}{4a\left(a+cx^4\right)} + \frac{\text{Subst}\left(\int \frac{x^6}{a+cx^8}\,dx, x, \sqrt{x}\right)}{4a}$$

$$= \frac{x^{7/2}}{4a\left(a+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{x^2}{\sqrt{-a}-\sqrt{c}x^4}\,dx, x, \sqrt{x}\right)}{8a\sqrt{c}} + \frac{\text{Subst}\left(\int \frac{x^2}{\sqrt{-a}+\sqrt{c}x^4}\,dx, x, \sqrt{x}\right)}{8a\sqrt{c}}$$

$$= \frac{x^{7/2}}{4a\left(a+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x^2}\,dx, x, \sqrt{x}\right)}{16ac^{3/4}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}+\sqrt[4]{c}x^2}\,dx, x, \sqrt{x}\right)}{16ac^{3/4}} - \frac{\text{Subst}\left(\int$$

$$= \frac{x^{7/2}}{4a\left(a+cx^4\right)} - \frac{\tan^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{9/8}c^{7/8}} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{9/8}c^{7/8}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[4]{-a}}{\sqrt[4]{c}} - \frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{c}} + x^2}\,dx, x, \sqrt{x}\right)}{32ac} + \frac{\text{Su}}{}$$

$$= \frac{x^{7/2}}{4a\left(a+cx^4\right)} - \frac{\tan^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{9/8}c^{7/8}} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{9/8}c^{7/8}} - \frac{\log\left(\sqrt[4]{-a} - \sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x} + \sqrt[4]{c}x\right)}{32\sqrt{2}(-a)^{9/8}c^{7/8}} + \frac{\text{lo}}{}$$

$$= \frac{x^{7/2}}{4a\left(a+cx^4\right)} + \frac{\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}(-a)^{9/8}c^{7/8}} - \frac{\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}(-a)^{9/8}c^{7/8}} - \frac{\tan^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{9/8}c^{7/8}} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{c}\sqrt{}}{\sqrt[8]{-a}}\right.}{16(-a)^{9/8}c^{7/}}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.09

$$\frac{2x^{7/2}\,{}_2F_1\left(\frac{7}{8}, 2; \frac{15}{8}; -\frac{cx^4}{a}\right)}{7a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + c*x^4)^2,x]
```

```
[Out] (2*x^(7/2)*Hypergeometric2F1[7/8, 2, 15/8, -((c*x^4)/a)])/(7*a^2)
```

fricas [B] time = 0.69, size = 542, normalized size = 1.76

$$\frac{16\,x^{\frac{7}{2}} - 4\sqrt{2}\left(acx^4+a^2\right)\left(-\frac{1}{a^9c^7}\right)^{\frac{1}{8}}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\,a^8c^6\sqrt{x}\left(-\frac{1}{a^9c^7}\right)^{\frac{7}{8}} - a^7c^5\left(-\frac{1}{a^9c^7}\right)^{\frac{3}{4}} + x}\,ac\left(-\frac{1}{a^9c^7}\right)^{\frac{1}{8}} - \sqrt{2}\,ac\sqrt{x}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/64*(16*x^(7/2) - 4*sqrt(2)*(a*c*x^4 + a^2)*(-1/(a^9*c^7))^(1/8)*arctan(sq
rt(2)*sqrt(sqrt(2)*a^8*c^6*sqrt(x)*(-1/(a^9*c^7))^(7/8) - a^7*c^5*(-1/(a^9*
c^7))^(3/4) + x)*a*c*(-1/(a^9*c^7))^(1/8) - sqrt(2)*a*c*sqrt(x)*(-1/(a^9*c^
7))^(1/8) + 1) - 4*sqrt(2)*(a*c*x^4 + a^2)*(-1/(a^9*c^7))^(1/8)*arctan(sqrt
(2)*sqrt(-sqrt(2)*a^8*c^6*sqrt(x)*(-1/(a^9*c^7))^(7/8) - a^7*c^5*(-1/(a^9*c
^7))^(3/4) + x)*a*c*(-1/(a^9*c^7))^(1/8) - sqrt(2)*a*c*sqrt(x)*(-1/(a^9*c^7
```

```
))^(1/8) - 1) + sqrt(2)*(a*c*x^4 + a^2)*(-1/(a^9*c^7))^(1/8)*log(sqrt(2)*a^
8*c^6*sqrt(x)*(-1/(a^9*c^7))^(7/8) - a^7*c^5*(-1/(a^9*c^7))^(3/4) + x) - sq
rt(2)*(a*c*x^4 + a^2)*(-1/(a^9*c^7))^(1/8)*log(-sqrt(2)*a^8*c^6*sqrt(x)*(-1
/(a^9*c^7))^(7/8) - a^7*c^5*(-1/(a^9*c^7))^(3/4) + x) - 8*(a*c*x^4 + a^2)*(
-1/(a^9*c^7))^(1/8)*arctan(sqrt(-a^7*c^5*(-1/(a^9*c^7))^(3/4) + x)*a*c*(-1/
(a^9*c^7))^(1/8) - a*c*sqrt(x)*(-1/(a^9*c^7))^(1/8)) + 2*(a*c*x^4 + a^2)*(-
1/(a^9*c^7))^(1/8)*log(a^8*c^6*(-1/(a^9*c^7))^(7/8) + sqrt(x)) - 2*(a*c*x^4
 + a^2)*(-1/(a^9*c^7))^(1/8)*log(-a^8*c^6*(-1/(a^9*c^7))^(7/8) + sqrt(x)))/
(a*c*x^4 + a^2)
```

giac [B] time = 0.73, size = 462, normalized size = 1.50

$$\frac{x^{\frac{7}{2}}}{4\left(cx^4+a\right)a}+\frac{\left(\frac{a}{c}\right)^{\frac{7}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16a^2\sqrt{-2\sqrt{2}+4}}+\frac{\left(\frac{a}{c}\right)^{\frac{7}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16a^2\sqrt{-2\sqrt{2}+4}}+\frac{\left(\frac{a}{c}\right)^{\frac{7}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right.}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*x^(7/2)/((c*x^4 + a)*a) + 1/16*(a/c)^(7/8)*arctan((sqrt(-sqrt(2) + 2)*(
a/c)^(1/8) + 2*sqrt(x))/(sqrt(sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*sqrt(-2*sqrt(
2) + 4)) + 1/16*(a/c)^(7/8)*arctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqr
t(x))/(sqrt(sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*sqrt(-2*sqrt(2) + 4)) + 1/16*(a
/c)^(7/8)*arctan((sqrt(sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2)
 + 2)*(a/c)^(1/8)))/(a^2*sqrt(2*sqrt(2) + 4)) + 1/16*(a/c)^(7/8)*arctan(-(s
qrt(sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))
/(a^2*sqrt(2*sqrt(2) + 4)) - 1/32*(a/c)^(7/8)*log(sqrt(x)*sqrt(sqrt(2) + 2)
*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*sqrt(-2*sqrt(2) + 4)) + 1/32*(a/c)^(7/
8)*log(-sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*sqrt(
-2*sqrt(2) + 4)) - 1/32*(a/c)^(7/8)*log(sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1
/8) + x + (a/c)^(1/4))/(a^2*sqrt(2*sqrt(2) + 4)) + 1/32*(a/c)^(7/8)*log(-sq
rt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*sqrt(2*sqrt(2)
 + 4))
```

maple [C] time = 0.01, size = 50, normalized size = 0.16

$$\frac{x^{\frac{7}{2}}}{4\left(c\,x^4+a\right)a}+\frac{\ln\left(-\operatorname{RootOf}\left(c_Z^8+a\right)+\sqrt{x}\right)}{32ac\operatorname{RootOf}\left(c_Z^8+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(c*x^4+a)^2,x)
```

```
[Out] 1/4*x^(7/2)/a/(c*x^4+a)+1/32/a/c*sum(1/_R*ln(-_R+x^(1/2)),_R=RootOf(_Z^8*c+
a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x^{\frac{7}{2}}}{4\left(acx^4+a^2\right)}+\int\frac{x^{\frac{5}{2}}}{8\left(acx^4+a^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^(7/2)/(a*c*x^4 + a^2) + integrate(1/8*x^(5/2)/(a*c*x^4 + a^2), x)
```

mupad [B] time = 0.13, size = 135, normalized size = 0.44

$$\frac{x^{7/2}}{4\,a\,\left(c\,x^4+a\right)}-\frac{\operatorname{atan}\left(\frac{c^{1/8}\,\sqrt{x}}{(-a)^{1/8}}\right)}{16\,(-a)^{9/8}\,c^{7/8}}-\frac{\operatorname{atan}\left(\frac{c^{1/8}\,\sqrt{x}\,1\mathrm{i}}{(-a)^{1/8}}\right)\,1\mathrm{i}}{16\,(-a)^{9/8}\,c^{7/8}}+\frac{\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(-\frac{1}{32}+\frac{1}{32}\mathrm{i}\right)}{(-a)^{9/8}\,c^{7/8}}+\frac{\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\right.\right.}{(-a)^{1/8}}\right.}{(-a)^{9/8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + c*x^4)^2,x)
```

```
[Out] x^(7/2)/(4*a*(a + c*x^4)) - atan((c^(1/8)*x^(1/2))/(-a)^(1/8))/(16*(-a)^(9/
8)*c^(7/8)) - (atan((c^(1/8)*x^(1/2)*1i)/(-a)^(1/8))*1i)/(16*(-a)^(9/8)*c^(
7/8)) - (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^(1/8))*(1
/32 - 1i/32))/((-a)^(9/8)*c^(7/8)) - (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)
*(1/2 + 1i/2))/(-a)^(1/8))*(1/32 + 1i/32))/((-a)^(9/8)*c^(7/8))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.751 $\int \frac{x^{3/2}}{\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=308

$$\frac{3\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{32\sqrt{2}\,(-a)^{11/8}c^{5/8}}-\frac{3\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{32\sqrt{2}\,(-a)^{11/8}c^{5/8}}+\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}\,(-a)^{11/8}c^{5/8}}-\frac{3\,\text{ta}}{1}$$

```
[Out] 1/4*x^(5/2)/a/(c*x^4+a)+3/16*arctan(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(11/8)
/c^(5/8)+3/16*arctanh(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(11/8)/c^(5/8)-3/32*
arctan(-1+c^(1/8)*2^(1/2)*x^(1/2)/(-a)^(1/8))/(-a)^(11/8)/c^(5/8)*2^(1/2)-3
/32*arctan(1+c^(1/8)*2^(1/2)*x^(1/2)/(-a)^(1/8))/(-a)^(11/8)/c^(5/8)*2^(1/2
)+3/64*ln((-a)^(1/4)+c^(1/4)*x-(-a)^(1/8)*c^(1/8)*2^(1/2)*x^(1/2))/(-a)^(11
/8)/c^(5/8)*2^(1/2)-3/64*ln((-a)^(1/4)+c^(1/4)*x+(-a)^(1/8)*c^(1/8)*2^(1/2)
*x^(1/2))/(-a)^(11/8)/c^(5/8)*2^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {290, 329, 301, 211, 1165, 628, 1162, 617, 204, 212, 208, 205}

$$\frac{3\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{32\sqrt{2}\,(-a)^{11/8}c^{5/8}}-\frac{3\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{32\sqrt{2}\,(-a)^{11/8}c^{5/8}}+\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}\,(-a)^{11/8}c^{5/8}}-\frac{3\,\text{ta}}{1}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + c*x^4)^2,x]
```

```
[Out] x^(5/2)/(4*a*(a + c*x^4)) + (3*ArcTan[1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1
/8)])/(16*Sqrt[2]*(-a)^(11/8)*c^(5/8)) - (3*ArcTan[1 + (Sqrt[2]*c^(1/8)*Sqr
t[x])/(-a)^(1/8)])/(16*Sqrt[2]*(-a)^(11/8)*c^(5/8)) + (3*ArcTan[(c^(1/8)*Sq
rt[x])/(-a)^(1/8)])/(16*(-a)^(11/8)*c^(5/8)) + (3*ArcTanh[(c^(1/8)*Sqrt[x])
/(-a)^(1/8)])/(16*(-a)^(11/8)*c^(5/8)) + (3*Log[(-a)^(1/4) - Sqrt[2]*(-a)^(
1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x])/(32*Sqrt[2]*(-a)^(11/8)*c^(5/8)) - (3*Lo
g[(-a)^(1/4) + Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x])/(32*Sqrt[2]
*(-a)^(11/8)*c^(5/8))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 301

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[x^(m - n/2
)/(r + s*x^(n/2)), x], x] - Dist[s/(2*b), Int[x^(m - n/2)/(r - s*x^(n/2)),
x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] &&
 LtQ[m, n] &&  !GtQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(a+cx^4\right)^2}\,dx = \frac{x^{5/2}}{4a\left(a+cx^4\right)} + \frac{3\int \frac{x^{3/2}}{a+cx^4}\,dx}{8a}$$

$$= \frac{x^{5/2}}{4a\left(a+cx^4\right)} + \frac{3\,\text{Subst}\left(\int \frac{x^4}{a+cx^8}\,dx, x, \sqrt{x}\right)}{4a}$$

$$= \frac{x^{5/2}}{4a\left(a+cx^4\right)} - \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt{-a}-\sqrt{c}\,x^4}\,dx, x, \sqrt{x}\right)}{8a\sqrt{c}} + \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt{-a}+\sqrt{c}\,x^4}\,dx, x, \sqrt{x}\right)}{8a\sqrt{c}}$$

$$= \frac{x^{5/2}}{4a\left(a+cx^4\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}\,x^2}\,dx, x, \sqrt{x}\right)}{16(-a)^{5/4}\sqrt{c}} + \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}+\sqrt[4]{c}\,x^2}\,dx, x, \sqrt{x}\right)}{16(-a)^{5/4}\sqrt{c}} - \frac{3\,\text{Su}}{}$$

$$= \frac{x^{5/2}}{4a\left(a+cx^4\right)} + \frac{3\tan^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{11/8}c^{5/8}} + \frac{3\tanh^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{11/8}c^{5/8}} - \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[4]{-a}}{\sqrt[4]{c}} - \frac{\sqrt{2}\sqrt[8]{-a}\,x}{\sqrt[8]{c}} + x^2}\,dx, x, \sqrt{}\right.}{32(-a)^{5/4}c^{3/4}}$$

$$= \frac{x^{5/2}}{4a\left(a+cx^4\right)} + \frac{3\tan^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{11/8}c^{5/8}} + \frac{3\tanh^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{11/8}c^{5/8}} + \frac{3\log\left(\sqrt[4]{-a} - \sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x} + \sqrt[4]{c}\right.}{32\sqrt{2}\,(-a)^{11/8}c^{5/8}}$$

$$= \frac{x^{5/2}}{4a\left(a+cx^4\right)} + \frac{3\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}\,(-a)^{11/8}c^{5/8}} - \frac{3\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}\,(-a)^{11/8}c^{5/8}} + \frac{3\tan^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{11/8}c^{5/8}} + \frac{3\tan}{16(}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.09

$$\frac{2x^{5/2}\,{}_2F_1\left(\frac{5}{8}, 2; \frac{13}{8}; -\frac{cx^4}{a}\right)}{5a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + c*x^4)^2,x]
[Out] (2*x^(5/2)*Hypergeometric2F1[5/8, 2, 13/8, -((c*x^4)/a)])/(5*a^2)
```

fricas [B] time = 0.79, size = 557, normalized size = 1.81

$$\frac{12\sqrt{2}\left(acx^4+a^2\right)\left(-\frac{1}{a^{11}c^5}\right)^{\frac{1}{8}}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\,a^7c^3\sqrt{x}\left(-\frac{1}{a^{11}c^5}\right)^{\frac{5}{8}} - a^3c\left(-\frac{1}{a^{11}c^5}\right)^{\frac{1}{4}} + x}\,a^4c^2\left(-\frac{1}{a^{11}c^5}\right)^{\frac{3}{8}} - \sqrt{2}\,a^4c^2\sqrt{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+a)^2,x, algorithm="fricas")
[Out] 1/64*(12*sqrt(2)*(a*c*x^4 + a^2)*(-1/(a^11*c^5))^(1/8)*arctan(sqrt(2)*sqrt(
sqrt(2)*a^7*c^3*sqrt(x)*(-1/(a^11*c^5))^(5/8) - a^3*c*(-1/(a^11*c^5))^(1/4)
 + x)*a^4*c^2*(-1/(a^11*c^5))^(3/8) - sqrt(2)*a^4*c^2*sqrt(x)*(-1/(a^11*c^5
))^(3/8) + 1) + 12*sqrt(2)*(a*c*x^4 + a^2)*(-1/(a^11*c^5))^(1/8)*arctan(sqr
```

```
t(2)*sqrt(-sqrt(2)*a^7*c^3*sqrt(x)*(-1/(a^11*c^5))^(5/8) - a^3*c*(-1/(a^11*
c^5))^(1/4) + x)*a^4*c^2*(-1/(a^11*c^5))^(3/8) - sqrt(2)*a^4*c^2*sqrt(x)*(-
1/(a^11*c^5))^(3/8) - 1) + 3*sqrt(2)*(a*c*x^4 + a^2)*(-1/(a^11*c^5))^(1/8)*
log(sqrt(2)*a^7*c^3*sqrt(x)*(-1/(a^11*c^5))^(5/8) - a^3*c*(-1/(a^11*c^5))^(
1/4) + x) - 3*sqrt(2)*(a*c*x^4 + a^2)*(-1/(a^11*c^5))^(1/8)*log(-sqrt(2)*a^
7*c^3*sqrt(x)*(-1/(a^11*c^5))^(5/8) - a^3*c*(-1/(a^11*c^5))^(1/4) + x) + 16
*x^(5/2) - 24*(a*c*x^4 + a^2)*(-1/(a^11*c^5))^(1/8)*arctan(sqrt(-a^3*c*(-1/
(a^11*c^5))^(1/4) + x)*a^4*c^2*(-1/(a^11*c^5))^(3/8) - a^4*c^2*sqrt(x)*(-1/
(a^11*c^5))^(3/8)) - 6*(a*c*x^4 + a^2)*(-1/(a^11*c^5))^(1/8)*log(a^7*c^3*(-
1/(a^11*c^5))^(5/8) + sqrt(x)) + 6*(a*c*x^4 + a^2)*(-1/(a^11*c^5))^(1/8)*lo
g(-a^7*c^3*(-1/(a^11*c^5))^(5/8) + sqrt(x)))/(a*c*x^4 + a^2)
```

giac [B] time = 0.77, size = 462, normalized size = 1.50

$$\frac{x^{\frac{5}{2}}}{4\left(cx^4+a\right)a}-\frac{3\left(\frac{a}{c}\right)^{\frac{5}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16a^2\sqrt{2\sqrt{2}+4}}-\frac{3\left(\frac{a}{c}\right)^{\frac{5}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16a^2\sqrt{2\sqrt{2}+4}}+\frac{3\left(\frac{a}{c}\right)^{\frac{5}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right.\right.}{\sqrt{-\sqrt{2}+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*x^(5/2)/((c*x^4 + a)*a) - 3/16*(a/c)^(5/8)*arctan((sqrt(-sqrt(2) + 2)*(
a/c)^(1/8) + 2*sqrt(x))/(sqrt(sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*sqrt(2*sqrt(2
) + 4)) - 3/16*(a/c)^(5/8)*arctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt
(x))/(sqrt(sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*sqrt(2*sqrt(2) + 4)) + 3/16*(a/c
)^(5/8)*arctan((sqrt(sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) +
 2)*(a/c)^(1/8)))/(a^2*sqrt(-2*sqrt(2) + 4)) + 3/16*(a/c)^(5/8)*arctan(-(sq
rt(sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/
(a^2*sqrt(-2*sqrt(2) + 4)) - 3/32*(a/c)^(5/8)*log(sqrt(x)*sqrt(sqrt(2) + 2)
*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*sqrt(2*sqrt(2) + 4)) + 3/32*(a/c)^(5/8
)*log(-sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*sqrt(2
*sqrt(2) + 4)) + 3/32*(a/c)^(5/8)*log(sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8
) + x + (a/c)^(1/4))/(a^2*sqrt(-2*sqrt(2) + 4)) - 3/32*(a/c)^(5/8)*log(-sqr
t(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*sqrt(-2*sqrt(2)
 + 4))
```

maple [C] time = 0.01, size = 50, normalized size = 0.16

$$\frac{x^{\frac{5}{2}}}{4\left(c\,x^4+a\right)a}+\frac{3\ln\left(-\operatorname{RootOf}\left(c_Z^8+a\right)+\sqrt{x}\right)}{32ac\operatorname{RootOf}\left(c_Z^8+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(c*x^4+a)^2,x)
```

```
[Out] 1/4*x^(5/2)/a/(c*x^4+a)+3/32/a/c*sum(1/_R^3*ln(-_R+x^(1/2)),_R=RootOf(_Z^8*
c+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x^{\frac{5}{2}}}{4\left(acx^4+a^2\right)}+3\int\frac{x^{\frac{3}{2}}}{8\left(acx^4+a^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^(5/2)/(a*c*x^4 + a^2) + 3*integrate(1/8*x^(3/2)/(a*c*x^4 + a^2), x)
```

mupad [B] time = 1.09, size = 135, normalized size = 0.44

$$\frac{x^{5/2}}{4\,a\,\left(c\,x^4+a\right)}+\frac{3\,\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}}{(-a)^{1/8}}\right)}{16\,(-a)^{11/8}\,c^{5/8}}-\frac{\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}\,1\mathrm{i}}{(-a)^{1/8}}\right)\,3\mathrm{i}}{16\,(-a)^{11/8}\,c^{5/8}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\left(-\frac{3}{32}-\frac{3}{32}\mathrm{i}\right)}{(-a)^{11/8}\,c^{5/8}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/}}{(-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + c*x^4)^2,x)
```

```
[Out] x^(5/2)/(4*a*(a + c*x^4)) + (3*atan((c^(1/8)*x^(1/2))/(-a)^(1/8)))/(16*(-a)
^(11/8)*c^(5/8)) - (atan((c^(1/8)*x^(1/2)*1i)/(-a)^(1/8))*3i)/(16*(-a)^(11/
8)*c^(5/8)) - (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^(1/
8))*(3/32 + 3i/32))/((-a)^(11/8)*c^(5/8)) - (2^(1/2)*atan((2^(1/2)*c^(1/8)*
x^(1/2)*(1/2 + 1i/2))/(-a)^(1/8))*(3/32 - 3i/32))/((-a)^(11/8)*c^(5/8))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.752 $\int \frac{\sqrt{x}}{\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=308

$$\frac{5\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{32\sqrt{2}\,(-a)^{13/8}c^{3/8}}-\frac{5\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{32\sqrt{2}\,(-a)^{13/8}c^{3/8}}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}\,(-a)^{13/8}c^{3/8}}+\frac{5\tan^{-}}{16\sqrt{}}$$

```
[Out] 1/4*x^(3/2)/a/(c*x^4+a)-5/16*arctan(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(13/8)
/c^(3/8)+5/16*arctanh(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(13/8)/c^(3/8)+5/32*
arctan(-1+c^(1/8)*2^(1/2)*x^(1/2)/(-a)^(1/8))/(-a)^(13/8)/c^(3/8)*2^(1/2)+5
/32*arctan(1+c^(1/8)*2^(1/2)*x^(1/2)/(-a)^(1/8))/(-a)^(13/8)/c^(3/8)*2^(1/2
)+5/64*ln((-a)^(1/4)+c^(1/4)*x-(-a)^(1/8)*c^(1/8)*2^(1/2)*x^(1/2))/(-a)^(13
/8)/c^(3/8)*2^(1/2)-5/64*ln((-a)^(1/4)+c^(1/4)*x+(-a)^(1/8)*c^(1/8)*2^(1/2)
*x^(1/2))/(-a)^(13/8)/c^(3/8)*2^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {290, 329, 300, 297, 1162, 617, 204, 1165, 628, 298, 205, 208}

$$\frac{5\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{32\sqrt{2}\,(-a)^{13/8}c^{3/8}}-\frac{5\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{32\sqrt{2}\,(-a)^{13/8}c^{3/8}}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}\,(-a)^{13/8}c^{3/8}}+\frac{5\tan^{-}}{16\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + c*x^4)^2,x]
```

```
[Out] x^(3/2)/(4*a*(a + c*x^4)) - (5*ArcTan[1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1
/8)])/(16*Sqrt[2]*(-a)^(13/8)*c^(3/8)) + (5*ArcTan[1 + (Sqrt[2]*c^(1/8)*Sqr
t[x])/(-a)^(1/8)])/(16*Sqrt[2]*(-a)^(13/8)*c^(3/8)) - (5*ArcTan[(c^(1/8)*Sq
rt[x])/(-a)^(1/8)])/(16*(-a)^(13/8)*c^(3/8)) + (5*ArcTanh[(c^(1/8)*Sqrt[x])
/(-a)^(1/8)])/(16*(-a)^(13/8)*c^(3/8)) + (5*Log[(-a)^(1/4) - Sqrt[2]*(-a)^(
1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x])/(32*Sqrt[2]*(-a)^(13/8)*c^(3/8)) - (5*Lo
g[(-a)^(1/4) + Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x])/(32*Sqrt[2]
*(-a)^(13/8)*c^(3/8))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
```

```
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 300

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[x^m/(r + s
*x^(n/2)), x], x] + Dist[r/(2*a), Int[x^m/(r - s*x^(n/2)), x], x]] /; FreeQ
[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n/2] &&  !GtQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\left(a+cx^4\right)^2}\,dx = \frac{x^{3/2}}{4a\left(a+cx^4\right)} + \frac{5\int \frac{\sqrt{x}}{a+cx^4}\,dx}{8a}$$

$$= \frac{x^{3/2}}{4a\left(a+cx^4\right)} + \frac{5\,\text{Subst}\left(\int \frac{x^2}{a+cx^8}\,dx, x, \sqrt{x}\right)}{4a}$$

$$= \frac{x^{3/2}}{4a\left(a+cx^4\right)} + \frac{5\,\text{Subst}\left(\int \frac{x^2}{\sqrt{-a}-\sqrt{c}\,x^4}\,dx, x, \sqrt{x}\right)}{8(-a)^{3/2}} + \frac{5\,\text{Subst}\left(\int \frac{x^2}{\sqrt{-a}+\sqrt{c}\,x^4}\,dx, x, \sqrt{x}\right)}{8(-a)^{3/2}}$$

$$= \frac{x^{3/2}}{4a\left(a+cx^4\right)} + \frac{5\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}\,x^2}\,dx, x, \sqrt{x}\right)}{16(-a)^{3/2}\sqrt[4]{c}} - \frac{5\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}+\sqrt[4]{c}\,x^2}\,dx, x, \sqrt{x}\right)}{16(-a)^{3/2}\sqrt[4]{c}} - \frac{5\,\text{Subs}}{}$$

$$= \frac{x^{3/2}}{4a\left(a+cx^4\right)} - \frac{5\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{13/8}c^{3/8}} + \frac{5\tanh^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{13/8}c^{3/8}} + \frac{5\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}-\frac{\sqrt{2}\,\sqrt[8]{-a}\,x}{\sqrt[8]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{32(-a)^{3/2}\sqrt{c}}$$

$$= \frac{x^{3/2}}{4a\left(a+cx^4\right)} - \frac{5\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{13/8}c^{3/8}} + \frac{5\tanh^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{13/8}c^{3/8}} + \frac{5\log\left(\sqrt[4]{-a}-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{c}\,x\right)}{32\sqrt{2}\,(-a)^{13/8}c^{3/8}}$$

$$= \frac{x^{3/2}}{4a\left(a+cx^4\right)} - \frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}\,(-a)^{13/8}c^{3/8}} + \frac{5\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}\,(-a)^{13/8}c^{3/8}} - \frac{5\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{13/8}c^{3/8}} + \frac{5\tanh^{-}}{16(-a}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.09

$$\frac{2x^{3/2}\,{}_2F_1\left(\frac{3}{8}, 2; \frac{11}{8}; -\frac{cx^4}{a}\right)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + c*x^4)^2,x]
```

```
[Out] (2*x^(3/2)*Hypergeometric2F1[3/8, 2, 11/8, -((c*x^4)/a)])/(3*a^2)
```

fricas [B] time = 0.68, size = 550, normalized size = 1.79

$$-\frac{20\sqrt{2}\left(acx^4+a^2\right)\left(-\frac{1}{a^{13}c^3}\right)^{\frac{1}{8}}\arctan\left(\sqrt{2}\sqrt{a^{10}c^2\left(-\frac{1}{a^{13}c^3}\right)^{\frac{3}{4}}+\sqrt{2}\,a^5c\sqrt{x}\left(-\frac{1}{a^{13}c^3}\right)^{\frac{3}{8}}+x}\,a^8c^2\left(-\frac{1}{a^{13}c^3}\right)^{\frac{5}{8}}-\sqrt{2}\,a^8c^2\sqrt{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] -1/64*(20*sqrt(2)*(a*c*x^4 + a^2)*(-1/(a^13*c^3))^(1/8)*arctan(sqrt(2)*sqrt
(a^10*c^2*(-1/(a^13*c^3))^(3/4) + sqrt(2)*a^5*c*sqrt(x)*(-1/(a^13*c^3))^(3/
8) + x)*a^8*c^2*(-1/(a^13*c^3))^(5/8) - sqrt(2)*a^8*c^2*sqrt(x)*(-1/(a^13*c
^3))^(5/8) + 1) + 20*sqrt(2)*(a*c*x^4 + a^2)*(-1/(a^13*c^3))^(1/8)*arctan(s
qrt(2)*sqrt(a^10*c^2*(-1/(a^13*c^3))^(3/4) - sqrt(2)*a^5*c*sqrt(x)*(-1/(a^1
3*c^3))^(3/8) + x)*a^8*c^2*(-1/(a^13*c^3))^(5/8) - sqrt(2)*a^8*c^2*sqrt(x)*
```

```
(-1/(a^13*c^3))^(5/8) - 1) - 5*sqrt(2)*(a*c*x^4 + a^2)*(-1/(a^13*c^3))^(1/8
)*log(a^10*c^2*(-1/(a^13*c^3))^(3/4) + sqrt(2)*a^5*c*sqrt(x)*(-1/(a^13*c^3)
)^(3/8) + x) + 5*sqrt(2)*(a*c*x^4 + a^2)*(-1/(a^13*c^3))^(1/8)*log(a^10*c^2
*(-1/(a^13*c^3))^(3/4) - sqrt(2)*a^5*c*sqrt(x)*(-1/(a^13*c^3))^(3/8) + x) -
 40*(a*c*x^4 + a^2)*(-1/(a^13*c^3))^(1/8)*arctan(sqrt(a^10*c^2*(-1/(a^13*c^
3))^(3/4) + x)*a^8*c^2*(-1/(a^13*c^3))^(5/8) - a^8*c^2*sqrt(x)*(-1/(a^13*c^
3))^(5/8)) + 10*(a*c*x^4 + a^2)*(-1/(a^13*c^3))^(1/8)*log(a^5*c*(-1/(a^13*c
^3))^(3/8) + sqrt(x)) - 10*(a*c*x^4 + a^2)*(-1/(a^13*c^3))^(1/8)*log(-a^5*c
*(-1/(a^13*c^3))^(3/8) + sqrt(x)) - 16*x^(3/2))/(a*c*x^4 + a^2)
```

giac [B] time = 0.72, size = 462, normalized size = 1.50

$$\frac{x^{\frac{3}{2}}}{4\left(cx^4+a\right)a}-\frac{5\left(\frac{a}{c}\right)^{\frac{3}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16a^2\sqrt{2\sqrt{2}+4}}-\frac{5\left(\frac{a}{c}\right)^{\frac{3}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16a^2\sqrt{2\sqrt{2}+4}}+\frac{5\left(\frac{a}{c}\right)^{\frac{3}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}+}}{\sqrt{-}\right.}\right.}{16a^2\sqrt{-2\sqrt{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*x^(3/2)/((c*x^4 + a)*a) - 5/16*(a/c)^(3/8)*arctan((sqrt(-sqrt(2) + 2)*(
a/c)^(1/8) + 2*sqrt(x))/(sqrt(sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*sqrt(2*sqrt(2
) + 4)) - 5/16*(a/c)^(3/8)*arctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt
(x))/(sqrt(sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*sqrt(2*sqrt(2) + 4)) + 5/16*(a/c
)^(3/8)*arctan((sqrt(sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) +
 2)*(a/c)^(1/8)))/(a^2*sqrt(-2*sqrt(2) + 4)) + 5/16*(a/c)^(3/8)*arctan(-(sq
rt(sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/
(a^2*sqrt(-2*sqrt(2) + 4)) + 5/32*(a/c)^(3/8)*log(sqrt(x)*sqrt(sqrt(2) + 2)
*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*sqrt(2*sqrt(2) + 4)) - 5/32*(a/c)^(3/8
)*log(-sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*sqrt(2
*sqrt(2) + 4)) - 5/32*(a/c)^(3/8)*log(sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8
) + x + (a/c)^(1/4))/(a^2*sqrt(-2*sqrt(2) + 4)) + 5/32*(a/c)^(3/8)*log(-sqr
t(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*sqrt(-2*sqrt(2)
 + 4))
```

maple [C] time = 0.01, size = 50, normalized size = 0.16

$$\frac{x^{\frac{3}{2}}}{4\left(c\,x^4+a\right)a}+\frac{5\ln\left(-\operatorname{RootOf}\left(c_Z^8+a\right)+\sqrt{x}\right)}{32ac\operatorname{RootOf}\left(c_Z^8+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(c*x^4+a)^2,x)
```

```
[Out] 1/4*x^(3/2)/a/(c*x^4+a)+5/32/a/c*sum(1/_R^5*ln(-_R+x^(1/2)),_R=RootOf(_Z^8*
c+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x^{\frac{3}{2}}}{4\left(acx^4+a^2\right)}+5\int\frac{\sqrt{x}}{8\left(acx^4+a^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^(3/2)/(a*c*x^4 + a^2) + 5*integrate(1/8*sqrt(x)/(a*c*x^4 + a^2), x)
```

mupad [B] time = 0.14, size = 135, normalized size = 0.44

$$\frac{x^{3/2}}{4\,a\,\left(c\,x^4+a\right)}-\frac{5\,\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}}{(-a)^{1/8}}\right)}{16\,(-a)^{13/8}\,c^{3/8}}-\frac{\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}\,1\mathrm{i}}{(-a)^{1/8}}\right)\,5\mathrm{i}}{16\,(-a)^{13/8}\,c^{3/8}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\left(\frac{5}{32}-\frac{5}{32}\mathrm{i}\right)}{(-a)^{13/8}\,c^{3/8}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\right.\right.}{(-a)^{1/8}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + c*x^4)^2,x)
```

```
[Out] x^(3/2)/(4*a*(a + c*x^4)) - (5*atan((c^(1/8)*x^(1/2))/(-a)^(1/8)))/(16*(-a)
^(13/8)*c^(3/8)) - (atan((c^(1/8)*x^(1/2)*1i)/(-a)^(1/8))*5i)/(16*(-a)^(13/
8)*c^(3/8)) + (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^(1/
8))*(5/32 - 5i/32))/((-a)^(13/8)*c^(3/8)) + (2^(1/2)*atan((2^(1/2)*c^(1/8)*
x^(1/2)*(1/2 + 1i/2))/(-a)^(1/8))*(5/32 + 5i/32))/((-a)^(13/8)*c^(3/8))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.753 $\int \frac{1}{\sqrt{x}\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=308

$$\frac{\sqrt{x}}{4a\left(a+cx^4\right)}-\frac{7\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{32\sqrt{2}\,(-a)^{15/8}\sqrt[8]{c}}+\frac{7\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{32\sqrt{2}\,(-a)^{15/8}\sqrt[8]{c}}-\frac{7\tan^{-1}\left(1-$$

```
[Out] 7/16*arctan(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(15/8)/c^(1/8)+7/16*arctanh(c^
(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(15/8)/c^(1/8)+7/32*arctan(-1+c^(1/8)*2^(1/2
)*x^(1/2)/(-a)^(1/8))/(-a)^(15/8)/c^(1/8)*2^(1/2)+7/32*arctan(1+c^(1/8)*2^(
1/2)*x^(1/2)/(-a)^(1/8))/(-a)^(15/8)/c^(1/8)*2^(1/2)-7/64*ln((-a)^(1/4)+c^(
1/4)*x-(-a)^(1/8)*c^(1/8)*2^(1/2)*x^(1/2))/(-a)^(15/8)/c^(1/8)*2^(1/2)+7/64
*ln((-a)^(1/4)+c^(1/4)*x+(-a)^(1/8)*c^(1/8)*2^(1/2)*x^(1/2))/(-a)^(15/8)/c^
(1/8)*2^(1/2)+1/4*x^(1/2)/a/(c*x^4+a)
```

Rubi [A] time = 0.27, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {290, 329, 214, 212, 208, 205, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt{x}}{4a\left(a+cx^4\right)}-\frac{7\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{32\sqrt{2}\,(-a)^{15/8}\sqrt[8]{c}}+\frac{7\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{32\sqrt{2}\,(-a)^{15/8}\sqrt[8]{c}}-\frac{7\tan^{-1}\left(1-$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + c*x^4)^2),x]
```

```
[Out] Sqrt[x]/(4*a*(a + c*x^4)) - (7*ArcTan[1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1
/8)])/(16*Sqrt[2]*(-a)^(15/8)*c^(1/8)) + (7*ArcTan[1 + (Sqrt[2]*c^(1/8)*Sqr
t[x])/(-a)^(1/8)])/(16*Sqrt[2]*(-a)^(15/8)*c^(1/8)) + (7*ArcTan[(c^(1/8)*Sq
rt[x])/(-a)^(1/8)])/(16*(-a)^(15/8)*c^(1/8)) + (7*ArcTanh[(c^(1/8)*Sqrt[x])
/(-a)^(1/8)])/(16*(-a)^(15/8)*c^(1/8)) - (7*Log[(-a)^(1/4) - Sqrt[2]*(-a)^(
1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x])/(32*Sqrt[2]*(-a)^(15/8)*c^(1/8)) + (7*Lo
g[(-a)^(1/4) + Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x])/(32*Sqrt[2]
*(-a)^(15/8)*c^(1/8))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(a+cx^4\right)^2}\,dx = \frac{\sqrt{x}}{4a\left(a+cx^4\right)} + \frac{7\int \frac{1}{\sqrt{x}\left(a+cx^4\right)}\,dx}{8a}$$

$$= \frac{\sqrt{x}}{4a\left(a+cx^4\right)} + \frac{7\,\text{Subst}\left(\int \frac{1}{a+cx^8}\,dx, x, \sqrt{x}\right)}{4a}$$

$$= \frac{\sqrt{x}}{4a\left(a+cx^4\right)} + \frac{7\,\text{Subst}\left(\int \frac{1}{\sqrt{-a}-\sqrt{c}\,x^4}\,dx, x, \sqrt{x}\right)}{8(-a)^{3/2}} + \frac{7\,\text{Subst}\left(\int \frac{1}{\sqrt{-a}+\sqrt{c}\,x^4}\,dx, x, \sqrt{x}\right)}{8(-a)^{3/2}}$$

$$= \frac{\sqrt{x}}{4a\left(a+cx^4\right)} + \frac{7\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}\,x^2}\,dx, x, \sqrt{x}\right)}{16(-a)^{7/4}} + \frac{7\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}+\sqrt[4]{c}\,x^2}\,dx, x, \sqrt{x}\right)}{16(-a)^{7/4}} +$$

$$= \frac{\sqrt{x}}{4a\left(a+cx^4\right)} + \frac{7\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{15/8}\sqrt[8]{c}} + \frac{7\tanh^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{15/8}\sqrt[8]{c}} + \frac{7\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[4]{-a}}{\sqrt[4]{c}} - \frac{\sqrt{2}\,\sqrt[8]{-a}\,x}{\sqrt[8]{c}} + x^2}\,dx, x\right.}{32(-a)^{7/4}\sqrt[4]{c}}$$

$$= \frac{\sqrt{x}}{4a\left(a+cx^4\right)} + \frac{7\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{15/8}\sqrt[8]{c}} + \frac{7\tanh^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{15/8}\sqrt[8]{c}} - \frac{7\log\left(\sqrt[4]{-a} - \sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x} +\right.}{32\sqrt{2}\,(-a)^{15/8}\sqrt[8]{c}}$$

$$= \frac{\sqrt{x}}{4a\left(a+cx^4\right)} - \frac{7\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}\,(-a)^{15/8}\sqrt[8]{c}} + \frac{7\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}\,(-a)^{15/8}\sqrt[8]{c}} + \frac{7\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{15/8}\sqrt[8]{c}} +$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.17

$$\frac{7\sqrt{x}\;{}_2F_1\left(\frac{1}{8},1;\frac{9}{8};-\frac{cx^4}{a}\right)}{4a^2} + \frac{\sqrt{x}}{4a\left(a+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + c*x^4)^2),x]
```

```
[Out] Sqrt[x]/(4*a*(a + c*x^4)) + (7*Sqrt[x]*Hypergeometric2F1[1/8, 1, 9/8, -((c*
x^4)/a)])/(4*a^2)
```

fricas [B] time = 0.71, size = 517, normalized size = 1.68

$$\frac{28\sqrt{2}\left(acx^4+a^2\right)\left(-\frac{1}{a^{15}c}\right)^{\frac{1}{8}}\arctan\left(\sqrt{2}\sqrt{a^4\left(-\frac{1}{a^{15}c}\right)^{\frac{1}{4}} + \sqrt{2}\,a^2\sqrt{x}\left(-\frac{1}{a^{15}c}\right)^{\frac{1}{8}} + x}\,a^{13}c\left(-\frac{1}{a^{15}c}\right)^{\frac{7}{8}} - \sqrt{2}\,a^{13}c\sqrt{x}\left(-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/64*(28*sqrt(2)*(a*c*x^4 + a^2)*(-1/(a^15*c))^(1/8)*arctan(sqrt(2)*sqrt(a^
4*(-1/(a^15*c))^(1/4) + sqrt(2)*a^2*sqrt(x)*(-1/(a^15*c))^(1/8) + x)*a^13*c
*(-1/(a^15*c))^(7/8) - sqrt(2)*a^13*c*sqrt(x)*(-1/(a^15*c))^(7/8) + 1) + 28
*sqrt(2)*(a*c*x^4 + a^2)*(-1/(a^15*c))^(1/8)*arctan(sqrt(2)*sqrt(a^4*(-1/(a
```

```
^15*c))^(1/4) - sqrt(2)*a^2*sqrt(x)*(-1/(a^15*c))^(1/8) + x)*a^13*c*(-1/(a^
15*c))^(7/8) - sqrt(2)*a^13*c*sqrt(x)*(-1/(a^15*c))^(7/8) - 1) + 7*sqrt(2)*
(a*c*x^4 + a^2)*(-1/(a^15*c))^(1/8)*log(a^4*(-1/(a^15*c))^(1/4) + sqrt(2)*a
^2*sqrt(x)*(-1/(a^15*c))^(1/8) + x) - 7*sqrt(2)*(a*c*x^4 + a^2)*(-1/(a^15*c
))^(1/8)*log(a^4*(-1/(a^15*c))^(1/4) - sqrt(2)*a^2*sqrt(x)*(-1/(a^15*c))^(1
/8) + x) + 56*(a*c*x^4 + a^2)*(-1/(a^15*c))^(1/8)*arctan(sqrt(a^4*(-1/(a^15
*c))^(1/4) + x)*a^13*c*(-1/(a^15*c))^(7/8) - a^13*c*sqrt(x)*(-1/(a^15*c))^(
7/8)) + 14*(a*c*x^4 + a^2)*(-1/(a^15*c))^(1/8)*log(a^2*(-1/(a^15*c))^(1/8)
+ sqrt(x)) - 14*(a*c*x^4 + a^2)*(-1/(a^15*c))^(1/8)*log(-a^2*(-1/(a^15*c))^
(1/8) + sqrt(x)) + 16*sqrt(x))/(a*c*x^4 + a^2)
```

giac [B] time = 0.57, size = 462, normalized size = 1.50

$$\frac{7\left(\frac{a}{c}\right)^{\frac{1}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16a^2\sqrt{-2\sqrt{2}+4}}+\frac{7\left(\frac{a}{c}\right)^{\frac{1}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16a^2\sqrt{-2\sqrt{2}+4}}+\frac{7\left(\frac{a}{c}\right)^{\frac{1}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16a^2\sqrt{2\sqrt{2}+4}}+\frac{7\left(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 7/16*(a/c)^(1/8)*arctan((sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(
sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*sqrt(-2*sqrt(2) + 4)) + 7/16*(a/c)^(1/8)*ar
ctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(sqrt(2) + 2)*(a/c)
^(1/8)))/(a^2*sqrt(-2*sqrt(2) + 4)) + 7/16*(a/c)^(1/8)*arctan((sqrt(sqrt(2)
 + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*sqrt(
2*sqrt(2) + 4)) + 7/16*(a/c)^(1/8)*arctan(-(sqrt(sqrt(2) + 2)*(a/c)^(1/8) -
 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*sqrt(2*sqrt(2) + 4)) + 7
/32*(a/c)^(1/8)*log(sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4)
)/(a^2*sqrt(-2*sqrt(2) + 4)) - 7/32*(a/c)^(1/8)*log(-sqrt(x)*sqrt(sqrt(2) +
 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*sqrt(-2*sqrt(2) + 4)) + 7/32*(a/c)^
(1/8)*log(sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*sq
rt(2*sqrt(2) + 4)) - 7/32*(a/c)^(1/8)*log(-sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)
^(1/8) + x + (a/c)^(1/4))/(a^2*sqrt(2*sqrt(2) + 4)) + 1/4*sqrt(x)/((c*x^4 +
 a)*a)
```

maple [C] time = 0.01, size = 50, normalized size = 0.16

$$\frac{\sqrt{x}}{4\left(c\,x^4+a\right)a}+\frac{7\ln\left(-\operatorname{RootOf}\left(c_Z^8+a\right)+\sqrt{x}\right)}{32ac\operatorname{RootOf}\left(c_Z^8+a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+a)^2/x^(1/2),x)
```

```
[Out] 1/4*x^(1/2)/a/(c*x^4+a)+7/32/a/c*sum(1/_R^7*ln(-_R+x^(1/2)),_R=RootOf(_Z^8*
c+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-7c\int\frac{x^{\frac{7}{2}}}{8\left(a^2cx^4+a^3\right)}\,dx+\frac{7cx^{\frac{9}{2}}+8a\sqrt{x}}{4\left(a^2cx^4+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] -7*c*integrate(1/8*x^(7/2)/(a^2*c*x^4 + a^3), x) + 1/4*(7*c*x^(9/2) + 8*a*s
qrt(x))/(a^2*c*x^4 + a^3)
```

mupad [B] time = 0.13, size = 135, normalized size = 0.44

$$\frac{\sqrt{x}}{4\,a\,\left(c\,x^4+a\right)}+\frac{7\,\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}}{(-a)^{1/8}}\right)}{16\,(-a)^{15/8}\,c^{1/8}}-\frac{\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}\,1\mathrm{i}}{(-a)^{1/8}}\right)\,7\mathrm{i}}{16\,(-a)^{15/8}\,c^{1/8}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\left(\frac{7}{32}+\frac{7}{32}\mathrm{i}\right)}{(-a)^{15/8}\,c^{1/8}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}}{(-a)^{1/8}}\right.}{(-a)^{15/8}\,c^{1/8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + c*x^4)^2),x)
```

```
[Out] x^(1/2)/(4*a*(a + c*x^4)) + (7*atan((c^(1/8)*x^(1/2))/(-a)^(1/8)))/(16*(-a)
^(15/8)*c^(1/8)) - (atan((c^(1/8)*x^(1/2)*1i)/(-a)^(1/8))*7i)/(16*(-a)^(15/
8)*c^(1/8)) + (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^(1/
8))*(7/32 + 7i/32))/((-a)^(15/8)*c^(1/8)) + (2^(1/2)*atan((2^(1/2)*c^(1/8)*
x^(1/2)*(1/2 + 1i/2))/(-a)^(1/8))*(7/32 - 7i/32))/((-a)^(15/8)*c^(1/8))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+a)**2/x**(1/2),x)
```

```
[Out] Timed out
```

3.754 $\int \frac{1}{x^{3/2}\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=320

$$-\frac{9}{4a^2\sqrt{x}}+\frac{1}{4a\sqrt{x}\left(a+cx^4\right)}-\frac{9\sqrt[8]{c}\log\left(-\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}x\right)}{32\sqrt{2}(-a)^{17/8}}+\frac{9\sqrt[8]{c}\log\left(\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\right.}{32\sqrt{2}(-a)^{17/8}}$$

```
[Out] -9/16*c^(1/8)*arctan(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(17/8)+9/16*c^(1/8)*a
rctanh(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(17/8)-9/32*c^(1/8)*arctan(-1+c^(1/
8)*2^(1/2)*x^(1/2)/(-a)^(1/8))/(-a)^(17/8)*2^(1/2)-9/32*c^(1/8)*arctan(1+c^
(1/8)*2^(1/2)*x^(1/2)/(-a)^(1/8))/(-a)^(17/8)*2^(1/2)-9/64*c^(1/8)*ln((-a)^
(1/4)+c^(1/4)*x-(-a)^(1/8)*c^(1/8)*2^(1/2)*x^(1/2))/(-a)^(17/8)*2^(1/2)+9/6
4*c^(1/8)*ln((-a)^(1/4)+c^(1/4)*x+(-a)^(1/8)*c^(1/8)*2^(1/2)*x^(1/2))/(-a)^
(17/8)*2^(1/2)-9/4/a^2/x^(1/2)+1/4/a/(c*x^4+a)/x^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.867$, Rules used = {290, 325, 329, 301, 297, 1162, 617, 204, 1165, 628, 298, 205, 208}

$$-\frac{9}{4a^2\sqrt{x}}+\frac{1}{4a\sqrt{x}\left(a+cx^4\right)}-\frac{9\sqrt[8]{c}\log\left(-\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}x\right)}{32\sqrt{2}(-a)^{17/8}}+\frac{9\sqrt[8]{c}\log\left(\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\right.}{32\sqrt{2}(-a)^{17/8}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + c*x^4)^2),x]
```

```
[Out] -9/(4*a^2*Sqrt[x]) + 1/(4*a*Sqrt[x]*(a + c*x^4)) + (9*c^(1/8)*ArcTan[1 - (S
qrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(16*Sqrt[2]*(-a)^(17/8)) - (9*c^(1/8)*
ArcTan[1 + (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(16*Sqrt[2]*(-a)^(17/8))
 - (9*c^(1/8)*ArcTan[(c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(16*(-a)^(17/8)) + (9*c^
(1/8)*ArcTanh[(c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(16*(-a)^(17/8)) - (9*c^(1/8)*
Log[(-a)^(1/4) - Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x])/(32*Sqrt[
2]*(-a)^(17/8)) + (9*c^(1/8)*Log[(-a)^(1/4) + Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sq
rt[x] + c^(1/4)*x])/(32*Sqrt[2]*(-a)^(17/8))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
```

```
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 301

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[x^(m - n/2
)/(r + s*x^(n/2)), x], x] - Dist[s/(2*b), Int[x^(m - n/2)/(r - s*x^(n/2)),
x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] &&
 LtQ[m, n] &&  !GtQ[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(a+cx^4\right)^2}\,dx = \frac{1}{4a\sqrt{x}\left(a+cx^4\right)} + \frac{9\int \frac{1}{x^{3/2}\left(a+cx^4\right)}\,dx}{8a}$$

$$= -\frac{9}{4a^2\sqrt{x}} + \frac{1}{4a\sqrt{x}\left(a+cx^4\right)} - \frac{(9c)\int \frac{x^{5/2}}{a+cx^4}\,dx}{8a^2}$$

$$= -\frac{9}{4a^2\sqrt{x}} + \frac{1}{4a\sqrt{x}\left(a+cx^4\right)} - \frac{(9c)\,\text{Subst}\left(\int \frac{x^6}{a+cx^8}\,dx, x, \sqrt{x}\right)}{4a^2}$$

$$= -\frac{9}{4a^2\sqrt{x}} + \frac{1}{4a\sqrt{x}\left(a+cx^4\right)} + \frac{\left(9\sqrt{c}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{-a}-\sqrt{c}\,x^4}\,dx, x, \sqrt{x}\right)}{8a^2} - \frac{\left(9\sqrt{c}\right)\text{Subst}\left(\int -\right.}{8}$$

$$= -\frac{9}{4a^2\sqrt{x}} + \frac{1}{4a\sqrt{x}\left(a+cx^4\right)} + \frac{\left(9\sqrt[4]{c}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}\,x^2}\,dx, x, \sqrt{x}\right)}{16a^2} - \frac{\left(9\sqrt[4]{c}\right)\text{Subst}\left(\int -\right.}{1}$$

$$= -\frac{9}{4a^2\sqrt{x}} + \frac{1}{4a\sqrt{x}\left(a+cx^4\right)} - \frac{9\sqrt[8]{c}\,\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{17/8}} + \frac{9\sqrt[8]{c}\,\tanh^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{17/8}} - \frac{9\,\text{Subst}\left(\int -\right.}{}$$

$$= -\frac{9}{4a^2\sqrt{x}} + \frac{1}{4a\sqrt{x}\left(a+cx^4\right)} - \frac{9\sqrt[8]{c}\,\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{17/8}} + \frac{9\sqrt[8]{c}\,\tanh^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{17/8}} - \frac{9\sqrt[8]{c}\,\log\left(\sqrt[4]{}\right.}{}$$

$$= -\frac{9}{4a^2\sqrt{x}} + \frac{1}{4a\sqrt{x}\left(a+cx^4\right)} + \frac{9\sqrt[8]{c}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}\,(-a)^{17/8}} - \frac{9\sqrt[8]{c}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}\,(-a)^{17/8}} - \frac{9}{}$$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.08

$$-\frac{2\,{}_2F_1\left(-\frac{1}{8}, 2; \frac{7}{8}; -\frac{cx^4}{a}\right)}{a^2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + c*x^4)^2),x]
[Out] (-2*Hypergeometric2F1[-1/8, 2, 7/8, -((c*x^4)/a)])/(a^2*Sqrt[x])
```

fricas [B] time = 0.68, size = 575, normalized size = 1.80

$$\frac{36\sqrt{2}\left(a^2cx^5+a^3x\right)\left(-\frac{c}{a^{17}}\right)^{\frac{1}{8}}\arctan\left(-\frac{4782969\sqrt{2}a^2c\sqrt{x}\left(-\frac{c}{a^{17}}\right)^{\frac{1}{8}}-\sqrt{2}\sqrt{-22876792454961\sqrt{2}a^{15}c\sqrt{x}\left(-\frac{c}{a^{17}}\right)^{\frac{7}{8}}-22876792454961}}{4782969\,c}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/64*(36*sqrt(2)*(a^2*c*x^5 + a^3*x)*(-c/a^17)^(1/8)*arctan(-1/4782969*(478
2969*sqrt(2)*a^2*c*sqrt(x)*(-c/a^17)^(1/8) - sqrt(2)*sqrt(-22876792454961*s
qrt(2)*a^15*c*sqrt(x)*(-c/a^17)^(7/8) - 22876792454961*a^13*c*(-c/a^17)^(3/
4) + 22876792454961*c^2*x)*a^2*(-c/a^17)^(1/8) + 4782969*c)/c) + 36*sqrt(2)
*(a^2*c*x^5 + a^3*x)*(-c/a^17)^(1/8)*arctan(-(sqrt(2)*a^2*c*sqrt(x)*(-c/a^1
7)^(1/8) - sqrt(2)*sqrt(sqrt(2)*a^15*c*sqrt(x)*(-c/a^17)^(7/8) - a^13*c*(-c
/a^17)^(3/4) + c^2*x)*a^2*(-c/a^17)^(1/8) - c)/c) - 9*sqrt(2)*(a^2*c*x^5 +
a^3*x)*(-c/a^17)^(1/8)*log(22876792454961*sqrt(2)*a^15*c*sqrt(x)*(-c/a^17)^
(7/8) - 22876792454961*a^13*c*(-c/a^17)^(3/4) + 22876792454961*c^2*x) + 9*s
qrt(2)*(a^2*c*x^5 + a^3*x)*(-c/a^17)^(1/8)*log(-22876792454961*sqrt(2)*a^15
*c*sqrt(x)*(-c/a^17)^(7/8) - 22876792454961*a^13*c*(-c/a^17)^(3/4) + 228767
92454961*c^2*x) + 72*(a^2*c*x^5 + a^3*x)*(-c/a^17)^(1/8)*arctan(-1/4782969*
(4782969*a^2*c*sqrt(x)*(-c/a^17)^(1/8) - sqrt(-22876792454961*a^13*c*(-c/a^
17)^(3/4) + 22876792454961*c^2*x)*a^2*(-c/a^17)^(1/8))/c) - 18*(a^2*c*x^5 +
 a^3*x)*(-c/a^17)^(1/8)*log(4782969*a^15*(-c/a^17)^(7/8) + 4782969*c*sqrt(x
)) + 18*(a^2*c*x^5 + a^3*x)*(-c/a^17)^(1/8)*log(-4782969*a^15*(-c/a^17)^(7/
8) + 4782969*c*sqrt(x)) - 16*(9*c*x^4 + 8*a)*sqrt(x))/(a^2*c*x^5 + a^3*x)
```

giac [B] time = 0.68, size = 481, normalized size = 1.50

$$-\frac{9c\left(\frac{a}{c}\right)^{\frac{7}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16a^3\sqrt{-2\sqrt{2}+4}}-\frac{9c\left(\frac{a}{c}\right)^{\frac{7}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16a^3\sqrt{-2\sqrt{2}+4}}-\frac{9c\left(\frac{a}{c}\right)^{\frac{7}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right.}{16a^3\sqrt{2\sqrt{2}+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -9/16*c*(a/c)^(7/8)*arctan((sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sq
rt(sqrt(2) + 2)*(a/c)^(1/8)))/(a^3*sqrt(-2*sqrt(2) + 4)) - 9/16*c*(a/c)^(7/
8)*arctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(sqrt(2) + 2)*
(a/c)^(1/8)))/(a^3*sqrt(-2*sqrt(2) + 4)) - 9/16*c*(a/c)^(7/8)*arctan((sqrt(
sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a^
3*sqrt(2*sqrt(2) + 4)) - 9/16*c*(a/c)^(7/8)*arctan(-(sqrt(sqrt(2) + 2)*(a/c
)^(1/8) - 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a^3*sqrt(2*sqrt(2)
+ 4)) + 9/32*c*(a/c)^(7/8)*log(sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x +
(a/c)^(1/4))/(a^3*sqrt(-2*sqrt(2) + 4)) - 9/32*c*(a/c)^(7/8)*log(-sqrt(x)*s
qrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^3*sqrt(-2*sqrt(2) + 4))
+ 9/32*c*(a/c)^(7/8)*log(sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)
^(1/4))/(a^3*sqrt(2*sqrt(2) + 4)) - 9/32*c*(a/c)^(7/8)*log(-sqrt(x)*sqrt(-s
qrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^3*sqrt(2*sqrt(2) + 4)) - 1/4*
(9*c*x^4 + 8*a)/((c*x^(9/2) + a*sqrt(x))*a^2)
```

maple [C] time = 0.02, size = 56, normalized size = 0.18

$$-\frac{c\,x^{\frac{7}{2}}}{4\left(c\,x^4+a\right)a^2}-\frac{9\ln\left(-\operatorname{RootOf}\left(c_Z^8+a\right)+\sqrt{x}\right)}{32a^2\operatorname{RootOf}\left(c_Z^8+a\right)}-\frac{2}{a^2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(c*x^4+a)^2,x)
```

```
[Out] -1/4/a^2*c*x^(7/2)/(c*x^4+a)-9/32/a^2*sum(1/_R*ln(-_R+x^(1/2)),_R=RootOf(_Z
^8*c+a))-2/a^2/x^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-9\,c\int \frac{x^{\frac{5}{2}}}{8\left(a^2cx^4+a^3\right)}\,dx-\frac{9\,cx^{\frac{7}{2}}+\frac{8\,a}{\sqrt{x}}}{4\left(a^2cx^4+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -9*c*integrate(1/8*x^(5/2)/(a^2*c*x^4 + a^3), x) - 1/4*(9*c*x^(7/2) + 8*a/s
qrt(x))/(a^2*c*x^4 + a^3)
```

mupad [B] time = 1.14, size = 148, normalized size = 0.46

$$-\frac{\frac{2}{a}+\frac{9\,c\,x^4}{4\,a^2}}{a\sqrt{x}+c\,x^{9/2}}-\frac{9\,(-c)^{1/8}\,\mathrm{atan}\left(\frac{(-c)^{1/8}\,\sqrt{x}}{a^{1/8}}\right)}{16\,a^{17/8}}-\frac{(-c)^{1/8}\,\mathrm{atan}\left(\frac{(-c)^{1/8}\,\sqrt{x}\,1\mathrm{i}}{a^{1/8}}\right)\,9\mathrm{i}}{16\,a^{17/8}}+\frac{\sqrt{2}\,(-c)^{1/8}\,\mathrm{atan}\left(\frac{\sqrt{2}\,(-c)^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{a^{1/8}}\right)\left(-\right.}{a^{17/8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + c*x^4)^2),x)
```

```
[Out] - (2/a + (9*c*x^4)/(4*a^2))/(a*x^(1/2) + c*x^(9/2)) - (9*(-c)^(1/8)*atan(((
-c)^(1/8)*x^(1/2))/a^(1/8)))/(16*a^(17/8)) - ((-c)^(1/8)*atan(((-c)^(1/8)*x
^(1/2)*1i)/a^(1/8))*9i)/(16*a^(17/8)) - (2^(1/2)*(-c)^(1/8)*atan((2^(1/2)*(
-c)^(1/8)*x^(1/2)*(1/2 - 1i/2))/a^(1/8))*(9/32 - 9i/32))/a^(17/8) - (2^(1/2
)*(-c)^(1/8)*atan((2^(1/2)*(-c)^(1/8)*x^(1/2)*(1/2 + 1i/2))/a^(1/8))*(9/32
+ 9i/32))/a^(17/8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.755 $\int \frac{x^{15/2}}{\left(a+cx^4\right)^3}\,dx$

Optimal. Leaf size=329

$$\frac{9\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{512\sqrt{2}\,(-a)^{7/8}c^{17/8}}-\frac{9\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{512\sqrt{2}\,(-a)^{7/8}c^{17/8}}+\frac{9\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}\,(-a)^{7/8}c^{17/8}}-\frac{9\,\text{ta}}{2}$$

```
[Out] -1/8*x^(9/2)/c/(c*x^4+a)^2-9/256*arctan(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(7
/8)/c^(17/8)-9/256*arctanh(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(7/8)/c^(17/8)-
9/512*arctan(-1+c^(1/8)*2^(1/2)*x^(1/2)/(-a)^(1/8))/(-a)^(7/8)/c^(17/8)*2^(
1/2)-9/512*arctan(1+c^(1/8)*2^(1/2)*x^(1/2)/(-a)^(1/8))/(-a)^(7/8)/c^(17/8)
*2^(1/2)+9/1024*ln((-a)^(1/4)+c^(1/4)*x-(-a)^(1/8)*c^(1/8)*2^(1/2)*x^(1/2))
/(-a)^(7/8)/c^(17/8)*2^(1/2)-9/1024*ln((-a)^(1/4)+c^(1/4)*x+(-a)^(1/8)*c^(1
/8)*2^(1/2)*x^(1/2))/(-a)^(7/8)/c^(17/8)*2^(1/2)-9/64*x^(1/2)/c^2/(c*x^4+a)
```

Rubi [A] time = 0.28, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {288, 329, 214, 212, 208, 205, 211, 1165, 628, 1162, 617, 204}

$$-\frac{9\sqrt{x}}{64c^2\left(a+cx^4\right)}+\frac{9\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{512\sqrt{2}\,(-a)^{7/8}c^{17/8}}-\frac{9\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{512\sqrt{2}\,(-a)^{7/8}c^{17/8}}+\frac{9\tan^{-1}\left(1\right.}{256\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(15/2)/(a + c*x^4)^3,x]
```

```
[Out] -x^(9/2)/(8*c*(a + c*x^4)^2) - (9*Sqrt[x])/(64*c^2*(a + c*x^4)) + (9*ArcTan
[1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(256*Sqrt[2]*(-a)^(7/8)*c^(17/8
)) - (9*ArcTan[1 + (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(256*Sqrt[2]*(-a)
^(7/8)*c^(17/8)) - (9*ArcTan[(c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(256*(-a)^(7/8)
*c^(17/8)) - (9*ArcTanh[(c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(256*(-a)^(7/8)*c^(1
7/8)) + (9*Log[(-a)^(1/4) - Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x]
)/(512*Sqrt[2]*(-a)^(7/8)*c^(17/8)) - (9*Log[(-a)^(1/4) + Sqrt[2]*(-a)^(1/8
)*c^(1/8)*Sqrt[x] + c^(1/4)*x])/(512*Sqrt[2]*(-a)^(7/8)*c^(17/8))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{15/2}}{\left(a+cx^4\right)^3}\,dx = -\frac{x^{9/2}}{8c\left(a+cx^4\right)^2} + \frac{9\int \frac{x^{7/2}}{\left(a+cx^4\right)^2}\,dx}{16c}$$

$$= -\frac{x^{9/2}}{8c\left(a+cx^4\right)^2} - \frac{9\sqrt{x}}{64c^2\left(a+cx^4\right)} + \frac{9\int \frac{1}{\sqrt{x}\left(a+cx^4\right)}\,dx}{128c^2}$$

$$= -\frac{x^{9/2}}{8c\left(a+cx^4\right)^2} - \frac{9\sqrt{x}}{64c^2\left(a+cx^4\right)} + \frac{9\,\text{Subst}\left(\int \frac{1}{a+cx^8}\,dx, x, \sqrt{x}\right)}{64c^2}$$

$$= -\frac{x^{9/2}}{8c\left(a+cx^4\right)^2} - \frac{9\sqrt{x}}{64c^2\left(a+cx^4\right)} - \frac{9\,\text{Subst}\left(\int \frac{1}{\sqrt{-a}-\sqrt{c}x^4}\,dx, x, \sqrt{x}\right)}{128\sqrt{-a}\,c^2} - \frac{9\,\text{Subst}\left(\int \frac{1}{\sqrt{-a}+\sqrt{c}x}\right.}{128\sqrt{-a}\,c}$$

$$= -\frac{x^{9/2}}{8c\left(a+cx^4\right)^2} - \frac{9\sqrt{x}}{64c^2\left(a+cx^4\right)} - \frac{9\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x^2}\,dx, x, \sqrt{x}\right)}{256(-a)^{3/4}c^2} - \frac{9\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}+\sqrt[4]{c}x}\right.}{256(-a)^{3/4}}$$

$$= -\frac{x^{9/2}}{8c\left(a+cx^4\right)^2} - \frac{9\sqrt{x}}{64c^2\left(a+cx^4\right)} - \frac{9\tan^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{7/8}c^{17/8}} - \frac{9\tanh^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{7/8}c^{17/8}} - \frac{9\,\text{Subst}\left(\int \frac{}{\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}\right.}{512($$

$$= -\frac{x^{9/2}}{8c\left(a+cx^4\right)^2} - \frac{9\sqrt{x}}{64c^2\left(a+cx^4\right)} - \frac{9\tan^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{7/8}c^{17/8}} - \frac{9\tanh^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{7/8}c^{17/8}} + \frac{9\log\left(\sqrt[4]{-a} - \sqrt{}\right.}{512\sqrt{2}}$$

$$= -\frac{x^{9/2}}{8c\left(a+cx^4\right)^2} - \frac{9\sqrt{x}}{64c^2\left(a+cx^4\right)} + \frac{9\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}\,(-a)^{7/8}c^{17/8}} - \frac{9\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}\,(-a)^{7/8}c^{17/8}} - \frac{9\tan}{256}$$

Mathematica [C] time = 0.02, size = 66, normalized size = 0.20

$$\frac{\sqrt{x}\left(9\left(a+cx^4\right)^2\,{}_2F_1\left(\frac{1}{8},1;\frac{9}{8};-\frac{cx^4}{a}\right) - a\left(9a+17cx^4\right)\right)}{64ac^2\left(a+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(15/2)/(a + c*x^4)^3,x]
```

```
[Out] (Sqrt[x]*(-(a*(9*a + 17*c*x^4)) + 9*(a + c*x^4)^2*Hypergeometric2F1[1/8, 1,
 9/8, -((c*x^4)/a)]))/(64*a*c^2*(a + c*x^4)^2)
```

fricas [B] time = 0.72, size = 672, normalized size = 2.04

$$\frac{36\sqrt{2}\left(c^4x^8+2ac^3x^4+a^2c^2\right)\left(-\frac{1}{a^7c^{17}}\right)^{\frac{1}{8}}\arctan\left(\sqrt{2}\sqrt{a^2c^4\left(-\frac{1}{a^7c^{17}}\right)^{\frac{1}{4}}+\sqrt{2}\,ac^2\sqrt{x}\left(-\frac{1}{a^7c^{17}}\right)^{\frac{1}{8}}+x}\,a^6c^{15}\left(-\frac{1}{a^7c^{17}}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 1/1024*(36*sqrt(2)*(c^4*x^8 + 2*a*c^3*x^4 + a^2*c^2)*(-1/(a^7*c^17))^(1/8)*
arctan(sqrt(2)*sqrt(a^2*c^4*(-1/(a^7*c^17))^(1/4) + sqrt(2)*a*c^2*sqrt(x)*(
-1/(a^7*c^17))^(1/8) + x)*a^6*c^15*(-1/(a^7*c^17))^(7/8) - sqrt(2)*a^6*c^15
*sqrt(x)*(-1/(a^7*c^17))^(7/8) + 1) + 36*sqrt(2)*(c^4*x^8 + 2*a*c^3*x^4 + a
^2*c^2)*(-1/(a^7*c^17))^(1/8)*arctan(sqrt(2)*sqrt(a^2*c^4*(-1/(a^7*c^17))^(
1/4) - sqrt(2)*a*c^2*sqrt(x)*(-1/(a^7*c^17))^(1/8) + x)*a^6*c^15*(-1/(a^7*c
^17))^(7/8) - sqrt(2)*a^6*c^15*sqrt(x)*(-1/(a^7*c^17))^(7/8) - 1) + 9*sqrt(
2)*(c^4*x^8 + 2*a*c^3*x^4 + a^2*c^2)*(-1/(a^7*c^17))^(1/8)*log(a^2*c^4*(-1/
(a^7*c^17))^(1/4) + sqrt(2)*a*c^2*sqrt(x)*(-1/(a^7*c^17))^(1/8) + x) - 9*sq
rt(2)*(c^4*x^8 + 2*a*c^3*x^4 + a^2*c^2)*(-1/(a^7*c^17))^(1/8)*log(a^2*c^4*(
-1/(a^7*c^17))^(1/4) - sqrt(2)*a*c^2*sqrt(x)*(-1/(a^7*c^17))^(1/8) + x) + 7
2*(c^4*x^8 + 2*a*c^3*x^4 + a^2*c^2)*(-1/(a^7*c^17))^(1/8)*arctan(sqrt(a^2*c
^4*(-1/(a^7*c^17))^(1/4) + x)*a^6*c^15*(-1/(a^7*c^17))^(7/8) - a^6*c^15*sqr
t(x)*(-1/(a^7*c^17))^(7/8)) + 18*(c^4*x^8 + 2*a*c^3*x^4 + a^2*c^2)*(-1/(a^7
*c^17))^(1/8)*log(a*c^2*(-1/(a^7*c^17))^(1/8) + sqrt(x)) - 18*(c^4*x^8 + 2*
a*c^3*x^4 + a^2*c^2)*(-1/(a^7*c^17))^(1/8)*log(-a*c^2*(-1/(a^7*c^17))^(1/8)
 + sqrt(x)) - 16*(17*c*x^4 + 9*a)*sqrt(x))/(c^4*x^8 + 2*a*c^3*x^4 + a^2*c^2
)
```

giac [B] time = 0.70, size = 496, normalized size = 1.51

$$\frac{9\left(\frac{a}{c}\right)^{\frac{1}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{256\,ac^2\sqrt{-2\sqrt{2}+4}}+\frac{9\left(\frac{a}{c}\right)^{\frac{1}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{256\,ac^2\sqrt{-2\sqrt{2}+4}}+\frac{9\left(\frac{a}{c}\right)^{\frac{1}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{256\,ac^2\sqrt{2\sqrt{2}+4}}+\frac{9\left(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 9/256*(a/c)^(1/8)*arctan((sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt
(sqrt(2) + 2)*(a/c)^(1/8)))/(a*c^2*sqrt(-2*sqrt(2) + 4)) + 9/256*(a/c)^(1/8
)*arctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(sqrt(2) + 2)*(
a/c)^(1/8)))/(a*c^2*sqrt(-2*sqrt(2) + 4)) + 9/256*(a/c)^(1/8)*arctan((sqrt(
sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a*
c^2*sqrt(2*sqrt(2) + 4)) + 9/256*(a/c)^(1/8)*arctan(-(sqrt(sqrt(2) + 2)*(a/
c)^(1/8) - 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a*c^2*sqrt(2*sqrt(
2) + 4)) + 9/512*(a/c)^(1/8)*log(sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x
 + (a/c)^(1/4))/(a*c^2*sqrt(-2*sqrt(2) + 4)) - 9/512*(a/c)^(1/8)*log(-sqrt(x
)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a*c^2*sqrt(-2*sqrt(2) +
 4)) + 9/512*(a/c)^(1/8)*log(sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (
a/c)^(1/4))/(a*c^2*sqrt(2*sqrt(2) + 4)) - 9/512*(a/c)^(1/8)*log(-sqrt(x)*sq
rt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a*c^2*sqrt(2*sqrt(2) + 4))
 - 1/64*(17*c*x^(9/2) + 9*a*sqrt(x))/((c*x^4 + a)^2*c^2)
```

maple [C] time = 0.02, size = 59, normalized size = 0.18

$$\frac{9\ln\left(-\operatorname{RootOf}\left(c_Z^8+a\right)+\sqrt{x}\right)}{512c^3\operatorname{RootOf}\left(c_Z^8+a\right)^7}+\frac{-\frac{17x^{\frac{9}{2}}}{64c}-\frac{9a\sqrt{x}}{64c^2}}{\left(cx^4+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)/(c*x^4+a)^3,x)
```

```
[Out] 2*(-9/128*a/c^2*x^(1/2)-17/128/c*x^(9/2))/(c*x^4+a)^2+9/512/c^3*sum(1/_R^7*
ln(-_R+x^(1/2)),_R=RootOf(_Z^8*c+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{9\,cx^{\frac{17}{2}} + ax^{\frac{9}{2}}}{64\left(ac^3x^8 + 2\,a^2c^2x^4 + a^3c\right)} - 9\int \frac{x^{\frac{7}{2}}}{128\left(ac^2x^4 + a^2c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/64*(9*c*x^(17/2) + a*x^(9/2))/(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c) - 9*int
egrate(1/128*x^(7/2)/(a*c^2*x^4 + a^2*c), x)
```

mupad [B] time = 1.14, size = 158, normalized size = 0.48

$$-\frac{\frac{17\,x^{9/2}}{64\,c} + \frac{9\,a\,\sqrt{x}}{64\,c^2}}{a^2 + 2\,a\,c\,x^4 + c^2\,x^8} - \frac{9\,\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}}{(-a)^{1/8}}\right)}{256\,(-a)^{7/8}\,c^{17/8}} + \frac{\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}\,1\mathrm{i}}{(-a)^{1/8}}\right)\,9\mathrm{i}}{256\,(-a)^{7/8}\,c^{17/8}} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\left(-\frac{9}{512}-\frac{9}{512}\mathrm{i}\right)}{(-a)^{7/8}\,c^{17/8}} + \frac{\sqrt{2}\,\mathrm{a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)/(a + c*x^4)^3,x)
```

```
[Out] (atan((c^(1/8)*x^(1/2)*1i)/(-a)^(1/8))*9i)/(256*(-a)^(7/8)*c^(17/8)) - (9*a
tan((c^(1/8)*x^(1/2))/(-a)^(1/8)))/(256*(-a)^(7/8)*c^(17/8)) - ((17*x^(9/2)
)/(64*c) + (9*a*x^(1/2))/(64*c^2))/(a^2 + c^2*x^8 + 2*a*c*x^4) - (2^(1/2)*a
tan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^(1/8))*(9/512 + 9i/512))/((
-a)^(7/8)*c^(17/8)) - (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 + 1i/2))/
(-a)^(1/8))*(9/512 - 9i/512))/((-a)^(7/8)*c^(17/8))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(15/2)/(c*x**4+a)**3,x)
```

```
[Out] Timed out
```

3.756 $\int \frac{x^{13/2}}{\left(a+cx^4\right)^3}\,dx$

Optimal. Leaf size=332

$$-\frac{7\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{512\sqrt{2}\,(-a)^{9/8}c^{15/8}}+\frac{7\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{512\sqrt{2}\,(-a)^{9/8}c^{15/8}}+\frac{7\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}\,(-a)^{9/8}c^{15/8}}-\frac{7\tan}{25}$$

```
[Out] -1/8*x^(7/2)/c/(c*x^4+a)^2+7/64*x^(7/2)/a/c/(c*x^4+a)-7/256*arctan(c^(1/8)*
x^(1/2)/(-a)^(1/8))/(-a)^(9/8)/c^(15/8)+7/256*arctanh(c^(1/8)*x^(1/2)/(-a)^
(1/8))/(-a)^(9/8)/c^(15/8)-7/512*arctan(-1+c^(1/8)*2^(1/2)*x^(1/2)/(-a)^(1/
8))/(-a)^(9/8)/c^(15/8)*2^(1/2)-7/512*arctan(1+c^(1/8)*2^(1/2)*x^(1/2)/(-a)
^(1/8))/(-a)^(9/8)/c^(15/8)*2^(1/2)-7/1024*ln((-a)^(1/4)+c^(1/4)*x-(-a)^(1/
8)*c^(1/8)*2^(1/2)*x^(1/2))/(-a)^(9/8)/c^(15/8)*2^(1/2)+7/1024*ln((-a)^(1/4
)+c^(1/4)*x+(-a)^(1/8)*c^(1/8)*2^(1/2)*x^(1/2))/(-a)^(9/8)/c^(15/8)*2^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.867$, Rules used = {288, 290, 329, 301, 297, 1162, 617, 204, 1165, 628, 298, 205, 208}

$$-\frac{7\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{512\sqrt{2}\,(-a)^{9/8}c^{15/8}}+\frac{7\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{512\sqrt{2}\,(-a)^{9/8}c^{15/8}}+\frac{7\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}\,(-a)^{9/8}c^{15/8}}-\frac{7\tan}{25}$$

Antiderivative was successfully verified.

```
[In] Int[x^(13/2)/(a + c*x^4)^3,x]
```

```
[Out] -x^(7/2)/(8*c*(a + c*x^4)^2) + (7*x^(7/2))/(64*a*c*(a + c*x^4)) + (7*ArcTan
[1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(256*Sqrt[2]*(-a)^(9/8)*c^(15/8
)) - (7*ArcTan[1 + (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(256*Sqrt[2]*(-a)
^(9/8)*c^(15/8)) - (7*ArcTan[(c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(256*(-a)^(9/8)
*c^(15/8)) + (7*ArcTanh[(c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(256*(-a)^(9/8)*c^(1
5/8)) - (7*Log[(-a)^(1/4) - Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x]
)/(512*Sqrt[2]*(-a)^(9/8)*c^(15/8)) + (7*Log[(-a)^(1/4) + Sqrt[2]*(-a)^(1/8
)*c^(1/8)*Sqrt[x] + c^(1/4)*x])/(512*Sqrt[2]*(-a)^(9/8)*c^(15/8))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
```

```
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 301

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[x^(m - n/2
)/(r + s*x^(n/2)), x], x] - Dist[s/(2*b), Int[x^(m - n/2)/(r - s*x^(n/2)),
x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] &&
 LtQ[m, n] &&  !GtQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{13/2}}{\left(a+cx^4\right)^3}\,dx = -\frac{x^{7/2}}{8c\left(a+cx^4\right)^2} + \frac{7\int \frac{x^{5/2}}{\left(a+cx^4\right)^2}\,dx}{16c}$$

$$= -\frac{x^{7/2}}{8c\left(a+cx^4\right)^2} + \frac{7x^{7/2}}{64ac\left(a+cx^4\right)} + \frac{7\int \frac{x^{5/2}}{a+cx^4}\,dx}{128ac}$$

$$= -\frac{x^{7/2}}{8c\left(a+cx^4\right)^2} + \frac{7x^{7/2}}{64ac\left(a+cx^4\right)} + \frac{7\,\text{Subst}\left(\int \frac{x^6}{a+cx^8}\,dx, x, \sqrt{x}\right)}{64ac}$$

$$= -\frac{x^{7/2}}{8c\left(a+cx^4\right)^2} + \frac{7x^{7/2}}{64ac\left(a+cx^4\right)} - \frac{7\,\text{Subst}\left(\int \frac{x^2}{\sqrt{-a}-\sqrt{c}x^4}\,dx, x, \sqrt{x}\right)}{128ac^{3/2}} + \frac{7\,\text{Subst}\left(\int \frac{x^2}{\sqrt{-a}+\sqrt{c}x^4}\,d\right.}{128ac^{3/2}}$$

$$= -\frac{x^{7/2}}{8c\left(a+cx^4\right)^2} + \frac{7x^{7/2}}{64ac\left(a+cx^4\right)} - \frac{7\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x^2}\,dx, x, \sqrt{x}\right)}{256ac^{7/4}} + \frac{7\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}+\sqrt[4]{c}x^2}\right.}{256ac^{7/4}}$$

$$= -\frac{x^{7/2}}{8c\left(a+cx^4\right)^2} + \frac{7x^{7/2}}{64ac\left(a+cx^4\right)} - \frac{7\tan^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{9/8}c^{15/8}} + \frac{7\tanh^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{9/8}c^{15/8}} + \frac{7\,\text{Subst}\left(\int \frac{}{\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}-\frac{\sqrt{}}{}}\right.}{512}$$

$$= -\frac{x^{7/2}}{8c\left(a+cx^4\right)^2} + \frac{7x^{7/2}}{64ac\left(a+cx^4\right)} - \frac{7\tan^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{9/8}c^{15/8}} + \frac{7\tanh^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{9/8}c^{15/8}} - \frac{7\log\left(\sqrt[4]{-a}-\sqrt{2}\right.}{512\sqrt{2}\,(}$$

$$= -\frac{x^{7/2}}{8c\left(a+cx^4\right)^2} + \frac{7x^{7/2}}{64ac\left(a+cx^4\right)} + \frac{7\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}\,(-a)^{9/8}c^{15/8}} - \frac{7\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}\,(-a)^{9/8}c^{15/8}} - \frac{7\tan^{-1}}{256(-}$$

Mathematica [C] time = 0.02, size = 45, normalized size = 0.14

$$\frac{2x^{7/2}\left(\frac{{}_2F_1\left(\frac{7}{8},3;\frac{15}{8};-\frac{cx^4}{a}\right)}{a^2} - \frac{1}{\left(a+cx^4\right)^2}\right)}{9c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(13/2)/(a + c*x^4)^3,x]
```

```
[Out] (2*x^(7/2)*(-(a + c*x^4)^(-2) + Hypergeometric2F1[7/8, 3, 15/8, -((c*x^4)/a
)]/a^2))/(9*c)
```

fricas [B] time = 0.84, size = 688, normalized size = 2.07

$$-\frac{28\sqrt{2}\left(ac^3x^8+2a^2c^2x^4+a^3c\right)\left(-\frac{1}{a^9c^{15}}\right)^{\frac{1}{8}}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}a^8c^{13}\sqrt{x}\left(-\frac{1}{a^9c^{15}}\right)^{\frac{7}{8}}-a^7c^{11}\left(-\frac{1}{a^9c^{15}}\right)^{\frac{3}{4}}+x}\,ac^2\left(-\frac{1}{a^9}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] -1/1024*(28*sqrt(2)*(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c)*(-1/(a^9*c^15))^(1/
8)*arctan(sqrt(2)*sqrt(sqrt(2)*a^8*c^13*sqrt(x)*(-1/(a^9*c^15))^(7/8) - a^7
*c^11*(-1/(a^9*c^15))^(3/4) + x)*a*c^2*(-1/(a^9*c^15))^(1/8) - sqrt(2)*a*c^
2*sqrt(x)*(-1/(a^9*c^15))^(1/8) + 1) + 28*sqrt(2)*(a*c^3*x^8 + 2*a^2*c^2*x^
4 + a^3*c)*(-1/(a^9*c^15))^(1/8)*arctan(sqrt(2)*sqrt(-sqrt(2)*a^8*c^13*sqrt
(x)*(-1/(a^9*c^15))^(7/8) - a^7*c^11*(-1/(a^9*c^15))^(3/4) + x)*a*c^2*(-1/(
a^9*c^15))^(1/8) - sqrt(2)*a*c^2*sqrt(x)*(-1/(a^9*c^15))^(1/8) - 1) - 7*sqr
t(2)*(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c)*(-1/(a^9*c^15))^(1/8)*log(sqrt(2)*
a^8*c^13*sqrt(x)*(-1/(a^9*c^15))^(7/8) - a^7*c^11*(-1/(a^9*c^15))^(3/4) + x
) + 7*sqrt(2)*(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c)*(-1/(a^9*c^15))^(1/8)*log
(-sqrt(2)*a^8*c^13*sqrt(x)*(-1/(a^9*c^15))^(7/8) - a^7*c^11*(-1/(a^9*c^15))
^(3/4) + x) + 56*(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c)*(-1/(a^9*c^15))^(1/8)*
arctan(sqrt(-a^7*c^11*(-1/(a^9*c^15))^(3/4) + x)*a*c^2*(-1/(a^9*c^15))^(1/8
) - a*c^2*sqrt(x)*(-1/(a^9*c^15))^(1/8)) - 14*(a*c^3*x^8 + 2*a^2*c^2*x^4 +
a^3*c)*(-1/(a^9*c^15))^(1/8)*log(a^8*c^13*(-1/(a^9*c^15))^(7/8) + sqrt(x))
+ 14*(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c)*(-1/(a^9*c^15))^(1/8)*log(-a^8*c^1
3*(-1/(a^9*c^15))^(7/8) + sqrt(x)) - 16*(7*c*x^7 - a*x^3)*sqrt(x))/(a*c^3*x
^8 + 2*a^2*c^2*x^4 + a^3*c)
```

giac [B] time = 0.88, size = 499, normalized size = 1.50

$$\frac{7\left(\frac{a}{c}\right)^{\frac{7}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{256a^2c\sqrt{-2\sqrt{2}+4}}+\frac{7\left(\frac{a}{c}\right)^{\frac{7}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{256a^2c\sqrt{-2\sqrt{2}+4}}+\frac{7\left(\frac{a}{c}\right)^{\frac{7}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{256a^2c\sqrt{2\sqrt{2}+4}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 7/256*(a/c)^(7/8)*arctan((sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt
(sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*c*sqrt(-2*sqrt(2) + 4)) + 7/256*(a/c)^(7/8
)*arctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(sqrt(2) + 2)*(
a/c)^(1/8)))/(a^2*c*sqrt(-2*sqrt(2) + 4)) + 7/256*(a/c)^(7/8)*arctan((sqrt(
sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a^
2*c*sqrt(2*sqrt(2) + 4)) + 7/256*(a/c)^(7/8)*arctan(-(sqrt(sqrt(2) + 2)*(a/
c)^(1/8) - 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*c*sqrt(2*sqrt(
2) + 4)) - 7/512*(a/c)^(7/8)*log(sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x
+ (a/c)^(1/4))/(a^2*c*sqrt(-2*sqrt(2) + 4)) + 7/512*(a/c)^(7/8)*log(-sqrt(x
)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*c*sqrt(-2*sqrt(2) +
 4)) - 7/512*(a/c)^(7/8)*log(sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (
a/c)^(1/4))/(a^2*c*sqrt(2*sqrt(2) + 4)) + 7/512*(a/c)^(7/8)*log(-sqrt(x)*sq
rt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*c*sqrt(2*sqrt(2) + 4))
 + 1/64*(7*c*x^(15/2) - a*x^(7/2))/((c*x^4 + a)^2*a*c)
```

maple [C] time = 0.02, size = 61, normalized size = 0.18

$$\frac{7\ln\left(-\operatorname{RootOf}\left(c_Z^8+a\right)+\sqrt{x}\right)}{512a\,c^2\operatorname{RootOf}\left(c_Z^8+a\right)}+\frac{\frac{7x^{\frac{15}{2}}}{64a}-\frac{x^{\frac{7}{2}}}{64c}}{\left(cx^4+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(c*x^4+a)^3,x)
```

```
[Out] 2*(-1/128/c*x^(7/2)+7/128/a*x^(15/2))/(c*x^4+a)^2+7/512/c^2/a*sum(1/_R*ln(-
_R+x^(1/2)),_R=RootOf(_Z^8*c+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{7\,c x^{\frac{15}{2}} - a x^{\frac{7}{2}}}{64\left(a c^3 x^8 + 2\,a^2 c^2 x^4 + a^3 c\right)} + 7\int \frac{x^{\frac{5}{2}}}{128\left(a c^2 x^4 + a^2 c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/64*(7*c*x^(15/2) - a*x^(7/2))/(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c) + 7*int
egrate(1/128*x^(5/2)/(a*c^2*x^4 + a^2*c), x)
```

mupad [B] time = 0.15, size = 156, normalized size = 0.47

$$\frac{\frac{7\,x^{15/2}}{64\,a} - \frac{x^{7/2}}{64\,c}}{a^2 + 2\,a\,c\,x^4 + c^2\,x^8} - \frac{7\,\mathrm{atan}\left(\frac{c^{1/8}\sqrt{x}}{(-a)^{1/8}}\right)}{256\,(-a)^{9/8}\,c^{15/8}} - \frac{\mathrm{atan}\left(\frac{c^{1/8}\sqrt{x}\,1\mathrm{i}}{(-a)^{1/8}}\right)7\mathrm{i}}{256\,(-a)^{9/8}\,c^{15/8}} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\sqrt{x}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\left(-\frac{7}{512}+\frac{7}{512}\mathrm{i}\right)}{(-a)^{9/8}\,c^{15/8}} + \frac{\sqrt{2}\,\mathrm{atan}\left($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(a + c*x^4)^3,x)
```

```
[Out] ((7*x^(15/2))/(64*a) - x^(7/2)/(64*c))/(a^2 + c^2*x^8 + 2*a*c*x^4) - (7*ata
n((c^(1/8)*x^(1/2))/(-a)^(1/8)))/(256*(-a)^(9/8)*c^(15/8)) - (atan((c^(1/8)
*x^(1/2)*1i)/(-a)^(1/8))*7i)/(256*(-a)^(9/8)*c^(15/8)) - (2^(1/2)*atan((2^(
1/2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^(1/8))*(7/512 - 7i/512))/((-a)^(9/8
)*c^(15/8)) - (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 + 1i/2))/(-a)^(1/
8))*(7/512 + 7i/512))/((-a)^(9/8)*c^(15/8))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)/(c*x**4+a)**3,x)
```

```
[Out] Timed out
```

3.757 $\int \frac{x^{11/2}}{\left(a+cx^4\right)^3}\,dx$

Optimal. Leaf size=332

$$\frac{15\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{512\sqrt{2}\,(-a)^{11/8}c^{13/8}}-\frac{15\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{512\sqrt{2}\,(-a)^{11/8}c^{13/8}}+\frac{15\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}\,(-a)^{11/8}c^{13/8}}-$$

```
[Out] -1/8*x^(5/2)/c/(c*x^4+a)^2+5/64*x^(5/2)/a/c/(c*x^4+a)+15/256*arctan(c^(1/8)
*x^(1/2)/(-a)^(1/8))/(-a)^(11/8)/c^(13/8)+15/256*arctanh(c^(1/8)*x^(1/2)/(-
a)^(1/8))/(-a)^(11/8)/c^(13/8)-15/512*arctan(-1+c^(1/8)*2^(1/2)*x^(1/2)/(-a
)^(1/8))/(-a)^(11/8)/c^(13/8)*2^(1/2)-15/512*arctan(1+c^(1/8)*2^(1/2)*x^(1/
2)/(-a)^(1/8))/(-a)^(11/8)/c^(13/8)*2^(1/2)+15/1024*ln((-a)^(1/4)+c^(1/4)*x
-(-a)^(1/8)*c^(1/8)*2^(1/2)*x^(1/2))/(-a)^(11/8)/c^(13/8)*2^(1/2)-15/1024*l
n((-a)^(1/4)+c^(1/4)*x+(-a)^(1/8)*c^(1/8)*2^(1/2)*x^(1/2))/(-a)^(11/8)/c^(1
3/8)*2^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.867$, Rules used = {288, 290, 329, 301, 211, 1165, 628, 1162, 617, 204, 212, 208, 205}

$$\frac{15\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{512\sqrt{2}\,(-a)^{11/8}c^{13/8}}-\frac{15\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{512\sqrt{2}\,(-a)^{11/8}c^{13/8}}+\frac{15\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}\,(-a)^{11/8}c^{13/8}}-$$

Antiderivative was successfully verified.

```
[In] Int[x^(11/2)/(a + c*x^4)^3,x]
```

```
[Out] -x^(5/2)/(8*c*(a + c*x^4)^2) + (5*x^(5/2))/(64*a*c*(a + c*x^4)) + (15*ArcTa
n[1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(256*Sqrt[2]*(-a)^(11/8)*c^(13
/8)) - (15*ArcTan[1 + (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(256*Sqrt[2]*(
-a)^(11/8)*c^(13/8)) + (15*ArcTan[(c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(256*(-a)^
(11/8)*c^(13/8)) + (15*ArcTanh[(c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(256*(-a)^(11
/8)*c^(13/8)) + (15*Log[(-a)^(1/4) - Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c
^(1/4)*x])/(512*Sqrt[2]*(-a)^(11/8)*c^(13/8)) - (15*Log[(-a)^(1/4) + Sqrt[2
]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x])/(512*Sqrt[2]*(-a)^(11/8)*c^(13/8
))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 301

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[x^(m - n/2
)/(r + s*x^(n/2)), x], x] - Dist[s/(2*b), Int[x^(m - n/2)/(r - s*x^(n/2)),
x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] &&
 LtQ[m, n] &&  !GtQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11/2}}{\left(a+cx^4\right)^3}\,dx &= -\frac{x^{5/2}}{8c\left(a+cx^4\right)^2} + \frac{5\int \frac{x^{3/2}}{\left(a+cx^4\right)^2}\,dx}{16c} \\
&= -\frac{x^{5/2}}{8c\left(a+cx^4\right)^2} + \frac{5x^{5/2}}{64ac\left(a+cx^4\right)} + \frac{15\int \frac{x^{3/2}}{a+cx^4}\,dx}{128ac} \\
&= -\frac{x^{5/2}}{8c\left(a+cx^4\right)^2} + \frac{5x^{5/2}}{64ac\left(a+cx^4\right)} + \frac{15\,\text{Subst}\left(\int \frac{x^4}{a+cx^8}\,dx, x, \sqrt{x}\right)}{64ac} \\
&= -\frac{x^{5/2}}{8c\left(a+cx^4\right)^2} + \frac{5x^{5/2}}{64ac\left(a+cx^4\right)} - \frac{15\,\text{Subst}\left(\int \frac{1}{\sqrt{-a}-\sqrt{c}x^4}\,dx, x, \sqrt{x}\right)}{128ac^{3/2}} + \frac{15\,\text{Subst}\left(\int \frac{1}{\sqrt{-a}+\sqrt{c}x^4}\,dx, x, \sqrt{x}\right)}{128ac^{3/2}} \\
&= -\frac{x^{5/2}}{8c\left(a+cx^4\right)^2} + \frac{5x^{5/2}}{64ac\left(a+cx^4\right)} + \frac{15\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x^2}\,dx, x, \sqrt{x}\right)}{256(-a)^{5/4}c^{3/2}} + \frac{15\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}+\sqrt[4]{c}x^2}\,dx, x, \sqrt{x}\right)}{256(-a)^{5/4}c^{3/2}} \\
&= -\frac{x^{5/2}}{8c\left(a+cx^4\right)^2} + \frac{5x^{5/2}}{64ac\left(a+cx^4\right)} + \frac{15\tan^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{11/8}c^{13/8}} + \frac{15\tanh^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{11/8}c^{13/8}} - \frac{15\,\text{Subst}\left(\int \frac{\sqrt[4]{-a}-\sqrt{c}x^2}{a+cx^8}\,dx, x, \sqrt{x}\right)}{256(-a)^{5/4}c^{3/2}} \\
&= -\frac{x^{5/2}}{8c\left(a+cx^4\right)^2} + \frac{5x^{5/2}}{64ac\left(a+cx^4\right)} + \frac{15\tan^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{11/8}c^{13/8}} + \frac{15\tanh^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{11/8}c^{13/8}} + \frac{15\log\left(\sqrt[4]{-a}-\sqrt{c}x^2\right)}{512(-a)^{11/8}c^{13/8}} \\
&= -\frac{x^{5/2}}{8c\left(a+cx^4\right)^2} + \frac{5x^{5/2}}{64ac\left(a+cx^4\right)} + \frac{15\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}(-a)^{11/8}c^{13/8}} - \frac{15\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}(-a)^{11/8}c^{13/8}} + \frac{15}{2}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 45, normalized size = 0.14

$$\frac{2x^{5/2}\left(\frac{{}_2F_1\left(\frac{5}{8},3;\frac{13}{8};-\frac{cx^4}{a}\right)}{a^2} - \frac{1}{\left(a+cx^4\right)^2}\right)}{11c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(11/2)/(a + c*x^4)^3,x]
```

```
[Out] (2*x^(5/2)*(-(a + c*x^4)^(-2) + Hypergeometric2F1[5/8, 3, 13/8, -((c*x^4)/a
)]/a^2))/(11*c)
```

fricas [B] time = 0.64, size = 700, normalized size = 2.11

$$\frac{60\sqrt{2}\left(ac^3x^8+2a^2c^2x^4+a^3c\right)\left(-\frac{1}{a^{11}c^{13}}\right)^{\frac{1}{8}}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}a^7c^8\sqrt{x}\left(-\frac{1}{a^{11}c^{13}}\right)^{\frac{5}{8}}-a^3c^3\left(-\frac{1}{a^{11}c^{13}}\right)^{\frac{1}{4}}+x}\,a^4c^5\left(-\frac{1}{a^{11}c^{13}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 1/1024*(60*sqrt(2)*(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c)*(-1/(a^11*c^13))^(1/
8)*arctan(sqrt(2)*sqrt(sqrt(2)*a^7*c^8*sqrt(x)*(-1/(a^11*c^13))^(5/8) - a^3
*c^3*(-1/(a^11*c^13))^(1/4) + x)*a^4*c^5*(-1/(a^11*c^13))^(3/8) - sqrt(2)*a
^4*c^5*sqrt(x)*(-1/(a^11*c^13))^(3/8) + 1) + 60*sqrt(2)*(a*c^3*x^8 + 2*a^2*
c^2*x^4 + a^3*c)*(-1/(a^11*c^13))^(1/8)*arctan(sqrt(2)*sqrt(-sqrt(2)*a^7*c^
8*sqrt(x)*(-1/(a^11*c^13))^(5/8) - a^3*c^3*(-1/(a^11*c^13))^(1/4) + x)*a^4*
c^5*(-1/(a^11*c^13))^(3/8) - sqrt(2)*a^4*c^5*sqrt(x)*(-1/(a^11*c^13))^(3/8)
 - 1) + 15*sqrt(2)*(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c)*(-1/(a^11*c^13))^(1/
8)*log(sqrt(2)*a^7*c^8*sqrt(x)*(-1/(a^11*c^13))^(5/8) - a^3*c^3*(-1/(a^11*c
^13))^(1/4) + x) - 15*sqrt(2)*(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c)*(-1/(a^11
*c^13))^(1/8)*log(-sqrt(2)*a^7*c^8*sqrt(x)*(-1/(a^11*c^13))^(5/8) - a^3*c^3
*(-1/(a^11*c^13))^(1/4) + x) - 120*(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c)*(-1/
(a^11*c^13))^(1/8)*arctan(sqrt(-a^3*c^3*(-1/(a^11*c^13))^(1/4) + x)*a^4*c^5
*(-1/(a^11*c^13))^(3/8) - a^4*c^5*sqrt(x)*(-1/(a^11*c^13))^(3/8)) - 30*(a*c
^3*x^8 + 2*a^2*c^2*x^4 + a^3*c)*(-1/(a^11*c^13))^(1/8)*log(a^7*c^8*(-1/(a^1
1*c^13))^(5/8) + sqrt(x)) + 30*(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c)*(-1/(a^1
1*c^13))^(1/8)*log(-a^7*c^8*(-1/(a^11*c^13))^(5/8) + sqrt(x)) + 16*(5*c*x^6
 - 3*a*x^2)*sqrt(x))/(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c)
```

giac [B] time = 0.87, size = 499, normalized size = 1.50

$$-\frac{15\left(\frac{a}{c}\right)^{\frac{5}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{256a^2c\sqrt{2\sqrt{2}+4}}-\frac{15\left(\frac{a}{c}\right)^{\frac{5}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{256a^2c\sqrt{2\sqrt{2}+4}}+\frac{15\left(\frac{a}{c}\right)^{\frac{5}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{256a^2c\sqrt{-2\sqrt{2}+4}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] -15/256*(a/c)^(5/8)*arctan((sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sq
rt(sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*c*sqrt(2*sqrt(2) + 4)) - 15/256*(a/c)^(5
/8)*arctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(sqrt(2) + 2)
*(a/c)^(1/8)))/(a^2*c*sqrt(2*sqrt(2) + 4)) + 15/256*(a/c)^(5/8)*arctan((sqr
t(sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(
a^2*c*sqrt(-2*sqrt(2) + 4)) + 15/256*(a/c)^(5/8)*arctan(-(sqrt(sqrt(2) + 2)
*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*c*sqrt(-2*
sqrt(2) + 4)) - 15/512*(a/c)^(5/8)*log(sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8
) + x + (a/c)^(1/4))/(a^2*c*sqrt(2*sqrt(2) + 4)) + 15/512*(a/c)^(5/8)*log(-
sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*c*sqrt(2*sqrt
(2) + 4)) + 15/512*(a/c)^(5/8)*log(sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) +
 x + (a/c)^(1/4))/(a^2*c*sqrt(-2*sqrt(2) + 4)) - 15/512*(a/c)^(5/8)*log(-sq
rt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*c*sqrt(-2*sqrt
(2) + 4)) + 1/64*(5*c*x^(13/2) - 3*a*x^(5/2))/((c*x^4 + a)^2*a*c)
```

maple [C] time = 0.02, size = 61, normalized size = 0.18

$$\frac{15\ln\left(-\text{RootOf}\left(c_Z^8+a\right)+\sqrt{x}\right)}{512a\,c^2\,\text{RootOf}\left(c_Z^8+a\right)^3}+\frac{\frac{5x^{\frac{13}{2}}}{64a}-\frac{3x^{\frac{5}{2}}}{64c}}{\left(c\,x^4+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(c*x^4+a)^3,x)
```

```
[Out] 2*(-3/128/c*x^(5/2)+5/128/a*x^(13/2))/(c*x^4+a)^2+15/512/c^2/a*sum(1/_R^3*l
n(-_R+x^(1/2)),_R=RootOf(_Z^8*c+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{5\,cx^{\frac{13}{2}}-3\,ax^{\frac{5}{2}}}{64\left(ac^3x^8+2\,a^2c^2x^4+a^3c\right)}+15\int\frac{x^{\frac{3}{2}}}{128\left(ac^2x^4+a^2c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/64*(5*c*x^(13/2) - 3*a*x^(5/2))/(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c) + 15*
integrate(1/128*x^(3/2)/(a*c^2*x^4 + a^2*c), x)
```

mupad [B] time = 1.13, size = 156, normalized size = 0.47

$$\frac{\frac{5\,x^{13/2}}{64\,a}-\frac{3\,x^{5/2}}{64\,c}}{a^2+2\,a\,c\,x^4+c^2\,x^8}+\frac{15\,\text{atan}\left(\frac{c^{1/8}\,\sqrt{x}}{(-a)^{1/8}}\right)}{256\,(-a)^{11/8}\,c^{13/8}}-\frac{\text{atan}\left(\frac{c^{1/8}\,\sqrt{x}\,1\text{i}}{(-a)^{1/8}}\right)15\text{i}}{256\,(-a)^{11/8}\,c^{13/8}}+\frac{\sqrt{2}\,\text{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\left(\frac{1}{2}-\frac{1}{2}\text{i}\right)}{(-a)^{1/8}}\right)\left(-\frac{15}{512}-\frac{15}{512}\text{i}\right)}{(-a)^{11/8}\,c^{13/8}}+\frac{\sqrt{2}\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(a + c*x^4)^3,x)
```

```
[Out] ((5*x^(13/2))/(64*a) - (3*x^(5/2))/(64*c))/(a^2 + c^2*x^8 + 2*a*c*x^4) + (1
5*atan((c^(1/8)*x^(1/2))/(-a)^(1/8)))/(256*(-a)^(11/8)*c^(13/8)) - (atan((c
^(1/8)*x^(1/2)*1i)/(-a)^(1/8))*15i)/(256*(-a)^(11/8)*c^(13/8)) - (2^(1/2)*a
tan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^(1/8))*(15/512 + 15i/512))/
((-a)^(11/8)*c^(13/8)) - (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 + 1i/2
))/(-a)^(1/8))*(15/512 - 15i/512))/((-a)^(11/8)*c^(13/8))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)/(c*x**4+a)**3,x)
```

```
[Out] Timed out
```

3.758 $\int \frac{x^{9/2}}{\left(a+cx^4\right)^3}\,dx$

Optimal. Leaf size=332

$$\frac{15\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{512\sqrt{2}\,(-a)^{13/8}c^{11/8}}-\frac{15\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{512\sqrt{2}\,(-a)^{13/8}c^{11/8}}-\frac{15\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}\,(-a)^{13/8}c^{11/8}}+\frac{15}{2}$$

```
[Out] -1/8*x^(3/2)/c/(c*x^4+a)^2+3/64*x^(3/2)/a/c/(c*x^4+a)-15/256*arctan(c^(1/8)
*x^(1/2)/(-a)^(1/8))/(-a)^(13/8)/c^(11/8)+15/256*arctanh(c^(1/8)*x^(1/2)/(-
a)^(1/8))/(-a)^(13/8)/c^(11/8)+15/512*arctan(-1+c^(1/8)*2^(1/2)*x^(1/2)/(-a
)^(1/8))/(-a)^(13/8)/c^(11/8)*2^(1/2)+15/512*arctan(1+c^(1/8)*2^(1/2)*x^(1/
2)/(-a)^(1/8))/(-a)^(13/8)/c^(11/8)*2^(1/2)+15/1024*ln((-a)^(1/4)+c^(1/4)*x
-(-a)^(1/8)*c^(1/8)*2^(1/2)*x^(1/2))/(-a)^(13/8)/c^(11/8)*2^(1/2)-15/1024*l
n((-a)^(1/4)+c^(1/4)*x+(-a)^(1/8)*c^(1/8)*2^(1/2)*x^(1/2))/(-a)^(13/8)/c^(1
1/8)*2^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.867$, Rules used = {288, 290, 329, 300, 297, 1162, 617, 204, 1165, 628, 298, 205, 208}

$$\frac{15\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{512\sqrt{2}\,(-a)^{13/8}c^{11/8}}-\frac{15\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{512\sqrt{2}\,(-a)^{13/8}c^{11/8}}-\frac{15\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}\,(-a)^{13/8}c^{11/8}}+\frac{15}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/(a + c*x^4)^3,x]
```

```
[Out] -x^(3/2)/(8*c*(a + c*x^4)^2) + (3*x^(3/2))/(64*a*c*(a + c*x^4)) - (15*ArcTa
n[1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(256*Sqrt[2]*(-a)^(13/8)*c^(11
/8)) + (15*ArcTan[1 + (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(256*Sqrt[2]*(
-a)^(13/8)*c^(11/8)) - (15*ArcTan[(c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(256*(-a)^
(13/8)*c^(11/8)) + (15*ArcTanh[(c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(256*(-a)^(13
/8)*c^(11/8)) + (15*Log[(-a)^(1/4) - Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c
^(1/4)*x])/(512*Sqrt[2]*(-a)^(13/8)*c^(11/8)) - (15*Log[(-a)^(1/4) + Sqrt[2
]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x])/(512*Sqrt[2]*(-a)^(13/8)*c^(11/8
))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 300

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[x^m/(r + s
*x^(n/2)), x], x] + Dist[r/(2*a), Int[x^m/(r - s*x^(n/2)), x], x]] /; FreeQ
[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n/2] &&  !GtQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
```

```
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{9/2}}{\left(a+cx^4\right)^3}\,dx = -\frac{x^{3/2}}{8c\left(a+cx^4\right)^2} + \frac{3\int \frac{\sqrt{x}}{\left(a+cx^4\right)^2}\,dx}{16c}$$

$$= -\frac{x^{3/2}}{8c\left(a+cx^4\right)^2} + \frac{3x^{3/2}}{64ac\left(a+cx^4\right)} + \frac{15\int \frac{\sqrt{x}}{a+cx^4}\,dx}{128ac}$$

$$= -\frac{x^{3/2}}{8c\left(a+cx^4\right)^2} + \frac{3x^{3/2}}{64ac\left(a+cx^4\right)} + \frac{15\,\text{Subst}\left(\int \frac{x^2}{a+cx^8}\,dx, x, \sqrt{x}\right)}{64ac}$$

$$= -\frac{x^{3/2}}{8c\left(a+cx^4\right)^2} + \frac{3x^{3/2}}{64ac\left(a+cx^4\right)} + \frac{15\,\text{Subst}\left(\int \frac{x^2}{\sqrt{-a}-\sqrt{c}\,x^4}\,dx, x, \sqrt{x}\right)}{128(-a)^{3/2}c} + \frac{15\,\text{Subst}\left(\int \frac{x^2}{\sqrt{-a}+\sqrt{c}\,x^4}\right.}{128(-a)^{3/2}}$$

$$= -\frac{x^{3/2}}{8c\left(a+cx^4\right)^2} + \frac{3x^{3/2}}{64ac\left(a+cx^4\right)} + \frac{15\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}\,x^2}\,dx, x, \sqrt{x}\right)}{256(-a)^{3/2}c^{5/4}} - \frac{15\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}+\sqrt[4]{c}\,x}\right.}{256(-a)^{3/2}c}$$

$$= -\frac{x^{3/2}}{8c\left(a+cx^4\right)^2} + \frac{3x^{3/2}}{64ac\left(a+cx^4\right)} - \frac{15\tan^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{13/8}c^{11/8}} + \frac{15\tanh^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{13/8}c^{11/8}} + \frac{15\,\text{Subst}\left(\int \frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[4]{-a}}\right.}{512}$$

$$= -\frac{x^{3/2}}{8c\left(a+cx^4\right)^2} + \frac{3x^{3/2}}{64ac\left(a+cx^4\right)} - \frac{15\tan^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{13/8}c^{11/8}} + \frac{15\tanh^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{13/8}c^{11/8}} + \frac{15\log\left(\sqrt[4]{-a}-\right.}{512\sqrt{2}}$$

$$= -\frac{x^{3/2}}{8c\left(a+cx^4\right)^2} + \frac{3x^{3/2}}{64ac\left(a+cx^4\right)} - \frac{15\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}(-a)^{13/8}c^{11/8}} + \frac{15\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}(-a)^{13/8}c^{11/8}} - \frac{15\,\text{ta}}{256}$$

Mathematica [C] time = 0.02, size = 45, normalized size = 0.14

$$\frac{2x^{3/2}\left(\frac{{}_2F_1\left(\frac{3}{8},3;\frac{11}{8};-\frac{cx^4}{a}\right)}{a^2} - \frac{1}{\left(a+cx^4\right)^2}\right)}{13c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/(a + c*x^4)^3,x]
```

```
[Out] (2*x^(3/2)*(-(a + c*x^4)^(-2) + Hypergeometric2F1[3/8, 3, 11/8, -((c*x^4)/a
)]/a^2))/(13*c)
```

fricas [B] time = 0.92, size = 693, normalized size = 2.09

$$-\frac{60\sqrt{2}\left(ac^3x^8+2a^2c^2x^4+a^3c\right)\left(-\frac{1}{a^{13}c^{11}}\right)^{\frac{1}{8}}\arctan\left(\sqrt{2}\sqrt{a^{10}c^8\left(-\frac{1}{a^{13}c^{11}}\right)^{\frac{3}{4}}+\sqrt{2}\,a^5c^4\sqrt{x}\left(-\frac{1}{a^{13}c^{11}}\right)^{\frac{3}{8}}+x}\,a^8c^7\left(-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] -1/1024*(60*sqrt(2)*(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c)*(-1/(a^13*c^11))^(1
/8)*arctan(sqrt(2)*sqrt(a^10*c^8*(-1/(a^13*c^11))^(3/4) + sqrt(2)*a^5*c^4*s
qrt(x)*(-1/(a^13*c^11))^(3/8) + x)*a^8*c^7*(-1/(a^13*c^11))^(5/8) - sqrt(2)
*a^8*c^7*sqrt(x)*(-1/(a^13*c^11))^(5/8) + 1) + 60*sqrt(2)*(a*c^3*x^8 + 2*a^
2*c^2*x^4 + a^3*c)*(-1/(a^13*c^11))^(1/8)*arctan(sqrt(2)*sqrt(a^10*c^8*(-1/
(a^13*c^11))^(3/4) - sqrt(2)*a^5*c^4*sqrt(x)*(-1/(a^13*c^11))^(3/8) + x)*a^
8*c^7*(-1/(a^13*c^11))^(5/8) - sqrt(2)*a^8*c^7*sqrt(x)*(-1/(a^13*c^11))^(5/
8) - 1) - 15*sqrt(2)*(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c)*(-1/(a^13*c^11))^(
1/8)*log(a^10*c^8*(-1/(a^13*c^11))^(3/4) + sqrt(2)*a^5*c^4*sqrt(x)*(-1/(a^1
3*c^11))^(3/8) + x) + 15*sqrt(2)*(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c)*(-1/(a
^13*c^11))^(1/8)*log(a^10*c^8*(-1/(a^13*c^11))^(3/4) - sqrt(2)*a^5*c^4*sqrt
(x)*(-1/(a^13*c^11))^(3/8) + x) - 120*(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c)*(
-1/(a^13*c^11))^(1/8)*arctan(sqrt(a^10*c^8*(-1/(a^13*c^11))^(3/4) + x)*a^8*
c^7*(-1/(a^13*c^11))^(5/8) - a^8*c^7*sqrt(x)*(-1/(a^13*c^11))^(5/8)) + 30*(
a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c)*(-1/(a^13*c^11))^(1/8)*log(a^5*c^4*(-1/(
a^13*c^11))^(3/8) + sqrt(x)) - 30*(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c)*(-1/(
a^13*c^11))^(1/8)*log(-a^5*c^4*(-1/(a^13*c^11))^(3/8) + sqrt(x)) - 16*(3*c*
x^5 - 5*a*x)*sqrt(x))/(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c)
```

giac [B] time = 0.86, size = 499, normalized size = 1.50

$$-\frac{15\left(\frac{a}{c}\right)^{\frac{3}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{256\,a^2c\sqrt{2\sqrt{2}+4}}-\frac{15\left(\frac{a}{c}\right)^{\frac{3}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{256\,a^2c\sqrt{2\sqrt{2}+4}}+\frac{15\left(\frac{a}{c}\right)^{\frac{3}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] -15/256*(a/c)^(3/8)*arctan((sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sq
rt(sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*c*sqrt(2*sqrt(2) + 4)) - 15/256*(a/c)^(3
/8)*arctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(sqrt(2) + 2)
*(a/c)^(1/8)))/(a^2*c*sqrt(2*sqrt(2) + 4)) + 15/256*(a/c)^(3/8)*arctan((sqr
t(sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(
a^2*c*sqrt(-2*sqrt(2) + 4)) + 15/256*(a/c)^(3/8)*arctan(-(sqrt(sqrt(2) + 2)
*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*c*sqrt(-2*
sqrt(2) + 4)) + 15/512*(a/c)^(3/8)*log(sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8
) + x + (a/c)^(1/4))/(a^2*c*sqrt(2*sqrt(2) + 4)) - 15/512*(a/c)^(3/8)*log(-
sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*c*sqrt(2*sqrt
(2) + 4)) - 15/512*(a/c)^(3/8)*log(sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) +
 x + (a/c)^(1/4))/(a^2*c*sqrt(-2*sqrt(2) + 4)) + 15/512*(a/c)^(3/8)*log(-sq
rt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*c*sqrt(-2*sqrt
(2) + 4)) + 1/64*(3*c*x^(11/2) - 5*a*x^(3/2))/((c*x^4 + a)^2*a*c)
```

maple [C] time = 0.02, size = 61, normalized size = 0.18

$$\frac{15\ln\left(-\text{RootOf}\left(c_Z^8+a\right)+\sqrt{x}\right)}{512a\,c^2\,\text{RootOf}\left(c_Z^8+a\right)^5}+\frac{\frac{3x^{\frac{11}{2}}}{64a}-\frac{5x^{\frac{3}{2}}}{64c}}{\left(c\,x^4+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(c*x^4+a)^3,x)
```

```
[Out] 2*(-5/128/c*x^(3/2)+3/128/a*x^(11/2))/(c*x^4+a)^2+15/512/c^2/a*sum(1/_R^5*l
n(-_R+x^(1/2)),_R=RootOf(_Z^8*c+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3\,cx^{\frac{11}{2}}-5\,ax^{\frac{3}{2}}}{64\left(ac^3x^8+2\,a^2c^2x^4+a^3c\right)}+15\int\frac{\sqrt{x}}{128\left(ac^2x^4+a^2c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/64*(3*c*x^(11/2) - 5*a*x^(3/2))/(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c) + 15*
integrate(1/128*sqrt(x)/(a*c^2*x^4 + a^2*c), x)
```

mupad [B] time = 1.08, size = 156, normalized size = 0.47

$$\frac{\frac{3x^{11/2}}{64a}-\frac{5x^{3/2}}{64c}}{a^2+2\,a\,c\,x^4+c^2\,x^8}-\frac{15\,\text{atan}\left(\frac{c^{1/8}\sqrt{x}}{(-a)^{1/8}}\right)}{256\,(-a)^{13/8}\,c^{11/8}}-\frac{\text{atan}\left(\frac{c^{1/8}\sqrt{x}\,1\text{i}}{(-a)^{1/8}}\right)15\text{i}}{256\,(-a)^{13/8}\,c^{11/8}}+\frac{\sqrt{2}\,\text{atan}\left(\frac{\sqrt{2}\,c^{1/8}\sqrt{x}\left(\frac{1}{2}-\frac{1}{2}\text{i}\right)}{(-a)^{1/8}}\right)\left(\frac{15}{512}-\frac{15}{512}\text{i}\right)}{(-a)^{13/8}\,c^{11/8}}+\frac{\sqrt{2}\,\text{atan}\left($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(a + c*x^4)^3,x)
```

```
[Out] ((3*x^(11/2))/(64*a) - (5*x^(3/2))/(64*c))/(a^2 + c^2*x^8 + 2*a*c*x^4) - (1
5*atan((c^(1/8)*x^(1/2))/(-a)^(1/8)))/(256*(-a)^(13/8)*c^(11/8)) - (atan((c
^(1/8)*x^(1/2)*1i)/(-a)^(1/8))*15i)/(256*(-a)^(13/8)*c^(11/8)) + (2^(1/2)*a
tan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^(1/8))*(15/512 - 15i/512))/
((-a)^(13/8)*c^(11/8)) + (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 + 1i/2
))/(-a)^(1/8))*(15/512 + 15i/512))/((-a)^(13/8)*c^(11/8))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(c*x**4+a)**3,x)
```

```
[Out] Timed out
```

3.759 $\int \frac{x^{7/2}}{\left(a+cx^4\right)^3}\,dx$

Optimal. Leaf size=332

$$-\frac{7\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{512\sqrt{2}\,(-a)^{15/8}c^{9/8}}+\frac{7\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{512\sqrt{2}\,(-a)^{15/8}c^{9/8}}-\frac{7\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}\,(-a)^{15/8}c^{9/8}}+\frac{7}{}$$

```
[Out] 7/256*arctan(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(15/8)/c^(9/8)+7/256*arctanh(
c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(15/8)/c^(9/8)+7/512*arctan(-1+c^(1/8)*2^(
1/2)*x^(1/2)/(-a)^(1/8))/(-a)^(15/8)/c^(9/8)*2^(1/2)+7/512*arctan(1+c^(1/8)
*2^(1/2)*x^(1/2)/(-a)^(1/8))/(-a)^(15/8)/c^(9/8)*2^(1/2)-7/1024*ln((-a)^(1/
4)+c^(1/4)*x-(-a)^(1/8)*c^(1/8)*2^(1/2)*x^(1/2))/(-a)^(15/8)/c^(9/8)*2^(1/2
)+7/1024*ln((-a)^(1/4)+c^(1/4)*x+(-a)^(1/8)*c^(1/8)*2^(1/2)*x^(1/2))/(-a)^(
15/8)/c^(9/8)*2^(1/2)-1/8*x^(1/2)/c/(c*x^4+a)^2+1/64*x^(1/2)/a/c/(c*x^4+a)
```

Rubi [A] time = 0.29, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.867$, Rules used = {288, 290, 329, 214, 212, 208, 205, 211, 1165, 628, 1162, 617, 204}

$$-\frac{7\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{512\sqrt{2}\,(-a)^{15/8}c^{9/8}}+\frac{7\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{512\sqrt{2}\,(-a)^{15/8}c^{9/8}}-\frac{7\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}\,(-a)^{15/8}c^{9/8}}+\frac{7}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(a + c*x^4)^3,x]
```

```
[Out] -Sqrt[x]/(8*c*(a + c*x^4)^2) + Sqrt[x]/(64*a*c*(a + c*x^4)) - (7*ArcTan[1 -
 (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(256*Sqrt[2]*(-a)^(15/8)*c^(9/8)) +
 (7*ArcTan[1 + (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(256*Sqrt[2]*(-a)^(15
/8)*c^(9/8)) + (7*ArcTan[(c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(256*(-a)^(15/8)*c^
(9/8)) + (7*ArcTanh[(c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(256*(-a)^(15/8)*c^(9/8)
) - (7*Log[(-a)^(1/4) - Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x])/(5
12*Sqrt[2]*(-a)^(15/8)*c^(9/8)) + (7*Log[(-a)^(1/4) + Sqrt[2]*(-a)^(1/8)*c^
(1/8)*Sqrt[x] + c^(1/4)*x])/(512*Sqrt[2]*(-a)^(15/8)*c^(9/8))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{7/2}}{\left(a+cx^4\right)^3}\,dx = -\frac{\sqrt{x}}{8c\left(a+cx^4\right)^2} + \frac{\int \frac{1}{\sqrt{x}\left(a+cx^4\right)^2}\,dx}{16c}$$

$$= -\frac{\sqrt{x}}{8c\left(a+cx^4\right)^2} + \frac{\sqrt{x}}{64ac\left(a+cx^4\right)} + \frac{7\int \frac{1}{\sqrt{x}\left(a+cx^4\right)}\,dx}{128ac}$$

$$= -\frac{\sqrt{x}}{8c\left(a+cx^4\right)^2} + \frac{\sqrt{x}}{64ac\left(a+cx^4\right)} + \frac{7\,\text{Subst}\left(\int \frac{1}{a+cx^8}\,dx, x, \sqrt{x}\right)}{64ac}$$

$$= -\frac{\sqrt{x}}{8c\left(a+cx^4\right)^2} + \frac{\sqrt{x}}{64ac\left(a+cx^4\right)} + \frac{7\,\text{Subst}\left(\int \frac{1}{\sqrt{-a}-\sqrt{c}\,x^4}\,dx, x, \sqrt{x}\right)}{128(-a)^{3/2}c} + \frac{7\,\text{Subst}\left(\int \frac{1}{\sqrt{-a}+\sqrt{c}\,x}\right.}{128(-a)^{3/}}$$

$$= -\frac{\sqrt{x}}{8c\left(a+cx^4\right)^2} + \frac{\sqrt{x}}{64ac\left(a+cx^4\right)} + \frac{7\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}\,x^2}\,dx, x, \sqrt{x}\right)}{256(-a)^{7/4}c} + \frac{7\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}+\sqrt[4]{c}}\right.}{256(-a)^{7}}$$

$$= -\frac{\sqrt{x}}{8c\left(a+cx^4\right)^2} + \frac{\sqrt{x}}{64ac\left(a+cx^4\right)} + \frac{7\tan^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{15/8}c^{9/8}} + \frac{7\tanh^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{15/8}c^{9/8}} + \frac{7\,\text{Subst}\left(\int \frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right.}{512}$$

$$= -\frac{\sqrt{x}}{8c\left(a+cx^4\right)^2} + \frac{\sqrt{x}}{64ac\left(a+cx^4\right)} + \frac{7\tan^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{15/8}c^{9/8}} + \frac{7\tanh^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{15/8}c^{9/8}} - \frac{7\log\left(\sqrt[4]{-a}-\sqrt{}\right.}{512\sqrt{}}$$

$$= -\frac{\sqrt{x}}{8c\left(a+cx^4\right)^2} + \frac{\sqrt{x}}{64ac\left(a+cx^4\right)} - \frac{7\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}\,(-a)^{15/8}c^{9/8}} + \frac{7\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}\,(-a)^{15/8}c^{9/8}} + \frac{7\,\text{ta}}{256}$$

Mathematica [C] time = 0.02, size = 64, normalized size = 0.19

$$\frac{\sqrt{x}\left(7\left(a+cx^4\right)^2\,{}_2F_1\left(\frac{1}{8},1;\frac{9}{8};-\frac{cx^4}{a}\right)+a\left(cx^4-7a\right)\right)}{64a^2c\left(a+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(a + c*x^4)^3,x]
```

```
[Out] (Sqrt[x]*(a*(-7*a + c*x^4) + 7*(a + c*x^4)^2*Hypergeometric2F1[1/8, 1, 9/8,
 -((c*x^4)/a)]))/(64*a^2*c*(a + c*x^4)^2)
```

fricas [B] time = 0.76, size = 679, normalized size = 2.05

$$\frac{28\sqrt{2}\left(ac^3x^8+2a^2c^2x^4+a^3c\right)\left(-\frac{1}{a^{15}c^9}\right)^{\frac{1}{8}}\arctan\left(\sqrt{2}\sqrt{a^4c^2\left(-\frac{1}{a^{15}c^9}\right)^{\frac{1}{4}}+\sqrt{2}\,a^2c\sqrt{x}\left(-\frac{1}{a^{15}c^9}\right)^{\frac{1}{8}}+x}\,a^{13}c^8\left(-\frac{1}{a^{15}c^9}\right)^{\frac{7}{8}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 1/1024*(28*sqrt(2)*(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c)*(-1/(a^15*c^9))^(1/8
)*arctan(sqrt(2)*sqrt(a^4*c^2*(-1/(a^15*c^9))^(1/4) + sqrt(2)*a^2*c*sqrt(x)
*(-1/(a^15*c^9))^(1/8) + x)*a^13*c^8*(-1/(a^15*c^9))^(7/8) - sqrt(2)*a^13*c
^8*sqrt(x)*(-1/(a^15*c^9))^(7/8) + 1) + 28*sqrt(2)*(a*c^3*x^8 + 2*a^2*c^2*x
^4 + a^3*c)*(-1/(a^15*c^9))^(1/8)*arctan(sqrt(2)*sqrt(a^4*c^2*(-1/(a^15*c^9
))^(1/4) - sqrt(2)*a^2*c*sqrt(x)*(-1/(a^15*c^9))^(1/8) + x)*a^13*c^8*(-1/(a
^15*c^9))^(7/8) - sqrt(2)*a^13*c^8*sqrt(x)*(-1/(a^15*c^9))^(7/8) - 1) + 7*s
qrt(2)*(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c)*(-1/(a^15*c^9))^(1/8)*log(a^4*c^
2*(-1/(a^15*c^9))^(1/4) + sqrt(2)*a^2*c*sqrt(x)*(-1/(a^15*c^9))^(1/8) + x)
- 7*sqrt(2)*(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c)*(-1/(a^15*c^9))^(1/8)*log(a
^4*c^2*(-1/(a^15*c^9))^(1/4) - sqrt(2)*a^2*c*sqrt(x)*(-1/(a^15*c^9))^(1/8)
+ x) + 56*(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c)*(-1/(a^15*c^9))^(1/8)*arctan(
sqrt(a^4*c^2*(-1/(a^15*c^9))^(1/4) + x)*a^13*c^8*(-1/(a^15*c^9))^(7/8) - a^
13*c^8*sqrt(x)*(-1/(a^15*c^9))^(7/8)) + 14*(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3
*c)*(-1/(a^15*c^9))^(1/8)*log(a^2*c*(-1/(a^15*c^9))^(1/8) + sqrt(x)) - 14*(
a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c)*(-1/(a^15*c^9))^(1/8)*log(-a^2*c*(-1/(a^
15*c^9))^(1/8) + sqrt(x)) + 16*(c*x^4 - 7*a)*sqrt(x))/(a*c^3*x^8 + 2*a^2*c^
2*x^4 + a^3*c)
```

giac [B] time = 0.73, size = 498, normalized size = 1.50

$$\frac{7\left(\frac{a}{c}\right)^{\frac{1}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{256\,a^2c\sqrt{-2\sqrt{2}+4}}+\frac{7\left(\frac{a}{c}\right)^{\frac{1}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{256\,a^2c\sqrt{-2\sqrt{2}+4}}+\frac{7\left(\frac{a}{c}\right)^{\frac{1}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{256\,a^2c\sqrt{2\sqrt{2}+4}}+\frac{7\left(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 7/256*(a/c)^(1/8)*arctan((sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt
(sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*c*sqrt(-2*sqrt(2) + 4)) + 7/256*(a/c)^(1/8
)*arctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(sqrt(2) + 2)*(
a/c)^(1/8)))/(a^2*c*sqrt(-2*sqrt(2) + 4)) + 7/256*(a/c)^(1/8)*arctan((sqrt(
sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a^
2*c*sqrt(2*sqrt(2) + 4)) + 7/256*(a/c)^(1/8)*arctan(-(sqrt(sqrt(2) + 2)*(a/
c)^(1/8) - 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*c*sqrt(2*sqrt(
2) + 4)) + 7/512*(a/c)^(1/8)*log(sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x
+ (a/c)^(1/4))/(a^2*c*sqrt(-2*sqrt(2) + 4)) - 7/512*(a/c)^(1/8)*log(-sqrt(x
)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*c*sqrt(-2*sqrt(2) +
 4)) + 7/512*(a/c)^(1/8)*log(sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (
a/c)^(1/4))/(a^2*c*sqrt(2*sqrt(2) + 4)) - 7/512*(a/c)^(1/8)*log(-sqrt(x)*sq
rt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*c*sqrt(2*sqrt(2) + 4))
 + 1/64*(c*x^(9/2) - 7*a*sqrt(x))/((c*x^4 + a)^2*a*c)
```

maple [C] time = 0.02, size = 61, normalized size = 0.18

$$\frac{7\ln\left(-\operatorname{RootOf}\left(c_Z^8+a\right)+\sqrt{x}\right)}{512a\,c^2\operatorname{RootOf}\left(c_Z^8+a\right)^7}+\frac{\frac{x^{\frac{9}{2}}}{64a}-\frac{7\sqrt{x}}{64c}}{\left(c\,x^4+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(c*x^4+a)^3,x)
```

```
[Out] 2*(-7/128/c*x^(1/2)+1/128/a*x^(9/2))/(c*x^4+a)^2+7/512/c^2/a*sum(1/_R^7*ln(
-_R+x^(1/2)),_R=RootOf(_Z^8*c+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{7\,c x^{\frac{17}{2}} + 15\,a x^{\frac{9}{2}}}{64\left(a^2c^2x^8 + 2\,a^3cx^4 + a^4\right)} - 7\int \frac{x^{\frac{7}{2}}}{128\left(a^2cx^4 + a^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/64*(7*c*x^(17/2) + 15*a*x^(9/2))/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4) - 7*in
tegrate(1/128*x^(7/2)/(a^2*c*x^4 + a^3), x)
```

mupad [B] time = 1.10, size = 156, normalized size = 0.47

$$\frac{\frac{x^{9/2}}{64\,a} - \frac{7\sqrt{x}}{64\,c}}{a^2 + 2\,a\,c\,x^4 + c^2\,x^8} + \frac{7\,\mathrm{atan}\left(\frac{c^{1/8}\sqrt{x}}{(-a)^{1/8}}\right)}{256\,(-a)^{15/8}\,c^{9/8}} - \frac{\mathrm{atan}\left(\frac{c^{1/8}\sqrt{x}\,1\mathrm{i}}{(-a)^{1/8}}\right)7\mathrm{i}}{256\,(-a)^{15/8}\,c^{9/8}} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\sqrt{x}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\left(\frac{7}{512}+\frac{7}{512}\mathrm{i}\right)}{(-a)^{15/8}\,c^{9/8}} + \frac{\sqrt{2}\,\mathrm{atan}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(a + c*x^4)^3,x)
```

```
[Out] (x^(9/2)/(64*a) - (7*x^(1/2))/(64*c))/(a^2 + c^2*x^8 + 2*a*c*x^4) + (7*atan
((c^(1/8)*x^(1/2))/(-a)^(1/8)))/(256*(-a)^(15/8)*c^(9/8)) - (atan((c^(1/8)*
x^(1/2)*1i)/(-a)^(1/8))*7i)/(256*(-a)^(15/8)*c^(9/8)) + (2^(1/2)*atan((2^(1
/2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^(1/8))*(7/512 + 7i/512))/((-a)^(15/8
)*c^(9/8)) + (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 + 1i/2))/(-a)^(1/8
))*(7/512 - 7i/512))/((-a)^(15/8)*c^(9/8))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(c*x**4+a)**3,x)
```

```
[Out] Timed out
```

3.760 $\int \frac{x^{5/2}}{\left(a+cx^4\right)^3}\,dx$

Optimal. Leaf size=329

$$\frac{9x^{7/2}}{64a^2\left(a+cx^4\right)}+\frac{9\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{512\sqrt{2}\,(-a)^{17/8}c^{7/8}}-\frac{9\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{512\sqrt{2}\,(-a)^{17/8}c^{7/8}}-\frac{9\tan^{-1}\left(1-\sqrt{2}\right.}{256\sqrt{2}\,(-a)}$$

```
[Out] 1/8*x^(7/2)/a/(c*x^4+a)^2+9/64*x^(7/2)/a^2/(c*x^4+a)+9/256*arctan(c^(1/8)*x
^(1/2)/(-a)^(1/8))/(-a)^(17/8)/c^(7/8)-9/256*arctanh(c^(1/8)*x^(1/2)/(-a)^(
1/8))/(-a)^(17/8)/c^(7/8)+9/512*arctan(-1+c^(1/8)*2^(1/2)*x^(1/2)/(-a)^(1/8
))/(-a)^(17/8)/c^(7/8)*2^(1/2)+9/512*arctan(1+c^(1/8)*2^(1/2)*x^(1/2)/(-a)^
(1/8))/(-a)^(17/8)/c^(7/8)*2^(1/2)+9/1024*ln((-a)^(1/4)+c^(1/4)*x-(-a)^(1/8
)*c^(1/8)*2^(1/2)*x^(1/2))/(-a)^(17/8)/c^(7/8)*2^(1/2)-9/1024*ln((-a)^(1/4)
+c^(1/4)*x+(-a)^(1/8)*c^(1/8)*2^(1/2)*x^(1/2))/(-a)^(17/8)/c^(7/8)*2^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {290, 329, 301, 297, 1162, 617, 204, 1165, 628, 298, 205, 208}

$$\frac{9x^{7/2}}{64a^2\left(a+cx^4\right)}+\frac{9\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{512\sqrt{2}\,(-a)^{17/8}c^{7/8}}-\frac{9\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{512\sqrt{2}\,(-a)^{17/8}c^{7/8}}-\frac{9\tan^{-1}\left(1-\sqrt{2}\right.}{256\sqrt{2}\,(-a)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + c*x^4)^3,x]
```

```
[Out] x^(7/2)/(8*a*(a + c*x^4)^2) + (9*x^(7/2))/(64*a^2*(a + c*x^4)) - (9*ArcTan[
1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(256*Sqrt[2]*(-a)^(17/8)*c^(7/8)
) + (9*ArcTan[1 + (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(256*Sqrt[2]*(-a)^
(17/8)*c^(7/8)) + (9*ArcTan[(c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(256*(-a)^(17/8)
*c^(7/8)) - (9*ArcTanh[(c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(256*(-a)^(17/8)*c^(7
/8)) + (9*Log[(-a)^(1/4) - Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x])
/(512*Sqrt[2]*(-a)^(17/8)*c^(7/8)) - (9*Log[(-a)^(1/4) + Sqrt[2]*(-a)^(1/8)
*c^(1/8)*Sqrt[x] + c^(1/4)*x])/(512*Sqrt[2]*(-a)^(17/8)*c^(7/8))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
```

```
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 301

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[x^(m - n/2
)/(r + s*x^(n/2)), x], x] - Dist[s/(2*b), Int[x^(m - n/2)/(r - s*x^(n/2)),
x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] &&
 LtQ[m, n] &&  !GtQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{5/2}}{\left(a+cx^4\right)^3}\,dx = \frac{x^{7/2}}{8a\left(a+cx^4\right)^2} + \frac{9\int \frac{x^{5/2}}{\left(a+cx^4\right)^2}\,dx}{16a}$$

$$= \frac{x^{7/2}}{8a\left(a+cx^4\right)^2} + \frac{9x^{7/2}}{64a^2\left(a+cx^4\right)} + \frac{9\int \frac{x^{5/2}}{a+cx^4}\,dx}{128a^2}$$

$$= \frac{x^{7/2}}{8a\left(a+cx^4\right)^2} + \frac{9x^{7/2}}{64a^2\left(a+cx^4\right)} + \frac{9\,\text{Subst}\left(\int \frac{x^6}{a+cx^8}\,dx, x, \sqrt{x}\right)}{64a^2}$$

$$= \frac{x^{7/2}}{8a\left(a+cx^4\right)^2} + \frac{9x^{7/2}}{64a^2\left(a+cx^4\right)} - \frac{9\,\text{Subst}\left(\int \frac{x^2}{\sqrt{-a}-\sqrt{c}\,x^4}\,dx, x, \sqrt{x}\right)}{128a^2\sqrt{c}} + \frac{9\,\text{Subst}\left(\int \frac{x^2}{\sqrt{-a}+\sqrt{c}\,x^4}\,dx\right.}{128a^2\sqrt{c}}$$

$$= \frac{x^{7/2}}{8a\left(a+cx^4\right)^2} + \frac{9x^{7/2}}{64a^2\left(a+cx^4\right)} - \frac{9\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}\,x^2}\,dx, x, \sqrt{x}\right)}{256a^2c^{3/4}} + \frac{9\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}+\sqrt[4]{c}\,x^2}\,dx\right.}{256a^2c^{3/4}}$$

$$= \frac{x^{7/2}}{8a\left(a+cx^4\right)^2} + \frac{9x^{7/2}}{64a^2\left(a+cx^4\right)} + \frac{9\tan^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{17/8}c^{7/8}} - \frac{9\tanh^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{17/8}c^{7/8}} + \frac{9\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}-\frac{\sqrt{2}}{}}\right.}{512a}$$

$$= \frac{x^{7/2}}{8a\left(a+cx^4\right)^2} + \frac{9x^{7/2}}{64a^2\left(a+cx^4\right)} + \frac{9\tan^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{17/8}c^{7/8}} - \frac{9\tanh^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{17/8}c^{7/8}} + \frac{9\log\left(\sqrt[4]{-a}-\sqrt{2}\sqrt[8]{}\right.}{512\sqrt{2}\,(-}$$

$$= \frac{x^{7/2}}{8a\left(a+cx^4\right)^2} + \frac{9x^{7/2}}{64a^2\left(a+cx^4\right)} - \frac{9\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}\,(-a)^{17/8}c^{7/8}} + \frac{9\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}\,(-a)^{17/8}c^{7/8}} + \frac{9\tan^{-1}}{256(-a)}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.09

$$\frac{2x^{7/2}\,{}_2F_1\left(\frac{7}{8}, 3; \frac{15}{8}; -\frac{cx^4}{a}\right)}{7a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + c*x^4)^3,x]
[Out] (2*x^(7/2)*Hypergeometric2F1[7/8, 3, 15/8, -((c*x^4)/a)])/(7*a^3)
```

fricas [B] time = 0.57, size = 672, normalized size = 2.04

$$-\frac{36\sqrt{2}\left(a^2c^2x^8+2a^3cx^4+a^4\right)\left(-\frac{1}{a^{17}c^7}\right)^{\frac{1}{8}}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\,a^{15}c^6\sqrt{x}\left(-\frac{1}{a^{17}c^7}\right)^{\frac{7}{8}}-a^{13}c^5\left(-\frac{1}{a^{17}c^7}\right)^{\frac{3}{4}}+x\,a^2c\left(-\frac{1}{a^{17}c^7}\right)^{\frac{1}{8}}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] -1/1024*(36*sqrt(2)*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^17*c^7))^(1/8)
*arctan(sqrt(2)*sqrt(sqrt(2)*a^15*c^6*sqrt(x)*(-1/(a^17*c^7))^(7/8) - a^13*
c^5*(-1/(a^17*c^7))^(3/4) + x)*a^2*c*(-1/(a^17*c^7))^(1/8) - sqrt(2)*a^2*c*
sqrt(x)*(-1/(a^17*c^7))^(1/8) + 1) + 36*sqrt(2)*(a^2*c^2*x^8 + 2*a^3*c*x^4
 + a^4)*(-1/(a^17*c^7))^(1/8)*arctan(sqrt(2)*sqrt(-sqrt(2)*a^15*c^6*sqrt(x)*
(-1/(a^17*c^7))^(7/8) - a^13*c^5*(-1/(a^17*c^7))^(3/4) + x)*a^2*c*(-1/(a^17
*c^7))^(1/8) - sqrt(2)*a^2*c*sqrt(x)*(-1/(a^17*c^7))^(1/8) - 1) - 9*sqrt(2)
*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^17*c^7))^(1/8)*log(sqrt(2)*a^15*c
^6*sqrt(x)*(-1/(a^17*c^7))^(7/8) - a^13*c^5*(-1/(a^17*c^7))^(3/4) + x) + 9*
sqrt(2)*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^17*c^7))^(1/8)*log(-sqrt(2
)*a^15*c^6*sqrt(x)*(-1/(a^17*c^7))^(7/8) - a^13*c^5*(-1/(a^17*c^7))^(3/4) +
 x) + 72*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^17*c^7))^(1/8)*arctan(sqr
t(-a^13*c^5*(-1/(a^17*c^7))^(3/4) + x)*a^2*c*(-1/(a^17*c^7))^(1/8) - a^2*c*
sqrt(x)*(-1/(a^17*c^7))^(1/8)) - 18*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(
a^17*c^7))^(1/8)*log(a^15*c^6*(-1/(a^17*c^7))^(7/8) + sqrt(x)) + 18*(a^2*c^
2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^17*c^7))^(1/8)*log(-a^15*c^6*(-1/(a^17*c^
7))^(7/8) + sqrt(x)) - 16*(9*c*x^7 + 17*a*x^3)*sqrt(x))/(a^2*c^2*x^8 + 2*a^
3*c*x^4 + a^4)
```

giac **[B]** time = 0.86, size = 472, normalized size = 1.43

$$\frac{9\left(\frac{a}{c}\right)^{\frac{7}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{256\,a^3\sqrt{-2\sqrt{2}+4}}+\frac{9\left(\frac{a}{c}\right)^{\frac{7}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{256\,a^3\sqrt{-2\sqrt{2}+4}}+\frac{9\left(\frac{a}{c}\right)^{\frac{7}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{256\,a^3\sqrt{2\sqrt{2}+4}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 9/256*(a/c)^(7/8)*arctan((sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt
(sqrt(2) + 2)*(a/c)^(1/8)))/(a^3*sqrt(-2*sqrt(2) + 4)) + 9/256*(a/c)^(7/8)*
arctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(sqrt(2) + 2)*(a/
c)^(1/8)))/(a^3*sqrt(-2*sqrt(2) + 4)) + 9/256*(a/c)^(7/8)*arctan((sqrt(sqrt
(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a^3*sq
rt(2*sqrt(2) + 4)) + 9/256*(a/c)^(7/8)*arctan(-(sqrt(sqrt(2) + 2)*(a/c)^(1/
8) - 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a^3*sqrt(2*sqrt(2) + 4))
 - 9/512*(a/c)^(7/8)*log(sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^
(1/4))/(a^3*sqrt(-2*sqrt(2) + 4)) + 9/512*(a/c)^(7/8)*log(-sqrt(x)*sqrt(sqr
t(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^3*sqrt(-2*sqrt(2) + 4)) - 9/512
*(a/c)^(7/8)*log(sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/
(a^3*sqrt(2*sqrt(2) + 4)) + 9/512*(a/c)^(7/8)*log(-sqrt(x)*sqrt(-sqrt(2) +
2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^3*sqrt(2*sqrt(2) + 4)) + 1/64*(9*c*x^(
15/2) + 17*a*x^(7/2))/((c*x^4 + a)^2*a^2)
```

maple **[C]** time = 0.02, size = 62, normalized size = 0.19

$$\frac{9\ln\left(-\operatorname{RootOf}\left(c_Z^8+a\right)+\sqrt{x}\right)}{512a^2c\operatorname{RootOf}\left(c_Z^8+a\right)}+\frac{\frac{9cx^{\frac{15}{2}}}{64a^2}+\frac{17x^{\frac{7}{2}}}{64a}}{\left(cx^4+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(c*x^4+a)^3,x)
```

```
[Out] 2*(17/128*x^(7/2)/a+9/128/a^2*c*x^(15/2))/(c*x^4+a)^2+9/512/a^2/c*sum(1/_R*
ln(-_R+x^(1/2)),_R=RootOf(_Z^8*c+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{9\,cx^{\frac{15}{2}}+17\,ax^{\frac{7}{2}}}{64\left(a^2c^2x^8+2\,a^3cx^4+a^4\right)}+9\int\frac{x^{\frac{5}{2}}}{128\left(a^2cx^4+a^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/64*(9*c*x^(15/2) + 17*a*x^(7/2))/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4) + 9*in
tegrate(1/128*x^(5/2)/(a^2*c*x^4 + a^3), x)
```

mupad [B] time = 1.10, size = 157, normalized size = 0.48

$$\frac{\frac{17\,x^{7/2}}{64\,a}+\frac{9\,c\,x^{15/2}}{64\,a^2}}{a^2+2\,a\,c\,x^4+c^2\,x^8}+\frac{9\,\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}}{(-a)^{1/8}}\right)}{256\,(-a)^{17/8}\,c^{7/8}}+\frac{\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}\,1\mathrm{i}}{(-a)^{1/8}}\right)\,9\mathrm{i}}{256\,(-a)^{17/8}\,c^{7/8}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\left(\frac{9}{512}-\frac{9}{512}\mathrm{i}\right)}{(-a)^{17/8}\,c^{7/8}}+\frac{\sqrt{2}\,\mathrm{atan}\left(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + c*x^4)^3,x)
```

```
[Out] ((17*x^(7/2))/(64*a) + (9*c*x^(15/2))/(64*a^2))/(a^2 + c^2*x^8 + 2*a*c*x^4)
 + (9*atan((c^(1/8)*x^(1/2))/(-a)^(1/8)))/(256*(-a)^(17/8)*c^(7/8)) + (atan
((c^(1/8)*x^(1/2)*1i)/(-a)^(1/8))*9i)/(256*(-a)^(17/8)*c^(7/8)) + (2^(1/2)*
atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^(1/8))*(9/512 - 9i/512))/(
(-a)^(17/8)*c^(7/8)) + (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 + 1i/2))
/(-a)^(1/8))*(9/512 + 9i/512))/((-a)^(17/8)*c^(7/8))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(c*x**4+a)**3,x)
```

```
[Out] Timed out
```

3.761 $\int \frac{x^{3/2}}{\left(a+cx^4\right)^3}\,dx$

Optimal. Leaf size=329

$$\frac{11x^{5/2}}{64a^2\left(a+cx^4\right)}-\frac{33\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{512\sqrt{2}\,(-a)^{19/8}c^{5/8}}+\frac{33\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{512\sqrt{2}\,(-a)^{19/8}c^{5/8}}-\frac{33\tan^{-1}}{256\sqrt{}}$$

```
[Out] 1/8*x^(5/2)/a/(c*x^4+a)^2+11/64*x^(5/2)/a^2/(c*x^4+a)-33/256*arctan(c^(1/8)
*x^(1/2)/(-a)^(1/8))/(-a)^(19/8)/c^(5/8)-33/256*arctanh(c^(1/8)*x^(1/2)/(-a
)^(1/8))/(-a)^(19/8)/c^(5/8)+33/512*arctan(-1+c^(1/8)*2^(1/2)*x^(1/2)/(-a)^
(1/8))/(-a)^(19/8)/c^(5/8)*2^(1/2)+33/512*arctan(1+c^(1/8)*2^(1/2)*x^(1/2)/
(-a)^(1/8))/(-a)^(19/8)/c^(5/8)*2^(1/2)-33/1024*ln((-a)^(1/4)+c^(1/4)*x-(-a
)^(1/8)*c^(1/8)*2^(1/2)*x^(1/2))/(-a)^(19/8)/c^(5/8)*2^(1/2)+33/1024*ln((-a
)^(1/4)+c^(1/4)*x+(-a)^(1/8)*c^(1/8)*2^(1/2)*x^(1/2))/(-a)^(19/8)/c^(5/8)*2
^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {290, 329, 301, 211, 1165, 628, 1162, 617, 204, 212, 208, 205}

$$\frac{11x^{5/2}}{64a^2\left(a+cx^4\right)}-\frac{33\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{512\sqrt{2}\,(-a)^{19/8}c^{5/8}}+\frac{33\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{512\sqrt{2}\,(-a)^{19/8}c^{5/8}}-\frac{33\tan^{-1}}{256\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + c*x^4)^3,x]
```

```
[Out] x^(5/2)/(8*a*(a + c*x^4)^2) + (11*x^(5/2))/(64*a^2*(a + c*x^4)) - (33*ArcTa
n[1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(256*Sqrt[2]*(-a)^(19/8)*c^(5/
8)) + (33*ArcTan[1 + (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(256*Sqrt[2]*(-
a)^(19/8)*c^(5/8)) - (33*ArcTan[(c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(256*(-a)^(1
9/8)*c^(5/8)) - (33*ArcTanh[(c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(256*(-a)^(19/8)
*c^(5/8)) - (33*Log[(-a)^(1/4) - Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/
4)*x])/(512*Sqrt[2]*(-a)^(19/8)*c^(5/8)) + (33*Log[(-a)^(1/4) + Sqrt[2]*(-a
)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x])/(512*Sqrt[2]*(-a)^(19/8)*c^(5/8))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 301

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[x^(m - n/2
)/(r + s*x^(n/2)), x], x] - Dist[s/(2*b), Int[x^(m - n/2)/(r - s*x^(n/2)),
x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] &&
 LtQ[m, n] &&  !GtQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(a+cx^4\right)^3}\,dx = \frac{x^{5/2}}{8a\left(a+cx^4\right)^2} + \frac{11\int \frac{x^{3/2}}{\left(a+cx^4\right)^2}\,dx}{16a}$$

$$= \frac{x^{5/2}}{8a\left(a+cx^4\right)^2} + \frac{11x^{5/2}}{64a^2\left(a+cx^4\right)} + \frac{33\int \frac{x^{3/2}}{a+cx^4}\,dx}{128a^2}$$

$$= \frac{x^{5/2}}{8a\left(a+cx^4\right)^2} + \frac{11x^{5/2}}{64a^2\left(a+cx^4\right)} + \frac{33\,\text{Subst}\left(\int \frac{x^4}{a+cx^8}\,dx, x, \sqrt{x}\right)}{64a^2}$$

$$= \frac{x^{5/2}}{8a\left(a+cx^4\right)^2} + \frac{11x^{5/2}}{64a^2\left(a+cx^4\right)} - \frac{33\,\text{Subst}\left(\int \frac{1}{\sqrt{-a}-\sqrt{c}\,x^4}\,dx, x, \sqrt{x}\right)}{128a^2\sqrt{c}} + \frac{33\,\text{Subst}\left(\int \frac{1}{\sqrt{-a}+\sqrt{c}}\right.}{128a^2\sqrt{}}$$

$$= \frac{x^{5/2}}{8a\left(a+cx^4\right)^2} + \frac{11x^{5/2}}{64a^2\left(a+cx^4\right)} - \frac{33\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}\,x^2}\,dx, x, \sqrt{x}\right)}{256(-a)^{9/4}\sqrt{c}} - \frac{33\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}+\sqrt[4]{}}\right.}{256(-a)^{9/}}$$

$$= \frac{x^{5/2}}{8a\left(a+cx^4\right)^2} + \frac{11x^{5/2}}{64a^2\left(a+cx^4\right)} - \frac{33\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{19/8}c^{5/8}} - \frac{33\tanh^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{19/8}c^{5/8}} + \frac{33\,\text{Subst}\left(\int \frac{}{\sqrt[4]{}}\right.}{51}$$

$$= \frac{x^{5/2}}{8a\left(a+cx^4\right)^2} + \frac{11x^{5/2}}{64a^2\left(a+cx^4\right)} - \frac{33\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{19/8}c^{5/8}} - \frac{33\tanh^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{19/8}c^{5/8}} - \frac{33\log\left(\sqrt[4]{-a}\right.}{512}$$

$$= \frac{x^{5/2}}{8a\left(a+cx^4\right)^2} + \frac{11x^{5/2}}{64a^2\left(a+cx^4\right)} - \frac{33\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}\,(-a)^{19/8}c^{5/8}} + \frac{33\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}\,(-a)^{19/8}c^{5/8}} - \frac{33}{25}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.09

$$\frac{2x^{5/2}\,{}_2F_1\left(\frac{5}{8}, 3; \frac{13}{8}; -\frac{cx^4}{a}\right)}{5a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + c*x^4)^3,x]
[Out] (2*x^(5/2)*Hypergeometric2F1[5/8, 3, 13/8, -((c*x^4)/a)])/(5*a^3)
```

fricas [B] time = 0.72, size = 674, normalized size = 2.05

$$\frac{132\sqrt{2}\left(a^2c^2x^8 + 2a^3cx^4 + a^4\right)\left(-\frac{1}{a^{19}c^5}\right)^{\frac{1}{8}}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\,a^{12}c^3\sqrt{x}\left(-\frac{1}{a^{19}c^5}\right)^{\frac{5}{8}} - a^5c\left(-\frac{1}{a^{19}c^5}\right)^{\frac{1}{4}} + x}\,a^7c^2\left(-\frac{1}{a^{19}c^5}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 1/1024*(132*sqrt(2)*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^19*c^5))^(1/8)
*arctan(sqrt(2)*sqrt(sqrt(2)*a^12*c^3*sqrt(x)*(-1/(a^19*c^5))^(5/8) - a^5*c
*(-1/(a^19*c^5))^(1/4) + x)*a^7*c^2*(-1/(a^19*c^5))^(3/8) - sqrt(2)*a^7*c^2
*sqrt(x)*(-1/(a^19*c^5))^(3/8) + 1) + 132*sqrt(2)*(a^2*c^2*x^8 + 2*a^3*c*x^
4 + a^4)*(-1/(a^19*c^5))^(1/8)*arctan(sqrt(2)*sqrt(-sqrt(2)*a^12*c^3*sqrt(x
)*(-1/(a^19*c^5))^(5/8) - a^5*c*(-1/(a^19*c^5))^(1/4) + x)*a^7*c^2*(-1/(a^1
9*c^5))^(3/8) - sqrt(2)*a^7*c^2*sqrt(x)*(-1/(a^19*c^5))^(3/8) - 1) + 33*sqr
t(2)*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^19*c^5))^(1/8)*log(sqrt(2)*a^
12*c^3*sqrt(x)*(-1/(a^19*c^5))^(5/8) - a^5*c*(-1/(a^19*c^5))^(1/4) + x) - 3
3*sqrt(2)*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^19*c^5))^(1/8)*log(-sqrt
(2)*a^12*c^3*sqrt(x)*(-1/(a^19*c^5))^(5/8) - a^5*c*(-1/(a^19*c^5))^(1/4) +
x) - 264*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^19*c^5))^(1/8)*arctan(sqr
t(-a^5*c*(-1/(a^19*c^5))^(1/4) + x)*a^7*c^2*(-1/(a^19*c^5))^(3/8) - a^7*c^2
*sqrt(x)*(-1/(a^19*c^5))^(3/8)) - 66*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/
(a^19*c^5))^(1/8)*log(a^12*c^3*(-1/(a^19*c^5))^(5/8) + sqrt(x)) + 66*(a^2*c
^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^19*c^5))^(1/8)*log(-a^12*c^3*(-1/(a^19*c
^5))^(5/8) + sqrt(x)) + 16*(11*c*x^6 + 19*a*x^2)*sqrt(x))/(a^2*c^2*x^8 + 2*
a^3*c*x^4 + a^4)
```

giac [B] time = 0.86, size = 472, normalized size = 1.43

$$-\frac{33\left(\frac{a}{c}\right)^{\frac{5}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{256\,a^3\sqrt{2\sqrt{2}+4}}-\frac{33\left(\frac{a}{c}\right)^{\frac{5}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{256\,a^3\sqrt{2\sqrt{2}+4}}+\frac{33\left(\frac{a}{c}\right)^{\frac{5}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{256\,a^3\sqrt{-2\sqrt{2}+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] -33/256*(a/c)^(5/8)*arctan((sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sq
rt(sqrt(2) + 2)*(a/c)^(1/8)))/(a^3*sqrt(2*sqrt(2) + 4)) - 33/256*(a/c)^(5/8
)*arctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(sqrt(2) + 2)*(
a/c)^(1/8)))/(a^3*sqrt(2*sqrt(2) + 4)) + 33/256*(a/c)^(5/8)*arctan((sqrt(sq
rt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a^3*
sqrt(-2*sqrt(2) + 4)) + 33/256*(a/c)^(5/8)*arctan(-(sqrt(sqrt(2) + 2)*(a/c)
^(1/8) - 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a^3*sqrt(-2*sqrt(2)
+ 4)) - 33/512*(a/c)^(5/8)*log(sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x +
(a/c)^(1/4))/(a^3*sqrt(2*sqrt(2) + 4)) + 33/512*(a/c)^(5/8)*log(-sqrt(x)*sq
rt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^3*sqrt(2*sqrt(2) + 4)) +
33/512*(a/c)^(5/8)*log(sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(
1/4))/(a^3*sqrt(-2*sqrt(2) + 4)) - 33/512*(a/c)^(5/8)*log(-sqrt(x)*sqrt(-sq
rt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^3*sqrt(-2*sqrt(2) + 4)) + 1/64
*(11*c*x^(13/2) + 19*a*x^(5/2))/((c*x^4 + a)^2*a^2)
```

maple [C] time = 0.02, size = 62, normalized size = 0.19

$$\frac{33\ln\left(-\operatorname{RootOf}\left(c_Z^8+a\right)+\sqrt{x}\right)}{512a^2c\operatorname{RootOf}\left(c_Z^8+a\right)^3}+\frac{\frac{11cx^{\frac{13}{2}}}{64a^2}+\frac{19x^{\frac{5}{2}}}{64a}}{\left(cx^4+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(c*x^4+a)^3,x)
```

```
[Out] 2*(19/128*x^(5/2)/a+11/128/a^2*c*x^(13/2))/(c*x^4+a)^2+33/512/a^2/c*sum(1/_
R^3*ln(-_R+x^(1/2)),_R=RootOf(_Z^8*c+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{11\,cx^{\frac{13}{2}}+19\,ax^{\frac{5}{2}}}{64\left(a^2c^2x^8+2\,a^3cx^4+a^4\right)}+33\int\frac{x^{\frac{3}{2}}}{128\left(a^2cx^4+a^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/64*(11*c*x^(13/2) + 19*a*x^(5/2))/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4) + 33*
integrate(1/128*x^(3/2)/(a^2*c*x^4 + a^3), x)
```

mupad [B] time = 0.14, size = 157, normalized size = 0.48

$$\frac{\frac{19\,x^{5/2}}{64\,a}+\frac{11\,c\,x^{13/2}}{64\,a^2}}{a^2+2\,a\,c\,x^4+c^2\,x^8}-\frac{33\,\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}}{(-a)^{1/8}}\right)}{256\,(-a)^{19/8}\,c^{5/8}}+\frac{\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}\,1\mathrm{i}}{(-a)^{1/8}}\right)\,33\mathrm{i}}{256\,(-a)^{19/8}\,c^{5/8}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\left(\frac{33}{512}+\frac{33}{512}\mathrm{i}\right)}{(-a)^{19/8}\,c^{5/8}}+\frac{\sqrt{2}\,\mathrm{ata}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + c*x^4)^3,x)
```

```
[Out] ((19*x^(5/2))/(64*a) + (11*c*x^(13/2))/(64*a^2))/(a^2 + c^2*x^8 + 2*a*c*x^4
) - (33*atan((c^(1/8)*x^(1/2))/(-a)^(1/8)))/(256*(-a)^(19/8)*c^(5/8)) + (at
an((c^(1/8)*x^(1/2)*1i)/(-a)^(1/8))*33i)/(256*(-a)^(19/8)*c^(5/8)) + (2^(1/
2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^(1/8))*(33/512 + 33i/51
2))/((-a)^(19/8)*c^(5/8)) + (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 + 1
i/2))/(-a)^(1/8))*(33/512 - 33i/512))/((-a)^(19/8)*c^(5/8))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(c*x**4+a)**3,x)
```

```
[Out] Timed out
```

3.762 $\int \frac{\sqrt{x}}{\left(a+cx^4\right)^3}\,dx$

Optimal. Leaf size=329

$$\frac{13x^{3/2}}{64a^2\left(a+cx^4\right)}-\frac{65\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{512\sqrt{2}\,(-a)^{21/8}c^{3/8}}+\frac{65\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{512\sqrt{2}\,(-a)^{21/8}c^{3/8}}+\frac{65\tan^{-1}\left(1\right.}{256\sqrt{2}\,(-}$$

```
[Out] 1/8*x^(3/2)/a/(c*x^4+a)^2+13/64*x^(3/2)/a^2/(c*x^4+a)+65/256*arctan(c^(1/8)
*x^(1/2)/(-a)^(1/8))/(-a)^(21/8)/c^(3/8)-65/256*arctanh(c^(1/8)*x^(1/2)/(-a
)^(1/8))/(-a)^(21/8)/c^(3/8)-65/512*arctan(-1+c^(1/8)*2^(1/2)*x^(1/2)/(-a)^
(1/8))/(-a)^(21/8)/c^(3/8)*2^(1/2)-65/512*arctan(1+c^(1/8)*2^(1/2)*x^(1/2)/
(-a)^(1/8))/(-a)^(21/8)/c^(3/8)*2^(1/2)-65/1024*ln((-a)^(1/4)+c^(1/4)*x-(-a
)^(1/8)*c^(1/8)*2^(1/2)*x^(1/2))/(-a)^(21/8)/c^(3/8)*2^(1/2)+65/1024*ln((-a
)^(1/4)+c^(1/4)*x+(-a)^(1/8)*c^(1/8)*2^(1/2)*x^(1/2))/(-a)^(21/8)/c^(3/8)*2
^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {290, 329, 300, 297, 1162, 617, 204, 1165, 628, 298, 205, 208}

$$\frac{13x^{3/2}}{64a^2\left(a+cx^4\right)}-\frac{65\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{512\sqrt{2}\,(-a)^{21/8}c^{3/8}}+\frac{65\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{512\sqrt{2}\,(-a)^{21/8}c^{3/8}}+\frac{65\tan^{-1}\left(1\right.}{256\sqrt{2}\,(-}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + c*x^4)^3,x]
```

```
[Out] x^(3/2)/(8*a*(a + c*x^4)^2) + (13*x^(3/2))/(64*a^2*(a + c*x^4)) + (65*ArcTa
n[1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(256*Sqrt[2]*(-a)^(21/8)*c^(3/
8)) - (65*ArcTan[1 + (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(256*Sqrt[2]*(-
a)^(21/8)*c^(3/8)) + (65*ArcTan[(c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(256*(-a)^(2
1/8)*c^(3/8)) - (65*ArcTanh[(c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(256*(-a)^(21/8)
*c^(3/8)) - (65*Log[(-a)^(1/4) - Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/
4)*x])/(512*Sqrt[2]*(-a)^(21/8)*c^(3/8)) + (65*Log[(-a)^(1/4) + Sqrt[2]*(-a
)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x])/(512*Sqrt[2]*(-a)^(21/8)*c^(3/8))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
```

```
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 300

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[x^m/(r + s
*x^(n/2)), x], x] + Dist[r/(2*a), Int[x^m/(r - s*x^(n/2)), x], x]] /; FreeQ
[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n/2] &&  !GtQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\left(a+cx^4\right)^3}\,dx = \frac{x^{3/2}}{8a\left(a+cx^4\right)^2} + \frac{13\int \frac{\sqrt{x}}{\left(a+cx^4\right)^2}\,dx}{16a}$$

$$= \frac{x^{3/2}}{8a\left(a+cx^4\right)^2} + \frac{13x^{3/2}}{64a^2\left(a+cx^4\right)} + \frac{65\int \frac{\sqrt{x}}{a+cx^4}\,dx}{128a^2}$$

$$= \frac{x^{3/2}}{8a\left(a+cx^4\right)^2} + \frac{13x^{3/2}}{64a^2\left(a+cx^4\right)} + \frac{65\,\text{Subst}\left(\int \frac{x^2}{a+cx^8}\,dx, x, \sqrt{x}\right)}{64a^2}$$

$$= \frac{x^{3/2}}{8a\left(a+cx^4\right)^2} + \frac{13x^{3/2}}{64a^2\left(a+cx^4\right)} - \frac{65\,\text{Subst}\left(\int \frac{x^2}{\sqrt{-a}-\sqrt{c}\,x^4}\,dx, x, \sqrt{x}\right)}{128(-a)^{5/2}} - \frac{65\,\text{Subst}\left(\int \frac{x^2}{\sqrt{-a}+\sqrt{c}\,x^4}\right.}{128(-a)^{5/2}}$$

$$= \frac{x^{3/2}}{8a\left(a+cx^4\right)^2} + \frac{13x^{3/2}}{64a^2\left(a+cx^4\right)} - \frac{65\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}\,x^2}\,dx, x, \sqrt{x}\right)}{256(-a)^{5/2}\sqrt[4]{c}} + \frac{65\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}+\sqrt[4]{c}\,x^2}\right.}{256(-a)^{5/2}\sqrt[4]{}}$$

$$= \frac{x^{3/2}}{8a\left(a+cx^4\right)^2} + \frac{13x^{3/2}}{64a^2\left(a+cx^4\right)} + \frac{65\tan^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{21/8}c^{3/8}} - \frac{65\tanh^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{21/8}c^{3/8}} - \frac{65\,\text{Subst}\left(\int \frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right.}{512(}$$

$$= \frac{x^{3/2}}{8a\left(a+cx^4\right)^2} + \frac{13x^{3/2}}{64a^2\left(a+cx^4\right)} + \frac{65\tan^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{21/8}c^{3/8}} - \frac{65\tanh^{-1}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{21/8}c^{3/8}} - \frac{65\log\left(\sqrt[4]{-a} - \sqrt{\right.}}{512\sqrt{2}}$$

$$= \frac{x^{3/2}}{8a\left(a+cx^4\right)^2} + \frac{13x^{3/2}}{64a^2\left(a+cx^4\right)} + \frac{65\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}(-a)^{21/8}c^{3/8}} - \frac{65\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}(-a)^{21/8}c^{3/8}} + \frac{65\tan}{256(}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.09

$$\frac{2x^{3/2}\,{}_2F_1\left(\frac{3}{8}, 3; \frac{11}{8}; -\frac{cx^4}{a}\right)}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + c*x^4)^3,x]
[Out] (2*x^(3/2)*Hypergeometric2F1[3/8, 3, 11/8, -((c*x^4)/a)])/(3*a^3)
```

fricas [B] time = 0.84, size = 665, normalized size = 2.02

$$-\frac{260\sqrt{2}\left(a^2c^2x^8 + 2a^3cx^4 + a^4\right)\left(-\frac{1}{a^{21}c^3}\right)^{\frac{1}{8}}\arctan\left(\sqrt{2}\sqrt{a^{16}c^2\left(-\frac{1}{a^{21}c^3}\right)^{\frac{3}{4}} + \sqrt{2}\,a^8c\sqrt{x}\left(-\frac{1}{a^{21}c^3}\right)^{\frac{3}{8}} + x}\,a^{13}c^2\left(-\frac{1}{a^{21}c^3}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] -1/1024*(260*sqrt(2)*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^21*c^3))^(1/8
)*arctan(sqrt(2)*sqrt(a^16*c^2*(-1/(a^21*c^3))^(3/4) + sqrt(2)*a^8*c*sqrt(x
)*(-1/(a^21*c^3))^(3/8) + x)*a^13*c^2*(-1/(a^21*c^3))^(5/8) - sqrt(2)*a^13*
c^2*sqrt(x)*(-1/(a^21*c^3))^(5/8) + 1) + 260*sqrt(2)*(a^2*c^2*x^8 + 2*a^3*c
*x^4 + a^4)*(-1/(a^21*c^3))^(1/8)*arctan(sqrt(2)*sqrt(a^16*c^2*(-1/(a^21*c^
3))^(3/4) - sqrt(2)*a^8*c*sqrt(x)*(-1/(a^21*c^3))^(3/8) + x)*a^13*c^2*(-1/(
a^21*c^3))^(5/8) - sqrt(2)*a^13*c^2*sqrt(x)*(-1/(a^21*c^3))^(5/8) - 1) - 65
*sqrt(2)*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^21*c^3))^(1/8)*log(a^16*c
^2*(-1/(a^21*c^3))^(3/4) + sqrt(2)*a^8*c*sqrt(x)*(-1/(a^21*c^3))^(3/8) + x)
 + 65*sqrt(2)*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^21*c^3))^(1/8)*log(a
^16*c^2*(-1/(a^21*c^3))^(3/4) - sqrt(2)*a^8*c*sqrt(x)*(-1/(a^21*c^3))^(3/8)
 + x) - 520*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^21*c^3))^(1/8)*arctan(
sqrt(a^16*c^2*(-1/(a^21*c^3))^(3/4) + x)*a^13*c^2*(-1/(a^21*c^3))^(5/8) - a
^13*c^2*sqrt(x)*(-1/(a^21*c^3))^(5/8)) + 130*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a
^4)*(-1/(a^21*c^3))^(1/8)*log(a^8*c*(-1/(a^21*c^3))^(3/8) + sqrt(x)) - 130*
(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^21*c^3))^(1/8)*log(-a^8*c*(-1/(a^2
1*c^3))^(3/8) + sqrt(x)) - 16*(13*c*x^5 + 21*a*x)*sqrt(x))/(a^2*c^2*x^8 + 2
*a^3*c*x^4 + a^4)
```

giac [B] time = 0.83, size = 472, normalized size = 1.43

$$-\frac{65\left(\frac{a}{c}\right)^{\frac{3}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{256\,a^3\sqrt{2\sqrt{2}+4}}-\frac{65\left(\frac{a}{c}\right)^{\frac{3}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{256\,a^3\sqrt{2\sqrt{2}+4}}+\frac{65\left(\frac{a}{c}\right)^{\frac{3}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right.}{256\,a^3\sqrt{-2\sqrt{2}+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] -65/256*(a/c)^(3/8)*arctan((sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sq
rt(sqrt(2) + 2)*(a/c)^(1/8)))/(a^3*sqrt(2*sqrt(2) + 4)) - 65/256*(a/c)^(3/8
)*arctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(sqrt(2) + 2)*(
a/c)^(1/8)))/(a^3*sqrt(2*sqrt(2) + 4)) + 65/256*(a/c)^(3/8)*arctan((sqrt(sq
rt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a^3*
sqrt(-2*sqrt(2) + 4)) + 65/256*(a/c)^(3/8)*arctan(-(sqrt(sqrt(2) + 2)*(a/c)
^(1/8) - 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a^3*sqrt(-2*sqrt(2)
+ 4)) + 65/512*(a/c)^(3/8)*log(sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x +
(a/c)^(1/4))/(a^3*sqrt(2*sqrt(2) + 4)) - 65/512*(a/c)^(3/8)*log(-sqrt(x)*sq
rt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^3*sqrt(2*sqrt(2) + 4)) -
65/512*(a/c)^(3/8)*log(sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(
1/4))/(a^3*sqrt(-2*sqrt(2) + 4)) + 65/512*(a/c)^(3/8)*log(-sqrt(x)*sqrt(-sq
rt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^3*sqrt(-2*sqrt(2) + 4)) + 1/64
*(13*c*x^(11/2) + 21*a*x^(3/2))/((c*x^4 + a)^2*a^2)
```

maple [C] time = 0.02, size = 62, normalized size = 0.19

$$\frac{65\ln\left(-\operatorname{RootOf}\left(c_Z^8+a\right)+\sqrt{x}\right)}{512a^2c\operatorname{RootOf}\left(c_Z^8+a\right)^5}+\frac{\frac{13cx^{\frac{11}{2}}}{64a^2}+\frac{21x^{\frac{3}{2}}}{64a}}{\left(cx^4+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(c*x^4+a)^3,x)
```

```
[Out] 2*(21/128*x^(3/2)/a+13/128/a^2*c*x^(11/2))/(c*x^4+a)^2+65/512/a^2/c*sum(1/_
R^5*ln(-_R+x^(1/2)),_R=RootOf(_Z^8*c+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{13\,cx^{\frac{11}{2}} + 21\,ax^{\frac{3}{2}}}{64\left(a^2c^2x^8 + 2\,a^3cx^4 + a^4\right)} + 65\int \frac{\sqrt{x}}{128\left(a^2cx^4 + a^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/64*(13*c*x^(11/2) + 21*a*x^(3/2))/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4) + 65*
integrate(1/128*sqrt(x)/(a^2*c*x^4 + a^3), x)
```

mupad [B] time = 1.08, size = 157, normalized size = 0.48

$$\frac{\frac{21\,x^{3/2}}{64\,a} + \frac{13\,c\,x^{11/2}}{64\,a^2}}{a^2 + 2\,a\,c\,x^4 + c^2\,x^8} + \frac{65\,\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}}{(-a)^{1/8}}\right)}{256\,(-a)^{21/8}\,c^{3/8}} + \frac{\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}\,1\mathrm{i}}{(-a)^{1/8}}\right)\,65\mathrm{i}}{256\,(-a)^{21/8}\,c^{3/8}} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\left(-\frac{65}{512}+\frac{65}{512}\mathrm{i}\right)}{(-a)^{21/8}\,c^{3/8}} + \frac{\sqrt{2}\,\mathrm{atan}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + c*x^4)^3,x)
```

```
[Out] ((21*x^(3/2))/(64*a) + (13*c*x^(11/2))/(64*a^2))/(a^2 + c^2*x^8 + 2*a*c*x^4
) + (65*atan((c^(1/8)*x^(1/2))/(-a)^(1/8)))/(256*(-a)^(21/8)*c^(3/8)) + (at
an((c^(1/8)*x^(1/2)*1i)/(-a)^(1/8))*65i)/(256*(-a)^(21/8)*c^(3/8)) - (2^(1/
2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^(1/8))*(65/512 - 65i/51
2))/((-a)^(21/8)*c^(3/8)) - (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 + 1
i/2))/(-a)^(1/8))*(65/512 + 65i/512))/((-a)^(21/8)*c^(3/8))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**4+a)**3,x)
```

```
[Out] Timed out
```

3.763 $\int \frac{1}{\sqrt{x}\left(a+cx^4\right)^3}\,dx$

Optimal. Leaf size=329

$$\frac{15\sqrt{x}}{64a^2\left(a+cx^4\right)}+\frac{\sqrt{x}}{8a\left(a+cx^4\right)^2}+\frac{105\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{512\sqrt{2}\,(-a)^{23/8}\sqrt[8]{c}}-\frac{105\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}-\right.}{512\sqrt{2}\,(-a)^{23/8}\sqrt[8]{c}}$$

```
[Out] -105/256*arctan(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(23/8)/c^(1/8)-105/256*arc
tanh(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(23/8)/c^(1/8)-105/512*arctan(-1+c^(1
/8)*2^(1/2)*x^(1/2)/(-a)^(1/8))/(-a)^(23/8)/c^(1/8)*2^(1/2)-105/512*arctan(
1+c^(1/8)*2^(1/2)*x^(1/2)/(-a)^(1/8))/(-a)^(23/8)/c^(1/8)*2^(1/2)+105/1024*
ln((-a)^(1/4)+c^(1/4)*x-(-a)^(1/8)*c^(1/8)*2^(1/2)*x^(1/2))/(-a)^(23/8)/c^(
1/8)*2^(1/2)-105/1024*ln((-a)^(1/4)+c^(1/4)*x+(-a)^(1/8)*c^(1/8)*2^(1/2)*x^
(1/2))/(-a)^(23/8)/c^(1/8)*2^(1/2)+1/8*x^(1/2)/a/(c*x^4+a)^2+15/64*x^(1/2)/
a^2/(c*x^4+a)
```

Rubi [A] time = 0.30, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {290, 329, 214, 212, 208, 205, 211, 1165, 628, 1162, 617, 204}

$$\frac{15\sqrt{x}}{64a^2\left(a+cx^4\right)}+\frac{\sqrt{x}}{8a\left(a+cx^4\right)^2}+\frac{105\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}\,x\right)}{512\sqrt{2}\,(-a)^{23/8}\sqrt[8]{c}}-\frac{105\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{c}\,\sqrt{x}+\sqrt[4]{-a}-\right.}{512\sqrt{2}\,(-a)^{23/8}\sqrt[8]{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + c*x^4)^3),x]
```

```
[Out] Sqrt[x]/(8*a*(a + c*x^4)^2) + (15*Sqrt[x])/(64*a^2*(a + c*x^4)) + (105*ArcT
an[1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(256*Sqrt[2]*(-a)^(23/8)*c^(1
/8)) - (105*ArcTan[1 + (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(256*Sqrt[2]*
(-a)^(23/8)*c^(1/8)) - (105*ArcTan[(c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(256*(-a)
^(23/8)*c^(1/8)) - (105*ArcTanh[(c^(1/8)*Sqrt[x])/(-a)^(1/8)])/(256*(-a)^(2
3/8)*c^(1/8)) + (105*Log[(-a)^(1/4) - Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] +
c^(1/4)*x])/(512*Sqrt[2]*(-a)^(23/8)*c^(1/8)) - (105*Log[(-a)^(1/4) + Sqrt[
2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x])/(512*Sqrt[2]*(-a)^(23/8)*c^(1/8
))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(a+cx^4\right)^3}\,dx = \frac{\sqrt{x}}{8a\left(a+cx^4\right)^2} + \frac{15\int \frac{1}{\sqrt{x}\left(a+cx^4\right)^2}\,dx}{16a}$$

$$= \frac{\sqrt{x}}{8a\left(a+cx^4\right)^2} + \frac{15\sqrt{x}}{64a^2\left(a+cx^4\right)} + \frac{105\int \frac{1}{\sqrt{x}\left(a+cx^4\right)}\,dx}{128a^2}$$

$$= \frac{\sqrt{x}}{8a\left(a+cx^4\right)^2} + \frac{15\sqrt{x}}{64a^2\left(a+cx^4\right)} + \frac{105\,\text{Subst}\left(\int \frac{1}{a+cx^8}\,dx, x, \sqrt{x}\right)}{64a^2}$$

$$= \frac{\sqrt{x}}{8a\left(a+cx^4\right)^2} + \frac{15\sqrt{x}}{64a^2\left(a+cx^4\right)} - \frac{105\,\text{Subst}\left(\int \frac{1}{\sqrt{-a}-\sqrt{c}\,x^4}\,dx, x, \sqrt{x}\right)}{128(-a)^{5/2}} - \frac{105\,\text{Subst}\left(\int -\right.}{128}$$

$$= \frac{\sqrt{x}}{8a\left(a+cx^4\right)^2} + \frac{15\sqrt{x}}{64a^2\left(a+cx^4\right)} - \frac{105\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}\,x^2}\,dx, x, \sqrt{x}\right)}{256(-a)^{11/4}} - \frac{105\,\text{Subst}\left(\int \right.}{256}$$

$$= \frac{\sqrt{x}}{8a\left(a+cx^4\right)^2} + \frac{15\sqrt{x}}{64a^2\left(a+cx^4\right)} - \frac{105\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{23/8}\sqrt[8]{c}} - \frac{105\tanh^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{23/8}\sqrt[8]{c}} - \frac{105\,\text{Sub}}{}$$

$$= \frac{\sqrt{x}}{8a\left(a+cx^4\right)^2} + \frac{15\sqrt{x}}{64a^2\left(a+cx^4\right)} - \frac{105\tan^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{23/8}\sqrt[8]{c}} - \frac{105\tanh^{-1}\left(\frac{\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{23/8}\sqrt[8]{c}} + \frac{105\log}{}$$

$$= \frac{\sqrt{x}}{8a\left(a+cx^4\right)^2} + \frac{15\sqrt{x}}{64a^2\left(a+cx^4\right)} + \frac{105\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}\,(-a)^{23/8}\sqrt[8]{c}} - \frac{105\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[8]{c}\,\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}\,(-a)^{23/8}\sqrt[8]{c}}$$

Mathematica [C] time = 0.02, size = 62, normalized size = 0.19

$$\frac{\sqrt{x}\left(105\left(a+cx^4\right)^2\,{}_2F_1\left(\frac{1}{8},1;\frac{9}{8};-\frac{cx^4}{a}\right)+a\left(23a+15cx^4\right)\right)}{64a^3\left(a+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + c*x^4)^3),x]
```

```
[Out] (Sqrt[x]*(a*(23*a + 15*c*x^4) + 105*(a + c*x^4)^2*Hypergeometric2F1[1/8, 1,
 9/8, -((c*x^4)/a)]))/(64*a^3*(a + c*x^4)^2)
```

fricas [B] time = 0.82, size = 631, normalized size = 1.92

$$\frac{420\sqrt{2}\left(a^2c^2x^8+2a^3cx^4+a^4\right)\left(-\frac{1}{a^{23}c}\right)^{\frac{1}{8}}\arctan\left(\sqrt{2}\sqrt{a^6\left(-\frac{1}{a^{23}c}\right)^{\frac{1}{4}}+\sqrt{2}\,a^3\sqrt{x}\left(-\frac{1}{a^{23}c}\right)^{\frac{1}{8}}+x}\,a^{20}c\left(-\frac{1}{a^{23}c}\right)^{\frac{7}{8}}-\sqrt{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/1024*(420*sqrt(2)*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^23*c))^(1/8)*a
rctan(sqrt(2)*sqrt(a^6*(-1/(a^23*c))^(1/4) + sqrt(2)*a^3*sqrt(x)*(-1/(a^23*
c))^(1/8) + x)*a^20*c*(-1/(a^23*c))^(7/8) - sqrt(2)*a^20*c*sqrt(x)*(-1/(a^2
3*c))^(7/8) + 1) + 420*sqrt(2)*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^23*
c))^(1/8)*arctan(sqrt(2)*sqrt(a^6*(-1/(a^23*c))^(1/4) - sqrt(2)*a^3*sqrt(x)
*(-1/(a^23*c))^(1/8) + x)*a^20*c*(-1/(a^23*c))^(7/8) - sqrt(2)*a^20*c*sqrt(
x)*(-1/(a^23*c))^(7/8) - 1) + 105*sqrt(2)*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)
*(-1/(a^23*c))^(1/8)*log(a^6*(-1/(a^23*c))^(1/4) + sqrt(2)*a^3*sqrt(x)*(-1/
(a^23*c))^(1/8) + x) - 105*sqrt(2)*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a
^23*c))^(1/8)*log(a^6*(-1/(a^23*c))^(1/4) - sqrt(2)*a^3*sqrt(x)*(-1/(a^23*c
))^(1/8) + x) + 840*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^23*c))^(1/8)*a
rctan(sqrt(a^6*(-1/(a^23*c))^(1/4) + x)*a^20*c*(-1/(a^23*c))^(7/8) - a^20*c
*sqrt(x)*(-1/(a^23*c))^(7/8)) + 210*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(
a^23*c))^(1/8)*log(a^3*(-1/(a^23*c))^(1/8) + sqrt(x)) - 210*(a^2*c^2*x^8 +
2*a^3*c*x^4 + a^4)*(-1/(a^23*c))^(1/8)*log(-a^3*(-1/(a^23*c))^(1/8) + sqrt(
x)) + 16*(15*c*x^4 + 23*a)*sqrt(x))/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)
```

giac [B] time = 0.65, size = 472, normalized size = 1.43

$$\frac{105\left(\frac{a}{c}\right)^{\frac{1}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{256a^3\sqrt{-2\sqrt{2}+4}}+\frac{105\left(\frac{a}{c}\right)^{\frac{1}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{256a^3\sqrt{-2\sqrt{2}+4}}+\frac{105\left(\frac{a}{c}\right)^{\frac{1}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{256a^3\sqrt{2\sqrt{2}+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 105/256*(a/c)^(1/8)*arctan((sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sq
rt(sqrt(2) + 2)*(a/c)^(1/8)))/(a^3*sqrt(-2*sqrt(2) + 4)) + 105/256*(a/c)^(1
/8)*arctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(sqrt(2) + 2)
*(a/c)^(1/8)))/(a^3*sqrt(-2*sqrt(2) + 4)) + 105/256*(a/c)^(1/8)*arctan((sqr
t(sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(
a^3*sqrt(2*sqrt(2) + 4)) + 105/256*(a/c)^(1/8)*arctan(-(sqrt(sqrt(2) + 2)*(
a/c)^(1/8) - 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a^3*sqrt(2*sqrt(
2) + 4)) + 105/512*(a/c)^(1/8)*log(sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) +
x + (a/c)^(1/4))/(a^3*sqrt(-2*sqrt(2) + 4)) - 105/512*(a/c)^(1/8)*log(-sqrt
(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^3*sqrt(-2*sqrt(2) +
 4)) + 105/512*(a/c)^(1/8)*log(sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x +
 (a/c)^(1/4))/(a^3*sqrt(2*sqrt(2) + 4)) - 105/512*(a/c)^(1/8)*log(-sqrt(x)*
sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^3*sqrt(2*sqrt(2) + 4))
 + 1/64*(15*c*x^(9/2) + 23*a*sqrt(x))/((c*x^4 + a)^2*a^2)
```

maple [C] time = 0.02, size = 62, normalized size = 0.19

$$\frac{105\ln\left(-\text{RootOf}\left(c_Z^8+a\right)+\sqrt{x}\right)}{512a^2c\,\text{RootOf}\left(c_Z^8+a\right)^7}+\frac{\frac{15cx^{\frac{9}{2}}}{64a^2}+\frac{23\sqrt{x}}{64a}}{\left(cx^4+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+a)^3/x^(1/2),x)
```

```
[Out] 2*(23/128*x^(1/2)/a+15/128/a^2*c*x^(9/2))/(c*x^4+a)^2+105/512/a^2/c*sum(1/_
R^7*ln(-_R+x^(1/2)),_R=RootOf(_Z^8*c+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-105\,c\int \frac{x^{\frac{7}{2}}}{128\left(a^3cx^4+a^4\right)}\,dx+\frac{105\,c^2x^{\frac{17}{2}}+225\,acx^{\frac{9}{2}}+128\,a^2\sqrt{x}}{64\left(a^3c^2x^8+2\,a^4cx^4+a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] -105*c*integrate(1/128*x^(7/2)/(a^3*c*x^4 + a^4), x) + 1/64*(105*c^2*x^(17/
2) + 225*a*c*x^(9/2) + 128*a^2*sqrt(x))/(a^3*c^2*x^8 + 2*a^4*c*x^4 + a^5)
```

mupad [B] time = 0.14, size = 157, normalized size = 0.48

$$\frac{\frac{23\sqrt{x}}{64\,a}+\frac{15\,c\,x^{9/2}}{64\,a^2}}{a^2+2\,a\,c\,x^4+c^2\,x^8}-\frac{105\,\mathrm{atan}\left(\frac{c^{1/8}\sqrt{x}}{(-a)^{1/8}}\right)}{256\,(-a)^{23/8}\,c^{1/8}}+\frac{\mathrm{atan}\left(\frac{c^{1/8}\sqrt{x}\,1\mathrm{i}}{(-a)^{1/8}}\right)105\mathrm{i}}{256\,(-a)^{23/8}\,c^{1/8}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\sqrt{x}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\left(-\frac{105}{512}-\frac{105}{512}\mathrm{i}\right)}{(-a)^{23/8}\,c^{1/8}}+\frac{\sqrt{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + c*x^4)^3),x)
```

```
[Out] ((23*x^(1/2))/(64*a) + (15*c*x^(9/2))/(64*a^2))/(a^2 + c^2*x^8 + 2*a*c*x^4)
 - (105*atan((c^(1/8)*x^(1/2))/(-a)^(1/8)))/(256*(-a)^(23/8)*c^(1/8)) + (at
an((c^(1/8)*x^(1/2)*1i)/(-a)^(1/8))*105i)/(256*(-a)^(23/8)*c^(1/8)) - (2^(1
/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^(1/8))*(105/512 + 105i
/512))/((-a)^(23/8)*c^(1/8)) - (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2
+ 1i/2))/(-a)^(1/8))*(105/512 - 105i/512))/((-a)^(23/8)*c^(1/8))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+a)**3/x**(1/2),x)
```

```
[Out] Timed out
```

3.764 $\int x^{11}\sqrt{a+cx^4}\,dx$

Optimal. Leaf size=59

$$\frac{a^2\left(a+cx^4\right)^{3/2}}{6c^3}+\frac{\left(a+cx^4\right)^{7/2}}{14c^3}-\frac{a\left(a+cx^4\right)^{5/2}}{5c^3}$$

```
[Out] 1/6*a^2*(c*x^4+a)^(3/2)/c^3-1/5*a*(c*x^4+a)^(5/2)/c^3+1/14*(c*x^4+a)^(7/2)/
c^3
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{a^2\left(a+cx^4\right)^{3/2}}{6c^3}+\frac{\left(a+cx^4\right)^{7/2}}{14c^3}-\frac{a\left(a+cx^4\right)^{5/2}}{5c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*Sqrt[a + c*x^4],x]
```

```
[Out] (a^2*(a + c*x^4)^(3/2))/(6*c^3) - (a*(a + c*x^4)^(5/2))/(5*c^3) + (a + c*x^
4)^(7/2)/(14*c^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{11}\sqrt{a+cx^4}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int x^2\sqrt{a+cx}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int\left(\frac{a^2\sqrt{a+cx}}{c^2}-\frac{2a(a+cx)^{3/2}}{c^2}+\frac{(a+cx)^{5/2}}{c^2}\right)dx, x, x^4\right)\\ &= \frac{a^2\left(a+cx^4\right)^{3/2}}{6c^3}-\frac{a\left(a+cx^4\right)^{5/2}}{5c^3}+\frac{\left(a+cx^4\right)^{7/2}}{14c^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 39, normalized size = 0.66

$$\frac{\left(a+cx^4\right)^{3/2}\left(8a^2-12acx^4+15c^2x^8\right)}{210c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*Sqrt[a + c*x^4],x]
```

```
[Out] ((a + c*x^4)^(3/2)*(8*a^2 - 12*a*c*x^4 + 15*c^2*x^8))/(210*c^3)
```

fricas [A] time = 0.70, size = 46, normalized size = 0.78

$$\frac{\left(15\,c^3x^{12}+3\,ac^2x^8-4\,a^2cx^4+8\,a^3\right)\sqrt{cx^4+a}}{210\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/210*(15*c^3*x^12 + 3*a*c^2*x^8 - 4*a^2*c*x^4 + 8*a^3)*sqrt(c*x^4 + a)/c^3
```

giac [A] time = 0.17, size = 43, normalized size = 0.73

$$\frac{15\left(cx^4+a\right)^{\frac{7}{2}}-42\left(cx^4+a\right)^{\frac{5}{2}}a+35\left(cx^4+a\right)^{\frac{3}{2}}a^2}{210\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/210*(15*(c*x^4 + a)^(7/2) - 42*(c*x^4 + a)^(5/2)*a + 35*(c*x^4 + a)^(3/2)
*a^2)/c^3
```

maple [A] time = 0.01, size = 36, normalized size = 0.61

$$\frac{\left(c\,x^4+a\right)^{\frac{3}{2}}\left(15c^2x^8-12ac\,x^4+8a^2\right)}{210c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(c*x^4+a)^(1/2),x)
```

```
[Out] 1/210*(c*x^4+a)^(3/2)*(15*c^2*x^8-12*a*c*x^4+8*a^2)/c^3
```

maxima [A] time = 1.37, size = 47, normalized size = 0.80

$$\frac{\left(cx^4+a\right)^{\frac{7}{2}}}{14\,c^3}-\frac{\left(cx^4+a\right)^{\frac{5}{2}}a}{5\,c^3}+\frac{\left(cx^4+a\right)^{\frac{3}{2}}a^2}{6\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/14*(c*x^4 + a)^(7/2)/c^3 - 1/5*(c*x^4 + a)^(5/2)*a/c^3 + 1/6*(c*x^4 + a)^
(3/2)*a^2/c^3
```

mupad [B] time = 1.17, size = 44, normalized size = 0.75

$$\sqrt{c\,x^4+a}\left(\frac{x^{12}}{14}+\frac{4\,a^3}{105\,c^3}+\frac{a\,x^8}{70\,c}-\frac{2\,a^2\,x^4}{105\,c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a + c*x^4)^(1/2),x)
```

```
[Out] (a + c*x^4)^(1/2)*(x^12/14 + (4*a^3)/(105*c^3) + (a*x^8)/(70*c) - (2*a^2*x^
4)/(105*c^2))
```

sympy [A] time = 3.79, size = 87, normalized size = 1.47

$$\begin{cases}\frac{4a^3\sqrt{a+cx^4}}{105c^3}-\frac{2a^2x^4\sqrt{a+cx^4}}{105c^2}+\frac{ax^8\sqrt{a+cx^4}}{70c}+\frac{x^{12}\sqrt{a+cx^4}}{14} & \text{for } c\neq 0\\ \frac{\sqrt{a}\,x^{12}}{12} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(c*x**4+a)**(1/2),x)
```

```
[Out] Piecewise((4*a**3*sqrt(a + c*x**4)/(105*c**3) - 2*a**2*x**4*sqrt(a + c*x**4
)/(105*c**2) + a*x**8*sqrt(a + c*x**4)/(70*c) + x**12*sqrt(a + c*x**4)/14,
Ne(c, 0)), (sqrt(a)*x**12/12, True))
```

3.765 $\int x^7\sqrt{a+cx^4}\,dx$

Optimal. Leaf size=38

$$\frac{\left(a+cx^4\right)^{5/2}}{10c^2}-\frac{a\left(a+cx^4\right)^{3/2}}{6c^2}$$

```
[Out] -1/6*a*(c*x^4+a)^(3/2)/c^2+1/10*(c*x^4+a)^(5/2)/c^2
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{\left(a+cx^4\right)^{5/2}}{10c^2}-\frac{a\left(a+cx^4\right)^{3/2}}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*Sqrt[a + c*x^4],x]
```

```
[Out] -(a*(a + c*x^4)^(3/2))/(6*c^2) + (a + c*x^4)^(5/2)/(10*c^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^7\sqrt{a+cx^4}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int x\sqrt{a+cx}\,dx, x, x^4\right)\\ &= \frac{1}{4}\operatorname{Subst}\left(\int\left(-\frac{a\sqrt{a+cx}}{c}+\frac{(a+cx)^{3/2}}{c}\right)dx, x, x^4\right)\\ &= -\frac{a\left(a+cx^4\right)^{3/2}}{6c^2}+\frac{\left(a+cx^4\right)^{5/2}}{10c^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 28, normalized size = 0.74

$$\frac{\left(a+cx^4\right)^{3/2}\left(3cx^4-2a\right)}{30c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*Sqrt[a + c*x^4],x]
```

```
[Out] ((a + c*x^4)^(3/2)*(-2*a + 3*c*x^4))/(30*c^2)
```

fricas [A] time = 0.62, size = 34, normalized size = 0.89

$$\frac{\left(3\,c^2x^8+acx^4-2\,a^2\right)\sqrt{cx^4+a}}{30\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*x^4+a)^(1/2),x, algorithm="fricas")
[Out] 1/30*(3*c^2*x^8 + a*c*x^4 - 2*a^2)*sqrt(c*x^4 + a)/c^2
```

giac [A] time = 0.15, size = 29, normalized size = 0.76

$$\frac{3\left(cx^4+a\right)^{\frac{5}{2}}-5\left(cx^4+a\right)^{\frac{3}{2}}a}{30\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*x^4+a)^(1/2),x, algorithm="giac")
[Out] 1/30*(3*(c*x^4 + a)^(5/2) - 5*(c*x^4 + a)^(3/2)*a)/c^2
```

maple [A] time = 0.00, size = 25, normalized size = 0.66

$$-\frac{\left(c\,x^4+a\right)^{\frac{3}{2}}\left(-3c\,x^4+2a\right)}{30c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(c*x^4+a)^(1/2),x)
[Out] -1/30*(c*x^4+a)^(3/2)*(-3*c*x^4+2*a)/c^2
```

maxima [A] time = 1.36, size = 30, normalized size = 0.79

$$\frac{\left(cx^4+a\right)^{\frac{5}{2}}}{10\,c^2}-\frac{\left(cx^4+a\right)^{\frac{3}{2}}a}{6\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*x^4+a)^(1/2),x, algorithm="maxima")
[Out] 1/10*(c*x^4 + a)^(5/2)/c^2 - 1/6*(c*x^4 + a)^(3/2)*a/c^2
```

mupad [B] time = 1.13, size = 33, normalized size = 0.87

$$\sqrt{c\,x^4+a}\,\left(\frac{x^8}{10}-\frac{a^2}{15\,c^2}+\frac{a\,x^4}{30\,c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + c*x^4)^(1/2),x)
[Out] (a + c*x^4)^(1/2)*(x^8/10 - a^2/(15*c^2) + (a*x^4)/(30*c))
```

sympy [A] time = 2.90, size = 61, normalized size = 1.61

$$\begin{cases} -\frac{a^2\sqrt{a+cx^4}}{15c^2}+\frac{ax^4\sqrt{a+cx^4}}{30c}+\frac{x^8\sqrt{a+cx^4}}{10} & \text{for } c\neq 0 \\ \frac{\sqrt{a}\,x^8}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(c*x**4+a)**(1/2),x)
[Out] Piecewise((-a**2*sqrt(a + c*x**4)/(15*c**2) + a*x**4*sqrt(a + c*x**4)/(30*c
) + x**8*sqrt(a + c*x**4)/10, Ne(c, 0)), (sqrt(a)*x**8/8, True))
```

3.766 $\int x^3\sqrt{a+cx^4}\,dx$

Optimal. Leaf size=18

$$\frac{\left(a+cx^4\right)^{3/2}}{6c}$$

```
[Out] 1/6*(c*x^4+a)^(3/2)/c
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$\frac{\left(a+cx^4\right)^{3/2}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + c*x^4],x]
```

```
[Out] (a + c*x^4)^(3/2)/(6*c)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^3\sqrt{a+cx^4}\,dx = \frac{\left(a+cx^4\right)^{3/2}}{6c}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{\left(a+cx^4\right)^{3/2}}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + c*x^4],x]
```

```
[Out] (a + c*x^4)^(3/2)/(6*c)
```

fricas [A] time = 0.83, size = 14, normalized size = 0.78

$$\frac{\left(cx^4+a\right)^{\frac{3}{2}}}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(c*x^4 + a)^(3/2)/c
```

giac [A] time = 0.18, size = 14, normalized size = 0.78

$$\frac{\left(cx^4+a\right)^{\frac{3}{2}}}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+a)^(1/2),x, algorithm="giac")
[Out] 1/6*(c*x^4 + a)^(3/2)/c
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$\frac{\left(c\,x^4+a\right)^{\frac{3}{2}}}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^4+a)^(1/2),x)
[Out] 1/6*(c*x^4+a)^(3/2)/c
```

maxima [A] time = 1.22, size = 14, normalized size = 0.78

$$\frac{\left(cx^4+a\right)^{\frac{3}{2}}}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+a)^(1/2),x, algorithm="maxima")
[Out] 1/6*(c*x^4 + a)^(3/2)/c
```

mupad [B] time = 1.12, size = 14, normalized size = 0.78

$$\frac{\left(c\,x^4+a\right)^{3/2}}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + c*x^4)^(1/2),x)
[Out] (a + c*x^4)^(3/2)/(6*c)
```

sympy [A] time = 0.40, size = 39, normalized size = 2.17

$$\begin{cases} \frac{a\sqrt{a+cx^4}}{6c} + \frac{x^4\sqrt{a+cx^4}}{6} & \text{for } c \neq 0 \\ \frac{\sqrt{a}\,x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**4+a)**(1/2),x)
[Out] Piecewise((a*sqrt(a + c*x**4)/(6*c) + x**4*sqrt(a + c*x**4)/6, Ne(c, 0)), (
sqrt(a)*x**4/4, True))
```

3.767 $\int \frac{\sqrt{a+cx^4}}{x}\,dx$

Optimal. Leaf size=43

$$\frac{1}{2}\sqrt{a+cx^4} - \frac{1}{2}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)$$

```
[Out] -1/2*arctanh((c*x^4+a)^(1/2)/a^(1/2))*a^(1/2)+1/2*(c*x^4+a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 50, 63, 208}

$$\frac{1}{2}\sqrt{a+cx^4} - \frac{1}{2}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^4]/x,x]
```

```
[Out] Sqrt[a + c*x^4]/2 - (Sqrt[a]*ArcTanh[Sqrt[a + c*x^4]/Sqrt[a]])/2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+cx^4}}{x}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x}\,dx, x, x^4\right)\\ &= \frac{1}{2}\sqrt{a+cx^4} + \frac{1}{4}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^4\right)\\ &= \frac{1}{2}\sqrt{a+cx^4} + \frac{a\,\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^4}\right)}{2c}\\ &= \frac{1}{2}\sqrt{a+cx^4} - \frac{1}{2}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 1.00

$$\frac{1}{2}\sqrt{a+cx^4} - \frac{1}{2}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^4]/x,x]
```

```
[Out] Sqrt[a + c*x^4]/2 - (Sqrt[a]*ArcTanh[Sqrt[a + c*x^4]/Sqrt[a]])/2
```

fricas [A] time = 0.77, size = 84, normalized size = 1.95

$$\left[\frac{1}{4}\sqrt{a}\,\log\left(\frac{cx^4 - 2\sqrt{cx^4+a}\sqrt{a} + 2a}{x^4}\right) + \frac{1}{2}\sqrt{cx^4+a}, \frac{1}{2}\sqrt{-a}\,\arctan\left(\frac{\sqrt{cx^4+a}\sqrt{-a}}{a}\right) + \frac{1}{2}\sqrt{cx^4+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(a)*log((c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(a) + 2*a)/x^4) + 1/2*sqrt(
c*x^4 + a), 1/2*sqrt(-a)*arctan(sqrt(c*x^4 + a)*sqrt(-a)/a) + 1/2*sqrt(c*x^
4 + a)]
```

giac [A] time = 0.15, size = 36, normalized size = 0.84

$$\frac{a\arctan\left(\frac{\sqrt{cx^4+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}} + \frac{1}{2}\sqrt{cx^4+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/2*a*arctan(sqrt(c*x^4 + a)/sqrt(-a))/sqrt(-a) + 1/2*sqrt(c*x^4 + a)
```

maple [A] time = 0.01, size = 41, normalized size = 0.95

$$-\frac{\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{c\,x^4+a}\,\sqrt{a}}{x^2}\right)}{2} + \frac{\sqrt{c\,x^4+a}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^(1/2)/x,x)
```

```
[Out] 1/2*(c*x^4+a)^(1/2)-1/2*a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^4+a)^(1/2))/x^2)
```

maxima [A] time = 3.04, size = 49, normalized size = 1.14

$$\frac{1}{4}\sqrt{a}\log\left(\frac{\sqrt{cx^4+a}-\sqrt{a}}{\sqrt{cx^4+a}+\sqrt{a}}\right)+\frac{1}{2}\sqrt{cx^4+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(a)*log((sqrt(c*x^4 + a) - sqrt(a))/(sqrt(c*x^4 + a) + sqrt(a))) +
1/2*sqrt(c*x^4 + a)
```

mupad [B] time = 1.20, size = 31, normalized size = 0.72

$$\frac{\sqrt{c\,x^4+a}}{2}-\frac{\sqrt{a}\,\operatorname{atanh}\left(\frac{\sqrt{c\,x^4+a}}{\sqrt{a}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^(1/2)/x,x)
```

```
[Out] (a + c*x^4)^(1/2)/2 - (a^(1/2)*atanh((a + c*x^4)^(1/2)/a^(1/2)))/2
```

sympy [A] time = 2.36, size = 66, normalized size = 1.53

$$-\frac{\sqrt{a}\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{c}\,x^2}\right)}{2}+\frac{a}{2\sqrt{c}\,x^2\sqrt{\frac{a}{cx^4}+1}}+\frac{\sqrt{c}\,x^2}{2\sqrt{\frac{a}{cx^4}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**(1/2)/x,x)
```

```
[Out] -sqrt(a)*asinh(sqrt(a)/(sqrt(c)*x**2))/2 + a/(2*sqrt(c)*x**2*sqrt(a/(c*x**4
) + 1)) + sqrt(c)*x**2/(2*sqrt(a/(c*x**4) + 1))
```

3.768 $\int \frac{\sqrt{a+cx^4}}{x^5}\,dx$

Optimal. Leaf size=47

$$-\frac{\sqrt{a+cx^4}}{4x^4}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

```
[Out] -1/4*c*arctanh((c*x^4+a)^(1/2)/a^(1/2))/a^(1/2)-1/4*(c*x^4+a)^(1/2)/x^4
```

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 47, 63, 208}

$$-\frac{\sqrt{a+cx^4}}{4x^4}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^4]/x^5,x]
```

```
[Out] -Sqrt[a + c*x^4]/(4*x^4) - (c*ArcTanh[Sqrt[a + c*x^4]/Sqrt[a]])/(4*Sqrt[a])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+cx^4}}{x^5}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x^2}\,dx, x, x^4\right)\\ &= -\frac{\sqrt{a+cx^4}}{4x^4} + \frac{1}{8}c\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^4\right)\\ &= -\frac{\sqrt{a+cx^4}}{4x^4} + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^4}\right)\\ &= -\frac{\sqrt{a+cx^4}}{4x^4} - \frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{4\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 59, normalized size = 1.26

$$-\frac{cx^4\sqrt{\frac{cx^4}{a}+1}\,\tanh^{-1}\left(\sqrt{\frac{cx^4}{a}+1}\right)+a+cx^4}{4x^4\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^4]/x^5,x]
```

```
[Out] -1/4*(a + c*x^4 + c*x^4*Sqrt[1 + (c*x^4)/a]*ArcTanh[Sqrt[1 + (c*x^4)/a]])/(
x^4*Sqrt[a + c*x^4])
```

fricas [A] time = 0.64, size = 108, normalized size = 2.30

$$\left[\frac{\sqrt{a}\,cx^4\log\left(\frac{cx^4-2\sqrt{cx^4+a}\sqrt{a}+2a}{x^4}\right)-2\sqrt{cx^4+a}\,a}{8ax^4}, \frac{\sqrt{-a}\,cx^4\arctan\left(\frac{\sqrt{cx^4+a}\sqrt{-a}}{a}\right)-\sqrt{cx^4+a}\,a}{4ax^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a)*c*x^4*log((c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(a) + 2*a)/x^4) - 2*
sqrt(c*x^4 + a)*a)/(a*x^4), 1/4*(sqrt(-a)*c*x^4*arctan(sqrt(c*x^4 + a)*sqrt
(-a)/a) - sqrt(c*x^4 + a)*a)/(a*x^4)]
```

giac [A] time = 0.16, size = 46, normalized size = 0.98

$$\frac{\frac{c^2\arctan\left(\frac{\sqrt{cx^4+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\sqrt{cx^4+a}\,c}{x^4}}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/4*(c^2*arctan(sqrt(c*x^4 + a)/sqrt(-a))/sqrt(-a) - sqrt(c*x^4 + a)*c/x^4)
/c
```

maple [A] time = 0.01, size = 63, normalized size = 1.34

$$-\frac{c\ln\left(\frac{2a+2\sqrt{cx^4+a}\sqrt{a}}{x^2}\right)}{4\sqrt{a}}+\frac{\sqrt{cx^4+a}\,c}{4a}-\frac{\left(cx^4+a\right)^{\frac{3}{2}}}{4ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^(1/2)/x^5,x)
```

```
[Out] -1/4/a/x^4*(c*x^4+a)^(3/2)-1/4*c/a^(1/2)*ln((2*a+2*(c*x^4+a)^(1/2)*a^(1/2))
/x^2)+1/4*c/a*(c*x^4+a)^(1/2)
```

maxima [A] time = 2.98, size = 53, normalized size = 1.13

$$\frac{c \log\left(\frac{\sqrt{cx^4+a}-\sqrt{a}}{\sqrt{cx^4+a}+\sqrt{a}}\right)}{8\sqrt{a}} - \frac{\sqrt{cx^4+a}}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*c*log((sqrt(c*x^4 + a) - sqrt(a))/(sqrt(c*x^4 + a) + sqrt(a)))/sqrt(a)
- 1/4*sqrt(c*x^4 + a)/x^4
```

mupad [B] time = 1.28, size = 35, normalized size = 0.74

$$-\frac{\sqrt{c\,x^4+a}}{4\,x^4} - \frac{c\,\operatorname{atanh}\left(\frac{\sqrt{c\,x^4+a}}{\sqrt{a}}\right)}{4\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^(1/2)/x^5,x)
```

```
[Out] - (a + c*x^4)^(1/2)/(4*x^4) - (c*atanh((a + c*x^4)^(1/2)/a^(1/2)))/(4*a^(1/
2))
```

sympy [A] time = 4.06, size = 46, normalized size = 0.98

$$-\frac{\sqrt{c}\sqrt{\frac{a}{cx^4}+1}}{4x^2} - \frac{c\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{c}\,x^2}\right)}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**(1/2)/x**5,x)
```

```
[Out] -sqrt(c)*sqrt(a/(c*x**4) + 1)/(4*x**2) - c*asinh(sqrt(a)/(sqrt(c)*x**2))/(4
*sqrt(a))
```

3.769 $\int \frac{\sqrt{a+cx^4}}{x^9}\,dx$

Optimal. Leaf size=71

$$\frac{c^2 \tanh^{-1}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{16a^{3/2}} - \frac{c\sqrt{a+cx^4}}{16ax^4} - \frac{\sqrt{a+cx^4}}{8x^8}$$

```
[Out] 1/16*c^2*arctanh((c*x^4+a)^(1/2)/a^(1/2))/a^(3/2)-1/8*(c*x^4+a)^(1/2)/x^8-1
/16*c*(c*x^4+a)^(1/2)/a/x^4
```

Rubi [A] time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {266, 47, 51, 63, 208}

$$\frac{c^2 \tanh^{-1}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{16a^{3/2}} - \frac{c\sqrt{a+cx^4}}{16ax^4} - \frac{\sqrt{a+cx^4}}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^4]/x^9,x]
```

```
[Out] -Sqrt[a + c*x^4]/(8*x^8) - (c*Sqrt[a + c*x^4])/(16*a*x^4) + (c^2*ArcTanh[Sq
rt[a + c*x^4]/Sqrt[a]])/(16*a^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+cx^4}}{x^9}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{\sqrt{a+cx}}{x^3}\,dx, x, x^4\right) \\
&= -\frac{\sqrt{a+cx^4}}{8x^8} + \frac{1}{16}c\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+cx}}\,dx, x, x^4\right) \\
&= -\frac{\sqrt{a+cx^4}}{8x^8} - \frac{c\sqrt{a+cx^4}}{16ax^4} - \frac{c^2\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^4\right)}{32a} \\
&= -\frac{\sqrt{a+cx^4}}{8x^8} - \frac{c\sqrt{a+cx^4}}{16ax^4} - \frac{c\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^4}\right)}{16a} \\
&= -\frac{\sqrt{a+cx^4}}{8x^8} - \frac{c\sqrt{a+cx^4}}{16ax^4} + \frac{c^2\tanh^{-1}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{16a^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.55

$$-\frac{c^2\left(a+cx^4\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{cx^4}{a}+1\right)}{6a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^4]/x^9,x]
```

```
[Out] -1/6*(c^2*(a + c*x^4)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + (c*x^4)/a])/
a^3
```

fricas [A] time = 0.56, size = 133, normalized size = 1.87

$$\left[\frac{\sqrt{a}\,c^2x^8\log\left(\frac{cx^4+2\sqrt{cx^4+a}\sqrt{a}+2a}{x^4}\right)-2\left(acx^4+2a^2\right)\sqrt{cx^4+a}}{32a^2x^8}, -\frac{\sqrt{-a}\,c^2x^8\arctan\left(\frac{\sqrt{cx^4+a}\sqrt{-a}}{a}\right)+\left(acx^4+2a^2\right)\sqrt{cx}}{16a^2x^8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/x^9,x, algorithm="fricas")
```

```
[Out] [1/32*(sqrt(a)*c^2*x^8*log((c*x^4 + 2*sqrt(c*x^4 + a)*sqrt(a) + 2*a)/x^4) -
 2*(a*c*x^4 + 2*a^2)*sqrt(c*x^4 + a))/(a^2*x^8), -1/16*(sqrt(-a)*c^2*x^8*ar
ctan(sqrt(c*x^4 + a)*sqrt(-a)/a) + (a*c*x^4 + 2*a^2)*sqrt(c*x^4 + a))/(a^2*
x^8)]
```

giac [A] time = 0.16, size = 72, normalized size = 1.01

$$-\frac{\frac{c^3\arctan\left(\frac{\sqrt{cx^4+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{\left(cx^4+a\right)^{\frac{3}{2}}c^3+\sqrt{cx^4+a}\,ac^3}{ac^2x^8}}{16\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] -1/16*(c^3*arctan(sqrt(c*x^4 + a)/sqrt(-a))/(sqrt(-a)*a) + ((c*x^4 + a)^(3/
2)*c^3 + sqrt(c*x^4 + a)*a*c^3)/(a*c^2*x^8))/c
```

maple [A] time = 0.01, size = 85, normalized size = 1.20

$$\frac{c^2 \ln\left(\frac{2a+2\sqrt{c\,x^4+a}\,\sqrt{a}}{x^2}\right)}{16a^{\frac{3}{2}}} - \frac{\sqrt{c\,x^4+a}\,c^2}{16a^2} + \frac{\left(c\,x^4+a\right)^{\frac{3}{2}}c}{16a^2x^4} - \frac{\left(c\,x^4+a\right)^{\frac{3}{2}}}{8a\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^(1/2)/x^9,x)
```

```
[Out] -1/8/a/x^8*(c*x^4+a)^(3/2)+1/16*c/a^2/x^4*(c*x^4+a)^(3/2)+1/16*c^2/a^(3/2)*
ln((2*a+2*(c*x^4+a)^(1/2)*a^(1/2))/x^2)-1/16*c^2/a^2*(c*x^4+a)^(1/2)
```

maxima [A] time = 2.97, size = 100, normalized size = 1.41

$$-\frac{c^2 \log\left(\frac{\sqrt{cx^4+a}-\sqrt{a}}{\sqrt{cx^4+a}+\sqrt{a}}\right)}{32\,a^{\frac{3}{2}}} - \frac{\left(cx^4+a\right)^{\frac{3}{2}}c^2 + \sqrt{cx^4+a}\,ac^2}{16\left(\left(cx^4+a\right)^2a - 2\left(cx^4+a\right)a^2 + a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/x^9,x, algorithm="maxima")
```

```
[Out] -1/32*c^2*log((sqrt(c*x^4 + a) - sqrt(a))/(sqrt(c*x^4 + a) + sqrt(a)))/a^(3
/2) - 1/16*((c*x^4 + a)^(3/2)*c^2 + sqrt(c*x^4 + a)*a*c^2)/((c*x^4 + a)^2*a
 - 2*(c*x^4 + a)*a^2 + a^3)
```

mupad [B] time = 1.44, size = 54, normalized size = 0.76

$$\frac{c^2\,\operatorname{atanh}\left(\frac{\sqrt{c\,x^4+a}}{\sqrt{a}}\right)}{16\,a^{3/2}} - \frac{\sqrt{c\,x^4+a}}{16\,x^8} - \frac{\left(c\,x^4+a\right)^{3/2}}{16\,a\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^(1/2)/x^9,x)
```

```
[Out] (c^2*atanh((a + c*x^4)^(1/2)/a^(1/2)))/(16*a^(3/2)) - (a + c*x^4)^(1/2)/(16
*x^8) - (a + c*x^4)^(3/2)/(16*a*x^8)
```

sympy [A] time = 5.09, size = 95, normalized size = 1.34

$$-\frac{a}{8\sqrt{c}\,x^{10}\sqrt{\frac{a}{cx^4}+1}} - \frac{3\sqrt{c}}{16x^6\sqrt{\frac{a}{cx^4}+1}} - \frac{c^{\frac{3}{2}}}{16ax^2\sqrt{\frac{a}{cx^4}+1}} + \frac{c^2\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{c}\,x^2}\right)}{16a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**(1/2)/x**9,x)
```

```
[Out] -a/(8*sqrt(c)*x**10*sqrt(a/(c*x**4) + 1)) - 3*sqrt(c)/(16*x**6*sqrt(a/(c*x*
*4) + 1)) - c**(3/2)/(16*a*x**2*sqrt(a/(c*x**4) + 1)) + c**2*asinh(sqrt(a)/
(sqrt(c)*x**2))/(16*a**(3/2))
```

3.770 $\int x^5\sqrt{a+cx^4}\,dx$

Optimal. Leaf size=74

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{16c^{3/2}}+\frac{1}{8}x^6\sqrt{a+cx^4}+\frac{ax^2\sqrt{a+cx^4}}{16c}$$

```
[Out] -1/16*a^2*arctanh(x^2*c^(1/2)/(c*x^4+a)^(1/2))/c^(3/2)+1/16*a*x^2*(c*x^4+a)
^(1/2)/c+1/8*x^6*(c*x^4+a)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {275, 279, 321, 217, 206}

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{16c^{3/2}}+\frac{1}{8}x^6\sqrt{a+cx^4}+\frac{ax^2\sqrt{a+cx^4}}{16c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[a + c*x^4],x]
```

```
[Out] (a*x^2*Sqrt[a + c*x^4])/(16*c) + (x^6*Sqrt[a + c*x^4])/8 - (a^2*ArcTanh[(Sq
rt[c]*x^2)/Sqrt[a + c*x^4]])/(16*c^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int x^5\sqrt{a+cx^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^2\sqrt{a+cx^2}\,dx, x, x^2\right)\\ &= \frac{1}{8}x^6\sqrt{a+cx^4} + \frac{1}{8}a\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{a+cx^2}}\,dx, x, x^2\right)\\ &= \frac{ax^2\sqrt{a+cx^4}}{16c} + \frac{1}{8}x^6\sqrt{a+cx^4} - \frac{a^2\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+cx^2}}\,dx, x, x^2\right)}{16c}\\ &= \frac{ax^2\sqrt{a+cx^4}}{16c} + \frac{1}{8}x^6\sqrt{a+cx^4} - \frac{a^2\operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{a+cx^4}}\right)}{16c}\\ &= \frac{ax^2\sqrt{a+cx^4}}{16c} + \frac{1}{8}x^6\sqrt{a+cx^4} - \frac{a^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{16c^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.14, size = 74, normalized size = 1.00

$$\frac{\sqrt{a+cx^4}\left(\sqrt{c}\,x^2\left(a+2cx^4\right) - \frac{a^{3/2}\sinh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{\sqrt{\frac{cx^4}{a}+1}}\right)}{16c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[a + c*x^4],x]
```

```
[Out] (Sqrt[a + c*x^4]*(Sqrt[c]*x^2*(a + 2*c*x^4) - (a^(3/2)*ArcSinh[(Sqrt[c]*x^2
)/Sqrt[a]])/Sqrt[1 + (c*x^4)/a]))/(16*c^(3/2))
```

fricas [A] time = 0.58, size = 127, normalized size = 1.72

$$\left[\frac{a^2\sqrt{c}\,\log\left(-2\,cx^4 + 2\,\sqrt{cx^4+a}\,\sqrt{c}\,x^2 - a\right) + 2\left(2\,c^2x^6 + acx^2\right)\sqrt{cx^4+a}}{32\,c^2}, \frac{a^2\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x^2}{\sqrt{cx^4+a}}\right) + \left(2\,c^2x^6 +\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*(a^2*sqrt(c)*log(-2*c*x^4 + 2*sqrt(c*x^4 + a)*sqrt(c)*x^2 - a) + 2*(2
*c^2*x^6 + a*c*x^2)*sqrt(c*x^4 + a))/c^2, 1/16*(a^2*sqrt(-c)*arctan(sqrt(-c
)*x^2/sqrt(c*x^4 + a)) + (2*c^2*x^6 + a*c*x^2)*sqrt(c*x^4 + a))/c^2]
```

giac [A] time = 0.18, size = 54, normalized size = 0.73

$$\frac{1}{16}\sqrt{cx^4+a}\left(2\,x^4 + \frac{a}{c}\right)x^2 + \frac{a^2\log\left(\left|-\sqrt{c}\,x^2 + \sqrt{cx^4+a}\right|\right)}{16\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(c*x^4 + a)*(2*x^4 + a/c)*x^2 + 1/16*a^2*log(abs(-sqrt(c)*x^2 + sq
rt(c*x^4 + a)))/c^(3/2)
```

maple [A] time = 0.01, size = 63, normalized size = 0.85

$$-\frac{\sqrt{c\,x^4+a}\;a\,x^2}{16c} - \frac{a^2\ln\left(\sqrt{c}\;x^2 + \sqrt{c\,x^4+a}\right)}{16c^{\frac{3}{2}}} + \frac{\left(c\,x^4+a\right)^{\frac{3}{2}}x^2}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*x^4+a)^(1/2),x)
```

```
[Out] 1/8*x^2*(c*x^4+a)^(3/2)/c-1/16*a*x^2*(c*x^4+a)^(1/2)/c-1/16*a^2/c^(3/2)*ln(
x^2*c^(1/2)+(c*x^4+a)^(1/2))
```

maxima [B] time = 2.99, size = 120, normalized size = 1.62

$$\frac{a^2 \log\left(-\frac{\sqrt{c}-\frac{\sqrt{cx^4+a}}{x^2}}{\sqrt{c}+\frac{\sqrt{cx^4+a}}{x^2}}\right)}{32\,c^{\frac{3}{2}}}+\frac{\frac{\sqrt{cx^4+a}\,a^2c}{x^2}+\frac{\left(cx^4+a\right)^{\frac{3}{2}}a^2}{x^6}}{16\left(c^3-\frac{2\left(cx^4+a\right)c^2}{x^4}+\frac{\left(cx^4+a\right)^2c}{x^8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/32*a^2*log(-(sqrt(c) - sqrt(c*x^4 + a)/x^2)/(sqrt(c) + sqrt(c*x^4 + a)/x^
2))/c^(3/2) + 1/16*(sqrt(c*x^4 + a)*a^2*c/x^2 + (c*x^4 + a)^(3/2)*a^2/x^6)/
(c^3 - 2*(c*x^4 + a)*c^2/x^4 + (c*x^4 + a)^2*c/x^8)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5\,\sqrt{c\,x^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + c*x^4)^(1/2),x)
```

```
[Out] int(x^5*(a + c*x^4)^(1/2), x)
```

sympy [A] time = 5.17, size = 95, normalized size = 1.28

$$\frac{a^{\frac{3}{2}}x^2}{16c\sqrt{1+\frac{cx^4}{a}}}+\frac{3\sqrt{a}\,x^6}{16\sqrt{1+\frac{cx^4}{a}}}-\frac{a^2\,\operatorname{asinh}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{16c^{\frac{3}{2}}}+\frac{cx^{10}}{8\sqrt{a}\,\sqrt{1+\frac{cx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*x**4+a)**(1/2),x)
```

```
[Out] a**(3/2)*x**2/(16*c*sqrt(1 + c*x**4/a)) + 3*sqrt(a)*x**6/(16*sqrt(1 + c*x**
4/a)) - a**2*asinh(sqrt(c)*x**2/sqrt(a))/(16*c**(3/2)) + c*x**10/(8*sqrt(a)
*sqrt(1 + c*x**4/a))
```

3.771 $\int x\sqrt{a+cx^4}\,dx$

Optimal. Leaf size=50

$$\frac{1}{4}x^2\sqrt{a+cx^4}+\frac{a\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{4\sqrt{c}}$$

```
[Out] 1/4*a*arctanh(x^2*c^(1/2)/(c*x^4+a)^(1/2))/c^(1/2)+1/4*x^2*(c*x^4+a)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {275, 195, 217, 206}

$$\frac{1}{4}x^2\sqrt{a+cx^4}+\frac{a\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + c*x^4],x]
```

```
[Out] (x^2*Sqrt[a + c*x^4])/4 + (a*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(4*Sqr
t[c])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{a+cx^4}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \sqrt{a+cx^2}\,dx, x, x^2\right)\\ &= \frac{1}{4}x^2\sqrt{a+cx^4} + \frac{1}{4}a\,\text{Subst}\left(\int \frac{1}{\sqrt{a+cx^2}}\,dx, x, x^2\right)\\ &= \frac{1}{4}x^2\sqrt{a+cx^4} + \frac{1}{4}a\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{a+cx^4}}\right)\\ &= \frac{1}{4}x^2\sqrt{a+cx^4} + \frac{a\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{4\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 72, normalized size = 1.44

$$\frac{a^{3/2}\sqrt{\frac{cx^4}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)+\sqrt{c}\,x^2\left(a+cx^4\right)}{4\sqrt{c}\,\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + c*x^4],x]
```

```
[Out] (Sqrt[c]*x^2*(a + c*x^4) + a^(3/2)*Sqrt[1 + (c*x^4)/a]*ArcSinh[(Sqrt[c]*x^2
)/Sqrt[a]])/(4*Sqrt[c]*Sqrt[a + c*x^4])
```

fricas [A] time = 0.78, size = 102, normalized size = 2.04

$$\left[\frac{2\sqrt{cx^4+a}\,cx^2+a\sqrt{c}\,\log\left(-2\,cx^4-2\sqrt{cx^4+a}\,\sqrt{c}\,x^2-a\right)}{8\,c}, \frac{\sqrt{cx^4+a}\,cx^2-a\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x^2}{\sqrt{cx^4+a}}\right)}{4\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(c*x^4 + a)*c*x^2 + a*sqrt(c)*log(-2*c*x^4 - 2*sqrt(c*x^4 + a)*
sqrt(c)*x^2 - a))/c, 1/4*(sqrt(c*x^4 + a)*c*x^2 - a*sqrt(-c)*arctan(sqrt(-c
)*x^2/sqrt(c*x^4 + a)))/c]
```

giac [A] time = 0.19, size = 41, normalized size = 0.82

$$\frac{1}{4}\sqrt{cx^4+a}\,x^2 - \frac{a\log\left(\left|-\sqrt{c}\,x^2+\sqrt{cx^4+a}\right|\right)}{4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(c*x^4 + a)*x^2 - 1/4*a*log(abs(-sqrt(c)*x^2 + sqrt(c*x^4 + a)))/sq
rt(c)
```

maple [A] time = 0.01, size = 40, normalized size = 0.80

$$\frac{\sqrt{c\,x^4+a}\;x^2}{4} + \frac{a\ln\left(\sqrt{c}\;x^2+\sqrt{c\,x^4+a}\right)}{4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+a)^(1/2),x)
```

```
[Out] 1/4*x^2*(c*x^4+a)^(1/2)+1/4*a/c^(1/2)*ln(c^(1/2)*x^2+(c*x^4+a)^(1/2))
```

maxima [B] time = 3.03, size = 78, normalized size = 1.56

$$-\frac{a\log\left(-\frac{\sqrt{c}-\frac{\sqrt{cx^4+a}}{x^2}}{\sqrt{c}+\frac{\sqrt{cx^4+a}}{x^2}}\right)}{8\sqrt{c}}-\frac{\sqrt{cx^4+a}\,a}{4\left(c-\frac{cx^4+a}{x^4}\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*a*log(-(sqrt(c) - sqrt(c*x^4 + a)/x^2)/(sqrt(c) + sqrt(c*x^4 + a)/x^2)
)/sqrt(c) - 1/4*sqrt(c*x^4 + a)*a/((c - (c*x^4 + a)/x^4)*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\sqrt{c\,x^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + c*x^4)^(1/2),x)
```

```
[Out] int(x*(a + c*x^4)^(1/2), x)
```

sympy [A] time = 3.53, size = 44, normalized size = 0.88

$$\frac{\sqrt{a}\,x^2\sqrt{1+\frac{cx^4}{a}}}{4}+\frac{a\,\operatorname{asinh}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*x**2*sqrt(1 + c*x**4/a)/4 + a*asinh(sqrt(c)*x**2/sqrt(a))/(4*sqrt(c
))
```

3.772 $\int \frac{\sqrt{a+cx^4}}{x^3}\,dx$

Optimal. Leaf size=49

$$\frac{1}{2}\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)-\frac{\sqrt{a+cx^4}}{2x^2}$$

```
[Out] 1/2*arctanh(x^2*c^(1/2)/(c*x^4+a)^(1/2))*c^(1/2)-1/2*(c*x^4+a)^(1/2)/x^2
```

Rubi [A] time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {275, 277, 217, 206}

$$\frac{1}{2}\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)-\frac{\sqrt{a+cx^4}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^4]/x^3,x]
```

```
[Out] -Sqrt[a + c*x^4]/(2*x^2) + (Sqrt[c]*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])
/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+cx^4}}{x^3}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{a+cx^2}}{x^2}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{a+cx^4}}{2x^2} + \frac{1}{2}c\,\text{Subst}\left(\int \frac{1}{\sqrt{a+cx^2}}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{a+cx^4}}{2x^2} + \frac{1}{2}c\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{a+cx^4}}\right)\\ &= -\frac{\sqrt{a+cx^4}}{2x^2} + \frac{1}{2}\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)\end{aligned}$$

Mathematica [A] time = 0.11, size = 69, normalized size = 1.41

$$-\frac{-\sqrt{a}\,\sqrt{c}\,x^2\sqrt{\frac{cx^4}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)+a+cx^4}{2x^2\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^4]/x^3,x]
```

```
[Out] -1/2*(a + c*x^4 - Sqrt[a]*Sqrt[c]*x^2*Sqrt[1 + (c*x^4)/a]*ArcSinh[(Sqrt[c]*
x^2)/Sqrt[a]])/(x^2*Sqrt[a + c*x^4])
```

fricas [A] time = 0.53, size = 96, normalized size = 1.96

$$\left[\frac{\sqrt{c}\,x^2\log\left(-2\,cx^4-2\,\sqrt{cx^4+a}\,\sqrt{c}\,x^2-a\right)-2\,\sqrt{cx^4+a}}{4\,x^2}, -\frac{\sqrt{-c}\,x^2\arctan\left(\frac{\sqrt{-c}\,x^2}{\sqrt{cx^4+a}}\right)+\sqrt{cx^4+a}}{2\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(c)*x^2*log(-2*c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(c)*x^2 - a) - 2*sqr
t(c*x^4 + a))/x^2, -1/2*(sqrt(-c)*x^2*arctan(sqrt(-c)*x^2/sqrt(c*x^4 + a))
+ sqrt(c*x^4 + a))/x^2]
```

giac [A] time = 0.21, size = 60, normalized size = 1.22

$$-\frac{1}{4}\sqrt{c}\,\log\left(\left(\sqrt{c}\,x^2-\sqrt{cx^4+a}\right)^2\right)+\frac{a\sqrt{c}}{\left(\sqrt{c}\,x^2-\sqrt{cx^4+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(c)*log((sqrt(c)*x^2 - sqrt(c*x^4 + a))^2) + a*sqrt(c)/((sqrt(c)*x
^2 - sqrt(c*x^4 + a))^2 - a)
```

maple [A] time = 0.01, size = 60, normalized size = 1.22

$$\frac{\sqrt{c\,x^4+a}\;c\,x^2}{2a}+\frac{\sqrt{c}\;\ln\left(\sqrt{c}\;x^2+\sqrt{c\,x^4+a}\right)}{2}-\frac{\left(c\,x^4+a\right)^{\frac{3}{2}}}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^(1/2)/x^3,x)
```

```
[Out] -1/2/a/x^2*(c*x^4+a)^(3/2)+1/2*c/a*x^2*(c*x^4+a)^(1/2)+1/2*c^(1/2)*ln(c^(1/2)*x^2+(c*x^4+a)^(1/2))
```

maxima [A] time = 2.93, size = 60, normalized size = 1.22

$$-\frac{1}{4}\sqrt{c}\,\log\left(-\frac{\sqrt{c}-\frac{\sqrt{cx^4+a}}{x^2}}{\sqrt{c}+\frac{\sqrt{cx^4+a}}{x^2}}\right)-\frac{\sqrt{cx^4+a}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(c)*log(-(sqrt(c) - sqrt(c*x^4 + a)/x^2)/(sqrt(c) + sqrt(c*x^4 + a)/x^2)) - 1/2*sqrt(c*x^4 + a)/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c\,x^4+a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^(1/2)/x^3,x)
```

```
[Out] int((a + c*x^4)^(1/2)/x^3, x)
```

sympy [A] time = 3.18, size = 66, normalized size = 1.35

$$-\frac{\sqrt{a}}{2x^2\sqrt{1+\frac{cx^4}{a}}}+\frac{\sqrt{c}\,\operatorname{asinh}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2}-\frac{cx^2}{2\sqrt{a}\,\sqrt{1+\frac{cx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**(1/2)/x**3,x)
```

```
[Out] -sqrt(a)/(2*x**2*sqrt(1 + c*x**4/a)) + sqrt(c)*asinh(sqrt(c)*x**2/sqrt(a))/2 - c*x**2/(2*sqrt(a)*sqrt(1 + c*x**4/a))
```

3.773 $\int \frac{\sqrt{a+cx^4}}{x^7}\,dx$

Optimal. Leaf size=21

$$-\frac{\left(a+cx^4\right)^{3/2}}{6ax^6}$$

```
[Out] -1/6*(c*x^4+a)^(3/2)/a/x^6
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {264}

$$-\frac{\left(a+cx^4\right)^{3/2}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^4]/x^7,x]
```

```
[Out] -(a + c*x^4)^(3/2)/(6*a*x^6)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^4}}{x^7}\,dx = -\frac{\left(a+cx^4\right)^{3/2}}{6ax^6}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$-\frac{\left(a+cx^4\right)^{3/2}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^4]/x^7,x]
```

```
[Out] -1/6*(a + c*x^4)^(3/2)/(a*x^6)
```

fricas [A] time = 0.58, size = 17, normalized size = 0.81

$$-\frac{\left(cx^4+a\right)^{\frac{3}{2}}}{6\,ax^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] -1/6*(c*x^4 + a)^(3/2)/(a*x^6)
```

giac [B] time = 0.18, size = 63, normalized size = 3.00

$$\frac{3\left(\sqrt{c}\,x^2-\sqrt{cx^4+a}\right)^4 c^{\frac{3}{2}}+a^2c^{\frac{3}{2}}}{3\left(\left(\sqrt{c}\,x^2-\sqrt{cx^4+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/3*(3*(sqrt(c)*x^2 - sqrt(c*x^4 + a))^4*c^(3/2) + a^2*c^(3/2))/((sqrt(c)*x
^2 - sqrt(c*x^4 + a))^2 - a)^3
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$-\frac{\left(c\,x^4+a\right)^{\frac{3}{2}}}{6a\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^(1/2)/x^7,x)
```

```
[Out] -1/6*(c*x^4+a)^(3/2)/x^6/a
```

maxima [A] time = 1.36, size = 17, normalized size = 0.81

$$-\frac{\left(cx^4+a\right)^{\frac{3}{2}}}{6\,ax^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/6*(c*x^4 + a)^(3/2)/(a*x^6)
```

mupad [B] time = 1.22, size = 17, normalized size = 0.81

$$-\frac{\left(c\,x^4+a\right)^{3/2}}{6\,a\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^(1/2)/x^7,x)
```

```
[Out] -(a + c*x^4)^(3/2)/(6*a*x^6)
```

sympy [B] time = 1.63, size = 42, normalized size = 2.00

$$-\frac{\sqrt{c}\sqrt{\frac{a}{cx^4}+1}}{6x^4}-\frac{c^{\frac{3}{2}}\sqrt{\frac{a}{cx^4}+1}}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**(1/2)/x**7,x)
```

```
[Out] -sqrt(c)*sqrt(a/(c*x**4) + 1)/(6*x**4) - c**(3/2)*sqrt(a/(c*x**4) + 1)/(6*a
)
```

3.774 $\int \frac{\sqrt{a+cx^4}}{x^{11}}\,dx$

Optimal. Leaf size=44

$$\frac{c\left(a+cx^4\right)^{3/2}}{15a^2x^6}-\frac{\left(a+cx^4\right)^{3/2}}{10ax^{10}}$$

```
[Out] -1/10*(c*x^4+a)^(3/2)/a/x^10+1/15*c*(c*x^4+a)^(3/2)/a^2/x^6
```

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$\frac{c\left(a+cx^4\right)^{3/2}}{15a^2x^6}-\frac{\left(a+cx^4\right)^{3/2}}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^4]/x^11,x]
```

```
[Out] -(a + c*x^4)^(3/2)/(10*a*x^10) + (c*(a + c*x^4)^(3/2))/(15*a^2*x^6)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+cx^4}}{x^{11}}\,dx &= -\frac{\left(a+cx^4\right)^{3/2}}{10ax^{10}}-\frac{(2c)\int \frac{\sqrt{a+cx^4}}{x^7}\,dx}{5a}\\ &= -\frac{\left(a+cx^4\right)^{3/2}}{10ax^{10}}+\frac{c\left(a+cx^4\right)^{3/2}}{15a^2x^6}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.70

$$\frac{\left(a+cx^4\right)^{3/2}\left(2cx^4-3a\right)}{30a^2x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^4]/x^11,x]
```

```
[Out] ((a + c*x^4)^(3/2)*(-3*a + 2*c*x^4))/(30*a^2*x^10)
```

fricas [A] time = 0.65, size = 38, normalized size = 0.86

$$\frac{\left(2\,c^2x^8-acx^4-3\,a^2\right)\sqrt{cx^4+a}}{30\,a^2x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/x^11,x, algorithm="fricas")
```

```
[Out] 1/30*(2*c^2*x^8 - a*c*x^4 - 3*a^2)*sqrt(c*x^4 + a)/(a^2*x^10)
```

giac [B] time = 0.21, size = 120, normalized size = 2.73

$$\frac{2\left(15\left(\sqrt{c}\,x^2-\sqrt{cx^4+a}\right)^6 c^{\frac{5}{2}}+5\left(\sqrt{c}\,x^2-\sqrt{cx^4+a}\right)^4 ac^{\frac{5}{2}}+5\left(\sqrt{c}\,x^2-\sqrt{cx^4+a}\right)^2 a^2c^{\frac{5}{2}}-a^3c^{\frac{5}{2}}\right)}{15\left(\left(\sqrt{c}\,x^2-\sqrt{cx^4+a}\right)^2-a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/x^11,x, algorithm="giac")
```

```
[Out] 2/15*(15*(sqrt(c)*x^2 - sqrt(c*x^4 + a))^6*c^(5/2) + 5*(sqrt(c)*x^2 - sqrt(
c*x^4 + a))^4*a*c^(5/2) + 5*(sqrt(c)*x^2 - sqrt(c*x^4 + a))^2*a^2*c^(5/2) -
 a^3*c^(5/2))/((sqrt(c)*x^2 - sqrt(c*x^4 + a))^2 - a)^5
```

maple [A] time = 0.00, size = 28, normalized size = 0.64

$$-\frac{\left(cx^4+a\right)^{\frac{3}{2}}\left(-2cx^4+3a\right)}{30a^2x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^(1/2)/x^11,x)
```

```
[Out] -1/30*(c*x^4+a)^(3/2)*(-2*c*x^4+3*a)/x^10/a^2
```

maxima [A] time = 1.36, size = 35, normalized size = 0.80

$$\frac{\frac{5\left(cx^4+a\right)^{\frac{3}{2}}c}{x^6}-\frac{3\left(cx^4+a\right)^{\frac{5}{2}}}{x^{10}}}{30\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/x^11,x, algorithm="maxima")
```

```
[Out] 1/30*(5*(c*x^4 + a)^(3/2)*c/x^6 - 3*(c*x^4 + a)^(5/2)/x^10)/a^2
```

mupad [B] time = 1.32, size = 37, normalized size = 0.84

$$-\frac{\sqrt{c\,x^4+a}\,\left(3\,a^2+a\,c\,x^4-2\,c^2\,x^8\right)}{30\,a^2\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^(1/2)/x^11,x)
```

```
[Out] -((a + c*x^4)^(1/2)*(3*a^2 - 2*c^2*x^8 + a*c*x^4))/(30*a^2*x^10)
```

sympy [A] time = 2.35, size = 66, normalized size = 1.50

$$-\frac{\sqrt{c}\sqrt{\frac{a}{cx^4}+1}}{10x^8}-\frac{c^{\frac{3}{2}}\sqrt{\frac{a}{cx^4}+1}}{30ax^4}+\frac{c^{\frac{5}{2}}\sqrt{\frac{a}{cx^4}+1}}{15a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**(1/2)/x**11,x)
```

```
[Out] -sqrt(c)*sqrt(a/(c*x**4) + 1)/(10*x**8) - c**(3/2)*sqrt(a/(c*x**4) + 1)/(30
*a*x**4) + c**(5/2)*sqrt(a/(c*x**4) + 1)/(15*a**2)
```

3.775 $\int \frac{\sqrt{a+cx^4}}{x^{15}}\,dx$

Optimal. Leaf size=68

$$-\frac{4c^2\left(a+cx^4\right)^{3/2}}{105a^3x^6}+\frac{2c\left(a+cx^4\right)^{3/2}}{35a^2x^{10}}-\frac{\left(a+cx^4\right)^{3/2}}{14ax^{14}}$$

```
[Out] -1/14*(c*x^4+a)^(3/2)/a/x^14+2/35*c*(c*x^4+a)^(3/2)/a^2/x^10-4/105*c^2*(c*x
^4+a)^(3/2)/a^3/x^6
```

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$-\frac{4c^2\left(a+cx^4\right)^{3/2}}{105a^3x^6}+\frac{2c\left(a+cx^4\right)^{3/2}}{35a^2x^{10}}-\frac{\left(a+cx^4\right)^{3/2}}{14ax^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^4]/x^15,x]
```

```
[Out] -(a + c*x^4)^(3/2)/(14*a*x^14) + (2*c*(a + c*x^4)^(3/2))/(35*a^2*x^10) - (4
*c^2*(a + c*x^4)^(3/2))/(105*a^3*x^6)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+cx^4}}{x^{15}}\,dx &= -\frac{\left(a+cx^4\right)^{3/2}}{14ax^{14}}-\frac{(4c)\int \frac{\sqrt{a+cx^4}}{x^{11}}\,dx}{7a}\\ &= -\frac{\left(a+cx^4\right)^{3/2}}{14ax^{14}}+\frac{2c\left(a+cx^4\right)^{3/2}}{35a^2x^{10}}+\frac{\left(8c^2\right)\int \frac{\sqrt{a+cx^4}}{x^7}\,dx}{35a^2}\\ &= -\frac{\left(a+cx^4\right)^{3/2}}{14ax^{14}}+\frac{2c\left(a+cx^4\right)^{3/2}}{35a^2x^{10}}-\frac{4c^2\left(a+cx^4\right)^{3/2}}{105a^3x^6}\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.62

$$-\frac{\left(a+cx^4\right)^{3/2}\left(15a^2-12acx^4+8c^2x^8\right)}{210a^3x^{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^4]/x^15,x]
```

```
[Out] -1/210*((a + c*x^4)^(3/2)*(15*a^2 - 12*a*c*x^4 + 8*c^2*x^8))/(a^3*x^14)
```

fricas [A] time = 0.63, size = 49, normalized size = 0.72

$$-\frac{\left(8\,c^3x^{12}-4\,ac^2x^8+3\,a^2cx^4+15\,a^3\right)\sqrt{cx^4+a}}{210\,a^3x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/x^15,x, algorithm="fricas")
```

```
[Out] -1/210*(8*c^3*x^12 - 4*a*c^2*x^8 + 3*a^2*c*x^4 + 15*a^3)*sqrt(c*x^4 + a)/(a
^3*x^14)
```

giac [B] time = 0.20, size = 148, normalized size = 2.18

$$\frac{8\left(70\left(\sqrt{c}\,x^2-\sqrt{cx^4+a}\right)^8c^{\frac{7}{2}}+35\left(\sqrt{c}\,x^2-\sqrt{cx^4+a}\right)^6ac^{\frac{7}{2}}+21\left(\sqrt{c}\,x^2-\sqrt{cx^4+a}\right)^4a^2c^{\frac{7}{2}}-7\left(\sqrt{c}\,x^2-\sqrt{cx^4+a}\right.\right.}{105\left(\left(\sqrt{c}\,x^2-\sqrt{cx^4+a}\right)^2-a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/x^15,x, algorithm="giac")
```

```
[Out] 8/105*(70*(sqrt(c)*x^2 - sqrt(c*x^4 + a))^8*c^(7/2) + 35*(sqrt(c)*x^2 - sqr
t(c*x^4 + a))^6*a*c^(7/2) + 21*(sqrt(c)*x^2 - sqrt(c*x^4 + a))^4*a^2*c^(7/2
) - 7*(sqrt(c)*x^2 - sqrt(c*x^4 + a))^2*a^3*c^(7/2) + a^4*c^(7/2))/((sqrt(c
)*x^2 - sqrt(c*x^4 + a))^2 - a)^7
```

maple [A] time = 0.00, size = 39, normalized size = 0.57

$$-\frac{\left(c\,x^4+a\right)^{\frac{3}{2}}\left(8c^2x^8-12ac\,x^4+15a^2\right)}{210a^3x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^(1/2)/x^15,x)
```

```
[Out] -1/210*(c*x^4+a)^(3/2)*(8*c^2*x^8-12*a*c*x^4+15*a^2)/x^14/a^3
```

maxima [A] time = 1.40, size = 52, normalized size = 0.76

$$-\frac{\frac{35\left(cx^4+a\right)^{\frac{3}{2}}c^2}{x^6}-\frac{42\left(cx^4+a\right)^{\frac{5}{2}}c}{x^{10}}+\frac{15\left(cx^4+a\right)^{\frac{7}{2}}}{x^{14}}}{210\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/x^15,x, algorithm="maxima")
```

```
[Out] -1/210*(35*(c*x^4 + a)^(3/2)*c^2/x^6 - 42*(c*x^4 + a)^(5/2)*c/x^10 + 15*(c*
x^4 + a)^(7/2)/x^14)/a^3
```

mupad [B] time = 1.51, size = 73, normalized size = 1.07

$$\frac{2\,c^2\sqrt{c\,x^4+a}}{105\,a^2\,x^6}-\frac{c\sqrt{c\,x^4+a}}{70\,a\,x^{10}}-\frac{4\,c^3\sqrt{c\,x^4+a}}{105\,a^3\,x^2}-\frac{\sqrt{c\,x^4+a}}{14\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^(1/2)/x^15,x)
```

```
[Out] (2*c^2*(a + c*x^4)^(1/2))/(105*a^2*x^6) - (c*(a + c*x^4)^(1/2))/(70*a*x^10)
 - (4*c^3*(a + c*x^4)^(1/2))/(105*a^3*x^2) - (a + c*x^4)^(1/2)/(14*x^14)
```

sympy [B] time = 3.43, size = 359, normalized size = 5.28

$$-\frac{15a^5c^{\frac{9}{2}}\sqrt{\frac{a}{cx^4}+1}}{210a^5c^4x^{12}+420a^4c^5x^{16}+210a^3c^6x^{20}}-\frac{33a^4c^{\frac{11}{2}}x^4\sqrt{\frac{a}{cx^4}+1}}{210a^5c^4x^{12}+420a^4c^5x^{16}+210a^3c^6x^{20}}-\frac{17a^3c^{\frac{13}{2}}x^8\sqrt{\frac{a}{cx^4}+}}{210a^5c^4x^{12}+420a^4c^5x^{16}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**(1/2)/x**15,x)
```

```
[Out] -15*a**5*c**(9/2)*sqrt(a/(c*x**4) + 1)/(210*a**5*c**4*x**12 + 420*a**4*c**5
*x**16 + 210*a**3*c**6*x**20) - 33*a**4*c**(11/2)*x**4*sqrt(a/(c*x**4) + 1)
/(210*a**5*c**4*x**12 + 420*a**4*c**5*x**16 + 210*a**3*c**6*x**20) - 17*a**
3*c**(13/2)*x**8*sqrt(a/(c*x**4) + 1)/(210*a**5*c**4*x**12 + 420*a**4*c**5*
x**16 + 210*a**3*c**6*x**20) - 3*a**2*c**(15/2)*x**12*sqrt(a/(c*x**4) + 1)/
(210*a**5*c**4*x**12 + 420*a**4*c**5*x**16 + 210*a**3*c**6*x**20) - 12*a*c*
*(17/2)*x**16*sqrt(a/(c*x**4) + 1)/(210*a**5*c**4*x**12 + 420*a**4*c**5*x**
16 + 210*a**3*c**6*x**20) - 8*c**(19/2)*x**20*sqrt(a/(c*x**4) + 1)/(210*a**
5*c**4*x**12 + 420*a**4*c**5*x**16 + 210*a**3*c**6*x**20)
```

3.776 $\int x^4\sqrt{a+cx^4}\,dx$

Optimal. Leaf size=127

$$-\frac{a^{7/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{21c^{5/4}\sqrt{a+cx^4}}+\frac{2ax\sqrt{a+cx^4}}{21c}+\frac{1}{7}x^5\sqrt{a+cx^4}$$

```
[Out] 2/21*a*x*(c*x^4+a)^(1/2)/c+1/7*x^5*(c*x^4+a)^(1/2)-1/21*a^(7/4)*(cos(2*arct
an(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(
sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4
+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(5/4)/(c*x^4+a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {279, 321, 220}

$$-\frac{a^{7/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{21c^{5/4}\sqrt{a+cx^4}}+\frac{1}{7}x^5\sqrt{a+cx^4}+\frac{2ax\sqrt{a+cx^4}}{21c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[a + c*x^4],x]
```

```
[Out] (2*a*x*Sqrt[a + c*x^4])/(21*c) + (x^5*Sqrt[a + c*x^4])/7 - (a^(7/4)*(Sqrt[a
] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*Ar
cTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(21*c^(5/4)*Sqrt[a + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^4\sqrt{a+cx^4}\,dx = \frac{1}{7}x^5\sqrt{a+cx^4} + \frac{1}{7}(2a)\int \frac{x^4}{\sqrt{a+cx^4}}\,dx$$
$$= \frac{2ax\sqrt{a+cx^4}}{21c} + \frac{1}{7}x^5\sqrt{a+cx^4} - \frac{\left(2a^2\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{21c}$$
$$= \frac{2ax\sqrt{a+cx^4}}{21c} + \frac{1}{7}x^5\sqrt{a+cx^4} - \frac{a^{7/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{21c^{5/4}\sqrt{a+cx^4}}$$

Mathematica [C] time = 0.06, size = 62, normalized size = 0.49

$$\frac{x\sqrt{a+cx^4}\left(-\frac{a\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4};-\frac{cx^4}{a}\right)}{\sqrt{\frac{cx^4}{a}+1}}+a+cx^4\right)}{7c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[a + c*x^4],x]
```

```
[Out] (x*Sqrt[a + c*x^4]*(a + c*x^4 - (a*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*x
^4)/a)])/Sqrt[1 + (c*x^4)/a]))/(7*c)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{cx^4+a}\,x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + a)*x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^4+a}\,x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + a)*x^4, x)
```

maple [C] time = 0.05, size = 108, normalized size = 0.85

$$\frac{\sqrt{c\,x^4+a}\;x^5}{7} - \frac{2\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\;\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\;a^2\,\text{EllipticF}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\;x, i\right)}{21\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\;\sqrt{c\,x^4+a}\;c} + \frac{2\sqrt{c\,x^4+a}\;ax}{21c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c*x^4+a)^(1/2),x)
```

```
[Out] 1/7*x^5*(c*x^4+a)^(1/2)+2/21*a*x*(c*x^4+a)^(1/2)/c-2/21*a^2/c/(I/a^(1/2)*c^
(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2
)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^4 + a}\, x^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + a)*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \sqrt{c\, x^4 + a}\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + c*x^4)^(1/2),x)
```

```
[Out] int(x^4*(a + c*x^4)^(1/2), x)
```

sympy [C] time = 2.03, size = 39, normalized size = 0.31

$$\frac{\sqrt{a}\, x^5 \Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{5}{4} \\ \frac{9}{4} \end{matrix} \middle| \frac{cx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), c*x**4*exp_polar(I*pi)/a
)/(4*gamma(9/4))
```

3.777 $\int \sqrt{a+cx^4}\,dx$

Optimal. Leaf size=105

$$\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{c}\,\sqrt{a+cx^4}}+\frac{1}{3}x\sqrt{a+cx^4}$$

```
[Out] 1/3*x*(c*x^4+a)^(1/2)+1/3*a^(3/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2
)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4)
)),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(
1/2)/c^(1/4)/(c*x^4+a)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {195, 220}

$$\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{c}\,\sqrt{a+cx^4}}+\frac{1}{3}x\sqrt{a+cx^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^4],x]
```

```
[Out] (x*Sqrt[a + c*x^4])/3 + (a^(3/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(
Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(3
*c^(1/4)*Sqrt[a + c*x^4])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+cx^4}\,dx &= \frac{1}{3}x\sqrt{a+cx^4}+\frac{1}{3}(2a)\int \frac{1}{\sqrt{a+cx^4}}\,dx\\ &= \frac{1}{3}x\sqrt{a+cx^4}+\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{c}\,\sqrt{a+cx^4}}\end{aligned}$$

Mathematica [C] time = 0.16, size = 89, normalized size = 0.85

$$\frac{x\left(a+cx^4\right)-\frac{2ia\sqrt{\frac{cx^4}{a}+1}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}}{3\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^4],x]
```

```
[Out] (x*(a + c*x^4) - ((2*I)*a*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*S
qrt[c])/Sqrt[a]]*x], -1])/Sqrt[(I*Sqrt[c])/Sqrt[a]])/(3*Sqrt[a + c*x^4])
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{cx^4+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + a), x)
```

maple [C] time = 0.01, size = 85, normalized size = 0.81

$$\frac{2\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,a\,\text{EllipticF}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x,i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{\sqrt{c\,x^4+a}\,x}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^(1/2),x)
```

```
[Out] 1/3*x*(c*x^4+a)^(1/2)+2/3*a/(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)*x
^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)*EllipticF((I/a^
(1/2)*c^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + a), x)
```

mupad [B] time = 1.05, size = 37, normalized size = 0.35

$$\frac{x\sqrt{c\,x^4+a}\;{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{1}{4};\;\frac{5}{4};\;-\frac{c\,x^4}{a}\right)}{\sqrt{\frac{c\,x^4}{a}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^(1/2),x)
```

```
[Out] (x*(a + c*x^4)^(1/2)*hypergeom([-1/2, 1/4], 5/4, -(c*x^4)/a))/((c*x^4)/a +
1)^(1/2)
```

sympy [C] time = 1.82, size = 37, normalized size = 0.35

$$\frac{\sqrt{a}\,x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(I*pi)/a)/(
4*gamma(5/4))
```

3.778 $\int \frac{\sqrt{a+cx^4}}{x^4}\,dx$

Optimal. Leaf size=107

$$\frac{c^{3/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}\,\sqrt{a+cx^4}}-\frac{\sqrt{a+cx^4}}{3x^3}$$

```
[Out] -1/3*(c*x^4+a)^(1/2)/x^3+1/3*c^(3/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(
1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1
/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2
)^(1/2)/a^(1/4)/(c*x^4+a)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {277, 220}

$$\frac{c^{3/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}\,\sqrt{a+cx^4}}-\frac{\sqrt{a+cx^4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^4]/x^4,x]
```

```
[Out] -Sqrt[a + c*x^4]/(3*x^3) + (c^(3/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4
)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])
/(3*a^(1/4)*Sqrt[a + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+cx^4}}{x^4}\,dx &= -\frac{\sqrt{a+cx^4}}{3x^3}+\frac{1}{3}(2c)\int \frac{1}{\sqrt{a+cx^4}}\,dx\\ &= -\frac{\sqrt{a+cx^4}}{3x^3}+\frac{c^{3/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}\,\sqrt{a+cx^4}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.48

$$-\frac{\sqrt{a+cx^4}\;{}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{4};-\frac{cx^4}{a}\right)}{3x^3\sqrt{\frac{cx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^4]/x^4,x]
```

```
[Out] -1/3*(Sqrt[a + c*x^4]*Hypergeometric2F1[-3/4, -1/2, 1/4, -((c*x^4)/a)])/(x^
3*Sqrt[1 + (c*x^4)/a])
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+a}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + a)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+a}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + a)/x^4, x)
```

maple [C] time = 0.01, size = 87, normalized size = 0.81

$$\frac{2\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,c\,\text{EllipticF}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x, i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}} - \frac{\sqrt{c\,x^4+a}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^(1/2)/x^4,x)
```

```
[Out] -1/3*(c*x^4+a)^(1/2)/x^3+2/3*c/(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2
)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)*EllipticF((I
/a^(1/2)*c^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+a}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + a)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4+a}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^(1/2)/x^4,x)
```

```
[Out] int((a + c*x^4)^(1/2)/x^4, x)
```

sympy [C] time = 1.62, size = 42, normalized size = 0.39

$$\frac{\sqrt{a}\,\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4}, -\frac{1}{2} \\ \frac{1}{4}\end{matrix}\middle| \frac{cx^4 e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**(1/2)/x**4,x)
```

```
[Out] sqrt(a)*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), c*x**4*exp_polar(I*pi)/a)/(
4*x**3*gamma(1/4))
```

3.779 $\int \frac{\sqrt{a+cx^4}}{x^8}\,dx$

Optimal. Leaf size=129

$$-\frac{c^{7/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{21a^{5/4}\sqrt{a+cx^4}}-\frac{\sqrt{a+cx^4}}{7x^7}-\frac{2c\sqrt{a+cx^4}}{21ax^3}$$

```
[Out] -1/7*(c*x^4+a)^(1/2)/x^7-2/21*c*(c*x^4+a)^(1/2)/a/x^3-1/21*c^(7/4)*(cos(2*a
rctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Ellipti
cF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*
x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(5/4)/(c*x^4+a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {277, 325, 220}

$$-\frac{c^{7/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{21a^{5/4}\sqrt{a+cx^4}}-\frac{2c\sqrt{a+cx^4}}{21ax^3}-\frac{\sqrt{a+cx^4}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^4]/x^8,x]
```

```
[Out] -Sqrt[a + c*x^4]/(7*x^7) - (2*c*Sqrt[a + c*x^4])/(21*a*x^3) - (c^(7/4)*(Sqr
t[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2
*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(21*a^(5/4)*Sqrt[a + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^4}}{x^8}\,dx = -\frac{\sqrt{a+cx^4}}{7x^7} + \frac{1}{7}(2c)\int \frac{1}{x^4\sqrt{a+cx^4}}\,dx$$
$$= -\frac{\sqrt{a+cx^4}}{7x^7} - \frac{2c\sqrt{a+cx^4}}{21ax^3} - \frac{\left(2c^2\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{21a}$$
$$= -\frac{\sqrt{a+cx^4}}{7x^7} - \frac{2c\sqrt{a+cx^4}}{21ax^3} - \frac{c^{7/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{21a^{5/4}\sqrt{a+cx^4}}$$

Mathematica [C] time = 0.02, size = 51, normalized size = 0.40

$$-\frac{\sqrt{a+cx^4}\;{}_2F_1\left(-\frac{7}{4},-\frac{1}{2};-\frac{3}{4};-\frac{cx^4}{a}\right)}{7x^7\sqrt{\frac{cx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^4]/x^8,x]
```

```
[Out] -1/7*(Sqrt[a + c*x^4]*Hypergeometric2F1[-7/4, -1/2, -3/4, -((c*x^4)/a)])/(x
^7*Sqrt[1 + (c*x^4)/a])
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+a}}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/x^8,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + a)/x^8, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+a}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/x^8,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + a)/x^8, x)
```

maple [C] time = 0.02, size = 110, normalized size = 0.85

$$-\frac{2\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,c^2\,\text{EllipticF}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x, i\right)}{21\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}\,a} - \frac{2\sqrt{c\,x^4+a}\,c}{21a\,x^3} - \frac{\sqrt{c\,x^4+a}}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^(1/2)/x^8,x)
```

```
[Out] -1/7*(c*x^4+a)^(1/2)/x^7-2/21*c*(c*x^4+a)^(1/2)/a/x^3-2/21*c^2/a/(I/a^(1/2)
*c^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^
(1/2)/(c*x^4+a)^(1/2)*EllipticF((I/a^(1/2)*c^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+a}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/x^8,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + a)/x^8, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4+a}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^(1/2)/x^8,x)
```

```
[Out] int((a + c*x^4)^(1/2)/x^8, x)
```

sympy [C] time = 1.39, size = 46, normalized size = 0.36

$$\frac{\sqrt{a}\,\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{7}{4}, -\frac{1}{2}\\ -\frac{3}{4}\end{matrix}\right| \frac{cx^4e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**(1/2)/x**8,x)
```

```
[Out] sqrt(a)*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), c*x**4*exp_polar(I*pi)/a)/
(4*x**7*gamma(-3/4))
```

3.780 $\int x^2\sqrt{a+cx^4}\,dx$

Optimal. Leaf size=234

$$\frac{a^{5/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{a+cx^4}}-\frac{2a^{5/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{a+cx^4}}+\frac{1}{5}x$$

```
[Out] 1/5*x^3*(c*x^4+a)^(1/2)+2/5*a*x*(c*x^4+a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2
))-2/5*a^(5/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1
/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^
(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4
+a)^(1/2)+1/5*a^(5/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arct
an(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/
2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)
/(c*x^4+a)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {279, 305, 220, 1196}

$$\frac{a^{5/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{a+cx^4}}-\frac{2a^{5/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{a+cx^4}}+\frac{1}{5}x$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + c*x^4],x]
```

```
[Out] (x^3*Sqrt[a + c*x^4])/5 + (2*a*x*Sqrt[a + c*x^4])/(5*Sqrt[c]*(Sqrt[a] + Sqr
t[c]*x^2)) - (2*a^(5/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] +
 Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(5*c^(3/4)*
Sqrt[a + c*x^4]) + (a^(5/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[
a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(5*c^(3
/4)*Sqrt[a + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int x^2\sqrt{a+cx^4}\,dx = \frac{1}{5}x^3\sqrt{a+cx^4} + \frac{1}{5}(2a)\int \frac{x^2}{\sqrt{a+cx^4}}\,dx$$

$$= \frac{1}{5}x^3\sqrt{a+cx^4} + \frac{\left(2a^{3/2}\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{5\sqrt{c}} - \frac{\left(2a^{3/2}\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{5\sqrt{c}}$$

$$= \frac{1}{5}x^3\sqrt{a+cx^4} + \frac{2ax\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{2a^{5/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\right)}{5c^{3/4}\sqrt{a+cx^4}}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.22

$$\frac{x^3\sqrt{a+cx^4}\;{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^4}{a}\right)}{3\sqrt{\frac{cx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + c*x^4],x]
```

```
[Out] (x^3*Sqrt[a + c*x^4]*Hypergeometric2F1[-1/2, 3/4, 7/4, -((c*x^4)/a)])/(3*Sq
rt[1 + (c*x^4)/a])
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{cx^4+a}\,x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + a)*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^4+a}\,x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + a)*x^2, x)
```

maple [C] time = 0.01, size = 112, normalized size = 0.48

$$\frac{\sqrt{c\,x^4+a}\;x^3}{5} + \frac{2i\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\;\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\;\left(-\text{EllipticE}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\;x,i\right)+\text{EllipticF}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\;x,i\right)\right)a^{\frac{3}{2}}}{5\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\;\sqrt{c\,x^4+a}\;\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^(1/2)*x^2,x)
```

```
[Out] 1/5*x^3*(c*x^4+a)^(1/2)+2/5*I*a^(3/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)
*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)/c^(1/
2)*(EllipticF((I/a^(1/2)*c^(1/2))^(1/2)*x,I)-EllipticE((I/a^(1/2)*c^(1/2))^
(1/2)*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^4 + a}\, x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + a)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \sqrt{c\, x^4 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + c*x^4)^(1/2),x)
```

```
[Out] int(x^2*(a + c*x^4)^(1/2), x)
```

sympy [C] time = 1.76, size = 39, normalized size = 0.17

$$\frac{\sqrt{a}\, x^3 \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{3}{4} \\ \frac{7}{4} \end{matrix} \middle| \frac{cx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), c*x**4*exp_polar(I*pi)/a
)/(4*gamma(7/4))
```

3.781 $\int \frac{\sqrt{a+cx^4}}{x^2}\,dx$

Optimal. Leaf size=224

$$-\frac{\sqrt{a+cx^4}}{x}+\frac{2\sqrt{c}\,x\sqrt{a+cx^4}}{\sqrt{a}+\sqrt{c}\,x^2}+\frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)\quad 2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right.}{\sqrt{a+cx^4}}-\frac{}{}$$

```
[Out] -(c*x^4+a)^(1/2)/x+2*x*c^(1/2)*(c*x^4+a)^(1/2)/(a^(1/2)+x^2*c^(1/2))-2*a^(1
/4)*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)
*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/
2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/(c*x^4+a)^(1/2)+a
^(1/4)*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1
/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^
(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/(c*x^4+a)^(1/2
)
```

Rubi [A] time = 0.07, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {277, 305, 220, 1196}

$$\frac{2\sqrt{c}\,x\sqrt{a+cx^4}}{\sqrt{a}+\sqrt{c}\,x^2}-\frac{\sqrt{a+cx^4}}{x}+\frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)\quad 2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}{\sqrt{a+cx^4}}-\frac{}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^4]/x^2,x]
```

```
[Out] -(Sqrt[a + c*x^4]/x) + (2*Sqrt[c]*x*Sqrt[a + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2
) - (2*a^(1/4)*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] +
Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/Sqrt[a + c*x
^4] + (a^(1/4)*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] +
Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/Sqrt[a + c*x
^4]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^4}}{x^2}\,dx = -\frac{\sqrt{a+cx^4}}{x} + (2c)\int \frac{x^2}{\sqrt{a+cx^4}}\,dx$$

$$= -\frac{\sqrt{a+cx^4}}{x} + \left(2\sqrt{a}\,\sqrt{c}\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx - \left(2\sqrt{a}\,\sqrt{c}\right)\int \frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx$$

$$= -\frac{\sqrt{a+cx^4}}{x} + \frac{2\sqrt{c}\,x\sqrt{a+cx^4}}{\sqrt{a}+\sqrt{c}\,x^2} - \frac{2\sqrt[4]{a}\,\sqrt[4]{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+cx^4}} + \cdots$$

Mathematica [C] time = 0.01, size = 49, normalized size = 0.22

$$-\frac{\sqrt{a+cx^4}\;{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};-\frac{cx^4}{a}\right)}{x\sqrt{\frac{cx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^4]/x^2,x]
```

```
[Out] -((Sqrt[a + c*x^4]*Hypergeometric2F1[-1/2, -1/4, 3/4, -((c*x^4)/a)])/(x*Sqr
t[1 + (c*x^4)/a]))
```

fricas [F] time = 1.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+a}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + a)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + a)/x^2, x)
```

maple [C] time = 0.01, size = 112, normalized size = 0.50

$$\frac{2i\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\left(-\text{EllipticE}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x,i\right)+\text{EllipticF}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x,i\right)\right)\sqrt{a}\,\sqrt{c}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}} - \frac{\sqrt{c\,x^4+a}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^(1/2)/x^2,x)
```

```
[Out] -(c*x^4+a)^(1/2)/x+2*I*c^(1/2)*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2
)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)*(Ell
ipticF((I/a^(1/2)*c^(1/2))^(1/2)*x,I)-EllipticE((I/a^(1/2)*c^(1/2))^(1/2)*x
,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + a)/x^2, x)
```

mupad [B] time = 1.28, size = 39, normalized size = 0.17

$$\frac{\sqrt{c\,x^4+a}\;{}_2\mathrm{F}_1\left(-\frac{1}{2},-\frac{1}{4};\;\frac{3}{4};\;-\frac{a}{c\,x^4}\right)}{x\sqrt{\frac{a}{c\,x^4}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^(1/2)/x^2,x)
```

```
[Out] ((a + c*x^4)^(1/2)*hypergeom([-1/2, -1/4], 3/4, -a/(c*x^4)))/(x*(a/(c*x^4)
+ 1)^(1/2))
```

sympy [C] time = 1.44, size = 41, normalized size = 0.18

$$\frac{\sqrt{a}\,\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\,\frac{cx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**(1/2)/x**2,x)
```

```
[Out] sqrt(a)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), c*x**4*exp_polar(I*pi)/a)/(
4*x*gamma(3/4))
```

3.782 $\int \frac{\sqrt{a+cx^4}}{x^6}\,dx$

Optimal. Leaf size=258

$$\frac{c^{5/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+cx^4}}-\frac{2c^{5/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+cx^4}}+\frac{2c}{5a}$$

```
[Out] -1/5*(c*x^4+a)^(1/2)/x^5-2/5*c*(c*x^4+a)^(1/2)/a/x+2/5*c^(3/2)*x*(c*x^4+a)^
(1/2)/a/(a^(1/2)+x^2*c^(1/2))-2/5*c^(5/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))
^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x
/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/
2))^2)^(1/2)/a^(3/4)/(c*x^4+a)^(1/2)+1/5*c^(5/4)*(cos(2*arctan(c^(1/4)*x/a^
(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^
(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^
2*c^(1/2))^2)^(1/2)/a^(3/4)/(c*x^4+a)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {277, 325, 305, 220, 1196}

$$\frac{c^{5/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+cx^4}}-\frac{2c^{5/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+cx^4}}+\frac{2c}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^4]/x^6,x]
```

```
[Out] -Sqrt[a + c*x^4]/(5*x^5) - (2*c*Sqrt[a + c*x^4])/(5*a*x) + (2*c^(3/2)*x*Sqr
t[a + c*x^4])/(5*a*(Sqrt[a] + Sqrt[c]*x^2)) - (2*c^(5/4)*(Sqrt[a] + Sqrt[c]
*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/
4)*x)/a^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[a + c*x^4]) + (c^(5/4)*(Sqrt[a] + Sqr
t[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c
^(1/4)*x)/a^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[a + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+cx^4}}{x^6}\,dx &= -\frac{\sqrt{a+cx^4}}{5x^5} + \frac{1}{5}(2c)\int \frac{1}{x^2\sqrt{a+cx^4}}\,dx \\
&= -\frac{\sqrt{a+cx^4}}{5x^5} - \frac{2c\sqrt{a+cx^4}}{5ax} + \frac{\left(2c^2\right)\int \frac{x^2}{\sqrt{a+cx^4}}\,dx}{5a} \\
&= -\frac{\sqrt{a+cx^4}}{5x^5} - \frac{2c\sqrt{a+cx^4}}{5ax} + \frac{\left(2c^{3/2}\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{5\sqrt{a}} - \frac{\left(2c^{3/2}\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{5\sqrt{a}} \\
&= -\frac{\sqrt{a+cx^4}}{5x^5} - \frac{2c\sqrt{a+cx^4}}{5ax} + \frac{2c^{3/2}x\sqrt{a+cx^4}}{5a\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{2c^{5/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan
\end{aligned}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.20

$$-\frac{\sqrt{a+cx^4}\;{}_2F_1\left(-\frac{5}{4},-\frac{1}{2};-\frac{1}{4};-\frac{cx^4}{a}\right)}{5x^5\sqrt{\frac{cx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^4]/x^6,x]
```

```
[Out] -1/5*(Sqrt[a + c*x^4]*Hypergeometric2F1[-5/4, -1/2, -1/4, -((c*x^4)/a)])/(x
^5*Sqrt[1 + (c*x^4)/a])
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cx^4+a}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + a)/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+a}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + a)/x^6, x)
```

maple [C] time = 0.01, size = 130, normalized size = 0.50

$$\frac{2i\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\left(-\text{EllipticE}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x,i\right)+\text{EllipticF}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x,i\right)\right)c^{\frac{3}{2}}}{5\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}\,\sqrt{a}}-\frac{2\sqrt{c\,x^4+a}\,c}{5ax}-\frac{\sqrt{c\,x^4+a}}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^(1/2)/x^6,x)
```

```
[Out] -1/5*(c*x^4+a)^(1/2)/x^5-2/5*c*(c*x^4+a)^(1/2)/a/x+2/5*I*c^(3/2)/a^(1/2)/(I
/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)
*x^2+1)^(1/2)/(c*x^4+a)^(1/2)*(EllipticF((I/a^(1/2)*c^(1/2))^(1/2)*x,I)-Ell
ipticE((I/a^(1/2)*c^(1/2))^(1/2)*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^4+a}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + a)/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^4+a}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^(1/2)/x^6,x)
```

```
[Out] int((a + c*x^4)^(1/2)/x^6, x)
```

sympy [C] time = 2.21, size = 46, normalized size = 0.18

$$\frac{\sqrt{a}\,\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\,\frac{cx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**(1/2)/x**6,x)
```

```
[Out] sqrt(a)*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), c*x**4*exp_polar(I*pi)/a)/
(4*x**5*gamma(-1/4))
```

3.783 $\int x^{11}\left(a+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=59

$$\frac{a^2\left(a+cx^4\right)^{5/2}}{10c^3}+\frac{\left(a+cx^4\right)^{9/2}}{18c^3}-\frac{a\left(a+cx^4\right)^{7/2}}{7c^3}$$

```
[Out] 1/10*a^2*(c*x^4+a)^(5/2)/c^3-1/7*a*(c*x^4+a)^(7/2)/c^3+1/18*(c*x^4+a)^(9/2)
/c^3
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{a^2\left(a+cx^4\right)^{5/2}}{10c^3}+\frac{\left(a+cx^4\right)^{9/2}}{18c^3}-\frac{a\left(a+cx^4\right)^{7/2}}{7c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a + c*x^4)^(3/2),x]
```

```
[Out] (a^2*(a + c*x^4)^(5/2))/(10*c^3) - (a*(a + c*x^4)^(7/2))/(7*c^3) + (a + c*x
^4)^(9/2)/(18*c^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{11}\left(a+cx^4\right)^{3/2}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int x^2(a+cx)^{3/2}\,dx,x,x^4\right)\\ &= \frac{1}{4}\operatorname{Subst}\left(\int\left(\frac{a^2(a+cx)^{3/2}}{c^2}-\frac{2a(a+cx)^{5/2}}{c^2}+\frac{(a+cx)^{7/2}}{c^2}\right)dx,x,x^4\right)\\ &= \frac{a^2\left(a+cx^4\right)^{5/2}}{10c^3}-\frac{a\left(a+cx^4\right)^{7/2}}{7c^3}+\frac{\left(a+cx^4\right)^{9/2}}{18c^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 39, normalized size = 0.66

$$\frac{\left(a+cx^4\right)^{5/2}\left(8a^2-20acx^4+35c^2x^8\right)}{630c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + c*x^4)^(3/2),x]
```

```
[Out] ((a + c*x^4)^(5/2)*(8*a^2 - 20*a*c*x^4 + 35*c^2*x^8))/(630*c^3)
```

fricas [A] time = 0.90, size = 57, normalized size = 0.97

$$\frac{\left(35\,c^4x^{16}+50\,ac^3x^{12}+3\,a^2c^2x^8-4\,a^3cx^4+8\,a^4\right)\sqrt{cx^4+a}}{630\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(c*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/630*(35*c^4*x^16 + 50*a*c^3*x^12 + 3*a^2*c^2*x^8 - 4*a^3*c*x^4 + 8*a^4)*s
qrt(c*x^4 + a)/c^3
```

giac [A] time = 0.17, size = 43, normalized size = 0.73

$$\frac{35\left(cx^4+a\right)^{\frac{9}{2}}-90\left(cx^4+a\right)^{\frac{7}{2}}a+63\left(cx^4+a\right)^{\frac{5}{2}}a^2}{630\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(c*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/630*(35*(c*x^4 + a)^(9/2) - 90*(c*x^4 + a)^(7/2)*a + 63*(c*x^4 + a)^(5/2)
*a^2)/c^3
```

maple [A] time = 0.01, size = 36, normalized size = 0.61

$$\frac{\left(c\,x^4+a\right)^{\frac{5}{2}}\left(35c^2x^8-20ac\,x^4+8a^2\right)}{630c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(c*x^4+a)^(3/2),x)
```

```
[Out] 1/630*(c*x^4+a)^(5/2)*(35*c^2*x^8-20*a*c*x^4+8*a^2)/c^3
```

maxima [A] time = 1.40, size = 47, normalized size = 0.80

$$\frac{\left(cx^4+a\right)^{\frac{9}{2}}}{18\,c^3}-\frac{\left(cx^4+a\right)^{\frac{7}{2}}a}{7\,c^3}+\frac{\left(cx^4+a\right)^{\frac{5}{2}}a^2}{10\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(c*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/18*(c*x^4 + a)^(9/2)/c^3 - 1/7*(c*x^4 + a)^(7/2)*a/c^3 + 1/10*(c*x^4 + a)
^(5/2)*a^2/c^3
```

mupad [B] time = 1.18, size = 53, normalized size = 0.90

$$\sqrt{c\,x^4+a}\left(\frac{5\,a\,x^{12}}{63}+\frac{c\,x^{16}}{18}+\frac{4\,a^4}{315\,c^3}-\frac{2\,a^3\,x^4}{315\,c^2}+\frac{a^2\,x^8}{210\,c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a + c*x^4)^(3/2),x)
```

```
[Out] (a + c*x^4)^(1/2)*((5*a*x^12)/63 + (c*x^16)/18 + (4*a^4)/(315*c^3) - (2*a^3
*x^4)/(315*c^2) + (a^2*x^8)/(210*c))
```

sympy [**A**] time = 11.03, size = 109, normalized size = 1.85

$$\begin{cases} \frac{4a^4\sqrt{a+cx^4}}{315c^3} - \frac{2a^3x^4\sqrt{a+cx^4}}{315c^2} + \frac{a^2x^8\sqrt{a+cx^4}}{210c} + \frac{5ax^{12}\sqrt{a+cx^4}}{63} + \frac{cx^{16}\sqrt{a+cx^4}}{18} & \text{for } c \neq 0 \\ \frac{a^{\frac{3}{2}}x^{12}}{12} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(c*x**4+a)**(3/2),x)
```

```
[Out] Piecewise((4*a**4*sqrt(a + c*x**4)/(315*c**3) - 2*a**3*x**4*sqrt(a + c*x**4
)/(315*c**2) + a**2*x**8*sqrt(a + c*x**4)/(210*c) + 5*a*x**12*sqrt(a + c*x*
*4)/63 + c*x**16*sqrt(a + c*x**4)/18, Ne(c, 0)), (a**(3/2)*x**12/12, True))
```

3.784 $\int x^7 \left(a+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=38

$$\frac{\left(a+cx^4\right)^{7/2}}{14c^2}-\frac{a\left(a+cx^4\right)^{5/2}}{10c^2}$$

```
[Out] -1/10*a*(c*x^4+a)^(5/2)/c^2+1/14*(c*x^4+a)^(7/2)/c^2
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{\left(a+cx^4\right)^{7/2}}{14c^2}-\frac{a\left(a+cx^4\right)^{5/2}}{10c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + c*x^4)^(3/2),x]
```

```
[Out] -(a*(a + c*x^4)^(5/2))/(10*c^2) + (a + c*x^4)^(7/2)/(14*c^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^7\left(a+cx^4\right)^{3/2}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int x(a+cx)^{3/2}\,dx, x, x^4\right)\\ &= \frac{1}{4}\operatorname{Subst}\left(\int\left(-\frac{a(a+cx)^{3/2}}{c}+\frac{(a+cx)^{5/2}}{c}\right)dx, x, x^4\right)\\ &= -\frac{a\left(a+cx^4\right)^{5/2}}{10c^2}+\frac{\left(a+cx^4\right)^{7/2}}{14c^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 0.74

$$\frac{\left(a+cx^4\right)^{5/2}\left(5cx^4-2a\right)}{70c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + c*x^4)^(3/2),x]
```

```
[Out] ((a + c*x^4)^(5/2)*(-2*a + 5*c*x^4))/(70*c^2)
```

fricas [A] time = 0.50, size = 45, normalized size = 1.18

$$\frac{\left(5\,c^3x^{12}+8\,ac^2x^8+a^2cx^4-2\,a^3\right)\sqrt{cx^4+a}}{70\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/70*(5*c^3*x^12 + 8*a*c^2*x^8 + a^2*c*x^4 - 2*a^3)*sqrt(c*x^4 + a)/c^2
```

giac [A] time = 0.16, size = 29, normalized size = 0.76

$$\frac{5\left(cx^4+a\right)^{\frac{7}{2}}-7\left(cx^4+a\right)^{\frac{5}{2}}a}{70\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/70*(5*(c*x^4 + a)^(7/2) - 7*(c*x^4 + a)^(5/2)*a)/c^2
```

maple [A] time = 0.00, size = 25, normalized size = 0.66

$$-\frac{\left(c\,x^4+a\right)^{\frac{5}{2}}\left(-5c\,x^4+2a\right)}{70c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(c*x^4+a)^(3/2),x)
```

```
[Out] -1/70*(c*x^4+a)^(5/2)*(-5*c*x^4+2*a)/c^2
```

maxima [A] time = 1.32, size = 30, normalized size = 0.79

$$\frac{\left(cx^4+a\right)^{\frac{7}{2}}}{14\,c^2}-\frac{\left(cx^4+a\right)^{\frac{5}{2}}a}{10\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/14*(c*x^4 + a)^(7/2)/c^2 - 1/10*(c*x^4 + a)^(5/2)*a/c^2
```

mupad [B] time = 1.14, size = 42, normalized size = 1.11

$$\sqrt{c\,x^4+a}\left(\frac{4\,a\,x^8}{35}+\frac{c\,x^{12}}{14}-\frac{a^3}{35\,c^2}+\frac{a^2\,x^4}{70\,c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + c*x^4)^(3/2),x)
```

```
[Out] (a + c*x^4)^(1/2)*((4*a*x^8)/35 + (c*x^12)/14 - a^3/(35*c^2) + (a^2*x^4)/(7
0*c))
```

sympy [A] time = 5.42, size = 83, normalized size = 2.18

$$\begin{cases} -\frac{a^3\sqrt{a+cx^4}}{35c^2}+\frac{a^2x^4\sqrt{a+cx^4}}{70c}+\frac{4ax^8\sqrt{a+cx^4}}{35}+\frac{cx^{12}\sqrt{a+cx^4}}{14} & \text{for } c\neq 0 \\ \frac{a^{\frac{3}{2}}x^8}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(c*x**4+a)**(3/2),x)
```

```
[Out] Piecewise((-a**3*sqrt(a + c*x**4)/(35*c**2) + a**2*x**4*sqrt(a + c*x**4)/(7
0*c) + 4*a*x**8*sqrt(a + c*x**4)/35 + c*x**12*sqrt(a + c*x**4)/14, Ne(c, 0)
), (a**(3/2)*x**8/8, True))
```

3.785 $\int x^3 \left(a + cx^4\right)^{3/2} dx$

Optimal. Leaf size=18

$$\frac{\left(a + cx^4\right)^{5/2}}{10c}$$

```
[Out] 1/10*(c*x^4+a)^(5/2)/c
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$\frac{\left(a + cx^4\right)^{5/2}}{10c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + c*x^4)^(3/2),x]
```

```
[Out] (a + c*x^4)^(5/2)/(10*c)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^3 \left(a + cx^4\right)^{3/2} dx = \frac{\left(a + cx^4\right)^{5/2}}{10c}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{\left(a + cx^4\right)^{5/2}}{10c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + c*x^4)^(3/2),x]
```

```
[Out] (a + c*x^4)^(5/2)/(10*c)
```

fricas [B] time = 0.80, size = 32, normalized size = 1.78

$$\frac{\left(c^2x^8 + 2\,acx^4 + a^2\right)\sqrt{cx^4 + a}}{10\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/10*(c^2*x^8 + 2*a*c*x^4 + a^2)*sqrt(c*x^4 + a)/c
```

giac [A] time = 0.15, size = 14, normalized size = 0.78

$$\frac{\left(cx^4 + a\right)^{\frac{5}{2}}}{10\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+a)^(3/2),x, algorithm="giac")
[Out] 1/10*(c*x^4 + a)^(5/2)/c
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$\frac{\left(c\,x^4+a\right)^{\frac{5}{2}}}{10c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^4+a)^(3/2),x)
[Out] 1/10*(c*x^4+a)^(5/2)/c
```

maxima [A] time = 1.33, size = 14, normalized size = 0.78

$$\frac{\left(cx^4+a\right)^{\frac{5}{2}}}{10\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+a)^(3/2),x, algorithm="maxima")
[Out] 1/10*(c*x^4 + a)^(5/2)/c
```

mupad [B] time = 1.14, size = 14, normalized size = 0.78

$$\frac{\left(c\,x^4+a\right)^{5/2}}{10\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + c*x^4)^(3/2),x)
[Out] (a + c*x^4)^(5/2)/(10*c)
```

sympy [A] time = 1.21, size = 60, normalized size = 3.33

$$\begin{cases} \frac{a^2\sqrt{a+cx^4}}{10c} + \frac{ax^4\sqrt{a+cx^4}}{5} + \frac{cx^8\sqrt{a+cx^4}}{10} & \text{for } c \neq 0 \\ \frac{a^{\frac{3}{2}}x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**4+a)**(3/2),x)
[Out] Piecewise((a**2*sqrt(a + c*x**4)/(10*c) + a*x**4*sqrt(a + c*x**4)/5 + c*x**
8*sqrt(a + c*x**4)/10, Ne(c, 0)), (a**(3/2)*x**4/4, True))
```

3.786 $\int \frac{\left(a+cx^4\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=59

$$-\frac{1}{2}a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)+\frac{1}{2}a\sqrt{a+cx^4}+\frac{1}{6}\left(a+cx^4\right)^{3/2}$$

```
[Out] 1/6*(c*x^4+a)^(3/2)-1/2*a^(3/2)*arctanh((c*x^4+a)^(1/2)/a^(1/2))+1/2*a*(c*x
^4+a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 50, 63, 208}

$$-\frac{1}{2}a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)+\frac{1}{2}a\sqrt{a+cx^4}+\frac{1}{6}\left(a+cx^4\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^(3/2)/x,x]
```

```
[Out] (a*Sqrt[a + c*x^4])/2 + (a + c*x^4)^(3/2)/6 - (a^(3/2)*ArcTanh[Sqrt[a + c*x
^4]/Sqrt[a]])/2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+cx^4\right)^{3/2}}{x}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{(a+cx)^{3/2}}{x}\,dx, x, x^4\right)\\ &= \frac{1}{6}\left(a+cx^4\right)^{3/2} + \frac{1}{4}a\operatorname{Subst}\left(\int \frac{\sqrt{a+cx}}{x}\,dx, x, x^4\right)\\ &= \frac{1}{2}a\sqrt{a+cx^4} + \frac{1}{6}\left(a+cx^4\right)^{3/2} + \frac{1}{4}a^2\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^4\right)\\ &= \frac{1}{2}a\sqrt{a+cx^4} + \frac{1}{6}\left(a+cx^4\right)^{3/2} + \frac{a^2\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^4}\right)}{2c}\\ &= \frac{1}{2}a\sqrt{a+cx^4} + \frac{1}{6}\left(a+cx^4\right)^{3/2} - \frac{1}{2}a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 0.86

$$\frac{1}{6}\left(\sqrt{a+cx^4}\left(4a+cx^4\right) - 3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^(3/2)/x,x]
```

```
[Out] (Sqrt[a + c*x^4]*(4*a + c*x^4) - 3*a^(3/2)*ArcTanh[Sqrt[a + c*x^4]/Sqrt[a]]
)/6
```

fricas [A] time = 0.65, size = 103, normalized size = 1.75

$$\left[\frac{1}{4}a^{\frac{3}{2}}\log\left(\frac{cx^4 - 2\sqrt{cx^4+a}\sqrt{a} + 2a}{x^4}\right) + \frac{1}{6}\left(cx^4+4a\right)\sqrt{cx^4+a}, \frac{1}{2}\sqrt{-a}a\arctan\left(\frac{\sqrt{cx^4+a}\sqrt{-a}}{a}\right) + \frac{1}{6}\left(cx^4+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*a^(3/2)*log((c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(a) + 2*a)/x^4) + 1/6*(c*x^
4 + 4*a)*sqrt(c*x^4 + a), 1/2*sqrt(-a)*a*arctan(sqrt(c*x^4 + a)*sqrt(-a)/a)
 + 1/6*(c*x^4 + 4*a)*sqrt(c*x^4 + a)]
```

giac [A] time = 0.16, size = 50, normalized size = 0.85

$$\frac{a^2\arctan\left(\frac{\sqrt{cx^4+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}} + \frac{1}{6}\left(cx^4+a\right)^{\frac{3}{2}} + \frac{1}{2}\sqrt{cx^4+a}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/2*a^2*arctan(sqrt(c*x^4 + a)/sqrt(-a))/sqrt(-a) + 1/6*(c*x^4 + a)^(3/2) +
 1/2*sqrt(c*x^4 + a)*a
```

maple [A] time = 0.02, size = 57, normalized size = 0.97

$$\frac{\sqrt{c\,x^4+a}\;c\,x^4}{6} - \frac{a^{\frac{3}{2}}\ln\left(\frac{2a+2\sqrt{c\,x^4+a}\;\sqrt{a}}{x^2}\right)}{2} + \frac{2\sqrt{c\,x^4+a}\;a}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^(3/2)/x,x)
```

```
[Out] 1/6*c*x^4*(c*x^4+a)^(1/2)+2/3*a*(c*x^4+a)^(1/2)-1/2*a^(3/2)*ln((2*a+2*(c*x^
4+a)^(1/2)*a^(1/2))/x^2)
```

maxima [A] time = 3.00, size = 61, normalized size = 1.03

$$\frac{1}{4}\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{cx^4+a}-\sqrt{a}}{\sqrt{cx^4+a}+\sqrt{a}}\right)+\frac{1}{6}\left(cx^4+a\right)^{\frac{3}{2}}+\frac{1}{2}\,\sqrt{cx^4+a}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 1/4*a^(3/2)*log((sqrt(c*x^4 + a) - sqrt(a))/(sqrt(c*x^4 + a) + sqrt(a))) +
1/6*(c*x^4 + a)^(3/2) + 1/2*sqrt(c*x^4 + a)*a
```

mupad [B] time = 1.17, size = 43, normalized size = 0.73

$$\frac{a\,\sqrt{c\,x^4+a}}{2}-\frac{a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{c\,x^4+a}}{\sqrt{a}}\right)}{2}+\frac{\left(c\,x^4+a\right)^{3/2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^(3/2)/x,x)
```

```
[Out] (a*(a + c*x^4)^(1/2))/2 - (a^(3/2)*atanh((a + c*x^4)^(1/2)/a^(1/2)))/2 + (a
 + c*x^4)^(3/2)/6
```

sympy [A] time = 3.64, size = 80, normalized size = 1.36

$$\frac{2a^{\frac{3}{2}}\sqrt{1+\frac{cx^4}{a}}}{3}+\frac{a^{\frac{3}{2}}\log\left(\frac{cx^4}{a}\right)}{4}-\frac{a^{\frac{3}{2}}\log\left(\sqrt{1+\frac{cx^4}{a}}+1\right)}{2}+\frac{\sqrt{a}\,cx^4\sqrt{1+\frac{cx^4}{a}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**(3/2)/x,x)
```

```
[Out] 2*a**(3/2)*sqrt(1 + c*x**4/a)/3 + a**(3/2)*log(c*x**4/a)/4 - a**(3/2)*log(s
qrt(1 + c*x**4/a) + 1)/2 + sqrt(a)*c*x**4*sqrt(1 + c*x**4/a)/6
```

3.787 $\int \frac{\left(a+cx^4\right)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=63

$$-\frac{\left(a+cx^4\right)^{3/2}}{4x^4}+\frac{3}{4}c\sqrt{a+cx^4}-\frac{3}{4}\sqrt{a}\,c\tanh^{-1}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)$$

```
[Out] -1/4*(c*x^4+a)^(3/2)/x^4-3/4*c*arctanh((c*x^4+a)^(1/2)/a^(1/2))*a^(1/2)+3/4
*c*(c*x^4+a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {266, 47, 50, 63, 208}

$$-\frac{\left(a+cx^4\right)^{3/2}}{4x^4}+\frac{3}{4}c\sqrt{a+cx^4}-\frac{3}{4}\sqrt{a}\,c\tanh^{-1}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^(3/2)/x^5,x]
```

```
[Out] (3*c*Sqrt[a + c*x^4])/4 - (a + c*x^4)^(3/2)/(4*x^4) - (3*Sqrt[a]*c*ArcTanh[
Sqrt[a + c*x^4]/Sqrt[a]])/4
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+cx^4\right)^{3/2}}{x^5}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{(a+cx)^{3/2}}{x^2}\,dx, x, x^4\right) \\
&= -\frac{\left(a+cx^4\right)^{3/2}}{4x^4} + \frac{1}{8}(3c)\operatorname{Subst}\left(\int \frac{\sqrt{a+cx}}{x}\,dx, x, x^4\right) \\
&= \frac{3}{4}c\sqrt{a+cx^4} - \frac{\left(a+cx^4\right)^{3/2}}{4x^4} + \frac{1}{8}(3ac)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^4\right) \\
&= \frac{3}{4}c\sqrt{a+cx^4} - \frac{\left(a+cx^4\right)^{3/2}}{4x^4} + \frac{1}{4}(3a)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^4}\right) \\
&= \frac{3}{4}c\sqrt{a+cx^4} - \frac{\left(a+cx^4\right)^{3/2}}{4x^4} - \frac{3}{4}\sqrt{a}\,c\tanh^{-1}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.59

$$\frac{c\left(a+cx^4\right)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{cx^4}{a}+1\right)}{10a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^(3/2)/x^5,x]
```

```
[Out] (c*(a + c*x^4)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, 1 + (c*x^4)/a])/(10*a^2
)
```

fricas [A] time = 0.84, size = 121, normalized size = 1.92

$$\left[\frac{3\sqrt{a}\,cx^4\log\left(\frac{cx^4-2\sqrt{cx^4+a}\sqrt{a}+2a}{x^4}\right)+2\left(2cx^4-a\right)\sqrt{cx^4+a}}{8x^4}, \frac{3\sqrt{-a}\,cx^4\arctan\left(\frac{\sqrt{cx^4+a}\sqrt{-a}}{a}\right)+\left(2cx^4-a\right)\sqrt{cx^4+a}}{4x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(a)*c*x^4*log((c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(a) + 2*a)/x^4) +
2*(2*c*x^4 - a)*sqrt(c*x^4 + a))/x^4, 1/4*(3*sqrt(-a)*c*x^4*arctan(sqrt(c*x
^4 + a)*sqrt(-a)/a) + (2*c*x^4 - a)*sqrt(c*x^4 + a))/x^4]
```

giac [A] time = 0.17, size = 63, normalized size = 1.00

$$\frac{\frac{3ac^2\arctan\left(\frac{\sqrt{cx^4+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{cx^4+a}\,c^2-\frac{\sqrt{cx^4+a}\,ac}{x^4}}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/4*(3*a*c^2*arctan(sqrt(c*x^4 + a)/sqrt(-a))/sqrt(-a) + 2*sqrt(c*x^4 + a)*
c^2 - sqrt(c*x^4 + a)*a*c/x^4)/c
```

maple [A] time = 0.02, size = 58, normalized size = 0.92

$$-\frac{3\sqrt{a}\,c\ln\left(\frac{2a+2\sqrt{c\,x^4+a}\,\sqrt{a}}{x^2}\right)}{4}+\frac{\sqrt{c\,x^4+a}\,c}{2}-\frac{\sqrt{c\,x^4+a}\,a}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^(3/2)/x^5,x)
```

```
[Out] 1/2*c*(c*x^4+a)^(1/2)-1/4*a/x^4*(c*x^4+a)^(1/2)-3/4*a^(1/2)*c*ln((2*a+2*(c*
x^4+a)^(1/2)*a^(1/2))/x^2)
```

maxima [A] time = 2.90, size = 66, normalized size = 1.05

$$\frac{3}{8}\sqrt{a}\,c\log\left(\frac{\sqrt{cx^4+a}-\sqrt{a}}{\sqrt{cx^4+a}+\sqrt{a}}\right)+\frac{1}{2}\sqrt{cx^4+a}\,c-\frac{\sqrt{cx^4+a}\,a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] 3/8*sqrt(a)*c*log((sqrt(c*x^4 + a) - sqrt(a))/(sqrt(c*x^4 + a) + sqrt(a)))
+ 1/2*sqrt(c*x^4 + a)*c - 1/4*sqrt(c*x^4 + a)*a/x^4
```

mupad [B] time = 1.36, size = 48, normalized size = 0.76

$$\frac{c\sqrt{c\,x^4+a}}{2}-\frac{a\sqrt{c\,x^4+a}}{4\,x^4}-\frac{3\sqrt{a}\,c\,\mathrm{atanh}\left(\frac{\sqrt{c\,x^4+a}}{\sqrt{a}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^(3/2)/x^5,x)
```

```
[Out] (c*(a + c*x^4)^(1/2))/2 - (a*(a + c*x^4)^(1/2))/(4*x^4) - (3*a^(1/2)*c*atan
h((a + c*x^4)^(1/2)/a^(1/2)))/4
```

sympy [A] time = 4.61, size = 95, normalized size = 1.51

$$-\frac{3\sqrt{a}\,c\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{c}\,x^2}\right)}{4}-\frac{a^2}{4\sqrt{c}\,x^6\sqrt{\frac{a}{cx^4}+1}}+\frac{a\sqrt{c}}{4x^2\sqrt{\frac{a}{cx^4}+1}}+\frac{c^{\frac{3}{2}}x^2}{2\sqrt{\frac{a}{cx^4}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**(3/2)/x**5,x)
```

```
[Out] -3*sqrt(a)*c*asinh(sqrt(a)/(sqrt(c)*x**2))/4 - a**2/(4*sqrt(c)*x**6*sqrt(a/
(c*x**4) + 1)) + a*sqrt(c)/(4*x**2*sqrt(a/(c*x**4) + 1)) + c**(3/2)*x**2/(2
*sqrt(a/(c*x**4) + 1))
```

3.788 $\int \frac{\left(a+cx^4\right)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=68

$$-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{16\sqrt{a}}-\frac{3c\sqrt{a+cx^4}}{16x^4}-\frac{\left(a+cx^4\right)^{3/2}}{8x^8}$$

```
[Out] -1/8*(c*x^4+a)^(3/2)/x^8-3/16*c^2*arctanh((c*x^4+a)^(1/2)/a^(1/2))/a^(1/2)-
3/16*c*(c*x^4+a)^(1/2)/x^4
```

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 47, 63, 208}

$$-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{16\sqrt{a}}-\frac{3c\sqrt{a+cx^4}}{16x^4}-\frac{\left(a+cx^4\right)^{3/2}}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^(3/2)/x^9,x]
```

```
[Out] (-3*c*Sqrt[a + c*x^4])/(16*x^4) - (a + c*x^4)^(3/2)/(8*x^8) - (3*c^2*ArcTan
h[Sqrt[a + c*x^4]/Sqrt[a]])/(16*Sqrt[a])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+cx^4\right)^{3/2}}{x^9}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{(a+cx)^{3/2}}{x^3}\,dx, x, x^4\right)\\ &= -\frac{\left(a+cx^4\right)^{3/2}}{8x^8} + \frac{1}{16}(3c)\,\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x^2}\,dx, x, x^4\right)\\ &= -\frac{3c\sqrt{a+cx^4}}{16x^4} - \frac{\left(a+cx^4\right)^{3/2}}{8x^8} + \frac{1}{32}\left(3c^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^4\right)\\ &= -\frac{3c\sqrt{a+cx^4}}{16x^4} - \frac{\left(a+cx^4\right)^{3/2}}{8x^8} + \frac{1}{16}(3c)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^4}\right)\\ &= -\frac{3c\sqrt{a+cx^4}}{16x^4} - \frac{\left(a+cx^4\right)^{3/2}}{8x^8} - \frac{3c^2\tanh^{-1}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{16\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 76, normalized size = 1.12

$$-\frac{2a^2+3c^2x^8\sqrt{\frac{cx^4}{a}+1}\,\tanh^{-1}\left(\sqrt{\frac{cx^4}{a}+1}\right)+7acx^4+5c^2x^8}{16x^8\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^(3/2)/x^9,x]
```

```
[Out] -1/16*(2*a^2 + 7*a*c*x^4 + 5*c^2*x^8 + 3*c^2*x^8*Sqrt[1 + (c*x^4)/a]*ArcTan
h[Sqrt[1 + (c*x^4)/a]])/(x^8*Sqrt[a + c*x^4])
```

fricas [A] time = 0.76, size = 138, normalized size = 2.03

$$\left[\frac{3\sqrt{a}\,c^2x^8\log\left(\frac{cx^4-2\sqrt{cx^4+a}\sqrt{a}+2a}{x^4}\right)-2\left(5acx^4+2a^2\right)\sqrt{cx^4+a}}{32ax^8},\ \frac{3\sqrt{-a}\,c^2x^8\arctan\left(\frac{\sqrt{cx^4+a}\sqrt{-a}}{a}\right)-\left(5acx^4+2\right.}{16ax^8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] [1/32*(3*sqrt(a)*c^2*x^8*log((c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(a) + 2*a)/x^4)
 - 2*(5*a*c*x^4 + 2*a^2)*sqrt(c*x^4 + a))/(a*x^8), 1/16*(3*sqrt(-a)*c^2*x^8
*arctan(sqrt(c*x^4 + a)*sqrt(-a)/a) - (5*a*c*x^4 + 2*a^2)*sqrt(c*x^4 + a))/
(a*x^8)]
```

giac [A] time = 0.16, size = 70, normalized size = 1.03

$$\frac{\frac{3c^3\arctan\left(\frac{\sqrt{cx^4+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{5\left(cx^4+a\right)^{\frac{3}{2}}c^3-3\sqrt{cx^4+a}\,ac^3}{c^2x^8}}{16c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] 1/16*(3*c^3*arctan(sqrt(c*x^4 + a)/sqrt(-a))/sqrt(-a) - (5*(c*x^4 + a)^(3/2
)*c^3 - 3*sqrt(c*x^4 + a)*a*c^3)/(c^2*x^8))/c
```

maple [A] time = 0.02, size = 63, normalized size = 0.93

$$-\frac{3c^2\ln\left(\frac{2a+2\sqrt{c\,x^4+a}\,\sqrt{a}}{x^2}\right)}{16\sqrt{a}}-\frac{5\sqrt{c\,x^4+a}\,c}{16x^4}-\frac{\sqrt{c\,x^4+a}\,a}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^(3/2)/x^9,x)
```

```
[Out] -5/16*c*(c*x^4+a)^(1/2)/x^4-3/16*c^2/a^(1/2)*ln((2*a+2*(c*x^4+a)^(1/2)*a^(1
/2))/x^2)-1/8*a/x^8*(c*x^4+a)^(1/2)
```

maxima [A] time = 2.94, size = 98, normalized size = 1.44

$$\frac{3\,c^2\log\left(\frac{\sqrt{cx^4+a}-\sqrt{a}}{\sqrt{cx^4+a}+\sqrt{a}}\right)}{32\,\sqrt{a}}-\frac{5\left(cx^4+a\right)^{\frac{3}{2}}c^2-3\,\sqrt{cx^4+a}\,ac^2}{16\left(\left(cx^4+a\right)^2-2\left(cx^4+a\right)a+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] 3/32*c^2*log((sqrt(c*x^4 + a) - sqrt(a))/(sqrt(c*x^4 + a) + sqrt(a)))/sqrt(
a) - 1/16*(5*(c*x^4 + a)^(3/2)*c^2 - 3*sqrt(c*x^4 + a)*a*c^2)/((c*x^4 + a)^
2 - 2*(c*x^4 + a)*a + a^2)
```

mupad [B] time = 1.49, size = 52, normalized size = 0.76

$$\frac{3\,a\,\sqrt{c\,x^4+a}}{16\,x^8}-\frac{3\,c^2\,\mathrm{atanh}\left(\frac{\sqrt{c\,x^4+a}}{\sqrt{a}}\right)}{16\,\sqrt{a}}-\frac{5\left(c\,x^4+a\right)^{3/2}}{16\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^(3/2)/x^9,x)
```

```
[Out] (3*a*(a + c*x^4)^(1/2))/(16*x^8) - (3*c^2*atanh((a + c*x^4)^(1/2)/a^(1/2)))
/(16*a^(1/2)) - (5*(a + c*x^4)^(3/2))/(16*x^8)
```

sympy [A] time = 4.98, size = 75, normalized size = 1.10

$$-\frac{a\sqrt{c}\,\sqrt{\frac{a}{cx^4}+1}}{8x^6}-\frac{5c^{\frac{3}{2}}\sqrt{\frac{a}{cx^4}+1}}{16x^2}-\frac{3c^2\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{c}\,x^2}\right)}{16\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**(3/2)/x**9,x)
```

```
[Out] -a*sqrt(c)*sqrt(a/(c*x**4) + 1)/(8*x**6) - 5*c**(3/2)*sqrt(a/(c*x**4) + 1)/
(16*x**2) - 3*c**2*asinh(sqrt(a)/(sqrt(c)*x**2))/(16*sqrt(a))
```

3.789 $\int x^5 \left(a+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=95

$$-\frac{a^3 \tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{32c^{3/2}}+\frac{a^2x^2\sqrt{a+cx^4}}{32c}+\frac{1}{12}x^6\left(a+cx^4\right)^{3/2}+\frac{1}{16}ax^6\sqrt{a+cx^4}$$

```
[Out] 1/12*x^6*(c*x^4+a)^(3/2)-1/32*a^3*arctanh(x^2*c^(1/2)/(c*x^4+a)^(1/2))/c^(3
/2)+1/32*a^2*x^2*(c*x^4+a)^(1/2)/c+1/16*a*x^6*(c*x^4+a)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {275, 279, 321, 217, 206}

$$-\frac{a^3 \tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{32c^{3/2}}+\frac{a^2x^2\sqrt{a+cx^4}}{32c}+\frac{1}{12}x^6\left(a+cx^4\right)^{3/2}+\frac{1}{16}ax^6\sqrt{a+cx^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + c*x^4)^(3/2),x]
```

```
[Out] (a^2*x^2*Sqrt[a + c*x^4])/(32*c) + (a*x^6*Sqrt[a + c*x^4])/16 + (x^6*(a + c
*x^4)^(3/2))/12 - (a^3*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(32*c^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int x^5\left(a+cx^4\right)^{3/2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^2\left(a+cx^2\right)^{3/2}\,dx,x,x^2\right)\\
&= \frac{1}{12}x^6\left(a+cx^4\right)^{3/2}+\frac{1}{4}a\operatorname{Subst}\left(\int x^2\sqrt{a+cx^2}\,dx,x,x^2\right)\\
&= \frac{1}{16}ax^6\sqrt{a+cx^4}\,+\frac{1}{12}x^6\left(a+cx^4\right)^{3/2}+\frac{1}{16}a^2\operatorname{Subst}\left(\int\frac{x^2}{\sqrt{a+cx^2}}\,dx,x,x^2\right)\\
&= \frac{a^2x^2\sqrt{a+cx^4}}{32c}+\frac{1}{16}ax^6\sqrt{a+cx^4}\,+\frac{1}{12}x^6\left(a+cx^4\right)^{3/2}-\frac{a^3\operatorname{Subst}\left(\int\frac{1}{\sqrt{a+cx^2}}\,dx,x,x^2\right)}{32c}\\
&= \frac{a^2x^2\sqrt{a+cx^4}}{32c}+\frac{1}{16}ax^6\sqrt{a+cx^4}\,+\frac{1}{12}x^6\left(a+cx^4\right)^{3/2}-\frac{a^3\operatorname{Subst}\left(\int\frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{a+cx^4}}\right)}{32c}\\
&= \frac{a^2x^2\sqrt{a+cx^4}}{32c}+\frac{1}{16}ax^6\sqrt{a+cx^4}\,+\frac{1}{12}x^6\left(a+cx^4\right)^{3/2}-\frac{a^3\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{32c^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 87, normalized size = 0.92

$$\frac{\sqrt{a+cx^4}\left(\sqrt{c}\,x^2\left(3a^2+14acx^4+8c^2x^8\right)-\frac{3a^{5/2}\sinh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{\sqrt{\frac{cx^4}{a}+1}}\right)}{96c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + c*x^4)^(3/2),x]
```

```
[Out] (Sqrt[a + c*x^4]*(Sqrt[c]*x^2*(3*a^2 + 14*a*c*x^4 + 8*c^2*x^8) - (3*a^(5/2)
*ArcSinh[(Sqrt[c]*x^2)/Sqrt[a]])/Sqrt[1 + (c*x^4)/a]))/(96*c^(3/2))
```

fricas [A] time = 0.64, size = 153, normalized size = 1.61

$$\left[\frac{3\,a^3\sqrt{c}\,\log\left(-2\,cx^4+2\,\sqrt{cx^4+a}\,\sqrt{c}\,x^2-a\right)+2\left(8\,c^3x^{10}+14\,ac^2x^6+3\,a^2cx^2\right)\sqrt{cx^4+a}}{192\,c^2},\ \frac{3\,a^3\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x}{\sqrt{cx^4+}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*a^3*sqrt(c)*log(-2*c*x^4 + 2*sqrt(c*x^4 + a)*sqrt(c)*x^2 - a) + 2
*(8*c^3*x^10 + 14*a*c^2*x^6 + 3*a^2*c*x^2)*sqrt(c*x^4 + a))/c^2, 1/96*(3*a^
3*sqrt(-c)*arctan(sqrt(-c)*x^2/sqrt(c*x^4 + a)) + (8*c^3*x^10 + 14*a*c^2*x^
6 + 3*a^2*c*x^2)*sqrt(c*x^4 + a))/c^2]
```

giac [A] time = 0.22, size = 126, normalized size = 1.33

$$\frac{1}{16}\left(\sqrt{cx^4+a}\left(2\,x^4+\frac{a}{c}\right)x^2+\frac{a^2\log\left(\left|-\sqrt{c}\,x^2+\sqrt{cx^4+a}\right|\right)}{c^{\frac{3}{2}}}\right)a+\frac{1}{96}\left(\left(2\left(4\,x^4+\frac{a}{c}\right)x^4-\frac{3\,a^2}{c^2}\right)\sqrt{cx^4+a}\,x^2-\frac{3\,a^3\log}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*(sqrt(c*x^4 + a)*(2*x^4 + a/c)*x^2 + a^2*log(abs(-sqrt(c)*x^2 + sqrt(c
*x^4 + a)))/c^(3/2))*a + 1/96*((2*(4*x^4 + a/c)*x^4 - 3*a^2/c^2)*sqrt(c*x^4
 + a)*x^2 - 3*a^3*log(abs(-sqrt(c)*x^2 + sqrt(c*x^4 + a)))/c^(5/2))*c
```

maple [A] time = 0.02, size = 78, normalized size = 0.82

$$\frac{\sqrt{c\,x^4+a}\;c\,x^{10}}{12}+\frac{7\sqrt{c\,x^4+a}\;a\,x^6}{48}+\frac{\sqrt{c\,x^4+a}\;a^2x^2}{32c}-\frac{a^3\ln\left(\sqrt{c}\;x^2+\sqrt{c\,x^4+a}\right)}{32c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*x^4+a)^(3/2),x)
```

```
[Out] 1/12*c*x^10*(c*x^4+a)^(1/2)+7/48*a*x^6*(c*x^4+a)^(1/2)+1/32*a^2*x^2*(c*x^4+
a)^(1/2)/c-1/32/c^(3/2)*a^3*ln(c^(1/2)*x^2+(c*x^4+a)^(1/2))
```

maxima [B] time = 2.98, size = 160, normalized size = 1.68

$$\frac{a^3\log\left(-\frac{\sqrt{c}-\frac{\sqrt{cx^4+a}}{x^2}}{\sqrt{c}+\frac{\sqrt{cx^4+a}}{x^2}}\right)}{64\,c^{\frac{3}{2}}}+\frac{\frac{3\sqrt{cx^4+a}\,a^3c^2}{x^2}-\frac{8\left(cx^4+a\right)^{\frac{3}{2}}a^3c}{x^6}-\frac{3\left(cx^4+a\right)^{\frac{5}{2}}a^3}{x^{10}}}{96\left(c^4-\frac{3\left(cx^4+a\right)c^3}{x^4}+\frac{3\left(cx^4+a\right)^2c^2}{x^8}-\frac{\left(cx^4+a\right)^3c}{x^{12}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/64*a^3*log(-(sqrt(c) - sqrt(c*x^4 + a)/x^2)/(sqrt(c) + sqrt(c*x^4 + a)/x^
2))/c^(3/2) + 1/96*(3*sqrt(c*x^4 + a)*a^3*c^2/x^2 - 8*(c*x^4 + a)^(3/2)*a^3
*c/x^6 - 3*(c*x^4 + a)^(5/2)*a^3/x^10)/(c^4 - 3*(c*x^4 + a)*c^3/x^4 + 3*(c*
x^4 + a)^2*c^2/x^8 - (c*x^4 + a)^3*c/x^12)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5\left(c\,x^4+a\right)^{3/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + c*x^4)^(3/2),x)
```

```
[Out] int(x^5*(a + c*x^4)^(3/2), x)
```

sympy [A] time = 6.34, size = 122, normalized size = 1.28

$$\frac{a^{\frac{5}{2}}x^2}{32c\sqrt{1+\frac{cx^4}{a}}}+\frac{17a^{\frac{3}{2}}x^6}{96\sqrt{1+\frac{cx^4}{a}}}+\frac{11\sqrt{a}\,cx^{10}}{48\sqrt{1+\frac{cx^4}{a}}}-\frac{a^3\operatorname{asinh}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{32c^{\frac{3}{2}}}+\frac{c^2x^{14}}{12\sqrt{a}\,\sqrt{1+\frac{cx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*x**4+a)**(3/2),x)
```

```
[Out] a**(5/2)*x**2/(32*c*sqrt(1 + c*x**4/a)) + 17*a**(3/2)*x**6/(96*sqrt(1 + c*x
**4/a)) + 11*sqrt(a)*c*x**10/(48*sqrt(1 + c*x**4/a)) - a**3*asinh(sqrt(c)*x
**2/sqrt(a))/(32*c**(3/2)) + c**2*x**14/(12*sqrt(a)*sqrt(1 + c*x**4/a))
```

3.790 $\int x\left(a+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=71

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{16\sqrt{c}}+\frac{3}{16}ax^2\sqrt{a+cx^4}+\frac{1}{8}x^2\left(a+cx^4\right)^{3/2}$$

```
[Out] 1/8*x^2*(c*x^4+a)^(3/2)+3/16*a^2*arctanh(x^2*c^(1/2)/(c*x^4+a)^(1/2))/c^(1/
2)+3/16*a*x^2*(c*x^4+a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {275, 195, 217, 206}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{16\sqrt{c}}+\frac{3}{16}ax^2\sqrt{a+cx^4}+\frac{1}{8}x^2\left(a+cx^4\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + c*x^4)^(3/2),x]
```

```
[Out] (3*a*x^2*Sqrt[a + c*x^4])/16 + (x^2*(a + c*x^4)^(3/2))/8 + (3*a^2*ArcTanh[(
Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(16*Sqrt[c])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int x\left(a+cx^4\right)^{3/2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \left(a+cx^2\right)^{3/2}\,dx,x,x^2\right)\\ &= \frac{1}{8}x^2\left(a+cx^4\right)^{3/2}+\frac{1}{8}(3a)\operatorname{Subst}\left(\int \sqrt{a+cx^2}\,dx,x,x^2\right)\\ &= \frac{3}{16}ax^2\sqrt{a+cx^4}+\frac{1}{8}x^2\left(a+cx^4\right)^{3/2}+\frac{1}{16}\left(3a^2\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+cx^2}}\,dx,x,x^2\right)\\ &= \frac{3}{16}ax^2\sqrt{a+cx^4}+\frac{1}{8}x^2\left(a+cx^4\right)^{3/2}+\frac{1}{16}\left(3a^2\right)\operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{a+cx^4}}\right)\\ &= \frac{3}{16}ax^2\sqrt{a+cx^4}+\frac{1}{8}x^2\left(a+cx^4\right)^{3/2}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{16\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.12, size = 69, normalized size = 0.97

$$\frac{1}{16}\sqrt{a+cx^4}\left(\frac{3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{\sqrt{c}\sqrt{\frac{cx^4}{a}+1}}+5ax^2+2cx^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + c*x^4)^(3/2),x]
```

```
[Out] (Sqrt[a + c*x^4]*(5*a*x^2 + 2*c*x^6 + (3*a^(3/2)*ArcSinh[(Sqrt[c]*x^2)/Sqrt
[a]])/(Sqrt[c]*Sqrt[1 + (c*x^4)/a])))/16
```

fricas [A] time = 0.59, size = 132, normalized size = 1.86

$$\left[\frac{3\,a^2\sqrt{c}\,\log\left(-2\,cx^4-2\,\sqrt{cx^4+a}\,\sqrt{c}\,x^2-a\right)+2\left(2\,c^2x^6+5\,acx^2\right)\sqrt{cx^4+a}}{32\,c},-\frac{3\,a^2\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x^2}{\sqrt{cx^4+a}}\right)-\left(2\,c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*a^2*sqrt(c)*log(-2*c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(c)*x^2 - a) + 2*
(2*c^2*x^6 + 5*a*c*x^2)*sqrt(c*x^4 + a))/c, -1/16*(3*a^2*sqrt(-c)*arctan(sq
rt(-c)*x^2/sqrt(c*x^4 + a)) - (2*c^2*x^6 + 5*a*c*x^2)*sqrt(c*x^4 + a))/c]
```

giac [A] time = 0.21, size = 99, normalized size = 1.39

$$\frac{1}{4}\left(\sqrt{cx^4+a}\,x^2-\frac{a\log\left(\left|-\sqrt{c}\,x^2+\sqrt{cx^4+a}\right|\right)}{\sqrt{c}}\right)a+\frac{1}{16}\left(\sqrt{cx^4+a}\left(2\,x^4+\frac{a}{c}\right)x^2+\frac{a^2\log\left(\left|-\sqrt{c}\,x^2+\sqrt{cx^4+a}\right|\right)}{c^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(c*x^4 + a)*x^2 - a*log(abs(-sqrt(c)*x^2 + sqrt(c*x^4 + a)))/sqrt(
c))*a + 1/16*(sqrt(c*x^4 + a)*(2*x^4 + a/c)*x^2 + a^2*log(abs(-sqrt(c)*x^2
+ sqrt(c*x^4 + a)))/c^(3/2))*c
```

maple [A] time = 0.01, size = 58, normalized size = 0.82

$$\frac{\sqrt{c\,x^4+a}\;c\,x^6}{8}+\frac{5\sqrt{c\,x^4+a}\;a\,x^2}{16}+\frac{3a^2\ln\left(\sqrt{c}\;x^2+\sqrt{c\,x^4+a}\right)}{16\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+a)^(3/2),x)
```

```
[Out] 1/8*c*x^6*(c*x^4+a)^(1/2)+5/16*a*x^2*(c*x^4+a)^(1/2)+3/16*a^2*ln(c^(1/2)*x^
2+(c*x^4+a)^(1/2))/c^(1/2)
```

maxima [B] time = 2.99, size = 119, normalized size = 1.68

$$-\frac{3\,a^2\log\left(-\frac{\sqrt{c}-\frac{\sqrt{cx^4+a}}{x^2}}{\sqrt{c}+\frac{\sqrt{cx^4+a}}{x^2}}\right)}{32\sqrt{c}}-\frac{\frac{3\sqrt{cx^4+a}\,a^2c}{x^2}-\frac{5\left(cx^4+a\right)^{\frac{3}{2}}a^2}{x^6}}{16\left(c^2-\frac{2\left(cx^4+a\right)c}{x^4}+\frac{\left(cx^4+a\right)^2}{x^8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -3/32*a^2*log(-(sqrt(c) - sqrt(c*x^4 + a)/x^2)/(sqrt(c) + sqrt(c*x^4 + a)/x
^2))/sqrt(c) - 1/16*(3*sqrt(c*x^4 + a)*a^2*c/x^2 - 5*(c*x^4 + a)^(3/2)*a^2/
x^6)/(c^2 - 2*(c*x^4 + a)*c/x^4 + (c*x^4 + a)^2/x^8)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(c\,x^4+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + c*x^4)^(3/2),x)
```

```
[Out] int(x*(a + c*x^4)^(3/2), x)
```

sympy [A] time = 4.59, size = 73, normalized size = 1.03

$$\frac{5a^{\frac{3}{2}}x^2\sqrt{1+\frac{cx^4}{a}}}{16}+\frac{\sqrt{a}\,cx^6\sqrt{1+\frac{cx^4}{a}}}{8}+\frac{3a^2\operatorname{asinh}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{16\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+a)**(3/2),x)
```

```
[Out] 5*a**(3/2)*x**2*sqrt(1 + c*x**4/a)/16 + sqrt(a)*c*x**6*sqrt(1 + c*x**4/a)/8
 + 3*a**2*asinh(sqrt(c)*x**2/sqrt(a))/(16*sqrt(c))
```

3.791 $\int \frac{\left(a+cx^4\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=69

$$\frac{3}{4}cx^2\sqrt{a+cx^4}-\frac{\left(a+cx^4\right)^{3/2}}{2x^2}+\frac{3}{4}a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)$$

```
[Out] -1/2*(c*x^4+a)^(3/2)/x^2+3/4*a*arctanh(x^2*c^(1/2)/(c*x^4+a)^(1/2))*c^(1/2)
+3/4*c*x^2*(c*x^4+a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {275, 277, 195, 217, 206}

$$\frac{3}{4}cx^2\sqrt{a+cx^4}-\frac{\left(a+cx^4\right)^{3/2}}{2x^2}+\frac{3}{4}a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^(3/2)/x^3,x]
```

```
[Out] (3*c*x^2*Sqrt[a + c*x^4])/4 - (a + c*x^4)^(3/2)/(2*x^2) + (3*a*Sqrt[c]*ArcT
anh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/4
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\left(a+cx^4\right)^{3/2}}{x^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a+cx^2\right)^{3/2}}{x^2}\,dx, x, x^2\right)$$
$$= -\frac{\left(a+cx^4\right)^{3/2}}{2x^2} + \frac{1}{2}(3c)\,\text{Subst}\left(\int \sqrt{a+cx^2}\,dx, x, x^2\right)$$
$$= \frac{3}{4}cx^2\sqrt{a+cx^4} - \frac{\left(a+cx^4\right)^{3/2}}{2x^2} + \frac{1}{4}(3ac)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+cx^2}}\,dx, x, x^2\right)$$
$$= \frac{3}{4}cx^2\sqrt{a+cx^4} - \frac{\left(a+cx^4\right)^{3/2}}{2x^2} + \frac{1}{4}(3ac)\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{a+cx^4}}\right)$$
$$= \frac{3}{4}cx^2\sqrt{a+cx^4} - \frac{\left(a+cx^4\right)^{3/2}}{2x^2} + \frac{3}{4}a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.75

$$-\frac{a\sqrt{a+cx^4}\;{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{cx^4}{a}\right)}{2x^2\sqrt{\frac{cx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^(3/2)/x^3,x]
```

```
[Out] -1/2*(a*Sqrt[a + c*x^4]*Hypergeometric2F1[-3/2, -1/2, 1/2, -((c*x^4)/a)])/(
x^2*Sqrt[1 + (c*x^4)/a])
```

fricas [A] time = 0.64, size = 120, normalized size = 1.74

$$\left[\frac{3\,a\sqrt{c}\,x^2\log\left(-2\,cx^4-2\sqrt{cx^4+a}\,\sqrt{c}\,x^2-a\right)+2\sqrt{cx^4+a}\left(cx^4-2\,a\right)}{8\,x^2}, -\frac{3\,a\sqrt{-c}\,x^2\arctan\left(\frac{\sqrt{-c}\,x^2}{\sqrt{cx^4+a}}\right)-\sqrt{cx^4+a}\,(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(3*a*sqrt(c)*x^2*log(-2*c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(c)*x^2 - a) + 2
*sqrt(c*x^4 + a)*(c*x^4 - 2*a))/x^2, -1/4*(3*a*sqrt(-c)*x^2*arctan(sqrt(-c)
*x^2/sqrt(c*x^4 + a)) - sqrt(c*x^4 + a)*(c*x^4 - 2*a))/x^2]
```

giac [A] time = 0.22, size = 78, normalized size = 1.13

$$\frac{1}{4}\sqrt{cx^4+a}\,cx^2 - \frac{3}{8}\,a\sqrt{c}\,\log\left(\left(\sqrt{c}\,x^2-\sqrt{cx^4+a}\right)^2\right) + \frac{a^2\sqrt{c}}{\left(\sqrt{c}\,x^2-\sqrt{cx^4+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(c*x^4 + a)*c*x^2 - 3/8*a*sqrt(c)*log((sqrt(c)*x^2 - sqrt(c*x^4 + a
))^2) + a^2*sqrt(c)/((sqrt(c)*x^2 - sqrt(c*x^4 + a))^2 - a)
```

maple [A] time = 0.02, size = 56, normalized size = 0.81

$$\frac{\sqrt{c\,x^4+a}\;c\,x^2}{4} + \frac{3a\sqrt{c}\;\ln\left(\sqrt{c}\;x^2+\sqrt{c\,x^4+a}\right)}{4} - \frac{\sqrt{c\,x^4+a}\;a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^(3/2)/x^3,x)
```

```
[Out] 1/4*c*x^2*(c*x^4+a)^(1/2)+3/4*a*c^(1/2)*ln(c^(1/2)*x^2+(c*x^4+a)^(1/2))-1/2
*a/x^2*(c*x^4+a)^(1/2)
```

maxima [A] time = 2.99, size = 94, normalized size = 1.36

$$-\frac{3}{8}\,a\sqrt{c}\,\log\left(-\frac{\sqrt{c}-\frac{\sqrt{cx^4+a}}{x^2}}{\sqrt{c}+\frac{\sqrt{cx^4+a}}{x^2}}\right)-\frac{\sqrt{cx^4+a}\,a}{2\,x^2}-\frac{\sqrt{cx^4+a}\,ac}{4\left(c-\frac{cx^4+a}{x^4}\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] -3/8*a*sqrt(c)*log(-(sqrt(c) - sqrt(c*x^4 + a)/x^2)/(sqrt(c) + sqrt(c*x^4 +
 a)/x^2)) - 1/2*sqrt(c*x^4 + a)*a/x^2 - 1/4*sqrt(c*x^4 + a)*a*c/((c - (c*x^
4 + a)/x^4)*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4+a\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^(3/2)/x^3,x)
```

```
[Out] int((a + c*x^4)^(3/2)/x^3, x)
```

sympy [A] time = 2.88, size = 95, normalized size = 1.38

$$-\frac{a^{\frac{3}{2}}}{2x^2\sqrt{1+\frac{cx^4}{a}}}-\frac{\sqrt{a}\,cx^2}{4\sqrt{1+\frac{cx^4}{a}}}+\frac{3a\sqrt{c}\,\operatorname{asinh}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4}+\frac{c^2x^6}{4\sqrt{a}\,\sqrt{1+\frac{cx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**(3/2)/x**3,x)
```

```
[Out] -a**(3/2)/(2*x**2*sqrt(1 + c*x**4/a)) - sqrt(a)*c*x**2/(4*sqrt(1 + c*x**4/a
)) + 3*a*sqrt(c)*asinh(sqrt(c)*x**2/sqrt(a))/4 + c**2*x**6/(4*sqrt(a)*sqrt(
1 + c*x**4/a))
```

3.792 $\int \frac{\left(a+cx^4\right)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=68

$$\frac{1}{2}c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)-\frac{\left(a+cx^4\right)^{3/2}}{6x^6}-\frac{c\sqrt{a+cx^4}}{2x^2}$$

```
[Out] -1/6*(c*x^4+a)^(3/2)/x^6+1/2*c^(3/2)*arctanh(x^2*c^(1/2)/(c*x^4+a)^(1/2))-1
/2*c*(c*x^4+a)^(1/2)/x^2
```

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {275, 277, 217, 206}

$$\frac{1}{2}c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)-\frac{c\sqrt{a+cx^4}}{2x^2}-\frac{\left(a+cx^4\right)^{3/2}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^(3/2)/x^7,x]
```

```
[Out] -(c*Sqrt[a + c*x^4])/(2*x^2) - (a + c*x^4)^(3/2)/(6*x^6) + (c^(3/2)*ArcTanh
[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+cx^4\right)^{3/2}}{x^7}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a+cx^2\right)^{3/2}}{x^4}\,dx, x, x^2\right)\\ &= -\frac{\left(a+cx^4\right)^{3/2}}{6x^6} + \frac{1}{2}c\,\text{Subst}\left(\int \frac{\sqrt{a+cx^2}}{x^2}\,dx, x, x^2\right)\\ &= -\frac{c\sqrt{a+cx^4}}{2x^2} - \frac{\left(a+cx^4\right)^{3/2}}{6x^6} + \frac{1}{2}c^2\,\text{Subst}\left(\int \frac{1}{\sqrt{a+cx^2}}\,dx, x, x^2\right)\\ &= -\frac{c\sqrt{a+cx^4}}{2x^2} - \frac{\left(a+cx^4\right)^{3/2}}{6x^6} + \frac{1}{2}c^2\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{a+cx^4}}\right)\\ &= -\frac{c\sqrt{a+cx^4}}{2x^2} - \frac{\left(a+cx^4\right)^{3/2}}{6x^6} + \frac{1}{2}c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.76

$$-\frac{a\sqrt{a+cx^4}\;{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};-\frac{cx^4}{a}\right)}{6x^6\sqrt{\frac{cx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^(3/2)/x^7,x]
```

```
[Out] -1/6*(a*Sqrt[a + c*x^4]*Hypergeometric2F1[-3/2, -3/2, -1/2, -((c*x^4)/a)])/
(x^6*Sqrt[1 + (c*x^4)/a])
```

fricas [A] time = 0.63, size = 116, normalized size = 1.71

$$\left[\frac{3\,c^{\frac{3}{2}}x^6\log\left(-2\,cx^4-2\sqrt{cx^4+a}\,\sqrt{c}\,x^2-a\right)-2\left(4\,cx^4+a\right)\sqrt{cx^4+a}}{12\,x^6}, -\frac{3\sqrt{-c}\,cx^6\arctan\left(\frac{\sqrt{-c}\,x^2}{\sqrt{cx^4+a}}\right)+\left(4\,cx^4+a\right.}{6\,x^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/12*(3*c^(3/2)*x^6*log(-2*c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(c)*x^2 - a) - 2*
(4*c*x^4 + a)*sqrt(c*x^4 + a))/x^6, -1/6*(3*sqrt(-c)*c*x^6*arctan(sqrt(-c)*
x^2/sqrt(c*x^4 + a)) + (4*c*x^4 + a)*sqrt(c*x^4 + a))/x^6]
```

giac [B] time = 0.23, size = 122, normalized size = 1.79

$$-\frac{1}{4}\,c^{\frac{3}{2}}\log\left(\left(\sqrt{c}\,x^2-\sqrt{cx^4+a}\right)^2\right)+\frac{2\left(3\left(\sqrt{c}\,x^2-\sqrt{cx^4+a}\right)^4 ac^{\frac{3}{2}}-3\left(\sqrt{c}\,x^2-\sqrt{cx^4+a}\right)^2 a^2c^{\frac{3}{2}}+2\,a^3c^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{c}\,x^2-\sqrt{cx^4+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/4*c^(3/2)*log((sqrt(c)*x^2 - sqrt(c*x^4 + a))^2) + 2/3*(3*(sqrt(c)*x^2 -
 sqrt(c*x^4 + a))^4*a*c^(3/2) - 3*(sqrt(c)*x^2 - sqrt(c*x^4 + a))^2*a^2*c^(
3/2) + 2*a^3*c^(3/2))/((sqrt(c)*x^2 - sqrt(c*x^4 + a))^2 - a)^3
```

maple [A] time = 0.02, size = 55, normalized size = 0.81

$$\frac{c^{\frac{3}{2}} \ln\left(\sqrt{c}\, x^2 + \sqrt{c\, x^4 + a}\right)}{2} - \frac{2\sqrt{c\, x^4 + a}\, c}{3x^2} - \frac{\sqrt{c\, x^4 + a}\, a}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^(3/2)/x^7,x)
```

```
[Out] 1/2*c^(3/2)*ln(c^(1/2)*x^2+(c*x^4+a)^(1/2))-1/6*a/x^6*(c*x^4+a)^(1/2)-2/3*c
*(c*x^4+a)^(1/2)/x^2
```

maxima [A] time = 3.04, size = 75, normalized size = 1.10

$$-\frac{1}{4} c^{\frac{3}{2}} \log\left(-\frac{\sqrt{c} - \frac{\sqrt{cx^4+a}}{x^2}}{\sqrt{c} + \frac{\sqrt{cx^4+a}}{x^2}}\right) - \frac{\sqrt{cx^4 + a}\, c}{2\, x^2} - \frac{\left(cx^4 + a\right)^{\frac{3}{2}}}{6\, x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/4*c^(3/2)*log(-(sqrt(c) - sqrt(c*x^4 + a)/x^2)/(sqrt(c) + sqrt(c*x^4 + a
)/x^2)) - 1/2*sqrt(c*x^4 + a)*c/x^2 - 1/6*(c*x^4 + a)^(3/2)/x^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\, x^4 + a\right)^{3/2}}{x^7}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^(3/2)/x^7,x)
```

```
[Out] int((a + c*x^4)^(3/2)/x^7, x)
```

sympy [A] time = 4.48, size = 80, normalized size = 1.18

$$-\frac{a\sqrt{c}\sqrt{\frac{a}{cx^4}+1}}{6x^4} - \frac{2c^{\frac{3}{2}}\sqrt{\frac{a}{cx^4}+1}}{3} - \frac{c^{\frac{3}{2}}\log\left(\frac{a}{cx^4}\right)}{4} + \frac{c^{\frac{3}{2}}\log\left(\sqrt{\frac{a}{cx^4}+1}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**(3/2)/x**7,x)
```

```
[Out] -a*sqrt(c)*sqrt(a/(c*x**4) + 1)/(6*x**4) - 2*c**(3/2)*sqrt(a/(c*x**4) + 1)/
3 - c**(3/2)*log(a/(c*x**4))/4 + c**(3/2)*log(sqrt(a/(c*x**4) + 1) + 1)/2
```

3.793 $\int \frac{\left(a+cx^4\right)^{3/2}}{x^{11}}\,dx$

Optimal. Leaf size=21

$$-\frac{\left(a+cx^4\right)^{5/2}}{10ax^{10}}$$

```
[Out] -1/10*(c*x^4+a)^(5/2)/a/x^10
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {264}

$$-\frac{\left(a+cx^4\right)^{5/2}}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^(3/2)/x^11,x]
```

```
[Out] -(a + c*x^4)^(5/2)/(10*a*x^10)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+cx^4\right)^{3/2}}{x^{11}}\,dx = -\frac{\left(a+cx^4\right)^{5/2}}{10ax^{10}}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$-\frac{\left(a+cx^4\right)^{5/2}}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^(3/2)/x^11,x]
```

```
[Out] -1/10*(a + c*x^4)^(5/2)/(a*x^10)
```

fricas [B] time = 0.87, size = 35, normalized size = 1.67

$$-\frac{\left(c^2x^8+2\,acx^4+a^2\right)\sqrt{cx^4+a}}{10\,ax^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(3/2)/x^11,x, algorithm="fricas")
```

```
[Out] -1/10*(c^2*x^8 + 2*a*c*x^4 + a^2)*sqrt(c*x^4 + a)/(a*x^10)
```

giac [B] time = 0.23, size = 92, normalized size = 4.38

$$\frac{5\left(\sqrt{c}\,x^2-\sqrt{cx^4+a}\right)^8 c^{\frac{5}{2}}+10\left(\sqrt{c}\,x^2-\sqrt{cx^4+a}\right)^4 a^2c^{\frac{5}{2}}+a^4c^{\frac{5}{2}}}{5\left(\left(\sqrt{c}\,x^2-\sqrt{cx^4+a}\right)^2-a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(3/2)/x^11,x, algorithm="giac")
```

```
[Out] 1/5*(5*(sqrt(c)*x^2 - sqrt(c*x^4 + a))^8*c^(5/2) + 10*(sqrt(c)*x^2 - sqrt(c
*x^4 + a))^4*a^2*c^(5/2) + a^4*c^(5/2))/((sqrt(c)*x^2 - sqrt(c*x^4 + a))^2
- a)^5
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$-\frac{\left(c\,x^4+a\right)^{\frac{5}{2}}}{10a\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^(3/2)/x^11,x)
```

```
[Out] -1/10*(c*x^4+a)^(5/2)/a/x^10
```

maxima [A] time = 1.35, size = 17, normalized size = 0.81

$$-\frac{\left(cx^4+a\right)^{\frac{5}{2}}}{10\,ax^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(3/2)/x^11,x, algorithm="maxima")
```

```
[Out] -1/10*(c*x^4 + a)^(5/2)/(a*x^10)
```

mupad [B] time = 1.52, size = 17, normalized size = 0.81

$$-\frac{\left(c\,x^4+a\right)^{5/2}}{10\,a\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^(3/2)/x^11,x)
```

```
[Out] -(a + c*x^4)^(5/2)/(10*a*x^10)
```

sympy [B] time = 2.50, size = 66, normalized size = 3.14

$$-\frac{a\sqrt{c}\sqrt{\frac{a}{cx^4}+1}}{10x^8}-\frac{c^{\frac{3}{2}}\sqrt{\frac{a}{cx^4}+1}}{5x^4}-\frac{c^{\frac{5}{2}}\sqrt{\frac{a}{cx^4}+1}}{10a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**(3/2)/x**11,x)
```

```
[Out] -a*sqrt(c)*sqrt(a/(c*x**4) + 1)/(10*x**8) - c**(3/2)*sqrt(a/(c*x**4) + 1)/(
5*x**4) - c**(5/2)*sqrt(a/(c*x**4) + 1)/(10*a)
```

3.794 $\int \frac{\left(a+cx^4\right)^{3/2}}{x^{15}}\,dx$

Optimal. Leaf size=44

$$\frac{c\left(a+cx^4\right)^{5/2}}{35a^2x^{10}}-\frac{\left(a+cx^4\right)^{5/2}}{14ax^{14}}$$

```
[Out] -1/14*(c*x^4+a)^(5/2)/a/x^14+1/35*c*(c*x^4+a)^(5/2)/a^2/x^10
```

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$\frac{c\left(a+cx^4\right)^{5/2}}{35a^2x^{10}}-\frac{\left(a+cx^4\right)^{5/2}}{14ax^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^(3/2)/x^15,x]
```

```
[Out] -(a + c*x^4)^(5/2)/(14*a*x^14) + (c*(a + c*x^4)^(5/2))/(35*a^2*x^10)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+cx^4\right)^{3/2}}{x^{15}}\,dx &= -\frac{\left(a+cx^4\right)^{5/2}}{14ax^{14}}-\frac{(2c)\int \frac{\left(a+cx^4\right)^{3/2}}{x^{11}}\,dx}{7a}\\ &= -\frac{\left(a+cx^4\right)^{5/2}}{14ax^{14}}+\frac{c\left(a+cx^4\right)^{5/2}}{35a^2x^{10}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 0.70

$$\frac{\left(a+cx^4\right)^{5/2}\left(2cx^4-5a\right)}{70a^2x^{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^(3/2)/x^15,x]
```

```
[Out] ((a + c*x^4)^(5/2)*(-5*a + 2*c*x^4))/(70*a^2*x^14)
```

fricas [A] time = 1.15, size = 49, normalized size = 1.11

$$\frac{\left(2\,c^3x^{12}-ac^2x^8-8\,a^2cx^4-5\,a^3\right)\sqrt{cx^4+a}}{70\,a^2x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(3/2)/x^15,x, algorithm="fricas")
```

```
[Out] 1/70*(2*c^3*x^12 - a*c^2*x^8 - 8*a^2*c*x^4 - 5*a^3)*sqrt(c*x^4 + a)/(a^2*x^
14)
```

giac [B] time = 0.24, size = 178, normalized size = 4.05

$$\frac{2\left(35\left(\sqrt{c}\,x^2-\sqrt{cx^4+a}\right)^{10}c^{\frac{7}{2}}+35\left(\sqrt{c}\,x^2-\sqrt{cx^4+a}\right)^{8}ac^{\frac{7}{2}}+70\left(\sqrt{c}\,x^2-\sqrt{cx^4+a}\right)^{6}a^2c^{\frac{7}{2}}+14\left(\sqrt{c}\,x^2-\sqrt{cx^4+\right.\right.}}{35\left(\left(\sqrt{c}\,x^2-\sqrt{cx^4+a}\right)^2-a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(3/2)/x^15,x, algorithm="giac")
```

```
[Out] 2/35*(35*(sqrt(c)*x^2 - sqrt(c*x^4 + a))^10*c^(7/2) + 35*(sqrt(c)*x^2 - sqr
t(c*x^4 + a))^8*a*c^(7/2) + 70*(sqrt(c)*x^2 - sqrt(c*x^4 + a))^6*a^2*c^(7/2
) + 14*(sqrt(c)*x^2 - sqrt(c*x^4 + a))^4*a^3*c^(7/2) + 7*(sqrt(c)*x^2 - sqr
t(c*x^4 + a))^2*a^4*c^(7/2) - a^5*c^(7/2))/((sqrt(c)*x^2 - sqrt(c*x^4 + a))
^2 - a)^7
```

maple [A] time = 0.00, size = 28, normalized size = 0.64

$$-\frac{\left(c\,x^4+a\right)^{\frac{5}{2}}\left(-2c\,x^4+5a\right)}{70a^2x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^(3/2)/x^15,x)
```

```
[Out] -1/70*(c*x^4+a)^(5/2)*(-2*c*x^4+5*a)/x^14/a^2
```

maxima [A] time = 1.40, size = 35, normalized size = 0.80

$$\frac{\frac{7\left(cx^4+a\right)^{\frac{5}{2}}c}{x^{10}}-\frac{5\left(cx^4+a\right)^{\frac{7}{2}}}{x^{14}}}{70\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(3/2)/x^15,x, algorithm="maxima")
```

```
[Out] 1/70*(7*(c*x^4 + a)^(5/2)*c/x^10 - 5*(c*x^4 + a)^(7/2)/x^14)/a^2
```

mupad [B] time = 1.76, size = 71, normalized size = 1.61

$$\frac{c^3\sqrt{c\,x^4+a}}{35\,a^2\,x^2}-\frac{4\,c\sqrt{c\,x^4+a}}{35\,x^{10}}-\frac{a\sqrt{c\,x^4+a}}{14\,x^{14}}-\frac{c^2\sqrt{c\,x^4+a}}{70\,a\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^(3/2)/x^15,x)
```

```
[Out] (c^3*(a + c*x^4)^(1/2))/(35*a^2*x^2) - (4*c*(a + c*x^4)^(1/2))/(35*x^10) -
(a*(a + c*x^4)^(1/2))/(14*x^14) - (c^2*(a + c*x^4)^(1/2))/(70*a*x^6)
```

sympy [B] time = 2.35, size = 92, normalized size = 2.09

$$-\frac{a\sqrt{c}\sqrt{\frac{a}{cx^4}+1}}{14x^{12}}-\frac{4c^{\frac{3}{2}}\sqrt{\frac{a}{cx^4}+1}}{35x^8}-\frac{c^{\frac{5}{2}}\sqrt{\frac{a}{cx^4}+1}}{70ax^4}+\frac{c^{\frac{7}{2}}\sqrt{\frac{a}{cx^4}+1}}{35a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**(3/2)/x**15,x)
```

```
[Out] -a*sqrt(c)*sqrt(a/(c*x**4) + 1)/(14*x**12) - 4*c**(3/2)*sqrt(a/(c*x**4) + 1
)/(35*x**8) - c**(5/2)*sqrt(a/(c*x**4) + 1)/(70*a*x**4) + c**(7/2)*sqrt(a/(
c*x**4) + 1)/(35*a**2)
```

3.795 $\int \frac{\left(a+cx^4\right)^{3/2}}{x^{19}}\,dx$

Optimal. Leaf size=68

$$-\frac{4c^2\left(a+cx^4\right)^{5/2}}{315a^3x^{10}}+\frac{2c\left(a+cx^4\right)^{5/2}}{63a^2x^{14}}-\frac{\left(a+cx^4\right)^{5/2}}{18ax^{18}}$$

```
[Out] -1/18*(c*x^4+a)^(5/2)/a/x^18+2/63*c*(c*x^4+a)^(5/2)/a^2/x^14-4/315*c^2*(c*x
^4+a)^(5/2)/a^3/x^10
```

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$-\frac{4c^2\left(a+cx^4\right)^{5/2}}{315a^3x^{10}}+\frac{2c\left(a+cx^4\right)^{5/2}}{63a^2x^{14}}-\frac{\left(a+cx^4\right)^{5/2}}{18ax^{18}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^(3/2)/x^19,x]
```

```
[Out] -(a + c*x^4)^(5/2)/(18*a*x^18) + (2*c*(a + c*x^4)^(5/2))/(63*a^2*x^14) - (4
*c^2*(a + c*x^4)^(5/2))/(315*a^3*x^10)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+cx^4\right)^{3/2}}{x^{19}}\,dx &= -\frac{\left(a+cx^4\right)^{5/2}}{18ax^{18}}-\frac{(4c)\int \frac{\left(a+cx^4\right)^{3/2}}{x^{15}}\,dx}{9a}\\ &= -\frac{\left(a+cx^4\right)^{5/2}}{18ax^{18}}+\frac{2c\left(a+cx^4\right)^{5/2}}{63a^2x^{14}}+\frac{\left(8c^2\right)\int \frac{\left(a+cx^4\right)^{3/2}}{x^{11}}\,dx}{63a^2}\\ &= -\frac{\left(a+cx^4\right)^{5/2}}{18ax^{18}}+\frac{2c\left(a+cx^4\right)^{5/2}}{63a^2x^{14}}-\frac{4c^2\left(a+cx^4\right)^{5/2}}{315a^3x^{10}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.62

$$-\frac{\left(a+cx^4\right)^{5/2}\left(35a^2-20acx^4+8c^2x^8\right)}{630a^3x^{18}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^(3/2)/x^19,x]
```

```
[Out] -1/630*((a + c*x^4)^(5/2)*(35*a^2 - 20*a*c*x^4 + 8*c^2*x^8))/(a^3*x^18)
```

fricas [A] time = 0.56, size = 60, normalized size = 0.88

$$-\frac{\left(8\,c^4x^{16}-4\,ac^3x^{12}+3\,a^2c^2x^8+50\,a^3cx^4+35\,a^4\right)\sqrt{cx^4+a}}{630\,a^3x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(3/2)/x^19,x, algorithm="fricas")
```

```
[Out] -1/630*(8*c^4*x^16 - 4*a*c^3*x^12 + 3*a^2*c^2*x^8 + 50*a^3*c*x^4 + 35*a^4)*
sqrt(c*x^4 + a)/(a^3*x^18)
```

giac [B] time = 0.25, size = 206, normalized size = 3.03

$$\frac{8\left(210\left(\sqrt{c}\,x^2-\sqrt{cx^4+a}\right)^{12}c^{\frac{9}{2}}+315\left(\sqrt{c}\,x^2-\sqrt{cx^4+a}\right)^{10}ac^{\frac{9}{2}}+441\left(\sqrt{c}\,x^2-\sqrt{cx^4+a}\right)^{8}a^2c^{\frac{9}{2}}+126\left(\sqrt{c}\,x^2\right.\right.}{315\left(\left(\sqrt{c}\,x^2-\sqrt{cx^4+a}\right)^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(3/2)/x^19,x, algorithm="giac")
```

```
[Out] 8/315*(210*(sqrt(c)*x^2 - sqrt(c*x^4 + a))^12*c^(9/2) + 315*(sqrt(c)*x^2 -
sqrt(c*x^4 + a))^10*a*c^(9/2) + 441*(sqrt(c)*x^2 - sqrt(c*x^4 + a))^8*a^2*c
^(9/2) + 126*(sqrt(c)*x^2 - sqrt(c*x^4 + a))^6*a^3*c^(9/2) + 36*(sqrt(c)*x^
2 - sqrt(c*x^4 + a))^4*a^4*c^(9/2) - 9*(sqrt(c)*x^2 - sqrt(c*x^4 + a))^2*a^
5*c^(9/2) + a^6*c^(9/2))/((sqrt(c)*x^2 - sqrt(c*x^4 + a))^2 - a)^9
```

maple [A] time = 0.01, size = 39, normalized size = 0.57

$$-\frac{\left(c\,x^4+a\right)^{\frac{5}{2}}\left(8c^2x^8-20ac\,x^4+35a^2\right)}{630a^3x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^(3/2)/x^19,x)
```

```
[Out] -1/630*(c*x^4+a)^(5/2)*(8*c^2*x^8-20*a*c*x^4+35*a^2)/x^18/a^3
```

maxima [A] time = 1.33, size = 52, normalized size = 0.76

$$-\frac{\frac{63\left(cx^4+a\right)^{\frac{5}{2}}c^2}{x^{10}}-\frac{90\left(cx^4+a\right)^{\frac{7}{2}}c}{x^{14}}+\frac{35\left(cx^4+a\right)^{\frac{9}{2}}}{x^{18}}}{630\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(3/2)/x^19,x, algorithm="maxima")
```

```
[Out] -1/630*(63*(c*x^4 + a)^(5/2)*c^2/x^10 - 90*(c*x^4 + a)^(7/2)*c/x^14 + 35*(c
*x^4 + a)^(9/2)/x^18)/a^3
```

mupad [B] time = 2.07, size = 91, normalized size = 1.34

$$\frac{2\,c^3\sqrt{c\,x^4+a}}{315\,a^2\,x^6}-\frac{5\,c\sqrt{c\,x^4+a}}{63\,x^{14}}-\frac{4\,c^4\sqrt{c\,x^4+a}}{315\,a^3\,x^2}-\frac{a\sqrt{c\,x^4+a}}{18\,x^{18}}-\frac{c^2\sqrt{c\,x^4+a}}{210\,a\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^(3/2)/x^19,x)
```

```
[Out] (2*c^3*(a + c*x^4)^(1/2))/(315*a^2*x^6) - (5*c*(a + c*x^4)^(1/2))/(63*x^14)
 - (4*c^4*(a + c*x^4)^(1/2))/(315*a^3*x^2) - (a*(a + c*x^4)^(1/2))/(18*x^18
) - (c^2*(a + c*x^4)^(1/2))/(210*a*x^10)
```

sympy [B] time = 5.41, size = 420, normalized size = 6.18

$$-\frac{35a^6c^{\frac{9}{2}}\sqrt{\frac{a}{cx^4}+1}}{630a^5c^4x^{16}+1260a^4c^5x^{20}+630a^3c^6x^{24}}-\frac{120a^5c^{\frac{11}{2}}x^4\sqrt{\frac{a}{cx^4}+1}}{630a^5c^4x^{16}+1260a^4c^5x^{20}+630a^3c^6x^{24}}-\frac{138a^4c^{\frac{13}{2}}x^8\sqrt{\frac{a}{cx^4}+}}{630a^5c^4x^{16}+1260a^4c^5x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**(3/2)/x**19,x)
```

```
[Out] -35*a**6*c**(9/2)*sqrt(a/(c*x**4) + 1)/(630*a**5*c**4*x**16 + 1260*a**4*c**
5*x**20 + 630*a**3*c**6*x**24) - 120*a**5*c**(11/2)*x**4*sqrt(a/(c*x**4) +
1)/(630*a**5*c**4*x**16 + 1260*a**4*c**5*x**20 + 630*a**3*c**6*x**24) - 138
*a**4*c**(13/2)*x**8*sqrt(a/(c*x**4) + 1)/(630*a**5*c**4*x**16 + 1260*a**4*
c**5*x**20 + 630*a**3*c**6*x**24) - 52*a**3*c**(15/2)*x**12*sqrt(a/(c*x**4)
 + 1)/(630*a**5*c**4*x**16 + 1260*a**4*c**5*x**20 + 630*a**3*c**6*x**24) -
3*a**2*c**(17/2)*x**16*sqrt(a/(c*x**4) + 1)/(630*a**5*c**4*x**16 + 1260*a**
4*c**5*x**20 + 630*a**3*c**6*x**24) - 12*a*c**(19/2)*x**20*sqrt(a/(c*x**4)
+ 1)/(630*a**5*c**4*x**16 + 1260*a**4*c**5*x**20 + 630*a**3*c**6*x**24) - 8
*c**(21/2)*x**24*sqrt(a/(c*x**4) + 1)/(630*a**5*c**4*x**16 + 1260*a**4*c**5
*x**20 + 630*a**3*c**6*x**24)
```

3.796 $\int x^4 \left(a+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=148

$$-\frac{2a^{11/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{77c^{5/4}\sqrt{a+cx^4}}+\frac{4a^2x\sqrt{a+cx^4}}{77c}+\frac{1}{11}x^5\left(a+cx^4\right)^{3/2}+\frac{6}{77}ax^5\sqrt{a+cx^4}$$

```
[Out] 1/11*x^5*(c*x^4+a)^(3/2)+4/77*a^2*x*(c*x^4+a)^(1/2)/c+6/77*a*x^5*(c*x^4+a)^
(1/2)-2/77*a^(11/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan
(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2)
)*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(5/4)/(
c*x^4+a)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {279, 321, 220}

$$-\frac{2a^{11/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{77c^{5/4}\sqrt{a+cx^4}}+\frac{4a^2x\sqrt{a+cx^4}}{77c}+\frac{1}{11}x^5\left(a+cx^4\right)^{3/2}+\frac{6}{77}ax^5\sqrt{a+cx^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + c*x^4)^(3/2),x]
```

```
[Out] (4*a^2*x*Sqrt[a + c*x^4])/(77*c) + (6*a*x^5*Sqrt[a + c*x^4])/77 + (x^5*(a +
 c*x^4)^(3/2))/11 - (2*a^(11/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(S
qrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(77
*c^(5/4)*Sqrt[a + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int x^4\left(a+cx^4\right)^{3/2}\,dx &= \frac{1}{11}x^5\left(a+cx^4\right)^{3/2}+\frac{1}{11}(6a)\int x^4\sqrt{a+cx^4}\,dx\\ &= \frac{6}{77}ax^5\sqrt{a+cx^4}+\frac{1}{11}x^5\left(a+cx^4\right)^{3/2}+\frac{1}{77}\left(12a^2\right)\int\frac{x^4}{\sqrt{a+cx^4}}\,dx\\ &= \frac{4a^2x\sqrt{a+cx^4}}{77c}+\frac{6}{77}ax^5\sqrt{a+cx^4}+\frac{1}{11}x^5\left(a+cx^4\right)^{3/2}-\frac{\left(4a^3\right)\int\frac{1}{\sqrt{a+cx^4}}\,dx}{77c}\\ &= \frac{4a^2x\sqrt{a+cx^4}}{77c}+\frac{6}{77}ax^5\sqrt{a+cx^4}+\frac{1}{11}x^5\left(a+cx^4\right)^{3/2}-\frac{2a^{11/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}}{77c^{5/4}\sqrt{a+cx}}\end{aligned}$$

Mathematica [C] time = 0.10, size = 67, normalized size = 0.45

$$\frac{x\sqrt{a+cx^4}\left(\left(a+cx^4\right)^2-\frac{a^2\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^4}{a}\right)}{\sqrt{\frac{cx^4}{a}+1}}\right)}{11c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + c*x^4)^(3/2),x]
```

```
[Out] (x*Sqrt[a + c*x^4]*((a + c*x^4)^2 - (a^2*Hypergeometric2F1[-3/2, 1/4, 5/4,
-((c*x^4)/a)])/Sqrt[1 + (c*x^4)/a]))/(11*c)
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^8+ax^4\right)\sqrt{cx^4+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^8 + a*x^4)*sqrt(c*x^4 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(cx^4+a\right)^{\frac{3}{2}}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + a)^(3/2)*x^4, x)
```

maple [C] time = 0.01, size = 126, normalized size = 0.85

$$\frac{\sqrt{c\,x^4+a}\;c\,x^9}{11}+\frac{13\sqrt{c\,x^4+a}\;a\,x^5}{77}-\frac{4\sqrt{-\frac{i\sqrt{c}\;x^2}{\sqrt{a}}+1}\;\sqrt{\frac{i\sqrt{c}\;x^2}{\sqrt{a}}+1}\;a^3\,\text{EllipticF}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\;x,i\right)}{77\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\;\sqrt{c\,x^4+a}\;c}+\frac{4\sqrt{c\,x^4+a}\;a^2x}{77c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c*x^4+a)^(3/2),x)
```

```
[Out] 1/11*c*x^9*(c*x^4+a)^(1/2)+13/77*a*x^5*(c*x^4+a)^(1/2)+4/77*a^2*x*(c*x^4+a)
^(1/2)/c-4/77*a^3/c/(I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1
/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)*EllipticF((I/a^(1/2)*c^
(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4 + a\right)^{\frac{3}{2}} x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + a)^(3/2)*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \left(c\,x^4 + a\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + c*x^4)^(3/2),x)
```

```
[Out] int(x^4*(a + c*x^4)^(3/2), x)
```

sympy [C] time = 1.77, size = 39, normalized size = 0.26

$$\frac{a^{\frac{3}{2}} x^5 \Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{5}{4} \\ \frac{9}{4} \end{matrix} \middle| \frac{cx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c*x**4+a)**(3/2),x)
```

```
[Out] a**(3/2)*x**5*gamma(5/4)*hyper((-3/2, 5/4), (9/4,), c*x**4*exp_polar(I*pi)/
a)/(4*gamma(9/4))
```

3.797 $\int \left(a+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=122

$$\frac{2a^{7/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{c}\,\sqrt{a+cx^4}}+\frac{2}{7}ax\sqrt{a+cx^4}+\frac{1}{7}x\left(a+cx^4\right)^{3/2}$$

```
[Out] 1/7*x*(c*x^4+a)^(3/2)+2/7*a*x*(c*x^4+a)^(1/2)+2/7*a^(7/4)*(cos(2*arctan(c^(
1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*
arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a
^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(1/4)/(c*x^4+a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {195, 220}

$$\frac{2a^{7/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{c}\,\sqrt{a+cx^4}}+\frac{2}{7}ax\sqrt{a+cx^4}+\frac{1}{7}x\left(a+cx^4\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^(3/2),x]
```

```
[Out] (2*a*x*Sqrt[a + c*x^4])/7 + (x*(a + c*x^4)^(3/2))/7 + (2*a^(7/4)*(Sqrt[a] +
 Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTa
n[(c^(1/4)*x)/a^(1/4)], 1/2])/(7*c^(1/4)*Sqrt[a + c*x^4])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \left(a+cx^4\right)^{3/2}\,dx &= \frac{1}{7}x\left(a+cx^4\right)^{3/2}+\frac{1}{7}(6a)\int\sqrt{a+cx^4}\,dx\\ &= \frac{2}{7}ax\sqrt{a+cx^4}+\frac{1}{7}x\left(a+cx^4\right)^{3/2}+\frac{1}{7}\left(4a^2\right)\int\frac{1}{\sqrt{a+cx^4}}\,dx\\ &= \frac{2}{7}ax\sqrt{a+cx^4}+\frac{1}{7}x\left(a+cx^4\right)^{3/2}+\frac{2a^{7/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{c}\,\sqrt{a+cx^4}}\end{aligned}$$

Mathematica [C] time = 0.02, size = 47, normalized size = 0.39

$$\frac{ax\sqrt{a+cx^4}\ {}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^4}{a}\right)}{\sqrt{\frac{cx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^(3/2),x]
```

```
[Out] (a*x*Sqrt[a + c*x^4]*Hypergeometric2F1[-3/2, 1/4, 5/4, -((c*x^4)/a)])/Sqrt[
1 + (c*x^4)/a]
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^4+a\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + a)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + a)^(3/2), x)
```

maple [C] time = 0.00, size = 103, normalized size = 0.84

$$\frac{\sqrt{c\,x^4+a}\ c\,x^5}{7}+\frac{4\sqrt{-\frac{i\sqrt{c}\ x^2}{\sqrt{a}}+1}\ \sqrt{\frac{i\sqrt{c}\ x^2}{\sqrt{a}}+1}\ a^2\,\text{EllipticF}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\ x,i\right)}{7\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\ \sqrt{c\,x^4+a}}+\frac{3\sqrt{c\,x^4+a}\ ax}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^(3/2),x)
```

```
[Out] 1/7*c*x^5*(c*x^4+a)^(1/2)+3/7*a*x*(c*x^4+a)^(1/2)+4/7*a^2/(I/a^(1/2)*c^(1/2
))^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/(
c*x^4+a)^(1/2)*EllipticF((I/a^(1/2)*c^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + a)^(3/2), x)
```

mupad [B] time = 1.04, size = 37, normalized size = 0.30

$$\frac{x\left(c\,x^4+a\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{4};\ \frac{5}{4};\ -\frac{c\,x^4}{a}\right)}{\left(\frac{c\,x^4}{a}+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^(3/2),x)
```

```
[Out] (x*(a + c*x^4)^(3/2)*hypergeom([-3/2, 1/4], 5/4, -(c*x^4)/a))/((c*x^4)/a +
1)^(3/2)
```

sympy [C] time = 1.25, size = 37, normalized size = 0.30

$$\frac{a^{\frac{3}{2}}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**(3/2),x)
```

```
[Out] a**(3/2)*x*gamma(1/4)*hyper((-3/2, 1/4), (5/4,), c*x**4*exp_polar(I*pi)/a)/
(4*gamma(5/4))
```

3.798 $\int \frac{\left(a+cx^4\right)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=124

$$\frac{2a^{3/4}c^{3/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt{a+cx^4}}+\frac{2}{3}cx\sqrt{a+cx^4}-\frac{\left(a+cx^4\right)^{3/2}}{3x^3}$$

```
[Out] -1/3*(c*x^4+a)^(3/2)/x^3+2/3*c*x*(c*x^4+a)^(1/2)+2/3*a^(3/4)*c^(3/4)*(cos(2
*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Ellip
ticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((
c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/(c*x^4+a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {277, 195, 220}

$$\frac{2a^{3/4}c^{3/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt{a+cx^4}}-\frac{\left(a+cx^4\right)^{3/2}}{3x^3}+\frac{2}{3}cx\sqrt{a+cx^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^(3/2)/x^4,x]
```

```
[Out] (2*c*x*Sqrt[a + c*x^4])/3 - (a + c*x^4)^(3/2)/(3*x^3) + (2*a^(3/4)*c^(3/4)*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipti
cF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(3*Sqrt[a + c*x^4])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\left(a+cx^4\right)^{3/2}}{x^4}\,dx = -\frac{\left(a+cx^4\right)^{3/2}}{3x^3} + (2c)\int \sqrt{a+cx^4}\,dx$$
$$= \frac{2}{3}cx\sqrt{a+cx^4} - \frac{\left(a+cx^4\right)^{3/2}}{3x^3} + \frac{1}{3}(4ac)\int \frac{1}{\sqrt{a+cx^4}}\,dx$$
$$= \frac{2}{3}cx\sqrt{a+cx^4} - \frac{\left(a+cx^4\right)^{3/2}}{3x^3} + \frac{2a^{3/4}c^{3/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt{a+cx^4}}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.42

$$-\frac{a\sqrt{a+cx^4}\;{}_2F_1\left(-\frac{3}{2},-\frac{3}{4};\frac{1}{4};-\frac{cx^4}{a}\right)}{3x^3\sqrt{\frac{cx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^(3/2)/x^4,x]
```

```
[Out] -1/3*(a*Sqrt[a + c*x^4]*Hypergeometric2F1[-3/2, -3/4, 1/4, -((c*x^4)/a)])/(
x^3*Sqrt[1 + (c*x^4)/a])
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(cx^4+a\right)^{\frac{3}{2}}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + a)^(3/2)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4+a\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + a)^(3/2)/x^4, x)
```

maple [C] time = 0.01, size = 102, normalized size = 0.82

$$\frac{4\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,ac\,\text{EllipticF}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x, i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}} + \frac{\sqrt{c\,x^4+a}\,cx}{3} - \frac{\sqrt{c\,x^4+a}\,a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^(3/2)/x^4,x)
```

```
[Out] -1/3*a*(c*x^4+a)^(1/2)/x^3+1/3*c*x*(c*x^4+a)^(1/2)+4/3*a*c/(I/a^(1/2)*c^(1/
2))^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)/
(c*x^4+a)^(1/2)*EllipticF((I/a^(1/2)*c^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4+a\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + a)^(3/2)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4+a\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^(3/2)/x^4,x)
```

```
[Out] int((a + c*x^4)^(3/2)/x^4, x)
```

sympy [C] time = 2.12, size = 42, normalized size = 0.34

$$\frac{a^{\frac{3}{2}}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{2}, -\frac{3}{4} \\ \frac{1}{4}\end{matrix}\right| \frac{cx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**(3/2)/x**4,x)
```

```
[Out] a**(3/2)*gamma(-3/4)*hyper((-3/2, -3/4), (1/4,), c*x**4*exp_polar(I*pi)/a)/
(4*x**3*gamma(1/4))
```

3.799 $\int \frac{\left(a+cx^4\right)^{3/2}}{x^8}\,dx$

Optimal. Leaf size=126

$$\frac{2c^{7/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{a}\,\sqrt{a+cx^4}}-\frac{\left(a+cx^4\right)^{3/2}}{7x^7}-\frac{2c\sqrt{a+cx^4}}{7x^3}$$

```
[Out] -1/7*(c*x^4+a)^(3/2)/x^7-2/7*c*(c*x^4+a)^(1/2)/x^3+2/7*c^(7/4)*(cos(2*arcta
n(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(s
in(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+
a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/(c*x^4+a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {277, 220}

$$\frac{2c^{7/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{a}\,\sqrt{a+cx^4}}-\frac{2c\sqrt{a+cx^4}}{7x^3}-\frac{\left(a+cx^4\right)^{3/2}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^(3/2)/x^8,x]
```

```
[Out] (-2*c*Sqrt[a + c*x^4])/(7*x^3) - (a + c*x^4)^(3/2)/(7*x^7) + (2*c^(7/4)*(Sq
rt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[
2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(7*a^(1/4)*Sqrt[a + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+cx^4\right)^{3/2}}{x^8}\,dx &= -\frac{\left(a+cx^4\right)^{3/2}}{7x^7}+\frac{1}{7}(6c)\int \frac{\sqrt{a+cx^4}}{x^4}\,dx\\ &= -\frac{2c\sqrt{a+cx^4}}{7x^3}-\frac{\left(a+cx^4\right)^{3/2}}{7x^7}+\frac{1}{7}\left(4c^2\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx\\ &= -\frac{2c\sqrt{a+cx^4}}{7x^3}-\frac{\left(a+cx^4\right)^{3/2}}{7x^7}+\frac{2c^{7/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{a}\,\sqrt{a+cx^4}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.41

$$-\frac{a\sqrt{a+cx^4}\ {}_2F_1\left(-\frac{7}{4},-\frac{3}{2};-\frac{3}{4};-\frac{cx^4}{a}\right)}{7x^7\sqrt{\frac{cx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^(3/2)/x^8,x]
```

```
[Out] -1/7*(a*Sqrt[a + c*x^4]*Hypergeometric2F1[-7/4, -3/2, -3/4, -((c*x^4)/a)])/
(x^7*Sqrt[1 + (c*x^4)/a])
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(cx^4+a\right)^{\frac{3}{2}}}{x^8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + a)^(3/2)/x^8, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4+a\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + a)^(3/2)/x^8, x)
```

maple [C] time = 0.01, size = 105, normalized size = 0.83

$$\frac{4\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\ \sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\ c^2\,\text{EllipticF}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\ x,i\right)}{7\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\ \sqrt{c\,x^4+a}}-\frac{3\sqrt{c\,x^4+a}\ c}{7x^3}-\frac{\sqrt{c\,x^4+a}\ a}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^(3/2)/x^8,x)
```

```
[Out] -1/7*a*(c*x^4+a)^(1/2)/x^7-3/7*c*(c*x^4+a)^(1/2)/x^3+4/7*c^2/(I/a^(1/2)*c^(
1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2+1)^(1/2
)/(c*x^4+a)^(1/2)*EllipticF((I/a^(1/2)*c^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4+a\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + a)^(3/2)/x^8, x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4+a\right)^{3/2}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^(3/2)/x^8,x)
```

```
[Out] int((a + c*x^4)^(3/2)/x^8, x)
```

sympy **[C]** time = 2.47, size = 46, normalized size = 0.37

$$\frac{a^{\frac{3}{2}}\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4}, -\frac{3}{2} \\ -\frac{3}{4}\end{matrix}\middle| \frac{cx^4e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**(3/2)/x**8,x)
```

```
[Out] a**(3/2)*gamma(-7/4)*hyper((-7/4, -3/2), (-3/4,), c*x**4*exp_polar(I*pi)/a)
/(4*x**7*gamma(-3/4))
```

3.800 $\int x^2 \left(a+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=255

$$\frac{2a^{9/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{a+cx^4}}-\frac{4a^{9/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{a+cx^4}}$$

```
[Out] 1/9*x^3*(c*x^4+a)^(3/2)+2/15*a*x^3*(c*x^4+a)^(1/2)+4/15*a^2*x*(c*x^4+a)^(1/
2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-4/15*a^(9/4)*(cos(2*arctan(c^(1/4)*x/a^(1/
4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/
4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c
^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+a)^(1/2)+2/15*a^(9/4)*(cos(2*arctan(c^(1/4)
*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arct
an(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/
2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+a)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {279, 305, 220, 1196}

$$\frac{2a^{9/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{a+cx^4}}-\frac{4a^{9/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + c*x^4)^(3/2),x]
```

```
[Out] (2*a*x^3*Sqrt[a + c*x^4])/15 + (4*a^2*x*Sqrt[a + c*x^4])/(15*Sqrt[c]*(Sqrt[
a] + Sqrt[c]*x^2)) + (x^3*(a + c*x^4)^(3/2))/9 - (4*a^(9/4)*(Sqrt[a] + Sqrt
[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^
(1/4)*x)/a^(1/4)], 1/2])/(15*c^(3/4)*Sqrt[a + c*x^4]) + (2*a^(9/4)*(Sqrt[a]
 + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*Arc
Tan[(c^(1/4)*x)/a^(1/4)], 1/2])/(15*c^(3/4)*Sqrt[a + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+cx^4\right)^{3/2}\,dx &= \frac{1}{9}x^3\left(a+cx^4\right)^{3/2}+\frac{1}{3}(2a)\int x^2\sqrt{a+cx^4}\,dx\\ &= \frac{2}{15}ax^3\sqrt{a+cx^4}+\frac{1}{9}x^3\left(a+cx^4\right)^{3/2}+\frac{1}{15}\left(4a^2\right)\int\frac{x^2}{\sqrt{a+cx^4}}\,dx\\ &= \frac{2}{15}ax^3\sqrt{a+cx^4}+\frac{1}{9}x^3\left(a+cx^4\right)^{3/2}+\frac{\left(4a^{5/2}\right)\int\frac{1}{\sqrt{a+cx^4}}\,dx}{15\sqrt{c}}-\frac{\left(4a^{5/2}\right)\int\frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{15\sqrt{c}}\\ &= \frac{2}{15}ax^3\sqrt{a+cx^4}+\frac{4a^2x\sqrt{a+cx^4}}{15\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{1}{9}x^3\left(a+cx^4\right)^{3/2}-\frac{4a^{9/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx}{(\sqrt{a}+\sqrt{}}}}{15c^{3/4}\sqrt{a}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.20

$$\frac{ax^3\sqrt{a+cx^4}\;{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^4}{a}\right)}{3\sqrt{\frac{cx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + c*x^4)^(3/2),x]
```

```
[Out] (a*x^3*Sqrt[a + c*x^4]*Hypergeometric2F1[-3/2, 3/4, 7/4, -((c*x^4)/a)])/(3*
Sqrt[1 + (c*x^4)/a])
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^6+ax^2\right)\sqrt{cx^4+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + a*x^2)*sqrt(c*x^4 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(cx^4+a\right)^{\frac{3}{2}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + a)^(3/2)*x^2, x)
```

maple [C] time = 0.01, size = 128, normalized size = 0.50

$$\frac{\sqrt{c\,x^4+a}\;c\,x^7}{9}+\frac{11\sqrt{c\,x^4+a}\;a\,x^3}{45}+\frac{4i\sqrt{-\frac{i\sqrt{c}\;x^2}{\sqrt{a}}+1}\;\sqrt{\frac{i\sqrt{c}\;x^2}{\sqrt{a}}+1}\;\left(-\text{EllipticE}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\;x,i\right)+\text{EllipticF}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\right.\right.}{15\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\;\sqrt{c\,x^4+a}\;\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+a)^(3/2),x)
```

```
[Out] 1/9*c*x^7*(c*x^4+a)^(1/2)+11/45*a*x^3*(c*x^4+a)^(1/2)+4/15*I*a^(5/2)/(I/a^(
1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^2
+1)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF((I/a^(1/2)*c^(1/2))^(1/2)*x,I)
-EllipticE((I/a^(1/2)*c^(1/2))^(1/2)*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^4+a\right)^{\frac{3}{2}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + a)^(3/2)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\left(c\,x^4+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + c*x^4)^(3/2),x)
```

```
[Out] int(x^2*(a + c*x^4)^(3/2), x)
```

sympy [C] time = 1.87, size = 39, normalized size = 0.15

$$\frac{a^{\frac{3}{2}}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+a)**(3/2),x)
```

```
[Out] a**(3/2)*x**3*gamma(3/4)*hyper((-3/2, 3/4), (7/4,), c*x**4*exp_polar(I*pi)/
a)/(4*gamma(7/4))
```

3.801 $\int \frac{\left(a+cx^4\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=251

$$\frac{6a^{5/4}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+cx^4}} - \frac{12a^{5/4}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\right.}{5\sqrt{a+cx^4}}$$

```
[Out] -(c*x^4+a)^(3/2)/x+6/5*c*x^3*(c*x^4+a)^(1/2)+12/5*a*x*c^(1/2)*(c*x^4+a)^(1/
2)/(a^(1/2)+x^2*c^(1/2))-12/5*a^(5/4)*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/
4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/
4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c
^(1/2))^2)^(1/2)/(c*x^4+a)^(1/2)+6/5*a^(5/4)*c^(1/4)*(cos(2*arctan(c^(1/4)*
x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arcta
n(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2
)+x^2*c^(1/2))^2)^(1/2)/(c*x^4+a)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {277, 279, 305, 220, 1196}

$$\frac{6a^{5/4}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+cx^4}} - \frac{12a^{5/4}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\right.}{5\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^(3/2)/x^2,x]
```

```
[Out] (6*c*x^3*Sqrt[a + c*x^4])/5 + (12*a*Sqrt[c]*x*Sqrt[a + c*x^4])/(5*(Sqrt[a]
+ Sqrt[c]*x^2)) - (a + c*x^4)^(3/2)/x - (12*a^(5/4)*c^(1/4)*(Sqrt[a] + Sqrt
[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^
(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + c*x^4]) + (6*a^(5/4)*c^(1/4)*(Sqrt[a]
+ Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcT
an[(c^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+cx^4\right)^{3/2}}{x^2}\,dx &= -\frac{\left(a+cx^4\right)^{3/2}}{x} + (6c)\int x^2\sqrt{a+cx^4}\,dx \\
&= \frac{6}{5}cx^3\sqrt{a+cx^4} - \frac{\left(a+cx^4\right)^{3/2}}{x} + \frac{1}{5}(12ac)\int \frac{x^2}{\sqrt{a+cx^4}}\,dx \\
&= \frac{6}{5}cx^3\sqrt{a+cx^4} - \frac{\left(a+cx^4\right)^{3/2}}{x} + \frac{1}{5}\left(12a^{3/2}\sqrt{c}\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx - \frac{1}{5}\left(12a^{3/2}\sqrt{c}\right)\int \frac{1-\frac{\sqrt{c}}{\sqrt{a}}}{\sqrt{a+c}} \\
&= \frac{6}{5}cx^3\sqrt{a+cx^4} + \frac{12a\sqrt{c}\,x\sqrt{a+cx^4}}{5\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{\left(a+cx^4\right)^{3/2}}{x} - \frac{12a^{5/4}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right.}}}{5\sqrt{a+cx^4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 50, normalized size = 0.20

$$-\frac{a\sqrt{a+cx^4}\;{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};\frac{3}{4};-\frac{cx^4}{a}\right)}{x\sqrt{\frac{cx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^(3/2)/x^2,x]
```

```
[Out] -((a*Sqrt[a + c*x^4]*Hypergeometric2F1[-3/2, -1/4, 3/4, -((c*x^4)/a)])/(x*S
qrt[1 + (c*x^4)/a]))
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(cx^4+a\right)^{\frac{3}{2}}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + a)^(3/2)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4+a\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + a)^(3/2)/x^2, x)
```

maple [C] time = 0.01, size = 128, normalized size = 0.51

$$\frac{\sqrt{c\,x^4+a}\;c\,x^3}{5}+\frac{12i\sqrt{-\frac{i\sqrt{c}\;x^2}{\sqrt{a}}+1}\;\sqrt{\frac{i\sqrt{c}\;x^2}{\sqrt{a}}+1}\left(-\text{EllipticE}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\;x,i\right)+\text{EllipticF}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\;x,i\right)\right)a^{\frac{3}{2}}\sqrt{c}}{5\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\;\sqrt{c\,x^4+a}}-\frac{\sqrt{c\,x^4+}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^(3/2)/x^2,x)
```

```
[Out] -a*(c*x^4+a)^(1/2)/x+1/5*c*x^3*(c*x^4+a)^(1/2)+12/5*I*a^(3/2)*c^(1/2)/(I/a^
(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2)*x^
2+1)^(1/2)/(c*x^4+a)^(1/2)*(EllipticF((I/a^(1/2)*c^(1/2))^(1/2)*x,I)-Ellipt
icE((I/a^(1/2)*c^(1/2))^(1/2)*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4+a\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + a)^(3/2)/x^2, x)
```

mupad [B] time = 1.50, size = 40, normalized size = 0.16

$$\frac{\left(c\,x^4+a\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{5}{4};\;-\frac{1}{4};\;-\frac{a}{c\,x^4}\right)}{5\,x\left(\frac{a}{c\,x^4}+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^(3/2)/x^2,x)
```

```
[Out] ((a + c*x^4)^(3/2)*hypergeom([-3/2, -5/4], -1/4, -a/(c*x^4)))/(5*x*(a/(c*x^
4) + 1)^(3/2))
```

sympy [C] time = 1.65, size = 41, normalized size = 0.16

$$\frac{a^{\frac{3}{2}}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**(3/2)/x**2,x)
```

```
[Out] a**(3/2)*gamma(-1/4)*hyper((-3/2, -1/4), (3/4,), c*x**4*exp_polar(I*pi)/a)/
(4*x*gamma(3/4))
```

3.802 $\int \frac{\left(a+cx^4\right)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=252

$$\frac{12c^{3/2}x\sqrt{a+cx^4}}{5\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{6\sqrt[4]{a}\,c^{5/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+cx^4}}-\frac{12\sqrt[4]{a}\,c^{5/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c}{(\sqrt{a}+}}}{5\sqrt{a+c}}$$

```
[Out] -1/5*(c*x^4+a)^(3/2)/x^5-6/5*c*(c*x^4+a)^(1/2)/x+12/5*c^(3/2)*x*(c*x^4+a)^(
1/2)/(a^(1/2)+x^2*c^(1/2))-12/5*a^(1/4)*c^(5/4)*(cos(2*arctan(c^(1/4)*x/a^(
1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(
1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2
*c^(1/2))^2)^(1/2)/(c*x^4+a)^(1/2)+6/5*a^(1/4)*c^(5/4)*(cos(2*arctan(c^(1/4
)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arc
tan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1
/2)+x^2*c^(1/2))^2)^(1/2)/(c*x^4+a)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {277, 305, 220, 1196}

$$\frac{12c^{3/2}x\sqrt{a+cx^4}}{5\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{6\sqrt[4]{a}\,c^{5/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+cx^4}}-\frac{12\sqrt[4]{a}\,c^{5/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c}{(\sqrt{a}+}}}{5\sqrt{a+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^(3/2)/x^6,x]
```

```
[Out] (-6*c*Sqrt[a + c*x^4])/(5*x) + (12*c^(3/2)*x*Sqrt[a + c*x^4])/(5*(Sqrt[a] +
 Sqrt[c]*x^2)) - (a + c*x^4)^(3/2)/(5*x^5) - (12*a^(1/4)*c^(5/4)*(Sqrt[a] +
 Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTa
n[(c^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + c*x^4]) + (6*a^(1/4)*c^(5/4)*(Sqr
t[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2
*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + c*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+cx^4\right)^{3/2}}{x^6}\,dx &= -\frac{\left(a+cx^4\right)^{3/2}}{5x^5} + \frac{1}{5}(6c)\int \frac{\sqrt{a+cx^4}}{x^2}\,dx \\
&= -\frac{6c\sqrt{a+cx^4}}{5x} - \frac{\left(a+cx^4\right)^{3/2}}{5x^5} + \frac{1}{5}\left(12c^2\right)\int \frac{x^2}{\sqrt{a+cx^4}}\,dx \\
&= -\frac{6c\sqrt{a+cx^4}}{5x} - \frac{\left(a+cx^4\right)^{3/2}}{5x^5} + \frac{1}{5}\left(12\sqrt{a}\,c^{3/2}\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx - \frac{1}{5}\left(12\sqrt{a}\,c^{3/2}\right)\int \frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,d \\
&= -\frac{6c\sqrt{a+cx^4}}{5x} + \frac{12c^{3/2}x\sqrt{a+cx^4}}{5\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{\left(a+cx^4\right)^{3/2}}{5x^5} - \frac{12\sqrt[4]{a}\,c^{5/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(}{5\sqrt{a+cx^4}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 52, normalized size = 0.21

$$-\frac{a\sqrt{a+cx^4}\;{}_2F_1\left(-\frac{3}{2},-\frac{5}{4};-\frac{1}{4};-\frac{cx^4}{a}\right)}{5x^5\sqrt{\frac{cx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^(3/2)/x^6,x]
```

```
[Out] -1/5*(a*Sqrt[a + c*x^4]*Hypergeometric2F1[-3/2, -5/4, -1/4, -((c*x^4)/a)])/
(x^5*Sqrt[1 + (c*x^4)/a])
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(cx^4+a\right)^{\frac{3}{2}}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + a)^(3/2)/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4+a\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + a)^(3/2)/x^6, x)
```

maple [C] time = 0.02, size = 128, normalized size = 0.51

$$\frac{12i\sqrt{-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{c}\,x^2}{\sqrt{a}}+1}\,\left(-\text{EllipticE}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x,i\right)+\text{EllipticF}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x,i\right)\right)\sqrt{a}\,c^{\frac{3}{2}}}{5\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}-\frac{7\sqrt{c\,x^4+a}\,c}{5x}-\frac{\sqrt{c\,x^4}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^(3/2)/x^6,x)
```

```
[Out] -1/5*a*(c*x^4+a)^(1/2)/x^5-7/5*c*(c*x^4+a)^(1/2)/x+12/5*I*c^(3/2)*a^(1/2)/(
I/a^(1/2)*c^(1/2))^(1/2)*(-I/a^(1/2)*c^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*c^(1/2
)*x^2+1)^(1/2)/(c*x^4+a)^(1/2)*(EllipticF((I/a^(1/2)*c^(1/2))^(1/2)*x,I)-El
lipticE((I/a^(1/2)*c^(1/2))^(1/2)*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^4+a\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + a)^(3/2)/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^4+a\right)^{3/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^(3/2)/x^6,x)
```

```
[Out] int((a + c*x^4)^(3/2)/x^6, x)
```

sympy [C] time = 1.44, size = 46, normalized size = 0.18

$$\frac{a^{\frac{3}{2}}\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{2},-\frac{5}{4}\\-\frac{1}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**(3/2)/x**6,x)
```

```
[Out] a**(3/2)*gamma(-5/4)*hyper((-3/2, -5/4), (-1/4,), c*x**4*exp_polar(I*pi)/a)
/(4*x**5*gamma(-1/4))
```

3.803 $\int \left(1+x^4\right)^{3/2}\,dx$

Optimal. Leaf size=72

$$\frac{1}{7}x\left(x^4+1\right)^{3/2}+\frac{2}{7}x\sqrt{x^4+1}+\frac{2\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{7\sqrt{x^4+1}}$$

```
[Out] 1/7*x*(x^4+1)^(3/2)+2/7*x*(x^4+1)^(1/2)+2/7*(x^2+1)*(cos(2*arctan(x))^2)^(1
/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2*2^(1/2))*((x^4+1)/(x^2+
1)^2)^(1/2)/(x^4+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {195, 220}

$$\frac{1}{7}x\left(x^4+1\right)^{3/2}+\frac{2}{7}x\sqrt{x^4+1}+\frac{2\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{7\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)^(3/2),x]
```

```
[Out] (2*x*Sqrt[1 + x^4])/7 + (x*(1 + x^4)^(3/2))/7 + (2*(1 + x^2)*Sqrt[(1 + x^4)
/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(7*Sqrt[1 + x^4])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \left(1+x^4\right)^{3/2}\,dx &= \frac{1}{7}x\left(1+x^4\right)^{3/2}+\frac{6}{7}\int \sqrt{1+x^4}\,dx\\ &= \frac{2}{7}x\sqrt{1+x^4}+\frac{1}{7}x\left(1+x^4\right)^{3/2}+\frac{4}{7}\int \frac{1}{\sqrt{1+x^4}}\,dx\\ &= \frac{2}{7}x\sqrt{1+x^4}+\frac{1}{7}x\left(1+x^4\right)^{3/2}+\frac{2\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{7\sqrt{1+x^4}}\end{aligned}$$

Mathematica [C] time = 0.00, size = 17, normalized size = 0.24

$$x\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)^(3/2),x]
```

```
[Out] x*Hypergeometric2F1[-3/2, 1/4, 5/4, -x^4]
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(x^4+1\right)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((x^4 + 1)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^4+1\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 1)^(3/2), x)
```

maple [C] time = 0.05, size = 84, normalized size = 1.17

$$\frac{\sqrt{x^4+1}\, x^5}{7} + \frac{3\sqrt{x^4+1}\, x}{7} + \frac{4\sqrt{-ix^2+1}\, \sqrt{ix^2+1}\, \text{EllipticF}\left(\left(\frac{\sqrt{2}}{2} + \frac{i\sqrt{2}}{2}\right)x, i\right)}{7\left(\frac{\sqrt{2}}{2} + \frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)^(3/2),x)
```

```
[Out] 1/7*x^5*(x^4+1)^(1/2)+3/7*x*(x^4+1)^(1/2)+4/7/(1/2*2^(1/2)+1/2*I*2^(1/2))*(
1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticF(x*(1/2*2^(1/2)+1/2*I
*2^(1/2)),I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^4+1\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)^(3/2), x)
```

mupad [B] time = 1.02, size = 12, normalized size = 0.17

$$x\,{}_2\mathrm{F}_1\left(-\frac{3}{2}, \frac{1}{4};\ \frac{5}{4};\ -x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)^(3/2),x)
```

```
[Out] x*hypergeom([-3/2, 1/4], 5/4, -x^4)
```

sympy [C] time = 1.01, size = 29, normalized size = 0.40

$$\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{2}, \frac{1}{4} \\ \frac{5}{4}\end{matrix}\right| x^4 e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)**(3/2),x)
```

```
[Out] x*gamma(1/4)*hyper((-3/2, 1/4), (5/4,), x**4*exp_polar(I*pi))/(4*gamma(5/4)
)
```

3.804 $\int \left(1-x^4\right)^{3/2}\,dx$

Optimal. Leaf size=41

$$\frac{1}{7}x\left(1-x^4\right)^{3/2}+\frac{2}{7}x\sqrt{1-x^4}+\frac{4}{7}F\left(\sin^{-1}(x)\middle|-1\right)$$

```
[Out] 1/7*x*(-x^4+1)^(3/2)+4/7*EllipticF(x,I)+2/7*x*(-x^4+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {195, 221}

$$\frac{1}{7}x\left(1-x^4\right)^{3/2}+\frac{2}{7}x\sqrt{1-x^4}+\frac{4}{7}F\left(\sin^{-1}(x)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)^(3/2),x]
```

```
[Out] (2*x*Sqrt[1 - x^4])/7 + (x*(1 - x^4)^(3/2))/7 + (4*EllipticF[ArcSin[x], -1]
)/7
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \left(1-x^4\right)^{3/2}\,dx &= \frac{1}{7}x\left(1-x^4\right)^{3/2}+\frac{6}{7}\int \sqrt{1-x^4}\,dx\\ &= \frac{2}{7}x\sqrt{1-x^4}+\frac{1}{7}x\left(1-x^4\right)^{3/2}+\frac{4}{7}\int \frac{1}{\sqrt{1-x^4}}\,dx\\ &= \frac{2}{7}x\sqrt{1-x^4}+\frac{1}{7}x\left(1-x^4\right)^{3/2}+\frac{4}{7}F\left(\sin^{-1}(x)\middle|-1\right)\end{aligned}$$

Mathematica [C] time = 0.00, size = 15, normalized size = 0.37

$$x\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)^(3/2),x]
```

```
[Out] x*Hypergeometric2F1[-3/2, 1/4, 5/4, x^4]
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-x^4+1\right)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((-x^4 + 1)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-x^4+1\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + 1)^(3/2), x)
```

maple [A] time = 0.01, size = 59, normalized size = 1.44

$$-\frac{\sqrt{-x^4+1}\, x^5}{7}+\frac{3\sqrt{-x^4+1}\, x}{7}+\frac{4\sqrt{-x^2+1}\, \sqrt{x^2+1}\, \text{EllipticF}\left(x, i\right)}{7\sqrt{-x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)^(3/2),x)
```

```
[Out] -1/7*x^5*(-x^4+1)^(1/2)+3/7*x*(-x^4+1)^(1/2)+4/7*(-x^2+1)^(1/2)*(x^2+1)^(1/
2)/(-x^4+1)^(1/2)*EllipticF(x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-x^4+1\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + 1)^(3/2), x)
```

mupad [B] time = 1.02, size = 10, normalized size = 0.24

$$x\, {}_2\mathrm{F}_1\left(-\frac{3}{2}, \frac{1}{4}; \frac{5}{4}; x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^4)^(3/2),x)
```

```
[Out] x*hypergeom([-3/2, 1/4], 5/4, x^4)
```

sympy [A] time = 1.31, size = 31, normalized size = 0.76

$$\frac{x\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{1}{4} \\ \frac{5}{4} \end{matrix}\middle| x^4 e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)**(3/2),x)
```

```
[Out] x*gamma(1/4)*hyper((-3/2, 1/4), (5/4,), x**4*exp_polar(2*I*pi))/(4*gamma(5/
4))
```

3.805 $\int x^7\sqrt{5+3x^4}\,dx$

Optimal. Leaf size=31

$$\frac{1}{90}\left(3x^4+5\right)^{5/2}-\frac{5}{54}\left(3x^4+5\right)^{3/2}$$

```
[Out] -5/54*(3*x^4+5)^(3/2)+1/90*(3*x^4+5)^(5/2)
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{1}{90}\left(3x^4+5\right)^{5/2}-\frac{5}{54}\left(3x^4+5\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*Sqrt[5 + 3*x^4],x]
```

```
[Out] (-5*(5 + 3*x^4)^(3/2))/54 + (5 + 3*x^4)^(5/2)/90
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^7\sqrt{5+3x^4}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int x\sqrt{5+3x}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int\left(-\frac{5}{3}\sqrt{5+3x}+\frac{1}{3}(5+3x)^{3/2}\right)dx, x, x^4\right)\\ &= -\frac{5}{54}\left(5+3x^4\right)^{3/2}+\frac{1}{90}\left(5+3x^4\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.71

$$\frac{1}{270}\left(3x^4+5\right)^{3/2}\left(9x^4-10\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*Sqrt[5 + 3*x^4],x]
```

```
[Out] ((5 + 3*x^4)^(3/2)*(-10 + 9*x^4))/270
```

fricas [A] time = 1.00, size = 23, normalized size = 0.74

$$\frac{1}{270}\left(27\,x^8+15\,x^4-50\right)\sqrt{3\,x^4+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(3*x^4+5)^(1/2),x, algorithm="fricas")
[Out] 1/270*(27*x^8 + 15*x^4 - 50)*sqrt(3*x^4 + 5)
```

giac [A] time = 0.15, size = 23, normalized size = 0.74

$$\frac{1}{90}\left(3x^4+5\right)^{\frac{5}{2}}-\frac{5}{54}\left(3x^4+5\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(3*x^4+5)^(1/2),x, algorithm="giac")
[Out] 1/90*(3*x^4 + 5)^(5/2) - 5/54*(3*x^4 + 5)^(3/2)
```

maple [A] time = 0.00, size = 19, normalized size = 0.61

$$\frac{\left(3x^4+5\right)^{\frac{3}{2}}\left(9x^4-10\right)}{270}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(3*x^4+5)^(1/2),x)
[Out] 1/270*(3*x^4+5)^(3/2)*(9*x^4-10)
```

maxima [A] time = 1.33, size = 23, normalized size = 0.74

$$\frac{1}{90}\left(3x^4+5\right)^{\frac{5}{2}}-\frac{5}{54}\left(3x^4+5\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(3*x^4+5)^(1/2),x, algorithm="maxima")
[Out] 1/90*(3*x^4 + 5)^(5/2) - 5/54*(3*x^4 + 5)^(3/2)
```

mupad [B] time = 1.13, size = 18, normalized size = 0.58

$$\frac{\left(3x^4+5\right)^{3/2}\left(9x^4-10\right)}{270}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(3*x^4 + 5)^(1/2),x)
[Out] ((3*x^4 + 5)^(3/2)*(9*x^4 - 10))/270
```

sympy [A] time = 0.85, size = 42, normalized size = 1.35

$$\frac{x^8\sqrt{3x^4+5}}{10}+\frac{x^4\sqrt{3x^4+5}}{18}-\frac{5\sqrt{3x^4+5}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(3*x**4+5)**(1/2),x)
[Out] x**8*sqrt(3*x**4 + 5)/10 + x**4*sqrt(3*x**4 + 5)/18 - 5*sqrt(3*x**4 + 5)/27
```

3.806 $\int x^3\sqrt{5+x^4}\,dx$

Optimal. Leaf size=13

$$\frac{1}{6}\left(x^4+5\right)^{3/2}$$

```
[Out] 1/6*(x^4+5)^(3/2)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$\frac{1}{6}\left(x^4+5\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[5 + x^4],x]
```

```
[Out] (5 + x^4)^(3/2)/6
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^3\sqrt{5+x^4}\,dx = \frac{1}{6}\left(5+x^4\right)^{3/2}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{1}{6}\left(x^4+5\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[5 + x^4],x]
```

```
[Out] (5 + x^4)^(3/2)/6
```

fricas [A] time = 0.85, size = 9, normalized size = 0.69

$$\frac{1}{6}\left(x^4+5\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(x^4 + 5)^(3/2)
```

giac [A] time = 0.16, size = 9, normalized size = 0.69

$$\frac{1}{6}\left(x^4+5\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^4+5)^(1/2),x, algorithm="giac")
[Out] 1/6*(x^4 + 5)^(3/2)
```

maple [A] time = 0.00, size = 10, normalized size = 0.77

$$\frac{\left(x^4+5\right)^{\frac{3}{2}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^4+5)^(1/2),x)
[Out] 1/6*(x^4+5)^(3/2)
```

maxima [A] time = 1.35, size = 9, normalized size = 0.69

$$\frac{1}{6}\left(x^4+5\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^4+5)^(1/2),x, algorithm="maxima")
[Out] 1/6*(x^4 + 5)^(3/2)
```

mupad [B] time = 1.12, size = 9, normalized size = 0.69

$$\frac{\left(x^4+5\right)^{3/2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^4 + 5)^(1/2),x)
[Out] (x^4 + 5)^(3/2)/6
```

sympy [B] time = 0.32, size = 24, normalized size = 1.85

$$\frac{x^4\sqrt{x^4+5}}{6}+\frac{5\sqrt{x^4+5}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(x**4+5)**(1/2),x)
[Out] x**4*sqrt(x**4 + 5)/6 + 5*sqrt(x**4 + 5)/6
```

3.807 $\int x\sqrt{3+2x^4}\,dx$

Optimal. Leaf size=40

$$\frac{3\sinh^{-1}\left(\sqrt{\frac{2}{3}}\,x^2\right)}{4\sqrt{2}}+\frac{1}{4}\sqrt{2x^4+3}\,x^2$$

```
[Out] 3/8*arcsinh(1/3*x^2*6^(1/2))*2^(1/2)+1/4*x^2*(2*x^4+3)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {275, 195, 215}

$$\frac{1}{4}\sqrt{2x^4+3}\,x^2+\frac{3\sinh^{-1}\left(\sqrt{\frac{2}{3}}\,x^2\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[3 + 2*x^4],x]
```

```
[Out] (x^2*Sqrt[3 + 2*x^4])/4 + (3*ArcSinh[Sqrt[2/3]*x^2])/(4*Sqrt[2])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{3+2x^4}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \sqrt{3+2x^2}\,dx, x, x^2\right)\\ &= \frac{1}{4}x^2\sqrt{3+2x^4}+\frac{3}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt{3+2x^2}}\,dx, x, x^2\right)\\ &= \frac{1}{4}x^2\sqrt{3+2x^4}+\frac{3\sinh^{-1}\left(\sqrt{\frac{2}{3}}\,x^2\right)}{4\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 40, normalized size = 1.00

$$\frac{1}{8}\left(3\sqrt{2}\,\sinh^{-1}\left(\sqrt{\frac{2}{3}}\,x^2\right)+2\sqrt{2x^4+3}\,x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[3 + 2*x^4],x]
```

```
[Out] (2*x^2*Sqrt[3 + 2*x^4] + 3*Sqrt[2]*ArcSinh[Sqrt[2/3]*x^2])/8
```

fricas [A] time = 1.08, size = 45, normalized size = 1.12

$$\frac{1}{4}\sqrt{2\,x^4+3}\,x^2+\frac{3}{16}\sqrt{2}\,\log\left(-4\,x^4-2\sqrt{2}\,\sqrt{2\,x^4+3}\,x^2-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^4+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2*x^4 + 3)*x^2 + 3/16*sqrt(2)*log(-4*x^4 - 2*sqrt(2)*sqrt(2*x^4 +
3)*x^2 - 3)
```

giac [A] time = 0.18, size = 39, normalized size = 0.98

$$\frac{1}{4}\sqrt{2\,x^4+3}\,x^2-\frac{3}{8}\sqrt{2}\,\log\left(-\sqrt{2}\,x^2+\sqrt{2\,x^4+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^4+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2*x^4 + 3)*x^2 - 3/8*sqrt(2)*log(-sqrt(2)*x^2 + sqrt(2*x^4 + 3))
```

maple [A] time = 0.01, size = 30, normalized size = 0.75

$$\frac{\sqrt{2x^4+3}\;x^2}{4}+\frac{3\sqrt{2}\;\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x^2}{3}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*x^4+3)^(1/2),x)
```

```
[Out] 3/8*arcsinh(1/3*6^(1/2)*x^2)*2^(1/2)+1/4*x^2*(2*x^4+3)^(1/2)
```

maxima [B] time = 2.98, size = 75, normalized size = 1.88

$$-\frac{3}{16}\sqrt{2}\,\log\left(-\frac{\sqrt{2}-\frac{\sqrt{2\,x^4+3}}{x^2}}{\sqrt{2}+\frac{\sqrt{2\,x^4+3}}{x^2}}\right)+\frac{3\,\sqrt{2\,x^4+3}}{4\,x^2\left(\frac{2\,x^4+3}{x^4}-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^4+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/16*sqrt(2)*log(-(sqrt(2) - sqrt(2*x^4 + 3)/x^2)/(sqrt(2) + sqrt(2*x^4 +
3)/x^2)) + 3/4*sqrt(2*x^4 + 3)/(x^2*((2*x^4 + 3)/x^4 - 2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\sqrt{2\,x^4+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*x^4 + 3)^(1/2),x)
```

```
[Out] int(x*(2*x^4 + 3)^(1/2), x)
```

sympy [**A**] time = 2.76, size = 51, normalized size = 1.28

$$\frac{x^6}{2\sqrt{2x^4+3}} + \frac{3x^2}{4\sqrt{2x^4+3}} + \frac{3\sqrt{2}\,\operatorname{asinh}\left(\frac{\sqrt{6}\,x^2}{3}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x**4+3)**(1/2),x)
```

```
[Out] x**6/(2*sqrt(2*x**4 + 3)) + 3*x**2/(4*sqrt(2*x**4 + 3)) + 3*sqrt(2)*asinh(s
qrt(6)*x**2/3)/8
```

3.808 $\int x\sqrt{-2+x^4}\,dx$

Optimal. Leaf size=35

$$\frac{1}{4}x^2\sqrt{x^4-2}-\frac{1}{2}\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4-2}}\right)$$

```
[Out] -1/2*arctanh(x^2/(x^4-2)^(1/2))+1/4*x^2*(x^4-2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {275, 195, 217, 206}

$$\frac{1}{4}x^2\sqrt{x^4-2}-\frac{1}{2}\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4-2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[-2 + x^4],x]
```

```
[Out] (x^2*Sqrt[-2 + x^4])/4 - ArcTanh[x^2/Sqrt[-2 + x^4]]/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int x\sqrt{-2+x^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \sqrt{-2+x^2}\,dx, x, x^2\right)\\
&= \frac{1}{4}x^2\sqrt{-2+x^4}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-2+x^2}}\,dx, x, x^2\right)\\
&= \frac{1}{4}x^2\sqrt{-2+x^4}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x^2}{\sqrt{-2+x^4}}\right)\\
&= \frac{1}{4}x^2\sqrt{-2+x^4}-\frac{1}{2}\tanh^{-1}\left(\frac{x^2}{\sqrt{-2+x^4}}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 1.43

$$\frac{\left(x^4-2\right)\left(2\sin^{-1}\left(\frac{x^2}{\sqrt{2}}\right)+\sqrt{2-x^4}\,x^2\right)}{4\sqrt{-\left(x^4-2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[-2 + x^4],x]
```

```
[Out] ((-2 + x^4)*(x^2*Sqrt[2 - x^4] + 2*ArcSin[x^2/Sqrt[2]]))/(4*Sqrt[-(-2 + x^4
)^2])
```

fricas [A] time = 0.90, size = 29, normalized size = 0.83

$$\frac{1}{4}\sqrt{x^4-2}\,x^2+\frac{1}{2}\log\left(-x^2+\sqrt{x^4-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^4-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(x^4 - 2)*x^2 + 1/2*log(-x^2 + sqrt(x^4 - 2))
```

giac [A] time = 0.18, size = 29, normalized size = 0.83

$$\frac{1}{4}\sqrt{x^4-2}\,x^2+\frac{1}{2}\log\left(x^2-\sqrt{x^4-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^4-2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(x^4 - 2)*x^2 + 1/2*log(x^2 - sqrt(x^4 - 2))
```

maple [A] time = 0.01, size = 28, normalized size = 0.80

$$\frac{\sqrt{x^4-2}\;x^2}{4}-\frac{\ln\left(x^2+\sqrt{x^4-2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^4-2)^(1/2),x)
```

```
[Out] 1/4*x^2*(x^4-2)^(1/2)-1/2*ln(x^2+(x^4-2)^(1/2))
```

maxima [B] time = 1.33, size = 58, normalized size = 1.66

$$-\frac{\sqrt{x^4-2}}{2\,x^2\left(\frac{x^4-2}{x^4}-1\right)}-\frac{1}{4}\log\left(\frac{\sqrt{x^4-2}}{x^2}+1\right)+\frac{1}{4}\log\left(\frac{\sqrt{x^4-2}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^4-2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(x^4 - 2)/(x^2*((x^4 - 2)/x^4 - 1)) - 1/4*log(sqrt(x^4 - 2)/x^2 +
1) + 1/4*log(sqrt(x^4 - 2)/x^2 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x\sqrt{x^4-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^4 - 2)^(1/2),x)
```

```
[Out] int(x*(x^4 - 2)^(1/2), x)
```

sympy [**A**] time = 2.96, size = 90, normalized size = 2.57

$$\begin{cases} \frac{x^6}{4\sqrt{x^4-2}} - \frac{x^2}{2\sqrt{x^4-2}} - \frac{\operatorname{acosh}\left(\frac{\sqrt{2}x^2}{2}\right)}{2} & \text{for } \frac{|x^4|}{2} > 1 \\ -\frac{ix^6}{4\sqrt{2-x^4}} + \frac{ix^2}{2\sqrt{2-x^4}} + \frac{i\operatorname{asin}\left(\frac{\sqrt{2}x^2}{2}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**4-2)**(1/2),x)
```

```
[Out] Piecewise((x**6/(4*sqrt(x**4 - 2)) - x**2/(2*sqrt(x**4 - 2)) - acosh(sqrt(2
)*x**2/2)/2, Abs(x**4)/2 > 1), (-I*x**6/(4*sqrt(2 - x**4)) + I*x**2/(2*sqrt
(2 - x**4)) + I*asin(sqrt(2)*x**2/2)/2, True))
```

3.809 $\int \sqrt{1+x^4}\,dx$

Optimal. Leaf size=58

$$\frac{1}{3}\sqrt{x^4+1}\,x+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{x^4+1}}$$

```
[Out] 1/3*x*(x^4+1)^(1/2)+1/3*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))
*EllipticF(sin(2*arctan(x)),1/2*2^(1/2))*((x^4+1)/(x^2+1)^2)^(1/2)/(x^4+1)^
(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {195, 220}

$$\frac{1}{3}\sqrt{x^4+1}\,x+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^4],x]
```

```
[Out] (x*Sqrt[1 + x^4])/3 + ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*Ar
cTan[x], 1/2])/(3*Sqrt[1 + x^4])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1+x^4}\,dx &= \frac{1}{3}x\sqrt{1+x^4}+\frac{2}{3}\int \frac{1}{\sqrt{1+x^4}}\,dx\\ &= \frac{1}{3}x\sqrt{1+x^4}+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{1+x^4}}\end{aligned}$$

Mathematica [C] time = 0.03, size = 48, normalized size = 0.83

$$\frac{x^5-2\sqrt[4]{-1}\,\sqrt{x^4+1}\,F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\,x\right)\middle|-1\right)+x}{3\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^4],x]
```

```
[Out] (x + x^5 - 2*(-1)^(1/4)*Sqrt[1 + x^4]*EllipticF[I*ArcSinh[(-1)^(1/4)*x], -1
])/(3*Sqrt[1 + x^4])
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{x^4+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 1), x)
```

maple [C] time = 0.01, size = 72, normalized size = 1.24

$$\frac{\sqrt{x^4+1}\, x}{3} + \frac{2\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\ \text{EllipticF}\left(\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)x, i\right)}{3\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)^(1/2),x)
```

```
[Out] 1/3*(x^4+1)^(1/2)*x+2/3/(1/2*2^(1/2)+1/2*I*2^(1/2))*(-I*x^2+1)^(1/2)*(I*x^2
+1)^(1/2)/(x^4+1)^(1/2)*EllipticF((1/2*2^(1/2)+1/2*I*2^(1/2))*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 1), x)
```

mupad [B] time = 0.04, size = 12, normalized size = 0.21

$$x\,{}_2\mathrm{F}_1\left(-\frac{1}{2}, \frac{1}{4}; \frac{5}{4}; -x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)^(1/2),x)
```

```
[Out] x*hypergeom([-1/2, 1/4], 5/4, -x^4)
```

sympy [C] time = 1.43, size = 29, normalized size = 0.50

$$\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2}, \frac{1}{4} \\ \frac{5}{4}\end{matrix}\right| x^4 e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)**(1/2),x)
```

```
[Out] x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), x**4*exp_polar(I*pi))/(4*gamma(5/4)
)
```

3.810 $\int \sqrt{1-x^4}\,dx$

Optimal. Leaf size=25

$$\frac{1}{3}\sqrt{1-x^4}\,x+\frac{2}{3}F\left(\sin^{-1}(x)\middle|-1\right)$$

```
[Out] 2/3*EllipticF(x,I)+1/3*x*(-x^4+1)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {195, 221}

$$\frac{1}{3}\sqrt{1-x^4}\,x+\frac{2}{3}F\left(\sin^{-1}(x)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^4],x]
```

```
[Out] (x*Sqrt[1 - x^4])/3 + (2*EllipticF[ArcSin[x], -1])/3
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1-x^4}\,dx &= \frac{1}{3}x\sqrt{1-x^4}+\frac{2}{3}\int \frac{1}{\sqrt{1-x^4}}\,dx\\ &= \frac{1}{3}x\sqrt{1-x^4}+\frac{2}{3}F\left(\sin^{-1}(x)\middle|-1\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 1.56

$$\frac{-x^5+2\sqrt{1-x^4}\,F\left(\sin^{-1}(x)\middle|-1\right)+x}{3\sqrt{1-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^4],x]
```

```
[Out] (x - x^5 + 2*Sqrt[1 - x^4]*EllipticF[ArcSin[x], -1])/(3*Sqrt[1 - x^4])
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-x^4+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^4+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + 1), x)
```

maple [B] time = 0.01, size = 45, normalized size = 1.80

$$\frac{\sqrt{-x^4+1}\, x}{3} + \frac{2\sqrt{-x^2+1}\, \sqrt{x^2+1}\ \text{EllipticF}\left(x, i\right)}{3\sqrt{-x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)^(1/2),x)
```

```
[Out] 1/3*(-x^4+1)^(1/2)*x+2/3*(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)*Ellipt
icF(x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^4+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + 1), x)
```

mupad [B] time = 1.01, size = 10, normalized size = 0.40

$$x\, {}_2\mathrm{F}_1\left(-\frac{1}{2}, \frac{1}{4};\ \frac{5}{4};\ x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^4)^(1/2),x)
```

```
[Out] x*hypergeom([-1/2, 1/4], 5/4, x^4)
```

sympy [A] time = 1.56, size = 31, normalized size = 1.24

$$\frac{x\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{4} \\ \frac{5}{4} \end{matrix} \middle| x^4 e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)**(1/2),x)
```

```
[Out] x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), x**4*exp_polar(2*I*pi))/(4*gamma(5/
4))
```

3.811 $\int \frac{x^{11}}{\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=59

$$\frac{a^2\sqrt{a+bx^4}}{2b^3}+\frac{\left(a+bx^4\right)^{5/2}}{10b^3}-\frac{a\left(a+bx^4\right)^{3/2}}{3b^3}$$

```
[Out] -1/3*a*(b*x^4+a)^(3/2)/b^3+1/10*(b*x^4+a)^(5/2)/b^3+1/2*a^2*(b*x^4+a)^(1/2)
/b^3
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{a^2\sqrt{a+bx^4}}{2b^3}+\frac{\left(a+bx^4\right)^{5/2}}{10b^3}-\frac{a\left(a+bx^4\right)^{3/2}}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/Sqrt[a + b*x^4],x]
```

```
[Out] (a^2*Sqrt[a + b*x^4])/(2*b^3) - (a*(a + b*x^4)^(3/2))/(3*b^3) + (a + b*x^4)
^(5/2)/(10*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{\sqrt{a+bx^4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx}}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int\left(\frac{a^2}{b^2\sqrt{a+bx}}-\frac{2a\sqrt{a+bx}}{b^2}+\frac{(a+bx)^{3/2}}{b^2}\right)dx, x, x^4\right)\\ &= \frac{a^2\sqrt{a+bx^4}}{2b^3}-\frac{a\left(a+bx^4\right)^{3/2}}{3b^3}+\frac{\left(a+bx^4\right)^{5/2}}{10b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.66

$$\frac{\sqrt{a+bx^4}\left(8a^2-4abx^4+3b^2x^8\right)}{30b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/Sqrt[a + b*x^4],x]
```

```
[Out] (Sqrt[a + b*x^4]*(8*a^2 - 4*a*b*x^4 + 3*b^2*x^8))/(30*b^3)
```

fricas [A] time = 0.82, size = 35, normalized size = 0.59

$$\frac{\left(3\,b^2x^8 - 4\,abx^4 + 8\,a^2\right)\sqrt{bx^4+a}}{30\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/30*(3*b^2*x^8 - 4*a*b*x^4 + 8*a^2)*sqrt(b*x^4 + a)/b^3
```

giac [A] time = 0.15, size = 47, normalized size = 0.80

$$\frac{\sqrt{bx^4+a}\,a^2}{2\,b^3} + \frac{3\left(bx^4+a\right)^{\frac{5}{2}} - 10\left(bx^4+a\right)^{\frac{3}{2}}a}{30\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^4 + a)*a^2/b^3 + 1/30*(3*(b*x^4 + a)^(5/2) - 10*(b*x^4 + a)^(3
/2)*a)/b^3
```

maple [A] time = 0.01, size = 36, normalized size = 0.61

$$\frac{\sqrt{b\,x^4+a}\,\left(3b^2x^8 - 4ab\,x^4 + 8a^2\right)}{30b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^4+a)^(1/2),x)
```

```
[Out] 1/30*(b*x^4+a)^(1/2)*(3*b^2*x^8-4*a*b*x^4+8*a^2)/b^3
```

maxima [A] time = 1.37, size = 47, normalized size = 0.80

$$\frac{\left(bx^4+a\right)^{\frac{5}{2}}}{10\,b^3} - \frac{\left(bx^4+a\right)^{\frac{3}{2}}a}{3\,b^3} + \frac{\sqrt{bx^4+a}\,a^2}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/10*(b*x^4 + a)^(5/2)/b^3 - 1/3*(b*x^4 + a)^(3/2)*a/b^3 + 1/2*sqrt(b*x^4 +
 a)*a^2/b^3
```

mupad [B] time = 1.20, size = 36, normalized size = 0.61

$$\sqrt{b\,x^4+a}\,\left(\frac{4\,a^2}{15\,b^3} + \frac{x^8}{10\,b} - \frac{2\,a\,x^4}{15\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a + b*x^4)^(1/2),x)
```

```
[Out] (a + b*x^4)^(1/2)*((4*a^2)/(15*b^3) + x^8/(10*b) - (2*a*x^4)/(15*b^2))
```

sympy [A] time = 5.09, size = 68, normalized size = 1.15

$$\begin{cases} \frac{4a^2\sqrt{a+bx^4}}{15b^3} - \frac{2ax^4\sqrt{a+bx^4}}{15b^2} + \frac{x^8\sqrt{a+bx^4}}{10b} & \text{for } b \neq 0 \\ \frac{x^{12}}{12\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**4+a)**(1/2),x)
```

```
[Out] Piecewise((4*a**2*sqrt(a + b*x**4)/(15*b**3) - 2*a*x**4*sqrt(a + b*x**4)/(1
5*b**2) + x**8*sqrt(a + b*x**4)/(10*b), Ne(b, 0)), (x**12/(12*sqrt(a)), Tru
e))
```

3.812 $\int \frac{x^7}{\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=38

$$\frac{\left(a+bx^4\right)^{3/2}}{6b^2}-\frac{a\sqrt{a+bx^4}}{2b^2}$$

```
[Out] 1/6*(b*x^4+a)^(3/2)/b^2-1/2*a*(b*x^4+a)^(1/2)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{\left(a+bx^4\right)^{3/2}}{6b^2}-\frac{a\sqrt{a+bx^4}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/Sqrt[a + b*x^4],x]
```

```
[Out] -(a*Sqrt[a + b*x^4])/(2*b^2) + (a + b*x^4)^(3/2)/(6*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{\sqrt{a+bx^4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int\left(-\frac{a}{b\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b}\right)dx, x, x^4\right)\\ &= -\frac{a\sqrt{a+bx^4}}{2b^2}+\frac{\left(a+bx^4\right)^{3/2}}{6b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.71

$$\frac{\left(bx^4-2a\right)\sqrt{a+bx^4}}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/Sqrt[a + b*x^4],x]
```

```
[Out] ((-2*a + b*x^4)*Sqrt[a + b*x^4])/(6*b^2)
```

fricas [A] time = 1.08, size = 23, normalized size = 0.61

$$\frac{\sqrt{bx^4+a}\left(bx^4-2a\right)}{6b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)^(1/2),x, algorithm="fricas")
[Out] 1/6*sqrt(b*x^4 + a)*(b*x^4 - 2*a)/b^2
```

giac [A] time = 0.22, size = 30, normalized size = 0.79

$$\frac{\left(bx^4+a\right)^{\frac{3}{2}}}{6b^2}-\frac{\sqrt{bx^4+a}\,a}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)^(1/2),x, algorithm="giac")
[Out] 1/6*(b*x^4 + a)^(3/2)/b^2 - 1/2*sqrt(b*x^4 + a)*a/b^2
```

maple [A] time = 0.01, size = 25, normalized size = 0.66

$$-\frac{\sqrt{b\,x^4+a}\,\left(-b\,x^4+2a\right)}{6b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^4+a)^(1/2),x)
[Out] -1/6*(b*x^4+a)^(1/2)*(-b*x^4+2*a)/b^2
```

maxima [A] time = 1.35, size = 30, normalized size = 0.79

$$\frac{\left(bx^4+a\right)^{\frac{3}{2}}}{6b^2}-\frac{\sqrt{bx^4+a}\,a}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)^(1/2),x, algorithm="maxima")
[Out] 1/6*(b*x^4 + a)^(3/2)/b^2 - 1/2*sqrt(b*x^4 + a)*a/b^2
```

mupad [B] time = 1.15, size = 24, normalized size = 0.63

$$-\frac{\sqrt{b\,x^4+a}\,\left(2\,a-b\,x^4\right)}{6\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^4)^(1/2),x)
[Out] -((a + b*x^4)^(1/2)*(2*a - b*x^4))/(6*b^2)
```

sympy [A] time = 1.94, size = 42, normalized size = 1.11

$$\begin{cases} -\frac{a\sqrt{a+bx^4}}{3b^2}+\frac{x^4\sqrt{a+bx^4}}{6b} & \text{for } b\neq 0 \\ \frac{x^8}{8\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**4+a)**(1/2),x)
```

```
[Out] Piecewise((-a*sqrt(a + b*x**4)/(3*b**2) + x**4*sqrt(a + b*x**4)/(6*b), Ne(b
, 0)), (x**8/(8*sqrt(a)), True))
```

3.813 $\int \frac{x^3}{\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=18

$$\frac{\sqrt{a+bx^4}}{2b}$$

```
[Out] 1/2*(b*x^4+a)^(1/2)/b
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$\frac{\sqrt{a+bx^4}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b*x^4],x]
```

```
[Out] Sqrt[a + b*x^4]/(2*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+bx^4}}\,dx = \frac{\sqrt{a+bx^4}}{2b}$$

Mathematica [A] time = 0.02, size = 18, normalized size = 1.00

$$\frac{\sqrt{a+bx^4}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + b*x^4],x]
```

```
[Out] Sqrt[a + b*x^4]/(2*b)
```

fricas [A] time = 1.08, size = 14, normalized size = 0.78

$$\frac{\sqrt{bx^4+a}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(b*x^4 + a)/b
```

giac [A] time = 0.15, size = 14, normalized size = 0.78

$$\frac{\sqrt{bx^4+a}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(b*x^4 + a)/b
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$\frac{\sqrt{b\,x^4+a}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^4+a)^(1/2),x)
[Out] 1/2*(b*x^4+a)^(1/2)/b
```

maxima [A] time = 1.31, size = 14, normalized size = 0.78

$$\frac{\sqrt{bx^4+a}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(b*x^4 + a)/b
```

mupad [B] time = 1.15, size = 14, normalized size = 0.78

$$\frac{\sqrt{b\,x^4+a}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^4)^(1/2),x)
[Out] (a + b*x^4)^(1/2)/(2*b)
```

sympy [A] time = 0.84, size = 22, normalized size = 1.22

$$\begin{cases} \frac{\sqrt{a+bx^4}}{2b} & \text{for } b \neq 0 \\ \frac{x^4}{4\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**4+a)**(1/2),x)
[Out] Piecewise((sqrt(a + b*x**4)/(2*b), Ne(b, 0)), (x**4/(4*sqrt(a)), True))
```

3.814 $\int \frac{1}{x\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=27

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

```
[Out] -1/2*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {266, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x^4]),x]
```

```
[Out] -ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]/(2*Sqrt[a])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{a+bx^4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^4\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^4}\right)}{2b}\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x^4]),x]
```

```
[Out] -1/2*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]/Sqrt[a]
```

fricas [A] time = 1.13, size = 63, normalized size = 2.33

$$\left[\frac{\log\left(\frac{bx^4-2\sqrt{bx^4+a}\sqrt{a}+2a}{x^4}\right)}{4\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{bx^4+a}\sqrt{-a}}{a}\right)}{2a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log((b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x^4)/sqrt(a), 1/2*sqrt(-
a)*arctan(sqrt(b*x^4 + a)*sqrt(-a)/a)/a]
```

giac [A] time = 0.17, size = 23, normalized size = 0.85

$$\frac{\arctan\left(\frac{\sqrt{bx^4+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*arctan(sqrt(b*x^4 + a)/sqrt(-a))/sqrt(-a)
```

maple [A] time = 0.01, size = 29, normalized size = 1.07

$$-\frac{\ln\left(\frac{2a+2\sqrt{b\,x^4+a}\,\sqrt{a}}{x^2}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^4+a)^(1/2),x)
```

```
[Out] -1/2/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)
```

maxima [A] time = 3.03, size = 37, normalized size = 1.37

$$\frac{\log\left(\frac{\sqrt{bx^4+a}-\sqrt{a}}{\sqrt{bx^4+a}+\sqrt{a}}\right)}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*log((sqrt(b*x^4 + a) - sqrt(a))/(sqrt(b*x^4 + a) + sqrt(a)))/sqrt(a)
```

mupad [B] time = 1.22, size = 19, normalized size = 0.70

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{b\,x^4+a}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^4)^(1/2)),x)
```

```
[Out] -atanh((a + b*x^4)^(1/2)/a^(1/2))/(2*a^(1/2))
```

sympy [A] time = 1.60, size = 22, normalized size = 0.81

$$-\frac{\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^2}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**4+a)**(1/2),x)
```

```
[Out] -asinh(sqrt(a)/(sqrt(b)*x**2))/(2*sqrt(a))
```

3.815 $\int \frac{1}{x^5\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=50

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4a^{3/2}} - \frac{\sqrt{a+bx^4}}{4ax^4}$$

```
[Out] 1/4*b*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)-1/4*(b*x^4+a)^(1/2)/a/x^4
```

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 51, 63, 208}

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4a^{3/2}} - \frac{\sqrt{a+bx^4}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[a + b*x^4]),x]
```

```
[Out] -Sqrt[a + b*x^4]/(4*a*x^4) + (b*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(4*a^(3/2
))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\sqrt{a+bx^4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^4\right)\\ &= -\frac{\sqrt{a+bx^4}}{4ax^4} - \frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^4\right)}{8a}\\ &= -\frac{\sqrt{a+bx^4}}{4ax^4} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^4}\right)}{4a}\\ &= -\frac{\sqrt{a+bx^4}}{4ax^4} + \frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4a^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.09, size = 64, normalized size = 1.28

$$\frac{b\sqrt{a+bx^4}\left(\frac{\tanh^{-1}\left(\sqrt{\frac{bx^4}{a}+1}\right)}{2\sqrt{\frac{bx^4}{a}+1}} - \frac{a}{2bx^4}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[a + b*x^4]),x]
```

```
[Out] (b*Sqrt[a + b*x^4]*(-1/2*a/(b*x^4) + ArcTanh[Sqrt[1 + (b*x^4)/a]]/(2*Sqrt[1
 + (b*x^4)/a])))/(2*a^2)
```

fricas [A] time = 1.00, size = 107, normalized size = 2.14

$$\left[\frac{\sqrt{a}\,bx^4\log\left(\frac{bx^4+2\sqrt{bx^4+a}\sqrt{a}+2a}{x^4}\right) - 2\sqrt{bx^4+a}\,a}{8a^2x^4}, -\frac{\sqrt{-a}\,bx^4\arctan\left(\frac{\sqrt{bx^4+a}\sqrt{-a}}{a}\right) + \sqrt{bx^4+a}\,a}{4a^2x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a)*b*x^4*log((b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x^4) - 2*
sqrt(b*x^4 + a)*a)/(a^2*x^4), -1/4*(sqrt(-a)*b*x^4*arctan(sqrt(b*x^4 + a)*s
qrt(-a)/a) + sqrt(b*x^4 + a)*a)/(a^2*x^4)]
```

giac [A] time = 0.17, size = 51, normalized size = 1.02

$$-\frac{\frac{b^2\arctan\left(\frac{\sqrt{bx^4+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a} + \frac{\sqrt{bx^4+a}\,b}{ax^4}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(b^2*arctan(sqrt(b*x^4 + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(b*x^4 + a)*b
/(a*x^4))/b
```

maple [A] time = 0.01, size = 48, normalized size = 0.96

$$\frac{b\ln\left(\frac{2a+2\sqrt{b\,x^4+a}\,\sqrt{a}}{x^2}\right)}{4a^{\frac{3}{2}}} - \frac{\sqrt{b\,x^4+a}}{4a\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^4+a)^(1/2),x)
```

```
[Out] -1/4*(b*x^4+a)^(1/2)/a/x^4+1/4*b/a^(3/2)*ln((2*a+2*(b*x^4+a)^(1/2)*a^(1/2))
/x^2)
```

maxima [A] time = 2.91, size = 68, normalized size = 1.36

$$-\frac{\sqrt{bx^4+a}\,b}{4\left(\left(bx^4+a\right)a-a^2\right)}-\frac{b\log\left(\frac{\sqrt{bx^4+a}-\sqrt{a}}{\sqrt{bx^4+a}+\sqrt{a}}\right)}{8\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(b*x^4 + a)*b/((b*x^4 + a)*a - a^2) - 1/8*b*log((sqrt(b*x^4 + a) -
 sqrt(a))/(sqrt(b*x^4 + a) + sqrt(a)))/a^(3/2)
```

mupad [B] time = 1.31, size = 38, normalized size = 0.76

$$\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^4+a}}{\sqrt{a}}\right)}{4\,a^{3/2}}-\frac{\sqrt{b\,x^4+a}}{4\,a\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^4)^(1/2)),x)
```

```
[Out] (b*atanh((a + b*x^4)^(1/2)/a^(1/2)))/(4*a^(3/2)) - (a + b*x^4)^(1/2)/(4*a*x
^4)
```

sympy [A] time = 4.22, size = 46, normalized size = 0.92

$$-\frac{\sqrt{b}\,\sqrt{\frac{a}{bx^4}+1}}{4ax^2}+\frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^2}\right)}{4a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**4+a)**(1/2),x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x**4) + 1)/(4*a*x**2) + b*asinh(sqrt(a)/(sqrt(b)*x**2))/
(4*a**(3/2))
```

3.816 $\int \frac{x^5}{\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=53

$$\frac{x^2\sqrt{a+bx^4}}{4b} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}}$$

```
[Out] -1/4*a*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(3/2)+1/4*x^2*(b*x^4+a)^(1/2)
/b
```

Rubi [A] time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {275, 321, 217, 206}

$$\frac{x^2\sqrt{a+bx^4}}{4b} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[a + b*x^4],x]
```

```
[Out] (x^2*Sqrt[a + b*x^4])/(4*b) - (a*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4
*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\sqrt{a+bx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^2}}\,dx, x, x^2\right)\\ &= \frac{x^2\sqrt{a+bx^4}}{4b} - \frac{a\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, x^2\right)}{4b}\\ &= \frac{x^2\sqrt{a+bx^4}}{4b} - \frac{a\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^2}{\sqrt{a+bx^4}}\right)}{4b}\\ &= \frac{x^2\sqrt{a+bx^4}}{4b} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 53, normalized size = 1.00

$$\frac{x^2\sqrt{a+bx^4}}{4b} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[a + b*x^4],x]
```

```
[Out] (x^2*Sqrt[a + b*x^4])/(4*b) - (a*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4
*b^(3/2))
```

fricas [A] time = 0.78, size = 101, normalized size = 1.91

$$\left[\frac{2\sqrt{bx^4+a}\,bx^2 + a\sqrt{b}\,\log\left(-2\,bx^4 + 2\sqrt{bx^4+a}\,\sqrt{b}\,x^2 - a\right)}{8\,b^2}, \frac{\sqrt{bx^4+a}\,bx^2 + a\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x^2}{\sqrt{bx^4+a}}\right)}{4\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(b*x^4 + a)*b*x^2 + a*sqrt(b)*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*
sqrt(b)*x^2 - a))/b^2, 1/4*(sqrt(b*x^4 + a)*b*x^2 + a*sqrt(-b)*arctan(sqrt(
-b)*x^2/sqrt(b*x^4 + a)))/b^2]
```

giac [A] time = 0.19, size = 44, normalized size = 0.83

$$\frac{\sqrt{bx^4+a}\,x^2}{4\,b} + \frac{a\log\left(\left|-\sqrt{b}\,x^2 + \sqrt{bx^4+a}\right|\right)}{4\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(b*x^4 + a)*x^2/b + 1/4*a*log(abs(-sqrt(b)*x^2 + sqrt(b*x^4 + a)))/
b^(3/2)
```

maple [A] time = 0.01, size = 43, normalized size = 0.81

$$\frac{\sqrt{b\,x^4+a}\;x^2}{4b} - \frac{a\ln\left(\sqrt{b}\;x^2 + \sqrt{b\,x^4+a}\right)}{4b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^4+a)^(1/2),x)
```

```
[Out] 1/4*x^2*(b*x^4+a)^(1/2)/b-1/4*a/b^(3/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))
```

maxima [A] time = 2.90, size = 81, normalized size = 1.53

$$\frac{a \log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^4+a}}{x^2}}{\sqrt{b}+\frac{\sqrt{bx^4+a}}{x^2}}\right)}{8\,b^{\frac{3}{2}}} - \frac{\sqrt{bx^4+a}\,a}{4\left(b^2-\frac{\left(bx^4+a\right)b}{x^4}\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*a*log(-(sqrt(b) - sqrt(b*x^4 + a)/x^2)/(sqrt(b) + sqrt(b*x^4 + a)/x^2))
/b^(3/2) - 1/4*sqrt(b*x^4 + a)*a/((b^2 - (b*x^4 + a)*b/x^4)*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^5}{\sqrt{b\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^4)^(1/2),x)
```

```
[Out] int(x^5/(a + b*x^4)^(1/2), x)
```

sympy [A] time = 4.17, size = 46, normalized size = 0.87

$$\frac{\sqrt{a}\,x^2\sqrt{1+\frac{bx^4}{a}}}{4b} - \frac{a\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*x**2*sqrt(1 + b*x**4/a)/(4*b) - a*asinh(sqrt(b)*x**2/sqrt(a))/(4*b*
*(3/2))
```

3.817 $\int \frac{x}{\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=30

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}$$

```
[Out] 1/2*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {275, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*x^4],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]]/(2*Sqrt[b])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{a+bx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^2}{\sqrt{a+bx^4}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b*x^4],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]]/(2*Sqrt[b])
```

fricas [A] time = 0.84, size = 63, normalized size = 2.10

$$\left[\frac{\log\left(-2\,bx^4 - 2\,\sqrt{bx^4+a}\,\sqrt{b}\,x^2 - a\right)}{4\,\sqrt{b}}, -\frac{\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x^2}{\sqrt{bx^4+a}}\right)}{2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a)/sqrt(b), -1/2*sqrt(-
b)*arctan(sqrt(-b)*x^2/sqrt(b*x^4 + a))/b]
```

giac [A] time = 0.18, size = 25, normalized size = 0.83

$$-\frac{\log\left(\left|-\sqrt{b}\,x^2 + \sqrt{bx^4+a}\right|\right)}{2\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(-sqrt(b)*x^2 + sqrt(b*x^4 + a)))/sqrt(b)
```

maple [A] time = 0.01, size = 24, normalized size = 0.80

$$\frac{\ln\left(\sqrt{b}\,x^2 + \sqrt{b\,x^4+a}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^4+a)^(1/2),x)
```

```
[Out] 1/2*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))/b^(1/2)
```

maxima [B] time = 3.03, size = 45, normalized size = 1.50

$$-\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^4+a}}{x^2}}{\sqrt{b}+\frac{\sqrt{bx^4+a}}{x^2}}\right)}{4\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*log(-(sqrt(b) - sqrt(b*x^4 + a)/x^2)/(sqrt(b) + sqrt(b*x^4 + a)/x^2))/
sqrt(b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{\sqrt{b\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^4)^(1/2),x)
```

```
[Out] int(x/(a + b*x^4)^(1/2), x)
```

sympy [**A**] time = 1.94, size = 20, normalized size = 0.67

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**4+a)**(1/2),x)
[Out] asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b))
```

3.818 $\int \frac{1}{x^3\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=21

$$-\frac{\sqrt{a+bx^4}}{2ax^2}$$

```
[Out] -1/2*(b*x^4+a)^(1/2)/a/x^2
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {264}

$$-\frac{\sqrt{a+bx^4}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*x^4]),x]
```

```
[Out] -Sqrt[a + b*x^4]/(2*a*x^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{a+bx^4}}{2ax^2}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$-\frac{\sqrt{a+bx^4}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + b*x^4]),x]
```

```
[Out] -1/2*Sqrt[a + b*x^4]/(a*x^2)
```

fricas [A] time = 0.70, size = 17, normalized size = 0.81

$$-\frac{\sqrt{bx^4+a}}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(b*x^4 + a)/(a*x^2)
```

giac [A] time = 0.19, size = 31, normalized size = 1.48

$$\frac{\sqrt{b}}{\left(\sqrt{b}\,x^2-\sqrt{bx^4+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)^(1/2),x, algorithm="giac")
[Out] sqrt(b)/((sqrt(b)*x^2 - sqrt(b*x^4 + a))^2 - a)
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$-\frac{\sqrt{b\,x^4+a}}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^4+a)^(1/2),x)
[Out] -1/2*(b*x^4+a)^(1/2)/a/x^2
```

maxima [A] time = 1.27, size = 17, normalized size = 0.81

$$-\frac{\sqrt{bx^4+a}}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)^(1/2),x, algorithm="maxima")
[Out] -1/2*sqrt(b*x^4 + a)/(a*x^2)
```

mupad [B] time = 1.14, size = 17, normalized size = 0.81

$$-\frac{\sqrt{b\,x^4+a}}{2\,a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^4)^(1/2)),x)
[Out] -(a + b*x^4)^(1/2)/(2*a*x^2)
```

sympy [A] time = 0.98, size = 20, normalized size = 0.95

$$-\frac{\sqrt{b}\,\sqrt{\frac{a}{bx^4}+1}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**4+a)**(1/2),x)
[Out] -sqrt(b)*sqrt(a/(b*x**4) + 1)/(2*a)
```

3.819 $\int \frac{1}{x^7\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=44

$$\frac{b\sqrt{a+bx^4}}{3a^2x^2}-\frac{\sqrt{a+bx^4}}{6ax^6}$$

```
[Out] -1/6*(b*x^4+a)^(1/2)/a/x^6+1/3*b*(b*x^4+a)^(1/2)/a^2/x^2
```

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$\frac{b\sqrt{a+bx^4}}{3a^2x^2}-\frac{\sqrt{a+bx^4}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*Sqrt[a + b*x^4]),x]
```

```
[Out] -Sqrt[a + b*x^4]/(6*a*x^6) + (b*Sqrt[a + b*x^4])/(3*a^2*x^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7\sqrt{a+bx^4}}\,dx &= -\frac{\sqrt{a+bx^4}}{6ax^6}-\frac{(2b)\int \frac{1}{x^3\sqrt{a+bx^4}}\,dx}{3a}\\ &= -\frac{\sqrt{a+bx^4}}{6ax^6}+\frac{b\sqrt{a+bx^4}}{3a^2x^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.66

$$-\frac{\left(a-2bx^4\right)\sqrt{a+bx^4}}{6a^2x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*Sqrt[a + b*x^4]),x]
```

```
[Out] -1/6*((a - 2*b*x^4)*Sqrt[a + b*x^4])/(a^2*x^6)
```

fricas [A] time = 0.89, size = 27, normalized size = 0.61

$$\frac{\left(2\,bx^4-a\right)\sqrt{bx^4+a}}{6\,a^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*x^4 - a)*sqrt(b*x^4 + a)/(a^2*x^6)
```

giac [A] time = 0.20, size = 59, normalized size = 1.34

$$\frac{2\left(3\left(\sqrt{b}\,x^2-\sqrt{bx^4+a}\right)^2-a\right)b^{\frac{3}{2}}}{3\left(\left(\sqrt{b}\,x^2-\sqrt{bx^4+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(3*(sqrt(b)*x^2 - sqrt(b*x^4 + a))^2 - a)*b^(3/2)/((sqrt(b)*x^2 - sqrt(
b*x^4 + a))^2 - a)^3
```

maple [A] time = 0.01, size = 26, normalized size = 0.59

$$-\frac{\sqrt{b\,x^4+a}\,\left(-2b\,x^4+a\right)}{6a^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^4+a)^(1/2),x)
```

```
[Out] -1/6*(b*x^4+a)^(1/2)*(-2*b*x^4+a)/a^2/x^6
```

maxima [A] time = 1.46, size = 35, normalized size = 0.80

$$\frac{\frac{3\sqrt{bx^4+a}\,b}{x^2}-\frac{\left(bx^4+a\right)^{\frac{3}{2}}}{x^6}}{6\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(3*sqrt(b*x^4 + a)*b/x^2 - (b*x^4 + a)^(3/2)/x^6)/a^2
```

mupad [B] time = 1.19, size = 25, normalized size = 0.57

$$-\frac{\sqrt{b\,x^4+a}\,\left(a-2\,b\,x^4\right)}{6\,a^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^4)^(1/2)),x)
```

```
[Out] -((a + b*x^4)^(1/2)*(a - 2*b*x^4))/(6*a^2*x^6)
```

sympy [A] time = 1.33, size = 44, normalized size = 1.00

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^4}+1}}{6ax^4}+\frac{b^{\frac{3}{2}}\sqrt{\frac{a}{bx^4}+1}}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**4+a)**(1/2),x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x**4) + 1)/(6*a*x**4) + b**(3/2)*sqrt(a/(b*x**4) + 1)/(3
*a**2)
```

3.820 $\int \frac{1}{x^{11}\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=68

$$-\frac{4b^2\sqrt{a+bx^4}}{15a^3x^2}+\frac{2b\sqrt{a+bx^4}}{15a^2x^6}-\frac{\sqrt{a+bx^4}}{10ax^{10}}$$

```
[Out] -1/10*(b*x^4+a)^(1/2)/a/x^10+2/15*b*(b*x^4+a)^(1/2)/a^2/x^6-4/15*b^2*(b*x^4
+a)^(1/2)/a^3/x^2
```

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$-\frac{4b^2\sqrt{a+bx^4}}{15a^3x^2}+\frac{2b\sqrt{a+bx^4}}{15a^2x^6}-\frac{\sqrt{a+bx^4}}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^11*Sqrt[a + b*x^4]),x]
```

```
[Out] -Sqrt[a + b*x^4]/(10*a*x^10) + (2*b*Sqrt[a + b*x^4])/(15*a^2*x^6) - (4*b^2*
Sqrt[a + b*x^4])/(15*a^3*x^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{11}\sqrt{a+bx^4}}\,dx &= -\frac{\sqrt{a+bx^4}}{10ax^{10}}-\frac{(4b)\int \frac{1}{x^7\sqrt{a+bx^4}}\,dx}{5a}\\ &= -\frac{\sqrt{a+bx^4}}{10ax^{10}}+\frac{2b\sqrt{a+bx^4}}{15a^2x^6}+\frac{\left(8b^2\right)\int \frac{1}{x^3\sqrt{a+bx^4}}\,dx}{15a^2}\\ &= -\frac{\sqrt{a+bx^4}}{10ax^{10}}+\frac{2b\sqrt{a+bx^4}}{15a^2x^6}-\frac{4b^2\sqrt{a+bx^4}}{15a^3x^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.62

$$-\frac{\sqrt{a+bx^4}\left(3a^2-4abx^4+8b^2x^8\right)}{30a^3x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^11*Sqrt[a + b*x^4]),x]
```

```
[Out] -1/30*(Sqrt[a + b*x^4]*(3*a^2 - 4*a*b*x^4 + 8*b^2*x^8))/(a^3*x^10)
```

fricas [A] time = 0.63, size = 38, normalized size = 0.56

$$-\frac{\left(8\,b^2x^8-4\,abx^4+3\,a^2\right)\sqrt{bx^4+a}}{30\,a^3x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/30*(8*b^2*x^8 - 4*a*b*x^4 + 3*a^2)*sqrt(b*x^4 + a)/(a^3*x^10)
```

giac [A] time = 0.21, size = 83, normalized size = 1.22

$$\frac{8\left(10\left(\sqrt{b}\,x^2-\sqrt{bx^4+a}\right)^4-5\left(\sqrt{b}\,x^2-\sqrt{bx^4+a}\right)^2a+a^2\right)b^{\frac{5}{2}}}{15\left(\left(\sqrt{b}\,x^2-\sqrt{bx^4+a}\right)^2-a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] 8/15*(10*(sqrt(b)*x^2 - sqrt(b*x^4 + a))^4 - 5*(sqrt(b)*x^2 - sqrt(b*x^4 +
a))^2*a + a^2)*b^(5/2)/((sqrt(b)*x^2 - sqrt(b*x^4 + a))^2 - a)^5
```

maple [A] time = 0.01, size = 39, normalized size = 0.57

$$-\frac{\sqrt{b\,x^4+a}\,\left(8b^2x^8-4ab\,x^4+3a^2\right)}{30a^3x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^11/(b*x^4+a)^(1/2),x)
```

```
[Out] -1/30*(b*x^4+a)^(1/2)*(8*b^2*x^8-4*a*b*x^4+3*a^2)/x^10/a^3
```

maxima [A] time = 1.34, size = 52, normalized size = 0.76

$$-\frac{\frac{15\,\sqrt{bx^4+a}\,b^2}{x^2}-\frac{10\left(bx^4+a\right)^{\frac{3}{2}}b}{x^6}+\frac{3\left(bx^4+a\right)^{\frac{5}{2}}}{x^{10}}}{30\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/30*(15*sqrt(b*x^4 + a)*b^2/x^2 - 10*(b*x^4 + a)^(3/2)*b/x^6 + 3*(b*x^4 +
 a)^(5/2)/x^10)/a^3
```

mupad [B] time = 1.28, size = 38, normalized size = 0.56

$$-\frac{\sqrt{b\,x^4+a}\,\left(3\,a^2-4\,a\,b\,x^4+8\,b^2\,x^8\right)}{30\,a^3\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^11*(a + b*x^4)^(1/2)),x)
```

```
[Out] -((a + b*x^4)^(1/2)*(3*a^2 + 8*b^2*x^8 - 4*a*b*x^4))/(30*a^3*x^10)
```

sympy [**B**] time = 2.92, size = 298, normalized size = 4.38

$$-\frac{3a^4b^{\frac{9}{2}}\sqrt{\frac{a}{bx^4}+1}}{30a^5b^4x^8+60a^4b^5x^{12}+30a^3b^6x^{16}}-\frac{2a^3b^{\frac{11}{2}}x^4\sqrt{\frac{a}{bx^4}+1}}{30a^5b^4x^8+60a^4b^5x^{12}+30a^3b^6x^{16}}-\frac{3a^2b^{\frac{13}{2}}x^8\sqrt{\frac{a}{bx^4}+1}}{30a^5b^4x^8+60a^4b^5x^{12}+30a^3b^6x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**11/(b*x**4+a)**(1/2),x)
```

```
[Out] -3*a**4*b**(9/2)*sqrt(a/(b*x**4) + 1)/(30*a**5*b**4*x**8 + 60*a**4*b**5*x**
12 + 30*a**3*b**6*x**16) - 2*a**3*b**(11/2)*x**4*sqrt(a/(b*x**4) + 1)/(30*a
**5*b**4*x**8 + 60*a**4*b**5*x**12 + 30*a**3*b**6*x**16) - 3*a**2*b**(13/2)
*x**8*sqrt(a/(b*x**4) + 1)/(30*a**5*b**4*x**8 + 60*a**4*b**5*x**12 + 30*a**
3*b**6*x**16) - 12*a*b**(15/2)*x**12*sqrt(a/(b*x**4) + 1)/(30*a**5*b**4*x**
8 + 60*a**4*b**5*x**12 + 30*a**3*b**6*x**16) - 8*b**(17/2)*x**16*sqrt(a/(b*
x**4) + 1)/(30*a**5*b**4*x**8 + 60*a**4*b**5*x**12 + 30*a**3*b**6*x**16)
```

3.821 $\int \frac{x^8}{\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=130

$$\frac{5a^{7/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{42b^{9/4}\sqrt{a+bx^4}}-\frac{5ax\sqrt{a+bx^4}}{21b^2}+\frac{x^5\sqrt{a+bx^4}}{7b}$$

```
[Out] -5/21*a*x*(b*x^4+a)^(1/2)/b^2+1/7*x^5*(b*x^4+a)^(1/2)/b+5/42*a^(7/4)*(cos(2
*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*Ellip
ticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((
b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(9/4)/(b*x^4+a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {321, 220}

$$\frac{5a^{7/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{42b^{9/4}\sqrt{a+bx^4}}-\frac{5ax\sqrt{a+bx^4}}{21b^2}+\frac{x^5\sqrt{a+bx^4}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/Sqrt[a + b*x^4],x]
```

```
[Out] (-5*a*x*Sqrt[a + b*x^4])/(21*b^2) + (x^5*Sqrt[a + b*x^4])/(7*b) + (5*a^(7/4
)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellip
ticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(42*b^(9/4)*Sqrt[a + b*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{\sqrt{a+bx^4}}\,dx &= \frac{x^5\sqrt{a+bx^4}}{7b}-\frac{(5a)\int \frac{x^4}{\sqrt{a+bx^4}}\,dx}{7b}\\ &= -\frac{5ax\sqrt{a+bx^4}}{21b^2}+\frac{x^5\sqrt{a+bx^4}}{7b}+\frac{\left(5a^2\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{21b^2}\\ &= -\frac{5ax\sqrt{a+bx^4}}{21b^2}+\frac{x^5\sqrt{a+bx^4}}{7b}+\frac{5a^{7/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{42b^{9/4}\sqrt{a+bx^4}}\end{aligned}$$

Mathematica [C] time = 0.02, size = 79, normalized size = 0.61

$$\frac{5a^2x\sqrt{\frac{bx^4}{a}+1}\ {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right)-5a^2x-2abx^5+3b^2x^9}{21b^2\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/Sqrt[a + b*x^4],x]
```

```
[Out] (-5*a^2*x - 2*a*b*x^5 + 3*b^2*x^9 + 5*a^2*x*Sqrt[1 + (b*x^4)/a]*Hypergeomet
ric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)])/(21*b^2*Sqrt[a + b*x^4])
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^8}{\sqrt{bx^4+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^8/sqrt(b*x^4 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\sqrt{bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^8/sqrt(b*x^4 + a), x)
```

maple [C] time = 0.05, size = 111, normalized size = 0.85

$$\frac{\sqrt{b\,x^4+a}\ x^5}{7b}+\frac{5\sqrt{-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\ \sqrt{\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\ a^2\,\text{EllipticF}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\ x,i\right)}{21\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\ \sqrt{b\,x^4+a}\ b^2}-\frac{5\sqrt{b\,x^4+a}\ ax}{21b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^4+a)^(1/2),x)
```

```
[Out] 1/7*x^5*(b*x^4+a)^(1/2)/b-5/21*a*x*(b*x^4+a)^(1/2)/b^2+5/21*a^2/b^2/(I/a^(1
/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2
)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\sqrt{bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^8/sqrt(b*x^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^8}{\sqrt{b\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^4)^(1/2),x)
```

```
[Out] int(x^8/(a + b*x^4)^(1/2), x)
```

sympy [C] time = 1.34, size = 37, normalized size = 0.28

$$\frac{x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**4+a)**(1/2),x)
```

```
[Out] x**9*gamma(9/4)*hyper((1/2, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqr
t(a)*gamma(13/4))
```

3.822 $\int \frac{x^4}{\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=108

$$\frac{x\sqrt{a+bx^4}}{3b} - \frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}}$$

```
[Out] 1/3*x*(b*x^4+a)^(1/2)/b-1/6*a^(3/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1
/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/
4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)
^(1/2)/b^(5/4)/(b*x^4+a)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {321, 220}

$$\frac{x\sqrt{a+bx^4}}{3b} - \frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a + b*x^4],x]
```

```
[Out] (x*Sqrt[a + b*x^4])/(3*b) - (a^(3/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^
4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2]
)/(6*b^(5/4)*Sqrt[a + b*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt{a+bx^4}}\,dx &= \frac{x\sqrt{a+bx^4}}{3b} - \frac{a\int \frac{1}{\sqrt{a+bx^4}}\,dx}{3b} \\ &= \frac{x\sqrt{a+bx^4}}{3b} - \frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}}\end{aligned}$$

Mathematica [C] time = 0.05, size = 62, normalized size = 0.57

$$\frac{x\left(-a\sqrt{\frac{bx^4}{a}+1}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right)+a+bx^4\right)}{3b\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a + b*x^4],x]
```

```
[Out] (x*(a + b*x^4 - a*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b
*x^4)/a)]))/(3*b*Sqrt[a + b*x^4])
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^4}{\sqrt{bx^4+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^4/sqrt(b*x^4 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(b*x^4 + a), x)
```

maple [C] time = 0.01, size = 91, normalized size = 0.84

$$-\frac{\sqrt{-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\,a\,\text{EllipticF}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x, i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,b}+\frac{\sqrt{b\,x^4+a}\,x}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^4+a)^(1/2),x)
```

```
[Out] 1/3*x*(b*x^4+a)^(1/2)/b-1/3*a/b/(I/a^(1/2)*b^(1/2))^(1/2)*(-I/a^(1/2)*b^(1/
2)*x^2+1)^(1/2)*(I/a^(1/2)*b^(1/2)*x^2+1)^(1/2)/(b*x^4+a)^(1/2)*EllipticF((
I/a^(1/2)*b^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(b*x^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sqrt{b\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^4)^(1/2),x)
```

```
[Out] int(x^4/(a + b*x^4)^(1/2), x)
```

sympy [C] time = 1.59, size = 37, normalized size = 0.34

$$\frac{x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{5}{4} \\ \frac{9}{4}\end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**4+a)**(1/2),x)
```

```
[Out] x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt
(a)*gamma(9/4))
```

3.823 $\int \frac{1}{\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=88

$$\frac{\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{a+bx^4}}$$

```
[Out] 1/2*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/
4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b
^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/(b*x^4+a)
^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {220}

$$\frac{\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*x^4],x]
```

```
[Out] ((Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellipt
icF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*Sqrt[a + b*x^4]
)
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^4}}\,dx = \frac{\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{a+bx^4}}$$

Mathematica [C] time = 0.03, size = 74, normalized size = 0.84

$$-\frac{i\sqrt{\frac{bx^4}{a}+1}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x],
 -1])/(Sqrt[(I*Sqrt[b])/Sqrt[a]]*Sqrt[a + b*x^4])
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{bx^4+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*x^4 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*x^4 + a), x)
```

maple [C] time = 0.00, size = 70, normalized size = 0.80

$$\frac{\sqrt{-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\;\text{EllipticF}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x, i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(1/2),x)
```

```
[Out] 1/(I/a^(1/2)*b^(1/2))^(1/2)*(-I/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*b^(
1/2)*x^2+1)^(1/2)/(b*x^4+a)^(1/2)*EllipticF((I/a^(1/2)*b^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*x^4 + a), x)
```

mupad [B] time = 1.08, size = 37, normalized size = 0.42

$$\frac{x\sqrt{\frac{b\,x^4}{a}+1}\;{}_2\mathrm{F}_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{b\,x^4}{a}\right)}{\sqrt{b\,x^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^4)^(1/2),x)
```

```
[Out] (x*((b*x^4)/a + 1)^(1/2)*hypergeom([1/4, 1/2], 5/4, -(b*x^4)/a))/(a + b*x^4
)^(1/2)
```

sympy [C] time = 1.49, size = 36, normalized size = 0.41

$$\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4}, \frac{1}{2} \\ \frac{5}{4}\end{matrix}\right| \frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(1/2),x)
```

```
[Out] x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)
*gamma(5/4))
```

3.824 $\int \frac{1}{x^4\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=110

$$-\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}}-\frac{\sqrt{a+bx^4}}{3ax^3}$$

```
[Out] -1/3*(b*x^4+a)^(1/2)/a/x^3-1/6*b^(3/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)
^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^
(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))
^2)^(1/2)/a^(5/4)/(b*x^4+a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {325, 220}

$$-\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}}-\frac{\sqrt{a+bx^4}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a + b*x^4]),x]
```

```
[Out] -Sqrt[a + b*x^4]/(3*a*x^3) - (b^(3/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x
^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2
])/(6*a^(5/4)*Sqrt[a + b*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{a+bx^4}}{3ax^3}-\frac{b\int \frac{1}{\sqrt{a+bx^4}}\,dx}{3a}$$

$$= -\frac{\sqrt{a+bx^4}}{3ax^3}-\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.46

$$-\frac{\sqrt{\frac{bx^4}{a}+1}\;{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-\frac{bx^4}{a}\right)}{3x^3\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a + b*x^4]),x]
```

```
[Out] -1/3*(Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-3/4, 1/2, 1/4, -((b*x^4)/a)])/
(x^3*Sqrt[a + b*x^4])
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^4+a}}{bx^8+ax^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)/(b*x^8 + a*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^4+a}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a)*x^4), x)
```

maple [C] time = 0.01, size = 93, normalized size = 0.85

$$-\frac{\sqrt{-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\,b\,\text{EllipticF}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x, i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,a} - \frac{\sqrt{b\,x^4+a}}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^4+a)^(1/2),x)
```

```
[Out] -1/3*(b*x^4+a)^(1/2)/a/x^3-1/3/a*b/(I/a^(1/2)*b^(1/2))^(1/2)*(-I/a^(1/2)*b^
(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*b^(1/2)*x^2+1)^(1/2)/(b*x^4+a)^(1/2)*Elliptic
F((I/a^(1/2)*b^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^4+a}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,\sqrt{b\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^4*(a + b*x^4)^(1/2)), x)
```

sympy [C] time = 2.05, size = 41, normalized size = 0.37

$$\frac{\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{4}, \frac{1}{2} \\ \frac{1}{4}\end{matrix}\right| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}\, x^3 \Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**4+a)**(1/2),x)
```

```
[Out] gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)
*x**3*gamma(1/4))
```

3.825 $\int \frac{1}{x^8\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=132

$$\frac{5b^{7/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{42a^{9/4}\sqrt{a+bx^4}}+\frac{5b\sqrt{a+bx^4}}{21a^2x^3}-\frac{\sqrt{a+bx^4}}{7ax^7}$$

```
[Out] -1/7*(b*x^4+a)^(1/2)/a/x^7+5/21*b*(b*x^4+a)^(1/2)/a^2/x^3+5/42*b^(7/4)*(cos
(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*Ell
ipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*
((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(9/4)/(b*x^4+a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {325, 220}

$$\frac{5b^{7/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{42a^{9/4}\sqrt{a+bx^4}}+\frac{5b\sqrt{a+bx^4}}{21a^2x^3}-\frac{\sqrt{a+bx^4}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*Sqrt[a + b*x^4]),x]
```

```
[Out] -Sqrt[a + b*x^4]/(7*a*x^7) + (5*b*Sqrt[a + b*x^4])/(21*a^2*x^3) + (5*b^(7/4
)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellip
ticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(42*a^(9/4)*Sqrt[a + b*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^8\sqrt{a+bx^4}}\,dx &= -\frac{\sqrt{a+bx^4}}{7ax^7}-\frac{(5b)\int \frac{1}{x^4\sqrt{a+bx^4}}\,dx}{7a}\\ &= -\frac{\sqrt{a+bx^4}}{7ax^7}+\frac{5b\sqrt{a+bx^4}}{21a^2x^3}+\frac{\left(5b^2\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{21a^2}\\ &= -\frac{\sqrt{a+bx^4}}{7ax^7}+\frac{5b\sqrt{a+bx^4}}{21a^2x^3}+\frac{5b^{7/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{42a^{9/4}\sqrt{a+bx^4}}\end{aligned}$$

Mathematica [C] time = 0.04, size = 51, normalized size = 0.39

$$-\frac{\sqrt{\frac{bx^4}{a}+1}\ {}_2F_1\left(-\frac{7}{4},\frac{1}{2};-\frac{3}{4};-\frac{bx^4}{a}\right)}{7x^7\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*Sqrt[a + b*x^4]),x]
```

```
[Out] -1/7*(Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-7/4, 1/2, -3/4, -((b*x^4)/a)])
/(x^7*Sqrt[a + b*x^4])
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^4+a}}{bx^{12}+ax^8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)/(b*x^12 + a*x^8), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^4+a}\,x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a)*x^8), x)
```

maple [C] time = 0.02, size = 113, normalized size = 0.86

$$\frac{5\sqrt{-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\ \sqrt{\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\ b^2\,\text{EllipticF}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\ x,i\right)}{21\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\ \sqrt{b\,x^4+a}\ a^2}+\frac{5\sqrt{b\,x^4+a}\ b}{21a^2x^3}-\frac{\sqrt{b\,x^4+a}}{7a\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(b*x^4+a)^(1/2),x)
```

```
[Out] -1/7*(b*x^4+a)^(1/2)/a/x^7+5/21*b*(b*x^4+a)^(1/2)/a^2/x^3+5/21/a^2*b^2/(I/a
^(1/2)*b^(1/2))^(1/2)*(-I/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*b^(1/2)*x
^2+1)^(1/2)/(b*x^4+a)^(1/2)*EllipticF((I/a^(1/2)*b^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^4+a}\,x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a)*x^8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^8 \sqrt{b\,x^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(a + b*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^8*(a + b*x^4)^(1/2)), x)
```

sympy [C] time = 2.28, size = 44, normalized size = 0.33

$$\frac{\Gamma\left(-\frac{7}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{7}{4}, \frac{1}{2} \\ -\frac{3}{4} \end{matrix} \middle| \frac{b x^4 e^{i\pi}}{a}\right)}{4\sqrt{a}\, x^7 \Gamma\left(-\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(b*x**4+a)**(1/2),x)
```

```
[Out] gamma(-7/4)*hyper((-7/4, 1/2), (-3/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a
)*x**7*gamma(-3/4))
```

3.826 $\int \frac{x^{10}}{\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=261

$$\frac{7a^{9/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{30b^{11/4}\sqrt{a+bx^4}}-\frac{7a^{9/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{11/4}\sqrt{a+bx^4}}$$

```
[Out] -7/45*a*x^3*(b*x^4+a)^(1/2)/b^2+1/9*x^7*(b*x^4+a)^(1/2)/b+7/15*a^2*x*(b*x^4
+a)^(1/2)/b^(5/2)/(a^(1/2)+x^2*b^(1/2))-7/15*a^(9/4)*(cos(2*arctan(b^(1/4)*
x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arcta
n(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2
)+x^2*b^(1/2))^2)^(1/2)/b^(11/4)/(b*x^4+a)^(1/2)+7/30*a^(9/4)*(cos(2*arctan
(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(si
n(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a
)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(11/4)/(b*x^4+a)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {321, 305, 220, 1196}

$$\frac{7a^2x\sqrt{a+bx^4}}{15b^{5/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{7a^{9/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{30b^{11/4}\sqrt{a+bx^4}}-\frac{7a^{9/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}}{15b^{11/4}\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/Sqrt[a + b*x^4],x]
```

```
[Out] (-7*a*x^3*Sqrt[a + b*x^4])/(45*b^2) + (x^7*Sqrt[a + b*x^4])/(9*b) + (7*a^2*
x*Sqrt[a + b*x^4])/(15*b^(5/2)*(Sqrt[a] + Sqrt[b]*x^2)) - (7*a^(9/4)*(Sqrt[
a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*A
rcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(11/4)*Sqrt[a + b*x^4]) + (7*a^(9/4
)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellip
ticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(30*b^(11/4)*Sqrt[a + b*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^{10}}{\sqrt{a+bx^4}}\,dx = \frac{x^7\sqrt{a+bx^4}}{9b} - \frac{(7a)\int \frac{x^6}{\sqrt{a+bx^4}}\,dx}{9b}$$

$$= -\frac{7ax^3\sqrt{a+bx^4}}{45b^2} + \frac{x^7\sqrt{a+bx^4}}{9b} + \frac{\left(7a^2\right)\int \frac{x^2}{\sqrt{a+bx^4}}\,dx}{15b^2}$$

$$= -\frac{7ax^3\sqrt{a+bx^4}}{45b^2} + \frac{x^7\sqrt{a+bx^4}}{9b} + \frac{\left(7a^{5/2}\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{15b^{5/2}} - \frac{\left(7a^{5/2}\right)\int \frac{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{15b^{5/2}}$$

$$= -\frac{7ax^3\sqrt{a+bx^4}}{45b^2} + \frac{x^7\sqrt{a+bx^4}}{9b} + \frac{7a^2x\sqrt{a+bx^4}}{15b^{5/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)} - \frac{7a^{9/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E}{15b^{11/4}\sqrt{a+bx^4}}$$

Mathematica [C] time = 0.03, size = 80, normalized size = 0.31

$$\frac{x^3\left(7a^2\sqrt{\frac{bx^4}{a}+1}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^4}{a}\right) - 7a^2 - 2abx^4 + 5b^2x^8\right)}{45b^2\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/Sqrt[a + b*x^4],x]
```

```
[Out] (x^3*(-7*a^2 - 2*a*b*x^4 + 5*b^2*x^8 + 7*a^2*Sqrt[1 + (b*x^4)/a]*Hypergeome
tric2F1[1/2, 3/4, 7/4, -((b*x^4)/a)]))/(45*b^2*Sqrt[a + b*x^4])
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^{10}}{\sqrt{bx^4+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^10/sqrt(b*x^4 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\sqrt{bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^10/sqrt(b*x^4 + a), x)
```

maple [C] time = 0.01, size = 133, normalized size = 0.51

$$\frac{\sqrt{b\,x^4+a}\;x^7}{9b}-\frac{7\sqrt{b\,x^4+a}\;a\,x^3}{45b^2}+\frac{7i\sqrt{-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\;\sqrt{\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\;\left(-\text{EllipticE}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\;x,i\right)+\text{EllipticF}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\;x,\right.\right.}{15\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\;\sqrt{b\,x^4+a}\;b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x^4+a)^(1/2),x)
```

```
[Out] 1/9*x^7*(b*x^4+a)^(1/2)/b-7/45*a*x^3*(b*x^4+a)^(1/2)/b^2+7/15*I*a^(5/2)/b^(
5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(-I/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*
b^(1/2)*x^2+1)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF((I/a^(1/2)*b^(1/2))^(1/2)*x
,I)-EllipticE((I/a^(1/2)*b^(1/2))^(1/2)*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\sqrt{bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^10/sqrt(b*x^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{10}}{\sqrt{b\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a + b*x^4)^(1/2),x)
```

```
[Out] int(x^10/(a + b*x^4)^(1/2), x)
```

sympy [C] time = 1.38, size = 37, normalized size = 0.14

$$\frac{x^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b*x**4+a)**(1/2),x)
```

```
[Out] x**11*gamma(11/4)*hyper((1/2, 11/4), (15/4,), b*x**4*exp_polar(I*pi)/a)/(4*
sqrt(a)*gamma(15/4))
```

3.827 $\int \frac{x^6}{\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=237

$$-\frac{3a^{5/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{10b^{7/4}\sqrt{a+bx^4}}+\frac{3a^{5/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^4}}-$$

```
[Out] 1/5*x^3*(b*x^4+a)^(1/2)/b-3/5*a*x*(b*x^4+a)^(1/2)/b^(3/2)/(a^(1/2)+x^2*b^(1
/2))+3/5*a^(5/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^
(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(
a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x
^4+a)^(1/2)-3/10*a^(5/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*a
rctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^
(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(7
/4)/(b*x^4+a)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {321, 305, 220, 1196}

$$-\frac{3a^{5/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{10b^{7/4}\sqrt{a+bx^4}}+\frac{3a^{5/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^4}}-$$

Antiderivative was successfully verified.

```
[In] Int[x^6/Sqrt[a + b*x^4],x]
```

```
[Out] (x^3*Sqrt[a + b*x^4])/(5*b) - (3*a*x*Sqrt[a + b*x^4])/(5*b^(3/2)*(Sqrt[a] +
 Sqrt[b]*x^2)) + (3*a^(5/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[
a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*b^(7
/4)*Sqrt[a + b*x^4]) - (3*a^(5/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/
(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(
10*b^(7/4)*Sqrt[a + b*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^6}{\sqrt{a+bx^4}}\,dx = \frac{x^3\sqrt{a+bx^4}}{5b} - \frac{(3a)\int \frac{x^2}{\sqrt{a+bx^4}}\,dx}{5b}$$

$$= \frac{x^3\sqrt{a+bx^4}}{5b} - \frac{\left(3a^{3/2}\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{5b^{3/2}} + \frac{\left(3a^{3/2}\right)\int \frac{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{5b^{3/2}}$$

$$= \frac{x^3\sqrt{a+bx^4}}{5b} - \frac{3ax\sqrt{a+bx^4}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)} + \frac{3a^{5/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right.}{5b^{7/4}\sqrt{a+bx^4}}$$

Mathematica [C] time = 0.02, size = 64, normalized size = 0.27

$$\frac{x^3\left(-a\sqrt{\frac{bx^4}{a}+1}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^4}{a}\right)+a+bx^4\right)}{5b\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/Sqrt[a + b*x^4],x]
```

```
[Out] (x^3*(a + b*x^4 - a*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -(
(b*x^4)/a)]))/(5*b*Sqrt[a + b*x^4])
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^6}{\sqrt{bx^4+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^6/sqrt(b*x^4 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt{bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/sqrt(b*x^4 + a), x)
```

maple [C] time = 0.01, size = 115, normalized size = 0.49

$$\frac{\sqrt{b\,x^4+a}\;x^3}{5b} - \frac{3i\sqrt{-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\;\sqrt{\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\;\left(-\text{EllipticE}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\;x,i\right)+\text{EllipticF}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\;x,i\right)\right)a^{\frac{3}{2}}}{5\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\;\sqrt{b\,x^4+a}\;b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^4+a)^(1/2),x)
```

```
[Out] 1/5*x^3*(b*x^4+a)^(1/2)/b-3/5*I*a^(3/2)/b^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(
-I/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*b^(1/2)*x^2+1)^(1/2)/(b*x^4+a)^(
1/2)*(EllipticF((I/a^(1/2)*b^(1/2))^(1/2)*x,I)-EllipticE((I/a^(1/2)*b^(1/2)
)^(1/2)*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt{bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/sqrt(b*x^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{\sqrt{b\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^4)^(1/2),x)
```

```
[Out] int(x^6/(a + b*x^4)^(1/2), x)
```

sympy [C] time = 1.71, size = 37, normalized size = 0.16

$$\frac{x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**4+a)**(1/2),x)
```

```
[Out] x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqr
t(a)*gamma(11/4))
```

3.828 $\int \frac{x^2}{\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=210

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}+$$

```
[Out] x*(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/2)+x^2*b^(1/2))-a^(1/4)*(cos(2*arctan(b^(1/
4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*ar
ctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(
1/2)+x^2*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^4+a)^(1/2)+1/2*a^(1/4)*(cos(2*arcta
n(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(s
in(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+
a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^4+a)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {305, 220, 1196}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}+$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*x^4],x]
```

```
[Out] (x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) - (a^(1/4)*(Sqrt[a] +
 Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTa
n[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(3/4)*Sqrt[a + b*x^4]) + (a^(1/4)*(Sqrt[a]
 + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*Arc
Tan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+bx^4}}\,dx = \frac{\sqrt{a}\int \frac{1}{\sqrt{a+bx^4}}\,dx}{\sqrt{b}} - \frac{\sqrt{a}\int \frac{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{\sqrt{b}}$$

$$= \frac{x\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)} - \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}} + \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\,x^2\right.}{}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.24

$$\frac{x^3\sqrt{\frac{bx^4}{a}+1}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^4}{a}\right)}{3\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b*x^4],x]
```

```
[Out] (x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x^4)/a)])/(3
*Sqrt[a + b*x^4])
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2}{\sqrt{bx^4+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^2/sqrt(b*x^4 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(b*x^4 + a), x)
```

maple [C] time = 0.01, size = 97, normalized size = 0.46

$$\frac{i\sqrt{-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\,\left(-\text{EllipticE}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x,i\right)+\text{EllipticF}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x,i\right)\right)\sqrt{a}}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^4+a)^(1/2),x)
```

```
[Out] I*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(-I/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(I/a^(
1/2)*b^(1/2)*x^2+1)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF((I/a^(1/2)*b^(
1/2))^(1/2)*x,I)-EllipticE((I/a^(1/2)*b^(1/2))^(1/2)*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(b*x^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{b\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^4)^(1/2),x)
```

```
[Out] int(x^2/(a + b*x^4)^(1/2), x)
```

sympy [C] time = 1.49, size = 37, normalized size = 0.18

$$\frac{x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**4+a)**(1/2),x)
```

```
[Out] x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt
(a)*gamma(7/4))
```

3.829 $\int \frac{1}{x^2\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=232

$$\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}-\frac{\sqrt{a}$$

```
[Out] -(b*x^4+a)^(1/2)/a/x+x*b^(1/2)*(b*x^4+a)^(1/2)/a/(a^(1/2)+x^2*b^(1/2))-b^(1
/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/
4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b
^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^4+a)^(1/2)+1
/2*b^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*
x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2
)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^4+a)^
(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {325, 305, 220, 1196}

$$\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}+\frac{\sqrt{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*x^4]),x]
```

```
[Out] -(Sqrt[a + b*x^4]/(a*x)) + (Sqrt[b]*x*Sqrt[a + b*x^4])/(a*(Sqrt[a] + Sqrt[b
]*x^2)) - (b^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt
[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a
+ b*x^4]) + (b^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sq
rt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*Sqr
t[a + b*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{a+bx^4}}{ax} + \frac{b\int \frac{x^2}{\sqrt{a+bx^4}}\,dx}{a}$$

$$= -\frac{\sqrt{a+bx^4}}{ax} + \frac{\sqrt{b}\int \frac{1}{\sqrt{a+bx^4}}\,dx}{\sqrt{a}} - \frac{\sqrt{b}\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{\sqrt{a}}$$

$$= -\frac{\sqrt{a+bx^4}}{ax} + \frac{\sqrt{b}\,x\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}\,x^2\right)} - \frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}} +$$

Mathematica [C] time = 0.01, size = 49, normalized size = 0.21

$$-\frac{\sqrt{\frac{bx^4}{a}+1}\;{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{bx^4}{a}\right)}{x\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x^4]),x]
```

```
[Out] -((Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-1/4, 1/2, 3/4, -((b*x^4)/a)])/(x*
Sqrt[a + b*x^4]))
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^4+a}}{bx^6+ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)/(b*x^6 + a*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^4+a}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a)*x^2), x)
```

maple [C] time = 0.01, size = 115, normalized size = 0.50

$$\frac{i\sqrt{-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\left(-\text{EllipticE}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x,i\right)+\text{EllipticF}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x,i\right)\right)\sqrt{b}}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{a}} - \frac{\sqrt{b\,x^4+a}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^4+a)^(1/2),x)
```

```
[Out] -(b*x^4+a)^(1/2)/a/x+I/a^(1/2)*b^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(-I/a^(1/2
)*b^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*b^(1/2)*x^2+1)^(1/2)/(b*x^4+a)^(1/2)*(Ell
ipticF((I/a^(1/2)*b^(1/2))^(1/2)*x,I)-EllipticE((I/a^(1/2)*b^(1/2))^(1/2)*x
,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^4+a}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a)*x^2), x)
```

mupad [B] time = 1.26, size = 40, normalized size = 0.17

$$-\frac{\sqrt{\frac{a}{b\,x^4}+1}\;{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\;\frac{7}{4};\;-\frac{a}{b\,x^4}\right)}{3\,x\,\sqrt{b\,x^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^4)^(1/2)),x)
```

```
[Out] -((a/(b*x^4) + 1)^(1/2)*hypergeom([1/2, 3/4], 7/4, -a/(b*x^4)))/(3*x*(a + b
*x^4)^(1/2))
```

sympy [C] time = 2.13, size = 39, normalized size = 0.17

$$\frac{\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,x\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**4+a)**(1/2),x)
```

```
[Out] gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)
*x*gamma(3/4))
```

3.830 $\int \frac{1}{x^6\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=261

$$-\frac{3b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{10a^{7/4}\sqrt{a+bx^4}}+\frac{3b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|$$

```
[Out] -1/5*(b*x^4+a)^(1/2)/a/x^5+3/5*b*(b*x^4+a)^(1/2)/a^2/x-3/5*b^(3/2)*x*(b*x^4
+a)^(1/2)/a^2/(a^(1/2)+x^2*b^(1/2))+3/5*b^(5/4)*(cos(2*arctan(b^(1/4)*x/a^(
1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(
1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2
*b^(1/2))^2)^(1/2)/a^(7/4)/(b*x^4+a)^(1/2)-3/10*b^(5/4)*(cos(2*arctan(b^(1/
4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*ar
ctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(
1/2)+x^2*b^(1/2))^2)^(1/2)/a^(7/4)/(b*x^4+a)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {325, 305, 220, 1196}

$$-\frac{3b^{3/2}x\sqrt{a+bx^4}}{5a^2\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{3b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{10a^{7/4}\sqrt{a+bx^4}}+\frac{3b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)}}}{5a^{7/4}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*Sqrt[a + b*x^4]),x]
```

```
[Out] -Sqrt[a + b*x^4]/(5*a*x^5) + (3*b*Sqrt[a + b*x^4])/(5*a^2*x) - (3*b^(3/2)*x
*Sqrt[a + b*x^4])/(5*a^2*(Sqrt[a] + Sqrt[b]*x^2)) + (3*b^(5/4)*(Sqrt[a] + S
qrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[
(b^(1/4)*x)/a^(1/4)], 1/2])/(5*a^(7/4)*Sqrt[a + b*x^4]) - (3*b^(5/4)*(Sqrt[
a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*A
rcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(10*a^(7/4)*Sqrt[a + b*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{x^6\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{a+bx^4}}{5ax^5} - \frac{(3b)\int \frac{1}{x^2\sqrt{a+bx^4}}\,dx}{5a}$$

$$= -\frac{\sqrt{a+bx^4}}{5ax^5} + \frac{3b\sqrt{a+bx^4}}{5a^2x} - \frac{\left(3b^2\right)\int \frac{x^2}{\sqrt{a+bx^4}}\,dx}{5a^2}$$

$$= -\frac{\sqrt{a+bx^4}}{5ax^5} + \frac{3b\sqrt{a+bx^4}}{5a^2x} - \frac{\left(3b^{3/2}\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{5a^{3/2}} + \frac{\left(3b^{3/2}\right)\int \frac{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{5a^{3/2}}$$

$$= -\frac{\sqrt{a+bx^4}}{5ax^5} + \frac{3b\sqrt{a+bx^4}}{5a^2x} - \frac{3b^{3/2}x\sqrt{a+bx^4}}{5a^2\left(\sqrt{a}+\sqrt{b}\,x^2\right)} + \frac{3b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\tan$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.20

$$-\frac{\sqrt{\frac{bx^4}{a}+1}\;{}_2F_1\left(-\frac{5}{4},\frac{1}{2};-\frac{1}{4};-\frac{bx^4}{a}\right)}{5x^5\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*Sqrt[a + b*x^4]),x]
```

```
[Out] -1/5*(Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-5/4, 1/2, -1/4, -((b*x^4)/a)])
/(x^5*Sqrt[a + b*x^4])
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^4+a}}{bx^{10}+ax^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)/(b*x^10 + a*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^4+a}\,x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a)*x^6), x)
```

maple [C] time = 0.01, size = 133, normalized size = 0.51

$$-\frac{3i\sqrt{-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\,\left(-\text{EllipticE}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x,i\right)+\text{EllipticF}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x,i\right)\right)b^{\frac{3}{2}}}{5\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,a^{\frac{3}{2}}}+\frac{3\sqrt{b\,x^4+a}\,b}{5a^2x}-\frac{\sqrt{b\,x^4+}}{5a\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^4+a)^(1/2),x)
```

```
[Out] -1/5*(b*x^4+a)^(1/2)/a/x^5+3/5*b*(b*x^4+a)^(1/2)/a^2/x-3/5*I/a^(3/2)*b^(3/2
)/(I/a^(1/2)*b^(1/2))^(1/2)*(-I/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*b^(
1/2)*x^2+1)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF((I/a^(1/2)*b^(1/2))^(1/2)*x,I)
-EllipticE((I/a^(1/2)*b^(1/2))^(1/2)*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^4+a}\,x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^6\,\sqrt{b\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^6*(a + b*x^4)^(1/2)), x)
```

sympy [C] time = 2.31, size = 44, normalized size = 0.17

$$\frac{\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{5}{4},\frac{1}{2}\\-\frac{1}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,x^5\Gamma\left(-\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**4+a)**(1/2),x)
```

```
[Out] gamma(-5/4)*hyper((-5/4, 1/2), (-1/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a
)*x**5*gamma(-1/4))
```

3.831 $\int \frac{x^{11}}{\sqrt{a-bx^4}}\,dx$

Optimal. Leaf size=62

$$-\frac{a^2\sqrt{a-bx^4}}{2b^3}-\frac{\left(a-bx^4\right)^{5/2}}{10b^3}+\frac{a\left(a-bx^4\right)^{3/2}}{3b^3}$$

```
[Out] 1/3*a*(-b*x^4+a)^(3/2)/b^3-1/10*(-b*x^4+a)^(5/2)/b^3-1/2*a^2*(-b*x^4+a)^(1/
2)/b^3
```

Rubi [A] time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {266, 43}

$$-\frac{a^2\sqrt{a-bx^4}}{2b^3}-\frac{\left(a-bx^4\right)^{5/2}}{10b^3}+\frac{a\left(a-bx^4\right)^{3/2}}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/Sqrt[a - b*x^4],x]
```

```
[Out] -(a^2*Sqrt[a - b*x^4])/(2*b^3) + (a*(a - b*x^4)^(3/2))/(3*b^3) - (a - b*x^4
)^(5/2)/(10*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{\sqrt{a-bx^4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a-bx}}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int\left(\frac{a^2}{b^2\sqrt{a-bx}}-\frac{2a\sqrt{a-bx}}{b^2}+\frac{(a-bx)^{3/2}}{b^2}\right)dx, x, x^4\right)\\ &= -\frac{a^2\sqrt{a-bx^4}}{2b^3}+\frac{a\left(a-bx^4\right)^{3/2}}{3b^3}-\frac{\left(a-bx^4\right)^{5/2}}{10b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 0.65

$$-\frac{\sqrt{a-bx^4}\left(8a^2+4abx^4+3b^2x^8\right)}{30b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/Sqrt[a - b*x^4],x]
```

```
[Out] -1/30*(Sqrt[a - b*x^4]*(8*a^2 + 4*a*b*x^4 + 3*b^2*x^8))/b^3
```

fricas [A] time = 0.81, size = 36, normalized size = 0.58

$$-\frac{\left(3\,b^2x^8+4\,abx^4+8\,a^2\right)\sqrt{-bx^4+a}}{30\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/30*(3*b^2*x^8 + 4*a*b*x^4 + 8*a^2)*sqrt(-b*x^4 + a)/b^3
```

giac [A] time = 0.17, size = 61, normalized size = 0.98

$$-\frac{\sqrt{-bx^4+a}\,a^2}{2\,b^3}-\frac{3\left(bx^4-a\right)^2\sqrt{-bx^4+a}-10\left(-bx^4+a\right)^{\frac{3}{2}}a}{30\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-b*x^4 + a)*a^2/b^3 - 1/30*(3*(b*x^4 - a)^2*sqrt(-b*x^4 + a) - 10
*(-b*x^4 + a)^(3/2)*a)/b^3
```

maple [A] time = 0.01, size = 37, normalized size = 0.60

$$-\frac{\sqrt{-b\,x^4+a}\,\left(3b^2x^8+4ab\,x^4+8a^2\right)}{30b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(-b*x^4+a)^(1/2),x)
```

```
[Out] -1/30*(-b*x^4+a)^(1/2)*(3*b^2*x^8+4*a*b*x^4+8*a^2)/b^3
```

maxima [A] time = 1.34, size = 50, normalized size = 0.81

$$-\frac{\left(-bx^4+a\right)^{\frac{5}{2}}}{10\,b^3}+\frac{\left(-bx^4+a\right)^{\frac{3}{2}}a}{3\,b^3}-\frac{\sqrt{-bx^4+a}\,a^2}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/10*(-b*x^4 + a)^(5/2)/b^3 + 1/3*(-b*x^4 + a)^(3/2)*a/b^3 - 1/2*sqrt(-b*x
^4 + a)*a^2/b^3
```

mupad [B] time = 1.21, size = 38, normalized size = 0.61

$$-\sqrt{a-b\,x^4}\,\left(\frac{4\,a^2}{15\,b^3}+\frac{x^8}{10\,b}+\frac{2\,a\,x^4}{15\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a - b*x^4)^(1/2),x)
```

```
[Out] -(a - b*x^4)^(1/2)*((4*a^2)/(15*b^3) + x^8/(10*b) + (2*a*x^4)/(15*b^2))
```

sympy [A] time = 3.25, size = 70, normalized size = 1.13

$$\begin{cases} -\frac{4a^2\sqrt{a-bx^4}}{15b^3}-\frac{2ax^4\sqrt{a-bx^4}}{15b^2}-\frac{x^8\sqrt{a-bx^4}}{10b} & \text{for } b\neq 0 \\ \frac{x^{12}}{12\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(-b*x**4+a)**(1/2),x)
```

```
[Out] Piecewise((-4*a**2*sqrt(a - b*x**4)/(15*b**3) - 2*a*x**4*sqrt(a - b*x**4)/(
15*b**2) - x**8*sqrt(a - b*x**4)/(10*b), Ne(b, 0)), (x**12/(12*sqrt(a)), Tr
ue))
```

3.832 $\int \frac{x^7}{\sqrt{a-bx^4}}\,dx$

Optimal. Leaf size=40

$$\frac{\left(a-bx^4\right)^{3/2}}{6b^2}-\frac{a\sqrt{a-bx^4}}{2b^2}$$

```
[Out] 1/6*(-b*x^4+a)^(3/2)/b^2-1/2*a*(-b*x^4+a)^(1/2)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {266, 43}

$$\frac{\left(a-bx^4\right)^{3/2}}{6b^2}-\frac{a\sqrt{a-bx^4}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/Sqrt[a - b*x^4],x]
```

```
[Out] -(a*Sqrt[a - b*x^4])/(2*b^2) + (a - b*x^4)^(3/2)/(6*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{\sqrt{a-bx^4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x}{\sqrt{a-bx}}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int \left(\frac{a}{b\sqrt{a-bx}}-\frac{\sqrt{a-bx}}{b}\right)dx, x, x^4\right)\\ &= -\frac{a\sqrt{a-bx^4}}{2b^2}+\frac{\left(a-bx^4\right)^{3/2}}{6b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.70

$$-\frac{\sqrt{a-bx^4}\left(2a+bx^4\right)}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/Sqrt[a - b*x^4],x]
```

```
[Out] -1/6*(Sqrt[a - b*x^4]*(2*a + b*x^4))/b^2
```

fricas [A] time = 0.87, size = 24, normalized size = 0.60

$$-\frac{\left(bx^4+2\,a\right)\sqrt{-bx^4+a}}{6\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-b*x^4+a)^(1/2),x, algorithm="fricas")
[Out] -1/6*(b*x^4 + 2*a)*sqrt(-b*x^4 + a)/b^2
```

giac [A] time = 0.15, size = 32, normalized size = 0.80

$$\frac{\left(-bx^4+a\right)^{\frac{3}{2}}}{6\,b^2}-\frac{\sqrt{-bx^4+a}\,a}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-b*x^4+a)^(1/2),x, algorithm="giac")
[Out] 1/6*(-b*x^4 + a)^(3/2)/b^2 - 1/2*sqrt(-b*x^4 + a)*a/b^2
```

maple [A] time = 0.01, size = 25, normalized size = 0.62

$$-\frac{\sqrt{-b\,x^4+a}\,\left(b\,x^4+2a\right)}{6b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-b*x^4+a)^(1/2),x)
[Out] -1/6*(-b*x^4+a)^(1/2)*(b*x^4+2*a)/b^2
```

maxima [A] time = 1.30, size = 32, normalized size = 0.80

$$\frac{\left(-bx^4+a\right)^{\frac{3}{2}}}{6\,b^2}-\frac{\sqrt{-bx^4+a}\,a}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-b*x^4+a)^(1/2),x, algorithm="maxima")
[Out] 1/6*(-b*x^4 + a)^(3/2)/b^2 - 1/2*sqrt(-b*x^4 + a)*a/b^2
```

mupad [B] time = 1.17, size = 24, normalized size = 0.60

$$-\frac{\sqrt{a-b\,x^4}\,\left(b\,x^4+2\,a\right)}{6\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a - b*x^4)^(1/2),x)
[Out] -((a - b*x^4)^(1/2)*(2*a + b*x^4))/(6*b^2)
```

sympy [A] time = 1.61, size = 44, normalized size = 1.10

$$\begin{cases} -\frac{a\sqrt{a-bx^4}}{3b^2}-\frac{x^4\sqrt{a-bx^4}}{6b} & \text{for } b\neq 0 \\ \frac{x^8}{8\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-b*x**4+a)**(1/2),x)
```

```
[Out] Piecewise((-a*sqrt(a - b*x**4)/(3*b**2) - x**4*sqrt(a - b*x**4)/(6*b), Ne(b
, 0)), (x**8/(8*sqrt(a)), True))
```

3.833 $\int \frac{x^3}{\sqrt{a-bx^4}}\,dx$

Optimal. Leaf size=19

$$-\frac{\sqrt{a-bx^4}}{2b}$$

```
[Out] -1/2*(-b*x^4+a)^(1/2)/b
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {261}

$$-\frac{\sqrt{a-bx^4}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a - b*x^4],x]
```

```
[Out] -Sqrt[a - b*x^4]/(2*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a-bx^4}}\,dx = -\frac{\sqrt{a-bx^4}}{2b}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$-\frac{\sqrt{a-bx^4}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a - b*x^4],x]
```

```
[Out] -1/2*Sqrt[a - b*x^4]/b
```

fricas [A] time = 0.83, size = 15, normalized size = 0.79

$$-\frac{\sqrt{-bx^4+a}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-b*x^4 + a)/b
```

giac [A] time = 0.19, size = 15, normalized size = 0.79

$$-\frac{\sqrt{-bx^4+a}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^4+a)^(1/2),x, algorithm="giac")
[Out] -1/2*sqrt(-b*x^4 + a)/b
```

maple [A] time = 0.00, size = 16, normalized size = 0.84

$$-\frac{\sqrt{-b\,x^4+a}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-b*x^4+a)^(1/2),x)
[Out] -1/2*(-b*x^4+a)^(1/2)/b
```

maxima [A] time = 1.30, size = 15, normalized size = 0.79

$$-\frac{\sqrt{-bx^4+a}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^4+a)^(1/2),x, algorithm="maxima")
[Out] -1/2*sqrt(-b*x^4 + a)/b
```

mupad [B] time = 1.19, size = 15, normalized size = 0.79

$$-\frac{\sqrt{a-b\,x^4}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a - b*x^4)^(1/2),x)
[Out] -(a - b*x^4)^(1/2)/(2*b)
```

sympy [A] time = 1.03, size = 24, normalized size = 1.26

$$\begin{cases} -\frac{\sqrt{a-bx^4}}{2b} & \text{for } b \neq 0 \\ \frac{x^4}{4\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-b*x**4+a)**(1/2),x)
[Out] Piecewise((-sqrt(a - b*x**4)/(2*b), Ne(b, 0)), (x**4/(4*sqrt(a)), True))
```

3.834 $\int \frac{1}{x\sqrt{a-bx^4}}\,dx$

Optimal. Leaf size=28

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a-bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

```
[Out] -1/2*arctanh((-b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {266, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a-bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a - b*x^4]),x]
```

```
[Out] -ArcTanh[Sqrt[a - b*x^4]/Sqrt[a]]/(2*Sqrt[a])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{a-bx^4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x\sqrt{a-bx}}\,dx, x, x^4\right)\\ &= -\frac{\text{Subst}\left(\int \frac{1}{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a-bx^4}\right)}{2b}\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{a-bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a-bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a - b*x^4]),x]
```

```
[Out] -1/2*ArcTanh[Sqrt[a - b*x^4]/Sqrt[a]]/Sqrt[a]
```

fricas [A] time = 0.78, size = 65, normalized size = 2.32

$$\left[\frac{\log\left(\frac{bx^4+2\sqrt{-bx^4+a}\sqrt{a}-2a}{x^4}\right)}{4\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{-bx^4+a}\sqrt{-a}}{a}\right)}{2a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log((b*x^4 + 2*sqrt(-b*x^4 + a)*sqrt(a) - 2*a)/x^4)/sqrt(a), 1/2*sqrt(
-a)*arctan(sqrt(-b*x^4 + a)*sqrt(-a)/a)/a]
```

giac [A] time = 0.16, size = 24, normalized size = 0.86

$$\frac{\arctan\left(\frac{\sqrt{-bx^4+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*arctan(sqrt(-b*x^4 + a)/sqrt(-a))/sqrt(-a)
```

maple [A] time = 0.01, size = 30, normalized size = 1.07

$$-\frac{\ln\left(\frac{2a+2\sqrt{-b\,x^4+a}\,\sqrt{a}}{x^2}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-b*x^4+a)^(1/2),x)
```

```
[Out] -1/2/a^(1/2)*ln((2*a+2*a^(1/2)*(-b*x^4+a)^(1/2))/x^2)
```

maxima [A] time = 2.95, size = 39, normalized size = 1.39

$$\frac{\log\left(\frac{\sqrt{-bx^4+a}-\sqrt{a}}{\sqrt{-bx^4+a}+\sqrt{a}}\right)}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*log((sqrt(-b*x^4 + a) - sqrt(a))/(sqrt(-b*x^4 + a) + sqrt(a)))/sqrt(a)
```

mupad [B] time = 1.21, size = 20, normalized size = 0.71

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{a-b\,x^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a - b*x^4)^(1/2)),x)
```

```
[Out] -atanh((a - b*x^4)^(1/2)/a^(1/2))/(2*a^(1/2))
```

sympy [A] time = 2.21, size = 53, normalized size = 1.89

$$\begin{cases} -\dfrac{\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^2}\right)}{2\sqrt{a}} & \text{for } \left|\frac{a}{bx^4}\right| > 1 \\ \dfrac{i\,\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^2}\right)}{2\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x**4+a)**(1/2),x)
```

```
[Out] Piecewise((-acosh(sqrt(a)/(sqrt(b)*x**2))/(2*sqrt(a)), Abs(a/(b*x**4)) > 1)
, (I*asin(sqrt(a)/(sqrt(b)*x**2))/(2*sqrt(a)), True))
```

3.835 $\int \frac{1}{x^5 \sqrt{a-bx^4}}\,dx$

Optimal. Leaf size=52

$$-\frac{b \tanh^{-1}\left(\frac{\sqrt{a-bx^4}}{\sqrt{a}}\right)}{4a^{3/2}} - \frac{\sqrt{a-bx^4}}{4ax^4}$$

```
[Out] -1/4*b*arctanh((-b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)-1/4*(-b*x^4+a)^(1/2)/a/x^4
```

Rubi [A] time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {266, 51, 63, 208}

$$-\frac{b \tanh^{-1}\left(\frac{\sqrt{a-bx^4}}{\sqrt{a}}\right)}{4a^{3/2}} - \frac{\sqrt{a-bx^4}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[a - b*x^4]),x]
```

```
[Out] -Sqrt[a - b*x^4]/(4*a*x^4) - (b*ArcTanh[Sqrt[a - b*x^4]/Sqrt[a]])/(4*a^(3/2
))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\sqrt{a-bx^4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a-bx}}\,dx, x, x^4\right)\\ &= -\frac{\sqrt{a-bx^4}}{4ax^4} + \frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a-bx}}\,dx, x, x^4\right)}{8a}\\ &= -\frac{\sqrt{a-bx^4}}{4ax^4} - \frac{\text{Subst}\left(\int \frac{1}{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a-bx^4}\right)}{4a}\\ &= -\frac{\sqrt{a-bx^4}}{4ax^4} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a-bx^4}}{\sqrt{a}}\right)}{4a^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 61, normalized size = 1.17

$$-\frac{b\sqrt{a-bx^4}\left(\frac{a}{bx^4} + \frac{\tanh^{-1}\left(\sqrt{1-\frac{bx^4}{a}}\right)}{\sqrt{1-\frac{bx^4}{a}}}\right)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[a - b*x^4]),x]
```

```
[Out] -1/4*(b*Sqrt[a - b*x^4]*(a/(b*x^4) + ArcTanh[Sqrt[1 - (b*x^4)/a]]/Sqrt[1 -
(b*x^4)/a]))/a^2
```

fricas [A] time = 0.85, size = 112, normalized size = 2.15

$$\left[\frac{\sqrt{a}\,bx^4\log\left(\frac{bx^4+2\sqrt{-bx^4+a}\sqrt{a}-2a}{x^4}\right) - 2\sqrt{-bx^4+a}\,a}{8a^2x^4}, \frac{\sqrt{-a}\,bx^4\arctan\left(\frac{\sqrt{-bx^4+a}\sqrt{-a}}{a}\right) - \sqrt{-bx^4+a}\,a}{4a^2x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a)*b*x^4*log((b*x^4 + 2*sqrt(-b*x^4 + a)*sqrt(a) - 2*a)/x^4) - 2
*sqrt(-b*x^4 + a)*a)/(a^2*x^4), 1/4*(sqrt(-a)*b*x^4*arctan(sqrt(-b*x^4 + a)
*sqrt(-a)/a) - sqrt(-b*x^4 + a)*a)/(a^2*x^4)]
```

giac [A] time = 0.16, size = 54, normalized size = 1.04

$$\frac{\frac{b^2\arctan\left(\frac{\sqrt{-bx^4+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a} - \frac{\sqrt{-bx^4+a}\,b}{ax^4}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(b^2*arctan(sqrt(-b*x^4 + a)/sqrt(-a))/(sqrt(-a)*a) - sqrt(-b*x^4 + a)*
b/(a*x^4))/b
```

maple [A] time = 0.01, size = 50, normalized size = 0.96

$$-\frac{b\ln\left(\frac{2a+2\sqrt{-b\,x^4+a}\,\sqrt{a}}{x^2}\right)}{4a^{\frac{3}{2}}} - \frac{\sqrt{-b\,x^4+a}}{4a\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-b*x^4+a)^(1/2),x)
```

```
[Out] -1/4*(-b*x^4+a)^(1/2)/a/x^4-1/4*b/a^(3/2)*ln((2*a+2*(-b*x^4+a)^(1/2)*a^(1/2
))/x^2)
```

maxima [A] time = 3.02, size = 71, normalized size = 1.37

$$-\frac{\sqrt{-bx^4+a}\,b}{4\left(\left(bx^4-a\right)a+a^2\right)}+\frac{b\log\left(\frac{\sqrt{-bx^4+a}-\sqrt{a}}{\sqrt{-bx^4+a}+\sqrt{a}}\right)}{8\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(-b*x^4 + a)*b/((b*x^4 - a)*a + a^2) + 1/8*b*log((sqrt(-b*x^4 + a)
 - sqrt(a))/(sqrt(-b*x^4 + a) + sqrt(a)))/a^(3/2)
```

mupad [B] time = 1.29, size = 40, normalized size = 0.77

$$-\frac{\sqrt{a-b\,x^4}}{4\,a\,x^4}-\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{a-b\,x^4}}{\sqrt{a}}\right)}{4\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a - b*x^4)^(1/2)),x)
```

```
[Out] - (a - b*x^4)^(1/2)/(4*a*x^4) - (b*atanh((a - b*x^4)^(1/2)/a^(1/2)))/(4*a^(
3/2))
```

sympy [A] time = 5.23, size = 129, normalized size = 2.48

$$\begin{cases} -\frac{\sqrt{b}\,\sqrt{\frac{a}{bx^4}-1}}{4ax^2}-\frac{b\,\mathrm{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^2}\right)}{4a^{\frac{3}{2}}} & \text{for } \left|\frac{a}{bx^4}\right|>1 \\ \frac{i}{4\sqrt{b}\,x^6\sqrt{-\frac{a}{bx^4}+1}}-\frac{i\sqrt{b}}{4ax^2\sqrt{-\frac{a}{bx^4}+1}}+\frac{ib\,\mathrm{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^2}\right)}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-b*x**4+a)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(b)*sqrt(a/(b*x**4) - 1)/(4*a*x**2) - b*acosh(sqrt(a)/(sqrt
(b)*x**2))/(4*a**(3/2)), Abs(a/(b*x**4)) > 1), (I/(4*sqrt(b)*x**6*sqrt(-a/(
b*x**4) + 1)) - I*sqrt(b)/(4*a*x**2*sqrt(-a/(b*x**4) + 1)) + I*b*asin(sqrt(
a)/(sqrt(b)*x**2))/(4*a**(3/2)), True))
```

3.836 $\int \frac{x^5}{\sqrt{a-bx^4}}\,dx$

Optimal. Leaf size=55

$$\frac{a\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}\right)}{4b^{3/2}} - \frac{x^2\sqrt{a-bx^4}}{4b}$$

```
[Out] 1/4*a*arctan(x^2*b^(1/2)/(-b*x^4+a)^(1/2))/b^(3/2)-1/4*x^2*(-b*x^4+a)^(1/2)
/b
```

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {275, 321, 217, 203}

$$\frac{a\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}\right)}{4b^{3/2}} - \frac{x^2\sqrt{a-bx^4}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[a - b*x^4],x]
```

```
[Out] -(x^2*Sqrt[a - b*x^4])/(4*b) + (a*ArcTan[(Sqrt[b]*x^2)/Sqrt[a - b*x^4]])/(4
*b^(3/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\sqrt{a-bx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a-bx^2}}\,dx, x, x^2\right)\\ &= -\frac{x^2\sqrt{a-bx^4}}{4b} + \frac{a\,\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx, x, x^2\right)}{4b}\\ &= -\frac{x^2\sqrt{a-bx^4}}{4b} + \frac{a\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x^2}{\sqrt{a-bx^4}}\right)}{4b}\\ &= -\frac{x^2\sqrt{a-bx^4}}{4b} + \frac{a\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}\right)}{4b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 55, normalized size = 1.00

$$\frac{a\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}\right)}{4b^{3/2}} - \frac{x^2\sqrt{a-bx^4}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[a - b*x^4],x]
```

```
[Out] -1/4*(x^2*Sqrt[a - b*x^4])/b + (a*ArcTan[(Sqrt[b]*x^2)/Sqrt[a - b*x^4]])/(4
*b^(3/2))
```

fricas [A] time = 0.89, size = 116, normalized size = 2.11

$$\left[-\frac{2\sqrt{-bx^4+a}\,bx^2 + a\sqrt{-b}\,\log\left(2\,bx^4 - 2\sqrt{-bx^4+a}\,\sqrt{-b}\,x^2 - a\right)}{8\,b^2}, -\frac{\sqrt{-bx^4+a}\,bx^2 + a\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx^4+a}\,\sqrt{b}}{bx^4-a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(2*sqrt(-b*x^4 + a)*b*x^2 + a*sqrt(-b)*log(2*b*x^4 - 2*sqrt(-b*x^4 +
a)*sqrt(-b)*x^2 - a))/b^2, -1/4*(sqrt(-b*x^4 + a)*b*x^2 + a*sqrt(b)*arctan(
sqrt(-b*x^4 + a)*sqrt(b)*x^2/(b*x^4 - a)))/b^2]
```

giac [A] time = 0.21, size = 53, normalized size = 0.96

$$-\frac{\sqrt{-bx^4+a}\,x^2}{4\,b} - \frac{a\log\left(\left|-\sqrt{-b}\,x^2 + \sqrt{-bx^4+a}\right|\right)}{4\sqrt{-b}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(-b*x^4 + a)*x^2/b - 1/4*a*log(abs(-sqrt(-b)*x^2 + sqrt(-b*x^4 + a
)))/(sqrt(-b)*b)
```

maple [A] time = 0.01, size = 44, normalized size = 0.80

$$-\frac{\sqrt{-b\,x^4+a}\;x^2}{4b} + \frac{a\arctan\left(\frac{\sqrt{b}\;x^2}{\sqrt{-b\,x^4+a}}\right)}{4b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-b*x^4+a)^(1/2),x)
```

```
[Out] 1/4*a*arctan(x^2*b^(1/2)/(-b*x^4+a)^(1/2))/b^(3/2)-1/4*x^2*(-b*x^4+a)^(1/2)
/b
```

maxima [A] time = 3.04, size = 62, normalized size = 1.13

$$-\frac{a \arctan\left(\frac{\sqrt{-bx^4+a}}{\sqrt{b}\,x^2}\right)}{4\,b^{\frac{3}{2}}} - \frac{\sqrt{-bx^4+a}\,a}{4\left(b^2 - \frac{\left(bx^4-a\right)b}{x^4}\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*a*arctan(sqrt(-b*x^4 + a)/(sqrt(b)*x^2))/b^(3/2) - 1/4*sqrt(-b*x^4 + a
)*a/((b^2 - (b*x^4 - a)*b/x^4)*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^5}{\sqrt{a - b\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a - b*x^4)^(1/2),x)
```

```
[Out] int(x^5/(a - b*x^4)^(1/2), x)
```

sympy [A] time = 4.38, size = 128, normalized size = 2.33

$$\begin{cases} -\frac{i\sqrt{a}\,x^2\sqrt{-1+\frac{bx^4}{a}}}{4b} - \frac{ia\,\operatorname{acosh}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}} & \text{for } \left|\frac{bx^4}{a}\right| > 1 \\ -\frac{\sqrt{a}\,x^2}{4b\sqrt{1-\frac{bx^4}{a}}} + \frac{a\,\operatorname{asin}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}} + \frac{x^6}{4\sqrt{a}\sqrt{1-\frac{bx^4}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-b*x**4+a)**(1/2),x)
```

```
[Out] Piecewise((-I*sqrt(a)*x**2*sqrt(-1 + b*x**4/a)/(4*b) - I*a*acosh(sqrt(b)*x*
*2/sqrt(a))/(4*b**(3/2)), Abs(b*x**4/a) > 1), (-sqrt(a)*x**2/(4*b*sqrt(1 -
b*x**4/a)) + a*asin(sqrt(b)*x**2/sqrt(a))/(4*b**(3/2)) + x**6/(4*sqrt(a)*sq
rt(1 - b*x**4/a)), True))
```

3.837 $\int \frac{x}{\sqrt{a-bx^4}}\,dx$

Optimal. Leaf size=31

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}\right)}{2\sqrt{b}}$$

```
[Out] 1/2*arctan(x^2*b^(1/2)/(-b*x^4+a)^(1/2))/b^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {275, 217, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a - b*x^4],x]
```

```
[Out] ArcTan[(Sqrt[b]*x^2)/Sqrt[a - b*x^4]]/(2*Sqrt[b])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{a-bx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x^2}{\sqrt{a-bx^4}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}\right)}{2\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 31, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a - b*x^4],x]
```

```
[Out] ArcTan[(Sqrt[b]*x^2)/Sqrt[a - b*x^4]]/(2*Sqrt[b])
```

fricas [A] time = 0.85, size = 76, normalized size = 2.45

$$\left[-\frac{\sqrt{-b}\,\log\left(2\,bx^4-2\,\sqrt{-bx^4+a}\,\sqrt{-b}\,x^2-a\right)}{4\,b},-\frac{\arctan\left(\frac{\sqrt{-bx^4+a}\,\sqrt{b}\,x^2}{bx^4-a}\right)}{2\,\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-b)*log(2*b*x^4 - 2*sqrt(-b*x^4 + a)*sqrt(-b)*x^2 - a)/b, -1/2*a
rctan(sqrt(-b*x^4 + a)*sqrt(b)*x^2/(b*x^4 - a))/sqrt(b)]
```

giac [A] time = 0.17, size = 30, normalized size = 0.97

$$-\frac{\log\left(\left|-\sqrt{-b}\,x^2+\sqrt{-bx^4+a}\right|\right)}{2\,\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(-sqrt(-b)*x^2 + sqrt(-b*x^4 + a)))/sqrt(-b)
```

maple [A] time = 0.01, size = 24, normalized size = 0.77

$$\frac{\arctan\left(\frac{\sqrt{b}\,x^2}{\sqrt{-b\,x^4+a}}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^4+a)^(1/2),x)
```

```
[Out] 1/2*arctan(1/(-b*x^4+a)^(1/2)*b^(1/2)*x^2)/b^(1/2)
```

maxima [A] time = 3.04, size = 23, normalized size = 0.74

$$-\frac{\arctan\left(\frac{\sqrt{-bx^4+a}}{\sqrt{b}\,x^2}\right)}{2\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*arctan(sqrt(-b*x^4 + a)/(sqrt(b)*x^2))/sqrt(b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{\sqrt{a-b\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a - b*x^4)^(1/2),x)
```

```
[Out] int(x/(a - b*x^4)^(1/2), x)
```

sympy [**A**] time = 1.64, size = 53, normalized size = 1.71

$$\begin{cases} -\dfrac{i\,\operatorname{acosh}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{b}} & \text{for } \left|\dfrac{bx^4}{a}\right| > 1 \\ \dfrac{\operatorname{asin}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**4+a)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)), Abs(b*x**4/a) > 1),
(asin(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)), True))
```

3.838 $\int \frac{1}{x^3\sqrt{a-bx^4}}\,dx$

Optimal. Leaf size=22

$$-\frac{\sqrt{a-bx^4}}{2ax^2}$$

```
[Out] -1/2*(-b*x^4+a)^(1/2)/a/x^2
```

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {264}

$$-\frac{\sqrt{a-bx^4}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a - b*x^4]),x]
```

```
[Out] -Sqrt[a - b*x^4]/(2*a*x^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{a-bx^4}}\,dx = -\frac{\sqrt{a-bx^4}}{2ax^2}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$-\frac{\sqrt{a-bx^4}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a - b*x^4]),x]
```

```
[Out] -1/2*Sqrt[a - b*x^4]/(a*x^2)
```

fricas [A] time = 0.91, size = 18, normalized size = 0.82

$$-\frac{\sqrt{-bx^4+a}}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-b*x^4 + a)/(a*x^2)
```

giac [A] time = 0.18, size = 36, normalized size = 1.64

$$\frac{\sqrt{-b}}{\left(\sqrt{-b}\,x^2-\sqrt{-bx^4+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(-b)/((sqrt(-b)*x^2 - sqrt(-b*x^4 + a))^2 - a)
```

maple [A] time = 0.00, size = 19, normalized size = 0.86

$$-\frac{\sqrt{-b\,x^4+a}}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-b*x^4+a)^(1/2),x)
```

```
[Out] -1/2*(-b*x^4+a)^(1/2)/a/x^2
```

maxima [A] time = 1.40, size = 18, normalized size = 0.82

$$-\frac{\sqrt{-bx^4+a}}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-b*x^4 + a)/(a*x^2)
```

mupad [B] time = 1.15, size = 18, normalized size = 0.82

$$-\frac{\sqrt{a-b\,x^4}}{2\,a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a - b*x^4)^(1/2)),x)
```

```
[Out] -(a - b*x^4)^(1/2)/(2*a*x^2)
```

sympy [A] time = 1.77, size = 51, normalized size = 2.32

$$\begin{cases} -\frac{\sqrt{b}\sqrt{\frac{a}{bx^4}-1}}{2a} & \text{for } \left|\frac{a}{bx^4}\right| > 1 \\ -\frac{i\sqrt{b}\sqrt{-\frac{a}{bx^4}+1}}{2a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-b*x**4+a)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(b)*sqrt(a/(b*x**4) - 1)/(2*a), Abs(a/(b*x**4)) > 1), (-I*s
qrt(b)*sqrt(-a/(b*x**4) + 1)/(2*a), True))
```

3.839 $\int \frac{1}{x^7\sqrt{a-bx^4}}\,dx$

Optimal. Leaf size=46

$$-\frac{b\sqrt{a-bx^4}}{3a^2x^2}-\frac{\sqrt{a-bx^4}}{6ax^6}$$

```
[Out] -1/6*(-b*x^4+a)^(1/2)/a/x^6-1/3*b*(-b*x^4+a)^(1/2)/a^2/x^2
```

Rubi [A] time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {271, 264}

$$-\frac{b\sqrt{a-bx^4}}{3a^2x^2}-\frac{\sqrt{a-bx^4}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*Sqrt[a - b*x^4]),x]
```

```
[Out] -Sqrt[a - b*x^4]/(6*a*x^6) - (b*Sqrt[a - b*x^4])/(3*a^2*x^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7\sqrt{a-bx^4}}\,dx &= -\frac{\sqrt{a-bx^4}}{6ax^6}+\frac{(2b)\int \frac{1}{x^3\sqrt{a-bx^4}}\,dx}{3a}\\ &= -\frac{\sqrt{a-bx^4}}{6ax^6}-\frac{b\sqrt{a-bx^4}}{3a^2x^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.65

$$-\frac{\sqrt{a-bx^4}\left(a+2bx^4\right)}{6a^2x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*Sqrt[a - b*x^4]),x]
```

```
[Out] -1/6*(Sqrt[a - b*x^4]*(a + 2*b*x^4))/(a^2*x^6)
```

fricas [A] time = 0.78, size = 26, normalized size = 0.57

$$-\frac{\left(2\,bx^4+a\right)\sqrt{-bx^4+a}}{6\,a^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(2*b*x^4 + a)*sqrt(-b*x^4 + a)/(a^2*x^6)
```

giac [A] time = 0.18, size = 68, normalized size = 1.48

$$-\frac{2\left(3\left(\sqrt{-b}\,x^2-\sqrt{-bx^4+a}\right)^2-a\right)\sqrt{-b}\,b}{3\left(\left(\sqrt{-b}\,x^2-\sqrt{-bx^4+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(3*(sqrt(-b)*x^2 - sqrt(-b*x^4 + a))^2 - a)*sqrt(-b)*b/((sqrt(-b)*x^2
- sqrt(-b*x^4 + a))^2 - a)^3
```

maple [A] time = 0.01, size = 27, normalized size = 0.59

$$-\frac{\sqrt{-b\,x^4+a}\,\left(2b\,x^4+a\right)}{6a^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(-b*x^4+a)^(1/2),x)
```

```
[Out] -1/6*(-b*x^4+a)^(1/2)*(2*b*x^4+a)/a^2/x^6
```

maxima [A] time = 1.37, size = 36, normalized size = 0.78

$$-\frac{\frac{3\sqrt{-bx^4+a}\,b}{x^2}+\frac{\left(-bx^4+a\right)^{\frac{3}{2}}}{x^6}}{6\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*(3*sqrt(-b*x^4 + a)*b/x^2 + (-b*x^4 + a)^(3/2)/x^6)/a^2
```

mupad [B] time = 1.19, size = 26, normalized size = 0.57

$$-\frac{\sqrt{a-b\,x^4}\,\left(2\,b\,x^4+a\right)}{6\,a^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a - b*x^4)^(1/2)),x)
```

```
[Out] -((a - b*x^4)^(1/2)*(a + 2*b*x^4))/(6*a^2*x^6)
```

sympy [A] time = 1.74, size = 189, normalized size = 4.11

$$\begin{cases} -\frac{\sqrt{b}\sqrt{\frac{a}{bx^4}-1}}{6ax^4}-\frac{b^{\frac{3}{2}}\sqrt{\frac{a}{bx^4}-1}}{3a^2} & \text{for } \left|\frac{a}{bx^4}\right|>1 \\ \frac{ia^2b^{\frac{3}{2}}\sqrt{-\frac{a}{bx^4}+1}}{-6a^3bx^4+6a^2b^2x^8}+\frac{iab^{\frac{5}{2}}x^4\sqrt{-\frac{a}{bx^4}+1}}{-6a^3bx^4+6a^2b^2x^8}-\frac{2ib^{\frac{7}{2}}x^8\sqrt{-\frac{a}{bx^4}+1}}{-6a^3bx^4+6a^2b^2x^8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(-b*x**4+a)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(b)*sqrt(a/(b*x**4) - 1)/(6*a*x**4) - b**(3/2)*sqrt(a/(b*x*
**4) - 1)/(3*a**2), Abs(a/(b*x**4)) > 1), (I*a**2*b**(3/2)*sqrt(-a/(b*x**4)
+ 1)/(-6*a**3*b*x**4 + 6*a**2*b**2*x**8) + I*a*b**(5/2)*x**4*sqrt(-a/(b*x**
4) + 1)/(-6*a**3*b*x**4 + 6*a**2*b**2*x**8) - 2*I*b**(7/2)*x**8*sqrt(-a/(b*
x**4) + 1)/(-6*a**3*b*x**4 + 6*a**2*b**2*x**8), True))
```

3.840 $\int \frac{1}{x^{11}\sqrt{a-bx^4}}\,dx$

Optimal. Leaf size=71

$$-\frac{4b^2\sqrt{a-bx^4}}{15a^3x^2}-\frac{2b\sqrt{a-bx^4}}{15a^2x^6}-\frac{\sqrt{a-bx^4}}{10ax^{10}}$$

```
[Out] -1/10*(-b*x^4+a)^(1/2)/a/x^10-2/15*b*(-b*x^4+a)^(1/2)/a^2/x^6-4/15*b^2*(-b*
x^4+a)^(1/2)/a^3/x^2
```

Rubi [A] time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {271, 264}

$$-\frac{4b^2\sqrt{a-bx^4}}{15a^3x^2}-\frac{2b\sqrt{a-bx^4}}{15a^2x^6}-\frac{\sqrt{a-bx^4}}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^11*Sqrt[a - b*x^4]),x]
```

```
[Out] -Sqrt[a - b*x^4]/(10*a*x^10) - (2*b*Sqrt[a - b*x^4])/(15*a^2*x^6) - (4*b^2*
Sqrt[a - b*x^4])/(15*a^3*x^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{11}\sqrt{a-bx^4}}\,dx &= -\frac{\sqrt{a-bx^4}}{10ax^{10}}+\frac{(4b)\int \frac{1}{x^7\sqrt{a-bx^4}}\,dx}{5a}\\ &= -\frac{\sqrt{a-bx^4}}{10ax^{10}}-\frac{2b\sqrt{a-bx^4}}{15a^2x^6}+\frac{\left(8b^2\right)\int \frac{1}{x^3\sqrt{a-bx^4}}\,dx}{15a^2}\\ &= -\frac{\sqrt{a-bx^4}}{10ax^{10}}-\frac{2b\sqrt{a-bx^4}}{15a^2x^6}-\frac{4b^2\sqrt{a-bx^4}}{15a^3x^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 0.61

$$-\frac{\sqrt{a-bx^4}\left(3a^2+4abx^4+8b^2x^8\right)}{30a^3x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^11*Sqrt[a - b*x^4]),x]
```

```
[Out] -1/30*(Sqrt[a - b*x^4]*(3*a^2 + 4*a*b*x^4 + 8*b^2*x^8))/(a^3*x^10)
```

fricas [A] time = 1.03, size = 39, normalized size = 0.55

$$-\frac{\left(8\,b^2x^8+4\,abx^4+3\,a^2\right)\sqrt{-bx^4+a}}{30\,a^3x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/30*(8*b^2*x^8 + 4*a*b*x^4 + 3*a^2)*sqrt(-b*x^4 + a)/(a^3*x^10)
```

giac [A] time = 0.18, size = 97, normalized size = 1.37

$$\frac{8\left(10\left(\sqrt{-b}\,x^2-\sqrt{-bx^4+a}\right)^4-5\left(\sqrt{-b}\,x^2-\sqrt{-bx^4+a}\right)^2a+a^2\right)\sqrt{-b}\,b^2}{15\left(\left(\sqrt{-b}\,x^2-\sqrt{-bx^4+a}\right)^2-a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] 8/15*(10*(sqrt(-b)*x^2 - sqrt(-b*x^4 + a))^4 - 5*(sqrt(-b)*x^2 - sqrt(-b*x^
4 + a))^2*a + a^2)*sqrt(-b)*b^2/((sqrt(-b)*x^2 - sqrt(-b*x^4 + a))^2 - a)^5
```

maple [A] time = 0.01, size = 40, normalized size = 0.56

$$-\frac{\sqrt{-b\,x^4+a}\,\left(8b^2x^8+4ab\,x^4+3a^2\right)}{30a^3x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^11/(-b*x^4+a)^(1/2),x)
```

```
[Out] -1/30*(-b*x^4+a)^(1/2)*(8*b^2*x^8+4*a*b*x^4+3*a^2)/x^10/a^3
```

maxima [A] time = 1.35, size = 55, normalized size = 0.77

$$-\frac{\frac{15\sqrt{-bx^4+a}\,b^2}{x^2}+\frac{10\left(-bx^4+a\right)^{\frac{3}{2}}b}{x^6}+\frac{3\left(-bx^4+a\right)^{\frac{5}{2}}}{x^{10}}}{30\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/30*(15*sqrt(-b*x^4 + a)*b^2/x^2 + 10*(-b*x^4 + a)^(3/2)*b/x^6 + 3*(-b*x^
4 + a)^(5/2)/x^10)/a^3
```

mupad [B] time = 1.31, size = 39, normalized size = 0.55

$$-\frac{\sqrt{a-b\,x^4}\,\left(3\,a^2+4\,a\,b\,x^4+8\,b^2\,x^8\right)}{30\,a^3\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^11*(a - b*x^4)^(1/2)),x)
```

```
[Out] -((a - b*x^4)^(1/2)*(3*a^2 + 8*b^2*x^8 + 4*a*b*x^4))/(30*a^3*x^10)
```

sympy [**B**] time = 5.28, size = 609, normalized size = 8.58

$$\begin{cases} -\frac{3a^4b^{\frac{9}{2}}\sqrt{\frac{a}{bx^4}-1}}{30a^5b^4x^8-60a^4b^5x^{12}+30a^3b^6x^{16}}+\frac{2a^3b^{\frac{11}{2}}x^4\sqrt{\frac{a}{bx^4}-1}}{30a^5b^4x^8-60a^4b^5x^{12}+30a^3b^6x^{16}}-\frac{3a^2b^{\frac{13}{2}}x^8\sqrt{\frac{a}{bx^4}-1}}{30a^5b^4x^8-60a^4b^5x^{12}+30a^3b^6x^{16}}+\frac{12ab^{\frac{15}{2}}x^{12}\sqrt{\frac{a}{bx^4}-1}}{30a^5b^4x^8-60a^4b^5x^{12}+30a^3b^6x^{16}} \\ -\frac{3ia^4b^{\frac{9}{2}}\sqrt{-\frac{a}{bx^4}+1}}{30a^5b^4x^8-60a^4b^5x^{12}+30a^3b^6x^{16}}+\frac{2ia^3b^{\frac{11}{2}}x^4\sqrt{-\frac{a}{bx^4}+1}}{30a^5b^4x^8-60a^4b^5x^{12}+30a^3b^6x^{16}}-\frac{3ia^2b^{\frac{13}{2}}x^8\sqrt{-\frac{a}{bx^4}+1}}{30a^5b^4x^8-60a^4b^5x^{12}+30a^3b^6x^{16}}+\frac{12iab^{\frac{15}{2}}x^{12}\sqrt{-\frac{a}{bx^4}+1}}{30a^5b^4x^8-60a^4b^5x^{12}+30a^3b^6x^{16}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**11/(-b*x**4+a)**(1/2),x)
```

```
[Out] Piecewise((-3*a**4*b**(9/2)*sqrt(a/(b*x**4) - 1)/(30*a**5*b**4*x**8 - 60*a*
*4*b**5*x**12 + 30*a**3*b**6*x**16) + 2*a**3*b**(11/2)*x**4*sqrt(a/(b*x**4)
 - 1)/(30*a**5*b**4*x**8 - 60*a**4*b**5*x**12 + 30*a**3*b**6*x**16) - 3*a**
2*b**(13/2)*x**8*sqrt(a/(b*x**4) - 1)/(30*a**5*b**4*x**8 - 60*a**4*b**5*x**
12 + 30*a**3*b**6*x**16) + 12*a*b**(15/2)*x**12*sqrt(a/(b*x**4) - 1)/(30*a*
*5*b**4*x**8 - 60*a**4*b**5*x**12 + 30*a**3*b**6*x**16) - 8*b**(17/2)*x**16
*sqrt(a/(b*x**4) - 1)/(30*a**5*b**4*x**8 - 60*a**4*b**5*x**12 + 30*a**3*b**
6*x**16), Abs(a/(b*x**4)) > 1), (-3*I*a**4*b**(9/2)*sqrt(-a/(b*x**4) + 1)/(
30*a**5*b**4*x**8 - 60*a**4*b**5*x**12 + 30*a**3*b**6*x**16) + 2*I*a**3*b**
(11/2)*x**4*sqrt(-a/(b*x**4) + 1)/(30*a**5*b**4*x**8 - 60*a**4*b**5*x**12 +
 30*a**3*b**6*x**16) - 3*I*a**2*b**(13/2)*x**8*sqrt(-a/(b*x**4) + 1)/(30*a*
*5*b**4*x**8 - 60*a**4*b**5*x**12 + 30*a**3*b**6*x**16) + 12*I*a*b**(15/2)*
x**12*sqrt(-a/(b*x**4) + 1)/(30*a**5*b**4*x**8 - 60*a**4*b**5*x**12 + 30*a*
*3*b**6*x**16) - 8*I*b**(17/2)*x**16*sqrt(-a/(b*x**4) + 1)/(30*a**5*b**4*x*
*8 - 60*a**4*b**5*x**12 + 30*a**3*b**6*x**16), True))
```

3.841 $\int \frac{x^8}{\sqrt{a-bx^4}}\,dx$

Optimal. Leaf size=100

$$\frac{5a^{9/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{21b^{9/4}\sqrt{a-bx^4}}-\frac{5ax\sqrt{a-bx^4}}{21b^2}-\frac{x^5\sqrt{a-bx^4}}{7b}$$

```
[Out] -5/21*a*x*(-b*x^4+a)^(1/2)/b^2-1/7*x^5*(-b*x^4+a)^(1/2)/b+5/21*a^(9/4)*Elli
pticF(b^(1/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/b^(9/4)/(-b*x^4+a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {321, 224, 221}

$$\frac{5a^{9/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{21b^{9/4}\sqrt{a-bx^4}}-\frac{5ax\sqrt{a-bx^4}}{21b^2}-\frac{x^5\sqrt{a-bx^4}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/Sqrt[a - b*x^4],x]
```

```
[Out] (-5*a*x*Sqrt[a - b*x^4])/(21*b^2) - (x^5*Sqrt[a - b*x^4])/(7*b) + (5*a^(9/4
)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(21*b^(9/
4)*Sqrt[a - b*x^4])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{\sqrt{a-bx^4}}\,dx &= -\frac{x^5\sqrt{a-bx^4}}{7b} + \frac{(5a)\int \frac{x^4}{\sqrt{a-bx^4}}\,dx}{7b} \\ &= -\frac{5ax\sqrt{a-bx^4}}{21b^2} - \frac{x^5\sqrt{a-bx^4}}{7b} + \frac{\left(5a^2\right)\int \frac{1}{\sqrt{a-bx^4}}\,dx}{21b^2} \\ &= -\frac{5ax\sqrt{a-bx^4}}{21b^2} - \frac{x^5\sqrt{a-bx^4}}{7b} + \frac{\left(5a^2\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}\,dx}{21b^2\sqrt{a-bx^4}} \\ &= -\frac{5ax\sqrt{a-bx^4}}{21b^2} - \frac{x^5\sqrt{a-bx^4}}{7b} + \frac{5a^{9/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{21b^{9/4}\sqrt{a-bx^4}}\end{aligned}$$

Mathematica [C] time = 0.02, size = 80, normalized size = 0.80

$$\frac{5a^2x\sqrt{1-\frac{bx^4}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{bx^4}{a}\right) - 5a^2x + 2abx^5 + 3b^2x^9}{21b^2\sqrt{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/Sqrt[a - b*x^4],x]
```

```
[Out] (-5*a^2*x + 2*a*b*x^5 + 3*b^2*x^9 + 5*a^2*x*Sqrt[1 - (b*x^4)/a]*Hypergeomet
ric2F1[1/4, 1/2, 5/4, (b*x^4)/a])/(21*b^2*Sqrt[a - b*x^4])
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-bx^4+a}\,x^8}{bx^4-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*x^4 + a)*x^8/(b*x^4 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\sqrt{-bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^8/sqrt(-b*x^4 + a), x)
```

maple [A] time = 0.02, size = 107, normalized size = 1.07

$$-\frac{\sqrt{-b\,x^4+a}\;x^5}{7b} + \frac{5\sqrt{-\frac{\sqrt{b}\;x^2}{\sqrt{a}}+1}\;\sqrt{\frac{\sqrt{b}\;x^2}{\sqrt{a}}+1}\;a^2\,\text{EllipticF}\left(\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\;x, i\right)}{21\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\;\sqrt{-b\,x^4+a}\;b^2} - \frac{5\sqrt{-b\,x^4+a}\;ax}{21b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-b*x^4+a)^(1/2),x)
```

```
[Out] -1/7*x^5*(-b*x^4+a)^(1/2)/b-5/21*a*x*(-b*x^4+a)^(1/2)/b^2+5/21*a^2/b^2/(1/a
^(1/2)*b^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))
^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\sqrt{-bx^4 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^8/sqrt(-b*x^4 + a), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^8}{\sqrt{a - b\,x^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a - b*x^4)^(1/2),x)
```

```
[Out] int(x^8/(a - b*x^4)^(1/2), x)
```

sympy **[A]** time = 2.11, size = 39, normalized size = 0.39

$$\frac{x^9 \Gamma\left(\frac{9}{4}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{9}{4} \\ \frac{13}{4}\end{matrix}\right| \frac{bx^4 e^{2i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-b*x**4+a)**(1/2),x)
```

```
[Out] x**9*gamma(9/4)*hyper((1/2, 9/4), (13/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*s
qrt(a)*gamma(13/4))
```

3.842 $\int \frac{x^4}{\sqrt{a-bx^4}}\,dx$

Optimal. Leaf size=77

$$\frac{a^{5/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{3b^{5/4}\sqrt{a-bx^4}}-\frac{x\sqrt{a-bx^4}}{3b}$$

```
[Out] -1/3*x*(-b*x^4+a)^(1/2)/b+1/3*a^(5/4)*EllipticF(b^(1/4)*x/a^(1/4),I)*(1-b*x
^4/a)^(1/2)/b^(5/4)/(-b*x^4+a)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {321, 224, 221}

$$\frac{a^{5/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{3b^{5/4}\sqrt{a-bx^4}}-\frac{x\sqrt{a-bx^4}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a - b*x^4],x]
```

```
[Out] -(x*Sqrt[a - b*x^4])/(3*b) + (a^(5/4)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[
(b^(1/4)*x)/a^(1/4)], -1])/(3*b^(5/4)*Sqrt[a - b*x^4])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{a-bx^4}}\,dx = -\frac{x\sqrt{a-bx^4}}{3b} + \frac{a\int \frac{1}{\sqrt{a-bx^4}}\,dx}{3b}$$
$$= -\frac{x\sqrt{a-bx^4}}{3b} + \frac{\left(a\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}\,dx}{3b\sqrt{a-bx^4}}$$
$$= -\frac{x\sqrt{a-bx^4}}{3b} + \frac{a^{5/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{3b^{5/4}\sqrt{a-bx^4}}$$

Mathematica [C] time = 0.02, size = 64, normalized size = 0.83

$$\frac{x\left(a\sqrt{1-\frac{bx^4}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{bx^4}{a}\right)-a+bx^4\right)}{3b\sqrt{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a - b*x^4],x]
```

```
[Out] (x*(-a + b*x^4 + a*Sqrt[1 - (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, (b*
x^4)/a]))/(3*b*Sqrt[a - b*x^4])
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-bx^4+a}\,x^4}{bx^4-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*x^4 + a)*x^4/(b*x^4 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{-bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(-b*x^4 + a), x)
```

maple [A] time = 0.01, size = 86, normalized size = 1.12

$$\frac{\sqrt{-\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\,a\,\text{EllipticF}\left(\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,x, i\right)}{3\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}\,b} - \frac{\sqrt{-b\,x^4+a}\,x}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-b*x^4+a)^(1/2),x)
```

```
[Out] -1/3*x*(-b*x^4+a)^(1/2)/b+1/3*a/b/(1/a^(1/2)*b^(1/2))^(1/2)*(-1/a^(1/2)*b^(
1/2)*x^2+1)^(1/2)*(1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)/(-b*x^4+a)^(1/2)*Elliptic
F((1/a^(1/2)*b^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{-bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(-b*x^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sqrt{a-b\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a - b*x^4)^(1/2),x)
```

```
[Out] int(x^4/(a - b*x^4)^(1/2), x)
```

sympy [A] time = 1.78, size = 39, normalized size = 0.51

$$\frac{x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-b*x**4+a)**(1/2),x)
```

```
[Out] x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*sq
rt(a)*gamma(9/4))
```

3.843 $\int \frac{1}{\sqrt{a-bx^4}}\,dx$

Optimal. Leaf size=53

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}$$

```
[Out] a^(1/4)*EllipticF(b^(1/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/(-b*x^4+a)
^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {224, 221}

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a - b*x^4],x]
```

```
[Out] (a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b
^(1/4)*Sqrt[a - b*x^4])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a-bx^4}}\,dx &= \frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}\,dx}{\sqrt{a-bx^4}} \\ &= \frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}\end{aligned}$$

Mathematica [C] time = 0.04, size = 72, normalized size = 1.36

$$-\frac{i\sqrt{1-\frac{bx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a - b*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[b]/Sqrt[a])]*x],
-1])/(Sqrt[-(Sqrt[b]/Sqrt[a])]*Sqrt[a - b*x^4])
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-bx^4+a}}{bx^4-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*x^4 + a)/(b*x^4 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-b*x^4 + a), x)
```

maple [A] time = 0.01, size = 64, normalized size = 1.21

$$\frac{\sqrt{-\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\,\text{EllipticF}\left(\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,x, i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^4+a)^(1/2),x)
```

```
[Out] 1/(1/a^(1/2)*b^(1/2))^(1/2)*(-1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*b^(
1/2)*x^2+1)^(1/2)/(-b*x^4+a)^(1/2)*EllipticF((1/a^(1/2)*b^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-b*x^4 + a), x)
```

mupad [B] time = 1.09, size = 38, normalized size = 0.72

$$\frac{x\sqrt{1-\frac{b\,x^4}{a}}\;{}_2\mathrm{F}_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; \frac{b\,x^4}{a}\right)}{\sqrt{a-b\,x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*x^4)^(1/2),x)
```

```
[Out] (x*(1 - (b*x^4)/a)^(1/2)*hypergeom([1/4, 1/2], 5/4, (b*x^4)/a))/(a - b*x^4)
^(1/2)
```

sympy [A] time = 1.00, size = 37, normalized size = 0.70

$$\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4}, \frac{1}{2} \\ \frac{5}{4}\end{matrix}\right| \frac{bx^4 e^{2i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**4+a)**(1/2),x)
```

```
[Out] x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(
a)*gamma(5/4))
```

3.844 $\int \frac{1}{x^4\sqrt{a-bx^4}}\,dx$

Optimal. Leaf size=79

$$\frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{3a^{3/4}\sqrt{a-bx^4}}-\frac{\sqrt{a-bx^4}}{3ax^3}$$

```
[Out] -1/3*(-b*x^4+a)^(1/2)/a/x^3+1/3*b^(3/4)*EllipticF(b^(1/4)*x/a^(1/4),I)*(1-b
*x^4/a)^(1/2)/a^(3/4)/(-b*x^4+a)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {325, 224, 221}

$$\frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{3a^{3/4}\sqrt{a-bx^4}}-\frac{\sqrt{a-bx^4}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a - b*x^4]),x]
```

```
[Out] -Sqrt[a - b*x^4]/(3*a*x^3) + (b^(3/4)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[
(b^(1/4)*x)/a^(1/4)], -1])/(3*a^(3/4)*Sqrt[a - b*x^4])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{a-bx^4}}\,dx &= -\frac{\sqrt{a-bx^4}}{3ax^3}+\frac{b\int \frac{1}{\sqrt{a-bx^4}}\,dx}{3a}\\ &= -\frac{\sqrt{a-bx^4}}{3ax^3}+\frac{\left(b\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}\,dx}{3a\sqrt{a-bx^4}}\\ &= -\frac{\sqrt{a-bx^4}}{3ax^3}+\frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{3a^{3/4}\sqrt{a-bx^4}}\end{aligned}$$

Mathematica [C] time = 0.03, size = 52, normalized size = 0.66

$$-\frac{\sqrt{1-\frac{bx^4}{a}}\ {}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};\frac{bx^4}{a}\right)}{3x^3\sqrt{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a - b*x^4]),x]
```

```
[Out] -1/3*(Sqrt[1 - (b*x^4)/a]*Hypergeometric2F1[-3/4, 1/2, 1/4, (b*x^4)/a])/(x^
3*Sqrt[a - b*x^4])
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-bx^4+a}}{bx^8-ax^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*x^4 + a)/(b*x^8 - a*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-bx^4+a}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-b*x^4 + a)*x^4), x)
```

maple [A] time = 0.01, size = 88, normalized size = 1.11

$$\frac{\sqrt{-\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\ \sqrt{\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\ b\,\text{EllipticF}\left(\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\ x,i\right)}{3\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\ \sqrt{-b\,x^4+a}\ a}-\frac{\sqrt{-b\,x^4+a}}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-b*x^4+a)^(1/2),x)
```

```
[Out] -1/3*(-b*x^4+a)^(1/2)/a/x^3+1/3/a*b/(1/a^(1/2)*b^(1/2))^(1/2)*(-1/a^(1/2)*b
^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)/(-b*x^4+a)^(1/2)*Ellipt
icF((1/a^(1/2)*b^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-bx^4+a}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-b*x^4 + a)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4 \sqrt{a - b\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a - b*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^4*(a - b*x^4)^(1/2)), x)
```

sympy [A] time = 1.98, size = 42, normalized size = 0.53

$$\frac{\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{1}{2} \\ \frac{1}{4} \end{matrix} \middle| \frac{b x^4 e^{2i\pi}}{a}\right)}{4\sqrt{a}\, x^3 \Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-b*x**4+a)**(1/2),x)
```

```
[Out] gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(
a)*x**3*gamma(1/4))
```

3.845 $\int \frac{1}{x^8\sqrt{a-bx^4}}\,dx$

Optimal. Leaf size=102

$$\frac{5b^{7/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{21a^{7/4}\sqrt{a-bx^4}}-\frac{5b\sqrt{a-bx^4}}{21a^2x^3}-\frac{\sqrt{a-bx^4}}{7ax^7}$$

```
[Out] -1/7*(-b*x^4+a)^(1/2)/a/x^7-5/21*b*(-b*x^4+a)^(1/2)/a^2/x^3+5/21*b^(7/4)*El
lipticF(b^(1/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/a^(7/4)/(-b*x^4+a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {325, 224, 221}

$$\frac{5b^{7/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{21a^{7/4}\sqrt{a-bx^4}}-\frac{5b\sqrt{a-bx^4}}{21a^2x^3}-\frac{\sqrt{a-bx^4}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*Sqrt[a - b*x^4]),x]
```

```
[Out] -Sqrt[a - b*x^4]/(7*a*x^7) - (5*b*Sqrt[a - b*x^4])/(21*a^2*x^3) + (5*b^(7/4
)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(21*a^(7/
4)*Sqrt[a - b*x^4])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^8\sqrt{a-bx^4}}\,dx &= -\frac{\sqrt{a-bx^4}}{7ax^7} + \frac{(5b)\int \frac{1}{x^4\sqrt{a-bx^4}}\,dx}{7a} \\ &= -\frac{\sqrt{a-bx^4}}{7ax^7} - \frac{5b\sqrt{a-bx^4}}{21a^2x^3} + \frac{\left(5b^2\right)\int \frac{1}{\sqrt{a-bx^4}}\,dx}{21a^2} \\ &= -\frac{\sqrt{a-bx^4}}{7ax^7} - \frac{5b\sqrt{a-bx^4}}{21a^2x^3} + \frac{\left(5b^2\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}\,dx}{21a^2\sqrt{a-bx^4}} \\ &= -\frac{\sqrt{a-bx^4}}{7ax^7} - \frac{5b\sqrt{a-bx^4}}{21a^2x^3} + \frac{5b^{7/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{21a^{7/4}\sqrt{a-bx^4}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.51

$$-\frac{\sqrt{1-\frac{bx^4}{a}}\;{}_2F_1\left(-\frac{7}{4},\frac{1}{2};-\frac{3}{4};\frac{bx^4}{a}\right)}{7x^7\sqrt{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*Sqrt[a - b*x^4]),x]
```

```
[Out] -1/7*(Sqrt[1 - (b*x^4)/a]*Hypergeometric2F1[-7/4, 1/2, -3/4, (b*x^4)/a])/(x
^7*Sqrt[a - b*x^4])
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-bx^4+a}}{bx^{12}-ax^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*x^4 + a)/(b*x^12 - a*x^8), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-bx^4+a}\,x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-b*x^4 + a)*x^8), x)
```

maple [A] time = 0.02, size = 109, normalized size = 1.07

$$\frac{5\sqrt{-\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\,b^2\,\text{EllipticF}\left(\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,x, i\right)}{21\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}\,a^2} - \frac{5\sqrt{-b\,x^4+a}\,b}{21a^2x^3} - \frac{\sqrt{-b\,x^4+a}}{7a\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(-b*x^4+a)^(1/2),x)
```

```
[Out] -1/7*(-b*x^4+a)^(1/2)/a/x^7-5/21*b*(-b*x^4+a)^(1/2)/a^2/x^3+5/21/a^2*b^2/(1
/a^(1/2)*b^(1/2))^(1/2)*(-1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*b^(1/2)
*x^2+1)^(1/2)/(-b*x^4+a)^(1/2)*EllipticF((1/a^(1/2)*b^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-bx^4 + a}\, x^8}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-b*x^4 + a)*x^8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^8 \sqrt{a - b\, x^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(a - b*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^8*(a - b*x^4)^(1/2)), x)
```

sympy [A] time = 1.40, size = 46, normalized size = 0.45

$$\frac{\Gamma\left(-\frac{7}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{7}{4}, \frac{1}{2} \\ -\frac{3}{4} \end{matrix} \middle| \frac{bx^4 e^{2i\pi}}{a}\right)}{4\sqrt{a}\, x^7 \Gamma\left(-\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(-b*x**4+a)**(1/2),x)
```

```
[Out] gamma(-7/4)*hyper((-7/4, 1/2), (-3/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*sqrt
(a)*x**7*gamma(-3/4))
```

3.846 $\int \frac{x^{10}}{\sqrt{a-bx^4}}\,dx$

Optimal. Leaf size=158

$$-\frac{7a^{11/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{15b^{11/4}\sqrt{a-bx^4}}+\frac{7a^{11/4}\sqrt{1-\frac{bx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{15b^{11/4}\sqrt{a-bx^4}}-\frac{7ax^3\sqrt{a-bx^4}}{45b^2}-\frac{x^7\sqrt{a-bx^4}}{9b}$$

```
[Out] -7/45*a*x^3*(-b*x^4+a)^(1/2)/b^2-1/9*x^7*(-b*x^4+a)^(1/2)/b+7/15*a^(11/4)*E
llipticE(b^(1/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/b^(11/4)/(-b*x^4+a)^(1/2)-7
/15*a^(11/4)*EllipticF(b^(1/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/b^(11/4)/(-b*
x^4+a)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {321, 307, 224, 221, 1200, 1199, 424}

$$-\frac{7a^{11/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{15b^{11/4}\sqrt{a-bx^4}}+\frac{7a^{11/4}\sqrt{1-\frac{bx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{15b^{11/4}\sqrt{a-bx^4}}-\frac{7ax^3\sqrt{a-bx^4}}{45b^2}-\frac{x^7\sqrt{a-bx^4}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/Sqrt[a - b*x^4],x]
```

```
[Out] (-7*a*x^3*Sqrt[a - b*x^4])/(45*b^2) - (x^7*Sqrt[a - b*x^4])/(9*b) + (7*a^(1
1/4)*Sqrt[1 - (b*x^4)/a]*EllipticE[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(15*b^
(11/4)*Sqrt[a - b*x^4]) - (7*a^(11/4)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[
(b^(1/4)*x)/a^(1/4)], -1])/(15*b^(11/4)*Sqrt[a - b*x^4])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
```

```
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{x^{10}}{\sqrt{a-bx^4}}\,dx = -\frac{x^7\sqrt{a-bx^4}}{9b} + \frac{(7a)\int \frac{x^6}{\sqrt{a-bx^4}}\,dx}{9b}$$

$$= -\frac{7ax^3\sqrt{a-bx^4}}{45b^2} - \frac{x^7\sqrt{a-bx^4}}{9b} + \frac{\left(7a^2\right)\int \frac{x^2}{\sqrt{a-bx^4}}\,dx}{15b^2}$$

$$= -\frac{7ax^3\sqrt{a-bx^4}}{45b^2} - \frac{x^7\sqrt{a-bx^4}}{9b} - \frac{\left(7a^{5/2}\right)\int \frac{1}{\sqrt{a-bx^4}}\,dx}{15b^{5/2}} + \frac{\left(7a^{5/2}\right)\int \frac{1+\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a-bx^4}}\,dx}{15b^{5/2}}$$

$$= -\frac{7ax^3\sqrt{a-bx^4}}{45b^2} - \frac{x^7\sqrt{a-bx^4}}{9b} - \frac{\left(7a^{5/2}\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}\,dx}{15b^{5/2}\sqrt{a-bx^4}} + \frac{\left(7a^{5/2}\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1+\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{1-\frac{bx^4}{a}}}\,dx}{15b^{5/2}\sqrt{a-bx^4}}$$

$$= -\frac{7ax^3\sqrt{a-bx^4}}{45b^2} - \frac{x^7\sqrt{a-bx^4}}{9b} - \frac{7a^{11/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{15b^{11/4}\sqrt{a-bx^4}} + \frac{\left(7a^{5/2}\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{\sqrt{}}{\sqrt{}}}{15b^{5/2}\sqrt{a-b}}$$

$$= -\frac{7ax^3\sqrt{a-bx^4}}{45b^2} - \frac{x^7\sqrt{a-bx^4}}{9b} + \frac{7a^{11/4}\sqrt{1-\frac{bx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{15b^{11/4}\sqrt{a-bx^4}} - \frac{7a^{11/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\text{si}\right.}{15b^{11/4}\sqrt{a}}$$

Mathematica [C] time = 0.04, size = 81, normalized size = 0.51

$$\frac{x^3\left(7a^2\sqrt{1-\frac{bx^4}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{bx^4}{a}\right) - 7a^2 + 2abx^4 + 5b^2x^8\right)}{45b^2\sqrt{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/Sqrt[a - b*x^4],x]
```

```
[Out] (x^3*(-7*a^2 + 2*a*b*x^4 + 5*b^2*x^8 + 7*a^2*Sqrt[1 - (b*x^4)/a]*Hypergeome
tric2F1[1/2, 3/4, 7/4, (b*x^4)/a]))/(45*b^2*Sqrt[a - b*x^4])
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-bx^4+a}\,x^{10}}{bx^4-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*x^4 + a)*x^10/(b*x^4 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\sqrt{-bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^10/sqrt(-b*x^4 + a), x)
```

maple [A] time = 0.01, size = 126, normalized size = 0.80

$$-\frac{\sqrt{-b\,x^4+a}\;x^7}{9b}-\frac{7\sqrt{-b\,x^4+a}\;a\,x^3}{45b^2}-\frac{7\sqrt{-\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\;\sqrt{\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\;\left(-\text{EllipticE}\left(\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\;x,i\right)+\text{EllipticF}\left(\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\;x\right.\right.}{15\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\;\sqrt{-b\,x^4+a}\;b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(-b*x^4+a)^(1/2),x)
```

```
[Out] -1/9*x^7*(-b*x^4+a)^(1/2)/b-7/45*a*x^3*(-b*x^4+a)^(1/2)/b^2-7/15*a^(5/2)/b^
(5/2)/(1/a^(1/2)*b^(1/2))^(1/2)*(-1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)
*b^(1/2)*x^2+1)^(1/2)/(-b*x^4+a)^(1/2)*(EllipticF((1/a^(1/2)*b^(1/2))^(1/2)
*x,I)-EllipticE((1/a^(1/2)*b^(1/2))^(1/2)*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\sqrt{-bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^10/sqrt(-b*x^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{10}}{\sqrt{a-b\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a - b*x^4)^(1/2),x)
```

```
[Out] int(x^10/(a - b*x^4)^(1/2), x)
```

sympy [A] time = 2.25, size = 39, normalized size = 0.25

$$\frac{x^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\right|\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(-b*x**4+a)**(1/2),x)
```

```
[Out] x**11*gamma(11/4)*hyper((1/2, 11/4), (15/4,), b*x**4*exp_polar(2*I*pi)/a)/(
4*sqrt(a)*gamma(15/4))
```

3.847 $\int \frac{x^6}{\sqrt{a-bx^4}}\,dx$

Optimal. Leaf size=135

$$-\frac{3a^{7/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5b^{7/4}\sqrt{a-bx^4}}+\frac{3a^{7/4}\sqrt{1-\frac{bx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5b^{7/4}\sqrt{a-bx^4}}-\frac{x^3\sqrt{a-bx^4}}{5b}$$

```
[Out] -1/5*x^3*(-b*x^4+a)^(1/2)/b+3/5*a^(7/4)*EllipticE(b^(1/4)*x/a^(1/4),I)*(1-b
*x^4/a)^(1/2)/b^(7/4)/(-b*x^4+a)^(1/2)-3/5*a^(7/4)*EllipticF(b^(1/4)*x/a^(1
/4),I)*(1-b*x^4/a)^(1/2)/b^(7/4)/(-b*x^4+a)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {321, 307, 224, 221, 1200, 1199, 424}

$$-\frac{3a^{7/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5b^{7/4}\sqrt{a-bx^4}}+\frac{3a^{7/4}\sqrt{1-\frac{bx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5b^{7/4}\sqrt{a-bx^4}}-\frac{x^3\sqrt{a-bx^4}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/Sqrt[a - b*x^4],x]
```

```
[Out] -(x^3*Sqrt[a - b*x^4])/(5*b) + (3*a^(7/4)*Sqrt[1 - (b*x^4)/a]*EllipticE[Arc
Sin[(b^(1/4)*x)/a^(1/4)], -1])/(5*b^(7/4)*Sqrt[a - b*x^4]) - (3*a^(7/4)*Sqr
t[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(5*b^(7/4)*Sqr
t[a - b*x^4])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
```

```
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{x^6}{\sqrt{a-bx^4}}\,dx = -\frac{x^3\sqrt{a-bx^4}}{5b} + \frac{(3a)\int \frac{x^2}{\sqrt{a-bx^4}}\,dx}{5b}$$

$$= -\frac{x^3\sqrt{a-bx^4}}{5b} - \frac{\left(3a^{3/2}\right)\int \frac{1}{\sqrt{a-bx^4}}\,dx}{5b^{3/2}} + \frac{\left(3a^{3/2}\right)\int \frac{1+\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a-bx^4}}\,dx}{5b^{3/2}}$$

$$= -\frac{x^3\sqrt{a-bx^4}}{5b} - \frac{\left(3a^{3/2}\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}\,dx}{5b^{3/2}\sqrt{a-bx^4}} + \frac{\left(3a^{3/2}\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1+\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{1-\frac{bx^4}{a}}}\,dx}{5b^{3/2}\sqrt{a-bx^4}}$$

$$= -\frac{x^3\sqrt{a-bx^4}}{5b} - \frac{3a^{7/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5b^{7/4}\sqrt{a-bx^4}} + \frac{\left(3a^{3/2}\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{\sqrt{1+\frac{\sqrt{b}x^2}{\sqrt{a}}}}{\sqrt{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}}\,dx}{5b^{3/2}\sqrt{a-bx^4}}$$

$$= -\frac{x^3\sqrt{a-bx^4}}{5b} + \frac{3a^{7/4}\sqrt{1-\frac{bx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5b^{7/4}\sqrt{a-bx^4}} - \frac{3a^{7/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5b^{7/4}\sqrt{a-bx^4}}$$

Mathematica [C] time = 0.03, size = 66, normalized size = 0.49

$$\frac{x^3\left(a\sqrt{1-\frac{bx^4}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{bx^4}{a}\right)-a+bx^4\right)}{5b\sqrt{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/Sqrt[a - b*x^4],x]
```

```
[Out] (x^3*(-a + b*x^4 + a*Sqrt[1 - (b*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, (
b*x^4)/a]))/(5*b*Sqrt[a - b*x^4])
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-bx^4+a}\,x^6}{bx^4-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*x^4 + a)*x^6/(b*x^4 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt{-bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/sqrt(-b*x^4 + a), x)
```

maple [A] time = 0.01, size = 107, normalized size = 0.79

$$-\frac{\sqrt{-b\,x^4+a}\;x^3}{5b}-\frac{3\sqrt{-\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\;\sqrt{\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\;\left(-\text{EllipticE}\left(\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\;x,i\right)+\text{EllipticF}\left(\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\;x,i\right)\right)a^{\frac{3}{2}}}{5\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\;\sqrt{-b\,x^4+a}\;b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-b*x^4+a)^(1/2),x)
```

```
[Out] -1/5*x^3*(-b*x^4+a)^(1/2)/b-3/5*a^(3/2)/b^(3/2)/(1/a^(1/2)*b^(1/2))^(1/2)*(
-1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)/(-b*x^4+a)^
(1/2)*(EllipticF((1/a^(1/2)*b^(1/2))^(1/2)*x,I)-EllipticE((1/a^(1/2)*b^(1/2
))^(1/2)*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt{-bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/sqrt(-b*x^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{\sqrt{a-b\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a - b*x^4)^(1/2),x)
```

```
[Out] int(x^6/(a - b*x^4)^(1/2), x)
```

sympy [A] time = 2.03, size = 39, normalized size = 0.29

$$\frac{x^7\,\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-b*x**4+a)**(1/2),x)

[Out] x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*s
qrt(a)*gamma(11/4))
```

3.848 $\int \frac{x^2}{\sqrt{a-bx^4}}\,dx$

Optimal. Leaf size=108

$$\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a-bx^4}}$$

```
[Out] a^(3/4)*EllipticE(b^(1/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/b^(3/4)/(-b*x^4+a)
^(1/2)-a^(3/4)*EllipticF(b^(1/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/b^(3/4)/(-b
*x^4+a)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {307, 224, 221, 1200, 1199, 424}

$$\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a - b*x^4],x]
```

```
[Out] (a^(3/4)*Sqrt[1 - (b*x^4)/a]*EllipticE[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b
^(3/4)*Sqrt[a - b*x^4]) - (a^(3/4)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^
(1/4)*x)/a^(1/4)], -1])/(b^(3/4)*Sqrt[a - b*x^4])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a-bx^4}}\,dx = -\frac{\sqrt{a}\int \frac{1}{\sqrt{a-bx^4}}\,dx}{\sqrt{b}} + \frac{\sqrt{a}\int \frac{1+\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a-bx^4}}\,dx}{\sqrt{b}}$$

$$= -\frac{\left(\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}\,dx}{\sqrt{b}\sqrt{a-bx^4}} + \frac{\left(\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1+\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{1-\frac{bx^4}{a}}}\,dx}{\sqrt{b}\sqrt{a-bx^4}}$$

$$= -\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a-bx^4}} + \frac{\left(\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{\sqrt{1+\frac{\sqrt{b}x^2}{\sqrt{a}}}}{\sqrt{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}}\,dx}{\sqrt{b}\sqrt{a-bx^4}}$$

$$= \frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a-bx^4}} - \frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a-bx^4}}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.48

$$\frac{x^3\sqrt{1-\frac{bx^4}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{bx^4}{a}\right)}{3\sqrt{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a - b*x^4],x]
```

```
[Out] (x^3*Sqrt[1 - (b*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, (b*x^4)/a])/(3*Sq
rt[a - b*x^4])
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-bx^4+a}\,x^2}{bx^4-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*x^4 + a)*x^2/(b*x^4 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(-b*x^4 + a), x)
```

maple [A] time = 0.01, size = 88, normalized size = 0.81

$$-\frac{\sqrt{-\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\,\left(-\text{EllipticE}\left(\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,x,i\right)+\text{EllipticF}\left(\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,x,i\right)\right)\sqrt{a}}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^4+a)^(1/2),x)
```

```
[Out] -a^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2)*(-1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(1/a^(1
/2)*b^(1/2)*x^2+1)^(1/2)/(-b*x^4+a)^(1/2)/b^(1/2)*(EllipticF((1/a^(1/2)*b^(
1/2))^(1/2)*x,I)-EllipticE((1/a^(1/2)*b^(1/2))^(1/2)*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(-b*x^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{a-b\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a - b*x^4)^(1/2),x)
```

```
[Out] int(x^2/(a - b*x^4)^(1/2), x)
```

sympy [A] time = 0.86, size = 39, normalized size = 0.36

$$\frac{x^3\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**4+a)**(1/2),x)
```

```
[Out] x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*sq
rt(a)*gamma(7/4))
```

3.849 $\int \frac{1}{x^2\sqrt{a-bx^4}}\,dx$

Optimal. Leaf size=128

$$-\frac{\sqrt{a-bx^4}}{ax}+\frac{\sqrt[4]{b}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{a}\sqrt{a-bx^4}}-\frac{\sqrt[4]{b}\sqrt{1-\frac{bx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{a}\sqrt{a-bx^4}}$$

```
[Out] -(-b*x^4+a)^(1/2)/a/x-b^(1/4)*EllipticE(b^(1/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1
/2)/a^(1/4)/(-b*x^4+a)^(1/2)+b^(1/4)*EllipticF(b^(1/4)*x/a^(1/4),I)*(1-b*x^
4/a)^(1/2)/a^(1/4)/(-b*x^4+a)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {325, 307, 224, 221, 1200, 1199, 424}

$$-\frac{\sqrt{a-bx^4}}{ax}+\frac{\sqrt[4]{b}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{a}\sqrt{a-bx^4}}-\frac{\sqrt[4]{b}\sqrt{1-\frac{bx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{a}\sqrt{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a - b*x^4]),x]
```

```
[Out] -(Sqrt[a - b*x^4]/(a*x)) - (b^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticE[ArcSin[(b
^(1/4)*x)/a^(1/4)], -1])/(a^(1/4)*Sqrt[a - b*x^4]) + (b^(1/4)*Sqrt[1 - (b*x
^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(a^(1/4)*Sqrt[a - b*x^4]
)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
```

```
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a-bx^4}}\,dx = -\frac{\sqrt{a-bx^4}}{ax} - \frac{b\int \frac{x^2}{\sqrt{a-bx^4}}\,dx}{a}$$

$$= -\frac{\sqrt{a-bx^4}}{ax} + \frac{\sqrt{b}\int \frac{1}{\sqrt{a-bx^4}}\,dx}{\sqrt{a}} - \frac{\sqrt{b}\int \frac{1+\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a-bx^4}}\,dx}{\sqrt{a}}$$

$$= -\frac{\sqrt{a-bx^4}}{ax} + \frac{\left(\sqrt{b}\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}\,dx}{\sqrt{a}\sqrt{a-bx^4}} - \frac{\left(\sqrt{b}\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1+\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{1-\frac{bx^4}{a}}}\,dx}{\sqrt{a}\sqrt{a-bx^4}}$$

$$= -\frac{\sqrt{a-bx^4}}{ax} + \frac{\sqrt[4]{b}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{a}\sqrt{a-bx^4}} - \frac{\left(\sqrt{b}\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{\sqrt{1+\frac{\sqrt{b}x^2}{\sqrt{a}}}}{\sqrt{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}}\,dx}{\sqrt{a}\sqrt{a-bx^4}}$$

$$= -\frac{\sqrt{a-bx^4}}{ax} - \frac{\sqrt[4]{b}\sqrt{1-\frac{bx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{a}\sqrt{a-bx^4}} + \frac{\sqrt[4]{b}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{a}\sqrt{a-bx^4}}$$

Mathematica [C] time = 0.01, size = 50, normalized size = 0.39

$$-\frac{\sqrt{1-\frac{bx^4}{a}}\;{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};\frac{bx^4}{a}\right)}{x\sqrt{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a - b*x^4]),x]
```

```
[Out] -((Sqrt[1 - (b*x^4)/a]*Hypergeometric2F1[-1/4, 1/2, 3/4, (b*x^4)/a])/(x*Sqr
t[a - b*x^4]))
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-bx^4+a}}{bx^6-ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*x^4 + a)/(b*x^6 - a*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-bx^4+a}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-b*x^4 + a)*x^2), x)
```

maple [A] time = 0.01, size = 106, normalized size = 0.83

$$\frac{\sqrt{-\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\left(-\text{EllipticE}\left(\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,x,i\right)+\text{EllipticF}\left(\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,x,i\right)\right)\sqrt{b}}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}\,\sqrt{a}}-\frac{\sqrt{-b\,x^4+a}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-b*x^4+a)^(1/2),x)
```

```
[Out] -(-b*x^4+a)^(1/2)/a/x+1/a^(1/2)*b^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2)*(-1/a^(1/
2)*b^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)/(-b*x^4+a)^(1/2)*(E
llipticF((1/a^(1/2)*b^(1/2))^(1/2)*x,I)-EllipticE((1/a^(1/2)*b^(1/2))^(1/2)
*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-bx^4+a}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-b*x^4 + a)*x^2), x)
```

mupad [B] time = 1.28, size = 41, normalized size = 0.32

$$-\frac{\sqrt{1-\frac{a}{b\,x^4}}\;{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\;\frac{7}{4};\;\frac{a}{b\,x^4}\right)}{3\,x\,\sqrt{a-b\,x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a - b*x^4)^(1/2)),x)
```

```
[Out] -((1 - a/(b*x^4))^(1/2)*hypergeom([1/2, 3/4], 7/4, a/(b*x^4)))/(3*x*(a - b*
x^4)^(1/2))
```

sympy [A] time = 2.04, size = 41, normalized size = 0.32

$$\frac{\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\right|\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\,x\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-b*x**4+a)**(1/2),x)
```

```
[Out] gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(
a)*x*gamma(3/4))
```

3.850 $\int \frac{1}{x^6\sqrt{a-bx^4}}\,dx$

Optimal. Leaf size=158

$$\frac{3b^{5/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5a^{5/4}\sqrt{a-bx^4}}-\frac{3b^{5/4}\sqrt{1-\frac{bx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5a^{5/4}\sqrt{a-bx^4}}-\frac{3b\sqrt{a-bx^4}}{5a^2x}-\frac{\sqrt{a-bx^4}}{5ax^5}$$

```
[Out] -1/5*(-b*x^4+a)^(1/2)/a/x^5-3/5*b*(-b*x^4+a)^(1/2)/a^2/x-3/5*b^(5/4)*Ellipt
icE(b^(1/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/a^(5/4)/(-b*x^4+a)^(1/2)+3/5*b^(
5/4)*EllipticF(b^(1/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/a^(5/4)/(-b*x^4+a)^(1
/2)
```

Rubi [A] time = 0.09, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {325, 307, 224, 221, 1200, 1199, 424}

$$\frac{3b^{5/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5a^{5/4}\sqrt{a-bx^4}}-\frac{3b^{5/4}\sqrt{1-\frac{bx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5a^{5/4}\sqrt{a-bx^4}}-\frac{3b\sqrt{a-bx^4}}{5a^2x}-\frac{\sqrt{a-bx^4}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*Sqrt[a - b*x^4]),x]
```

```
[Out] -Sqrt[a - b*x^4]/(5*a*x^5) - (3*b*Sqrt[a - b*x^4])/(5*a^2*x) - (3*b^(5/4)*S
qrt[1 - (b*x^4)/a]*EllipticE[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(5*a^(5/4)*S
qrt[a - b*x^4]) + (3*b^(5/4)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*
x)/a^(1/4)], -1])/(5*a^(5/4)*Sqrt[a - b*x^4])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
```

```
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1200

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + (c*x^4)/a], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{1}{x^6\sqrt{a-bx^4}}\,dx = -\frac{\sqrt{a-bx^4}}{5ax^5} + \frac{(3b)\int \frac{1}{x^2\sqrt{a-bx^4}}\,dx}{5a}$$

$$= -\frac{\sqrt{a-bx^4}}{5ax^5} - \frac{3b\sqrt{a-bx^4}}{5a^2x} - \frac{\left(3b^2\right)\int \frac{x^2}{\sqrt{a-bx^4}}\,dx}{5a^2}$$

$$= -\frac{\sqrt{a-bx^4}}{5ax^5} - \frac{3b\sqrt{a-bx^4}}{5a^2x} + \frac{\left(3b^{3/2}\right)\int \frac{1}{\sqrt{a-bx^4}}\,dx}{5a^{3/2}} - \frac{\left(3b^{3/2}\right)\int \frac{1+\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a-bx^4}}\,dx}{5a^{3/2}}$$

$$= -\frac{\sqrt{a-bx^4}}{5ax^5} - \frac{3b\sqrt{a-bx^4}}{5a^2x} + \frac{\left(3b^{3/2}\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}\,dx}{5a^{3/2}\sqrt{a-bx^4}} - \frac{\left(3b^{3/2}\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1+\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{1-\frac{bx^4}{a}}}}{5a^{3/2}\sqrt{a-bx^4}}$$

$$= -\frac{\sqrt{a-bx^4}}{5ax^5} - \frac{3b\sqrt{a-bx^4}}{5a^2x} + \frac{3b^{5/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5a^{5/4}\sqrt{a-bx^4}} - \frac{\left(3b^{3/2}\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{\sqrt{}}{\sqrt{}}}{5a^{3/2}\sqrt{a-bx}}$$

$$= -\frac{\sqrt{a-bx^4}}{5ax^5} - \frac{3b\sqrt{a-bx^4}}{5a^2x} - \frac{3b^{5/4}\sqrt{1-\frac{bx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5a^{5/4}\sqrt{a-bx^4}} + \frac{3b^{5/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin\right.}{5a^{5/4}\sqrt{a-}}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.33

$$-\frac{\sqrt{1-\frac{bx^4}{a}}\;{}_2F_1\left(-\frac{5}{4},\frac{1}{2};-\frac{1}{4};\frac{bx^4}{a}\right)}{5x^5\sqrt{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*Sqrt[a - b*x^4]),x]
```

```
[Out] -1/5*(Sqrt[1 - (b*x^4)/a]*Hypergeometric2F1[-5/4, 1/2, -1/4, (b*x^4)/a])/(x
^5*Sqrt[a - b*x^4])
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-bx^4+a}}{bx^{10}-ax^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*x^4 + a)/(b*x^10 - a*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-bx^4+a}\,x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-b*x^4 + a)*x^6), x)
```

maple [A] time = 0.01, size = 126, normalized size = 0.80

$$\frac{3\sqrt{-\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\left(-\text{EllipticE}\left(\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,x,i\right)+\text{EllipticF}\left(\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,x,i\right)\right)b^{\frac{3}{2}}}{5\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}\,a^{\frac{3}{2}}}-\frac{3\sqrt{-b\,x^4+a}\,b}{5a^2x}-\frac{\sqrt{-b\,x^4+a}}{5a\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-b*x^4+a)^(1/2),x)
```

```
[Out] -1/5*(-b*x^4+a)^(1/2)/a/x^5-3/5*b*(-b*x^4+a)^(1/2)/a^2/x+3/5/a^(3/2)*b^(3/2
)/(1/a^(1/2)*b^(1/2))^(1/2)*(-1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*b^(
1/2)*x^2+1)^(1/2)/(-b*x^4+a)^(1/2)*(EllipticF((1/a^(1/2)*b^(1/2))^(1/2)*x,I
)-EllipticE((1/a^(1/2)*b^(1/2))^(1/2)*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-bx^4+a}\,x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-b*x^4 + a)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^6\,\sqrt{a-b\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a - b*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^6*(a - b*x^4)^(1/2)), x)
```

sympy [A] time = 1.48, size = 39, normalized size = 0.25

$$-\frac{i\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{a}{bx^4}\right)}{4\sqrt{b}\,x^7\Gamma\left(-\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-b*x**4+a)**(1/2),x)
```

```
[Out] -I*gamma(-7/4)*hyper((1/2, 7/4), (11/4,), a/(b*x**4))/(4*sqrt(b)*x**7*gamma
(-3/4))
```

3.851 $\int \frac{x^{11}}{\left(a+bx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=57

$$-\frac{a^2}{2b^3\sqrt{a+bx^4}}-\frac{a\sqrt{a+bx^4}}{b^3}+\frac{\left(a+bx^4\right)^{3/2}}{6b^3}$$

```
[Out] 1/6*(b*x^4+a)^(3/2)/b^3-1/2*a^2/b^3/(b*x^4+a)^(1/2)-a*(b*x^4+a)^(1/2)/b^3
```

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{a^2}{2b^3\sqrt{a+bx^4}}-\frac{a\sqrt{a+bx^4}}{b^3}+\frac{\left(a+bx^4\right)^{3/2}}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + b*x^4)^(3/2),x]
```

```
[Out] -a^2/(2*b^3*Sqrt[a + b*x^4]) - (a*Sqrt[a + b*x^4])/b^3 + (a + b*x^4)^(3/2)/
(6*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{\left(a+bx^4\right)^{3/2}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x^2}{(a+bx)^{3/2}}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int \left(\frac{a^2}{b^2(a+bx)^{3/2}}-\frac{2a}{b^2\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b^2}\right)dx, x, x^4\right)\\ &= -\frac{a^2}{2b^3\sqrt{a+bx^4}}-\frac{a\sqrt{a+bx^4}}{b^3}+\frac{\left(a+bx^4\right)^{3/2}}{6b^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 38, normalized size = 0.67

$$\frac{-8a^2-4abx^4+b^2x^8}{6b^3\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + b*x^4)^(3/2),x]
```

```
[Out] (-8*a^2 - 4*a*b*x^4 + b^2*x^8)/(6*b^3*Sqrt[a + b*x^4])
```

fricas [A] time = 0.89, size = 46, normalized size = 0.81

$$\frac{\left(b^2x^8 - 4\,abx^4 - 8\,a^2\right)\sqrt{bx^4 + a}}{6\left(b^4x^4 + ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(b^2*x^8 - 4*a*b*x^4 - 8*a^2)*sqrt(b*x^4 + a)/(b^4*x^4 + a*b^3)
```

giac [A] time = 0.20, size = 52, normalized size = 0.91

$$-\frac{a^2}{2\sqrt{bx^4 + a}\,b^3} + \frac{\left(bx^4 + a\right)^{\frac{3}{2}}b^6 - 6\sqrt{bx^4 + a}\,ab^6}{6\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*a^2/(sqrt(b*x^4 + a)*b^3) + 1/6*((b*x^4 + a)^(3/2)*b^6 - 6*sqrt(b*x^4
+ a)*a*b^6)/b^9
```

maple [A] time = 0.01, size = 36, normalized size = 0.63

$$-\frac{-b^2x^8 + 4ab\,x^4 + 8a^2}{6\sqrt{b\,x^4 + a}\;b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^4+a)^(3/2),x)
```

```
[Out] -1/6*(-b^2*x^8+4*a*b*x^4+8*a^2)/(b*x^4+a)^(1/2)/b^3
```

maxima [A] time = 1.30, size = 47, normalized size = 0.82

$$\frac{\left(bx^4 + a\right)^{\frac{3}{2}}}{6\,b^3} - \frac{\sqrt{bx^4 + a}\,a}{b^3} - \frac{a^2}{2\sqrt{bx^4 + a}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^4 + a)^(3/2)/b^3 - sqrt(b*x^4 + a)*a/b^3 - 1/2*a^2/(sqrt(b*x^4 + a
)*b^3)
```

mupad [B] time = 1.30, size = 41, normalized size = 0.72

$$-\frac{6\,a\,\left(b\,x^4 + a\right) - \left(b\,x^4 + a\right)^2 + 3\,a^2}{6\,b^3\,\sqrt{b\,x^4 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a + b*x^4)^(3/2),x)
```

```
[Out] -(6*a*(a + b*x^4) - (a + b*x^4)^2 + 3*a^2)/(6*b^3*(a + b*x^4)^(1/2))
```

sympy [**A**] time = 3.57, size = 68, normalized size = 1.19

$$\begin{cases} -\frac{4a^2}{3b^3\sqrt{a+bx^4}} - \frac{2ax^4}{3b^2\sqrt{a+bx^4}} + \frac{x^8}{6b\sqrt{a+bx^4}} & \text{for } b \neq 0 \\ \frac{x^{12}}{12a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**4+a)**(3/2),x)
```

```
[Out] Piecewise((-4*a**2/(3*b**3*sqrt(a + b*x**4)) - 2*a*x**4/(3*b**2*sqrt(a + b*
x**4)) + x**8/(6*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**12/(12*a**(3/2)), True
))
```

3.852 $\int \frac{x^7}{\left(a+bx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=38

$$\frac{a}{2b^2\sqrt{a+bx^4}}+\frac{\sqrt{a+bx^4}}{2b^2}$$

```
[Out] 1/2*a/b^2/(b*x^4+a)^(1/2)+1/2*(b*x^4+a)^(1/2)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{a}{2b^2\sqrt{a+bx^4}}+\frac{\sqrt{a+bx^4}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^4)^(3/2),x]
```

```
[Out] a/(2*b^2*Sqrt[a + b*x^4]) + Sqrt[a + b*x^4]/(2*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{\left(a+bx^4\right)^{3/2}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x}{(a+bx)^{3/2}}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^{3/2}}+\frac{1}{b\sqrt{a+bx}}\right)dx, x, x^4\right)\\ &= \frac{a}{2b^2\sqrt{a+bx^4}}+\frac{\sqrt{a+bx^4}}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 27, normalized size = 0.71

$$\frac{2a+bx^4}{2b^2\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^4)^(3/2),x]
```

```
[Out] (2*a + b*x^4)/(2*b^2*Sqrt[a + b*x^4])
```

fricas [A] time = 0.98, size = 35, normalized size = 0.92

$$\frac{\left(bx^4+2\,a\right)\sqrt{bx^4+a}}{2\left(b^3x^4+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)^(3/2),x, algorithm="fricas")
[Out] 1/2*(b*x^4 + 2*a)*sqrt(b*x^4 + a)/(b^3*x^4 + a*b^2)
```

giac [A] time = 0.17, size = 33, normalized size = 0.87

$$\frac{\frac{\sqrt{bx^4+a}}{b}+\frac{a}{\sqrt{bx^4+a}\,b}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)^(3/2),x, algorithm="giac")
[Out] 1/2*(sqrt(b*x^4 + a)/b + a/(sqrt(b*x^4 + a)*b))/b
```

maple [A] time = 0.01, size = 24, normalized size = 0.63

$$\frac{b\,x^4+2a}{2\sqrt{b\,x^4+a}\;b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^4+a)^(3/2),x)
[Out] 1/2*(b*x^4+2*a)/(b*x^4+a)^(1/2)/b^2
```

maxima [A] time = 1.30, size = 30, normalized size = 0.79

$$\frac{\sqrt{bx^4+a}}{2\,b^2}+\frac{a}{2\,\sqrt{bx^4+a}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)^(3/2),x, algorithm="maxima")
[Out] 1/2*sqrt(b*x^4 + a)/b^2 + 1/2*a/(sqrt(b*x^4 + a)*b^2)
```

mupad [B] time = 1.21, size = 21, normalized size = 0.55

$$\frac{\frac{b\,x^4}{2}+a}{b^2\,\sqrt{b\,x^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^4)^(3/2),x)
[Out] (a + (b*x^4)/2)/(b^2*(a + b*x^4)^(1/2))
```

sympy [A] time = 2.48, size = 41, normalized size = 1.08

$$\begin{cases} \frac{a}{b^2\sqrt{a+bx^4}}+\frac{x^4}{2b\sqrt{a+bx^4}} & \text{for } b\neq 0 \\ \frac{x^8}{8a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**4+a)**(3/2),x)
```

```
[Out] Piecewise((a/(b**2*sqrt(a + b*x**4)) + x**4/(2*b*sqrt(a + b*x**4)), Ne(b, 0
)), (x**8/(8*a**(3/2)), True))
```

3.853 $\int \frac{x^3}{\left(a+bx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=18

$$-\frac{1}{2b\sqrt{a+bx^4}}$$

```
[Out] -1/2/b/(b*x^4+a)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$-\frac{1}{2b\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^4)^(3/2),x]
```

```
[Out] -1/(2*b*Sqrt[a + b*x^4])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^4\right)^{3/2}}\,dx = -\frac{1}{2b\sqrt{a+bx^4}}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$-\frac{1}{2b\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^4)^(3/2),x]
```

```
[Out] -1/2*1/(b*Sqrt[a + b*x^4])
```

fricas [A] time = 0.74, size = 24, normalized size = 1.33

$$-\frac{\sqrt{bx^4+a}}{2\left(b^2x^4+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(b*x^4 + a)/(b^2*x^4 + a*b)
```

giac [A] time = 0.15, size = 14, normalized size = 0.78

$$-\frac{1}{2\sqrt{bx^4+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)^(3/2),x, algorithm="giac")
[Out] -1/2/(sqrt(b*x^4 + a)*b)
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$-\frac{1}{2\sqrt{b\,x^4+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^4+a)^(3/2),x)
[Out] -1/2/b/(b*x^4+a)^(1/2)
```

maxima [A] time = 1.30, size = 14, normalized size = 0.78

$$-\frac{1}{2\,\sqrt{b x^4+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)^(3/2),x, algorithm="maxima")
[Out] -1/2/(sqrt(b*x^4 + a)*b)
```

mupad [B] time = 1.13, size = 14, normalized size = 0.78

$$-\frac{1}{2\,b\,\sqrt{b\,x^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^4)^(3/2),x)
[Out] -1/(2*b*(a + b*x^4)^(1/2))
```

sympy [A] time = 1.12, size = 26, normalized size = 1.44

$$\begin{cases} -\frac{1}{2b\sqrt{a+bx^4}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**4+a)**(3/2),x)
[Out] Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)), True))
```

3.854 $\int \frac{1}{x\left(a+bx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=46

$$\frac{1}{2a\sqrt{a+bx^4}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}}$$

```
[Out] -1/2*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)+1/2/a/(b*x^4+a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 51, 63, 208}

$$\frac{1}{2a\sqrt{a+bx^4}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^4)^(3/2)),x]
```

```
[Out] 1/(2*a*Sqrt[a + b*x^4]) - ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]/(2*a^(3/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^4\right)^{3/2}}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, x^4\right)$$
$$= \frac{1}{2a\sqrt{a+bx^4}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^4\right)}{4a}$$
$$= \frac{1}{2a\sqrt{a+bx^4}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^4}\right)}{2ab}$$
$$= \frac{1}{2a\sqrt{a+bx^4}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}}$$

Mathematica [C] time = 0.01, size = 36, normalized size = 0.78

$$\frac{{}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{bx^4}{a}+1\right)}{2a\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^4)^(3/2)),x]
```

```
[Out] Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^4)/a]/(2*a*Sqrt[a + b*x^4])
```

fricas [A] time = 0.85, size = 129, normalized size = 2.80

$$\left[\frac{\left(bx^4+a\right)\sqrt{a}\,\log\left(\frac{bx^4-2\,\sqrt{bx^4+a}\,\sqrt{a}+2\,a}{x^4}\right)+2\,\sqrt{bx^4+a}\,a}{4\left(a^2bx^4+a^3\right)}, \frac{\left(bx^4+a\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx^4+a}\,\sqrt{-a}}{a}\right)+\sqrt{bx^4+a}\,a}{2\left(a^2bx^4+a^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((b*x^4 + a)*sqrt(a)*log((b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x^4
) + 2*sqrt(b*x^4 + a)*a)/(a^2*b*x^4 + a^3), 1/2*((b*x^4 + a)*sqrt(-a)*arcta
n(sqrt(b*x^4 + a)*sqrt(-a)/a) + sqrt(b*x^4 + a)*a)/(a^2*b*x^4 + a^3)]
```

giac [A] time = 0.16, size = 41, normalized size = 0.89

$$\frac{\arctan\left(\frac{\sqrt{bx^4+a}}{\sqrt{-a}}\right)}{2\,\sqrt{-a}\,a} + \frac{1}{2\,\sqrt{bx^4+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*arctan(sqrt(b*x^4 + a)/sqrt(-a))/(sqrt(-a)*a) + 1/2/(sqrt(b*x^4 + a)*a)
```

maple [A] time = 0.01, size = 44, normalized size = 0.96

$$-\frac{\ln\left(\frac{2a+2\sqrt{b\,x^4+a}\,\sqrt{a}}{x^2}\right)}{2a^{\frac{3}{2}}} + \frac{1}{2\sqrt{b\,x^4+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^4+a)^(3/2),x)
```

```
[Out] 1/2/a/(b*x^4+a)^(1/2)-1/2/a^(3/2)*ln((2*a+2*(b*x^4+a)^(1/2)*a^(1/2))/x^2)
```

maxima [A] time = 3.01, size = 52, normalized size = 1.13

$$\frac{\log\left(\frac{\sqrt{bx^4+a}-\sqrt{a}}{\sqrt{bx^4+a}+\sqrt{a}}\right)}{4\,a^{\frac{3}{2}}}+\frac{1}{2\,\sqrt{bx^4+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*log((sqrt(b*x^4 + a) - sqrt(a))/(sqrt(b*x^4 + a) + sqrt(a)))/a^(3/2) +
1/2/(sqrt(b*x^4 + a)*a)
```

mupad [B] time = 1.25, size = 34, normalized size = 0.74

$$\frac{1}{2\,a\,\sqrt{b\,x^4+a}}-\frac{\operatorname{atanh}\left(\frac{\sqrt{b\,x^4+a}}{\sqrt{a}}\right)}{2\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^4)^(3/2)),x)
```

```
[Out] 1/(2*a*(a + b*x^4)^(1/2)) - atanh((a + b*x^4)^(1/2)/a^(1/2))/(2*a^(3/2))
```

sympy [B] time = 3.79, size = 184, normalized size = 4.00

$$\frac{2a^3\sqrt{1+\frac{bx^4}{a}}}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}+\frac{a^3\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}+\frac{a^2bx^4\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}-\frac{2a^2bx^4\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**4+a)**(3/2),x)
```

```
[Out] 2*a**3*sqrt(1 + b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**3*log(b*x**
4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a**3*log(sqrt(1 + b*x**4/a) + 1)/
(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**2*b*x**4*log(b*x**4/a)/(4*a**(9/2) +
4*a**(7/2)*b*x**4) - 2*a**2*b*x**4*log(sqrt(1 + b*x**4/a) + 1)/(4*a**(9/2)
+ 4*a**(7/2)*b*x**4)
```

3.855 $\int \frac{1}{x^5\left(a+bx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=69

$$\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4a^{5/2}} - \frac{3b}{4a^2\sqrt{a+bx^4}} - \frac{1}{4ax^4\sqrt{a+bx^4}}$$

```
[Out] 3/4*b*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(5/2)-3/4*b/a^2/(b*x^4+a)^(1/2)-1/
4/a/x^4/(b*x^4+a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 51, 63, 208}

$$-\frac{3\sqrt{a+bx^4}}{4a^2x^4} + \frac{3b\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4a^{5/2}} + \frac{1}{2ax^4\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^4)^(3/2)),x]
```

```
[Out] 1/(2*a*x^4*Sqrt[a + b*x^4]) - (3*Sqrt[a + b*x^4])/(4*a^2*x^4) + (3*b*ArcTan
h[Sqrt[a + b*x^4]/Sqrt[a]])/(4*a^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^5\left(a+bx^4\right)^{3/2}}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx, x, x^4\right)$$

$$= \frac{1}{2ax^4\sqrt{a+bx^4}} + \frac{3\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^4\right)}{4a}$$

$$= \frac{1}{2ax^4\sqrt{a+bx^4}} - \frac{3\sqrt{a+bx^4}}{4a^2x^4} - \frac{(3b)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^4\right)}{8a^2}$$

$$= \frac{1}{2ax^4\sqrt{a+bx^4}} - \frac{3\sqrt{a+bx^4}}{4a^2x^4} - \frac{3\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^4}\right)}{4a^2}$$

$$= \frac{1}{2ax^4\sqrt{a+bx^4}} - \frac{3\sqrt{a+bx^4}}{4a^2x^4} + \frac{3b\tanh^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4a^{5/2}}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.54

$$-\frac{b\,{}_2F_1\left(-\frac{1}{2}, 2; \frac{1}{2}; \frac{bx^4}{a}+1\right)}{2a^2\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^4)^(3/2)),x]
```

```
[Out] -1/2*(b*Hypergeometric2F1[-1/2, 2, 1/2, 1 + (b*x^4)/a])/(a^2*Sqrt[a + b*x^4
])
```

fricas [A] time = 0.97, size = 173, normalized size = 2.51

$$\left[\frac{3\left(b^2x^8+abx^4\right)\sqrt{a}\,\log\left(\frac{bx^4+2\sqrt{bx^4+a}\sqrt{a}+2a}{x^4}\right) - 2\left(3abx^4+a^2\right)\sqrt{bx^4+a}}{8\left(a^3bx^8+a^4x^4\right)}, -\frac{3\left(b^2x^8+abx^4\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx^4+a}\sqrt{-}\right.}{a}\right.}{4\left(a^3bx^8+a^{4}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*(b^2*x^8 + a*b*x^4)*sqrt(a)*log((b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(a)
+ 2*a)/x^4) - 2*(3*a*b*x^4 + a^2)*sqrt(b*x^4 + a))/(a^3*b*x^8 + a^4*x^4), -
1/4*(3*(b^2*x^8 + a*b*x^4)*sqrt(-a)*arctan(sqrt(b*x^4 + a)*sqrt(-a)/a) + (3
*a*b*x^4 + a^2)*sqrt(b*x^4 + a))/(a^3*b*x^8 + a^4*x^4)]
```

giac [A] time = 0.17, size = 72, normalized size = 1.04

$$-\frac{3\,b\arctan\left(\frac{\sqrt{bx^4+a}}{\sqrt{-a}}\right)}{4\sqrt{-a}\,a^2} - \frac{3\left(bx^4+a\right)b - 2\,ab}{4\left(\left(bx^4+a\right)^{\frac{3}{2}} - \sqrt{bx^4+a}\,a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] -3/4*b*arctan(sqrt(b*x^4 + a)/sqrt(-a))/(sqrt(-a)*a^2) - 1/4*(3*(b*x^4 + a)
*b - 2*a*b)/(((b*x^4 + a)^(3/2) - sqrt(b*x^4 + a)*a)*a^2)
```

maple [A] time = 0.02, size = 63, normalized size = 0.91

$$\frac{3b\ln\left(\frac{2a+2\sqrt{b\,x^4+a}\,\sqrt{a}}{x^2}\right)}{4a^{\frac{5}{2}}}-\frac{3b}{4\sqrt{b\,x^4+a}\,a^2}-\frac{1}{4\sqrt{b\,x^4+a}\,a\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^4+a)^(3/2),x)
```

```
[Out] -1/4/a/x^4/(b*x^4+a)^(1/2)-3/4*b/a^2/(b*x^4+a)^(1/2)+3/4/a^(5/2)*b*ln((2*a+
2*(b*x^4+a)^(1/2)*a^(1/2))/x^2)
```

maxima [A] time = 2.98, size = 86, normalized size = 1.25

$$-\frac{3\left(bx^4+a\right)b-2\,ab}{4\left(\left(bx^4+a\right)^{\frac{3}{2}}a^2-\sqrt{bx^4+a}\,a^3\right)}-\frac{3\,b\log\left(\frac{\sqrt{bx^4+a}-\sqrt{a}}{\sqrt{bx^4+a}+\sqrt{a}}\right)}{8\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(3*(b*x^4 + a)*b - 2*a*b)/((b*x^4 + a)^(3/2)*a^2 - sqrt(b*x^4 + a)*a^3
) - 3/8*b*log((sqrt(b*x^4 + a) - sqrt(a))/(sqrt(b*x^4 + a) + sqrt(a)))/a^(5
/2)
```

mupad [B] time = 1.37, size = 53, normalized size = 0.77

$$\frac{3\,b\,\operatorname{atanh}\left(\frac{\sqrt{b\,x^4+a}}{\sqrt{a}}\right)}{4\,a^{5/2}}-\frac{1}{4\,a\,x^4\,\sqrt{b\,x^4+a}}-\frac{3\,b}{4\,a^2\,\sqrt{b\,x^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^4)^(3/2)),x)
```

```
[Out] (3*b*atanh((a + b*x^4)^(1/2)/a^(1/2)))/(4*a^(5/2)) - 1/(4*a*x^4*(a + b*x^4)
^(1/2)) - (3*b)/(4*a^2*(a + b*x^4)^(1/2))
```

sympy [A] time = 6.88, size = 76, normalized size = 1.10

$$-\frac{1}{4a\sqrt{b}\,x^6\sqrt{\frac{a}{bx^4}+1}}-\frac{3\sqrt{b}}{4a^2x^2\sqrt{\frac{a}{bx^4}+1}}+\frac{3b\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^2}\right)}{4a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**4+a)**(3/2),x)
```

```
[Out] -1/(4*a*sqrt(b)*x**6*sqrt(a/(b*x**4) + 1)) - 3*sqrt(b)/(4*a**2*x**2*sqrt(a/
(b*x**4) + 1)) + 3*b*asinh(sqrt(a)/(sqrt(b)*x**2))/(4*a**(5/2))
```

3.856 $\int \frac{x^9}{\left(a+bx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=74

$$-\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{4b^{5/2}}+\frac{3x^2\sqrt{a+bx^4}}{4b^2}-\frac{x^6}{2b\sqrt{a+bx^4}}$$

```
[Out] -3/4*a*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(5/2)-1/2*x^6/b/(b*x^4+a)^(1/
2)+3/4*x^2*(b*x^4+a)^(1/2)/b^2
```

Rubi [A] time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {275, 288, 321, 217, 206}

$$\frac{3x^2\sqrt{a+bx^4}}{4b^2}-\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{4b^{5/2}}-\frac{x^6}{2b\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x^4)^(3/2),x]
```

```
[Out] -x^6/(2*b*Sqrt[a + b*x^4]) + (3*x^2*Sqrt[a + b*x^4])/(4*b^2) - (3*a*ArcTanh
[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{\left(a+bx^4\right)^{3/2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^2\right)^{3/2}}\,dx, x, x^2\right)\\ &= -\frac{x^6}{2b\sqrt{a+bx^4}} + \frac{3\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^2}}\,dx, x, x^2\right)}{2b}\\ &= -\frac{x^6}{2b\sqrt{a+bx^4}} + \frac{3x^2\sqrt{a+bx^4}}{4b^2} - \frac{(3a)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, x^2\right)}{4b^2}\\ &= -\frac{x^6}{2b\sqrt{a+bx^4}} + \frac{3x^2\sqrt{a+bx^4}}{4b^2} - \frac{(3a)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^2}{\sqrt{a+bx^4}}\right)}{4b^2}\\ &= -\frac{x^6}{2b\sqrt{a+bx^4}} + \frac{3x^2\sqrt{a+bx^4}}{4b^2} - \frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{4b^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 75, normalized size = 1.01

$$\frac{\sqrt{b}\,x^2\left(3a+bx^4\right) - 3a^{3/2}\sqrt{\frac{bx^4}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4b^{5/2}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x^4)^(3/2),x]
```

```
[Out] (Sqrt[b]*x^2*(3*a + b*x^4) - 3*a^(3/2)*Sqrt[1 + (b*x^4)/a]*ArcSinh[(Sqrt[b]
*x^2)/Sqrt[a]])/(4*b^(5/2)*Sqrt[a + b*x^4])
```

fricas [A] time = 0.60, size = 167, normalized size = 2.26

$$\left[\frac{3\left(abx^4+a^2\right)\sqrt{b}\,\log\left(-2\,bx^4+2\,\sqrt{bx^4+a}\,\sqrt{b}\,x^2-a\right)+2\left(b^2x^6+3\,abx^2\right)\sqrt{bx^4+a}}{8\left(b^4x^4+ab^3\right)}, \frac{3\left(abx^4+a^2\right)\sqrt{-b}\,\text{arcta}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*(a*b*x^4 + a^2)*sqrt(b)*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b)*x^
2 - a) + 2*(b^2*x^6 + 3*a*b*x^2)*sqrt(b*x^4 + a))/(b^4*x^4 + a*b^3), 1/4*(3
*(a*b*x^4 + a^2)*sqrt(-b)*arctan(sqrt(-b)*x^2/sqrt(b*x^4 + a)) + (b^2*x^6 +
 3*a*b*x^2)*sqrt(b*x^4 + a))/(b^4*x^4 + a*b^3)]
```

giac [A] time = 0.19, size = 55, normalized size = 0.74

$$\frac{\left(\frac{x^4}{b}+\frac{3\,a}{b^2}\right)x^2}{4\sqrt{bx^4+a}} + \frac{3\,a\log\left(\left|-\sqrt{b}\,x^2+\sqrt{bx^4+a}\right|\right)}{4\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(x^4/b + 3*a/b^2)*x^2/sqrt(b*x^4 + a) + 3/4*a*log(abs(-sqrt(b)*x^2 + sq
rt(b*x^4 + a)))/b^(5/2)
```

maple [A] time = 0.01, size = 61, normalized size = 0.82

$$\frac{x^6}{4\sqrt{b\,x^4+a}\;b}+\frac{3a\,x^2}{4\sqrt{b\,x^4+a}\;b^2}-\frac{3a\ln\left(\sqrt{b}\;x^2+\sqrt{b\,x^4+a}\right)}{4b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^4+a)^(3/2),x)
```

```
[Out] 1/4*x^6/b/(b*x^4+a)^(1/2)+3/4*a/b^2*x^2/(b*x^4+a)^(1/2)-3/4*a/b^(5/2)*ln(b^
(1/2)*x^2+(b*x^4+a)^(1/2))
```

maxima [A] time = 3.01, size = 103, normalized size = 1.39

$$\frac{2\,ab-\frac{3\left(bx^4+a\right)a}{x^4}}{4\left(\frac{\sqrt{bx^4+a}\,b^3}{x^2}-\frac{\left(bx^4+a\right)^{\frac{3}{2}}b^2}{x^6}\right)}+\frac{3\,a\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^4+a}}{x^2}}{\sqrt{b}+\frac{\sqrt{bx^4+a}}{x^2}}\right)}{8\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*a*b - 3*(b*x^4 + a)*a/x^4)/(sqrt(b*x^4 + a)*b^3/x^2 - (b*x^4 + a)^(3
/2)*b^2/x^6) + 3/8*a*log(-(sqrt(b) - sqrt(b*x^4 + a)/x^2)/(sqrt(b) + sqrt(b
*x^4 + a)/x^2))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^9}{\left(b\,x^4+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + b*x^4)^(3/2),x)
```

```
[Out] int(x^9/(a + b*x^4)^(3/2), x)
```

sympy [A] time = 4.27, size = 75, normalized size = 1.01

$$\frac{3\sqrt{a}\,x^2}{4b^2\sqrt{1+\frac{bx^4}{a}}}-\frac{3a\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{4b^{\frac{5}{2}}}+\frac{x^6}{4\sqrt{a}\,b\sqrt{1+\frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**4+a)**(3/2),x)
```

```
[Out] 3*sqrt(a)*x**2/(4*b**2*sqrt(1 + b*x**4/a)) - 3*a*asinh(sqrt(b)*x**2/sqrt(a)
)/(4*b**(5/2)) + x**6/(4*sqrt(a)*b*sqrt(1 + b*x**4/a))
```

3.857 $\int \frac{x^5}{\left(a+bx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=52

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}} - \frac{x^2}{2b\sqrt{a+bx^4}}$$

```
[Out] 1/2*arctanh(x^2*b^(1/2)/(b*x^4+a)^(1/2))/b^(3/2)-1/2*x^2/b/(b*x^4+a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {275, 288, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}} - \frac{x^2}{2b\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^4)^(3/2),x]
```

```
[Out] -x^2/(2*b*Sqrt[a + b*x^4]) + ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]]/(2*b^(3
/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\left(a+bx^4\right)^{3/2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^2\right)^{3/2}}\,dx, x, x^2\right)\\ &= -\frac{x^2}{2b\sqrt{a+bx^4}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, x^2\right)}{2b}\\ &= -\frac{x^2}{2b\sqrt{a+bx^4}} + \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^2}{\sqrt{a+bx^4}}\right)}{2b}\\ &= -\frac{x^2}{2b\sqrt{a+bx^4}} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.09, size = 66, normalized size = 1.27

$$\frac{\sqrt{a}\,\sqrt{\frac{bx^4}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right) - \sqrt{b}\,x^2}{2b^{3/2}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^4)^(3/2),x]
```

```
[Out] (-(Sqrt[b]*x^2) + Sqrt[a]*Sqrt[1 + (b*x^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]
])/(2*b^(3/2)*Sqrt[a + b*x^4])
```

fricas [A] time = 0.90, size = 138, normalized size = 2.65

$$\left[-\frac{2\sqrt{bx^4+a}\,bx^2 - \left(bx^4+a\right)\sqrt{b}\,\log\left(-2\,bx^4 - 2\sqrt{bx^4+a}\,\sqrt{b}\,x^2 - a\right)}{4\left(b^3x^4+ab^2\right)}, -\frac{\sqrt{bx^4+a}\,bx^2 + \left(bx^4+a\right)\sqrt{-b}\,\arctan\left(-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*sqrt(b*x^4 + a)*b*x^2 - (b*x^4 + a)*sqrt(b)*log(-2*b*x^4 - 2*sqrt(
b*x^4 + a)*sqrt(b)*x^2 - a))/(b^3*x^4 + a*b^2), -1/2*(sqrt(b*x^4 + a)*b*x^2
 + (b*x^4 + a)*sqrt(-b)*arctan(sqrt(-b)*x^2/sqrt(b*x^4 + a)))/(b^3*x^4 + a*
b^2)]
```

giac [A] time = 0.19, size = 43, normalized size = 0.83

$$-\frac{x^2}{2\sqrt{bx^4+a}\,b} - \frac{\log\left(\left|-\sqrt{b}\,x^2 + \sqrt{bx^4+a}\right|\right)}{2\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*x^2/(sqrt(b*x^4 + a)*b) - 1/2*log(abs(-sqrt(b)*x^2 + sqrt(b*x^4 + a)))
/b^(3/2)
```

maple [A] time = 0.01, size = 42, normalized size = 0.81

$$-\frac{x^2}{2\sqrt{b\,x^4+a}\,b} + \frac{\ln\left(\sqrt{b}\,x^2 + \sqrt{b\,x^4+a}\right)}{2b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^4+a)^(3/2),x)
```

```
[Out] -1/2*x^2/b/(b*x^4+a)^(1/2)+1/2/b^(3/2)*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))
```

maxima [A] time = 2.88, size = 63, normalized size = 1.21

$$-\frac{x^2}{2\sqrt{bx^4+a}\,b}-\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^4+a}}{x^2}}{\sqrt{b}+\frac{\sqrt{bx^4+a}}{x^2}}\right)}{4\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*x^2/(sqrt(b*x^4 + a)*b) - 1/4*log(-(sqrt(b) - sqrt(b*x^4 + a)/x^2)/(sq
rt(b) + sqrt(b*x^4 + a)/x^2))/b^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^5}{\left(b\,x^4+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^4)^(3/2),x)
```

```
[Out] int(x^5/(a + b*x^4)^(3/2), x)
```

sympy [A] time = 3.32, size = 44, normalized size = 0.85

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}-\frac{x^2}{2\sqrt{a}\,b\sqrt{1+\frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**4+a)**(3/2),x)
```

```
[Out] asinh(sqrt(b)*x**2/sqrt(a))/(2*b**(3/2)) - x**2/(2*sqrt(a)*b*sqrt(1 + b*x**
4/a))
```

3.858 $\int \frac{x}{\left(a+bx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=21

$$\frac{x^2}{2a\sqrt{a+bx^4}}$$

```
[Out] 1/2*x^2/a/(b*x^4+a)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {264}

$$\frac{x^2}{2a\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^4)^(3/2),x]
```

```
[Out] x^2/(2*a*Sqrt[a + b*x^4])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^4\right)^{3/2}}\,dx = \frac{x^2}{2a\sqrt{a+bx^4}}$$

Mathematica [A] time = 0.02, size = 21, normalized size = 1.00

$$\frac{x^2}{2a\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^4)^(3/2),x]
```

```
[Out] x^2/(2*a*Sqrt[a + b*x^4])
```

fricas [A] time = 1.14, size = 26, normalized size = 1.24

$$\frac{\sqrt{bx^4+a}\,x^2}{2\left(abx^4+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(b*x^4 + a)*x^2/(a*b*x^4 + a^2)
```

giac [A] time = 0.17, size = 17, normalized size = 0.81

$$\frac{x^2}{2\sqrt{bx^4+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)^(3/2),x, algorithm="giac")
[Out] 1/2*x^2/(sqrt(b*x^4 + a)*a)
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$\frac{x^2}{2\sqrt{b\,x^4+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^4+a)^(3/2),x)
[Out] 1/2*x^2/a/(b*x^4+a)^(1/2)
```

maxima [A] time = 1.30, size = 17, normalized size = 0.81

$$\frac{x^2}{2\,\sqrt{b x^4+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)^(3/2),x, algorithm="maxima")
[Out] 1/2*x^2/(sqrt(b*x^4 + a)*a)
```

mupad [B] time = 1.03, size = 17, normalized size = 0.81

$$\frac{x^2}{2\,a\,\sqrt{b\,x^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^4)^(3/2),x)
[Out] x^2/(2*a*(a + b*x^4)^(1/2))
```

sympy [A] time = 1.04, size = 20, normalized size = 0.95

$$\frac{x^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{bx^4}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**4+a)**(3/2),x)
[Out] x**2/(2*a**(3/2)*sqrt(1 + b*x**4/a))
```

3.859 $\int \frac{1}{x^3\left(a+bx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=42

$$-\frac{bx^2}{a^2\sqrt{a+bx^4}}-\frac{1}{2ax^2\sqrt{a+bx^4}}$$

```
[Out] -1/2/a/x^2/(b*x^4+a)^(1/2)-b*x^2/a^2/(b*x^4+a)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$-\frac{bx^2}{a^2\sqrt{a+bx^4}}-\frac{1}{2ax^2\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^4)^(3/2)),x]
```

```
[Out] -1/(2*a*x^2*Sqrt[a + b*x^4]) - (b*x^2)/(a^2*Sqrt[a + b*x^4])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a+bx^4\right)^{3/2}}\,dx &= -\frac{1}{2ax^2\sqrt{a+bx^4}}-\frac{(2b)\int \frac{x}{\left(a+bx^4\right)^{3/2}}\,dx}{a}\\ &= -\frac{1}{2ax^2\sqrt{a+bx^4}}-\frac{bx^2}{a^2\sqrt{a+bx^4}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.69

$$-\frac{a+2bx^4}{2a^2x^2\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^4)^(3/2)),x]
```

```
[Out] -1/2*(a + 2*b*x^4)/(a^2*x^2*Sqrt[a + b*x^4])
```

fricas [A] time = 0.89, size = 37, normalized size = 0.88

$$-\frac{\left(2\,bx^4+a\right)\sqrt{bx^4+a}}{2\left(a^2bx^6+a^3x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*x^4 + a)*sqrt(b*x^4 + a)/(a^2*b*x^6 + a^3*x^2)
```

giac [A] time = 0.19, size = 53, normalized size = 1.26

$$-\frac{bx^2}{2\sqrt{bx^4+a}\,a^2}+\frac{\sqrt{b}}{\left(\left(\sqrt{b}\,x^2-\sqrt{bx^4+a}\right)^2-a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*b*x^2/(sqrt(b*x^4 + a)*a^2) + sqrt(b)/(((sqrt(b)*x^2 - sqrt(b*x^4 + a)
)^2 - a)*a)
```

maple [A] time = 0.00, size = 26, normalized size = 0.62

$$-\frac{2b\,x^4+a}{2\sqrt{b\,x^4+a}\;a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^4+a)^(3/2),x)
```

```
[Out] -1/2*(2*b*x^4+a)/x^2/(b*x^4+a)^(1/2)/a^2
```

maxima [A] time = 1.32, size = 36, normalized size = 0.86

$$-\frac{bx^2}{2\sqrt{bx^4+a}\,a^2}-\frac{\sqrt{bx^4+a}}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*b*x^2/(sqrt(b*x^4 + a)*a^2) - 1/2*sqrt(b*x^4 + a)/(a^2*x^2)
```

mupad [B] time = 1.15, size = 25, normalized size = 0.60

$$-\frac{2\,b\,x^4+a}{2\,a^2\,x^2\,\sqrt{b\,x^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^4)^(3/2)),x)
```

```
[Out] -(a + 2*b*x^4)/(2*a^2*x^2*(a + b*x^4)^(1/2))
```

sympy [A] time = 1.34, size = 46, normalized size = 1.10

$$-\frac{1}{2a\sqrt{b}\,x^4\sqrt{\frac{a}{bx^4}+1}}-\frac{\sqrt{b}}{a^2\sqrt{\frac{a}{bx^4}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**4+a)**(3/2),x)
```

```
[Out] -1/(2*a*sqrt(b)*x**4*sqrt(a/(b*x**4) + 1)) - sqrt(b)/(a**2*sqrt(a/(b*x**4)
+ 1))
```

3.860 $\int \frac{1}{x^7\left(a+bx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=68

$$\frac{4b^2x^2}{3a^3\sqrt{a+bx^4}}+\frac{2b}{3a^2x^2\sqrt{a+bx^4}}-\frac{1}{6ax^6\sqrt{a+bx^4}}$$

```
[Out] -1/6/a/x^6/(b*x^4+a)^(1/2)+2/3*b/a^2/x^2/(b*x^4+a)^(1/2)+4/3*b^2*x^2/a^3/(b
*x^4+a)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$\frac{4b^2x^2}{3a^3\sqrt{a+bx^4}}+\frac{2b}{3a^2x^2\sqrt{a+bx^4}}-\frac{1}{6ax^6\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^4)^(3/2)),x]
```

```
[Out] -1/(6*a*x^6*Sqrt[a + b*x^4]) + (2*b)/(3*a^2*x^2*Sqrt[a + b*x^4]) + (4*b^2*x
^2)/(3*a^3*Sqrt[a + b*x^4])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7\left(a+bx^4\right)^{3/2}}\,dx &= -\frac{1}{6ax^6\sqrt{a+bx^4}}-\frac{(4b)\int \frac{1}{x^3\left(a+bx^4\right)^{3/2}}\,dx}{3a}\\ &= -\frac{1}{6ax^6\sqrt{a+bx^4}}+\frac{2b}{3a^2x^2\sqrt{a+bx^4}}+\frac{\left(8b^2\right)\int \frac{x}{\left(a+bx^4\right)^{3/2}}\,dx}{3a^2}\\ &= -\frac{1}{6ax^6\sqrt{a+bx^4}}+\frac{2b}{3a^2x^2\sqrt{a+bx^4}}+\frac{4b^2x^2}{3a^3\sqrt{a+bx^4}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 0.59

$$-\frac{a^2-4abx^4-8b^2x^8}{6a^3x^6\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^4)^(3/2)),x]
```

```
[Out] -1/6*(a^2 - 4*a*b*x^4 - 8*b^2*x^8)/(a^3*x^6*Sqrt[a + b*x^4])
```

fricas [A] time = 0.84, size = 50, normalized size = 0.74

$$\frac{\left(8\,b^2x^8+4\,abx^4-a^2\right)\sqrt{bx^4+a}}{6\left(a^3bx^{10}+a^4x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(8*b^2*x^8 + 4*a*b*x^4 - a^2)*sqrt(b*x^4 + a)/(a^3*b*x^10 + a^4*x^6)
```

giac [B] time = 0.22, size = 115, normalized size = 1.69

$$\frac{b^2x^2}{2\sqrt{bx^4+a}\,a^3}-\frac{3\left(\sqrt{b}\,x^2-\sqrt{bx^4+a}\right)^4b^{\frac{3}{2}}-12\left(\sqrt{b}\,x^2-\sqrt{bx^4+a}\right)^2ab^{\frac{3}{2}}+5\,a^2b^{\frac{3}{2}}}{3\left(\left(\sqrt{b}\,x^2-\sqrt{bx^4+a}\right)^2-a\right)^3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*b^2*x^2/(sqrt(b*x^4 + a)*a^3) - 1/3*(3*(sqrt(b)*x^2 - sqrt(b*x^4 + a))^
4*b^(3/2) - 12*(sqrt(b)*x^2 - sqrt(b*x^4 + a))^2*a*b^(3/2) + 5*a^2*b^(3/2))
/(((sqrt(b)*x^2 - sqrt(b*x^4 + a))^2 - a)^3*a^2)
```

maple [A] time = 0.01, size = 37, normalized size = 0.54

$$-\frac{-8b^2x^8-4ab\,x^4+a^2}{6\sqrt{b\,x^4+a}\;a^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^4+a)^(3/2),x)
```

```
[Out] -1/6*(-8*b^2*x^8-4*a*b*x^4+a^2)/x^6/(b*x^4+a)^(1/2)/a^3
```

maxima [A] time = 1.34, size = 56, normalized size = 0.82

$$\frac{b^2x^2}{2\sqrt{bx^4+a}\,a^3}+\frac{\frac{6\sqrt{bx^4+a}\,b}{x^2}-\frac{\left(bx^4+a\right)^{\frac{3}{2}}}{x^6}}{6\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2/(sqrt(b*x^4 + a)*a^3) + 1/6*(6*sqrt(b*x^4 + a)*b/x^2 - (b*x^4 +
a)^(3/2)/x^6)/a^3
```

mupad [B] time = 1.28, size = 70, normalized size = 1.03

$$-\frac{8\left(b\,x^4+a\right)^2-12\,a\,\left(b\,x^4+a\right)+3\,a^2}{\left(\frac{6\,a^4\,x^2}{b}-\frac{6\,a^3\,x^2\left(b\,x^4+a\right)}{b}\right)\sqrt{b\,x^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^4)^(3/2)),x)
```

```
[Out] -(8*(a + b*x^4)^2 - 12*a*(a + b*x^4) + 3*a^2)/(((6*a^4*x^2)/b - (6*a^3*x^2*
(a + b*x^4))/b)*(a + b*x^4)^(1/2))
```

sympy [**B**] time = 2.66, size = 233, normalized size = 3.43

$$-\frac{a^3 b^{\frac{9}{2}}\sqrt{\frac{a}{bx^4}+1}}{6a^5b^4x^4+12a^4b^5x^8+6a^3b^6x^{12}}+\frac{3a^2b^{\frac{11}{2}}x^4\sqrt{\frac{a}{bx^4}+1}}{6a^5b^4x^4+12a^4b^5x^8+6a^3b^6x^{12}}+\frac{12ab^{\frac{13}{2}}x^8\sqrt{\frac{a}{bx^4}+1}}{6a^5b^4x^4+12a^4b^5x^8+6a^3b^6x^{12}}+\frac{8b^{\frac{15}{2}}}{6a^5b^4x^4+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**4+a)**(3/2),x)
```

```
[Out] -a**3*b**(9/2)*sqrt(a/(b*x**4) + 1)/(6*a**5*b**4*x**4 + 12*a**4*b**5*x**8 +
 6*a**3*b**6*x**12) + 3*a**2*b**(11/2)*x**4*sqrt(a/(b*x**4) + 1)/(6*a**5*b*
*4*x**4 + 12*a**4*b**5*x**8 + 6*a**3*b**6*x**12) + 12*a*b**(13/2)*x**8*sqrt
(a/(b*x**4) + 1)/(6*a**5*b**4*x**4 + 12*a**4*b**5*x**8 + 6*a**3*b**6*x**12)
 + 8*b**(15/2)*x**12*sqrt(a/(b*x**4) + 1)/(6*a**5*b**4*x**4 + 12*a**4*b**5*
x**8 + 6*a**3*b**6*x**12)
```

3.861 $\int \frac{x^{12}}{\left(a+bx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=151

$$\frac{15a^{7/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{28b^{13/4}\sqrt{a+bx^4}}-\frac{15ax\sqrt{a+bx^4}}{14b^3}+\frac{9x^5\sqrt{a+bx^4}}{14b^2}-\frac{x^9}{2b\sqrt{a+bx^4}}$$

```
[Out] -1/2*x^9/b/(b*x^4+a)^(1/2)-15/14*a*x*(b*x^4+a)^(1/2)/b^3+9/14*x^5*(b*x^4+a)
^(1/2)/b^2+15/28*a^(7/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*a
rctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^
(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(1
3/4)/(b*x^4+a)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {288, 321, 220}

$$\frac{15a^{7/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{28b^{13/4}\sqrt{a+bx^4}}+\frac{9x^5\sqrt{a+bx^4}}{14b^2}-\frac{15ax\sqrt{a+bx^4}}{14b^3}-\frac{x^9}{2b\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^12/(a + b*x^4)^(3/2),x]
```

```
[Out] -x^9/(2*b*Sqrt[a + b*x^4]) - (15*a*x*Sqrt[a + b*x^4])/(14*b^3) + (9*x^5*Sqr
t[a + b*x^4])/(14*b^2) + (15*a^(7/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^
4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2]
)/(28*b^(13/4)*Sqrt[a + b*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^{12}}{\left(a+bx^4\right)^{3/2}}\,dx = -\frac{x^9}{2b\sqrt{a+bx^4}} + \frac{9\int \frac{x^8}{\sqrt{a+bx^4}}\,dx}{2b}$$

$$= -\frac{x^9}{2b\sqrt{a+bx^4}} + \frac{9x^5\sqrt{a+bx^4}}{14b^2} - \frac{(45a)\int \frac{x^4}{\sqrt{a+bx^4}}\,dx}{14b^2}$$

$$= -\frac{x^9}{2b\sqrt{a+bx^4}} - \frac{15ax\sqrt{a+bx^4}}{14b^3} + \frac{9x^5\sqrt{a+bx^4}}{14b^2} + \frac{\left(15a^2\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{14b^3}$$

$$= -\frac{x^9}{2b\sqrt{a+bx^4}} - \frac{15ax\sqrt{a+bx^4}}{14b^3} + \frac{9x^5\sqrt{a+bx^4}}{14b^2} + \frac{15a^{7/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\,\text{ta}\right.}{28b^{13/4}\sqrt{a+bx^4}}$$

Mathematica [C] time = 0.04, size = 79, normalized size = 0.52

$$\frac{15a^2x\sqrt{\frac{bx^4}{a}+1}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right) - 15a^2x - 6abx^5 + 2b^2x^9}{14b^3\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12/(a + b*x^4)^(3/2),x]
```

```
[Out] (-15*a^2*x - 6*a*b*x^5 + 2*b^2*x^9 + 15*a^2*x*Sqrt[1 + (b*x^4)/a]*Hypergeom
etric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)])/(14*b^3*Sqrt[a + b*x^4])
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^4+a}\,x^{12}}{b^2x^8+2\,abx^4+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*x^12/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{12}}{\left(bx^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^12/(b*x^4 + a)^(3/2), x)
```

maple [C] time = 0.02, size = 133, normalized size = 0.88

$$\frac{\sqrt{b\,x^4+a}\;x^5}{7b^2} - \frac{a^2x}{2\sqrt{\left(x^4+\frac{a}{b}\right)b}\;b^3} + \frac{15\sqrt{-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\;\sqrt{\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\;a^2\,\text{EllipticF}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\;x, i\right)}{14\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\;\sqrt{b\,x^4+a}\;b^3} - \frac{4\sqrt{b\,x^4+a}\;ax}{7b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(b*x^4+a)^(3/2),x)
```

```
[Out] -1/2/b^3*a^2*x/((x^4+a/b)*b)^(1/2)+1/7*x^5*(b*x^4+a)^(1/2)/b^2-4/7*a*x*(b*x
^4+a)^(1/2)/b^3+15/14*a^2/b^3/(I/a^(1/2)*b^(1/2))^(1/2)*(-I/a^(1/2)*b^(1/2)
*x^2+1)^(1/2)*(I/a^(1/2)*b^(1/2)*x^2+1)^(1/2)/(b*x^4+a)^(1/2)*EllipticF((I/
a^(1/2)*b^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{12}}{\left(bx^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^12/(b*x^4 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{12}}{\left(b\,x^4+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(a + b*x^4)^(3/2),x)
```

```
[Out] int(x^12/(a + b*x^4)^(3/2), x)
```

sympy [C] time = 2.47, size = 37, normalized size = 0.25

$$\frac{x^{13}\Gamma\left(\frac{13}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2}, \frac{13}{4} \\ \frac{17}{4}\end{matrix}\right| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{17}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12/(b*x**4+a)**(3/2),x)
```

```
[Out] x**13*gamma(13/4)*hyper((3/2, 13/4), (17/4,), b*x**4*exp_polar(I*pi)/a)/(4*
a**(3/2)*gamma(17/4))
```

3.862 $\int \frac{x^8}{\left(a+bx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=129

$$-\frac{5a^{3/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{12b^{9/4}\sqrt{a+bx^4}}+\frac{5x\sqrt{a+bx^4}}{6b^2}-\frac{x^5}{2b\sqrt{a+bx^4}}$$

```
[Out] -1/2*x^5/b/(b*x^4+a)^(1/2)+5/6*x*(b*x^4+a)^(1/2)/b^2-5/12*a^(3/4)*(cos(2*ar
ctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*Elliptic
F(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x
^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(9/4)/(b*x^4+a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {288, 321, 220}

$$-\frac{5a^{3/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{12b^{9/4}\sqrt{a+bx^4}}+\frac{5x\sqrt{a+bx^4}}{6b^2}-\frac{x^5}{2b\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^4)^(3/2),x]
```

```
[Out] -x^5/(2*b*Sqrt[a + b*x^4]) + (5*x*Sqrt[a + b*x^4])/(6*b^2) - (5*a^(3/4)*(Sq
rt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[
2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(12*b^(9/4)*Sqrt[a + b*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^8}{\left(a+bx^4\right)^{3/2}}\,dx = -\frac{x^5}{2b\sqrt{a+bx^4}} + \frac{5\int \frac{x^4}{\sqrt{a+bx^4}}\,dx}{2b}$$

$$= -\frac{x^5}{2b\sqrt{a+bx^4}} + \frac{5x\sqrt{a+bx^4}}{6b^2} - \frac{(5a)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{6b^2}$$

$$= -\frac{x^5}{2b\sqrt{a+bx^4}} + \frac{5x\sqrt{a+bx^4}}{6b^2} - \frac{5a^{3/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{12b^{9/4}\sqrt{a+bx^4}}$$

Mathematica [C] time = 0.04, size = 66, normalized size = 0.51

$$\frac{-5ax\sqrt{\frac{bx^4}{a}+1}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right)+5ax+2bx^5}{6b^2\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^4)^(3/2),x]
```

```
[Out] (5*a*x + 2*b*x^5 - 5*a*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/
4, -((b*x^4)/a)])/(6*b^2*Sqrt[a + b*x^4])
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^4+a}\,x^8}{b^2x^8+2\,abx^4+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*x^8/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(bx^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^8/(b*x^4 + a)^(3/2), x)
```

maple [C] time = 0.01, size = 111, normalized size = 0.86

$$\frac{ax}{2\sqrt{\left(x^4+\frac{a}{b}\right)b}\;b^2} - \frac{5\sqrt{-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\;\sqrt{\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\;a\,\text{EllipticF}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\;x, i\right)}{6\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\;\sqrt{b\,x^4+a}\;b^2} + \frac{\sqrt{b\,x^4+a}\;x}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^4+a)^(3/2),x)
```

```
[Out] 1/2/b^2*a*x/((x^4+a/b)*b)^(1/2)+1/3*x*(b*x^4+a)^(1/2)/b^2-5/6*a/b^2/(I/a^(1
/2)*b^(1/2))^(1/2)*(-I/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*b^(1/2)*x^2+
1)^(1/2)/(b*x^4+a)^(1/2)*EllipticF((I/a^(1/2)*b^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(bx^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^8/(b*x^4 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^8}{\left(b\,x^4+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^4)^(3/2),x)
```

```
[Out] int(x^8/(a + b*x^4)^(3/2), x)
```

sympy [C] time = 2.21, size = 37, normalized size = 0.29

$$\frac{x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2}, \frac{9}{4} \\ \frac{13}{4}\end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**4+a)**(3/2),x)
```

```
[Out] x**9*gamma(9/4)*hyper((3/2, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**
(3/2)*gamma(13/4))
```

3.863 $\int \frac{x^4}{\left(a+bx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=108

$$\frac{\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\,b^{5/4}\sqrt{a+bx^4}}-\frac{x}{2b\sqrt{a+bx^4}}$$

```
[Out] -1/2*x/b/(b*x^4+a)^(1/2)+1/4*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos
(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/
2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/
a^(1/4)/b^(5/4)/(b*x^4+a)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {288, 220}

$$\frac{\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\,b^{5/4}\sqrt{a+bx^4}}-\frac{x}{2b\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^4)^(3/2),x]
```

```
[Out] -x/(2*b*Sqrt[a + b*x^4]) + ((Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[
a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1
/4)*b^(5/4)*Sqrt[a + b*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^4\right)^{3/2}}\,dx = -\frac{x}{2b\sqrt{a+bx^4}}+\frac{\int \frac{1}{\sqrt{a+bx^4}}\,dx}{2b}$$

$$= -\frac{x}{2b\sqrt{a+bx^4}}+\frac{\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\,b^{5/4}\sqrt{a+bx^4}}$$

Mathematica [C] time = 0.02, size = 55, normalized size = 0.51

$$\frac{x\left(\sqrt{\frac{bx^4}{a}+1}\ {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right)-1\right)}{2b\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^4)^(3/2),x]
```

```
[Out] (x*(-1 + Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)]
))/(2*b*Sqrt[a + b*x^4])
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^4+a}\,x^4}{b^2x^8+2\,abx^4+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*x^4/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/(b*x^4 + a)^(3/2), x)
```

maple [C] time = 0.01, size = 94, normalized size = 0.87

$$-\frac{x}{2\sqrt{\left(x^4+\frac{a}{b}\right)b}\ b}+\frac{\sqrt{-\frac{i\sqrt{b}\ x^2}{\sqrt{a}}+1}\ \sqrt{\frac{i\sqrt{b}\ x^2}{\sqrt{a}}+1}\ \text{EllipticF}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\ x,i\right)}{2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\ \sqrt{b\,x^4+a}\ b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^4+a)^(3/2),x)
```

```
[Out] -1/2/b*x/((x^4+a/b)*b)^(1/2)+1/2/b/(I/a^(1/2)*b^(1/2))^(1/2)*(-I/a^(1/2)*b^
(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*b^(1/2)*x^2+1)^(1/2)/(b*x^4+a)^(1/2)*Elliptic
F((I/a^(1/2)*b^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(b*x^4 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\left(b\,x^4+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^4)^(3/2),x)
```

```
[Out] int(x^4/(a + b*x^4)^(3/2), x)
```

sympy [C] time = 1.99, size = 37, normalized size = 0.34

$$\frac{x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\,\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**4+a)**(3/2),x)
```

```
[Out] x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
3/2)*gamma(9/4))
```

3.864 $\int \frac{1}{\left(a+bx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=108

$$\frac{\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{b}\,\sqrt{a+bx^4}}+\frac{x}{2a\sqrt{a+bx^4}}$$

```
[Out] 1/2*x/a/(b*x^4+a)^(1/2)+1/4*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(
2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2
*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a
^(5/4)/b^(1/4)/(b*x^4+a)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {199, 220}

$$\frac{\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{b}\,\sqrt{a+bx^4}}+\frac{x}{2a\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(-3/2),x]
```

```
[Out] x/(2*a*Sqrt[a + b*x^4]) + ((Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a
] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/
4)*b^(1/4)*Sqrt[a + b*x^4])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)^{3/2}}\,dx = \frac{x}{2a\sqrt{a+bx^4}}+\frac{\int \frac{1}{\sqrt{a+bx^4}}\,dx}{2a}$$

$$= \frac{x}{2a\sqrt{a+bx^4}}+\frac{\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{b}\,\sqrt{a+bx^4}}$$

Mathematica [C] time = 0.02, size = 55, normalized size = 0.51

$$\frac{x\sqrt{\frac{bx^4}{a}+1}\ {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right)+x}{2a\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(-3/2),x]
```

```
[Out] (x + x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)])/
(2*a*Sqrt[a + b*x^4])
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^4+a}}{b^2x^8+2\,abx^4+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(-3/2), x)
```

maple [C] time = 0.01, size = 94, normalized size = 0.87

$$\frac{x}{2\sqrt{\left(x^4+\frac{a}{b}\right)b}\ a}+\frac{\sqrt{-\frac{i\sqrt{b}\ x^2}{\sqrt{a}}+1}\ \sqrt{\frac{i\sqrt{b}\ x^2}{\sqrt{a}}+1}\ \text{EllipticF}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\ x,i\right)}{2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\ \sqrt{b\,x^4+a}\ a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(3/2),x)
```

```
[Out] 1/2/a*x/((x^4+a/b)*b)^(1/2)+1/2/a/(I/a^(1/2)*b^(1/2))^(1/2)*(-I/a^(1/2)*b^(
1/2)*x^2+1)^(1/2)*(I/a^(1/2)*b^(1/2)*x^2+1)^(1/2)/(b*x^4+a)^(1/2)*EllipticF
((I/a^(1/2)*b^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(-3/2), x)
```

mupad [B] time = 1.11, size = 37, normalized size = 0.34

$$\frac{x\left(\frac{b\,x^4}{a}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{2};\ \frac{5}{4};\ -\frac{b\,x^4}{a}\right)}{\left(b\,x^4+a\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^4)^(3/2),x)
```

```
[Out] (x*((b*x^4)/a + 1)^(3/2)*hypergeom([1/4, 3/2], 5/4, -(b*x^4)/a))/(a + b*x^4
)^(3/2)
```

sympy [C] time = 0.90, size = 36, normalized size = 0.33

$$\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(3/2),x)
```

```
[Out] x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2
)*gamma(5/4))
```

3.865 $\int \frac{1}{x^4\left(a+bx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=131

$$-\frac{5b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{12a^{9/4}\sqrt{a+bx^4}}-\frac{5\sqrt{a+bx^4}}{6a^2x^3}+\frac{1}{2ax^3\sqrt{a+bx^4}}$$

```
[Out] 1/2/a/x^3/(b*x^4+a)^(1/2)-5/6*(b*x^4+a)^(1/2)/a^2/x^3-5/12*b^(3/4)*(cos(2*a
rctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*Ellipti
cF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*
x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(9/4)/(b*x^4+a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {290, 325, 220}

$$-\frac{5b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{12a^{9/4}\sqrt{a+bx^4}}-\frac{5\sqrt{a+bx^4}}{6a^2x^3}+\frac{1}{2ax^3\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^4)^(3/2)),x]
```

```
[Out] 1/(2*a*x^3*Sqrt[a + b*x^4]) - (5*Sqrt[a + b*x^4])/(6*a^2*x^3) - (5*b^(3/4)*
(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellipti
cF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(12*a^(9/4)*Sqrt[a + b*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^4\right)^{3/2}}\,dx = \frac{1}{2ax^3\sqrt{a+bx^4}} + \frac{5\int \frac{1}{x^4\sqrt{a+bx^4}}\,dx}{2a}$$

$$= \frac{1}{2ax^3\sqrt{a+bx^4}} - \frac{5\sqrt{a+bx^4}}{6a^2x^3} - \frac{(5b)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{6a^2}$$

$$= \frac{1}{2ax^3\sqrt{a+bx^4}} - \frac{5\sqrt{a+bx^4}}{6a^2x^3} - \frac{5b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{12a^{9/4}\sqrt{a+bx^4}}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.41

$$-\frac{\sqrt{\frac{bx^4}{a}+1}\;{}_2F_1\left(-\frac{3}{4},\frac{3}{2};\frac{1}{4};-\frac{bx^4}{a}\right)}{3ax^3\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^4)^(3/2)),x]
```

```
[Out] -1/3*(Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-3/4, 3/2, 1/4, -((b*x^4)/a)])/
(a*x^3*Sqrt[a + b*x^4])
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^4+a}}{b^2x^{12}+2\,abx^8+a^2x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)/(b^2*x^12 + 2*a*b*x^8 + a^2*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(3/2)*x^4), x)
```

maple [C] time = 0.02, size = 113, normalized size = 0.86

$$-\frac{bx}{2\sqrt{\left(x^4+\frac{a}{b}\right)b}\;a^2} - \frac{5\sqrt{-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\;\sqrt{\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\;b\,\text{EllipticF}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\;x, i\right)}{6\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\;\sqrt{b\,x^4+a}\;a^2} - \frac{\sqrt{b\,x^4+a}}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^4+a)^(3/2),x)
```

```
[Out] -1/3*(b*x^4+a)^(1/2)/a^2/x^3-1/2*b/a^2*x/((x^4+a/b)*b)^(1/2)-5/6/a^2*b/(I/a
^(1/2)*b^(1/2))^(1/2)*(-I/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*b^(1/2)*x
^2+1)^(1/2)/(b*x^4+a)^(1/2)*EllipticF((I/a^(1/2)*b^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(3/2)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\left(b\,x^4+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^4)^(3/2)),x)
```

```
[Out] int(1/(x^4*(a + b*x^4)^(3/2)), x)
```

sympy [C] time = 1.91, size = 41, normalized size = 0.31

$$\frac{\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{4},\frac{3}{2}\\ \frac{1}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}x^3\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**4+a)**(3/2),x)
```

```
[Out] gamma(-3/4)*hyper((-3/4, 3/2), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2
)*x**3*gamma(1/4))
```

3.866 $\int \frac{1}{x^8\left(a+bx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=153

$$\frac{15b^{7/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{28a^{13/4}\sqrt{a+bx^4}}+\frac{15b\sqrt{a+bx^4}}{14a^3x^3}-\frac{9\sqrt{a+bx^4}}{14a^2x^7}+\frac{1}{2ax^7\sqrt{a+bx^4}}$$

```
[Out] 1/2/a/x^7/(b*x^4+a)^(1/2)-9/14*(b*x^4+a)^(1/2)/a^2/x^7+15/14*b*(b*x^4+a)^(1
/2)/a^3/x^3+15/28*b^(7/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*
arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2
^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(
13/4)/(b*x^4+a)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {290, 325, 220}

$$\frac{15b^{7/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{28a^{13/4}\sqrt{a+bx^4}}+\frac{15b\sqrt{a+bx^4}}{14a^3x^3}-\frac{9\sqrt{a+bx^4}}{14a^2x^7}+\frac{1}{2ax^7\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(a + b*x^4)^(3/2)),x]
```

```
[Out] 1/(2*a*x^7*Sqrt[a + b*x^4]) - (9*Sqrt[a + b*x^4])/(14*a^2*x^7) + (15*b*Sqrt
[a + b*x^4])/(14*a^3*x^3) + (15*b^(7/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b
*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1
/2])/(28*a^(13/4)*Sqrt[a + b*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^8\left(a+bx^4\right)^{3/2}}\,dx = \frac{1}{2ax^7\sqrt{a+bx^4}} + \frac{9\int \frac{1}{x^8\sqrt{a+bx^4}}\,dx}{2a}$$

$$= \frac{1}{2ax^7\sqrt{a+bx^4}} - \frac{9\sqrt{a+bx^4}}{14a^2x^7} - \frac{(45b)\int \frac{1}{x^4\sqrt{a+bx^4}}\,dx}{14a^2}$$

$$= \frac{1}{2ax^7\sqrt{a+bx^4}} - \frac{9\sqrt{a+bx^4}}{14a^2x^7} + \frac{15b\sqrt{a+bx^4}}{14a^3x^3} + \frac{\left(15b^2\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{14a^3}$$

$$= \frac{1}{2ax^7\sqrt{a+bx^4}} - \frac{9\sqrt{a+bx^4}}{14a^2x^7} + \frac{15b\sqrt{a+bx^4}}{14a^3x^3} + \frac{15b^{7/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(\right.}{28a^{13/4}\sqrt{a+bx^4}}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.35

$$-\frac{\sqrt{\frac{bx^4}{a}+1}\;{}_2F_1\left(-\frac{7}{4},\frac{3}{2};-\frac{3}{4};-\frac{bx^4}{a}\right)}{7ax^7\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(a + b*x^4)^(3/2)),x]
```

```
[Out] -1/7*(Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-7/4, 3/2, -3/4, -((b*x^4)/a)])
/(a*x^7*Sqrt[a + b*x^4])
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^4+a}}{b^2x^{16}+2\,abx^{12}+a^2x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)/(b^2*x^16 + 2*a*b*x^12 + a^2*x^8), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{3}{2}}x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(3/2)*x^8), x)
```

maple [C] time = 0.02, size = 135, normalized size = 0.88

$$\frac{b^2x}{2\sqrt{\left(x^4+\frac{a}{b}\right)b}\;a^3} + \frac{15\sqrt{-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\;\sqrt{\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\;b^2\,\text{EllipticF}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\;x, i\right)}{14\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\;\sqrt{b\,x^4+a}\;a^3} + \frac{4\sqrt{b\,x^4+a}\;b}{7a^3x^3} - \frac{\sqrt{b\,x^4+a}}{7a^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(b*x^4+a)^(3/2),x)
```

```
[Out] -1/7*(b*x^4+a)^(1/2)/a^2/x^7+4/7*b*(b*x^4+a)^(1/2)/a^3/x^3+1/2*b^2/a^3*x/((
x^4+a/b)*b)^(1/2)+15/14/a^3*b^2/(I/a^(1/2)*b^(1/2))^(1/2)*(-I/a^(1/2)*b^(1/
2)*x^2+1)^(1/2)*(I/a^(1/2)*b^(1/2)*x^2+1)^(1/2)/(b*x^4+a)^(1/2)*EllipticF((
I/a^(1/2)*b^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{3}{2}}x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(3/2)*x^8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^8\left(b\,x^4+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(a + b*x^4)^(3/2)),x)
```

```
[Out] int(1/(x^8*(a + b*x^4)^(3/2)), x)
```

sympy [C] time = 1.56, size = 44, normalized size = 0.29

$$\frac{\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4}, \frac{3}{2} \\ -\frac{3}{4}\end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}x^7\Gamma\left(-\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(b*x**4+a)**(3/2),x)
```

```
[Out] gamma(-7/4)*hyper((-7/4, 3/2), (-3/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/
2)*x**7*gamma(-3/4))
```

3.867 $\int \frac{x^{14}}{\left(a+bx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=282

$$\frac{77a^{9/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{60b^{15/4}\sqrt{a+bx^4}}-\frac{77a^{9/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\right.}{30b^{15/4}\sqrt{a+bx^4}}$$

```
[Out] -1/2*x^11/b/(b*x^4+a)^(1/2)-77/90*a*x^3*(b*x^4+a)^(1/2)/b^3+11/18*x^7*(b*x^
4+a)^(1/2)/b^2+77/30*a^2*x*(b*x^4+a)^(1/2)/b^(7/2)/(a^(1/2)+x^2*b^(1/2))-77
/30*a^(9/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)
*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/
2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(15/4)/(b*x^4+a
)^(1/2)+77/60*a^(9/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arct
an(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/
2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(15/4
)/(b*x^4+a)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {288, 321, 305, 220, 1196}

$$\frac{77a^2x\sqrt{a+bx^4}}{30b^{7/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{77a^{9/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{60b^{15/4}\sqrt{a+bx^4}}-\frac{77a^{9/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b}{(\sqrt{a}+\sqrt{}}}}{30b^{15/4}\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/(a + b*x^4)^(3/2),x]
```

```
[Out] -x^11/(2*b*Sqrt[a + b*x^4]) - (77*a*x^3*Sqrt[a + b*x^4])/(90*b^3) + (11*x^7
*Sqrt[a + b*x^4])/(18*b^2) + (77*a^2*x*Sqrt[a + b*x^4])/(30*b^(7/2)*(Sqrt[a
] + Sqrt[b]*x^2)) - (77*a^(9/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(S
qrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(30
*b^(15/4)*Sqrt[a + b*x^4]) + (77*a^(9/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a +
b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)],
1/2])/(60*b^(15/4)*Sqrt[a + b*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^{14}}{\left(a+bx^4\right)^{3/2}}\,dx = -\frac{x^{11}}{2b\sqrt{a+bx^4}} + \frac{11\int \frac{x^{10}}{\sqrt{a+bx^4}}\,dx}{2b}$$

$$= -\frac{x^{11}}{2b\sqrt{a+bx^4}} + \frac{11x^7\sqrt{a+bx^4}}{18b^2} - \frac{(77a)\int \frac{x^6}{\sqrt{a+bx^4}}\,dx}{18b^2}$$

$$= -\frac{x^{11}}{2b\sqrt{a+bx^4}} - \frac{77ax^3\sqrt{a+bx^4}}{90b^3} + \frac{11x^7\sqrt{a+bx^4}}{18b^2} + \frac{\left(77a^2\right)\int \frac{x^2}{\sqrt{a+bx^4}}\,dx}{30b^3}$$

$$= -\frac{x^{11}}{2b\sqrt{a+bx^4}} - \frac{77ax^3\sqrt{a+bx^4}}{90b^3} + \frac{11x^7\sqrt{a+bx^4}}{18b^2} + \frac{\left(77a^{5/2}\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{30b^{7/2}} - \frac{\left(77a^{5/2}\right)\int \frac{1-\frac{\sqrt{}}{\sqrt{a+}}}{\sqrt{a+}}}{30b^{7/2}}$$

$$= -\frac{x^{11}}{2b\sqrt{a+bx^4}} - \frac{77ax^3\sqrt{a+bx^4}}{90b^3} + \frac{11x^7\sqrt{a+bx^4}}{18b^2} + \frac{77a^2x\sqrt{a+bx^4}}{30b^{7/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)} - \frac{77a^{9/4}\left(\sqrt{a}+\sqrt{}\right.}{}$$

Mathematica [C] time = 0.03, size = 80, normalized size = 0.28

$$\frac{x^3\left(-77a^2\sqrt{\frac{bx^4}{a}+1}\;{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{bx^4}{a}\right)+77a^2-11abx^4+5b^2x^8\right)}{45b^3\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/(a + b*x^4)^(3/2),x]
```

```
[Out] (x^3*(77*a^2 - 11*a*b*x^4 + 5*b^2*x^8 - 77*a^2*Sqrt[1 + (b*x^4)/a]*Hypergeo
metric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)]))/(45*b^3*Sqrt[a + b*x^4])
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^4+a}\,x^{14}}{b^2x^8+2\,abx^4+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*x^14/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(bx^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^14/(b*x^4 + a)^(3/2), x)
```

maple [C] time = 0.01, size = 157, normalized size = 0.56

$$\frac{\sqrt{b\,x^4+a}\;x^7}{9b^2}-\frac{a^2x^3}{2\sqrt{\left(x^4+\frac{a}{b}\right)b}\;b^3}-\frac{16\sqrt{b\,x^4+a}\;a\,x^3}{45b^3}+\frac{77i\sqrt{-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\;\sqrt{\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\;\left(-\text{EllipticE}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\;x,i\right)+\right.}{30\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\;\sqrt{b\,x^4+a}\;b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(b*x^4+a)^(3/2),x)
```

```
[Out] -1/2/b^3*a^2*x^3/((x^4+a/b)*b)^(1/2)+1/9*x^7*(b*x^4+a)^(1/2)/b^2-16/45*a*x^
3*(b*x^4+a)^(1/2)/b^3+77/30*I*a^(5/2)/b^(7/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(-I
/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*b^(1/2)*x^2+1)^(1/2)/(b*x^4+a)^(1/
2)*(EllipticF((I/a^(1/2)*b^(1/2))^(1/2)*x,I)-EllipticE((I/a^(1/2)*b^(1/2))^
(1/2)*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(bx^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^14/(b*x^4 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{14}}{\left(b\,x^4+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(a + b*x^4)^(3/2),x)
```

```
[Out] int(x^14/(a + b*x^4)^(3/2), x)
```

sympy [C] time = 1.88, size = 37, normalized size = 0.13

$$\frac{x^{15}\Gamma\left(\frac{15}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{15}{4}\\ \frac{19}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{19}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(b*x**4+a)**(3/2),x)
```

```
[Out] x**15*gamma(15/4)*hyper((3/2, 15/4), (19/4,), b*x**4*exp_polar(I*pi)/a)/(4*
a**(3/2)*gamma(19/4))
```

3.868 $\int \frac{x^{10}}{\left(a+bx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=258

$$-\frac{21a^{5/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{20b^{11/4}\sqrt{a+bx^4}}+\frac{21a^{5/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\right.}{10b^{11/4}\sqrt{a+bx^4}}$$

```
[Out] -1/2*x^7/b/(b*x^4+a)^(1/2)+7/10*x^3*(b*x^4+a)^(1/2)/b^2-21/10*a*x*(b*x^4+a)
^(1/2)/b^(5/2)/(a^(1/2)+x^2*b^(1/2))+21/10*a^(5/4)*(cos(2*arctan(b^(1/4)*x/
a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(
b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+
x^2*b^(1/2))^2)^(1/2)/b^(11/4)/(b*x^4+a)^(1/2)-21/20*a^(5/4)*(cos(2*arctan(
b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin
(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)
/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(11/4)/(b*x^4+a)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {288, 321, 305, 220, 1196}

$$-\frac{21a^{5/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{20b^{11/4}\sqrt{a+bx^4}}+\frac{21a^{5/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\right.}{10b^{11/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x^4)^(3/2),x]
```

```
[Out] -x^7/(2*b*Sqrt[a + b*x^4]) + (7*x^3*Sqrt[a + b*x^4])/(10*b^2) - (21*a*x*Sqr
t[a + b*x^4])/(10*b^(5/2)*(Sqrt[a] + Sqrt[b]*x^2)) + (21*a^(5/4)*(Sqrt[a] +
 Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTa
n[(b^(1/4)*x)/a^(1/4)], 1/2])/(10*b^(11/4)*Sqrt[a + b*x^4]) - (21*a^(5/4)*(
Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Elliptic
F[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(20*b^(11/4)*Sqrt[a + b*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^{10}}{\left(a+bx^4\right)^{3/2}}\,dx = -\frac{x^7}{2b\sqrt{a+bx^4}} + \frac{7\int \frac{x^6}{\sqrt{a+bx^4}}\,dx}{2b}$$

$$= -\frac{x^7}{2b\sqrt{a+bx^4}} + \frac{7x^3\sqrt{a+bx^4}}{10b^2} - \frac{(21a)\int \frac{x^2}{\sqrt{a+bx^4}}\,dx}{10b^2}$$

$$= -\frac{x^7}{2b\sqrt{a+bx^4}} + \frac{7x^3\sqrt{a+bx^4}}{10b^2} - \frac{\left(21a^{3/2}\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{10b^{5/2}} + \frac{\left(21a^{3/2}\right)\int \frac{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{10b^{5/2}}$$

$$= -\frac{x^7}{2b\sqrt{a+bx^4}} + \frac{7x^3\sqrt{a+bx^4}}{10b^2} - \frac{21ax\sqrt{a+bx^4}}{10b^{5/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)} + \frac{21a^{5/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}}{10b^{11/4}\sqrt{a+bx}}$$

Mathematica [C] time = 0.03, size = 66, normalized size = 0.26

$$\frac{x^3\left(7a\sqrt{\frac{bx^4}{a}+1}\;{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{bx^4}{a}\right)-7a+bx^4\right)}{5b^2\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x^4)^(3/2),x]
```

```
[Out] (x^3*(-7*a + b*x^4 + 7*a*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/
4, -((b*x^4)/a)]))/(5*b^2*Sqrt[a + b*x^4])
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^4+a}\,x^{10}}{b^2x^8+2\,abx^4+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*x^10/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\left(bx^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^10/(b*x^4 + a)^(3/2), x)
```

maple [C] time = 0.01, size = 137, normalized size = 0.53

$$\frac{a\,x^3}{2\sqrt{\left(x^4+\frac{a}{b}\right)b}\;b^2}+\frac{\sqrt{b\,x^4+a}\;x^3}{5b^2}-\frac{21i\sqrt{-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\;\sqrt{\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\;\left(-\text{EllipticE}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\;x,i\right)+\text{EllipticF}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\right.\right.}{10\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\;\sqrt{b\,x^4+a}\;b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x^4+a)^(3/2),x)
```

```
[Out] 1/2/b^2*a*x^3/((x^4+a/b)*b)^(1/2)+1/5*x^3*(b*x^4+a)^(1/2)/b^2-21/10*I*a^(3/
2)/b^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(-I/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(I/a^
(1/2)*b^(1/2)*x^2+1)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF((I/a^(1/2)*b^(1/2))^(
1/2)*x,I)-EllipticE((I/a^(1/2)*b^(1/2))^(1/2)*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\left(bx^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^10/(b*x^4 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{10}}{\left(b\,x^4+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a + b*x^4)^(3/2),x)
```

```
[Out] int(x^10/(a + b*x^4)^(3/2), x)
```

sympy [C] time = 1.63, size = 37, normalized size = 0.14

$$\frac{x^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b*x**4+a)**(3/2),x)
```

```
[Out] x**11*gamma(11/4)*hyper((3/2, 11/4), (15/4,), b*x**4*exp_polar(I*pi)/a)/(4*
a**(3/2)*gamma(15/4))
```

3.869 $\int \frac{x^6}{\left(a+bx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=236

$$\frac{3x\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{3\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4b^{7/4}\sqrt{a+bx^4}}-\frac{3\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}E}{2b^{7/4}\sqrt{a+bx^4}}$$

```
[Out] -1/2*x^3/b/(b*x^4+a)^(1/2)+3/2*x*(b*x^4+a)^(1/2)/b^(3/2)/(a^(1/2)+x^2*b^(1/
2))-3/2*a^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(
1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a
^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^
4+a)^(1/2)+3/4*a^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arc
tan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1
/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/b^(7/4
)/(b*x^4+a)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {288, 305, 220, 1196}

$$\frac{3x\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{3\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4b^{7/4}\sqrt{a+bx^4}}-\frac{3\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}E}{2b^{7/4}\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^4)^(3/2),x]
```

```
[Out] -x^3/(2*b*Sqrt[a + b*x^4]) + (3*x*Sqrt[a + b*x^4])/(2*b^(3/2)*(Sqrt[a] + Sq
rt[b]*x^2)) - (3*a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a]
+ Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(7/4)
*Sqrt[a + b*x^4]) + (3*a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sq
rt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*b
^(7/4)*Sqrt[a + b*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^6}{\left(a+bx^4\right)^{3/2}}\,dx = -\frac{x^3}{2b\sqrt{a+bx^4}} + \frac{3\int \frac{x^2}{\sqrt{a+bx^4}}\,dx}{2b}$$

$$= -\frac{x^3}{2b\sqrt{a+bx^4}} + \frac{\left(3\sqrt{a}\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{2b^{3/2}} - \frac{\left(3\sqrt{a}\right)\int \frac{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{2b^{3/2}}$$

$$= -\frac{x^3}{2b\sqrt{a+bx^4}} + \frac{3x\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)} - \frac{3\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right.\right.}{2b^{7/4}\sqrt{a+bx^4}}$$

Mathematica [C] time = 0.02, size = 57, normalized size = 0.24

$$\frac{x^3 - x^3\sqrt{\frac{bx^4}{a}+1}\;{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{bx^4}{a}\right)}{b\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^4)^(3/2),x]
```

```
[Out] (x^3 - x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^4)/a
)])/(b*Sqrt[a + b*x^4])
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^4+a}\,x^6}{b^2x^8+2\,abx^4+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*x^6/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(bx^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/(b*x^4 + a)^(3/2), x)
```

maple [C] time = 0.01, size = 119, normalized size = 0.50

$$-\frac{x^3}{2\sqrt{\left(x^4+\frac{a}{b}\right)b}\,b}+\frac{3i\sqrt{-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\,\left(-\text{EllipticE}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x,i\right)+\text{EllipticF}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x,i\right)\right)\sqrt{a}}{2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^4+a)^(3/2),x)
```

```
[Out] -1/2/b*x^3/((x^4+a/b)*b)^(1/2)+3/2*I/b^(3/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1
/2)*(-I/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*b^(1/2)*x^2+1)^(1/2)/(b*x^4
+a)^(1/2)*(EllipticF((I/a^(1/2)*b^(1/2))^(1/2)*x,I)-EllipticE((I/a^(1/2)*b^
(1/2))^(1/2)*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(bx^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(b*x^4 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{\left(b\,x^4+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^4)^(3/2),x)
```

```
[Out] int(x^6/(a + b*x^4)^(3/2), x)
```

sympy [C] time = 1.22, size = 37, normalized size = 0.16

$$\frac{x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**4+a)**(3/2),x)
```

```
[Out] x**7*gamma(7/4)*hyper((3/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**
(3/2)*gamma(11/4))
```

3.870 $\int \frac{x^2}{\left(a+bx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=239

$$-\frac{\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{3/4}b^{3/4}\sqrt{a+bx^4}}+\frac{\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}}+\frac{x}{2a\sqrt{a}}$$

```
[Out] 1/2*x^3/a/(b*x^4+a)^(1/2)-1/2*x*(b*x^4+a)^(1/2)/a/b^(1/2)/(a^(1/2)+x^2*b^(1
/2))+1/2*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/
a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+
x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)/b^(3/4)/(b*x
^4+a)^(1/2)-1/4*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(
1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a
^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(3/4)/b^(3/
4)/(b*x^4+a)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {290, 305, 220, 1196}

$$-\frac{\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{3/4}b^{3/4}\sqrt{a+bx^4}}+\frac{\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}}+\frac{x}{2a\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^4)^(3/2),x]
```

```
[Out] x^3/(2*a*Sqrt[a + b*x^4]) - (x*Sqrt[a + b*x^4])/(2*a*Sqrt[b]*(Sqrt[a] + Sqr
t[b]*x^2)) + ((Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x
^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*b^(3/4)*Sq
rt[a + b*x^4]) - ((Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[
b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(3/4)*b^(3/4
)*Sqrt[a + b*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^4\right)^{3/2}}\,dx = \frac{x^3}{2a\sqrt{a+bx^4}} - \frac{\int \frac{x^2}{\sqrt{a+bx^4}}\,dx}{2a}$$

$$= \frac{x^3}{2a\sqrt{a+bx^4}} - \frac{\int \frac{1}{\sqrt{a+bx^4}}\,dx}{2\sqrt{a}\,\sqrt{b}} + \frac{\int \frac{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{2\sqrt{a}\,\sqrt{b}}$$

$$= \frac{x^3}{2a\sqrt{a+bx^4}} - \frac{x\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)} + \frac{\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}} - -$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.23

$$\frac{x^3\sqrt{\frac{bx^4}{a}+1}\;{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{bx^4}{a}\right)}{3a\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^4)^(3/2),x]
```

```
[Out] (x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)])/(3
*a*Sqrt[a + b*x^4])
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^4+a}\,x^2}{b^2x^8+2\,abx^4+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)*x^2/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*x^4 + a)^(3/2), x)
```

maple [C] time = 0.01, size = 119, normalized size = 0.50

$$\frac{x^3}{2\sqrt{\left(x^4+\frac{a}{b}\right)b}\,a}-\frac{i\sqrt{-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\left(-\text{EllipticE}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x,i\right)+\text{EllipticF}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x,i\right)\right)}{2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^4+a)^(3/2),x)
```

```
[Out] 1/2/a*x^3/((x^4+a/b)*b)^(1/2)-1/2*I/a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(-I/a
^(1/2)*b^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*b^(1/2)*x^2+1)^(1/2)/(b*x^4+a)^(1/2)
/b^(1/2)*(EllipticF((I/a^(1/2)*b^(1/2))^(1/2)*x,I)-EllipticE((I/a^(1/2)*b^(
1/2))^(1/2)*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^4+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*x^4 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(b\,x^4+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^4)^(3/2),x)
```

```
[Out] int(x^2/(a + b*x^4)^(3/2), x)
```

sympy [C] time = 0.99, size = 37, normalized size = 0.15

$$\frac{x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**4+a)**(3/2),x)
```

```
[Out] x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
3/2)*gamma(7/4))
```

3.871 $\int \frac{1}{x^2\left(a+bx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=260

$$\frac{3\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{7/4}\sqrt{a+bx^4}}-\frac{3\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{7/4}\sqrt{a+bx^4}}-\frac{3}{}$$

```
[Out] 1/2/a/x/(b*x^4+a)^(1/2)-3/2*(b*x^4+a)^(1/2)/a^2/x+3/2*x*b^(1/2)*(b*x^4+a)^(
1/2)/a^2/(a^(1/2)+x^2*b^(1/2))-3/2*b^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4))
)^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*
x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1
/2))^2)^(1/2)/a^(7/4)/(b*x^4+a)^(1/2)+3/4*b^(1/4)*(cos(2*arctan(b^(1/4)*x/a
^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b
^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x
^2*b^(1/2))^2)^(1/2)/a^(7/4)/(b*x^4+a)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {290, 325, 305, 220, 1196}

$$\frac{3\sqrt{b}\,x\sqrt{a+bx^4}}{2a^2\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{3\sqrt{a+bx^4}}{2a^2x}+\frac{3\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{7/4}\sqrt{a+bx^4}}-\frac{3\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{}{(}}}{2a^{7/}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^4)^(3/2)),x]
```

```
[Out] 1/(2*a*x*Sqrt[a + b*x^4]) - (3*Sqrt[a + b*x^4])/(2*a^2*x) + (3*Sqrt[b]*x*Sq
rt[a + b*x^4])/(2*a^2*(Sqrt[a] + Sqrt[b]*x^2)) - (3*b^(1/4)*(Sqrt[a] + Sqrt
[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^
(1/4)*x)/a^(1/4)], 1/2])/(2*a^(7/4)*Sqrt[a + b*x^4]) + (3*b^(1/4)*(Sqrt[a]
 + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcT
an[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(7/4)*Sqrt[a + b*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^4\right)^{3/2}}\,dx = \frac{1}{2ax\sqrt{a+bx^4}} + \frac{3\int \frac{1}{x^2\sqrt{a+bx^4}}\,dx}{2a}$$

$$= \frac{1}{2ax\sqrt{a+bx^4}} - \frac{3\sqrt{a+bx^4}}{2a^2x} + \frac{(3b)\int \frac{x^2}{\sqrt{a+bx^4}}\,dx}{2a^2}$$

$$= \frac{1}{2ax\sqrt{a+bx^4}} - \frac{3\sqrt{a+bx^4}}{2a^2x} + \frac{\left(3\sqrt{b}\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{2a^{3/2}} - \frac{\left(3\sqrt{b}\right)\int \frac{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{2a^{3/2}}$$

$$= \frac{1}{2ax\sqrt{a+bx^4}} - \frac{3\sqrt{a+bx^4}}{2a^2x} + \frac{3\sqrt{b}\,x\sqrt{a+bx^4}}{2a^2\left(\sqrt{a}+\sqrt{b}\,x^2\right)} - \frac{3\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E}{2a^{7/4}\sqrt{a+bx^4}}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.20

$$-\frac{\sqrt{\frac{bx^4}{a}+1}\;{}_2F_1\left(-\frac{1}{4},\frac{3}{2};\frac{3}{4};-\frac{bx^4}{a}\right)}{ax\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^4)^(3/2)),x]
```

```
[Out] -((Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-1/4, 3/2, 3/4, -((b*x^4)/a)])/(a*
x*Sqrt[a + b*x^4]))
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^4+a}}{b^2x^{10}+2\,abx^6+a^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)/(b^2*x^10 + 2*a*b*x^6 + a^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(3/2)*x^2), x)
```

maple [C] time = 0.02, size = 137, normalized size = 0.53

$$-\frac{b\,x^3}{2\sqrt{\left(x^4+\frac{a}{b}\right)b}\;a^2}+\frac{3i\sqrt{-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\;\sqrt{\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\;\left(-\text{EllipticE}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\;x,i\right)+\text{EllipticF}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\;x,i\right)\right)\sqrt{b}}{2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\;\sqrt{b\,x^4+a}\;a^{\frac{3}{2}}}-\frac{\sqrt{b\,x^4}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^4+a)^(3/2),x)
```

```
[Out] -1/2*b/a^2*x^3/((x^4+a/b)*b)^(1/2)-(b*x^4+a)^(1/2)/a^2/x+3/2*I/a^(3/2)*b^(1
/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(-I/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*b
^(1/2)*x^2+1)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF((I/a^(1/2)*b^(1/2))^(1/2)*x,
I)-EllipticE((I/a^(1/2)*b^(1/2))^(1/2)*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(3/2)*x^2), x)
```

mupad [B] time = 1.35, size = 40, normalized size = 0.15

$$-\frac{\left(\frac{a}{b\,x^4}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{7}{4};\;\frac{11}{4};\;-\frac{a}{b\,x^4}\right)}{7\,x\left(b\,x^4+a\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^4)^(3/2)),x)
```

```
[Out] -((a/(b*x^4) + 1)^(3/2)*hypergeom([3/2, 7/4], 11/4, -a/(b*x^4)))/(7*x*(a +
b*x^4)^(3/2))
```

sympy [C] time = 1.79, size = 39, normalized size = 0.15

$$\frac{\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{3}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}x\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**4+a)**(3/2),x)
```

```
[Out] gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2
)*x*gamma(3/4))
```

3.872 $\int \frac{1}{x^6\left(a+bx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=282

$$-\frac{21b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{20a^{11/4}\sqrt{a+bx^4}}+\frac{21b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\right.}{10a^{11/4}\sqrt{a+bx^4}}$$

```
[Out] 1/2/a/x^5/(b*x^4+a)^(1/2)-7/10*(b*x^4+a)^(1/2)/a^2/x^5+21/10*b*(b*x^4+a)^(1
/2)/a^3/x-21/10*b^(3/2)*x*(b*x^4+a)^(1/2)/a^3/(a^(1/2)+x^2*b^(1/2))+21/10*b
^(5/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^
(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^
2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(11/4)/(b*x^4+a)^(1/
2)-21/20*b^(5/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^
(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(
a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(11/4)/(b*
x^4+a)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {290, 325, 305, 220, 1196}

$$-\frac{21b^{3/2}x\sqrt{a+bx^4}}{10a^3\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{21b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{20a^{11/4}\sqrt{a+bx^4}}+\frac{21b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx}{(\sqrt{a}+\sqrt{b}}}}{10a^{11/4}\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^4)^(3/2)),x]
```

```
[Out] 1/(2*a*x^5*Sqrt[a + b*x^4]) - (7*Sqrt[a + b*x^4])/(10*a^2*x^5) + (21*b*Sqrt
[a + b*x^4])/(10*a^3*x) - (21*b^(3/2)*x*Sqrt[a + b*x^4])/(10*a^3*(Sqrt[a] +
 Sqrt[b]*x^2)) + (21*b^(5/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt
[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(10*a^
(11/4)*Sqrt[a + b*x^4]) - (21*b^(5/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x
^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2
])/(20*a^(11/4)*Sqrt[a + b*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{x^6\left(a+bx^4\right)^{3/2}}\,dx = \frac{1}{2ax^5\sqrt{a+bx^4}} + \frac{7\int \frac{1}{x^6\sqrt{a+bx^4}}\,dx}{2a}$$

$$= \frac{1}{2ax^5\sqrt{a+bx^4}} - \frac{7\sqrt{a+bx^4}}{10a^2x^5} - \frac{(21b)\int \frac{1}{x^2\sqrt{a+bx^4}}\,dx}{10a^2}$$

$$= \frac{1}{2ax^5\sqrt{a+bx^4}} - \frac{7\sqrt{a+bx^4}}{10a^2x^5} + \frac{21b\sqrt{a+bx^4}}{10a^3x} - \frac{\left(21b^2\right)\int \frac{x^2}{\sqrt{a+bx^4}}\,dx}{10a^3}$$

$$= \frac{1}{2ax^5\sqrt{a+bx^4}} - \frac{7\sqrt{a+bx^4}}{10a^2x^5} + \frac{21b\sqrt{a+bx^4}}{10a^3x} - \frac{\left(21b^{3/2}\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{10a^{5/2}} + \frac{\left(21b^{3/2}\right)\int \frac{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}}{10a^{5/2}}$$

$$= \frac{1}{2ax^5\sqrt{a+bx^4}} - \frac{7\sqrt{a+bx^4}}{10a^2x^5} + \frac{21b\sqrt{a+bx^4}}{10a^3x} - \frac{21b^{3/2}x\sqrt{a+bx^4}}{10a^3\left(\sqrt{a}+\sqrt{b}\,x^2\right)} + \frac{21b^{5/4}\left(\sqrt{a}+\sqrt{b}\,x\right.}{}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.19

$$-\frac{\sqrt{\frac{bx^4}{a}+1}\;{}_2F_1\left(-\frac{5}{4},\frac{3}{2};-\frac{1}{4};-\frac{bx^4}{a}\right)}{5ax^5\sqrt{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^4)^(3/2)),x]
```

```
[Out] -1/5*(Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-5/4, 3/2, -1/4, -((b*x^4)/a)])
/(a*x^5*Sqrt[a + b*x^4])
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^4+a}}{b^2x^{14}+2\,abx^{10}+a^2x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)/(b^2*x^14 + 2*a*b*x^10 + a^2*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{3}{2}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^4+a)^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^(3/2)*x^6), x)
```

maple [C] time = 0.02, size = 157, normalized size = 0.56

$$\frac{b^2x^3}{2\sqrt{\left(x^4+\frac{a}{b}\right)b}\,a^3}-\frac{21i\sqrt{-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\left(-\text{EllipticE}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x,i\right)+\text{EllipticF}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x,i\right)\right)b^{\frac{3}{2}}}{10\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,a^{\frac{5}{2}}}+\frac{8\sqrt{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^4+a)^(3/2),x)
[Out] -1/5*(b*x^4+a)^(1/2)/a^2/x^5+8/5*b*(b*x^4+a)^(1/2)/a^3/x+1/2*b^2/a^3*x^3/((
x^4+a/b)*b)^(1/2)-21/10*I/a^(5/2)*b^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(-I/a^(
1/2)*b^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*b^(1/2)*x^2+1)^(1/2)/(b*x^4+a)^(1/2)*(
EllipticF((I/a^(1/2)*b^(1/2))^(1/2)*x,I)-EllipticE((I/a^(1/2)*b^(1/2))^(1/2
)*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{3}{2}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^4+a)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(3/2)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^6\left(b\,x^4+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^4)^(3/2)),x)
[Out] int(1/(x^6*(a + b*x^4)^(3/2)), x)
```

sympy [C] time = 1.80, size = 44, normalized size = 0.16

$$\frac{\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{5}{4},\frac{3}{2}\\-\frac{1}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}x^5\Gamma\left(-\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**4+a)**(3/2),x)
```

```
[Out] gamma(-5/4)*hyper((-5/4, 3/2), (-1/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/
2)*x**5*gamma(-1/4))
```

3.873 $\int \frac{1}{\left(a+bx^4\right)^{5/2}}\,dx$

Optimal. Leaf size=127

$$\frac{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{24a^{9/4}\sqrt[4]{b}\,\sqrt{a+bx^4}}+\frac{5x}{12a^2\sqrt{a+bx^4}}+\frac{x}{6a\left(a+bx^4\right)^{3/2}}$$

```
[Out] 1/6*x/a/(b*x^4+a)^(3/2)+5/12*x/a^2/(b*x^4+a)^(1/2)+5/24*(cos(2*arctan(b^(1/
4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*ar
ctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(
1/2)+x^2*b^(1/2))^2)^(1/2)/a^(9/4)/b^(1/4)/(b*x^4+a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {199, 220}

$$\frac{5x}{12a^2\sqrt{a+bx^4}}+\frac{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{24a^{9/4}\sqrt[4]{b}\,\sqrt{a+bx^4}}+\frac{x}{6a\left(a+bx^4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(-5/2),x]
```

```
[Out] x/(6*a*(a + b*x^4)^(3/2)) + (5*x)/(12*a^2*Sqrt[a + b*x^4]) + (5*(Sqrt[a] +
Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan
[(b^(1/4)*x)/a^(1/4)], 1/2])/(24*a^(9/4)*b^(1/4)*Sqrt[a + b*x^4])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)^{5/2}}\,dx = \frac{x}{6a\left(a+bx^4\right)^{3/2}} + \frac{5\int \frac{1}{\left(a+bx^4\right)^{3/2}}\,dx}{6a}$$

$$= \frac{x}{6a\left(a+bx^4\right)^{3/2}} + \frac{5x}{12a^2\sqrt{a+bx^4}} + \frac{5\int \frac{1}{\sqrt{a+bx^4}}\,dx}{12a^2}$$

$$= \frac{x}{6a\left(a+bx^4\right)^{3/2}} + \frac{5x}{12a^2\sqrt{a+bx^4}} + \frac{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{24a^{9/4}\sqrt[4]{b}\,\sqrt{a+bx^4}}$$

Mathematica [C] time = 0.04, size = 72, normalized size = 0.57

$$\frac{5x\left(a+bx^4\right)\sqrt{\frac{bx^4}{a}+1}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^4}{a}\right)+7ax+5bx^5}{12a^2\left(a+bx^4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(-5/2),x]
```

```
[Out] (7*a*x + 5*b*x^5 + 5*x*(a + b*x^4)*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/
4, 1/2, 5/4, -((b*x^4)/a)])/(12*a^2*(a + b*x^4)^(3/2))
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^4+a}}{b^3x^{12}+3\,ab^2x^8+3\,a^2bx^4+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a)/(b^3*x^12 + 3*a*b^2*x^8 + 3*a^2*b*x^4 + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(-5/2), x)
```

maple [C] time = 0.01, size = 123, normalized size = 0.97

$$\frac{5x}{12\sqrt{\left(x^4+\frac{a}{b}\right)b}\;a^2} + \frac{5\sqrt{-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\;\sqrt{\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\;\text{EllipticF}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\;x,i\right)}{12\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\;\sqrt{b\,x^4+a}\;a^2} + \frac{\sqrt{b\,x^4+a}\;x}{6\left(x^4+\frac{a}{b}\right)^2 a\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(5/2),x)
```

```
[Out] 1/6/a*x/b^2*(b*x^4+a)^(1/2)/(x^4+a/b)^2+5/12/a^2*x/((x^4+a/b)*b)^(1/2)+5/12
/a^2/(I/a^(1/2)*b^(1/2))^(1/2)*(-I/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*
b^(1/2)*x^2+1)^(1/2)/(b*x^4+a)^(1/2)*EllipticF((I/a^(1/2)*b^(1/2))^(1/2)*x,
I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(-5/2), x)
```

mupad [B] time = 1.16, size = 37, normalized size = 0.29

$$\frac{x\left(\frac{b\,x^4}{a}+1\right)^{5/2}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{5}{2};\ \frac{5}{4};\ -\frac{b\,x^4}{a}\right)}{\left(b\,x^4+a\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^4)^(5/2),x)
```

```
[Out] (x*((b*x^4)/a + 1)^(5/2)*hypergeom([1/4, 5/2], 5/4, -(b*x^4)/a))/(a + b*x^4
)^(5/2)
```

sympy [C] time = 1.33, size = 36, normalized size = 0.28

$$\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{5}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(5/2),x)
```

```
[Out] x*gamma(1/4)*hyper((1/4, 5/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(5/2
)*gamma(5/4))
```

3.874 $\int \frac{x^{11}}{\sqrt{1-x^4}}\,dx$

Optimal. Leaf size=46

$$-\frac{1}{10}\left(1-x^4\right)^{5/2}+\frac{1}{3}\left(1-x^4\right)^{3/2}-\frac{\sqrt{1-x^4}}{2}$$

```
[Out] 1/3*(-x^4+1)^(3/2)-1/10*(-x^4+1)^(5/2)-1/2*(-x^4+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{1}{10}\left(1-x^4\right)^{5/2}+\frac{1}{3}\left(1-x^4\right)^{3/2}-\frac{\sqrt{1-x^4}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/Sqrt[1 - x^4],x]
```

```
[Out] -Sqrt[1 - x^4]/2 + (1 - x^4)^(3/2)/3 - (1 - x^4)^(5/2)/10
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{\sqrt{1-x^4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1-x}}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int \left(\frac{1}{\sqrt{1-x}}-2\sqrt{1-x}+(1-x)^{3/2}\right)dx, x, x^4\right)\\ &= -\frac{1}{2}\sqrt{1-x^4}+\frac{1}{3}\left(1-x^4\right)^{3/2}-\frac{1}{10}\left(1-x^4\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.59

$$-\frac{1}{30}\sqrt{1-x^4}\left(3x^8+4x^4+8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/Sqrt[1 - x^4],x]
```

```
[Out] -1/30*(Sqrt[1 - x^4]*(8 + 4*x^4 + 3*x^8))
```

fricas [A] time = 0.71, size = 23, normalized size = 0.50

$$-\frac{1}{30}\left(3\,x^8+4\,x^4+8\right)\sqrt{-x^4+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^4+1)^(1/2),x, algorithm="fricas")
[Out] -1/30*(3*x^8 + 4*x^4 + 8)*sqrt(-x^4 + 1)
```

giac [A] time = 0.17, size = 41, normalized size = 0.89

$$-\frac{1}{10}\left(x^4-1\right)^2\sqrt{-x^4+1}+\frac{1}{3}\left(-x^4+1\right)^{\frac{3}{2}}-\frac{1}{2}\sqrt{-x^4+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^4+1)^(1/2),x, algorithm="giac")
[Out] -1/10*(x^4 - 1)^2*sqrt(-x^4 + 1) + 1/3*(-x^4 + 1)^(3/2) - 1/2*sqrt(-x^4 + 1
)
```

maple [A] time = 0.01, size = 35, normalized size = 0.76

$$\frac{(x-1)(x+1)\left(x^2+1\right)\left(3x^8+4x^4+8\right)}{30\sqrt{-x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(-x^4+1)^(1/2),x)
[Out] 1/30*(x-1)*(x+1)*(x^2+1)*(3*x^8+4*x^4+8)/(-x^4+1)^(1/2)
```

maxima [A] time = 1.38, size = 34, normalized size = 0.74

$$-\frac{1}{10}\left(-x^4+1\right)^{\frac{5}{2}}+\frac{1}{3}\left(-x^4+1\right)^{\frac{3}{2}}-\frac{1}{2}\sqrt{-x^4+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^4+1)^(1/2),x, algorithm="maxima")
[Out] -1/10*(-x^4 + 1)^(5/2) + 1/3*(-x^4 + 1)^(3/2) - 1/2*sqrt(-x^4 + 1)
```

mupad [B] time = 1.20, size = 23, normalized size = 0.50

$$-\sqrt{1-x^4}\left(\frac{x^8}{10}+\frac{2x^4}{15}+\frac{4}{15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(1 - x^4)^(1/2),x)
[Out] -(1 - x^4)^(1/2)*((2*x^4)/15 + x^8/10 + 4/15)
```

sympy [A] time = 1.56, size = 41, normalized size = 0.89

$$-\frac{x^8\sqrt{1-x^4}}{10}-\frac{2x^4\sqrt{1-x^4}}{15}-\frac{4\sqrt{1-x^4}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(-x**4+1)**(1/2),x)
[Out] -x**8*sqrt(1 - x**4)/10 - 2*x**4*sqrt(1 - x**4)/15 - 4*sqrt(1 - x**4)/15
```

3.875 $\int \frac{x^7}{\sqrt{1-x^4}}\,dx$

Optimal. Leaf size=31

$$\frac{1}{6}\left(1-x^4\right)^{3/2}-\frac{\sqrt{1-x^4}}{2}$$

```
[Out] 1/6*(-x^4+1)^(3/2)-1/2*(-x^4+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{1}{6}\left(1-x^4\right)^{3/2}-\frac{\sqrt{1-x^4}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/Sqrt[1 - x^4],x]
```

```
[Out] -Sqrt[1 - x^4]/2 + (1 - x^4)^(3/2)/6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{\sqrt{1-x^4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x}{\sqrt{1-x}}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int \left(\frac{1}{\sqrt{1-x}}-\sqrt{1-x}\right)dx, x, x^4\right)\\ &= -\frac{1}{2}\sqrt{1-x^4}+\frac{1}{6}\left(1-x^4\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.65

$$-\frac{1}{6}\sqrt{1-x^4}\left(x^4+2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/Sqrt[1 - x^4],x]
```

```
[Out] -1/6*(Sqrt[1 - x^4]*(2 + x^4))
```

fricas [A] time = 0.64, size = 16, normalized size = 0.52

$$-\frac{1}{6}\left(x^4+2\right)\sqrt{-x^4+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^4+1)^(1/2),x, algorithm="fricas")
[Out] -1/6*(x^4 + 2)*sqrt(-x^4 + 1)
```

giac [A] time = 0.15, size = 23, normalized size = 0.74

$$\frac{1}{6}\left(-x^4+1\right)^{\frac{3}{2}}-\frac{1}{2}\sqrt{-x^4+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^4+1)^(1/2),x, algorithm="giac")
[Out] 1/6*(-x^4 + 1)^(3/2) - 1/2*sqrt(-x^4 + 1)
```

maple [A] time = 0.00, size = 28, normalized size = 0.90

$$\frac{(x-1)(x+1)\left(x^2+1\right)\left(x^4+2\right)}{6\sqrt{-x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-x^4+1)^(1/2),x)
[Out] 1/6*(x-1)*(x+1)*(x^2+1)*(x^4+2)/(-x^4+1)^(1/2)
```

maxima [A] time = 1.19, size = 23, normalized size = 0.74

$$\frac{1}{6}\left(-x^4+1\right)^{\frac{3}{2}}-\frac{1}{2}\sqrt{-x^4+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^4+1)^(1/2),x, algorithm="maxima")
[Out] 1/6*(-x^4 + 1)^(3/2) - 1/2*sqrt(-x^4 + 1)
```

mupad [B] time = 1.13, size = 16, normalized size = 0.52

$$-\frac{\sqrt{1-x^4}\,\left(x^4+2\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(1 - x^4)^(1/2),x)
[Out] -((1 - x^4)^(1/2)*(x^4 + 2))/6
```

sympy [A] time = 1.13, size = 24, normalized size = 0.77

$$-\frac{x^4\sqrt{1-x^4}}{6}-\frac{\sqrt{1-x^4}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-x**4+1)**(1/2),x)
[Out] -x**4*sqrt(1 - x**4)/6 - sqrt(1 - x**4)/3
```

3.876 $\int \frac{x^3}{\sqrt{1-x^4}}\,dx$

Optimal. Leaf size=15

$$-\frac{1}{2}\sqrt{1-x^4}$$

```
[Out] -1/2*(-x^4+1)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$-\frac{1}{2}\sqrt{1-x^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[1 - x^4],x]
```

```
[Out] -Sqrt[1 - x^4]/2
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{1-x^4}}\,dx = -\frac{1}{2}\sqrt{1-x^4}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$-\frac{1}{2}\sqrt{1-x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[1 - x^4],x]
```

```
[Out] -1/2*Sqrt[1 - x^4]
```

fricas [A] time = 0.77, size = 11, normalized size = 0.73

$$-\frac{1}{2}\sqrt{-x^4+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-x^4 + 1)
```

giac [A] time = 0.15, size = 11, normalized size = 0.73

$$-\frac{1}{2}\sqrt{-x^4+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^4+1)^(1/2),x, algorithm="giac")
[Out] -1/2*sqrt(-x^4 + 1)
```

maple [A] time = 0.01, size = 23, normalized size = 1.53

$$\frac{(x-1)(x+1)\left(x^2+1\right)}{2\sqrt{-x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-x^4+1)^(1/2),x)
[Out] 1/2*(x-1)*(x+1)*(x^2+1)/(-x^4+1)^(1/2)
```

maxima [A] time = 1.25, size = 11, normalized size = 0.73

$$-\frac{1}{2}\sqrt{-x^4+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^4+1)^(1/2),x, algorithm="maxima")
[Out] -1/2*sqrt(-x^4 + 1)
```

mupad [B] time = 1.22, size = 11, normalized size = 0.73

$$-\frac{\sqrt{1-x^4}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(1 - x^4)^(1/2),x)
[Out] -(1 - x^4)^(1/2)/2
```

sympy [A] time = 0.34, size = 10, normalized size = 0.67

$$-\frac{\sqrt{1-x^4}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-x**4+1)**(1/2),x)
[Out] -sqrt(1 - x**4)/2
```

3.877 $\int \frac{1}{x\sqrt{1-x^4}}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{2}\tanh^{-1}\left(\sqrt{1-x^4}\right)$$

```
[Out] -1/2*arctanh((-x^4+1)^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {266, 63, 206}

$$-\frac{1}{2}\tanh^{-1}\left(\sqrt{1-x^4}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[1 - x^4]),x]
```

```
[Out] -ArcTanh[Sqrt[1 - x^4]]/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{1-x^4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, x^4\right)\\ &= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-x^4}\right)\right)\\ &= -\frac{1}{2}\tanh^{-1}\left(\sqrt{1-x^4}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{2}\tanh^{-1}\left(\sqrt{1-x^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[1 - x^4]),x]
[Out] -1/2*ArcTanh[Sqrt[1 - x^4]]
```

fricas [B] time = 0.68, size = 29, normalized size = 1.81

$$-\frac{1}{4}\log\left(\sqrt{-x^4+1}+1\right)+\frac{1}{4}\log\left(\sqrt{-x^4+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^4+1)^(1/2),x, algorithm="fricas")
[Out] -1/4*log(sqrt(-x^4 + 1) + 1) + 1/4*log(sqrt(-x^4 + 1) - 1)
```

giac [B] time = 0.15, size = 31, normalized size = 1.94

$$-\frac{1}{4}\log\left(\sqrt{-x^4+1}+1\right)+\frac{1}{4}\log\left(-\sqrt{-x^4+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^4+1)^(1/2),x, algorithm="giac")
[Out] -1/4*log(sqrt(-x^4 + 1) + 1) + 1/4*log(-sqrt(-x^4 + 1) + 1)
```

maple [A] time = 0.01, size = 13, normalized size = 0.81

$$-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^4+1}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^4+1)^(1/2),x)
[Out] -1/2*arctanh(1/(-x^4+1)^(1/2))
```

maxima [B] time = 1.36, size = 29, normalized size = 1.81

$$-\frac{1}{4}\log\left(\sqrt{-x^4+1}+1\right)+\frac{1}{4}\log\left(\sqrt{-x^4+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^4+1)^(1/2),x, algorithm="maxima")
[Out] -1/4*log(sqrt(-x^4 + 1) + 1) + 1/4*log(sqrt(-x^4 + 1) - 1)
```

mupad [B] time = 1.31, size = 12, normalized size = 0.75

$$-\frac{\operatorname{atanh}\left(\sqrt{1-x^4}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1 - x^4)^(1/2)),x)
[Out] -atanh((1 - x^4)^(1/2))/2
```

sympy [A] time = 1.92, size = 24, normalized size = 1.50

$$\begin{cases} -\frac{\operatorname{acosh}\left(\frac{1}{x^2}\right)}{2} & \text{for } \frac{1}{|x^4|} > 1 \\ \frac{i\operatorname{asin}\left(\frac{1}{x^2}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**4+1)**(1/2),x)
[Out] Piecewise((-acosh(x**(-2))/2, 1/Abs(x**4) > 1), (I*asin(x**(-2))/2, True))
```

3.878 $\int \frac{1}{x^5\sqrt{1-x^4}}\,dx$

Optimal. Leaf size=35

$$-\frac{\sqrt{1-x^4}}{4x^4}-\frac{1}{4}\tanh^{-1}\left(\sqrt{1-x^4}\right)$$

```
[Out] -1/4*arctanh((-x^4+1)^(1/2))-1/4*(-x^4+1)^(1/2)/x^4
```

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 51, 63, 206}

$$-\frac{\sqrt{1-x^4}}{4x^4}-\frac{1}{4}\tanh^{-1}\left(\sqrt{1-x^4}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[1 - x^4]),x]
```

```
[Out] -Sqrt[1 - x^4]/(4*x^4) - ArcTanh[Sqrt[1 - x^4]]/4
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\sqrt{1-x^4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x^2}\,dx, x, x^4\right)\\ &= -\frac{\sqrt{1-x^4}}{4x^4} + \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, x^4\right)\\ &= -\frac{\sqrt{1-x^4}}{4x^4} - \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-x^4}\right)\\ &= -\frac{\sqrt{1-x^4}}{4x^4} - \frac{1}{4}\tanh^{-1}\left(\sqrt{1-x^4}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.00

$$-\frac{\sqrt{1-x^4}}{4x^4} - \frac{1}{4}\tanh^{-1}\left(\sqrt{1-x^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[1 - x^4]),x]
```

```
[Out] -1/4*Sqrt[1 - x^4]/x^4 - ArcTanh[Sqrt[1 - x^4]]/4
```

fricas [A] time = 0.84, size = 50, normalized size = 1.43

$$-\frac{x^4\log\left(\sqrt{-x^4+1}+1\right) - x^4\log\left(\sqrt{-x^4+1}-1\right) + 2\sqrt{-x^4+1}}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*(x^4*log(sqrt(-x^4 + 1) + 1) - x^4*log(sqrt(-x^4 + 1) - 1) + 2*sqrt(-x
^4 + 1))/x^4
```

giac [A] time = 0.15, size = 45, normalized size = 1.29

$$-\frac{\sqrt{-x^4+1}}{4\,x^4} - \frac{1}{8}\log\left(\sqrt{-x^4+1}+1\right) + \frac{1}{8}\log\left(-\sqrt{-x^4+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(-x^4 + 1)/x^4 - 1/8*log(sqrt(-x^4 + 1) + 1) + 1/8*log(-sqrt(-x^4
+ 1) + 1)
```

maple [A] time = 0.01, size = 28, normalized size = 0.80

$$-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^4+1}}\right)}{4} - \frac{\sqrt{-x^4+1}}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-x^4+1)^(1/2),x)
```

```
[Out] -1/4*(-x^4+1)^(1/2)/x^4-1/4*arctanh(1/(-x^4+1)^(1/2))
```

maxima [A] time = 1.33, size = 43, normalized size = 1.23

$$-\frac{\sqrt{-x^4+1}}{4\,x^4}-\frac{1}{8}\log\left(\sqrt{-x^4+1}+1\right)+\frac{1}{8}\log\left(\sqrt{-x^4+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(-x^4 + 1)/x^4 - 1/8*log(sqrt(-x^4 + 1) + 1) + 1/8*log(sqrt(-x^4 +
 1) - 1)
```

mupad [B] time = 1.21, size = 27, normalized size = 0.77

$$-\frac{\operatorname{atanh}\left(\sqrt{1-x^4}\right)}{4}-\frac{\sqrt{1-x^4}}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(1 - x^4)^(1/2)),x)
```

```
[Out] - atanh((1 - x^4)^(1/2))/4 - (1 - x^4)^(1/2)/(4*x^4)
```

sympy [A] time = 2.20, size = 73, normalized size = 2.09

$$\begin{cases} -\frac{\operatorname{acosh}\left(\frac{1}{x^2}\right)}{4}-\frac{\sqrt{-1+\frac{1}{x^4}}}{4x^2} & \text{for } \frac{1}{|x^4|}>1 \\ \frac{i\operatorname{asin}\left(\frac{1}{x^2}\right)}{4}-\frac{i}{4x^2\sqrt{1-\frac{1}{x^4}}}+\frac{i}{4x^6\sqrt{1-\frac{1}{x^4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-x**4+1)**(1/2),x)
```

```
[Out] Piecewise((-acosh(x**(-2))/4 - sqrt(-1 + x**(-4))/(4*x**2), 1/Abs(x**4) > 1
), (I*asin(x**(-2))/4 - I/(4*x**2*sqrt(1 - 1/x**4)) + I/(4*x**6*sqrt(1 - 1/
x**4)), True))
```

3.879 $\int \frac{x^5}{\sqrt{1-x^4}}\,dx$

Optimal. Leaf size=27

$$\frac{1}{4}\sin^{-1}\left(x^2\right)-\frac{1}{4}x^2\sqrt{1-x^4}$$

```
[Out] 1/4*arcsin(x^2)-1/4*x^2*(-x^4+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {275, 321, 216}

$$\frac{1}{4}\sin^{-1}\left(x^2\right)-\frac{1}{4}x^2\sqrt{1-x^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[1 - x^4],x]
```

```
[Out] -(x^2*Sqrt[1 - x^4])/4 + ArcSin[x^2]/4
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\sqrt{1-x^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1-x^2}}\,dx, x, x^2\right)\\ &= -\frac{1}{4}x^2\sqrt{1-x^4}+\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, x^2\right)\\ &= -\frac{1}{4}x^2\sqrt{1-x^4}+\frac{1}{4}\sin^{-1}\left(x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{1}{4}\sin^{-1}\left(x^2\right)-\frac{1}{4}x^2\sqrt{1-x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[1 - x^4],x]
```

```
[Out] -1/4*(x^2*Sqrt[1 - x^4]) + ArcSin[x^2]/4
```

fricas [A] time = 0.86, size = 33, normalized size = 1.22

$$-\frac{1}{4}\sqrt{-x^4+1}\,x^2-\frac{1}{2}\arctan\left(\frac{\sqrt{-x^4+1}-1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-x^4 + 1)*x^2 - 1/2*arctan((sqrt(-x^4 + 1) - 1)/x^2)
```

giac [A] time = 0.18, size = 21, normalized size = 0.78

$$-\frac{1}{4}\sqrt{-x^4+1}\,x^2+\frac{1}{4}\arcsin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(-x^4 + 1)*x^2 + 1/4*arcsin(x^2)
```

maple [A] time = 0.01, size = 22, normalized size = 0.81

$$-\frac{\sqrt{-x^4+1}\,x^2}{4}+\frac{\arcsin\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-x^4+1)^(1/2),x)
```

```
[Out] 1/4*arcsin(x^2)-1/4*x^2*(-x^4+1)^(1/2)
```

maxima [B] time = 2.97, size = 44, normalized size = 1.63

$$\frac{\sqrt{-x^4+1}}{4\,x^2\left(\frac{x^4-1}{x^4}-1\right)}-\frac{1}{4}\arctan\left(\frac{\sqrt{-x^4+1}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(-x^4 + 1)/(x^2*((x^4 - 1)/x^4 - 1)) - 1/4*arctan(sqrt(-x^4 + 1)/x^
2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^5}{\sqrt{1-x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(1 - x^4)^(1/2),x)
```

```
[Out] int(x^5/(1 - x^4)^(1/2), x)
```

sympy [A] time = 3.18, size = 61, normalized size = 2.26

$$\begin{cases} -\frac{ix^2\sqrt{x^4-1}}{4}-\frac{i\operatorname{acosh}\left(x^2\right)}{4} & \text{for } \left|x^4\right|>1 \\ \frac{x^6}{4\sqrt{1-x^4}}-\frac{x^2}{4\sqrt{1-x^4}}+\frac{\operatorname{asin}\left(x^2\right)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-x**4+1)**(1/2),x)
```

```
[Out] Piecewise((-I*x**2*sqrt(x**4 - 1)/4 - I*acosh(x**2)/4, Abs(x**4) > 1), (x**
6/(4*sqrt(1 - x**4)) - x**2/(4*sqrt(1 - x**4)) + asin(x**2)/4, True))
```

3.880 $\int \frac{x}{\sqrt{1-x^4}}\,dx$

Optimal. Leaf size=8

$$\frac{1}{2}\sin^{-1}\left(x^2\right)$$

```
[Out] 1/2*arcsin(x^2)
```

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {275, 216}

$$\frac{1}{2}\sin^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[1 - x^4],x]
```

```
[Out] ArcSin[x^2]/2
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{1-x^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\sin^{-1}\left(x^2\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\frac{1}{2}\sin^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[1 - x^4],x]
```

```
[Out] ArcSin[x^2]/2
```

fricas [B] time = 0.82, size = 18, normalized size = 2.25

$$-\arctan\left(\frac{\sqrt{-x^4+1}-1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan((sqrt(-x^4 + 1) - 1)/x^2)
```

giac [A] time = 0.16, size = 6, normalized size = 0.75

$$\frac{1}{2}\arcsin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+1)^(1/2),x, algorithm="giac")
[Out] 1/2*arcsin(x^2)
```

maple [A] time = 0.01, size = 7, normalized size = 0.88

$$\frac{\arcsin\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^4+1)^(1/2),x)
[Out] 1/2*arcsin(x^2)
```

maxima [B] time = 2.86, size = 16, normalized size = 2.00

$$-\frac{1}{2}\arctan\left(\frac{\sqrt{-x^4+1}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+1)^(1/2),x, algorithm="maxima")
[Out] -1/2*arctan(sqrt(-x^4 + 1)/x^2)
```

mupad [B] time = 0.16, size = 16, normalized size = 2.00

$$\frac{\operatorname{atan}\left(\frac{x^2}{\sqrt{1-x^4}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1 - x^4)^(1/2),x)
[Out] atan(x^2/(1 - x^4)^(1/2))/2
```

sympy [A] time = 1.80, size = 19, normalized size = 2.38

$$\begin{cases} -\frac{i\operatorname{acosh}\left(x^2\right)}{2} & \text{for } \left|x^4\right| > 1 \\ \frac{\operatorname{asin}\left(x^2\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**4+1)**(1/2),x)
[Out] Piecewise((-I*acosh(x**2)/2, Abs(x**4) > 1), (asin(x**2)/2, True))
```

3.881 $\int \frac{1}{x^3\sqrt{1-x^4}}\,dx$

Optimal. Leaf size=18

$$-\frac{\sqrt{1-x^4}}{2x^2}$$

```
[Out] -1/2*(-x^4+1)^(1/2)/x^2
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {264}

$$-\frac{\sqrt{1-x^4}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[1 - x^4]),x]
```

```
[Out] -Sqrt[1 - x^4]/(2*x^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{1-x^4}}\,dx = -\frac{\sqrt{1-x^4}}{2x^2}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$-\frac{\sqrt{1-x^4}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[1 - x^4]),x]
```

```
[Out] -1/2*Sqrt[1 - x^4]/x^2
```

fricas [A] time = 0.71, size = 14, normalized size = 0.78

$$-\frac{\sqrt{-x^4+1}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-x^4 + 1)/x^2
```

giac [B] time = 0.17, size = 35, normalized size = 1.94

$$\frac{x^2}{4\left(\sqrt{-x^4+1}-1\right)} - \frac{\sqrt{-x^4+1}-1}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*x^2/(sqrt(-x^4 + 1) - 1) - 1/4*(sqrt(-x^4 + 1) - 1)/x^2
```

maple [A] time = 0.01, size = 26, normalized size = 1.44

$$\frac{(x-1)(x+1)\left(x^2+1\right)}{2\sqrt{-x^4+1}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-x^4+1)^(1/2),x)
```

```
[Out] 1/2/x^2*(x-1)*(x+1)*(x^2+1)/(-x^4+1)^(1/2)
```

maxima [A] time = 1.33, size = 14, normalized size = 0.78

$$-\frac{\sqrt{-x^4+1}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-x^4 + 1)/x^2
```

mupad [B] time = 1.12, size = 14, normalized size = 0.78

$$-\frac{\sqrt{1-x^4}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(1 - x^4)^(1/2)),x)
```

```
[Out] -(1 - x^4)^(1/2)/(2*x^2)
```

sympy [A] time = 1.18, size = 34, normalized size = 1.89

$$\begin{cases} -\frac{i\sqrt{x^4-1}}{2x^2} & \text{for } \left|x^4\right| > 1 \\ -\frac{\sqrt{1-x^4}}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-x**4+1)**(1/2),x)
```

```
[Out] Piecewise((-I*sqrt(x**4 - 1)/(2*x**2), Abs(x**4) > 1), (-sqrt(1 - x**4)/(2*
x**2), True))
```

3.882 $\int \frac{1}{x^7\sqrt{1-x^4}}\,dx$

Optimal. Leaf size=37

$$-\frac{\sqrt{1-x^4}}{6x^6}-\frac{\sqrt{1-x^4}}{3x^2}$$

```
[Out] -1/6*(-x^4+1)^(1/2)/x^6-1/3*(-x^4+1)^(1/2)/x^2
```

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$-\frac{\sqrt{1-x^4}}{3x^2}-\frac{\sqrt{1-x^4}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*Sqrt[1 - x^4]),x]
```

```
[Out] -Sqrt[1 - x^4]/(6*x^6) - Sqrt[1 - x^4]/(3*x^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7\sqrt{1-x^4}}\,dx &= -\frac{\sqrt{1-x^4}}{6x^6}+\frac{2}{3}\int \frac{1}{x^3\sqrt{1-x^4}}\,dx\\ &= -\frac{\sqrt{1-x^4}}{6x^6}-\frac{\sqrt{1-x^4}}{3x^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.68

$$-\frac{\sqrt{1-x^4}\left(2x^4+1\right)}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*Sqrt[1 - x^4]),x]
```

```
[Out] -1/6*(Sqrt[1 - x^4]*(1 + 2*x^4))/x^6
```

fricas [A] time = 0.88, size = 21, normalized size = 0.57

$$-\frac{\left(2\,x^4+1\right)\sqrt{-x^4+1}}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(2*x^4 + 1)*sqrt(-x^4 + 1)/x^6
```

giac [B] time = 0.20, size = 73, normalized size = 1.97

$$\frac{x^6\left(\frac{9\left(\sqrt{-x^4+1}-1\right)^2}{x^4}+1\right)}{48\left(\sqrt{-x^4+1}-1\right)^3}-\frac{3\left(\sqrt{-x^4+1}-1\right)}{16\,x^2}-\frac{\left(\sqrt{-x^4+1}-1\right)^3}{48\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*x^6*(9*(sqrt(-x^4 + 1) - 1)^2/x^4 + 1)/(sqrt(-x^4 + 1) - 1)^3 - 3/16*(
sqrt(-x^4 + 1) - 1)/x^2 - 1/48*(sqrt(-x^4 + 1) - 1)^3/x^6
```

maple [A] time = 0.00, size = 33, normalized size = 0.89

$$\frac{(x-1)\,(x+1)\left(x^2+1\right)\left(2x^4+1\right)}{6\sqrt{-x^4+1}\;x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(-x^4+1)^(1/2),x)
```

```
[Out] 1/6*(x-1)*(x+1)*(x^2+1)*(2*x^4+1)/x^6/(-x^4+1)^(1/2)
```

maxima [A] time = 1.33, size = 29, normalized size = 0.78

$$-\frac{\sqrt{-x^4+1}}{2\,x^2}-\frac{\left(-x^4+1\right)^{\frac{3}{2}}}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-x^4 + 1)/x^2 - 1/6*(-x^4 + 1)^(3/2)/x^6
```

mupad [B] time = 1.15, size = 21, normalized size = 0.57

$$-\frac{\sqrt{1-x^4}\;\left(2\,x^4+1\right)}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(1 - x^4)^(1/2)),x)
```

```
[Out] -((1 - x^4)^(1/2)*(2*x^4 + 1))/(6*x^6)
```

sympy [A] time = 2.25, size = 63, normalized size = 1.70

$$\begin{cases} -\frac{i\sqrt{x^4-1}}{3x^2}-\frac{i\sqrt{x^4-1}}{6x^6} & \text{for } \left|x^4\right|>1 \\ -\frac{\sqrt{1-x^4}}{3x^2}-\frac{\sqrt{1-x^4}}{6x^6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(-x**4+1)**(1/2),x)

[Out] Piecewise((-I*sqrt(x**4 - 1)/(3*x**2) - I*sqrt(x**4 - 1)/(6*x**6), Abs(x**4
) > 1), (-sqrt(1 - x**4)/(3*x**2) - sqrt(1 - x**4)/(6*x**6), True))
```

3.883 $\int \frac{1}{x^{11}\sqrt{1-x^4}}\,dx$

Optimal. Leaf size=55

$$-\frac{\sqrt{1-x^4}}{10x^{10}}-\frac{2\sqrt{1-x^4}}{15x^6}-\frac{4\sqrt{1-x^4}}{15x^2}$$

```
[Out] -1/10*(-x^4+1)^(1/2)/x^10-2/15*(-x^4+1)^(1/2)/x^6-4/15*(-x^4+1)^(1/2)/x^2
```

Rubi [A] time = 0.01, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$-\frac{4\sqrt{1-x^4}}{15x^2}-\frac{2\sqrt{1-x^4}}{15x^6}-\frac{\sqrt{1-x^4}}{10x^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^11*Sqrt[1 - x^4]),x]
```

```
[Out] -Sqrt[1 - x^4]/(10*x^10) - (2*Sqrt[1 - x^4])/(15*x^6) - (4*Sqrt[1 - x^4])/(
15*x^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{11}\sqrt{1-x^4}}\,dx &= -\frac{\sqrt{1-x^4}}{10x^{10}}+\frac{4}{5}\int \frac{1}{x^7\sqrt{1-x^4}}\,dx\\ &= -\frac{\sqrt{1-x^4}}{10x^{10}}-\frac{2\sqrt{1-x^4}}{15x^6}+\frac{8}{15}\int \frac{1}{x^3\sqrt{1-x^4}}\,dx\\ &= -\frac{\sqrt{1-x^4}}{10x^{10}}-\frac{2\sqrt{1-x^4}}{15x^6}-\frac{4\sqrt{1-x^4}}{15x^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.55

$$-\frac{\sqrt{1-x^4}\left(8x^8+4x^4+3\right)}{30x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^11*Sqrt[1 - x^4]),x]
```

```
[Out] -1/30*(Sqrt[1 - x^4]*(3 + 4*x^4 + 8*x^8))/x^10
```

fricas [A] time = 0.85, size = 26, normalized size = 0.47

$$-\frac{\left(8\,x^8+4\,x^4+3\right)\sqrt{-x^4+1}}{30\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(-x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/30*(8*x^8 + 4*x^4 + 3)*sqrt(-x^4 + 1)/x^10
```

giac [B] time = 0.17, size = 109, normalized size = 1.98

$$\frac{x^{10}\left(\frac{25\left(\sqrt{-x^4+1}-1\right)^2}{x^4}+\frac{150\left(\sqrt{-x^4+1}-1\right)^4}{x^8}+3\right)}{960\left(\sqrt{-x^4+1}-1\right)^5}-\frac{5\left(\sqrt{-x^4+1}-1\right)}{32\,x^2}-\frac{5\left(\sqrt{-x^4+1}-1\right)^3}{192\,x^6}-\frac{\left(\sqrt{-x^4+1}-1\right)^5}{320\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(-x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/960*x^10*(25*(sqrt(-x^4 + 1) - 1)^2/x^4 + 150*(sqrt(-x^4 + 1) - 1)^4/x^8
+ 3)/(sqrt(-x^4 + 1) - 1)^5 - 5/32*(sqrt(-x^4 + 1) - 1)/x^2 - 5/192*(sqrt(-
x^4 + 1) - 1)^3/x^6 - 1/320*(sqrt(-x^4 + 1) - 1)^5/x^10
```

maple [A] time = 0.00, size = 38, normalized size = 0.69

$$\frac{(x-1)\,(x+1)\left(x^2+1\right)\left(8x^8+4x^4+3\right)}{30\sqrt{-x^4+1}\;x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^11/(-x^4+1)^(1/2),x)
```

```
[Out] 1/30*(x-1)*(x+1)*(x^2+1)*(8*x^8+4*x^4+3)/x^10/(-x^4+1)^(1/2)
```

maxima [A] time = 1.26, size = 43, normalized size = 0.78

$$-\frac{\sqrt{-x^4+1}}{2\,x^2}-\frac{\left(-x^4+1\right)^{\frac{3}{2}}}{3\,x^6}-\frac{\left(-x^4+1\right)^{\frac{5}{2}}}{10\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(-x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-x^4 + 1)/x^2 - 1/3*(-x^4 + 1)^(3/2)/x^6 - 1/10*(-x^4 + 1)^(5/2)/
x^10
```

mupad [B] time = 1.19, size = 26, normalized size = 0.47

$$-\frac{\sqrt{1-x^4}\;\left(8\,x^8+4\,x^4+3\right)}{30\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^11*(1 - x^4)^(1/2)),x)
```

```
[Out] -((1 - x^4)^(1/2)*(4*x^4 + 8*x^8 + 3))/(30*x^10)
```

sympy [**A**] time = 1.69, size = 104, normalized size = 1.89

$$\begin{cases} -\frac{4\sqrt{-1+\frac{1}{x^4}}}{15} - \frac{2\sqrt{-1+\frac{1}{x^4}}}{15x^4} - \frac{\sqrt{-1+\frac{1}{x^4}}}{10x^8} & \text{for } \frac{1}{|x^4|} > 1 \\ -\frac{4i\sqrt{1-\frac{1}{x^4}}}{15} - \frac{2i\sqrt{1-\frac{1}{x^4}}}{15x^4} - \frac{i\sqrt{1-\frac{1}{x^4}}}{10x^8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**11/(-x**4+1)**(1/2),x)
```

```
[Out] Piecewise((-4*sqrt(-1 + x**(-4))/15 - 2*sqrt(-1 + x**(-4))/(15*x**4) - sqrt
(-1 + x**(-4))/(10*x**8), 1/Abs(x**4) > 1), (-4*I*sqrt(1 - 1/x**4)/15 - 2*I
*sqrt(1 - 1/x**4)/(15*x**4) - I*sqrt(1 - 1/x**4)/(10*x**8), True))
```

3.884 $\int \frac{x^8}{\sqrt{1-x^4}}\,dx$

Optimal. Leaf size=43

$$-\frac{5}{21}\sqrt{1-x^4}\,x-\frac{1}{7}\sqrt{1-x^4}\,x^5+\frac{5}{21}F\left(\sin^{-1}(x)\middle|-1\right)$$

```
[Out] 5/21*EllipticF(x,I)-5/21*x*(-x^4+1)^(1/2)-1/7*x^5*(-x^4+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {321, 221}

$$-\frac{1}{7}\sqrt{1-x^4}\,x^5-\frac{5}{21}\sqrt{1-x^4}\,x+\frac{5}{21}F\left(\sin^{-1}(x)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^8/Sqrt[1 - x^4],x]
```

```
[Out] (-5*x*Sqrt[1 - x^4])/21 - (x^5*Sqrt[1 - x^4])/7 + (5*EllipticF[ArcSin[x], -
1])/21
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{\sqrt{1-x^4}}\,dx &= -\frac{1}{7}x^5\sqrt{1-x^4}+\frac{5}{7}\int \frac{x^4}{\sqrt{1-x^4}}\,dx\\ &= -\frac{5}{21}x\sqrt{1-x^4}-\frac{1}{7}x^5\sqrt{1-x^4}+\frac{5}{21}\int \frac{1}{\sqrt{1-x^4}}\,dx\\ &= -\frac{5}{21}x\sqrt{1-x^4}-\frac{1}{7}x^5\sqrt{1-x^4}+\frac{5}{21}F\left(\sin^{-1}(x)\middle|-1\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 42, normalized size = 0.98

$$\frac{1}{21}\left(5x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};x^4\right)-x\sqrt{1-x^4}\left(3x^4+5\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/Sqrt[1 - x^4],x]
```

```
[Out] (-(x*Sqrt[1 - x^4]*(5 + 3*x^4)) + 5*x*Hypergeometric2F1[1/4, 1/2, 5/4, x^4]
)/21
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^4+1}\,x^8}{x^4-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 1)*x^8/(x^4 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\sqrt{-x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^8/sqrt(-x^4 + 1), x)
```

maple [A] time = 0.01, size = 59, normalized size = 1.37

$$-\frac{\sqrt{-x^4+1}\,x^5}{7} - \frac{5\sqrt{-x^4+1}\,x}{21} + \frac{5\sqrt{-x^2+1}\,\sqrt{x^2+1}\,\text{EllipticF}(x,i)}{21\sqrt{-x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-x^4+1)^(1/2),x)
```

```
[Out] -1/7*(-x^4+1)^(1/2)*x^5-5/21*(-x^4+1)^(1/2)*x+5/21*(-x^2+1)^(1/2)*(x^2+1)^(
1/2)/(-x^4+1)^(1/2)*EllipticF(x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\sqrt{-x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^8/sqrt(-x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^8}{\sqrt{1-x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(1 - x^4)^(1/2),x)
```

```
[Out] int(x^8/(1 - x^4)^(1/2), x)
```

sympy [A] time = 1.01, size = 31, normalized size = 0.72

$$\frac{x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{9}{4} \\ \frac{13}{4}\end{matrix}\right| x^4 e^{2i\pi}\right)}{4\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-x**4+1)**(1/2),x)
```

```
[Out] x**9*gamma(9/4)*hyper((1/2, 9/4), (13/4,), x**4*exp_polar(2*I*pi))/(4*gamma
(13/4))
```

3.885 $\int \frac{x^4}{\sqrt{1-x^4}}\,dx$

Optimal. Leaf size=25

$$\frac{1}{3}F\left(\sin^{-1}(x)\middle|-1\right)-\frac{1}{3}x\sqrt{1-x^4}$$

```
[Out] 1/3*EllipticF(x,I)-1/3*x*(-x^4+1)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {321, 221}

$$\frac{1}{3}F\left(\sin^{-1}(x)\middle|-1\right)-\frac{1}{3}x\sqrt{1-x^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[1 - x^4],x]
```

```
[Out] -(x*Sqrt[1 - x^4])/3 + EllipticF[ArcSin[x], -1]/3
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt{1-x^4}}\,dx &= -\frac{1}{3}x\sqrt{1-x^4}+\frac{1}{3}\int \frac{1}{\sqrt{1-x^4}}\,dx\\ &= -\frac{1}{3}x\sqrt{1-x^4}+\frac{1}{3}F\left(\sin^{-1}(x)\middle|-1\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 32, normalized size = 1.28

$$\frac{1}{3}x\left({}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};x^4\right)-\sqrt{1-x^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[1 - x^4],x]
```

```
[Out] (x*(-Sqrt[1 - x^4] + Hypergeometric2F1[1/4, 1/2, 5/4, x^4]))/3
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^4+1}\,x^4}{x^4-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 1)*x^4/(x^4 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{-x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(-x^4 + 1), x)
```

maple [B] time = 0.01, size = 45, normalized size = 1.80

$$-\frac{\sqrt{-x^4+1}\,x}{3}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,\text{EllipticF}\left(x,i\right)}{3\sqrt{-x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^4+1)^(1/2),x)
```

```
[Out] -1/3*(-x^4+1)^(1/2)*x+1/3*(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)*Ellip
ticF(x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{-x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(-x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^4}{\sqrt{1-x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(1 - x^4)^(1/2),x)
```

```
[Out] int(x^4/(1 - x^4)^(1/2), x)
```

sympy [B] time = 1.17, size = 31, normalized size = 1.24

$$\frac{x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**4+1)**(1/2),x)
```

```
[Out] x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), x**4*exp_polar(2*I*pi))/(4*gamma(
9/4))
```

3.886 $\int \frac{1}{\sqrt{1-x^4}}\,dx$

Optimal. Leaf size=4

$$F\left(\sin^{-1}(x)\middle|-1\right)$$

```
[Out] EllipticF(x,I)
```

Rubi [A] time = 0.00, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {221}

$$F\left(\sin^{-1}(x)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 - x^4],x]
```

```
[Out] EllipticF[ArcSin[x], -1]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-x^4}}\,dx = F\left(\sin^{-1}(x)\middle|-1\right)$$

Mathematica [A] time = 0.01, size = 4, normalized size = 1.00

$$F\left(\sin^{-1}(x)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 - x^4],x]
```

```
[Out] EllipticF[ArcSin[x], -1]
```

fricas [F] time = 1.23, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^4+1}}{x^4-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 1)/(x^4 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-x^4 + 1), x)
```

maple [B] time = 0.00, size = 31, normalized size = 7.75

$$\frac{\sqrt{-x^2+1}\ \sqrt{x^2+1}\ \text{EllipticF}\left(x, i\right)}{\sqrt{-x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+1)^(1/2),x)
```

```
[Out] (-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)*EllipticF(x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^4+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-x^4 + 1), x)
```

mupad [B] time = 1.07, size = 10, normalized size = 2.50

$$x\,{}_2\mathrm{F}_1\left(\frac{1}{4}, \frac{1}{2};\ \frac{5}{4};\ x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1 - x^4)^(1/2),x)
```

```
[Out] x*hypergeom([1/4, 1/2], 5/4, x^4)
```

sympy [B] time = 0.83, size = 29, normalized size = 7.25

$$\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4}, \frac{1}{2} \\ \frac{5}{4}\end{matrix}\middle| x^4 e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+1)**(1/2),x)
```

```
[Out] x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), x**4*exp_polar(2*I*pi))/(4*gamma(5/4
))
```

3.887 $\int \frac{1}{x^4\sqrt{1-x^4}}\,dx$

Optimal. Leaf size=27

$$\frac{1}{3}F\left(\sin^{-1}(x)\middle|-1\right)-\frac{\sqrt{1-x^4}}{3x^3}$$

```
[Out] 1/3*EllipticF(x,I)-1/3*(-x^4+1)^(1/2)/x^3
```

Rubi [A] time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {325, 221}

$$\frac{1}{3}F\left(\sin^{-1}(x)\middle|-1\right)-\frac{\sqrt{1-x^4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[1 - x^4]),x]
```

```
[Out] -Sqrt[1 - x^4]/(3*x^3) + EllipticF[ArcSin[x], -1]/3
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{1-x^4}}\,dx &= -\frac{\sqrt{1-x^4}}{3x^3}+\frac{1}{3}\int \frac{1}{\sqrt{1-x^4}}\,dx\\ &= -\frac{\sqrt{1-x^4}}{3x^3}+\frac{1}{3}F\left(\sin^{-1}(x)\middle|-1\right)\end{aligned}$$

Mathematica [C] time = 0.00, size = 20, normalized size = 0.74

$$-\frac{{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};x^4\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[1 - x^4]),x]
```

```
[Out] -1/3*Hypergeometric2F1[-3/4, 1/2, 1/4, x^4]/x^3
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^4+1}}{x^8-x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 1)/(x^8 - x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^4+1}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^4 + 1)*x^4), x)
```

maple [B] time = 0.01, size = 47, normalized size = 1.74

$$\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,\text{EllipticF}\left(x,i\right)}{3\sqrt{-x^4+1}}-\frac{\sqrt{-x^4+1}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-x^4+1)^(1/2),x)
```

```
[Out] -1/3*(-x^4+1)^(1/2)/x^3+1/3*(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)*Ell
ipticF(x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^4+1}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^4 + 1)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^4\,\sqrt{1-x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(1 - x^4)^(1/2)),x)
```

```
[Out] int(1/(x^4*(1 - x^4)^(1/2)), x)
```

sympy [A] time = 0.97, size = 34, normalized size = 1.26

$$\frac{\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{1}{4}\end{matrix}\right|x^4e^{2i\pi}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-x**4+1)**(1/2),x)
```

```
[Out] gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), x**4*exp_polar(2*I*pi))/(4*x**3*gamm
a(1/4))
```

3.888 $\int \frac{1}{x^8\sqrt{1-x^4}}\,dx$

Optimal. Leaf size=45

$$-\frac{\sqrt{1-x^4}}{7x^7}-\frac{5\sqrt{1-x^4}}{21x^3}+\frac{5}{21}F\left(\sin^{-1}(x)\middle|-1\right)$$

```
[Out] 5/21*EllipticF(x,I)-1/7*(-x^4+1)^(1/2)/x^7-5/21*(-x^4+1)^(1/2)/x^3
```

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {325, 221}

$$-\frac{5\sqrt{1-x^4}}{21x^3}-\frac{\sqrt{1-x^4}}{7x^7}+\frac{5}{21}F\left(\sin^{-1}(x)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*Sqrt[1 - x^4]),x]
```

```
[Out] -Sqrt[1 - x^4]/(7*x^7) - (5*Sqrt[1 - x^4])/(21*x^3) + (5*EllipticF[ArcSin[x
], -1])/21
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^8\sqrt{1-x^4}}\,dx &= -\frac{\sqrt{1-x^4}}{7x^7}+\frac{5}{7}\int \frac{1}{x^4\sqrt{1-x^4}}\,dx\\ &= -\frac{\sqrt{1-x^4}}{7x^7}-\frac{5\sqrt{1-x^4}}{21x^3}+\frac{5}{21}\int \frac{1}{\sqrt{1-x^4}}\,dx\\ &= -\frac{\sqrt{1-x^4}}{7x^7}-\frac{5\sqrt{1-x^4}}{21x^3}+\frac{5}{21}F\left(\sin^{-1}(x)\middle|-1\right)\end{aligned}$$

Mathematica [C] time = 0.00, size = 20, normalized size = 0.44

$$-\frac{{}_2F_1\left(-\frac{7}{4},\frac{1}{2};-\frac{3}{4};x^4\right)}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*Sqrt[1 - x^4]),x]
```

```
[Out] -1/7*Hypergeometric2F1[-7/4, 1/2, -3/4, x^4]/x^7
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^4+1}}{x^{12}-x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 1)/(x^12 - x^8), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^4+1}\,x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^4 + 1)*x^8), x)
```

maple [A] time = 0.01, size = 61, normalized size = 1.36

$$\frac{5\sqrt{-x^2+1}\,\sqrt{x^2+1}\,\text{EllipticF}\left(x,i\right)}{21\sqrt{-x^4+1}}-\frac{5\sqrt{-x^4+1}}{21x^3}-\frac{\sqrt{-x^4+1}}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(-x^4+1)^(1/2),x)
```

```
[Out] -1/7*(-x^4+1)^(1/2)/x^7-5/21*(-x^4+1)^(1/2)/x^3+5/21*(-x^2+1)^(1/2)*(x^2+1)
^(1/2)/(-x^4+1)^(1/2)*EllipticF(x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^4+1}\,x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^4 + 1)*x^8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^8\sqrt{1-x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(1 - x^4)^(1/2)),x)
```

```
[Out] int(1/(x^8*(1 - x^4)^(1/2)), x)
```

sympy [A] time = 1.96, size = 37, normalized size = 0.82

$$\frac{\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{7}{4}, \frac{1}{2}\\ -\frac{3}{4}\end{matrix}\right| x^4 e^{2i\pi}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(-x**4+1)**(1/2),x)
```

```
[Out] gamma(-7/4)*hyper((-7/4, 1/2), (-3/4,), x**4*exp_polar(2*I*pi))/(4*x**7*gam
ma(-3/4))
```

3.889 $\int \frac{x^{10}}{\sqrt{1-x^4}}\,dx$

Optimal. Leaf size=53

$$-\frac{1}{9}\sqrt{1-x^4}\,x^7-\frac{7}{45}\sqrt{1-x^4}\,x^3-\frac{7}{15}F\left(\sin^{-1}(x)\middle|-1\right)+\frac{7}{15}E\left(\sin^{-1}(x)\middle|-1\right)$$

```
[Out] 7/15*EllipticE(x,I)-7/15*EllipticF(x,I)-7/45*x^3*(-x^4+1)^(1/2)-1/9*x^7*(-x
^4+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {321, 307, 221, 1181, 424}

$$-\frac{1}{9}\sqrt{1-x^4}\,x^7-\frac{7}{45}\sqrt{1-x^4}\,x^3-\frac{7}{15}F\left(\sin^{-1}(x)\middle|-1\right)+\frac{7}{15}E\left(\sin^{-1}(x)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^10/Sqrt[1 - x^4],x]
```

```
[Out] (-7*x^3*Sqrt[1 - x^4])/45 - (x^7*Sqrt[1 - x^4])/9 + (7*EllipticE[ArcSin[x],
 -1])/15 - (7*EllipticF[ArcSin[x], -1])/15
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[(d + e*x^2)/(Sqrt[q + c*x^2]*Sqrt[q - c
*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{10}}{\sqrt{1-x^4}}\,dx &= -\frac{1}{9}x^7\sqrt{1-x^4} + \frac{7}{9}\int \frac{x^6}{\sqrt{1-x^4}}\,dx \\ &= -\frac{7}{45}x^3\sqrt{1-x^4} - \frac{1}{9}x^7\sqrt{1-x^4} + \frac{7}{15}\int \frac{x^2}{\sqrt{1-x^4}}\,dx \\ &= -\frac{7}{45}x^3\sqrt{1-x^4} - \frac{1}{9}x^7\sqrt{1-x^4} - \frac{7}{15}\int \frac{1}{\sqrt{1-x^4}}\,dx + \frac{7}{15}\int \frac{1+x^2}{\sqrt{1-x^4}}\,dx \\ &= -\frac{7}{45}x^3\sqrt{1-x^4} - \frac{1}{9}x^7\sqrt{1-x^4} - \frac{7}{15}F\left(\sin^{-1}(x)\middle|-1\right) + \frac{7}{15}\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx \\ &= -\frac{7}{45}x^3\sqrt{1-x^4} - \frac{1}{9}x^7\sqrt{1-x^4} + \frac{7}{15}E\left(\sin^{-1}(x)\middle|-1\right) - \frac{7}{15}F\left(\sin^{-1}(x)\middle|-1\right)\end{aligned}$$

Mathematica [C] time = 0.03, size = 43, normalized size = 0.81

$$\frac{1}{45}x^3\left(7\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};x^4\right) - \sqrt{1-x^4}\left(5x^4+7\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/Sqrt[1 - x^4],x]
```

```
[Out] (x^3*(-(Sqrt[1 - x^4]*(7 + 5*x^4)) + 7*Hypergeometric2F1[1/2, 3/4, 7/4, x^4
])/45
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^4+1}\,x^{10}}{x^4-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(-x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 1)*x^10/(x^4 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\sqrt{-x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(-x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^10/sqrt(-x^4 + 1), x)
```

maple [A] time = 0.01, size = 68, normalized size = 1.28

$$-\frac{\sqrt{-x^4+1}\,x^7}{9} - \frac{7\sqrt{-x^4+1}\,x^3}{45} - \frac{7\sqrt{-x^2+1}\,\sqrt{x^2+1}\,\left(-\text{EllipticE}\left(x,i\right)+\text{EllipticF}\left(x,i\right)\right)}{15\sqrt{-x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(-x^4+1)^(1/2),x)
```

```
[Out] -1/9*x^7*(-x^4+1)^(1/2)-7/45*x^3*(-x^4+1)^(1/2)-7/15*(-x^2+1)^(1/2)*(x^2+1)
^(1/2)/(-x^4+1)^(1/2)*(EllipticF(x,I)-EllipticE(x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\sqrt{-x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(-x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^10/sqrt(-x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{10}}{\sqrt{1-x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(1 - x^4)^(1/2),x)
```

```
[Out] int(x^10/(1 - x^4)^(1/2), x)
```

sympy [A] time = 1.21, size = 31, normalized size = 0.58

$$\frac{x^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{11}{4} \\ \frac{15}{4}\end{matrix}\right| x^4 e^{2i\pi}\right)}{4\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(-x**4+1)**(1/2),x)
```

```
[Out] x**11*gamma(11/4)*hyper((1/2, 11/4), (15/4,), x**4*exp_polar(2*I*pi))/(4*ga
mma(15/4))
```

3.890 $\int \frac{x^6}{\sqrt{1-x^4}}\,dx$

Optimal. Leaf size=35

$$-\frac{1}{5}\sqrt{1-x^4}\,x^3-\frac{3}{5}F\left(\sin^{-1}(x)\middle|-1\right)+\frac{3}{5}E\left(\sin^{-1}(x)\middle|-1\right)$$

```
[Out] 3/5*EllipticE(x,I)-3/5*EllipticF(x,I)-1/5*x^3*(-x^4+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {321, 307, 221, 1181, 424}

$$-\frac{1}{5}\sqrt{1-x^4}\,x^3-\frac{3}{5}F\left(\sin^{-1}(x)\middle|-1\right)+\frac{3}{5}E\left(\sin^{-1}(x)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^6/Sqrt[1 - x^4],x]
```

```
[Out] -(x^3*Sqrt[1 - x^4])/5 + (3*EllipticE[ArcSin[x], -1])/5 - (3*EllipticF[ArcS
in[x], -1])/5
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[(d + e*x^2)/(Sqrt[q + c*x^2]*Sqrt[q - c
*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{\sqrt{1-x^4}}\,dx &= -\frac{1}{5}x^3\sqrt{1-x^4} + \frac{3}{5}\int \frac{x^2}{\sqrt{1-x^4}}\,dx \\ &= -\frac{1}{5}x^3\sqrt{1-x^4} - \frac{3}{5}\int \frac{1}{\sqrt{1-x^4}}\,dx + \frac{3}{5}\int \frac{1+x^2}{\sqrt{1-x^4}}\,dx \\ &= -\frac{1}{5}x^3\sqrt{1-x^4} - \frac{3}{5}F\left(\sin^{-1}(x)\middle|-1\right) + \frac{3}{5}\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx \\ &= -\frac{1}{5}x^3\sqrt{1-x^4} + \frac{3}{5}E\left(\sin^{-1}(x)\middle|-1\right) - \frac{3}{5}F\left(\sin^{-1}(x)\middle|-1\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.97

$$\frac{1}{5}x^3\left({}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};x^4\right) - \sqrt{1-x^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/Sqrt[1 - x^4],x]
```

```
[Out] (x^3*(-Sqrt[1 - x^4] + Hypergeometric2F1[1/2, 3/4, 7/4, x^4]))/5
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^4+1}\,x^6}{x^4-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 1)*x^6/(x^4 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt{-x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/sqrt(-x^4 + 1), x)
```

maple [A] time = 0.01, size = 54, normalized size = 1.54

$$-\frac{\sqrt{-x^4+1}\,x^3}{5} - \frac{3\sqrt{-x^2+1}\,\sqrt{x^2+1}\,\left(-\text{EllipticE}\left(x,i\right)+\text{EllipticF}\left(x,i\right)\right)}{5\sqrt{-x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-x^4+1)^(1/2),x)
```

```
[Out] -1/5*(-x^4+1)^(1/2)*x^3-3/5*(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)*(El
lipticF(x,I)-EllipticE(x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt{-x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/sqrt(-x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^6}{\sqrt{1-x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(1 - x^4)^(1/2),x)
```

```
[Out] int(x^6/(1 - x^4)^(1/2), x)
```

sympy [A] time = 1.55, size = 31, normalized size = 0.89

$$\frac{x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{7}{4} \\ \frac{11}{4}\end{matrix}\right| x^4 e^{2i\pi}\right)}{4\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-x**4+1)**(1/2),x)
```

```
[Out] x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), x**4*exp_polar(2*I*pi))/(4*gamma
(11/4))
```

3.891 $\int \frac{x^2}{\sqrt{1-x^4}}\,dx$

Optimal. Leaf size=11

$$E\left(\sin^{-1}(x)\middle|-1\right)-F\left(\sin^{-1}(x)\middle|-1\right)$$

```
[Out] EllipticE(x,I)-EllipticF(x,I)
```

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {307, 221, 1181, 424}

$$E\left(\sin^{-1}(x)\middle|-1\right)-F\left(\sin^{-1}(x)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[1 - x^4],x]
```

```
[Out] EllipticE[ArcSin[x], -1] - EllipticF[ArcSin[x], -1]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[(d + e*x^2)/(Sqrt[q + c*x^2]*Sqrt[q - c
*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{1-x^4}}\,dx &= -\int \frac{1}{\sqrt{1-x^4}}\,dx + \int \frac{1+x^2}{\sqrt{1-x^4}}\,dx \\ &= -F\left(\sin^{-1}(x)\middle|-1\right) + \int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx \\ &= E\left(\sin^{-1}(x)\middle|-1\right) - F\left(\sin^{-1}(x)\middle|-1\right)\end{aligned}$$

Mathematica [C] time = 0.00, size = 20, normalized size = 1.82

$$\frac{1}{3}x^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[1 - x^4],x]
[Out] (x^3*Hypergeometric2F1[1/2, 3/4, 7/4, x^4])/3
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^4+1}\,x^2}{x^4-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-x^4 + 1)*x^2/(x^4 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/sqrt(-x^4 + 1), x)
```

maple [B] time = 0.01, size = 39, normalized size = 3.55

$$-\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,\left(-\text{EllipticE}\left(x,i\right)+\text{EllipticF}\left(x,i\right)\right)}{\sqrt{-x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^4+1)^(1/2),x)
[Out] -(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)*(EllipticF(x,I)-EllipticE(x,I)
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/sqrt(-x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.09

$$\int \frac{x^2}{\sqrt{1-x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1 - x^4)^(1/2),x)
[Out] int(x^2/(1 - x^4)^(1/2), x)
```

sympy [**B**] time = 1.54, size = 31, normalized size = 2.82

$$\frac{x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{3}{4} \\ \frac{7}{4}\end{matrix}\right| x^4 e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**4+1)**(1/2),x)
```

```
[Out] x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**4*exp_polar(2*I*pi))/(4*gamma(
7/4))
```

3.892 $\int \frac{1}{x^2\sqrt{1-x^4}}\,dx$

Optimal. Leaf size=27

$$-\frac{\sqrt{1-x^4}}{x}+F\left(\sin^{-1}(x)\middle|-1\right)-E\left(\sin^{-1}(x)\middle|-1\right)$$

```
[Out] -EllipticE(x,I)+EllipticF(x,I)-(-x^4+1)^(1/2)/x
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {325, 307, 221, 1181, 424}

$$-\frac{\sqrt{1-x^4}}{x}+F\left(\sin^{-1}(x)\middle|-1\right)-E\left(\sin^{-1}(x)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[1 - x^4]),x]
```

```
[Out] -(Sqrt[1 - x^4]/x) - EllipticE[ArcSin[x], -1] + EllipticF[ArcSin[x], -1]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
 + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[(d + e*x^2)/(Sqrt[q + c*x^2]*Sqrt[q - c
*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{1-x^4}}\,dx &= -\frac{\sqrt{1-x^4}}{x} - \int \frac{x^2}{\sqrt{1-x^4}}\,dx \\ &= -\frac{\sqrt{1-x^4}}{x} + \int \frac{1}{\sqrt{1-x^4}}\,dx - \int \frac{1+x^2}{\sqrt{1-x^4}}\,dx \\ &= -\frac{\sqrt{1-x^4}}{x} + F\left(\sin^{-1}(x)\middle|-1\right) - \int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx \\ &= -\frac{\sqrt{1-x^4}}{x} - E\left(\sin^{-1}(x)\middle|-1\right) + F\left(\sin^{-1}(x)\middle|-1\right)\end{aligned}$$

Mathematica [C] time = 0.00, size = 18, normalized size = 0.67

$$-\frac{{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};x^4\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[1 - x^4]),x]
[Out] -(Hypergeometric2F1[-1/4, 1/2, 3/4, x^4]/x)
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^4+1}}{x^6-x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^4+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-x^4 + 1)/(x^6 - x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^4+1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-x^4 + 1)*x^2), x)
```

maple [B] time = 0.01, size = 53, normalized size = 1.96

$$-\frac{\sqrt{-x^4+1}}{x} + \frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,\left(-\text{EllipticE}\left(x,i\right)+\text{EllipticF}\left(x,i\right)\right)}{\sqrt{-x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-x^4+1)^(1/2),x)
[Out] -(-x^4+1)^(1/2)/x+(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)*(EllipticF(x,
I)-EllipticE(x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^4+1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^4 + 1)*x^2), x)
```

mupad [**B**] time = 1.12, size = 13, normalized size = 0.48

$$-\frac{{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ x^4\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(1 - x^4)^(1/2)),x)
```

```
[Out] -hypergeom([-1/4, 1/2], 3/4, x^4)/x
```

sympy [**B**] time = 1.13, size = 32, normalized size = 1.19

$$\frac{\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\right| x^4 e^{2i\pi}\right)}{4x\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-x**4+1)**(1/2),x)
```

```
[Out] gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), x**4*exp_polar(2*I*pi))/(4*x*gamma(3
/4))
```

3.893 $\int \frac{1}{x^6\sqrt{1-x^4}}\,dx$

Optimal. Leaf size=53

$$-\frac{3\sqrt{1-x^4}}{5x}-\frac{\sqrt{1-x^4}}{5x^5}+\frac{3}{5}F\left(\sin^{-1}(x)\middle|-1\right)-\frac{3}{5}E\left(\sin^{-1}(x)\middle|-1\right)$$

```
[Out] -3/5*EllipticE(x,I)+3/5*EllipticF(x,I)-1/5*(-x^4+1)^(1/2)/x^5-3/5*(-x^4+1)^
(1/2)/x
```

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {325, 307, 221, 1181, 424}

$$-\frac{3\sqrt{1-x^4}}{5x}-\frac{\sqrt{1-x^4}}{5x^5}+\frac{3}{5}F\left(\sin^{-1}(x)\middle|-1\right)-\frac{3}{5}E\left(\sin^{-1}(x)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*Sqrt[1 - x^4]),x]
```

```
[Out] -Sqrt[1 - x^4]/(5*x^5) - (3*Sqrt[1 - x^4])/(5*x) - (3*EllipticE[ArcSin[x],
-1])/5 + (3*EllipticF[ArcSin[x], -1])/5
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[(d + e*x^2)/(Sqrt[q + c*x^2]*Sqrt[q - c
*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^6\sqrt{1-x^4}}\,dx &= -\frac{\sqrt{1-x^4}}{5x^5} + \frac{3}{5}\int \frac{1}{x^2\sqrt{1-x^4}}\,dx \\ &= -\frac{\sqrt{1-x^4}}{5x^5} - \frac{3\sqrt{1-x^4}}{5x} - \frac{3}{5}\int \frac{x^2}{\sqrt{1-x^4}}\,dx \\ &= -\frac{\sqrt{1-x^4}}{5x^5} - \frac{3\sqrt{1-x^4}}{5x} + \frac{3}{5}\int \frac{1}{\sqrt{1-x^4}}\,dx - \frac{3}{5}\int \frac{1+x^2}{\sqrt{1-x^4}}\,dx \\ &= -\frac{\sqrt{1-x^4}}{5x^5} - \frac{3\sqrt{1-x^4}}{5x} + \frac{3}{5}F\left(\sin^{-1}(x)\middle|-1\right) - \frac{3}{5}\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx \\ &= -\frac{\sqrt{1-x^4}}{5x^5} - \frac{3\sqrt{1-x^4}}{5x} - \frac{3}{5}E\left(\sin^{-1}(x)\middle|-1\right) + \frac{3}{5}F\left(\sin^{-1}(x)\middle|-1\right)\end{aligned}$$

Mathematica [C] time = 0.00, size = 20, normalized size = 0.38

$$-\frac{{}_2F_1\left(-\frac{5}{4},\frac{1}{2};-\frac{1}{4};x^4\right)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*Sqrt[1 - x^4]),x]
[Out] -1/5*Hypergeometric2F1[-5/4, 1/2, -1/4, x^4]/x^5
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^4+1}}{x^{10}-x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^4+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-x^4 + 1)/(x^10 - x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^4+1}\,x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-x^4 + 1)*x^6), x)
```

maple [A] time = 0.01, size = 68, normalized size = 1.28

$$-\frac{3\sqrt{-x^4+1}}{5x} - \frac{\sqrt{-x^4+1}}{5x^5} + \frac{3\sqrt{-x^2+1}\,\sqrt{x^2+1}\,\left(-\text{EllipticE}\left(x,i\right)+\text{EllipticF}\left(x,i\right)\right)}{5\sqrt{-x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-x^4+1)^(1/2),x)
[Out] -1/5*(-x^4+1)^(1/2)/x^5-3/5*(-x^4+1)^(1/2)/x+3/5*(-x^2+1)^(1/2)*(x^2+1)^(1/
2)/(-x^4+1)^(1/2)*(EllipticF(x,I)-EllipticE(x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^4+1}\,x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^4 + 1)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^6\sqrt{1-x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(1 - x^4)^(1/2)),x)
```

```
[Out] int(1/(x^6*(1 - x^4)^(1/2)), x)
```

sympy [A] time = 1.01, size = 37, normalized size = 0.70

$$\frac{\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{5}{4}, \frac{1}{2} \\ -\frac{1}{4}\end{matrix}\right| x^4 e^{2i\pi}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-x**4+1)**(1/2),x)
```

```
[Out] gamma(-5/4)*hyper((-5/4, 1/2), (-1/4,), x**4*exp_polar(2*I*pi))/(4*x**5*gam
ma(-1/4))
```

3.894 $\int \frac{x^{11}}{\left(1-x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=42

$$-\frac{1}{6}\left(1-x^4\right)^{3/2}+\sqrt{1-x^4}+\frac{1}{2\sqrt{1-x^4}}$$

```
[Out] -1/6*(-x^4+1)^(3/2)+1/2/(-x^4+1)^(1/2)+(-x^4+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{1}{6}\left(1-x^4\right)^{3/2}+\sqrt{1-x^4}+\frac{1}{2\sqrt{1-x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(1 - x^4)^(3/2),x]
```

```
[Out] 1/(2*Sqrt[1 - x^4]) + Sqrt[1 - x^4] - (1 - x^4)^(3/2)/6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{\left(1-x^4\right)^{3/2}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x^2}{(1-x)^{3/2}}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int \left(\frac{1}{(1-x)^{3/2}}-\frac{2}{\sqrt{1-x}}+\sqrt{1-x}\right)\,dx, x, x^4\right)\\ &= \frac{1}{2\sqrt{1-x^4}}+\sqrt{1-x^4}-\frac{1}{6}\left(1-x^4\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.64

$$\frac{-x^8-4x^4+8}{6\sqrt{1-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(1 - x^4)^(3/2),x]
```

```
[Out] (8 - 4*x^4 - x^8)/(6*Sqrt[1 - x^4])
```

fricas [A] time = 0.85, size = 28, normalized size = 0.67

$$\frac{\left(x^8+4\,x^4-8\right)\sqrt{-x^4+1}}{6\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^4+1)^(3/2),x, algorithm="fricas")
[Out] 1/6*(x^8 + 4*x^4 - 8)*sqrt(-x^4 + 1)/(x^4 - 1)
```

giac [A] time = 0.17, size = 32, normalized size = 0.76

$$-\frac{1}{6}\left(-x^4+1\right)^{\frac{3}{2}}+\sqrt{-x^4+1}+\frac{1}{2\sqrt{-x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^4+1)^(3/2),x, algorithm="giac")
[Out] -1/6*(-x^4 + 1)^(3/2) + sqrt(-x^4 + 1) + 1/2/sqrt(-x^4 + 1)
```

maple [A] time = 0.01, size = 33, normalized size = 0.79

$$\frac{(x-1)\,(x+1)\left(x^2+1\right)\left(x^8+4x^4-8\right)}{6\left(-x^4+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(-x^4+1)^(3/2),x)
[Out] 1/6*(x-1)*(x+1)*(x^2+1)*(x^8+4*x^4-8)/(-x^4+1)^(3/2)
```

maxima [A] time = 1.33, size = 32, normalized size = 0.76

$$-\frac{1}{6}\left(-x^4+1\right)^{\frac{3}{2}}+\sqrt{-x^4+1}+\frac{1}{2\sqrt{-x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^4+1)^(3/2),x, algorithm="maxima")
[Out] -1/6*(-x^4 + 1)^(3/2) + sqrt(-x^4 + 1) + 1/2/sqrt(-x^4 + 1)
```

mupad [B] time = 1.19, size = 25, normalized size = 0.60

$$-\frac{\left(x^4-1\right)^2+6\,x^4-9}{6\sqrt{1-x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(1 - x^4)^(3/2),x)
[Out] -((x^4 - 1)^2 + 6*x^4 - 9)/(6*(1 - x^4)^(1/2))
```

sympy [A] time = 1.72, size = 39, normalized size = 0.93

$$-\frac{x^8}{6\sqrt{1-x^4}}-\frac{2x^4}{3\sqrt{1-x^4}}+\frac{4}{3\sqrt{1-x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(-x**4+1)**(3/2),x)
[Out] -x**8/(6*sqrt(1 - x**4)) - 2*x**4/(3*sqrt(1 - x**4)) + 4/(3*sqrt(1 - x**4))
```

3.895 $\int \frac{x^7}{\left(1-x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=31

$$\frac{\sqrt{1-x^4}}{2}+\frac{1}{2\sqrt{1-x^4}}$$

```
[Out] 1/2/(-x^4+1)^(1/2)+1/2*(-x^4+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{\sqrt{1-x^4}}{2}+\frac{1}{2\sqrt{1-x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(1 - x^4)^(3/2),x]
```

```
[Out] 1/(2*Sqrt[1 - x^4]) + Sqrt[1 - x^4]/2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{\left(1-x^4\right)^{3/2}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x}{(1-x)^{3/2}}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int \left(\frac{1}{(1-x)^{3/2}}-\frac{1}{\sqrt{1-x}}\right)dx, x, x^4\right)\\ &= \frac{1}{2\sqrt{1-x^4}}+\frac{\sqrt{1-x^4}}{2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.71

$$\frac{2-x^4}{2\sqrt{1-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(1 - x^4)^(3/2),x]
```

```
[Out] (2 - x^4)/(2*Sqrt[1 - x^4])
```

fricas [A] time = 0.84, size = 23, normalized size = 0.74

$$\frac{\left(x^4-2\right)\sqrt{-x^4+1}}{2\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^4+1)^(3/2),x, algorithm="fricas")
[Out] 1/2*(x^4 - 2)*sqrt(-x^4 + 1)/(x^4 - 1)
```

giac [A] time = 0.15, size = 23, normalized size = 0.74

$$\frac{1}{2}\sqrt{-x^4+1}+\frac{1}{2\sqrt{-x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^4+1)^(3/2),x, algorithm="giac")
[Out] 1/2*sqrt(-x^4 + 1) + 1/2/sqrt(-x^4 + 1)
```

maple [A] time = 0.00, size = 28, normalized size = 0.90

$$\frac{(x-1)(x+1)\left(x^2+1\right)\left(x^4-2\right)}{2\left(-x^4+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-x^4+1)^(3/2),x)
[Out] 1/2*(x-1)*(x+1)*(x^2+1)*(x^4-2)/(-x^4+1)^(3/2)
```

maxima [A] time = 1.30, size = 23, normalized size = 0.74

$$\frac{1}{2}\sqrt{-x^4+1}+\frac{1}{2\sqrt{-x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^4+1)^(3/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-x^4 + 1) + 1/2/sqrt(-x^4 + 1)
```

mupad [B] time = 1.17, size = 16, normalized size = 0.52

$$-\frac{x^4-2}{2\sqrt{1-x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(1 - x^4)^(3/2),x)
[Out] -(x^4 - 2)/(2*(1 - x^4)^(1/2))
```

sympy [A] time = 1.20, size = 22, normalized size = 0.71

$$-\frac{x^4}{2\sqrt{1-x^4}}+\frac{1}{\sqrt{1-x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-x**4+1)**(3/2),x)
[Out] -x**4/(2*sqrt(1 - x**4)) + 1/sqrt(1 - x**4)
```

3.896 $\int \frac{x^3}{\left(1-x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=15

$$\frac{1}{2\sqrt{1-x^4}}$$

```
[Out] 1/2/(-x^4+1)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$\frac{1}{2\sqrt{1-x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 - x^4)^(3/2),x]
```

```
[Out] 1/(2*Sqrt[1 - x^4])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^3}{\left(1-x^4\right)^{3/2}}\,dx = \frac{1}{2\sqrt{1-x^4}}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{1}{2\sqrt{1-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 - x^4)^(3/2),x]
```

```
[Out] 1/(2*Sqrt[1 - x^4])
```

fricas [A] time = 0.78, size = 18, normalized size = 1.20

$$-\frac{\sqrt{-x^4+1}}{2\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^4+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-x^4 + 1)/(x^4 - 1)
```

giac [A] time = 0.19, size = 11, normalized size = 0.73

$$\frac{1}{2\sqrt{-x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^4+1)^(3/2),x, algorithm="giac")
[Out] 1/2/sqrt(-x^4 + 1)
```

maple [A] time = 0.01, size = 23, normalized size = 1.53

$$-\frac{(x-1)(x+1)\left(x^2+1\right)}{2\left(-x^4+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-x^4+1)^(3/2),x)
[Out] -1/2*(x-1)*(x+1)*(x^2+1)/(-x^4+1)^(3/2)
```

maxima [A] time = 1.36, size = 11, normalized size = 0.73

$$\frac{1}{2\sqrt{-x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^4+1)^(3/2),x, algorithm="maxima")
[Out] 1/2/sqrt(-x^4 + 1)
```

mupad [B] time = 1.19, size = 11, normalized size = 0.73

$$\frac{1}{2\sqrt{1-x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(1 - x^4)^(3/2),x)
[Out] 1/(2*(1 - x^4)^(1/2))
```

sympy [A] time = 0.59, size = 10, normalized size = 0.67

$$\frac{1}{2\sqrt{1-x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-x**4+1)**(3/2),x)
[Out] 1/(2*sqrt(1 - x**4))
```

3.897 $\int \frac{1}{x\left(1-x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=32

$$\frac{1}{2\sqrt{1-x^4}} - \frac{1}{2}\tanh^{-1}\left(\sqrt{1-x^4}\right)$$

```
[Out] -1/2*arctanh((-x^4+1)^(1/2))+1/2/(-x^4+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 51, 63, 206}

$$\frac{1}{2\sqrt{1-x^4}} - \frac{1}{2}\tanh^{-1}\left(\sqrt{1-x^4}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^4)^(3/2)),x]
```

```
[Out] 1/(2*Sqrt[1 - x^4]) - ArcTanh[Sqrt[1 - x^4]]/2
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(1-x^4\right)^{3/2}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{(1-x)^{3/2}x}\,dx, x, x^4\right)\\ &= \frac{1}{2\sqrt{1-x^4}} + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, x^4\right)\\ &= \frac{1}{2\sqrt{1-x^4}} - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-x^4}\right)\\ &= \frac{1}{2\sqrt{1-x^4}} - \frac{1}{2}\tanh^{-1}\left(\sqrt{1-x^4}\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 30, normalized size = 0.94

$$\frac{{}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; 1-x^4\right)}{2\sqrt{1-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^4)^(3/2)),x]
```

```
[Out] Hypergeometric2F1[-1/2, 1, 1/2, 1 - x^4]/(2*Sqrt[1 - x^4])
```

fricas [B] time = 0.99, size = 58, normalized size = 1.81

$$-\frac{\left(x^4-1\right)\log\left(\sqrt{-x^4+1}+1\right) - \left(x^4-1\right)\log\left(\sqrt{-x^4+1}-1\right) + 2\sqrt{-x^4+1}}{4\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^4+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*((x^4 - 1)*log(sqrt(-x^4 + 1) + 1) - (x^4 - 1)*log(sqrt(-x^4 + 1) - 1)
 + 2*sqrt(-x^4 + 1))/(x^4 - 1)
```

giac [A] time = 0.15, size = 42, normalized size = 1.31

$$\frac{1}{2\sqrt{-x^4+1}} - \frac{1}{4}\log\left(\sqrt{-x^4+1}+1\right) + \frac{1}{4}\log\left(-\sqrt{-x^4+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^4+1)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2/sqrt(-x^4 + 1) - 1/4*log(sqrt(-x^4 + 1) + 1) + 1/4*log(-sqrt(-x^4 + 1)
+ 1)
```

maple [B] time = 0.02, size = 68, normalized size = 2.12

$$-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^4+1}}\right)}{2} - \frac{\sqrt{-2x^2-\left(x^2-1\right)^2+2}}{4\left(x^2-1\right)} + \frac{\sqrt{2x^2-\left(x^2+1\right)^2+2}}{4x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^4+1)^(3/2),x)
```

```
[Out] -1/4/(x^2-1)*(-(x^2-1)^2-2*x^2+2)^(1/2)+1/4/(x^2+1)*(-(x^2+1)^2+2*x^2+2)^(1
/2)-1/2*arctanh(1/(-x^4+1)^(1/2))
```

maxima [A] time = 1.31, size = 40, normalized size = 1.25

$$\frac{1}{2\sqrt{-x^4+1}} - \frac{1}{4}\log\left(\sqrt{-x^4+1}+1\right) + \frac{1}{4}\log\left(\sqrt{-x^4+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^4+1)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2/sqrt(-x^4 + 1) - 1/4*log(sqrt(-x^4 + 1) + 1) + 1/4*log(sqrt(-x^4 + 1) -
 1)
```

mupad [B] time = 1.22, size = 24, normalized size = 0.75

$$\frac{1}{2\sqrt{1-x^4}} - \frac{\operatorname{atanh}\left(\sqrt{1-x^4}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1 - x^4)^(3/2)),x)
```

```
[Out] 1/(2*(1 - x^4)^(1/2)) - atanh((1 - x^4)^(1/2))/2
```

sympy [C] time = 2.70, size = 228, normalized size = 7.12

$$\begin{cases} \frac{2x^4\log\left(x^2\right)}{4-4x^4} - \frac{x^4\log\left(x^4\right)}{4-4x^4} - \frac{2ix^4\operatorname{asin}\left(\frac{1}{x^2}\right)}{4-4x^4} + \frac{2i\sqrt{x^4-1}}{4-4x^4} - \frac{2\log\left(x^2\right)}{4-4x^4} + \frac{\log\left(x^4\right)}{4-4x^4} + \frac{2i\operatorname{asin}\left(\frac{1}{x^2}\right)}{4-4x^4} & \text{for } \left|x^4\right| > 1 \\ -\frac{x^4\log\left(x^4\right)}{4-4x^4} + \frac{2x^4\log\left(\sqrt{1-x^4}+1\right)}{4-4x^4} - \frac{i\pi x^4}{4-4x^4} + \frac{2\sqrt{1-x^4}}{4-4x^4} + \frac{\log\left(x^4\right)}{4-4x^4} - \frac{2\log\left(\sqrt{1-x^4}+1\right)}{4-4x^4} + \frac{i\pi}{4-4x^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**4+1)**(3/2),x)
```

```
[Out] Piecewise((2*x**4*log(x**2)/(4 - 4*x**4) - x**4*log(x**4)/(4 - 4*x**4) - 2*
I*x**4*asin(x**(-2))/(4 - 4*x**4) + 2*I*sqrt(x**4 - 1)/(4 - 4*x**4) - 2*log
(x**2)/(4 - 4*x**4) + log(x**4)/(4 - 4*x**4) + 2*I*asin(x**(-2))/(4 - 4*x**
4), Abs(x**4) > 1), (-x**4*log(x**4)/(4 - 4*x**4) + 2*x**4*log(sqrt(1 - x**
4) + 1)/(4 - 4*x**4) - I*pi*x**4/(4 - 4*x**4) + 2*sqrt(1 - x**4)/(4 - 4*x**
4) + log(x**4)/(4 - 4*x**4) - 2*log(sqrt(1 - x**4) + 1)/(4 - 4*x**4) + I*pi
/(4 - 4*x**4), True))
```

3.898 $\int \frac{1}{x^5\left(1-x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=50

$$-\frac{1}{4x^4\sqrt{1-x^4}}+\frac{3}{4\sqrt{1-x^4}}-\frac{3}{4}\tanh^{-1}\left(\sqrt{1-x^4}\right)$$

```
[Out] -3/4*arctanh((-x^4+1)^(1/2))+3/4/(-x^4+1)^(1/2)-1/4/x^4/(-x^4+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 51, 63, 206}

$$-\frac{3\sqrt{1-x^4}}{4x^4}+\frac{1}{2x^4\sqrt{1-x^4}}-\frac{3}{4}\tanh^{-1}\left(\sqrt{1-x^4}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 - x^4)^(3/2)),x]
```

```
[Out] 1/(2*x^4*Sqrt[1 - x^4]) - (3*Sqrt[1 - x^4])/(4*x^4) - (3*ArcTanh[Sqrt[1 - x
^4]])/4
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\left(1-x^4\right)^{3/2}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{(1-x)^{3/2}x^2}\,dx, x, x^4\right)\\ &= \frac{1}{2x^4\sqrt{1-x^4}} + \frac{3}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x^2}\,dx, x, x^4\right)\\ &= \frac{1}{2x^4\sqrt{1-x^4}} - \frac{3\sqrt{1-x^4}}{4x^4} + \frac{3}{8}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, x^4\right)\\ &= \frac{1}{2x^4\sqrt{1-x^4}} - \frac{3\sqrt{1-x^4}}{4x^4} - \frac{3}{4}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-x^4}\right)\\ &= \frac{1}{2x^4\sqrt{1-x^4}} - \frac{3\sqrt{1-x^4}}{4x^4} - \frac{3}{4}\tanh^{-1}\left(\sqrt{1-x^4}\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 30, normalized size = 0.60

$$\frac{{}_2F_1\left(-\frac{1}{2}, 2; \frac{1}{2}; 1-x^4\right)}{2\sqrt{1-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 - x^4)^(3/2)),x]
```

```
[Out] Hypergeometric2F1[-1/2, 2, 1/2, 1 - x^4]/(2*Sqrt[1 - x^4])
```

fricas [B] time = 0.63, size = 78, normalized size = 1.56

$$-\frac{3\left(x^8-x^4\right)\log\left(\sqrt{-x^4+1}+1\right) - 3\left(x^8-x^4\right)\log\left(\sqrt{-x^4+1}-1\right) + 2\left(3x^4-1\right)\sqrt{-x^4+1}}{8\left(x^8-x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^4+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/8*(3*(x^8 - x^4)*log(sqrt(-x^4 + 1) + 1) - 3*(x^8 - x^4)*log(sqrt(-x^4 +
 1) - 1) + 2*(3*x^4 - 1)*sqrt(-x^4 + 1))/(x^8 - x^4)
```

giac [A] time = 0.17, size = 63, normalized size = 1.26

$$-\frac{3x^4-1}{4\left(\left(-x^4+1\right)^{\frac{3}{2}} - \sqrt{-x^4+1}\right)} - \frac{3}{8}\log\left(\sqrt{-x^4+1}+1\right) + \frac{3}{8}\log\left(-\sqrt{-x^4+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^4+1)^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*(3*x^4 - 1)/((-x^4 + 1)^(3/2) - sqrt(-x^4 + 1)) - 3/8*log(sqrt(-x^4 +
1) + 1) + 3/8*log(-sqrt(-x^4 + 1) + 1)
```

maple [B] time = 0.02, size = 82, normalized size = 1.64

$$-\frac{3\,\text{arctanh}\left(\frac{1}{\sqrt{-x^4+1}}\right)}{4} - \frac{\sqrt{-x^4+1}}{4x^4} - \frac{\sqrt{-2x^2-\left(x^2-1\right)^2+2}}{4\left(x^2-1\right)} + \frac{\sqrt{2x^2-\left(x^2+1\right)^2+2}}{4x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-x^4+1)^(3/2),x)
```

```
[Out] -1/4/(x^2-1)*(-2*x^2-(x^2-1)^2+2)^(1/2)-1/4*(-x^4+1)^(1/2)/x^4-3/4*arctanh(
1/(-x^4+1)^(1/2))+1/4/(x^2+1)*(2*x^2-(x^2+1)^2+2)^(1/2)
```

maxima [A] time = 1.37, size = 61, normalized size = 1.22

$$-\frac{3\,x^4-1}{4\left(\left(-x^4+1\right)^{\frac{3}{2}}-\sqrt{-x^4+1}\right)}-\frac{3}{8}\log\left(\sqrt{-x^4+1}+1\right)+\frac{3}{8}\log\left(\sqrt{-x^4+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^4+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(3*x^4 - 1)/((-x^4 + 1)^(3/2) - sqrt(-x^4 + 1)) - 3/8*log(sqrt(-x^4 +
1) + 1) + 3/8*log(sqrt(-x^4 + 1) - 1)
```

mupad [B] time = 1.23, size = 38, normalized size = 0.76

$$\frac{3}{4\sqrt{1-x^4}}-\frac{1}{4\,x^4\sqrt{1-x^4}}-\frac{3\,\mathrm{atanh}\left(\sqrt{1-x^4}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(1 - x^4)^(3/2)),x)
```

```
[Out] 3/(4*(1 - x^4)^(1/2)) - 1/(4*x^4*(1 - x^4)^(1/2)) - (3*atanh((1 - x^4)^(1/2
)))/4
```

sympy [A] time = 4.87, size = 95, normalized size = 1.90

$$\begin{cases}-\frac{3\,\mathrm{acosh}\left(\frac{1}{x^2}\right)}{4}+\frac{3}{4x^2\sqrt{-1+\frac{1}{x^4}}}-\frac{1}{4x^6\sqrt{-1+\frac{1}{x^4}}} & \text{for } \frac{1}{|x^4|}>1\\ \frac{3i\,\mathrm{asin}\left(\frac{1}{x^2}\right)}{4}-\frac{3i}{4x^2\sqrt{1-\frac{1}{x^4}}}+\frac{i}{4x^6\sqrt{1-\frac{1}{x^4}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-x**4+1)**(3/2),x)
```

```
[Out] Piecewise((-3*acosh(x**(-2))/4 + 3/(4*x**2*sqrt(-1 + x**(-4))) - 1/(4*x**6*
sqrt(-1 + x**(-4))), 1/Abs(x**4) > 1), (3*I*asin(x**(-2))/4 - 3*I/(4*x**2*s
qrt(1 - 1/x**4)) + I/(4*x**6*sqrt(1 - 1/x**4)), True))
```

3.899 $\int \frac{x^9}{\left(1-x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=45

$$-\frac{3}{4}\sin^{-1}\left(x^2\right)+\frac{x^6}{2\sqrt{1-x^4}}+\frac{3}{4}\sqrt{1-x^4}\,x^2$$

```
[Out] -3/4*arcsin(x^2)+1/2*x^6/(-x^4+1)^(1/2)+3/4*x^2*(-x^4+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {275, 288, 321, 216}

$$\frac{x^6}{2\sqrt{1-x^4}}+\frac{3}{4}\sqrt{1-x^4}\,x^2-\frac{3}{4}\sin^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(1 - x^4)^(3/2),x]
```

```
[Out] x^6/(2*Sqrt[1 - x^4]) + (3*x^2*Sqrt[1 - x^4])/4 - (3*ArcSin[x^2])/4
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{\left(1-x^4\right)^{3/2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)^{3/2}}\,dx, x, x^2\right)\\ &= \frac{x^6}{2\sqrt{1-x^4}} - \frac{3}{2}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1-x^2}}\,dx, x, x^2\right)\\ &= \frac{x^6}{2\sqrt{1-x^4}} + \frac{3}{4}x^2\sqrt{1-x^4} - \frac{3}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, x^2\right)\\ &= \frac{x^6}{2\sqrt{1-x^4}} + \frac{3}{4}x^2\sqrt{1-x^4} - \frac{3}{4}\sin^{-1}\left(x^2\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 0.91

$$-\frac{x^6 - 3x^2 + 3\sqrt{1-x^4}\,\sin^{-1}\left(x^2\right)}{4\sqrt{1-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(1 - x^4)^(3/2),x]
```

```
[Out] -1/4*(-3*x^2 + x^6 + 3*Sqrt[1 - x^4]*ArcSin[x^2])/Sqrt[1 - x^4]
```

fricas [A] time = 0.86, size = 52, normalized size = 1.16

$$\frac{6\left(x^4-1\right)\arctan\left(\frac{\sqrt{-x^4+1}-1}{x^2}\right) + \left(x^6-3\,x^2\right)\sqrt{-x^4+1}}{4\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(-x^4+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(6*(x^4 - 1)*arctan((sqrt(-x^4 + 1) - 1)/x^2) + (x^6 - 3*x^2)*sqrt(-x^4
 + 1))/(x^4 - 1)
```

giac [A] time = 0.19, size = 33, normalized size = 0.73

$$\frac{\left(x^4-3\right)\sqrt{-x^4+1}\,x^2}{4\left(x^4-1\right)} - \frac{3}{4}\,\arcsin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(-x^4+1)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(x^4 - 3)*sqrt(-x^4 + 1)*x^2/(x^4 - 1) - 3/4*arcsin(x^2)
```

maple [B] time = 0.02, size = 76, normalized size = 1.69

$$\frac{\sqrt{-x^4+1}\,x^2}{4} - \frac{3\arcsin\left(x^2\right)}{4} - \frac{\sqrt{-2x^2-\left(x^2-1\right)^2+2}}{4\left(x^2-1\right)} - \frac{\sqrt{2x^2-\left(x^2+1\right)^2+2}}{4\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(-x^4+1)^(3/2),x)
```

```
[Out] 1/4*(-x^4+1)^(1/2)*x^2-3/4*arcsin(x^2)-1/4/(x^2-1)*(-2*x^2-(x^2-1)^2+2)^(1/
2)-1/4/(x^2+1)*(2*x^2-(x^2+1)^2+2)^(1/2)
```

maxima [A] time = 2.95, size = 60, normalized size = 1.33

$$-\frac{\frac{3\left(x^4-1\right)}{x^4}-2}{4\left(\frac{\sqrt{-x^4+1}}{x^2}+\frac{\left(-x^4+1\right)^{\frac{3}{2}}}{x^6}\right)}+\frac{3}{4}\arctan\left(\frac{\sqrt{-x^4+1}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(-x^4+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(3*(x^4 - 1)/x^4 - 2)/(sqrt(-x^4 + 1)/x^2 + (-x^4 + 1)^(3/2)/x^6) + 3/
4*arctan(sqrt(-x^4 + 1)/x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^9}{\left(1-x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(1 - x^4)^(3/2),x)
```

```
[Out] int(x^9/(1 - x^4)^(3/2), x)
```

sympy [A] time = 4.16, size = 82, normalized size = 1.82

$$\begin{cases} \frac{ix^6}{4\sqrt{x^4-1}}-\frac{3ix^2}{4\sqrt{x^4-1}}+\frac{3i\,\operatorname{acosh}\left(x^2\right)}{4} & \text{for } \left|x^4\right|>1 \\ -\frac{x^6}{4\sqrt{1-x^4}}+\frac{3x^2}{4\sqrt{1-x^4}}-\frac{3\operatorname{asin}\left(x^2\right)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(-x**4+1)**(3/2),x)
```

```
[Out] Piecewise((I*x**6/(4*sqrt(x**4 - 1)) - 3*I*x**2/(4*sqrt(x**4 - 1)) + 3*I*ac
osh(x**2)/4, Abs(x**4) > 1), (-x**6/(4*sqrt(1 - x**4)) + 3*x**2/(4*sqrt(1 -
 x**4)) - 3*asin(x**2)/4, True))
```

3.900 $\int \frac{x^5}{\left(1-x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=27

$$\frac{x^2}{2\sqrt{1-x^4}} - \frac{1}{2}\sin^{-1}\left(x^2\right)$$

```
[Out] -1/2*arcsin(x^2)+1/2*x^2/(-x^4+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {275, 288, 216}

$$\frac{x^2}{2\sqrt{1-x^4}} - \frac{1}{2}\sin^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(1 - x^4)^(3/2),x]
```

```
[Out] x^2/(2*Sqrt[1 - x^4]) - ArcSin[x^2]/2
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\left(1-x^4\right)^{3/2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)^{3/2}}\,dx, x, x^2\right)\\ &= \frac{x^2}{2\sqrt{1-x^4}} - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, x^2\right)\\ &= \frac{x^2}{2\sqrt{1-x^4}} - \frac{1}{2}\sin^{-1}\left(x^2\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 0.96

$$\frac{1}{2}\left(\frac{x^2}{\sqrt{1-x^4}} - \sin^{-1}\left(x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(1 - x^4)^(3/2),x]
```

```
[Out] (x^2/Sqrt[1 - x^4] - ArcSin[x^2])/2
```

fricas [B] time = 1.09, size = 46, normalized size = 1.70

$$-\frac{\sqrt{-x^4+1}\,x^2-2\left(x^4-1\right)\arctan\left(\frac{\sqrt{-x^4+1}-1}{x^2}\right)}{2\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^4+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(-x^4 + 1)*x^2 - 2*(x^4 - 1)*arctan((sqrt(-x^4 + 1) - 1)/x^2))/(x
^4 - 1)
```

giac [A] time = 0.18, size = 28, normalized size = 1.04

$$-\frac{\sqrt{-x^4+1}\,x^2}{2\left(x^4-1\right)}-\frac{1}{2}\arcsin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^4+1)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-x^4 + 1)*x^2/(x^4 - 1) - 1/2*arcsin(x^2)
```

maple [B] time = 0.02, size = 62, normalized size = 2.30

$$-\frac{\arcsin\left(x^2\right)}{2}-\frac{\sqrt{-2x^2-\left(x^2-1\right)^2+2}}{4\left(x^2-1\right)}-\frac{\sqrt{2x^2-\left(x^2+1\right)^2+2}}{4\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-x^4+1)^(3/2),x)
```

```
[Out] -1/2*arcsin(x^2)-1/4/(x^2-1)*(-2*x^2-(x^2-1)^2+2)^(1/2)-1/4/(x^2+1)*(2*x^2-
(x^2+1)^2+2)^(1/2)
```

maxima [A] time = 2.91, size = 31, normalized size = 1.15

$$\frac{x^2}{2\sqrt{-x^4+1}}+\frac{1}{2}\arctan\left(\frac{\sqrt{-x^4+1}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^4+1)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/sqrt(-x^4 + 1) + 1/2*arctan(sqrt(-x^4 + 1)/x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^5}{\left(1-x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(1 - x^4)^(3/2),x)
```

```
[Out] int(x^5/(1 - x^4)^(3/2), x)
```

sympy [**A**] time = 2.29, size = 46, normalized size = 1.70

$$\begin{cases} -\frac{ix^2}{2\sqrt{x^4-1}} + \frac{i\,\operatorname{acosh}\left(x^2\right)}{2} & \text{for } \left|x^4\right| > 1 \\ \frac{x^2}{2\sqrt{1-x^4}} - \frac{\operatorname{asin}\left(x^2\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-x**4+1)**(3/2),x)
```

```
[Out] Piecewise((-I*x**2/(2*sqrt(x**4 - 1)) + I*acosh(x**2)/2, Abs(x**4) > 1), (x
**2/(2*sqrt(1 - x**4)) - asin(x**2)/2, True))
```

3.901 $\int \frac{x}{\left(1-x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=18

$$\frac{x^2}{2\sqrt{1-x^4}}$$

```
[Out] 1/2*x^2/(-x^4+1)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {264}

$$\frac{x^2}{2\sqrt{1-x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 - x^4)^(3/2),x]
```

```
[Out] x^2/(2*Sqrt[1 - x^4])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x}{\left(1-x^4\right)^{3/2}}\,dx = \frac{x^2}{2\sqrt{1-x^4}}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{x^2}{2\sqrt{1-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 - x^4)^(3/2),x]
```

```
[Out] x^2/(2*Sqrt[1 - x^4])
```

fricas [A] time = 0.86, size = 21, normalized size = 1.17

$$-\frac{\sqrt{-x^4+1}\,x^2}{2\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-x^4 + 1)*x^2/(x^4 - 1)
```

giac [A] time = 0.18, size = 21, normalized size = 1.17

$$-\frac{\sqrt{-x^4+1}\,x^2}{2\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+1)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-x^4 + 1)*x^2/(x^4 - 1)
```

maple [A] time = 0.00, size = 26, normalized size = 1.44

$$-\frac{(x-1)(x+1)\left(x^2+1\right)x^2}{2\left(-x^4+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^4+1)^(3/2),x)
```

```
[Out] -1/2*(x-1)*(x+1)*(x^2+1)*x^2/(-x^4+1)^(3/2)
```

maxima [A] time = 1.28, size = 14, normalized size = 0.78

$$\frac{x^2}{2\sqrt{-x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+1)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/sqrt(-x^4 + 1)
```

mupad [B] time = 1.05, size = 14, normalized size = 0.78

$$\frac{x^2}{2\sqrt{1-x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1 - x^4)^(3/2),x)
```

```
[Out] x^2/(2*(1 - x^4)^(1/2))
```

sympy [A] time = 1.40, size = 32, normalized size = 1.78

$$\begin{cases} -\frac{ix^2}{2\sqrt{x^4-1}} & \text{for } \left|x^4\right| > 1 \\ \frac{x^2}{2\sqrt{1-x^4}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**4+1)**(3/2),x)
```

```
[Out] Piecewise((-I*x**2/(2*sqrt(x**4 - 1)), Abs(x**4) > 1), (x**2/(2*sqrt(1 - x*
**4)), True))
```

3.902 $\int \frac{1}{x^3\left(1-x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=34

$$\frac{x^2}{\sqrt{1-x^4}}-\frac{1}{2x^2\sqrt{1-x^4}}$$

```
[Out] -1/2/x^2/(-x^4+1)^(1/2)+x^2/(-x^4+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$\frac{x^2}{\sqrt{1-x^4}}-\frac{1}{2x^2\sqrt{1-x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 - x^4)^(3/2)),x]
```

```
[Out] -1/(2*x^2*Sqrt[1 - x^4]) + x^2/Sqrt[1 - x^4]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(1-x^4\right)^{3/2}}\,dx &= -\frac{1}{2x^2\sqrt{1-x^4}}+2\int \frac{x}{\left(1-x^4\right)^{3/2}}\,dx\\ &= -\frac{1}{2x^2\sqrt{1-x^4}}+\frac{x^2}{\sqrt{1-x^4}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.74

$$-\frac{1-2x^4}{2x^2\sqrt{1-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 - x^4)^(3/2)),x]
```

```
[Out] -1/2*(1 - 2*x^4)/(x^2*Sqrt[1 - x^4])
```

fricas [A] time = 0.88, size = 29, normalized size = 0.85

$$-\frac{\left(2x^4-1\right)\sqrt{-x^4+1}}{2\left(x^6-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^4+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*x^4 - 1)*sqrt(-x^4 + 1)/(x^6 - x^2)
```

giac [A] time = 0.19, size = 56, normalized size = 1.65

$$-\frac{\sqrt{-x^4+1}\,x^2}{2\left(x^4-1\right)}+\frac{x^2}{4\left(\sqrt{-x^4+1}-1\right)}-\frac{\sqrt{-x^4+1}-1}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^4+1)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-x^4 + 1)*x^2/(x^4 - 1) + 1/4*x^2/(sqrt(-x^4 + 1) - 1) - 1/4*(sqr
t(-x^4 + 1) - 1)/x^2
```

maple [A] time = 0.00, size = 33, normalized size = 0.97

$$-\frac{(x-1)\,(x+1)\left(x^2+1\right)\left(2x^4-1\right)}{2\left(-x^4+1\right)^{\frac{3}{2}}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-x^4+1)^(3/2),x)
```

```
[Out] -1/2*(x-1)*(x+1)*(x^2+1)*(2*x^4-1)/x^2/(-x^4+1)^(3/2)
```

maxima [A] time = 1.32, size = 29, normalized size = 0.85

$$\frac{x^2}{2\sqrt{-x^4+1}}-\frac{\sqrt{-x^4+1}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^4+1)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/sqrt(-x^4 + 1) - 1/2*sqrt(-x^4 + 1)/x^2
```

mupad [B] time = 1.14, size = 18, normalized size = 0.53

$$\frac{x^4-\frac{1}{2}}{x^2\sqrt{1-x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(1 - x^4)^(3/2)),x)
```

```
[Out] (x^4 - 1/2)/(x^2*(1 - x^4)^(1/2))
```

sympy [A] time = 1.86, size = 90, normalized size = 2.65

$$\begin{cases} -\frac{2ix^4\sqrt{x^4-1}}{2x^6-2x^2}+\frac{i\sqrt{x^4-1}}{2x^6-2x^2} & \text{for } \left|x^4\right|>1 \\ -\frac{2x^4\sqrt{1-x^4}}{2x^6-2x^2}+\frac{\sqrt{1-x^4}}{2x^6-2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-x**4+1)**(3/2),x)
```

```
[Out] Piecewise((-2*I*x**4*sqrt(x**4 - 1)/(2*x**6 - 2*x**2) + I*sqrt(x**4 - 1)/(2
*x**6 - 2*x**2), Abs(x**4) > 1), (-2*x**4*sqrt(1 - x**4)/(2*x**6 - 2*x**2)
+ sqrt(1 - x**4)/(2*x**6 - 2*x**2), True))
```

3.903 $\int \frac{1}{x^7\left(1-x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=55

$$-\frac{1}{6\sqrt{1-x^4}\,x^6}+\frac{4x^2}{3\sqrt{1-x^4}}-\frac{2}{3\sqrt{1-x^4}\,x^2}$$

```
[Out] -1/6/x^6/(-x^4+1)^(1/2)-2/3/x^2/(-x^4+1)^(1/2)+4/3*x^2/(-x^4+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$\frac{4x^2}{3\sqrt{1-x^4}}-\frac{2}{3\sqrt{1-x^4}\,x^2}-\frac{1}{6\sqrt{1-x^4}\,x^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(1 - x^4)^(3/2)),x]
```

```
[Out] -1/(6*x^6*Sqrt[1 - x^4]) - 2/(3*x^2*Sqrt[1 - x^4]) + (4*x^2)/(3*Sqrt[1 - x^
4])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7\left(1-x^4\right)^{3/2}}\,dx &= -\frac{1}{6x^6\sqrt{1-x^4}}+\frac{4}{3}\int \frac{1}{x^3\left(1-x^4\right)^{3/2}}\,dx\\ &= -\frac{1}{6x^6\sqrt{1-x^4}}-\frac{2}{3x^2\sqrt{1-x^4}}+\frac{8}{3}\int \frac{x}{\left(1-x^4\right)^{3/2}}\,dx\\ &= -\frac{1}{6x^6\sqrt{1-x^4}}-\frac{2}{3x^2\sqrt{1-x^4}}+\frac{4x^2}{3\sqrt{1-x^4}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.55

$$-\frac{-8x^8+4x^4+1}{6x^6\sqrt{1-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(1 - x^4)^(3/2)),x]
```

```
[Out] -1/6*(1 + 4*x^4 - 8*x^8)/(x^6*Sqrt[1 - x^4])
```

fricas [A] time = 1.12, size = 34, normalized size = 0.62

$$-\frac{\left(8x^8-4x^4-1\right)\sqrt{-x^4+1}}{6\left(x^{10}-x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-x^4+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*(8*x^8 - 4*x^4 - 1)*sqrt(-x^4 + 1)/(x^10 - x^6)
```

giac [B] time = 0.17, size = 94, normalized size = 1.71

$$\frac{x^6\left(\frac{21\left(\sqrt{-x^4+1}-1\right)^2}{x^4}+1\right)}{48\left(\sqrt{-x^4+1}-1\right)^3}-\frac{\sqrt{-x^4+1}\,x^2}{2\left(x^4-1\right)}-\frac{7\left(\sqrt{-x^4+1}-1\right)}{16x^2}-\frac{\left(\sqrt{-x^4+1}-1\right)^3}{48x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-x^4+1)^(3/2),x, algorithm="giac")
```

```
[Out] 1/48*x^6*(21*(sqrt(-x^4 + 1) - 1)^2/x^4 + 1)/(sqrt(-x^4 + 1) - 1)^3 - 1/2*s
qrt(-x^4 + 1)*x^2/(x^4 - 1) - 7/16*(sqrt(-x^4 + 1) - 1)/x^2 - 1/48*(sqrt(-x
^4 + 1) - 1)^3/x^6
```

maple [A] time = 0.00, size = 38, normalized size = 0.69

$$-\frac{(x-1)(x+1)\left(x^2+1\right)\left(8x^8-4x^4-1\right)}{6\left(-x^4+1\right)^{\frac{3}{2}}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(-x^4+1)^(3/2),x)
```

```
[Out] -1/6*(x-1)*(x+1)*(x^2+1)*(8*x^8-4*x^4-1)/x^6/(-x^4+1)^(3/2)
```

maxima [A] time = 1.34, size = 43, normalized size = 0.78

$$\frac{x^2}{2\sqrt{-x^4+1}}-\frac{\sqrt{-x^4+1}}{x^2}-\frac{\left(-x^4+1\right)^{\frac{3}{2}}}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-x^4+1)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/sqrt(-x^4 + 1) - sqrt(-x^4 + 1)/x^2 - 1/6*(-x^4 + 1)^(3/2)/x^6
```

mupad [B] time = 1.20, size = 30, normalized size = 0.55

$$\frac{8\left(x^4-1\right)^2+12x^4-9}{6x^6\sqrt{1-x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(1 - x^4)^(3/2)),x)
```

```
[Out] (8*(x^4 - 1)^2 + 12*x^4 - 9)/(6*x^6*(1 - x^4)^(1/2))
```

sympy [**A**] time = 2.36, size = 151, normalized size = 2.75

$$\begin{cases} -\frac{8x^8\sqrt{-1+\frac{1}{x^4}}}{6x^8-6x^4}+\frac{4x^4\sqrt{-1+\frac{1}{x^4}}}{6x^8-6x^4}+\frac{\sqrt{-1+\frac{1}{x^4}}}{6x^8-6x^4} & \text{for } \frac{1}{|x^4|}>1 \\ -\frac{8ix^8\sqrt{1-\frac{1}{x^4}}}{6x^8-6x^4}+\frac{4ix^4\sqrt{1-\frac{1}{x^4}}}{6x^8-6x^4}+\frac{i\sqrt{1-\frac{1}{x^4}}}{6x^8-6x^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(-x**4+1)**(3/2),x)
```

```
[Out] Piecewise((-8*x**8*sqrt(-1 + x**(-4))/(6*x**8 - 6*x**4) + 4*x**4*sqrt(-1 +
x**(-4))/(6*x**8 - 6*x**4) + sqrt(-1 + x**(-4))/(6*x**8 - 6*x**4), 1/Abs(x*
*4) > 1), (-8*I*x**8*sqrt(1 - 1/x**4)/(6*x**8 - 6*x**4) + 4*I*x**4*sqrt(1 -
 1/x**4)/(6*x**8 - 6*x**4) + I*sqrt(1 - 1/x**4)/(6*x**8 - 6*x**4), True))
```

3.904 $\int \frac{x^{12}}{\left(1-x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=61

$$\frac{15}{14}\sqrt{1-x^4}\,x+\frac{x^9}{2\sqrt{1-x^4}}+\frac{9}{14}\sqrt{1-x^4}\,x^5-\frac{15}{14}F\left(\sin^{-1}(x)\middle|-1\right)$$

```
[Out] -15/14*EllipticF(x,I)+1/2*x^9/(-x^4+1)^(1/2)+15/14*x*(-x^4+1)^(1/2)+9/14*x^
5*(-x^4+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {288, 321, 221}

$$\frac{x^9}{2\sqrt{1-x^4}}+\frac{9}{14}\sqrt{1-x^4}\,x^5+\frac{15}{14}\sqrt{1-x^4}\,x-\frac{15}{14}F\left(\sin^{-1}(x)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^12/(1 - x^4)^(3/2),x]
```

```
[Out] x^9/(2*Sqrt[1 - x^4]) + (15*x*Sqrt[1 - x^4])/14 + (9*x^5*Sqrt[1 - x^4])/14
 - (15*EllipticF[ArcSin[x], -1])/14
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{12}}{\left(1-x^4\right)^{3/2}}\,dx &= \frac{x^9}{2\sqrt{1-x^4}}-\frac{9}{2}\int \frac{x^8}{\sqrt{1-x^4}}\,dx \\
&= \frac{x^9}{2\sqrt{1-x^4}}+\frac{9}{14}x^5\sqrt{1-x^4}-\frac{45}{14}\int \frac{x^4}{\sqrt{1-x^4}}\,dx \\
&= \frac{x^9}{2\sqrt{1-x^4}}+\frac{15}{14}x\sqrt{1-x^4}+\frac{9}{14}x^5\sqrt{1-x^4}-\frac{15}{14}\int \frac{1}{\sqrt{1-x^4}}\,dx \\
&= \frac{x^9}{2\sqrt{1-x^4}}+\frac{15}{14}x\sqrt{1-x^4}+\frac{9}{14}x^5\sqrt{1-x^4}-\frac{15}{14}F\left(\sin^{-1}(x)\middle|-1\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.89

$$-\frac{x\left(15\sqrt{1-x^4}\ {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};x^4\right)+2x^8+6x^4-15\right)}{14\sqrt{1-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12/(1 - x^4)^(3/2),x]
```

```
[Out] -1/14*(x*(-15 + 6*x^4 + 2*x^8 + 15*Sqrt[1 - x^4]*Hypergeometric2F1[1/4, 1/2
, 5/4, x^4]))/Sqrt[1 - x^4]
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-x^4+1}\,x^{12}}{x^8-2\,x^4+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(-x^4+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + 1)*x^12/(x^8 - 2*x^4 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{12}}{\left(-x^4+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(-x^4+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^12/(-x^4 + 1)^(3/2), x)
```

maple [A] time = 0.01, size = 71, normalized size = 1.16

$$\frac{\sqrt{-x^4+1}\ x^5}{7}+\frac{x}{2\sqrt{-x^4+1}}+\frac{4\sqrt{-x^4+1}\ x}{7}-\frac{15\sqrt{-x^2+1}\ \sqrt{x^2+1}\ \text{EllipticF}\left(x,i\right)}{14\sqrt{-x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(-x^4+1)^(3/2),x)
```

```
[Out] 1/2*x/(-x^4+1)^(1/2)+1/7*(-x^4+1)^(1/2)*x^5+4/7*(-x^4+1)^(1/2)*x-15/14*(-x^
2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)*EllipticF(x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{12}}{\left(-x^4+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(-x^4+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^12/(-x^4 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{12}}{\left(1-x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(1 - x^4)^(3/2),x)
```

```
[Out] int(x^12/(1 - x^4)^(3/2), x)
```

sympy [**A**] time = 1.74, size = 31, normalized size = 0.51

$$\frac{x^{13}\Gamma\left(\frac{13}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2}, \frac{13}{4} \\ \frac{17}{4}\end{matrix}\right| x^4 e^{2i\pi}\right)}{4\Gamma\left(\frac{17}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12/(-x**4+1)**(3/2),x)
```

```
[Out] x**13*gamma(13/4)*hyper((3/2, 13/4), (17/4,), x**4*exp_polar(2*I*pi))/(4*ga
mma(17/4))
```

3.905 $\int \frac{x^8}{\left(1-x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=43

$$\frac{5}{6}\sqrt{1-x^4}\,x+\frac{x^5}{2\sqrt{1-x^4}}-\frac{5}{6}F\left(\sin^{-1}(x)\middle|-1\right)$$

```
[Out] -5/6*EllipticF(x,I)+1/2*x^5/(-x^4+1)^(1/2)+5/6*x*(-x^4+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {288, 321, 221}

$$\frac{x^5}{2\sqrt{1-x^4}}+\frac{5}{6}\sqrt{1-x^4}\,x-\frac{5}{6}F\left(\sin^{-1}(x)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(1 - x^4)^(3/2),x]
```

```
[Out] x^5/(2*Sqrt[1 - x^4]) + (5*x*Sqrt[1 - x^4])/6 - (5*EllipticF[ArcSin[x], -1]
)/6
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{\left(1-x^4\right)^{3/2}}\,dx &= \frac{x^5}{2\sqrt{1-x^4}}-\frac{5}{2}\int \frac{x^4}{\sqrt{1-x^4}}\,dx\\ &= \frac{x^5}{2\sqrt{1-x^4}}+\frac{5}{6}x\sqrt{1-x^4}-\frac{5}{6}\int \frac{1}{\sqrt{1-x^4}}\,dx\\ &= \frac{x^5}{2\sqrt{1-x^4}}+\frac{5}{6}x\sqrt{1-x^4}-\frac{5}{6}F\left(\sin^{-1}(x)\middle|-1\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 49, normalized size = 1.14

$$-\frac{x\left(5\sqrt{1-x^4}\ {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};x^4\right)+2x^4-5\right)}{6\sqrt{1-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(1 - x^4)^(3/2),x]
```

```
[Out] -1/6*(x*(-5 + 2*x^4 + 5*Sqrt[1 - x^4]*Hypergeometric2F1[1/4, 1/2, 5/4, x^4]
))/Sqrt[1 - x^4]
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-x^4+1}\,x^8}{x^8-2\,x^4+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^4+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + 1)*x^8/(x^8 - 2*x^4 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(-x^4+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^4+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^8/(-x^4 + 1)^(3/2), x)
```

maple [A] time = 0.01, size = 57, normalized size = 1.33

$$\frac{x}{2\sqrt{-x^4+1}}+\frac{\sqrt{-x^4+1}\ x}{3}-\frac{5\sqrt{-x^2+1}\ \sqrt{x^2+1}\ \text{EllipticF}\left(x,i\right)}{6\sqrt{-x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-x^4+1)^(3/2),x)
```

```
[Out] 1/2/(-x^4+1)^(1/2)*x+1/3*(-x^4+1)^(1/2)*x-5/6*(-x^2+1)^(1/2)*(x^2+1)^(1/2)/
(-x^4+1)^(1/2)*EllipticF(x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(-x^4+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^4+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^8/(-x^4 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^8}{\left(1-x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(1 - x^4)^(3/2),x)
```

```
[Out] int(x^8/(1 - x^4)^(3/2), x)
```

sympy [**A**] time = 1.14, size = 31, normalized size = 0.72

$$\frac{x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2}, \frac{9}{4} \\ \frac{13}{4}\end{matrix}\right| x^4 e^{2i\pi}\right)}{4\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-x**4+1)**(3/2),x)
```

```
[Out] x**9*gamma(9/4)*hyper((3/2, 9/4), (13/4,), x**4*exp_polar(2*I*pi))/(4*gamma
(13/4))
```

3.906 $\int \frac{x^4}{\left(1-x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=25

$$\frac{x}{2\sqrt{1-x^4}} - \frac{1}{2}F\left(\sin^{-1}(x)\middle|-1\right)$$

```
[Out] -1/2*EllipticF(x,I)+1/2*x/(-x^4+1)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {288, 221}

$$\frac{x}{2\sqrt{1-x^4}} - \frac{1}{2}F\left(\sin^{-1}(x)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 - x^4)^(3/2),x]
```

```
[Out] x/(2*Sqrt[1 - x^4]) - EllipticF[ArcSin[x], -1]/2
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\left(1-x^4\right)^{3/2}}\,dx &= \frac{x}{2\sqrt{1-x^4}} - \frac{1}{2}\int \frac{1}{\sqrt{1-x^4}}\,dx \\ &= \frac{x}{2\sqrt{1-x^4}} - \frac{1}{2}F\left(\sin^{-1}(x)\middle|-1\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 32, normalized size = 1.28

$$\frac{1}{2}x\left(\frac{1}{\sqrt{1-x^4}} - {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};x^4\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 - x^4)^(3/2),x]
```

```
[Out] (x*(1/Sqrt[1 - x^4] - Hypergeometric2F1[1/4, 1/2, 5/4, x^4]))/2
```

fricas [F] time = 1.12, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-x^4+1}\,x^4}{x^8-2\,x^4+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^4+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + 1)*x^4/(x^8 - 2*x^4 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-x^4+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^4+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/(-x^4 + 1)^(3/2), x)
```

maple [B] time = 0.01, size = 45, normalized size = 1.80

$$\frac{x}{2\sqrt{-x^4+1}} - \frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,\text{EllipticF}\left(x,i\right)}{2\sqrt{-x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^4+1)^(3/2),x)
```

```
[Out] 1/2/(-x^4+1)^(1/2)*x-1/2*(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)*Ellipt
icF(x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-x^4+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^4+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(-x^4 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^4}{\left(1-x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(1 - x^4)^(3/2),x)
```

```
[Out] int(x^4/(1 - x^4)^(3/2), x)
```

sympy [B] time = 1.28, size = 31, normalized size = 1.24

$$\frac{x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\right| x^4 e^{2i\pi}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**4+1)**(3/2),x)

[Out] x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), x**4*exp_polar(2*I*pi))/(4*gamma(
9/4))
```

3.907 $\int \frac{1}{\left(1-x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=25

$$\frac{x}{2\sqrt{1-x^4}}+\frac{1}{2}F\left(\sin^{-1}(x)\middle|-1\right)$$

```
[Out] 1/2*EllipticF(x,I)+1/2*x/(-x^4+1)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {199, 221}

$$\frac{x}{2\sqrt{1-x^4}}+\frac{1}{2}F\left(\sin^{-1}(x)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)^(-3/2),x]
```

```
[Out] x/(2*Sqrt[1 - x^4]) + EllipticF[ArcSin[x], -1]/2
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(1-x^4\right)^{3/2}}\,dx &= \frac{x}{2\sqrt{1-x^4}}+\frac{1}{2}\int \frac{1}{\sqrt{1-x^4}}\,dx\\ &= \frac{x}{2\sqrt{1-x^4}}+\frac{1}{2}F\left(\sin^{-1}(x)\middle|-1\right)\end{aligned}$$

Mathematica [C] time = 0.02, size = 30, normalized size = 1.20

$$\frac{1}{2}x\left({}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};x^4\right)+\frac{1}{\sqrt{1-x^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)^(-3/2),x]
```

```
[Out] (x*(1/Sqrt[1 - x^4] + Hypergeometric2F1[1/4, 1/2, 5/4, x^4]))/2
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-x^4+1}}{x^8-2\,x^4+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + 1)/(x^8 - 2*x^4 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^4+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + 1)^(-3/2), x)
```

maple [B] time = 0.01, size = 45, normalized size = 1.80

$$\frac{x}{2\sqrt{-x^4+1}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,\operatorname{EllipticF}(x,i)}{2\sqrt{-x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+1)^(3/2),x)
```

```
[Out] 1/2/(-x^4+1)^(1/2)*x+1/2*(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)*Ellipt
icF(x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^4+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + 1)^(-3/2), x)
```

mupad [B] time = 1.07, size = 10, normalized size = 0.40

$$x\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{2};\ \frac{5}{4};\ x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1 - x^4)^(3/2),x)
```

```
[Out] x*hypergeom([1/4, 3/2], 5/4, x^4)
```

sympy [A] time = 1.02, size = 29, normalized size = 1.16

$$\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\right|x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+1)**(3/2),x)
```

```
[Out] x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), x**4*exp_polar(2*I*pi))/(4*gamma(5/4
))
```

3.908 $\int \frac{1}{x^4\left(1-x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=45

$$-\frac{5\sqrt{1-x^4}}{6x^3}+\frac{1}{2x^3\sqrt{1-x^4}}+\frac{5}{6}F\left(\sin^{-1}(x)\middle|-1\right)$$

```
[Out] 5/6*EllipticF(x,I)+1/2/x^3/(-x^4+1)^(1/2)-5/6*(-x^4+1)^(1/2)/x^3
```

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {290, 325, 221}

$$-\frac{5\sqrt{1-x^4}}{6x^3}+\frac{1}{2x^3\sqrt{1-x^4}}+\frac{5}{6}F\left(\sin^{-1}(x)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 - x^4)^(3/2)),x]
```

```
[Out] 1/(2*x^3*Sqrt[1 - x^4]) - (5*Sqrt[1 - x^4])/(6*x^3) + (5*EllipticF[ArcSin[x
], -1])/6
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(1-x^4\right)^{3/2}}\,dx &= \frac{1}{2x^3\sqrt{1-x^4}}+\frac{5}{2}\int \frac{1}{x^4\sqrt{1-x^4}}\,dx\\ &= \frac{1}{2x^3\sqrt{1-x^4}}-\frac{5\sqrt{1-x^4}}{6x^3}+\frac{5}{6}\int \frac{1}{\sqrt{1-x^4}}\,dx\\ &= \frac{1}{2x^3\sqrt{1-x^4}}-\frac{5\sqrt{1-x^4}}{6x^3}+\frac{5}{6}F\left(\sin^{-1}(x)\middle|-1\right)\end{aligned}$$

Mathematica [C] time = 0.00, size = 20, normalized size = 0.44

$$-\frac{{}_2F_1\left(-\frac{3}{4},\frac{3}{2};\frac{1}{4};x^4\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 - x^4)^(3/2)),x]
[Out] -1/3*Hypergeometric2F1[-3/4, 3/2, 1/4, x^4]/x^3
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-x^4+1}}{x^{12}-2\,x^8+x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^4+1)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(-x^4 + 1)/(x^12 - 2*x^8 + x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^4+1\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^4+1)^(3/2),x, algorithm="giac")
[Out] integrate(1/((-x^4 + 1)^(3/2)*x^4), x)
```

maple [A] time = 0.02, size = 59, normalized size = 1.31

$$\frac{x}{2\sqrt{-x^4+1}}+\frac{5\sqrt{-x^2+1}\,\sqrt{x^2+1}\,\text{EllipticF}\left(x,i\right)}{6\sqrt{-x^4+1}}-\frac{\sqrt{-x^4+1}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-x^4+1)^(3/2),x)
[Out] -1/3*(-x^4+1)^(1/2)/x^3+1/2/(-x^4+1)^(1/2)*x+5/6*(-x^2+1)^(1/2)*(x^2+1)^(1/
2)/(-x^4+1)^(1/2)*EllipticF(x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^4+1\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^4+1)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((-x^4 + 1)^(3/2)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^4\left(1-x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(1 - x^4)^(3/2)),x)
```

```
[Out] int(1/(x^4*(1 - x^4)^(3/2)), x)
```

sympy [**A**] time = 1.39, size = 34, normalized size = 0.76

$$\frac{\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{4}, \frac{3}{2} \\ \frac{1}{4}\end{matrix}\right| x^4 e^{2i\pi}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-x**4+1)**(3/2),x)
```

```
[Out] gamma(-3/4)*hyper((-3/4, 3/2), (1/4,), x**4*exp_polar(2*I*pi))/(4*x**3*gamm
a(1/4))
```

3.909 $\int \frac{1}{x^8\left(1-x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=63

$$-\frac{9\sqrt{1-x^4}}{14x^7}+\frac{1}{2x^7\sqrt{1-x^4}}-\frac{15\sqrt{1-x^4}}{14x^3}+\frac{15}{14}F\left(\sin^{-1}(x)\middle|-1\right)$$

```
[Out] 15/14*EllipticF(x,I)+1/2/x^7/(-x^4+1)^(1/2)-9/14*(-x^4+1)^(1/2)/x^7-15/14*(
-x^4+1)^(1/2)/x^3
```

Rubi [A] time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {290, 325, 221}

$$-\frac{15\sqrt{1-x^4}}{14x^3}-\frac{9\sqrt{1-x^4}}{14x^7}+\frac{1}{2x^7\sqrt{1-x^4}}+\frac{15}{14}F\left(\sin^{-1}(x)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(1 - x^4)^(3/2)),x]
```

```
[Out] 1/(2*x^7*Sqrt[1 - x^4]) - (9*Sqrt[1 - x^4])/(14*x^7) - (15*Sqrt[1 - x^4])/(
14*x^3) + (15*EllipticF[ArcSin[x], -1])/14
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^8\left(1-x^4\right)^{3/2}}\,dx &= \frac{1}{2x^7\sqrt{1-x^4}} + \frac{9}{2}\int \frac{1}{x^8\sqrt{1-x^4}}\,dx \\ &= \frac{1}{2x^7\sqrt{1-x^4}} - \frac{9\sqrt{1-x^4}}{14x^7} + \frac{45}{14}\int \frac{1}{x^4\sqrt{1-x^4}}\,dx \\ &= \frac{1}{2x^7\sqrt{1-x^4}} - \frac{9\sqrt{1-x^4}}{14x^7} - \frac{15\sqrt{1-x^4}}{14x^3} + \frac{15}{14}\int \frac{1}{\sqrt{1-x^4}}\,dx \\ &= \frac{1}{2x^7\sqrt{1-x^4}} - \frac{9\sqrt{1-x^4}}{14x^7} - \frac{15\sqrt{1-x^4}}{14x^3} + \frac{15}{14}F\left(\sin^{-1}(x)\middle|-1\right)\end{aligned}$$

Mathematica [C] time = 0.00, size = 20, normalized size = 0.32

$$-\frac{{}_2F_1\left(-\frac{7}{4},\frac{3}{2};-\frac{3}{4};x^4\right)}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(1 - x^4)^(3/2)),x]
[Out] -1/7*Hypergeometric2F1[-7/4, 3/2, -3/4, x^4]/x^7
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-x^4+1}}{x^{16}-2\,x^{12}+x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-x^4+1)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(-x^4 + 1)/(x^16 - 2*x^12 + x^8), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^4+1\right)^{\frac{3}{2}}x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-x^4+1)^(3/2),x, algorithm="giac")
[Out] integrate(1/((-x^4 + 1)^(3/2)*x^8), x)
```

maple [A] time = 0.02, size = 73, normalized size = 1.16

$$\frac{x}{2\sqrt{-x^4+1}} + \frac{15\sqrt{-x^2+1}\,\sqrt{x^2+1}\,\text{EllipticF}\left(x,i\right)}{14\sqrt{-x^4+1}} - \frac{4\sqrt{-x^4+1}}{7x^3} - \frac{\sqrt{-x^4+1}}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(-x^4+1)^(3/2),x)
[Out] -1/7*(-x^4+1)^(1/2)/x^7-4/7*(-x^4+1)^(1/2)/x^3+1/2/(-x^4+1)^(1/2)*x+15/14*(
-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)*EllipticF(x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^4+1\right)^{\frac{3}{2}} x^8}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-x^4+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-x^4 + 1)^(3/2)*x^8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^8 \left(1-x^4\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(1 - x^4)^(3/2)),x)
```

```
[Out] int(1/(x^8*(1 - x^4)^(3/2)), x)
```

sympy [A] time = 1.50, size = 37, normalized size = 0.59

$$\frac{\Gamma\left(-\frac{7}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{7}{4}, \frac{3}{2} \\ -\frac{3}{4} \end{matrix} \middle| x^4 e^{2i\pi}\right)}{4x^7 \Gamma\left(-\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(-x**4+1)**(3/2),x)
```

```
[Out] gamma(-7/4)*hyper((-7/4, 3/2), (-3/4,), x**4*exp_polar(2*I*pi))/(4*x**7*gam
ma(-3/4))
```

3.910 $\int \frac{x^{14}}{\left(1-x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=71

$$\frac{x^{11}}{2\sqrt{1-x^4}}+\frac{11}{18}\sqrt{1-x^4}\,x^7+\frac{77}{90}\sqrt{1-x^4}\,x^3+\frac{77}{30}F\left(\sin^{-1}(x)\middle|-1\right)-\frac{77}{30}E\left(\sin^{-1}(x)\middle|-1\right)$$

```
[Out] -77/30*EllipticE(x,I)+77/30*EllipticF(x,I)+1/2*x^11/(-x^4+1)^(1/2)+77/90*x^
3*(-x^4+1)^(1/2)+11/18*x^7*(-x^4+1)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {288, 321, 307, 221, 1181, 424}

$$\frac{x^{11}}{2\sqrt{1-x^4}}+\frac{11}{18}\sqrt{1-x^4}\,x^7+\frac{77}{90}\sqrt{1-x^4}\,x^3+\frac{77}{30}F\left(\sin^{-1}(x)\middle|-1\right)-\frac{77}{30}E\left(\sin^{-1}(x)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^14/(1 - x^4)^(3/2),x]
```

```
[Out] x^11/(2*Sqrt[1 - x^4]) + (77*x^3*Sqrt[1 - x^4])/90 + (11*x^7*Sqrt[1 - x^4])
/18 - (77*EllipticE[ArcSin[x], -1])/30 + (77*EllipticF[ArcSin[x], -1])/30
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[(d + e*x^2)/(Sqrt[q + c*x^2]*Sqrt[q - c
*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{14}}{\left(1-x^4\right)^{3/2}}\,dx &= \frac{x^{11}}{2\sqrt{1-x^4}} - \frac{11}{2}\int \frac{x^{10}}{\sqrt{1-x^4}}\,dx \\
&= \frac{x^{11}}{2\sqrt{1-x^4}} + \frac{11}{18}x^7\sqrt{1-x^4} - \frac{77}{18}\int \frac{x^6}{\sqrt{1-x^4}}\,dx \\
&= \frac{x^{11}}{2\sqrt{1-x^4}} + \frac{77}{90}x^3\sqrt{1-x^4} + \frac{11}{18}x^7\sqrt{1-x^4} - \frac{77}{30}\int \frac{x^2}{\sqrt{1-x^4}}\,dx \\
&= \frac{x^{11}}{2\sqrt{1-x^4}} + \frac{77}{90}x^3\sqrt{1-x^4} + \frac{11}{18}x^7\sqrt{1-x^4} + \frac{77}{30}\int \frac{1}{\sqrt{1-x^4}}\,dx - \frac{77}{30}\int \frac{1+x^2}{\sqrt{1-x^4}}\,dx \\
&= \frac{x^{11}}{2\sqrt{1-x^4}} + \frac{77}{90}x^3\sqrt{1-x^4} + \frac{11}{18}x^7\sqrt{1-x^4} + \frac{77}{30}F\left(\sin^{-1}(x)\middle|-1\right) - \frac{77}{30}\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx \\
&= \frac{x^{11}}{2\sqrt{1-x^4}} + \frac{77}{90}x^3\sqrt{1-x^4} + \frac{11}{18}x^7\sqrt{1-x^4} - \frac{77}{30}E\left(\sin^{-1}(x)\middle|-1\right) + \frac{77}{30}F\left(\sin^{-1}(x)\middle|-1\right)
\end{aligned}$$

Mathematica [C] time = 0.02, size = 56, normalized size = 0.79

$$-\frac{x^3\left(-77\sqrt{1-x^4}\;{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};x^4\right)+5x^8+11x^4+77\right)}{45\sqrt{1-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/(1 - x^4)^(3/2),x]
```

```
[Out] -1/45*(x^3*(77 + 11*x^4 + 5*x^8 - 77*Sqrt[1 - x^4]*Hypergeometric2F1[3/4, 3
/2, 7/4, x^4]))/Sqrt[1 - x^4]
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-x^4+1}\,x^{14}}{x^8-2\,x^4+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(-x^4+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + 1)*x^14/(x^8 - 2*x^4 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(-x^4+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(-x^4+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^14/(-x^4 + 1)^(3/2), x)
```

maple [A] time = 0.01, size = 82, normalized size = 1.15

$$\frac{\sqrt{-x^4+1}\,x^7}{9}+\frac{x^3}{2\sqrt{-x^4+1}}+\frac{16\sqrt{-x^4+1}\,x^3}{45}+\frac{77\sqrt{-x^2+1}\,\sqrt{x^2+1}\,\left(-\text{EllipticE}\left(x,i\right)+\text{EllipticF}\left(x,i\right)\right)}{30\sqrt{-x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(-x^4+1)^(3/2),x)
```

```
[Out] 1/2*x^3/(-x^4+1)^(1/2)+1/9*(-x^4+1)^(1/2)*x^7+16/45*(-x^4+1)^(1/2)*x^3+77/3
0*(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)*(EllipticF(x,I)-EllipticE(x,I
))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(-x^4+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(-x^4+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^14/(-x^4 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{14}}{\left(1-x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(1 - x^4)^(3/2),x)
```

```
[Out] int(x^14/(1 - x^4)^(3/2), x)
```

sympy [A] time = 1.66, size = 31, normalized size = 0.44

$$\frac{x^{15}\Gamma\left(\frac{15}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2}, \frac{15}{4} \\ \frac{19}{4}\end{matrix}\right| x^4 e^{2i\pi}\right)}{4\Gamma\left(\frac{19}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(-x**4+1)**(3/2),x)
```

```
[Out] x**15*gamma(15/4)*hyper((3/2, 15/4), (19/4,), x**4*exp_polar(2*I*pi))/(4*ga
mma(19/4))
```

3.911 $\int \frac{x^{10}}{\left(1-x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=53

$$\frac{x^7}{2\sqrt{1-x^4}}+\frac{7}{10}\sqrt{1-x^4}\,x^3+\frac{21}{10}F\left(\sin^{-1}(x)\middle|-1\right)-\frac{21}{10}E\left(\sin^{-1}(x)\middle|-1\right)$$

```
[Out] -21/10*EllipticE(x,I)+21/10*EllipticF(x,I)+1/2*x^7/(-x^4+1)^(1/2)+7/10*x^3*
(-x^4+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {288, 321, 307, 221, 1181, 424}

$$\frac{x^7}{2\sqrt{1-x^4}}+\frac{7}{10}\sqrt{1-x^4}\,x^3+\frac{21}{10}F\left(\sin^{-1}(x)\middle|-1\right)-\frac{21}{10}E\left(\sin^{-1}(x)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(1 - x^4)^(3/2),x]
```

```
[Out] x^7/(2*Sqrt[1 - x^4]) + (7*x^3*Sqrt[1 - x^4])/10 - (21*EllipticE[ArcSin[x],
 -1])/10 + (21*EllipticF[ArcSin[x], -1])/10
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[(d + e*x^2)/(Sqrt[q + c*x^2]*Sqrt[q - c
*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{10}}{\left(1-x^4\right)^{3/2}}\,dx &= \frac{x^7}{2\sqrt{1-x^4}} - \frac{7}{2}\int \frac{x^6}{\sqrt{1-x^4}}\,dx \\
&= \frac{x^7}{2\sqrt{1-x^4}} + \frac{7}{10}x^3\sqrt{1-x^4} - \frac{21}{10}\int \frac{x^2}{\sqrt{1-x^4}}\,dx \\
&= \frac{x^7}{2\sqrt{1-x^4}} + \frac{7}{10}x^3\sqrt{1-x^4} + \frac{21}{10}\int \frac{1}{\sqrt{1-x^4}}\,dx - \frac{21}{10}\int \frac{1+x^2}{\sqrt{1-x^4}}\,dx \\
&= \frac{x^7}{2\sqrt{1-x^4}} + \frac{7}{10}x^3\sqrt{1-x^4} + \frac{21}{10}F\left(\sin^{-1}(x)\middle|-1\right) - \frac{21}{10}\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx \\
&= \frac{x^7}{2\sqrt{1-x^4}} + \frac{7}{10}x^3\sqrt{1-x^4} - \frac{21}{10}E\left(\sin^{-1}(x)\middle|-1\right) + \frac{21}{10}F\left(\sin^{-1}(x)\middle|-1\right)
\end{aligned}$$

Mathematica [C] time = 0.03, size = 49, normalized size = 0.92

$$-\frac{x^3\left(-7\sqrt{1-x^4}\;{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};x^4\right)+x^4+7\right)}{5\sqrt{1-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(1 - x^4)^(3/2),x]
```

```
[Out] -1/5*(x^3*(7 + x^4 - 7*Sqrt[1 - x^4]*Hypergeometric2F1[3/4, 3/2, 7/4, x^4])
)/Sqrt[1 - x^4]
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-x^4+1}\,x^{10}}{x^8-2\,x^4+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(-x^4+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + 1)*x^10/(x^8 - 2*x^4 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\left(-x^4+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(-x^4+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^10/(-x^4 + 1)^(3/2), x)
```

maple [A] time = 0.01, size = 68, normalized size = 1.28

$$\frac{x^3}{2\sqrt{-x^4+1}} + \frac{\sqrt{-x^4+1}\,x^3}{5} + \frac{21\sqrt{-x^2+1}\,\sqrt{x^2+1}\,\left(-\text{EllipticE}\left(x,i\right)+\text{EllipticF}\left(x,i\right)\right)}{10\sqrt{-x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(-x^4+1)^(3/2),x)
```

```
[Out] 1/2/(-x^4+1)^(1/2)*x^3+1/5*(-x^4+1)^(1/2)*x^3+21/10*(-x^2+1)^(1/2)*(x^2+1)^
(1/2)/(-x^4+1)^(1/2)*(EllipticF(x,I)-EllipticE(x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\left(-x^4+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(-x^4+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^10/(-x^4 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{10}}{\left(1-x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(1 - x^4)^(3/2),x)
```

```
[Out] int(x^10/(1 - x^4)^(3/2), x)
```

sympy [A] time = 1.63, size = 31, normalized size = 0.58

$$\frac{x^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2}, \frac{11}{4} \\ \frac{15}{4}\end{matrix}\right| x^4 e^{2i\pi}\right)}{4\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(-x**4+1)**(3/2),x)
```

```
[Out] x**11*gamma(11/4)*hyper((3/2, 11/4), (15/4,), x**4*exp_polar(2*I*pi))/(4*ga
mma(15/4))
```

3.912 $\int \frac{x^6}{\left(1-x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=35

$$\frac{x^3}{2\sqrt{1-x^4}}+\frac{3}{2}F\left(\sin^{-1}(x)\middle|-1\right)-\frac{3}{2}E\left(\sin^{-1}(x)\middle|-1\right)$$

```
[Out] -3/2*EllipticE(x,I)+3/2*EllipticF(x,I)+1/2*x^3/(-x^4+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {288, 307, 221, 1181, 424}

$$\frac{x^3}{2\sqrt{1-x^4}}+\frac{3}{2}F\left(\sin^{-1}(x)\middle|-1\right)-\frac{3}{2}E\left(\sin^{-1}(x)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(1 - x^4)^(3/2),x]
```

```
[Out] x^3/(2*Sqrt[1 - x^4]) - (3*EllipticE[ArcSin[x], -1])/2 + (3*EllipticF[ArcSi
n[x], -1])/2
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[(d + e*x^2)/(Sqrt[q + c*x^2]*Sqrt[q - c
*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{\left(1-x^4\right)^{3/2}}\,dx &= \frac{x^3}{2\sqrt{1-x^4}} - \frac{3}{2}\int \frac{x^2}{\sqrt{1-x^4}}\,dx \\ &= \frac{x^3}{2\sqrt{1-x^4}} + \frac{3}{2}\int \frac{1}{\sqrt{1-x^4}}\,dx - \frac{3}{2}\int \frac{1+x^2}{\sqrt{1-x^4}}\,dx \\ &= \frac{x^3}{2\sqrt{1-x^4}} + \frac{3}{2}F\left(\sin^{-1}(x)\middle|-1\right) - \frac{3}{2}\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx \\ &= \frac{x^3}{2\sqrt{1-x^4}} - \frac{3}{2}E\left(\sin^{-1}(x)\middle|-1\right) + \frac{3}{2}F\left(\sin^{-1}(x)\middle|-1\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 31, normalized size = 0.89

$$x^3\left({}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};x^4\right) - \frac{1}{\sqrt{1-x^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(1 - x^4)^(3/2),x]
[Out] x^3*(-(1/Sqrt[1 - x^4]) + Hypergeometric2F1[3/4, 3/2, 7/4, x^4])
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-x^4+1}\,x^6}{x^8-2\,x^4+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^4+1)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(-x^4 + 1)*x^6/(x^8 - 2*x^4 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(-x^4+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^4+1)^(3/2),x, algorithm="giac")
[Out] integrate(x^6/(-x^4 + 1)^(3/2), x)
```

maple [A] time = 0.01, size = 54, normalized size = 1.54

$$\frac{x^3}{2\sqrt{-x^4+1}} + \frac{3\sqrt{-x^2+1}\,\sqrt{x^2+1}\,\left(-\text{EllipticE}\left(x,i\right)+\text{EllipticF}\left(x,i\right)\right)}{2\sqrt{-x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-x^4+1)^(3/2),x)
[Out] 1/2/(-x^4+1)^(1/2)*x^3+3/2*(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)*(Ell
ipticF(x,I)-EllipticE(x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(-x^4+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^4+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(-x^4 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^6}{\left(1-x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(1 - x^4)^(3/2),x)
```

```
[Out] int(x^6/(1 - x^4)^(3/2), x)
```

sympy [A] time = 1.99, size = 31, normalized size = 0.89

$$\frac{x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2}, \frac{7}{4} \\ \frac{11}{4}\end{matrix}\right| x^4 e^{2i\pi}\right)}{4\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-x**4+1)**(3/2),x)
```

```
[Out] x**7*gamma(7/4)*hyper((3/2, 7/4), (11/4,), x**4*exp_polar(2*I*pi))/(4*gamma
(11/4))
```

3.913 $\int \frac{x^2}{\left(1-x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=35

$$\frac{x^3}{2\sqrt{1-x^4}}+\frac{1}{2}F\left(\sin^{-1}(x)\middle|-1\right)-\frac{1}{2}E\left(\sin^{-1}(x)\middle|-1\right)$$

```
[Out] -1/2*EllipticE(x,I)+1/2*EllipticF(x,I)+1/2*x^3/(-x^4+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {290, 307, 221, 1181, 424}

$$\frac{x^3}{2\sqrt{1-x^4}}+\frac{1}{2}F\left(\sin^{-1}(x)\middle|-1\right)-\frac{1}{2}E\left(\sin^{-1}(x)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 - x^4)^(3/2),x]
```

```
[Out] x^3/(2*Sqrt[1 - x^4]) - EllipticE[ArcSin[x], -1]/2 + EllipticF[ArcSin[x], -
1]/2
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[(d + e*x^2)/(Sqrt[q + c*x^2]*Sqrt[q - c
*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(1-x^4\right)^{3/2}}\,dx &= \frac{x^3}{2\sqrt{1-x^4}} - \frac{1}{2}\int \frac{x^2}{\sqrt{1-x^4}}\,dx \\ &= \frac{x^3}{2\sqrt{1-x^4}} + \frac{1}{2}\int \frac{1}{\sqrt{1-x^4}}\,dx - \frac{1}{2}\int \frac{1+x^2}{\sqrt{1-x^4}}\,dx \\ &= \frac{x^3}{2\sqrt{1-x^4}} + \frac{1}{2}F\left(\sin^{-1}(x)\middle|-1\right) - \frac{1}{2}\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx \\ &= \frac{x^3}{2\sqrt{1-x^4}} - \frac{1}{2}E\left(\sin^{-1}(x)\middle|-1\right) + \frac{1}{2}F\left(\sin^{-1}(x)\middle|-1\right)\end{aligned}$$

Mathematica [C] time = 0.00, size = 20, normalized size = 0.57

$$\frac{1}{3}x^3\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 - x^4)^(3/2),x]
[Out] (x^3*Hypergeometric2F1[3/4, 3/2, 7/4, x^4])/3
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-x^4+1}\,x^2}{x^8-2\,x^4+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+1)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(-x^4 + 1)*x^2/(x^8 - 2*x^4 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-x^4+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+1)^(3/2),x, algorithm="giac")
[Out] integrate(x^2/(-x^4 + 1)^(3/2), x)
```

maple [A] time = 0.01, size = 54, normalized size = 1.54

$$\frac{x^3}{2\sqrt{-x^4+1}} + \frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,\left(-\text{EllipticE}\left(x,i\right)+\text{EllipticF}\left(x,i\right)\right)}{2\sqrt{-x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^4+1)^(3/2),x)
[Out] 1/2/(-x^4+1)^(1/2)*x^3+1/2*(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)*(Ell
ipticF(x,I)-EllipticE(x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-x^4+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(-x^4 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2}{\left(1-x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1 - x^4)^(3/2),x)
```

```
[Out] int(x^2/(1 - x^4)^(3/2), x)
```

sympy [A] time = 0.86, size = 31, normalized size = 0.89

$$\frac{x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4}, \frac{3}{2} \\ \frac{7}{4}\end{matrix}\right| x^4 e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**4+1)**(3/2),x)
```

```
[Out] x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), x**4*exp_polar(2*I*pi))/(4*gamma(
7/4))
```

3.914 $\int \frac{1}{x^2\left(1-x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=53

$$-\frac{3\sqrt{1-x^4}}{2x}+\frac{1}{2x\sqrt{1-x^4}}+\frac{3}{2}F\left(\sin^{-1}(x)\middle|-1\right)-\frac{3}{2}E\left(\sin^{-1}(x)\middle|-1\right)$$

```
[Out] -3/2*EllipticE(x,I)+3/2*EllipticF(x,I)+1/2/x/(-x^4+1)^(1/2)-3/2*(-x^4+1)^(1
/2)/x
```

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {290, 325, 307, 221, 1181, 424}

$$-\frac{3\sqrt{1-x^4}}{2x}+\frac{1}{2x\sqrt{1-x^4}}+\frac{3}{2}F\left(\sin^{-1}(x)\middle|-1\right)-\frac{3}{2}E\left(\sin^{-1}(x)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 - x^4)^(3/2)),x]
```

```
[Out] 1/(2*x*Sqrt[1 - x^4]) - (3*Sqrt[1 - x^4])/(2*x) - (3*EllipticE[ArcSin[x], -
1])/2 + (3*EllipticF[ArcSin[x], -1])/2
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[(d + e*x^2)/(Sqrt[q + c*x^2]*Sqrt[q - c
*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\left(1-x^4\right)^{3/2}}\,dx &= \frac{1}{2x\sqrt{1-x^4}} + \frac{3}{2}\int \frac{1}{x^2\sqrt{1-x^4}}\,dx \\
&= \frac{1}{2x\sqrt{1-x^4}} - \frac{3\sqrt{1-x^4}}{2x} - \frac{3}{2}\int \frac{x^2}{\sqrt{1-x^4}}\,dx \\
&= \frac{1}{2x\sqrt{1-x^4}} - \frac{3\sqrt{1-x^4}}{2x} + \frac{3}{2}\int \frac{1}{\sqrt{1-x^4}}\,dx - \frac{3}{2}\int \frac{1+x^2}{\sqrt{1-x^4}}\,dx \\
&= \frac{1}{2x\sqrt{1-x^4}} - \frac{3\sqrt{1-x^4}}{2x} + \frac{3}{2}F\left(\sin^{-1}(x)\middle|-1\right) - \frac{3}{2}\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx \\
&= \frac{1}{2x\sqrt{1-x^4}} - \frac{3\sqrt{1-x^4}}{2x} - \frac{3}{2}E\left(\sin^{-1}(x)\middle|-1\right) + \frac{3}{2}F\left(\sin^{-1}(x)\middle|-1\right)
\end{aligned}$$

Mathematica [C] time = 0.00, size = 18, normalized size = 0.34

$$-\frac{{}_2F_1\left(-\frac{1}{4},\frac{3}{2};\frac{3}{4};x^4\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 - x^4)^(3/2)),x]
```

```
[Out] -(Hypergeometric2F1[-1/4, 3/2, 3/4, x^4]/x)
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-x^4+1}}{x^{10}-2\,x^6+x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^4+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + 1)/(x^10 - 2*x^6 + x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^4+1\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^4+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((-x^4 + 1)^(3/2)*x^2), x)
```

maple [A] time = 0.02, size = 68, normalized size = 1.28

$$\frac{x^3}{2\sqrt{-x^4+1}} - \frac{\sqrt{-x^4+1}}{x} + \frac{3\sqrt{-x^2+1}\,\sqrt{x^2+1}\,\left(-\text{EllipticE}\left(x,i\right)+\text{EllipticF}\left(x,i\right)\right)}{2\sqrt{-x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-x^4+1)^(3/2),x)
```

```
[Out] 1/2/(-x^4+1)^(1/2)*x^3-(-x^4+1)^(1/2)/x+3/2*(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)*(EllipticF(x,I)-EllipticE(x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^4+1\right)^{\frac{3}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^4+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-x^4 + 1)^(3/2)*x^2), x)
```

mupad [B] time = 1.16, size = 13, normalized size = 0.25

$$-\frac{{}_2\mathrm{F}_1\left(-\frac{1}{4}, \frac{3}{2}; \frac{3}{4}; x^4\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(1 - x^4)^(3/2)),x)
```

```
[Out] -hypergeom([-1/4, 3/2], 3/4, x^4)/x
```

sympy [A] time = 1.26, size = 32, normalized size = 0.60

$$\frac{\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{3}{2} \\ \frac{3}{4} \end{matrix} \middle| x^4 e^{2i\pi}\right)}{4x\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-x**4+1)**(3/2),x)
```

```
[Out] gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), x**4*exp_polar(2*I*pi))/(4*x*gamma(3/4))
```

3.915 $\int \frac{1}{x^6\left(1-x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=71

$$-\frac{21\sqrt{1-x^4}}{10x}-\frac{7\sqrt{1-x^4}}{10x^5}+\frac{1}{2x^5\sqrt{1-x^4}}+\frac{21}{10}F\left(\sin^{-1}(x)\middle|-1\right)-\frac{21}{10}E\left(\sin^{-1}(x)\middle|-1\right)$$

```
[Out] -21/10*EllipticE(x,I)+21/10*EllipticF(x,I)+1/2/x^5/(-x^4+1)^(1/2)-7/10*(-x^
4+1)^(1/2)/x^5-21/10*(-x^4+1)^(1/2)/x
```

Rubi [A] time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {290, 325, 307, 221, 1181, 424}

$$-\frac{21\sqrt{1-x^4}}{10x}-\frac{7\sqrt{1-x^4}}{10x^5}+\frac{1}{2x^5\sqrt{1-x^4}}+\frac{21}{10}F\left(\sin^{-1}(x)\middle|-1\right)-\frac{21}{10}E\left(\sin^{-1}(x)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(1 - x^4)^(3/2)),x]
```

```
[Out] 1/(2*x^5*Sqrt[1 - x^4]) - (7*Sqrt[1 - x^4])/(10*x^5) - (21*Sqrt[1 - x^4])/(
10*x) - (21*EllipticE[ArcSin[x], -1])/10 + (21*EllipticF[ArcSin[x], -1])/10
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[(d + e*x^2)/(Sqrt[q + c*x^2]*Sqrt[q - c
*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^6\left(1-x^4\right)^{3/2}}\,dx &= \frac{1}{2x^5\sqrt{1-x^4}} + \frac{7}{2}\int \frac{1}{x^6\sqrt{1-x^4}}\,dx \\
&= \frac{1}{2x^5\sqrt{1-x^4}} - \frac{7\sqrt{1-x^4}}{10x^5} + \frac{21}{10}\int \frac{1}{x^2\sqrt{1-x^4}}\,dx \\
&= \frac{1}{2x^5\sqrt{1-x^4}} - \frac{7\sqrt{1-x^4}}{10x^5} - \frac{21\sqrt{1-x^4}}{10x} - \frac{21}{10}\int \frac{x^2}{\sqrt{1-x^4}}\,dx \\
&= \frac{1}{2x^5\sqrt{1-x^4}} - \frac{7\sqrt{1-x^4}}{10x^5} - \frac{21\sqrt{1-x^4}}{10x} + \frac{21}{10}\int \frac{1}{\sqrt{1-x^4}}\,dx - \frac{21}{10}\int \frac{1+x^2}{\sqrt{1-x^4}}\,dx \\
&= \frac{1}{2x^5\sqrt{1-x^4}} - \frac{7\sqrt{1-x^4}}{10x^5} - \frac{21\sqrt{1-x^4}}{10x} + \frac{21}{10}F\left(\sin^{-1}(x)\middle|-1\right) - \frac{21}{10}\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx \\
&= \frac{1}{2x^5\sqrt{1-x^4}} - \frac{7\sqrt{1-x^4}}{10x^5} - \frac{21\sqrt{1-x^4}}{10x} - \frac{21}{10}E\left(\sin^{-1}(x)\middle|-1\right) + \frac{21}{10}F\left(\sin^{-1}(x)\middle|-1\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 20, normalized size = 0.28

$$-\frac{{}_2F_1\left(-\frac{5}{4},\frac{3}{2};-\frac{1}{4};x^4\right)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(1 - x^4)^(3/2)),x]
```

```
[Out] -1/5*Hypergeometric2F1[-5/4, 3/2, -1/4, x^4]/x^5
```

fricas [F] time = 1.21, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-x^4+1}}{x^{14}-2\,x^{10}+x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^4+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + 1)/(x^14 - 2*x^10 + x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^4+1\right)^{\frac{3}{2}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^4+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((-x^4 + 1)^(3/2)*x^6), x)
```

maple [A] time = 0.02, size = 82, normalized size = 1.15

$$\frac{x^3}{2\sqrt{-x^4+1}}-\frac{8\sqrt{-x^4+1}}{5x}-\frac{\sqrt{-x^4+1}}{5x^5}+\frac{21\sqrt{-x^2+1}\ \sqrt{x^2+1}\ \left(-\text{EllipticE}\left(x,i\right)+\text{EllipticF}\left(x,i\right)\right)}{10\sqrt{-x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-x^4+1)^(3/2),x)
```

```
[Out] -1/5*(-x^4+1)^(1/2)/x^5-8/5*(-x^4+1)^(1/2)/x+1/2/(-x^4+1)^(1/2)*x^3+21/10*(
-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)*(EllipticF(x,I)-EllipticE(x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^4+1\right)^{\frac{3}{2}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^4+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-x^4 + 1)^(3/2)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^6\left(1-x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(1 - x^4)^(3/2)),x)
```

```
[Out] int(1/(x^6*(1 - x^4)^(3/2)), x)
```

sympy [A] time = 1.48, size = 37, normalized size = 0.52

$$\frac{\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{5}{4},\frac{3}{2}\\-\frac{1}{4}\end{matrix}\right|x^4e^{2i\pi}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-x**4+1)**(3/2),x)
```

```
[Out] gamma(-5/4)*hyper((-5/4, 3/2), (-1/4,), x**4*exp_polar(2*I*pi))/(4*x**5*gam
ma(-1/4))
```

3.916 $\int \frac{1}{\left(1-x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=41

$$\frac{5x}{12\sqrt{1-x^4}}+\frac{x}{6\left(1-x^4\right)^{3/2}}+\frac{5}{12}F\left(\sin^{-1}(x)\middle|-1\right)$$

```
[Out] 1/6*x/(-x^4+1)^(3/2)+5/12*EllipticF(x,I)+5/12*x/(-x^4+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {199, 221}

$$\frac{5x}{12\sqrt{1-x^4}}+\frac{x}{6\left(1-x^4\right)^{3/2}}+\frac{5}{12}F\left(\sin^{-1}(x)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)^(-5/2),x]
```

```
[Out] x/(6*(1 - x^4)^(3/2)) + (5*x)/(12*Sqrt[1 - x^4]) + (5*EllipticF[ArcSin[x],
-1])/12
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(1-x^4\right)^{5/2}}\,dx &= \frac{x}{6\left(1-x^4\right)^{3/2}}+\frac{5}{6}\int \frac{1}{\left(1-x^4\right)^{3/2}}\,dx\\ &= \frac{x}{6\left(1-x^4\right)^{3/2}}+\frac{5x}{12\sqrt{1-x^4}}+\frac{5}{12}\int \frac{1}{\sqrt{1-x^4}}\,dx\\ &= \frac{x}{6\left(1-x^4\right)^{3/2}}+\frac{5x}{12\sqrt{1-x^4}}+\frac{5}{12}F\left(\sin^{-1}(x)\middle|-1\right)\end{aligned}$$

Mathematica [C] time = 0.02, size = 51, normalized size = 1.24

$$\frac{5}{12}x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};x^4\right)+\frac{5x}{12\sqrt{1-x^4}}+\frac{x}{6\left(1-x^4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)^(-5/2),x]
```

```
[Out] x/(6*(1 - x^4)^(3/2)) + (5*x)/(12*Sqrt[1 - x^4]) + (5*x*Hypergeometric2F1[1
/4, 1/2, 5/4, x^4])/12
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^4+1}}{x^{12}-3\,x^8+3\,x^4-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+1)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 1)/(x^12 - 3*x^8 + 3*x^4 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^4+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+1)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + 1)^(-5/2), x)
```

maple [B] time = 0.01, size = 64, normalized size = 1.56

$$\frac{\sqrt{-x^4+1}\,x}{6\left(x^4-1\right)^2}+\frac{5x}{12\sqrt{-x^4+1}}+\frac{5\sqrt{-x^2+1}\,\sqrt{x^2+1}\,\text{EllipticF}\left(x,i\right)}{12\sqrt{-x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+1)^(5/2),x)
```

```
[Out] 1/6*x*(-x^4+1)^(1/2)/(x^4-1)^2+5/12/(-x^4+1)^(1/2)*x+5/12*(-x^2+1)^(1/2)*(x
^2+1)^(1/2)/(-x^4+1)^(1/2)*EllipticF(x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^4+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+1)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + 1)^(-5/2), x)
```

mupad [B] time = 1.07, size = 10, normalized size = 0.24

$$x\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{5}{2};\ \frac{5}{4};\ x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1 - x^4)^(5/2),x)
```

```
[Out] x*hypergeom([1/4, 5/2], 5/4, x^4)
```

sympy [**A**] time = 1.36, size = 29, normalized size = 0.71

$$\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4}, \frac{5}{2} \\ \frac{5}{4}\end{matrix}\right| x^4 e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+1)**(5/2),x)
```

```
[Out] x*gamma(1/4)*hyper((1/4, 5/2), (5/4,), x**4*exp_polar(2*I*pi))/(4*gamma(5/4
))
```

3.917 $\int \frac{x^{11}}{\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=40

$$\frac{1}{10}\left(x^4+1\right)^{5/2}-\frac{1}{3}\left(x^4+1\right)^{3/2}+\frac{\sqrt{x^4+1}}{2}$$

```
[Out] -1/3*(x^4+1)^(3/2)+1/10*(x^4+1)^(5/2)+1/2*(x^4+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{1}{10}\left(x^4+1\right)^{5/2}-\frac{1}{3}\left(x^4+1\right)^{3/2}+\frac{\sqrt{x^4+1}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/Sqrt[1 + x^4],x]
```

```
[Out] Sqrt[1 + x^4]/2 - (1 + x^4)^(3/2)/3 + (1 + x^4)^(5/2)/10
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{\sqrt{1+x^4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x}}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int \left(\frac{1}{\sqrt{1+x}}-2\sqrt{1+x}+(1+x)^{3/2}\right)dx, x, x^4\right)\\ &= \frac{\sqrt{1+x^4}}{2}-\frac{1}{3}\left(1+x^4\right)^{3/2}+\frac{1}{10}\left(1+x^4\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.62

$$\frac{1}{30}\sqrt{x^4+1}\left(3x^8-4x^4+8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/Sqrt[1 + x^4],x]
```

```
[Out] (Sqrt[1 + x^4]*(8 - 4*x^4 + 3*x^8))/30
```

fricas [A] time = 0.85, size = 21, normalized size = 0.52

$$\frac{1}{30}\left(3\,x^8-4\,x^4+8\right)\sqrt{x^4+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^4+1)^(1/2),x, algorithm="fricas")
[Out] 1/30*(3*x^8 - 4*x^4 + 8)*sqrt(x^4 + 1)
```

giac [A] time = 0.15, size = 28, normalized size = 0.70

$$\frac{1}{10}\left(x^4+1\right)^{\frac{5}{2}}-\frac{1}{3}\left(x^4+1\right)^{\frac{3}{2}}+\frac{1}{2}\sqrt{x^4+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^4+1)^(1/2),x, algorithm="giac")
[Out] 1/10*(x^4 + 1)^(5/2) - 1/3*(x^4 + 1)^(3/2) + 1/2*sqrt(x^4 + 1)
```

maple [A] time = 0.00, size = 22, normalized size = 0.55

$$\frac{\sqrt{x^4+1}\,\left(3x^8-4x^4+8\right)}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^4+1)^(1/2),x)
[Out] 1/30*(x^4+1)^(1/2)*(3*x^8-4*x^4+8)
```

maxima [A] time = 1.35, size = 28, normalized size = 0.70

$$\frac{1}{10}\left(x^4+1\right)^{\frac{5}{2}}-\frac{1}{3}\left(x^4+1\right)^{\frac{3}{2}}+\frac{1}{2}\sqrt{x^4+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^4+1)^(1/2),x, algorithm="maxima")
[Out] 1/10*(x^4 + 1)^(5/2) - 1/3*(x^4 + 1)^(3/2) + 1/2*sqrt(x^4 + 1)
```

mupad [B] time = 1.16, size = 20, normalized size = 0.50

$$\sqrt{x^4+1}\,\left(\frac{x^8}{10}-\frac{2\,x^4}{15}+\frac{4}{15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^4 + 1)^(1/2),x)
[Out] (x^4 + 1)^(1/2)*(x^8/10 - (2*x^4)/15 + 4/15)
```

sympy [A] time = 2.19, size = 39, normalized size = 0.98

$$\frac{x^8\sqrt{x^4+1}}{10}-\frac{2x^4\sqrt{x^4+1}}{15}+\frac{4\sqrt{x^4+1}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(x**4+1)**(1/2),x)
[Out] x**8*sqrt(x**4 + 1)/10 - 2*x**4*sqrt(x**4 + 1)/15 + 4*sqrt(x**4 + 1)/15
```

3.918 $\int \frac{x^7}{\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=27

$$\frac{1}{6}\left(x^4+1\right)^{3/2}-\frac{\sqrt{x^4+1}}{2}$$

```
[Out] 1/6*(x^4+1)^(3/2)-1/2*(x^4+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{1}{6}\left(x^4+1\right)^{3/2}-\frac{\sqrt{x^4+1}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/Sqrt[1 + x^4],x]
```

```
[Out] -Sqrt[1 + x^4]/2 + (1 + x^4)^(3/2)/6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{\sqrt{1+x^4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x}{\sqrt{1+x}}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int \left(-\frac{1}{\sqrt{1+x}}+\sqrt{1+x}\right)\,dx, x, x^4\right)\\ &= -\frac{1}{2}\sqrt{1+x^4}+\frac{1}{6}\left(1+x^4\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 0.67

$$\frac{1}{6}\left(x^4-2\right)\sqrt{x^4+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/Sqrt[1 + x^4],x]
```

```
[Out] ((-2 + x^4)*Sqrt[1 + x^4])/6
```

fricas [A] time = 0.80, size = 14, normalized size = 0.52

$$\frac{1}{6}\sqrt{x^4+1}\left(x^4-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^4+1)^(1/2),x, algorithm="fricas")
[Out] 1/6*sqrt(x^4 + 1)*(x^4 - 2)
```

giac [**A**] time = 0.15, size = 19, normalized size = 0.70

$$\frac{1}{6}\left(x^4+1\right)^{\frac{3}{2}}-\frac{1}{2}\sqrt{x^4+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^4+1)^(1/2),x, algorithm="giac")
[Out] 1/6*(x^4 + 1)^(3/2) - 1/2*sqrt(x^4 + 1)
```

maple [**A**] time = 0.00, size = 15, normalized size = 0.56

$$\frac{\sqrt{x^4+1}\,\left(x^4-2\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^4+1)^(1/2),x)
[Out] 1/6*(x^4+1)^(1/2)*(x^4-2)
```

maxima [**A**] time = 1.35, size = 19, normalized size = 0.70

$$\frac{1}{6}\left(x^4+1\right)^{\frac{3}{2}}-\frac{1}{2}\sqrt{x^4+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^4+1)^(1/2),x, algorithm="maxima")
[Out] 1/6*(x^4 + 1)^(3/2) - 1/2*sqrt(x^4 + 1)
```

mupad [**B**] time = 1.15, size = 14, normalized size = 0.52

$$\frac{\sqrt{x^4+1}\,\left(x^4-2\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^4 + 1)^(1/2),x)
[Out] ((x^4 + 1)^(1/2)*(x^4 - 2))/6
```

sympy [**A**] time = 0.83, size = 22, normalized size = 0.81

$$\frac{x^4\sqrt{x^4+1}}{6}-\frac{\sqrt{x^4+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**4+1)**(1/2),x)
[Out] x**4*sqrt(x**4 + 1)/6 - sqrt(x**4 + 1)/3
```

3.919 $\int \frac{x^3}{\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=13

$$\frac{\sqrt{x^4+1}}{2}$$

```
[Out] 1/2*(x^4+1)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$\frac{\sqrt{x^4+1}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[1 + x^4],x]
```

```
[Out] Sqrt[1 + x^4]/2
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{1+x^4}}\,dx = \frac{\sqrt{1+x^4}}{2}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{\sqrt{x^4+1}}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[1 + x^4],x]
```

```
[Out] Sqrt[1 + x^4]/2
```

fricas [A] time = 0.81, size = 9, normalized size = 0.69

$$\frac{1}{2}\sqrt{x^4+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(x^4 + 1)
```

giac [A] time = 0.18, size = 9, normalized size = 0.69

$$\frac{1}{2}\sqrt{x^4+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4+1)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(x^4 + 1)
```

maple [A] time = 0.00, size = 10, normalized size = 0.77

$$\frac{\sqrt{x^4+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^4+1)^(1/2),x)
[Out] 1/2*(x^4+1)^(1/2)
```

maxima [A] time = 1.31, size = 9, normalized size = 0.69

$$\frac{1}{2}\sqrt{x^4+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4+1)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(x^4 + 1)
```

mupad [B] time = 1.04, size = 9, normalized size = 0.69

$$\frac{\sqrt{x^4+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^4 + 1)^(1/2),x)
[Out] (x^4 + 1)^(1/2)/2
```

sympy [A] time = 0.19, size = 8, normalized size = 0.62

$$\frac{\sqrt{x^4+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**4+1)**(1/2),x)
[Out] sqrt(x**4 + 1)/2
```

3.920 $\int \frac{1}{x\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{2}\tanh^{-1}\left(\sqrt{x^4+1}\right)$$

```
[Out] -1/2*arctanh((x^4+1)^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {266, 63, 207}

$$-\frac{1}{2}\tanh^{-1}\left(\sqrt{x^4+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[1 + x^4]),x]
```

```
[Out] -ArcTanh[Sqrt[1 + x^4]]/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{1+x^4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^4\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^4}\right)\\ &= -\frac{1}{2}\tanh^{-1}\left(\sqrt{1+x^4}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{2}\tanh^{-1}\left(\sqrt{x^4+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[1 + x^4]),x]
[Out] -1/2*ArcTanh[Sqrt[1 + x^4]]
```

fricas [B] time = 0.91, size = 25, normalized size = 1.79

$$-\frac{1}{4}\log\left(\sqrt{x^4+1}+1\right)+\frac{1}{4}\log\left(\sqrt{x^4+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4+1)^(1/2),x, algorithm="fricas")
[Out] -1/4*log(sqrt(x^4 + 1) + 1) + 1/4*log(sqrt(x^4 + 1) - 1)
```

giac [B] time = 0.15, size = 25, normalized size = 1.79

$$-\frac{1}{4}\log\left(\sqrt{x^4+1}+1\right)+\frac{1}{4}\log\left(\sqrt{x^4+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4+1)^(1/2),x, algorithm="giac")
[Out] -1/4*log(sqrt(x^4 + 1) + 1) + 1/4*log(sqrt(x^4 + 1) - 1)
```

maple [A] time = 0.01, size = 11, normalized size = 0.79

$$-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^4+1)^(1/2),x)
[Out] -1/2*arctanh(1/(x^4+1)^(1/2))
```

maxima [B] time = 1.29, size = 25, normalized size = 1.79

$$-\frac{1}{4}\log\left(\sqrt{x^4+1}+1\right)+\frac{1}{4}\log\left(\sqrt{x^4+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4+1)^(1/2),x, algorithm="maxima")
[Out] -1/4*log(sqrt(x^4 + 1) + 1) + 1/4*log(sqrt(x^4 + 1) - 1)
```

mupad [B] time = 0.05, size = 10, normalized size = 0.71

$$-\frac{\operatorname{atanh}\left(\sqrt{x^4+1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^4 + 1)^(1/2)),x)
[Out] -atanh((x^4 + 1)^(1/2))/2
```

sympy [A] time = 1.11, size = 8, normalized size = 0.57

$$-\frac{\operatorname{asinh}\left(\frac{1}{x^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**4+1)**(1/2),x)
[Out] -asinh(x**(-2))/2
```

3.921 $\int \frac{1}{x^5\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=31

$$\frac{1}{4}\tanh^{-1}\left(\sqrt{x^4+1}\right)-\frac{\sqrt{x^4+1}}{4x^4}$$

```
[Out] 1/4*arctanh((x^4+1)^(1/2))-1/4*(x^4+1)^(1/2)/x^4
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {266, 51, 63, 207}

$$\frac{1}{4}\tanh^{-1}\left(\sqrt{x^4+1}\right)-\frac{\sqrt{x^4+1}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[1 + x^4]),x]
```

```
[Out] -Sqrt[1 + x^4]/(4*x^4) + ArcTanh[Sqrt[1 + x^4]]/4
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\sqrt{1+x^4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x}}\,dx, x, x^4\right)\\ &= -\frac{\sqrt{1+x^4}}{4x^4} - \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^4\right)\\ &= -\frac{\sqrt{1+x^4}}{4x^4} - \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^4}\right)\\ &= -\frac{\sqrt{1+x^4}}{4x^4} + \frac{1}{4}\tanh^{-1}\left(\sqrt{1+x^4}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 1.00

$$\frac{1}{4}\tanh^{-1}\left(\sqrt{x^4+1}\right) - \frac{\sqrt{x^4+1}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[1 + x^4]),x]
```

```
[Out] -1/4*Sqrt[1 + x^4]/x^4 + ArcTanh[Sqrt[1 + x^4]]/4
```

fricas [A] time = 0.72, size = 44, normalized size = 1.42

$$\frac{x^4\log\left(\sqrt{x^4+1}+1\right) - x^4\log\left(\sqrt{x^4+1}-1\right) - 2\sqrt{x^4+1}}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(x^4*log(sqrt(x^4 + 1) + 1) - x^4*log(sqrt(x^4 + 1) - 1) - 2*sqrt(x^4 +
 1))/x^4
```

giac [A] time = 0.15, size = 37, normalized size = 1.19

$$-\frac{\sqrt{x^4+1}}{4\,x^4} + \frac{1}{8}\,\log\left(\sqrt{x^4+1}+1\right) - \frac{1}{8}\,\log\left(\sqrt{x^4+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(x^4 + 1)/x^4 + 1/8*log(sqrt(x^4 + 1) + 1) - 1/8*log(sqrt(x^4 + 1)
 - 1)
```

maple [A] time = 0.01, size = 24, normalized size = 0.77

$$\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{4} - \frac{\sqrt{x^4+1}}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^4+1)^(1/2),x)
```

```
[Out] -1/4*(x^4+1)^(1/2)/x^4+1/4*arctanh(1/(x^4+1)^(1/2))
```

maxima [A] time = 1.38, size = 37, normalized size = 1.19

$$-\frac{\sqrt{x^4+1}}{4\,x^4}+\frac{1}{8}\,\log\left(\sqrt{x^4+1}\,+1\right)-\frac{1}{8}\,\log\left(\sqrt{x^4+1}\,-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(x^4 + 1)/x^4 + 1/8*log(sqrt(x^4 + 1) + 1) - 1/8*log(sqrt(x^4 + 1)
 - 1)
```

mupad [B] time = 1.21, size = 23, normalized size = 0.74

$$\frac{\operatorname{atanh}\left(\sqrt{x^4+1}\right)}{4}-\frac{\sqrt{x^4+1}}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(x^4 + 1)^(1/2)),x)
```

```
[Out] atanh((x^4 + 1)^(1/2))/4 - (x^4 + 1)^(1/2)/(4*x^4)
```

sympy [A] time = 2.10, size = 22, normalized size = 0.71

$$\frac{\operatorname{asinh}\left(\frac{1}{x^2}\right)}{4}-\frac{\sqrt{1+\frac{1}{x^4}}}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**4+1)**(1/2),x)
```

```
[Out] asinh(x**(-2))/4 - sqrt(1 + x**(-4))/(4*x**2)
```

3.922 $\int \frac{x^5}{\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=25

$$\frac{1}{4}x^2\sqrt{x^4+1}-\frac{1}{4}\sinh^{-1}\left(x^2\right)$$

```
[Out] -1/4*arcsinh(x^2)+1/4*x^2*(x^4+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {275, 321, 215}

$$\frac{1}{4}x^2\sqrt{x^4+1}-\frac{1}{4}\sinh^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[1 + x^4],x]
```

```
[Out] (x^2*Sqrt[1 + x^4])/4 - ArcSinh[x^2]/4
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\sqrt{1+x^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}}\,dx, x, x^2\right)\\ &= \frac{1}{4}x^2\sqrt{1+x^4}-\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, x^2\right)\\ &= \frac{1}{4}x^2\sqrt{1+x^4}-\frac{1}{4}\sinh^{-1}\left(x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{1}{4}x^2\sqrt{x^4+1}-\frac{1}{4}\sinh^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[1 + x^4],x]
```

```
[Out] (x^2*Sqrt[1 + x^4])/4 - ArcSinh[x^2]/4
```

fricas [A] time = 0.89, size = 29, normalized size = 1.16

$$\frac{1}{4}\sqrt{x^4+1}\,x^2+\frac{1}{4}\log\left(-x^2+\sqrt{x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(x^4 + 1)*x^2 + 1/4*log(-x^2 + sqrt(x^4 + 1))
```

giac [A] time = 0.16, size = 29, normalized size = 1.16

$$\frac{1}{4}\sqrt{x^4+1}\,x^2+\frac{1}{4}\log\left(-x^2+\sqrt{x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(x^4 + 1)*x^2 + 1/4*log(-x^2 + sqrt(x^4 + 1))
```

maple [A] time = 0.01, size = 20, normalized size = 0.80

$$\frac{\sqrt{x^4+1}\;x^2}{4}-\frac{\operatorname{arcsinh}\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^4+1)^(1/2),x)
```

```
[Out] -1/4*arcsinh(x^2)+1/4*x^2*(x^4+1)^(1/2)
```

maxima [B] time = 1.42, size = 58, normalized size = 2.32

$$\frac{\sqrt{x^4+1}}{4\,x^2\left(\frac{x^4+1}{x^4}-1\right)}-\frac{1}{8}\log\left(\frac{\sqrt{x^4+1}}{x^2}+1\right)+\frac{1}{8}\log\left(\frac{\sqrt{x^4+1}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(x^4 + 1)/(x^2*((x^4 + 1)/x^4 - 1)) - 1/8*log(sqrt(x^4 + 1)/x^2 + 1
) + 1/8*log(sqrt(x^4 + 1)/x^2 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^5}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^4 + 1)^(1/2),x)
```

```
[Out] int(x^5/(x^4 + 1)^(1/2), x)
```

sympy [A] time = 2.12, size = 19, normalized size = 0.76

$$\frac{x^2\sqrt{x^4+1}}{4}-\frac{\operatorname{asinh}\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**4+1)**(1/2),x)
```

```
[Out] x**2*sqrt(x**4 + 1)/4 - asinh(x**2)/4
```

3.923 $\int \frac{x}{\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=8

$$\frac{1}{2}\sinh^{-1}\left(x^2\right)$$

```
[Out] 1/2*arcsinh(x^2)
```

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {275, 215}

$$\frac{1}{2}\sinh^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[1 + x^4],x]
```

```
[Out] ArcSinh[x^2]/2
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{1+x^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\sinh^{-1}\left(x^2\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\frac{1}{2}\sinh^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[1 + x^4],x]
```

```
[Out] ArcSinh[x^2]/2
```

fricas [B] time = 0.87, size = 16, normalized size = 2.00

$$-\frac{1}{2}\log\left(-x^2+\sqrt{x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*log(-x^2 + sqrt(x^4 + 1))
```

giac [B] time = 0.16, size = 16, normalized size = 2.00

$$-\frac{1}{2}\log\left(-x^2+\sqrt{x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+1)^(1/2),x, algorithm="giac")
[Out] -1/2*log(-x^2 + sqrt(x^4 + 1))
```

maple [A] time = 0.01, size = 7, normalized size = 0.88

$$\frac{\operatorname{arcsinh}\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4+1)^(1/2),x)
[Out] 1/2*arcsinh(x^2)
```

maxima [B] time = 1.37, size = 33, normalized size = 4.12

$$\frac{1}{4}\log\left(\frac{\sqrt{x^4+1}}{x^2}+1\right)-\frac{1}{4}\log\left(\frac{\sqrt{x^4+1}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+1)^(1/2),x, algorithm="maxima")
[Out] 1/4*log(sqrt(x^4 + 1)/x^2 + 1) - 1/4*log(sqrt(x^4 + 1)/x^2 - 1)
```

mupad [B] time = 1.02, size = 6, normalized size = 0.75

$$\frac{\operatorname{asinh}\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4 + 1)^(1/2),x)
[Out] asinh(x^2)/2
```

sympy [A] time = 1.02, size = 5, normalized size = 0.62

$$\frac{\operatorname{asinh}\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4+1)**(1/2),x)
[Out] asinh(x**2)/2
```

3.924 $\int \frac{1}{x^3\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=16

$$-\frac{\sqrt{x^4+1}}{2x^2}$$

```
[Out] -1/2*(x^4+1)^(1/2)/x^2
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {264}

$$-\frac{\sqrt{x^4+1}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[1 + x^4]),x]
```

```
[Out] -Sqrt[1 + x^4]/(2*x^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{1+x^4}}\,dx = -\frac{\sqrt{1+x^4}}{2x^2}$$

Mathematica [A] time = 0.03, size = 16, normalized size = 1.00

$$-\frac{\sqrt{x^4+1}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[1 + x^4]),x]
```

```
[Out] -1/2*Sqrt[1 + x^4]/x^2
```

fricas [A] time = 0.72, size = 16, normalized size = 1.00

$$-\frac{x^2+\sqrt{x^4+1}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(x^2 + sqrt(x^4 + 1))/x^2
```

giac [A] time = 0.16, size = 19, normalized size = 1.19

$$\frac{1}{\left(x^2-\sqrt{x^4+1}\right)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^4+1)^(1/2),x, algorithm="giac")
[Out] 1/((x^2 - sqrt(x^4 + 1))^2 - 1)
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$-\frac{\sqrt{x^4+1}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^4+1)^(1/2),x)
[Out] -1/2*(x^4+1)^(1/2)/x^2
```

maxima [A] time = 1.35, size = 12, normalized size = 0.75

$$-\frac{\sqrt{x^4+1}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^4+1)^(1/2),x, algorithm="maxima")
[Out] -1/2*sqrt(x^4 + 1)/x^2
```

mupad [B] time = 1.11, size = 12, normalized size = 0.75

$$-\frac{\sqrt{x^4+1}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x^4 + 1)^(1/2)),x)
[Out] -(x^4 + 1)^(1/2)/(2*x^2)
```

sympy [A] time = 0.79, size = 12, normalized size = 0.75

$$-\frac{\sqrt{1+\frac{1}{x^4}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**4+1)**(1/2),x)
[Out] -sqrt(1 + x**(-4))/2
```

3.925 $\int \frac{1}{x^7\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=33

$$\frac{\sqrt{x^4+1}}{3x^2}-\frac{\sqrt{x^4+1}}{6x^6}$$

```
[Out] -1/6*(x^4+1)^(1/2)/x^6+1/3*(x^4+1)^(1/2)/x^2
```

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {271, 264}

$$\frac{\sqrt{x^4+1}}{3x^2}-\frac{\sqrt{x^4+1}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*Sqrt[1 + x^4]),x]
```

```
[Out] -Sqrt[1 + x^4]/(6*x^6) + Sqrt[1 + x^4]/(3*x^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7\sqrt{1+x^4}}\,dx &= -\frac{\sqrt{1+x^4}}{6x^6}-\frac{2}{3}\int \frac{1}{x^3\sqrt{1+x^4}}\,dx\\ &= -\frac{\sqrt{1+x^4}}{6x^6}+\frac{\sqrt{1+x^4}}{3x^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 0.70

$$-\frac{\left(1-2x^4\right)\sqrt{x^4+1}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*Sqrt[1 + x^4]),x]
```

```
[Out] -1/6*((1 - 2*x^4)*Sqrt[1 + x^4])/x^6
```

fricas [A] time = 0.77, size = 26, normalized size = 0.79

$$\frac{2\,x^6+\left(2\,x^4-1\right)\sqrt{x^4+1}}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^4+1)^(1/2),x, algorithm="fricas")
[Out] 1/6*(2*x^6 + (2*x^4 - 1)*sqrt(x^4 + 1))/x^6
```

giac [A] time = 0.16, size = 40, normalized size = 1.21

$$\frac{2\left(3\left(x^2-\sqrt{x^4+1}\right)^2-1\right)}{3\left(\left(x^2-\sqrt{x^4+1}\right)^2-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^4+1)^(1/2),x, algorithm="giac")
[Out] 2/3*(3*(x^2 - sqrt(x^4 + 1))^2 - 1)/((x^2 - sqrt(x^4 + 1))^2 - 1)^3
```

maple [A] time = 0.00, size = 20, normalized size = 0.61

$$\frac{\sqrt{x^4+1}\,\left(2x^4-1\right)}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^4+1)^(1/2),x)
[Out] 1/6*(x^4+1)^(1/2)*(2*x^4-1)/x^6
```

maxima [A] time = 1.32, size = 25, normalized size = 0.76

$$\frac{\sqrt{x^4+1}}{2\,x^2}-\frac{\left(x^4+1\right)^{\frac{3}{2}}}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^4+1)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(x^4 + 1)/x^2 - 1/6*(x^4 + 1)^(3/2)/x^6
```

mupad [B] time = 1.15, size = 19, normalized size = 0.58

$$\frac{\sqrt{x^4+1}\,\left(2\,x^4-1\right)}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(x^4 + 1)^(1/2)),x)
[Out] ((x^4 + 1)^(1/2)*(2*x^4 - 1))/(6*x^6)
```

sympy [A] time = 1.07, size = 26, normalized size = 0.79

$$\frac{\sqrt{1+\frac{1}{x^4}}}{3}-\frac{\sqrt{1+\frac{1}{x^4}}}{6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**4+1)**(1/2),x)
[Out] sqrt(1 + x**(-4))/3 - sqrt(1 + x**(-4))/(6*x**4)
```

3.926 $\int \frac{1}{x^{11}\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=49

$$-\frac{\sqrt{x^4+1}}{10x^{10}}+\frac{2\sqrt{x^4+1}}{15x^6}-\frac{4\sqrt{x^4+1}}{15x^2}$$

```
[Out] -1/10*(x^4+1)^(1/2)/x^10+2/15*(x^4+1)^(1/2)/x^6-4/15*(x^4+1)^(1/2)/x^2
```

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {271, 264}

$$-\frac{4\sqrt{x^4+1}}{15x^2}+\frac{2\sqrt{x^4+1}}{15x^6}-\frac{\sqrt{x^4+1}}{10x^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^11*Sqrt[1 + x^4]),x]
```

```
[Out] -Sqrt[1 + x^4]/(10*x^10) + (2*Sqrt[1 + x^4])/(15*x^6) - (4*Sqrt[1 + x^4])/(
15*x^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{11}\sqrt{1+x^4}}\,dx &= -\frac{\sqrt{1+x^4}}{10x^{10}}-\frac{4}{5}\int \frac{1}{x^7\sqrt{1+x^4}}\,dx\\ &= -\frac{\sqrt{1+x^4}}{10x^{10}}+\frac{2\sqrt{1+x^4}}{15x^6}+\frac{8}{15}\int \frac{1}{x^3\sqrt{1+x^4}}\,dx\\ &= -\frac{\sqrt{1+x^4}}{10x^{10}}+\frac{2\sqrt{1+x^4}}{15x^6}-\frac{4\sqrt{1+x^4}}{15x^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.57

$$-\frac{\sqrt{x^4+1}\left(8x^8-4x^4+3\right)}{30x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^11*Sqrt[1 + x^4]),x]
```

```
[Out] -1/30*(Sqrt[1 + x^4]*(3 - 4*x^4 + 8*x^8))/x^10
```

fricas [A] time = 0.61, size = 31, normalized size = 0.63

$$-\frac{8x^{10}+\left(8x^8-4x^4+3\right)\sqrt{x^4+1}}{30x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/30*(8*x^10 + (8*x^8 - 4*x^4 + 3)*sqrt(x^4 + 1))/x^10
```

giac [A] time = 0.18, size = 57, normalized size = 1.16

$$\frac{8\left(10\left(x^2-\sqrt{x^4+1}\right)^4-5\left(x^2-\sqrt{x^4+1}\right)^2+1\right)}{15\left(\left(x^2-\sqrt{x^4+1}\right)^2-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] 8/15*(10*(x^2 - sqrt(x^4 + 1))^4 - 5*(x^2 - sqrt(x^4 + 1))^2 + 1)/((x^2 - s
qrt(x^4 + 1))^2 - 1)^5
```

maple [A] time = 0.01, size = 25, normalized size = 0.51

$$-\frac{\sqrt{x^4+1}\ \left(8x^8-4x^4+3\right)}{30x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^11/(x^4+1)^(1/2),x)
```

```
[Out] -1/30*(x^4+1)^(1/2)*(8*x^8-4*x^4+3)/x^10
```

maxima [A] time = 1.32, size = 37, normalized size = 0.76

$$-\frac{\sqrt{x^4+1}}{2x^2}+\frac{\left(x^4+1\right)^{\frac{3}{2}}}{3x^6}-\frac{\left(x^4+1\right)^{\frac{5}{2}}}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(x^4 + 1)/x^2 + 1/3*(x^4 + 1)^(3/2)/x^6 - 1/10*(x^4 + 1)^(5/2)/x^1
0
```

mupad [B] time = 1.22, size = 24, normalized size = 0.49

$$-\frac{\sqrt{x^4+1}\ \left(8x^8-4x^4+3\right)}{30x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^11*(x^4 + 1)^(1/2)),x)
```

```
[Out] -((x^4 + 1)^(1/2)*(8*x^8 - 4*x^4 + 3))/(30*x^10)
```

sympy [A] time = 1.54, size = 44, normalized size = 0.90

$$-\frac{4\sqrt{1+\frac{1}{x^4}}}{15}+\frac{2\sqrt{1+\frac{1}{x^4}}}{15x^4}-\frac{\sqrt{1+\frac{1}{x^4}}}{10x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**11/(x**4+1)**(1/2),x)
```

```
[Out] -4*sqrt(1 + x**(-4))/15 + 2*sqrt(1 + x**(-4))/(15*x**4) - sqrt(1 + x**(-4))
/(10*x**8)
```

3.927 $\int \frac{x^8}{\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=74

$$-\frac{5}{21}\sqrt{x^4+1}\,x+\frac{1}{7}\sqrt{x^4+1}\,x^5+\frac{5\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{42\sqrt{x^4+1}}$$

```
[Out] -5/21*x*(x^4+1)^(1/2)+1/7*x^5*(x^4+1)^(1/2)+5/42*(x^2+1)*(cos(2*arctan(x))^
2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2*2^(1/2))*((x^4+1)/
(x^2+1)^2)^(1/2)/(x^4+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {321, 220}

$$\frac{1}{7}\sqrt{x^4+1}\,x^5-\frac{5}{21}\sqrt{x^4+1}\,x+\frac{5\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{42\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/Sqrt[1 + x^4],x]
```

```
[Out] (-5*x*Sqrt[1 + x^4])/21 + (x^5*Sqrt[1 + x^4])/7 + (5*(1 + x^2)*Sqrt[(1 + x^
4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(42*Sqrt[1 + x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{\sqrt{1+x^4}}\,dx &= \frac{1}{7}x^5\sqrt{1+x^4}-\frac{5}{7}\int \frac{x^4}{\sqrt{1+x^4}}\,dx\\ &= -\frac{5}{21}x\sqrt{1+x^4}+\frac{1}{7}x^5\sqrt{1+x^4}+\frac{5}{21}\int \frac{1}{\sqrt{1+x^4}}\,dx\\ &= -\frac{5}{21}x\sqrt{1+x^4}+\frac{1}{7}x^5\sqrt{1+x^4}+\frac{5\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{42\sqrt{1+x^4}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.54

$$\frac{1}{21}x\left(5\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-x^4\right)+\sqrt{x^4+1}\,\left(3x^4-5\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/Sqrt[1 + x^4],x]
```

```
[Out] (x*(Sqrt[1 + x^4]*(-5 + 3*x^4) + 5*Hypergeometric2F1[1/4, 1/2, 5/4, -x^4]))
/21
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^8}{\sqrt{x^4+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^8/sqrt(x^4 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^8/sqrt(x^4 + 1), x)
```

maple [C] time = 0.01, size = 84, normalized size = 1.14

$$\frac{\sqrt{x^4+1}\,x^5}{7} - \frac{5\sqrt{x^4+1}\,x}{21} + \frac{5\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\text{EllipticF}\left(\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)x, i\right)}{21\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^4+1)^(1/2),x)
```

```
[Out] 1/7*(x^4+1)^(1/2)*x^5-5/21*(x^4+1)^(1/2)*x+5/21/(1/2*2^(1/2)+1/2*I*2^(1/2))
*(-I*x^2+1)^(1/2)*(I*x^2+1)^(1/2)/(x^4+1)^(1/2)*EllipticF((1/2*2^(1/2)+1/2*
I*2^(1/2))*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^8/sqrt(x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^8}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^4 + 1)^(1/2),x)
```

```
[Out] int(x^8/(x^4 + 1)^(1/2), x)
```

sympy [C] time = 1.54, size = 29, normalized size = 0.39

$$\frac{x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{9}{4} \\ \frac{13}{4}\end{matrix}\right| x^4 e^{i\pi}\right)}{4\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**4+1)**(1/2),x)
```

```
[Out] x**9*gamma(9/4)*hyper((1/2, 9/4), (13/4,), x**4*exp_polar(I*pi))/(4*gamma(1
3/4))
```

3.928 $\int \frac{x^4}{\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=58

$$\frac{1}{3}x\sqrt{x^4+1} - \frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{6\sqrt{x^4+1}}$$

```
[Out] 1/3*x*(x^4+1)^(1/2)-1/6*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))
*EllipticF(sin(2*arctan(x)),1/2*2^(1/2))*((x^4+1)/(x^2+1)^2)^(1/2)/(x^4+1)^
(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {321, 220}

$$\frac{1}{3}x\sqrt{x^4+1} - \frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{6\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[1 + x^4],x]
```

```
[Out] (x*Sqrt[1 + x^4])/3 - ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*Ar
cTan[x], 1/2])/(6*Sqrt[1 + x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt{1+x^4}}\,dx &= \frac{1}{3}x\sqrt{1+x^4} - \frac{1}{3}\int \frac{1}{\sqrt{1+x^4}}\,dx \\ &= \frac{1}{3}x\sqrt{1+x^4} - \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{6\sqrt{1+x^4}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.55

$$\frac{1}{3}x\left(\sqrt{x^4+1} - {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-x^4\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[1 + x^4],x]
[Out] (x*(Sqrt[1 + x^4] - Hypergeometric2F1[1/4, 1/2, 5/4, -x^4]))/3
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^4}{\sqrt{x^4+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4+1)^(1/2),x, algorithm="fricas")
[Out] integral(x^4/sqrt(x^4 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^4/sqrt(x^4 + 1), x)
```

maple [C] time = 0.01, size = 72, normalized size = 1.24

$$\frac{\sqrt{x^4+1}\,x}{3} - \frac{\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\ \text{EllipticF}\left(\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)x, i\right)}{3\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^4+1)^(1/2),x)
[Out] 1/3*(x^4+1)^(1/2)*x-1/3/(1/2*2^(1/2)+1/2*I*2^(1/2))*(-I*x^2+1)^(1/2)*(I*x^2
+1)^(1/2)/(x^4+1)^(1/2)*EllipticF((1/2*2^(1/2)+1/2*I*2^(1/2))*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^4/sqrt(x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^4 + 1)^(1/2),x)
[Out] int(x^4/(x^4 + 1)^(1/2), x)
```

sympy [C] time = 0.77, size = 29, normalized size = 0.50

$$\frac{x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{5}{4} \\ \frac{9}{4}\end{matrix}\right| x^4 e^{i\pi}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**4+1)**(1/2),x)
```

```
[Out] x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), x**4*exp_polar(I*pi))/(4*gamma(9/
4))
```

3.929 $\int \frac{1}{\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=43

$$\frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{x^4+1}}$$

```
[Out] 1/2*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arc
tan(x)),1/2*2^(1/2))*((x^4+1)/(x^2+1)^2)^(1/2)/(x^4+1)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {220}

$$\frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + x^4],x]
```

```
[Out] ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(2*Sqrt
[1 + x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+x^4}}\,dx = \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{1+x^4}}$$

Mathematica [C] time = 0.02, size = 21, normalized size = 0.49

$$-\sqrt[4]{-1}\,F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\,x\right)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + x^4],x]
```

```
[Out] -((-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*x], -1])
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{x^4+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(x^4 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(x^4 + 1), x)
```

maple [C] time = 0.00, size = 60, normalized size = 1.40

$$\frac{\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\;\text{EllipticF}\left(\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)x, i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+1)^(1/2),x)
```

```
[Out] 1/(1/2*2^(1/2)+1/2*I*2^(1/2))*(-I*x^2+1)^(1/2)*(I*x^2+1)^(1/2)/(x^4+1)^(1/2
)*EllipticF((1/2*2^(1/2)+1/2*I*2^(1/2))*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(x^4 + 1), x)
```

mupad [B] time = 1.05, size = 12, normalized size = 0.28

$$x\,{}_2\mathrm{F}_1\left(\frac{1}{4}, \frac{1}{2};\ \frac{5}{4};\ -x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4 + 1)^(1/2),x)
```

```
[Out] x*hypergeom([1/4, 1/2], 5/4, -x^4)
```

sympy [C] time = 0.79, size = 27, normalized size = 0.63

$$\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4}, \frac{1}{2}\\ \frac{5}{4}\end{matrix}\right| x^4 e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+1)**(1/2),x)
```

```
[Out] x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), x**4*exp_polar(I*pi))/(4*gamma(5/4))
```

3.930 $\int \frac{1}{x^4\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=60

$$-\frac{\sqrt{x^4+1}}{3x^3}-\frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{6\sqrt{x^4+1}}$$

```
[Out] -1/3*(x^4+1)^(1/2)/x^3-1/6*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(
x))*EllipticF(sin(2*arctan(x)),1/2*2^(1/2))*((x^4+1)/(x^2+1)^2)^(1/2)/(x^4+
1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {325, 220}

$$-\frac{\sqrt{x^4+1}}{3x^3}-\frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{6\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[1 + x^4]),x]
```

```
[Out] -Sqrt[1 + x^4]/(3*x^3) - ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2
*ArcTan[x], 1/2])/(6*Sqrt[1 + x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{1+x^4}}\,dx &= -\frac{\sqrt{1+x^4}}{3x^3}-\frac{1}{3}\int \frac{1}{\sqrt{1+x^4}}\,dx\\ &= -\frac{\sqrt{1+x^4}}{3x^3}-\frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{6\sqrt{1+x^4}}\end{aligned}$$

Mathematica [C] time = 0.00, size = 22, normalized size = 0.37

$$-\frac{{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-x^4\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[1 + x^4]),x]
[Out] -1/3*Hypergeometric2F1[-3/4, 1/2, 1/4, -x^4]/x^3
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+1}}{x^8+x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^4+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x^4 + 1)/(x^8 + x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4+1}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^4 + 1)*x^4), x)
```

maple [C] time = 0.01, size = 74, normalized size = 1.23

$$-\frac{\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\text{EllipticF}\left(\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)x, i\right)}{3\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}} - \frac{\sqrt{x^4+1}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^4+1)^(1/2),x)
[Out] -1/3*(x^4+1)^(1/2)/x^3-1/3/(1/2*2^(1/2)+1/2*I*2^(1/2))*(-I*x^2+1)^(1/2)*(I*
x^2+1)^(1/2)/(x^4+1)^(1/2)*EllipticF((1/2*2^(1/2)+1/2*I*2^(1/2))*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4+1}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^4+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^4 + 1)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^4\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x^4 + 1)^(1/2)),x)
[Out] int(1/(x^4*(x^4 + 1)^(1/2)), x)
```

sympy [C] time = 0.90, size = 32, normalized size = 0.53

$$\frac{\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{4}, \frac{1}{2} \\ \frac{1}{4}\end{matrix}\right| x^4 e^{i\pi}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**4+1)**(1/2),x)
```

```
[Out] gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), x**4*exp_polar(I*pi))/(4*x**3*gamma(
1/4))
```

3.931 $\int \frac{1}{x^8\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=76

$$-\frac{\sqrt{x^4+1}}{7x^7}+\frac{5\sqrt{x^4+1}}{21x^3}+\frac{5\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{42\sqrt{x^4+1}}$$

```
[Out] -1/7*(x^4+1)^(1/2)/x^7+5/21*(x^4+1)^(1/2)/x^3+5/42*(x^2+1)*(cos(2*arctan(x)
)^2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2*2^(1/2))*((x^4+1
)/(x^2+1)^2)^(1/2)/(x^4+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {325, 220}

$$\frac{5\sqrt{x^4+1}}{21x^3}-\frac{\sqrt{x^4+1}}{7x^7}+\frac{5\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{42\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*Sqrt[1 + x^4]),x]
```

```
[Out] -Sqrt[1 + x^4]/(7*x^7) + (5*Sqrt[1 + x^4])/(21*x^3) + (5*(1 + x^2)*Sqrt[(1
+ x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(42*Sqrt[1 + x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^8\sqrt{1+x^4}}\,dx &= -\frac{\sqrt{1+x^4}}{7x^7}-\frac{5}{7}\int \frac{1}{x^4\sqrt{1+x^4}}\,dx\\ &= -\frac{\sqrt{1+x^4}}{7x^7}+\frac{5\sqrt{1+x^4}}{21x^3}+\frac{5}{21}\int \frac{1}{\sqrt{1+x^4}}\,dx\\ &= -\frac{\sqrt{1+x^4}}{7x^7}+\frac{5\sqrt{1+x^4}}{21x^3}+\frac{5\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{42\sqrt{1+x^4}}\end{aligned}$$

Mathematica [C] time = 0.00, size = 22, normalized size = 0.29

$$-\frac{{}_2F_1\left(-\frac{7}{4},\frac{1}{2};-\frac{3}{4};-x^4\right)}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*Sqrt[1 + x^4]),x]
```

```
[Out] -1/7*Hypergeometric2F1[-7/4, 1/2, -3/4, -x^4]/x^7
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+1}}{x^{12}+x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 1)/(x^12 + x^8), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4+1}\,x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + 1)*x^8), x)
```

maple [C] time = 0.01, size = 86, normalized size = 1.13

$$\frac{5\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\;\text{EllipticF}\left(\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)x, i\right)}{21\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}} + \frac{5\sqrt{x^4+1}}{21x^3} - \frac{\sqrt{x^4+1}}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(x^4+1)^(1/2),x)
```

```
[Out] -1/7*(x^4+1)^(1/2)/x^7+5/21*(x^4+1)^(1/2)/x^3+5/21/(1/2*2^(1/2)+1/2*I*2^(1/
2))*(-I*x^2+1)^(1/2)*(I*x^2+1)^(1/2)/(x^4+1)^(1/2)*EllipticF((1/2*2^(1/2)+1
/2*I*2^(1/2))*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4+1}\,x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + 1)*x^8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^8\,\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(x^4 + 1)^(1/2)),x)
```

```
[Out] int(1/(x^8*(x^4 + 1)^(1/2)), x)
```

sympy [C] time = 1.68, size = 36, normalized size = 0.47

$$\frac{\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{7}{4}, \frac{1}{2} \\ -\frac{3}{4}\end{matrix}\right| x^4 e^{i\pi}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(x**4+1)**(1/2),x)
```

```
[Out] gamma(-7/4)*hyper((-7/4, 1/2), (-3/4,), x**4*exp_polar(I*pi))/(4*x**7*gamma
(-3/4))
```

3.932 $\int \frac{x^{10}}{\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=140

$$\frac{1}{9}\sqrt{x^4+1}\,x^7-\frac{7}{45}\sqrt{x^4+1}\,x^3+\frac{7\sqrt{x^4+1}\,x}{15\left(x^2+1\right)}+\frac{7\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{30\sqrt{x^4+1}}-\frac{7\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,E\left(2\tan^{-1}(x)\right.}{15\sqrt{x^4+1}}$$

```
[Out] -7/45*x^3*(x^4+1)^(1/2)+1/9*x^7*(x^4+1)^(1/2)+7/15*x*(x^4+1)^(1/2)/(x^2+1)-
7/15*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*ar
ctan(x)),1/2*2^(1/2))*((x^4+1)/(x^2+1)^2)^(1/2)/(x^4+1)^(1/2)+7/30*(x^2+1)*
(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2*
2^(1/2))*((x^4+1)/(x^2+1)^2)^(1/2)/(x^4+1)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {321, 305, 220, 1196}

$$\frac{1}{9}\sqrt{x^4+1}\,x^7-\frac{7}{45}\sqrt{x^4+1}\,x^3+\frac{7\sqrt{x^4+1}\,x}{15\left(x^2+1\right)}+\frac{7\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{30\sqrt{x^4+1}}-\frac{7\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,E\left(2\tan^{-1}(x)\right.}{15\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/Sqrt[1 + x^4],x]
```

```
[Out] (-7*x^3*Sqrt[1 + x^4])/45 + (x^7*Sqrt[1 + x^4])/9 + (7*x*Sqrt[1 + x^4])/(15
*(1 + x^2)) - (7*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x
], 1/2])/(15*Sqrt[1 + x^4]) + (7*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*Elli
pticF[2*ArcTan[x], 1/2])/(30*Sqrt[1 + x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
```

```
x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{10}}{\sqrt{1+x^4}}\,dx &= \frac{1}{9}x^7\sqrt{1+x^4} - \frac{7}{9}\int \frac{x^6}{\sqrt{1+x^4}}\,dx \\ &= -\frac{7}{45}x^3\sqrt{1+x^4} + \frac{1}{9}x^7\sqrt{1+x^4} + \frac{7}{15}\int \frac{x^2}{\sqrt{1+x^4}}\,dx \\ &= -\frac{7}{45}x^3\sqrt{1+x^4} + \frac{1}{9}x^7\sqrt{1+x^4} + \frac{7}{15}\int \frac{1}{\sqrt{1+x^4}}\,dx - \frac{7}{15}\int \frac{1-x^2}{\sqrt{1+x^4}}\,dx \\ &= -\frac{7}{45}x^3\sqrt{1+x^4} + \frac{1}{9}x^7\sqrt{1+x^4} + \frac{7x\sqrt{1+x^4}}{15\left(1+x^2\right)} - \frac{7\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{15\sqrt{1+x^4}} + \frac{7\left(1+\right.}{}\end{aligned}$$

Mathematica [C] time = 0.02, size = 42, normalized size = 0.30

$$\frac{1}{45}x^3\left(7\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-x^4\right)+\sqrt{x^4+1}\left(5x^4-7\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/Sqrt[1 + x^4],x]
```

```
[Out] (x^3*(Sqrt[1 + x^4]*(-7 + 5*x^4) + 7*Hypergeometric2F1[1/2, 3/4, 7/4, -x^4]
))/45
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^{10}}{\sqrt{x^4+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^10/sqrt(x^4 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^10/sqrt(x^4 + 1), x)
```

maple [C] time = 0.01, size = 107, normalized size = 0.76

$$\frac{\sqrt{x^4+1}\,x^7}{9} - \frac{7\sqrt{x^4+1}\,x^3}{45} + \frac{7i\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\left(-\text{EllipticE}\left(\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)x,i\right)+\text{EllipticF}\left(\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)x,i\right.\right.}{15\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(x^4+1)^(1/2),x)
```

```
[Out] 1/9*x^7*(x^4+1)^(1/2)-7/45*x^3*(x^4+1)^(1/2)+7/15*I/(1/2*2^(1/2)+1/2*I*2^(1
/2))*(-I*x^2+1)^(1/2)*(I*x^2+1)^(1/2)/(x^4+1)^(1/2)*(EllipticF((1/2*2^(1/2)
+1/2*I*2^(1/2))*x,I)-EllipticE((1/2*2^(1/2)+1/2*I*2^(1/2))*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^10/sqrt(x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{10}}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(x^4 + 1)^(1/2),x)
```

```
[Out] int(x^10/(x^4 + 1)^(1/2), x)
```

sympy [C] time = 1.16, size = 29, normalized size = 0.21

$$\frac{x^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{11}{4} \\ \frac{15}{4}\end{matrix}\right| x^4 e^{i\pi}\right)}{4\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(x**4+1)**(1/2),x)
```

```
[Out] x**11*gamma(11/4)*hyper((1/2, 11/4), (15/4,), x**4*exp_polar(I*pi))/(4*gamm
a(15/4))
```

3.933 $\int \frac{x^6}{\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=124

$$\frac{1}{5}\sqrt{x^4+1}\,x^3-\frac{3\sqrt{x^4+1}\,x}{5\left(x^2+1\right)}-\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{10\sqrt{x^4+1}}+\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{5\sqrt{x^4+1}}$$

```
[Out] 1/5*x^3*(x^4+1)^(1/2)-3/5*x*(x^4+1)^(1/2)/(x^2+1)+3/5*(x^2+1)*(cos(2*arctan
(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x)),1/2*2^(1/2))*((x^
4+1)/(x^2+1)^2)^(1/2)/(x^4+1)^(1/2)-3/10*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)
/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2*2^(1/2))*((x^4+1)/(x^2+1)^
2)^(1/2)/(x^4+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {321, 305, 220, 1196}

$$\frac{1}{5}\sqrt{x^4+1}\,x^3-\frac{3\sqrt{x^4+1}\,x}{5\left(x^2+1\right)}-\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{10\sqrt{x^4+1}}+\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{5\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/Sqrt[1 + x^4],x]
```

```
[Out] (x^3*Sqrt[1 + x^4])/5 - (3*x*Sqrt[1 + x^4])/(5*(1 + x^2)) + (3*(1 + x^2)*Sq
rt[(1 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/2])/(5*Sqrt[1 + x^4]) -
(3*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(10*S
qrt[1 + x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
```

```
x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{\sqrt{1+x^4}}\,dx &= \frac{1}{5}x^3\sqrt{1+x^4} - \frac{3}{5}\int \frac{x^2}{\sqrt{1+x^4}}\,dx \\ &= \frac{1}{5}x^3\sqrt{1+x^4} - \frac{3}{5}\int \frac{1}{\sqrt{1+x^4}}\,dx + \frac{3}{5}\int \frac{1-x^2}{\sqrt{1+x^4}}\,dx \\ &= \frac{1}{5}x^3\sqrt{1+x^4} - \frac{3x\sqrt{1+x^4}}{5\left(1+x^2\right)} + \frac{3\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{5\sqrt{1+x^4}} - \frac{3\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan\right.}{10\sqrt{1+x^4}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.27

$$\frac{1}{5}x^3\left(\sqrt{x^4+1} - {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-x^4\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/Sqrt[1 + x^4],x]
[Out] (x^3*(Sqrt[1 + x^4] - Hypergeometric2F1[1/2, 3/4, 7/4, -x^4]))/5
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^6}{\sqrt{x^4+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^4+1)^(1/2),x, algorithm="fricas")
[Out] integral(x^6/sqrt(x^4 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^6/sqrt(x^4 + 1), x)
```

maple [C] time = 0.01, size = 95, normalized size = 0.77

$$\frac{\sqrt{x^4+1}\,x^3}{5} - \frac{3i\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\left(-\text{EllipticE}\left(\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)x, i\right) + \text{EllipticF}\left(\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)x, i\right)\right)}{5\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^4+1)^(1/2),x)
```

```
[Out] 1/5*(x^4+1)^(1/2)*x^3-3/5*I/(1/2*2^(1/2)+1/2*I*2^(1/2))*(-I*x^2+1)^(1/2)*(I
*x^2+1)^(1/2)/(x^4+1)^(1/2)*(EllipticF((1/2*2^(1/2)+1/2*I*2^(1/2))*x,I)-Ell
ipticE((1/2*2^(1/2)+1/2*I*2^(1/2))*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/sqrt(x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^4 + 1)^(1/2),x)
```

```
[Out] int(x^6/(x^4 + 1)^(1/2), x)
```

sympy [C] time = 0.91, size = 29, normalized size = 0.23

$$\frac{x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{7}{4} \\ \frac{11}{4}\end{matrix}\right| x^4 e^{i\pi}\right)}{4\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**4+1)**(1/2),x)
```

```
[Out] x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), x**4*exp_polar(I*pi))/(4*gamma(1
1/4))
```

3.934 $\int \frac{x^2}{\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=103

$$\frac{\sqrt{x^4+1}\,x}{x^2+1}+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{x^4+1}}-\frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+1}}$$

```
[Out] x*(x^4+1)^(1/2)/(x^2+1)-(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))
*EllipticE(sin(2*arctan(x)),1/2*2^(1/2))*((x^4+1)/(x^2+1)^2)^(1/2)/(x^4+1)^
(1/2)+1/2*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticF(sin
(2*arctan(x)),1/2*2^(1/2))*((x^4+1)/(x^2+1)^2)^(1/2)/(x^4+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {305, 220, 1196}

$$\frac{\sqrt{x^4+1}\,x}{x^2+1}+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{x^4+1}}-\frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[1 + x^4],x]
```

```
[Out] (x*Sqrt[1 + x^4])/(1 + x^2) - ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*Ellipt
icE[2*ArcTan[x], 1/2])/Sqrt[1 + x^4] + ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^
2]*EllipticF[2*ArcTan[x], 1/2])/(2*Sqrt[1 + x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{1+x^4}}\,dx = \int \frac{1}{\sqrt{1+x^4}}\,dx - \int \frac{1-x^2}{\sqrt{1+x^4}}\,dx$$

$$= \frac{x\sqrt{1+x^4}}{1+x^2} - \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{1+x^4}} + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{1+x^4}}$$

Mathematica [C] time = 0.00, size = 22, normalized size = 0.21

$$\frac{1}{3}x^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[1 + x^4],x]
[Out] (x^3*Hypergeometric2F1[1/2, 3/4, 7/4, -x^4])/3
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2}{\sqrt{x^4+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+1)^(1/2),x, algorithm="fricas")
[Out] integral(x^2/sqrt(x^4 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/sqrt(x^4 + 1), x)
```

maple [C] time = 0.01, size = 82, normalized size = 0.80

$$\frac{i\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\left(-\text{EllipticE}\left(\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)x,i\right)+\text{EllipticF}\left(\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)x,i\right)\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4+1)^(1/2),x)
[Out] I/(1/2*2^(1/2)+1/2*I*2^(1/2))*(-I*x^2+1)^(1/2)*(I*x^2+1)^(1/2)/(x^4+1)^(1/2
)*(EllipticF((1/2*2^(1/2)+1/2*I*2^(1/2))*x,I)-EllipticE((1/2*2^(1/2)+1/2*I*
2^(1/2))*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/sqrt(x^4 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4 + 1)^(1/2),x)
```

```
[Out] int(x^2/(x^4 + 1)^(1/2), x)
```

sympy [C] time = 0.94, size = 29, normalized size = 0.28

$$\frac{x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{3}{4} \\ \frac{7}{4}\end{matrix}\right| x^4 e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**4+1)**(1/2),x)
```

```
[Out] x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**4*exp_polar(I*pi))/(4*gamma(7/
4))
```

3.935 $\int \frac{1}{x^2\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=117

$$-\frac{\sqrt{x^4+1}}{x}+\frac{\sqrt{x^4+1}\,x}{x^2+1}+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{x^4+1}}-\frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+1}}$$

```
[Out] -(x^4+1)^(1/2)/x+x*(x^4+1)^(1/2)/(x^2+1)-(x^2+1)*(cos(2*arctan(x))^2)^(1/2)
/cos(2*arctan(x))*EllipticE(sin(2*arctan(x)),1/2*2^(1/2))*((x^4+1)/(x^2+1)^
2)^(1/2)/(x^4+1)^(1/2)+1/2*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(
x))*EllipticF(sin(2*arctan(x)),1/2*2^(1/2))*((x^4+1)/(x^2+1)^2)^(1/2)/(x^4+
1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {325, 305, 220, 1196}

$$\frac{\sqrt{x^4+1}\,x}{x^2+1}-\frac{\sqrt{x^4+1}}{x}+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{x^4+1}}-\frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[1 + x^4]),x]
```

```
[Out] -(Sqrt[1 + x^4]/x) + (x*Sqrt[1 + x^4])/(1 + x^2) - ((1 + x^2)*Sqrt[(1 + x^4
)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/2])/Sqrt[1 + x^4] + ((1 + x^2)*Sqrt
[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(2*Sqrt[1 + x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{1+x^4}}\,dx &= -\frac{\sqrt{1+x^4}}{x} + \int \frac{x^2}{\sqrt{1+x^4}}\,dx \\ &= -\frac{\sqrt{1+x^4}}{x} + \int \frac{1}{\sqrt{1+x^4}}\,dx - \int \frac{1-x^2}{\sqrt{1+x^4}}\,dx \\ &= -\frac{\sqrt{1+x^4}}{x} + \frac{x\sqrt{1+x^4}}{1+x^2} - \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{1+x^4}} + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(\right.}{2\sqrt{1+x^4}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 20, normalized size = 0.17

$$-\frac{{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-x^4\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[1 + x^4]),x]
[Out] -(Hypergeometric2F1[-1/4, 1/2, 3/4, -x^4]/x)
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+1}}{x^6+x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^4+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x^4 + 1)/(x^6 + x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4+1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^4 + 1)*x^2), x)
```

maple [C] time = 0.01, size = 95, normalized size = 0.81

$$-\frac{\sqrt{x^4+1}}{x} + \frac{i\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\left(-\text{EllipticE}\left(\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)x, i\right) + \text{EllipticF}\left(\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)x, i\right)\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^4+1)^(1/2),x)
[Out] -(x^4+1)^(1/2)/x+I/(1/2*2^(1/2)+1/2*I*2^(1/2))*(-I*x^2+1)^(1/2)*(I*x^2+1)^(
1/2)/(x^4+1)^(1/2)*(EllipticF((1/2*2^(1/2)+1/2*I*2^(1/2))*x,I)-EllipticE((1
/2*2^(1/2)+1/2*I*2^(1/2))*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4+1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + 1)*x^2), x)
```

mupad [B] time = 1.14, size = 15, normalized size = 0.13

$$-\frac{{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ -x^4\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^4 + 1)^(1/2)),x)
```

```
[Out] -hypergeom([-1/4, 1/2], 3/4, -x^4)/x
```

sympy [C] time = 1.28, size = 31, normalized size = 0.26

$$\frac{\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\right| x^4 e^{i\pi}\right)}{4x\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**4+1)**(1/2),x)
```

```
[Out] gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), x**4*exp_polar(I*pi))/(4*x*gamma(3/4
))
```

3.936 $\int \frac{1}{x^6\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=140

$$\frac{3\sqrt{x^4+1}}{5x}-\frac{\sqrt{x^4+1}}{5x^5}-\frac{3\sqrt{x^4+1}\,x}{5\left(x^2+1\right)}-\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{10\sqrt{x^4+1}}+\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{5\sqrt{x^4+1}}$$

```
[Out] -1/5*(x^4+1)^(1/2)/x^5+3/5*(x^4+1)^(1/2)/x-3/5*x*(x^4+1)^(1/2)/(x^2+1)+3/5*
(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(
x)),1/2*2^(1/2))*((x^4+1)/(x^2+1)^2)^(1/2)/(x^4+1)^(1/2)-3/10*(x^2+1)*(cos(
2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2*2^(1/
2))*((x^4+1)/(x^2+1)^2)^(1/2)/(x^4+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {325, 305, 220, 1196}

$$-\frac{3\sqrt{x^4+1}\,x}{5\left(x^2+1\right)}+\frac{3\sqrt{x^4+1}}{5x}-\frac{\sqrt{x^4+1}}{5x^5}-\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{10\sqrt{x^4+1}}+\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{5\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*Sqrt[1 + x^4]),x]
```

```
[Out] -Sqrt[1 + x^4]/(5*x^5) + (3*Sqrt[1 + x^4])/(5*x) - (3*x*Sqrt[1 + x^4])/(5*(
1 + x^2)) + (3*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x],
 1/2])/(5*Sqrt[1 + x^4]) - (3*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*Ellipti
cF[2*ArcTan[x], 1/2])/(10*Sqrt[1 + x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
```

```
x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^6\sqrt{1+x^4}}\,dx &= -\frac{\sqrt{1+x^4}}{5x^5} - \frac{3}{5}\int \frac{1}{x^2\sqrt{1+x^4}}\,dx \\ &= -\frac{\sqrt{1+x^4}}{5x^5} + \frac{3\sqrt{1+x^4}}{5x} - \frac{3}{5}\int \frac{x^2}{\sqrt{1+x^4}}\,dx \\ &= -\frac{\sqrt{1+x^4}}{5x^5} + \frac{3\sqrt{1+x^4}}{5x} - \frac{3}{5}\int \frac{1}{\sqrt{1+x^4}}\,dx + \frac{3}{5}\int \frac{1-x^2}{\sqrt{1+x^4}}\,dx \\ &= -\frac{\sqrt{1+x^4}}{5x^5} + \frac{3\sqrt{1+x^4}}{5x} - \frac{3x\sqrt{1+x^4}}{5\left(1+x^2\right)} + \frac{3\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{5\sqrt{1+x^4}} - \frac{3\left(1+x^2\right)}{}\end{aligned}$$

Mathematica [C] time = 0.00, size = 22, normalized size = 0.16

$$-\frac{{}_2F_1\left(-\frac{5}{4},\frac{1}{2};-\frac{1}{4};-x^4\right)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*Sqrt[1 + x^4]),x]
[Out] -1/5*Hypergeometric2F1[-5/4, 1/2, -1/4, -x^4]/x^5
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+1}}{x^{10}+x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^4+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x^4 + 1)/(x^10 + x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4+1}\,x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^4 + 1)*x^6), x)
```

maple [C] time = 0.01, size = 107, normalized size = 0.76

$$\frac{3\sqrt{x^4+1}}{5x} - \frac{\sqrt{x^4+1}}{5x^5} - \frac{3i\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\left(-\text{EllipticE}\left(\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)x, i\right)+\text{EllipticF}\left(\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)x, i\right)\right)}{5\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^4+1)^(1/2),x)
```

```
[Out] -1/5*(x^4+1)^(1/2)/x^5+3/5*(x^4+1)^(1/2)/x-3/5*I/(1/2*2^(1/2)+1/2*I*2^(1/2)
)*(-I*x^2+1)^(1/2)*(I*x^2+1)^(1/2)/(x^4+1)^(1/2)*(EllipticF((1/2*2^(1/2)+1/
2*I*2^(1/2))*x,I)-EllipticE((1/2*2^(1/2)+1/2*I*2^(1/2))*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4+1}\,x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + 1)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^6\,\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(x^4 + 1)^(1/2)),x)
```

```
[Out] int(1/(x^6*(x^4 + 1)^(1/2)), x)
```

sympy [C] time = 1.12, size = 36, normalized size = 0.26

$$\frac{\Gamma\left(-\frac{5}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{5}{4}, \frac{1}{2} \\ -\frac{1}{4}\end{matrix}\right| x^4 e^{i\pi}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**4+1)**(1/2),x)
```

```
[Out] gamma(-5/4)*hyper((-5/4, 1/2), (-1/4,), x**4*exp_polar(I*pi))/(4*x**5*gamma
(-1/4))
```

3.937 $\int \frac{x^{11}}{\left(1+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=38

$$\frac{1}{6}\left(x^4+1\right)^{3/2}-\sqrt{x^4+1}-\frac{1}{2\sqrt{x^4+1}}$$

```
[Out] 1/6*(x^4+1)^(3/2)-1/2/(x^4+1)^(1/2)-(x^4+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{1}{6}\left(x^4+1\right)^{3/2}-\sqrt{x^4+1}-\frac{1}{2\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(1 + x^4)^(3/2),x]
```

```
[Out] -1/(2*Sqrt[1 + x^4]) - Sqrt[1 + x^4] + (1 + x^4)^(3/2)/6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{\left(1+x^4\right)^{3/2}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x^2}{(1+x)^{3/2}}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int \left(\frac{1}{(1+x)^{3/2}}-\frac{2}{\sqrt{1+x}}+\sqrt{1+x}\right)\,dx, x, x^4\right)\\ &= -\frac{1}{2\sqrt{1+x^4}}-\sqrt{1+x^4}+\frac{1}{6}\left(1+x^4\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 23, normalized size = 0.61

$$\frac{x^8-4x^4-8}{6\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(1 + x^4)^(3/2),x]
```

```
[Out] (-8 - 4*x^4 + x^8)/(6*Sqrt[1 + x^4])
```

fricas [A] time = 0.81, size = 19, normalized size = 0.50

$$\frac{x^8 - 4\,x^4 - 8}{6\,\sqrt{x^4 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^4+1)^(3/2),x, algorithm="fricas")
[Out] 1/6*(x^8 - 4*x^4 - 8)/sqrt(x^4 + 1)
```

giac [A] time = 0.15, size = 28, normalized size = 0.74

$$\frac{1}{6}\left(x^4 + 1\right)^{\frac{3}{2}} - \sqrt{x^4 + 1} - \frac{1}{2\,\sqrt{x^4 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^4+1)^(3/2),x, algorithm="giac")
[Out] 1/6*(x^4 + 1)^(3/2) - sqrt(x^4 + 1) - 1/2/sqrt(x^4 + 1)
```

maple [A] time = 0.00, size = 20, normalized size = 0.53

$$\frac{x^8 - 4x^4 - 8}{6\sqrt{x^4 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^4+1)^(3/2),x)
[Out] 1/6*(x^8-4*x^4-8)/(x^4+1)^(1/2)
```

maxima [A] time = 1.27, size = 28, normalized size = 0.74

$$\frac{1}{6}\left(x^4 + 1\right)^{\frac{3}{2}} - \sqrt{x^4 + 1} - \frac{1}{2\,\sqrt{x^4 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^4+1)^(3/2),x, algorithm="maxima")
[Out] 1/6*(x^4 + 1)^(3/2) - sqrt(x^4 + 1) - 1/2/sqrt(x^4 + 1)
```

mupad [B] time = 1.32, size = 25, normalized size = 0.66

$$-\frac{6\,x^4 - \left(x^4 + 1\right)^2 + 9}{6\,\sqrt{x^4 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^4 + 1)^(3/2),x)
[Out] -(6*x^4 - (x^4 + 1)^2 + 9)/(6*(x^4 + 1)^(1/2))
```

sympy [A] time = 2.26, size = 39, normalized size = 1.03

$$\frac{x^8}{6\sqrt{x^4 + 1}} - \frac{2x^4}{3\sqrt{x^4 + 1}} - \frac{4}{3\sqrt{x^4 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(x**4+1)**(3/2),x)
[Out] x**8/(6*sqrt(x**4 + 1)) - 2*x**4/(3*sqrt(x**4 + 1)) - 4/(3*sqrt(x**4 + 1))
```

3.938 $\int \frac{x^7}{\left(1+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=27

$$\frac{\sqrt{x^4+1}}{2}+\frac{1}{2\sqrt{x^4+1}}$$

```
[Out] 1/2/(x^4+1)^(1/2)+1/2*(x^4+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{\sqrt{x^4+1}}{2}+\frac{1}{2\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(1 + x^4)^(3/2),x]
```

```
[Out] 1/(2*Sqrt[1 + x^4]) + Sqrt[1 + x^4]/2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{\left(1+x^4\right)^{3/2}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x}{(1+x)^{3/2}}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int \left(-\frac{1}{(1+x)^{3/2}}+\frac{1}{\sqrt{1+x}}\right)dx, x, x^4\right)\\ &= \frac{1}{2\sqrt{1+x^4}}+\frac{\sqrt{1+x^4}}{2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 0.67

$$\frac{x^4+2}{2\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(1 + x^4)^(3/2),x]
```

```
[Out] (2 + x^4)/(2*Sqrt[1 + x^4])
```

fricas [A] time = 0.87, size = 14, normalized size = 0.52

$$\frac{x^4+2}{2\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^4+1)^(3/2),x, algorithm="fricas")
[Out] 1/2*(x^4 + 2)/sqrt(x^4 + 1)
```

giac [A] time = 0.15, size = 19, normalized size = 0.70

$$\frac{1}{2}\sqrt{x^4+1}+\frac{1}{2\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^4+1)^(3/2),x, algorithm="giac")
[Out] 1/2*sqrt(x^4 + 1) + 1/2/sqrt(x^4 + 1)
```

maple [A] time = 0.00, size = 15, normalized size = 0.56

$$\frac{x^4+2}{2\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^4+1)^(3/2),x)
[Out] 1/2*(x^4+2)/(x^4+1)^(1/2)
```

maxima [A] time = 1.31, size = 19, normalized size = 0.70

$$\frac{1}{2}\sqrt{x^4+1}+\frac{1}{2\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^4+1)^(3/2),x, algorithm="maxima")
[Out] 1/2*sqrt(x^4 + 1) + 1/2/sqrt(x^4 + 1)
```

mupad [B] time = 1.16, size = 14, normalized size = 0.52

$$\frac{x^4+2}{2\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^4 + 1)^(3/2),x)
[Out] (x^4 + 2)/(2*(x^4 + 1)^(1/2))
```

sympy [A] time = 1.17, size = 22, normalized size = 0.81

$$\frac{x^4}{2\sqrt{x^4+1}}+\frac{1}{\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**4+1)**(3/2),x)
[Out] x**4/(2*sqrt(x**4 + 1)) + 1/sqrt(x**4 + 1)
```

3.939 $\int \frac{x^3}{\left(1+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{2\sqrt{x^4+1}}$$

```
[Out] -1/2/(x^4+1)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$-\frac{1}{2\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 + x^4)^(3/2),x]
```

```
[Out] -1/(2*Sqrt[1 + x^4])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^3}{\left(1+x^4\right)^{3/2}}\,dx = -\frac{1}{2\sqrt{1+x^4}}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$-\frac{1}{2\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 + x^4)^(3/2),x]
```

```
[Out] -1/2*1/Sqrt[1 + x^4]
```

fricas [A] time = 0.66, size = 9, normalized size = 0.69

$$-\frac{1}{2\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2/sqrt(x^4 + 1)
```

giac [A] time = 0.15, size = 9, normalized size = 0.69

$$-\frac{1}{2\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4+1)^(3/2),x, algorithm="giac")
[Out] -1/2/sqrt(x^4 + 1)
```

maple [A] time = 0.00, size = 10, normalized size = 0.77

$$-\frac{1}{2\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^4+1)^(3/2),x)
[Out] -1/2/(x^4+1)^(1/2)
```

maxima [A] time = 1.28, size = 9, normalized size = 0.69

$$-\frac{1}{2\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4+1)^(3/2),x, algorithm="maxima")
[Out] -1/2/sqrt(x^4 + 1)
```

mupad [B] time = 1.17, size = 9, normalized size = 0.69

$$-\frac{1}{2\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^4 + 1)^(3/2),x)
[Out] -1/(2*(x^4 + 1)^(1/2))
```

sympy [A] time = 0.43, size = 12, normalized size = 0.92

$$-\frac{1}{2\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**4+1)**(3/2),x)
[Out] -1/(2*sqrt(x**4 + 1))
```

3.940 $\int \frac{1}{x\left(1+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=28

$$\frac{1}{2\sqrt{x^4+1}}-\frac{1}{2}\tanh^{-1}\left(\sqrt{x^4+1}\right)$$

```
[Out] -1/2*arctanh((x^4+1)^(1/2))+1/2/(x^4+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {266, 51, 63, 207}

$$\frac{1}{2\sqrt{x^4+1}}-\frac{1}{2}\tanh^{-1}\left(\sqrt{x^4+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + x^4)^(3/2)),x]
```

```
[Out] 1/(2*Sqrt[1 + x^4]) - ArcTanh[Sqrt[1 + x^4]]/2
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(1+x^4\right)^{3/2}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x(1+x)^{3/2}}\,dx, x, x^4\right)\\ &= \frac{1}{2\sqrt{1+x^4}} + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^4\right)\\ &= \frac{1}{2\sqrt{1+x^4}} + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^4}\right)\\ &= \frac{1}{2\sqrt{1+x^4}} - \frac{1}{2}\tanh^{-1}\left(\sqrt{1+x^4}\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 26, normalized size = 0.93

$$\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};x^4+1\right)}{2\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + x^4)^(3/2)),x]
```

```
[Out] Hypergeometric2F1[-1/2, 1, 1/2, 1 + x^4]/(2*Sqrt[1 + x^4])
```

fricas [B] time = 0.87, size = 52, normalized size = 1.86

$$-\frac{\left(x^4+1\right)\log\left(\sqrt{x^4+1}+1\right)-\left(x^4+1\right)\log\left(\sqrt{x^4+1}-1\right)-2\sqrt{x^4+1}}{4\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*((x^4 + 1)*log(sqrt(x^4 + 1) + 1) - (x^4 + 1)*log(sqrt(x^4 + 1) - 1) -
 2*sqrt(x^4 + 1))/(x^4 + 1)
```

giac [A] time = 0.17, size = 34, normalized size = 1.21

$$\frac{1}{2\sqrt{x^4+1}} - \frac{1}{4}\log\left(\sqrt{x^4+1}+1\right) + \frac{1}{4}\log\left(\sqrt{x^4+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4+1)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2/sqrt(x^4 + 1) - 1/4*log(sqrt(x^4 + 1) + 1) + 1/4*log(sqrt(x^4 + 1) - 1)
```

maple [A] time = 0.01, size = 21, normalized size = 0.75

$$-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{2} + \frac{1}{2\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^4+1)^(3/2),x)
```

```
[Out] 1/2/(x^4+1)^(1/2)-1/2*arctanh(1/(x^4+1)^(1/2))
```

maxima [A] time = 1.28, size = 34, normalized size = 1.21

$$\frac{1}{2\sqrt{x^4+1}} - \frac{1}{4}\log\left(\sqrt{x^4+1}+1\right) + \frac{1}{4}\log\left(\sqrt{x^4+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4+1)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2/sqrt(x^4 + 1) - 1/4*log(sqrt(x^4 + 1) + 1) + 1/4*log(sqrt(x^4 + 1) - 1)
```

mupad [B] time = 1.19, size = 20, normalized size = 0.71

$$\frac{1}{2\sqrt{x^4+1}} - \frac{\operatorname{atanh}\left(\sqrt{x^4+1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^4 + 1)^(3/2)),x)
```

```
[Out] 1/(2*(x^4 + 1)^(1/2)) - atanh((x^4 + 1)^(1/2))/2
```

sympy [B] time = 1.89, size = 87, normalized size = 3.11

$$\frac{x^4\log\left(x^4\right)}{4x^4+4} - \frac{2x^4\log\left(\sqrt{x^4+1}+1\right)}{4x^4+4} + \frac{2\sqrt{x^4+1}}{4x^4+4} + \frac{\log\left(x^4\right)}{4x^4+4} - \frac{2\log\left(\sqrt{x^4+1}+1\right)}{4x^4+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**4+1)**(3/2),x)
```

```
[Out] x**4*log(x**4)/(4*x**4 + 4) - 2*x**4*log(sqrt(x**4 + 1) + 1)/(4*x**4 + 4) +
 2*sqrt(x**4 + 1)/(4*x**4 + 4) + log(x**4)/(4*x**4 + 4) - 2*log(sqrt(x**4 +
 1) + 1)/(4*x**4 + 4)
```

3.941 $\int \frac{1}{x^5\left(1+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=44

$$-\frac{1}{4x^4\sqrt{x^4+1}}-\frac{3}{4\sqrt{x^4+1}}+\frac{3}{4}\tanh^{-1}\left(\sqrt{x^4+1}\right)$$

```
[Out] 3/4*arctanh((x^4+1)^(1/2))-3/4/(x^4+1)^(1/2)-1/4/x^4/(x^4+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {266, 51, 63, 207}

$$-\frac{3\sqrt{x^4+1}}{4x^4}+\frac{1}{2x^4\sqrt{x^4+1}}+\frac{3}{4}\tanh^{-1}\left(\sqrt{x^4+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 + x^4)^(3/2)),x]
```

```
[Out] 1/(2*x^4*Sqrt[1 + x^4]) - (3*Sqrt[1 + x^4])/(4*x^4) + (3*ArcTanh[Sqrt[1 + x
^4]])/4
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\left(1+x^4\right)^{3/2}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x^2(1+x)^{3/2}}\,dx, x, x^4\right)\\ &= \frac{1}{2x^4\sqrt{1+x^4}} + \frac{3}{4}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x}}\,dx, x, x^4\right)\\ &= \frac{1}{2x^4\sqrt{1+x^4}} - \frac{3\sqrt{1+x^4}}{4x^4} - \frac{3}{8}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^4\right)\\ &= \frac{1}{2x^4\sqrt{1+x^4}} - \frac{3\sqrt{1+x^4}}{4x^4} - \frac{3}{4}\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^4}\right)\\ &= \frac{1}{2x^4\sqrt{1+x^4}} - \frac{3\sqrt{1+x^4}}{4x^4} + \frac{3}{4}\tanh^{-1}\left(\sqrt{1+x^4}\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 26, normalized size = 0.59

$$-\frac{{}_2F_1\left(-\frac{1}{2}, 2; \frac{1}{2}; x^4+1\right)}{2\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 + x^4)^(3/2)),x]
```

```
[Out] -1/2*Hypergeometric2F1[-1/2, 2, 1/2, 1 + x^4]/Sqrt[1 + x^4]
```

fricas [B] time = 0.83, size = 66, normalized size = 1.50

$$\frac{3\left(x^8+x^4\right)\log\left(\sqrt{x^4+1}+1\right) - 3\left(x^8+x^4\right)\log\left(\sqrt{x^4+1}-1\right) - 2\left(3x^4+1\right)\sqrt{x^4+1}}{8\left(x^8+x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^4+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(3*(x^8 + x^4)*log(sqrt(x^4 + 1) + 1) - 3*(x^8 + x^4)*log(sqrt(x^4 + 1)
 - 1) - 2*(3*x^4 + 1)*sqrt(x^4 + 1))/(x^8 + x^4)
```

giac [A] time = 0.15, size = 53, normalized size = 1.20

$$-\frac{3x^4+1}{4\left(\left(x^4+1\right)^{\frac{3}{2}} - \sqrt{x^4+1}\right)} + \frac{3}{8}\log\left(\sqrt{x^4+1}+1\right) - \frac{3}{8}\log\left(\sqrt{x^4+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^4+1)^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*(3*x^4 + 1)/((x^4 + 1)^(3/2) - sqrt(x^4 + 1)) + 3/8*log(sqrt(x^4 + 1)
+ 1) - 3/8*log(sqrt(x^4 + 1) - 1)
```

maple [A] time = 0.01, size = 33, normalized size = 0.75

$$\frac{3\,\text{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{4} - \frac{1}{4\sqrt{x^4+1}\,x^4} - \frac{3}{4\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^4+1)^(3/2),x)
```

```
[Out] -1/4/x^4/(x^4+1)^(1/2)-3/4/(x^4+1)^(1/2)+3/4*arctanh(1/(x^4+1)^(1/2))
```

maxima [A] time = 1.32, size = 53, normalized size = 1.20

$$-\frac{3x^4+1}{4\left(\left(x^4+1\right)^{\frac{3}{2}}-\sqrt{x^4+1}\right)}+\frac{3}{8}\log\left(\sqrt{x^4+1}+1\right)-\frac{3}{8}\log\left(\sqrt{x^4+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^4+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(3*x^4 + 1)/((x^4 + 1)^(3/2) - sqrt(x^4 + 1)) + 3/8*log(sqrt(x^4 + 1)
+ 1) - 3/8*log(sqrt(x^4 + 1) - 1)
```

mupad [B] time = 1.24, size = 32, normalized size = 0.73

$$\frac{3\,\mathrm{atanh}\left(\sqrt{x^4+1}\right)}{4}-\frac{3}{4\sqrt{x^4+1}}-\frac{1}{4x^4\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(x^4 + 1)^(3/2)),x)
```

```
[Out] (3*atanh((x^4 + 1)^(1/2)))/4 - 3/(4*(x^4 + 1)^(1/2)) - 1/(4*x^4*(x^4 + 1)^(
1/2))
```

sympy [A] time = 3.07, size = 42, normalized size = 0.95

$$\frac{3\,\mathrm{asinh}\left(\frac{1}{x^2}\right)}{4}-\frac{3}{4x^2\sqrt{1+\frac{1}{x^4}}}-\frac{1}{4x^6\sqrt{1+\frac{1}{x^4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**4+1)**(3/2),x)
```

```
[Out] 3*asinh(x**(-2))/4 - 3/(4*x**2*sqrt(1 + x**(-4))) - 1/(4*x**6*sqrt(1 + x**(
-4)))
```

3.942 $\int \frac{x^9}{\left(1+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{3}{4}\sinh^{-1}\left(x^2\right)-\frac{x^6}{2\sqrt{x^4+1}}+\frac{3}{4}\sqrt{x^4+1}\,x^2$$

```
[Out] -3/4*arcsinh(x^2)-1/2*x^6/(x^4+1)^(1/2)+3/4*x^2*(x^4+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {275, 288, 321, 215}

$$-\frac{x^6}{2\sqrt{x^4+1}}+\frac{3}{4}\sqrt{x^4+1}\,x^2-\frac{3}{4}\sinh^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(1 + x^4)^(3/2),x]
```

```
[Out] -x^6/(2*Sqrt[1 + x^4]) + (3*x^2*Sqrt[1 + x^4])/4 - (3*ArcSinh[x^2])/4
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{\left(1+x^4\right)^{3/2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^{3/2}}\,dx, x, x^2\right)\\ &= -\frac{x^6}{2\sqrt{1+x^4}} + \frac{3}{2}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}}\,dx, x, x^2\right)\\ &= -\frac{x^6}{2\sqrt{1+x^4}} + \frac{3}{4}x^2\sqrt{1+x^4} - \frac{3}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, x^2\right)\\ &= -\frac{x^6}{2\sqrt{1+x^4}} + \frac{3}{4}x^2\sqrt{1+x^4} - \frac{3}{4}\sinh^{-1}\left(x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 0.90

$$\frac{x^6 + 3x^2 - 3\sqrt{x^4+1}\,\sinh^{-1}\left(x^2\right)}{4\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(1 + x^4)^(3/2),x]
```

```
[Out] (3*x^2 + x^6 - 3*Sqrt[1 + x^4]*ArcSinh[x^2])/(4*Sqrt[1 + x^4])
```

fricas [A] time = 0.95, size = 54, normalized size = 1.32

$$\frac{2\,x^4 + 3\left(x^4+1\right)\log\left(-x^2+\sqrt{x^4+1}\right) + \left(x^6+3\,x^2\right)\sqrt{x^4+1} + 2}{4\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^4+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*x^4 + 3*(x^4 + 1)*log(-x^2 + sqrt(x^4 + 1)) + (x^6 + 3*x^2)*sqrt(x^4
 + 1) + 2)/(x^4 + 1)
```

giac [A] time = 0.16, size = 34, normalized size = 0.83

$$\frac{\left(x^4+3\right)x^2}{4\sqrt{x^4+1}} + \frac{3}{4}\log\left(-x^2+\sqrt{x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^4+1)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(x^4 + 3)*x^2/sqrt(x^4 + 1) + 3/4*log(-x^2 + sqrt(x^4 + 1))
```

maple [A] time = 0.01, size = 32, normalized size = 0.78

$$\frac{x^6}{4\sqrt{x^4+1}} + \frac{3x^2}{4\sqrt{x^4+1}} - \frac{3\,\text{arcsinh}\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^4+1)^(3/2),x)
```

```
[Out] 1/4*x^6/(x^4+1)^(1/2)+3/4*x^2/(x^4+1)^(1/2)-3/4*arcsinh(x^2)
```

maxima [B] time = 1.38, size = 73, normalized size = 1.78

$$-\frac{\frac{3\left(x^4+1\right)}{x^4}-2}{4\left(\frac{\sqrt{x^4+1}}{x^2}-\frac{\left(x^4+1\right)^{\frac{3}{2}}}{x^6}\right)}-\frac{3}{8}\log\left(\frac{\sqrt{x^4+1}}{x^2}+1\right)+\frac{3}{8}\log\left(\frac{\sqrt{x^4+1}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^4+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(3*(x^4 + 1)/x^4 - 2)/(sqrt(x^4 + 1)/x^2 - (x^4 + 1)^(3/2)/x^6) - 3/8*
log(sqrt(x^4 + 1)/x^2 + 1) + 3/8*log(sqrt(x^4 + 1)/x^2 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^9}{\left(x^4+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^4 + 1)^(3/2),x)
```

```
[Out] int(x^9/(x^4 + 1)^(3/2), x)
```

sympy [A] time = 4.01, size = 36, normalized size = 0.88

$$\frac{x^6}{4\sqrt{x^4+1}}+\frac{3x^2}{4\sqrt{x^4+1}}-\frac{3\,\mathrm{asinh}\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(x**4+1)**(3/2),x)
```

```
[Out] x**6/(4*sqrt(x**4 + 1)) + 3*x**2/(4*sqrt(x**4 + 1)) - 3*asinh(x**2)/4
```

3.943 $\int \frac{x^5}{\left(1+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=25

$$\frac{1}{2}\sinh^{-1}\left(x^2\right)-\frac{x^2}{2\sqrt{x^4+1}}$$

```
[Out] 1/2*arcsinh(x^2)-1/2*x^2/(x^4+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {275, 288, 215}

$$\frac{1}{2}\sinh^{-1}\left(x^2\right)-\frac{x^2}{2\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(1 + x^4)^(3/2),x]
```

```
[Out] -x^2/(2*Sqrt[1 + x^4]) + ArcSinh[x^2]/2
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\left(1+x^4\right)^{3/2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^{3/2}}\,dx, x, x^2\right)\\ &= -\frac{x^2}{2\sqrt{1+x^4}}+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, x^2\right)\\ &= -\frac{x^2}{2\sqrt{1+x^4}}+\frac{1}{2}\sinh^{-1}\left(x^2\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 23, normalized size = 0.92

$$\frac{1}{2}\left(\sinh^{-1}\left(x^2\right)-\frac{x^2}{\sqrt{x^4+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(1 + x^4)^(3/2),x]
```

```
[Out] (-(x^2/Sqrt[1 + x^4]) + ArcSinh[x^2])/2
```

fricas [B] time = 0.95, size = 45, normalized size = 1.80

$$-\frac{x^4+\sqrt{x^4+1}\,x^2+\left(x^4+1\right)\log\left(-x^2+\sqrt{x^4+1}\right)+1}{2\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^4+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(x^4 + sqrt(x^4 + 1)*x^2 + (x^4 + 1)*log(-x^2 + sqrt(x^4 + 1)) + 1)/(x
^4 + 1)
```

giac [A] time = 0.18, size = 29, normalized size = 1.16

$$-\frac{x^2}{2\sqrt{x^4+1}}-\frac{1}{2}\log\left(-x^2+\sqrt{x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^4+1)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*x^2/sqrt(x^4 + 1) - 1/2*log(-x^2 + sqrt(x^4 + 1))
```

maple [A] time = 0.01, size = 20, normalized size = 0.80

$$-\frac{x^2}{2\sqrt{x^4+1}}+\frac{\operatorname{arcsinh}\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^4+1)^(3/2),x)
```

```
[Out] 1/2*arcsinh(x^2)-1/2/(x^4+1)^(1/2)*x^2
```

maxima [B] time = 1.30, size = 45, normalized size = 1.80

$$-\frac{x^2}{2\sqrt{x^4+1}}+\frac{1}{4}\log\left(\frac{\sqrt{x^4+1}}{x^2}+1\right)-\frac{1}{4}\log\left(\frac{\sqrt{x^4+1}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^4+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*x^2/sqrt(x^4 + 1) + 1/4*log(sqrt(x^4 + 1)/x^2 + 1) - 1/4*log(sqrt(x^4
+ 1)/x^2 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^5}{\left(x^4+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^4 + 1)^(3/2),x)
```

```
[Out] int(x^5/(x^4 + 1)^(3/2), x)
```

sympy [**A**] time = 2.30, size = 19, normalized size = 0.76

$$-\frac{x^2}{2\sqrt{x^4+1}}+\frac{\operatorname{asinh}\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**4+1)**(3/2),x)
[Out] -x**2/(2*sqrt(x**4 + 1)) + asinh(x**2)/2
```

3.944 $\int \frac{x}{\left(1+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=16

$$\frac{x^2}{2\sqrt{x^4+1}}$$

```
[Out] 1/2*x^2/(x^4+1)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {264}

$$\frac{x^2}{2\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + x^4)^(3/2),x]
```

```
[Out] x^2/(2*Sqrt[1 + x^4])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x}{\left(1+x^4\right)^{3/2}}\,dx = \frac{x^2}{2\sqrt{1+x^4}}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{x^2}{2\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + x^4)^(3/2),x]
```

```
[Out] x^2/(2*Sqrt[1 + x^4])
```

fricas [B] time = 0.84, size = 25, normalized size = 1.56

$$\frac{x^4+\sqrt{x^4+1}\,x^2+1}{2\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(x^4 + sqrt(x^4 + 1)*x^2 + 1)/(x^4 + 1)
```

giac [A] time = 0.16, size = 12, normalized size = 0.75

$$\frac{x^2}{2\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+1)^(3/2),x, algorithm="giac")
[Out] 1/2*x^2/sqrt(x^4 + 1)
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{x^2}{2\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4+1)^(3/2),x)
[Out] 1/2/(x^4+1)^(1/2)*x^2
```

maxima [A] time = 1.28, size = 12, normalized size = 0.75

$$\frac{x^2}{2\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+1)^(3/2),x, algorithm="maxima")
[Out] 1/2*x^2/sqrt(x^4 + 1)
```

mupad [B] time = 1.05, size = 12, normalized size = 0.75

$$\frac{x^2}{2\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4 + 1)^(3/2),x)
[Out] x^2/(2*(x^4 + 1)^(1/2))
```

sympy [A] time = 0.66, size = 12, normalized size = 0.75

$$\frac{x^2}{2\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4+1)**(3/2),x)
[Out] x**2/(2*sqrt(x**4 + 1))
```

3.945 $\int \frac{1}{x^3\left(1+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=31

$$-\frac{x^2}{\sqrt{x^4+1}}-\frac{1}{2\sqrt{x^4+1}\,x^2}$$

```
[Out] -1/2/x^2/(x^4+1)^(1/2)-x^2/(x^4+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {271, 264}

$$-\frac{x^2}{\sqrt{x^4+1}}-\frac{1}{2\sqrt{x^4+1}\,x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 + x^4)^(3/2)),x]
```

```
[Out] -1/(2*x^2*Sqrt[1 + x^4]) - x^2/Sqrt[1 + x^4]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(1+x^4\right)^{3/2}}\,dx &= -\frac{1}{2x^2\sqrt{1+x^4}}-2\int \frac{x}{\left(1+x^4\right)^{3/2}}\,dx\\ &= -\frac{1}{2x^2\sqrt{1+x^4}}-\frac{x^2}{\sqrt{1+x^4}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 0.74

$$-\frac{2x^4+1}{2x^2\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 + x^4)^(3/2)),x]
```

```
[Out] -1/2*(1 + 2*x^4)/(x^2*Sqrt[1 + x^4])
```

fricas [A] time = 0.85, size = 37, normalized size = 1.19

$$-\frac{2\,x^6+2\,x^2+\left(2\,x^4+1\right)\sqrt{x^4+1}}{2\left(x^6+x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^4+1)^(3/2),x, algorithm="fricas")
[Out] -1/2*(2*x^6 + 2*x^2 + (2*x^4 + 1)*sqrt(x^4 + 1))/(x^6 + x^2)
```

giac [A] time = 0.17, size = 32, normalized size = 1.03

$$-\frac{x^2}{2\sqrt{x^4+1}}+\frac{1}{\left(x^2-\sqrt{x^4+1}\right)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^4+1)^(3/2),x, algorithm="giac")
[Out] -1/2*x^2/sqrt(x^4 + 1) + 1/((x^2 - sqrt(x^4 + 1))^2 - 1)
```

maple [A] time = 0.00, size = 20, normalized size = 0.65

$$-\frac{2x^4+1}{2\sqrt{x^4+1}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^4+1)^(3/2),x)
[Out] -1/2*(2*x^4+1)/x^2/(x^4+1)^(1/2)
```

maxima [A] time = 1.36, size = 25, normalized size = 0.81

$$-\frac{x^2}{2\sqrt{x^4+1}}-\frac{\sqrt{x^4+1}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^4+1)^(3/2),x, algorithm="maxima")
[Out] -1/2*x^2/sqrt(x^4 + 1) - 1/2*sqrt(x^4 + 1)/x^2
```

mupad [B] time = 1.14, size = 17, normalized size = 0.55

$$-\frac{x^4+\frac{1}{2}}{x^2\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x^4 + 1)^(3/2)),x)
[Out] -(x^4 + 1/2)/(x^2*(x^4 + 1)^(1/2))
```

sympy [A] time = 0.90, size = 42, normalized size = 1.35

$$-\frac{2x^4\sqrt{x^4+1}}{2x^6+2x^2}-\frac{\sqrt{x^4+1}}{2x^6+2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**4+1)**(3/2),x)
[Out] -2*x**4*sqrt(x**4 + 1)/(2*x**6 + 2*x**2) - sqrt(x**4 + 1)/(2*x**6 + 2*x**2)
```

3.946 $\int \frac{1}{x^7\left(1+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=49

$$-\frac{1}{6\sqrt{x^4+1}\,x^6}+\frac{4x^2}{3\sqrt{x^4+1}}+\frac{2}{3\sqrt{x^4+1}\,x^2}$$

```
[Out] -1/6/x^6/(x^4+1)^(1/2)+2/3/x^2/(x^4+1)^(1/2)+4/3*x^2/(x^4+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {271, 264}

$$\frac{4x^2}{3\sqrt{x^4+1}}+\frac{2}{3\sqrt{x^4+1}\,x^2}-\frac{1}{6\sqrt{x^4+1}\,x^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(1 + x^4)^(3/2)),x]
```

```
[Out] -1/(6*x^6*Sqrt[1 + x^4]) + 2/(3*x^2*Sqrt[1 + x^4]) + (4*x^2)/(3*Sqrt[1 + x^
4])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7\left(1+x^4\right)^{3/2}}\,dx &= -\frac{1}{6x^6\sqrt{1+x^4}}-\frac{4}{3}\int \frac{1}{x^3\left(1+x^4\right)^{3/2}}\,dx\\ &= -\frac{1}{6x^6\sqrt{1+x^4}}+\frac{2}{3x^2\sqrt{1+x^4}}+\frac{8}{3}\int \frac{x}{\left(1+x^4\right)^{3/2}}\,dx\\ &= -\frac{1}{6x^6\sqrt{1+x^4}}+\frac{2}{3x^2\sqrt{1+x^4}}+\frac{4x^2}{3\sqrt{1+x^4}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.57

$$-\frac{-8x^8-4x^4+1}{6x^6\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(1 + x^4)^(3/2)),x]
```

```
[Out] -1/6*(1 - 4*x^4 - 8*x^8)/(x^6*Sqrt[1 + x^4])
```

fricas [A] time = 0.87, size = 42, normalized size = 0.86

$$\frac{8x^{10}+8x^6+\left(8x^8+4x^4-1\right)\sqrt{x^4+1}}{6\left(x^{10}+x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^4+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(8*x^10 + 8*x^6 + (8*x^8 + 4*x^4 - 1)*sqrt(x^4 + 1))/(x^10 + x^6)
```

giac [A] time = 0.18, size = 70, normalized size = 1.43

$$\frac{x^2}{2\sqrt{x^4+1}}-\frac{3\left(x^2-\sqrt{x^4+1}\right)^4-12\left(x^2-\sqrt{x^4+1}\right)^2+5}{3\left(\left(x^2-\sqrt{x^4+1}\right)^2-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^4+1)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*x^2/sqrt(x^4 + 1) - 1/3*(3*(x^2 - sqrt(x^4 + 1))^4 - 12*(x^2 - sqrt(x^4
 + 1))^2 + 5)/((x^2 - sqrt(x^4 + 1))^2 - 1)^3
```

maple [A] time = 0.00, size = 25, normalized size = 0.51

$$\frac{8x^8+4x^4-1}{6\sqrt{x^4+1}\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^4+1)^(3/2),x)
```

```
[Out] 1/6*(8*x^8+4*x^4-1)/x^6/(x^4+1)^(1/2)
```

maxima [A] time = 1.26, size = 36, normalized size = 0.73

$$\frac{x^2}{2\sqrt{x^4+1}}+\frac{\sqrt{x^4+1}}{x^2}-\frac{\left(x^4+1\right)^{\frac{3}{2}}}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^4+1)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/sqrt(x^4 + 1) + sqrt(x^4 + 1)/x^2 - 1/6*(x^4 + 1)^(3/2)/x^6
```

mupad [B] time = 1.23, size = 28, normalized size = 0.57

$$-\frac{12x^4-8\left(x^4+1\right)^2+9}{6x^6\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(x^4 + 1)^(3/2)),x)
```

```
[Out] -(12*x^4 - 8*(x^4 + 1)^2 + 9)/(6*x^6*(x^4 + 1)^(1/2))
```

sympy [A] time = 2.25, size = 70, normalized size = 1.43

$$\frac{8x^8\sqrt{1+\frac{1}{x^4}}}{6x^8+6x^4}+\frac{4x^4\sqrt{1+\frac{1}{x^4}}}{6x^8+6x^4}-\frac{\sqrt{1+\frac{1}{x^4}}}{6x^8+6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**4+1)**(3/2),x)
```

```
[Out] 8*x**8*sqrt(1 + x**(-4))/(6*x**8 + 6*x**4) + 4*x**4*sqrt(1 + x**(-4))/(6*x*
*8 + 6*x**4) - sqrt(1 + x**(-4))/(6*x**8 + 6*x**4)
```

3.947 $\int \frac{x^{12}}{\left(1+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=90

$$-\frac{15}{14}\sqrt{x^4+1}\,x-\frac{x^9}{2\sqrt{x^4+1}}+\frac{9}{14}\sqrt{x^4+1}\,x^5+\frac{15\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{28\sqrt{x^4+1}}$$

```
[Out] -1/2*x^9/(x^4+1)^(1/2)-15/14*x*(x^4+1)^(1/2)+9/14*x^5*(x^4+1)^(1/2)+15/28*(
x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x
)),1/2*2^(1/2))*((x^4+1)/(x^2+1)^2)^(1/2)/(x^4+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {288, 321, 220}

$$-\frac{x^9}{2\sqrt{x^4+1}}+\frac{9}{14}\sqrt{x^4+1}\,x^5-\frac{15}{14}\sqrt{x^4+1}\,x+\frac{15\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{28\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^12/(1 + x^4)^(3/2),x]
```

```
[Out] -x^9/(2*Sqrt[1 + x^4]) - (15*x*Sqrt[1 + x^4])/14 + (9*x^5*Sqrt[1 + x^4])/14
 + (15*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(
28*Sqrt[1 + x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{12}}{\left(1+x^4\right)^{3/2}}\,dx &= -\frac{x^9}{2\sqrt{1+x^4}} + \frac{9}{2}\int \frac{x^8}{\sqrt{1+x^4}}\,dx \\ &= -\frac{x^9}{2\sqrt{1+x^4}} + \frac{9}{14}x^5\sqrt{1+x^4} - \frac{45}{14}\int \frac{x^4}{\sqrt{1+x^4}}\,dx \\ &= -\frac{x^9}{2\sqrt{1+x^4}} - \frac{15}{14}x\sqrt{1+x^4} + \frac{9}{14}x^5\sqrt{1+x^4} + \frac{15}{14}\int \frac{1}{\sqrt{1+x^4}}\,dx \\ &= -\frac{x^9}{2\sqrt{1+x^4}} - \frac{15}{14}x\sqrt{1+x^4} + \frac{9}{14}x^5\sqrt{1+x^4} + \frac{15\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{28\sqrt{1+x^4}}\end{aligned}$$

Mathematica [C] time = 0.02, size = 52, normalized size = 0.58

$$\frac{x\left(15\sqrt{x^4+1}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-x^4\right)+2x^8-6x^4-15\right)}{14\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12/(1 + x^4)^(3/2),x]
```

```
[Out] (x*(-15 - 6*x^4 + 2*x^8 + 15*Sqrt[1 + x^4]*Hypergeometric2F1[1/4, 1/2, 5/4,
 -x^4]))/(14*Sqrt[1 + x^4])
```

fricas [F] time = 1.09, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+1}\,x^{12}}{x^8+2\,x^4+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(x^4+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 1)*x^12/(x^8 + 2*x^4 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{12}}{\left(x^4+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(x^4+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^12/(x^4 + 1)^(3/2), x)
```

maple [C] time = 0.01, size = 94, normalized size = 1.04

$$\frac{\sqrt{x^4+1}\;x^5}{7} - \frac{x}{2\sqrt{x^4+1}} - \frac{4\sqrt{x^4+1}\;x}{7} + \frac{15\sqrt{-ix^2+1}\;\sqrt{ix^2+1}\;\text{EllipticF}\left(\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)x, i\right)}{14\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(x^4+1)^(3/2),x)
```

```
[Out] -1/2*x/(x^4+1)^(1/2)+1/7*(x^4+1)^(1/2)*x^5-4/7*(x^4+1)^(1/2)*x+15/14/(1/2*2
^(1/2)+1/2*I*2^(1/2))*(-I*x^2+1)^(1/2)*(I*x^2+1)^(1/2)/(x^4+1)^(1/2)*Ellipt
icF((1/2*2^(1/2)+1/2*I*2^(1/2))*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{12}}{\left(x^4+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(x^4+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^12/(x^4 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{12}}{\left(x^4+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(x^4 + 1)^(3/2),x)
```

```
[Out] int(x^12/(x^4 + 1)^(3/2), x)
```

sympy [C] time = 1.94, size = 29, normalized size = 0.32

$$\frac{x^{13}\Gamma\left(\frac{13}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2}, \frac{13}{4} \\ \frac{17}{4}\end{matrix}\right| x^4 e^{i\pi}\right)}{4\Gamma\left(\frac{17}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12/(x**4+1)**(3/2),x)
```

```
[Out] x**13*gamma(13/4)*hyper((3/2, 13/4), (17/4,), x**4*exp_polar(I*pi))/(4*gamm
a(17/4))
```

3.948 $\int \frac{x^8}{\left(1+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=74

$$\frac{5}{6}\sqrt{x^4+1}\,x-\frac{x^5}{2\sqrt{x^4+1}}-\frac{5\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{12\sqrt{x^4+1}}$$

```
[Out] -1/2*x^5/(x^4+1)^(1/2)+5/6*x*(x^4+1)^(1/2)-5/12*(x^2+1)*(cos(2*arctan(x))^2
)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2*2^(1/2))*((x^4+1)/(
x^2+1)^2)^(1/2)/(x^4+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {288, 321, 220}

$$-\frac{x^5}{2\sqrt{x^4+1}}+\frac{5}{6}\sqrt{x^4+1}\,x-\frac{5\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{12\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(1 + x^4)^(3/2),x]
```

```
[Out] -x^5/(2*Sqrt[1 + x^4]) + (5*x*Sqrt[1 + x^4])/6 - (5*(1 + x^2)*Sqrt[(1 + x^4
)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(12*Sqrt[1 + x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^8}{\left(1+x^4\right)^{3/2}}\,dx = -\frac{x^5}{2\sqrt{1+x^4}} + \frac{5}{2}\int \frac{x^4}{\sqrt{1+x^4}}\,dx$$
$$= -\frac{x^5}{2\sqrt{1+x^4}} + \frac{5}{6}x\sqrt{1+x^4} - \frac{5}{6}\int \frac{1}{\sqrt{1+x^4}}\,dx$$
$$= -\frac{x^5}{2\sqrt{1+x^4}} + \frac{5}{6}x\sqrt{1+x^4} - \frac{5\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{12\sqrt{1+x^4}}$$

Mathematica [C] time = 0.01, size = 47, normalized size = 0.64

$$\frac{x\left(-5\sqrt{x^4+1}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-x^4\right)+2x^4+5\right)}{6\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(1 + x^4)^(3/2),x]
```

```
[Out] (x*(5 + 2*x^4 - 5*Sqrt[1 + x^4]*Hypergeometric2F1[1/4, 1/2, 5/4, -x^4]))/(6
*Sqrt[1 + x^4])
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+1}\,x^8}{x^8+2\,x^4+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^4+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 1)*x^8/(x^8 + 2*x^4 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(x^4+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^4+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^8/(x^4 + 1)^(3/2), x)
```

maple [C] time = 0.01, size = 82, normalized size = 1.11

$$\frac{x}{2\sqrt{x^4+1}} + \frac{\sqrt{x^4+1}\,x}{3} - \frac{5\sqrt{-ix^2+1}\;\sqrt{ix^2+1}\;\text{EllipticF}\left(\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)x, i\right)}{6\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^4+1)^(3/2),x)
```

```
[Out] 1/2/(x^4+1)^(1/2)*x+1/3*(x^4+1)^(1/2)*x-5/6/(1/2*2^(1/2)+1/2*I*2^(1/2))*(-I
*x^2+1)^(1/2)*(I*x^2+1)^(1/2)/(x^4+1)^(1/2)*EllipticF((1/2*2^(1/2)+1/2*I*2^
(1/2))*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(x^4+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^4+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^8/(x^4 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^8}{\left(x^4+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^4 + 1)^(3/2),x)
```

```
[Out] int(x^8/(x^4 + 1)^(3/2), x)
```

sympy [C] time = 1.40, size = 29, normalized size = 0.39

$$\frac{x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2}, \frac{9}{4} \\ \frac{13}{4}\end{matrix}\right| x^4 e^{i\pi}\right)}{4\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**4+1)**(3/2),x)
```

```
[Out] x**9*gamma(9/4)*hyper((3/2, 9/4), (13/4,), x**4*exp_polar(I*pi))/(4*gamma(1
3/4))
```

3.949 $\int \frac{x^4}{\left(1+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=58

$$\frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{x^4+1}}-\frac{x}{2\sqrt{x^4+1}}$$

```
[Out] -1/2*x/(x^4+1)^(1/2)+1/4*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x)
)*EllipticF(sin(2*arctan(x)),1/2*2^(1/2))*((x^4+1)/(x^2+1)^2)^(1/2)/(x^4+1)
^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {288, 220}

$$\frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{x^4+1}}-\frac{x}{2\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 + x^4)^(3/2),x]
```

```
[Out] -x/(2*Sqrt[1 + x^4]) + ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*A
rcTan[x], 1/2])/(4*Sqrt[1 + x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^4}{\left(1+x^4\right)^{3/2}}\,dx = -\frac{x}{2\sqrt{1+x^4}}+\frac{1}{2}\int \frac{1}{\sqrt{1+x^4}}\,dx$$

$$= -\frac{x}{2\sqrt{1+x^4}}+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{1+x^4}}$$

Mathematica [C] time = 0.02, size = 32, normalized size = 0.55

$$\frac{1}{2}x\left({}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-x^4\right)-\frac{1}{\sqrt{x^4+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 + x^4)^(3/2),x]
[Out] (x*(-(1/Sqrt[1 + x^4]) + Hypergeometric2F1[1/4, 1/2, 5/4, -x^4]))/2
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+1}\,x^4}{x^8+2\,x^4+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4+1)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(x^4 + 1)*x^4/(x^8 + 2*x^4 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(x^4+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4+1)^(3/2),x, algorithm="giac")
[Out] integrate(x^4/(x^4 + 1)^(3/2), x)
```

maple [C] time = 0.01, size = 72, normalized size = 1.24

$$-\frac{x}{2\sqrt{x^4+1}}+\frac{\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\;\text{EllipticF}\left(\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)x, i\right)}{2\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^4+1)^(3/2),x)
[Out] -1/2/(x^4+1)^(1/2)*x+1/2/(1/2*2^(1/2)+1/2*I*2^(1/2))*(-I*x^2+1)^(1/2)*(I*x^
2+1)^(1/2)/(x^4+1)^(1/2)*EllipticF((1/2*2^(1/2)+1/2*I*2^(1/2))*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(x^4+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4+1)^(3/2),x, algorithm="maxima")
[Out] integrate(x^4/(x^4 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{\left(x^4+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^4 + 1)^(3/2),x)
```

```
[Out] int(x^4/(x^4 + 1)^(3/2), x)
```

sympy [C] time = 0.83, size = 29, normalized size = 0.50

$$\frac{x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{4}, \frac{3}{2} \\ \frac{9}{4}\end{matrix}\right| x^4 e^{i\pi}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**4+1)**(3/2),x)
```

```
[Out] x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), x**4*exp_polar(I*pi))/(4*gamma(9/
4))
```

3.950 $\int \frac{1}{\left(1+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=58

$$\frac{x}{2\sqrt{x^4+1}}+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{x^4+1}}$$

```
[Out] 1/2*x/(x^4+1)^(1/2)+1/4*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))
*EllipticF(sin(2*arctan(x)),1/2*2^(1/2))*((x^4+1)/(x^2+1)^2)^(1/2)/(x^4+1)^
(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {199, 220}

$$\frac{x}{2\sqrt{x^4+1}}+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)^(-3/2),x]
```

```
[Out] x/(2*Sqrt[1 + x^4]) + ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*Ar
cTan[x], 1/2])/(4*Sqrt[1 + x^4])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(1+x^4\right)^{3/2}}\,dx &= \frac{x}{2\sqrt{1+x^4}}+\frac{1}{2}\int \frac{1}{\sqrt{1+x^4}}\,dx\\ &= \frac{x}{2\sqrt{1+x^4}}+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{1+x^4}}\end{aligned}$$

Mathematica [C] time = 0.02, size = 30, normalized size = 0.52

$$\frac{1}{2}x\left({}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-x^4\right)+\frac{1}{\sqrt{x^4+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)^(-3/2),x]
```

```
[Out] (x*(1/Sqrt[1 + x^4] + Hypergeometric2F1[1/4, 1/2, 5/4, -x^4]))/2
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+1}}{x^8+2\,x^4+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 1)/(x^8 + 2*x^4 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 1)^(-3/2), x)
```

maple [C] time = 0.00, size = 72, normalized size = 1.24

$$\frac{x}{2\sqrt{x^4+1}}+\frac{\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\;\text{EllipticF}\left(\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)x, i\right)}{2\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+1)^(3/2),x)
```

```
[Out] 1/2/(x^4+1)^(1/2)*x+1/2/(1/2*2^(1/2)+1/2*I*2^(1/2))*(-I*x^2+1)^(1/2)*(I*x^2
+1)^(1/2)/(x^4+1)^(1/2)*EllipticF((1/2*2^(1/2)+1/2*I*2^(1/2))*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)^(-3/2), x)
```

mupad [B] time = 0.08, size = 12, normalized size = 0.21

$$x\,{}_2\mathrm{F}_1\left(\frac{1}{4}, \frac{3}{2}; \frac{5}{4}; -x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4 + 1)^(3/2),x)
```

```
[Out] x*hypergeom([1/4, 3/2], 5/4, -x^4)
```

sympy [C] time = 0.86, size = 27, normalized size = 0.47

$$\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4}, \frac{3}{2} \\ \frac{5}{4}\end{matrix}\right| x^4 e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+1)**(3/2),x)
[Out] x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), x**4*exp_polar(I*pi))/(4*gamma(5/4))
```

3.951 $\int \frac{1}{x^4\left(1+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=76

$$-\frac{5\sqrt{x^4+1}}{6x^3}+\frac{1}{2x^3\sqrt{x^4+1}}-\frac{5\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{12\sqrt{x^4+1}}$$

```
[Out] 1/2/x^3/(x^4+1)^(1/2)-5/6*(x^4+1)^(1/2)/x^3-5/12*(x^2+1)*(cos(2*arctan(x))^
2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2*2^(1/2))*((x^4+1)/
(x^2+1)^2)^(1/2)/(x^4+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {290, 325, 220}

$$-\frac{5\sqrt{x^4+1}}{6x^3}+\frac{1}{2x^3\sqrt{x^4+1}}-\frac{5\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{12\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 + x^4)^(3/2)),x]
```

```
[Out] 1/(2*x^3*Sqrt[1 + x^4]) - (5*Sqrt[1 + x^4])/(6*x^3) - (5*(1 + x^2)*Sqrt[(1
+ x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(12*Sqrt[1 + x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^4\left(1+x^4\right)^{3/2}}\,dx = \frac{1}{2x^3\sqrt{1+x^4}} + \frac{5}{2}\int \frac{1}{x^4\sqrt{1+x^4}}\,dx$$
$$= \frac{1}{2x^3\sqrt{1+x^4}} - \frac{5\sqrt{1+x^4}}{6x^3} - \frac{5}{6}\int \frac{1}{\sqrt{1+x^4}}\,dx$$
$$= \frac{1}{2x^3\sqrt{1+x^4}} - \frac{5\sqrt{1+x^4}}{6x^3} - \frac{5\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{12\sqrt{1+x^4}}$$

Mathematica [C] time = 0.00, size = 22, normalized size = 0.29

$$-\frac{{}_2F_1\left(-\frac{3}{4},\frac{3}{2};\frac{1}{4};-x^4\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 + x^4)^(3/2)),x]
[Out] -1/3*Hypergeometric2F1[-3/4, 3/2, 1/4, -x^4]/x^3
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+1}}{x^{12}+2\,x^8+x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^4+1)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(x^4 + 1)/(x^12 + 2*x^8 + x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+1\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^4+1)^(3/2),x, algorithm="giac")
[Out] integrate(1/((x^4 + 1)^(3/2)*x^4), x)
```

maple [C] time = 0.01, size = 84, normalized size = 1.11

$$-\frac{x}{2\sqrt{x^4+1}} - \frac{5\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\text{EllipticF}\left(\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)x, i\right)}{6\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}} - \frac{\sqrt{x^4+1}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^4+1)^(3/2),x)
[Out] -1/3*(x^4+1)^(1/2)/x^3-1/2/(x^4+1)^(1/2)*x-5/6/(1/2*2^(1/2)+1/2*I*2^(1/2))*
(-I*x^2+1)^(1/2)*(I*x^2+1)^(1/2)/(x^4+1)^(1/2)*EllipticF((1/2*2^(1/2)+1/2*I
*2^(1/2))*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+1\right)^{\frac{3}{2}} x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^4+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 1)^(3/2)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4 \left(x^4+1\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x^4 + 1)^(3/2)),x)
```

```
[Out] int(1/(x^4*(x^4 + 1)^(3/2)), x)
```

sympy [C] time = 1.39, size = 32, normalized size = 0.42

$$\frac{\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(\left.\begin{matrix} -\frac{3}{4}, \frac{3}{2} \\ \frac{1}{4} \end{matrix}\right| x^4 e^{i\pi}\right)}{4x^3 \Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**4+1)**(3/2),x)
```

```
[Out] gamma(-3/4)*hyper((-3/4, 3/2), (1/4,), x**4*exp_polar(I*pi))/(4*x**3*gamma(
1/4))
```

3.952 $\int \frac{1}{x^8\left(1+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=92

$$-\frac{9\sqrt{x^4+1}}{14x^7}+\frac{1}{2x^7\sqrt{x^4+1}}+\frac{15\sqrt{x^4+1}}{14x^3}+\frac{15\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{28\sqrt{x^4+1}}$$

```
[Out] 1/2/x^7/(x^4+1)^(1/2)-9/14*(x^4+1)^(1/2)/x^7+15/14*(x^4+1)^(1/2)/x^3+15/28*
(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(
x)),1/2*2^(1/2))*((x^4+1)/(x^2+1)^2)^(1/2)/(x^4+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {290, 325, 220}

$$\frac{15\sqrt{x^4+1}}{14x^3}-\frac{9\sqrt{x^4+1}}{14x^7}+\frac{1}{2x^7\sqrt{x^4+1}}+\frac{15\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{28\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(1 + x^4)^(3/2)),x]
```

```
[Out] 1/(2*x^7*Sqrt[1 + x^4]) - (9*Sqrt[1 + x^4])/(14*x^7) + (15*Sqrt[1 + x^4])/(
14*x^3) + (15*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x],
1/2])/(28*Sqrt[1 + x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^8\left(1+x^4\right)^{3/2}}\,dx = \frac{1}{2x^7\sqrt{1+x^4}} + \frac{9}{2}\int \frac{1}{x^8\sqrt{1+x^4}}\,dx$$
$$= \frac{1}{2x^7\sqrt{1+x^4}} - \frac{9\sqrt{1+x^4}}{14x^7} - \frac{45}{14}\int \frac{1}{x^4\sqrt{1+x^4}}\,dx$$
$$= \frac{1}{2x^7\sqrt{1+x^4}} - \frac{9\sqrt{1+x^4}}{14x^7} + \frac{15\sqrt{1+x^4}}{14x^3} + \frac{15}{14}\int \frac{1}{\sqrt{1+x^4}}\,dx$$
$$= \frac{1}{2x^7\sqrt{1+x^4}} - \frac{9\sqrt{1+x^4}}{14x^7} + \frac{15\sqrt{1+x^4}}{14x^3} + \frac{15\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{28\sqrt{1+x^4}}$$

Mathematica [C] time = 0.00, size = 22, normalized size = 0.24

$$-\frac{{}_2F_1\left(-\frac{7}{4},\frac{3}{2};-\frac{3}{4};-x^4\right)}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(1 + x^4)^(3/2)),x]
[Out] -1/7*Hypergeometric2F1[-7/4, 3/2, -3/4, -x^4]/x^7
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+1}}{x^{16}+2\,x^{12}+x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^4+1)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(x^4 + 1)/(x^16 + 2*x^12 + x^8), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+1\right)^{\frac{3}{2}}x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^4+1)^(3/2),x, algorithm="giac")
[Out] integrate(1/((x^4 + 1)^(3/2)*x^8), x)
```

maple [C] time = 0.02, size = 96, normalized size = 1.04

$$\frac{x}{2\sqrt{x^4+1}} + \frac{15\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\;\text{EllipticF}\left(\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)x, i\right)}{14\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}} + \frac{4\sqrt{x^4+1}}{7x^3} - \frac{\sqrt{x^4+1}}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(x^4+1)^(3/2),x)
```

```
[Out] -1/7*(x^4+1)^(1/2)/x^7+4/7*(x^4+1)^(1/2)/x^3+1/2/(x^4+1)^(1/2)*x+15/14/(1/2
*2^(1/2)+1/2*I*2^(1/2))*(-I*x^2+1)^(1/2)*(I*x^2+1)^(1/2)/(x^4+1)^(1/2)*Elli
pticF((1/2*2^(1/2)+1/2*I*2^(1/2))*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+1\right)^{\frac{3}{2}} x^8}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^4+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 1)^(3/2)*x^8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^8 \left(x^4+1\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(x^4 + 1)^(3/2)),x)
```

```
[Out] int(1/(x^8*(x^4 + 1)^(3/2)), x)
```

sympy [C] time = 2.06, size = 36, normalized size = 0.39

$$\frac{\Gamma\left(-\frac{7}{4}\right) {}_2F_1\left(\left.\begin{matrix} -\frac{7}{4}, \frac{3}{2} \\ -\frac{3}{4} \end{matrix}\right| x^4 e^{i\pi}\right)}{4x^7 \Gamma\left(-\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(x**4+1)**(3/2),x)
```

```
[Out] gamma(-7/4)*hyper((-7/4, 3/2), (-3/4,), x**4*exp_polar(I*pi))/(4*x**7*gamma
(-3/4))
```

3.953 $\int \frac{x^{14}}{\left(1+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=156

$$-\frac{x^{11}}{2\sqrt{x^4+1}}+\frac{11}{18}\sqrt{x^4+1}\,x^7-\frac{77}{90}\sqrt{x^4+1}\,x^3+\frac{77\sqrt{x^4+1}\,x}{30\left(x^2+1\right)}+\frac{77\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{60\sqrt{x^4+1}}-\frac{77\left(x^2+1\right)\sqrt{\frac{}{(x}}}{30}$$

```
[Out] -1/2*x^11/(x^4+1)^(1/2)-77/90*x^3*(x^4+1)^(1/2)+11/18*x^7*(x^4+1)^(1/2)+77/
30*x*(x^4+1)^(1/2)/(x^2+1)-77/30*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*a
rctan(x))*EllipticE(sin(2*arctan(x)),1/2*2^(1/2))*((x^4+1)/(x^2+1)^2)^(1/2)
/(x^4+1)^(1/2)+77/60*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*El
lipticF(sin(2*arctan(x)),1/2*2^(1/2))*((x^4+1)/(x^2+1)^2)^(1/2)/(x^4+1)^(1/
2)
```

Rubi [A] time = 0.04, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {288, 321, 305, 220, 1196}

$$-\frac{x^{11}}{2\sqrt{x^4+1}}+\frac{11}{18}\sqrt{x^4+1}\,x^7-\frac{77}{90}\sqrt{x^4+1}\,x^3+\frac{77\sqrt{x^4+1}\,x}{30\left(x^2+1\right)}+\frac{77\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{60\sqrt{x^4+1}}-\frac{77\left(x^2+1\right)\sqrt{\frac{}{(x}}}{30}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/(1 + x^4)^(3/2),x]
```

```
[Out] -x^11/(2*Sqrt[1 + x^4]) - (77*x^3*Sqrt[1 + x^4])/90 + (11*x^7*Sqrt[1 + x^4]
)/18 + (77*x*Sqrt[1 + x^4])/(30*(1 + x^2)) - (77*(1 + x^2)*Sqrt[(1 + x^4)/(
1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/2])/(30*Sqrt[1 + x^4]) + (77*(1 + x^2)
*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(60*Sqrt[1 + x^4]
)
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{14}}{\left(1+x^4\right)^{3/2}}\,dx &= -\frac{x^{11}}{2\sqrt{1+x^4}} + \frac{11}{2}\int \frac{x^{10}}{\sqrt{1+x^4}}\,dx \\
&= -\frac{x^{11}}{2\sqrt{1+x^4}} + \frac{11}{18}x^7\sqrt{1+x^4} - \frac{77}{18}\int \frac{x^6}{\sqrt{1+x^4}}\,dx \\
&= -\frac{x^{11}}{2\sqrt{1+x^4}} - \frac{77}{90}x^3\sqrt{1+x^4} + \frac{11}{18}x^7\sqrt{1+x^4} + \frac{77}{30}\int \frac{x^2}{\sqrt{1+x^4}}\,dx \\
&= -\frac{x^{11}}{2\sqrt{1+x^4}} - \frac{77}{90}x^3\sqrt{1+x^4} + \frac{11}{18}x^7\sqrt{1+x^4} + \frac{77}{30}\int \frac{1}{\sqrt{1+x^4}}\,dx - \frac{77}{30}\int \frac{1-x^2}{\sqrt{1+x^4}}\,dx \\
&= -\frac{x^{11}}{2\sqrt{1+x^4}} - \frac{77}{90}x^3\sqrt{1+x^4} + \frac{11}{18}x^7\sqrt{1+x^4} + \frac{77x\sqrt{1+x^4}}{30\left(1+x^2\right)} - \frac{77\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,E\left(2\,\text{ta}\right.}{30\sqrt{1+x^4}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.35

$$\frac{x^3\left(-77\sqrt{x^4+1}\;{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-x^4\right)+5x^8-11x^4+77\right)}{45\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/(1 + x^4)^(3/2),x]
```

```
[Out] (x^3*(77 - 11*x^4 + 5*x^8 - 77*Sqrt[1 + x^4]*Hypergeometric2F1[3/4, 3/2, 7/
4, -x^4]))/(45*Sqrt[1 + x^4])
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+1}\,x^{14}}{x^8+2\,x^4+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(x^4+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 1)*x^14/(x^8 + 2*x^4 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(x^4+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(x^4+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^14/(x^4 + 1)^(3/2), x)
```

maple [C] time = 0.01, size = 119, normalized size = 0.76

$$\frac{\sqrt{x^4+1}\,x^7}{9}-\frac{x^3}{2\sqrt{x^4+1}}-\frac{16\sqrt{x^4+1}\,x^3}{45}+\frac{77i\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\left(-\text{EllipticE}\left(\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)x,i\right)+\text{EllipticF}\left(\left(\frac{\sqrt{2}}{2}\right.\right.\right.}{30\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(x^4+1)^(3/2),x)
```

```
[Out] -1/2*x^3/(x^4+1)^(1/2)+1/9*(x^4+1)^(1/2)*x^7-16/45*(x^4+1)^(1/2)*x^3+77/30*
I/(1/2*2^(1/2)+1/2*I*2^(1/2))*(-I*x^2+1)^(1/2)*(I*x^2+1)^(1/2)/(x^4+1)^(1/2
)*(EllipticF((1/2*2^(1/2)+1/2*I*2^(1/2))*x,I)-EllipticE((1/2*2^(1/2)+1/2*I*
2^(1/2))*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(x^4+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(x^4+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^14/(x^4 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{14}}{\left(x^4+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(x^4 + 1)^(3/2),x)
```

```
[Out] int(x^14/(x^4 + 1)^(3/2), x)
```

sympy [C] time = 2.24, size = 29, normalized size = 0.19

$$\frac{x^{15}\Gamma\left(\frac{15}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{15}{4}\\ \frac{19}{4}\end{matrix}\right|x^4e^{i\pi}\right)}{4\Gamma\left(\frac{19}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(x**4+1)**(3/2),x)
```

```
[Out] x**15*gamma(15/4)*hyper((3/2, 15/4), (19/4,), x**4*exp_polar(I*pi))/(4*gamm
a(19/4))
```

3.954 $\int \frac{x^{10}}{\left(1+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=140

$$-\frac{x^7}{2\sqrt{x^4+1}}+\frac{7}{10}\sqrt{x^4+1}\,x^3-\frac{21\sqrt{x^4+1}\,x}{10\left(x^2+1\right)}-\frac{21\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{20\sqrt{x^4+1}}+\frac{21\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}E\left(2\tan\right.}{10\sqrt{x^4+1}}$$

```
[Out] -1/2*x^7/(x^4+1)^(1/2)+7/10*x^3*(x^4+1)^(1/2)-21/10*x*(x^4+1)^(1/2)/(x^2+1)
+21/10*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*
arctan(x)),1/2*2^(1/2))*((x^4+1)/(x^2+1)^2)^(1/2)/(x^4+1)^(1/2)-21/20*(x^2+
1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1
/2*2^(1/2))*((x^4+1)/(x^2+1)^2)^(1/2)/(x^4+1)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {288, 321, 305, 220, 1196}

$$-\frac{x^7}{2\sqrt{x^4+1}}+\frac{7}{10}\sqrt{x^4+1}\,x^3-\frac{21\sqrt{x^4+1}\,x}{10\left(x^2+1\right)}-\frac{21\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{20\sqrt{x^4+1}}+\frac{21\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}E\left(2\tan\right.}{10\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(1 + x^4)^(3/2),x]
```

```
[Out] -x^7/(2*Sqrt[1 + x^4]) + (7*x^3*Sqrt[1 + x^4])/10 - (21*x*Sqrt[1 + x^4])/(1
0*(1 + x^2)) + (21*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan
[x], 1/2])/(10*Sqrt[1 + x^4]) - (21*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*E
llipticF[2*ArcTan[x], 1/2])/(20*Sqrt[1 + x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{10}}{\left(1+x^4\right)^{3/2}}\,dx &= -\frac{x^7}{2\sqrt{1+x^4}} + \frac{7}{2}\int \frac{x^6}{\sqrt{1+x^4}}\,dx \\
&= -\frac{x^7}{2\sqrt{1+x^4}} + \frac{7}{10}x^3\sqrt{1+x^4} - \frac{21}{10}\int \frac{x^2}{\sqrt{1+x^4}}\,dx \\
&= -\frac{x^7}{2\sqrt{1+x^4}} + \frac{7}{10}x^3\sqrt{1+x^4} - \frac{21}{10}\int \frac{1}{\sqrt{1+x^4}}\,dx + \frac{21}{10}\int \frac{1-x^2}{\sqrt{1+x^4}}\,dx \\
&= -\frac{x^7}{2\sqrt{1+x^4}} + \frac{7}{10}x^3\sqrt{1+x^4} - \frac{21x\sqrt{1+x^4}}{10\left(1+x^2\right)} + \frac{21\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x)\middle|\frac{1}{2}\right)}{10\sqrt{1+x^4}} - \frac{21\left(1+\right.}{}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 47, normalized size = 0.34

$$\frac{x^3\left(7\sqrt{x^4+1}\;{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-x^4\right)+x^4-7\right)}{5\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(1 + x^4)^(3/2),x]
```

```
[Out] (x^3*(-7 + x^4 + 7*Sqrt[1 + x^4]*Hypergeometric2F1[3/4, 3/2, 7/4, -x^4]))/(
5*Sqrt[1 + x^4])
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+1}\,x^{10}}{x^8+2\,x^4+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(x^4+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 1)*x^10/(x^8 + 2*x^4 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\left(x^4+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(x^4+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^10/(x^4 + 1)^(3/2), x)
```

maple [C] time = 0.01, size = 107, normalized size = 0.76

$$\frac{x^3}{2\sqrt{x^4+1}}+\frac{\sqrt{x^4+1}\,x^3}{5}-\frac{21i\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\left(-\text{EllipticE}\left(\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)x,i\right)+\text{EllipticF}\left(\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)x,i\right)\right)}{10\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(x^4+1)^(3/2),x)
```

```
[Out] 1/2/(x^4+1)^(1/2)*x^3+1/5*(x^4+1)^(1/2)*x^3-21/10*I/(1/2*2^(1/2)+1/2*I*2^(1
/2))*(-I*x^2+1)^(1/2)*(I*x^2+1)^(1/2)/(x^4+1)^(1/2)*(EllipticF((1/2*2^(1/2)
+1/2*I*2^(1/2))*x,I)-EllipticE((1/2*2^(1/2)+1/2*I*2^(1/2))*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\left(x^4+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(x^4+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^10/(x^4 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{10}}{\left(x^4+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(x^4 + 1)^(3/2),x)
```

```
[Out] int(x^10/(x^4 + 1)^(3/2), x)
```

sympy [C] time = 1.20, size = 29, normalized size = 0.21

$$\frac{x^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\right|x^4e^{i\pi}\right)}{4\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(x**4+1)**(3/2),x)
```

```
[Out] x**11*gamma(11/4)*hyper((3/2, 11/4), (15/4,), x**4*exp_polar(I*pi))/(4*gamm
a(15/4))
```

3.955 $\int \frac{x^6}{\left(1+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=124

$$-\frac{x^3}{2\sqrt{x^4+1}}+\frac{3\sqrt{x^4+1}\,x}{2\left(x^2+1\right)}+\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{x^4+1}}-\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{x^4+1}}$$

```
[Out] -1/2*x^3/(x^4+1)^(1/2)+3/2*x*(x^4+1)^(1/2)/(x^2+1)-3/2*(x^2+1)*(cos(2*arcta
n(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x)),1/2*2^(1/2))*((x
^4+1)/(x^2+1)^2)^(1/2)/(x^4+1)^(1/2)+3/4*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)
/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2*2^(1/2))*((x^4+1)/(x^2+1)^
2)^(1/2)/(x^4+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {288, 305, 220, 1196}

$$-\frac{x^3}{2\sqrt{x^4+1}}+\frac{3\sqrt{x^4+1}\,x}{2\left(x^2+1\right)}+\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{x^4+1}}-\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(1 + x^4)^(3/2),x]
```

```
[Out] -x^3/(2*Sqrt[1 + x^4]) + (3*x*Sqrt[1 + x^4])/(2*(1 + x^2)) - (3*(1 + x^2)*S
qrt[(1 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/2])/(2*Sqrt[1 + x^4]) +
 (3*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(4*S
qrt[1 + x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
```

```
x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{\left(1+x^4\right)^{3/2}}\,dx &= -\frac{x^3}{2\sqrt{1+x^4}} + \frac{3}{2}\int \frac{x^2}{\sqrt{1+x^4}}\,dx \\ &= -\frac{x^3}{2\sqrt{1+x^4}} + \frac{3}{2}\int \frac{1}{\sqrt{1+x^4}}\,dx - \frac{3}{2}\int \frac{1-x^2}{\sqrt{1+x^4}}\,dx \\ &= -\frac{x^3}{2\sqrt{1+x^4}} + \frac{3x\sqrt{1+x^4}}{2\left(1+x^2\right)} - \frac{3\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{1+x^4}} + \frac{3\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(\right.}{4\sqrt{1+x^4}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 31, normalized size = 0.25

$$x^3\left(\frac{1}{\sqrt{x^4+1}} - {}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-x^4\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(1 + x^4)^(3/2),x]
[Out] x^3*(1/Sqrt[1 + x^4] - Hypergeometric2F1[3/4, 3/2, 7/4, -x^4])
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+1}\,x^6}{x^8+2\,x^4+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^4+1)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(x^4 + 1)*x^6/(x^8 + 2*x^4 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(x^4+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^4+1)^(3/2),x, algorithm="giac")
[Out] integrate(x^6/(x^4 + 1)^(3/2), x)
```

maple [C] time = 0.01, size = 95, normalized size = 0.77

$$-\frac{x^3}{2\sqrt{x^4+1}} + \frac{3i\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\left(-\text{EllipticE}\left(\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)x, i\right) + \text{EllipticF}\left(\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)x, i\right)\right)}{2\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^4+1)^(3/2),x)
```

```
[Out] -1/2/(x^4+1)^(1/2)*x^3+3/2*I/(1/2*2^(1/2)+1/2*I*2^(1/2))*(-I*x^2+1)^(1/2)*(
I*x^2+1)^(1/2)/(x^4+1)^(1/2)*(EllipticF((1/2*2^(1/2)+1/2*I*2^(1/2))*x,I)-El
lipticE((1/2*2^(1/2)+1/2*I*2^(1/2))*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(x^4+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^4+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(x^4 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{\left(x^4+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^4 + 1)^(3/2),x)
```

```
[Out] int(x^6/(x^4 + 1)^(3/2), x)
```

sympy [C] time = 0.97, size = 29, normalized size = 0.23

$$\frac{x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2}, \frac{7}{4} \\ \frac{11}{4}\end{matrix}\right| x^4 e^{i\pi}\right)}{4\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**4+1)**(3/2),x)
```

```
[Out] x**7*gamma(7/4)*hyper((3/2, 7/4), (11/4,), x**4*exp_polar(I*pi))/(4*gamma(1
1/4))
```

3.956 $\int \frac{x^2}{\left(1+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=124

$$\frac{x^3}{2\sqrt{x^4+1}}-\frac{\sqrt{x^4+1}\,x}{2\left(x^2+1\right)}-\frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{x^4+1}}+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{x^4+1}}$$

```
[Out] 1/2*x^3/(x^4+1)^(1/2)-1/2*x*(x^4+1)^(1/2)/(x^2+1)+1/2*(x^2+1)*(cos(2*arctan
(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x)),1/2*2^(1/2))*((x^
4+1)/(x^2+1)^2)^(1/2)/(x^4+1)^(1/2)-1/4*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/
cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2*2^(1/2))*((x^4+1)/(x^2+1)^2
)^(1/2)/(x^4+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {290, 305, 220, 1196}

$$\frac{x^3}{2\sqrt{x^4+1}}-\frac{\sqrt{x^4+1}\,x}{2\left(x^2+1\right)}-\frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{x^4+1}}+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + x^4)^(3/2),x]
```

```
[Out] x^3/(2*Sqrt[1 + x^4]) - (x*Sqrt[1 + x^4])/(2*(1 + x^2)) + ((1 + x^2)*Sqrt[(
1 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/2])/(2*Sqrt[1 + x^4]) - ((1
+ x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(4*Sqrt[1 +
 x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
```

```
x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(1+x^4\right)^{3/2}}\,dx &= \frac{x^3}{2\sqrt{1+x^4}} - \frac{1}{2}\int \frac{x^2}{\sqrt{1+x^4}}\,dx \\ &= \frac{x^3}{2\sqrt{1+x^4}} - \frac{1}{2}\int \frac{1}{\sqrt{1+x^4}}\,dx + \frac{1}{2}\int \frac{1-x^2}{\sqrt{1+x^4}}\,dx \\ &= \frac{x^3}{2\sqrt{1+x^4}} - \frac{x\sqrt{1+x^4}}{2\left(1+x^2\right)} + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{1+x^4}} - \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x\right.}{4\sqrt{1+x^4}}\end{aligned}$$

Mathematica [C] time = 0.00, size = 22, normalized size = 0.18

$$\frac{1}{3}x^3\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + x^4)^(3/2),x]
[Out] (x^3*Hypergeometric2F1[3/4, 3/2, 7/4, -x^4])/3
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+1}\,x^2}{x^8+2\,x^4+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+1)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(x^4 + 1)*x^2/(x^8 + 2*x^4 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^4+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+1)^(3/2),x, algorithm="giac")
[Out] integrate(x^2/(x^4 + 1)^(3/2), x)
```

maple [C] time = 0.01, size = 95, normalized size = 0.77

$$\frac{x^3}{2\sqrt{x^4+1}} - \frac{i\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\left(-\text{EllipticE}\left(\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)x, i\right)+\text{EllipticF}\left(\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)x, i\right)\right)}{2\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4+1)^(3/2),x)
```

```
[Out] 1/2/(x^4+1)^(1/2)*x^3-1/2*I/(1/2*2^(1/2)+1/2*I*2^(1/2))*(-I*x^2+1)^(1/2)*(I
*x^2+1)^(1/2)/(x^4+1)^(1/2)*(EllipticF((1/2*2^(1/2)+1/2*I*2^(1/2))*x,I)-Ell
ipticE((1/2*2^(1/2)+1/2*I*2^(1/2))*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^4+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(x^4 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(x^4+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4 + 1)^(3/2),x)
```

```
[Out] int(x^2/(x^4 + 1)^(3/2), x)
```

sympy [C] time = 0.89, size = 29, normalized size = 0.23

$$\frac{x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4}, \frac{3}{2} \\ \frac{7}{4}\end{matrix}\right| x^4 e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**4+1)**(3/2),x)
```

```
[Out] x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), x**4*exp_polar(I*pi))/(4*gamma(7/
4))
```

3.957 $\int \frac{1}{x^2\left(1+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=140

$$-\frac{3\sqrt{x^4+1}}{2x}+\frac{1}{2\sqrt{x^4+1}\,x}+\frac{3\sqrt{x^4+1}\,x}{2\left(x^2+1\right)}+\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{x^4+1}}-\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{x^4+1}}$$

```
[Out] 1/2/x/(x^4+1)^(1/2)-3/2*(x^4+1)^(1/2)/x+3/2*x*(x^4+1)^(1/2)/(x^2+1)-3/2*(x^
2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x))
,1/2*2^(1/2))*((x^4+1)/(x^2+1)^2)^(1/2)/(x^4+1)^(1/2)+3/4*(x^2+1)*(cos(2*ar
ctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2*2^(1/2))*
((x^4+1)/(x^2+1)^2)^(1/2)/(x^4+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {290, 325, 305, 220, 1196}

$$\frac{3\sqrt{x^4+1}\,x}{2\left(x^2+1\right)}-\frac{3\sqrt{x^4+1}}{2x}+\frac{1}{2\sqrt{x^4+1}\,x}+\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{x^4+1}}-\frac{3\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 + x^4)^(3/2)),x]
```

```
[Out] 1/(2*x*Sqrt[1 + x^4]) - (3*Sqrt[1 + x^4])/(2*x) + (3*x*Sqrt[1 + x^4])/(2*(1
 + x^2)) - (3*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x],
1/2])/(2*Sqrt[1 + x^4]) + (3*(1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*Elliptic
F[2*ArcTan[x], 1/2])/(4*Sqrt[1 + x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\left(1+x^4\right)^{3/2}}\,dx &= \frac{1}{2x\sqrt{1+x^4}} + \frac{3}{2}\int \frac{1}{x^2\sqrt{1+x^4}}\,dx \\
&= \frac{1}{2x\sqrt{1+x^4}} - \frac{3\sqrt{1+x^4}}{2x} + \frac{3}{2}\int \frac{x^2}{\sqrt{1+x^4}}\,dx \\
&= \frac{1}{2x\sqrt{1+x^4}} - \frac{3\sqrt{1+x^4}}{2x} + \frac{3}{2}\int \frac{1}{\sqrt{1+x^4}}\,dx - \frac{3}{2}\int \frac{1-x^2}{\sqrt{1+x^4}}\,dx \\
&= \frac{1}{2x\sqrt{1+x^4}} - \frac{3\sqrt{1+x^4}}{2x} + \frac{3x\sqrt{1+x^4}}{2\left(1+x^2\right)} - \frac{3\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x)\middle|\frac{1}{2}\right)}{2\sqrt{1+x^4}} + \frac{3\left(1+\right.}{}
\end{aligned}$$

Mathematica [C] time = 0.00, size = 20, normalized size = 0.14

$$-\frac{{}_2F_1\left(-\frac{1}{4},\frac{3}{2};\frac{3}{4};-x^4\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 + x^4)^(3/2)),x]
```

```
[Out] -(Hypergeometric2F1[-1/4, 3/2, 3/4, -x^4]/x)
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+1}}{x^{10}+2\,x^6+x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^4+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 1)/(x^10 + 2*x^6 + x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+1\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^4+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + 1)^(3/2)*x^2), x)
```

maple [C] time = 0.01, size = 107, normalized size = 0.76

$$-\frac{x^3}{2\sqrt{x^4+1}}-\frac{\sqrt{x^4+1}}{x}+\frac{3i\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\left(-\text{EllipticE}\left(\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)x,i\right)+\text{EllipticF}\left(\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)x,i\right)\right)}{2\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^4+1)^(3/2),x)
```

```
[Out] -1/2/(x^4+1)^(1/2)*x^3-(x^4+1)^(1/2)/x+3/2*I/(1/2*2^(1/2)+1/2*I*2^(1/2))*(-
I*x^2+1)^(1/2)*(I*x^2+1)^(1/2)/(x^4+1)^(1/2)*(EllipticF((1/2*2^(1/2)+1/2*I*
2^(1/2))*x,I)-EllipticE((1/2*2^(1/2)+1/2*I*2^(1/2))*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+1\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^4+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 1)^(3/2)*x^2), x)
```

mupad [B] time = 1.14, size = 15, normalized size = 0.11

$$-\frac{{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{3}{2};\,\frac{3}{4};\,-x^4\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^4 + 1)^(3/2)),x)
```

```
[Out] -hypergeom([-1/4, 3/2], 3/4, -x^4)/x
```

sympy [C] time = 1.46, size = 31, normalized size = 0.22

$$\frac{\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{3}{4}\end{matrix}\right|x^4e^{i\pi}\right)}{4x\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**4+1)**(3/2),x)
```

```
[Out] gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), x**4*exp_polar(I*pi))/(4*x*gamma(3/4
))
```

3.958 $\int \frac{1}{x^6\left(1+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=156

$$\frac{21\sqrt{x^4+1}}{10x}-\frac{7\sqrt{x^4+1}}{10x^5}+\frac{1}{2\sqrt{x^4+1}\,x^5}-\frac{21\sqrt{x^4+1}\,x}{10\left(x^2+1\right)}-\frac{21\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{20\sqrt{x^4+1}}+\frac{21\left(x^2+1\right)\sqrt{\frac{x^4+}{(x^2+}}}{10\sqrt{x}}$$

```
[Out] 1/2/x^5/(x^4+1)^(1/2)-7/10*(x^4+1)^(1/2)/x^5+21/10*(x^4+1)^(1/2)/x-21/10*x*
(x^4+1)^(1/2)/(x^2+1)+21/10*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan
(x))*EllipticE(sin(2*arctan(x)),1/2*2^(1/2))*((x^4+1)/(x^2+1)^2)^(1/2)/(x^4
+1)^(1/2)-21/20*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*Ellipti
cF(sin(2*arctan(x)),1/2*2^(1/2))*((x^4+1)/(x^2+1)^2)^(1/2)/(x^4+1)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {290, 325, 305, 220, 1196}

$$-\frac{21\sqrt{x^4+1}\,x}{10\left(x^2+1\right)}+\frac{21\sqrt{x^4+1}}{10x}-\frac{7\sqrt{x^4+1}}{10x^5}+\frac{1}{2\sqrt{x^4+1}\,x^5}-\frac{21\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{20\sqrt{x^4+1}}+\frac{21\left(x^2+1\right)\sqrt{\frac{x^}{(x^2}}}{10\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(1 + x^4)^(3/2)),x]
```

```
[Out] 1/(2*x^5*Sqrt[1 + x^4]) - (7*Sqrt[1 + x^4])/(10*x^5) + (21*Sqrt[1 + x^4])/(
10*x) - (21*x*Sqrt[1 + x^4])/(10*(1 + x^2)) + (21*(1 + x^2)*Sqrt[(1 + x^4)/
(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/2])/(10*Sqrt[1 + x^4]) - (21*(1 + x^2
)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(20*Sqrt[1 + x^4
])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^6\left(1+x^4\right)^{3/2}}\,dx &= \frac{1}{2x^5\sqrt{1+x^4}} + \frac{7}{2}\int \frac{1}{x^6\sqrt{1+x^4}}\,dx \\
&= \frac{1}{2x^5\sqrt{1+x^4}} - \frac{7\sqrt{1+x^4}}{10x^5} - \frac{21}{10}\int \frac{1}{x^2\sqrt{1+x^4}}\,dx \\
&= \frac{1}{2x^5\sqrt{1+x^4}} - \frac{7\sqrt{1+x^4}}{10x^5} + \frac{21\sqrt{1+x^4}}{10x} - \frac{21}{10}\int \frac{x^2}{\sqrt{1+x^4}}\,dx \\
&= \frac{1}{2x^5\sqrt{1+x^4}} - \frac{7\sqrt{1+x^4}}{10x^5} + \frac{21\sqrt{1+x^4}}{10x} - \frac{21}{10}\int \frac{1}{\sqrt{1+x^4}}\,dx + \frac{21}{10}\int \frac{1-x^2}{\sqrt{1+x^4}}\,dx \\
&= \frac{1}{2x^5\sqrt{1+x^4}} - \frac{7\sqrt{1+x^4}}{10x^5} + \frac{21\sqrt{1+x^4}}{10x} - \frac{21x\sqrt{1+x^4}}{10\left(1+x^2\right)} + \frac{21\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x\right.}{10\sqrt{1+x^4}}
\end{aligned}$$

Mathematica [C] time = 0.00, size = 22, normalized size = 0.14

$$-\frac{{}_2F_1\left(-\frac{5}{4},\frac{3}{2};-\frac{1}{4};-x^4\right)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(1 + x^4)^(3/2)),x]
```

```
[Out] -1/5*Hypergeometric2F1[-5/4, 3/2, -1/4, -x^4]/x^5
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+1}}{x^{14}+2\,x^{10}+x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^4+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 1)/(x^14 + 2*x^10 + x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+1\right)^{\frac{3}{2}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^4+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + 1)^(3/2)*x^6), x)
```

maple [C] time = 0.01, size = 119, normalized size = 0.76

$$\frac{x^3}{2\sqrt{x^4+1}}+\frac{8\sqrt{x^4+1}}{5x}-\frac{\sqrt{x^4+1}}{5x^5}-\frac{21i\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\left(-\text{EllipticE}\left(\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)x,i\right)+\text{EllipticF}\left(\left(\frac{\sqrt{2}}{2}+\frac{i}{}\right.\right.\right.}{10\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^4+1)^(3/2),x)
```

```
[Out] -1/5*(x^4+1)^(1/2)/x^5+8/5*(x^4+1)^(1/2)/x+1/2/(x^4+1)^(1/2)*x^3-21/10*I/(1
/2*2^(1/2)+1/2*I*2^(1/2))*(-I*x^2+1)^(1/2)*(I*x^2+1)^(1/2)/(x^4+1)^(1/2)*(E
llipticF((1/2*2^(1/2)+1/2*I*2^(1/2))*x,I)-EllipticE((1/2*2^(1/2)+1/2*I*2^(1
/2))*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+1\right)^{\frac{3}{2}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^4+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 1)^(3/2)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^6\left(x^4+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(x^4 + 1)^(3/2)),x)
```

```
[Out] int(1/(x^6*(x^4 + 1)^(3/2)), x)
```

sympy [C] time = 1.82, size = 36, normalized size = 0.23

$$\frac{\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{5}{4},\frac{3}{2}\\-\frac{1}{4}\end{matrix}\right|x^4e^{i\pi}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**4+1)**(3/2),x)
```

```
[Out] gamma(-5/4)*hyper((-5/4, 3/2), (-1/4,), x**4*exp_polar(I*pi))/(4*x**5*gamma
(-1/4))
```

3.959 $\int \frac{1}{\left(1+x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=72

$$\frac{5x}{12\sqrt{x^4+1}}+\frac{x}{6\left(x^4+1\right)^{3/2}}+\frac{5\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{24\sqrt{x^4+1}}$$

```
[Out] 1/6*x/(x^4+1)^(3/2)+5/12*x/(x^4+1)^(1/2)+5/24*(x^2+1)*(cos(2*arctan(x))^2)^
(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2*2^(1/2))*((x^4+1)/(x^
2+1)^2)^(1/2)/(x^4+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {199, 220}

$$\frac{5x}{12\sqrt{x^4+1}}+\frac{x}{6\left(x^4+1\right)^{3/2}}+\frac{5\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{24\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)^(-5/2),x]
```

```
[Out] x/(6*(1 + x^4)^(3/2)) + (5*x)/(12*Sqrt[1 + x^4]) + (5*(1 + x^2)*Sqrt[(1 + x
^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(24*Sqrt[1 + x^4])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(1+x^4\right)^{5/2}}\,dx &= \frac{x}{6\left(1+x^4\right)^{3/2}}+\frac{5}{6}\int \frac{1}{\left(1+x^4\right)^{3/2}}\,dx\\ &= \frac{x}{6\left(1+x^4\right)^{3/2}}+\frac{5x}{12\sqrt{1+x^4}}+\frac{5}{12}\int \frac{1}{\sqrt{1+x^4}}\,dx\\ &= \frac{x}{6\left(1+x^4\right)^{3/2}}+\frac{5x}{12\sqrt{1+x^4}}+\frac{5\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{24\sqrt{1+x^4}}\end{aligned}$$

Mathematica [C] time = 0.02, size = 49, normalized size = 0.68

$$\frac{5}{12}x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-x^4\right)+\frac{5x}{12\sqrt{x^4+1}}+\frac{x}{6\left(x^4+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)^(-5/2),x]
```

```
[Out] x/(6*(1 + x^4)^(3/2)) + (5*x)/(12*Sqrt[1 + x^4]) + (5*x*Hypergeometric2F1[1
/4, 1/2, 5/4, -x^4])/12
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4+1}}{x^{12}+3\,x^8+3\,x^4+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 1)/(x^12 + 3*x^8 + 3*x^4 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 1)^(-5/2), x)
```

maple [C] time = 0.01, size = 82, normalized size = 1.14

$$\frac{x}{6\left(x^4+1\right)^{\frac{3}{2}}}+\frac{5x}{12\sqrt{x^4+1}}+\frac{5\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\text{EllipticF}\left(\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)x,i\right)}{12\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+1)^(5/2),x)
```

```
[Out] 1/6*x/(x^4+1)^(3/2)+5/12/(x^4+1)^(1/2)*x+5/12/(1/2*2^(1/2)+1/2*I*2^(1/2))*(
-I*x^2+1)^(1/2)*(I*x^2+1)^(1/2)/(x^4+1)^(1/2)*EllipticF((1/2*2^(1/2)+1/2*I*
2^(1/2))*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)^(-5/2), x)
```

mupad [B] time = 1.07, size = 12, normalized size = 0.17

$$x\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{5}{2};\ \frac{5}{4};\ -x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4 + 1)^(5/2),x)
[Out] x*hypergeom([1/4, 5/2], 5/4, -x^4)
```

sympy [C] time = 1.03, size = 27, normalized size = 0.38

$$\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{5}{2}\\ \frac{5}{4}\end{matrix}\right|x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+1)**(5/2),x)
[Out] x*gamma(1/4)*hyper((1/4, 5/2), (5/4,), x**4*exp_polar(I*pi))/(4*gamma(5/4))
```

3.960 $\int \frac{x^7}{\sqrt{16-x^4}}\,dx$

Optimal. Leaf size=29

$$\frac{1}{6}\left(16-x^4\right)^{3/2}-8\sqrt{16-x^4}$$

```
[Out] 1/6*(-x^4+16)^(3/2)-8*(-x^4+16)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{1}{6}\left(16-x^4\right)^{3/2}-8\sqrt{16-x^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/Sqrt[16 - x^4],x]
```

```
[Out] -8*Sqrt[16 - x^4] + (16 - x^4)^(3/2)/6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{\sqrt{16-x^4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x}{\sqrt{16-x}}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int \left(\frac{16}{\sqrt{16-x}}-\sqrt{16-x}\right)dx, x, x^4\right)\\ &= -8\sqrt{16-x^4}+\frac{1}{6}\left(16-x^4\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.69

$$-\frac{1}{6}\sqrt{16-x^4}\left(x^4+32\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/Sqrt[16 - x^4],x]
```

```
[Out] -1/6*(Sqrt[16 - x^4]*(32 + x^4))
```

fricas [A] time = 0.74, size = 16, normalized size = 0.55

$$-\frac{1}{6}\left(x^4+32\right)\sqrt{-x^4+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^4+16)^(1/2),x, algorithm="fricas")
[Out] -1/6*(x^4 + 32)*sqrt(-x^4 + 16)
```

giac [A] time = 0.15, size = 23, normalized size = 0.79

$$\frac{1}{6}\left(-x^4+16\right)^{\frac{3}{2}}-8\sqrt{-x^4+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^4+16)^(1/2),x, algorithm="giac")
[Out] 1/6*(-x^4 + 16)^(3/2) - 8*sqrt(-x^4 + 16)
```

maple [A] time = 0.01, size = 28, normalized size = 0.97

$$\frac{(x-2)(x+2)\left(x^2+4\right)\left(x^4+32\right)}{6\sqrt{-x^4+16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-x^4+16)^(1/2),x)
[Out] 1/6*(x-2)*(2+x)*(x^2+4)*(x^4+32)/(-x^4+16)^(1/2)
```

maxima [A] time = 1.28, size = 23, normalized size = 0.79

$$\frac{1}{6}\left(-x^4+16\right)^{\frac{3}{2}}-8\sqrt{-x^4+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^4+16)^(1/2),x, algorithm="maxima")
[Out] 1/6*(-x^4 + 16)^(3/2) - 8*sqrt(-x^4 + 16)
```

mupad [B] time = 1.19, size = 16, normalized size = 0.55

$$-\frac{\sqrt{16-x^4}\,\left(x^4+32\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(16 - x^4)^(1/2),x)
[Out] -((16 - x^4)^(1/2)*(x^4 + 32))/6
```

sympy [A] time = 0.66, size = 26, normalized size = 0.90

$$-\frac{x^4\sqrt{16-x^4}}{6}-\frac{16\sqrt{16-x^4}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-x**4+16)**(1/2),x)
[Out] -x**4*sqrt(16 - x**4)/6 - 16*sqrt(16 - x**4)/3
```

3.961 $\int \frac{x^5}{\sqrt{16-x^4}}\,dx$

Optimal. Leaf size=29

$$4\sin^{-1}\left(\frac{x^2}{4}\right)-\frac{1}{4}x^2\sqrt{16-x^4}$$

```
[Out] 4*arcsin(1/4*x^2)-1/4*x^2*(-x^4+16)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {275, 321, 216}

$$4\sin^{-1}\left(\frac{x^2}{4}\right)-\frac{1}{4}x^2\sqrt{16-x^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[16 - x^4],x]
```

```
[Out] -(x^2*Sqrt[16 - x^4])/4 + 4*ArcSin[x^2/4]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\sqrt{16-x^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{16-x^2}}\,dx, x, x^2\right)\\ &= -\frac{1}{4}x^2\sqrt{16-x^4}+4\,\text{Subst}\left(\int \frac{1}{\sqrt{16-x^2}}\,dx, x, x^2\right)\\ &= -\frac{1}{4}x^2\sqrt{16-x^4}+4\sin^{-1}\left(\frac{x^2}{4}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 29, normalized size = 1.00

$$4\sin^{-1}\left(\frac{x^2}{4}\right)-\frac{1}{4}x^2\sqrt{16-x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[16 - x^4],x]
[Out] -1/4*(x^2*Sqrt[16 - x^4]) + 4*ArcSin[x^2/4]
```

fricas [A] time = 0.79, size = 33, normalized size = 1.14

$$-\frac{1}{4}\sqrt{-x^4+16}\,x^2 - 8\,\arctan\left(\frac{\sqrt{-x^4+16}-4}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^4+16)^(1/2),x, algorithm="fricas")
[Out] -1/4*sqrt(-x^4 + 16)*x^2 - 8*arctan((sqrt(-x^4 + 16) - 4)/x^2)
```

giac [A] time = 0.17, size = 23, normalized size = 0.79

$$-\frac{1}{4}\sqrt{-x^4+16}\,x^2 + 4\,\arcsin\left(\frac{1}{4}\,x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^4+16)^(1/2),x, algorithm="giac")
[Out] -1/4*sqrt(-x^4 + 16)*x^2 + 4*arcsin(1/4*x^2)
```

maple [A] time = 0.02, size = 24, normalized size = 0.83

$$-\frac{\sqrt{-x^4+16}\,x^2}{4} + 4\arcsin\left(\frac{x^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-x^4+16)^(1/2),x)
[Out] 4*arcsin(1/4*x^2)-1/4*x^2*(-x^4+16)^(1/2)
```

maxima [A] time = 2.97, size = 44, normalized size = 1.52

$$\frac{4\sqrt{-x^4+16}}{x^2\left(\frac{x^4-16}{x^4}-1\right)} - 4\,\arctan\left(\frac{\sqrt{-x^4+16}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^4+16)^(1/2),x, algorithm="maxima")
[Out] 4*sqrt(-x^4 + 16)/(x^2*((x^4 - 16)/x^4 - 1)) - 4*arctan(sqrt(-x^4 + 16)/x^2
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^5}{\sqrt{16-x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(16 - x^4)^(1/2),x)
[Out] int(x^5/(16 - x^4)^(1/2), x)
```

sympy [**A**] time = 2.89, size = 80, normalized size = 2.76

$$\begin{cases} -\frac{ix^6}{4\sqrt{x^4-16}} + \frac{4ix^2}{\sqrt{x^4-16}} - 4i\,\operatorname{acosh}\left(\frac{x^2}{4}\right) & \text{for } \frac{|x^4|}{16} > 1 \\ \frac{x^6}{4\sqrt{16-x^4}} - \frac{4x^2}{\sqrt{16-x^4}} + 4\,\operatorname{asin}\left(\frac{x^2}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-x**4+16)**(1/2),x)
```

```
[Out] Piecewise((-I*x**6/(4*sqrt(x**4 - 16)) + 4*I*x**2/sqrt(x**4 - 16) - 4*I*aco
sh(x**2/4), Abs(x**4)/16 > 1), (x**6/(4*sqrt(16 - x**4)) - 4*x**2/sqrt(16 -
 x**4) + 4*asin(x**2/4), True))
```

3.962 $\int \frac{x^3}{\sqrt{16-x^4}}\,dx$

Optimal. Leaf size=15

$$-\frac{1}{2}\sqrt{16-x^4}$$

```
[Out] -1/2*(-x^4+16)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$-\frac{1}{2}\sqrt{16-x^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[16 - x^4],x]
```

```
[Out] -Sqrt[16 - x^4]/2
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{16-x^4}}\,dx = -\frac{1}{2}\sqrt{16-x^4}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$-\frac{1}{2}\sqrt{16-x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[16 - x^4],x]
```

```
[Out] -1/2*Sqrt[16 - x^4]
```

fricas [A] time = 0.85, size = 11, normalized size = 0.73

$$-\frac{1}{2}\sqrt{-x^4+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^4+16)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-x^4 + 16)
```

giac [A] time = 0.16, size = 11, normalized size = 0.73

$$-\frac{1}{2}\sqrt{-x^4+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^4+16)^(1/2),x, algorithm="giac")
[Out] -1/2*sqrt(-x^4 + 16)
```

maple [A] time = 0.01, size = 23, normalized size = 1.53

$$\frac{(x-2)(x+2)\left(x^2+4\right)}{2\sqrt{-x^4+16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-x^4+16)^(1/2),x)
[Out] 1/2*(x-2)*(x+2)*(x^2+4)/(-x^4+16)^(1/2)
```

maxima [A] time = 1.30, size = 11, normalized size = 0.73

$$-\frac{1}{2}\sqrt{-x^4+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^4+16)^(1/2),x, algorithm="maxima")
[Out] -1/2*sqrt(-x^4 + 16)
```

mupad [B] time = 1.23, size = 11, normalized size = 0.73

$$-\frac{\sqrt{16-x^4}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(16 - x^4)^(1/2),x)
[Out] -(16 - x^4)^(1/2)/2
```

sympy [A] time = 0.32, size = 10, normalized size = 0.67

$$-\frac{\sqrt{16-x^4}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-x**4+16)**(1/2),x)
[Out] -sqrt(16 - x**4)/2
```

3.963 $\int \frac{x}{\sqrt{16-x^4}}\,dx$

Optimal. Leaf size=12

$$\frac{1}{2}\sin^{-1}\left(\frac{x^2}{4}\right)$$

```
[Out] 1/2*arcsin(1/4*x^2)
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {275, 216}

$$\frac{1}{2}\sin^{-1}\left(\frac{x^2}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[16 - x^4],x]
```

```
[Out] ArcSin[x^2/4]/2
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{16-x^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{16-x^2}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\sin^{-1}\left(\frac{x^2}{4}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\frac{1}{2}\sin^{-1}\left(\frac{x^2}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[16 - x^4],x]
```

```
[Out] ArcSin[x^2/4]/2
```

fricas [B] time = 0.86, size = 18, normalized size = 1.50

$$-\arctan\left(\frac{\sqrt{-x^4+16}-4}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+16)^(1/2),x, algorithm="fricas")
[Out] -arctan((sqrt(-x^4 + 16) - 4)/x^2)
```

giac [A] time = 0.17, size = 8, normalized size = 0.67

$$\frac{1}{2}\arcsin\left(\frac{1}{4}x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+16)^(1/2),x, algorithm="giac")
[Out] 1/2*arcsin(1/4*x^2)
```

maple [A] time = 0.01, size = 9, normalized size = 0.75

$$\frac{\arcsin\left(\frac{x^2}{4}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^4+16)^(1/2),x)
[Out] 1/2*arcsin(1/4*x^2)
```

maxima [A] time = 2.96, size = 16, normalized size = 1.33

$$-\frac{1}{2}\arctan\left(\frac{\sqrt{-x^4+16}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+16)^(1/2),x, algorithm="maxima")
[Out] -1/2*arctan(sqrt(-x^4 + 16)/x^2)
```

mupad [B] time = 0.17, size = 16, normalized size = 1.33

$$\frac{\operatorname{atan}\left(\frac{x^2}{\sqrt{16-x^4}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(16 - x^4)^(1/2),x)
[Out] atan(x^2/(16 - x^4)^(1/2))/2
```

sympy [A] time = 1.19, size = 24, normalized size = 2.00

$$\begin{cases} -\frac{i\operatorname{acosh}\left(\frac{x^2}{4}\right)}{2} & \text{for } \frac{|x^4|}{16} > 1 \\ \frac{\operatorname{asin}\left(\frac{x^2}{4}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**4+16)**(1/2),x)
[Out] Piecewise((-I*acosh(x**2/4)/2, Abs(x**4)/16 > 1), (asin(x**2/4)/2, True))
```

3.964 $\int \frac{1}{x\sqrt{16-x^4}}\,dx$

Optimal. Leaf size=20

$$-\frac{1}{8}\tanh^{-1}\left(\frac{\sqrt{16-x^4}}{4}\right)$$

```
[Out] -1/8*arctanh(1/4*(-x^4+16)^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {266, 63, 206}

$$-\frac{1}{8}\tanh^{-1}\left(\frac{\sqrt{16-x^4}}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[16 - x^4]),x]
```

```
[Out] -ArcTanh[Sqrt[16 - x^4]/4]/8
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{16-x^4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt{16-x}\,x}\,dx, x, x^4\right)\\ &= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{16-x^2}\,dx, x, \sqrt{16-x^4}\right)\right)\\ &= -\frac{1}{8}\tanh^{-1}\left(\frac{\sqrt{16-x^4}}{4}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$-\frac{1}{8}\tanh^{-1}\left(\frac{\sqrt{16-x^4}}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[16 - x^4]),x]
[Out] -1/8*ArcTanh[Sqrt[16 - x^4]/4]
```

fricas [B] time = 0.84, size = 29, normalized size = 1.45

$$-\frac{1}{16}\log\left(\sqrt{-x^4+16}+4\right)+\frac{1}{16}\log\left(\sqrt{-x^4+16}-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^4+16)^(1/2),x, algorithm="fricas")
[Out] -1/16*log(sqrt(-x^4 + 16) + 4) + 1/16*log(sqrt(-x^4 + 16) - 4)
```

giac [B] time = 0.15, size = 31, normalized size = 1.55

$$-\frac{1}{16}\log\left(\sqrt{-x^4+16}+4\right)+\frac{1}{16}\log\left(-\sqrt{-x^4+16}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^4+16)^(1/2),x, algorithm="giac")
[Out] -1/16*log(sqrt(-x^4 + 16) + 4) + 1/16*log(-sqrt(-x^4 + 16) + 4)
```

maple [A] time = 0.01, size = 15, normalized size = 0.75

$$-\frac{\operatorname{arctanh}\left(\frac{4}{\sqrt{-x^4+16}}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^4+16)^(1/2),x)
[Out] -1/8*arctanh(4/(-x^4+16)^(1/2))
```

maxima [B] time = 1.31, size = 29, normalized size = 1.45

$$-\frac{1}{16}\log\left(\sqrt{-x^4+16}+4\right)+\frac{1}{16}\log\left(\sqrt{-x^4+16}-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^4+16)^(1/2),x, algorithm="maxima")
[Out] -1/16*log(sqrt(-x^4 + 16) + 4) + 1/16*log(sqrt(-x^4 + 16) - 4)
```

mupad [B] time = 1.28, size = 14, normalized size = 0.70

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{16-x^4}}{4}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(16 - x^4)^(1/2)),x)
[Out] -atanh((16 - x^4)^(1/2)/4)/8
```

sympy [**A**] time = 1.87, size = 24, normalized size = 1.20

$$\begin{cases} -\dfrac{\operatorname{acosh}\left(\frac{4}{x^2}\right)}{8} & \text{for } \dfrac{16}{|x^4|} > 1 \\ \dfrac{i \operatorname{asin}\left(\frac{4}{x^2}\right)}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**4+16)**(1/2),x)
```

```
[Out] Piecewise((-acosh(4/x**2)/8, 16/Abs(x**4) > 1), (I*asin(4/x**2)/8, True))
```

3.965 $\int \frac{1}{x^3\sqrt{16-x^4}}\,dx$

Optimal. Leaf size=18

$$-\frac{\sqrt{16-x^4}}{32x^2}$$

```
[Out] -1/32*(-x^4+16)^(1/2)/x^2
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {264}

$$-\frac{\sqrt{16-x^4}}{32x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[16 - x^4]),x]
```

```
[Out] -Sqrt[16 - x^4]/(32*x^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{16-x^4}}\,dx = -\frac{\sqrt{16-x^4}}{32x^2}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$-\frac{\sqrt{16-x^4}}{32x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[16 - x^4]),x]
```

```
[Out] -1/32*Sqrt[16 - x^4]/x^2
```

fricas [A] time = 0.89, size = 14, normalized size = 0.78

$$-\frac{\sqrt{-x^4+16}}{32\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^4+16)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/32*sqrt(-x^4 + 16)/x^2
```

giac [B] time = 0.17, size = 35, normalized size = 1.94

$$\frac{x^2}{64\left(\sqrt{-x^4+16}-4\right)} - \frac{\sqrt{-x^4+16}-4}{64\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^4+16)^(1/2),x, algorithm="giac")
```

```
[Out] 1/64*x^2/(sqrt(-x^4 + 16) - 4) - 1/64*(sqrt(-x^4 + 16) - 4)/x^2
```

maple [A] time = 0.00, size = 26, normalized size = 1.44

$$\frac{(x-2)(x+2)\left(x^2+4\right)}{32\sqrt{-x^4+16}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-x^4+16)^(1/2),x)
```

```
[Out] 1/32/x^2*(x-2)*(x+2)*(x^2+4)/(-x^4+16)^(1/2)
```

maxima [A] time = 1.35, size = 14, normalized size = 0.78

$$-\frac{\sqrt{-x^4+16}}{32\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^4+16)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/32*sqrt(-x^4 + 16)/x^2
```

mupad [B] time = 1.14, size = 14, normalized size = 0.78

$$-\frac{\sqrt{16-x^4}}{32\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(16 - x^4)^(1/2)),x)
```

```
[Out] -(16 - x^4)^(1/2)/(32*x^2)
```

sympy [A] time = 0.87, size = 32, normalized size = 1.78

$$\begin{cases} -\frac{\sqrt{-1+\frac{16}{x^4}}}{32} & \text{for } \frac{16}{|x^4|} > 1 \\ -\frac{i\sqrt{1-\frac{16}{x^4}}}{32} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-x**4+16)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(-1 + 16/x**4)/32, 16/Abs(x**4) > 1), (-I*sqrt(1 - 16/x**4)
/32, True))
```

3.966 $\int \frac{1}{x^5\sqrt{16-x^4}}\,dx$

Optimal. Leaf size=39

$$-\frac{\sqrt{16-x^4}}{64x^4}-\frac{1}{256}\tanh^{-1}\left(\frac{\sqrt{16-x^4}}{4}\right)$$

```
[Out] -1/256*arctanh(1/4*(-x^4+16)^(1/2))-1/64*(-x^4+16)^(1/2)/x^4
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 51, 63, 206}

$$-\frac{\sqrt{16-x^4}}{64x^4}-\frac{1}{256}\tanh^{-1}\left(\frac{\sqrt{16-x^4}}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[16 - x^4]),x]
```

```
[Out] -Sqrt[16 - x^4]/(64*x^4) - ArcTanh[Sqrt[16 - x^4]/4]/256
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\sqrt{16-x^4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt{16-x}\,x^2}\,dx, x, x^4\right)\\ &= -\frac{\sqrt{16-x^4}}{64x^4} + \frac{1}{128}\,\text{Subst}\left(\int \frac{1}{\sqrt{16-x}\,x}\,dx, x, x^4\right)\\ &= -\frac{\sqrt{16-x^4}}{64x^4} - \frac{1}{64}\,\text{Subst}\left(\int \frac{1}{16-x^2}\,dx, x, \sqrt{16-x^4}\right)\\ &= -\frac{\sqrt{16-x^4}}{64x^4} - \frac{1}{256}\tanh^{-1}\left(\frac{\sqrt{16-x^4}}{4}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 0.95

$$\frac{1}{256}\left(-\frac{4\sqrt{16-x^4}}{x^4} - \tanh^{-1}\left(\sqrt{1-\frac{x^4}{16}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[16 - x^4]),x]
```

```
[Out] ((-4*Sqrt[16 - x^4])/x^4 - ArcTanh[Sqrt[1 - x^4/16]])/256
```

fricas [A] time = 0.88, size = 50, normalized size = 1.28

$$-\frac{x^4\log\left(\sqrt{-x^4+16}+4\right) - x^4\log\left(\sqrt{-x^4+16}-4\right) + 8\sqrt{-x^4+16}}{512\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^4+16)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/512*(x^4*log(sqrt(-x^4 + 16) + 4) - x^4*log(sqrt(-x^4 + 16) - 4) + 8*sqr
t(-x^4 + 16))/x^4
```

giac [A] time = 0.15, size = 45, normalized size = 1.15

$$-\frac{\sqrt{-x^4+16}}{64\,x^4} - \frac{1}{512}\log\left(\sqrt{-x^4+16}+4\right) + \frac{1}{512}\log\left(-\sqrt{-x^4+16}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^4+16)^(1/2),x, algorithm="giac")
```

```
[Out] -1/64*sqrt(-x^4 + 16)/x^4 - 1/512*log(sqrt(-x^4 + 16) + 4) + 1/512*log(-sqr
t(-x^4 + 16) + 4)
```

maple [A] time = 0.01, size = 30, normalized size = 0.77

$$-\frac{\operatorname{arctanh}\left(\frac{4}{\sqrt{-x^4+16}}\right)}{256} - \frac{\sqrt{-x^4+16}}{64x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-x^4+16)^(1/2),x)
```

```
[Out] -1/64*(-x^4+16)^(1/2)/x^4-1/256*arctanh(4/(-x^4+16)^(1/2))
```

maxima [A] time = 1.32, size = 43, normalized size = 1.10

$$-\frac{\sqrt{-x^4+16}}{64\,x^4}-\frac{1}{512}\log\left(\sqrt{-x^4+16}+4\right)+\frac{1}{512}\log\left(\sqrt{-x^4+16}-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^4+16)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/64*sqrt(-x^4 + 16)/x^4 - 1/512*log(sqrt(-x^4 + 16) + 4) + 1/512*log(sqrt
(-x^4 + 16) - 4)
```

mupad [B] time = 1.19, size = 29, normalized size = 0.74

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{16-x^4}}{4}\right)}{256}-\frac{\sqrt{16-x^4}}{64\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(16 - x^4)^(1/2)),x)
```

```
[Out] - atanh((16 - x^4)^(1/2)/4)/256 - (16 - x^4)^(1/2)/(64*x^4)
```

sympy [A] time = 3.01, size = 73, normalized size = 1.87

$$\begin{cases} -\frac{\operatorname{acosh}\left(\frac{4}{x^2}\right)}{256}-\frac{\sqrt{-1+\frac{16}{x^4}}}{64x^2} & \text{for } \frac{16}{|x^4|}>1 \\ \frac{i\operatorname{asin}\left(\frac{4}{x^2}\right)}{256}-\frac{i}{64x^2\sqrt{1-\frac{16}{x^4}}}+\frac{i}{4x^6\sqrt{1-\frac{16}{x^4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-x**4+16)**(1/2),x)
```

```
[Out] Piecewise((-acosh(4/x**2)/256 - sqrt(-1 + 16/x**4)/(64*x**2), 16/Abs(x**4)
> 1), (I*asin(4/x**2)/256 - I/(64*x**2*sqrt(1 - 16/x**4)) + I/(4*x**6*sqrt(
1 - 16/x**4)), True))
```

3.967 $\int \frac{1}{x^7\sqrt{16-x^4}}\,dx$

Optimal. Leaf size=37

$$-\frac{\sqrt{16-x^4}}{96x^6}-\frac{\sqrt{16-x^4}}{768x^2}$$

```
[Out] -1/96*(-x^4+16)^(1/2)/x^6-1/768*(-x^4+16)^(1/2)/x^2
```

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$-\frac{\sqrt{16-x^4}}{768x^2}-\frac{\sqrt{16-x^4}}{96x^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*Sqrt[16 - x^4]),x]
```

```
[Out] -Sqrt[16 - x^4]/(96*x^6) - Sqrt[16 - x^4]/(768*x^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7\sqrt{16-x^4}}\,dx &= -\frac{\sqrt{16-x^4}}{96x^6}+\frac{1}{24}\int \frac{1}{x^3\sqrt{16-x^4}}\,dx\\ &= -\frac{\sqrt{16-x^4}}{96x^6}-\frac{\sqrt{16-x^4}}{768x^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.68

$$-\frac{\sqrt{1-\frac{x^4}{16}}\left(x^4+8\right)}{192x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*Sqrt[16 - x^4]),x]
```

```
[Out] -1/192*(Sqrt[1 - x^4/16]*(8 + x^4))/x^6
```

fricas [A] time = 0.83, size = 19, normalized size = 0.51

$$-\frac{\left(x^4+8\right)\sqrt{-x^4+16}}{768\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-x^4+16)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/768*(x^4 + 8)*sqrt(-x^4 + 16)/x^6
```

giac [B] time = 0.17, size = 73, normalized size = 1.97

$$\frac{x^6\left(\frac{9\left(\sqrt{-x^4+16}-4\right)^2}{x^4}+1\right)}{12288\left(\sqrt{-x^4+16}-4\right)^3}-\frac{3\left(\sqrt{-x^4+16}-4\right)}{4096\,x^2}-\frac{\left(\sqrt{-x^4+16}-4\right)^3}{12288\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-x^4+16)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12288*x^6*(9*(sqrt(-x^4 + 16) - 4)^2/x^4 + 1)/(sqrt(-x^4 + 16) - 4)^3 - 3
/4096*(sqrt(-x^4 + 16) - 4)/x^2 - 1/12288*(sqrt(-x^4 + 16) - 4)^3/x^6
```

maple [A] time = 0.00, size = 31, normalized size = 0.84

$$\frac{(x-2)\,(x+2)\left(x^2+4\right)\left(x^4+8\right)}{768\sqrt{-x^4+16}\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(-x^4+16)^(1/2),x)
```

```
[Out] 1/768*(x-2)*(x+2)*(x^2+4)*(x^4+8)/x^6/(-x^4+16)^(1/2)
```

maxima [A] time = 1.31, size = 29, normalized size = 0.78

$$-\frac{\sqrt{-x^4+16}}{512\,x^2}-\frac{\left(-x^4+16\right)^{\frac{3}{2}}}{1536\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-x^4+16)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/512*sqrt(-x^4 + 16)/x^2 - 1/1536*(-x^4 + 16)^(3/2)/x^6
```

mupad [B] time = 1.14, size = 19, normalized size = 0.51

$$-\frac{\left(x^4+8\right)\,\sqrt{16-x^4}}{768\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(16 - x^4)^(1/2)),x)
```

```
[Out] -((x^4 + 8)*(16 - x^4)^(1/2))/(768*x^6)
```

sympy [A] time = 1.58, size = 65, normalized size = 1.76

$$\begin{cases} -\frac{\sqrt{-1+\frac{16}{x^4}}}{768}-\frac{\sqrt{-1+\frac{16}{x^4}}}{96x^4} & \text{for } \frac{16}{|x^4|}>1 \\ -\frac{i\sqrt{1-\frac{16}{x^4}}}{768}-\frac{i\sqrt{1-\frac{16}{x^4}}}{96x^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(-x**4+16)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(-1 + 16/x**4)/768 - sqrt(-1 + 16/x**4)/(96*x**4), 16/Abs(x
**4) > 1), (-I*sqrt(1 - 16/x**4)/768 - I*sqrt(1 - 16/x**4)/(96*x**4), True)
)
```

3.968 $\int \frac{x^6}{\sqrt{16-x^4}}\,dx$

Optimal. Leaf size=43

$$-\frac{1}{5}\sqrt{16-x^4}\,x^3-\frac{96}{5}F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-1\right)+\frac{96}{5}E\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-1\right)$$

```
[Out] 96/5*EllipticE(1/2*x,I)-96/5*EllipticF(1/2*x,I)-1/5*x^3*(-x^4+16)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {321, 307, 221, 1181, 21, 424}

$$-\frac{1}{5}\sqrt{16-x^4}\,x^3-\frac{96}{5}F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-1\right)+\frac{96}{5}E\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^6/Sqrt[16 - x^4],x]
```

```
[Out] -(x^3*Sqrt[16 - x^4])/5 + (96*EllipticE[ArcSin[x/2], -1])/5 - (96*EllipticF
[ArcSin[x/2], -1])/5
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[(d + e*x^2)/(Sqrt[q + c*x^2]*Sqrt[q - c
```

```
*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{\sqrt{16-x^4}}\,dx &= -\frac{1}{5}x^3\sqrt{16-x^4} + \frac{48}{5}\int \frac{x^2}{\sqrt{16-x^4}}\,dx \\
&= -\frac{1}{5}x^3\sqrt{16-x^4} - \frac{192}{5}\int \frac{1}{\sqrt{16-x^4}}\,dx + \frac{192}{5}\int \frac{1+\frac{x^2}{4}}{\sqrt{16-x^4}}\,dx \\
&= -\frac{1}{5}x^3\sqrt{16-x^4} - \frac{96}{5}F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle| -1\right) + \frac{192}{5}\int \frac{1+\frac{x^2}{4}}{\sqrt{4-x^2}\sqrt{4+x^2}}\,dx \\
&= -\frac{1}{5}x^3\sqrt{16-x^4} - \frac{96}{5}F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle| -1\right) + \frac{48}{5}\int \frac{\sqrt{4+x^2}}{\sqrt{4-x^2}}\,dx \\
&= -\frac{1}{5}x^3\sqrt{16-x^4} + \frac{96}{5}E\left(\sin^{-1}\left(\frac{x}{2}\right)\middle| -1\right) - \frac{96}{5}F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle| -1\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 38, normalized size = 0.88

$$-\frac{1}{5}x^3\left(\sqrt{16-x^4} - 4\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{x^4}{16}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/Sqrt[16 - x^4],x]
[Out] -1/5*(x^3*(Sqrt[16 - x^4] - 4*Hypergeometric2F1[1/2, 3/4, 7/4, x^4/16]))
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^4+16}\,x^6}{x^4-16}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^4+16)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-x^4 + 16)*x^6/(x^4 - 16), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt{-x^4+16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^4+16)^(1/2),x, algorithm="giac")
[Out] integrate(x^6/sqrt(-x^4 + 16), x)
```

maple [A] time = 0.01, size = 58, normalized size = 1.35

$$-\frac{\sqrt{-x^4+16}\,x^3}{5} - \frac{96\sqrt{-x^2+4}\,\sqrt{x^2+4}\,\left(-\text{EllipticE}\left(\frac{x}{2},i\right)+\text{EllipticF}\left(\frac{x}{2},i\right)\right)}{5\sqrt{-x^4+16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-x^4+16)^(1/2),x)
```

```
[Out] -1/5*x^3*(-x^4+16)^(1/2)-96/5*(-x^2+4)^(1/2)*(x^2+4)^(1/2)/(-x^4+16)^(1/2)*
(EllipticF(1/2*x,I)-EllipticE(1/2*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt{-x^4+16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^4+16)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/sqrt(-x^4 + 16), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^6}{\sqrt{16-x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(16 - x^4)^(1/2),x)
```

```
[Out] int(x^6/(16 - x^4)^(1/2), x)
```

sympy [A] time = 0.99, size = 32, normalized size = 0.74

$$\frac{x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{x^4e^{2i\pi}}{16}\right)}{16\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-x**4+16)**(1/2),x)
```

```
[Out] x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), x**4*exp_polar(2*I*pi)/16)/(16*g
amma(11/4))
```

3.969 $\int \frac{x^4}{\sqrt{16-x^4}}\,dx$

Optimal. Leaf size=29

$$\frac{8}{3}F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-1\right)-\frac{1}{3}x\sqrt{16-x^4}$$

```
[Out] 8/3*EllipticF(1/2*x,I)-1/3*x*(-x^4+16)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {321, 221}

$$\frac{8}{3}F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-1\right)-\frac{1}{3}x\sqrt{16-x^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[16 - x^4],x]
```

```
[Out] -(x*Sqrt[16 - x^4])/3 + (8*EllipticF[ArcSin[x/2], -1])/3
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt{16-x^4}}\,dx &= -\frac{1}{3}x\sqrt{16-x^4}+\frac{16}{3}\int \frac{1}{\sqrt{16-x^4}}\,dx\\ &= -\frac{1}{3}x\sqrt{16-x^4}+\frac{8}{3}F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-1\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 36, normalized size = 1.24

$$-\frac{1}{3}x\left(\sqrt{16-x^4}-4\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{x^4}{16}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[16 - x^4],x]
```

```
[Out] -1/3*(x*(Sqrt[16 - x^4] - 4*Hypergeometric2F1[1/4, 1/2, 5/4, x^4/16]))
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^4+16}\,x^4}{x^4-16},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^4+16)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 16)*x^4/(x^4 - 16), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{-x^4+16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^4+16)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(-x^4 + 16), x)
```

maple [B] time = 0.01, size = 47, normalized size = 1.62

$$-\frac{\sqrt{-x^4+16}\,x}{3}+\frac{8\sqrt{-x^2+4}\,\sqrt{x^2+4}\,\text{EllipticF}\left(\frac{x}{2},i\right)}{3\sqrt{-x^4+16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^4+16)^(1/2),x)
```

```
[Out] -1/3*x*(-x^4+16)^(1/2)+8/3*(-x^2+4)^(1/2)*(x^2+4)^(1/2)/(-x^4+16)^(1/2)*Ell
ipticF(1/2*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{-x^4+16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^4+16)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(-x^4 + 16), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^4}{\sqrt{16-x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(16 - x^4)^(1/2),x)
```

```
[Out] int(x^4/(16 - x^4)^(1/2), x)
```

sympy [A] time = 0.99, size = 32, normalized size = 1.10

$$\frac{x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{x^4e^{2i\pi}}{16}\right)}{16\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**4+16)**(1/2),x)
```

```
[Out] x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), x**4*exp_polar(2*I*pi)/16)/(16*ga
mma(9/4))
```

3.970 $\int \frac{x^2}{\sqrt{16-x^4}}\,dx$

Optimal. Leaf size=21

$$2E\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-1\right)-2F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-1\right)$$

```
[Out] 2*EllipticE(1/2*x,I)-2*EllipticF(1/2*x,I)
```

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {307, 221, 1181, 21, 424}

$$2E\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-1\right)-2F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[16 - x^4],x]
```

```
[Out] 2*EllipticE[ArcSin[x/2], -1] - 2*EllipticF[ArcSin[x/2], -1]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[(d + e*x^2)/(Sqrt[q + c*x^2]*Sqrt[q - c
*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{16-x^4}}\,dx &= -\left(4\int \frac{1}{\sqrt{16-x^4}}\,dx\right) + 4\int \frac{1+\frac{x^2}{4}}{\sqrt{16-x^4}}\,dx \\ &= -2F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-1\right) + 4\int \frac{1+\frac{x^2}{4}}{\sqrt{4-x^2}\sqrt{4+x^2}}\,dx \\ &= -2F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-1\right) + \int \frac{\sqrt{4+x^2}}{\sqrt{4-x^2}}\,dx \\ &= 2E\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-1\right) - 2F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-1\right)\end{aligned}$$

Mathematica [C] time = 0.00, size = 24, normalized size = 1.14

$$\frac{1}{12}x^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{x^4}{16}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[16 - x^4],x]
[Out] (x^3*Hypergeometric2F1[1/2, 3/4, 7/4, x^4/16])/12
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^4+16}\,x^2}{x^4-16}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+16)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-x^4 + 16)*x^2/(x^4 - 16), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-x^4+16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+16)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/sqrt(-x^4 + 16), x)
```

maple [B] time = 0.01, size = 43, normalized size = 2.05

$$-\frac{2\sqrt{-x^2+4}\,\sqrt{x^2+4}\,\left(-\text{EllipticE}\left(\frac{x}{2},i\right)+\text{EllipticF}\left(\frac{x}{2},i\right)\right)}{\sqrt{-x^4+16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^4+16)^(1/2),x)
[Out] -2*(-x^2+4)^(1/2)*(x^2+4)^(1/2)/(-x^4+16)^(1/2)*(EllipticF(1/2*x,I)-Ellipti
cE(1/2*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-x^4+16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+16)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(-x^4 + 16), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^2}{\sqrt{16 - x^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(16 - x^4)^(1/2),x)
```

```
[Out] int(x^2/(16 - x^4)^(1/2), x)
```

sympy **[B]** time = 1.31, size = 32, normalized size = 1.52

$$\frac{x^3 \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{3}{4} \\ \frac{7}{4}\end{matrix} \middle| \frac{x^4 e^{2i\pi}}{16}\right)}{16\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**4+16)**(1/2),x)
```

```
[Out] x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**4*exp_polar(2*I*pi)/16)/(16*ga
mma(7/4))
```

3.971 $\int \frac{1}{\sqrt{16-x^4}}\,dx$

Optimal. Leaf size=12

$$\frac{1}{2}F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-1\right)$$

```
[Out] 1/2*EllipticF(1/2*x,I)
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {221}

$$\frac{1}{2}F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[16 - x^4],x]
```

```
[Out] EllipticF[ArcSin[x/2], -1]/2
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{16-x^4}}\,dx = \frac{1}{2}F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-1\right)$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$\frac{1}{2}F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[16 - x^4],x]
```

```
[Out] EllipticF[ArcSin[x/2], -1]/2
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^4+16}}{x^4-16}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+16)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 16)/(x^4 - 16), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^4+16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+16)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-x^4 + 16), x)
```

maple [B] time = 0.01, size = 34, normalized size = 2.83

$$\frac{\sqrt{-x^2+4}\ \sqrt{x^2+4}\ \text{EllipticF}\left(\frac{x}{2}, i\right)}{2\sqrt{-x^4+16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+16)^(1/2),x)
```

```
[Out] 1/2*(-x^2+4)^(1/2)*(x^2+4)^(1/2)/(-x^4+16)^(1/2)*EllipticF(1/2*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^4+16}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+16)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-x^4 + 16), x)
```

mupad [B] time = 0.07, size = 13, normalized size = 1.08

$$\frac{x\,{}_2\mathrm{F}_1\left(\frac{1}{4}, \frac{1}{2};\ \frac{5}{4};\ \frac{x^4}{16}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(16 - x^4)^(1/2),x)
```

```
[Out] (x*hypergeom([1/4, 1/2], 5/4, x^4/16))/4
```

sympy [B] time = 0.92, size = 31, normalized size = 2.58

$$\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4}, \frac{1}{2} \\ \frac{5}{4}\end{matrix}\middle| \frac{x^4 e^{2i\pi}}{16}\right)}{16\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+16)**(1/2),x)
```

```
[Out] x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), x**4*exp_polar(2*I*pi)/16)/(16*gamma
(5/4))
```

3.972 $\int \frac{1}{x^2\sqrt{16-x^4}}\,dx$

Optimal. Leaf size=43

$$-\frac{\sqrt{16-x^4}}{16x}+\frac{1}{8}F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-1\right)-\frac{1}{8}E\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-1\right)$$

```
[Out] -1/8*EllipticE(1/2*x,I)+1/8*EllipticF(1/2*x,I)-1/16*(-x^4+16)^(1/2)/x
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {325, 307, 221, 1181, 21, 424}

$$-\frac{\sqrt{16-x^4}}{16x}+\frac{1}{8}F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-1\right)-\frac{1}{8}E\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[16 - x^4]),x]
```

```
[Out] -Sqrt[16 - x^4]/(16*x) - EllipticE[ArcSin[x/2], -1]/8 + EllipticF[ArcSin[x/
2], -1]/8
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[(d + e*x^2)/(Sqrt[q + c*x^2]*Sqrt[q - c
```

```
*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\sqrt{16-x^4}}\,dx &= -\frac{\sqrt{16-x^4}}{16x} - \frac{1}{16}\int \frac{x^2}{\sqrt{16-x^4}}\,dx \\
&= -\frac{\sqrt{16-x^4}}{16x} + \frac{1}{4}\int \frac{1}{\sqrt{16-x^4}}\,dx - \frac{1}{4}\int \frac{1+\frac{x^2}{4}}{\sqrt{16-x^4}}\,dx \\
&= -\frac{\sqrt{16-x^4}}{16x} + \frac{1}{8}F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-1\right) - \frac{1}{4}\int \frac{1+\frac{x^2}{4}}{\sqrt{4-x^2}\sqrt{4+x^2}}\,dx \\
&= -\frac{\sqrt{16-x^4}}{16x} + \frac{1}{8}F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-1\right) - \frac{1}{16}\int \frac{\sqrt{4+x^2}}{\sqrt{4-x^2}}\,dx \\
&= -\frac{\sqrt{16-x^4}}{16x} - \frac{1}{8}E\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-1\right) + \frac{1}{8}F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-1\right)
\end{aligned}$$

Mathematica [C] time = 0.00, size = 24, normalized size = 0.56

$$-\frac{{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};\frac{x^4}{16}\right)}{4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[16 - x^4]),x]
[Out] -1/4*Hypergeometric2F1[-1/4, 1/2, 3/4, x^4/16]/x
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^4+16}}{x^6-16\,x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^4+16)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-x^4 + 16)/(x^6 - 16*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^4+16}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^4+16)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-x^4 + 16)*x^2), x)
```

maple [A] time = 0.01, size = 58, normalized size = 1.35

$$-\frac{\sqrt{-x^4+16}}{16x} + \frac{\sqrt{-x^2+4}\,\sqrt{x^2+4}\,\left(-\text{EllipticE}\left(\frac{x}{2},i\right)+\text{EllipticF}\left(\frac{x}{2},i\right)\right)}{8\sqrt{-x^4+16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-x^4+16)^(1/2),x)
```

```
[Out] -1/16*(-x^4+16)^(1/2)/x+1/8*(-x^2+4)^(1/2)*(x^2+4)^(1/2)/(-x^4+16)^(1/2)*(E
llipticF(1/2*x,I)-EllipticE(1/2*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^4+16}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^4+16)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^4 + 16)*x^2), x)
```

mupad [B] time = 1.25, size = 33, normalized size = 0.77

$$-\frac{\sqrt{1-\frac{16}{x^4}}\;{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\;\frac{7}{4};\;\frac{16}{x^4}\right)}{3\,x\,\sqrt{16-x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(16 - x^4)^(1/2)),x)
```

```
[Out] -((1 - 16/x^4)^(1/2)*hypergeom([1/2, 3/4], 7/4, 16/x^4))/(3*x*(16 - x^4)^(1
/2))
```

sympy [A] time = 0.98, size = 34, normalized size = 0.79

$$\frac{\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\right|\frac{x^4e^{2i\pi}}{16}\right)}{16x\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-x**4+16)**(1/2),x)
```

```
[Out] gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), x**4*exp_polar(2*I*pi)/16)/(16*x*gam
ma(3/4))
```

3.973 $\int \frac{1}{x^4\sqrt{16-x^4}}\,dx$

Optimal. Leaf size=31

$$\frac{1}{96}F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-1\right)-\frac{\sqrt{16-x^4}}{48x^3}$$

```
[Out] 1/96*EllipticF(1/2*x,I)-1/48*(-x^4+16)^(1/2)/x^3
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {325, 221}

$$\frac{1}{96}F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-1\right)-\frac{\sqrt{16-x^4}}{48x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[16 - x^4]),x]
```

```
[Out] -Sqrt[16 - x^4]/(48*x^3) + EllipticF[ArcSin[x/2], -1]/96
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{16-x^4}}\,dx &= -\frac{\sqrt{16-x^4}}{48x^3}+\frac{1}{48}\int \frac{1}{\sqrt{16-x^4}}\,dx\\ &= -\frac{\sqrt{16-x^4}}{48x^3}+\frac{1}{96}F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-1\right)\end{aligned}$$

Mathematica [C] time = 0.00, size = 24, normalized size = 0.77

$$-\frac{{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};\frac{x^4}{16}\right)}{12x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[16 - x^4]),x]
```

```
[Out] -1/12*Hypergeometric2F1[-3/4, 1/2, 1/4, x^4/16]/x^3
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-x^4+16}}{x^8-16\,x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^4+16)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 16)/(x^8 - 16*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^4+16}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^4+16)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^4 + 16)*x^4), x)
```

maple [B] time = 0.01, size = 49, normalized size = 1.58

$$\frac{\sqrt{-x^2+4}\,\sqrt{x^2+4}\,\text{EllipticF}\left(\frac{x}{2},i\right)}{96\sqrt{-x^4+16}}-\frac{\sqrt{-x^4+16}}{48x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-x^4+16)^(1/2),x)
```

```
[Out] -1/48*(-x^4+16)^(1/2)/x^3+1/96*(-x^2+4)^(1/2)*(x^2+4)^(1/2)/(-x^4+16)^(1/2)
*EllipticF(1/2*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^4+16}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^4+16)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^4 + 16)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x^4\,\sqrt{16-x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(16 - x^4)^(1/2)),x)
```

```
[Out] int(1/(x^4*(16 - x^4)^(1/2)), x)
```

sympy [A] time = 1.11, size = 36, normalized size = 1.16

$$\frac{\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{1}{4}\end{matrix}\right|\frac{x^4e^{2i\pi}}{16}\right)}{16x^3\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-x**4+16)**(1/2),x)
```

```
[Out] gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), x**4*exp_polar(2*I*pi)/16)/(16*x**3*
gamma(1/4))
```

3.974 $\int \frac{x}{\sqrt{-4+x^4}}\,dx$

Optimal. Leaf size=18

$$\frac{1}{2}\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4-4}}\right)$$

```
[Out] 1/2*arctanh(x^2/(x^4-4)^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {275, 217, 206}

$$\frac{1}{2}\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4-4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[-4 + x^4],x]
```

```
[Out] ArcTanh[x^2/Sqrt[-4 + x^4]]/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{-4+x^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-4+x^2}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x^2}{\sqrt{-4+x^4}}\right)\\ &= \frac{1}{2}\tanh^{-1}\left(\frac{x^2}{\sqrt{-4+x^4}}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{1}{2}\tanh^{-1}\left(\frac{x^2}{\sqrt{x^4-4}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[-4 + x^4],x]
```

```
[Out] ArcTanh[x^2/Sqrt[-4 + x^4]]/2
```

fricas [A] time = 1.14, size = 16, normalized size = 0.89

$$-\frac{1}{2}\log\left(-x^2+\sqrt{x^4-4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4-4)^(1/2),x, algorithm="fricas")
[Out] -1/2*log(-x^2 + sqrt(x^4 - 4))
```

giac [A] time = 0.16, size = 16, normalized size = 0.89

$$-\frac{1}{2}\log\left(x^2-\sqrt{x^4-4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4-4)^(1/2),x, algorithm="giac")
[Out] -1/2*log(x^2 - sqrt(x^4 - 4))
```

maple [A] time = 0.01, size = 15, normalized size = 0.83

$$\frac{\ln\left(x^2+\sqrt{x^4-4}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4-4)^(1/2),x)
[Out] 1/2*ln(x^2+(x^4-4)^(1/2))
```

maxima [B] time = 1.35, size = 33, normalized size = 1.83

$$\frac{1}{4}\log\left(\frac{\sqrt{x^4-4}}{x^2}+1\right)-\frac{1}{4}\log\left(\frac{\sqrt{x^4-4}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4-4)^(1/2),x, algorithm="maxima")
[Out] 1/4*log(sqrt(x^4 - 4)/x^2 + 1) - 1/4*log(sqrt(x^4 - 4)/x^2 - 1)
```

mupad [B] time = 0.08, size = 14, normalized size = 0.78

$$\frac{\ln\left(\sqrt{x^4-4}+x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4 - 4)^(1/2),x)
[Out] log((x^4 - 4)^(1/2) + x^2)/2
```

sympy [A] time = 1.96, size = 24, normalized size = 1.33

$$\begin{cases} \frac{\operatorname{acosh}\left(\frac{x^2}{2}\right)}{2} & \text{for } \frac{|x^4|}{4} > 1 \\ -\frac{i\operatorname{asin}\left(\frac{x^2}{2}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4-4)**(1/2),x)
[Out] Piecewise((acosh(x**2/2)/2, Abs(x**4)/4 > 1), (-I*asin(x**2/2)/2, True))
```

3.975 $\int \frac{x}{\sqrt{4+x^4}}\,dx$

Optimal. Leaf size=12

$$\frac{1}{2}\sinh^{-1}\left(\frac{x^2}{2}\right)$$

```
[Out] 1/2*arcsinh(1/2*x^2)
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {275, 215}

$$\frac{1}{2}\sinh^{-1}\left(\frac{x^2}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[4 + x^4],x]
```

```
[Out] ArcSinh[x^2/2]/2
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{4+x^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{4+x^2}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\sinh^{-1}\left(\frac{x^2}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\frac{1}{2}\sinh^{-1}\left(\frac{x^2}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[4 + x^4],x]
```

```
[Out] ArcSinh[x^2/2]/2
```

fricas [A] time = 0.98, size = 16, normalized size = 1.33

$$-\frac{1}{2}\log\left(-x^2+\sqrt{x^4+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+4)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*log(-x^2 + sqrt(x^4 + 4))
```

giac [A] time = 0.20, size = 16, normalized size = 1.33

$$-\frac{1}{2}\log\left(-x^2+\sqrt{x^4+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+4)^(1/2),x, algorithm="giac")
[Out] -1/2*log(-x^2 + sqrt(x^4 + 4))
```

maple [A] time = 0.01, size = 9, normalized size = 0.75

$$\frac{\operatorname{arcsinh}\left(\frac{x^2}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4+4)^(1/2),x)
[Out] 1/2*arcsinh(1/2*x^2)
```

maxima [B] time = 1.36, size = 33, normalized size = 2.75

$$\frac{1}{4}\log\left(\frac{\sqrt{x^4+4}}{x^2}+1\right)-\frac{1}{4}\log\left(\frac{\sqrt{x^4+4}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+4)^(1/2),x, algorithm="maxima")
[Out] 1/4*log(sqrt(x^4 + 4)/x^2 + 1) - 1/4*log(sqrt(x^4 + 4)/x^2 - 1)
```

mupad [B] time = 0.03, size = 8, normalized size = 0.67

$$\frac{\operatorname{asinh}\left(\frac{x^2}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4 + 4)^(1/2),x)
[Out] asinh(x^2/2)/2
```

sympy [A] time = 1.48, size = 7, normalized size = 0.58

$$\frac{\operatorname{asinh}\left(\frac{x^2}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4+4)**(1/2),x)
[Out] asinh(x**2/2)/2
```

3.976 $\int \frac{1}{x\sqrt{-1+x^4}}\,dx$

Optimal. Leaf size=14

$$\frac{1}{2}\tan^{-1}\left(\sqrt{x^4-1}\right)$$

```
[Out] 1/2*arctan((x^4-1)^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {266, 63, 203}

$$\frac{1}{2}\tan^{-1}\left(\sqrt{x^4-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-1 + x^4]),x]
```

```
[Out] ArcTan[Sqrt[-1 + x^4]]/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{-1+x^4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx, x, x^4\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x^4}\right)\\ &= \frac{1}{2}\tan^{-1}\left(\sqrt{-1+x^4}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{1}{2}\tan^{-1}\left(\sqrt{x^4-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-1 + x^4]),x]
[Out] ArcTan[Sqrt[-1 + x^4]]/2
```

fricas [A] time = 1.06, size = 10, normalized size = 0.71

$$\frac{1}{2}\arctan\left(\sqrt{x^4-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4-1)^(1/2),x, algorithm="fricas")
[Out] 1/2*arctan(sqrt(x^4 - 1))
```

giac [A] time = 0.19, size = 10, normalized size = 0.71

$$\frac{1}{2}\arctan\left(\sqrt{x^4-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4-1)^(1/2),x, algorithm="giac")
[Out] 1/2*arctan(sqrt(x^4 - 1))
```

maple [A] time = 0.01, size = 11, normalized size = 0.79

$$-\frac{\arctan\left(\frac{1}{\sqrt{x^4-1}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^4-1)^(1/2),x)
[Out] -1/2*arctan(1/(x^4-1)^(1/2))
```

maxima [A] time = 3.00, size = 10, normalized size = 0.71

$$\frac{1}{2}\arctan\left(\sqrt{x^4-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4-1)^(1/2),x, algorithm="maxima")
[Out] 1/2*arctan(sqrt(x^4 - 1))
```

mupad [B] time = 0.15, size = 10, normalized size = 0.71

$$\frac{\operatorname{atan}\left(\sqrt{x^4-1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^4 - 1)^(1/2)),x)
[Out] atan((x^4 - 1)^(1/2))/2
```

sympy [A] time = 1.56, size = 24, normalized size = 1.71

$$\begin{cases} \frac{i\operatorname{acosh}\left(\frac{1}{x^2}\right)}{2} & \text{for } \frac{1}{|x^4|} > 1 \\ -\frac{\operatorname{asin}\left(\frac{1}{x^2}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**4-1)**(1/2),x)
[Out] Piecewise((I*acosh(x**(-2))/2, 1/Abs(x**4) > 1), (-asin(x**(-2))/2, True))
```

3.977 $\int \frac{x^4}{\sqrt{-1+x^4}}\,dx$

Optimal. Leaf size=72

$$\frac{1}{3}\sqrt{x^4-1}\,x+\frac{\sqrt{x^2-1}\,\sqrt{x^2+1}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{3\sqrt{2}\,\sqrt{x^4-1}}$$

```
[Out] 1/6*EllipticF(x*2^(1/2)/(x^2-1)^(1/2),1/2*2^(1/2))*(x^2-1)^(1/2)*(x^2+1)^(1
/2)*2^(1/2)/(x^4-1)^(1/2)+1/3*x*(x^4-1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {321, 222}

$$\frac{1}{3}\sqrt{x^4-1}\,x+\frac{\sqrt{x^2-1}\,\sqrt{x^2+1}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{3\sqrt{2}\,\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[-1 + x^4],x]
```

```
[Out] (x*Sqrt[-1 + x^4])/3 + (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*EllipticF[ArcSin[(Sqrt
[2]*x)/Sqrt[-1 + x^2]], 1/2])/(3*Sqrt[2]*Sqrt[-1 + x^4])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(a*b), 2]}, Sim
p[(Sqrt[-a + q*x^2]*Sqrt[(a + q*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)
/(2*q)]], 1/2])/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]), x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt{-1+x^4}}\,dx &= \frac{1}{3}x\sqrt{-1+x^4}+\frac{1}{3}\int \frac{1}{\sqrt{-1+x^4}}\,dx\\ &= \frac{1}{3}x\sqrt{-1+x^4}+\frac{\sqrt{-1+x^2}\,\sqrt{1+x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{-1+x^2}}\right)|\frac{1}{2}\right)}{3\sqrt{2}\,\sqrt{-1+x^4}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 44, normalized size = 0.61

$$\frac{x\left(\sqrt{1-x^4}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};x^4\right)+x^4-1\right)}{3\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[-1 + x^4],x]
```

```
[Out] (x*(-1 + x^4 + Sqrt[1 - x^4]*Hypergeometric2F1[1/4, 1/2, 5/4, x^4]))/(3*Sqr
t[-1 + x^4])
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^4}{\sqrt{x^4-1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^4/sqrt(x^4 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(x^4 - 1), x)
```

maple [C] time = 0.01, size = 45, normalized size = 0.62

$$\frac{\sqrt{x^4-1}\,x}{3} - \frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\ \text{EllipticF}\left(ix, i\right)}{3\sqrt{x^4-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^4-1)^(1/2),x)
```

```
[Out] 1/3*x*(x^4-1)^(1/2)-1/3*I*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)^(1/2)*Ellipt
icF(I*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(x^4 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^4 - 1)^(1/2),x)
```

```
[Out] int(x^4/(x^4 - 1)^(1/2), x)
```

sympy [C] time = 1.46, size = 27, normalized size = 0.38

$$-\frac{ix^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{5}{4} \\ \frac{9}{4}\end{matrix}\right| x^4\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**4-1)**(1/2),x)
[Out] -I*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), x**4)/(4*gamma(9/4))
```

3.978 $\int \frac{1}{\sqrt{-1+x^4}}\,dx$

Optimal. Leaf size=54

$$\frac{\sqrt{x^2-1}\,\sqrt{x^2+1}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{x^4-1}}$$

```
[Out] 1/2*EllipticF(x*2^(1/2)/(x^2-1)^(1/2),1/2*2^(1/2))*(x^2-1)^(1/2)*(x^2+1)^(1
/2)*2^(1/2)/(x^4-1)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {222}

$$\frac{\sqrt{x^2-1}\,\sqrt{x^2+1}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-1 + x^4],x]
```

```
[Out] (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*EllipticF[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]],
 1/2])/(Sqrt[2]*Sqrt[-1 + x^4])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(a*b), 2]}, Sim
p[(Sqrt[-a + q*x^2]*Sqrt[(a + q*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)
/(2*q)]], 1/2])/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]), x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+x^4}}\,dx = \frac{\sqrt{-1+x^2}\,\sqrt{1+x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{-1+x^2}}\right)|\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{-1+x^4}}$$

Mathematica [A] time = 0.03, size = 25, normalized size = 0.46

$$\frac{\sqrt{1-x^4}\,F\left(\sin^{-1}(x)\middle|-1\right)}{\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-1 + x^4],x]
```

```
[Out] (Sqrt[1 - x^4]*EllipticF[ArcSin[x], -1])/Sqrt[-1 + x^4]
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{x^4-1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(x^4 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-1)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(x^4 - 1), x)
```

maple [C] time = 0.01, size = 34, normalized size = 0.63

$$-\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,\operatorname{EllipticF}(ix,i)}{\sqrt{x^4-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-1)^(1/2),x)
[Out] -I*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)^(1/2)*EllipticF(I*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(x^4 - 1), x)
```

mupad [B] time = 1.11, size = 26, normalized size = 0.48

$$\frac{x\sqrt{1-x^4}\;{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\;\frac{5}{4};\;x^4\right)}{\sqrt{x^4-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4 - 1)^(1/2),x)
[Out] (x*(1 - x^4)^(1/2)*hypergeom([1/4, 1/2], 5/4, x^4))/(x^4 - 1)^(1/2)
```

sympy [C] time = 0.77, size = 26, normalized size = 0.48

$$-\frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right|x^4\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-1)**(1/2),x)
[Out] -I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), x**4)/(4*gamma(5/4))
```

3.979 $\int \frac{1}{x^4\sqrt{-1+x^4}}\,dx$

Optimal. Leaf size=74

$$\frac{\sqrt{x^4-1}}{3x^3}+\frac{\sqrt{x^2-1}\,\sqrt{x^2+1}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{3\sqrt{2}\,\sqrt{x^4-1}}$$

```
[Out] 1/6*EllipticF(x*2^(1/2)/(x^2-1)^(1/2),1/2*2^(1/2))*(x^2-1)^(1/2)*(x^2+1)^(1
/2)*2^(1/2)/(x^4-1)^(1/2)+1/3*(x^4-1)^(1/2)/x^3
```

Rubi [A] time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {325, 222}

$$\frac{\sqrt{x^4-1}}{3x^3}+\frac{\sqrt{x^2-1}\,\sqrt{x^2+1}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{3\sqrt{2}\,\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[-1 + x^4]),x]
```

```
[Out] Sqrt[-1 + x^4]/(3*x^3) + (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*EllipticF[ArcSin[(Sq
rt[2]*x)/Sqrt[-1 + x^2]], 1/2])/(3*Sqrt[2]*Sqrt[-1 + x^4])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(a*b), 2]}, Sim
p[(Sqrt[-a + q*x^2]*Sqrt[(a + q*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)
/(2*q)]], 1/2])/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]), x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{-1+x^4}}\,dx &= \frac{\sqrt{-1+x^4}}{3x^3}+\frac{1}{3}\int \frac{1}{\sqrt{-1+x^4}}\,dx\\ &= \frac{\sqrt{-1+x^4}}{3x^3}+\frac{\sqrt{-1+x^2}\,\sqrt{1+x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{-1+x^2}}\right)|\frac{1}{2}\right)}{3\sqrt{2}\,\sqrt{-1+x^4}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.54

$$-\frac{\sqrt{1-x^4}\;{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};x^4\right)}{3x^3\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[-1 + x^4]),x]
```

```
[Out] -1/3*(Sqrt[1 - x^4]*Hypergeometric2F1[-3/4, 1/2, 1/4, x^4])/(x^3*Sqrt[-1 +
x^4])
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4-1}}{x^8-x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 - 1)/(x^8 - x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4-1}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 - 1)*x^4), x)
```

maple [C] time = 0.01, size = 47, normalized size = 0.64

$$-\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\;\text{EllipticF}\left(ix, i\right)}{3\sqrt{x^4-1}} + \frac{\sqrt{x^4-1}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^4-1)^(1/2),x)
```

```
[Out] 1/3*(x^4-1)^(1/2)/x^3-1/3*I*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)^(1/2)*Elli
pticF(I*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4-1}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 - 1)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x^4 - 1)^(1/2)),x)
```

```
[Out] int(1/(x^4*(x^4 - 1)^(1/2)), x)
```

sympy [C] time = 1.58, size = 31, normalized size = 0.42

$$-\frac{i\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{4}, \frac{1}{2} \\ \frac{1}{4}\end{matrix}\right| x^4\right)}{4x^3\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**4-1)**(1/2),x)
```

```
[Out] -I*gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), x**4)/(4*x**3*gamma(1/4))
```

3.980 $\int \frac{x^6}{\sqrt{-1+x^4}}\,dx$

Optimal. Leaf size=150

$$\frac{1}{5}\sqrt{x^4-1}\,x^3+\frac{3\left(x^2+1\right)x}{5\sqrt{x^4-1}}+\frac{3\sqrt{x^2-1}\,\sqrt{x^2+1}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{5\sqrt{2}\,\sqrt{x^4-1}}-\frac{3\sqrt{2}\,\sqrt{x^2-1}\,\sqrt{x^2+1}\,E\left(\sin^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{5\sqrt{x^4-1}}$$

```
[Out] 3/5*x*(x^2+1)/(x^4-1)^(1/2)+3/10*EllipticF(x*2^(1/2)/(x^2-1)^(1/2),1/2*2^(1
/2))*(x^2-1)^(1/2)*(x^2+1)^(1/2)*2^(1/2)/(x^4-1)^(1/2)-3/5*EllipticE(x*2^(1
/2)/(x^2-1)^(1/2),1/2*2^(1/2))*2^(1/2)*(x^2-1)^(1/2)*(x^2+1)^(1/2)/(x^4-1)^
(1/2)+1/5*x^3*(x^4-1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {321, 306, 222, 1185}

$$\frac{1}{5}\sqrt{x^4-1}\,x^3+\frac{3\left(x^2+1\right)x}{5\sqrt{x^4-1}}+\frac{3\sqrt{x^2-1}\,\sqrt{x^2+1}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{5\sqrt{2}\,\sqrt{x^4-1}}-\frac{3\sqrt{2}\,\sqrt{x^2-1}\,\sqrt{x^2+1}\,E\left(\sin^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{5\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/Sqrt[-1 + x^4],x]
```

```
[Out] (3*x*(1 + x^2))/(5*Sqrt[-1 + x^4]) + (x^3*Sqrt[-1 + x^4])/5 - (3*Sqrt[2]*Sq
rt[-1 + x^2]*Sqrt[1 + x^2]*EllipticE[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/
2])/(5*Sqrt[-1 + x^4]) + (3*Sqrt[-1 + x^2]*Sqrt[1 + x^2]*EllipticF[ArcSin[(
Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2])/(5*Sqrt[2]*Sqrt[-1 + x^4])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(a*b), 2]}, Sim
p[(Sqrt[-a + q*x^2]*Sqrt[(a + q*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)
/(2*q)]], 1/2])/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]), x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rule 306

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, Dist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[
a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1185

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(a*c), 2]}, Simp[(e*x*(q + c*x^2))/(c*Sqrt[a + c*x^4]), x] - Simp[(Sqr
t[2]*e*q*Sqrt[-a + q*x^2]*Sqrt[(a + q*x^2)/q]*EllipticE[ArcSin[x/Sqrt[(a +
q*x^2)/(2*q)]], 1/2])/(Sqrt[-a]*c*Sqrt[a + c*x^4]), x] /; EqQ[c*d + e*q, 0]
 && IntegerQ[q]] /; FreeQ[{a, c, d, e}, x] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{\sqrt{-1+x^4}}\,dx &= \frac{1}{5}x^3\sqrt{-1+x^4} + \frac{3}{5}\int \frac{x^2}{\sqrt{-1+x^4}}\,dx \\ &= \frac{1}{5}x^3\sqrt{-1+x^4} + \frac{3}{5}\int \frac{1}{\sqrt{-1+x^4}}\,dx - \frac{3}{5}\int \frac{1-x^2}{\sqrt{-1+x^4}}\,dx \\ &= \frac{3x\left(1+x^2\right)}{5\sqrt{-1+x^4}} + \frac{1}{5}x^3\sqrt{-1+x^4} - \frac{3\sqrt{2}\,\sqrt{-1+x^2}\,\sqrt{1+x^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{-1+x^2}}\right)|\frac{1}{2}\right)}{5\sqrt{-1+x^4}} + \frac{3\sqrt{-1+x^2}}{}\end{aligned}$$

Mathematica [C] time = 0.04, size = 46, normalized size = 0.31

$$\frac{x^3\left(\sqrt{1-x^4}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};x^4\right)+x^4-1\right)}{5\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/Sqrt[-1 + x^4],x]
```

```
[Out] (x^3*(-1 + x^4 + Sqrt[1 - x^4]*Hypergeometric2F1[1/2, 3/4, 7/4, x^4]))/(5*S
qrt[-1 + x^4])
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^6}{\sqrt{x^4-1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^6/sqrt(x^4 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/sqrt(x^4 - 1), x)
```

maple [C] time = 0.01, size = 57, normalized size = 0.38

$$\frac{\sqrt{x^4-1}\;x^3}{5} - \frac{3i\sqrt{x^2+1}\;\sqrt{-x^2+1}\;\left(-\text{EllipticE}\left(ix,i\right)+\text{EllipticF}\left(ix,i\right)\right)}{5\sqrt{x^4-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^4-1)^(1/2),x)
```

```
[Out] 1/5*x^3*(x^4-1)^(1/2)-3/5*I*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)^(1/2)*(Ell
ipticF(I*x,I)-EllipticE(I*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^4-1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^6/sqrt(x^4 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^4 - 1)^(1/2),x)
[Out] int(x^6/(x^4 - 1)^(1/2), x)
```

sympy [C] time = 1.01, size = 27, normalized size = 0.18

$$-\frac{ix^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{7}{4} \\ \frac{11}{4}\end{matrix}\right| x^4\right)}{4\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**4-1)**(1/2),x)
[Out] -I*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), x**4)/(4*gamma(11/4))
```

3.981 $\int \frac{x^2}{\sqrt{-1+x^4}}\,dx$

Optimal. Leaf size=126

$$\frac{x\left(x^2+1\right)}{\sqrt{x^4-1}}+\frac{\sqrt{x^2-1}\,\sqrt{x^2+1}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{x^4-1}}-\frac{\sqrt{2}\,\sqrt{x^2-1}\,\sqrt{x^2+1}\,E\left(\sin^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{\sqrt{x^4-1}}$$

```
[Out] x*(x^2+1)/(x^4-1)^(1/2)+1/2*EllipticF(x*2^(1/2)/(x^2-1)^(1/2),1/2*2^(1/2))*
(x^2-1)^(1/2)*(x^2+1)^(1/2)*2^(1/2)/(x^4-1)^(1/2)-EllipticE(x*2^(1/2)/(x^2-
1)^(1/2),1/2*2^(1/2))*2^(1/2)*(x^2-1)^(1/2)*(x^2+1)^(1/2)/(x^4-1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {306, 222, 1185}

$$\frac{x\left(x^2+1\right)}{\sqrt{x^4-1}}+\frac{\sqrt{x^2-1}\,\sqrt{x^2+1}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{x^4-1}}-\frac{\sqrt{2}\,\sqrt{x^2-1}\,\sqrt{x^2+1}\,E\left(\sin^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[-1 + x^4],x]
```

```
[Out] (x*(1 + x^2))/Sqrt[-1 + x^4] - (Sqrt[2]*Sqrt[-1 + x^2]*Sqrt[1 + x^2]*Ellipt
icE[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2])/Sqrt[-1 + x^4] + (Sqrt[-1 + x
^2]*Sqrt[1 + x^2]*EllipticF[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2])/(Sqrt
[2]*Sqrt[-1 + x^4])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(a*b), 2]}, Sim
p[(Sqrt[-a + q*x^2]*Sqrt[(a + q*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)
/(2*q)]], 1/2])/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]), x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rule 306

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, Dist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[
a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rule 1185

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(a*c), 2]}, Simp[(e*x*(q + c*x^2))/(c*Sqrt[a + c*x^4]), x] - Simp[(Sqr
t[2]*e*q*Sqrt[-a + q*x^2]*Sqrt[(a + q*x^2)/q]*EllipticE[ArcSin[x/Sqrt[(a +
q*x^2)/(2*q)]], 1/2])/(Sqrt[-a]*c*Sqrt[a + c*x^4]), x] /; EqQ[c*d + e*q, 0]
 && IntegerQ[q]] /; FreeQ[{a, c, d, e}, x] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{-1+x^4}}\,dx = \int \frac{1}{\sqrt{-1+x^4}}\,dx - \int \frac{1-x^2}{\sqrt{-1+x^4}}\,dx$$

$$= \frac{x\left(1+x^2\right)}{\sqrt{-1+x^4}} - \frac{\sqrt{2}\,\sqrt{-1+x^2}\,\sqrt{1+x^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{-1+x^2}}\right)|\frac{1}{2}\right)}{\sqrt{-1+x^4}} + \frac{\sqrt{-1+x^2}\,\sqrt{1+x^2}\,F\left(\sin^{-1}\left(\frac{}{}\right.\right.}{\sqrt{2}\,\sqrt{-1+x^4}}$$

Mathematica [C] time = 0.00, size = 40, normalized size = 0.32

$$\frac{x^3\sqrt{1-x^4}\ {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};x^4\right)}{3\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[-1 + x^4],x]
```

```
[Out] (x^3*Sqrt[1 - x^4]*Hypergeometric2F1[1/2, 3/4, 7/4, x^4])/(3*Sqrt[-1 + x^4]
)
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2}{\sqrt{x^4-1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^2/sqrt(x^4 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(x^4 - 1), x)
```

maple [C] time = 0.01, size = 44, normalized size = 0.35

$$-\frac{i\sqrt{x^2+1}\ \sqrt{-x^2+1}\ \left(-\text{EllipticE}\left(ix,i\right)+\text{EllipticF}\left(ix,i\right)\right)}{\sqrt{x^4-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4-1)^(1/2),x)
```

```
[Out] -I*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)^(1/2)*(EllipticF(I*x,I)-EllipticE(I
*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(x^4 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4 - 1)^(1/2),x)
```

```
[Out] int(x^2/(x^4 - 1)^(1/2), x)
```

sympy [**C**] time = 0.94, size = 27, normalized size = 0.21

$$-\frac{ix^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{3}{4} \\ \frac{7}{4}\end{matrix}\right| x^4\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**4-1)**(1/2),x)
```

```
[Out] -I*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**4)/(4*gamma(7/4))
```

3.982 $\int \frac{1}{x^2\sqrt{-1+x^4}}\,dx$

Optimal. Leaf size=140

$$\frac{\sqrt{x^4-1}}{x}-\frac{x\left(x^2+1\right)}{\sqrt{x^4-1}}-\frac{\sqrt{x^2-1}\,\sqrt{x^2+1}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{x^4-1}}+\frac{\sqrt{2}\,\sqrt{x^2-1}\,\sqrt{x^2+1}\,E\left(\sin^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{\sqrt{x^4-1}}$$

```
[Out] -x*(x^2+1)/(x^4-1)^(1/2)-1/2*EllipticF(x*2^(1/2)/(x^2-1)^(1/2),1/2*2^(1/2))
*(x^2-1)^(1/2)*(x^2+1)^(1/2)*2^(1/2)/(x^4-1)^(1/2)+EllipticE(x*2^(1/2)/(x^2
-1)^(1/2),1/2*2^(1/2))*2^(1/2)*(x^2-1)^(1/2)*(x^2+1)^(1/2)/(x^4-1)^(1/2)+(x
^4-1)^(1/2)/x
```

Rubi [A] time = 0.02, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {325, 306, 222, 1185}

$$-\frac{x\left(x^2+1\right)}{\sqrt{x^4-1}}+\frac{\sqrt{x^4-1}}{x}-\frac{\sqrt{x^2-1}\,\sqrt{x^2+1}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{x^4-1}}+\frac{\sqrt{2}\,\sqrt{x^2-1}\,\sqrt{x^2+1}\,E\left(\sin^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[-1 + x^4]),x]
```

```
[Out] -((x*(1 + x^2))/Sqrt[-1 + x^4]) + Sqrt[-1 + x^4]/x + (Sqrt[2]*Sqrt[-1 + x^2
]*Sqrt[1 + x^2]*EllipticE[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2])/Sqrt[-1
 + x^4] - (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*EllipticF[ArcSin[(Sqrt[2]*x)/Sqrt[-
1 + x^2]], 1/2])/(Sqrt[2]*Sqrt[-1 + x^4])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(a*b), 2]}, Sim
p[(Sqrt[-a + q*x^2]*Sqrt[(a + q*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)
/(2*q)]], 1/2])/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]), x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rule 306

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, Dist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[
a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1185

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(a*c), 2]}, Simp[(e*x*(q + c*x^2))/(c*Sqrt[a + c*x^4]), x] - Simp[(Sqr
t[2]*e*q*Sqrt[-a + q*x^2]*Sqrt[(a + q*x^2)/q]*EllipticE[ArcSin[x/Sqrt[(a +
q*x^2)/(2*q)]], 1/2])/(Sqrt[-a]*c*Sqrt[a + c*x^4]), x] /; EqQ[c*d + e*q, 0]
 && IntegerQ[q]] /; FreeQ[{a, c, d, e}, x] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{-1+x^4}}\,dx &= \frac{\sqrt{-1+x^4}}{x} - \int \frac{x^2}{\sqrt{-1+x^4}}\,dx \\ &= \frac{\sqrt{-1+x^4}}{x} - \int \frac{1}{\sqrt{-1+x^4}}\,dx + \int \frac{1-x^2}{\sqrt{-1+x^4}}\,dx \\ &= -\frac{x\left(1+x^2\right)}{\sqrt{-1+x^4}} + \frac{\sqrt{-1+x^4}}{x} + \frac{\sqrt{2}\,\sqrt{-1+x^2}\,\sqrt{1+x^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{-1+x^2}}\right)|\frac{1}{2}\right)}{\sqrt{-1+x^4}} - \frac{\sqrt{-1+x^2}\,\sqrt{1}}{}\end{aligned}$$

Mathematica [C] time = 0.01, size = 38, normalized size = 0.27

$$-\frac{\sqrt{1-x^4}\;{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};x^4\right)}{x\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[-1 + x^4]),x]
```

```
[Out] -((Sqrt[1 - x^4]*Hypergeometric2F1[-1/4, 1/2, 3/4, x^4])/(x*Sqrt[-1 + x^4])
)
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^4-1}}{x^6-x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 - 1)/(x^6 - x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4-1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 - 1)*x^2), x)
```

maple [C] time = 0.01, size = 56, normalized size = 0.40

$$\frac{\sqrt{x^4-1}}{x} + \frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,\left(-\text{EllipticE}\left(ix,i\right)+\text{EllipticF}\left(ix,i\right)\right)}{\sqrt{x^4-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^4-1)^(1/2),x)
```

```
[Out] (x^4-1)^(1/2)/x+I*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)^(1/2)*(EllipticF(I*x
,I)-EllipticE(I*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4-1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^4-1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^4 - 1)*x^2), x)
```

mupad [**B**] time = 1.21, size = 18, normalized size = 0.13

$$-\frac{\sqrt{\frac{1}{x^4}}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\ \frac{7}{4};\ \frac{1}{x^4}\right)}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^4 - 1)^(1/2)),x)
[Out] -((1/x^4)^(1/2)*hypergeom([1/2, 3/4], 7/4, 1/x^4))/(3*x)
```

sympy [**C**] time = 0.97, size = 29, normalized size = 0.21

$$-\frac{i\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\right|x^4\right)}{4x\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**4-1)**(1/2),x)
[Out] -I*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), x**4)/(4*x*gamma(3/4))
```

3.983 $\int \frac{x^2}{\sqrt{3-2x^4}}\,dx$

Optimal. Leaf size=48

$$\frac{\sqrt[4]{3}\,E\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)\middle|-1\right)}{2^{3/4}}-\frac{\sqrt[4]{3}\,F\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)\middle|-1\right)}{2^{3/4}}$$

```
[Out] 1/2*3^(1/4)*EllipticE(1/3*2^(1/4)*3^(3/4)*x,I)*2^(1/4)-1/2*3^(1/4)*Elliptic
F(1/3*2^(1/4)*3^(3/4)*x,I)*2^(1/4)
```

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {307, 221, 1181, 21, 424}

$$\frac{\sqrt[4]{3}\,E\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)\middle|-1\right)}{2^{3/4}}-\frac{\sqrt[4]{3}\,F\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)\middle|-1\right)}{2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[3 - 2*x^4],x]
```

```
[Out] (3^(1/4)*EllipticE[ArcSin[(2/3)^(1/4)*x], -1])/2^(3/4) - (3^(1/4)*EllipticF
[ArcSin[(2/3)^(1/4)*x], -1])/2^(3/4)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[(d + e*x^2)/(Sqrt[q + c*x^2]*Sqrt[q - c
*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{3-2x^4}}\,dx &= -\left(\sqrt{\frac{3}{2}}\int \frac{1}{\sqrt{3-2x^4}}\,dx\right)+\sqrt{\frac{3}{2}}\int \frac{1+\sqrt{\frac{2}{3}}\,x^2}{\sqrt{3-2x^4}}\,dx \\ &= -\frac{\sqrt[4]{3}\,F\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)\middle|-1\right)}{2^{3/4}}+\sqrt{3}\int \frac{1+\sqrt{\frac{2}{3}}\,x^2}{\sqrt{\sqrt{6}-2x^2}\,\sqrt{\sqrt{6}+2x^2}}\,dx \\ &= -\frac{\sqrt[4]{3}\,F\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)\middle|-1\right)}{2^{3/4}}+\frac{\int \frac{\sqrt{\sqrt{6}+2x^2}}{\sqrt{\sqrt{6}-2x^2}}\,dx}{\sqrt{2}} \\ &= \frac{\sqrt[4]{3}\,E\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)\middle|-1\right)}{2^{3/4}}-\frac{\sqrt[4]{3}\,F\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)\middle|-1\right)}{2^{3/4}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.60

$$\frac{x^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{2x^4}{3}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[3 - 2*x^4],x]
[Out] (x^3*Hypergeometric2F1[1/2, 3/4, 7/4, (2*x^4)/3])/(3*Sqrt[3])
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4+3}\,x^2}{2\,x^4-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-2*x^4+3)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-2*x^4 + 3)*x^2/(2*x^4 - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-2\,x^4+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-2*x^4+3)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/sqrt(-2*x^4 + 3), x)
```

maple [A] time = 0.04, size = 69, normalized size = 1.44

$$-\frac{\sqrt{3}\;6^{\frac{1}{4}}\sqrt{-3\sqrt{6}\;x^2+9}\;\sqrt{3\sqrt{6}\;x^2+9}\;\left(-\text{EllipticE}\left(\frac{\sqrt{3}\;6^{\frac{1}{4}}x}{3},i\right)+\text{EllipticF}\left(\frac{\sqrt{3}\;6^{\frac{1}{4}}x}{3},i\right)\right)}{18\sqrt{-2x^4+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-2*x^4+3)^(1/2),x)
```

```
[Out] -1/18*3^(1/2)*6^(1/4)*(9-3*6^(1/2)*x^2)^(1/2)*(9+3*6^(1/2)*x^2)^(1/2)/(-2*x
^4+3)^(1/2)*(EllipticF(1/3*x*3^(1/2)*6^(1/4),I)-EllipticE(1/3*x*3^(1/2)*6^(
1/4),I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-2\,x^4+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-2*x^4+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(-2*x^4 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\sqrt{3-2\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3 - 2*x^4)^(1/2),x)
```

```
[Out] int(x^2/(3 - 2*x^4)^(1/2), x)
```

sympy [A] time = 1.28, size = 39, normalized size = 0.81

$$\frac{\sqrt{3}\,x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{2x^4e^{2i\pi}}{3}\right)}{12\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-2*x**4+3)**(1/2),x)
```

```
[Out] sqrt(3)*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), 2*x**4*exp_polar(2*I*pi)/
3)/(12*gamma(7/4))
```

3.984 $\int \frac{x^2}{\sqrt{3-bx^4}}\,dx$

Optimal. Leaf size=54

$$\frac{\sqrt[4]{3}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{3}}\right)\middle|-1\right)}{b^{3/4}}-\frac{\sqrt[4]{3}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{3}}\right)\middle|-1\right)}{b^{3/4}}$$

```
[Out] 3^(1/4)*EllipticE(1/3*b^(1/4)*x*3^(3/4),I)/b^(3/4)-3^(1/4)*EllipticF(1/3*b^
(1/4)*x*3^(3/4),I)/b^(3/4)
```

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {307, 221, 1199, 424}

$$\frac{\sqrt[4]{3}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{3}}\right)\middle|-1\right)}{b^{3/4}}-\frac{\sqrt[4]{3}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{3}}\right)\middle|-1\right)}{b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[3 - b*x^4],x]
```

```
[Out] (3^(1/4)*EllipticE[ArcSin[(b^(1/4)*x)/3^(1/4)], -1])/b^(3/4) - (3^(1/4)*Ell
ipticF[ArcSin[(b^(1/4)*x)/3^(1/4)], -1])/b^(3/4)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 307

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(b/a), 2]}
, -Dist[q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1199

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + (e*x^2)/d]/Sqrt[1 - (e*x^2)/d], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{3-bx^4}}\,dx = -\frac{\sqrt{3}\int \frac{1}{\sqrt{3-bx^4}}\,dx}{\sqrt{b}} + \frac{\sqrt{3}\int \frac{1+\frac{\sqrt{b}x^2}{\sqrt{3}}}{\sqrt{3-bx^4}}\,dx}{\sqrt{b}}$$

$$= -\frac{\sqrt[4]{3}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{3}}\right)\middle| -1\right)}{b^{3/4}} + \frac{\int \frac{\sqrt{1+\frac{\sqrt{b}x^2}{\sqrt{3}}}}{\sqrt{1-\frac{\sqrt{b}x^2}{\sqrt{3}}}}\,dx}{\sqrt{b}}$$

$$= \frac{\sqrt[4]{3}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{3}}\right)\middle| -1\right)}{b^{3/4}} - \frac{\sqrt[4]{3}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{3}}\right)\middle| -1\right)}{b^{3/4}}$$

Mathematica [C] time = 0.01, size = 30, normalized size = 0.56

$$\frac{x^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{bx^4}{3}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[3 - b*x^4],x]
[Out] (x^3*Hypergeometric2F1[1/2, 3/4, 7/4, (b*x^4)/3])/(3*Sqrt[3])
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-bx^4+3}\,x^2}{bx^4-3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^4+3)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-b*x^4 + 3)*x^2/(b*x^4 - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-bx^4+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^4+3)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/sqrt(-b*x^4 + 3), x)
```

maple [B] time = 0.01, size = 94, normalized size = 1.74

$$-\frac{\sqrt{-3\sqrt{3}\,\sqrt{b}\,x^2+9}\,\sqrt{3\sqrt{3}\,\sqrt{b}\,x^2+9}\left(-\text{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{\sqrt{3}\,\sqrt{b}}\,x}{3}, i\right)+\text{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{\sqrt{3}\,\sqrt{b}}\,x}{3}, i\right)\right)}{3\sqrt{\sqrt{3}\,\sqrt{b}}\,\sqrt{-bx^4+3}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^4+3)^(1/2),x)
```

```
[Out] -1/3/(3^(1/2)*b^(1/2))^(1/2)*(9-3*3^(1/2)*b^(1/2)*x^2)^(1/2)*(9+3*3^(1/2)*b
^(1/2)*x^2)^(1/2)/(-b*x^4+3)^(1/2)/b^(1/2)*(EllipticF(1/3*x*3^(1/2)*(3^(1/2
)*b^(1/2))^(1/2),I)-EllipticE(1/3*x*3^(1/2)*(3^(1/2)*b^(1/2))^(1/2),I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-bx^4+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^4+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(-b*x^4 + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\sqrt{3-b\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3 - b*x^4)^(1/2),x)
```

```
[Out] int(x^2/(3 - b*x^4)^(1/2), x)
```

sympy [A] time = 1.39, size = 39, normalized size = 0.72

$$\frac{\sqrt{3}\,x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{bx^4e^{2i\pi}}{3}\right)}{12\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**4+3)**(1/2),x)
```

```
[Out] sqrt(3)*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b*x**4*exp_polar(2*I*pi)/
3)/(12*gamma(7/4))
```

3.985 $\int x^7 \sqrt[3]{1+x^4}\, dx$

Optimal. Leaf size=27

$$\frac{3}{28}\left(x^4+1\right)^{7/3}-\frac{3}{16}\left(x^4+1\right)^{4/3}$$

```
[Out] -3/16*(x^4+1)^(4/3)+3/28*(x^4+1)^(7/3)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{3}{28}\left(x^4+1\right)^{7/3}-\frac{3}{16}\left(x^4+1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(1 + x^4)^(1/3),x]
```

```
[Out] (-3*(1 + x^4)^(4/3))/16 + (3*(1 + x^4)^(7/3))/28
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^7 \sqrt[3]{1+x^4}\, dx &= \frac{1}{4}\text{Subst}\left(\int x\sqrt[3]{1+x}\, dx, x, x^4\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \left(-\sqrt[3]{1+x}+(1+x)^{4/3}\right) dx, x, x^4\right)\\ &= -\frac{3}{16}\left(1+x^4\right)^{4/3}+\frac{3}{28}\left(1+x^4\right)^{7/3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.74

$$\frac{3}{112}\left(x^4+1\right)^{4/3}\left(4x^4-3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(1 + x^4)^(1/3),x]
```

```
[Out] (3*(1 + x^4)^(4/3)*(-3 + 4*x^4))/112
```

fricas [A] time = 0.73, size = 19, normalized size = 0.70

$$\frac{3}{112}\left(4x^8+x^4-3\right)\left(x^4+1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(x^4+1)^(1/3),x, algorithm="fricas")
[Out] 3/112*(4*x^8 + x^4 - 3)*(x^4 + 1)^(1/3)
```

giac [A] time = 0.15, size = 19, normalized size = 0.70

$$\frac{3}{28}\left(x^4+1\right)^{\frac{7}{3}}-\frac{3}{16}\left(x^4+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(x^4+1)^(1/3),x, algorithm="giac")
[Out] 3/28*(x^4 + 1)^(7/3) - 3/16*(x^4 + 1)^(4/3)
```

maple [A] time = 0.00, size = 17, normalized size = 0.63

$$\frac{3\left(x^4+1\right)^{\frac{4}{3}}\left(4x^4-3\right)}{112}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(x^4+1)^(1/3),x)
[Out] 3/112*(x^4+1)^(4/3)*(4*x^4-3)
```

maxima [A] time = 1.36, size = 19, normalized size = 0.70

$$\frac{3}{28}\left(x^4+1\right)^{\frac{7}{3}}-\frac{3}{16}\left(x^4+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(x^4+1)^(1/3),x, algorithm="maxima")
[Out] 3/28*(x^4 + 1)^(7/3) - 3/16*(x^4 + 1)^(4/3)
```

mupad [B] time = 1.10, size = 16, normalized size = 0.59

$$\frac{3\left(x^4+1\right)^{4/3}\left(4\,x^4-3\right)}{112}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(x^4 + 1)^(1/3),x)
[Out] (3*(x^4 + 1)^(4/3)*(4*x^4 - 3))/112
```

sympy [A] time = 1.10, size = 41, normalized size = 1.52

$$\frac{3x^8\sqrt[3]{x^4+1}}{28}+\frac{3x^4\sqrt[3]{x^4+1}}{112}-\frac{9\sqrt[3]{x^4+1}}{112}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(x**4+1)**(1/3),x)
[Out] 3*x**8*(x**4 + 1)**(1/3)/28 + 3*x**4*(x**4 + 1)**(1/3)/112 - 9*(x**4 + 1)**
(1/3)/112
```

3.986 $\int \frac{x^3}{\left(1+x^4\right)^{4/3}}\,dx$

Optimal. Leaf size=13

$$-\frac{3}{4\sqrt[3]{x^4+1}}$$

```
[Out] -3/4/(x^4+1)^(1/3)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$-\frac{3}{4\sqrt[3]{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 + x^4)^(4/3),x]
```

```
[Out] -3/(4*(1 + x^4)^(1/3))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^3}{\left(1+x^4\right)^{4/3}}\,dx = -\frac{3}{4\sqrt[3]{1+x^4}}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$-\frac{3}{4\sqrt[3]{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 + x^4)^(4/3),x]
```

```
[Out] -3/(4*(1 + x^4)^(1/3))
```

fricas [A] time = 1.05, size = 9, normalized size = 0.69

$$-\frac{3}{4\left(x^4+1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4+1)^(4/3),x, algorithm="fricas")
```

```
[Out] -3/4/(x^4 + 1)^(1/3)
```

giac [A] time = 0.16, size = 9, normalized size = 0.69

$$-\frac{3}{4\left(x^4+1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4+1)^(4/3),x, algorithm="giac")
[Out] -3/4/(x^4 + 1)^(1/3)
```

maple [A] time = 0.00, size = 10, normalized size = 0.77

$$-\frac{3}{4\left(x^4+1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^4+1)^(4/3),x)
[Out] -3/4/(x^4+1)^(1/3)
```

maxima [A] time = 1.35, size = 9, normalized size = 0.69

$$-\frac{3}{4\left(x^4+1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4+1)^(4/3),x, algorithm="maxima")
[Out] -3/4/(x^4 + 1)^(1/3)
```

mupad [B] time = 1.10, size = 9, normalized size = 0.69

$$-\frac{3}{4\left(x^4+1\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^4 + 1)^(4/3),x)
[Out] -3/(4*(x^4 + 1)^(1/3))
```

sympy [A] time = 0.56, size = 12, normalized size = 0.92

$$-\frac{3}{4\sqrt[3]{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**4+1)**(4/3),x)
[Out] -3/(4*(x**4 + 1)**(1/3))
```

3.987 $\int \frac{x^3}{\sqrt[3]{1+x^4}}\,dx$

Optimal. Leaf size=13

$$\frac{3}{8}\left(x^4+1\right)^{2/3}$$

```
[Out] 3/8*(x^4+1)^(2/3)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$\frac{3}{8}\left(x^4+1\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 + x^4)^(1/3),x]
```

```
[Out] (3*(1 + x^4)^(2/3))/8
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^3}{\sqrt[3]{1+x^4}}\,dx = \frac{3}{8}\left(1+x^4\right)^{2/3}$$

Mathematica [A] time = 0.02, size = 13, normalized size = 1.00

$$\frac{3}{8}\left(x^4+1\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 + x^4)^(1/3),x]
```

```
[Out] (3*(1 + x^4)^(2/3))/8
```

fricas [A] time = 0.58, size = 9, normalized size = 0.69

$$\frac{3}{8}\left(x^4+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/8*(x^4 + 1)^(2/3)
```

giac [A] time = 0.15, size = 9, normalized size = 0.69

$$\frac{3}{8}\left(x^4+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4+1)^(1/3),x, algorithm="giac")
[Out] 3/8*(x^4 + 1)^(2/3)
```

maple [A] time = 0.00, size = 10, normalized size = 0.77

$$\frac{3\left(x^4+1\right)^{\frac{2}{3}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^4+1)^(1/3),x)
[Out] 3/8*(x^4+1)^(2/3)
```

maxima [A] time = 1.30, size = 9, normalized size = 0.69

$$\frac{3}{8}\left(x^4+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4+1)^(1/3),x, algorithm="maxima")
[Out] 3/8*(x^4 + 1)^(2/3)
```

mupad [B] time = 0.07, size = 9, normalized size = 0.69

$$\frac{3\left(x^4+1\right)^{2/3}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^4 + 1)^(1/3),x)
[Out] (3*(x^4 + 1)^(2/3))/8
```

sympy [A] time = 0.20, size = 10, normalized size = 0.77

$$\frac{3\left(x^4+1\right)^{\frac{2}{3}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**4+1)**(1/3),x)
[Out] 3*(x**4 + 1)**(2/3)/8
```

3.988 $\int x^{19}\sqrt[4]{a+bx^4}\,dx$

Optimal. Leaf size=101

$$\frac{a^4\left(a+bx^4\right)^{5/4}}{5b^5}-\frac{4a^3\left(a+bx^4\right)^{9/4}}{9b^5}+\frac{6a^2\left(a+bx^4\right)^{13/4}}{13b^5}+\frac{\left(a+bx^4\right)^{21/4}}{21b^5}-\frac{4a\left(a+bx^4\right)^{17/4}}{17b^5}$$

```
[Out] 1/5*a^4*(b*x^4+a)^(5/4)/b^5-4/9*a^3*(b*x^4+a)^(9/4)/b^5+6/13*a^2*(b*x^4+a)^
(13/4)/b^5-4/17*a*(b*x^4+a)^(17/4)/b^5+1/21*(b*x^4+a)^(21/4)/b^5
```

Rubi [A] time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{6a^2\left(a+bx^4\right)^{13/4}}{13b^5}-\frac{4a^3\left(a+bx^4\right)^{9/4}}{9b^5}+\frac{a^4\left(a+bx^4\right)^{5/4}}{5b^5}+\frac{\left(a+bx^4\right)^{21/4}}{21b^5}-\frac{4a\left(a+bx^4\right)^{17/4}}{17b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^19*(a + b*x^4)^(1/4),x]
```

```
[Out] (a^4*(a + b*x^4)^(5/4))/(5*b^5) - (4*a^3*(a + b*x^4)^(9/4))/(9*b^5) + (6*a^
2*(a + b*x^4)^(13/4))/(13*b^5) - (4*a*(a + b*x^4)^(17/4))/(17*b^5) + (a + b
*x^4)^(21/4)/(21*b^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{19}\sqrt[4]{a+bx^4}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int x^4\sqrt[4]{a+bx}\,dx, x, x^4\right)\\ &= \frac{1}{4}\operatorname{Subst}\left(\int\left(\frac{a^4\sqrt[4]{a+bx}}{b^4}-\frac{4a^3(a+bx)^{5/4}}{b^4}+\frac{6a^2(a+bx)^{9/4}}{b^4}-\frac{4a(a+bx)^{13/4}}{b^4}+\frac{(a+bx)^{1}}{b^4}\right.\right.\\ &= \frac{a^4\left(a+bx^4\right)^{5/4}}{5b^5}-\frac{4a^3\left(a+bx^4\right)^{9/4}}{9b^5}+\frac{6a^2\left(a+bx^4\right)^{13/4}}{13b^5}-\frac{4a\left(a+bx^4\right)^{17/4}}{17b^5}+\frac{\left(a+bx^4\right)^{21/4}}{21b^5}\end{aligned}$$

Mathematica [A] time = 0.03, size = 61, normalized size = 0.60

$$\frac{\left(a+bx^4\right)^{5/4}\left(2048a^4-2560a^3bx^4+2880a^2b^2x^8-3120ab^3x^{12}+3315b^4x^{16}\right)}{69615b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^19*(a + b*x^4)^(1/4),x]
```

```
[Out] ((a + b*x^4)^(5/4)*(2048*a^4 - 2560*a^3*b*x^4 + 2880*a^2*b^2*x^8 - 3120*a*b
^3*x^12 + 3315*b^4*x^16))/(69615*b^5)
```

fricas [A] time = 0.86, size = 68, normalized size = 0.67

$$\frac{\left(3315\,b^5x^{20}+195\,ab^4x^{16}-240\,a^2b^3x^{12}+320\,a^3b^2x^8-512\,a^4bx^4+2048\,a^5\right)\left(bx^4+a\right)^{\frac{1}{4}}}{69615\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19*(b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/69615*(3315*b^5*x^20 + 195*a*b^4*x^16 - 240*a^2*b^3*x^12 + 320*a^3*b^2*x^
8 - 512*a^4*b*x^4 + 2048*a^5)*(b*x^4 + a)^(1/4)/b^5
```

giac [A] time = 0.17, size = 71, normalized size = 0.70

$$\frac{3315\left(bx^4+a\right)^{\frac{21}{4}}-16380\left(bx^4+a\right)^{\frac{17}{4}}a+32130\left(bx^4+a\right)^{\frac{13}{4}}a^2-30940\left(bx^4+a\right)^{\frac{9}{4}}a^3+13923\left(bx^4+a\right)^{\frac{5}{4}}a^4}{69615\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19*(b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] 1/69615*(3315*(b*x^4 + a)^(21/4) - 16380*(b*x^4 + a)^(17/4)*a + 32130*(b*x^
4 + a)^(13/4)*a^2 - 30940*(b*x^4 + a)^(9/4)*a^3 + 13923*(b*x^4 + a)^(5/4)*a
^4)/b^5
```

maple [A] time = 0.01, size = 58, normalized size = 0.57

$$\frac{\left(b\,x^4+a\right)^{\frac{5}{4}}\left(3315x^{16}b^4-3120a\,x^{12}b^3+2880a^2x^8b^2-2560a^3x^4b+2048a^4\right)}{69615b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19*(b*x^4+a)^(1/4),x)
```

```
[Out] 1/69615*(b*x^4+a)^(5/4)*(3315*b^4*x^16-3120*a*b^3*x^12+2880*a^2*b^2*x^8-256
0*a^3*b*x^4+2048*a^4)/b^5
```

maxima [A] time = 1.34, size = 81, normalized size = 0.80

$$\frac{\left(bx^4+a\right)^{\frac{21}{4}}}{21\,b^5}-\frac{4\left(bx^4+a\right)^{\frac{17}{4}}a}{17\,b^5}+\frac{6\left(bx^4+a\right)^{\frac{13}{4}}a^2}{13\,b^5}-\frac{4\left(bx^4+a\right)^{\frac{9}{4}}a^3}{9\,b^5}+\frac{\left(bx^4+a\right)^{\frac{5}{4}}a^4}{5\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19*(b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] 1/21*(b*x^4 + a)^(21/4)/b^5 - 4/17*(b*x^4 + a)^(17/4)*a/b^5 + 6/13*(b*x^4 +
 a)^(13/4)*a^2/b^5 - 4/9*(b*x^4 + a)^(9/4)*a^3/b^5 + 1/5*(b*x^4 + a)^(5/4)*
a^4/b^5
```

mupad [B] time = 1.15, size = 66, normalized size = 0.65

$$\left(b\,x^4+a\right)^{1/4}\left(\frac{x^{20}}{21}+\frac{2048\,a^5}{69615\,b^5}+\frac{a\,x^{16}}{357\,b}-\frac{512\,a^4\,x^4}{69615\,b^4}+\frac{64\,a^3\,x^8}{13923\,b^3}-\frac{16\,a^2\,x^{12}}{4641\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19*(a + b*x^4)^(1/4),x)
```

```
[Out] (a + b*x^4)^(1/4)*(x^20/21 + (2048*a^5)/(69615*b^5) + (a*x^16)/(357*b) - (5
12*a^4*x^4)/(69615*b^4) + (64*a^3*x^8)/(13923*b^3) - (16*a^2*x^12)/(4641*b^
2))
```

sympy [A] time = 22.69, size = 134, normalized size = 1.33

$$\begin{cases} \frac{2048a^5\sqrt[4]{a+bx^4}}{69615b^5} - \frac{512a^4x^4\sqrt[4]{a+bx^4}}{69615b^4} + \frac{64a^3x^8\sqrt[4]{a+bx^4}}{13923b^3} - \frac{16a^2x^{12}\sqrt[4]{a+bx^4}}{4641b^2} + \frac{ax^{16}\sqrt[4]{a+bx^4}}{357b} + \frac{x^{20}\sqrt[4]{a+bx^4}}{21} & \text{for } b \neq 0 \\ \frac{\sqrt[4]{a}\,x^{20}}{20} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**19*(b*x**4+a)**(1/4),x)
```

```
[Out] Piecewise((2048*a**5*(a + b*x**4)**(1/4)/(69615*b**5) - 512*a**4*x**4*(a +
b*x**4)**(1/4)/(69615*b**4) + 64*a**3*x**8*(a + b*x**4)**(1/4)/(13923*b**3)
 - 16*a**2*x**12*(a + b*x**4)**(1/4)/(4641*b**2) + a*x**16*(a + b*x**4)**(1
/4)/(357*b) + x**20*(a + b*x**4)**(1/4)/21, Ne(b, 0)), (a**(1/4)*x**20/20,
True))
```

3.989 $\int x^{15}\sqrt[4]{a+bx^4}\,dx$

Optimal. Leaf size=80

$$-\frac{a^3\left(a+bx^4\right)^{5/4}}{5b^4}+\frac{a^2\left(a+bx^4\right)^{9/4}}{3b^4}+\frac{\left(a+bx^4\right)^{17/4}}{17b^4}-\frac{3a\left(a+bx^4\right)^{13/4}}{13b^4}$$

```
[Out] -1/5*a^3*(b*x^4+a)^(5/4)/b^4+1/3*a^2*(b*x^4+a)^(9/4)/b^4-3/13*a*(b*x^4+a)^(
13/4)/b^4+1/17*(b*x^4+a)^(17/4)/b^4
```

Rubi [A] time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{a^2\left(a+bx^4\right)^{9/4}}{3b^4}-\frac{a^3\left(a+bx^4\right)^{5/4}}{5b^4}+\frac{\left(a+bx^4\right)^{17/4}}{17b^4}-\frac{3a\left(a+bx^4\right)^{13/4}}{13b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^15*(a + b*x^4)^(1/4),x]
```

```
[Out] -(a^3*(a + b*x^4)^(5/4))/(5*b^4) + (a^2*(a + b*x^4)^(9/4))/(3*b^4) - (3*a*(
a + b*x^4)^(13/4))/(13*b^4) + (a + b*x^4)^(17/4)/(17*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{15}\sqrt[4]{a+bx^4}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int x^3\sqrt[4]{a+bx}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int\left(-\frac{a^3\sqrt[4]{a+bx}}{b^3}+\frac{3a^2(a+bx)^{5/4}}{b^3}-\frac{3a(a+bx)^{9/4}}{b^3}+\frac{(a+bx)^{13/4}}{b^3}\right)dx, x, x^4\right)\\ &= -\frac{a^3\left(a+bx^4\right)^{5/4}}{5b^4}+\frac{a^2\left(a+bx^4\right)^{9/4}}{3b^4}-\frac{3a\left(a+bx^4\right)^{13/4}}{13b^4}+\frac{\left(a+bx^4\right)^{17/4}}{17b^4}\end{aligned}$$

Mathematica [A] time = 0.04, size = 50, normalized size = 0.62

$$\frac{\left(a+bx^4\right)^{5/4}\left(-128a^3+160a^2bx^4-180ab^2x^8+195b^3x^{12}\right)}{3315b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15*(a + b*x^4)^(1/4),x]
```

```
[Out] ((a + b*x^4)^(5/4)*(-128*a^3 + 160*a^2*b*x^4 - 180*a*b^2*x^8 + 195*b^3*x^12
))/(3315*b^4)
```

fricas [A] time = 0.85, size = 57, normalized size = 0.71

$$\frac{\left(195\,b^4x^{16}+15\,ab^3x^{12}-20\,a^2b^2x^8+32\,a^3bx^4-128\,a^4\right)\left(bx^4+a\right)^{\frac{1}{4}}}{3315\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/3315*(195*b^4*x^16 + 15*a*b^3*x^12 - 20*a^2*b^2*x^8 + 32*a^3*b*x^4 - 128*
a^4)*(b*x^4 + a)^(1/4)/b^4
```

giac [A] time = 0.16, size = 57, normalized size = 0.71

$$\frac{195\left(bx^4+a\right)^{\frac{17}{4}}-765\left(bx^4+a\right)^{\frac{13}{4}}a+1105\left(bx^4+a\right)^{\frac{9}{4}}a^2-663\left(bx^4+a\right)^{\frac{5}{4}}a^3}{3315\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] 1/3315*(195*(b*x^4 + a)^(17/4) - 765*(b*x^4 + a)^(13/4)*a + 1105*(b*x^4 + a
)^(9/4)*a^2 - 663*(b*x^4 + a)^(5/4)*a^3)/b^4
```

maple [A] time = 0.01, size = 47, normalized size = 0.59

$$-\frac{\left(b\,x^4+a\right)^{\frac{5}{4}}\left(-195b^3x^{12}+180a\,b^2x^8-160a^2b\,x^4+128a^3\right)}{3315b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15*(b*x^4+a)^(1/4),x)
```

```
[Out] -1/3315*(b*x^4+a)^(5/4)*(-195*b^3*x^12+180*a*b^2*x^8-160*a^2*b*x^4+128*a^3)
/b^4
```

maxima [A] time = 1.30, size = 64, normalized size = 0.80

$$\frac{\left(bx^4+a\right)^{\frac{17}{4}}}{17\,b^4}-\frac{3\left(bx^4+a\right)^{\frac{13}{4}}a}{13\,b^4}+\frac{\left(bx^4+a\right)^{\frac{9}{4}}a^2}{3\,b^4}-\frac{\left(bx^4+a\right)^{\frac{5}{4}}a^3}{5\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] 1/17*(b*x^4 + a)^(17/4)/b^4 - 3/13*(b*x^4 + a)^(13/4)*a/b^4 + 1/3*(b*x^4 +
a)^(9/4)*a^2/b^4 - 1/5*(b*x^4 + a)^(5/4)*a^3/b^4
```

mupad [B] time = 1.13, size = 55, normalized size = 0.69

$$\left(b\,x^4+a\right)^{1/4}\left(\frac{x^{16}}{17}-\frac{128\,a^4}{3315\,b^4}+\frac{a\,x^{12}}{221\,b}+\frac{32\,a^3\,x^4}{3315\,b^3}-\frac{4\,a^2\,x^8}{663\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15*(a + b*x^4)^(1/4),x)
```

```
[Out] (a + b*x^4)^(1/4)*(x^16/17 - (128*a^4)/(3315*b^4) + (a*x^12)/(221*b) + (32*
a^3*x^4)/(3315*b^3) - (4*a^2*x^8)/(663*b^2))
```

sympy [A] time = 15.77, size = 110, normalized size = 1.38

$$\begin{cases} -\frac{128a^4\sqrt[4]{a+bx^4}}{3315b^4} + \frac{32a^3x^4\sqrt[4]{a+bx^4}}{3315b^3} - \frac{4a^2x^8\sqrt[4]{a+bx^4}}{663b^2} + \frac{ax^{12}\sqrt[4]{a+bx^4}}{221b} + \frac{x^{16}\sqrt[4]{a+bx^4}}{17} & \text{for } b \neq 0 \\ \frac{\sqrt[4]{a}\,x^{16}}{16} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15*(b*x**4+a)**(1/4),x)
```

```
[Out] Piecewise((-128*a**4*(a + b*x**4)**(1/4)/(3315*b**4) + 32*a**3*x**4*(a + b*
x**4)**(1/4)/(3315*b**3) - 4*a**2*x**8*(a + b*x**4)**(1/4)/(663*b**2) + a*x
**12*(a + b*x**4)**(1/4)/(221*b) + x**16*(a + b*x**4)**(1/4)/17, Ne(b, 0)),
 (a**(1/4)*x**16/16, True))
```

3.990 $\int x^{11}\sqrt[4]{a+bx^4}\,dx$

Optimal. Leaf size=59

$$\frac{a^2\left(a+bx^4\right)^{5/4}}{5b^3}+\frac{\left(a+bx^4\right)^{13/4}}{13b^3}-\frac{2a\left(a+bx^4\right)^{9/4}}{9b^3}$$

```
[Out] 1/5*a^2*(b*x^4+a)^(5/4)/b^3-2/9*a*(b*x^4+a)^(9/4)/b^3+1/13*(b*x^4+a)^(13/4)
/b^3
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{a^2\left(a+bx^4\right)^{5/4}}{5b^3}+\frac{\left(a+bx^4\right)^{13/4}}{13b^3}-\frac{2a\left(a+bx^4\right)^{9/4}}{9b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a + b*x^4)^(1/4),x]
```

```
[Out] (a^2*(a + b*x^4)^(5/4))/(5*b^3) - (2*a*(a + b*x^4)^(9/4))/(9*b^3) + (a + b*
x^4)^(13/4)/(13*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{11}\sqrt[4]{a+bx^4}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int x^2\sqrt[4]{a+bx}\,dx, x, x^4\right)\\ &= \frac{1}{4}\operatorname{Subst}\left(\int\left(\frac{a^2\sqrt[4]{a+bx}}{b^2}-\frac{2a(a+bx)^{5/4}}{b^2}+\frac{(a+bx)^{9/4}}{b^2}\right)dx, x, x^4\right)\\ &= \frac{a^2\left(a+bx^4\right)^{5/4}}{5b^3}-\frac{2a\left(a+bx^4\right)^{9/4}}{9b^3}+\frac{\left(a+bx^4\right)^{13/4}}{13b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.66

$$\frac{\left(a+bx^4\right)^{5/4}\left(32a^2-40abx^4+45b^2x^8\right)}{585b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*x^4)^(1/4),x]
```

```
[Out] ((a + b*x^4)^(5/4)*(32*a^2 - 40*a*b*x^4 + 45*b^2*x^8))/(585*b^3)
```

fricas [A] time = 0.47, size = 46, normalized size = 0.78

$$\frac{\left(45\,b^3x^{12}+5\,ab^2x^8-8\,a^2bx^4+32\,a^3\right)\left(bx^4+a\right)^{\frac{1}{4}}}{585\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/585*(45*b^3*x^12 + 5*a*b^2*x^8 - 8*a^2*b*x^4 + 32*a^3)*(b*x^4 + a)^(1/4)/
b^3
```

giac [A] time = 0.16, size = 43, normalized size = 0.73

$$\frac{45\left(bx^4+a\right)^{\frac{13}{4}}-130\left(bx^4+a\right)^{\frac{9}{4}}a+117\left(bx^4+a\right)^{\frac{5}{4}}a^2}{585\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] 1/585*(45*(b*x^4 + a)^(13/4) - 130*(b*x^4 + a)^(9/4)*a + 117*(b*x^4 + a)^(5
/4)*a^2)/b^3
```

maple [A] time = 0.01, size = 36, normalized size = 0.61

$$\frac{\left(b\,x^4+a\right)^{\frac{5}{4}}\left(45b^2x^8-40ab\,x^4+32a^2\right)}{585b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x^4+a)^(1/4),x)
```

```
[Out] 1/585*(b*x^4+a)^(5/4)*(45*b^2*x^8-40*a*b*x^4+32*a^2)/b^3
```

maxima [A] time = 1.37, size = 47, normalized size = 0.80

$$\frac{\left(bx^4+a\right)^{\frac{13}{4}}}{13\,b^3}-\frac{2\left(bx^4+a\right)^{\frac{9}{4}}a}{9\,b^3}+\frac{\left(bx^4+a\right)^{\frac{5}{4}}a^2}{5\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] 1/13*(b*x^4 + a)^(13/4)/b^3 - 2/9*(b*x^4 + a)^(9/4)*a/b^3 + 1/5*(b*x^4 + a)
^(5/4)*a^2/b^3
```

mupad [B] time = 1.12, size = 44, normalized size = 0.75

$$\left(b\,x^4+a\right)^{1/4}\left(\frac{x^{12}}{13}+\frac{32\,a^3}{585\,b^3}+\frac{a\,x^8}{117\,b}-\frac{8\,a^2\,x^4}{585\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a + b*x^4)^(1/4),x)
```

```
[Out] (a + b*x^4)^(1/4)*(x^12/13 + (32*a^3)/(585*b^3) + (a*x^8)/(117*b) - (8*a^2*
x^4)/(585*b^2))
```

sympy [**A**] time = 5.62, size = 87, normalized size = 1.47

$$\begin{cases} \frac{32a^3\sqrt[4]{a+bx^4}}{585b^3} - \frac{8a^2x^4\sqrt[4]{a+bx^4}}{585b^2} + \frac{ax^8\sqrt[4]{a+bx^4}}{117b} + \frac{x^{12}\sqrt[4]{a+bx^4}}{13} & \text{for } b \neq 0 \\ \frac{\sqrt[4]{a}\,x^{12}}{12} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x**4+a)**(1/4),x)
```

```
[Out] Piecewise((32*a**3*(a + b*x**4)**(1/4)/(585*b**3) - 8*a**2*x**4*(a + b*x**4
)**(1/4)/(585*b**2) + a*x**8*(a + b*x**4)**(1/4)/(117*b) + x**12*(a + b*x**
4)**(1/4)/13, Ne(b, 0)), (a**(1/4)*x**12/12, True))
```

3.991 $\int x^7 \sqrt[4]{a+bx^4}\, dx$

Optimal. Leaf size=38

$$\frac{\left(a+bx^4\right)^{9/4}}{9b^2} - \frac{a\left(a+bx^4\right)^{5/4}}{5b^2}$$

```
[Out] -1/5*a*(b*x^4+a)^(5/4)/b^2+1/9*(b*x^4+a)^(9/4)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{\left(a+bx^4\right)^{9/4}}{9b^2} - \frac{a\left(a+bx^4\right)^{5/4}}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^4)^(1/4),x]
```

```
[Out] -(a*(a + b*x^4)^(5/4))/(5*b^2) + (a + b*x^4)^(9/4)/(9*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^7 \sqrt[4]{a+bx^4}\, dx &= \frac{1}{4}\,\text{Subst}\left(\int x\sqrt[4]{a+bx}\, dx, x, x^4\right) \\
&= \frac{1}{4}\,\text{Subst}\left(\int \left(-\frac{a\sqrt[4]{a+bx}}{b} + \frac{(a+bx)^{5/4}}{b}\right) dx, x, x^4\right) \\
&= -\frac{a\left(a+bx^4\right)^{5/4}}{5b^2} + \frac{\left(a+bx^4\right)^{9/4}}{9b^2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 0.74

$$\frac{\left(a+bx^4\right)^{5/4}\left(5bx^4-4a\right)}{45b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^4)^(1/4),x]
```

```
[Out] ((a + b*x^4)^(5/4)*(-4*a + 5*b*x^4))/(45*b^2)
```

fricas [A] time = 0.87, size = 34, normalized size = 0.89

$$\frac{\left(5\,b^2x^8+abx^4-4\,a^2\right)\left(bx^4+a\right)^{\frac{1}{4}}}{45\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^4+a)^(1/4),x, algorithm="fricas")
[Out] 1/45*(5*b^2*x^8 + a*b*x^4 - 4*a^2)*(b*x^4 + a)^(1/4)/b^2
```

giac [A] time = 0.17, size = 29, normalized size = 0.76

$$\frac{5\left(bx^4+a\right)^{\frac{9}{4}}-9\left(bx^4+a\right)^{\frac{5}{4}}a}{45\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^4+a)^(1/4),x, algorithm="giac")
[Out] 1/45*(5*(b*x^4 + a)^(9/4) - 9*(b*x^4 + a)^(5/4)*a)/b^2
```

maple [A] time = 0.01, size = 25, normalized size = 0.66

$$-\frac{\left(b\,x^4+a\right)^{\frac{5}{4}}\left(-5b\,x^4+4a\right)}{45b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^4+a)^(1/4),x)
[Out] -1/45*(b*x^4+a)^(5/4)*(-5*b*x^4+4*a)/b^2
```

maxima [A] time = 1.36, size = 30, normalized size = 0.79

$$\frac{\left(bx^4+a\right)^{\frac{9}{4}}}{9\,b^2}-\frac{\left(bx^4+a\right)^{\frac{5}{4}}a}{5\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^4+a)^(1/4),x, algorithm="maxima")
[Out] 1/9*(b*x^4 + a)^(9/4)/b^2 - 1/5*(b*x^4 + a)^(5/4)*a/b^2
```

mupad [B] time = 1.10, size = 33, normalized size = 0.87

$$\left(b\,x^4+a\right)^{1/4}\left(\frac{x^8}{9}-\frac{4\,a^2}{45\,b^2}+\frac{a\,x^4}{45\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^4)^(1/4),x)
[Out] (a + b*x^4)^(1/4)*(x^8/9 - (4*a^2)/(45*b^2) + (a*x^4)/(45*b))
```

sympy [A] time = 2.88, size = 63, normalized size = 1.66

$$\begin{cases} -\frac{4a^2\sqrt[4]{a+bx^4}}{45b^2}+\frac{ax^4\sqrt[4]{a+bx^4}}{45b}+\frac{x^8\sqrt[4]{a+bx^4}}{9} & \text{for } b\neq 0 \\ \frac{\sqrt[4]{a}\,x^8}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**4+a)**(1/4),x)
```

```
[Out] Piecewise((-4*a**2*(a + b*x**4)**(1/4)/(45*b**2) + a*x**4*(a + b*x**4)**(1/
4)/(45*b) + x**8*(a + b*x**4)**(1/4)/9, Ne(b, 0)), (a**(1/4)*x**8/8, True))
```

3.992 $\int x^3 \sqrt[4]{a+bx^4}\, dx$

Optimal. Leaf size=18

$$\frac{\left(a+bx^4\right)^{5/4}}{5b}$$

```
[Out] 1/5*(b*x^4+a)^(5/4)/b
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$\frac{\left(a+bx^4\right)^{5/4}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^4)^(1/4),x]
```

```
[Out] (a + b*x^4)^(5/4)/(5*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^3 \sqrt[4]{a+bx^4}\, dx = \frac{\left(a+bx^4\right)^{5/4}}{5b}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{\left(a+bx^4\right)^{5/4}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^4)^(1/4),x]
```

```
[Out] (a + b*x^4)^(5/4)/(5*b)
```

fricas [A] time = 0.76, size = 14, normalized size = 0.78

$$\frac{\left(bx^4+a\right)^{\frac{5}{4}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/5*(b*x^4 + a)^(5/4)/b
```

giac [A] time = 0.17, size = 14, normalized size = 0.78

$$\frac{\left(bx^4+a\right)^{\frac{5}{4}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] 1/5*(b*x^4 + a)^(5/4)/b
```

maple [A] time = 0.01, size = 15, normalized size = 0.83

$$\frac{\left(b\,x^4+a\right)^{\frac{5}{4}}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^4+a)^(1/4),x)
```

```
[Out] 1/5*(b*x^4+a)^(5/4)/b
```

maxima [A] time = 1.34, size = 14, normalized size = 0.78

$$\frac{\left(bx^4+a\right)^{\frac{5}{4}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] 1/5*(b*x^4 + a)^(5/4)/b
```

mupad [B] time = 1.13, size = 14, normalized size = 0.78

$$\frac{\left(b\,x^4+a\right)^{5/4}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^4)^(1/4),x)
```

```
[Out] (a + b*x^4)^(5/4)/(5*b)
```

sympy [A] time = 0.90, size = 39, normalized size = 2.17

$$\begin{cases} \frac{a\sqrt[4]{a+bx^4}}{5b}+\frac{x^4\sqrt[4]{a+bx^4}}{5} & \text{for } b\neq 0 \\ \frac{\sqrt[4]{a}\,x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**4+a)**(1/4),x)
```

```
[Out] Piecewise((a*(a + b*x**4)**(1/4)/(5*b) + x**4*(a + b*x**4)**(1/4)/5, Ne(b,
0)), (a**(1/4)*x**4/4, True))
```

3.993 $\int \frac{\sqrt[4]{a+bx^4}}{x}\,dx$

Optimal. Leaf size=66

$$\sqrt[4]{a+bx^4} - \frac{1}{2}\sqrt[4]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right) - \frac{1}{2}\sqrt[4]{a}\,\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)$$

```
[Out] (b*x^4+a)^(1/4)-1/2*a^(1/4)*arctan((b*x^4+a)^(1/4)/a^(1/4))-1/2*a^(1/4)*arc
tanh((b*x^4+a)^(1/4)/a^(1/4))
```

Rubi [A] time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {266, 50, 63, 212, 206, 203}

$$\sqrt[4]{a+bx^4} - \frac{1}{2}\sqrt[4]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right) - \frac{1}{2}\sqrt[4]{a}\,\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(1/4)/x,x]
```

```
[Out] (a + b*x^4)^(1/4) - (a^(1/4)*ArcTan[(a + b*x^4)^(1/4)/a^(1/4)])/2 - (a^(1/4
)*ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)])/2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{a+bx^4}}{x}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{\sqrt[4]{a+bx}}{x}\,dx, x, x^4\right) \\
&= \sqrt[4]{a+bx^4} + \frac{1}{4}a\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/4}}\,dx, x, x^4\right) \\
&= \sqrt[4]{a+bx^4} + \frac{a\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^4}{b}}\,dx, x, \sqrt[4]{a+bx^4}\right)}{b} \\
&= \sqrt[4]{a+bx^4} - \frac{1}{2}\sqrt{a}\,\text{Subst}\left(\int \frac{1}{\sqrt{a}-x^2}\,dx, x, \sqrt[4]{a+bx^4}\right) - \frac{1}{2}\sqrt{a}\,\text{Subst}\left(\int \frac{1}{\sqrt{a}+x^2}\,dx, x, \sqrt[4]{a+\right. \\
&= \sqrt[4]{a+bx^4} - \frac{1}{2}\sqrt[4]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right) - \frac{1}{2}\sqrt[4]{a}\,\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 1.00

$$\sqrt[4]{a+bx^4} - \frac{1}{2}\sqrt[4]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right) - \frac{1}{2}\sqrt[4]{a}\,\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(1/4)/x,x]
```

```
[Out] (a + b*x^4)^(1/4) - (a^(1/4)*ArcTan[(a + b*x^4)^(1/4)/a^(1/4)])/2 - (a^(1/4
)*ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)])/2
```

fricas [A] time = 0.83, size = 93, normalized size = 1.41

$$a^{\frac{1}{4}}\arctan\left(\frac{a^{\frac{3}{4}}\sqrt{\sqrt{bx^4+a}+\sqrt{a}} - \left(bx^4+a\right)^{\frac{1}{4}}a^{\frac{3}{4}}}{a}\right) - \frac{1}{4}a^{\frac{1}{4}}\log\left(\left(bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}\right) + \frac{1}{4}a^{\frac{1}{4}}\log\left(\left(bx^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}\right) + \left(bx^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x,x, algorithm="fricas")
```

```
[Out] a^(1/4)*arctan((a^(3/4)*sqrt(sqrt(b*x^4 + a) + sqrt(a)) - (b*x^4 + a)^(1/4)
*a^(3/4))/a) - 1/4*a^(1/4)*log((b*x^4 + a)^(1/4) + a^(1/4)) + 1/4*a^(1/4)*l
og((b*x^4 + a)^(1/4) - a^(1/4)) + (b*x^4 + a)^(1/4)
```

giac [B] time = 0.16, size = 183, normalized size = 2.77

$$-\frac{1}{4}\sqrt{2}\,(-a)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}+2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right) - \frac{1}{4}\sqrt{2}\,(-a)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}-2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right) - \frac{1}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(-a)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x^4 +
 a)^(1/4))/(-a)^(1/4)) - 1/4*sqrt(2)*(-a)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2
)*(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4)) - 1/8*sqrt(2)*(-a)^(1/4)*lo
g(sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt(-a)) + 1/8*
sqrt(2)*(-a)^(1/4)*log(-sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 +
 a) + sqrt(-a)) + (b*x^4 + a)^(1/4)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{1}{4}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(1/4)/x,x)
```

```
[Out] int((b*x^4+a)^(1/4)/x,x)
```

maxima [A] time = 2.92, size = 66, normalized size = 1.00

$$-\frac{1}{2}\,a^{\frac{1}{4}}\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)+\frac{1}{4}\,a^{\frac{1}{4}}\log\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{\left(bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)+\left(bx^4+a\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x,x, algorithm="maxima")
```

```
[Out] -1/2*a^(1/4)*arctan((b*x^4 + a)^(1/4)/a^(1/4)) + 1/4*a^(1/4)*log(((b*x^4 +
a)^(1/4) - a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/4))) + (b*x^4 + a)^(1/4)
```

mupad [B] time = 1.18, size = 48, normalized size = 0.73

$$\left(b\,x^4+a\right)^{1/4}-\frac{a^{1/4}\,\operatorname{atanh}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)}{2}-\frac{a^{1/4}\,\operatorname{atan}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(1/4)/x,x)
```

```
[Out] (a + b*x^4)^(1/4) - (a^(1/4)*atanh((a + b*x^4)^(1/4)/a^(1/4)))/2 - (a^(1/4)
*atan((a + b*x^4)^(1/4)/a^(1/4)))/2
```

sympy [C] time = 2.25, size = 42, normalized size = 0.64

$$-\frac{\sqrt[4]{b}\,x\,\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{4},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\right|\frac{ae^{i\pi}}{bx^4}\right)}{4\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(1/4)/x,x)
```

```
[Out] -b**(1/4)*x*gamma(-1/4)*hyper((-1/4, -1/4), (3/4,), a*exp_polar(I*pi)/(b*x*
*4))/(4*gamma(3/4))
```

3.994 $\int \frac{\sqrt[4]{a+bx^4}}{x^5}\,dx$

Optimal. Leaf size=75

$$-\frac{b\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{3/4}}-\frac{b\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{4x^4}$$

```
[Out] -1/4*(b*x^4+a)^(1/4)/x^4-1/8*b*arctan((b*x^4+a)^(1/4)/a^(1/4))/a^(3/4)-1/8*
b*arctanh((b*x^4+a)^(1/4)/a^(1/4))/a^(3/4)
```

Rubi [A] time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {266, 47, 63, 212, 206, 203}

$$-\frac{b\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{3/4}}-\frac{b\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(1/4)/x^5,x]
```

```
[Out] -(a + b*x^4)^(1/4)/(4*x^4) - (b*ArcTan[(a + b*x^4)^(1/4)/a^(1/4)])/(8*a^(3/
4)) - (b*ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)])/(8*a^(3/4))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{a+bx^4}}{x^5}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{\sqrt[4]{a+bx}}{x^2}\,dx, x, x^4\right) \\
&= -\frac{\sqrt[4]{a+bx^4}}{4x^4} + \frac{1}{16}b\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/4}}\,dx, x, x^4\right) \\
&= -\frac{\sqrt[4]{a+bx^4}}{4x^4} + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^4}{b}}\,dx, x, \sqrt[4]{a+bx^4}\right) \\
&= -\frac{\sqrt[4]{a+bx^4}}{4x^4} - \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a}-x^2}\,dx, x, \sqrt[4]{a+bx^4}\right)}{8\sqrt{a}} - \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a}+x^2}\,dx, x, \sqrt[4]{a+bx^4}\right)}{8\sqrt{a}} \\
&= -\frac{\sqrt[4]{a+bx^4}}{4x^4} - \frac{b\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{3/4}} - \frac{b\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 37, normalized size = 0.49

$$\frac{b\left(a+bx^4\right)^{5/4}\,{}_2F_1\left(\frac{5}{4},2;\frac{9}{4};\frac{bx^4}{a}+1\right)}{5a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(1/4)/x^5,x]
```

```
[Out] (b*(a + b*x^4)^(5/4)*Hypergeometric2F1[5/4, 2, 9/4, 1 + (b*x^4)/a])/(5*a^2)
```

fricas [B] time = 0.71, size = 180, normalized size = 2.40

$$\frac{4\left(\frac{b^4}{a^3}\right)^{\frac{1}{4}}x^4\arctan\left(-\frac{\left(bx^4+a\right)^{\frac{1}{4}}a^2b\left(\frac{b^4}{a^3}\right)^{\frac{3}{4}}-\sqrt{\sqrt{bx^4+a}\,b^2+a^2\sqrt{\frac{b^4}{a^3}}}\,a^2\left(\frac{b^4}{a^3}\right)^{\frac{3}{4}}}{b^4}\right)-\left(\frac{b^4}{a^3}\right)^{\frac{1}{4}}x^4\log\left(\left(bx^4+a\right)^{\frac{1}{4}}b+a\left(\frac{b^4}{a^3}\right)^{\frac{1}{4}}\right)+\left(\frac{b^4}{a^3}\right)^{\frac{1}{4}}}{16\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^5,x, algorithm="fricas")
```

```
[Out] 1/16*(4*(b^4/a^3)^(1/4)*x^4*arctan(-((b*x^4 + a)^(1/4)*a^2*b*(b^4/a^3)^(3/4
) - sqrt(sqrt(b*x^4 + a)*b^2 + a^2*sqrt(b^4/a^3))*a^2*(b^4/a^3)^(3/4))/b^4)
 - (b^4/a^3)^(1/4)*x^4*log((b*x^4 + a)^(1/4)*b + a*(b^4/a^3)^(1/4)) + (b^4/
a^3)^(1/4)*x^4*log((b*x^4 + a)^(1/4)*b - a*(b^4/a^3)^(1/4)) - 4*(b*x^4 + a)
^(1/4))/x^4
```

giac [B] time = 0.18, size = 207, normalized size = 2.76

$$\frac{2\sqrt{2}\,b^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{3}{4}}}+\frac{2\sqrt{2}\,b^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{3}{4}}}+\frac{\sqrt{2}\,b^2\log\left(\sqrt{2}\left(bx^4+a\right)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^4+a}+\sqrt{-a}\right)}{(-a)^{\frac{3}{4}}}+\frac{\sqrt{2}}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^5,x, algorithm="giac")
```

```
[Out] 1/32*(2*sqrt(2)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x^4 + a)^
(1/4))/(-a)^(1/4))/(-a)^(3/4) + 2*sqrt(2)*b^2*arctan(-1/2*sqrt(2)*(sqrt(2)*
(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4))/(-a)^(3/4) + sqrt(2)*b^2*log(
sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt(-a))/(-a)^(3/
4) + sqrt(2)*(-a)^(1/4)*b^2*log(-sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqr
t(b*x^4 + a) + sqrt(-a))/a - 8*(b*x^4 + a)^(1/4)*b/x^4)/b
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{1}{4}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(1/4)/x^5,x)
```

```
[Out] int((b*x^4+a)^(1/4)/x^5,x)
```

maxima [A] time = 3.02, size = 73, normalized size = 0.97

$$-\frac{b\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{8\,a^{\frac{3}{4}}}+\frac{b\log\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{\left(bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{16\,a^{\frac{3}{4}}}-\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^5,x, algorithm="maxima")
```

```
[Out] -1/8*b*arctan((b*x^4 + a)^(1/4)/a^(1/4))/a^(3/4) + 1/16*b*log(((b*x^4 + a)^
(1/4) - a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/4)))/a^(3/4) - 1/4*(b*x^4 + a)^(
1/4)/x^4
```

mupad [B] time = 1.28, size = 55, normalized size = 0.73

$$-\frac{\left(b\,x^4+a\right)^{1/4}}{4\,x^4}-\frac{b\,\mathrm{atan}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)}{8\,a^{3/4}}-\frac{b\,\mathrm{atanh}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)}{8\,a^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(1/4)/x^5,x)
```

```
[Out] - (a + b*x^4)^(1/4)/(4*x^4) - (b*atan((a + b*x^4)^(1/4)/a^(1/4)))/(8*a^(3/4
)) - (b*atanh((a + b*x^4)^(1/4)/a^(1/4)))/(8*a^(3/4))
```

sympy [C] time = 2.86, size = 41, normalized size = 0.55

$$-\frac{\sqrt[4]{b}\,\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{4}, \frac{3}{4} \\ \frac{7}{4}\end{matrix}\right| \frac{ae^{i\pi}}{bx^4}\right)}{4x^3\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(1/4)/x**5,x)
```

```
[Out] -b**(1/4)*gamma(3/4)*hyper((-1/4, 3/4), (7/4,), a*exp_polar(I*pi)/(b*x**4))
/(4*x**3*gamma(7/4))
```

3.995 $\int \frac{\sqrt[4]{a+bx^4}}{x^9}\,dx$

Optimal. Leaf size=101

$$\frac{3b^2\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{7/4}}+\frac{3b^2\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{7/4}}-\frac{b\sqrt[4]{a+bx^4}}{32ax^4}-\frac{\sqrt[4]{a+bx^4}}{8x^8}$$

```
[Out] -1/8*(b*x^4+a)^(1/4)/x^8-1/32*b*(b*x^4+a)^(1/4)/a/x^4+3/64*b^2*arctan((b*x^
4+a)^(1/4)/a^(1/4))/a^(7/4)+3/64*b^2*arctanh((b*x^4+a)^(1/4)/a^(1/4))/a^(7/
4)
```

Rubi [A] time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {266, 47, 51, 63, 212, 206, 203}

$$\frac{3b^2\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{7/4}}+\frac{3b^2\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{7/4}}-\frac{b\sqrt[4]{a+bx^4}}{32ax^4}-\frac{\sqrt[4]{a+bx^4}}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(1/4)/x^9,x]
```

```
[Out] -(a + b*x^4)^(1/4)/(8*x^8) - (b*(a + b*x^4)^(1/4))/(32*a*x^4) + (3*b^2*ArcT
an[(a + b*x^4)^(1/4)/a^(1/4)])/(64*a^(7/4)) + (3*b^2*ArcTanh[(a + b*x^4)^(1
/4)/a^(1/4)])/(64*a^(7/4))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx^4}}{x^9}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{\sqrt[4]{a+bx}}{x^3}\,dx, x, x^4\right)$$

$$= -\frac{\sqrt[4]{a+bx^4}}{8x^8} + \frac{1}{32}b\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/4}}\,dx, x, x^4\right)$$

$$= -\frac{\sqrt[4]{a+bx^4}}{8x^8} - \frac{b\sqrt[4]{a+bx^4}}{32ax^4} - \frac{\left(3b^2\right)\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/4}}\,dx, x, x^4\right)}{128a}$$

$$= -\frac{\sqrt[4]{a+bx^4}}{8x^8} - \frac{b\sqrt[4]{a+bx^4}}{32ax^4} - \frac{(3b)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^4}{b}}\,dx, x, \sqrt[4]{a+bx^4}\right)}{32a}$$

$$= -\frac{\sqrt[4]{a+bx^4}}{8x^8} - \frac{b\sqrt[4]{a+bx^4}}{32ax^4} + \frac{\left(3b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a}-x^2}\,dx, x, \sqrt[4]{a+bx^4}\right)}{64a^{3/2}} + \frac{\left(3b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a}+x}\right.}{64a}$$

$$= -\frac{\sqrt[4]{a+bx^4}}{8x^8} - \frac{b\sqrt[4]{a+bx^4}}{32ax^4} + \frac{3b^2\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{7/4}} + \frac{3b^2\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{7/4}}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.39

$$-\frac{b^2\left(a+bx^4\right)^{5/4}\,{}_2F_1\left(\frac{5}{4}, 3; \frac{9}{4}; \frac{bx^4}{a}+1\right)}{5a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(1/4)/x^9,x]
```

```
[Out] -1/5*(b^2*(a + b*x^4)^(5/4)*Hypergeometric2F1[5/4, 3, 9/4, 1 + (b*x^4)/a])/
a^3
```

fricas [B] time = 0.69, size = 209, normalized size = 2.07

$$-\frac{12\,a\left(\frac{b^8}{a^7}\right)^{\frac{1}{4}}x^8\arctan\left(-\frac{\left(bx^4+a\right)^{\frac{1}{4}}a^5b^2\left(\frac{b^8}{a^7}\right)^{\frac{3}{4}}-\sqrt{\sqrt{bx^4+a}\,b^4+a^4\sqrt{\frac{b^8}{a^7}}}\,a^5\left(\frac{b^8}{a^7}\right)^{\frac{3}{4}}}{b^8}\right)-3\,a\left(\frac{b^8}{a^7}\right)^{\frac{1}{4}}x^8\log\left(3\left(bx^4+a\right)^{\frac{1}{4}}b^2+3\,a^2\left(\right.\right.}{128\,ax^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^9,x, algorithm="fricas")
```

```
[Out] -1/128*(12*a*(b^8/a^7)^(1/4)*x^8*arctan(-((b*x^4 + a)^(1/4)*a^5*b^2*(b^8/a^
7)^(3/4) - sqrt(sqrt(b*x^4 + a)*b^4 + a^4*sqrt(b^8/a^7))*a^5*(b^8/a^7)^(3/4
))/b^8) - 3*a*(b^8/a^7)^(1/4)*x^8*log(3*(b*x^4 + a)^(1/4)*b^2 + 3*a^2*(b^8/
a^7)^(1/4)) + 3*a*(b^8/a^7)^(1/4)*x^8*log(3*(b*x^4 + a)^(1/4)*b^2 - 3*a^2*(
b^8/a^7)^(1/4)) + 4*(b*x^4 + 4*a)*(b*x^4 + a)^(1/4))/(a*x^8)
```

giac [B] time = 0.18, size = 243, normalized size = 2.41

$$\frac{6\sqrt{2}(-a)^{\frac{1}{4}}b^3\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a^2}+\frac{6\sqrt{2}(-a)^{\frac{1}{4}}b^3\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a^2}+\frac{3\sqrt{2}(-a)^{\frac{1}{4}}b^3\log\left(\sqrt{2}\left(bx^4+a\right)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^4}\right.}{a^2}$$

$$256\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^9,x, algorithm="giac")
```

```
[Out] 1/256*(6*sqrt(2)*(-a)^(1/4)*b^3*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*
(b*x^4 + a)^(1/4))/(-a)^(1/4))/a^2 + 6*sqrt(2)*(-a)^(1/4)*b^3*arctan(-1/2*s
qrt(2)*(sqrt(2)*(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4))/a^2 + 3*sqrt(
2)*(-a)^(1/4)*b^3*log(sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a
) + sqrt(-a))/a^2 + 3*sqrt(2)*b^3*log(-sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4)
 + sqrt(b*x^4 + a) + sqrt(-a))/((-a)^(3/4)*a) - 8*((b*x^4 + a)^(5/4)*b^3 +
3*(b*x^4 + a)^(1/4)*a*b^3)/(a*b^2*x^8))/b
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{1}{4}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(1/4)/x^9,x)
```

```
[Out] int((b*x^4+a)^(1/4)/x^9,x)
```

maxima [A] time = 2.93, size = 129, normalized size = 1.28

$$-\frac{\left(bx^4+a\right)^{\frac{5}{4}}b^2+3\left(bx^4+a\right)^{\frac{1}{4}}ab^2}{32\left(\left(bx^4+a\right)^2a-2\left(bx^4+a\right)a^2+a^3\right)}+\frac{3\left(\frac{2\,b^2\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{3}{4}}}-\frac{b^2\log\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{\left(bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{3}{4}}}\right)}{128\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^9,x, algorithm="maxima")
```

```
[Out] -1/32*((b*x^4 + a)^(5/4)*b^2 + 3*(b*x^4 + a)^(1/4)*a*b^2)/((b*x^4 + a)^2*a
- 2*(b*x^4 + a)*a^2 + a^3) + 3/128*(2*b^2*arctan((b*x^4 + a)^(1/4)/a^(1/4))
/a^(3/4) - b^2*log(((b*x^4 + a)^(1/4) - a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/
4)))/a^(3/4))/a
```

mupad [B] time = 1.39, size = 79, normalized size = 0.78

$$\frac{3\,b^2\,\mathrm{atan}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)}{64\,a^{7/4}}-\frac{3\left(b\,x^4+a\right)^{1/4}}{32\,x^8}-\frac{\left(b\,x^4+a\right)^{5/4}}{32\,a\,x^8}-\frac{b^2\,\mathrm{atan}\left(\frac{\left(b\,x^4+a\right)^{1/4}\,1\mathrm{i}}{a^{1/4}}\right)\,3\mathrm{i}}{64\,a^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(1/4)/x^9,x)
```

```
[Out] (3*b^2*atan((a + b*x^4)^(1/4)/a^(1/4)))/(64*a^(7/4)) - (3*(a + b*x^4)^(1/4)
)/(32*x^8) - (b^2*atan(((a + b*x^4)^(1/4)*1i)/a^(1/4))*3i)/(64*a^(7/4)) - (
a + b*x^4)^(5/4)/(32*a*x^8)
```

sympy [C] time = 2.66, size = 41, normalized size = 0.41

$$-\frac{\sqrt[4]{b}\,\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{a e^{i\pi}}{b x^4}\right)}{4x^7\,\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(1/4)/x**9,x)
```

```
[Out] -b**(1/4)*gamma(7/4)*hyper((-1/4, 7/4), (11/4,), a*exp_polar(I*pi)/(b*x**4)
)/(4*x**7*gamma(11/4))
```

3.996 $\int x^9 \sqrt[4]{a+bx^4}\,dx$

Optimal. Leaf size=125

$$\frac{4a^{7/2}\left(\frac{bx^4}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{77b^{5/2}\left(a+bx^4\right)^{3/4}}-\frac{2a^2x^2\sqrt[4]{a+bx^4}}{77b^2}+\frac{1}{11}x^{10}\sqrt[4]{a+bx^4}+\frac{ax^6\sqrt[4]{a+bx^4}}{77b}$$

```
[Out] -2/77*a^2*x^2*(b*x^4+a)^(1/4)/b^2+1/77*a*x^6*(b*x^4+a)^(1/4)/b+1/11*x^10*(b
*x^4+a)^(1/4)+4/77*a^(7/2)*(1+b*x^4/a)^(3/4)*(cos(1/2*arctan(x^2*b^(1/2)/a^
(1/2)))^2)^(1/2)/cos(1/2*arctan(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arc
tan(x^2*b^(1/2)/a^(1/2))),2^(1/2))/b^(5/2)/(b*x^4+a)^(3/4)
```

Rubi [A] time = 0.08, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {275, 279, 321, 233, 231}

$$-\frac{2a^2x^2\sqrt[4]{a+bx^4}}{77b^2}+\frac{4a^{7/2}\left(\frac{bx^4}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{77b^{5/2}\left(a+bx^4\right)^{3/4}}+\frac{1}{11}x^{10}\sqrt[4]{a+bx^4}+\frac{ax^6\sqrt[4]{a+bx^4}}{77b}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a + b*x^4)^(1/4),x]
```

```
[Out] (-2*a^2*x^2*(a + b*x^4)^(1/4))/(77*b^2) + (a*x^6*(a + b*x^4)^(1/4))/(77*b)
+ (x^10*(a + b*x^4)^(1/4))/11 + (4*a^(7/2)*(1 + (b*x^4)/a)^(3/4)*EllipticF[
ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(77*b^(5/2)*(a + b*x^4)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^9 \sqrt[4]{a+bx^4}\,dx = \frac{1}{2}\text{Subst}\left(\int x^4 \sqrt[4]{a+bx^2}\,dx, x, x^2\right)$$

$$= \frac{1}{11}x^{10}\sqrt[4]{a+bx^4} + \frac{1}{22}a\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^2\right)^{3/4}}\,dx, x, x^2\right)$$

$$= \frac{ax^6\sqrt[4]{a+bx^4}}{77b} + \frac{1}{11}x^{10}\sqrt[4]{a+bx^4} - \frac{\left(3a^2\right)\text{Subst}\left(\int \frac{x^2}{\left(a+bx^2\right)^{3/4}}\,dx, x, x^2\right)}{77b}$$

$$= -\frac{2a^2x^2\sqrt[4]{a+bx^4}}{77b^2} + \frac{ax^6\sqrt[4]{a+bx^4}}{77b} + \frac{1}{11}x^{10}\sqrt[4]{a+bx^4} + \frac{\left(2a^3\right)\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx, x, x^2\right)}{77b^2}$$

$$= -\frac{2a^2x^2\sqrt[4]{a+bx^4}}{77b^2} + \frac{ax^6\sqrt[4]{a+bx^4}}{77b} + \frac{1}{11}x^{10}\sqrt[4]{a+bx^4} + \frac{\left(2a^3\left(1+\frac{bx^4}{a}\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\right.}{77b^2\left(a+bx^4\right)^{3/4}}$$

$$= -\frac{2a^2x^2\sqrt[4]{a+bx^4}}{77b^2} + \frac{ax^6\sqrt[4]{a+bx^4}}{77b} + \frac{1}{11}x^{10}\sqrt[4]{a+bx^4} + \frac{4a^{7/2}\left(1+\frac{bx^4}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right.\right.}{77b^{5/2}\left(a+bx^4\right)^{3/4}}$$

Mathematica [C] time = 0.08, size = 95, normalized size = 0.76

$$\frac{x^2\sqrt[4]{a+bx^4}\left(\sqrt[4]{\frac{bx^4}{a}+1}\left(-6a^2+abx^4+7b^2x^8\right)+6a^2\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^4}{a}\right)\right)}{77b^2\sqrt[4]{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a + b*x^4)^(1/4),x]
```

```
[Out] (x^2*(a + b*x^4)^(1/4)*((1 + (b*x^4)/a)^(1/4)*(-6*a^2 + a*b*x^4 + 7*b^2*x^8
) + 6*a^2*Hypergeometric2F1[-1/4, 1/2, 3/2, -((b*x^4)/a)]))/(77*b^2*(1 + (b
*x^4)/a)^(1/4))
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^4+a\right)^{\frac{1}{4}}x^9, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(1/4)*x^9, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4+a\right)^{\frac{1}{4}}x^9\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^4+a)^(1/4),x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(1/4)*x^9, x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \left(b\,x^4+a\right)^{\frac{1}{4}} x^9\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b*x^4+a)^(1/4),x)
[Out] int(x^9*(b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4+a\right)^{\frac{1}{4}} x^9\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^4+a)^(1/4),x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(1/4)*x^9, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^9\left(b\,x^4+a\right)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(a + b*x^4)^(1/4),x)
[Out] int(x^9*(a + b*x^4)^(1/4), x)
```

sympy [C] time = 1.78, size = 29, normalized size = 0.23

$$\frac{\sqrt[4]{a}\,x^{10}\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{4}, \frac{5}{2} \\ \frac{7}{2}\end{matrix}\right| \frac{bx^4 e^{i\pi}}{a}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b*x**4+a)**(1/4),x)
[Out] a**(1/4)*x**10*hyper((-1/4, 5/2), (7/2,), b*x**4*exp_polar(I*pi)/a)/10
```

3.997 $\int x^5 \sqrt[4]{a+bx^4}\,dx$

Optimal. Leaf size=101

$$-\frac{2a^{5/2}\left(\frac{bx^4}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{21b^{3/2}\left(a+bx^4\right)^{3/4}}+\frac{1}{7}x^6\sqrt[4]{a+bx^4}+\frac{ax^2\sqrt[4]{a+bx^4}}{21b}$$

```
[Out] 1/21*a*x^2*(b*x^4+a)^(1/4)/b+1/7*x^6*(b*x^4+a)^(1/4)-2/21*a^(5/2)*(1+b*x^4/
a)^(3/4)*(cos(1/2*arctan(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x^2*
b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arctan(x^2*b^(1/2)/a^(1/2))),2^(1/2))/b
^(3/2)/(b*x^4+a)^(3/4)
```

Rubi [A] time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {275, 279, 321, 233, 231}

$$-\frac{2a^{5/2}\left(\frac{bx^4}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{21b^{3/2}\left(a+bx^4\right)^{3/4}}+\frac{1}{7}x^6\sqrt[4]{a+bx^4}+\frac{ax^2\sqrt[4]{a+bx^4}}{21b}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^4)^(1/4),x]
```

```
[Out] (a*x^2*(a + b*x^4)^(1/4))/(21*b) + (x^6*(a + b*x^4)^(1/4))/7 - (2*a^(5/2)*(
1 + (b*x^4)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(21*b^(
3/2)*(a + b*x^4)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int x^5 \sqrt[4]{a+bx^4}\, dx &= \frac{1}{2}\,\text{Subst}\left(\int x^2 \sqrt[4]{a+bx^2}\, dx, x, x^2\right) \\
&= \frac{1}{7}x^6\sqrt[4]{a+bx^4} + \frac{1}{14}a\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^2\right)^{3/4}}\, dx, x, x^2\right) \\
&= \frac{ax^2\sqrt[4]{a+bx^4}}{21b} + \frac{1}{7}x^6\sqrt[4]{a+bx^4} - \frac{a^2\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{3/4}}\, dx, x, x^2\right)}{21b} \\
&= \frac{ax^2\sqrt[4]{a+bx^4}}{21b} + \frac{1}{7}x^6\sqrt[4]{a+bx^4} - \frac{\left(a^2\left(1+\frac{bx^4}{a}\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\, dx, x, x^2\right)}{21b\left(a+bx^4\right)^{3/4}} \\
&= \frac{ax^2\sqrt[4]{a+bx^4}}{21b} + \frac{1}{7}x^6\sqrt[4]{a+bx^4} - \frac{2a^{5/2}\left(1+\frac{bx^4}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle| 2\right)}{21b^{3/2}\left(a+bx^4\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.09, size = 64, normalized size = 0.63

$$\frac{x^2\sqrt[4]{a+bx^4}\left(-\frac{a\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^4}{a}\right)}{\sqrt[4]{\frac{bx^4}{a}+1}} + a + bx^4\right)}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^4)^(1/4),x]
```

```
[Out] (x^2*(a + b*x^4)^(1/4)*(a + b*x^4 - (a*Hypergeometric2F1[-1/4, 1/2, 3/2, -(
(b*x^4)/a)])/(1 + (b*x^4)/a)^(1/4)))/(7*b)
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^4+a\right)^{\frac{1}{4}}x^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(1/4)*x^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4+a\right)^{\frac{1}{4}}x^5\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(1/4)*x^5, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \left(b\,x^4+a\right)^{\frac{1}{4}}\,x^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^4+a)^(1/4),x)
```

```
[Out] int(x^5*(b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4+a\right)^{\frac{1}{4}}x^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(1/4)*x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5\left(b\,x^4+a\right)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^4)^(1/4),x)
```

```
[Out] int(x^5*(a + b*x^4)^(1/4), x)
```

sympy [C] time = 1.32, size = 29, normalized size = 0.29

$$\frac{\sqrt[4]{a}\,x^6\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**4+a)**(1/4),x)
```

```
[Out] a**(1/4)*x**6*hyper((-1/4, 3/2), (5/2,), b*x**4*exp_polar(I*pi)/a)/6
```

3.998 $\int x\sqrt[4]{a+bx^4}\,dx$

Optimal. Leaf size=79

$$\frac{a^{3/2}\left(\frac{bx^4}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{b}\left(a+bx^4\right)^{3/4}}+\frac{1}{3}x^2\sqrt[4]{a+bx^4}$$

```
[Out] 1/3*x^2*(b*x^4+a)^(1/4)+1/3*a^(3/2)*(1+b*x^4/a)^(3/4)*(cos(1/2*arctan(x^2*b
^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x^2*b^(1/2)/a^(1/2)))*EllipticF(si
n(1/2*arctan(x^2*b^(1/2)/a^(1/2))),2^(1/2))/(b*x^4+a)^(3/4)/b^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {275, 195, 233, 231}

$$\frac{a^{3/2}\left(\frac{bx^4}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{b}\left(a+bx^4\right)^{3/4}}+\frac{1}{3}x^2\sqrt[4]{a+bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^4)^(1/4),x]
```

```
[Out] (x^2*(a + b*x^4)^(1/4))/3 + (a^(3/2)*(1 + (b*x^4)/a)^(3/4)*EllipticF[ArcTan
[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(3*Sqrt[b]*(a + b*x^4)^(3/4))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int x\sqrt[4]{a+bx^4}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \sqrt[4]{a+bx^2}\,dx, x, x^2\right)$$
$$= \frac{1}{3}x^2\sqrt[4]{a+bx^4} + \frac{1}{6}a\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx, x, x^2\right)$$
$$= \frac{1}{3}x^2\sqrt[4]{a+bx^4} + \frac{\left(a\left(1+\frac{bx^4}{a}\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx, x, x^2\right)}{6\left(a+bx^4\right)^{3/4}}$$
$$= \frac{1}{3}x^2\sqrt[4]{a+bx^4} + \frac{a^{3/2}\left(1+\frac{bx^4}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle| 2\right)}{3\sqrt{b}\,\left(a+bx^4\right)^{3/4}}$$

Mathematica [C] time = 0.02, size = 51, normalized size = 0.65

$$\frac{x^2\sqrt[4]{a+bx^4}\;{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^4}{a}\right)}{2\sqrt[4]{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^4)^(1/4),x]
```

```
[Out] (x^2*(a + b*x^4)^(1/4)*Hypergeometric2F1[-1/4, 1/2, 3/2, -((b*x^4)/a)])/(2*
(1 + (b*x^4)/a)^(1/4))
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^4+a\right)^{\frac{1}{4}}x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(1/4)*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4+a\right)^{\frac{1}{4}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(1/4)*x, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \left(b\,x^4+a\right)^{\frac{1}{4}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^4+a)^(1/4),x)
```

```
[Out] int(x*(b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4 + a\right)^{\frac{1}{4}} x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(1/4)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \left(b\,x^4 + a\right)^{1/4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^4)^(1/4),x)
```

```
[Out] int(x*(a + b*x^4)^(1/4), x)
```

sympy [C] time = 1.85, size = 29, normalized size = 0.37

$$\frac{\sqrt[4]{a}\, x^2 {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**4+a)**(1/4),x)
```

```
[Out] a**(1/4)*x**2*hyper((-1/4, 1/2), (3/2,), b*x**4*exp_polar(I*pi)/a)/2
```

3.999 $\int \frac{\sqrt[4]{a+bx^4}}{x^3}\,dx$

Optimal. Leaf size=79

$$\frac{\sqrt{a}\,\sqrt{b}\left(\frac{bx^4}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{2\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{2x^2}$$

```
[Out] -1/2*(b*x^4+a)^(1/4)/x^2+1/2*(1+b*x^4/a)^(3/4)*(cos(1/2*arctan(x^2*b^(1/2)/
a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*a
rctan(x^2*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)*b^(1/2)/(b*x^4+a)^(3/4)
```

Rubi [A] time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {275, 277, 233, 231}

$$\frac{\sqrt{a}\,\sqrt{b}\left(\frac{bx^4}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{2\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(1/4)/x^3,x]
```

```
[Out] -(a + b*x^4)^(1/4)/(2*x^2) + (Sqrt[a]*Sqrt[b]*(1 + (b*x^4)/a)^(3/4)*Ellipti
cF[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(2*(a + b*x^4)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx^4}}{x^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt[4]{a+bx^2}}{x^2}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt[4]{a+bx^4}}{2x^2} + \frac{1}{4}b\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt[4]{a+bx^4}}{2x^2} + \frac{\left(b\left(1+\frac{bx^4}{a}\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx, x, x^2\right)}{4\left(a+bx^4\right)^{3/4}}$$
$$= -\frac{\sqrt[4]{a+bx^4}}{2x^2} + \frac{\sqrt{a}\,\sqrt{b}\,\left(1+\frac{bx^4}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle| 2\right)}{2\left(a+bx^4\right)^{3/4}}$$

Mathematica [C] time = 0.02, size = 51, normalized size = 0.65

$$-\frac{\sqrt[4]{a+bx^4}\;{}_2F_1\left(-\frac{1}{2}, -\frac{1}{4}; \frac{1}{2}; -\frac{bx^4}{a}\right)}{2x^2\sqrt[4]{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(1/4)/x^3,x]
```

```
[Out] -1/2*((a + b*x^4)^(1/4)*Hypergeometric2F1[-1/2, -1/4, 1/2, -((b*x^4)/a)])/(
x^2*(1 + (b*x^4)/a)^(1/4))
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(1/4)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(1/4)/x^3, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{1}{4}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(1/4)/x^3,x)
[Out] int((b*x^4+a)^(1/4)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^3,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(1/4)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^4+a\right)^{1/4}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(1/4)/x^3,x)
[Out] int((a + b*x^4)^(1/4)/x^3, x)
```

sympy [C] time = 1.10, size = 32, normalized size = 0.41

$$-\frac{\sqrt[4]{a}\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2}, -\frac{1}{4} \\ \frac{1}{2}\end{matrix}\right| \frac{bx^4 e^{i\pi}}{a}\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(1/4)/x**3,x)
[Out] -a**(1/4)*hyper((-1/2, -1/4), (1/2,), b*x**4*exp_polar(I*pi)/a)/(2*x**2)
```

3.1000 $\int \frac{\sqrt[4]{a+bx^4}}{x^7}\,dx$

Optimal. Leaf size=101

$$-\frac{b^{3/2}\left(\frac{bx^4}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{12\sqrt{a}\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{6x^6}-\frac{b\sqrt[4]{a+bx^4}}{12ax^2}$$

```
[Out] -1/6*(b*x^4+a)^(1/4)/x^6-1/12*b*(b*x^4+a)^(1/4)/a/x^2-1/12*b^(3/2)*(1+b*x^4
/a)^(3/4)*(cos(1/2*arctan(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x^2
*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arctan(x^2*b^(1/2)/a^(1/2))),2^(1/2))/
(b*x^4+a)^(3/4)/a^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {275, 277, 325, 233, 231}

$$-\frac{b^{3/2}\left(\frac{bx^4}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{12\sqrt{a}\left(a+bx^4\right)^{3/4}}-\frac{b\sqrt[4]{a+bx^4}}{12ax^2}-\frac{\sqrt[4]{a+bx^4}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(1/4)/x^7,x]
```

```
[Out] -(a + b*x^4)^(1/4)/(6*x^6) - (b*(a + b*x^4)^(1/4))/(12*a*x^2) - (b^(3/2)*(1
 + (b*x^4)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(12*Sqrt
[a]*(a + b*x^4)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[4]{a+bx^4}}{x^7}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt[4]{a+bx^2}}{x^4}\,dx, x, x^2\right)\\ &= -\frac{\sqrt[4]{a+bx^4}}{6x^6} + \frac{1}{12}b\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^{3/4}}\,dx, x, x^2\right)\\ &= -\frac{\sqrt[4]{a+bx^4}}{6x^6} - \frac{b\sqrt[4]{a+bx^4}}{12ax^2} - \frac{b^2\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx, x, x^2\right)}{24a}\\ &= -\frac{\sqrt[4]{a+bx^4}}{6x^6} - \frac{b\sqrt[4]{a+bx^4}}{12ax^2} - \frac{\left(b^2\left(1+\frac{bx^4}{a}\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx, x, x^2\right)}{24a\left(a+bx^4\right)^{3/4}}\\ &= -\frac{\sqrt[4]{a+bx^4}}{6x^6} - \frac{b\sqrt[4]{a+bx^4}}{12ax^2} - \frac{b^{3/2}\left(1+\frac{bx^4}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|\,2\right)}{12\sqrt{a}\left(a+bx^4\right)^{3/4}}\end{aligned}$$

Mathematica [C] time = 0.03, size = 51, normalized size = 0.50

$$-\frac{\sqrt[4]{a+bx^4}\;{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};-\frac{1}{2};-\frac{bx^4}{a}\right)}{6x^6\sqrt[4]{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(1/4)/x^7,x]
```

```
[Out] -1/6*((a + b*x^4)^(1/4)*Hypergeometric2F1[-3/2, -1/4, -1/2, -((b*x^4)/a)])/
(x^6*(1 + (b*x^4)/a)^(1/4))
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^7,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(1/4)/x^7, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^7,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(1/4)/x^7, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{1}{4}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(1/4)/x^7,x)
[Out] int((b*x^4+a)^(1/4)/x^7,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^7,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(1/4)/x^7, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^4+a\right)^{1/4}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(1/4)/x^7,x)
[Out] int((a + b*x^4)^(1/4)/x^7, x)
```

sympy [C] time = 2.78, size = 34, normalized size = 0.34

$$-\frac{\sqrt[4]{a}\,{}_2F_1\left(\left.\begin{matrix}-\frac{3}{2}, -\frac{1}{4} \\ -\frac{1}{2}\end{matrix}\right| \frac{bx^4e^{i\pi}}{a}\right)}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(1/4)/x**7,x)
[Out] -a**(1/4)*hyper((-3/2, -1/4), (-1/2,), b*x**4*exp_polar(I*pi)/a)/(6*x**6)
```

3.1001 $\int \frac{\sqrt[4]{a+bx^4}}{x^{11}}\,dx$

Optimal. Leaf size=125

$$\frac{b^{5/2}\left(\frac{bx^4}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{24a^{3/2}\left(a+bx^4\right)^{3/4}}+\frac{b^2\sqrt[4]{a+bx^4}}{24a^2x^2}-\frac{\sqrt[4]{a+bx^4}}{10x^{10}}-\frac{b\sqrt[4]{a+bx^4}}{60ax^6}$$

```
[Out] -1/10*(b*x^4+a)^(1/4)/x^10-1/60*b*(b*x^4+a)^(1/4)/a/x^6+1/24*b^2*(b*x^4+a)^
(1/4)/a^2/x^2+1/24*b^(5/2)*(1+b*x^4/a)^(3/4)*(cos(1/2*arctan(x^2*b^(1/2)/a^
(1/2)))^2)^(1/2)/cos(1/2*arctan(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arc
tan(x^2*b^(1/2)/a^(1/2))),2^(1/2))/a^(3/2)/(b*x^4+a)^(3/4)
```

Rubi [A] time = 0.08, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {275, 277, 325, 233, 231}

$$\frac{b^2\sqrt[4]{a+bx^4}}{24a^2x^2}+\frac{b^{5/2}\left(\frac{bx^4}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{24a^{3/2}\left(a+bx^4\right)^{3/4}}-\frac{b\sqrt[4]{a+bx^4}}{60ax^6}-\frac{\sqrt[4]{a+bx^4}}{10x^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(1/4)/x^11,x]
```

```
[Out] -(a + b*x^4)^(1/4)/(10*x^10) - (b*(a + b*x^4)^(1/4))/(60*a*x^6) + (b^2*(a +
 b*x^4)^(1/4))/(24*a^2*x^2) + (b^(5/2)*(1 + (b*x^4)/a)^(3/4)*EllipticF[ArcT
an[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(24*a^(3/2)*(a + b*x^4)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{a+bx^4}}{x^{11}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt[4]{a+bx^2}}{x^6}\,dx, x, x^2\right)\\
&= -\frac{\sqrt[4]{a+bx^4}}{10x^{10}} + \frac{1}{20}b\,\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^2\right)^{3/4}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt[4]{a+bx^4}}{10x^{10}} - \frac{b\sqrt[4]{a+bx^4}}{60ax^6} - \frac{b^2\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^{3/4}}\,dx, x, x^2\right)}{24a}\\
&= -\frac{\sqrt[4]{a+bx^4}}{10x^{10}} - \frac{b\sqrt[4]{a+bx^4}}{60ax^6} + \frac{b^2\sqrt[4]{a+bx^4}}{24a^2x^2} + \frac{b^3\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx, x, x^2\right)}{48a^2}\\
&= -\frac{\sqrt[4]{a+bx^4}}{10x^{10}} - \frac{b\sqrt[4]{a+bx^4}}{60ax^6} + \frac{b^2\sqrt[4]{a+bx^4}}{24a^2x^2} + \frac{\left(b^3\left(1+\frac{bx^4}{a}\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx, x, x^2\right)}{48a^2\left(a+bx^4\right)^{3/4}}\\
&= -\frac{\sqrt[4]{a+bx^4}}{10x^{10}} - \frac{b\sqrt[4]{a+bx^4}}{60ax^6} + \frac{b^2\sqrt[4]{a+bx^4}}{24a^2x^2} + \frac{b^{5/2}\left(1+\frac{bx^4}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{24a^{3/2}\left(a+bx^4\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 51, normalized size = 0.41

$$-\frac{\sqrt[4]{a+bx^4}\;{}_2F_1\left(-\frac{5}{2},-\frac{1}{4};-\frac{3}{2};-\frac{bx^4}{a}\right)}{10x^{10}\sqrt[4]{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(1/4)/x^11,x]
```

```
[Out] -1/10*((a + b*x^4)^(1/4)*Hypergeometric2F1[-5/2, -1/4, -3/2, -((b*x^4)/a)])
/(x^10*(1 + (b*x^4)/a)^(1/4))
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x^{11}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^11,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(1/4)/x^11, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^11,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(1/4)/x^11, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{1}{4}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(1/4)/x^11,x)
[Out] int((b*x^4+a)^(1/4)/x^11,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^11,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(1/4)/x^11, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^4+a\right)^{1/4}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(1/4)/x^11,x)
[Out] int((a + b*x^4)^(1/4)/x^11, x)
```

sympy [C] time = 3.45, size = 34, normalized size = 0.27

$$-\frac{\sqrt[4]{a}\,{}_2F_1\left(\left.\begin{matrix}-\frac{5}{2}, -\frac{1}{4} \\ -\frac{3}{2}\end{matrix}\right| \frac{bx^4e^{i\pi}}{a}\right)}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(1/4)/x**11,x)
[Out] -a**(1/4)*hyper((-5/2, -1/4), (-3/2,), b*x**4*exp_polar(I*pi)/a)/(10*x**10)
```

3.1002 $\int x^6 \sqrt[4]{a+bx^4}\,dx$

Optimal. Leaf size=103

$$\frac{3a^2 \tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{64b^{7/4}} - \frac{3a^2 \tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{64b^{7/4}} + \frac{1}{8}x^7\sqrt[4]{a+bx^4} + \frac{ax^3\sqrt[4]{a+bx^4}}{32b}$$

```
[Out] 1/32*a*x^3*(b*x^4+a)^(1/4)/b+1/8*x^7*(b*x^4+a)^(1/4)+3/64*a^2*arctan(b^(1/4
)*x/(b*x^4+a)^(1/4))/b^(7/4)-3/64*a^2*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^
(7/4)
```

Rubi [A] time = 0.04, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {279, 321, 331, 298, 203, 206}

$$\frac{3a^2 \tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{64b^{7/4}} - \frac{3a^2 \tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{64b^{7/4}} + \frac{1}{8}x^7\sqrt[4]{a+bx^4} + \frac{ax^3\sqrt[4]{a+bx^4}}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x^4)^(1/4),x]
```

```
[Out] (a*x^3*(a + b*x^4)^(1/4))/(32*b) + (x^7*(a + b*x^4)^(1/4))/8 + (3*a^2*ArcTa
n[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(64*b^(7/4)) - (3*a^2*ArcTanh[(b^(1/4)*x)
/(a + b*x^4)^(1/4)])/(64*b^(7/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int x^6 \sqrt[4]{a+bx^4}\,dx = \frac{1}{8}x^7\sqrt[4]{a+bx^4} + \frac{1}{8}a\int \frac{x^6}{\left(a+bx^4\right)^{3/4}}\,dx$$

$$= \frac{ax^3\sqrt[4]{a+bx^4}}{32b} + \frac{1}{8}x^7\sqrt[4]{a+bx^4} - \frac{\left(3a^2\right)\int \frac{x^2}{\left(a+bx^4\right)^{3/4}}\,dx}{32b}$$

$$= \frac{ax^3\sqrt[4]{a+bx^4}}{32b} + \frac{1}{8}x^7\sqrt[4]{a+bx^4} - \frac{\left(3a^2\right)\text{Subst}\left(\int \frac{x^2}{1-bx^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{32b}$$

$$= \frac{ax^3\sqrt[4]{a+bx^4}}{32b} + \frac{1}{8}x^7\sqrt[4]{a+bx^4} - \frac{\left(3a^2\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{b}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{64b^{3/2}} + \frac{\left(3a^2\right)\text{Subst}\left(\int \right.}{}$$

$$= \frac{ax^3\sqrt[4]{a+bx^4}}{32b} + \frac{1}{8}x^7\sqrt[4]{a+bx^4} + \frac{3a^2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{64b^{7/4}} - \frac{3a^2\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{64b^{7/4}}$$

Mathematica [C] time = 0.06, size = 64, normalized size = 0.62

$$\frac{x^3\sqrt[4]{a+bx^4}\left(-\frac{a\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^4}{a}\right)}{\sqrt[4]{\frac{bx^4}{a}+1}} + a + bx^4\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x^4)^(1/4),x]
```

```
[Out] (x^3*(a + b*x^4)^(1/4)*(a + b*x^4 - (a*Hypergeometric2F1[-1/4, 3/4, 7/4, -(
(b*x^4)/a)])/(1 + (b*x^4)/a)^(1/4)))/(8*b)
```

fricas [B] time = 0.69, size = 220, normalized size = 2.14

$$\frac{12\left(\frac{a^8}{b^7}\right)^{\frac{1}{4}} b\arctan\left(-\frac{\left(bx^4+a\right)^{\frac{1}{4}}a^2\left(\frac{a^8}{b^7}\right)^{\frac{3}{4}}b^5 - \left(\frac{a^8}{b^7}\right)^{\frac{3}{4}}b^5x\sqrt{\frac{\sqrt{\frac{a^8}{b^7}}\,b^4x^2+\sqrt{bx^4+a}\,a^4}{x^2}}}{a^8x}\right) - 3\left(\frac{a^8}{b^7}\right)^{\frac{1}{4}} b\log\left(\frac{3\left(\left(\frac{a^8}{b^7}\right)^{\frac{1}{4}}b^2x+\left(bx^4+a\right)^{\frac{1}{4}}a^2\right)}{x}\right) + 3\left(}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/128*(12*(a^8/b^7)^(1/4)*b*arctan(-((b*x^4 + a)^(1/4)*a^2*(a^8/b^7)^(3/4)*
b^5 - (a^8/b^7)^(3/4)*b^5*x*sqrt((sqrt(a^8/b^7)*b^4*x^2 + sqrt(b*x^4 + a)*a
```

```
^4)/x^2))/(a^8*x)) - 3*(a^8/b^7)^(1/4)*b*log(3*((a^8/b^7)^(1/4)*b^2*x + (b*
x^4 + a)^(1/4)*a^2)/x) + 3*(a^8/b^7)^(1/4)*b*log(-3*((a^8/b^7)^(1/4)*b^2*x
- (b*x^4 + a)^(1/4)*a^2)/x) + 4*(4*b*x^7 + a*x^3)*(b*x^4 + a)^(1/4))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4 + a\right)^{\frac{1}{4}} x^6 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(1/4)*x^6, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \left(b\,x^4 + a\right)^{\frac{1}{4}} x^6 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^4+a)^(1/4),x)
```

```
[Out] int(x^6*(b*x^4+a)^(1/4),x)
```

maxima [A] time = 2.98, size = 152, normalized size = 1.48

$$\frac{\frac{3\left(bx^4+a\right)^{\frac{1}{4}}a^2b}{x} + \frac{\left(bx^4+a\right)^{\frac{5}{4}}a^2}{x^5}}{32\left(b^3 - \frac{2\left(bx^4+a\right)b^2}{x^4} + \frac{\left(bx^4+a\right)^2 b}{x^8}\right)} - \frac{3\left(\frac{2\,a^2\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{3}{4}}} - \frac{a^2\log\left(-\frac{b^{\frac{1}{4}} - \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}} + \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{3}{4}}}\right)}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] 1/32*(3*(b*x^4 + a)^(1/4)*a^2*b/x + (b*x^4 + a)^(5/4)*a^2/x^5)/(b^3 - 2*(b*
x^4 + a)*b^2/x^4 + (b*x^4 + a)^2*b/x^8) - 3/128*(2*a^2*arctan((b*x^4 + a)^(
1/4)/(b^(1/4)*x))/b^(3/4) - a^2*log(-(b^(1/4) - (b*x^4 + a)^(1/4)/x)/(b^(1/
4) + (b*x^4 + a)^(1/4)/x))/b^(3/4))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^6 \left(b\,x^4 + a\right)^{1/4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b*x^4)^(1/4),x)
```

```
[Out] int(x^6*(a + b*x^4)^(1/4), x)
```

sympy [C] time = 3.10, size = 39, normalized size = 0.38

$$\frac{\sqrt[4]{a}\,x^7\,\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4}, \frac{7}{4} \\ \frac{11}{4}\end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**4+a)**(1/4),x)
```

```
[Out] a**(1/4)*x**7*gamma(7/4)*hyper((-1/4, 7/4), (11/4,), b*x**4*exp_polar(I*pi)
/a)/(4*gamma(11/4))
```

3.1003 $\int x^2 \sqrt[4]{a+bx^4}\, dx$

Optimal. Leaf size=77

$$-\frac{a \tan^{-1}\left(\frac{\sqrt[4]{b}\, x}{\sqrt[4]{a+bx^4}}\right)}{8b^{3/4}} + \frac{a \tanh^{-1}\left(\frac{\sqrt[4]{b}\, x}{\sqrt[4]{a+bx^4}}\right)}{8b^{3/4}} + \frac{1}{4}x^3 \sqrt[4]{a+bx^4}$$

```
[Out] 1/4*x^3*(b*x^4+a)^(1/4)-1/8*a*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(3/4)+1/8
*a*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(3/4)
```

Rubi [A] time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {279, 331, 298, 203, 206}

$$-\frac{a \tan^{-1}\left(\frac{\sqrt[4]{b}\, x}{\sqrt[4]{a+bx^4}}\right)}{8b^{3/4}} + \frac{a \tanh^{-1}\left(\frac{\sqrt[4]{b}\, x}{\sqrt[4]{a+bx^4}}\right)}{8b^{3/4}} + \frac{1}{4}x^3 \sqrt[4]{a+bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^4)^(1/4),x]
```

```
[Out] (x^3*(a + b*x^4)^(1/4))/4 - (a*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(8*b^
(3/4)) + (a*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(8*b^(3/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt[4]{a+bx^4}\,dx &= \frac{1}{4}x^3\sqrt[4]{a+bx^4} + \frac{1}{4}a\int \frac{x^2}{\left(a+bx^4\right)^{3/4}}\,dx \\ &= \frac{1}{4}x^3\sqrt[4]{a+bx^4} + \frac{1}{4}a\,\text{Subst}\left(\int \frac{x^2}{1-bx^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right) \\ &= \frac{1}{4}x^3\sqrt[4]{a+bx^4} + \frac{a\,\text{Subst}\left(\int \frac{1}{1-\sqrt{b}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{8\sqrt{b}} - \frac{a\,\text{Subst}\left(\int \frac{1}{1+\sqrt{b}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{8\sqrt{b}} \\ &= \frac{1}{4}x^3\sqrt[4]{a+bx^4} - \frac{a\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8b^{3/4}} + \frac{a\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8b^{3/4}}\end{aligned}$$

Mathematica [C] time = 0.05, size = 51, normalized size = 0.66

$$\frac{x^3\sqrt[4]{a+bx^4}\;{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^4}{a}\right)}{3\sqrt[4]{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^4)^(1/4),x]
```

```
[Out] (x^3*(a + b*x^4)^(1/4)*Hypergeometric2F1[-1/4, 3/4, 7/4, -((b*x^4)/a)])/(3*
(1 + (b*x^4)/a)^(1/4))
```

fricas [B] time = 0.79, size = 191, normalized size = 2.48

$$\frac{1}{4}\left(bx^4+a\right)^{\frac{1}{4}}x^3 - \frac{1}{4}\left(\frac{a^4}{b^3}\right)^{\frac{1}{4}}\arctan\left(\frac{\left(\frac{a^4}{b^3}\right)^{\frac{3}{4}}b^2x\sqrt{\frac{\sqrt{\frac{a^4}{b^3}}\,b^2x^2+\sqrt{bx^4+a}\,a^2}{x^2}} - \left(bx^4+a\right)^{\frac{1}{4}}a\left(\frac{a^4}{b^3}\right)^{\frac{3}{4}}b^2}{a^4x}\right) + \frac{1}{16}\left(\frac{a^4}{b^3}\right)^{\frac{1}{4}}\log\left(\frac{\left(\frac{a^4}{b^3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/4*(b*x^4 + a)^(1/4)*x^3 - 1/4*(a^4/b^3)^(1/4)*arctan(((a^4/b^3)^(3/4)*b^2
*x*sqrt((sqrt(a^4/b^3)*b^2*x^2 + sqrt(b*x^4 + a)*a^2)/x^2) - (b*x^4 + a)^(1
/4)*a*(a^4/b^3)^(3/4)*b^2)/(a^4*x)) + 1/16*(a^4/b^3)^(1/4)*log(((a^4/b^3)^(
1/4)*b*x + (b*x^4 + a)^(1/4)*a)/x) - 1/16*(a^4/b^3)^(1/4)*log(-((a^4/b^3)^(
1/4)*b*x - (b*x^4 + a)^(1/4)*a)/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4+a\right)^{\frac{1}{4}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(1/4)*x^2, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \left(b\,x^4+a\right)^{\frac{1}{4}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^4+a)^(1/4),x)
```

```
[Out] int(x^2*(b*x^4+a)^(1/4),x)
```

maxima [A] time = 3.00, size = 101, normalized size = 1.31

$$\frac{a\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{8b^{\frac{3}{4}}}-\frac{a\log\left(-\frac{b^{\frac{1}{4}}-\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}\right)}{16b^{\frac{3}{4}}}-\frac{\left(bx^4+a\right)^{\frac{1}{4}}a}{4\left(b-\frac{bx^4+a}{x^4}\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] 1/8*a*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(3/4) - 1/16*a*log(-(b^(1/4)
- (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(3/4) - 1/4*(b*x^
4 + a)^(1/4)*a/((b - (b*x^4 + a)/x^4)*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(b\,x^4+a\right)^{1/4}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^4)^(1/4),x)
```

```
[Out] int(x^2*(a + b*x^4)^(1/4), x)
```

sympy [C] time = 1.65, size = 39, normalized size = 0.51

$$\frac{\sqrt[4]{a}\,x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**4+a)**(1/4),x)
```

```
[Out] a**(1/4)*x**3*gamma(3/4)*hyper((-1/4, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/
a)/(4*gamma(7/4))
```

3.1004 $\int \frac{\sqrt[4]{a+bx^4}}{x^2}\,dx$

Optimal. Leaf size=73

$$-\frac{\sqrt[4]{a+bx^4}}{x}-\frac{1}{2}\sqrt[4]{b}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)+\frac{1}{2}\sqrt[4]{b}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)$$

```
[Out] -(b*x^4+a)^(1/4)/x-1/2*b^(1/4)*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))+1/2*b^(1/4
)*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))
```

Rubi [A] time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {277, 331, 298, 203, 206}

$$-\frac{\sqrt[4]{a+bx^4}}{x}-\frac{1}{2}\sqrt[4]{b}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)+\frac{1}{2}\sqrt[4]{b}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(1/4)/x^2,x]
```

```
[Out] -((a + b*x^4)^(1/4)/x) - (b^(1/4)*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/2
+ (b^(1/4)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[4]{a+bx^4}}{x^2}\,dx &= -\frac{\sqrt[4]{a+bx^4}}{x} + b\int \frac{x^2}{\left(a+bx^4\right)^{3/4}}\,dx \\ &= -\frac{\sqrt[4]{a+bx^4}}{x} + b\,\text{Subst}\left(\int \frac{x^2}{1-bx^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right) \\ &= -\frac{\sqrt[4]{a+bx^4}}{x} + \frac{1}{2}\sqrt{b}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{b}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right) - \frac{1}{2}\sqrt{b}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{b}\,x^2}\,dx, x, \right. \\ &= -\frac{\sqrt[4]{a+bx^4}}{x} - \frac{1}{2}\sqrt[4]{b}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right) + \frac{1}{2}\sqrt[4]{b}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 49, normalized size = 0.67

$$-\frac{\sqrt[4]{a+bx^4}\;{}_2F_1\left(-\frac{1}{4},-\frac{1}{4};\frac{3}{4};-\frac{bx^4}{a}\right)}{x\sqrt[4]{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(1/4)/x^2,x]
```

```
[Out] -(((a + b*x^4)^(1/4)*Hypergeometric2F1[-1/4, -1/4, 3/4, -((b*x^4)/a)])/(x*(
1 + (b*x^4)/a)^(1/4)))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(1/4)/x^2, x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{1}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(1/4)/x^2,x)
```

```
[Out] int((b*x^4+a)^(1/4)/x^2,x)
```

maxima [A] time = 2.87, size = 82, normalized size = 1.12

$$\frac{1}{2}\, b^{\frac{1}{4}} \arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right) - \frac{1}{4}\, b^{\frac{1}{4}} \log\left(-\frac{b^{\frac{1}{4}} - \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}} + \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}\right) - \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^(1/4)*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x)) - 1/4*b^(1/4)*log(-(b^(1/
4) - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x)) - (b*x^4 + a)^(1
/4)/x
```

mupad [B] time = 1.23, size = 40, normalized size = 0.55

$$-\frac{\left(b\,x^4+a\right)^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},-\frac{1}{4};\ \frac{3}{4};\ -\frac{b\,x^4}{a}\right)}{x\left(\frac{b\,x^4}{a}+1\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(1/4)/x^2,x)
```

```
[Out] -((a + b*x^4)^(1/4)*hypergeom([-1/4, -1/4], 3/4, -(b*x^4)/a))/(x*((b*x^4)/a
 + 1)^(1/4))
```

sympy [C] time = 1.95, size = 41, normalized size = 0.56

$$\frac{\sqrt[4]{a}\,\Gamma\left(-\frac{1}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{4},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(1/4)/x**2,x)
```

```
[Out] a**(1/4)*gamma(-1/4)*hyper((-1/4, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/
(4*x*gamma(3/4))
```

3.1005 $\int \frac{\sqrt[4]{a+bx^4}}{x^6}\,dx$

Optimal. Leaf size=21

$$-\frac{\left(a+bx^4\right)^{5/4}}{5ax^5}$$

```
[Out] -1/5*(b*x^4+a)^(5/4)/a/x^5
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {264}

$$-\frac{\left(a+bx^4\right)^{5/4}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(1/4)/x^6,x]
```

```
[Out] -(a + b*x^4)^(5/4)/(5*a*x^5)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx^4}}{x^6}\,dx = -\frac{\left(a+bx^4\right)^{5/4}}{5ax^5}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$-\frac{\left(a+bx^4\right)^{5/4}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(1/4)/x^6,x]
```

```
[Out] -1/5*(a + b*x^4)^(5/4)/(a*x^5)
```

fricas [A] time = 1.13, size = 17, normalized size = 0.81

$$-\frac{\left(bx^4+a\right)^{\frac{5}{4}}}{5\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^6,x, algorithm="fricas")
```

```
[Out] -1/5*(b*x^4 + a)^(5/4)/(a*x^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(1/4)/x^6, x)
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$-\frac{\left(b\,x^4+a\right)^{\frac{5}{4}}}{5a\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(1/4)/x^6,x)
```

```
[Out] -1/5*(b*x^4+a)^(5/4)/a/x^5
```

maxima [A] time = 1.33, size = 17, normalized size = 0.81

$$-\frac{\left(bx^4+a\right)^{\frac{5}{4}}}{5\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^6,x, algorithm="maxima")
```

```
[Out] -1/5*(b*x^4 + a)^(5/4)/(a*x^5)
```

mupad [B] time = 1.29, size = 17, normalized size = 0.81

$$-\frac{\left(b\,x^4+a\right)^{5/4}}{5\,a\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(1/4)/x^6,x)
```

```
[Out] -(a + b*x^4)^(5/4)/(5*a*x^5)
```

sympy [B] time = 1.77, size = 68, normalized size = 3.24

$$\frac{\sqrt[4]{b}\,\sqrt[4]{\frac{a}{bx^4}+1}\,\Gamma\left(-\frac{5}{4}\right)}{4x^4\Gamma\left(-\frac{1}{4}\right)}+\frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx^4}+1}\,\Gamma\left(-\frac{5}{4}\right)}{4a\Gamma\left(-\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(1/4)/x**6,x)
```

```
[Out] b**(1/4)*(a/(b*x**4) + 1)**(1/4)*gamma(-5/4)/(4*x**4*gamma(-1/4)) + b**(5/4
)*(a/(b*x**4) + 1)**(1/4)*gamma(-5/4)/(4*a*gamma(-1/4))
```

3.1006 $\int \frac{\sqrt[4]{a+bx^4}}{x^{10}}\,dx$

Optimal. Leaf size=44

$$\frac{4b\left(a+bx^4\right)^{5/4}}{45a^2x^5}-\frac{\left(a+bx^4\right)^{5/4}}{9ax^9}$$

```
[Out] -1/9*(b*x^4+a)^(5/4)/a/x^9+4/45*b*(b*x^4+a)^(5/4)/a^2/x^5
```

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$\frac{4b\left(a+bx^4\right)^{5/4}}{45a^2x^5}-\frac{\left(a+bx^4\right)^{5/4}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(1/4)/x^10,x]
```

```
[Out] -(a + b*x^4)^(5/4)/(9*a*x^9) + (4*b*(a + b*x^4)^(5/4))/(45*a^2*x^5)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[4]{a+bx^4}}{x^{10}}\,dx &= -\frac{\left(a+bx^4\right)^{5/4}}{9ax^9}-\frac{(4b)\int \frac{\sqrt[4]{a+bx^4}}{x^6}\,dx}{9a}\\ &= -\frac{\left(a+bx^4\right)^{5/4}}{9ax^9}+\frac{4b\left(a+bx^4\right)^{5/4}}{45a^2x^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.70

$$\frac{\left(a+bx^4\right)^{5/4}\left(4bx^4-5a\right)}{45a^2x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(1/4)/x^10,x]
```

```
[Out] ((a + b*x^4)^(5/4)*(-5*a + 4*b*x^4))/(45*a^2*x^9)
```

fricas [A] time = 1.03, size = 38, normalized size = 0.86

$$\frac{\left(4\,b^2x^8-abx^4-5\,a^2\right)\left(bx^4+a\right)^{\frac{1}{4}}}{45\,a^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^10,x, algorithm="fricas")
```

```
[Out] 1/45*(4*b^2*x^8 - a*b*x^4 - 5*a^2)*(b*x^4 + a)^(1/4)/(a^2*x^9)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^10,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(1/4)/x^10, x)
```

maple [A] time = 0.00, size = 28, normalized size = 0.64

$$-\frac{\left(b\,x^4+a\right)^{\frac{5}{4}}\left(-4b\,x^4+5a\right)}{45a^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(1/4)/x^10,x)
```

```
[Out] -1/45*(b*x^4+a)^(5/4)*(-4*b*x^4+5*a)/a^2/x^9
```

maxima [A] time = 1.34, size = 35, normalized size = 0.80

$$\frac{\frac{9\left(bx^4+a\right)^{\frac{5}{4}}b}{x^5}-\frac{5\left(bx^4+a\right)^{\frac{9}{4}}}{x^9}}{45\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^10,x, algorithm="maxima")
```

```
[Out] 1/45*(9*(b*x^4 + a)^(5/4)*b/x^5 - 5*(b*x^4 + a)^(9/4)/x^9)/a^2
```

mupad [B] time = 1.30, size = 37, normalized size = 0.84

$$-\frac{\left(b\,x^4+a\right)^{1/4}\,\left(5\,a^2+a\,b\,x^4-4\,b^2\,x^8\right)}{45\,a^2\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(1/4)/x^10,x)
```

```
[Out] -((a + b*x^4)^(1/4)*(5*a^2 - 4*b^2*x^8 + a*b*x^4))/(45*a^2*x^9)
```

sympy [B] time = 2.84, size = 109, normalized size = 2.48

$$-\frac{5\sqrt[4]{b}\,\sqrt[4]{\frac{a}{bx^4}+1}\,\Gamma\left(-\frac{9}{4}\right)}{16x^8\Gamma\left(-\frac{1}{4}\right)}-\frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx^4}+1}\,\Gamma\left(-\frac{9}{4}\right)}{16ax^4\Gamma\left(-\frac{1}{4}\right)}+\frac{b^{\frac{9}{4}}\sqrt[4]{\frac{a}{bx^4}+1}\,\Gamma\left(-\frac{9}{4}\right)}{4a^2\Gamma\left(-\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(1/4)/x**10,x)
```

```
[Out] -5*b**(1/4)*(a/(b*x**4) + 1)**(1/4)*gamma(-9/4)/(16*x**8*gamma(-1/4)) - b**
(5/4)*(a/(b*x**4) + 1)**(1/4)*gamma(-9/4)/(16*a*x**4*gamma(-1/4)) + b**(9/4
)*(a/(b*x**4) + 1)**(1/4)*gamma(-9/4)/(4*a**2*gamma(-1/4))
```

3.1007 $\int \frac{\sqrt[4]{a+bx^4}}{x^{14}}\,dx$

Optimal. Leaf size=68

$$-\frac{32b^2\left(a+bx^4\right)^{5/4}}{585a^3x^5}+\frac{8b\left(a+bx^4\right)^{5/4}}{117a^2x^9}-\frac{\left(a+bx^4\right)^{5/4}}{13ax^{13}}$$

```
[Out] -1/13*(b*x^4+a)^(5/4)/a/x^13+8/117*b*(b*x^4+a)^(5/4)/a^2/x^9-32/585*b^2*(b*
x^4+a)^(5/4)/a^3/x^5
```

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$-\frac{32b^2\left(a+bx^4\right)^{5/4}}{585a^3x^5}+\frac{8b\left(a+bx^4\right)^{5/4}}{117a^2x^9}-\frac{\left(a+bx^4\right)^{5/4}}{13ax^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(1/4)/x^14,x]
```

```
[Out] -(a + b*x^4)^(5/4)/(13*a*x^13) + (8*b*(a + b*x^4)^(5/4))/(117*a^2*x^9) - (3
2*b^2*(a + b*x^4)^(5/4))/(585*a^3*x^5)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[4]{a+bx^4}}{x^{14}}\,dx &= -\frac{\left(a+bx^4\right)^{5/4}}{13ax^{13}}-\frac{(8b)\int \frac{\sqrt[4]{a+bx^4}}{x^{10}}\,dx}{13a}\\ &= -\frac{\left(a+bx^4\right)^{5/4}}{13ax^{13}}+\frac{8b\left(a+bx^4\right)^{5/4}}{117a^2x^9}+\frac{\left(32b^2\right)\int \frac{\sqrt[4]{a+bx^4}}{x^6}\,dx}{117a^2}\\ &= -\frac{\left(a+bx^4\right)^{5/4}}{13ax^{13}}+\frac{8b\left(a+bx^4\right)^{5/4}}{117a^2x^9}-\frac{32b^2\left(a+bx^4\right)^{5/4}}{585a^3x^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.62

$$-\frac{\left(a+bx^4\right)^{5/4}\left(45a^2-40abx^4+32b^2x^8\right)}{585a^3x^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(1/4)/x^14,x]
```

```
[Out] -1/585*((a + b*x^4)^(5/4)*(45*a^2 - 40*a*b*x^4 + 32*b^2*x^8))/(a^3*x^13)
```

fricas [A] time = 0.98, size = 49, normalized size = 0.72

$$-\frac{\left(32\,b^3x^{12}-8\,ab^2x^8+5\,a^2bx^4+45\,a^3\right)\left(bx^4+a\right)^{\frac{1}{4}}}{585\,a^3x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^14,x, algorithm="fricas")
```

```
[Out] -1/585*(32*b^3*x^12 - 8*a*b^2*x^8 + 5*a^2*b*x^4 + 45*a^3)*(b*x^4 + a)^(1/4)
/(a^3*x^13)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^14,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(1/4)/x^14, x)
```

maple [A] time = 0.00, size = 39, normalized size = 0.57

$$-\frac{\left(b\,x^4+a\right)^{\frac{5}{4}}\left(32b^2x^8-40ab\,x^4+45a^2\right)}{585a^3x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(1/4)/x^14,x)
```

```
[Out] -1/585*(b*x^4+a)^(5/4)*(32*b^2*x^8-40*a*b*x^4+45*a^2)/x^13/a^3
```

maxima [A] time = 1.33, size = 52, normalized size = 0.76

$$-\frac{\frac{117\left(bx^4+a\right)^{\frac{5}{4}}b^2}{x^5}-\frac{130\left(bx^4+a\right)^{\frac{9}{4}}b}{x^9}+\frac{45\left(bx^4+a\right)^{\frac{13}{4}}}{x^{13}}}{585\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^14,x, algorithm="maxima")
```

```
[Out] -1/585*(117*(b*x^4 + a)^(5/4)*b^2/x^5 - 130*(b*x^4 + a)^(9/4)*b/x^9 + 45*(b
*x^4 + a)^(13/4)/x^13)/a^3
```

mupad [B] time = 1.48, size = 73, normalized size = 1.07

$$\frac{8\,b^2\left(b\,x^4+a\right)^{1/4}}{585\,a^2\,x^5}-\frac{b\left(b\,x^4+a\right)^{1/4}}{117\,a\,x^9}-\frac{32\,b^3\left(b\,x^4+a\right)^{1/4}}{585\,a^3\,x}-\frac{\left(b\,x^4+a\right)^{1/4}}{13\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(1/4)/x^14,x)
```

```
[Out] (8*b^2*(a + b*x^4)^(1/4))/(585*a^2*x^5) - (b*(a + b*x^4)^(1/4))/(117*a*x^9)
 - (32*b^3*(a + b*x^4)^(1/4))/(585*a^3*x) - (a + b*x^4)^(1/4)/(13*x^13)
```

sympy [**B**] time = 3.26, size = 520, normalized size = 7.65

$$\frac{45a^5b^{\frac{17}{4}}\sqrt[4]{\frac{a}{bx^4}+1}\,\Gamma\left(-\frac{13}{4}\right)}{64a^5b^4x^{12}\Gamma\left(-\frac{1}{4}\right)+128a^4b^5x^{16}\Gamma\left(-\frac{1}{4}\right)+64a^3b^6x^{20}\Gamma\left(-\frac{1}{4}\right)}+\frac{95a^4b^{\frac{21}{4}}x^4\sqrt[4]{\frac{a}{bx^4}+1}\,\Gamma\left(-\frac{13}{4}\right)}{64a^5b^4x^{12}\Gamma\left(-\frac{1}{4}\right)+128a^4b^5x^{16}\Gamma\left(-\frac{1}{4}\right)+64a^3b^6x^{20}\Gamma\left(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(1/4)/x**14,x)
```

```
[Out] 45*a**5*b**(17/4)*(a/(b*x**4) + 1)**(1/4)*gamma(-13/4)/(64*a**5*b**4*x**12*
gamma(-1/4) + 128*a**4*b**5*x**16*gamma(-1/4) + 64*a**3*b**6*x**20*gamma(-1
/4)) + 95*a**4*b**(21/4)*x**4*(a/(b*x**4) + 1)**(1/4)*gamma(-13/4)/(64*a**5
*b**4*x**12*gamma(-1/4) + 128*a**4*b**5*x**16*gamma(-1/4) + 64*a**3*b**6*x*
*20*gamma(-1/4)) + 47*a**3*b**(25/4)*x**8*(a/(b*x**4) + 1)**(1/4)*gamma(-13
/4)/(64*a**5*b**4*x**12*gamma(-1/4) + 128*a**4*b**5*x**16*gamma(-1/4) + 64*
a**3*b**6*x**20*gamma(-1/4)) + 21*a**2*b**(29/4)*x**12*(a/(b*x**4) + 1)**(1
/4)*gamma(-13/4)/(64*a**5*b**4*x**12*gamma(-1/4) + 128*a**4*b**5*x**16*gamm
a(-1/4) + 64*a**3*b**6*x**20*gamma(-1/4)) + 56*a*b**(33/4)*x**16*(a/(b*x**4
) + 1)**(1/4)*gamma(-13/4)/(64*a**5*b**4*x**12*gamma(-1/4) + 128*a**4*b**5*
x**16*gamma(-1/4) + 64*a**3*b**6*x**20*gamma(-1/4)) + 32*b**(37/4)*x**20*(a
/(b*x**4) + 1)**(1/4)*gamma(-13/4)/(64*a**5*b**4*x**12*gamma(-1/4) + 128*a*
*4*b**5*x**16*gamma(-1/4) + 64*a**3*b**6*x**20*gamma(-1/4))
```

3.1008 $\int \frac{\sqrt[4]{a+bx^4}}{x^{18}}\,dx$

Optimal. Leaf size=92

$$\frac{128b^3\left(a+bx^4\right)^{5/4}}{3315a^4x^5}-\frac{32b^2\left(a+bx^4\right)^{5/4}}{663a^3x^9}+\frac{12b\left(a+bx^4\right)^{5/4}}{221a^2x^{13}}-\frac{\left(a+bx^4\right)^{5/4}}{17ax^{17}}$$

```
[Out] -1/17*(b*x^4+a)^(5/4)/a/x^17+12/221*b*(b*x^4+a)^(5/4)/a^2/x^13-32/663*b^2*(
b*x^4+a)^(5/4)/a^3/x^9+128/3315*b^3*(b*x^4+a)^(5/4)/a^4/x^5
```

Rubi [A] time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$\frac{128b^3\left(a+bx^4\right)^{5/4}}{3315a^4x^5}-\frac{32b^2\left(a+bx^4\right)^{5/4}}{663a^3x^9}+\frac{12b\left(a+bx^4\right)^{5/4}}{221a^2x^{13}}-\frac{\left(a+bx^4\right)^{5/4}}{17ax^{17}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(1/4)/x^18,x]
```

```
[Out] -(a + b*x^4)^(5/4)/(17*a*x^17) + (12*b*(a + b*x^4)^(5/4))/(221*a^2*x^13) -
(32*b^2*(a + b*x^4)^(5/4))/(663*a^3*x^9) + (128*b^3*(a + b*x^4)^(5/4))/(331
5*a^4*x^5)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{a+bx^4}}{x^{18}}\,dx &= -\frac{\left(a+bx^4\right)^{5/4}}{17ax^{17}}-\frac{(12b)\int \frac{\sqrt[4]{a+bx^4}}{x^{14}}\,dx}{17a}\\
&= -\frac{\left(a+bx^4\right)^{5/4}}{17ax^{17}}+\frac{12b\left(a+bx^4\right)^{5/4}}{221a^2x^{13}}+\frac{\left(96b^2\right)\int \frac{\sqrt[4]{a+bx^4}}{x^{10}}\,dx}{221a^2}\\
&= -\frac{\left(a+bx^4\right)^{5/4}}{17ax^{17}}+\frac{12b\left(a+bx^4\right)^{5/4}}{221a^2x^{13}}-\frac{32b^2\left(a+bx^4\right)^{5/4}}{663a^3x^9}-\frac{\left(128b^3\right)\int \frac{\sqrt[4]{a+bx^4}}{x^6}\,dx}{663a^3}\\
&= -\frac{\left(a+bx^4\right)^{5/4}}{17ax^{17}}+\frac{12b\left(a+bx^4\right)^{5/4}}{221a^2x^{13}}-\frac{32b^2\left(a+bx^4\right)^{5/4}}{663a^3x^9}+\frac{128b^3\left(a+bx^4\right)^{5/4}}{3315a^4x^5}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 53, normalized size = 0.58

$$\frac{\left(a+bx^4\right)^{5/4}\left(-195a^3+180a^2bx^4-160ab^2x^8+128b^3x^{12}\right)}{3315a^4x^{17}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(1/4)/x^18,x]
```

```
[Out] ((a + b*x^4)^(5/4)*(-195*a^3 + 180*a^2*b*x^4 - 160*a*b^2*x^8 + 128*b^3*x^12
))/(3315*a^4*x^17)
```

fricas [A] time = 0.83, size = 60, normalized size = 0.65

$$\frac{\left(128\,b^4x^{16} - 32\,ab^3x^{12} + 20\,a^2b^2x^8 - 15\,a^3bx^4 - 195\,a^4\right)\left(bx^4 + a\right)^{\frac{1}{4}}}{3315\,a^4x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^18,x, algorithm="fricas")
```

```
[Out] 1/3315*(128*b^4*x^16 - 32*a*b^3*x^12 + 20*a^2*b^2*x^8 - 15*a^3*b*x^4 - 195*
a^4)*(b*x^4 + a)^(1/4)/(a^4*x^17)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4 + a\right)^{\frac{1}{4}}}{x^{18}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^18,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(1/4)/x^18, x)
```

maple [A] time = 0.01, size = 50, normalized size = 0.54

$$-\frac{\left(b\,x^4 + a\right)^{\frac{5}{4}}\left(-128b^3x^{12} + 160a\,b^2x^8 - 180a^2b\,x^4 + 195a^3\right)}{3315a^4x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(1/4)/x^18,x)
```

```
[Out] -1/3315*(b*x^4+a)^(5/4)*(-128*b^3*x^12+160*a*b^2*x^8-180*a^2*b*x^4+195*a^3)
/x^17/a^4
```

maxima [A] time = 1.36, size = 69, normalized size = 0.75

$$\frac{\frac{663\left(bx^4+a\right)^{\frac{5}{4}}b^3}{x^5} - \frac{1105\left(bx^4+a\right)^{\frac{9}{4}}b^2}{x^9} + \frac{765\left(bx^4+a\right)^{\frac{13}{4}}b}{x^{13}} - \frac{195\left(bx^4+a\right)^{\frac{17}{4}}}{x^{17}}}{3315\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^18,x, algorithm="maxima")
```

```
[Out] 1/3315*(663*(b*x^4 + a)^(5/4)*b^3/x^5 - 1105*(b*x^4 + a)^(9/4)*b^2/x^9 + 76
5*(b*x^4 + a)^(13/4)*b/x^13 - 195*(b*x^4 + a)^(17/4)/x^17)/a^4
```

mupad [B] time = 1.70, size = 93, normalized size = 1.01

$$\frac{128\,b^4\left(b\,x^4 + a\right)^{1/4}}{3315\,a^4\,x} - \frac{b\left(b\,x^4 + a\right)^{1/4}}{221\,a\,x^{13}} - \frac{\left(b\,x^4 + a\right)^{1/4}}{17\,x^{17}} - \frac{32\,b^3\left(b\,x^4 + a\right)^{1/4}}{3315\,a^3\,x^5} + \frac{4\,b^2\left(b\,x^4 + a\right)^{1/4}}{663\,a^2\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(1/4)/x^18,x)
```

```
[Out] (128*b^4*(a + b*x^4)^(1/4))/(3315*a^4*x) - (b*(a + b*x^4)^(1/4))/(221*a*x^1
3) - (a + b*x^4)^(1/4)/(17*x^17) - (32*b^3*(a + b*x^4)^(1/4))/(3315*a^3*x^5
) + (4*b^2*(a + b*x^4)^(1/4))/(663*a^2*x^9)
```

sympy [**B**] time = 5.85, size = 847, normalized size = 9.21

$$-\frac{585a^7b^{\frac{37}{4}}\sqrt[4]{\frac{a}{bx^4}+1}\,\Gamma\left(-\frac{17}{4}\right)}{256a^7b^9x^{16}\Gamma\left(-\frac{1}{4}\right)+768a^6b^{10}x^{20}\Gamma\left(-\frac{1}{4}\right)+768a^5b^{11}x^{24}\Gamma\left(-\frac{1}{4}\right)+256a^4b^{12}x^{28}\Gamma\left(-\frac{1}{4}\right)}-\frac{}{256a^7b^9x^{16}\Gamma\left(-\frac{1}{4}\right)+76}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(1/4)/x**18,x)
```

```
[Out] -585*a**7*b**(37/4)*(a/(b*x**4) + 1)**(1/4)*gamma(-17/4)/(256*a**7*b**9*x**
16*gamma(-1/4) + 768*a**6*b**10*x**20*gamma(-1/4) + 768*a**5*b**11*x**24*ga
mma(-1/4) + 256*a**4*b**12*x**28*gamma(-1/4)) - 1800*a**6*b**(41/4)*x**4*(a
/(b*x**4) + 1)**(1/4)*gamma(-17/4)/(256*a**7*b**9*x**16*gamma(-1/4) + 768*a
**6*b**10*x**20*gamma(-1/4) + 768*a**5*b**11*x**24*gamma(-1/4) + 256*a**4*b
**12*x**28*gamma(-1/4)) - 1830*a**5*b**(45/4)*x**8*(a/(b*x**4) + 1)**(1/4)*
gamma(-17/4)/(256*a**7*b**9*x**16*gamma(-1/4) + 768*a**6*b**10*x**20*gamma(
-1/4) + 768*a**5*b**11*x**24*gamma(-1/4) + 256*a**4*b**12*x**28*gamma(-1/4)
) - 636*a**4*b**(49/4)*x**12*(a/(b*x**4) + 1)**(1/4)*gamma(-17/4)/(256*a**7
*b**9*x**16*gamma(-1/4) + 768*a**6*b**10*x**20*gamma(-1/4) + 768*a**5*b**11
*x**24*gamma(-1/4) + 256*a**4*b**12*x**28*gamma(-1/4)) + 231*a**3*b**(53/4)
*x**16*(a/(b*x**4) + 1)**(1/4)*gamma(-17/4)/(256*a**7*b**9*x**16*gamma(-1/4
) + 768*a**6*b**10*x**20*gamma(-1/4) + 768*a**5*b**11*x**24*gamma(-1/4) + 2
56*a**4*b**12*x**28*gamma(-1/4)) + 924*a**2*b**(57/4)*x**20*(a/(b*x**4) + 1
)**(1/4)*gamma(-17/4)/(256*a**7*b**9*x**16*gamma(-1/4) + 768*a**6*b**10*x**
20*gamma(-1/4) + 768*a**5*b**11*x**24*gamma(-1/4) + 256*a**4*b**12*x**28*ga
mma(-1/4)) + 1056*a*b**(61/4)*x**24*(a/(b*x**4) + 1)**(1/4)*gamma(-17/4)/(2
56*a**7*b**9*x**16*gamma(-1/4) + 768*a**6*b**10*x**20*gamma(-1/4) + 768*a**
5*b**11*x**24*gamma(-1/4) + 256*a**4*b**12*x**28*gamma(-1/4)) + 384*b**(65/
4)*x**28*(a/(b*x**4) + 1)**(1/4)*gamma(-17/4)/(256*a**7*b**9*x**16*gamma(-1
/4) + 768*a**6*b**10*x**20*gamma(-1/4) + 768*a**5*b**11*x**24*gamma(-1/4) +
 256*a**4*b**12*x**28*gamma(-1/4))
```

3.1009 $\int x^{12}\sqrt[4]{a+bx^4}\,dx$

Optimal. Leaf size=150

$$\frac{3a^{7/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{112b^{5/2}\left(a+bx^4\right)^{3/4}}+\frac{3a^3x\sqrt[4]{a+bx^4}}{112b^3}-\frac{3a^2x^5\sqrt[4]{a+bx^4}}{280b^2}+\frac{1}{14}x^{13}\sqrt[4]{a+bx^4}+\frac{ax^9\sqrt[4]{a+bx^4}}{140b}$$

```
[Out] 3/112*a^3*x*(b*x^4+a)^(1/4)/b^3-3/280*a^2*x^5*(b*x^4+a)^(1/4)/b^2+1/140*a*x
^9*(b*x^4+a)^(1/4)/b+1/14*x^13*(b*x^4+a)^(1/4)+3/112*a^(7/2)*(1+a/b/x^4)^(3
/4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b
^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))/b^
(5/2)/(b*x^4+a)^(3/4)
```

Rubi [A] time = 0.07, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {279, 321, 237, 335, 275, 231}

$$-\frac{3a^2x^5\sqrt[4]{a+bx^4}}{280b^2}+\frac{3a^3x\sqrt[4]{a+bx^4}}{112b^3}+\frac{3a^{7/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{112b^{5/2}\left(a+bx^4\right)^{3/4}}+\frac{1}{14}x^{13}\sqrt[4]{a+bx^4}+\frac{ax^9\sqrt[4]{a+bx^4}}{140b}$$

Antiderivative was successfully verified.

```
[In] Int[x^12*(a + b*x^4)^(1/4),x]
```

```
[Out] (3*a^3*x*(a + b*x^4)^(1/4))/(112*b^3) - (3*a^2*x^5*(a + b*x^4)^(1/4))/(280*
b^2) + (a*x^9*(a + b*x^4)^(1/4))/(140*b) + (x^13*(a + b*x^4)^(1/4))/14 + (3
*a^(7/2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/
2, 2])/(112*b^(5/2)*(a + b*x^4)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int x^{12}\sqrt[4]{a+bx^4}\,dx = \frac{1}{14}x^{13}\sqrt[4]{a+bx^4} + \frac{1}{14}a\int \frac{x^{12}}{\left(a+bx^4\right)^{3/4}}\,dx$$

$$= \frac{ax^9\sqrt[4]{a+bx^4}}{140b} + \frac{1}{14}x^{13}\sqrt[4]{a+bx^4} - \frac{\left(9a^2\right)\int \frac{x^8}{\left(a+bx^4\right)^{3/4}}\,dx}{140b}$$

$$= -\frac{3a^2x^5\sqrt[4]{a+bx^4}}{280b^2} + \frac{ax^9\sqrt[4]{a+bx^4}}{140b} + \frac{1}{14}x^{13}\sqrt[4]{a+bx^4} + \frac{\left(3a^3\right)\int \frac{x^4}{\left(a+bx^4\right)^{3/4}}\,dx}{56b^2}$$

$$= \frac{3a^3x\sqrt[4]{a+bx^4}}{112b^3} - \frac{3a^2x^5\sqrt[4]{a+bx^4}}{280b^2} + \frac{ax^9\sqrt[4]{a+bx^4}}{140b} + \frac{1}{14}x^{13}\sqrt[4]{a+bx^4} - \frac{\left(3a^4\right)\int \frac{1}{\left(a+bx^4\right)^{3/4}}}{112b^3}$$

$$= \frac{3a^3x\sqrt[4]{a+bx^4}}{112b^3} - \frac{3a^2x^5\sqrt[4]{a+bx^4}}{280b^2} + \frac{ax^9\sqrt[4]{a+bx^4}}{140b} + \frac{1}{14}x^{13}\sqrt[4]{a+bx^4} - \frac{\left(3a^4\left(1+\frac{a}{bx^4}\right)^{3/4}\right.}{112b^3\left(\right.}$$

$$= \frac{3a^3x\sqrt[4]{a+bx^4}}{112b^3} - \frac{3a^2x^5\sqrt[4]{a+bx^4}}{280b^2} + \frac{ax^9\sqrt[4]{a+bx^4}}{140b} + \frac{1}{14}x^{13}\sqrt[4]{a+bx^4} + \frac{\left(3a^4\left(1+\frac{a}{bx^4}\right)^{3/4}\right.}{11}$$

$$= \frac{3a^3x\sqrt[4]{a+bx^4}}{112b^3} - \frac{3a^2x^5\sqrt[4]{a+bx^4}}{280b^2} + \frac{ax^9\sqrt[4]{a+bx^4}}{140b} + \frac{1}{14}x^{13}\sqrt[4]{a+bx^4} + \frac{\left(3a^4\left(1+\frac{a}{bx^4}\right)^{3/4}\right.}{2}$$

$$= \frac{3a^3x\sqrt[4]{a+bx^4}}{112b^3} - \frac{3a^2x^5\sqrt[4]{a+bx^4}}{280b^2} + \frac{ax^9\sqrt[4]{a+bx^4}}{140b} + \frac{1}{14}x^{13}\sqrt[4]{a+bx^4} + \frac{3a^{7/2}\left(1+\frac{a}{bx^4}\right)^{3/4}}{112b^{5/}}$$

Mathematica [C] time = 0.08, size = 105, normalized size = 0.70

$$\frac{x\sqrt[4]{a+bx^4}\left(\sqrt[4]{\frac{bx^4}{a}+1}\left(15a^3-3a^2bx^4+2ab^2x^8+20b^3x^{12}\right)-15a^3\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{5}{4};-\frac{bx^4}{a}\right)\right)}{280b^3\sqrt[4]{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12*(a + b*x^4)^(1/4),x]
```

```
[Out] (x*(a + b*x^4)^(1/4)*((1 + (b*x^4)/a)^(1/4)*(15*a^3 - 3*a^2*b*x^4 + 2*a*b^2
*x^8 + 20*b^3*x^12) - 15*a^3*Hypergeometric2F1[-1/4, 1/4, 5/4, -((b*x^4)/a)
]))/(280*b^3*(1 + (b*x^4)/a)^(1/4))
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^4+a\right)^{\frac{1}{4}}x^{12}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(1/4)*x^12, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4+a\right)^{\frac{1}{4}}x^{12}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(1/4)*x^12, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \left(b\,x^4+a\right)^{\frac{1}{4}}x^{12}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(b*x^4+a)^(1/4),x)
```

```
[Out] int(x^12*(b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4+a\right)^{\frac{1}{4}}x^{12}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(1/4)*x^12, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{12}\left(b\,x^4+a\right)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(a + b*x^4)^(1/4),x)
```

```
[Out] int(x^12*(a + b*x^4)^(1/4), x)
```

sympy [C] time = 2.91, size = 39, normalized size = 0.26

$$\frac{\sqrt[4]{a}\,x^{13}\Gamma\left(\frac{13}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4}, \frac{13}{4}\\ \frac{17}{4}\end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{17}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12*(b*x**4+a)**(1/4),x)
```

```
[Out] a**(1/4)*x**13*gamma(13/4)*hyper((-1/4, 13/4), (17/4,), b*x**4*exp_polar(I*
pi)/a)/(4*gamma(17/4))
```

3.1010 $\int x^8 \sqrt[4]{a+bx^4}\,dx$

Optimal. Leaf size=126

$$-\frac{a^{5/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{24b^{3/2}\left(a+bx^4\right)^{3/4}}-\frac{a^2x\sqrt[4]{a+bx^4}}{24b^2}+\frac{1}{10}x^9\sqrt[4]{a+bx^4}+\frac{ax^5\sqrt[4]{a+bx^4}}{60b}$$

```
[Out] -1/24*a^2*x*(b*x^4+a)^(1/4)/b^2+1/60*a*x^5*(b*x^4+a)^(1/4)/b+1/10*x^9*(b*x^
4+a)^(1/4)-1/24*a^(5/2)*(1+a/b/x^4)^(3/4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)/a
^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*ar
ccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))/b^(3/2)/(b*x^4+a)^(3/4)
```

Rubi [A] time = 0.06, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {279, 321, 237, 335, 275, 231}

$$-\frac{a^2x\sqrt[4]{a+bx^4}}{24b^2}-\frac{a^{5/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{24b^{3/2}\left(a+bx^4\right)^{3/4}}+\frac{1}{10}x^9\sqrt[4]{a+bx^4}+\frac{ax^5\sqrt[4]{a+bx^4}}{60b}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x^4)^(1/4),x]
```

```
[Out] -(a^2*x*(a + b*x^4)^(1/4))/(24*b^2) + (a*x^5*(a + b*x^4)^(1/4))/(60*b) + (x
^9*(a + b*x^4)^(1/4))/10 - (a^(5/2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[Arc
Cot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(24*b^(3/2)*(a + b*x^4)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int x^8 \sqrt[4]{a+bx^4}\,dx = \frac{1}{10}x^9\sqrt[4]{a+bx^4} + \frac{1}{10}a\int \frac{x^8}{\left(a+bx^4\right)^{3/4}}\,dx$$

$$= \frac{ax^5\sqrt[4]{a+bx^4}}{60b} + \frac{1}{10}x^9\sqrt[4]{a+bx^4} - \frac{a^2\int \frac{x^4}{\left(a+bx^4\right)^{3/4}}\,dx}{12b}$$

$$= -\frac{a^2x\sqrt[4]{a+bx^4}}{24b^2} + \frac{ax^5\sqrt[4]{a+bx^4}}{60b} + \frac{1}{10}x^9\sqrt[4]{a+bx^4} + \frac{a^3\int \frac{1}{\left(a+bx^4\right)^{3/4}}\,dx}{24b^2}$$

$$= -\frac{a^2x\sqrt[4]{a+bx^4}}{24b^2} + \frac{ax^5\sqrt[4]{a+bx^4}}{60b} + \frac{1}{10}x^9\sqrt[4]{a+bx^4} + \frac{\left(a^3\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{24b^2\left(a+bx^4\right)^{3/4}}$$

$$= -\frac{a^2x\sqrt[4]{a+bx^4}}{24b^2} + \frac{ax^5\sqrt[4]{a+bx^4}}{60b} + \frac{1}{10}x^9\sqrt[4]{a+bx^4} - \frac{\left(a^3\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/}}\right.}{24b^2\left(a+bx^4\right)^{3/4}}$$

$$= -\frac{a^2x\sqrt[4]{a+bx^4}}{24b^2} + \frac{ax^5\sqrt[4]{a+bx^4}}{60b} + \frac{1}{10}x^9\sqrt[4]{a+bx^4} - \frac{\left(a^3\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{3/}}\right.}{48b^2\left(a+bx^4\right)^{3/4}}$$

$$= -\frac{a^2x\sqrt[4]{a+bx^4}}{24b^2} + \frac{ax^5\sqrt[4]{a+bx^4}}{60b} + \frac{1}{10}x^9\sqrt[4]{a+bx^4} - \frac{a^{5/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\right|}{24b^{3/2}\left(a+bx^4\right)^{3/4}}$$

Mathematica [C] time = 0.06, size = 93, normalized size = 0.74

$$\frac{x\sqrt[4]{a+bx^4}\left(\sqrt[4]{\frac{bx^4}{a}+1}\left(-5a^2+abx^4+6b^2x^8\right)+5a^2\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{5}{4};-\frac{bx^4}{a}\right)\right)}{60b^2\sqrt[4]{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x^4)^(1/4),x]
```

```
[Out] (x*(a + b*x^4)^(1/4)*((1 + (b*x^4)/a)^(1/4)*(-5*a^2 + a*b*x^4 + 6*b^2*x^8)
+ 5*a^2*Hypergeometric2F1[-1/4, 1/4, 5/4, -((b*x^4)/a)]))/(60*b^2*(1 + (b*x
^4)/a)^(1/4))
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^4+a\right)^{\frac{1}{4}}x^8, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^4+a)^(1/4),x, algorithm="fricas")
[Out] integral((b*x^4 + a)^(1/4)*x^8, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4+a\right)^{\frac{1}{4}}x^8\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^4+a)^(1/4),x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(1/4)*x^8, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \left(b\,x^4+a\right)^{\frac{1}{4}}x^8\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^4+a)^(1/4),x)
[Out] int(x^8*(b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4+a\right)^{\frac{1}{4}}x^8\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^4+a)^(1/4),x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(1/4)*x^8, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^8\left(b\,x^4+a\right)^{1/4}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*x^4)^(1/4),x)
[Out] int(x^8*(a + b*x^4)^(1/4), x)
```

sympy [C] time = 2.83, size = 39, normalized size = 0.31

$$\frac{\sqrt[4]{a}\,x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{4}, \frac{9}{4}\\ \frac{13}{4}\end{matrix}\right| \frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**4+a)**(1/4),x)
```

```
[Out] a**(1/4)*x**9*gamma(9/4)*hyper((-1/4, 9/4), (13/4,), b*x**4*exp_polar(I*pi)
/a)/(4*gamma(13/4))
```

3.1011 $\int x^4 \sqrt[4]{a+bx^4}\,dx$

Optimal. Leaf size=102

$$\frac{a^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{12\sqrt{b}\left(a+bx^4\right)^{3/4}}+\frac{ax\sqrt[4]{a+bx^4}}{12b}+\frac{1}{6}x^5\sqrt[4]{a+bx^4}$$

```
[Out] 1/12*a*x*(b*x^4+a)^(1/4)/b+1/6*x^5*(b*x^4+a)^(1/4)+1/12*a^(3/2)*(1+a/b/x^4)
^(3/4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^
2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))
/(b*x^4+a)^(3/4)/b^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {279, 321, 237, 335, 275, 231}

$$\frac{a^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{12\sqrt{b}\left(a+bx^4\right)^{3/4}}+\frac{1}{6}x^5\sqrt[4]{a+bx^4}+\frac{ax\sqrt[4]{a+bx^4}}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^4)^(1/4),x]
```

```
[Out] (a*x*(a + b*x^4)^(1/4))/(12*b) + (x^5*(a + b*x^4)^(1/4))/6 + (a^(3/2)*(1 +
a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(12*Sqr
t[b]*(a + b*x^4)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int x^4 \sqrt[4]{a+bx^4}\,dx = \frac{1}{6}x^5\sqrt[4]{a+bx^4} + \frac{1}{6}a\int \frac{x^4}{\left(a+bx^4\right)^{3/4}}\,dx$$

$$= \frac{ax\sqrt[4]{a+bx^4}}{12b} + \frac{1}{6}x^5\sqrt[4]{a+bx^4} - \frac{a^2\int \frac{1}{\left(a+bx^4\right)^{3/4}}\,dx}{12b}$$

$$= \frac{ax\sqrt[4]{a+bx^4}}{12b} + \frac{1}{6}x^5\sqrt[4]{a+bx^4} - \frac{\left(a^2\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{12b\left(a+bx^4\right)^{3/4}}$$

$$= \frac{ax\sqrt[4]{a+bx^4}}{12b} + \frac{1}{6}x^5\sqrt[4]{a+bx^4} + \frac{\left(a^2\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{12b\left(a+bx^4\right)^{3/4}}$$

$$= \frac{ax\sqrt[4]{a+bx^4}}{12b} + \frac{1}{6}x^5\sqrt[4]{a+bx^4} + \frac{\left(a^2\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{x^2}\right)}{24b\left(a+bx^4\right)^{3/4}}$$

$$= \frac{ax\sqrt[4]{a+bx^4}}{12b} + \frac{1}{6}x^5\sqrt[4]{a+bx^4} + \frac{a^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{12\sqrt{b}\,\left(a+bx^4\right)^{3/4}}$$

Mathematica [C] time = 0.07, size = 62, normalized size = 0.61

$$\frac{x\sqrt[4]{a+bx^4}\left(-\frac{a\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{5}{4};-\frac{bx^4}{a}\right)}{\sqrt[4]{\frac{bx^4}{a}+1}} + a + bx^4\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^4)^(1/4),x]
```

```
[Out] (x*(a + b*x^4)^(1/4)*(a + b*x^4 - (a*Hypergeometric2F1[-1/4, 1/4, 5/4, -((b
*x^4)/a)])/(1 + (b*x^4)/a)^(1/4)))/(6*b)
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^4+a\right)^{\frac{1}{4}}x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^4+a)^(1/4),x, algorithm="fricas")
[Out] integral((b*x^4 + a)^(1/4)*x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4 + a\right)^{\frac{1}{4}} x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^4+a)^(1/4),x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(1/4)*x^4, x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \left(b\,x^4 + a\right)^{\frac{1}{4}} x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^4+a)^(1/4),x)
[Out] int(x^4*(b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4 + a\right)^{\frac{1}{4}} x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^4+a)^(1/4),x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(1/4)*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \left(b\,x^4 + a\right)^{1/4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^4)^(1/4),x)
[Out] int(x^4*(a + b*x^4)^(1/4), x)
```

sympy [C] time = 2.04, size = 39, normalized size = 0.38

$$\frac{\sqrt[4]{a}\, x^5 \Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{5}{4} \\ \frac{9}{4} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**4+a)**(1/4),x)
[Out] a**(1/4)*x**5*gamma(5/4)*hyper((-1/4, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/
a)/(4*gamma(9/4))
```

3.1012 $\int \sqrt[4]{a+bx^4}\,dx$

Optimal. Leaf size=80

$$\frac{1}{2}x\sqrt[4]{a+bx^4}-\frac{\sqrt{a}\,\sqrt{b}\,x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{2\left(a+bx^4\right)^{3/4}}$$

```
[Out] 1/2*x*(b*x^4+a)^(1/4)-1/2*(1+a/b/x^4)^(3/4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)
/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*
arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)*b^(1/2)/(b*x^4+a)^(3/4)
```

Rubi [A] time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {195, 237, 335, 275, 231}

$$\frac{1}{2}x\sqrt[4]{a+bx^4}-\frac{\sqrt{a}\,\sqrt{b}\,x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{2\left(a+bx^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(1/4),x]
```

```
[Out] (x*(a + b*x^4)^(1/4))/2 - (Sqrt[a]*Sqrt[b]*(1 + a/(b*x^4))^(3/4)*x^3*Ellipt
icF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(2*(a + b*x^4)^(3/4))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \sqrt[4]{a+bx^4}\,dx &= \frac{1}{2}x\sqrt[4]{a+bx^4} + \frac{1}{2}a\int \frac{1}{\left(a+bx^4\right)^{3/4}}\,dx \\
&= \frac{1}{2}x\sqrt[4]{a+bx^4} + \frac{\left(a\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{2\left(a+bx^4\right)^{3/4}} \\
&= \frac{1}{2}x\sqrt[4]{a+bx^4} - \frac{\left(a\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{2\left(a+bx^4\right)^{3/4}} \\
&= \frac{1}{2}x\sqrt[4]{a+bx^4} - \frac{\left(a\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{x^2}\right)}{4\left(a+bx^4\right)^{3/4}} \\
&= \frac{1}{2}x\sqrt[4]{a+bx^4} - \frac{\sqrt{a}\sqrt{b}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{2\left(a+bx^4\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 46, normalized size = 0.58

$$\frac{x\sqrt[4]{a+bx^4}\;{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{5}{4};-\frac{bx^4}{a}\right)}{\sqrt[4]{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(1/4),x]
```

```
[Out] (x*(a + b*x^4)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 5/4, -((b*x^4)/a)])/(1 +
(b*x^4)/a)^(1/4)
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^4+a\right)^{\frac{1}{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(1/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4+a\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(1/4), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \left(b\,x^4+a\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(1/4),x)
```

```
[Out] int((b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4+a\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(1/4), x)
```

mupad [B] time = 1.03, size = 37, normalized size = 0.46

$$\frac{x\left(b\,x^4+a\right)^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ -\frac{b\,x^4}{a}\right)}{\left(\frac{b\,x^4}{a}+1\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(1/4),x)
```

```
[Out] (x*(a + b*x^4)^(1/4)*hypergeom([-1/4, 1/4], 5/4, -(b*x^4)/a))/((b*x^4)/a +
1)^(1/4)
```

sympy [C] time = 1.45, size = 37, normalized size = 0.46

$$\frac{\sqrt[4]{a}\,x\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\,\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(1/4),x)
```

```
[Out] a**(1/4)*x*gamma(1/4)*hyper((-1/4, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/
(4*gamma(5/4))
```

3.1013 $\int \frac{\sqrt[4]{a+bx^4}}{x^4}\,dx$

Optimal. Leaf size=82

$$-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3x^3}$$

```
[Out] -1/3*(b*x^4+a)^(1/4)/x^3-1/3*b^(3/2)*(1+a/b/x^4)^(3/4)*x^3*(cos(1/2*arccot(
x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))*Ellipti
cF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))/(b*x^4+a)^(3/4)/a^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {277, 237, 335, 275, 231}

$$-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(1/4)/x^4,x]
```

```
[Out] -(a + b*x^4)^(1/4)/(3*x^3) - (b^(3/2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[A
rcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(3*Sqrt[a]*(a + b*x^4)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{a+bx^4}}{x^4}\,dx &= -\frac{\sqrt[4]{a+bx^4}}{3x^3} + \frac{1}{3}b\int \frac{1}{\left(a+bx^4\right)^{3/4}}\,dx \\
&= -\frac{\sqrt[4]{a+bx^4}}{3x^3} + \frac{\left(b\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{3\left(a+bx^4\right)^{3/4}} \\
&= -\frac{\sqrt[4]{a+bx^4}}{3x^3} - \frac{\left(b\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{3\left(a+bx^4\right)^{3/4}} \\
&= -\frac{\sqrt[4]{a+bx^4}}{3x^3} - \frac{\left(b\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{x^2}\right)}{6\left(a+bx^4\right)^{3/4}} \\
&= -\frac{\sqrt[4]{a+bx^4}}{3x^3} - \frac{b^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}\,\left(a+bx^4\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.62

$$-\frac{\sqrt[4]{a+bx^4}\;{}_2F_1\left(-\frac{3}{4},-\frac{1}{4};\frac{1}{4};-\frac{bx^4}{a}\right)}{3x^3\sqrt[4]{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(1/4)/x^4,x]
```

```
[Out] -1/3*((a + b*x^4)^(1/4)*Hypergeometric2F1[-3/4, -1/4, 1/4, -((b*x^4)/a)])/(
x^3*(1 + (b*x^4)/a)^(1/4))
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^4,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(1/4)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^4,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(1/4)/x^4, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{1}{4}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(1/4)/x^4,x)
[Out] int((b*x^4+a)^(1/4)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^4,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(1/4)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^4+a\right)^{1/4}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(1/4)/x^4,x)
[Out] int((a + b*x^4)^(1/4)/x^4, x)
```

sympy [C] time = 1.79, size = 31, normalized size = 0.38

$$-\frac{\sqrt[4]{b}\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\right|\frac{ae^{i\pi}}{bx^4}\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(1/4)/x**4,x)
[Out] -b**(1/4)*hyper((-1/4, 1/2), (3/2,), a*exp_polar(I*pi)/(b*x**4))/(2*x**2)
```

3.1014 $\int \frac{\sqrt[4]{a+bx^4}}{x^8}\,dx$

Optimal. Leaf size=104

$$\frac{2b^{5/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{21a^{3/2}\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{7x^7}-\frac{b\sqrt[4]{a+bx^4}}{21ax^3}$$

```
[Out] -1/7*(b*x^4+a)^(1/4)/x^7-1/21*b*(b*x^4+a)^(1/4)/a/x^3+2/21*b^(5/2)*(1+a/b/x
^4)^(3/4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot
(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/
2))/a^(3/2)/(b*x^4+a)^(3/4)
```

Rubi [A] time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {277, 325, 237, 335, 275, 231}

$$\frac{2b^{5/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{21a^{3/2}\left(a+bx^4\right)^{3/4}}-\frac{b\sqrt[4]{a+bx^4}}{21ax^3}-\frac{\sqrt[4]{a+bx^4}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(1/4)/x^8,x]
```

```
[Out] -(a + b*x^4)^(1/4)/(7*x^7) - (b*(a + b*x^4)^(1/4))/(21*a*x^3) + (2*b^(5/2)*
(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(2
1*a^(3/2)*(a + b*x^4)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{a+bx^4}}{x^8}\,dx &= -\frac{\sqrt[4]{a+bx^4}}{7x^7} + \frac{1}{7}b\int \frac{1}{x^4\left(a+bx^4\right)^{3/4}}\,dx \\
&= -\frac{\sqrt[4]{a+bx^4}}{7x^7} - \frac{b\sqrt[4]{a+bx^4}}{21ax^3} - \frac{\left(2b^2\right)\int \frac{1}{\left(a+bx^4\right)^{3/4}}\,dx}{21a} \\
&= -\frac{\sqrt[4]{a+bx^4}}{7x^7} - \frac{b\sqrt[4]{a+bx^4}}{21ax^3} - \frac{\left(2b^2\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{21a\left(a+bx^4\right)^{3/4}} \\
&= -\frac{\sqrt[4]{a+bx^4}}{7x^7} - \frac{b\sqrt[4]{a+bx^4}}{21ax^3} + \frac{\left(2b^2\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{21a\left(a+bx^4\right)^{3/4}} \\
&= -\frac{\sqrt[4]{a+bx^4}}{7x^7} - \frac{b\sqrt[4]{a+bx^4}}{21ax^3} + \frac{\left(b^2\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{x^2}\right)}{21a\left(a+bx^4\right)^{3/4}} \\
&= -\frac{\sqrt[4]{a+bx^4}}{7x^7} - \frac{b\sqrt[4]{a+bx^4}}{21ax^3} + \frac{2b^{5/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{21a^{3/2}\left(a+bx^4\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.49

$$-\frac{\sqrt[4]{a+bx^4}\;{}_2F_1\left(-\frac{7}{4},-\frac{1}{4};-\frac{3}{4};-\frac{bx^4}{a}\right)}{7x^7\sqrt[4]{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(1/4)/x^8,x]
```

```
[Out] -1/7*((a + b*x^4)^(1/4)*Hypergeometric2F1[-7/4, -1/4, -3/4, -((b*x^4)/a)])/
(x^7*(1 + (b*x^4)/a)^(1/4))
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^8,x, algorithm="fricas")
[Out] integral((b*x^4 + a)^(1/4)/x^8, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^8,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(1/4)/x^8, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{1}{4}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(1/4)/x^8,x)
[Out] int((b*x^4+a)^(1/4)/x^8,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^8,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(1/4)/x^8, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^4+a\right)^{1/4}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(1/4)/x^8,x)
[Out] int((a + b*x^4)^(1/4)/x^8, x)
```

sympy [C] time = 2.43, size = 31, normalized size = 0.30

$$-\frac{\sqrt[4]{b}\,{}_2F_1\left(\begin{matrix}-\frac{1}{4}, \frac{3}{2} \\ \frac{5}{2}\end{matrix}\middle| \frac{ae^{i\pi}}{bx^4}\right)}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(1/4)/x**8,x)
[Out] -b**(1/4)*hyper((-1/4, 3/2), (5/2,), a*exp_polar(I*pi)/(b*x**4))/(6*x**6)
```

3.1015 $\int \frac{\sqrt[4]{a+bx^4}}{x^{12}}\,dx$

Optimal. Leaf size=128

$$-\frac{4b^{7/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{77a^{5/2}\left(a+bx^4\right)^{3/4}}+\frac{2b^2\sqrt[4]{a+bx^4}}{77a^2x^3}-\frac{\sqrt[4]{a+bx^4}}{11x^{11}}-\frac{b\sqrt[4]{a+bx^4}}{77ax^7}$$

```
[Out] -1/11*(b*x^4+a)^(1/4)/x^11-1/77*b*(b*x^4+a)^(1/4)/a/x^7+2/77*b^2*(b*x^4+a)^
(1/4)/a^2/x^3-4/77*b^(7/2)*(1+a/b/x^4)^(3/4)*x^3*(cos(1/2*arccot(x^2*b^(1/2
)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2
*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))/a^(5/2)/(b*x^4+a)^(3/4)
```

Rubi [A] time = 0.06, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {277, 325, 237, 335, 275, 231}

$$\frac{2b^2\sqrt[4]{a+bx^4}}{77a^2x^3}-\frac{4b^{7/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{77a^{5/2}\left(a+bx^4\right)^{3/4}}-\frac{b\sqrt[4]{a+bx^4}}{77ax^7}-\frac{\sqrt[4]{a+bx^4}}{11x^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(1/4)/x^12,x]
```

```
[Out] -(a + b*x^4)^(1/4)/(11*x^11) - (b*(a + b*x^4)^(1/4))/(77*a*x^7) + (2*b^2*(a
 + b*x^4)^(1/4))/(77*a^2*x^3) - (4*b^(7/2)*(1 + a/(b*x^4))^(3/4)*x^3*Ellipt
icF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(77*a^(5/2)*(a + b*x^4)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{12}}\,dx = -\frac{\sqrt[4]{a+bx^4}}{11x^{11}} + \frac{1}{11}b\int \frac{1}{x^8\left(a+bx^4\right)^{3/4}}\,dx$$

$$= -\frac{\sqrt[4]{a+bx^4}}{11x^{11}} - \frac{b\sqrt[4]{a+bx^4}}{77ax^7} - \frac{\left(6b^2\right)\int \frac{1}{x^4\left(a+bx^4\right)^{3/4}}\,dx}{77a}$$

$$= -\frac{\sqrt[4]{a+bx^4}}{11x^{11}} - \frac{b\sqrt[4]{a+bx^4}}{77ax^7} + \frac{2b^2\sqrt[4]{a+bx^4}}{77a^2x^3} + \frac{\left(4b^3\right)\int \frac{1}{\left(a+bx^4\right)^{3/4}}\,dx}{77a^2}$$

$$= -\frac{\sqrt[4]{a+bx^4}}{11x^{11}} - \frac{b\sqrt[4]{a+bx^4}}{77ax^7} + \frac{2b^2\sqrt[4]{a+bx^4}}{77a^2x^3} + \frac{\left(4b^3\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{77a^2\left(a+bx^4\right)^{3/4}}$$

$$= -\frac{\sqrt[4]{a+bx^4}}{11x^{11}} - \frac{b\sqrt[4]{a+bx^4}}{77ax^7} + \frac{2b^2\sqrt[4]{a+bx^4}}{77a^2x^3} - \frac{\left(4b^3\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}\,dx, x,\right.}{77a^2\left(a+bx^4\right)^{3/4}}$$

$$= -\frac{\sqrt[4]{a+bx^4}}{11x^{11}} - \frac{b\sqrt[4]{a+bx^4}}{77ax^7} + \frac{2b^2\sqrt[4]{a+bx^4}}{77a^2x^3} - \frac{\left(2b^3\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{3/4}}\,dx, x,\right.}{77a^2\left(a+bx^4\right)^{3/4}}$$

$$= -\frac{\sqrt[4]{a+bx^4}}{11x^{11}} - \frac{b\sqrt[4]{a+bx^4}}{77ax^7} + \frac{2b^2\sqrt[4]{a+bx^4}}{77a^2x^3} - \frac{4b^{7/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\left.\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\right|2\right)}{77a^{5/2}\left(a+bx^4\right)^{3/4}}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.40

$$-\frac{\sqrt[4]{a+bx^4}\;{}_2F_1\left(-\frac{11}{4},-\frac{1}{4};-\frac{7}{4};-\frac{bx^4}{a}\right)}{11x^{11}\sqrt[4]{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(1/4)/x^12,x]
```

```
[Out] -1/11*((a + b*x^4)^(1/4)*Hypergeometric2F1[-11/4, -1/4, -7/4, -((b*x^4)/a)]
)/(x^11*(1 + (b*x^4)/a)^(1/4))
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x^{12}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^12,x, algorithm="fricas")
[Out] integral((b*x^4 + a)^(1/4)/x^12, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^12,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(1/4)/x^12, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{1}{4}}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(1/4)/x^12,x)
[Out] int((b*x^4+a)^(1/4)/x^12,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^12,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(1/4)/x^12, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^4+a\right)^{1/4}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(1/4)/x^12,x)
[Out] int((a + b*x^4)^(1/4)/x^12, x)
```

sympy [C] time = 2.84, size = 46, normalized size = 0.36

$$\frac{\sqrt[4]{a}\,\Gamma\left(-\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{11}{4}, -\frac{1}{4}\\ -\frac{7}{4}\end{matrix}\middle|\,\frac{bx^4e^{i\pi}}{a}\right)}{4x^{11}\Gamma\left(-\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(1/4)/x**12,x)
```

```
[Out] a**(1/4)*gamma(-11/4)*hyper((-11/4, -1/4), (-7/4,), b*x**4*exp_polar(I*pi)/
a)/(4*x**11*gamma(-7/4))
```

3.1016 $\int \frac{\sqrt[4]{a+bx^4}}{x^{16}}\,dx$

Optimal. Leaf size=152

$$\frac{8b^{9/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{231a^{7/2}\left(a+bx^4\right)^{3/4}}-\frac{4b^3\sqrt[4]{a+bx^4}}{231a^3x^3}+\frac{2b^2\sqrt[4]{a+bx^4}}{231a^2x^7}-\frac{\sqrt[4]{a+bx^4}}{15x^{15}}-\frac{b\sqrt[4]{a+bx^4}}{165ax^{11}}$$

```
[Out] -1/15*(b*x^4+a)^(1/4)/x^15-1/165*b*(b*x^4+a)^(1/4)/a/x^11+2/231*b^2*(b*x^4+
a)^(1/4)/a^2/x^7-4/231*b^3*(b*x^4+a)^(1/4)/a^3/x^3+8/231*b^(9/2)*(1+a/b/x^4
)^(3/4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x
^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2)
)/a^(7/2)/(b*x^4+a)^(3/4)
```

Rubi [A] time = 0.08, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {277, 325, 237, 335, 275, 231}

$$-\frac{4b^3\sqrt[4]{a+bx^4}}{231a^3x^3}+\frac{2b^2\sqrt[4]{a+bx^4}}{231a^2x^7}+\frac{8b^{9/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{231a^{7/2}\left(a+bx^4\right)^{3/4}}-\frac{b\sqrt[4]{a+bx^4}}{165ax^{11}}-\frac{\sqrt[4]{a+bx^4}}{15x^{15}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(1/4)/x^16,x]
```

```
[Out] -(a + b*x^4)^(1/4)/(15*x^15) - (b*(a + b*x^4)^(1/4))/(165*a*x^11) + (2*b^2*
(a + b*x^4)^(1/4))/(231*a^2*x^7) - (4*b^3*(a + b*x^4)^(1/4))/(231*a^3*x^3)
+ (8*b^(9/2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[
a]]/2, 2])/(231*a^(7/2)*(a + b*x^4)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{16}}\,dx = -\frac{\sqrt[4]{a+bx^4}}{15x^{15}} + \frac{1}{15}b\int \frac{1}{x^{12}\left(a+bx^4\right)^{3/4}}\,dx$$

$$= -\frac{\sqrt[4]{a+bx^4}}{15x^{15}} - \frac{b\sqrt[4]{a+bx^4}}{165ax^{11}} - \frac{\left(2b^2\right)\int \frac{1}{x^8\left(a+bx^4\right)^{3/4}}\,dx}{33a}$$

$$= -\frac{\sqrt[4]{a+bx^4}}{15x^{15}} - \frac{b\sqrt[4]{a+bx^4}}{165ax^{11}} + \frac{2b^2\sqrt[4]{a+bx^4}}{231a^2x^7} + \frac{\left(4b^3\right)\int \frac{1}{x^4\left(a+bx^4\right)^{3/4}}\,dx}{77a^2}$$

$$= -\frac{\sqrt[4]{a+bx^4}}{15x^{15}} - \frac{b\sqrt[4]{a+bx^4}}{165ax^{11}} + \frac{2b^2\sqrt[4]{a+bx^4}}{231a^2x^7} - \frac{4b^3\sqrt[4]{a+bx^4}}{231a^3x^3} - \frac{\left(8b^4\right)\int \frac{1}{\left(a+bx^4\right)^{3/4}}\,dx}{231a^3}$$

$$= -\frac{\sqrt[4]{a+bx^4}}{15x^{15}} - \frac{b\sqrt[4]{a+bx^4}}{165ax^{11}} + \frac{2b^2\sqrt[4]{a+bx^4}}{231a^2x^7} - \frac{4b^3\sqrt[4]{a+bx^4}}{231a^3x^3} - \frac{\left(8b^4\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}}}{231a^3\left(a+bx^4\right)^{3/4}}$$

$$= -\frac{\sqrt[4]{a+bx^4}}{15x^{15}} - \frac{b\sqrt[4]{a+bx^4}}{165ax^{11}} + \frac{2b^2\sqrt[4]{a+bx^4}}{231a^2x^7} - \frac{4b^3\sqrt[4]{a+bx^4}}{231a^3x^3} + \frac{\left(8b^4\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int$$

$$= -\frac{\sqrt[4]{a+bx^4}}{15x^{15}} - \frac{b\sqrt[4]{a+bx^4}}{165ax^{11}} + \frac{2b^2\sqrt[4]{a+bx^4}}{231a^2x^7} - \frac{4b^3\sqrt[4]{a+bx^4}}{231a^3x^3} + \frac{\left(4b^4\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int$$

$$= -\frac{\sqrt[4]{a+bx^4}}{15x^{15}} - \frac{b\sqrt[4]{a+bx^4}}{165ax^{11}} + \frac{2b^2\sqrt[4]{a+bx^4}}{231a^2x^7} - \frac{4b^3\sqrt[4]{a+bx^4}}{231a^3x^3} + \frac{8b^{9/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\right.$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.34

$$-\frac{\sqrt[4]{a+bx^4}\;{}_2F_1\left(-\frac{15}{4},-\frac{1}{4};-\frac{11}{4};-\frac{bx^4}{a}\right)}{15x^{15}\sqrt[4]{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(1/4)/x^16,x]
```

```
[Out] -1/15*((a + b*x^4)^(1/4)*Hypergeometric2F1[-15/4, -1/4, -11/4, -((b*x^4)/a)
])/(x^15*(1 + (b*x^4)/a)^(1/4))
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x^{16}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^16,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(1/4)/x^16, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^16,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(1/4)/x^16, x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{1}{4}}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(1/4)/x^16,x)
```

```
[Out] int((b*x^4+a)^(1/4)/x^16,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/x^16,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(1/4)/x^16, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^4+a\right)^{1/4}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(1/4)/x^16,x)
```

```
[Out] int((a + b*x^4)^(1/4)/x^16, x)
```

sympy [C] time = 4.38, size = 46, normalized size = 0.30

$$\frac{\sqrt[4]{a}\,\Gamma\left(-\frac{15}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{15}{4}, -\frac{1}{4} \\ -\frac{11}{4}\end{matrix}\right| \frac{bx^4 e^{i\pi}}{a}\right)}{4x^{15}\Gamma\left(-\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(1/4)/x**16,x)
```

```
[Out] a**(1/4)*gamma(-15/4)*hyper((-15/4, -1/4), (-11/4,), b*x**4*exp_polar(I*pi)
/a)/(4*x**15*gamma(-11/4))
```

3.1017 $\int x^{19}\left(a+bx^4\right)^{3/4}\,dx$

Optimal. Leaf size=101

$$\frac{a^4\left(a+bx^4\right)^{7/4}}{7b^5}-\frac{4a^3\left(a+bx^4\right)^{11/4}}{11b^5}+\frac{2a^2\left(a+bx^4\right)^{15/4}}{5b^5}+\frac{\left(a+bx^4\right)^{23/4}}{23b^5}-\frac{4a\left(a+bx^4\right)^{19/4}}{19b^5}$$

```
[Out] 1/7*a^4*(b*x^4+a)^(7/4)/b^5-4/11*a^3*(b*x^4+a)^(11/4)/b^5+2/5*a^2*(b*x^4+a)
^(15/4)/b^5-4/19*a*(b*x^4+a)^(19/4)/b^5+1/23*(b*x^4+a)^(23/4)/b^5
```

Rubi [A] time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{2a^2\left(a+bx^4\right)^{15/4}}{5b^5}-\frac{4a^3\left(a+bx^4\right)^{11/4}}{11b^5}+\frac{a^4\left(a+bx^4\right)^{7/4}}{7b^5}+\frac{\left(a+bx^4\right)^{23/4}}{23b^5}-\frac{4a\left(a+bx^4\right)^{19/4}}{19b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^19*(a + b*x^4)^(3/4),x]
```

```
[Out] (a^4*(a + b*x^4)^(7/4))/(7*b^5) - (4*a^3*(a + b*x^4)^(11/4))/(11*b^5) + (2*
a^2*(a + b*x^4)^(15/4))/(5*b^5) - (4*a*(a + b*x^4)^(19/4))/(19*b^5) + (a +
b*x^4)^(23/4)/(23*b^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{19}\left(a+bx^4\right)^{3/4}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int x^4(a+bx)^{3/4}\,dx,x,x^4\right)\\ &= \frac{1}{4}\operatorname{Subst}\left(\int\left(\frac{a^4(a+bx)^{3/4}}{b^4}-\frac{4a^3(a+bx)^{7/4}}{b^4}+\frac{6a^2(a+bx)^{11/4}}{b^4}-\frac{4a(a+bx)^{15/4}}{b^4}+\frac{(a+b\right.\right.\\ &= \frac{a^4\left(a+bx^4\right)^{7/4}}{7b^5}-\frac{4a^3\left(a+bx^4\right)^{11/4}}{11b^5}+\frac{2a^2\left(a+bx^4\right)^{15/4}}{5b^5}-\frac{4a\left(a+bx^4\right)^{19/4}}{19b^5}+\frac{\left(a+bx^4\right)^{23/}\end{aligned}$$

Mathematica [A] time = 0.05, size = 61, normalized size = 0.60

$$\frac{\left(a+bx^4\right)^{7/4}\left(2048a^4-3584a^3bx^4+4928a^2b^2x^8-6160ab^3x^{12}+7315b^4x^{16}\right)}{168245b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^19*(a + b*x^4)^(3/4),x]
```

```
[Out] ((a + b*x^4)^(7/4)*(2048*a^4 - 3584*a^3*b*x^4 + 4928*a^2*b^2*x^8 - 6160*a*b
^3*x^12 + 7315*b^4*x^16))/(168245*b^5)
```

fricas [A] time = 0.56, size = 68, normalized size = 0.67

$$\frac{\left(7315\,b^5x^{20}+1155\,ab^4x^{16}-1232\,a^2b^3x^{12}+1344\,a^3b^2x^8-1536\,a^4bx^4+2048\,a^5\right)\left(bx^4+a\right)^{\frac{3}{4}}}{168245\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19*(b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/168245*(7315*b^5*x^20 + 1155*a*b^4*x^16 - 1232*a^2*b^3*x^12 + 1344*a^3*b^
2*x^8 - 1536*a^4*b*x^4 + 2048*a^5)*(b*x^4 + a)^(3/4)/b^5
```

giac [B] time = 0.17, size = 163, normalized size = 1.61

$$\frac{23\left(1155\left(bx^4+a\right)^{\frac{19}{4}}-5852\left(bx^4+a\right)^{\frac{15}{4}}a+11970\left(bx^4+a\right)^{\frac{11}{4}}a^2-12540\left(bx^4+a\right)^{\frac{7}{4}}a^3+7315\left(bx^4+a\right)^{\frac{3}{4}}a^4\right)a}{b^4}+\frac{5\left(4389\left(bx^4+a\right)^{\frac{23}{4}}-26565\left(bx^4+a\right)^{\frac{19}{4}}a+\right.}{504735\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19*(b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] 1/504735*(23*(1155*(b*x^4 + a)^(19/4) - 5852*(b*x^4 + a)^(15/4)*a + 11970*(
b*x^4 + a)^(11/4)*a^2 - 12540*(b*x^4 + a)^(7/4)*a^3 + 7315*(b*x^4 + a)^(3/4
)*a^4)*a/b^4 + 5*(4389*(b*x^4 + a)^(23/4) - 26565*(b*x^4 + a)^(19/4)*a + 67
298*(b*x^4 + a)^(15/4)*a^2 - 91770*(b*x^4 + a)^(11/4)*a^3 + 72105*(b*x^4 +
a)^(7/4)*a^4 - 33649*(b*x^4 + a)^(3/4)*a^5)/b^4)/b
```

maple [A] time = 0.01, size = 58, normalized size = 0.57

$$\frac{\left(b\,x^4+a\right)^{\frac{7}{4}}\left(7315x^{16}b^4-6160a\,x^{12}b^3+4928a^2x^8b^2-3584a^3x^4b+2048a^4\right)}{168245b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19*(b*x^4+a)^(3/4),x)
```

```
[Out] 1/168245*(b*x^4+a)^(7/4)*(7315*b^4*x^16-6160*a*b^3*x^12+4928*a^2*b^2*x^8-35
84*a^3*b*x^4+2048*a^4)/b^5
```

maxima [A] time = 1.35, size = 81, normalized size = 0.80

$$\frac{\left(bx^4+a\right)^{\frac{23}{4}}}{23\,b^5}-\frac{4\left(bx^4+a\right)^{\frac{19}{4}}a}{19\,b^5}+\frac{2\left(bx^4+a\right)^{\frac{15}{4}}a^2}{5\,b^5}-\frac{4\left(bx^4+a\right)^{\frac{11}{4}}a^3}{11\,b^5}+\frac{\left(bx^4+a\right)^{\frac{7}{4}}a^4}{7\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19*(b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] 1/23*(b*x^4 + a)^(23/4)/b^5 - 4/19*(b*x^4 + a)^(19/4)*a/b^5 + 2/5*(b*x^4 +
a)^(15/4)*a^2/b^5 - 4/11*(b*x^4 + a)^(11/4)*a^3/b^5 + 1/7*(b*x^4 + a)^(7/4)
*a^4/b^5
```

mupad [B] time = 1.13, size = 66, normalized size = 0.65

$$\left(b\,x^4+a\right)^{3/4}\left(\frac{x^{20}}{23}+\frac{2048\,a^5}{168245\,b^5}+\frac{3\,a\,x^{16}}{437\,b}-\frac{1536\,a^4\,x^4}{168245\,b^4}+\frac{192\,a^3\,x^8}{24035\,b^3}-\frac{16\,a^2\,x^{12}}{2185\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19*(a + b*x^4)^(3/4),x)
```

```
[Out] (a + b*x^4)^(3/4)*(x^20/23 + (2048*a^5)/(168245*b^5) + (3*a*x^16)/(437*b) -
 (1536*a^4*x^4)/(168245*b^4) + (192*a^3*x^8)/(24035*b^3) - (16*a^2*x^12)/(2
185*b^2))
```

sympy [A] time = 39.72, size = 136, normalized size = 1.35

$$\begin{cases} \frac{2048a^5\left(a+bx^4\right)^{\frac{3}{4}}}{168245b^5} - \frac{1536a^4x^4\left(a+bx^4\right)^{\frac{3}{4}}}{168245b^4} + \frac{192a^3x^8\left(a+bx^4\right)^{\frac{3}{4}}}{24035b^3} - \frac{16a^2x^{12}\left(a+bx^4\right)^{\frac{3}{4}}}{2185b^2} + \frac{3ax^{16}\left(a+bx^4\right)^{\frac{3}{4}}}{437b} + \frac{x^{20}\left(a+bx^4\right)^{\frac{3}{4}}}{23} & \text{for } b \neq 0 \\ \frac{a^{\frac{3}{4}}x^{20}}{20} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**19*(b*x**4+a)**(3/4),x)
```

```
[Out] Piecewise((2048*a**5*(a + b*x**4)**(3/4)/(168245*b**5) - 1536*a**4*x**4*(a
 + b*x**4)**(3/4)/(168245*b**4) + 192*a**3*x**8*(a + b*x**4)**(3/4)/(24035*b
 **3) - 16*a**2*x**12*(a + b*x**4)**(3/4)/(2185*b**2) + 3*a*x**16*(a + b*x**
 4)**(3/4)/(437*b) + x**20*(a + b*x**4)**(3/4)/23, Ne(b, 0)), (a**(3/4)*x**2
 0/20, True))
```

3.1018 $\int x^{15}\left(a+bx^4\right)^{3/4}\,dx$

Optimal. Leaf size=80

$$-\frac{a^3\left(a+bx^4\right)^{7/4}}{7b^4}+\frac{3a^2\left(a+bx^4\right)^{11/4}}{11b^4}+\frac{\left(a+bx^4\right)^{19/4}}{19b^4}-\frac{a\left(a+bx^4\right)^{15/4}}{5b^4}$$

```
[Out] -1/7*a^3*(b*x^4+a)^(7/4)/b^4+3/11*a^2*(b*x^4+a)^(11/4)/b^4-1/5*a*(b*x^4+a)^
(15/4)/b^4+1/19*(b*x^4+a)^(19/4)/b^4
```

Rubi [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{3a^2\left(a+bx^4\right)^{11/4}}{11b^4}-\frac{a^3\left(a+bx^4\right)^{7/4}}{7b^4}+\frac{\left(a+bx^4\right)^{19/4}}{19b^4}-\frac{a\left(a+bx^4\right)^{15/4}}{5b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^15*(a + b*x^4)^(3/4),x]
```

```
[Out] -(a^3*(a + b*x^4)^(7/4))/(7*b^4) + (3*a^2*(a + b*x^4)^(11/4))/(11*b^4) - (a
*(a + b*x^4)^(15/4))/(5*b^4) + (a + b*x^4)^(19/4)/(19*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{15}\left(a+bx^4\right)^{3/4}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int x^3(a+bx)^{3/4}\,dx,x,x^4\right)\\ &= \frac{1}{4}\operatorname{Subst}\left(\int\left(-\frac{a^3(a+bx)^{3/4}}{b^3}+\frac{3a^2(a+bx)^{7/4}}{b^3}-\frac{3a(a+bx)^{11/4}}{b^3}+\frac{(a+bx)^{15/4}}{b^3}\right)dx,x,\\ &= -\frac{a^3\left(a+bx^4\right)^{7/4}}{7b^4}+\frac{3a^2\left(a+bx^4\right)^{11/4}}{11b^4}-\frac{a\left(a+bx^4\right)^{15/4}}{5b^4}+\frac{\left(a+bx^4\right)^{19/4}}{19b^4}\end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.62

$$\frac{\left(a+bx^4\right)^{7/4}\left(-128a^3+224a^2bx^4-308ab^2x^8+385b^3x^{12}\right)}{7315b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15*(a + b*x^4)^(3/4),x]
```

```
[Out] ((a + b*x^4)^(7/4)*(-128*a^3 + 224*a^2*b*x^4 - 308*a*b^2*x^8 + 385*b^3*x^12
))/(7315*b^4)
```

fricas [A] time = 0.65, size = 57, normalized size = 0.71

$$\frac{\left(385\,b^4x^{16}+77\,ab^3x^{12}-84\,a^2b^2x^8+96\,a^3bx^4-128\,a^4\right)\left(bx^4+a\right)^{\frac{3}{4}}}{7315\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/7315*(385*b^4*x^16 + 77*a*b^3*x^12 - 84*a^2*b^2*x^8 + 96*a^3*b*x^4 - 128*
a^4)*(b*x^4 + a)^(3/4)/b^4
```

giac [B] time = 0.20, size = 134, normalized size = 1.68

$$\frac{19\left(77\left(bx^4+a\right)^{\frac{15}{4}}-315\left(bx^4+a\right)^{\frac{11}{4}}a+495\left(bx^4+a\right)^{\frac{7}{4}}a^2-385\left(bx^4+a\right)^{\frac{3}{4}}a^3\right)a}{b^3}+\frac{1155\left(bx^4+a\right)^{\frac{19}{4}}-5852\left(bx^4+a\right)^{\frac{15}{4}}a+11970\left(bx^4+a\right)^{\frac{11}{4}}a^2-12540\left(bx^4+a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] 1/21945*(19*(77*(b*x^4 + a)^(15/4) - 315*(b*x^4 + a)^(11/4)*a + 495*(b*x^4
+ a)^(7/4)*a^2 - 385*(b*x^4 + a)^(3/4)*a^3)*a/b^3 + (1155*(b*x^4 + a)^(19/4
) - 5852*(b*x^4 + a)^(15/4)*a + 11970*(b*x^4 + a)^(11/4)*a^2 - 12540*(b*x^4
 + a)^(7/4)*a^3 + 7315*(b*x^4 + a)^(3/4)*a^4)/b^3)/b
```

maple [A] time = 0.01, size = 47, normalized size = 0.59

$$-\frac{\left(b\,x^4+a\right)^{\frac{7}{4}}\left(-385b^3x^{12}+308a\,b^2x^8-224a^2b\,x^4+128a^3\right)}{7315b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15*(b*x^4+a)^(3/4),x)
```

```
[Out] -1/7315*(b*x^4+a)^(7/4)*(-385*b^3*x^12+308*a*b^2*x^8-224*a^2*b*x^4+128*a^3)
/b^4
```

maxima [A] time = 1.33, size = 64, normalized size = 0.80

$$\frac{\left(bx^4+a\right)^{\frac{19}{4}}}{19\,b^4}-\frac{\left(bx^4+a\right)^{\frac{15}{4}}a}{5\,b^4}+\frac{3\left(bx^4+a\right)^{\frac{11}{4}}a^2}{11\,b^4}-\frac{\left(bx^4+a\right)^{\frac{7}{4}}a^3}{7\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] 1/19*(b*x^4 + a)^(19/4)/b^4 - 1/5*(b*x^4 + a)^(15/4)*a/b^4 + 3/11*(b*x^4 +
a)^(11/4)*a^2/b^4 - 1/7*(b*x^4 + a)^(7/4)*a^3/b^4
```

mupad [B] time = 1.09, size = 55, normalized size = 0.69

$$\left(b\,x^4+a\right)^{3/4}\left(\frac{x^{16}}{19}-\frac{128\,a^4}{7315\,b^4}+\frac{a\,x^{12}}{95\,b}+\frac{96\,a^3\,x^4}{7315\,b^3}-\frac{12\,a^2\,x^8}{1045\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15*(a + b*x^4)^(3/4),x)
```

```
[Out] (a + b*x^4)^(3/4)*(x^16/19 - (128*a^4)/(7315*b^4) + (a*x^12)/(95*b) + (96*a
^3*x^4)/(7315*b^3) - (12*a^2*x^8)/(1045*b^2))
```

sympy [A] time = 22.76, size = 110, normalized size = 1.38

$$\begin{cases} -\frac{128a^4\left(a+bx^4\right)^{\frac{3}{4}}}{7315b^4} + \frac{96a^3x^4\left(a+bx^4\right)^{\frac{3}{4}}}{7315b^3} - \frac{12a^2x^8\left(a+bx^4\right)^{\frac{3}{4}}}{1045b^2} + \frac{ax^{12}\left(a+bx^4\right)^{\frac{3}{4}}}{95b} + \frac{x^{16}\left(a+bx^4\right)^{\frac{3}{4}}}{19} & \text{for } b \neq 0 \\ \frac{a^{\frac{3}{4}}x^{16}}{16} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15*(b*x**4+a)**(3/4),x)
```

```
[Out] Piecewise((-128*a**4*(a + b*x**4)**(3/4)/(7315*b**4) + 96*a**3*x**4*(a + b*
x**4)**(3/4)/(7315*b**3) - 12*a**2*x**8*(a + b*x**4)**(3/4)/(1045*b**2) + a
*x**12*(a + b*x**4)**(3/4)/(95*b) + x**16*(a + b*x**4)**(3/4)/19, Ne(b, 0))
, (a**(3/4)*x**16/16, True))
```

3.1019 $\int x^{11}\left(a+bx^4\right)^{3/4}\,dx$

Optimal. Leaf size=59

$$\frac{a^2\left(a+bx^4\right)^{7/4}}{7b^3}+\frac{\left(a+bx^4\right)^{15/4}}{15b^3}-\frac{2a\left(a+bx^4\right)^{11/4}}{11b^3}$$

```
[Out] 1/7*a^2*(b*x^4+a)^(7/4)/b^3-2/11*a*(b*x^4+a)^(11/4)/b^3+1/15*(b*x^4+a)^(15/
4)/b^3
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{a^2\left(a+bx^4\right)^{7/4}}{7b^3}+\frac{\left(a+bx^4\right)^{15/4}}{15b^3}-\frac{2a\left(a+bx^4\right)^{11/4}}{11b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a + b*x^4)^(3/4),x]
```

```
[Out] (a^2*(a + b*x^4)^(7/4))/(7*b^3) - (2*a*(a + b*x^4)^(11/4))/(11*b^3) + (a +
b*x^4)^(15/4)/(15*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{11}\left(a+bx^4\right)^{3/4}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int x^2(a+bx)^{3/4}\,dx,x,x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int\left(\frac{a^2(a+bx)^{3/4}}{b^2}-\frac{2a(a+bx)^{7/4}}{b^2}+\frac{(a+bx)^{11/4}}{b^2}\right)dx,x,x^4\right)\\ &= \frac{a^2\left(a+bx^4\right)^{7/4}}{7b^3}-\frac{2a\left(a+bx^4\right)^{11/4}}{11b^3}+\frac{\left(a+bx^4\right)^{15/4}}{15b^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 39, normalized size = 0.66

$$\frac{\left(a+bx^4\right)^{7/4}\left(32a^2-56abx^4+77b^2x^8\right)}{1155b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*x^4)^(3/4),x]
```

```
[Out] ((a + b*x^4)^(7/4)*(32*a^2 - 56*a*b*x^4 + 77*b^2*x^8))/(1155*b^3)
```

fricas [A] time = 0.77, size = 46, normalized size = 0.78

$$\frac{\left(77\,b^3x^{12}+21\,ab^2x^8-24\,a^2bx^4+32\,a^3\right)\left(bx^4+a\right)^{\frac{3}{4}}}{1155\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/1155*(77*b^3*x^12 + 21*a*b^2*x^8 - 24*a^2*b*x^4 + 32*a^3)*(b*x^4 + a)^(3/
4)/b^3
```

giac [B] time = 0.16, size = 106, normalized size = 1.80

$$\frac{\frac{5\left(21\left(bx^4+a\right)^{\frac{11}{4}}-66\left(bx^4+a\right)^{\frac{7}{4}}a+77\left(bx^4+a\right)^{\frac{3}{4}}a^2\right)a}{b^2}+\frac{77\left(bx^4+a\right)^{\frac{15}{4}}-315\left(bx^4+a\right)^{\frac{11}{4}}a+495\left(bx^4+a\right)^{\frac{7}{4}}a^2-385\left(bx^4+a\right)^{\frac{3}{4}}a^3}{b^2}}{1155\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] 1/1155*(5*(21*(b*x^4 + a)^(11/4) - 66*(b*x^4 + a)^(7/4)*a + 77*(b*x^4 + a)^
(3/4)*a^2)*a/b^2 + (77*(b*x^4 + a)^(15/4) - 315*(b*x^4 + a)^(11/4)*a + 495*
(b*x^4 + a)^(7/4)*a^2 - 385*(b*x^4 + a)^(3/4)*a^3)/b^2)/b
```

maple [A] time = 0.01, size = 36, normalized size = 0.61

$$\frac{\left(b\,x^4+a\right)^{\frac{7}{4}}\left(77b^2x^8-56ab\,x^4+32a^2\right)}{1155b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x^4+a)^(3/4),x)
```

```
[Out] 1/1155*(b*x^4+a)^(7/4)*(77*b^2*x^8-56*a*b*x^4+32*a^2)/b^3
```

maxima [A] time = 1.30, size = 47, normalized size = 0.80

$$\frac{\left(bx^4+a\right)^{\frac{15}{4}}}{15\,b^3}-\frac{2\left(bx^4+a\right)^{\frac{11}{4}}a}{11\,b^3}+\frac{\left(bx^4+a\right)^{\frac{7}{4}}a^2}{7\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] 1/15*(b*x^4 + a)^(15/4)/b^3 - 2/11*(b*x^4 + a)^(11/4)*a/b^3 + 1/7*(b*x^4 +
a)^(7/4)*a^2/b^3
```

mupad [B] time = 1.08, size = 44, normalized size = 0.75

$$\left(b\,x^4+a\right)^{3/4}\left(\frac{x^{12}}{15}+\frac{32\,a^3}{1155\,b^3}+\frac{a\,x^8}{55\,b}-\frac{8\,a^2\,x^4}{385\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a + b*x^4)^(3/4),x)
```

```
[Out] (a + b*x^4)^(3/4)*(x^12/15 + (32*a^3)/(1155*b^3) + (a*x^8)/(55*b) - (8*a^2*
x^4)/(385*b^2))
```

sympy [**A**] time = 15.89, size = 87, normalized size = 1.47

$$\begin{cases} \frac{32a^3\left(a+bx^4\right)^{\frac{3}{4}}}{1155b^3} - \frac{8a^2x^4\left(a+bx^4\right)^{\frac{3}{4}}}{385b^2} + \frac{ax^8\left(a+bx^4\right)^{\frac{3}{4}}}{55b} + \frac{x^{12}\left(a+bx^4\right)^{\frac{3}{4}}}{15} & \text{for } b \neq 0 \\ \frac{a^{\frac{3}{4}}x^{12}}{12} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x**4+a)**(3/4),x)
```

```
[Out] Piecewise((32*a**3*(a + b*x**4)**(3/4)/(1155*b**3) - 8*a**2*x**4*(a + b*x**
4)**(3/4)/(385*b**2) + a*x**8*(a + b*x**4)**(3/4)/(55*b) + x**12*(a + b*x**
4)**(3/4)/15, Ne(b, 0)), (a**(3/4)*x**12/12, True))
```

3.1020 $\int x^7 \left(a+bx^4\right)^{3/4}\,dx$

Optimal. Leaf size=38

$$\frac{\left(a+bx^4\right)^{11/4}}{11b^2}-\frac{a\left(a+bx^4\right)^{7/4}}{7b^2}$$

```
[Out] -1/7*a*(b*x^4+a)^(7/4)/b^2+1/11*(b*x^4+a)^(11/4)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{\left(a+bx^4\right)^{11/4}}{11b^2}-\frac{a\left(a+bx^4\right)^{7/4}}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^4)^(3/4),x]
```

```
[Out] -(a*(a + b*x^4)^(7/4))/(7*b^2) + (a + b*x^4)^(11/4)/(11*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^7\left(a+bx^4\right)^{3/4}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int x(a+bx)^{3/4}\,dx, x, x^4\right)\\ &= \frac{1}{4}\operatorname{Subst}\left(\int\left(-\frac{a(a+bx)^{3/4}}{b}+\frac{(a+bx)^{7/4}}{b}\right)dx, x, x^4\right)\\ &= -\frac{a\left(a+bx^4\right)^{7/4}}{7b^2}+\frac{\left(a+bx^4\right)^{11/4}}{11b^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 0.74

$$\frac{\left(a+bx^4\right)^{7/4}\left(7bx^4-4a\right)}{77b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^4)^(3/4),x]
```

```
[Out] ((a + b*x^4)^(7/4)*(-4*a + 7*b*x^4))/(77*b^2)
```

fricas [A] time = 0.81, size = 35, normalized size = 0.92

$$\frac{\left(7\,b^2x^8+3\,abx^4-4\,a^2\right)\left(bx^4+a\right)^{\frac{3}{4}}}{77\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/77*(7*b^2*x^8 + 3*a*b*x^4 - 4*a^2)*(b*x^4 + a)^(3/4)/b^2
```

giac [B] time = 0.16, size = 78, normalized size = 2.05

$$\frac{\frac{11\left(3\left(bx^4+a\right)^{\frac{7}{4}}-7\left(bx^4+a\right)^{\frac{3}{4}}a\right)a}{b}+\frac{21\left(bx^4+a\right)^{\frac{11}{4}}-66\left(bx^4+a\right)^{\frac{7}{4}}a+77\left(bx^4+a\right)^{\frac{3}{4}}a^2}{b}}{231\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] 1/231*(11*(3*(b*x^4 + a)^(7/4) - 7*(b*x^4 + a)^(3/4)*a)*a/b + (21*(b*x^4 +
a)^(11/4) - 66*(b*x^4 + a)^(7/4)*a + 77*(b*x^4 + a)^(3/4)*a^2)/b)/b
```

maple [A] time = 0.00, size = 25, normalized size = 0.66

$$-\frac{\left(b\,x^4+a\right)^{\frac{7}{4}}\left(-7b\,x^4+4a\right)}{77b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^4+a)^(3/4),x)
```

```
[Out] -1/77*(b*x^4+a)^(7/4)*(-7*b*x^4+4*a)/b^2
```

maxima [A] time = 1.36, size = 30, normalized size = 0.79

$$\frac{\left(bx^4+a\right)^{\frac{11}{4}}}{11\,b^2}-\frac{\left(bx^4+a\right)^{\frac{7}{4}}a}{7\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] 1/11*(b*x^4 + a)^(11/4)/b^2 - 1/7*(b*x^4 + a)^(7/4)*a/b^2
```

mupad [B] time = 1.08, size = 33, normalized size = 0.87

$$\left(b\,x^4+a\right)^{3/4}\left(\frac{x^8}{11}-\frac{4\,a^2}{77\,b^2}+\frac{3\,a\,x^4}{77\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^4)^(3/4),x)
```

```
[Out] (a + b*x^4)^(3/4)*(x^8/11 - (4*a^2)/(77*b^2) + (3*a*x^4)/(77*b))
```

sympy [A] time = 8.26, size = 65, normalized size = 1.71

$$\begin{cases}-\frac{4a^2\left(a+bx^4\right)^{\frac{3}{4}}}{77b^2}+\frac{3ax^4\left(a+bx^4\right)^{\frac{3}{4}}}{77b}+\frac{x^8\left(a+bx^4\right)^{\frac{3}{4}}}{11} & \text{for } b\neq 0\\ \frac{a^{\frac{3}{4}}x^8}{8} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**4+a)**(3/4),x)
```

```
[Out] Piecewise((-4*a**2*(a + b*x**4)**(3/4)/(77*b**2) + 3*a*x**4*(a + b*x**4)**(
3/4)/(77*b) + x**8*(a + b*x**4)**(3/4)/11, Ne(b, 0)), (a**(3/4)*x**8/8, Tru
e))
```

3.1021 $\int x^3 \left(a + bx^4\right)^{3/4} \, dx$

Optimal. Leaf size=18

$$\frac{\left(a + bx^4\right)^{7/4}}{7b}$$

```
[Out] 1/7*(b*x^4+a)^(7/4)/b
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$\frac{\left(a + bx^4\right)^{7/4}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^4)^(3/4),x]
```

```
[Out] (a + b*x^4)^(7/4)/(7*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^3 \left(a + bx^4\right)^{3/4} \, dx = \frac{\left(a + bx^4\right)^{7/4}}{7b}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{\left(a + bx^4\right)^{7/4}}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^4)^(3/4),x]
```

```
[Out] (a + b*x^4)^(7/4)/(7*b)
```

fricas [A] time = 0.89, size = 14, normalized size = 0.78

$$\frac{\left(bx^4 + a\right)^{\frac{7}{4}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/7*(b*x^4 + a)^(7/4)/b
```

giac [A] time = 0.18, size = 14, normalized size = 0.78

$$\frac{\left(bx^4 + a\right)^{\frac{7}{4}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^4+a)^(3/4),x, algorithm="giac")
[Out] 1/7*(b*x^4 + a)^(7/4)/b
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$\frac{\left(b\,x^4+a\right)^{\frac{7}{4}}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^4+a)^(3/4),x)
[Out] 1/7*(b*x^4+a)^(7/4)/b
```

maxima [A] time = 1.27, size = 14, normalized size = 0.78

$$\frac{\left(bx^4+a\right)^{\frac{7}{4}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^4+a)^(3/4),x, algorithm="maxima")
[Out] 1/7*(b*x^4 + a)^(7/4)/b
```

mupad [B] time = 1.07, size = 14, normalized size = 0.78

$$\frac{\left(b\,x^4+a\right)^{7/4}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^4)^(3/4),x)
[Out] (a + b*x^4)^(7/4)/(7*b)
```

sympy [A] time = 1.80, size = 39, normalized size = 2.17

$$\begin{cases} \frac{a\left(a+bx^4\right)^{\frac{3}{4}}}{7b}+\frac{x^4\left(a+bx^4\right)^{\frac{3}{4}}}{7} & \text{for } b\neq 0 \\ \frac{a^{\frac{3}{4}}x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**4+a)**(3/4),x)
[Out] Piecewise((a*(a + b*x**4)**(3/4)/(7*b) + x**4*(a + b*x**4)**(3/4)/7, Ne(b,
0)), (a**(3/4)*x**4/4, True))
```

3.1022 $\int \frac{\left(a+bx^4\right)^{3/4}}{x}\,dx$

Optimal. Leaf size=70

$$\frac{1}{2}a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)-\frac{1}{2}a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)+\frac{1}{3}\left(a+bx^4\right)^{3/4}$$

```
[Out] 1/3*(b*x^4+a)^(3/4)+1/2*a^(3/4)*arctan((b*x^4+a)^(1/4)/a^(1/4))-1/2*a^(3/4)
*arctanh((b*x^4+a)^(1/4)/a^(1/4))
```

Rubi [A] time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {266, 50, 63, 298, 203, 206}

$$\frac{1}{2}a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)-\frac{1}{2}a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)+\frac{1}{3}\left(a+bx^4\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(3/4)/x,x]
```

```
[Out] (a + b*x^4)^(3/4)/3 + (a^(3/4)*ArcTan[(a + b*x^4)^(1/4)/a^(1/4)])/2 - (a^(3
/4)*ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)])/2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^4\right)^{3/4}}{x}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{(a+bx)^{3/4}}{x}\,dx, x, x^4\right)\\
&= \frac{1}{3}\left(a+bx^4\right)^{3/4} + \frac{1}{4}a\,\text{Subst}\left(\int \frac{1}{x\sqrt[4]{a+bx}}\,dx, x, x^4\right)\\
&= \frac{1}{3}\left(a+bx^4\right)^{3/4} + \frac{a\,\text{Subst}\left(\int \frac{x^2}{-\frac{a}{b}+\frac{x^4}{b}}\,dx, x, \sqrt[4]{a+bx^4}\right)}{b}\\
&= \frac{1}{3}\left(a+bx^4\right)^{3/4} - \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{\sqrt{a}-x^2}\,dx, x, \sqrt[4]{a+bx^4}\right) + \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{\sqrt{a}+x^2}\,dx, x, \right.\\
&= \frac{1}{3}\left(a+bx^4\right)^{3/4} + \frac{1}{2}a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right) - \frac{1}{2}a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 70, normalized size = 1.00

$$\frac{1}{2}a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right) - \frac{1}{2}a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right) + \frac{1}{3}\left(a+bx^4\right)^{3/4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(3/4)/x,x]
```

```
[Out] (a + b*x^4)^(3/4)/3 + (a^(3/4)*ArcTan[(a + b*x^4)^(1/4)/a^(1/4)])/2 - (a^(3
/4)*ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)])/2
```

fricas [B] time = 0.93, size = 132, normalized size = 1.89

$$-\left(a^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(bx^4+a\right)^{\frac{1}{4}}\left(a^3\right)^{\frac{1}{4}}a^2 - \sqrt{\sqrt{bx^4+a}\,a^4+\sqrt{a^3}\,a^3}\left(a^3\right)^{\frac{1}{4}}}{a^3}\right) - \frac{1}{4}\left(a^3\right)^{\frac{1}{4}}\log\left(\left(bx^4+a\right)^{\frac{1}{4}}a^2+\left(a^3\right)^{\frac{3}{4}}\right)+\frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x,x, algorithm="fricas")
```

```
[Out] -(a^3)^(1/4)*arctan(-((b*x^4 + a)^(1/4)*(a^3)^(1/4)*a^2 - sqrt(sqrt(b*x^4 +
 a)*a^4 + sqrt(a^3)*a^3)*(a^3)^(1/4))/a^3) - 1/4*(a^3)^(1/4)*log((b*x^4 + a
)^(1/4)*a^2 + (a^3)^(3/4)) + 1/4*(a^3)^(1/4)*log((b*x^4 + a)^(1/4)*a^2 - (a
^3)^(3/4)) + 1/3*(b*x^4 + a)^(3/4)
```

giac [B] time = 0.17, size = 185, normalized size = 2.64

$$-\frac{1}{4}\sqrt{2}\,(-a)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}+2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right) - \frac{1}{4}\sqrt{2}\,(-a)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}-2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(-a)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x^4 +
 a)^(1/4))/(-a)^(1/4)) - 1/4*sqrt(2)*(-a)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2
)*(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4)) + 1/8*sqrt(2)*(-a)^(3/4)*lo
g(sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt(-a)) - 1/8*
sqrt(2)*(-a)^(3/4)*log(-sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 +
 a) + sqrt(-a)) + 1/3*(b*x^4 + a)^(3/4)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{3}{4}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(3/4)/x,x)
```

```
[Out] int((b*x^4+a)^(3/4)/x,x)
```

maxima [A] time = 2.92, size = 71, normalized size = 1.01

$$\frac{1}{4}\,a\left(\frac{2\,\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}+\frac{\log\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{\left(bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}\right)+\frac{1}{3}\left(bx^4+a\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x,x, algorithm="maxima")
```

```
[Out] 1/4*a*(2*arctan((b*x^4 + a)^(1/4)/a^(1/4))/a^(1/4) + log(((b*x^4 + a)^(1/4)
 - a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/4)))/a^(1/4)) + 1/3*(b*x^4 + a)^(3/4)
```

mupad [B] time = 1.14, size = 50, normalized size = 0.71

$$\frac{a^{3/4}\,\mathrm{atan}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)}{2}-\frac{a^{3/4}\,\mathrm{atanh}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)}{2}+\frac{\left(b\,x^4+a\right)^{3/4}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(3/4)/x,x)
```

```
[Out] (a^(3/4)*atan((a + b*x^4)^(1/4)/a^(1/4)))/2 - (a^(3/4)*atanh((a + b*x^4)^(1
/4)/a^(1/4)))/2 + (a + b*x^4)^(3/4)/3
```

sympy [C] time = 1.99, size = 44, normalized size = 0.63

$$-\frac{b^{\frac{3}{4}}x^3\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{4},-\frac{3}{4}\\ \frac{1}{4}\end{matrix}\right|\frac{ae^{i\pi}}{bx^4}\right)}{4\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(3/4)/x,x)

[Out] -b**(3/4)*x**3*gamma(-3/4)*hyper((-3/4, -3/4), (1/4,), a*exp_polar(I*pi)/(b
*x**4))/(4*gamma(1/4))
```

3.1023 $\int \frac{\left(a+bx^4\right)^{3/4}}{x^5}\,dx$

Optimal. Leaf size=75

$$-\frac{\left(a+bx^4\right)^{3/4}}{4x^4}+\frac{3b\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}}-\frac{3b\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}}$$

```
[Out] -1/4*(b*x^4+a)^(3/4)/x^4+3/8*b*arctan((b*x^4+a)^(1/4)/a^(1/4))/a^(1/4)-3/8*
b*arctanh((b*x^4+a)^(1/4)/a^(1/4))/a^(1/4)
```

Rubi [A] time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {266, 47, 63, 298, 203, 206}

$$-\frac{\left(a+bx^4\right)^{3/4}}{4x^4}+\frac{3b\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}}-\frac{3b\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(3/4)/x^5,x]
```

```
[Out] -(a + b*x^4)^(3/4)/(4*x^4) + (3*b*ArcTan[(a + b*x^4)^(1/4)/a^(1/4)])/(8*a^(
1/4)) - (3*b*ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)])/(8*a^(1/4))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^4\right)^{3/4}}{x^5}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{(a+bx)^{3/4}}{x^2}\,dx, x, x^4\right)\\
&= -\frac{\left(a+bx^4\right)^{3/4}}{4x^4} + \frac{1}{16}(3b)\,\text{Subst}\left(\int \frac{1}{x\sqrt[4]{a+bx}}\,dx, x, x^4\right)\\
&= -\frac{\left(a+bx^4\right)^{3/4}}{4x^4} + \frac{3}{4}\,\text{Subst}\left(\int \frac{x^2}{-\frac{a}{b}+\frac{x^4}{b}}\,dx, x, \sqrt[4]{a+bx^4}\right)\\
&= -\frac{\left(a+bx^4\right)^{3/4}}{4x^4} - \frac{1}{8}(3b)\,\text{Subst}\left(\int \frac{1}{\sqrt{a}-x^2}\,dx, x, \sqrt[4]{a+bx^4}\right) + \frac{1}{8}(3b)\,\text{Subst}\left(\int \frac{1}{\sqrt{a}+x^2}\,d\right.\\
&= -\frac{\left(a+bx^4\right)^{3/4}}{4x^4} + \frac{3b\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}} - \frac{3b\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 37, normalized size = 0.49

$$\frac{b\left(a+bx^4\right)^{7/4}\,{}_2F_1\left(\frac{7}{4},2;\frac{11}{4};\frac{bx^4}{a}+1\right)}{7a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(3/4)/x^5,x]
```

```
[Out] (b*(a + b*x^4)^(7/4)*Hypergeometric2F1[7/4, 2, 11/4, 1 + (b*x^4)/a])/(7*a^2
)
```

fricas [B] time = 0.66, size = 185, normalized size = 2.47

$$-\frac{12\left(\frac{b^4}{a}\right)^{\frac{1}{4}}x^4\arctan\left(-\frac{\left(bx^4+a\right)^{\frac{1}{4}}\left(\frac{b^4}{a}\right)^{\frac{1}{4}}b^3-\sqrt{\sqrt{bx^4+a}\,b^6+\sqrt{\frac{b^4}{a}}ab^4}\left(\frac{b^4}{a}\right)^{\frac{1}{4}}}{b^4}\right)+3\left(\frac{b^4}{a}\right)^{\frac{1}{4}}x^4\log\left(27\left(bx^4+a\right)^{\frac{1}{4}}b^3+27\left(\frac{b^4}{a}\right)^{\frac{3}{4}}\right.}{16\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^5,x, algorithm="fricas")
```

```
[Out] -1/16*(12*(b^4/a)^(1/4)*x^4*arctan(-((b*x^4 + a)^(1/4)*(b^4/a)^(1/4)*b^3 -
sqrt(sqrt(b*x^4 + a)*b^6 + sqrt(b^4/a)*a*b^4)*(b^4/a)^(1/4))/b^4) + 3*(b^4/
a)^(1/4)*x^4*log(27*(b*x^4 + a)^(1/4)*b^3 + 27*(b^4/a)^(3/4)*a) - 3*(b^4/a)
^(1/4)*x^4*log(27*(b*x^4 + a)^(1/4)*b^3 - 27*(b^4/a)^(3/4)*a) + 4*(b*x^4 +
a)^(3/4))/x^4
```

giac [B] time = 0.21, size = 209, normalized size = 2.79

$$\frac{6\sqrt{2}\,b^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{1}{4}}}+\frac{6\sqrt{2}\,b^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{1}{4}}}+\frac{3\sqrt{2}\,(-a)^{\frac{3}{4}}b^2\log\left(\sqrt{2}\left(bx^4+a\right)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^4+a}+\sqrt{-a}\right)}{a}}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^5,x, algorithm="giac")
```

```
[Out] 1/32*(6*sqrt(2)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x^4 + a)^
(1/4))/(-a)^(1/4))/(-a)^(1/4) + 6*sqrt(2)*b^2*arctan(-1/2*sqrt(2)*(sqrt(2)*
(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4))/(-a)^(1/4) + 3*sqrt(2)*(-a)^(
3/4)*b^2*log(sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt(
-a))/a + 3*sqrt(2)*b^2*log(-sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x
^4 + a) + sqrt(-a))/(-a)^(1/4) - 8*(b*x^4 + a)^(3/4)*b/x^4)/b
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{3}{4}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(3/4)/x^5,x)
```

```
[Out] int((b*x^4+a)^(3/4)/x^5,x)
```

maxima [A] time = 3.01, size = 74, normalized size = 0.99

$$\frac{3}{16}\,b\left(\frac{2\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}+\frac{\log\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{\left(bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}\right)-\frac{\left(bx^4+a\right)^{\frac{3}{4}}}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^5,x, algorithm="maxima")
```

```
[Out] 3/16*b*(2*arctan((b*x^4 + a)^(1/4)/a^(1/4))/a^(1/4) + log(((b*x^4 + a)^(1/4
) - a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/4)))/a^(1/4)) - 1/4*(b*x^4 + a)^(3/4
)/x^4
```

mupad [B] time = 1.26, size = 55, normalized size = 0.73

$$\frac{3\,b\,\operatorname{atan}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)}{8\,a^{1/4}}-\frac{\left(b\,x^4+a\right)^{3/4}}{4\,x^4}-\frac{3\,b\,\operatorname{atanh}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)}{8\,a^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(3/4)/x^5,x)
```

```
[Out] (3*b*atan((a + b*x^4)^(1/4)/a^(1/4)))/(8*a^(1/4)) - (a + b*x^4)^(3/4)/(4*x^
4) - (3*b*atanh((a + b*x^4)^(1/4)/a^(1/4)))/(8*a^(1/4))
```

sympy [C] time = 3.05, size = 39, normalized size = 0.52

$$-\frac{b^{\frac{3}{4}}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{4}, \frac{1}{4} \\ \frac{5}{4}\end{matrix}\right| \frac{ae^{i\pi}}{bx^4}\right)}{4x\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(3/4)/x**5,x)
```

```
[Out] -b**(3/4)*gamma(1/4)*hyper((-3/4, 1/4), (5/4,), a*exp_polar(I*pi)/(b*x**4))
/(4*x*gamma(5/4))
```

3.1024 $\int \frac{\left(a+bx^4\right)^{3/4}}{x^9}\,dx$

Optimal. Leaf size=101

$$-\frac{3b^2\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{5/4}}+\frac{3b^2\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{5/4}}-\frac{3b\left(a+bx^4\right)^{3/4}}{32ax^4}-\frac{\left(a+bx^4\right)^{3/4}}{8x^8}$$

```
[Out] -1/8*(b*x^4+a)^(3/4)/x^8-3/32*b*(b*x^4+a)^(3/4)/a/x^4-3/64*b^2*arctan((b*x^
4+a)^(1/4)/a^(1/4))/a^(5/4)+3/64*b^2*arctanh((b*x^4+a)^(1/4)/a^(1/4))/a^(5/
4)
```

Rubi [A] time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {266, 47, 51, 63, 298, 203, 206}

$$-\frac{3b^2\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{5/4}}+\frac{3b^2\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{5/4}}-\frac{3b\left(a+bx^4\right)^{3/4}}{32ax^4}-\frac{\left(a+bx^4\right)^{3/4}}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(3/4)/x^9,x]
```

```
[Out] -(a + b*x^4)^(3/4)/(8*x^8) - (3*b*(a + b*x^4)^(3/4))/(32*a*x^4) - (3*b^2*Ar
cTan[(a + b*x^4)^(1/4)/a^(1/4)])/(64*a^(5/4)) + (3*b^2*ArcTanh[(a + b*x^4)^
(1/4)/a^(1/4)])/(64*a^(5/4))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^4\right)^{3/4}}{x^9}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{(a+bx)^{3/4}}{x^3}\,dx, x, x^4\right)\\
&= -\frac{\left(a+bx^4\right)^{3/4}}{8x^8} + \frac{1}{32}(3b)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt[4]{a+bx}}\,dx, x, x^4\right)\\
&= -\frac{\left(a+bx^4\right)^{3/4}}{8x^8} - \frac{3b\left(a+bx^4\right)^{3/4}}{32ax^4} - \frac{\left(3b^2\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt[4]{a+bx}}\,dx, x, x^4\right)}{128a}\\
&= -\frac{\left(a+bx^4\right)^{3/4}}{8x^8} - \frac{3b\left(a+bx^4\right)^{3/4}}{32ax^4} - \frac{(3b)\operatorname{Subst}\left(\int \frac{x^2}{-\frac{a}{b}+\frac{x^4}{b}}\,dx, x, \sqrt[4]{a+bx^4}\right)}{32a}\\
&= -\frac{\left(a+bx^4\right)^{3/4}}{8x^8} - \frac{3b\left(a+bx^4\right)^{3/4}}{32ax^4} + \frac{\left(3b^2\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a}-x^2}\,dx, x, \sqrt[4]{a+bx^4}\right)}{64a} - \frac{\left(3b^2\right)\operatorname{Subst}}{}\\
&= -\frac{\left(a+bx^4\right)^{3/4}}{8x^8} - \frac{3b\left(a+bx^4\right)^{3/4}}{32ax^4} - \frac{3b^2\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{5/4}} + \frac{3b^2\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{5/4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.39

$$-\frac{b^2\left(a+bx^4\right)^{7/4}\,{}_2F_1\left(\frac{7}{4},3;\frac{11}{4};\frac{bx^4}{a}+1\right)}{7a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(3/4)/x^9,x]
```

```
[Out] -1/7*(b^2*(a + b*x^4)^(7/4)*Hypergeometric2F1[7/4, 3, 11/4, 1 + (b*x^4)/a])
/a^3
```

fricas [B] time = 0.94, size = 209, normalized size = 2.07

$$\frac{12\left(\frac{b^8}{a^5}\right)^{\frac{1}{4}} ax^8 \arctan\left(-\frac{\left(bx^4+a\right)^{\frac{1}{4}}\left(\frac{b^8}{a^5}\right)^{\frac{1}{4}} ab^6 - \sqrt{\sqrt{bx^4+a}\,b^{12}+\sqrt{\frac{b^8}{a^5}}\,a^3b^8}\left(\frac{b^8}{a^5}\right)^{\frac{1}{4}} a}{b^8}\right) + 3\left(\frac{b^8}{a^5}\right)^{\frac{1}{4}} ax^8 \log\left(27\left(bx^4+a\right)^{\frac{1}{4}}b^6 + 27\left(\frac{b^8}{a^5}\right.\right.}{128\,ax^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^9,x, algorithm="fricas")
```

```
[Out] 1/128*(12*(b^8/a^5)^(1/4)*a*x^8*arctan(-((b*x^4 + a)^(1/4)*(b^8/a^5)^(1/4)*
a*b^6 - sqrt(sqrt(b*x^4 + a)*b^12 + sqrt(b^8/a^5)*a^3*b^8)*(b^8/a^5)^(1/4)*
a)/b^8) + 3*(b^8/a^5)^(1/4)*a*x^8*log(27*(b*x^4 + a)^(1/4)*b^6 + 27*(b^8/a^
5)^(3/4)*a^4) - 3*(b^8/a^5)^(1/4)*a*x^8*log(27*(b*x^4 + a)^(1/4)*b^6 - 27*(
b^8/a^5)^(3/4)*a^4) - 4*(3*b*x^4 + 4*a)*(b*x^4 + a)^(3/4))/(a*x^8)
```

giac [B] time = 0.18, size = 243, normalized size = 2.41

$$\frac{6\sqrt{2}(-a)^{\frac{3}{4}}b^3\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a^2}+\frac{6\sqrt{2}(-a)^{\frac{3}{4}}b^3\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a^2}+\frac{3\sqrt{2}b^3\log\left(\sqrt{2}\left(bx^4+a\right)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^4+a}+\right.}{(-a)^{\frac{1}{4}}a}$$

$$256\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^9,x, algorithm="giac")
```

```
[Out] 1/256*(6*sqrt(2)*(-a)^(3/4)*b^3*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*
(b*x^4 + a)^(1/4))/(-a)^(1/4))/a^2 + 6*sqrt(2)*(-a)^(3/4)*b^3*arctan(-1/2*s
qrt(2)*(sqrt(2)*(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4))/a^2 + 3*sqrt(
2)*b^3*log(sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt(-a
))/((-a)^(1/4)*a) + 3*sqrt(2)*(-a)^(3/4)*b^3*log(-sqrt(2)*(b*x^4 + a)^(1/4)
*(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt(-a))/a^2 - 8*(3*(b*x^4 + a)^(7/4)*b^3
 + (b*x^4 + a)^(3/4)*a*b^3)/(a*b^2*x^8))/b
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{3}{4}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(3/4)/x^9,x)
```

```
[Out] int((b*x^4+a)^(3/4)/x^9,x)
```

maxima [A] time = 2.99, size = 125, normalized size = 1.24

$$-\frac{3\,b^2\left(\frac{2\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}+\frac{\log\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{\left(bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}\right)}{128\,a}-\frac{3\left(bx^4+a\right)^{\frac{7}{4}}b^2+\left(bx^4+a\right)^{\frac{3}{4}}ab^2}{32\left(\left(bx^4+a\right)^2a-2\left(bx^4+a\right)a^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^9,x, algorithm="maxima")
```

```
[Out] -3/128*b^2*(2*arctan((b*x^4 + a)^(1/4)/a^(1/4))/a^(1/4) + log(((b*x^4 + a)^
(1/4) - a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/4)))/a^(1/4))/a - 1/32*(3*(b*x^4
 + a)^(7/4)*b^2 + (b*x^4 + a)^(3/4)*a*b^2)/((b*x^4 + a)^2*a - 2*(b*x^4 + a)
*a^2 + a^3)
```

mupad [B] time = 1.42, size = 79, normalized size = 0.78

$$-\frac{\left(b\,x^4+a\right)^{3/4}}{32\,x^8}-\frac{3\,b^2\,\mathrm{atan}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)}{64\,a^{5/4}}-\frac{3\left(b\,x^4+a\right)^{7/4}}{32\,a\,x^8}-\frac{b^2\,\mathrm{atan}\left(\frac{\left(b\,x^4+a\right)^{1/4}\,1\mathrm{i}}{a^{1/4}}\right)\,3\mathrm{i}}{64\,a^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(3/4)/x^9,x)
```

```
[Out] - (a + b*x^4)^(3/4)/(32*x^8) - (3*b^2*atan((a + b*x^4)^(1/4)/a^(1/4)))/(64*
a^(5/4)) - (b^2*atan(((a + b*x^4)^(1/4)*1i)/a^(1/4))*3i)/(64*a^(5/4)) - (3*
(a + b*x^4)^(7/4))/(32*a*x^8)
```

sympy [C] time = 2.98, size = 41, normalized size = 0.41

$$-\frac{b^{\frac{3}{4}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{ae^{i\pi}}{bx^4}\right)}{4x^5\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(3/4)/x**9,x)
```

```
[Out] -b**(3/4)*gamma(5/4)*hyper((-3/4, 5/4), (9/4,), a*exp_polar(I*pi)/(b*x**4))
/(4*x**5*gamma(9/4))
```

3.1025 $\int x^9 \left(a+bx^4\right)^{3/4}\,dx$

Optimal. Leaf size=149

$$-\frac{4a^{7/2}\sqrt[4]{\frac{bx^4}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{65b^{5/2}\sqrt[4]{a+bx^4}}+\frac{4a^3x^2}{65b^2\sqrt[4]{a+bx^4}}-\frac{2a^2x^2\left(a+bx^4\right)^{3/4}}{65b^2}+\frac{1}{13}x^{10}\left(a+bx^4\right)^{3/4}+\frac{ax^6\left(a+bx^4\right)^{3/4}}{39b}$$

```
[Out] 4/65*a^3*x^2/b^2/(b*x^4+a)^(1/4)-2/65*a^2*x^2*(b*x^4+a)^(3/4)/b^2+1/39*a*x^
6*(b*x^4+a)^(3/4)/b+1/13*x^10*(b*x^4+a)^(3/4)-4/65*a^(7/2)*(1+b*x^4/a)^(1/4
)*(cos(1/2*arctan(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x^2*b^(1/2)
/a^(1/2)))*EllipticE(sin(1/2*arctan(x^2*b^(1/2)/a^(1/2))),2^(1/2))/b^(5/2)/
(b*x^4+a)^(1/4)
```

Rubi [A] time = 0.10, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {275, 279, 321, 229, 227, 196}

$$-\frac{2a^2x^2\left(a+bx^4\right)^{3/4}}{65b^2}+\frac{4a^3x^2}{65b^2\sqrt[4]{a+bx^4}}-\frac{4a^{7/2}\sqrt[4]{\frac{bx^4}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{65b^{5/2}\sqrt[4]{a+bx^4}}+\frac{1}{13}x^{10}\left(a+bx^4\right)^{3/4}+\frac{ax^6\left(a+bx^4\right)^{3/4}}{39b}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a + b*x^4)^(3/4),x]
```

```
[Out] (4*a^3*x^2)/(65*b^2*(a + b*x^4)^(1/4)) - (2*a^2*x^2*(a + b*x^4)^(3/4))/(65*
b^2) + (a*x^6*(a + b*x^4)^(3/4))/(39*b) + (x^10*(a + b*x^4)^(3/4))/13 - (4*
a^(7/2)*(1 + (b*x^4)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2]
)/(65*b^(5/2)*(a + b*x^4)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
```

```
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int x^9 \left(a+bx^4\right)^{3/4}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int x^4 \left(a+bx^2\right)^{3/4}\,dx, x, x^2\right) \\
&= \frac{1}{13}x^{10}\left(a+bx^4\right)^{3/4} + \frac{1}{26}(3a)\,\text{Subst}\left(\int \frac{x^4}{\sqrt[4]{a+bx^2}}\,dx, x, x^2\right) \\
&= \frac{ax^6\left(a+bx^4\right)^{3/4}}{39b} + \frac{1}{13}x^{10}\left(a+bx^4\right)^{3/4} - \frac{a^2\,\text{Subst}\left(\int \frac{x^2}{\sqrt[4]{a+bx^2}}\,dx, x, x^2\right)}{13b} \\
&= -\frac{2a^2x^2\left(a+bx^4\right)^{3/4}}{65b^2} + \frac{ax^6\left(a+bx^4\right)^{3/4}}{39b} + \frac{1}{13}x^{10}\left(a+bx^4\right)^{3/4} + \frac{\left(2a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{a+bx^2}}\right.}{65b^2} \\
&= -\frac{2a^2x^2\left(a+bx^4\right)^{3/4}}{65b^2} + \frac{ax^6\left(a+bx^4\right)^{3/4}}{39b} + \frac{1}{13}x^{10}\left(a+bx^4\right)^{3/4} + \frac{\left(2a^3\sqrt[4]{1+\frac{bx^4}{a}}\right)\text{Subst}\left(\right.}{65b^2\sqrt[4]{a+} \\
&= \frac{4a^3x^2}{65b^2\sqrt[4]{a+bx^4}} - \frac{2a^2x^2\left(a+bx^4\right)^{3/4}}{65b^2} + \frac{ax^6\left(a+bx^4\right)^{3/4}}{39b} + \frac{1}{13}x^{10}\left(a+bx^4\right)^{3/4} - \frac{\left(2a^3\sqrt[4]{1}\right.}{} \\
&= \frac{4a^3x^2}{65b^2\sqrt[4]{a+bx^4}} - \frac{2a^2x^2\left(a+bx^4\right)^{3/4}}{65b^2} + \frac{ax^6\left(a+bx^4\right)^{3/4}}{39b} + \frac{1}{13}x^{10}\left(a+bx^4\right)^{3/4} - \frac{4a^{7/2}\sqrt[4]{}}{}
\end{aligned}$$

Mathematica [C] time = 0.09, size = 95, normalized size = 0.64

$$\frac{x^2\left(a+bx^4\right)^{3/4}\left(\left(\frac{bx^4}{a}+1\right)^{3/4}\left(-2a^2+abx^4+3b^2x^8\right)+2a^2\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^4}{a}\right)\right)}{39b^2\left(\frac{bx^4}{a}+1\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a + b*x^4)^(3/4),x]
```

```
[Out] (x^2*(a + b*x^4)^(3/4)*((1 + (b*x^4)/a)^(3/4)*(-2*a^2 + a*b*x^4 + 3*b^2*x^8
) + 2*a^2*Hypergeometric2F1[-3/4, 1/2, 3/2, -((b*x^4)/a)]))/(39*b^2*(1 + (b
*x^4)/a)^(3/4))
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^4+a\right)^{\frac{3}{4}}x^9, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^4+a)^(3/4),x, algorithm="fricas")
[Out] integral((b*x^4 + a)^(3/4)*x^9, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4 + a\right)^{\frac{3}{4}} x^9 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^4+a)^(3/4),x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(3/4)*x^9, x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \left(b\,x^4 + a\right)^{\frac{3}{4}} x^9 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b*x^4+a)^(3/4),x)
[Out] int(x^9*(b*x^4+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4 + a\right)^{\frac{3}{4}} x^9 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^4+a)^(3/4),x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(3/4)*x^9, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^9 \left(b\,x^4 + a\right)^{3/4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(a + b*x^4)^(3/4),x)
[Out] int(x^9*(a + b*x^4)^(3/4), x)
```

sympy [C] time = 2.52, size = 29, normalized size = 0.19

$$\frac{a^{\frac{3}{4}} x^{10} {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b*x**4+a)**(3/4),x)
[Out] a**(3/4)*x**10*hyper((-3/4, 5/2), (7/2,), b*x**4*exp_polar(I*pi)/a)/10
```

3.1026 $\int x^5 \left(a+bx^4\right)^{3/4}\,dx$

Optimal. Leaf size=125

$$\frac{2a^{5/2}\sqrt[4]{\frac{bx^4}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{15b^{3/2}\sqrt[4]{a+bx^4}}-\frac{2a^2x^2}{15b\sqrt[4]{a+bx^4}}+\frac{1}{9}x^6\left(a+bx^4\right)^{3/4}+\frac{ax^2\left(a+bx^4\right)^{3/4}}{15b}$$

```
[Out] -2/15*a^2*x^2/b/(b*x^4+a)^(1/4)+1/15*a*x^2*(b*x^4+a)^(3/4)/b+1/9*x^6*(b*x^4
+a)^(3/4)+2/15*a^(5/2)*(1+b*x^4/a)^(1/4)*(cos(1/2*arctan(x^2*b^(1/2)/a^(1/2
)))^2)^(1/2)/cos(1/2*arctan(x^2*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(
x^2*b^(1/2)/a^(1/2))),2^(1/2))/b^(3/2)/(b*x^4+a)^(1/4)
```

Rubi [A] time = 0.08, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {275, 279, 321, 229, 227, 196}

$$\frac{2a^{5/2}\sqrt[4]{\frac{bx^4}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{15b^{3/2}\sqrt[4]{a+bx^4}}-\frac{2a^2x^2}{15b\sqrt[4]{a+bx^4}}+\frac{1}{9}x^6\left(a+bx^4\right)^{3/4}+\frac{ax^2\left(a+bx^4\right)^{3/4}}{15b}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^4)^(3/4),x]
```

```
[Out] (-2*a^2*x^2)/(15*b*(a + b*x^4)^(1/4)) + (a*x^2*(a + b*x^4)^(3/4))/(15*b) +
(x^6*(a + b*x^4)^(3/4))/9 + (2*a^(5/2)*(1 + (b*x^4)/a)^(1/4)*EllipticE[ArcT
an[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(15*b^(3/2)*(a + b*x^4)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int x^5 \left(a+bx^4\right)^{3/4}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int x^2\left(a+bx^2\right)^{3/4}\,dx, x, x^2\right)\\
&= \frac{1}{9}x^6\left(a+bx^4\right)^{3/4} + \frac{1}{6}a\,\text{Subst}\left(\int \frac{x^2}{\sqrt[4]{a+bx^2}}\,dx, x, x^2\right)\\
&= \frac{ax^2\left(a+bx^4\right)^{3/4}}{15b} + \frac{1}{9}x^6\left(a+bx^4\right)^{3/4} - \frac{a^2\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx, x, x^2\right)}{15b}\\
&= \frac{ax^2\left(a+bx^4\right)^{3/4}}{15b} + \frac{1}{9}x^6\left(a+bx^4\right)^{3/4} - \frac{\left(a^2\sqrt[4]{1+\frac{bx^4}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx, x, x^2\right)}{15b\sqrt[4]{a+bx^4}}\\
&= -\frac{2a^2x^2}{15b\sqrt[4]{a+bx^4}} + \frac{ax^2\left(a+bx^4\right)^{3/4}}{15b} + \frac{1}{9}x^6\left(a+bx^4\right)^{3/4} + \frac{\left(a^2\sqrt[4]{1+\frac{bx^4}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\right.}{15b\sqrt[4]{a+bx^4}}\\
&= -\frac{2a^2x^2}{15b\sqrt[4]{a+bx^4}} + \frac{ax^2\left(a+bx^4\right)^{3/4}}{15b} + \frac{1}{9}x^6\left(a+bx^4\right)^{3/4} + \frac{2a^{5/2}\sqrt[4]{1+\frac{bx^4}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\right.}{15b^{3/2}\sqrt[4]{a+bx^4}}
\end{aligned}$$

Mathematica [C] time = 0.09, size = 64, normalized size = 0.51

$$\frac{x^2\left(a+bx^4\right)^{3/4}\left(-\frac{a\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^4}{a}\right)}{\left(\frac{bx^4}{a}+1\right)^{3/4}} + a + bx^4\right)}{9b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^4)^(3/4),x]
```

```
[Out] (x^2*(a + b*x^4)^(3/4)*(a + b*x^4 - (a*Hypergeometric2F1[-3/4, 1/2, 3/2, -(
(b*x^4)/a)])/(1 + (b*x^4)/a)^(3/4)))/(9*b)
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^4+a\right)^{\frac{3}{4}}x^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(3/4)*x^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4 + a\right)^{\frac{3}{4}} x^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^4+a)^(3/4),x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(3/4)*x^5, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \left(b\,x^4 + a\right)^{\frac{3}{4}} x^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^4+a)^(3/4),x)
[Out] int(x^5*(b*x^4+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4 + a\right)^{\frac{3}{4}} x^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^4+a)^(3/4),x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(3/4)*x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5 \left(b\,x^4 + a\right)^{3/4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^4)^(3/4),x)
[Out] int(x^5*(a + b*x^4)^(3/4), x)
```

sympy [C] time = 3.89, size = 29, normalized size = 0.23

$$\frac{a^{\frac{3}{4}} x^6 {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**4+a)**(3/4),x)
[Out] a**(3/4)*x**6*hyper((-3/4, 3/2), (5/2,), b*x**4*exp_polar(I*pi)/a)/6
```

3.1027 $\int x\left(a+bx^4\right)^{3/4}\,dx$

Optimal. Leaf size=98

$$-\frac{3a^{3/2}\sqrt[4]{\frac{bx^4}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{b}\,\sqrt[4]{a+bx^4}}+\frac{1}{5}x^2\left(a+bx^4\right)^{3/4}+\frac{3ax^2}{5\sqrt[4]{a+bx^4}}$$

```
[Out] 3/5*a*x^2/(b*x^4+a)^(1/4)+1/5*x^2*(b*x^4+a)^(3/4)-3/5*a^(3/2)*(1+b*x^4/a)^(
1/4)*(cos(1/2*arctan(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x^2*b^(1
/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x^2*b^(1/2)/a^(1/2))),2^(1/2))/(b*x^
4+a)^(1/4)/b^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {275, 195, 229, 227, 196}

$$-\frac{3a^{3/2}\sqrt[4]{\frac{bx^4}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{b}\,\sqrt[4]{a+bx^4}}+\frac{1}{5}x^2\left(a+bx^4\right)^{3/4}+\frac{3ax^2}{5\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^4)^(3/4),x]
```

```
[Out] (3*a*x^2)/(5*(a + b*x^4)^(1/4)) + (x^2*(a + b*x^4)^(3/4))/5 - (3*a^(3/2)*(1
 + (b*x^4)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(5*Sqrt[
b]*(a + b*x^4)^(1/4))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int x\left(a+bx^4\right)^{3/4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \left(a+bx^2\right)^{3/4}\,dx, x, x^2\right)\\
&= \frac{1}{5}x^2\left(a+bx^4\right)^{3/4} + \frac{1}{10}(3a)\operatorname{Subst}\left(\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx, x, x^2\right)\\
&= \frac{1}{5}x^2\left(a+bx^4\right)^{3/4} + \frac{\left(3a\sqrt[4]{1+\frac{bx^4}{a}}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx, x, x^2\right)}{10\sqrt[4]{a+bx^4}}\\
&= \frac{3ax^2}{5\sqrt[4]{a+bx^4}} + \frac{1}{5}x^2\left(a+bx^4\right)^{3/4} - \frac{\left(3a\sqrt[4]{1+\frac{bx^4}{a}}\right)\operatorname{Subst}\left(\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx, x, x^2\right)}{10\sqrt[4]{a+bx^4}}\\
&= \frac{3ax^2}{5\sqrt[4]{a+bx^4}} + \frac{1}{5}x^2\left(a+bx^4\right)^{3/4} - \frac{3a^{3/2}\sqrt[4]{1+\frac{bx^4}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|\,2\right)}{5\sqrt{b}\,\sqrt[4]{a+bx^4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.52

$$\frac{x^2\left(a+bx^4\right)^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^4}{a}\right)}{2\left(\frac{bx^4}{a}+1\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^4)^(3/4),x]
```

```
[Out] (x^2*(a + b*x^4)^(3/4)*Hypergeometric2F1[-3/4, 1/2, 3/2, -((b*x^4)/a)])/(2*
(1 + (b*x^4)/a)^(3/4))
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^4+a\right)^{\frac{3}{4}}x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(3/4)*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4+a\right)^{\frac{3}{4}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/4)*x, x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \left(b\,x^4+a\right)^{\frac{3}{4}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^4+a)^(3/4),x)
[Out] int(x*(b*x^4+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4+a\right)^{\frac{3}{4}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^4+a)^(3/4),x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(3/4)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(b\,x^4+a\right)^{3/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^4)^(3/4),x)
[Out] int(x*(a + b*x^4)^(3/4), x)
```

sympy [C] time = 1.90, size = 29, normalized size = 0.30

$$\frac{a^{\frac{3}{4}}x^2\,{}_2F_1\left(\left.\begin{matrix}-\frac{3}{4}, \frac{1}{2}\\ \frac{3}{2}\end{matrix}\right| \frac{bx^4e^{i\pi}}{a}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**4+a)**(3/4),x)
[Out] a**(3/4)*x**2*hyper((-3/4, 1/2), (3/2,), b*x**4*exp_polar(I*pi)/a)/2
```

3.1028 $\int \frac{\left(a+bx^4\right)^{3/4}}{x^3}\,dx$

Optimal. Leaf size=98

$$\frac{3bx^2}{2\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{2x^2} - \frac{3\sqrt{a}\,\sqrt{b}\,\sqrt[4]{\frac{bx^4}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt[4]{a+bx^4}}$$

```
[Out] 3/2*b*x^2/(b*x^4+a)^(1/4)-1/2*(b*x^4+a)^(3/4)/x^2-3/2*(1+b*x^4/a)^(1/4)*(co
s(1/2*arctan(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x^2*b^(1/2)/a^(1
/2)))*EllipticE(sin(1/2*arctan(x^2*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)*b^(1/
2)/(b*x^4+a)^(1/4)
```

Rubi [A] time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {275, 277, 229, 227, 196}

$$\frac{3bx^2}{2\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{2x^2} - \frac{3\sqrt{a}\,\sqrt{b}\,\sqrt[4]{\frac{bx^4}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(3/4)/x^3,x]
```

```
[Out] (3*b*x^2)/(2*(a + b*x^4)^(1/4)) - (a + b*x^4)^(3/4)/(2*x^2) - (3*Sqrt[a]*Sq
rt[b]*(1 + (b*x^4)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/
(2*(a + b*x^4)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

```
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^4\right)^{3/4}}{x^3}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/4}}{x^2}\,dx, x, x^2\right) \\
&= -\frac{\left(a+bx^4\right)^{3/4}}{2x^2} + \frac{1}{4}(3b)\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx, x, x^2\right) \\
&= -\frac{\left(a+bx^4\right)^{3/4}}{2x^2} + \frac{\left(3b\sqrt[4]{1+\frac{bx^4}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx, x, x^2\right)}{4\sqrt[4]{a+bx^4}} \\
&= \frac{3bx^2}{2\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{2x^2} - \frac{\left(3b\sqrt[4]{1+\frac{bx^4}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx, x, x^2\right)}{4\sqrt[4]{a+bx^4}} \\
&= \frac{3bx^2}{2\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{2x^2} - \frac{3\sqrt{a}\,\sqrt{b}\,\sqrt[4]{1+\frac{bx^4}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|\,2\right)}{2\sqrt[4]{a+bx^4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.52

$$-\frac{\left(a+bx^4\right)^{3/4}\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{2};-\frac{bx^4}{a}\right)}{2x^2\left(\frac{bx^4}{a}+1\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(3/4)/x^3,x]
```

```
[Out] -1/2*((a + b*x^4)^(3/4)*Hypergeometric2F1[-3/4, -1/2, 1/2, -((b*x^4)/a)])/(
x^2*(1 + (b*x^4)/a)^(3/4))
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{3}{4}}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(3/4)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{4}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^3,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(3/4)/x^3, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{3}{4}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(3/4)/x^3,x)
[Out] int((b*x^4+a)^(3/4)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{4}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^3,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(3/4)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^4+a\right)^{3/4}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(3/4)/x^3,x)
[Out] int((a + b*x^4)^(3/4)/x^3, x)
```

sympy [C] time = 1.25, size = 32, normalized size = 0.33

$$-\frac{a^{\frac{3}{4}}\,{}_2F_1\left(\left.\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{2}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(3/4)/x**3,x)
[Out] -a**(3/4)*hyper((-3/4, -1/2), (1/2,), b*x**4*exp_polar(I*pi)/a)/(2*x**2)
```

3.1029 $\int \frac{\left(a+bx^4\right)^{3/4}}{x^7}\,dx$

Optimal. Leaf size=125

$$-\frac{b^{3/2}\sqrt[4]{\frac{bx^4}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{4\sqrt{a}\,\sqrt[4]{a+bx^4}}+\frac{b^2x^2}{4a\sqrt[4]{a+bx^4}}-\frac{\left(a+bx^4\right)^{3/4}}{6x^6}-\frac{b\left(a+bx^4\right)^{3/4}}{4ax^2}$$

```
[Out] 1/4*b^2*x^2/a/(b*x^4+a)^(1/4)-1/6*(b*x^4+a)^(3/4)/x^6-1/4*b*(b*x^4+a)^(3/4)
/a/x^2-1/4*b^(3/2)*(1+b*x^4/a)^(1/4)*(cos(1/2*arctan(x^2*b^(1/2)/a^(1/2)))^
2)^(1/2)/cos(1/2*arctan(x^2*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x^2*
b^(1/2)/a^(1/2))),2^(1/2))/(b*x^4+a)^(1/4)/a^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {275, 277, 325, 229, 227, 196}

$$\frac{b^2x^2}{4a\sqrt[4]{a+bx^4}}-\frac{b^{3/2}\sqrt[4]{\frac{bx^4}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{4\sqrt{a}\,\sqrt[4]{a+bx^4}}-\frac{b\left(a+bx^4\right)^{3/4}}{4ax^2}-\frac{\left(a+bx^4\right)^{3/4}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(3/4)/x^7,x]
```

```
[Out] (b^2*x^2)/(4*a*(a + b*x^4)^(1/4)) - (a + b*x^4)^(3/4)/(6*x^6) - (b*(a + b*x
^4)^(3/4))/(4*a*x^2) - (b^(3/2)*(1 + (b*x^4)/a)^(1/4)*EllipticE[ArcTan[(Sqr
t[b]*x^2)/Sqrt[a]]/2, 2])/(4*Sqrt[a]*(a + b*x^4)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

```
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^4\right)^{3/4}}{x^7}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/4}}{x^4}\,dx, x, x^2\right) \\
&= -\frac{\left(a+bx^4\right)^{3/4}}{6x^6} + \frac{1}{4}b\,\text{Subst}\left(\int \frac{1}{x^2\sqrt[4]{a+bx^2}}\,dx, x, x^2\right) \\
&= -\frac{\left(a+bx^4\right)^{3/4}}{6x^6} - \frac{b\left(a+bx^4\right)^{3/4}}{4ax^2} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx, x, x^2\right)}{8a} \\
&= -\frac{\left(a+bx^4\right)^{3/4}}{6x^6} - \frac{b\left(a+bx^4\right)^{3/4}}{4ax^2} + \frac{\left(b^2\sqrt[4]{1+\frac{bx^4}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx, x, x^2\right)}{8a\sqrt[4]{a+bx^4}} \\
&= \frac{b^2x^2}{4a\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{6x^6} - \frac{b\left(a+bx^4\right)^{3/4}}{4ax^2} - \frac{\left(b^2\sqrt[4]{1+\frac{bx^4}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx, x, x^2\right.}{8a\sqrt[4]{a+bx^4}} \\
&= \frac{b^2x^2}{4a\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{6x^6} - \frac{b\left(a+bx^4\right)^{3/4}}{4ax^2} - \frac{b^{3/2}\sqrt[4]{1+\frac{bx^4}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{4\sqrt{a}\,\sqrt[4]{a+bx^4}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 51, normalized size = 0.41

$$-\frac{\left(a+bx^4\right)^{3/4}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{4};-\frac{1}{2};-\frac{bx^4}{a}\right)}{6x^6\left(\frac{bx^4}{a}+1\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(3/4)/x^7,x]
```

```
[Out] -1/6*((a + b*x^4)^(3/4)*Hypergeometric2F1[-3/2, -3/4, -1/2, -((b*x^4)/a)])/
(x^6*(1 + (b*x^4)/a)^(3/4))
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{3}{4}}}{x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^7,x, algorithm="fricas")
[Out] integral((b*x^4 + a)^(3/4)/x^7, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{4}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^7,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(3/4)/x^7, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{3}{4}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(3/4)/x^7,x)
[Out] int((b*x^4+a)^(3/4)/x^7,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{4}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^7,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(3/4)/x^7, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^4+a\right)^{3/4}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(3/4)/x^7,x)
[Out] int((a + b*x^4)^(3/4)/x^7, x)
```

sympy [C] time = 3.17, size = 34, normalized size = 0.27

$$-\frac{a^{\frac{3}{4}}\,{}_2F_1\left(\left.\begin{matrix}-\frac{3}{2}, -\frac{3}{4} \\ -\frac{1}{2}\end{matrix}\right| \frac{bx^4e^{i\pi}}{a}\right)}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(3/4)/x**7,x)
[Out] -a**(3/4)*hyper((-3/2, -3/4), (-1/2,), b*x**4*exp_polar(I*pi)/a)/(6*x**6)
```

3.1030 $\int \frac{\left(a+bx^4\right)^{3/4}}{x^{11}}\,dx$

Optimal. Leaf size=149

$$\frac{3b^{5/2}\sqrt[4]{\frac{bx^4}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{40a^{3/2}\sqrt[4]{a+bx^4}}-\frac{3b^3x^2}{40a^2\sqrt[4]{a+bx^4}}+\frac{3b^2\left(a+bx^4\right)^{3/4}}{40a^2x^2}-\frac{\left(a+bx^4\right)^{3/4}}{10x^{10}}-\frac{b\left(a+bx^4\right)^{3/4}}{20ax^6}$$

```
[Out] -3/40*b^3*x^2/a^2/(b*x^4+a)^(1/4)-1/10*(b*x^4+a)^(3/4)/x^10-1/20*b*(b*x^4+a
)^(3/4)/a/x^6+3/40*b^2*(b*x^4+a)^(3/4)/a^2/x^2+3/40*b^(5/2)*(1+b*x^4/a)^(1/
4)*(cos(1/2*arctan(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x^2*b^(1/2
)/a^(1/2)))*EllipticE(sin(1/2*arctan(x^2*b^(1/2)/a^(1/2))),2^(1/2))/a^(3/2)
/(b*x^4+a)^(1/4)
```

Rubi [A] time = 0.09, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {275, 277, 325, 229, 227, 196}

$$-\frac{3b^3x^2}{40a^2\sqrt[4]{a+bx^4}}+\frac{3b^2\left(a+bx^4\right)^{3/4}}{40a^2x^2}+\frac{3b^{5/2}\sqrt[4]{\frac{bx^4}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{40a^{3/2}\sqrt[4]{a+bx^4}}-\frac{b\left(a+bx^4\right)^{3/4}}{20ax^6}-\frac{\left(a+bx^4\right)^{3/4}}{10x^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(3/4)/x^11,x]
```

```
[Out] (-3*b^3*x^2)/(40*a^2*(a + b*x^4)^(1/4)) - (a + b*x^4)^(3/4)/(10*x^10) - (b*
(a + b*x^4)^(3/4))/(20*a*x^6) + (3*b^2*(a + b*x^4)^(3/4))/(40*a^2*x^2) + (3
*b^(5/2)*(1 + (b*x^4)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2
])/(40*a^(3/2)*(a + b*x^4)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
```

```
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{\left(a+bx^4\right)^{3/4}}{x^{11}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/4}}{x^6}\,dx, x, x^2\right)$$

$$= -\frac{\left(a+bx^4\right)^{3/4}}{10x^{10}} + \frac{1}{20}(3b)\,\text{Subst}\left(\int \frac{1}{x^4\sqrt[4]{a+bx^2}}\,dx, x, x^2\right)$$

$$= -\frac{\left(a+bx^4\right)^{3/4}}{10x^{10}} - \frac{b\left(a+bx^4\right)^{3/4}}{20ax^6} - \frac{\left(3b^2\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt[4]{a+bx^2}}\,dx, x, x^2\right)}{40a}$$

$$= -\frac{\left(a+bx^4\right)^{3/4}}{10x^{10}} - \frac{b\left(a+bx^4\right)^{3/4}}{20ax^6} + \frac{3b^2\left(a+bx^4\right)^{3/4}}{40a^2x^2} - \frac{\left(3b^3\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx, x, x^2\right)}{80a^2}$$

$$= -\frac{\left(a+bx^4\right)^{3/4}}{10x^{10}} - \frac{b\left(a+bx^4\right)^{3/4}}{20ax^6} + \frac{3b^2\left(a+bx^4\right)^{3/4}}{40a^2x^2} - \frac{\left(3b^3\sqrt[4]{1+\frac{bx^4}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx, x, \right.}{80a^2\sqrt[4]{a+bx^4}}$$

$$= -\frac{3b^3x^2}{40a^2\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{10x^{10}} - \frac{b\left(a+bx^4\right)^{3/4}}{20ax^6} + \frac{3b^2\left(a+bx^4\right)^{3/4}}{40a^2x^2} + \frac{\left(3b^3\sqrt[4]{1+\frac{bx^4}{a}}\right)\text{Subst}}{80a^2\sqrt[4]{}}$$

$$= -\frac{3b^3x^2}{40a^2\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{10x^{10}} - \frac{b\left(a+bx^4\right)^{3/4}}{20ax^6} + \frac{3b^2\left(a+bx^4\right)^{3/4}}{40a^2x^2} + \frac{3b^{5/2}\sqrt[4]{1+\frac{bx^4}{a}}\,E\left(\frac{1}{2}\,\text{ta}\right.}{40a^{3/2}\sqrt[4]{a+}}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.34

$$-\frac{\left(a+bx^4\right)^{3/4}\,{}_2F_1\left(-\frac{5}{2}, -\frac{3}{4}; -\frac{3}{2}; -\frac{bx^4}{a}\right)}{10x^{10}\left(\frac{bx^4}{a}+1\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(3/4)/x^11,x]
```

```
[Out] -1/10*((a + b*x^4)^(3/4)*Hypergeometric2F1[-5/2, -3/4, -3/2, -((b*x^4)/a)])
/(x^10*(1 + (b*x^4)/a)^(3/4))
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{3}{4}}}{x^{11}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^11,x, algorithm="fricas")
[Out] integral((b*x^4 + a)^(3/4)/x^11, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{4}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^11,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(3/4)/x^11, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{3}{4}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(3/4)/x^11,x)
[Out] int((b*x^4+a)^(3/4)/x^11,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{4}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^11,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(3/4)/x^11, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^4+a\right)^{3/4}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(3/4)/x^11,x)
[Out] int((a + b*x^4)^(3/4)/x^11, x)
```

sympy [C] time = 2.22, size = 34, normalized size = 0.23

$$-\frac{a^{\frac{3}{4}}\,{}_2F_1\left(\left.\begin{matrix}-\frac{5}{2}, -\frac{3}{4}\\ -\frac{3}{2}\end{matrix}\right| \frac{bx^4e^{i\pi}}{a}\right)}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(3/4)/x**11,x)
[Out] -a**(3/4)*hyper((-5/2, -3/4), (-3/2,), b*x**4*exp_polar(I*pi)/a)/(10*x**10)
```

3.1031 $\int x^{12}\left(a+bx^4\right)^{3/4}\,dx$

Optimal. Leaf size=149

$$-\frac{45a^4\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{4096b^{13/4}}-\frac{45a^4\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{4096b^{13/4}}+\frac{45a^3x\left(a+bx^4\right)^{3/4}}{2048b^3}-\frac{9a^2x^5\left(a+bx^4\right)^{3/4}}{512b^2}+\frac{1}{16}x^{13}\left(a+bx^4\right)^{3/4}+\frac{ax^9(}{}$$

```
[Out] 45/2048*a^3*x*(b*x^4+a)^(3/4)/b^3-9/512*a^2*x^5*(b*x^4+a)^(3/4)/b^2+1/64*a*
x^9*(b*x^4+a)^(3/4)/b+1/16*x^13*(b*x^4+a)^(3/4)-45/4096*a^4*arctan(b^(1/4)*
x/(b*x^4+a)^(1/4))/b^(13/4)-45/4096*a^4*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))/
b^(13/4)
```

Rubi [A] time = 0.06, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {279, 321, 240, 212, 206, 203}

$$-\frac{9a^2x^5\left(a+bx^4\right)^{3/4}}{512b^2}+\frac{45a^3x\left(a+bx^4\right)^{3/4}}{2048b^3}-\frac{45a^4\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{4096b^{13/4}}-\frac{45a^4\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{4096b^{13/4}}+\frac{1}{16}x^{13}\left(a+bx^4\right)^{3/4}+\frac{ax^9(}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^12*(a + b*x^4)^(3/4),x]
```

```
[Out] (45*a^3*x*(a + b*x^4)^(3/4))/(2048*b^3) - (9*a^2*x^5*(a + b*x^4)^(3/4))/(51
2*b^2) + (a*x^9*(a + b*x^4)^(3/4))/(64*b) + (x^13*(a + b*x^4)^(3/4))/16 - (
45*a^4*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(4096*b^(13/4)) - (45*a^4*Arc
Tanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(4096*b^(13/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
```

```
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^{12}\left(a+bx^4\right)^{3/4}\,dx = \frac{1}{16}x^{13}\left(a+bx^4\right)^{3/4} + \frac{1}{16}(3a)\int \frac{x^{12}}{\sqrt[4]{a+bx^4}}\,dx$$

$$= \frac{ax^9\left(a+bx^4\right)^{3/4}}{64b} + \frac{1}{16}x^{13}\left(a+bx^4\right)^{3/4} - \frac{\left(9a^2\right)\int \frac{x^8}{\sqrt[4]{a+bx^4}}\,dx}{64b}$$

$$= -\frac{9a^2x^5\left(a+bx^4\right)^{3/4}}{512b^2} + \frac{ax^9\left(a+bx^4\right)^{3/4}}{64b} + \frac{1}{16}x^{13}\left(a+bx^4\right)^{3/4} + \frac{\left(45a^3\right)\int \frac{x^4}{\sqrt[4]{a+bx^4}}\,dx}{512b^2}$$

$$= \frac{45a^3x\left(a+bx^4\right)^{3/4}}{2048b^3} - \frac{9a^2x^5\left(a+bx^4\right)^{3/4}}{512b^2} + \frac{ax^9\left(a+bx^4\right)^{3/4}}{64b} + \frac{1}{16}x^{13}\left(a+bx^4\right)^{3/4} - \frac{\left(45$$

$$= \frac{45a^3x\left(a+bx^4\right)^{3/4}}{2048b^3} - \frac{9a^2x^5\left(a+bx^4\right)^{3/4}}{512b^2} + \frac{ax^9\left(a+bx^4\right)^{3/4}}{64b} + \frac{1}{16}x^{13}\left(a+bx^4\right)^{3/4} - \frac{\left(45$$

$$= \frac{45a^3x\left(a+bx^4\right)^{3/4}}{2048b^3} - \frac{9a^2x^5\left(a+bx^4\right)^{3/4}}{512b^2} + \frac{ax^9\left(a+bx^4\right)^{3/4}}{64b} + \frac{1}{16}x^{13}\left(a+bx^4\right)^{3/4} - \frac{\left(45$$

$$= \frac{45a^3x\left(a+bx^4\right)^{3/4}}{2048b^3} - \frac{9a^2x^5\left(a+bx^4\right)^{3/4}}{512b^2} + \frac{ax^9\left(a+bx^4\right)^{3/4}}{64b} + \frac{1}{16}x^{13}\left(a+bx^4\right)^{3/4} - \frac{45$$

Mathematica [C] time = 0.10, size = 105, normalized size = 0.70

$$\frac{x\left(a+bx^4\right)^{3/4}\left(\left(\frac{bx^4}{a}+1\right)^{3/4}\left(15a^3-9a^2bx^4+8ab^2x^8+32b^3x^{12}\right)-15a^3\,{}_2F_1\left(-\frac{3}{4},\frac{1}{4};\frac{5}{4};-\frac{bx^4}{a}\right)\right)}{512b^3\left(\frac{bx^4}{a}+1\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12*(a + b*x^4)^(3/4),x]
```

```
[Out] (x*(a + b*x^4)^(3/4)*((1 + (b*x^4)/a)^(3/4)*(15*a^3 - 9*a^2*b*x^4 + 8*a*b^2
*x^8 + 32*b^3*x^12) - 15*a^3*Hypergeometric2F1[-3/4, 1/4, 5/4, -((b*x^4)/a)
]))/(512*b^3*(1 + (b*x^4)/a)^(3/4))
```

fricas [B] time = 0.90, size = 250, normalized size = 1.68

$$180\left(\frac{a^{16}}{b^{13}}\right)^{\frac{1}{4}}b^3\arctan\left(-\frac{\left(bx^4+a\right)^{\frac{1}{4}}\left(\frac{a^{16}}{b^{13}}\right)^{\frac{1}{4}}a^{12}b^3-\left(\frac{a^{16}}{b^{13}}\right)^{\frac{1}{4}}b^3x\sqrt{\frac{\sqrt{\frac{a^{16}}{b^{13}}}a^{16}b^7x^2+\sqrt{bx^4+a}\,a^{24}}{x^2}}}{a^{16}x}\right)+45\left(\frac{a^{16}}{b^{13}}\right)^{\frac{1}{4}}b^3\log\left(\frac{91125\left(\left(bx^4+a\right)^{\frac{1}{4}}a\right.}{x}\right.$$

$$-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/8192*(180*(a^16/b^13)^(1/4)*b^3*arctan(-((b*x^4 + a)^(1/4)*(a^16/b^13)^(
1/4)*a^12*b^3 - (a^16/b^13)^(1/4)*b^3*x*sqrt((sqrt(a^16/b^13)*a^16*b^7*x^2
+ sqrt(b*x^4 + a)*a^24)/x^2))/(a^16*x)) + 45*(a^16/b^13)^(1/4)*b^3*log(9112
5*((b*x^4 + a)^(1/4)*a^12 + (a^16/b^13)^(3/4)*b^10*x)/x) - 45*(a^16/b^13)^(
1/4)*b^3*log(91125*((b*x^4 + a)^(1/4)*a^12 - (a^16/b^13)^(3/4)*b^10*x)/x) -
 4*(128*b^3*x^13 + 32*a*b^2*x^9 - 36*a^2*b*x^5 + 45*a^3*x)*(b*x^4 + a)^(3/4
))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4 + a\right)^{\frac{3}{4}} x^{12}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/4)*x^12, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \left(b\,x^4 + a\right)^{\frac{3}{4}} x^{12}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(b*x^4+a)^(3/4),x)
```

```
[Out] int(x^12*(b*x^4+a)^(3/4),x)
```

maxima [A] time = 3.08, size = 225, normalized size = 1.51

$$\frac{45\,a^4\left(\frac{2\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)}{8192\,b^3}+\frac{\frac{15\left(bx^4+a\right)^{\frac{3}{4}}a^4b^3}{x^3}+\frac{239\left(bx^4+a\right)^{\frac{7}{4}}a^4b^2}{x^7}-\frac{171\left(bx^4+a\right)^{\frac{11}{4}}a^4b}{x^{11}}+\frac{45\left(bx^4+a\right)^{\frac{15}{4}}a^4}{x^{15}}}{2048\left(b^7-\frac{4\left(bx^4+a\right)b^6}{x^4}+\frac{6\left(bx^4+a\right)^2b^5}{x^8}-\frac{4\left(bx^4+a\right)^3b^4}{x^{12}}+\frac{\left(bx^4+a\right)^4b^3}{x^{16}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] 45/8192*a^4*(2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) + log(-(b^(1/4
) - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/4))/b^3 + 1/
2048*(15*(b*x^4 + a)^(3/4)*a^4*b^3/x^3 + 239*(b*x^4 + a)^(7/4)*a^4*b^2/x^7
- 171*(b*x^4 + a)^(11/4)*a^4*b/x^11 + 45*(b*x^4 + a)^(15/4)*a^4/x^15)/(b^7
- 4*(b*x^4 + a)*b^6/x^4 + 6*(b*x^4 + a)^2*b^5/x^8 - 4*(b*x^4 + a)^3*b^4/x^1
2 + (b*x^4 + a)^4*b^3/x^16)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{12}\left(b\,x^4 + a\right)^{3/4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(a + b*x^4)^(3/4),x)
```

```
[Out] int(x^12*(a + b*x^4)^(3/4), x)
```

sympy [C] time = 10.40, size = 39, normalized size = 0.26

$$\frac{a^{\frac{3}{4}} x^{13} \Gamma\left(\frac{13}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{13}{4} \\ \frac{17}{4} \end{matrix} \middle| \frac{b x^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{17}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12*(b*x**4+a)**(3/4),x)
```

```
[Out] a**(3/4)*x**13*gamma(13/4)*hyper((-3/4, 13/4), (17/4,), b*x**4*exp_polar(I*
pi)/a)/(4*gamma(17/4))
```

3.1032 $\int x^8 \left(a+bx^4\right)^{3/4}\,dx$

Optimal. Leaf size=125

$$\frac{5a^3\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{256b^{9/4}}+\frac{5a^3\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{256b^{9/4}}-\frac{5a^2x\left(a+bx^4\right)^{3/4}}{128b^2}+\frac{1}{12}x^9\left(a+bx^4\right)^{3/4}+\frac{ax^5\left(a+bx^4\right)^{3/4}}{32b}$$

```
[Out] -5/128*a^2*x*(b*x^4+a)^(3/4)/b^2+1/32*a*x^5*(b*x^4+a)^(3/4)/b+1/12*x^9*(b*x
^4+a)^(3/4)+5/256*a^3*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(9/4)+5/256*a^3*a
rctanh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(9/4)
```

Rubi [A] time = 0.05, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {279, 321, 240, 212, 206, 203}

$$-\frac{5a^2x\left(a+bx^4\right)^{3/4}}{128b^2}+\frac{5a^3\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{256b^{9/4}}+\frac{5a^3\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{256b^{9/4}}+\frac{1}{12}x^9\left(a+bx^4\right)^{3/4}+\frac{ax^5\left(a+bx^4\right)^{3/4}}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x^4)^(3/4),x]
```

```
[Out] (-5*a^2*x*(a + b*x^4)^(3/4))/(128*b^2) + (a*x^5*(a + b*x^4)^(3/4))/(32*b) +
 (x^9*(a + b*x^4)^(3/4))/12 + (5*a^3*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)])
/(256*b^(9/4)) + (5*a^3*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(256*b^(9/4
))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
```

```
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^8 \left(a+bx^4\right)^{3/4}\,dx = \frac{1}{12}x^9\left(a+bx^4\right)^{3/4} + \frac{1}{4}a\int \frac{x^8}{\sqrt[4]{a+bx^4}}\,dx$$

$$= \frac{ax^5\left(a+bx^4\right)^{3/4}}{32b} + \frac{1}{12}x^9\left(a+bx^4\right)^{3/4} - \frac{\left(5a^2\right)\int \frac{x^4}{\sqrt[4]{a+bx^4}}\,dx}{32b}$$

$$= -\frac{5a^2x\left(a+bx^4\right)^{3/4}}{128b^2} + \frac{ax^5\left(a+bx^4\right)^{3/4}}{32b} + \frac{1}{12}x^9\left(a+bx^4\right)^{3/4} + \frac{\left(5a^3\right)\int \frac{1}{\sqrt[4]{a+bx^4}}\,dx}{128b^2}$$

$$= -\frac{5a^2x\left(a+bx^4\right)^{3/4}}{128b^2} + \frac{ax^5\left(a+bx^4\right)^{3/4}}{32b} + \frac{1}{12}x^9\left(a+bx^4\right)^{3/4} + \frac{\left(5a^3\right)\text{Subst}\left(\int \frac{1}{1-bx^4}\,dx,\right.}{128b^2}$$

$$= -\frac{5a^2x\left(a+bx^4\right)^{3/4}}{128b^2} + \frac{ax^5\left(a+bx^4\right)^{3/4}}{32b} + \frac{1}{12}x^9\left(a+bx^4\right)^{3/4} + \frac{\left(5a^3\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{b}\,x^2}\,dx\right.}{256b^2}$$

$$= -\frac{5a^2x\left(a+bx^4\right)^{3/4}}{128b^2} + \frac{ax^5\left(a+bx^4\right)^{3/4}}{32b} + \frac{1}{12}x^9\left(a+bx^4\right)^{3/4} + \frac{5a^3\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{256b^{9/4}} + \frac{5a}{}$$

Mathematica [C] time = 0.05, size = 94, normalized size = 0.75

$$\frac{x\left(a+bx^4\right)^{3/4}\left(\left(\frac{bx^4}{a}+1\right)^{3/4}\left(-5a^2+3abx^4+8b^2x^8\right)+5a^2\,{}_2F_1\left(-\frac{3}{4},\frac{1}{4};\frac{5}{4};-\frac{bx^4}{a}\right)\right)}{96b^2\left(\frac{bx^4}{a}+1\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x^4)^(3/4),x]
```

```
[Out] (x*(a + b*x^4)^(3/4)*((1 + (b*x^4)/a)^(3/4)*(-5*a^2 + 3*a*b*x^4 + 8*b^2*x^8
) + 5*a^2*Hypergeometric2F1[-3/4, 1/4, 5/4, -((b*x^4)/a)]))/(96*b^2*(1 + (b
*x^4)/a)^(3/4))
```

fricas [B] time = 0.94, size = 239, normalized size = 1.91

$$\frac{60\left(\frac{a^{12}}{b^9}\right)^{\frac{1}{4}}b^2\arctan\left(-\frac{\left(bx^4+a\right)^{\frac{1}{4}}\left(\frac{a^{12}}{b^9}\right)^{\frac{1}{4}}a^9b^2-\left(\frac{a^{12}}{b^9}\right)^{\frac{1}{4}}b^2x\sqrt{\frac{\sqrt{\frac{a^{12}}{b^9}}\,a^{12}b^5x^2+\sqrt{bx^4+a}\,a^{18}}{x^2}}}{a^{12}x}\right)+15\left(\frac{a^{12}}{b^9}\right)^{\frac{1}{4}}b^2\log\left(\frac{125\left(\left(bx^4+a\right)^{\frac{1}{4}}a^9+\left(\frac{a^{12}}{b^9}\right.\right.\right.}{x}$$

$$1536\,b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/1536*(60*(a^12/b^9)^(1/4)*b^2*arctan(-((b*x^4 + a)^(1/4)*(a^12/b^9)^(1/4)
*a^9*b^2 - (a^12/b^9)^(1/4)*b^2*x*sqrt((sqrt(a^12/b^9)*a^12*b^5*x^2 + sqrt(
b*x^4 + a)*a^18)/x^2))/(a^12*x)) + 15*(a^12/b^9)^(1/4)*b^2*log(125*((b*x^4
+ a)^(1/4)*a^9 + (a^12/b^9)^(3/4)*b^7*x)/x) - 15*(a^12/b^9)^(1/4)*b^2*log(1
25*((b*x^4 + a)^(1/4)*a^9 - (a^12/b^9)^(3/4)*b^7*x)/x) + 4*(32*b^2*x^9 + 12
*a*b*x^5 - 15*a^2*x)*(b*x^4 + a)^(3/4))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4 + a\right)^{\frac{3}{4}} x^8 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/4)*x^8, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \left(b\,x^4 + a\right)^{\frac{3}{4}} x^8 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^4+a)^(3/4),x)
```

```
[Out] int(x^8*(b*x^4+a)^(3/4),x)
```

maxima [A] time = 2.95, size = 189, normalized size = 1.51

$$-\frac{5\,a^3\left(\frac{2\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)}{512\,b^2}-\frac{\frac{5\left(bx^4+a\right)^{\frac{3}{4}}a^3b^2}{x^3}+\frac{42\left(bx^4+a\right)^{\frac{7}{4}}a^3b}{x^7}-\frac{15\left(bx^4+a\right)^{\frac{11}{4}}a^3}{x^{11}}}{384\left(b^5-\frac{3\left(bx^4+a\right)b^4}{x^4}+\frac{3\left(bx^4+a\right)^2b^3}{x^8}-\frac{\left(bx^4+a\right)^3b^2}{x^{12}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] -5/512*a^3*(2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) + log(-(b^(1/4)
 - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/4))/b^2 - 1/3
84*(5*(b*x^4 + a)^(3/4)*a^3*b^2/x^3 + 42*(b*x^4 + a)^(7/4)*a^3*b/x^7 - 15*(
b*x^4 + a)^(11/4)*a^3/x^11)/(b^5 - 3*(b*x^4 + a)*b^4/x^4 + 3*(b*x^4 + a)^2*
b^3/x^8 - (b*x^4 + a)^3*b^2/x^12)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^8 \left(b\,x^4 + a\right)^{3/4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*x^4)^(3/4),x)
```

```
[Out] int(x^8*(a + b*x^4)^(3/4), x)
```

sympy [C] time = 4.69, size = 39, normalized size = 0.31

$$\frac{a^{\frac{3}{4}} x^9 \Gamma\left(\frac{9}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{9}{4} \\ \frac{13}{4} \end{matrix} \middle| \frac{b x^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**4+a)**(3/4),x)
```

```
[Out] a**(3/4)*x**9*gamma(9/4)*hyper((-3/4, 9/4), (13/4,), b*x**4*exp_polar(I*pi)
/a)/(4*gamma(13/4))
```

3.1033 $\int x^4 \left(a+bx^4\right)^{3/4}\,dx$

Optimal. Leaf size=101

$$-\frac{3a^2 \tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{5/4}}-\frac{3a^2 \tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{5/4}}+\frac{3ax\left(a+bx^4\right)^{3/4}}{32b}+\frac{1}{8}x^5\left(a+bx^4\right)^{3/4}$$

```
[Out] 3/32*a*x*(b*x^4+a)^(3/4)/b+1/8*x^5*(b*x^4+a)^(3/4)-3/64*a^2*arctan(b^(1/4)*
x/(b*x^4+a)^(1/4))/b^(5/4)-3/64*a^2*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(5
/4)
```

Rubi [A] time = 0.03, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {279, 321, 240, 212, 206, 203}

$$-\frac{3a^2 \tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{5/4}}-\frac{3a^2 \tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{5/4}}+\frac{1}{8}x^5\left(a+bx^4\right)^{3/4}+\frac{3ax\left(a+bx^4\right)^{3/4}}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^4)^(3/4),x]
```

```
[Out] (3*a*x*(a + b*x^4)^(3/4))/(32*b) + (x^5*(a + b*x^4)^(3/4))/8 - (3*a^2*ArcTa
n[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(64*b^(5/4)) - (3*a^2*ArcTanh[(b^(1/4)*x)
/(a + b*x^4)^(1/4)])/(64*b^(5/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int x^4\left(a+bx^4\right)^{3/4}\,dx &= \frac{1}{8}x^5\left(a+bx^4\right)^{3/4}+\frac{1}{8}(3a)\int \frac{x^4}{\sqrt[4]{a+bx^4}}\,dx \\
&= \frac{3ax\left(a+bx^4\right)^{3/4}}{32b}+\frac{1}{8}x^5\left(a+bx^4\right)^{3/4}-\frac{\left(3a^2\right)\int \frac{1}{\sqrt[4]{a+bx^4}}\,dx}{32b} \\
&= \frac{3ax\left(a+bx^4\right)^{3/4}}{32b}+\frac{1}{8}x^5\left(a+bx^4\right)^{3/4}-\frac{\left(3a^2\right)\text{Subst}\left(\int \frac{1}{1-bx^4}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{32b} \\
&= \frac{3ax\left(a+bx^4\right)^{3/4}}{32b}+\frac{1}{8}x^5\left(a+bx^4\right)^{3/4}-\frac{\left(3a^2\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{b}x^2}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{64b}-\frac{\left(3a^2\right)\text{Su}}{} \\
&= \frac{3ax\left(a+bx^4\right)^{3/4}}{32b}+\frac{1}{8}x^5\left(a+bx^4\right)^{3/4}-\frac{3a^2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{5/4}}-\frac{3a^2\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{5/4}}
\end{aligned}$$

Mathematica [C] time = 0.07, size = 62, normalized size = 0.61

$$\frac{x\left(a+bx^4\right)^{3/4}\left(-\frac{a\,{}_2F_1\left(-\frac{3}{4},\frac{1}{4};\frac{5}{4};-\frac{bx^4}{a}\right)}{\left(\frac{bx^4}{a}+1\right)^{3/4}}+a+bx^4\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^4)^(3/4),x]
```

```
[Out] (x*(a + b*x^4)^(3/4)*(a + b*x^4 - (a*Hypergeometric2F1[-3/4, 1/4, 5/4, -((b
*x^4)/a)])/(1 + (b*x^4)/a)^(3/4)))/(8*b)
```

fricas [B] time = 0.94, size = 218, normalized size = 2.16

$$-\frac{12\left(\frac{a^8}{b^5}\right)^{\frac{1}{4}}b\arctan\left(-\frac{\left(bx^4+a\right)^{\frac{1}{4}}\left(\frac{a^8}{b^5}\right)^{\frac{1}{4}}a^6b-\left(\frac{a^8}{b^5}\right)^{\frac{1}{4}}bx\sqrt{\frac{\sqrt{\frac{a^8}{b^5}}a^8b^3x^2+\sqrt{bx^4+a}a^{12}}{x^2}}}{a^8x}\right)+3\left(\frac{a^8}{b^5}\right)^{\frac{1}{4}}b\log\left(\frac{27\left(\left(bx^4+a\right)^{\frac{1}{4}}a^6+\left(\frac{a^8}{b^5}\right)^{\frac{3}{4}}b^4x\right)}{x}\right)-}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/128*(12*(a^8/b^5)^(1/4)*b*arctan(-((b*x^4 + a)^(1/4)*(a^8/b^5)^(1/4)*a^6
*b - (a^8/b^5)^(1/4)*b*x*sqrt((sqrt(a^8/b^5)*a^8*b^3*x^2 + sqrt(b*x^4 + a)*
a^12)/x^2))/(a^8*x)) + 3*(a^8/b^5)^(1/4)*b*log(27*((b*x^4 + a)^(1/4)*a^6 +
```

```
(a^8/b^5)^(3/4)*b^4*x)/x) - 3*(a^8/b^5)^(1/4)*b*log(27*((b*x^4 + a)^(1/4)*a
^6 - (a^8/b^5)^(3/4)*b^4*x)/x) - 4*(4*b*x^5 + 3*a*x)*(b*x^4 + a)^(3/4))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4 + a\right)^{\frac{3}{4}} x^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/4)*x^4, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \left(b\,x^4 + a\right)^{\frac{3}{4}} x^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^4+a)^(3/4),x)
```

```
[Out] int(x^4*(b*x^4+a)^(3/4),x)
```

maxima [A] time = 3.03, size = 148, normalized size = 1.47

$$\frac{3\,a^2\left(\frac{2\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}} + \frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)}{128\,b} + \frac{\frac{\left(bx^4+a\right)^{\frac{3}{4}}a^2b}{x^3} + \frac{3\left(bx^4+a\right)^{\frac{7}{4}}a^2}{x^7}}{32\left(b^3 - \frac{2\left(bx^4+a\right)b^2}{x^4} + \frac{\left(bx^4+a\right)^2 b}{x^8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] 3/128*a^2*(2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) + log(-(b^(1/4)
- (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/4))/b + 1/32*(
(b*x^4 + a)^(3/4)*a^2*b/x^3 + 3*(b*x^4 + a)^(7/4)*a^2/x^7)/(b^3 - 2*(b*x^4
+ a)*b^2/x^4 + (b*x^4 + a)^2*b/x^8)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \left(b\,x^4 + a\right)^{3/4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^4)^(3/4),x)
```

```
[Out] int(x^4*(a + b*x^4)^(3/4), x)
```

sympy [C] time = 4.29, size = 39, normalized size = 0.39

$$\frac{a^{\frac{3}{4}}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{4}, \frac{5}{4} \\ \frac{9}{4}\end{matrix}\right| \frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**4+a)**(3/4),x)
```

```
[Out] a**(3/4)*x**5*gamma(5/4)*hyper((-3/4, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/
a)/(4*gamma(9/4))
```

3.1034 $\int \left(a + bx^4\right)^{3/4}\, dx$

Optimal. Leaf size=75

$$\frac{1}{4}x\left(a+bx^4\right)^{3/4} + \frac{3a\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8\sqrt[4]{b}} + \frac{3a\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8\sqrt[4]{b}}$$

```
[Out] 1/4*x*(b*x^4+a)^(3/4)+3/8*a*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(1/4)+3/8*a
*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(1/4)
```

Rubi [A] time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {195, 240, 212, 206, 203}

$$\frac{1}{4}x\left(a+bx^4\right)^{3/4} + \frac{3a\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8\sqrt[4]{b}} + \frac{3a\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(3/4),x]
```

```
[Out] (x*(a + b*x^4)^(3/4))/4 + (3*a*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(8*b^
(1/4)) + (3*a*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(8*b^(1/4))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^4\right)^{3/4}\,dx &= \frac{1}{4}x\left(a+bx^4\right)^{3/4}+\frac{1}{4}(3a)\int \frac{1}{\sqrt[4]{a+bx^4}}\,dx\\ &= \frac{1}{4}x\left(a+bx^4\right)^{3/4}+\frac{1}{4}(3a)\,\text{Subst}\left(\int \frac{1}{1-bx^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)\\ &= \frac{1}{4}x\left(a+bx^4\right)^{3/4}+\frac{1}{8}(3a)\,\text{Subst}\left(\int \frac{1}{1-\sqrt{b}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)+\frac{1}{8}(3a)\,\text{Subst}\left(\int \frac{1}{1+\sqrt{b}}\right.\\ &= \frac{1}{4}x\left(a+bx^4\right)^{3/4}+\frac{3a\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8\sqrt[4]{b}}+\frac{3a\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8\sqrt[4]{b}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 46, normalized size = 0.61

$$\frac{x\left(a+bx^4\right)^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{4};\frac{5}{4};-\frac{bx^4}{a}\right)}{\left(\frac{bx^4}{a}+1\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(3/4),x]
```

```
[Out] (x*(a + b*x^4)^(3/4)*Hypergeometric2F1[-3/4, 1/4, 5/4, -((b*x^4)/a)])/(1 +
(b*x^4)/a)^(3/4)
```

fricas [B] time = 0.94, size = 192, normalized size = 2.56

$$\frac{1}{4}\left(bx^4+a\right)^{\frac{3}{4}}x+\frac{3}{4}\left(\frac{a^4}{b}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(bx^4+a\right)^{\frac{1}{4}}\left(\frac{a^4}{b}\right)^{\frac{1}{4}}a^3-\left(\frac{a^4}{b}\right)^{\frac{1}{4}}x\sqrt{\frac{\sqrt{\frac{a^4}{b}}\,a^4bx^2+\sqrt{bx^4+a}\,a^6}{x^2}}}{a^4x}\right)+\frac{3}{16}\left(\frac{a^4}{b}\right)^{\frac{1}{4}}\log\left(\frac{27\left((b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/4*(b*x^4 + a)^(3/4)*x + 3/4*(a^4/b)^(1/4)*arctan(-((b*x^4 + a)^(1/4)*(a^4
/b)^(1/4)*a^3 - (a^4/b)^(1/4)*x*sqrt((sqrt(a^4/b)*a^4*b*x^2 + sqrt(b*x^4 +
a)*a^6)/x^2))/(a^4*x)) + 3/16*(a^4/b)^(1/4)*log(27*((b*x^4 + a)^(1/4)*a^3 +
 (a^4/b)^(3/4)*b*x)/x) - 3/16*(a^4/b)^(1/4)*log(27*((b*x^4 + a)^(1/4)*a^3 -
 (a^4/b)^(3/4)*b*x)/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4+a\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/4), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \left(b\,x^4+a\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(3/4),x)
```

```
[Out] int((b*x^4+a)^(3/4),x)
```

maxima [A] time = 2.90, size = 102, normalized size = 1.36

$$-\frac{3}{16}a\left(\frac{2\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)-\frac{\left(bx^4+a\right)^{\frac{3}{4}}a}{4\left(b-\frac{bx^4+a}{x^4}\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] -3/16*a*(2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) + log(-(b^(1/4) -
(b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/4)) - 1/4*(b*x^4
 + a)^(3/4)*a/((b - (b*x^4 + a)/x^4)*x^3)
```

mupad [B] time = 1.06, size = 37, normalized size = 0.49

$$\frac{x\left(b\,x^4+a\right)^{3/4}\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{4};\ \frac{5}{4};\ -\frac{b\,x^4}{a}\right)}{\left(\frac{b\,x^4}{a}+1\right)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(3/4),x)
```

```
[Out] (x*(a + b*x^4)^(3/4)*hypergeom([-3/4, 1/4], 5/4, -(b*x^4)/a))/((b*x^4)/a +
1)^(3/4)
```

sympy [C] time = 1.79, size = 37, normalized size = 0.49

$$\frac{a^{\frac{3}{4}}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(3/4),x)
```

```
[Out] a**(3/4)*x*gamma(1/4)*hyper((-3/4, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/
(4*gamma(5/4))
```

3.1035 $\int \frac{\left(a+bx^4\right)^{3/4}}{x^4}\,dx$

Optimal. Leaf size=75

$$\frac{1}{2}b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)+\frac{1}{2}b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)-\frac{\left(a+bx^4\right)^{3/4}}{3x^3}$$

```
[Out] -1/3*(b*x^4+a)^(3/4)/x^3+1/2*b^(3/4)*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))+1/2*
b^(3/4)*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))
```

Rubi [A] time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {277, 240, 212, 206, 203}

$$\frac{1}{2}b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)+\frac{1}{2}b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)-\frac{\left(a+bx^4\right)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(3/4)/x^4,x]
```

```
[Out] -(a + b*x^4)^(3/4)/(3*x^3) + (b^(3/4)*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)]
)/2 + (b^(3/4)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^4\right)^{3/4}}{x^4}\,dx &= -\frac{\left(a+bx^4\right)^{3/4}}{3x^3} + b\int \frac{1}{\sqrt[4]{a+bx^4}}\,dx \\ &= -\frac{\left(a+bx^4\right)^{3/4}}{3x^3} + b\,\text{Subst}\left(\int \frac{1}{1-bx^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right) \\ &= -\frac{\left(a+bx^4\right)^{3/4}}{3x^3} + \frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{1-\sqrt{b}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right) + \frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{1+\sqrt{b}\,x^2}\,dx, x, \right. \\ &= -\frac{\left(a+bx^4\right)^{3/4}}{3x^3} + \frac{1}{2}b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right) + \frac{1}{2}b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.68

$$-\frac{\left(a+bx^4\right)^{3/4}\,{}_2F_1\left(-\frac{3}{4},-\frac{3}{4};\frac{1}{4};-\frac{bx^4}{a}\right)}{3x^3\left(\frac{bx^4}{a}+1\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(3/4)/x^4,x]
```

```
[Out] -1/3*((a + b*x^4)^(3/4)*Hypergeometric2F1[-3/4, -3/4, 1/4, -((b*x^4)/a)])/(
x^3*(1 + (b*x^4)/a)^(3/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{4}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/4)/x^4, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{3}{4}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(3/4)/x^4,x)
```

```
[Out] int((b*x^4+a)^(3/4)/x^4,x)
```

maxima [A] time = 3.03, size = 85, normalized size = 1.13

$$-\frac{1}{4}\,b\left(\frac{2\,\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)-\frac{\left(bx^4+a\right)^{\frac{3}{4}}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^4,x, algorithm="maxima")
```

```
[Out] -1/4*b*(2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) + log(-(b^(1/4) - (
b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/4)) - 1/3*(b*x^4
+ a)^(3/4)/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^4+a\right)^{3/4}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(3/4)/x^4,x)
```

```
[Out] int((a + b*x^4)^(3/4)/x^4, x)
```

sympy [C] time = 3.34, size = 42, normalized size = 0.56

$$\frac{a^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{4},-\frac{3}{4}\\ \frac{1}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(3/4)/x**4,x)
```

```
[Out] a**(3/4)*gamma(-3/4)*hyper((-3/4, -3/4), (1/4,), b*x**4*exp_polar(I*pi)/a)/
(4*x**3*gamma(1/4))
```

3.1036 $\int \frac{\left(a+bx^4\right)^{3/4}}{x^8}\,dx$

Optimal. Leaf size=21

$$-\frac{\left(a+bx^4\right)^{7/4}}{7ax^7}$$

```
[Out] -1/7*(b*x^4+a)^(7/4)/a/x^7
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {264}

$$-\frac{\left(a+bx^4\right)^{7/4}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(3/4)/x^8,x]
```

```
[Out] -(a + b*x^4)^(7/4)/(7*a*x^7)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^4\right)^{3/4}}{x^8}\,dx = -\frac{\left(a+bx^4\right)^{7/4}}{7ax^7}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$-\frac{\left(a+bx^4\right)^{7/4}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(3/4)/x^8,x]
```

```
[Out] -1/7*(a + b*x^4)^(7/4)/(a*x^7)
```

fricas [A] time = 1.23, size = 17, normalized size = 0.81

$$-\frac{\left(bx^4+a\right)^{\frac{7}{4}}}{7\,ax^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^8,x, algorithm="fricas")
```

```
[Out] -1/7*(b*x^4 + a)^(7/4)/(a*x^7)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{4}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^8,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(3/4)/x^8, x)
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$-\frac{\left(b\,x^4+a\right)^{\frac{7}{4}}}{7a\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(3/4)/x^8,x)
[Out] -1/7*(b*x^4+a)^(7/4)/a/x^7
```

maxima [A] time = 1.36, size = 17, normalized size = 0.81

$$-\frac{\left(bx^4+a\right)^{\frac{7}{4}}}{7\,ax^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^8,x, algorithm="maxima")
[Out] -1/7*(b*x^4 + a)^(7/4)/(a*x^7)
```

mupad [B] time = 1.25, size = 17, normalized size = 0.81

$$-\frac{\left(b\,x^4+a\right)^{7/4}}{7\,a\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(3/4)/x^8,x)
[Out] -(a + b*x^4)^(7/4)/(7*a*x^7)
```

sympy [B] time = 3.18, size = 68, normalized size = 3.24

$$\frac{b^{\frac{3}{4}}\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{4x^4\Gamma\left(-\frac{3}{4}\right)}+\frac{b^{\frac{7}{4}}\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{4a\Gamma\left(-\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(3/4)/x**8,x)
[Out] b**(3/4)*(a/(b*x**4) + 1)**(3/4)*gamma(-7/4)/(4*x**4*gamma(-3/4)) + b**(7/4
)*(a/(b*x**4) + 1)**(3/4)*gamma(-7/4)/(4*a*gamma(-3/4))
```

3.1037 $\int \frac{\left(a+bx^4\right)^{3/4}}{x^{12}}\,dx$

Optimal. Leaf size=44

$$\frac{4b\left(a+bx^4\right)^{7/4}}{77a^2x^7}-\frac{\left(a+bx^4\right)^{7/4}}{11ax^{11}}$$

```
[Out] -1/11*(b*x^4+a)^(7/4)/a/x^11+4/77*b*(b*x^4+a)^(7/4)/a^2/x^7
```

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$\frac{4b\left(a+bx^4\right)^{7/4}}{77a^2x^7}-\frac{\left(a+bx^4\right)^{7/4}}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(3/4)/x^12,x]
```

```
[Out] -(a + b*x^4)^(7/4)/(11*a*x^11) + (4*b*(a + b*x^4)^(7/4))/(77*a^2*x^7)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^4\right)^{3/4}}{x^{12}}\,dx &= -\frac{\left(a+bx^4\right)^{7/4}}{11ax^{11}}-\frac{(4b)\int \frac{\left(a+bx^4\right)^{3/4}}{x^8}\,dx}{11a}\\ &= -\frac{\left(a+bx^4\right)^{7/4}}{11ax^{11}}+\frac{4b\left(a+bx^4\right)^{7/4}}{77a^2x^7}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.70

$$\frac{\left(a+bx^4\right)^{7/4}\left(4bx^4-7a\right)}{77a^2x^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(3/4)/x^12,x]
```

```
[Out] ((a + b*x^4)^(7/4)*(-7*a + 4*b*x^4))/(77*a^2*x^11)
```

fricas [A] time = 1.17, size = 38, normalized size = 0.86

$$\frac{\left(4\,b^2x^8-3\,abx^4-7\,a^2\right)\left(bx^4+a\right)^{\frac{3}{4}}}{77\,a^2x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^12,x, algorithm="fricas")
```

```
[Out] 1/77*(4*b^2*x^8 - 3*a*b*x^4 - 7*a^2)*(b*x^4 + a)^(3/4)/(a^2*x^11)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{4}}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^12,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/4)/x^12, x)
```

maple [A] time = 0.00, size = 28, normalized size = 0.64

$$-\frac{\left(b\,x^4+a\right)^{\frac{7}{4}}\left(-4b\,x^4+7a\right)}{77a^2x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(3/4)/x^12,x)
```

```
[Out] -1/77*(b*x^4+a)^(7/4)*(-4*b*x^4+7*a)/x^11/a^2
```

maxima [A] time = 1.34, size = 35, normalized size = 0.80

$$\frac{\frac{11\left(bx^4+a\right)^{\frac{7}{4}}b}{x^7}-\frac{7\left(bx^4+a\right)^{\frac{11}{4}}}{x^{11}}}{77\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^12,x, algorithm="maxima")
```

```
[Out] 1/77*(11*(b*x^4 + a)^(7/4)*b/x^7 - 7*(b*x^4 + a)^(11/4)/x^11)/a^2
```

mupad [B] time = 1.33, size = 56, normalized size = 1.27

$$-\frac{7\,a^2\left(b\,x^4+a\right)^{3/4}-4\,b^2\,x^8\left(b\,x^4+a\right)^{3/4}+3\,a\,b\,x^4\left(b\,x^4+a\right)^{3/4}}{77\,a^2\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(3/4)/x^12,x)
```

```
[Out] -(7*a^2*(a + b*x^4)^(3/4) - 4*b^2*x^8*(a + b*x^4)^(3/4) + 3*a*b*x^4*(a + b*
x^4)^(3/4))/(77*a^2*x^11)
```

sympy [B] time = 3.09, size = 110, normalized size = 2.50

$$-\frac{7b^{\frac{3}{4}}\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{11}{4}\right)}{16x^8\Gamma\left(-\frac{3}{4}\right)}-\frac{3b^{\frac{7}{4}}\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{11}{4}\right)}{16ax^4\Gamma\left(-\frac{3}{4}\right)}+\frac{b^{\frac{11}{4}}\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{11}{4}\right)}{4a^2\Gamma\left(-\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(3/4)/x**12,x)
```

```
[Out] -7*b**(3/4)*(a/(b*x**4) + 1)**(3/4)*gamma(-11/4)/(16*x**8*gamma(-3/4)) - 3*
b**(7/4)*(a/(b*x**4) + 1)**(3/4)*gamma(-11/4)/(16*a*x**4*gamma(-3/4)) + b**
(11/4)*(a/(b*x**4) + 1)**(3/4)*gamma(-11/4)/(4*a**2*gamma(-3/4))
```

3.1038 $\int \frac{\left(a+bx^4\right)^{3/4}}{x^{16}}\,dx$

Optimal. Leaf size=68

$$-\frac{32b^2\left(a+bx^4\right)^{7/4}}{1155a^3x^7}+\frac{8b\left(a+bx^4\right)^{7/4}}{165a^2x^{11}}-\frac{\left(a+bx^4\right)^{7/4}}{15ax^{15}}$$

```
[Out] -1/15*(b*x^4+a)^(7/4)/a/x^15+8/165*b*(b*x^4+a)^(7/4)/a^2/x^11-32/1155*b^2*(
b*x^4+a)^(7/4)/a^3/x^7
```

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$-\frac{32b^2\left(a+bx^4\right)^{7/4}}{1155a^3x^7}+\frac{8b\left(a+bx^4\right)^{7/4}}{165a^2x^{11}}-\frac{\left(a+bx^4\right)^{7/4}}{15ax^{15}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(3/4)/x^16,x]
```

```
[Out] -(a + b*x^4)^(7/4)/(15*a*x^15) + (8*b*(a + b*x^4)^(7/4))/(165*a^2*x^11) - (
32*b^2*(a + b*x^4)^(7/4))/(1155*a^3*x^7)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^4\right)^{3/4}}{x^{16}}\,dx &= -\frac{\left(a+bx^4\right)^{7/4}}{15ax^{15}}-\frac{(8b)\int \frac{\left(a+bx^4\right)^{3/4}}{x^{12}}\,dx}{15a}\\ &= -\frac{\left(a+bx^4\right)^{7/4}}{15ax^{15}}+\frac{8b\left(a+bx^4\right)^{7/4}}{165a^2x^{11}}+\frac{\left(32b^2\right)\int \frac{\left(a+bx^4\right)^{3/4}}{x^8}\,dx}{165a^2}\\ &= -\frac{\left(a+bx^4\right)^{7/4}}{15ax^{15}}+\frac{8b\left(a+bx^4\right)^{7/4}}{165a^2x^{11}}-\frac{32b^2\left(a+bx^4\right)^{7/4}}{1155a^3x^7}\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.62

$$-\frac{\left(a+bx^4\right)^{7/4}\left(77a^2-56abx^4+32b^2x^8\right)}{1155a^3x^{15}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(3/4)/x^16,x]
```

```
[Out] -1/1155*((a + b*x^4)^(7/4)*(77*a^2 - 56*a*b*x^4 + 32*b^2*x^8))/(a^3*x^15)
```

fricas [A] time = 1.05, size = 49, normalized size = 0.72

$$-\frac{\left(32\,b^3x^{12}-24\,ab^2x^8+21\,a^2bx^4+77\,a^3\right)\left(bx^4+a\right)^{\frac{3}{4}}}{1155\,a^3x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^16,x, algorithm="fricas")
```

```
[Out] -1/1155*(32*b^3*x^12 - 24*a*b^2*x^8 + 21*a^2*b*x^4 + 77*a^3)*(b*x^4 + a)^(3
/4)/(a^3*x^15)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{4}}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^16,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/4)/x^16, x)
```

maple [A] time = 0.01, size = 39, normalized size = 0.57

$$-\frac{\left(b\,x^4+a\right)^{\frac{7}{4}}\left(32b^2x^8-56ab\,x^4+77a^2\right)}{1155a^3x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(3/4)/x^16,x)
```

```
[Out] -1/1155*(b*x^4+a)^(7/4)*(32*b^2*x^8-56*a*b*x^4+77*a^2)/a^3/x^15
```

maxima [A] time = 1.30, size = 52, normalized size = 0.76

$$-\frac{\frac{165\left(bx^4+a\right)^{\frac{7}{4}}b^2}{x^7}-\frac{210\left(bx^4+a\right)^{\frac{11}{4}}b}{x^{11}}+\frac{77\left(bx^4+a\right)^{\frac{15}{4}}}{x^{15}}}{1155\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^16,x, algorithm="maxima")
```

```
[Out] -1/1155*(165*(b*x^4 + a)^(7/4)*b^2/x^7 - 210*(b*x^4 + a)^(11/4)*b/x^11 + 77
*(b*x^4 + a)^(15/4)/x^15)/a^3
```

mupad [B] time = 1.55, size = 73, normalized size = 1.07

$$\frac{8\,b^2\left(b\,x^4+a\right)^{3/4}}{385\,a^2\,x^7}-\frac{b\left(b\,x^4+a\right)^{3/4}}{55\,a\,x^{11}}-\frac{32\,b^3\left(b\,x^4+a\right)^{3/4}}{1155\,a^3\,x^3}-\frac{\left(b\,x^4+a\right)^{3/4}}{15\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(3/4)/x^16,x)
```

```
[Out] (8*b^2*(a + b*x^4)^(3/4))/(385*a^2*x^7) - (b*(a + b*x^4)^(3/4))/(55*a*x^11)
 - (32*b^3*(a + b*x^4)^(3/4))/(1155*a^3*x^3) - (a + b*x^4)^(3/4)/(15*x^15)
```

sympy [**B**] time = 7.16, size = 520, normalized size = 7.65

$$\frac{77a^5b^{\frac{19}{4}}\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{15}{4}\right)}{64a^5b^4x^{12}\Gamma\left(-\frac{3}{4}\right)+128a^4b^5x^{16}\Gamma\left(-\frac{3}{4}\right)+64a^3b^6x^{20}\Gamma\left(-\frac{3}{4}\right)}+\frac{175a^4b^{\frac{23}{4}}x^4\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{15}{4}\right)}{64a^5b^4x^{12}\Gamma\left(-\frac{3}{4}\right)+128a^4b^5x^{16}\Gamma\left(-\frac{3}{4}\right)+64a^3b^6x^{20}\Gamma\left(-\frac{3}{4}\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(3/4)/x**16,x)
```

```
[Out] 77*a**5*b**(19/4)*(a/(b*x**4) + 1)**(3/4)*gamma(-15/4)/(64*a**5*b**4*x**12*
gamma(-3/4) + 128*a**4*b**5*x**16*gamma(-3/4) + 64*a**3*b**6*x**20*gamma(-3
/4)) + 175*a**4*b**(23/4)*x**4*(a/(b*x**4) + 1)**(3/4)*gamma(-15/4)/(64*a**
5*b**4*x**12*gamma(-3/4) + 128*a**4*b**5*x**16*gamma(-3/4) + 64*a**3*b**6*x
**20*gamma(-3/4)) + 95*a**3*b**(27/4)*x**8*(a/(b*x**4) + 1)**(3/4)*gamma(-1
5/4)/(64*a**5*b**4*x**12*gamma(-3/4) + 128*a**4*b**5*x**16*gamma(-3/4) + 64
*a**3*b**6*x**20*gamma(-3/4)) + 5*a**2*b**(31/4)*x**12*(a/(b*x**4) + 1)**(3
/4)*gamma(-15/4)/(64*a**5*b**4*x**12*gamma(-3/4) + 128*a**4*b**5*x**16*gamm
a(-3/4) + 64*a**3*b**6*x**20*gamma(-3/4)) + 40*a*b**(35/4)*x**16*(a/(b*x**4
) + 1)**(3/4)*gamma(-15/4)/(64*a**5*b**4*x**12*gamma(-3/4) + 128*a**4*b**5*
x**16*gamma(-3/4) + 64*a**3*b**6*x**20*gamma(-3/4)) + 32*b**(39/4)*x**20*(a
/(b*x**4) + 1)**(3/4)*gamma(-15/4)/(64*a**5*b**4*x**12*gamma(-3/4) + 128*a*
*4*b**5*x**16*gamma(-3/4) + 64*a**3*b**6*x**20*gamma(-3/4))
```

3.1039 $\int \frac{\left(a+bx^4\right)^{3/4}}{x^{20}}\,dx$

Optimal. Leaf size=92

$$\frac{128b^3\left(a+bx^4\right)^{7/4}}{7315a^4x^7}-\frac{32b^2\left(a+bx^4\right)^{7/4}}{1045a^3x^{11}}+\frac{4b\left(a+bx^4\right)^{7/4}}{95a^2x^{15}}-\frac{\left(a+bx^4\right)^{7/4}}{19ax^{19}}$$

```
[Out] -1/19*(b*x^4+a)^(7/4)/a/x^19+4/95*b*(b*x^4+a)^(7/4)/a^2/x^15-32/1045*b^2*(b
*x^4+a)^(7/4)/a^3/x^11+128/7315*b^3*(b*x^4+a)^(7/4)/a^4/x^7
```

Rubi [A] time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$\frac{128b^3\left(a+bx^4\right)^{7/4}}{7315a^4x^7}-\frac{32b^2\left(a+bx^4\right)^{7/4}}{1045a^3x^{11}}+\frac{4b\left(a+bx^4\right)^{7/4}}{95a^2x^{15}}-\frac{\left(a+bx^4\right)^{7/4}}{19ax^{19}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(3/4)/x^20,x]
```

```
[Out] -(a + b*x^4)^(7/4)/(19*a*x^19) + (4*b*(a + b*x^4)^(7/4))/(95*a^2*x^15) - (3
2*b^2*(a + b*x^4)^(7/4))/(1045*a^3*x^11) + (128*b^3*(a + b*x^4)^(7/4))/(731
5*a^4*x^7)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^4\right)^{3/4}}{x^{20}}\,dx &= -\frac{\left(a+bx^4\right)^{7/4}}{19ax^{19}}-\frac{(12b)\int \frac{\left(a+bx^4\right)^{3/4}}{x^{16}}\,dx}{19a}\\
&= -\frac{\left(a+bx^4\right)^{7/4}}{19ax^{19}}+\frac{4b\left(a+bx^4\right)^{7/4}}{95a^2x^{15}}+\frac{\left(32b^2\right)\int \frac{\left(a+bx^4\right)^{3/4}}{x^{12}}\,dx}{95a^2}\\
&= -\frac{\left(a+bx^4\right)^{7/4}}{19ax^{19}}+\frac{4b\left(a+bx^4\right)^{7/4}}{95a^2x^{15}}-\frac{32b^2\left(a+bx^4\right)^{7/4}}{1045a^3x^{11}}-\frac{\left(128b^3\right)\int \frac{\left(a+bx^4\right)^{3/4}}{x^8}\,dx}{1045a^3}\\
&= -\frac{\left(a+bx^4\right)^{7/4}}{19ax^{19}}+\frac{4b\left(a+bx^4\right)^{7/4}}{95a^2x^{15}}-\frac{32b^2\left(a+bx^4\right)^{7/4}}{1045a^3x^{11}}+\frac{128b^3\left(a+bx^4\right)^{7/4}}{7315a^4x^7}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 53, normalized size = 0.58

$$\frac{\left(a+bx^4\right)^{7/4}\left(-385a^3+308a^2bx^4-224ab^2x^8+128b^3x^{12}\right)}{7315a^4x^{19}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(3/4)/x^20,x]
```

```
[Out] ((a + b*x^4)^(7/4)*(-385*a^3 + 308*a^2*b*x^4 - 224*a*b^2*x^8 + 128*b^3*x^12
))/(7315*a^4*x^19)
```

fricas [A] time = 1.06, size = 60, normalized size = 0.65

$$\frac{\left(128\,b^4x^{16} - 96\,ab^3x^{12} + 84\,a^2b^2x^8 - 77\,a^3bx^4 - 385\,a^4\right)\left(bx^4 + a\right)^{\frac{3}{4}}}{7315\,a^4x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^20,x, algorithm="fricas")
```

```
[Out] 1/7315*(128*b^4*x^16 - 96*a*b^3*x^12 + 84*a^2*b^2*x^8 - 77*a^3*b*x^4 - 385*
a^4)*(b*x^4 + a)^(3/4)/(a^4*x^19)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4 + a\right)^{\frac{3}{4}}}{x^{20}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^20,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/4)/x^20, x)
```

maple [A] time = 0.01, size = 50, normalized size = 0.54

$$-\frac{\left(b\,x^4 + a\right)^{\frac{7}{4}}\left(-128b^3x^{12} + 224a\,b^2x^8 - 308a^2b\,x^4 + 385a^3\right)}{7315a^4x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(3/4)/x^20,x)
```

```
[Out] -1/7315*(b*x^4+a)^(7/4)*(-128*b^3*x^12+224*a*b^2*x^8-308*a^2*b*x^4+385*a^3)
/x^19/a^4
```

maxima [A] time = 1.37, size = 69, normalized size = 0.75

$$\frac{\frac{1045\left(bx^4+a\right)^{\frac{7}{4}}b^3}{x^7} - \frac{1995\left(bx^4+a\right)^{\frac{11}{4}}b^2}{x^{11}} + \frac{1463\left(bx^4+a\right)^{\frac{15}{4}}b}{x^{15}} - \frac{385\left(bx^4+a\right)^{\frac{19}{4}}}{x^{19}}}{7315\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^20,x, algorithm="maxima")
```

```
[Out] 1/7315*(1045*(b*x^4 + a)^(7/4)*b^3/x^7 - 1995*(b*x^4 + a)^(11/4)*b^2/x^11 +
 1463*(b*x^4 + a)^(15/4)*b/x^15 - 385*(b*x^4 + a)^(19/4)/x^19)/a^4
```

mupad [B] time = 1.81, size = 93, normalized size = 1.01

$$\frac{128\,b^4\left(b\,x^4 + a\right)^{3/4}}{7315\,a^4\,x^3} - \frac{b\left(b\,x^4 + a\right)^{3/4}}{95\,a\,x^{15}} - \frac{\left(b\,x^4 + a\right)^{3/4}}{19\,x^{19}} - \frac{96\,b^3\left(b\,x^4 + a\right)^{3/4}}{7315\,a^3\,x^7} + \frac{12\,b^2\left(b\,x^4 + a\right)^{3/4}}{1045\,a^2\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(3/4)/x^20,x)
```

```
[Out] (128*b^4*(a + b*x^4)^(3/4))/(7315*a^4*x^3) - (b*(a + b*x^4)^(3/4))/(95*a*x^
15) - (a + b*x^4)^(3/4)/(19*x^19) - (96*b^3*(a + b*x^4)^(3/4))/(7315*a^3*x^
7) + (12*b^2*(a + b*x^4)^(3/4))/(1045*a^2*x^11)
```

sympy [**B**] time = 14.53, size = 847, normalized size = 9.21

$$-\frac{1155a^7b^{\frac{39}{4}}\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{19}{4}\right)}{256a^7b^9x^{16}\Gamma\left(-\frac{3}{4}\right)+768a^6b^{10}x^{20}\Gamma\left(-\frac{3}{4}\right)+768a^5b^{11}x^{24}\Gamma\left(-\frac{3}{4}\right)+256a^4b^{12}x^{28}\Gamma\left(-\frac{3}{4}\right)}-\frac{}{256a^7b^9x^{16}\Gamma\left(-\frac{3}{4}\right)+76}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(3/4)/x**20,x)
```

```
[Out] -1155*a**7*b**(39/4)*(a/(b*x**4) + 1)**(3/4)*gamma(-19/4)/(256*a**7*b**9*x**
*16*gamma(-3/4) + 768*a**6*b**10*x**20*gamma(-3/4) + 768*a**5*b**11*x**24*g
amma(-3/4) + 256*a**4*b**12*x**28*gamma(-3/4)) - 3696*a**6*b**(43/4)*x**4*(
a/(b*x**4) + 1)**(3/4)*gamma(-19/4)/(256*a**7*b**9*x**16*gamma(-3/4) + 768*
a**6*b**10*x**20*gamma(-3/4) + 768*a**5*b**11*x**24*gamma(-3/4) + 256*a**4*
b**12*x**28*gamma(-3/4)) - 3906*a**5*b**(47/4)*x**8*(a/(b*x**4) + 1)**(3/4)
*gamma(-19/4)/(256*a**7*b**9*x**16*gamma(-3/4) + 768*a**6*b**10*x**20*gamma
(-3/4) + 768*a**5*b**11*x**24*gamma(-3/4) + 256*a**4*b**12*x**28*gamma(-3/4
)) - 1380*a**4*b**(51/4)*x**12*(a/(b*x**4) + 1)**(3/4)*gamma(-19/4)/(256*a*
*7*b**9*x**16*gamma(-3/4) + 768*a**6*b**10*x**20*gamma(-3/4) + 768*a**5*b**
11*x**24*gamma(-3/4) + 256*a**4*b**12*x**28*gamma(-3/4)) + 45*a**3*b**(55/4
)*x**16*(a/(b*x**4) + 1)**(3/4)*gamma(-19/4)/(256*a**7*b**9*x**16*gamma(-3/
4) + 768*a**6*b**10*x**20*gamma(-3/4) + 768*a**5*b**11*x**24*gamma(-3/4) +
256*a**4*b**12*x**28*gamma(-3/4)) + 540*a**2*b**(59/4)*x**20*(a/(b*x**4) +
1)**(3/4)*gamma(-19/4)/(256*a**7*b**9*x**16*gamma(-3/4) + 768*a**6*b**10*x*
*20*gamma(-3/4) + 768*a**5*b**11*x**24*gamma(-3/4) + 256*a**4*b**12*x**28*g
amma(-3/4)) + 864*a*b**(63/4)*x**24*(a/(b*x**4) + 1)**(3/4)*gamma(-19/4)/(2
56*a**7*b**9*x**16*gamma(-3/4) + 768*a**6*b**10*x**20*gamma(-3/4) + 768*a**
5*b**11*x**24*gamma(-3/4) + 256*a**4*b**12*x**28*gamma(-3/4)) + 384*b**(67/
4)*x**28*(a/(b*x**4) + 1)**(3/4)*gamma(-19/4)/(256*a**7*b**9*x**16*gamma(-3
/4) + 768*a**6*b**10*x**20*gamma(-3/4) + 768*a**5*b**11*x**24*gamma(-3/4) +
 256*a**4*b**12*x**28*gamma(-3/4))
```

3.1040 $\int x^{10}\left(a+bx^4\right)^{3/4}\,dx$

Optimal. Leaf size=150

$$\frac{3a^{7/2}x\sqrt[4]{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{80b^{5/2}\sqrt[4]{a+bx^4}}+\frac{3a^3x^3}{80b^2\sqrt[4]{a+bx^4}}-\frac{a^2x^3\left(a+bx^4\right)^{3/4}}{40b^2}+\frac{1}{14}x^{11}\left(a+bx^4\right)^{3/4}+\frac{3ax^7\left(a+bx^4\right)^{3/4}}{140b}$$

```
[Out] 3/80*a^3*x^3/b^2/(b*x^4+a)^(1/4)-1/40*a^2*x^3*(b*x^4+a)^(3/4)/b^2+3/140*a*x
^7*(b*x^4+a)^(3/4)/b+1/14*x^11*(b*x^4+a)^(3/4)+3/80*a^(7/2)*(1+a/b/x^4)^(1/
4)*x*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1
/2)/a^(1/2)))*EllipticE(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))/b^(5/
2)/(b*x^4+a)^(1/4)
```

Rubi [A] time = 0.07, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {279, 321, 310, 281, 335, 275, 196}

$$-\frac{a^2x^3\left(a+bx^4\right)^{3/4}}{40b^2}+\frac{3a^3x^3}{80b^2\sqrt[4]{a+bx^4}}+\frac{3a^{7/2}x\sqrt[4]{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{80b^{5/2}\sqrt[4]{a+bx^4}}+\frac{1}{14}x^{11}\left(a+bx^4\right)^{3/4}+\frac{3ax^7\left(a+bx^4\right)^{3/4}}{140b}$$

Antiderivative was successfully verified.

```
[In] Int[x^10*(a + b*x^4)^(3/4),x]
```

```
[Out] (3*a^3*x^3)/(80*b^2*(a + b*x^4)^(1/4)) - (a^2*x^3*(a + b*x^4)^(3/4))/(40*b^
2) + (3*a*x^7*(a + b*x^4)^(3/4))/(140*b) + (x^11*(a + b*x^4)^(3/4))/14 + (3
*a^(7/2)*(1 + a/(b*x^4))^(1/4)*x*EllipticE[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2,
 2])/(80*b^(5/2)*(a + b*x^4)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 281

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Dist[(x*(1 + a/(b*x^4)
)^(1/4))/(b*(a + b*x^4)^(1/4)), Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x] /
; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 310

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Dist[a/2, Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int x^{10}\left(a+bx^4\right)^{3/4}\,dx = \frac{1}{14}x^{11}\left(a+bx^4\right)^{3/4} + \frac{1}{14}(3a)\int \frac{x^{10}}{\sqrt[4]{a+bx^4}}\,dx$$

$$= \frac{3ax^7\left(a+bx^4\right)^{3/4}}{140b} + \frac{1}{14}x^{11}\left(a+bx^4\right)^{3/4} - \frac{\left(3a^2\right)\int \frac{x^6}{\sqrt[4]{a+bx^4}}\,dx}{20b}$$

$$= -\frac{a^2x^3\left(a+bx^4\right)^{3/4}}{40b^2} + \frac{3ax^7\left(a+bx^4\right)^{3/4}}{140b} + \frac{1}{14}x^{11}\left(a+bx^4\right)^{3/4} + \frac{\left(3a^3\right)\int \frac{x^2}{\sqrt[4]{a+bx^4}}\,dx}{40b^2}$$

$$= \frac{3a^3x^3}{80b^2\sqrt[4]{a+bx^4}} - \frac{a^2x^3\left(a+bx^4\right)^{3/4}}{40b^2} + \frac{3ax^7\left(a+bx^4\right)^{3/4}}{140b} + \frac{1}{14}x^{11}\left(a+bx^4\right)^{3/4} - \frac{\left(3a^4\right)}{}$$

$$= \frac{3a^3x^3}{80b^2\sqrt[4]{a+bx^4}} - \frac{a^2x^3\left(a+bx^4\right)^{3/4}}{40b^2} + \frac{3ax^7\left(a+bx^4\right)^{3/4}}{140b} + \frac{1}{14}x^{11}\left(a+bx^4\right)^{3/4} - \frac{\left(3a^4\sqrt[4]{}\right.}{}$$

$$= \frac{3a^3x^3}{80b^2\sqrt[4]{a+bx^4}} - \frac{a^2x^3\left(a+bx^4\right)^{3/4}}{40b^2} + \frac{3ax^7\left(a+bx^4\right)^{3/4}}{140b} + \frac{1}{14}x^{11}\left(a+bx^4\right)^{3/4} + \frac{\left(3a^4\sqrt[4]{}\right.}{}$$

$$= \frac{3a^3x^3}{80b^2\sqrt[4]{a+bx^4}} - \frac{a^2x^3\left(a+bx^4\right)^{3/4}}{40b^2} + \frac{3ax^7\left(a+bx^4\right)^{3/4}}{140b} + \frac{1}{14}x^{11}\left(a+bx^4\right)^{3/4} + \frac{\left(3a^4\sqrt[4]{}\right.}{}$$

$$= \frac{3a^3x^3}{80b^2\sqrt[4]{a+bx^4}} - \frac{a^2x^3\left(a+bx^4\right)^{3/4}}{40b^2} + \frac{3ax^7\left(a+bx^4\right)^{3/4}}{140b} + \frac{1}{14}x^{11}\left(a+bx^4\right)^{3/4} + \frac{3a^{7/2}\sqrt[4]{}}{}$$

Mathematica [C] time = 0.07, size = 96, normalized size = 0.64

$$\frac{x^3\left(a+bx^4\right)^{3/4}\left(\left(\frac{bx^4}{a}+1\right)^{3/4}\left(-7a^2+3abx^4+10b^2x^8\right)+7a^2\,{}_2F_1\left(-\frac{3}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^4}{a}\right)\right)}{140b^2\left(\frac{bx^4}{a}+1\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10*(a + b*x^4)^(3/4),x]
```

```
[Out] (x^3*(a + b*x^4)^(3/4)*((1 + (b*x^4)/a)^(3/4)*(-7*a^2 + 3*a*b*x^4 + 10*b^2*
x^8) + 7*a^2*Hypergeometric2F1[-3/4, 3/4, 7/4, -((b*x^4)/a)]))/(140*b^2*(1
+ (b*x^4)/a)^(3/4))
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^4+a\right)^{\frac{3}{4}}x^{10},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(3/4)*x^10, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4+a\right)^{\frac{3}{4}}x^{10}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/4)*x^10, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \left(b\,x^4+a\right)^{\frac{3}{4}}x^{10}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(b*x^4+a)^(3/4),x)
```

```
[Out] int(x^10*(b*x^4+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4+a\right)^{\frac{3}{4}}x^{10}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/4)*x^10, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{10}\left(b\,x^4+a\right)^{3/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(a + b*x^4)^(3/4),x)
```

```
[Out] int(x^10*(a + b*x^4)^(3/4), x)
```

sympy [C] time = 4.84, size = 39, normalized size = 0.26

$$\frac{a^{\frac{3}{4}} x^{11} \Gamma\left(\frac{11}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{11}{4} \\ \frac{15}{4} \end{matrix} \middle| \frac{b x^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(b*x**4+a)**(3/4),x)
```

```
[Out] a**(3/4)*x**11*gamma(11/4)*hyper((-3/4, 11/4), (15/4,), b*x**4*exp_polar(I*
pi)/a)/(4*gamma(15/4))
```

3.1041 $\int x^6 \left(a+bx^4\right)^{3/4}\,dx$

Optimal. Leaf size=126

$$-\frac{3a^{5/2}x\sqrt[4]{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{40b^{3/2}\sqrt[4]{a+bx^4}}-\frac{3a^2x^3}{40b\sqrt[4]{a+bx^4}}+\frac{1}{10}x^7\left(a+bx^4\right)^{3/4}+\frac{ax^3\left(a+bx^4\right)^{3/4}}{20b}$$

```
[Out] -3/40*a^2*x^3/b/(b*x^4+a)^(1/4)+1/20*a*x^3*(b*x^4+a)^(3/4)/b+1/10*x^7*(b*x^
4+a)^(3/4)-3/40*a^(5/2)*(1+a/b/x^4)^(1/4)*x*(cos(1/2*arccot(x^2*b^(1/2)/a^(
1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arcc
ot(x^2*b^(1/2)/a^(1/2))),2^(1/2))/b^(3/2)/(b*x^4+a)^(1/4)
```

Rubi [A] time = 0.06, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {279, 321, 310, 281, 335, 275, 196}

$$-\frac{3a^{5/2}x\sqrt[4]{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{40b^{3/2}\sqrt[4]{a+bx^4}}-\frac{3a^2x^3}{40b\sqrt[4]{a+bx^4}}+\frac{1}{10}x^7\left(a+bx^4\right)^{3/4}+\frac{ax^3\left(a+bx^4\right)^{3/4}}{20b}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x^4)^(3/4),x]
```

```
[Out] (-3*a^2*x^3)/(40*b*(a + b*x^4)^(1/4)) + (a*x^3*(a + b*x^4)^(3/4))/(20*b) +
(x^7*(a + b*x^4)^(3/4))/10 - (3*a^(5/2)*(1 + a/(b*x^4))^(1/4)*x*EllipticE[A
rcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(40*b^(3/2)*(a + b*x^4)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 281

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Dist[(x*(1 + a/(b*x^4)
)^(1/4))/(b*(a + b*x^4)^(1/4)), Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x] /
; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 310

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Dist[a/2, Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b},
```

```
x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int x^6 \left(a+bx^4\right)^{3/4}\,dx = \frac{1}{10}x^7\left(a+bx^4\right)^{3/4} + \frac{1}{10}(3a)\int \frac{x^6}{\sqrt[4]{a+bx^4}}\,dx$$

$$= \frac{ax^3\left(a+bx^4\right)^{3/4}}{20b} + \frac{1}{10}x^7\left(a+bx^4\right)^{3/4} - \frac{\left(3a^2\right)\int \frac{x^2}{\sqrt[4]{a+bx^4}}\,dx}{20b}$$

$$= -\frac{3a^2x^3}{40b\sqrt[4]{a+bx^4}} + \frac{ax^3\left(a+bx^4\right)^{3/4}}{20b} + \frac{1}{10}x^7\left(a+bx^4\right)^{3/4} + \frac{\left(3a^3\right)\int \frac{x^2}{\left(a+bx^4\right)^{5/4}}\,dx}{40b}$$

$$= -\frac{3a^2x^3}{40b\sqrt[4]{a+bx^4}} + \frac{ax^3\left(a+bx^4\right)^{3/4}}{20b} + \frac{1}{10}x^7\left(a+bx^4\right)^{3/4} + \frac{\left(3a^3\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{5/4}x}}{40b^2\sqrt[4]{a+bx^4}}$$

$$= -\frac{3a^2x^3}{40b\sqrt[4]{a+bx^4}} + \frac{ax^3\left(a+bx^4\right)^{3/4}}{20b} + \frac{1}{10}x^7\left(a+bx^4\right)^{3/4} - \frac{\left(3a^3\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{}{\left(1\right.}\right.}{40b^2\sqrt[4]{a+bx^4}}$$

$$= -\frac{3a^2x^3}{40b\sqrt[4]{a+bx^4}} + \frac{ax^3\left(a+bx^4\right)^{3/4}}{20b} + \frac{1}{10}x^7\left(a+bx^4\right)^{3/4} - \frac{\left(3a^3\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{}{\left(1\right.}\right.}{80b^2\sqrt[4]{a+bx^4}}$$

$$= -\frac{3a^2x^3}{40b\sqrt[4]{a+bx^4}} + \frac{ax^3\left(a+bx^4\right)^{3/4}}{20b} + \frac{1}{10}x^7\left(a+bx^4\right)^{3/4} - \frac{3a^{5/2}\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\right.\right.}{40b^{3/2}\sqrt[4]{a+bx^4}}$$

Mathematica [C] time = 0.06, size = 64, normalized size = 0.51

$$\frac{x^3\left(a+bx^4\right)^{3/4}\left(-\frac{a\,{}_2F_1\left(-\frac{3}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^4}{a}\right)}{\left(\frac{bx^4}{a}+1\right)^{3/4}} + a + bx^4\right)}{10b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x^4)^(3/4),x]
```

```
[Out] (x^3*(a + b*x^4)^(3/4)*(a + b*x^4 - (a*Hypergeometric2F1[-3/4, 3/4, 7/4, -(
(b*x^4)/a)])/(1 + (b*x^4)/a)^(3/4)))/(10*b)
```

fricas [F] time = 1.09, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^4+a\right)^{\frac{3}{4}}x^6, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(3/4)*x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4+a\right)^{\frac{3}{4}}x^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/4)*x^6, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \left(b\,x^4+a\right)^{\frac{3}{4}}x^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^4+a)^(3/4),x)
```

```
[Out] int(x^6*(b*x^4+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4+a\right)^{\frac{3}{4}}x^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/4)*x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^6\left(b\,x^4+a\right)^{3/4}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b*x^4)^(3/4),x)
```

```
[Out] int(x^6*(a + b*x^4)^(3/4), x)
```

sympy [C] time = 3.86, size = 39, normalized size = 0.31

$$\frac{a^{\frac{3}{4}}x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4}, \frac{7}{4} \\ \frac{11}{4}\end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**4+a)**(3/4),x)
```

```
[Out] a**(3/4)*x**7*gamma(7/4)*hyper((-3/4, 7/4), (11/4,), b*x**4*exp_polar(I*pi)
/a)/(4*gamma(11/4))
```

3.1042 $\int x^2 \left(a+bx^4\right)^{3/4}\,dx$

Optimal. Leaf size=99

$$\frac{a^{3/2}x\sqrt{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{4\sqrt{b}\,\sqrt[4]{a+bx^4}}+\frac{1}{6}x^3\left(a+bx^4\right)^{3/4}+\frac{ax^3}{4\sqrt[4]{a+bx^4}}$$

```
[Out] 1/4*a*x^3/(b*x^4+a)^(1/4)+1/6*x^3*(b*x^4+a)^(3/4)+1/4*a^(3/2)*(1+a/b/x^4)^(
1/4)*x*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^
(1/2)/a^(1/2)))*EllipticE(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))/(b*
x^4+a)^(1/4)/b^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {279, 310, 281, 335, 275, 196}

$$\frac{a^{3/2}x\sqrt{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{4\sqrt{b}\,\sqrt[4]{a+bx^4}}+\frac{1}{6}x^3\left(a+bx^4\right)^{3/4}+\frac{ax^3}{4\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^4)^(3/4),x]
```

```
[Out] (a*x^3)/(4*(a + b*x^4)^(1/4)) + (x^3*(a + b*x^4)^(3/4))/6 + (a^(3/2)*(1 + a
/(b*x^4))^(1/4)*x*EllipticE[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(4*Sqrt[b]
*(a + b*x^4)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 281

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Dist[(x*(1 + a/(b*x^4)
)^(1/4))/(b*(a + b*x^4)^(1/4)), Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x] /
; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 310

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Dist[a/2, Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b},
```

```
x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int x^2\left(a+bx^4\right)^{3/4}\,dx &= \frac{1}{6}x^3\left(a+bx^4\right)^{3/4}+\frac{1}{2}a\int\frac{x^2}{\sqrt[4]{a+bx^4}}\,dx \\
&= \frac{ax^3}{4\sqrt[4]{a+bx^4}}+\frac{1}{6}x^3\left(a+bx^4\right)^{3/4}-\frac{1}{4}a^2\int\frac{x^2}{\left(a+bx^4\right)^{5/4}}\,dx \\
&= \frac{ax^3}{4\sqrt[4]{a+bx^4}}+\frac{1}{6}x^3\left(a+bx^4\right)^{3/4}-\frac{\left(a^2\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\int\frac{1}{\left(1+\frac{a}{bx^4}\right)^{5/4}x^3}\,dx}{4b\sqrt[4]{a+bx^4}} \\
&= \frac{ax^3}{4\sqrt[4]{a+bx^4}}+\frac{1}{6}x^3\left(a+bx^4\right)^{3/4}+\frac{\left(a^2\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int\frac{x}{\left(1+\frac{ax^4}{b}\right)^{5/4}}\,dx,x,\frac{1}{x}\right)}{4b\sqrt[4]{a+bx^4}} \\
&= \frac{ax^3}{4\sqrt[4]{a+bx^4}}+\frac{1}{6}x^3\left(a+bx^4\right)^{3/4}+\frac{\left(a^2\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int\frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx,x,\frac{1}{x^2}\right)}{8b\sqrt[4]{a+bx^4}} \\
&= \frac{ax^3}{4\sqrt[4]{a+bx^4}}+\frac{1}{6}x^3\left(a+bx^4\right)^{3/4}+\frac{a^{3/2}\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{4\sqrt{b}\,\sqrt[4]{a+bx^4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.52

$$\frac{x^3\left(a+bx^4\right)^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^4}{a}\right)}{3\left(\frac{bx^4}{a}+1\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^4)^(3/4),x]
```

```
[Out] (x^3*(a + b*x^4)^(3/4)*Hypergeometric2F1[-3/4, 3/4, 7/4, -((b*x^4)/a)])/(3*
(1 + (b*x^4)/a)^(3/4))
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^4+a\right)^{\frac{3}{4}}x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(3/4)*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4 + a\right)^{\frac{3}{4}} x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^4+a)^(3/4),x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(3/4)*x^2, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \left(b\,x^4 + a\right)^{\frac{3}{4}} x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^4+a)^(3/4),x)
[Out] int(x^2*(b*x^4+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4 + a\right)^{\frac{3}{4}} x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^4+a)^(3/4),x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(3/4)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left(b\,x^4 + a\right)^{3/4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^4)^(3/4),x)
[Out] int(x^2*(a + b*x^4)^(3/4), x)
```

sympy [C] time = 2.50, size = 39, normalized size = 0.39

$$\frac{a^{\frac{3}{4}} x^3 \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{3}{4} \\ \frac{7}{4} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**4+a)**(3/4),x)
[Out] a**(3/4)*x**3*gamma(3/4)*hyper((-3/4, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/
a)/(4*gamma(7/4))
```

3.1043 $\int \frac{\left(a+bx^4\right)^{3/4}}{x^2}\,dx$

Optimal. Leaf size=97

$$-\frac{\left(a+bx^4\right)^{3/4}}{x}+\frac{3bx^3}{2\sqrt[4]{a+bx^4}}+\frac{3\sqrt{a}\,\sqrt{b}\,x\sqrt[4]{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt[4]{a+bx^4}}$$

```
[Out] 3/2*b*x^3/(b*x^4+a)^(1/4)-(b*x^4+a)^(3/4)/x+3/2*(1+a/b/x^4)^(1/4)*x*(cos(1/
2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))
)*EllipticE(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)*b^(1/2)/(
b*x^4+a)^(1/4)
```

Rubi [A] time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {277, 310, 281, 335, 275, 196}

$$\frac{3bx^3}{2\sqrt[4]{a+bx^4}}-\frac{\left(a+bx^4\right)^{3/4}}{x}+\frac{3\sqrt{a}\,\sqrt{b}\,x\sqrt[4]{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(3/4)/x^2,x]
```

```
[Out] (3*b*x^3)/(2*(a + b*x^4)^(1/4)) - (a + b*x^4)^(3/4)/x + (3*Sqrt[a]*Sqrt[b]*
(1 + a/(b*x^4))^(1/4)*x*EllipticE[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(2*(
a + b*x^4)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 281

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Dist[(x*(1 + a/(b*x^4)
)^(1/4))/(b*(a + b*x^4)^(1/4)), Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x] /
; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 310

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Dist[a/2, Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b},
```

```
x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^4\right)^{3/4}}{x^2}\,dx &= -\frac{\left(a+bx^4\right)^{3/4}}{x} + (3b)\int \frac{x^2}{\sqrt[4]{a+bx^4}}\,dx \\
&= \frac{3bx^3}{2\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{x} - \frac{1}{2}(3ab)\int \frac{x^2}{\left(a+bx^4\right)^{5/4}}\,dx \\
&= \frac{3bx^3}{2\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{x} - \frac{\left(3a\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{5/4}x^3}\,dx}{2\sqrt[4]{a+bx^4}} \\
&= \frac{3bx^3}{2\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{x} + \frac{\left(3a\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{2\sqrt[4]{a+bx^4}} \\
&= \frac{3bx^3}{2\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{x} + \frac{\left(3a\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx, x, \frac{1}{x^2}\right)}{4\sqrt[4]{a+bx^4}} \\
&= \frac{3bx^3}{2\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{x} + \frac{3\sqrt{a}\,\sqrt{b}\,\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt[4]{a+bx^4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 49, normalized size = 0.51

$$-\frac{\left(a+bx^4\right)^{3/4}\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{4};\frac{3}{4};-\frac{bx^4}{a}\right)}{x\left(\frac{bx^4}{a}+1\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(3/4)/x^2,x]
```

```
[Out] -(((a + b*x^4)^(3/4)*Hypergeometric2F1[-3/4, -1/4, 3/4, -((b*x^4)/a)])/(x*(
1 + (b*x^4)/a)^(3/4)))
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{3}{4}}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(3/4)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/4)/x^2, x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{3}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(3/4)/x^2,x)
```

```
[Out] int((b*x^4+a)^(3/4)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/4)/x^2, x)
```

mupad [B] time = 1.35, size = 40, normalized size = 0.41

$$\frac{\left(b\,x^4+a\right)^{3/4}\,{}_2\mathrm{F}_1\left(-\frac{3}{4},-\frac{1}{2};\ \frac{1}{2};\ -\frac{a}{b\,x^4}\right)}{2\,x\left(\frac{a}{b\,x^4}+1\right)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(3/4)/x^2,x)
```

```
[Out] ((a + b*x^4)^(3/4)*hypergeom([-3/4, -1/2], 1/2, -a/(b*x^4)))/(2*x*(a/(b*x^4
) + 1)^(3/4))
```

sympy [C] time = 2.04, size = 41, normalized size = 0.42

$$\frac{a^{\frac{3}{4}}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\,\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(3/4)/x**2,x)
```

```
[Out] a**(3/4)*gamma(-1/4)*hyper((-3/4, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/
(4*x*gamma(3/4))
```

3.1044 $\int \frac{\left(a+bx^4\right)^{3/4}}{x^6}\,dx$

Optimal. Leaf size=99

$$\frac{3b^{3/2}x\sqrt[4]{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{a}\,\sqrt[4]{a+bx^4}}-\frac{3b}{5x\sqrt[4]{a+bx^4}}-\frac{\left(a+bx^4\right)^{3/4}}{5x^5}$$

```
[Out] -3/5*b/x/(b*x^4+a)^(1/4)-1/5*(b*x^4+a)^(3/4)/x^5+3/5*b^(3/2)*(1+a/b/x^4)^(1
/4)*x*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(
1/2)/a^(1/2)))*EllipticE(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))/(b*x
^4+a)^(1/4)/a^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {277, 312, 281, 335, 275, 196}

$$\frac{3b^{3/2}x\sqrt[4]{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{a}\,\sqrt[4]{a+bx^4}}-\frac{3b}{5x\sqrt[4]{a+bx^4}}-\frac{\left(a+bx^4\right)^{3/4}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(3/4)/x^6,x]
```

```
[Out] (-3*b)/(5*x*(a + b*x^4)^(1/4)) - (a + b*x^4)^(3/4)/(5*x^5) + (3*b^(3/2)*(1
+ a/(b*x^4))^(1/4)*x*EllipticE[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(5*Sqrt
[a]*(a + b*x^4)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 281

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Dist[(x*(1 + a/(b*x^4)
)^(1/4))/(b*(a + b*x^4)^(1/4)), Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x] /
; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 312

```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> -Simp[(x*(a + b*x^
4)^(1/4))^(-1), x] - Dist[b, Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a,
```

```
b}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^4\right)^{3/4}}{x^6}\,dx &= -\frac{\left(a+bx^4\right)^{3/4}}{5x^5} + \frac{1}{5}(3b)\int \frac{1}{x^2\sqrt[4]{a+bx^4}}\,dx \\
&= -\frac{3b}{5x\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{5x^5} - \frac{1}{5}\left(3b^2\right)\int \frac{x^2}{\left(a+bx^4\right)^{5/4}}\,dx \\
&= -\frac{3b}{5x\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{5x^5} - \frac{\left(3b\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{5/4}x^3}\,dx}{5\sqrt[4]{a+bx^4}} \\
&= -\frac{3b}{5x\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{5x^5} + \frac{\left(3b\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{5\sqrt[4]{a+bx^4}} \\
&= -\frac{3b}{5x\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{5x^5} + \frac{\left(3b\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx, x, \frac{1}{x^2}\right)}{10\sqrt[4]{a+bx^4}} \\
&= -\frac{3b}{5x\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{5x^5} + \frac{3b^{3/2}\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{a}\,\sqrt[4]{a+bx^4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.52

$$-\frac{\left(a+bx^4\right)^{3/4}\,{}_2F_1\left(-\frac{5}{4},-\frac{3}{4};-\frac{1}{4};-\frac{bx^4}{a}\right)}{5x^5\left(\frac{bx^4}{a}+1\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(3/4)/x^6,x]
```

```
[Out] -1/5*((a + b*x^4)^(3/4)*Hypergeometric2F1[-5/4, -3/4, -1/4, -((b*x^4)/a)])/
(x^5*(1 + (b*x^4)/a)^(3/4))
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{3}{4}}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^6,x, algorithm="fricas")
[Out] integral((b*x^4 + a)^(3/4)/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{4}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^6,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(3/4)/x^6, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{3}{4}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(3/4)/x^6,x)
[Out] int((b*x^4+a)^(3/4)/x^6,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{4}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^6,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(3/4)/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^4+a\right)^{3/4}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(3/4)/x^6,x)
[Out] int((a + b*x^4)^(3/4)/x^6, x)
```

sympy [C] time = 3.58, size = 31, normalized size = 0.31

$$-\frac{b^{\frac{3}{4}}\,{}_2F_1\left(\left.\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\right|\frac{ae^{i\pi}}{bx^4}\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(3/4)/x**6,x)
[Out] -b**(3/4)*hyper((-3/4, 1/2), (3/2,), a*exp_polar(I*pi)/(b*x**4))/(2*x**2)
```

3.1045 $\int \frac{\left(a+bx^4\right)^{3/4}}{x^{10}}\,dx$

Optimal. Leaf size=126

$$-\frac{2b^{5/2}x\sqrt[4]{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{15a^{3/2}\sqrt[4]{a+bx^4}}+\frac{2b^2}{15ax\sqrt[4]{a+bx^4}}-\frac{\left(a+bx^4\right)^{3/4}}{9x^9}-\frac{b\left(a+bx^4\right)^{3/4}}{15ax^5}$$

```
[Out] 2/15*b^2/a/x/(b*x^4+a)^(1/4)-1/9*(b*x^4+a)^(3/4)/x^9-1/15*b*(b*x^4+a)^(3/4)
/a/x^5-2/15*b^(5/2)*(1+a/b/x^4)^(1/4)*x*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)
))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arccot(x
^2*b^(1/2)/a^(1/2))),2^(1/2))/a^(3/2)/(b*x^4+a)^(1/4)
```

Rubi [A] time = 0.06, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {277, 325, 312, 281, 335, 275, 196}

$$-\frac{2b^{5/2}x\sqrt[4]{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{15a^{3/2}\sqrt[4]{a+bx^4}}+\frac{2b^2}{15ax\sqrt[4]{a+bx^4}}-\frac{b\left(a+bx^4\right)^{3/4}}{15ax^5}-\frac{\left(a+bx^4\right)^{3/4}}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(3/4)/x^10,x]
```

```
[Out] (2*b^2)/(15*a*x*(a + b*x^4)^(1/4)) - (a + b*x^4)^(3/4)/(9*x^9) - (b*(a + b*
x^4)^(3/4))/(15*a*x^5) - (2*b^(5/2)*(1 + a/(b*x^4))^(1/4)*x*EllipticE[ArcCo
t[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(15*a^(3/2)*(a + b*x^4)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 281

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Dist[(x*(1 + a/(b*x^4)
)^(1/4))/(b*(a + b*x^4)^(1/4)), Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x] /
; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 312

```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> -Simp[(x*(a + b*x^
4)^(1/4))^(-1), x] - Dist[b, Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a,
```

```
b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{\left(a+bx^4\right)^{3/4}}{x^{10}}\,dx = -\frac{\left(a+bx^4\right)^{3/4}}{9x^9} + \frac{1}{3}b\int \frac{1}{x^6\sqrt[4]{a+bx^4}}\,dx$$

$$= -\frac{\left(a+bx^4\right)^{3/4}}{9x^9} - \frac{b\left(a+bx^4\right)^{3/4}}{15ax^5} - \frac{\left(2b^2\right)\int \frac{1}{x^2\sqrt[4]{a+bx^4}}\,dx}{15a}$$

$$= \frac{2b^2}{15ax\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{9x^9} - \frac{b\left(a+bx^4\right)^{3/4}}{15ax^5} + \frac{\left(2b^3\right)\int \frac{x^2}{\left(a+bx^4\right)^{5/4}}\,dx}{15a}$$

$$= \frac{2b^2}{15ax\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{9x^9} - \frac{b\left(a+bx^4\right)^{3/4}}{15ax^5} + \frac{\left(2b^2\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{5/4}x^3}\,dx}{15a\sqrt[4]{a+bx^4}}$$

$$= \frac{2b^2}{15ax\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{9x^9} - \frac{b\left(a+bx^4\right)^{3/4}}{15ax^5} - \frac{\left(2b^2\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{15a\sqrt[4]{a+bx^4}}$$

$$= \frac{2b^2}{15ax\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{9x^9} - \frac{b\left(a+bx^4\right)^{3/4}}{15ax^5} - \frac{\left(b^2\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx, x, \frac{1}{x^2}\right)}{15a\sqrt[4]{a+bx^4}}$$

$$= \frac{2b^2}{15ax\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{9x^9} - \frac{b\left(a+bx^4\right)^{3/4}}{15ax^5} - \frac{2b^{5/2}\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{15a^{3/2}\sqrt[4]{a+bx^4}}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.40

$$-\frac{\left(a+bx^4\right)^{3/4}\,{}_2F_1\left(-\frac{9}{4},-\frac{3}{4};-\frac{5}{4};-\frac{bx^4}{a}\right)}{9x^9\left(\frac{bx^4}{a}+1\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(3/4)/x^10,x]
```

```
[Out] -1/9*((a + b*x^4)^(3/4)*Hypergeometric2F1[-9/4, -3/4, -5/4, -((b*x^4)/a)])/
(x^9*(1 + (b*x^4)/a)^(3/4))
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{3}{4}}}{x^{10}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^10,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(3/4)/x^10, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{4}}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^10,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/4)/x^10, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{3}{4}}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(3/4)/x^10,x)
```

```
[Out] int((b*x^4+a)^(3/4)/x^10,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{4}}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^10,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/4)/x^10, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^4+a\right)^{3/4}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(3/4)/x^10,x)
```

```
[Out] int((a + b*x^4)^(3/4)/x^10, x)
```

sympy [C] time = 4.52, size = 31, normalized size = 0.25

$$-\frac{b^{\frac{3}{4}}\,{}_2F_1\left(\left.\begin{matrix}-\frac{3}{4}, \frac{3}{2} \\ \frac{5}{2}\end{matrix}\right| \frac{ae^{i\pi}}{bx^4}\right)}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(3/4)/x**10,x)
```

```
[Out] -b**(3/4)*hyper((-3/4, 3/2), (5/2,), a*exp_polar(I*pi)/(b*x**4))/(6*x**6)
```

3.1046 $\int \frac{\left(a+bx^4\right)^{3/4}}{x^{14}}\,dx$

Optimal. Leaf size=150

$$\frac{4b^{7/2}x\sqrt[4]{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{65a^{5/2}\sqrt[4]{a+bx^4}}-\frac{4b^3}{65a^2x\sqrt[4]{a+bx^4}}+\frac{2b^2\left(a+bx^4\right)^{3/4}}{65a^2x^5}-\frac{\left(a+bx^4\right)^{3/4}}{13x^{13}}-\frac{b\left(a+bx^4\right)^{3/4}}{39ax^9}$$

```
[Out] -4/65*b^3/a^2/x/(b*x^4+a)^(1/4)-1/13*(b*x^4+a)^(3/4)/x^13-1/39*b*(b*x^4+a)^
(3/4)/a/x^9+2/65*b^2*(b*x^4+a)^(3/4)/a^2/x^5+4/65*b^(7/2)*(1+a/b/x^4)^(1/4)
*x*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2
)/a^(1/2)))*EllipticE(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))/a^(5/2)
/(b*x^4+a)^(1/4)
```

Rubi [A] time = 0.07, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {277, 325, 312, 281, 335, 275, 196}

$$-\frac{4b^3}{65a^2x\sqrt[4]{a+bx^4}}+\frac{2b^2\left(a+bx^4\right)^{3/4}}{65a^2x^5}+\frac{4b^{7/2}x\sqrt[4]{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{65a^{5/2}\sqrt[4]{a+bx^4}}-\frac{b\left(a+bx^4\right)^{3/4}}{39ax^9}-\frac{\left(a+bx^4\right)^{3/4}}{13x^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(3/4)/x^14,x]
```

```
[Out] (-4*b^3)/(65*a^2*x*(a + b*x^4)^(1/4)) - (a + b*x^4)^(3/4)/(13*x^13) - (b*(a
 + b*x^4)^(3/4))/(39*a*x^9) + (2*b^2*(a + b*x^4)^(3/4))/(65*a^2*x^5) + (4*b
^(7/2)*(1 + a/(b*x^4))^(1/4)*x*EllipticE[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2
])/(65*a^(5/2)*(a + b*x^4)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 281

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Dist[(x*(1 + a/(b*x^4)
)^(1/4))/(b*(a + b*x^4)^(1/4)), Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x] /
; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 312

```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> -Simp[(x*(a + b*x^
4)^(1/4))^(-1), x] - Dist[b, Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a,
 b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{\left(a+bx^4\right)^{3/4}}{x^{14}}\,dx = -\frac{\left(a+bx^4\right)^{3/4}}{13x^{13}} + \frac{1}{13}(3b)\int \frac{1}{x^{10}\sqrt[4]{a+bx^4}}\,dx$$

$$= -\frac{\left(a+bx^4\right)^{3/4}}{13x^{13}} - \frac{b\left(a+bx^4\right)^{3/4}}{39ax^9} - \frac{\left(2b^2\right)\int \frac{1}{x^6\sqrt[4]{a+bx^4}}\,dx}{13a}$$

$$= -\frac{\left(a+bx^4\right)^{3/4}}{13x^{13}} - \frac{b\left(a+bx^4\right)^{3/4}}{39ax^9} + \frac{2b^2\left(a+bx^4\right)^{3/4}}{65a^2x^5} + \frac{\left(4b^3\right)\int \frac{1}{x^2\sqrt[4]{a+bx^4}}\,dx}{65a^2}$$

$$= -\frac{4b^3}{65a^2x\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{13x^{13}} - \frac{b\left(a+bx^4\right)^{3/4}}{39ax^9} + \frac{2b^2\left(a+bx^4\right)^{3/4}}{65a^2x^5} - \frac{\left(4b^4\right)\int \frac{x^2}{\left(a+bx^4\right)^{5/4}}\,dx}{65a^2}$$

$$= -\frac{4b^3}{65a^2x\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{13x^{13}} - \frac{b\left(a+bx^4\right)^{3/4}}{39ax^9} + \frac{2b^2\left(a+bx^4\right)^{3/4}}{65a^2x^5} - \frac{\left(4b^3\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\int -}{65a^2\sqrt[4]{a+b}}$$

$$= -\frac{4b^3}{65a^2x\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{13x^{13}} - \frac{b\left(a+bx^4\right)^{3/4}}{39ax^9} + \frac{2b^2\left(a+bx^4\right)^{3/4}}{65a^2x^5} + \frac{\left(4b^3\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\text{Sub}}{65a^2}$$

$$= -\frac{4b^3}{65a^2x\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{13x^{13}} - \frac{b\left(a+bx^4\right)^{3/4}}{39ax^9} + \frac{2b^2\left(a+bx^4\right)^{3/4}}{65a^2x^5} + \frac{\left(2b^3\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\text{Sub}}{65a^2}$$

$$= -\frac{4b^3}{65a^2x\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{13x^{13}} - \frac{b\left(a+bx^4\right)^{3/4}}{39ax^9} + \frac{2b^2\left(a+bx^4\right)^{3/4}}{65a^2x^5} + \frac{4b^{7/2}\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\right.}{65a^{5/2}\sqrt[4]{a}}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.34

$$-\frac{\left(a+bx^4\right)^{3/4}\,{}_2F_1\left(-\frac{13}{4},-\frac{3}{4};-\frac{9}{4};-\frac{bx^4}{a}\right)}{13x^{13}\left(\frac{bx^4}{a}+1\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(3/4)/x^14,x]
```

```
[Out] -1/13*((a + b*x^4)^(3/4)*Hypergeometric2F1[-13/4, -3/4, -9/4, -((b*x^4)/a)]
)/(x^13*(1 + (b*x^4)/a)^(3/4))
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{3}{4}}}{x^{14}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^14,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(3/4)/x^14, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{4}}}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^14,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/4)/x^14, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{3}{4}}}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(3/4)/x^14,x)
```

```
[Out] int((b*x^4+a)^(3/4)/x^14,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{4}}}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/x^14,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/4)/x^14, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^4+a\right)^{3/4}}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(3/4)/x^14,x)
```

```
[Out] int((a + b*x^4)^(3/4)/x^14, x)
```

sympy [C] time = 4.45, size = 46, normalized size = 0.31

$$\frac{a^{\frac{3}{4}}\Gamma\left(-\frac{13}{4}\right){}_2F_1\left(\begin{matrix}-\frac{13}{4}, -\frac{3}{4} \\ -\frac{9}{4}\end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4x^{13}\Gamma\left(-\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(3/4)/x**14,x)
```

```
[Out] a**(3/4)*gamma(-13/4)*hyper((-13/4, -3/4), (-9/4,), b*x**4*exp_polar(I*pi)/
a)/(4*x**13*gamma(-9/4))
```

3.1047 $\int x^{19}\left(a+bx^4\right)^{5/4}\,dx$

Optimal. Leaf size=101

$$\frac{a^4\left(a+bx^4\right)^{9/4}}{9b^5}-\frac{4a^3\left(a+bx^4\right)^{13/4}}{13b^5}+\frac{6a^2\left(a+bx^4\right)^{17/4}}{17b^5}+\frac{\left(a+bx^4\right)^{25/4}}{25b^5}-\frac{4a\left(a+bx^4\right)^{21/4}}{21b^5}$$

```
[Out] 1/9*a^4*(b*x^4+a)^(9/4)/b^5-4/13*a^3*(b*x^4+a)^(13/4)/b^5+6/17*a^2*(b*x^4+a
)^(17/4)/b^5-4/21*a*(b*x^4+a)^(21/4)/b^5+1/25*(b*x^4+a)^(25/4)/b^5
```

Rubi [A] time = 0.05, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{6a^2\left(a+bx^4\right)^{17/4}}{17b^5}-\frac{4a^3\left(a+bx^4\right)^{13/4}}{13b^5}+\frac{a^4\left(a+bx^4\right)^{9/4}}{9b^5}+\frac{\left(a+bx^4\right)^{25/4}}{25b^5}-\frac{4a\left(a+bx^4\right)^{21/4}}{21b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^19*(a + b*x^4)^(5/4),x]
```

```
[Out] (a^4*(a + b*x^4)^(9/4))/(9*b^5) - (4*a^3*(a + b*x^4)^(13/4))/(13*b^5) + (6*
a^2*(a + b*x^4)^(17/4))/(17*b^5) - (4*a*(a + b*x^4)^(21/4))/(21*b^5) + (a +
 b*x^4)^(25/4)/(25*b^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{19}\left(a+bx^4\right)^{5/4}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int x^4(a+bx)^{5/4}\,dx,x,x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int\left(\frac{a^4(a+bx)^{5/4}}{b^4}-\frac{4a^3(a+bx)^{9/4}}{b^4}+\frac{6a^2(a+bx)^{13/4}}{b^4}-\frac{4a(a+bx)^{17/4}}{b^4}+\frac{(a+\right.\right.\\ &= \frac{a^4\left(a+bx^4\right)^{9/4}}{9b^5}-\frac{4a^3\left(a+bx^4\right)^{13/4}}{13b^5}+\frac{6a^2\left(a+bx^4\right)^{17/4}}{17b^5}-\frac{4a\left(a+bx^4\right)^{21/4}}{21b^5}+\frac{\left(a+bx^4\right.}{25b^5}\end{aligned}$$

Mathematica [A] time = 0.04, size = 61, normalized size = 0.60

$$\frac{\left(a+bx^4\right)^{9/4}\left(2048a^4-4608a^3bx^4+7488a^2b^2x^8-10608ab^3x^{12}+13923b^4x^{16}\right)}{348075b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^19*(a + b*x^4)^(5/4),x]
```

```
[Out] ((a + b*x^4)^(9/4)*(2048*a^4 - 4608*a^3*b*x^4 + 7488*a^2*b^2*x^8 - 10608*a*
b^3*x^12 + 13923*b^4*x^16))/(348075*b^5)
```

fricas [A] time = 0.87, size = 79, normalized size = 0.78

$$\frac{\left(13923\,b^6x^{24}+17238\,ab^5x^{20}+195\,a^2b^4x^{16}-240\,a^3b^3x^{12}+320\,a^4b^2x^8-512\,a^5bx^4+2048\,a^6\right)\left(bx^4+a\right)^{\frac{1}{4}}}{348075\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19*(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] 1/348075*(13923*b^6*x^24 + 17238*a*b^5*x^20 + 195*a^2*b^4*x^16 - 240*a^3*b^
3*x^12 + 320*a^4*b^2*x^8 - 512*a^5*b*x^4 + 2048*a^6)*(b*x^4 + a)^(1/4)/b^5
```

giac [A] time = 0.20, size = 71, normalized size = 0.70

$$\frac{13923\left(bx^4+a\right)^{\frac{25}{4}}-66300\left(bx^4+a\right)^{\frac{21}{4}}a+122850\left(bx^4+a\right)^{\frac{17}{4}}a^2-107100\left(bx^4+a\right)^{\frac{13}{4}}a^3+38675\left(bx^4+a\right)^{\frac{9}{4}}a^4}{348075\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19*(b*x^4+a)^(5/4),x, algorithm="giac")
```

```
[Out] 1/348075*(13923*(b*x^4 + a)^(25/4) - 66300*(b*x^4 + a)^(21/4)*a + 122850*(b
*x^4 + a)^(17/4)*a^2 - 107100*(b*x^4 + a)^(13/4)*a^3 + 38675*(b*x^4 + a)^(9
/4)*a^4)/b^5
```

maple [A] time = 0.01, size = 58, normalized size = 0.57

$$\frac{\left(b\,x^4+a\right)^{\frac{9}{4}}\left(13923x^{16}b^4-10608a\,x^{12}b^3+7488a^2x^8b^2-4608a^3x^4b+2048a^4\right)}{348075b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19*(b*x^4+a)^(5/4),x)
```

```
[Out] 1/348075*(b*x^4+a)^(9/4)*(13923*b^4*x^16-10608*a*b^3*x^12+7488*a^2*b^2*x^8-
4608*a^3*b*x^4+2048*a^4)/b^5
```

maxima [A] time = 1.35, size = 81, normalized size = 0.80

$$\frac{\left(bx^4+a\right)^{\frac{25}{4}}}{25\,b^5}-\frac{4\left(bx^4+a\right)^{\frac{21}{4}}a}{21\,b^5}+\frac{6\left(bx^4+a\right)^{\frac{17}{4}}a^2}{17\,b^5}-\frac{4\left(bx^4+a\right)^{\frac{13}{4}}a^3}{13\,b^5}+\frac{\left(bx^4+a\right)^{\frac{9}{4}}a^4}{9\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19*(b*x^4+a)^(5/4),x, algorithm="maxima")
```

```
[Out] 1/25*(b*x^4 + a)^(25/4)/b^5 - 4/21*(b*x^4 + a)^(21/4)*a/b^5 + 6/17*(b*x^4 +
 a)^(17/4)*a^2/b^5 - 4/13*(b*x^4 + a)^(13/4)*a^3/b^5 + 1/9*(b*x^4 + a)^(9/4
)*a^4/b^5
```

mupad [B] time = 1.18, size = 75, normalized size = 0.74

$$\left(b\,x^4+a\right)^{1/4}\left(\frac{26\,a\,x^{20}}{525}+\frac{b\,x^{24}}{25}+\frac{2048\,a^6}{348075\,b^5}-\frac{512\,a^5\,x^4}{348075\,b^4}+\frac{64\,a^4\,x^8}{69615\,b^3}-\frac{16\,a^3\,x^{12}}{23205\,b^2}+\frac{a^2\,x^{16}}{1785\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19*(a + b*x^4)^(5/4),x)
```

```
[Out] (a + b*x^4)^(1/4)*((26*a*x^20)/525 + (b*x^24)/25 + (2048*a^6)/(348075*b^5)
- (512*a^5*x^4)/(348075*b^4) + (64*a^4*x^8)/(69615*b^3) - (16*a^3*x^12)/(23
205*b^2) + (a^2*x^16)/(1785*b))
```

sympy [A] time = 87.31, size = 156, normalized size = 1.54

$$\begin{cases} \frac{2048a^6\sqrt[4]{a+bx^4}}{348075b^5} - \frac{512a^5x^4\sqrt[4]{a+bx^4}}{348075b^4} + \frac{64a^4x^8\sqrt[4]{a+bx^4}}{69615b^3} - \frac{16a^3x^{12}\sqrt[4]{a+bx^4}}{23205b^2} + \frac{a^2x^{16}\sqrt[4]{a+bx^4}}{1785b} + \frac{26ax^{20}\sqrt[4]{a+bx^4}}{525} + \frac{bx^{24}\sqrt[4]{a+bx^4}}{25} & \text{for } b \\ \frac{a^{\frac{5}{4}}x^{20}}{20} & \text{othe} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**19*(b*x**4+a)**(5/4),x)
```

```
[Out] Piecewise((2048*a**6*(a + b*x**4)**(1/4)/(348075*b**5) - 512*a**5*x**4*(a +
 b*x**4)**(1/4)/(348075*b**4) + 64*a**4*x**8*(a + b*x**4)**(1/4)/(69615*b**
3) - 16*a**3*x**12*(a + b*x**4)**(1/4)/(23205*b**2) + a**2*x**16*(a + b*x**
4)**(1/4)/(1785*b) + 26*a*x**20*(a + b*x**4)**(1/4)/525 + b*x**24*(a + b*x*
*4)**(1/4)/25, Ne(b, 0)), (a**(5/4)*x**20/20, True))
```

3.1048 $\int x^{15}\left(a+bx^4\right)^{5/4}\,dx$

Optimal. Leaf size=80

$$-\frac{a^3\left(a+bx^4\right)^{9/4}}{9b^4}+\frac{3a^2\left(a+bx^4\right)^{13/4}}{13b^4}+\frac{\left(a+bx^4\right)^{21/4}}{21b^4}-\frac{3a\left(a+bx^4\right)^{17/4}}{17b^4}$$

```
[Out] -1/9*a^3*(b*x^4+a)^(9/4)/b^4+3/13*a^2*(b*x^4+a)^(13/4)/b^4-3/17*a*(b*x^4+a)
^(17/4)/b^4+1/21*(b*x^4+a)^(21/4)/b^4
```

Rubi [A] time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{3a^2\left(a+bx^4\right)^{13/4}}{13b^4}-\frac{a^3\left(a+bx^4\right)^{9/4}}{9b^4}+\frac{\left(a+bx^4\right)^{21/4}}{21b^4}-\frac{3a\left(a+bx^4\right)^{17/4}}{17b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^15*(a + b*x^4)^(5/4),x]
```

```
[Out] -(a^3*(a + b*x^4)^(9/4))/(9*b^4) + (3*a^2*(a + b*x^4)^(13/4))/(13*b^4) - (3
*a*(a + b*x^4)^(17/4))/(17*b^4) + (a + b*x^4)^(21/4)/(21*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{15}\left(a+bx^4\right)^{5/4}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int x^3(a+bx)^{5/4}\,dx,x,x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int\left(-\frac{a^3(a+bx)^{5/4}}{b^3}+\frac{3a^2(a+bx)^{9/4}}{b^3}-\frac{3a(a+bx)^{13/4}}{b^3}+\frac{(a+bx)^{17/4}}{b^3}\right)dx,x,x^4\right)\\ &= -\frac{a^3\left(a+bx^4\right)^{9/4}}{9b^4}+\frac{3a^2\left(a+bx^4\right)^{13/4}}{13b^4}-\frac{3a\left(a+bx^4\right)^{17/4}}{17b^4}+\frac{\left(a+bx^4\right)^{21/4}}{21b^4}\end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.62

$$\frac{\left(a+bx^4\right)^{9/4}\left(-128a^3+288a^2bx^4-468ab^2x^8+663b^3x^{12}\right)}{13923b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15*(a + b*x^4)^(5/4),x]
```

```
[Out] ((a + b*x^4)^(9/4)*(-128*a^3 + 288*a^2*b*x^4 - 468*a*b^2*x^8 + 663*b^3*x^12
))/(13923*b^4)
```

fricas [A] time = 0.71, size = 68, normalized size = 0.85

$$\frac{\left(663\,b^5x^{20}+858\,ab^4x^{16}+15\,a^2b^3x^{12}-20\,a^3b^2x^8+32\,a^4bx^4-128\,a^5\right)\left(bx^4+a\right)^{\frac{1}{4}}}{13923\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] 1/13923*(663*b^5*x^20 + 858*a*b^4*x^16 + 15*a^2*b^3*x^12 - 20*a^3*b^2*x^8 +
 32*a^4*b*x^4 - 128*a^5)*(b*x^4 + a)^(1/4)/b^4
```

giac [A] time = 0.18, size = 57, normalized size = 0.71

$$\frac{663\left(bx^4+a\right)^{\frac{21}{4}}-2457\left(bx^4+a\right)^{\frac{17}{4}}a+3213\left(bx^4+a\right)^{\frac{13}{4}}a^2-1547\left(bx^4+a\right)^{\frac{9}{4}}a^3}{13923\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(b*x^4+a)^(5/4),x, algorithm="giac")
```

```
[Out] 1/13923*(663*(b*x^4 + a)^(21/4) - 2457*(b*x^4 + a)^(17/4)*a + 3213*(b*x^4 +
 a)^(13/4)*a^2 - 1547*(b*x^4 + a)^(9/4)*a^3)/b^4
```

maple [A] time = 0.01, size = 47, normalized size = 0.59

$$-\frac{\left(b\,x^4+a\right)^{\frac{9}{4}}\left(-663b^3x^{12}+468a\,b^2x^8-288a^2b\,x^4+128a^3\right)}{13923b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15*(b*x^4+a)^(5/4),x)
```

```
[Out] -1/13923*(b*x^4+a)^(9/4)*(-663*b^3*x^12+468*a*b^2*x^8-288*a^2*b*x^4+128*a^3
)/b^4
```

maxima [A] time = 1.31, size = 64, normalized size = 0.80

$$\frac{\left(bx^4+a\right)^{\frac{21}{4}}}{21\,b^4}-\frac{3\left(bx^4+a\right)^{\frac{17}{4}}a}{17\,b^4}+\frac{3\left(bx^4+a\right)^{\frac{13}{4}}a^2}{13\,b^4}-\frac{\left(bx^4+a\right)^{\frac{9}{4}}a^3}{9\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(b*x^4+a)^(5/4),x, algorithm="maxima")
```

```
[Out] 1/21*(b*x^4 + a)^(21/4)/b^4 - 3/17*(b*x^4 + a)^(17/4)*a/b^4 + 3/13*(b*x^4 +
 a)^(13/4)*a^2/b^4 - 1/9*(b*x^4 + a)^(9/4)*a^3/b^4
```

mupad [B] time = 1.14, size = 64, normalized size = 0.80

$$\left(b\,x^4+a\right)^{1/4}\left(\frac{22\,a\,x^{16}}{357}+\frac{b\,x^{20}}{21}-\frac{128\,a^5}{13923\,b^4}+\frac{32\,a^4\,x^4}{13923\,b^3}-\frac{20\,a^3\,x^8}{13923\,b^2}+\frac{5\,a^2\,x^{12}}{4641\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15*(a + b*x^4)^(5/4),x)
```

```
[Out] (a + b*x^4)^(1/4)*((22*a*x^16)/357 + (b*x^20)/21 - (128*a^5)/(13923*b^4) +
(32*a^4*x^4)/(13923*b^3) - (20*a^3*x^8)/(13923*b^2) + (5*a^2*x^12)/(4641*b)
)
```

sympy [**A**] time = 38.26, size = 134, normalized size = 1.68

$$\begin{cases} -\frac{128a^5\sqrt[4]{a+bx^4}}{13923b^4} + \frac{32a^4x^4\sqrt[4]{a+bx^4}}{13923b^3} - \frac{20a^3x^8\sqrt[4]{a+bx^4}}{13923b^2} + \frac{5a^2x^{12}\sqrt[4]{a+bx^4}}{4641b} + \frac{22ax^{16}\sqrt[4]{a+bx^4}}{357} + \frac{bx^{20}\sqrt[4]{a+bx^4}}{21} & \text{for } b \neq 0 \\ \frac{a^{\frac{5}{4}}x^{16}}{16} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15*(b*x**4+a)**(5/4),x)
```

```
[Out] Piecewise((-128*a**5*(a + b*x**4)**(1/4)/(13923*b**4) + 32*a**4*x**4*(a + b
*x**4)**(1/4)/(13923*b**3) - 20*a**3*x**8*(a + b*x**4)**(1/4)/(13923*b**2)
+ 5*a**2*x**12*(a + b*x**4)**(1/4)/(4641*b) + 22*a*x**16*(a + b*x**4)**(1/4
)/357 + b*x**20*(a + b*x**4)**(1/4)/21, Ne(b, 0)), (a**(5/4)*x**16/16, True
))
```

3.1049 $\int x^{11}\left(a+bx^4\right)^{5/4}\,dx$

Optimal. Leaf size=59

$$\frac{a^2\left(a+bx^4\right)^{9/4}}{9b^3}+\frac{\left(a+bx^4\right)^{17/4}}{17b^3}-\frac{2a\left(a+bx^4\right)^{13/4}}{13b^3}$$

```
[Out] 1/9*a^2*(b*x^4+a)^(9/4)/b^3-2/13*a*(b*x^4+a)^(13/4)/b^3+1/17*(b*x^4+a)^(17/
4)/b^3
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{a^2\left(a+bx^4\right)^{9/4}}{9b^3}+\frac{\left(a+bx^4\right)^{17/4}}{17b^3}-\frac{2a\left(a+bx^4\right)^{13/4}}{13b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a + b*x^4)^(5/4),x]
```

```
[Out] (a^2*(a + b*x^4)^(9/4))/(9*b^3) - (2*a*(a + b*x^4)^(13/4))/(13*b^3) + (a +
b*x^4)^(17/4)/(17*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{11}\left(a+bx^4\right)^{5/4}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int x^2(a+bx)^{5/4}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int\left(\frac{a^2(a+bx)^{5/4}}{b^2}-\frac{2a(a+bx)^{9/4}}{b^2}+\frac{(a+bx)^{13/4}}{b^2}\right)dx, x, x^4\right)\\ &= \frac{a^2\left(a+bx^4\right)^{9/4}}{9b^3}-\frac{2a\left(a+bx^4\right)^{13/4}}{13b^3}+\frac{\left(a+bx^4\right)^{17/4}}{17b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.66

$$\frac{\left(a+bx^4\right)^{9/4}\left(32a^2-72abx^4+117b^2x^8\right)}{1989b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*x^4)^(5/4),x]
```

```
[Out] ((a + b*x^4)^(9/4)*(32*a^2 - 72*a*b*x^4 + 117*b^2*x^8))/(1989*b^3)
```

fricas [A] time = 0.78, size = 57, normalized size = 0.97

$$\frac{\left(117\,b^4x^{16}+162\,ab^3x^{12}+5\,a^2b^2x^8-8\,a^3bx^4+32\,a^4\right)\left(bx^4+a\right)^{\frac{1}{4}}}{1989\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] 1/1989*(117*b^4*x^16 + 162*a*b^3*x^12 + 5*a^2*b^2*x^8 - 8*a^3*b*x^4 + 32*a^
4)*(b*x^4 + a)^(1/4)/b^3
```

giac [A] time = 0.16, size = 43, normalized size = 0.73

$$\frac{117\left(bx^4+a\right)^{\frac{17}{4}}-306\left(bx^4+a\right)^{\frac{13}{4}}a+221\left(bx^4+a\right)^{\frac{9}{4}}a^2}{1989\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^4+a)^(5/4),x, algorithm="giac")
```

```
[Out] 1/1989*(117*(b*x^4 + a)^(17/4) - 306*(b*x^4 + a)^(13/4)*a + 221*(b*x^4 + a)
^(9/4)*a^2)/b^3
```

maple [A] time = 0.00, size = 36, normalized size = 0.61

$$\frac{\left(b\,x^4+a\right)^{\frac{9}{4}}\left(117b^2x^8-72ab\,x^4+32a^2\right)}{1989b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x^4+a)^(5/4),x)
```

```
[Out] 1/1989*(b*x^4+a)^(9/4)*(117*b^2*x^8-72*a*b*x^4+32*a^2)/b^3
```

maxima [A] time = 1.29, size = 47, normalized size = 0.80

$$\frac{\left(bx^4+a\right)^{\frac{17}{4}}}{17\,b^3}-\frac{2\left(bx^4+a\right)^{\frac{13}{4}}a}{13\,b^3}+\frac{\left(bx^4+a\right)^{\frac{9}{4}}a^2}{9\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^4+a)^(5/4),x, algorithm="maxima")
```

```
[Out] 1/17*(b*x^4 + a)^(17/4)/b^3 - 2/13*(b*x^4 + a)^(13/4)*a/b^3 + 1/9*(b*x^4 +
a)^(9/4)*a^2/b^3
```

mupad [B] time = 1.14, size = 53, normalized size = 0.90

$$\left(b\,x^4+a\right)^{1/4}\left(\frac{18\,a\,x^{12}}{221}+\frac{b\,x^{16}}{17}+\frac{32\,a^4}{1989\,b^3}-\frac{8\,a^3\,x^4}{1989\,b^2}+\frac{5\,a^2\,x^8}{1989\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a + b*x^4)^(5/4),x)
```

```
[Out] (a + b*x^4)^(1/4)*((18*a*x^12)/221 + (b*x^16)/17 + (32*a^4)/(1989*b^3) - (8
*a^3*x^4)/(1989*b^2) + (5*a^2*x^8)/(1989*b))
```

sympy [**A**] time = 25.29, size = 110, normalized size = 1.86

$$\begin{cases} \frac{32a^4\sqrt[4]{a+bx^4}}{1989b^3} - \frac{8a^3x^4\sqrt[4]{a+bx^4}}{1989b^2} + \frac{5a^2x^8\sqrt[4]{a+bx^4}}{1989b} + \frac{18ax^{12}\sqrt[4]{a+bx^4}}{221} + \frac{bx^{16}\sqrt[4]{a+bx^4}}{17} & \text{for } b \neq 0 \\ \frac{a^{\frac{5}{4}}x^{12}}{12} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x**4+a)**(5/4),x)
```

```
[Out] Piecewise((32*a**4*(a + b*x**4)**(1/4)/(1989*b**3) - 8*a**3*x**4*(a + b*x**
4)**(1/4)/(1989*b**2) + 5*a**2*x**8*(a + b*x**4)**(1/4)/(1989*b) + 18*a*x**
12*(a + b*x**4)**(1/4)/221 + b*x**16*(a + b*x**4)**(1/4)/17, Ne(b, 0)), (a*
*(5/4)*x**12/12, True))
```

3.1050 $\int x^7 \left(a+bx^4\right)^{5/4}\,dx$

Optimal. Leaf size=38

$$\frac{\left(a+bx^4\right)^{13/4}}{13b^2}-\frac{a\left(a+bx^4\right)^{9/4}}{9b^2}$$

```
[Out] -1/9*a*(b*x^4+a)^(9/4)/b^2+1/13*(b*x^4+a)^(13/4)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{\left(a+bx^4\right)^{13/4}}{13b^2}-\frac{a\left(a+bx^4\right)^{9/4}}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^4)^(5/4),x]
```

```
[Out] -(a*(a + b*x^4)^(9/4))/(9*b^2) + (a + b*x^4)^(13/4)/(13*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^7\left(a+bx^4\right)^{5/4}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int x(a+bx)^{5/4}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int\left(-\frac{a(a+bx)^{5/4}}{b}+\frac{(a+bx)^{9/4}}{b}\right)dx, x, x^4\right)\\ &= -\frac{a\left(a+bx^4\right)^{9/4}}{9b^2}+\frac{\left(a+bx^4\right)^{13/4}}{13b^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 0.74

$$\frac{\left(a+bx^4\right)^{9/4}\left(9bx^4-4a\right)}{117b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^4)^(5/4),x]
```

```
[Out] ((a + b*x^4)^(9/4)*(-4*a + 9*b*x^4))/(117*b^2)
```

fricas [A] time = 0.88, size = 45, normalized size = 1.18

$$\frac{\left(9\,b^3x^{12}+14\,ab^2x^8+a^2bx^4-4\,a^3\right)\left(bx^4+a\right)^{\frac{1}{4}}}{117\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] 1/117*(9*b^3*x^12 + 14*a*b^2*x^8 + a^2*b*x^4 - 4*a^3)*(b*x^4 + a)^(1/4)/b^2
```

giac [A] time = 0.16, size = 29, normalized size = 0.76

$$\frac{9\left(bx^4+a\right)^{\frac{13}{4}}-13\left(bx^4+a\right)^{\frac{9}{4}}a}{117\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^4+a)^(5/4),x, algorithm="giac")
```

```
[Out] 1/117*(9*(b*x^4 + a)^(13/4) - 13*(b*x^4 + a)^(9/4)*a)/b^2
```

maple [A] time = 0.00, size = 25, normalized size = 0.66

$$-\frac{\left(b\,x^4+a\right)^{\frac{9}{4}}\left(-9b\,x^4+4a\right)}{117b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^4+a)^(5/4),x)
```

```
[Out] -1/117*(b*x^4+a)^(9/4)*(-9*b*x^4+4*a)/b^2
```

maxima [A] time = 1.41, size = 30, normalized size = 0.79

$$\frac{\left(bx^4+a\right)^{\frac{13}{4}}}{13\,b^2}-\frac{\left(bx^4+a\right)^{\frac{9}{4}}a}{9\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^4+a)^(5/4),x, algorithm="maxima")
```

```
[Out] 1/13*(b*x^4 + a)^(13/4)/b^2 - 1/9*(b*x^4 + a)^(9/4)*a/b^2
```

mupad [B] time = 1.11, size = 42, normalized size = 1.11

$$\left(b\,x^4+a\right)^{1/4}\left(\frac{14\,a\,x^8}{117}+\frac{b\,x^{12}}{13}-\frac{4\,a^3}{117\,b^2}+\frac{a^2\,x^4}{117\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^4)^(5/4),x)
```

```
[Out] (a + b*x^4)^(1/4)*((14*a*x^8)/117 + (b*x^12)/13 - (4*a^3)/(117*b^2) + (a^2*
x^4)/(117*b))
```

sympy [A] time = 18.34, size = 85, normalized size = 2.24

$$\begin{cases} -\frac{4a^3\sqrt[4]{a+bx^4}}{117b^2}+\frac{a^2x^4\sqrt[4]{a+bx^4}}{117b}+\frac{14ax^8\sqrt[4]{a+bx^4}}{117}+\frac{bx^{12}\sqrt[4]{a+bx^4}}{13} & \text{for } b\neq 0 \\ \frac{a^{\frac{5}{4}}x^8}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**4+a)**(5/4),x)
```

```
[Out] Piecewise((-4*a**3*(a + b*x**4)**(1/4)/(117*b**2) + a**2*x**4*(a + b*x**4)*
*(1/4)/(117*b) + 14*a*x**8*(a + b*x**4)**(1/4)/117 + b*x**12*(a + b*x**4)**
(1/4)/13, Ne(b, 0)), (a**(5/4)*x**8/8, True))
```

3.1051 $\int x^3 \left(a + bx^4\right)^{5/4} \, dx$

Optimal. Leaf size=18

$$\frac{\left(a + bx^4\right)^{9/4}}{9b}$$

```
[Out] 1/9*(b*x^4+a)^(9/4)/b
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$\frac{\left(a + bx^4\right)^{9/4}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^4)^(5/4),x]
```

```
[Out] (a + b*x^4)^(9/4)/(9*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^3 \left(a + bx^4\right)^{5/4} \, dx = \frac{\left(a + bx^4\right)^{9/4}}{9b}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{\left(a + bx^4\right)^{9/4}}{9b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^4)^(5/4),x]
```

```
[Out] (a + b*x^4)^(9/4)/(9*b)
```

fricas [B] time = 0.77, size = 32, normalized size = 1.78

$$\frac{\left(b^2x^8 + 2\,abx^4 + a^2\right)\left(bx^4 + a\right)^{\frac{1}{4}}}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] 1/9*(b^2*x^8 + 2*a*b*x^4 + a^2)*(b*x^4 + a)^(1/4)/b
```

giac [A] time = 0.17, size = 14, normalized size = 0.78

$$\frac{\left(bx^4 + a\right)^{\frac{9}{4}}}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^4+a)^(5/4),x, algorithm="giac")
[Out] 1/9*(b*x^4 + a)^(9/4)/b
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$\frac{\left(b\,x^4+a\right)^{\frac{9}{4}}}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^4+a)^(5/4),x)
[Out] 1/9*(b*x^4+a)^(9/4)/b
```

maxima [A] time = 1.62, size = 14, normalized size = 0.78

$$\frac{\left(bx^4+a\right)^{\frac{9}{4}}}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^4+a)^(5/4),x, algorithm="maxima")
[Out] 1/9*(b*x^4 + a)^(9/4)/b
```

mupad [B] time = 1.11, size = 14, normalized size = 0.78

$$\frac{\left(b\,x^4+a\right)^{9/4}}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^4)^(5/4),x)
[Out] (a + b*x^4)^(9/4)/(9*b)
```

sympy [A] time = 11.15, size = 61, normalized size = 3.39

$$\begin{cases} \frac{a^2\sqrt[4]{a+bx^4}}{9b} + \frac{2ax^4\sqrt[4]{a+bx^4}}{9} + \frac{bx^8\sqrt[4]{a+bx^4}}{9} & \text{for } b \neq 0 \\ \frac{a^{\frac{5}{4}}x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**4+a)**(5/4),x)
[Out] Piecewise((a**2*(a + b*x**4)**(1/4)/(9*b) + 2*a*x**4*(a + b*x**4)**(1/4)/9
+ b*x**8*(a + b*x**4)**(1/4)/9, Ne(b, 0)), (a**(5/4)*x**4/4, True))
```

3.1052 $\int \frac{\left(a+bx^4\right)^{5/4}}{x}\,dx$

Optimal. Leaf size=83

$$-\frac{1}{2}a^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)-\frac{1}{2}a^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)+a\sqrt[4]{a+bx^4}+\frac{1}{5}\left(a+bx^4\right)^{5/4}$$

```
[Out] a*(b*x^4+a)^(1/4)+1/5*(b*x^4+a)^(5/4)-1/2*a^(5/4)*arctan((b*x^4+a)^(1/4)/a^
(1/4))-1/2*a^(5/4)*arctanh((b*x^4+a)^(1/4)/a^(1/4))
```

Rubi [A] time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {266, 50, 63, 212, 206, 203}

$$-\frac{1}{2}a^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)-\frac{1}{2}a^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)+a\sqrt[4]{a+bx^4}+\frac{1}{5}\left(a+bx^4\right)^{5/4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(5/4)/x,x]
```

```
[Out] a*(a + b*x^4)^(1/4) + (a + b*x^4)^(5/4)/5 - (a^(5/4)*ArcTan[(a + b*x^4)^(1/
4)/a^(1/4)])/2 - (a^(5/4)*ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)])/2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^4\right)^{5/4}}{x}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{(a+bx)^{5/4}}{x}\,dx, x, x^4\right) \\
&= \frac{1}{5}\left(a+bx^4\right)^{5/4} + \frac{1}{4}a\,\text{Subst}\left(\int \frac{\sqrt[4]{a+bx}}{x}\,dx, x, x^4\right) \\
&= a\sqrt[4]{a+bx^4} + \frac{1}{5}\left(a+bx^4\right)^{5/4} + \frac{1}{4}a^2\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/4}}\,dx, x, x^4\right) \\
&= a\sqrt[4]{a+bx^4} + \frac{1}{5}\left(a+bx^4\right)^{5/4} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^4}{b}}\,dx, x, \sqrt[4]{a+bx^4}\right)}{b} \\
&= a\sqrt[4]{a+bx^4} + \frac{1}{5}\left(a+bx^4\right)^{5/4} - \frac{1}{2}a^{3/2}\,\text{Subst}\left(\int \frac{1}{\sqrt{a}-x^2}\,dx, x, \sqrt[4]{a+bx^4}\right) - \frac{1}{2}a^{3/2}\,\text{Subst}\left(\int \right. \\
&= a\sqrt[4]{a+bx^4} + \frac{1}{5}\left(a+bx^4\right)^{5/4} - \frac{1}{2}a^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right) - \frac{1}{2}a^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 77, normalized size = 0.93

$$\frac{1}{10}\left(-5a^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right) - 5a^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right) + 2\sqrt[4]{a+bx^4}\left(6a+bx^4\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(5/4)/x,x]
```

```
[Out] (2*(a + b*x^4)^(1/4)*(6*a + b*x^4) - 5*a^(5/4)*ArcTan[(a + b*x^4)^(1/4)/a^(
1/4)] - 5*a^(5/4)*ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)])/10
```

fricas [B] time = 0.55, size = 130, normalized size = 1.57

$$\frac{1}{5}\left(bx^4+6\,a\right)\left(bx^4+a\right)^{\frac{1}{4}} + \left(a^5\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(a^5\right)^{\frac{3}{4}}\left(bx^4+a\right)^{\frac{1}{4}}a - \left(a^5\right)^{\frac{3}{4}}\sqrt{\sqrt{bx^4+a}\,a^2+\sqrt{a^5}}}{a^5}\right) - \frac{1}{4}\left(a^5\right)^{\frac{1}{4}}\log\left(\left(bx^4+a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x,x, algorithm="fricas")
```

```
[Out] 1/5*(b*x^4 + 6*a)*(b*x^4 + a)^(1/4) + (a^5)^(1/4)*arctan(-((a^5)^(3/4)*(b*x
^4 + a)^(1/4)*a - (a^5)^(3/4)*sqrt(sqrt(b*x^4 + a)*a^2 + sqrt(a^5)))/a^5) -
 1/4*(a^5)^(1/4)*log((b*x^4 + a)^(1/4)*a + (a^5)^(1/4)) + 1/4*(a^5)^(1/4)*l
og((b*x^4 + a)^(1/4)*a - (a^5)^(1/4))
```

giac [B] time = 0.18, size = 200, normalized size = 2.41

$$-\frac{1}{4}\sqrt{2}\,(-a)^{\frac{1}{4}}\,a\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}+2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)-\frac{1}{4}\sqrt{2}\,(-a)^{\frac{1}{4}}\,a\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}-2\left(bx^4+a\right)^{\frac{1}{4}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(-a)^(1/4)*a*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x^4
 + a)^(1/4))/(-a)^(1/4)) - 1/4*sqrt(2)*(-a)^(1/4)*a*arctan(-1/2*sqrt(2)*(sq
rt(2)*(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4)) - 1/8*sqrt(2)*(-a)^(1/4
)*a*log(sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt(-a))
 + 1/8*sqrt(2)*(-a)^(1/4)*a*log(-sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt
(b*x^4 + a) + sqrt(-a)) + 1/5*(b*x^4 + a)^(5/4) + (b*x^4 + a)^(1/4)*a
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{5}{4}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(5/4)/x,x)
```

```
[Out] int((b*x^4+a)^(5/4)/x,x)
```

maxima [A] time = 2.95, size = 79, normalized size = 0.95

$$-\frac{1}{2}\,a^{\frac{5}{4}}\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)+\frac{1}{4}\,a^{\frac{5}{4}}\log\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{\left(bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)+\frac{1}{5}\left(bx^4+a\right)^{\frac{5}{4}}+\left(bx^4+a\right)^{\frac{1}{4}}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x,x, algorithm="maxima")
```

```
[Out] -1/2*a^(5/4)*arctan((b*x^4 + a)^(1/4)/a^(1/4)) + 1/4*a^(5/4)*log(((b*x^4 +
 a)^(1/4) - a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/4))) + 1/5*(b*x^4 + a)^(5/4)
 + (b*x^4 + a)^(1/4)*a
```

mupad [B] time = 1.16, size = 64, normalized size = 0.77

$$a\left(b\,x^4+a\right)^{1/4}-\frac{a^{5/4}\,\mathrm{atan}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)}{2}+\frac{\left(b\,x^4+a\right)^{5/4}}{5}+\frac{a^{5/4}\,\mathrm{atan}\left(\frac{\left(b\,x^4+a\right)^{1/4}\,1\mathrm{i}}{a^{1/4}}\right)\,1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(5/4)/x,x)
```

```
[Out] (a^(5/4)*atan(((a + b*x^4)^(1/4)*1i)/a^(1/4))*1i)/2 - (a^(5/4)*atan((a + b*
x^4)^(1/4)/a^(1/4)))/2 + a*(a + b*x^4)^(1/4) + (a + b*x^4)^(5/4)/5
```

sympy [C] time = 2.76, size = 48, normalized size = 0.58

$$-\frac{b^{\frac{5}{4}}x^5\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{5}{4},-\frac{5}{4}\\-\frac{1}{4}\end{matrix}\right|\frac{ae^{i\pi}}{bx^4}\right)}{4\Gamma\left(-\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(5/4)/x,x)
```

```
[Out] -b**(5/4)*x**5*gamma(-5/4)*hyper((-5/4, -5/4), (-1/4,), a*exp_polar(I*pi)/(
b*x**4))/(4*gamma(-1/4))
```

3.1053 $\int \frac{\left(a+bx^4\right)^{5/4}}{x^5}\,dx$

Optimal. Leaf size=91

$$-\frac{\left(a+bx^4\right)^{5/4}}{4x^4}+\frac{5}{4}b\sqrt[4]{a+bx^4}-\frac{5}{8}\sqrt[4]{a}\,b\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)-\frac{5}{8}\sqrt[4]{a}\,b\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)$$

```
[Out] 5/4*b*(b*x^4+a)^(1/4)-1/4*(b*x^4+a)^(5/4)/x^4-5/8*a^(1/4)*b*arctan((b*x^4+a
)^(1/4)/a^(1/4))-5/8*a^(1/4)*b*arctanh((b*x^4+a)^(1/4)/a^(1/4))
```

Rubi [A] time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {266, 47, 50, 63, 212, 206, 203}

$$-\frac{\left(a+bx^4\right)^{5/4}}{4x^4}+\frac{5}{4}b\sqrt[4]{a+bx^4}-\frac{5}{8}\sqrt[4]{a}\,b\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)-\frac{5}{8}\sqrt[4]{a}\,b\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(5/4)/x^5,x]
```

```
[Out] (5*b*(a + b*x^4)^(1/4))/4 - (a + b*x^4)^(5/4)/(4*x^4) - (5*a^(1/4)*b*ArcTan
[(a + b*x^4)^(1/4)/a^(1/4)])/8 - (5*a^(1/4)*b*ArcTanh[(a + b*x^4)^(1/4)/a^(
1/4)])/8
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^4\right)^{5/4}}{x^5}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{(a+bx)^{5/4}}{x^2}\,dx, x, x^4\right)\\
&= -\frac{\left(a+bx^4\right)^{5/4}}{4x^4} + \frac{1}{16}(5b)\operatorname{Subst}\left(\int \frac{\sqrt[4]{a+bx}}{x}\,dx, x, x^4\right)\\
&= \frac{5}{4}b\sqrt[4]{a+bx^4} - \frac{\left(a+bx^4\right)^{5/4}}{4x^4} + \frac{1}{16}(5ab)\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{3/4}}\,dx, x, x^4\right)\\
&= \frac{5}{4}b\sqrt[4]{a+bx^4} - \frac{\left(a+bx^4\right)^{5/4}}{4x^4} + \frac{1}{4}(5a)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^4}{b}}\,dx, x, \sqrt[4]{a+bx^4}\right)\\
&= \frac{5}{4}b\sqrt[4]{a+bx^4} - \frac{\left(a+bx^4\right)^{5/4}}{4x^4} - \frac{1}{8}\left(5\sqrt{a}\,b\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a}-x^2}\,dx, x, \sqrt[4]{a+bx^4}\right) - \frac{1}{8}\left(5\sqrt{a}\,b\right)\mathrm{S}\\
&= \frac{5}{4}b\sqrt[4]{a+bx^4} - \frac{\left(a+bx^4\right)^{5/4}}{4x^4} - \frac{5}{8}\sqrt[4]{a}\,b\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right) - \frac{5}{8}\sqrt[4]{a}\,b\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.41

$$\frac{b\left(a+bx^4\right)^{9/4}\,{}_2F_1\left(2,\frac{9}{4};\frac{13}{4};\frac{bx^4}{a}+1\right)}{9a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(5/4)/x^5,x]
```

```
[Out] (b*(a + b*x^4)^(9/4)*Hypergeometric2F1[2, 9/4, 13/4, 1 + (b*x^4)/a])/(9*a^2
)
```

fricas [B] time = 0.88, size = 169, normalized size = 1.86

$$\frac{20\left(ab^4\right)^{\frac{1}{4}}x^4\arctan\left(-\frac{\left(ab^4\right)^{\frac{3}{4}}\left(bx^4+a\right)^{\frac{1}{4}}b-\left(ab^4\right)^{\frac{3}{4}}\sqrt{\sqrt{bx^4+a}\,b^2+\sqrt{ab^4}}}{ab^4}\right) - 5\left(ab^4\right)^{\frac{1}{4}}x^4\log\left(5\left(bx^4+a\right)^{\frac{1}{4}}b+5\left(ab^4\right)^{\frac{1}{4}}\right)+5\left(a}{16\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^5,x, algorithm="fricas")
```

```
[Out] 1/16*(20*(a*b^4)^(1/4)*x^4*arctan(-((a*b^4)^(3/4)*(b*x^4 + a)^(1/4)*b - (a*
b^4)^(3/4)*sqrt(sqrt(b*x^4 + a)*b^2 + sqrt(a*b^4)))/(a*b^4)) - 5*(a*b^4)^(1
/4)*x^4*log(5*(b*x^4 + a)^(1/4)*b + 5*(a*b^4)^(1/4)) + 5*(a*b^4)^(1/4)*x^4*
log(5*(b*x^4 + a)^(1/4)*b - 5*(a*b^4)^(1/4)) + 4*(4*b*x^4 - a)*(b*x^4 + a)^
(1/4))/x^4
```

giac [B] time = 0.18, size = 221, normalized size = 2.43

$$-\frac{10\sqrt{2}\,(-a)^{\frac{1}{4}}\,b^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}+2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)+10\sqrt{2}\,(-a)^{\frac{1}{4}}\,b^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}-2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)+5\sqrt{2}\,(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^5,x, algorithm="giac")
```

```
[Out] -1/32*(10*sqrt(2)*(-a)^(1/4)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2
*(b*x^4 + a)^(1/4))/(-a)^(1/4)) + 10*sqrt(2)*(-a)^(1/4)*b^2*arctan(-1/2*sqr
t(2)*(sqrt(2)*(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4)) + 5*sqrt(2)*(-a
)^(1/4)*b^2*log(sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) + sq
rt(-a)) - 5*sqrt(2)*(-a)^(1/4)*b^2*log(-sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4
) + sqrt(b*x^4 + a) + sqrt(-a)) - 32*(b*x^4 + a)^(1/4)*b^2 + 8*(b*x^4 + a)^
(1/4)*a*b/x^4)/b
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{5}{4}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(5/4)/x^5,x)
```

```
[Out] int((b*x^4+a)^(5/4)/x^5,x)
```

maxima [A] time = 2.97, size = 89, normalized size = 0.98

$$-\frac{5}{16}\left(\frac{2\,b\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{3}{4}}}-\frac{b\log\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{\left(bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{3}{4}}}\right)a+\left(bx^4+a\right)^{\frac{1}{4}}b-\frac{\left(bx^4+a\right)^{\frac{1}{4}}a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^5,x, algorithm="maxima")
```

```
[Out] -5/16*(2*b*arctan((b*x^4 + a)^(1/4)/a^(1/4))/a^(3/4) - b*log(((b*x^4 + a)^(
1/4) - a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/4)))/a^(3/4))*a + (b*x^4 + a)^(1/
4)*b - 1/4*(b*x^4 + a)^(1/4)*a/x^4
```

mupad [B] time = 1.40, size = 70, normalized size = 0.77

$$b\left(b\,x^4+a\right)^{1/4}-\frac{a\left(b\,x^4+a\right)^{1/4}}{4\,x^4}-\frac{5\,a^{1/4}\,b\,\mathrm{atan}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)}{8}+\frac{a^{1/4}\,b\,\mathrm{atan}\left(\frac{\left(b\,x^4+a\right)^{1/4}\,1\mathrm{i}}{a^{1/4}}\right)\,5\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(5/4)/x^5,x)
```

```
[Out] b*(a + b*x^4)^(1/4) - (a*(a + b*x^4)^(1/4))/(4*x^4) - (5*a^(1/4)*b*atan((a
+ b*x^4)^(1/4)/a^(1/4)))/8 + (a^(1/4)*b*atan(((a + b*x^4)^(1/4)*1i)/a^(1/4)
)*5i)/8
```

sympy [C] time = 3.89, size = 42, normalized size = 0.46

$$-\frac{b^{\frac{5}{4}} x \Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{4}, -\frac{1}{4} \\ \frac{3}{4} \end{matrix} \middle| \frac{a e^{i\pi}}{b x^4}\right)}{4\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(5/4)/x**5,x)
```

```
[Out] -b**(5/4)*x*gamma(-1/4)*hyper((-5/4, -1/4), (3/4,), a*exp_polar(I*pi)/(b*x*
**4))/(4*gamma(3/4))
```

3.1054 $\int \frac{\left(a+bx^4\right)^{5/4}}{x^9}\,dx$

Optimal. Leaf size=98

$$-\frac{5b^2\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{3/4}}-\frac{5b^2\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{3/4}}-\frac{5b\sqrt[4]{a+bx^4}}{32x^4}-\frac{\left(a+bx^4\right)^{5/4}}{8x^8}$$

```
[Out] -5/32*b*(b*x^4+a)^(1/4)/x^4-1/8*(b*x^4+a)^(5/4)/x^8-5/64*b^2*arctan((b*x^4+
a)^(1/4)/a^(1/4))/a^(3/4)-5/64*b^2*arctanh((b*x^4+a)^(1/4)/a^(1/4))/a^(3/4)
```

Rubi [A] time = 0.06, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {266, 47, 63, 212, 206, 203}

$$-\frac{5b^2\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{3/4}}-\frac{5b^2\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{3/4}}-\frac{5b\sqrt[4]{a+bx^4}}{32x^4}-\frac{\left(a+bx^4\right)^{5/4}}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(5/4)/x^9,x]
```

```
[Out] (-5*b*(a + b*x^4)^(1/4))/(32*x^4) - (a + b*x^4)^(5/4)/(8*x^8) - (5*b^2*ArcT
an[(a + b*x^4)^(1/4)/a^(1/4)])/(64*a^(3/4)) - (5*b^2*ArcTanh[(a + b*x^4)^(1
/4)/a^(1/4)])/(64*a^(3/4))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
```

```
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^4\right)^{5/4}}{x^9}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{(a+bx)^{5/4}}{x^3}\,dx, x, x^4\right) \\
&= -\frac{\left(a+bx^4\right)^{5/4}}{8x^8} + \frac{1}{32}(5b)\operatorname{Subst}\left(\int \frac{\sqrt[4]{a+bx}}{x^2}\,dx, x, x^4\right) \\
&= -\frac{5b\sqrt[4]{a+bx^4}}{32x^4} - \frac{\left(a+bx^4\right)^{5/4}}{8x^8} + \frac{1}{128}\left(5b^2\right)\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{3/4}}\,dx, x, x^4\right) \\
&= -\frac{5b\sqrt[4]{a+bx^4}}{32x^4} - \frac{\left(a+bx^4\right)^{5/4}}{8x^8} + \frac{1}{32}(5b)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^4}{b}}\,dx, x, \sqrt[4]{a+bx^4}\right) \\
&= -\frac{5b\sqrt[4]{a+bx^4}}{32x^4} - \frac{\left(a+bx^4\right)^{5/4}}{8x^8} - \frac{\left(5b^2\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a}-x^2}\,dx, x, \sqrt[4]{a+bx^4}\right)}{64\sqrt{a}} - \frac{\left(5b^2\right)\operatorname{Subst}\left(\int \frac{}{\sqrt{a}}\right.}{64} \\
&= -\frac{5b\sqrt[4]{a+bx^4}}{32x^4} - \frac{\left(a+bx^4\right)^{5/4}}{8x^8} - \frac{5b^2\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{3/4}} - \frac{5b^2\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.40

$$-\frac{b^2\left(a+bx^4\right)^{9/4}\,{}_2F_1\left(\frac{9}{4},3;\frac{13}{4};\frac{bx^4}{a}+1\right)}{9a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(5/4)/x^9,x]
```

```
[Out] -1/9*(b^2*(a + b*x^4)^(9/4)*Hypergeometric2F1[9/4, 3, 13/4, 1 + (b*x^4)/a])
/a^3
```

fricas [B] time = 0.81, size = 200, normalized size = 2.04

$$\frac{20\left(\frac{b^8}{a^3}\right)^{\frac{1}{4}}x^8\arctan\left(-\frac{\left(\frac{b^8}{a^3}\right)^{\frac{3}{4}}\left(bx^4+a\right)^{\frac{1}{4}}a^2b^2-\left(\frac{b^8}{a^3}\right)^{\frac{3}{4}}\sqrt{\sqrt{bx^4+a}\,b^4+\sqrt{\frac{b^8}{a^3}}\,a^2}\,a^2}{b^8}\right)-5\left(\frac{b^8}{a^3}\right)^{\frac{1}{4}}x^8\log\left(5\left(bx^4+a\right)^{\frac{1}{4}}b^2+5\left(\frac{b^8}{a^3}\right)^{\frac{1}{4}}a\right)+}{128\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^9,x, algorithm="fricas")
```

```
[Out] 1/128*(20*(b^8/a^3)^(1/4)*x^8*arctan(-((b^8/a^3)^(3/4)*(b*x^4 + a)^(1/4)*a^
2*b^2 - (b^8/a^3)^(3/4)*sqrt(sqrt(b*x^4 + a)*b^4 + sqrt(b^8/a^3)*a^2)*a^2)/
```

```
b^8) - 5*(b^8/a^3)^(1/4)*x^8*log(5*(b*x^4 + a)^(1/4)*b^2 + 5*(b^8/a^3)^(1/4
)*a) + 5*(b^8/a^3)^(1/4)*x^8*log(5*(b*x^4 + a)^(1/4)*b^2 - 5*(b^8/a^3)^(1/4
)*a) - 4*(9*b*x^4 + 4*a)*(b*x^4 + a)^(1/4))/x^8
```

giac [B] time = 0.19, size = 232, normalized size = 2.37

$$\frac{10\sqrt{2}\,b^3\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}+2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{3}{4}}}+\frac{10\sqrt{2}\,b^3\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}-2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{3}{4}}}+\frac{5\sqrt{2}\,b^3\log\left(\sqrt{2}\left(bx^4+a\right)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^4+a}+\sqrt{-a}\right)}{(-a)^{\frac{3}{4}}}}{256\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^9,x, algorithm="giac")
```

```
[Out] 1/256*(10*sqrt(2)*b^3*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x^4 + a
)^(1/4))/(-a)^(1/4))/(-a)^(3/4) + 10*sqrt(2)*b^3*arctan(-1/2*sqrt(2)*(sqrt(
2)*(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4))/(-a)^(3/4) + 5*sqrt(2)*b^3
*log(sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt(-a))/(-a
)^(3/4) + 5*sqrt(2)*(-a)^(1/4)*b^3*log(-sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4
) + sqrt(b*x^4 + a) + sqrt(-a))/a - 8*(9*(b*x^4 + a)^(5/4)*b^3 - 5*(b*x^4 +
 a)^(1/4)*a*b^3)/(b^2*x^8))/b
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{5}{4}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(5/4)/x^9,x)
```

```
[Out] int((b*x^4+a)^(5/4)/x^9,x)
```

maxima [A] time = 2.89, size = 120, normalized size = 1.22

$$-\frac{5\,b^2\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{64\,a^{\frac{3}{4}}}+\frac{5\,b^2\log\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{\left(bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{128\,a^{\frac{3}{4}}}-\frac{9\left(bx^4+a\right)^{\frac{5}{4}}b^2-5\left(bx^4+a\right)^{\frac{1}{4}}ab^2}{32\left(\left(bx^4+a\right)^2-2\left(bx^4+a\right)a+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^9,x, algorithm="maxima")
```

```
[Out] -5/64*b^2*arctan((b*x^4 + a)^(1/4)/a^(1/4))/a^(3/4) + 5/128*b^2*log(((b*x^4
 + a)^(1/4) - a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/4)))/a^(3/4) - 1/32*(9*(b*
x^4 + a)^(5/4)*b^2 - 5*(b*x^4 + a)^(1/4)*a*b^2)/((b*x^4 + a)^2 - 2*(b*x^4 +
 a)*a + a^2)
```

mupad [B] time = 1.50, size = 77, normalized size = 0.79

$$\frac{5\,a\left(b\,x^4+a\right)^{1/4}}{32\,x^8}-\frac{5\,b^2\,\mathrm{atan}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)}{64\,a^{3/4}}-\frac{9\left(b\,x^4+a\right)^{5/4}}{32\,x^8}+\frac{b^2\,\mathrm{atan}\left(\frac{\left(b\,x^4+a\right)^{1/4}\,1\mathrm{i}}{a^{1/4}}\right)5\mathrm{i}}{64\,a^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(5/4)/x^9,x)
```

```
[Out] (b^2*atan(((a + b*x^4)^(1/4)*1i)/a^(1/4))*5i)/(64*a^(3/4)) - (5*b^2*atan((a
 + b*x^4)^(1/4)/a^(1/4)))/(64*a^(3/4)) - (9*(a + b*x^4)^(5/4))/(32*x^8) + (
5*a*(a + b*x^4)^(1/4))/(32*x^8)
```

sympy [C] time = 4.19, size = 41, normalized size = 0.42

$$-\frac{b^{\frac{5}{4}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4}, \frac{3}{4} \\ \frac{7}{4}\end{matrix}\middle| \frac{ae^{i\pi}}{bx^4}\right)}{4x^3\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(5/4)/x**9,x)
```

```
[Out] -b**(5/4)*gamma(3/4)*hyper((-5/4, 3/4), (7/4,), a*exp_polar(I*pi)/(b*x**4))
/(4*x**3*gamma(7/4))
```

3.1055 $\int x^9 \left(a+bx^4\right)^{5/4}\,dx$

Optimal. Leaf size=146

$$\frac{4a^{9/2}\left(\frac{bx^4}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{231b^{5/2}\left(a+bx^4\right)^{3/4}}-\frac{2a^3x^2\sqrt[4]{a+bx^4}}{231b^2}+\frac{a^2x^6\sqrt[4]{a+bx^4}}{231b}+\frac{1}{15}x^{10}\left(a+bx^4\right)^{5/4}+\frac{1}{33}ax^{10}\sqrt[4]{a+bx}$$

```
[Out] -2/231*a^3*x^2*(b*x^4+a)^(1/4)/b^2+1/231*a^2*x^6*(b*x^4+a)^(1/4)/b+1/33*a*x
^10*(b*x^4+a)^(1/4)+1/15*x^10*(b*x^4+a)^(5/4)+4/231*a^(9/2)*(1+b*x^4/a)^(3/
4)*(cos(1/2*arctan(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x^2*b^(1/2
)/a^(1/2)))*EllipticF(sin(1/2*arctan(x^2*b^(1/2)/a^(1/2))),2^(1/2))/b^(5/2)
/(b*x^4+a)^(3/4)
```

Rubi [A] time = 0.10, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {275, 279, 321, 233, 231}

$$-\frac{2a^3x^2\sqrt[4]{a+bx^4}}{231b^2}+\frac{4a^{9/2}\left(\frac{bx^4}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{231b^{5/2}\left(a+bx^4\right)^{3/4}}+\frac{a^2x^6\sqrt[4]{a+bx^4}}{231b}+\frac{1}{15}x^{10}\left(a+bx^4\right)^{5/4}+\frac{1}{33}ax^{10}\sqrt[4]{a+}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a + b*x^4)^(5/4),x]
```

```
[Out] (-2*a^3*x^2*(a + b*x^4)^(1/4))/(231*b^2) + (a^2*x^6*(a + b*x^4)^(1/4))/(231
*b) + (a*x^10*(a + b*x^4)^(1/4))/33 + (x^10*(a + b*x^4)^(5/4))/15 + (4*a^(9
/2)*(1 + (b*x^4)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(2
31*b^(5/2)*(a + b*x^4)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^9 \left(a+bx^4\right)^{5/4}\,dx = \frac{1}{2}\,\text{Subst}\left(\int x^4 \left(a+bx^2\right)^{5/4}\,dx, x, x^2\right)$$

$$= \frac{1}{15}x^{10}\left(a+bx^4\right)^{5/4} + \frac{1}{6}a\,\text{Subst}\left(\int x^4\sqrt[4]{a+bx^2}\,dx, x, x^2\right)$$

$$= \frac{1}{33}ax^{10}\sqrt[4]{a+bx^4} + \frac{1}{15}x^{10}\left(a+bx^4\right)^{5/4} + \frac{1}{66}a^2\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^2\right)^{3/4}}\,dx, x, x^2\right)$$

$$= \frac{a^2x^6\sqrt[4]{a+bx^4}}{231b} + \frac{1}{33}ax^{10}\sqrt[4]{a+bx^4} + \frac{1}{15}x^{10}\left(a+bx^4\right)^{5/4} - \frac{a^3\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^2\right)^{3/4}}\,dx, x, x^2\right)}{77b}$$

$$= -\frac{2a^3x^2\sqrt[4]{a+bx^4}}{231b^2} + \frac{a^2x^6\sqrt[4]{a+bx^4}}{231b} + \frac{1}{33}ax^{10}\sqrt[4]{a+bx^4} + \frac{1}{15}x^{10}\left(a+bx^4\right)^{5/4} + \frac{\left(2a^4\right)\text{Sub}}{}$$

$$= -\frac{2a^3x^2\sqrt[4]{a+bx^4}}{231b^2} + \frac{a^2x^6\sqrt[4]{a+bx^4}}{231b} + \frac{1}{33}ax^{10}\sqrt[4]{a+bx^4} + \frac{1}{15}x^{10}\left(a+bx^4\right)^{5/4} + \frac{\left(2a^4\left(1+\right.\right.}{}$$

$$= -\frac{2a^3x^2\sqrt[4]{a+bx^4}}{231b^2} + \frac{a^2x^6\sqrt[4]{a+bx^4}}{231b} + \frac{1}{33}ax^{10}\sqrt[4]{a+bx^4} + \frac{1}{15}x^{10}\left(a+bx^4\right)^{5/4} + \frac{4a^{9/2}\left(1+\right.}{2}$$

Mathematica [C] time = 0.10, size = 81, normalized size = 0.55

$$\frac{x^2\sqrt[4]{a+bx^4}\left(\frac{6a^3\,{}_2F_1\left(-\frac{5}{4},\frac{1}{2},\frac{3}{2};-\frac{bx^4}{a}\right)}{\sqrt[4]{\frac{bx^4}{a}+1}} - \left(6a-11bx^4\right)\left(a+bx^4\right)^2\right)}{165b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a + b*x^4)^(5/4),x]
```

```
[Out] (x^2*(a + b*x^4)^(1/4)*(-((6*a - 11*b*x^4)*(a + b*x^4)^2) + (6*a^3*Hypergeo
metric2F1[-5/4, 1/2, 3/2, -((b*x^4)/a)])/(1 + (b*x^4)/a)^(1/4)))/(165*b^2)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^{13}+ax^9\right)\left(bx^4+a\right)^{\frac{1}{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^13 + a*x^9)*(b*x^4 + a)^(1/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4 + a\right)^{\frac{5}{4}} x^9 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^4+a)^(5/4),x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(5/4)*x^9, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \left(b\,x^4 + a\right)^{\frac{5}{4}} x^9 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b*x^4+a)^(5/4),x)
[Out] int(x^9*(b*x^4+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4 + a\right)^{\frac{5}{4}} x^9 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^4+a)^(5/4),x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(5/4)*x^9, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^9 \left(b\,x^4 + a\right)^{5/4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(a + b*x^4)^(5/4),x)
[Out] int(x^9*(a + b*x^4)^(5/4), x)
```

sympy [C] time = 4.83, size = 29, normalized size = 0.20

$$\frac{a^{\frac{5}{4}} x^{10} {}_2F_1\left(\begin{matrix} -\frac{5}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b*x**4+a)**(5/4),x)
[Out] a**(5/4)*x**10*hyper((-5/4, 5/2), (7/2,), b*x**4*exp_polar(I*pi)/a)/10
```

3.1056 $\int x^5 \left(a+bx^4\right)^{5/4}\,dx$

Optimal. Leaf size=122

$$-\frac{10a^{7/2}\left(\frac{bx^4}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{231b^{3/2}\left(a+bx^4\right)^{3/4}}+\frac{5a^2x^2\sqrt[4]{a+bx^4}}{231b}+\frac{1}{11}x^6\left(a+bx^4\right)^{5/4}+\frac{5}{77}ax^6\sqrt[4]{a+bx^4}$$

```
[Out] 5/231*a^2*x^2*(b*x^4+a)^(1/4)/b+5/77*a*x^6*(b*x^4+a)^(1/4)+1/11*x^6*(b*x^4+
a)^(5/4)-10/231*a^(7/2)*(1+b*x^4/a)^(3/4)*(cos(1/2*arctan(x^2*b^(1/2)/a^(1/
2)))^2)^(1/2)/cos(1/2*arctan(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arctan
(x^2*b^(1/2)/a^(1/2))),2^(1/2))/b^(3/2)/(b*x^4+a)^(3/4)
```

Rubi [A] time = 0.08, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {275, 279, 321, 233, 231}

$$-\frac{10a^{7/2}\left(\frac{bx^4}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{231b^{3/2}\left(a+bx^4\right)^{3/4}}+\frac{5a^2x^2\sqrt[4]{a+bx^4}}{231b}+\frac{1}{11}x^6\left(a+bx^4\right)^{5/4}+\frac{5}{77}ax^6\sqrt[4]{a+bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^4)^(5/4),x]
```

```
[Out] (5*a^2*x^2*(a + b*x^4)^(1/4))/(231*b) + (5*a*x^6*(a + b*x^4)^(1/4))/77 + (x
^6*(a + b*x^4)^(5/4))/11 - (10*a^(7/2)*(1 + (b*x^4)/a)^(3/4)*EllipticF[ArcT
an[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(231*b^(3/2)*(a + b*x^4)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int x^5 \left(a+bx^4\right)^{5/4}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int x^2\left(a+bx^2\right)^{5/4}\,dx, x, x^2\right)\\
&= \frac{1}{11}x^6\left(a+bx^4\right)^{5/4} + \frac{1}{22}(5a)\,\text{Subst}\left(\int x^2\sqrt[4]{a+bx^2}\,dx, x, x^2\right)\\
&= \frac{5}{77}ax^6\sqrt[4]{a+bx^4} + \frac{1}{11}x^6\left(a+bx^4\right)^{5/4} + \frac{1}{154}\left(5a^2\right)\text{Subst}\left(\int \frac{x^2}{\left(a+bx^2\right)^{3/4}}\,dx, x, x^2\right)\\
&= \frac{5a^2x^2\sqrt[4]{a+bx^4}}{231b} + \frac{5}{77}ax^6\sqrt[4]{a+bx^4} + \frac{1}{11}x^6\left(a+bx^4\right)^{5/4} - \frac{\left(5a^3\right)\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx,\right.}{231b}\\
&= \frac{5a^2x^2\sqrt[4]{a+bx^4}}{231b} + \frac{5}{77}ax^6\sqrt[4]{a+bx^4} + \frac{1}{11}x^6\left(a+bx^4\right)^{5/4} - \frac{\left(5a^3\left(1+\frac{bx^4}{a}\right)^{3/4}\right)\text{Subst}\left(\int \right.}{231b\left(a+bx^4\right.}\\
&= \frac{5a^2x^2\sqrt[4]{a+bx^4}}{231b} + \frac{5}{77}ax^6\sqrt[4]{a+bx^4} + \frac{1}{11}x^6\left(a+bx^4\right)^{5/4} - \frac{10a^{7/2}\left(1+\frac{bx^4}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\right.}{231b^{3/2}\left(a+bx^4\right)^{3/}}
\end{aligned}$$

Mathematica [C] time = 0.10, size = 69, normalized size = 0.57

$$\frac{x^2\sqrt[4]{a+bx^4}\left(\left(a+bx^4\right)^2 - \frac{a^2\,{}_2F_1\left(-\frac{5}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^4}{a}\right)}{\sqrt[4]{\frac{bx^4}{a}+1}}\right)}{11b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^4)^(5/4),x]
```

```
[Out] (x^2*(a + b*x^4)^(1/4)*((a + b*x^4)^2 - (a^2*Hypergeometric2F1[-5/4, 1/2, 3
/2, -((b*x^4)/a)])/(1 + (b*x^4)/a)^(1/4)))/(11*b)
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^9+ax^5\right)\left(bx^4+a\right)^{\frac{1}{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^9 + a*x^5)*(b*x^4 + a)^(1/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4+a\right)^{\frac{5}{4}}x^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^4+a)^(5/4),x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(5/4)*x^5, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \left(b\,x^4+a\right)^{\frac{5}{4}} x^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^4+a)^(5/4),x)
[Out] int(x^5*(b*x^4+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4+a\right)^{\frac{5}{4}} x^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^4+a)^(5/4),x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(5/4)*x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5\left(b\,x^4+a\right)^{5/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^4)^(5/4),x)
[Out] int(x^5*(a + b*x^4)^(5/4), x)
```

sympy [C] time = 4.29, size = 29, normalized size = 0.24

$$\frac{a^{\frac{5}{4}} x^6\, {}_2F_1\left(\begin{matrix} -\frac{5}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**4+a)**(5/4),x)
[Out] a**(5/4)*x**6*hyper((-5/4, 3/2), (5/2,), b*x**4*exp_polar(I*pi)/a)/6
```

3.1057 $\int x\left(a+bx^4\right)^{5/4}\,dx$

Optimal. Leaf size=98

$$\frac{5a^{5/2}\left(\frac{bx^4}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{21\sqrt{b}\left(a+bx^4\right)^{3/4}}+\frac{5}{21}ax^2\sqrt[4]{a+bx^4}+\frac{1}{7}x^2\left(a+bx^4\right)^{5/4}$$

```
[Out] 5/21*a*x^2*(b*x^4+a)^(1/4)+1/7*x^2*(b*x^4+a)^(5/4)+5/21*a^(5/2)*(1+b*x^4/a)
^(3/4)*(cos(1/2*arctan(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x^2*b^
(1/2)/a^(1/2)))*EllipticF(sin(1/2*arctan(x^2*b^(1/2)/a^(1/2))),2^(1/2))/(b*
x^4+a)^(3/4)/b^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {275, 195, 233, 231}

$$\frac{5a^{5/2}\left(\frac{bx^4}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{21\sqrt{b}\left(a+bx^4\right)^{3/4}}+\frac{5}{21}ax^2\sqrt[4]{a+bx^4}+\frac{1}{7}x^2\left(a+bx^4\right)^{5/4}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^4)^(5/4),x]
```

```
[Out] (5*a*x^2*(a + b*x^4)^(1/4))/21 + (x^2*(a + b*x^4)^(5/4))/7 + (5*a^(5/2)*(1
 + (b*x^4)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(21*Sqrt[
b]*(a + b*x^4)^(3/4))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int x\left(a+bx^4\right)^{5/4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \left(a+bx^2\right)^{5/4}\,dx,x,x^2\right)\\
&= \frac{1}{7}x^2\left(a+bx^4\right)^{5/4} + \frac{1}{14}(5a)\operatorname{Subst}\left(\int \sqrt[4]{a+bx^2}\,dx,x,x^2\right)\\
&= \frac{5}{21}ax^2\sqrt[4]{a+bx^4} + \frac{1}{7}x^2\left(a+bx^4\right)^{5/4} + \frac{1}{42}\left(5a^2\right)\operatorname{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx,x,x^2\right)\\
&= \frac{5}{21}ax^2\sqrt[4]{a+bx^4} + \frac{1}{7}x^2\left(a+bx^4\right)^{5/4} + \frac{\left(5a^2\left(1+\frac{bx^4}{a}\right)^{3/4}\right)\operatorname{Subst}\left(\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx,x,x^2\right)}{42\left(a+bx^4\right)^{3/4}}\\
&= \frac{5}{21}ax^2\sqrt[4]{a+bx^4} + \frac{1}{7}x^2\left(a+bx^4\right)^{5/4} + \frac{5a^{5/2}\left(1+\frac{bx^4}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{21\sqrt{b}\left(a+bx^4\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.53

$$\frac{ax^2\sqrt[4]{a+bx^4}\;{}_2F_1\left(-\frac{5}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^4}{a}\right)}{2\sqrt[4]{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^4)^(5/4),x]
```

```
[Out] (a*x^2*(a + b*x^4)^(1/4)*Hypergeometric2F1[-5/4, 1/2, 3/2, -((b*x^4)/a)])/(
2*(1 + (b*x^4)/a)^(1/4))
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^5+ax\right)\left(bx^4+a\right)^{\frac{1}{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^5 + a*x)*(b*x^4 + a)^(1/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4+a\right)^{\frac{5}{4}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^4+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(5/4)*x, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \left(b\,x^4+a\right)^{\frac{5}{4}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^4+a)^(5/4),x)
```

```
[Out] int(x*(b*x^4+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4 + a\right)^{\frac{5}{4}} x\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^4+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(5/4)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \left(b\, x^4 + a\right)^{5/4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^4)^(5/4),x)
```

```
[Out] int(x*(a + b*x^4)^(5/4), x)
```

sympy [C] time = 2.25, size = 29, normalized size = 0.30

$$\frac{a^{\frac{5}{4}} x^2 {}_2F_1\left(\begin{matrix} -\frac{5}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**4+a)**(5/4),x)
```

```
[Out] a**(5/4)*x**2*hyper((-5/4, 1/2), (3/2,), b*x**4*exp_polar(I*pi)/a)/2
```

3.1058 $\int \frac{\left(a+bx^4\right)^{5/4}}{x^3}\,dx$

Optimal. Leaf size=98

$$\frac{5a^{3/2}\sqrt{b}\left(\frac{bx^4}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{6\left(a+bx^4\right)^{3/4}}+\frac{5}{6}bx^2\sqrt[4]{a+bx^4}-\frac{\left(a+bx^4\right)^{5/4}}{2x^2}$$

```
[Out] 5/6*b*x^2*(b*x^4+a)^(1/4)-1/2*(b*x^4+a)^(5/4)/x^2+5/6*a^(3/2)*(1+b*x^4/a)^(
3/4)*(cos(1/2*arctan(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x^2*b^(1
/2)/a^(1/2)))*EllipticF(sin(1/2*arctan(x^2*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/
2)/(b*x^4+a)^(3/4)
```

Rubi [A] time = 0.06, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {275, 277, 195, 233, 231}

$$\frac{5a^{3/2}\sqrt{b}\left(\frac{bx^4}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{6\left(a+bx^4\right)^{3/4}}+\frac{5}{6}bx^2\sqrt[4]{a+bx^4}-\frac{\left(a+bx^4\right)^{5/4}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(5/4)/x^3,x]
```

```
[Out] (5*b*x^2*(a + b*x^4)^(1/4))/6 - (a + b*x^4)^(5/4)/(2*x^2) + (5*a^(3/2)*Sqrt
[b]*(1 + (b*x^4)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(6
*(a + b*x^4)^(3/4))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^4\right)^{5/4}}{x^3}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{5/4}}{x^2}\,dx, x, x^2\right) \\
&= -\frac{\left(a+bx^4\right)^{5/4}}{2x^2} + \frac{1}{4}(5b)\,\text{Subst}\left(\int \sqrt[4]{a+bx^2}\,dx, x, x^2\right) \\
&= \frac{5}{6}bx^2\sqrt[4]{a+bx^4} - \frac{\left(a+bx^4\right)^{5/4}}{2x^2} + \frac{1}{12}(5ab)\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx, x, x^2\right) \\
&= \frac{5}{6}bx^2\sqrt[4]{a+bx^4} - \frac{\left(a+bx^4\right)^{5/4}}{2x^2} + \frac{\left(5ab\left(1+\frac{bx^4}{a}\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx, x, x^2\right)}{12\left(a+bx^4\right)^{3/4}} \\
&= \frac{5}{6}bx^2\sqrt[4]{a+bx^4} - \frac{\left(a+bx^4\right)^{5/4}}{2x^2} + \frac{5a^{3/2}\sqrt{b}\left(1+\frac{bx^4}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{6\left(a+bx^4\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.04, size = 52, normalized size = 0.53

$$-\frac{a\sqrt[4]{a+bx^4}\;{}_2F_1\left(-\frac{5}{4},-\frac{1}{2};\frac{1}{2};-\frac{bx^4}{a}\right)}{2x^2\sqrt[4]{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(5/4)/x^3,x]
```

```
[Out] -1/2*(a*(a + b*x^4)^(1/4)*Hypergeometric2F1[-5/4, -1/2, 1/2, -((b*x^4)/a)])
/(x^2*(1 + (b*x^4)/a)^(1/4))
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{5}{4}}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(5/4)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{5}{4}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^3,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(5/4)/x^3, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{5}{4}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(5/4)/x^3,x)
[Out] int((b*x^4+a)^(5/4)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{5}{4}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^3,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(5/4)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^4+a\right)^{5/4}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(5/4)/x^3,x)
[Out] int((a + b*x^4)^(5/4)/x^3, x)
```

sympy [C] time = 1.48, size = 32, normalized size = 0.33

$$-\frac{a^{\frac{5}{4}}\,{}_2F_1\left(\left.\begin{matrix}-\frac{5}{4}, -\frac{1}{2}\\ \frac{1}{2}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(5/4)/x**3,x)
[Out] -a**(5/4)*hyper((-5/4, -1/2), (1/2,), b*x**4*exp_polar(I*pi)/a)/(2*x**2)
```

3.1059 $\int \frac{\left(a+bx^4\right)^{5/4}}{x^7}\,dx$

Optimal. Leaf size=98

$$\frac{5\sqrt{a}\,b^{3/2}\left(\frac{bx^4}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{12\left(a+bx^4\right)^{3/4}}-\frac{\left(a+bx^4\right)^{5/4}}{6x^6}-\frac{5b\sqrt[4]{a+bx^4}}{12x^2}$$

```
[Out] -5/12*b*(b*x^4+a)^(1/4)/x^2-1/6*(b*x^4+a)^(5/4)/x^6+5/12*b^(3/2)*(1+b*x^4/a
)^(3/4)*(cos(1/2*arctan(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x^2*b
^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arctan(x^2*b^(1/2)/a^(1/2))),2^(1/2))*a^
(1/2)/(b*x^4+a)^(3/4)
```

Rubi [A] time = 0.06, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {275, 277, 233, 231}

$$\frac{5\sqrt{a}\,b^{3/2}\left(\frac{bx^4}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{12\left(a+bx^4\right)^{3/4}}-\frac{5b\sqrt[4]{a+bx^4}}{12x^2}-\frac{\left(a+bx^4\right)^{5/4}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(5/4)/x^7,x]
```

```
[Out] (-5*b*(a + b*x^4)^(1/4))/(12*x^2) - (a + b*x^4)^(5/4)/(6*x^6) + (5*Sqrt[a]*
b^(3/2)*(1 + (b*x^4)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2]
)/(12*(a + b*x^4)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^4\right)^{5/4}}{x^7}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{5/4}}{x^4}\,dx, x, x^2\right) \\
&= -\frac{\left(a+bx^4\right)^{5/4}}{6x^6} + \frac{1}{12}(5b)\,\text{Subst}\left(\int \frac{\sqrt[4]{a+bx^2}}{x^2}\,dx, x, x^2\right) \\
&= -\frac{5b\sqrt[4]{a+bx^4}}{12x^2} - \frac{\left(a+bx^4\right)^{5/4}}{6x^6} + \frac{1}{24}\left(5b^2\right)\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx, x, x^2\right) \\
&= -\frac{5b\sqrt[4]{a+bx^4}}{12x^2} - \frac{\left(a+bx^4\right)^{5/4}}{6x^6} + \frac{\left(5b^2\left(1+\frac{bx^4}{a}\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx, x, x^2\right)}{24\left(a+bx^4\right)^{3/4}} \\
&= -\frac{5b\sqrt[4]{a+bx^4}}{12x^2} - \frac{\left(a+bx^4\right)^{5/4}}{6x^6} + \frac{5\sqrt{a}\,b^{3/2}\left(1+\frac{bx^4}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle| 2\right)}{12\left(a+bx^4\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.53

$$-\frac{a\sqrt[4]{a+bx^4}\;{}_2F_1\left(-\frac{3}{2},-\frac{5}{4};-\frac{1}{2};-\frac{bx^4}{a}\right)}{6x^6\sqrt[4]{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(5/4)/x^7,x]
```

```
[Out] -1/6*(a*(a + b*x^4)^(1/4)*Hypergeometric2F1[-3/2, -5/4, -1/2, -((b*x^4)/a)]
)/(x^6*(1 + (b*x^4)/a)^(1/4))
```

fricas [F] time = 1.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{5}{4}}}{x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^7,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(5/4)/x^7, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{5}{4}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^7,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(5/4)/x^7, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{5}{4}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(5/4)/x^7,x)
[Out] int((b*x^4+a)^(5/4)/x^7,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{5}{4}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^7,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(5/4)/x^7, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^4+a\right)^{5/4}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(5/4)/x^7,x)
[Out] int((a + b*x^4)^(5/4)/x^7, x)
```

sympy [C] time = 3.13, size = 34, normalized size = 0.35

$$-\frac{a^{\frac{5}{4}}\,{}_2F_1\left(\left.\begin{matrix}-\frac{3}{2}, -\frac{5}{4}\\ -\frac{1}{2}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(5/4)/x**7,x)
[Out] -a**(5/4)*hyper((-3/2, -5/4), (-1/2,), b*x**4*exp_polar(I*pi)/a)/(6*x**6)
```

3.1060 $\int \frac{\left(a+bx^4\right)^{5/4}}{x^{11}}\,dx$

Optimal. Leaf size=122

$$-\frac{b^{5/2}\left(\frac{bx^4}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{24\sqrt{a}\left(a+bx^4\right)^{3/4}}-\frac{b^2\sqrt[4]{a+bx^4}}{24ax^2}-\frac{\left(a+bx^4\right)^{5/4}}{10x^{10}}-\frac{b\sqrt[4]{a+bx^4}}{12x^6}$$

```
[Out] -1/12*b*(b*x^4+a)^(1/4)/x^6-1/24*b^2*(b*x^4+a)^(1/4)/a/x^2-1/10*(b*x^4+a)^(
5/4)/x^10-1/24*b^(5/2)*(1+b*x^4/a)^(3/4)*(cos(1/2*arctan(x^2*b^(1/2)/a^(1/2
)))^2)^(1/2)/cos(1/2*arctan(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arctan(
x^2*b^(1/2)/a^(1/2))),2^(1/2))/(b*x^4+a)^(3/4)/a^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {275, 277, 325, 233, 231}

$$-\frac{b^2\sqrt[4]{a+bx^4}}{24ax^2}-\frac{b^{5/2}\left(\frac{bx^4}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{24\sqrt{a}\left(a+bx^4\right)^{3/4}}-\frac{b\sqrt[4]{a+bx^4}}{12x^6}-\frac{\left(a+bx^4\right)^{5/4}}{10x^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(5/4)/x^11,x]
```

```
[Out] -(b*(a + b*x^4)^(1/4))/(12*x^6) - (b^2*(a + b*x^4)^(1/4))/(24*a*x^2) - (a +
 b*x^4)^(5/4)/(10*x^10) - (b^(5/2)*(1 + (b*x^4)/a)^(3/4)*EllipticF[ArcTan[(
Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(24*Sqrt[a]*(a + b*x^4)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^4\right)^{5/4}}{x^{11}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{5/4}}{x^6}\,dx, x, x^2\right)\\
&= -\frac{\left(a+bx^4\right)^{5/4}}{10x^{10}} + \frac{1}{4}b\,\text{Subst}\left(\int \frac{\sqrt[4]{a+bx^2}}{x^4}\,dx, x, x^2\right)\\
&= -\frac{b\sqrt[4]{a+bx^4}}{12x^6} - \frac{\left(a+bx^4\right)^{5/4}}{10x^{10}} + \frac{1}{24}b^2\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^{3/4}}\,dx, x, x^2\right)\\
&= -\frac{b\sqrt[4]{a+bx^4}}{12x^6} - \frac{b^2\sqrt[4]{a+bx^4}}{24ax^2} - \frac{\left(a+bx^4\right)^{5/4}}{10x^{10}} - \frac{b^3\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx, x, x^2\right)}{48a}\\
&= -\frac{b\sqrt[4]{a+bx^4}}{12x^6} - \frac{b^2\sqrt[4]{a+bx^4}}{24ax^2} - \frac{\left(a+bx^4\right)^{5/4}}{10x^{10}} - \frac{\left(b^3\left(1+\frac{bx^4}{a}\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx, x, x\right.}{48a\left(a+bx^4\right)^{3/4}}\\
&= -\frac{b\sqrt[4]{a+bx^4}}{12x^6} - \frac{b^2\sqrt[4]{a+bx^4}}{24ax^2} - \frac{\left(a+bx^4\right)^{5/4}}{10x^{10}} - \frac{b^{5/2}\left(1+\frac{bx^4}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{24\sqrt{a}\left(a+bx^4\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.43

$$-\frac{a\sqrt[4]{a+bx^4}\;{}_2F_1\left(-\frac{5}{2},-\frac{5}{4};-\frac{3}{2};-\frac{bx^4}{a}\right)}{10x^{10}\sqrt[4]{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(5/4)/x^11,x]
```

```
[Out] -1/10*(a*(a + b*x^4)^(1/4)*Hypergeometric2F1[-5/2, -5/4, -3/2, -((b*x^4)/a)
])/(x^10*(1 + (b*x^4)/a)^(1/4))
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{5}{4}}}{x^{11}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^11,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(5/4)/x^11, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{5}{4}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^11,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(5/4)/x^11, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{5}{4}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(5/4)/x^11,x)
[Out] int((b*x^4+a)^(5/4)/x^11,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{5}{4}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^11,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(5/4)/x^11, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^4+a\right)^{5/4}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(5/4)/x^11,x)
[Out] int((a + b*x^4)^(5/4)/x^11, x)
```

sympy [C] time = 6.72, size = 34, normalized size = 0.28

$$-\frac{a^{\frac{5}{4}}\,{}_2F_1\left(\left.\begin{matrix}-\frac{5}{2}, -\frac{5}{4}\\ -\frac{3}{2}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(5/4)/x**11,x)
[Out] -a**(5/4)*hyper((-5/2, -5/4), (-3/2,), b*x**4*exp_polar(I*pi)/a)/(10*x**10)
```

3.1061 $\int \frac{\left(a+bx^4\right)^{5/4}}{x^{15}}\,dx$

Optimal. Leaf size=146

$$\frac{5b^{7/2}\left(\frac{bx^4}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{336a^{3/2}\left(a+bx^4\right)^{3/4}}+\frac{5b^3\sqrt[4]{a+bx^4}}{336a^2x^2}-\frac{b^2\sqrt[4]{a+bx^4}}{168ax^6}-\frac{\left(a+bx^4\right)^{5/4}}{14x^{14}}-\frac{b\sqrt[4]{a+bx^4}}{28x^{10}}$$

```
[Out] -1/28*b*(b*x^4+a)^(1/4)/x^10-1/168*b^2*(b*x^4+a)^(1/4)/a/x^6+5/336*b^3*(b*x
^4+a)^(1/4)/a^2/x^2-1/14*(b*x^4+a)^(5/4)/x^14+5/336*b^(7/2)*(1+b*x^4/a)^(3/
4)*(cos(1/2*arctan(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x^2*b^(1/2
)/a^(1/2)))*EllipticF(sin(1/2*arctan(x^2*b^(1/2)/a^(1/2))),2^(1/2))/a^(3/2)
/(b*x^4+a)^(3/4)
```

Rubi [A] time = 0.10, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {275, 277, 325, 233, 231}

$$\frac{5b^3\sqrt[4]{a+bx^4}}{336a^2x^2}+\frac{5b^{7/2}\left(\frac{bx^4}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{336a^{3/2}\left(a+bx^4\right)^{3/4}}-\frac{b^2\sqrt[4]{a+bx^4}}{168ax^6}-\frac{b\sqrt[4]{a+bx^4}}{28x^{10}}-\frac{\left(a+bx^4\right)^{5/4}}{14x^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(5/4)/x^15,x]
```

```
[Out] -(b*(a + b*x^4)^(1/4))/(28*x^10) - (b^2*(a + b*x^4)^(1/4))/(168*a*x^6) + (5
*b^3*(a + b*x^4)^(1/4))/(336*a^2*x^2) - (a + b*x^4)^(5/4)/(14*x^14) + (5*b^
(7/2)*(1 + (b*x^4)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/
(336*a^(3/2)*(a + b*x^4)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{\left(a+bx^4\right)^{5/4}}{x^{15}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{5/4}}{x^8}\,dx, x, x^2\right)$$

$$= -\frac{\left(a+bx^4\right)^{5/4}}{14x^{14}} + \frac{1}{28}(5b)\,\text{Subst}\left(\int \frac{\sqrt[4]{a+bx^2}}{x^6}\,dx, x, x^2\right)$$

$$= -\frac{b\sqrt[4]{a+bx^4}}{28x^{10}} - \frac{\left(a+bx^4\right)^{5/4}}{14x^{14}} + \frac{1}{56}b^2\,\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^2\right)^{3/4}}\,dx, x, x^2\right)$$

$$= -\frac{b\sqrt[4]{a+bx^4}}{28x^{10}} - \frac{b^2\sqrt[4]{a+bx^4}}{168ax^6} - \frac{\left(a+bx^4\right)^{5/4}}{14x^{14}} - \frac{\left(5b^3\right)\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^{3/4}}\,dx, x, x^2\right)}{336a}$$

$$= -\frac{b\sqrt[4]{a+bx^4}}{28x^{10}} - \frac{b^2\sqrt[4]{a+bx^4}}{168ax^6} + \frac{5b^3\sqrt[4]{a+bx^4}}{336a^2x^2} - \frac{\left(a+bx^4\right)^{5/4}}{14x^{14}} + \frac{\left(5b^4\right)\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx, x\right.}{672a^2}$$

$$= -\frac{b\sqrt[4]{a+bx^4}}{28x^{10}} - \frac{b^2\sqrt[4]{a+bx^4}}{168ax^6} + \frac{5b^3\sqrt[4]{a+bx^4}}{336a^2x^2} - \frac{\left(a+bx^4\right)^{5/4}}{14x^{14}} + \frac{\left(5b^4\left(1+\frac{bx^4}{a}\right)^{3/4}\right)\text{Subst}\left(\int -\right.}{672a^2\left(a+bx^4\right)}$$

$$= -\frac{b\sqrt[4]{a+bx^4}}{28x^{10}} - \frac{b^2\sqrt[4]{a+bx^4}}{168ax^6} + \frac{5b^3\sqrt[4]{a+bx^4}}{336a^2x^2} - \frac{\left(a+bx^4\right)^{5/4}}{14x^{14}} + \frac{5b^{7/2}\left(1+\frac{bx^4}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\right.}{336a^{3/2}\left(a+bx^4\right)^{3/4}}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.36

$$-\frac{a\sqrt[4]{a+bx^4}\;{}_2F_1\left(-\frac{7}{2},-\frac{5}{4};-\frac{5}{2};-\frac{bx^4}{a}\right)}{14x^{14}\sqrt[4]{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(5/4)/x^15,x]
```

```
[Out] -1/14*(a*(a + b*x^4)^(1/4)*Hypergeometric2F1[-7/2, -5/4, -5/2, -((b*x^4)/a)
])/(x^14*(1 + (b*x^4)/a)^(1/4))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{5}{4}}}{x^{15}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^15,x, algorithm="fricas")
[Out] integral((b*x^4 + a)^(5/4)/x^15, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{5}{4}}}{x^{15}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^15,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(5/4)/x^15, x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{5}{4}}}{x^{15}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(5/4)/x^15,x)
[Out] int((b*x^4+a)^(5/4)/x^15,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{5}{4}}}{x^{15}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^15,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(5/4)/x^15, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^4+a\right)^{5/4}}{x^{15}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(5/4)/x^15,x)
[Out] int((a + b*x^4)^(5/4)/x^15, x)
```

sympy [C] time = 5.55, size = 34, normalized size = 0.23

$$-\frac{a^{\frac{5}{4}}\,{}_2F_1\left(\left.\begin{matrix}-\frac{7}{2}, -\frac{5}{4}\\ -\frac{5}{2}\end{matrix}\right| \frac{bx^4e^{i\pi}}{a}\right)}{14x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(5/4)/x**15,x)
[Out] -a**(5/4)*hyper((-7/2, -5/4), (-5/2,), b*x**4*exp_polar(I*pi)/a)/(14*x**14)
```

3.1062 $\int x^{10}\left(a+bx^4\right)^{5/4}\,dx$

Optimal. Leaf size=148

$$-\frac{35a^4\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{4096b^{11/4}}+\frac{35a^4\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{4096b^{11/4}}-\frac{35a^3x^3\sqrt[4]{a+bx^4}}{6144b^2}+\frac{5a^2x^7\sqrt[4]{a+bx^4}}{1536b}+\frac{1}{16}x^{11}\left(a+bx^4\right)^{5/4}+\frac{5}{192}ax^{11}$$

```
[Out] -35/6144*a^3*x^3*(b*x^4+a)^(1/4)/b^2+5/1536*a^2*x^7*(b*x^4+a)^(1/4)/b+5/192
*a*x^11*(b*x^4+a)^(1/4)+1/16*x^11*(b*x^4+a)^(5/4)-35/4096*a^4*arctan(b^(1/4
)*x/(b*x^4+a)^(1/4))/b^(11/4)+35/4096*a^4*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4)
)/b^(11/4)
```

Rubi [A] time = 0.06, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {279, 321, 331, 298, 203, 206}

$$-\frac{35a^3x^3\sqrt[4]{a+bx^4}}{6144b^2}-\frac{35a^4\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{4096b^{11/4}}+\frac{35a^4\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{4096b^{11/4}}+\frac{5a^2x^7\sqrt[4]{a+bx^4}}{1536b}+\frac{1}{16}x^{11}\left(a+bx^4\right)^{5/4}+\frac{5}{192}ax^{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^10*(a + b*x^4)^(5/4),x]
```

```
[Out] (-35*a^3*x^3*(a + b*x^4)^(1/4))/(6144*b^2) + (5*a^2*x^7*(a + b*x^4)^(1/4))/
(1536*b) + (5*a*x^11*(a + b*x^4)^(1/4))/192 + (x^11*(a + b*x^4)^(5/4))/16 -
 (35*a^4*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(4096*b^(11/4)) + (35*a^4*A
rcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(4096*b^(11/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\begin{aligned}
\int x^{10}\left(a+bx^4\right)^{5/4}\,dx &= \frac{1}{16}x^{11}\left(a+bx^4\right)^{5/4} + \frac{1}{16}(5a)\int x^{10}\sqrt[4]{a+bx^4}\,dx \\
&= \frac{5}{192}ax^{11}\sqrt[4]{a+bx^4} + \frac{1}{16}x^{11}\left(a+bx^4\right)^{5/4} + \frac{1}{192}\left(5a^2\right)\int \frac{x^{10}}{\left(a+bx^4\right)^{3/4}}\,dx \\
&= \frac{5a^2x^7\sqrt[4]{a+bx^4}}{1536b} + \frac{5}{192}ax^{11}\sqrt[4]{a+bx^4} + \frac{1}{16}x^{11}\left(a+bx^4\right)^{5/4} - \frac{\left(35a^3\right)\int \frac{x^6}{\left(a+bx^4\right)^{3/4}}\,dx}{1536b} \\
&= -\frac{35a^3x^3\sqrt[4]{a+bx^4}}{6144b^2} + \frac{5a^2x^7\sqrt[4]{a+bx^4}}{1536b} + \frac{5}{192}ax^{11}\sqrt[4]{a+bx^4} + \frac{1}{16}x^{11}\left(a+bx^4\right)^{5/4} + \frac{(35}{} \\
&= -\frac{35a^3x^3\sqrt[4]{a+bx^4}}{6144b^2} + \frac{5a^2x^7\sqrt[4]{a+bx^4}}{1536b} + \frac{5}{192}ax^{11}\sqrt[4]{a+bx^4} + \frac{1}{16}x^{11}\left(a+bx^4\right)^{5/4} + \frac{(35}{} \\
&= -\frac{35a^3x^3\sqrt[4]{a+bx^4}}{6144b^2} + \frac{5a^2x^7\sqrt[4]{a+bx^4}}{1536b} + \frac{5}{192}ax^{11}\sqrt[4]{a+bx^4} + \frac{1}{16}x^{11}\left(a+bx^4\right)^{5/4} + \frac{(35}{} \\
&= -\frac{35a^3x^3\sqrt[4]{a+bx^4}}{6144b^2} + \frac{5a^2x^7\sqrt[4]{a+bx^4}}{1536b} + \frac{5}{192}ax^{11}\sqrt[4]{a+bx^4} + \frac{1}{16}x^{11}\left(a+bx^4\right)^{5/4} - \frac{35a}{}
\end{aligned}$$

Mathematica [C] time = 0.11, size = 81, normalized size = 0.55

$$\frac{x^3\sqrt[4]{a+bx^4}\left(\frac{7a^3\,{}_2F_1\left(-\frac{5}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^4}{a}\right)}{\sqrt[4]{\frac{bx^4}{a}+1}} - \left(7a-12bx^4\right)\left(a+bx^4\right)^2\right)}{192b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10*(a + b*x^4)^(5/4),x]
```

```
[Out] (x^3*(a + b*x^4)^(1/4)*(-((7*a - 12*b*x^4)*(a + b*x^4)^2) + (7*a^3*Hypergeo
metric2F1[-5/4, 3/4, 7/4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^(1/4)))/(192*b^2)
```

fricas [B] time = 1.02, size = 249, normalized size = 1.68

$$420\left(\frac{a^{16}}{b^{11}}\right)^{\frac{1}{4}}b^2\arctan\left(-\frac{\left(\frac{a^{16}}{b^{11}}\right)^{\frac{3}{4}}\left(bx^4+a\right)^{\frac{1}{4}}a^4b^8-\left(\frac{a^{16}}{b^{11}}\right)^{\frac{3}{4}}b^8x\sqrt{\frac{\sqrt{bx^4+a}\,a^8+\sqrt{\frac{a^{16}}{b^{11}}}\,b^6x^2}{x^2}}}{a^{16}x}\right) - 105\left(\frac{a^{16}}{b^{11}}\right)^{\frac{1}{4}}b^2\log\left(\frac{35\left(\left(bx^4+a\right)^{\frac{1}{4}}a^4+\left(\frac{a^{16}}{b^{11}}\right.\right.}{x}\right.$$

$$-\frac{}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] -1/24576*(420*(a^16/b^11)^(1/4)*b^2*arctan(-((a^16/b^11)^(3/4)*(b*x^4 + a)^
(1/4)*a^4*b^8 - (a^16/b^11)^(3/4)*b^8*x*sqrt((sqrt(b*x^4 + a)*a^8 + sqrt(a^
16/b^11)*b^6*x^2)/x^2))/(a^16*x)) - 105*(a^16/b^11)^(1/4)*b^2*log(35*((b*x^
4 + a)^(1/4)*a^4 + (a^16/b^11)^(1/4)*b^3*x)/x) + 105*(a^16/b^11)^(1/4)*b^2*
log(35*((b*x^4 + a)^(1/4)*a^4 - (a^16/b^11)^(1/4)*b^3*x)/x) - 4*(384*b^3*x^
15 + 544*a*b^2*x^11 + 20*a^2*b*x^7 - 35*a^3*x^3)*(b*x^4 + a)^(1/4))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4 + a\right)^{\frac{5}{4}} x^{10}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b*x^4+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(5/4)*x^10, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \left(b\,x^4 + a\right)^{\frac{5}{4}} x^{10}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(b*x^4+a)^(5/4),x)
```

```
[Out] int(x^10*(b*x^4+a)^(5/4),x)
```

maxima [A] time = 2.97, size = 229, normalized size = 1.55

$$-\frac{\frac{105\left(bx^4+a\right)^{\frac{1}{4}}a^4b^3}{x} - \frac{399\left(bx^4+a\right)^{\frac{5}{4}}a^4b^2}{x^5} - \frac{125\left(bx^4+a\right)^{\frac{9}{4}}a^4b}{x^9} + \frac{35\left(bx^4+a\right)^{\frac{13}{4}}a^4}{x^{13}}}{6144\left(b^6 - \frac{4\left(bx^4+a\right)b^5}{x^4} + \frac{6\left(bx^4+a\right)^2b^4}{x^8} - \frac{4\left(bx^4+a\right)^3b^3}{x^{12}} + \frac{\left(bx^4+a\right)^4b^2}{x^{16}}\right)} + \frac{35\left(\frac{2\,a^4\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{3}{4}}} - \frac{a^4\log\left(-\frac{b^{\frac{1}{4}} - \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}} + \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{3}{4}}}\right)}{8192\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b*x^4+a)^(5/4),x, algorithm="maxima")
```

```
[Out] -1/6144*(105*(b*x^4 + a)^(1/4)*a^4*b^3/x - 399*(b*x^4 + a)^(5/4)*a^4*b^2/x^
5 - 125*(b*x^4 + a)^(9/4)*a^4*b/x^9 + 35*(b*x^4 + a)^(13/4)*a^4/x^13)/(b^6
- 4*(b*x^4 + a)*b^5/x^4 + 6*(b*x^4 + a)^2*b^4/x^8 - 4*(b*x^4 + a)^3*b^3/x^1
2 + (b*x^4 + a)^4*b^2/x^16) + 35/8192*(2*a^4*arctan((b*x^4 + a)^(1/4)/(b^(1
/4)*x))/b^(3/4) - a^4*log(-(b^(1/4) - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^
4 + a)^(1/4)/x))/b^(3/4))/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{10} \left(b\,x^4 + a\right)^{5/4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(a + b*x^4)^(5/4),x)
```

```
[Out] int(x^10*(a + b*x^4)^(5/4), x)
```

sympy [C] time = 11.44, size = 39, normalized size = 0.26

$$\frac{a^{\frac{5}{4}} x^{11} \Gamma\left(\frac{11}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{4}, \frac{11}{4} \\ \frac{15}{4} \end{matrix} \middle| \frac{b x^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(b*x**4+a)**(5/4),x)
```

```
[Out] a**(5/4)*x**11*gamma(11/4)*hyper((-5/4, 11/4), (15/4,), b*x**4*exp_polar(I*
pi)/a)/(4*gamma(15/4))
```

3.1063 $\int x^6 \left(a+bx^4\right)^{5/4}\,dx$

Optimal. Leaf size=124

$$\frac{5a^3\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{256b^{7/4}}-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{256b^{7/4}}+\frac{5a^2x^3\sqrt[4]{a+bx^4}}{384b}+\frac{1}{12}x^7\left(a+bx^4\right)^{5/4}+\frac{5}{96}ax^7\sqrt[4]{a+bx^4}$$

```
[Out] 5/384*a^2*x^3*(b*x^4+a)^(1/4)/b+5/96*a*x^7*(b*x^4+a)^(1/4)+1/12*x^7*(b*x^4+
a)^(5/4)+5/256*a^3*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(7/4)-5/256*a^3*arct
anh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(7/4)
```

Rubi [A] time = 0.05, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {279, 321, 331, 298, 203, 206}

$$\frac{5a^3\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{256b^{7/4}}-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{256b^{7/4}}+\frac{5a^2x^3\sqrt[4]{a+bx^4}}{384b}+\frac{1}{12}x^7\left(a+bx^4\right)^{5/4}+\frac{5}{96}ax^7\sqrt[4]{a+bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x^4)^(5/4),x]
```

```
[Out] (5*a^2*x^3*(a + b*x^4)^(1/4))/(384*b) + (5*a*x^7*(a + b*x^4)^(1/4))/96 + (x
^7*(a + b*x^4)^(5/4))/12 + (5*a^3*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(2
56*b^(7/4)) - (5*a^3*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(256*b^(7/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\begin{aligned}
\int x^6\left(a+bx^4\right)^{5/4}\,dx &= \frac{1}{12}x^7\left(a+bx^4\right)^{5/4}+\frac{1}{12}(5a)\int x^6\sqrt[4]{a+bx^4}\,dx \\
&= \frac{5}{96}ax^7\sqrt[4]{a+bx^4}+\frac{1}{12}x^7\left(a+bx^4\right)^{5/4}+\frac{1}{96}\left(5a^2\right)\int \frac{x^6}{\left(a+bx^4\right)^{3/4}}\,dx \\
&= \frac{5a^2x^3\sqrt[4]{a+bx^4}}{384b}+\frac{5}{96}ax^7\sqrt[4]{a+bx^4}+\frac{1}{12}x^7\left(a+bx^4\right)^{5/4}-\frac{\left(5a^3\right)\int \frac{x^2}{\left(a+bx^4\right)^{3/4}}\,dx}{128b} \\
&= \frac{5a^2x^3\sqrt[4]{a+bx^4}}{384b}+\frac{5}{96}ax^7\sqrt[4]{a+bx^4}+\frac{1}{12}x^7\left(a+bx^4\right)^{5/4}-\frac{\left(5a^3\right)\text{Subst}\left(\int \frac{x^2}{1-bx^4}\,dx,x,\right.}{128b} \\
&= \frac{5a^2x^3\sqrt[4]{a+bx^4}}{384b}+\frac{5}{96}ax^7\sqrt[4]{a+bx^4}+\frac{1}{12}x^7\left(a+bx^4\right)^{5/4}-\frac{\left(5a^3\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{b}\,x^2}\,dx,x,\right.}{256b^{3/2}} \\
&= \frac{5a^2x^3\sqrt[4]{a+bx^4}}{384b}+\frac{5}{96}ax^7\sqrt[4]{a+bx^4}+\frac{1}{12}x^7\left(a+bx^4\right)^{5/4}+\frac{5a^3\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{256b^{7/4}}-\frac{5a^3\,\text{ta}}{}
\end{aligned}$$

Mathematica [C] time = 0.08, size = 69, normalized size = 0.56

$$\frac{x^3\sqrt[4]{a+bx^4}\left(\left(a+bx^4\right)^2-\frac{a^2\,{}_2F_1\left(-\frac{5}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^4}{a}\right)}{\sqrt[4]{\frac{bx^4}{a}+1}}\right)}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x^4)^(5/4),x]
```

```
[Out] (x^3*(a + b*x^4)^(1/4)*((a + b*x^4)^2 - (a^2*Hypergeometric2F1[-5/4, 3/4, 7
/4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^(1/4)))/(12*b)
```

fricas [B] time = 0.91, size = 232, normalized size = 1.87

$$\frac{60\left(\frac{a^{12}}{b^7}\right)^{\frac{1}{4}}b\arctan\left(-\frac{\left(\frac{a^{12}}{b^7}\right)^{\frac{3}{4}}\left(bx^4+a\right)^{\frac{1}{4}}a^3b^5-\left(\frac{a^{12}}{b^7}\right)^{\frac{3}{4}}b^5x\sqrt{\frac{\sqrt{bx^4+a}\,a^6+\sqrt{\frac{a^{12}}{b^7}}\,b^4x^2}{x^2}}}{a^{12}x}\right)-15\left(\frac{a^{12}}{b^7}\right)^{\frac{1}{4}}b\log\left(\frac{5\left(\left(bx^4+a\right)^{\frac{1}{4}}a^3+\left(\frac{a^{12}}{b^7}\right)^{\frac{1}{4}}b^2x\right)}{x}\right.}{1536\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] 1/1536*(60*(a^12/b^7)^(1/4)*b*arctan(-((a^12/b^7)^(3/4)*(b*x^4 + a)^(1/4)*a
^3*b^5 - (a^12/b^7)^(3/4)*b^5*x*sqrt((sqrt(b*x^4 + a)*a^6 + sqrt(a^12/b^7)*
b^4*x^2)/x^2))/(a^12*x)) - 15*(a^12/b^7)^(1/4)*b*log(5*((b*x^4 + a)^(1/4)*a
^3 + (a^12/b^7)^(1/4)*b^2*x)/x) + 15*(a^12/b^7)^(1/4)*b*log(5*((b*x^4 + a)^
(1/4)*a^3 - (a^12/b^7)^(1/4)*b^2*x)/x) + 4*(32*b^2*x^11 + 52*a*b*x^7 + 5*a^
2*x^3)*(b*x^4 + a)^(1/4))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4 + a\right)^{\frac{5}{4}} x^6 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^4+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(5/4)*x^6, x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \left(b\,x^4 + a\right)^{\frac{5}{4}} x^6 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^4+a)^(5/4),x)
```

```
[Out] int(x^6*(b*x^4+a)^(5/4),x)
```

maxima [A] time = 3.07, size = 191, normalized size = 1.54

$$\frac{\frac{15\left(bx^4+a\right)^{\frac{1}{4}}a^3b^2}{x} - \frac{42\left(bx^4+a\right)^{\frac{5}{4}}a^3b}{x^5} - \frac{5\left(bx^4+a\right)^{\frac{9}{4}}a^3}{x^9}}{384\left(b^4 - \frac{3\left(bx^4+a\right)b^3}{x^4} + \frac{3\left(bx^4+a\right)^2b^2}{x^8} - \frac{\left(bx^4+a\right)^3b}{x^{12}}\right)} - \frac{5\left(\frac{2\,a^3\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{3}{4}}} - \frac{a^3\log\left(-\frac{b^{\frac{1}{4}} - \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}} + \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{3}{4}}}\right)}{512\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^4+a)^(5/4),x, algorithm="maxima")
```

```
[Out] 1/384*(15*(b*x^4 + a)^(1/4)*a^3*b^2/x - 42*(b*x^4 + a)^(5/4)*a^3*b/x^5 - 5*
(b*x^4 + a)^(9/4)*a^3/x^9)/(b^4 - 3*(b*x^4 + a)*b^3/x^4 + 3*(b*x^4 + a)^2*b
^2/x^8 - (b*x^4 + a)^3*b/x^12) - 5/512*(2*a^3*arctan((b*x^4 + a)^(1/4)/(b^(
1/4)*x))/b^(3/4) - a^3*log(-(b^(1/4) - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x
^4 + a)^(1/4)/x))/b^(3/4))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^6 \left(b\,x^4 + a\right)^{5/4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b*x^4)^(5/4),x)
```

```
[Out] int(x^6*(a + b*x^4)^(5/4), x)
```

sympy [C] time = 5.83, size = 39, normalized size = 0.31

$$\frac{a^{\frac{5}{4}} x^7 \Gamma\left(\frac{7}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{4}, \frac{7}{4} \\ \frac{11}{4} \end{matrix} \middle| \frac{b x^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**4+a)**(5/4),x)
```

```
[Out] a**(5/4)*x**7*gamma(7/4)*hyper((-5/4, 7/4), (11/4,), b*x**4*exp_polar(I*pi)
/a)/(4*gamma(11/4))
```

3.1064 $\int x^2 \left(a+bx^4\right)^{5/4}\,dx$

Optimal. Leaf size=100

$$-\frac{5a^2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{3/4}}+\frac{5a^2\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{3/4}}+\frac{1}{8}x^3\left(a+bx^4\right)^{5/4}+\frac{5}{32}ax^3\sqrt[4]{a+bx^4}$$

```
[Out] 5/32*a*x^3*(b*x^4+a)^(1/4)+1/8*x^3*(b*x^4+a)^(5/4)-5/64*a^2*arctan(b^(1/4)*
x/(b*x^4+a)^(1/4))/b^(3/4)+5/64*a^2*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(3
/4)
```

Rubi [A] time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {279, 331, 298, 203, 206}

$$-\frac{5a^2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{3/4}}+\frac{5a^2\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{3/4}}+\frac{1}{8}x^3\left(a+bx^4\right)^{5/4}+\frac{5}{32}ax^3\sqrt[4]{a+bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^4)^(5/4),x]
```

```
[Out] (5*a*x^3*(a + b*x^4)^(1/4))/32 + (x^3*(a + b*x^4)^(5/4))/8 - (5*a^2*ArcTan[
(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(64*b^(3/4)) + (5*a^2*ArcTanh[(b^(1/4)*x)/(
a + b*x^4)^(1/4)])/(64*b^(3/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
```

```
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^4\right)^{5/4}\,dx &= \frac{1}{8}x^3\left(a+bx^4\right)^{5/4}+\frac{1}{8}(5a)\int x^2\sqrt[4]{a+bx^4}\,dx\\ &= \frac{5}{32}ax^3\sqrt[4]{a+bx^4}+\frac{1}{8}x^3\left(a+bx^4\right)^{5/4}+\frac{1}{32}\left(5a^2\right)\int\frac{x^2}{\left(a+bx^4\right)^{3/4}}\,dx\\ &= \frac{5}{32}ax^3\sqrt[4]{a+bx^4}+\frac{1}{8}x^3\left(a+bx^4\right)^{5/4}+\frac{1}{32}\left(5a^2\right)\text{Subst}\left(\int\frac{x^2}{1-bx^4}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)\\ &= \frac{5}{32}ax^3\sqrt[4]{a+bx^4}+\frac{1}{8}x^3\left(a+bx^4\right)^{5/4}+\frac{\left(5a^2\right)\text{Subst}\left(\int\frac{1}{1-\sqrt{b}x^2}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{64\sqrt{b}}-\frac{\left(5a^2\right)\text{Su}}{}\\ &= \frac{5}{32}ax^3\sqrt[4]{a+bx^4}+\frac{1}{8}x^3\left(a+bx^4\right)^{5/4}-\frac{5a^2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{3/4}}+\frac{5a^2\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{3/4}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.52

$$\frac{ax^3\sqrt[4]{a+bx^4}\;{}_2F_1\left(-\frac{5}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^4}{a}\right)}{3\sqrt[4]{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^4)^(5/4),x]
```

```
[Out] (a*x^3*(a + b*x^4)^(1/4)*Hypergeometric2F1[-5/4, 3/4, 7/4, -((b*x^4)/a)])/(
3*(1 + (b*x^4)/a)^(1/4))
```

fricas [B] time = 0.83, size = 209, normalized size = 2.09

$$\frac{1}{32}\left(4\,bx^7+9\,ax^3\right)\left(bx^4+a\right)^{\frac{1}{4}}-\frac{5}{32}\left(\frac{a^8}{b^3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(\frac{a^8}{b^3}\right)^{\frac{3}{4}}\left(bx^4+a\right)^{\frac{1}{4}}a^2b^2-\left(\frac{a^8}{b^3}\right)^{\frac{3}{4}}b^2x\sqrt{\frac{\sqrt{bx^4+a}\,a^4+\sqrt{\frac{a^8}{b^3}}\,b^2x^2}{x^2}}}{a^8x}\right)+\frac{5}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] 1/32*(4*b*x^7 + 9*a*x^3)*(b*x^4 + a)^(1/4) - 5/32*(a^8/b^3)^(1/4)*arctan(-(
(a^8/b^3)^(3/4)*(b*x^4 + a)^(1/4)*a^2*b^2 - (a^8/b^3)^(3/4)*b^2*x*sqrt((sqr
t(b*x^4 + a)*a^4 + sqrt(a^8/b^3)*b^2*x^2)/x^2))/(a^8*x)) + 5/128*(a^8/b^3)^
(1/4)*log(5*((b*x^4 + a)^(1/4)*a^2 + (a^8/b^3)^(1/4)*b*x)/x) - 5/128*(a^8/b
^3)^(1/4)*log(5*((b*x^4 + a)^(1/4)*a^2 - (a^8/b^3)^(1/4)*b*x)/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx^4+a\right)^{\frac{5}{4}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^4+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(5/4)*x^2, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \left(b\,x^4+a\right)^{\frac{5}{4}} x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^4+a)^(5/4),x)
```

```
[Out] int(x^2*(b*x^4+a)^(5/4),x)
```

maxima [A] time = 2.96, size = 144, normalized size = 1.44

$$\frac{5\,a^2\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{64\,b^{\frac{3}{4}}}-\frac{5\,a^2\log\left(-\frac{b^{\frac{1}{4}}-\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}\right)}{128\,b^{\frac{3}{4}}}-\frac{\frac{5\left(bx^4+a\right)^{\frac{1}{4}}a^2b}{x}-\frac{9\left(bx^4+a\right)^{\frac{5}{4}}a^2}{x^5}}{32\left(b^2-\frac{2\left(bx^4+a\right)b}{x^4}+\frac{\left(bx^4+a\right)^2}{x^8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^4+a)^(5/4),x, algorithm="maxima")
```

```
[Out] 5/64*a^2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(3/4) - 5/128*a^2*log(-(b^
(1/4) - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(3/4) - 1/3
2*(5*(b*x^4 + a)^(1/4)*a^2*b/x - 9*(b*x^4 + a)^(5/4)*a^2/x^5)/(b^2 - 2*(b*x
^4 + a)*b/x^4 + (b*x^4 + a)^2/x^8)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(b\,x^4+a\right)^{5/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^4)^(5/4),x)
```

```
[Out] int(x^2*(a + b*x^4)^(5/4), x)
```

sympy [C] time = 1.94, size = 39, normalized size = 0.39

$$\frac{a^{\frac{5}{4}}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{5}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**4+a)**(5/4),x)
```

```
[Out] a**(5/4)*x**3*gamma(3/4)*hyper((-5/4, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/
a)/(4*gamma(7/4))
```

3.1065 $\int \frac{\left(a+bx^4\right)^{5/4}}{x^2}\,dx$

Optimal. Leaf size=94

$$-\frac{\left(a+bx^4\right)^{5/4}}{x}-\frac{5}{8}a\sqrt[4]{b}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)+\frac{5}{8}a\sqrt[4]{b}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)+\frac{5}{4}bx^3\sqrt[4]{a+bx^4}$$

```
[Out] 5/4*b*x^3*(b*x^4+a)^(1/4)-(b*x^4+a)^(5/4)/x-5/8*a*b^(1/4)*arctan(b^(1/4)*x/
(b*x^4+a)^(1/4))+5/8*a*b^(1/4)*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))
```

Rubi [A] time = 0.04, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {277, 279, 331, 298, 203, 206}

$$\frac{5}{4}bx^3\sqrt[4]{a+bx^4}-\frac{\left(a+bx^4\right)^{5/4}}{x}-\frac{5}{8}a\sqrt[4]{b}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)+\frac{5}{8}a\sqrt[4]{b}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(5/4)/x^2,x]
```

```
[Out] (5*b*x^3*(a + b*x^4)^(1/4))/4 - (a + b*x^4)^(5/4)/x - (5*a*b^(1/4)*ArcTan[(
b^(1/4)*x)/(a + b*x^4)^(1/4)])/8 + (5*a*b^(1/4)*ArcTanh[(b^(1/4)*x)/(a + b*
x^4)^(1/4)])/8
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
```

```
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^4\right)^{5/4}}{x^2}\,dx &= -\frac{\left(a+bx^4\right)^{5/4}}{x} + (5b)\int x^2\sqrt[4]{a+bx^4}\,dx \\
&= \frac{5}{4}bx^3\sqrt[4]{a+bx^4} - \frac{\left(a+bx^4\right)^{5/4}}{x} + \frac{1}{4}(5ab)\int \frac{x^2}{\left(a+bx^4\right)^{3/4}}\,dx \\
&= \frac{5}{4}bx^3\sqrt[4]{a+bx^4} - \frac{\left(a+bx^4\right)^{5/4}}{x} + \frac{1}{4}(5ab)\,\text{Subst}\left(\int \frac{x^2}{1-bx^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right) \\
&= \frac{5}{4}bx^3\sqrt[4]{a+bx^4} - \frac{\left(a+bx^4\right)^{5/4}}{x} + \frac{1}{8}\left(5a\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{b}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right) - \frac{1}{8}\left(5a\sqrt{b}\right) \\
&= \frac{5}{4}bx^3\sqrt[4]{a+bx^4} - \frac{\left(a+bx^4\right)^{5/4}}{x} - \frac{5}{8}a\sqrt[4]{b}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right) + \frac{5}{8}a\sqrt[4]{b}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 50, normalized size = 0.53

$$-\frac{a\sqrt[4]{a+bx^4}\;{}_2F_1\left(-\frac{5}{4},-\frac{1}{4};\frac{3}{4};-\frac{bx^4}{a}\right)}{x\sqrt[4]{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(5/4)/x^2,x]
```

```
[Out] -((a*(a + b*x^4)^(1/4)*Hypergeometric2F1[-5/4, -1/4, 3/4, -((b*x^4)/a)])/(x
*(1 + (b*x^4)/a)^(1/4)))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{5}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(5/4)/x^2, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{5}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(5/4)/x^2,x)
```

```
[Out] int((b*x^4+a)^(5/4)/x^2,x)
```

maxima [A] time = 3.02, size = 119, normalized size = 1.27

$$\frac{5}{16}\left(2\,b^{\frac{1}{4}}\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)-b^{\frac{1}{4}}\log\left(-\frac{b^{\frac{1}{4}}-\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}\right)\right)a-\frac{\left(bx^4+a\right)^{\frac{1}{4}}a}{x}-\frac{\left(bx^4+a\right)^{\frac{1}{4}}ab}{4\left(b-\frac{bx^4+a}{x^4}\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^2,x, algorithm="maxima")
```

```
[Out] 5/16*(2*b^(1/4)*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x)) - b^(1/4)*log(-(b^(1/
4) - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x)))*a - (b*x^4 + a)
^(1/4)*a/x - 1/4*(b*x^4 + a)^(1/4)*a*b/((b - (b*x^4 + a)/x^4)*x)
```

mupad [B] time = 1.53, size = 40, normalized size = 0.43

$$-\frac{\left(b\,x^4+a\right)^{5/4}\,{}_2\mathrm{F}_1\left(-\frac{5}{4},-\frac{1}{4};\,\frac{3}{4};\,-\frac{b\,x^4}{a}\right)}{x\left(\frac{b\,x^4}{a}+1\right)^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(5/4)/x^2,x)
```

```
[Out] -((a + b*x^4)^(5/4)*hypergeom([-5/4, -1/4], 3/4, -(b*x^4)/a))/(x*((b*x^4)/a
 + 1)^(5/4))
```

sympy [C] time = 3.85, size = 41, normalized size = 0.44

$$\frac{a^{\frac{5}{4}}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\,\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(5/4)/x**2,x)
```

```
[Out] a**(5/4)*gamma(-1/4)*hyper((-5/4, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/
(4*x*gamma(3/4))
```

3.1066 $\int \frac{\left(a+bx^4\right)^{5/4}}{x^6}\,dx$

Optimal. Leaf size=92

$$-\frac{1}{2}b^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)+\frac{1}{2}b^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)-\frac{b\sqrt[4]{a+bx^4}}{x}-\frac{\left(a+bx^4\right)^{5/4}}{5x^5}$$

```
[Out] -b*(b*x^4+a)^(1/4)/x-1/5*(b*x^4+a)^(5/4)/x^5-1/2*b^(5/4)*arctan(b^(1/4)*x/(
b*x^4+a)^(1/4))+1/2*b^(5/4)*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))
```

Rubi [A] time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {277, 331, 298, 203, 206}

$$-\frac{1}{2}b^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)+\frac{1}{2}b^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)-\frac{b\sqrt[4]{a+bx^4}}{x}-\frac{\left(a+bx^4\right)^{5/4}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(5/4)/x^6,x]
```

```
[Out] -((b*(a + b*x^4)^(1/4))/x) - (a + b*x^4)^(5/4)/(5*x^5) - (b^(5/4)*ArcTan[(b
^(1/4)*x)/(a + b*x^4)^(1/4)])/2 + (b^(5/4)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^
(1/4)])/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
```

```
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^4\right)^{5/4}}{x^6}\,dx &= -\frac{\left(a+bx^4\right)^{5/4}}{5x^5} + b\int \frac{\sqrt[4]{a+bx^4}}{x^2}\,dx \\
&= -\frac{b\sqrt[4]{a+bx^4}}{x} - \frac{\left(a+bx^4\right)^{5/4}}{5x^5} + b^2\int \frac{x^2}{\left(a+bx^4\right)^{3/4}}\,dx \\
&= -\frac{b\sqrt[4]{a+bx^4}}{x} - \frac{\left(a+bx^4\right)^{5/4}}{5x^5} + b^2\,\text{Subst}\left(\int \frac{x^2}{1-bx^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right) \\
&= -\frac{b\sqrt[4]{a+bx^4}}{x} - \frac{\left(a+bx^4\right)^{5/4}}{5x^5} + \frac{1}{2}b^{3/2}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{b}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right) - \frac{1}{2}b^{3/2}\,\text{Subst} \\
&= -\frac{b\sqrt[4]{a+bx^4}}{x} - \frac{\left(a+bx^4\right)^{5/4}}{5x^5} - \frac{1}{2}b^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right) + \frac{1}{2}b^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.57

$$-\frac{a\sqrt[4]{a+bx^4}\;{}_2F_1\left(-\frac{5}{4},-\frac{5}{4};-\frac{1}{4};-\frac{bx^4}{a}\right)}{5x^5\sqrt[4]{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(5/4)/x^6,x]
```

```
[Out] -1/5*(a*(a + b*x^4)^(1/4)*Hypergeometric2F1[-5/4, -5/4, -1/4, -((b*x^4)/a)]
)/(x^5*(1 + (b*x^4)/a)^(1/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^6,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{5}{4}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(5/4)/x^6, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{5}{4}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(5/4)/x^6,x)
```

```
[Out] int((b*x^4+a)^(5/4)/x^6,x)
```

maxima [A] time = 2.95, size = 97, normalized size = 1.05

$$\frac{1}{2}\,b^{\frac{5}{4}}\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)-\frac{1}{4}\,b^{\frac{5}{4}}\log\left(-\frac{b^{\frac{1}{4}}-\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}\right)-\frac{\left(bx^4+a\right)^{\frac{1}{4}}b}{x}-\frac{\left(bx^4+a\right)^{\frac{5}{4}}}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^6,x, algorithm="maxima")
```

```
[Out] 1/2*b^(5/4)*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x)) - 1/4*b^(5/4)*log(-(b^(1/
4) - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x)) - (b*x^4 + a)^(1
/4)*b/x - 1/5*(b*x^4 + a)^(5/4)/x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^4+a\right)^{5/4}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(5/4)/x^6,x)
```

```
[Out] int((a + b*x^4)^(5/4)/x^6, x)
```

sympy [C] time = 4.83, size = 46, normalized size = 0.50

$$\frac{a^{\frac{5}{4}}\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{5}{4},-\frac{5}{4}\\-\frac{1}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(5/4)/x**6,x)
```

```
[Out] a**(5/4)*gamma(-5/4)*hyper((-5/4, -5/4), (-1/4,), b*x**4*exp_polar(I*pi)/a)
/(4*x**5*gamma(-1/4))
```

3.1067 $\int \frac{\left(a+bx^4\right)^{5/4}}{x^{10}}\,dx$

Optimal. Leaf size=21

$$-\frac{\left(a+bx^4\right)^{9/4}}{9ax^9}$$

```
[Out] -1/9*(b*x^4+a)^(9/4)/a/x^9
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {264}

$$-\frac{\left(a+bx^4\right)^{9/4}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(5/4)/x^10,x]
```

```
[Out] -(a + b*x^4)^(9/4)/(9*a*x^9)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^4\right)^{5/4}}{x^{10}}\,dx = -\frac{\left(a+bx^4\right)^{9/4}}{9ax^9}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$-\frac{\left(a+bx^4\right)^{9/4}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(5/4)/x^10,x]
```

```
[Out] -1/9*(a + b*x^4)^(9/4)/(a*x^9)
```

fricas [B] time = 1.20, size = 35, normalized size = 1.67

$$-\frac{\left(b^2x^8+2\,abx^4+a^2\right)\left(bx^4+a\right)^{\frac{1}{4}}}{9\,ax^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^10,x, algorithm="fricas")
```

```
[Out] -1/9*(b^2*x^8 + 2*a*b*x^4 + a^2)*(b*x^4 + a)^(1/4)/(a*x^9)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{5}{4}}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^10,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(5/4)/x^10, x)
```

maple [A] time = 0.01, size = 18, normalized size = 0.86

$$-\frac{\left(b\,x^4+a\right)^{\frac{9}{4}}}{9a\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(5/4)/x^10,x)
[Out] -1/9*(b*x^4+a)^(9/4)/a/x^9
```

maxima [A] time = 1.38, size = 17, normalized size = 0.81

$$-\frac{\left(bx^4+a\right)^{\frac{9}{4}}}{9\,ax^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^10,x, algorithm="maxima")
[Out] -1/9*(b*x^4 + a)^(9/4)/(a*x^9)
```

mupad [B] time = 1.46, size = 17, normalized size = 0.81

$$-\frac{\left(b\,x^4+a\right)^{9/4}}{9\,a\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(5/4)/x^10,x)
[Out] -(a + b*x^4)^(9/4)/(9*a*x^9)
```

sympy [B] time = 3.35, size = 105, normalized size = 5.00

$$\frac{a\sqrt[4]{b}\,\sqrt[4]{\frac{a}{bx^4}+1}\,\Gamma\left(-\frac{9}{4}\right)}{4x^8\Gamma\left(-\frac{5}{4}\right)}+\frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx^4}+1}\,\Gamma\left(-\frac{9}{4}\right)}{2x^4\Gamma\left(-\frac{5}{4}\right)}+\frac{b^{\frac{9}{4}}\sqrt[4]{\frac{a}{bx^4}+1}\,\Gamma\left(-\frac{9}{4}\right)}{4a\Gamma\left(-\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(5/4)/x**10,x)
[Out] a*b**(1/4)*(a/(b*x**4) + 1)**(1/4)*gamma(-9/4)/(4*x**8*gamma(-5/4)) + b**(5
/4)*(a/(b*x**4) + 1)**(1/4)*gamma(-9/4)/(2*x**4*gamma(-5/4)) + b**(9/4)*(a/
(b*x**4) + 1)**(1/4)*gamma(-9/4)/(4*a*gamma(-5/4))
```

3.1068 $\int \frac{\left(a+bx^4\right)^{5/4}}{x^{14}}\,dx$

Optimal. Leaf size=44

$$\frac{4b\left(a+bx^4\right)^{9/4}}{117a^2x^9}-\frac{\left(a+bx^4\right)^{9/4}}{13ax^{13}}$$

```
[Out] -1/13*(b*x^4+a)^(9/4)/a/x^13+4/117*b*(b*x^4+a)^(9/4)/a^2/x^9
```

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$\frac{4b\left(a+bx^4\right)^{9/4}}{117a^2x^9}-\frac{\left(a+bx^4\right)^{9/4}}{13ax^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(5/4)/x^14,x]
```

```
[Out] -(a + b*x^4)^(9/4)/(13*a*x^13) + (4*b*(a + b*x^4)^(9/4))/(117*a^2*x^9)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^4\right)^{5/4}}{x^{14}}\,dx &= -\frac{\left(a+bx^4\right)^{9/4}}{13ax^{13}}-\frac{(4b)\int \frac{\left(a+bx^4\right)^{5/4}}{x^{10}}\,dx}{13a}\\ &= -\frac{\left(a+bx^4\right)^{9/4}}{13ax^{13}}+\frac{4b\left(a+bx^4\right)^{9/4}}{117a^2x^9}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.70

$$\frac{\left(a+bx^4\right)^{9/4}\left(4bx^4-9a\right)}{117a^2x^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(5/4)/x^14,x]
```

```
[Out] ((a + b*x^4)^(9/4)*(-9*a + 4*b*x^4))/(117*a^2*x^13)
```

fricas [A] time = 0.88, size = 49, normalized size = 1.11

$$\frac{\left(4\,b^3x^{12}-ab^2x^8-14\,a^2bx^4-9\,a^3\right)\left(bx^4+a\right)^{\frac{1}{4}}}{117\,a^2x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^14,x, algorithm="fricas")
```

```
[Out] 1/117*(4*b^3*x^12 - a*b^2*x^8 - 14*a^2*b*x^4 - 9*a^3)*(b*x^4 + a)^(1/4)/(a^
2*x^13)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{5}{4}}}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^14,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(5/4)/x^14, x)
```

maple [A] time = 0.01, size = 28, normalized size = 0.64

$$-\frac{\left(b\,x^4+a\right)^{\frac{9}{4}}\left(-4b\,x^4+9a\right)}{117a^2x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(5/4)/x^14,x)
```

```
[Out] -1/117*(b*x^4+a)^(9/4)*(-4*b*x^4+9*a)/x^13/a^2
```

maxima [A] time = 1.31, size = 35, normalized size = 0.80

$$\frac{\frac{13\left(bx^4+a\right)^{\frac{9}{4}}b}{x^9}-\frac{9\left(bx^4+a\right)^{\frac{13}{4}}}{x^{13}}}{117\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^14,x, algorithm="maxima")
```

```
[Out] 1/117*(13*(b*x^4 + a)^(9/4)*b/x^9 - 9*(b*x^4 + a)^(13/4)/x^13)/a^2
```

mupad [B] time = 1.74, size = 71, normalized size = 1.61

$$\frac{4\,b^3\left(b\,x^4+a\right)^{1/4}}{117\,a^2\,x}-\frac{14\,b\left(b\,x^4+a\right)^{1/4}}{117\,x^9}-\frac{a\left(b\,x^4+a\right)^{1/4}}{13\,x^{13}}-\frac{b^2\left(b\,x^4+a\right)^{1/4}}{117\,a\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(5/4)/x^14,x)
```

```
[Out] (4*b^3*(a + b*x^4)^(1/4))/(117*a^2*x) - (14*b*(a + b*x^4)^(1/4))/(117*x^9)
- (a*(a + b*x^4)^(1/4))/(13*x^13) - (b^2*(a + b*x^4)^(1/4))/(117*a*x^5)
```

sympy [B] time = 5.33, size = 148, normalized size = 3.36

$$-\frac{9a\sqrt[4]{b}\,\sqrt[4]{\frac{a}{bx^4}+1}\,\Gamma\left(-\frac{13}{4}\right)}{16x^{12}\Gamma\left(-\frac{5}{4}\right)}-\frac{7b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx^4}+1}\,\Gamma\left(-\frac{13}{4}\right)}{8x^8\Gamma\left(-\frac{5}{4}\right)}-\frac{b^{\frac{9}{4}}\sqrt[4]{\frac{a}{bx^4}+1}\,\Gamma\left(-\frac{13}{4}\right)}{16ax^4\Gamma\left(-\frac{5}{4}\right)}+\frac{b^{\frac{13}{4}}\sqrt[4]{\frac{a}{bx^4}+1}\,\Gamma\left(-\frac{13}{4}\right)}{4a^2\Gamma\left(-\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(5/4)/x**14,x)
```

```
[Out] -9*a*b**(1/4)*(a/(b*x**4) + 1)**(1/4)*gamma(-13/4)/(16*x**12*gamma(-5/4)) -
 7*b**(5/4)*(a/(b*x**4) + 1)**(1/4)*gamma(-13/4)/(8*x**8*gamma(-5/4)) - b**
(9/4)*(a/(b*x**4) + 1)**(1/4)*gamma(-13/4)/(16*a*x**4*gamma(-5/4)) + b**(13
/4)*(a/(b*x**4) + 1)**(1/4)*gamma(-13/4)/(4*a**2*gamma(-5/4))
```

3.1069 $\int \frac{\left(a+bx^4\right)^{5/4}}{x^{18}}\,dx$

Optimal. Leaf size=68

$$-\frac{32b^2\left(a+bx^4\right)^{9/4}}{1989a^3x^9}+\frac{8b\left(a+bx^4\right)^{9/4}}{221a^2x^{13}}-\frac{\left(a+bx^4\right)^{9/4}}{17ax^{17}}$$

```
[Out] -1/17*(b*x^4+a)^(9/4)/a/x^17+8/221*b*(b*x^4+a)^(9/4)/a^2/x^13-32/1989*b^2*(
b*x^4+a)^(9/4)/a^3/x^9
```

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$-\frac{32b^2\left(a+bx^4\right)^{9/4}}{1989a^3x^9}+\frac{8b\left(a+bx^4\right)^{9/4}}{221a^2x^{13}}-\frac{\left(a+bx^4\right)^{9/4}}{17ax^{17}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(5/4)/x^18,x]
```

```
[Out] -(a + b*x^4)^(9/4)/(17*a*x^17) + (8*b*(a + b*x^4)^(9/4))/(221*a^2*x^13) - (
32*b^2*(a + b*x^4)^(9/4))/(1989*a^3*x^9)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^4\right)^{5/4}}{x^{18}}\,dx &= -\frac{\left(a+bx^4\right)^{9/4}}{17ax^{17}}-\frac{(8b)\int \frac{\left(a+bx^4\right)^{5/4}}{x^{14}}\,dx}{17a}\\ &= -\frac{\left(a+bx^4\right)^{9/4}}{17ax^{17}}+\frac{8b\left(a+bx^4\right)^{9/4}}{221a^2x^{13}}+\frac{\left(32b^2\right)\int \frac{\left(a+bx^4\right)^{5/4}}{x^{10}}\,dx}{221a^2}\\ &= -\frac{\left(a+bx^4\right)^{9/4}}{17ax^{17}}+\frac{8b\left(a+bx^4\right)^{9/4}}{221a^2x^{13}}-\frac{32b^2\left(a+bx^4\right)^{9/4}}{1989a^3x^9}\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.62

$$-\frac{\left(a+bx^4\right)^{9/4}\left(117a^2-72abx^4+32b^2x^8\right)}{1989a^3x^{17}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(5/4)/x^18,x]
```

```
[Out] -1/1989*((a + b*x^4)^(9/4)*(117*a^2 - 72*a*b*x^4 + 32*b^2*x^8))/(a^3*x^17)
```

fricas [A] time = 0.80, size = 60, normalized size = 0.88

$$-\frac{\left(32\,b^4x^{16}-8\,ab^3x^{12}+5\,a^2b^2x^8+162\,a^3bx^4+117\,a^4\right)\left(bx^4+a\right)^{\frac{1}{4}}}{1989\,a^3x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^18,x, algorithm="fricas")
```

```
[Out] -1/1989*(32*b^4*x^16 - 8*a*b^3*x^12 + 5*a^2*b^2*x^8 + 162*a^3*b*x^4 + 117*a
^4)*(b*x^4 + a)^(1/4)/(a^3*x^17)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{5}{4}}}{x^{18}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^18,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(5/4)/x^18, x)
```

maple [A] time = 0.01, size = 39, normalized size = 0.57

$$-\frac{\left(b\,x^4+a\right)^{\frac{9}{4}}\left(32b^2x^8-72ab\,x^4+117a^2\right)}{1989a^3x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(5/4)/x^18,x)
```

```
[Out] -1/1989*(b*x^4+a)^(9/4)*(32*b^2*x^8-72*a*b*x^4+117*a^2)/x^17/a^3
```

maxima [A] time = 1.33, size = 52, normalized size = 0.76

$$-\frac{\frac{221\left(bx^4+a\right)^{\frac{9}{4}}b^2}{x^9}-\frac{306\left(bx^4+a\right)^{\frac{13}{4}}b}{x^{13}}+\frac{117\left(bx^4+a\right)^{\frac{17}{4}}}{x^{17}}}{1989\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^18,x, algorithm="maxima")
```

```
[Out] -1/1989*(221*(b*x^4 + a)^(9/4)*b^2/x^9 - 306*(b*x^4 + a)^(13/4)*b/x^13 + 11
7*(b*x^4 + a)^(17/4)/x^17)/a^3
```

mupad [B] time = 2.07, size = 91, normalized size = 1.34

$$\frac{8\,b^3\left(b\,x^4+a\right)^{1/4}}{1989\,a^2\,x^5}-\frac{18\,b\left(b\,x^4+a\right)^{1/4}}{221\,x^{13}}-\frac{32\,b^4\left(b\,x^4+a\right)^{1/4}}{1989\,a^3\,x}-\frac{a\left(b\,x^4+a\right)^{1/4}}{17\,x^{17}}-\frac{5\,b^2\left(b\,x^4+a\right)^{1/4}}{1989\,a\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(5/4)/x^18,x)
```

```
[Out] (8*b^3*(a + b*x^4)^(1/4))/(1989*a^2*x^5) - (18*b*(a + b*x^4)^(1/4))/(221*x^
13) - (32*b^4*(a + b*x^4)^(1/4))/(1989*a^3*x) - (a*(a + b*x^4)^(1/4))/(17*x
^17) - (5*b^2*(a + b*x^4)^(1/4))/(1989*a*x^9)
```

sympy [**B**] time = 11.92, size = 609, normalized size = 8.96

$$\frac{117a^6b^{\frac{17}{4}}\sqrt[4]{\frac{a}{bx^4}+1}\,\Gamma\left(-\frac{17}{4}\right)}{64a^5b^4x^{16}\Gamma\left(-\frac{5}{4}\right)+128a^4b^5x^{20}\Gamma\left(-\frac{5}{4}\right)+64a^3b^6x^{24}\Gamma\left(-\frac{5}{4}\right)}+\frac{396a^5b^{\frac{21}{4}}x^4\sqrt[4]{\frac{a}{bx^4}+1}\,\Gamma\left(-\frac{17}{4}\right)}{64a^5b^4x^{16}\Gamma\left(-\frac{5}{4}\right)+128a^4b^5x^{20}\Gamma\left(-\frac{5}{4}\right)+64a^3b^6x^{24}\Gamma\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(5/4)/x**18,x)
```

```
[Out] 117*a**6*b**(17/4)*(a/(b*x**4) + 1)**(1/4)*gamma(-17/4)/(64*a**5*b**4*x**16
*gamma(-5/4) + 128*a**4*b**5*x**20*gamma(-5/4) + 64*a**3*b**6*x**24*gamma(-
5/4)) + 396*a**5*b**(21/4)*x**4*(a/(b*x**4) + 1)**(1/4)*gamma(-17/4)/(64*a*
*5*b**4*x**16*gamma(-5/4) + 128*a**4*b**5*x**20*gamma(-5/4) + 64*a**3*b**6*
x**24*gamma(-5/4)) + 446*a**4*b**(25/4)*x**8*(a/(b*x**4) + 1)**(1/4)*gamma(
-17/4)/(64*a**5*b**4*x**16*gamma(-5/4) + 128*a**4*b**5*x**20*gamma(-5/4) +
64*a**3*b**6*x**24*gamma(-5/4)) + 164*a**3*b**(29/4)*x**12*(a/(b*x**4) + 1)
**(1/4)*gamma(-17/4)/(64*a**5*b**4*x**16*gamma(-5/4) + 128*a**4*b**5*x**20*
gamma(-5/4) + 64*a**3*b**6*x**24*gamma(-5/4)) + 21*a**2*b**(33/4)*x**16*(a/
(b*x**4) + 1)**(1/4)*gamma(-17/4)/(64*a**5*b**4*x**16*gamma(-5/4) + 128*a**
4*b**5*x**20*gamma(-5/4) + 64*a**3*b**6*x**24*gamma(-5/4)) + 56*a*b**(37/4)
*x**20*(a/(b*x**4) + 1)**(1/4)*gamma(-17/4)/(64*a**5*b**4*x**16*gamma(-5/4)
 + 128*a**4*b**5*x**20*gamma(-5/4) + 64*a**3*b**6*x**24*gamma(-5/4)) + 32*b
**(41/4)*x**24*(a/(b*x**4) + 1)**(1/4)*gamma(-17/4)/(64*a**5*b**4*x**16*gam
ma(-5/4) + 128*a**4*b**5*x**20*gamma(-5/4) + 64*a**3*b**6*x**24*gamma(-5/4)
)
```

3.1070 $\int \frac{\left(a+bx^4\right)^{5/4}}{x^{22}}\,dx$

Optimal. Leaf size=92

$$\frac{128b^3\left(a+bx^4\right)^{9/4}}{13923a^4x^9}-\frac{32b^2\left(a+bx^4\right)^{9/4}}{1547a^3x^{13}}+\frac{4b\left(a+bx^4\right)^{9/4}}{119a^2x^{17}}-\frac{\left(a+bx^4\right)^{9/4}}{21ax^{21}}$$

```
[Out] -1/21*(b*x^4+a)^(9/4)/a/x^21+4/119*b*(b*x^4+a)^(9/4)/a^2/x^17-32/1547*b^2*(
b*x^4+a)^(9/4)/a^3/x^13+128/13923*b^3*(b*x^4+a)^(9/4)/a^4/x^9
```

Rubi [A] time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$\frac{128b^3\left(a+bx^4\right)^{9/4}}{13923a^4x^9}-\frac{32b^2\left(a+bx^4\right)^{9/4}}{1547a^3x^{13}}+\frac{4b\left(a+bx^4\right)^{9/4}}{119a^2x^{17}}-\frac{\left(a+bx^4\right)^{9/4}}{21ax^{21}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(5/4)/x^22,x]
```

```
[Out] -(a + b*x^4)^(9/4)/(21*a*x^21) + (4*b*(a + b*x^4)^(9/4))/(119*a^2*x^17) - (
32*b^2*(a + b*x^4)^(9/4))/(1547*a^3*x^13) + (128*b^3*(a + b*x^4)^(9/4))/(13
923*a^4*x^9)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^4\right)^{5/4}}{x^{22}}\,dx &= -\frac{\left(a+bx^4\right)^{9/4}}{21ax^{21}}-\frac{(4b)\int \frac{\left(a+bx^4\right)^{5/4}}{x^{18}}\,dx}{7a}\\
&= -\frac{\left(a+bx^4\right)^{9/4}}{21ax^{21}}+\frac{4b\left(a+bx^4\right)^{9/4}}{119a^2x^{17}}+\frac{\left(32b^2\right)\int \frac{\left(a+bx^4\right)^{5/4}}{x^{14}}\,dx}{119a^2}\\
&= -\frac{\left(a+bx^4\right)^{9/4}}{21ax^{21}}+\frac{4b\left(a+bx^4\right)^{9/4}}{119a^2x^{17}}-\frac{32b^2\left(a+bx^4\right)^{9/4}}{1547a^3x^{13}}-\frac{\left(128b^3\right)\int \frac{\left(a+bx^4\right)^{5/4}}{x^{10}}\,dx}{1547a^3}\\
&= -\frac{\left(a+bx^4\right)^{9/4}}{21ax^{21}}+\frac{4b\left(a+bx^4\right)^{9/4}}{119a^2x^{17}}-\frac{32b^2\left(a+bx^4\right)^{9/4}}{1547a^3x^{13}}+\frac{128b^3\left(a+bx^4\right)^{9/4}}{13923a^4x^9}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 53, normalized size = 0.58

$$\frac{\left(a+bx^4\right)^{9/4}\left(-663a^3+468a^2bx^4-288ab^2x^8+128b^3x^{12}\right)}{13923a^4x^{21}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(5/4)/x^22,x]
```

```
[Out] ((a + b*x^4)^(9/4)*(-663*a^3 + 468*a^2*b*x^4 - 288*a*b^2*x^8 + 128*b^3*x^12
))/(13923*a^4*x^21)
```

fricas [A] time = 0.82, size = 71, normalized size = 0.77

$$\frac{\left(128\,b^5x^{20} - 32\,ab^4x^{16} + 20\,a^2b^3x^{12} - 15\,a^3b^2x^8 - 858\,a^4bx^4 - 663\,a^5\right)\left(bx^4 + a\right)^{\frac{1}{4}}}{13923\,a^4x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^22,x, algorithm="fricas")
```

```
[Out] 1/13923*(128*b^5*x^20 - 32*a*b^4*x^16 + 20*a^2*b^3*x^12 - 15*a^3*b^2*x^8 -
858*a^4*b*x^4 - 663*a^5)*(b*x^4 + a)^(1/4)/(a^4*x^21)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4 + a\right)^{\frac{5}{4}}}{x^{22}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^22,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(5/4)/x^22, x)
```

maple [A] time = 0.01, size = 50, normalized size = 0.54

$$-\frac{\left(b\,x^4 + a\right)^{\frac{9}{4}}\left(-128b^3x^{12} + 288a\,b^2x^8 - 468a^2b\,x^4 + 663a^3\right)}{13923a^4x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(5/4)/x^22,x)
```

```
[Out] -1/13923*(b*x^4+a)^(9/4)*(-128*b^3*x^12+288*a*b^2*x^8-468*a^2*b*x^4+663*a^3
)/x^21/a^4
```

maxima [A] time = 1.37, size = 69, normalized size = 0.75

$$\frac{\frac{1547\left(bx^4+a\right)^{\frac{9}{4}}b^3}{x^9} - \frac{3213\left(bx^4+a\right)^{\frac{13}{4}}b^2}{x^{13}} + \frac{2457\left(bx^4+a\right)^{\frac{17}{4}}b}{x^{17}} - \frac{663\left(bx^4+a\right)^{\frac{21}{4}}}{x^{21}}}{13923\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^22,x, algorithm="maxima")
```

```
[Out] 1/13923*(1547*(b*x^4 + a)^(9/4)*b^3/x^9 - 3213*(b*x^4 + a)^(13/4)*b^2/x^13
+ 2457*(b*x^4 + a)^(17/4)*b/x^17 - 663*(b*x^4 + a)^(21/4)/x^21)/a^4
```

mupad [B] time = 2.52, size = 111, normalized size = 1.21

$$\frac{128\,b^5\left(b\,x^4 + a\right)^{1/4}}{13923\,a^4\,x} - \frac{22\,b\left(b\,x^4 + a\right)^{1/4}}{357\,x^{17}} - \frac{a\left(b\,x^4 + a\right)^{1/4}}{21\,x^{21}} - \frac{32\,b^4\left(b\,x^4 + a\right)^{1/4}}{13923\,a^3\,x^5} + \frac{20\,b^3\left(b\,x^4 + a\right)^{1/4}}{13923\,a^2\,x^9} - \frac{5\,b^2\left(b\,x^4 + a\right)^{1/4}}{4641\,a\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(5/4)/x^22,x)
```

```
[Out] (128*b^5*(a + b*x^4)^(1/4))/(13923*a^4*x) - (22*b*(a + b*x^4)^(1/4))/(357*x
^17) - (a*(a + b*x^4)^(1/4))/(21*x^21) - (32*b^4*(a + b*x^4)^(1/4))/(13923*
a^3*x^5) + (20*b^3*(a + b*x^4)^(1/4))/(13923*a^2*x^9) - (5*b^2*(a + b*x^4)^
(1/4))/(4641*a*x^13)
```

sympy [**B**] time = 15.29, size = 954, normalized size = 10.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(5/4)/x**22,x)
```

```
[Out] -1989*a**8*b**(37/4)*(a/(b*x**4) + 1)**(1/4)*gamma(-21/4)/(256*a**7*b**9*x*
*20*gamma(-5/4) + 768*a**6*b**10*x**24*gamma(-5/4) + 768*a**5*b**11*x**28*g
amma(-5/4) + 256*a**4*b**12*x**32*gamma(-5/4)) - 8541*a**7*b**(41/4)*x**4*(
a/(b*x**4) + 1)**(1/4)*gamma(-21/4)/(256*a**7*b**9*x**20*gamma(-5/4) + 768*
a**6*b**10*x**24*gamma(-5/4) + 768*a**5*b**11*x**28*gamma(-5/4) + 256*a**4*
b**12*x**32*gamma(-5/4)) - 13734*a**6*b**(45/4)*x**8*(a/(b*x**4) + 1)**(1/4
)*gamma(-21/4)/(256*a**7*b**9*x**20*gamma(-5/4) + 768*a**6*b**10*x**24*gamm
a(-5/4) + 768*a**5*b**11*x**28*gamma(-5/4) + 256*a**4*b**12*x**32*gamma(-5/
4)) - 9786*a**5*b**(49/4)*x**12*(a/(b*x**4) + 1)**(1/4)*gamma(-21/4)/(256*a
**7*b**9*x**20*gamma(-5/4) + 768*a**6*b**10*x**24*gamma(-5/4) + 768*a**5*b*
*11*x**28*gamma(-5/4) + 256*a**4*b**12*x**32*gamma(-5/4)) - 2625*a**4*b**(5
3/4)*x**16*(a/(b*x**4) + 1)**(1/4)*gamma(-21/4)/(256*a**7*b**9*x**20*gamma(
-5/4) + 768*a**6*b**10*x**24*gamma(-5/4) + 768*a**5*b**11*x**28*gamma(-5/4)
 + 256*a**4*b**12*x**32*gamma(-5/4)) + 231*a**3*b**(57/4)*x**20*(a/(b*x**4)
 + 1)**(1/4)*gamma(-21/4)/(256*a**7*b**9*x**20*gamma(-5/4) + 768*a**6*b**10
*x**24*gamma(-5/4) + 768*a**5*b**11*x**28*gamma(-5/4) + 256*a**4*b**12*x**3
2*gamma(-5/4)) + 924*a**2*b**(61/4)*x**24*(a/(b*x**4) + 1)**(1/4)*gamma(-21
/4)/(256*a**7*b**9*x**20*gamma(-5/4) + 768*a**6*b**10*x**24*gamma(-5/4) + 7
68*a**5*b**11*x**28*gamma(-5/4) + 256*a**4*b**12*x**32*gamma(-5/4)) + 1056*
a*b**(65/4)*x**28*(a/(b*x**4) + 1)**(1/4)*gamma(-21/4)/(256*a**7*b**9*x**20
*gamma(-5/4) + 768*a**6*b**10*x**24*gamma(-5/4) + 768*a**5*b**11*x**28*gamm
a(-5/4) + 256*a**4*b**12*x**32*gamma(-5/4)) + 384*b**(69/4)*x**32*(a/(b*x**
4) + 1)**(1/4)*gamma(-21/4)/(256*a**7*b**9*x**20*gamma(-5/4) + 768*a**6*b**
10*x**24*gamma(-5/4) + 768*a**5*b**11*x**28*gamma(-5/4) + 256*a**4*b**12*x*
*32*gamma(-5/4))
```

3.1071 $\int x^{12}\left(a+bx^4\right)^{5/4}\,dx$

Optimal. Leaf size=171

$$\frac{5a^{9/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{672b^{5/2}\left(a+bx^4\right)^{3/4}}+\frac{5a^4x\sqrt[4]{a+bx^4}}{672b^3}-\frac{a^3x^5\sqrt[4]{a+bx^4}}{336b^2}+\frac{a^2x^9\sqrt[4]{a+bx^4}}{504b}+\frac{1}{18}x^{13}\left(a+bx^4\right)^{5/4}+\frac{}{2}$$

```
[Out] 5/672*a^4*x*(b*x^4+a)^(1/4)/b^3-1/336*a^3*x^5*(b*x^4+a)^(1/4)/b^2+1/504*a^2
*x^9*(b*x^4+a)^(1/4)/b+5/252*a*x^13*(b*x^4+a)^(1/4)+1/18*x^13*(b*x^4+a)^(5/
4)+5/672*a^(9/2)*(1+a/b/x^4)^(3/4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2))
)^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x^
2*b^(1/2)/a^(1/2))),2^(1/2))/b^(5/2)/(b*x^4+a)^(3/4)
```

Rubi [A] time = 0.09, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {279, 321, 237, 335, 275, 231}

$$-\frac{a^3x^5\sqrt[4]{a+bx^4}}{336b^2}+\frac{5a^4x\sqrt[4]{a+bx^4}}{672b^3}+\frac{5a^{9/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{672b^{5/2}\left(a+bx^4\right)^{3/4}}+\frac{a^2x^9\sqrt[4]{a+bx^4}}{504b}+\frac{1}{18}x^{13}\left(a+bx^4\right)^{5/4}+$$

Antiderivative was successfully verified.

```
[In] Int[x^12*(a + b*x^4)^(5/4),x]
```

```
[Out] (5*a^4*x*(a + b*x^4)^(1/4))/(672*b^3) - (a^3*x^5*(a + b*x^4)^(1/4))/(336*b^
2) + (a^2*x^9*(a + b*x^4)^(1/4))/(504*b) + (5*a*x^13*(a + b*x^4)^(1/4))/252
 + (x^13*(a + b*x^4)^(5/4))/18 + (5*a^(9/2)*(1 + a/(b*x^4))^(3/4)*x^3*Ellip
ticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(672*b^(5/2)*(a + b*x^4)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int x^{12}\left(a+bx^4\right)^{5/4}\,dx = \frac{1}{18}x^{13}\left(a+bx^4\right)^{5/4} + \frac{1}{18}(5a)\int x^{12}\sqrt[4]{a+bx^4}\,dx$$

$$= \frac{5}{252}ax^{13}\sqrt[4]{a+bx^4} + \frac{1}{18}x^{13}\left(a+bx^4\right)^{5/4} + \frac{1}{252}\left(5a^2\right)\int \frac{x^{12}}{\left(a+bx^4\right)^{3/4}}\,dx$$

$$= \frac{a^2x^9\sqrt[4]{a+bx^4}}{504b} + \frac{5}{252}ax^{13}\sqrt[4]{a+bx^4} + \frac{1}{18}x^{13}\left(a+bx^4\right)^{5/4} - \frac{a^3\int \frac{x^8}{\left(a+bx^4\right)^{3/4}}\,dx}{56b}$$

$$= -\frac{a^3x^5\sqrt[4]{a+bx^4}}{336b^2} + \frac{a^2x^9\sqrt[4]{a+bx^4}}{504b} + \frac{5}{252}ax^{13}\sqrt[4]{a+bx^4} + \frac{1}{18}x^{13}\left(a+bx^4\right)^{5/4} + \frac{\left(5a^4\right).}{}$$

$$= \frac{5a^4x\sqrt[4]{a+bx^4}}{672b^3} - \frac{a^3x^5\sqrt[4]{a+bx^4}}{336b^2} + \frac{a^2x^9\sqrt[4]{a+bx^4}}{504b} + \frac{5}{252}ax^{13}\sqrt[4]{a+bx^4} + \frac{1}{18}x^{13}\left(a+\right.$$

$$= \frac{5a^4x\sqrt[4]{a+bx^4}}{672b^3} - \frac{a^3x^5\sqrt[4]{a+bx^4}}{336b^2} + \frac{a^2x^9\sqrt[4]{a+bx^4}}{504b} + \frac{5}{252}ax^{13}\sqrt[4]{a+bx^4} + \frac{1}{18}x^{13}\left(a+\right.$$

$$= \frac{5a^4x\sqrt[4]{a+bx^4}}{672b^3} - \frac{a^3x^5\sqrt[4]{a+bx^4}}{336b^2} + \frac{a^2x^9\sqrt[4]{a+bx^4}}{504b} + \frac{5}{252}ax^{13}\sqrt[4]{a+bx^4} + \frac{1}{18}x^{13}\left(a+\right.$$

$$= \frac{5a^4x\sqrt[4]{a+bx^4}}{672b^3} - \frac{a^3x^5\sqrt[4]{a+bx^4}}{336b^2} + \frac{a^2x^9\sqrt[4]{a+bx^4}}{504b} + \frac{5}{252}ax^{13}\sqrt[4]{a+bx^4} + \frac{1}{18}x^{13}\left(a+\right.$$

Mathematica [C] time = 0.13, size = 89, normalized size = 0.52

$$\frac{x\sqrt[4]{a+bx^4}\left(\left(a+bx^4\right)^2\left(9a^2-18abx^4+28b^2x^8\right) - \frac{9a^4\,{}_2F_1\left(-\frac{5}{4},\frac{1}{4};\frac{5}{4};-\frac{bx^4}{a}\right)}{\sqrt[4]{\frac{bx^4}{a}+1}}\right)}{504b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12*(a + b*x^4)^(5/4),x]
```

```
[Out] (x*(a + b*x^4)^(1/4)*((a + b*x^4)^2*(9*a^2 - 18*a*b*x^4 + 28*b^2*x^8) - (9*
a^4*Hypergeometric2F1[-5/4, 1/4, 5/4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^(1/4))
)/(504*b^3)
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^{16} + ax^{12}\right)\left(bx^4 + a\right)^{\frac{1}{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^16 + a*x^12)*(b*x^4 + a)^(1/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4 + a\right)^{\frac{5}{4}} x^{12}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^4+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(5/4)*x^12, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \left(b\,x^4 + a\right)^{\frac{5}{4}} x^{12}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(b*x^4+a)^(5/4),x)
```

```
[Out] int(x^12*(b*x^4+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4 + a\right)^{\frac{5}{4}} x^{12}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^4+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(5/4)*x^12, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{12} \left(b\,x^4 + a\right)^{5/4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(a + b*x^4)^(5/4),x)
```

```
[Out] int(x^12*(a + b*x^4)^(5/4), x)
```

sympy [C] time = 3.99, size = 39, normalized size = 0.23

$$\frac{a^{\frac{5}{4}} x^{13} \Gamma\left(\frac{13}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{4}, \frac{13}{4} \\ \frac{17}{4} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{17}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12*(b*x**4+a)**(5/4),x)
```

```
[Out] a**(5/4)*x**13*gamma(13/4)*hyper((-5/4, 13/4), (17/4,), b*x**4*exp_polar(I*
pi)/a)/(4*gamma(17/4))
```

3.1072 $\int x^8 \left(a+bx^4\right)^{5/4}\,dx$

Optimal. Leaf size=147

$$-\frac{5a^{7/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{336b^{3/2}\left(a+bx^4\right)^{3/4}}-\frac{5a^3x\sqrt[4]{a+bx^4}}{336b^2}+\frac{a^2x^5\sqrt[4]{a+bx^4}}{168b}+\frac{1}{14}x^9\left(a+bx^4\right)^{5/4}+\frac{1}{28}ax^9\sqrt[4]{a+bx^4}$$

```
[Out] -5/336*a^3*x*(b*x^4+a)^(1/4)/b^2+1/168*a^2*x^5*(b*x^4+a)^(1/4)/b+1/28*a*x^9
*(b*x^4+a)^(1/4)+1/14*x^9*(b*x^4+a)^(5/4)-5/336*a^(7/2)*(1+a/b/x^4)^(3/4)*x
^3*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2
)/a^(1/2)))*EllipticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))/b^(3/2)
/(b*x^4+a)^(3/4)
```

Rubi [A] time = 0.07, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {279, 321, 237, 335, 275, 231}

$$-\frac{5a^3x\sqrt[4]{a+bx^4}}{336b^2}-\frac{5a^{7/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{336b^{3/2}\left(a+bx^4\right)^{3/4}}+\frac{a^2x^5\sqrt[4]{a+bx^4}}{168b}+\frac{1}{14}x^9\left(a+bx^4\right)^{5/4}+\frac{1}{28}ax^9\sqrt[4]{a+bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x^4)^(5/4),x]
```

```
[Out] (-5*a^3*x*(a + b*x^4)^(1/4))/(336*b^2) + (a^2*x^5*(a + b*x^4)^(1/4))/(168*b
) + (a*x^9*(a + b*x^4)^(1/4))/28 + (x^9*(a + b*x^4)^(5/4))/14 - (5*a^(7/2)*
(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(3
36*b^(3/2)*(a + b*x^4)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int x^8 \left(a+bx^4\right)^{5/4}\,dx = \frac{1}{14}x^9\left(a+bx^4\right)^{5/4} + \frac{1}{14}(5a)\int x^8\sqrt[4]{a+bx^4}\,dx$$

$$= \frac{1}{28}ax^9\sqrt[4]{a+bx^4} + \frac{1}{14}x^9\left(a+bx^4\right)^{5/4} + \frac{1}{28}a^2\int \frac{x^8}{\left(a+bx^4\right)^{3/4}}\,dx$$

$$= \frac{a^2x^5\sqrt[4]{a+bx^4}}{168b} + \frac{1}{28}ax^9\sqrt[4]{a+bx^4} + \frac{1}{14}x^9\left(a+bx^4\right)^{5/4} - \frac{\left(5a^3\right)\int \frac{x^4}{\left(a+bx^4\right)^{3/4}}\,dx}{168b}$$

$$= -\frac{5a^3x\sqrt[4]{a+bx^4}}{336b^2} + \frac{a^2x^5\sqrt[4]{a+bx^4}}{168b} + \frac{1}{28}ax^9\sqrt[4]{a+bx^4} + \frac{1}{14}x^9\left(a+bx^4\right)^{5/4} + \frac{\left(5a^4\right)\int \frac{}{(a}}{336}$$

$$= -\frac{5a^3x\sqrt[4]{a+bx^4}}{336b^2} + \frac{a^2x^5\sqrt[4]{a+bx^4}}{168b} + \frac{1}{28}ax^9\sqrt[4]{a+bx^4} + \frac{1}{14}x^9\left(a+bx^4\right)^{5/4} + \frac{\left(5a^4\left(1+\right.\right.}{3}$$

$$= -\frac{5a^3x\sqrt[4]{a+bx^4}}{336b^2} + \frac{a^2x^5\sqrt[4]{a+bx^4}}{168b} + \frac{1}{28}ax^9\sqrt[4]{a+bx^4} + \frac{1}{14}x^9\left(a+bx^4\right)^{5/4} - \frac{\left(5a^4\left(1+\right.\right.}{}$$

$$= -\frac{5a^3x\sqrt[4]{a+bx^4}}{336b^2} + \frac{a^2x^5\sqrt[4]{a+bx^4}}{168b} + \frac{1}{28}ax^9\sqrt[4]{a+bx^4} + \frac{1}{14}x^9\left(a+bx^4\right)^{5/4} - \frac{\left(5a^4\left(1+\right.\right.}{}$$

$$= -\frac{5a^3x\sqrt[4]{a+bx^4}}{336b^2} + \frac{a^2x^5\sqrt[4]{a+bx^4}}{168b} + \frac{1}{28}ax^9\sqrt[4]{a+bx^4} + \frac{1}{14}x^9\left(a+bx^4\right)^{5/4} - \frac{5a^{7/2}\left(1+\right.}{}$$

Mathematica [C] time = 0.12, size = 76, normalized size = 0.52

$$\frac{x\sqrt[4]{a+bx^4}\left(\frac{a^3\,{}_2F_1\left(-\frac{5}{4},\frac{1}{4};\frac{5}{4};-\frac{bx^4}{a}\right)}{\sqrt[4]{\frac{bx^4}{a}+1}} - \left(a-2bx^4\right)\left(a+bx^4\right)^2\right)}{28b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x^4)^(5/4),x]
```

```
[Out] (x*(a + b*x^4)^(1/4)*(-((a - 2*b*x^4)*(a + b*x^4)^2) + (a^3*Hypergeometric2
F1[-5/4, 1/4, 5/4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^(1/4)))/(28*b^2)
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^{12}+ax^{8}\right)\left(bx^{4}+a\right)^{\frac{1}{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^12 + a*x^8)*(b*x^4 + a)^(1/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^{4}+a\right)^{\frac{5}{4}}x^{8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^4+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(5/4)*x^8, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \left(b\,x^{4}+a\right)^{\frac{5}{4}}x^{8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^4+a)^(5/4),x)
```

```
[Out] int(x^8*(b*x^4+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^{4}+a\right)^{\frac{5}{4}}x^{8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^4+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(5/4)*x^8, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{8}\left(b\,x^{4}+a\right)^{5/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*x^4)^(5/4),x)
```

```
[Out] int(x^8*(a + b*x^4)^(5/4), x)
```

sympy [C] time = 3.37, size = 39, normalized size = 0.27

$$\frac{a^{\frac{5}{4}}x^{9}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{5}{4},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\right|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**4+a)**(5/4),x)
```

```
[Out] a**(5/4)*x**9*gamma(9/4)*hyper((-5/4, 9/4), (13/4,), b*x**4*exp_polar(I*pi)
/a)/(4*gamma(13/4))
```

3.1073 $\int x^4 \left(a+bx^4\right)^{5/4}\,dx$

Optimal. Leaf size=123

$$\frac{a^{5/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{24\sqrt{b}\left(a+bx^4\right)^{3/4}}+\frac{a^2x\sqrt[4]{a+bx^4}}{24b}+\frac{1}{10}x^5\left(a+bx^4\right)^{5/4}+\frac{1}{12}ax^5\sqrt[4]{a+bx^4}$$

```
[Out] 1/24*a^2*x*(b*x^4+a)^(1/4)/b+1/12*a*x^5*(b*x^4+a)^(1/4)+1/10*x^5*(b*x^4+a)^
(5/4)+1/24*a^(5/2)*(1+a/b/x^4)^(3/4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2
)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(
x^2*b^(1/2)/a^(1/2))),2^(1/2))/(b*x^4+a)^(3/4)/b^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {279, 321, 237, 335, 275, 231}

$$\frac{a^2x\sqrt[4]{a+bx^4}}{24b}+\frac{a^{5/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{24\sqrt{b}\left(a+bx^4\right)^{3/4}}+\frac{1}{10}x^5\left(a+bx^4\right)^{5/4}+\frac{1}{12}ax^5\sqrt[4]{a+bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^4)^(5/4),x]
```

```
[Out] (a^2*x*(a + b*x^4)^(1/4))/(24*b) + (a*x^5*(a + b*x^4)^(1/4))/12 + (x^5*(a +
 b*x^4)^(5/4))/10 + (a^(5/2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sq
rt[b]*x^2)/Sqrt[a]]/2, 2])/(24*Sqrt[b]*(a + b*x^4)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int x^4\left(a+bx^4\right)^{5/4}\,dx &= \frac{1}{10}x^5\left(a+bx^4\right)^{5/4}+\frac{1}{2}a\int x^4\sqrt[4]{a+bx^4}\,dx\\
&= \frac{1}{12}ax^5\sqrt[4]{a+bx^4}+\frac{1}{10}x^5\left(a+bx^4\right)^{5/4}+\frac{1}{12}a^2\int\frac{x^4}{\left(a+bx^4\right)^{3/4}}\,dx\\
&= \frac{a^2x\sqrt[4]{a+bx^4}}{24b}+\frac{1}{12}ax^5\sqrt[4]{a+bx^4}+\frac{1}{10}x^5\left(a+bx^4\right)^{5/4}-\frac{a^3\int\frac{1}{\left(a+bx^4\right)^{3/4}}\,dx}{24b}\\
&= \frac{a^2x\sqrt[4]{a+bx^4}}{24b}+\frac{1}{12}ax^5\sqrt[4]{a+bx^4}+\frac{1}{10}x^5\left(a+bx^4\right)^{5/4}-\frac{\left(a^3\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int\frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x}}{24b\left(a+bx^4\right)^{3/4}}\\
&= \frac{a^2x\sqrt[4]{a+bx^4}}{24b}+\frac{1}{12}ax^5\sqrt[4]{a+bx^4}+\frac{1}{10}x^5\left(a+bx^4\right)^{5/4}+\frac{\left(a^3\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int-\right.}{24b\left(a+bx^4\right)^{3/}}\\
&= \frac{a^2x\sqrt[4]{a+bx^4}}{24b}+\frac{1}{12}ax^5\sqrt[4]{a+bx^4}+\frac{1}{10}x^5\left(a+bx^4\right)^{5/4}+\frac{\left(a^3\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int-\right.}{48b\left(a+bx^4\right)^{3/}}\\
&= \frac{a^2x\sqrt[4]{a+bx^4}}{24b}+\frac{1}{12}ax^5\sqrt[4]{a+bx^4}+\frac{1}{10}x^5\left(a+bx^4\right)^{5/4}+\frac{a^{5/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\right.}{24\sqrt{b}\left(a+bx^4\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.09, size = 67, normalized size = 0.54

$$\frac{x\sqrt[4]{a+bx^4}\left(\left(a+bx^4\right)^2-\frac{a^2\,{}_2F_1\left(-\frac{5}{4},\frac{1}{4},\frac{5}{4};-\frac{bx^4}{a}\right)}{\sqrt[4]{\frac{bx^4}{a}+1}}\right)}{10b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^4)^(5/4),x]
```

```
[Out] (x*(a + b*x^4)^(1/4)*((a + b*x^4)^2 - (a^2*Hypergeometric2F1[-5/4, 1/4, 5/4
, -((b*x^4)/a)])/(1 + (b*x^4)/a)^(1/4)))/(10*b)
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^8+ax^4\right)\left(bx^4+a\right)^{\frac{1}{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^4+a)^(5/4),x, algorithm="fricas")
[Out] integral((b*x^8 + a*x^4)*(b*x^4 + a)^(1/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4 + a\right)^{\frac{5}{4}} x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^4+a)^(5/4),x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(5/4)*x^4, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \left(b\,x^4 + a\right)^{\frac{5}{4}} x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^4+a)^(5/4),x)
[Out] int(x^4*(b*x^4+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4 + a\right)^{\frac{5}{4}} x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^4+a)^(5/4),x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(5/4)*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \left(b\,x^4 + a\right)^{5/4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^4)^(5/4),x)
[Out] int(x^4*(a + b*x^4)^(5/4), x)
```

sympy [C] time = 1.71, size = 39, normalized size = 0.32

$$\frac{a^{\frac{5}{4}} x^5 \Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{4}, \frac{5}{4} \\ \frac{9}{4} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**4+a)**(5/4),x)
[Out] a**(5/4)*x**5*gamma(5/4)*hyper((-5/4, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/
a)/(4*gamma(9/4))
```

3.1074 $\int \left(a+bx^4\right)^{5/4}\,dx$

Optimal. Leaf size=97

$$-\frac{5a^{3/2}\sqrt{b}\,x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{12\left(a+bx^4\right)^{3/4}}+\frac{1}{6}x\left(a+bx^4\right)^{5/4}+\frac{5}{12}ax\sqrt[4]{a+bx^4}$$

```
[Out] 5/12*a*x*(b*x^4+a)^(1/4)+1/6*x*(b*x^4+a)^(5/4)-5/12*a^(3/2)*(1+a/b/x^4)^(3/
4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^
(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))*b^(
1/2)/(b*x^4+a)^(3/4)
```

Rubi [A] time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {195, 237, 335, 275, 231}

$$-\frac{5a^{3/2}\sqrt{b}\,x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{12\left(a+bx^4\right)^{3/4}}+\frac{1}{6}x\left(a+bx^4\right)^{5/4}+\frac{5}{12}ax\sqrt[4]{a+bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(5/4),x]
```

```
[Out] (5*a*x*(a + b*x^4)^(1/4))/12 + (x*(a + b*x^4)^(5/4))/6 - (5*a^(3/2)*Sqrt[b]
*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(
12*(a + b*x^4)^(3/4))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
```

egerQ[m]

Rubi steps

$$\begin{aligned}
\int \left(a+bx^4\right)^{5/4}\,dx &= \frac{1}{6}x\left(a+bx^4\right)^{5/4} + \frac{1}{6}(5a)\int \sqrt[4]{a+bx^4}\,dx \\
&= \frac{5}{12}ax\sqrt[4]{a+bx^4} + \frac{1}{6}x\left(a+bx^4\right)^{5/4} + \frac{1}{12}\left(5a^2\right)\int \frac{1}{\left(a+bx^4\right)^{3/4}}\,dx \\
&= \frac{5}{12}ax\sqrt[4]{a+bx^4} + \frac{1}{6}x\left(a+bx^4\right)^{5/4} + \frac{\left(5a^2\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{12\left(a+bx^4\right)^{3/4}} \\
&= \frac{5}{12}ax\sqrt[4]{a+bx^4} + \frac{1}{6}x\left(a+bx^4\right)^{5/4} - \frac{\left(5a^2\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{12\left(a+bx^4\right)^{3/4}} \\
&= \frac{5}{12}ax\sqrt[4]{a+bx^4} + \frac{1}{6}x\left(a+bx^4\right)^{5/4} - \frac{\left(5a^2\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{x^2}\right)}{24\left(a+bx^4\right)^{3/4}} \\
&= \frac{5}{12}ax\sqrt[4]{a+bx^4} + \frac{1}{6}x\left(a+bx^4\right)^{5/4} - \frac{5a^{3/2}\sqrt{b}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{12\left(a+bx^4\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 47, normalized size = 0.48

$$\frac{ax\sqrt[4]{a+bx^4}\;{}_2F_1\left(-\frac{5}{4},\frac{1}{4};\frac{5}{4};-\frac{bx^4}{a}\right)}{\sqrt[4]{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(5/4),x]
```

```
[Out] (a*x*(a + b*x^4)^(1/4)*Hypergeometric2F1[-5/4, 1/4, 5/4, -((b*x^4)/a)])/(1
+ (b*x^4)/a)^(1/4)
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^4+a\right)^{\frac{5}{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(5/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4+a\right)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(5/4), x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \left(b\,x^4+a\right)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(5/4),x)
```

```
[Out] int((b*x^4+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4+a\right)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(5/4), x)
```

mupad [B] time = 1.07, size = 37, normalized size = 0.38

$$\frac{x\left(b\,x^4+a\right)^{5/4}\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{4};\ \frac{5}{4};\ -\frac{b\,x^4}{a}\right)}{\left(\frac{b\,x^4}{a}+1\right)^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(5/4),x)
```

```
[Out] (x*(a + b*x^4)^(5/4)*hypergeom([-5/4, 1/4], 5/4, -(b*x^4)/a))/((b*x^4)/a +
1)^(5/4)
```

sympy [C] time = 2.70, size = 37, normalized size = 0.38

$$\frac{a^{\frac{5}{4}}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\,\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(5/4),x)
```

```
[Out] a**(5/4)*x*gamma(1/4)*hyper((-5/4, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/
(4*gamma(5/4))
```

3.1075 $\int \frac{\left(a+bx^4\right)^{5/4}}{x^4}\,dx$

Optimal. Leaf size=99

$$-\frac{5\sqrt{a}\,b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{6\left(a+bx^4\right)^{3/4}}+\frac{5}{6}bx\sqrt[4]{a+bx^4}-\frac{\left(a+bx^4\right)^{5/4}}{3x^3}$$

```
[Out] 5/6*b*x*(b*x^4+a)^(1/4)-1/3*(b*x^4+a)^(5/4)/x^3-5/6*b^(3/2)*(1+a/b/x^4)^(3/
4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^
(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))*a^(
1/2)/(b*x^4+a)^(3/4)
```

Rubi [A] time = 0.04, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {277, 195, 237, 335, 275, 231}

$$-\frac{5\sqrt{a}\,b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{6\left(a+bx^4\right)^{3/4}}+\frac{5}{6}bx\sqrt[4]{a+bx^4}-\frac{\left(a+bx^4\right)^{5/4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(5/4)/x^4,x]
```

```
[Out] (5*b*x*(a + b*x^4)^(1/4))/6 - (a + b*x^4)^(5/4)/(3*x^3) - (5*Sqrt[a]*b^(3/2
)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/
(6*(a + b*x^4)^(3/4))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{\left(a+bx^4\right)^{5/4}}{x^4}\,dx = -\frac{\left(a+bx^4\right)^{5/4}}{3x^3} + \frac{1}{3}(5b)\int \sqrt[4]{a+bx^4}\,dx$$

$$= \frac{5}{6}bx\sqrt[4]{a+bx^4} - \frac{\left(a+bx^4\right)^{5/4}}{3x^3} + \frac{1}{6}(5ab)\int \frac{1}{\left(a+bx^4\right)^{3/4}}\,dx$$

$$= \frac{5}{6}bx\sqrt[4]{a+bx^4} - \frac{\left(a+bx^4\right)^{5/4}}{3x^3} + \frac{\left(5ab\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{6\left(a+bx^4\right)^{3/4}}$$

$$= \frac{5}{6}bx\sqrt[4]{a+bx^4} - \frac{\left(a+bx^4\right)^{5/4}}{3x^3} - \frac{\left(5ab\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{6\left(a+bx^4\right)^{3/4}}$$

$$= \frac{5}{6}bx\sqrt[4]{a+bx^4} - \frac{\left(a+bx^4\right)^{5/4}}{3x^3} - \frac{\left(5ab\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{x^2}\right)}{12\left(a+bx^4\right)^{3/4}}$$

$$= \frac{5}{6}bx\sqrt[4]{a+bx^4} - \frac{\left(a+bx^4\right)^{5/4}}{3x^3} - \frac{5\sqrt{a}\,b^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{6\left(a+bx^4\right)^{3/4}}$$

Mathematica [C] time = 0.03, size = 52, normalized size = 0.53

$$-\frac{a\sqrt[4]{a+bx^4}\;{}_2F_1\left(-\frac{5}{4},-\frac{3}{4};\frac{1}{4};-\frac{bx^4}{a}\right)}{3x^3\sqrt[4]{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(5/4)/x^4,x]
```

```
[Out] -1/3*(a*(a + b*x^4)^(1/4)*Hypergeometric2F1[-5/4, -3/4, 1/4, -((b*x^4)/a)])
/(x^3*(1 + (b*x^4)/a)^(1/4))
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{5}{4}}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^4,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(5/4)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{5}{4}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(5/4)/x^4, x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{5}{4}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(5/4)/x^4,x)
```

```
[Out] int((b*x^4+a)^(5/4)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{5}{4}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(5/4)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^4+a\right)^{5/4}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(5/4)/x^4,x)
```

```
[Out] int((a + b*x^4)^(5/4)/x^4, x)
```

sympy [C] time = 2.59, size = 42, normalized size = 0.42

$$\frac{a^{\frac{5}{4}}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4}, -\frac{3}{4} \\ \frac{1}{4}\end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(5/4)/x**4,x)
```

```
[Out] a**(5/4)*gamma(-3/4)*hyper((-5/4, -3/4), (1/4,), b*x**4*exp_polar(I*pi)/a)/
(4*x**3*gamma(1/4))
```

3.1076 $\int \frac{\left(a+bx^4\right)^{5/4}}{x^8}\,dx$

Optimal. Leaf size=101

$$-\frac{5b^{5/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{21\sqrt{a}\left(a+bx^4\right)^{3/4}}-\frac{\left(a+bx^4\right)^{5/4}}{7x^7}-\frac{5b\sqrt[4]{a+bx^4}}{21x^3}$$

```
[Out] -5/21*b*(b*x^4+a)^(1/4)/x^3-1/7*(b*x^4+a)^(5/4)/x^7-5/21*b^(5/2)*(1+a/b/x^4
)^(3/4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x
^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2)
)/(b*x^4+a)^(3/4)/a^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {277, 237, 335, 275, 231}

$$-\frac{5b^{5/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{21\sqrt{a}\left(a+bx^4\right)^{3/4}}-\frac{5b\sqrt[4]{a+bx^4}}{21x^3}-\frac{\left(a+bx^4\right)^{5/4}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(5/4)/x^8,x]
```

```
[Out] (-5*b*(a + b*x^4)^(1/4))/(21*x^3) - (a + b*x^4)^(5/4)/(7*x^7) - (5*b^(5/2)*
(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(2
1*Sqrt[a]*(a + b*x^4)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^4\right)^{5/4}}{x^8}\,dx &= -\frac{\left(a+bx^4\right)^{5/4}}{7x^7} + \frac{1}{7}(5b)\int \frac{\sqrt[4]{a+bx^4}}{x^4}\,dx \\
&= -\frac{5b\sqrt[4]{a+bx^4}}{21x^3} - \frac{\left(a+bx^4\right)^{5/4}}{7x^7} + \frac{1}{21}\left(5b^2\right)\int \frac{1}{\left(a+bx^4\right)^{3/4}}\,dx \\
&= -\frac{5b\sqrt[4]{a+bx^4}}{21x^3} - \frac{\left(a+bx^4\right)^{5/4}}{7x^7} + \frac{\left(5b^2\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{21\left(a+bx^4\right)^{3/4}} \\
&= -\frac{5b\sqrt[4]{a+bx^4}}{21x^3} - \frac{\left(a+bx^4\right)^{5/4}}{7x^7} - \frac{\left(5b^2\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{21\left(a+bx^4\right)^{3/4}} \\
&= -\frac{5b\sqrt[4]{a+bx^4}}{21x^3} - \frac{\left(a+bx^4\right)^{5/4}}{7x^7} - \frac{\left(5b^2\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{x^2}\right)}{42\left(a+bx^4\right)^{3/4}} \\
&= -\frac{5b\sqrt[4]{a+bx^4}}{21x^3} - \frac{\left(a+bx^4\right)^{5/4}}{7x^7} - \frac{5b^{5/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{21\sqrt{a}\,\left(a+bx^4\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.51

$$-\frac{a\sqrt[4]{a+bx^4}\;{}_2F_1\left(-\frac{7}{4},-\frac{5}{4};-\frac{3}{4};-\frac{bx^4}{a}\right)}{7x^7\sqrt[4]{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(5/4)/x^8,x]
```

```
[Out] -1/7*(a*(a + b*x^4)^(1/4)*Hypergeometric2F1[-7/4, -5/4, -3/4, -((b*x^4)/a)]
)/(x^7*(1 + (b*x^4)/a)^(1/4))
```

fricas [F] time = 1.03, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{5}{4}}}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^8,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(5/4)/x^8, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{5}{4}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^8,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(5/4)/x^8, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{5}{4}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(5/4)/x^8,x)
[Out] int((b*x^4+a)^(5/4)/x^8,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{5}{4}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^8,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(5/4)/x^8, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^4+a\right)^{5/4}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(5/4)/x^8,x)
[Out] int((a + b*x^4)^(5/4)/x^8, x)
```

sympy [C] time = 3.39, size = 31, normalized size = 0.31

$$-\frac{b^{\frac{5}{4}}\,{}_2F_1\left(\left.\begin{matrix}-\frac{5}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\right|\frac{ae^{i\pi}}{bx^4}\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(5/4)/x**8,x)
[Out] -b**(5/4)*hyper((-5/4, 1/2), (3/2,), a*exp_polar(I*pi)/(b*x**4))/(2*x**2)
```

3.1077 $\int \frac{\left(a+bx^4\right)^{5/4}}{x^{12}}\,dx$

Optimal. Leaf size=125

$$\frac{10b^{7/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{231a^{3/2}\left(a+bx^4\right)^{3/4}}-\frac{5b^2\sqrt[4]{a+bx^4}}{231ax^3}-\frac{\left(a+bx^4\right)^{5/4}}{11x^{11}}-\frac{5b\sqrt[4]{a+bx^4}}{77x^7}$$

```
[Out] -5/77*b*(b*x^4+a)^(1/4)/x^7-5/231*b^2*(b*x^4+a)^(1/4)/a/x^3-1/11*(b*x^4+a)^
(5/4)/x^11+10/231*b^(7/2)*(1+a/b/x^4)^(3/4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)
/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*
arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))/a^(3/2)/(b*x^4+a)^(3/4)
```

Rubi [A] time = 0.05, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {277, 325, 237, 335, 275, 231}

$$\frac{10b^{7/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{231a^{3/2}\left(a+bx^4\right)^{3/4}}-\frac{5b^2\sqrt[4]{a+bx^4}}{231ax^3}-\frac{5b\sqrt[4]{a+bx^4}}{77x^7}-\frac{\left(a+bx^4\right)^{5/4}}{11x^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(5/4)/x^12,x]
```

```
[Out] (-5*b*(a + b*x^4)^(1/4))/(77*x^7) - (5*b^2*(a + b*x^4)^(1/4))/(231*a*x^3) -
 (a + b*x^4)^(5/4)/(11*x^11) + (10*b^(7/2)*(1 + a/(b*x^4))^(3/4)*x^3*Ellipt
icF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(231*a^(3/2)*(a + b*x^4)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^4\right)^{5/4}}{x^{12}}\,dx &= -\frac{\left(a+bx^4\right)^{5/4}}{11x^{11}} + \frac{1}{11}(5b)\int \frac{\sqrt[4]{a+bx^4}}{x^8}\,dx \\
&= -\frac{5b\sqrt[4]{a+bx^4}}{77x^7} - \frac{\left(a+bx^4\right)^{5/4}}{11x^{11}} + \frac{1}{77}\left(5b^2\right)\int \frac{1}{x^4\left(a+bx^4\right)^{3/4}}\,dx \\
&= -\frac{5b\sqrt[4]{a+bx^4}}{77x^7} - \frac{5b^2\sqrt[4]{a+bx^4}}{231ax^3} - \frac{\left(a+bx^4\right)^{5/4}}{11x^{11}} - \frac{\left(10b^3\right)\int \frac{1}{\left(a+bx^4\right)^{3/4}}\,dx}{231a} \\
&= -\frac{5b\sqrt[4]{a+bx^4}}{77x^7} - \frac{5b^2\sqrt[4]{a+bx^4}}{231ax^3} - \frac{\left(a+bx^4\right)^{5/4}}{11x^{11}} - \frac{\left(10b^3\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{231a\left(a+bx^4\right)^{3/4}} \\
&= -\frac{5b\sqrt[4]{a+bx^4}}{77x^7} - \frac{5b^2\sqrt[4]{a+bx^4}}{231ax^3} - \frac{\left(a+bx^4\right)^{5/4}}{11x^{11}} + \frac{\left(10b^3\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}\right.}{231a\left(a+bx^4\right)^{3/4}} \\
&= -\frac{5b\sqrt[4]{a+bx^4}}{77x^7} - \frac{5b^2\sqrt[4]{a+bx^4}}{231ax^3} - \frac{\left(a+bx^4\right)^{5/4}}{11x^{11}} + \frac{\left(5b^3\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{3/4}}\right.}{231a\left(a+bx^4\right)^{3/4}} \\
&= -\frac{5b\sqrt[4]{a+bx^4}}{77x^7} - \frac{5b^2\sqrt[4]{a+bx^4}}{231ax^3} - \frac{\left(a+bx^4\right)^{5/4}}{11x^{11}} + \frac{10b^{7/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\right|2}{231a^{3/2}\left(a+bx^4\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.42

$$-\frac{a\sqrt[4]{a+bx^4}\;{}_2F_1\left(-\frac{11}{4},-\frac{5}{4};-\frac{7}{4};-\frac{bx^4}{a}\right)}{11x^{11}\sqrt[4]{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(5/4)/x^12,x]
```

```
[Out] -1/11*(a*(a + b*x^4)^(1/4)*Hypergeometric2F1[-11/4, -5/4, -7/4, -((b*x^4)/a
)])/(x^11*(1 + (b*x^4)/a)^(1/4))
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{5}{4}}}{x^{12}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^12,x, algorithm="fricas")
[Out] integral((b*x^4 + a)^(5/4)/x^12, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{5}{4}}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^12,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(5/4)/x^12, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{5}{4}}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(5/4)/x^12,x)
[Out] int((b*x^4+a)^(5/4)/x^12,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{5}{4}}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^12,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(5/4)/x^12, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^4+a\right)^{5/4}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(5/4)/x^12,x)
[Out] int((a + b*x^4)^(5/4)/x^12, x)
```

sympy [C] time = 2.40, size = 31, normalized size = 0.25

$$-\frac{b^{\frac{5}{4}}\,{}_2F_1\left(\left.\begin{matrix}-\frac{5}{4}, \frac{3}{2} \\ \frac{5}{2}\end{matrix}\right| \frac{ae^{i\pi}}{bx^4}\right)}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(5/4)/x**12,x)
[Out] -b**(5/4)*hyper((-5/4, 3/2), (5/2,), a*exp_polar(I*pi)/(b*x**4))/(6*x**6)
```

3.1078 $\int \frac{\left(a+bx^4\right)^{5/4}}{x^{16}}\,dx$

Optimal. Leaf size=149

$$-\frac{4b^{9/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{231a^{5/2}\left(a+bx^4\right)^{3/4}}+\frac{2b^3\sqrt[4]{a+bx^4}}{231a^2x^3}-\frac{b^2\sqrt[4]{a+bx^4}}{231ax^7}-\frac{\left(a+bx^4\right)^{5/4}}{15x^{15}}-\frac{b\sqrt[4]{a+bx^4}}{33x^{11}}$$

```
[Out] -1/33*b*(b*x^4+a)^(1/4)/x^11-1/231*b^2*(b*x^4+a)^(1/4)/a/x^7+2/231*b^3*(b*x
^4+a)^(1/4)/a^2/x^3-1/15*(b*x^4+a)^(5/4)/x^15-4/231*b^(9/2)*(1+a/b/x^4)^(3/
4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^
(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))/a^(
5/2)/(b*x^4+a)^(3/4)
```

Rubi [A] time = 0.07, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {277, 325, 237, 335, 275, 231}

$$\frac{2b^3\sqrt[4]{a+bx^4}}{231a^2x^3}-\frac{4b^{9/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{231a^{5/2}\left(a+bx^4\right)^{3/4}}-\frac{b^2\sqrt[4]{a+bx^4}}{231ax^7}-\frac{b\sqrt[4]{a+bx^4}}{33x^{11}}-\frac{\left(a+bx^4\right)^{5/4}}{15x^{15}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(5/4)/x^16,x]
```

```
[Out] -(b*(a + b*x^4)^(1/4))/(33*x^11) - (b^2*(a + b*x^4)^(1/4))/(231*a*x^7) + (2
*b^3*(a + b*x^4)^(1/4))/(231*a^2*x^3) - (a + b*x^4)^(5/4)/(15*x^15) - (4*b^
(9/2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2,
2])/(231*a^(5/2)*(a + b*x^4)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{\left(a+bx^4\right)^{5/4}}{x^{16}}\,dx = -\frac{\left(a+bx^4\right)^{5/4}}{15x^{15}} + \frac{1}{3}b\int \frac{\sqrt[4]{a+bx^4}}{x^{12}}\,dx$$

$$= -\frac{b\sqrt[4]{a+bx^4}}{33x^{11}} - \frac{\left(a+bx^4\right)^{5/4}}{15x^{15}} + \frac{1}{33}b^2\int \frac{1}{x^8\left(a+bx^4\right)^{3/4}}\,dx$$

$$= -\frac{b\sqrt[4]{a+bx^4}}{33x^{11}} - \frac{b^2\sqrt[4]{a+bx^4}}{231ax^7} - \frac{\left(a+bx^4\right)^{5/4}}{15x^{15}} - \frac{\left(2b^3\right)\int \frac{1}{x^4\left(a+bx^4\right)^{3/4}}\,dx}{77a}$$

$$= -\frac{b\sqrt[4]{a+bx^4}}{33x^{11}} - \frac{b^2\sqrt[4]{a+bx^4}}{231ax^7} + \frac{2b^3\sqrt[4]{a+bx^4}}{231a^2x^3} - \frac{\left(a+bx^4\right)^{5/4}}{15x^{15}} + \frac{\left(4b^4\right)\int \frac{1}{\left(a+bx^4\right)^{3/4}}\,dx}{231a^2}$$

$$= -\frac{b\sqrt[4]{a+bx^4}}{33x^{11}} - \frac{b^2\sqrt[4]{a+bx^4}}{231ax^7} + \frac{2b^3\sqrt[4]{a+bx^4}}{231a^2x^3} - \frac{\left(a+bx^4\right)^{5/4}}{15x^{15}} + \frac{\left(4b^4\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{}{\left(1+\frac{}{}\right.}$$

$$= -\frac{b\sqrt[4]{a+bx^4}}{33x^{11}} - \frac{b^2\sqrt[4]{a+bx^4}}{231ax^7} + \frac{2b^3\sqrt[4]{a+bx^4}}{231a^2x^3} - \frac{\left(a+bx^4\right)^{5/4}}{15x^{15}} - \frac{\left(4b^4\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}}{231a^2\left(a+\right.}$$

$$= -\frac{b\sqrt[4]{a+bx^4}}{33x^{11}} - \frac{b^2\sqrt[4]{a+bx^4}}{231ax^7} + \frac{2b^3\sqrt[4]{a+bx^4}}{231a^2x^3} - \frac{\left(a+bx^4\right)^{5/4}}{15x^{15}} - \frac{\left(2b^4\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}}{231a^2\left(a+\right.}$$

$$= -\frac{b\sqrt[4]{a+bx^4}}{33x^{11}} - \frac{b^2\sqrt[4]{a+bx^4}}{231ax^7} + \frac{2b^3\sqrt[4]{a+bx^4}}{231a^2x^3} - \frac{\left(a+bx^4\right)^{5/4}}{15x^{15}} - \frac{4b^{9/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\text{co}\right.}{231a^{5/2}\left(a+bx^4\right.}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.35

$$-\frac{a\sqrt[4]{a+bx^4}\;{}_2F_1\left(-\frac{15}{4},-\frac{5}{4};-\frac{11}{4};-\frac{bx^4}{a}\right)}{15x^{15}\sqrt[4]{\frac{bx^4}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(5/4)/x^16,x]
```

```
[Out] -1/15*(a*(a + b*x^4)^(1/4)*Hypergeometric2F1[-15/4, -5/4, -11/4, -((b*x^4)/
a)])/(x^15*(1 + (b*x^4)/a)^(1/4))
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{5}{4}}}{x^{16}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^16,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(5/4)/x^16, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{5}{4}}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^16,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(5/4)/x^16, x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{5}{4}}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(5/4)/x^16,x)
```

```
[Out] int((b*x^4+a)^(5/4)/x^16,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{5}{4}}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/x^16,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(5/4)/x^16, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^4+a\right)^{5/4}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(5/4)/x^16,x)
```

```
[Out] int((a + b*x^4)^(5/4)/x^16, x)
```

sympy [C] time = 6.85, size = 46, normalized size = 0.31

$$\frac{a^{\frac{5}{4}}\Gamma\left(-\frac{15}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{15}{4}, -\frac{5}{4} \\ -\frac{11}{4}\end{matrix}\right| \frac{bx^4e^{i\pi}}{a}\right)}{4x^{15}\Gamma\left(-\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(5/4)/x**16,x)
```

```
[Out] a**(5/4)*gamma(-15/4)*hyper((-15/4, -5/4), (-11/4,), b*x**4*exp_polar(I*pi)
/a)/(4*x**15*gamma(-11/4))
```

3.1079 $\int \left(a+bx^4\right)^{7/4}\,dx$

Optimal. Leaf size=96

$$\frac{21a^2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{64\sqrt[4]{b}}+\frac{21a^2\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{64\sqrt[4]{b}}+\frac{7}{32}ax\left(a+bx^4\right)^{3/4}+\frac{1}{8}x\left(a+bx^4\right)^{7/4}$$

```
[Out] 7/32*a*x*(b*x^4+a)^(3/4)+1/8*x*(b*x^4+a)^(7/4)+21/64*a^2*arctan(b^(1/4)*x/(
b*x^4+a)^(1/4))/b^(1/4)+21/64*a^2*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(1/4
)
```

Rubi [A] time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {195, 240, 212, 206, 203}

$$\frac{21a^2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{64\sqrt[4]{b}}+\frac{21a^2\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{64\sqrt[4]{b}}+\frac{7}{32}ax\left(a+bx^4\right)^{3/4}+\frac{1}{8}x\left(a+bx^4\right)^{7/4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(7/4),x]
```

```
[Out] (7*a*x*(a + b*x^4)^(3/4))/32 + (x*(a + b*x^4)^(7/4))/8 + (21*a^2*ArcTan[(b^
(1/4)*x)/(a + b*x^4)^(1/4)])/(64*b^(1/4)) + (21*a^2*ArcTanh[(b^(1/4)*x)/(a
+ b*x^4)^(1/4)])/(64*b^(1/4))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
```

n]

Rubi steps

$$\begin{aligned}\int \left(a+bx^4\right)^{7/4}\,dx &= \frac{1}{8}x\left(a+bx^4\right)^{7/4}+\frac{1}{8}(7a)\int \left(a+bx^4\right)^{3/4}\,dx\\ &= \frac{7}{32}ax\left(a+bx^4\right)^{3/4}+\frac{1}{8}x\left(a+bx^4\right)^{7/4}+\frac{1}{32}\left(21a^2\right)\int \frac{1}{\sqrt[4]{a+bx^4}}\,dx\\ &= \frac{7}{32}ax\left(a+bx^4\right)^{3/4}+\frac{1}{8}x\left(a+bx^4\right)^{7/4}+\frac{1}{32}\left(21a^2\right)\text{Subst}\left(\int \frac{1}{1-bx^4}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)\\ &= \frac{7}{32}ax\left(a+bx^4\right)^{3/4}+\frac{1}{8}x\left(a+bx^4\right)^{7/4}+\frac{1}{64}\left(21a^2\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{b}\,x^2}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)+\\ &= \frac{7}{32}ax\left(a+bx^4\right)^{3/4}+\frac{1}{8}x\left(a+bx^4\right)^{7/4}+\frac{21a^2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{64\sqrt[4]{b}}+\frac{21a^2\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{64\sqrt[4]{b}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 47, normalized size = 0.49

$$\frac{ax\left(a+bx^4\right)^{3/4}\,{}_2F_1\left(-\frac{7}{4},\frac{1}{4};\frac{5}{4};-\frac{bx^4}{a}\right)}{\left(\frac{bx^4}{a}+1\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(7/4),x]
```

```
[Out] (a*x*(a + b*x^4)^(3/4)*Hypergeometric2F1[-7/4, 1/4, 5/4, -((b*x^4)/a)])/(1
+ (b*x^4)/a)^(3/4)
```

fricas [B] time = 0.91, size = 202, normalized size = 2.10

$$\frac{1}{32}\left(4\,bx^5+11\,ax\right)\left(bx^4+a\right)^{\frac{3}{4}}+\frac{21}{32}\left(\frac{a^8}{b}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(\frac{a^8}{b}\right)^{\frac{1}{4}}\left(bx^4+a\right)^{\frac{1}{4}}a^6-\left(\frac{a^8}{b}\right)^{\frac{1}{4}}x\sqrt{\frac{\sqrt{bx^4+a}\,a^{12}+\sqrt{\frac{a^8}{b}}\,a^8bx^2}{x^2}}}{a^8x}\right)+\frac{21}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(7/4),x, algorithm="fricas")
```

```
[Out] 1/32*(4*b*x^5 + 11*a*x)*(b*x^4 + a)^(3/4) + 21/32*(a^8/b)^(1/4)*arctan(-((a
^8/b)^(1/4)*(b*x^4 + a)^(1/4)*a^6 - (a^8/b)^(1/4)*x*sqrt((sqrt(b*x^4 + a)*a
^12 + sqrt(a^8/b)*a^8*b*x^2)/x^2))/(a^8*x)) + 21/128*(a^8/b)^(1/4)*log(9261
*((b*x^4 + a)^(1/4)*a^6 + (a^8/b)^(3/4)*b*x)/x) - 21/128*(a^8/b)^(1/4)*log(
9261*((b*x^4 + a)^(1/4)*a^6 - (a^8/b)^(3/4)*b*x)/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4+a\right)^{\frac{7}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(7/4), x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \left(b\,x^4+a\right)^{\frac{7}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(7/4),x)
```

```
[Out] int((b*x^4+a)^(7/4),x)
```

maxima [A] time = 3.03, size = 143, normalized size = 1.49

$$-\frac{21}{128}\,a^2\left(\frac{2\,\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)-\frac{\frac{7\left(bx^4+a\right)^{\frac{3}{4}}a^2b}{x^3}-\frac{11\left(bx^4+a\right)^{\frac{7}{4}}a^2}{x^7}}{32\left(b^2-\frac{2\left(bx^4+a\right)b}{x^4}+\frac{\left(bx^4+a\right)^2}{x^8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(7/4),x, algorithm="maxima")
```

```
[Out] -21/128*a^2*(2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) + log(-(b^(1/4
) - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/4)) - 1/32*(
7*(b*x^4 + a)^(3/4)*a^2*b/x^3 - 11*(b*x^4 + a)^(7/4)*a^2/x^7)/(b^2 - 2*(b*x
^4 + a)*b/x^4 + (b*x^4 + a)^2/x^8)
```

mupad [B] time = 1.06, size = 37, normalized size = 0.39

$$\frac{x\left(b\,x^4+a\right)^{7/4}\,{}_2\mathrm{F}_1\left(-\frac{7}{4},\frac{1}{4};\,\frac{5}{4};\,-\frac{b\,x^4}{a}\right)}{\left(\frac{b\,x^4}{a}+1\right)^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(7/4),x)
```

```
[Out] (x*(a + b*x^4)^(7/4)*hypergeom([-7/4, 1/4], 5/4, -(b*x^4)/a))/((b*x^4)/a +
1)^(7/4)
```

sympy [C] time = 2.00, size = 37, normalized size = 0.39

$$\frac{a^{\frac{7}{4}}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{7}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(7/4),x)
```

```
[Out] a**(7/4)*x*gamma(1/4)*hyper((-7/4, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/
(4*gamma(5/4))
```

3.1080 $\int \frac{x^{19}}{\sqrt[4]{a+bx^4}}\,dx$

Optimal. Leaf size=101

$$\frac{a^4\left(a+bx^4\right)^{3/4}}{3b^5}-\frac{4a^3\left(a+bx^4\right)^{7/4}}{7b^5}+\frac{6a^2\left(a+bx^4\right)^{11/4}}{11b^5}+\frac{\left(a+bx^4\right)^{19/4}}{19b^5}-\frac{4a\left(a+bx^4\right)^{15/4}}{15b^5}$$

```
[Out] 1/3*a^4*(b*x^4+a)^(3/4)/b^5-4/7*a^3*(b*x^4+a)^(7/4)/b^5+6/11*a^2*(b*x^4+a)^
(11/4)/b^5-4/15*a*(b*x^4+a)^(15/4)/b^5+1/19*(b*x^4+a)^(19/4)/b^5
```

Rubi [A] time = 0.05, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{6a^2\left(a+bx^4\right)^{11/4}}{11b^5}-\frac{4a^3\left(a+bx^4\right)^{7/4}}{7b^5}+\frac{a^4\left(a+bx^4\right)^{3/4}}{3b^5}+\frac{\left(a+bx^4\right)^{19/4}}{19b^5}-\frac{4a\left(a+bx^4\right)^{15/4}}{15b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^19/(a + b*x^4)^(1/4),x]
```

```
[Out] (a^4*(a + b*x^4)^(3/4))/(3*b^5) - (4*a^3*(a + b*x^4)^(7/4))/(7*b^5) + (6*a^
2*(a + b*x^4)^(11/4))/(11*b^5) - (4*a*(a + b*x^4)^(15/4))/(15*b^5) + (a + b
*x^4)^(19/4)/(19*b^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{19}}{\sqrt[4]{a+bx^4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x^4}{\sqrt[4]{a+bx}}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int\left(\frac{a^4}{b^4\sqrt[4]{a+bx}}-\frac{4a^3(a+bx)^{3/4}}{b^4}+\frac{6a^2(a+bx)^{7/4}}{b^4}-\frac{4a(a+bx)^{11/4}}{b^4}+\frac{(a+bx)^{15/4}}{b^4}\right.\right.\\ &= \frac{a^4\left(a+bx^4\right)^{3/4}}{3b^5}-\frac{4a^3\left(a+bx^4\right)^{7/4}}{7b^5}+\frac{6a^2\left(a+bx^4\right)^{11/4}}{11b^5}-\frac{4a\left(a+bx^4\right)^{15/4}}{15b^5}+\frac{\left(a+bx^4\right)^{19/4}}{19b^5}\end{aligned}$$

Mathematica [A] time = 0.05, size = 61, normalized size = 0.60

$$\frac{\left(a+bx^4\right)^{3/4}\left(2048a^4-1536a^3bx^4+1344a^2b^2x^8-1232ab^3x^{12}+1155b^4x^{16}\right)}{21945b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^19/(a + b*x^4)^(1/4),x]
```

```
[Out] ((a + b*x^4)^(3/4)*(2048*a^4 - 1536*a^3*b*x^4 + 1344*a^2*b^2*x^8 - 1232*a*b
^3*x^12 + 1155*b^4*x^16))/(21945*b^5)
```

fricas [A] time = 0.96, size = 57, normalized size = 0.56

$$\frac{\left(1155\,b^4x^{16}-1232\,ab^3x^{12}+1344\,a^2b^2x^8-1536\,a^3bx^4+2048\,a^4\right)\left(bx^4+a\right)^{\frac{3}{4}}}{21945\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/21945*(1155*b^4*x^16 - 1232*a*b^3*x^12 + 1344*a^2*b^2*x^8 - 1536*a^3*b*x^
4 + 2048*a^4)*(b*x^4 + a)^(3/4)/b^5
```

giac [A] time = 0.16, size = 71, normalized size = 0.70

$$\frac{1155\left(bx^4+a\right)^{\frac{19}{4}}-5852\left(bx^4+a\right)^{\frac{15}{4}}a+11970\left(bx^4+a\right)^{\frac{11}{4}}a^2-12540\left(bx^4+a\right)^{\frac{7}{4}}a^3+7315\left(bx^4+a\right)^{\frac{3}{4}}a^4}{21945\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] 1/21945*(1155*(b*x^4 + a)^(19/4) - 5852*(b*x^4 + a)^(15/4)*a + 11970*(b*x^4
 + a)^(11/4)*a^2 - 12540*(b*x^4 + a)^(7/4)*a^3 + 7315*(b*x^4 + a)^(3/4)*a^4
)/b^5
```

maple [A] time = 0.01, size = 58, normalized size = 0.57

$$\frac{\left(b\,x^4+a\right)^{\frac{3}{4}}\left(1155x^{16}b^4-1232a\,x^{12}b^3+1344a^2x^8b^2-1536a^3x^4b+2048a^4\right)}{21945b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19/(b*x^4+a)^(1/4),x)
```

```
[Out] 1/21945*(b*x^4+a)^(3/4)*(1155*b^4*x^16-1232*a*b^3*x^12+1344*a^2*b^2*x^8-153
6*a^3*b*x^4+2048*a^4)/b^5
```

maxima [A] time = 1.38, size = 81, normalized size = 0.80

$$\frac{\left(bx^4+a\right)^{\frac{19}{4}}}{19\,b^5}-\frac{4\left(bx^4+a\right)^{\frac{15}{4}}a}{15\,b^5}+\frac{6\left(bx^4+a\right)^{\frac{11}{4}}a^2}{11\,b^5}-\frac{4\left(bx^4+a\right)^{\frac{7}{4}}a^3}{7\,b^5}+\frac{\left(bx^4+a\right)^{\frac{3}{4}}a^4}{3\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] 1/19*(b*x^4 + a)^(19/4)/b^5 - 4/15*(b*x^4 + a)^(15/4)*a/b^5 + 6/11*(b*x^4 +
 a)^(11/4)*a^2/b^5 - 4/7*(b*x^4 + a)^(7/4)*a^3/b^5 + 1/3*(b*x^4 + a)^(3/4)*
a^4/b^5
```

mupad [B] time = 1.16, size = 58, normalized size = 0.57

$$\left(b\,x^4+a\right)^{3/4}\left(\frac{2048\,a^4}{21945\,b^5}+\frac{x^{16}}{19\,b}-\frac{16\,a\,x^{12}}{285\,b^2}-\frac{512\,a^3\,x^4}{7315\,b^4}+\frac{64\,a^2\,x^8}{1045\,b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19/(a + b*x^4)^(1/4),x)
```

```
[Out] (a + b*x^4)^(3/4)*((2048*a^4)/(21945*b^5) + x^16/(19*b) - (16*a*x^12)/(285*
b^2) - (512*a^3*x^4)/(7315*b^4) + (64*a^2*x^8)/(1045*b^3))
```

sympy [A] time = 24.29, size = 116, normalized size = 1.15

$$\begin{cases} \frac{2048a^4\left(a+bx^4\right)^{\frac{3}{4}}}{21945b^5} - \frac{512a^3x^4\left(a+bx^4\right)^{\frac{3}{4}}}{7315b^4} + \frac{64a^2x^8\left(a+bx^4\right)^{\frac{3}{4}}}{1045b^3} - \frac{16ax^{12}\left(a+bx^4\right)^{\frac{3}{4}}}{285b^2} + \frac{x^{16}\left(a+bx^4\right)^{\frac{3}{4}}}{19b} & \text{for } b \neq 0 \\ \frac{x^{20}}{20\sqrt[4]{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**19/(b*x**4+a)**(1/4),x)
```

```
[Out] Piecewise((2048*a**4*(a + b*x**4)**(3/4)/(21945*b**5) - 512*a**3*x**4*(a +
b*x**4)**(3/4)/(7315*b**4) + 64*a**2*x**8*(a + b*x**4)**(3/4)/(1045*b**3) -
 16*a*x**12*(a + b*x**4)**(3/4)/(285*b**2) + x**16*(a + b*x**4)**(3/4)/(19*
b), Ne(b, 0)), (x**20/(20*a**(1/4)), True))
```

3.1081 $\int \frac{x^{15}}{\sqrt[4]{a+bx^4}}\,dx$

Optimal. Leaf size=80

$$-\frac{a^3\left(a+bx^4\right)^{3/4}}{3b^4}+\frac{3a^2\left(a+bx^4\right)^{7/4}}{7b^4}+\frac{\left(a+bx^4\right)^{15/4}}{15b^4}-\frac{3a\left(a+bx^4\right)^{11/4}}{11b^4}$$

```
[Out] -1/3*a^3*(b*x^4+a)^(3/4)/b^4+3/7*a^2*(b*x^4+a)^(7/4)/b^4-3/11*a*(b*x^4+a)^(
11/4)/b^4+1/15*(b*x^4+a)^(15/4)/b^4
```

Rubi [A] time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{3a^2\left(a+bx^4\right)^{7/4}}{7b^4}-\frac{a^3\left(a+bx^4\right)^{3/4}}{3b^4}+\frac{\left(a+bx^4\right)^{15/4}}{15b^4}-\frac{3a\left(a+bx^4\right)^{11/4}}{11b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^15/(a + b*x^4)^(1/4),x]
```

```
[Out] -(a^3*(a + b*x^4)^(3/4))/(3*b^4) + (3*a^2*(a + b*x^4)^(7/4))/(7*b^4) - (3*a
*(a + b*x^4)^(11/4))/(11*b^4) + (a + b*x^4)^(15/4)/(15*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{15}}{\sqrt[4]{a+bx^4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x^3}{\sqrt[4]{a+bx}}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int \left(-\frac{a^3}{b^3\sqrt[4]{a+bx}}+\frac{3a^2(a+bx)^{3/4}}{b^3}-\frac{3a(a+bx)^{7/4}}{b^3}+\frac{(a+bx)^{11/4}}{b^3}\right)dx, x, x^4\right)\\ &= -\frac{a^3\left(a+bx^4\right)^{3/4}}{3b^4}+\frac{3a^2\left(a+bx^4\right)^{7/4}}{7b^4}-\frac{3a\left(a+bx^4\right)^{11/4}}{11b^4}+\frac{\left(a+bx^4\right)^{15/4}}{15b^4}\end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.62

$$\frac{\left(a+bx^4\right)^{3/4}\left(-128a^3+96a^2bx^4-84ab^2x^8+77b^3x^{12}\right)}{1155b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15/(a + b*x^4)^(1/4),x]
```

```
[Out] ((a + b*x^4)^(3/4)*(-128*a^3 + 96*a^2*b*x^4 - 84*a*b^2*x^8 + 77*b^3*x^12))/
(1155*b^4)
```

fricas [A] time = 0.86, size = 46, normalized size = 0.58

$$\frac{\left(77\,b^3x^{12}-84\,ab^2x^8+96\,a^2bx^4-128\,a^3\right)\left(bx^4+a\right)^{\frac{3}{4}}}{1155\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/1155*(77*b^3*x^12 - 84*a*b^2*x^8 + 96*a^2*b*x^4 - 128*a^3)*(b*x^4 + a)^(3
/4)/b^4
```

giac [A] time = 0.17, size = 57, normalized size = 0.71

$$\frac{77\left(bx^4+a\right)^{\frac{15}{4}}-315\left(bx^4+a\right)^{\frac{11}{4}}a+495\left(bx^4+a\right)^{\frac{7}{4}}a^2-385\left(bx^4+a\right)^{\frac{3}{4}}a^3}{1155\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] 1/1155*(77*(b*x^4 + a)^(15/4) - 315*(b*x^4 + a)^(11/4)*a + 495*(b*x^4 + a)^
(7/4)*a^2 - 385*(b*x^4 + a)^(3/4)*a^3)/b^4
```

maple [A] time = 0.01, size = 47, normalized size = 0.59

$$-\frac{\left(b\,x^4+a\right)^{\frac{3}{4}}\left(-77b^3x^{12}+84a\,b^2x^8-96a^2b\,x^4+128a^3\right)}{1155b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(b*x^4+a)^(1/4),x)
```

```
[Out] -1/1155*(b*x^4+a)^(3/4)*(-77*b^3*x^12+84*a*b^2*x^8-96*a^2*b*x^4+128*a^3)/b^
4
```

maxima [A] time = 1.34, size = 64, normalized size = 0.80

$$\frac{\left(bx^4+a\right)^{\frac{15}{4}}}{15\,b^4}-\frac{3\left(bx^4+a\right)^{\frac{11}{4}}a}{11\,b^4}+\frac{3\left(bx^4+a\right)^{\frac{7}{4}}a^2}{7\,b^4}-\frac{\left(bx^4+a\right)^{\frac{3}{4}}a^3}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] 1/15*(b*x^4 + a)^(15/4)/b^4 - 3/11*(b*x^4 + a)^(11/4)*a/b^4 + 3/7*(b*x^4 +
a)^(7/4)*a^2/b^4 - 1/3*(b*x^4 + a)^(3/4)*a^3/b^4
```

mupad [B] time = 1.13, size = 48, normalized size = 0.60

$$-\left(b\,x^4+a\right)^{3/4}\left(\frac{128\,a^3}{1155\,b^4}-\frac{x^{12}}{15\,b}+\frac{4\,a\,x^8}{55\,b^2}-\frac{32\,a^2\,x^4}{385\,b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(a + b*x^4)^(1/4),x)
```

```
[Out] -(a + b*x^4)^(3/4)*((128*a^3)/(1155*b^4) - x^12/(15*b) + (4*a*x^8)/(55*b^2)
 - (32*a^2*x^4)/(385*b^3))
```

sympy [**A**] time = 14.85, size = 92, normalized size = 1.15

$$\begin{cases} -\frac{128a^3\left(a+bx^4\right)^{\frac{3}{4}}}{1155b^4} + \frac{32a^2x^4\left(a+bx^4\right)^{\frac{3}{4}}}{385b^3} - \frac{4ax^8\left(a+bx^4\right)^{\frac{3}{4}}}{55b^2} + \frac{x^{12}\left(a+bx^4\right)^{\frac{3}{4}}}{15b} & \text{for } b \neq 0 \\ \frac{x^{16}}{16\sqrt[4]{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15/(b*x**4+a)**(1/4),x)
```

```
[Out] Piecewise((-128*a**3*(a + b*x**4)**(3/4)/(1155*b**4) + 32*a**2*x**4*(a + b*
x**4)**(3/4)/(385*b**3) - 4*a*x**8*(a + b*x**4)**(3/4)/(55*b**2) + x**12*(a
 + b*x**4)**(3/4)/(15*b), Ne(b, 0)), (x**16/(16*a**(1/4)), True))
```

3.1082 $\int \frac{x^{11}}{\sqrt[4]{a+bx^4}}\,dx$

Optimal. Leaf size=59

$$\frac{a^2\left(a+bx^4\right)^{3/4}}{3b^3}+\frac{\left(a+bx^4\right)^{11/4}}{11b^3}-\frac{2a\left(a+bx^4\right)^{7/4}}{7b^3}$$

```
[Out] 1/3*a^2*(b*x^4+a)^(3/4)/b^3-2/7*a*(b*x^4+a)^(7/4)/b^3+1/11*(b*x^4+a)^(11/4)
/b^3
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{a^2\left(a+bx^4\right)^{3/4}}{3b^3}+\frac{\left(a+bx^4\right)^{11/4}}{11b^3}-\frac{2a\left(a+bx^4\right)^{7/4}}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + b*x^4)^(1/4),x]
```

```
[Out] (a^2*(a + b*x^4)^(3/4))/(3*b^3) - (2*a*(a + b*x^4)^(7/4))/(7*b^3) + (a + b*
x^4)^(11/4)/(11*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{\sqrt[4]{a+bx^4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x^2}{\sqrt[4]{a+bx}}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int \left(\frac{a^2}{b^2\sqrt[4]{a+bx}}-\frac{2a(a+bx)^{3/4}}{b^2}+\frac{(a+bx)^{7/4}}{b^2}\right)dx, x, x^4\right)\\ &= \frac{a^2\left(a+bx^4\right)^{3/4}}{3b^3}-\frac{2a\left(a+bx^4\right)^{7/4}}{7b^3}+\frac{\left(a+bx^4\right)^{11/4}}{11b^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 39, normalized size = 0.66

$$\frac{\left(a+bx^4\right)^{3/4}\left(32a^2-24abx^4+21b^2x^8\right)}{231b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + b*x^4)^(1/4),x]
```

```
[Out] ((a + b*x^4)^(3/4)*(32*a^2 - 24*a*b*x^4 + 21*b^2*x^8))/(231*b^3)
```

fricas [A] time = 0.66, size = 35, normalized size = 0.59

$$\frac{\left(21\,b^2x^8 - 24\,abx^4 + 32\,a^2\right)\left(bx^4 + a\right)^{\frac{3}{4}}}{231\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/231*(21*b^2*x^8 - 24*a*b*x^4 + 32*a^2)*(b*x^4 + a)^(3/4)/b^3
```

giac [A] time = 0.17, size = 43, normalized size = 0.73

$$\frac{21\left(bx^4 + a\right)^{\frac{11}{4}} - 66\left(bx^4 + a\right)^{\frac{7}{4}}a + 77\left(bx^4 + a\right)^{\frac{3}{4}}a^2}{231\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] 1/231*(21*(b*x^4 + a)^(11/4) - 66*(b*x^4 + a)^(7/4)*a + 77*(b*x^4 + a)^(3/4
)*a^2)/b^3
```

maple [A] time = 0.01, size = 36, normalized size = 0.61

$$\frac{\left(b\,x^4 + a\right)^{\frac{3}{4}}\left(21b^2x^8 - 24ab\,x^4 + 32a^2\right)}{231b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^4+a)^(1/4),x)
```

```
[Out] 1/231*(b*x^4+a)^(3/4)*(21*b^2*x^8-24*a*b*x^4+32*a^2)/b^3
```

maxima [A] time = 1.32, size = 47, normalized size = 0.80

$$\frac{\left(bx^4 + a\right)^{\frac{11}{4}}}{11\,b^3} - \frac{2\left(bx^4 + a\right)^{\frac{7}{4}}a}{7\,b^3} + \frac{\left(bx^4 + a\right)^{\frac{3}{4}}a^2}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] 1/11*(b*x^4 + a)^(11/4)/b^3 - 2/7*(b*x^4 + a)^(7/4)*a/b^3 + 1/3*(b*x^4 + a)
^(3/4)*a^2/b^3
```

mupad [B] time = 1.13, size = 36, normalized size = 0.61

$$\left(b\,x^4 + a\right)^{3/4}\left(\frac{32\,a^2}{231\,b^3} + \frac{x^8}{11\,b} - \frac{8\,a\,x^4}{77\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a + b*x^4)^(1/4),x)
```

```
[Out] (a + b*x^4)^(3/4)*((32*a^2)/(231*b^3) + x^8/(11*b) - (8*a*x^4)/(77*b^2))
```

sympy [**A**] time = 6.35, size = 68, normalized size = 1.15

$$\begin{cases} \frac{32a^2\left(a+bx^4\right)^{\frac{3}{4}}}{231b^3} - \frac{8ax^4\left(a+bx^4\right)^{\frac{3}{4}}}{77b^2} + \frac{x^8\left(a+bx^4\right)^{\frac{3}{4}}}{11b} & \text{for } b \neq 0 \\ \frac{x^{12}}{12\sqrt[4]{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**4+a)**(1/4),x)
```

```
[Out] Piecewise((32*a**2*(a + b*x**4)**(3/4)/(231*b**3) - 8*a*x**4*(a + b*x**4)**
(3/4)/(77*b**2) + x**8*(a + b*x**4)**(3/4)/(11*b), Ne(b, 0)), (x**12/(12*a*
*(1/4)), True))
```

3.1083 $\int \frac{x^7}{\sqrt[4]{a+bx^4}}\,dx$

Optimal. Leaf size=38

$$\frac{\left(a+bx^4\right)^{7/4}}{7b^2}-\frac{a\left(a+bx^4\right)^{3/4}}{3b^2}$$

```
[Out] -1/3*a*(b*x^4+a)^(3/4)/b^2+1/7*(b*x^4+a)^(7/4)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{\left(a+bx^4\right)^{7/4}}{7b^2}-\frac{a\left(a+bx^4\right)^{3/4}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^4)^(1/4),x]
```

```
[Out] -(a*(a + b*x^4)^(3/4))/(3*b^2) + (a + b*x^4)^(7/4)/(7*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{\sqrt[4]{a+bx^4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x}{\sqrt[4]{a+bx}}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int \left(-\frac{a}{b\sqrt[4]{a+bx}}+\frac{(a+bx)^{3/4}}{b}\right)dx, x, x^4\right)\\ &= -\frac{a\left(a+bx^4\right)^{3/4}}{3b^2}+\frac{\left(a+bx^4\right)^{7/4}}{7b^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 0.74

$$\frac{\left(a+bx^4\right)^{3/4}\left(3bx^4-4a\right)}{21b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^4)^(1/4),x]
```

```
[Out] ((a + b*x^4)^(3/4)*(-4*a + 3*b*x^4))/(21*b^2)
```

fricas [A] time = 0.88, size = 24, normalized size = 0.63

$$\frac{\left(3\,bx^4-4\,a\right)\left(bx^4+a\right)^{\frac{3}{4}}}{21\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)^(1/4),x, algorithm="fricas")
[Out] 1/21*(3*b*x^4 - 4*a)*(b*x^4 + a)^(3/4)/b^2
```

giac [A] time = 0.15, size = 29, normalized size = 0.76

$$\frac{3\left(bx^4+a\right)^{\frac{7}{4}}-7\left(bx^4+a\right)^{\frac{3}{4}}a}{21\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)^(1/4),x, algorithm="giac")
[Out] 1/21*(3*(b*x^4 + a)^(7/4) - 7*(b*x^4 + a)^(3/4)*a)/b^2
```

maple [A] time = 0.01, size = 25, normalized size = 0.66

$$-\frac{\left(b\,x^4+a\right)^{\frac{3}{4}}\left(-3b\,x^4+4a\right)}{21b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^4+a)^(1/4),x)
[Out] -1/21*(b*x^4+a)^(3/4)*(-3*b*x^4+4*a)/b^2
```

maxima [A] time = 1.34, size = 30, normalized size = 0.79

$$\frac{\left(bx^4+a\right)^{\frac{7}{4}}}{7\,b^2}-\frac{\left(bx^4+a\right)^{\frac{3}{4}}a}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)^(1/4),x, algorithm="maxima")
[Out] 1/7*(b*x^4 + a)^(7/4)/b^2 - 1/3*(b*x^4 + a)^(3/4)*a/b^2
```

mupad [B] time = 1.11, size = 26, normalized size = 0.68

$$-\left(b\,x^4+a\right)^{3/4}\left(\frac{4\,a}{21\,b^2}-\frac{x^4}{7\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^4)^(1/4),x)
[Out] -(a + b*x^4)^(3/4)*((4*a)/(21*b^2) - x^4/(7*b))
```

sympy [A] time = 2.27, size = 44, normalized size = 1.16

$$\begin{cases} -\frac{4a\left(a+bx^4\right)^{\frac{3}{4}}}{21b^2}+\frac{x^4\left(a+bx^4\right)^{\frac{3}{4}}}{7b} & \text{for } b\neq 0 \\ \frac{x^8}{8\sqrt[4]{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**4+a)**(1/4),x)
```

```
[Out] Piecewise((-4*a*(a + b*x**4)**(3/4)/(21*b**2) + x**4*(a + b*x**4)**(3/4)/(7
*b), Ne(b, 0)), (x**8/(8*a**(1/4)), True))
```

3.1084 $\int \frac{x^3}{\sqrt[4]{a+bx^4}}\,dx$

Optimal. Leaf size=18

$$\frac{\left(a+bx^4\right)^{3/4}}{3b}$$

```
[Out] 1/3*(b*x^4+a)^(3/4)/b
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$\frac{\left(a+bx^4\right)^{3/4}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^4)^(1/4),x]
```

```
[Out] (a + b*x^4)^(3/4)/(3*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^3}{\sqrt[4]{a+bx^4}}\,dx = \frac{\left(a+bx^4\right)^{3/4}}{3b}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{\left(a+bx^4\right)^{3/4}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^4)^(1/4),x]
```

```
[Out] (a + b*x^4)^(3/4)/(3*b)
```

fricas [A] time = 0.78, size = 14, normalized size = 0.78

$$\frac{\left(bx^4+a\right)^{\frac{3}{4}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/3*(b*x^4 + a)^(3/4)/b
```

giac [A] time = 0.17, size = 14, normalized size = 0.78

$$\frac{\left(bx^4+a\right)^{\frac{3}{4}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)^(1/4),x, algorithm="giac")
[Out] 1/3*(b*x^4 + a)^(3/4)/b
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$\frac{\left(b\,x^4+a\right)^{\frac{3}{4}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^4+a)^(1/4),x)
[Out] 1/3*(b*x^4+a)^(3/4)/b
```

maxima [A] time = 1.31, size = 14, normalized size = 0.78

$$\frac{\left(bx^4+a\right)^{\frac{3}{4}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)^(1/4),x, algorithm="maxima")
[Out] 1/3*(b*x^4 + a)^(3/4)/b
```

mupad [B] time = 1.10, size = 14, normalized size = 0.78

$$\frac{\left(b\,x^4+a\right)^{3/4}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^4)^(1/4),x)
[Out] (a + b*x^4)^(3/4)/(3*b)
```

sympy [A] time = 0.90, size = 22, normalized size = 1.22

$$\begin{cases} \frac{\left(a+bx^4\right)^{\frac{3}{4}}}{3b} & \text{for } b \neq 0 \\ \frac{x^4}{4\sqrt[4]{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**4+a)**(1/4),x)
[Out] Piecewise(((a + b*x**4)**(3/4)/(3*b), Ne(b, 0)), (x**4/(4*a**(1/4)), True))
```

3.1085 $\int \frac{1}{x\sqrt[4]{a+bx^4}}\,dx$

Optimal. Leaf size=55

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}$$

```
[Out] 1/2*arctan((b*x^4+a)^(1/4)/a^(1/4))/a^(1/4)-1/2*arctanh((b*x^4+a)^(1/4)/a^(
1/4))/a^(1/4)
```

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {266, 63, 298, 203, 206}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^4)^(1/4)),x]
```

```
[Out] ArcTan[(a + b*x^4)^(1/4)/a^(1/4)]/(2*a^(1/4)) - ArcTanh[(a + b*x^4)^(1/4)/a
^(1/4)]/(2*a^(1/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt[4]{a+bx^4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x\sqrt[4]{a+bx}}\,dx, x, x^4\right)\\ &= \frac{\text{Subst}\left(\int \frac{x^2}{-\frac{a}{b}+\frac{x^4}{b}}\,dx, x, \sqrt[4]{a+bx^4}\right)}{b}\\ &= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{a}-x^2}\,dx, x, \sqrt[4]{a+bx^4}\right)\right)+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{a}+x^2}\,dx, x, \sqrt[4]{a+bx^4}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 48, normalized size = 0.87

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)-\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^4)^(1/4)),x]
```

```
[Out] (ArcTan[(a + b*x^4)^(1/4)/a^(1/4)] - ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)])/(2
*a^(1/4))
```

fricas [B] time = 0.97, size = 81, normalized size = 1.47

$$-\frac{\arctan\left(\frac{\sqrt{\sqrt{bx^4+a}+\sqrt{a}}}{a^{\frac{1}{4}}}-\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}-\frac{\log\left(\left(bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}\right)}{4\,a^{\frac{1}{4}}}+\frac{\log\left(\left(bx^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}\right)}{4\,a^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -arctan(sqrt(sqrt(b*x^4 + a) + sqrt(a))/a^(1/4) - (b*x^4 + a)^(1/4)/a^(1/4)
)/a^(1/4) - 1/4*log((b*x^4 + a)^(1/4) + a^(1/4))/a^(1/4) + 1/4*log((b*x^4 +
 a)^(1/4) - a^(1/4))/a^(1/4)
```

giac [B] time = 0.17, size = 186, normalized size = 3.38

$$-\frac{\sqrt{2}\,(-a)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}+2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{4\,a}-\frac{\sqrt{2}\,(-a)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}-2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{4\,a}+\frac{\sqrt{2}\,(-a)^{\frac{3}{4}}\log\left(\sqrt{2}\left(bx^4\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(-a)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x^4 +
 a)^(1/4))/(-a)^(1/4))/a - 1/4*sqrt(2)*(-a)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4))/a + 1/8*sqrt(2)*(-a)^(3/4
)*log(sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt(-a))/a
```

```
- 1/8*sqrt(2)*(-a)^(3/4)*log(-sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b
*x^4 + a) + sqrt(-a))/a
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^4+a)^(1/4),x)
```

```
[Out] int(1/x/(b*x^4+a)^(1/4),x)
```

maxima [A] time = 2.89, size = 57, normalized size = 1.04

$$\frac{\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{2\,a^{\frac{1}{4}}}+\frac{\log\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{\left(bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{4\,a^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] 1/2*arctan((b*x^4 + a)^(1/4)/a^(1/4))/a^(1/4) + 1/4*log(((b*x^4 + a)^(1/4)
- a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/4)))/a^(1/4)
```

mupad [B] time = 1.16, size = 36, normalized size = 0.65

$$\frac{\operatorname{atan}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)-\operatorname{atanh}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)}{2\,a^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^4)^(1/4)),x)
```

```
[Out] (atan((a + b*x^4)^(1/4)/a^(1/4)) - atanh((a + b*x^4)^(1/4)/a^(1/4)))/(2*a^(
1/4))
```

sympy [C] time = 1.25, size = 37, normalized size = 0.67

$$-\frac{\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\right|\frac{ae^{i\pi}}{bx^4}\right)}{4\sqrt[4]{b}\,x\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**4+a)**(1/4),x)
```

```
[Out] -gamma(1/4)*hyper((1/4, 1/4), (5/4,), a*exp_polar(I*pi)/(b*x**4))/(4*b**(1/
4)*x*gamma(5/4))
```

3.1086 $\int \frac{1}{x^5 \sqrt[4]{a+bx^4}}\,dx$

Optimal. Leaf size=78

$$-\frac{b\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{5/4}}+\frac{b\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{5/4}}-\frac{\left(a+bx^4\right)^{3/4}}{4ax^4}$$

```
[Out] -1/4*(b*x^4+a)^(3/4)/a/x^4-1/8*b*arctan((b*x^4+a)^(1/4)/a^(1/4))/a^(5/4)+1/
8*b*arctanh((b*x^4+a)^(1/4)/a^(1/4))/a^(5/4)
```

Rubi [A] time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {266, 51, 63, 298, 203, 206}

$$-\frac{b\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{5/4}}+\frac{b\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{5/4}}-\frac{\left(a+bx^4\right)^{3/4}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^4)^(1/4)),x]
```

```
[Out] -(a + b*x^4)^(3/4)/(4*a*x^4) - (b*ArcTan[(a + b*x^4)^(1/4)/a^(1/4)])/(8*a^(
5/4)) + (b*ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)])/(8*a^(5/4))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5 \sqrt[4]{a+bx^4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x^2 \sqrt[4]{a+bx}}\,dx, x, x^4\right) \\
&= -\frac{\left(a+bx^4\right)^{3/4}}{4ax^4} - \frac{b\,\text{Subst}\left(\int \frac{1}{x \sqrt[4]{a+bx}}\,dx, x, x^4\right)}{16a} \\
&= -\frac{\left(a+bx^4\right)^{3/4}}{4ax^4} - \frac{\text{Subst}\left(\int \frac{x^2}{-\frac{a}{b}+\frac{x^4}{b}}\,dx, x, \sqrt[4]{a+bx^4}\right)}{4a} \\
&= -\frac{\left(a+bx^4\right)^{3/4}}{4ax^4} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a}-x^2}\,dx, x, \sqrt[4]{a+bx^4}\right)}{8a} - \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a}+x^2}\,dx, x, \sqrt[4]{a+bx^4}\right)}{8a} \\
&= -\frac{\left(a+bx^4\right)^{3/4}}{4ax^4} - \frac{b\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{5/4}} + \frac{b\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{5/4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.47

$$\frac{b\left(a+bx^4\right)^{3/4}\,{}_2F_1\left(\frac{3}{4}, 2; \frac{7}{4}; \frac{bx^4}{a}+1\right)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^4)^(1/4)),x]
```

```
[Out] (b*(a + b*x^4)^(3/4)*Hypergeometric2F1[3/4, 2, 7/4, 1 + (b*x^4)/a])/(3*a^2)
```

fricas [B] time = 0.87, size = 195, normalized size = 2.50

$$\frac{4\,ax^4\left(\frac{b^4}{a^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(bx^4+a\right)^{\frac{1}{4}}ab^3\left(\frac{b^4}{a^5}\right)^{\frac{1}{4}}-\sqrt{a^3b^4\sqrt{\frac{b^4}{a^5}}+\sqrt{bx^4+a}\,b^6}\,a\left(\frac{b^4}{a^5}\right)^{\frac{1}{4}}}{b^4}\right)+ax^4\left(\frac{b^4}{a^5}\right)^{\frac{1}{4}}\log\left(a^4\left(\frac{b^4}{a^5}\right)^{\frac{3}{4}}+\left(bx^4+a\right)^{\frac{1}{4}}b^3\right)-}{16\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/16*(4*a*x^4*(b^4/a^5)^(1/4)*arctan(-((b*x^4 + a)^(1/4)*a*b^3*(b^4/a^5)^(1
/4) - sqrt(a^3*b^4*sqrt(b^4/a^5) + sqrt(b*x^4 + a)*b^6)*a*(b^4/a^5)^(1/4))/
b^4) + a*x^4*(b^4/a^5)^(1/4)*log(a^4*(b^4/a^5)^(3/4) + (b*x^4 + a)^(1/4)*b^
3) - a*x^4*(b^4/a^5)^(1/4)*log(-a^4*(b^4/a^5)^(3/4) + (b*x^4 + a)^(1/4)*b^3
) - 4*(b*x^4 + a)^(3/4))/(a*x^4)
```

giac [B] time = 0.20, size = 219, normalized size = 2.81

$$\frac{2\sqrt{2}(-a)^{\frac{3}{4}}b^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a^2}+\frac{2\sqrt{2}(-a)^{\frac{3}{4}}b^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a^2}+\frac{\sqrt{2}b^2\log\left(\sqrt{2}\left(bx^4+a\right)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^4+a}+\sqrt{}\right.}{(-a)^{\frac{1}{4}}a}}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] 1/32*(2*sqrt(2)*(-a)^(3/4)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(
b*x^4 + a)^(1/4))/(-a)^(1/4))/a^2 + 2*sqrt(2)*(-a)^(3/4)*b^2*arctan(-1/2*sq
rt(2)*(sqrt(2)*(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4))/a^2 + sqrt(2)*
b^2*log(sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt(-a))/
((-a)^(1/4)*a) + sqrt(2)*(-a)^(3/4)*b^2*log(-sqrt(2)*(b*x^4 + a)^(1/4)*(-a)
^(1/4) + sqrt(b*x^4 + a) + sqrt(-a))/a^2 - 8*(b*x^4 + a)^(3/4)*b/(a*x^4))/b
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^4+a)^(1/4),x)
```

```
[Out] int(1/x^5/(b*x^4+a)^(1/4),x)
```

maxima [A] time = 2.94, size = 92, normalized size = 1.18

$$-\frac{b\left(\frac{2\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}+\frac{\log\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{\left(bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}\right)}{16\,a}-\frac{\left(bx^4+a\right)^{\frac{3}{4}}b}{4\left(\left(bx^4+a\right)a-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] -1/16*b*(2*arctan((b*x^4 + a)^(1/4)/a^(1/4))/a^(1/4) + log(((b*x^4 + a)^(1/
4) - a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/4)))/a^(1/4))/a - 1/4*(b*x^4 + a)^(
3/4)*b/((b*x^4 + a)*a - a^2)
```

mupad [B] time = 1.28, size = 58, normalized size = 0.74

$$\frac{b\,\mathrm{atanh}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)}{8\,a^{5/4}}-\frac{b\,\mathrm{atan}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)}{8\,a^{5/4}}-\frac{\left(b\,x^4+a\right)^{3/4}}{4\,a\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^4)^(1/4)),x)
```

```
[Out] (b*atanh((a + b*x^4)^(1/4)/a^(1/4)))/(8*a^(5/4)) - (b*atan((a + b*x^4)^(1/4
)/a^(1/4)))/(8*a^(5/4)) - (a + b*x^4)^(3/4)/(4*a*x^4)
```

sympy [C] time = 2.72, size = 39, normalized size = 0.50

$$-\frac{\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4}, \frac{5}{4} \\ \frac{9}{4}\end{matrix}\right| \frac{ae^{i\pi}}{bx^4}\right)}{4\sqrt[4]{b}\, x^5 \Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**4+a)**(1/4),x)
```

```
[Out] -gamma(5/4)*hyper((1/4, 5/4), (9/4,), a*exp_polar(I*pi)/(b*x**4))/(4*b**(1/
4)*x**5*gamma(9/4))
```

3.1087 $\int \frac{1}{x^9 \sqrt[4]{a+bx^4}}\,dx$

Optimal. Leaf size=104

$$\frac{5b^2 \tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{9/4}} - \frac{5b^2 \tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{9/4}} + \frac{5b\left(a+bx^4\right)^{3/4}}{32a^2x^4} - \frac{\left(a+bx^4\right)^{3/4}}{8ax^8}$$

```
[Out] -1/8*(b*x^4+a)^(3/4)/a/x^8+5/32*b*(b*x^4+a)^(3/4)/a^2/x^4+5/64*b^2*arctan((
b*x^4+a)^(1/4)/a^(1/4))/a^(9/4)-5/64*b^2*arctanh((b*x^4+a)^(1/4)/a^(1/4))/a
^(9/4)
```

Rubi [A] time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {266, 51, 63, 298, 203, 206}

$$\frac{5b^2 \tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{9/4}} - \frac{5b^2 \tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{9/4}} + \frac{5b\left(a+bx^4\right)^{3/4}}{32a^2x^4} - \frac{\left(a+bx^4\right)^{3/4}}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(a + b*x^4)^(1/4)),x]
```

```
[Out] -(a + b*x^4)^(3/4)/(8*a*x^8) + (5*b*(a + b*x^4)^(3/4))/(32*a^2*x^4) + (5*b^
2*ArcTan[(a + b*x^4)^(1/4)/a^(1/4)])/(64*a^(9/4)) - (5*b^2*ArcTanh[(a + b*x
^4)^(1/4)/a^(1/4)])/(64*a^(9/4))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$\int \frac{1}{x^9 \sqrt[4]{a+bx^4}}\,dx = \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x^3 \sqrt[4]{a+bx}}\,dx, x, x^4\right)$$

$$= -\frac{\left(a+bx^4\right)^{3/4}}{8ax^8} - \frac{(5b)\operatorname{Subst}\left(\int \frac{1}{x^2 \sqrt[4]{a+bx}}\,dx, x, x^4\right)}{32a}$$

$$= -\frac{\left(a+bx^4\right)^{3/4}}{8ax^8} + \frac{5b\left(a+bx^4\right)^{3/4}}{32a^2x^4} + \frac{\left(5b^2\right)\operatorname{Subst}\left(\int \frac{1}{x \sqrt[4]{a+bx}}\,dx, x, x^4\right)}{128a^2}$$

$$= -\frac{\left(a+bx^4\right)^{3/4}}{8ax^8} + \frac{5b\left(a+bx^4\right)^{3/4}}{32a^2x^4} + \frac{(5b)\operatorname{Subst}\left(\int \frac{x^2}{-\frac{a}{b}+\frac{x^4}{b}}\,dx, x, \sqrt[4]{a+bx^4}\right)}{32a^2}$$

$$= -\frac{\left(a+bx^4\right)^{3/4}}{8ax^8} + \frac{5b\left(a+bx^4\right)^{3/4}}{32a^2x^4} - \frac{\left(5b^2\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a}-x^2}\,dx, x, \sqrt[4]{a+bx^4}\right)}{64a^2} + \frac{\left(5b^2\right)\text{Subst}}{}$$

$$= -\frac{\left(a+bx^4\right)^{3/4}}{8ax^8} + \frac{5b\left(a+bx^4\right)^{3/4}}{32a^2x^4} + \frac{5b^2\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{9/4}} - \frac{5b^2\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{9/4}}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.38

$$-\frac{b^2\left(a+bx^4\right)^{3/4}\,{}_2F_1\left(\frac{3}{4}, 3; \frac{7}{4}; \frac{bx^4}{a}+1\right)}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^9*(a + b*x^4)^(1/4)),x]
```

```
[Out] -1/3*(b^2*(a + b*x^4)^(3/4)*Hypergeometric2F1[3/4, 3, 7/4, 1 + (b*x^4)/a])/
a^3
```

fricas [B] time = 0.75, size = 219, normalized size = 2.11

$$-\frac{20\,a^2x^8\left(\frac{b^8}{a^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(bx^4+a\right)^{\frac{1}{4}}a^2b^6\left(\frac{b^8}{a^9}\right)^{\frac{1}{4}}-\sqrt{a^5b^8\sqrt{\frac{b^8}{a^9}}+\sqrt{bx^4+a}\,b^{12}}\,a^2\left(\frac{b^8}{a^9}\right)^{\frac{1}{4}}}{b^8}\right)+5\,a^2x^8\left(\frac{b^8}{a^9}\right)^{\frac{1}{4}}\log\left(125\,a^7\left(\frac{b^8}{a^9}\right)^{\frac{3}{4}}+125\right.}{128\,a^2x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/128*(20*a^2*x^8*(b^8/a^9)^(1/4)*arctan(-((b*x^4 + a)^(1/4)*a^2*b^6*(b^8/
a^9)^(1/4) - sqrt(a^5*b^8*sqrt(b^8/a^9) + sqrt(b*x^4 + a)*b^12)*a^2*(b^8/a^
```

```
9)^(1/4))/b^8) + 5*a^2*x^8*(b^8/a^9)^(1/4)*log(125*a^7*(b^8/a^9)^(3/4) + 12
5*(b*x^4 + a)^(1/4)*b^6) - 5*a^2*x^8*(b^8/a^9)^(1/4)*log(-125*a^7*(b^8/a^9)
^(3/4) + 125*(b*x^4 + a)^(1/4)*b^6) - 4*(5*b*x^4 - 4*a)*(b*x^4 + a)^(3/4))/
(a^2*x^8)
```

giac [B] time = 0.24, size = 244, normalized size = 2.35

$$\frac{10\sqrt{2}\,b^3\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}+2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{1}{4}}a^2}+\frac{10\sqrt{2}\,b^3\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}-2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{1}{4}}a^2}+\frac{5\sqrt{2}\,(-a)^{\frac{3}{4}}b^3\log\left(\sqrt{2}\left(bx^4+a\right)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^4+a}+\sqrt{-a}\right.}{a^3}$$

$$256\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] 1/256*(10*sqrt(2)*b^3*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x^4 + a
)^(1/4))/(-a)^(1/4))/((-a)^(1/4)*a^2) + 10*sqrt(2)*b^3*arctan(-1/2*sqrt(2)*
(sqrt(2)*(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4))/((-a)^(1/4)*a^2) + 5
*sqrt(2)*(-a)^(3/4)*b^3*log(sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x
^4 + a) + sqrt(-a))/a^3 + 5*sqrt(2)*b^3*log(-sqrt(2)*(b*x^4 + a)^(1/4)*(-a)
^(1/4) + sqrt(b*x^4 + a) + sqrt(-a))/((-a)^(1/4)*a^2) + 8*(5*(b*x^4 + a)^(7
/4)*b^3 - 9*(b*x^4 + a)^(3/4)*a*b^3)/(a^2*b^2*x^8))/b
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}}x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(b*x^4+a)^(1/4),x)
```

```
[Out] int(1/x^9/(b*x^4+a)^(1/4),x)
```

maxima [A] time = 2.99, size = 128, normalized size = 1.23

$$\frac{5\,b^2\left(\frac{2\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}+\frac{\log\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{\left(bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}\right)}{128\,a^2}+\frac{5\left(bx^4+a\right)^{\frac{7}{4}}b^2-9\left(bx^4+a\right)^{\frac{3}{4}}ab^2}{32\left(\left(bx^4+a\right)^2a^2-2\left(bx^4+a\right)a^3+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] 5/128*b^2*(2*arctan((b*x^4 + a)^(1/4)/a^(1/4))/a^(1/4) + log(((b*x^4 + a)^(
1/4) - a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/4)))/a^(1/4))/a^2 + 1/32*(5*(b*x^
4 + a)^(7/4)*b^2 - 9*(b*x^4 + a)^(3/4)*a*b^2)/((b*x^4 + a)^2*a^2 - 2*(b*x^4
 + a)*a^3 + a^4)
```

mupad [B] time = 1.36, size = 82, normalized size = 0.79

$$\frac{5\,b^2\operatorname{atan}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)}{64\,a^{9/4}}-\frac{9\left(b\,x^4+a\right)^{3/4}}{32\,a\,x^8}+\frac{5\left(b\,x^4+a\right)^{7/4}}{32\,a^2\,x^8}+\frac{b^2\operatorname{atan}\left(\frac{\left(b\,x^4+a\right)^{1/4}\,1\mathrm{i}}{a^{1/4}}\right)5\mathrm{i}}{64\,a^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(a + b*x^4)^(1/4)),x)
```

```
[Out] (5*b^2*atan((a + b*x^4)^(1/4)/a^(1/4)))/(64*a^(9/4)) + (b^2*atan(((a + b*x^
4)^(1/4)*1i)/a^(1/4))*5i)/(64*a^(9/4)) - (9*(a + b*x^4)^(3/4))/(32*a*x^8) +
 (5*(a + b*x^4)^(7/4))/(32*a^2*x^8)
```

sympy [C] time = 3.38, size = 39, normalized size = 0.38

$$-\frac{\Gamma\left(\frac{9}{4}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{4}, \frac{9}{4} \\ \frac{13}{4}\end{matrix}\right| \frac{ae^{i\pi}}{bx^4}\right)}{4\sqrt[4]{b}\, x^9 \Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(b*x**4+a)**(1/4),x)
```

```
[Out] -gamma(9/4)*hyper((1/4, 9/4), (13/4,), a*exp_polar(I*pi)/(b*x**4))/(4*b**(1
/4)*x**9*gamma(13/4))
```

3.1088 $\int \frac{x^{13}}{\sqrt[4]{a+bx^4}}\,dx$

Optimal. Leaf size=152

$$\frac{8a^{7/2}\sqrt[4]{\frac{bx^4}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{39b^{7/2}\sqrt[4]{a+bx^4}}-\frac{8a^3x^2}{39b^3\sqrt[4]{a+bx^4}}+\frac{4a^2x^2\left(a+bx^4\right)^{3/4}}{39b^3}-\frac{10ax^6\left(a+bx^4\right)^{3/4}}{117b^2}+\frac{x^{10}\left(a+bx^4\right)^{3/4}}{13b}$$

```
[Out] -8/39*a^3*x^2/b^3/(b*x^4+a)^(1/4)+4/39*a^2*x^2*(b*x^4+a)^(3/4)/b^3-10/117*a
*x^6*(b*x^4+a)^(3/4)/b^2+1/13*x^10*(b*x^4+a)^(3/4)/b+8/39*a^(7/2)*(1+b*x^4/
a)^(1/4)*(cos(1/2*arctan(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x^2*
b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x^2*b^(1/2)/a^(1/2))),2^(1/2))/b
^(7/2)/(b*x^4+a)^(1/4)
```

Rubi [A] time = 0.10, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {275, 321, 229, 227, 196}

$$\frac{4a^2x^2\left(a+bx^4\right)^{3/4}}{39b^3}-\frac{8a^3x^2}{39b^3\sqrt[4]{a+bx^4}}+\frac{8a^{7/2}\sqrt[4]{\frac{bx^4}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{39b^{7/2}\sqrt[4]{a+bx^4}}-\frac{10ax^6\left(a+bx^4\right)^{3/4}}{117b^2}+\frac{x^{10}\left(a+bx^4\right)^{3/4}}{13b}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/(a + b*x^4)^(1/4),x]
```

```
[Out] (-8*a^3*x^2)/(39*b^3*(a + b*x^4)^(1/4)) + (4*a^2*x^2*(a + b*x^4)^(3/4))/(39
*b^3) - (10*a*x^6*(a + b*x^4)^(3/4))/(117*b^2) + (x^10*(a + b*x^4)^(3/4))/(
13*b) + (8*a^(7/2)*(1 + (b*x^4)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x^2)/Sqr
t[a]]/2, 2])/(39*b^(7/2)*(a + b*x^4)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^{13}}{\sqrt[4]{a+bx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^6}{\sqrt[4]{a+bx^2}}\,dx, x, x^2\right)$$

$$= \frac{x^{10}\left(a+bx^4\right)^{3/4}}{13b} - \frac{(5a)\,\text{Subst}\left(\int \frac{x^4}{\sqrt[4]{a+bx^2}}\,dx, x, x^2\right)}{13b}$$

$$= -\frac{10ax^6\left(a+bx^4\right)^{3/4}}{117b^2} + \frac{x^{10}\left(a+bx^4\right)^{3/4}}{13b} + \frac{\left(10a^2\right)\text{Subst}\left(\int \frac{x^2}{\sqrt[4]{a+bx^2}}\,dx, x, x^2\right)}{39b^2}$$

$$= \frac{4a^2x^2\left(a+bx^4\right)^{3/4}}{39b^3} - \frac{10ax^6\left(a+bx^4\right)^{3/4}}{117b^2} + \frac{x^{10}\left(a+bx^4\right)^{3/4}}{13b} - \frac{\left(4a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx, x, x\right.}{39b^3}$$

$$= \frac{4a^2x^2\left(a+bx^4\right)^{3/4}}{39b^3} - \frac{10ax^6\left(a+bx^4\right)^{3/4}}{117b^2} + \frac{x^{10}\left(a+bx^4\right)^{3/4}}{13b} - \frac{\left(4a^3\sqrt[4]{1+\frac{bx^4}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+}}\right.}{39b^3\sqrt[4]{a+bx^4}}$$

$$= -\frac{8a^3x^2}{39b^3\sqrt[4]{a+bx^4}} + \frac{4a^2x^2\left(a+bx^4\right)^{3/4}}{39b^3} - \frac{10ax^6\left(a+bx^4\right)^{3/4}}{117b^2} + \frac{x^{10}\left(a+bx^4\right)^{3/4}}{13b} + \frac{\left(4a^3\sqrt[4]{1+\right.}\right.}{}$$

$$= -\frac{8a^3x^2}{39b^3\sqrt[4]{a+bx^4}} + \frac{4a^2x^2\left(a+bx^4\right)^{3/4}}{39b^3} - \frac{10ax^6\left(a+bx^4\right)^{3/4}}{117b^2} + \frac{x^{10}\left(a+bx^4\right)^{3/4}}{13b} + \frac{8a^{7/2}\sqrt[4]{1+}}{}$$

Mathematica [C] time = 0.04, size = 91, normalized size = 0.60

$$\frac{x^2\left(-12a^3\sqrt[4]{\frac{bx^4}{a}+1}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^4}{a}\right)+12a^3+2a^2bx^4-ab^2x^8+9b^3x^{12}\right)}{117b^3\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13/(a + b*x^4)^(1/4),x]
```

```
[Out] (x^2*(12*a^3 + 2*a^2*b*x^4 - a*b^2*x^8 + 9*b^3*x^12 - 12*a^3*(1 + (b*x^4)/a
)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -((b*x^4)/a)]))/(117*b^3*(a + b*x^
4)^(1/4))
```

fricas [F] time = 1.18, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^{13}}{\left(bx^4+a\right)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(x^13/(b*x^4 + a)^(1/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{\left(bx^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^4+a)^(1/4),x, algorithm="giac")
[Out] integrate(x^13/(b*x^4 + a)^(1/4), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{\left(b\,x^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(b*x^4+a)^(1/4),x)
[Out] int(x^13/(b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{\left(bx^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^4+a)^(1/4),x, algorithm="maxima")
[Out] integrate(x^13/(b*x^4 + a)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{13}}{\left(b\,x^4+a\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(a + b*x^4)^(1/4),x)
[Out] int(x^13/(a + b*x^4)^(1/4), x)
```

sympy [C] time = 1.63, size = 27, normalized size = 0.18

$$\frac{x^{14}\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{4}, \frac{7}{2} \\ \frac{9}{2}\end{matrix}\right| \frac{bx^4 e^{i\pi}}{a}\right)}{14\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(b*x**4+a)**(1/4),x)
[Out] x**14*hyper((1/4, 7/2), (9/2,), b*x**4*exp_polar(I*pi)/a)/(14*a**(1/4))
```

3.1089 $\int \frac{x^9}{\sqrt[4]{a+bx^4}}\,dx$

Optimal. Leaf size=128

$$-\frac{4a^{5/2}\sqrt[4]{\frac{bx^4}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{15b^{5/2}\sqrt[4]{a+bx^4}}+\frac{4a^2x^2}{15b^2\sqrt[4]{a+bx^4}}-\frac{2ax^2\left(a+bx^4\right)^{3/4}}{15b^2}+\frac{x^6\left(a+bx^4\right)^{3/4}}{9b}$$

```
[Out] 4/15*a^2*x^2/b^2/(b*x^4+a)^(1/4)-2/15*a*x^2*(b*x^4+a)^(3/4)/b^2+1/9*x^6*(b*
x^4+a)^(3/4)/b-4/15*a^(5/2)*(1+b*x^4/a)^(1/4)*(cos(1/2*arctan(x^2*b^(1/2)/a
^(1/2)))^2)^(1/2)/cos(1/2*arctan(x^2*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*ar
ctan(x^2*b^(1/2)/a^(1/2))),2^(1/2))/b^(5/2)/(b*x^4+a)^(1/4)
```

Rubi [A] time = 0.07, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {275, 321, 229, 227, 196}

$$\frac{4a^2x^2}{15b^2\sqrt[4]{a+bx^4}}-\frac{4a^{5/2}\sqrt[4]{\frac{bx^4}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{15b^{5/2}\sqrt[4]{a+bx^4}}-\frac{2ax^2\left(a+bx^4\right)^{3/4}}{15b^2}+\frac{x^6\left(a+bx^4\right)^{3/4}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x^4)^(1/4),x]
```

```
[Out] (4*a^2*x^2)/(15*b^2*(a + b*x^4)^(1/4)) - (2*a*x^2*(a + b*x^4)^(3/4))/(15*b^
2) + (x^6*(a + b*x^4)^(3/4))/(9*b) - (4*a^(5/2)*(1 + (b*x^4)/a)^(1/4)*Ellip
ticE[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(15*b^(5/2)*(a + b*x^4)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9}{\sqrt[4]{a+bx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{\sqrt[4]{a+bx^2}}\,dx, x, x^2\right) \\
&= \frac{x^6\left(a+bx^4\right)^{3/4}}{9b} - \frac{a\,\text{Subst}\left(\int \frac{x^2}{\sqrt[4]{a+bx^2}}\,dx, x, x^2\right)}{3b} \\
&= -\frac{2ax^2\left(a+bx^4\right)^{3/4}}{15b^2} + \frac{x^6\left(a+bx^4\right)^{3/4}}{9b} + \frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx, x, x^2\right)}{15b^2} \\
&= -\frac{2ax^2\left(a+bx^4\right)^{3/4}}{15b^2} + \frac{x^6\left(a+bx^4\right)^{3/4}}{9b} + \frac{\left(2a^2\sqrt[4]{1+\frac{bx^4}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx, x, x^2\right)}{15b^2\sqrt[4]{a+bx^4}} \\
&= \frac{4a^2x^2}{15b^2\sqrt[4]{a+bx^4}} - \frac{2ax^2\left(a+bx^4\right)^{3/4}}{15b^2} + \frac{x^6\left(a+bx^4\right)^{3/4}}{9b} - \frac{\left(2a^2\sqrt[4]{1+\frac{bx^4}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx,\right.}{15b^2\sqrt[4]{a+bx^4}} \\
&= \frac{4a^2x^2}{15b^2\sqrt[4]{a+bx^4}} - \frac{2ax^2\left(a+bx^4\right)^{3/4}}{15b^2} + \frac{x^6\left(a+bx^4\right)^{3/4}}{9b} - \frac{4a^{5/2}\sqrt[4]{1+\frac{bx^4}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{15b^{5/2}\sqrt[4]{a+bx^4}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 80, normalized size = 0.62

$$\frac{x^2\left(6a^2\sqrt[4]{\frac{bx^4}{a}+1}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^4}{a}\right)-6a^2-abx^4+5b^2x^8\right)}{45b^2\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x^4)^(1/4),x]
```

```
[Out] (x^2*(-6*a^2 - a*b*x^4 + 5*b^2*x^8 + 6*a^2*(1 + (b*x^4)/a)^(1/4)*Hypergeome
tric2F1[1/4, 1/2, 3/2, -((b*x^4)/a)]))/(45*b^2*(a + b*x^4)^(1/4))
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^9}{\left(bx^4+a\right)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(x^9/(b*x^4 + a)^(1/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(bx^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)^(1/4),x, algorithm="giac")
[Out] integrate(x^9/(b*x^4 + a)^(1/4), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(b\,x^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^4+a)^(1/4),x)
[Out] int(x^9/(b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(bx^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)^(1/4),x, algorithm="maxima")
[Out] integrate(x^9/(b*x^4 + a)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^9}{\left(b\,x^4+a\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + b*x^4)^(1/4),x)
[Out] int(x^9/(a + b*x^4)^(1/4), x)
```

sympy [C] time = 1.21, size = 27, normalized size = 0.21

$$\frac{x^{10}\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{4}, \frac{5}{2} \\ \frac{7}{2}\end{matrix}\right| \frac{bx^4 e^{i\pi}}{a}\right)}{10\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**4+a)**(1/4),x)
[Out] x**10*hyper((1/4, 5/2), (7/2,), b*x**4*exp_polar(I*pi)/a)/(10*a**(1/4))
```

3.1090 $\int \frac{x^5}{\sqrt[4]{a+bx^4}}\,dx$

Optimal. Leaf size=104

$$\frac{2a^{3/2}\sqrt{\frac{bx^4}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{5b^{3/2}\sqrt[4]{a+bx^4}}+\frac{x^2\left(a+bx^4\right)^{3/4}}{5b}-\frac{2ax^2}{5b\sqrt[4]{a+bx^4}}$$

```
[Out] -2/5*a*x^2/b/(b*x^4+a)^(1/4)+1/5*x^2*(b*x^4+a)^(3/4)/b+2/5*a^(3/2)*(1+b*x^4
/a)^(1/4)*(cos(1/2*arctan(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x^2
*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x^2*b^(1/2)/a^(1/2))),2^(1/2))/
b^(3/2)/(b*x^4+a)^(1/4)
```

Rubi [A] time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {275, 321, 229, 227, 196}

$$\frac{2a^{3/2}\sqrt{\frac{bx^4}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{5b^{3/2}\sqrt[4]{a+bx^4}}+\frac{x^2\left(a+bx^4\right)^{3/4}}{5b}-\frac{2ax^2}{5b\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^4)^(1/4),x]
```

```
[Out] (-2*a*x^2)/(5*b*(a + b*x^4)^(1/4)) + (x^2*(a + b*x^4)^(3/4))/(5*b) + (2*a^(
3/2)*(1 + (b*x^4)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(
5*b^(3/2)*(a + b*x^4)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\sqrt[4]{a+bx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\sqrt[4]{a+bx^2}}\,dx, x, x^2\right) \\
&= \frac{x^2\left(a+bx^4\right)^{3/4}}{5b} - \frac{a\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx, x, x^2\right)}{5b} \\
&= \frac{x^2\left(a+bx^4\right)^{3/4}}{5b} - \frac{\left(a\sqrt[4]{1+\frac{bx^4}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx, x, x^2\right)}{5b\sqrt[4]{a+bx^4}} \\
&= -\frac{2ax^2}{5b\sqrt[4]{a+bx^4}} + \frac{x^2\left(a+bx^4\right)^{3/4}}{5b} + \frac{\left(a\sqrt[4]{1+\frac{bx^4}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx, x, x^2\right)}{5b\sqrt[4]{a+bx^4}} \\
&= -\frac{2ax^2}{5b\sqrt[4]{a+bx^4}} + \frac{x^2\left(a+bx^4\right)^{3/4}}{5b} + \frac{2a^{3/2}\sqrt[4]{1+\frac{bx^4}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|\,2\right)}{5b^{3/2}\sqrt[4]{a+bx^4}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 64, normalized size = 0.62

$$\frac{x^2\left(-a\sqrt[4]{\frac{bx^4}{a}+1}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^4}{a}\right)+a+bx^4\right)}{5b\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^4)^(1/4),x]
```

```
[Out] (x^2*(a + b*x^4 - a*(1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2,
-((b*x^4)/a)]))/(5*b*(a + b*x^4)^(1/4))
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^5}{\left(bx^4+a\right)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(x^5/(b*x^4 + a)^(1/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(bx^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^5/(b*x^4 + a)^(1/4), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(b\,x^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^4+a)^(1/4),x)
[Out] int(x^5/(b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(bx^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)^(1/4),x, algorithm="maxima")
[Out] integrate(x^5/(b*x^4 + a)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\left(b\,x^4+a\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^4)^(1/4),x)
[Out] int(x^5/(a + b*x^4)^(1/4), x)
```

sympy [C] time = 1.55, size = 27, normalized size = 0.26

$$\frac{x^6\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{6\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**4+a)**(1/4),x)
[Out] x**6*hyper((1/4, 3/2), (5/2,), b*x**4*exp_polar(I*pi)/a)/(6*a**(1/4))
```

3.1091 $\int \frac{x}{\sqrt[4]{a+bx^4}}\,dx$

Optimal. Leaf size=74

$$\frac{x^2}{\sqrt[4]{a+bx^4}} - \frac{\sqrt{a}\,\sqrt[4]{\frac{bx^4}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\,\sqrt[4]{a+bx^4}}$$

```
[Out] x^2/(b*x^4+a)^(1/4)-(1+b*x^4/a)^(1/4)*(cos(1/2*arctan(x^2*b^(1/2)/a^(1/2)))
^2)^(1/2)/cos(1/2*arctan(x^2*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x^2
*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)/(b*x^4+a)^(1/4)/b^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {275, 229, 227, 196}

$$\frac{x^2}{\sqrt[4]{a+bx^4}} - \frac{\sqrt{a}\,\sqrt[4]{\frac{bx^4}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\,\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^4)^(1/4),x]
```

```
[Out] x^2/(a + b*x^4)^(1/4) - (Sqrt[a]*(1 + (b*x^4)/a)^(1/4)*EllipticE[ArcTan[(Sq
rt[b]*x^2)/Sqrt[a]]/2, 2])/(Sqrt[b]*(a + b*x^4)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x}{\sqrt[4]{a+bx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx, x, x^2\right)$$
$$= \frac{\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx, x, x^2\right)}{2\sqrt[4]{a+bx^4}}$$
$$= \frac{x^2}{\sqrt[4]{a+bx^4}} - \frac{\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Subst}\left(\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx, x, x^2\right)}{2\sqrt[4]{a+bx^4}}$$
$$= \frac{x^2}{\sqrt[4]{a+bx^4}} - \frac{\sqrt{a}\,\sqrt[4]{1+\frac{bx^4}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\,\sqrt[4]{a+bx^4}}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.69

$$\frac{x^2\sqrt[4]{\frac{bx^4}{a}+1}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^4}{a}\right)}{2\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^4)^(1/4),x]
```

```
[Out] (x^2*(1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -((b*x^4)/a)])/
(2*(a + b*x^4)^(1/4))
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{\left(bx^4+a\right)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(x/(b*x^4 + a)^(1/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x/(b*x^4 + a)^(1/4), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(b\,x^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^4+a)^(1/4),x)
[Out] int(x/(b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)^(1/4),x, algorithm="maxima")
[Out] integrate(x/(b*x^4 + a)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\left(b\,x^4+a\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^4)^(1/4),x)
[Out] int(x/(a + b*x^4)^(1/4), x)
```

sympy [C] time = 1.54, size = 27, normalized size = 0.36

$$\frac{x^2\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{2\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**4+a)**(1/4),x)
[Out] x**2*hyper((1/4, 1/2), (3/2,), b*x**4*exp_polar(I*pi)/a)/(2*a**(1/4))
```

3.1092 $\int \frac{1}{x^3 \sqrt[4]{a+bx^4}}\,dx$

Optimal. Leaf size=104

$$\frac{bx^2}{2a\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{2ax^2} - \frac{\sqrt{b}\sqrt[4]{\frac{bx^4}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}\sqrt[4]{a+bx^4}}$$

```
[Out] 1/2*b*x^2/a/(b*x^4+a)^(1/4)-1/2*(b*x^4+a)^(3/4)/a/x^2-1/2*(1+b*x^4/a)^(1/4)
*(cos(1/2*arctan(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x^2*b^(1/2)/
a^(1/2)))*EllipticE(sin(1/2*arctan(x^2*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)/(
b*x^4+a)^(1/4)/a^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {275, 325, 229, 227, 196}

$$\frac{bx^2}{2a\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{2ax^2} - \frac{\sqrt{b}\sqrt[4]{\frac{bx^4}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^4)^(1/4)),x]
```

```
[Out] (b*x^2)/(2*a*(a + b*x^4)^(1/4)) - (a + b*x^4)^(3/4)/(2*a*x^2) - (Sqrt[b]*(1
 + (b*x^4)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(2*Sqrt[
a]*(a + b*x^4)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

x]

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt[4]{a+bx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt[4]{a+bx^2}}\,dx, x, x^2\right)\\ &= -\frac{\left(a+bx^4\right)^{3/4}}{2ax^2} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx, x, x^2\right)}{4a}\\ &= -\frac{\left(a+bx^4\right)^{3/4}}{2ax^2} + \frac{\left(b\sqrt[4]{1+\frac{bx^4}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx, x, x^2\right)}{4a\sqrt[4]{a+bx^4}}\\ &= \frac{bx^2}{2a\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{2ax^2} - \frac{\left(b\sqrt[4]{1+\frac{bx^4}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx, x, x^2\right)}{4a\sqrt[4]{a+bx^4}}\\ &= \frac{bx^2}{2a\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{2ax^2} - \frac{\sqrt{b}\sqrt[4]{1+\frac{bx^4}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}\sqrt[4]{a+bx^4}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.49

$$-\frac{\sqrt[4]{\frac{bx^4}{a}+1}\;{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};-\frac{bx^4}{a}\right)}{2x^2\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^4)^(1/4)),x]
```

```
[Out] -1/2*((1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, -((b*x^4)/a)]
)/(x^2*(a + b*x^4)^(1/4))
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{3}{4}}}{bx^7+ax^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(3/4)/(b*x^7 + a*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{1}{4}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(1/4)*x^3), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^4+a)^(1/4),x)
[Out] int(1/x^3/(b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{1}{4}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(1/4)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\left(b\,x^4+a\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^4)^(1/4)),x)
[Out] int(1/(x^3*(a + b*x^4)^(1/4)), x)
```

sympy [C] time = 1.77, size = 31, normalized size = 0.30

$$-\frac{{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{1}{2}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**4+a)**(1/4),x)
[Out] -hyper((-1/2, 1/4), (1/2,), b*x**4*exp_polar(I*pi)/a)/(2*a**(1/4)*x**2)
```

3.1093 $\int \frac{1}{x^7 \sqrt[4]{a+bx^4}}\,dx$

Optimal. Leaf size=128

$$\frac{b^{3/2}\sqrt[4]{\frac{bx^4}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{4a^{3/2}\sqrt[4]{a+bx^4}}-\frac{b^2x^2}{4a^2\sqrt[4]{a+bx^4}}+\frac{b\left(a+bx^4\right)^{3/4}}{4a^2x^2}-\frac{\left(a+bx^4\right)^{3/4}}{6ax^6}$$

```
[Out] -1/4*b^2*x^2/a^2/(b*x^4+a)^(1/4)-1/6*(b*x^4+a)^(3/4)/a/x^6+1/4*b*(b*x^4+a)^
(3/4)/a^2/x^2+1/4*b^(3/2)*(1+b*x^4/a)^(1/4)*(cos(1/2*arctan(x^2*b^(1/2)/a^(
1/2)))^2)^(1/2)/cos(1/2*arctan(x^2*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arct
an(x^2*b^(1/2)/a^(1/2))),2^(1/2))/a^(3/2)/(b*x^4+a)^(1/4)
```

Rubi [A] time = 0.07, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {275, 325, 229, 227, 196}

$$-\frac{b^2x^2}{4a^2\sqrt[4]{a+bx^4}}+\frac{b^{3/2}\sqrt[4]{\frac{bx^4}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{4a^{3/2}\sqrt[4]{a+bx^4}}+\frac{b\left(a+bx^4\right)^{3/4}}{4a^2x^2}-\frac{\left(a+bx^4\right)^{3/4}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^4)^(1/4)),x]
```

```
[Out] -(b^2*x^2)/(4*a^2*(a + b*x^4)^(1/4)) - (a + b*x^4)^(3/4)/(6*a*x^6) + (b*(a
+ b*x^4)^(3/4))/(4*a^2*x^2) + (b^(3/2)*(1 + (b*x^4)/a)^(1/4)*EllipticE[ArcT
an[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(4*a^(3/2)*(a + b*x^4)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^7 \sqrt[4]{a+bx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^4 \sqrt[4]{a+bx^2}}\,dx, x, x^2\right) \\
&= -\frac{\left(a+bx^4\right)^{3/4}}{6ax^6} - \frac{b\,\text{Subst}\left(\int \frac{1}{x^2 \sqrt[4]{a+bx^2}}\,dx, x, x^2\right)}{4a} \\
&= -\frac{\left(a+bx^4\right)^{3/4}}{6ax^6} + \frac{b\left(a+bx^4\right)^{3/4}}{4a^2x^2} - \frac{b^2\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx, x, x^2\right)}{8a^2} \\
&= -\frac{\left(a+bx^4\right)^{3/4}}{6ax^6} + \frac{b\left(a+bx^4\right)^{3/4}}{4a^2x^2} - \frac{\left(b^2 \sqrt[4]{1+\frac{bx^4}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx, x, x^2\right)}{8a^2 \sqrt[4]{a+bx^4}} \\
&= -\frac{b^2x^2}{4a^2 \sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{6ax^6} + \frac{b\left(a+bx^4\right)^{3/4}}{4a^2x^2} + \frac{\left(b^2 \sqrt[4]{1+\frac{bx^4}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx, x, x^2\right.}{8a^2 \sqrt[4]{a+bx^4}} \\
&= -\frac{b^2x^2}{4a^2 \sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{6ax^6} + \frac{b\left(a+bx^4\right)^{3/4}}{4a^2x^2} + \frac{b^{3/2} \sqrt[4]{1+\frac{bx^4}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{4a^{3/2} \sqrt[4]{a+bx^4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.40

$$-\frac{\sqrt[4]{\frac{bx^4}{a}+1}\;{}_2F_1\left(-\frac{3}{2}, \frac{1}{4}; -\frac{1}{2}; -\frac{bx^4}{a}\right)}{6x^6 \sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^4)^(1/4)),x]
```

```
[Out] -1/6*((1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[-3/2, 1/4, -1/2, -((b*x^4)/a)
])/(x^6*(a + b*x^4)^(1/4))
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{3}{4}}}{bx^{11}+ax^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(3/4)/(b*x^11 + a*x^7), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{1}{4}}x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)^(1/4),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^(1/4)*x^7), x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}}\,x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^4+a)^(1/4),x)
[Out] int(1/x^7/(b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{1}{4}}x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(1/4)*x^7), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^7\left(b\,x^4+a\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^4)^(1/4)),x)
[Out] int(1/(x^7*(a + b*x^4)^(1/4)), x)
```

sympy [C] time = 2.48, size = 32, normalized size = 0.25

$$-\frac{{}_2F_1\left(\left.\begin{matrix}-\frac{3}{2}, \frac{1}{4} \\ -\frac{1}{2}\end{matrix}\right| \frac{bx^4 e^{i\pi}}{a}\right)}{6\sqrt[4]{a}\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**4+a)**(1/4),x)
[Out] -hyper((-3/2, 1/4), (-1/2,), b*x**4*exp_polar(I*pi)/a)/(6*a**(1/4)*x**6)
```

3.1094 $\int \frac{1}{x^{11}\sqrt[4]{a+bx^4}}\,dx$

Optimal. Leaf size=152

$$-\frac{7b^{5/2}\sqrt[4]{\frac{bx^4}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{40a^{5/2}\sqrt[4]{a+bx^4}}+\frac{7b^3x^2}{40a^3\sqrt[4]{a+bx^4}}-\frac{7b^2\left(a+bx^4\right)^{3/4}}{40a^3x^2}+\frac{7b\left(a+bx^4\right)^{3/4}}{60a^2x^6}-\frac{\left(a+bx^4\right)^{3/4}}{10ax^{10}}$$

```
[Out] 7/40*b^3*x^2/a^3/(b*x^4+a)^(1/4)-1/10*(b*x^4+a)^(3/4)/a/x^10+7/60*b*(b*x^4+
a)^(3/4)/a^2/x^6-7/40*b^2*(b*x^4+a)^(3/4)/a^3/x^2-7/40*b^(5/2)*(1+b*x^4/a)^
(1/4)*(cos(1/2*arctan(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x^2*b^(
1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x^2*b^(1/2)/a^(1/2))),2^(1/2))/a^(5
/2)/(b*x^4+a)^(1/4)
```

Rubi [A] time = 0.10, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {275, 325, 229, 227, 196}

$$\frac{7b^3x^2}{40a^3\sqrt[4]{a+bx^4}}-\frac{7b^2\left(a+bx^4\right)^{3/4}}{40a^3x^2}-\frac{7b^{5/2}\sqrt[4]{\frac{bx^4}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{40a^{5/2}\sqrt[4]{a+bx^4}}+\frac{7b\left(a+bx^4\right)^{3/4}}{60a^2x^6}-\frac{\left(a+bx^4\right)^{3/4}}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^11*(a + b*x^4)^(1/4)),x]
```

```
[Out] (7*b^3*x^2)/(40*a^3*(a + b*x^4)^(1/4)) - (a + b*x^4)^(3/4)/(10*a*x^10) + (7
*b*(a + b*x^4)^(3/4))/(60*a^2*x^6) - (7*b^2*(a + b*x^4)^(3/4))/(40*a^3*x^2)
 - (7*b^(5/2)*(1 + (b*x^4)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]
/2, 2])/(40*a^(5/2)*(a + b*x^4)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{11}\sqrt[4]{a+bx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^6\sqrt[4]{a+bx^2}}\,dx, x, x^2\right)\\
&= -\frac{\left(a+bx^4\right)^{3/4}}{10ax^{10}} - \frac{(7b)\,\text{Subst}\left(\int \frac{1}{x^4\sqrt[4]{a+bx^2}}\,dx, x, x^2\right)}{20a}\\
&= -\frac{\left(a+bx^4\right)^{3/4}}{10ax^{10}} + \frac{7b\left(a+bx^4\right)^{3/4}}{60a^2x^6} + \frac{\left(7b^2\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt[4]{a+bx^2}}\,dx, x, x^2\right)}{40a^2}\\
&= -\frac{\left(a+bx^4\right)^{3/4}}{10ax^{10}} + \frac{7b\left(a+bx^4\right)^{3/4}}{60a^2x^6} - \frac{7b^2\left(a+bx^4\right)^{3/4}}{40a^3x^2} + \frac{\left(7b^3\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx, x, x^2\right)}{80a^3}\\
&= -\frac{\left(a+bx^4\right)^{3/4}}{10ax^{10}} + \frac{7b\left(a+bx^4\right)^{3/4}}{60a^2x^6} - \frac{7b^2\left(a+bx^4\right)^{3/4}}{40a^3x^2} + \frac{\left(7b^3\sqrt[4]{1+\frac{bx^4}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx, x, x^2\right)}{80a^3\sqrt[4]{a+bx^4}}\\
&= \frac{7b^3x^2}{40a^3\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{10ax^{10}} + \frac{7b\left(a+bx^4\right)^{3/4}}{60a^2x^6} - \frac{7b^2\left(a+bx^4\right)^{3/4}}{40a^3x^2} - \frac{\left(7b^3\sqrt[4]{1+\frac{bx^4}{a}}\right)\text{Su}}{80a}\\
&= \frac{7b^3x^2}{40a^3\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{10ax^{10}} + \frac{7b\left(a+bx^4\right)^{3/4}}{60a^2x^6} - \frac{7b^2\left(a+bx^4\right)^{3/4}}{40a^3x^2} - \frac{7b^{5/2}\sqrt[4]{1+\frac{bx^4}{a}}\,E\left(}{40a^{5/2}\sqrt[4]{}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.34

$$-\frac{\sqrt[4]{\frac{bx^4}{a}+1}\;{}_2F_1\left(-\frac{5}{2},\frac{1}{4};-\frac{3}{2};-\frac{bx^4}{a}\right)}{10x^{10}\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^11*(a + b*x^4)^(1/4)),x]
```

```
[Out] -1/10*((1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[-5/2, 1/4, -3/2, -((b*x^4)/a
)])/(x^10*(a + b*x^4)^(1/4))
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{3}{4}}}{bx^{15}+ax^{11}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(3/4)/(b*x^15 + a*x^11), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{1}{4}} x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(b*x^4+a)^(1/4),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^(1/4)*x^11), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}} x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^11/(b*x^4+a)^(1/4),x)
[Out] int(1/x^11/(b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{1}{4}} x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(b*x^4+a)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(1/4)*x^11), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{11}\left(b\,x^4+a\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^11*(a + b*x^4)^(1/4)),x)
[Out] int(1/(x^11*(a + b*x^4)^(1/4)), x)
```

sympy [C] time = 2.42, size = 32, normalized size = 0.21

$$-\frac{{}_2F_1\left(\left.\begin{matrix}-\frac{5}{2}, \frac{1}{4} \\ -\frac{3}{2}\end{matrix}\right| \frac{bx^4 e^{i\pi}}{a}\right)}{10\sqrt[4]{a}\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**11/(b*x**4+a)**(1/4),x)
[Out] -hyper((-5/2, 1/4), (-3/2,), b*x**4*exp_polar(I*pi)/a)/(10*a**(1/4)*x**10)
```

3.1095 $\int \frac{x^8}{\sqrt[4]{a+bx^4}}\,dx$

Optimal. Leaf size=104

$$\frac{5a^2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{9/4}}+\frac{5a^2\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{9/4}}-\frac{5ax\left(a+bx^4\right)^{3/4}}{32b^2}+\frac{x^5\left(a+bx^4\right)^{3/4}}{8b}$$

```
[Out] -5/32*a*x*(b*x^4+a)^(3/4)/b^2+1/8*x^5*(b*x^4+a)^(3/4)/b+5/64*a^2*arctan(b^(
1/4)*x/(b*x^4+a)^(1/4))/b^(9/4)+5/64*a^2*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))
/b^(9/4)
```

Rubi [A] time = 0.03, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {321, 240, 212, 206, 203}

$$\frac{5a^2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{9/4}}+\frac{5a^2\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{9/4}}-\frac{5ax\left(a+bx^4\right)^{3/4}}{32b^2}+\frac{x^5\left(a+bx^4\right)^{3/4}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^4)^(1/4),x]
```

```
[Out] (-5*a*x*(a + b*x^4)^(3/4))/(32*b^2) + (x^5*(a + b*x^4)^(3/4))/(8*b) + (5*a^
2*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(64*b^(9/4)) + (5*a^2*ArcTanh[(b^(
1/4)*x)/(a + b*x^4)^(1/4)])/(64*b^(9/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{\sqrt[4]{a+bx^4}}\,dx &= \frac{x^5\left(a+bx^4\right)^{3/4}}{8b} - \frac{(5a)\int \frac{x^4}{\sqrt[4]{a+bx^4}}\,dx}{8b} \\ &= -\frac{5ax\left(a+bx^4\right)^{3/4}}{32b^2} + \frac{x^5\left(a+bx^4\right)^{3/4}}{8b} + \frac{\left(5a^2\right)\int \frac{1}{\sqrt[4]{a+bx^4}}\,dx}{32b^2} \\ &= -\frac{5ax\left(a+bx^4\right)^{3/4}}{32b^2} + \frac{x^5\left(a+bx^4\right)^{3/4}}{8b} + \frac{\left(5a^2\right)\text{Subst}\left(\int \frac{1}{1-bx^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{32b^2} \\ &= -\frac{5ax\left(a+bx^4\right)^{3/4}}{32b^2} + \frac{x^5\left(a+bx^4\right)^{3/4}}{8b} + \frac{\left(5a^2\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{b}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{64b^2} + \frac{\left(5a^2\right)\text{Subst}\left(\int \right.}{} \\ &= -\frac{5ax\left(a+bx^4\right)^{3/4}}{32b^2} + \frac{x^5\left(a+bx^4\right)^{3/4}}{8b} + \frac{5a^2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{64b^{9/4}} + \frac{5a^2\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{64b^{9/4}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 87, normalized size = 0.84

$$\frac{5a^2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right) + 5a^2\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right) + 2\sqrt[4]{b}\,x\left(a+bx^4\right)^{3/4}\left(4bx^4-5a\right)}{64b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^4)^(1/4),x]
```

```
[Out] (2*b^(1/4)*x*(a + b*x^4)^(3/4)*(-5*a + 4*b*x^4) + 5*a^2*ArcTan[(b^(1/4)*x)/
(a + b*x^4)^(1/4)] + 5*a^2*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(64*b^(9
/4))
```

fricas [B] time = 0.87, size = 228, normalized size = 2.19

$$\frac{20\,b^2\left(\frac{a^8}{b^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(bx^4+a\right)^{\frac{1}{4}}a^6b^2\left(\frac{a^8}{b^9}\right)^{\frac{1}{4}} - b^2x\left(\frac{a^8}{b^9}\right)^{\frac{1}{4}}\sqrt{\frac{a^8b^5x^2\sqrt{\frac{a^8}{b^9}} + \sqrt{bx^4+a}\,a^{12}}{x^2}}}{a^8x}\right) + 5\,b^2\left(\frac{a^8}{b^9}\right)^{\frac{1}{4}}\log\left(\frac{125\left(b^7x\left(\frac{a^8}{b^9}\right)^{\frac{3}{4}} + \left(bx^4+a\right)^{\frac{1}{4}}a^6\right)}{x}\right) - 5}{128\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/128*(20*b^2*(a^8/b^9)^(1/4)*arctan(-((b*x^4 + a)^(1/4)*a^6*b^2*(a^8/b^9)^
(1/4) - b^2*x*(a^8/b^9)^(1/4)*sqrt((a^8*b^5*x^2*sqrt(a^8/b^9) + sqrt(b*x^4
+ a)*a^12)/x^2))/(a^8*x)) + 5*b^2*(a^8/b^9)^(1/4)*log(125*(b^7*x*(a^8/b^9)^
(3/4) + (b*x^4 + a)^(1/4)*a^6)/x) - 5*b^2*(a^8/b^9)^(1/4)*log(-125*(b^7*x*(
a^8/b^9)^(3/4) - (b*x^4 + a)^(1/4)*a^6)/x) + 4*(4*b*x^5 - 5*a*x)*(b*x^4 + a
)^(3/4))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(bx^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^8/(b*x^4 + a)^(1/4), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(b\,x^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^4+a)^(1/4),x)
```

```
[Out] int(x^8/(b*x^4+a)^(1/4),x)
```

maxima [A] time = 3.13, size = 151, normalized size = 1.45

$$-\frac{5\,a^2\left(\frac{2\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)}{128\,b^2}+\frac{\frac{9\left(bx^4+a\right)^{\frac{3}{4}}a^2b}{x^3}-\frac{5\left(bx^4+a\right)^{\frac{7}{4}}a^2}{x^7}}{32\left(b^4-\frac{2\left(bx^4+a\right)b^3}{x^4}+\frac{\left(bx^4+a\right)^2b^2}{x^8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] -5/128*a^2*(2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) + log(-(b^(1/4)
 - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/4))/b^2 + 1/3
2*(9*(b*x^4 + a)^(3/4)*a^2*b/x^3 - 5*(b*x^4 + a)^(7/4)*a^2/x^7)/(b^4 - 2*(b
*x^4 + a)*b^3/x^4 + (b*x^4 + a)^2*b^2/x^8)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^8}{\left(b\,x^4+a\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^4)^(1/4),x)
```

```
[Out] int(x^8/(a + b*x^4)^(1/4), x)
```

sympy [C] time = 2.17, size = 37, normalized size = 0.36

$$\frac{x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[4]{a}\,\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**4+a)**(1/4),x)
```

```
[Out] x**9*gamma(9/4)*hyper((1/4, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**
(1/4)*gamma(13/4))
```

3.1096 $\int \frac{x^4}{\sqrt[4]{a+bx^4}}\,dx$

Optimal. Leaf size=78

$$-\frac{a\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8b^{5/4}}-\frac{a\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8b^{5/4}}+\frac{x\left(a+bx^4\right)^{3/4}}{4b}$$

```
[Out] 1/4*x*(b*x^4+a)^(3/4)/b-1/8*a*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(5/4)-1/8
*a*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(5/4)
```

Rubi [A] time = 0.02, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {321, 240, 212, 206, 203}

$$-\frac{a\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8b^{5/4}}-\frac{a\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8b^{5/4}}+\frac{x\left(a+bx^4\right)^{3/4}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^4)^(1/4),x]
```

```
[Out] (x*(a + b*x^4)^(3/4))/(4*b) - (a*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(8*
b^(5/4)) - (a*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(8*b^(5/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt[4]{a+bx^4}}\,dx &= \frac{x\left(a+bx^4\right)^{3/4}}{4b} - \frac{a\int \frac{1}{\sqrt[4]{a+bx^4}}\,dx}{4b}\\ &= \frac{x\left(a+bx^4\right)^{3/4}}{4b} - \frac{a\,\text{Subst}\left(\int \frac{1}{1-bx^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{4b}\\ &= \frac{x\left(a+bx^4\right)^{3/4}}{4b} - \frac{a\,\text{Subst}\left(\int \frac{1}{1-\sqrt{b}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{8b} - \frac{a\,\text{Subst}\left(\int \frac{1}{1+\sqrt{b}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{8b}\\ &= \frac{x\left(a+bx^4\right)^{3/4}}{4b} - \frac{a\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8b^{5/4}} - \frac{a\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8b^{5/4}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 73, normalized size = 0.94

$$\frac{2\sqrt[4]{b}x\left(a+bx^4\right)^{3/4} - a\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right) - a\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^4)^(1/4),x]
```

```
[Out] (2*b^(1/4)*x*(a + b*x^4)^(3/4) - a*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)] -
a*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(8*b^(5/4))
```

fricas [B] time = 1.06, size = 206, normalized size = 2.64

$$-\frac{4\,b\left(\frac{a^4}{b^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(bx^4+a\right)^{\frac{1}{4}}a^3b\left(\frac{a^4}{b^5}\right)^{\frac{1}{4}} - bx\left(\frac{a^4}{b^5}\right)^{\frac{1}{4}}\sqrt{\frac{a^4b^3x^2\sqrt{\frac{a^4}{b^5}}+\sqrt{bx^4+a}\,a^6}{x^2}}}{a^4x}\right) + b\left(\frac{a^4}{b^5}\right)^{\frac{1}{4}}\log\left(\frac{b^4x\left(\frac{a^4}{b^5}\right)^{\frac{3}{4}}+\left(bx^4+a\right)^{\frac{1}{4}}a^3}{x}\right) - b\left(\frac{a^4}{b^5}\right)^{\frac{1}{4}}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/16*(4*b*(a^4/b^5)^(1/4)*arctan(-((b*x^4 + a)^(1/4)*a^3*b*(a^4/b^5)^(1/4)
 - b*x*(a^4/b^5)^(1/4)*sqrt((a^4*b^3*x^2*sqrt(a^4/b^5) + sqrt(b*x^4 + a)*a^
6)/x^2))/(a^4*x)) + b*(a^4/b^5)^(1/4)*log((b^4*x*(a^4/b^5)^(3/4) + (b*x^4 +
 a)^(1/4)*a^3)/x) - b*(a^4/b^5)^(1/4)*log(-(b^4*x*(a^4/b^5)^(3/4) - (b*x^4
+ a)^(1/4)*a^3)/x) - 4*(b*x^4 + a)^(3/4)*x)/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(b*x^4 + a)^(1/4), x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(b\,x^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^4+a)^(1/4),x)
```

```
[Out] int(x^4/(b*x^4+a)^(1/4),x)
```

maxima [A] time = 2.93, size = 108, normalized size = 1.38

$$\frac{a\left(\frac{2\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)}{16\,b}-\frac{\left(bx^4+a\right)^{\frac{3}{4}}a}{4\left(b^2-\frac{\left(bx^4+a\right)b}{x^4}\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] 1/16*a*(2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) + log(-(b^(1/4) - (
b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/4))/b - 1/4*(b*x^
4 + a)^(3/4)*a/((b^2 - (b*x^4 + a)*b/x^4)*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\left(b\,x^4+a\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^4)^(1/4),x)
```

```
[Out] int(x^4/(a + b*x^4)^(1/4), x)
```

sympy [C] time = 1.55, size = 37, normalized size = 0.47

$$\frac{x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[4]{a}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**4+a)**(1/4),x)
```

```
[Out] x**5*gamma(5/4)*hyper((1/4, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
1/4)*gamma(9/4))
```

3.1097 $\int \frac{1}{\sqrt[4]{a+bx^4}}\,dx$

Optimal. Leaf size=57

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}$$

```
[Out] 1/2*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(1/4)+1/2*arctanh(b^(1/4)*x/(b*x^4+
a)^(1/4))/b^(1/4)
```

Rubi [A] time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {240, 212, 206, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(-1/4),x]
```

```
[Out] ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(1/4)) + ArcTanh[(b^(1/4)*x)/(a
+ b*x^4)^(1/4)]/(2*b^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a+bx^4}}\,dx = \text{Subst}\left(\int \frac{1}{1-bx^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt{b}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{b}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}$$

Mathematica [A] time = 0.03, size = 76, normalized size = 1.33

$$\frac{-\log\left(1-\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)+\log\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}+1\right)+2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{4\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(-1/4),x]
```

```
[Out] (2*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)] - Log[1 - (b^(1/4)*x)/(a + b*x^4)^
(1/4)] + Log[1 + (b^(1/4)*x)/(a + b*x^4)^(1/4)])/(4*b^(1/4))
```

fricas [B] time = 0.61, size = 106, normalized size = 1.86

$$\frac{\arctan\left(\frac{x\sqrt{\frac{\sqrt{b}x^2+\sqrt{bx^4+a}}{x^2}}}{b^{\frac{1}{4}}} - \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}}}{x}\right)}{b^{\frac{1}{4}}} + \frac{\log\left(\frac{b^{\frac{1}{4}}x+\left(bx^4+a\right)^{\frac{1}{4}}}{x}\right)}{4\,b^{\frac{1}{4}}} - \frac{\log\left(-\frac{b^{\frac{1}{4}}x-\left(bx^4+a\right)^{\frac{1}{4}}}{x}\right)}{4\,b^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] arctan((x*sqrt((sqrt(b)*x^2 + sqrt(b*x^4 + a))/x^2)/b^(1/4) - (b*x^4 + a)^(
1/4)/b^(1/4))/x)/b^(1/4) + 1/4*log((b^(1/4)*x + (b*x^4 + a)^(1/4))/x)/b^(1/
4) - 1/4*log(-(b^(1/4)*x - (b*x^4 + a)^(1/4))/x)/b^(1/4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(-1/4), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(1/4),x)
```

```
[Out] int(1/(b*x^4+a)^(1/4),x)
```

maxima [A] time = 2.99, size = 68, normalized size = 1.19

$$-\frac{\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{2\,b^{\frac{1}{4}}}-\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}\right)}{4\,b^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] -1/2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) - 1/4*log(-(b^(1/4) - (b
*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/4)
```

mupad [B] time = 1.07, size = 37, normalized size = 0.65

$$\frac{x\left(\frac{b\,x^4}{a}+1\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ -\frac{b\,x^4}{a}\right)}{\left(b\,x^4+a\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^4)^(1/4),x)
```

```
[Out] (x*((b*x^4)/a + 1)^(1/4)*hypergeom([1/4, 1/4], 5/4, -(b*x^4)/a))/(a + b*x^4
)^(1/4)
```

sympy [C] time = 1.23, size = 36, normalized size = 0.63

$$\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[4]{a}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(1/4),x)
```

```
[Out] x*gamma(1/4)*hyper((1/4, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(1/4
)*gamma(5/4))
```

3.1098 $\int \frac{1}{x^4 \sqrt[4]{a+bx^4}}\,dx$

Optimal. Leaf size=21

$$-\frac{\left(a+bx^4\right)^{3/4}}{3ax^3}$$

```
[Out] -1/3*(b*x^4+a)^(3/4)/a/x^3
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {264}

$$-\frac{\left(a+bx^4\right)^{3/4}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^4)^(1/4)),x]
```

```
[Out] -(a + b*x^4)^(3/4)/(3*a*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4 \sqrt[4]{a+bx^4}}\,dx = -\frac{\left(a+bx^4\right)^{3/4}}{3ax^3}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$-\frac{\left(a+bx^4\right)^{3/4}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^4)^(1/4)),x]
```

```
[Out] -1/3*(a + b*x^4)^(3/4)/(a*x^3)
```

fricas [A] time = 0.77, size = 17, normalized size = 0.81

$$-\frac{\left(bx^4+a\right)^{\frac{3}{4}}}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/3*(b*x^4 + a)^(3/4)/(a*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)^(1/4),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^(1/4)*x^4), x)
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$-\frac{\left(b\,x^4+a\right)^{\frac{3}{4}}}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^4+a)^(1/4),x)
[Out] -1/3*(b*x^4+a)^(3/4)/a/x^3
```

maxima [A] time = 1.27, size = 17, normalized size = 0.81

$$-\frac{\left(bx^4+a\right)^{\frac{3}{4}}}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)^(1/4),x, algorithm="maxima")
[Out] -1/3*(b*x^4 + a)^(3/4)/(a*x^3)
```

mupad [B] time = 1.09, size = 17, normalized size = 0.81

$$-\frac{\left(b\,x^4+a\right)^{3/4}}{3\,a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^4)^(1/4)),x)
[Out] -(a + b*x^4)^(3/4)/(3*a*x^3)
```

sympy [A] time = 1.06, size = 31, normalized size = 1.48

$$\frac{b^{\frac{3}{4}}\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{4a\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**4+a)**(1/4),x)
[Out] b**(3/4)*(a/(b*x**4) + 1)**(3/4)*gamma(-3/4)/(4*a*gamma(1/4))
```

3.1099 $\int \frac{1}{x^8 \sqrt[4]{a+bx^4}}\,dx$

Optimal. Leaf size=44

$$\frac{4b\left(a+bx^4\right)^{3/4}}{21a^2x^3}-\frac{\left(a+bx^4\right)^{3/4}}{7ax^7}$$

```
[Out] -1/7*(b*x^4+a)^(3/4)/a/x^7+4/21*b*(b*x^4+a)^(3/4)/a^2/x^3
```

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$\frac{4b\left(a+bx^4\right)^{3/4}}{21a^2x^3}-\frac{\left(a+bx^4\right)^{3/4}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(a + b*x^4)^(1/4)),x]
```

```
[Out] -(a + b*x^4)^(3/4)/(7*a*x^7) + (4*b*(a + b*x^4)^(3/4))/(21*a^2*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^8 \sqrt[4]{a+bx^4}}\,dx &= -\frac{\left(a+bx^4\right)^{3/4}}{7ax^7}-\frac{(4b)\int \frac{1}{x^4 \sqrt[4]{a+bx^4}}\,dx}{7a}\\ &= -\frac{\left(a+bx^4\right)^{3/4}}{7ax^7}+\frac{4b\left(a+bx^4\right)^{3/4}}{21a^2x^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.70

$$\frac{\left(a+bx^4\right)^{3/4}\left(4bx^4-3a\right)}{21a^2x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(a + b*x^4)^(1/4)),x]
```

```
[Out] ((a + b*x^4)^(3/4)*(-3*a + 4*b*x^4))/(21*a^2*x^7)
```

fricas [A] time = 0.56, size = 27, normalized size = 0.61

$$\frac{\left(4\,bx^4-3\,a\right)\left(bx^4+a\right)^{\frac{3}{4}}}{21\,a^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/21*(4*b*x^4 - 3*a)*(b*x^4 + a)^(3/4)/(a^2*x^7)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{1}{4}} x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(1/4)*x^8), x)
```

maple [A] time = 0.00, size = 28, normalized size = 0.64

$$-\frac{\left(b\,x^4+a\right)^{\frac{3}{4}}\left(-4b\,x^4+3a\right)}{21a^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(b*x^4+a)^(1/4),x)
```

```
[Out] -1/21*(b*x^4+a)^(3/4)*(-4*b*x^4+3*a)/a^2/x^7
```

maxima [A] time = 1.34, size = 35, normalized size = 0.80

$$\frac{\frac{7\left(bx^4+a\right)^{\frac{3}{4}}b}{x^3}-\frac{3\left(bx^4+a\right)^{\frac{7}{4}}}{x^7}}{21\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] 1/21*(7*(b*x^4 + a)^(3/4)*b/x^3 - 3*(b*x^4 + a)^(7/4)/x^7)/a^2
```

mupad [B] time = 1.16, size = 36, normalized size = 0.82

$$-\frac{3\,a\left(b\,x^4+a\right)^{3/4}-4\,b\,x^4\left(b\,x^4+a\right)^{3/4}}{21\,a^2\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(a + b*x^4)^(1/4)),x)
```

```
[Out] -(3*a*(a + b*x^4)^(3/4) - 4*b*x^4*(a + b*x^4)^(3/4))/(21*a^2*x^7)
```

sympy [A] time = 2.77, size = 70, normalized size = 1.59

$$-\frac{3b^{\frac{3}{4}}\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{16ax^4\Gamma\left(\frac{1}{4}\right)}+\frac{b^{\frac{7}{4}}\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{4a^2\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(b*x**4+a)**(1/4),x)
```

```
[Out] -3*b**(3/4)*(a/(b*x**4) + 1)**(3/4)*gamma(-7/4)/(16*a*x**4*gamma(1/4)) + b*
**(7/4)*(a/(b*x**4) + 1)**(3/4)*gamma(-7/4)/(4*a**2*gamma(1/4))
```

3.1100 $\int \frac{1}{x^{12}\sqrt[4]{a+bx^4}}\,dx$

Optimal. Leaf size=68

$$-\frac{32b^2\left(a+bx^4\right)^{3/4}}{231a^3x^3}+\frac{8b\left(a+bx^4\right)^{3/4}}{77a^2x^7}-\frac{\left(a+bx^4\right)^{3/4}}{11ax^{11}}$$

```
[Out] -1/11*(b*x^4+a)^(3/4)/a/x^11+8/77*b*(b*x^4+a)^(3/4)/a^2/x^7-32/231*b^2*(b*x
^4+a)^(3/4)/a^3/x^3
```

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$-\frac{32b^2\left(a+bx^4\right)^{3/4}}{231a^3x^3}+\frac{8b\left(a+bx^4\right)^{3/4}}{77a^2x^7}-\frac{\left(a+bx^4\right)^{3/4}}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^12*(a + b*x^4)^(1/4)),x]
```

```
[Out] -(a + b*x^4)^(3/4)/(11*a*x^11) + (8*b*(a + b*x^4)^(3/4))/(77*a^2*x^7) - (32
*b^2*(a + b*x^4)^(3/4))/(231*a^3*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{12}\sqrt[4]{a+bx^4}}\,dx &= -\frac{\left(a+bx^4\right)^{3/4}}{11ax^{11}}-\frac{(8b)\int \frac{1}{x^8\sqrt[4]{a+bx^4}}\,dx}{11a}\\ &= -\frac{\left(a+bx^4\right)^{3/4}}{11ax^{11}}+\frac{8b\left(a+bx^4\right)^{3/4}}{77a^2x^7}+\frac{\left(32b^2\right)\int \frac{1}{x^4\sqrt[4]{a+bx^4}}\,dx}{77a^2}\\ &= -\frac{\left(a+bx^4\right)^{3/4}}{11ax^{11}}+\frac{8b\left(a+bx^4\right)^{3/4}}{77a^2x^7}-\frac{32b^2\left(a+bx^4\right)^{3/4}}{231a^3x^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.62

$$-\frac{\left(a+bx^4\right)^{3/4}\left(21a^2-24abx^4+32b^2x^8\right)}{231a^3x^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^12*(a + b*x^4)^(1/4)),x]
```

```
[Out] -1/231*((a + b*x^4)^(3/4)*(21*a^2 - 24*a*b*x^4 + 32*b^2*x^8))/(a^3*x^11)
```

fricas [A] time = 0.81, size = 38, normalized size = 0.56

$$-\frac{\left(32\,b^2x^8-24\,abx^4+21\,a^2\right)\left(bx^4+a\right)^{\frac{3}{4}}}{231\,a^3x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^12/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/231*(32*b^2*x^8 - 24*a*b*x^4 + 21*a^2)*(b*x^4 + a)^(3/4)/(a^3*x^11)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{1}{4}}x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^12/(b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(1/4)*x^12), x)
```

maple [A] time = 0.01, size = 39, normalized size = 0.57

$$-\frac{\left(b\,x^4+a\right)^{\frac{3}{4}}\left(32b^2x^8-24ab\,x^4+21a^2\right)}{231a^3x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^12/(b*x^4+a)^(1/4),x)
```

```
[Out] -1/231*(b*x^4+a)^(3/4)*(32*b^2*x^8-24*a*b*x^4+21*a^2)/a^3/x^11
```

maxima [A] time = 1.33, size = 52, normalized size = 0.76

$$-\frac{\frac{77\left(bx^4+a\right)^{\frac{3}{4}}b^2}{x^3}-\frac{66\left(bx^4+a\right)^{\frac{7}{4}}b}{x^7}+\frac{21\left(bx^4+a\right)^{\frac{11}{4}}}{x^{11}}}{231\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^12/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] -1/231*(77*(b*x^4 + a)^(3/4)*b^2/x^3 - 66*(b*x^4 + a)^(7/4)*b/x^7 + 21*(b*x
^4 + a)^(11/4)/x^11)/a^3
```

mupad [B] time = 1.24, size = 56, normalized size = 0.82

$$-\frac{21\,a^2\left(b\,x^4+a\right)^{3/4}+32\,b^2\,x^8\left(b\,x^4+a\right)^{3/4}-24\,a\,b\,x^4\left(b\,x^4+a\right)^{3/4}}{231\,a^3\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^12*(a + b*x^4)^(1/4)),x)
```

```
[Out] -(21*a^2*(a + b*x^4)^(3/4) + 32*b^2*x^8*(a + b*x^4)^(3/4) - 24*a*b*x^4*(a +
 b*x^4)^(3/4))/(231*a^3*x^11)
```

sympy [**B**] time = 3.07, size = 406, normalized size = 5.97

$$\frac{21a^4b^{\frac{19}{4}}\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{11}{4}\right)}{64a^5b^4x^8\Gamma\left(\frac{1}{4}\right)+128a^4b^5x^{12}\Gamma\left(\frac{1}{4}\right)+64a^3b^6x^{16}\Gamma\left(\frac{1}{4}\right)}+\frac{18a^3b^{\frac{23}{4}}x^4\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{11}{4}\right)}{64a^5b^4x^8\Gamma\left(\frac{1}{4}\right)+128a^4b^5x^{12}\Gamma\left(\frac{1}{4}\right)+64a^3b^6x^{16}\Gamma\left(\frac{1}{4}\right)}+\frac{}{64a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**12/(b*x**4+a)**(1/4),x)
```

```
[Out] 21*a**4*b**(19/4)*(a/(b*x**4) + 1)**(3/4)*gamma(-11/4)/(64*a**5*b**4*x**8*g
amma(1/4) + 128*a**4*b**5*x**12*gamma(1/4) + 64*a**3*b**6*x**16*gamma(1/4))
 + 18*a**3*b**(23/4)*x**4*(a/(b*x**4) + 1)**(3/4)*gamma(-11/4)/(64*a**5*b**
4*x**8*gamma(1/4) + 128*a**4*b**5*x**12*gamma(1/4) + 64*a**3*b**6*x**16*gam
ma(1/4)) + 5*a**2*b**(27/4)*x**8*(a/(b*x**4) + 1)**(3/4)*gamma(-11/4)/(64*a
**5*b**4*x**8*gamma(1/4) + 128*a**4*b**5*x**12*gamma(1/4) + 64*a**3*b**6*x*
*16*gamma(1/4)) + 40*a*b**(31/4)*x**12*(a/(b*x**4) + 1)**(3/4)*gamma(-11/4)
/(64*a**5*b**4*x**8*gamma(1/4) + 128*a**4*b**5*x**12*gamma(1/4) + 64*a**3*b
**6*x**16*gamma(1/4)) + 32*b**(35/4)*x**16*(a/(b*x**4) + 1)**(3/4)*gamma(-1
1/4)/(64*a**5*b**4*x**8*gamma(1/4) + 128*a**4*b**5*x**12*gamma(1/4) + 64*a*
*3*b**6*x**16*gamma(1/4))
```

3.1101 $\int \frac{1}{x^{16}\sqrt[4]{a+bx^4}}\,dx$

Optimal. Leaf size=92

$$\frac{128b^3\left(a+bx^4\right)^{3/4}}{1155a^4x^3}-\frac{32b^2\left(a+bx^4\right)^{3/4}}{385a^3x^7}+\frac{4b\left(a+bx^4\right)^{3/4}}{55a^2x^{11}}-\frac{\left(a+bx^4\right)^{3/4}}{15ax^{15}}$$

```
[Out] -1/15*(b*x^4+a)^(3/4)/a/x^15+4/55*b*(b*x^4+a)^(3/4)/a^2/x^11-32/385*b^2*(b*
x^4+a)^(3/4)/a^3/x^7+128/1155*b^3*(b*x^4+a)^(3/4)/a^4/x^3
```

Rubi [A] time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$\frac{128b^3\left(a+bx^4\right)^{3/4}}{1155a^4x^3}-\frac{32b^2\left(a+bx^4\right)^{3/4}}{385a^3x^7}+\frac{4b\left(a+bx^4\right)^{3/4}}{55a^2x^{11}}-\frac{\left(a+bx^4\right)^{3/4}}{15ax^{15}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^16*(a + b*x^4)^(1/4)),x]
```

```
[Out] -(a + b*x^4)^(3/4)/(15*a*x^15) + (4*b*(a + b*x^4)^(3/4))/(55*a^2*x^11) - (3
2*b^2*(a + b*x^4)^(3/4))/(385*a^3*x^7) + (128*b^3*(a + b*x^4)^(3/4))/(1155*
a^4*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{16}\sqrt[4]{a+bx^4}}\,dx &= -\frac{\left(a+bx^4\right)^{3/4}}{15ax^{15}}-\frac{(4b)\int \frac{1}{x^{12}\sqrt[4]{a+bx^4}}\,dx}{5a}\\ &= -\frac{\left(a+bx^4\right)^{3/4}}{15ax^{15}}+\frac{4b\left(a+bx^4\right)^{3/4}}{55a^2x^{11}}+\frac{\left(32b^2\right)\int \frac{1}{x^8\sqrt[4]{a+bx^4}}\,dx}{55a^2}\\ &= -\frac{\left(a+bx^4\right)^{3/4}}{15ax^{15}}+\frac{4b\left(a+bx^4\right)^{3/4}}{55a^2x^{11}}-\frac{32b^2\left(a+bx^4\right)^{3/4}}{385a^3x^7}-\frac{\left(128b^3\right)\int \frac{1}{x^4\sqrt[4]{a+bx^4}}\,dx}{385a^3}\\ &= -\frac{\left(a+bx^4\right)^{3/4}}{15ax^{15}}+\frac{4b\left(a+bx^4\right)^{3/4}}{55a^2x^{11}}-\frac{32b^2\left(a+bx^4\right)^{3/4}}{385a^3x^7}+\frac{128b^3\left(a+bx^4\right)^{3/4}}{1155a^4x^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.58

$$\frac{\left(a+bx^4\right)^{3/4}\left(-77a^3+84a^2bx^4-96ab^2x^8+128b^3x^{12}\right)}{1155a^4x^{15}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^16*(a + b*x^4)^(1/4)),x]
```

```
[Out] ((a + b*x^4)^(3/4)*(-77*a^3 + 84*a^2*b*x^4 - 96*a*b^2*x^8 + 128*b^3*x^12))/
(1155*a^4*x^15)
```

fricas [A] time = 0.79, size = 49, normalized size = 0.53

$$\frac{\left(128\,b^3x^{12} - 96\,ab^2x^8 + 84\,a^2bx^4 - 77\,a^3\right)\left(bx^4 + a\right)^{\frac{3}{4}}}{1155\,a^4x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^16/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/1155*(128*b^3*x^12 - 96*a*b^2*x^8 + 84*a^2*b*x^4 - 77*a^3)*(b*x^4 + a)^(3
/4)/(a^4*x^15)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4 + a\right)^{\frac{1}{4}}x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^16/(b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(1/4)*x^16), x)
```

maple [A] time = 0.01, size = 50, normalized size = 0.54

$$-\frac{\left(b\,x^4 + a\right)^{\frac{3}{4}}\left(-128b^3x^{12} + 96a\,b^2x^8 - 84a^2b\,x^4 + 77a^3\right)}{1155a^4x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^16/(b*x^4+a)^(1/4),x)
```

```
[Out] -1/1155*(b*x^4+a)^(3/4)*(-128*b^3*x^12+96*a*b^2*x^8-84*a^2*b*x^4+77*a^3)/x^
15/a^4
```

maxima [A] time = 1.35, size = 69, normalized size = 0.75

$$\frac{\frac{385\left(bx^4+a\right)^{\frac{3}{4}}b^3}{x^3} - \frac{495\left(bx^4+a\right)^{\frac{7}{4}}b^2}{x^7} + \frac{315\left(bx^4+a\right)^{\frac{11}{4}}b}{x^{11}} - \frac{77\left(bx^4+a\right)^{\frac{15}{4}}}{x^{15}}}{1155\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^16/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] 1/1155*(385*(b*x^4 + a)^(3/4)*b^3/x^3 - 495*(b*x^4 + a)^(7/4)*b^2/x^7 + 315
*(b*x^4 + a)^(11/4)*b/x^11 - 77*(b*x^4 + a)^(15/4)/x^15)/a^4
```

mupad [B] time = 1.35, size = 76, normalized size = 0.83

$$\frac{4\,b\left(b\,x^4 + a\right)^{3/4}}{55\,a^2\,x^{11}} - \frac{\left(b\,x^4 + a\right)^{3/4}}{15\,a\,x^{15}} + \frac{128\,b^3\left(b\,x^4 + a\right)^{3/4}}{1155\,a^4\,x^3} - \frac{32\,b^2\left(b\,x^4 + a\right)^{3/4}}{385\,a^3\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^16*(a + b*x^4)^(1/4)),x)
```

```
[Out] (4*b*(a + b*x^4)^(3/4))/(55*a^2*x^11) - (a + b*x^4)^(3/4)/(15*a*x^15) + (12
8*b^3*(a + b*x^4)^(3/4))/(1155*a^4*x^3) - (32*b^2*(a + b*x^4)^(3/4))/(385*a
^3*x^7)
```

sympy [B] time = 5.36, size = 692, normalized size = 7.52

$$-\frac{231a^6b^{\frac{39}{4}}\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{15}{4}\right)}{256a^7b^9x^{12}\Gamma\left(\frac{1}{4}\right)+768a^6b^{10}x^{16}\Gamma\left(\frac{1}{4}\right)+768a^5b^{11}x^{20}\Gamma\left(\frac{1}{4}\right)+256a^4b^{12}x^{24}\Gamma\left(\frac{1}{4}\right)}-\frac{441a}{256a^7b^9x^{12}\Gamma\left(\frac{1}{4}\right)+768a^6b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**16/(b*x**4+a)**(1/4),x)
```

```
[Out] -231*a**6*b**(39/4)*(a/(b*x**4) + 1)**(3/4)*gamma(-15/4)/(256*a**7*b**9*x**
12*gamma(1/4) + 768*a**6*b**10*x**16*gamma(1/4) + 768*a**5*b**11*x**20*gamm
a(1/4) + 256*a**4*b**12*x**24*gamma(1/4)) - 441*a**5*b**(43/4)*x**4*(a/(b*x
**4) + 1)**(3/4)*gamma(-15/4)/(256*a**7*b**9*x**12*gamma(1/4) + 768*a**6*b*
*10*x**16*gamma(1/4) + 768*a**5*b**11*x**20*gamma(1/4) + 256*a**4*b**12*x**
24*gamma(1/4)) - 225*a**4*b**(47/4)*x**8*(a/(b*x**4) + 1)**(3/4)*gamma(-15/
4)/(256*a**7*b**9*x**12*gamma(1/4) + 768*a**6*b**10*x**16*gamma(1/4) + 768*
a**5*b**11*x**20*gamma(1/4) + 256*a**4*b**12*x**24*gamma(1/4)) + 45*a**3*b*
*(51/4)*x**12*(a/(b*x**4) + 1)**(3/4)*gamma(-15/4)/(256*a**7*b**9*x**12*gam
ma(1/4) + 768*a**6*b**10*x**16*gamma(1/4) + 768*a**5*b**11*x**20*gamma(1/4)
 + 256*a**4*b**12*x**24*gamma(1/4)) + 540*a**2*b**(55/4)*x**16*(a/(b*x**4)
+ 1)**(3/4)*gamma(-15/4)/(256*a**7*b**9*x**12*gamma(1/4) + 768*a**6*b**10*x
**16*gamma(1/4) + 768*a**5*b**11*x**20*gamma(1/4) + 256*a**4*b**12*x**24*ga
mma(1/4)) + 864*a*b**(59/4)*x**20*(a/(b*x**4) + 1)**(3/4)*gamma(-15/4)/(256
*a**7*b**9*x**12*gamma(1/4) + 768*a**6*b**10*x**16*gamma(1/4) + 768*a**5*b*
*11*x**20*gamma(1/4) + 256*a**4*b**12*x**24*gamma(1/4)) + 384*b**(63/4)*x**
24*(a/(b*x**4) + 1)**(3/4)*gamma(-15/4)/(256*a**7*b**9*x**12*gamma(1/4) + 7
68*a**6*b**10*x**16*gamma(1/4) + 768*a**5*b**11*x**20*gamma(1/4) + 256*a**4
*b**12*x**24*gamma(1/4))
```

3.1102 $\int \frac{1}{x^{20}\sqrt[4]{a+bx^4}}\,dx$

Optimal. Leaf size=116

$$-\frac{2048b^4\left(a+bx^4\right)^{3/4}}{21945a^5x^3}+\frac{512b^3\left(a+bx^4\right)^{3/4}}{7315a^4x^7}-\frac{64b^2\left(a+bx^4\right)^{3/4}}{1045a^3x^{11}}+\frac{16b\left(a+bx^4\right)^{3/4}}{285a^2x^{15}}-\frac{\left(a+bx^4\right)^{3/4}}{19ax^{19}}$$

```
[Out] -1/19*(b*x^4+a)^(3/4)/a/x^19+16/285*b*(b*x^4+a)^(3/4)/a^2/x^15-64/1045*b^2*
(b*x^4+a)^(3/4)/a^3/x^11+512/7315*b^3*(b*x^4+a)^(3/4)/a^4/x^7-2048/21945*b^
4*(b*x^4+a)^(3/4)/a^5/x^3
```

Rubi [A] time = 0.04, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$-\frac{2048b^4\left(a+bx^4\right)^{3/4}}{21945a^5x^3}+\frac{512b^3\left(a+bx^4\right)^{3/4}}{7315a^4x^7}-\frac{64b^2\left(a+bx^4\right)^{3/4}}{1045a^3x^{11}}+\frac{16b\left(a+bx^4\right)^{3/4}}{285a^2x^{15}}-\frac{\left(a+bx^4\right)^{3/4}}{19ax^{19}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^20*(a + b*x^4)^(1/4)),x]
```

```
[Out] -(a + b*x^4)^(3/4)/(19*a*x^19) + (16*b*(a + b*x^4)^(3/4))/(285*a^2*x^15) -
(64*b^2*(a + b*x^4)^(3/4))/(1045*a^3*x^11) + (512*b^3*(a + b*x^4)^(3/4))/(7
315*a^4*x^7) - (2048*b^4*(a + b*x^4)^(3/4))/(21945*a^5*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{20}\sqrt[4]{a+bx^4}}\,dx &= -\frac{\left(a+bx^4\right)^{3/4}}{19ax^{19}}-\frac{(16b)\int \frac{1}{x^{16}\sqrt[4]{a+bx^4}}\,dx}{19a}\\
&= -\frac{\left(a+bx^4\right)^{3/4}}{19ax^{19}}+\frac{16b\left(a+bx^4\right)^{3/4}}{285a^2x^{15}}+\frac{\left(64b^2\right)\int \frac{1}{x^{12}\sqrt[4]{a+bx^4}}\,dx}{95a^2}\\
&= -\frac{\left(a+bx^4\right)^{3/4}}{19ax^{19}}+\frac{16b\left(a+bx^4\right)^{3/4}}{285a^2x^{15}}-\frac{64b^2\left(a+bx^4\right)^{3/4}}{1045a^3x^{11}}-\frac{\left(512b^3\right)\int \frac{1}{x^8\sqrt[4]{a+bx^4}}\,dx}{1045a^3}\\
&= -\frac{\left(a+bx^4\right)^{3/4}}{19ax^{19}}+\frac{16b\left(a+bx^4\right)^{3/4}}{285a^2x^{15}}-\frac{64b^2\left(a+bx^4\right)^{3/4}}{1045a^3x^{11}}+\frac{512b^3\left(a+bx^4\right)^{3/4}}{7315a^4x^7}+\frac{\left(2048b^4\right)\int -}{7315}\\
&= -\frac{\left(a+bx^4\right)^{3/4}}{19ax^{19}}+\frac{16b\left(a+bx^4\right)^{3/4}}{285a^2x^{15}}-\frac{64b^2\left(a+bx^4\right)^{3/4}}{1045a^3x^{11}}+\frac{512b^3\left(a+bx^4\right)^{3/4}}{7315a^4x^7}-\frac{2048b^4\left(a+b\right.}{21945a^5x}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 64, normalized size = 0.55

$$-\frac{\left(a+bx^4\right)^{3/4}\left(1155a^4-1232a^3bx^4+1344a^2b^2x^8-1536ab^3x^{12}+2048b^4x^{16}\right)}{21945a^5x^{19}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^20*(a + b*x^4)^(1/4)),x]
```

```
[Out] -1/21945*((a + b*x^4)^(3/4)*(1155*a^4 - 1232*a^3*b*x^4 + 1344*a^2*b^2*x^8 -
 1536*a*b^3*x^12 + 2048*b^4*x^16))/(a^5*x^19)
```

fricas [A] time = 0.81, size = 60, normalized size = 0.52

$$-\frac{\left(2048\,b^4x^{16}-1536\,ab^3x^{12}+1344\,a^2b^2x^8-1232\,a^3bx^4+1155\,a^4\right)\left(bx^4+a\right)^{\frac{3}{4}}}{21945\,a^5x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^20/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/21945*(2048*b^4*x^16 - 1536*a*b^3*x^12 + 1344*a^2*b^2*x^8 - 1232*a^3*b*x
^4 + 1155*a^4)*(b*x^4 + a)^(3/4)/(a^5*x^19)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{1}{4}}x^{20}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^20/(b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(1/4)*x^20), x)
```

maple [A] time = 0.01, size = 61, normalized size = 0.53

$$-\frac{\left(b\,x^4+a\right)^{\frac{3}{4}}\left(2048x^{16}b^4-1536a\,x^{12}b^3+1344a^2x^8b^2-1232a^3x^4b+1155a^4\right)}{21945a^5x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^20/(b*x^4+a)^(1/4),x)
```

```
[Out] -1/21945*(b*x^4+a)^(3/4)*(2048*b^4*x^16-1536*a*b^3*x^12+1344*a^2*b^2*x^8-12
32*a^3*b*x^4+1155*a^4)/x^19/a^5
```

maxima [A] time = 1.33, size = 86, normalized size = 0.74

$$-\frac{\frac{7315\left(bx^4+a\right)^{\frac{3}{4}}b^4}{x^3}-\frac{12540\left(bx^4+a\right)^{\frac{7}{4}}b^3}{x^7}+\frac{11970\left(bx^4+a\right)^{\frac{11}{4}}b^2}{x^{11}}-\frac{5852\left(bx^4+a\right)^{\frac{15}{4}}b}{x^{15}}+\frac{1155\left(bx^4+a\right)^{\frac{19}{4}}}{x^{19}}}{21945\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^20/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] -1/21945*(7315*(b*x^4 + a)^(3/4)*b^4/x^3 - 12540*(b*x^4 + a)^(7/4)*b^3/x^7
 + 11970*(b*x^4 + a)^(11/4)*b^2/x^11 - 5852*(b*x^4 + a)^(15/4)*b/x^15 + 1155
 *(b*x^4 + a)^(19/4)/x^19)/a^5
```

mupad [B] time = 1.49, size = 96, normalized size = 0.83

$$\frac{16\,b\left(b\,x^4+a\right)^{3/4}}{285\,a^2\,x^{15}}-\frac{\left(b\,x^4+a\right)^{3/4}}{19\,a\,x^{19}}-\frac{2048\,b^4\left(b\,x^4+a\right)^{3/4}}{21945\,a^5\,x^3}+\frac{512\,b^3\left(b\,x^4+a\right)^{3/4}}{7315\,a^4\,x^7}-\frac{64\,b^2\left(b\,x^4+a\right)^{3/4}}{1045\,a^3\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^20*(a + b*x^4)^(1/4)),x)
```

```
[Out] (16*b*(a + b*x^4)^(3/4))/(285*a^2*x^15) - (a + b*x^4)^(3/4)/(19*a*x^19) - (
2048*b^4*(a + b*x^4)^(3/4))/(21945*a^5*x^3) + (512*b^3*(a + b*x^4)^(3/4))/(
7315*a^4*x^7) - (64*b^2*(a + b*x^4)^(3/4))/(1045*a^3*x^11)
```

sympy [B] time = 8.48, size = 1046, normalized size = 9.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**20/(b*x**4+a)**(1/4),x)
```

```
[Out] 3465*a**8*b**(67/4)*(a/(b*x**4) + 1)**(3/4)*gamma(-19/4)/(1024*a**9*b**16*x
**16*gamma(1/4) + 4096*a**8*b**17*x**20*gamma(1/4) + 6144*a**7*b**18*x**24*
gamma(1/4) + 4096*a**6*b**19*x**28*gamma(1/4) + 1024*a**5*b**20*x**32*gamma
(1/4)) + 10164*a**7*b**(71/4)*x**4*(a/(b*x**4) + 1)**(3/4)*gamma(-19/4)/(10
24*a**9*b**16*x**16*gamma(1/4) + 4096*a**8*b**17*x**20*gamma(1/4) + 6144*a*
*7*b**18*x**24*gamma(1/4) + 4096*a**6*b**19*x**28*gamma(1/4) + 1024*a**5*b*
*20*x**32*gamma(1/4)) + 10038*a**6*b**(75/4)*x**8*(a/(b*x**4) + 1)**(3/4)*g
amma(-19/4)/(1024*a**9*b**16*x**16*gamma(1/4) + 4096*a**8*b**17*x**20*gamma
(1/4) + 6144*a**7*b**18*x**24*gamma(1/4) + 4096*a**6*b**19*x**28*gamma(1/4)
 + 1024*a**5*b**20*x**32*gamma(1/4)) + 3204*a**5*b**(79/4)*x**12*(a/(b*x**4
) + 1)**(3/4)*gamma(-19/4)/(1024*a**9*b**16*x**16*gamma(1/4) + 4096*a**8*b*
*17*x**20*gamma(1/4) + 6144*a**7*b**18*x**24*gamma(1/4) + 4096*a**6*b**19*x
**28*gamma(1/4) + 1024*a**5*b**20*x**32*gamma(1/4)) + 585*a**4*b**(83/4)*x*
*16*(a/(b*x**4) + 1)**(3/4)*gamma(-19/4)/(1024*a**9*b**16*x**16*gamma(1/4)
+ 4096*a**8*b**17*x**20*gamma(1/4) + 6144*a**7*b**18*x**24*gamma(1/4) + 409
6*a**6*b**19*x**28*gamma(1/4) + 1024*a**5*b**20*x**32*gamma(1/4)) + 9360*a*
*3*b**(87/4)*x**20*(a/(b*x**4) + 1)**(3/4)*gamma(-19/4)/(1024*a**9*b**16*x*
*16*gamma(1/4) + 4096*a**8*b**17*x**20*gamma(1/4) + 6144*a**7*b**18*x**24*g
amma(1/4) + 4096*a**6*b**19*x**28*gamma(1/4) + 1024*a**5*b**20*x**32*gamma(
1/4)) + 22464*a**2*b**(91/4)*x**24*(a/(b*x**4) + 1)**(3/4)*gamma(-19/4)/(10
24*a**9*b**16*x**16*gamma(1/4) + 4096*a**8*b**17*x**20*gamma(1/4) + 6144*a*
*7*b**18*x**24*gamma(1/4) + 4096*a**6*b**19*x**28*gamma(1/4) + 1024*a**5*b*
*20*x**32*gamma(1/4)) + 19968*a*b**(95/4)*x**28*(a/(b*x**4) + 1)**(3/4)*gam
ma(-19/4)/(1024*a**9*b**16*x**16*gamma(1/4) + 4096*a**8*b**17*x**20*gamma(1
/4) + 6144*a**7*b**18*x**24*gamma(1/4) + 4096*a**6*b**19*x**28*gamma(1/4) +
 1024*a**5*b**20*x**32*gamma(1/4)) + 6144*b**(99/4)*x**32*(a/(b*x**4) + 1)*
*(3/4)*gamma(-19/4)/(1024*a**9*b**16*x**16*gamma(1/4) + 4096*a**8*b**17*x**
20*gamma(1/4) + 6144*a**7*b**18*x**24*gamma(1/4) + 4096*a**6*b**19*x**28*ga
mma(1/4) + 1024*a**5*b**20*x**32*gamma(1/4))
```

3.1103 $\int \frac{x^{10}}{\sqrt[4]{a+bx^4}}\,dx$

Optimal. Leaf size=129

$$\frac{7a^{5/2}x\sqrt[4]{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{40b^{5/2}\sqrt[4]{a+bx^4}}+\frac{7a^2x^3}{40b^2\sqrt[4]{a+bx^4}}-\frac{7ax^3\left(a+bx^4\right)^{3/4}}{60b^2}+\frac{x^7\left(a+bx^4\right)^{3/4}}{10b}$$

```
[Out] 7/40*a^2*x^3/b^2/(b*x^4+a)^(1/4)-7/60*a*x^3*(b*x^4+a)^(3/4)/b^2+1/10*x^7*(b
*x^4+a)^(3/4)/b+7/40*a^(5/2)*(1+a/b/x^4)^(1/4)*x*(cos(1/2*arccot(x^2*b^(1/2
)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2
*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))/b^(5/2)/(b*x^4+a)^(1/4)
```

Rubi [A] time = 0.06, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {321, 310, 281, 335, 275, 196}

$$\frac{7a^2x^3}{40b^2\sqrt[4]{a+bx^4}}+\frac{7a^{5/2}x\sqrt[4]{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{40b^{5/2}\sqrt[4]{a+bx^4}}-\frac{7ax^3\left(a+bx^4\right)^{3/4}}{60b^2}+\frac{x^7\left(a+bx^4\right)^{3/4}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x^4)^(1/4),x]
```

```
[Out] (7*a^2*x^3)/(40*b^2*(a + b*x^4)^(1/4)) - (7*a*x^3*(a + b*x^4)^(3/4))/(60*b^
2) + (x^7*(a + b*x^4)^(3/4))/(10*b) + (7*a^(5/2)*(1 + a/(b*x^4))^(1/4)*x*El
lipticE[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(40*b^(5/2)*(a + b*x^4)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 281

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Dist[(x*(1 + a/(b*x^4)
)^(1/4))/(b*(a + b*x^4)^(1/4)), Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x] /
; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 310

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Dist[a/2, Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{x^{10}}{\sqrt[4]{a+bx^4}}\,dx = \frac{x^7\left(a+bx^4\right)^{3/4}}{10b} - \frac{(7a)\int \frac{x^6}{\sqrt[4]{a+bx^4}}\,dx}{10b}$$

$$= -\frac{7ax^3\left(a+bx^4\right)^{3/4}}{60b^2} + \frac{x^7\left(a+bx^4\right)^{3/4}}{10b} + \frac{\left(7a^2\right)\int \frac{x^2}{\sqrt[4]{a+bx^4}}\,dx}{20b^2}$$

$$= \frac{7a^2x^3}{40b^2\sqrt[4]{a+bx^4}} - \frac{7ax^3\left(a+bx^4\right)^{3/4}}{60b^2} + \frac{x^7\left(a+bx^4\right)^{3/4}}{10b} - \frac{\left(7a^3\right)\int \frac{x^2}{\left(a+bx^4\right)^{5/4}}\,dx}{40b^2}$$

$$= \frac{7a^2x^3}{40b^2\sqrt[4]{a+bx^4}} - \frac{7ax^3\left(a+bx^4\right)^{3/4}}{60b^2} + \frac{x^7\left(a+bx^4\right)^{3/4}}{10b} - \frac{\left(7a^3\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{5/4}x^3}\,dx}{40b^3\sqrt[4]{a+bx^4}}$$

$$= \frac{7a^2x^3}{40b^2\sqrt[4]{a+bx^4}} - \frac{7ax^3\left(a+bx^4\right)^{3/4}}{60b^2} + \frac{x^7\left(a+bx^4\right)^{3/4}}{10b} + \frac{\left(7a^3\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{5/4}}\,dx\right.}{40b^3\sqrt[4]{a+bx^4}}$$

$$= \frac{7a^2x^3}{40b^2\sqrt[4]{a+bx^4}} - \frac{7ax^3\left(a+bx^4\right)^{3/4}}{60b^2} + \frac{x^7\left(a+bx^4\right)^{3/4}}{10b} + \frac{\left(7a^3\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx\right.}{80b^3\sqrt[4]{a+bx^4}}$$

$$= \frac{7a^2x^3}{40b^2\sqrt[4]{a+bx^4}} - \frac{7ax^3\left(a+bx^4\right)^{3/4}}{60b^2} + \frac{x^7\left(a+bx^4\right)^{3/4}}{10b} + \frac{7a^{5/2}\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{40b^{5/2}\sqrt[4]{a+bx^4}}$$

Mathematica [C] time = 0.04, size = 80, normalized size = 0.62

$$\frac{x^3\left(7a^2\sqrt[4]{\frac{bx^4}{a}+1}\;{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^4}{a}\right) - 7a^2 - abx^4 + 6b^2x^8\right)}{60b^2\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x^4)^(1/4),x]
```

```
[Out] (x^3*(-7*a^2 - a*b*x^4 + 6*b^2*x^8 + 7*a^2*(1 + (b*x^4)/a)^(1/4)*Hypergeome
tric2F1[1/4, 3/4, 7/4, -((b*x^4)/a)]))/(60*b^2*(a + b*x^4)^(1/4))
```

fricas [F] time = 1.18, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^{10}}{\left(bx^4+a\right)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(x^10/(b*x^4 + a)^(1/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\left(bx^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^10/(b*x^4 + a)^(1/4), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\left(b\,x^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x^4+a)^(1/4),x)
```

```
[Out] int(x^10/(b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\left(bx^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^10/(b*x^4 + a)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{10}}{\left(b\,x^4+a\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a + b*x^4)^(1/4),x)
```

```
[Out] int(x^10/(a + b*x^4)^(1/4), x)
```

sympy [C] time = 1.41, size = 37, normalized size = 0.29

$$\frac{x^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4}, \frac{11}{4} \\ \frac{15}{4}\end{matrix}\right| \frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[4]{a}\,\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b*x**4+a)**(1/4),x)
```

```
[Out] x**11*gamma(11/4)*hyper((1/4, 11/4), (15/4,), b*x**4*exp_polar(I*pi)/a)/(4*
a**(1/4)*gamma(15/4))
```

3.1104 $\int \frac{x^6}{\sqrt[4]{a+bx^4}}\,dx$

Optimal. Leaf size=105

$$-\frac{a^{3/2}x\sqrt[4]{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{4b^{3/2}\sqrt[4]{a+bx^4}}+\frac{x^3\left(a+bx^4\right)^{3/4}}{6b}-\frac{ax^3}{4b\sqrt[4]{a+bx^4}}$$

```
[Out] -1/4*a*x^3/b/(b*x^4+a)^(1/4)+1/6*x^3*(b*x^4+a)^(3/4)/b-1/4*a^(3/2)*(1+a/b/x
^4)^(1/4)*x*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x
^2*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2)
)/b^(3/2)/(b*x^4+a)^(1/4)
```

Rubi [A] time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {321, 310, 281, 335, 275, 196}

$$-\frac{a^{3/2}x\sqrt[4]{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{4b^{3/2}\sqrt[4]{a+bx^4}}+\frac{x^3\left(a+bx^4\right)^{3/4}}{6b}-\frac{ax^3}{4b\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^4)^(1/4),x]
```

```
[Out] -(a*x^3)/(4*b*(a + b*x^4)^(1/4)) + (x^3*(a + b*x^4)^(3/4))/(6*b) - (a^(3/2)
*(1 + a/(b*x^4))^(1/4)*x*EllipticE[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(4*
b^(3/2)*(a + b*x^4)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 281

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Dist[(x*(1 + a/(b*x^4)
)^(1/4))/(b*(a + b*x^4)^(1/4)), Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x] /
; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 310

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Dist[a/2, Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{x^6}{\sqrt[4]{a+bx^4}}\,dx = \frac{x^3\left(a+bx^4\right)^{3/4}}{6b} - \frac{a\int \frac{x^2}{\sqrt[4]{a+bx^4}}\,dx}{2b}$$

$$= -\frac{ax^3}{4b\sqrt[4]{a+bx^4}} + \frac{x^3\left(a+bx^4\right)^{3/4}}{6b} + \frac{a^2\int \frac{x^2}{\left(a+bx^4\right)^{5/4}}\,dx}{4b}$$

$$= -\frac{ax^3}{4b\sqrt[4]{a+bx^4}} + \frac{x^3\left(a+bx^4\right)^{3/4}}{6b} + \frac{\left(a^2\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{5/4}x^3}\,dx}{4b^2\sqrt[4]{a+bx^4}}$$

$$= -\frac{ax^3}{4b\sqrt[4]{a+bx^4}} + \frac{x^3\left(a+bx^4\right)^{3/4}}{6b} - \frac{\left(a^2\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{4b^2\sqrt[4]{a+bx^4}}$$

$$= -\frac{ax^3}{4b\sqrt[4]{a+bx^4}} + \frac{x^3\left(a+bx^4\right)^{3/4}}{6b} - \frac{\left(a^2\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx, x, \frac{1}{x^2}\right)}{8b^2\sqrt[4]{a+bx^4}}$$

$$= -\frac{ax^3}{4b\sqrt[4]{a+bx^4}} + \frac{x^3\left(a+bx^4\right)^{3/4}}{6b} - \frac{a^{3/2}\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{4b^{3/2}\sqrt[4]{a+bx^4}}$$

Mathematica [C] time = 0.02, size = 64, normalized size = 0.61

$$\frac{x^3\left(-a\sqrt[4]{\frac{bx^4}{a}+1}\;{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^4}{a}\right)+a+bx^4\right)}{6b\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^4)^(1/4),x]
```

```
[Out] (x^3*(a + b*x^4 - a*(1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[1/4, 3/4, 7/4,
-((b*x^4)/a)]))/(6*b*(a + b*x^4)^(1/4))
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^6}{\left(bx^4+a\right)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(x^6/(b*x^4 + a)^(1/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(bx^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/(b*x^4 + a)^(1/4), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(b\,x^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^4+a)^(1/4),x)
```

```
[Out] int(x^6/(b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(bx^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(b*x^4 + a)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{\left(b\,x^4+a\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^4)^(1/4),x)
```

```
[Out] int(x^6/(a + b*x^4)^(1/4), x)
```

sympy [C] time = 1.77, size = 37, normalized size = 0.35

$$\frac{x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[4]{a}\,\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**4+a)**(1/4),x)
```

```
[Out] x**7*gamma(7/4)*hyper((1/4, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**
(1/4)*gamma(11/4))
```

3.1105 $\int \frac{x^2}{\sqrt[4]{a+bx^4}}\,dx$

Optimal. Leaf size=80

$$\frac{x^3}{2\sqrt[4]{a+bx^4}}+\frac{\sqrt{a}\,x\sqrt[4]{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{b}\,\sqrt[4]{a+bx^4}}$$

```
[Out] 1/2*x^3/(b*x^4+a)^(1/4)+1/2*(1+a/b/x^4)^(1/4)*x*(cos(1/2*arccot(x^2*b^(1/2)
/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*
arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)/(b*x^4+a)^(1/4)/b^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {310, 281, 335, 275, 196}

$$\frac{x^3}{2\sqrt[4]{a+bx^4}}+\frac{\sqrt{a}\,x\sqrt[4]{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{b}\,\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^4)^(1/4),x]
```

```
[Out] x^3/(2*(a + b*x^4)^(1/4)) + (Sqrt[a]*(1 + a/(b*x^4))^(1/4)*x*EllipticE[ArcC
ot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(2*Sqrt[b]*(a + b*x^4)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 281

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Dist[(x*(1 + a/(b*x^4)
)^(1/4))/(b*(a + b*x^4)^(1/4)), Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x] /
; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 310

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Dist[a/2, Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt[4]{a+bx^4}}\,dx &= \frac{x^3}{2\sqrt[4]{a+bx^4}} - \frac{1}{2}a\int \frac{x^2}{\left(a+bx^4\right)^{5/4}}\,dx \\
&= \frac{x^3}{2\sqrt[4]{a+bx^4}} - \frac{\left(a\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{5/4}x^3}\,dx}{2b\sqrt[4]{a+bx^4}} \\
&= \frac{x^3}{2\sqrt[4]{a+bx^4}} + \frac{\left(a\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{2b\sqrt[4]{a+bx^4}} \\
&= \frac{x^3}{2\sqrt[4]{a+bx^4}} + \frac{\left(a\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx, x, \frac{1}{x^2}\right)}{4b\sqrt[4]{a+bx^4}} \\
&= \frac{x^3}{2\sqrt[4]{a+bx^4}} + \frac{\sqrt{a}\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a+bx^4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.64

$$\frac{x^3\sqrt[4]{\frac{bx^4}{a}+1}\;{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^4}{a}\right)}{3\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^4)^(1/4),x]
```

```
[Out] (x^3*(1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[1/4, 3/4, 7/4, -((b*x^4)/a)])/
(3*(a + b*x^4)^(1/4))
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2}{\left(bx^4+a\right)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(x^2/(b*x^4 + a)^(1/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*x^4 + a)^(1/4), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(b\,x^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^4+a)^(1/4),x)
```

```
[Out] int(x^2/(b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*x^4 + a)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(b\,x^4+a\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^4)^(1/4),x)
```

```
[Out] int(x^2/(a + b*x^4)^(1/4), x)
```

sympy [C] time = 1.45, size = 37, normalized size = 0.46

$$\frac{x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\,\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[4]{a}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**4+a)**(1/4),x)
```

```
[Out] x**3*gamma(3/4)*hyper((1/4, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
1/4)*gamma(7/4))
```

3.1106 $\int \frac{1}{x^2 \sqrt[4]{a+bx^4}}\,dx$

Optimal. Leaf size=75

$$\frac{\sqrt{b}\,x\sqrt[4]{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\sqrt[4]{a+bx^4}}-\frac{1}{x\sqrt[4]{a+bx^4}}$$

```
[Out] -1/x/(b*x^4+a)^(1/4)+(1+a/b/x^4)^(1/4)*x*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2
))))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arccot(
x^2*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)/(b*x^4+a)^(1/4)/a^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {312, 281, 335, 275, 196}

$$\frac{\sqrt{b}\,x\sqrt[4]{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\sqrt[4]{a+bx^4}}-\frac{1}{x\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^4)^(1/4)),x]
```

```
[Out] -(1/(x*(a + b*x^4)^(1/4))) + (Sqrt[b]*(1 + a/(b*x^4))^(1/4)*x*EllipticE[Arc
Cot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a + b*x^4)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 281

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Dist[(x*(1 + a/(b*x^4)
)^(1/4))/(b*(a + b*x^4)^(1/4)), Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x] /
; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 312

```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> -Simp[(x*(a + b*x^
4)^(1/4))^(-1), x] - Dist[b, Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a,
 b}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt[4]{a+bx^4}}\,dx &= -\frac{1}{x\sqrt[4]{a+bx^4}} - b\int \frac{x^2}{\left(a+bx^4\right)^{5/4}}\,dx \\ &= -\frac{1}{x\sqrt[4]{a+bx^4}} - \frac{\left(\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{5/4}x^3}\,dx}{\sqrt[4]{a+bx^4}} \\ &= -\frac{1}{x\sqrt[4]{a+bx^4}} + \frac{\left(\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{\sqrt[4]{a+bx^4}} \\ &= -\frac{1}{x\sqrt[4]{a+bx^4}} + \frac{\left(\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx, x, \frac{1}{x^2}\right)}{2\sqrt[4]{a+bx^4}} \\ &= -\frac{1}{x\sqrt[4]{a+bx^4}} + \frac{\sqrt{b}\,\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\sqrt[4]{a+bx^4}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 49, normalized size = 0.65

$$-\frac{\sqrt[4]{\frac{bx^4}{a}+1}\;{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{3}{4};-\frac{bx^4}{a}\right)}{x\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^4)^(1/4)),x]
```

```
[Out] -(((1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 3/4, -((b*x^4)/a)])/(
x*(a + b*x^4)^(1/4)))
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{3}{4}}}{bx^6+ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(3/4)/(b*x^6 + a*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{1}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(1/4)*x^2), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^4+a)^(1/4),x)
```

```
[Out] int(1/x^2/(b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{1}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(1/4)*x^2), x)
```

mupad [B] time = 1.46, size = 40, normalized size = 0.53

$$-\frac{\left(\frac{a}{b\,x^4}+1\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ -\frac{a}{b\,x^4}\right)}{2\,x\left(b\,x^4+a\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^4)^(1/4)),x)
```

```
[Out] -((a/(b*x^4) + 1)^(1/4)*hypergeom([1/4, 1/2], 3/2, -a/(b*x^4)))/(2*x*(a + b
*x^4)^(1/4))
```

sympy [C] time = 1.55, size = 39, normalized size = 0.52

$$\frac{\Gamma\left(-\frac{1}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{4},\frac{1}{4}\\ \frac{3}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[4]{a}\,x\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**4+a)**(1/4),x)
```

```
[Out] gamma(-1/4)*hyper((-1/4, 1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(1/4
)*x*gamma(3/4))
```

3.1107 $\int \frac{1}{x^6 \sqrt[4]{a+bx^4}}\,dx$

Optimal. Leaf size=105

$$-\frac{2b^{3/2}x\sqrt[4]{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}\sqrt[4]{a+bx^4}}+\frac{2b}{5ax\sqrt[4]{a+bx^4}}-\frac{\left(a+bx^4\right)^{3/4}}{5ax^5}$$

```
[Out] 2/5*b/a/x/(b*x^4+a)^(1/4)-1/5*(b*x^4+a)^(3/4)/a/x^5-2/5*b^(3/2)*(1+a/b/x^4)
^(1/4)*x*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*
b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))/a
^(3/2)/(b*x^4+a)^(1/4)
```

Rubi [A] time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {325, 312, 281, 335, 275, 196}

$$-\frac{2b^{3/2}x\sqrt[4]{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}\sqrt[4]{a+bx^4}}+\frac{2b}{5ax\sqrt[4]{a+bx^4}}-\frac{\left(a+bx^4\right)^{3/4}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^4)^(1/4)),x]
```

```
[Out] (2*b)/(5*a*x*(a + b*x^4)^(1/4)) - (a + b*x^4)^(3/4)/(5*a*x^5) - (2*b^(3/2)*
(1 + a/(b*x^4))^(1/4)*x*EllipticE[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(5*a
^(3/2)*(a + b*x^4)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 281

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Dist[(x*(1 + a/(b*x^4)
)^(1/4))/(b*(a + b*x^4)^(1/4)), Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x] /
; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 312

```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> -Simp[(x*(a + b*x^
4)^(1/4))^(-1), x] - Dist[b, Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a,
 b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^6 \sqrt[4]{a+bx^4}}\,dx &= -\frac{\left(a+bx^4\right)^{3/4}}{5ax^5} - \frac{(2b)\int \frac{1}{x^2 \sqrt[4]{a+bx^4}}\,dx}{5a} \\
&= \frac{2b}{5ax\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{5ax^5} + \frac{\left(2b^2\right)\int \frac{x^2}{\left(a+bx^4\right)^{5/4}}\,dx}{5a} \\
&= \frac{2b}{5ax\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{5ax^5} + \frac{\left(2b\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{5/4}x^3}\,dx}{5a\sqrt[4]{a+bx^4}} \\
&= \frac{2b}{5ax\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{5ax^5} - \frac{\left(2b\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{5a\sqrt[4]{a+bx^4}} \\
&= \frac{2b}{5ax\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{5ax^5} - \frac{\left(b\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx, x, \frac{1}{x^2}\right)}{5a\sqrt[4]{a+bx^4}} \\
&= \frac{2b}{5ax\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{5ax^5} - \frac{2b^{3/2}\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}\sqrt[4]{a+bx^4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.49

$$-\frac{\sqrt[4]{\frac{bx^4}{a}+1}\;{}_2F_1\left(-\frac{5}{4},\frac{1}{4};-\frac{1}{4};-\frac{bx^4}{a}\right)}{5x^5\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^4)^(1/4)),x]
```

```
[Out] -1/5*((1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[-5/4, 1/4, -1/4, -((b*x^4)/a)
])/(x^5*(a + b*x^4)^(1/4))
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{3}{4}}}{bx^{10}+ax^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(3/4)/(b*x^10 + a*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{1}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^4+a)^(1/4),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^(1/4)*x^6), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^4+a)^(1/4),x)
[Out] int(1/x^6/(b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{1}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^4+a)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(1/4)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^6\left(b\,x^4+a\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^4)^(1/4)),x)
[Out] int(1/(x^6*(a + b*x^4)^(1/4)), x)
```

sympy [C] time = 1.73, size = 29, normalized size = 0.28

$$-\frac{{}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\right|\frac{ae^{i\pi}}{bx^4}\right)}{6\sqrt[4]{b}\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**4+a)**(1/4),x)
[Out] -hyper((1/4, 3/2), (5/2,), a*exp_polar(I*pi)/(b*x**4))/(6*b**(1/4)*x**6)
```

3.1108 $\int \frac{1}{x^{10}\sqrt[4]{a+bx^4}}\,dx$

Optimal. Leaf size=129

$$\frac{4b^{5/2}x\sqrt{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{15a^{5/2}\sqrt[4]{a+bx^4}}-\frac{4b^2}{15a^2x\sqrt[4]{a+bx^4}}+\frac{2b\left(a+bx^4\right)^{3/4}}{15a^2x^5}-\frac{\left(a+bx^4\right)^{3/4}}{9ax^9}$$

```
[Out] -4/15*b^2/a^2/x/(b*x^4+a)^(1/4)-1/9*(b*x^4+a)^(3/4)/a/x^9+2/15*b*(b*x^4+a)^
(3/4)/a^2/x^5+4/15*b^(5/2)*(1+a/b/x^4)^(1/4)*x*(cos(1/2*arccot(x^2*b^(1/2)/
a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*a
rccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))/a^(5/2)/(b*x^4+a)^(1/4)
```

Rubi [A] time = 0.06, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {325, 312, 281, 335, 275, 196}

$$-\frac{4b^2}{15a^2x\sqrt[4]{a+bx^4}}+\frac{4b^{5/2}x\sqrt{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{15a^{5/2}\sqrt[4]{a+bx^4}}+\frac{2b\left(a+bx^4\right)^{3/4}}{15a^2x^5}-\frac{\left(a+bx^4\right)^{3/4}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*(a + b*x^4)^(1/4)),x]
```

```
[Out] (-4*b^2)/(15*a^2*x*(a + b*x^4)^(1/4)) - (a + b*x^4)^(3/4)/(9*a*x^9) + (2*b*
(a + b*x^4)^(3/4))/(15*a^2*x^5) + (4*b^(5/2)*(1 + a/(b*x^4))^(1/4)*x*Ellipt
icE[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(15*a^(5/2)*(a + b*x^4)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 281

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Dist[(x*(1 + a/(b*x^4)
)^(1/4))/(b*(a + b*x^4)^(1/4)), Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x] /
; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 312

```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> -Simp[(x*(a + b*x^
4)^(1/4))^(-1), x] - Dist[b, Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a,
 b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{10}\sqrt[4]{a+bx^4}}\,dx &= -\frac{\left(a+bx^4\right)^{3/4}}{9ax^9} - \frac{(2b)\int \frac{1}{x^6\sqrt[4]{a+bx^4}}\,dx}{3a} \\
&= -\frac{\left(a+bx^4\right)^{3/4}}{9ax^9} + \frac{2b\left(a+bx^4\right)^{3/4}}{15a^2x^5} + \frac{\left(4b^2\right)\int \frac{1}{x^2\sqrt[4]{a+bx^4}}\,dx}{15a^2} \\
&= -\frac{4b^2}{15a^2x\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{9ax^9} + \frac{2b\left(a+bx^4\right)^{3/4}}{15a^2x^5} - \frac{\left(4b^3\right)\int \frac{x^2}{\left(a+bx^4\right)^{5/4}}\,dx}{15a^2} \\
&= -\frac{4b^2}{15a^2x\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{9ax^9} + \frac{2b\left(a+bx^4\right)^{3/4}}{15a^2x^5} - \frac{\left(4b^2\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{5/4}x^3}\,dx}{15a^2\sqrt[4]{a+bx^4}} \\
&= -\frac{4b^2}{15a^2x\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{9ax^9} + \frac{2b\left(a+bx^4\right)^{3/4}}{15a^2x^5} + \frac{\left(4b^2\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{5/}}\right.}{15a^2\sqrt[4]{a+bx^4}} \\
&= -\frac{4b^2}{15a^2x\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{9ax^9} + \frac{2b\left(a+bx^4\right)^{3/4}}{15a^2x^5} + \frac{\left(2b^2\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/}}\right.}{15a^2\sqrt[4]{a+bx^4}} \\
&= -\frac{4b^2}{15a^2x\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{9ax^9} + \frac{2b\left(a+bx^4\right)^{3/4}}{15a^2x^5} + \frac{4b^{5/2}\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\right|}{15a^{5/2}\sqrt[4]{a+bx^4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.40

$$-\frac{\sqrt[4]{\frac{bx^4}{a}+1}\;{}_2F_1\left(-\frac{9}{4},\frac{1}{4};-\frac{5}{4};-\frac{bx^4}{a}\right)}{9x^9\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^10*(a + b*x^4)^(1/4)),x]
```

```
[Out] -1/9*((1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[-9/4, 1/4, -5/4, -((b*x^4)/a)
])/(x^9*(a + b*x^4)^(1/4))
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{3}{4}}}{bx^{14}+ax^{10}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(3/4)/(b*x^14 + a*x^10), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{1}{4}} x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(1/4)*x^10), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}} x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(b*x^4+a)^(1/4),x)
```

```
[Out] int(1/x^10/(b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{1}{4}} x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(1/4)*x^10), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{10}\left(b\,x^4+a\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^10*(a + b*x^4)^(1/4)),x)
```

```
[Out] int(1/(x^10*(a + b*x^4)^(1/4)), x)
```

sympy [C] time = 1.82, size = 44, normalized size = 0.34

$$\frac{\Gamma\left(-\frac{9}{4}\right) {}_2F_1\left(\begin{matrix}-\frac{9}{4}, \frac{1}{4} \\ -\frac{5}{4}\end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt[4]{a}\, x^9 \Gamma\left(-\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(b*x**4+a)**(1/4),x)
```

```
[Out] gamma(-9/4)*hyper((-9/4, 1/4), (-5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(1/
4)*x**9*gamma(-5/4))
```

3.1109 $\int \frac{1}{x^{14}\sqrt[4]{a+bx^4}}\,dx$

Optimal. Leaf size=153

$$-\frac{8b^{7/2}x\sqrt{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{39a^{7/2}\sqrt[4]{a+bx^4}}+\frac{8b^3}{39a^3x\sqrt[4]{a+bx^4}}-\frac{4b^2\left(a+bx^4\right)^{3/4}}{39a^3x^5}+\frac{10b\left(a+bx^4\right)^{3/4}}{117a^2x^9}-\frac{\left(a+bx^4\right)^{3/4}}{13ax^{13}}$$

```
[Out] 8/39*b^3/a^3/x/(b*x^4+a)^(1/4)-1/13*(b*x^4+a)^(3/4)/a/x^13+10/117*b*(b*x^4+
a)^(3/4)/a^2/x^9-4/39*b^2*(b*x^4+a)^(3/4)/a^3/x^5-8/39*b^(7/2)*(1+a/b/x^4)^
(1/4)*x*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b
^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))/a^
(7/2)/(b*x^4+a)^(1/4)
```

Rubi [A] time = 0.07, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {325, 312, 281, 335, 275, 196}

$$\frac{8b^3}{39a^3x\sqrt[4]{a+bx^4}}-\frac{4b^2\left(a+bx^4\right)^{3/4}}{39a^3x^5}-\frac{8b^{7/2}x\sqrt{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{39a^{7/2}\sqrt[4]{a+bx^4}}+\frac{10b\left(a+bx^4\right)^{3/4}}{117a^2x^9}-\frac{\left(a+bx^4\right)^{3/4}}{13ax^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^14*(a + b*x^4)^(1/4)),x]
```

```
[Out] (8*b^3)/(39*a^3*x*(a + b*x^4)^(1/4)) - (a + b*x^4)^(3/4)/(13*a*x^13) + (10*
b*(a + b*x^4)^(3/4))/(117*a^2*x^9) - (4*b^2*(a + b*x^4)^(3/4))/(39*a^3*x^5)
 - (8*b^(7/2)*(1 + a/(b*x^4))^(1/4)*x*EllipticE[ArcCot[(Sqrt[b]*x^2)/Sqrt[a
]]/2, 2])/(39*a^(7/2)*(a + b*x^4)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 281

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Dist[(x*(1 + a/(b*x^4)
)^(1/4))/(b*(a + b*x^4)^(1/4)), Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x] /
; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 312

```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> -Simp[(x*(a + b*x^
4)^(1/4))^(-1), x] - Dist[b, Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a,
 b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{14}\sqrt[4]{a+bx^4}}\,dx &= -\frac{\left(a+bx^4\right)^{3/4}}{13ax^{13}} - \frac{(10b)\int \frac{1}{x^{10}\sqrt[4]{a+bx^4}}\,dx}{13a} \\
&= -\frac{\left(a+bx^4\right)^{3/4}}{13ax^{13}} + \frac{10b\left(a+bx^4\right)^{3/4}}{117a^2x^9} + \frac{\left(20b^2\right)\int \frac{1}{x^6\sqrt[4]{a+bx^4}}\,dx}{39a^2} \\
&= -\frac{\left(a+bx^4\right)^{3/4}}{13ax^{13}} + \frac{10b\left(a+bx^4\right)^{3/4}}{117a^2x^9} - \frac{4b^2\left(a+bx^4\right)^{3/4}}{39a^3x^5} - \frac{\left(8b^3\right)\int \frac{1}{x^2\sqrt[4]{a+bx^4}}\,dx}{39a^3} \\
&= \frac{8b^3}{39a^3x\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{13ax^{13}} + \frac{10b\left(a+bx^4\right)^{3/4}}{117a^2x^9} - \frac{4b^2\left(a+bx^4\right)^{3/4}}{39a^3x^5} + \frac{\left(8b^4\right)\int \frac{x^2}{\left(a+bx^4\right)^{5/4}}\,dx}{39a^3} \\
&= \frac{8b^3}{39a^3x\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{13ax^{13}} + \frac{10b\left(a+bx^4\right)^{3/4}}{117a^2x^9} - \frac{4b^2\left(a+bx^4\right)^{3/4}}{39a^3x^5} + \frac{\left(8b^3\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\int \frac{x^2}{\left(1+\frac{a}{bx^4}\right)^{5/4}}\,dx}{39a^3\sqrt[4]{a+bx^4}} \\
&= \frac{8b^3}{39a^3x\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{13ax^{13}} + \frac{10b\left(a+bx^4\right)^{3/4}}{117a^2x^9} - \frac{4b^2\left(a+bx^4\right)^{3/4}}{39a^3x^5} - \frac{\left(8b^3\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{5/4}x^3}\,dx}{39a^3\sqrt[4]{a+bx^4}} \\
&= \frac{8b^3}{39a^3x\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{13ax^{13}} + \frac{10b\left(a+bx^4\right)^{3/4}}{117a^2x^9} - \frac{4b^2\left(a+bx^4\right)^{3/4}}{39a^3x^5} - \frac{\left(4b^3\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{5/4}x^3}\,dx}{39a^3\sqrt[4]{a+bx^4}} \\
&= \frac{8b^3}{39a^3x\sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{13ax^{13}} + \frac{10b\left(a+bx^4\right)^{3/4}}{117a^2x^9} - \frac{4b^2\left(a+bx^4\right)^{3/4}}{39a^3x^5} - \frac{8b^{7/2}\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\right.}{39a^{7/2}\sqrt[4]{}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.33

$$-\frac{\sqrt[4]{\frac{bx^4}{a}+1}\;{}_2F_1\left(-\frac{13}{4},\frac{1}{4};-\frac{9}{4};-\frac{bx^4}{a}\right)}{13x^{13}\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^14*(a + b*x^4)^(1/4)),x]
```

```
[Out] -1/13*((1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[-13/4, 1/4, -9/4, -((b*x^4)/
a)])/(x^13*(a + b*x^4)^(1/4))
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{3}{4}}}{bx^{18}+ax^{14}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^14/(b*x^4+a)^(1/4),x, algorithm="fricas")
[Out] integral((b*x^4 + a)^(3/4)/(b*x^18 + a*x^14), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{1}{4}}x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^14/(b*x^4+a)^(1/4),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^(1/4)*x^14), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}}x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^14/(b*x^4+a)^(1/4),x)
[Out] int(1/x^14/(b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{1}{4}}x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^14/(b*x^4+a)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(1/4)*x^14), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{14}\left(b\,x^4+a\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^14*(a + b*x^4)^(1/4)),x)
[Out] int(1/(x^14*(a + b*x^4)^(1/4)), x)
```

sympy [C] time = 3.61, size = 44, normalized size = 0.29

$$\frac{\Gamma\left(-\frac{13}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{13}{4}, \frac{1}{4} \\ -\frac{9}{4}\end{matrix}\right| \frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[4]{a}\,x^{13}\Gamma\left(-\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**14/(b*x**4+a)**(1/4),x)
```

```
[Out] gamma(-13/4)*hyper((-13/4, 1/4), (-9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
1/4)*x**13*gamma(-9/4))
```

3.1110 $\int \frac{x^{19}}{\left(a+bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=98

$$\frac{a^4\sqrt[4]{a+bx^4}}{b^5}-\frac{4a^3\left(a+bx^4\right)^{5/4}}{5b^5}+\frac{2a^2\left(a+bx^4\right)^{9/4}}{3b^5}+\frac{\left(a+bx^4\right)^{17/4}}{17b^5}-\frac{4a\left(a+bx^4\right)^{13/4}}{13b^5}$$

```
[Out] a^4*(b*x^4+a)^(1/4)/b^5-4/5*a^3*(b*x^4+a)^(5/4)/b^5+2/3*a^2*(b*x^4+a)^(9/4)
/b^5-4/13*a*(b*x^4+a)^(13/4)/b^5+1/17*(b*x^4+a)^(17/4)/b^5
```

Rubi [A] time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{2a^2\left(a+bx^4\right)^{9/4}}{3b^5}-\frac{4a^3\left(a+bx^4\right)^{5/4}}{5b^5}+\frac{a^4\sqrt[4]{a+bx^4}}{b^5}+\frac{\left(a+bx^4\right)^{17/4}}{17b^5}-\frac{4a\left(a+bx^4\right)^{13/4}}{13b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^19/(a + b*x^4)^(3/4),x]
```

```
[Out] (a^4*(a + b*x^4)^(1/4))/b^5 - (4*a^3*(a + b*x^4)^(5/4))/(5*b^5) + (2*a^2*(a
 + b*x^4)^(9/4))/(3*b^5) - (4*a*(a + b*x^4)^(13/4))/(13*b^5) + (a + b*x^4)^
(17/4)/(17*b^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{19}}{\left(a+bx^4\right)^{3/4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x^4}{(a+bx)^{3/4}}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int\left(\frac{a^4}{b^4(a+bx)^{3/4}}-\frac{4a^3\sqrt[4]{a+bx}}{b^4}+\frac{6a^2(a+bx)^{5/4}}{b^4}-\frac{4a(a+bx)^{9/4}}{b^4}+\frac{(a+bx)^{13/}}{b^4}\right.\right.\\ &= \frac{a^4\sqrt[4]{a+bx^4}}{b^5}-\frac{4a^3\left(a+bx^4\right)^{5/4}}{5b^5}+\frac{2a^2\left(a+bx^4\right)^{9/4}}{3b^5}-\frac{4a\left(a+bx^4\right)^{13/4}}{13b^5}+\frac{\left(a+bx^4\right)^{17/4}}{17b^5}\end{aligned}$$

Mathematica [A] time = 0.04, size = 61, normalized size = 0.62

$$\frac{\sqrt[4]{a+bx^4}\left(2048a^4-512a^3bx^4+320a^2b^2x^8-240ab^3x^{12}+195b^4x^{16}\right)}{3315b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^19/(a + b*x^4)^(3/4),x]
```

```
[Out] ((a + b*x^4)^(1/4)*(2048*a^4 - 512*a^3*b*x^4 + 320*a^2*b^2*x^8 - 240*a*b^3*
x^12 + 195*b^4*x^16))/(3315*b^5)
```

fricas [A] time = 0.65, size = 57, normalized size = 0.58

$$\frac{\left(195\,b^4x^{16} - 240\,ab^3x^{12} + 320\,a^2b^2x^8 - 512\,a^3bx^4 + 2048\,a^4\right)\left(bx^4 + a\right)^{\frac{1}{4}}}{3315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/3315*(195*b^4*x^16 - 240*a*b^3*x^12 + 320*a^2*b^2*x^8 - 512*a^3*b*x^4 + 2
048*a^4)*(b*x^4 + a)^(1/4)/b^5
```

giac [A] time = 0.16, size = 74, normalized size = 0.76

$$\frac{\left(bx^4 + a\right)^{\frac{1}{4}}a^4}{b^5} + \frac{195\left(bx^4 + a\right)^{\frac{17}{4}} - 1020\left(bx^4 + a\right)^{\frac{13}{4}}a + 2210\left(bx^4 + a\right)^{\frac{9}{4}}a^2 - 2652\left(bx^4 + a\right)^{\frac{5}{4}}a^3}{3315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] (b*x^4 + a)^(1/4)*a^4/b^5 + 1/3315*(195*(b*x^4 + a)^(17/4) - 1020*(b*x^4 +
a)^(13/4)*a + 2210*(b*x^4 + a)^(9/4)*a^2 - 2652*(b*x^4 + a)^(5/4)*a^3)/b^5
```

maple [A] time = 0.01, size = 58, normalized size = 0.59

$$\frac{\left(b\,x^4 + a\right)^{\frac{1}{4}}\left(195x^{16}b^4 - 240a\,x^{12}b^3 + 320a^2x^8b^2 - 512a^3x^4b + 2048a^4\right)}{3315b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19/(b*x^4+a)^(3/4),x)
```

```
[Out] 1/3315*(b*x^4+a)^(1/4)*(195*b^4*x^16-240*a*b^3*x^12+320*a^2*b^2*x^8-512*a^3
*b*x^4+2048*a^4)/b^5
```

maxima [A] time = 1.36, size = 80, normalized size = 0.82

$$\frac{\left(bx^4 + a\right)^{\frac{17}{4}}}{17\,b^5} - \frac{4\left(bx^4 + a\right)^{\frac{13}{4}}a}{13\,b^5} + \frac{2\left(bx^4 + a\right)^{\frac{9}{4}}a^2}{3\,b^5} - \frac{4\left(bx^4 + a\right)^{\frac{5}{4}}a^3}{5\,b^5} + \frac{\left(bx^4 + a\right)^{\frac{1}{4}}a^4}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] 1/17*(b*x^4 + a)^(17/4)/b^5 - 4/13*(b*x^4 + a)^(13/4)*a/b^5 + 2/3*(b*x^4 +
a)^(9/4)*a^2/b^5 - 4/5*(b*x^4 + a)^(5/4)*a^3/b^5 + (b*x^4 + a)^(1/4)*a^4/b^
5
```

mupad [B] time = 1.17, size = 58, normalized size = 0.59

$$\left(b\,x^4 + a\right)^{1/4}\left(\frac{2048\,a^4}{3315\,b^5} + \frac{x^{16}}{17\,b} - \frac{16\,a\,x^{12}}{221\,b^2} - \frac{512\,a^3\,x^4}{3315\,b^4} + \frac{64\,a^2\,x^8}{663\,b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19/(a + b*x^4)^(3/4),x)
```

```
[Out] (a + b*x^4)^(1/4)*((2048*a^4)/(3315*b^5) + x^16/(17*b) - (16*a*x^12)/(221*b
^2) - (512*a^3*x^4)/(3315*b^4) + (64*a^2*x^8)/(663*b^3))
```

sympy [A] time = 25.21, size = 116, normalized size = 1.18

$$\begin{cases} \frac{2048a^4\sqrt[4]{a+bx^4}}{3315b^5} - \frac{512a^3x^4\sqrt[4]{a+bx^4}}{3315b^4} + \frac{64a^2x^8\sqrt[4]{a+bx^4}}{663b^3} - \frac{16ax^{12}\sqrt[4]{a+bx^4}}{221b^2} + \frac{x^{16}\sqrt[4]{a+bx^4}}{17b} & \text{for } b \neq 0 \\ \frac{x^{20}}{20a^{\frac{3}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**19/(b*x**4+a)**(3/4),x)
```

```
[Out] Piecewise((2048*a**4*(a + b*x**4)**(1/4)/(3315*b**5) - 512*a**3*x**4*(a + b
*x**4)**(1/4)/(3315*b**4) + 64*a**2*x**8*(a + b*x**4)**(1/4)/(663*b**3) - 1
6*a*x**12*(a + b*x**4)**(1/4)/(221*b**2) + x**16*(a + b*x**4)**(1/4)/(17*b)
, Ne(b, 0)), (x**20/(20*a**(3/4)), True))
```

3.1111 $\int \frac{x^{15}}{\left(a+bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=78

$$-\frac{a^3\sqrt[4]{a+bx^4}}{b^4}+\frac{3a^2\left(a+bx^4\right)^{5/4}}{5b^4}+\frac{\left(a+bx^4\right)^{13/4}}{13b^4}-\frac{a\left(a+bx^4\right)^{9/4}}{3b^4}$$

```
[Out] -a^3*(b*x^4+a)^(1/4)/b^4+3/5*a^2*(b*x^4+a)^(5/4)/b^4-1/3*a*(b*x^4+a)^(9/4)/
b^4+1/13*(b*x^4+a)^(13/4)/b^4
```

Rubi [A] time = 0.04, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{3a^2\left(a+bx^4\right)^{5/4}}{5b^4}-\frac{a^3\sqrt[4]{a+bx^4}}{b^4}+\frac{\left(a+bx^4\right)^{13/4}}{13b^4}-\frac{a\left(a+bx^4\right)^{9/4}}{3b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^15/(a + b*x^4)^(3/4),x]
```

```
[Out] -((a^3*(a + b*x^4)^(1/4))/b^4) + (3*a^2*(a + b*x^4)^(5/4))/(5*b^4) - (a*(a
+ b*x^4)^(9/4))/(3*b^4) + (a + b*x^4)^(13/4)/(13*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{15}}{\left(a+bx^4\right)^{3/4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x^3}{(a+bx)^{3/4}}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int\left(-\frac{a^3}{b^3(a+bx)^{3/4}}+\frac{3a^2\sqrt[4]{a+bx}}{b^3}-\frac{3a(a+bx)^{5/4}}{b^3}+\frac{(a+bx)^{9/4}}{b^3}\right)dx, x, x^4\right)\\ &= -\frac{a^3\sqrt[4]{a+bx^4}}{b^4}+\frac{3a^2\left(a+bx^4\right)^{5/4}}{5b^4}-\frac{a\left(a+bx^4\right)^{9/4}}{3b^4}+\frac{\left(a+bx^4\right)^{13/4}}{13b^4}\end{aligned}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 0.64

$$\frac{\sqrt[4]{a+bx^4}\left(-128a^3+32a^2bx^4-20ab^2x^8+15b^3x^{12}\right)}{195b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15/(a + b*x^4)^(3/4),x]
```

```
[Out] ((a + b*x^4)^(1/4)*(-128*a^3 + 32*a^2*b*x^4 - 20*a*b^2*x^8 + 15*b^3*x^12))/
(195*b^4)
```

fricas [A] time = 0.87, size = 46, normalized size = 0.59

$$\frac{\left(15\,b^3x^{12} - 20\,ab^2x^8 + 32\,a^2bx^4 - 128\,a^3\right)\left(bx^4 + a\right)^{\frac{1}{4}}}{195\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/195*(15*b^3*x^12 - 20*a*b^2*x^8 + 32*a^2*b*x^4 - 128*a^3)*(b*x^4 + a)^(1/
4)/b^4
```

giac [A] time = 0.17, size = 61, normalized size = 0.78

$$-\frac{\left(bx^4 + a\right)^{\frac{1}{4}}a^3}{b^4} + \frac{15\left(bx^4 + a\right)^{\frac{13}{4}} - 65\left(bx^4 + a\right)^{\frac{9}{4}}a + 117\left(bx^4 + a\right)^{\frac{5}{4}}a^2}{195\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] -(b*x^4 + a)^(1/4)*a^3/b^4 + 1/195*(15*(b*x^4 + a)^(13/4) - 65*(b*x^4 + a)^
(9/4)*a + 117*(b*x^4 + a)^(5/4)*a^2)/b^4
```

maple [A] time = 0.01, size = 47, normalized size = 0.60

$$-\frac{\left(b\,x^4 + a\right)^{\frac{1}{4}}\left(-15b^3x^{12} + 20a\,b^2x^8 - 32a^2b\,x^4 + 128a^3\right)}{195b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(b*x^4+a)^(3/4),x)
```

```
[Out] -1/195*(b*x^4+a)^(1/4)*(-15*b^3*x^12+20*a*b^2*x^8-32*a^2*b*x^4+128*a^3)/b^4
```

maxima [A] time = 1.30, size = 64, normalized size = 0.82

$$\frac{\left(bx^4 + a\right)^{\frac{13}{4}}}{13\,b^4} - \frac{\left(bx^4 + a\right)^{\frac{9}{4}}a}{3\,b^4} + \frac{3\left(bx^4 + a\right)^{\frac{5}{4}}a^2}{5\,b^4} - \frac{\left(bx^4 + a\right)^{\frac{1}{4}}a^3}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] 1/13*(b*x^4 + a)^(13/4)/b^4 - 1/3*(b*x^4 + a)^(9/4)*a/b^4 + 3/5*(b*x^4 + a)
^(5/4)*a^2/b^4 - (b*x^4 + a)^(1/4)*a^3/b^4
```

mupad [B] time = 1.18, size = 48, normalized size = 0.62

$$-\left(b\,x^4 + a\right)^{1/4}\left(\frac{128\,a^3}{195\,b^4} - \frac{x^{12}}{13\,b} + \frac{4\,a\,x^8}{39\,b^2} - \frac{32\,a^2\,x^4}{195\,b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(a + b*x^4)^(3/4),x)
```

```
[Out] -(a + b*x^4)^(1/4)*((128*a^3)/(195*b^4) - x^12/(13*b) + (4*a*x^8)/(39*b^2)
- (32*a^2*x^4)/(195*b^3))
```

sympy [A] time = 10.23, size = 92, normalized size = 1.18

$$\begin{cases} -\frac{128a^3\sqrt[4]{a+bx^4}}{195b^4} + \frac{32a^2x^4\sqrt[4]{a+bx^4}}{195b^3} - \frac{4ax^8\sqrt[4]{a+bx^4}}{39b^2} + \frac{x^{12}\sqrt[4]{a+bx^4}}{13b} & \text{for } b \neq 0 \\ \frac{x^{16}}{16a^{\frac{3}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15/(b*x**4+a)**(3/4),x)
```

```
[Out] Piecewise((-128*a**3*(a + b*x**4)**(1/4)/(195*b**4) + 32*a**2*x**4*(a + b*x
**4)**(1/4)/(195*b**3) - 4*a*x**8*(a + b*x**4)**(1/4)/(39*b**2) + x**12*(a
+ b*x**4)**(1/4)/(13*b), Ne(b, 0)), (x**16/(16*a**(3/4)), True))
```

3.1112 $\int \frac{x^{11}}{\left(a+bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=56

$$\frac{a^2\sqrt[4]{a+bx^4}}{b^3}+\frac{\left(a+bx^4\right)^{9/4}}{9b^3}-\frac{2a\left(a+bx^4\right)^{5/4}}{5b^3}$$

```
[Out] a^2*(b*x^4+a)^(1/4)/b^3-2/5*a*(b*x^4+a)^(5/4)/b^3+1/9*(b*x^4+a)^(9/4)/b^3
```

Rubi [A] time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{a^2\sqrt[4]{a+bx^4}}{b^3}+\frac{\left(a+bx^4\right)^{9/4}}{9b^3}-\frac{2a\left(a+bx^4\right)^{5/4}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + b*x^4)^(3/4),x]
```

```
[Out] (a^2*(a + b*x^4)^(1/4))/b^3 - (2*a*(a + b*x^4)^(5/4))/(5*b^3) + (a + b*x^4)
^(9/4)/(9*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{\left(a+bx^4\right)^{3/4}}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{x^2}{(a+bx)^{3/4}}\,dx, x, x^4\right)\\ &= \frac{1}{4}\text{Subst}\left(\int\left(\frac{a^2}{b^2(a+bx)^{3/4}}-\frac{2a\sqrt[4]{a+bx}}{b^2}+\frac{(a+bx)^{5/4}}{b^2}\right)dx, x, x^4\right)\\ &= \frac{a^2\sqrt[4]{a+bx^4}}{b^3}-\frac{2a\left(a+bx^4\right)^{5/4}}{5b^3}+\frac{\left(a+bx^4\right)^{9/4}}{9b^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 39, normalized size = 0.70

$$\frac{\sqrt[4]{a+bx^4}\left(32a^2-8abx^4+5b^2x^8\right)}{45b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + b*x^4)^(3/4),x]
```

```
[Out] ((a + b*x^4)^(1/4)*(32*a^2 - 8*a*b*x^4 + 5*b^2*x^8))/(45*b^3)
```

fricas [A] time = 0.87, size = 35, normalized size = 0.62

$$\frac{\left(5\,b^2x^8 - 8\,abx^4 + 32\,a^2\right)\left(bx^4 + a\right)^{\frac{1}{4}}}{45\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/45*(5*b^2*x^8 - 8*a*b*x^4 + 32*a^2)*(b*x^4 + a)^(1/4)/b^3
```

giac [A] time = 0.15, size = 46, normalized size = 0.82

$$\frac{\left(bx^4 + a\right)^{\frac{1}{4}}a^2}{b^3} + \frac{5\left(bx^4 + a\right)^{\frac{9}{4}} - 18\left(bx^4 + a\right)^{\frac{5}{4}}a}{45\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] (b*x^4 + a)^(1/4)*a^2/b^3 + 1/45*(5*(b*x^4 + a)^(9/4) - 18*(b*x^4 + a)^(5/4
)*a)/b^3
```

maple [A] time = 0.00, size = 36, normalized size = 0.64

$$\frac{\left(b\,x^4 + a\right)^{\frac{1}{4}}\left(5b^2x^8 - 8ab\,x^4 + 32a^2\right)}{45b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^4+a)^(3/4),x)
```

```
[Out] 1/45*(b*x^4+a)^(1/4)*(5*b^2*x^8-8*a*b*x^4+32*a^2)/b^3
```

maxima [A] time = 1.33, size = 46, normalized size = 0.82

$$\frac{\left(bx^4 + a\right)^{\frac{9}{4}}}{9\,b^3} - \frac{2\left(bx^4 + a\right)^{\frac{5}{4}}a}{5\,b^3} + \frac{\left(bx^4 + a\right)^{\frac{1}{4}}a^2}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] 1/9*(b*x^4 + a)^(9/4)/b^3 - 2/5*(b*x^4 + a)^(5/4)*a/b^3 + (b*x^4 + a)^(1/4)
*a^2/b^3
```

mupad [B] time = 1.14, size = 36, normalized size = 0.64

$$\left(b\,x^4 + a\right)^{1/4}\left(\frac{32\,a^2}{45\,b^3} + \frac{x^8}{9\,b} - \frac{8\,a\,x^4}{45\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a + b*x^4)^(3/4),x)
```

```
[Out] (a + b*x^4)^(1/4)*((32*a^2)/(45*b^3) + x^8/(9*b) - (8*a*x^4)/(45*b^2))
```

sympy [**A**] time = 4.24, size = 68, normalized size = 1.21

$$\begin{cases} \frac{32a^2\sqrt[4]{a+bx^4}}{45b^3} - \frac{8ax^4\sqrt[4]{a+bx^4}}{45b^2} + \frac{x^8\sqrt[4]{a+bx^4}}{9b} & \text{for } b \neq 0 \\ \frac{x^{12}}{12a^{\frac{3}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**4+a)**(3/4),x)
```

```
[Out] Piecewise((32*a**2*(a + b*x**4)**(1/4)/(45*b**3) - 8*a*x**4*(a + b*x**4)**(
1/4)/(45*b**2) + x**8*(a + b*x**4)**(1/4)/(9*b), Ne(b, 0)), (x**12/(12*a**(
3/4)), True))
```

3.1113 $\int \frac{x^7}{\left(a+bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=36

$$\frac{\left(a+bx^4\right)^{5/4}}{5b^2}-\frac{a\sqrt[4]{a+bx^4}}{b^2}$$

```
[Out] -a*(b*x^4+a)^(1/4)/b^2+1/5*(b*x^4+a)^(5/4)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{\left(a+bx^4\right)^{5/4}}{5b^2}-\frac{a\sqrt[4]{a+bx^4}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^4)^(3/4),x]
```

```
[Out] -((a*(a + b*x^4)^(1/4))/b^2) + (a + b*x^4)^(5/4)/(5*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{\left(a+bx^4\right)^{3/4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x}{(a+bx)^{3/4}}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int\left(-\frac{a}{b(a+bx)^{3/4}}+\frac{\sqrt[4]{a+bx}}{b}\right)dx, x, x^4\right)\\ &= -\frac{a\sqrt[4]{a+bx^4}}{b^2}+\frac{\left(a+bx^4\right)^{5/4}}{5b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.75

$$\frac{\left(bx^4-4a\right)\sqrt[4]{a+bx^4}}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^4)^(3/4),x]
```

```
[Out] ((-4*a + b*x^4)*(a + b*x^4)^(1/4))/(5*b^2)
```

fricas [A] time = 1.12, size = 23, normalized size = 0.64

$$\frac{\left(bx^4+a\right)^{\frac{1}{4}}\left(bx^4-4\,a\right)}{5\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)^(3/4),x, algorithm="fricas")
[Out] 1/5*(b*x^4 + a)^(1/4)*(b*x^4 - 4*a)/b^2
```

giac [A] time = 0.15, size = 30, normalized size = 0.83

$$\frac{\left(bx^4+a\right)^{\frac{5}{4}}}{5\,b^2}-\frac{\left(bx^4+a\right)^{\frac{1}{4}}a}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)^(3/4),x, algorithm="giac")
[Out] 1/5*(b*x^4 + a)^(5/4)/b^2 - (b*x^4 + a)^(1/4)*a/b^2
```

maple [A] time = 0.00, size = 25, normalized size = 0.69

$$-\frac{\left(b\,x^4+a\right)^{\frac{1}{4}}\left(-b\,x^4+4a\right)}{5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^4+a)^(3/4),x)
[Out] -1/5*(b*x^4+a)^(1/4)*(-b*x^4+4*a)/b^2
```

maxima [A] time = 1.27, size = 30, normalized size = 0.83

$$\frac{\left(bx^4+a\right)^{\frac{5}{4}}}{5\,b^2}-\frac{\left(bx^4+a\right)^{\frac{1}{4}}a}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)^(3/4),x, algorithm="maxima")
[Out] 1/5*(b*x^4 + a)^(5/4)/b^2 - (b*x^4 + a)^(1/4)*a/b^2
```

mupad [B] time = 1.13, size = 24, normalized size = 0.67

$$-\frac{\left(b\,x^4+a\right)^{1/4}\left(4\,a-b\,x^4\right)}{5\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^4)^(3/4),x)
[Out] -((a + b*x^4)^(1/4)*(4*a - b*x^4))/(5*b^2)
```

sympy [A] time = 2.28, size = 44, normalized size = 1.22

$$\begin{cases} -\frac{4a\sqrt[4]{a+bx^4}}{5b^2}+\frac{x^4\sqrt[4]{a+bx^4}}{5b} & \text{for } b\neq 0 \\ \frac{x^8}{8a^{\frac{3}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**4+a)**(3/4),x)
```

```
[Out] Piecewise((-4*a*(a + b*x**4)**(1/4)/(5*b**2) + x**4*(a + b*x**4)**(1/4)/(5*
b), Ne(b, 0)), (x**8/(8*a**(3/4)), True))
```

3.1114 $\int \frac{x^3}{\left(a+bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=15

$$\frac{\sqrt[4]{a+bx^4}}{b}$$

```
[Out] (b*x^4+a)^(1/4)/b
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$\frac{\sqrt[4]{a+bx^4}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^4)^(3/4),x]
```

```
[Out] (a + b*x^4)^(1/4)/b
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^4\right)^{3/4}}\,dx = \frac{\sqrt[4]{a+bx^4}}{b}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\sqrt[4]{a+bx^4}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^4)^(3/4),x]
```

```
[Out] (a + b*x^4)^(1/4)/b
```

fricas [A] time = 0.89, size = 13, normalized size = 0.87

$$\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] (b*x^4 + a)^(1/4)/b
```

giac [A] time = 0.15, size = 13, normalized size = 0.87

$$\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)^(3/4),x, algorithm="giac")
[Out] (b*x^4 + a)^(1/4)/b
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{\left(b\,x^4+a\right)^{\frac{1}{4}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^4+a)^(3/4),x)
[Out] (b*x^4+a)^(1/4)/b
```

maxima [A] time = 1.35, size = 13, normalized size = 0.87

$$\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)^(3/4),x, algorithm="maxima")
[Out] (b*x^4 + a)^(1/4)/b
```

mupad [B] time = 1.12, size = 13, normalized size = 0.87

$$\frac{\left(b\,x^4+a\right)^{1/4}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^4)^(3/4),x)
[Out] (a + b*x^4)^(1/4)/b
```

sympy [A] time = 0.93, size = 20, normalized size = 1.33

$$\begin{cases} \frac{\sqrt[4]{a+bx^4}}{b} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{3}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**4+a)**(3/4),x)
[Out] Piecewise(((a + b*x**4)**(1/4)/b, Ne(b, 0)), (x**4/(4*a**(3/4)), True))
```

3.1115 $\int \frac{1}{x\left(a+bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=55

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}$$

```
[Out] -1/2*arctan((b*x^4+a)^(1/4)/a^(1/4))/a^(3/4)-1/2*arctanh((b*x^4+a)^(1/4)/a^
(1/4))/a^(3/4)
```

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {266, 63, 212, 206, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^4)^(3/4)),x]
```

```
[Out] -ArcTan[(a + b*x^4)^(1/4)/a^(1/4)]/(2*a^(3/4)) - ArcTanh[(a + b*x^4)^(1/4)/
a^(1/4)]/(2*a^(3/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^4\right)^{3/4}}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/4}}\,dx, x, x^4\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^4}{b}}\,dx, x, \sqrt[4]{a+bx^4}\right)}{b}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{a}-x^2}\,dx, x, \sqrt[4]{a+bx^4}\right)}{2\sqrt{a}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a}+x^2}\,dx, x, \sqrt[4]{a+bx^4}\right)}{2\sqrt{a}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 0.84

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right) + \tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^4)^(3/4)),x]
```

```
[Out] -1/2*(ArcTan[(a + b*x^4)^(1/4)/a^(1/4)] + ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)
])/a^(3/4)
```

fricas [B] time = 0.91, size = 109, normalized size = 1.98

$$\frac{1}{a^3}^{\frac{1}{4}} \arctan\left(\sqrt{a^2\sqrt{\frac{1}{a^3}} + \sqrt{bx^4+a}}\; a^2 \frac{1}{a^3}^{\frac{3}{4}} - \left(bx^4+a\right)^{\frac{1}{4}} a^2 \frac{1}{a^3}^{\frac{3}{4}}\right) - \frac{1}{4}\frac{1}{a^3}^{\frac{1}{4}} \log\left(a\frac{1}{a^3}^{\frac{1}{4}} + \left(bx^4+a\right)^{\frac{1}{4}}\right) + \frac{1}{4}\frac{1}{a^3}^{\frac{1}{4}} \log\left(-a\frac{1}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] (a^(-3))^(1/4)*arctan(sqrt(a^2*sqrt(a^(-3)) + sqrt(b*x^4 + a))*a^2*(a^(-3))
^(3/4) - (b*x^4 + a)^(1/4)*a^2*(a^(-3))^(3/4)) - 1/4*(a^(-3))^(1/4)*log(a*(
a^(-3))^(1/4) + (b*x^4 + a)^(1/4)) + 1/4*(a^(-3))^(1/4)*log(-a*(a^(-3))^(1/
4) + (b*x^4 + a)^(1/4))
```

giac [B] time = 0.16, size = 186, normalized size = 3.38

$$-\frac{\sqrt{2}\,(-a)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}} + 2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{4\,a} - \frac{\sqrt{2}\,(-a)^{\frac{1}{4}} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}} - 2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{4\,a} - \frac{\sqrt{2}\,(-a)^{\frac{1}{4}} \log\left(\sqrt{2}\left(bx^4\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(-a)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x^4 +
 a)^(1/4))/(-a)^(1/4))/a - 1/4*sqrt(2)*(-a)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4))/a - 1/8*sqrt(2)*(-a)^(1/4
```

```
)*log(sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt(-a))/a
+ 1/8*sqrt(2)*(-a)^(1/4)*log(-sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b
*x^4 + a) + sqrt(-a))/a
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{3}{4}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^4+a)^(3/4),x)
```

```
[Out] int(1/x/(b*x^4+a)^(3/4),x)
```

maxima [A] time = 3.01, size = 57, normalized size = 1.04

$$-\frac{\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{2\,a^{\frac{3}{4}}}+\frac{\log\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{\left(bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{4\,a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] -1/2*arctan((b*x^4 + a)^(1/4)/a^(1/4))/a^(3/4) + 1/4*log(((b*x^4 + a)^(1/4)
 - a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/4)))/a^(3/4)
```

mupad [B] time = 1.16, size = 34, normalized size = 0.62

$$-\frac{\operatorname{atan}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)+\operatorname{atanh}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)}{2\,a^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^4)^(3/4)),x)
```

```
[Out] -(atan((a + b*x^4)^(1/4)/a^(1/4)) + atanh((a + b*x^4)^(1/4)/a^(1/4)))/(2*a^
(3/4))
```

sympy [C] time = 2.09, size = 39, normalized size = 0.71

$$-\frac{\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{ae^{i\pi}}{bx^4}\right)}{4b^{\frac{3}{4}}x^3\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**4+a)**(3/4),x)
```

```
[Out] -gamma(3/4)*hyper((3/4, 3/4), (7/4,), a*exp_polar(I*pi)/(b*x**4))/(4*b**(3/
4)*x**3*gamma(7/4))
```

3.1116 $\int \frac{1}{x^5\left(a+bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=78

$$\frac{3b\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{7/4}}+\frac{3b\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{7/4}}-\frac{\sqrt[4]{a+bx^4}}{4ax^4}$$

```
[Out] -1/4*(b*x^4+a)^(1/4)/a/x^4+3/8*b*arctan((b*x^4+a)^(1/4)/a^(1/4))/a^(7/4)+3/
8*b*arctanh((b*x^4+a)^(1/4)/a^(1/4))/a^(7/4)
```

Rubi [A] time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {266, 51, 63, 212, 206, 203}

$$\frac{3b\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{7/4}}+\frac{3b\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{7/4}}-\frac{\sqrt[4]{a+bx^4}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^4)^(3/4)),x]
```

```
[Out] -(a + b*x^4)^(1/4)/(4*a*x^4) + (3*b*ArcTan[(a + b*x^4)^(1/4)/a^(1/4)])/(8*a
^(7/4)) + (3*b*ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)])/(8*a^(7/4))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
```

```
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^5\left(a+bx^4\right)^{3/4}}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/4}}\,dx, x, x^4\right)$$

$$= -\frac{\sqrt[4]{a+bx^4}}{4ax^4} - \frac{(3b)\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/4}}\,dx, x, x^4\right)}{16a}$$

$$= -\frac{\sqrt[4]{a+bx^4}}{4ax^4} - \frac{3\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^4}{b}}\,dx, x, \sqrt[4]{a+bx^4}\right)}{4a}$$

$$= -\frac{\sqrt[4]{a+bx^4}}{4ax^4} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{\sqrt{a}-x^2}\,dx, x, \sqrt[4]{a+bx^4}\right)}{8a^{3/2}} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{\sqrt{a}+x^2}\,dx, x, \sqrt[4]{a}\right)}{8a^{3/2}}$$

$$= -\frac{\sqrt[4]{a+bx^4}}{4ax^4} + \frac{3b\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{7/4}} + \frac{3b\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{7/4}}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.44

$$\frac{b\sqrt[4]{a+bx^4}\;{}_2F_1\left(\frac{1}{4},2;\frac{5}{4};\frac{bx^4}{a}+1\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^4)^(3/4)),x]
```

```
[Out] (b*(a + b*x^4)^(1/4)*Hypergeometric2F1[1/4, 2, 5/4, 1 + (b*x^4)/a])/a^2
```

fricas [B] time = 0.89, size = 194, normalized size = 2.49

$$-\frac{12\,ax^4\left(\frac{b^4}{a^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(bx^4+a\right)^{\frac{1}{4}}a^5b\left(\frac{b^4}{a^7}\right)^{\frac{3}{4}}-\sqrt{a^4\sqrt{\frac{b^4}{a^7}}+\sqrt{bx^4+a}\,b^2}\,a^5\left(\frac{b^4}{a^7}\right)^{\frac{3}{4}}}{b^4}\right)-3\,ax^4\left(\frac{b^4}{a^7}\right)^{\frac{1}{4}}\log\left(3\,a^2\left(\frac{b^4}{a^7}\right)^{\frac{1}{4}}+3\left(bx^4+a\right)^{\frac{1}{4}}\right.}{16\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/16*(12*a*x^4*(b^4/a^7)^(1/4)*arctan(-((b*x^4 + a)^(1/4)*a^5*b*(b^4/a^7)^
(3/4) - sqrt(a^4*sqrt(b^4/a^7) + sqrt(b*x^4 + a)*b^2)*a^5*(b^4/a^7)^(3/4))/
b^4) - 3*a*x^4*(b^4/a^7)^(1/4)*log(3*a^2*(b^4/a^7)^(1/4) + 3*(b*x^4 + a)^(1
/4)*b) + 3*a*x^4*(b^4/a^7)^(1/4)*log(-3*a^2*(b^4/a^7)^(1/4) + 3*(b*x^4 + a)
^(1/4)*b) + 4*(b*x^4 + a)^(1/4))/(a*x^4)
```

giac [B] time = 0.16, size = 221, normalized size = 2.83

$$\frac{6\sqrt{2}(-a)^{\frac{1}{4}}b^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a^2}+\frac{6\sqrt{2}(-a)^{\frac{1}{4}}b^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a^2}+\frac{3\sqrt{2}(-a)^{\frac{1}{4}}b^2\log\left(\sqrt{2}\left(bx^4+a\right)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^4}\right.}{a^2}$$

$$32\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] 1/32*(6*sqrt(2)*(-a)^(1/4)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(
b*x^4 + a)^(1/4))/(-a)^(1/4))/a^2 + 6*sqrt(2)*(-a)^(1/4)*b^2*arctan(-1/2*sq
rt(2)*(sqrt(2)*(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4))/a^2 + 3*sqrt(2
)*(-a)^(1/4)*b^2*log(sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a)
 + sqrt(-a))/a^2 + 3*sqrt(2)*b^2*log(-sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4)
+ sqrt(b*x^4 + a) + sqrt(-a))/((-a)^(3/4)*a) - 8*(b*x^4 + a)^(1/4)*b/(a*x^4
))/b
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{3}{4}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^4+a)^(3/4),x)
```

```
[Out] int(1/x^5/(b*x^4+a)^(3/4),x)
```

maxima [A] time = 3.03, size = 94, normalized size = 1.21

$$-\frac{\left(bx^4+a\right)^{\frac{1}{4}}b}{4\left(\left(bx^4+a\right)a-a^2\right)}+\frac{3\left(\frac{2\,b\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{3}{4}}}-\frac{b\log\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{\left(bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{3}{4}}}\right)}{16\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] -1/4*(b*x^4 + a)^(1/4)*b/((b*x^4 + a)*a - a^2) + 3/16*(2*b*arctan((b*x^4 +
a)^(1/4)/a^(1/4))/a^(3/4) - b*log(((b*x^4 + a)^(1/4) - a^(1/4))/((b*x^4 + a
)^(1/4) + a^(1/4)))/a^(3/4))/a
```

mupad [B] time = 1.29, size = 58, normalized size = 0.74

$$\frac{3\,b\,\mathrm{atan}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)}{8\,a^{7/4}}-\frac{\left(b\,x^4+a\right)^{1/4}}{4\,a\,x^4}+\frac{3\,b\,\mathrm{atanh}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)}{8\,a^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^4)^(3/4)),x)
```

```
[Out] (3*b*atan((a + b*x^4)^(1/4)/a^(1/4)))/(8*a^(7/4)) - (a + b*x^4)^(1/4)/(4*a*
x^4) + (3*b*atanh((a + b*x^4)^(1/4)/a^(1/4)))/(8*a^(7/4))
```

sympy [C] time = 2.47, size = 39, normalized size = 0.50

$$-\frac{\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4}, \frac{7}{4} \\ \frac{11}{4}\end{matrix}\right| \frac{ae^{i\pi}}{bx^4}\right)}{4b^{\frac{3}{4}}x^7\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**4+a)**(3/4),x)
```

```
[Out] -gamma(7/4)*hyper((3/4, 7/4), (11/4,), a*exp_polar(I*pi)/(b*x**4))/(4*b**(3
/4)*x**7*gamma(11/4))
```

3.1117 $\int \frac{1}{x^9\left(a+bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=104

$$-\frac{21b^2\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{11/4}}-\frac{21b^2\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{11/4}}+\frac{7b\sqrt[4]{a+bx^4}}{32a^2x^4}-\frac{\sqrt[4]{a+bx^4}}{8ax^8}$$

```
[Out] -1/8*(b*x^4+a)^(1/4)/a/x^8+7/32*b*(b*x^4+a)^(1/4)/a^2/x^4-21/64*b^2*arctan(
(b*x^4+a)^(1/4)/a^(1/4))/a^(11/4)-21/64*b^2*arctanh((b*x^4+a)^(1/4)/a^(1/4)
)/a^(11/4)
```

Rubi [A] time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {266, 51, 63, 212, 206, 203}

$$-\frac{21b^2\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{11/4}}-\frac{21b^2\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{11/4}}+\frac{7b\sqrt[4]{a+bx^4}}{32a^2x^4}-\frac{\sqrt[4]{a+bx^4}}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(a + b*x^4)^(3/4)),x]
```

```
[Out] -(a + b*x^4)^(1/4)/(8*a*x^8) + (7*b*(a + b*x^4)^(1/4))/(32*a^2*x^4) - (21*b
^2*ArcTan[(a + b*x^4)^(1/4)/a^(1/4)])/(64*a^(11/4)) - (21*b^2*ArcTanh[(a +
b*x^4)^(1/4)/a^(1/4)])/(64*a^(11/4))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^9\left(a+bx^4\right)^{3/4}}\,dx = \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x^3(a+bx)^{3/4}}\,dx, x, x^4\right)$$

$$= -\frac{\sqrt[4]{a+bx^4}}{8ax^8} - \frac{(7b)\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/4}}\,dx, x, x^4\right)}{32a}$$

$$= -\frac{\sqrt[4]{a+bx^4}}{8ax^8} + \frac{7b\sqrt[4]{a+bx^4}}{32a^2x^4} + \frac{\left(21b^2\right)\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{3/4}}\,dx, x, x^4\right)}{128a^2}$$

$$= -\frac{\sqrt[4]{a+bx^4}}{8ax^8} + \frac{7b\sqrt[4]{a+bx^4}}{32a^2x^4} + \frac{(21b)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^4}{b}}\,dx, x, \sqrt[4]{a+bx^4}\right)}{32a^2}$$

$$= -\frac{\sqrt[4]{a+bx^4}}{8ax^8} + \frac{7b\sqrt[4]{a+bx^4}}{32a^2x^4} - \frac{\left(21b^2\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a}-x^2}\,dx, x, \sqrt[4]{a+bx^4}\right)}{64a^{5/2}} - \frac{\left(21b^2\right)\operatorname{Subst}}{}$$

$$= -\frac{\sqrt[4]{a+bx^4}}{8ax^8} + \frac{7b\sqrt[4]{a+bx^4}}{32a^2x^4} - \frac{21b^2\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{11/4}} - \frac{21b^2\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{11/4}}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.36

$$-\frac{b^2\sqrt[4]{a+bx^4}\;{}_2F_1\left(\frac{1}{4},3;\frac{5}{4};\frac{bx^4}{a}+1\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^9*(a + b*x^4)^(3/4)),x]
```

```
[Out] -((b^2*(a + b*x^4)^(1/4)*Hypergeometric2F1[1/4, 3, 5/4, 1 + (b*x^4)/a])/a^3
)
```

fricas [B] time = 0.79, size = 216, normalized size = 2.08

$$\frac{84\,a^2x^8\left(\frac{b^8}{a^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(bx^4+a\right)^{\frac{1}{4}}a^8b^2\left(\frac{b^8}{a^{11}}\right)^{\frac{3}{4}}-\sqrt{a^6\sqrt{\frac{b^8}{a^{11}}}+\sqrt{bx^4+a}\,b^4}\,a^8\left(\frac{b^8}{a^{11}}\right)^{\frac{3}{4}}}{b^8}\right)-21\,a^2x^8\left(\frac{b^8}{a^{11}}\right)^{\frac{1}{4}}\log\left(21\,a^3\left(\frac{b^8}{a^{11}}\right)^{\frac{1}{4}}+21\,\left(\right.\right.}{128\,a^2x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/128*(84*a^2*x^8*(b^8/a^11)^(1/4)*arctan(-((b*x^4 + a)^(1/4)*a^8*b^2*(b^8/
a^11)^(3/4) - sqrt(a^6*sqrt(b^8/a^11) + sqrt(b*x^4 + a)*b^4)*a^8*(b^8/a^11)
^(3/4))/b^8) - 21*a^2*x^8*(b^8/a^11)^(1/4)*log(21*a^3*(b^8/a^11)^(1/4) + 21
*(b*x^4 + a)^(1/4)*b^2) + 21*a^2*x^8*(b^8/a^11)^(1/4)*log(-21*a^3*(b^8/a^11
)^(1/4) + 21*(b*x^4 + a)^(1/4)*b^2) + 4*(7*b*x^4 - 4*a)*(b*x^4 + a)^(1/4))/
(a^2*x^8)
```

giac [B] time = 0.18, size = 244, normalized size = 2.35

$$\frac{42\sqrt{2}\,b^3\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}+2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{3}{4}}a^2}+\frac{42\sqrt{2}\,b^3\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}-2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{3}{4}}a^2}+\frac{21\sqrt{2}\,b^3\log\left(\sqrt{2}\left(bx^4+a\right)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^4+a}+\sqrt{-a}\right)}{(-a)^{\frac{3}{4}}a^2}+$$

$$256\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] 1/256*(42*sqrt(2)*b^3*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x^4 + a
)^(1/4))/(-a)^(1/4))/((-a)^(3/4)*a^2) + 42*sqrt(2)*b^3*arctan(-1/2*sqrt(2)*
(sqrt(2)*(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4))/((-a)^(3/4)*a^2) + 2
1*sqrt(2)*b^3*log(sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) +
sqrt(-a))/((-a)^(3/4)*a^2) + 21*sqrt(2)*(-a)^(1/4)*b^3*log(-sqrt(2)*(b*x^4
+ a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt(-a))/a^3 + 8*(7*(b*x^4 + a)^
(5/4)*b^3 - 11*(b*x^4 + a)^(1/4)*a*b^3)/(a^2*b^2*x^8))/b
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{3}{4}}x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(b*x^4+a)^(3/4),x)
```

```
[Out] int(1/x^9/(b*x^4+a)^(3/4),x)
```

maxima [A] time = 3.10, size = 132, normalized size = 1.27

$$\frac{7\left(bx^4+a\right)^{\frac{5}{4}}b^2-11\left(bx^4+a\right)^{\frac{1}{4}}ab^2}{32\left(\left(bx^4+a\right)^2a^2-2\left(bx^4+a\right)a^3+a^4\right)}-\frac{21\left(\frac{2\,b^2\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{3}{4}}}-\frac{b^2\log\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{\left(bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{3}{4}}}\right)}{128\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] 1/32*(7*(b*x^4 + a)^(5/4)*b^2 - 11*(b*x^4 + a)^(1/4)*a*b^2)/((b*x^4 + a)^2*
a^2 - 2*(b*x^4 + a)*a^3 + a^4) - 21/128*(2*b^2*arctan((b*x^4 + a)^(1/4)/a^(
1/4))/a^(3/4) - b^2*log(((b*x^4 + a)^(1/4) - a^(1/4))/((b*x^4 + a)^(1/4) +
a^(1/4)))/a^(3/4))/a^2
```

mupad [B] time = 1.36, size = 82, normalized size = 0.79

$$\frac{7\left(b\,x^4+a\right)^{5/4}}{32\,a^2\,x^8}-\frac{11\left(b\,x^4+a\right)^{1/4}}{32\,a\,x^8}-\frac{21\,b^2\,\mathrm{atan}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)}{64\,a^{11/4}}+\frac{b^2\,\mathrm{atan}\left(\frac{\left(b\,x^4+a\right)^{1/4}\,1\mathrm{i}}{a^{1/4}}\right)21\mathrm{i}}{64\,a^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(a + b*x^4)^(3/4)),x)
```

```
[Out] (b^2*atan(((a + b*x^4)^(1/4)*1i)/a^(1/4))*21i)/(64*a^(11/4)) - (21*b^2*atan
((a + b*x^4)^(1/4)/a^(1/4)))/(64*a^(11/4)) - (11*(a + b*x^4)^(1/4))/(32*a*x
^8) + (7*(a + b*x^4)^(5/4))/(32*a^2*x^8)
```

sympy [C] time = 3.51, size = 39, normalized size = 0.38

$$-\frac{\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4}, \frac{11}{4} \\ \frac{15}{4}\end{matrix}\middle| \frac{ae^{i\pi}}{bx^4}\right)}{4b^{\frac{3}{4}}x^{11}\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(b*x**4+a)**(3/4),x)
```

```
[Out] -gamma(11/4)*hyper((3/4, 11/4), (15/4,), a*exp_polar(I*pi)/(b*x**4))/(4*b**
(3/4)*x**11*gamma(15/4))
```

3.1118 $\int \frac{x^{13}}{\left(a+bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=128

$$-\frac{40a^{7/2}\left(\frac{bx^4}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{77b^{7/2}\left(a+bx^4\right)^{3/4}}+\frac{20a^2x^2\sqrt[4]{a+bx^4}}{77b^3}-\frac{10ax^6\sqrt[4]{a+bx^4}}{77b^2}+\frac{x^{10}\sqrt[4]{a+bx^4}}{11b}$$

```
[Out] 20/77*a^2*x^2*(b*x^4+a)^(1/4)/b^3-10/77*a*x^6*(b*x^4+a)^(1/4)/b^2+1/11*x^10
*(b*x^4+a)^(1/4)/b-40/77*a^(7/2)*(1+b*x^4/a)^(3/4)*(cos(1/2*arctan(x^2*b^(1
/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1
/2*arctan(x^2*b^(1/2)/a^(1/2))),2^(1/2))/b^(7/2)/(b*x^4+a)^(3/4)
```

Rubi [A] time = 0.08, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {275, 321, 233, 231}

$$\frac{20a^2x^2\sqrt[4]{a+bx^4}}{77b^3}-\frac{40a^{7/2}\left(\frac{bx^4}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{77b^{7/2}\left(a+bx^4\right)^{3/4}}-\frac{10ax^6\sqrt[4]{a+bx^4}}{77b^2}+\frac{x^{10}\sqrt[4]{a+bx^4}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/(a + b*x^4)^(3/4),x]
```

```
[Out] (20*a^2*x^2*(a + b*x^4)^(1/4))/(77*b^3) - (10*a*x^6*(a + b*x^4)^(1/4))/(77*
b^2) + (x^10*(a + b*x^4)^(1/4))/(11*b) - (40*a^(7/2)*(1 + (b*x^4)/a)^(3/4)*
EllipticF[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(77*b^(7/2)*(a + b*x^4)^(3/4
))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^{13}}{\left(a+bx^4\right)^{3/4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^6}{\left(a+bx^2\right)^{3/4}}\,dx, x, x^2\right)$$

$$= \frac{x^{10}\sqrt[4]{a+bx^4}}{11b} - \frac{(5a)\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^2\right)^{3/4}}\,dx, x, x^2\right)}{11b}$$

$$= -\frac{10ax^6\sqrt[4]{a+bx^4}}{77b^2} + \frac{x^{10}\sqrt[4]{a+bx^4}}{11b} + \frac{\left(30a^2\right)\text{Subst}\left(\int \frac{x^2}{\left(a+bx^2\right)^{3/4}}\,dx, x, x^2\right)}{77b^2}$$

$$= \frac{20a^2x^2\sqrt[4]{a+bx^4}}{77b^3} - \frac{10ax^6\sqrt[4]{a+bx^4}}{77b^2} + \frac{x^{10}\sqrt[4]{a+bx^4}}{11b} - \frac{\left(20a^3\right)\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx, x, x^2\right.}{77b^3}$$

$$= \frac{20a^2x^2\sqrt[4]{a+bx^4}}{77b^3} - \frac{10ax^6\sqrt[4]{a+bx^4}}{77b^2} + \frac{x^{10}\sqrt[4]{a+bx^4}}{11b} - \frac{\left(20a^3\left(1+\frac{bx^4}{a}\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{bx}{a}\right.}\right.}{77b^3\left(a+bx^4\right)^{3/4}}$$

$$= \frac{20a^2x^2\sqrt[4]{a+bx^4}}{77b^3} - \frac{10ax^6\sqrt[4]{a+bx^4}}{77b^2} + \frac{x^{10}\sqrt[4]{a+bx^4}}{11b} - \frac{40a^{7/2}\left(1+\frac{bx^4}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}}\right.\right.}{77b^{7/2}\left(a+bx^4\right)^{3/4}}$$

Mathematica [C] time = 0.05, size = 91, normalized size = 0.71

$$\frac{x^2\left(-20a^3\left(\frac{bx^4}{a}+1\right)^{3/4}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{bx^4}{a}\right)+20a^3+10a^2bx^4-3ab^2x^8+7b^3x^{12}\right)}{77b^3\left(a+bx^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13/(a + b*x^4)^(3/4),x]
```

```
[Out] (x^2*(20*a^3 + 10*a^2*b*x^4 - 3*a*b^2*x^8 + 7*b^3*x^12 - 20*a^3*(1 + (b*x^4
)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, -((b*x^4)/a)]))/(77*b^3*(a + b*
x^4)^(3/4))
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^{13}}{\left(bx^4+a\right)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(x^13/(b*x^4 + a)^(3/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{\left(bx^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^4+a)^(3/4),x, algorithm="giac")
[Out] integrate(x^13/(b*x^4 + a)^(3/4), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{\left(b\,x^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(b*x^4+a)^(3/4),x)
[Out] int(x^13/(b*x^4+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{\left(bx^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^4+a)^(3/4),x, algorithm="maxima")
[Out] integrate(x^13/(b*x^4 + a)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{13}}{\left(b\,x^4+a\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(a + b*x^4)^(3/4),x)
[Out] int(x^13/(a + b*x^4)^(3/4), x)
```

sympy [C] time = 2.72, size = 27, normalized size = 0.21

$$\frac{x^{14}\,{}_2F_1\left(\left.\begin{matrix}\frac{3}{4}, \frac{7}{2} \\ \frac{9}{2}\end{matrix}\right| \frac{bx^4e^{i\pi}}{a}\right)}{14a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(b*x**4+a)**(3/4),x)
[Out] x**14*hyper((3/4, 7/2), (9/2,), b*x**4*exp_polar(I*pi)/a)/(14*a**(3/4))
```

3.1119 $\int \frac{x^9}{\left(a+bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=104

$$\frac{4a^{5/2}\left(\frac{bx^4}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{7b^{5/2}\left(a+bx^4\right)^{3/4}}-\frac{2ax^2\sqrt[4]{a+bx^4}}{7b^2}+\frac{x^6\sqrt[4]{a+bx^4}}{7b}$$

```
[Out] -2/7*a*x^2*(b*x^4+a)^(1/4)/b^2+1/7*x^6*(b*x^4+a)^(1/4)/b+4/7*a^(5/2)*(1+b*x
^4/a)^(3/4)*(cos(1/2*arctan(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x
^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arctan(x^2*b^(1/2)/a^(1/2))),2^(1/2)
)/b^(5/2)/(b*x^4+a)^(3/4)
```

Rubi [A] time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {275, 321, 233, 231}

$$\frac{4a^{5/2}\left(\frac{bx^4}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{7b^{5/2}\left(a+bx^4\right)^{3/4}}-\frac{2ax^2\sqrt[4]{a+bx^4}}{7b^2}+\frac{x^6\sqrt[4]{a+bx^4}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x^4)^(3/4),x]
```

```
[Out] (-2*a*x^2*(a + b*x^4)^(1/4))/(7*b^2) + (x^6*(a + b*x^4)^(1/4))/(7*b) + (4*a
^(5/2)*(1 + (b*x^4)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])
/(7*b^(5/2)*(a + b*x^4)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^9}{\left(a+bx^4\right)^{3/4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^2\right)^{3/4}}\,dx, x, x^2\right)$$

$$= \frac{x^6\sqrt[4]{a+bx^4}}{7b} - \frac{(3a)\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^2\right)^{3/4}}\,dx, x, x^2\right)}{7b}$$

$$= -\frac{2ax^2\sqrt[4]{a+bx^4}}{7b^2} + \frac{x^6\sqrt[4]{a+bx^4}}{7b} + \frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx, x, x^2\right)}{7b^2}$$

$$= -\frac{2ax^2\sqrt[4]{a+bx^4}}{7b^2} + \frac{x^6\sqrt[4]{a+bx^4}}{7b} + \frac{\left(2a^2\left(1+\frac{bx^4}{a}\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx, x, x^2\right)}{7b^2\left(a+bx^4\right)^{3/4}}$$

$$= -\frac{2ax^2\sqrt[4]{a+bx^4}}{7b^2} + \frac{x^6\sqrt[4]{a+bx^4}}{7b} + \frac{4a^{5/2}\left(1+\frac{bx^4}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|\,2\right)}{7b^{5/2}\left(a+bx^4\right)^{3/4}}$$

Mathematica [C] time = 0.05, size = 79, normalized size = 0.76

$$\frac{x^2\left(2a^2\left(\frac{bx^4}{a}+1\right)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{bx^4}{a}\right) - 2a^2 - abx^4 + b^2x^8\right)}{7b^2\left(a+bx^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x^4)^(3/4),x]
```

```
[Out] (x^2*(-2*a^2 - a*b*x^4 + b^2*x^8 + 2*a^2*(1 + (b*x^4)/a)^(3/4)*Hypergeometr
ic2F1[1/2, 3/4, 3/2, -((b*x^4)/a)]))/(7*b^2*(a + b*x^4)^(3/4))
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^9}{\left(bx^4+a\right)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(x^9/(b*x^4 + a)^(3/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(bx^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^9/(b*x^4 + a)^(3/4), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(b\,x^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^4+a)^(3/4),x)
[Out] int(x^9/(b*x^4+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(bx^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)^(3/4),x, algorithm="maxima")
[Out] integrate(x^9/(b*x^4 + a)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^9}{\left(b\,x^4+a\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + b*x^4)^(3/4),x)
[Out] int(x^9/(a + b*x^4)^(3/4), x)
```

sympy [C] time = 2.32, size = 27, normalized size = 0.26

$$\frac{x^{10}\,{}_2F_1\left(\left.\begin{matrix}\frac{3}{4}, \frac{5}{2} \\ \frac{7}{2}\end{matrix}\right| \frac{bx^4e^{i\pi}}{a}\right)}{10a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**4+a)**(3/4),x)
[Out] x**10*hyper((3/4, 5/2), (7/2,), b*x**4*exp_polar(I*pi)/a)/(10*a**(3/4))
```

3.1120 $\int \frac{x^5}{\left(a+bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=82

$$\frac{x^2\sqrt[4]{a+bx^4}}{3b}-\frac{2a^{3/2}\left(\frac{bx^4}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{3b^{3/2}\left(a+bx^4\right)^{3/4}}$$

```
[Out] 1/3*x^2*(b*x^4+a)^(1/4)/b-2/3*a^(3/2)*(1+b*x^4/a)^(3/4)*(cos(1/2*arctan(x^2
*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x^2*b^(1/2)/a^(1/2)))*EllipticF(
sin(1/2*arctan(x^2*b^(1/2)/a^(1/2))),2^(1/2))/b^(3/2)/(b*x^4+a)^(3/4)
```

Rubi [A] time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {275, 321, 233, 231}

$$\frac{x^2\sqrt[4]{a+bx^4}}{3b}-\frac{2a^{3/2}\left(\frac{bx^4}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{3b^{3/2}\left(a+bx^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^4)^(3/4),x]
```

```
[Out] (x^2*(a + b*x^4)^(1/4))/(3*b) - (2*a^(3/2)*(1 + (b*x^4)/a)^(3/4)*EllipticF[
ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(3*b^(3/2)*(a + b*x^4)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^5}{\left(a+bx^4\right)^{3/4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^2\right)^{3/4}}\,dx, x, x^2\right)$$
$$= \frac{x^2\sqrt[4]{a+bx^4}}{3b} - \frac{a\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx, x, x^2\right)}{3b}$$
$$= \frac{x^2\sqrt[4]{a+bx^4}}{3b} - \frac{\left(a\left(1+\frac{bx^4}{a}\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx, x, x^2\right)}{3b\left(a+bx^4\right)^{3/4}}$$
$$= \frac{x^2\sqrt[4]{a+bx^4}}{3b} - \frac{2a^{3/2}\left(1+\frac{bx^4}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle| 2\right)}{3b^{3/2}\left(a+bx^4\right)^{3/4}}$$

Mathematica [C] time = 0.02, size = 64, normalized size = 0.78

$$\frac{x^2\left(-a\left(\frac{bx^4}{a}+1\right)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{bx^4}{a}\right)+a+bx^4\right)}{3b\left(a+bx^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^4)^(3/4),x]
```

```
[Out] (x^2*(a + b*x^4 - a*(1 + (b*x^4)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2,
-((b*x^4)/a)]))/(3*b*(a + b*x^4)^(3/4))
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^5}{\left(bx^4+a\right)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(x^5/(b*x^4 + a)^(3/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(bx^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^5/(b*x^4 + a)^(3/4), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(b\,x^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^4+a)^(3/4),x)
[Out] int(x^5/(b*x^4+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(bx^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)^(3/4),x, algorithm="maxima")
[Out] integrate(x^5/(b*x^4 + a)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\left(b\,x^4+a\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^4)^(3/4),x)
[Out] int(x^5/(a + b*x^4)^(3/4), x)
```

sympy [C] time = 1.30, size = 27, normalized size = 0.33

$$\frac{x^6\,{}_2F_1\left(\begin{matrix}\frac{3}{4}, \frac{3}{2} \\ \frac{5}{2}\end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{6a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**4+a)**(3/4),x)
[Out] x**6*hyper((3/4, 3/2), (5/2,), b*x**4*exp_polar(I*pi)/a)/(6*a**(3/4))
```

3.1121 $\int \frac{x}{\left(a+bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=57

$$\frac{\sqrt{a}\left(\frac{bx^4}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\left(a+bx^4\right)^{3/4}}$$

```
[Out] (1+b*x^4/a)^(3/4)*(cos(1/2*arctan(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*ar
ctan(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arctan(x^2*b^(1/2)/a^(1/2))),2
^(1/2))*a^(1/2)/(b*x^4+a)^(3/4)/b^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {275, 233, 231}

$$\frac{\sqrt{a}\left(\frac{bx^4}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\left(a+bx^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^4)^(3/4),x]
```

```
[Out] (Sqrt[a]*(1 + (b*x^4)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2
])/(Sqrt[b]*(a + b*x^4)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^4\right)^{3/4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx, x, x^2\right)$$
$$= \frac{\left(1+\frac{bx^4}{a}\right)^{3/4}\,\text{Subst}\left(\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx, x, x^2\right)}{2\left(a+bx^4\right)^{3/4}}$$
$$= \frac{\sqrt{a}\left(1+\frac{bx^4}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|\,2\right)}{\sqrt{b}\left(a+bx^4\right)^{3/4}}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.89

$$\frac{x^2\left(\frac{bx^4}{a}+1\right)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\frac{bx^4}{a}\right)}{2\left(a+bx^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^4)^(3/4),x]
```

```
[Out] (x^2*(1 + (b*x^4)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, -((b*x^4)/a)])/
(2*(a + b*x^4)^(3/4))
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{\left(bx^4+a\right)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(x/(b*x^4 + a)^(3/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x/(b*x^4 + a)^(3/4), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(b\,x^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^4+a)^(3/4),x)
```

```
[Out] int(x/(b*x^4+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*x^4 + a)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\left(b\,x^4+a\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^4)^(3/4),x)
```

```
[Out] int(x/(a + b*x^4)^(3/4), x)
```

sympy [C] time = 1.21, size = 27, normalized size = 0.47

$$\frac{x^2\,{}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{3}{4} \\ \frac{3}{2}\end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**4+a)**(3/4),x)
```

```
[Out] x**2*hyper((1/2, 3/4), (3/2,), b*x**4*exp_polar(I*pi)/a)/(2*a**(3/4))
```

3.1122 $\int \frac{1}{x^3\left(a+bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=82

$$-\frac{\sqrt[4]{a+bx^4}}{2ax^2}-\frac{\sqrt{b}\left(\frac{bx^4}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}\left(a+bx^4\right)^{3/4}}$$

```
[Out] -1/2*(b*x^4+a)^(1/4)/a/x^2-1/2*(1+b*x^4/a)^(3/4)*(cos(1/2*arctan(x^2*b^(1/2
)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2
*arctan(x^2*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)/(b*x^4+a)^(3/4)/a^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {275, 325, 233, 231}

$$-\frac{\sqrt[4]{a+bx^4}}{2ax^2}-\frac{\sqrt{b}\left(\frac{bx^4}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}\left(a+bx^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^4)^(3/4)),x]
```

```
[Out] -(a + b*x^4)^(1/4)/(2*a*x^2) - (Sqrt[b]*(1 + (b*x^4)/a)^(3/4)*EllipticF[Arc
Tan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(2*Sqrt[a]*(a + b*x^4)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^4\right)^{3/4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^{3/4}}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt[4]{a+bx^4}}{2ax^2} - \frac{b\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx, x, x^2\right)}{4a}$$

$$= -\frac{\sqrt[4]{a+bx^4}}{2ax^2} - \frac{\left(b\left(1+\frac{bx^4}{a}\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx, x, x^2\right)}{4a\left(a+bx^4\right)^{3/4}}$$

$$= -\frac{\sqrt[4]{a+bx^4}}{2ax^2} - \frac{\sqrt{b}\left(1+\frac{bx^4}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle| 2\right)}{2\sqrt{a}\left(a+bx^4\right)^{3/4}}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.62

$$-\frac{\left(\frac{bx^4}{a}+1\right)^{3/4}\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{1}{2};-\frac{bx^4}{a}\right)}{2x^2\left(a+bx^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^4)^(3/4)),x]
```

```
[Out] -1/2*((1 + (b*x^4)/a)^(3/4)*Hypergeometric2F1[-1/2, 3/4, 1/2, -((b*x^4)/a)]
)/(x^2*(a + b*x^4)^(3/4))
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{bx^7+ax^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(1/4)/(b*x^7 + a*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{3}{4}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(3/4)*x^3), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{3}{4}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^4+a)^(3/4),x)
[Out] int(1/x^3/(b*x^4+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{3}{4}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(3/4)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\left(b\,x^4+a\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^4)^(3/4)),x)
[Out] int(1/(x^3*(a + b*x^4)^(3/4)), x)
```

sympy [C] time = 1.47, size = 31, normalized size = 0.38

$$-\frac{{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{1}{2}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**4+a)**(3/4),x)
[Out] -hyper((-1/2, 3/4), (1/2,), b*x**4*exp_polar(I*pi)/a)/(2*a**(3/4)*x**2)
```

3.1123 $\int \frac{1}{x^7\left(a+bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=104

$$\frac{5b^{3/2}\left(\frac{bx^4}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{12a^{3/2}\left(a+bx^4\right)^{3/4}}+\frac{5b\sqrt[4]{a+bx^4}}{12a^2x^2}-\frac{\sqrt[4]{a+bx^4}}{6ax^6}$$

```
[Out] -1/6*(b*x^4+a)^(1/4)/a/x^6+5/12*b*(b*x^4+a)^(1/4)/a^2/x^2+5/12*b^(3/2)*(1+b
*x^4/a)^(3/4)*(cos(1/2*arctan(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan
(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arctan(x^2*b^(1/2)/a^(1/2))),2^(1/
2))/a^(3/2)/(b*x^4+a)^(3/4)
```

Rubi [A] time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {275, 325, 233, 231}

$$\frac{5b^{3/2}\left(\frac{bx^4}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{12a^{3/2}\left(a+bx^4\right)^{3/4}}+\frac{5b\sqrt[4]{a+bx^4}}{12a^2x^2}-\frac{\sqrt[4]{a+bx^4}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^4)^(3/4)),x]
```

```
[Out] -(a + b*x^4)^(1/4)/(6*a*x^6) + (5*b*(a + b*x^4)^(1/4))/(12*a^2*x^2) + (5*b^
(3/2)*(1 + (b*x^4)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/
(12*a^(3/2)*(a + b*x^4)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^7\left(a+bx^4\right)^{3/4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^2\right)^{3/4}}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt[4]{a+bx^4}}{6ax^6} - \frac{(5b)\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^{3/4}}\,dx, x, x^2\right)}{12a}$$

$$= -\frac{\sqrt[4]{a+bx^4}}{6ax^6} + \frac{5b\sqrt[4]{a+bx^4}}{12a^2x^2} + \frac{\left(5b^2\right)\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx, x, x^2\right)}{24a^2}$$

$$= -\frac{\sqrt[4]{a+bx^4}}{6ax^6} + \frac{5b\sqrt[4]{a+bx^4}}{12a^2x^2} + \frac{\left(5b^2\left(1+\frac{bx^4}{a}\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx, x, x^2\right)}{24a^2\left(a+bx^4\right)^{3/4}}$$

$$= -\frac{\sqrt[4]{a+bx^4}}{6ax^6} + \frac{5b\sqrt[4]{a+bx^4}}{12a^2x^2} + \frac{5b^{3/2}\left(1+\frac{bx^4}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|\,2\right)}{12a^{3/2}\left(a+bx^4\right)^{3/4}}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.49

$$-\frac{\left(\frac{bx^4}{a}+1\right)^{3/4}\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};-\frac{1}{2};-\frac{bx^4}{a}\right)}{6x^6\left(a+bx^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^4)^(3/4)),x]
```

```
[Out] -1/6*((1 + (b*x^4)/a)^(3/4)*Hypergeometric2F1[-3/2, 3/4, -1/2, -((b*x^4)/a)
])/(x^6*(a + b*x^4)^(3/4))
```

fricas [F] time = 1.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{bx^{11}+ax^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(1/4)/(b*x^11 + a*x^7), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{3}{4}}x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(3/4)*x^7), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{3}{4}}\,x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^4+a)^(3/4),x)
[Out] int(1/x^7/(b*x^4+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{3}{4}}x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(3/4)*x^7), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^7\left(b\,x^4+a\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^4)^(3/4)),x)
[Out] int(1/(x^7*(a + b*x^4)^(3/4)), x)
```

sympy [C] time = 2.20, size = 32, normalized size = 0.31

$$-\frac{{}_2F_1\left(\left.\begin{matrix}-\frac{3}{2}, \frac{3}{4} \\ -\frac{1}{2}\end{matrix}\right| \frac{bx^4e^{i\pi}}{a}\right)}{6a^{\frac{3}{4}}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**4+a)**(3/4),x)
[Out] -hyper((-3/2, 3/4), (-1/2,), b*x**4*exp_polar(I*pi)/a)/(6*a**(3/4)*x**6)
```

3.1124 $\int \frac{1}{x^{11}\left(a+bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=128

$$-\frac{3b^{5/2}\left(\frac{bx^4}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{8a^{5/2}\left(a+bx^4\right)^{3/4}}-\frac{3b^2\sqrt[4]{a+bx^4}}{8a^3x^2}+\frac{3b\sqrt[4]{a+bx^4}}{20a^2x^6}-\frac{\sqrt[4]{a+bx^4}}{10ax^{10}}$$

```
[Out] -1/10*(b*x^4+a)^(1/4)/a/x^10+3/20*b*(b*x^4+a)^(1/4)/a^2/x^6-3/8*b^2*(b*x^4+
a)^(1/4)/a^3/x^2-3/8*b^(5/2)*(1+b*x^4/a)^(3/4)*(cos(1/2*arctan(x^2*b^(1/2)/
a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*a
rctan(x^2*b^(1/2)/a^(1/2))),2^(1/2))/a^(5/2)/(b*x^4+a)^(3/4)
```

Rubi [A] time = 0.08, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {275, 325, 233, 231}

$$-\frac{3b^2\sqrt[4]{a+bx^4}}{8a^3x^2}-\frac{3b^{5/2}\left(\frac{bx^4}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{8a^{5/2}\left(a+bx^4\right)^{3/4}}+\frac{3b\sqrt[4]{a+bx^4}}{20a^2x^6}-\frac{\sqrt[4]{a+bx^4}}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^11*(a + b*x^4)^(3/4)),x]
```

```
[Out] -(a + b*x^4)^(1/4)/(10*a*x^10) + (3*b*(a + b*x^4)^(1/4))/(20*a^2*x^6) - (3*
b^2*(a + b*x^4)^(1/4))/(8*a^3*x^2) - (3*b^(5/2)*(1 + (b*x^4)/a)^(3/4)*Ellip
ticF[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(8*a^(5/2)*(a + b*x^4)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^{11}\left(a+bx^4\right)^{3/4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^6\left(a+bx^2\right)^{3/4}}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt[4]{a+bx^4}}{10ax^{10}} - \frac{(9b)\,\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^2\right)^{3/4}}\,dx, x, x^2\right)}{20a}$$

$$= -\frac{\sqrt[4]{a+bx^4}}{10ax^{10}} + \frac{3b\sqrt[4]{a+bx^4}}{20a^2x^6} + \frac{\left(3b^2\right)\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^{3/4}}\,dx, x, x^2\right)}{8a^2}$$

$$= -\frac{\sqrt[4]{a+bx^4}}{10ax^{10}} + \frac{3b\sqrt[4]{a+bx^4}}{20a^2x^6} - \frac{3b^2\sqrt[4]{a+bx^4}}{8a^3x^2} - \frac{\left(3b^3\right)\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx, x, x^2\right)}{16a^3}$$

$$= -\frac{\sqrt[4]{a+bx^4}}{10ax^{10}} + \frac{3b\sqrt[4]{a+bx^4}}{20a^2x^6} - \frac{3b^2\sqrt[4]{a+bx^4}}{8a^3x^2} - \frac{\left(3b^3\left(1+\frac{bx^4}{a}\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx\right.}{16a^3\left(a+bx^4\right)^{3/4}}$$

$$= -\frac{\sqrt[4]{a+bx^4}}{10ax^{10}} + \frac{3b\sqrt[4]{a+bx^4}}{20a^2x^6} - \frac{3b^2\sqrt[4]{a+bx^4}}{8a^3x^2} - \frac{3b^{5/2}\left(1+\frac{bx^4}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{8a^{5/2}\left(a+bx^4\right)^{3/4}}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.40

$$-\frac{\left(\frac{bx^4}{a}+1\right)^{3/4}\,{}_2F_1\left(-\frac{5}{2},\frac{3}{4};-\frac{3}{2};-\frac{bx^4}{a}\right)}{10x^{10}\left(a+bx^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^11*(a + b*x^4)^(3/4)),x]
```

```
[Out] -1/10*((1 + (b*x^4)/a)^(3/4)*Hypergeometric2F1[-5/2, 3/4, -3/2, -((b*x^4)/a
)])/(x^10*(a + b*x^4)^(3/4))
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{bx^{15}+ax^{11}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(1/4)/(b*x^15 + a*x^11), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{3}{4}}x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(b*x^4+a)^(3/4),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^(3/4)*x^11), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{3}{4}}\,x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^11/(b*x^4+a)^(3/4),x)
[Out] int(1/x^11/(b*x^4+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{3}{4}}x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(b*x^4+a)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(3/4)*x^11), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{11}\left(b\,x^4+a\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^11*(a + b*x^4)^(3/4)),x)
[Out] int(1/(x^11*(a + b*x^4)^(3/4)), x)
```

sympy [C] time = 2.71, size = 32, normalized size = 0.25

$$-\frac{{}_2F_1\left(\begin{matrix}-\frac{5}{2}, \frac{3}{4} \\ -\frac{3}{2}\end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{10a^{\frac{3}{4}}x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**11/(b*x**4+a)**(3/4),x)
[Out] -hyper((-5/2, 3/4), (-3/2,), b*x**4*exp_polar(I*pi)/a)/(10*a**(3/4)*x**10)
```

3.1125 $\int \frac{x^{10}}{\left(a+bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=106

$$-\frac{21a^2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{11/4}}+\frac{21a^2\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{11/4}}-\frac{7ax^3\sqrt[4]{a+bx^4}}{32b^2}+\frac{x^7\sqrt[4]{a+bx^4}}{8b}$$

```
[Out] -7/32*a*x^3*(b*x^4+a)^(1/4)/b^2+1/8*x^7*(b*x^4+a)^(1/4)/b-21/64*a^2*arctan(
b^(1/4)*x/(b*x^4+a)^(1/4))/b^(11/4)+21/64*a^2*arctanh(b^(1/4)*x/(b*x^4+a)^(
1/4))/b^(11/4)
```

Rubi [A] time = 0.04, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {321, 331, 298, 203, 206}

$$-\frac{21a^2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{11/4}}+\frac{21a^2\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{11/4}}-\frac{7ax^3\sqrt[4]{a+bx^4}}{32b^2}+\frac{x^7\sqrt[4]{a+bx^4}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x^4)^(3/4),x]
```

```
[Out] (-7*a*x^3*(a + b*x^4)^(1/4))/(32*b^2) + (x^7*(a + b*x^4)^(1/4))/(8*b) - (21
*a^2*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(64*b^(11/4)) + (21*a^2*ArcTanh
[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(64*b^(11/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
```

```
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int \frac{x^{10}}{\left(a+bx^4\right)^{3/4}}\,dx = \frac{x^7\sqrt[4]{a+bx^4}}{8b} - \frac{(7a)\int \frac{x^6}{\left(a+bx^4\right)^{3/4}}\,dx}{8b}$$

$$= -\frac{7ax^3\sqrt[4]{a+bx^4}}{32b^2} + \frac{x^7\sqrt[4]{a+bx^4}}{8b} + \frac{\left(21a^2\right)\int \frac{x^2}{\left(a+bx^4\right)^{3/4}}\,dx}{32b^2}$$

$$= -\frac{7ax^3\sqrt[4]{a+bx^4}}{32b^2} + \frac{x^7\sqrt[4]{a+bx^4}}{8b} + \frac{\left(21a^2\right)\text{Subst}\left(\int \frac{x^2}{1-bx^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{32b^2}$$

$$= -\frac{7ax^3\sqrt[4]{a+bx^4}}{32b^2} + \frac{x^7\sqrt[4]{a+bx^4}}{8b} + \frac{\left(21a^2\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{b}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{64b^{5/2}} - \frac{\left(21a^2\right)\text{Subst}\left(\int$$

$$= -\frac{7ax^3\sqrt[4]{a+bx^4}}{32b^2} + \frac{x^7\sqrt[4]{a+bx^4}}{8b} - \frac{21a^2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{64b^{11/4}} + \frac{21a^2\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{64b^{11/4}}$$

Mathematica [A] time = 0.04, size = 89, normalized size = 0.84

$$\frac{-21a^2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right) + 21a^2\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right) + 2b^{3/4}x^3\sqrt[4]{a+bx^4}\left(4bx^4-7a\right)}{64b^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x^4)^(3/4),x]
```

```
[Out] (2*b^(3/4)*x^3*(a + b*x^4)^(1/4)*(-7*a + 4*b*x^4) - 21*a^2*ArcTan[(b^(1/4)*
x)/(a + b*x^4)^(1/4)] + 21*a^2*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(64*
b^(11/4))
```

fricas [B] time = 1.01, size = 227, normalized size = 2.14

$$-\frac{84\,b^2\left(\frac{a^8}{b^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(bx^4+a\right)^{\frac{1}{4}}a^2b^8\left(\frac{a^8}{b^{11}}\right)^{\frac{3}{4}} - b^8x\sqrt{\frac{b^6x^2\sqrt{\frac{a^8}{b^{11}}}+\sqrt{bx^4+a}\,a^4}{x^2}}\left(\frac{a^8}{b^{11}}\right)^{\frac{3}{4}}}{a^8x}\right) - 21\,b^2\left(\frac{a^8}{b^{11}}\right)^{\frac{1}{4}}\log\left(\frac{21\left(b^3x\left(\frac{a^8}{b^{11}}\right)^{\frac{1}{4}}+\left(bx^4+a\right)^{\frac{1}{4}}a^2\right)}{x}\right)}{128\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/128*(84*b^2*(a^8/b^11)^(1/4)*arctan(-((b*x^4 + a)^(1/4)*a^2*b^8*(a^8/b^1
1)^(3/4) - b^8*x*sqrt((b^6*x^2*sqrt(a^8/b^11) + sqrt(b*x^4 + a)*a^4)/x^2)*(
a^8/b^11)^(3/4))/(a^8*x)) - 21*b^2*(a^8/b^11)^(1/4)*log(21*(b^3*x*(a^8/b^11
)^(1/4) + (b*x^4 + a)^(1/4)*a^2)/x) + 21*b^2*(a^8/b^11)^(1/4)*log(-21*(b^3*
x*(a^8/b^11)^(1/4) - (b*x^4 + a)^(1/4)*a^2)/x) - 4*(4*b*x^7 - 7*a*x^3)*(b*x
^4 + a)^(1/4))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\left(bx^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^4+a)^(3/4),x, algorithm="giac")
[Out] integrate(x^10/(b*x^4 + a)^(3/4), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\left(b\,x^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x^4+a)^(3/4),x)
[Out] int(x^10/(b*x^4+a)^(3/4),x)
```

maxima [A] time = 3.00, size = 155, normalized size = 1.46

$$\frac{\frac{11\left(bx^4+a\right)^{\frac{1}{4}}a^2b}{x}-\frac{7\left(bx^4+a\right)^{\frac{5}{4}}a^2}{x^5}}{32\left(b^4-\frac{2\left(bx^4+a\right)b^3}{x^4}+\frac{\left(bx^4+a\right)^2b^2}{x^8}\right)}+\frac{21\left(\frac{2\,a^2\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{3}{4}}}-\frac{a^2\log\left(-\frac{b^{\frac{1}{4}}-\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{3}{4}}}\right)}{128\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^4+a)^(3/4),x, algorithm="maxima")
[Out] 1/32*(11*(b*x^4 + a)^(1/4)*a^2*b/x - 7*(b*x^4 + a)^(5/4)*a^2/x^5)/(b^4 - 2*
(b*x^4 + a)*b^3/x^4 + (b*x^4 + a)^2*b^2/x^8) + 21/128*(2*a^2*arctan((b*x^4
+ a)^(1/4)/(b^(1/4)*x))/b^(3/4) - a^2*log(-(b^(1/4) - (b*x^4 + a)^(1/4)/x)/
(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(3/4))/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{10}}{\left(b\,x^4+a\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a + b*x^4)^(3/4),x)
[Out] int(x^10/(a + b*x^4)^(3/4), x)
```

sympy [C] time = 2.35, size = 37, normalized size = 0.35

$$\frac{x^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{4}}\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b*x**4+a)**(3/4),x)
```

```
[Out] x**11*gamma(11/4)*hyper((3/4, 11/4), (15/4,), b*x**4*exp_polar(I*pi)/a)/(4*
a**(3/4)*gamma(15/4))
```

3.1126 $\int \frac{x^6}{\left(a+bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=80

$$\frac{3a\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8b^{7/4}}-\frac{3a\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8b^{7/4}}+\frac{x^3\sqrt[4]{a+bx^4}}{4b}$$

```
[Out] 1/4*x^3*(b*x^4+a)^(1/4)/b+3/8*a*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(7/4)-3
/8*a*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(7/4)
```

Rubi [A] time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {321, 331, 298, 203, 206}

$$\frac{3a\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8b^{7/4}}-\frac{3a\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8b^{7/4}}+\frac{x^3\sqrt[4]{a+bx^4}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^4)^(3/4),x]
```

```
[Out] (x^3*(a + b*x^4)^(1/4))/(4*b) + (3*a*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)])
/(8*b^(7/4)) - (3*a*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(8*b^(7/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
```

```
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int \frac{x^6}{\left(a+bx^4\right)^{3/4}}\,dx = \frac{x^3\sqrt[4]{a+bx^4}}{4b} - \frac{(3a)\int \frac{x^2}{\left(a+bx^4\right)^{3/4}}\,dx}{4b}$$

$$= \frac{x^3\sqrt[4]{a+bx^4}}{4b} - \frac{(3a)\,\text{Subst}\left(\int \frac{x^2}{1-bx^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{4b}$$

$$= \frac{x^3\sqrt[4]{a+bx^4}}{4b} - \frac{(3a)\,\text{Subst}\left(\int \frac{1}{1-\sqrt{b}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{8b^{3/2}} + \frac{(3a)\,\text{Subst}\left(\int \frac{1}{1+\sqrt{b}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{8b^{3/2}}$$

$$= \frac{x^3\sqrt[4]{a+bx^4}}{4b} + \frac{3a\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8b^{7/4}} - \frac{3a\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8b^{7/4}}$$

Mathematica [A] time = 0.02, size = 75, normalized size = 0.94

$$\frac{2b^{3/4}x^3\sqrt[4]{a+bx^4} + 3a\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right) - 3a\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^4)^(3/4),x]
```

```
[Out] (2*b^(3/4)*x^3*(a + b*x^4)^(1/4) + 3*a*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)
] - 3*a*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(8*b^(7/4))
```

fricas [B] time = 0.53, size = 204, normalized size = 2.55

$$\frac{4\left(bx^4+a\right)^{\frac{1}{4}}x^3 + 12\,b\left(\frac{a^4}{b^7}\right)^{\frac{1}{4}}\arctan\left(\frac{b^5x\sqrt{\frac{b^4x^2\sqrt{\frac{a^4}{b^7}}+\sqrt{bx^4+a}\,a^2}{x^2}}\left(\frac{a^4}{b^7}\right)^{\frac{3}{4}} - \left(bx^4+a\right)^{\frac{1}{4}}ab^5\left(\frac{a^4}{b^7}\right)^{\frac{3}{4}}}{a^4x}\right) - 3\,b\left(\frac{a^4}{b^7}\right)^{\frac{1}{4}}\log\left(\frac{3\left(b^2x\left(\frac{a^4}{b^7}\right)^{\frac{1}{4}}+\left(bx^4+a\right.\right.\right.}{x}$$

$$16\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/16*(4*(b*x^4 + a)^(1/4)*x^3 + 12*b*(a^4/b^7)^(1/4)*arctan((b^5*x*sqrt((b^
4*x^2*sqrt(a^4/b^7) + sqrt(b*x^4 + a)*a^2)/x^2)*(a^4/b^7)^(3/4) - (b*x^4 +
a)^(1/4)*a*b^5*(a^4/b^7)^(3/4))/(a^4*x)) - 3*b*(a^4/b^7)^(1/4)*log(3*(b^2*x
*(a^4/b^7)^(1/4) + (b*x^4 + a)^(1/4)*a)/x) + 3*b*(a^4/b^7)^(1/4)*log(-3*(b^
2*x*(a^4/b^7)^(1/4) - (b*x^4 + a)^(1/4)*a)/x))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(bx^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/(b*x^4 + a)^(3/4), x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(b\,x^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^4+a)^(3/4),x)
```

```
[Out] int(x^6/(b*x^4+a)^(3/4),x)
```

maxima [A] time = 3.00, size = 110, normalized size = 1.38

$$-\frac{3\left(\frac{2\,a\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{3}{4}}}-\frac{a\log\left(-\frac{b^{\frac{1}{4}}-\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{3}{4}}}\right)}{16\,b}-\frac{\left(bx^4+a\right)^{\frac{1}{4}}a}{4\left(b^2-\frac{\left(bx^4+a\right)b}{x^4}\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] -3/16*(2*a*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(3/4) - a*log(-(b^(1/4)
- (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(3/4))/b - 1/4*(b
*x^4 + a)^(1/4)*a/((b^2 - (b*x^4 + a)*b/x^4)*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{\left(b\,x^4+a\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^4)^(3/4),x)
```

```
[Out] int(x^6/(a + b*x^4)^(3/4), x)
```

sympy [C] time = 1.64, size = 37, normalized size = 0.46

$$\frac{x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{4}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**4+a)**(3/4),x)
```

```
[Out] x**7*gamma(7/4)*hyper((3/4, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**
(3/4)*gamma(11/4))
```

3.1127 $\int \frac{x^2}{\left(a+bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=57

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}$$

```
[Out] -1/2*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(3/4)+1/2*arctanh(b^(1/4)*x/(b*x^4
+a)^(1/4))/b^(3/4)
```

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {331, 298, 203, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^4)^(3/4),x]
```

```
[Out] -ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(3/4)) + ArcTanh[(b^(1/4)*x)/(a
 + b*x^4)^(1/4)]/(2*b^(3/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^4\right)^{3/4}}\,dx = \text{Subst}\left(\int \frac{x^2}{1-bx^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{b}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt{b}} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{b}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt{b}}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}$$

Mathematica [A] time = 0.01, size = 50, normalized size = 0.88

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right) - \tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^4)^(3/4),x]
```

```
[Out] (-ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)] + ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(
1/4)])/(2*b^(3/4))
```

fricas [B] time = 0.91, size = 134, normalized size = 2.35

$$-\frac{1}{b^3}^{\frac{1}{4}} \arctan\left(\frac{b^2 x \sqrt{\frac{b^2\sqrt{\frac{1}{b^3}}\,x^2+\sqrt{bx^4+a}}{x^2}}\,\frac{1}{b^3}^{\frac{3}{4}} - \left(bx^4+a\right)^{\frac{1}{4}} b^2 \frac{1}{b^3}^{\frac{3}{4}}}{x}\right) + \frac{1}{4}\frac{1}{b^3}^{\frac{1}{4}} \log\left(\frac{b\frac{1}{b^3}^{\frac{1}{4}}x + \left(bx^4+a\right)^{\frac{1}{4}}}{x}\right) - \frac{1}{4}\frac{1}{b^3}^{\frac{1}{4}} \log\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] -(b^(-3))^(1/4)*arctan((b^2*x*sqrt((b^2*sqrt(b^(-3))*x^2 + sqrt(b*x^4 + a))
/x^2)*(b^(-3))^(3/4) - (b*x^4 + a)^(1/4)*b^2*(b^(-3))^(3/4))/x) + 1/4*(b^(-
3))^(1/4)*log((b*(b^(-3))^(1/4)*x + (b*x^4 + a)^(1/4))/x) - 1/4*(b^(-3))^(1
/4)*log(-(b*(b^(-3))^(1/4)*x - (b*x^4 + a)^(1/4))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*x^4 + a)^(3/4), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(b\,x^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^4+a)^(3/4),x)
```

```
[Out] int(x^2/(b*x^4+a)^(3/4),x)
```

maxima [A] time = 2.90, size = 68, normalized size = 1.19

$$\frac{\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{2\,b^{\frac{3}{4}}}-\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}\right)}{4\,b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] 1/2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(3/4) - 1/4*log(-(b^(1/4) - (b*
x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(3/4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\left(b\,x^4+a\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^4)^(3/4),x)
```

```
[Out] int(x^2/(a + b*x^4)^(3/4), x)
```

sympy [C] time = 1.62, size = 37, normalized size = 0.65

$$\frac{x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**4+a)**(3/4),x)
```

```
[Out] x**3*gamma(3/4)*hyper((3/4, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
3/4)*gamma(7/4))
```

3.1128 $\int \frac{1}{x^2\left(a+bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=19

$$-\frac{\sqrt[4]{a+bx^4}}{ax}$$

```
[Out] -(b*x^4+a)^(1/4)/a/x
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {264}

$$-\frac{\sqrt[4]{a+bx^4}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^4)^(3/4)),x]
```

```
[Out] -((a + b*x^4)^(1/4)/(a*x))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^4\right)^{3/4}}\,dx = -\frac{\sqrt[4]{a+bx^4}}{ax}$$

Mathematica [A] time = 0.02, size = 19, normalized size = 1.00

$$-\frac{\sqrt[4]{a+bx^4}}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^4)^(3/4)),x]
```

```
[Out] -((a + b*x^4)^(1/4)/(a*x))
```

fricas [A] time = 1.09, size = 17, normalized size = 0.89

$$-\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] -(b*x^4 + a)^(1/4)/(a*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)^(3/4),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^(3/4)*x^2), x)
```

maple [A] time = 0.00, size = 18, normalized size = 0.95

$$-\frac{\left(b\,x^4+a\right)^{\frac{1}{4}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^4+a)^(3/4),x)
[Out] -(b*x^4+a)^(1/4)/a/x
```

maxima [A] time = 1.33, size = 17, normalized size = 0.89

$$-\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)^(3/4),x, algorithm="maxima")
[Out] -(b*x^4 + a)^(1/4)/(a*x)
```

mupad [B] time = 1.10, size = 17, normalized size = 0.89

$$-\frac{\left(b\,x^4+a\right)^{1/4}}{a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^4)^(3/4)),x)
[Out] -(a + b*x^4)^(1/4)/(a*x)
```

sympy [B] time = 1.21, size = 31, normalized size = 1.63

$$\frac{\sqrt[4]{b}\,\sqrt[4]{\frac{a}{bx^4}+1}\,\Gamma\left(-\frac{1}{4}\right)}{4a\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**4+a)**(3/4),x)
[Out] b**(1/4)*(a/(b*x**4) + 1)**(1/4)*gamma(-1/4)/(4*a*gamma(3/4))
```

3.1129 $\int \frac{1}{x^6\left(a+bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=44

$$\frac{4b\sqrt[4]{a+bx^4}}{5a^2x}-\frac{\sqrt[4]{a+bx^4}}{5ax^5}$$

```
[Out] -1/5*(b*x^4+a)^(1/4)/a/x^5+4/5*b*(b*x^4+a)^(1/4)/a^2/x
```

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$\frac{4b\sqrt[4]{a+bx^4}}{5a^2x}-\frac{\sqrt[4]{a+bx^4}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^4)^(3/4)),x]
```

```
[Out] -(a + b*x^4)^(1/4)/(5*a*x^5) + (4*b*(a + b*x^4)^(1/4))/(5*a^2*x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^6\left(a+bx^4\right)^{3/4}}\,dx &= -\frac{\sqrt[4]{a+bx^4}}{5ax^5}-\frac{(4b)\int \frac{1}{x^2\left(a+bx^4\right)^{3/4}}\,dx}{5a}\\ &= -\frac{\sqrt[4]{a+bx^4}}{5ax^5}+\frac{4b\sqrt[4]{a+bx^4}}{5a^2x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.66

$$-\frac{\left(a-4bx^4\right)\sqrt[4]{a+bx^4}}{5a^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^4)^(3/4)),x]
```

```
[Out] -1/5*((a - 4*b*x^4)*(a + b*x^4)^(1/4))/(a^2*x^5)
```

fricas [A] time = 0.91, size = 27, normalized size = 0.61

$$\frac{\left(4\,bx^4-a\right)\left(bx^4+a\right)^{\frac{1}{4}}}{5\,a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^4+a)^(3/4),x, algorithm="fricas")
[Out] 1/5*(4*b*x^4 - a)*(b*x^4 + a)^(1/4)/(a^2*x^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{3}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^4+a)^(3/4),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^(3/4)*x^6), x)
```

maple [A] time = 0.01, size = 26, normalized size = 0.59

$$-\frac{\left(b\,x^4+a\right)^{\frac{1}{4}}\left(-4b\,x^4+a\right)}{5a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^4+a)^(3/4),x)
[Out] -1/5*(b*x^4+a)^(1/4)*(-4*b*x^4+a)/a^2/x^5
```

maxima [A] time = 1.33, size = 35, normalized size = 0.80

$$\frac{\frac{5\left(bx^4+a\right)^{\frac{1}{4}}b}{x}-\frac{\left(bx^4+a\right)^{\frac{5}{4}}}{x^5}}{5\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^4+a)^(3/4),x, algorithm="maxima")
[Out] 1/5*(5*(b*x^4 + a)^(1/4)*b/x - (b*x^4 + a)^(5/4)/x^5)/a^2
```

mupad [B] time = 1.17, size = 25, normalized size = 0.57

$$-\frac{\left(b\,x^4+a\right)^{1/4}\left(a-4\,b\,x^4\right)}{5\,a^2\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^4)^(3/4)),x)
[Out] -((a + b*x^4)^(1/4)*(a - 4*b*x^4))/(5*a^2*x^5)
```

sympy [A] time = 1.32, size = 68, normalized size = 1.55

$$-\frac{\sqrt[4]{b}\,\sqrt[4]{\frac{a}{bx^4}+1}\,\Gamma\left(-\frac{5}{4}\right)}{16ax^4\Gamma\left(\frac{3}{4}\right)}+\frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx^4}+1}\,\Gamma\left(-\frac{5}{4}\right)}{4a^2\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**4+a)**(3/4),x)
[Out] -b**(1/4)*(a/(b*x**4) + 1)**(1/4)*gamma(-5/4)/(16*a*x**4*gamma(3/4)) + b**(
5/4)*(a/(b*x**4) + 1)**(1/4)*gamma(-5/4)/(4*a**2*gamma(3/4))
```

3.1130 $\int \frac{1}{x^{10}\left(a+bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=68

$$-\frac{32b^2\sqrt[4]{a+bx^4}}{45a^3x}+\frac{8b\sqrt[4]{a+bx^4}}{45a^2x^5}-\frac{\sqrt[4]{a+bx^4}}{9ax^9}$$

```
[Out] -1/9*(b*x^4+a)^(1/4)/a/x^9+8/45*b*(b*x^4+a)^(1/4)/a^2/x^5-32/45*b^2*(b*x^4+
a)^(1/4)/a^3/x
```

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$-\frac{32b^2\sqrt[4]{a+bx^4}}{45a^3x}+\frac{8b\sqrt[4]{a+bx^4}}{45a^2x^5}-\frac{\sqrt[4]{a+bx^4}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*(a + b*x^4)^(3/4)),x]
```

```
[Out] -(a + b*x^4)^(1/4)/(9*a*x^9) + (8*b*(a + b*x^4)^(1/4))/(45*a^2*x^5) - (32*b
^2*(a + b*x^4)^(1/4))/(45*a^3*x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{10}\left(a+bx^4\right)^{3/4}}\,dx &= -\frac{\sqrt[4]{a+bx^4}}{9ax^9}-\frac{(8b)\int \frac{1}{x^6\left(a+bx^4\right)^{3/4}}\,dx}{9a}\\ &= -\frac{\sqrt[4]{a+bx^4}}{9ax^9}+\frac{8b\sqrt[4]{a+bx^4}}{45a^2x^5}+\frac{\left(32b^2\right)\int \frac{1}{x^2\left(a+bx^4\right)^{3/4}}\,dx}{45a^2}\\ &= -\frac{\sqrt[4]{a+bx^4}}{9ax^9}+\frac{8b\sqrt[4]{a+bx^4}}{45a^2x^5}-\frac{32b^2\sqrt[4]{a+bx^4}}{45a^3x}\end{aligned}$$

Mathematica [A] time = 0.03, size = 42, normalized size = 0.62

$$-\frac{\sqrt[4]{a+bx^4}\left(5a^2-8abx^4+32b^2x^8\right)}{45a^3x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^10*(a + b*x^4)^(3/4)),x]
```

```
[Out] -1/45*((a + b*x^4)^(1/4)*(5*a^2 - 8*a*b*x^4 + 32*b^2*x^8))/(a^3*x^9)
```

fricas [A] time = 0.92, size = 38, normalized size = 0.56

$$-\frac{\left(32\,b^2x^8-8\,abx^4+5\,a^2\right)\left(bx^4+a\right)^{\frac{1}{4}}}{45\,a^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/45*(32*b^2*x^8 - 8*a*b*x^4 + 5*a^2)*(b*x^4 + a)^(1/4)/(a^3*x^9)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{3}{4}}x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(3/4)*x^10), x)
```

maple [A] time = 0.01, size = 39, normalized size = 0.57

$$-\frac{\left(b\,x^4+a\right)^{\frac{1}{4}}\left(32b^2x^8-8ab\,x^4+5a^2\right)}{45a^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(b*x^4+a)^(3/4),x)
```

```
[Out] -1/45*(b*x^4+a)^(1/4)*(32*b^2*x^8-8*a*b*x^4+5*a^2)/a^3/x^9
```

maxima [A] time = 1.37, size = 52, normalized size = 0.76

$$-\frac{\frac{45\left(bx^4+a\right)^{\frac{1}{4}}b^2}{x}-\frac{18\left(bx^4+a\right)^{\frac{5}{4}}b}{x^5}+\frac{5\left(bx^4+a\right)^{\frac{9}{4}}}{x^9}}{45\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] -1/45*(45*(b*x^4 + a)^(1/4)*b^2/x - 18*(b*x^4 + a)^(5/4)*b/x^5 + 5*(b*x^4 +
 a)^(9/4)/x^9)/a^3
```

mupad [B] time = 1.27, size = 38, normalized size = 0.56

$$-\frac{\left(b\,x^4+a\right)^{1/4}\left(5\,a^2-8\,a\,b\,x^4+32\,b^2\,x^8\right)}{45\,a^3\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^10*(a + b*x^4)^(3/4)),x)
```

```
[Out] -((a + b*x^4)^(1/4)*(5*a^2 + 32*b^2*x^8 - 8*a*b*x^4))/(45*a^3*x^9)
```

sympy [**B**] time = 2.63, size = 406, normalized size = 5.97

$$\frac{5a^4b^{\frac{17}{4}}\sqrt[4]{\frac{a}{bx^4}+1}\,\Gamma\left(-\frac{9}{4}\right)}{64a^5b^4x^8\Gamma\left(\frac{3}{4}\right)+128a^4b^5x^{12}\Gamma\left(\frac{3}{4}\right)+64a^3b^6x^{16}\Gamma\left(\frac{3}{4}\right)}+\frac{2a^3b^{\frac{21}{4}}x^4\sqrt[4]{\frac{a}{bx^4}+1}\,\Gamma\left(-\frac{9}{4}\right)}{64a^5b^4x^8\Gamma\left(\frac{3}{4}\right)+128a^4b^5x^{12}\Gamma\left(\frac{3}{4}\right)+64a^3b^6x^{16}\Gamma\left(\frac{3}{4}\right)}+-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(b*x**4+a)**(3/4),x)
```

```
[Out] 5*a**4*b**(17/4)*(a/(b*x**4) + 1)**(1/4)*gamma(-9/4)/(64*a**5*b**4*x**8*gam
ma(3/4) + 128*a**4*b**5*x**12*gamma(3/4) + 64*a**3*b**6*x**16*gamma(3/4)) +
 2*a**3*b**(21/4)*x**4*(a/(b*x**4) + 1)**(1/4)*gamma(-9/4)/(64*a**5*b**4*x*
*8*gamma(3/4) + 128*a**4*b**5*x**12*gamma(3/4) + 64*a**3*b**6*x**16*gamma(3
/4)) + 21*a**2*b**(25/4)*x**8*(a/(b*x**4) + 1)**(1/4)*gamma(-9/4)/(64*a**5*
b**4*x**8*gamma(3/4) + 128*a**4*b**5*x**12*gamma(3/4) + 64*a**3*b**6*x**16*
gamma(3/4)) + 56*a*b**(29/4)*x**12*(a/(b*x**4) + 1)**(1/4)*gamma(-9/4)/(64*
a**5*b**4*x**8*gamma(3/4) + 128*a**4*b**5*x**12*gamma(3/4) + 64*a**3*b**6*x
**16*gamma(3/4)) + 32*b**(33/4)*x**16*(a/(b*x**4) + 1)**(1/4)*gamma(-9/4)/(
64*a**5*b**4*x**8*gamma(3/4) + 128*a**4*b**5*x**12*gamma(3/4) + 64*a**3*b**
6*x**16*gamma(3/4))
```

3.1131 $\int \frac{1}{x^{14}\left(a+bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=92

$$\frac{128b^3\sqrt[4]{a+bx^4}}{195a^4x}-\frac{32b^2\sqrt[4]{a+bx^4}}{195a^3x^5}+\frac{4b\sqrt[4]{a+bx^4}}{39a^2x^9}-\frac{\sqrt[4]{a+bx^4}}{13ax^{13}}$$

```
[Out] -1/13*(b*x^4+a)^(1/4)/a/x^13+4/39*b*(b*x^4+a)^(1/4)/a^2/x^9-32/195*b^2*(b*x
^4+a)^(1/4)/a^3/x^5+128/195*b^3*(b*x^4+a)^(1/4)/a^4/x
```

Rubi [A] time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$\frac{128b^3\sqrt[4]{a+bx^4}}{195a^4x}-\frac{32b^2\sqrt[4]{a+bx^4}}{195a^3x^5}+\frac{4b\sqrt[4]{a+bx^4}}{39a^2x^9}-\frac{\sqrt[4]{a+bx^4}}{13ax^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^14*(a + b*x^4)^(3/4)),x]
```

```
[Out] -(a + b*x^4)^(1/4)/(13*a*x^13) + (4*b*(a + b*x^4)^(1/4))/(39*a^2*x^9) - (32
*b^2*(a + b*x^4)^(1/4))/(195*a^3*x^5) + (128*b^3*(a + b*x^4)^(1/4))/(195*a^
4*x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{14}\left(a+bx^4\right)^{3/4}}\,dx &= -\frac{\sqrt[4]{a+bx^4}}{13ax^{13}}-\frac{(12b)\int \frac{1}{x^{10}\left(a+bx^4\right)^{3/4}}\,dx}{13a}\\ &= -\frac{\sqrt[4]{a+bx^4}}{13ax^{13}}+\frac{4b\sqrt[4]{a+bx^4}}{39a^2x^9}+\frac{\left(32b^2\right)\int \frac{1}{x^6\left(a+bx^4\right)^{3/4}}\,dx}{39a^2}\\ &= -\frac{\sqrt[4]{a+bx^4}}{13ax^{13}}+\frac{4b\sqrt[4]{a+bx^4}}{39a^2x^9}-\frac{32b^2\sqrt[4]{a+bx^4}}{195a^3x^5}-\frac{\left(128b^3\right)\int \frac{1}{x^2\left(a+bx^4\right)^{3/4}}\,dx}{195a^3}\\ &= -\frac{\sqrt[4]{a+bx^4}}{13ax^{13}}+\frac{4b\sqrt[4]{a+bx^4}}{39a^2x^9}-\frac{32b^2\sqrt[4]{a+bx^4}}{195a^3x^5}+\frac{128b^3\sqrt[4]{a+bx^4}}{195a^4x}\end{aligned}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.58

$$\frac{\sqrt[4]{a+bx^4}\left(-15a^3+20a^2bx^4-32ab^2x^8+128b^3x^{12}\right)}{195a^4x^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^14*(a + b*x^4)^(3/4)),x]
```

```
[Out] ((a + b*x^4)^(1/4)*(-15*a^3 + 20*a^2*b*x^4 - 32*a*b^2*x^8 + 128*b^3*x^12))/
(195*a^4*x^13)
```

fricas [A] time = 1.02, size = 49, normalized size = 0.53

$$\frac{\left(128\,b^3x^{12} - 32\,ab^2x^8 + 20\,a^2bx^4 - 15\,a^3\right)\left(bx^4 + a\right)^{\frac{1}{4}}}{195\,a^4x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^14/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/195*(128*b^3*x^12 - 32*a*b^2*x^8 + 20*a^2*b*x^4 - 15*a^3)*(b*x^4 + a)^(1/
4)/(a^4*x^13)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4 + a\right)^{\frac{3}{4}}x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^14/(b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(3/4)*x^14), x)
```

maple [A] time = 0.01, size = 50, normalized size = 0.54

$$-\frac{\left(b\,x^4 + a\right)^{\frac{1}{4}}\left(-128b^3x^{12} + 32a\,b^2x^8 - 20a^2b\,x^4 + 15a^3\right)}{195a^4x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^14/(b*x^4+a)^(3/4),x)
```

```
[Out] -1/195*(b*x^4+a)^(1/4)*(-128*b^3*x^12+32*a*b^2*x^8-20*a^2*b*x^4+15*a^3)/x^1
3/a^4
```

maxima [A] time = 1.36, size = 69, normalized size = 0.75

$$\frac{\frac{195\left(bx^4+a\right)^{\frac{1}{4}}b^3}{x} - \frac{117\left(bx^4+a\right)^{\frac{5}{4}}b^2}{x^5} + \frac{65\left(bx^4+a\right)^{\frac{9}{4}}b}{x^9} - \frac{15\left(bx^4+a\right)^{\frac{13}{4}}}{x^{13}}}{195\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^14/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] 1/195*(195*(b*x^4 + a)^(1/4)*b^3/x - 117*(b*x^4 + a)^(5/4)*b^2/x^5 + 65*(b*
x^4 + a)^(9/4)*b/x^9 - 15*(b*x^4 + a)^(13/4)/x^13)/a^4
```

mupad [B] time = 1.36, size = 76, normalized size = 0.83

$$\frac{4\,b\left(b\,x^4 + a\right)^{1/4}}{39\,a^2\,x^9} - \frac{\left(b\,x^4 + a\right)^{1/4}}{13\,a\,x^{13}} + \frac{128\,b^3\left(b\,x^4 + a\right)^{1/4}}{195\,a^4\,x} - \frac{32\,b^2\left(b\,x^4 + a\right)^{1/4}}{195\,a^3\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^14*(a + b*x^4)^(3/4)),x)
```

```
[Out] (4*b*(a + b*x^4)^(1/4))/(39*a^2*x^9) - (a + b*x^4)^(1/4)/(13*a*x^13) + (128
*b^3*(a + b*x^4)^(1/4))/(195*a^4*x) - (32*b^2*(a + b*x^4)^(1/4))/(195*a^3*x
^5)
```

sympy [**B**] time = 3.26, size = 692, normalized size = 7.52

$$-\frac{45a^6b^{\frac{37}{4}}\sqrt[4]{\frac{a}{bx^4}+1}\,\Gamma\left(-\frac{13}{4}\right)}{256a^7b^9x^{12}\Gamma\left(\frac{3}{4}\right)+768a^6b^{10}x^{16}\Gamma\left(\frac{3}{4}\right)+768a^5b^{11}x^{20}\Gamma\left(\frac{3}{4}\right)+256a^4b^{12}x^{24}\Gamma\left(\frac{3}{4}\right)}-\frac{75a^5b^{\frac{4}{}}}{256a^7b^9x^{12}\Gamma\left(\frac{3}{4}\right)+768a^6b^{10}x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**14/(b*x**4+a)**(3/4),x)
```

```
[Out] -45*a**6*b**(37/4)*(a/(b*x**4) + 1)**(1/4)*gamma(-13/4)/(256*a**7*b**9*x**1
2*gamma(3/4) + 768*a**6*b**10*x**16*gamma(3/4) + 768*a**5*b**11*x**20*gamma
(3/4) + 256*a**4*b**12*x**24*gamma(3/4)) - 75*a**5*b**(41/4)*x**4*(a/(b*x**
4) + 1)**(1/4)*gamma(-13/4)/(256*a**7*b**9*x**12*gamma(3/4) + 768*a**6*b**1
0*x**16*gamma(3/4) + 768*a**5*b**11*x**20*gamma(3/4) + 256*a**4*b**12*x**24
*gamma(3/4)) - 51*a**4*b**(45/4)*x**8*(a/(b*x**4) + 1)**(1/4)*gamma(-13/4)/
(256*a**7*b**9*x**12*gamma(3/4) + 768*a**6*b**10*x**16*gamma(3/4) + 768*a**
5*b**11*x**20*gamma(3/4) + 256*a**4*b**12*x**24*gamma(3/4)) + 231*a**3*b**(
49/4)*x**12*(a/(b*x**4) + 1)**(1/4)*gamma(-13/4)/(256*a**7*b**9*x**12*gamma
(3/4) + 768*a**6*b**10*x**16*gamma(3/4) + 768*a**5*b**11*x**20*gamma(3/4) +
 256*a**4*b**12*x**24*gamma(3/4)) + 924*a**2*b**(53/4)*x**16*(a/(b*x**4) +
1)**(1/4)*gamma(-13/4)/(256*a**7*b**9*x**12*gamma(3/4) + 768*a**6*b**10*x**
16*gamma(3/4) + 768*a**5*b**11*x**20*gamma(3/4) + 256*a**4*b**12*x**24*gamm
a(3/4)) + 1056*a*b**(57/4)*x**20*(a/(b*x**4) + 1)**(1/4)*gamma(-13/4)/(256*
a**7*b**9*x**12*gamma(3/4) + 768*a**6*b**10*x**16*gamma(3/4) + 768*a**5*b**
11*x**20*gamma(3/4) + 256*a**4*b**12*x**24*gamma(3/4)) + 384*b**(61/4)*x**2
4*(a/(b*x**4) + 1)**(1/4)*gamma(-13/4)/(256*a**7*b**9*x**12*gamma(3/4) + 76
8*a**6*b**10*x**16*gamma(3/4) + 768*a**5*b**11*x**20*gamma(3/4) + 256*a**4*
b**12*x**24*gamma(3/4))
```

3.1132 $\int \frac{x^{12}}{\left(a+bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=129

$$\frac{3a^{5/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{8b^{5/2}\left(a+bx^4\right)^{3/4}}+\frac{3a^2x\sqrt[4]{a+bx^4}}{8b^3}-\frac{3ax^5\sqrt[4]{a+bx^4}}{20b^2}+\frac{x^9\sqrt[4]{a+bx^4}}{10b}$$

```
[Out] 3/8*a^2*x*(b*x^4+a)^(1/4)/b^3-3/20*a*x^5*(b*x^4+a)^(1/4)/b^2+1/10*x^9*(b*x^
4+a)^(1/4)/b+3/8*a^(5/2)*(1+a/b/x^4)^(3/4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)/
a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*a
rccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))/b^(5/2)/(b*x^4+a)^(3/4)
```

Rubi [A] time = 0.06, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {321, 237, 335, 275, 231}

$$\frac{3a^2x\sqrt[4]{a+bx^4}}{8b^3}+\frac{3a^{5/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{8b^{5/2}\left(a+bx^4\right)^{3/4}}-\frac{3ax^5\sqrt[4]{a+bx^4}}{20b^2}+\frac{x^9\sqrt[4]{a+bx^4}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[x^12/(a + b*x^4)^(3/4),x]
```

```
[Out] (3*a^2*x*(a + b*x^4)^(1/4))/(8*b^3) - (3*a*x^5*(a + b*x^4)^(1/4))/(20*b^2)
+ (x^9*(a + b*x^4)^(1/4))/(10*b) + (3*a^(5/2)*(1 + a/(b*x^4))^(3/4)*x^3*Ell
ipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(8*b^(5/2)*(a + b*x^4)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{x^{12}}{\left(a+bx^4\right)^{3/4}}\,dx = \frac{x^9\sqrt[4]{a+bx^4}}{10b} - \frac{(9a)\int \frac{x^8}{\left(a+bx^4\right)^{3/4}}\,dx}{10b}$$

$$= -\frac{3ax^5\sqrt[4]{a+bx^4}}{20b^2} + \frac{x^9\sqrt[4]{a+bx^4}}{10b} + \frac{\left(3a^2\right)\int \frac{x^4}{\left(a+bx^4\right)^{3/4}}\,dx}{4b^2}$$

$$= \frac{3a^2x\sqrt[4]{a+bx^4}}{8b^3} - \frac{3ax^5\sqrt[4]{a+bx^4}}{20b^2} + \frac{x^9\sqrt[4]{a+bx^4}}{10b} - \frac{\left(3a^3\right)\int \frac{1}{\left(a+bx^4\right)^{3/4}}\,dx}{8b^3}$$

$$= \frac{3a^2x\sqrt[4]{a+bx^4}}{8b^3} - \frac{3ax^5\sqrt[4]{a+bx^4}}{20b^2} + \frac{x^9\sqrt[4]{a+bx^4}}{10b} - \frac{\left(3a^3\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{8b^3\left(a+bx^4\right)^{3/4}}$$

$$= \frac{3a^2x\sqrt[4]{a+bx^4}}{8b^3} - \frac{3ax^5\sqrt[4]{a+bx^4}}{20b^2} + \frac{x^9\sqrt[4]{a+bx^4}}{10b} + \frac{\left(3a^3\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}\,d\right.}{8b^3\left(a+bx^4\right)^{3/4}}$$

$$= \frac{3a^2x\sqrt[4]{a+bx^4}}{8b^3} - \frac{3ax^5\sqrt[4]{a+bx^4}}{20b^2} + \frac{x^9\sqrt[4]{a+bx^4}}{10b} + \frac{\left(3a^3\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{3/4}}\,d\right.}{16b^3\left(a+bx^4\right)^{3/4}}$$

$$= \frac{3a^2x\sqrt[4]{a+bx^4}}{8b^3} - \frac{3ax^5\sqrt[4]{a+bx^4}}{20b^2} + \frac{x^9\sqrt[4]{a+bx^4}}{10b} + \frac{3a^{5/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{8b^{5/2}\left(a+bx^4\right)^{3/4}}$$

Mathematica [C] time = 0.03, size = 90, normalized size = 0.70

$$\frac{-15a^3x\left(\frac{bx^4}{a}+1\right)^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{bx^4}{a}\right)+15a^3x+9a^2bx^5-2ab^2x^9+4b^3x^{13}}{40b^3\left(a+bx^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12/(a + b*x^4)^(3/4),x]
```

```
[Out] (15*a^3*x + 9*a^2*b*x^5 - 2*a*b^2*x^9 + 4*b^3*x^13 - 15*a^3*x*(1 + (b*x^4)/
a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, -((b*x^4)/a)])/(40*b^3*(a + b*x^4
)^(3/4))
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^{12}}{\left(bx^4+a\right)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(x^12/(b*x^4 + a)^(3/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{12}}{\left(bx^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^12/(b*x^4 + a)^(3/4), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^{12}}{\left(b\,x^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(b*x^4+a)^(3/4),x)
```

```
[Out] int(x^12/(b*x^4+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{12}}{\left(bx^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^12/(b*x^4 + a)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{12}}{\left(b\,x^4+a\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(a + b*x^4)^(3/4),x)
```

```
[Out] int(x^12/(a + b*x^4)^(3/4), x)
```

sympy [C] time = 2.14, size = 37, normalized size = 0.29

$$\frac{x^{13}\Gamma\left(\frac{13}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4}, \frac{13}{4} \\ \frac{17}{4}\end{matrix}\right| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{4}}\Gamma\left(\frac{17}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12/(b*x**4+a)**(3/4),x)
```

```
[Out] x**13*gamma(13/4)*hyper((3/4, 13/4), (17/4,), b*x**4*exp_polar(I*pi)/a)/(4*
a**(3/4)*gamma(17/4))
```

3.1133 $\int \frac{x^8}{\left(a+bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=105

$$-\frac{5a^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{12b^{3/2}\left(a+bx^4\right)^{3/4}}-\frac{5ax\sqrt[4]{a+bx^4}}{12b^2}+\frac{x^5\sqrt[4]{a+bx^4}}{6b}$$

```
[Out] -5/12*a*x*(b*x^4+a)^(1/4)/b^2+1/6*x^5*(b*x^4+a)^(1/4)/b-5/12*a^(3/2)*(1+a/b
/x^4)^(3/4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcc
ot(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(
1/2))/b^(3/2)/(b*x^4+a)^(3/4)
```

Rubi [A] time = 0.04, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {321, 237, 335, 275, 231}

$$-\frac{5a^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{12b^{3/2}\left(a+bx^4\right)^{3/4}}-\frac{5ax\sqrt[4]{a+bx^4}}{12b^2}+\frac{x^5\sqrt[4]{a+bx^4}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^4)^(3/4),x]
```

```
[Out] (-5*a*x*(a + b*x^4)^(1/4))/(12*b^2) + (x^5*(a + b*x^4)^(1/4))/(6*b) - (5*a^
(3/2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2,
2])/(12*b^(3/2)*(a + b*x^4)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{x^8}{\left(a+bx^4\right)^{3/4}}\,dx = \frac{x^5\sqrt[4]{a+bx^4}}{6b} - \frac{(5a)\int \frac{x^4}{\left(a+bx^4\right)^{3/4}}\,dx}{6b}$$

$$= -\frac{5ax\sqrt[4]{a+bx^4}}{12b^2} + \frac{x^5\sqrt[4]{a+bx^4}}{6b} + \frac{\left(5a^2\right)\int \frac{1}{\left(a+bx^4\right)^{3/4}}\,dx}{12b^2}$$

$$= -\frac{5ax\sqrt[4]{a+bx^4}}{12b^2} + \frac{x^5\sqrt[4]{a+bx^4}}{6b} + \frac{\left(5a^2\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{12b^2\left(a+bx^4\right)^{3/4}}$$

$$= -\frac{5ax\sqrt[4]{a+bx^4}}{12b^2} + \frac{x^5\sqrt[4]{a+bx^4}}{6b} - \frac{\left(5a^2\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{12b^2\left(a+bx^4\right)^{3/4}}$$

$$= -\frac{5ax\sqrt[4]{a+bx^4}}{12b^2} + \frac{x^5\sqrt[4]{a+bx^4}}{6b} - \frac{\left(5a^2\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{x^2}\right)}{24b^2\left(a+bx^4\right)^{3/4}}$$

$$= -\frac{5ax\sqrt[4]{a+bx^4}}{12b^2} + \frac{x^5\sqrt[4]{a+bx^4}}{6b} - \frac{5a^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{12b^{3/2}\left(a+bx^4\right)^{3/4}}$$

Mathematica [C] time = 0.02, size = 79, normalized size = 0.75

$$\frac{5a^2x\left(\frac{bx^4}{a}+1\right)^{3/4}{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{bx^4}{a}\right) - 5a^2x - 3abx^5 + 2b^2x^9}{12b^2\left(a+bx^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^4)^(3/4),x]
```

```
[Out] (-5*a^2*x - 3*a*b*x^5 + 2*b^2*x^9 + 5*a^2*x*(1 + (b*x^4)/a)^(3/4)*Hypergeom
etric2F1[1/4, 3/4, 5/4, -((b*x^4)/a)])/(12*b^2*(a + b*x^4)^(3/4))
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^8}{\left(bx^4+a\right)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(x^8/(b*x^4 + a)^(3/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(bx^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)^(3/4),x, algorithm="giac")
[Out] integrate(x^8/(b*x^4 + a)^(3/4), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(b\,x^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^4+a)^(3/4),x)
[Out] int(x^8/(b*x^4+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(bx^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)^(3/4),x, algorithm="maxima")
[Out] integrate(x^8/(b*x^4 + a)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^8}{\left(b\,x^4+a\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^4)^(3/4),x)
[Out] int(x^8/(a + b*x^4)^(3/4), x)
```

sympy [C] time = 1.55, size = 37, normalized size = 0.35

$$\frac{x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{4}}\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**4+a)**(3/4),x)
[Out] x**9*gamma(9/4)*hyper((3/4, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**
(3/4)*gamma(13/4))
```

3.1134 $\int \frac{x^4}{\left(a+bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=83

$$\frac{x\sqrt[4]{a+bx^4}}{2b}+\frac{\sqrt{a}\,x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{b}\left(a+bx^4\right)^{3/4}}$$

```
[Out] 1/2*x*(b*x^4+a)^(1/4)/b+1/2*(1+a/b/x^4)^(3/4)*x^3*(cos(1/2*arccot(x^2*b^(1/
2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/
2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)/(b*x^4+a)^(3/4)/b^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {321, 237, 335, 275, 231}

$$\frac{x\sqrt[4]{a+bx^4}}{2b}+\frac{\sqrt{a}\,x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{b}\left(a+bx^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^4)^(3/4),x]
```

```
[Out] (x*(a + b*x^4)^(1/4))/(2*b) + (Sqrt[a]*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[
ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(2*Sqrt[b]*(a + b*x^4)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
```

egerQ[m]

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\left(a+bx^4\right)^{3/4}}\,dx &= \frac{x\sqrt[4]{a+bx^4}}{2b} - \frac{a\int \frac{1}{\left(a+bx^4\right)^{3/4}}\,dx}{2b} \\ &= \frac{x\sqrt[4]{a+bx^4}}{2b} - \frac{\left(a\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{2b\left(a+bx^4\right)^{3/4}} \\ &= \frac{x\sqrt[4]{a+bx^4}}{2b} + \frac{\left(a\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{2b\left(a+bx^4\right)^{3/4}} \\ &= \frac{x\sqrt[4]{a+bx^4}}{2b} + \frac{\left(a\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{x^2}\right)}{4b\left(a+bx^4\right)^{3/4}} \\ &= \frac{x\sqrt[4]{a+bx^4}}{2b} + \frac{\sqrt{a}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{b}\left(a+bx^4\right)^{3/4}}\end{aligned}$$

Mathematica [C] time = 0.02, size = 62, normalized size = 0.75

$$\frac{x\left(-a\left(\frac{bx^4}{a}+1\right)^{3/4}{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{bx^4}{a}\right)+a+bx^4\right)}{2b\left(a+bx^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^4)^(3/4),x]
```

```
[Out] (x*(a + b*x^4 - a*(1 + (b*x^4)/a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, -(
(b*x^4)/a)]))/(2*b*(a + b*x^4)^(3/4))
```

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^4}{\left(bx^4+a\right)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(x^4/(b*x^4 + a)^(3/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(b*x^4 + a)^(3/4), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(b\,x^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^4+a)^(3/4),x)
```

```
[Out] int(x^4/(b*x^4+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(b*x^4 + a)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\left(b\,x^4+a\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^4)^(3/4),x)
```

```
[Out] int(x^4/(a + b*x^4)^(3/4), x)
```

sympy [C] time = 0.97, size = 37, normalized size = 0.45

$$\frac{x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4}, \frac{5}{4}\\ \frac{9}{4}\end{matrix}\right| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{4}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**4+a)**(3/4),x)
```

```
[Out] x**5*gamma(5/4)*hyper((3/4, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
3/4)*gamma(9/4))
```

3.1135 $\int \frac{1}{\left(a+bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=61

$$-\frac{\sqrt{b}\,x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\left(a+bx^4\right)^{3/4}}$$

```
[Out] -(1+a/b/x^4)^(3/4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1
/2*arccot(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2
))),2^(1/2))*b^(1/2)/(b*x^4+a)^(3/4)/a^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {237, 335, 275, 231}

$$-\frac{\sqrt{b}\,x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\left(a+bx^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(-3/4),x]
```

```
[Out] -((Sqrt[b]*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]
]/2, 2])/(Sqrt[a]*(a + b*x^4)^(3/4)))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)^{3/4}}\,dx = \frac{\left(\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{\left(a+bx^4\right)^{3/4}}$$

$$= -\frac{\left(\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{\left(a+bx^4\right)^{3/4}}$$

$$= -\frac{\left(\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{x^2}\right)}{2\left(a+bx^4\right)^{3/4}}$$

$$= -\frac{\sqrt{b}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\left(a+bx^4\right)^{3/4}}$$

Mathematica [C] time = 0.01, size = 46, normalized size = 0.75

$$\frac{x\left(\frac{bx^4}{a}+1\right)^{3/4}{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{bx^4}{a}\right)}{\left(a+bx^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(-3/4),x]
```

```
[Out] (x*(1 + (b*x^4)/a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, -((b*x^4)/a)])/(a
 + b*x^4)^(3/4)
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\left(bx^4+a\right)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(-3/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(-3/4), x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(3/4),x)
```

```
[Out] int(1/(b*x^4+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(-3/4), x)
```

mupad [B] time = 1.08, size = 37, normalized size = 0.61

$$\frac{x\left(\frac{b\,x^4}{a}+1\right)^{3/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{4};\ \frac{5}{4};\ -\frac{b\,x^4}{a}\right)}{\left(b\,x^4+a\right)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^4)^(3/4),x)
```

```
[Out] (x*((b*x^4)/a + 1)^(3/4)*hypergeom([1/4, 3/4], 5/4, -(b*x^4)/a))/(a + b*x^4
)^(3/4)
```

sympy [C] time = 0.91, size = 36, normalized size = 0.59

$$\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{5}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{4}}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(3/4),x)
```

```
[Out] x*gamma(1/4)*hyper((1/4, 3/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/4
)*gamma(5/4))
```

3.1136 $\int \frac{1}{x^4\left(a+bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=85

$$\frac{2b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}$$

```
[Out] -1/3*(b*x^4+a)^(1/4)/a/x^3+2/3*b^(3/2)*(1+a/b/x^4)^(3/4)*x^3*(cos(1/2*arcco
t(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))*Ellip
ticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))/a^(3/2)/(b*x^4+a)^(3/4)
```

Rubi [A] time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {325, 237, 335, 275, 231}

$$\frac{2b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^4)^(3/4)),x]
```

```
[Out] -(a + b*x^4)^(1/4)/(3*a*x^3) + (2*b^(3/2)*(1 + a/(b*x^4))^(3/4)*x^3*Ellipti
cF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(3*a^(3/2)*(a + b*x^4)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
 + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
```

`egerQ[m]`

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(a+bx^4\right)^{3/4}}\,dx &= -\frac{\sqrt[4]{a+bx^4}}{3ax^3} - \frac{(2b)\int \frac{1}{\left(a+bx^4\right)^{3/4}}\,dx}{3a} \\ &= -\frac{\sqrt[4]{a+bx^4}}{3ax^3} - \frac{\left(2b\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{3a\left(a+bx^4\right)^{3/4}} \\ &= -\frac{\sqrt[4]{a+bx^4}}{3ax^3} + \frac{\left(2b\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{3a\left(a+bx^4\right)^{3/4}} \\ &= -\frac{\sqrt[4]{a+bx^4}}{3ax^3} + \frac{\left(b\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{x^2}\right)}{3a\left(a+bx^4\right)^{3/4}} \\ &= -\frac{\sqrt[4]{a+bx^4}}{3ax^3} + \frac{2b^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}\left(a+bx^4\right)^{3/4}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.60

$$-\frac{\left(\frac{bx^4}{a}+1\right)^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{3}{4};\frac{1}{4};-\frac{bx^4}{a}\right)}{3x^3\left(a+bx^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^4)^(3/4)),x]
```

```
[Out] -1/3*((1 + (b*x^4)/a)^(3/4)*Hypergeometric2F1[-3/4, 3/4, 1/4, -((b*x^4)/a)]
)/(x^3*(a + b*x^4)^(3/4))
```

fricas [F] time = 1.22, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{bx^8+ax^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(1/4)/(b*x^8 + a*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{3}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(3/4)*x^4), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{3}{4}}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^4+a)^(3/4),x)
```

```
[Out] int(1/x^4/(b*x^4+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{3}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(3/4)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\left(b\,x^4+a\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^4)^(3/4)),x)
```

```
[Out] int(1/(x^4*(a + b*x^4)^(3/4)), x)
```

sympy [C] time = 1.56, size = 41, normalized size = 0.48

$$\frac{\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{4},\frac{3}{4}\\ \frac{1}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{4}}x^3\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**4+a)**(3/4),x)
```

```
[Out] gamma(-3/4)*hyper((-3/4, 3/4), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/4
)*x**3*gamma(1/4))
```

3.1137 $\int \frac{1}{x^8\left(a+bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=107

$$-\frac{4b^{5/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{7a^{5/2}\left(a+bx^4\right)^{3/4}}+\frac{2b\sqrt[4]{a+bx^4}}{7a^2x^3}-\frac{\sqrt[4]{a+bx^4}}{7ax^7}$$

```
[Out] -1/7*(b*x^4+a)^(1/4)/a/x^7+2/7*b*(b*x^4+a)^(1/4)/a^2/x^3-4/7*b^(5/2)*(1+a/b
/x^4)^(3/4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcc
ot(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(
1/2))/a^(5/2)/(b*x^4+a)^(3/4)
```

Rubi [A] time = 0.05, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {325, 237, 335, 275, 231}

$$-\frac{4b^{5/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{7a^{5/2}\left(a+bx^4\right)^{3/4}}+\frac{2b\sqrt[4]{a+bx^4}}{7a^2x^3}-\frac{\sqrt[4]{a+bx^4}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(a + b*x^4)^(3/4)),x]
```

```
[Out] -(a + b*x^4)^(1/4)/(7*a*x^7) + (2*b*(a + b*x^4)^(1/4))/(7*a^2*x^3) - (4*b^(
5/2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2
])/(7*a^(5/2)*(a + b*x^4)^(3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^8\left(a+bx^4\right)^{3/4}}\,dx &= -\frac{\sqrt[4]{a+bx^4}}{7ax^7} - \frac{(6b)\int \frac{1}{x^4\left(a+bx^4\right)^{3/4}}\,dx}{7a} \\
&= -\frac{\sqrt[4]{a+bx^4}}{7ax^7} + \frac{2b\sqrt[4]{a+bx^4}}{7a^2x^3} + \frac{\left(4b^2\right)\int \frac{1}{\left(a+bx^4\right)^{3/4}}\,dx}{7a^2} \\
&= -\frac{\sqrt[4]{a+bx^4}}{7ax^7} + \frac{2b\sqrt[4]{a+bx^4}}{7a^2x^3} + \frac{\left(4b^2\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{7a^2\left(a+bx^4\right)^{3/4}} \\
&= -\frac{\sqrt[4]{a+bx^4}}{7ax^7} + \frac{2b\sqrt[4]{a+bx^4}}{7a^2x^3} - \frac{\left(4b^2\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{7a^2\left(a+bx^4\right)^{3/4}} \\
&= -\frac{\sqrt[4]{a+bx^4}}{7ax^7} + \frac{2b\sqrt[4]{a+bx^4}}{7a^2x^3} - \frac{\left(2b^2\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{x^2}\right)}{7a^2\left(a+bx^4\right)^{3/4}} \\
&= -\frac{\sqrt[4]{a+bx^4}}{7ax^7} + \frac{2b\sqrt[4]{a+bx^4}}{7a^2x^3} - \frac{4b^{5/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{7a^{5/2}\left(a+bx^4\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.48

$$-\frac{\left(\frac{bx^4}{a}+1\right)^{3/4}\,{}_2F_1\left(-\frac{7}{4},\frac{3}{4};-\frac{3}{4};-\frac{bx^4}{a}\right)}{7x^7\left(a+bx^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(a + b*x^4)^(3/4)),x]
```

```
[Out] -1/7*((1 + (b*x^4)/a)^(3/4)*Hypergeometric2F1[-7/4, 3/4, -3/4, -((b*x^4)/a)
])/(x^7*(a + b*x^4)^(3/4))
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{bx^{12}+ax^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(1/4)/(b*x^12 + a*x^8), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{3}{4}}x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^4+a)^(3/4),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^(3/4)*x^8), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{3}{4}}x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(b*x^4+a)^(3/4),x)
[Out] int(1/x^8/(b*x^4+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{3}{4}}x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^4+a)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(3/4)*x^8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^8\left(b\,x^4+a\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(a + b*x^4)^(3/4)),x)
[Out] int(1/(x^8*(a + b*x^4)^(3/4)), x)
```

sympy [C] time = 1.99, size = 44, normalized size = 0.41

$$\frac{\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4}, \frac{3}{4} \\ -\frac{3}{4}\end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{4}}x^7\Gamma\left(-\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(b*x**4+a)**(3/4),x)
[Out] gamma(-7/4)*hyper((-7/4, 3/4), (-3/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/
4)*x**7*gamma(-3/4))
```

3.1138 $\int \frac{1}{x^{12}\left(a+bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=131

$$\frac{40b^{7/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{77a^{7/2}\left(a+bx^4\right)^{3/4}}-\frac{20b^2\sqrt[4]{a+bx^4}}{77a^3x^3}+\frac{10b\sqrt[4]{a+bx^4}}{77a^2x^7}-\frac{\sqrt[4]{a+bx^4}}{11ax^{11}}$$

```
[Out] -1/11*(b*x^4+a)^(1/4)/a/x^11+10/77*b*(b*x^4+a)^(1/4)/a^2/x^7-20/77*b^2*(b*x
^4+a)^(1/4)/a^3/x^3+40/77*b^(7/2)*(1+a/b/x^4)^(3/4)*x^3*(cos(1/2*arccot(x^2
*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))*EllipticF(
sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))/a^(7/2)/(b*x^4+a)^(3/4)
```

Rubi [A] time = 0.06, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {325, 237, 335, 275, 231}

$$-\frac{20b^2\sqrt[4]{a+bx^4}}{77a^3x^3}+\frac{40b^{7/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{77a^{7/2}\left(a+bx^4\right)^{3/4}}+\frac{10b\sqrt[4]{a+bx^4}}{77a^2x^7}-\frac{\sqrt[4]{a+bx^4}}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^12*(a + b*x^4)^(3/4)),x]
```

```
[Out] -(a + b*x^4)^(1/4)/(11*a*x^11) + (10*b*(a + b*x^4)^(1/4))/(77*a^2*x^7) - (2
0*b^2*(a + b*x^4)^(1/4))/(77*a^3*x^3) + (40*b^(7/2)*(1 + a/(b*x^4))^(3/4)*x
^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(77*a^(7/2)*(a + b*x^4)^(
3/4))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{x^{12}\left(a+bx^4\right)^{3/4}}\,dx = -\frac{\sqrt[4]{a+bx^4}}{11ax^{11}} - \frac{(10b)\int \frac{1}{x^8\left(a+bx^4\right)^{3/4}}\,dx}{11a}$$

$$= -\frac{\sqrt[4]{a+bx^4}}{11ax^{11}} + \frac{10b\sqrt[4]{a+bx^4}}{77a^2x^7} + \frac{\left(60b^2\right)\int \frac{1}{x^4\left(a+bx^4\right)^{3/4}}\,dx}{77a^2}$$

$$= -\frac{\sqrt[4]{a+bx^4}}{11ax^{11}} + \frac{10b\sqrt[4]{a+bx^4}}{77a^2x^7} - \frac{20b^2\sqrt[4]{a+bx^4}}{77a^3x^3} - \frac{\left(40b^3\right)\int \frac{1}{\left(a+bx^4\right)^{3/4}}\,dx}{77a^3}$$

$$= -\frac{\sqrt[4]{a+bx^4}}{11ax^{11}} + \frac{10b\sqrt[4]{a+bx^4}}{77a^2x^7} - \frac{20b^2\sqrt[4]{a+bx^4}}{77a^3x^3} - \frac{\left(40b^3\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{77a^3\left(a+bx^4\right)^{3/4}}$$

$$= -\frac{\sqrt[4]{a+bx^4}}{11ax^{11}} + \frac{10b\sqrt[4]{a+bx^4}}{77a^2x^7} - \frac{20b^2\sqrt[4]{a+bx^4}}{77a^3x^3} + \frac{\left(40b^3\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/}}\right.}{77a^3\left(a+bx^4\right)^{3/4}}$$

$$= -\frac{\sqrt[4]{a+bx^4}}{11ax^{11}} + \frac{10b\sqrt[4]{a+bx^4}}{77a^2x^7} - \frac{20b^2\sqrt[4]{a+bx^4}}{77a^3x^3} + \frac{\left(20b^3\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{3/}}\right.}{77a^3\left(a+bx^4\right)^{3/4}}$$

$$= -\frac{\sqrt[4]{a+bx^4}}{11ax^{11}} + \frac{10b\sqrt[4]{a+bx^4}}{77a^2x^7} - \frac{20b^2\sqrt[4]{a+bx^4}}{77a^3x^3} + \frac{40b^{7/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\right|}{77a^{7/2}\left(a+bx^4\right)^{3/4}}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.39

$$-\frac{\left(\frac{bx^4}{a}+1\right)^{3/4}{}_2F_1\left(-\frac{11}{4},\frac{3}{4};-\frac{7}{4};-\frac{bx^4}{a}\right)}{11x^{11}\left(a+bx^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^12*(a + b*x^4)^(3/4)),x]
```

```
[Out] -1/11*((1 + (b*x^4)/a)^(3/4)*Hypergeometric2F1[-11/4, 3/4, -7/4, -((b*x^4)/
a)])/(x^11*(a + b*x^4)^(3/4))
```

fricas [F] time = 1.18, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{bx^{16}+ax^{12}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^12/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(1/4)/(b*x^16 + a*x^12), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{3}{4}}x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^12/(b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(3/4)*x^12), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{3}{4}}x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^12/(b*x^4+a)^(3/4),x)
```

```
[Out] int(1/x^12/(b*x^4+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{3}{4}}x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^12/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(3/4)*x^12), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{12}\left(b\,x^4+a\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^12*(a + b*x^4)^(3/4)),x)
```

```
[Out] int(1/(x^12*(a + b*x^4)^(3/4)), x)
```

sympy [C] time = 2.57, size = 44, normalized size = 0.34

$$\frac{\Gamma\left(-\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{11}{4}, \frac{3}{4} \\ -\frac{7}{4}\end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{4}}x^{11}\Gamma\left(-\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**12/(b*x**4+a)**(3/4),x)
```

```
[Out] gamma(-11/4)*hyper((-11/4, 3/4), (-7/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
3/4)*x**11*gamma(-7/4))
```

3.1139 $\int \frac{x^{19}}{\left(a+bx^4\right)^{5/4}}\,dx$

Optimal. Leaf size=99

$$-\frac{a^4}{b^5\sqrt[4]{a+bx^4}}-\frac{4a^3\left(a+bx^4\right)^{3/4}}{3b^5}+\frac{6a^2\left(a+bx^4\right)^{7/4}}{7b^5}-\frac{4a\left(a+bx^4\right)^{11/4}}{11b^5}+\frac{\left(a+bx^4\right)^{15/4}}{15b^5}$$

```
[Out] -a^4/b^5/(b*x^4+a)^(1/4)-4/3*a^3*(b*x^4+a)^(3/4)/b^5+6/7*a^2*(b*x^4+a)^(7/4
)/b^5-4/11*a*(b*x^4+a)^(11/4)/b^5+1/15*(b*x^4+a)^(15/4)/b^5
```

Rubi [A] time = 0.05, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{a^4}{b^5\sqrt[4]{a+bx^4}}-\frac{4a^3\left(a+bx^4\right)^{3/4}}{3b^5}+\frac{6a^2\left(a+bx^4\right)^{7/4}}{7b^5}-\frac{4a\left(a+bx^4\right)^{11/4}}{11b^5}+\frac{\left(a+bx^4\right)^{15/4}}{15b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^19/(a + b*x^4)^(5/4),x]
```

```
[Out] -(a^4/(b^5*(a + b*x^4)^(1/4))) - (4*a^3*(a + b*x^4)^(3/4))/(3*b^5) + (6*a^2
*(a + b*x^4)^(7/4))/(7*b^5) - (4*a*(a + b*x^4)^(11/4))/(11*b^5) + (a + b*x^
4)^(15/4)/(15*b^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{19}}{\left(a+bx^4\right)^{5/4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x^4}{(a+bx)^{5/4}}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int \left(\frac{a^4}{b^4(a+bx)^{5/4}}-\frac{4a^3}{b^4\sqrt[4]{a+bx}}+\frac{6a^2(a+bx)^{3/4}}{b^4}-\frac{4a(a+bx)^{7/4}}{b^4}+\frac{(a+bx)^{11/4}}{b^4}\right)dx\right)\\ &= -\frac{a^4}{b^5\sqrt[4]{a+bx^4}}-\frac{4a^3\left(a+bx^4\right)^{3/4}}{3b^5}+\frac{6a^2\left(a+bx^4\right)^{7/4}}{7b^5}-\frac{4a\left(a+bx^4\right)^{11/4}}{11b^5}+\frac{\left(a+bx^4\right)^{15/4}}{15b^5}\end{aligned}$$

Mathematica [A] time = 0.05, size = 61, normalized size = 0.62

$$\frac{-2048a^4-512a^3bx^4+192a^2b^2x^8-112ab^3x^{12}+77b^4x^{16}}{1155b^5\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^19/(a + b*x^4)^(5/4),x]
```

```
[Out] (-2048*a^4 - 512*a^3*b*x^4 + 192*a^2*b^2*x^8 - 112*a*b^3*x^12 + 77*b^4*x^16
)/(1155*b^5*(a + b*x^4)^(1/4))
```

fricas [A] time = 0.85, size = 69, normalized size = 0.70

$$\frac{\left(77\,b^4x^{16}-112\,ab^3x^{12}+192\,a^2b^2x^8-512\,a^3bx^4-2048\,a^4\right)\left(bx^4+a\right)^{\frac{3}{4}}}{1155\left(b^6x^4+ab^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] 1/1155*(77*b^4*x^16 - 112*a*b^3*x^12 + 192*a^2*b^2*x^8 - 512*a^3*b*x^4 - 20
48*a^4)*(b*x^4 + a)^(3/4)/(b^6*x^4 + a*b^5)
```

giac [A] time = 0.22, size = 87, normalized size = 0.88

$$-\frac{a^4}{\left(bx^4+a\right)^{\frac{1}{4}}b^5}+\frac{77\left(bx^4+a\right)^{\frac{15}{4}}b^{70}-420\left(bx^4+a\right)^{\frac{11}{4}}ab^{70}+990\left(bx^4+a\right)^{\frac{7}{4}}a^2b^{70}-1540\left(bx^4+a\right)^{\frac{3}{4}}a^3b^{70}}{1155\,b^{75}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^4+a)^(5/4),x, algorithm="giac")
```

```
[Out] -a^4/((b*x^4 + a)^(1/4)*b^5) + 1/1155*(77*(b*x^4 + a)^(15/4)*b^70 - 420*(b*
x^4 + a)^(11/4)*a*b^70 + 990*(b*x^4 + a)^(7/4)*a^2*b^70 - 1540*(b*x^4 + a)^
(3/4)*a^3*b^70)/b^75
```

maple [A] time = 0.01, size = 58, normalized size = 0.59

$$-\frac{-77x^{16}b^4+112a\,x^{12}b^3-192a^2x^8b^2+512a^3x^4b+2048a^4}{1155\left(b\,x^4+a\right)^{\frac{1}{4}}b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19/(b*x^4+a)^(5/4),x)
```

```
[Out] -1/1155*(-77*b^4*x^16+112*a*b^3*x^12-192*a^2*b^2*x^8+512*a^3*b*x^4+2048*a^4
)/(b*x^4+a)^(1/4)/b^5
```

maxima [A] time = 1.12, size = 81, normalized size = 0.82

$$\frac{\left(bx^4+a\right)^{\frac{15}{4}}}{15\,b^5}-\frac{4\left(bx^4+a\right)^{\frac{11}{4}}a}{11\,b^5}+\frac{6\left(bx^4+a\right)^{\frac{7}{4}}a^2}{7\,b^5}-\frac{4\left(bx^4+a\right)^{\frac{3}{4}}a^3}{3\,b^5}-\frac{a^4}{\left(bx^4+a\right)^{\frac{1}{4}}b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

```
[Out] 1/15*(b*x^4 + a)^(15/4)/b^5 - 4/11*(b*x^4 + a)^(11/4)*a/b^5 + 6/7*(b*x^4 +
a)^(7/4)*a^2/b^5 - 4/3*(b*x^4 + a)^(3/4)*a^3/b^5 - a^4/((b*x^4 + a)^(1/4)*b
^5)
```

mupad [B] time = 1.29, size = 69, normalized size = 0.70

$$-\frac{420\,a\left(b\,x^4+a\right)^3+1540\,a^3\left(b\,x^4+a\right)-77\left(b\,x^4+a\right)^4+1155\,a^4-990\,a^2\left(b\,x^4+a\right)^2}{1155\,b^5\left(b\,x^4+a\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19/(a + b*x^4)^(5/4),x)
```

```
[Out] -(420*a*(a + b*x^4)^3 + 1540*a^3*(a + b*x^4) - 77*(a + b*x^4)^4 + 1155*a^4
- 990*a^2*(a + b*x^4)^2)/(1155*b^5*(a + b*x^4)^(1/4))
```

sympy [A] time = 18.12, size = 116, normalized size = 1.17

$$\begin{cases} -\frac{2048a^4}{1155b^5\sqrt[4]{a+bx^4}} - \frac{512a^3x^4}{1155b^4\sqrt[4]{a+bx^4}} + \frac{64a^2x^8}{385b^3\sqrt[4]{a+bx^4}} - \frac{16ax^{12}}{165b^2\sqrt[4]{a+bx^4}} + \frac{x^{16}}{15b\sqrt[4]{a+bx^4}} & \text{for } b \neq 0 \\ \frac{x^{20}}{20a^{\frac{5}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**19/(b*x**4+a)**(5/4),x)
```

```
[Out] Piecewise((-2048*a**4/(1155*b**5*(a + b*x**4)**(1/4)) - 512*a**3*x**4/(1155
*b**4*(a + b*x**4)**(1/4)) + 64*a**2*x**8/(385*b**3*(a + b*x**4)**(1/4)) -
16*a*x**12/(165*b**2*(a + b*x**4)**(1/4)) + x**16/(15*b*(a + b*x**4)**(1/4)
), Ne(b, 0)), (x**20/(20*a**(5/4)), True))
```

3.1140 $\int \frac{x^{15}}{\left(a+bx^4\right)^{5/4}}\,dx$

Optimal. Leaf size=74

$$\frac{a^3}{b^4\sqrt[4]{a+bx^4}}+\frac{a^2\left(a+bx^4\right)^{3/4}}{b^4}-\frac{3a\left(a+bx^4\right)^{7/4}}{7b^4}+\frac{\left(a+bx^4\right)^{11/4}}{11b^4}$$

```
[Out] a^3/b^4/(b*x^4+a)^(1/4)+a^2*(b*x^4+a)^(3/4)/b^4-3/7*a*(b*x^4+a)^(7/4)/b^4+1
/11*(b*x^4+a)^(11/4)/b^4
```

Rubi [A] time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{a^3}{b^4\sqrt[4]{a+bx^4}}+\frac{a^2\left(a+bx^4\right)^{3/4}}{b^4}-\frac{3a\left(a+bx^4\right)^{7/4}}{7b^4}+\frac{\left(a+bx^4\right)^{11/4}}{11b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^15/(a + b*x^4)^(5/4),x]
```

```
[Out] a^3/(b^4*(a + b*x^4)^(1/4)) + (a^2*(a + b*x^4)^(3/4))/b^4 - (3*a*(a + b*x^4
)^(7/4))/(7*b^4) + (a + b*x^4)^(11/4)/(11*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{15}}{\left(a+bx^4\right)^{5/4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x^3}{(a+bx)^{5/4}}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int \left(-\frac{a^3}{b^3(a+bx)^{5/4}}+\frac{3a^2}{b^3\sqrt[4]{a+bx}}-\frac{3a(a+bx)^{3/4}}{b^3}+\frac{(a+bx)^{7/4}}{b^3}\right)dx, x, x^4\right)\\ &= \frac{a^3}{b^4\sqrt[4]{a+bx^4}}+\frac{a^2\left(a+bx^4\right)^{3/4}}{b^4}-\frac{3a\left(a+bx^4\right)^{7/4}}{7b^4}+\frac{\left(a+bx^4\right)^{11/4}}{11b^4}\end{aligned}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 0.68

$$\frac{128a^3+32a^2bx^4-12ab^2x^8+7b^3x^{12}}{77b^4\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15/(a + b*x^4)^(5/4),x]
```

```
[Out] (128*a^3 + 32*a^2*b*x^4 - 12*a*b^2*x^8 + 7*b^3*x^12)/(77*b^4*(a + b*x^4)^(1
/4))
```

fricas [A] time = 1.04, size = 58, normalized size = 0.78

$$\frac{\left(7\,b^3x^{12}-12\,ab^2x^8+32\,a^2bx^4+128\,a^3\right)\left(bx^4+a\right)^{\frac{3}{4}}}{77\left(b^5x^4+ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] 1/77*(7*b^3*x^12 - 12*a*b^2*x^8 + 32*a^2*b*x^4 + 128*a^3)*(b*x^4 + a)^(3/4)
/(b^5*x^4 + a*b^4)
```

giac [A] time = 0.16, size = 69, normalized size = 0.93

$$\frac{a^3}{\left(bx^4+a\right)^{\frac{1}{4}}b^4}+\frac{7\left(bx^4+a\right)^{\frac{11}{4}}b^{40}-33\left(bx^4+a\right)^{\frac{7}{4}}ab^{40}+77\left(bx^4+a\right)^{\frac{3}{4}}a^2b^{40}}{77\,b^{44}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^4+a)^(5/4),x, algorithm="giac")
```

```
[Out] a^3/((b*x^4 + a)^(1/4)*b^4) + 1/77*(7*(b*x^4 + a)^(11/4)*b^40 - 33*(b*x^4 +
 a)^(7/4)*a*b^40 + 77*(b*x^4 + a)^(3/4)*a^2*b^40)/b^44
```

maple [A] time = 0.01, size = 47, normalized size = 0.64

$$\frac{7b^3x^{12}-12a\,b^2x^8+32a^2b\,x^4+128a^3}{77\left(b\,x^4+a\right)^{\frac{1}{4}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(b*x^4+a)^(5/4),x)
```

```
[Out] 1/77*(7*b^3*x^12-12*a*b^2*x^8+32*a^2*b*x^4+128*a^3)/(b*x^4+a)^(1/4)/b^4
```

maxima [A] time = 1.36, size = 62, normalized size = 0.84

$$\frac{\left(bx^4+a\right)^{\frac{11}{4}}}{11\,b^4}-\frac{3\left(bx^4+a\right)^{\frac{7}{4}}a}{7\,b^4}+\frac{\left(bx^4+a\right)^{\frac{3}{4}}a^2}{b^4}+\frac{a^3}{\left(bx^4+a\right)^{\frac{1}{4}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

```
[Out] 1/11*(b*x^4 + a)^(11/4)/b^4 - 3/7*(b*x^4 + a)^(7/4)*a/b^4 + (b*x^4 + a)^(3/
4)*a^2/b^4 + a^3/((b*x^4 + a)^(1/4)*b^4)
```

mupad [B] time = 1.23, size = 55, normalized size = 0.74

$$\frac{77\,a^2\left(b\,x^4+a\right)-33\,a\left(b\,x^4+a\right)^2+7\left(b\,x^4+a\right)^3+77\,a^3}{77\,b^4\left(b\,x^4+a\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(a + b*x^4)^(5/4),x)
```

```
[Out] (77*a^2*(a + b*x^4) - 33*a*(a + b*x^4)^2 + 7*(a + b*x^4)^3 + 77*a^3)/(77*b^
4*(a + b*x^4)^(1/4))
```

sympy [**A**] time = 12.75, size = 92, normalized size = 1.24

$$\begin{cases} \frac{128a^3}{77b^4\sqrt[4]{a+bx^4}} + \frac{32a^2x^4}{77b^3\sqrt[4]{a+bx^4}} - \frac{12ax^8}{77b^2\sqrt[4]{a+bx^4}} + \frac{x^{12}}{11b\sqrt[4]{a+bx^4}} & \text{for } b \neq 0 \\ \frac{x^{16}}{16a^{\frac{5}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15/(b*x**4+a)**(5/4),x)
```

```
[Out] Piecewise((128*a**3/(77*b**4*(a + b*x**4)**(1/4)) + 32*a**2*x**4/(77*b**3*(
a + b*x**4)**(1/4)) - 12*a*x**8/(77*b**2*(a + b*x**4)**(1/4)) + x**12/(11*b
*(a + b*x**4)**(1/4)), Ne(b, 0)), (x**16/(16*a**(5/4)), True))
```

3.1141 $\int \frac{x^{11}}{\left(a+bx^4\right)^{5/4}}\,dx$

Optimal. Leaf size=57

$$-\frac{a^2}{b^3\sqrt[4]{a+bx^4}}-\frac{2a\left(a+bx^4\right)^{3/4}}{3b^3}+\frac{\left(a+bx^4\right)^{7/4}}{7b^3}$$

```
[Out] -a^2/b^3/(b*x^4+a)^(1/4)-2/3*a*(b*x^4+a)^(3/4)/b^3+1/7*(b*x^4+a)^(7/4)/b^3
```

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{a^2}{b^3\sqrt[4]{a+bx^4}}-\frac{2a\left(a+bx^4\right)^{3/4}}{3b^3}+\frac{\left(a+bx^4\right)^{7/4}}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + b*x^4)^(5/4),x]
```

```
[Out] -(a^2/(b^3*(a + b*x^4)^(1/4))) - (2*a*(a + b*x^4)^(3/4))/(3*b^3) + (a + b*x
^4)^(7/4)/(7*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{\left(a+bx^4\right)^{5/4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x^2}{(a+bx)^{5/4}}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int \left(\frac{a^2}{b^2(a+bx)^{5/4}}-\frac{2a}{b^2\sqrt[4]{a+bx}}+\frac{(a+bx)^{3/4}}{b^2}\right)dx, x, x^4\right)\\ &= -\frac{a^2}{b^3\sqrt[4]{a+bx^4}}-\frac{2a\left(a+bx^4\right)^{3/4}}{3b^3}+\frac{\left(a+bx^4\right)^{7/4}}{7b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.68

$$\frac{-32a^2-8abx^4+3b^2x^8}{21b^3\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + b*x^4)^(5/4),x]
```

```
[Out] (-32*a^2 - 8*a*b*x^4 + 3*b^2*x^8)/(21*b^3*(a + b*x^4)^(1/4))
```

fricas [A] time = 0.96, size = 47, normalized size = 0.82

$$\frac{\left(3\,b^2x^8 - 8\,abx^4 - 32\,a^2\right)\left(bx^4 + a\right)^{\frac{3}{4}}}{21\left(b^4x^4 + ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] 1/21*(3*b^2*x^8 - 8*a*b*x^4 - 32*a^2)*(b*x^4 + a)^(3/4)/(b^4*x^4 + a*b^3)
```

giac [A] time = 0.18, size = 53, normalized size = 0.93

$$-\frac{a^2}{\left(bx^4 + a\right)^{\frac{1}{4}}b^3} + \frac{3\left(bx^4 + a\right)^{\frac{7}{4}}b^{18} - 14\left(bx^4 + a\right)^{\frac{3}{4}}ab^{18}}{21\,b^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)^(5/4),x, algorithm="giac")
```

```
[Out] -a^2/((b*x^4 + a)^(1/4)*b^3) + 1/21*(3*(b*x^4 + a)^(7/4)*b^18 - 14*(b*x^4 +
 a)^(3/4)*a*b^18)/b^21
```

maple [A] time = 0.01, size = 36, normalized size = 0.63

$$-\frac{-3b^2x^8 + 8ab\,x^4 + 32a^2}{21\left(b\,x^4 + a\right)^{\frac{1}{4}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^4+a)^(5/4),x)
```

```
[Out] -1/21*(-3*b^2*x^8+8*a*b*x^4+32*a^2)/(b*x^4+a)^(1/4)/b^3
```

maxima [A] time = 1.20, size = 47, normalized size = 0.82

$$\frac{\left(bx^4 + a\right)^{\frac{7}{4}}}{7\,b^3} - \frac{2\left(bx^4 + a\right)^{\frac{3}{4}}a}{3\,b^3} - \frac{a^2}{\left(bx^4 + a\right)^{\frac{1}{4}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

```
[Out] 1/7*(b*x^4 + a)^(7/4)/b^3 - 2/3*(b*x^4 + a)^(3/4)*a/b^3 - a^2/((b*x^4 + a)^
(1/4)*b^3)
```

mupad [B] time = 1.21, size = 39, normalized size = 0.68

$$-\frac{\frac{2\,a\left(b\,x^4+a\right)}{3} - \frac{\left(b\,x^4+a\right)^2}{7} + a^2}{b^3\left(b\,x^4 + a\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a + b*x^4)^(5/4),x)
```

```
[Out] -((2*a*(a + b*x^4))/3 - (a + b*x^4)^2/7 + a^2)/(b^3*(a + b*x^4)^(1/4))
```

sympy [**A**] time = 5.03, size = 68, normalized size = 1.19

$$\begin{cases} -\frac{32a^2}{21b^3\sqrt[4]{a+bx^4}} - \frac{8ax^4}{21b^2\sqrt[4]{a+bx^4}} + \frac{x^8}{7b\sqrt[4]{a+bx^4}} & \text{for } b \neq 0 \\ \frac{x^{12}}{12a^{\frac{5}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**4+a)**(5/4),x)
```

```
[Out] Piecewise((-32*a**2/(21*b**3*(a + b*x**4)**(1/4)) - 8*a*x**4/(21*b**2*(a +
b*x**4)**(1/4)) + x**8/(7*b*(a + b*x**4)**(1/4)), Ne(b, 0)), (x**12/(12*a**
(5/4)), True))
```

3.1142 $\int \frac{x^7}{\left(a+bx^4\right)^{5/4}}\,dx$

Optimal. Leaf size=35

$$\frac{a}{b^2\sqrt[4]{a+bx^4}}+\frac{\left(a+bx^4\right)^{3/4}}{3b^2}$$

```
[Out] a/b^2/(b*x^4+a)^(1/4)+1/3*(b*x^4+a)^(3/4)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{a}{b^2\sqrt[4]{a+bx^4}}+\frac{\left(a+bx^4\right)^{3/4}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^4)^(5/4),x]
```

```
[Out] a/(b^2*(a + b*x^4)^(1/4)) + (a + b*x^4)^(3/4)/(3*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{\left(a+bx^4\right)^{5/4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x}{(a+bx)^{5/4}}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^{5/4}}+\frac{1}{b\sqrt[4]{a+bx}}\right)\,dx, x, x^4\right)\\ &= \frac{a}{b^2\sqrt[4]{a+bx^4}}+\frac{\left(a+bx^4\right)^{3/4}}{3b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.77

$$\frac{4a+bx^4}{3b^2\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^4)^(5/4),x]
```

```
[Out] (4*a + b*x^4)/(3*b^2*(a + b*x^4)^(1/4))
```

fricas [A] time = 0.84, size = 35, normalized size = 1.00

$$\frac{\left(bx^4+4\,a\right)\left(bx^4+a\right)^{\frac{3}{4}}}{3\left(b^3x^4+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)^(5/4),x, algorithm="fricas")
[Out] 1/3*(b*x^4 + 4*a)*(b*x^4 + a)^(3/4)/(b^3*x^4 + a*b^2)
```

giac [A] time = 0.19, size = 34, normalized size = 0.97

$$\frac{\frac{\left(bx^4+a\right)^{\frac{3}{4}}}{b}+\frac{3\,a}{\left(bx^4+a\right)^{\frac{1}{4}}b}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)^(5/4),x, algorithm="giac")
[Out] 1/3*((b*x^4 + a)^(3/4)/b + 3*a/((b*x^4 + a)^(1/4)*b))/b
```

maple [A] time = 0.01, size = 24, normalized size = 0.69

$$\frac{b\,x^4+4a}{3\left(b\,x^4+a\right)^{\frac{1}{4}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^4+a)^(5/4),x)
[Out] 1/3*(b*x^4+4*a)/(b*x^4+a)^(1/4)/b^2
```

maxima [A] time = 1.32, size = 29, normalized size = 0.83

$$\frac{\left(bx^4+a\right)^{\frac{3}{4}}}{3\,b^2}+\frac{a}{\left(bx^4+a\right)^{\frac{1}{4}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)^(5/4),x, algorithm="maxima")
[Out] 1/3*(b*x^4 + a)^(3/4)/b^2 + a/((b*x^4 + a)^(1/4)*b^2)
```

mupad [B] time = 1.15, size = 23, normalized size = 0.66

$$\frac{b\,x^4+4\,a}{3\,b^2\left(b\,x^4+a\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^4)^(5/4),x)
[Out] (4*a + b*x^4)/(3*b^2*(a + b*x^4)^(1/4))
```

sympy [**A**] time = 2.10, size = 44, normalized size = 1.26

$$\begin{cases} \frac{4a}{3b^2\sqrt[4]{a+bx^4}} + \frac{x^4}{3b\sqrt[4]{a+bx^4}} & \text{for } b \neq 0 \\ \frac{x^8}{8a^{\frac{5}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**4+a)**(5/4),x)
```

```
[Out] Piecewise((4*a/(3*b**2*(a + b*x**4)**(1/4)) + x**4/(3*b*(a + b*x**4)**(1/4)
), Ne(b, 0)), (x**8/(8*a**(5/4)), True))
```

3.1143 $\int \frac{x^3}{\left(a+bx^4\right)^{5/4}}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{b\sqrt[4]{a+bx^4}}$$

```
[Out] -1/b/(b*x^4+a)^(1/4)
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$-\frac{1}{b\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^4)^(5/4),x]
```

```
[Out] -(1/(b*(a + b*x^4)^(1/4)))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^4\right)^{5/4}}\,dx = -\frac{1}{b\sqrt[4]{a+bx^4}}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{b\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^4)^(5/4),x]
```

```
[Out] -(1/(b*(a + b*x^4)^(1/4)))
```

fricas [A] time = 0.83, size = 24, normalized size = 1.50

$$-\frac{\left(bx^4+a\right)^{\frac{3}{4}}}{b^2x^4+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] -(b*x^4 + a)^(3/4)/(b^2*x^4 + a*b)
```

giac [A] time = 0.16, size = 14, normalized size = 0.88

$$-\frac{1}{\left(bx^4+a\right)^{\frac{1}{4}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)^(5/4),x, algorithm="giac")
[Out] -1/((b*x^4 + a)^(1/4)*b)
```

maple [A] time = 0.01, size = 15, normalized size = 0.94

$$-\frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^4+a)^(5/4),x)
[Out] -1/b/(b*x^4+a)^(1/4)
```

maxima [A] time = 1.30, size = 14, normalized size = 0.88

$$-\frac{1}{\left(bx^4+a\right)^{\frac{1}{4}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)^(5/4),x, algorithm="maxima")
[Out] -1/((b*x^4 + a)^(1/4)*b)
```

mupad [B] time = 1.10, size = 14, normalized size = 0.88

$$-\frac{1}{b\left(b\,x^4+a\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^4)^(5/4),x)
[Out] -1/(b*(a + b*x^4)^(1/4))
```

sympy [A] time = 1.30, size = 24, normalized size = 1.50

$$\begin{cases} -\frac{1}{b\sqrt[4]{a+bx^4}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{5}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**4+a)**(5/4),x)
[Out] Piecewise((-1/(b*(a + b*x**4)**(1/4)), Ne(b, 0)), (x**4/(4*a**(5/4)), True)
)
```

3.1144 $\int \frac{1}{x\left(a+bx^4\right)^{5/4}}\,dx$

Optimal. Leaf size=70

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{5/4}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{5/4}} + \frac{1}{a\sqrt[4]{a+bx^4}}$$

```
[Out] 1/a/(b*x^4+a)^(1/4)+1/2*arctan((b*x^4+a)^(1/4)/a^(1/4))/a^(5/4)-1/2*arctanh
((b*x^4+a)^(1/4)/a^(1/4))/a^(5/4)
```

Rubi [A] time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {266, 51, 63, 298, 203, 206}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{5/4}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{5/4}} + \frac{1}{a\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^4)^(5/4)),x]
```

```
[Out] 1/(a*(a + b*x^4)^(1/4)) + ArcTan[(a + b*x^4)^(1/4)/a^(1/4)]/(2*a^(5/4)) - A
rcTanh[(a + b*x^4)^(1/4)/a^(1/4)]/(2*a^(5/4))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+bx^4\right)^{5/4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/4}}\,dx, x, x^4\right) \\
&= \frac{1}{a\sqrt[4]{a+bx^4}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt[4]{a+bx}}\,dx, x, x^4\right)}{4a} \\
&= \frac{1}{a\sqrt[4]{a+bx^4}} + \frac{\text{Subst}\left(\int \frac{x^2}{-\frac{a}{b}+\frac{x^4}{b}}\,dx, x, \sqrt[4]{a+bx^4}\right)}{ab} \\
&= \frac{1}{a\sqrt[4]{a+bx^4}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a}-x^2}\,dx, x, \sqrt[4]{a+bx^4}\right)}{2a} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a}+x^2}\,dx, x, \sqrt[4]{a+bx^4}\right)}{2a} \\
&= \frac{1}{a\sqrt[4]{a+bx^4}} + \frac{\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{5/4}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{5/4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 33, normalized size = 0.47

$$\frac{{}_2F_1\left(-\frac{1}{4}, 1; \frac{3}{4}; \frac{bx^4}{a}+1\right)}{a\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^4)^(5/4)),x]
```

```
[Out] Hypergeometric2F1[-1/4, 1, 3/4, 1 + (b*x^4)/a]/(a*(a + b*x^4)^(1/4))
```

fricas [B] time = 0.89, size = 164, normalized size = 2.34

$$-\frac{4\left(abx^4+a^2\right)\frac{1}{a^5}^{\frac{1}{4}}\arctan\left(\sqrt{a^3\sqrt{\frac{1}{a^5}}+\sqrt{bx^4+a}}\,a\frac{1}{a^5}^{\frac{1}{4}}-\left(bx^4+a\right)^{\frac{1}{4}}a\frac{1}{a^5}^{\frac{1}{4}}\right)+\left(abx^4+a^2\right)\frac{1}{a^5}^{\frac{1}{4}}\log\left(a^4\frac{1}{a^5}^{\frac{3}{4}}+\left(bx^4\right.\right.}{4\left(abx^4+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] -1/4*(4*(a*b*x^4 + a^2)*(a^(-5))^(1/4)*arctan(sqrt(a^3*sqrt(a^(-5)) + sqrt(
b*x^4 + a))*a*(a^(-5))^(1/4) - (b*x^4 + a)^(1/4)*a*(a^(-5))^(1/4)) + (a*b*x
^4 + a^2)*(a^(-5))^(1/4)*log(a^4*(a^(-5))^(3/4) + (b*x^4 + a)^(1/4)) - (a*b
*x^4 + a^2)*(a^(-5))^(1/4)*log(-a^4*(a^(-5))^(3/4) + (b*x^4 + a)^(1/4)) - 4
*(b*x^4 + a)^(3/4))/(a*b*x^4 + a^2)
```

giac [B] time = 0.17, size = 199, normalized size = 2.84

$$-\frac{\sqrt{2}\,(-a)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}+2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{4\,a^2}-\frac{\sqrt{2}\,(-a)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}-2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{4\,a^2}+\frac{\sqrt{2}\,(-a)^{\frac{3}{4}}\log\left(\sqrt{2}\left(bx^4\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)^(5/4),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(-a)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x^4 +
 a)^(1/4))/(-a)^(1/4))/a^2 - 1/4*sqrt(2)*(-a)^(3/4)*arctan(-1/2*sqrt(2)*(sq
rt(2)*(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4))/a^2 + 1/8*sqrt(2)*(-a)^
(3/4)*log(sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt(-a)
)/a^2 - 1/8*sqrt(2)*(-a)^(3/4)*log(-sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) +
sqrt(b*x^4 + a) + sqrt(-a))/a^2 + 1/((b*x^4 + a)^(1/4)*a)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{5}{4}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^4+a)^(5/4),x)
```

```
[Out] int(1/x/(b*x^4+a)^(5/4),x)
```

maxima [A] time = 3.09, size = 75, normalized size = 1.07

$$\frac{\frac{2\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}+\frac{\log\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{\left(bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}}{4\,a}+\frac{1}{\left(bx^4+a\right)^{\frac{1}{4}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

```
[Out] 1/4*(2*arctan((b*x^4 + a)^(1/4)/a^(1/4))/a^(1/4) + log(((b*x^4 + a)^(1/4) -
 a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/4)))/a^(1/4))/a + 1/((b*x^4 + a)^(1/4)*
a)
```

mupad [B] time = 1.23, size = 52, normalized size = 0.74

$$\frac{\operatorname{atan}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)}{2\,a^{5/4}}-\frac{\operatorname{atanh}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)}{2\,a^{5/4}}+\frac{1}{a\left(b\,x^4+a\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^4)^(5/4)),x)
```

```
[Out] atan((a + b*x^4)^(1/4)/a^(1/4))/(2*a^(5/4)) - atanh((a + b*x^4)^(1/4)/a^(1/
4))/(2*a^(5/4)) + 1/(a*(a + b*x^4)^(1/4))
```

sympy [C] time = 1.82, size = 39, normalized size = 0.56

$$-\frac{\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{4}, \frac{5}{4} \\ \frac{9}{4}\end{matrix}\right| \frac{ae^{i\pi}}{bx^4}\right)}{4b^{\frac{5}{4}}x^5\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**4+a)**(5/4),x)
```

```
[Out] -gamma(5/4)*hyper((5/4, 5/4), (9/4,), a*exp_polar(I*pi)/(b*x**4))/(4*b**(5/
4)*x**5*gamma(9/4))
```

3.1145 $\int \frac{1}{x^5\left(a+bx^4\right)^{5/4}}\,dx$

Optimal. Leaf size=97

$$-\frac{5b\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{9/4}}+\frac{5b\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{9/4}}-\frac{5b}{4a^2\sqrt[4]{a+bx^4}}-\frac{1}{4ax^4\sqrt[4]{a+bx^4}}$$

```
[Out] -5/4*b/a^2/(b*x^4+a)^(1/4)-1/4/a/x^4/(b*x^4+a)^(1/4)-5/8*b*arctan((b*x^4+a)
^(1/4)/a^(1/4))/a^(9/4)+5/8*b*arctanh((b*x^4+a)^(1/4)/a^(1/4))/a^(9/4)
```

Rubi [A] time = 0.06, antiderivative size = 96, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {266, 51, 63, 298, 203, 206}

$$-\frac{5\left(a+bx^4\right)^{3/4}}{4a^2x^4}-\frac{5b\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{9/4}}+\frac{5b\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{9/4}}+\frac{1}{ax^4\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^4)^(5/4)),x]
```

```
[Out] 1/(a*x^4*(a + b*x^4)^(1/4)) - (5*(a + b*x^4)^(3/4))/(4*a^2*x^4) - (5*b*ArcT
an[(a + b*x^4)^(1/4)/a^(1/4)])/(8*a^(9/4)) + (5*b*ArcTanh[(a + b*x^4)^(1/4)
/a^(1/4)])/(8*a^(9/4))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$\int \frac{1}{x^5\left(a+bx^4\right)^{5/4}}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{5/4}}\,dx, x, x^4\right)$$

$$= \frac{1}{ax^4\sqrt[4]{a+bx^4}} + \frac{5\,\text{Subst}\left(\int \frac{1}{x^2\sqrt[4]{a+bx}}\,dx, x, x^4\right)}{4a}$$

$$= \frac{1}{ax^4\sqrt[4]{a+bx^4}} - \frac{5\left(a+bx^4\right)^{3/4}}{4a^2x^4} - \frac{(5b)\,\text{Subst}\left(\int \frac{1}{x\sqrt[4]{a+bx}}\,dx, x, x^4\right)}{16a^2}$$

$$= \frac{1}{ax^4\sqrt[4]{a+bx^4}} - \frac{5\left(a+bx^4\right)^{3/4}}{4a^2x^4} - \frac{5\,\text{Subst}\left(\int \frac{x^2}{-\frac{a}{b}+\frac{x^4}{b}}\,dx, x, \sqrt[4]{a+bx^4}\right)}{4a^2}$$

$$= \frac{1}{ax^4\sqrt[4]{a+bx^4}} - \frac{5\left(a+bx^4\right)^{3/4}}{4a^2x^4} + \frac{(5b)\,\text{Subst}\left(\int \frac{1}{\sqrt{a}-x^2}\,dx, x, \sqrt[4]{a+bx^4}\right)}{8a^2} - \frac{(5b)\,\text{Subst}\left(\right.}{}$$

$$= \frac{1}{ax^4\sqrt[4]{a+bx^4}} - \frac{5\left(a+bx^4\right)^{3/4}}{4a^2x^4} - \frac{5b\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{9/4}} + \frac{5b\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{9/4}}$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.36

$$-\frac{b\,{}_2F_1\left(-\frac{1}{4}, 2; \frac{3}{4}; \frac{bx^4}{a}+1\right)}{a^2\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^4)^(5/4)),x]
```

```
[Out] -((b*Hypergeometric2F1[-1/4, 2, 3/4, 1 + (b*x^4)/a])/(a^2*(a + b*x^4)^(1/4)
))
```

fricas [B] time = 0.92, size = 259, normalized size = 2.67

$$\frac{20\left(a^2bx^8+a^3x^4\right)\left(\frac{b^4}{a^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(bx^4+a\right)^{\frac{1}{4}}a^2b^3\left(\frac{b^4}{a^9}\right)^{\frac{1}{4}}-\sqrt{a^5b^4\sqrt{\frac{b^4}{a^9}}+\sqrt{bx^4+a}\,b^6}\,a^2\left(\frac{b^4}{a^9}\right)^{\frac{1}{4}}}{b^4}\right)+5\left(a^2bx^8+a^3x^4\right)\left(\frac{b^4}{a^9}\right)^{\frac{1}{4}}\log\left(1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] 1/16*(20*(a^2*b*x^8 + a^3*x^4)*(b^4/a^9)^(1/4)*arctan(-((b*x^4 + a)^(1/4)*a
^2*b^3*(b^4/a^9)^(1/4) - sqrt(a^5*b^4*sqrt(b^4/a^9) + sqrt(b*x^4 + a)*b^6)*
a^2*(b^4/a^9)^(1/4))/b^4) + 5*(a^2*b*x^8 + a^3*x^4)*(b^4/a^9)^(1/4)*log(125
*a^7*(b^4/a^9)^(3/4) + 125*(b*x^4 + a)^(1/4)*b^3) - 5*(a^2*b*x^8 + a^3*x^4)
*(b^4/a^9)^(1/4)*log(-125*a^7*(b^4/a^9)^(3/4) + 125*(b*x^4 + a)^(1/4)*b^3)
- 4*(5*b*x^4 + a)*(b*x^4 + a)^(3/4))/(a^2*b*x^8 + a^3*x^4)
```

giac [B] time = 0.18, size = 234, normalized size = 2.41

$$\frac{5\sqrt{2}\,(-a)^{\frac{3}{4}}\,b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}+2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{16\,a^3}+\frac{5\sqrt{2}\,(-a)^{\frac{3}{4}}\,b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}-2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{16\,a^3}+\frac{5\sqrt{2}\,b\log\left(\sqrt{2}\left(b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)^(5/4),x, algorithm="giac")
```

```
[Out] 5/16*sqrt(2)*(-a)^(3/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x^4
 + a)^(1/4))/(-a)^(1/4))/a^3 + 5/16*sqrt(2)*(-a)^(3/4)*b*arctan(-1/2*sqrt(2
)*(sqrt(2)*(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4))/a^3 + 5/32*sqrt(2)
*b*log(sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt(-a))/(
(-a)^(1/4)*a^2) + 5/32*sqrt(2)*(-a)^(3/4)*b*log(-sqrt(2)*(b*x^4 + a)^(1/4)*
(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt(-a))/a^3 - 1/4*(5*(b*x^4 + a)*b - 4*a*b
)/(((b*x^4 + a)^(5/4) - (b*x^4 + a)^(1/4)*a)*a^2)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{5}{4}}\,x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^4+a)^(5/4),x)
```

```
[Out] int(1/x^5/(b*x^4+a)^(5/4),x)
```

maxima [A] time = 2.98, size = 110, normalized size = 1.13

$$-\frac{5\left(bx^4+a\right)b-4\,ab}{4\left(\left(bx^4+a\right)^{\frac{5}{4}}a^2-\left(bx^4+a\right)^{\frac{1}{4}}a^3\right)}-\frac{5\,b\left(\frac{2\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}+\frac{\log\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{\left(bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}\right)}{16\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

```
[Out] -1/4*(5*(b*x^4 + a)*b - 4*a*b)/((b*x^4 + a)^(5/4)*a^2 - (b*x^4 + a)^(1/4)*a
^3) - 5/16*b*(2*arctan((b*x^4 + a)^(1/4)/a^(1/4))/a^(1/4) + log(((b*x^4 + a
)^(1/4) - a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/4)))/a^(1/4))/a^2
```

mupad [B] time = 1.39, size = 87, normalized size = 0.90

$$\frac{5\,b\,\operatorname{atanh}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)}{8\,a^{9/4}}-\frac{5\,b\,\operatorname{atan}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)}{8\,a^{9/4}}-\frac{\frac{b}{a}-\frac{5\,b\left(b\,x^4+a\right)}{4\,a^2}}{a\left(b\,x^4+a\right)^{1/4}-\left(b\,x^4+a\right)^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^4)^(5/4)),x)
```

```
[Out] (5*b*atanh((a + b*x^4)^(1/4)/a^(1/4)))/(8*a^(9/4)) - (5*b*atan((a + b*x^4)^
(1/4)/a^(1/4)))/(8*a^(9/4)) - (b/a - (5*b*(a + b*x^4))/(4*a^2))/(a*(a + b*x
^4)^(1/4) - (a + b*x^4)^(5/4))
```

sympy [C] time = 1.83, size = 39, normalized size = 0.40

$$-\frac{\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{4},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\right|\frac{ae^{i\pi}}{bx^4}\right)}{4b^{\frac{5}{4}}x^9\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**4+a)**(5/4),x)
```

```
[Out] -gamma(9/4)*hyper((5/4, 9/4), (13/4,), a*exp_polar(I*pi)/(b*x**4))/(4*b**(5
/4)*x**9*gamma(13/4))
```

3.1146 $\int \frac{1}{x^9\left(a+bx^4\right)^{5/4}}\,dx$

Optimal. Leaf size=125

$$\frac{45b^2\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{13/4}}-\frac{45b^2\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{13/4}}+\frac{45b^2}{32a^3\sqrt[4]{a+bx^4}}+\frac{9b}{32a^2x^4\sqrt[4]{a+bx^4}}-\frac{1}{8ax^8\sqrt[4]{a+bx^4}}$$

```
[Out] 45/32*b^2/a^3/(b*x^4+a)^(1/4)-1/8/a/x^8/(b*x^4+a)^(1/4)+9/32*b/a^2/x^4/(b*x
^4+a)^(1/4)+45/64*b^2*arctan((b*x^4+a)^(1/4)/a^(1/4))/a^(13/4)-45/64*b^2*ar
ctanh((b*x^4+a)^(1/4)/a^(1/4))/a^(13/4)
```

Rubi [A] time = 0.08, antiderivative size = 122, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {266, 51, 63, 298, 203, 206}

$$\frac{45b^2\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{13/4}}-\frac{45b^2\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{13/4}}+\frac{45b\left(a+bx^4\right)^{3/4}}{32a^3x^4}-\frac{9\left(a+bx^4\right)^{3/4}}{8a^2x^8}+\frac{1}{ax^8\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(a + b*x^4)^(5/4)),x]
```

```
[Out] 1/(a*x^8*(a + b*x^4)^(1/4)) - (9*(a + b*x^4)^(3/4))/(8*a^2*x^8) + (45*b*(a
+ b*x^4)^(3/4))/(32*a^3*x^4) + (45*b^2*ArcTan[(a + b*x^4)^(1/4)/a^(1/4)])/(
64*a^(13/4)) - (45*b^2*ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)])/(64*a^(13/4))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$\int \frac{1}{x^9\left(a+bx^4\right)^{5/4}}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x^3(a+bx)^{5/4}}\,dx, x, x^4\right)$$

$$= \frac{1}{ax^8\sqrt[4]{a+bx^4}} + \frac{9\,\text{Subst}\left(\int \frac{1}{x^3\sqrt[4]{a+bx}}\,dx, x, x^4\right)}{4a}$$

$$= \frac{1}{ax^8\sqrt[4]{a+bx^4}} - \frac{9\left(a+bx^4\right)^{3/4}}{8a^2x^8} - \frac{(45b)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt[4]{a+bx}}\,dx, x, x^4\right)}{32a^2}$$

$$= \frac{1}{ax^8\sqrt[4]{a+bx^4}} - \frac{9\left(a+bx^4\right)^{3/4}}{8a^2x^8} + \frac{45b\left(a+bx^4\right)^{3/4}}{32a^3x^4} + \frac{\left(45b^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt[4]{a+bx}}\,dx, x, x^4\right)}{128a^3}$$

$$= \frac{1}{ax^8\sqrt[4]{a+bx^4}} - \frac{9\left(a+bx^4\right)^{3/4}}{8a^2x^8} + \frac{45b\left(a+bx^4\right)^{3/4}}{32a^3x^4} + \frac{(45b)\,\text{Subst}\left(\int \frac{x^2}{-\frac{a}{b}+\frac{x^4}{b}}\,dx, x, \sqrt[4]{a+}\right.}{32a^3}$$

$$= \frac{1}{ax^8\sqrt[4]{a+bx^4}} - \frac{9\left(a+bx^4\right)^{3/4}}{8a^2x^8} + \frac{45b\left(a+bx^4\right)^{3/4}}{32a^3x^4} - \frac{\left(45b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a}-x^2}\,dx, x, \sqrt[4]{a+}\right.}{64a^3}$$

$$= \frac{1}{ax^8\sqrt[4]{a+bx^4}} - \frac{9\left(a+bx^4\right)^{3/4}}{8a^2x^8} + \frac{45b\left(a+bx^4\right)^{3/4}}{32a^3x^4} + \frac{45b^2\tan^{-1}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{13/4}} - \frac{45b^2\tanh}{64}$$

Mathematica [C] time = 0.01, size = 36, normalized size = 0.29

$$\frac{b^2\,{}_2F_1\left(-\frac{1}{4}, 3; \frac{3}{4}; \frac{bx^4}{a}+1\right)}{a^3\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^9*(a + b*x^4)^(5/4)),x]
```

```
[Out] (b^2*Hypergeometric2F1[-1/4, 3, 3/4, 1 + (b*x^4)/a])/(a^3*(a + b*x^4)^(1/4)
)
```

fricas [B] time = 0.87, size = 272, normalized size = 2.18

$$180\left(a^3bx^{12}+a^4x^8\right)\left(\frac{b^8}{a^{13}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(bx^4+a\right)^{\frac{1}{4}}a^3b^6\left(\frac{b^8}{a^{13}}\right)^{\frac{1}{4}}-\sqrt{a^7b^8\sqrt{\frac{b^8}{a^{13}}}+\sqrt{bx^4+a}\,b^{12}}\,a^3\left(\frac{b^8}{a^{13}}\right)^{\frac{1}{4}}}{b^8}\right)+45\left(a^3bx^{12}+a^4x^8\right)\left(\frac{b}{a}\right.$$

$$-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] -1/128*(180*(a^3*b*x^12 + a^4*x^8)*(b^8/a^13)^(1/4)*arctan(-((b*x^4 + a)^(1
/4)*a^3*b^6*(b^8/a^13)^(1/4) - sqrt(a^7*b^8*sqrt(b^8/a^13) + sqrt(b*x^4 + a
)*b^12)*a^3*(b^8/a^13)^(1/4))/b^8) + 45*(a^3*b*x^12 + a^4*x^8)*(b^8/a^13)^(
1/4)*log(91125*a^10*(b^8/a^13)^(3/4) + 91125*(b*x^4 + a)^(1/4)*b^6) - 45*(a
^3*b*x^12 + a^4*x^8)*(b^8/a^13)^(1/4)*log(-91125*a^10*(b^8/a^13)^(3/4) + 91
125*(b*x^4 + a)^(1/4)*b^6) - 4*(45*b^2*x^8 + 9*a*b*x^4 - 4*a^2)*(b*x^4 + a)
^(3/4))/(a^3*b*x^12 + a^4*x^8)
```

giac [B] time = 0.18, size = 255, normalized size = 2.04

$$\frac{45\sqrt{2}\,b^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}+2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{128\,(-a)^{\frac{1}{4}}a^3}+\frac{45\sqrt{2}\,b^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}-2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{128\,(-a)^{\frac{1}{4}}a^3}+\frac{45\sqrt{2}\,(-a)^{\frac{3}{4}}b^2\log\left(\sqrt{2}\left(b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^4+a)^(5/4),x, algorithm="giac")
```

```
[Out] 45/128*sqrt(2)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x^4 + a)^(
1/4))/(-a)^(1/4))/((-a)^(1/4)*a^3) + 45/128*sqrt(2)*b^2*arctan(-1/2*sqrt(2)
*(sqrt(2)*(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4))/((-a)^(1/4)*a^3) +
45/256*sqrt(2)*(-a)^(3/4)*b^2*log(sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sq
rt(b*x^4 + a) + sqrt(-a))/a^4 + 45/256*sqrt(2)*b^2*log(-sqrt(2)*(b*x^4 + a)
^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt(-a))/((-a)^(1/4)*a^3) + b^2/((b*
x^4 + a)^(1/4)*a^3) + 1/32*(13*(b*x^4 + a)^(7/4)*b^2 - 17*(b*x^4 + a)^(3/4)
*a*b^2)/(a^3*b^2*x^8)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{5}{4}}x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(b*x^4+a)^(5/4),x)
```

```
[Out] int(1/x^9/(b*x^4+a)^(5/4),x)
```

maxima [A] time = 2.99, size = 146, normalized size = 1.17

$$\frac{45\left(bx^4+a\right)^2b^2-81\left(bx^4+a\right)ab^2+32\,a^2b^2}{32\left(\left(bx^4+a\right)^{\frac{9}{4}}a^3-2\left(bx^4+a\right)^{\frac{5}{4}}a^4+\left(bx^4+a\right)^{\frac{1}{4}}a^5\right)}+\frac{45\,b^2\left(\frac{2\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}+\frac{\log\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{\left(bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}\right)}{128\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

```
[Out] 1/32*(45*(b*x^4 + a)^2*b^2 - 81*(b*x^4 + a)*a*b^2 + 32*a^2*b^2)/((b*x^4 + a
)^(9/4)*a^3 - 2*(b*x^4 + a)^(5/4)*a^4 + (b*x^4 + a)^(1/4)*a^5) + 45/128*b^2
```

```
*(2*arctan((b*x^4 + a)^(1/4)/a^(1/4))/a^(1/4) + log(((b*x^4 + a)^(1/4) - a^
(1/4))/((b*x^4 + a)^(1/4) + a^(1/4)))/a^(1/4))/a^3
```

mupad [B] time = 1.51, size = 123, normalized size = 0.98

$$\frac{\frac{b^2}{a}-\frac{81\,b^2\left(b\,x^4+a\right)}{32\,a^2}+\frac{45\,b^2\left(b\,x^4+a\right)^2}{32\,a^3}}{\left(b\,x^4+a\right)^{9/4}-2\,a\left(b\,x^4+a\right)^{5/4}+a^2\left(b\,x^4+a\right)^{1/4}}+\frac{45\,b^2\,\mathrm{atan}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)}{64\,a^{13/4}}-\frac{45\,b^2\,\mathrm{atanh}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)}{64\,a^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(a + b*x^4)^(5/4)),x)
```

```
[Out] (b^2/a - (81*b^2*(a + b*x^4))/(32*a^2) + (45*b^2*(a + b*x^4)^2)/(32*a^3))/(
(a + b*x^4)^(9/4) - 2*a*(a + b*x^4)^(5/4) + a^2*(a + b*x^4)^(1/4)) + (45*b^
2*atan((a + b*x^4)^(1/4)/a^(1/4)))/(64*a^(13/4)) - (45*b^2*atanh((a + b*x^4
)^(1/4)/a^(1/4)))/(64*a^(13/4))
```

sympy [C] time = 3.21, size = 39, normalized size = 0.31

$$-\frac{\Gamma\left(\frac{13}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{4},\frac{13}{4}\\ \frac{17}{4}\end{matrix}\right|\frac{ae^{i\pi}}{bx^4}\right)}{4b^{\frac{5}{4}}x^{13}\Gamma\left(\frac{17}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(b*x**4+a)**(5/4),x)
```

```
[Out] -gamma(13/4)*hyper((5/4, 13/4), (17/4,), a*exp_polar(I*pi)/(b*x**4))/(4*b**
(5/4)*x**13*gamma(17/4))
```

3.1147 $\int \frac{x^{13}}{\left(a+bx^4\right)^{5/4}}\,dx$

Optimal. Leaf size=128

$$-\frac{8a^{5/2}\sqrt[4]{\frac{bx^4}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{3b^{7/2}\sqrt[4]{a+bx^4}}+\frac{4a^2x^2}{3b^3\sqrt[4]{a+bx^4}}-\frac{2ax^6}{9b^2\sqrt[4]{a+bx^4}}+\frac{x^{10}}{9b\sqrt[4]{a+bx^4}}$$

```
[Out] 4/3*a^2*x^2/b^3/(b*x^4+a)^(1/4)-2/9*a*x^6/b^2/(b*x^4+a)^(1/4)+1/9*x^10/b/(b
*x^4+a)^(1/4)-8/3*a^(5/2)*(1+b*x^4/a)^(1/4)*(cos(1/2*arctan(x^2*b^(1/2)/a^(
1/2)))^2)^(1/2)/cos(1/2*arctan(x^2*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arct
an(x^2*b^(1/2)/a^(1/2))),2^(1/2))/b^(7/2)/(b*x^4+a)^(1/4)
```

Rubi [A] time = 0.09, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {275, 285, 197, 196}

$$\frac{4a^2x^2}{3b^3\sqrt[4]{a+bx^4}}-\frac{8a^{5/2}\sqrt[4]{\frac{bx^4}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{3b^{7/2}\sqrt[4]{a+bx^4}}-\frac{2ax^6}{9b^2\sqrt[4]{a+bx^4}}+\frac{x^{10}}{9b\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/(a + b*x^4)^(5/4),x]
```

```
[Out] (4*a^2*x^2)/(3*b^3*(a + b*x^4)^(1/4)) - (2*a*x^6)/(9*b^2*(a + b*x^4)^(1/4))
 + x^10/(9*b*(a + b*x^4)^(1/4)) - (8*a^(5/2)*(1 + (b*x^4)/a)^(1/4)*Elliptic
E[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(3*b^(7/2)*(a + b*x^4)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + (b*x^2)/a)^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 285

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(2*c*(
c*x)^(m - 1))/(b*(2*m - 3)*(a + b*x^2)^(1/4)), x] - Dist[(2*a*c^2*(m - 1))/
(b*(2*m - 3)), Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Rubi steps

$$\int \frac{x^{13}}{\left(a+bx^4\right)^{5/4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^6}{\left(a+bx^2\right)^{5/4}}\,dx, x, x^2\right)$$

$$= \frac{x^{10}}{9b\sqrt[4]{a+bx^4}} - \frac{(5a)\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^2\right)^{5/4}}\,dx, x, x^2\right)}{9b}$$

$$= -\frac{2ax^6}{9b^2\sqrt[4]{a+bx^4}} + \frac{x^{10}}{9b\sqrt[4]{a+bx^4}} + \frac{\left(2a^2\right)\text{Subst}\left(\int \frac{x^2}{\left(a+bx^2\right)^{5/4}}\,dx, x, x^2\right)}{3b^2}$$

$$= \frac{4a^2x^2}{3b^3\sqrt[4]{a+bx^4}} - \frac{2ax^6}{9b^2\sqrt[4]{a+bx^4}} + \frac{x^{10}}{9b\sqrt[4]{a+bx^4}} - \frac{\left(4a^3\right)\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{5/4}}\,dx, x, x^2\right)}{3b^3}$$

$$= \frac{4a^2x^2}{3b^3\sqrt[4]{a+bx^4}} - \frac{2ax^6}{9b^2\sqrt[4]{a+bx^4}} + \frac{x^{10}}{9b\sqrt[4]{a+bx^4}} - \frac{\left(4a^2\sqrt[4]{1+\frac{bx^4}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx, x,\right.}{3b^3\sqrt[4]{a+bx^4}}$$

$$= \frac{4a^2x^2}{3b^3\sqrt[4]{a+bx^4}} - \frac{2ax^6}{9b^2\sqrt[4]{a+bx^4}} + \frac{x^{10}}{9b\sqrt[4]{a+bx^4}} - \frac{8a^{5/2}\sqrt[4]{1+\frac{bx^4}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{3b^{7/2}\sqrt[4]{a+bx^4}}$$

Mathematica [C] time = 0.05, size = 79, normalized size = 0.62

$$\frac{x^2\left(12a^2\sqrt[4]{\frac{bx^4}{a}+1}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^4}{a}\right) - 12a^2 - 2abx^4 + b^2x^8\right)}{9b^3\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13/(a + b*x^4)^(5/4),x]
```

```
[Out] (x^2*(-12*a^2 - 2*a*b*x^4 + b^2*x^8 + 12*a^2*(1 + (b*x^4)/a)^(1/4)*Hypergeo
metric2F1[1/4, 1/2, 3/2, -((b*x^4)/a)]))/(9*b^3*(a + b*x^4)^(1/4))
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{3}{4}}x^{13}}{b^2x^8+2\,abx^4+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(3/4)*x^13/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{\left(bx^4+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^4+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(x^13/(b*x^4 + a)^(5/4), x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{\left(b\,x^4+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(b*x^4+a)^(5/4),x)
```

```
[Out] int(x^13/(b*x^4+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{\left(bx^4+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(x^13/(b*x^4 + a)^(5/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{13}}{\left(b\,x^4+a\right)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(a + b*x^4)^(5/4),x)
```

```
[Out] int(x^13/(a + b*x^4)^(5/4), x)
```

sympy [C] time = 2.29, size = 27, normalized size = 0.21

$$\frac{x^{14}\,{}_2F_1\left(\left.\begin{matrix}\frac{5}{4},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{14a^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(b*x**4+a)**(5/4),x)
```

```
[Out] x**14*hyper((5/4, 7/2), (9/2,), b*x**4*exp_polar(I*pi)/a)/(14*a**(5/4))
```

3.1148 $\int \frac{x^9}{\left(a+bx^4\right)^{5/4}}\,dx$

Optimal. Leaf size=104

$$\frac{12a^{3/2}\sqrt[4]{\frac{bx^4}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{5b^{5/2}\sqrt[4]{a+bx^4}}-\frac{6ax^2}{5b^2\sqrt[4]{a+bx^4}}+\frac{x^6}{5b\sqrt[4]{a+bx^4}}$$

```
[Out] -6/5*a*x^2/b^2/(b*x^4+a)^(1/4)+1/5*x^6/b/(b*x^4+a)^(1/4)+12/5*a^(3/2)*(1+b*
x^4/a)^(1/4)*(cos(1/2*arctan(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(
x^2*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x^2*b^(1/2)/a^(1/2))),2^(1/2
))/b^(5/2)/(b*x^4+a)^(1/4)
```

Rubi [A] time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {275, 285, 197, 196}

$$\frac{12a^{3/2}\sqrt[4]{\frac{bx^4}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{5b^{5/2}\sqrt[4]{a+bx^4}}-\frac{6ax^2}{5b^2\sqrt[4]{a+bx^4}}+\frac{x^6}{5b\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x^4)^(5/4),x]
```

```
[Out] (-6*a*x^2)/(5*b^2*(a + b*x^4)^(1/4)) + x^6/(5*b*(a + b*x^4)^(1/4)) + (12*a^
(3/2)*(1 + (b*x^4)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/
(5*b^(5/2)*(a + b*x^4)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + (b*x^2)/a)^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 285

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(2*c*(
c*x)^(m - 1))/(b*(2*m - 3)*(a + b*x^2)^(1/4)), x] - Dist[(2*a*c^2*(m - 1))/
(b*(2*m - 3)), Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Rubi steps

$$\int \frac{x^9}{\left(a+bx^4\right)^{5/4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^2\right)^{5/4}}\,dx, x, x^2\right)$$

$$= \frac{x^6}{5b\sqrt[4]{a+bx^4}} - \frac{(3a)\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^2\right)^{5/4}}\,dx, x, x^2\right)}{5b}$$

$$= -\frac{6ax^2}{5b^2\sqrt[4]{a+bx^4}} + \frac{x^6}{5b\sqrt[4]{a+bx^4}} + \frac{\left(6a^2\right)\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{5/4}}\,dx, x, x^2\right)}{5b^2}$$

$$= -\frac{6ax^2}{5b^2\sqrt[4]{a+bx^4}} + \frac{x^6}{5b\sqrt[4]{a+bx^4}} + \frac{\left(6a\sqrt[4]{1+\frac{bx^4}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx, x, x^2\right)}{5b^2\sqrt[4]{a+bx^4}}$$

$$= -\frac{6ax^2}{5b^2\sqrt[4]{a+bx^4}} + \frac{x^6}{5b\sqrt[4]{a+bx^4}} + \frac{12a^{3/2}\sqrt[4]{1+\frac{bx^4}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{5b^{5/2}\sqrt[4]{a+bx^4}}$$

Mathematica [C] time = 0.02, size = 66, normalized size = 0.63

$$\frac{x^2\left(-6a\sqrt[4]{\frac{bx^4}{a}+1}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^4}{a}\right)+6a+bx^4\right)}{5b^2\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x^4)^(5/4),x]
```

```
[Out] (x^2*(6*a + b*x^4 - 6*a*(1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3
/2, -((b*x^4)/a)]))/(5*b^2*(a + b*x^4)^(1/4))
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{3}{4}}x^9}{b^2x^8+2\,abx^4+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(3/4)*x^9/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(bx^4+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(x^9/(b*x^4 + a)^(5/4), x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(b\,x^4+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^4+a)^(5/4),x)
[Out] int(x^9/(b*x^4+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(bx^4+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)^(5/4),x, algorithm="maxima")
[Out] integrate(x^9/(b*x^4 + a)^(5/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^9}{\left(b\,x^4+a\right)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + b*x^4)^(5/4),x)
[Out] int(x^9/(a + b*x^4)^(5/4), x)
```

sympy [C] time = 1.73, size = 27, normalized size = 0.26

$$\frac{x^{10}\,{}_2F_1\left(\left.\begin{matrix}\frac{5}{4}, \frac{5}{2} \\ \frac{7}{2}\end{matrix}\right| \frac{bx^4e^{i\pi}}{a}\right)}{10a^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**4+a)**(5/4),x)
[Out] x**10*hyper((5/4, 5/2), (7/2,), b*x**4*exp_polar(I*pi)/a)/(10*a**(5/4))
```

3.1149 $\int \frac{x^5}{\left(a+bx^4\right)^{5/4}}\,dx$

Optimal. Leaf size=77

$$\frac{x^2}{b\sqrt[4]{a+bx^4}}-\frac{2\sqrt{a}\,\sqrt[4]{\frac{bx^4}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^4}}$$

```
[Out] x^2/b/(b*x^4+a)^(1/4)-2*(1+b*x^4/a)^(1/4)*(cos(1/2*arctan(x^2*b^(1/2)/a^(1/
2)))^2)^(1/2)/cos(1/2*arctan(x^2*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan
(x^2*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)/b^(3/2)/(b*x^4+a)^(1/4)
```

Rubi [A] time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {275, 285, 197, 196}

$$\frac{x^2}{b\sqrt[4]{a+bx^4}}-\frac{2\sqrt{a}\,\sqrt[4]{\frac{bx^4}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^4)^(5/4),x]
```

```
[Out] x^2/(b*(a + b*x^4)^(1/4)) - (2*Sqrt[a]*(1 + (b*x^4)/a)^(1/4)*EllipticE[ArcT
an[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(b^(3/2)*(a + b*x^4)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + (b*x^2)/a)^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 285

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(2*c*(
c*x)^(m - 1))/(b*(2*m - 3)*(a + b*x^2)^(1/4)), x] - Dist[(2*a*c^2*(m - 1))/
(b*(2*m - 3)), Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Rubi steps

$$\int \frac{x^5}{\left(a+bx^4\right)^{5/4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^2\right)^{5/4}}\,dx, x, x^2\right)$$
$$= \frac{x^2}{b\sqrt[4]{a+bx^4}} - \frac{a\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{5/4}}\,dx, x, x^2\right)}{b}$$
$$= \frac{x^2}{b\sqrt[4]{a+bx^4}} - \frac{\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Subst}\left(\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx, x, x^2\right)}{b\sqrt[4]{a+bx^4}}$$
$$= \frac{x^2}{b\sqrt[4]{a+bx^4}} - \frac{2\sqrt{a}\,\sqrt[4]{1+\frac{bx^4}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^4}}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.70

$$\frac{x^2\left(\sqrt[4]{\frac{bx^4}{a}+1}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^4}{a}\right)-1\right)}{b\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^4)^(5/4),x]
```

```
[Out] (x^2*(-1 + (1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -((b*x^4)
/a)]))/(b*(a + b*x^4)^(1/4))
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{3}{4}}x^5}{b^2x^8+2\,abx^4+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(3/4)*x^5/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(bx^4+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(x^5/(b*x^4 + a)^(5/4), x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(b\,x^4+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^4+a)^(5/4),x)
[Out] int(x^5/(b*x^4+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(bx^4+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)^(5/4),x, algorithm="maxima")
[Out] integrate(x^5/(b*x^4 + a)^(5/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\left(b\,x^4+a\right)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^4)^(5/4),x)
[Out] int(x^5/(a + b*x^4)^(5/4), x)
```

sympy [C] time = 1.24, size = 27, normalized size = 0.35

$$\frac{x^6\,{}_2F_1\left(\left.\begin{matrix}\frac{5}{4}, \frac{3}{2} \\ \frac{5}{2}\end{matrix}\right| \frac{bx^4e^{i\pi}}{a}\right)}{6a^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**4+a)**(5/4),x)
[Out] x**6*hyper((5/4, 3/2), (5/2,), b*x**4*exp_polar(I*pi)/a)/(6*a**(5/4))
```

3.1150 $\int \frac{x}{\left(a+bx^4\right)^{5/4}}\,dx$

Optimal. Leaf size=57

$$\frac{\sqrt[4]{\frac{bx^4}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt[4]{a+bx^4}}$$

```
[Out] (1+b*x^4/a)^(1/4)*(cos(1/2*arctan(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*ar
ctan(x^2*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x^2*b^(1/2)/a^(1/2))),2
^(1/2))/(b*x^4+a)^(1/4)/a^(1/2)/b^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {275, 197, 196}

$$\frac{\sqrt[4]{\frac{bx^4}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^4)^(5/4),x]
```

```
[Out] ((1 + (b*x^4)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(Sqrt
[a]*Sqrt[b]*(a + b*x^4)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + (b*x^2)/a)^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^4\right)^{5/4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{5/4}}\,dx, x, x^2\right)$$
$$= \frac{\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Subst}\left(\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx, x, x^2\right)}{2a\sqrt[4]{a+bx^4}}$$
$$= \frac{\sqrt[4]{1+\frac{bx^4}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|\,2\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt[4]{a+bx^4}}$$

Mathematica [C] time = 0.02, size = 57, normalized size = 1.00

$$-\frac{x^2\left(\sqrt[4]{\frac{bx^4}{a}+1}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^4}{a}\right)-2\right)}{2a\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^4)^(5/4),x]
```

```
[Out] -1/2*(x^2*(-2 + (1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -((b
*x^4)/a)]))/(a*(a + b*x^4)^(1/4))
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{3}{4}}x}{b^2x^8+2\,abx^4+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(3/4)*x/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^4+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(x/(b*x^4 + a)^(5/4), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(b\,x^4+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^4+a)^(5/4),x)
```

```
[Out] int(x/(b*x^4+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^4+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*x^4 + a)^(5/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\left(b\,x^4+a\right)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^4)^(5/4),x)
```

```
[Out] int(x/(a + b*x^4)^(5/4), x)
```

sympy [C] time = 0.94, size = 27, normalized size = 0.47

$$\frac{x^2\,{}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{5}{4} \\ \frac{3}{2}\end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**4+a)**(5/4),x)
```

```
[Out] x**2*hyper((1/2, 5/4), (3/2,), b*x**4*exp_polar(I*pi)/a)/(2*a**(5/4))
```

3.1151 $\int \frac{1}{x^3\left(a+bx^4\right)^{5/4}}\,dx$

Optimal. Leaf size=82

$$-\frac{3\sqrt{b}\,\sqrt[4]{\frac{bx^4}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{2a^{3/2}\sqrt[4]{a+bx^4}}-\frac{1}{2ax^2\sqrt[4]{a+bx^4}}$$

```
[Out] -1/2/a/x^2/(b*x^4+a)^(1/4)-3/2*(1+b*x^4/a)^(1/4)*(cos(1/2*arctan(x^2*b^(1/2
)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x^2*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2
*arctan(x^2*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)/a^(3/2)/(b*x^4+a)^(1/4)
```

Rubi [A] time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {275, 286, 197, 196}

$$-\frac{3\sqrt{b}\,\sqrt[4]{\frac{bx^4}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{2a^{3/2}\sqrt[4]{a+bx^4}}-\frac{1}{2ax^2\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^4)^(5/4)),x]
```

```
[Out] -1/(2*a*x^2*(a + b*x^4)^(1/4)) - (3*Sqrt[b]*(1 + (b*x^4)/a)^(1/4)*EllipticE
[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(2*a^(3/2)*(a + b*x^4)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + (b*x^2)/a)^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 286

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Dist[(b*(2*m + 1))/(2*a*c^2*(
m + 1)), Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x]
 && PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^4\right)^{5/4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^{5/4}}\,dx, x, x^2\right)$$
$$= -\frac{1}{2ax^2\sqrt[4]{a+bx^4}} - \frac{(3b)\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{5/4}}\,dx, x, x^2\right)}{4a}$$
$$= -\frac{1}{2ax^2\sqrt[4]{a+bx^4}} - \frac{\left(3b\sqrt[4]{1+\frac{bx^4}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx, x, x^2\right)}{4a^2\sqrt[4]{a+bx^4}}$$
$$= -\frac{1}{2ax^2\sqrt[4]{a+bx^4}} - \frac{3\sqrt{b}\,\sqrt[4]{1+\frac{bx^4}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|\,2\right)}{2a^{3/2}\sqrt[4]{a+bx^4}}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.66

$$-\frac{\sqrt[4]{\frac{bx^4}{a}+1}\;{}_2F_1\left(-\frac{1}{2},\frac{5}{4};\frac{1}{2};-\frac{bx^4}{a}\right)}{2ax^2\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^4)^(5/4)),x]
```

```
[Out] -1/2*((1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[-1/2, 5/4, 1/2, -((b*x^4)/a)]
)/(a*x^2*(a + b*x^4)^(1/4))
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{3}{4}}}{b^2x^{11}+2\,abx^7+a^2x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(3/4)/(b^2*x^11 + 2*a*b*x^7 + a^2*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{5}{4}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(5/4)*x^3), x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{5}{4}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^4+a)^(5/4),x)
[Out] int(1/x^3/(b*x^4+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{5}{4}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)^(5/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(5/4)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\left(b\,x^4+a\right)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^4)^(5/4)),x)
[Out] int(1/(x^3*(a + b*x^4)^(5/4)), x)
```

sympy [C] time = 1.70, size = 31, normalized size = 0.38

$$-\frac{{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{1}{2}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**4+a)**(5/4),x)
[Out] -hyper((-1/2, 5/4), (1/2,), b*x**4*exp_polar(I*pi)/a)/(2*a**(5/4)*x**2)
```

3.1152 $\int \frac{1}{x^7\left(a+bx^4\right)^{5/4}}\,dx$

Optimal. Leaf size=104

$$\frac{7b^{3/2}\sqrt[4]{\frac{bx^4}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{4a^{5/2}\sqrt[4]{a+bx^4}}+\frac{7b}{12a^2x^2\sqrt[4]{a+bx^4}}-\frac{1}{6ax^6\sqrt[4]{a+bx^4}}$$

```
[Out] -1/6/a/x^6/(b*x^4+a)^(1/4)+7/12*b/a^2/x^2/(b*x^4+a)^(1/4)+7/4*b^(3/2)*(1+b*
x^4/a)^(1/4)*(cos(1/2*arctan(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(
x^2*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x^2*b^(1/2)/a^(1/2))),2^(1/2
))/a^(5/2)/(b*x^4+a)^(1/4)
```

Rubi [A] time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {275, 286, 197, 196}

$$\frac{7b^{3/2}\sqrt[4]{\frac{bx^4}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{4a^{5/2}\sqrt[4]{a+bx^4}}+\frac{7b}{12a^2x^2\sqrt[4]{a+bx^4}}-\frac{1}{6ax^6\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^4)^(5/4)),x]
```

```
[Out] -1/(6*a*x^6*(a + b*x^4)^(1/4)) + (7*b)/(12*a^2*x^2*(a + b*x^4)^(1/4)) + (7*
b^(3/2)*(1 + (b*x^4)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2]
)/(4*a^(5/2)*(a + b*x^4)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + (b*x^2)/a)^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 286

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Dist[(b*(2*m + 1))/(2*a*c^2*(
m + 1)), Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x]
 && PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^7\left(a+bx^4\right)^{5/4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^2\right)^{5/4}}\,dx, x, x^2\right)$$

$$= -\frac{1}{6ax^6\sqrt[4]{a+bx^4}} - \frac{(7b)\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^{5/4}}\,dx, x, x^2\right)}{12a}$$

$$= -\frac{1}{6ax^6\sqrt[4]{a+bx^4}} + \frac{7b}{12a^2x^2\sqrt[4]{a+bx^4}} + \frac{\left(7b^2\right)\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{5/4}}\,dx, x, x^2\right)}{8a^2}$$

$$= -\frac{1}{6ax^6\sqrt[4]{a+bx^4}} + \frac{7b}{12a^2x^2\sqrt[4]{a+bx^4}} + \frac{\left(7b^2\sqrt[4]{1+\frac{bx^4}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx, x, x^2\right)}{8a^3\sqrt[4]{a+bx^4}}$$

$$= -\frac{1}{6ax^6\sqrt[4]{a+bx^4}} + \frac{7b}{12a^2x^2\sqrt[4]{a+bx^4}} + \frac{7b^{3/2}\sqrt[4]{1+\frac{bx^4}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|\,2\right)}{4a^{5/2}\sqrt[4]{a+bx^4}}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.52

$$-\frac{\sqrt[4]{\frac{bx^4}{a}+1}\;{}_2F_1\left(-\frac{3}{2},\frac{5}{4};-\frac{1}{2};-\frac{bx^4}{a}\right)}{6ax^6\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^4)^(5/4)),x]
```

```
[Out] -1/6*((1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[-3/2, 5/4, -1/2, -((b*x^4)/a)
])/(a*x^6*(a + b*x^4)^(1/4))
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{3}{4}}}{b^2x^{15}+2\,abx^{11}+a^2x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(3/4)/(b^2*x^15 + 2*a*b*x^11 + a^2*x^7), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{5}{4}}x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(5/4)*x^7), x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{5}{4}}\,x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^4+a)^(5/4),x)
[Out] int(1/x^7/(b*x^4+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{5}{4}}x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)^(5/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(5/4)*x^7), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^7\left(b\,x^4+a\right)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^4)^(5/4)),x)
[Out] int(1/(x^7*(a + b*x^4)^(5/4)), x)
```

sympy [C] time = 1.61, size = 32, normalized size = 0.31

$$-\frac{{}_2F_1\left(\left.\begin{matrix}-\frac{3}{2}, \frac{5}{4} \\ -\frac{1}{2}\end{matrix}\right| \frac{bx^4e^{i\pi}}{a}\right)}{6a^{\frac{5}{4}}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**4+a)**(5/4),x)
[Out] -hyper((-3/2, 5/4), (-1/2,), b*x**4*exp_polar(I*pi)/a)/(6*a**(5/4)*x**6)
```

3.1153 $\int \frac{1}{x^{11}\left(a+bx^4\right)^{5/4}}\,dx$

Optimal. Leaf size=128

$$-\frac{77b^{5/2}\sqrt[4]{\frac{bx^4}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{40a^{7/2}\sqrt[4]{a+bx^4}}-\frac{77b^2}{120a^3x^2\sqrt[4]{a+bx^4}}+\frac{11b}{60a^2x^6\sqrt[4]{a+bx^4}}-\frac{1}{10ax^{10}\sqrt[4]{a+bx^4}}$$

```
[Out] -1/10/a/x^10/(b*x^4+a)^(1/4)+11/60*b/a^2/x^6/(b*x^4+a)^(1/4)-77/120*b^2/a^3
/x^2/(b*x^4+a)^(1/4)-77/40*b^(5/2)*(1+b*x^4/a)^(1/4)*(cos(1/2*arctan(x^2*b^
(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x^2*b^(1/2)/a^(1/2)))*EllipticE(sin
(1/2*arctan(x^2*b^(1/2)/a^(1/2))),2^(1/2))/a^(7/2)/(b*x^4+a)^(1/4)
```

Rubi [A] time = 0.08, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {275, 286, 197, 196}

$$-\frac{77b^2}{120a^3x^2\sqrt[4]{a+bx^4}}-\frac{77b^{5/2}\sqrt[4]{\frac{bx^4}{a}+1}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{40a^{7/2}\sqrt[4]{a+bx^4}}+\frac{11b}{60a^2x^6\sqrt[4]{a+bx^4}}-\frac{1}{10ax^{10}\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^11*(a + b*x^4)^(5/4)),x]
```

```
[Out] -1/(10*a*x^10*(a + b*x^4)^(1/4)) + (11*b)/(60*a^2*x^6*(a + b*x^4)^(1/4)) -
(77*b^2)/(120*a^3*x^2*(a + b*x^4)^(1/4)) - (77*b^(5/2)*(1 + (b*x^4)/a)^(1/4
)*EllipticE[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(40*a^(7/2)*(a + b*x^4)^(1
/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + (b*x^2)/a)^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 286

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Dist[(b*(2*m + 1))/(2*a*c^2*(
m + 1)), Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x]
 && PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{11}\left(a+bx^4\right)^{5/4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^6\left(a+bx^2\right)^{5/4}}\,dx, x, x^2\right)$$

$$= -\frac{1}{10ax^{10}\sqrt[4]{a+bx^4}} - \frac{(11b)\,\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^2\right)^{5/4}}\,dx, x, x^2\right)}{20a}$$

$$= -\frac{1}{10ax^{10}\sqrt[4]{a+bx^4}} + \frac{11b}{60a^2x^6\sqrt[4]{a+bx^4}} + \frac{\left(77b^2\right)\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^{5/4}}\,dx, x, x^2\right)}{120a^2}$$

$$= -\frac{1}{10ax^{10}\sqrt[4]{a+bx^4}} + \frac{11b}{60a^2x^6\sqrt[4]{a+bx^4}} - \frac{77b^2}{120a^3x^2\sqrt[4]{a+bx^4}} - \frac{\left(77b^3\right)\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)}\right.}{80a^3}$$

$$= -\frac{1}{10ax^{10}\sqrt[4]{a+bx^4}} + \frac{11b}{60a^2x^6\sqrt[4]{a+bx^4}} - \frac{77b^2}{120a^3x^2\sqrt[4]{a+bx^4}} - \frac{\left(77b^3\sqrt[4]{1+\frac{bx^4}{a}}\right)\text{Subst}}{80a^4\sqrt[4]{a}}$$

$$= -\frac{1}{10ax^{10}\sqrt[4]{a+bx^4}} + \frac{11b}{60a^2x^6\sqrt[4]{a+bx^4}} - \frac{77b^2}{120a^3x^2\sqrt[4]{a+bx^4}} - \frac{77b^{5/2}\sqrt[4]{1+\frac{bx^4}{a}}\,E\left(\frac{1}{2}\,\text{ta}\right.}{40a^{7/2}\sqrt[4]{a+}}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.42

$$-\frac{\sqrt[4]{\frac{bx^4}{a}+1}\;{}_2F_1\left(-\frac{5}{2},\frac{5}{4};-\frac{3}{2};-\frac{bx^4}{a}\right)}{10ax^{10}\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^11*(a + b*x^4)^(5/4)),x]
```

```
[Out] -1/10*((1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[-5/2, 5/4, -3/2, -((b*x^4)/a
)])/(a*x^10*(a + b*x^4)^(1/4))
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{3}{4}}}{b^2x^{19}+2\,abx^{15}+a^2x^{11}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(3/4)/(b^2*x^19 + 2*a*b*x^15 + a^2*x^11), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{5}{4}}x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(b*x^4+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(5/4)*x^11), x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{5}{4}}\,x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^11/(b*x^4+a)^(5/4),x)
```

```
[Out] int(1/x^11/(b*x^4+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{5}{4}}x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(5/4)*x^11), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{11}\left(b\,x^4+a\right)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^11*(a + b*x^4)^(5/4)),x)
```

```
[Out] int(1/(x^11*(a + b*x^4)^(5/4)), x)
```

sympy [C] time = 3.15, size = 32, normalized size = 0.25

$$-\frac{{}_2F_1\left(\left.\begin{matrix}-\frac{5}{2}, \frac{5}{4} \\ -\frac{3}{2}\end{matrix}\right| \frac{bx^4e^{i\pi}}{a}\right)}{10a^{\frac{5}{4}}x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**11/(b*x**4+a)**(5/4),x)
```

```
[Out] -hyper((-5/2, 5/4), (-3/2,), b*x**4*exp_polar(I*pi)/a)/(10*a**(5/4)*x**10)
```

3.1154 $\int \frac{x^{12}}{\left(a+bx^4\right)^{5/4}}\,dx$

Optimal. Leaf size=123

$$\frac{45a^2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{64b^{13/4}}+\frac{45a^2\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{64b^{13/4}}-\frac{45ax\left(a+bx^4\right)^{3/4}}{32b^3}+\frac{9x^5\left(a+bx^4\right)^{3/4}}{8b^2}-\frac{x^9}{b\sqrt[4]{a+bx^4}}$$

```
[Out] -x^9/b/(b*x^4+a)^(1/4)-45/32*a*x*(b*x^4+a)^(3/4)/b^3+9/8*x^5*(b*x^4+a)^(3/4
)/b^2+45/64*a^2*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(13/4)+45/64*a^2*arctan
h(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(13/4)
```

Rubi [A] time = 0.05, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {288, 321, 240, 212, 206, 203}

$$\frac{45a^2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{64b^{13/4}}+\frac{45a^2\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{64b^{13/4}}+\frac{9x^5\left(a+bx^4\right)^{3/4}}{8b^2}-\frac{45ax\left(a+bx^4\right)^{3/4}}{32b^3}-\frac{x^9}{b\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^12/(a + b*x^4)^(5/4),x]
```

```
[Out] -(x^9/(b*(a + b*x^4)^(1/4))) - (45*a*x*(a + b*x^4)^(3/4))/(32*b^3) + (9*x^5
*(a + b*x^4)^(3/4))/(8*b^2) + (45*a^2*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)]
)/(64*b^(13/4)) + (45*a^2*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(64*b^(13
/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
```

```
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^{12}}{\left(a+bx^4\right)^{5/4}}\,dx = -\frac{x^9}{b\sqrt[4]{a+bx^4}} + \frac{9\int \frac{x^8}{\sqrt[4]{a+bx^4}}\,dx}{b}$$

$$= -\frac{x^9}{b\sqrt[4]{a+bx^4}} + \frac{9x^5\left(a+bx^4\right)^{3/4}}{8b^2} - \frac{(45a)\int \frac{x^4}{\sqrt[4]{a+bx^4}}\,dx}{8b^2}$$

$$= -\frac{x^9}{b\sqrt[4]{a+bx^4}} - \frac{45ax\left(a+bx^4\right)^{3/4}}{32b^3} + \frac{9x^5\left(a+bx^4\right)^{3/4}}{8b^2} + \frac{\left(45a^2\right)\int \frac{1}{\sqrt[4]{a+bx^4}}\,dx}{32b^3}$$

$$= -\frac{x^9}{b\sqrt[4]{a+bx^4}} - \frac{45ax\left(a+bx^4\right)^{3/4}}{32b^3} + \frac{9x^5\left(a+bx^4\right)^{3/4}}{8b^2} + \frac{\left(45a^2\right)\text{Subst}\left(\int \frac{1}{1-bx^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{32b^3}$$

$$= -\frac{x^9}{b\sqrt[4]{a+bx^4}} - \frac{45ax\left(a+bx^4\right)^{3/4}}{32b^3} + \frac{9x^5\left(a+bx^4\right)^{3/4}}{8b^2} + \frac{\left(45a^2\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{b}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{64b^3}$$

$$= -\frac{x^9}{b\sqrt[4]{a+bx^4}} - \frac{45ax\left(a+bx^4\right)^{3/4}}{32b^3} + \frac{9x^5\left(a+bx^4\right)^{3/4}}{8b^2} + \frac{45a^2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{64b^{13/4}} + \frac{45a^2\tanh^{-1}}{64b^{13}}$$

Mathematica [C] time = 0.03, size = 67, normalized size = 0.54

$$\frac{x^5\left(9a\sqrt[4]{\frac{bx^4}{a}+1}\;{}_2F_1\left(\frac{5}{4},\frac{5}{4};\frac{9}{4};-\frac{bx^4}{a}\right)-9a+4bx^4\right)}{32b^2\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12/(a + b*x^4)^(5/4),x]
```

```
[Out] (x^5*(-9*a + 4*b*x^4 + 9*a*(1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[5/4, 5/4
, 9/4, -((b*x^4)/a)]))/(32*b^2*(a + b*x^4)^(1/4))
```

fricas [B] time = 0.71, size = 281, normalized size = 2.28

$$180\left(b^4x^4+ab^3\right)\left(\frac{a^8}{b^{13}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(bx^4+a\right)^{\frac{1}{4}}a^6b^3\left(\frac{a^8}{b^{13}}\right)^{\frac{1}{4}}-b^3x\sqrt{\frac{a^8b^7x^2\sqrt{\frac{a^8}{b^{13}}}+\sqrt{bx^4+a}\,a^{12}}{x^2}}\left(\frac{a^8}{b^{13}}\right)^{\frac{1}{4}}}{a^8x}\right)+45\left(b^4x^4+ab^3\right)\left(\frac{a^8}{b^{13}}\right)^{\frac{1}{4}}\log\left($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] 1/128*(180*(b^4*x^4 + a*b^3)*(a^8/b^13)^(1/4)*arctan(-((b*x^4 + a)^(1/4)*a^
6*b^3*(a^8/b^13)^(1/4) - b^3*x*sqrt((a^8*b^7*x^2*sqrt(a^8/b^13) + sqrt(b*x^
4 + a)*a^12)/x^2)*(a^8/b^13)^(1/4))/(a^8*x)) + 45*(b^4*x^4 + a*b^3)*(a^8/b^
13)^(1/4)*log(91125*(b^10*x*(a^8/b^13)^(3/4) + (b*x^4 + a)^(1/4)*a^6)/x) -
45*(b^4*x^4 + a*b^3)*(a^8/b^13)^(1/4)*log(-91125*(b^10*x*(a^8/b^13)^(3/4) -
 (b*x^4 + a)^(1/4)*a^6)/x) + 4*(4*b^2*x^9 - 9*a*b*x^5 - 45*a^2*x)*(b*x^4 +
a)^(3/4))/(b^4*x^4 + a*b^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{12}}{\left(bx^4+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^4+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(x^12/(b*x^4 + a)^(5/4), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{x^{12}}{\left(b\,x^4+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(b*x^4+a)^(5/4),x)
```

```
[Out] int(x^12/(b*x^4+a)^(5/4),x)
```

maxima [A] time = 3.19, size = 172, normalized size = 1.40

$$-\frac{32\,a^2b^2-\frac{81\left(bx^4+a\right)a^2b}{x^4}+\frac{45\left(bx^4+a\right)^2a^2}{x^8}}{32\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}b^5}{x}-\frac{2\left(bx^4+a\right)^{\frac{5}{4}}b^4}{x^5}+\frac{\left(bx^4+a\right)^{\frac{9}{4}}b^3}{x^9}\right)}-\frac{45\,a^2\left(\frac{2\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)}{128\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

```
[Out] -1/32*(32*a^2*b^2 - 81*(b*x^4 + a)*a^2*b/x^4 + 45*(b*x^4 + a)^2*a^2/x^8)/((
b*x^4 + a)^(1/4)*b^5/x - 2*(b*x^4 + a)^(5/4)*b^4/x^5 + (b*x^4 + a)^(9/4)*b^
3/x^9) - 45/128*a^2*(2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) + log(
-(b^(1/4) - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/4))/
b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{12}}{\left(b\,x^4+a\right)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(a + b*x^4)^(5/4),x)
```

```
[Out] int(x^12/(a + b*x^4)^(5/4), x)
```

sympy [C] time = 2.76, size = 37, normalized size = 0.30

$$\frac{x^{13}\Gamma\left(\frac{13}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4}, \frac{13}{4} \\ \frac{17}{4}\end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{4}}\Gamma\left(\frac{17}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12/(b*x**4+a)**(5/4),x)
```

```
[Out] x**13*gamma(13/4)*hyper((5/4, 13/4), (17/4,), b*x**4*exp_polar(I*pi)/a)/(4*
a**(5/4)*gamma(17/4))
```

3.1155 $\int \frac{x^8}{\left(a+bx^4\right)^{5/4}}\,dx$

Optimal. Leaf size=97

$$-\frac{5a\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8b^{9/4}}-\frac{5a\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8b^{9/4}}+\frac{5x\left(a+bx^4\right)^{3/4}}{4b^2}-\frac{x^5}{b\sqrt[4]{a+bx^4}}$$

```
[Out] -x^5/b/(b*x^4+a)^(1/4)+5/4*x*(b*x^4+a)^(3/4)/b^2-5/8*a*arctan(b^(1/4)*x/(b*
x^4+a)^(1/4))/b^(9/4)-5/8*a*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(9/4)
```

Rubi [A] time = 0.03, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {288, 321, 240, 212, 206, 203}

$$\frac{5x\left(a+bx^4\right)^{3/4}}{4b^2}-\frac{5a\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8b^{9/4}}-\frac{5a\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8b^{9/4}}-\frac{x^5}{b\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^4)^(5/4),x]
```

```
[Out] -(x^5/(b*(a + b*x^4)^(1/4))) + (5*x*(a + b*x^4)^(3/4))/(4*b^2) - (5*a*ArcTa
n[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(8*b^(9/4)) - (5*a*ArcTanh[(b^(1/4)*x)/(a
 + b*x^4)^(1/4)])/(8*b^(9/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^8}{\left(a+bx^4\right)^{5/4}}\,dx = -\frac{x^5}{b\sqrt[4]{a+bx^4}} + \frac{5\int \frac{x^4}{\sqrt[4]{a+bx^4}}\,dx}{b}$$

$$= -\frac{x^5}{b\sqrt[4]{a+bx^4}} + \frac{5x\left(a+bx^4\right)^{3/4}}{4b^2} - \frac{(5a)\int \frac{1}{\sqrt[4]{a+bx^4}}\,dx}{4b^2}$$

$$= -\frac{x^5}{b\sqrt[4]{a+bx^4}} + \frac{5x\left(a+bx^4\right)^{3/4}}{4b^2} - \frac{(5a)\,\text{Subst}\left(\int \frac{1}{1-bx^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{4b^2}$$

$$= -\frac{x^5}{b\sqrt[4]{a+bx^4}} + \frac{5x\left(a+bx^4\right)^{3/4}}{4b^2} - \frac{(5a)\,\text{Subst}\left(\int \frac{1}{1-\sqrt{b}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{8b^2} - \frac{(5a)\,\text{Subst}\left(\int \frac{1}{1+\sqrt{b}}\right.}{8b^2}$$

$$= -\frac{x^5}{b\sqrt[4]{a+bx^4}} + \frac{5x\left(a+bx^4\right)^{3/4}}{4b^2} - \frac{5a\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8b^{9/4}} - \frac{5a\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8b^{9/4}}$$

Mathematica [C] time = 0.02, size = 60, normalized size = 0.62

$$\frac{x^5 - x^5\sqrt[4]{\frac{bx^4}{a}+1}\;{}_2F_1\left(\frac{5}{4},\frac{5}{4};\frac{9}{4};-\frac{bx^4}{a}\right)}{4b\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^4)^(5/4),x]
```

```
[Out] (x^5 - x^5*(1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[5/4, 5/4, 9/4, -((b*x^4)
/a)])/(4*b*(a + b*x^4)^(1/4))
```

fricas [B] time = 0.87, size = 269, normalized size = 2.77

$$-\frac{20\left(b^3x^4+ab^2\right)\left(\frac{a^4}{b^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(bx^4+a\right)^{\frac{1}{4}}a^3b^2\left(\frac{a^4}{b^9}\right)^{\frac{1}{4}} - b^2x\sqrt{\frac{a^4b^5x^2\sqrt{\frac{a^4}{b^9}}+\sqrt{bx^4+a}\,a^6}{x^2}}\left(\frac{a^4}{b^9}\right)^{\frac{1}{4}}}{a^4x}\right) + 5\left(b^3x^4+ab^2\right)\left(\frac{a^4}{b^9}\right)^{\frac{1}{4}}\log\left(\frac{125\left(b^7\right.\right.}{16\left(b^3x^4+ab^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] -1/16*(20*(b^3*x^4 + a*b^2)*(a^4/b^9)^(1/4)*arctan(-((b*x^4 + a)^(1/4)*a^3*
b^2*(a^4/b^9)^(1/4) - b^2*x*sqrt((a^4*b^5*x^2*sqrt(a^4/b^9) + sqrt(b*x^4 +
```

```
a)*a^6)/x^2)*(a^4/b^9)^(1/4))/(a^4*x)) + 5*(b^3*x^4 + a*b^2)*(a^4/b^9)^(1/4
)*log(125*(b^7*x*(a^4/b^9)^(3/4) + (b*x^4 + a)^(1/4)*a^3)/x) - 5*(b^3*x^4 +
 a*b^2)*(a^4/b^9)^(1/4)*log(-125*(b^7*x*(a^4/b^9)^(3/4) - (b*x^4 + a)^(1/4)
*a^3)/x) - 4*(b*x^5 + 5*a*x)*(b*x^4 + a)^(3/4))/(b^3*x^4 + a*b^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(bx^4+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(x^8/(b*x^4 + a)^(5/4), x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(b\,x^4+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^4+a)^(5/4),x)
```

```
[Out] int(x^8/(b*x^4+a)^(5/4),x)
```

maxima [A] time = 2.88, size = 130, normalized size = 1.34

$$\frac{4\,ab-\frac{5\left(bx^4+a\right)a}{x^4}}{4\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}b^3}{x}-\frac{\left(bx^4+a\right)^{\frac{5}{4}}b^2}{x^5}\right)}+\frac{5\,a\left(\frac{2\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)}{16\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

```
[Out] 1/4*(4*a*b - 5*(b*x^4 + a)*a/x^4)/((b*x^4 + a)^(1/4)*b^3/x - (b*x^4 + a)^(5
/4)*b^2/x^5) + 5/16*a*(2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) + lo
g(-(b^(1/4) - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/4)
)/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^8}{\left(b\,x^4+a\right)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^4)^(5/4),x)
```

```
[Out] int(x^8/(a + b*x^4)^(5/4), x)
```

sympy [C] time = 1.57, size = 37, normalized size = 0.38

$$\frac{x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{4}, \frac{9}{4} \\ \frac{13}{4}\end{matrix}\right| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{4}}\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**4+a)**(5/4),x)
```

```
[Out] x**9*gamma(9/4)*hyper((5/4, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**
(5/4)*gamma(13/4))
```

3.1156 $\int \frac{x^4}{\left(a+bx^4\right)^{5/4}}\,dx$

Optimal. Leaf size=74

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2b^{5/4}}-\frac{x}{b\sqrt[4]{a+bx^4}}$$

```
[Out] -x/b/(b*x^4+a)^(1/4)+1/2*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(5/4)+1/2*arct
anh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(5/4)
```

Rubi [A] time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {288, 240, 212, 206, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2b^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2b^{5/4}}-\frac{x}{b\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^4)^(5/4),x]
```

```
[Out] -(x/(b*(a + b*x^4)^(1/4))) + ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(5/
4)) + ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(5/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\left(a+bx^4\right)^{5/4}}\,dx &= -\frac{x}{b\sqrt[4]{a+bx^4}} + \frac{\int \frac{1}{\sqrt[4]{a+bx^4}}\,dx}{b} \\ &= -\frac{x}{b\sqrt[4]{a+bx^4}} + \frac{\text{Subst}\left(\int \frac{1}{1-bx^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{b} \\ &= -\frac{x}{b\sqrt[4]{a+bx^4}} + \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{b}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{b}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2b} \\ &= -\frac{x}{b\sqrt[4]{a+bx^4}} + \frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2b^{5/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2b^{5/4}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.73

$$\frac{x^5\sqrt[4]{\frac{bx^4}{a}+1}\;{}_2F_1\left(\frac{5}{4},\frac{5}{4};\frac{9}{4};-\frac{bx^4}{a}\right)}{5a\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^4)^(5/4),x]
```

```
[Out] (x^5*(1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[5/4, 5/4, 9/4, -((b*x^4)/a)])/
(5*a*(a + b*x^4)^(1/4))
```

fricas [B] time = 0.70, size = 193, normalized size = 2.61

$$\frac{4\left(b^2x^4+ab\right)\frac{1}{b^5}^{\frac{1}{4}}\arctan\left(\frac{bx\sqrt{\frac{b^3\sqrt{\frac{1}{b^5}}x^2+\sqrt{bx^4+a}}{x^2}}\,\frac{1}{b^5}^{\frac{1}{4}}-\left(bx^4+a\right)^{\frac{1}{4}}b\,\frac{1}{b^5}^{\frac{1}{4}}}{x}\right)+\left(b^2x^4+ab\right)\frac{1}{b^5}^{\frac{1}{4}}\log\left(\frac{b^4\frac{1}{b^5}^{\frac{3}{4}}x+\left(bx^4+a\right)^{\frac{1}{4}}}{x}\right)-\left(b^2x^4+a\right.}{4\left(b^2x^4+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] 1/4*(4*(b^2*x^4 + a*b)*(b^(-5))^(1/4)*arctan((b*x*sqrt((b^3*sqrt(b^(-5))*x^
2 + sqrt(b*x^4 + a))/x^2)*(b^(-5))^(1/4) - (b*x^4 + a)^(1/4)*b*(b^(-5))^(1/
4))/x) + (b^2*x^4 + a*b)*(b^(-5))^(1/4)*log((b^4*(b^(-5))^(3/4)*x + (b*x^4
+ a)^(1/4))/x) - (b^2*x^4 + a*b)*(b^(-5))^(1/4)*log(-(b^4*(b^(-5))^(3/4)*x
- (b*x^4 + a)^(1/4))/x) - 4*(b*x^4 + a)^(3/4)*x)/(b^2*x^4 + a*b)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^4+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(b*x^4 + a)^(5/4), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(b\,x^4+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^4+a)^(5/4),x)
```

```
[Out] int(x^4/(b*x^4+a)^(5/4),x)
```

maxima [A] time = 3.20, size = 88, normalized size = 1.19

$$-\frac{\frac{2\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}}{4\,b}-\frac{x}{\left(bx^4+a\right)^{\frac{1}{4}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

```
[Out] -1/4*(2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) + log(-(b^(1/4) - (b*
x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/4))/b - x/((b*x^4 +
 a)^(1/4)*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\left(b\,x^4+a\right)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^4)^(5/4),x)
```

```
[Out] int(x^4/(a + b*x^4)^(5/4), x)
```

sympy [C] time = 1.77, size = 37, normalized size = 0.50

$$\frac{x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{4}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**4+a)**(5/4),x)
```

```
[Out] x**5*gamma(5/4)*hyper((5/4, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
5/4)*gamma(9/4))
```

3.1157 $\int \frac{1}{\left(a+bx^4\right)^{5/4}}\,dx$

Optimal. Leaf size=16

$$\frac{x}{a\sqrt[4]{a+bx^4}}$$

```
[Out] x/a/(b*x^4+a)^(1/4)
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {191}

$$\frac{x}{a\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(-5/4),x]
```

```
[Out] x/(a*(a + b*x^4)^(1/4))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)^{5/4}}\,dx = \frac{x}{a\sqrt[4]{a+bx^4}}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{x}{a\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(-5/4),x]
```

```
[Out] x/(a*(a + b*x^4)^(1/4))
```

fricas [A] time = 0.82, size = 23, normalized size = 1.44

$$\frac{\left(bx^4+a\right)^{\frac{3}{4}}x}{abx^4+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] (b*x^4 + a)^(3/4)*x/(a*b*x^4 + a^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(5/4),x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(-5/4), x)
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$\frac{x}{\left(b\,x^4+a\right)^{\frac{1}{4}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(5/4),x)
[Out] x/a/(b*x^4+a)^(1/4)
```

maxima [A] time = 1.36, size = 14, normalized size = 0.88

$$\frac{x}{\left(bx^4+a\right)^{\frac{1}{4}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(5/4),x, algorithm="maxima")
[Out] x/((b*x^4 + a)^(1/4)*a)
```

mupad [B] time = 1.02, size = 14, normalized size = 0.88

$$\frac{x}{a\left(b\,x^4+a\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^4)^(5/4),x)
[Out] x/(a*(a + b*x^4)^(1/4))
```

sympy [B] time = 1.08, size = 29, normalized size = 1.81

$$\frac{x\Gamma\left(\frac{1}{4}\right)}{4a^{\frac{5}{4}}\sqrt[4]{1+\frac{bx^4}{a}}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(5/4),x)
[Out] x*gamma(1/4)/(4*a**(5/4)*(1 + b*x**4/a)**(1/4)*gamma(5/4))
```

3.1158 $\int \frac{1}{x^4\left(a+bx^4\right)^{5/4}}\,dx$

Optimal. Leaf size=42

$$-\frac{4bx}{3a^2\sqrt[4]{a+bx^4}}-\frac{1}{3ax^3\sqrt[4]{a+bx^4}}$$

```
[Out] -1/3/a/x^3/(b*x^4+a)^(1/4)-4/3*b*x/a^2/(b*x^4+a)^(1/4)
```

Rubi [A] time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 191}

$$-\frac{4bx}{3a^2\sqrt[4]{a+bx^4}}-\frac{1}{3ax^3\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^4)^(5/4)),x]
```

```
[Out] -1/(3*a*x^3*(a + b*x^4)^(1/4)) - (4*b*x)/(3*a^2*(a + b*x^4)^(1/4))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(a+bx^4\right)^{5/4}}\,dx &= -\frac{1}{3ax^3\sqrt[4]{a+bx^4}}-\frac{(4b)\int \frac{1}{\left(a+bx^4\right)^{5/4}}\,dx}{3a}\\ &= -\frac{1}{3ax^3\sqrt[4]{a+bx^4}}-\frac{4bx}{3a^2\sqrt[4]{a+bx^4}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.69

$$-\frac{a+4bx^4}{3a^2x^3\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^4)^(5/4)),x]
```

```
[Out] -1/3*(a + 4*b*x^4)/(a^2*x^3*(a + b*x^4)^(1/4))
```

fricas [A] time = 0.84, size = 37, normalized size = 0.88

$$-\frac{\left(4\,bx^4+a\right)\left(bx^4+a\right)^{\frac{3}{4}}}{3\left(a^2bx^7+a^3x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] -1/3*(4*b*x^4 + a)*(b*x^4 + a)^(3/4)/(a^2*b*x^7 + a^3*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{5}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(5/4)*x^4), x)
```

maple [A] time = 0.00, size = 26, normalized size = 0.62

$$-\frac{4b\,x^4+a}{3\left(b\,x^4+a\right)^{\frac{1}{4}}a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^4+a)^(5/4),x)
```

```
[Out] -1/3*(4*b*x^4+a)/x^3/(b*x^4+a)^(1/4)/a^2
```

maxima [A] time = 1.43, size = 34, normalized size = 0.81

$$-\frac{bx}{\left(bx^4+a\right)^{\frac{1}{4}}a^2}-\frac{\left(bx^4+a\right)^{\frac{3}{4}}}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

```
[Out] -b*x/((b*x^4 + a)^(1/4)*a^2) - 1/3*(b*x^4 + a)^(3/4)/(a^2*x^3)
```

mupad [B] time = 1.16, size = 25, normalized size = 0.60

$$-\frac{4\,b\,x^4+a}{3\,a^2\,x^3\left(b\,x^4+a\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^4)^(5/4)),x)
```

```
[Out] -(a + 4*b*x^4)/(3*a^2*x^3*(a + b*x^4)^(1/4))
```

sympy [A] time = 1.67, size = 68, normalized size = 1.62

$$\frac{\Gamma\left(-\frac{3}{4}\right)}{16a\sqrt[4]{b}\,x^4\sqrt[4]{\frac{a}{bx^4}+1}\,\Gamma\left(\frac{5}{4}\right)}+\frac{b^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{4a^2\sqrt[4]{\frac{a}{bx^4}+1}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**4+a)**(5/4),x)
```

```
[Out] gamma(-3/4)/(16*a*b**(1/4)*x**4*(a/(b*x**4) + 1)**(1/4)*gamma(5/4)) + b**(3
/4)*gamma(-3/4)/(4*a**2*(a/(b*x**4) + 1)**(1/4)*gamma(5/4))
```

3.1159 $\int \frac{1}{x^8\left(a+bx^4\right)^{5/4}}\,dx$

Optimal. Leaf size=66

$$\frac{32b^2x}{21a^3\sqrt[4]{a+bx^4}}+\frac{8b}{21a^2x^3\sqrt[4]{a+bx^4}}-\frac{1}{7ax^7\sqrt[4]{a+bx^4}}$$

```
[Out] -1/7/a/x^7/(b*x^4+a)^(1/4)+8/21*b/a^2/x^3/(b*x^4+a)^(1/4)+32/21*b^2*x/a^3/(
b*x^4+a)^(1/4)
```

Rubi [A] time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 191}

$$\frac{32b^2x}{21a^3\sqrt[4]{a+bx^4}}+\frac{8b}{21a^2x^3\sqrt[4]{a+bx^4}}-\frac{1}{7ax^7\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(a + b*x^4)^(5/4)),x]
```

```
[Out] -1/(7*a*x^7*(a + b*x^4)^(1/4)) + (8*b)/(21*a^2*x^3*(a + b*x^4)^(1/4)) + (32
*b^2*x)/(21*a^3*(a + b*x^4)^(1/4))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^8\left(a+bx^4\right)^{5/4}}\,dx &= -\frac{1}{7ax^7\sqrt[4]{a+bx^4}}-\frac{(8b)\int \frac{1}{x^4\left(a+bx^4\right)^{5/4}}\,dx}{7a}\\ &= -\frac{1}{7ax^7\sqrt[4]{a+bx^4}}+\frac{8b}{21a^2x^3\sqrt[4]{a+bx^4}}+\frac{\left(32b^2\right)\int \frac{1}{\left(a+bx^4\right)^{5/4}}\,dx}{21a^2}\\ &= -\frac{1}{7ax^7\sqrt[4]{a+bx^4}}+\frac{8b}{21a^2x^3\sqrt[4]{a+bx^4}}+\frac{32b^2x}{21a^3\sqrt[4]{a+bx^4}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.64

$$-\frac{3a^2-8abx^4-32b^2x^8}{21a^3x^7\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(a + b*x^4)^(5/4)),x]
```

```
[Out] -1/21*(3*a^2 - 8*a*b*x^4 - 32*b^2*x^8)/(a^3*x^7*(a + b*x^4)^(1/4))
```

fricas [A] time = 0.89, size = 50, normalized size = 0.76

$$\frac{\left(32\,b^2x^8+8\,abx^4-3\,a^2\right)\left(bx^4+a\right)^{\frac{3}{4}}}{21\left(a^3bx^{11}+a^4x^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] 1/21*(32*b^2*x^8 + 8*a*b*x^4 - 3*a^2)*(b*x^4 + a)^(3/4)/(a^3*b*x^11 + a^4*x
^7)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{5}{4}}x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^4+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(5/4)*x^8), x)
```

maple [A] time = 0.00, size = 39, normalized size = 0.59

$$-\frac{-32b^2x^8-8ab\,x^4+3a^2}{21\left(b\,x^4+a\right)^{\frac{1}{4}}a^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(b*x^4+a)^(5/4),x)
```

```
[Out] -1/21*(-32*b^2*x^8-8*a*b*x^4+3*a^2)/x^7/(b*x^4+a)^(1/4)/a^3
```

maxima [A] time = 1.59, size = 53, normalized size = 0.80

$$\frac{b^2x}{\left(bx^4+a\right)^{\frac{1}{4}}a^3}+\frac{\frac{14\left(bx^4+a\right)^{\frac{3}{4}}b}{x^3}-\frac{3\left(bx^4+a\right)^{\frac{7}{4}}}{x^7}}{21\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

```
[Out] b^2*x/((b*x^4 + a)^(1/4)*a^3) + 1/21*(14*(b*x^4 + a)^(3/4)*b/x^3 - 3*(b*x^4
 + a)^(7/4)/x^7)/a^3
```

mupad [B] time = 1.31, size = 70, normalized size = 1.06

$$-\frac{32\left(b\,x^4+a\right)^2-56\,a\,\left(b\,x^4+a\right)+21\,a^2}{\left(\frac{21\,a^4\,x^3}{b}-\frac{21\,a^3\,x^3\left(b\,x^4+a\right)}{b}\right)\left(b\,x^4+a\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(a + b*x^4)^(5/4)),x)
```

```
[Out] -(32*(a + b*x^4)^2 - 56*a*(a + b*x^4) + 21*a^2)/(((21*a^4*x^3)/b - (21*a^3*
x^3*(a + b*x^4))/b)*(a + b*x^4)^(1/4))
```

sympy [**B**] time = 2.04, size = 323, normalized size = 4.89

$$-\frac{3a^3b^{\frac{19}{4}}\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{64a^5b^4x^4\Gamma\left(\frac{5}{4}\right)+128a^4b^5x^8\Gamma\left(\frac{5}{4}\right)+64a^3b^6x^{12}\Gamma\left(\frac{5}{4}\right)}+\frac{5a^2b^{\frac{23}{4}}x^4\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{64a^5b^4x^4\Gamma\left(\frac{5}{4}\right)+128a^4b^5x^8\Gamma\left(\frac{5}{4}\right)+64a^3b^6x^{12}\Gamma\left(\frac{5}{4}\right)}+\frac{}{64a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(b*x**4+a)**(5/4),x)
```

```
[Out] -3*a**3*b**(19/4)*(a/(b*x**4) + 1)**(3/4)*gamma(-7/4)/(64*a**5*b**4*x**4*ga
mma(5/4) + 128*a**4*b**5*x**8*gamma(5/4) + 64*a**3*b**6*x**12*gamma(5/4)) +
 5*a**2*b**(23/4)*x**4*(a/(b*x**4) + 1)**(3/4)*gamma(-7/4)/(64*a**5*b**4*x*
*4*gamma(5/4) + 128*a**4*b**5*x**8*gamma(5/4) + 64*a**3*b**6*x**12*gamma(5/
4)) + 40*a*b**(27/4)*x**8*(a/(b*x**4) + 1)**(3/4)*gamma(-7/4)/(64*a**5*b**4
*x**4*gamma(5/4) + 128*a**4*b**5*x**8*gamma(5/4) + 64*a**3*b**6*x**12*gamma
(5/4)) + 32*b**(31/4)*x**12*(a/(b*x**4) + 1)**(3/4)*gamma(-7/4)/(64*a**5*b*
*4*x**4*gamma(5/4) + 128*a**4*b**5*x**8*gamma(5/4) + 64*a**3*b**6*x**12*gam
ma(5/4))
```

3.1160 $\int \frac{1}{x^{12}\left(a+bx^4\right)^{5/4}}\,dx$

Optimal. Leaf size=90

$$-\frac{128b^3x}{77a^4\sqrt[4]{a+bx^4}}-\frac{32b^2}{77a^3x^3\sqrt[4]{a+bx^4}}+\frac{12b}{77a^2x^7\sqrt[4]{a+bx^4}}-\frac{1}{11ax^{11}\sqrt[4]{a+bx^4}}$$

```
[Out] -1/11/a/x^11/(b*x^4+a)^(1/4)+12/77*b/a^2/x^7/(b*x^4+a)^(1/4)-32/77*b^2/a^3/
x^3/(b*x^4+a)^(1/4)-128/77*b^3*x/a^4/(b*x^4+a)^(1/4)
```

Rubi [A] time = 0.03, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 191}

$$-\frac{128b^3x}{77a^4\sqrt[4]{a+bx^4}}-\frac{32b^2}{77a^3x^3\sqrt[4]{a+bx^4}}+\frac{12b}{77a^2x^7\sqrt[4]{a+bx^4}}-\frac{1}{11ax^{11}\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^12*(a + b*x^4)^(5/4)),x]
```

```
[Out] -1/(11*a*x^11*(a + b*x^4)^(1/4)) + (12*b)/(77*a^2*x^7*(a + b*x^4)^(1/4)) -
(32*b^2)/(77*a^3*x^3*(a + b*x^4)^(1/4)) - (128*b^3*x)/(77*a^4*(a + b*x^4)^(
1/4))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{12}\left(a+bx^4\right)^{5/4}}\,dx &= -\frac{1}{11ax^{11}\sqrt[4]{a+bx^4}}-\frac{(12b)\int \frac{1}{x^8\left(a+bx^4\right)^{5/4}}\,dx}{11a}\\
&= -\frac{1}{11ax^{11}\sqrt[4]{a+bx^4}}+\frac{12b}{77a^2x^7\sqrt[4]{a+bx^4}}+\frac{\left(96b^2\right)\int \frac{1}{x^4\left(a+bx^4\right)^{5/4}}\,dx}{77a^2}\\
&= -\frac{1}{11ax^{11}\sqrt[4]{a+bx^4}}+\frac{12b}{77a^2x^7\sqrt[4]{a+bx^4}}-\frac{32b^2}{77a^3x^3\sqrt[4]{a+bx^4}}-\frac{\left(128b^3\right)\int \frac{1}{\left(a+bx^4\right)^{5/4}}\,dx}{77a^3}\\
&= -\frac{1}{11ax^{11}\sqrt[4]{a+bx^4}}+\frac{12b}{77a^2x^7\sqrt[4]{a+bx^4}}-\frac{32b^2}{77a^3x^3\sqrt[4]{a+bx^4}}-\frac{128b^3x}{77a^4\sqrt[4]{a+bx^4}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 53, normalized size = 0.59

$$-\frac{7a^3-12a^2bx^4+32ab^2x^8+128b^3x^{12}}{77a^4x^{11}\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^12*(a + b*x^4)^(5/4)),x]
```

```
[Out] -1/77*(7*a^3 - 12*a^2*b*x^4 + 32*a*b^2*x^8 + 128*b^3*x^12)/(a^4*x^11*(a + b
*x^4)^(1/4))
```

fricas [A] time = 0.66, size = 61, normalized size = 0.68

$$-\frac{\left(128\,b^3x^{12}+32\,ab^2x^8-12\,a^2bx^4+7\,a^3\right)\left(bx^4+a\right)^{\frac{3}{4}}}{77\left(a^4bx^{15}+a^5x^{11}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^12/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] -1/77*(128*b^3*x^12 + 32*a*b^2*x^8 - 12*a^2*b*x^4 + 7*a^3)*(b*x^4 + a)^(3/4
)/(a^4*b*x^15 + a^5*x^11)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{5}{4}}x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^12/(b*x^4+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(5/4)*x^12), x)
```

maple [A] time = 0.01, size = 50, normalized size = 0.56

$$-\frac{128b^3x^{12}+32a\,b^2x^8-12a^2b\,x^4+7a^3}{77\left(b\,x^4+a\right)^{\frac{1}{4}}a^4x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^12/(b*x^4+a)^(5/4),x)
```

```
[Out] -1/77*(128*b^3*x^12+32*a*b^2*x^8-12*a^2*b*x^4+7*a^3)/x^11/(b*x^4+a)^(1/4)/a
^4
```

maxima [A] time = 1.29, size = 71, normalized size = 0.79

$$-\frac{b^3x}{\left(bx^4+a\right)^{\frac{1}{4}}a^4}-\frac{\frac{77\left(bx^4+a\right)^{\frac{3}{4}}b^2}{x^3}-\frac{33\left(bx^4+a\right)^{\frac{7}{4}}b}{x^7}+\frac{7\left(bx^4+a\right)^{\frac{11}{4}}}{x^{11}}}{77\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^12/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

```
[Out] -b^3*x/((b*x^4 + a)^(1/4)*a^4) - 1/77*(77*(b*x^4 + a)^(3/4)*b^2/x^3 - 33*(b
*x^4 + a)^(7/4)*b/x^7 + 7*(b*x^4 + a)^(11/4)/x^11)/a^4
```

mupad [B] time = 1.46, size = 74, normalized size = 0.82

$$\frac{19\,b\left(b\,x^4+a\right)^{3/4}}{77\,a^3\,x^7}-\frac{b^3\,x}{a^4\left(b\,x^4+a\right)^{1/4}}-\frac{\left(b\,x^4+a\right)^{3/4}}{11\,a^2\,x^{11}}-\frac{51\,b^2\left(b\,x^4+a\right)^{3/4}}{77\,a^4\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^12*(a + b*x^4)^(5/4)),x)
```

```
[Out] (19*b*(a + b*x^4)^(3/4))/(77*a^3*x^7) - (b^3*x)/(a^4*(a + b*x^4)^(1/4)) - (
a + b*x^4)^(3/4)/(11*a^2*x^11) - (51*b^2*(a + b*x^4)^(3/4))/(77*a^4*x^3)
```

sympy [B] time = 4.09, size = 592, normalized size = 6.58

$$\frac{21a^5b^{\frac{39}{4}}\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{11}{4}\right)}{256a^7b^9x^8\Gamma\left(\frac{5}{4}\right)+768a^6b^{10}x^{12}\Gamma\left(\frac{5}{4}\right)+768a^5b^{11}x^{16}\Gamma\left(\frac{5}{4}\right)+256a^4b^{12}x^{20}\Gamma\left(\frac{5}{4}\right)}+\frac{6a^4b^{\frac{43}{4}}}{256a^7b^9x^8\Gamma\left(\frac{5}{4}\right)+768a^6b^{10}x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**12/(b*x**4+a)**(5/4),x)
```

```
[Out] 21*a**5*b**(39/4)*(a/(b*x**4) + 1)**(3/4)*gamma(-11/4)/(256*a**7*b**9*x**8*
gamma(5/4) + 768*a**6*b**10*x**12*gamma(5/4) + 768*a**5*b**11*x**16*gamma(5
/4) + 256*a**4*b**12*x**20*gamma(5/4)) + 6*a**4*b**(43/4)*x**4*(a/(b*x**4)
+ 1)**(3/4)*gamma(-11/4)/(256*a**7*b**9*x**8*gamma(5/4) + 768*a**6*b**10*x*
*12*gamma(5/4) + 768*a**5*b**11*x**16*gamma(5/4) + 256*a**4*b**12*x**20*gam
ma(5/4)) + 45*a**3*b**(47/4)*x**8*(a/(b*x**4) + 1)**(3/4)*gamma(-11/4)/(256
*a**7*b**9*x**8*gamma(5/4) + 768*a**6*b**10*x**12*gamma(5/4) + 768*a**5*b**
11*x**16*gamma(5/4) + 256*a**4*b**12*x**20*gamma(5/4)) + 540*a**2*b**(51/4)
*x**12*(a/(b*x**4) + 1)**(3/4)*gamma(-11/4)/(256*a**7*b**9*x**8*gamma(5/4)
+ 768*a**6*b**10*x**12*gamma(5/4) + 768*a**5*b**11*x**16*gamma(5/4) + 256*a
**4*b**12*x**20*gamma(5/4)) + 864*a*b**(55/4)*x**16*(a/(b*x**4) + 1)**(3/4)
*gamma(-11/4)/(256*a**7*b**9*x**8*gamma(5/4) + 768*a**6*b**10*x**12*gamma(5
/4) + 768*a**5*b**11*x**16*gamma(5/4) + 256*a**4*b**12*x**20*gamma(5/4)) +
384*b**(59/4)*x**20*(a/(b*x**4) + 1)**(3/4)*gamma(-11/4)/(256*a**7*b**9*x**
8*gamma(5/4) + 768*a**6*b**10*x**12*gamma(5/4) + 768*a**5*b**11*x**16*gamma
(5/4) + 256*a**4*b**12*x**20*gamma(5/4))
```

3.1161 $\int \frac{1}{x^{16}\left(a+bx^4\right)^{5/4}}\,dx$

Optimal. Leaf size=114

$$\frac{2048b^4x}{1155a^5\sqrt[4]{a+bx^4}}+\frac{512b^3}{1155a^4x^3\sqrt[4]{a+bx^4}}-\frac{64b^2}{385a^3x^7\sqrt[4]{a+bx^4}}+\frac{16b}{165a^2x^{11}\sqrt[4]{a+bx^4}}-\frac{1}{15ax^{15}\sqrt[4]{a+bx^4}}$$

```
[Out] -1/15/a/x^15/(b*x^4+a)^(1/4)+16/165*b/a^2/x^11/(b*x^4+a)^(1/4)-64/385*b^2/a
^3/x^7/(b*x^4+a)^(1/4)+512/1155*b^3/a^4/x^3/(b*x^4+a)^(1/4)+2048/1155*b^4*x
/a^5/(b*x^4+a)^(1/4)
```

Rubi [A] time = 0.04, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 191}

$$\frac{2048b^4x}{1155a^5\sqrt[4]{a+bx^4}}+\frac{512b^3}{1155a^4x^3\sqrt[4]{a+bx^4}}-\frac{64b^2}{385a^3x^7\sqrt[4]{a+bx^4}}+\frac{16b}{165a^2x^{11}\sqrt[4]{a+bx^4}}-\frac{1}{15ax^{15}\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^16*(a + b*x^4)^(5/4)),x]
```

```
[Out] -1/(15*a*x^15*(a + b*x^4)^(1/4)) + (16*b)/(165*a^2*x^11*(a + b*x^4)^(1/4))
- (64*b^2)/(385*a^3*x^7*(a + b*x^4)^(1/4)) + (512*b^3)/(1155*a^4*x^3*(a + b
*x^4)^(1/4)) + (2048*b^4*x)/(1155*a^5*(a + b*x^4)^(1/4))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{16}\left(a+bx^4\right)^{5/4}}\,dx &= -\frac{1}{15ax^{15}\sqrt[4]{a+bx^4}}-\frac{(16b)\int \frac{1}{x^{12}\left(a+bx^4\right)^{5/4}}\,dx}{15a}\\
&= -\frac{1}{15ax^{15}\sqrt[4]{a+bx^4}}+\frac{16b}{165a^2x^{11}\sqrt[4]{a+bx^4}}+\frac{\left(64b^2\right)\int \frac{1}{x^8\left(a+bx^4\right)^{5/4}}\,dx}{55a^2}\\
&= -\frac{1}{15ax^{15}\sqrt[4]{a+bx^4}}+\frac{16b}{165a^2x^{11}\sqrt[4]{a+bx^4}}-\frac{64b^2}{385a^3x^7\sqrt[4]{a+bx^4}}-\frac{\left(512b^3\right)\int \frac{1}{x^4\left(a+bx^4\right)^{5/4}}\,dx}{385a^3}\\
&= -\frac{1}{15ax^{15}\sqrt[4]{a+bx^4}}+\frac{16b}{165a^2x^{11}\sqrt[4]{a+bx^4}}-\frac{64b^2}{385a^3x^7\sqrt[4]{a+bx^4}}+\frac{512b^3}{1155a^4x^3\sqrt[4]{a+bx^4}}+\frac{\left(2\right.}{}\\
&= -\frac{1}{15ax^{15}\sqrt[4]{a+bx^4}}+\frac{16b}{165a^2x^{11}\sqrt[4]{a+bx^4}}-\frac{64b^2}{385a^3x^7\sqrt[4]{a+bx^4}}+\frac{512b^3}{1155a^4x^3\sqrt[4]{a+bx^4}}+\frac{}{1}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 64, normalized size = 0.56

$$-\frac{77a^4 - 112a^3bx^4 + 192a^2b^2x^8 - 512ab^3x^{12} - 2048b^4x^{16}}{1155a^5x^{15}\sqrt[4]{a + bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^16*(a + b*x^4)^(5/4)),x]
```

```
[Out] -1/1155*(77*a^4 - 112*a^3*b*x^4 + 192*a^2*b^2*x^8 - 512*a*b^3*x^12 - 2048*b
^4*x^16)/(a^5*x^15*(a + b*x^4)^(1/4))
```

fricas [A] time = 0.75, size = 72, normalized size = 0.63

$$\frac{\left(2048\,b^4x^{16} + 512\,ab^3x^{12} - 192\,a^2b^2x^8 + 112\,a^3bx^4 - 77\,a^4\right)\left(bx^4 + a\right)^{\frac{3}{4}}}{1155\left(a^5bx^{19} + a^6x^{15}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^16/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] 1/1155*(2048*b^4*x^16 + 512*a*b^3*x^12 - 192*a^2*b^2*x^8 + 112*a^3*b*x^4 -
77*a^4)*(b*x^4 + a)^(3/4)/(a^5*b*x^19 + a^6*x^15)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4 + a\right)^{\frac{5}{4}}x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^16/(b*x^4+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(5/4)*x^16), x)
```

maple [A] time = 0.01, size = 61, normalized size = 0.54

$$-\frac{-2048x^{16}b^4 - 512a\,x^{12}b^3 + 192a^2x^8b^2 - 112a^3x^4b + 77a^4}{1155\left(b\,x^4 + a\right)^{\frac{1}{4}}a^5x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^16/(b*x^4+a)^(5/4),x)
```

```
[Out] -1/1155*(-2048*b^4*x^16-512*a*b^3*x^12+192*a^2*b^2*x^8-112*a^3*b*x^4+77*a^4
)/x^15/(b*x^4+a)^(1/4)/a^5
```

maxima [A] time = 1.36, size = 87, normalized size = 0.76

$$\frac{b^4x}{\left(bx^4 + a\right)^{\frac{1}{4}}a^5} + \frac{\frac{1540\left(bx^4+a\right)^{\frac{3}{4}}b^3}{x^3} - \frac{990\left(bx^4+a\right)^{\frac{7}{4}}b^2}{x^7} + \frac{420\left(bx^4+a\right)^{\frac{11}{4}}b}{x^{11}} - \frac{77\left(bx^4+a\right)^{\frac{15}{4}}}{x^{15}}}{1155\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^16/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

```
[Out] b^4*x/((b*x^4 + a)^(1/4)*a^5) + 1/1155*(1540*(b*x^4 + a)^(3/4)*b^3/x^3 - 99
0*(b*x^4 + a)^(7/4)*b^2/x^7 + 420*(b*x^4 + a)^(11/4)*b/x^11 - 77*(b*x^4 + a
)^(15/4)/x^15)/a^5
```

mupad [B] time = 1.65, size = 93, normalized size = 0.82

$$\frac{b^4 x}{a^5 \left(b x^4+a\right)^{1/4}}-\frac{\left(b x^4+a\right)^{3/4}}{15 a^2 x^{15}}+\frac{9 b \left(b x^4+a\right)^{3/4}}{55 a^3 x^{11}}+\frac{893 b^3 \left(b x^4+a\right)^{3/4}}{1155 a^5 x^3}-\frac{127 b^2 \left(b x^4+a\right)^{3/4}}{385 a^4 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^16*(a + b*x^4)^(5/4)),x)
```

```
[Out] (b^4*x)/(a^5*(a + b*x^4)^(1/4)) - (a + b*x^4)^(3/4)/(15*a^2*x^15) + (9*b*(a
 + b*x^4)^(3/4))/(55*a^3*x^11) + (893*b^3*(a + b*x^4)^(3/4))/(1155*a^5*x^3)
 - (127*b^2*(a + b*x^4)^(3/4))/(385*a^4*x^7)
```

sympy [B] time = 6.41, size = 928, normalized size = 8.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**16/(b*x**4+a)**(5/4),x)
```

```
[Out] -231*a**7*b**(67/4)*(a/(b*x**4) + 1)**(3/4)*gamma(-15/4)/(1024*a**9*b**16*x
**12*gamma(5/4) + 4096*a**8*b**17*x**16*gamma(5/4) + 6144*a**7*b**18*x**20*
gamma(5/4) + 4096*a**6*b**19*x**24*gamma(5/4) + 1024*a**5*b**20*x**28*gamma
(5/4)) - 357*a**6*b**(71/4)*x**4*(a/(b*x**4) + 1)**(3/4)*gamma(-15/4)/(1024
*a**9*b**16*x**12*gamma(5/4) + 4096*a**8*b**17*x**16*gamma(5/4) + 6144*a**7
*b**18*x**20*gamma(5/4) + 4096*a**6*b**19*x**24*gamma(5/4) + 1024*a**5*b**2
0*x**28*gamma(5/4)) - 261*a**5*b**(75/4)*x**8*(a/(b*x**4) + 1)**(3/4)*gamma
(-15/4)/(1024*a**9*b**16*x**12*gamma(5/4) + 4096*a**8*b**17*x**16*gamma(5/4
) + 6144*a**7*b**18*x**20*gamma(5/4) + 4096*a**6*b**19*x**24*gamma(5/4) + 1
024*a**5*b**20*x**28*gamma(5/4)) + 585*a**4*b**(79/4)*x**12*(a/(b*x**4) + 1
)**(3/4)*gamma(-15/4)/(1024*a**9*b**16*x**12*gamma(5/4) + 4096*a**8*b**17*x
**16*gamma(5/4) + 6144*a**7*b**18*x**20*gamma(5/4) + 4096*a**6*b**19*x**24*
gamma(5/4) + 1024*a**5*b**20*x**28*gamma(5/4)) + 9360*a**3*b**(83/4)*x**16*
(a/(b*x**4) + 1)**(3/4)*gamma(-15/4)/(1024*a**9*b**16*x**12*gamma(5/4) + 40
96*a**8*b**17*x**16*gamma(5/4) + 6144*a**7*b**18*x**20*gamma(5/4) + 4096*a*
*6*b**19*x**24*gamma(5/4) + 1024*a**5*b**20*x**28*gamma(5/4)) + 22464*a**2*
b**(87/4)*x**20*(a/(b*x**4) + 1)**(3/4)*gamma(-15/4)/(1024*a**9*b**16*x**12
*gamma(5/4) + 4096*a**8*b**17*x**16*gamma(5/4) + 6144*a**7*b**18*x**20*gamm
a(5/4) + 4096*a**6*b**19*x**24*gamma(5/4) + 1024*a**5*b**20*x**28*gamma(5/4
)) + 19968*a*b**(91/4)*x**24*(a/(b*x**4) + 1)**(3/4)*gamma(-15/4)/(1024*a**
9*b**16*x**12*gamma(5/4) + 4096*a**8*b**17*x**16*gamma(5/4) + 6144*a**7*b**
18*x**20*gamma(5/4) + 4096*a**6*b**19*x**24*gamma(5/4) + 1024*a**5*b**20*x*
*28*gamma(5/4)) + 6144*b**(95/4)*x**28*(a/(b*x**4) + 1)**(3/4)*gamma(-15/4)
/(1024*a**9*b**16*x**12*gamma(5/4) + 4096*a**8*b**17*x**16*gamma(5/4) + 614
4*a**7*b**18*x**20*gamma(5/4) + 4096*a**6*b**19*x**24*gamma(5/4) + 1024*a**
5*b**20*x**28*gamma(5/4))
```

3.1162 $\int \frac{x^{14}}{\left(a+bx^4\right)^{5/4}}\,dx$

Optimal. Leaf size=129

$$\frac{77a^{5/2}x\sqrt[4]{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{40b^{7/2}\sqrt[4]{a+bx^4}}+\frac{77a^2x^3}{120b^3\sqrt[4]{a+bx^4}}-\frac{11ax^7}{60b^2\sqrt[4]{a+bx^4}}+\frac{x^{11}}{10b\sqrt[4]{a+bx^4}}$$

```
[Out] 77/120*a^2*x^3/b^3/(b*x^4+a)^(1/4)-11/60*a*x^7/b^2/(b*x^4+a)^(1/4)+1/10*x^1
1/b/(b*x^4+a)^(1/4)+77/40*a^(5/2)*(1+a/b/x^4)^(1/4)*x*(cos(1/2*arccot(x^2*b
^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))*EllipticE(si
n(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))/b^(7/2)/(b*x^4+a)^(1/4)
```

Rubi [A] time = 0.06, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {282, 281, 335, 275, 196}

$$\frac{77a^2x^3}{120b^3\sqrt[4]{a+bx^4}}+\frac{77a^{5/2}x\sqrt[4]{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{40b^{7/2}\sqrt[4]{a+bx^4}}-\frac{11ax^7}{60b^2\sqrt[4]{a+bx^4}}+\frac{x^{11}}{10b\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/(a + b*x^4)^(5/4),x]
```

```
[Out] (77*a^2*x^3)/(120*b^3*(a + b*x^4)^(1/4)) - (11*a*x^7)/(60*b^2*(a + b*x^4)^(
1/4)) + x^11/(10*b*(a + b*x^4)^(1/4)) + (77*a^(5/2)*(1 + a/(b*x^4))^(1/4)*x
*EllipticE[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(40*b^(7/2)*(a + b*x^4)^(1/
4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 281

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Dist[(x*(1 + a/(b*x^4)
)^(1/4))/(b*(a + b*x^4)^(1/4)), Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x] /
; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 282

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x^(m - 3)/(b*(
m - 4)*(a + b*x^4)^(1/4)), x] - Dist[(a*(m - 3))/(b*(m - 4)), Int[x^(m - 4)
/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}, x] && PosQ[b/a] && IGtQ[(m - 2)
/4, 0]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
```

egerQ[m]

Rubi steps

$$\int \frac{x^{14}}{\left(a+bx^4\right)^{5/4}}\,dx = \frac{x^{11}}{10b\sqrt[4]{a+bx^4}} - \frac{(11a)\int \frac{x^{10}}{\left(a+bx^4\right)^{5/4}}\,dx}{10b}$$

$$= -\frac{11ax^7}{60b^2\sqrt[4]{a+bx^4}} + \frac{x^{11}}{10b\sqrt[4]{a+bx^4}} + \frac{\left(77a^2\right)\int \frac{x^6}{\left(a+bx^4\right)^{5/4}}\,dx}{60b^2}$$

$$= \frac{77a^2x^3}{120b^3\sqrt[4]{a+bx^4}} - \frac{11ax^7}{60b^2\sqrt[4]{a+bx^4}} + \frac{x^{11}}{10b\sqrt[4]{a+bx^4}} - \frac{\left(77a^3\right)\int \frac{x^2}{\left(a+bx^4\right)^{5/4}}\,dx}{40b^3}$$

$$= \frac{77a^2x^3}{120b^3\sqrt[4]{a+bx^4}} - \frac{11ax^7}{60b^2\sqrt[4]{a+bx^4}} + \frac{x^{11}}{10b\sqrt[4]{a+bx^4}} - \frac{\left(77a^3\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{5/4}x^3}\,dx}{40b^4\sqrt[4]{a+bx^4}}$$

$$= \frac{77a^2x^3}{120b^3\sqrt[4]{a+bx^4}} - \frac{11ax^7}{60b^2\sqrt[4]{a+bx^4}} + \frac{x^{11}}{10b\sqrt[4]{a+bx^4}} + \frac{\left(77a^3\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{5/4}}\,d\right.}{40b^4\sqrt[4]{a+bx^4}}$$

$$= \frac{77a^2x^3}{120b^3\sqrt[4]{a+bx^4}} - \frac{11ax^7}{60b^2\sqrt[4]{a+bx^4}} + \frac{x^{11}}{10b\sqrt[4]{a+bx^4}} + \frac{\left(77a^3\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,d\right.}{80b^4\sqrt[4]{a+bx^4}}$$

$$= \frac{77a^2x^3}{120b^3\sqrt[4]{a+bx^4}} - \frac{11ax^7}{60b^2\sqrt[4]{a+bx^4}} + \frac{x^{11}}{10b\sqrt[4]{a+bx^4}} + \frac{77a^{5/2}\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{40b^{7/2}\sqrt[4]{a+bx^4}}$$

Mathematica [C] time = 0.04, size = 80, normalized size = 0.62

$$\frac{x^3\left(-77a^2\sqrt[4]{\frac{bx^4}{a}+1}\;{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{7}{4};-\frac{bx^4}{a}\right)+77a^2-22abx^4+12b^2x^8\right)}{120b^3\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/(a + b*x^4)^(5/4),x]
```

```
[Out] (x^3*(77*a^2 - 22*a*b*x^4 + 12*b^2*x^8 - 77*a^2*(1 + (b*x^4)/a)^(1/4)*Hyper
geometric2F1[3/4, 5/4, 7/4, -((b*x^4)/a)]))/(120*b^3*(a + b*x^4)^(1/4))
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{3}{4}}x^{14}}{b^2x^8+2\,abx^4+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(3/4)*x^14/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(bx^4+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^4+a)^(5/4),x, algorithm="giac")
[Out] integrate(x^14/(b*x^4 + a)^(5/4), x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(b\,x^4+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(b*x^4+a)^(5/4),x)
[Out] int(x^14/(b*x^4+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(bx^4+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^4+a)^(5/4),x, algorithm="maxima")
[Out] integrate(x^14/(b*x^4 + a)^(5/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{14}}{\left(b\,x^4+a\right)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(a + b*x^4)^(5/4),x)
[Out] int(x^14/(a + b*x^4)^(5/4), x)
```

sympy [C] time = 2.27, size = 37, normalized size = 0.29

$$\frac{x^{15}\Gamma\left(\frac{15}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{4}, \frac{15}{4} \\ \frac{19}{4}\end{matrix}\right| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{4}}\Gamma\left(\frac{19}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(b*x**4+a)**(5/4),x)
[Out] x**15*gamma(15/4)*hyper((5/4, 15/4), (19/4,), b*x**4*exp_polar(I*pi)/a)/(4*
a**(5/4)*gamma(19/4))
```

3.1163 $\int \frac{x^{10}}{\left(a+bx^4\right)^{5/4}}\,dx$

Optimal. Leaf size=105

$$-\frac{7a^{3/2}x\sqrt[4]{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{4b^{5/2}\sqrt[4]{a+bx^4}}-\frac{7ax^3}{12b^2\sqrt[4]{a+bx^4}}+\frac{x^7}{6b\sqrt[4]{a+bx^4}}$$

```
[Out] -7/12*a*x^3/b^2/(b*x^4+a)^(1/4)+1/6*x^7/b/(b*x^4+a)^(1/4)-7/4*a^(3/2)*(1+a/
b/x^4)^(1/4)*x*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcco
t(x^2*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1
/2))/b^(5/2)/(b*x^4+a)^(1/4)
```

Rubi [A] time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {282, 281, 335, 275, 196}

$$-\frac{7a^{3/2}x\sqrt[4]{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{4b^{5/2}\sqrt[4]{a+bx^4}}-\frac{7ax^3}{12b^2\sqrt[4]{a+bx^4}}+\frac{x^7}{6b\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x^4)^(5/4),x]
```

```
[Out] (-7*a*x^3)/(12*b^2*(a + b*x^4)^(1/4)) + x^7/(6*b*(a + b*x^4)^(1/4)) - (7*a^
(3/2)*(1 + a/(b*x^4))^(1/4)*x*EllipticE[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2]
)/(4*b^(5/2)*(a + b*x^4)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 281

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Dist[(x*(1 + a/(b*x^4)
)^(1/4))/(b*(a + b*x^4)^(1/4)), Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x] /
; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 282

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x^(m - 3)/(b*(
m - 4)*(a + b*x^4)^(1/4)), x] - Dist[(a*(m - 3))/(b*(m - 4)), Int[x^(m - 4)
/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}, x] && PosQ[b/a] && IGtQ[(m - 2)
/4, 0]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
```

egerQ[m]

Rubi steps

$$\int \frac{x^{10}}{\left(a+bx^4\right)^{5/4}}\,dx = \frac{x^7}{6b\sqrt[4]{a+bx^4}} - \frac{(7a)\int \frac{x^6}{\left(a+bx^4\right)^{5/4}}\,dx}{6b}$$

$$= -\frac{7ax^3}{12b^2\sqrt[4]{a+bx^4}} + \frac{x^7}{6b\sqrt[4]{a+bx^4}} + \frac{\left(7a^2\right)\int \frac{x^2}{\left(a+bx^4\right)^{5/4}}\,dx}{4b^2}$$

$$= -\frac{7ax^3}{12b^2\sqrt[4]{a+bx^4}} + \frac{x^7}{6b\sqrt[4]{a+bx^4}} + \frac{\left(7a^2\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{5/4}x^3}\,dx}{4b^3\sqrt[4]{a+bx^4}}$$

$$= -\frac{7ax^3}{12b^2\sqrt[4]{a+bx^4}} + \frac{x^7}{6b\sqrt[4]{a+bx^4}} - \frac{\left(7a^2\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{4b^3\sqrt[4]{a+bx^4}}$$

$$= -\frac{7ax^3}{12b^2\sqrt[4]{a+bx^4}} + \frac{x^7}{6b\sqrt[4]{a+bx^4}} - \frac{\left(7a^2\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx, x, \frac{1}{x^2}\right)}{8b^3\sqrt[4]{a+bx^4}}$$

$$= -\frac{7ax^3}{12b^2\sqrt[4]{a+bx^4}} + \frac{x^7}{6b\sqrt[4]{a+bx^4}} - \frac{7a^{3/2}\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{4b^{5/2}\sqrt[4]{a+bx^4}}$$

Mathematica [C] time = 0.02, size = 67, normalized size = 0.64

$$\frac{x^3\left(7a\sqrt[4]{\frac{bx^4}{a}+1}\;{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{7}{4};-\frac{bx^4}{a}\right)-7a+2bx^4\right)}{12b^2\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x^4)^(5/4),x]
```

```
[Out] (x^3*(-7*a + 2*b*x^4 + 7*a*(1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[3/4, 5/4
, 7/4, -((b*x^4)/a)]))/(12*b^2*(a + b*x^4)^(1/4))
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{3}{4}}x^{10}}{b^2x^8+2\,abx^4+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(3/4)*x^10/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\left(bx^4+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^4+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(x^10/(b*x^4 + a)^(5/4), x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\left(b\,x^4+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x^4+a)^(5/4),x)
```

```
[Out] int(x^10/(b*x^4+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\left(bx^4+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(x^10/(b*x^4 + a)^(5/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{10}}{\left(b\,x^4+a\right)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a + b*x^4)^(5/4),x)
```

```
[Out] int(x^10/(a + b*x^4)^(5/4), x)
```

sympy [C] time = 1.69, size = 37, normalized size = 0.35

$$\frac{x^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{4}, \frac{11}{4} \\ \frac{15}{4}\end{matrix}\right| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{4}}\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b*x**4+a)**(5/4),x)
```

```
[Out] x**11*gamma(11/4)*hyper((5/4, 11/4), (15/4,), b*x**4*exp_polar(I*pi)/a)/(4*
a**(5/4)*gamma(15/4))
```

3.1164 $\int \frac{x^6}{\left(a+bx^4\right)^{5/4}}\,dx$

Optimal. Leaf size=83

$$\frac{3\sqrt{a}\,x\sqrt[4]{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{2b^{3/2}\sqrt[4]{a+bx^4}}+\frac{x^3}{2b\sqrt[4]{a+bx^4}}$$

```
[Out] 1/2*x^3/b/(b*x^4+a)^(1/4)+3/2*(1+a/b/x^4)^(1/4)*x*(cos(1/2*arccot(x^2*b^(1/
2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))*EllipticE(sin(1/
2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)/b^(3/2)/(b*x^4+a)^(1/4)
```

Rubi [A] time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {282, 281, 335, 275, 196}

$$\frac{3\sqrt{a}\,x\sqrt[4]{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{2b^{3/2}\sqrt[4]{a+bx^4}}+\frac{x^3}{2b\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^4)^(5/4),x]
```

```
[Out] x^3/(2*b*(a + b*x^4)^(1/4)) + (3*Sqrt[a]*(1 + a/(b*x^4))^(1/4)*x*EllipticE[
ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(2*b^(3/2)*(a + b*x^4)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 281

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Dist[(x*(1 + a/(b*x^4)
)^(1/4))/(b*(a + b*x^4)^(1/4)), Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x] /
; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 282

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x^(m - 3)/(b*(
m - 4)*(a + b*x^4)^(1/4)), x] - Dist[(a*(m - 3))/(b*(m - 4)), Int[x^(m - 4)
/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}, x] && PosQ[b/a] && IGtQ[(m - 2)
/4, 0]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{\left(a+bx^4\right)^{5/4}}\,dx &= \frac{x^3}{2b\sqrt[4]{a+bx^4}} - \frac{(3a)\int \frac{x^2}{\left(a+bx^4\right)^{5/4}}\,dx}{2b} \\
&= \frac{x^3}{2b\sqrt[4]{a+bx^4}} - \frac{\left(3a\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{5/4}x^3}\,dx}{2b^2\sqrt[4]{a+bx^4}} \\
&= \frac{x^3}{2b\sqrt[4]{a+bx^4}} + \frac{\left(3a\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{2b^2\sqrt[4]{a+bx^4}} \\
&= \frac{x^3}{2b\sqrt[4]{a+bx^4}} + \frac{\left(3a\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx, x, \frac{1}{x^2}\right)}{4b^2\sqrt[4]{a+bx^4}} \\
&= \frac{x^3}{2b\sqrt[4]{a+bx^4}} + \frac{3\sqrt{a}\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{2b^{3/2}\sqrt[4]{a+bx^4}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 60, normalized size = 0.72

$$\frac{x^3 - x^3\sqrt[4]{\frac{bx^4}{a}+1}\;{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{7}{4};-\frac{bx^4}{a}\right)}{2b\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^4)^(5/4),x]
```

```
[Out] (x^3 - x^3*(1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[3/4, 5/4, 7/4, -((b*x^4)
/a)])/(2*b*(a + b*x^4)^(1/4))
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{3}{4}}x^6}{b^2x^8+2\,abx^4+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(3/4)*x^6/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(bx^4+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/(b*x^4 + a)^(5/4), x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(b\,x^4+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^4+a)^(5/4),x)
```

```
[Out] int(x^6/(b*x^4+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(bx^4+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(b*x^4 + a)^(5/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{\left(b\,x^4+a\right)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^4)^(5/4),x)
```

```
[Out] int(x^6/(a + b*x^4)^(5/4), x)
```

sympy [C] time = 1.33, size = 37, normalized size = 0.45

$$\frac{x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{4}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**4+a)**(5/4),x)
```

```
[Out] x**7*gamma(7/4)*hyper((5/4, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**
(5/4)*gamma(11/4))
```

3.1165 $\int \frac{x^2}{\left(a+bx^4\right)^{5/4}}\,dx$

Optimal. Leaf size=59

$$-\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt[4]{a+bx^4}}$$

```
[Out] -(1+a/b/x^4)^(1/4)*x*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2
*arccot(x^2*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))
),2^(1/2))/(b*x^4+a)^(1/4)/a^(1/2)/b^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {281, 335, 275, 196}

$$-\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^4)^(5/4),x]
```

```
[Out] -(((1 + a/(b*x^4))^(1/4)*x*EllipticE[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(
Sqrt[a]*Sqrt[b]*(a + b*x^4)^(1/4)))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 281

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Dist[(x*(1 + a/(b*x^4)
)^(1/4))/(b*(a + b*x^4)^(1/4)), Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x] /
; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^4\right)^{5/4}}\,dx = \frac{\left(\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{5/4}x^3}\,dx}{b\sqrt[4]{a+bx^4}}$$

$$= -\frac{\left(\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{b\sqrt[4]{a+bx^4}}$$

$$= -\frac{\left(\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx, x, \frac{1}{x^2}\right)}{2b\sqrt[4]{a+bx^4}}$$

$$= -\frac{\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|\,2\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt[4]{a+bx^4}}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.92

$$\frac{x^3\sqrt[4]{\frac{bx^4}{a}+1}\;{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{7}{4};-\frac{bx^4}{a}\right)}{3a\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^4)^(5/4),x]
```

```
[Out] (x^3*(1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[3/4, 5/4, 7/4, -((b*x^4)/a)])/
(3*a*(a + b*x^4)^(1/4))
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{3}{4}}x^2}{b^2x^8+2\,abx^4+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(3/4)*x^2/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^4+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*x^4 + a)^(5/4), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(b\,x^4+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^4+a)^(5/4),x)
```

```
[Out] int(x^2/(b*x^4+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^4+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*x^4 + a)^(5/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\left(b\,x^4+a\right)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^4)^(5/4),x)
```

```
[Out] int(x^2/(a + b*x^4)^(5/4), x)
```

sympy [C] time = 0.97, size = 37, normalized size = 0.63

$$\frac{x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4}, \frac{5}{4} \\ \frac{7}{4}\end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{4}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**4+a)**(5/4),x)
```

```
[Out] x**3*gamma(3/4)*hyper((3/4, 5/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
5/4)*gamma(7/4))
```

3.1166 $\int \frac{1}{x^2\left(a+bx^4\right)^{5/4}}\,dx$

Optimal. Leaf size=79

$$\frac{2\sqrt{b}\,x\sqrt[4]{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a+bx^4}}-\frac{1}{ax\sqrt[4]{a+bx^4}}$$

```
[Out] -1/a/x/(b*x^4+a)^(1/4)+2*(1+a/b/x^4)^(1/4)*x*(cos(1/2*arccot(x^2*b^(1/2)/a^
(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arc
cot(x^2*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)/a^(3/2)/(b*x^4+a)^(1/4)
```

Rubi [A] time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {283, 281, 335, 275, 196}

$$\frac{2\sqrt{b}\,x\sqrt[4]{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a+bx^4}}-\frac{1}{ax\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^4)^(5/4)),x]
```

```
[Out] -(1/(a*x*(a + b*x^4)^(1/4))) + (2*Sqrt[b]*(1 + a/(b*x^4))^(1/4)*x*EllipticE
[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(a^(3/2)*(a + b*x^4)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 281

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Dist[(x*(1 + a/(b*x^4)
)^(1/4))/(b*(a + b*x^4)^(1/4)), Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x] /
; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 283

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x^(m + 1)/(a*(
m + 1)*(a + b*x^4)^(1/4)), x] - Dist[(b*m)/(a*(m + 1)), Int[x^(m + 4)/(a +
b*x^4)^(5/4), x], x] /; FreeQ[{a, b}, x] && PosQ[b/a] && ILtQ[(m - 2)/4, 0]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^4\right)^{5/4}}\,dx = -\frac{1}{ax\sqrt[4]{a+bx^4}} - \frac{(2b)\int \frac{x^2}{\left(a+bx^4\right)^{5/4}}\,dx}{a}$$

$$= -\frac{1}{ax\sqrt[4]{a+bx^4}} - \frac{\left(2\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{5/4}x^3}\,dx}{a\sqrt[4]{a+bx^4}}$$

$$= -\frac{1}{ax\sqrt[4]{a+bx^4}} + \frac{\left(2\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{a\sqrt[4]{a+bx^4}}$$

$$= -\frac{1}{ax\sqrt[4]{a+bx^4}} + \frac{\left(\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx, x, \frac{1}{x^2}\right)}{a\sqrt[4]{a+bx^4}}$$

$$= -\frac{1}{ax\sqrt[4]{a+bx^4}} + \frac{2\sqrt{b}\,\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a+bx^4}}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.66

$$-\frac{\sqrt[4]{\frac{bx^4}{a}+1}\;{}_2F_1\left(-\frac{1}{4},\frac{5}{4};\frac{3}{4};-\frac{bx^4}{a}\right)}{ax\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^4)^(5/4)),x]
```

```
[Out] -(((1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[-1/4, 5/4, 3/4, -((b*x^4)/a)])/(
a*x*(a + b*x^4)^(1/4)))
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{3}{4}}}{b^2x^{10}+2\,abx^6+a^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(3/4)/(b^2*x^10 + 2*a*b*x^6 + a^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{5}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(5/4)*x^2), x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{5}{4}}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^4+a)^(5/4),x)
```

```
[Out] int(1/x^2/(b*x^4+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{5}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(5/4)*x^2), x)
```

mupad [B] time = 1.35, size = 40, normalized size = 0.51

$$-\frac{\left(\frac{a}{b\,x^4}+1\right)^{5/4}\,{}_2\mathrm{F}_1\left(\frac{5}{4},\frac{3}{2};\ \frac{5}{2};\ -\frac{a}{b\,x^4}\right)}{6\,x\left(b\,x^4+a\right)^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^4)^(5/4)),x)
```

```
[Out] -((a/(b*x^4) + 1)^(5/4)*hypergeom([5/4, 3/2], 5/2, -a/(b*x^4)))/(6*x*(a + b
*x^4)^(5/4))
```

sympy [C] time = 1.60, size = 39, normalized size = 0.49

$$\frac{\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{4},\frac{5}{4}\\ \frac{3}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{4}}x\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**4+a)**(5/4),x)
```

```
[Out] gamma(-1/4)*hyper((-1/4, 5/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(5/4
)*x*gamma(3/4))
```

3.1167 $\int \frac{1}{x^6\left(a+bx^4\right)^{5/4}}\,dx$

Optimal. Leaf size=105

$$-\frac{12b^{3/2}x\sqrt[4]{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{5a^{5/2}\sqrt[4]{a+bx^4}}+\frac{6b}{5a^2x\sqrt[4]{a+bx^4}}-\frac{1}{5ax^5\sqrt[4]{a+bx^4}}$$

```
[Out] -1/5/a/x^5/(b*x^4+a)^(1/4)+6/5*b/a^2/x/(b*x^4+a)^(1/4)-12/5*b^(3/2)*(1+a/b/
x^4)^(1/4)*x*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(
x^2*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2
))/a^(5/2)/(b*x^4+a)^(1/4)
```

Rubi [A] time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {283, 281, 335, 275, 196}

$$-\frac{12b^{3/2}x\sqrt[4]{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{5a^{5/2}\sqrt[4]{a+bx^4}}+\frac{6b}{5a^2x\sqrt[4]{a+bx^4}}-\frac{1}{5ax^5\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^4)^(5/4)),x]
```

```
[Out] -1/(5*a*x^5*(a + b*x^4)^(1/4)) + (6*b)/(5*a^2*x*(a + b*x^4)^(1/4)) - (12*b^
(3/2)*(1 + a/(b*x^4))^(1/4)*x*EllipticE[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2]
)/(5*a^(5/2)*(a + b*x^4)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 281

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Dist[(x*(1 + a/(b*x^4)
)^(1/4))/(b*(a + b*x^4)^(1/4)), Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x] /
; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 283

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x^(m + 1)/(a*(
m + 1)*(a + b*x^4)^(1/4)), x] - Dist[(b*m)/(a*(m + 1)), Int[x^(m + 4)/(a +
b*x^4)^(5/4), x], x] /; FreeQ[{a, b}, x] && PosQ[b/a] && ILtQ[(m - 2)/4, 0]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{x^6\left(a+bx^4\right)^{5/4}}\,dx = -\frac{1}{5ax^5\sqrt[4]{a+bx^4}} - \frac{(6b)\int \frac{1}{x^2\left(a+bx^4\right)^{5/4}}\,dx}{5a}$$

$$= -\frac{1}{5ax^5\sqrt[4]{a+bx^4}} + \frac{6b}{5a^2x\sqrt[4]{a+bx^4}} + \frac{\left(12b^2\right)\int \frac{x^2}{\left(a+bx^4\right)^{5/4}}\,dx}{5a^2}$$

$$= -\frac{1}{5ax^5\sqrt[4]{a+bx^4}} + \frac{6b}{5a^2x\sqrt[4]{a+bx^4}} + \frac{\left(12b\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{5/4}x^3}\,dx}{5a^2\sqrt[4]{a+bx^4}}$$

$$= -\frac{1}{5ax^5\sqrt[4]{a+bx^4}} + \frac{6b}{5a^2x\sqrt[4]{a+bx^4}} - \frac{\left(12b\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{5a^2\sqrt[4]{a+bx^4}}$$

$$= -\frac{1}{5ax^5\sqrt[4]{a+bx^4}} + \frac{6b}{5a^2x\sqrt[4]{a+bx^4}} - \frac{\left(6b\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{5/4}}\,dx, x, \frac{1}{x^2}\right)}{5a^2\sqrt[4]{a+bx^4}}$$

$$= -\frac{1}{5ax^5\sqrt[4]{a+bx^4}} + \frac{6b}{5a^2x\sqrt[4]{a+bx^4}} - \frac{12b^{3/2}\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{5a^{5/2}\sqrt[4]{a+bx^4}}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.51

$$-\frac{\sqrt[4]{\frac{bx^4}{a}+1}\;{}_2F_1\left(-\frac{5}{4},\frac{5}{4};-\frac{1}{4};-\frac{bx^4}{a}\right)}{5ax^5\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^4)^(5/4)),x]
```

```
[Out] -1/5*((1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[-5/4, 5/4, -1/4, -((b*x^4)/a)
])/(a*x^5*(a + b*x^4)^(1/4))
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{3}{4}}}{b^2x^{14}+2\,abx^{10}+a^2x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(3/4)/(b^2*x^14 + 2*a*b*x^10 + a^2*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{5}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^4+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(5/4)*x^6), x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{5}{4}}\,x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^4+a)^(5/4),x)
```

```
[Out] int(1/x^6/(b*x^4+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{5}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(5/4)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^6\left(b\,x^4+a\right)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^4)^(5/4)),x)
```

```
[Out] int(1/(x^6*(a + b*x^4)^(5/4)), x)
```

sympy [C] time = 2.06, size = 44, normalized size = 0.42

$$\frac{\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4}, \frac{5}{4} \\ -\frac{1}{4}\end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{4}}x^5\Gamma\left(-\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**4+a)**(5/4),x)
```

```
[Out] gamma(-5/4)*hyper((-5/4, 5/4), (-1/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(5/
4)*x**5*gamma(-1/4))
```

3.1168 $\int \frac{1}{x^{10}\left(a+bx^4\right)^{5/4}}\,dx$

Optimal. Leaf size=129

$$\frac{8b^{5/2}x\sqrt[4]{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{3a^{7/2}\sqrt[4]{a+bx^4}}-\frac{4b^2}{3a^3x\sqrt[4]{a+bx^4}}+\frac{2b}{9a^2x^5\sqrt[4]{a+bx^4}}-\frac{1}{9ax^9\sqrt[4]{a+bx^4}}$$

```
[Out] -1/9/a/x^9/(b*x^4+a)^(1/4)+2/9*b/a^2/x^5/(b*x^4+a)^(1/4)-4/3*b^2/a^3/x/(b*x
^4+a)^(1/4)+8/3*b^(5/2)*(1+a/b/x^4)^(1/4)*x*(cos(1/2*arccot(x^2*b^(1/2)/a^(
1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arcc
ot(x^2*b^(1/2)/a^(1/2))),2^(1/2))/a^(7/2)/(b*x^4+a)^(1/4)
```

Rubi [A] time = 0.06, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {283, 281, 335, 275, 196}

$$-\frac{4b^2}{3a^3x\sqrt[4]{a+bx^4}}+\frac{8b^{5/2}x\sqrt[4]{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{3a^{7/2}\sqrt[4]{a+bx^4}}+\frac{2b}{9a^2x^5\sqrt[4]{a+bx^4}}-\frac{1}{9ax^9\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*(a + b*x^4)^(5/4)),x]
```

```
[Out] -1/(9*a*x^9*(a + b*x^4)^(1/4)) + (2*b)/(9*a^2*x^5*(a + b*x^4)^(1/4)) - (4*b
^2)/(3*a^3*x*(a + b*x^4)^(1/4)) + (8*b^(5/2)*(1 + a/(b*x^4))^(1/4)*x*Ellipt
icE[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(3*a^(7/2)*(a + b*x^4)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 281

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Dist[(x*(1 + a/(b*x^4)
)^(1/4))/(b*(a + b*x^4)^(1/4)), Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x] /
; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 283

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x^(m + 1)/(a*(
m + 1)*(a + b*x^4)^(1/4)), x] - Dist[(b*m)/(a*(m + 1)), Int[x^(m + 4)/(a +
b*x^4)^(5/4), x], x] /; FreeQ[{a, b}, x] && PosQ[b/a] && ILtQ[(m - 2)/4, 0]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{x^{10}\left(a+bx^4\right)^{5/4}}\,dx = -\frac{1}{9ax^9\sqrt[4]{a+bx^4}} - \frac{(10b)\int \frac{1}{x^6\left(a+bx^4\right)^{5/4}}\,dx}{9a}$$

$$= -\frac{1}{9ax^9\sqrt[4]{a+bx^4}} + \frac{2b}{9a^2x^5\sqrt[4]{a+bx^4}} + \frac{\left(4b^2\right)\int \frac{1}{x^2\left(a+bx^4\right)^{5/4}}\,dx}{3a^2}$$

$$= -\frac{1}{9ax^9\sqrt[4]{a+bx^4}} + \frac{2b}{9a^2x^5\sqrt[4]{a+bx^4}} - \frac{4b^2}{3a^3x\sqrt[4]{a+bx^4}} - \frac{\left(8b^3\right)\int \frac{x^2}{\left(a+bx^4\right)^{5/4}}\,dx}{3a^3}$$

$$= -\frac{1}{9ax^9\sqrt[4]{a+bx^4}} + \frac{2b}{9a^2x^5\sqrt[4]{a+bx^4}} - \frac{4b^2}{3a^3x\sqrt[4]{a+bx^4}} - \frac{\left(8b^2\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{5/4}x^3}\,dx}{3a^3\sqrt[4]{a+bx^4}}$$

$$= -\frac{1}{9ax^9\sqrt[4]{a+bx^4}} + \frac{2b}{9a^2x^5\sqrt[4]{a+bx^4}} - \frac{4b^2}{3a^3x\sqrt[4]{a+bx^4}} + \frac{\left(8b^2\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right.}\right.}{3a^3\sqrt[4]{a+bx^4}}$$

$$= -\frac{1}{9ax^9\sqrt[4]{a+bx^4}} + \frac{2b}{9a^2x^5\sqrt[4]{a+bx^4}} - \frac{4b^2}{3a^3x\sqrt[4]{a+bx^4}} + \frac{\left(4b^2\sqrt[4]{1+\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right.}\right.}{3a^3\sqrt[4]{a+bx^4}}$$

$$= -\frac{1}{9ax^9\sqrt[4]{a+bx^4}} + \frac{2b}{9a^2x^5\sqrt[4]{a+bx^4}} - \frac{4b^2}{3a^3x\sqrt[4]{a+bx^4}} + \frac{8b^{5/2}\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right.\right.}{3a^{7/2}\sqrt[4]{a+bx^4}}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.42

$$-\frac{\sqrt[4]{\frac{bx^4}{a}+1}\;{}_2F_1\left(-\frac{9}{4},\frac{5}{4};-\frac{5}{4};-\frac{bx^4}{a}\right)}{9ax^9\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^10*(a + b*x^4)^(5/4)),x]
```

```
[Out] -1/9*((1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[-9/4, 5/4, -5/4, -((b*x^4)/a)
])/(a*x^9*(a + b*x^4)^(1/4))
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{3}{4}}}{b^2x^{18}+2\,abx^{14}+a^2x^{10}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(3/4)/(b^2*x^18 + 2*a*b*x^14 + a^2*x^10), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{5}{4}}x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x^4+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(5/4)*x^10), x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{5}{4}}\,x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(b*x^4+a)^(5/4),x)
```

```
[Out] int(1/x^10/(b*x^4+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{5}{4}}x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(5/4)*x^10), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{10}\left(b\,x^4+a\right)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^10*(a + b*x^4)^(5/4)),x)
```

```
[Out] int(1/(x^10*(a + b*x^4)^(5/4)), x)
```

sympy [C] time = 3.16, size = 44, normalized size = 0.34

$$\frac{\Gamma\left(-\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{9}{4}, \frac{5}{4} \\ -\frac{5}{4}\end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{4}}x^9\Gamma\left(-\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(b*x**4+a)**(5/4),x)
```

```
[Out] gamma(-9/4)*hyper((-9/4, 5/4), (-5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(5/
4)*x**9*gamma(-5/4))
```

3.1169 $\int \frac{1}{x^{14}\left(a+bx^4\right)^{5/4}}\,dx$

Optimal. Leaf size=153

$$-\frac{112b^{7/2}x\sqrt[4]{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{39a^{9/2}\sqrt[4]{a+bx^4}}+\frac{56b^3}{39a^4x\sqrt[4]{a+bx^4}}-\frac{28b^2}{117a^3x^5\sqrt[4]{a+bx^4}}+\frac{14b}{117a^2x^9\sqrt[4]{a+bx^4}}-\frac{1}{13ax^{13}\sqrt[4]{a+}}$$

```
[Out] -1/13/a/x^13/(b*x^4+a)^(1/4)+14/117*b/a^2/x^9/(b*x^4+a)^(1/4)-28/117*b^2/a^
3/x^5/(b*x^4+a)^(1/4)+56/39*b^3/a^4/x/(b*x^4+a)^(1/4)-112/39*b^(7/2)*(1+a/b
/x^4)^(1/4)*x*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot
(x^2*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/
2))/a^(9/2)/(b*x^4+a)^(1/4)
```

Rubi [A] time = 0.08, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {283, 281, 335, 275, 196}

$$\frac{56b^3}{39a^4x\sqrt[4]{a+bx^4}}-\frac{28b^2}{117a^3x^5\sqrt[4]{a+bx^4}}-\frac{112b^{7/2}x\sqrt[4]{\frac{a}{bx^4}+1}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{39a^{9/2}\sqrt[4]{a+bx^4}}+\frac{14b}{117a^2x^9\sqrt[4]{a+bx^4}}-\frac{1}{13ax^{13}\sqrt[4]{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^14*(a + b*x^4)^(5/4)),x]
```

```
[Out] -1/(13*a*x^13*(a + b*x^4)^(1/4)) + (14*b)/(117*a^2*x^9*(a + b*x^4)^(1/4)) -
 (28*b^2)/(117*a^3*x^5*(a + b*x^4)^(1/4)) + (56*b^3)/(39*a^4*x*(a + b*x^4)^
(1/4)) - (112*b^(7/2)*(1 + a/(b*x^4))^(1/4)*x*EllipticE[ArcCot[(Sqrt[b]*x^2
)/Sqrt[a]]/2, 2])/(39*a^(9/2)*(a + b*x^4)^(1/4))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 281

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Dist[(x*(1 + a/(b*x^4)
)^(1/4))/(b*(a + b*x^4)^(1/4)), Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x] /
; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 283

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x^(m + 1)/(a*(
m + 1)*(a + b*x^4)^(1/4)), x] - Dist[(b*m)/(a*(m + 1)), Int[x^(m + 4)/(a +
b*x^4)^(5/4), x], x] /; FreeQ[{a, b}, x] && PosQ[b/a] && ILtQ[(m - 2)/4, 0]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
```

egerQ[m]

Rubi steps

$$\int \frac{1}{x^{14}\left(a+bx^4\right)^{5/4}}\,dx = -\frac{1}{13ax^{13}\sqrt[4]{a+bx^4}} - \frac{(14b)\int \frac{1}{x^{10}\left(a+bx^4\right)^{5/4}}\,dx}{13a}$$

$$= -\frac{1}{13ax^{13}\sqrt[4]{a+bx^4}} + \frac{14b}{117a^2x^9\sqrt[4]{a+bx^4}} + \frac{\left(140b^2\right)\int \frac{1}{x^6\left(a+bx^4\right)^{5/4}}\,dx}{117a^2}$$

$$= -\frac{1}{13ax^{13}\sqrt[4]{a+bx^4}} + \frac{14b}{117a^2x^9\sqrt[4]{a+bx^4}} - \frac{28b^2}{117a^3x^5\sqrt[4]{a+bx^4}} - \frac{\left(56b^3\right)\int \frac{1}{x^2\left(a+bx^4\right)^{5/4}}\,dx}{39a^3}$$

$$= -\frac{1}{13ax^{13}\sqrt[4]{a+bx^4}} + \frac{14b}{117a^2x^9\sqrt[4]{a+bx^4}} - \frac{28b^2}{117a^3x^5\sqrt[4]{a+bx^4}} + \frac{56b^3}{39a^4x\sqrt[4]{a+bx^4}} + \frac{\left(11\right.}{}$$

$$= -\frac{1}{13ax^{13}\sqrt[4]{a+bx^4}} + \frac{14b}{117a^2x^9\sqrt[4]{a+bx^4}} - \frac{28b^2}{117a^3x^5\sqrt[4]{a+bx^4}} + \frac{56b^3}{39a^4x\sqrt[4]{a+bx^4}} + \frac{\left(11\right.}{}$$

$$= -\frac{1}{13ax^{13}\sqrt[4]{a+bx^4}} + \frac{14b}{117a^2x^9\sqrt[4]{a+bx^4}} - \frac{28b^2}{117a^3x^5\sqrt[4]{a+bx^4}} + \frac{56b^3}{39a^4x\sqrt[4]{a+bx^4}} - \frac{\left(11\right.}{}$$

$$= -\frac{1}{13ax^{13}\sqrt[4]{a+bx^4}} + \frac{14b}{117a^2x^9\sqrt[4]{a+bx^4}} - \frac{28b^2}{117a^3x^5\sqrt[4]{a+bx^4}} + \frac{56b^3}{39a^4x\sqrt[4]{a+bx^4}} - \frac{\left(56\right.}{}$$

$$= -\frac{1}{13ax^{13}\sqrt[4]{a+bx^4}} + \frac{14b}{117a^2x^9\sqrt[4]{a+bx^4}} - \frac{28b^2}{117a^3x^5\sqrt[4]{a+bx^4}} + \frac{56b^3}{39a^4x\sqrt[4]{a+bx^4}} - \frac{112}{}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.35

$$-\frac{\sqrt[4]{\frac{bx^4}{a}+1}\;{}_2F_1\left(-\frac{13}{4},\frac{5}{4};-\frac{9}{4};-\frac{bx^4}{a}\right)}{13ax^{13}\sqrt[4]{a+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^14*(a + b*x^4)^(5/4)),x]
```

```
[Out] -1/13*((1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[-13/4, 5/4, -9/4, -((b*x^4)/
a)])/(a*x^13*(a + b*x^4)^(1/4))
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{3}{4}}}{b^2x^{22}+2\,abx^{18}+a^2x^{14}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^14/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(3/4)/(b^2*x^22 + 2*a*b*x^18 + a^2*x^14), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{5}{4}} x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^14/(b*x^4+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(5/4)*x^14), x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{5}{4}} x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^14/(b*x^4+a)^(5/4),x)
```

```
[Out] int(1/x^14/(b*x^4+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{5}{4}} x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^14/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(5/4)*x^14), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{14}\left(b\,x^4+a\right)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^14*(a + b*x^4)^(5/4)),x)
```

```
[Out] int(1/(x^14*(a + b*x^4)^(5/4)), x)
```

sympy [C] time = 3.96, size = 44, normalized size = 0.29

$$\frac{\Gamma\left(-\frac{13}{4}\right){}_2F_1\left(\begin{matrix}-\frac{13}{4}, \frac{5}{4} \\ -\frac{9}{4}\end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4a^{\frac{5}{4}}x^{13}\Gamma\left(-\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**14/(b*x**4+a)**(5/4),x)
```

```
[Out] gamma(-13/4)*hyper((-13/4, 5/4), (-9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
5/4)*x**13*gamma(-9/4))
```

3.1170 $\int \frac{1}{\left(a+bx^4\right)^{7/4}}\,dx$

Optimal. Leaf size=83

$$\frac{x}{3a\left(a+bx^4\right)^{3/4}}-\frac{2\sqrt{b}\,x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}\left(a+bx^4\right)^{3/4}}$$

```
[Out] 1/3*x/a/(b*x^4+a)^(3/4)-2/3*(1+a/b/x^4)^(3/4)*x^3*(cos(1/2*arccot(x^2*b^(1/
2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/
2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)/a^(3/2)/(b*x^4+a)^(3/4)
```

Rubi [A] time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {199, 237, 335, 275, 231}

$$\frac{x}{3a\left(a+bx^4\right)^{3/4}}-\frac{2\sqrt{b}\,x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}\left(a+bx^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(-7/4),x]
```

```
[Out] x/(3*a*(a + b*x^4)^(3/4)) - (2*Sqrt[b]*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[
ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(3*a^(3/2)*(a + b*x^4)^(3/4))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
```

egerQ[m]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+bx^4\right)^{7/4}}\,dx &= \frac{x}{3a\left(a+bx^4\right)^{3/4}} + \frac{2\int \frac{1}{\left(a+bx^4\right)^{3/4}}\,dx}{3a} \\
&= \frac{x}{3a\left(a+bx^4\right)^{3/4}} + \frac{\left(2\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{3a\left(a+bx^4\right)^{3/4}} \\
&= \frac{x}{3a\left(a+bx^4\right)^{3/4}} - \frac{\left(2\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{3a\left(a+bx^4\right)^{3/4}} \\
&= \frac{x}{3a\left(a+bx^4\right)^{3/4}} - \frac{\left(\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{x^2}\right)}{3a\left(a+bx^4\right)^{3/4}} \\
&= \frac{x}{3a\left(a+bx^4\right)^{3/4}} - \frac{2\sqrt{b}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}\left(a+bx^4\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.04, size = 56, normalized size = 0.67

$$\frac{2x\left(\frac{bx^4}{a}+1\right)^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{bx^4}{a}\right)+x}{3a\left(a+bx^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(-7/4),x]
```

```
[Out] (x + 2*x*(1 + (b*x^4)/a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, -((b*x^4)/a
)])/(3*a*(a + b*x^4)^(3/4))
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^2x^8+2\,abx^4+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(7/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(1/4)/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(-7/4), x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(7/4),x)
```

```
[Out] int(1/(b*x^4+a)^(7/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(-7/4), x)
```

mupad [B] time = 1.14, size = 37, normalized size = 0.45

$$\frac{x\left(\frac{b\,x^4}{a}+1\right)^{7/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{7}{4};\ \frac{5}{4};\ -\frac{b\,x^4}{a}\right)}{\left(b\,x^4+a\right)^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^4)^(7/4),x)
```

```
[Out] (x*((b*x^4)/a + 1)^(7/4)*hypergeom([1/4, 7/4], 5/4, -(b*x^4)/a))/(a + b*x^4
)^(7/4)
```

sympy [C] time = 2.12, size = 36, normalized size = 0.43

$$\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{7}{4}\\ \frac{5}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{7}{4}}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(7/4),x)
```

```
[Out] x*gamma(1/4)*hyper((1/4, 7/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(7/4
)*gamma(5/4))
```

3.1171 $\int \frac{1}{\left(a+bx^4\right)^{9/4}}\,dx$

Optimal. Leaf size=39

$$\frac{4x}{5a^2\sqrt[4]{a+bx^4}}+\frac{x}{5a\left(a+bx^4\right)^{5/4}}$$

```
[Out] 1/5*x/a/(b*x^4+a)^(5/4)+4/5*x/a^2/(b*x^4+a)^(1/4)
```

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {192, 191}

$$\frac{4x}{5a^2\sqrt[4]{a+bx^4}}+\frac{x}{5a\left(a+bx^4\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(-9/4),x]
```

```
[Out] x/(5*a*(a + b*x^4)^(5/4)) + (4*x)/(5*a^2*(a + b*x^4)^(1/4))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+bx^4\right)^{9/4}}\,dx &= \frac{x}{5a\left(a+bx^4\right)^{5/4}}+\frac{4\int \frac{1}{\left(a+bx^4\right)^{5/4}}\,dx}{5a}\\ &= \frac{x}{5a\left(a+bx^4\right)^{5/4}}+\frac{4x}{5a^2\sqrt[4]{a+bx^4}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.74

$$\frac{x\left(5a+4bx^4\right)}{5a^2\left(a+bx^4\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(-9/4),x]
```

```
[Out] (x*(5*a + 4*b*x^4))/(5*a^2*(a + b*x^4)^(5/4))
```

fricas [A] time = 0.84, size = 47, normalized size = 1.21

$$\frac{\left(4\,bx^5+5\,ax\right)\left(bx^4+a\right)^{\frac{3}{4}}}{5\left(a^2b^2x^8+2\,a^3bx^4+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(9/4),x, algorithm="fricas")
[Out] 1/5*(4*b*x^5 + 5*a*x)*(b*x^4 + a)^(3/4)/(a^2*b^2*x^8 + 2*a^3*b*x^4 + a^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(9/4),x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(-9/4), x)
```

maple [A] time = 0.00, size = 26, normalized size = 0.67

$$\frac{\left(4b\,x^4+5a\right)x}{5\left(b\,x^4+a\right)^{\frac{5}{4}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(9/4),x)
[Out] 1/5*x*(4*b*x^4+5*a)/(b*x^4+a)^(5/4)/a^2
```

maxima [A] time = 1.44, size = 31, normalized size = 0.79

$$-\frac{\left(b-\frac{5\left(bx^4+a\right)}{x^4}\right)x^5}{5\left(bx^4+a\right)^{\frac{5}{4}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(9/4),x, algorithm="maxima")
[Out] -1/5*(b - 5*(b*x^4 + a)/x^4)*x^5/((b*x^4 + a)^(5/4)*a^2)
```

mupad [B] time = 1.06, size = 28, normalized size = 0.72

$$\frac{4\,x\left(b\,x^4+a\right)+a\,x}{5\,a^2\left(b\,x^4+a\right)^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^4)^(9/4),x)
[Out] (4*x*(a + b*x^4) + a*x)/(5*a^2*(a + b*x^4)^(5/4))
```

sympy [**B**] time = 2.53, size = 126, normalized size = 3.23

$$\frac{5ax\Gamma\left(\frac{1}{4}\right)}{16a^{\frac{13}{4}}\sqrt[4]{1+\frac{bx^4}{a}}\,\Gamma\left(\frac{9}{4}\right)+16a^{\frac{9}{4}}bx^4\sqrt[4]{1+\frac{bx^4}{a}}\,\Gamma\left(\frac{9}{4}\right)}+\frac{4bx^5\Gamma\left(\frac{1}{4}\right)}{16a^{\frac{13}{4}}\sqrt[4]{1+\frac{bx^4}{a}}\,\Gamma\left(\frac{9}{4}\right)+16a^{\frac{9}{4}}bx^4\sqrt[4]{1+\frac{bx^4}{a}}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(9/4),x)
```

```
[Out] 5*a*x*gamma(1/4)/(16*a**(13/4)*(1 + b*x**4/a)**(1/4)*gamma(9/4) + 16*a**(9/
4)*b*x**4*(1 + b*x**4/a)**(1/4)*gamma(9/4)) + 4*b*x**5*gamma(1/4)/(16*a**(1
3/4)*(1 + b*x**4/a)**(1/4)*gamma(9/4) + 16*a**(9/4)*b*x**4*(1 + b*x**4/a)**
(1/4)*gamma(9/4))
```

3.1172 $\int \frac{1}{\left(a+bx^4\right)^{11/4}}\,dx$

Optimal. Leaf size=102

$$-\frac{4\sqrt{b}\,x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{7a^{5/2}\left(a+bx^4\right)^{3/4}}+\frac{2x}{7a^2\left(a+bx^4\right)^{3/4}}+\frac{x}{7a\left(a+bx^4\right)^{7/4}}$$

```
[Out] 1/7*x/a/(b*x^4+a)^(7/4)+2/7*x/a^2/(b*x^4+a)^(3/4)-4/7*(1+a/b/x^4)^(3/4)*x^3
*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2)/
a^(1/2)))*EllipticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)/a
^(5/2)/(b*x^4+a)^(3/4)
```

Rubi [A] time = 0.04, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {199, 237, 335, 275, 231}

$$\frac{2x}{7a^2\left(a+bx^4\right)^{3/4}}-\frac{4\sqrt{b}\,x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{7a^{5/2}\left(a+bx^4\right)^{3/4}}+\frac{x}{7a\left(a+bx^4\right)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(-11/4),x]
```

```
[Out] x/(7*a*(a + b*x^4)^(7/4)) + (2*x)/(7*a^2*(a + b*x^4)^(3/4)) - (4*Sqrt[b]*(1
 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(7*a
^(5/2)*(a + b*x^4)^(3/4))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+bx^4\right)^{11/4}}\,dx &= \frac{x}{7a\left(a+bx^4\right)^{7/4}} + \frac{6\int \frac{1}{\left(a+bx^4\right)^{7/4}}\,dx}{7a} \\
&= \frac{x}{7a\left(a+bx^4\right)^{7/4}} + \frac{2x}{7a^2\left(a+bx^4\right)^{3/4}} + \frac{4\int \frac{1}{\left(a+bx^4\right)^{3/4}}\,dx}{7a^2} \\
&= \frac{x}{7a\left(a+bx^4\right)^{7/4}} + \frac{2x}{7a^2\left(a+bx^4\right)^{3/4}} + \frac{\left(4\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{7a^2\left(a+bx^4\right)^{3/4}} \\
&= \frac{x}{7a\left(a+bx^4\right)^{7/4}} + \frac{2x}{7a^2\left(a+bx^4\right)^{3/4}} - \frac{\left(4\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{7a^2\left(a+bx^4\right)^{3/4}} \\
&= \frac{x}{7a\left(a+bx^4\right)^{7/4}} + \frac{2x}{7a^2\left(a+bx^4\right)^{3/4}} - \frac{\left(2\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{ax^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{x^2}\right)}{7a^2\left(a+bx^4\right)^{3/4}} \\
&= \frac{x}{7a\left(a+bx^4\right)^{7/4}} + \frac{2x}{7a^2\left(a+bx^4\right)^{3/4}} - \frac{4\sqrt{b}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3 F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{7a^{5/2}\left(a+bx^4\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.05, size = 72, normalized size = 0.71

$$\frac{4x\left(a+bx^4\right)\left(\frac{bx^4}{a}+1\right)^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{bx^4}{a}\right)+3ax+2bx^5}{7a^2\left(a+bx^4\right)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(-11/4),x]
```

```
[Out] (3*a*x + 2*b*x^5 + 4*x*(a + b*x^4)*(1 + (b*x^4)/a)^(3/4)*Hypergeometric2F1[
1/4, 3/4, 5/4, -((b*x^4)/a)])/(7*a^2*(a + b*x^4)^(7/4))
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^3x^{12}+3\,ab^2x^8+3\,a^2bx^4+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(11/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^(1/4)/(b^3*x^12 + 3*a*b^2*x^8 + 3*a^2*b*x^4 + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(11/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(-11/4), x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(11/4),x)
```

```
[Out] int(1/(b*x^4+a)^(11/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(11/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(-11/4), x)
```

mupad [B] time = 1.13, size = 37, normalized size = 0.36

$$\frac{x\left(\frac{b\,x^4}{a}+1\right)^{11/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{11}{4};\ \frac{5}{4};\ -\frac{b\,x^4}{a}\right)}{\left(b\,x^4+a\right)^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^4)^(11/4),x)
```

```
[Out] (x*((b*x^4)/a + 1)^(11/4)*hypergeom([1/4, 11/4], 5/4, -(b*x^4)/a))/(a + b*x
^4)^(11/4)
```

sympy [C] time = 3.53, size = 36, normalized size = 0.35

$$\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{11}{4}\\ \frac{5}{4}\end{matrix}\middle|\,\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{11}{4}}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(11/4),x)
```

```
[Out] x*gamma(1/4)*hyper((1/4, 11/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(11
/4)*gamma(5/4))
```

3.1173 $\int \frac{1}{\left(a+bx^4\right)^{13/4}}\,dx$

Optimal. Leaf size=58

$$\frac{32x}{45a^3\sqrt[4]{a+bx^4}}+\frac{8x}{45a^2\left(a+bx^4\right)^{5/4}}+\frac{x}{9a\left(a+bx^4\right)^{9/4}}$$

```
[Out] 1/9*x/a/(b*x^4+a)^(9/4)+8/45*x/a^2/(b*x^4+a)^(5/4)+32/45*x/a^3/(b*x^4+a)^(1
/4)
```

Rubi [A] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {192, 191}

$$\frac{32x}{45a^3\sqrt[4]{a+bx^4}}+\frac{8x}{45a^2\left(a+bx^4\right)^{5/4}}+\frac{x}{9a\left(a+bx^4\right)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(-13/4),x]
```

```
[Out] x/(9*a*(a + b*x^4)^(9/4)) + (8*x)/(45*a^2*(a + b*x^4)^(5/4)) + (32*x)/(45*a
^3*(a + b*x^4)^(1/4))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+bx^4\right)^{13/4}}\,dx &= \frac{x}{9a\left(a+bx^4\right)^{9/4}}+\frac{8\int \frac{1}{\left(a+bx^4\right)^{9/4}}\,dx}{9a}\\ &= \frac{x}{9a\left(a+bx^4\right)^{9/4}}+\frac{8x}{45a^2\left(a+bx^4\right)^{5/4}}+\frac{32\int \frac{1}{\left(a+bx^4\right)^{5/4}}\,dx}{45a^2}\\ &= \frac{x}{9a\left(a+bx^4\right)^{9/4}}+\frac{8x}{45a^2\left(a+bx^4\right)^{5/4}}+\frac{32x}{45a^3\sqrt[4]{a+bx^4}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 0.69

$$\frac{x\left(45a^2+72abx^4+32b^2x^8\right)}{45a^3\left(a+bx^4\right)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(-13/4),x]
```

```
[Out] (x*(45*a^2 + 72*a*b*x^4 + 32*b^2*x^8))/(45*a^3*(a + b*x^4)^(9/4))
```

fricas [A] time = 0.79, size = 69, normalized size = 1.19

$$\frac{\left(32\,b^2x^9+72\,abx^5+45\,a^2x\right)\left(bx^4+a\right)^{\frac{3}{4}}}{45\left(a^3b^3x^{12}+3\,a^4b^2x^8+3\,a^5bx^4+a^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(13/4),x, algorithm="fricas")
```

```
[Out] 1/45*(32*b^2*x^9 + 72*a*b*x^5 + 45*a^2*x)*(b*x^4 + a)^(3/4)/(a^3*b^3*x^12 +
 3*a^4*b^2*x^8 + 3*a^5*b*x^4 + a^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{13}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(13/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(-13/4), x)
```

maple [A] time = 0.00, size = 37, normalized size = 0.64

$$\frac{\left(32b^2x^8+72ab\,x^4+45a^2\right)x}{45\left(b\,x^4+a\right)^{\frac{9}{4}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(13/4),x)
```

```
[Out] 1/45*x*(32*b^2*x^8+72*a*b*x^4+45*a^2)/(b*x^4+a)^(9/4)/a^3
```

maxima [A] time = 1.40, size = 50, normalized size = 0.86

$$\frac{\left(5\,b^2-\frac{18\left(bx^4+a\right)b}{x^4}+\frac{45\left(bx^4+a\right)^2}{x^8}\right)x^9}{45\left(bx^4+a\right)^{\frac{9}{4}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(13/4),x, algorithm="maxima")
```

```
[Out] 1/45*(5*b^2 - 18*(b*x^4 + a)*b/x^4 + 45*(b*x^4 + a)^2/x^8)*x^9/((b*x^4 + a)
^(9/4)*a^3)
```

mupad [B] time = 1.07, size = 44, normalized size = 0.76

$$\frac{32\,x\left(b\,x^4+a\right)^2+5\,a^2\,x+8\,a\,x\,\left(b\,x^4+a\right)}{45\,a^3\left(b\,x^4+a\right)^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^4)^(13/4),x)
```

```
[Out] (32*x*(a + b*x^4)^2 + 5*a^2*x + 8*a*x*(a + b*x^4))/(45*a^3*(a + b*x^4)^(9/4
))
```

sympy [**B**] time = 5.22, size = 515, normalized size = 8.88

$$\frac{45a^5x\Gamma\left(\frac{1}{4}\right)}{64a^{\frac{33}{4}}\sqrt[4]{1+\frac{bx^4}{a}}\,\Gamma\left(\frac{13}{4}\right)+192a^{\frac{29}{4}}bx^4\sqrt[4]{1+\frac{bx^4}{a}}\,\Gamma\left(\frac{13}{4}\right)+192a^{\frac{25}{4}}b^2x^8\sqrt[4]{1+\frac{bx^4}{a}}\,\Gamma\left(\frac{13}{4}\right)+64a^{\frac{21}{4}}b^3x^{12}\sqrt[4]{1+\frac{bx^4}{a}}\,\Gamma\left(\frac{13}{4}\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(13/4),x)
```

```
[Out] 45*a**5*x*gamma(1/4)/(64*a**(33/4)*(1 + b*x**4/a)**(1/4)*gamma(13/4) + 192*
a**(29/4)*b*x**4*(1 + b*x**4/a)**(1/4)*gamma(13/4) + 192*a**(25/4)*b**2*x**
8*(1 + b*x**4/a)**(1/4)*gamma(13/4) + 64*a**(21/4)*b**3*x**12*(1 + b*x**4/a
)**(1/4)*gamma(13/4)) + 117*a**4*b*x**5*gamma(1/4)/(64*a**(33/4)*(1 + b*x**
4/a)**(1/4)*gamma(13/4) + 192*a**(29/4)*b*x**4*(1 + b*x**4/a)**(1/4)*gamma(
13/4) + 192*a**(25/4)*b**2*x**8*(1 + b*x**4/a)**(1/4)*gamma(13/4) + 64*a**(
21/4)*b**3*x**12*(1 + b*x**4/a)**(1/4)*gamma(13/4)) + 104*a**3*b**2*x**9*ga
mma(1/4)/(64*a**(33/4)*(1 + b*x**4/a)**(1/4)*gamma(13/4) + 192*a**(29/4)*b*
x**4*(1 + b*x**4/a)**(1/4)*gamma(13/4) + 192*a**(25/4)*b**2*x**8*(1 + b*x**
4/a)**(1/4)*gamma(13/4) + 64*a**(21/4)*b**3*x**12*(1 + b*x**4/a)**(1/4)*gam
ma(13/4)) + 32*a**2*b**3*x**13*gamma(1/4)/(64*a**(33/4)*(1 + b*x**4/a)**(1/
4)*gamma(13/4) + 192*a**(29/4)*b*x**4*(1 + b*x**4/a)**(1/4)*gamma(13/4) + 1
92*a**(25/4)*b**2*x**8*(1 + b*x**4/a)**(1/4)*gamma(13/4) + 64*a**(21/4)*b**
3*x**12*(1 + b*x**4/a)**(1/4)*gamma(13/4))
```

3.1174 $\int \frac{1}{\left(a+bx^4\right)^{17/4}}\,dx$

Optimal. Leaf size=77

$$\frac{128x}{195a^4\sqrt[4]{a+bx^4}}+\frac{32x}{195a^3\left(a+bx^4\right)^{5/4}}+\frac{4x}{39a^2\left(a+bx^4\right)^{9/4}}+\frac{x}{13a\left(a+bx^4\right)^{13/4}}$$

```
[Out] 1/13*x/a/(b*x^4+a)^(13/4)+4/39*x/a^2/(b*x^4+a)^(9/4)+32/195*x/a^3/(b*x^4+a)
^(5/4)+128/195*x/a^4/(b*x^4+a)^(1/4)
```

Rubi [A] time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {192, 191}

$$\frac{128x}{195a^4\sqrt[4]{a+bx^4}}+\frac{32x}{195a^3\left(a+bx^4\right)^{5/4}}+\frac{4x}{39a^2\left(a+bx^4\right)^{9/4}}+\frac{x}{13a\left(a+bx^4\right)^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(-17/4),x]
```

```
[Out] x/(13*a*(a + b*x^4)^(13/4)) + (4*x)/(39*a^2*(a + b*x^4)^(9/4)) + (32*x)/(19
5*a^3*(a + b*x^4)^(5/4)) + (128*x)/(195*a^4*(a + b*x^4)^(1/4))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+bx^4\right)^{17/4}}\,dx &= \frac{x}{13a\left(a+bx^4\right)^{13/4}}+\frac{12\int \frac{1}{\left(a+bx^4\right)^{13/4}}\,dx}{13a}\\
&= \frac{x}{13a\left(a+bx^4\right)^{13/4}}+\frac{4x}{39a^2\left(a+bx^4\right)^{9/4}}+\frac{32\int \frac{1}{\left(a+bx^4\right)^{9/4}}\,dx}{39a^2}\\
&= \frac{x}{13a\left(a+bx^4\right)^{13/4}}+\frac{4x}{39a^2\left(a+bx^4\right)^{9/4}}+\frac{32x}{195a^3\left(a+bx^4\right)^{5/4}}+\frac{128\int \frac{1}{\left(a+bx^4\right)^{5/4}}\,dx}{195a^3}\\
&= \frac{x}{13a\left(a+bx^4\right)^{13/4}}+\frac{4x}{39a^2\left(a+bx^4\right)^{9/4}}+\frac{32x}{195a^3\left(a+bx^4\right)^{5/4}}+\frac{128x}{195a^4\sqrt[4]{a+bx^4}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 0.66

$$\frac{x\left(195a^3+468a^2bx^4+416ab^2x^8+128b^3x^{12}\right)}{195a^4\left(a+bx^4\right)^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^(-17/4),x]
```

```
[Out] (x*(195*a^3 + 468*a^2*b*x^4 + 416*a*b^2*x^8 + 128*b^3*x^12))/(195*a^4*(a +
b*x^4)^(13/4))
```

fricas [A] time = 0.87, size = 91, normalized size = 1.18

$$\frac{\left(128\,b^3x^{13}+416\,ab^2x^9+468\,a^2bx^5+195\,a^3x\right)\left(bx^4+a\right)^{\frac{3}{4}}}{195\left(a^4b^4x^{16}+4\,a^5b^3x^{12}+6\,a^6b^2x^8+4\,a^7bx^4+a^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(17/4),x, algorithm="fricas")
```

```
[Out] 1/195*(128*b^3*x^13 + 416*a*b^2*x^9 + 468*a^2*b*x^5 + 195*a^3*x)*(b*x^4 + a
)^(3/4)/(a^4*b^4*x^16 + 4*a^5*b^3*x^12 + 6*a^6*b^2*x^8 + 4*a^7*b*x^4 + a^8)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{17}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(17/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(-17/4), x)
```

maple [A] time = 0.01, size = 48, normalized size = 0.62

$$\frac{\left(128b^3x^{12}+416a\,b^2x^8+468a^2b\,x^4+195a^3\right)x}{195\left(b\,x^4+a\right)^{\frac{13}{4}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(17/4),x)
```

```
[Out] 1/195*x*(128*b^3*x^12+416*a*b^2*x^8+468*a^2*b*x^4+195*a^3)/(b*x^4+a)^(13/4)
/a^4
```

maxima [A] time = 1.33, size = 67, normalized size = 0.87

$$-\frac{\left(15\,b^3-\frac{65\left(bx^4+a\right)b^2}{x^4}+\frac{117\left(bx^4+a\right)^2b}{x^8}-\frac{195\left(bx^4+a\right)^3}{x^{12}}\right)x^{13}}{195\left(bx^4+a\right)^{\frac{13}{4}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(17/4),x, algorithm="maxima")
```

```
[Out] -1/195*(15*b^3 - 65*(b*x^4 + a)*b^2/x^4 + 117*(b*x^4 + a)^2*b/x^8 - 195*(b*
x^4 + a)^3/x^12)*x^13/((b*x^4 + a)^(13/4)*a^4)
```

mupad [B] time = 1.08, size = 61, normalized size = 0.79

$$\frac{128\,x}{195\,a^4\left(b\,x^4+a\right)^{1/4}}+\frac{32\,x}{195\,a^3\left(b\,x^4+a\right)^{5/4}}+\frac{4\,x}{39\,a^2\left(b\,x^4+a\right)^{9/4}}+\frac{x}{13\,a\left(b\,x^4+a\right)^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^4)^(17/4),x)
```

```
[Out] (128*x)/(195*a^4*(a + b*x^4)^(1/4)) + (32*x)/(195*a^3*(a + b*x^4)^(5/4)) +
(4*x)/(39*a^2*(a + b*x^4)^(9/4)) + x/(13*a*(a + b*x^4)^(13/4))
```

sympy [**B**] time = 7.86, size = 1550, normalized size = 20.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(17/4),x)
```

```
[Out] 585*a**14*x*gamma(1/4)/(256*a**(73/4)*(1 + b*x**4/a)**(1/4)*gamma(17/4) + 1
536*a**(69/4)*b*x**4*(1 + b*x**4/a)**(1/4)*gamma(17/4) + 3840*a**(65/4)*b**
2*x**8*(1 + b*x**4/a)**(1/4)*gamma(17/4) + 5120*a**(61/4)*b**3*x**12*(1 + b
*x**4/a)**(1/4)*gamma(17/4) + 3840*a**(57/4)*b**4*x**16*(1 + b*x**4/a)**(1/
4)*gamma(17/4) + 1536*a**(53/4)*b**5*x**20*(1 + b*x**4/a)**(1/4)*gamma(17/4
) + 256*a**(49/4)*b**6*x**24*(1 + b*x**4/a)**(1/4)*gamma(17/4)) + 3159*a**1
3*b*x**5*gamma(1/4)/(256*a**(73/4)*(1 + b*x**4/a)**(1/4)*gamma(17/4) + 1536
*a**(69/4)*b*x**4*(1 + b*x**4/a)**(1/4)*gamma(17/4) + 3840*a**(65/4)*b**2*x
**8*(1 + b*x**4/a)**(1/4)*gamma(17/4) + 5120*a**(61/4)*b**3*x**12*(1 + b*x*
*4/a)**(1/4)*gamma(17/4) + 3840*a**(57/4)*b**4*x**16*(1 + b*x**4/a)**(1/4)*
gamma(17/4) + 1536*a**(53/4)*b**5*x**20*(1 + b*x**4/a)**(1/4)*gamma(17/4) +
 256*a**(49/4)*b**6*x**24*(1 + b*x**4/a)**(1/4)*gamma(17/4)) + 7215*a**12*b
**2*x**9*gamma(1/4)/(256*a**(73/4)*(1 + b*x**4/a)**(1/4)*gamma(17/4) + 1536
*a**(69/4)*b*x**4*(1 + b*x**4/a)**(1/4)*gamma(17/4) + 3840*a**(65/4)*b**2*x
**8*(1 + b*x**4/a)**(1/4)*gamma(17/4) + 5120*a**(61/4)*b**3*x**12*(1 + b*x*
*4/a)**(1/4)*gamma(17/4) + 3840*a**(57/4)*b**4*x**16*(1 + b*x**4/a)**(1/4)*
gamma(17/4) + 1536*a**(53/4)*b**5*x**20*(1 + b*x**4/a)**(1/4)*gamma(17/4) +
 256*a**(49/4)*b**6*x**24*(1 + b*x**4/a)**(1/4)*gamma(17/4)) + 8925*a**11*b
**3*x**13*gamma(1/4)/(256*a**(73/4)*(1 + b*x**4/a)**(1/4)*gamma(17/4) + 153
6*a**(69/4)*b*x**4*(1 + b*x**4/a)**(1/4)*gamma(17/4) + 3840*a**(65/4)*b**2*
x**8*(1 + b*x**4/a)**(1/4)*gamma(17/4) + 5120*a**(61/4)*b**3*x**12*(1 + b*x
**4/a)**(1/4)*gamma(17/4) + 3840*a**(57/4)*b**4*x**16*(1 + b*x**4/a)**(1/4)
*gamma(17/4) + 1536*a**(53/4)*b**5*x**20*(1 + b*x**4/a)**(1/4)*gamma(17/4)
+ 256*a**(49/4)*b**6*x**24*(1 + b*x**4/a)**(1/4)*gamma(17/4)) + 6300*a**10*
b**4*x**17*gamma(1/4)/(256*a**(73/4)*(1 + b*x**4/a)**(1/4)*gamma(17/4) + 15
36*a**(69/4)*b*x**4*(1 + b*x**4/a)**(1/4)*gamma(17/4) + 3840*a**(65/4)*b**2
*x**8*(1 + b*x**4/a)**(1/4)*gamma(17/4) + 5120*a**(61/4)*b**3*x**12*(1 + b*
x**4/a)**(1/4)*gamma(17/4) + 3840*a**(57/4)*b**4*x**16*(1 + b*x**4/a)**(1/4
)*gamma(17/4) + 1536*a**(53/4)*b**5*x**20*(1 + b*x**4/a)**(1/4)*gamma(17/4)
 + 256*a**(49/4)*b**6*x**24*(1 + b*x**4/a)**(1/4)*gamma(17/4)) + 2400*a**9*
b**5*x**21*gamma(1/4)/(256*a**(73/4)*(1 + b*x**4/a)**(1/4)*gamma(17/4) + 15
36*a**(69/4)*b*x**4*(1 + b*x**4/a)**(1/4)*gamma(17/4) + 3840*a**(65/4)*b**2
*x**8*(1 + b*x**4/a)**(1/4)*gamma(17/4) + 5120*a**(61/4)*b**3*x**12*(1 + b*
x**4/a)**(1/4)*gamma(17/4) + 3840*a**(57/4)*b**4*x**16*(1 + b*x**4/a)**(1/4
)*gamma(17/4) + 1536*a**(53/4)*b**5*x**20*(1 + b*x**4/a)**(1/4)*gamma(17/4)
 + 256*a**(49/4)*b**6*x**24*(1 + b*x**4/a)**(1/4)*gamma(17/4)) + 384*a**8*b
**6*x**25*gamma(1/4)/(256*a**(73/4)*(1 + b*x**4/a)**(1/4)*gamma(17/4) + 153
6*a**(69/4)*b*x**4*(1 + b*x**4/a)**(1/4)*gamma(17/4) + 3840*a**(65/4)*b**2*
x**8*(1 + b*x**4/a)**(1/4)*gamma(17/4) + 5120*a**(61/4)*b**3*x**12*(1 + b*x
**4/a)**(1/4)*gamma(17/4) + 3840*a**(57/4)*b**4*x**16*(1 + b*x**4/a)**(1/4)
*gamma(17/4) + 1536*a**(53/4)*b**5*x**20*(1 + b*x**4/a)**(1/4)*gamma(17/4)
+ 256*a**(49/4)*b**6*x**24*(1 + b*x**4/a)**(1/4)*gamma(17/4))
```

3.1175 $\int x^{19}\sqrt[4]{a-bx^4}\,dx$

Optimal. Leaf size=106

$$-\frac{a^4\left(a-bx^4\right)^{5/4}}{5b^5}+\frac{4a^3\left(a-bx^4\right)^{9/4}}{9b^5}-\frac{6a^2\left(a-bx^4\right)^{13/4}}{13b^5}-\frac{\left(a-bx^4\right)^{21/4}}{21b^5}+\frac{4a\left(a-bx^4\right)^{17/4}}{17b^5}$$

```
[Out] -1/5*a^4*(-b*x^4+a)^(5/4)/b^5+4/9*a^3*(-b*x^4+a)^(9/4)/b^5-6/13*a^2*(-b*x^4
+a)^(13/4)/b^5+4/17*a*(-b*x^4+a)^(17/4)/b^5-1/21*(-b*x^4+a)^(21/4)/b^5
```

Rubi [A] time = 0.06, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {266, 43}

$$-\frac{6a^2\left(a-bx^4\right)^{13/4}}{13b^5}+\frac{4a^3\left(a-bx^4\right)^{9/4}}{9b^5}-\frac{a^4\left(a-bx^4\right)^{5/4}}{5b^5}-\frac{\left(a-bx^4\right)^{21/4}}{21b^5}+\frac{4a\left(a-bx^4\right)^{17/4}}{17b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^19*(a - b*x^4)^(1/4),x]
```

```
[Out] -(a^4*(a - b*x^4)^(5/4))/(5*b^5) + (4*a^3*(a - b*x^4)^(9/4))/(9*b^5) - (6*a
^2*(a - b*x^4)^(13/4))/(13*b^5) + (4*a*(a - b*x^4)^(17/4))/(17*b^5) - (a -
b*x^4)^(21/4)/(21*b^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{19}\sqrt[4]{a-bx^4}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int x^4\sqrt[4]{a-bx}\,dx, x, x^4\right)\\ &= \frac{1}{4}\operatorname{Subst}\left(\int\left(\frac{a^4\sqrt[4]{a-bx}}{b^4}-\frac{4a^3(a-bx)^{5/4}}{b^4}+\frac{6a^2(a-bx)^{9/4}}{b^4}-\frac{4a(a-bx)^{13/4}}{b^4}+\frac{(a-bx)^{17/4}}{b^4}\right)\right)\\ &= -\frac{a^4\left(a-bx^4\right)^{5/4}}{5b^5}+\frac{4a^3\left(a-bx^4\right)^{9/4}}{9b^5}-\frac{6a^2\left(a-bx^4\right)^{13/4}}{13b^5}+\frac{4a\left(a-bx^4\right)^{17/4}}{17b^5}-\frac{\left(a-bx^4\right)^{21/4}}{21b^5}\end{aligned}$$

Mathematica [A] time = 0.05, size = 62, normalized size = 0.58

$$-\frac{\left(a-bx^4\right)^{5/4}\left(2048a^4+2560a^3bx^4+2880a^2b^2x^8+3120ab^3x^{12}+3315b^4x^{16}\right)}{69615b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^19*(a - b*x^4)^(1/4),x]
```

```
[Out] -1/69615*((a - b*x^4)^(5/4)*(2048*a^4 + 2560*a^3*b*x^4 + 2880*a^2*b^2*x^8 +
 3120*a*b^3*x^12 + 3315*b^4*x^16))/b^5
```

fricas [A] time = 0.80, size = 69, normalized size = 0.65

$$\frac{\left(3315\,b^5x^{20}-195\,ab^4x^{16}-240\,a^2b^3x^{12}-320\,a^3b^2x^8-512\,a^4bx^4-2048\,a^5\right)\left(-bx^4+a\right)^{\frac{1}{4}}}{69615\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19*(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/69615*(3315*b^5*x^20 - 195*a*b^4*x^16 - 240*a^2*b^3*x^12 - 320*a^3*b^2*x^
8 - 512*a^4*b*x^4 - 2048*a^5)*(-b*x^4 + a)^(1/4)/b^5
```

giac [A] time = 0.18, size = 120, normalized size = 1.13

$$\frac{3315\left(bx^4-a\right)^5\left(-bx^4+a\right)^{\frac{1}{4}}+16380\left(bx^4-a\right)^4\left(-bx^4+a\right)^{\frac{1}{4}}a+32130\left(bx^4-a\right)^3\left(-bx^4+a\right)^{\frac{1}{4}}a^2+30940\left(bx^4-a\right.}{69615\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19*(-b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] 1/69615*(3315*(b*x^4 - a)^5*(-b*x^4 + a)^(1/4) + 16380*(b*x^4 - a)^4*(-b*x^
4 + a)^(1/4)*a + 32130*(b*x^4 - a)^3*(-b*x^4 + a)^(1/4)*a^2 + 30940*(b*x^4
 - a)^2*(-b*x^4 + a)^(1/4)*a^3 - 13923*(-b*x^4 + a)^(5/4)*a^4)/b^5
```

maple [A] time = 0.01, size = 59, normalized size = 0.56

$$-\frac{\left(-b\,x^4+a\right)^{\frac{5}{4}}\left(3315x^{16}b^4+3120a\,x^{12}b^3+2880a^2x^8b^2+2560a^3x^4b+2048a^4\right)}{69615b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19*(-b*x^4+a)^(1/4),x)
```

```
[Out] -1/69615*(-b*x^4+a)^(5/4)*(3315*b^4*x^16+3120*a*b^3*x^12+2880*a^2*b^2*x^8+2
560*a^3*b*x^4+2048*a^4)/b^5
```

maxima [A] time = 1.33, size = 86, normalized size = 0.81

$$-\frac{\left(-bx^4+a\right)^{\frac{21}{4}}}{21\,b^5}+\frac{4\left(-bx^4+a\right)^{\frac{17}{4}}a}{17\,b^5}-\frac{6\left(-bx^4+a\right)^{\frac{13}{4}}a^2}{13\,b^5}+\frac{4\left(-bx^4+a\right)^{\frac{9}{4}}a^3}{9\,b^5}-\frac{\left(-bx^4+a\right)^{\frac{5}{4}}a^4}{5\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19*(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] -1/21*(-b*x^4 + a)^(21/4)/b^5 + 4/17*(-b*x^4 + a)^(17/4)*a/b^5 - 6/13*(-b*x
^4 + a)^(13/4)*a^2/b^5 + 4/9*(-b*x^4 + a)^(9/4)*a^3/b^5 - 1/5*(-b*x^4 + a)^
(5/4)*a^4/b^5
```

mupad [B] time = 1.15, size = 68, normalized size = 0.64

$$-\left(a-b\,x^4\right)^{1/4}\left(\frac{2048\,a^5}{69615\,b^5}-\frac{x^{20}}{21}+\frac{a\,x^{16}}{357\,b}+\frac{512\,a^4\,x^4}{69615\,b^4}+\frac{64\,a^3\,x^8}{13923\,b^3}+\frac{16\,a^2\,x^{12}}{4641\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19*(a - b*x^4)^(1/4),x)
```

```
[Out] -(a - b*x^4)^(1/4)*((2048*a^5)/(69615*b^5) - x^20/21 + (a*x^16)/(357*b) + (
512*a^4*x^4)/(69615*b^4) + (64*a^3*x^8)/(13923*b^3) + (16*a^2*x^12)/(4641*b
^2))
```

sympy [**A**] time = 31.77, size = 134, normalized size = 1.26

$$\begin{cases} -\frac{2048a^5\sqrt[4]{a-bx^4}}{69615b^5} - \frac{512a^4x^4\sqrt[4]{a-bx^4}}{69615b^4} - \frac{64a^3x^8\sqrt[4]{a-bx^4}}{13923b^3} - \frac{16a^2x^{12}\sqrt[4]{a-bx^4}}{4641b^2} - \frac{ax^{16}\sqrt[4]{a-bx^4}}{357b} + \frac{x^{20}\sqrt[4]{a-bx^4}}{21} & \text{for } b \neq 0 \\ \frac{\sqrt[4]{a}\,x^{20}}{20} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**19*(-b*x**4+a)**(1/4),x)
```

```
[Out] Piecewise((-2048*a**5*(a - b*x**4)**(1/4)/(69615*b**5) - 512*a**4*x**4*(a -
 b*x**4)**(1/4)/(69615*b**4) - 64*a**3*x**8*(a - b*x**4)**(1/4)/(13923*b**3
) - 16*a**2*x**12*(a - b*x**4)**(1/4)/(4641*b**2) - a*x**16*(a - b*x**4)**(
1/4)/(357*b) + x**20*(a - b*x**4)**(1/4)/21, Ne(b, 0)), (a**(1/4)*x**20/20,
 True))
```

3.1176 $\int x^{15}\sqrt[4]{a-bx^4}\,dx$

Optimal. Leaf size=84

$$-\frac{a^3\left(a-bx^4\right)^{5/4}}{5b^4}+\frac{a^2\left(a-bx^4\right)^{9/4}}{3b^4}+\frac{\left(a-bx^4\right)^{17/4}}{17b^4}-\frac{3a\left(a-bx^4\right)^{13/4}}{13b^4}$$

```
[Out] -1/5*a^3*(-b*x^4+a)^(5/4)/b^4+1/3*a^2*(-b*x^4+a)^(9/4)/b^4-3/13*a*(-b*x^4+a
)^(13/4)/b^4+1/17*(-b*x^4+a)^(17/4)/b^4
```

Rubi [A] time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {266, 43}

$$\frac{a^2\left(a-bx^4\right)^{9/4}}{3b^4}-\frac{a^3\left(a-bx^4\right)^{5/4}}{5b^4}+\frac{\left(a-bx^4\right)^{17/4}}{17b^4}-\frac{3a\left(a-bx^4\right)^{13/4}}{13b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^15*(a - b*x^4)^(1/4),x]
```

```
[Out] -(a^3*(a - b*x^4)^(5/4))/(5*b^4) + (a^2*(a - b*x^4)^(9/4))/(3*b^4) - (3*a*(
a - b*x^4)^(13/4))/(13*b^4) + (a - b*x^4)^(17/4)/(17*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{15}\sqrt[4]{a-bx^4}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int x^3\sqrt[4]{a-bx}\,dx, x, x^4\right)\\ &= \frac{1}{4}\operatorname{Subst}\left(\int\left(\frac{a^3\sqrt[4]{a-bx}}{b^3}-\frac{3a^2(a-bx)^{5/4}}{b^3}+\frac{3a(a-bx)^{9/4}}{b^3}-\frac{(a-bx)^{13/4}}{b^3}\right)dx, x, x^4\right)\\ &= -\frac{a^3\left(a-bx^4\right)^{5/4}}{5b^4}+\frac{a^2\left(a-bx^4\right)^{9/4}}{3b^4}-\frac{3a\left(a-bx^4\right)^{13/4}}{13b^4}+\frac{\left(a-bx^4\right)^{17/4}}{17b^4}\end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 0.61

$$-\frac{\left(a-bx^4\right)^{5/4}\left(128a^3+160a^2bx^4+180ab^2x^8+195b^3x^{12}\right)}{3315b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15*(a - b*x^4)^(1/4),x]
```

```
[Out] -1/3315*((a - b*x^4)^(5/4)*(128*a^3 + 160*a^2*b*x^4 + 180*a*b^2*x^8 + 195*b
^3*x^12))/b^4
```

fricas [A] time = 0.87, size = 58, normalized size = 0.69

$$\frac{\left(195\,b^4x^{16}-15\,ab^3x^{12}-20\,a^2b^2x^8-32\,a^3bx^4-128\,a^4\right)\left(-bx^4+a\right)^{\frac{1}{4}}}{3315\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/3315*(195*b^4*x^16 - 15*a*b^3*x^12 - 20*a^2*b^2*x^8 - 32*a^3*b*x^4 - 128*
a^4)*(-b*x^4 + a)^(1/4)/b^4
```

giac [A] time = 0.16, size = 94, normalized size = 1.12

$$\frac{195\left(bx^4-a\right)^4\left(-bx^4+a\right)^{\frac{1}{4}}+765\left(bx^4-a\right)^3\left(-bx^4+a\right)^{\frac{1}{4}}a+1105\left(bx^4-a\right)^2\left(-bx^4+a\right)^{\frac{1}{4}}a^2-663\left(-bx^4+a\right)^{\frac{5}{4}}a^3}{3315\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(-b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] 1/3315*(195*(b*x^4 - a)^4*(-b*x^4 + a)^(1/4) + 765*(b*x^4 - a)^3*(-b*x^4 +
a)^(1/4)*a + 1105*(b*x^4 - a)^2*(-b*x^4 + a)^(1/4)*a^2 - 663*(-b*x^4 + a)^(
5/4)*a^3)/b^4
```

maple [A] time = 0.01, size = 48, normalized size = 0.57

$$-\frac{\left(-b\,x^4+a\right)^{\frac{5}{4}}\left(195b^3x^{12}+180a\,b^2x^8+160a^2b\,x^4+128a^3\right)}{3315b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15*(-b*x^4+a)^(1/4),x)
```

```
[Out] -1/3315*(-b*x^4+a)^(5/4)*(195*b^3*x^12+180*a*b^2*x^8+160*a^2*b*x^4+128*a^3)
/b^4
```

maxima [A] time = 1.41, size = 68, normalized size = 0.81

$$\frac{\left(-bx^4+a\right)^{\frac{17}{4}}}{17\,b^4}-\frac{3\left(-bx^4+a\right)^{\frac{13}{4}}a}{13\,b^4}+\frac{\left(-bx^4+a\right)^{\frac{9}{4}}a^2}{3\,b^4}-\frac{\left(-bx^4+a\right)^{\frac{5}{4}}a^3}{5\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] 1/17*(-b*x^4 + a)^(17/4)/b^4 - 3/13*(-b*x^4 + a)^(13/4)*a/b^4 + 1/3*(-b*x^4
 + a)^(9/4)*a^2/b^4 - 1/5*(-b*x^4 + a)^(5/4)*a^3/b^4
```

mupad [B] time = 1.12, size = 57, normalized size = 0.68

$$-\left(a-b\,x^4\right)^{1/4}\left(\frac{128\,a^4}{3315\,b^4}-\frac{x^{16}}{17}+\frac{a\,x^{12}}{221\,b}+\frac{32\,a^3\,x^4}{3315\,b^3}+\frac{4\,a^2\,x^8}{663\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15*(a - b*x^4)^(1/4),x)
```

```
[Out] -(a - b*x^4)^(1/4)*((128*a^4)/(3315*b^4) - x^16/17 + (a*x^12)/(221*b) + (32
*a^3*x^4)/(3315*b^3) + (4*a^2*x^8)/(663*b^2))
```

sympy [A] time = 13.33, size = 110, normalized size = 1.31

$$\begin{cases} -\frac{128a^4\sqrt[4]{a-bx^4}}{3315b^4} - \frac{32a^3x^4\sqrt[4]{a-bx^4}}{3315b^3} - \frac{4a^2x^8\sqrt[4]{a-bx^4}}{663b^2} - \frac{ax^{12}\sqrt[4]{a-bx^4}}{221b} + \frac{x^{16}\sqrt[4]{a-bx^4}}{17} & \text{for } b \neq 0 \\ \frac{\sqrt[4]{a}\,x^{16}}{16} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15*(-b*x**4+a)**(1/4),x)
```

```
[Out] Piecewise((-128*a**4*(a - b*x**4)**(1/4)/(3315*b**4) - 32*a**3*x**4*(a - b*
x**4)**(1/4)/(3315*b**3) - 4*a**2*x**8*(a - b*x**4)**(1/4)/(663*b**2) - a*x
**12*(a - b*x**4)**(1/4)/(221*b) + x**16*(a - b*x**4)**(1/4)/17, Ne(b, 0)),
 (a**(1/4)*x**16/16, True))
```

3.1177 $\int x^{11}\sqrt[4]{a-bx^4}\,dx$

Optimal. Leaf size=62

$$-\frac{a^2\left(a-bx^4\right)^{5/4}}{5b^3}-\frac{\left(a-bx^4\right)^{13/4}}{13b^3}+\frac{2a\left(a-bx^4\right)^{9/4}}{9b^3}$$

```
[Out] -1/5*a^2*(-b*x^4+a)^(5/4)/b^3+2/9*a*(-b*x^4+a)^(9/4)/b^3-1/13*(-b*x^4+a)^(1
3/4)/b^3
```

Rubi [A] time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {266, 43}

$$-\frac{a^2\left(a-bx^4\right)^{5/4}}{5b^3}-\frac{\left(a-bx^4\right)^{13/4}}{13b^3}+\frac{2a\left(a-bx^4\right)^{9/4}}{9b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a - b*x^4)^(1/4),x]
```

```
[Out] -(a^2*(a - b*x^4)^(5/4))/(5*b^3) + (2*a*(a - b*x^4)^(9/4))/(9*b^3) - (a - b
*x^4)^(13/4)/(13*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{11}\sqrt[4]{a-bx^4}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int x^2\sqrt[4]{a-bx}\,dx, x, x^4\right)\\ &= \frac{1}{4}\operatorname{Subst}\left(\int\left(\frac{a^2\sqrt[4]{a-bx}}{b^2}-\frac{2a(a-bx)^{5/4}}{b^2}+\frac{(a-bx)^{9/4}}{b^2}\right)dx, x, x^4\right)\\ &= -\frac{a^2\left(a-bx^4\right)^{5/4}}{5b^3}+\frac{2a\left(a-bx^4\right)^{9/4}}{9b^3}-\frac{\left(a-bx^4\right)^{13/4}}{13b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 0.65

$$-\frac{\left(a-bx^4\right)^{5/4}\left(32a^2+40abx^4+45b^2x^8\right)}{585b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a - b*x^4)^(1/4),x]
```

```
[Out] -1/585*((a - b*x^4)^(5/4)*(32*a^2 + 40*a*b*x^4 + 45*b^2*x^8))/b^3
```

fricas [A] time = 0.85, size = 47, normalized size = 0.76

$$\frac{\left(45\,b^3x^{12}-5\,ab^2x^8-8\,a^2bx^4-32\,a^3\right)\left(-bx^4+a\right)^{\frac{1}{4}}}{585\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/585*(45*b^3*x^12 - 5*a*b^2*x^8 - 8*a^2*b*x^4 - 32*a^3)*(-b*x^4 + a)^(1/4)
/b^3
```

giac [A] time = 0.16, size = 68, normalized size = 1.10

$$\frac{45\left(bx^4-a\right)^3\left(-bx^4+a\right)^{\frac{1}{4}}+130\left(bx^4-a\right)^2\left(-bx^4+a\right)^{\frac{1}{4}}a-117\left(-bx^4+a\right)^{\frac{5}{4}}a^2}{585\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(-b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] 1/585*(45*(b*x^4 - a)^3*(-b*x^4 + a)^(1/4) + 130*(b*x^4 - a)^2*(-b*x^4 + a)
^(1/4)*a - 117*(-b*x^4 + a)^(5/4)*a^2)/b^3
```

maple [A] time = 0.01, size = 37, normalized size = 0.60

$$-\frac{\left(-b\,x^4+a\right)^{\frac{5}{4}}\left(45b^2x^8+40ab\,x^4+32a^2\right)}{585b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(-b*x^4+a)^(1/4),x)
```

```
[Out] -1/585*(-b*x^4+a)^(5/4)*(45*b^2*x^8+40*a*b*x^4+32*a^2)/b^3
```

maxima [A] time = 1.35, size = 50, normalized size = 0.81

$$-\frac{\left(-bx^4+a\right)^{\frac{13}{4}}}{13\,b^3}+\frac{2\left(-bx^4+a\right)^{\frac{9}{4}}a}{9\,b^3}-\frac{\left(-bx^4+a\right)^{\frac{5}{4}}a^2}{5\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] -1/13*(-b*x^4 + a)^(13/4)/b^3 + 2/9*(-b*x^4 + a)^(9/4)*a/b^3 - 1/5*(-b*x^4
+ a)^(5/4)*a^2/b^3
```

mupad [B] time = 1.13, size = 46, normalized size = 0.74

$$-\left(a-b\,x^4\right)^{1/4}\left(\frac{32\,a^3}{585\,b^3}-\frac{x^{12}}{13}+\frac{a\,x^8}{117\,b}+\frac{8\,a^2\,x^4}{585\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a - b*x^4)^(1/4),x)
```

```
[Out] -(a - b*x^4)^(1/4)*((32*a^3)/(585*b^3) - x^12/13 + (a*x^8)/(117*b) + (8*a^2
*x^4)/(585*b^2))
```

sympy [**A**] time = 6.25, size = 87, normalized size = 1.40

$$\begin{cases} -\frac{32a^3\sqrt[4]{a-bx^4}}{585b^3} - \frac{8a^2x^4\sqrt[4]{a-bx^4}}{585b^2} - \frac{ax^8\sqrt[4]{a-bx^4}}{117b} + \frac{x^{12}\sqrt[4]{a-bx^4}}{13} & \text{for } b \neq 0 \\ \frac{\sqrt[4]{a}\,x^{12}}{12} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(-b*x**4+a)**(1/4),x)
```

```
[Out] Piecewise((-32*a**3*(a - b*x**4)**(1/4)/(585*b**3) - 8*a**2*x**4*(a - b*x**
4)**(1/4)/(585*b**2) - a*x**8*(a - b*x**4)**(1/4)/(117*b) + x**12*(a - b*x*
*4)**(1/4)/13, Ne(b, 0)), (a**(1/4)*x**12/12, True))
```

3.1178 $\int x^7 \sqrt[4]{a-bx^4}\,dx$

Optimal. Leaf size=40

$$\frac{\left(a-bx^4\right)^{9/4}}{9b^2}-\frac{a\left(a-bx^4\right)^{5/4}}{5b^2}$$

```
[Out] -1/5*a*(-b*x^4+a)^(5/4)/b^2+1/9*(-b*x^4+a)^(9/4)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {266, 43}

$$\frac{\left(a-bx^4\right)^{9/4}}{9b^2}-\frac{a\left(a-bx^4\right)^{5/4}}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a - b*x^4)^(1/4),x]
```

```
[Out] -(a*(a - b*x^4)^(5/4))/(5*b^2) + (a - b*x^4)^(9/4)/(9*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^7 \sqrt[4]{a-bx^4}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int x\sqrt[4]{a-bx}\,dx, x, x^4\right)\\ &= \frac{1}{4}\operatorname{Subst}\left(\int\left(\frac{a\sqrt[4]{a-bx}}{b}-\frac{(a-bx)^{5/4}}{b}\right)dx, x, x^4\right)\\ &= -\frac{a\left(a-bx^4\right)^{5/4}}{5b^2}+\frac{\left(a-bx^4\right)^{9/4}}{9b^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 29, normalized size = 0.72

$$-\frac{\left(a-bx^4\right)^{5/4}\left(4a+5bx^4\right)}{45b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a - b*x^4)^(1/4),x]
```

```
[Out] -1/45*((a - b*x^4)^(5/4)*(4*a + 5*b*x^4))/b^2
```

fricas [A] time = 0.83, size = 36, normalized size = 0.90

$$\frac{\left(5\,b^2x^8 - abx^4 - 4\,a^2\right)\left(-bx^4 + a\right)^{\frac{1}{4}}}{45\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(-b*x^4+a)^(1/4),x, algorithm="fricas")
[Out] 1/45*(5*b^2*x^8 - a*b*x^4 - 4*a^2)*(-b*x^4 + a)^(1/4)/b^2
```

giac [A] time = 0.17, size = 42, normalized size = 1.05

$$\frac{5\left(bx^4 - a\right)^2\left(-bx^4 + a\right)^{\frac{1}{4}} - 9\left(-bx^4 + a\right)^{\frac{5}{4}}a}{45\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(-b*x^4+a)^(1/4),x, algorithm="giac")
[Out] 1/45*(5*(b*x^4 - a)^2*(-b*x^4 + a)^(1/4) - 9*(-b*x^4 + a)^(5/4)*a)/b^2
```

maple [A] time = 0.01, size = 26, normalized size = 0.65

$$-\frac{\left(-b\,x^4 + a\right)^{\frac{5}{4}}\left(5b\,x^4 + 4a\right)}{45b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(-b*x^4+a)^(1/4),x)
[Out] -1/45*(-b*x^4+a)^(5/4)*(5*b*x^4+4*a)/b^2
```

maxima [A] time = 1.30, size = 32, normalized size = 0.80

$$\frac{\left(-bx^4 + a\right)^{\frac{9}{4}}}{9\,b^2} - \frac{\left(-bx^4 + a\right)^{\frac{5}{4}}a}{5\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(-b*x^4+a)^(1/4),x, algorithm="maxima")
[Out] 1/9*(-b*x^4 + a)^(9/4)/b^2 - 1/5*(-b*x^4 + a)^(5/4)*a/b^2
```

mupad [B] time = 1.12, size = 35, normalized size = 0.88

$$-\left(a - b\,x^4\right)^{1/4}\left(\frac{4\,a^2}{45\,b^2} - \frac{x^8}{9} + \frac{a\,x^4}{45\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a - b*x^4)^(1/4),x)
[Out] -(a - b*x^4)^(1/4)*((4*a^2)/(45*b^2) - x^8/9 + (a*x^4)/(45*b))
```

sympy [A] time = 1.84, size = 63, normalized size = 1.58

$$\begin{cases} -\frac{4a^2\sqrt[4]{a-bx^4}}{45b^2} - \frac{ax^4\sqrt[4]{a-bx^4}}{45b} + \frac{x^8\sqrt[4]{a-bx^4}}{9} & \text{for } b \neq 0 \\ \frac{\sqrt[4]{a}\,x^8}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(-b*x**4+a)**(1/4),x)
```

```
[Out] Piecewise((-4*a**2*(a - b*x**4)**(1/4)/(45*b**2) - a*x**4*(a - b*x**4)**(1/
4)/(45*b) + x**8*(a - b*x**4)**(1/4)/9, Ne(b, 0)), (a**(1/4)*x**8/8, True))
```

3.1179 $\int x^3 \sqrt[4]{a - bx^4}\, dx$

Optimal. Leaf size=19

$$-\frac{\left(a - bx^4\right)^{5/4}}{5b}$$

```
[Out] -1/5*(-b*x^4+a)^(5/4)/b
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {261}

$$-\frac{\left(a - bx^4\right)^{5/4}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a - b*x^4)^(1/4),x]
```

```
[Out] -(a - b*x^4)^(5/4)/(5*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^3 \sqrt[4]{a - bx^4}\, dx = -\frac{\left(a - bx^4\right)^{5/4}}{5b}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$-\frac{\left(a - bx^4\right)^{5/4}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a - b*x^4)^(1/4),x]
```

```
[Out] -1/5*(a - b*x^4)^(5/4)/b
```

fricas [A] time = 1.04, size = 24, normalized size = 1.26

$$\frac{\left(bx^4 - a\right)\left(-bx^4 + a\right)^{\frac{1}{4}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/5*(b*x^4 - a)*(-b*x^4 + a)^(1/4)/b
```

giac [A] time = 0.15, size = 15, normalized size = 0.79

$$-\frac{\left(-bx^4 + a\right)^{\frac{5}{4}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-b*x^4+a)^(1/4),x, algorithm="giac")
[Out] -1/5*(-b*x^4 + a)^(5/4)/b
```

maple [A] time = 0.00, size = 16, normalized size = 0.84

$$-\frac{\left(-b\,x^4+a\right)^{\frac{5}{4}}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-b*x^4+a)^(1/4),x)
[Out] -1/5*(-b*x^4+a)^(5/4)/b
```

maxima [A] time = 1.28, size = 15, normalized size = 0.79

$$-\frac{\left(-bx^4+a\right)^{\frac{5}{4}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-b*x^4+a)^(1/4),x, algorithm="maxima")
[Out] -1/5*(-b*x^4 + a)^(5/4)/b
```

mupad [B] time = 1.11, size = 15, normalized size = 0.79

$$-\frac{\left(a-b\,x^4\right)^{5/4}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a - b*x^4)^(1/4),x)
[Out] -(a - b*x^4)^(5/4)/(5*b)
```

sympy [A] time = 0.69, size = 39, normalized size = 2.05

$$\begin{cases} -\frac{a\sqrt[4]{a-bx^4}}{5b}+\frac{x^4\sqrt[4]{a-bx^4}}{5} & \text{for } b\neq 0 \\ \frac{\sqrt[4]{a}\,x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-b*x**4+a)**(1/4),x)
[Out] Piecewise((-a*(a - b*x**4)**(1/4)/(5*b) + x**4*(a - b*x**4)**(1/4)/5, Ne(b,
 0)), (a**(1/4)*x**4/4, True))
```

3.1180 $\int \frac{\sqrt[4]{a-bx^4}}{x}\,dx$

Optimal. Leaf size=69

$$\sqrt[4]{a-bx^4} - \frac{1}{2}\sqrt[4]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right) - \frac{1}{2}\sqrt[4]{a}\,\tanh^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)$$

```
[Out] (-b*x^4+a)^(1/4)-1/2*a^(1/4)*arctan((-b*x^4+a)^(1/4)/a^(1/4))-1/2*a^(1/4)*a
rctanh((-b*x^4+a)^(1/4)/a^(1/4))
```

Rubi [A] time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {266, 50, 63, 212, 206, 203}

$$\sqrt[4]{a-bx^4} - \frac{1}{2}\sqrt[4]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right) - \frac{1}{2}\sqrt[4]{a}\,\tanh^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^4)^(1/4)/x,x]
```

```
[Out] (a - b*x^4)^(1/4) - (a^(1/4)*ArcTan[(a - b*x^4)^(1/4)/a^(1/4)])/2 - (a^(1/4
)*ArcTanh[(a - b*x^4)^(1/4)/a^(1/4)])/2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{a-bx^4}}{x}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{\sqrt[4]{a-bx}}{x}\,dx, x, x^4\right) \\
&= \sqrt[4]{a-bx^4} + \frac{1}{4}a\,\text{Subst}\left(\int \frac{1}{x(a-bx)^{3/4}}\,dx, x, x^4\right) \\
&= \sqrt[4]{a-bx^4} - \frac{a\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}-\frac{x^4}{b}}\,dx, x, \sqrt[4]{a-bx^4}\right)}{b} \\
&= \sqrt[4]{a-bx^4} - \frac{1}{2}\sqrt{a}\,\text{Subst}\left(\int \frac{1}{\sqrt{a}-x^2}\,dx, x, \sqrt[4]{a-bx^4}\right) - \frac{1}{2}\sqrt{a}\,\text{Subst}\left(\int \frac{1}{\sqrt{a}+x^2}\,dx, x, \sqrt[4]{a}\right. \\
&= \sqrt[4]{a-bx^4} - \frac{1}{2}\sqrt[4]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right) - \frac{1}{2}\sqrt[4]{a}\,\tanh^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 69, normalized size = 1.00

$$\sqrt[4]{a-bx^4} - \frac{1}{2}\sqrt[4]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right) - \frac{1}{2}\sqrt[4]{a}\,\tanh^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^4)^(1/4)/x,x]
```

```
[Out] (a - b*x^4)^(1/4) - (a^(1/4)*ArcTan[(a - b*x^4)^(1/4)/a^(1/4)])/2 - (a^(1/4
)*ArcTanh[(a - b*x^4)^(1/4)/a^(1/4)])/2
```

fricas [A] time = 0.69, size = 98, normalized size = 1.42

$$a^{\frac{1}{4}}\arctan\left(\frac{a^{\frac{3}{4}}\sqrt{\sqrt{-bx^4+a}+\sqrt{a}} - \left(-bx^4+a\right)^{\frac{1}{4}}a^{\frac{3}{4}}}{a}\right) - \frac{1}{4}a^{\frac{1}{4}}\log\left(\left(-bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}\right) + \frac{1}{4}a^{\frac{1}{4}}\log\left(\left(-bx^4+a\right)^{\frac{1}{4}} -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x,x, algorithm="fricas")
```

```
[Out] a^(1/4)*arctan((a^(3/4)*sqrt(sqrt(-b*x^4 + a) + sqrt(a)) - (-b*x^4 + a)^(1/
4)*a^(3/4))/a) - 1/4*a^(1/4)*log((-b*x^4 + a)^(1/4) + a^(1/4)) + 1/4*a^(1/4
)*log((-b*x^4 + a)^(1/4) - a^(1/4)) + (-b*x^4 + a)^(1/4)
```

giac [B] time = 0.17, size = 190, normalized size = 2.75

$$-\frac{1}{4}\sqrt{2}\,(-a)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}+2\left(-bx^4+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right) - \frac{1}{4}\sqrt{2}\,(-a)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}-2\left(-bx^4+a\right)^{\frac{1}{4}}\right.\right.}{2\,(-a)^{\frac{1}{4}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(-a)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(-b*x^4
+ a)^(1/4))/(-a)^(1/4)) - 1/4*sqrt(2)*(-a)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(
2)*(-a)^(1/4) - 2*(-b*x^4 + a)^(1/4))/(-a)^(1/4)) - 1/8*sqrt(2)*(-a)^(1/4)*
log(sqrt(2)*(-b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(-b*x^4 + a) + sqrt(-a)) +
1/8*sqrt(2)*(-a)^(1/4)*log(-sqrt(2)*(-b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(-b
*x^4 + a) + sqrt(-a)) + (-b*x^4 + a)^(1/4)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^4+a\right)^{\frac{1}{4}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(1/4)/x,x)
```

```
[Out] int((-b*x^4+a)^(1/4)/x,x)
```

maxima [A] time = 3.14, size = 70, normalized size = 1.01

$$-\frac{1}{2}\,a^{\frac{1}{4}}\arctan\left(\frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)+\frac{1}{4}\,a^{\frac{1}{4}}\log\left(\frac{\left(-bx^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{\left(-bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)+\left(-bx^4+a\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x,x, algorithm="maxima")
```

```
[Out] -1/2*a^(1/4)*arctan((-b*x^4 + a)^(1/4)/a^(1/4)) + 1/4*a^(1/4)*log(((-b*x^4
+ a)^(1/4) - a^(1/4))/((-b*x^4 + a)^(1/4) + a^(1/4))) + (-b*x^4 + a)^(1/4)
```

mupad [B] time = 1.21, size = 51, normalized size = 0.74

$$\left(a-b\,x^4\right)^{1/4}-\frac{a^{1/4}\,\mathrm{atanh}\left(\frac{\left(a-b\,x^4\right)^{1/4}}{a^{1/4}}\right)}{2}-\frac{a^{1/4}\,\mathrm{atan}\left(\frac{\left(a-b\,x^4\right)^{1/4}}{a^{1/4}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^4)^(1/4)/x,x)
```

```
[Out] (a - b*x^4)^(1/4) - (a^(1/4)*atanh((a - b*x^4)^(1/4)/a^(1/4)))/2 - (a^(1/4)
*atan((a - b*x^4)^(1/4)/a^(1/4)))/2
```

sympy [C] time = 1.52, size = 44, normalized size = 0.64

$$-\frac{\sqrt[4]{b}\,x e^{\frac{i\pi}{4}}\,\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{4},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\right|\frac{a}{bx^4}\right)}{4\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(1/4)/x,x)
```

```
[Out] -b**(1/4)*x*exp(I*pi/4)*gamma(-1/4)*hyper((-1/4, -1/4), (3/4,), a/(b*x**4))
/(4*gamma(3/4))
```

3.1181 $\int \frac{\sqrt[4]{a-bx^4}}{x^5}\,dx$

Optimal. Leaf size=78

$$\frac{b\tan^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{8a^{3/4}}+\frac{b\tanh^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{8a^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{4x^4}$$

```
[Out] -1/4*(-b*x^4+a)^(1/4)/x^4+1/8*b*arctan((-b*x^4+a)^(1/4)/a^(1/4))/a^(3/4)+1/
8*b*arctanh((-b*x^4+a)^(1/4)/a^(1/4))/a^(3/4)
```

Rubi [A] time = 0.04, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {266, 47, 63, 212, 206, 203}

$$\frac{b\tan^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{8a^{3/4}}+\frac{b\tanh^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{8a^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^4)^(1/4)/x^5,x]
```

```
[Out] -(a - b*x^4)^(1/4)/(4*x^4) + (b*ArcTan[(a - b*x^4)^(1/4)/a^(1/4)])/(8*a^(3/
4)) + (b*ArcTanh[(a - b*x^4)^(1/4)/a^(1/4)])/(8*a^(3/4))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{a-bx^4}}{x^5}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{\sqrt[4]{a-bx}}{x^2}\,dx, x, x^4\right)\\
&= -\frac{\sqrt[4]{a-bx^4}}{4x^4} - \frac{1}{16}b\,\text{Subst}\left(\int \frac{1}{x(a-bx)^{3/4}}\,dx, x, x^4\right)\\
&= -\frac{\sqrt[4]{a-bx^4}}{4x^4} + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}-\frac{x^4}{b}}\,dx, x, \sqrt[4]{a-bx^4}\right)\\
&= -\frac{\sqrt[4]{a-bx^4}}{4x^4} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a}-x^2}\,dx, x, \sqrt[4]{a-bx^4}\right)}{8\sqrt{a}} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a}+x^2}\,dx, x, \sqrt[4]{a-bx^4}\right)}{8\sqrt{a}}\\
&= -\frac{\sqrt[4]{a-bx^4}}{4x^4} + \frac{b\tan^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{8a^{3/4}} + \frac{b\tanh^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{8a^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.50

$$-\frac{b\left(a-bx^4\right)^{5/4}\,{}_2F_1\left(\frac{5}{4},2;\frac{9}{4};1-\frac{bx^4}{a}\right)}{5a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^4)^(1/4)/x^5,x]
```

```
[Out] -1/5*(b*(a - b*x^4)^(5/4)*Hypergeometric2F1[5/4, 2, 9/4, 1 - (b*x^4)/a])/a^
2
```

fricas [B] time = 0.89, size = 185, normalized size = 2.37

$$-\frac{4\left(\frac{b^4}{a^3}\right)^{\frac{1}{4}}x^4\arctan\left(-\frac{\left(-bx^4+a\right)^{\frac{1}{4}}a^2b\left(\frac{b^4}{a^3}\right)^{\frac{3}{4}}-\sqrt{\sqrt{-bx^4+a}\,b^2+a^2\sqrt{\frac{b^4}{a^3}}}\,a^2\left(\frac{b^4}{a^3}\right)^{\frac{3}{4}}}{b^4}\right)-\left(\frac{b^4}{a^3}\right)^{\frac{1}{4}}x^4\log\left(\left(-bx^4+a\right)^{\frac{1}{4}}b+a\left(\frac{b^4}{a^3}\right)^{\frac{1}{4}}\right)+\left(\frac{b^4}{a^3}\right.}{16\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^5,x, algorithm="fricas")
```

```
[Out] -1/16*(4*(b^4/a^3)^(1/4)*x^4*arctan(-((-b*x^4 + a)^(1/4)*a^2*b*(b^4/a^3)^(3
/4) - sqrt(sqrt(-b*x^4 + a)*b^2 + a^2*sqrt(b^4/a^3))*a^2*(b^4/a^3)^(3/4))/b
^4) - (b^4/a^3)^(1/4)*x^4*log((-b*x^4 + a)^(1/4)*b + a*(b^4/a^3)^(1/4)) + (
b^4/a^3)^(1/4)*x^4*log((-b*x^4 + a)^(1/4)*b - a*(b^4/a^3)^(1/4)) + 4*(-b*x^
4 + a)^(1/4))/x^4
```

giac [B] time = 0.18, size = 220, normalized size = 2.82

$$\frac{2\sqrt{2}(-a)^{\frac{1}{4}}b^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(-bx^4+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a}+\frac{2\sqrt{2}(-a)^{\frac{1}{4}}b^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(-bx^4+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a}+\frac{\sqrt{2}(-a)^{\frac{1}{4}}b^2\log\left(\sqrt{2}\left(-bx^4+a\right)^{\frac{1}{4}}(-a)^{\frac{1}{4}}\right.}{a}$$

$$\frac{}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^5,x, algorithm="giac")
```

```
[Out] 1/32*(2*sqrt(2)*(-a)^(1/4)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(
-b*x^4 + a)^(1/4))/(-a)^(1/4))/a + 2*sqrt(2)*(-a)^(1/4)*b^2*arctan(-1/2*sqr
t(2)*(sqrt(2)*(-a)^(1/4) - 2*(-b*x^4 + a)^(1/4))/(-a)^(1/4))/a + sqrt(2)*(-
a)^(1/4)*b^2*log(sqrt(2)*(-b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(-b*x^4 + a) +
 sqrt(-a))/a + sqrt(2)*b^2*log(-sqrt(2)*(-b*x^4 + a)^(1/4)*(-a)^(1/4) + sqr
t(-b*x^4 + a) + sqrt(-a))/(-a)^(3/4) - 8*(-b*x^4 + a)^(1/4)*b/x^4)/b
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^4+a\right)^{\frac{1}{4}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(1/4)/x^5,x)
```

```
[Out] int((-b*x^4+a)^(1/4)/x^5,x)
```

maxima [A] time = 3.01, size = 77, normalized size = 0.99

$$\frac{b\arctan\left(\frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{8\,a^{\frac{3}{4}}}-\frac{b\log\left(\frac{\left(-bx^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{\left(-bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{16\,a^{\frac{3}{4}}}-\frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*b*arctan((-b*x^4 + a)^(1/4)/a^(1/4))/a^(3/4) - 1/16*b*log(((-b*x^4 + a)
^(1/4) - a^(1/4))/((-b*x^4 + a)^(1/4) + a^(1/4)))/a^(3/4) - 1/4*(-b*x^4 + a
)^(1/4)/x^4
```

mupad [B] time = 1.29, size = 58, normalized size = 0.74

$$\frac{b\,\mathrm{atan}\left(\frac{\left(a-b\,x^4\right)^{1/4}}{a^{1/4}}\right)}{8\,a^{3/4}}-\frac{\left(a-b\,x^4\right)^{1/4}}{4\,x^4}+\frac{b\,\mathrm{atanh}\left(\frac{\left(a-b\,x^4\right)^{1/4}}{a^{1/4}}\right)}{8\,a^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^4)^(1/4)/x^5,x)
```

```
[Out] (b*atan((a - b*x^4)^(1/4)/a^(1/4)))/(8*a^(3/4)) - (a - b*x^4)^(1/4)/(4*x^4)
 + (b*atanh((a - b*x^4)^(1/4)/a^(1/4)))/(8*a^(3/4))
```

sympy [C] time = 1.95, size = 42, normalized size = 0.54

$$\frac{\sqrt[4]{b}\, e^{-\frac{3i\pi}{4}} \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\left.\begin{matrix} -\frac{1}{4}, \frac{3}{4} \\ \frac{7}{4} \end{matrix}\right| \frac{a}{bx^4}\right)}{4x^3 \Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(1/4)/x**5,x)
```

```
[Out] b**(1/4)*exp(-3*I*pi/4)*gamma(3/4)*hyper((-1/4, 3/4), (7/4,), a/(b*x**4))/(
4*x**3*gamma(7/4))
```

3.1182 $\int \frac{\sqrt[4]{a-bx^4}}{x^9}\,dx$

Optimal. Leaf size=105

$$\frac{3b^2\tan^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{64a^{7/4}}+\frac{3b^2\tanh^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{64a^{7/4}}+\frac{b\sqrt[4]{a-bx^4}}{32ax^4}-\frac{\sqrt[4]{a-bx^4}}{8x^8}$$

```
[Out] -1/8*(-b*x^4+a)^(1/4)/x^8+1/32*b*(-b*x^4+a)^(1/4)/a/x^4+3/64*b^2*arctan((-b
*x^4+a)^(1/4)/a^(1/4))/a^(7/4)+3/64*b^2*arctanh((-b*x^4+a)^(1/4)/a^(1/4))/a
^(7/4)
```

Rubi [A] time = 0.06, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {266, 47, 51, 63, 212, 206, 203}

$$\frac{3b^2\tan^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{64a^{7/4}}+\frac{3b^2\tanh^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{64a^{7/4}}+\frac{b\sqrt[4]{a-bx^4}}{32ax^4}-\frac{\sqrt[4]{a-bx^4}}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^4)^(1/4)/x^9,x]
```

```
[Out] -(a - b*x^4)^(1/4)/(8*x^8) + (b*(a - b*x^4)^(1/4))/(32*a*x^4) + (3*b^2*ArcT
an[(a - b*x^4)^(1/4)/a^(1/4)])/(64*a^(7/4)) + (3*b^2*ArcTanh[(a - b*x^4)^(1
/4)/a^(1/4)])/(64*a^(7/4))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{a-bx^4}}{x^9}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{\sqrt[4]{a-bx}}{x^3}\,dx, x, x^4\right)\\
&= -\frac{\sqrt[4]{a-bx^4}}{8x^8} - \frac{1}{32}b\,\text{Subst}\left(\int \frac{1}{x^2(a-bx)^{3/4}}\,dx, x, x^4\right)\\
&= -\frac{\sqrt[4]{a-bx^4}}{8x^8} + \frac{b\sqrt[4]{a-bx^4}}{32ax^4} - \frac{\left(3b^2\right)\text{Subst}\left(\int \frac{1}{x(a-bx)^{3/4}}\,dx, x, x^4\right)}{128a}\\
&= -\frac{\sqrt[4]{a-bx^4}}{8x^8} + \frac{b\sqrt[4]{a-bx^4}}{32ax^4} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}-\frac{x^4}{b}}\,dx, x, \sqrt[4]{a-bx^4}\right)}{32a}\\
&= -\frac{\sqrt[4]{a-bx^4}}{8x^8} + \frac{b\sqrt[4]{a-bx^4}}{32ax^4} + \frac{\left(3b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a}-x^2}\,dx, x, \sqrt[4]{a-bx^4}\right)}{64a^{3/2}} + \frac{\left(3b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a}+x^2}\,dx\right.}{64a^{3/2}}\\
&= -\frac{\sqrt[4]{a-bx^4}}{8x^8} + \frac{b\sqrt[4]{a-bx^4}}{32ax^4} + \frac{3b^2\tan^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{64a^{7/4}} + \frac{3b^2\tanh^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{64a^{7/4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 41, normalized size = 0.39

$$-\frac{b^2\left(a-bx^4\right)^{5/4}\,{}_2F_1\left(\frac{5}{4},3;\frac{9}{4};1-\frac{bx^4}{a}\right)}{5a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^4)^(1/4)/x^9,x]
```

```
[Out] -1/5*(b^2*(a - b*x^4)^(5/4)*Hypergeometric2F1[5/4, 3, 9/4, 1 - (b*x^4)/a])/
a^3
```

fricas [B] time = 1.00, size = 214, normalized size = 2.04

$$-\frac{12\,a\left(\frac{b^8}{a^7}\right)^{\frac{1}{4}}x^8\arctan\left(-\frac{\left(-bx^4+a\right)^{\frac{1}{4}}a^5b^2\left(\frac{b^8}{a^7}\right)^{\frac{3}{4}}-\sqrt{\sqrt{-bx^4+a}\,b^4+a^4\sqrt{\frac{b^8}{a^7}}}\,a^5\left(\frac{b^8}{a^7}\right)^{\frac{3}{4}}}{b^8}\right)-3\,a\left(\frac{b^8}{a^7}\right)^{\frac{1}{4}}x^8\log\left(3\left(-bx^4+a\right)^{\frac{1}{4}}b^2+3\,a^2\left(\right.\right.}{128\,ax^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^9,x, algorithm="fricas")
```

```
[Out] -1/128*(12*a*(b^8/a^7)^(1/4)*x^8*arctan(-((-b*x^4 + a)^(1/4)*a^5*b^2*(b^8/a
^7)^(3/4) - sqrt(sqrt(-b*x^4 + a)*b^4 + a^4*sqrt(b^8/a^7))*a^5*(b^8/a^7)^(3
/4))/b^8) - 3*a*(b^8/a^7)^(1/4)*x^8*log(3*(-b*x^4 + a)^(1/4)*b^2 + 3*a^2*(b
^8/a^7)^(1/4)) + 3*a*(b^8/a^7)^(1/4)*x^8*log(3*(-b*x^4 + a)^(1/4)*b^2 - 3*a
^2*(b^8/a^7)^(1/4)) - 4*(b*x^4 - 4*a)*(-b*x^4 + a)^(1/4))/(a*x^8)
```

giac [B] time = 0.21, size = 251, normalized size = 2.39

$$\frac{6\sqrt{2}(-a)^{\frac{1}{4}}b^3\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(-bx^4+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a^2}+\frac{6\sqrt{2}(-a)^{\frac{1}{4}}b^3\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(-bx^4+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a^2}+\frac{3\sqrt{2}(-a)^{\frac{1}{4}}b^3\log\left(\sqrt{2}\left(-bx^4+a\right)^{\frac{1}{4}}(-a)\right.}{a^2}$$

$$256\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^9,x, algorithm="giac")
```

```
[Out] 1/256*(6*sqrt(2)*(-a)^(1/4)*b^3*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*
(-b*x^4 + a)^(1/4))/(-a)^(1/4))/a^2 + 6*sqrt(2)*(-a)^(1/4)*b^3*arctan(-1/2*
sqrt(2)*(sqrt(2)*(-a)^(1/4) - 2*(-b*x^4 + a)^(1/4))/(-a)^(1/4))/a^2 + 3*sqr
t(2)*(-a)^(1/4)*b^3*log(sqrt(2)*(-b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(-b*x^4
 + a) + sqrt(-a))/a^2 + 3*sqrt(2)*b^3*log(-sqrt(2)*(-b*x^4 + a)^(1/4)*(-a)^
(1/4) + sqrt(-b*x^4 + a) + sqrt(-a))/((-a)^(3/4)*a) - 8*((-b*x^4 + a)^(5/4)
*b^3 + 3*(-b*x^4 + a)^(1/4)*a*b^3)/(a*b^2*x^8))/b
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^4+a\right)^{\frac{1}{4}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(1/4)/x^9,x)
```

```
[Out] int((-b*x^4+a)^(1/4)/x^9,x)
```

maxima [A] time = 2.97, size = 138, normalized size = 1.31

$$-\frac{\left(-bx^4+a\right)^{\frac{5}{4}}b^2+3\left(-bx^4+a\right)^{\frac{1}{4}}ab^2}{32\left(\left(bx^4-a\right)^2a+2\left(bx^4-a\right)a^2+a^3\right)}+\frac{3\left(\frac{2\,b^2\arctan\left(\frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{3}{4}}}-\frac{b^2\log\left(\frac{\left(-bx^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{\left(-bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{3}{4}}}\right)}{128\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^9,x, algorithm="maxima")
```

```
[Out] -1/32*((-b*x^4 + a)^(5/4)*b^2 + 3*(-b*x^4 + a)^(1/4)*a*b^2)/((b*x^4 - a)^2*
a + 2*(b*x^4 - a)*a^2 + a^3) + 3/128*(2*b^2*arctan((-b*x^4 + a)^(1/4)/a^(1/
4))/a^(3/4) - b^2*log(((-b*x^4 + a)^(1/4) - a^(1/4))/((-b*x^4 + a)^(1/4) +
a^(1/4)))/a^(3/4))/a
```

mupad [B] time = 1.45, size = 83, normalized size = 0.79

$$\frac{3\,b^2\,\mathrm{atan}\left(\frac{\left(a-b\,x^4\right)^{1/4}}{a^{1/4}}\right)}{64\,a^{7/4}}-\frac{3\left(a-b\,x^4\right)^{1/4}}{32\,x^8}-\frac{\left(a-b\,x^4\right)^{5/4}}{32\,a\,x^8}-\frac{b^2\,\mathrm{atan}\left(\frac{\left(a-b\,x^4\right)^{1/4}\,1\mathrm{i}}{a^{1/4}}\right)\,3\mathrm{i}}{64\,a^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^4)^(1/4)/x^9,x)
```

```
[Out] (3*b^2*atan((a - b*x^4)^(1/4)/a^(1/4)))/(64*a^(7/4)) - (3*(a - b*x^4)^(1/4)
)/(32*x^8) - (b^2*atan(((a - b*x^4)^(1/4)*1i)/a^(1/4))*3i)/(64*a^(7/4)) - (
a - b*x^4)^(5/4)/(32*a*x^8)
```

sympy [C] time = 2.09, size = 42, normalized size = 0.40

$$-\frac{\sqrt[4]{b}\,e^{\frac{i\pi}{4}}\,\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\,\frac{a}{bx^4}\right)}{4x^7\,\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(1/4)/x**9,x)
```

```
[Out] -b**(1/4)*exp(I*pi/4)*gamma(7/4)*hyper((-1/4, 7/4), (11/4,), a/(b*x**4))/(4
*x**7*gamma(11/4))
```

3.1183 $\int x^9 \sqrt[4]{a-bx^4}\,dx$

Optimal. Leaf size=130

$$\frac{4a^{7/2}\left(1-\frac{bx^4}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{77b^{5/2}\left(a-bx^4\right)^{3/4}}-\frac{2a^2x^2\sqrt[4]{a-bx^4}}{77b^2}+\frac{1}{11}x^{10}\sqrt[4]{a-bx^4}-\frac{ax^6\sqrt[4]{a-bx^4}}{77b}$$

```
[Out] -2/77*a^2*x^2*(-b*x^4+a)^(1/4)/b^2-1/77*a*x^6*(-b*x^4+a)^(1/4)/b+1/11*x^10*
(-b*x^4+a)^(1/4)+4/77*a^(7/2)*(1-b*x^4/a)^(3/4)*(cos(1/2*arcsin(x^2*b^(1/2)
/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*
arcsin(x^2*b^(1/2)/a^(1/2))),2^(1/2))/b^(5/2)/(-b*x^4+a)^(3/4)
```

Rubi [A] time = 0.08, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {275, 279, 321, 233, 232}

$$-\frac{2a^2x^2\sqrt[4]{a-bx^4}}{77b^2}+\frac{4a^{7/2}\left(1-\frac{bx^4}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{77b^{5/2}\left(a-bx^4\right)^{3/4}}+\frac{1}{11}x^{10}\sqrt[4]{a-bx^4}-\frac{ax^6\sqrt[4]{a-bx^4}}{77b}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a - b*x^4)^(1/4),x]
```

```
[Out] (-2*a^2*x^2*(a - b*x^4)^(1/4))/(77*b^2) - (a*x^6*(a - b*x^4)^(1/4))/(77*b)
+ (x^10*(a - b*x^4)^(1/4))/11 + (4*a^(7/2)*(1 - (b*x^4)/a)^(3/4)*EllipticF[
ArcSin[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(77*b^(5/2)*(a - b*x^4)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int x^9 \sqrt[4]{a-bx^4}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int x^4 \sqrt[4]{a-bx^2}\,dx, x, x^2\right) \\
&= \frac{1}{11}x^{10}\sqrt[4]{a-bx^4} + \frac{1}{22}a\,\text{Subst}\left(\int \frac{x^4}{\left(a-bx^2\right)^{3/4}}\,dx, x, x^2\right) \\
&= -\frac{ax^6\sqrt[4]{a-bx^4}}{77b} + \frac{1}{11}x^{10}\sqrt[4]{a-bx^4} + \frac{\left(3a^2\right)\text{Subst}\left(\int \frac{x^2}{\left(a-bx^2\right)^{3/4}}\,dx, x, x^2\right)}{77b} \\
&= -\frac{2a^2x^2\sqrt[4]{a-bx^4}}{77b^2} - \frac{ax^6\sqrt[4]{a-bx^4}}{77b} + \frac{1}{11}x^{10}\sqrt[4]{a-bx^4} + \frac{\left(2a^3\right)\text{Subst}\left(\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx, x, x^2\right)}{77b^2} \\
&= -\frac{2a^2x^2\sqrt[4]{a-bx^4}}{77b^2} - \frac{ax^6\sqrt[4]{a-bx^4}}{77b} + \frac{1}{11}x^{10}\sqrt[4]{a-bx^4} + \frac{\left(2a^3\left(1-\frac{bx^4}{a}\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}} d\right.}{77b^2\left(a-bx^4\right)^{3/4}} \\
&= -\frac{2a^2x^2\sqrt[4]{a-bx^4}}{77b^2} - \frac{ax^6\sqrt[4]{a-bx^4}}{77b} + \frac{1}{11}x^{10}\sqrt[4]{a-bx^4} + \frac{4a^{7/2}\left(1-\frac{bx^4}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\right|2}{77b^{5/2}\left(a-bx^4\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.09, size = 98, normalized size = 0.75

$$\frac{x^2\sqrt[4]{a-bx^4}\left(6a^2\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^4}{a}\right) - \sqrt[4]{1-\frac{bx^4}{a}}\left(6a^2+abx^4-7b^2x^8\right)\right)}{77b^2\sqrt[4]{1-\frac{bx^4}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a - b*x^4)^(1/4),x]
```

```
[Out] (x^2*(a - b*x^4)^(1/4)*(-((1 - (b*x^4)/a)^(1/4)*(6*a^2 + a*b*x^4 - 7*b^2*x^
8)) + 6*a^2*Hypergeometric2F1[-1/4, 1/2, 3/2, (b*x^4)/a]))/(77*b^2*(1 - (b*
x^4)/a)^(1/4))
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-bx^4+a\right)^{\frac{1}{4}}x^9, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((-b*x^4 + a)^(1/4)*x^9, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-bx^4+a\right)^{\frac{1}{4}}x^9\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(-b*x^4+a)^(1/4),x, algorithm="giac")
[Out] integrate((-b*x^4 + a)^(1/4)*x^9, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \left(-b\,x^4+a\right)^{\frac{1}{4}} x^9\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(-b*x^4+a)^(1/4),x)
[Out] int(x^9*(-b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-bx^4+a\right)^{\frac{1}{4}} x^9\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(-b*x^4+a)^(1/4),x, algorithm="maxima")
[Out] integrate((-b*x^4 + a)^(1/4)*x^9, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^9 \left(a-b\,x^4\right)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(a - b*x^4)^(1/4),x)
[Out] int(x^9*(a - b*x^4)^(1/4), x)
```

sympy [C] time = 1.94, size = 31, normalized size = 0.24

$$\frac{\sqrt[4]{a}\,x^{10}\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{4}, \frac{5}{2} \\ \frac{7}{2}\end{matrix}\right| \frac{bx^4 e^{2i\pi}}{a}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(-b*x**4+a)**(1/4),x)
[Out] a**(1/4)*x**10*hyper((-1/4, 5/2), (7/2,), b*x**4*exp_polar(2*I*pi)/a)/10
```

3.1184 $\int x^5 \sqrt[4]{a-bx^4}\,dx$

Optimal. Leaf size=105

$$\frac{2a^{5/2}\left(1-\frac{bx^4}{a}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{21b^{3/2}\left(a-bx^4\right)^{3/4}} + \frac{1}{7}x^6\sqrt[4]{a-bx^4} - \frac{ax^2\sqrt[4]{a-bx^4}}{21b}$$

```
[Out] -1/21*a*x^2*(-b*x^4+a)^(1/4)/b+1/7*x^6*(-b*x^4+a)^(1/4)+2/21*a^(5/2)*(1-b*x
^4/a)^(3/4)*(cos(1/2*arcsin(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x
^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arcsin(x^2*b^(1/2)/a^(1/2))),2^(1/2)
)/b^(3/2)/(-b*x^4+a)^(3/4)
```

Rubi [A] time = 0.06, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {275, 279, 321, 233, 232}

$$\frac{2a^{5/2}\left(1-\frac{bx^4}{a}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{21b^{3/2}\left(a-bx^4\right)^{3/4}} + \frac{1}{7}x^6\sqrt[4]{a-bx^4} - \frac{ax^2\sqrt[4]{a-bx^4}}{21b}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a - b*x^4)^(1/4),x]
```

```
[Out] -(a*x^2*(a - b*x^4)^(1/4))/(21*b) + (x^6*(a - b*x^4)^(1/4))/7 + (2*a^(5/2)*
(1 - (b*x^4)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(21*b^
(3/2)*(a - b*x^4)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int x^5 \sqrt[4]{a-bx^4}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int x^2 \sqrt[4]{a-bx^2}\,dx, x, x^2\right) \\
&= \frac{1}{7}x^6\sqrt[4]{a-bx^4} + \frac{1}{14}a\,\text{Subst}\left(\int \frac{x^2}{\left(a-bx^2\right)^{3/4}}\,dx, x, x^2\right) \\
&= -\frac{ax^2\sqrt[4]{a-bx^4}}{21b} + \frac{1}{7}x^6\sqrt[4]{a-bx^4} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx, x, x^2\right)}{21b} \\
&= -\frac{ax^2\sqrt[4]{a-bx^4}}{21b} + \frac{1}{7}x^6\sqrt[4]{a-bx^4} + \frac{\left(a^2\left(1-\frac{bx^4}{a}\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx, x, x^2\right)}{21b\left(a-bx^4\right)^{3/4}} \\
&= -\frac{ax^2\sqrt[4]{a-bx^4}}{21b} + \frac{1}{7}x^6\sqrt[4]{a-bx^4} + \frac{2a^{5/2}\left(1-\frac{bx^4}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{21b^{3/2}\left(a-bx^4\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.11, size = 66, normalized size = 0.63

$$\frac{x^2\sqrt[4]{a-bx^4}\left(\frac{a\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^4}{a}\right)}{\sqrt[4]{1-\frac{bx^4}{a}}} - a + bx^4\right)}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a - b*x^4)^(1/4),x]
```

```
[Out] (x^2*(a - b*x^4)^(1/4)*(-a + b*x^4 + (a*Hypergeometric2F1[-1/4, 1/2, 3/2, (
b*x^4)/a])/(1 - (b*x^4)/a)^(1/4)))/(7*b)
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-bx^4+a\right)^{\frac{1}{4}}x^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((-b*x^4 + a)^(1/4)*x^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-bx^4+a\right)^{\frac{1}{4}}x^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^4 + a)^(1/4)*x^5, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \left(-b\,x^4+a\right)^{\frac{1}{4}} x^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(-b*x^4+a)^(1/4),x)
[Out] int(x^5*(-b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-bx^4+a\right)^{\frac{1}{4}} x^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-b*x^4+a)^(1/4),x, algorithm="maxima")
[Out] integrate((-b*x^4 + a)^(1/4)*x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5\left(a-b\,x^4\right)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a - b*x^4)^(1/4),x)
[Out] int(x^5*(a - b*x^4)^(1/4), x)
```

sympy [C] time = 1.90, size = 31, normalized size = 0.30

$$\frac{\sqrt[4]{a}\,x^6\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{4}, \frac{3}{2} \\ \frac{5}{2}\end{matrix}\right| \frac{bx^4e^{2i\pi}}{a}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(-b*x**4+a)**(1/4),x)
[Out] a**(1/4)*x**6*hyper((-1/4, 3/2), (5/2,), b*x**4*exp_polar(2*I*pi)/a)/6
```

3.1185 $\int x\sqrt[4]{a-bx^4}\,dx$

Optimal. Leaf size=82

$$\frac{a^{3/2}\left(1-\frac{bx^4}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{b}\left(a-bx^4\right)^{3/4}}+\frac{1}{3}x^2\sqrt[4]{a-bx^4}$$

```
[Out] 1/3*x^2*(-b*x^4+a)^(1/4)+1/3*a^(3/2)*(1-b*x^4/a)^(3/4)*(cos(1/2*arcsin(x^2*
b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x^2*b^(1/2)/a^(1/2)))*EllipticF(s
in(1/2*arcsin(x^2*b^(1/2)/a^(1/2))),2^(1/2))/(-b*x^4+a)^(3/4)/b^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {275, 195, 233, 232}

$$\frac{a^{3/2}\left(1-\frac{bx^4}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{b}\left(a-bx^4\right)^{3/4}}+\frac{1}{3}x^2\sqrt[4]{a-bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a - b*x^4)^(1/4),x]
```

```
[Out] (x^2*(a - b*x^4)^(1/4))/3 + (a^(3/2)*(1 - (b*x^4)/a)^(3/4)*EllipticF[ArcSin
[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(3*Sqrt[b]*(a - b*x^4)^(3/4))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int x\sqrt[4]{a-bx^4}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \sqrt[4]{a-bx^2}\,dx, x, x^2\right)$$
$$= \frac{1}{3}x^2\sqrt[4]{a-bx^4} + \frac{1}{6}a\,\text{Subst}\left(\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx, x, x^2\right)$$
$$= \frac{1}{3}x^2\sqrt[4]{a-bx^4} + \frac{\left(a\left(1-\frac{bx^4}{a}\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx, x, x^2\right)}{6\left(a-bx^4\right)^{3/4}}$$
$$= \frac{1}{3}x^2\sqrt[4]{a-bx^4} + \frac{a^{3/2}\left(1-\frac{bx^4}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|\,2\right)}{3\sqrt{b}\,\left(a-bx^4\right)^{3/4}}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.63

$$\frac{x^2\sqrt[4]{a-bx^4}\;{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^4}{a}\right)}{2\sqrt[4]{1-\frac{bx^4}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a - b*x^4)^(1/4),x]
```

```
[Out] (x^2*(a - b*x^4)^(1/4)*Hypergeometric2F1[-1/4, 1/2, 3/2, (b*x^4)/a])/(2*(1
- (b*x^4)/a)^(1/4))
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-bx^4+a\right)^{\frac{1}{4}}x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((-b*x^4 + a)^(1/4)*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-bx^4+a\right)^{\frac{1}{4}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^4 + a)^(1/4)*x, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \left(-b\,x^4+a\right)^{\frac{1}{4}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-b*x^4+a)^(1/4),x)
```

```
[Out] int(x*(-b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-bx^4 + a\right)^{\frac{1}{4}} x\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^4 + a)^(1/4)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(a - b\,x^4\right)^{1/4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a - b*x^4)^(1/4),x)
```

```
[Out] int(x*(a - b*x^4)^(1/4), x)
```

sympy [C] time = 1.29, size = 31, normalized size = 0.38

$$\frac{\sqrt[4]{a}\, x^2\, {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix} \middle| \frac{bx^4 e^{2i\pi}}{a}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x**4+a)**(1/4),x)
```

```
[Out] a**(1/4)*x**2*hyper((-1/4, 1/2), (3/2,), b*x**4*exp_polar(2*I*pi)/a)/2
```

3.1186 $\int \frac{\sqrt[4]{a-bx^4}}{x^3}\,dx$

Optimal. Leaf size=82

$$-\frac{\sqrt[4]{a-bx^4}}{2x^2}-\frac{\sqrt{a}\sqrt{b}\left(1-\frac{bx^4}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{2\left(a-bx^4\right)^{3/4}}$$

```
[Out] -1/2*(-b*x^4+a)^(1/4)/x^2-1/2*(1-b*x^4/a)^(3/4)*(cos(1/2*arcsin(x^2*b^(1/2)
/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*
arcsin(x^2*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)*b^(1/2)/(-b*x^4+a)^(3/4)
```

Rubi [A] time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {275, 277, 233, 232}

$$-\frac{\sqrt[4]{a-bx^4}}{2x^2}-\frac{\sqrt{a}\sqrt{b}\left(1-\frac{bx^4}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{2\left(a-bx^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^4)^(1/4)/x^3,x]
```

```
[Out] -(a - b*x^4)^(1/4)/(2*x^2) - (Sqrt[a]*Sqrt[b]*(1 - (b*x^4)/a)^(3/4)*Ellipti
cF[ArcSin[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(2*(a - b*x^4)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\sqrt[4]{a-bx^4}}{x^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt[4]{a-bx^2}}{x^2}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt[4]{a-bx^4}}{2x^2} - \frac{1}{4}b\,\text{Subst}\left(\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt[4]{a-bx^4}}{2x^2} - \frac{\left(b\left(1-\frac{bx^4}{a}\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx, x, x^2\right)}{4\left(a-bx^4\right)^{3/4}}$$
$$= -\frac{\sqrt[4]{a-bx^4}}{2x^2} - \frac{\sqrt{a}\,\sqrt{b}\,\left(1-\frac{bx^4}{a}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle| 2\right)}{2\left(a-bx^4\right)^{3/4}}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.63

$$-\frac{\sqrt[4]{a-bx^4}\;{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{1}{2};\frac{bx^4}{a}\right)}{2x^2\sqrt[4]{1-\frac{bx^4}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^4)^(1/4)/x^3,x]
```

```
[Out] -1/2*((a - b*x^4)^(1/4)*Hypergeometric2F1[-1/2, -1/4, 1/2, (b*x^4)/a])/(x^2
*(1 - (b*x^4)/a)^(1/4))
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^3,x, algorithm="fricas")
```

```
[Out] integral((-b*x^4 + a)^(1/4)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^3,x, algorithm="giac")
```

```
[Out] integrate((-b*x^4 + a)^(1/4)/x^3, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^4+a\right)^{\frac{1}{4}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(1/4)/x^3,x)
[Out] int((-b*x^4+a)^(1/4)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^3,x, algorithm="maxima")
[Out] integrate((-b*x^4 + a)^(1/4)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a-b\,x^4\right)^{1/4}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^4)^(1/4)/x^3,x)
[Out] int((a - b*x^4)^(1/4)/x^3, x)
```

sympy [C] time = 1.66, size = 34, normalized size = 0.41

$$-\frac{\sqrt[4]{a}\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2}, -\frac{1}{4}\\ \frac{1}{2}\end{matrix}\right| \frac{bx^4e^{2i\pi}}{a}\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(1/4)/x**3,x)
[Out] -a**(1/4)*hyper((-1/2, -1/4), (1/2,), b*x**4*exp_polar(2*I*pi)/a)/(2*x**2)
```

3.1187 $\int \frac{\sqrt[4]{a-bx^4}}{x^7}\,dx$

Optimal. Leaf size=105

$$-\frac{b^{3/2}\left(1-\frac{bx^4}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{12\sqrt{a}\left(a-bx^4\right)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{6x^6}+\frac{b\sqrt[4]{a-bx^4}}{12ax^2}$$

```
[Out] -1/6*(-b*x^4+a)^(1/4)/x^6+1/12*b*(-b*x^4+a)^(1/4)/a/x^2-1/12*b^(3/2)*(1-b*x
^4/a)^(3/4)*(cos(1/2*arcsin(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x
^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arcsin(x^2*b^(1/2)/a^(1/2))),2^(1/2)
)/(-b*x^4+a)^(3/4)/a^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {275, 277, 325, 233, 232}

$$-\frac{b^{3/2}\left(1-\frac{bx^4}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{12\sqrt{a}\left(a-bx^4\right)^{3/4}}+\frac{b\sqrt[4]{a-bx^4}}{12ax^2}-\frac{\sqrt[4]{a-bx^4}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^4)^(1/4)/x^7,x]
```

```
[Out] -(a - b*x^4)^(1/4)/(6*x^6) + (b*(a - b*x^4)^(1/4))/(12*a*x^2) - (b^(3/2)*(1
 - (b*x^4)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(12*Sqrt
[a]*(a - b*x^4)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{a-bx^4}}{x^7}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt[4]{a-bx^2}}{x^4}\,dx, x, x^2\right) \\
&= -\frac{\sqrt[4]{a-bx^4}}{6x^6} - \frac{1}{12}b\,\text{Subst}\left(\int \frac{1}{x^2\left(a-bx^2\right)^{3/4}}\,dx, x, x^2\right) \\
&= -\frac{\sqrt[4]{a-bx^4}}{6x^6} + \frac{b\sqrt[4]{a-bx^4}}{12ax^2} - \frac{b^2\,\text{Subst}\left(\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx, x, x^2\right)}{24a} \\
&= -\frac{\sqrt[4]{a-bx^4}}{6x^6} + \frac{b\sqrt[4]{a-bx^4}}{12ax^2} - \frac{\left(b^2\left(1-\frac{bx^4}{a}\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx, x, x^2\right)}{24a\left(a-bx^4\right)^{3/4}} \\
&= -\frac{\sqrt[4]{a-bx^4}}{6x^6} + \frac{b\sqrt[4]{a-bx^4}}{12ax^2} - \frac{b^{3/2}\left(1-\frac{bx^4}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|\,2\right)}{12\sqrt{a}\,\left(a-bx^4\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.50

$$-\frac{\sqrt[4]{a-bx^4}\;{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};-\frac{1}{2};\frac{bx^4}{a}\right)}{6x^6\sqrt[4]{1-\frac{bx^4}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^4)^(1/4)/x^7,x]
```

```
[Out] -1/6*((a - b*x^4)^(1/4)*Hypergeometric2F1[-3/2, -1/4, -1/2, (b*x^4)/a])/(x^
6*(1 - (b*x^4)/a)^(1/4))
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^7,x, algorithm="fricas")
```

```
[Out] integral((-b*x^4 + a)^(1/4)/x^7, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^7,x, algorithm="giac")
[Out] integrate((-b*x^4 + a)^(1/4)/x^7, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^4+a\right)^{\frac{1}{4}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(1/4)/x^7,x)
[Out] int((-b*x^4+a)^(1/4)/x^7,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^7,x, algorithm="maxima")
[Out] integrate((-b*x^4 + a)^(1/4)/x^7, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a-b\,x^4\right)^{1/4}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^4)^(1/4)/x^7,x)
[Out] int((a - b*x^4)^(1/4)/x^7, x)
```

sympy [C] time = 1.85, size = 36, normalized size = 0.34

$$-\frac{\sqrt[4]{a}\,{}_2F_1\left(\left.\begin{matrix}-\frac{3}{2}, -\frac{1}{4} \\ -\frac{1}{2}\end{matrix}\right| \frac{bx^4e^{2i\pi}}{a}\right)}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(1/4)/x**7,x)
[Out] -a**(1/4)*hyper((-3/2, -1/4), (-1/2,), b*x**4*exp_polar(2*I*pi)/a)/(6*x**6)
```

3.1188 $\int \frac{\sqrt[4]{a-bx^4}}{x^{11}}\,dx$

Optimal. Leaf size=130

$$-\frac{b^{5/2}\left(1-\frac{bx^4}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{24a^{3/2}\left(a-bx^4\right)^{3/4}}+\frac{b^2\sqrt[4]{a-bx^4}}{24a^2x^2}-\frac{\sqrt[4]{a-bx^4}}{10x^{10}}+\frac{b\sqrt[4]{a-bx^4}}{60ax^6}$$

```
[Out] -1/10*(-b*x^4+a)^(1/4)/x^10+1/60*b*(-b*x^4+a)^(1/4)/a/x^6+1/24*b^2*(-b*x^4+
a)^(1/4)/a^2/x^2-1/24*b^(5/2)*(1-b*x^4/a)^(3/4)*(cos(1/2*arcsin(x^2*b^(1/2)
/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*
arcsin(x^2*b^(1/2)/a^(1/2))),2^(1/2))/a^(3/2)/(-b*x^4+a)^(3/4)
```

Rubi [A] time = 0.09, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {275, 277, 325, 233, 232}

$$\frac{b^2\sqrt[4]{a-bx^4}}{24a^2x^2}-\frac{b^{5/2}\left(1-\frac{bx^4}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{24a^{3/2}\left(a-bx^4\right)^{3/4}}+\frac{b\sqrt[4]{a-bx^4}}{60ax^6}-\frac{\sqrt[4]{a-bx^4}}{10x^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^4)^(1/4)/x^11,x]
```

```
[Out] -(a - b*x^4)^(1/4)/(10*x^10) + (b*(a - b*x^4)^(1/4))/(60*a*x^6) + (b^2*(a -
 b*x^4)^(1/4))/(24*a^2*x^2) - (b^(5/2)*(1 - (b*x^4)/a)^(3/4)*EllipticF[ArcS
in[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(24*a^(3/2)*(a - b*x^4)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{a-bx^4}}{x^{11}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt[4]{a-bx^2}}{x^6}\,dx, x, x^2\right)\\
&= -\frac{\sqrt[4]{a-bx^4}}{10x^{10}} - \frac{1}{20}b\,\text{Subst}\left(\int \frac{1}{x^4\left(a-bx^2\right)^{3/4}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt[4]{a-bx^4}}{10x^{10}} + \frac{b\sqrt[4]{a-bx^4}}{60ax^6} - \frac{b^2\,\text{Subst}\left(\int \frac{1}{x^2\left(a-bx^2\right)^{3/4}}\,dx, x, x^2\right)}{24a}\\
&= -\frac{\sqrt[4]{a-bx^4}}{10x^{10}} + \frac{b\sqrt[4]{a-bx^4}}{60ax^6} + \frac{b^2\sqrt[4]{a-bx^4}}{24a^2x^2} - \frac{b^3\,\text{Subst}\left(\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx, x, x^2\right)}{48a^2}\\
&= -\frac{\sqrt[4]{a-bx^4}}{10x^{10}} + \frac{b\sqrt[4]{a-bx^4}}{60ax^6} + \frac{b^2\sqrt[4]{a-bx^4}}{24a^2x^2} - \frac{\left(b^3\left(1-\frac{bx^4}{a}\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx, x, x^2\right)}{48a^2\left(a-bx^4\right)^{3/4}}\\
&= -\frac{\sqrt[4]{a-bx^4}}{10x^{10}} + \frac{b\sqrt[4]{a-bx^4}}{60ax^6} + \frac{b^2\sqrt[4]{a-bx^4}}{24a^2x^2} - \frac{b^{5/2}\left(1-\frac{bx^4}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{24a^{3/2}\left(a-bx^4\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 52, normalized size = 0.40

$$-\frac{\sqrt[4]{a-bx^4}\;{}_2F_1\left(-\frac{5}{2},-\frac{1}{4};-\frac{3}{2};\frac{bx^4}{a}\right)}{10x^{10}\sqrt[4]{1-\frac{bx^4}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^4)^(1/4)/x^11,x]
```

```
[Out] -1/10*((a - b*x^4)^(1/4)*Hypergeometric2F1[-5/2, -1/4, -3/2, (b*x^4)/a])/(x
^10*(1 - (b*x^4)/a)^(1/4))
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{x^{11}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^11,x, algorithm="fricas")
```

```
[Out] integral((-b*x^4 + a)^(1/4)/x^11, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^11,x, algorithm="giac")
```

```
[Out] integrate((-b*x^4 + a)^(1/4)/x^11, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^4+a\right)^{\frac{1}{4}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(1/4)/x^11,x)
```

```
[Out] int((-b*x^4+a)^(1/4)/x^11,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^11,x, algorithm="maxima")
```

```
[Out] integrate((-b*x^4 + a)^(1/4)/x^11, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a-b\,x^4\right)^{1/4}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^4)^(1/4)/x^11,x)
```

```
[Out] int((a - b*x^4)^(1/4)/x^11, x)
```

sympy [C] time = 2.39, size = 36, normalized size = 0.28

$$-\frac{\sqrt[4]{a}\,{}_2F_1\left(\left.\begin{matrix}-\frac{5}{2}, -\frac{1}{4} \\ -\frac{3}{2}\end{matrix}\right| \frac{bx^4e^{2i\pi}}{a}\right)}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(1/4)/x**11,x)
```

```
[Out] -a**(1/4)*hyper((-5/2, -1/4), (-3/2,), b*x**4*exp_polar(2*I*pi)/a)/(10*x**1
0)
```

3.1189 $\int x^6 \sqrt[4]{a-bx^4}\,dx$

Optimal. Leaf size=263

$$-\frac{3a^2\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{64\sqrt{2}\,b^{7/4}}+\frac{3a^2\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+1\right)}{64\sqrt{2}\,b^{7/4}}+\frac{3a^2\log\left(-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}+1\right)}{128\sqrt{2}\,b^{7/4}}-\frac{3a^2\log\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+\frac{\sqrt{b}\,x^2}{\sqrt{a-bx}}\right.}{128\sqrt{2}\,b^{7/4}}$$

```
[Out] -1/32*a*x^3*(-b*x^4+a)^(1/4)/b+1/8*x^7*(-b*x^4+a)^(1/4)+3/128*a^2*arctan(-1
+b^(1/4)*x*2^(1/2)/(-b*x^4+a)^(1/4))/b^(7/4)*2^(1/2)+3/128*a^2*arctan(1+b^(
1/4)*x*2^(1/2)/(-b*x^4+a)^(1/4))/b^(7/4)*2^(1/2)+3/256*a^2*ln(1-b^(1/4)*x*2
^(1/2)/(-b*x^4+a)^(1/4)+x^2*b^(1/2)/(-b*x^4+a)^(1/2))/b^(7/4)*2^(1/2)-3/256
*a^2*ln(1+b^(1/4)*x*2^(1/2)/(-b*x^4+a)^(1/4)+x^2*b^(1/2)/(-b*x^4+a)^(1/2))/
b^(7/4)*2^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {279, 321, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{3a^2\log\left(\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+1\right)}{128\sqrt{2}\,b^{7/4}}-\frac{3a^2\log\left(\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+1\right)}{128\sqrt{2}\,b^{7/4}}-\frac{3a^2\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{64\sqrt{2}\,b^{7/4}}+\frac{3a^2\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+\right.}{64\sqrt{2}\,b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a - b*x^4)^(1/4),x]
```

```
[Out] -(a*x^3*(a - b*x^4)^(1/4))/(32*b) + (x^7*(a - b*x^4)^(1/4))/8 - (3*a^2*ArcT
an[1 - (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)])/(64*Sqrt[2]*b^(7/4)) + (3*a^
2*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)])/(64*Sqrt[2]*b^(7/4)) +
 (3*a^2*Log[1 + (Sqrt[b]*x^2)/Sqrt[a - b*x^4] - (Sqrt[2]*b^(1/4)*x)/(a - b*
x^4)^(1/4)])/(128*Sqrt[2]*b^(7/4)) - (3*a^2*Log[1 + (Sqrt[b]*x^2)/Sqrt[a -
b*x^4] + (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)])/(128*Sqrt[2]*b^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int x^6 \sqrt[4]{a-bx^4}\,dx = \frac{1}{8}x^7\sqrt[4]{a-bx^4} + \frac{1}{8}a\int \frac{x^6}{\left(a-bx^4\right)^{3/4}}\,dx$$

$$= -\frac{ax^3\sqrt[4]{a-bx^4}}{32b} + \frac{1}{8}x^7\sqrt[4]{a-bx^4} + \frac{\left(3a^2\right)\int \frac{x^2}{\left(a-bx^4\right)^{3/4}}\,dx}{32b}$$

$$= -\frac{ax^3\sqrt[4]{a-bx^4}}{32b} + \frac{1}{8}x^7\sqrt[4]{a-bx^4} + \frac{\left(3a^2\right)\text{Subst}\left(\int \frac{x^2}{1+bx^4}\,dx, x, \frac{x}{\sqrt[4]{a-bx^4}}\right)}{32b}$$

$$= -\frac{ax^3\sqrt[4]{a-bx^4}}{32b} + \frac{1}{8}x^7\sqrt[4]{a-bx^4} - \frac{\left(3a^2\right)\text{Subst}\left(\int \frac{1-\sqrt{b}\,x^2}{1+bx^4}\,dx, x, \frac{x}{\sqrt[4]{a-bx^4}}\right)}{64b^{3/2}} + \frac{\left(3a^2\right)\text{Subst}\left(\int$$

$$= -\frac{ax^3\sqrt[4]{a-bx^4}}{32b} + \frac{1}{8}x^7\sqrt[4]{a-bx^4} + \frac{\left(3a^2\right)\text{Subst}\left(\int \frac{1}{\frac{1}{\sqrt{b}}-\frac{\sqrt{2}x}{\sqrt[4]{b}}+x^2}\,dx, x, \frac{x}{\sqrt[4]{a-bx^4}}\right)}{128b^2} + \frac{\left(3a^2\right)\text{Subs}}{}$$

$$= -\frac{ax^3\sqrt[4]{a-bx^4}}{32b} + \frac{1}{8}x^7\sqrt[4]{a-bx^4} + \frac{3a^2\log\left(1+\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{128\sqrt{2}\,b^{7/4}} - \frac{3a^2\log\left(1+\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}+\right.}{128\sqrt{2}\,b^{7/4}}$$

$$= -\frac{ax^3\sqrt[4]{a-bx^4}}{32b} + \frac{1}{8}x^7\sqrt[4]{a-bx^4} - \frac{3a^2\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{64\sqrt{2}\,b^{7/4}} + \frac{3a^2\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{64\sqrt{2}\,b^{7/4}} + \frac{3a}{}$$

Mathematica [C] time = 0.08, size = 66, normalized size = 0.25

$$\frac{x^3\sqrt[4]{a-bx^4}\left(\frac{a\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};\frac{bx^4}{a}\right)}{\sqrt[4]{1-\frac{bx^4}{a}}} - a + bx^4\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a - b*x^4)^(1/4),x]
```

```
[Out] (x^3*(a - b*x^4)^(1/4)*(-a + b*x^4 + (a*Hypergeometric2F1[-1/4, 3/4, 7/4, (
b*x^4)/a])/(1 - (b*x^4)/a)^(1/4)))/(8*b)
```

fricas [A] time = 0.90, size = 234, normalized size = 0.89

$$-\frac{12\left(-\frac{a^8}{b^7}\right)^{\frac{1}{4}} b\arctan\left(-\frac{\left(-bx^4+a\right)^{\frac{1}{4}}a^2\left(-\frac{a^8}{b^7}\right)^{\frac{3}{4}}b^5-\left(-\frac{a^8}{b^7}\right)^{\frac{3}{4}}b^5x\sqrt{\frac{\sqrt{-\frac{a^8}{b^7}}\,b^4x^2+\sqrt{-bx^4+a}\,a^4}{x^2}}}{a^8x}\right) + 3\left(-\frac{a^8}{b^7}\right)^{\frac{1}{4}} b\log\left(\frac{3\left(\left(-\frac{a^8}{b^7}\right)^{\frac{1}{4}}b^2x+\left(-bx^4+a\right.\right.}{x}\right.}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/128*(12*(-a^8/b^7)^(1/4)*b*arctan(-((-b*x^4 + a)^(1/4)*a^2*(-a^8/b^7)^(3
/4)*b^5 - (-a^8/b^7)^(3/4)*b^5*x*sqrt((sqrt(-a^8/b^7)*b^4*x^2 + sqrt(-b*x^4
 + a)*a^4)/x^2))/(a^8*x)) + 3*(-a^8/b^7)^(1/4)*b*log(3*((-a^8/b^7)^(1/4)*b^
2*x + (-b*x^4 + a)^(1/4)*a^2)/x) - 3*(-a^8/b^7)^(1/4)*b*log(-3*((-a^8/b^7)^
(1/4)*b^2*x - (-b*x^4 + a)^(1/4)*a^2)/x) - 4*(4*b*x^7 - a*x^3)*(-b*x^4 + a)
^(1/4))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-bx^4 + a\right)^{\frac{1}{4}} x^6\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(-b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^4 + a)^(1/4)*x^6, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \left(-b\,x^4 + a\right)^{\frac{1}{4}} x^6\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(-b*x^4+a)^(1/4),x)
```

```
[Out] int(x^6*(-b*x^4+a)^(1/4),x)
```

maxima [A] time = 3.09, size = 272, normalized size = 1.03

$$\frac{\frac{3\left(-bx^4+a\right)^{\frac{1}{4}}a^2b}{x} - \frac{\left(-bx^4+a\right)^{\frac{5}{4}}a^2}{x^5}}{32\left(b^3 - \frac{2\left(bx^4-a\right)b^2}{x^4} + \frac{\left(bx^4-a\right)^2 b}{x^8}\right)} - \frac{3\left(\frac{2\sqrt{2}\,a^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}} + \frac{2\left(-bx^4+a\right)^{\frac{1}{4}}}{x}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}} + \frac{2\sqrt{2}\,a^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}} - \frac{2\left(-bx^4+a\right)^{\frac{1}{4}}}{x}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}} + \frac{\sqrt{2}\,a^2\log($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] 1/32*(3*(-b*x^4 + a)^(1/4)*a^2*b/x - (-b*x^4 + a)^(5/4)*a^2/x^5)/(b^3 - 2*(
b*x^4 - a)*b^2/x^4 + (b*x^4 - a)^2*b/x^8) - 3/256*(2*sqrt(2)*a^2*arctan(1/2
*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(-b*x^4 + a)^(1/4)/x)/b^(1/4))/b^(3/4) + 2*sq
rt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(-b*x^4 + a)^(1/4)/x)/b^
(1/4))/b^(3/4) + sqrt(2)*a^2*log(sqrt(b) + sqrt(2)*(-b*x^4 + a)^(1/4)*b^(1/
4)/x + sqrt(-b*x^4 + a)/x^2)/b^(3/4) - sqrt(2)*a^2*log(sqrt(b) - sqrt(2)*(-
b*x^4 + a)^(1/4)*b^(1/4)/x + sqrt(-b*x^4 + a)/x^2)/b^(3/4))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^6 \left(a - b\,x^4\right)^{1/4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a - b*x^4)^(1/4),x)
```

```
[Out] int(x^6*(a - b*x^4)^(1/4), x)
```

sympy [C] time = 3.08, size = 41, normalized size = 0.16

$$\frac{\sqrt[4]{a}\, x^7 \Gamma\left(\frac{7}{4}\right) {}_2F_1\left(\left.\begin{matrix} -\frac{1}{4}, \frac{7}{4} \\ \frac{11}{4} \end{matrix}\right| \frac{b x^4 e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(-b*x**4+a)**(1/4),x)
```

```
[Out] a**(1/4)*x**7*gamma(7/4)*hyper((-1/4, 7/4), (11/4,), b*x**4*exp_polar(2*I*p
i)/a)/(4*gamma(11/4))
```

3.1190 $\int x^2 \sqrt[4]{a-bx^4}\,dx$

Optimal. Leaf size=232

$$-\frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{8\sqrt{2}\,b^{3/4}}+\frac{a\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+1\right)}{8\sqrt{2}\,b^{3/4}}+\frac{a\log\left(-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}+1\right)}{16\sqrt{2}\,b^{3/4}}-\frac{a\log\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}+1\right)}{16\sqrt{2}\,b^{3/4}}+\frac{1}{4}x^3$$

```
[Out] 1/4*x^3*(-b*x^4+a)^(1/4)+1/16*a*arctan(-1+b^(1/4)*x*2^(1/2)/(-b*x^4+a)^(1/4
))/b^(3/4)*2^(1/2)+1/16*a*arctan(1+b^(1/4)*x*2^(1/2)/(-b*x^4+a)^(1/4))/b^(3
/4)*2^(1/2)+1/32*a*ln(1-b^(1/4)*x*2^(1/2)/(-b*x^4+a)^(1/4)+x^2*b^(1/2)/(-b*
x^4+a)^(1/2))/b^(3/4)*2^(1/2)-1/32*a*ln(1+b^(1/4)*x*2^(1/2)/(-b*x^4+a)^(1/4
)+x^2*b^(1/2)/(-b*x^4+a)^(1/2))/b^(3/4)*2^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {279, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{a\log\left(\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+1\right)}{16\sqrt{2}\,b^{3/4}}-\frac{a\log\left(\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+1\right)}{16\sqrt{2}\,b^{3/4}}-\frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{8\sqrt{2}\,b^{3/4}}+\frac{a\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+1\right)}{8\sqrt{2}\,b^{3/4}}+\frac{1}{4}x^3\sqrt[4]{a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a - b*x^4)^(1/4),x]
```

```
[Out] (x^3*(a - b*x^4)^(1/4))/4 - (a*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(
1/4)])/(8*Sqrt[2]*b^(3/4)) + (a*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^
(1/4)])/(8*Sqrt[2]*b^(3/4)) + (a*Log[1 + (Sqrt[b]*x^2)/Sqrt[a - b*x^4] - (S
qrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)])/(16*Sqrt[2]*b^(3/4)) - (a*Log[1 + (Sq
rt[b]*x^2)/Sqrt[a - b*x^4] + (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)])/(16*Sq
rt[2]*b^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
```

```
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int x^2 \sqrt[4]{a-bx^4}\,dx &= \frac{1}{4}x^3\sqrt[4]{a-bx^4} + \frac{1}{4}a\int \frac{x^2}{\left(a-bx^4\right)^{3/4}}\,dx \\
&= \frac{1}{4}x^3\sqrt[4]{a-bx^4} + \frac{1}{4}a\,\text{Subst}\left(\int \frac{x^2}{1+bx^4}\,dx, x, \frac{x}{\sqrt[4]{a-bx^4}}\right) \\
&= \frac{1}{4}x^3\sqrt[4]{a-bx^4} - \frac{a\,\text{Subst}\left(\int \frac{1-\sqrt{b}x^2}{1+bx^4}\,dx, x, \frac{x}{\sqrt[4]{a-bx^4}}\right)}{8\sqrt{b}} + \frac{a\,\text{Subst}\left(\int \frac{1+\sqrt{b}x^2}{1+bx^4}\,dx, x, \frac{x}{\sqrt[4]{a-bx^4}}\right)}{8\sqrt{b}} \\
&= \frac{1}{4}x^3\sqrt[4]{a-bx^4} + \frac{a\,\text{Subst}\left(\int \frac{1}{\frac{1}{\sqrt{b}}-\frac{\sqrt{2}x}{\sqrt[4]{b}}+x^2}\,dx, x, \frac{x}{\sqrt[4]{a-bx^4}}\right)}{16b} + \frac{a\,\text{Subst}\left(\int \frac{1}{\frac{1}{\sqrt{b}}+\frac{\sqrt{2}x}{\sqrt[4]{b}}+x^2}\,dx, x, \frac{x}{\sqrt[4]{a-b}}\right.}{16b} \\
&= \frac{1}{4}x^3\sqrt[4]{a-bx^4} + \frac{a\log\left(1+\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{16\sqrt{2}b^{3/4}} - \frac{a\log\left(1+\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{16\sqrt{2}b^{3/4}} + \frac{a\,\text{Subst}\left(\right.}{} \\
&= \frac{1}{4}x^3\sqrt[4]{a-bx^4} - \frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{8\sqrt{2}b^{3/4}} + \frac{a\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{8\sqrt{2}b^{3/4}} + \frac{a\log\left(1+\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{a-b}}\right.}{16\sqrt{2}b^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.22

$$\frac{x^3\sqrt[4]{a-bx^4}\;{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};\frac{bx^4}{a}\right)}{3\sqrt[4]{1-\frac{bx^4}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a - b*x^4)^(1/4),x]
```

```
[Out] (x^3*(a - b*x^4)^(1/4)*Hypergeometric2F1[-1/4, 3/4, 7/4, (b*x^4)/a])/(3*(1
- (b*x^4)/a)^(1/4))
```

fricas [A] time = 0.85, size = 204, normalized size = 0.88

$$\frac{1}{4}\left(-bx^4+a\right)^{\frac{1}{4}}x^3-\frac{1}{4}\left(-\frac{a^4}{b^3}\right)^{\frac{1}{4}}\arctan\left(\frac{\left(-\frac{a^4}{b^3}\right)^{\frac{3}{4}}b^2x\sqrt{\frac{\sqrt{-\frac{a^4}{b^3}}\,b^2x^2+\sqrt{-bx^4+a}\,a^2}{x^2}}-\left(-bx^4+a\right)^{\frac{1}{4}}a\left(-\frac{a^4}{b^3}\right)^{\frac{3}{4}}b^2}{a^4x}\right)-\frac{1}{16}\left(-\frac{a^4}{b^3}\right)^{\frac{1}{4}}l$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/4*(-b*x^4 + a)^(1/4)*x^3 - 1/4*(-a^4/b^3)^(1/4)*arctan(((-a^4/b^3)^(3/4)*
b^2*x*sqrt((sqrt(-a^4/b^3)*b^2*x^2 + sqrt(-b*x^4 + a)*a^2)/x^2) - (-b*x^4 +
 a)^(1/4)*a*(-a^4/b^3)^(3/4)*b^2)/(a^4*x)) - 1/16*(-a^4/b^3)^(1/4)*log(((-a
^4/b^3)^(1/4)*b*x + (-b*x^4 + a)^(1/4)*a)/x) + 1/16*(-a^4/b^3)^(1/4)*log(-(
(-a^4/b^3)^(1/4)*b*x - (-b*x^4 + a)^(1/4)*a)/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-bx^4+a\right)^{\frac{1}{4}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^4 + a)^(1/4)*x^2, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \left(-b\,x^4+a\right)^{\frac{1}{4}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-b*x^4+a)^(1/4),x)
```

```
[Out] int(x^2*(-b*x^4+a)^(1/4),x)
```

maxima [A] time = 2.96, size = 214, normalized size = 0.92

$$-\frac{\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}+\frac{2\left(-bx^4+a\right)^{\frac{1}{4}}}{x}\right)}{2\,b^{\frac{1}{4}}}\right)}{16\,b^{\frac{3}{4}}}-\frac{\sqrt{2}\,a\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}-\frac{2\left(-bx^4+a\right)^{\frac{1}{4}}}{x}\right)}{2\,b^{\frac{1}{4}}}\right)}{16\,b^{\frac{3}{4}}}-\frac{\sqrt{2}\,a\log\left(\sqrt{b}+\frac{\sqrt{2}\left(-bx^4+a\right)^{\frac{1}{4}}b^{\frac{1}{4}}}{x}+\frac{\sqrt{-b}}{x}\right.}{32\,b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] -1/16*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(-b*x^4 + a)^(1/4)/
x)/b^(1/4))/b^(3/4) - 1/16*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) -
```

```
 2*(-b*x^4 + a)^(1/4)/x)/b^(1/4))/b^(3/4) - 1/32*sqrt(2)*a*log(sqrt(b) + sq
rt(2)*(-b*x^4 + a)^(1/4)*b^(1/4)/x + sqrt(-b*x^4 + a)/x^2)/b^(3/4) + 1/32*s
qrt(2)*a*log(sqrt(b) - sqrt(2)*(-b*x^4 + a)^(1/4)*b^(1/4)/x + sqrt(-b*x^4 +
 a)/x^2)/b^(3/4) + 1/4*(-b*x^4 + a)^(1/4)*a/((b - (b*x^4 - a)/x^4)*x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(a - b\,x^4\right)^{1/4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a - b*x^4)^(1/4),x)
```

```
[Out] int(x^2*(a - b*x^4)^(1/4), x)
```

sympy **[C]** time = 1.85, size = 41, normalized size = 0.18

$$\frac{\sqrt[4]{a}\, x^3 \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{3}{4} \\ \frac{7}{4} \end{matrix} \middle| \frac{b x^4 e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-b*x**4+a)**(1/4),x)
```

```
[Out] a**(1/4)*x**3*gamma(3/4)*hyper((-1/4, 3/4), (7/4,), b*x**4*exp_polar(2*I*pi
)/a)/(4*gamma(7/4))
```

3.1191 $\int \frac{\sqrt[4]{a-bx^4}}{x^2}\,dx$

Optimal. Leaf size=226

$$-\frac{\sqrt[4]{a-bx^4}}{x}+\frac{\sqrt[4]{b}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{2\sqrt{2}}-\frac{\sqrt[4]{b}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+1\right)}{2\sqrt{2}}-\frac{\sqrt[4]{b}\,\log\left(-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}+1\right)}{4\sqrt{2}}+\frac{\sqrt[4]{b}\,\log\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right.}{4\sqrt{}}$$

```
[Out] -(-b*x^4+a)^(1/4)/x-1/4*b^(1/4)*arctan(-1+b^(1/4)*x*2^(1/2)/(-b*x^4+a)^(1/4
))*2^(1/2)-1/4*b^(1/4)*arctan(1+b^(1/4)*x*2^(1/2)/(-b*x^4+a)^(1/4))*2^(1/2)
-1/8*b^(1/4)*ln(1-b^(1/4)*x*2^(1/2)/(-b*x^4+a)^(1/4)+x^2*b^(1/2)/(-b*x^4+a)
^(1/2))*2^(1/2)+1/8*b^(1/4)*ln(1+b^(1/4)*x*2^(1/2)/(-b*x^4+a)^(1/4)+x^2*b^(
1/2)/(-b*x^4+a)^(1/2))*2^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {277, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{a-bx^4}}{x}-\frac{\sqrt[4]{b}\,\log\left(\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+1\right)}{4\sqrt{2}}+\frac{\sqrt[4]{b}\,\log\left(\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+1\right)}{4\sqrt{2}}+\frac{\sqrt[4]{b}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{2\sqrt{2}}-\frac{\sqrt[4]{b}\,\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^4)^(1/4)/x^2,x]
```

```
[Out] -((a - b*x^4)^(1/4)/x) + (b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/(a - b*x^4
)^(1/4)])/(2*Sqrt[2]) - (b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)
^(1/4)])/(2*Sqrt[2]) - (b^(1/4)*Log[1 + (Sqrt[b]*x^2)/Sqrt[a - b*x^4] - (Sq
rt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)])/(4*Sqrt[2]) + (b^(1/4)*Log[1 + (Sqrt[b
]*x^2)/Sqrt[a - b*x^4] + (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)])/(4*Sqrt[2]
)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
```

```
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt[4]{a-bx^4}}{x^2}\,dx = -\frac{\sqrt[4]{a-bx^4}}{x} - b\int \frac{x^2}{\left(a-bx^4\right)^{3/4}}\,dx$$

$$= -\frac{\sqrt[4]{a-bx^4}}{x} - b\,\text{Subst}\left(\int \frac{x^2}{1+bx^4}\,dx, x, \frac{x}{\sqrt[4]{a-bx^4}}\right)$$

$$= -\frac{\sqrt[4]{a-bx^4}}{x} + \frac{1}{2}\sqrt{b}\,\text{Subst}\left(\int \frac{1-\sqrt{b}\,x^2}{1+bx^4}\,dx, x, \frac{x}{\sqrt[4]{a-bx^4}}\right) - \frac{1}{2}\sqrt{b}\,\text{Subst}\left(\int \frac{1+\sqrt{b}\,x^2}{1+bx^4}\,dx, x\right.$$

$$= -\frac{\sqrt[4]{a-bx^4}}{x} - \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{\frac{1}{\sqrt{b}} - \frac{\sqrt{2}\,x}{\sqrt[4]{b}} + x^2}\,dx, x, \frac{x}{\sqrt[4]{a-bx^4}}\right) - \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{\frac{1}{\sqrt{b}} + \frac{\sqrt{2}\,x}{\sqrt[4]{b}} + x^2}\right.$$

$$= -\frac{\sqrt[4]{a-bx^4}}{x} - \frac{\sqrt[4]{b}\,\log\left(1 + \frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}} - \frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{4\sqrt{2}} + \frac{\sqrt[4]{b}\,\log\left(1 + \frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}} + \frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{4\sqrt{2}} - \frac{\sqrt[4]{b}\,\text{Subs}}{}$$

$$= -\frac{\sqrt[4]{a-bx^4}}{x} + \frac{\sqrt[4]{b}\,\tan^{-1}\left(1 - \frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{2\sqrt{2}} - \frac{\sqrt[4]{b}\,\tan^{-1}\left(1 + \frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{2\sqrt{2}} - \frac{\sqrt[4]{b}\,\log\left(1 + \frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}} - \frac{\sqrt{}}{\sqrt[4]{}}\right.}{4\sqrt{2}}$$

Mathematica [C] time = 0.03, size = 50, normalized size = 0.22

$$-\frac{\sqrt[4]{a-bx^4}\ {}_2F_1\left(-\frac{1}{4},-\frac{1}{4};\frac{3}{4};\frac{bx^4}{a}\right)}{x\sqrt[4]{1-\frac{bx^4}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^4)^(1/4)/x^2,x]
```

```
[Out] -(((a - b*x^4)^(1/4)*Hypergeometric2F1[-1/4, -1/4, 3/4, (b*x^4)/a])/(x*(1 -
 (b*x^4)/a)^(1/4)))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^2,x, algorithm="giac")
```

```
[Out] integrate((-b*x^4 + a)^(1/4)/x^2, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^4+a\right)^{\frac{1}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(1/4)/x^2,x)
```

```
[Out] int((-b*x^4+a)^(1/4)/x^2,x)
```

maxima [A] time = 3.05, size = 191, normalized size = 0.85

$$\frac{1}{4}\sqrt{2}\,b^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}+\frac{2\left(-bx^4+a\right)^{\frac{1}{4}}}{x}\right)}{2\,b^{\frac{1}{4}}}\right)+\frac{1}{4}\sqrt{2}\,b^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}-\frac{2\left(-bx^4+a\right)^{\frac{1}{4}}}{x}\right)}{2\,b^{\frac{1}{4}}}\right)+\frac{1}{8}\sqrt{2}\,b^{\frac{1}{4}}\log\left(\sqrt{b}+\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*b^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(-b*x^4 + a)^(1
/4)/x)/b^(1/4)) + 1/4*sqrt(2)*b^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)
- 2*(-b*x^4 + a)^(1/4)/x)/b^(1/4)) + 1/8*sqrt(2)*b^(1/4)*log(sqrt(b) + sqrt
(2)*(-b*x^4 + a)^(1/4)*b^(1/4)/x + sqrt(-b*x^4 + a)/x^2) - 1/8*sqrt(2)*b^(1
```

```
/4)*log(sqrt(b) - sqrt(2)*(-b*x^4 + a)^(1/4)*b^(1/4)/x + sqrt(-b*x^4 + a)/x
^2) - (-b*x^4 + a)^(1/4)/x
```

mupad [B] time = 1.28, size = 41, normalized size = 0.18

$$-\frac{\left(a-b\,x^4\right)^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},-\frac{1}{4};\ \frac{3}{4};\ \frac{b\,x^4}{a}\right)}{x\left(1-\frac{b\,x^4}{a}\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^4)^(1/4)/x^2,x)
```

```
[Out] -((a - b*x^4)^(1/4)*hypergeom([-1/4, -1/4], 3/4, (b*x^4)/a))/(x*(1 - (b*x^4
)/a)^(1/4))
```

sympy [C] time = 1.86, size = 42, normalized size = 0.19

$$\frac{\sqrt[4]{a}\,\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{4},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\right|\frac{b x^4 e^{2i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(1/4)/x**2,x)
```

```
[Out] a**(1/4)*gamma(-1/4)*hyper((-1/4, -1/4), (3/4,), b*x**4*exp_polar(2*I*pi)/a
)/(4*x*gamma(3/4))
```

3.1192 $\int \frac{\sqrt[4]{a-bx^4}}{x^6}\,dx$

Optimal. Leaf size=22

$$-\frac{\left(a-bx^4\right)^{5/4}}{5ax^5}$$

```
[Out] -1/5*(-b*x^4+a)^(5/4)/a/x^5
```

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {264}

$$-\frac{\left(a-bx^4\right)^{5/4}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^4)^(1/4)/x^6,x]
```

```
[Out] -(a - b*x^4)^(5/4)/(5*a*x^5)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt[4]{a-bx^4}}{x^6}\,dx = -\frac{\left(a-bx^4\right)^{5/4}}{5ax^5}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 1.00

$$-\frac{\left(a-bx^4\right)^{5/4}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^4)^(1/4)/x^6,x]
```

```
[Out] -1/5*(a - b*x^4)^(5/4)/(a*x^5)
```

fricas [A] time = 0.63, size = 27, normalized size = 1.23

$$\frac{\left(bx^4-a\right)\left(-bx^4+a\right)^{\frac{1}{4}}}{5\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^6,x, algorithm="fricas")
```

```
[Out] 1/5*(b*x^4 - a)*(-b*x^4 + a)^(1/4)/(a*x^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^6,x, algorithm="giac")
[Out] integrate((-b*x^4 + a)^(1/4)/x^6, x)
```

maple [A] time = 0.00, size = 19, normalized size = 0.86

$$-\frac{\left(-b\,x^4+a\right)^{\frac{5}{4}}}{5a\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(1/4)/x^6,x)
[Out] -1/5*(-b*x^4+a)^(5/4)/a/x^5
```

maxima [A] time = 1.42, size = 18, normalized size = 0.82

$$-\frac{\left(-bx^4+a\right)^{\frac{5}{4}}}{5\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^6,x, algorithm="maxima")
[Out] -1/5*(-b*x^4 + a)^(5/4)/(a*x^5)
```

mupad [B] time = 1.21, size = 18, normalized size = 0.82

$$-\frac{\left(a-b\,x^4\right)^{5/4}}{5\,a\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^4)^(1/4)/x^6,x)
[Out] -(a - b*x^4)^(5/4)/(5*a*x^5)
```

sympy [B] time = 1.31, size = 158, normalized size = 7.18

$$\begin{cases} \frac{\sqrt[4]{b}\,\sqrt[4]{\frac{a}{bx^4}-1}\,\Gamma\left(-\frac{5}{4}\right)}{4x^4\Gamma\left(-\frac{1}{4}\right)}-\frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx^4}-1}\,\Gamma\left(-\frac{5}{4}\right)}{4a\Gamma\left(-\frac{1}{4}\right)} & \text{for } \left|\frac{a}{bx^4}\right|>1 \\ \frac{\sqrt[4]{b}\,\sqrt[4]{-\frac{a}{bx^4}+1}\,e^{\frac{i\pi}{4}}\,\Gamma\left(-\frac{5}{4}\right)}{4x^4\Gamma\left(-\frac{1}{4}\right)}-\frac{b^{\frac{5}{4}}\sqrt[4]{-\frac{a}{bx^4}+1}\,e^{\frac{i\pi}{4}}\,\Gamma\left(-\frac{5}{4}\right)}{4a\Gamma\left(-\frac{1}{4}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(1/4)/x**6,x)
[Out] Piecewise((b**(1/4)*(a/(b*x**4) - 1)**(1/4)*gamma(-5/4)/(4*x**4*gamma(-1/4)
) - b**(5/4)*(a/(b*x**4) - 1)**(1/4)*gamma(-5/4)/(4*a*gamma(-1/4)), Abs(a/(
b*x**4)) > 1), (b**(1/4)*(-a/(b*x**4) + 1)**(1/4)*exp(I*pi/4)*gamma(-5/4)/(
4*x**4*gamma(-1/4)) - b**(5/4)*(-a/(b*x**4) + 1)**(1/4)*exp(I*pi/4)*gamma(-
5/4)/(4*a*gamma(-1/4)), True))
```

3.1193 $\int \frac{\sqrt[4]{a-bx^4}}{x^{10}}\,dx$

Optimal. Leaf size=46

$$-\frac{4b\left(a-bx^4\right)^{5/4}}{45a^2x^5}-\frac{\left(a-bx^4\right)^{5/4}}{9ax^9}$$

```
[Out] -1/9*(-b*x^4+a)^(5/4)/a/x^9-4/45*b*(-b*x^4+a)^(5/4)/a^2/x^5
```

Rubi [A] time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {271, 264}

$$-\frac{4b\left(a-bx^4\right)^{5/4}}{45a^2x^5}-\frac{\left(a-bx^4\right)^{5/4}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^4)^(1/4)/x^10,x]
```

```
[Out] -(a - b*x^4)^(5/4)/(9*a*x^9) - (4*b*(a - b*x^4)^(5/4))/(45*a^2*x^5)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[4]{a-bx^4}}{x^{10}}\,dx &= -\frac{\left(a-bx^4\right)^{5/4}}{9ax^9}+\frac{(4b)\int \frac{\sqrt[4]{a-bx^4}}{x^6}\,dx}{9a}\\ &= -\frac{\left(a-bx^4\right)^{5/4}}{9ax^9}-\frac{4b\left(a-bx^4\right)^{5/4}}{45a^2x^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 0.70

$$-\frac{\left(a-bx^4\right)^{5/4}\left(5a+4bx^4\right)}{45a^2x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^4)^(1/4)/x^10,x]
```

```
[Out] -1/45*((a - b*x^4)^(5/4)*(5*a + 4*b*x^4))/(a^2*x^9)
```

fricas [A] time = 0.85, size = 38, normalized size = 0.83

$$\frac{\left(4\,b^2x^8+abx^4-5\,a^2\right)\left(-bx^4+a\right)^{\frac{1}{4}}}{45\,a^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^10,x, algorithm="fricas")
[Out] 1/45*(4*b^2*x^8 + a*b*x^4 - 5*a^2)*(-b*x^4 + a)^(1/4)/(a^2*x^9)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^10,x, algorithm="giac")
[Out] integrate((-b*x^4 + a)^(1/4)/x^10, x)
```

maple [A] time = 0.01, size = 29, normalized size = 0.63

$$-\frac{\left(-b\,x^4+a\right)^{\frac{5}{4}}\left(4b\,x^4+5a\right)}{45a^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(1/4)/x^10,x)
[Out] -1/45*(-b*x^4+a)^(5/4)*(4*b*x^4+5*a)/a^2/x^9
```

maxima [A] time = 1.34, size = 37, normalized size = 0.80

$$-\frac{\frac{9\left(-bx^4+a\right)^{\frac{5}{4}}b}{x^5}+\frac{5\left(-bx^4+a\right)^{\frac{9}{4}}}{x^9}}{45\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^10,x, algorithm="maxima")
[Out] -1/45*(9*(-b*x^4 + a)^(5/4)*b/x^5 + 5*(-b*x^4 + a)^(9/4)/x^9)/a^2
```

mupad [B] time = 1.30, size = 38, normalized size = 0.83

$$\frac{\left(a-b\,x^4\right)^{1/4}\left(-5\,a^2+a\,b\,x^4+4\,b^2\,x^8\right)}{45\,a^2\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^4)^(1/4)/x^10,x)
[Out] ((a - b*x^4)^(1/4)*(4*b^2*x^8 - 5*a^2 + a*b*x^4))/(45*a^2*x^9)
```

sympy [B] time = 2.06, size = 406, normalized size = 8.83

$$\begin{cases} -\frac{5\sqrt[4]{b}\,\sqrt[4]{\frac{a}{bx^4}-1}\,\Gamma\left(-\frac{9}{4}\right)}{16x^8\Gamma\left(-\frac{1}{4}\right)}+\frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx^4}-1}\,\Gamma\left(-\frac{9}{4}\right)}{16ax^4\Gamma\left(-\frac{1}{4}\right)}+\frac{b^{\frac{9}{4}}\sqrt[4]{\frac{a}{bx^4}-1}\,\Gamma\left(-\frac{9}{4}\right)}{4a^2\Gamma\left(-\frac{1}{4}\right)} \\ \frac{5a^3b^{\frac{5}{4}}\sqrt[4]{-\frac{a}{bx^4}+1}\,e^{\frac{i\pi}{4}}\Gamma\left(-\frac{9}{4}\right)}{x^4\left(-16a^3bx^4\Gamma\left(-\frac{1}{4}\right)+16a^2b^2x^8\Gamma\left(-\frac{1}{4}\right)\right)}-\frac{6a^2b^{\frac{9}{4}}\sqrt[4]{-\frac{a}{bx^4}+1}\,e^{\frac{i\pi}{4}}\Gamma\left(-\frac{9}{4}\right)}{-16a^3bx^4\Gamma\left(-\frac{1}{4}\right)+16a^2b^2x^8\Gamma\left(-\frac{1}{4}\right)}-\frac{3ab^{\frac{13}{4}}x^4\sqrt[4]{-\frac{a}{bx^4}+1}\,e^{\frac{i\pi}{4}}\Gamma\left(-\frac{9}{4}\right)}{-16a^3bx^4\Gamma\left(-\frac{1}{4}\right)+16a^2b^2x^8\Gamma\left(-\frac{1}{4}\right)}+\frac{4b^{\frac{17}{4}}x^8\sqrt[4]{-\frac{a}{bx^4}+1}\,e^{\frac{i\pi}{4}}\Gamma\left(\right.}{-16a^3bx^4\Gamma\left(-\frac{1}{4}\right)+16a^2b^2x^8} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(1/4)/x**10,x)
```

```
[Out] Piecewise((-5*b**(1/4)*(a/(b*x**4) - 1)**(1/4)*gamma(-9/4)/(16*x**8*gamma(-
1/4)) + b**(5/4)*(a/(b*x**4) - 1)**(1/4)*gamma(-9/4)/(16*a*x**4*gamma(-1/4)
) + b**(9/4)*(a/(b*x**4) - 1)**(1/4)*gamma(-9/4)/(4*a**2*gamma(-1/4)), Abs(
a/(b*x**4)) > 1), (5*a**3*b**(5/4)*(-a/(b*x**4) + 1)**(1/4)*exp(I*pi/4)*gam
ma(-9/4)/(x**4*(-16*a**3*b*x**4*gamma(-1/4) + 16*a**2*b**2*x**8*gamma(-1/4)
)) - 6*a**2*b**(9/4)*(-a/(b*x**4) + 1)**(1/4)*exp(I*pi/4)*gamma(-9/4)/(-16*
a**3*b*x**4*gamma(-1/4) + 16*a**2*b**2*x**8*gamma(-1/4)) - 3*a*b**(13/4)*x*
*4*(-a/(b*x**4) + 1)**(1/4)*exp(I*pi/4)*gamma(-9/4)/(-16*a**3*b*x**4*gamma(
-1/4) + 16*a**2*b**2*x**8*gamma(-1/4)) + 4*b**(17/4)*x**8*(-a/(b*x**4) + 1)
**(1/4)*exp(I*pi/4)*gamma(-9/4)/(-16*a**3*b*x**4*gamma(-1/4) + 16*a**2*b**2
*x**8*gamma(-1/4)), True))
```

3.1194 $\int \frac{\sqrt[4]{a-bx^4}}{x^{14}}\,dx$

Optimal. Leaf size=71

$$-\frac{32b^2\left(a-bx^4\right)^{5/4}}{585a^3x^5}-\frac{8b\left(a-bx^4\right)^{5/4}}{117a^2x^9}-\frac{\left(a-bx^4\right)^{5/4}}{13ax^{13}}$$

```
[Out] -1/13*(-b*x^4+a)^(5/4)/a/x^13-8/117*b*(-b*x^4+a)^(5/4)/a^2/x^9-32/585*b^2*(
-b*x^4+a)^(5/4)/a^3/x^5
```

Rubi [A] time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {271, 264}

$$-\frac{32b^2\left(a-bx^4\right)^{5/4}}{585a^3x^5}-\frac{8b\left(a-bx^4\right)^{5/4}}{117a^2x^9}-\frac{\left(a-bx^4\right)^{5/4}}{13ax^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^4)^(1/4)/x^14,x]
```

```
[Out] -(a - b*x^4)^(5/4)/(13*a*x^13) - (8*b*(a - b*x^4)^(5/4))/(117*a^2*x^9) - (3
2*b^2*(a - b*x^4)^(5/4))/(585*a^3*x^5)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[4]{a-bx^4}}{x^{14}}\,dx &= -\frac{\left(a-bx^4\right)^{5/4}}{13ax^{13}}+\frac{(8b)\int \frac{\sqrt[4]{a-bx^4}}{x^{10}}\,dx}{13a}\\ &= -\frac{\left(a-bx^4\right)^{5/4}}{13ax^{13}}-\frac{8b\left(a-bx^4\right)^{5/4}}{117a^2x^9}+\frac{\left(32b^2\right)\int \frac{\sqrt[4]{a-bx^4}}{x^6}\,dx}{117a^2}\\ &= -\frac{\left(a-bx^4\right)^{5/4}}{13ax^{13}}-\frac{8b\left(a-bx^4\right)^{5/4}}{117a^2x^9}-\frac{32b^2\left(a-bx^4\right)^{5/4}}{585a^3x^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 0.61

$$-\frac{\left(a-bx^4\right)^{5/4}\left(45a^2+40abx^4+32b^2x^8\right)}{585a^3x^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^4)^(1/4)/x^14,x]
```

```
[Out] -1/585*((a - b*x^4)^(5/4)*(45*a^2 + 40*a*b*x^4 + 32*b^2*x^8))/(a^3*x^13)
```

fricas [A] time = 0.90, size = 50, normalized size = 0.70

$$\frac{\left(32\,b^3x^{12}+8\,ab^2x^8+5\,a^2bx^4-45\,a^3\right)\left(-bx^4+a\right)^{\frac{1}{4}}}{585\,a^3x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^14,x, algorithm="fricas")
```

```
[Out] 1/585*(32*b^3*x^12 + 8*a*b^2*x^8 + 5*a^2*b*x^4 - 45*a^3)*(-b*x^4 + a)^(1/4)
/(a^3*x^13)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^14,x, algorithm="giac")
```

```
[Out] integrate((-b*x^4 + a)^(1/4)/x^14, x)
```

maple [A] time = 0.01, size = 40, normalized size = 0.56

$$-\frac{\left(-b\,x^4+a\right)^{\frac{5}{4}}\left(32b^2x^8+40ab\,x^4+45a^2\right)}{585a^3x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(1/4)/x^14,x)
```

```
[Out] -1/585*(-b*x^4+a)^(5/4)*(32*b^2*x^8+40*a*b*x^4+45*a^2)/x^13/a^3
```

maxima [A] time = 1.42, size = 55, normalized size = 0.77

$$-\frac{\frac{117\left(-bx^4+a\right)^{\frac{5}{4}}b^2}{x^5}+\frac{130\left(-bx^4+a\right)^{\frac{9}{4}}b}{x^9}+\frac{45\left(-bx^4+a\right)^{\frac{13}{4}}}{x^{13}}}{585\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^14,x, algorithm="maxima")
```

```
[Out] -1/585*(117*(-b*x^4 + a)^(5/4)*b^2/x^5 + 130*(-b*x^4 + a)^(9/4)*b/x^9 + 45*
(-b*x^4 + a)^(13/4)/x^13)/a^3
```

mupad [B] time = 1.48, size = 77, normalized size = 1.08

$$\frac{b\left(a-b\,x^4\right)^{1/4}}{117\,a\,x^9}-\frac{\left(a-b\,x^4\right)^{1/4}}{13\,x^{13}}+\frac{32\,b^3\left(a-b\,x^4\right)^{1/4}}{585\,a^3\,x}+\frac{8\,b^2\left(a-b\,x^4\right)^{1/4}}{585\,a^2\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^4)^(1/4)/x^14,x)
```

```
[Out] (b*(a - b*x^4)^(1/4))/(117*a*x^9) - (a - b*x^4)^(1/4)/(13*x^13) + (32*b^3*(
a - b*x^4)^(1/4))/(585*a^3*x) + (8*b^2*(a - b*x^4)^(1/4))/(585*a^2*x^5)
```

sympy [**B**] time = 4.23, size = 1090, normalized size = 15.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(1/4)/x**14,x)
```

```
[Out] Piecewise((45*a**5*b**(17/4)*(a/(b*x**4) - 1)**(1/4)*gamma(-13/4)/(64*a**5*
b**4*x**12*gamma(-1/4) - 128*a**4*b**5*x**16*gamma(-1/4) + 64*a**3*b**6*x**
20*gamma(-1/4)) - 95*a**4*b**(21/4)*x**4*(a/(b*x**4) - 1)**(1/4)*gamma(-13/
4)/(64*a**5*b**4*x**12*gamma(-1/4) - 128*a**4*b**5*x**16*gamma(-1/4) + 64*a
**3*b**6*x**20*gamma(-1/4)) + 47*a**3*b**(25/4)*x**8*(a/(b*x**4) - 1)**(1/4
)*gamma(-13/4)/(64*a**5*b**4*x**12*gamma(-1/4) - 128*a**4*b**5*x**16*gamma(
-1/4) + 64*a**3*b**6*x**20*gamma(-1/4)) - 21*a**2*b**(29/4)*x**12*(a/(b*x**
4) - 1)**(1/4)*gamma(-13/4)/(64*a**5*b**4*x**12*gamma(-1/4) - 128*a**4*b**5
*x**16*gamma(-1/4) + 64*a**3*b**6*x**20*gamma(-1/4)) + 56*a*b**(33/4)*x**16
*(a/(b*x**4) - 1)**(1/4)*gamma(-13/4)/(64*a**5*b**4*x**12*gamma(-1/4) - 128
*a**4*b**5*x**16*gamma(-1/4) + 64*a**3*b**6*x**20*gamma(-1/4)) - 32*b**(37/
4)*x**20*(a/(b*x**4) - 1)**(1/4)*gamma(-13/4)/(64*a**5*b**4*x**12*gamma(-1/
4) - 128*a**4*b**5*x**16*gamma(-1/4) + 64*a**3*b**6*x**20*gamma(-1/4)), Abs
(a/(b*x**4)) > 1), (45*a**5*b**(17/4)*(-a/(b*x**4) + 1)**(1/4)*exp(I*pi/4)*
gamma(-13/4)/(64*a**5*b**4*x**12*gamma(-1/4) - 128*a**4*b**5*x**16*gamma(-1
/4) + 64*a**3*b**6*x**20*gamma(-1/4)) - 95*a**4*b**(21/4)*x**4*(-a/(b*x**4)
 + 1)**(1/4)*exp(I*pi/4)*gamma(-13/4)/(64*a**5*b**4*x**12*gamma(-1/4) - 128
*a**4*b**5*x**16*gamma(-1/4) + 64*a**3*b**6*x**20*gamma(-1/4)) + 47*a**3*b*
*(25/4)*x**8*(-a/(b*x**4) + 1)**(1/4)*exp(I*pi/4)*gamma(-13/4)/(64*a**5*b**
4*x**12*gamma(-1/4) - 128*a**4*b**5*x**16*gamma(-1/4) + 64*a**3*b**6*x**20*
gamma(-1/4)) - 21*a**2*b**(29/4)*x**12*(-a/(b*x**4) + 1)**(1/4)*exp(I*pi/4)
*gamma(-13/4)/(64*a**5*b**4*x**12*gamma(-1/4) - 128*a**4*b**5*x**16*gamma(-
1/4) + 64*a**3*b**6*x**20*gamma(-1/4)) + 56*a*b**(33/4)*x**16*(-a/(b*x**4)
+ 1)**(1/4)*exp(I*pi/4)*gamma(-13/4)/(64*a**5*b**4*x**12*gamma(-1/4) - 128*
a**4*b**5*x**16*gamma(-1/4) + 64*a**3*b**6*x**20*gamma(-1/4)) - 32*b**(37/4
)*x**20*(-a/(b*x**4) + 1)**(1/4)*exp(I*pi/4)*gamma(-13/4)/(64*a**5*b**4*x**
12*gamma(-1/4) - 128*a**4*b**5*x**16*gamma(-1/4) + 64*a**3*b**6*x**20*gamma
(-1/4)), True))
```

3.1195 $\int \frac{\sqrt[4]{a-bx^4}}{x^{18}}\,dx$

Optimal. Leaf size=96

$$-\frac{128b^3\left(a-bx^4\right)^{5/4}}{3315a^4x^5}-\frac{32b^2\left(a-bx^4\right)^{5/4}}{663a^3x^9}-\frac{12b\left(a-bx^4\right)^{5/4}}{221a^2x^{13}}-\frac{\left(a-bx^4\right)^{5/4}}{17ax^{17}}$$

```
[Out] -1/17*(-b*x^4+a)^(5/4)/a/x^17-12/221*b*(-b*x^4+a)^(5/4)/a^2/x^13-32/663*b^2
*(-b*x^4+a)^(5/4)/a^3/x^9-128/3315*b^3*(-b*x^4+a)^(5/4)/a^4/x^5
```

Rubi [A] time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {271, 264}

$$-\frac{128b^3\left(a-bx^4\right)^{5/4}}{3315a^4x^5}-\frac{32b^2\left(a-bx^4\right)^{5/4}}{663a^3x^9}-\frac{12b\left(a-bx^4\right)^{5/4}}{221a^2x^{13}}-\frac{\left(a-bx^4\right)^{5/4}}{17ax^{17}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^4)^(1/4)/x^18,x]
```

```
[Out] -(a - b*x^4)^(5/4)/(17*a*x^17) - (12*b*(a - b*x^4)^(5/4))/(221*a^2*x^13) -
(32*b^2*(a - b*x^4)^(5/4))/(663*a^3*x^9) - (128*b^3*(a - b*x^4)^(5/4))/(331
5*a^4*x^5)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{a-bx^4}}{x^{18}}\,dx &= -\frac{\left(a-bx^4\right)^{5/4}}{17ax^{17}}+\frac{(12b)\int \frac{\sqrt[4]{a-bx^4}}{x^{14}}\,dx}{17a}\\
&= -\frac{\left(a-bx^4\right)^{5/4}}{17ax^{17}}-\frac{12b\left(a-bx^4\right)^{5/4}}{221a^2x^{13}}+\frac{\left(96b^2\right)\int \frac{\sqrt[4]{a-bx^4}}{x^{10}}\,dx}{221a^2}\\
&= -\frac{\left(a-bx^4\right)^{5/4}}{17ax^{17}}-\frac{12b\left(a-bx^4\right)^{5/4}}{221a^2x^{13}}-\frac{32b^2\left(a-bx^4\right)^{5/4}}{663a^3x^9}+\frac{\left(128b^3\right)\int \frac{\sqrt[4]{a-bx^4}}{x^6}\,dx}{663a^3}\\
&= -\frac{\left(a-bx^4\right)^{5/4}}{17ax^{17}}-\frac{12b\left(a-bx^4\right)^{5/4}}{221a^2x^{13}}-\frac{32b^2\left(a-bx^4\right)^{5/4}}{663a^3x^9}-\frac{128b^3\left(a-bx^4\right)^{5/4}}{3315a^4x^5}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 0.56

$$-\frac{\left(a-bx^4\right)^{5/4}\left(195a^3+180a^2bx^4+160ab^2x^8+128b^3x^{12}\right)}{3315a^4x^{17}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^4)^(1/4)/x^18,x]
```

```
[Out] -1/3315*((a - b*x^4)^(5/4)*(195*a^3 + 180*a^2*b*x^4 + 160*a*b^2*x^8 + 128*b
^3*x^12))/(a^4*x^17)
```

fricas [A] time = 1.19, size = 61, normalized size = 0.64

$$\frac{\left(128\,b^4x^{16}+32\,ab^3x^{12}+20\,a^2b^2x^8+15\,a^3bx^4-195\,a^4\right)\left(-bx^4+a\right)^{\frac{1}{4}}}{3315\,a^4x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^18,x, algorithm="fricas")
```

```
[Out] 1/3315*(128*b^4*x^16 + 32*a*b^3*x^12 + 20*a^2*b^2*x^8 + 15*a^3*b*x^4 - 195*
a^4)*(-b*x^4 + a)^(1/4)/(a^4*x^17)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{x^{18}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^18,x, algorithm="giac")
```

```
[Out] integrate((-b*x^4 + a)^(1/4)/x^18, x)
```

maple [A] time = 0.01, size = 51, normalized size = 0.53

$$-\frac{\left(-b\,x^4+a\right)^{\frac{5}{4}}\left(128b^3x^{12}+160a\,b^2x^8+180a^2b\,x^4+195a^3\right)}{3315a^4x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(1/4)/x^18,x)
```

```
[Out] -1/3315*(-b*x^4+a)^(5/4)*(128*b^3*x^12+160*a*b^2*x^8+180*a^2*b*x^4+195*a^3)
/x^17/a^4
```

maxima [A] time = 1.32, size = 73, normalized size = 0.76

$$-\frac{\frac{663\left(-bx^4+a\right)^{\frac{5}{4}}b^3}{x^5}+\frac{1105\left(-bx^4+a\right)^{\frac{9}{4}}b^2}{x^9}+\frac{765\left(-bx^4+a\right)^{\frac{13}{4}}b}{x^{13}}+\frac{195\left(-bx^4+a\right)^{\frac{17}{4}}}{x^{17}}}{3315\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^18,x, algorithm="maxima")
```

```
[Out] -1/3315*(663*(-b*x^4 + a)^(5/4)*b^3/x^5 + 1105*(-b*x^4 + a)^(9/4)*b^2/x^9 +
 765*(-b*x^4 + a)^(13/4)*b/x^13 + 195*(-b*x^4 + a)^(17/4)/x^17)/a^4
```

mupad [B] time = 1.76, size = 98, normalized size = 1.02

$$\frac{b\left(a-b\,x^4\right)^{1/4}}{221\,a\,x^{13}}-\frac{\left(a-b\,x^4\right)^{1/4}}{17\,x^{17}}+\frac{128\,b^4\left(a-b\,x^4\right)^{1/4}}{3315\,a^4\,x}+\frac{32\,b^3\left(a-b\,x^4\right)^{1/4}}{3315\,a^3\,x^5}+\frac{4\,b^2\left(a-b\,x^4\right)^{1/4}}{663\,a^2\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^4)^(1/4)/x^18,x)
```

```
[Out] (b*(a - b*x^4)^(1/4))/(221*a*x^13) - (a - b*x^4)^(1/4)/(17*x^17) + (128*b^4
*(a - b*x^4)^(1/4))/(3315*a^4*x) + (32*b^3*(a - b*x^4)^(1/4))/(3315*a^3*x^5
) + (4*b^2*(a - b*x^4)^(1/4))/(663*a^2*x^9)
```

sympy [**B**] time = 4.92, size = 1756, normalized size = 18.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(1/4)/x**18,x)
```

```
[Out] Piecewise((585*a**7*b**(37/4)*(a/(b*x**4) - 1)**(1/4)*gamma(-17/4)/(-256*a*
*7*b**9*x**16*gamma(-1/4) + 768*a**6*b**10*x**20*gamma(-1/4) - 768*a**5*b**
11*x**24*gamma(-1/4) + 256*a**4*b**12*x**28*gamma(-1/4)) - 1800*a**6*b**(41
/4)*x**4*(a/(b*x**4) - 1)**(1/4)*gamma(-17/4)/(-256*a**7*b**9*x**16*gamma(-
1/4) + 768*a**6*b**10*x**20*gamma(-1/4) - 768*a**5*b**11*x**24*gamma(-1/4)
+ 256*a**4*b**12*x**28*gamma(-1/4)) + 1830*a**5*b**(45/4)*x**8*(a/(b*x**4)
- 1)**(1/4)*gamma(-17/4)/(-256*a**7*b**9*x**16*gamma(-1/4) + 768*a**6*b**10
*x**20*gamma(-1/4) - 768*a**5*b**11*x**24*gamma(-1/4) + 256*a**4*b**12*x**2
8*gamma(-1/4)) - 636*a**4*b**(49/4)*x**12*(a/(b*x**4) - 1)**(1/4)*gamma(-17
/4)/(-256*a**7*b**9*x**16*gamma(-1/4) + 768*a**6*b**10*x**20*gamma(-1/4) -
768*a**5*b**11*x**24*gamma(-1/4) + 256*a**4*b**12*x**28*gamma(-1/4)) - 231*
a**3*b**(53/4)*x**16*(a/(b*x**4) - 1)**(1/4)*gamma(-17/4)/(-256*a**7*b**9*x
**16*gamma(-1/4) + 768*a**6*b**10*x**20*gamma(-1/4) - 768*a**5*b**11*x**24*
gamma(-1/4) + 256*a**4*b**12*x**28*gamma(-1/4)) + 924*a**2*b**(57/4)*x**20*
(a/(b*x**4) - 1)**(1/4)*gamma(-17/4)/(-256*a**7*b**9*x**16*gamma(-1/4) + 76
8*a**6*b**10*x**20*gamma(-1/4) - 768*a**5*b**11*x**24*gamma(-1/4) + 256*a**
4*b**12*x**28*gamma(-1/4)) - 1056*a*b**(61/4)*x**24*(a/(b*x**4) - 1)**(1/4)
*gamma(-17/4)/(-256*a**7*b**9*x**16*gamma(-1/4) + 768*a**6*b**10*x**20*gamm
a(-1/4) - 768*a**5*b**11*x**24*gamma(-1/4) + 256*a**4*b**12*x**28*gamma(-1/
4)) + 384*b**(65/4)*x**28*(a/(b*x**4) - 1)**(1/4)*gamma(-17/4)/(-256*a**7*b
**9*x**16*gamma(-1/4) + 768*a**6*b**10*x**20*gamma(-1/4) - 768*a**5*b**11*x
**24*gamma(-1/4) + 256*a**4*b**12*x**28*gamma(-1/4)), Abs(a/(b*x**4)) > 1),
 (585*a**7*b**(37/4)*(-a/(b*x**4) + 1)**(1/4)*exp(I*pi/4)*gamma(-17/4)/(-25
6*a**7*b**9*x**16*gamma(-1/4) + 768*a**6*b**10*x**20*gamma(-1/4) - 768*a**5
*b**11*x**24*gamma(-1/4) + 256*a**4*b**12*x**28*gamma(-1/4)) - 1800*a**6*b*
*(41/4)*x**4*(-a/(b*x**4) + 1)**(1/4)*exp(I*pi/4)*gamma(-17/4)/(-256*a**7*b
**9*x**16*gamma(-1/4) + 768*a**6*b**10*x**20*gamma(-1/4) - 768*a**5*b**11*x
**24*gamma(-1/4) + 256*a**4*b**12*x**28*gamma(-1/4)) + 1830*a**5*b**(45/4)*
x**8*(-a/(b*x**4) + 1)**(1/4)*exp(I*pi/4)*gamma(-17/4)/(-256*a**7*b**9*x**1
6*gamma(-1/4) + 768*a**6*b**10*x**20*gamma(-1/4) - 768*a**5*b**11*x**24*gam
ma(-1/4) + 256*a**4*b**12*x**28*gamma(-1/4)) - 636*a**4*b**(49/4)*x**12*(-a
/(b*x**4) + 1)**(1/4)*exp(I*pi/4)*gamma(-17/4)/(-256*a**7*b**9*x**16*gamma(
-1/4) + 768*a**6*b**10*x**20*gamma(-1/4) - 768*a**5*b**11*x**24*gamma(-1/4)
 + 256*a**4*b**12*x**28*gamma(-1/4)) - 231*a**3*b**(53/4)*x**16*(-a/(b*x**4
) + 1)**(1/4)*exp(I*pi/4)*gamma(-17/4)/(-256*a**7*b**9*x**16*gamma(-1/4) +
768*a**6*b**10*x**20*gamma(-1/4) - 768*a**5*b**11*x**24*gamma(-1/4) + 256*a
**4*b**12*x**28*gamma(-1/4)) + 924*a**2*b**(57/4)*x**20*(-a/(b*x**4) + 1)**
(1/4)*exp(I*pi/4)*gamma(-17/4)/(-256*a**7*b**9*x**16*gamma(-1/4) + 768*a**6
*b**10*x**20*gamma(-1/4) - 768*a**5*b**11*x**24*gamma(-1/4) + 256*a**4*b**1
2*x**28*gamma(-1/4)) - 1056*a*b**(61/4)*x**24*(-a/(b*x**4) + 1)**(1/4)*exp(
I*pi/4)*gamma(-17/4)/(-256*a**7*b**9*x**16*gamma(-1/4) + 768*a**6*b**10*x**
20*gamma(-1/4) - 768*a**5*b**11*x**24*gamma(-1/4) + 256*a**4*b**12*x**28*ga
mma(-1/4)) + 384*b**(65/4)*x**28*(-a/(b*x**4) + 1)**(1/4)*exp(I*pi/4)*gamma
(-17/4)/(-256*a**7*b**9*x**16*gamma(-1/4) + 768*a**6*b**10*x**20*gamma(-1/4
) - 768*a**5*b**11*x**24*gamma(-1/4) + 256*a**4*b**12*x**28*gamma(-1/4)), T
rue))
```

3.1196 $\int x^{12}\sqrt[4]{a-bx^4}\,dx$

Optimal. Leaf size=156

$$-\frac{3a^{7/2}x^3\left(1-\frac{a}{bx^4}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{112b^{5/2}\left(a-bx^4\right)^{3/4}}-\frac{3a^3x\sqrt[4]{a-bx^4}}{112b^3}-\frac{3a^2x^5\sqrt[4]{a-bx^4}}{280b^2}+\frac{1}{14}x^{13}\sqrt[4]{a-bx^4}-\frac{ax^9\sqrt[4]{a-bx^4}}{140b}$$

```
[Out] -3/112*a^3*x*(-b*x^4+a)^(1/4)/b^3-3/280*a^2*x^5*(-b*x^4+a)^(1/4)/b^2-1/140*
a*x^9*(-b*x^4+a)^(1/4)/b+1/14*x^13*(-b*x^4+a)^(1/4)-3/112*a^(7/2)*(1-a/b/x^
4)^(3/4)*x^3*(cos(1/2*arccsc(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccsc(
x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccsc(x^2*b^(1/2)/a^(1/2))),2^(1/2
))/b^(5/2)/(-b*x^4+a)^(3/4)
```

Rubi [A] time = 0.08, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {279, 321, 237, 335, 275, 232}

$$-\frac{3a^2x^5\sqrt[4]{a-bx^4}}{280b^2}-\frac{3a^3x\sqrt[4]{a-bx^4}}{112b^3}-\frac{3a^{7/2}x^3\left(1-\frac{a}{bx^4}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{112b^{5/2}\left(a-bx^4\right)^{3/4}}+\frac{1}{14}x^{13}\sqrt[4]{a-bx^4}-\frac{ax^9\sqrt[4]{a-bx^4}}{140b}$$

Antiderivative was successfully verified.

```
[In] Int[x^12*(a - b*x^4)^(1/4),x]
```

```
[Out] (-3*a^3*x*(a - b*x^4)^(1/4))/(112*b^3) - (3*a^2*x^5*(a - b*x^4)^(1/4))/(280
*b^2) - (a*x^9*(a - b*x^4)^(1/4))/(140*b) + (x^13*(a - b*x^4)^(1/4))/14 - (
3*a^(7/2)*(1 - a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCsc[(Sqrt[b]*x^2)/Sqrt[a]]
/2, 2])/(112*b^(5/2)*(a - b*x^4)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int x^{12}\sqrt[4]{a-bx^4}\,dx = \frac{1}{14}x^{13}\sqrt[4]{a-bx^4} + \frac{1}{14}a\int \frac{x^{12}}{\left(a-bx^4\right)^{3/4}}\,dx$$

$$= -\frac{ax^9\sqrt[4]{a-bx^4}}{140b} + \frac{1}{14}x^{13}\sqrt[4]{a-bx^4} + \frac{\left(9a^2\right)\int \frac{x^8}{\left(a-bx^4\right)^{3/4}}\,dx}{140b}$$

$$= -\frac{3a^2x^5\sqrt[4]{a-bx^4}}{280b^2} - \frac{ax^9\sqrt[4]{a-bx^4}}{140b} + \frac{1}{14}x^{13}\sqrt[4]{a-bx^4} + \frac{\left(3a^3\right)\int \frac{x^4}{\left(a-bx^4\right)^{3/4}}\,dx}{56b^2}$$

$$= -\frac{3a^3x\sqrt[4]{a-bx^4}}{112b^3} - \frac{3a^2x^5\sqrt[4]{a-bx^4}}{280b^2} - \frac{ax^9\sqrt[4]{a-bx^4}}{140b} + \frac{1}{14}x^{13}\sqrt[4]{a-bx^4} + \frac{\left(3a^4\right)\int \frac{1}{\left(a-bx^4\right)^{3/4}}\,dx}{112b^3}$$

$$= -\frac{3a^3x\sqrt[4]{a-bx^4}}{112b^3} - \frac{3a^2x^5\sqrt[4]{a-bx^4}}{280b^2} - \frac{ax^9\sqrt[4]{a-bx^4}}{140b} + \frac{1}{14}x^{13}\sqrt[4]{a-bx^4} + \frac{\left(3a^4\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right.}{112b^3\left(a-\right.}$$

$$= -\frac{3a^3x\sqrt[4]{a-bx^4}}{112b^3} - \frac{3a^2x^5\sqrt[4]{a-bx^4}}{280b^2} - \frac{ax^9\sqrt[4]{a-bx^4}}{140b} + \frac{1}{14}x^{13}\sqrt[4]{a-bx^4} - \frac{\left(3a^4\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right.}{112b}$$

$$= -\frac{3a^3x\sqrt[4]{a-bx^4}}{112b^3} - \frac{3a^2x^5\sqrt[4]{a-bx^4}}{280b^2} - \frac{ax^9\sqrt[4]{a-bx^4}}{140b} + \frac{1}{14}x^{13}\sqrt[4]{a-bx^4} - \frac{\left(3a^4\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right.}{224}$$

$$= -\frac{3a^3x\sqrt[4]{a-bx^4}}{112b^3} - \frac{3a^2x^5\sqrt[4]{a-bx^4}}{280b^2} - \frac{ax^9\sqrt[4]{a-bx^4}}{140b} + \frac{1}{14}x^{13}\sqrt[4]{a-bx^4} - \frac{3a^{7/2}\left(1-\frac{a}{bx^4}\right)^{3/4}x^3}{112b^{5/2}\left(\right.}$$

Mathematica [C] time = 0.09, size = 108, normalized size = 0.69

$$\frac{x\sqrt[4]{a-bx^4}\left(15a^3\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{bx^4}{a}\right) - \sqrt[4]{1-\frac{bx^4}{a}}\left(15a^3+3a^2bx^4+2ab^2x^8-20b^3x^{12}\right)\right)}{280b^3\sqrt[4]{1-\frac{bx^4}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12*(a - b*x^4)^(1/4),x]
```

```
[Out] (x*(a - b*x^4)^(1/4)*(-((1 - (b*x^4)/a)^(1/4)*(15*a^3 + 3*a^2*b*x^4 + 2*a*b
^2*x^8 - 20*b^3*x^12)) + 15*a^3*Hypergeometric2F1[-1/4, 1/4, 5/4, (b*x^4)/a
]))/(280*b^3*(1 - (b*x^4)/a)^(1/4))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-bx^4+a\right)^{\frac{1}{4}}x^{12}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((-b*x^4 + a)^(1/4)*x^12, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-bx^4+a\right)^{\frac{1}{4}}x^{12}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(-b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^4 + a)^(1/4)*x^12, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \left(-b\,x^4+a\right)^{\frac{1}{4}}x^{12}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(-b*x^4+a)^(1/4),x)
```

```
[Out] int(x^12*(-b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-bx^4+a\right)^{\frac{1}{4}}x^{12}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^4 + a)^(1/4)*x^12, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{12}\left(a-b\,x^4\right)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(a - b*x^4)^(1/4),x)
```

```
[Out] int(x^12*(a - b*x^4)^(1/4), x)
```

sympy [C] time = 2.25, size = 41, normalized size = 0.26

$$\frac{\sqrt[4]{a}\,x^{13}\Gamma\left(\frac{13}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4}, \frac{13}{4}\\ \frac{17}{4}\end{matrix}\middle|\,\frac{bx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{17}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12*(-b*x**4+a)**(1/4),x)
```

```
[Out] a**(1/4)*x**13*gamma(13/4)*hyper((-1/4, 13/4), (17/4,), b*x**4*exp_polar(2*
I*pi)/a)/(4*gamma(17/4))
```

3.1197 $\int x^8 \sqrt[4]{a-bx^4}\,dx$

Optimal. Leaf size=131

$$-\frac{a^{5/2}x^3\left(1-\frac{a}{bx^4}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{24b^{3/2}\left(a-bx^4\right)^{3/4}}-\frac{a^2x\sqrt[4]{a-bx^4}}{24b^2}+\frac{1}{10}x^9\sqrt[4]{a-bx^4}-\frac{ax^5\sqrt[4]{a-bx^4}}{60b}$$

```
[Out] -1/24*a^2*x*(-b*x^4+a)^(1/4)/b^2-1/60*a*x^5*(-b*x^4+a)^(1/4)/b+1/10*x^9*(-b
*x^4+a)^(1/4)-1/24*a^(5/2)*(1-a/b/x^4)^(3/4)*x^3*(cos(1/2*arccsc(x^2*b^(1/2
)/a^(1/2)))^2)^(1/2)/cos(1/2*arccsc(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2
*arccsc(x^2*b^(1/2)/a^(1/2))),2^(1/2))/b^(3/2)/(-b*x^4+a)^(3/4)
```

Rubi [A] time = 0.06, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {279, 321, 237, 335, 275, 232}

$$-\frac{a^2x\sqrt[4]{a-bx^4}}{24b^2}-\frac{a^{5/2}x^3\left(1-\frac{a}{bx^4}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{24b^{3/2}\left(a-bx^4\right)^{3/4}}+\frac{1}{10}x^9\sqrt[4]{a-bx^4}-\frac{ax^5\sqrt[4]{a-bx^4}}{60b}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a - b*x^4)^(1/4),x]
```

```
[Out] -(a^2*x*(a - b*x^4)^(1/4))/(24*b^2) - (a*x^5*(a - b*x^4)^(1/4))/(60*b) + (x
^9*(a - b*x^4)^(1/4))/10 - (a^(5/2)*(1 - a/(b*x^4))^(3/4)*x^3*EllipticF[Arc
Csc[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(24*b^(3/2)*(a - b*x^4)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int x^8 \sqrt[4]{a-bx^4}\,dx = \frac{1}{10}x^9\sqrt[4]{a-bx^4} + \frac{1}{10}a\int \frac{x^8}{\left(a-bx^4\right)^{3/4}}\,dx$$

$$= -\frac{ax^5\sqrt[4]{a-bx^4}}{60b} + \frac{1}{10}x^9\sqrt[4]{a-bx^4} + \frac{a^2\int \frac{x^4}{\left(a-bx^4\right)^{3/4}}\,dx}{12b}$$

$$= -\frac{a^2x\sqrt[4]{a-bx^4}}{24b^2} - \frac{ax^5\sqrt[4]{a-bx^4}}{60b} + \frac{1}{10}x^9\sqrt[4]{a-bx^4} + \frac{a^3\int \frac{1}{\left(a-bx^4\right)^{3/4}}\,dx}{24b^2}$$

$$= -\frac{a^2x\sqrt[4]{a-bx^4}}{24b^2} - \frac{ax^5\sqrt[4]{a-bx^4}}{60b} + \frac{1}{10}x^9\sqrt[4]{a-bx^4} + \frac{\left(a^3\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{24b^2\left(a-bx^4\right)^{3/4}}$$

$$= -\frac{a^2x\sqrt[4]{a-bx^4}}{24b^2} - \frac{ax^5\sqrt[4]{a-bx^4}}{60b} + \frac{1}{10}x^9\sqrt[4]{a-bx^4} - \frac{\left(a^3\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1-\frac{ax^4}{b}\right)^{3/4}}\,dx\right.}{24b^2\left(a-bx^4\right)^{3/4}}$$

$$= -\frac{a^2x\sqrt[4]{a-bx^4}}{24b^2} - \frac{ax^5\sqrt[4]{a-bx^4}}{60b} + \frac{1}{10}x^9\sqrt[4]{a-bx^4} - \frac{\left(a^3\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{ax^2}{b}\right)^{3/4}}\,dx\right.}{48b^2\left(a-bx^4\right)^{3/4}}$$

$$= -\frac{a^2x\sqrt[4]{a-bx^4}}{24b^2} - \frac{ax^5\sqrt[4]{a-bx^4}}{60b} + \frac{1}{10}x^9\sqrt[4]{a-bx^4} - \frac{a^{5/2}\left(1-\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{24b^{3/2}\left(a-bx^4\right)^{3/4}}$$

Mathematica [C] time = 0.07, size = 96, normalized size = 0.73

$$\frac{x\sqrt[4]{a-bx^4}\left(5a^2\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{bx^4}{a}\right) - \sqrt[4]{1-\frac{bx^4}{a}}\left(5a^2+abx^4-6b^2x^8\right)\right)}{60b^2\sqrt[4]{1-\frac{bx^4}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a - b*x^4)^(1/4),x]
```

```
[Out] (x*(a - b*x^4)^(1/4)*(-((1 - (b*x^4)/a)^(1/4)*(5*a^2 + a*b*x^4 - 6*b^2*x^8)
) + 5*a^2*Hypergeometric2F1[-1/4, 1/4, 5/4, (b*x^4)/a]))/(60*b^2*(1 - (b*x^
4)/a)^(1/4))
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-bx^4+a\right)^{\frac{1}{4}}x^8, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(-b*x^4+a)^(1/4),x, algorithm="fricas")
[Out] integral((-b*x^4 + a)^(1/4)*x^8, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-bx^4+a\right)^{\frac{1}{4}}x^8\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(-b*x^4+a)^(1/4),x, algorithm="giac")
[Out] integrate((-b*x^4 + a)^(1/4)*x^8, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \left(-b\,x^4+a\right)^{\frac{1}{4}}x^8\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(-b*x^4+a)^(1/4),x)
[Out] int(x^8*(-b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-bx^4+a\right)^{\frac{1}{4}}x^8\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(-b*x^4+a)^(1/4),x, algorithm="maxima")
[Out] integrate((-b*x^4 + a)^(1/4)*x^8, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^8\left(a-b\,x^4\right)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a - b*x^4)^(1/4),x)
[Out] int(x^8*(a - b*x^4)^(1/4), x)
```

sympy [C] time = 2.05, size = 41, normalized size = 0.31

$$\frac{\sqrt[4]{a}\,x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4}, \frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle| \frac{bx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(-b*x**4+a)**(1/4),x)
```

```
[Out] a**(1/4)*x**9*gamma(9/4)*hyper((-1/4, 9/4), (13/4,), b*x**4*exp_polar(2*I*p
i)/a)/(4*gamma(13/4))
```

3.1198 $\int x^4 \sqrt[4]{a-bx^4}\,dx$

Optimal. Leaf size=106

$$-\frac{a^{3/2}x^3\left(1-\frac{a}{bx^4}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{12\sqrt{b}\left(a-bx^4\right)^{3/4}}-\frac{ax\sqrt[4]{a-bx^4}}{12b}+\frac{1}{6}x^5\sqrt[4]{a-bx^4}$$

```
[Out] -1/12*a*x*(-b*x^4+a)^(1/4)/b+1/6*x^5*(-b*x^4+a)^(1/4)-1/12*a^(3/2)*(1-a/b/x
^4)^(3/4)*x^3*(cos(1/2*arccsc(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccsc
(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccsc(x^2*b^(1/2)/a^(1/2))),2^(1/
2))/(-b*x^4+a)^(3/4)/b^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {279, 321, 237, 335, 275, 232}

$$-\frac{a^{3/2}x^3\left(1-\frac{a}{bx^4}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{12\sqrt{b}\left(a-bx^4\right)^{3/4}}+\frac{1}{6}x^5\sqrt[4]{a-bx^4}-\frac{ax\sqrt[4]{a-bx^4}}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a - b*x^4)^(1/4),x]
```

```
[Out] -(a*x*(a - b*x^4)^(1/4))/(12*b) + (x^5*(a - b*x^4)^(1/4))/6 - (a^(3/2)*(1 -
 a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCsc[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(12*Sq
rt[b]*(a - b*x^4)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int x^4 \sqrt[4]{a-bx^4}\,dx = \frac{1}{6}x^5\sqrt[4]{a-bx^4} + \frac{1}{6}a\int \frac{x^4}{\left(a-bx^4\right)^{3/4}}\,dx$$

$$= -\frac{ax\sqrt[4]{a-bx^4}}{12b} + \frac{1}{6}x^5\sqrt[4]{a-bx^4} + \frac{a^2\int \frac{1}{\left(a-bx^4\right)^{3/4}}\,dx}{12b}$$

$$= -\frac{ax\sqrt[4]{a-bx^4}}{12b} + \frac{1}{6}x^5\sqrt[4]{a-bx^4} + \frac{\left(a^2\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{12b\left(a-bx^4\right)^{3/4}}$$

$$= -\frac{ax\sqrt[4]{a-bx^4}}{12b} + \frac{1}{6}x^5\sqrt[4]{a-bx^4} - \frac{\left(a^2\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1-\frac{ax^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{12b\left(a-bx^4\right)^{3/4}}$$

$$= -\frac{ax\sqrt[4]{a-bx^4}}{12b} + \frac{1}{6}x^5\sqrt[4]{a-bx^4} - \frac{\left(a^2\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{ax^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{x^2}\right)}{24b\left(a-bx^4\right)^{3/4}}$$

$$= -\frac{ax\sqrt[4]{a-bx^4}}{12b} + \frac{1}{6}x^5\sqrt[4]{a-bx^4} - \frac{a^{3/2}\left(1-\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{12\sqrt{b}\left(a-bx^4\right)^{3/4}}$$

Mathematica [C] time = 0.05, size = 64, normalized size = 0.60

$$\frac{x\sqrt[4]{a-bx^4}\left(\frac{a\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{bx^4}{a}\right)}{\sqrt[4]{1-\frac{bx^4}{a}}} - a + bx^4\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a - b*x^4)^(1/4),x]
```

```
[Out] (x*(a - b*x^4)^(1/4)*(-a + b*x^4 + (a*Hypergeometric2F1[-1/4, 1/4, 5/4, (b*
x^4)/a])/(1 - (b*x^4)/a)^(1/4)))/(6*b)
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-bx^4+a\right)^{\frac{1}{4}}x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((-b*x^4 + a)^(1/4)*x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-bx^4 + a\right)^{\frac{1}{4}} x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^4 + a)^(1/4)*x^4, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \left(-b\,x^4 + a\right)^{\frac{1}{4}} x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-b*x^4+a)^(1/4),x)
```

```
[Out] int(x^4*(-b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-bx^4 + a\right)^{\frac{1}{4}} x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^4 + a)^(1/4)*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \left(a - b\,x^4\right)^{1/4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a - b*x^4)^(1/4),x)
```

```
[Out] int(x^4*(a - b*x^4)^(1/4), x)
```

sympy [C] time = 1.37, size = 41, normalized size = 0.39

$$\frac{\sqrt[4]{a}\, x^5 \Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{5}{4} \\ \frac{9}{4} \end{matrix} \middle| \frac{bx^4 e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-b*x**4+a)**(1/4),x)
```

```
[Out] a**(1/4)*x**5*gamma(5/4)*hyper((-1/4, 5/4), (9/4,), b*x**4*exp_polar(2*I*pi
)/a)/(4*gamma(9/4))
```

3.1199 $\int \sqrt[4]{a - bx^4}\, dx$

Optimal. Leaf size=83

$$\frac{1}{2}x\sqrt[4]{a-bx^4} - \frac{\sqrt{a}\,\sqrt{b}\,x^3\left(1-\frac{a}{bx^4}\right)^{3/4} F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|\, 2\right)}{2\left(a-bx^4\right)^{3/4}}$$

```
[Out] 1/2*x*(-b*x^4+a)^(1/4)-1/2*(1-a/b/x^4)^(3/4)*x^3*(cos(1/2*arccsc(x^2*b^(1/2
)/a^(1/2)))^2)^(1/2)/cos(1/2*arccsc(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2
*arccsc(x^2*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)*b^(1/2)/(-b*x^4+a)^(3/4)
```

Rubi [A] time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {195, 237, 335, 275, 232}

$$\frac{1}{2}x\sqrt[4]{a-bx^4} - \frac{\sqrt{a}\,\sqrt{b}\,x^3\left(1-\frac{a}{bx^4}\right)^{3/4} F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|\, 2\right)}{2\left(a-bx^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^4)^(1/4),x]
```

```
[Out] (x*(a - b*x^4)^(1/4))/2 - (Sqrt[a]*Sqrt[b]*(1 - a/(b*x^4))^(3/4)*x^3*Ellipt
icF[ArcCsc[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(2*(a - b*x^4)^(3/4))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \sqrt[4]{a-bx^4}\,dx &= \frac{1}{2}x\sqrt[4]{a-bx^4} + \frac{1}{2}a\int \frac{1}{\left(a-bx^4\right)^{3/4}}\,dx \\
&= \frac{1}{2}x\sqrt[4]{a-bx^4} + \frac{\left(a\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{2\left(a-bx^4\right)^{3/4}} \\
&= \frac{1}{2}x\sqrt[4]{a-bx^4} - \frac{\left(a\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1-\frac{ax^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{2\left(a-bx^4\right)^{3/4}} \\
&= \frac{1}{2}x\sqrt[4]{a-bx^4} - \frac{\left(a\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{ax^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{x^2}\right)}{4\left(a-bx^4\right)^{3/4}} \\
&= \frac{1}{2}x\sqrt[4]{a-bx^4} - \frac{\sqrt{a}\sqrt{b}\left(1-\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{2\left(a-bx^4\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 47, normalized size = 0.57

$$\frac{x\sqrt[4]{a-bx^4}\;{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{bx^4}{a}\right)}{\sqrt[4]{1-\frac{bx^4}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^4)^(1/4),x]
```

```
[Out] (x*(a - b*x^4)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 5/4, (b*x^4)/a])/(1 - (b*
x^4)/a)^(1/4)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-bx^4+a\right)^{\frac{1}{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((-b*x^4 + a)^(1/4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-bx^4+a\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^4 + a)^(1/4), x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \left(-b\,x^4+a\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(1/4),x)
```

```
[Out] int((-b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-bx^4+a\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^4 + a)^(1/4), x)
```

mupad [B] time = 1.07, size = 38, normalized size = 0.46

$$\frac{x\left(a-b\,x^4\right)^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ \frac{b\,x^4}{a}\right)}{\left(1-\frac{b\,x^4}{a}\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^4)^(1/4),x)
```

```
[Out] (x*(a - b*x^4)^(1/4)*hypergeom([-1/4, 1/4], 5/4, (b*x^4)/a))/(1 - (b*x^4)/a
)^(1/4)
```

sympy [C] time = 0.97, size = 39, normalized size = 0.47

$$\frac{\sqrt[4]{a}\,x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\right|\frac{bx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(1/4),x)
```

```
[Out] a**(1/4)*x*gamma(1/4)*hyper((-1/4, 1/4), (5/4,), b*x**4*exp_polar(2*I*pi)/a
)/(4*gamma(5/4))
```

3.1200 $\int \frac{\sqrt[4]{a-bx^4}}{x^4}\,dx$

Optimal. Leaf size=85

$$\frac{b^{3/2}x^3\left(1-\frac{a}{bx^4}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}\left(a-bx^4\right)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{3x^3}$$

```
[Out] -1/3*(-b*x^4+a)^(1/4)/x^3+1/3*b^(3/2)*(1-a/b/x^4)^(3/4)*x^3*(cos(1/2*arccsc
(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccsc(x^2*b^(1/2)/a^(1/2)))*Ellipt
icF(sin(1/2*arccsc(x^2*b^(1/2)/a^(1/2))),2^(1/2))/(-b*x^4+a)^(3/4)/a^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {277, 237, 335, 275, 232}

$$\frac{b^{3/2}x^3\left(1-\frac{a}{bx^4}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}\left(a-bx^4\right)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^4)^(1/4)/x^4,x]
```

```
[Out] -(a - b*x^4)^(1/4)/(3*x^3) + (b^(3/2)*(1 - a/(b*x^4))^(3/4)*x^3*EllipticF[A
rcCsc[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(3*Sqrt[a]*(a - b*x^4)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{a-bx^4}}{x^4}\,dx &= -\frac{\sqrt[4]{a-bx^4}}{3x^3} - \frac{1}{3}b\int \frac{1}{\left(a-bx^4\right)^{3/4}}\,dx \\
&= -\frac{\sqrt[4]{a-bx^4}}{3x^3} - \frac{\left(b\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{3\left(a-bx^4\right)^{3/4}} \\
&= -\frac{\sqrt[4]{a-bx^4}}{3x^3} + \frac{\left(b\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1-\frac{ax^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{3\left(a-bx^4\right)^{3/4}} \\
&= -\frac{\sqrt[4]{a-bx^4}}{3x^3} + \frac{\left(b\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{ax^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{x^2}\right)}{6\left(a-bx^4\right)^{3/4}} \\
&= -\frac{\sqrt[4]{a-bx^4}}{3x^3} + \frac{b^{3/2}\left(1-\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}\,\left(a-bx^4\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.61

$$-\frac{\sqrt[4]{a-bx^4}\;{}_2F_1\left(-\frac{3}{4},-\frac{1}{4};\frac{1}{4};\frac{bx^4}{a}\right)}{3x^3\sqrt[4]{1-\frac{bx^4}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^4)^(1/4)/x^4,x]
```

```
[Out] -1/3*((a - b*x^4)^(1/4)*Hypergeometric2F1[-3/4, -1/4, 1/4, (b*x^4)/a])/(x^3
*(1 - (b*x^4)/a)^(1/4))
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^4,x, algorithm="fricas")
```

```
[Out] integral((-b*x^4 + a)^(1/4)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^4,x, algorithm="giac")
[Out] integrate((-b*x^4 + a)^(1/4)/x^4, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^4+a\right)^{\frac{1}{4}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(1/4)/x^4,x)
[Out] int((-b*x^4+a)^(1/4)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^4,x, algorithm="maxima")
[Out] integrate((-b*x^4 + a)^(1/4)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a-b\,x^4\right)^{1/4}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^4)^(1/4)/x^4,x)
[Out] int((a - b*x^4)^(1/4)/x^4, x)
```

sympy [C] time = 1.99, size = 34, normalized size = 0.40

$$-\frac{i\sqrt[4]{b}\,e^{-\frac{i\pi}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\,\frac{a}{bx^4}\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(1/4)/x**4,x)
[Out] -I*b**(1/4)*exp(-I*pi/4)*hyper((-1/4, 1/2), (3/2,), a/(b*x**4))/(2*x**2)
```

3.1201 $\int \frac{\sqrt[4]{a-bx^4}}{x^8}\,dx$

Optimal. Leaf size=108

$$\frac{2b^{5/2}x^3\left(1-\frac{a}{bx^4}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{21a^{3/2}\left(a-bx^4\right)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{7x^7}+\frac{b\sqrt[4]{a-bx^4}}{21ax^3}$$

```
[Out] -1/7*(-b*x^4+a)^(1/4)/x^7+1/21*b*(-b*x^4+a)^(1/4)/a/x^3+2/21*b^(5/2)*(1-a/b
/x^4)^(3/4)*x^3*(cos(1/2*arccsc(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcc
sc(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccsc(x^2*b^(1/2)/a^(1/2))),2^(
1/2))/a^(3/2)/(-b*x^4+a)^(3/4)
```

Rubi [A] time = 0.05, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {277, 325, 237, 335, 275, 232}

$$\frac{2b^{5/2}x^3\left(1-\frac{a}{bx^4}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{21a^{3/2}\left(a-bx^4\right)^{3/4}}+\frac{b\sqrt[4]{a-bx^4}}{21ax^3}-\frac{\sqrt[4]{a-bx^4}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^4)^(1/4)/x^8,x]
```

```
[Out] -(a - b*x^4)^(1/4)/(7*x^7) + (b*(a - b*x^4)^(1/4))/(21*a*x^3) + (2*b^(5/2)*
(1 - a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCsc[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(2
1*a^(3/2)*(a - b*x^4)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{a-bx^4}}{x^8}\,dx &= -\frac{\sqrt[4]{a-bx^4}}{7x^7} - \frac{1}{7}b\int \frac{1}{x^4\left(a-bx^4\right)^{3/4}}\,dx \\
&= -\frac{\sqrt[4]{a-bx^4}}{7x^7} + \frac{b\sqrt[4]{a-bx^4}}{21ax^3} - \frac{\left(2b^2\right)\int \frac{1}{\left(a-bx^4\right)^{3/4}}\,dx}{21a} \\
&= -\frac{\sqrt[4]{a-bx^4}}{7x^7} + \frac{b\sqrt[4]{a-bx^4}}{21ax^3} - \frac{\left(2b^2\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{21a\left(a-bx^4\right)^{3/4}} \\
&= -\frac{\sqrt[4]{a-bx^4}}{7x^7} + \frac{b\sqrt[4]{a-bx^4}}{21ax^3} + \frac{\left(2b^2\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1-\frac{ax^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{21a\left(a-bx^4\right)^{3/4}} \\
&= -\frac{\sqrt[4]{a-bx^4}}{7x^7} + \frac{b\sqrt[4]{a-bx^4}}{21ax^3} + \frac{\left(b^2\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{ax^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{x^2}\right)}{21a\left(a-bx^4\right)^{3/4}} \\
&= -\frac{\sqrt[4]{a-bx^4}}{7x^7} + \frac{b\sqrt[4]{a-bx^4}}{21ax^3} + \frac{2b^{5/2}\left(1-\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{21a^{3/2}\left(a-bx^4\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.48

$$-\frac{\sqrt[4]{a-bx^4}\;{}_2F_1\left(-\frac{7}{4},-\frac{1}{4};-\frac{3}{4};\frac{bx^4}{a}\right)}{7x^7\sqrt[4]{1-\frac{bx^4}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^4)^(1/4)/x^8,x]
```

```
[Out] -1/7*((a - b*x^4)^(1/4)*Hypergeometric2F1[-7/4, -1/4, -3/4, (b*x^4)/a])/(x^
7*(1 - (b*x^4)/a)^(1/4))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^8,x, algorithm="fricas")
[Out] integral((-b*x^4 + a)^(1/4)/x^8, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^8,x, algorithm="giac")
[Out] integrate((-b*x^4 + a)^(1/4)/x^8, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^4+a\right)^{\frac{1}{4}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(1/4)/x^8,x)
[Out] int((-b*x^4+a)^(1/4)/x^8,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^8,x, algorithm="maxima")
[Out] integrate((-b*x^4 + a)^(1/4)/x^8, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a-b\,x^4\right)^{1/4}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^4)^(1/4)/x^8,x)
[Out] int((a - b*x^4)^(1/4)/x^8, x)
```

sympy [C] time = 2.39, size = 34, normalized size = 0.31

$$\frac{i\sqrt[4]{b}\,e^{\frac{3i\pi}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\,\frac{a}{bx^4}\right)}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(1/4)/x**8,x)
[Out] I*b**(1/4)*exp(3*I*pi/4)*hyper((-1/4, 3/2), (5/2,), a/(b*x**4))/(6*x**6)
```

3.1202 $\int \frac{\sqrt[4]{a-bx^4}}{x^{12}}\,dx$

Optimal. Leaf size=133

$$\frac{4b^{7/2}x^3\left(1-\frac{a}{bx^4}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{77a^{5/2}\left(a-bx^4\right)^{3/4}}+\frac{2b^2\sqrt[4]{a-bx^4}}{77a^2x^3}-\frac{\sqrt[4]{a-bx^4}}{11x^{11}}+\frac{b\sqrt[4]{a-bx^4}}{77ax^7}$$

```
[Out] -1/11*(-b*x^4+a)^(1/4)/x^11+1/77*b*(-b*x^4+a)^(1/4)/a/x^7+2/77*b^2*(-b*x^4+
a)^(1/4)/a^2/x^3+4/77*b^(7/2)*(1-a/b/x^4)^(3/4)*x^3*(cos(1/2*arccsc(x^2*b^(
1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccsc(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(
1/2*arccsc(x^2*b^(1/2)/a^(1/2))),2^(1/2))/a^(5/2)/(-b*x^4+a)^(3/4)
```

Rubi [A] time = 0.06, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {277, 325, 237, 335, 275, 232}

$$\frac{2b^2\sqrt[4]{a-bx^4}}{77a^2x^3}+\frac{4b^{7/2}x^3\left(1-\frac{a}{bx^4}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{77a^{5/2}\left(a-bx^4\right)^{3/4}}+\frac{b\sqrt[4]{a-bx^4}}{77ax^7}-\frac{\sqrt[4]{a-bx^4}}{11x^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^4)^(1/4)/x^12,x]
```

```
[Out] -(a - b*x^4)^(1/4)/(11*x^11) + (b*(a - b*x^4)^(1/4))/(77*a*x^7) + (2*b^2*(a
 - b*x^4)^(1/4))/(77*a^2*x^3) + (4*b^(7/2)*(1 - a/(b*x^4))^(3/4)*x^3*Elliptic
F[ArcCsc[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(77*a^(5/2)*(a - b*x^4)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{a-bx^4}}{x^{12}}\,dx &= -\frac{\sqrt[4]{a-bx^4}}{11x^{11}} - \frac{1}{11}b\int \frac{1}{x^8\left(a-bx^4\right)^{3/4}}\,dx \\
&= -\frac{\sqrt[4]{a-bx^4}}{11x^{11}} + \frac{b\sqrt[4]{a-bx^4}}{77ax^7} - \frac{\left(6b^2\right)\int \frac{1}{x^4\left(a-bx^4\right)^{3/4}}\,dx}{77a} \\
&= -\frac{\sqrt[4]{a-bx^4}}{11x^{11}} + \frac{b\sqrt[4]{a-bx^4}}{77ax^7} + \frac{2b^2\sqrt[4]{a-bx^4}}{77a^2x^3} - \frac{\left(4b^3\right)\int \frac{1}{\left(a-bx^4\right)^{3/4}}\,dx}{77a^2} \\
&= -\frac{\sqrt[4]{a-bx^4}}{11x^{11}} + \frac{b\sqrt[4]{a-bx^4}}{77ax^7} + \frac{2b^2\sqrt[4]{a-bx^4}}{77a^2x^3} - \frac{\left(4b^3\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{77a^2\left(a-bx^4\right)^{3/4}} \\
&= -\frac{\sqrt[4]{a-bx^4}}{11x^{11}} + \frac{b\sqrt[4]{a-bx^4}}{77ax^7} + \frac{2b^2\sqrt[4]{a-bx^4}}{77a^2x^3} + \frac{\left(4b^3\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1-\frac{ax^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{77a^2\left(a-bx^4\right)^{3/4}} \\
&= -\frac{\sqrt[4]{a-bx^4}}{11x^{11}} + \frac{b\sqrt[4]{a-bx^4}}{77ax^7} + \frac{2b^2\sqrt[4]{a-bx^4}}{77a^2x^3} + \frac{\left(2b^3\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{ax^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{x^2}\right)}{77a^2\left(a-bx^4\right)^{3/4}} \\
&= -\frac{\sqrt[4]{a-bx^4}}{11x^{11}} + \frac{b\sqrt[4]{a-bx^4}}{77ax^7} + \frac{2b^2\sqrt[4]{a-bx^4}}{77a^2x^3} + \frac{4b^{7/2}\left(1-\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{77a^{5/2}\left(a-bx^4\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.39

$$-\frac{\sqrt[4]{a-bx^4}\;{}_2F_1\left(-\frac{11}{4},-\frac{1}{4};-\frac{7}{4};\frac{bx^4}{a}\right)}{11x^{11}\sqrt[4]{1-\frac{bx^4}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^4)^(1/4)/x^12,x]
```

```
[Out] -1/11*((a - b*x^4)^(1/4)*Hypergeometric2F1[-11/4, -1/4, -7/4, (b*x^4)/a])/(
x^11*(1 - (b*x^4)/a)^(1/4))
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{x^{12}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^12,x, algorithm="fricas")
[Out] integral((-b*x^4 + a)^(1/4)/x^12, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^12,x, algorithm="giac")
[Out] integrate((-b*x^4 + a)^(1/4)/x^12, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^4+a\right)^{\frac{1}{4}}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(1/4)/x^12,x)
[Out] int((-b*x^4+a)^(1/4)/x^12,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^12,x, algorithm="maxima")
[Out] integrate((-b*x^4 + a)^(1/4)/x^12, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a-b\,x^4\right)^{1/4}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^4)^(1/4)/x^12,x)
[Out] int((a - b*x^4)^(1/4)/x^12, x)
```

sympy [C] time = 1.88, size = 34, normalized size = 0.26

$$-\frac{i\sqrt[4]{b}\,e^{-\frac{i\pi}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{1}{4}, \frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle| \frac{a}{bx^4}\right)}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(1/4)/x**12,x)
[Out] -I*b**(1/4)*exp(-I*pi/4)*hyper((-1/4, 5/2), (7/2,), a/(b*x**4))/(10*x**10)
```

3.1203 $\int \frac{\sqrt[4]{a-bx^4}}{x^{16}}\,dx$

Optimal. Leaf size=158

$$\frac{8b^{9/2}x^3\left(1-\frac{a}{bx^4}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{231a^{7/2}\left(a-bx^4\right)^{3/4}}+\frac{4b^3\sqrt[4]{a-bx^4}}{231a^3x^3}+\frac{2b^2\sqrt[4]{a-bx^4}}{231a^2x^7}-\frac{\sqrt[4]{a-bx^4}}{15x^{15}}+\frac{b\sqrt[4]{a-bx^4}}{165ax^{11}}$$

```
[Out] -1/15*(-b*x^4+a)^(1/4)/x^15+1/165*b*(-b*x^4+a)^(1/4)/a/x^11+2/231*b^2*(-b*x
^4+a)^(1/4)/a^2/x^7+4/231*b^3*(-b*x^4+a)^(1/4)/a^3/x^3+8/231*b^(9/2)*(1-a/b
/x^4)^(3/4)*x^3*(cos(1/2*arccsc(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcc
sc(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccsc(x^2*b^(1/2)/a^(1/2))),2^(
1/2))/a^(7/2)/(-b*x^4+a)^(3/4)
```

Rubi [A] time = 0.08, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {277, 325, 237, 335, 275, 232}

$$\frac{4b^3\sqrt[4]{a-bx^4}}{231a^3x^3}+\frac{2b^2\sqrt[4]{a-bx^4}}{231a^2x^7}+\frac{8b^{9/2}x^3\left(1-\frac{a}{bx^4}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{231a^{7/2}\left(a-bx^4\right)^{3/4}}+\frac{b\sqrt[4]{a-bx^4}}{165ax^{11}}-\frac{\sqrt[4]{a-bx^4}}{15x^{15}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^4)^(1/4)/x^16,x]
```

```
[Out] -(a - b*x^4)^(1/4)/(15*x^15) + (b*(a - b*x^4)^(1/4))/(165*a*x^11) + (2*b^2*
(a - b*x^4)^(1/4))/(231*a^2*x^7) + (4*b^3*(a - b*x^4)^(1/4))/(231*a^3*x^3)
+ (8*b^(9/2)*(1 - a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCsc[(Sqrt[b]*x^2)/Sqrt[
a]]/2, 2])/(231*a^(7/2)*(a - b*x^4)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{16}}\,dx = -\frac{\sqrt[4]{a-bx^4}}{15x^{15}} - \frac{1}{15}b\int \frac{1}{x^{12}\left(a-bx^4\right)^{3/4}}\,dx$$

$$= -\frac{\sqrt[4]{a-bx^4}}{15x^{15}} + \frac{b\sqrt[4]{a-bx^4}}{165ax^{11}} - \frac{\left(2b^2\right)\int \frac{1}{x^8\left(a-bx^4\right)^{3/4}}\,dx}{33a}$$

$$= -\frac{\sqrt[4]{a-bx^4}}{15x^{15}} + \frac{b\sqrt[4]{a-bx^4}}{165ax^{11}} + \frac{2b^2\sqrt[4]{a-bx^4}}{231a^2x^7} - \frac{\left(4b^3\right)\int \frac{1}{x^4\left(a-bx^4\right)^{3/4}}\,dx}{77a^2}$$

$$= -\frac{\sqrt[4]{a-bx^4}}{15x^{15}} + \frac{b\sqrt[4]{a-bx^4}}{165ax^{11}} + \frac{2b^2\sqrt[4]{a-bx^4}}{231a^2x^7} + \frac{4b^3\sqrt[4]{a-bx^4}}{231a^3x^3} - \frac{\left(8b^4\right)\int \frac{1}{\left(a-bx^4\right)^{3/4}}\,dx}{231a^3}$$

$$= -\frac{\sqrt[4]{a-bx^4}}{15x^{15}} + \frac{b\sqrt[4]{a-bx^4}}{165ax^{11}} + \frac{2b^2\sqrt[4]{a-bx^4}}{231a^2x^7} + \frac{4b^3\sqrt[4]{a-bx^4}}{231a^3x^3} - \frac{\left(8b^4\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x^3}}{231a^3\left(a-bx^4\right)^{3/4}}$$

$$= -\frac{\sqrt[4]{a-bx^4}}{15x^{15}} + \frac{b\sqrt[4]{a-bx^4}}{165ax^{11}} + \frac{2b^2\sqrt[4]{a-bx^4}}{231a^2x^7} + \frac{4b^3\sqrt[4]{a-bx^4}}{231a^3x^3} + \frac{\left(8b^4\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{}{\left(1-\right.}\right.}{231a^3\left(a-bx^4\right)^{3/4}}$$

$$= -\frac{\sqrt[4]{a-bx^4}}{15x^{15}} + \frac{b\sqrt[4]{a-bx^4}}{165ax^{11}} + \frac{2b^2\sqrt[4]{a-bx^4}}{231a^2x^7} + \frac{4b^3\sqrt[4]{a-bx^4}}{231a^3x^3} + \frac{\left(4b^4\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{}{\left(1-\right.}\right.}{231a^3\left(a-bx^4\right)^{3/4}}$$

$$= -\frac{\sqrt[4]{a-bx^4}}{15x^{15}} + \frac{b\sqrt[4]{a-bx^4}}{165ax^{11}} + \frac{2b^2\sqrt[4]{a-bx^4}}{231a^2x^7} + \frac{4b^3\sqrt[4]{a-bx^4}}{231a^3x^3} + \frac{8b^{9/2}\left(1-\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{}}{}\right.\right.}{231a^{7/2}\left(a-bx^4\right)^{3/4}}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.33

$$-\frac{\sqrt[4]{a-bx^4}\;{}_2F_1\left(-\frac{15}{4},-\frac{1}{4};-\frac{11}{4};\frac{bx^4}{a}\right)}{15x^{15}\sqrt[4]{1-\frac{bx^4}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^4)^(1/4)/x^16,x]
```

```
[Out] -1/15*((a - b*x^4)^(1/4)*Hypergeometric2F1[-15/4, -1/4, -11/4, (b*x^4)/a])/
(x^15*(1 - (b*x^4)/a)^(1/4))
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{x^{16}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^16,x, algorithm="fricas")
[Out] integral((-b*x^4 + a)^(1/4)/x^16, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^16,x, algorithm="giac")
[Out] integrate((-b*x^4 + a)^(1/4)/x^16, x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^4+a\right)^{\frac{1}{4}}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(1/4)/x^16,x)
[Out] int((-b*x^4+a)^(1/4)/x^16,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/4)/x^16,x, algorithm="maxima")
[Out] integrate((-b*x^4 + a)^(1/4)/x^16, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a-b\,x^4\right)^{1/4}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^4)^(1/4)/x^16,x)
[Out] int((a - b*x^4)^(1/4)/x^16, x)
```

sympy [C] time = 3.13, size = 34, normalized size = 0.22

$$\frac{i\sqrt[4]{b}\, e^{\frac{3i\pi}{4}}\, {}_2F_1\left(\left.\begin{matrix} -\frac{1}{4}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\right| \frac{a}{bx^4}\right)}{14x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(1/4)/x**16,x)
```

```
[Out] I*b**(1/4)*exp(3*I*pi/4)*hyper((-1/4, 7/2), (9/2,), a/(b*x**4))/(14*x**14)
```

3.1204 $\int \frac{x^{19}}{\sqrt[4]{a-bx^4}}\,dx$

Optimal. Leaf size=106

$$-\frac{a^4\left(a-bx^4\right)^{3/4}}{3b^5}+\frac{4a^3\left(a-bx^4\right)^{7/4}}{7b^5}-\frac{6a^2\left(a-bx^4\right)^{11/4}}{11b^5}-\frac{\left(a-bx^4\right)^{19/4}}{19b^5}+\frac{4a\left(a-bx^4\right)^{15/4}}{15b^5}$$

```
[Out] -1/3*a^4*(-b*x^4+a)^(3/4)/b^5+4/7*a^3*(-b*x^4+a)^(7/4)/b^5-6/11*a^2*(-b*x^4
+a)^(11/4)/b^5+4/15*a*(-b*x^4+a)^(15/4)/b^5-1/19*(-b*x^4+a)^(19/4)/b^5
```

Rubi [A] time = 0.06, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {266, 43}

$$-\frac{6a^2\left(a-bx^4\right)^{11/4}}{11b^5}+\frac{4a^3\left(a-bx^4\right)^{7/4}}{7b^5}-\frac{a^4\left(a-bx^4\right)^{3/4}}{3b^5}-\frac{\left(a-bx^4\right)^{19/4}}{19b^5}+\frac{4a\left(a-bx^4\right)^{15/4}}{15b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^19/(a - b*x^4)^(1/4),x]
```

```
[Out] -(a^4*(a - b*x^4)^(3/4))/(3*b^5) + (4*a^3*(a - b*x^4)^(7/4))/(7*b^5) - (6*a
^2*(a - b*x^4)^(11/4))/(11*b^5) + (4*a*(a - b*x^4)^(15/4))/(15*b^5) - (a -
b*x^4)^(19/4)/(19*b^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{19}}{\sqrt[4]{a-bx^4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x^4}{\sqrt[4]{a-bx}}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int\left(\frac{a^4}{b^4\sqrt[4]{a-bx}}-\frac{4a^3(a-bx)^{3/4}}{b^4}+\frac{6a^2(a-bx)^{7/4}}{b^4}-\frac{4a(a-bx)^{11/4}}{b^4}+\frac{(a-bx)^{15/4}}{b^4}\right)\right.\\ &= -\frac{a^4\left(a-bx^4\right)^{3/4}}{3b^5}+\frac{4a^3\left(a-bx^4\right)^{7/4}}{7b^5}-\frac{6a^2\left(a-bx^4\right)^{11/4}}{11b^5}+\frac{4a\left(a-bx^4\right)^{15/4}}{15b^5}-\frac{\left(a-bx^4\right)^{19/4}}{19b^5}\end{aligned}$$

Mathematica [A] time = 0.03, size = 62, normalized size = 0.58

$$-\frac{\left(a-bx^4\right)^{3/4}\left(2048a^4+1536a^3bx^4+1344a^2b^2x^8+1232ab^3x^{12}+1155b^4x^{16}\right)}{21945b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^19/(a - b*x^4)^(1/4),x]
```

```
[Out] -1/21945*((a - b*x^4)^(3/4)*(2048*a^4 + 1536*a^3*b*x^4 + 1344*a^2*b^2*x^8 +
 1232*a*b^3*x^12 + 1155*b^4*x^16))/b^5
```

fricas [A] time = 0.56, size = 58, normalized size = 0.55

$$-\frac{\left(1155\,b^4x^{16}+1232\,ab^3x^{12}+1344\,a^2b^2x^8+1536\,a^3bx^4+2048\,a^4\right)\left(-bx^4+a\right)^{\frac{3}{4}}}{21945\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/21945*(1155*b^4*x^16 + 1232*a*b^3*x^12 + 1344*a^2*b^2*x^8 + 1536*a^3*b*x
^4 + 2048*a^4)*(-b*x^4 + a)^(3/4)/b^5
```

giac [A] time = 0.16, size = 109, normalized size = 1.03

$$-\frac{1155\left(bx^4-a\right)^4\left(-bx^4+a\right)^{\frac{3}{4}}+5852\left(bx^4-a\right)^3\left(-bx^4+a\right)^{\frac{3}{4}}a+11970\left(bx^4-a\right)^2\left(-bx^4+a\right)^{\frac{3}{4}}a^2-12540\left(-bx^4+\right.}{21945\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] -1/21945*(1155*(b*x^4 - a)^4*(-b*x^4 + a)^(3/4) + 5852*(b*x^4 - a)^3*(-b*x^
4 + a)^(3/4)*a + 11970*(b*x^4 - a)^2*(-b*x^4 + a)^(3/4)*a^2 - 12540*(-b*x^4
 + a)^(7/4)*a^3 + 7315*(-b*x^4 + a)^(3/4)*a^4)/b^5
```

maple [A] time = 0.01, size = 59, normalized size = 0.56

$$-\frac{\left(-b\,x^4+a\right)^{\frac{3}{4}}\left(1155x^{16}b^4+1232a\,x^{12}b^3+1344a^2x^8b^2+1536a^3x^4b+2048a^4\right)}{21945b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19/(-b*x^4+a)^(1/4),x)
```

```
[Out] -1/21945*(-b*x^4+a)^(3/4)*(1155*b^4*x^16+1232*a*b^3*x^12+1344*a^2*b^2*x^8+1
536*a^3*b*x^4+2048*a^4)/b^5
```

maxima [A] time = 1.38, size = 86, normalized size = 0.81

$$-\frac{\left(-bx^4+a\right)^{\frac{19}{4}}}{19\,b^5}+\frac{4\left(-bx^4+a\right)^{\frac{15}{4}}a}{15\,b^5}-\frac{6\left(-bx^4+a\right)^{\frac{11}{4}}a^2}{11\,b^5}+\frac{4\left(-bx^4+a\right)^{\frac{7}{4}}a^3}{7\,b^5}-\frac{\left(-bx^4+a\right)^{\frac{3}{4}}a^4}{3\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] -1/19*(-b*x^4 + a)^(19/4)/b^5 + 4/15*(-b*x^4 + a)^(15/4)*a/b^5 - 6/11*(-b*x
^4 + a)^(11/4)*a^2/b^5 + 4/7*(-b*x^4 + a)^(7/4)*a^3/b^5 - 1/3*(-b*x^4 + a)^
(3/4)*a^4/b^5
```

mupad [B] time = 1.20, size = 60, normalized size = 0.57

$$-\left(a-b\,x^4\right)^{3/4}\left(\frac{2048\,a^4}{21945\,b^5}+\frac{x^{16}}{19\,b}+\frac{16\,a\,x^{12}}{285\,b^2}+\frac{512\,a^3\,x^4}{7315\,b^4}+\frac{64\,a^2\,x^8}{1045\,b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19/(a - b*x^4)^(1/4),x)
```

```
[Out] -(a - b*x^4)^(3/4)*((2048*a^4)/(21945*b^5) + x^16/(19*b) + (16*a*x^12)/(285
*b^2) + (512*a^3*x^4)/(7315*b^4) + (64*a^2*x^8)/(1045*b^3))
```

sympy **[A]** time = 16.94, size = 117, normalized size = 1.10

$$\begin{cases} -\frac{2048a^4\left(a-bx^4\right)^{\frac{3}{4}}}{21945b^5} - \frac{512a^3x^4\left(a-bx^4\right)^{\frac{3}{4}}}{7315b^4} - \frac{64a^2x^8\left(a-bx^4\right)^{\frac{3}{4}}}{1045b^3} - \frac{16ax^{12}\left(a-bx^4\right)^{\frac{3}{4}}}{285b^2} - \frac{x^{16}\left(a-bx^4\right)^{\frac{3}{4}}}{19b} & \text{for } b \neq 0 \\ \frac{x^{20}}{20\sqrt[4]{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**19/(-b*x**4+a)**(1/4),x)
```

```
[Out] Piecewise((-2048*a**4*(a - b*x**4)**(3/4)/(21945*b**5) - 512*a**3*x**4*(a -
 b*x**4)**(3/4)/(7315*b**4) - 64*a**2*x**8*(a - b*x**4)**(3/4)/(1045*b**3)
- 16*a*x**12*(a - b*x**4)**(3/4)/(285*b**2) - x**16*(a - b*x**4)**(3/4)/(19
*b), Ne(b, 0)), (x**20/(20*a**(1/4)), True))
```

3.1205 $\int \frac{x^{15}}{\sqrt[4]{a-bx^4}}\,dx$

Optimal. Leaf size=84

$$-\frac{a^3\left(a-bx^4\right)^{3/4}}{3b^4}+\frac{3a^2\left(a-bx^4\right)^{7/4}}{7b^4}+\frac{\left(a-bx^4\right)^{15/4}}{15b^4}-\frac{3a\left(a-bx^4\right)^{11/4}}{11b^4}$$

```
[Out] -1/3*a^3*(-b*x^4+a)^(3/4)/b^4+3/7*a^2*(-b*x^4+a)^(7/4)/b^4-3/11*a*(-b*x^4+a
)^(11/4)/b^4+1/15*(-b*x^4+a)^(15/4)/b^4
```

Rubi [A] time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {266, 43}

$$\frac{3a^2\left(a-bx^4\right)^{7/4}}{7b^4}-\frac{a^3\left(a-bx^4\right)^{3/4}}{3b^4}+\frac{\left(a-bx^4\right)^{15/4}}{15b^4}-\frac{3a\left(a-bx^4\right)^{11/4}}{11b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^15/(a - b*x^4)^(1/4),x]
```

```
[Out] -(a^3*(a - b*x^4)^(3/4))/(3*b^4) + (3*a^2*(a - b*x^4)^(7/4))/(7*b^4) - (3*a
*(a - b*x^4)^(11/4))/(11*b^4) + (a - b*x^4)^(15/4)/(15*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{15}}{\sqrt[4]{a-bx^4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x^3}{\sqrt[4]{a-bx}}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int \left(\frac{a^3}{b^3\sqrt[4]{a-bx}}-\frac{3a^2(a-bx)^{3/4}}{b^3}+\frac{3a(a-bx)^{7/4}}{b^3}-\frac{(a-bx)^{11/4}}{b^3}\right)dx, x, x^4\right)\\ &= -\frac{a^3\left(a-bx^4\right)^{3/4}}{3b^4}+\frac{3a^2\left(a-bx^4\right)^{7/4}}{7b^4}-\frac{3a\left(a-bx^4\right)^{11/4}}{11b^4}+\frac{\left(a-bx^4\right)^{15/4}}{15b^4}\end{aligned}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.61

$$-\frac{\left(a-bx^4\right)^{3/4}\left(128a^3+96a^2bx^4+84ab^2x^8+77b^3x^{12}\right)}{1155b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15/(a - b*x^4)^(1/4),x]
```

```
[Out] -1/1155*((a - b*x^4)^(3/4)*(128*a^3 + 96*a^2*b*x^4 + 84*a*b^2*x^8 + 77*b^3*
x^12))/b^4
```

fricas [A] time = 0.60, size = 47, normalized size = 0.56

$$-\frac{\left(77\,b^3x^{12}+84\,ab^2x^8+96\,a^2bx^4+128\,a^3\right)\left(-bx^4+a\right)^{\frac{3}{4}}}{1155\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/1155*(77*b^3*x^12 + 84*a*b^2*x^8 + 96*a^2*b*x^4 + 128*a^3)*(-b*x^4 + a)^
(3/4)/b^4
```

giac [A] time = 0.16, size = 83, normalized size = 0.99

$$-\frac{77\left(bx^4-a\right)^3\left(-bx^4+a\right)^{\frac{3}{4}}+315\left(bx^4-a\right)^2\left(-bx^4+a\right)^{\frac{3}{4}}a-495\left(-bx^4+a\right)^{\frac{7}{4}}a^2+385\left(-bx^4+a\right)^{\frac{3}{4}}a^3}{1155\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] -1/1155*(77*(b*x^4 - a)^3*(-b*x^4 + a)^(3/4) + 315*(b*x^4 - a)^2*(-b*x^4 +
a)^(3/4)*a - 495*(-b*x^4 + a)^(7/4)*a^2 + 385*(-b*x^4 + a)^(3/4)*a^3)/b^4
```

maple [A] time = 0.01, size = 48, normalized size = 0.57

$$-\frac{\left(-b\,x^4+a\right)^{\frac{3}{4}}\left(77b^3x^{12}+84a\,b^2x^8+96a^2b\,x^4+128a^3\right)}{1155b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(-b*x^4+a)^(1/4),x)
```

```
[Out] -1/1155*(-b*x^4+a)^(3/4)*(77*b^3*x^12+84*a*b^2*x^8+96*a^2*b*x^4+128*a^3)/b^
4
```

maxima [A] time = 1.33, size = 68, normalized size = 0.81

$$\frac{\left(-bx^4+a\right)^{\frac{15}{4}}}{15\,b^4}-\frac{3\left(-bx^4+a\right)^{\frac{11}{4}}a}{11\,b^4}+\frac{3\left(-bx^4+a\right)^{\frac{7}{4}}a^2}{7\,b^4}-\frac{\left(-bx^4+a\right)^{\frac{3}{4}}a^3}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] 1/15*(-b*x^4 + a)^(15/4)/b^4 - 3/11*(-b*x^4 + a)^(11/4)*a/b^4 + 3/7*(-b*x^4
 + a)^(7/4)*a^2/b^4 - 1/3*(-b*x^4 + a)^(3/4)*a^3/b^4
```

mupad [B] time = 1.17, size = 49, normalized size = 0.58

$$-\left(a-b\,x^4\right)^{3/4}\left(\frac{128\,a^3}{1155\,b^4}+\frac{x^{12}}{15\,b}+\frac{4\,a\,x^8}{55\,b^2}+\frac{32\,a^2\,x^4}{385\,b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(a - b*x^4)^(1/4),x)
```

```
[Out] -(a - b*x^4)^(3/4)*((128*a^3)/(1155*b^4) + x^12/(15*b) + (4*a*x^8)/(55*b^2)
 + (32*a^2*x^4)/(385*b^3))
```

sympy [A] time = 9.90, size = 94, normalized size = 1.12

$$\begin{cases} -\frac{128a^3\left(a-bx^4\right)^{\frac{3}{4}}}{1155b^4} - \frac{32a^2x^4\left(a-bx^4\right)^{\frac{3}{4}}}{385b^3} - \frac{4ax^8\left(a-bx^4\right)^{\frac{3}{4}}}{55b^2} - \frac{x^{12}\left(a-bx^4\right)^{\frac{3}{4}}}{15b} & \text{for } b \neq 0 \\ \frac{x^{16}}{16\sqrt[4]{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15/(-b*x**4+a)**(1/4),x)
```

```
[Out] Piecewise((-128*a**3*(a - b*x**4)**(3/4)/(1155*b**4) - 32*a**2*x**4*(a - b*
x**4)**(3/4)/(385*b**3) - 4*a*x**8*(a - b*x**4)**(3/4)/(55*b**2) - x**12*(a
 - b*x**4)**(3/4)/(15*b), Ne(b, 0)), (x**16/(16*a**(1/4)), True))
```

3.1206 $\int \frac{x^{11}}{\sqrt[4]{a-bx^4}}\,dx$

Optimal. Leaf size=62

$$-\frac{a^2\left(a-bx^4\right)^{3/4}}{3b^3}-\frac{\left(a-bx^4\right)^{11/4}}{11b^3}+\frac{2a\left(a-bx^4\right)^{7/4}}{7b^3}$$

```
[Out] -1/3*a^2*(-b*x^4+a)^(3/4)/b^3+2/7*a*(-b*x^4+a)^(7/4)/b^3-1/11*(-b*x^4+a)^(1
1/4)/b^3
```

Rubi [A] time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {266, 43}

$$-\frac{a^2\left(a-bx^4\right)^{3/4}}{3b^3}-\frac{\left(a-bx^4\right)^{11/4}}{11b^3}+\frac{2a\left(a-bx^4\right)^{7/4}}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a - b*x^4)^(1/4),x]
```

```
[Out] -(a^2*(a - b*x^4)^(3/4))/(3*b^3) + (2*a*(a - b*x^4)^(7/4))/(7*b^3) - (a - b
*x^4)^(11/4)/(11*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{\sqrt[4]{a-bx^4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x^2}{\sqrt[4]{a-bx}}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int \left(\frac{a^2}{b^2\sqrt[4]{a-bx}}-\frac{2a(a-bx)^{3/4}}{b^2}+\frac{(a-bx)^{7/4}}{b^2}\right)dx, x, x^4\right)\\ &= -\frac{a^2\left(a-bx^4\right)^{3/4}}{3b^3}+\frac{2a\left(a-bx^4\right)^{7/4}}{7b^3}-\frac{\left(a-bx^4\right)^{11/4}}{11b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 0.65

$$-\frac{\left(a-bx^4\right)^{3/4}\left(32a^2+24abx^4+21b^2x^8\right)}{231b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a - b*x^4)^(1/4),x]
```

```
[Out] -1/231*((a - b*x^4)^(3/4)*(32*a^2 + 24*a*b*x^4 + 21*b^2*x^8))/b^3
```

fricas [A] time = 0.72, size = 36, normalized size = 0.58

$$-\frac{\left(21\,b^2x^8+24\,abx^4+32\,a^2\right)\left(-bx^4+a\right)^{\frac{3}{4}}}{231\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/231*(21*b^2*x^8 + 24*a*b*x^4 + 32*a^2)*(-b*x^4 + a)^(3/4)/b^3
```

giac [A] time = 0.15, size = 57, normalized size = 0.92

$$-\frac{21\left(bx^4-a\right)^2\left(-bx^4+a\right)^{\frac{3}{4}}-66\left(-bx^4+a\right)^{\frac{7}{4}}a+77\left(-bx^4+a\right)^{\frac{3}{4}}a^2}{231\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] -1/231*(21*(b*x^4 - a)^2*(-b*x^4 + a)^(3/4) - 66*(-b*x^4 + a)^(7/4)*a + 77*
(-b*x^4 + a)^(3/4)*a^2)/b^3
```

maple [A] time = 0.00, size = 37, normalized size = 0.60

$$-\frac{\left(-b\,x^4+a\right)^{\frac{3}{4}}\left(21b^2x^8+24ab\,x^4+32a^2\right)}{231b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(-b*x^4+a)^(1/4),x)
```

```
[Out] -1/231*(-b*x^4+a)^(3/4)*(21*b^2*x^8+24*a*b*x^4+32*a^2)/b^3
```

maxima [A] time = 1.36, size = 50, normalized size = 0.81

$$-\frac{\left(-bx^4+a\right)^{\frac{11}{4}}}{11\,b^3}+\frac{2\left(-bx^4+a\right)^{\frac{7}{4}}a}{7\,b^3}-\frac{\left(-bx^4+a\right)^{\frac{3}{4}}a^2}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] -1/11*(-b*x^4 + a)^(11/4)/b^3 + 2/7*(-b*x^4 + a)^(7/4)*a/b^3 - 1/3*(-b*x^4
+ a)^(3/4)*a^2/b^3
```

mupad [B] time = 1.16, size = 38, normalized size = 0.61

$$-\left(a-b\,x^4\right)^{3/4}\left(\frac{32\,a^2}{231\,b^3}+\frac{x^8}{11\,b}+\frac{8\,a\,x^4}{77\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a - b*x^4)^(1/4),x)
```

```
[Out] -(a - b*x^4)^(3/4)*((32*a^2)/(231*b^3) + x^8/(11*b) + (8*a*x^4)/(77*b^2))
```

sympy [**A**] time = 4.23, size = 70, normalized size = 1.13

$$\begin{cases} -\frac{32a^2\left(a-bx^4\right)^{\frac{3}{4}}}{231b^3} - \frac{8ax^4\left(a-bx^4\right)^{\frac{3}{4}}}{77b^2} - \frac{x^8\left(a-bx^4\right)^{\frac{3}{4}}}{11b} & \text{for } b \neq 0 \\ \frac{x^{12}}{12\sqrt[4]{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(-b*x**4+a)**(1/4),x)
```

```
[Out] Piecewise((-32*a**2*(a - b*x**4)**(3/4)/(231*b**3) - 8*a*x**4*(a - b*x**4)*
*(3/4)/(77*b**2) - x**8*(a - b*x**4)**(3/4)/(11*b), Ne(b, 0)), (x**12/(12*a
**(1/4)), True))
```

3.1207 $\int \frac{x^7}{\sqrt[4]{a-bx^4}}\,dx$

Optimal. Leaf size=40

$$\frac{\left(a-bx^4\right)^{7/4}}{7b^2}-\frac{a\left(a-bx^4\right)^{3/4}}{3b^2}$$

```
[Out] -1/3*a*(-b*x^4+a)^(3/4)/b^2+1/7*(-b*x^4+a)^(7/4)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {266, 43}

$$\frac{\left(a-bx^4\right)^{7/4}}{7b^2}-\frac{a\left(a-bx^4\right)^{3/4}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a - b*x^4)^(1/4),x]
```

```
[Out] -(a*(a - b*x^4)^(3/4))/(3*b^2) + (a - b*x^4)^(7/4)/(7*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{\sqrt[4]{a-bx^4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x}{\sqrt[4]{a-bx}}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int \left(\frac{a}{b\sqrt[4]{a-bx}}-\frac{(a-bx)^{3/4}}{b}\right)dx, x, x^4\right)\\ &= -\frac{a\left(a-bx^4\right)^{3/4}}{3b^2}+\frac{\left(a-bx^4\right)^{7/4}}{7b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.72

$$-\frac{\left(a-bx^4\right)^{3/4}\left(4a+3bx^4\right)}{21b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a - b*x^4)^(1/4),x]
```

```
[Out] -1/21*((a - b*x^4)^(3/4)*(4*a + 3*b*x^4))/b^2
```

fricas [A] time = 0.68, size = 25, normalized size = 0.62

$$-\frac{\left(3\,bx^4+4\,a\right)\left(-bx^4+a\right)^{\frac{3}{4}}}{21\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-b*x^4+a)^(1/4),x, algorithm="fricas")
[Out] -1/21*(3*b*x^4 + 4*a)*(-b*x^4 + a)^(3/4)/b^2
```

giac [A] time = 0.15, size = 31, normalized size = 0.78

$$\frac{3\left(-bx^4+a\right)^{\frac{7}{4}}-7\left(-bx^4+a\right)^{\frac{3}{4}}a}{21\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-b*x^4+a)^(1/4),x, algorithm="giac")
[Out] 1/21*(3*(-b*x^4 + a)^(7/4) - 7*(-b*x^4 + a)^(3/4)*a)/b^2
```

maple [A] time = 0.01, size = 26, normalized size = 0.65

$$-\frac{\left(-b\,x^4+a\right)^{\frac{3}{4}}\left(3b\,x^4+4a\right)}{21b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-b*x^4+a)^(1/4),x)
[Out] -1/21*(-b*x^4+a)^(3/4)*(3*b*x^4+4*a)/b^2
```

maxima [A] time = 1.29, size = 32, normalized size = 0.80

$$\frac{\left(-bx^4+a\right)^{\frac{7}{4}}}{7\,b^2}-\frac{\left(-bx^4+a\right)^{\frac{3}{4}}a}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-b*x^4+a)^(1/4),x, algorithm="maxima")
[Out] 1/7*(-b*x^4 + a)^(7/4)/b^2 - 1/3*(-b*x^4 + a)^(3/4)*a/b^2
```

mupad [B] time = 1.15, size = 27, normalized size = 0.68

$$-\left(a-b\,x^4\right)^{3/4}\left(\frac{4\,a}{21\,b^2}+\frac{x^4}{7\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a - b*x^4)^(1/4),x)
[Out] -(a - b*x^4)^(3/4)*((4*a)/(21*b^2) + x^4/(7*b))
```

sympy [A] time = 1.87, size = 46, normalized size = 1.15

$$\begin{cases} -\frac{4a\left(a-bx^4\right)^{\frac{3}{4}}}{21b^2}-\frac{x^4\left(a-bx^4\right)^{\frac{3}{4}}}{7b} & \text{for } b\neq 0 \\ \frac{x^8}{8\sqrt[4]{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-b*x**4+a)**(1/4),x)
```

```
[Out] Piecewise((-4*a*(a - b*x**4)**(3/4)/(21*b**2) - x**4*(a - b*x**4)**(3/4)/(7
*b), Ne(b, 0)), (x**8/(8*a**(1/4)), True))
```

3.1208 $\int \frac{x^3}{\sqrt[4]{a-bx^4}}\,dx$

Optimal. Leaf size=19

$$-\frac{\left(a-bx^4\right)^{3/4}}{3b}$$

```
[Out] -1/3*(-b*x^4+a)^(3/4)/b
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {261}

$$-\frac{\left(a-bx^4\right)^{3/4}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a - b*x^4)^(1/4),x]
```

```
[Out] -(a - b*x^4)^(3/4)/(3*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^3}{\sqrt[4]{a-bx^4}}\,dx = -\frac{\left(a-bx^4\right)^{3/4}}{3b}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$-\frac{\left(a-bx^4\right)^{3/4}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a - b*x^4)^(1/4),x]
```

```
[Out] -1/3*(a - b*x^4)^(3/4)/b
```

fricas [A] time = 0.57, size = 15, normalized size = 0.79

$$-\frac{\left(-bx^4+a\right)^{\frac{3}{4}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/3*(-b*x^4 + a)^(3/4)/b
```

giac [A] time = 0.15, size = 15, normalized size = 0.79

$$-\frac{\left(-bx^4+a\right)^{\frac{3}{4}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^4+a)^(1/4),x, algorithm="giac")
[Out] -1/3*(-b*x^4 + a)^(3/4)/b
```

maple [A] time = 0.00, size = 16, normalized size = 0.84

$$-\frac{\left(-b\,x^4+a\right)^{\frac{3}{4}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-b*x^4+a)^(1/4),x)
[Out] -1/3*(-b*x^4+a)^(3/4)/b
```

maxima [A] time = 1.24, size = 15, normalized size = 0.79

$$-\frac{\left(-bx^4+a\right)^{\frac{3}{4}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^4+a)^(1/4),x, algorithm="maxima")
[Out] -1/3*(-b*x^4 + a)^(3/4)/b
```

mupad [B] time = 1.13, size = 15, normalized size = 0.79

$$-\frac{\left(a-b\,x^4\right)^{3/4}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a - b*x^4)^(1/4),x)
[Out] -(a - b*x^4)^(3/4)/(3*b)
```

sympy [A] time = 0.58, size = 24, normalized size = 1.26

$$\begin{cases} -\frac{\left(a-bx^4\right)^{\frac{3}{4}}}{3b} & \text{for } b \neq 0 \\ \frac{x^4}{4\sqrt[4]{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-b*x**4+a)**(1/4),x)
[Out] Piecewise((-(a - b*x**4)**(3/4)/(3*b), Ne(b, 0)), (x**4/(4*a**(1/4)), True)
)
```

3.1209 $\int \frac{1}{x\sqrt[4]{a-bx^4}}\,dx$

Optimal. Leaf size=57

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}$$

```
[Out] 1/2*arctan((-b*x^4+a)^(1/4)/a^(1/4))/a^(1/4)-1/2*arctanh((-b*x^4+a)^(1/4)/a
^(1/4))/a^(1/4)
```

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {266, 63, 298, 203, 206}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a - b*x^4)^(1/4)),x]
```

```
[Out] ArcTan[(a - b*x^4)^(1/4)/a^(1/4)]/(2*a^(1/4)) - ArcTanh[(a - b*x^4)^(1/4)/a
^(1/4)]/(2*a^(1/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt[4]{a-bx^4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x\sqrt[4]{a-bx}}\,dx, x, x^4\right)\\ &= -\frac{\text{Subst}\left(\int \frac{x^2}{\frac{a}{b}-\frac{x^4}{b}}\,dx, x, \sqrt[4]{a-bx^4}\right)}{b}\\ &= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{a}-x^2}\,dx, x, \sqrt[4]{a-bx^4}\right)\right)+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{a}+x^2}\,dx, x, \sqrt[4]{a-bx^4}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 50, normalized size = 0.88

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)-\tanh^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a - b*x^4)^(1/4)),x]
```

```
[Out] (ArcTan[(a - b*x^4)^(1/4)/a^(1/4)] - ArcTanh[(a - b*x^4)^(1/4)/a^(1/4)])/(2
*a^(1/4))
```

fricas [B] time = 0.79, size = 85, normalized size = 1.49

$$-\frac{\arctan\left(\frac{\sqrt{\sqrt{-bx^4+a}+\sqrt{a}}}{a^{\frac{1}{4}}}-\frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}-\frac{\log\left(\left(-bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}\right)}{4\,a^{\frac{1}{4}}}+\frac{\log\left(\left(-bx^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}\right)}{4\,a^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -arctan(sqrt(sqrt(-b*x^4 + a) + sqrt(a))/a^(1/4) - (-b*x^4 + a)^(1/4)/a^(1/
4))/a^(1/4) - 1/4*log((-b*x^4 + a)^(1/4) + a^(1/4))/a^(1/4) + 1/4*log((-b*x
^4 + a)^(1/4) - a^(1/4))/a^(1/4)
```

giac [B] time = 0.17, size = 192, normalized size = 3.37

$$-\frac{\sqrt{2}\,(-a)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}+2\left(-bx^4+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{4\,a}-\frac{\sqrt{2}\,(-a)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}-2\left(-bx^4+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{4\,a}+\frac{\sqrt{2}\,(-a)^{\frac{3}{4}}\log\left(\sqrt{2}\left(-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(-a)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(-b*x^4
+ a)^(1/4))/(-a)^(1/4))/a - 1/4*sqrt(2)*(-a)^(3/4)*arctan(-1/2*sqrt(2)*(sqr
t(2)*(-a)^(1/4) - 2*(-b*x^4 + a)^(1/4))/(-a)^(1/4))/a + 1/8*sqrt(2)*(-a)^(3
/4)*log(sqrt(2)*(-b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(-b*x^4 + a) + sqrt(-a)
```

```
)/a - 1/8*sqrt(2)*(-a)^(3/4)*log(-sqrt(2)*(-b*x^4 + a)^(1/4)*(-a)^(1/4) + s
qrt(-b*x^4 + a) + sqrt(-a))/a
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^4+a\right)^{\frac{1}{4}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-b*x^4+a)^(1/4),x)
```

```
[Out] int(1/x/(-b*x^4+a)^(1/4),x)
```

maxima [A] time = 2.85, size = 60, normalized size = 1.05

$$\frac{\arctan\left(\frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{2\,a^{\frac{1}{4}}}+\frac{\log\left(\frac{\left(-bx^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{\left(-bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{4\,a^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] 1/2*arctan((-b*x^4 + a)^(1/4)/a^(1/4))/a^(1/4) + 1/4*log(((-b*x^4 + a)^(1/4
) - a^(1/4))/((-b*x^4 + a)^(1/4) + a^(1/4)))/a^(1/4)
```

mupad [B] time = 1.18, size = 38, normalized size = 0.67

$$\frac{\operatorname{atan}\left(\frac{\left(a-b\,x^4\right)^{1/4}}{a^{1/4}}\right)-\operatorname{atanh}\left(\frac{\left(a-b\,x^4\right)^{1/4}}{a^{1/4}}\right)}{2\,a^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a - b*x^4)^(1/4)),x)
```

```
[Out] (atan((a - b*x^4)^(1/4)/a^(1/4)) - atanh((a - b*x^4)^(1/4)/a^(1/4)))/(2*a^(
1/4))
```

sympy [C] time = 1.33, size = 39, normalized size = 0.68

$$-\frac{e^{-\frac{i\pi}{4}}\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\right|\frac{a}{bx^4}\right)}{4\sqrt[4]{b}\,x\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x**4+a)**(1/4),x)
```

```
[Out] -exp(-I*pi/4)*gamma(1/4)*hyper((1/4, 1/4), (5/4,), a/(b*x**4))/(4*b**(1/4)*
x*gamma(5/4))
```

3.1210 $\int \frac{1}{x^5 \sqrt[4]{a-bx^4}}\,dx$

Optimal. Leaf size=81

$$\frac{b \tan^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{8a^{5/4}} - \frac{b \tanh^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{8a^{5/4}} - \frac{\left(a-bx^4\right)^{3/4}}{4ax^4}$$

```
[Out] -1/4*(-b*x^4+a)^(3/4)/a/x^4+1/8*b*arctan((-b*x^4+a)^(1/4)/a^(1/4))/a^(5/4)-
1/8*b*arctanh((-b*x^4+a)^(1/4)/a^(1/4))/a^(5/4)
```

Rubi [A] time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {266, 51, 63, 298, 203, 206}

$$\frac{b \tan^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{8a^{5/4}} - \frac{b \tanh^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{8a^{5/4}} - \frac{\left(a-bx^4\right)^{3/4}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a - b*x^4)^(1/4)),x]
```

```
[Out] -(a - b*x^4)^(3/4)/(4*a*x^4) + (b*ArcTan[(a - b*x^4)^(1/4)/a^(1/4)])/(8*a^(
5/4)) - (b*ArcTanh[(a - b*x^4)^(1/4)/a^(1/4)])/(8*a^(5/4))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5 \sqrt[4]{a-bx^4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x^2 \sqrt[4]{a-bx}}\,dx, x, x^4\right) \\
&= -\frac{\left(a-bx^4\right)^{3/4}}{4ax^4} + \frac{b\,\text{Subst}\left(\int \frac{1}{x \sqrt[4]{a-bx}}\,dx, x, x^4\right)}{16a} \\
&= -\frac{\left(a-bx^4\right)^{3/4}}{4ax^4} - \frac{\text{Subst}\left(\int \frac{x^2}{\frac{a}{b}-\frac{x^4}{b}}\,dx, x, \sqrt[4]{a-bx^4}\right)}{4a} \\
&= -\frac{\left(a-bx^4\right)^{3/4}}{4ax^4} - \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a}-x^2}\,dx, x, \sqrt[4]{a-bx^4}\right)}{8a} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a}+x^2}\,dx, x, \sqrt[4]{a-bx^4}\right)}{8a} \\
&= -\frac{\left(a-bx^4\right)^{3/4}}{4ax^4} + \frac{b\tan^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{8a^{5/4}} - \frac{b\tanh^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{8a^{5/4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.48

$$-\frac{b\left(a-bx^4\right)^{3/4}\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};1-\frac{bx^4}{a}\right)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a - b*x^4)^(1/4)),x]
```

```
[Out] -1/3*(b*(a - b*x^4)^(3/4)*Hypergeometric2F1[3/4, 2, 7/4, 1 - (b*x^4)/a])/a^
2
```

fricas [B] time = 0.56, size = 200, normalized size = 2.47

$$-\frac{4\,ax^4\left(\frac{b^4}{a^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-bx^4+a\right)^{\frac{1}{4}}ab^3\left(\frac{b^4}{a^5}\right)^{\frac{1}{4}}-\sqrt{a^3b^4\sqrt{\frac{b^4}{a^5}}+\sqrt{-bx^4+a}\,b^6}\,a\left(\frac{b^4}{a^5}\right)^{\frac{1}{4}}}{b^4}\right)+ax^4\left(\frac{b^4}{a^5}\right)^{\frac{1}{4}}\log\left(a^4\left(\frac{b^4}{a^5}\right)^{\frac{3}{4}}+\left(-bx^4+a\right)^{\frac{1}{4}}b\right.}{16\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/16*(4*a*x^4*(b^4/a^5)^(1/4)*arctan(-((-b*x^4 + a)^(1/4)*a*b^3*(b^4/a^5)^
(1/4) - sqrt(a^3*b^4*sqrt(b^4/a^5) + sqrt(-b*x^4 + a)*b^6)*a*(b^4/a^5)^(1/4
))/b^4) + a*x^4*(b^4/a^5)^(1/4)*log(a^4*(b^4/a^5)^(3/4) + (-b*x^4 + a)^(1/4
)*b^3) - a*x^4*(b^4/a^5)^(1/4)*log(-a^4*(b^4/a^5)^(3/4) + (-b*x^4 + a)^(1/4
)*b^3) + 4*(-b*x^4 + a)^(3/4))/(a*x^4)
```

giac [B] time = 0.18, size = 226, normalized size = 2.79

$$\frac{2\sqrt{2}(-a)^{\frac{3}{4}}b^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(-bx^4+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a^2}+\frac{2\sqrt{2}(-a)^{\frac{3}{4}}b^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(-bx^4+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a^2}+\frac{\sqrt{2}b^2\log\left(\sqrt{2}\left(-bx^4+a\right)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{-bx^4}\right.}{(-a)^{\frac{1}{4}}a}$$
$$-\frac{}{32b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] -1/32*(2*sqrt(2)*(-a)^(3/4)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*
(-b*x^4 + a)^(1/4))/(-a)^(1/4))/a^2 + 2*sqrt(2)*(-a)^(3/4)*b^2*arctan(-1/2*
sqrt(2)*(sqrt(2)*(-a)^(1/4) - 2*(-b*x^4 + a)^(1/4))/(-a)^(1/4))/a^2 + sqrt(
2)*b^2*log(sqrt(2)*(-b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(-b*x^4 + a) + sqrt(
-a))/((-a)^(1/4)*a) + sqrt(2)*(-a)^(3/4)*b^2*log(-sqrt(2)*(-b*x^4 + a)^(1/4
)*(-a)^(1/4) + sqrt(-b*x^4 + a) + sqrt(-a))/a^2 + 8*(-b*x^4 + a)^(3/4)*b/(a
*x^4))/b
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^4+a\right)^{\frac{1}{4}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-b*x^4+a)^(1/4),x)
```

```
[Out] int(1/x^5/(-b*x^4+a)^(1/4),x)
```

maxima [A] time = 2.97, size = 96, normalized size = 1.19

$$\frac{b\left(\frac{2\arctan\left(\frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}+\frac{\log\left(\frac{\left(-bx^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{\left(-bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}\right)}{16a}-\frac{\left(-bx^4+a\right)^{\frac{3}{4}}b}{4\left(\left(bx^4-a\right)a+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] 1/16*b*(2*arctan((-b*x^4 + a)^(1/4)/a^(1/4))/a^(1/4) + log(((-b*x^4 + a)^(1
/4) - a^(1/4))/((-b*x^4 + a)^(1/4) + a^(1/4)))/a^(1/4))/a - 1/4*(-b*x^4 + a
)^(3/4)*b/((b*x^4 - a)*a + a^2)
```

mupad [B] time = 1.30, size = 61, normalized size = 0.75

$$\frac{b\,\mathrm{atan}\left(\frac{\left(a-b\,x^4\right)^{1/4}}{a^{1/4}}\right)}{8\,a^{5/4}}-\frac{\left(a-b\,x^4\right)^{3/4}}{4\,a\,x^4}-\frac{b\,\mathrm{atanh}\left(\frac{\left(a-b\,x^4\right)^{1/4}}{a^{1/4}}\right)}{8\,a^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a - b*x^4)^(1/4)),x)
```

```
[Out] (b*atan((a - b*x^4)^(1/4)/a^(1/4)))/(8*a^(5/4)) - (a - b*x^4)^(3/4)/(4*a*x^
4) - (b*atanh((a - b*x^4)^(1/4)/a^(1/4)))/(8*a^(5/4))
```

sympy [C] time = 1.72, size = 41, normalized size = 0.51

$$\frac{e^{\frac{3i\pi}{4}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4}, \frac{5}{4} \\ \frac{9}{4}\end{matrix}\right| \frac{a}{bx^4}\right)}{4\sqrt[4]{b}\,x^5\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-b*x**4+a)**(1/4),x)
```

```
[Out] exp(3*I*pi/4)*gamma(5/4)*hyper((1/4, 5/4), (9/4,), a/(b*x**4))/(4*b**(1/4)*
x**5*gamma(9/4))
```

3.1211 $\int \frac{1}{x^9 \sqrt[4]{a-bx^4}}\,dx$

Optimal. Leaf size=108

$$\frac{5b^2 \tan^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{64a^{9/4}} - \frac{5b^2 \tanh^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{64a^{9/4}} - \frac{5b\left(a-bx^4\right)^{3/4}}{32a^2x^4} - \frac{\left(a-bx^4\right)^{3/4}}{8ax^8}$$

```
[Out] -1/8*(-b*x^4+a)^(3/4)/a/x^8-5/32*b*(-b*x^4+a)^(3/4)/a^2/x^4+5/64*b^2*arctan
((-b*x^4+a)^(1/4)/a^(1/4))/a^(9/4)-5/64*b^2*arctanh((-b*x^4+a)^(1/4)/a^(1/4
))/a^(9/4)
```

Rubi [A] time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {266, 51, 63, 298, 203, 206}

$$\frac{5b^2 \tan^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{64a^{9/4}} - \frac{5b^2 \tanh^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{64a^{9/4}} - \frac{5b\left(a-bx^4\right)^{3/4}}{32a^2x^4} - \frac{\left(a-bx^4\right)^{3/4}}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(a - b*x^4)^(1/4)),x]
```

```
[Out] -(a - b*x^4)^(3/4)/(8*a*x^8) - (5*b*(a - b*x^4)^(3/4))/(32*a^2*x^4) + (5*b^
2*ArcTan[(a - b*x^4)^(1/4)/a^(1/4)])/(64*a^(9/4)) - (5*b^2*ArcTanh[(a - b*x
^4)^(1/4)/a^(1/4)])/(64*a^(9/4))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$\int \frac{1}{x^9 \sqrt[4]{a-bx^4}}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x^3 \sqrt[4]{a-bx}}\,dx, x, x^4\right)$$

$$= -\frac{\left(a-bx^4\right)^{3/4}}{8ax^8} + \frac{(5b)\,\text{Subst}\left(\int \frac{1}{x^2 \sqrt[4]{a-bx}}\,dx, x, x^4\right)}{32a}$$

$$= -\frac{\left(a-bx^4\right)^{3/4}}{8ax^8} - \frac{5b\left(a-bx^4\right)^{3/4}}{32a^2x^4} + \frac{\left(5b^2\right)\text{Subst}\left(\int \frac{1}{x \sqrt[4]{a-bx}}\,dx, x, x^4\right)}{128a^2}$$

$$= -\frac{\left(a-bx^4\right)^{3/4}}{8ax^8} - \frac{5b\left(a-bx^4\right)^{3/4}}{32a^2x^4} - \frac{(5b)\,\text{Subst}\left(\int \frac{x^2}{\frac{a}{b}-\frac{x^4}{b}}\,dx, x, \sqrt[4]{a-bx^4}\right)}{32a^2}$$

$$= -\frac{\left(a-bx^4\right)^{3/4}}{8ax^8} - \frac{5b\left(a-bx^4\right)^{3/4}}{32a^2x^4} - \frac{\left(5b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a}-x^2}\,dx, x, \sqrt[4]{a-bx^4}\right)}{64a^2} + \frac{\left(5b^2\right)\text{Subst}\left($$

$$= -\frac{\left(a-bx^4\right)^{3/4}}{8ax^8} - \frac{5b\left(a-bx^4\right)^{3/4}}{32a^2x^4} + \frac{5b^2\tan^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{64a^{9/4}} - \frac{5b^2\tanh^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{64a^{9/4}}$$

Mathematica [C] time = 0.01, size = 41, normalized size = 0.38

$$-\frac{b^2\left(a-bx^4\right)^{3/4}\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};1-\frac{bx^4}{a}\right)}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^9*(a - b*x^4)^(1/4)),x]
```

```
[Out] -1/3*(b^2*(a - b*x^4)^(3/4)*Hypergeometric2F1[3/4, 3, 7/4, 1 - (b*x^4)/a])/
a^3
```

fricas [B] time = 0.57, size = 224, normalized size = 2.07

$$-\frac{20\,a^2x^8\left(\frac{b^8}{a^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-bx^4+a\right)^{\frac{1}{4}}a^2b^6\left(\frac{b^8}{a^9}\right)^{\frac{1}{4}}-\sqrt{a^5b^8\sqrt{\frac{b^8}{a^9}}+\sqrt{-bx^4+a}\,b^{12}}\,a^2\left(\frac{b^8}{a^9}\right)^{\frac{1}{4}}}{b^8}\right)+5\,a^2x^8\left(\frac{b^8}{a^9}\right)^{\frac{1}{4}}\log\left(125\,a^7\left(\frac{b^8}{a^9}\right)^{\frac{3}{4}}+12\right.}{128\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/128*(20*a^2*x^8*(b^8/a^9)^(1/4)*arctan(-((-b*x^4 + a)^(1/4)*a^2*b^6*(b^8
/a^9)^(1/4) - sqrt(a^5*b^8*sqrt(b^8/a^9) + sqrt(-b*x^4 + a)*b^12)*a^2*(b^8/
```

```
a^9)^(1/4))/b^8) + 5*a^2*x^8*(b^8/a^9)^(1/4)*log(125*a^7*(b^8/a^9)^(3/4) +
125*(-b*x^4 + a)^(1/4)*b^6) - 5*a^2*x^8*(b^8/a^9)^(1/4)*log(-125*a^7*(b^8/a
^9)^(3/4) + 125*(-b*x^4 + a)^(1/4)*b^6) + 4*(5*b*x^4 + 4*a)*(-b*x^4 + a)^(3
/4))/(a^2*x^8)
```

giac [B] time = 0.18, size = 252, normalized size = 2.33

$$\frac{10\sqrt{2}(-a)^{\frac{3}{4}}b^3\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(-bx^4+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a^3}+\frac{10\sqrt{2}(-a)^{\frac{3}{4}}b^3\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(-bx^4+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a^3}+\frac{5\sqrt{2}b^3\log\left(\sqrt{2}\left(-bx^4+a\right)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{}\right.}{(-a)^{\frac{1}{4}}a^2}$$
$$-\frac{}{256\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] -1/256*(10*sqrt(2)*(-a)^(3/4)*b^3*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) +
2*(-b*x^4 + a)^(1/4))/(-a)^(1/4))/a^3 + 10*sqrt(2)*(-a)^(3/4)*b^3*arctan(-1
/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) - 2*(-b*x^4 + a)^(1/4))/(-a)^(1/4))/a^3 + 5*
sqrt(2)*b^3*log(sqrt(2)*(-b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(-b*x^4 + a) +
sqrt(-a))/((-a)^(1/4)*a^2) + 5*sqrt(2)*(-a)^(3/4)*b^3*log(-sqrt(2)*(-b*x^4
+ a)^(1/4)*(-a)^(1/4) + sqrt(-b*x^4 + a) + sqrt(-a))/a^3 - 8*(5*(-b*x^4 + a
)^(7/4)*b^3 - 9*(-b*x^4 + a)^(3/4)*a*b^3)/(a^2*b^2*x^8))/b
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^4+a\right)^{\frac{1}{4}}x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(-b*x^4+a)^(1/4),x)
```

```
[Out] int(1/x^9/(-b*x^4+a)^(1/4),x)
```

maxima [A] time = 2.41, size = 137, normalized size = 1.27

$$\frac{5\,b^2\left(\frac{2\arctan\left(\frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}+\frac{\log\left(\frac{\left(-bx^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{\left(-bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}\right)}{128\,a^2}+\frac{5\left(-bx^4+a\right)^{\frac{7}{4}}b^2-9\left(-bx^4+a\right)^{\frac{3}{4}}ab^2}{32\left(\left(bx^4-a\right)^2a^2+2\left(bx^4-a\right)a^3+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] 5/128*b^2*(2*arctan((-b*x^4 + a)^(1/4)/a^(1/4))/a^(1/4) + log(((-b*x^4 + a)
^(1/4) - a^(1/4))/((-b*x^4 + a)^(1/4) + a^(1/4)))/a^(1/4))/a^2 + 1/32*(5*(-
b*x^4 + a)^(7/4)*b^2 - 9*(-b*x^4 + a)^(3/4)*a*b^2)/((b*x^4 - a)^2*a^2 + 2*(
b*x^4 - a)*a^3 + a^4)
```

mupad [B] time = 1.39, size = 86, normalized size = 0.80

$$\frac{5\,b^2\,\mathrm{atan}\left(\frac{\left(a-b\,x^4\right)^{1/4}}{a^{1/4}}\right)}{64\,a^{9/4}}-\frac{9\left(a-b\,x^4\right)^{3/4}}{32\,a\,x^8}+\frac{5\left(a-b\,x^4\right)^{7/4}}{32\,a^2\,x^8}+\frac{b^2\,\mathrm{atan}\left(\frac{\left(a-b\,x^4\right)^{1/4}1\mathrm{i}}{a^{1/4}}\right)5\mathrm{i}}{64\,a^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(a - b*x^4)^(1/4)),x)
```

```
[Out] (5*b^2*atan((a - b*x^4)^(1/4)/a^(1/4)))/(64*a^(9/4)) + (b^2*atan(((a - b*x^
4)^(1/4)*1i)/a^(1/4))*5i)/(64*a^(9/4)) - (9*(a - b*x^4)^(3/4))/(32*a*x^8) +
 (5*(a - b*x^4)^(7/4))/(32*a^2*x^8)
```

sympy [C] time = 2.12, size = 41, normalized size = 0.38

$$-\frac{e^{-\frac{i\pi}{4}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4}, \frac{9}{4} \\ \frac{13}{4}\end{matrix}\middle| \frac{a}{bx^4}\right)}{4\sqrt[4]{b}\, x^9\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(-b*x**4+a)**(1/4),x)
```

```
[Out] -exp(-I*pi/4)*gamma(9/4)*hyper((1/4, 9/4), (13/4,), a/(b*x**4))/(4*b**(1/4)
*x**9*gamma(13/4))
```

3.1212 $\int \frac{x^{13}}{\sqrt[4]{a-bx^4}}\,dx$

Optimal. Leaf size=133

$$\frac{8a^{7/2}\sqrt[4]{1-\frac{bx^4}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{39b^{7/2}\sqrt[4]{a-bx^4}}-\frac{4a^2x^2\left(a-bx^4\right)^{3/4}}{39b^3}-\frac{10ax^6\left(a-bx^4\right)^{3/4}}{117b^2}-\frac{x^{10}\left(a-bx^4\right)^{3/4}}{13b}$$

```
[Out] -4/39*a^2*x^2*(-b*x^4+a)^(3/4)/b^3-10/117*a*x^6*(-b*x^4+a)^(3/4)/b^2-1/13*x
^10*(-b*x^4+a)^(3/4)/b+8/39*a^(7/2)*(1-b*x^4/a)^(1/4)*(cos(1/2*arcsin(x^2*b
^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x^2*b^(1/2)/a^(1/2)))*EllipticE(si
n(1/2*arcsin(x^2*b^(1/2)/a^(1/2))),2^(1/2))/b^(7/2)/(-b*x^4+a)^(1/4)
```

Rubi [A] time = 0.08, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {275, 321, 229, 228}

$$-\frac{4a^2x^2\left(a-bx^4\right)^{3/4}}{39b^3}+\frac{8a^{7/2}\sqrt[4]{1-\frac{bx^4}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{39b^{7/2}\sqrt[4]{a-bx^4}}-\frac{10ax^6\left(a-bx^4\right)^{3/4}}{117b^2}-\frac{x^{10}\left(a-bx^4\right)^{3/4}}{13b}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/(a - b*x^4)^(1/4),x]
```

```
[Out] (-4*a^2*x^2*(a - b*x^4)^(3/4))/(39*b^3) - (10*a*x^6*(a - b*x^4)^(3/4))/(117
*b^2) - (x^10*(a - b*x^4)^(3/4))/(13*b) + (8*a^(7/2)*(1 - (b*x^4)/a)^(1/4)*
EllipticE[ArcSin[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(39*b^(7/2)*(a - b*x^4)^(1/4
))
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{13}}{\sqrt[4]{a-bx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^6}{\sqrt[4]{a-bx^2}}\,dx, x, x^2\right)\\
&= -\frac{x^{10}\left(a-bx^4\right)^{3/4}}{13b} + \frac{(5a)\,\text{Subst}\left(\int \frac{x^4}{\sqrt[4]{a-bx^2}}\,dx, x, x^2\right)}{13b}\\
&= -\frac{10ax^6\left(a-bx^4\right)^{3/4}}{117b^2} - \frac{x^{10}\left(a-bx^4\right)^{3/4}}{13b} + \frac{\left(10a^2\right)\text{Subst}\left(\int \frac{x^2}{\sqrt[4]{a-bx^2}}\,dx, x, x^2\right)}{39b^2}\\
&= -\frac{4a^2x^2\left(a-bx^4\right)^{3/4}}{39b^3} - \frac{10ax^6\left(a-bx^4\right)^{3/4}}{117b^2} - \frac{x^{10}\left(a-bx^4\right)^{3/4}}{13b} + \frac{\left(4a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx, x,\right.}{39b^3}\\
&= -\frac{4a^2x^2\left(a-bx^4\right)^{3/4}}{39b^3} - \frac{10ax^6\left(a-bx^4\right)^{3/4}}{117b^2} - \frac{x^{10}\left(a-bx^4\right)^{3/4}}{13b} + \frac{\left(4a^3\sqrt[4]{1-\frac{bx^4}{a}}\right)\text{Subst}\left(\int \frac{}{\sqrt[4]{1-}}\right.}{39b^3\sqrt[4]{a-bx^4}}\\
&= -\frac{4a^2x^2\left(a-bx^4\right)^{3/4}}{39b^3} - \frac{10ax^6\left(a-bx^4\right)^{3/4}}{117b^2} - \frac{x^{10}\left(a-bx^4\right)^{3/4}}{13b} + \frac{8a^{7/2}\sqrt[4]{1-\frac{bx^4}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}}{\sqrt{}}\right.\right.}{39b^{7/2}\sqrt[4]{a-bx^4}}
\end{aligned}$$

Mathematica [C] time = 0.07, size = 91, normalized size = 0.68

$$\frac{x^2\left(12a^3\sqrt[4]{1-\frac{bx^4}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^4}{a}\right) - 12a^3 + 2a^2bx^4 + ab^2x^8 + 9b^3x^{12}\right)}{117b^3\sqrt[4]{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13/(a - b*x^4)^(1/4),x]
```

```
[Out] (x^2*(-12*a^3 + 2*a^2*b*x^4 + a*b^2*x^8 + 9*b^3*x^12 + 12*a^3*(1 - (b*x^4)/
a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (b*x^4)/a]))/(117*b^3*(a - b*x^4)
^(1/4))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^4+a\right)^{\frac{3}{4}}x^{13}}{bx^4-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^4 + a)^(3/4)*x^13/(b*x^4 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{\left(-bx^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^13/(-b*x^4 + a)^(1/4), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{\left(-b\,x^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(-b*x^4+a)^(1/4),x)
[Out] int(x^13/(-b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{\left(-bx^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(-b*x^4+a)^(1/4),x, algorithm="maxima")
[Out] integrate(x^13/(-b*x^4 + a)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{13}}{\left(a-b\,x^4\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(a - b*x^4)^(1/4),x)
[Out] int(x^13/(a - b*x^4)^(1/4), x)
```

sympy [C] time = 1.68, size = 29, normalized size = 0.22

$$\frac{x^{14}\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{4}, \frac{7}{2} \\ \frac{9}{2}\end{matrix}\right| \frac{bx^4e^{2i\pi}}{a}\right)}{14\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(-b*x**4+a)**(1/4),x)
[Out] x**14*hyper((1/4, 7/2), (9/2,), b*x**4*exp_polar(2*I*pi)/a)/(14*a**(1/4))
```

3.1213 $\int \frac{x^9}{\sqrt[4]{a-bx^4}}\,dx$

Optimal. Leaf size=108

$$\frac{4a^{5/2}\sqrt[4]{1-\frac{bx^4}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{15b^{5/2}\sqrt[4]{a-bx^4}}-\frac{2ax^2\left(a-bx^4\right)^{3/4}}{15b^2}-\frac{x^6\left(a-bx^4\right)^{3/4}}{9b}$$

```
[Out] -2/15*a*x^2*(-b*x^4+a)^(3/4)/b^2-1/9*x^6*(-b*x^4+a)^(3/4)/b+4/15*a^(5/2)*(1
-b*x^4/a)^(1/4)*(cos(1/2*arcsin(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcs
in(x^2*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arcsin(x^2*b^(1/2)/a^(1/2))),2^(
1/2))/b^(5/2)/(-b*x^4+a)^(1/4)
```

Rubi [A] time = 0.07, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {275, 321, 229, 228}

$$\frac{4a^{5/2}\sqrt[4]{1-\frac{bx^4}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{15b^{5/2}\sqrt[4]{a-bx^4}}-\frac{2ax^2\left(a-bx^4\right)^{3/4}}{15b^2}-\frac{x^6\left(a-bx^4\right)^{3/4}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a - b*x^4)^(1/4),x]
```

```
[Out] (-2*a*x^2*(a - b*x^4)^(3/4))/(15*b^2) - (x^6*(a - b*x^4)^(3/4))/(9*b) + (4*
a^(5/2)*(1 - (b*x^4)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2]
)/(15*b^(5/2)*(a - b*x^4)^(1/4))
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{\sqrt[4]{a-bx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{\sqrt[4]{a-bx^2}}\,dx, x, x^2\right)\\ &= -\frac{x^6\left(a-bx^4\right)^{3/4}}{9b} + \frac{a\,\text{Subst}\left(\int \frac{x^2}{\sqrt[4]{a-bx^2}}\,dx, x, x^2\right)}{3b}\\ &= -\frac{2ax^2\left(a-bx^4\right)^{3/4}}{15b^2} - \frac{x^6\left(a-bx^4\right)^{3/4}}{9b} + \frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx, x, x^2\right)}{15b^2}\\ &= -\frac{2ax^2\left(a-bx^4\right)^{3/4}}{15b^2} - \frac{x^6\left(a-bx^4\right)^{3/4}}{9b} + \frac{\left(2a^2\sqrt[4]{1-\frac{bx^4}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx, x, x^2\right)}{15b^2\sqrt[4]{a-bx^4}}\\ &= -\frac{2ax^2\left(a-bx^4\right)^{3/4}}{15b^2} - \frac{x^6\left(a-bx^4\right)^{3/4}}{9b} + \frac{4a^{5/2}\sqrt[4]{1-\frac{bx^4}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|\,2\right)}{15b^{5/2}\sqrt[4]{a-bx^4}}\end{aligned}$$

Mathematica [C] time = 0.04, size = 80, normalized size = 0.74

$$\frac{x^2\left(6a^2\sqrt[4]{1-\frac{bx^4}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^4}{a}\right)-6a^2+abx^4+5b^2x^8\right)}{45b^2\sqrt[4]{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a - b*x^4)^(1/4),x]
```

```
[Out] (x^2*(-6*a^2 + a*b*x^4 + 5*b^2*x^8 + 6*a^2*(1 - (b*x^4)/a)^(1/4)*Hypergeome
tric2F1[1/4, 1/2, 3/2, (b*x^4)/a]))/(45*b^2*(a - b*x^4)^(1/4))
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^4+a\right)^{\frac{3}{4}}x^9}{bx^4-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^4 + a)^(3/4)*x^9/(b*x^4 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(-bx^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^9/(-b*x^4 + a)^(1/4), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(-b\,x^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(-b*x^4+a)^(1/4),x)
[Out] int(x^9/(-b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(-bx^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(-b*x^4+a)^(1/4),x, algorithm="maxima")
[Out] integrate(x^9/(-b*x^4 + a)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^9}{\left(a-b\,x^4\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a - b*x^4)^(1/4),x)
[Out] int(x^9/(a - b*x^4)^(1/4), x)
```

sympy [C] time = 1.54, size = 29, normalized size = 0.27

$$\frac{x^{10}\,{}_2F_1\left(\begin{matrix}\frac{1}{4}, \frac{5}{2} \\ \frac{7}{2}\end{matrix}\middle| \frac{bx^4e^{2i\pi}}{a}\right)}{10\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(-b*x**4+a)**(1/4),x)
[Out] x**10*hyper((1/4, 5/2), (7/2,), b*x**4*exp_polar(2*I*pi)/a)/(10*a**(1/4))
```

3.1214 $\int \frac{x^5}{\sqrt[4]{a-bx^4}}\,dx$

Optimal. Leaf size=85

$$\frac{2a^{3/2}\sqrt[4]{1-\frac{bx^4}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{5b^{3/2}\sqrt[4]{a-bx^4}}-\frac{x^2\left(a-bx^4\right)^{3/4}}{5b}$$

```
[Out] -1/5*x^2*(-b*x^4+a)^(3/4)/b+2/5*a^(3/2)*(1-b*x^4/a)^(1/4)*(cos(1/2*arcsin(x
^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x^2*b^(1/2)/a^(1/2)))*Elliptic
E(sin(1/2*arcsin(x^2*b^(1/2)/a^(1/2))),2^(1/2))/b^(3/2)/(-b*x^4+a)^(1/4)
```

Rubi [A] time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {275, 321, 229, 228}

$$\frac{2a^{3/2}\sqrt[4]{1-\frac{bx^4}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{5b^{3/2}\sqrt[4]{a-bx^4}}-\frac{x^2\left(a-bx^4\right)^{3/4}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a - b*x^4)^(1/4),x]
```

```
[Out] -(x^2*(a - b*x^4)^(3/4))/(5*b) + (2*a^(3/2)*(1 - (b*x^4)/a)^(1/4)*EllipticE
[ArcSin[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(5*b^(3/2)*(a - b*x^4)^(1/4))
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\sqrt[4]{a-bx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\sqrt[4]{a-bx^2}}\,dx, x, x^2\right)\\ &= -\frac{x^2\left(a-bx^4\right)^{3/4}}{5b} + \frac{a\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx, x, x^2\right)}{5b}\\ &= -\frac{x^2\left(a-bx^4\right)^{3/4}}{5b} + \frac{\left(a\sqrt[4]{1-\frac{bx^4}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx, x, x^2\right)}{5b\sqrt[4]{a-bx^4}}\\ &= -\frac{x^2\left(a-bx^4\right)^{3/4}}{5b} + \frac{2a^{3/2}\sqrt[4]{1-\frac{bx^4}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{5b^{3/2}\sqrt[4]{a-bx^4}}\end{aligned}$$

Mathematica [C] time = 0.02, size = 66, normalized size = 0.78

$$\frac{x^2\left(a\sqrt[4]{1-\frac{bx^4}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^4}{a}\right)-a+bx^4\right)}{5b\sqrt[4]{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a - b*x^4)^(1/4),x]
```

```
[Out] (x^2*(-a + b*x^4 + a*(1 - (b*x^4)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2,
 (b*x^4)/a]))/(5*b*(a - b*x^4)^(1/4))
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^4+a\right)^{\frac{3}{4}}x^5}{bx^4-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^4 + a)^(3/4)*x^5/(b*x^4 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(-bx^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^5/(-b*x^4 + a)^(1/4), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(-b\,x^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-b*x^4+a)^(1/4),x)
```

```
[Out] int(x^5/(-b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(-bx^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^5/(-b*x^4 + a)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\left(a-b\,x^4\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a - b*x^4)^(1/4),x)
```

```
[Out] int(x^5/(a - b*x^4)^(1/4), x)
```

sympy [C] time = 1.70, size = 29, normalized size = 0.34

$$\frac{x^6\,{}_2F_1\left(\begin{matrix}\frac{1}{4}, \frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle| \frac{bx^4e^{2i\pi}}{a}\right)}{6\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-b*x**4+a)**(1/4),x)
```

```
[Out] x**6*hyper((1/4, 3/2), (5/2,), b*x**4*exp_polar(2*I*pi)/a)/(6*a**(1/4))
```

3.1215 $\int \frac{x}{\sqrt[4]{a-bx^4}}\,dx$

Optimal. Leaf size=59

$$\frac{\sqrt{a}\,\sqrt[4]{1-\frac{bx^4}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\,\sqrt[4]{a-bx^4}}$$

```
[Out] (1-b*x^4/a)^(1/4)*(cos(1/2*arcsin(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*ar
csin(x^2*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arcsin(x^2*b^(1/2)/a^(1/2))),2
^(1/2))*a^(1/2)/(-b*x^4+a)^(1/4)/b^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {275, 229, 228}

$$\frac{\sqrt{a}\,\sqrt[4]{1-\frac{bx^4}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\,\sqrt[4]{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a - b*x^4)^(1/4),x]
```

```
[Out] (Sqrt[a]*(1 - (b*x^4)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2
])/(Sqrt[b]*(a - b*x^4)^(1/4))
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt[4]{a-bx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx, x, x^2\right)\\ &= \frac{\sqrt[4]{1-\frac{bx^4}{a}}\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx, x, x^2\right)}{2\sqrt[4]{a-bx^4}}\\ &= \frac{\sqrt{a}\,\sqrt[4]{1-\frac{bx^4}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\,\sqrt[4]{a-bx^4}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.88

$$\frac{x^2 \sqrt[4]{1-\frac{bx^4}{a}}\ {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{bx^4}{a}\right)}{2\sqrt[4]{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a - b*x^4)^(1/4),x]
```

```
[Out] (x^2*(1 - (b*x^4)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (b*x^4)/a])/(2*
(a - b*x^4)^(1/4))
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^4+a\right)^{\frac{3}{4}}x}{bx^4-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^4 + a)^(3/4)*x/(b*x^4 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-bx^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x/(-b*x^4 + a)^(1/4), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-b\,x^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^4+a)^(1/4),x)
```

```
[Out] int(x/(-b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-bx^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x/(-b*x^4 + a)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\left(a-b\,x^4\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a - b*x^4)^(1/4),x)
```

```
[Out] int(x/(a - b*x^4)^(1/4), x)
```

sympy [C] time = 1.28, size = 29, normalized size = 0.49

$$\frac{x^2 {}_2F_1\left(\left.\begin{matrix}\frac{1}{4}, \frac{1}{2} \\ \frac{3}{2}\end{matrix}\right| \frac{b x^4 e^{2 i \pi}}{a}\right)}{2 \sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**4+a)**(1/4),x)
```

```
[Out] x**2*hyper((1/4, 1/2), (3/2,), b*x**4*exp_polar(2*I*pi)/a)/(2*a**(1/4))
```

3.1216 $\int \frac{1}{x^3 \sqrt[4]{a-bx^4}}\,dx$

Optimal. Leaf size=85

$$-\frac{\left(a-bx^4\right)^{3/4}}{2ax^2}-\frac{\sqrt{b}\,\sqrt[4]{1-\frac{bx^4}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}\,\sqrt[4]{a-bx^4}}$$

```
[Out] -1/2*(-b*x^4+a)^(3/4)/a/x^2-1/2*(1-b*x^4/a)^(1/4)*(cos(1/2*arcsin(x^2*b^(1/
2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x^2*b^(1/2)/a^(1/2)))*EllipticE(sin(1/
2*arcsin(x^2*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)/(-b*x^4+a)^(1/4)/a^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {275, 325, 229, 228}

$$-\frac{\left(a-bx^4\right)^{3/4}}{2ax^2}-\frac{\sqrt{b}\,\sqrt[4]{1-\frac{bx^4}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}\,\sqrt[4]{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a - b*x^4)^(1/4)),x]
```

```
[Out] -(a - b*x^4)^(3/4)/(2*a*x^2) - (Sqrt[b]*(1 - (b*x^4)/a)^(1/4)*EllipticE[Arc
Sin[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(2*Sqrt[a]*(a - b*x^4)^(1/4))
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt[4]{a-bx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt[4]{a-bx^2}}\,dx, x, x^2\right)$$
$$= -\frac{\left(a-bx^4\right)^{3/4}}{2ax^2} - \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx, x, x^2\right)}{4a}$$
$$= -\frac{\left(a-bx^4\right)^{3/4}}{2ax^2} - \frac{\left(b\sqrt[4]{1-\frac{bx^4}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx, x, x^2\right)}{4a\sqrt[4]{a-bx^4}}$$
$$= -\frac{\left(a-bx^4\right)^{3/4}}{2ax^2} - \frac{\sqrt{b}\,\sqrt[4]{1-\frac{bx^4}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|\,2\right)}{2\sqrt{a}\,\sqrt[4]{a-bx^4}}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.61

$$-\frac{\sqrt[4]{1-\frac{bx^4}{a}}\;{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};\frac{bx^4}{a}\right)}{2x^2\sqrt[4]{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a - b*x^4)^(1/4)),x]
```

```
[Out] -1/2*((1 - (b*x^4)/a)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, (b*x^4)/a])/(
x^2*(a - b*x^4)^(1/4))
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^4+a\right)^{\frac{3}{4}}}{bx^7-ax^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^4 + a)^(3/4)/(b*x^7 - a*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{1}{4}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^4 + a)^(1/4)*x^3), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^4+a\right)^{\frac{1}{4}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-b*x^4+a)^(1/4),x)
```

```
[Out] int(1/x^3/(-b*x^4+a)^(1/4),x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{1}{4}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^4 + a)^(1/4)*x^3), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\left(a-b\,x^4\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a - b*x^4)^(1/4)),x)
```

```
[Out] int(1/(x^3*(a - b*x^4)^(1/4)), x)
```

sympy **[C]** time = 1.41, size = 32, normalized size = 0.38

$$-\frac{{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{1}{2}\end{matrix}\right|\frac{bx^4e^{2i\pi}}{a}\right)}{2\sqrt[4]{a}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-b*x**4+a)**(1/4),x)
```

```
[Out] -hyper((-1/2, 1/4), (1/2,), b*x**4*exp_polar(2*I*pi)/a)/(2*a**(1/4)*x**2)
```

3.1217 $\int \frac{1}{x^7 \sqrt[4]{a-bx^4}}\,dx$

Optimal. Leaf size=108

$$-\frac{b^{3/2}\sqrt[4]{1-\frac{bx^4}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{4a^{3/2}\sqrt[4]{a-bx^4}}-\frac{b\left(a-bx^4\right)^{3/4}}{4a^2x^2}-\frac{\left(a-bx^4\right)^{3/4}}{6ax^6}$$

```
[Out] -1/6*(-b*x^4+a)^(3/4)/a/x^6-1/4*b*(-b*x^4+a)^(3/4)/a^2/x^2-1/4*b^(3/2)*(1-b
*x^4/a)^(1/4)*(cos(1/2*arcsin(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin
(x^2*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arcsin(x^2*b^(1/2)/a^(1/2))),2^(1/
2))/a^(3/2)/(-b*x^4+a)^(1/4)
```

Rubi [A] time = 0.07, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {275, 325, 229, 228}

$$-\frac{b^{3/2}\sqrt[4]{1-\frac{bx^4}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{4a^{3/2}\sqrt[4]{a-bx^4}}-\frac{b\left(a-bx^4\right)^{3/4}}{4a^2x^2}-\frac{\left(a-bx^4\right)^{3/4}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a - b*x^4)^(1/4)),x]
```

```
[Out] -(a - b*x^4)^(3/4)/(6*a*x^6) - (b*(a - b*x^4)^(3/4))/(4*a^2*x^2) - (b^(3/2)
*(1 - (b*x^4)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(4*a^
(3/2)*(a - b*x^4)^(1/4))
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7 \sqrt[4]{a-bx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^4 \sqrt[4]{a-bx^2}}\,dx, x, x^2\right)\\ &= -\frac{\left(a-bx^4\right)^{3/4}}{6ax^6} + \frac{b\,\text{Subst}\left(\int \frac{1}{x^2 \sqrt[4]{a-bx^2}}\,dx, x, x^2\right)}{4a}\\ &= -\frac{\left(a-bx^4\right)^{3/4}}{6ax^6} - \frac{b\left(a-bx^4\right)^{3/4}}{4a^2x^2} - \frac{b^2\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx, x, x^2\right)}{8a^2}\\ &= -\frac{\left(a-bx^4\right)^{3/4}}{6ax^6} - \frac{b\left(a-bx^4\right)^{3/4}}{4a^2x^2} - \frac{\left(b^2 \sqrt[4]{1-\frac{bx^4}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx, x, x^2\right)}{8a^2 \sqrt[4]{a-bx^4}}\\ &= -\frac{\left(a-bx^4\right)^{3/4}}{6ax^6} - \frac{b\left(a-bx^4\right)^{3/4}}{4a^2x^2} - \frac{b^{3/2} \sqrt[4]{1-\frac{bx^4}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|\,2\right)}{4a^{3/2} \sqrt[4]{a-bx^4}}\end{aligned}$$

Mathematica [C] time = 0.03, size = 52, normalized size = 0.48

$$-\frac{\sqrt[4]{1-\frac{bx^4}{a}}\;{}_2F_1\left(-\frac{3}{2}, \frac{1}{4}; -\frac{1}{2}; \frac{bx^4}{a}\right)}{6x^6 \sqrt[4]{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a - b*x^4)^(1/4)),x]
```

```
[Out] -1/6*((1 - (b*x^4)/a)^(1/4)*Hypergeometric2F1[-3/2, 1/4, -1/2, (b*x^4)/a])/
(x^6*(a - b*x^4)^(1/4))
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^4+a\right)^{\frac{3}{4}}}{bx^{11}-ax^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^4 + a)^(3/4)/(b*x^11 - a*x^7), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{1}{4}} x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^4 + a)^(1/4)*x^7), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^4+a\right)^{\frac{1}{4}} x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(-b*x^4+a)^(1/4),x)
[Out] int(1/x^7/(-b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{1}{4}}x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-b*x^4+a)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((-b*x^4 + a)^(1/4)*x^7), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^7\left(a-b\,x^4\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a - b*x^4)^(1/4)),x)
[Out] int(1/(x^7*(a - b*x^4)^(1/4)), x)
```

sympy [C] time = 1.91, size = 34, normalized size = 0.31

$$-\frac{{}_2F_1\left(\left.\begin{matrix}-\frac{3}{2}, \frac{1}{4} \\ -\frac{1}{2}\end{matrix}\right| \frac{bx^4e^{2i\pi}}{a}\right)}{6\sqrt[4]{a}\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(-b*x**4+a)**(1/4),x)
[Out] -hyper((-3/2, 1/4), (-1/2,), b*x**4*exp_polar(2*I*pi)/a)/(6*a**(1/4)*x**6)
```

3.1218 $\int \frac{1}{x^{11}\sqrt[4]{a-bx^4}}\,dx$

Optimal. Leaf size=133

$$-\frac{7b^{5/2}\sqrt[4]{1-\frac{bx^4}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{40a^{5/2}\sqrt[4]{a-bx^4}}-\frac{7b^2\left(a-bx^4\right)^{3/4}}{40a^3x^2}-\frac{7b\left(a-bx^4\right)^{3/4}}{60a^2x^6}-\frac{\left(a-bx^4\right)^{3/4}}{10ax^{10}}$$

```
[Out] -1/10*(-b*x^4+a)^(3/4)/a/x^10-7/60*b*(-b*x^4+a)^(3/4)/a^2/x^6-7/40*b^2*(-b*
x^4+a)^(3/4)/a^3/x^2-7/40*b^(5/2)*(1-b*x^4/a)^(1/4)*(cos(1/2*arcsin(x^2*b^(
1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x^2*b^(1/2)/a^(1/2)))*EllipticE(sin(
1/2*arcsin(x^2*b^(1/2)/a^(1/2))),2^(1/2))/a^(5/2)/(-b*x^4+a)^(1/4)
```

Rubi [A] time = 0.09, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {275, 325, 229, 228}

$$-\frac{7b^2\left(a-bx^4\right)^{3/4}}{40a^3x^2}-\frac{7b^{5/2}\sqrt[4]{1-\frac{bx^4}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{40a^{5/2}\sqrt[4]{a-bx^4}}-\frac{7b\left(a-bx^4\right)^{3/4}}{60a^2x^6}-\frac{\left(a-bx^4\right)^{3/4}}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^11*(a - b*x^4)^(1/4)),x]
```

```
[Out] -(a - b*x^4)^(3/4)/(10*a*x^10) - (7*b*(a - b*x^4)^(3/4))/(60*a^2*x^6) - (7*
b^2*(a - b*x^4)^(3/4))/(40*a^3*x^2) - (7*b^(5/2)*(1 - (b*x^4)/a)^(1/4)*Elli
pticE[ArcSin[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(40*a^(5/2)*(a - b*x^4)^(1/4))
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + (b*x^2)/a)^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{11}\sqrt[4]{a-bx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^6\sqrt[4]{a-bx^2}}\,dx, x, x^2\right)\\
&= -\frac{\left(a-bx^4\right)^{3/4}}{10ax^{10}} + \frac{(7b)\,\text{Subst}\left(\int \frac{1}{x^4\sqrt[4]{a-bx^2}}\,dx, x, x^2\right)}{20a}\\
&= -\frac{\left(a-bx^4\right)^{3/4}}{10ax^{10}} - \frac{7b\left(a-bx^4\right)^{3/4}}{60a^2x^6} + \frac{\left(7b^2\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt[4]{a-bx^2}}\,dx, x, x^2\right)}{40a^2}\\
&= -\frac{\left(a-bx^4\right)^{3/4}}{10ax^{10}} - \frac{7b\left(a-bx^4\right)^{3/4}}{60a^2x^6} - \frac{7b^2\left(a-bx^4\right)^{3/4}}{40a^3x^2} - \frac{\left(7b^3\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx, x, x^2\right)}{80a^3}\\
&= -\frac{\left(a-bx^4\right)^{3/4}}{10ax^{10}} - \frac{7b\left(a-bx^4\right)^{3/4}}{60a^2x^6} - \frac{7b^2\left(a-bx^4\right)^{3/4}}{40a^3x^2} - \frac{\left(7b^3\sqrt[4]{1-\frac{bx^4}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}\,dx\right.}{80a^3\sqrt[4]{a-bx^4}}\\
&= -\frac{\left(a-bx^4\right)^{3/4}}{10ax^{10}} - \frac{7b\left(a-bx^4\right)^{3/4}}{60a^2x^6} - \frac{7b^2\left(a-bx^4\right)^{3/4}}{40a^3x^2} - \frac{7b^{5/2}\sqrt[4]{1-\frac{bx^4}{a}}\,E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right.}{40a^{5/2}\sqrt[4]{a-bx^4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.39

$$-\frac{\sqrt[4]{1-\frac{bx^4}{a}}\;{}_2F_1\left(-\frac{5}{2},\frac{1}{4};-\frac{3}{2};\frac{bx^4}{a}\right)}{10x^{10}\sqrt[4]{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^11*(a - b*x^4)^(1/4)),x]
```

```
[Out] -1/10*((1 - (b*x^4)/a)^(1/4)*Hypergeometric2F1[-5/2, 1/4, -3/2, (b*x^4)/a])
/(x^10*(a - b*x^4)^(1/4))
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^4+a\right)^{\frac{3}{4}}}{bx^{15}-ax^{11}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^4 + a)^(3/4)/(b*x^15 - a*x^11), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{1}{4}}x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^4 + a)^(1/4)*x^11), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^4+a\right)^{\frac{1}{4}}\,x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^11/(-b*x^4+a)^(1/4),x)
```

```
[Out] int(1/x^11/(-b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{1}{4}}\,x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^4 + a)^(1/4)*x^11), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{11}\left(a-b\,x^4\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^11*(a - b*x^4)^(1/4)),x)
```

```
[Out] int(1/(x^11*(a - b*x^4)^(1/4)), x)
```

sympy [C] time = 4.30, size = 34, normalized size = 0.26

$$-\frac{{}_2F_1\left(\left.\begin{matrix}-\frac{5}{2}, \frac{1}{4} \\ -\frac{3}{2}\end{matrix}\right| \frac{bx^4e^{2i\pi}}{a}\right)}{10\sqrt[4]{a}\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**11/(-b*x**4+a)**(1/4),x)
```

```
[Out] -hyper((-5/2, 1/4), (-3/2,), b*x**4*exp_polar(2*I*pi)/a)/(10*a**(1/4)*x**10
)
```

3.1219 $\int \frac{x^8}{\sqrt[4]{a-bx^4}}\,dx$

Optimal. Leaf size=264

$$-\frac{5a^2\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{64\sqrt{2}\,b^{9/4}}+\frac{5a^2\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+1\right)}{64\sqrt{2}\,b^{9/4}}-\frac{5a^2\log\left(-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}+1\right)}{128\sqrt{2}\,b^{9/4}}+\frac{5a^2\log\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+\frac{\sqrt{b}\,x^2}{\sqrt{a-bx}}\right.}{128\sqrt{2}\,b^{9/4}}$$

```
[Out] -5/32*a*x*(-b*x^4+a)^(3/4)/b^2-1/8*x^5*(-b*x^4+a)^(3/4)/b+5/128*a^2*arctan(
-1+b^(1/4)*x*2^(1/2)/(-b*x^4+a)^(1/4))/b^(9/4)*2^(1/2)+5/128*a^2*arctan(1+b
^(1/4)*x*2^(1/2)/(-b*x^4+a)^(1/4))/b^(9/4)*2^(1/2)-5/256*a^2*ln(1-b^(1/4)*x
*2^(1/2)/(-b*x^4+a)^(1/4)+x^2*b^(1/2)/(-b*x^4+a)^(1/2))/b^(9/4)*2^(1/2)+5/2
56*a^2*ln(1+b^(1/4)*x*2^(1/2)/(-b*x^4+a)^(1/4)+x^2*b^(1/2)/(-b*x^4+a)^(1/2)
)/b^(9/4)*2^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {321, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{5a^2\log\left(\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+1\right)}{128\sqrt{2}\,b^{9/4}}+\frac{5a^2\log\left(\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+1\right)}{128\sqrt{2}\,b^{9/4}}-\frac{5a^2\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{64\sqrt{2}\,b^{9/4}}+\frac{5a^2\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right.}{64\sqrt{2}\,b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a - b*x^4)^(1/4),x]
```

```
[Out] (-5*a*x*(a - b*x^4)^(3/4))/(32*b^2) - (x^5*(a - b*x^4)^(3/4))/(8*b) - (5*a^
2*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)])/(64*Sqrt[2]*b^(9/4)) +
 (5*a^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)])/(64*Sqrt[2]*b^(9
/4)) - (5*a^2*Log[1 + (Sqrt[b]*x^2)/Sqrt[a - b*x^4] - (Sqrt[2]*b^(1/4)*x)/(
a - b*x^4)^(1/4)])/(128*Sqrt[2]*b^(9/4)) + (5*a^2*Log[1 + (Sqrt[b]*x^2)/Sqr
t[a - b*x^4] + (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)])/(128*Sqrt[2]*b^(9/4)
)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^8}{\sqrt[4]{a-bx^4}}\,dx = -\frac{x^5\left(a-bx^4\right)^{3/4}}{8b} + \frac{(5a)\int \frac{x^4}{\sqrt[4]{a-bx^4}}\,dx}{8b}$$

$$= -\frac{5ax\left(a-bx^4\right)^{3/4}}{32b^2} - \frac{x^5\left(a-bx^4\right)^{3/4}}{8b} + \frac{\left(5a^2\right)\int \frac{1}{\sqrt[4]{a-bx^4}}\,dx}{32b^2}$$

$$= -\frac{5ax\left(a-bx^4\right)^{3/4}}{32b^2} - \frac{x^5\left(a-bx^4\right)^{3/4}}{8b} + \frac{\left(5a^2\right)\text{Subst}\left(\int \frac{1}{1+bx^4}\,dx, x, \frac{x}{\sqrt[4]{a-bx^4}}\right)}{32b^2}$$

$$= -\frac{5ax\left(a-bx^4\right)^{3/4}}{32b^2} - \frac{x^5\left(a-bx^4\right)^{3/4}}{8b} + \frac{\left(5a^2\right)\text{Subst}\left(\int \frac{1-\sqrt{b}\,x^2}{1+bx^4}\,dx, x, \frac{x}{\sqrt[4]{a-bx^4}}\right)}{64b^2} + \frac{\left(5a^2\right)\text{Subst}\left(\int$$

$$= -\frac{5ax\left(a-bx^4\right)^{3/4}}{32b^2} - \frac{x^5\left(a-bx^4\right)^{3/4}}{8b} + \frac{\left(5a^2\right)\text{Subst}\left(\int \frac{1}{\frac{1}{\sqrt{b}}-\frac{\sqrt{2}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \frac{x}{\sqrt[4]{a-bx^4}}\right)}{128b^{5/2}} + \frac{\left(5a^2\right)\text{Subst}$$

$$= -\frac{5ax\left(a-bx^4\right)^{3/4}}{32b^2} - \frac{x^5\left(a-bx^4\right)^{3/4}}{8b} - \frac{5a^2\log\left(1+\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{128\sqrt{2}\,b^{9/4}} + \frac{5a^2\log\left(1+\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}+\right.}{128\sqrt{2}\,b^{9/4}}$$

$$= -\frac{5ax\left(a-bx^4\right)^{3/4}}{32b^2} - \frac{x^5\left(a-bx^4\right)^{3/4}}{8b} - \frac{5a^2\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{64\sqrt{2}\,b^{9/4}} + \frac{5a^2\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{64\sqrt{2}\,b^{9/4}} - \frac{5a^2}{}$$

Mathematica [A] time = 0.18, size = 245, normalized size = 0.93

$$-\frac{10\sqrt{2}\,a^2\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)-10\sqrt{2}\,a^2\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+1\right)+5\sqrt{2}\,a^2\log\left(-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}+1\right)-5\sqrt{2}\,a^2\,\text{lo}}{256b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a - b*x^4)^(1/4),x]
```

```
[Out] -1/256*(40*a*b^(1/4)*x*(a - b*x^4)^(3/4) + 32*b^(5/4)*x^5*(a - b*x^4)^(3/4)
 + 10*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)] - 10*Sq
rt[2]*a^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)] + 5*Sqrt[2]*a^2
*Log[1 + (Sqrt[b]*x^2)/Sqrt[a - b*x^4] - (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1
/4)] - 5*Sqrt[2]*a^2*Log[1 + (Sqrt[b]*x^2)/Sqrt[a - b*x^4] + (Sqrt[2]*b^(1/
4)*x)/(a - b*x^4)^(1/4)])/b^(9/4)
```

fricas [A] time = 0.80, size = 243, normalized size = 0.92

$$\frac{20\,b^2\left(-\frac{a^8}{b^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-bx^4+a\right)^{\frac{1}{4}}a^6b^2\left(-\frac{a^8}{b^9}\right)^{\frac{1}{4}}-b^2x\left(-\frac{a^8}{b^9}\right)^{\frac{1}{4}}\sqrt{-\frac{a^8b^5x^2\sqrt{-\frac{a^8}{b^9}}-\sqrt{-bx^4+a}\,a^{12}}{x^2}}}{a^8x}\right)-5\,b^2\left(-\frac{a^8}{b^9}\right)^{\frac{1}{4}}\log\left(\frac{125\left(b^7x\left(-\frac{a^8}{b^9}\right)^{\frac{3}{4}}+\left(-\right.\right.\right.}{x}}{128\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/128*(20*b^2*(-a^8/b^9)^(1/4)*arctan(-((-b*x^4 + a)^(1/4)*a^6*b^2*(-a^8/b^
9)^(1/4) - b^2*x*(-a^8/b^9)^(1/4)*sqrt(-(a^8*b^5*x^2*sqrt(-a^8/b^9) - sqrt(
-b*x^4 + a)*a^12)/x^2))/(a^8*x)) - 5*b^2*(-a^8/b^9)^(1/4)*log(125*(b^7*x*(-
a^8/b^9)^(3/4) + (-b*x^4 + a)^(1/4)*a^6)/x) + 5*b^2*(-a^8/b^9)^(1/4)*log(-1
25*(b^7*x*(-a^8/b^9)^(3/4) - (-b*x^4 + a)^(1/4)*a^6)/x) - 4*(4*b*x^5 + 5*a*
x)*(-b*x^4 + a)^(3/4))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(-bx^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^8/(-b*x^4 + a)^(1/4), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(-b\,x^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-b*x^4+a)^(1/4),x)
```

```
[Out] int(x^8/(-b*x^4+a)^(1/4),x)
```

maxima [A] time = 2.46, size = 265, normalized size = 1.00

$$-\frac{5\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+\frac{2\left(-bx^4+a\right)^{\frac{1}{4}}}{x}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-\frac{2\left(-bx^4+a\right)^{\frac{1}{4}}}{x}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{b}+\frac{\sqrt{2}\left(-bx^4+a\right)^{\frac{1}{4}}b^{\frac{1}{4}}}{x}+\frac{\sqrt{-bx^4+a}}{x^2}\right)}{b^{\frac{1}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{}\right.}{}\right.}{256\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] -5/256*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(-b*x^4 + a)^(1/4
)/x)/b^(1/4))/b^(1/4) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*
(-b*x^4 + a)^(1/4)/x)/b^(1/4))/b^(1/4) - sqrt(2)*log(sqrt(b) + sqrt(2)*(-b*
x^4 + a)^(1/4)*b^(1/4)/x + sqrt(-b*x^4 + a)/x^2)/b^(1/4) + sqrt(2)*log(sqrt
(b) - sqrt(2)*(-b*x^4 + a)^(1/4)*b^(1/4)/x + sqrt(-b*x^4 + a)/x^2)/b^(1/4))
*a^2/b^2 - 1/32*(9*(-b*x^4 + a)^(3/4)*a^2*b/x^3 + 5*(-b*x^4 + a)^(7/4)*a^2/
x^7)/(b^4 - 2*(b*x^4 - a)*b^3/x^4 + (b*x^4 - a)^2*b^2/x^8)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^8}{\left(a-b\,x^4\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a - b*x^4)^(1/4),x)
```

```
[Out] int(x^8/(a - b*x^4)^(1/4), x)
```

sympy [C] time = 2.30, size = 39, normalized size = 0.15

$$\frac{x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\right|\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt[4]{a}\,\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-b*x**4+a)**(1/4),x)
```

```
[Out] x**9*gamma(9/4)*hyper((1/4, 9/4), (13/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*a
**(1/4)*gamma(13/4))
```

3.1220 $\int \frac{x^4}{\sqrt[4]{a-bx^4}}\,dx$

Optimal. Leaf size=233

$$-\frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{8\sqrt{2}\,b^{5/4}}+\frac{a\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+1\right)}{8\sqrt{2}\,b^{5/4}}-\frac{a\log\left(-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}+1\right)}{16\sqrt{2}\,b^{5/4}}+\frac{a\log\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}+1\right)}{16\sqrt{2}\,b^{5/4}}-\frac{x}{}$$

```
[Out] -1/4*x*(-b*x^4+a)^(3/4)/b+1/16*a*arctan(-1+b^(1/4)*x*2^(1/2)/(-b*x^4+a)^(1/
4))/b^(5/4)*2^(1/2)+1/16*a*arctan(1+b^(1/4)*x*2^(1/2)/(-b*x^4+a)^(1/4))/b^(
5/4)*2^(1/2)-1/32*a*ln(1-b^(1/4)*x*2^(1/2)/(-b*x^4+a)^(1/4)+x^2*b^(1/2)/(-b
*x^4+a)^(1/2))/b^(5/4)*2^(1/2)+1/32*a*ln(1+b^(1/4)*x*2^(1/2)/(-b*x^4+a)^(1/
4)+x^2*b^(1/2)/(-b*x^4+a)^(1/2))/b^(5/4)*2^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {321, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{a\log\left(\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+1\right)}{16\sqrt{2}\,b^{5/4}}+\frac{a\log\left(\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+1\right)}{16\sqrt{2}\,b^{5/4}}-\frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{8\sqrt{2}\,b^{5/4}}+\frac{a\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+1\right)}{8\sqrt{2}\,b^{5/4}}-\frac{x\left(}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a - b*x^4)^(1/4),x]
```

```
[Out] -(x*(a - b*x^4)^(3/4))/(4*b) - (a*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/(a - b*x^4
)^(1/4)])/(8*Sqrt[2]*b^(5/4)) + (a*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/(a - b*x^
4)^(1/4)])/(8*Sqrt[2]*b^(5/4)) - (a*Log[1 + (Sqrt[b]*x^2)/Sqrt[a - b*x^4] -
 (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)])/(16*Sqrt[2]*b^(5/4)) + (a*Log[1 +
(Sqrt[b]*x^2)/Sqrt[a - b*x^4] + (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)])/(16
*Sqrt[2]*b^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^4}{\sqrt[4]{a-bx^4}}\,dx = -\frac{x\left(a-bx^4\right)^{3/4}}{4b} + \frac{a\int \frac{1}{\sqrt[4]{a-bx^4}}\,dx}{4b}$$

$$= -\frac{x\left(a-bx^4\right)^{3/4}}{4b} + \frac{a\,\text{Subst}\left(\int \frac{1}{1+bx^4}\,dx, x, \frac{x}{\sqrt[4]{a-bx^4}}\right)}{4b}$$

$$= -\frac{x\left(a-bx^4\right)^{3/4}}{4b} + \frac{a\,\text{Subst}\left(\int \frac{1-\sqrt{b}\,x^2}{1+bx^4}\,dx, x, \frac{x}{\sqrt[4]{a-bx^4}}\right)}{8b} + \frac{a\,\text{Subst}\left(\int \frac{1+\sqrt{b}\,x^2}{1+bx^4}\,dx, x, \frac{x}{\sqrt[4]{a-bx^4}}\right)}{8b}$$

$$= -\frac{x\left(a-bx^4\right)^{3/4}}{4b} + \frac{a\,\text{Subst}\left(\int \frac{1}{\frac{1}{\sqrt{b}}-\frac{\sqrt{2}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \frac{x}{\sqrt[4]{a-bx^4}}\right)}{16b^{3/2}} + \frac{a\,\text{Subst}\left(\int \frac{1}{\frac{1}{\sqrt{b}}+\frac{\sqrt{2}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \frac{x}{\sqrt[4]{a-bx^4}}\right.}{16b^{3/2}}$$

$$= -\frac{x\left(a-bx^4\right)^{3/4}}{4b} - \frac{a\log\left(1+\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{16\sqrt{2}\,b^{5/4}} + \frac{a\log\left(1+\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{16\sqrt{2}\,b^{5/4}} + \frac{a\,\text{Subst}\left(\int -\right.}{}$$

$$= -\frac{x\left(a-bx^4\right)^{3/4}}{4b} - \frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{8\sqrt{2}\,b^{5/4}} + \frac{a\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{8\sqrt{2}\,b^{5/4}} - \frac{a\log\left(1+\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{16\sqrt{2}\,b^{5/4}}$$

Mathematica [A] time = 0.09, size = 213, normalized size = 0.91

$$-\frac{8\sqrt[4]{b}\,x\left(a-bx^4\right)^{3/4}+2\sqrt{2}\,a\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)-2\sqrt{2}\,a\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+1\right)+\sqrt{2}\,a\log\left(-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}+\right.}{32b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a - b*x^4)^(1/4),x]
```

```
[Out] -1/32*(8*b^(1/4)*x*(a - b*x^4)^(3/4) + 2*Sqrt[2]*a*ArcTan[1 - (Sqrt[2]*b^(1
/4)*x)/(a - b*x^4)^(1/4)] - 2*Sqrt[2]*a*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/(a -
 b*x^4)^(1/4)] + Sqrt[2]*a*Log[1 + (Sqrt[b]*x^2)/Sqrt[a - b*x^4] - (Sqrt[2]
*b^(1/4)*x)/(a - b*x^4)^(1/4)] - Sqrt[2]*a*Log[1 + (Sqrt[b]*x^2)/Sqrt[a - b
*x^4] + (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)])/b^(5/4)
```

fricas [A] time = 0.70, size = 221, normalized size = 0.95

$$\frac{4\,b\left(-\frac{a^4}{b^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-bx^4+a\right)^{\frac{1}{4}}a^3b\left(-\frac{a^4}{b^5}\right)^{\frac{1}{4}}-bx\left(-\frac{a^4}{b^5}\right)^{\frac{1}{4}}\sqrt{-\frac{a^4b^3x^2\sqrt{-\frac{a^4}{b^5}}-\sqrt{-bx^4+a}\,a^6}{x^2}}}{a^4x}\right)-b\left(-\frac{a^4}{b^5}\right)^{\frac{1}{4}}\log\left(\frac{b^4x\left(-\frac{a^4}{b^5}\right)^{\frac{3}{4}}+\left(-bx^4+a\right)^{\frac{1}{4}}a^3}{x}\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/16*(4*b*(-a^4/b^5)^(1/4)*arctan(-((-b*x^4 + a)^(1/4)*a^3*b*(-a^4/b^5)^(1/
4) - b*x*(-a^4/b^5)^(1/4)*sqrt(-(a^4*b^3*x^2*sqrt(-a^4/b^5) - sqrt(-b*x^4 +
 a)*a^6)/x^2))/(a^4*x)) - b*(-a^4/b^5)^(1/4)*log((b^4*x*(-a^4/b^5)^(3/4) +
(-b*x^4 + a)^(1/4)*a^3)/x) + b*(-a^4/b^5)^(1/4)*log(-(b^4*x*(-a^4/b^5)^(3/4
) - (-b*x^4 + a)^(1/4)*a^3)/x) - 4*(-b*x^4 + a)^(3/4)*x)/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-bx^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(-b*x^4 + a)^(1/4), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-b\,x^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-b*x^4+a)^(1/4),x)
```

```
[Out] int(x^4/(-b*x^4+a)^(1/4),x)
```

maxima [A] time = 2.42, size = 219, normalized size = 0.94

$$-\frac{\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}+\frac{2\left(-bx^4+a\right)^{\frac{1}{4}}}{x}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}-\frac{2\left(-bx^4+a\right)^{\frac{1}{4}}}{x}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{b}+\frac{\sqrt{2}\left(-bx^4+a\right)^{\frac{1}{4}}b^{\frac{1}{4}}}{x}+\frac{\sqrt{-bx^4+a}}{x^2}\right)}{b^{\frac{1}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}-\right.\right.}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] -1/32*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(-b*x^4 + a)^(1/4)
/x)/b^(1/4))/b^(1/4) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(
-b*x^4 + a)^(1/4)/x)/b^(1/4))/b^(1/4) - sqrt(2)*log(sqrt(b) + sqrt(2)*(-b*x
^4 + a)^(1/4)*b^(1/4)/x + sqrt(-b*x^4 + a)/x^2)/b^(1/4) + sqrt(2)*log(sqrt(
b) - sqrt(2)*(-b*x^4 + a)^(1/4)*b^(1/4)/x + sqrt(-b*x^4 + a)/x^2)/b^(1/4))*
a/b - 1/4*(-b*x^4 + a)^(3/4)*a/((b^2 - (b*x^4 - a)*b/x^4)*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(a-b\,x^4\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a - b*x^4)^(1/4),x)
```

```
[Out] int(x^4/(a - b*x^4)^(1/4), x)
```

sympy [C] time = 1.37, size = 39, normalized size = 0.17

$$\frac{x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt[4]{a}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-b*x**4+a)**(1/4),x)
```

```
[Out] x**5*gamma(5/4)*hyper((1/4, 5/4), (9/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*a*
*(1/4)*gamma(9/4))
```

3.1221 $\int \frac{1}{\sqrt[4]{a-bx^4}}\,dx$

Optimal. Leaf size=209

$$-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{2\sqrt{2}\,\sqrt[4]{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+1\right)}{2\sqrt{2}\,\sqrt[4]{b}}-\frac{\log\left(-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}+1\right)}{4\sqrt{2}\,\sqrt[4]{b}}+\frac{\log\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}+1\right)}{4\sqrt{2}\,\sqrt[4]{b}}$$

```
[Out] 1/4*arctan(-1+b^(1/4)*x*2^(1/2)/(-b*x^4+a)^(1/4))/b^(1/4)*2^(1/2)+1/4*arcta
n(1+b^(1/4)*x*2^(1/2)/(-b*x^4+a)^(1/4))/b^(1/4)*2^(1/2)-1/8*ln(1-b^(1/4)*x*
2^(1/2)/(-b*x^4+a)^(1/4)+x^2*b^(1/2)/(-b*x^4+a)^(1/2))/b^(1/4)*2^(1/2)+1/8*
ln(1+b^(1/4)*x*2^(1/2)/(-b*x^4+a)^(1/4)+x^2*b^(1/2)/(-b*x^4+a)^(1/2))/b^(1/
4)*2^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+1\right)}{4\sqrt{2}\,\sqrt[4]{b}}+\frac{\log\left(\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+1\right)}{4\sqrt{2}\,\sqrt[4]{b}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{2\sqrt{2}\,\sqrt[4]{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+1\right)}{2\sqrt{2}\,\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^4)^(-1/4),x]
```

```
[Out] -ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)]/(2*Sqrt[2]*b^(1/4)) + Ar
cTan[1 + (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)]/(2*Sqrt[2]*b^(1/4)) - Log[1
 + (Sqrt[b]*x^2)/Sqrt[a - b*x^4] - (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)]/(
4*Sqrt[2]*b^(1/4)) + Log[1 + (Sqrt[b]*x^2)/Sqrt[a - b*x^4] + (Sqrt[2]*b^(1/
4)*x)/(a - b*x^4)^(1/4)]/(4*Sqrt[2]*b^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt[4]{a-bx^4}}\,dx &= \text{Subst}\left(\int \frac{1}{1+bx^4}\,dx, x, \frac{x}{\sqrt[4]{a-bx^4}}\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1-\sqrt{b}\,x^2}{1+bx^4}\,dx, x, \frac{x}{\sqrt[4]{a-bx^4}}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1+\sqrt{b}\,x^2}{1+bx^4}\,dx, x, \frac{x}{\sqrt[4]{a-bx^4}}\right) \\
&= \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{\sqrt{b}}-\frac{\sqrt{2}x}{\sqrt[4]{b}}+x^2}\,dx, x, \frac{x}{\sqrt[4]{a-bx^4}}\right)}{4\sqrt{b}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{\sqrt{b}}+\frac{\sqrt{2}x}{\sqrt[4]{b}}+x^2}\,dx, x, \frac{x}{\sqrt[4]{a-bx^4}}\right)}{4\sqrt{b}} - \frac{\text{Subst}\left(\int \frac{\frac{\sqrt{2}}{\sqrt[4]{b}}+2}{-\frac{1}{\sqrt{b}}-\frac{\sqrt{2}}{\sqrt[4]{b}}}\right.}{4\sqrt{2}} \\
&= -\frac{\log\left(1+\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{4\sqrt{2}\,\sqrt[4]{b}} + \frac{\log\left(1+\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{4\sqrt{2}\,\sqrt[4]{b}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right.}{2\sqrt{2}\,\sqrt[4]{b}} \\
&= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{2\sqrt{2}\,\sqrt[4]{b}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{2\sqrt{2}\,\sqrt[4]{b}} - \frac{\log\left(1+\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{4\sqrt{2}\,\sqrt[4]{b}} + \frac{\log\left(1+\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}+\right.}{4\sqrt{2}\,\sqrt[4]{b}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 173, normalized size = 0.83

$$\frac{-2\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)+2\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+1\right)-\log\left(-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}+1\right)+\log\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}+1\right)}{4\sqrt{2}\,\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^4)^(-1/4),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)] + 2*ArcTan[1 + (Sqrt[
2]*b^(1/4)*x)/(a - b*x^4)^(1/4)] - Log[1 + (Sqrt[b]*x^2)/Sqrt[a - b*x^4] -
(Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)] + Log[1 + (Sqrt[b]*x^2)/Sqrt[a - b*x
^4] + (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)])/(4*Sqrt[2]*b^(1/4))
```

fricas [A] time = 0.63, size = 148, normalized size = 0.71

$$\left(-\frac{1}{b}\right)^{\frac{1}{4}} \arctan\left(\frac{x\left(-\frac{1}{b}\right)^{\frac{1}{4}}\sqrt{-\frac{bx^2\sqrt{-\frac{1}{b}}-\sqrt{-bx^4+a}}{x^2}}-\left(-bx^4+a\right)^{\frac{1}{4}}\left(-\frac{1}{b}\right)^{\frac{1}{4}}}{x}\right)-\frac{1}{4}\left(-\frac{1}{b}\right)^{\frac{1}{4}}\log\left(\frac{bx\left(-\frac{1}{b}\right)^{\frac{3}{4}}+\left(-bx^4+a\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] (-1/b)^(1/4)*arctan((x*(-1/b)^(1/4)*sqrt(-(b*x^2*sqrt(-1/b) - sqrt(-b*x^4 +
 a))/x^2) - (-b*x^4 + a)^(1/4)*(-1/b)^(1/4))/x) - 1/4*(-1/b)^(1/4)*log((b*x
*(-1/b)^(3/4) + (-b*x^4 + a)^(1/4))/x) + 1/4*(-1/b)^(1/4)*log(-(b*x*(-1/b)^
(3/4) - (-b*x^4 + a)^(1/4))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^4 + a)^(-1/4), x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^4+a)^(1/4),x)
```

```
[Out] int(1/(-b*x^4+a)^(1/4),x)
```

maxima [A] time = 2.32, size = 176, normalized size = 0.84

$$-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}+\frac{2\left(-bx^4+a\right)^{\frac{1}{4}}}{x}\right)}{2\,b^{\frac{1}{4}}}\right)}{4\,b^{\frac{1}{4}}}-\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}-\frac{2\left(-bx^4+a\right)^{\frac{1}{4}}}{x}\right)}{2\,b^{\frac{1}{4}}}\right)}{4\,b^{\frac{1}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}+\frac{\sqrt{2}\left(-bx^4+a\right)^{\frac{1}{4}}b^{\frac{1}{4}}}{x}+\frac{\sqrt{-bx}}{x^2}\right.}{8\,b^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(-b*x^4 + a)^(1/4)/x)/
b^(1/4))/b^(1/4) - 1/4*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(-b
*x^4 + a)^(1/4)/x)/b^(1/4))/b^(1/4) + 1/8*sqrt(2)*log(sqrt(b) + sqrt(2)*(-b
*x^4 + a)^(1/4)*b^(1/4)/x + sqrt(-b*x^4 + a)/x^2)/b^(1/4) - 1/8*sqrt(2)*log
(sqrt(b) - sqrt(2)*(-b*x^4 + a)^(1/4)*b^(1/4)/x + sqrt(-b*x^4 + a)/x^2)/b^(
1/4)
```

mupad [B] time = 1.10, size = 38, normalized size = 0.18

$$\frac{x\left(1-\frac{b\,x^4}{a}\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ \frac{b\,x^4}{a}\right)}{\left(a-b\,x^4\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*x^4)^(1/4),x)
```

```
[Out] (x*(1 - (b*x^4)/a)^(1/4)*hypergeom([1/4, 1/4], 5/4, (b*x^4)/a))/(a - b*x^4)
^(1/4)
```

sympy [C] time = 1.58, size = 37, normalized size = 0.18

$$\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\right|\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt[4]{a}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**4+a)**(1/4),x)
```

```
[Out] x*gamma(1/4)*hyper((1/4, 1/4), (5/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*a**(1
/4)*gamma(5/4))
```

3.1222 $\int \frac{1}{x^4 \sqrt[4]{a-bx^4}}\,dx$

Optimal. Leaf size=22

$$-\frac{\left(a-bx^4\right)^{3/4}}{3ax^3}$$

```
[Out] -1/3*(-b*x^4+a)^(3/4)/a/x^3
```

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {264}

$$-\frac{\left(a-bx^4\right)^{3/4}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a - b*x^4)^(1/4)),x]
```

```
[Out] -(a - b*x^4)^(3/4)/(3*a*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4 \sqrt[4]{a-bx^4}}\,dx = -\frac{\left(a-bx^4\right)^{3/4}}{3ax^3}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$-\frac{\left(a-bx^4\right)^{3/4}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a - b*x^4)^(1/4)),x]
```

```
[Out] -1/3*(a - b*x^4)^(3/4)/(a*x^3)
```

fricas [A] time = 0.52, size = 18, normalized size = 0.82

$$-\frac{\left(-bx^4+a\right)^{\frac{3}{4}}}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/3*(-b*x^4 + a)^(3/4)/(a*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{1}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-b*x^4+a)^(1/4),x, algorithm="giac")
[Out] integrate(1/((-b*x^4 + a)^(1/4)*x^4), x)
```

maple [A] time = 0.01, size = 19, normalized size = 0.86

$$-\frac{\left(-b\,x^4+a\right)^{\frac{3}{4}}}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-b*x^4+a)^(1/4),x)
[Out] -1/3*(-b*x^4+a)^(3/4)/a/x^3
```

maxima [A] time = 0.99, size = 18, normalized size = 0.82

$$-\frac{\left(-bx^4+a\right)^{\frac{3}{4}}}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-b*x^4+a)^(1/4),x, algorithm="maxima")
[Out] -1/3*(-b*x^4 + a)^(3/4)/(a*x^3)
```

mupad [B] time = 1.09, size = 18, normalized size = 0.82

$$-\frac{\left(a-b\,x^4\right)^{3/4}}{3\,a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a - b*x^4)^(1/4)),x)
[Out] -(a - b*x^4)^(3/4)/(3*a*x^3)
```

sympy [A] time = 1.55, size = 78, normalized size = 3.55

$$\begin{cases} \frac{b^{\frac{3}{4}}\left(\frac{a}{bx^4}-1\right)^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{4a\Gamma\left(\frac{1}{4}\right)} & \text{for } \left|\frac{a}{bx^4}\right| > 1 \\ -\frac{b^{\frac{3}{4}}\left(-\frac{a}{bx^4}+1\right)^{\frac{3}{4}}e^{-\frac{i\pi}{4}}\Gamma\left(-\frac{3}{4}\right)}{4a\Gamma\left(\frac{1}{4}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-b*x**4+a)**(1/4),x)
[Out] Piecewise((b**(3/4)*(a/(b*x**4) - 1)**(3/4)*gamma(-3/4)/(4*a*gamma(1/4)), A
bs(a/(b*x**4)) > 1), (-b**(3/4)*(-a/(b*x**4) + 1)**(3/4)*exp(-I*pi/4)*gamma
(-3/4)/(4*a*gamma(1/4)), True))
```

3.1223 $\int \frac{1}{x^8 \sqrt[4]{a-bx^4}}\,dx$

Optimal. Leaf size=46

$$-\frac{4b\left(a-bx^4\right)^{3/4}}{21a^2x^3}-\frac{\left(a-bx^4\right)^{3/4}}{7ax^7}$$

```
[Out] -1/7*(-b*x^4+a)^(3/4)/a/x^7-4/21*b*(-b*x^4+a)^(3/4)/a^2/x^3
```

Rubi [A] time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {271, 264}

$$-\frac{4b\left(a-bx^4\right)^{3/4}}{21a^2x^3}-\frac{\left(a-bx^4\right)^{3/4}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(a - b*x^4)^(1/4)),x]
```

```
[Out] -(a - b*x^4)^(3/4)/(7*a*x^7) - (4*b*(a - b*x^4)^(3/4))/(21*a^2*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^8 \sqrt[4]{a-bx^4}}\,dx &= -\frac{\left(a-bx^4\right)^{3/4}}{7ax^7}+\frac{(4b)\int \frac{1}{x^4 \sqrt[4]{a-bx^4}}\,dx}{7a}\\ &= -\frac{\left(a-bx^4\right)^{3/4}}{7ax^7}-\frac{4b\left(a-bx^4\right)^{3/4}}{21a^2x^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 0.70

$$-\frac{\left(a-bx^4\right)^{3/4}\left(3a+4bx^4\right)}{21a^2x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(a - b*x^4)^(1/4)),x]
```

```
[Out] -1/21*((a - b*x^4)^(3/4)*(3*a + 4*b*x^4))/(a^2*x^7)
```

fricas [A] time = 0.65, size = 28, normalized size = 0.61

$$-\frac{\left(4\,bx^4+3\,a\right)\left(-bx^4+a\right)^{\frac{3}{4}}}{21\,a^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-b*x^4+a)^(1/4),x, algorithm="fricas")
[Out] -1/21*(4*b*x^4 + 3*a)*(-b*x^4 + a)^(3/4)/(a^2*x^7)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{1}{4}}x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-b*x^4+a)^(1/4),x, algorithm="giac")
[Out] integrate(1/((-b*x^4 + a)^(1/4)*x^8), x)
```

maple [A] time = 0.00, size = 29, normalized size = 0.63

$$-\frac{\left(-b\,x^4+a\right)^{\frac{3}{4}}\left(4b\,x^4+3a\right)}{21a^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(-b*x^4+a)^(1/4),x)
[Out] -1/21*(-b*x^4+a)^(3/4)*(4*b*x^4+3*a)/a^2/x^7
```

maxima [A] time = 0.99, size = 37, normalized size = 0.80

$$-\frac{\frac{7\left(-bx^4+a\right)^{\frac{3}{4}}b}{x^3}+\frac{3\left(-bx^4+a\right)^{\frac{7}{4}}}{x^7}}{21\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-b*x^4+a)^(1/4),x, algorithm="maxima")
[Out] -1/21*(7*(-b*x^4 + a)^(3/4)*b/x^3 + 3*(-b*x^4 + a)^(7/4)/x^7)/a^2
```

mupad [B] time = 1.17, size = 38, normalized size = 0.83

$$-\frac{3\,a\left(a-b\,x^4\right)^{3/4}+4\,b\,x^4\left(a-b\,x^4\right)^{3/4}}{21\,a^2\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(a - b*x^4)^(1/4)),x)
[Out] -(3*a*(a - b*x^4)^(3/4) + 4*b*x^4*(a - b*x^4)^(3/4))/(21*a^2*x^7)
```

sympy [A] time = 2.00, size = 303, normalized size = 6.59

$$\begin{cases} -\frac{3b^{\frac{3}{4}}\left(\frac{a}{bx^4}-1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{16ax^4\Gamma\left(\frac{1}{4}\right)}-\frac{b^{\frac{7}{4}}\left(\frac{a}{bx^4}-1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{4a^2\Gamma\left(\frac{1}{4}\right)} & \text{for } \left|\frac{a}{bx^4}\right|>1 \\ -\frac{3a^2b^{\frac{7}{4}}\left(-\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{-16a^3bx^4e^{\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right)+16a^2b^2x^8e^{\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right)}-\frac{ab^{\frac{11}{4}}x^4\left(-\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{-16a^3bx^4e^{\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right)+16a^2b^2x^8e^{\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right)}+\frac{4b^{\frac{15}{4}}x^8\left(-\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{-16a^3bx^4e^{\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right)+16a^2b^2x^8e^{\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(-b*x**4+a)**(1/4),x)
```

```
[Out] Piecewise((-3*b**(3/4)*(a/(b*x**4) - 1)**(3/4)*gamma(-7/4)/(16*a*x**4*gamma
(1/4)) - b**(7/4)*(a/(b*x**4) - 1)**(3/4)*gamma(-7/4)/(4*a**2*gamma(1/4)),
Abs(a/(b*x**4)) > 1), (-3*a**2*b**(7/4)*(-a/(b*x**4) + 1)**(3/4)*gamma(-7/4
)/(-16*a**3*b*x**4*exp(I*pi/4)*gamma(1/4) + 16*a**2*b**2*x**8*exp(I*pi/4)*g
amma(1/4)) - a*b**(11/4)*x**4*(-a/(b*x**4) + 1)**(3/4)*gamma(-7/4)/(-16*a**
3*b*x**4*exp(I*pi/4)*gamma(1/4) + 16*a**2*b**2*x**8*exp(I*pi/4)*gamma(1/4))
 + 4*b**(15/4)*x**8*(-a/(b*x**4) + 1)**(3/4)*gamma(-7/4)/(-16*a**3*b*x**4*e
xp(I*pi/4)*gamma(1/4) + 16*a**2*b**2*x**8*exp(I*pi/4)*gamma(1/4)), True))
```

3.1224 $\int \frac{1}{x^{12}\sqrt[4]{a-bx^4}}\,dx$

Optimal. Leaf size=71

$$-\frac{32b^2\left(a-bx^4\right)^{3/4}}{231a^3x^3}-\frac{8b\left(a-bx^4\right)^{3/4}}{77a^2x^7}-\frac{\left(a-bx^4\right)^{3/4}}{11ax^{11}}$$

```
[Out] -1/11*(-b*x^4+a)^(3/4)/a/x^11-8/77*b*(-b*x^4+a)^(3/4)/a^2/x^7-32/231*b^2*(-
b*x^4+a)^(3/4)/a^3/x^3
```

Rubi [A] time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {271, 264}

$$-\frac{32b^2\left(a-bx^4\right)^{3/4}}{231a^3x^3}-\frac{8b\left(a-bx^4\right)^{3/4}}{77a^2x^7}-\frac{\left(a-bx^4\right)^{3/4}}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^12*(a - b*x^4)^(1/4)),x]
```

```
[Out] -(a - b*x^4)^(3/4)/(11*a*x^11) - (8*b*(a - b*x^4)^(3/4))/(77*a^2*x^7) - (32
*b^2*(a - b*x^4)^(3/4))/(231*a^3*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{12}\sqrt[4]{a-bx^4}}\,dx &= -\frac{\left(a-bx^4\right)^{3/4}}{11ax^{11}}+\frac{(8b)\int \frac{1}{x^8\sqrt[4]{a-bx^4}}\,dx}{11a}\\ &= -\frac{\left(a-bx^4\right)^{3/4}}{11ax^{11}}-\frac{8b\left(a-bx^4\right)^{3/4}}{77a^2x^7}+\frac{\left(32b^2\right)\int \frac{1}{x^4\sqrt[4]{a-bx^4}}\,dx}{77a^2}\\ &= -\frac{\left(a-bx^4\right)^{3/4}}{11ax^{11}}-\frac{8b\left(a-bx^4\right)^{3/4}}{77a^2x^7}-\frac{32b^2\left(a-bx^4\right)^{3/4}}{231a^3x^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 43, normalized size = 0.61

$$-\frac{\left(a-bx^4\right)^{3/4}\left(21a^2+24abx^4+32b^2x^8\right)}{231a^3x^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^12*(a - b*x^4)^(1/4)),x]
```

```
[Out] -1/231*((a - b*x^4)^(3/4)*(21*a^2 + 24*a*b*x^4 + 32*b^2*x^8))/(a^3*x^11)
```

fricas [A] time = 0.51, size = 39, normalized size = 0.55

$$-\frac{\left(32\,b^2x^8+24\,abx^4+21\,a^2\right)\left(-bx^4+a\right)^{\frac{3}{4}}}{231\,a^3x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^12/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/231*(32*b^2*x^8 + 24*a*b*x^4 + 21*a^2)*(-b*x^4 + a)^(3/4)/(a^3*x^11)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{1}{4}}x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^12/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^4 + a)^(1/4)*x^12), x)
```

maple [A] time = 0.00, size = 40, normalized size = 0.56

$$-\frac{\left(-b\,x^4+a\right)^{\frac{3}{4}}\left(32b^2x^8+24ab\,x^4+21a^2\right)}{231a^3x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^12/(-b*x^4+a)^(1/4),x)
```

```
[Out] -1/231*(-b*x^4+a)^(3/4)*(32*b^2*x^8+24*a*b*x^4+21*a^2)/a^3/x^11
```

maxima [A] time = 1.07, size = 55, normalized size = 0.77

$$-\frac{\frac{77\left(-bx^4+a\right)^{\frac{3}{4}}b^2}{x^3}+\frac{66\left(-bx^4+a\right)^{\frac{7}{4}}b}{x^7}+\frac{21\left(-bx^4+a\right)^{\frac{11}{4}}}{x^{11}}}{231\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^12/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] -1/231*(77*(-b*x^4 + a)^(3/4)*b^2/x^3 + 66*(-b*x^4 + a)^(7/4)*b/x^7 + 21*(-
b*x^4 + a)^(11/4)/x^11)/a^3
```

mupad [B] time = 1.26, size = 59, normalized size = 0.83

$$-\frac{21\,a^2\left(a-b\,x^4\right)^{3/4}+32\,b^2\,x^8\left(a-b\,x^4\right)^{3/4}+24\,a\,b\,x^4\left(a-b\,x^4\right)^{3/4}}{231\,a^3\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^12*(a - b*x^4)^(1/4)),x)
```

```
[Out] -(21*a^2*(a - b*x^4)^(3/4) + 32*b^2*x^8*(a - b*x^4)^(3/4) + 24*a*b*x^4*(a -
 b*x^4)^(3/4))/(231*a^3*x^11)
```

sympy [C] time = 3.66, size = 1068, normalized size = 15.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**12/(-b*x**4+a)**(1/4),x)
```

```
[Out] Piecewise((-21*a**4*b**(19/4)*(a/(b*x**4) - 1)**(3/4)*exp(-3*I*pi/4)*gamma(
-11/4)/(64*a**5*b**4*x**8*exp(I*pi/4)*gamma(1/4) - 128*a**4*b**5*x**12*exp(
I*pi/4)*gamma(1/4) + 64*a**3*b**6*x**16*exp(I*pi/4)*gamma(1/4)) + 18*a**3*b
**(23/4)*x**4*(a/(b*x**4) - 1)**(3/4)*exp(-3*I*pi/4)*gamma(-11/4)/(64*a**5*
b**4*x**8*exp(I*pi/4)*gamma(1/4) - 128*a**4*b**5*x**12*exp(I*pi/4)*gamma(1/
4) + 64*a**3*b**6*x**16*exp(I*pi/4)*gamma(1/4)) - 5*a**2*b**(27/4)*x**8*(a/
(b*x**4) - 1)**(3/4)*exp(-3*I*pi/4)*gamma(-11/4)/(64*a**5*b**4*x**8*exp(I*p
i/4)*gamma(1/4) - 128*a**4*b**5*x**12*exp(I*pi/4)*gamma(1/4) + 64*a**3*b**6
*x**16*exp(I*pi/4)*gamma(1/4)) + 40*a*b**(31/4)*x**12*(a/(b*x**4) - 1)**(3/
4)*exp(-3*I*pi/4)*gamma(-11/4)/(64*a**5*b**4*x**8*exp(I*pi/4)*gamma(1/4) -
128*a**4*b**5*x**12*exp(I*pi/4)*gamma(1/4) + 64*a**3*b**6*x**16*exp(I*pi/4)
*gamma(1/4)) - 32*b**(35/4)*x**16*(a/(b*x**4) - 1)**(3/4)*exp(-3*I*pi/4)*ga
mma(-11/4)/(64*a**5*b**4*x**8*exp(I*pi/4)*gamma(1/4) - 128*a**4*b**5*x**12*
exp(I*pi/4)*gamma(1/4) + 64*a**3*b**6*x**16*exp(I*pi/4)*gamma(1/4)), Abs(a/
(b*x**4)) > 1), (-21*a**4*b**(19/4)*(-a/(b*x**4) + 1)**(3/4)*gamma(-11/4)/(
64*a**5*b**4*x**8*exp(I*pi/4)*gamma(1/4) - 128*a**4*b**5*x**12*exp(I*pi/4)*
gamma(1/4) + 64*a**3*b**6*x**16*exp(I*pi/4)*gamma(1/4)) + 18*a**3*b**(23/4)
*x**4*(-a/(b*x**4) + 1)**(3/4)*gamma(-11/4)/(64*a**5*b**4*x**8*exp(I*pi/4)*
gamma(1/4) - 128*a**4*b**5*x**12*exp(I*pi/4)*gamma(1/4) + 64*a**3*b**6*x**1
6*exp(I*pi/4)*gamma(1/4)) - 5*a**2*b**(27/4)*x**8*(-a/(b*x**4) + 1)**(3/4)*
gamma(-11/4)/(64*a**5*b**4*x**8*exp(I*pi/4)*gamma(1/4) - 128*a**4*b**5*x**1
2*exp(I*pi/4)*gamma(1/4) + 64*a**3*b**6*x**16*exp(I*pi/4)*gamma(1/4)) + 40*
a*b**(31/4)*x**12*(-a/(b*x**4) + 1)**(3/4)*gamma(-11/4)/(64*a**5*b**4*x**8*
exp(I*pi/4)*gamma(1/4) - 128*a**4*b**5*x**12*exp(I*pi/4)*gamma(1/4) + 64*a*
*3*b**6*x**16*exp(I*pi/4)*gamma(1/4)) - 32*b**(35/4)*x**16*(-a/(b*x**4) + 1
)**(3/4)*gamma(-11/4)/(64*a**5*b**4*x**8*exp(I*pi/4)*gamma(1/4) - 128*a**4*
b**5*x**12*exp(I*pi/4)*gamma(1/4) + 64*a**3*b**6*x**16*exp(I*pi/4)*gamma(1/
4)), True))
```

3.1225 $\int \frac{1}{x^{16}\sqrt[4]{a-bx^4}}\,dx$

Optimal. Leaf size=96

$$-\frac{128b^3\left(a-bx^4\right)^{3/4}}{1155a^4x^3}-\frac{32b^2\left(a-bx^4\right)^{3/4}}{385a^3x^7}-\frac{4b\left(a-bx^4\right)^{3/4}}{55a^2x^{11}}-\frac{\left(a-bx^4\right)^{3/4}}{15ax^{15}}$$

```
[Out] -1/15*(-b*x^4+a)^(3/4)/a/x^15-4/55*b*(-b*x^4+a)^(3/4)/a^2/x^11-32/385*b^2*(
-b*x^4+a)^(3/4)/a^3/x^7-128/1155*b^3*(-b*x^4+a)^(3/4)/a^4/x^3
```

Rubi [A] time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {271, 264}

$$-\frac{128b^3\left(a-bx^4\right)^{3/4}}{1155a^4x^3}-\frac{32b^2\left(a-bx^4\right)^{3/4}}{385a^3x^7}-\frac{4b\left(a-bx^4\right)^{3/4}}{55a^2x^{11}}-\frac{\left(a-bx^4\right)^{3/4}}{15ax^{15}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^16*(a - b*x^4)^(1/4)),x]
```

```
[Out] -(a - b*x^4)^(3/4)/(15*a*x^15) - (4*b*(a - b*x^4)^(3/4))/(55*a^2*x^11) - (3
2*b^2*(a - b*x^4)^(3/4))/(385*a^3*x^7) - (128*b^3*(a - b*x^4)^(3/4))/(1155*
a^4*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{16}\sqrt[4]{a-bx^4}}\,dx &= -\frac{\left(a-bx^4\right)^{3/4}}{15ax^{15}}+\frac{(4b)\int \frac{1}{x^{12}\sqrt[4]{a-bx^4}}\,dx}{5a}\\
&= -\frac{\left(a-bx^4\right)^{3/4}}{15ax^{15}}-\frac{4b\left(a-bx^4\right)^{3/4}}{55a^2x^{11}}+\frac{\left(32b^2\right)\int \frac{1}{x^8\sqrt[4]{a-bx^4}}\,dx}{55a^2}\\
&= -\frac{\left(a-bx^4\right)^{3/4}}{15ax^{15}}-\frac{4b\left(a-bx^4\right)^{3/4}}{55a^2x^{11}}-\frac{32b^2\left(a-bx^4\right)^{3/4}}{385a^3x^7}+\frac{\left(128b^3\right)\int \frac{1}{x^4\sqrt[4]{a-bx^4}}\,dx}{385a^3}\\
&= -\frac{\left(a-bx^4\right)^{3/4}}{15ax^{15}}-\frac{4b\left(a-bx^4\right)^{3/4}}{55a^2x^{11}}-\frac{32b^2\left(a-bx^4\right)^{3/4}}{385a^3x^7}-\frac{128b^3\left(a-bx^4\right)^{3/4}}{1155a^4x^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 0.56

$$-\frac{\left(a-bx^4\right)^{3/4}\left(77a^3+84a^2bx^4+96ab^2x^8+128b^3x^{12}\right)}{1155a^4x^{15}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^16*(a - b*x^4)^(1/4)),x]
```

```
[Out] -1/1155*((a - b*x^4)^(3/4)*(77*a^3 + 84*a^2*b*x^4 + 96*a*b^2*x^8 + 128*b^3*
x^12))/(a^4*x^15)
```

fricas [A] time = 0.45, size = 50, normalized size = 0.52

$$-\frac{\left(128\,b^3x^{12}+96\,ab^2x^8+84\,a^2bx^4+77\,a^3\right)\left(-bx^4+a\right)^{\frac{3}{4}}}{1155\,a^4x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^16/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/1155*(128*b^3*x^12 + 96*a*b^2*x^8 + 84*a^2*b*x^4 + 77*a^3)*(-b*x^4 + a)^
(3/4)/(a^4*x^15)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{1}{4}}x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^16/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^4 + a)^(1/4)*x^16), x)
```

maple [A] time = 0.01, size = 51, normalized size = 0.53

$$-\frac{\left(-b\,x^4+a\right)^{\frac{3}{4}}\left(128b^3x^{12}+96a\,b^2x^8+84a^2b\,x^4+77a^3\right)}{1155a^4x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^16/(-b*x^4+a)^(1/4),x)
```

```
[Out] -1/1155*(-b*x^4+a)^(3/4)*(128*b^3*x^12+96*a*b^2*x^8+84*a^2*b*x^4+77*a^3)/x^
15/a^4
```

maxima [A] time = 0.97, size = 73, normalized size = 0.76

$$-\frac{\frac{385\left(-bx^4+a\right)^{\frac{3}{4}}b^3}{x^3}+\frac{495\left(-bx^4+a\right)^{\frac{7}{4}}b^2}{x^7}+\frac{315\left(-bx^4+a\right)^{\frac{11}{4}}b}{x^{11}}+\frac{77\left(-bx^4+a\right)^{\frac{15}{4}}}{x^{15}}}{1155\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^16/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] -1/1155*(385*(-b*x^4 + a)^(3/4)*b^3/x^3 + 495*(-b*x^4 + a)^(7/4)*b^2/x^7 +
315*(-b*x^4 + a)^(11/4)*b/x^11 + 77*(-b*x^4 + a)^(15/4)/x^15)/a^4
```

mupad [B] time = 1.36, size = 80, normalized size = 0.83

$$-\frac{\left(a-b\,x^4\right)^{3/4}}{15\,a\,x^{15}}-\frac{4\,b\left(a-b\,x^4\right)^{3/4}}{55\,a^2\,x^{11}}-\frac{128\,b^3\left(a-b\,x^4\right)^{3/4}}{1155\,a^4\,x^3}-\frac{32\,b^2\left(a-b\,x^4\right)^{3/4}}{385\,a^3\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^16*(a - b*x^4)^(1/4)),x)
```

```
[Out] - (a - b*x^4)^(3/4)/(15*a*x^15) - (4*b*(a - b*x^4)^(3/4))/(55*a^2*x^11) - (
128*b^3*(a - b*x^4)^(3/4))/(1155*a^4*x^3) - (32*b^2*(a - b*x^4)^(3/4))/(385
*a^3*x^7)
```

sympy [C] time = 6.84, size = 1821, normalized size = 18.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**16/(-b*x**4+a)**(1/4),x)
```

```
[Out] Piecewise((231*a**6*b**(39/4)*(a/(b*x**4) - 1)**(3/4)*exp(I*pi/4)*gamma(-15
/4)/(-256*a**7*b**9*x**12*exp(I*pi/4)*gamma(1/4) + 768*a**6*b**10*x**16*exp
(I*pi/4)*gamma(1/4) - 768*a**5*b**11*x**20*exp(I*pi/4)*gamma(1/4) + 256*a**
4*b**12*x**24*exp(I*pi/4)*gamma(1/4)) - 441*a**5*b**(43/4)*x**4*(a/(b*x**4)
 - 1)**(3/4)*exp(I*pi/4)*gamma(-15/4)/(-256*a**7*b**9*x**12*exp(I*pi/4)*gam
ma(1/4) + 768*a**6*b**10*x**16*exp(I*pi/4)*gamma(1/4) - 768*a**5*b**11*x**2
0*exp(I*pi/4)*gamma(1/4) + 256*a**4*b**12*x**24*exp(I*pi/4)*gamma(1/4)) + 2
25*a**4*b**(47/4)*x**8*(a/(b*x**4) - 1)**(3/4)*exp(I*pi/4)*gamma(-15/4)/(-2
56*a**7*b**9*x**12*exp(I*pi/4)*gamma(1/4) + 768*a**6*b**10*x**16*exp(I*pi/4
)*gamma(1/4) - 768*a**5*b**11*x**20*exp(I*pi/4)*gamma(1/4) + 256*a**4*b**12
*x**24*exp(I*pi/4)*gamma(1/4)) + 45*a**3*b**(51/4)*x**12*(a/(b*x**4) - 1)**
(3/4)*exp(I*pi/4)*gamma(-15/4)/(-256*a**7*b**9*x**12*exp(I*pi/4)*gamma(1/4)
 + 768*a**6*b**10*x**16*exp(I*pi/4)*gamma(1/4) - 768*a**5*b**11*x**20*exp(I
*pi/4)*gamma(1/4) + 256*a**4*b**12*x**24*exp(I*pi/4)*gamma(1/4)) - 540*a**2
*b**(55/4)*x**16*(a/(b*x**4) - 1)**(3/4)*exp(I*pi/4)*gamma(-15/4)/(-256*a**
7*b**9*x**12*exp(I*pi/4)*gamma(1/4) + 768*a**6*b**10*x**16*exp(I*pi/4)*gamm
a(1/4) - 768*a**5*b**11*x**20*exp(I*pi/4)*gamma(1/4) + 256*a**4*b**12*x**24
*exp(I*pi/4)*gamma(1/4)) + 864*a*b**(59/4)*x**20*(a/(b*x**4) - 1)**(3/4)*ex
p(I*pi/4)*gamma(-15/4)/(-256*a**7*b**9*x**12*exp(I*pi/4)*gamma(1/4) + 768*a
**6*b**10*x**16*exp(I*pi/4)*gamma(1/4) - 768*a**5*b**11*x**20*exp(I*pi/4)*g
amma(1/4) + 256*a**4*b**12*x**24*exp(I*pi/4)*gamma(1/4)) - 384*b**(63/4)*x*
*24*(a/(b*x**4) - 1)**(3/4)*exp(I*pi/4)*gamma(-15/4)/(-256*a**7*b**9*x**12*
exp(I*pi/4)*gamma(1/4) + 768*a**6*b**10*x**16*exp(I*pi/4)*gamma(1/4) - 768*
a**5*b**11*x**20*exp(I*pi/4)*gamma(1/4) + 256*a**4*b**12*x**24*exp(I*pi/4)*
gamma(1/4)), Abs(a/(b*x**4)) > 1), (-231*a**6*b**(39/4)*(-a/(b*x**4) + 1)**
(3/4)*gamma(-15/4)/(-256*a**7*b**9*x**12*exp(I*pi/4)*gamma(1/4) + 768*a**6*
b**10*x**16*exp(I*pi/4)*gamma(1/4) - 768*a**5*b**11*x**20*exp(I*pi/4)*gamma
(1/4) + 256*a**4*b**12*x**24*exp(I*pi/4)*gamma(1/4)) + 441*a**5*b**(43/4)*x
**4*(-a/(b*x**4) + 1)**(3/4)*gamma(-15/4)/(-256*a**7*b**9*x**12*exp(I*pi/4)
*gamma(1/4) + 768*a**6*b**10*x**16*exp(I*pi/4)*gamma(1/4) - 768*a**5*b**11*
x**20*exp(I*pi/4)*gamma(1/4) + 256*a**4*b**12*x**24*exp(I*pi/4)*gamma(1/4))
 - 225*a**4*b**(47/4)*x**8*(-a/(b*x**4) + 1)**(3/4)*gamma(-15/4)/(-256*a**7
*b**9*x**12*exp(I*pi/4)*gamma(1/4) + 768*a**6*b**10*x**16*exp(I*pi/4)*gamma
(1/4) - 768*a**5*b**11*x**20*exp(I*pi/4)*gamma(1/4) + 256*a**4*b**12*x**24*
exp(I*pi/4)*gamma(1/4)) - 45*a**3*b**(51/4)*x**12*(-a/(b*x**4) + 1)**(3/4)*
gamma(-15/4)/(-256*a**7*b**9*x**12*exp(I*pi/4)*gamma(1/4) + 768*a**6*b**10*
x**16*exp(I*pi/4)*gamma(1/4) - 768*a**5*b**11*x**20*exp(I*pi/4)*gamma(1/4)
+ 256*a**4*b**12*x**24*exp(I*pi/4)*gamma(1/4)) + 540*a**2*b**(55/4)*x**16*(
-a/(b*x**4) + 1)**(3/4)*gamma(-15/4)/(-256*a**7*b**9*x**12*exp(I*pi/4)*gamm
a(1/4) + 768*a**6*b**10*x**16*exp(I*pi/4)*gamma(1/4) - 768*a**5*b**11*x**20
*exp(I*pi/4)*gamma(1/4) + 256*a**4*b**12*x**24*exp(I*pi/4)*gamma(1/4)) - 86
4*a*b**(59/4)*x**20*(-a/(b*x**4) + 1)**(3/4)*gamma(-15/4)/(-256*a**7*b**9*x
**12*exp(I*pi/4)*gamma(1/4) + 768*a**6*b**10*x**16*exp(I*pi/4)*gamma(1/4) -
 768*a**5*b**11*x**20*exp(I*pi/4)*gamma(1/4) + 256*a**4*b**12*x**24*exp(I*p
i/4)*gamma(1/4)) + 384*b**(63/4)*x**24*(-a/(b*x**4) + 1)**(3/4)*gamma(-15/4
)/(-256*a**7*b**9*x**12*exp(I*pi/4)*gamma(1/4) + 768*a**6*b**10*x**16*exp(I
*pi/4)*gamma(1/4) - 768*a**5*b**11*x**20*exp(I*pi/4)*gamma(1/4) + 256*a**4*
b**12*x**24*exp(I*pi/4)*gamma(1/4)), True))
```

3.1226 $\int \frac{1}{x^{20}\sqrt[4]{a-bx^4}}\,dx$

Optimal. Leaf size=121

$$-\frac{2048b^4\left(a-bx^4\right)^{3/4}}{21945a^5x^3}-\frac{512b^3\left(a-bx^4\right)^{3/4}}{7315a^4x^7}-\frac{64b^2\left(a-bx^4\right)^{3/4}}{1045a^3x^{11}}-\frac{16b\left(a-bx^4\right)^{3/4}}{285a^2x^{15}}-\frac{\left(a-bx^4\right)^{3/4}}{19ax^{19}}$$

```
[Out] -1/19*(-b*x^4+a)^(3/4)/a/x^19-16/285*b*(-b*x^4+a)^(3/4)/a^2/x^15-64/1045*b^
2*(-b*x^4+a)^(3/4)/a^3/x^11-512/7315*b^3*(-b*x^4+a)^(3/4)/a^4/x^7-2048/2194
5*b^4*(-b*x^4+a)^(3/4)/a^5/x^3
```

Rubi [A] time = 0.04, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {271, 264}

$$-\frac{2048b^4\left(a-bx^4\right)^{3/4}}{21945a^5x^3}-\frac{512b^3\left(a-bx^4\right)^{3/4}}{7315a^4x^7}-\frac{64b^2\left(a-bx^4\right)^{3/4}}{1045a^3x^{11}}-\frac{16b\left(a-bx^4\right)^{3/4}}{285a^2x^{15}}-\frac{\left(a-bx^4\right)^{3/4}}{19ax^{19}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^20*(a - b*x^4)^(1/4)),x]
```

```
[Out] -(a - b*x^4)^(3/4)/(19*a*x^19) - (16*b*(a - b*x^4)^(3/4))/(285*a^2*x^15) -
(64*b^2*(a - b*x^4)^(3/4))/(1045*a^3*x^11) - (512*b^3*(a - b*x^4)^(3/4))/(7
315*a^4*x^7) - (2048*b^4*(a - b*x^4)^(3/4))/(21945*a^5*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{20}\sqrt[4]{a-bx^4}}\,dx &= -\frac{\left(a-bx^4\right)^{3/4}}{19ax^{19}}+\frac{(16b)\int \frac{1}{x^{16}\sqrt[4]{a-bx^4}}\,dx}{19a}\\
&= -\frac{\left(a-bx^4\right)^{3/4}}{19ax^{19}}-\frac{16b\left(a-bx^4\right)^{3/4}}{285a^2x^{15}}+\frac{\left(64b^2\right)\int \frac{1}{x^{12}\sqrt[4]{a-bx^4}}\,dx}{95a^2}\\
&= -\frac{\left(a-bx^4\right)^{3/4}}{19ax^{19}}-\frac{16b\left(a-bx^4\right)^{3/4}}{285a^2x^{15}}-\frac{64b^2\left(a-bx^4\right)^{3/4}}{1045a^3x^{11}}+\frac{\left(512b^3\right)\int \frac{1}{x^8\sqrt[4]{a-bx^4}}\,dx}{1045a^3}\\
&= -\frac{\left(a-bx^4\right)^{3/4}}{19ax^{19}}-\frac{16b\left(a-bx^4\right)^{3/4}}{285a^2x^{15}}-\frac{64b^2\left(a-bx^4\right)^{3/4}}{1045a^3x^{11}}-\frac{512b^3\left(a-bx^4\right)^{3/4}}{7315a^4x^7}+\frac{\left(2048b^4\right)\int \frac{}{x^4\sqrt[4]{}}}{7315a^{}}\\
&= -\frac{\left(a-bx^4\right)^{3/4}}{19ax^{19}}-\frac{16b\left(a-bx^4\right)^{3/4}}{285a^2x^{15}}-\frac{64b^2\left(a-bx^4\right)^{3/4}}{1045a^3x^{11}}-\frac{512b^3\left(a-bx^4\right)^{3/4}}{7315a^4x^7}-\frac{2048b^4\left(a-bx\right.}{21945a^5x^3}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 65, normalized size = 0.54

$$-\frac{\left(a-bx^4\right)^{3/4}\left(1155a^4+1232a^3bx^4+1344a^2b^2x^8+1536ab^3x^{12}+2048b^4x^{16}\right)}{21945a^5x^{19}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^20*(a - b*x^4)^(1/4)),x]
```

```
[Out] -1/21945*((a - b*x^4)^(3/4)*(1155*a^4 + 1232*a^3*b*x^4 + 1344*a^2*b^2*x^8 +
 1536*a*b^3*x^12 + 2048*b^4*x^16))/(a^5*x^19)
```

fricas [A] time = 0.59, size = 61, normalized size = 0.50

$$-\frac{\left(2048\,b^4x^{16}+1536\,ab^3x^{12}+1344\,a^2b^2x^8+1232\,a^3bx^4+1155\,a^4\right)\left(-bx^4+a\right)^{\frac{3}{4}}}{21945\,a^5x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^20/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/21945*(2048*b^4*x^16 + 1536*a*b^3*x^12 + 1344*a^2*b^2*x^8 + 1232*a^3*b*x
^4 + 1155*a^4)*(-b*x^4 + a)^(3/4)/(a^5*x^19)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{1}{4}}x^{20}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^20/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^4 + a)^(1/4)*x^20), x)
```

maple [A] time = 0.01, size = 62, normalized size = 0.51

$$-\frac{\left(-b\,x^4+a\right)^{\frac{3}{4}}\left(2048x^{16}b^4+1536a\,x^{12}b^3+1344a^2x^8b^2+1232a^3x^4b+1155a^4\right)}{21945a^5x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^20/(-b*x^4+a)^(1/4),x)
```

```
[Out] -1/21945*(-b*x^4+a)^(3/4)*(2048*b^4*x^16+1536*a*b^3*x^12+1344*a^2*b^2*x^8+1
232*a^3*b*x^4+1155*a^4)/x^19/a^5
```

maxima [A] time = 1.07, size = 91, normalized size = 0.75

$$-\frac{\frac{7315\left(-bx^4+a\right)^{\frac{3}{4}}b^4}{x^3}+\frac{12540\left(-bx^4+a\right)^{\frac{7}{4}}b^3}{x^7}+\frac{11970\left(-bx^4+a\right)^{\frac{11}{4}}b^2}{x^{11}}+\frac{5852\left(-bx^4+a\right)^{\frac{15}{4}}b}{x^{15}}+\frac{1155\left(-bx^4+a\right)^{\frac{19}{4}}}{x^{19}}}{21945\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^20/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] -1/21945*(7315*(-b*x^4 + a)^(3/4)*b^4/x^3 + 12540*(-b*x^4 + a)^(7/4)*b^3/x^
7 + 11970*(-b*x^4 + a)^(11/4)*b^2/x^11 + 5852*(-b*x^4 + a)^(15/4)*b/x^15 +
1155*(-b*x^4 + a)^(19/4)/x^19)/a^5
```

mupad [B] time = 1.44, size = 101, normalized size = 0.83

$$-\frac{\left(a-b\,x^4\right)^{3/4}}{19\,a\,x^{19}}-\frac{16\,b\left(a-b\,x^4\right)^{3/4}}{285\,a^2\,x^{15}}-\frac{2048\,b^4\left(a-b\,x^4\right)^{3/4}}{21945\,a^5\,x^3}-\frac{512\,b^3\left(a-b\,x^4\right)^{3/4}}{7315\,a^4\,x^7}-\frac{64\,b^2\left(a-b\,x^4\right)^{3/4}}{1045\,a^3\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^20*(a - b*x^4)^(1/4)),x)
```

```
[Out] - (a - b*x^4)^(3/4)/(19*a*x^19) - (16*b*(a - b*x^4)^(3/4))/(285*a^2*x^15) -
 (2048*b^4*(a - b*x^4)^(3/4))/(21945*a^5*x^3) - (512*b^3*(a - b*x^4)^(3/4))
/(7315*a^4*x^7) - (64*b^2*(a - b*x^4)^(3/4))/(1045*a^3*x^11)
```

sympy [C] time = 9.88, size = 2788, normalized size = 23.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**20/(-b*x**4+a)**(1/4),x)
```

```
[Out] Piecewise((-3465*a**8*b**(67/4)*(a/(b*x**4) - 1)**(3/4)*exp(-3*I*pi/4)*gamm
a(-19/4)/(1024*a**9*b**16*x**16*exp(I*pi/4)*gamma(1/4) - 4096*a**8*b**17*x*
*20*exp(I*pi/4)*gamma(1/4) + 6144*a**7*b**18*x**24*exp(I*pi/4)*gamma(1/4) -
 4096*a**6*b**19*x**28*exp(I*pi/4)*gamma(1/4) + 1024*a**5*b**20*x**32*exp(I
*pi/4)*gamma(1/4)) + 10164*a**7*b**(71/4)*x**4*(a/(b*x**4) - 1)**(3/4)*exp(
-3*I*pi/4)*gamma(-19/4)/(1024*a**9*b**16*x**16*exp(I*pi/4)*gamma(1/4) - 409
6*a**8*b**17*x**20*exp(I*pi/4)*gamma(1/4) + 6144*a**7*b**18*x**24*exp(I*pi/
4)*gamma(1/4) - 4096*a**6*b**19*x**28*exp(I*pi/4)*gamma(1/4) + 1024*a**5*b*
*20*x**32*exp(I*pi/4)*gamma(1/4)) - 10038*a**6*b**(75/4)*x**8*(a/(b*x**4) -
 1)**(3/4)*exp(-3*I*pi/4)*gamma(-19/4)/(1024*a**9*b**16*x**16*exp(I*pi/4)*g
amma(1/4) - 4096*a**8*b**17*x**20*exp(I*pi/4)*gamma(1/4) + 6144*a**7*b**18*
x**24*exp(I*pi/4)*gamma(1/4) - 4096*a**6*b**19*x**28*exp(I*pi/4)*gamma(1/4)
 + 1024*a**5*b**20*x**32*exp(I*pi/4)*gamma(1/4)) + 3204*a**5*b**(79/4)*x**1
2*(a/(b*x**4) - 1)**(3/4)*exp(-3*I*pi/4)*gamma(-19/4)/(1024*a**9*b**16*x**1
6*exp(I*pi/4)*gamma(1/4) - 4096*a**8*b**17*x**20*exp(I*pi/4)*gamma(1/4) + 6
144*a**7*b**18*x**24*exp(I*pi/4)*gamma(1/4) - 4096*a**6*b**19*x**28*exp(I*p
i/4)*gamma(1/4) + 1024*a**5*b**20*x**32*exp(I*pi/4)*gamma(1/4)) - 585*a**4*
b**(83/4)*x**16*(a/(b*x**4) - 1)**(3/4)*exp(-3*I*pi/4)*gamma(-19/4)/(1024*a
**9*b**16*x**16*exp(I*pi/4)*gamma(1/4) - 4096*a**8*b**17*x**20*exp(I*pi/4)*
gamma(1/4) + 6144*a**7*b**18*x**24*exp(I*pi/4)*gamma(1/4) - 4096*a**6*b**19
*x**28*exp(I*pi/4)*gamma(1/4) + 1024*a**5*b**20*x**32*exp(I*pi/4)*gamma(1/4
)) + 9360*a**3*b**(87/4)*x**20*(a/(b*x**4) - 1)**(3/4)*exp(-3*I*pi/4)*gamma
(-19/4)/(1024*a**9*b**16*x**16*exp(I*pi/4)*gamma(1/4) - 4096*a**8*b**17*x**
20*exp(I*pi/4)*gamma(1/4) + 6144*a**7*b**18*x**24*exp(I*pi/4)*gamma(1/4) -
4096*a**6*b**19*x**28*exp(I*pi/4)*gamma(1/4) + 1024*a**5*b**20*x**32*exp(I*
pi/4)*gamma(1/4)) - 22464*a**2*b**(91/4)*x**24*(a/(b*x**4) - 1)**(3/4)*exp(
-3*I*pi/4)*gamma(-19/4)/(1024*a**9*b**16*x**16*exp(I*pi/4)*gamma(1/4) - 409
6*a**8*b**17*x**20*exp(I*pi/4)*gamma(1/4) + 6144*a**7*b**18*x**24*exp(I*pi/
4)*gamma(1/4) - 4096*a**6*b**19*x**28*exp(I*pi/4)*gamma(1/4) + 1024*a**5*b*
*20*x**32*exp(I*pi/4)*gamma(1/4)) + 19968*a*b**(95/4)*x**28*(a/(b*x**4) - 1
)**(3/4)*exp(-3*I*pi/4)*gamma(-19/4)/(1024*a**9*b**16*x**16*exp(I*pi/4)*gam
ma(1/4) - 4096*a**8*b**17*x**20*exp(I*pi/4)*gamma(1/4) + 6144*a**7*b**18*x*
*24*exp(I*pi/4)*gamma(1/4) - 4096*a**6*b**19*x**28*exp(I*pi/4)*gamma(1/4) +
 1024*a**5*b**20*x**32*exp(I*pi/4)*gamma(1/4)) - 6144*b**(99/4)*x**32*(a/(b
*x**4) - 1)**(3/4)*exp(-3*I*pi/4)*gamma(-19/4)/(1024*a**9*b**16*x**16*exp(I
*pi/4)*gamma(1/4) - 4096*a**8*b**17*x**20*exp(I*pi/4)*gamma(1/4) + 6144*a**
7*b**18*x**24*exp(I*pi/4)*gamma(1/4) - 4096*a**6*b**19*x**28*exp(I*pi/4)*ga
mma(1/4) + 1024*a**5*b**20*x**32*exp(I*pi/4)*gamma(1/4)), Abs(a/(b*x**4)) >
 1), (-3465*a**8*b**(67/4)*(-a/(b*x**4) + 1)**(3/4)*gamma(-19/4)/(1024*a**9
```

```
*b**16*x**16*exp(I*pi/4)*gamma(1/4) - 4096*a**8*b**17*x**20*exp(I*pi/4)*gam
ma(1/4) + 6144*a**7*b**18*x**24*exp(I*pi/4)*gamma(1/4) - 4096*a**6*b**19*x*
*28*exp(I*pi/4)*gamma(1/4) + 1024*a**5*b**20*x**32*exp(I*pi/4)*gamma(1/4))
+ 10164*a**7*b**(71/4)*x**4*(-a/(b*x**4) + 1)**(3/4)*gamma(-19/4)/(1024*a**
9*b**16*x**16*exp(I*pi/4)*gamma(1/4) - 4096*a**8*b**17*x**20*exp(I*pi/4)*ga
mma(1/4) + 6144*a**7*b**18*x**24*exp(I*pi/4)*gamma(1/4) - 4096*a**6*b**19*x
**28*exp(I*pi/4)*gamma(1/4) + 1024*a**5*b**20*x**32*exp(I*pi/4)*gamma(1/4))
 - 10038*a**6*b**(75/4)*x**8*(-a/(b*x**4) + 1)**(3/4)*gamma(-19/4)/(1024*a*
*9*b**16*x**16*exp(I*pi/4)*gamma(1/4) - 4096*a**8*b**17*x**20*exp(I*pi/4)*g
amma(1/4) + 6144*a**7*b**18*x**24*exp(I*pi/4)*gamma(1/4) - 4096*a**6*b**19*
x**28*exp(I*pi/4)*gamma(1/4) + 1024*a**5*b**20*x**32*exp(I*pi/4)*gamma(1/4)
) + 3204*a**5*b**(79/4)*x**12*(-a/(b*x**4) + 1)**(3/4)*gamma(-19/4)/(1024*a
**9*b**16*x**16*exp(I*pi/4)*gamma(1/4) - 4096*a**8*b**17*x**20*exp(I*pi/4)*
gamma(1/4) + 6144*a**7*b**18*x**24*exp(I*pi/4)*gamma(1/4) - 4096*a**6*b**19
*x**28*exp(I*pi/4)*gamma(1/4) + 1024*a**5*b**20*x**32*exp(I*pi/4)*gamma(1/4
)) - 585*a**4*b**(83/4)*x**16*(-a/(b*x**4) + 1)**(3/4)*gamma(-19/4)/(1024*a
**9*b**16*x**16*exp(I*pi/4)*gamma(1/4) - 4096*a**8*b**17*x**20*exp(I*pi/4)*
gamma(1/4) + 6144*a**7*b**18*x**24*exp(I*pi/4)*gamma(1/4) - 4096*a**6*b**19
*x**28*exp(I*pi/4)*gamma(1/4) + 1024*a**5*b**20*x**32*exp(I*pi/4)*gamma(1/4
)) + 9360*a**3*b**(87/4)*x**20*(-a/(b*x**4) + 1)**(3/4)*gamma(-19/4)/(1024*
a**9*b**16*x**16*exp(I*pi/4)*gamma(1/4) - 4096*a**8*b**17*x**20*exp(I*pi/4)
*gamma(1/4) + 6144*a**7*b**18*x**24*exp(I*pi/4)*gamma(1/4) - 4096*a**6*b**1
9*x**28*exp(I*pi/4)*gamma(1/4) + 1024*a**5*b**20*x**32*exp(I*pi/4)*gamma(1/
4)) - 22464*a**2*b**(91/4)*x**24*(-a/(b*x**4) + 1)**(3/4)*gamma(-19/4)/(102
4*a**9*b**16*x**16*exp(I*pi/4)*gamma(1/4) - 4096*a**8*b**17*x**20*exp(I*pi/
4)*gamma(1/4) + 6144*a**7*b**18*x**24*exp(I*pi/4)*gamma(1/4) - 4096*a**6*b*
*19*x**28*exp(I*pi/4)*gamma(1/4) + 1024*a**5*b**20*x**32*exp(I*pi/4)*gamma(
1/4)) + 19968*a*b**(95/4)*x**28*(-a/(b*x**4) + 1)**(3/4)*gamma(-19/4)/(1024
*a**9*b**16*x**16*exp(I*pi/4)*gamma(1/4) - 4096*a**8*b**17*x**20*exp(I*pi/4
)*gamma(1/4) + 6144*a**7*b**18*x**24*exp(I*pi/4)*gamma(1/4) - 4096*a**6*b**
19*x**28*exp(I*pi/4)*gamma(1/4) + 1024*a**5*b**20*x**32*exp(I*pi/4)*gamma(1
/4)) - 6144*b**(99/4)*x**32*(-a/(b*x**4) + 1)**(3/4)*gamma(-19/4)/(1024*a**
9*b**16*x**16*exp(I*pi/4)*gamma(1/4) - 4096*a**8*b**17*x**20*exp(I*pi/4)*ga
mma(1/4) + 6144*a**7*b**18*x**24*exp(I*pi/4)*gamma(1/4) - 4096*a**6*b**19*x
**28*exp(I*pi/4)*gamma(1/4) + 1024*a**5*b**20*x**32*exp(I*pi/4)*gamma(1/4))
, True))
```

3.1227 $\int \frac{x^{10}}{\sqrt[4]{a-bx^4}}\,dx$

Optimal. Leaf size=134

$$\frac{7a^{5/2}x\sqrt[4]{1-\frac{a}{bx^4}}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{40b^{5/2}\sqrt[4]{a-bx^4}}-\frac{7a^2\left(a-bx^4\right)^{3/4}}{40b^3x}-\frac{7ax^3\left(a-bx^4\right)^{3/4}}{60b^2}-\frac{x^7\left(a-bx^4\right)^{3/4}}{10b}$$

```
[Out] -7/40*a^2*(-b*x^4+a)^(3/4)/b^3/x-7/60*a*x^3*(-b*x^4+a)^(3/4)/b^2-1/10*x^7*(
-b*x^4+a)^(3/4)/b+7/40*a^(5/2)*(1-a/b/x^4)^(1/4)*x*(cos(1/2*arccsc(x^2*b^(1
/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccsc(x^2*b^(1/2)/a^(1/2)))*EllipticE(sin(1
/2*arccsc(x^2*b^(1/2)/a^(1/2))),2^(1/2))/b^(5/2)/(-b*x^4+a)^(1/4)
```

Rubi [A] time = 0.06, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {321, 311, 313, 335, 275, 228}

$$-\frac{7a^2\left(a-bx^4\right)^{3/4}}{40b^3x}+\frac{7a^{5/2}x\sqrt[4]{1-\frac{a}{bx^4}}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{40b^{5/2}\sqrt[4]{a-bx^4}}-\frac{7ax^3\left(a-bx^4\right)^{3/4}}{60b^2}-\frac{x^7\left(a-bx^4\right)^{3/4}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a - b*x^4)^(1/4),x]
```

```
[Out] (-7*a^2*(a - b*x^4)^(3/4))/(40*b^3*x) - (7*a*x^3*(a - b*x^4)^(3/4))/(60*b^2
) - (x^7*(a - b*x^4)^(3/4))/(10*b) + (7*a^(5/2)*(1 - a/(b*x^4))^(1/4)*x*Ell
ipticE[ArcCsc[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(40*b^(5/2)*(a - b*x^4)^(1/4))
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 311

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[(a + b*x^4)^(3/4)
/(2*b*x), x] + Dist[a/(2*b), Int[1/(x^2*(a + b*x^4)^(1/4)), x], x] /; FreeQ
[{a, b}, x] && NegQ[b/a]
```

Rule 313

```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> Dist[(x*(1 + a/(b*
x^4))^(1/4))/(a + b*x^4)^(1/4), Int[1/(x^3*(1 + a/(b*x^4))^(1/4)), x], x] /
; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{x^{10}}{\sqrt[4]{a-bx^4}}\,dx = -\frac{x^7\left(a-bx^4\right)^{3/4}}{10b} + \frac{(7a)\int \frac{x^6}{\sqrt[4]{a-bx^4}}\,dx}{10b}$$

$$= -\frac{7ax^3\left(a-bx^4\right)^{3/4}}{60b^2} - \frac{x^7\left(a-bx^4\right)^{3/4}}{10b} + \frac{\left(7a^2\right)\int \frac{x^2}{\sqrt[4]{a-bx^4}}\,dx}{20b^2}$$

$$= -\frac{7a^2\left(a-bx^4\right)^{3/4}}{40b^3x} - \frac{7ax^3\left(a-bx^4\right)^{3/4}}{60b^2} - \frac{x^7\left(a-bx^4\right)^{3/4}}{10b} - \frac{\left(7a^3\right)\int \frac{1}{x^2\sqrt[4]{a-bx^4}}\,dx}{40b^3}$$

$$= -\frac{7a^2\left(a-bx^4\right)^{3/4}}{40b^3x} - \frac{7ax^3\left(a-bx^4\right)^{3/4}}{60b^2} - \frac{x^7\left(a-bx^4\right)^{3/4}}{10b} - \frac{\left(7a^3\sqrt[4]{1-\frac{a}{bx^4}}\,x\right)\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^4}}\,x^3}\,dx}{40b^3\sqrt[4]{a-bx^4}}$$

$$= -\frac{7a^2\left(a-bx^4\right)^{3/4}}{40b^3x} - \frac{7ax^3\left(a-bx^4\right)^{3/4}}{60b^2} - \frac{x^7\left(a-bx^4\right)^{3/4}}{10b} + \frac{\left(7a^3\sqrt[4]{1-\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{x}{\sqrt[4]{1-\frac{ax^4}{b}}}\right.}{40b^3\sqrt[4]{a-bx^4}}$$

$$= -\frac{7a^2\left(a-bx^4\right)^{3/4}}{40b^3x} - \frac{7ax^3\left(a-bx^4\right)^{3/4}}{60b^2} - \frac{x^7\left(a-bx^4\right)^{3/4}}{10b} + \frac{\left(7a^3\sqrt[4]{1-\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{ax^2}{b}}}\right.}{80b^3\sqrt[4]{a-bx^4}}$$

$$= -\frac{7a^2\left(a-bx^4\right)^{3/4}}{40b^3x} - \frac{7ax^3\left(a-bx^4\right)^{3/4}}{60b^2} - \frac{x^7\left(a-bx^4\right)^{3/4}}{10b} + \frac{7a^{5/2}\sqrt[4]{1-\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\right.}{40b^{5/2}\sqrt[4]{a-bx^4}}$$

Mathematica [C] time = 0.03, size = 80, normalized size = 0.60

$$\frac{x^3\left(7a^2\sqrt[4]{1-\frac{bx^4}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};\frac{bx^4}{a}\right) - 7a^2 + abx^4 + 6b^2x^8\right)}{60b^2\sqrt[4]{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a - b*x^4)^(1/4),x]
```

```
[Out] (x^3*(-7*a^2 + a*b*x^4 + 6*b^2*x^8 + 7*a^2*(1 - (b*x^4)/a)^(1/4)*Hypergeome
tric2F1[1/4, 3/4, 7/4, (b*x^4)/a]))/(60*b^2*(a - b*x^4)^(1/4))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^4+a\right)^{\frac{3}{4}}x^{10}}{bx^4-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(-b*x^4+a)^(1/4),x, algorithm="fricas")
[Out] integral(-(-b*x^4 + a)^(3/4)*x^10/(b*x^4 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\left(-bx^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(-b*x^4+a)^(1/4),x, algorithm="giac")
[Out] integrate(x^10/(-b*x^4 + a)^(1/4), x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\left(-b\,x^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(-b*x^4+a)^(1/4),x)
[Out] int(x^10/(-b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\left(-bx^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(-b*x^4+a)^(1/4),x, algorithm="maxima")
[Out] integrate(x^10/(-b*x^4 + a)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{10}}{\left(a-b\,x^4\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a - b*x^4)^(1/4),x)
[Out] int(x^10/(a - b*x^4)^(1/4), x)
```

sympy [C] time = 2.46, size = 39, normalized size = 0.29

$$\frac{x^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4}, \frac{11}{4} \\ \frac{15}{4}\end{matrix}\right| \frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt[4]{a}\,\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(-b*x**4+a)**(1/4),x)
[Out] x**11*gamma(11/4)*hyper((1/4, 11/4), (15/4,), b*x**4*exp_polar(2*I*pi)/a)/(
4*a**(1/4)*gamma(15/4))
```

3.1228 $\int \frac{x^6}{\sqrt[4]{a-bx^4}}\,dx$

Optimal. Leaf size=109

$$\frac{a^{3/2}x\sqrt[4]{1-\frac{a}{bx^4}}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{4b^{3/2}\sqrt[4]{a-bx^4}}-\frac{a\left(a-bx^4\right)^{3/4}}{4b^2x}-\frac{x^3\left(a-bx^4\right)^{3/4}}{6b}$$

```
[Out] -1/4*a*(-b*x^4+a)^(3/4)/b^2/x-1/6*x^3*(-b*x^4+a)^(3/4)/b+1/4*a^(3/2)*(1-a/b
/x^4)^(1/4)*x*(cos(1/2*arccsc(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccsc
(x^2*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arccsc(x^2*b^(1/2)/a^(1/2))),2^(1/
2))/b^(3/2)/(-b*x^4+a)^(1/4)
```

Rubi [A] time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {321, 311, 313, 335, 275, 228}

$$\frac{a^{3/2}x\sqrt[4]{1-\frac{a}{bx^4}}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{4b^{3/2}\sqrt[4]{a-bx^4}}-\frac{a\left(a-bx^4\right)^{3/4}}{4b^2x}-\frac{x^3\left(a-bx^4\right)^{3/4}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a - b*x^4)^(1/4),x]
```

```
[Out] -(a*(a - b*x^4)^(3/4))/(4*b^2*x) - (x^3*(a - b*x^4)^(3/4))/(6*b) + (a^(3/2)
*(1 - a/(b*x^4))^(1/4)*x*EllipticE[ArcCsc[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(4*
b^(3/2)*(a - b*x^4)^(1/4))
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 311

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[(a + b*x^4)^(3/4)
/(2*b*x), x] + Dist[a/(2*b), Int[1/(x^2*(a + b*x^4)^(1/4)), x], x] /; FreeQ
[{a, b}, x] && NegQ[b/a]
```

Rule 313

```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> Dist[(x*(1 + a/(b*
x^4))^(1/4))/(a + b*x^4)^(1/4), Int[1/(x^3*(1 + a/(b*x^4))^(1/4)), x], x] /
; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{\sqrt[4]{a-bx^4}}\,dx &= -\frac{x^3\left(a-bx^4\right)^{3/4}}{6b} + \frac{a\int \frac{x^2}{\sqrt[4]{a-bx^4}}\,dx}{2b} \\
&= -\frac{a\left(a-bx^4\right)^{3/4}}{4b^2x} - \frac{x^3\left(a-bx^4\right)^{3/4}}{6b} - \frac{a^2\int \frac{1}{x^2\sqrt[4]{a-bx^4}}\,dx}{4b^2} \\
&= -\frac{a\left(a-bx^4\right)^{3/4}}{4b^2x} - \frac{x^3\left(a-bx^4\right)^{3/4}}{6b} - \frac{\left(a^2\sqrt[4]{1-\frac{a}{bx^4}}\,x\right)\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^4}}\,x^3}\,dx}{4b^2\sqrt[4]{a-bx^4}} \\
&= -\frac{a\left(a-bx^4\right)^{3/4}}{4b^2x} - \frac{x^3\left(a-bx^4\right)^{3/4}}{6b} + \frac{\left(a^2\sqrt[4]{1-\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{x}{\sqrt[4]{1-\frac{ax^4}{b}}}\,dx, x, \frac{1}{x}\right)}{4b^2\sqrt[4]{a-bx^4}} \\
&= -\frac{a\left(a-bx^4\right)^{3/4}}{4b^2x} - \frac{x^3\left(a-bx^4\right)^{3/4}}{6b} + \frac{\left(a^2\sqrt[4]{1-\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{ax^2}{b}}}\,dx, x, \frac{1}{x^2}\right)}{8b^2\sqrt[4]{a-bx^4}} \\
&= -\frac{a\left(a-bx^4\right)^{3/4}}{4b^2x} - \frac{x^3\left(a-bx^4\right)^{3/4}}{6b} + \frac{a^{3/2}\sqrt[4]{1-\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{4b^{3/2}\sqrt[4]{a-bx^4}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 66, normalized size = 0.61

$$\frac{x^3\left(a\sqrt[4]{1-\frac{bx^4}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};\frac{bx^4}{a}\right)-a+bx^4\right)}{6b\sqrt[4]{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a - b*x^4)^(1/4),x]
```

```
[Out] (x^3*(-a + b*x^4 + a*(1 - (b*x^4)/a)^(1/4)*Hypergeometric2F1[1/4, 3/4, 7/4,
 (b*x^4)/a]))/(6*b*(a - b*x^4)^(1/4))
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^4+a\right)^{\frac{3}{4}}x^6}{bx^4-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^4 + a)^(3/4)*x^6/(b*x^4 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(-bx^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-b*x^4+a)^(1/4),x, algorithm="giac")
[Out] integrate(x^6/(-b*x^4 + a)^(1/4), x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(-b\,x^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-b*x^4+a)^(1/4),x)
[Out] int(x^6/(-b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(-bx^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-b*x^4+a)^(1/4),x, algorithm="maxima")
[Out] integrate(x^6/(-b*x^4 + a)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{\left(a-b\,x^4\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a - b*x^4)^(1/4),x)
[Out] int(x^6/(a - b*x^4)^(1/4), x)
```

sympy [C] time = 1.86, size = 39, normalized size = 0.36

$$\frac{x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt[4]{a}\,\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-b*x**4+a)**(1/4),x)
[Out] x**7*gamma(7/4)*hyper((1/4, 7/4), (11/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*a
**(1/4)*gamma(11/4))
```

3.1229 $\int \frac{x^2}{\sqrt[4]{a-bx^4}}\,dx$

Optimal. Leaf size=86

$$\frac{\sqrt{a}\,x\sqrt[4]{1-\frac{a}{bx^4}}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{b}\,\sqrt[4]{a-bx^4}}-\frac{\left(a-bx^4\right)^{3/4}}{2bx}$$

```
[Out] -1/2*(-b*x^4+a)^(3/4)/b/x+1/2*(1-a/b/x^4)^(1/4)*x*(cos(1/2*arccsc(x^2*b^(1/
2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccsc(x^2*b^(1/2)/a^(1/2)))*EllipticE(sin(1/
2*arccsc(x^2*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)/(-b*x^4+a)^(1/4)/b^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {311, 313, 335, 275, 228}

$$\frac{\sqrt{a}\,x\sqrt[4]{1-\frac{a}{bx^4}}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{b}\,\sqrt[4]{a-bx^4}}-\frac{\left(a-bx^4\right)^{3/4}}{2bx}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a - b*x^4)^(1/4),x]
```

```
[Out] -(a - b*x^4)^(3/4)/(2*b*x) + (Sqrt[a]*(1 - a/(b*x^4))^(1/4)*x*EllipticE[Arc
Csc[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(2*Sqrt[b]*(a - b*x^4)^(1/4))
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 311

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[(a + b*x^4)^(3/4)
/(2*b*x), x] + Dist[a/(2*b), Int[1/(x^2*(a + b*x^4)^(1/4)), x], x] /; FreeQ
[{a, b}, x] && NegQ[b/a]
```

Rule 313

```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> Dist[(x*(1 + a/(b*
x^4))^(1/4))/(a + b*x^4)^(1/4), Int[1/(x^3*(1 + a/(b*x^4))^(1/4)), x], x] /
; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt[4]{a-bx^4}}\,dx &= -\frac{\left(a-bx^4\right)^{3/4}}{2bx} - \frac{a\int \frac{1}{x^2\sqrt[4]{a-bx^4}}\,dx}{2b}\\ &= -\frac{\left(a-bx^4\right)^{3/4}}{2bx} - \frac{\left(a\sqrt[4]{1-\frac{a}{bx^4}}\,x\right)\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^4}}\,x^3}\,dx}{2b\sqrt[4]{a-bx^4}}\\ &= -\frac{\left(a-bx^4\right)^{3/4}}{2bx} + \frac{\left(a\sqrt[4]{1-\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{x}{\sqrt[4]{1-\frac{ax^4}{b}}}\,dx, x, \frac{1}{x}\right)}{2b\sqrt[4]{a-bx^4}}\\ &= -\frac{\left(a-bx^4\right)^{3/4}}{2bx} + \frac{\left(a\sqrt[4]{1-\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{ax^2}{b}}}\,dx, x, \frac{1}{x^2}\right)}{4b\sqrt[4]{a-bx^4}}\\ &= -\frac{\left(a-bx^4\right)^{3/4}}{2bx} + \frac{\sqrt{a}\sqrt[4]{1-\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a-bx^4}}\end{aligned}$$

Mathematica [C] time = 0.07, size = 52, normalized size = 0.60

$$\frac{x^3\sqrt[4]{1-\frac{bx^4}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};\frac{bx^4}{a}\right)}{3\sqrt[4]{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a - b*x^4)^(1/4),x]
```

```
[Out] (x^3*(1 - (b*x^4)/a)^(1/4)*Hypergeometric2F1[1/4, 3/4, 7/4, (b*x^4)/a])/(3*
(a - b*x^4)^(1/4))
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^4+a\right)^{\frac{3}{4}}x^2}{bx^4-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^4 + a)^(3/4)*x^2/(b*x^4 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-bx^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(-b*x^4 + a)^(1/4), x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-b\,x^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^4+a)^(1/4),x)
```

```
[Out] int(x^2/(-b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-bx^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(-b*x^4 + a)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(a-b\,x^4\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a - b*x^4)^(1/4),x)
```

```
[Out] int(x^2/(a - b*x^4)^(1/4), x)
```

sympy [C] time = 1.33, size = 39, normalized size = 0.45

$$\frac{x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt[4]{a}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**4+a)**(1/4),x)
```

```
[Out] x**3*gamma(3/4)*hyper((1/4, 3/4), (7/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*a*
*(1/4)*gamma(7/4))
```

3.1230 $\int \frac{1}{x^2 \sqrt[4]{a-bx^4}}\,dx$

Optimal. Leaf size=61

$$-\frac{\sqrt{b}\,x\sqrt[4]{1-\frac{a}{bx^4}}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\sqrt[4]{a-bx^4}}$$

```
[Out] -(1-a/b/x^4)^(1/4)*x*(cos(1/2*arccsc(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2
*arccsc(x^2*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arccsc(x^2*b^(1/2)/a^(1/2))
),2^(1/2))*b^(1/2)/(-b*x^4+a)^(1/4)/a^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {313, 335, 275, 228}

$$-\frac{\sqrt{b}\,x\sqrt[4]{1-\frac{a}{bx^4}}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\sqrt[4]{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a - b*x^4)^(1/4)),x]
```

```
[Out] -((Sqrt[b]*(1 - a/(b*x^4))^(1/4)*x*EllipticE[ArcCsc[(Sqrt[b]*x^2)/Sqrt[a]]/
2, 2])/(Sqrt[a]*(a - b*x^4)^(1/4)))
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 313

```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> Dist[(x*(1 + a/(b*
x^4))^(1/4))/(a + b*x^4)^(1/4), Int[1/(x^3*(1 + a/(b*x^4))^(1/4)), x], x] /
; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt[4]{a-bx^4}}\,dx = \frac{\left(\sqrt[4]{1-\frac{a}{bx^4}}\,x\right)\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^4}}\,x^3}\,dx}{\sqrt[4]{a-bx^4}}$$

$$= -\frac{\left(\sqrt[4]{1-\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{x}{\sqrt[4]{1-\frac{ax^4}{b}}}\,dx, x, \frac{1}{x}\right)}{\sqrt[4]{a-bx^4}}$$

$$= -\frac{\left(\sqrt[4]{1-\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{ax^2}{b}}}\,dx, x, \frac{1}{x^2}\right)}{2\sqrt[4]{a-bx^4}}$$

$$= -\frac{\sqrt{b}\,\sqrt[4]{1-\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|\,2\right)}{\sqrt{a}\,\sqrt[4]{a-bx^4}}$$

Mathematica [C] time = 0.01, size = 50, normalized size = 0.82

$$-\frac{\sqrt[4]{1-\frac{bx^4}{a}}\;{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{3}{4};\frac{bx^4}{a}\right)}{x\sqrt[4]{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a - b*x^4)^(1/4)),x]
```

```
[Out] -(((1 - (b*x^4)/a)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 3/4, (b*x^4)/a])/(x*(
a - b*x^4)^(1/4)))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^4+a\right)^{\frac{3}{4}}}{bx^6-ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^4 + a)^(3/4)/(b*x^6 - a*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{1}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^4 + a)^(1/4)*x^2), x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^4+a\right)^{\frac{1}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-b*x^4+a)^(1/4),x)
```

```
[Out] int(1/x^2/(-b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{1}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^4 + a)^(1/4)*x^2), x)
```

mupad [B] time = 1.34, size = 41, normalized size = 0.67

$$-\frac{\left(1-\frac{a}{b\,x^4}\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ \frac{a}{b\,x^4}\right)}{2\,x\left(a-b\,x^4\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a - b*x^4)^(1/4)),x)
```

```
[Out] -((1 - a/(b*x^4))^(1/4)*hypergeom([1/4, 1/2], 3/2, a/(b*x^4)))/(2*x*(a - b*
x^4)^(1/4))
```

sympy [C] time = 1.66, size = 31, normalized size = 0.51

$$\frac{ie^{\frac{i\pi}{4}}\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\,\frac{a}{bx^4}\right)}{2\sqrt[4]{b}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-b*x**4+a)**(1/4),x)
```

```
[Out] I*exp(I*pi/4)*hyper((1/4, 1/2), (3/2,), a/(b*x**4))/(2*b**(1/4)*x**2)
```

3.1231 $\int \frac{1}{x^6 \sqrt[4]{a-bx^4}}\,dx$

Optimal. Leaf size=86

$$-\frac{2b^{3/2}x\sqrt[4]{1-\frac{a}{bx^4}}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}\sqrt[4]{a-bx^4}}-\frac{\left(a-bx^4\right)^{3/4}}{5ax^5}$$

```
[Out] -1/5*(-b*x^4+a)^(3/4)/a/x^5-2/5*b^(3/2)*(1-a/b/x^4)^(1/4)*x*(cos(1/2*arccsc
(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccsc(x^2*b^(1/2)/a^(1/2)))*Ellipt
icE(sin(1/2*arccsc(x^2*b^(1/2)/a^(1/2))),2^(1/2))/a^(3/2)/(-b*x^4+a)^(1/4)
```

Rubi [A] time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {325, 313, 335, 275, 228}

$$-\frac{2b^{3/2}x\sqrt[4]{1-\frac{a}{bx^4}}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}\sqrt[4]{a-bx^4}}-\frac{\left(a-bx^4\right)^{3/4}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a - b*x^4)^(1/4)),x]
```

```
[Out] -(a - b*x^4)^(3/4)/(5*a*x^5) - (2*b^(3/2)*(1 - a/(b*x^4))^(1/4)*x*EllipticE
[ArcCsc[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(5*a^(3/2)*(a - b*x^4)^(1/4))
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 313

```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> Dist[(x*(1 + a/(b*
x^4))^(1/4))/(a + b*x^4)^(1/4), Int[1/(x^3*(1 + a/(b*x^4))^(1/4)), x], x] /
; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^6\sqrt[4]{a-bx^4}}\,dx &= -\frac{\left(a-bx^4\right)^{3/4}}{5ax^5} + \frac{(2b)\int \frac{1}{x^2\sqrt[4]{a-bx^4}}\,dx}{5a} \\ &= -\frac{\left(a-bx^4\right)^{3/4}}{5ax^5} + \frac{\left(2b\sqrt[4]{1-\frac{a}{bx^4}}\,x\right)\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^4}}\,x^3}\,dx}{5a\sqrt[4]{a-bx^4}} \\ &= -\frac{\left(a-bx^4\right)^{3/4}}{5ax^5} - \frac{\left(2b\sqrt[4]{1-\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{x}{\sqrt[4]{1-\frac{ax^4}{b}}}\,dx, x, \frac{1}{x}\right)}{5a\sqrt[4]{a-bx^4}} \\ &= -\frac{\left(a-bx^4\right)^{3/4}}{5ax^5} - \frac{\left(b\sqrt[4]{1-\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{ax^2}{b}}}\,dx, x, \frac{1}{x^2}\right)}{5a\sqrt[4]{a-bx^4}} \\ &= -\frac{\left(a-bx^4\right)^{3/4}}{5ax^5} - \frac{2b^{3/2}\sqrt[4]{1-\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|\,2\right)}{5a^{3/2}\sqrt[4]{a-bx^4}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.60

$$-\frac{\sqrt[4]{1-\frac{bx^4}{a}}\;{}_2F_1\left(-\frac{5}{4}, \frac{1}{4}; -\frac{1}{4}; \frac{bx^4}{a}\right)}{5x^5\sqrt[4]{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a - b*x^4)^(1/4)),x]
```

```
[Out] -1/5*((1 - (b*x^4)/a)^(1/4)*Hypergeometric2F1[-5/4, 1/4, -1/4, (b*x^4)/a])/
(x^5*(a - b*x^4)^(1/4))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^4+a\right)^{\frac{3}{4}}}{bx^{10}-ax^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^4 + a)^(3/4)/(b*x^10 - a*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{1}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^4 + a)^(1/4)*x^6), x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^4+a\right)^{\frac{1}{4}}\,x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-b*x^4+a)^(1/4),x)
[Out] int(1/x^6/(-b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{1}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-b*x^4+a)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((-b*x^4 + a)^(1/4)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^6\left(a-b\,x^4\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a - b*x^4)^(1/4)),x)
[Out] int(1/(x^6*(a - b*x^4)^(1/4)), x)
```

sympy [C] time = 2.55, size = 34, normalized size = 0.40

$$-\frac{i e^{-\frac{3i\pi}{4}}\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\right|\frac{a}{bx^4}\right)}{6\sqrt[4]{b}\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-b*x**4+a)**(1/4),x)
[Out] -I*exp(-3*I*pi/4)*hyper((1/4, 3/2), (5/2,), a/(b*x**4))/(6*b**(1/4)*x**6)
```

3.1232 $\int \frac{1}{x^{10}\sqrt[4]{a-bx^4}}\,dx$

Optimal. Leaf size=109

$$-\frac{4b^{5/2}x\sqrt{1-\frac{a}{bx^4}}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{15a^{5/2}\sqrt[4]{a-bx^4}}-\frac{2b\left(a-bx^4\right)^{3/4}}{15a^2x^5}-\frac{\left(a-bx^4\right)^{3/4}}{9ax^9}$$

```
[Out] -1/9*(-b*x^4+a)^(3/4)/a/x^9-2/15*b*(-b*x^4+a)^(3/4)/a^2/x^5-4/15*b^(5/2)*(1
-a/b/x^4)^(1/4)*x*(cos(1/2*arccsc(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*ar
ccsc(x^2*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arccsc(x^2*b^(1/2)/a^(1/2))),2
^(1/2))/a^(5/2)/(-b*x^4+a)^(1/4)
```

Rubi [A] time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {325, 313, 335, 275, 228}

$$-\frac{4b^{5/2}x\sqrt{1-\frac{a}{bx^4}}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{15a^{5/2}\sqrt[4]{a-bx^4}}-\frac{2b\left(a-bx^4\right)^{3/4}}{15a^2x^5}-\frac{\left(a-bx^4\right)^{3/4}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*(a - b*x^4)^(1/4)),x]
```

```
[Out] -(a - b*x^4)^(3/4)/(9*a*x^9) - (2*b*(a - b*x^4)^(3/4))/(15*a^2*x^5) - (4*b^
(5/2)*(1 - a/(b*x^4))^(1/4)*x*EllipticE[ArcCsc[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2]
)/(15*a^(5/2)*(a - b*x^4)^(1/4))
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 313

```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> Dist[(x*(1 + a/(b*
x^4))^(1/4))/(a + b*x^4)^(1/4), Int[1/(x^3*(1 + a/(b*x^4))^(1/4)), x], x] /
; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
```

egerQ[m]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{10}\sqrt[4]{a-bx^4}}\,dx &= -\frac{\left(a-bx^4\right)^{3/4}}{9ax^9} + \frac{(2b)\int \frac{1}{x^6\sqrt[4]{a-bx^4}}\,dx}{3a} \\
&= -\frac{\left(a-bx^4\right)^{3/4}}{9ax^9} - \frac{2b\left(a-bx^4\right)^{3/4}}{15a^2x^5} + \frac{\left(4b^2\right)\int \frac{1}{x^2\sqrt[4]{a-bx^4}}\,dx}{15a^2} \\
&= -\frac{\left(a-bx^4\right)^{3/4}}{9ax^9} - \frac{2b\left(a-bx^4\right)^{3/4}}{15a^2x^5} + \frac{\left(4b^2\sqrt[4]{1-\frac{a}{bx^4}}\,x\right)\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^4}}\,x^3}\,dx}{15a^2\sqrt[4]{a-bx^4}} \\
&= -\frac{\left(a-bx^4\right)^{3/4}}{9ax^9} - \frac{2b\left(a-bx^4\right)^{3/4}}{15a^2x^5} - \frac{\left(4b^2\sqrt[4]{1-\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{x}{\sqrt[4]{1-\frac{ax^4}{b}}}\,dx, x, \frac{1}{x}\right)}{15a^2\sqrt[4]{a-bx^4}} \\
&= -\frac{\left(a-bx^4\right)^{3/4}}{9ax^9} - \frac{2b\left(a-bx^4\right)^{3/4}}{15a^2x^5} - \frac{\left(2b^2\sqrt[4]{1-\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{ax^2}{b}}}\,dx, x, \frac{1}{x^2}\right)}{15a^2\sqrt[4]{a-bx^4}} \\
&= -\frac{\left(a-bx^4\right)^{3/4}}{9ax^9} - \frac{2b\left(a-bx^4\right)^{3/4}}{15a^2x^5} - \frac{4b^{5/2}\sqrt[4]{1-\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{15a^{5/2}\sqrt[4]{a-bx^4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.48

$$-\frac{\sqrt[4]{1-\frac{bx^4}{a}}\;{}_2F_1\left(-\frac{9}{4},\frac{1}{4};-\frac{5}{4};\frac{bx^4}{a}\right)}{9x^9\sqrt[4]{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^10*(a - b*x^4)^(1/4)),x]
```

```
[Out] -1/9*((1 - (b*x^4)/a)^(1/4)*Hypergeometric2F1[-9/4, 1/4, -5/4, (b*x^4)/a])/
(x^9*(a - b*x^4)^(1/4))
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^4+a\right)^{\frac{3}{4}}}{bx^{14}-ax^{10}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^4 + a)^(3/4)/(b*x^14 - a*x^10), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{1}{4}}x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(-b*x^4+a)^(1/4),x, algorithm="giac")
[Out] integrate(1/((-b*x^4 + a)^(1/4)*x^10), x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^4+a\right)^{\frac{1}{4}}\,x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(-b*x^4+a)^(1/4),x)
[Out] int(1/x^10/(-b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{1}{4}}x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(-b*x^4+a)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((-b*x^4 + a)^(1/4)*x^10), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{10}\left(a-b\,x^4\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^10*(a - b*x^4)^(1/4)),x)
[Out] int(1/(x^10*(a - b*x^4)^(1/4)), x)
```

sympy [C] time = 2.91, size = 31, normalized size = 0.28

$$\frac{ie^{\frac{i\pi}{4}}\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\right|\frac{a}{bx^4}\right)}{10\sqrt[4]{b}\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(-b*x**4+a)**(1/4),x)
[Out] I*exp(I*pi/4)*hyper((1/4, 5/2), (7/2,), a/(b*x**4))/(10*b**(1/4)*x**10)
```

3.1233 $\int \frac{1}{x^{14}\sqrt[4]{a-bx^4}}\,dx$

Optimal. Leaf size=134

$$-\frac{8b^{7/2}x\sqrt[4]{1-\frac{a}{bx^4}}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{39a^{7/2}\sqrt[4]{a-bx^4}}-\frac{4b^2\left(a-bx^4\right)^{3/4}}{39a^3x^5}-\frac{10b\left(a-bx^4\right)^{3/4}}{117a^2x^9}-\frac{\left(a-bx^4\right)^{3/4}}{13ax^{13}}$$

```
[Out] -1/13*(-b*x^4+a)^(3/4)/a/x^13-10/117*b*(-b*x^4+a)^(3/4)/a^2/x^9-4/39*b^2*(-
b*x^4+a)^(3/4)/a^3/x^5-8/39*b^(7/2)*(1-a/b/x^4)^(1/4)*x*(cos(1/2*arccsc(x^2
*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccsc(x^2*b^(1/2)/a^(1/2)))*EllipticE(
sin(1/2*arccsc(x^2*b^(1/2)/a^(1/2))),2^(1/2))/a^(7/2)/(-b*x^4+a)^(1/4)
```

Rubi [A] time = 0.06, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {325, 313, 335, 275, 228}

$$-\frac{4b^2\left(a-bx^4\right)^{3/4}}{39a^3x^5}-\frac{8b^{7/2}x\sqrt[4]{1-\frac{a}{bx^4}}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{39a^{7/2}\sqrt[4]{a-bx^4}}-\frac{10b\left(a-bx^4\right)^{3/4}}{117a^2x^9}-\frac{\left(a-bx^4\right)^{3/4}}{13ax^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^14*(a - b*x^4)^(1/4)),x]
```

```
[Out] -(a - b*x^4)^(3/4)/(13*a*x^13) - (10*b*(a - b*x^4)^(3/4))/(117*a^2*x^9) - (
4*b^2*(a - b*x^4)^(3/4))/(39*a^3*x^5) - (8*b^(7/2)*(1 - a/(b*x^4))^(1/4)*x*
EllipticE[ArcCsc[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(39*a^(7/2)*(a - b*x^4)^(1/4
))
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 313

```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> Dist[(x*(1 + a/(b*
x^4))^(1/4))/(a + b*x^4)^(1/4), Int[1/(x^3*(1 + a/(b*x^4))^(1/4)), x], x] /
; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
```

egerQ[m]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{14}\sqrt[4]{a-bx^4}}\,dx &= -\frac{\left(a-bx^4\right)^{3/4}}{13ax^{13}} + \frac{(10b)\int \frac{1}{x^{10}\sqrt[4]{a-bx^4}}\,dx}{13a} \\
&= -\frac{\left(a-bx^4\right)^{3/4}}{13ax^{13}} - \frac{10b\left(a-bx^4\right)^{3/4}}{117a^2x^9} + \frac{\left(20b^2\right)\int \frac{1}{x^6\sqrt[4]{a-bx^4}}\,dx}{39a^2} \\
&= -\frac{\left(a-bx^4\right)^{3/4}}{13ax^{13}} - \frac{10b\left(a-bx^4\right)^{3/4}}{117a^2x^9} - \frac{4b^2\left(a-bx^4\right)^{3/4}}{39a^3x^5} + \frac{\left(8b^3\right)\int \frac{1}{x^2\sqrt[4]{a-bx^4}}\,dx}{39a^3} \\
&= -\frac{\left(a-bx^4\right)^{3/4}}{13ax^{13}} - \frac{10b\left(a-bx^4\right)^{3/4}}{117a^2x^9} - \frac{4b^2\left(a-bx^4\right)^{3/4}}{39a^3x^5} + \frac{\left(8b^3\sqrt[4]{1-\frac{a}{bx^4}}\,x\right)\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^4}}\,x^3}\,dx}{39a^3\sqrt[4]{a-bx^4}} \\
&= -\frac{\left(a-bx^4\right)^{3/4}}{13ax^{13}} - \frac{10b\left(a-bx^4\right)^{3/4}}{117a^2x^9} - \frac{4b^2\left(a-bx^4\right)^{3/4}}{39a^3x^5} - \frac{\left(8b^3\sqrt[4]{1-\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{x}{\sqrt[4]{1-\frac{ax^4}{b}}}\right)}{39a^3\sqrt[4]{a-bx^4}} \\
&= -\frac{\left(a-bx^4\right)^{3/4}}{13ax^{13}} - \frac{10b\left(a-bx^4\right)^{3/4}}{117a^2x^9} - \frac{4b^2\left(a-bx^4\right)^{3/4}}{39a^3x^5} - \frac{\left(4b^3\sqrt[4]{1-\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{ax^2}{b}}}\right)}{39a^3\sqrt[4]{a-bx^4}} \\
&= -\frac{\left(a-bx^4\right)^{3/4}}{13ax^{13}} - \frac{10b\left(a-bx^4\right)^{3/4}}{117a^2x^9} - \frac{4b^2\left(a-bx^4\right)^{3/4}}{39a^3x^5} - \frac{8b^{7/2}\sqrt[4]{1-\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\right|}{39a^{7/2}\sqrt[4]{a-bx^4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.39

$$-\frac{\sqrt[4]{1-\frac{bx^4}{a}}\;{}_2F_1\left(-\frac{13}{4},\frac{1}{4};-\frac{9}{4};\frac{bx^4}{a}\right)}{13x^{13}\sqrt[4]{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^14*(a - b*x^4)^(1/4)),x]
```

```
[Out] -1/13*((1 - (b*x^4)/a)^(1/4)*Hypergeometric2F1[-13/4, 1/4, -9/4, (b*x^4)/a]
)/(x^13*(a - b*x^4)^(1/4))
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^4+a\right)^{\frac{3}{4}}}{bx^{18}-ax^{14}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^14/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^4 + a)^(3/4)/(b*x^18 - a*x^14), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{1}{4}}x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^14/(-b*x^4+a)^(1/4),x, algorithm="giac")
[Out] integrate(1/((-b*x^4 + a)^(1/4)*x^14), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^4+a\right)^{\frac{1}{4}}\,x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^14/(-b*x^4+a)^(1/4),x)
[Out] int(1/x^14/(-b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{1}{4}}x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^14/(-b*x^4+a)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((-b*x^4 + a)^(1/4)*x^14), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{14}\left(a-b\,x^4\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^14*(a - b*x^4)^(1/4)),x)
[Out] int(1/(x^14*(a - b*x^4)^(1/4)), x)
```

sympy [C] time = 5.43, size = 34, normalized size = 0.25

$$-\frac{ie^{-\frac{3i\pi}{4}}\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\right|\frac{a}{bx^4}\right)}{14\sqrt[4]{b}\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**14/(-b*x**4+a)**(1/4),x)
[Out] -I*exp(-3*I*pi/4)*hyper((1/4, 7/2), (9/2,), a/(b*x**4))/(14*b**(1/4)*x**14)
```

3.1234 $\int \frac{x^{19}}{\left(a-bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=104

$$-\frac{a^4\sqrt[4]{a-bx^4}}{b^5}+\frac{4a^3\left(a-bx^4\right)^{5/4}}{5b^5}-\frac{2a^2\left(a-bx^4\right)^{9/4}}{3b^5}-\frac{\left(a-bx^4\right)^{17/4}}{17b^5}+\frac{4a\left(a-bx^4\right)^{13/4}}{13b^5}$$

```
[Out] -a^4*(-b*x^4+a)^(1/4)/b^5+4/5*a^3*(-b*x^4+a)^(5/4)/b^5-2/3*a^2*(-b*x^4+a)^(
9/4)/b^5+4/13*a*(-b*x^4+a)^(13/4)/b^5-1/17*(-b*x^4+a)^(17/4)/b^5
```

Rubi [A] time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {266, 43}

$$-\frac{2a^2\left(a-bx^4\right)^{9/4}}{3b^5}+\frac{4a^3\left(a-bx^4\right)^{5/4}}{5b^5}-\frac{a^4\sqrt[4]{a-bx^4}}{b^5}-\frac{\left(a-bx^4\right)^{17/4}}{17b^5}+\frac{4a\left(a-bx^4\right)^{13/4}}{13b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^19/(a - b*x^4)^(3/4),x]
```

```
[Out] -((a^4*(a - b*x^4)^(1/4))/b^5) + (4*a^3*(a - b*x^4)^(5/4))/(5*b^5) - (2*a^2
*(a - b*x^4)^(9/4))/(3*b^5) + (4*a*(a - b*x^4)^(13/4))/(13*b^5) - (a - b*x^
4)^(17/4)/(17*b^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{19}}{\left(a-bx^4\right)^{3/4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x^4}{(a-bx)^{3/4}}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int\left(\frac{a^4}{b^4(a-bx)^{3/4}}-\frac{4a^3\sqrt[4]{a-bx}}{b^4}+\frac{6a^2(a-bx)^{5/4}}{b^4}-\frac{4a(a-bx)^{9/4}}{b^4}+\frac{(a-bx)^{13/4}}{b^4}\right.\right.\\ &= -\frac{a^4\sqrt[4]{a-bx^4}}{b^5}+\frac{4a^3\left(a-bx^4\right)^{5/4}}{5b^5}-\frac{2a^2\left(a-bx^4\right)^{9/4}}{3b^5}+\frac{4a\left(a-bx^4\right)^{13/4}}{13b^5}-\frac{\left(a-bx^4\right)^{17/4}}{17b^5}\end{aligned}$$

Mathematica [A] time = 0.03, size = 62, normalized size = 0.60

$$-\frac{\sqrt[4]{a-bx^4}\left(2048a^4+512a^3bx^4+320a^2b^2x^8+240ab^3x^{12}+195b^4x^{16}\right)}{3315b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^19/(a - b*x^4)^(3/4),x]
```

```
[Out] -1/3315*((a - b*x^4)^(1/4)*(2048*a^4 + 512*a^3*b*x^4 + 320*a^2*b^2*x^8 + 24
0*a*b^3*x^12 + 195*b^4*x^16))/b^5
```

fricas [A] time = 0.53, size = 58, normalized size = 0.56

$$-\frac{\left(195\,b^4x^{16}+240\,ab^3x^{12}+320\,a^2b^2x^8+512\,a^3bx^4+2048\,a^4\right)\left(-bx^4+a\right)^{\frac{1}{4}}}{3315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/3315*(195*b^4*x^16 + 240*a*b^3*x^12 + 320*a^2*b^2*x^8 + 512*a^3*b*x^4 +
2048*a^4)*(-b*x^4 + a)^(1/4)/b^5
```

giac [A] time = 0.16, size = 113, normalized size = 1.09

$$-\frac{\left(-bx^4+a\right)^{\frac{1}{4}}a^4}{b^5}-\frac{195\left(bx^4-a\right)^4\left(-bx^4+a\right)^{\frac{1}{4}}+1020\left(bx^4-a\right)^3\left(-bx^4+a\right)^{\frac{1}{4}}a+2210\left(bx^4-a\right)^2\left(-bx^4+a\right)^{\frac{1}{4}}a^2-2}{3315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] -(-b*x^4 + a)^(1/4)*a^4/b^5 - 1/3315*(195*(b*x^4 - a)^4*(-b*x^4 + a)^(1/4)
+ 1020*(b*x^4 - a)^3*(-b*x^4 + a)^(1/4)*a + 2210*(b*x^4 - a)^2*(-b*x^4 + a)
^(1/4)*a^2 - 2652*(-b*x^4 + a)^(5/4)*a^3)/b^5
```

maple [A] time = 0.01, size = 59, normalized size = 0.57

$$-\frac{\left(-b\,x^4+a\right)^{\frac{1}{4}}\left(195x^{16}b^4+240a\,x^{12}b^3+320a^2x^8b^2+512a^3x^4b+2048a^4\right)}{3315b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19/(-b*x^4+a)^(3/4),x)
```

```
[Out] -1/3315*(-b*x^4+a)^(1/4)*(195*b^4*x^16+240*a*b^3*x^12+320*a^2*b^2*x^8+512*a
^3*b*x^4+2048*a^4)/b^5
```

maxima [A] time = 0.97, size = 86, normalized size = 0.83

$$-\frac{\left(-bx^4+a\right)^{\frac{17}{4}}}{17\,b^5}+\frac{4\left(-bx^4+a\right)^{\frac{13}{4}}a}{13\,b^5}-\frac{2\left(-bx^4+a\right)^{\frac{9}{4}}a^2}{3\,b^5}+\frac{4\left(-bx^4+a\right)^{\frac{5}{4}}a^3}{5\,b^5}-\frac{\left(-bx^4+a\right)^{\frac{1}{4}}a^4}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] -1/17*(-b*x^4 + a)^(17/4)/b^5 + 4/13*(-b*x^4 + a)^(13/4)*a/b^5 - 2/3*(-b*x^
4 + a)^(9/4)*a^2/b^5 + 4/5*(-b*x^4 + a)^(5/4)*a^3/b^5 - (-b*x^4 + a)^(1/4)*
a^4/b^5
```

mupad [B] time = 1.22, size = 60, normalized size = 0.58

$$-\left(a-b\,x^4\right)^{1/4}\left(\frac{2048\,a^4}{3315\,b^5}+\frac{x^{16}}{17\,b}+\frac{16\,a\,x^{12}}{221\,b^2}+\frac{512\,a^3\,x^4}{3315\,b^4}+\frac{64\,a^2\,x^8}{663\,b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19/(a - b*x^4)^(3/4),x)
```

```
[Out] -(a - b*x^4)^(1/4)*((2048*a^4)/(3315*b^5) + x^16/(17*b) + (16*a*x^12)/(221*
b^2) + (512*a^3*x^4)/(3315*b^4) + (64*a^2*x^8)/(663*b^3))
```

sympy [A] time = 26.21, size = 117, normalized size = 1.12

$$\begin{cases} -\frac{2048a^4\sqrt[4]{a-bx^4}}{3315b^5} - \frac{512a^3x^4\sqrt[4]{a-bx^4}}{3315b^4} - \frac{64a^2x^8\sqrt[4]{a-bx^4}}{663b^3} - \frac{16ax^{12}\sqrt[4]{a-bx^4}}{221b^2} - \frac{x^{16}\sqrt[4]{a-bx^4}}{17b} & \text{for } b \neq 0 \\ \frac{x^{20}}{20a^{\frac{3}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**19/(-b*x**4+a)**(3/4),x)
```

```
[Out] Piecewise((-2048*a**4*(a - b*x**4)**(1/4)/(3315*b**5) - 512*a**3*x**4*(a -
b*x**4)**(1/4)/(3315*b**4) - 64*a**2*x**8*(a - b*x**4)**(1/4)/(663*b**3) -
16*a*x**12*(a - b*x**4)**(1/4)/(221*b**2) - x**16*(a - b*x**4)**(1/4)/(17*b
), Ne(b, 0)), (x**20/(20*a**(3/4)), True))
```

3.1235 $\int \frac{x^{15}}{\left(a-bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=82

$$-\frac{a^3\sqrt[4]{a-bx^4}}{b^4}+\frac{3a^2\left(a-bx^4\right)^{5/4}}{5b^4}+\frac{\left(a-bx^4\right)^{13/4}}{13b^4}-\frac{a\left(a-bx^4\right)^{9/4}}{3b^4}$$

```
[Out] -a^3*(-b*x^4+a)^(1/4)/b^4+3/5*a^2*(-b*x^4+a)^(5/4)/b^4-1/3*a*(-b*x^4+a)^(9/
4)/b^4+1/13*(-b*x^4+a)^(13/4)/b^4
```

Rubi [A] time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {266, 43}

$$\frac{3a^2\left(a-bx^4\right)^{5/4}}{5b^4}-\frac{a^3\sqrt[4]{a-bx^4}}{b^4}+\frac{\left(a-bx^4\right)^{13/4}}{13b^4}-\frac{a\left(a-bx^4\right)^{9/4}}{3b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^15/(a - b*x^4)^(3/4),x]
```

```
[Out] -((a^3*(a - b*x^4)^(1/4))/b^4) + (3*a^2*(a - b*x^4)^(5/4))/(5*b^4) - (a*(a
- b*x^4)^(9/4))/(3*b^4) + (a - b*x^4)^(13/4)/(13*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{15}}{\left(a-bx^4\right)^{3/4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x^3}{(a-bx)^{3/4}}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int\left(\frac{a^3}{b^3(a-bx)^{3/4}}-\frac{3a^2\sqrt[4]{a-bx}}{b^3}+\frac{3a(a-bx)^{5/4}}{b^3}-\frac{(a-bx)^{9/4}}{b^3}\right)dx, x, x^4\right)\\ &= -\frac{a^3\sqrt[4]{a-bx^4}}{b^4}+\frac{3a^2\left(a-bx^4\right)^{5/4}}{5b^4}-\frac{a\left(a-bx^4\right)^{9/4}}{3b^4}+\frac{\left(a-bx^4\right)^{13/4}}{13b^4}\end{aligned}$$

Mathematica [A] time = 0.04, size = 51, normalized size = 0.62

$$-\frac{\sqrt[4]{a-bx^4}\left(128a^3+32a^2bx^4+20ab^2x^8+15b^3x^{12}\right)}{195b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15/(a - b*x^4)^(3/4),x]
```

```
[Out] -1/195*((a - b*x^4)^(1/4)*(128*a^3 + 32*a^2*b*x^4 + 20*a*b^2*x^8 + 15*b^3*x
^12))/b^4
```

fricas [A] time = 0.58, size = 47, normalized size = 0.57

$$-\frac{\left(15\,b^3x^{12}+20\,ab^2x^8+32\,a^2bx^4+128\,a^3\right)\left(-bx^4+a\right)^{\frac{1}{4}}}{195\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/195*(15*b^3*x^12 + 20*a*b^2*x^8 + 32*a^2*b*x^4 + 128*a^3)*(-b*x^4 + a)^(
1/4)/b^4
```

giac [A] time = 0.16, size = 87, normalized size = 1.06

$$-\frac{\left(-bx^4+a\right)^{\frac{1}{4}}a^3}{b^4}-\frac{15\left(bx^4-a\right)^3\left(-bx^4+a\right)^{\frac{1}{4}}+65\left(bx^4-a\right)^2\left(-bx^4+a\right)^{\frac{1}{4}}a-117\left(-bx^4+a\right)^{\frac{5}{4}}a^2}{195\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] -(-b*x^4 + a)^(1/4)*a^3/b^4 - 1/195*(15*(b*x^4 - a)^3*(-b*x^4 + a)^(1/4) +
65*(b*x^4 - a)^2*(-b*x^4 + a)^(1/4)*a - 117*(-b*x^4 + a)^(5/4)*a^2)/b^4
```

maple [A] time = 0.01, size = 48, normalized size = 0.59

$$-\frac{\left(-b\,x^4+a\right)^{\frac{1}{4}}\left(15b^3x^{12}+20a\,b^2x^8+32a^2b\,x^4+128a^3\right)}{195b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(-b*x^4+a)^(3/4),x)
```

```
[Out] -1/195*(-b*x^4+a)^(1/4)*(15*b^3*x^12+20*a*b^2*x^8+32*a^2*b*x^4+128*a^3)/b^4
```

maxima [A] time = 1.08, size = 68, normalized size = 0.83

$$\frac{\left(-bx^4+a\right)^{\frac{13}{4}}}{13\,b^4}-\frac{\left(-bx^4+a\right)^{\frac{9}{4}}a}{3\,b^4}+\frac{3\left(-bx^4+a\right)^{\frac{5}{4}}a^2}{5\,b^4}-\frac{\left(-bx^4+a\right)^{\frac{1}{4}}a^3}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] 1/13*(-b*x^4 + a)^(13/4)/b^4 - 1/3*(-b*x^4 + a)^(9/4)*a/b^4 + 3/5*(-b*x^4 +
 a)^(5/4)*a^2/b^4 - (-b*x^4 + a)^(1/4)*a^3/b^4
```

mupad [B] time = 1.17, size = 49, normalized size = 0.60

$$-\left(a-b\,x^4\right)^{1/4}\left(\frac{128\,a^3}{195\,b^4}+\frac{x^{12}}{13\,b}+\frac{4\,a\,x^8}{39\,b^2}+\frac{32\,a^2\,x^4}{195\,b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(a - b*x^4)^(3/4),x)
```

```
[Out] -(a - b*x^4)^(1/4)*((128*a^3)/(195*b^4) + x^12/(13*b) + (4*a*x^8)/(39*b^2)
+ (32*a^2*x^4)/(195*b^3))
```

sympy [**A**] time = 12.76, size = 94, normalized size = 1.15

$$\begin{cases} -\frac{128a^3\sqrt[4]{a-bx^4}}{195b^4} - \frac{32a^2x^4\sqrt[4]{a-bx^4}}{195b^3} - \frac{4ax^8\sqrt[4]{a-bx^4}}{39b^2} - \frac{x^{12}\sqrt[4]{a-bx^4}}{13b} & \text{for } b \neq 0 \\ \frac{x^{16}}{16a^{\frac{3}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15/(-b*x**4+a)**(3/4),x)
```

```
[Out] Piecewise((-128*a**3*(a - b*x**4)**(1/4)/(195*b**4) - 32*a**2*x**4*(a - b*x
**4)**(1/4)/(195*b**3) - 4*a*x**8*(a - b*x**4)**(1/4)/(39*b**2) - x**12*(a
- b*x**4)**(1/4)/(13*b), Ne(b, 0)), (x**16/(16*a**(3/4)), True))
```

3.1236 $\int \frac{x^{11}}{\left(a-bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=60

$$-\frac{a^2\sqrt[4]{a-bx^4}}{b^3}-\frac{\left(a-bx^4\right)^{9/4}}{9b^3}+\frac{2a\left(a-bx^4\right)^{5/4}}{5b^3}$$

```
[Out] -a^2*(-b*x^4+a)^(1/4)/b^3+2/5*a*(-b*x^4+a)^(5/4)/b^3-1/9*(-b*x^4+a)^(9/4)/b
^3
```

Rubi [A] time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {266, 43}

$$-\frac{a^2\sqrt[4]{a-bx^4}}{b^3}-\frac{\left(a-bx^4\right)^{9/4}}{9b^3}+\frac{2a\left(a-bx^4\right)^{5/4}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a - b*x^4)^(3/4),x]
```

```
[Out] -((a^2*(a - b*x^4)^(1/4))/b^3) + (2*a*(a - b*x^4)^(5/4))/(5*b^3) - (a - b*x
^4)^(9/4)/(9*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{\left(a-bx^4\right)^{3/4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x^2}{(a-bx)^{3/4}}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int\left(\frac{a^2}{b^2(a-bx)^{3/4}}-\frac{2a\sqrt[4]{a-bx}}{b^2}+\frac{(a-bx)^{5/4}}{b^2}\right)dx, x, x^4\right)\\ &= -\frac{a^2\sqrt[4]{a-bx^4}}{b^3}+\frac{2a\left(a-bx^4\right)^{5/4}}{5b^3}-\frac{\left(a-bx^4\right)^{9/4}}{9b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 0.67

$$-\frac{\sqrt[4]{a-bx^4}\left(32a^2+8abx^4+5b^2x^8\right)}{45b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a - b*x^4)^(3/4),x]
```

```
[Out] -1/45*((a - b*x^4)^(1/4)*(32*a^2 + 8*a*b*x^4 + 5*b^2*x^8))/b^3
```

fricas [A] time = 0.69, size = 36, normalized size = 0.60

$$-\frac{\left(5\,b^2x^8+8\,abx^4+32\,a^2\right)\left(-bx^4+a\right)^{\frac{1}{4}}}{45\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/45*(5*b^2*x^8 + 8*a*b*x^4 + 32*a^2)*(-b*x^4 + a)^(1/4)/b^3
```

giac [A] time = 0.16, size = 61, normalized size = 1.02

$$-\frac{\left(-bx^4+a\right)^{\frac{1}{4}}a^2}{b^3}-\frac{5\left(bx^4-a\right)^2\left(-bx^4+a\right)^{\frac{1}{4}}-18\left(-bx^4+a\right)^{\frac{5}{4}}a}{45\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] -(-b*x^4 + a)^(1/4)*a^2/b^3 - 1/45*(5*(b*x^4 - a)^2*(-b*x^4 + a)^(1/4) - 18
*(-b*x^4 + a)^(5/4)*a)/b^3
```

maple [A] time = 0.01, size = 37, normalized size = 0.62

$$-\frac{\left(-b\,x^4+a\right)^{\frac{1}{4}}\left(5b^2x^8+8ab\,x^4+32a^2\right)}{45b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(-b*x^4+a)^(3/4),x)
```

```
[Out] -1/45*(-b*x^4+a)^(1/4)*(5*b^2*x^8+8*a*b*x^4+32*a^2)/b^3
```

maxima [A] time = 1.06, size = 50, normalized size = 0.83

$$-\frac{\left(-bx^4+a\right)^{\frac{9}{4}}}{9\,b^3}+\frac{2\left(-bx^4+a\right)^{\frac{5}{4}}a}{5\,b^3}-\frac{\left(-bx^4+a\right)^{\frac{1}{4}}a^2}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] -1/9*(-b*x^4 + a)^(9/4)/b^3 + 2/5*(-b*x^4 + a)^(5/4)*a/b^3 - (-b*x^4 + a)^(
1/4)*a^2/b^3
```

mupad [B] time = 1.15, size = 38, normalized size = 0.63

$$-\left(a-b\,x^4\right)^{1/4}\left(\frac{32\,a^2}{45\,b^3}+\frac{x^8}{9\,b}+\frac{8\,a\,x^4}{45\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a - b*x^4)^(3/4),x)
```

```
[Out] -(a - b*x^4)^(1/4)*((32*a^2)/(45*b^3) + x^8/(9*b) + (8*a*x^4)/(45*b^2))
```

sympy [**A**] time = 5.67, size = 70, normalized size = 1.17

$$\begin{cases} -\frac{32a^2\sqrt[4]{a-bx^4}}{45b^3} - \frac{8ax^4\sqrt[4]{a-bx^4}}{45b^2} - \frac{x^8\sqrt[4]{a-bx^4}}{9b} & \text{for } b \neq 0 \\ \frac{x^{12}}{12a^{\frac{3}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(-b*x**4+a)**(3/4),x)
```

```
[Out] Piecewise((-32*a**2*(a - b*x**4)**(1/4)/(45*b**3) - 8*a*x**4*(a - b*x**4)**
(1/4)/(45*b**2) - x**8*(a - b*x**4)**(1/4)/(9*b), Ne(b, 0)), (x**12/(12*a**
(3/4)), True))
```

3.1237 $\int \frac{x^7}{\left(a-bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=38

$$\frac{\left(a-bx^4\right)^{5/4}}{5b^2}-\frac{a\sqrt[4]{a-bx^4}}{b^2}$$

```
[Out] -a*(-b*x^4+a)^(1/4)/b^2+1/5*(-b*x^4+a)^(5/4)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {266, 43}

$$\frac{\left(a-bx^4\right)^{5/4}}{5b^2}-\frac{a\sqrt[4]{a-bx^4}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a - b*x^4)^(3/4),x]
```

```
[Out] -((a*(a - b*x^4)^(1/4))/b^2) + (a - b*x^4)^(5/4)/(5*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{\left(a-bx^4\right)^{3/4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x}{(a-bx)^{3/4}}\,dx, x, x^4\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int\left(\frac{a}{b(a-bx)^{3/4}}-\frac{\sqrt[4]{a-bx}}{b}\right)dx, x, x^4\right)\\ &= -\frac{a\sqrt[4]{a-bx^4}}{b^2}+\frac{\left(a-bx^4\right)^{5/4}}{5b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.74

$$-\frac{\sqrt[4]{a-bx^4}\left(4a+bx^4\right)}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a - b*x^4)^(3/4),x]
```

```
[Out] -1/5*((a - b*x^4)^(1/4)*(4*a + b*x^4))/b^2
```

fricas [A] time = 0.58, size = 24, normalized size = 0.63

$$-\frac{\left(bx^4+4\,a\right)\left(-bx^4+a\right)^{\frac{1}{4}}}{5\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-b*x^4+a)^(3/4),x, algorithm="fricas")
[Out] -1/5*(b*x^4 + 4*a)*(-b*x^4 + a)^(1/4)/b^2
```

giac [A] time = 0.16, size = 32, normalized size = 0.84

$$\frac{\left(-bx^4+a\right)^{\frac{5}{4}}}{5\,b^2}-\frac{\left(-bx^4+a\right)^{\frac{1}{4}}a}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-b*x^4+a)^(3/4),x, algorithm="giac")
[Out] 1/5*(-b*x^4 + a)^(5/4)/b^2 - (-b*x^4 + a)^(1/4)*a/b^2
```

maple [A] time = 0.00, size = 25, normalized size = 0.66

$$-\frac{\left(-b\,x^4+a\right)^{\frac{1}{4}}\left(b\,x^4+4a\right)}{5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-b*x^4+a)^(3/4),x)
[Out] -1/5*(-b*x^4+a)^(1/4)*(b*x^4+4*a)/b^2
```

maxima [A] time = 1.03, size = 32, normalized size = 0.84

$$\frac{\left(-bx^4+a\right)^{\frac{5}{4}}}{5\,b^2}-\frac{\left(-bx^4+a\right)^{\frac{1}{4}}a}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-b*x^4+a)^(3/4),x, algorithm="maxima")
[Out] 1/5*(-b*x^4 + a)^(5/4)/b^2 - (-b*x^4 + a)^(1/4)*a/b^2
```

mupad [B] time = 1.15, size = 24, normalized size = 0.63

$$-\frac{\left(a-b\,x^4\right)^{1/4}\left(b\,x^4+4\,a\right)}{5\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a - b*x^4)^(3/4),x)
[Out] -((a - b*x^4)^(1/4)*(4*a + b*x^4))/(5*b^2)
```

sympy [A] time = 1.23, size = 46, normalized size = 1.21

$$\begin{cases} -\frac{4a\sqrt[4]{a-bx^4}}{5b^2}-\frac{x^4\sqrt[4]{a-bx^4}}{5b} & \text{for } b\neq 0 \\ \frac{x^8}{8a^{\frac{3}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-b*x**4+a)**(3/4),x)
```

```
[Out] Piecewise((-4*a*(a - b*x**4)**(1/4)/(5*b**2) - x**4*(a - b*x**4)**(1/4)/(5*
b), Ne(b, 0)), (x**8/(8*a**(3/4)), True))
```

3.1238 $\int \frac{x^3}{\left(a-bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=17

$$-\frac{\sqrt[4]{a-bx^4}}{b}$$

```
[Out] -(-b*x^4+a)^(1/4)/b
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {261}

$$-\frac{\sqrt[4]{a-bx^4}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a - b*x^4)^(3/4),x]
```

```
[Out] -((a - b*x^4)^(1/4)/b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^3}{\left(a-bx^4\right)^{3/4}}\,dx = -\frac{\sqrt[4]{a-bx^4}}{b}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$-\frac{\sqrt[4]{a-bx^4}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a - b*x^4)^(3/4),x]
```

```
[Out] -((a - b*x^4)^(1/4)/b)
```

fricas [A] time = 0.53, size = 15, normalized size = 0.88

$$-\frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] -(-b*x^4 + a)^(1/4)/b
```

giac [A] time = 0.19, size = 15, normalized size = 0.88

$$-\frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^4+a)^(3/4),x, algorithm="giac")
[Out] -(-b*x^4 + a)^(1/4)/b
```

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$-\frac{\left(-b\,x^4+a\right)^{\frac{1}{4}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-b*x^4+a)^(3/4),x)
[Out] -(-b*x^4+a)^(1/4)/b
```

maxima [A] time = 0.98, size = 15, normalized size = 0.88

$$-\frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^4+a)^(3/4),x, algorithm="maxima")
[Out] -(-b*x^4 + a)^(1/4)/b
```

mupad [B] time = 1.13, size = 15, normalized size = 0.88

$$-\frac{\left(a-b\,x^4\right)^{1/4}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a - b*x^4)^(3/4),x)
[Out] -(a - b*x^4)^(1/4)/b
```

sympy [A] time = 0.82, size = 22, normalized size = 1.29

$$\begin{cases} -\frac{\sqrt[4]{a-bx^4}}{b} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{3}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-b*x**4+a)**(3/4),x)
[Out] Piecewise((-(a - b*x**4)**(1/4)/b, Ne(b, 0)), (x**4/(4*a**(3/4)), True))
```

3.1239 $\int \frac{1}{x\left(a-bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=57

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}$$

```
[Out] -1/2*arctan((-b*x^4+a)^(1/4)/a^(1/4))/a^(3/4)-1/2*arctanh((-b*x^4+a)^(1/4)/
a^(1/4))/a^(3/4)
```

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {266, 63, 212, 206, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a - b*x^4)^(3/4)),x]
```

```
[Out] -ArcTan[(a - b*x^4)^(1/4)/a^(1/4)]/(2*a^(3/4)) - ArcTanh[(a - b*x^4)^(1/4)/
a^(1/4)]/(2*a^(3/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a-bx^4\right)^{3/4}}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x(a-bx)^{3/4}}\,dx, x, x^4\right)$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{\frac{a}{b}-\frac{x^4}{b}}\,dx, x, \sqrt[4]{a-bx^4}\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{a}-x^2}\,dx, x, \sqrt[4]{a-bx^4}\right)}{2\sqrt{a}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a}+x^2}\,dx, x, \sqrt[4]{a-bx^4}\right)}{2\sqrt{a}}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}$$

Mathematica [A] time = 0.01, size = 48, normalized size = 0.84

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right) + \tanh^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a - b*x^4)^(3/4)),x]
```

```
[Out] -1/2*(ArcTan[(a - b*x^4)^(1/4)/a^(1/4)] + ArcTanh[(a - b*x^4)^(1/4)/a^(1/4)
])/a^(3/4)
```

fricas [B] time = 0.66, size = 113, normalized size = 1.98

$$\frac{1}{a^3}^{\frac{1}{4}} \arctan\left(\sqrt{a^2\sqrt{\frac{1}{a^3}} + \sqrt{-bx^4+a}}\; a^2 \frac{1}{a^3}^{\frac{3}{4}} - \left(-bx^4+a\right)^{\frac{1}{4}} a^2 \frac{1}{a^3}^{\frac{3}{4}}\right) - \frac{1}{4}\,\frac{1}{a^3}^{\frac{1}{4}} \log\left(a \frac{1}{a^3}^{\frac{1}{4}} + \left(-bx^4+a\right)^{\frac{1}{4}}\right) + \frac{1}{4}\,\frac{1}{a^3}^{\frac{1}{4}} \log($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] (a^(-3))^(1/4)*arctan(sqrt(a^2*sqrt(a^(-3)) + sqrt(-b*x^4 + a))*a^2*(a^(-3)
)^(3/4) - (-b*x^4 + a)^(1/4)*a^2*(a^(-3))^(3/4)) - 1/4*(a^(-3))^(1/4)*log(a
*(a^(-3))^(1/4) + (-b*x^4 + a)^(1/4)) + 1/4*(a^(-3))^(1/4)*log(-a*(a^(-3))^
(1/4) + (-b*x^4 + a)^(1/4))
```

giac [B] time = 0.16, size = 192, normalized size = 3.37

$$-\frac{\sqrt{2}\,(-a)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}} + 2\left(-bx^4+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{4\,a} - \frac{\sqrt{2}\,(-a)^{\frac{1}{4}} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}} - 2\left(-bx^4+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{4\,a} - \frac{\sqrt{2}\,(-a)^{\frac{1}{4}} \log\left(\sqrt{2}\,(-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(-a)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(-b*x^4
+ a)^(1/4))/(-a)^(1/4))/a - 1/4*sqrt(2)*(-a)^(1/4)*arctan(-1/2*sqrt(2)*(sqr
t(2)*(-a)^(1/4) - 2*(-b*x^4 + a)^(1/4))/(-a)^(1/4))/a - 1/8*sqrt(2)*(-a)^(1
```

```
/4)*log(sqrt(2)*(-b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(-b*x^4 + a) + sqrt(-a)
)/a + 1/8*sqrt(2)*(-a)^(1/4)*log(-sqrt(2)*(-b*x^4 + a)^(1/4)*(-a)^(1/4) + s
qrt(-b*x^4 + a) + sqrt(-a))/a
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^4+a\right)^{\frac{3}{4}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-b*x^4+a)^(3/4),x)
```

```
[Out] int(1/x/(-b*x^4+a)^(3/4),x)
```

maxima [A] time = 2.28, size = 60, normalized size = 1.05

$$-\frac{\arctan\left(\frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{2\,a^{\frac{3}{4}}}+\frac{\log\left(\frac{\left(-bx^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{\left(-bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{4\,a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] -1/2*arctan((-b*x^4 + a)^(1/4)/a^(1/4))/a^(3/4) + 1/4*log(((-b*x^4 + a)^(1/
4) - a^(1/4))/((-b*x^4 + a)^(1/4) + a^(1/4)))/a^(3/4)
```

mupad [B] time = 1.16, size = 36, normalized size = 0.63

$$-\frac{\operatorname{atan}\left(\frac{\left(a-b\,x^4\right)^{1/4}}{a^{1/4}}\right)+\operatorname{atanh}\left(\frac{\left(a-b\,x^4\right)^{1/4}}{a^{1/4}}\right)}{2\,a^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a - b*x^4)^(3/4)),x)
```

```
[Out] -(atan((a - b*x^4)^(1/4)/a^(1/4)) + atanh((a - b*x^4)^(1/4)/a^(1/4)))/(2*a^
(3/4))
```

sympy [C] time = 2.39, size = 42, normalized size = 0.74

$$-\frac{e^{-\frac{3i\pi}{4}}\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{a}{bx^4}\right)}{4b^{\frac{3}{4}}x^3\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x**4+a)**(3/4),x)
```

```
[Out] -exp(-3*I*pi/4)*gamma(3/4)*hyper((3/4, 3/4), (7/4,), a/(b*x**4))/(4*b**(3/4
)*x**3*gamma(7/4))
```

3.1240 $\int \frac{1}{x^5\left(a-bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=81

$$-\frac{3b\tan^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{8a^{7/4}}-\frac{3b\tanh^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{8a^{7/4}}-\frac{\sqrt[4]{a-bx^4}}{4ax^4}$$

```
[Out] -1/4*(-b*x^4+a)^(1/4)/a/x^4-3/8*b*arctan((-b*x^4+a)^(1/4)/a^(1/4))/a^(7/4)-
3/8*b*arctanh((-b*x^4+a)^(1/4)/a^(1/4))/a^(7/4)
```

Rubi [A] time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {266, 51, 63, 212, 206, 203}

$$-\frac{3b\tan^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{8a^{7/4}}-\frac{3b\tanh^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{8a^{7/4}}-\frac{\sqrt[4]{a-bx^4}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a - b*x^4)^(3/4)),x]
```

```
[Out] -(a - b*x^4)^(1/4)/(4*a*x^4) - (3*b*ArcTan[(a - b*x^4)^(1/4)/a^(1/4)])/(8*a
^(7/4)) - (3*b*ArcTanh[(a - b*x^4)^(1/4)/a^(1/4)])/(8*a^(7/4))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
```

```
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^5\left(a-bx^4\right)^{3/4}}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x^2(a-bx)^{3/4}}\,dx, x, x^4\right)$$

$$= -\frac{\sqrt[4]{a-bx^4}}{4ax^4} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{x(a-bx)^{3/4}}\,dx, x, x^4\right)}{16a}$$

$$= -\frac{\sqrt[4]{a-bx^4}}{4ax^4} - \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}-\frac{x^4}{b}}\,dx, x, \sqrt[4]{a-bx^4}\right)}{4a}$$

$$= -\frac{\sqrt[4]{a-bx^4}}{4ax^4} - \frac{(3b)\,\text{Subst}\left(\int \frac{1}{\sqrt{a}-x^2}\,dx, x, \sqrt[4]{a-bx^4}\right)}{8a^{3/2}} - \frac{(3b)\,\text{Subst}\left(\int \frac{1}{\sqrt{a}+x^2}\,dx, x, \sqrt[4]{a-}\right.}{8a^{3/2}}$$

$$= -\frac{\sqrt[4]{a-bx^4}}{4ax^4} - \frac{3b\tan^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{8a^{7/4}} - \frac{3b\tanh^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{8a^{7/4}}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.46

$$-\frac{b\sqrt[4]{a-bx^4}\;{}_2F_1\left(\frac{1}{4},2;\frac{5}{4};1-\frac{bx^4}{a}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a - b*x^4)^(3/4)),x]
```

```
[Out] -((b*(a - b*x^4)^(1/4)*Hypergeometric2F1[1/4, 2, 5/4, 1 - (b*x^4)/a])/a^2)
```

fricas [B] time = 0.73, size = 199, normalized size = 2.46

$$\frac{12\,ax^4\left(\frac{b^4}{a^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-bx^4+a\right)^{\frac{1}{4}}a^5b\left(\frac{b^4}{a^7}\right)^{\frac{3}{4}}-\sqrt{a^4\sqrt{\frac{b^4}{a^7}}+\sqrt{-bx^4+a}\,b^2}\,a^5\left(\frac{b^4}{a^7}\right)^{\frac{3}{4}}}{b^4}\right)-3\,ax^4\left(\frac{b^4}{a^7}\right)^{\frac{1}{4}}\log\left(3\,a^2\left(\frac{b^4}{a^7}\right)^{\frac{1}{4}}+3\left(-bx^4+a\right.\right.}{16\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/16*(12*a*x^4*(b^4/a^7)^(1/4)*arctan(-((-b*x^4 + a)^(1/4)*a^5*b*(b^4/a^7)^
(3/4) - sqrt(a^4*sqrt(b^4/a^7) + sqrt(-b*x^4 + a)*b^2)*a^5*(b^4/a^7)^(3/4))
/b^4) - 3*a*x^4*(b^4/a^7)^(1/4)*log(3*a^2*(b^4/a^7)^(1/4) + 3*(-b*x^4 + a)^
(1/4)*b) + 3*a*x^4*(b^4/a^7)^(1/4)*log(-3*a^2*(b^4/a^7)^(1/4) + 3*(-b*x^4 +
 a)^(1/4)*b) - 4*(-b*x^4 + a)^(1/4))/(a*x^4)
```

giac [B] time = 0.17, size = 228, normalized size = 2.81

$$\frac{6\sqrt{2}(-a)^{\frac{1}{4}}b^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(-bx^4+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a^2}+\frac{6\sqrt{2}(-a)^{\frac{1}{4}}b^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(-bx^4+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a^2}+\frac{3\sqrt{2}(-a)^{\frac{1}{4}}b^2\log\left(\sqrt{2}\left(-bx^4+a\right)^{\frac{1}{4}}(-a)^{\frac{1}{4}}-\right.}{a^2}$$

$$-\frac{}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] -1/32*(6*sqrt(2)*(-a)^(1/4)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*
(-b*x^4 + a)^(1/4))/(-a)^(1/4))/a^2 + 6*sqrt(2)*(-a)^(1/4)*b^2*arctan(-1/2*
sqrt(2)*(sqrt(2)*(-a)^(1/4) - 2*(-b*x^4 + a)^(1/4))/(-a)^(1/4))/a^2 + 3*sqr
t(2)*(-a)^(1/4)*b^2*log(sqrt(2)*(-b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(-b*x^4
 + a) + sqrt(-a))/a^2 + 3*sqrt(2)*b^2*log(-sqrt(2)*(-b*x^4 + a)^(1/4)*(-a)^
(1/4) + sqrt(-b*x^4 + a) + sqrt(-a))/((-a)^(3/4)*a) + 8*(-b*x^4 + a)^(1/4)*
b/(a*x^4))/b
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^4+a\right)^{\frac{3}{4}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-b*x^4+a)^(3/4),x)
```

```
[Out] int(1/x^5/(-b*x^4+a)^(3/4),x)
```

maxima [A] time = 2.42, size = 98, normalized size = 1.21

$$-\frac{\left(-bx^4+a\right)^{\frac{1}{4}}b}{4\left(\left(bx^4-a\right)a+a^2\right)}-\frac{3\left(\frac{2\,b\arctan\left(\frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{3}{4}}}-\frac{b\log\left(\frac{\left(-bx^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{\left(-bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{3}{4}}}\right)}{16\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] -1/4*(-b*x^4 + a)^(1/4)*b/((b*x^4 - a)*a + a^2) - 3/16*(2*b*arctan((-b*x^4
+ a)^(1/4)/a^(1/4))/a^(3/4) - b*log(((-b*x^4 + a)^(1/4) - a^(1/4))/((-b*x^4
 + a)^(1/4) + a^(1/4)))/a^(3/4))/a
```

mupad [B] time = 1.30, size = 61, normalized size = 0.75

$$-\frac{\left(a-b\,x^4\right)^{1/4}}{4\,a\,x^4}-\frac{3\,b\,\mathrm{atan}\left(\frac{\left(a-b\,x^4\right)^{1/4}}{a^{1/4}}\right)}{8\,a^{7/4}}-\frac{3\,b\,\mathrm{atanh}\left(\frac{\left(a-b\,x^4\right)^{1/4}}{a^{1/4}}\right)}{8\,a^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a - b*x^4)^(3/4)),x)
```

```
[Out] - (a - b*x^4)^(1/4)/(4*a*x^4) - (3*b*atan((a - b*x^4)^(1/4)/a^(1/4)))/(8*a^
(7/4)) - (3*b*atanh((a - b*x^4)^(1/4)/a^(1/4)))/(8*a^(7/4))
```

sympy [C] time = 2.31, size = 39, normalized size = 0.48

$$\frac{e^{\frac{i\pi}{4}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4}, \frac{7}{4} \\ \frac{11}{4}\end{matrix}\right| \frac{a}{bx^4}\right)}{4b^{\frac{3}{4}}x^7\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-b*x**4+a)**(3/4),x)
```

```
[Out] exp(I*pi/4)*gamma(7/4)*hyper((3/4, 7/4), (11/4,), a/(b*x**4))/(4*b**(3/4)*x
**7*gamma(11/4))
```

3.1241 $\int \frac{1}{x^9\left(a-bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=108

$$-\frac{21b^2\tan^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{64a^{11/4}}-\frac{21b^2\tanh^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{64a^{11/4}}-\frac{7b\sqrt[4]{a-bx^4}}{32a^2x^4}-\frac{\sqrt[4]{a-bx^4}}{8ax^8}$$

```
[Out] -1/8*(-b*x^4+a)^(1/4)/a/x^8-7/32*b*(-b*x^4+a)^(1/4)/a^2/x^4-21/64*b^2*arcta
n((-b*x^4+a)^(1/4)/a^(1/4))/a^(11/4)-21/64*b^2*arctanh((-b*x^4+a)^(1/4)/a^(
1/4))/a^(11/4)
```

Rubi [A] time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {266, 51, 63, 212, 206, 203}

$$-\frac{21b^2\tan^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{64a^{11/4}}-\frac{21b^2\tanh^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{64a^{11/4}}-\frac{7b\sqrt[4]{a-bx^4}}{32a^2x^4}-\frac{\sqrt[4]{a-bx^4}}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(a - b*x^4)^(3/4)),x]
```

```
[Out] -(a - b*x^4)^(1/4)/(8*a*x^8) - (7*b*(a - b*x^4)^(1/4))/(32*a^2*x^4) - (21*b
^2*ArcTan[(a - b*x^4)^(1/4)/a^(1/4)])/(64*a^(11/4)) - (21*b^2*ArcTanh[(a -
b*x^4)^(1/4)/a^(1/4)])/(64*a^(11/4))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^9\left(a-bx^4\right)^{3/4}}\,dx = \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x^3(a-bx)^{3/4}}\,dx, x, x^4\right)$$

$$= -\frac{\sqrt[4]{a-bx^4}}{8ax^8} + \frac{(7b)\operatorname{Subst}\left(\int \frac{1}{x^2(a-bx)^{3/4}}\,dx, x, x^4\right)}{32a}$$

$$= -\frac{\sqrt[4]{a-bx^4}}{8ax^8} - \frac{7b\sqrt[4]{a-bx^4}}{32a^2x^4} + \frac{\left(21b^2\right)\operatorname{Subst}\left(\int \frac{1}{x(a-bx)^{3/4}}\,dx, x, x^4\right)}{128a^2}$$

$$= -\frac{\sqrt[4]{a-bx^4}}{8ax^8} - \frac{7b\sqrt[4]{a-bx^4}}{32a^2x^4} - \frac{(21b)\operatorname{Subst}\left(\int \frac{1}{\frac{a}{b}-\frac{x^4}{b}}\,dx, x, \sqrt[4]{a-bx^4}\right)}{32a^2}$$

$$= -\frac{\sqrt[4]{a-bx^4}}{8ax^8} - \frac{7b\sqrt[4]{a-bx^4}}{32a^2x^4} - \frac{\left(21b^2\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a}-x^2}\,dx, x, \sqrt[4]{a-bx^4}\right)}{64a^{5/2}} - \frac{\left(21b^2\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a}+x^2}\,dx, x, \sqrt[4]{a-bx^4}\right)}{64a^{5/2}}$$

$$= -\frac{\sqrt[4]{a-bx^4}}{8ax^8} - \frac{7b\sqrt[4]{a-bx^4}}{32a^2x^4} - \frac{21b^2\tan^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{64a^{11/4}} - \frac{21b^2\tanh^{-1}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{64a^{11/4}}$$

Mathematica [C] time = 0.02, size = 39, normalized size = 0.36

$$-\frac{b^2\sqrt[4]{a-bx^4}\;{}_2F_1\left(\frac{1}{4},3;\frac{5}{4};1-\frac{bx^4}{a}\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^9*(a - b*x^4)^(3/4)),x]
```

```
[Out] -((b^2*(a - b*x^4)^(1/4)*Hypergeometric2F1[1/4, 3, 5/4, 1 - (b*x^4)/a])/a^3
)
```

fricas [B] time = 0.70, size = 221, normalized size = 2.05

$$\frac{84\,a^2x^8\left(\frac{b^8}{a^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-bx^4+a\right)^{\frac{1}{4}}a^8b^2\left(\frac{b^8}{a^{11}}\right)^{\frac{3}{4}}-\sqrt{a^6\sqrt{\frac{b^8}{a^{11}}}+\sqrt{-bx^4+a}\,b^4\,a^8\left(\frac{b^8}{a^{11}}\right)^{\frac{3}{4}}}}{b^8}\right) - 21\,a^2x^8\left(\frac{b^8}{a^{11}}\right)^{\frac{1}{4}}\log\left(21\,a^3\left(\frac{b^8}{a^{11}}\right)^{\frac{1}{4}}+21\right.}{128\,a^2x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/128*(84*a^2*x^8*(b^8/a^11)^(1/4)*arctan(-((-b*x^4 + a)^(1/4)*a^8*b^2*(b^8
/a^11)^(3/4) - sqrt(a^6*sqrt(b^8/a^11) + sqrt(-b*x^4 + a)*b^4)*a^8*(b^8/a^1
1)^(3/4))/b^8) - 21*a^2*x^8*(b^8/a^11)^(1/4)*log(21*a^3*(b^8/a^11)^(1/4) +
21*(-b*x^4 + a)^(1/4)*b^2) + 21*a^2*x^8*(b^8/a^11)^(1/4)*log(-21*a^3*(b^8/a
^11)^(1/4) + 21*(-b*x^4 + a)^(1/4)*b^2) - 4*(7*b*x^4 + 4*a)*(-b*x^4 + a)^(1
/4))/(a^2*x^8)
```

giac [B] time = 0.17, size = 252, normalized size = 2.33

$$\frac{42\sqrt{2}\,(-a)^{\frac{1}{4}}b^3\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}+2\left(-bx^4+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{a^3}+\frac{42\sqrt{2}\,(-a)^{\frac{1}{4}}b^3\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}-2\left(-bx^4+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{a^3}+\frac{21\sqrt{2}\,(-a)^{\frac{1}{4}}b^3\log\left(\sqrt{2}\left(-bx^4+a\right)^{\frac{1}{4}}(-$$
$$-\frac{}{256\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] -1/256*(42*sqrt(2)*(-a)^(1/4)*b^3*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) +
2*(-b*x^4 + a)^(1/4))/(-a)^(1/4))/a^3 + 42*sqrt(2)*(-a)^(1/4)*b^3*arctan(-1
/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) - 2*(-b*x^4 + a)^(1/4))/(-a)^(1/4))/a^3 + 21
*sqrt(2)*(-a)^(1/4)*b^3*log(sqrt(2)*(-b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(-b
*x^4 + a) + sqrt(-a))/a^3 + 21*sqrt(2)*b^3*log(-sqrt(2)*(-b*x^4 + a)^(1/4)*
(-a)^(1/4) + sqrt(-b*x^4 + a) + sqrt(-a))/((-a)^(3/4)*a^2) - 8*(7*(-b*x^4 +
 a)^(5/4)*b^3 - 11*(-b*x^4 + a)^(1/4)*a*b^3)/(a^2*b^2*x^8))/b
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^4+a\right)^{\frac{3}{4}}x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(-b*x^4+a)^(3/4),x)
```

```
[Out] int(1/x^9/(-b*x^4+a)^(3/4),x)
```

maxima [A] time = 2.38, size = 141, normalized size = 1.31

$$\frac{7\left(-bx^4+a\right)^{\frac{5}{4}}b^2-11\left(-bx^4+a\right)^{\frac{1}{4}}ab^2}{32\left(\left(bx^4-a\right)^2a^2+2\left(bx^4-a\right)a^3+a^4\right)}-\frac{21\left(\frac{2\,b^2\arctan\left(\frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{3}{4}}}-\frac{b^2\log\left(\frac{\left(-bx^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{\left(-bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{3}{4}}}\right)}{128\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] 1/32*(7*(-b*x^4 + a)^(5/4)*b^2 - 11*(-b*x^4 + a)^(1/4)*a*b^2)/((b*x^4 - a)^
2*a^2 + 2*(b*x^4 - a)*a^3 + a^4) - 21/128*(2*b^2*arctan((-b*x^4 + a)^(1/4)/
a^(1/4))/a^(3/4) - b^2*log(((-b*x^4 + a)^(1/4) - a^(1/4))/((-b*x^4 + a)^(1/
4) + a^(1/4)))/a^(3/4))/a^2
```

mupad [B] time = 1.39, size = 86, normalized size = 0.80

$$\frac{7\left(a-b\,x^4\right)^{5/4}}{32\,a^2\,x^8}-\frac{11\left(a-b\,x^4\right)^{1/4}}{32\,a\,x^8}-\frac{21\,b^2\,\mathrm{atan}\left(\frac{\left(a-b\,x^4\right)^{1/4}}{a^{1/4}}\right)}{64\,a^{11/4}}+\frac{b^2\,\mathrm{atan}\left(\frac{\left(a-b\,x^4\right)^{1/4}1\mathrm{i}}{a^{1/4}}\right)21\mathrm{i}}{64\,a^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(a - b*x^4)^(3/4)),x)
```

```
[Out] (b^2*atan(((a - b*x^4)^(1/4)*1i)/a^(1/4))*21i)/(64*a^(11/4)) - (21*b^2*atan
((a - b*x^4)^(1/4)/a^(1/4)))/(64*a^(11/4)) - (11*(a - b*x^4)^(1/4))/(32*a*x
^8) + (7*(a - b*x^4)^(5/4))/(32*a^2*x^8)
```

sympy [C] time = 3.26, size = 42, normalized size = 0.39

$$-\frac{e^{-\frac{3i\pi}{4}}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4}, \frac{11}{4} \\ \frac{15}{4}\end{matrix}\middle| \frac{a}{bx^4}\right)}{4b^{\frac{3}{4}}x^{11}\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(-b*x**4+a)**(3/4),x)
```

```
[Out] -exp(-3*I*pi/4)*gamma(11/4)*hyper((3/4, 11/4), (15/4,), a/(b*x**4))/(4*b**(
3/4)*x**11*gamma(15/4))
```

3.1242 $\int \frac{x^{13}}{\left(a-bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=133

$$\frac{40a^{7/2}\left(1-\frac{bx^4}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{77b^{7/2}\left(a-bx^4\right)^{3/4}}-\frac{20a^2x^2\sqrt[4]{a-bx^4}}{77b^3}-\frac{10ax^6\sqrt[4]{a-bx^4}}{77b^2}-\frac{x^{10}\sqrt[4]{a-bx^4}}{11b}$$

```
[Out] -20/77*a^2*x^2*(-b*x^4+a)^(1/4)/b^3-10/77*a*x^6*(-b*x^4+a)^(1/4)/b^2-1/11*x
^10*(-b*x^4+a)^(1/4)/b+40/77*a^(7/2)*(1-b*x^4/a)^(3/4)*(cos(1/2*arcsin(x^2*
b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x^2*b^(1/2)/a^(1/2)))*EllipticF(s
in(1/2*arcsin(x^2*b^(1/2)/a^(1/2))),2^(1/2))/b^(7/2)/(-b*x^4+a)^(3/4)
```

Rubi [A] time = 0.09, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {275, 321, 233, 232}

$$-\frac{20a^2x^2\sqrt[4]{a-bx^4}}{77b^3}+\frac{40a^{7/2}\left(1-\frac{bx^4}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{77b^{7/2}\left(a-bx^4\right)^{3/4}}-\frac{10ax^6\sqrt[4]{a-bx^4}}{77b^2}-\frac{x^{10}\sqrt[4]{a-bx^4}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/(a - b*x^4)^(3/4),x]
```

```
[Out] (-20*a^2*x^2*(a - b*x^4)^(1/4))/(77*b^3) - (10*a*x^6*(a - b*x^4)^(1/4))/(77
*b^2) - (x^10*(a - b*x^4)^(1/4))/(11*b) + (40*a^(7/2)*(1 - (b*x^4)/a)^(3/4)
*EllipticF[ArcSin[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(77*b^(7/2)*(a - b*x^4)^(3/
4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^{13}}{\left(a-bx^4\right)^{3/4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^6}{\left(a-bx^2\right)^{3/4}}\,dx, x, x^2\right)$$

$$= -\frac{x^{10}\sqrt[4]{a-bx^4}}{11b} + \frac{(5a)\,\text{Subst}\left(\int \frac{x^4}{\left(a-bx^2\right)^{3/4}}\,dx, x, x^2\right)}{11b}$$

$$= -\frac{10ax^6\sqrt[4]{a-bx^4}}{77b^2} - \frac{x^{10}\sqrt[4]{a-bx^4}}{11b} + \frac{\left(30a^2\right)\text{Subst}\left(\int \frac{x^2}{\left(a-bx^2\right)^{3/4}}\,dx, x, x^2\right)}{77b^2}$$

$$= -\frac{20a^2x^2\sqrt[4]{a-bx^4}}{77b^3} - \frac{10ax^6\sqrt[4]{a-bx^4}}{77b^2} - \frac{x^{10}\sqrt[4]{a-bx^4}}{11b} + \frac{\left(20a^3\right)\text{Subst}\left(\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx, x, x\right.}{77b^3}$$

$$= -\frac{20a^2x^2\sqrt[4]{a-bx^4}}{77b^3} - \frac{10ax^6\sqrt[4]{a-bx^4}}{77b^2} - \frac{x^{10}\sqrt[4]{a-bx^4}}{11b} + \frac{\left(20a^3\left(1-\frac{bx^4}{a}\right)^{3/4}\right)\text{Subst}\left(\int \frac{}{\left(1-\frac{b}{}\right.}\right.}{77b^3\left(a-bx^4\right)^{3/4}}$$

$$= -\frac{20a^2x^2\sqrt[4]{a-bx^4}}{77b^3} - \frac{10ax^6\sqrt[4]{a-bx^4}}{77b^2} - \frac{x^{10}\sqrt[4]{a-bx^4}}{11b} + \frac{40a^{7/2}\left(1-\frac{bx^4}{a}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}}{\sqrt{}}\right.\right.}{77b^{7/2}\left(a-bx^4\right)^{3/4}}$$

Mathematica [C] time = 0.04, size = 92, normalized size = 0.69

$$\frac{x^2\left(20a^3\left(1-\frac{bx^4}{a}\right)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{bx^4}{a}\right) - 20a^3 + 10a^2bx^4 + 3ab^2x^8 + 7b^3x^{12}\right)}{77b^3\left(a-bx^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13/(a - b*x^4)^(3/4),x]
```

```
[Out] (x^2*(-20*a^3 + 10*a^2*b*x^4 + 3*a*b^2*x^8 + 7*b^3*x^12 + 20*a^3*(1 - (b*x^
4)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, (b*x^4)/a]))/(77*b^3*(a - b*x^
4)^(3/4))
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^4+a\right)^{\frac{1}{4}}x^{13}}{bx^4-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^4 + a)^(1/4)*x^13/(b*x^4 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{\left(-bx^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(-b*x^4+a)^(3/4),x, algorithm="giac")
[Out] integrate(x^13/(-b*x^4 + a)^(3/4), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{\left(-b\,x^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(-b*x^4+a)^(3/4),x)
[Out] int(x^13/(-b*x^4+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{\left(-bx^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(-b*x^4+a)^(3/4),x, algorithm="maxima")
[Out] integrate(x^13/(-b*x^4 + a)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{13}}{\left(a-b\,x^4\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(a - b*x^4)^(3/4),x)
[Out] int(x^13/(a - b*x^4)^(3/4), x)
```

sympy [C] time = 1.76, size = 29, normalized size = 0.22

$$\frac{x^{14}\,{}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\right|\frac{bx^4e^{2i\pi}}{a}\right)}{14a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(-b*x**4+a)**(3/4),x)
[Out] x**14*hyper((3/4, 7/2), (9/2,), b*x**4*exp_polar(2*I*pi)/a)/(14*a**(3/4))
```

3.1243 $\int \frac{x^9}{\left(a-bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=108

$$\frac{4a^{5/2}\left(1-\frac{bx^4}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{7b^{5/2}\left(a-bx^4\right)^{3/4}}-\frac{2ax^2\sqrt[4]{a-bx^4}}{7b^2}-\frac{x^6\sqrt[4]{a-bx^4}}{7b}$$

```
[Out] -2/7*a*x^2*(-b*x^4+a)^(1/4)/b^2-1/7*x^6*(-b*x^4+a)^(1/4)/b+4/7*a^(5/2)*(1-b
*x^4/a)^(3/4)*(cos(1/2*arcsin(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin
(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arcsin(x^2*b^(1/2)/a^(1/2))),2^(1/
2))/b^(5/2)/(-b*x^4+a)^(3/4)
```

Rubi [A] time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {275, 321, 233, 232}

$$\frac{4a^{5/2}\left(1-\frac{bx^4}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{7b^{5/2}\left(a-bx^4\right)^{3/4}}-\frac{2ax^2\sqrt[4]{a-bx^4}}{7b^2}-\frac{x^6\sqrt[4]{a-bx^4}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a - b*x^4)^(3/4),x]
```

```
[Out] (-2*a*x^2*(a - b*x^4)^(1/4))/(7*b^2) - (x^6*(a - b*x^4)^(1/4))/(7*b) + (4*a
^(5/2)*(1 - (b*x^4)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])
/(7*b^(5/2)*(a - b*x^4)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^9}{\left(a-bx^4\right)^{3/4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{\left(a-bx^2\right)^{3/4}}\,dx, x, x^2\right)$$

$$= -\frac{x^6\sqrt[4]{a-bx^4}}{7b} + \frac{(3a)\,\text{Subst}\left(\int \frac{x^2}{\left(a-bx^2\right)^{3/4}}\,dx, x, x^2\right)}{7b}$$

$$= -\frac{2ax^2\sqrt[4]{a-bx^4}}{7b^2} - \frac{x^6\sqrt[4]{a-bx^4}}{7b} + \frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx, x, x^2\right)}{7b^2}$$

$$= -\frac{2ax^2\sqrt[4]{a-bx^4}}{7b^2} - \frac{x^6\sqrt[4]{a-bx^4}}{7b} + \frac{\left(2a^2\left(1-\frac{bx^4}{a}\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx, x, x^2\right)}{7b^2\left(a-bx^4\right)^{3/4}}$$

$$= -\frac{2ax^2\sqrt[4]{a-bx^4}}{7b^2} - \frac{x^6\sqrt[4]{a-bx^4}}{7b} + \frac{4a^{5/2}\left(1-\frac{bx^4}{a}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|\,2\right)}{7b^{5/2}\left(a-bx^4\right)^{3/4}}$$

Mathematica [C] time = 0.03, size = 79, normalized size = 0.73

$$\frac{x^2\left(2a^2\left(1-\frac{bx^4}{a}\right)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{bx^4}{a}\right) - 2a^2 + abx^4 + b^2x^8\right)}{7b^2\left(a-bx^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a - b*x^4)^(3/4),x]
```

```
[Out] (x^2*(-2*a^2 + a*b*x^4 + b^2*x^8 + 2*a^2*(1 - (b*x^4)/a)^(3/4)*Hypergeometr
ic2F1[1/2, 3/4, 3/2, (b*x^4)/a]))/(7*b^2*(a - b*x^4)^(3/4))
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^4+a\right)^{\frac{1}{4}}x^9}{bx^4-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^4 + a)^(1/4)*x^9/(b*x^4 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(-bx^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^9/(-b*x^4 + a)^(3/4), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(-b\,x^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(-b*x^4+a)^(3/4),x)
[Out] int(x^9/(-b*x^4+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(-bx^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(-b*x^4+a)^(3/4),x, algorithm="maxima")
[Out] integrate(x^9/(-b*x^4 + a)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^9}{\left(a-b\,x^4\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a - b*x^4)^(3/4),x)
[Out] int(x^9/(a - b*x^4)^(3/4), x)
```

sympy [C] time = 1.41, size = 29, normalized size = 0.27

$$\frac{x^{10}\,{}_2F_1\left(\left.\begin{matrix}\frac{3}{4}, \frac{5}{2}\\ \frac{7}{2}\end{matrix}\right| \frac{bx^4e^{2i\pi}}{a}\right)}{10a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(-b*x**4+a)**(3/4),x)
[Out] x**10*hyper((3/4, 5/2), (7/2,), b*x**4*exp_polar(2*I*pi)/a)/(10*a**(3/4))
```

3.1244 $\int \frac{x^5}{\left(a-bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=85

$$\frac{2a^{3/2}\left(1-\frac{bx^4}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{3b^{3/2}\left(a-bx^4\right)^{3/4}}-\frac{x^2\sqrt[4]{a-bx^4}}{3b}$$

```
[Out] -1/3*x^2*(-b*x^4+a)^(1/4)/b+2/3*a^(3/2)*(1-b*x^4/a)^(3/4)*(cos(1/2*arcsin(x
^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x^2*b^(1/2)/a^(1/2)))*Elliptic
F(sin(1/2*arcsin(x^2*b^(1/2)/a^(1/2))),2^(1/2))/b^(3/2)/(-b*x^4+a)^(3/4)
```

Rubi [A] time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {275, 321, 233, 232}

$$\frac{2a^{3/2}\left(1-\frac{bx^4}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{3b^{3/2}\left(a-bx^4\right)^{3/4}}-\frac{x^2\sqrt[4]{a-bx^4}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a - b*x^4)^(3/4),x]
```

```
[Out] -(x^2*(a - b*x^4)^(1/4))/(3*b) + (2*a^(3/2)*(1 - (b*x^4)/a)^(3/4)*EllipticF
[ArcSin[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(3*b^(3/2)*(a - b*x^4)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^5}{\left(a-bx^4\right)^{3/4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\left(a-bx^2\right)^{3/4}}\,dx, x, x^2\right)$$

$$= -\frac{x^2\sqrt[4]{a-bx^4}}{3b} + \frac{a\,\text{Subst}\left(\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx, x, x^2\right)}{3b}$$

$$= -\frac{x^2\sqrt[4]{a-bx^4}}{3b} + \frac{\left(a\left(1-\frac{bx^4}{a}\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx, x, x^2\right)}{3b\left(a-bx^4\right)^{3/4}}$$

$$= -\frac{x^2\sqrt[4]{a-bx^4}}{3b} + \frac{2a^{3/2}\left(1-\frac{bx^4}{a}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle| 2\right)}{3b^{3/2}\left(a-bx^4\right)^{3/4}}$$

Mathematica [C] time = 0.02, size = 66, normalized size = 0.78

$$\frac{x^2\left(a\left(1-\frac{bx^4}{a}\right)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{bx^4}{a}\right)-a+bx^4\right)}{3b\left(a-bx^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a - b*x^4)^(3/4),x]
```

```
[Out] (x^2*(-a + b*x^4 + a*(1 - (b*x^4)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2,
 (b*x^4)/a]))/(3*b*(a - b*x^4)^(3/4))
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^4+a\right)^{\frac{1}{4}}x^5}{bx^4-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^4 + a)^(1/4)*x^5/(b*x^4 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(-bx^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^5/(-b*x^4 + a)^(3/4), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(-b\,x^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-b*x^4+a)^(3/4),x)
[Out] int(x^5/(-b*x^4+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(-bx^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-b*x^4+a)^(3/4),x, algorithm="maxima")
[Out] integrate(x^5/(-b*x^4 + a)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\left(a-b\,x^4\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a - b*x^4)^(3/4),x)
[Out] int(x^5/(a - b*x^4)^(3/4), x)
```

sympy [C] time = 1.88, size = 29, normalized size = 0.34

$$\frac{x^6\,{}_2F_1\left(\begin{matrix}\frac{3}{4}, \frac{3}{2} \\ \frac{5}{2}\end{matrix}\middle| \frac{bx^4e^{2i\pi}}{a}\right)}{6a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-b*x**4+a)**(3/4),x)
[Out] x**6*hyper((3/4, 3/2), (5/2,), b*x**4*exp_polar(2*I*pi)/a)/(6*a**(3/4))
```

3.1245 $\int \frac{x}{\left(a-bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=59

$$\frac{\sqrt{a}\left(1-\frac{bx^4}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\left(a-bx^4\right)^{3/4}}$$

```
[Out] (1-b*x^4/a)^(3/4)*(cos(1/2*arcsin(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*ar
csin(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arcsin(x^2*b^(1/2)/a^(1/2))),2
^(1/2))*a^(1/2)/(-b*x^4+a)^(3/4)/b^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {275, 233, 232}

$$\frac{\sqrt{a}\left(1-\frac{bx^4}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\left(a-bx^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a - b*x^4)^(3/4),x]
```

```
[Out] (Sqrt[a]*(1 - (b*x^4)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2
])/(Sqrt[b]*(a - b*x^4)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x}{\left(a-bx^4\right)^{3/4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx, x, x^2\right)$$
$$= \frac{\left(1-\frac{bx^4}{a}\right)^{3/4}\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx, x, x^2\right)}{2\left(a-bx^4\right)^{3/4}}$$
$$= \frac{\sqrt{a}\left(1-\frac{bx^4}{a}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|\,2\right)}{\sqrt{b}\left(a-bx^4\right)^{3/4}}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.88

$$\frac{x^2\left(1-\frac{bx^4}{a}\right)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{bx^4}{a}\right)}{2\left(a-bx^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a - b*x^4)^(3/4),x]
```

```
[Out] (x^2*(1 - (b*x^4)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, (b*x^4)/a])/(2*
(a - b*x^4)^(3/4))
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^4+a\right)^{\frac{1}{4}}x}{bx^4-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^4 + a)^(1/4)*x/(b*x^4 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-bx^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x/(-b*x^4 + a)^(3/4), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-b\,x^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^4+a)^(3/4),x)
```

```
[Out] int(x/(-b*x^4+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-bx^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x/(-b*x^4 + a)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\left(a-b\,x^4\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a - b*x^4)^(3/4),x)
```

```
[Out] int(x/(a - b*x^4)^(3/4), x)
```

sympy [C] time = 1.45, size = 29, normalized size = 0.49

$$\frac{x^2\,{}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{3}{4} \\ \frac{3}{2}\end{matrix}\middle| \frac{bx^4e^{2i\pi}}{a}\right)}{2a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**4+a)**(3/4),x)
```

```
[Out] x**2*hyper((1/2, 3/4), (3/2,), b*x**4*exp_polar(2*I*pi)/a)/(2*a**(3/4))
```

3.1246 $\int \frac{1}{x^3\left(a-bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=85

$$\frac{\sqrt{b}\left(1-\frac{bx^4}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}\left(a-bx^4\right)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{2ax^2}$$

```
[Out] -1/2*(-b*x^4+a)^(1/4)/a/x^2+1/2*(1-b*x^4/a)^(3/4)*(cos(1/2*arcsin(x^2*b^(1/
2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/
2*arcsin(x^2*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)/(-b*x^4+a)^(3/4)/a^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {275, 325, 233, 232}

$$\frac{\sqrt{b}\left(1-\frac{bx^4}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}\left(a-bx^4\right)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a - b*x^4)^(3/4)),x]
```

```
[Out] -(a - b*x^4)^(1/4)/(2*a*x^2) + (Sqrt[b]*(1 - (b*x^4)/a)^(3/4)*EllipticF[Arc
Sin[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(2*Sqrt[a]*(a - b*x^4)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^3\left(a-bx^4\right)^{3/4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\left(a-bx^2\right)^{3/4}}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt[4]{a-bx^4}}{2ax^2} + \frac{b\,\text{Subst}\left(\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx, x, x^2\right)}{4a}$$

$$= -\frac{\sqrt[4]{a-bx^4}}{2ax^2} + \frac{\left(b\left(1-\frac{bx^4}{a}\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx, x, x^2\right)}{4a\left(a-bx^4\right)^{3/4}}$$

$$= -\frac{\sqrt[4]{a-bx^4}}{2ax^2} + \frac{\sqrt{b}\left(1-\frac{bx^4}{a}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|\, 2\right)}{2\sqrt{a}\left(a-bx^4\right)^{3/4}}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.61

$$-\frac{\left(1-\frac{bx^4}{a}\right)^{3/4}\,{}_2F_1\left(-\frac{1}{2}, \frac{3}{4}; \frac{1}{2}; \frac{bx^4}{a}\right)}{2x^2\left(a-bx^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a - b*x^4)^(3/4)),x]
```

```
[Out] -1/2*((1 - (b*x^4)/a)^(3/4)*Hypergeometric2F1[-1/2, 3/4, 1/2, (b*x^4)/a])/(
x^2*(a - b*x^4)^(3/4))
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{bx^7-ax^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^4 + a)^(1/4)/(b*x^7 - a*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{3}{4}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^4 + a)^(3/4)*x^3), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^4+a\right)^{\frac{3}{4}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-b*x^4+a)^(3/4),x)
[Out] int(1/x^3/(-b*x^4+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{3}{4}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-b*x^4+a)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((-b*x^4 + a)^(3/4)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\left(a-b\,x^4\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a - b*x^4)^(3/4)),x)
[Out] int(1/(x^3*(a - b*x^4)^(3/4)), x)
```

sympy [C] time = 2.07, size = 32, normalized size = 0.38

$$-\frac{{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2}, \frac{3}{4} \\ \frac{1}{2}\end{matrix}\right| \frac{bx^4e^{2i\pi}}{a}\right)}{2a^{\frac{3}{4}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-b*x**4+a)**(3/4),x)
[Out] -hyper((-1/2, 3/4), (1/2,), b*x**4*exp_polar(2*I*pi)/a)/(2*a**(3/4)*x**2)
```

3.1247 $\int \frac{1}{x^7\left(a-bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=108

$$\frac{5b^{3/2}\left(1-\frac{bx^4}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{12a^{3/2}\left(a-bx^4\right)^{3/4}}-\frac{5b\sqrt[4]{a-bx^4}}{12a^2x^2}-\frac{\sqrt[4]{a-bx^4}}{6ax^6}$$

```
[Out] -1/6*(-b*x^4+a)^(1/4)/a/x^6-5/12*b*(-b*x^4+a)^(1/4)/a^2/x^2+5/12*b^(3/2)*(1
-b*x^4/a)^(3/4)*(cos(1/2*arcsin(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcs
in(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arcsin(x^2*b^(1/2)/a^(1/2))),2^(
1/2))/a^(3/2)/(-b*x^4+a)^(3/4)
```

Rubi [A] time = 0.07, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {275, 325, 233, 232}

$$\frac{5b^{3/2}\left(1-\frac{bx^4}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{12a^{3/2}\left(a-bx^4\right)^{3/4}}-\frac{5b\sqrt[4]{a-bx^4}}{12a^2x^2}-\frac{\sqrt[4]{a-bx^4}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a - b*x^4)^(3/4)),x]
```

```
[Out] -(a - b*x^4)^(1/4)/(6*a*x^6) - (5*b*(a - b*x^4)^(1/4))/(12*a^2*x^2) + (5*b^
(3/2)*(1 - (b*x^4)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/
(12*a^(3/2)*(a - b*x^4)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^7\left(a-bx^4\right)^{3/4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^4\left(a-bx^2\right)^{3/4}}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt[4]{a-bx^4}}{6ax^6} + \frac{(5b)\,\text{Subst}\left(\int \frac{1}{x^2\left(a-bx^2\right)^{3/4}}\,dx, x, x^2\right)}{12a}$$

$$= -\frac{\sqrt[4]{a-bx^4}}{6ax^6} - \frac{5b\sqrt[4]{a-bx^4}}{12a^2x^2} + \frac{\left(5b^2\right)\text{Subst}\left(\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx, x, x^2\right)}{24a^2}$$

$$= -\frac{\sqrt[4]{a-bx^4}}{6ax^6} - \frac{5b\sqrt[4]{a-bx^4}}{12a^2x^2} + \frac{\left(5b^2\left(1-\frac{bx^4}{a}\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx, x, x^2\right)}{24a^2\left(a-bx^4\right)^{3/4}}$$

$$= -\frac{\sqrt[4]{a-bx^4}}{6ax^6} - \frac{5b\sqrt[4]{a-bx^4}}{12a^2x^2} + \frac{5b^{3/2}\left(1-\frac{bx^4}{a}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|\,2\right)}{12a^{3/2}\left(a-bx^4\right)^{3/4}}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.48

$$-\frac{\left(1-\frac{bx^4}{a}\right)^{3/4}\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};-\frac{1}{2};\frac{bx^4}{a}\right)}{6x^6\left(a-bx^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a - b*x^4)^(3/4)),x]
```

```
[Out] -1/6*((1 - (b*x^4)/a)^(3/4)*Hypergeometric2F1[-3/2, 3/4, -1/2, (b*x^4)/a])/
(x^6*(a - b*x^4)^(3/4))
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{bx^{11}-ax^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^4 + a)^(1/4)/(b*x^11 - a*x^7), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{3}{4}}x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^4 + a)^(3/4)*x^7), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^4+a\right)^{\frac{3}{4}}\,x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(-b*x^4+a)^(3/4),x)
[Out] int(1/x^7/(-b*x^4+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{3}{4}}x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-b*x^4+a)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((-b*x^4 + a)^(3/4)*x^7), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^7\left(a-b\,x^4\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a - b*x^4)^(3/4)),x)
[Out] int(1/(x^7*(a - b*x^4)^(3/4)), x)
```

sympy [C] time = 2.77, size = 34, normalized size = 0.31

$$-\frac{{}_2F_1\left(\left.\begin{matrix}-\frac{3}{2}, \frac{3}{4} \\ -\frac{1}{2}\end{matrix}\right| \frac{bx^4e^{2i\pi}}{a}\right)}{6a^{\frac{3}{4}}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(-b*x**4+a)**(3/4),x)
[Out] -hyper((-3/2, 3/4), (-1/2,), b*x**4*exp_polar(2*I*pi)/a)/(6*a**(3/4)*x**6)
```

3.1248 $\int \frac{1}{x^{11}\left(a-bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=133

$$\frac{3b^{5/2}\left(1-\frac{bx^4}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{8a^{5/2}\left(a-bx^4\right)^{3/4}}-\frac{3b^2\sqrt[4]{a-bx^4}}{8a^3x^2}-\frac{3b\sqrt[4]{a-bx^4}}{20a^2x^6}-\frac{\sqrt[4]{a-bx^4}}{10ax^{10}}$$

```
[Out] -1/10*(-b*x^4+a)^(1/4)/a/x^10-3/20*b*(-b*x^4+a)^(1/4)/a^2/x^6-3/8*b^2*(-b*x
^4+a)^(1/4)/a^3/x^2+3/8*b^(5/2)*(1-b*x^4/a)^(3/4)*(cos(1/2*arcsin(x^2*b^(1/
2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcsin(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/
2*arcsin(x^2*b^(1/2)/a^(1/2))),2^(1/2))/a^(5/2)/(-b*x^4+a)^(3/4)
```

Rubi [A] time = 0.09, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {275, 325, 233, 232}

$$-\frac{3b^2\sqrt[4]{a-bx^4}}{8a^3x^2}+\frac{3b^{5/2}\left(1-\frac{bx^4}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{8a^{5/2}\left(a-bx^4\right)^{3/4}}-\frac{3b\sqrt[4]{a-bx^4}}{20a^2x^6}-\frac{\sqrt[4]{a-bx^4}}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^11*(a - b*x^4)^(3/4)),x]
```

```
[Out] -(a - b*x^4)^(1/4)/(10*a*x^10) - (3*b*(a - b*x^4)^(1/4))/(20*a^2*x^6) - (3*
b^2*(a - b*x^4)^(1/4))/(8*a^3*x^2) + (3*b^(5/2)*(1 - (b*x^4)/a)^(3/4)*Ellip
ticF[ArcSin[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(8*a^(5/2)*(a - b*x^4)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + (b*x^2)/a)^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + (b*x^2)/a)^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^{11}\left(a-bx^4\right)^{3/4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^6\left(a-bx^2\right)^{3/4}}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt[4]{a-bx^4}}{10ax^{10}} + \frac{(9b)\,\text{Subst}\left(\int \frac{1}{x^4\left(a-bx^2\right)^{3/4}}\,dx, x, x^2\right)}{20a}$$

$$= -\frac{\sqrt[4]{a-bx^4}}{10ax^{10}} - \frac{3b\sqrt[4]{a-bx^4}}{20a^2x^6} + \frac{\left(3b^2\right)\text{Subst}\left(\int \frac{1}{x^2\left(a-bx^2\right)^{3/4}}\,dx, x, x^2\right)}{8a^2}$$

$$= -\frac{\sqrt[4]{a-bx^4}}{10ax^{10}} - \frac{3b\sqrt[4]{a-bx^4}}{20a^2x^6} - \frac{3b^2\sqrt[4]{a-bx^4}}{8a^3x^2} + \frac{\left(3b^3\right)\text{Subst}\left(\int \frac{1}{\left(a-bx^2\right)^{3/4}}\,dx, x, x^2\right)}{16a^3}$$

$$= -\frac{\sqrt[4]{a-bx^4}}{10ax^{10}} - \frac{3b\sqrt[4]{a-bx^4}}{20a^2x^6} - \frac{3b^2\sqrt[4]{a-bx^4}}{8a^3x^2} + \frac{\left(3b^3\left(1-\frac{bx^4}{a}\right)^{3/4}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\,dx\right.}{16a^3\left(a-bx^4\right)^{3/4}}$$

$$= -\frac{\sqrt[4]{a-bx^4}}{10ax^{10}} - \frac{3b\sqrt[4]{a-bx^4}}{20a^2x^6} - \frac{3b^2\sqrt[4]{a-bx^4}}{8a^3x^2} + \frac{3b^{5/2}\left(1-\frac{bx^4}{a}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{8a^{5/2}\left(a-bx^4\right)^{3/4}}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.39

$$-\frac{\left(1-\frac{bx^4}{a}\right)^{3/4}\,{}_2F_1\left(-\frac{5}{2},\frac{3}{4};-\frac{3}{2};\frac{bx^4}{a}\right)}{10x^{10}\left(a-bx^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^11*(a - b*x^4)^(3/4)),x]
```

```
[Out] -1/10*((1 - (b*x^4)/a)^(3/4)*Hypergeometric2F1[-5/2, 3/4, -3/2, (b*x^4)/a])
/(x^10*(a - b*x^4)^(3/4))
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{bx^{15}-ax^{11}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^4 + a)^(1/4)/(b*x^15 - a*x^11), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{3}{4}}x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^4 + a)^(3/4)*x^11), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^4+a\right)^{\frac{3}{4}}\,x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^11/(-b*x^4+a)^(3/4),x)
```

```
[Out] int(1/x^11/(-b*x^4+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{3}{4}}x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^4 + a)^(3/4)*x^11), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{11}\left(a-b\,x^4\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^11*(a - b*x^4)^(3/4)),x)
```

```
[Out] int(1/(x^11*(a - b*x^4)^(3/4)), x)
```

sympy [C] time = 2.18, size = 34, normalized size = 0.26

$$-\frac{{}_2F_1\left(\begin{matrix}-\frac{5}{2}, \frac{3}{4} \\ -\frac{3}{2}\end{matrix}\middle| \frac{bx^4e^{2i\pi}}{a}\right)}{10a^{\frac{3}{4}}x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**11/(-b*x**4+a)**(3/4),x)
```

```
[Out] -hyper((-5/2, 3/4), (-3/2,), b*x**4*exp_polar(2*I*pi)/a)/(10*a**(3/4)*x**10
)
```

3.1249 $\int \frac{x^{10}}{\left(a-bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=266

$$-\frac{21a^2\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{64\sqrt{2}\,b^{11/4}}+\frac{21a^2\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+1\right)}{64\sqrt{2}\,b^{11/4}}+\frac{21a^2\log\left(-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}+1\right)}{128\sqrt{2}\,b^{11/4}}-\frac{21a^2\log\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+\right.}{128\sqrt{2}\,b^{11/}}$$

```
[Out] -7/32*a*x^3*(-b*x^4+a)^(1/4)/b^2-1/8*x^7*(-b*x^4+a)^(1/4)/b+21/128*a^2*arct
an(-1+b^(1/4)*x*2^(1/2)/(-b*x^4+a)^(1/4))/b^(11/4)*2^(1/2)+21/128*a^2*arcta
n(1+b^(1/4)*x*2^(1/2)/(-b*x^4+a)^(1/4))/b^(11/4)*2^(1/2)+21/256*a^2*ln(1-b^
(1/4)*x*2^(1/2)/(-b*x^4+a)^(1/4)+x^2*b^(1/2)/(-b*x^4+a)^(1/2))/b^(11/4)*2^(
1/2)-21/256*a^2*ln(1+b^(1/4)*x*2^(1/2)/(-b*x^4+a)^(1/4)+x^2*b^(1/2)/(-b*x^4
+a)^(1/2))/b^(11/4)*2^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {321, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{21a^2\log\left(\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+1\right)}{128\sqrt{2}\,b^{11/4}}-\frac{21a^2\log\left(\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+1\right)}{128\sqrt{2}\,b^{11/4}}-\frac{21a^2\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{64\sqrt{2}\,b^{11/4}}+\frac{21a^2\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{}}{\sqrt[4]{a-b}}\right.}{64\sqrt{2}\,b^{11/}}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a - b*x^4)^(3/4),x]
```

```
[Out] (-7*a*x^3*(a - b*x^4)^(1/4))/(32*b^2) - (x^7*(a - b*x^4)^(1/4))/(8*b) - (21
*a^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)])/(64*Sqrt[2]*b^(11/4
)) + (21*a^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)])/(64*Sqrt[2]
*b^(11/4)) + (21*a^2*Log[1 + (Sqrt[b]*x^2)/Sqrt[a - b*x^4] - (Sqrt[2]*b^(1/
4)*x)/(a - b*x^4)^(1/4)])/(128*Sqrt[2]*b^(11/4)) - (21*a^2*Log[1 + (Sqrt[b]
*x^2)/Sqrt[a - b*x^4] + (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)])/(128*Sqrt[2
]*b^(11/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{10}}{\left(a-bx^4\right)^{3/4}}\,dx = -\frac{x^7\sqrt[4]{a-bx^4}}{8b} + \frac{(7a)\int \frac{x^6}{\left(a-bx^4\right)^{3/4}}\,dx}{8b}$$

$$= -\frac{7ax^3\sqrt[4]{a-bx^4}}{32b^2} - \frac{x^7\sqrt[4]{a-bx^4}}{8b} + \frac{\left(21a^2\right)\int \frac{x^2}{\left(a-bx^4\right)^{3/4}}\,dx}{32b^2}$$

$$= -\frac{7ax^3\sqrt[4]{a-bx^4}}{32b^2} - \frac{x^7\sqrt[4]{a-bx^4}}{8b} + \frac{\left(21a^2\right)\text{Subst}\left(\int \frac{x^2}{1+bx^4}\,dx, x, \frac{x}{\sqrt[4]{a-bx^4}}\right)}{32b^2}$$

$$= -\frac{7ax^3\sqrt[4]{a-bx^4}}{32b^2} - \frac{x^7\sqrt[4]{a-bx^4}}{8b} - \frac{\left(21a^2\right)\text{Subst}\left(\int \frac{1-\sqrt{b}\,x^2}{1+bx^4}\,dx, x, \frac{x}{\sqrt[4]{a-bx^4}}\right)}{64b^{5/2}} + \frac{\left(21a^2\right)\text{Subst}}{}$$

$$= -\frac{7ax^3\sqrt[4]{a-bx^4}}{32b^2} - \frac{x^7\sqrt[4]{a-bx^4}}{8b} + \frac{\left(21a^2\right)\text{Subst}\left(\int \frac{1}{\frac{1}{\sqrt{b}}-\frac{\sqrt{2}x}{\sqrt[4]{b}}+x^2}\,dx, x, \frac{x}{\sqrt[4]{a-bx^4}}\right)}{128b^3} + \frac{\left(21a^2\right)\text{Su}}{}$$

$$= -\frac{7ax^3\sqrt[4]{a-bx^4}}{32b^2} - \frac{x^7\sqrt[4]{a-bx^4}}{8b} + \frac{21a^2\log\left(1+\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{128\sqrt{2}\,b^{11/4}} - \frac{21a^2\log\left(1+\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}\right.}{128\sqrt{2}\,b^{11}}$$

$$= -\frac{7ax^3\sqrt[4]{a-bx^4}}{32b^2} - \frac{x^7\sqrt[4]{a-bx^4}}{8b} - \frac{21a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{64\sqrt{2}\,b^{11/4}} + \frac{21a^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{64\sqrt{2}\,b^{11/4}} + \text{-}$$

Mathematica [A] time = 0.05, size = 104, normalized size = 0.39

$$\frac{21a^2\sqrt[4]{-b}\,\tan^{-1}\left(\frac{\sqrt[4]{-b}\,x}{\sqrt[4]{a-bx^4}}\right) - 21a^2\sqrt[4]{-b}\,\tanh^{-1}\left(\frac{\sqrt[4]{-b}\,x}{\sqrt[4]{a-bx^4}}\right) - 2bx^3\sqrt[4]{a-bx^4}\,\left(7a+4bx^4\right)}{64b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a - b*x^4)^(3/4),x]
```

```
[Out] (-2*b*x^3*(a - b*x^4)^(1/4)*(7*a + 4*b*x^4) + 21*a^2*(-b)^(1/4)*ArcTan[((-b
)^(1/4)*x)/(a - b*x^4)^(1/4)] - 21*a^2*(-b)^(1/4)*ArcTanh[((-b)^(1/4)*x)/(a
 - b*x^4)^(1/4)])/(64*b^3)
```

fricas [A] time = 0.78, size = 240, normalized size = 0.90

$$84\,b^2\left(-\frac{a^8}{b^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-bx^4+a\right)^{\frac{1}{4}}a^2b^8\left(-\frac{a^8}{b^{11}}\right)^{\frac{3}{4}}-b^8x\sqrt{\frac{b^6x^2\sqrt{-\frac{a^8}{b^{11}}}+\sqrt{-bx^4+a}\,a^4}{x^2}}\left(-\frac{a^8}{b^{11}}\right)^{\frac{3}{4}}}{a^8x}\right) + 21\,b^2\left(-\frac{a^8}{b^{11}}\right)^{\frac{1}{4}}\log\left(\frac{21\left(b^3x\left(-\frac{a^8}{b^{11}}\right)^{\frac{1}{4}}\right.\right.$$

$$-\frac{}{128\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/128*(84*b^2*(-a^8/b^11)^(1/4)*arctan(-((-b*x^4 + a)^(1/4)*a^2*b^8*(-a^8/
b^11)^(3/4) - b^8*x*sqrt((b^6*x^2*sqrt(-a^8/b^11) + sqrt(-b*x^4 + a)*a^4)/x
^2)*(-a^8/b^11)^(3/4))/(a^8*x)) + 21*b^2*(-a^8/b^11)^(1/4)*log(21*(b^3*x*(-
a^8/b^11)^(1/4) + (-b*x^4 + a)^(1/4)*a^2)/x) - 21*b^2*(-a^8/b^11)^(1/4)*log
```

```
(-21*(b^3*x*(-a^8/b^11)^(1/4) - (-b*x^4 + a)^(1/4)*a^2)/x) + 4*(4*b*x^7 + 7
*a*x^3)*(-b*x^4 + a)^(1/4))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\left(-bx^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^10/(-b*x^4 + a)^(3/4), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\left(-b\,x^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(-b*x^4+a)^(3/4),x)
```

```
[Out] int(x^10/(-b*x^4+a)^(3/4),x)
```

maxima [A] time = 2.30, size = 274, normalized size = 1.03

$$-\frac{\frac{11\left(-bx^4+a\right)^{\frac{1}{4}}a^2b}{x}+\frac{7\left(-bx^4+a\right)^{\frac{5}{4}}a^2}{x^5}}{32\left(b^4-\frac{2\left(bx^4-a\right)b^3}{x^4}+\frac{\left(bx^4-a\right)^2b^2}{x^8}\right)}-\frac{21\left(\frac{2\sqrt{2}\,a^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}+\frac{2\left(-bx^4+a\right)^{\frac{1}{4}}}{x}\right)}{2\,b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}+\frac{2\sqrt{2}\,a^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}-\frac{2\left(-bx^4+a\right)^{\frac{1}{4}}}{x}\right)}{2\,b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}+\frac{\sqrt{2}\,a^2}{}\right.}{256\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] -1/32*(11*(-b*x^4 + a)^(1/4)*a^2*b/x + 7*(-b*x^4 + a)^(5/4)*a^2/x^5)/(b^4 -
 2*(b*x^4 - a)*b^3/x^4 + (b*x^4 - a)^2*b^2/x^8) - 21/256*(2*sqrt(2)*a^2*arc
tan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(-b*x^4 + a)^(1/4)/x)/b^(1/4))/b^(3/4)
 + 2*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(-b*x^4 + a)^(1/4
)/x)/b^(1/4))/b^(3/4) + sqrt(2)*a^2*log(sqrt(b) + sqrt(2)*(-b*x^4 + a)^(1/4
)*b^(1/4)/x + sqrt(-b*x^4 + a)/x^2)/b^(3/4) - sqrt(2)*a^2*log(sqrt(b) - sqr
t(2)*(-b*x^4 + a)^(1/4)*b^(1/4)/x + sqrt(-b*x^4 + a)/x^2)/b^(3/4))/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{10}}{\left(a-b\,x^4\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a - b*x^4)^(3/4),x)
```

```
[Out] int(x^10/(a - b*x^4)^(3/4), x)
```

sympy [C] time = 1.77, size = 39, normalized size = 0.15

$$\frac{x^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4}, \frac{11}{4} \\ \frac{15}{4}\end{matrix}\middle| \frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{4}}\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(-b*x**4+a)**(3/4),x)
```

```
[Out] x**11*gamma(11/4)*hyper((3/4, 11/4), (15/4,), b*x**4*exp_polar(2*I*pi)/a)/(
4*a**(3/4)*gamma(15/4))
```

3.1250 $\int \frac{x^6}{\left(a-bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=235

$$-\frac{3a\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{8\sqrt{2}\,b^{7/4}}+\frac{3a\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+1\right)}{8\sqrt{2}\,b^{7/4}}+\frac{3a\log\left(-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}+1\right)}{16\sqrt{2}\,b^{7/4}}-\frac{3a\log\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}+1\right)}{16\sqrt{2}\,b^{7/4}}$$

```
[Out] -1/4*x^3*(-b*x^4+a)^(1/4)/b+3/16*a*arctan(-1+b^(1/4)*x*2^(1/2)/(-b*x^4+a)^(
1/4))/b^(7/4)*2^(1/2)+3/16*a*arctan(1+b^(1/4)*x*2^(1/2)/(-b*x^4+a)^(1/4))/b
^(7/4)*2^(1/2)+3/32*a*ln(1-b^(1/4)*x*2^(1/2)/(-b*x^4+a)^(1/4)+x^2*b^(1/2)/(
-b*x^4+a)^(1/2))/b^(7/4)*2^(1/2)-3/32*a*ln(1+b^(1/4)*x*2^(1/2)/(-b*x^4+a)^(
1/4)+x^2*b^(1/2)/(-b*x^4+a)^(1/2))/b^(7/4)*2^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {321, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{3a\log\left(\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+1\right)}{16\sqrt{2}\,b^{7/4}}-\frac{3a\log\left(\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+1\right)}{16\sqrt{2}\,b^{7/4}}-\frac{3a\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{8\sqrt{2}\,b^{7/4}}+\frac{3a\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+1\right)}{8\sqrt{2}\,b^{7/4}}-\frac{x^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a - b*x^4)^(3/4),x]
```

```
[Out] -(x^3*(a - b*x^4)^(1/4))/(4*b) - (3*a*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/(a - b
*x^4)^(1/4)])/(8*Sqrt[2]*b^(7/4)) + (3*a*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/(a
 - b*x^4)^(1/4)])/(8*Sqrt[2]*b^(7/4)) + (3*a*Log[1 + (Sqrt[b]*x^2)/Sqrt[a -
b*x^4] - (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)])/(16*Sqrt[2]*b^(7/4)) - (3*
a*Log[1 + (Sqrt[b]*x^2)/Sqrt[a - b*x^4] + (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(
1/4)])/(16*Sqrt[2]*b^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
```

```
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^6}{\left(a-bx^4\right)^{3/4}}\,dx = -\frac{x^3\sqrt[4]{a-bx^4}}{4b} + \frac{(3a)\int \frac{x^2}{\left(a-bx^4\right)^{3/4}}\,dx}{4b}$$

$$= -\frac{x^3\sqrt[4]{a-bx^4}}{4b} + \frac{(3a)\,\text{Subst}\left(\int \frac{x^2}{1+bx^4}\,dx, x, \frac{x}{\sqrt[4]{a-bx^4}}\right)}{4b}$$

$$= -\frac{x^3\sqrt[4]{a-bx^4}}{4b} - \frac{(3a)\,\text{Subst}\left(\int \frac{1-\sqrt{b}x^2}{1+bx^4}\,dx, x, \frac{x}{\sqrt[4]{a-bx^4}}\right)}{8b^{3/2}} + \frac{(3a)\,\text{Subst}\left(\int \frac{1+\sqrt{b}x^2}{1+bx^4}\,dx, x, \frac{x}{\sqrt[4]{a-bx^4}}\right.}{8b^{3/2}}$$

$$= -\frac{x^3\sqrt[4]{a-bx^4}}{4b} + \frac{(3a)\,\text{Subst}\left(\int \frac{1}{\frac{1}{\sqrt{b}}-\frac{\sqrt{2}x}{\sqrt[4]{b}}+x^2}\,dx, x, \frac{x}{\sqrt[4]{a-bx^4}}\right)}{16b^2} + \frac{(3a)\,\text{Subst}\left(\int \frac{1}{\frac{1}{\sqrt{b}}+\frac{\sqrt{2}x}{\sqrt[4]{b}}+x^2}\,dx,\right.}{16b^2}$$

$$= -\frac{x^3\sqrt[4]{a-bx^4}}{4b} + \frac{3a\log\left(1+\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{16\sqrt{2}\,b^{7/4}} - \frac{3a\log\left(1+\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{16\sqrt{2}\,b^{7/4}} + \frac{(3a)\,\text{Su}}{}$$

$$= -\frac{x^3\sqrt[4]{a-bx^4}}{4b} - \frac{3a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{8\sqrt{2}\,b^{7/4}} + \frac{3a\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{8\sqrt{2}\,b^{7/4}} + \frac{3a\log\left(1+\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}-\right.}{16\sqrt{2}\,b^{7/4}}$$

Mathematica [A] time = 0.03, size = 90, normalized size = 0.38

$$-\frac{-3a\sqrt[4]{-b}\,\tan^{-1}\left(\frac{\sqrt[4]{-b}\,x}{\sqrt[4]{a-bx^4}}\right)+3a\sqrt[4]{-b}\,\tanh^{-1}\left(\frac{\sqrt[4]{-b}\,x}{\sqrt[4]{a-bx^4}}\right)+2bx^3\sqrt[4]{a-bx^4}}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a - b*x^4)^(3/4),x]
```

```
[Out] -1/8*(2*b*x^3*(a - b*x^4)^(1/4) - 3*a*(-b)^(1/4)*ArcTan[((-b)^(1/4)*x)/(a -
 b*x^4)^(1/4)] + 3*a*(-b)^(1/4)*ArcTanh[((-b)^(1/4)*x)/(a - b*x^4)^(1/4)])/
b^2
```

fricas [A] time = 0.68, size = 217, normalized size = 0.92

$$-\frac{4\left(-bx^4+a\right)^{\frac{1}{4}}x^3+12\,b\left(-\frac{a^4}{b^7}\right)^{\frac{1}{4}}\arctan\left(\frac{b^5x\sqrt{\frac{b^4x^2\sqrt{-\frac{a^4}{b^7}}+\sqrt{-bx^4+a}\,a^2}{x^2}}\left(-\frac{a^4}{b^7}\right)^{\frac{3}{4}}-\left(-bx^4+a\right)^{\frac{1}{4}}ab^5\left(-\frac{a^4}{b^7}\right)^{\frac{3}{4}}}{a^4x}\right)+3\,b\left(-\frac{a^4}{b^7}\right)^{\frac{1}{4}}\log\left(\frac{3\left(b^2x\right.\right.}{}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/16*(4*(-b*x^4 + a)^(1/4)*x^3 + 12*b*(-a^4/b^7)^(1/4)*arctan((b^5*x*sqrt(
(b^4*x^2*sqrt(-a^4/b^7) + sqrt(-b*x^4 + a)*a^2)/x^2)*(-a^4/b^7)^(3/4) - (-b
*x^4 + a)^(1/4)*a*b^5*(-a^4/b^7)^(3/4))/(a^4*x)) + 3*b*(-a^4/b^7)^(1/4)*log
(3*(b^2*x*(-a^4/b^7)^(1/4) + (-b*x^4 + a)^(1/4)*a)/x) - 3*b*(-a^4/b^7)^(1/4
)*log(-3*(b^2*x*(-a^4/b^7)^(1/4) - (-b*x^4 + a)^(1/4)*a)/x))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(-bx^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/(-b*x^4 + a)^(3/4), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(-b\,x^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-b*x^4+a)^(3/4),x)
```

```
[Out] int(x^6/(-b*x^4+a)^(3/4),x)
```

maxima [A] time = 2.48, size = 222, normalized size = 0.94

$$-\frac{3\left(\frac{2\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}+\frac{2\left(-bx^4+a\right)^{\frac{1}{4}}}{x}\right)}{2\,b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}+\frac{2\sqrt{2}\,a\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}-\frac{2\left(-bx^4+a\right)^{\frac{1}{4}}}{x}\right)}{2\,b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}+\frac{\sqrt{2}\,a\log\left(\sqrt{b}+\frac{\sqrt{2}\left(-bx^4+a\right)^{\frac{1}{4}}b^{\frac{1}{4}}}{x}+\frac{\sqrt{-bx^4+a}}{x^2}\right)}{b^{\frac{3}{4}}}-\frac{\sqrt{2}}{}\right.}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] -3/32*(2*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(-b*x^4 + a)^(1/
4)/x)/b^(1/4))/b^(3/4) + 2*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) -
 2*(-b*x^4 + a)^(1/4)/x)/b^(1/4))/b^(3/4) + sqrt(2)*a*log(sqrt(b) + sqrt(2)
*(-b*x^4 + a)^(1/4)*b^(1/4)/x + sqrt(-b*x^4 + a)/x^2)/b^(3/4) - sqrt(2)*a*l
og(sqrt(b) - sqrt(2)*(-b*x^4 + a)^(1/4)*b^(1/4)/x + sqrt(-b*x^4 + a)/x^2)/b
^(3/4))/b - 1/4*(-b*x^4 + a)^(1/4)*a/((b^2 - (b*x^4 - a)*b/x^4)*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{\left(a-b\,x^4\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a - b*x^4)^(3/4),x)
```

```
[Out] int(x^6/(a - b*x^4)^(3/4), x)
```

sympy [C] time = 1.95, size = 39, normalized size = 0.17

$$\frac{x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{4}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-b*x**4+a)**(3/4),x)
```

```
[Out] x**7*gamma(7/4)*hyper((3/4, 7/4), (11/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*a
**(3/4)*gamma(11/4))
```

3.1251 $\int \frac{x^2}{\left(a-bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=209

$$-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{2\sqrt{2}\,b^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+1\right)}{2\sqrt{2}\,b^{3/4}}+\frac{\log\left(-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}+1\right)}{4\sqrt{2}\,b^{3/4}}-\frac{\log\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}+1\right)}{4\sqrt{2}\,b^{3/4}}$$

```
[Out] 1/4*arctan(-1+b^(1/4)*x*2^(1/2)/(-b*x^4+a)^(1/4))/b^(3/4)*2^(1/2)+1/4*arcta
n(1+b^(1/4)*x*2^(1/2)/(-b*x^4+a)^(1/4))/b^(3/4)*2^(1/2)+1/8*ln(1-b^(1/4)*x*
2^(1/2)/(-b*x^4+a)^(1/4)+x^2*b^(1/2)/(-b*x^4+a)^(1/2))/b^(3/4)*2^(1/2)-1/8*
ln(1+b^(1/4)*x*2^(1/2)/(-b*x^4+a)^(1/4)+x^2*b^(1/2)/(-b*x^4+a)^(1/2))/b^(3/
4)*2^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {331, 297, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+1\right)}{4\sqrt{2}\,b^{3/4}}-\frac{\log\left(\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+1\right)}{4\sqrt{2}\,b^{3/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{2\sqrt{2}\,b^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}+1\right)}{2\sqrt{2}\,b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a - b*x^4)^(3/4),x]
```

```
[Out] -ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)]/(2*Sqrt[2]*b^(3/4)) + Ar
cTan[1 + (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)]/(2*Sqrt[2]*b^(3/4)) + Log[1
 + (Sqrt[b]*x^2)/Sqrt[a - b*x^4] - (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)]/(
4*Sqrt[2]*b^(3/4)) - Log[1 + (Sqrt[b]*x^2)/Sqrt[a - b*x^4] + (Sqrt[2]*b^(1/
4)*x)/(a - b*x^4)^(1/4)]/(4*Sqrt[2]*b^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^2}{\left(a-bx^4\right)^{3/4}}\,dx = \text{Subst}\left(\int \frac{x^2}{1+bx^4}\,dx, x, \frac{x}{\sqrt[4]{a-bx^4}}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{1-\sqrt{b}\,x^2}{1+bx^4}\,dx, x, \frac{x}{\sqrt[4]{a-bx^4}}\right)}{2\sqrt{b}} + \frac{\text{Subst}\left(\int \frac{1+\sqrt{b}\,x^2}{1+bx^4}\,dx, x, \frac{x}{\sqrt[4]{a-bx^4}}\right)}{2\sqrt{b}}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{\sqrt{b}}-\frac{\sqrt{2}x}{\sqrt[4]{b}}+x^2}\,dx, x, \frac{x}{\sqrt[4]{a-bx^4}}\right)}{4b} + \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{\sqrt{b}}+\frac{\sqrt{2}x}{\sqrt[4]{b}}+x^2}\,dx, x, \frac{x}{\sqrt[4]{a-bx^4}}\right)}{4b} + \frac{\text{Subst}\left(\int \frac{}{}\right.}{}$$

$$= \frac{\log\left(1+\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{4\sqrt{2}\,b^{3/4}} - \frac{\log\left(1+\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{4\sqrt{2}\,b^{3/4}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\,\sqrt[4]{}}{\sqrt[4]{a-}}\right.}{2\sqrt{2}\,b^{3/4}}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{2\sqrt{2}\,b^{3/4}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{2\sqrt{2}\,b^{3/4}} + \frac{\log\left(1+\frac{\sqrt{b}\,x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a-bx^4}}\right)}{4\sqrt{2}\,b^{3/4}} - \frac{\log\left(1+\frac{\sqrt{b}}{\sqrt{a-}}\right.}{4\sqrt{2}}$$

Mathematica [A] time = 0.03, size = 58, normalized size = 0.28

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{-b}\,x}{\sqrt[4]{a-bx^4}}\right)-\tan^{-1}\left(\frac{\sqrt[4]{-b}\,x}{\sqrt[4]{a-bx^4}}\right)}{2(-b)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a - b*x^4)^(3/4),x]
```

```
[Out] (-ArcTan[((-b)^(1/4)*x)/(a - b*x^4)^(1/4)] + ArcTanh[((-b)^(1/4)*x)/(a - b*
x^4)^(1/4)])/(2*(-b)^(3/4))
```

fricas [A] time = 0.85, size = 154, normalized size = 0.74

$$-\left(-\frac{1}{b^3}\right)^{\frac{1}{4}}\arctan\left(\frac{b^2x\sqrt{\frac{b^2x^2\sqrt{-\frac{1}{b^3}}+\sqrt{-bx^4+a}}{x^2}}\left(-\frac{1}{b^3}\right)^{\frac{3}{4}}-\left(-bx^4+a\right)^{\frac{1}{4}}b^2\left(-\frac{1}{b^3}\right)^{\frac{3}{4}}}{x}\right)-\frac{1}{4}\left(-\frac{1}{b^3}\right)^{\frac{1}{4}}\log\left(\frac{bx\left(-\frac{1}{b^3}\right)^{\frac{1}{4}}+\left(-bx^4\right.\right.}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] -(-1/b^3)^(1/4)*arctan((b^2*x*sqrt((b^2*x^2*sqrt(-1/b^3) + sqrt(-b*x^4 + a)
)/x^2)*(-1/b^3)^(3/4) - (-b*x^4 + a)^(1/4)*b^2*(-1/b^3)^(3/4))/x) - 1/4*(-1
/b^3)^(1/4)*log((b*x*(-1/b^3)^(1/4) + (-b*x^4 + a)^(1/4))/x) + 1/4*(-1/b^3)
^(1/4)*log(-(b*x*(-1/b^3)^(1/4) - (-b*x^4 + a)^(1/4))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-bx^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(-b*x^4 + a)^(3/4), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-b\,x^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^4+a)^(3/4),x)
```

```
[Out] int(x^2/(-b*x^4+a)^(3/4),x)
```

maxima [A] time = 2.41, size = 176, normalized size = 0.84

$$-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}+\frac{2\left(-bx^4+a\right)^{\frac{1}{4}}}{x}\right)}{2\,b^{\frac{1}{4}}}\right)}{4\,b^{\frac{3}{4}}}-\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}-\frac{2\left(-bx^4+a\right)^{\frac{1}{4}}}{x}\right)}{2\,b^{\frac{1}{4}}}\right)}{4\,b^{\frac{3}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{b}+\frac{\sqrt{2}\left(-bx^4+a\right)^{\frac{1}{4}}b^{\frac{1}{4}}}{x}+\frac{\sqrt{-bx^4+a}}{x^2}\right.}{8\,b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(-b*x^4 + a)^(1/4)/x)/
b^(1/4))/b^(3/4) - 1/4*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(-b
*x^4 + a)^(1/4)/x)/b^(1/4))/b^(3/4) - 1/8*sqrt(2)*log(sqrt(b) + sqrt(2)*(-b
*x^4 + a)^(1/4)*b^(1/4)/x + sqrt(-b*x^4 + a)/x^2)/b^(3/4) + 1/8*sqrt(2)*log
(sqrt(b) - sqrt(2)*(-b*x^4 + a)^(1/4)*b^(1/4)/x + sqrt(-b*x^4 + a)/x^2)/b^(
3/4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(a - b\,x^4\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a - b*x^4)^(3/4),x)
```

```
[Out] int(x^2/(a - b*x^4)^(3/4), x)
```

sympy [C] time = 2.00, size = 39, normalized size = 0.19

$$\frac{x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4}, \frac{3}{4} \\ \frac{7}{4}\end{matrix}\middle| \frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**4+a)**(3/4),x)
```

```
[Out] x**3*gamma(3/4)*hyper((3/4, 3/4), (7/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*a*
*(3/4)*gamma(7/4))
```

3.1252 $\int \frac{1}{x^2\left(a-bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=20

$$-\frac{\sqrt[4]{a-bx^4}}{ax}$$

```
[Out] -(-b*x^4+a)^(1/4)/a/x
```

Rubi [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {264}

$$-\frac{\sqrt[4]{a-bx^4}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a - b*x^4)^(3/4)),x]
```

```
[Out] -((a - b*x^4)^(1/4)/(a*x))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\left(a-bx^4\right)^{3/4}}\,dx = -\frac{\sqrt[4]{a-bx^4}}{ax}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$-\frac{\sqrt[4]{a-bx^4}}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a - b*x^4)^(3/4)),x]
```

```
[Out] -((a - b*x^4)^(1/4)/(a*x))
```

fricas [A] time = 0.69, size = 18, normalized size = 0.90

$$-\frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] -(-b*x^4 + a)^(1/4)/(a*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{3}{4}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^4 + a)^(3/4)*x^2), x)
```

maple [A] time = 0.00, size = 19, normalized size = 0.95

$$-\frac{\left(-b\,x^4+a\right)^{\frac{1}{4}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-b*x^4+a)^(3/4),x)
```

```
[Out] -(-b*x^4+a)^(1/4)/a/x
```

maxima [A] time = 1.18, size = 18, normalized size = 0.90

$$-\frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] -(-b*x^4 + a)^(1/4)/(a*x)
```

mupad [B] time = 1.11, size = 18, normalized size = 0.90

$$-\frac{\left(a-b\,x^4\right)^{1/4}}{a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a - b*x^4)^(3/4)),x)
```

```
[Out] -(a - b*x^4)^(1/4)/(a*x)
```

sympy [B] time = 1.52, size = 80, normalized size = 4.00

$$\begin{cases} \frac{\sqrt[4]{b}\,\sqrt[4]{\frac{a}{bx^4}-1}\,\Gamma\left(-\frac{1}{4}\right)}{4a\Gamma\left(\frac{3}{4}\right)} & \text{for } \left|\frac{a}{bx^4}\right|>1 \\ -\frac{\sqrt[4]{b}\,\sqrt[4]{-\frac{a}{bx^4}+1}\,e^{-\frac{3i\pi}{4}}\,\Gamma\left(-\frac{1}{4}\right)}{4a\Gamma\left(\frac{3}{4}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-b*x**4+a)**(3/4),x)
```

```
[Out] Piecewise((b**(1/4)*(a/(b*x**4) - 1)**(1/4)*gamma(-1/4)/(4*a*gamma(3/4)), A
bs(a/(b*x**4)) > 1), (-b**(1/4)*(-a/(b*x**4) + 1)**(1/4)*exp(-3*I*pi/4)*gam
ma(-1/4)/(4*a*gamma(3/4)), True))
```

3.1253 $\int \frac{1}{x^6\left(a-bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=46

$$-\frac{4b\sqrt[4]{a-bx^4}}{5a^2x}-\frac{\sqrt[4]{a-bx^4}}{5ax^5}$$

```
[Out] -1/5*(-b*x^4+a)^(1/4)/a/x^5-4/5*b*(-b*x^4+a)^(1/4)/a^2/x
```

Rubi [A] time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {271, 264}

$$-\frac{4b\sqrt[4]{a-bx^4}}{5a^2x}-\frac{\sqrt[4]{a-bx^4}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a - b*x^4)^(3/4)),x]
```

```
[Out] -(a - b*x^4)^(1/4)/(5*a*x^5) - (4*b*(a - b*x^4)^(1/4))/(5*a^2*x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^6\left(a-bx^4\right)^{3/4}}\,dx &= -\frac{\sqrt[4]{a-bx^4}}{5ax^5}+\frac{(4b)\int \frac{1}{x^2\left(a-bx^4\right)^{3/4}}\,dx}{5a}\\ &= -\frac{\sqrt[4]{a-bx^4}}{5ax^5}-\frac{4b\sqrt[4]{a-bx^4}}{5a^2x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.65

$$-\frac{\sqrt[4]{a-bx^4}\left(a+4bx^4\right)}{5a^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a - b*x^4)^(3/4)),x]
```

```
[Out] -1/5*((a - b*x^4)^(1/4)*(a + 4*b*x^4))/(a^2*x^5)
```

fricas [A] time = 0.64, size = 26, normalized size = 0.57

$$-\frac{\left(4\,bx^4+a\right)\left(-bx^4+a\right)^{\frac{1}{4}}}{5\,a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-b*x^4+a)^(3/4),x, algorithm="fricas")
[Out] -1/5*(4*b*x^4 + a)*(-b*x^4 + a)^(1/4)/(a^2*x^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{3}{4}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-b*x^4+a)^(3/4),x, algorithm="giac")
[Out] integrate(1/((-b*x^4 + a)^(3/4)*x^6), x)
```

maple [A] time = 0.00, size = 27, normalized size = 0.59

$$-\frac{\left(-b\,x^4+a\right)^{\frac{1}{4}}\left(4b\,x^4+a\right)}{5a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-b*x^4+a)^(3/4),x)
[Out] -1/5*(-b*x^4+a)^(1/4)*(4*b*x^4+a)/a^2/x^5
```

maxima [A] time = 1.15, size = 36, normalized size = 0.78

$$-\frac{\frac{5\left(-bx^4+a\right)^{\frac{1}{4}}b}{x}+\frac{\left(-bx^4+a\right)^{\frac{5}{4}}}{x^5}}{5\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-b*x^4+a)^(3/4),x, algorithm="maxima")
[Out] -1/5*(5*(-b*x^4 + a)^(1/4)*b/x + (-b*x^4 + a)^(5/4)/x^5)/a^2
```

mupad [B] time = 1.19, size = 26, normalized size = 0.57

$$-\frac{\left(a-b\,x^4\right)^{1/4}\left(4\,b\,x^4+a\right)}{5\,a^2\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a - b*x^4)^(3/4)),x)
[Out] -((a - b*x^4)^(1/4)*(a + 4*b*x^4))/(5*a^2*x^5)
```

sympy [A] time = 2.35, size = 311, normalized size = 6.76

$$\begin{cases} -\frac{\sqrt[4]{b}\,\sqrt[4]{\frac{a}{bx^4}-1}\,\Gamma\left(-\frac{5}{4}\right)}{16ax^4\Gamma\left(\frac{3}{4}\right)}-\frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx^4}-1}\,\Gamma\left(-\frac{5}{4}\right)}{4a^2\Gamma\left(\frac{3}{4}\right)} & \text{for } \left|\frac{a}{bx^4}\right| \\ -\frac{a^2b^{\frac{5}{4}}\sqrt[4]{-\frac{a}{bx^4}+1}\,\Gamma\left(-\frac{5}{4}\right)}{-16a^3bx^4e^{\frac{3i\pi}{4}}\Gamma\left(\frac{3}{4}\right)+16a^2b^2x^8e^{\frac{3i\pi}{4}}\Gamma\left(\frac{3}{4}\right)}-\frac{3ab^{\frac{9}{4}}x^4\sqrt[4]{-\frac{a}{bx^4}+1}\,\Gamma\left(-\frac{5}{4}\right)}{-16a^3bx^4e^{\frac{3i\pi}{4}}\Gamma\left(\frac{3}{4}\right)+16a^2b^2x^8e^{\frac{3i\pi}{4}}\Gamma\left(\frac{3}{4}\right)}+\frac{4b^{\frac{13}{4}}x^8\sqrt[4]{-\frac{a}{bx^4}+1}\,\Gamma\left(-\frac{5}{4}\right)}{-16a^3bx^4e^{\frac{3i\pi}{4}}\Gamma\left(\frac{3}{4}\right)+16a^2b^2x^8e^{\frac{3i\pi}{4}}\Gamma\left(\frac{3}{4}\right)} & \text{otherw} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-b*x**4+a)**(3/4),x)
```

```
[Out] Piecewise((-b**(1/4)*(a/(b*x**4) - 1)**(1/4)*gamma(-5/4)/(16*a*x**4*gamma(3
/4)) - b**(5/4)*(a/(b*x**4) - 1)**(1/4)*gamma(-5/4)/(4*a**2*gamma(3/4)), Ab
s(a/(b*x**4)) > 1), (-a**2*b**(5/4)*(-a/(b*x**4) + 1)**(1/4)*gamma(-5/4)/(-
16*a**3*b*x**4*exp(3*I*pi/4)*gamma(3/4) + 16*a**2*b**2*x**8*exp(3*I*pi/4)*g
amma(3/4)) - 3*a*b**(9/4)*x**4*(-a/(b*x**4) + 1)**(1/4)*gamma(-5/4)/(-16*a*
*3*b*x**4*exp(3*I*pi/4)*gamma(3/4) + 16*a**2*b**2*x**8*exp(3*I*pi/4)*gamma(
3/4)) + 4*b**(13/4)*x**8*(-a/(b*x**4) + 1)**(1/4)*gamma(-5/4)/(-16*a**3*b*x
**4*exp(3*I*pi/4)*gamma(3/4) + 16*a**2*b**2*x**8*exp(3*I*pi/4)*gamma(3/4)),
 True))
```

3.1254 $\int \frac{1}{x^{10}\left(a-bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=71

$$-\frac{32b^2\sqrt[4]{a-bx^4}}{45a^3x}-\frac{8b\sqrt[4]{a-bx^4}}{45a^2x^5}-\frac{\sqrt[4]{a-bx^4}}{9ax^9}$$

```
[Out] -1/9*(-b*x^4+a)^(1/4)/a/x^9-8/45*b*(-b*x^4+a)^(1/4)/a^2/x^5-32/45*b^2*(-b*x
^4+a)^(1/4)/a^3/x
```

Rubi [A] time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {271, 264}

$$-\frac{32b^2\sqrt[4]{a-bx^4}}{45a^3x}-\frac{8b\sqrt[4]{a-bx^4}}{45a^2x^5}-\frac{\sqrt[4]{a-bx^4}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*(a - b*x^4)^(3/4)),x]
```

```
[Out] -(a - b*x^4)^(1/4)/(9*a*x^9) - (8*b*(a - b*x^4)^(1/4))/(45*a^2*x^5) - (32*b
^2*(a - b*x^4)^(1/4))/(45*a^3*x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{10}\left(a-bx^4\right)^{3/4}}\,dx &= -\frac{\sqrt[4]{a-bx^4}}{9ax^9}+\frac{(8b)\int \frac{1}{x^6\left(a-bx^4\right)^{3/4}}\,dx}{9a}\\ &= -\frac{\sqrt[4]{a-bx^4}}{9ax^9}-\frac{8b\sqrt[4]{a-bx^4}}{45a^2x^5}+\frac{\left(32b^2\right)\int \frac{1}{x^2\left(a-bx^4\right)^{3/4}}\,dx}{45a^2}\\ &= -\frac{\sqrt[4]{a-bx^4}}{9ax^9}-\frac{8b\sqrt[4]{a-bx^4}}{45a^2x^5}-\frac{32b^2\sqrt[4]{a-bx^4}}{45a^3x}\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 0.61

$$-\frac{\sqrt[4]{a-bx^4}\left(5a^2+8abx^4+32b^2x^8\right)}{45a^3x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^10*(a - b*x^4)^(3/4)),x]
[Out] -1/45*((a - b*x^4)^(1/4)*(5*a^2 + 8*a*b*x^4 + 32*b^2*x^8))/(a^3*x^9)
```

fricas [A] time = 0.62, size = 39, normalized size = 0.55

$$-\frac{\left(32\,b^2x^8+8\,abx^4+5\,a^2\right)\left(-bx^4+a\right)^{\frac{1}{4}}}{45\,a^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(-b*x^4+a)^(3/4),x, algorithm="fricas")
[Out] -1/45*(32*b^2*x^8 + 8*a*b*x^4 + 5*a^2)*(-b*x^4 + a)^(1/4)/(a^3*x^9)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{3}{4}}x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(-b*x^4+a)^(3/4),x, algorithm="giac")
[Out] integrate(1/((-b*x^4 + a)^(3/4)*x^10), x)
```

maple [A] time = 0.00, size = 40, normalized size = 0.56

$$-\frac{\left(-b\,x^4+a\right)^{\frac{1}{4}}\left(32b^2x^8+8ab\,x^4+5a^2\right)}{45a^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(-b*x^4+a)^(3/4),x)
[Out] -1/45*(-b*x^4+a)^(1/4)*(32*b^2*x^8+8*a*b*x^4+5*a^2)/a^3/x^9
```

maxima [A] time = 1.11, size = 55, normalized size = 0.77

$$-\frac{\frac{45\left(-bx^4+a\right)^{\frac{1}{4}}b^2}{x}+\frac{18\left(-bx^4+a\right)^{\frac{5}{4}}b}{x^5}+\frac{5\left(-bx^4+a\right)^{\frac{9}{4}}}{x^9}}{45\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(-b*x^4+a)^(3/4),x, algorithm="maxima")
[Out] -1/45*(45*(-b*x^4 + a)^(1/4)*b^2/x + 18*(-b*x^4 + a)^(5/4)*b/x^5 + 5*(-b*x^
4 + a)^(9/4)/x^9)/a^3
```

mupad [B] time = 1.29, size = 39, normalized size = 0.55

$$-\frac{\left(a-b\,x^4\right)^{1/4}\left(5\,a^2+8\,a\,b\,x^4+32\,b^2\,x^8\right)}{45\,a^3\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^10*(a - b*x^4)^(3/4)),x)
[Out] -((a - b*x^4)^(1/4)*(5*a^2 + 32*b^2*x^8 + 8*a*b*x^4))/(45*a^3*x^9)
```

sympy [C] time = 2.36, size = 1110, normalized size = 15.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(-b*x**4+a)**(3/4),x)
```

```
[Out] Piecewise((-5*a**4*b**(17/4)*(a/(b*x**4) - 1)**(1/4)*exp(-I*pi/4)*gamma(-9/
4)/(64*a**5*b**4*x**8*exp(3*I*pi/4)*gamma(3/4) - 128*a**4*b**5*x**12*exp(3*
I*pi/4)*gamma(3/4) + 64*a**3*b**6*x**16*exp(3*I*pi/4)*gamma(3/4)) + 2*a**3*
b**(21/4)*x**4*(a/(b*x**4) - 1)**(1/4)*exp(-I*pi/4)*gamma(-9/4)/(64*a**5*b*
*4*x**8*exp(3*I*pi/4)*gamma(3/4) - 128*a**4*b**5*x**12*exp(3*I*pi/4)*gamma(
3/4) + 64*a**3*b**6*x**16*exp(3*I*pi/4)*gamma(3/4)) - 21*a**2*b**(25/4)*x**
8*(a/(b*x**4) - 1)**(1/4)*exp(-I*pi/4)*gamma(-9/4)/(64*a**5*b**4*x**8*exp(3
*I*pi/4)*gamma(3/4) - 128*a**4*b**5*x**12*exp(3*I*pi/4)*gamma(3/4) + 64*a**
3*b**6*x**16*exp(3*I*pi/4)*gamma(3/4)) + 56*a*b**(29/4)*x**12*(a/(b*x**4) -
 1)**(1/4)*exp(-I*pi/4)*gamma(-9/4)/(64*a**5*b**4*x**8*exp(3*I*pi/4)*gamma(
3/4) - 128*a**4*b**5*x**12*exp(3*I*pi/4)*gamma(3/4) + 64*a**3*b**6*x**16*ex
p(3*I*pi/4)*gamma(3/4)) - 32*b**(33/4)*x**16*(a/(b*x**4) - 1)**(1/4)*exp(-I
*pi/4)*gamma(-9/4)/(64*a**5*b**4*x**8*exp(3*I*pi/4)*gamma(3/4) - 128*a**4*b
**5*x**12*exp(3*I*pi/4)*gamma(3/4) + 64*a**3*b**6*x**16*exp(3*I*pi/4)*gamma
(3/4)), Abs(a/(b*x**4)) > 1), (-5*a**4*b**(17/4)*(-a/(b*x**4) + 1)**(1/4)*g
amma(-9/4)/(64*a**5*b**4*x**8*exp(3*I*pi/4)*gamma(3/4) - 128*a**4*b**5*x**1
2*exp(3*I*pi/4)*gamma(3/4) + 64*a**3*b**6*x**16*exp(3*I*pi/4)*gamma(3/4)) +
 2*a**3*b**(21/4)*x**4*(-a/(b*x**4) + 1)**(1/4)*gamma(-9/4)/(64*a**5*b**4*x
**8*exp(3*I*pi/4)*gamma(3/4) - 128*a**4*b**5*x**12*exp(3*I*pi/4)*gamma(3/4)
 + 64*a**3*b**6*x**16*exp(3*I*pi/4)*gamma(3/4)) - 21*a**2*b**(25/4)*x**8*(-
a/(b*x**4) + 1)**(1/4)*gamma(-9/4)/(64*a**5*b**4*x**8*exp(3*I*pi/4)*gamma(3
/4) - 128*a**4*b**5*x**12*exp(3*I*pi/4)*gamma(3/4) + 64*a**3*b**6*x**16*exp
(3*I*pi/4)*gamma(3/4)) + 56*a*b**(29/4)*x**12*(-a/(b*x**4) + 1)**(1/4)*gamm
a(-9/4)/(64*a**5*b**4*x**8*exp(3*I*pi/4)*gamma(3/4) - 128*a**4*b**5*x**12*e
xp(3*I*pi/4)*gamma(3/4) + 64*a**3*b**6*x**16*exp(3*I*pi/4)*gamma(3/4)) - 32
*b**(33/4)*x**16*(-a/(b*x**4) + 1)**(1/4)*gamma(-9/4)/(64*a**5*b**4*x**8*ex
p(3*I*pi/4)*gamma(3/4) - 128*a**4*b**5*x**12*exp(3*I*pi/4)*gamma(3/4) + 64*
a**3*b**6*x**16*exp(3*I*pi/4)*gamma(3/4)), True))
```

3.1255 $\int \frac{1}{x^{14}\left(a-bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=96

$$-\frac{128b^3\sqrt[4]{a-bx^4}}{195a^4x}-\frac{32b^2\sqrt[4]{a-bx^4}}{195a^3x^5}-\frac{4b\sqrt[4]{a-bx^4}}{39a^2x^9}-\frac{\sqrt[4]{a-bx^4}}{13ax^{13}}$$

```
[Out] -1/13*(-b*x^4+a)^(1/4)/a/x^13-4/39*b*(-b*x^4+a)^(1/4)/a^2/x^9-32/195*b^2*(-
b*x^4+a)^(1/4)/a^3/x^5-128/195*b^3*(-b*x^4+a)^(1/4)/a^4/x
```

Rubi [A] time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {271, 264}

$$-\frac{128b^3\sqrt[4]{a-bx^4}}{195a^4x}-\frac{32b^2\sqrt[4]{a-bx^4}}{195a^3x^5}-\frac{4b\sqrt[4]{a-bx^4}}{39a^2x^9}-\frac{\sqrt[4]{a-bx^4}}{13ax^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^14*(a - b*x^4)^(3/4)),x]
```

```
[Out] -(a - b*x^4)^(1/4)/(13*a*x^13) - (4*b*(a - b*x^4)^(1/4))/(39*a^2*x^9) - (32
*b^2*(a - b*x^4)^(1/4))/(195*a^3*x^5) - (128*b^3*(a - b*x^4)^(1/4))/(195*a^
4*x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{14}\left(a-bx^4\right)^{3/4}}\,dx &= -\frac{\sqrt[4]{a-bx^4}}{13ax^{13}}+\frac{(12b)\int \frac{1}{x^{10}\left(a-bx^4\right)^{3/4}}\,dx}{13a}\\ &= -\frac{\sqrt[4]{a-bx^4}}{13ax^{13}}-\frac{4b\sqrt[4]{a-bx^4}}{39a^2x^9}+\frac{\left(32b^2\right)\int \frac{1}{x^6\left(a-bx^4\right)^{3/4}}\,dx}{39a^2}\\ &= -\frac{\sqrt[4]{a-bx^4}}{13ax^{13}}-\frac{4b\sqrt[4]{a-bx^4}}{39a^2x^9}-\frac{32b^2\sqrt[4]{a-bx^4}}{195a^3x^5}+\frac{\left(128b^3\right)\int \frac{1}{x^2\left(a-bx^4\right)^{3/4}}\,dx}{195a^3}\\ &= -\frac{\sqrt[4]{a-bx^4}}{13ax^{13}}-\frac{4b\sqrt[4]{a-bx^4}}{39a^2x^9}-\frac{32b^2\sqrt[4]{a-bx^4}}{195a^3x^5}-\frac{128b^3\sqrt[4]{a-bx^4}}{195a^4x}\end{aligned}$$

Mathematica [A] time = 0.03, size = 54, normalized size = 0.56

$$-\frac{\sqrt[4]{a-bx^4}\left(15a^3+20a^2bx^4+32ab^2x^8+128b^3x^{12}\right)}{195a^4x^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^14*(a - b*x^4)^(3/4)),x]
```

```
[Out] -1/195*((a - b*x^4)^(1/4)*(15*a^3 + 20*a^2*b*x^4 + 32*a*b^2*x^8 + 128*b^3*x
^12))/(a^4*x^13)
```

fricas [A] time = 0.46, size = 50, normalized size = 0.52

$$-\frac{\left(128\,b^3x^{12}+32\,ab^2x^8+20\,a^2bx^4+15\,a^3\right)\left(-bx^4+a\right)^{\frac{1}{4}}}{195\,a^4x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^14/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/195*(128*b^3*x^12 + 32*a*b^2*x^8 + 20*a^2*b*x^4 + 15*a^3)*(-b*x^4 + a)^(
1/4)/(a^4*x^13)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{3}{4}}x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^14/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^4 + a)^(3/4)*x^14), x)
```

maple [A] time = 0.01, size = 51, normalized size = 0.53

$$-\frac{\left(-b\,x^4+a\right)^{\frac{1}{4}}\left(128b^3x^{12}+32a\,b^2x^8+20a^2b\,x^4+15a^3\right)}{195a^4x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^14/(-b*x^4+a)^(3/4),x)
```

```
[Out] -1/195*(-b*x^4+a)^(1/4)*(128*b^3*x^12+32*a*b^2*x^8+20*a^2*b*x^4+15*a^3)/x^1
3/a^4
```

maxima [A] time = 1.05, size = 73, normalized size = 0.76

$$-\frac{\frac{195\left(-bx^4+a\right)^{\frac{1}{4}}b^3}{x}+\frac{117\left(-bx^4+a\right)^{\frac{5}{4}}b^2}{x^5}+\frac{65\left(-bx^4+a\right)^{\frac{9}{4}}b}{x^9}+\frac{15\left(-bx^4+a\right)^{\frac{13}{4}}}{x^{13}}}{195\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^14/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] -1/195*(195*(-b*x^4 + a)^(1/4)*b^3/x + 117*(-b*x^4 + a)^(5/4)*b^2/x^5 + 65*
(-b*x^4 + a)^(9/4)*b/x^9 + 15*(-b*x^4 + a)^(13/4)/x^13)/a^4
```

mupad [B] time = 1.37, size = 80, normalized size = 0.83

$$-\frac{\left(a-b\,x^4\right)^{1/4}}{13\,a\,x^{13}}-\frac{4\,b\left(a-b\,x^4\right)^{1/4}}{39\,a^2\,x^9}-\frac{128\,b^3\left(a-b\,x^4\right)^{1/4}}{195\,a^4\,x}-\frac{32\,b^2\left(a-b\,x^4\right)^{1/4}}{195\,a^3\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^14*(a - b*x^4)^(3/4)),x)
```

```
[Out] - (a - b*x^4)^(1/4)/(13*a*x^13) - (4*b*(a - b*x^4)^(1/4))/(39*a^2*x^9) - (1
28*b^3*(a - b*x^4)^(1/4))/(195*a^4*x) - (32*b^2*(a - b*x^4)^(1/4))/(195*a^3
*x^5)
```

sympy [C] time = 5.65, size = 1928, normalized size = 20.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**14/(-b*x**4+a)**(3/4),x)
```

```
[Out] Piecewise((45*a**6*b**(37/4)*(a/(b*x**4) - 1)**(1/4)*exp(3*I*pi/4)*gamma(-1
3/4)/(-256*a**7*b**9*x**12*exp(3*I*pi/4)*gamma(3/4) + 768*a**6*b**10*x**16*
exp(3*I*pi/4)*gamma(3/4) - 768*a**5*b**11*x**20*exp(3*I*pi/4)*gamma(3/4) +
256*a**4*b**12*x**24*exp(3*I*pi/4)*gamma(3/4)) - 75*a**5*b**(41/4)*x**4*(a/
(b*x**4) - 1)**(1/4)*exp(3*I*pi/4)*gamma(-13/4)/(-256*a**7*b**9*x**12*exp(3
*I*pi/4)*gamma(3/4) + 768*a**6*b**10*x**16*exp(3*I*pi/4)*gamma(3/4) - 768*a
**5*b**11*x**20*exp(3*I*pi/4)*gamma(3/4) + 256*a**4*b**12*x**24*exp(3*I*pi/
4)*gamma(3/4)) + 51*a**4*b**(45/4)*x**8*(a/(b*x**4) - 1)**(1/4)*exp(3*I*pi/
4)*gamma(-13/4)/(-256*a**7*b**9*x**12*exp(3*I*pi/4)*gamma(3/4) + 768*a**6*b
**10*x**16*exp(3*I*pi/4)*gamma(3/4) - 768*a**5*b**11*x**20*exp(3*I*pi/4)*ga
mma(3/4) + 256*a**4*b**12*x**24*exp(3*I*pi/4)*gamma(3/4)) + 231*a**3*b**(49
/4)*x**12*(a/(b*x**4) - 1)**(1/4)*exp(3*I*pi/4)*gamma(-13/4)/(-256*a**7*b**
9*x**12*exp(3*I*pi/4)*gamma(3/4) + 768*a**6*b**10*x**16*exp(3*I*pi/4)*gamma
(3/4) - 768*a**5*b**11*x**20*exp(3*I*pi/4)*gamma(3/4) + 256*a**4*b**12*x**2
4*exp(3*I*pi/4)*gamma(3/4)) - 924*a**2*b**(53/4)*x**16*(a/(b*x**4) - 1)**(1
/4)*exp(3*I*pi/4)*gamma(-13/4)/(-256*a**7*b**9*x**12*exp(3*I*pi/4)*gamma(3/
4) + 768*a**6*b**10*x**16*exp(3*I*pi/4)*gamma(3/4) - 768*a**5*b**11*x**20*e
xp(3*I*pi/4)*gamma(3/4) + 256*a**4*b**12*x**24*exp(3*I*pi/4)*gamma(3/4)) +
1056*a*b**(57/4)*x**20*(a/(b*x**4) - 1)**(1/4)*exp(3*I*pi/4)*gamma(-13/4)/(
-256*a**7*b**9*x**12*exp(3*I*pi/4)*gamma(3/4) + 768*a**6*b**10*x**16*exp(3*
I*pi/4)*gamma(3/4) - 768*a**5*b**11*x**20*exp(3*I*pi/4)*gamma(3/4) + 256*a*
*4*b**12*x**24*exp(3*I*pi/4)*gamma(3/4)) - 384*b**(61/4)*x**24*(a/(b*x**4)
- 1)**(1/4)*exp(3*I*pi/4)*gamma(-13/4)/(-256*a**7*b**9*x**12*exp(3*I*pi/4)*
gamma(3/4) + 768*a**6*b**10*x**16*exp(3*I*pi/4)*gamma(3/4) - 768*a**5*b**11
*x**20*exp(3*I*pi/4)*gamma(3/4) + 256*a**4*b**12*x**24*exp(3*I*pi/4)*gamma(
3/4)), Abs(a/(b*x**4)) > 1), (-45*a**6*b**(37/4)*(-a/(b*x**4) + 1)**(1/4)*g
amma(-13/4)/(-256*a**7*b**9*x**12*exp(3*I*pi/4)*gamma(3/4) + 768*a**6*b**10
*x**16*exp(3*I*pi/4)*gamma(3/4) - 768*a**5*b**11*x**20*exp(3*I*pi/4)*gamma(
3/4) + 256*a**4*b**12*x**24*exp(3*I*pi/4)*gamma(3/4)) + 75*a**5*b**(41/4)*x
**4*(-a/(b*x**4) + 1)**(1/4)*gamma(-13/4)/(-256*a**7*b**9*x**12*exp(3*I*pi/
4)*gamma(3/4) + 768*a**6*b**10*x**16*exp(3*I*pi/4)*gamma(3/4) - 768*a**5*b*
*11*x**20*exp(3*I*pi/4)*gamma(3/4) + 256*a**4*b**12*x**24*exp(3*I*pi/4)*gam
ma(3/4)) - 51*a**4*b**(45/4)*x**8*(-a/(b*x**4) + 1)**(1/4)*gamma(-13/4)/(-2
56*a**7*b**9*x**12*exp(3*I*pi/4)*gamma(3/4) + 768*a**6*b**10*x**16*exp(3*I*
pi/4)*gamma(3/4) - 768*a**5*b**11*x**20*exp(3*I*pi/4)*gamma(3/4) + 256*a**4
*b**12*x**24*exp(3*I*pi/4)*gamma(3/4)) - 231*a**3*b**(49/4)*x**12*(-a/(b*x*
*4) + 1)**(1/4)*gamma(-13/4)/(-256*a**7*b**9*x**12*exp(3*I*pi/4)*gamma(3/4)
 + 768*a**6*b**10*x**16*exp(3*I*pi/4)*gamma(3/4) - 768*a**5*b**11*x**20*exp
(3*I*pi/4)*gamma(3/4) + 256*a**4*b**12*x**24*exp(3*I*pi/4)*gamma(3/4)) + 92
4*a**2*b**(53/4)*x**16*(-a/(b*x**4) + 1)**(1/4)*gamma(-13/4)/(-256*a**7*b**
9*x**12*exp(3*I*pi/4)*gamma(3/4) + 768*a**6*b**10*x**16*exp(3*I*pi/4)*gamma
(3/4) - 768*a**5*b**11*x**20*exp(3*I*pi/4)*gamma(3/4) + 256*a**4*b**12*x**2
4*exp(3*I*pi/4)*gamma(3/4)) - 1056*a*b**(57/4)*x**20*(-a/(b*x**4) + 1)**(1/
4)*gamma(-13/4)/(-256*a**7*b**9*x**12*exp(3*I*pi/4)*gamma(3/4) + 768*a**6*b
**10*x**16*exp(3*I*pi/4)*gamma(3/4) - 768*a**5*b**11*x**20*exp(3*I*pi/4)*ga
mma(3/4) + 256*a**4*b**12*x**24*exp(3*I*pi/4)*gamma(3/4)) + 384*b**(61/4)*x
**24*(-a/(b*x**4) + 1)**(1/4)*gamma(-13/4)/(-256*a**7*b**9*x**12*exp(3*I*pi
```

```
/4)*gamma(3/4) + 768*a**6*b**10*x**16*exp(3*I*pi/4)*gamma(3/4) - 768*a**5*b
**11*x**20*exp(3*I*pi/4)*gamma(3/4) + 256*a**4*b**12*x**24*exp(3*I*pi/4)*ga
mma(3/4)), True))
```

3.1256 $\int \frac{x^{12}}{\left(a-bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=134

$$-\frac{3a^{5/2}x^3\left(1-\frac{a}{bx^4}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{8b^{5/2}\left(a-bx^4\right)^{3/4}}-\frac{3a^2x\sqrt[4]{a-bx^4}}{8b^3}-\frac{3ax^5\sqrt[4]{a-bx^4}}{20b^2}-\frac{x^9\sqrt[4]{a-bx^4}}{10b}$$

```
[Out] -3/8*a^2*x*(-b*x^4+a)^(1/4)/b^3-3/20*a*x^5*(-b*x^4+a)^(1/4)/b^2-1/10*x^9*(-
b*x^4+a)^(1/4)/b-3/8*a^(5/2)*(1-a/b/x^4)^(3/4)*x^3*(cos(1/2*arccsc(x^2*b^(1
/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccsc(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1
/2*arccsc(x^2*b^(1/2)/a^(1/2))),2^(1/2))/b^(5/2)/(-b*x^4+a)^(3/4)
```

Rubi [A] time = 0.06, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {321, 237, 335, 275, 232}

$$-\frac{3a^2x\sqrt[4]{a-bx^4}}{8b^3}-\frac{3a^{5/2}x^3\left(1-\frac{a}{bx^4}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{8b^{5/2}\left(a-bx^4\right)^{3/4}}-\frac{3ax^5\sqrt[4]{a-bx^4}}{20b^2}-\frac{x^9\sqrt[4]{a-bx^4}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[x^12/(a - b*x^4)^(3/4),x]
```

```
[Out] (-3*a^2*x*(a - b*x^4)^(1/4))/(8*b^3) - (3*a*x^5*(a - b*x^4)^(1/4))/(20*b^2)
 - (x^9*(a - b*x^4)^(1/4))/(10*b) - (3*a^(5/2)*(1 - a/(b*x^4))^(3/4)*x^3*El
lipticF[ArcCsc[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(8*b^(5/2)*(a - b*x^4)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{x^{12}}{\left(a-bx^4\right)^{3/4}}\,dx = -\frac{x^9\sqrt[4]{a-bx^4}}{10b} + \frac{(9a)\int \frac{x^8}{\left(a-bx^4\right)^{3/4}}\,dx}{10b}$$

$$= -\frac{3ax^5\sqrt[4]{a-bx^4}}{20b^2} - \frac{x^9\sqrt[4]{a-bx^4}}{10b} + \frac{\left(3a^2\right)\int \frac{x^4}{\left(a-bx^4\right)^{3/4}}\,dx}{4b^2}$$

$$= -\frac{3a^2x\sqrt[4]{a-bx^4}}{8b^3} - \frac{3ax^5\sqrt[4]{a-bx^4}}{20b^2} - \frac{x^9\sqrt[4]{a-bx^4}}{10b} + \frac{\left(3a^3\right)\int \frac{1}{\left(a-bx^4\right)^{3/4}}\,dx}{8b^3}$$

$$= -\frac{3a^2x\sqrt[4]{a-bx^4}}{8b^3} - \frac{3ax^5\sqrt[4]{a-bx^4}}{20b^2} - \frac{x^9\sqrt[4]{a-bx^4}}{10b} + \frac{\left(3a^3\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{8b^3\left(a-bx^4\right)^{3/4}}$$

$$= -\frac{3a^2x\sqrt[4]{a-bx^4}}{8b^3} - \frac{3ax^5\sqrt[4]{a-bx^4}}{20b^2} - \frac{x^9\sqrt[4]{a-bx^4}}{10b} - \frac{\left(3a^3\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1-\frac{ax^4}{b}\right)^{3}}\right.}{8b^3\left(a-bx^4\right)^{3/4}}$$

$$= -\frac{3a^2x\sqrt[4]{a-bx^4}}{8b^3} - \frac{3ax^5\sqrt[4]{a-bx^4}}{20b^2} - \frac{x^9\sqrt[4]{a-bx^4}}{10b} - \frac{\left(3a^3\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{ax^2}{b}\right)^{3}}\right.}{16b^3\left(a-bx^4\right)^{3/4}}$$

$$= -\frac{3a^2x\sqrt[4]{a-bx^4}}{8b^3} - \frac{3ax^5\sqrt[4]{a-bx^4}}{20b^2} - \frac{x^9\sqrt[4]{a-bx^4}}{10b} - \frac{3a^{5/2}\left(1-\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\right.}{8b^{5/2}\left(a-bx^4\right)^{3/4}}$$

Mathematica [C] time = 0.03, size = 91, normalized size = 0.68

$$\frac{15a^3x\left(1-\frac{bx^4}{a}\right)^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};\frac{bx^4}{a}\right) - 15a^3x + 9a^2bx^5 + 2ab^2x^9 + 4b^3x^{13}}{40b^3\left(a-bx^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12/(a - b*x^4)^(3/4),x]
```

```
[Out] (-15*a^3*x + 9*a^2*b*x^5 + 2*a*b^2*x^9 + 4*b^3*x^13 + 15*a^3*x*(1 - (b*x^4)
/a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, (b*x^4)/a])/(40*b^3*(a - b*x^4)^
(3/4))
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^4+a\right)^{\frac{1}{4}}x^{12}}{bx^4-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^4 + a)^(1/4)*x^12/(b*x^4 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{12}}{\left(-bx^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^12/(-b*x^4 + a)^(3/4), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^{12}}{\left(-b\,x^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(-b*x^4+a)^(3/4),x)
```

```
[Out] int(x^12/(-b*x^4+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{12}}{\left(-bx^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^12/(-b*x^4 + a)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{12}}{\left(a-b\,x^4\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(a - b*x^4)^(3/4),x)
```

```
[Out] int(x^12/(a - b*x^4)^(3/4), x)
```

sympy [C] time = 2.67, size = 39, normalized size = 0.29

$$\frac{x^{13}\Gamma\left(\frac{13}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{13}{4}\\ \frac{17}{4}\end{matrix}\right|\frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{4}}\Gamma\left(\frac{17}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12/(-b*x**4+a)**(3/4),x)
```

```
[Out] x**13*gamma(13/4)*hyper((3/4, 13/4), (17/4,), b*x**4*exp_polar(2*I*pi)/a)/(
4*a**(3/4)*gamma(17/4))
```

3.1257 $\int \frac{x^8}{\left(a-bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=109

$$-\frac{5a^{3/2}x^3\left(1-\frac{a}{bx^4}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{12b^{3/2}\left(a-bx^4\right)^{3/4}}-\frac{5ax\sqrt[4]{a-bx^4}}{12b^2}-\frac{x^5\sqrt[4]{a-bx^4}}{6b}$$

```
[Out] -5/12*a*x*(-b*x^4+a)^(1/4)/b^2-1/6*x^5*(-b*x^4+a)^(1/4)/b-5/12*a^(3/2)*(1-a
/b/x^4)^(3/4)*x^3*(cos(1/2*arccsc(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*ar
ccsc(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccsc(x^2*b^(1/2)/a^(1/2))),2
^(1/2))/b^(3/2)/(-b*x^4+a)^(3/4)
```

Rubi [A] time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {321, 237, 335, 275, 232}

$$-\frac{5a^{3/2}x^3\left(1-\frac{a}{bx^4}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{12b^{3/2}\left(a-bx^4\right)^{3/4}}-\frac{5ax\sqrt[4]{a-bx^4}}{12b^2}-\frac{x^5\sqrt[4]{a-bx^4}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a - b*x^4)^(3/4),x]
```

```
[Out] (-5*a*x*(a - b*x^4)^(1/4))/(12*b^2) - (x^5*(a - b*x^4)^(1/4))/(6*b) - (5*a^
(3/2)*(1 - a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCsc[(Sqrt[b]*x^2)/Sqrt[a]]/2,
2])/(12*b^(3/2)*(a - b*x^4)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{x^8}{\left(a-bx^4\right)^{3/4}}\,dx = -\frac{x^5\sqrt[4]{a-bx^4}}{6b} + \frac{(5a)\int \frac{x^4}{\left(a-bx^4\right)^{3/4}}\,dx}{6b}$$

$$= -\frac{5ax\sqrt[4]{a-bx^4}}{12b^2} - \frac{x^5\sqrt[4]{a-bx^4}}{6b} + \frac{\left(5a^2\right)\int \frac{1}{\left(a-bx^4\right)^{3/4}}\,dx}{12b^2}$$

$$= -\frac{5ax\sqrt[4]{a-bx^4}}{12b^2} - \frac{x^5\sqrt[4]{a-bx^4}}{6b} + \frac{\left(5a^2\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{12b^2\left(a-bx^4\right)^{3/4}}$$

$$= -\frac{5ax\sqrt[4]{a-bx^4}}{12b^2} - \frac{x^5\sqrt[4]{a-bx^4}}{6b} - \frac{\left(5a^2\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1-\frac{ax^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{12b^2\left(a-bx^4\right)^{3/4}}$$

$$= -\frac{5ax\sqrt[4]{a-bx^4}}{12b^2} - \frac{x^5\sqrt[4]{a-bx^4}}{6b} - \frac{\left(5a^2\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{ax^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{x^2}\right)}{24b^2\left(a-bx^4\right)^{3/4}}$$

$$= -\frac{5ax\sqrt[4]{a-bx^4}}{12b^2} - \frac{x^5\sqrt[4]{a-bx^4}}{6b} - \frac{5a^{3/2}\left(1-\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{12b^{3/2}\left(a-bx^4\right)^{3/4}}$$

Mathematica [C] time = 0.02, size = 80, normalized size = 0.73

$$\frac{5a^2x\left(1-\frac{bx^4}{a}\right)^{3/4}{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};\frac{bx^4}{a}\right) - 5a^2x + 3abx^5 + 2b^2x^9}{12b^2\left(a-bx^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a - b*x^4)^(3/4),x]
```

```
[Out] (-5*a^2*x + 3*a*b*x^5 + 2*b^2*x^9 + 5*a^2*x*(1 - (b*x^4)/a)^(3/4)*Hypergeom
etric2F1[1/4, 3/4, 5/4, (b*x^4)/a])/(12*b^2*(a - b*x^4)^(3/4))
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^4+a\right)^{\frac{1}{4}}x^8}{bx^4-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^4 + a)^(1/4)*x^8/(b*x^4 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(-bx^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-b*x^4+a)^(3/4),x, algorithm="giac")
[Out] integrate(x^8/(-b*x^4 + a)^(3/4), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(-b\,x^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-b*x^4+a)^(3/4),x)
[Out] int(x^8/(-b*x^4+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(-bx^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-b*x^4+a)^(3/4),x, algorithm="maxima")
[Out] integrate(x^8/(-b*x^4 + a)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^8}{\left(a-b\,x^4\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a - b*x^4)^(3/4),x)
[Out] int(x^8/(a - b*x^4)^(3/4), x)
```

sympy [C] time = 1.93, size = 39, normalized size = 0.36

$$\frac{x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4}, \frac{9}{4} \\ \frac{13}{4}\end{matrix}\right| \frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{4}}\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-b*x**4+a)**(3/4),x)
[Out] x**9*gamma(9/4)*hyper((3/4, 9/4), (13/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*a
**(3/4)*gamma(13/4))
```

3.1258 $\int \frac{x^4}{\left(a-bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=86

$$-\frac{x\sqrt[4]{a-bx^4}}{2b}-\frac{\sqrt{a}\,x^3\left(1-\frac{a}{bx^4}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{b}\left(a-bx^4\right)^{3/4}}$$

```
[Out] -1/2*x*(-b*x^4+a)^(1/4)/b-1/2*(1-a/b/x^4)^(3/4)*x^3*(cos(1/2*arccsc(x^2*b^(
1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccsc(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(
1/2*arccsc(x^2*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)/(-b*x^4+a)^(3/4)/b^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {321, 237, 335, 275, 232}

$$-\frac{x\sqrt[4]{a-bx^4}}{2b}-\frac{\sqrt{a}\,x^3\left(1-\frac{a}{bx^4}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{b}\left(a-bx^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a - b*x^4)^(3/4),x]
```

```
[Out] -(x*(a - b*x^4)^(1/4))/(2*b) - (Sqrt[a]*(1 - a/(b*x^4))^(3/4)*x^3*EllipticF
[ArcCsc[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(2*Sqrt[b]*(a - b*x^4)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
```

egerQ[m]

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\left(a-bx^4\right)^{3/4}}\,dx &= -\frac{x\sqrt[4]{a-bx^4}}{2b} + \frac{a\int \frac{1}{\left(a-bx^4\right)^{3/4}}\,dx}{2b} \\
&= -\frac{x\sqrt[4]{a-bx^4}}{2b} + \frac{\left(a\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{2b\left(a-bx^4\right)^{3/4}} \\
&= -\frac{x\sqrt[4]{a-bx^4}}{2b} - \frac{\left(a\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1-\frac{ax^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{2b\left(a-bx^4\right)^{3/4}} \\
&= -\frac{x\sqrt[4]{a-bx^4}}{2b} - \frac{\left(a\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{ax^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{x^2}\right)}{4b\left(a-bx^4\right)^{3/4}} \\
&= -\frac{x\sqrt[4]{a-bx^4}}{2b} - \frac{\sqrt{a}\left(1-\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{b}\left(a-bx^4\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 64, normalized size = 0.74

$$\frac{x\left(a\left(1-\frac{bx^4}{a}\right)^{3/4}{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};\frac{bx^4}{a}\right)-a+bx^4\right)}{2b\left(a-bx^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a - b*x^4)^(3/4),x]
```

```
[Out] (x*(-a + b*x^4 + a*(1 - (b*x^4)/a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, (
b*x^4)/a]))/(2*b*(a - b*x^4)^(3/4))
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^4+a\right)^{\frac{1}{4}}x^4}{bx^4-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^4 + a)^(1/4)*x^4/(b*x^4 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-bx^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(-b*x^4 + a)^(3/4), x)
```

maple **[F]** time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-b\,x^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-b*x^4+a)^(3/4),x)
```

```
[Out] int(x^4/(-b*x^4+a)^(3/4),x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-bx^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(-b*x^4 + a)^(3/4), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\left(a-b\,x^4\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a - b*x^4)^(3/4),x)
```

```
[Out] int(x^4/(a - b*x^4)^(3/4), x)
```

sympy **[C]** time = 1.56, size = 39, normalized size = 0.45

$$\frac{x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{4}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-b*x**4+a)**(3/4),x)
```

```
[Out] x**5*gamma(5/4)*hyper((3/4, 5/4), (9/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*a*
**(3/4)*gamma(9/4))
```

3.1259 $\int \frac{1}{\left(a-bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=63

$$-\frac{\sqrt{b}\,x^3\left(1-\frac{a}{bx^4}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\left(a-bx^4\right)^{3/4}}$$

```
[Out] -(1-a/b/x^4)^(3/4)*x^3*(cos(1/2*arccsc(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1
/2*arccsc(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccsc(x^2*b^(1/2)/a^(1/2
))),2^(1/2))*b^(1/2)/(-b*x^4+a)^(3/4)/a^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {237, 335, 275, 232}

$$-\frac{\sqrt{b}\,x^3\left(1-\frac{a}{bx^4}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\left(a-bx^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^4)^(-3/4),x]
```

```
[Out] -((Sqrt[b]*(1 - a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCsc[(Sqrt[b]*x^2)/Sqrt[a]
]/2, 2])/(Sqrt[a]*(a - b*x^4)^(3/4)))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{\left(a-bx^4\right)^{3/4}}\,dx = \frac{\left(\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{\left(a-bx^4\right)^{3/4}}$$

$$= -\frac{\left(\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1-\frac{ax^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{\left(a-bx^4\right)^{3/4}}$$

$$= -\frac{\left(\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{ax^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{x^2}\right)}{2\left(a-bx^4\right)^{3/4}}$$

$$= -\frac{\sqrt{b}\left(1-\frac{a}{bx^4}\right)^{3/4}x^3 F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|\,2\right)}{\sqrt{a}\left(a-bx^4\right)^{3/4}}$$

Mathematica [C] time = 0.01, size = 47, normalized size = 0.75

$$\frac{x\left(1-\frac{bx^4}{a}\right)^{3/4}{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};\frac{bx^4}{a}\right)}{\left(a-bx^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^4)^(-3/4),x]
```

```
[Out] (x*(1 - (b*x^4)/a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, (b*x^4)/a])/(a -
b*x^4)^(3/4)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{bx^4-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^4 + a)^(1/4)/(b*x^4 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^4 + a)^(-3/4), x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^4+a)^(3/4),x)
```

```
[Out] int(1/(-b*x^4+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^4 + a)^(-3/4), x)
```

mupad [B] time = 1.10, size = 38, normalized size = 0.60

$$\frac{x\left(1-\frac{b\,x^4}{a}\right)^{3/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{4};\ \frac{5}{4};\ \frac{b\,x^4}{a}\right)}{\left(a-b\,x^4\right)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*x^4)^(3/4),x)
```

```
[Out] (x*(1 - (b*x^4)/a)^(3/4)*hypergeom([1/4, 3/4], 5/4, (b*x^4)/a))/(a - b*x^4)
^(3/4)
```

sympy [C] time = 1.57, size = 37, normalized size = 0.59

$$\frac{x\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{5}{4}\end{matrix}\right|\frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{4}}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**4+a)**(3/4),x)
```

```
[Out] x*gamma(1/4)*hyper((1/4, 3/4), (5/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*a**(3
/4)*gamma(5/4))
```

3.1260 $\int \frac{1}{x^4\left(a-bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=88

$$-\frac{2b^{3/2}x^3\left(1-\frac{a}{bx^4}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}\left(a-bx^4\right)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{3ax^3}$$

```
[Out] -1/3*(-b*x^4+a)^(1/4)/a/x^3-2/3*b^(3/2)*(1-a/b/x^4)^(3/4)*x^3*(cos(1/2*arcc
sc(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccsc(x^2*b^(1/2)/a^(1/2)))*Elli
pticF(sin(1/2*arccsc(x^2*b^(1/2)/a^(1/2))),2^(1/2))/a^(3/2)/(-b*x^4+a)^(3/4
)
```

Rubi [A] time = 0.04, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {325, 237, 335, 275, 232}

$$-\frac{2b^{3/2}x^3\left(1-\frac{a}{bx^4}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}\left(a-bx^4\right)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a - b*x^4)^(3/4)),x]
```

```
[Out] -(a - b*x^4)^(1/4)/(3*a*x^3) - (2*b^(3/2)*(1 - a/(b*x^4))^(3/4)*x^3*Ellipti
cF[ArcCsc[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(3*a^(3/2)*(a - b*x^4)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
```

`egerQ[m]`

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(a-bx^4\right)^{3/4}}\,dx &= -\frac{\sqrt[4]{a-bx^4}}{3ax^3} + \frac{(2b)\int \frac{1}{\left(a-bx^4\right)^{3/4}}\,dx}{3a} \\
&= -\frac{\sqrt[4]{a-bx^4}}{3ax^3} + \frac{\left(2b\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{3a\left(a-bx^4\right)^{3/4}} \\
&= -\frac{\sqrt[4]{a-bx^4}}{3ax^3} - \frac{\left(2b\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1-\frac{ax^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{3a\left(a-bx^4\right)^{3/4}} \\
&= -\frac{\sqrt[4]{a-bx^4}}{3ax^3} - \frac{\left(b\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{ax^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{x^2}\right)}{3a\left(a-bx^4\right)^{3/4}} \\
&= -\frac{\sqrt[4]{a-bx^4}}{3ax^3} - \frac{2b^{3/2}\left(1-\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}\left(a-bx^4\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.59

$$-\frac{\left(1-\frac{bx^4}{a}\right)^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{3}{4};\frac{1}{4};\frac{bx^4}{a}\right)}{3x^3\left(a-bx^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a - b*x^4)^(3/4)),x]
```

```
[Out] -1/3*((1 - (b*x^4)/a)^(3/4)*Hypergeometric2F1[-3/4, 3/4, 1/4, (b*x^4)/a])/(
x^3*(a - b*x^4)^(3/4))
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{bx^8-ax^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^4 + a)^(1/4)/(b*x^8 - a*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{3}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-b*x^4+a)^(3/4),x, algorithm="giac")
[Out] integrate(1/((-b*x^4 + a)^(3/4)*x^4), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^4+a\right)^{\frac{3}{4}}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-b*x^4+a)^(3/4),x)
[Out] int(1/x^4/(-b*x^4+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{3}{4}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-b*x^4+a)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((-b*x^4 + a)^(3/4)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\left(a-b\,x^4\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a - b*x^4)^(3/4)),x)
[Out] int(1/(x^4*(a - b*x^4)^(3/4)), x)
```

sympy [C] time = 2.30, size = 34, normalized size = 0.39

$$-\frac{i e^{\frac{3i\pi}{4}}\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\,\frac{a}{bx^4}\right)}{6b^{\frac{3}{4}}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-b*x**4+a)**(3/4),x)
[Out] -I*exp(3*I*pi/4)*hyper((3/4, 3/2), (5/2,), a/(b*x**4))/(6*b**(3/4)*x**6)
```

3.1261 $\int \frac{1}{x^8\left(a-bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=111

$$-\frac{4b^{5/2}x^3\left(1-\frac{a}{bx^4}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{7a^{5/2}\left(a-bx^4\right)^{3/4}}-\frac{2b\sqrt[4]{a-bx^4}}{7a^2x^3}-\frac{\sqrt[4]{a-bx^4}}{7ax^7}$$

```
[Out] -1/7*(-b*x^4+a)^(1/4)/a/x^7-2/7*b*(-b*x^4+a)^(1/4)/a^2/x^3-4/7*b^(5/2)*(1-a
/b/x^4)^(3/4)*x^3*(cos(1/2*arccsc(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*ar
ccsc(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccsc(x^2*b^(1/2)/a^(1/2))),2
^(1/2))/a^(5/2)/(-b*x^4+a)^(3/4)
```

Rubi [A] time = 0.05, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {325, 237, 335, 275, 232}

$$-\frac{4b^{5/2}x^3\left(1-\frac{a}{bx^4}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{7a^{5/2}\left(a-bx^4\right)^{3/4}}-\frac{2b\sqrt[4]{a-bx^4}}{7a^2x^3}-\frac{\sqrt[4]{a-bx^4}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(a - b*x^4)^(3/4)),x]
```

```
[Out] -(a - b*x^4)^(1/4)/(7*a*x^7) - (2*b*(a - b*x^4)^(1/4))/(7*a^2*x^3) - (4*b^(
5/2)*(1 - a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCsc[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2
])/(7*a^(5/2)*(a - b*x^4)^(3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{x^8\left(a-bx^4\right)^{3/4}}\,dx = -\frac{\sqrt[4]{a-bx^4}}{7ax^7} + \frac{(6b)\int \frac{1}{x^4\left(a-bx^4\right)^{3/4}}\,dx}{7a}$$

$$= -\frac{\sqrt[4]{a-bx^4}}{7ax^7} - \frac{2b\sqrt[4]{a-bx^4}}{7a^2x^3} + \frac{\left(4b^2\right)\int \frac{1}{\left(a-bx^4\right)^{3/4}}\,dx}{7a^2}$$

$$= -\frac{\sqrt[4]{a-bx^4}}{7ax^7} - \frac{2b\sqrt[4]{a-bx^4}}{7a^2x^3} + \frac{\left(4b^2\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{7a^2\left(a-bx^4\right)^{3/4}}$$

$$= -\frac{\sqrt[4]{a-bx^4}}{7ax^7} - \frac{2b\sqrt[4]{a-bx^4}}{7a^2x^3} - \frac{\left(4b^2\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1-\frac{ax^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{7a^2\left(a-bx^4\right)^{3/4}}$$

$$= -\frac{\sqrt[4]{a-bx^4}}{7ax^7} - \frac{2b\sqrt[4]{a-bx^4}}{7a^2x^3} - \frac{\left(2b^2\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{ax^2}{b}\right)^{3/4}}\,dx, x, \frac{1}{x^2}\right)}{7a^2\left(a-bx^4\right)^{3/4}}$$

$$= -\frac{\sqrt[4]{a-bx^4}}{7ax^7} - \frac{2b\sqrt[4]{a-bx^4}}{7a^2x^3} - \frac{4b^{5/2}\left(1-\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{7a^{5/2}\left(a-bx^4\right)^{3/4}}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.47

$$-\frac{\left(1-\frac{bx^4}{a}\right)^{3/4}{}_2F_1\left(-\frac{7}{4},\frac{3}{4};-\frac{3}{4};\frac{bx^4}{a}\right)}{7x^7\left(a-bx^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(a - b*x^4)^(3/4)),x]
```

```
[Out] -1/7*((1 - (b*x^4)/a)^(3/4)*Hypergeometric2F1[-7/4, 3/4, -3/4, (b*x^4)/a])/
(x^7*(a - b*x^4)^(3/4))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{bx^{12}-ax^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(-(-b*x^4 + a)^(1/4)/(b*x^12 - a*x^8), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{3}{4}}x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-b*x^4+a)^(3/4),x, algorithm="giac")
[Out] integrate(1/((-b*x^4 + a)^(3/4)*x^8), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^4+a\right)^{\frac{3}{4}}x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(-b*x^4+a)^(3/4),x)
[Out] int(1/x^8/(-b*x^4+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{3}{4}}x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-b*x^4+a)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((-b*x^4 + a)^(3/4)*x^8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^8\left(a-b\,x^4\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(a - b*x^4)^(3/4)),x)
[Out] int(1/(x^8*(a - b*x^4)^(3/4)), x)
```

sympy [C] time = 2.61, size = 31, normalized size = 0.28

$$\frac{ie^{-\frac{i\pi}{4}}{}_2F_1\left(\begin{matrix}\frac{3}{4}, \frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle| \frac{a}{bx^4}\right)}{10b^{\frac{3}{4}}x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(-b*x**4+a)**(3/4),x)
[Out] I*exp(-I*pi/4)*hyper((3/4, 5/2), (7/2,), a/(b*x**4))/(10*b**(3/4)*x**10)
```

3.1262 $\int \frac{1}{x^{12}\left(a-bx^4\right)^{3/4}}\,dx$

Optimal. Leaf size=136

$$-\frac{40b^{7/2}x^3\left(1-\frac{a}{bx^4}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{77a^{7/2}\left(a-bx^4\right)^{3/4}}-\frac{20b^2\sqrt[4]{a-bx^4}}{77a^3x^3}-\frac{10b\sqrt[4]{a-bx^4}}{77a^2x^7}-\frac{\sqrt[4]{a-bx^4}}{11ax^{11}}$$

```
[Out] -1/11*(-b*x^4+a)^(1/4)/a/x^11-10/77*b*(-b*x^4+a)^(1/4)/a^2/x^7-20/77*b^2*(-
b*x^4+a)^(1/4)/a^3/x^3-40/77*b^(7/2)*(1-a/b/x^4)^(3/4)*x^3*(cos(1/2*arccsc(
x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccsc(x^2*b^(1/2)/a^(1/2)))*Ellipti
cF(sin(1/2*arccsc(x^2*b^(1/2)/a^(1/2))),2^(1/2))/a^(7/2)/(-b*x^4+a)^(3/4)
```

Rubi [A] time = 0.06, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {325, 237, 335, 275, 232}

$$-\frac{20b^2\sqrt[4]{a-bx^4}}{77a^3x^3}-\frac{40b^{7/2}x^3\left(1-\frac{a}{bx^4}\right)^{3/4}F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{77a^{7/2}\left(a-bx^4\right)^{3/4}}-\frac{10b\sqrt[4]{a-bx^4}}{77a^2x^7}-\frac{\sqrt[4]{a-bx^4}}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^12*(a - b*x^4)^(3/4)),x]
```

```
[Out] -(a - b*x^4)^(1/4)/(11*a*x^11) - (10*b*(a - b*x^4)^(1/4))/(77*a^2*x^7) - (2
0*b^2*(a - b*x^4)^(1/4))/(77*a^3*x^3) - (40*b^(7/2)*(1 - a/(b*x^4))^(3/4)*x
^3*EllipticF[ArcCsc[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(77*a^(7/2)*(a - b*x^4)^(
3/4))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{x^{12}\left(a-bx^4\right)^{3/4}}\,dx = -\frac{\sqrt[4]{a-bx^4}}{11ax^{11}} + \frac{(10b)\int \frac{1}{x^8\left(a-bx^4\right)^{3/4}}\,dx}{11a}$$

$$= -\frac{\sqrt[4]{a-bx^4}}{11ax^{11}} - \frac{10b\sqrt[4]{a-bx^4}}{77a^2x^7} + \frac{\left(60b^2\right)\int \frac{1}{x^4\left(a-bx^4\right)^{3/4}}\,dx}{77a^2}$$

$$= -\frac{\sqrt[4]{a-bx^4}}{11ax^{11}} - \frac{10b\sqrt[4]{a-bx^4}}{77a^2x^7} - \frac{20b^2\sqrt[4]{a-bx^4}}{77a^3x^3} + \frac{\left(40b^3\right)\int \frac{1}{\left(a-bx^4\right)^{3/4}}\,dx}{77a^3}$$

$$= -\frac{\sqrt[4]{a-bx^4}}{11ax^{11}} - \frac{10b\sqrt[4]{a-bx^4}}{77a^2x^7} - \frac{20b^2\sqrt[4]{a-bx^4}}{77a^3x^3} + \frac{\left(40b^3\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{77a^3\left(a-bx^4\right)^{3/4}}$$

$$= -\frac{\sqrt[4]{a-bx^4}}{11ax^{11}} - \frac{10b\sqrt[4]{a-bx^4}}{77a^2x^7} - \frac{20b^2\sqrt[4]{a-bx^4}}{77a^3x^3} - \frac{\left(40b^3\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1-\frac{ax^4}{b}\right)^{3/4}}\right.}{77a^3\left(a-bx^4\right)^{3/4}}$$

$$= -\frac{\sqrt[4]{a-bx^4}}{11ax^{11}} - \frac{10b\sqrt[4]{a-bx^4}}{77a^2x^7} - \frac{20b^2\sqrt[4]{a-bx^4}}{77a^3x^3} - \frac{\left(20b^3\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{ax^2}{b}\right)^{3/4}}\right.}{77a^3\left(a-bx^4\right)^{3/4}}$$

$$= -\frac{\sqrt[4]{a-bx^4}}{11ax^{11}} - \frac{10b\sqrt[4]{a-bx^4}}{77a^2x^7} - \frac{20b^2\sqrt[4]{a-bx^4}}{77a^3x^3} - \frac{40b^{7/2}\left(1-\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right.\right.}{77a^{7/2}\left(a-bx^4\right)^{3/4}}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.38

$$-\frac{\left(1-\frac{bx^4}{a}\right)^{3/4}{}_2F_1\left(-\frac{11}{4},\frac{3}{4};-\frac{7}{4};\frac{bx^4}{a}\right)}{11x^{11}\left(a-bx^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^12*(a - b*x^4)^(3/4)),x]
```

```
[Out] -1/11*((1 - (b*x^4)/a)^(3/4)*Hypergeometric2F1[-11/4, 3/4, -7/4, (b*x^4)/a]
)/(x^11*(a - b*x^4)^(3/4))
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{bx^{16}-ax^{12}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^12/(-b*x^4+a)^(3/4),x, algorithm="fricas")
[Out] integral(-(-b*x^4 + a)^(1/4)/(b*x^16 - a*x^12), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{3}{4}}x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^12/(-b*x^4+a)^(3/4),x, algorithm="giac")
[Out] integrate(1/((-b*x^4 + a)^(3/4)*x^12), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^4+a\right)^{\frac{3}{4}}x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^12/(-b*x^4+a)^(3/4),x)
[Out] int(1/x^12/(-b*x^4+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{3}{4}}x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^12/(-b*x^4+a)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((-b*x^4 + a)^(3/4)*x^12), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{12}\left(a-b\,x^4\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^12*(a - b*x^4)^(3/4)),x)
[Out] int(1/(x^12*(a - b*x^4)^(3/4)), x)
```

sympy [C] time = 2.94, size = 34, normalized size = 0.25

$$-\frac{ie^{\frac{3i\pi}{4}}{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{a}{bx^4}\right)}{14b^{\frac{3}{4}}x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**12/(-b*x**4+a)**(3/4),x)
[Out] -I*exp(3*I*pi/4)*hyper((3/4, 7/2), (9/2,), a/(b*x**4))/(14*b**(3/4)*x**14)
```

3.1263 $\int \frac{x^2}{\left(a-bx^4\right)^{5/4}}\,dx$

Optimal. Leaf size=81

$$\frac{1}{bx\sqrt[4]{a-bx^4}}-\frac{x\sqrt[4]{1-\frac{a}{bx^4}}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt[4]{a-bx^4}}$$

```
[Out] 1/b/x/(-b*x^4+a)^(1/4)-(1-a/b/x^4)^(1/4)*x*(cos(1/2*arccsc(x^2*b^(1/2)/a^(1
/2)))^2)^(1/2)/cos(1/2*arccsc(x^2*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arccs
c(x^2*b^(1/2)/a^(1/2))),2^(1/2))/(-b*x^4+a)^(1/4)/a^(1/2)/b^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {287, 313, 335, 275, 228}

$$\frac{1}{bx\sqrt[4]{a-bx^4}}-\frac{x\sqrt[4]{1-\frac{a}{bx^4}}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt[4]{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a - b*x^4)^(5/4),x]
```

```
[Out] 1/(b*x*(a - b*x^4)^(1/4)) - ((1 - a/(b*x^4))^(1/4)*x*EllipticE[ArcCsc[(Sqrt
[b]*x^2)/Sqrt[a]]/2, 2])/(Sqrt[a]*Sqrt[b]*(a - b*x^4)^(1/4))
```

Rule 228

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 287

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> -Simp[(b*x*(a + b*x^4)
^(1/4))^(-1), x] - Dist[1/b, Int[1/(x^2*(a + b*x^4)^(1/4)), x], x] /; FreeQ
[{a, b}, x] && NegQ[b/a]
```

Rule 313

```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> Dist[(x*(1 + a/(b*
x^4))^(1/4))/(a + b*x^4)^(1/4), Int[1/(x^3*(1 + a/(b*x^4))^(1/4)), x], x] /
; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a-bx^4\right)^{5/4}}\,dx &= \frac{1}{bx\sqrt[4]{a-bx^4}} + \frac{\int \frac{1}{x^2\sqrt[4]{a-bx^4}}\,dx}{b} \\ &= \frac{1}{bx\sqrt[4]{a-bx^4}} + \frac{\left(\sqrt[4]{1-\frac{a}{bx^4}}\,x\right)\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^4}}\,x^3}\,dx}{b\sqrt[4]{a-bx^4}} \\ &= \frac{1}{bx\sqrt[4]{a-bx^4}} - \frac{\left(\sqrt[4]{1-\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{x}{\sqrt[4]{1-\frac{ax^4}{b}}}\,dx, x, \frac{1}{x}\right)}{b\sqrt[4]{a-bx^4}} \\ &= \frac{1}{bx\sqrt[4]{a-bx^4}} - \frac{\left(\sqrt[4]{1-\frac{a}{bx^4}}\,x\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{ax^2}{b}}}\,dx, x, \frac{1}{x^2}\right)}{2b\sqrt[4]{a-bx^4}} \\ &= \frac{1}{bx\sqrt[4]{a-bx^4}} - \frac{\sqrt[4]{1-\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt[4]{a-bx^4}}\end{aligned}$$

Mathematica [C] time = 0.04, size = 55, normalized size = 0.68

$$\frac{x^3\sqrt[4]{1-\frac{bx^4}{a}}\;{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{7}{4};\frac{bx^4}{a}\right)}{3a\sqrt[4]{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a - b*x^4)^(5/4),x]
```

```
[Out] (x^3*(1 - (b*x^4)/a)^(1/4)*Hypergeometric2F1[3/4, 5/4, 7/4, (b*x^4)/a])/(3*
a*(a - b*x^4)^(1/4))
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-bx^4+a\right)^{\frac{3}{4}}x^2}{b^2x^8-2\,abx^4+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^4+a)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((-b*x^4 + a)^(3/4)*x^2/(b^2*x^8 - 2*a*b*x^4 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-bx^4+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^4+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(-b*x^4 + a)^(5/4), x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-b\,x^4+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^4+a)^(5/4),x)
```

```
[Out] int(x^2/(-b*x^4+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-bx^4+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^4+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(-b*x^4 + a)^(5/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(a-b\,x^4\right)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a - b*x^4)^(5/4),x)
```

```
[Out] int(x^2/(a - b*x^4)^(5/4), x)
```

sympy [C] time = 2.27, size = 39, normalized size = 0.48

$$\frac{x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{5}{4}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**4+a)**(5/4),x)
```

```
[Out] x**3*gamma(3/4)*hyper((3/4, 5/4), (7/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*a*
**(5/4)*gamma(7/4))
```

3.1264 $\int x^7 \left(a+bx^4\right)^p \, dx$

Optimal. Leaf size=48

$$\frac{\left(a+bx^4\right)^{p+2}}{4b^2(p+2)} - \frac{a\left(a+bx^4\right)^{p+1}}{4b^2(p+1)}$$

```
[Out] -1/4*a*(b*x^4+a)^(1+p)/b^2/(1+p)+1/4*(b*x^4+a)^(2+p)/b^2/(2+p)
```

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{\left(a+bx^4\right)^{p+2}}{4b^2(p+2)} - \frac{a\left(a+bx^4\right)^{p+1}}{4b^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^4)^p,x]
```

```
[Out] -(a*(a + b*x^4)^(1 + p))/(4*b^2*(1 + p)) + (a + b*x^4)^(2 + p)/(4*b^2*(2 +
p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^7 \left(a+bx^4\right)^p \, dx &= \frac{1}{4}\,\text{Subst}\left(\int x(a+bx)^p \, dx, x, x^4\right) \\
&= \frac{1}{4}\,\text{Subst}\left(\int \left(-\frac{a(a+bx)^p}{b} + \frac{(a+bx)^{1+p}}{b}\right) dx, x, x^4\right) \\
&= -\frac{a\left(a+bx^4\right)^{1+p}}{4b^2(1+p)} + \frac{\left(a+bx^4\right)^{2+p}}{4b^2(2+p)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 0.83

$$\frac{\left(a+bx^4\right)^{p+1}\left(b(p+1)x^4 - a\right)}{4b^2(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^4)^p,x]
```

```
[Out] ((a + b*x^4)^(1 + p)*(-a + b*(1 + p)*x^4))/(4*b^2*(1 + p)*(2 + p))
```

fricas [A] time = 0.50, size = 58, normalized size = 1.21

$$\frac{\left(\left(b^2p+b^2\right)x^8+abpx^4-a^2\right)\left(bx^4+a\right)^p}{4\left(b^2p^2+3\,b^2p+2\,b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^4+a)^p,x, algorithm="fricas")
```

```
[Out] 1/4*((b^2*p + b^2)*x^8 + a*b*p*x^4 - a^2)*(b*x^4 + a)^p/(b^2*p^2 + 3*b^2*p
+ 2*b^2)
```

giac [B] time = 0.17, size = 94, normalized size = 1.96

$$\frac{\left(bx^4+a\right)^2\left(bx^4+a\right)^p p-\left(bx^4+a\right)\left(bx^4+a\right)^p ap+\left(bx^4+a\right)^2\left(bx^4+a\right)^p-2\left(bx^4+a\right)\left(bx^4+a\right)^p a}{4\left(p^2+3\,p+2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^4+a)^p,x, algorithm="giac")
```

```
[Out] 1/4*((b*x^4 + a)^2*(b*x^4 + a)^p*p - (b*x^4 + a)*(b*x^4 + a)^p*a*p + (b*x^4
 + a)^2*(b*x^4 + a)^p - 2*(b*x^4 + a)*(b*x^4 + a)^p*a)/((p^2 + 3*p + 2)*b^2
)
```

maple [A] time = 0.01, size = 42, normalized size = 0.88

$$-\frac{\left(-x^4pb-b\,x^4+a\right)\left(b\,x^4+a\right)^{p+1}}{4\left(p^2+3p+2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^4+a)^p,x)
```

```
[Out] -1/4*(b*x^4+a)^(p+1)*(-b*p*x^4-b*x^4+a)/b^2/(p^2+3*p+2)
```

maxima [A] time = 1.04, size = 47, normalized size = 0.98

$$\frac{\left(b^2\left(p+1\right)x^8+abpx^4-a^2\right)\left(bx^4+a\right)^p}{4\left(p^2+3\,p+2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^4+a)^p,x, algorithm="maxima")
```

```
[Out] 1/4*(b^2*(p + 1)*x^8 + a*b*p*x^4 - a^2)*(b*x^4 + a)^p/((p^2 + 3*p + 2)*b^2)
```

mupad [B] time = 1.14, size = 68, normalized size = 1.42

$$\left(b\,x^4+a\right)^p\left(\frac{x^8\,\left(p+1\right)}{4\,\left(p^2+3\,p+2\right)}-\frac{a^2}{4\,b^2\,\left(p^2+3\,p+2\right)}+\frac{a\,p\,x^4}{4\,b\,\left(p^2+3\,p+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^4)^p,x)
```

```
[Out] (a + b*x^4)^p*((x^8*(p + 1))/(4*(3*p + p^2 + 2)) - a^2/(4*b^2*(3*p + p^2 +
2)) + (a*p*x^4)/(4*b*(3*p + p^2 + 2)))
```

sympy [**A**] time = 8.39, size = 495, normalized size = 10.31

$$\begin{cases} \frac{a^p x^8}{8} \\ \frac{a \log\left(-\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}+x\right)}{4ab^2+4b^3x^4}+\frac{a \log\left(\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}+x\right)}{4ab^2+4b^3x^4}+\frac{a \log\left(i\sqrt{a}\,\sqrt{\frac{1}{b}}+x^2\right)}{4ab^2+4b^3x^4}+\frac{a}{4ab^2+4b^3x^4}+\frac{bx^4 \log\left(-\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}+x\right)}{4ab^2+4b^3x^4}+\frac{bx^4 \log\left(\sqrt[4]{-1}\,\sqrt[4]{a} \right.}{4ab^2+4b^3x} \\ -\frac{a \log\left(-\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}+x\right)}{4b^2}-\frac{a \log\left(\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}+x\right)}{4b^2}-\frac{a \log\left(i\sqrt{a}\,\sqrt{\frac{1}{b}}+x^2\right)}{4b^2}+\frac{x^4}{4b} \\ -\frac{a^2\left(a+bx^4\right)^p}{4b^2p^2+12b^2p+8b^2}+\frac{abpx^4\left(a+bx^4\right)^p}{4b^2p^2+12b^2p+8b^2}+\frac{b^2px^8\left(a+bx^4\right)^p}{4b^2p^2+12b^2p+8b^2}+\frac{b^2x^8\left(a+bx^4\right)^p}{4b^2p^2+12b^2p+8b^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**4+a)**p,x)
```

```
[Out] Piecewise((a**p*x**8/8, Eq(b, 0)), (a*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4
) + x)/(4*a*b**2 + 4*b**3*x**4) + a*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) +
 x)/(4*a*b**2 + 4*b**3*x**4) + a*log(I*sqrt(a)*sqrt(1/b) + x**2)/(4*a*b**2
+ 4*b**3*x**4) + a/(4*a*b**2 + 4*b**3*x**4) + b*x**4*log(-(-1)**(1/4)*a**(1
/4)*(1/b)**(1/4) + x)/(4*a*b**2 + 4*b**3*x**4) + b*x**4*log((-1)**(1/4)*a**
(1/4)*(1/b)**(1/4) + x)/(4*a*b**2 + 4*b**3*x**4) + b*x**4*log(I*sqrt(a)*sqr
t(1/b) + x**2)/(4*a*b**2 + 4*b**3*x**4), Eq(p, -2)), (-a*log(-(-1)**(1/4)*a
**(1/4)*(1/b)**(1/4) + x)/(4*b**2) - a*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4
) + x)/(4*b**2) - a*log(I*sqrt(a)*sqrt(1/b) + x**2)/(4*b**2) + x**4/(4*b),
Eq(p, -1)), (-a**2*(a + b*x**4)**p/(4*b**2*p**2 + 12*b**2*p + 8*b**2) + a*b
*p*x**4*(a + b*x**4)**p/(4*b**2*p**2 + 12*b**2*p + 8*b**2) + b**2*p*x**8*(a
 + b*x**4)**p/(4*b**2*p**2 + 12*b**2*p + 8*b**2) + b**2*x**8*(a + b*x**4)**
p/(4*b**2*p**2 + 12*b**2*p + 8*b**2), True))
```

3.1265 $\int x^3 \left(a + bx^4\right)^p \, dx$

Optimal. Leaf size=23

$$\frac{\left(a + bx^4\right)^{p+1}}{4b(p+1)}$$

```
[Out] 1/4*(b*x^4+a)^(1+p)/b/(1+p)
```

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$\frac{\left(a + bx^4\right)^{p+1}}{4b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^4)^p,x]
```

```
[Out] (a + b*x^4)^(1 + p)/(4*b*(1 + p))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^3 \left(a + bx^4\right)^p \, dx = \frac{\left(a + bx^4\right)^{1+p}}{4b(1+p)}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{\left(a + bx^4\right)^{p+1}}{4b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^4)^p,x]
```

```
[Out] (a + b*x^4)^(1 + p)/(4*b*(1 + p))
```

fricas [A] time = 0.91, size = 25, normalized size = 1.09

$$\frac{\left(bx^4 + a\right)\left(bx^4 + a\right)^p}{4\left(bp + b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^4+a)^p,x, algorithm="fricas")
```

```
[Out] 1/4*(b*x^4 + a)*(b*x^4 + a)^p/(b*p + b)
```

giac [A] time = 0.15, size = 21, normalized size = 0.91

$$\frac{\left(bx^4+a\right)^{p+1}}{4\,b\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^4+a)^p,x, algorithm="giac")
[Out] 1/4*(b*x^4 + a)^(p + 1)/(b*(p + 1))
```

maple [A] time = 0.00, size = 22, normalized size = 0.96

$$\frac{\left(b\,x^4+a\right)^{p+1}}{4\left(p+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^4+a)^p,x)
[Out] 1/4*(b*x^4+a)^(p+1)/b/(p+1)
```

maxima [A] time = 1.15, size = 21, normalized size = 0.91

$$\frac{\left(bx^4+a\right)^{p+1}}{4\,b\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^4+a)^p,x, algorithm="maxima")
[Out] 1/4*(b*x^4 + a)^(p + 1)/(b*(p + 1))
```

mupad [B] time = 1.12, size = 21, normalized size = 0.91

$$\frac{\left(b\,x^4+a\right)^{p+1}}{4\,b\,\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^4)^p,x)
[Out] (a + b*x^4)^(p + 1)/(4*b*(p + 1))
```

sympy [A] time = 3.36, size = 129, normalized size = 5.61

$$\begin{cases} \frac{x^4}{4a} & \text{for } b = 0 \wedge p = -1 \\ \frac{a^p x^4}{4} & \text{for } b = 0 \\ \frac{\log\left(-\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}+x\right)}{4b} + \frac{\log\left(\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}+x\right)}{4b} + \frac{\log\left(i\sqrt{a}\,\sqrt{\frac{1}{b}}+x^2\right)}{4b} & \text{for } p = -1 \\ \frac{a\left(a+bx^4\right)^p}{4bp+4b} + \frac{bx^4\left(a+bx^4\right)^p}{4bp+4b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**4+a)**p,x)
```

```
[Out] Piecewise((x**4/(4*a), Eq(b, 0) & Eq(p, -1)), (a**p*x**4/4, Eq(b, 0)), (log
(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + x)/(4*b) + log((-1)**(1/4)*a**(1/4)*(
1/b)**(1/4) + x)/(4*b) + log(I*sqrt(a)*sqrt(1/b) + x**2)/(4*b), Eq(p, -1)),
 (a*(a + b*x**4)**p/(4*b*p + 4*b) + b*x**4*(a + b*x**4)**p/(4*b*p + 4*b), T
rue))
```

3.1266 $\int \frac{\left(a+bx^4\right)^p}{x}\,dx$

Optimal. Leaf size=41

$$-\frac{\left(a+bx^4\right)^{p+1}\ {}_2F_1\left(1,p+1;p+2;\frac{bx^4}{a}+1\right)}{4a(p+1)}$$

```
[Out] -1/4*(b*x^4+a)^(1+p)*hypergeom([1, 1+p],[2+p],1+b*x^4/a)/a/(1+p)
```

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 65}

$$-\frac{\left(a+bx^4\right)^{p+1}\ {}_2F_1\left(1,p+1;p+2;\frac{bx^4}{a}+1\right)}{4a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^p/x,x]
```

```
[Out] -((a + b*x^4)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^4)/a])/(4
*a*(1 + p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^4\right)^p}{x}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, x^4\right)\\ &= -\frac{\left(a+bx^4\right)^{1+p}\ {}_2F_1\left(1,1+p;2+p;1+\frac{bx^4}{a}\right)}{4a(1+p)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 1.00

$$-\frac{\left(a+bx^4\right)^{p+1}\ {}_2F_1\left(1,p+1;p+2;\frac{bx^4}{a}+1\right)}{4a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^p/x,x]
```

```
[Out] -1/4*((a + b*x^4)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^4)/a]
)/(a*(1 + p))
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)^p}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^p/x,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^p/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^p/x,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^p/x, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^p/x,x)
```

```
[Out] int((b*x^4+a)^p/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^p/x,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^p/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,x^4+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^p/x,x)
```

```
[Out] int((a + b*x^4)^p/x, x)
```

sympy [C] time = 7.82, size = 39, normalized size = 0.95

$$-\frac{b^p x^{4p}\Gamma\left(-p\right){}_2F_1\left(\begin{matrix}-p,-p\\1-p\end{matrix}\middle|\frac{ae^{i\pi}}{bx^4}\right)}{4\Gamma\left(1-p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**p/x,x)
```

```
[Out] -b**p*x**(4*p)*gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*x**
4))/(4*gamma(1 - p))
```

3.1267 $\int \frac{x^{24}}{a+bx^5}\,dx$

Optimal. Leaf size=66

$$\frac{a^4\log\left(a+bx^5\right)}{5b^5}-\frac{a^3x^5}{5b^4}+\frac{a^2x^{10}}{10b^3}-\frac{ax^{15}}{15b^2}+\frac{x^{20}}{20b}$$

```
[Out] -1/5*a^3*x^5/b^4+1/10*a^2*x^10/b^3-1/15*a*x^15/b^2+1/20*x^20/b+1/5*a^4*ln(b
*x^5+a)/b^5
```

Rubi [A] time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{a^2x^{10}}{10b^3}-\frac{a^3x^5}{5b^4}+\frac{a^4\log\left(a+bx^5\right)}{5b^5}-\frac{ax^{15}}{15b^2}+\frac{x^{20}}{20b}$$

Antiderivative was successfully verified.

```
[In] Int[x^24/(a + b*x^5),x]
```

```
[Out] -(a^3*x^5)/(5*b^4) + (a^2*x^10)/(10*b^3) - (a*x^15)/(15*b^2) + x^20/(20*b)
+ (a^4*Log[a + b*x^5])/(5*b^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{24}}{a+bx^5}\,dx &= \frac{1}{5}\operatorname{Subst}\left(\int \frac{x^4}{a+bx}\,dx, x, x^5\right)\\ &= \frac{1}{5}\operatorname{Subst}\left(\int \left(-\frac{a^3}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{b^2}+\frac{x^3}{b}+\frac{a^4}{b^4(a+bx)}\right)dx, x, x^5\right)\\ &= -\frac{a^3x^5}{5b^4}+\frac{a^2x^{10}}{10b^3}-\frac{ax^{15}}{15b^2}+\frac{x^{20}}{20b}+\frac{a^4\log\left(a+bx^5\right)}{5b^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 66, normalized size = 1.00

$$\frac{a^4\log\left(a+bx^5\right)}{5b^5}-\frac{a^3x^5}{5b^4}+\frac{a^2x^{10}}{10b^3}-\frac{ax^{15}}{15b^2}+\frac{x^{20}}{20b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^24/(a + b*x^5),x]
```

```
[Out] -1/5*(a^3*x^5)/b^4 + (a^2*x^10)/(10*b^3) - (a*x^15)/(15*b^2) + x^20/(20*b)
+ (a^4*Log[a + b*x^5])/(5*b^5)
```

fricas [A] time = 0.59, size = 56, normalized size = 0.85

$$\frac{3\,b^4x^{20} - 4\,ab^3x^{15} + 6\,a^2b^2x^{10} - 12\,a^3bx^5 + 12\,a^4\log\left(bx^5 + a\right)}{60\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24/(b*x^5+a),x, algorithm="fricas")
```

```
[Out] 1/60*(3*b^4*x^20 - 4*a*b^3*x^15 + 6*a^2*b^2*x^10 - 12*a^3*b*x^5 + 12*a^4*lo
g(b*x^5 + a))/b^5
```

giac [A] time = 0.17, size = 58, normalized size = 0.88

$$\frac{a^4\log\left(\left|bx^5 + a\right|\right)}{5\,b^5} + \frac{3\,b^3x^{20} - 4\,ab^2x^{15} + 6\,a^2bx^{10} - 12\,a^3x^5}{60\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24/(b*x^5+a),x, algorithm="giac")
```

```
[Out] 1/5*a^4*log(abs(b*x^5 + a))/b^5 + 1/60*(3*b^3*x^20 - 4*a*b^2*x^15 + 6*a^2*b
*x^10 - 12*a^3*x^5)/b^4
```

maple [A] time = 0.00, size = 57, normalized size = 0.86

$$\frac{x^{20}}{20b} - \frac{a\,x^{15}}{15b^2} + \frac{a^2x^{10}}{10b^3} - \frac{a^3x^5}{5b^4} + \frac{a^4\ln\left(b\,x^5 + a\right)}{5b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^24/(b*x^5+a),x)
```

```
[Out] -1/5*a^3*x^5/b^4+1/10*a^2*x^10/b^3-1/15*a*x^15/b^2+1/20*x^20/b+1/5*a^4*ln(b
*x^5+a)/b^5
```

maxima [A] time = 1.02, size = 57, normalized size = 0.86

$$\frac{a^4\log\left(bx^5 + a\right)}{5\,b^5} + \frac{3\,b^3x^{20} - 4\,ab^2x^{15} + 6\,a^2bx^{10} - 12\,a^3x^5}{60\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24/(b*x^5+a),x, algorithm="maxima")
```

```
[Out] 1/5*a^4*log(b*x^5 + a)/b^5 + 1/60*(3*b^3*x^20 - 4*a*b^2*x^15 + 6*a^2*b*x^10
 - 12*a^3*x^5)/b^4
```

mupad [B] time = 0.08, size = 56, normalized size = 0.85

$$\frac{x^{20}}{20\,b} - \frac{a\,x^{15}}{15\,b^2} + \frac{a^4\,\ln\left(b\,x^5 + a\right)}{5\,b^5} - \frac{a^3\,x^5}{5\,b^4} + \frac{a^2\,x^{10}}{10\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^24/(a + b*x^5),x)
```

```
[Out] x^20/(20*b) - (a*x^15)/(15*b^2) + (a^4*log(a + b*x^5))/(5*b^5) - (a^3*x^5)/
(5*b^4) + (a^2*x^10)/(10*b^3)
```

sympy [A] time = 0.32, size = 56, normalized size = 0.85

$$\frac{a^4\log\left(a + bx^5\right)}{5b^5} - \frac{a^3x^5}{5b^4} + \frac{a^2x^{10}}{10b^3} - \frac{ax^{15}}{15b^2} + \frac{x^{20}}{20b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**24/(b*x**5+a),x)
```

```
[Out] a**4*log(a + b*x**5)/(5*b**5) - a**3*x**5/(5*b**4) + a**2*x**10/(10*b**3) -
 a*x**15/(15*b**2) + x**20/(20*b)
```

3.1268 $\int \frac{x^{19}}{a+bx^5}\,dx$

Optimal. Leaf size=53

$$-\frac{a^3 \log\left(a+bx^5\right)}{5b^4}+\frac{a^2x^5}{5b^3}-\frac{ax^{10}}{10b^2}+\frac{x^{15}}{15b}$$

```
[Out] 1/5*a^2*x^5/b^3-1/10*a*x^10/b^2+1/15*x^15/b-1/5*a^3*ln(b*x^5+a)/b^4
```

Rubi [A] time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{a^2x^5}{5b^3}-\frac{a^3 \log\left(a+bx^5\right)}{5b^4}-\frac{ax^{10}}{10b^2}+\frac{x^{15}}{15b}$$

Antiderivative was successfully verified.

```
[In] Int[x^19/(a + b*x^5),x]
```

```
[Out] (a^2*x^5)/(5*b^3) - (a*x^10)/(10*b^2) + x^15/(15*b) - (a^3*Log[a + b*x^5])/
(5*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{19}}{a+bx^5}\,dx &= \frac{1}{5}\,\text{Subst}\left(\int \frac{x^3}{a+bx}\,dx, x, x^5\right)\\ &= \frac{1}{5}\,\text{Subst}\left(\int \left(\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right)dx, x, x^5\right)\\ &= \frac{a^2x^5}{5b^3}-\frac{ax^{10}}{10b^2}+\frac{x^{15}}{15b}-\frac{a^3 \log\left(a+bx^5\right)}{5b^4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 53, normalized size = 1.00

$$-\frac{a^3 \log\left(a+bx^5\right)}{5b^4}+\frac{a^2x^5}{5b^3}-\frac{ax^{10}}{10b^2}+\frac{x^{15}}{15b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^19/(a + b*x^5),x]
```

```
[Out] (a^2*x^5)/(5*b^3) - (a*x^10)/(10*b^2) + x^15/(15*b) - (a^3*Log[a + b*x^5])/
(5*b^4)
```

fricas [A] time = 0.60, size = 45, normalized size = 0.85

$$\frac{2\,b^3x^{15} - 3\,ab^2x^{10} + 6\,a^2bx^5 - 6\,a^3\log\left(bx^5 + a\right)}{30\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^5+a),x, algorithm="fricas")
```

```
[Out] 1/30*(2*b^3*x^15 - 3*a*b^2*x^10 + 6*a^2*b*x^5 - 6*a^3*log(b*x^5 + a))/b^4
```

giac [A] time = 0.21, size = 47, normalized size = 0.89

$$-\frac{a^3\log\left(\left|bx^5 + a\right|\right)}{5\,b^4} + \frac{2\,b^2x^{15} - 3\,abx^{10} + 6\,a^2x^5}{30\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^5+a),x, algorithm="giac")
```

```
[Out] -1/5*a^3*log(abs(b*x^5 + a))/b^4 + 1/30*(2*b^2*x^15 - 3*a*b*x^10 + 6*a^2*x^
5)/b^3
```

maple [A] time = 0.00, size = 46, normalized size = 0.87

$$\frac{x^{15}}{15b} - \frac{a\,x^{10}}{10b^2} + \frac{a^2x^5}{5b^3} - \frac{a^3\ln\left(b\,x^5 + a\right)}{5b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19/(b*x^5+a),x)
```

```
[Out] 1/5*a^2*x^5/b^3-1/10*a*x^10/b^2+1/15*x^15/b-1/5*a^3*ln(b*x^5+a)/b^4
```

maxima [A] time = 1.01, size = 46, normalized size = 0.87

$$-\frac{a^3\log\left(bx^5 + a\right)}{5\,b^4} + \frac{2\,b^2x^{15} - 3\,abx^{10} + 6\,a^2x^5}{30\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^5+a),x, algorithm="maxima")
```

```
[Out] -1/5*a^3*log(b*x^5 + a)/b^4 + 1/30*(2*b^2*x^15 - 3*a*b*x^10 + 6*a^2*x^5)/b^
3
```

mupad [B] time = 0.05, size = 45, normalized size = 0.85

$$\frac{x^{15}}{15\,b} - \frac{a\,x^{10}}{10\,b^2} - \frac{a^3\,\ln\left(b\,x^5 + a\right)}{5\,b^4} + \frac{a^2\,x^5}{5\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19/(a + b*x^5),x)
```

```
[Out] x^15/(15*b) - (a*x^10)/(10*b^2) - (a^3*log(a + b*x^5))/(5*b^4) + (a^2*x^5)/
(5*b^3)
```

sympy [A] time = 0.67, size = 44, normalized size = 0.83

$$-\frac{a^3\log\left(a + bx^5\right)}{5b^4} + \frac{a^2x^5}{5b^3} - \frac{ax^{10}}{10b^2} + \frac{x^{15}}{15b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**19/(b*x**5+a),x)
```

```
[Out] -a**3*log(a + b*x**5)/(5*b**4) + a**2*x**5/(5*b**3) - a*x**10/(10*b**2) + x
**15/(15*b)
```

3.1269 $\int \frac{x^{14}}{a+bx^5}\,dx$

Optimal. Leaf size=40

$$\frac{a^2\log\left(a+bx^5\right)}{5b^3}-\frac{ax^5}{5b^2}+\frac{x^{10}}{10b}$$

```
[Out] -1/5*a*x^5/b^2+1/10*x^10/b+1/5*a^2*ln(b*x^5+a)/b^3
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{a^2\log\left(a+bx^5\right)}{5b^3}-\frac{ax^5}{5b^2}+\frac{x^{10}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/(a + b*x^5),x]
```

```
[Out] -(a*x^5)/(5*b^2) + x^10/(10*b) + (a^2*Log[a + b*x^5])/(5*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{14}}{a+bx^5}\,dx &= \frac{1}{5}\,\text{Subst}\left(\int \frac{x^2}{a+bx}\,dx, x, x^5\right)\\ &= \frac{1}{5}\,\text{Subst}\left(\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx, x, x^5\right)\\ &= -\frac{ax^5}{5b^2}+\frac{x^{10}}{10b}+\frac{a^2\log\left(a+bx^5\right)}{5b^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.00

$$\frac{a^2\log\left(a+bx^5\right)}{5b^3}-\frac{ax^5}{5b^2}+\frac{x^{10}}{10b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/(a + b*x^5),x]
```

```
[Out] -1/5*(a*x^5)/b^2 + x^10/(10*b) + (a^2*Log[a + b*x^5])/(5*b^3)
```

fricas [A] time = 0.55, size = 33, normalized size = 0.82

$$\frac{b^2x^{10}-2\,abx^5+2\,a^2\log\left(bx^5+a\right)}{10\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^5+a),x, algorithm="fricas")
[Out] 1/10*(b^2*x^10 - 2*a*b*x^5 + 2*a^2*log(b*x^5 + a))/b^3
```

giac [A] time = 0.17, size = 35, normalized size = 0.88

$$\frac{a^2 \log\left(\left|bx^5+a\right|\right)}{5\,b^3}+\frac{bx^{10}-2\,ax^5}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^5+a),x, algorithm="giac")
[Out] 1/5*a^2*log(abs(b*x^5 + a))/b^3 + 1/10*(b*x^10 - 2*a*x^5)/b^2
```

maple [A] time = 0.00, size = 35, normalized size = 0.88

$$\frac{x^{10}}{10b}-\frac{a\,x^5}{5b^2}+\frac{a^2\ln\left(b\,x^5+a\right)}{5b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(b*x^5+a),x)
[Out] -1/5*a*x^5/b^2+1/10*x^10/b+1/5*a^2*ln(b*x^5+a)/b^3
```

maxima [A] time = 1.06, size = 34, normalized size = 0.85

$$\frac{a^2 \log\left(bx^5+a\right)}{5\,b^3}+\frac{bx^{10}-2\,ax^5}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^5+a),x, algorithm="maxima")
[Out] 1/5*a^2*log(b*x^5 + a)/b^3 + 1/10*(b*x^10 - 2*a*x^5)/b^2
```

mupad [B] time = 1.06, size = 33, normalized size = 0.82

$$\frac{2\,a^2\,\ln\left(b\,x^5+a\right)+b^2\,x^{10}-2\,a\,b\,x^5}{10\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(a + b*x^5),x)
[Out] (2*a^2*log(a + b*x^5) + b^2*x^10 - 2*a*b*x^5)/(10*b^3)
```

sympy [A] time = 0.60, size = 32, normalized size = 0.80

$$\frac{a^2 \log\left(a+bx^5\right)}{5b^3}-\frac{ax^5}{5b^2}+\frac{x^{10}}{10b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(b*x**5+a),x)
[Out] a**2*log(a + b*x**5)/(5*b**3) - a*x**5/(5*b**2) + x**10/(10*b)
```

3.1270 $\int \frac{x^9}{a+bx^5}\,dx$

Optimal. Leaf size=27

$$\frac{x^5}{5b} - \frac{a\log\left(a+bx^5\right)}{5b^2}$$

```
[Out] 1/5*x^5/b-1/5*a*ln(b*x^5+a)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{x^5}{5b} - \frac{a\log\left(a+bx^5\right)}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x^5),x]
```

```
[Out] x^5/(5*b) - (a*Log[a + b*x^5])/(5*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{a+bx^5}\,dx &= \frac{1}{5}\,\text{Subst}\left(\int \frac{x}{a+bx}\,dx, x, x^5\right)\\ &= \frac{1}{5}\,\text{Subst}\left(\int \left(\frac{1}{b} - \frac{a}{b(a+bx)}\right)\,dx, x, x^5\right)\\ &= \frac{x^5}{5b} - \frac{a\log\left(a+bx^5\right)}{5b^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.00

$$\frac{x^5}{5b} - \frac{a\log\left(a+bx^5\right)}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x^5),x]
```

```
[Out] x^5/(5*b) - (a*Log[a + b*x^5])/(5*b^2)
```

fricas [A] time = 0.67, size = 22, normalized size = 0.81

$$\frac{bx^5 - a\log\left(bx^5+a\right)}{5\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^5+a),x, algorithm="fricas")
[Out] 1/5*(b*x^5 - a*log(b*x^5 + a))/b^2
```

giac [A] time = 0.18, size = 24, normalized size = 0.89

$$\frac{x^5}{5\,b} - \frac{a\log\left(\left|bx^5+a\right|\right)}{5\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^5+a),x, algorithm="giac")
[Out] 1/5*x^5/b - 1/5*a*log(abs(b*x^5 + a))/b^2
```

maple [A] time = 0.00, size = 24, normalized size = 0.89

$$\frac{x^5}{5b} - \frac{a\ln\left(b\,x^5+a\right)}{5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^5+a),x)
[Out] 1/5/b*x^5-1/5*a*ln(b*x^5+a)/b^2
```

maxima [A] time = 1.08, size = 23, normalized size = 0.85

$$\frac{x^5}{5\,b} - \frac{a\log\left(bx^5+a\right)}{5\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^5+a),x, algorithm="maxima")
[Out] 1/5*x^5/b - 1/5*a*log(b*x^5 + a)/b^2
```

mupad [B] time = 1.05, size = 22, normalized size = 0.81

$$-\frac{a\,\ln\left(b\,x^5+a\right)-b\,x^5}{5\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + b*x^5),x)
[Out] -(a*log(a + b*x^5) - b*x^5)/(5*b^2)
```

sympy [A] time = 0.41, size = 20, normalized size = 0.74

$$-\frac{a\log\left(a+bx^5\right)}{5b^2} + \frac{x^5}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**5+a),x)
[Out] -a*log(a + b*x**5)/(5*b**2) + x**5/(5*b)
```

3.1271 $\int \frac{x^4}{a+bx^5}\,dx$

Optimal. Leaf size=15

$$\frac{\log\left(a+bx^5\right)}{5b}$$

```
[Out] 1/5*ln(b*x^5+a)/b
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {260}

$$\frac{\log\left(a+bx^5\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^5),x]
```

```
[Out] Log[a + b*x^5]/(5*b)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^4}{a+bx^5}\,dx = \frac{\log\left(a+bx^5\right)}{5b}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log\left(a+bx^5\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^5),x]
```

```
[Out] Log[a + b*x^5]/(5*b)
```

fricas [A] time = 0.71, size = 13, normalized size = 0.87

$$\frac{\log\left(bx^5+a\right)}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^5+a),x, algorithm="fricas")
```

```
[Out] 1/5*log(b*x^5 + a)/b
```

giac [A] time = 0.17, size = 14, normalized size = 0.93

$$\frac{\log\left(\left|bx^5+a\right|\right)}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^5+a),x, algorithm="giac")
[Out] 1/5*log(abs(b*x^5 + a))/b
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{\ln\left(b\,x^5+a\right)}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^5+a),x)
[Out] 1/5*ln(b*x^5+a)/b
```

maxima [A] time = 1.11, size = 13, normalized size = 0.87

$$\frac{\log\left(bx^5+a\right)}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^5+a),x, algorithm="maxima")
[Out] 1/5*log(b*x^5 + a)/b
```

mupad [B] time = 0.03, size = 13, normalized size = 0.87

$$\frac{\ln\left(b\,x^5+a\right)}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^5),x)
[Out] log(a + b*x^5)/(5*b)
```

sympy [A] time = 0.29, size = 10, normalized size = 0.67

$$\frac{\log\left(a+bx^5\right)}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**5+a),x)
[Out] log(a + b*x**5)/(5*b)
```

3.1272 $\int \frac{1}{x\left(a+bx^5\right)}\,dx$

Optimal. Leaf size=22

$$\frac{\log(x)}{a}-\frac{\log\left(a+bx^5\right)}{5a}$$

```
[Out] ln(x)/a-1/5*ln(b*x^5+a)/a
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {266, 36, 29, 31}

$$\frac{\log(x)}{a}-\frac{\log\left(a+bx^5\right)}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^5)),x]
```

```
[Out] Log[x]/a - Log[a + b*x^5]/(5*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a+bx^5\right)}\,dx &= \frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{x(a+bx)}\,dx, x, x^5\right)\\ &= \frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, x^5\right)}{5a}-\frac{b\operatorname{Subst}\left(\int \frac{1}{a+bx}\,dx, x, x^5\right)}{5a}\\ &= \frac{\log(x)}{a}-\frac{\log\left(a+bx^5\right)}{5a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{\log(x)}{a}-\frac{\log\left(a+bx^5\right)}{5a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^5)),x]
[Out] Log[x]/a - Log[a + b*x^5]/(5*a)
```

fricas [A] time = 0.80, size = 18, normalized size = 0.82

$$-\frac{\log\left(bx^5+a\right)-5\,\log(x)}{5\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^5+a),x, algorithm="fricas")
[Out] -1/5*(log(b*x^5 + a) - 5*log(x))/a
```

giac [A] time = 0.21, size = 22, normalized size = 1.00

$$-\frac{\log\left(\left|bx^5+a\right|\right)}{5\,a}+\frac{\log\left(|x|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^5+a),x, algorithm="giac")
[Out] -1/5*log(abs(b*x^5 + a))/a + log(abs(x))/a
```

maple [A] time = 0.00, size = 21, normalized size = 0.95

$$\frac{\ln(x)}{a}-\frac{\ln\left(b\,x^5+a\right)}{5a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^5+a),x)
[Out] 1/a*ln(x)-1/5*ln(b*x^5+a)/a
```

maxima [A] time = 0.97, size = 23, normalized size = 1.05

$$-\frac{\log\left(bx^5+a\right)}{5\,a}+\frac{\log\left(x^5\right)}{5\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^5+a),x, algorithm="maxima")
[Out] -1/5*log(b*x^5 + a)/a + 1/5*log(x^5)/a
```

mupad [B] time = 0.09, size = 18, normalized size = 0.82

$$-\frac{\ln\left(b\,x^5+a\right)-5\,\ln(x)}{5\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^5)),x)
[Out] -(log(a + b*x^5) - 5*log(x))/(5*a)
```

sympy [A] time = 0.53, size = 15, normalized size = 0.68

$$\frac{\log(x)}{a}-\frac{\log\left(\frac{a}{b}+x^5\right)}{5a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**5+a),x)
[Out] log(x)/a - log(a/b + x**5)/(5*a)
```

3.1273 $\int \frac{1}{x^6\left(a+bx^5\right)}\,dx$

Optimal. Leaf size=35

$$\frac{b\log\left(a+bx^5\right)}{5a^2}-\frac{b\log(x)}{a^2}-\frac{1}{5ax^5}$$

```
[Out] -1/5/a/x^5-b*ln(x)/a^2+1/5*b*ln(b*x^5+a)/a^2
```

Rubi [A] time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 44}

$$\frac{b\log\left(a+bx^5\right)}{5a^2}-\frac{b\log(x)}{a^2}-\frac{1}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^5)),x]
```

```
[Out] -1/(5*a*x^5) - (b*Log[x])/a^2 + (b*Log[a + b*x^5])/(5*a^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^6\left(a+bx^5\right)}\,dx &= \frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)}\,dx,x,x^5\right)\\ &= \frac{1}{5}\operatorname{Subst}\left(\int\left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx,x,x^5\right)\\ &= -\frac{1}{5ax^5}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^5\right)}{5a^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.00

$$\frac{b\log\left(a+bx^5\right)}{5a^2}-\frac{b\log(x)}{a^2}-\frac{1}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^5)),x]
```

```
[Out] -1/5*1/(a*x^5) - (b*Log[x])/a^2 + (b*Log[a + b*x^5])/(5*a^2)
```

fricas [A] time = 0.68, size = 33, normalized size = 0.94

$$\frac{bx^5\log\left(bx^5+a\right)-5\,bx^5\log(x)-a}{5\,a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^5+a),x, algorithm="fricas")
```

```
[Out] 1/5*(b*x^5*log(b*x^5 + a) - 5*b*x^5*log(x) - a)/(a^2*x^5)
```

giac [A] time = 0.17, size = 42, normalized size = 1.20

$$\frac{b\log\left(\left|bx^5+a\right|\right)}{5\,a^2}-\frac{b\log\left(|x|\right)}{a^2}+\frac{bx^5-a}{5\,a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^5+a),x, algorithm="giac")
```

```
[Out] 1/5*b*log(abs(b*x^5 + a))/a^2 - b*log(abs(x))/a^2 + 1/5*(b*x^5 - a)/(a^2*x^
5)
```

maple [A] time = 0.01, size = 32, normalized size = 0.91

$$-\frac{b\ln(x)}{a^2}+\frac{b\ln\left(b\,x^5+a\right)}{5a^2}-\frac{1}{5a\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^5+a),x)
```

```
[Out] -1/5/a/x^5-1/a^2*b*ln(x)+1/5*b*ln(b*x^5+a)/a^2
```

maxima [A] time = 0.97, size = 33, normalized size = 0.94

$$\frac{b\log\left(bx^5+a\right)}{5\,a^2}-\frac{b\log\left(x^5\right)}{5\,a^2}-\frac{1}{5\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^5+a),x, algorithm="maxima")
```

```
[Out] 1/5*b*log(b*x^5 + a)/a^2 - 1/5*b*log(x^5)/a^2 - 1/5/(a*x^5)
```

mupad [B] time = 1.07, size = 31, normalized size = 0.89

$$\frac{b\,\ln\left(b\,x^5+a\right)}{5\,a^2}-\frac{1}{5\,a\,x^5}-\frac{b\,\ln(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^5)),x)
```

```
[Out] (b*log(a + b*x^5))/(5*a^2) - 1/(5*a*x^5) - (b*log(x))/a^2
```

sympy [A] time = 0.70, size = 31, normalized size = 0.89

$$-\frac{1}{5ax^5}-\frac{b\log\left(x\right)}{a^2}+\frac{b\log\left(\frac{a}{b}+x^5\right)}{5a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**5+a),x)
```

```
[Out] -1/(5*a*x**5) - b*log(x)/a**2 + b*log(a/b + x**5)/(5*a**2)
```

3.1274 $\int \frac{1}{x^{11}\left(a+bx^5\right)}\,dx$

Optimal. Leaf size=49

$$-\frac{b^2\log\left(a+bx^5\right)}{5a^3}+\frac{b^2\log(x)}{a^3}+\frac{b}{5a^2x^5}-\frac{1}{10ax^{10}}$$

```
[Out] -1/10/a/x^10+1/5*b/a^2/x^5+b^2*ln(x)/a^3-1/5*b^2*ln(b*x^5+a)/a^3
```

Rubi [A] time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 44}

$$-\frac{b^2\log\left(a+bx^5\right)}{5a^3}+\frac{b^2\log(x)}{a^3}+\frac{b}{5a^2x^5}-\frac{1}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^11*(a + b*x^5)),x]
```

```
[Out] -1/(10*a*x^10) + b/(5*a^2*x^5) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x^5])/(5
*a^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{11}\left(a+bx^5\right)}\,dx &= \frac{1}{5}\,\text{Subst}\left(\int \frac{1}{x^3(a+bx)}\,dx, x, x^5\right)\\ &= \frac{1}{5}\,\text{Subst}\left(\int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)dx, x, x^5\right)\\ &= -\frac{1}{10ax^{10}}+\frac{b}{5a^2x^5}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log\left(a+bx^5\right)}{5a^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.00

$$-\frac{b^2\log\left(a+bx^5\right)}{5a^3}+\frac{b^2\log(x)}{a^3}+\frac{b}{5a^2x^5}-\frac{1}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^11*(a + b*x^5)),x]
```

```
[Out] -1/10*1/(a*x^10) + b/(5*a^2*x^5) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x^5])/
(5*a^3)
```

fricas [A] time = 0.80, size = 45, normalized size = 0.92

$$-\frac{2\,b^2x^{10}\log\left(bx^5+a\right)-10\,b^2x^{10}\log(x)-2\,abx^5+a^2}{10\,a^3x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(b*x^5+a),x, algorithm="fricas")
```

```
[Out] -1/10*(2*b^2*x^10*log(b*x^5 + a) - 10*b^2*x^10*log(x) - 2*a*b*x^5 + a^2)/(a
^3*x^10)
```

giac [A] time = 0.17, size = 55, normalized size = 1.12

$$-\frac{b^2\log\left(\left|bx^5+a\right|\right)}{5\,a^3}+\frac{b^2\log\left(|x|\right)}{a^3}-\frac{3\,b^2x^{10}-2\,abx^5+a^2}{10\,a^3x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(b*x^5+a),x, algorithm="giac")
```

```
[Out] -1/5*b^2*log(abs(b*x^5 + a))/a^3 + b^2*log(abs(x))/a^3 - 1/10*(3*b^2*x^10 -
 2*a*b*x^5 + a^2)/(a^3*x^10)
```

maple [A] time = 0.01, size = 44, normalized size = 0.90

$$\frac{b^2\ln(x)}{a^3}-\frac{b^2\ln\left(b\,x^5+a\right)}{5a^3}+\frac{b}{5a^2x^5}-\frac{1}{10a\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^11/(b*x^5+a),x)
```

```
[Out] -1/10/a/x^10+1/5*b/a^2/x^5+b^2*ln(x)/a^3-1/5*b^2*ln(b*x^5+a)/a^3
```

maxima [A] time = 1.15, size = 47, normalized size = 0.96

$$-\frac{b^2\log\left(bx^5+a\right)}{5\,a^3}+\frac{b^2\log\left(x^5\right)}{5\,a^3}+\frac{2\,bx^5-a}{10\,a^2x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(b*x^5+a),x, algorithm="maxima")
```

```
[Out] -1/5*b^2*log(b*x^5 + a)/a^3 + 1/5*b^2*log(x^5)/a^3 + 1/10*(2*b*x^5 - a)/(a^
2*x^10)
```

mupad [B] time = 0.09, size = 46, normalized size = 0.94

$$\frac{b^2\,\ln(x)}{a^3}-\frac{b^2\,\ln\left(b\,x^5+a\right)}{5\,a^3}-\frac{\frac{1}{10\,a}-\frac{b\,x^5}{5\,a^2}}{x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^11*(a + b*x^5)),x)
```

```
[Out] (b^2*log(x))/a^3 - (b^2*log(a + b*x^5))/(5*a^3) - (1/(10*a) - (b*x^5)/(5*a^
2))/x^10
```

sympy [A] time = 1.26, size = 42, normalized size = 0.86

$$\frac{-a+2bx^5}{10a^2x^{10}}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log\left(\frac{a}{b}+x^5\right)}{5a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**11/(b*x**5+a),x)
```

```
[Out] (-a + 2*b*x**5)/(10*a**2*x**10) + b**2*log(x)/a**3 - b**2*log(a/b + x**5)/(
5*a**3)
```

3.1275 $\int \frac{1}{x^{16}\left(a+bx^5\right)}\,dx$

Optimal. Leaf size=63

$$\frac{b^3\log\left(a+bx^5\right)}{5a^4}-\frac{b^3\log(x)}{a^4}-\frac{b^2}{5a^3x^5}+\frac{b}{10a^2x^{10}}-\frac{1}{15ax^{15}}$$

```
[Out] -1/15/a/x^15+1/10*b/a^2/x^10-1/5*b^2/a^3/x^5-b^3*ln(x)/a^4+1/5*b^3*ln(b*x^5
+a)/a^4
```

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 44}

$$-\frac{b^2}{5a^3x^5}+\frac{b^3\log\left(a+bx^5\right)}{5a^4}-\frac{b^3\log(x)}{a^4}+\frac{b}{10a^2x^{10}}-\frac{1}{15ax^{15}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^16*(a + b*x^5)),x]
```

```
[Out] -1/(15*a*x^15) + b/(10*a^2*x^10) - b^2/(5*a^3*x^5) - (b^3*Log[x])/a^4 + (b^
3*Log[a + b*x^5])/(5*a^4)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{16}\left(a+bx^5\right)}\,dx &= \frac{1}{5}\,\text{Subst}\left(\int \frac{1}{x^4(a+bx)}\,dx,x,x^5\right)\\ &= \frac{1}{5}\,\text{Subst}\left(\int\left(\frac{1}{ax^4}-\frac{b}{a^2x^3}+\frac{b^2}{a^3x^2}-\frac{b^3}{a^4x}+\frac{b^4}{a^4(a+bx)}\right)dx,x,x^5\right)\\ &= -\frac{1}{15ax^{15}}+\frac{b}{10a^2x^{10}}-\frac{b^2}{5a^3x^5}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log\left(a+bx^5\right)}{5a^4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 63, normalized size = 1.00

$$\frac{b^3\log\left(a+bx^5\right)}{5a^4}-\frac{b^3\log(x)}{a^4}-\frac{b^2}{5a^3x^5}+\frac{b}{10a^2x^{10}}-\frac{1}{15ax^{15}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^16*(a + b*x^5)),x]
```

```
[Out] -1/15*1/(a*x^15) + b/(10*a^2*x^10) - b^2/(5*a^3*x^5) - (b^3*Log[x])/a^4 + (
b^3*Log[a + b*x^5])/(5*a^4)
```

fricas [A] time = 0.60, size = 58, normalized size = 0.92

$$\frac{6\,b^3x^{15}\log\left(bx^5+a\right)-30\,b^3x^{15}\log(x)-6\,ab^2x^{10}+3\,a^2bx^5-2\,a^3}{30\,a^4x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^16/(b*x^5+a),x, algorithm="fricas")
```

```
[Out] 1/30*(6*b^3*x^15*log(b*x^5 + a) - 30*b^3*x^15*log(x) - 6*a*b^2*x^10 + 3*a^2
*b*x^5 - 2*a^3)/(a^4*x^15)
```

giac [A] time = 0.17, size = 69, normalized size = 1.10

$$\frac{b^3\log\left(\left|bx^5+a\right|\right)}{5\,a^4}-\frac{b^3\log\left(|x|\right)}{a^4}+\frac{11\,b^3x^{15}-6\,ab^2x^{10}+3\,a^2bx^5-2\,a^3}{30\,a^4x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^16/(b*x^5+a),x, algorithm="giac")
```

```
[Out] 1/5*b^3*log(abs(b*x^5 + a))/a^4 - b^3*log(abs(x))/a^4 + 1/30*(11*b^3*x^15 -
 6*a*b^2*x^10 + 3*a^2*b*x^5 - 2*a^3)/(a^4*x^15)
```

maple [A] time = 0.01, size = 56, normalized size = 0.89

$$-\frac{b^3\ln(x)}{a^4}+\frac{b^3\ln\left(b\,x^5+a\right)}{5a^4}-\frac{b^2}{5a^3x^5}+\frac{b}{10a^2x^{10}}-\frac{1}{15a\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^16/(b*x^5+a),x)
```

```
[Out] -1/15/a/x^15+1/10*b/a^2/x^10-1/5*b^2/a^3/x^5-b^3*ln(x)/a^4+1/5*b^3*ln(b*x^5
+a)/a^4
```

maxima [A] time = 1.09, size = 58, normalized size = 0.92

$$\frac{b^3\log\left(bx^5+a\right)}{5\,a^4}-\frac{b^3\log\left(x^5\right)}{5\,a^4}-\frac{6\,b^2x^{10}-3\,abx^5+2\,a^2}{30\,a^3x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^16/(b*x^5+a),x, algorithm="maxima")
```

```
[Out] 1/5*b^3*log(b*x^5 + a)/a^4 - 1/5*b^3*log(x^5)/a^4 - 1/30*(6*b^2*x^10 - 3*a*
b*x^5 + 2*a^2)/(a^3*x^15)
```

mupad [B] time = 1.09, size = 58, normalized size = 0.92

$$\frac{b^3\,\ln\left(b\,x^5+a\right)}{5\,a^4}-\frac{\frac{1}{15\,a}-\frac{b\,x^5}{10\,a^2}+\frac{b^2\,x^{10}}{5\,a^3}}{x^{15}}-\frac{b^3\,\ln(x)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^16*(a + b*x^5)),x)
```

```
[Out] (b^3*log(a + b*x^5))/(5*a^4) - (1/(15*a) - (b*x^5)/(10*a^2) + (b^2*x^10)/(5
*a^3))/x^15 - (b^3*log(x))/a^4
```

sympy [**A**] time = 1.03, size = 56, normalized size = 0.89

$$\frac{-2a^2+3abx^5-6b^2x^{10}}{30a^3x^{15}}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log\left(\frac{a}{b}+x^5\right)}{5a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**16/(b*x**5+a),x)
```

```
[Out] (-2*a**2 + 3*a*b*x**5 - 6*b**2*x**10)/(30*a**3*x**15) - b**3*log(x)/a**4 +
b**3*log(a/b + x**5)/(5*a**4)
```

3.1276 $\int \frac{1}{a+bx^5}\,dx$

Optimal. Leaf size=301

$$-\frac{\left(1-\sqrt{5}\right)\log\left(2a^{2/5}-\left(1-\sqrt{5}\right)\sqrt[5]{a}\,\sqrt[5]{b}\,x+2b^{2/5}x^2\right)}{20a^{4/5}\sqrt[5]{b}}-\frac{\left(1+\sqrt{5}\right)\log\left(2a^{2/5}-\left(1+\sqrt{5}\right)\sqrt[5]{a}\,\sqrt[5]{b}\,x+2b^{2/5}x^2\right)}{20a^{4/5}\sqrt[5]{b}}+\frac{\log(}{5}$$

```
[Out] 1/5*ln(a^(1/5)+b^(1/5)*x)/a^(4/5)/b^(1/5)-1/20*ln(2*a^(2/5)+2*b^(2/5)*x^2-a
^(1/5)*b^(1/5)*x*(-5^(1/2)+1))*(-5^(1/2)+1)/a^(4/5)/b^(1/5)-1/20*ln(2*a^(2/
5)+2*b^(2/5)*x^2-a^(1/5)*b^(1/5)*x*(5^(1/2)+1))*(5^(1/2)+1)/a^(4/5)/b^(1/5)
-1/10*arctan((-4*b^(1/5)*x+a^(1/5)*(5^(1/2)+1))/a^(1/5)/(10-2*5^(1/2))^(1/2
))*(10-2*5^(1/2))^(1/2)/a^(4/5)/b^(1/5)-1/10*arctan((-4*b^(1/5)*x+a^(1/5)*(
-5^(1/2)+1))/a^(1/5)/(10+2*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)/a^(4/5)/b^(
1/5)
```

Rubi [A] time = 0.64, antiderivative size = 305, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {201, 634, 618, 204, 628, 31}

$$-\frac{\left(1-\sqrt{5}\right)\log\left(a^{2/5}-\frac{1}{2}\left(1-\sqrt{5}\right)\sqrt[5]{a}\,\sqrt[5]{b}\,x+b^{2/5}x^2\right)}{20a^{4/5}\sqrt[5]{b}}-\frac{\left(1+\sqrt{5}\right)\log\left(a^{2/5}-\frac{1}{2}\left(1+\sqrt{5}\right)\sqrt[5]{a}\,\sqrt[5]{b}\,x+b^{2/5}x^2\right)}{20a^{4/5}\sqrt[5]{b}}+\frac{\log(}{5}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^5)^(-1),x]
```

```
[Out] (Sqrt[(5 + Sqrt[5])/2]*ArcTan[Sqrt[(5 - 2*Sqrt[5])/5] + (2*Sqrt[2/(5 + Sqrt
[5])]*b^(1/5)*x)/a^(1/5)])/(5*a^(4/5)*b^(1/5)) - (Sqrt[(5 - Sqrt[5])/2]*Arc
Tan[Sqrt[(5 + 2*Sqrt[5])/5] - (Sqrt[(2*(5 + Sqrt[5]))/5]*b^(1/5)*x)/a^(1/5)
])/(5*a^(4/5)*b^(1/5)) + Log[a^(1/5) + b^(1/5)*x]/(5*a^(4/5)*b^(1/5)) - ((1
 - Sqrt[5])*Log[a^(2/5) - ((1 - Sqrt[5])*a^(1/5)*b^(1/5)*x)/2 + b^(2/5)*x^2
])/(20*a^(4/5)*b^(1/5)) - ((1 + Sqrt[5])*Log[a^(2/5) - ((1 + Sqrt[5])*a^(1/
5)*b^(1/5)*x)/2 + b^(2/5)*x^2])/(20*a^(4/5)*b^(1/5))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[((2*k -
 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]; (r*Int[1
/(r + s*x), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 1)/2}], x], x]
] /; FreeQ[{a, b}, x] && IGtQ[(n - 3)/2, 0] && PosQ[a/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{a+bx^5}\,dx = \frac{2\int \frac{\sqrt[5]{a}-\frac{1}{4}\left(1-\sqrt{5}\right)\sqrt[5]{b}\,x}{a^{2/5}-\frac{1}{2}\left(1-\sqrt{5}\right)\sqrt[5]{a}\,\sqrt[5]{b}\,x+b^{2/5}x^2}\,dx}{5a^{4/5}} + \frac{2\int \frac{\sqrt[5]{a}-\frac{1}{4}\left(1+\sqrt{5}\right)\sqrt[5]{b}\,x}{a^{2/5}-\frac{1}{2}\left(1+\sqrt{5}\right)\sqrt[5]{a}\,\sqrt[5]{b}\,x+b^{2/5}x^2}\,dx}{5a^{4/5}} + \frac{\int \frac{1}{\sqrt[5]{a}+\sqrt[5]{b}\,x}\,dx}{5a^{4/5}}$$

$$= \frac{\log\left(\sqrt[5]{a}+\sqrt[5]{b}\,x\right)}{5a^{4/5}\sqrt[5]{b}} + \frac{\left(5-\sqrt{5}\right)\int \frac{1}{a^{2/5}-\frac{1}{2}\left(1+\sqrt{5}\right)\sqrt[5]{a}\,\sqrt[5]{b}\,x+b^{2/5}x^2}\,dx}{20a^{3/5}} + \frac{\left(5+\sqrt{5}\right)\int \frac{1}{a^{2/5}-\frac{1}{2}\left(1-\sqrt{5}\right)\sqrt[5]{a}\,\sqrt[5]{b}\,x}}$$

$$= \frac{\log\left(\sqrt[5]{a}+\sqrt[5]{b}\,x\right)}{5a^{4/5}\sqrt[5]{b}} - \frac{\left(1+\sqrt{5}\right)\log\left(2a^{2/5}-\sqrt[5]{a}\,\sqrt[5]{b}\,x-\sqrt{5}\,\sqrt[5]{a}\,\sqrt[5]{b}\,x+2b^{2/5}x^2\right)}{20a^{4/5}\sqrt[5]{b}} - \frac{\left(1-\sqrt{5}\right)\log}$$

$$= -\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\,\tan^{-1}\left(\frac{\sqrt{5+\sqrt{5}}\left(\left(1+\sqrt{5}\right)\sqrt[5]{a}-4\sqrt[5]{b}\,x\right)}{2\sqrt{10}\,\sqrt[5]{a}}\right)}{5a^{4/5}\sqrt[5]{b}} + \frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\,\tan^{-1}\left(\sqrt{\frac{1}{5}\left(5-2\sqrt{5}\right)}+\frac{2\sqrt{}}{}\right.}{5a^{4/5}\sqrt[5]{b}}$$

Mathematica [A] time = 0.23, size = 311, normalized size = 1.03

$$-\frac{\left(1-\sqrt{5}\right)\log\left(a^{2/5}-\frac{1}{2}\left(1-\sqrt{5}\right)\sqrt[5]{a}\,\sqrt[5]{b}\,x+b^{2/5}x^2\right)}{20a^{4/5}\sqrt[5]{b}} - \frac{\left(1+\sqrt{5}\right)\log\left(a^{2/5}-\frac{1}{2}\left(1+\sqrt{5}\right)\sqrt[5]{a}\,\sqrt[5]{b}\,x+b^{2/5}x^2\right)}{20a^{4/5}\sqrt[5]{b}} + \frac{\log}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^5)^(-1),x]
```

```
[Out] (2*Sqrt[5/8 + Sqrt[5]/8]*ArcTan[(-1/4*((1 - Sqrt[5])*a^(1/5)) + b^(1/5)*x)/
(Sqrt[5/8 + Sqrt[5]/8]*a^(1/5))])/(5*a^(4/5)*b^(1/5)) + (2*Sqrt[5/8 - Sqrt[
5]/8]*ArcTan[(-1/4*((1 + Sqrt[5])*a^(1/5)) + b^(1/5)*x)/(Sqrt[5/8 - Sqrt[5]
/8]*a^(1/5))])/(5*a^(4/5)*b^(1/5)) + Log[a^(1/5) + b^(1/5)*x]/(5*a^(4/5)*b^
(1/5)) - ((1 - Sqrt[5])*Log[a^(2/5) - ((1 - Sqrt[5])*a^(1/5)*b^(1/5)*x)/2 +
 b^(2/5)*x^2])/(20*a^(4/5)*b^(1/5)) - ((1 + Sqrt[5])*Log[a^(2/5) - ((1 + Sq
rt[5])*a^(1/5)*b^(1/5)*x)/2 + b^(2/5)*x^2])/(20*a^(4/5)*b^(1/5))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.17, size = 275, normalized size = 0.91

$$-\frac{\left(-\frac{a}{b}\right)^{\frac{1}{5}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{5}}\right|\right)}{5a}+\frac{\left(-ab^4\right)^{\frac{1}{5}}\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{\left(\sqrt{5}-1\right)\left(-\frac{a}{b}\right)^{\frac{1}{5}}-4x}{\sqrt{2\sqrt{5}+10}\left(-\frac{a}{b}\right)^{\frac{1}{5}}}\right)}{10ab}+\frac{\left(-ab^4\right)^{\frac{1}{5}}\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{(\sqrt{}}{\sqrt{-}}\right.}{10ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5+a),x, algorithm="giac")
```

```
[Out] -1/5*(-a/b)^(1/5)*log(abs(x - (-a/b)^(1/5)))/a + 1/10*(-a*b^4)^(1/5)*sqrt(2
*sqrt(5) + 10)*arctan(-((sqrt(5) - 1)*(-a/b)^(1/5) - 4*x)/(sqrt(2*sqrt(5) +
 10)*(-a/b)^(1/5)))/(a*b) + 1/10*(-a*b^4)^(1/5)*sqrt(-2*sqrt(5) + 10)*arcta
n(((sqrt(5) + 1)*(-a/b)^(1/5) + 4*x)/(sqrt(-2*sqrt(5) + 10)*(-a/b)^(1/5)))/
(a*b) + 1/5*(-a*b^4)^(1/5)*log(x^2 + 1/2*x*(sqrt(5)*(-a/b)^(1/5) + (-a/b)^(
1/5)) + (-a/b)^(2/5))/(a*b*(sqrt(5) - 1)) - 1/5*(-a*b^4)^(1/5)*log(x^2 - 1/
2*x*(sqrt(5)*(-a/b)^(1/5) - (-a/b)^(1/5)) + (-a/b)^(2/5))/(a*b*(sqrt(5) + 1
))
```

maple [B] time = 0.15, size = 895, normalized size = 2.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^5+a),x)
```

```
[Out] -1/b/(a/b)^(4/5)/(5^(1/2)-5)/(5+5^(1/2))*ln((a/b)^(1/5)*x*5^(1/2)+2*(a/b)^(
2/5)-(a/b)^(1/5)*x+2*x^2)*5^(1/2)+1/b/(a/b)^(4/5)/(5^(1/2)-5)/(5+5^(1/2))*l
n((a/b)^(1/5)*x*5^(1/2)+2*(a/b)^(2/5)-(a/b)^(1/5)*x+2*x^2)-20/b/(a/b)^(3/5)
/(5^(1/2)-5)/(5+5^(1/2))/(10*(a/b)^(2/5)+2*(a/b)^(2/5)*5^(1/2))^(1/2)*arcta
n(1/(10*(a/b)^(2/5)+2*(a/b)^(2/5)*5^(1/2))^(1/2)*(a/b)^(1/5)*5^(1/2)-1/(10*
(a/b)^(2/5)+2*(a/b)^(2/5)*5^(1/2))^(1/2)*(a/b)^(1/5)+4*x/(10*(a/b)^(2/5)+2*
(a/b)^(2/5)*5^(1/2))^(1/2))-4/b/(a/b)^(3/5)/(5^(1/2)-5)/(5+5^(1/2))/(10*(a/
b)^(2/5)+2*(a/b)^(2/5)*5^(1/2))^(1/2)*arctan(1/(10*(a/b)^(2/5)+2*(a/b)^(2/5
)*5^(1/2))^(1/2)*(a/b)^(1/5)*5^(1/2)-1/(10*(a/b)^(2/5)+2*(a/b)^(2/5)*5^(1/2
))^(1/2)*(a/b)^(1/5)+4*x/(10*(a/b)^(2/5)+2*(a/b)^(2/5)*5^(1/2))^(1/2))*5^(1
/2)-4/b/(a/b)^(4/5)/(5^(1/2)-5)/(5+5^(1/2))*ln(x+(a/b)^(1/5))+1/b/(a/b)^(4/
5)/(5^(1/2)-5)/(5+5^(1/2))*ln(-(a/b)^(1/5)*x*5^(1/2)+2*(a/b)^(2/5)-(a/b)^(1
/5)*x+2*x^2)*5^(1/2)+1/b/(a/b)^(4/5)/(5^(1/2)-5)/(5+5^(1/2))*ln(-(a/b)^(1/5
)*x*5^(1/2)+2*(a/b)^(2/5)-(a/b)^(1/5)*x+2*x^2)-20/b/(a/b)^(3/5)/(5^(1/2)-5)
/(5+5^(1/2))/(10*(a/b)^(2/5)-2*(a/b)^(2/5)*5^(1/2))^(1/2)*arctan(-1/(10*(a/
b)^(2/5)-2*(a/b)^(2/5)*5^(1/2))^(1/2)*(a/b)^(1/5)*5^(1/2)-1/(10*(a/b)^(2/5)
-2*(a/b)^(2/5)*5^(1/2))^(1/2)*(a/b)^(1/5)+4*x/(10*(a/b)^(2/5)-2*(a/b)^(2/5)
*5^(1/2))^(1/2))+4/b/(a/b)^(3/5)/(5^(1/2)-5)/(5+5^(1/2))/(10*(a/b)^(2/5)-2*
(a/b)^(2/5)*5^(1/2))^(1/2)*arctan(-1/(10*(a/b)^(2/5)-2*(a/b)^(2/5)*5^(1/2))
^(1/2)*(a/b)^(1/5)*5^(1/2)-1/(10*(a/b)^(2/5)-2*(a/b)^(2/5)*5^(1/2))^(1/2)*(
a/b)^(1/5)+4*x/(10*(a/b)^(2/5)-2*(a/b)^(2/5)*5^(1/2))^(1/2))*5^(1/2)
```

maxima [A] time = 2.49, size = 239, normalized size = 0.79

$$\frac{\sqrt{5}\left(\sqrt{5}+1\right)\arctan\left(\frac{4b^{\frac{2}{5}}x+a^{\frac{1}{5}}b^{\frac{1}{5}}\left(\sqrt{5}-1\right)}{a^{\frac{1}{5}}b^{\frac{1}{5}}\sqrt{2\sqrt{5}+10}}\right)}{5a^{\frac{4}{5}}b^{\frac{1}{5}}\sqrt{2\sqrt{5}+10}}+\frac{\sqrt{5}\left(\sqrt{5}-1\right)\arctan\left(\frac{4b^{\frac{2}{5}}x-a^{\frac{1}{5}}b^{\frac{1}{5}}\left(\sqrt{5}+1\right)}{a^{\frac{1}{5}}b^{\frac{1}{5}}\sqrt{-2\sqrt{5}+10}}\right)}{5a^{\frac{4}{5}}b^{\frac{1}{5}}\sqrt{-2\sqrt{5}+10}}-\frac{\left(\sqrt{5}+3\right)\log\left(2b^{\frac{2}{5}}x^2-a^{\frac{1}{5}}b^{\frac{1}{5}}x\right.}{10a^{\frac{4}{5}}b^{\frac{1}{5}}\left(\sqrt{5}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5+a),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(5)*(sqrt(5) + 1)*arctan((4*b^(2/5)*x + a^(1/5)*b^(1/5)*(sqrt(5) -
1))/(a^(1/5)*b^(1/5)*sqrt(2*sqrt(5) + 10)))/(a^(4/5)*b^(1/5)*sqrt(2*sqrt(5)
 + 10)) + 1/5*sqrt(5)*(sqrt(5) - 1)*arctan((4*b^(2/5)*x - a^(1/5)*b^(1/5)*(
sqrt(5) + 1))/(a^(1/5)*b^(1/5)*sqrt(-2*sqrt(5) + 10)))/(a^(4/5)*b^(1/5)*sqr
t(-2*sqrt(5) + 10)) - 1/10*(sqrt(5) + 3)*log(2*b^(2/5)*x^2 - a^(1/5)*b^(1/5
)*x*(sqrt(5) + 1) + 2*a^(2/5))/(a^(4/5)*b^(1/5)*(sqrt(5) + 1)) - 1/10*(sqrt
(5) - 3)*log(2*b^(2/5)*x^2 + a^(1/5)*b^(1/5)*x*(sqrt(5) - 1) + 2*a^(2/5))/(
a^(4/5)*b^(1/5)*(sqrt(5) - 1)) + 1/5*log(b^(1/5)*x + a^(1/5))/(a^(4/5)*b^(1
/5))
```

mupad **[B]** time = 1.53, size = 235, normalized size = 0.78

$$\frac{\ln\left(b^{1/5}x+a^{1/5}\right)}{5a^{4/5}b^{1/5}}-\frac{\ln\left(5b^4x-\frac{5a^{1/5}b^{19/5}\left(\sqrt{5}-\sqrt{2\sqrt{5}-10}+1\right)}{4}\right)\left(\sqrt{5}-\sqrt{2\sqrt{5}-10}+1\right)}{20a^{4/5}b^{1/5}}-\frac{\ln\left(5b^4x-\frac{5a^{1/5}b^{19/5}\left(\sqrt{-2}\right.\right.}{4}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^5),x)
```

```
[Out] log(b^(1/5)*x + a^(1/5))/(5*a^(4/5)*b^(1/5)) - (log(5*b^4*x - (5*a^(1/5)*b^
(19/5)*(5^(1/2) - (2*5^(1/2) - 10)^(1/2) + 1))/4)*(5^(1/2) - (2*5^(1/2) - 1
0)^(1/2) + 1))/(20*a^(4/5)*b^(1/5)) - (log(5*b^4*x - (5*a^(1/5)*b^(19/5)*((
- 2*5^(1/2) - 10)^(1/2) - 5^(1/2) + 1))/4)*((- 2*5^(1/2) - 10)^(1/2) - 5^(1
/2) + 1))/(20*a^(4/5)*b^(1/5)) + (log(5*b^4*x + (5*a^(1/5)*b^(19/5)*(5^(1/2
) + (- 2*5^(1/2) - 10)^(1/2) - 1))/4)*(5^(1/2) + (- 2*5^(1/2) - 10)^(1/2) -
 1))/(20*a^(4/5)*b^(1/5)) - (log(5*b^4*x - (5*a^(1/5)*b^(19/5)*(5^(1/2) + (
2*5^(1/2) - 10)^(1/2) + 1))/4)*(5^(1/2) + (2*5^(1/2) - 10)^(1/2) + 1))/(20*
a^(4/5)*b^(1/5))
```

sympy **[A]** time = 0.26, size = 20, normalized size = 0.07

$$\operatorname{RootSum}\left(3125t^5a^4b-1,\left(t\mapsto t\log\left(5ta+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**5+a),x)
```

```
[Out] RootSum(3125*_t**5*a**4*b - 1, Lambda(_t, _t*log(5*_t*a + x)))
```

3.1277 $\int \frac{x^{24}}{\left(a+bx^5\right)^2}\,dx$

Optimal. Leaf size=72

$$-\frac{a^4}{5b^5\left(a+bx^5\right)}-\frac{4a^3\log\left(a+bx^5\right)}{5b^5}+\frac{3a^2x^5}{5b^4}-\frac{ax^{10}}{5b^3}+\frac{x^{15}}{15b^2}$$

```
[Out] 3/5*a^2*x^5/b^4-1/5*a*x^10/b^3+1/15*x^15/b^2-1/5*a^4/b^5/(b*x^5+a)-4/5*a^3*
ln(b*x^5+a)/b^5
```

Rubi [A] time = 0.06, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{3a^2x^5}{5b^4}-\frac{a^4}{5b^5\left(a+bx^5\right)}-\frac{4a^3\log\left(a+bx^5\right)}{5b^5}-\frac{ax^{10}}{5b^3}+\frac{x^{15}}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^24/(a + b*x^5)^2,x]
```

```
[Out] (3*a^2*x^5)/(5*b^4) - (a*x^10)/(5*b^3) + x^15/(15*b^2) - a^4/(5*b^5*(a + b*
x^5)) - (4*a^3*Log[a + b*x^5])/(5*b^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{24}}{\left(a+bx^5\right)^2}\,dx &= \frac{1}{5}\operatorname{Subst}\left(\int \frac{x^4}{(a+bx)^2}\,dx, x, x^5\right)\\ &= \frac{1}{5}\operatorname{Subst}\left(\int\left(\frac{3a^2}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{b^2}+\frac{a^4}{b^4(a+bx)^2}-\frac{4a^3}{b^4(a+bx)}\right)dx, x, x^5\right)\\ &= \frac{3a^2x^5}{5b^4}-\frac{ax^{10}}{5b^3}+\frac{x^{15}}{15b^2}-\frac{a^4}{5b^5\left(a+bx^5\right)}-\frac{4a^3\log\left(a+bx^5\right)}{5b^5}\end{aligned}$$

Mathematica [A] time = 0.03, size = 60, normalized size = 0.83

$$\frac{-\frac{3a^4}{a+bx^5}-12a^3\log\left(a+bx^5\right)+9a^2bx^5-3ab^2x^{10}+b^3x^{15}}{15b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^24/(a + b*x^5)^2,x]
```

```
[Out] (9*a^2*b*x^5 - 3*a*b^2*x^10 + b^3*x^15 - (3*a^4)/(a + b*x^5) - 12*a^3*Log[a
 + b*x^5])/(15*b^5)
```

fricas [A] time = 0.80, size = 81, normalized size = 1.12

$$\frac{b^4x^{20} - 2\,ab^3x^{15} + 6\,a^2b^2x^{10} + 9\,a^3bx^5 - 3\,a^4 - 12\left(a^3bx^5 + a^4\right)\log\left(bx^5 + a\right)}{15\left(b^6x^5 + ab^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24/(b*x^5+a)^2,x, algorithm="fricas")
```

```
[Out] 1/15*(b^4*x^20 - 2*a*b^3*x^15 + 6*a^2*b^2*x^10 + 9*a^3*b*x^5 - 3*a^4 - 12*(
a^3*b*x^5 + a^4)*log(b*x^5 + a))/(b^6*x^5 + a*b^5)
```

giac [A] time = 0.17, size = 80, normalized size = 1.11

$$-\frac{4\,a^3\log\left(\left|bx^5 + a\right|\right)}{5\,b^5} + \frac{b^4x^{15} - 3\,ab^3x^{10} + 9\,a^2b^2x^5}{15\,b^6} + \frac{4\,a^3bx^5 + 3\,a^4}{5\left(bx^5 + a\right)b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24/(b*x^5+a)^2,x, algorithm="giac")
```

```
[Out] -4/5*a^3*log(abs(b*x^5 + a))/b^5 + 1/15*(b^4*x^15 - 3*a*b^3*x^10 + 9*a^2*b^
2*x^5)/b^6 + 1/5*(4*a^3*b*x^5 + 3*a^4)/((b*x^5 + a)*b^5)
```

maple [A] time = 0.01, size = 63, normalized size = 0.88

$$\frac{x^{15}}{15b^2} - \frac{a\,x^{10}}{5b^3} + \frac{3a^2x^5}{5b^4} - \frac{a^4}{5\left(b\,x^5 + a\right)b^5} - \frac{4a^3\ln\left(b\,x^5 + a\right)}{5b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^24/(b*x^5+a)^2,x)
```

```
[Out] 3/5*a^2*x^5/b^4-1/5*a*x^10/b^3+1/15*x^15/b^2-1/5*a^4/b^5/(b*x^5+a)-4/5*a^3*
ln(b*x^5+a)/b^5
```

maxima [A] time = 1.02, size = 65, normalized size = 0.90

$$-\frac{a^4}{5\left(b^6x^5 + ab^5\right)} - \frac{4\,a^3\log\left(bx^5 + a\right)}{5\,b^5} + \frac{b^2x^{15} - 3\,abx^{10} + 9\,a^2x^5}{15\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24/(b*x^5+a)^2,x, algorithm="maxima")
```

```
[Out] -1/5*a^4/(b^6*x^5 + a*b^5) - 4/5*a^3*log(b*x^5 + a)/b^5 + 1/15*(b^2*x^15 -
3*a*b*x^10 + 9*a^2*x^5)/b^4
```

mupad [B] time = 0.09, size = 68, normalized size = 0.94

$$\frac{x^{15}}{15\,b^2} - \frac{a^4}{5\,b\left(b^5\,x^5 + a\,b^4\right)} - \frac{a\,x^{10}}{5\,b^3} - \frac{4\,a^3\,\ln\left(b\,x^5 + a\right)}{5\,b^5} + \frac{3\,a^2\,x^5}{5\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^24/(a + b*x^5)^2,x)
```

```
[Out] x^15/(15*b^2) - a^4/(5*b*(a*b^4 + b^5*x^5)) - (a*x^10)/(5*b^3) - (4*a^3*log
(a + b*x^5))/(5*b^5) + (3*a^2*x^5)/(5*b^4)
```

sympy [**A**] time = 1.02, size = 68, normalized size = 0.94

$$-\frac{a^4}{5ab^5+5b^6x^5}-\frac{4a^3\log\left(a+bx^5\right)}{5b^5}+\frac{3a^2x^5}{5b^4}-\frac{ax^{10}}{5b^3}+\frac{x^{15}}{15b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**24/(b*x**5+a)**2,x)
```

```
[Out] -a**4/(5*a*b**5 + 5*b**6*x**5) - 4*a**3*log(a + b*x**5)/(5*b**5) + 3*a**2*x
**5/(5*b**4) - a*x**10/(5*b**3) + x**15/(15*b**2)
```

3.1278 $\int \frac{x^{19}}{\left(a+bx^5\right)^2}\,dx$

Optimal. Leaf size=59

$$\frac{a^3}{5b^4\left(a+bx^5\right)}+\frac{3a^2\log\left(a+bx^5\right)}{5b^4}-\frac{2ax^5}{5b^3}+\frac{x^{10}}{10b^2}$$

```
[Out] -2/5*a*x^5/b^3+1/10*x^10/b^2+1/5*a^3/b^4/(b*x^5+a)+3/5*a^2*ln(b*x^5+a)/b^4
```

Rubi [A] time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{a^3}{5b^4\left(a+bx^5\right)}+\frac{3a^2\log\left(a+bx^5\right)}{5b^4}-\frac{2ax^5}{5b^3}+\frac{x^{10}}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^19/(a + b*x^5)^2,x]
```

```
[Out] (-2*a*x^5)/(5*b^3) + x^10/(10*b^2) + a^3/(5*b^4*(a + b*x^5)) + (3*a^2*Log[a
 + b*x^5])/(5*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{19}}{\left(a+bx^5\right)^2}\,dx &= \frac{1}{5}\,\text{Subst}\left(\int \frac{x^3}{(a+bx)^2}\,dx, x, x^5\right)\\ &= \frac{1}{5}\,\text{Subst}\left(\int \left(-\frac{2a}{b^3}+\frac{x}{b^2}-\frac{a^3}{b^3(a+bx)^2}+\frac{3a^2}{b^3(a+bx)}\right)dx, x, x^5\right)\\ &= -\frac{2ax^5}{5b^3}+\frac{x^{10}}{10b^2}+\frac{a^3}{5b^4\left(a+bx^5\right)}+\frac{3a^2\log\left(a+bx^5\right)}{5b^4}\end{aligned}$$

Mathematica [A] time = 0.03, size = 49, normalized size = 0.83

$$\frac{\frac{2a^3}{a+bx^5}+6a^2\log\left(a+bx^5\right)-4abx^5+b^2x^{10}}{10b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^19/(a + b*x^5)^2,x]
```

```
[Out] (-4*a*b*x^5 + b^2*x^10 + (2*a^3)/(a + b*x^5) + 6*a^2*Log[a + b*x^5])/(10*b^
4)
```

fricas [A] time = 0.60, size = 70, normalized size = 1.19

$$\frac{b^3x^{15} - 3\,ab^2x^{10} - 4\,a^2bx^5 + 2\,a^3 + 6\left(a^2bx^5 + a^3\right)\log\left(bx^5 + a\right)}{10\left(b^5x^5 + ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^5+a)^2,x, algorithm="fricas")
```

```
[Out] 1/10*(b^3*x^15 - 3*a*b^2*x^10 - 4*a^2*b*x^5 + 2*a^3 + 6*(a^2*b*x^5 + a^3)*l
og(b*x^5 + a))/(b^5*x^5 + a*b^4)
```

giac [A] time = 0.20, size = 67, normalized size = 1.14

$$\frac{3\,a^2\log\left(\left|bx^5 + a\right|\right)}{5\,b^4} + \frac{b^2x^{10} - 4\,abx^5}{10\,b^4} - \frac{3\,a^2bx^5 + 2\,a^3}{5\left(bx^5 + a\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^5+a)^2,x, algorithm="giac")
```

```
[Out] 3/5*a^2*log(abs(b*x^5 + a))/b^4 + 1/10*(b^2*x^10 - 4*a*b*x^5)/b^4 - 1/5*(3*
a^2*b*x^5 + 2*a^3)/((b*x^5 + a)*b^4)
```

maple [A] time = 0.01, size = 52, normalized size = 0.88

$$\frac{x^{10}}{10b^2} - \frac{2a\,x^5}{5b^3} + \frac{a^3}{5\left(b\,x^5 + a\right)b^4} + \frac{3a^2\ln\left(b\,x^5 + a\right)}{5b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19/(b*x^5+a)^2,x)
```

```
[Out] -2/5*a*x^5/b^3+1/10*x^10/b^2+1/5*a^3/b^4/(b*x^5+a)+3/5*a^2*ln(b*x^5+a)/b^4
```

maxima [A] time = 1.15, size = 54, normalized size = 0.92

$$\frac{a^3}{5\left(b^5x^5 + ab^4\right)} + \frac{3\,a^2\log\left(bx^5 + a\right)}{5\,b^4} + \frac{bx^{10} - 4\,ax^5}{10\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^5+a)^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^3/(b^5*x^5 + a*b^4) + 3/5*a^2*log(b*x^5 + a)/b^4 + 1/10*(b*x^10 - 4*a
*x^5)/b^3
```

mupad [B] time = 0.09, size = 57, normalized size = 0.97

$$\frac{x^{10}}{10\,b^2} + \frac{a^3}{5\,b\,\left(b^4\,x^5 + a\,b^3\right)} - \frac{2\,a\,x^5}{5\,b^3} + \frac{3\,a^2\,\ln\left(b\,x^5 + a\right)}{5\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19/(a + b*x^5)^2,x)
```

```
[Out] x^10/(10*b^2) + a^3/(5*b*(a*b^3 + b^4*x^5)) - (2*a*x^5)/(5*b^3) + (3*a^2*lo
g(a + b*x^5))/(5*b^4)
```

sympy [**A**] time = 0.97, size = 56, normalized size = 0.95

$$\frac{a^3}{5ab^4 + 5b^5x^5} + \frac{3a^2 \log\left(a + bx^5\right)}{5b^4} - \frac{2ax^5}{5b^3} + \frac{x^{10}}{10b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**19/(b*x**5+a)**2,x)
```

```
[Out] a**3/(5*a*b**4 + 5*b**5*x**5) + 3*a**2*log(a + b*x**5)/(5*b**4) - 2*a*x**5/
(5*b**3) + x**10/(10*b**2)
```

3.1279 $\int \frac{x^{14}}{\left(a+bx^5\right)^2}\,dx$

Optimal. Leaf size=46

$$-\frac{a^2}{5b^3\left(a+bx^5\right)}-\frac{2a\log\left(a+bx^5\right)}{5b^3}+\frac{x^5}{5b^2}$$

```
[Out] 1/5*x^5/b^2-1/5*a^2/b^3/(b*x^5+a)-2/5*a*ln(b*x^5+a)/b^3
```

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$-\frac{a^2}{5b^3\left(a+bx^5\right)}-\frac{2a\log\left(a+bx^5\right)}{5b^3}+\frac{x^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/(a + b*x^5)^2,x]
```

```
[Out] x^5/(5*b^2) - a^2/(5*b^3*(a + b*x^5)) - (2*a*Log[a + b*x^5])/(5*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{14}}{\left(a+bx^5\right)^2}\,dx &= \frac{1}{5}\operatorname{Subst}\left(\int \frac{x^2}{(a+bx)^2}\,dx, x, x^5\right)\\ &= \frac{1}{5}\operatorname{Subst}\left(\int \left(\frac{1}{b^2}+\frac{a^2}{b^2(a+bx)^2}-\frac{2a}{b^2(a+bx)}\right)dx, x, x^5\right)\\ &= \frac{x^5}{5b^2}-\frac{a^2}{5b^3\left(a+bx^5\right)}-\frac{2a\log\left(a+bx^5\right)}{5b^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 38, normalized size = 0.83

$$\frac{-\frac{a^2}{a+bx^5}-2a\log\left(a+bx^5\right)+bx^5}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/(a + b*x^5)^2,x]
```

```
[Out] (b*x^5 - a^2/(a + b*x^5) - 2*a*Log[a + b*x^5])/(5*b^3)
```

fricas [A] time = 0.67, size = 56, normalized size = 1.22

$$\frac{b^2x^{10} + abx^5 - a^2 - 2\left(abx^5 + a^2\right)\log\left(bx^5 + a\right)}{5\left(b^4x^5 + ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^5+a)^2,x, algorithm="fricas")
```

```
[Out] 1/5*(b^2*x^10 + a*b*x^5 - a^2 - 2*(a*b*x^5 + a^2)*log(b*x^5 + a))/(b^4*x^5
+ a*b^3)
```

giac [A] time = 0.17, size = 49, normalized size = 1.07

$$\frac{x^5}{5\,b^2} - \frac{2\,a\log\left(\left|bx^5 + a\right|\right)}{5\,b^3} + \frac{2\,abx^5 + a^2}{5\left(bx^5 + a\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^5+a)^2,x, algorithm="giac")
```

```
[Out] 1/5*x^5/b^2 - 2/5*a*log(abs(b*x^5 + a))/b^3 + 1/5*(2*a*b*x^5 + a^2)/((b*x^5
 + a)*b^3)
```

maple [A] time = 0.01, size = 41, normalized size = 0.89

$$\frac{x^5}{5b^2} - \frac{a^2}{5\left(b\,x^5 + a\right)b^3} - \frac{2a\ln\left(b\,x^5 + a\right)}{5b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(b*x^5+a)^2,x)
```

```
[Out] 1/5*x^5/b^2-1/5*a^2/b^3/(b*x^5+a)-2/5*a*ln(b*x^5+a)/b^3
```

maxima [A] time = 1.07, size = 43, normalized size = 0.93

$$\frac{x^5}{5\,b^2} - \frac{a^2}{5\left(b^4x^5 + ab^3\right)} - \frac{2\,a\log\left(bx^5 + a\right)}{5\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^5+a)^2,x, algorithm="maxima")
```

```
[Out] 1/5*x^5/b^2 - 1/5*a^2/(b^4*x^5 + a*b^3) - 2/5*a*log(b*x^5 + a)/b^3
```

mupad [B] time = 1.05, size = 45, normalized size = 0.98

$$\frac{x^5}{5\,b^2} - \frac{a^2}{5\,\left(b^4\,x^5 + a\,b^3\right)} - \frac{2\,a\,\ln\left(b\,x^5 + a\right)}{5\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(a + b*x^5)^2,x)
```

```
[Out] x^5/(5*b^2) - a^2/(5*(a*b^3 + b^4*x^5)) - (2*a*log(a + b*x^5))/(5*b^3)
```

sympy [**A**] time = 0.78, size = 42, normalized size = 0.91

$$-\frac{a^2}{5ab^3+5b^4x^5}-\frac{2a\log\left(a+bx^5\right)}{5b^3}+\frac{x^5}{5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(b*x**5+a)**2,x)
```

```
[Out] -a**2/(5*a*b**3 + 5*b**4*x**5) - 2*a*log(a + b*x**5)/(5*b**3) + x**5/(5*b**
2)
```

3.1280 $\int \frac{x^9}{\left(a+bx^5\right)^2}\,dx$

Optimal. Leaf size=33

$$\frac{a}{5b^2\left(a+bx^5\right)}+\frac{\log\left(a+bx^5\right)}{5b^2}$$

```
[Out] 1/5*a/b^2/(b*x^5+a)+1/5*ln(b*x^5+a)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{a}{5b^2\left(a+bx^5\right)}+\frac{\log\left(a+bx^5\right)}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x^5)^2,x]
```

```
[Out] a/(5*b^2*(a + b*x^5)) + Log[a + b*x^5]/(5*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{\left(a+bx^5\right)^2}\,dx &= \frac{1}{5}\,\text{Subst}\left(\int \frac{x}{(a+bx)^2}\,dx, x, x^5\right)\\ &= \frac{1}{5}\,\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^2}+\frac{1}{b(a+bx)}\right)dx, x, x^5\right)\\ &= \frac{a}{5b^2\left(a+bx^5\right)}+\frac{\log\left(a+bx^5\right)}{5b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.82

$$\frac{\frac{a}{a+bx^5}+\log\left(a+bx^5\right)}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x^5)^2,x]
```

```
[Out] (a/(a + b*x^5) + Log[a + b*x^5])/(5*b^2)
```

fricas [A] time = 0.81, size = 35, normalized size = 1.06

$$\frac{\left(bx^5+a\right)\log\left(bx^5+a\right)+a}{5\left(b^3x^5+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^5+a)^2,x, algorithm="fricas")
[Out] 1/5*((b*x^5 + a)*log(b*x^5 + a) + a)/(b^3*x^5 + a*b^2)
```

giac [A] time = 0.17, size = 48, normalized size = 1.45

$$-\frac{\frac{\log\left(\frac{\left|bx^5+a\right|}{\left(bx^5+a\right)^2|b|}\right)}{b}-\frac{a}{\left(bx^5+a\right)b}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^5+a)^2,x, algorithm="giac")
[Out] -1/5*(log(abs(b*x^5 + a)/((b*x^5 + a)^2*abs(b)))/b - a/((b*x^5 + a)*b))/b
```

maple [A] time = 0.01, size = 30, normalized size = 0.91

$$\frac{a}{5\left(b\,x^5+a\right)b^2}+\frac{\ln\left(b\,x^5+a\right)}{5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^5+a)^2,x)
[Out] 1/5*a/b^2/(b*x^5+a)+1/5*ln(b*x^5+a)/b^2
```

maxima [A] time = 0.98, size = 32, normalized size = 0.97

$$\frac{a}{5\left(b^3x^5+ab^2\right)}+\frac{\log\left(bx^5+a\right)}{5\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^5+a)^2,x, algorithm="maxima")
[Out] 1/5*a/(b^3*x^5 + a*b^2) + 1/5*log(b*x^5 + a)/b^2
```

mupad [B] time = 0.05, size = 29, normalized size = 0.88

$$\frac{\ln\left(b\,x^5+a\right)}{5\,b^2}+\frac{a}{5\,b^2\,\left(b\,x^5+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + b*x^5)^2,x)
[Out] log(a + b*x^5)/(5*b^2) + a/(5*b^2*(a + b*x^5))
```

sympy [A] time = 0.56, size = 29, normalized size = 0.88

$$\frac{a}{5ab^2+5b^3x^5}+\frac{\log\left(a+bx^5\right)}{5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**5+a)**2,x)
[Out] a/(5*a*b**2 + 5*b**3*x**5) + log(a + b*x**5)/(5*b**2)
```

3.1281 $\int \frac{x^4}{\left(a+bx^5\right)^2}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{5b\left(a+bx^5\right)}$$

```
[Out] -1/5/b/(b*x^5+a)
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$-\frac{1}{5b\left(a+bx^5\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^5)^2,x]
```

```
[Out] -1/(5*b*(a + b*x^5))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^5\right)^2}\,dx = -\frac{1}{5b\left(a+bx^5\right)}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{5b\left(a+bx^5\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^5)^2,x]
```

```
[Out] -1/5*1/(b*(a + b*x^5))
```

fricas [A] time = 0.86, size = 15, normalized size = 0.94

$$-\frac{1}{5\left(b^2x^5+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^5+a)^2,x, algorithm="fricas")
```

```
[Out] -1/5/(b^2*x^5 + a*b)
```

giac [A] time = 0.17, size = 14, normalized size = 0.88

$$-\frac{1}{5\left(bx^5+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^5+a)^2,x, algorithm="giac")
[Out] -1/5/((b*x^5 + a)*b)
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$-\frac{1}{5\left(b\,x^5+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^5+a)^2,x)
[Out] -1/5/b/(b*x^5+a)
```

maxima [A] time = 1.01, size = 14, normalized size = 0.88

$$-\frac{1}{5\left(bx^5+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^5+a)^2,x, algorithm="maxima")
[Out] -1/5/((b*x^5 + a)*b)
```

mupad [B] time = 0.03, size = 14, normalized size = 0.88

$$-\frac{1}{5\,b\,\left(b\,x^5+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^5)^2,x)
[Out] -1/(5*b*(a + b*x^5))
```

sympy [A] time = 0.60, size = 15, normalized size = 0.94

$$-\frac{1}{5ab+5b^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**5+a)**2,x)
[Out] -1/(5*a*b + 5*b**2*x**5)
```

3.1282 $\int \frac{1}{x\left(a+bx^5\right)^2}\,dx$

Optimal. Leaf size=38

$$-\frac{\log\left(a+bx^5\right)}{5a^2}+\frac{\log(x)}{a^2}+\frac{1}{5a\left(a+bx^5\right)}$$

```
[Out] 1/5/a/(b*x^5+a)+ln(x)/a^2-1/5*ln(b*x^5+a)/a^2
```

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 44}

$$-\frac{\log\left(a+bx^5\right)}{5a^2}+\frac{\log(x)}{a^2}+\frac{1}{5a\left(a+bx^5\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^5)^2),x]
```

```
[Out] 1/(5*a*(a + b*x^5)) + Log[x]/a^2 - Log[a + b*x^5]/(5*a^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a+bx^5\right)^2}\,dx &= \frac{1}{5}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^2}\,dx, x, x^5\right)\\ &= \frac{1}{5}\,\text{Subst}\left(\int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right)dx, x, x^5\right)\\ &= \frac{1}{5a\left(a+bx^5\right)}+\frac{\log(x)}{a^2}-\frac{\log\left(a+bx^5\right)}{5a^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 33, normalized size = 0.87

$$\frac{\frac{a}{a+bx^5}-\log\left(a+bx^5\right)+5\log(x)}{5a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^5)^2),x]
```

```
[Out] (a/(a + b*x^5) + 5*Log[x] - Log[a + b*x^5])/(5*a^2)
```

fricas [A] time = 0.83, size = 47, normalized size = 1.24

$$-\frac{\left(bx^5+a\right)\log\left(bx^5+a\right)-5\left(bx^5+a\right)\log(x)-a}{5\left(a^2bx^5+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^5+a)^2,x, algorithm="fricas")
```

```
[Out] -1/5*((b*x^5 + a)*log(b*x^5 + a) - 5*(b*x^5 + a)*log(x) - a)/(a^2*b*x^5 + a
^3)
```

giac [A] time = 0.17, size = 45, normalized size = 1.18

$$-\frac{\log\left(\left|bx^5+a\right|\right)}{5\,a^2}+\frac{\log\left(|x|\right)}{a^2}+\frac{bx^5+2\,a}{5\left(bx^5+a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^5+a)^2,x, algorithm="giac")
```

```
[Out] -1/5*log(abs(b*x^5 + a))/a^2 + log(abs(x))/a^2 + 1/5*(b*x^5 + 2*a)/((b*x^5
+ a)*a^2)
```

maple [A] time = 0.01, size = 35, normalized size = 0.92

$$\frac{1}{5\left(b\,x^5+a\right)a}+\frac{\ln(x)}{a^2}-\frac{\ln\left(b\,x^5+a\right)}{5a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^5+a)^2,x)
```

```
[Out] 1/5/a/(b*x^5+a)+1/a^2*ln(x)-1/5*ln(b*x^5+a)/a^2
```

maxima [A] time = 1.12, size = 37, normalized size = 0.97

$$\frac{1}{5\left(abx^5+a^2\right)}-\frac{\log\left(bx^5+a\right)}{5\,a^2}+\frac{\log\left(x^5\right)}{5\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^5+a)^2,x, algorithm="maxima")
```

```
[Out] 1/5/(a*b*x^5 + a^2) - 1/5*log(b*x^5 + a)/a^2 + 1/5*log(x^5)/a^2
```

mupad [B] time = 0.07, size = 34, normalized size = 0.89

$$\frac{\ln(x)}{a^2}+\frac{1}{5\,a\,\left(b\,x^5+a\right)}-\frac{\ln\left(b\,x^5+a\right)}{5\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^5)^2),x)
```

```
[Out] log(x)/a^2 + 1/(5*a*(a + b*x^5)) - log(a + b*x^5)/(5*a^2)
```

sympy [A] time = 0.57, size = 34, normalized size = 0.89

$$\frac{1}{5a^2+5abx^5}+\frac{\log(x)}{a^2}-\frac{\log\left(\frac{a}{b}+x^5\right)}{5a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**5+a)**2,x)
[Out] 1/(5*a**2 + 5*a*b*x**5) + log(x)/a**2 - log(a/b + x**5)/(5*a**2)
```

3.1283 $\int \frac{1}{x^6\left(a+bx^5\right)^2}\,dx$

Optimal. Leaf size=52

$$\frac{2b\log\left(a+bx^5\right)}{5a^3}-\frac{2b\log(x)}{a^3}-\frac{b}{5a^2\left(a+bx^5\right)}-\frac{1}{5a^2x^5}$$

```
[Out] -1/5/a^2/x^5-1/5*b/a^2/(b*x^5+a)-2*b*ln(x)/a^3+2/5*b*ln(b*x^5+a)/a^3
```

Rubi [A] time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 44}

$$-\frac{b}{5a^2\left(a+bx^5\right)}+\frac{2b\log\left(a+bx^5\right)}{5a^3}-\frac{2b\log(x)}{a^3}-\frac{1}{5a^2x^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^5)^2),x]
```

```
[Out] -1/(5*a^2*x^5) - b/(5*a^2*(a + b*x^5)) - (2*b*Log[x])/a^3 + (2*b*Log[a + b*
x^5])/(5*a^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^6\left(a+bx^5\right)^2}\,dx &= \frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)^2}\,dx,x,x^5\right)\\ &= \frac{1}{5}\operatorname{Subst}\left(\int\left(\frac{1}{a^2x^2}-\frac{2b}{a^3x}+\frac{b^2}{a^2(a+bx)^2}+\frac{2b^2}{a^3(a+bx)}\right)dx,x,x^5\right)\\ &= -\frac{1}{5a^2x^5}-\frac{b}{5a^2\left(a+bx^5\right)}-\frac{2b\log(x)}{a^3}+\frac{2b\log\left(a+bx^5\right)}{5a^3}\end{aligned}$$

Mathematica [A] time = 0.06, size = 41, normalized size = 0.79

$$-\frac{a\left(\frac{b}{a+bx^5}+\frac{1}{x^5}\right)-2b\log\left(a+bx^5\right)+10b\log(x)}{5a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^5)^2),x]
```

```
[Out] -1/5*(a*(x^(-5) + b/(a + b*x^5)) + 10*b*Log[x] - 2*b*Log[a + b*x^5])/a^3
```

fricas [A] time = 0.68, size = 73, normalized size = 1.40

$$-\frac{2\,abx^5 + a^2 - 2\left(b^2x^{10} + abx^5\right)\log\left(bx^5 + a\right) + 10\left(b^2x^{10} + abx^5\right)\log(x)}{5\left(a^3bx^{10} + a^4x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^5+a)^2,x, algorithm="fricas")
```

```
[Out] -1/5*(2*a*b*x^5 + a^2 - 2*(b^2*x^10 + a*b*x^5)*log(b*x^5 + a) + 10*(b^2*x^1
0 + a*b*x^5)*log(x))/(a^3*b*x^10 + a^4*x^5)
```

giac [A] time = 0.24, size = 51, normalized size = 0.98

$$\frac{2\,b\log\left(\left|bx^5 + a\right|\right)}{5\,a^3} - \frac{2\,b\log\left(|x|\right)}{a^3} - \frac{2\,bx^5 + a}{5\left(bx^{10} + ax^5\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^5+a)^2,x, algorithm="giac")
```

```
[Out] 2/5*b*log(abs(b*x^5 + a))/a^3 - 2*b*log(abs(x))/a^3 - 1/5*(2*b*x^5 + a)/((b
*x^10 + a*x^5)*a^2)
```

maple [A] time = 0.01, size = 47, normalized size = 0.90

$$-\frac{b}{5\left(b\,x^5 + a\right)a^2} - \frac{2b\ln(x)}{a^3} + \frac{2b\ln\left(b\,x^5 + a\right)}{5a^3} - \frac{1}{5a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^5+a)^2,x)
```

```
[Out] -1/5/a^2/x^5-1/5*b/a^2/(b*x^5+a)-2/a^3*b*ln(x)+2/5*b*ln(b*x^5+a)/a^3
```

maxima [A] time = 1.12, size = 53, normalized size = 1.02

$$-\frac{2\,bx^5 + a}{5\left(a^2bx^{10} + a^3x^5\right)} + \frac{2\,b\log\left(bx^5 + a\right)}{5\,a^3} - \frac{2\,b\log\left(x^5\right)}{5\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^5+a)^2,x, algorithm="maxima")
```

```
[Out] -1/5*(2*b*x^5 + a)/(a^2*b*x^10 + a^3*x^5) + 2/5*b*log(b*x^5 + a)/a^3 - 2/5*
b*log(x^5)/a^3
```

mupad [B] time = 1.07, size = 53, normalized size = 1.02

$$\frac{2\,b\,\ln\left(b\,x^5 + a\right)}{5\,a^3} - \frac{\frac{1}{5\,a} + \frac{2\,b\,x^5}{5\,a^2}}{b\,x^{10} + a\,x^5} - \frac{2\,b\,\ln(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^5)^2),x)
```

```
[Out] (2*b*log(a + b*x^5))/(5*a^3) - (1/(5*a) + (2*b*x^5)/(5*a^2))/(a*x^5 + b*x^1
0) - (2*b*log(x))/a^3
```

sympy [**A**] time = 1.27, size = 54, normalized size = 1.04

$$\frac{-a-2bx^5}{5a^3x^5+5a^2bx^{10}}-\frac{2b\log(x)}{a^3}+\frac{2b\log\left(\frac{a}{b}+x^5\right)}{5a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**5+a)**2,x)
```

```
[Out] (-a - 2*b*x**5)/(5*a**3*x**5 + 5*a**2*b*x**10) - 2*b*log(x)/a**3 + 2*b*log(
a/b + x**5)/(5*a**3)
```

3.1284 $\int \frac{1}{x^{11}\left(a+bx^5\right)^2}\,dx$

Optimal. Leaf size=69

$$-\frac{3b^2\log\left(a+bx^5\right)}{5a^4}+\frac{3b^2\log(x)}{a^4}+\frac{b^2}{5a^3\left(a+bx^5\right)}+\frac{2b}{5a^3x^5}-\frac{1}{10a^2x^{10}}$$

```
[Out] -1/10/a^2/x^10+2/5*b/a^3/x^5+1/5*b^2/a^3/(b*x^5+a)+3*b^2*ln(x)/a^4-3/5*b^2*
ln(b*x^5+a)/a^4
```

Rubi [A] time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 44}

$$\frac{b^2}{5a^3\left(a+bx^5\right)}-\frac{3b^2\log\left(a+bx^5\right)}{5a^4}+\frac{3b^2\log(x)}{a^4}+\frac{2b}{5a^3x^5}-\frac{1}{10a^2x^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^11*(a + b*x^5)^2),x]
```

```
[Out] -1/(10*a^2*x^10) + (2*b)/(5*a^3*x^5) + b^2/(5*a^3*(a + b*x^5)) + (3*b^2*Log
[x])/a^4 - (3*b^2*Log[a + b*x^5])/(5*a^4)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{11}\left(a+bx^5\right)^2}\,dx &= \frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{x^3(a+bx)^2}\,dx,x,x^5\right)\\ &= \frac{1}{5}\operatorname{Subst}\left(\int\left(\frac{1}{a^2x^3}-\frac{2b}{a^3x^2}+\frac{3b^2}{a^4x}-\frac{b^3}{a^3(a+bx)^2}-\frac{3b^3}{a^4(a+bx)}\right)dx,x,x^5\right)\\ &= -\frac{1}{10a^2x^{10}}+\frac{2b}{5a^3x^5}+\frac{b^2}{5a^3\left(a+bx^5\right)}+\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log\left(a+bx^5\right)}{5a^4}\end{aligned}$$

Mathematica [A] time = 0.08, size = 57, normalized size = 0.83

$$\frac{-6b^2\log\left(a+bx^5\right)+a\left(\frac{2b^2}{a+bx^5}-\frac{a}{x^{10}}+\frac{4b}{x^5}\right)+30b^2\log(x)}{10a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^11*(a + b*x^5)^2),x]
```

```
[Out] (a*(-(a/x^10) + (4*b)/x^5 + (2*b^2)/(a + b*x^5)) + 30*b^2*Log[x] - 6*b^2*Lo
g[a + b*x^5])/(10*a^4)
```

fricas [A] time = 0.91, size = 90, normalized size = 1.30

$$\frac{6\,ab^2x^{10} + 3\,a^2bx^5 - a^3 - 6\left(b^3x^{15} + ab^2x^{10}\right)\log\left(bx^5 + a\right) + 30\left(b^3x^{15} + ab^2x^{10}\right)\log(x)}{10\left(a^4bx^{15} + a^5x^{10}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(b*x^5+a)^2,x, algorithm="fricas")
```

```
[Out] 1/10*(6*a*b^2*x^10 + 3*a^2*b*x^5 - a^3 - 6*(b^3*x^15 + a*b^2*x^10)*log(b*x^
5 + a) + 30*(b^3*x^15 + a*b^2*x^10)*log(x))/(a^4*b*x^15 + a^5*x^10)
```

giac [A] time = 0.18, size = 85, normalized size = 1.23

$$-\frac{3\,b^2\log\left(\left|bx^5 + a\right|\right)}{5\,a^4} + \frac{3\,b^2\log\left(|x|\right)}{a^4} + \frac{3\,b^3x^5 + 4\,ab^2}{5\left(bx^5 + a\right)a^4} - \frac{9\,b^2x^{10} - 4\,abx^5 + a^2}{10\,a^4x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(b*x^5+a)^2,x, algorithm="giac")
```

```
[Out] -3/5*b^2*log(abs(b*x^5 + a))/a^4 + 3*b^2*log(abs(x))/a^4 + 1/5*(3*b^3*x^5 +
 4*a*b^2)/((b*x^5 + a)*a^4) - 1/10*(9*b^2*x^10 - 4*a*b*x^5 + a^2)/(a^4*x^10
)
```

maple [A] time = 0.02, size = 62, normalized size = 0.90

$$\frac{b^2}{5\left(b\,x^5 + a\right)a^3} + \frac{3b^2\ln(x)}{a^4} - \frac{3b^2\ln\left(b\,x^5 + a\right)}{5a^4} + \frac{2b}{5a^3x^5} - \frac{1}{10a^2x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^11/(b*x^5+a)^2,x)
```

```
[Out] -1/10/a^2/x^10+2/5*b/a^3/x^5+1/5*b^2/a^3/(b*x^5+a)+3*b^2*ln(x)/a^4-3/5*b^2*
ln(b*x^5+a)/a^4
```

maxima [A] time = 1.02, size = 70, normalized size = 1.01

$$\frac{6\,b^2x^{10} + 3\,abx^5 - a^2}{10\left(a^3bx^{15} + a^4x^{10}\right)} - \frac{3\,b^2\log\left(bx^5 + a\right)}{5\,a^4} + \frac{3\,b^2\log\left(x^5\right)}{5\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^11/(b*x^5+a)^2,x, algorithm="maxima")
```

```
[Out] 1/10*(6*b^2*x^10 + 3*a*b*x^5 - a^2)/(a^3*b*x^15 + a^4*x^10) - 3/5*b^2*log(b
*x^5 + a)/a^4 + 3/5*b^2*log(x^5)/a^4
```

mupad [B] time = 1.07, size = 67, normalized size = 0.97

$$\frac{\frac{3\,b\,x^5}{10\,a^2} - \frac{1}{10\,a} + \frac{3\,b^2\,x^{10}}{5\,a^3}}{b\,x^{15} + a\,x^{10}} - \frac{3\,b^2\,\ln\left(b\,x^5 + a\right)}{5\,a^4} + \frac{3\,b^2\,\ln(x)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^11*(a + b*x^5)^2),x)
```

```
[Out] ((3*b*x^5)/(10*a^2) - 1/(10*a) + (3*b^2*x^10)/(5*a^3))/(a*x^10 + b*x^15) -
(3*b^2*log(a + b*x^5))/(5*a^4) + (3*b^2*log(x))/a^4
```

sympy [**A**] time = 1.60, size = 68, normalized size = 0.99

$$\frac{-a^2 + 3abx^5 + 6b^2x^{10}}{10a^4x^{10} + 10a^3bx^{15}} + \frac{3b^2 \log(x)}{a^4} - \frac{3b^2 \log\left(\frac{a}{b} + x^5\right)}{5a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**11/(b*x**5+a)**2,x)
```

```
[Out] (-a**2 + 3*a*b*x**5 + 6*b**2*x**10)/(10*a**4*x**10 + 10*a**3*b*x**15) + 3*b
**2*log(x)/a**4 - 3*b**2*log(a/b + x**5)/(5*a**4)
```

3.1285 $\int \frac{x^{14}}{2b+bx^5}\,dx$

Optimal. Leaf size=34

$$\frac{x^{10}}{10b}-\frac{2x^5}{5b}+\frac{4\log\left(x^5+2\right)}{5b}$$

```
[Out] -2/5*x^5/b+1/10*x^10/b+4/5*ln(x^5+2)/b
```

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{x^{10}}{10b}-\frac{2x^5}{5b}+\frac{4\log\left(x^5+2\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/(2*b + b*x^5),x]
```

```
[Out] (-2*x^5)/(5*b) + x^10/(10*b) + (4*Log[2 + x^5])/(5*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{14}}{2b+bx^5}\,dx &= \frac{1}{5}\,\text{Subst}\left(\int \frac{x^2}{2b+bx}\,dx, x, x^5\right)\\ &= \frac{1}{5}\,\text{Subst}\left(\int \left(-\frac{2}{b}+\frac{x}{b}+\frac{4}{b(2+x)}\right)\,dx, x, x^5\right)\\ &= -\frac{2x^5}{5b}+\frac{x^{10}}{10b}+\frac{4\log\left(2+x^5\right)}{5b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.74

$$\frac{x^{10}-4x^5+8\log\left(x^5+2\right)-12}{10b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/(2*b + b*x^5),x]
```

```
[Out] (-12 - 4*x^5 + x^10 + 8*Log[2 + x^5])/(10*b)
```

fricas [A] time = 0.70, size = 22, normalized size = 0.65

$$\frac{x^{10}-4\,x^5+8\,\log\left(x^5+2\right)}{10\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^5+2*b),x, algorithm="fricas")
[Out] 1/10*(x^10 - 4*x^5 + 8*log(x^5 + 2))/b
```

giac [A] time = 0.20, size = 30, normalized size = 0.88

$$\frac{4\,\log\left(\left|x^5+2\right|\right)}{5\,b}+\frac{bx^{10}-4\,bx^5}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^5+2*b),x, algorithm="giac")
[Out] 4/5*log(abs(x^5 + 2))/b + 1/10*(b*x^10 - 4*b*x^5)/b^2
```

maple [A] time = 0.00, size = 29, normalized size = 0.85

$$\frac{x^{10}}{10b}-\frac{2x^5}{5b}+\frac{4\ln\left(x^5+2\right)}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(b*x^5+2*b),x)
[Out] -2/5/b*x^5+1/10/b*x^10+4/5*ln(x^5+2)/b
```

maxima [A] time = 1.04, size = 26, normalized size = 0.76

$$\frac{x^{10}-4\,x^5}{10\,b}+\frac{4\,\log\left(x^5+2\right)}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^5+2*b),x, algorithm="maxima")
[Out] 1/10*(x^10 - 4*x^5)/b + 4/5*log(x^5 + 2)/b
```

mupad [B] time = 0.06, size = 22, normalized size = 0.65

$$\frac{8\,\ln\left(x^5+2\right)-4\,x^5+x^{10}}{10\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(2*b + b*x^5),x)
[Out] (8*log(x^5 + 2) - 4*x^5 + x^10)/(10*b)
```

sympy [A] time = 0.43, size = 26, normalized size = 0.76

$$\frac{x^{10}}{10b}-\frac{2x^5}{5b}+\frac{4\log\left(x^5+2\right)}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(b*x**5+2*b),x)
[Out] x**10/(10*b) - 2*x**5/(5*b) + 4*log(x**5 + 2)/(5*b)
```

3.1286 $\int \frac{x^9}{2b+bx^5}\,dx$

Optimal. Leaf size=24

$$\frac{x^5}{5b}-\frac{2\log\left(x^5+2\right)}{5b}$$

```
[Out] 1/5*x^5/b-2/5*ln(x^5+2)/b
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{x^5}{5b}-\frac{2\log\left(x^5+2\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(2*b + b*x^5),x]
```

```
[Out] x^5/(5*b) - (2*Log[2 + x^5])/(5*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{2b+bx^5}\,dx &= \frac{1}{5}\text{Subst}\left(\int \frac{x}{2b+bx}\,dx, x, x^5\right)\\ &= \frac{1}{5}\text{Subst}\left(\int \left(\frac{1}{b}-\frac{2}{b(2+x)}\right)\,dx, x, x^5\right)\\ &= \frac{x^5}{5b}-\frac{2\log\left(2+x^5\right)}{5b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$\frac{x^5}{5b}-\frac{2\log\left(x^5+2\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(2*b + b*x^5),x]
```

```
[Out] x^5/(5*b) - (2*Log[2 + x^5])/(5*b)
```

fricas [A] time = 0.77, size = 17, normalized size = 0.71

$$\frac{x^5-2\,\log\left(x^5+2\right)}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^5+2*b),x, algorithm="fricas")
[Out] 1/5*(x^5 - 2*log(x^5 + 2))/b
```

giac [A] time = 0.23, size = 21, normalized size = 0.88

$$\frac{x^5}{5\,b} - \frac{2\,\log\left(\left|x^5+2\right|\right)}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^5+2*b),x, algorithm="giac")
[Out] 1/5*x^5/b - 2/5*log(abs(x^5 + 2))/b
```

maple [A] time = 0.00, size = 21, normalized size = 0.88

$$\frac{x^5}{5b} - \frac{2\ln\left(x^5+2\right)}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^5+2*b),x)
[Out] 1/5/b*x^5-2/5/b*ln(x^5+2)
```

maxima [A] time = 0.97, size = 20, normalized size = 0.83

$$\frac{x^5}{5\,b} - \frac{2\,\log\left(x^5+2\right)}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^5+2*b),x, algorithm="maxima")
[Out] 1/5*x^5/b - 2/5*log(x^5 + 2)/b
```

mupad [B] time = 0.04, size = 19, normalized size = 0.79

$$-\frac{2\,\ln\left(x^5+2\right)-x^5}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(2*b + b*x^5),x)
[Out] -(2*log(x^5 + 2) - x^5)/(5*b)
```

sympy [A] time = 0.28, size = 17, normalized size = 0.71

$$\frac{x^5}{5b} - \frac{2\log\left(x^5+2\right)}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**5+2*b),x)
[Out] x**5/(5*b) - 2*log(x**5 + 2)/(5*b)
```

3.1287 $\int \frac{x^4}{2b+bx^5}\,dx$

Optimal. Leaf size=13

$$\frac{\log\left(x^5+2\right)}{5b}$$

```
[Out] 1/5*ln(x^5+2)/b
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {260}

$$\frac{\log\left(x^5+2\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(2*b + b*x^5),x]
```

```
[Out] Log[2 + x^5]/(5*b)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^4}{2b+bx^5}\,dx = \frac{\log\left(2+x^5\right)}{5b}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.31

$$\frac{\log\left(bx^5+2b\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(2*b + b*x^5),x]
```

```
[Out] Log[2*b + b*x^5]/(5*b)
```

fricas [A] time = 0.80, size = 11, normalized size = 0.85

$$\frac{\log\left(x^5+2\right)}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^5+2*b),x, algorithm="fricas")
```

```
[Out] 1/5*log(x^5 + 2)/b
```

giac [A] time = 0.20, size = 16, normalized size = 1.23

$$\frac{\log\left(\left|bx^5+2\,b\right|\right)}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^5+2*b),x, algorithm="giac")
[Out] 1/5*log(abs(b*x^5 + 2*b))/b
```

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$\frac{\ln\left(x^5+2\right)}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^5+2*b),x)
[Out] 1/5/b*ln(x^5+2)
```

maxima [A] time = 1.06, size = 15, normalized size = 1.15

$$\frac{\log\left(bx^5+2\,b\right)}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^5+2*b),x, algorithm="maxima")
[Out] 1/5*log(b*x^5 + 2*b)/b
```

mupad [B] time = 0.04, size = 11, normalized size = 0.85

$$\frac{\ln\left(x^5+2\right)}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(2*b + b*x^5),x)
[Out] log(x^5 + 2)/(5*b)
```

sympy [A] time = 0.37, size = 8, normalized size = 0.62

$$\frac{\log\left(x^5+2\right)}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**5+2*b),x)
[Out] log(x**5 + 2)/(5*b)
```

3.1288 $\int \frac{1}{x\left(2b+bx^5\right)}\,dx$

Optimal. Leaf size=23

$$\frac{\log(x)}{2b}-\frac{\log\left(x^5+2\right)}{10b}$$

```
[Out] 1/2*ln(x)/b-1/10*ln(x^5+2)/b
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 36, 29, 31}

$$\frac{\log(x)}{2b}-\frac{\log\left(x^5+2\right)}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(2*b + b*x^5)),x]
```

```
[Out] Log[x]/(2*b) - Log[2 + x^5]/(10*b)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(2b+bx^5\right)}\,dx &= \frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{x(2b+bx)}\,dx,x,x^5\right)\\ &= -\left(\frac{1}{10}\operatorname{Subst}\left(\int \frac{1}{2b+bx}\,dx,x,x^5\right)\right)+\frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,x^5\right)}{10b}\\ &= \frac{\log(x)}{2b}-\frac{\log\left(2+x^5\right)}{10b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{\log(x)}{2b}-\frac{\log\left(x^5+2\right)}{10b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(2*b + b*x^5)),x]
[Out] Log[x]/(2*b) - Log[2 + x^5]/(10*b)
```

fricas [A] time = 0.84, size = 16, normalized size = 0.70

$$-\frac{\log\left(x^5+2\right)-5\,\log(x)}{10\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^5+2*b),x, algorithm="fricas")
[Out] -1/10*(log(x^5 + 2) - 5*log(x))/b
```

giac [A] time = 0.24, size = 21, normalized size = 0.91

$$-\frac{\log\left(\left|x^5+2\right|\right)}{10\,b}+\frac{\log\left(|x|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^5+2*b),x, algorithm="giac")
[Out] -1/10*log(abs(x^5 + 2))/b + 1/2*log(abs(x))/b
```

maple [A] time = 0.00, size = 20, normalized size = 0.87

$$\frac{\ln(x)}{2b}-\frac{\ln\left(x^5+2\right)}{10b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^5+2*b),x)
[Out] 1/2*ln(x)/b-1/10/b*ln(x^5+2)
```

maxima [A] time = 1.10, size = 21, normalized size = 0.91

$$-\frac{\log\left(x^5+2\right)}{10\,b}+\frac{\log\left(x^5\right)}{10\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^5+2*b),x, algorithm="maxima")
[Out] -1/10*log(x^5 + 2)/b + 1/10*log(x^5)/b
```

mupad [B] time = 1.09, size = 19, normalized size = 0.83

$$\frac{\ln(x)}{2\,b}-\frac{\ln\left(x^5+2\right)}{10\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(2*b + b*x^5)),x)
[Out] log(x)/(2*b) - log(x^5 + 2)/(10*b)
```

sympy [A] time = 0.83, size = 15, normalized size = 0.65

$$\frac{\log(x)}{2b}-\frac{\log\left(x^5+2\right)}{10b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**5+2*b),x)
[Out] log(x)/(2*b) - log(x**5 + 2)/(10*b)
```

3.1289 $\int \frac{1}{x^6\left(2b+bx^5\right)}\,dx$

Optimal. Leaf size=33

$$-\frac{1}{10bx^5}+\frac{\log\left(x^5+2\right)}{20b}-\frac{\log(x)}{4b}$$

```
[Out] -1/10/b/x^5-1/4*ln(x)/b+1/20*ln(x^5+2)/b
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 44}

$$-\frac{1}{10bx^5}+\frac{\log\left(x^5+2\right)}{20b}-\frac{\log(x)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(2*b + b*x^5)),x]
```

```
[Out] -1/(10*b*x^5) - Log[x]/(4*b) + Log[2 + x^5]/(20*b)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^6\left(2b+bx^5\right)}\,dx &= \frac{1}{5}\,\text{Subst}\left(\int \frac{1}{x^2(2b+bx)}\,dx, x, x^5\right)\\ &= \frac{1}{5}\,\text{Subst}\left(\int \left(\frac{1}{2bx^2}-\frac{1}{4bx}+\frac{1}{4b(2+x)}\right)\,dx, x, x^5\right)\\ &= -\frac{1}{10bx^5}-\frac{\log(x)}{4b}+\frac{\log\left(2+x^5\right)}{20b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 33, normalized size = 1.00

$$-\frac{1}{10bx^5}+\frac{\log\left(x^5+2\right)}{20b}-\frac{\log(x)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(2*b + b*x^5)),x]
```

```
[Out] -1/10*1/(b*x^5) - Log[x]/(4*b) + Log[2 + x^5]/(20*b)
```

fricas [A] time = 0.89, size = 27, normalized size = 0.82

$$\frac{x^5\log\left(x^5+2\right)-5\,x^5\log(x)-2}{20\,bx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^5+2*b),x, algorithm="fricas")
[Out] 1/20*(x^5*log(x^5 + 2) - 5*x^5*log(x) - 2)/(b*x^5)
```

giac [A] time = 0.23, size = 34, normalized size = 1.03

$$\frac{\log\left(\left|x^5+2\right|\right)}{20\,b}-\frac{\log\left(|x|\right)}{4\,b}+\frac{x^5-2}{20\,bx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^5+2*b),x, algorithm="giac")
[Out] 1/20*log(abs(x^5 + 2))/b - 1/4*log(abs(x))/b + 1/20*(x^5 - 2)/(b*x^5)
```

maple [A] time = 0.01, size = 28, normalized size = 0.85

$$-\frac{\ln(x)}{4b}+\frac{\ln\left(x^5+2\right)}{20b}-\frac{1}{10b\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^5+2*b),x)
[Out] -1/10/b/x^5-1/4/b*ln(x)+1/20/b*ln(x^5+2)
```

maxima [A] time = 1.07, size = 29, normalized size = 0.88

$$\frac{\log\left(x^5+2\right)}{20\,b}-\frac{\log\left(x^5\right)}{20\,b}-\frac{1}{10\,bx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^5+2*b),x, algorithm="maxima")
[Out] 1/20*log(x^5 + 2)/b - 1/20*log(x^5)/b - 1/10/(b*x^5)
```

mupad [B] time = 1.07, size = 27, normalized size = 0.82

$$\frac{\ln\left(x^5+2\right)}{20\,b}-\frac{\ln(x)}{4\,b}-\frac{1}{10\,b\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(2*b + b*x^5)),x)
[Out] log(x^5 + 2)/(20*b) - log(x)/(4*b) - 1/(10*b*x^5)
```

sympy [A] time = 0.42, size = 24, normalized size = 0.73

$$-\frac{\log(x)}{4b}+\frac{\log\left(x^5+2\right)}{20b}-\frac{1}{10bx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**5+2*b),x)
[Out] -log(x)/(4*b) + log(x**5 + 2)/(20*b) - 1/(10*b*x**5)
```

3.1290 $\int \frac{x^{14}}{3+bx^5}\,dx$

Optimal. Leaf size=36

$$\frac{9\log\left(bx^5+3\right)}{5b^3}-\frac{3x^5}{5b^2}+\frac{x^{10}}{10b}$$

```
[Out] -3/5*x^5/b^2+1/10*x^10/b+9/5*ln(b*x^5+3)/b^3
```

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$-\frac{3x^5}{5b^2}+\frac{9\log\left(bx^5+3\right)}{5b^3}+\frac{x^{10}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/(3 + b*x^5),x]
```

```
[Out] (-3*x^5)/(5*b^2) + x^10/(10*b) + (9*Log[3 + b*x^5])/(5*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{14}}{3+bx^5}\,dx &= \frac{1}{5}\,\text{Subst}\left(\int \frac{x^2}{3+bx}\,dx, x, x^5\right)\\ &= \frac{1}{5}\,\text{Subst}\left(\int \left(-\frac{3}{b^2}+\frac{x}{b}+\frac{9}{b^2(3+bx)}\right)\,dx, x, x^5\right)\\ &= -\frac{3x^5}{5b^2}+\frac{x^{10}}{10b}+\frac{9\log\left(3+bx^5\right)}{5b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 36, normalized size = 1.00

$$\frac{9\log\left(bx^5+3\right)}{5b^3}-\frac{3x^5}{5b^2}+\frac{x^{10}}{10b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/(3 + b*x^5),x]
```

```
[Out] (-3*x^5)/(5*b^2) + x^10/(10*b) + (9*Log[3 + b*x^5])/(5*b^3)
```

fricas [A] time = 0.85, size = 29, normalized size = 0.81

$$\frac{b^2x^{10}-6\,bx^5+18\,\log\left(bx^5+3\right)}{10\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^5+3),x, algorithm="fricas")
[Out] 1/10*(b^2*x^10 - 6*b*x^5 + 18*log(b*x^5 + 3))/b^3
```

giac [A] time = 0.67, size = 31, normalized size = 0.86

$$\frac{bx^{10} - 6\,x^5}{10\,b^2} + \frac{9\,\log\left(\left|bx^5 + 3\right|\right)}{5\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^5+3),x, algorithm="giac")
[Out] 1/10*(b*x^10 - 6*x^5)/b^2 + 9/5*log(abs(b*x^5 + 3))/b^3
```

maple [A] time = 0.00, size = 31, normalized size = 0.86

$$\frac{x^{10}}{10b} - \frac{3x^5}{5b^2} + \frac{9\ln\left(b\,x^5 + 3\right)}{5b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(b*x^5+3),x)
[Out] -3/5/b^2*x^5+1/10/b*x^10+9/5*ln(b*x^5+3)/b^3
```

maxima [A] time = 0.96, size = 30, normalized size = 0.83

$$\frac{bx^{10} - 6\,x^5}{10\,b^2} + \frac{9\,\log\left(bx^5 + 3\right)}{5\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^5+3),x, algorithm="maxima")
[Out] 1/10*(b*x^10 - 6*x^5)/b^2 + 9/5*log(b*x^5 + 3)/b^3
```

mupad [B] time = 0.05, size = 29, normalized size = 0.81

$$\frac{18\,\ln\left(b\,x^5 + 3\right) - 6\,b\,x^5 + b^2\,x^{10}}{10\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(b*x^5 + 3),x)
[Out] (18*log(b*x^5 + 3) - 6*b*x^5 + b^2*x^10)/(10*b^3)
```

sympy [A] time = 0.44, size = 31, normalized size = 0.86

$$\frac{x^{10}}{10b} - \frac{3x^5}{5b^2} + \frac{9\log\left(bx^5 + 3\right)}{5b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(b*x**5+3),x)
[Out] x**10/(10*b) - 3*x**5/(5*b**2) + 9*log(b*x**5 + 3)/(5*b**3)
```

3.1291 $\int \frac{x^9}{3+bx^5}\,dx$

Optimal. Leaf size=26

$$\frac{x^5}{5b}-\frac{3\log\left(bx^5+3\right)}{5b^2}$$

```
[Out] 1/5*x^5/b-3/5*ln(b*x^5+3)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{x^5}{5b}-\frac{3\log\left(bx^5+3\right)}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(3 + b*x^5),x]
```

```
[Out] x^5/(5*b) - (3*Log[3 + b*x^5])/(5*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{3+bx^5}\,dx &= \frac{1}{5}\,\text{Subst}\left(\int \frac{x}{3+bx}\,dx, x, x^5\right)\\ &= \frac{1}{5}\,\text{Subst}\left(\int \left(\frac{1}{b}-\frac{3}{b(3+bx)}\right)\,dx, x, x^5\right)\\ &= \frac{x^5}{5b}-\frac{3\log\left(3+bx^5\right)}{5b^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 26, normalized size = 1.00

$$\frac{x^5}{5b}-\frac{3\log\left(bx^5+3\right)}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(3 + b*x^5),x]
```

```
[Out] x^5/(5*b) - (3*Log[3 + b*x^5])/(5*b^2)
```

fricas [A] time = 0.79, size = 21, normalized size = 0.81

$$\frac{bx^5-3\,\log\left(bx^5+3\right)}{5\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^5+3),x, algorithm="fricas")
[Out] 1/5*(b*x^5 - 3*log(b*x^5 + 3))/b^2
```

giac [A] time = 0.67, size = 23, normalized size = 0.88

$$\frac{x^5}{5\,b} - \frac{3\,\log\left(\left|b x^5+3\right|\right)}{5\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^5+3),x, algorithm="giac")
[Out] 1/5*x^5/b - 3/5*log(abs(b*x^5 + 3))/b^2
```

maple [A] time = 0.00, size = 23, normalized size = 0.88

$$\frac{x^5}{5b} - \frac{3\ln\left(b\,x^5+3\right)}{5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^5+3),x)
[Out] 1/5/b*x^5-3/5*ln(b*x^5+3)/b^2
```

maxima [A] time = 1.01, size = 22, normalized size = 0.85

$$\frac{x^5}{5\,b} - \frac{3\,\log\left(b x^5+3\right)}{5\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^5+3),x, algorithm="maxima")
[Out] 1/5*x^5/b - 3/5*log(b*x^5 + 3)/b^2
```

mupad [B] time = 1.02, size = 22, normalized size = 0.85

$$-\frac{3\,\ln\left(b\,x^5+3\right)-b\,x^5}{5\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^5 + 3),x)
[Out] -(3*log(b*x^5 + 3) - b*x^5)/(5*b^2)
```

sympy [A] time = 0.25, size = 20, normalized size = 0.77

$$\frac{x^5}{5b} - \frac{3\log\left(bx^5+3\right)}{5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**5+3),x)
[Out] x**5/(5*b) - 3*log(b*x**5 + 3)/(5*b**2)
```

3.1292 $\int \frac{x^4}{3+bx^5}\,dx$

Optimal. Leaf size=15

$$\frac{\log\left(bx^5+3\right)}{5b}$$

```
[Out] 1/5*ln(b*x^5+3)/b
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {260}

$$\frac{\log\left(bx^5+3\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(3 + b*x^5),x]
```

```
[Out] Log[3 + b*x^5]/(5*b)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^4}{3+bx^5}\,dx = \frac{\log\left(3+bx^5\right)}{5b}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log\left(bx^5+3\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(3 + b*x^5),x]
```

```
[Out] Log[3 + b*x^5]/(5*b)
```

fricas [A] time = 0.91, size = 13, normalized size = 0.87

$$\frac{\log\left(bx^5+3\right)}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^5+3),x, algorithm="fricas")
```

```
[Out] 1/5*log(b*x^5 + 3)/b
```

giac [A] time = 0.67, size = 14, normalized size = 0.93

$$\frac{\log\left(\left|bx^5+3\right|\right)}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^5+3),x, algorithm="giac")
[Out] 1/5*log(abs(b*x^5 + 3))/b
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{\ln\left(b\,x^5+3\right)}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^5+3),x)
[Out] 1/5*ln(b*x^5+3)/b
```

maxima [A] time = 0.99, size = 13, normalized size = 0.87

$$\frac{\log\left(bx^5+3\right)}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^5+3),x, algorithm="maxima")
[Out] 1/5*log(b*x^5 + 3)/b
```

mupad [B] time = 1.02, size = 13, normalized size = 0.87

$$\frac{\ln\left(b\,x^5+3\right)}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^5 + 3),x)
[Out] log(b*x^5 + 3)/(5*b)
```

sympy [A] time = 0.40, size = 10, normalized size = 0.67

$$\frac{\log\left(bx^5+3\right)}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**5+3),x)
[Out] log(b*x**5 + 3)/(5*b)
```

3.1293 $\int \frac{1}{x\left(3+bx^5\right)}\,dx$

Optimal. Leaf size=19

$$\frac{\log(x)}{3}-\frac{1}{15}\log\left(bx^5+3\right)$$

```
[Out] 1/3*ln(x)-1/15*ln(b*x^5+3)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {266, 36, 29, 31}

$$\frac{\log(x)}{3}-\frac{1}{15}\log\left(bx^5+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(3 + b*x^5)),x]
```

```
[Out] Log[x]/3 - Log[3 + b*x^5]/15
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(3+bx^5\right)}\,dx &= \frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{x(3+bx)}\,dx,x,x^5\right)\\ &= \frac{1}{15}\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,x^5\right)-\frac{1}{15}b\operatorname{Subst}\left(\int \frac{1}{3+bx}\,dx,x,x^5\right)\\ &= \frac{\log(x)}{3}-\frac{1}{15}\log\left(3+bx^5\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{\log(x)}{3}-\frac{1}{15}\log\left(bx^5+3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(3 + b*x^5)),x]
[Out] Log[x]/3 - Log[3 + b*x^5]/15
```

fricas [A] time = 0.78, size = 15, normalized size = 0.79

$$-\frac{1}{15}\log\left(bx^5+3\right)+\frac{1}{3}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^5+3),x, algorithm="fricas")
[Out] -1/15*log(b*x^5 + 3) + 1/3*log(x)
```

giac [A] time = 0.34, size = 17, normalized size = 0.89

$$-\frac{1}{15}\log\left(\left|bx^5+3\right|\right)+\frac{1}{3}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^5+3),x, algorithm="giac")
[Out] -1/15*log(abs(b*x^5 + 3)) + 1/3*log(abs(x))
```

maple [A] time = 0.00, size = 16, normalized size = 0.84

$$\frac{\ln(x)}{3}-\frac{\ln\left(b\,x^5+3\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^5+3),x)
[Out] 1/3*ln(x)-1/15*ln(b*x^5+3)
```

maxima [A] time = 0.95, size = 17, normalized size = 0.89

$$-\frac{1}{15}\log\left(bx^5+3\right)+\frac{1}{15}\log\left(x^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^5+3),x, algorithm="maxima")
[Out] -1/15*log(b*x^5 + 3) + 1/15*log(x^5)
```

mupad [B] time = 0.06, size = 16, normalized size = 0.84

$$\frac{\ln(x)}{3}-\frac{\ln\left(\frac{2\,b\,x^5}{5}+\frac{6}{5}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^5 + 3)),x)
[Out] log(x)/3 - log((2*b*x^5)/5 + 6/5)/15
```

sympy [A] time = 0.49, size = 14, normalized size = 0.74

$$\frac{\log(x)}{3}-\frac{\log\left(x^5+\frac{3}{b}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**5+3),x)
[Out] log(x)/3 - log(x**5 + 3/b)/15
```

3.1294 $\int \frac{1}{x^6\left(3+bx^5\right)}\,dx$

Optimal. Leaf size=28

$$\frac{1}{45}b\log\left(bx^5+3\right)-\frac{1}{9}b\log(x)-\frac{1}{15x^5}$$

```
[Out] -1/15/x^5-1/9*b*ln(x)+1/45*b*ln(b*x^5+3)
```

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 44}

$$\frac{1}{45}b\log\left(bx^5+3\right)-\frac{1}{9}b\log(x)-\frac{1}{15x^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(3 + b*x^5)),x]
```

```
[Out] -1/(15*x^5) - (b*Log[x])/9 + (b*Log[3 + b*x^5])/45
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^6\left(3+bx^5\right)}\,dx &= \frac{1}{5}\,\text{Subst}\left(\int \frac{1}{x^2(3+bx)}\,dx, x, x^5\right)\\ &= \frac{1}{5}\,\text{Subst}\left(\int \left(\frac{1}{3x^2}-\frac{b}{9x}+\frac{b^2}{9(3+bx)}\right)\,dx, x, x^5\right)\\ &= -\frac{1}{15x^5}-\frac{1}{9}b\log(x)+\frac{1}{45}b\log\left(3+bx^5\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$\frac{1}{45}b\log\left(bx^5+3\right)-\frac{1}{9}b\log(x)-\frac{1}{15x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(3 + b*x^5)),x]
```

```
[Out] -1/15*1/x^5 - (b*Log[x])/9 + (b*Log[3 + b*x^5])/45
```

fricas [A] time = 0.82, size = 28, normalized size = 1.00

$$\frac{bx^5\log\left(bx^5+3\right)-5\,bx^5\log(x)-3}{45\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^5+3),x, algorithm="fricas")
[Out] 1/45*(b*x^5*log(b*x^5 + 3) - 5*b*x^5*log(x) - 3)/x^5
```

giac [A] time = 0.52, size = 31, normalized size = 1.11

$$\frac{1}{45}\,b\log\left(\left|bx^5+3\right|\right)-\frac{1}{9}\,b\log\left(|x|\right)+\frac{bx^5-3}{45\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^5+3),x, algorithm="giac")
[Out] 1/45*b*log(abs(b*x^5 + 3)) - 1/9*b*log(abs(x)) + 1/45*(b*x^5 - 3)/x^5
```

maple [A] time = 0.01, size = 23, normalized size = 0.82

$$-\frac{b\ln(x)}{9}+\frac{b\ln\left(b\,x^5+3\right)}{45}-\frac{1}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^5+3),x)
[Out] -1/15/x^5-1/9*b*ln(x)+1/45*b*ln(b*x^5+3)
```

maxima [A] time = 1.02, size = 24, normalized size = 0.86

$$\frac{1}{45}\,b\log\left(bx^5+3\right)-\frac{1}{45}\,b\log\left(x^5\right)-\frac{1}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^5+3),x, algorithm="maxima")
[Out] 1/45*b*log(b*x^5 + 3) - 1/45*b*log(x^5) - 1/15/x^5
```

mupad [B] time = 0.06, size = 22, normalized size = 0.79

$$\frac{b\,\ln\left(b\,x^5+3\right)}{45}-\frac{b\,\ln(x)}{9}-\frac{1}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(b*x^5 + 3)),x)
[Out] (b*log(b*x^5 + 3))/45 - (b*log(x))/9 - 1/(15*x^5)
```

sympy [A] time = 0.88, size = 24, normalized size = 0.86

$$-\frac{b\log(x)}{9}+\frac{b\log\left(x^5+\frac{3}{b}\right)}{45}-\frac{1}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**5+3),x)
[Out] -b*log(x)/9 + b*log(x**5 + 3/b)/45 - 1/(15*x**5)
```

3.1295 $\int \frac{x^{14}}{1+x^5}\,dx$

Optimal. Leaf size=25

$$\frac{x^{10}}{10}-\frac{x^5}{5}+\frac{1}{5}\log\left(x^5+1\right)$$

```
[Out] -1/5*x^5+1/10*x^10+1/5*ln(x^5+1)
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {266, 43}

$$\frac{x^{10}}{10}-\frac{x^5}{5}+\frac{1}{5}\log\left(x^5+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^14/(1 + x^5),x]
```

```
[Out] -x^5/5 + x^10/10 + Log[1 + x^5]/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{14}}{1+x^5}\,dx &= \frac{1}{5}\,\text{Subst}\left(\int \frac{x^2}{1+x}\,dx, x, x^5\right)\\ &= \frac{1}{5}\,\text{Subst}\left(\int \left(-1+x+\frac{1}{1+x}\right)\,dx, x, x^5\right)\\ &= -\frac{x^5}{5}+\frac{x^{10}}{10}+\frac{1}{5}\log\left(1+x^5\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.88

$$\frac{1}{10}\left(x^{10}-2x^5+2\log\left(x^5+1\right)-3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/(1 + x^5),x]
```

```
[Out] (-3 - 2*x^5 + x^10 + 2*Log[1 + x^5])/10
```

fricas [A] time = 0.81, size = 19, normalized size = 0.76

$$\frac{1}{10}x^{10}-\frac{1}{5}x^5+\frac{1}{5}\log\left(x^5+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(x^5+1),x, algorithm="fricas")
[Out] 1/10*x^10 - 1/5*x^5 + 1/5*log(x^5 + 1)
```

giac [A] time = 0.16, size = 20, normalized size = 0.80

$$\frac{1}{10}x^{10} - \frac{1}{5}x^5 + \frac{1}{5}\log\left(\left|x^5+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(x^5+1),x, algorithm="giac")
[Out] 1/10*x^10 - 1/5*x^5 + 1/5*log(abs(x^5 + 1))
```

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$\frac{x^{10}}{10} - \frac{x^5}{5} + \frac{\ln\left(x^5+1\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(x^5+1),x)
[Out] -1/5*x^5+1/10*x^10+1/5*ln(x^5+1)
```

maxima [A] time = 1.08, size = 19, normalized size = 0.76

$$\frac{1}{10}x^{10} - \frac{1}{5}x^5 + \frac{1}{5}\log\left(x^5+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(x^5+1),x, algorithm="maxima")
[Out] 1/10*x^10 - 1/5*x^5 + 1/5*log(x^5 + 1)
```

mupad [B] time = 0.04, size = 19, normalized size = 0.76

$$\frac{\ln\left(x^5+1\right)}{5} - \frac{x^5}{5} + \frac{x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(x^5 + 1),x)
[Out] log(x^5 + 1)/5 - x^5/5 + x^10/10
```

sympy [A] time = 0.19, size = 17, normalized size = 0.68

$$\frac{x^{10}}{10} - \frac{x^5}{5} + \frac{\log\left(x^5+1\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(x**5+1),x)
[Out] x**10/10 - x**5/5 + log(x**5 + 1)/5
```

3.1296 $\int \frac{x^9}{1+x^5}\,dx$

Optimal. Leaf size=18

$$\frac{x^5}{5}-\frac{1}{5}\log\left(x^5+1\right)$$

```
[Out] 1/5*x^5-1/5*ln(x^5+1)
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {266, 43}

$$\frac{x^5}{5}-\frac{1}{5}\log\left(x^5+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(1 + x^5),x]
```

```
[Out] x^5/5 - Log[1 + x^5]/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{1+x^5}\,dx &= \frac{1}{5}\operatorname{Subst}\left(\int \frac{x}{1+x}\,dx, x, x^5\right)\\ &= \frac{1}{5}\operatorname{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)\,dx, x, x^5\right)\\ &= \frac{x^5}{5}-\frac{1}{5}\log\left(1+x^5\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{x^5}{5}-\frac{1}{5}\log\left(x^5+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(1 + x^5),x]
```

```
[Out] x^5/5 - Log[1 + x^5]/5
```

fricas [A] time = 0.86, size = 14, normalized size = 0.78

$$\frac{1}{5}x^5-\frac{1}{5}\log\left(x^5+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^5+1),x, algorithm="fricas")
[Out] 1/5*x^5 - 1/5*log(x^5 + 1)
```

giac [A] time = 0.16, size = 15, normalized size = 0.83

$$\frac{1}{5}x^5 - \frac{1}{5}\log\left(\left|x^5+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^5+1),x, algorithm="giac")
[Out] 1/5*x^5 - 1/5*log(abs(x^5 + 1))
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$\frac{x^5}{5} - \frac{\ln\left(x^5+1\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^5+1),x)
[Out] 1/5*x^5-1/5*ln(x^5+1)
```

maxima [A] time = 1.02, size = 14, normalized size = 0.78

$$\frac{1}{5}x^5 - \frac{1}{5}\log\left(x^5+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^5+1),x, algorithm="maxima")
[Out] 1/5*x^5 - 1/5*log(x^5 + 1)
```

mupad [B] time = 0.03, size = 14, normalized size = 0.78

$$\frac{x^5}{5} - \frac{\ln\left(x^5+1\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^5 + 1),x)
[Out] x^5/5 - log(x^5 + 1)/5
```

sympy [A] time = 0.21, size = 12, normalized size = 0.67

$$\frac{x^5}{5} - \frac{\log\left(x^5+1\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(x**5+1),x)
[Out] x**5/5 - log(x**5 + 1)/5
```

3.1297 $\int \frac{x^4}{1+x^5}\,dx$

Optimal. Leaf size=10

$$\frac{1}{5}\log\left(x^5+1\right)$$

```
[Out] 1/5*ln(x^5+1)
```

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {260}

$$\frac{1}{5}\log\left(x^5+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 + x^5),x]
```

```
[Out] Log[1 + x^5]/5
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^4}{1+x^5}\,dx = \frac{1}{5}\log\left(1+x^5\right)$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{5}\log\left(x^5+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 + x^5),x]
```

```
[Out] Log[1 + x^5]/5
```

fricas [A] time = 0.84, size = 8, normalized size = 0.80

$$\frac{1}{5}\log\left(x^5+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^5+1),x, algorithm="fricas")
```

```
[Out] 1/5*log(x^5 + 1)
```

giac [A] time = 0.21, size = 9, normalized size = 0.90

$$\frac{1}{5}\log\left(\left|x^5+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^5+1),x, algorithm="giac")
[Out] 1/5*log(abs(x^5 + 1))
```

maple [A] time = 0.00, size = 9, normalized size = 0.90

$$\frac{\ln\left(x^5+1\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^5+1),x)
[Out] 1/5*ln(x^5+1)
```

maxima [A] time = 1.03, size = 8, normalized size = 0.80

$$\frac{1}{5}\log\left(x^5+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^5+1),x, algorithm="maxima")
[Out] 1/5*log(x^5 + 1)
```

mupad [B] time = 0.03, size = 8, normalized size = 0.80

$$\frac{\ln\left(x^5+1\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^5 + 1),x)
[Out] log(x^5 + 1)/5
```

sympy [A] time = 0.17, size = 7, normalized size = 0.70

$$\frac{\log\left(x^5+1\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**5+1),x)
[Out] log(x**5 + 1)/5
```

3.1298 $\int \frac{1}{x\left(1+x^5\right)}\,dx$

Optimal. Leaf size=13

$$\log(x)-\frac{1}{5}\log\left(x^5+1\right)$$

```
[Out] ln(x)-1/5*ln(x^5+1)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {266, 36, 29, 31}

$$\log(x)-\frac{1}{5}\log\left(x^5+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + x^5)),x]
```

```
[Out] Log[x] - Log[1 + x^5]/5
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(1+x^5\right)}\,dx &= \frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{x(1+x)}\,dx,x,x^5\right)\\ &= \frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,x^5\right)-\frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx,x,x^5\right)\\ &= \log(x)-\frac{1}{5}\log\left(1+x^5\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\log(x)-\frac{1}{5}\log\left(x^5+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + x^5)),x]
[Out] Log[x] - Log[1 + x^5]/5
```

fricas [A] time = 0.88, size = 11, normalized size = 0.85

$$-\frac{1}{5}\log\left(x^5+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^5+1),x, algorithm="fricas")
[Out] -1/5*log(x^5 + 1) + log(x)
```

giac [A] time = 0.16, size = 13, normalized size = 1.00

$$-\frac{1}{5}\log\left(\left|x^5+1\right|\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^5+1),x, algorithm="giac")
[Out] -1/5*log(abs(x^5 + 1)) + log(abs(x))
```

maple [B] time = 0.01, size = 29, normalized size = 2.23

$$\ln(x)-\frac{\ln(x+1)}{5}-\frac{\ln\left(x^4-x^3+x^2-x+1\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^5+1),x)
[Out] -1/5*ln(x^4-x^3+x^2-x+1)-1/5*ln(x+1)+ln(x)
```

maxima [A] time = 1.08, size = 15, normalized size = 1.15

$$-\frac{1}{5}\log\left(x^5+1\right)+\frac{1}{5}\log\left(x^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^5+1),x, algorithm="maxima")
[Out] -1/5*log(x^5 + 1) + 1/5*log(x^5)
```

mupad [B] time = 1.05, size = 11, normalized size = 0.85

$$\ln(x)-\frac{\ln\left(x^5+1\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^5 + 1)),x)
[Out] log(x) - log(x^5 + 1)/5
```

sympy [A] time = 0.17, size = 10, normalized size = 0.77

$$\log(x)-\frac{\log\left(x^5+1\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**5+1),x)
[Out] log(x) - log(x**5 + 1)/5
```

3.1299 $\int \frac{1}{x^6\left(1+x^5\right)}\,dx$

Optimal. Leaf size=22

$$-\frac{1}{5x^5}+\frac{1}{5}\log\left(x^5+1\right)-\log(x)$$

```
[Out] -1/5/x^5-ln(x)+1/5*ln(x^5+1)
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {266, 44}

$$-\frac{1}{5x^5}+\frac{1}{5}\log\left(x^5+1\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(1 + x^5)),x]
```

```
[Out] -1/(5*x^5) - Log[x] + Log[1 + x^5]/5
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^6\left(1+x^5\right)}\,dx &= \frac{1}{5}\,\text{Subst}\left(\int \frac{1}{x^2(1+x)}\,dx, x, x^5\right)\\ &= \frac{1}{5}\,\text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{1}{x}+\frac{1}{1+x}\right)dx, x, x^5\right)\\ &= -\frac{1}{5x^5}-\log(x)+\frac{1}{5}\log\left(1+x^5\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$-\frac{1}{5x^5}+\frac{1}{5}\log\left(x^5+1\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(1 + x^5)),x]
```

```
[Out] -1/5*1/x^5 - Log[x] + Log[1 + x^5]/5
```

fricas [A] time = 0.77, size = 24, normalized size = 1.09

$$\frac{x^5\log\left(x^5+1\right)-5\,x^5\log(x)-1}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^5+1),x, algorithm="fricas")
[Out] 1/5*(x^5*log(x^5 + 1) - 5*x^5*log(x) - 1)/x^5
```

giac [A] time = 0.16, size = 25, normalized size = 1.14

$$\frac{x^5-1}{5\,x^5}+\frac{1}{5}\,\log\left(\left|x^5+1\right|\right)-\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^5+1),x, algorithm="giac")
[Out] 1/5*(x^5 - 1)/x^5 + 1/5*log(abs(x^5 + 1)) - log(abs(x))
```

maple [A] time = 0.01, size = 36, normalized size = 1.64

$$-\ln(x)+\frac{\ln(x+1)}{5}+\frac{\ln\left(x^4-x^3+x^2-x+1\right)}{5}-\frac{1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^5+1),x)
[Out] 1/5*ln(x^4-x^3+x^2-x+1)+1/5*ln(x+1)-1/5/x^5-ln(x)
```

maxima [A] time = 0.98, size = 20, normalized size = 0.91

$$-\frac{1}{5\,x^5}+\frac{1}{5}\,\log\left(x^5+1\right)-\frac{1}{5}\,\log\left(x^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^5+1),x, algorithm="maxima")
[Out] -1/5/x^5 + 1/5*log(x^5 + 1) - 1/5*log(x^5)
```

mupad [B] time = 0.05, size = 18, normalized size = 0.82

$$\frac{\ln\left(x^5+1\right)}{5}-\ln(x)-\frac{1}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(x^5 + 1)),x)
[Out] log(x^5 + 1)/5 - log(x) - 1/(5*x^5)
```

sympy [A] time = 0.21, size = 17, normalized size = 0.77

$$-\log(x)+\frac{\log\left(x^5+1\right)}{5}-\frac{1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**5+1),x)
[Out] -log(x) + log(x**5 + 1)/5 - 1/(5*x**5)
```

3.1300 $\int \frac{x^5}{1+x^5}\,dx$

Optimal. Leaf size=186

$$\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1-\sqrt{5}\right)x+1\right)+\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1+\sqrt{5}\right)x+1\right)+x-\frac{1}{5}\log(x+1)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\right.}$$

```
[Out] x-1/5*ln(1+x)+1/20*ln(1+x^2-1/2*x*(-5^(1/2)+1))*(-5^(1/2)+1)+1/20*ln(1+x^2-
1/2*x*(5^(1/2)+1))*(5^(1/2)+1)-1/10*arctan(1/5*x*(50+10*5^(1/2))^(1/2)-1/5*
(25+10*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)-1/10*arctan(1/5*(25-10*5^(1/2))
^(1/2)+2*x*2^(1/2)/(5+5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {321, 201, 634, 618, 204, 628, 31}

$$\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1-\sqrt{5}\right)x+1\right)+\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1+\sqrt{5}\right)x+1\right)+x-\frac{1}{5}\log(x+1)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(1 + x^5),x]
```

```
[Out] x - (Sqrt[(5 + Sqrt[5])/2]*ArcTan[Sqrt[(5 - 2*Sqrt[5])/5] + 2*Sqrt[2/(5 + S
qrt[5])]*x])/5 + (Sqrt[(5 - Sqrt[5])/2]*ArcTan[Sqrt[(5 + 2*Sqrt[5])/5] - Sq
rt[(2*(5 + Sqrt[5]))/5]*x])/5 - Log[1 + x]/5 + ((1 - Sqrt[5])*Log[1 - ((1 -
 Sqrt[5])*x)/2 + x^2])/20 + ((1 + Sqrt[5])*Log[1 - ((1 + Sqrt[5])*x)/2 + x^
2])/20
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[((2*k -
 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]; (r*Int[1
/(r + s*x), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 1)/2}], x], x]
] /; FreeQ[{a, b}, x] && IGtQ[(n - 3)/2, 0] && PosQ[a/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^5}{1+x^5}\,dx = x - \int \frac{1}{1+x^5}\,dx$$

$$= x - \frac{2}{5}\int \frac{1-\frac{1}{4}\left(1-\sqrt{5}\right)x}{1-\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx - \frac{2}{5}\int \frac{1-\frac{1}{4}\left(1+\sqrt{5}\right)x}{1-\frac{1}{2}\left(1+\sqrt{5}\right)x+x^2}\,dx - \frac{1}{5}\int \frac{1}{1+x}\,dx$$

$$= x - \frac{1}{5}\log(1+x) - \frac{1}{20}\left(-1-\sqrt{5}\right)\int \frac{\frac{1}{2}\left(-1-\sqrt{5}\right)+2x}{1+\frac{1}{2}\left(-1-\sqrt{5}\right)x+x^2}\,dx - \frac{1}{20}\left(5-\sqrt{5}\right)\int \frac{1}{1+\frac{1}{2}\left(-1-\right.}$$

$$= x - \frac{1}{5}\log(1+x) + \frac{1}{20}\left(1+\sqrt{5}\right)\log\left(2-x-\sqrt{5}\,x+2x^2\right) + \frac{1}{20}\left(1-\sqrt{5}\right)\log\left(2-x+\sqrt{5}\,x+2\right.$$

$$= x + \frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\,\tan^{-1}\left(\frac{1-\sqrt{5}-4x}{\sqrt{2\left(5+\sqrt{5}\right)}}\right) + \frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\,\tan^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(1+\sqrt{\right.}\right.$$

Mathematica [A] time = 0.14, size = 147, normalized size = 0.79

$$\frac{1}{20}\left(-\left(\sqrt{5}-1\right)\log\left(x^2+\frac{1}{2}\left(\sqrt{5}-1\right)x+1\right)+\left(1+\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1+\sqrt{5}\right)x+1\right)+20x-4\log(x+1)+2\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(1 + x^5),x]
```

```
[Out] (20*x + 2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[(1 + Sqrt[5] - 4*x)/Sqrt[10 - 2*Sqrt[
5]]] - 2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(-1 + Sqrt[5] + 4*x)/Sqrt[2*(5 + Sqrt
[5])]] - 4*Log[1 + x] - (-1 + Sqrt[5])*Log[1 + ((-1 + Sqrt[5])*x)/2 + x^2]
+ (1 + Sqrt[5])*Log[1 - ((1 + Sqrt[5])*x)/2 + x^2])/20
```

fricas [B] time = 2.58, size = 502, normalized size = 2.70

$$\frac{1}{20}\left(\sqrt{5}-2\sqrt{-\frac{3}{16}\left(\sqrt{5}+10\sqrt{-\frac{1}{50}\sqrt{5}-\frac{1}{10}}-1\right)^2-\frac{1}{8}\left(\sqrt{5}+10\sqrt{-\frac{1}{50}\sqrt{5}-\frac{1}{10}}+3\right)\left(\sqrt{5}-10\sqrt{-\frac{1}{50}\sqrt{5}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^5+1),x, algorithm="fricas")
```

```
[Out] 1/20*(sqrt(5) - 2*sqrt(-3/16*(sqrt(5) + 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)^
2 - 1/8*(sqrt(5) + 10*sqrt(-1/50*sqrt(5) - 1/10) + 3)*(sqrt(5) - 10*sqrt(-1
/50*sqrt(5) - 1/10) - 1) - 3/16*(sqrt(5) - 10*sqrt(-1/50*sqrt(5) - 1/10) -
1)^2 - 1/2*sqrt(5) - 5*sqrt(-1/50*sqrt(5) - 1/10) - 5/2) + 1)*log(2*x - 1/2
*sqrt(5) + sqrt(-3/16*(sqrt(5) + 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)^2 - 1/8
*(sqrt(5) + 10*sqrt(-1/50*sqrt(5) - 1/10) + 3)*(sqrt(5) - 10*sqrt(-1/50*sqr
t(5) - 1/10) - 1) - 3/16*(sqrt(5) - 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)^2 -
1/2*sqrt(5) - 5*sqrt(-1/50*sqrt(5) - 1/10) - 5/2) - 1/2) + 1/20*(sqrt(5) +
2*sqrt(-3/16*(sqrt(5) + 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)^2 - 1/8*(sqrt(5)
 + 10*sqrt(-1/50*sqrt(5) - 1/10) + 3)*(sqrt(5) - 10*sqrt(-1/50*sqrt(5) - 1/
10) - 1) - 3/16*(sqrt(5) - 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)^2 - 1/2*sqrt(
5) - 5*sqrt(-1/50*sqrt(5) - 1/10) - 5/2) + 1)*log(2*x - 1/2*sqrt(5) - sqrt(
-3/16*(sqrt(5) + 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)^2 - 1/8*(sqrt(5) + 10*s
qrt(-1/50*sqrt(5) - 1/10) + 3)*(sqrt(5) - 10*sqrt(-1/50*sqrt(5) - 1/10) - 1
) - 3/16*(sqrt(5) - 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)^2 - 1/2*sqrt(5) - 5*
sqrt(-1/50*sqrt(5) - 1/10) - 5/2) - 1/2) - 1/20*(sqrt(5) + 10*sqrt(-1/50*sq
rt(5) - 1/10) - 1)*log(x + 1/4*sqrt(5) + 5/2*sqrt(-1/50*sqrt(5) - 1/10) - 1
/4) - 1/20*(sqrt(5) - 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)*log(x + 1/4*sqrt(5
) - 5/2*sqrt(-1/50*sqrt(5) - 1/10) - 1/4) + x - 1/5*log(x + 1)
```

giac [A] time = 0.18, size = 128, normalized size = 0.69

$$-\frac{1}{10}\sqrt{2\sqrt{5}+10}\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{2\sqrt{5}+10}}\right)-\frac{1}{10}\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{-2\sqrt{5}+10}}\right)+\frac{1}{20}\sqrt{5}\log\left(x^2-\frac{1}{2}x\left(\sqrt{5}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^5+1),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(2*sqrt(5) + 10)*arctan((4*x + sqrt(5) - 1)/sqrt(2*sqrt(5) + 10))
 - 1/10*sqrt(-2*sqrt(5) + 10)*arctan((4*x - sqrt(5) - 1)/sqrt(-2*sqrt(5) +
10)) + 1/20*sqrt(5)*log(x^2 - 1/2*x*(sqrt(5) + 1) + 1) - 1/20*sqrt(5)*log(x
^2 + 1/2*x*(sqrt(5) - 1) + 1) + x + 1/20*log(x^4 - x^3 + x^2 - x + 1) - 1/5
*log(abs(x + 1))
```

maple [A] time = 0.05, size = 219, normalized size = 1.18

$$x-\frac{\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{10-2\sqrt{5}}}\right)}{\sqrt{10-2\sqrt{5}}}+\frac{\sqrt{5}\ \arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{10-2\sqrt{5}}}\right)}{5\sqrt{10-2\sqrt{5}}}-\frac{\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{10+2\sqrt{5}}}\right)}{\sqrt{10+2\sqrt{5}}}-\frac{\sqrt{5}\ \arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{10+2\sqrt{5}}}\right)}{5\sqrt{10+2\sqrt{5}}}-\frac{\ln(x+1)}{5}+\frac{\sqrt{5}\ \ln($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^5+1),x)
```

```
[Out] x-1/20*ln(x*5^(1/2)+2*x^2-x+2)*5^(1/2)+1/20*ln(x*5^(1/2)+2*x^2-x+2)-1/(10+2
*5^(1/2))^(1/2)*arctan((5^(1/2)+4*x-1)/(10+2*5^(1/2))^(1/2))-1/5/(10+2*5^(1
/2))^(1/2)*arctan((5^(1/2)+4*x-1)/(10+2*5^(1/2))^(1/2))*5^(1/2)+1/20*ln(-x*
5^(1/2)+2*x^2-x+2)*5^(1/2)+1/20*ln(-x*5^(1/2)+2*x^2-x+2)-1/(10-2*5^(1/2))^(
1/2)*arctan((-5^(1/2)+4*x-1)/(10-2*5^(1/2))^(1/2))+1/5/(10-2*5^(1/2))^(1/2)
*arctan((-5^(1/2)+4*x-1)/(10-2*5^(1/2))^(1/2))*5^(1/2)-1/5*ln(x+1)
```

maxima [A] time = 2.40, size = 145, normalized size = 0.78

$$-\frac{\sqrt{5}\left(\sqrt{5}+1\right)\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{2\sqrt{5}+10}}\right)}{5\sqrt{2\sqrt{5}+10}}-\frac{\sqrt{5}\left(\sqrt{5}-1\right)\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{-2\sqrt{5}+10}}\right)}{5\sqrt{-2\sqrt{5}+10}}+x+\frac{\left(\sqrt{5}+3\right)\log\left(2x^2-x\left(\sqrt{5}+1\right)+2\right)}{10\left(\sqrt{5}+1\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^5+1),x, algorithm="maxima")
```

```
[Out] -1/5*sqrt(5)*(sqrt(5) + 1)*arctan((4*x + sqrt(5) - 1)/sqrt(2*sqrt(5) + 10))
/sqrt(2*sqrt(5) + 10) - 1/5*sqrt(5)*(sqrt(5) - 1)*arctan((4*x - sqrt(5) - 1
)/sqrt(-2*sqrt(5) + 10))/sqrt(-2*sqrt(5) + 10) + x + 1/10*(sqrt(5) + 3)*log
(2*x^2 - x*(sqrt(5) + 1) + 2)/(sqrt(5) + 1) + 1/10*(sqrt(5) - 3)*log(2*x^2
+ x*(sqrt(5) - 1) + 2)/(sqrt(5) - 1) - 1/5*log(x + 1)
```

mupad **[B]** time = 0.29, size = 181, normalized size = 0.97

$$x-\frac{\ln(x+1)}{5}+\ln\left(x-\frac{\sqrt{5}}{4}-\frac{\sqrt{2}\,\sqrt{\sqrt{5}-5}}{4}-\frac{1}{4}\right)\left(\frac{\sqrt{5}}{20}+\frac{\sqrt{2}\,\sqrt{\sqrt{5}-5}}{20}+\frac{1}{20}\right)+\ln\left(x-\frac{\sqrt{5}}{4}+\frac{\sqrt{2}\,\sqrt{\sqrt{5}-5}}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^5 + 1),x)
```

```
[Out] x - log(x + 1)/5 + log(x - 5^(1/2)/4 - (2^(1/2)*(5^(1/2) - 5)^(1/2))/4 - 1/
4)*(5^(1/2)/20 + (2^(1/2)*(5^(1/2) - 5)^(1/2))/20 + 1/20) + log(x - 5^(1/2)
/4 + (2^(1/2)*(5^(1/2) - 5)^(1/2))/4 - 1/4)*(5^(1/2)/20 - (2^(1/2)*(5^(1/2)
 - 5)^(1/2))/20 + 1/20) + log(x - (2^(1/2)*(- 5^(1/2) - 5)^(1/2))/4 + 5^(1/
2)/4 - 1/4)*((2^(1/2)*(- 5^(1/2) - 5)^(1/2))/20 - 5^(1/2)/20 + 1/20) - log(
x + (2^(1/2)*(- 5^(1/2) - 5)^(1/2))/4 + 5^(1/2)/4 - 1/4)*((2^(1/2)*(- 5^(1/
2) - 5)^(1/2))/20 + 5^(1/2)/20 - 1/20)
```

sympy **[A]** time = 1.07, size = 36, normalized size = 0.19

$$x-\frac{\log(x+1)}{5}+\operatorname{RootSum}\left(625t^4-125t^3+25t^2-5t+1,\left(t\mapsto t\log(-5t+x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**5+1),x)
```

```
[Out] x - log(x + 1)/5 + RootSum(625*_t**4 - 125*_t**3 + 25*_t**2 - 5*_t + 1, Lam
bda(_t, _t*log(-5*_t + x)))
```

3.1301 $\int \frac{x^3}{1+x^5}\,dx$

Optimal. Leaf size=185

$$\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1-\sqrt{5}\right)x+1\right)+\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1+\sqrt{5}\right)x+1\right)-\frac{1}{5}\log(x+1)+\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right.}$$

```
[Out] -1/5*ln(1+x)+1/20*ln(1+x^2-1/2*x*(-5^(1/2)+1))*(-5^(1/2)+1)+1/20*ln(1+x^2-1
/2*x*(5^(1/2)+1))*(5^(1/2)+1)+1/10*arctan(1/5*x*(50+10*5^(1/2))^(1/2)-1/5*(
25+10*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)+1/10*arctan(1/5*(25-10*5^(1/2))^
(1/2)+2*x*2^(1/2)/(5+5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {293, 634, 618, 204, 628, 31}

$$\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1-\sqrt{5}\right)x+1\right)+\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1+\sqrt{5}\right)x+1\right)-\frac{1}{5}\log(x+1)+\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 + x^5),x]
```

```
[Out] (Sqrt[(5 + Sqrt[5])/2]*ArcTan[Sqrt[(5 - 2*Sqrt[5])/5] + 2*Sqrt[2/(5 + Sqrt[
5])]*x])/5 - (Sqrt[(5 - Sqrt[5])/2]*ArcTan[Sqrt[(5 + 2*Sqrt[5])/5] - Sqrt[(
2*(5 + Sqrt[5]))/5]*x])/5 - Log[1 + x]/5 + ((1 - Sqrt[5])*Log[1 - ((1 - Sqr
t[5])*x)/2 + x^2])/20 + ((1 + Sqrt[5])*Log[1 - ((1 + Sqrt[5])*x)/2 + x^2])/
20
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 293

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x]; -(((-r)^(m + 1)*Int[1/(r + s*x), x])/(a*n*s^m
)) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 1)/2}], x], x]] /; Fr
eeQ[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ
[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^3}{1+x^5}\,dx = \frac{2}{5}\int \frac{\frac{1}{4}\left(1+\sqrt{5}\right)-\frac{1}{4}\left(-1+\sqrt{5}\right)x}{1-\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx + \frac{2}{5}\int \frac{\frac{1}{4}\left(1-\sqrt{5}\right)-\frac{1}{4}\left(-1-\sqrt{5}\right)x}{1-\frac{1}{2}\left(1+\sqrt{5}\right)x+x^2}\,dx - \frac{1}{5}\int \frac{1}{1+x}\,dx$$

$$= -\frac{1}{5}\log(1+x) + \frac{1}{20}\left(1-\sqrt{5}\right)\int \frac{\frac{1}{2}\left(-1+\sqrt{5}\right)+2x}{1+\frac{1}{2}\left(-1+\sqrt{5}\right)x+x^2}\,dx + \frac{1}{20}\left(5-\sqrt{5}\right)\int \frac{1}{1+\frac{1}{2}\left(-1-\sqrt{5}\right.}$$

$$= -\frac{1}{5}\log(1+x) + \frac{1}{20}\left(1+\sqrt{5}\right)\log\left(2-x-\sqrt{5}\,x+2x^2\right) + \frac{1}{20}\left(1-\sqrt{5}\right)\log\left(2-x+\sqrt{5}\,x+2x^2\right.$$

$$= -\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\,\tan^{-1}\left(\frac{1-\sqrt{5}-4x}{\sqrt{2\left(5+\sqrt{5}\right)}}\right) - \frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\,\tan^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(1+\sqrt{5}\right.\right.$$

Mathematica [A] time = 0.10, size = 144, normalized size = 0.78

$$\frac{1}{20}\left(-\left(\sqrt{5}-1\right)\log\left(x^2+\frac{1}{2}\left(\sqrt{5}-1\right)x+1\right)+\left(1+\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1+\sqrt{5}\right)x+1\right)-4\log(x+1)-2\sqrt{10-\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 + x^5),x]
```

```
[Out] (-2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[(1 + Sqrt[5] - 4*x)/Sqrt[10 - 2*Sqrt[5]]] +
 2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(-1 + Sqrt[5] + 4*x)/Sqrt[2*(5 + Sqrt[5])]]
 - 4*Log[1 + x] - (-1 + Sqrt[5])*Log[1 + ((-1 + Sqrt[5])*x)/2 + x^2] + (1 +
 Sqrt[5])*Log[1 - ((1 + Sqrt[5])*x)/2 + x^2])/20
```

fricas [B] time = 3.08, size = 905, normalized size = 4.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^5+1),x, algorithm="fricas")
```

```
[Out] -1/20*(sqrt(5) - 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)*log(1/64*(sqrt(5) + 10*
sqrt(-1/50*sqrt(5) - 1/10) - 1)^3 + 1/64*(sqrt(5) + 10*sqrt(-1/50*sqrt(5) -
 1/10) - 1)*(sqrt(5) - 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)^2 + 1/16*(sqrt(5)
 + 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)^2 + 1/64*((sqrt(5) + 10*sqrt(-1/50*sq
rt(5) - 1/10) - 1)^2 + 4*sqrt(5) + 40*sqrt(-1/50*sqrt(5) - 1/10) - 4)*(sqrt
(5) - 10*sqrt(-1/50*sqrt(5) - 1/10) - 1) + x + 1/4*sqrt(5) + 5/2*sqrt(-1/50
*sqrt(5) - 1/10) - 1/4) - 1/20*(sqrt(5) + 10*sqrt(-1/50*sqrt(5) - 1/10) - 1
)*log(-1/64*(sqrt(5) + 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)^3 - 1/16*(sqrt(5)
```

```
 + 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)^2 + x - 1/4*sqrt(5) - 5/2*sqrt(-1/50*
sqrt(5) - 1/10) - 3/4) + 1/20*(sqrt(5) + 2*sqrt(-3/16*(sqrt(5) + 10*sqrt(-1
/50*sqrt(5) - 1/10) - 1)^2 - 1/8*(sqrt(5) + 10*sqrt(-1/50*sqrt(5) - 1/10) +
 3)*(sqrt(5) - 10*sqrt(-1/50*sqrt(5) - 1/10) - 1) - 3/16*(sqrt(5) - 10*sqrt
(-1/50*sqrt(5) - 1/10) - 1)^2 - 1/2*sqrt(5) - 5*sqrt(-1/50*sqrt(5) - 1/10)
- 5/2) + 1)*log(-1/64*(sqrt(5) + 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)*(sqrt(5
) - 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)^2 + 1/16*sqrt(-3/16*(sqrt(5) + 10*sq
rt(-1/50*sqrt(5) - 1/10) - 1)^2 - 1/8*(sqrt(5) + 10*sqrt(-1/50*sqrt(5) - 1/
10) + 3)*(sqrt(5) - 10*sqrt(-1/50*sqrt(5) - 1/10) - 1) - 3/16*(sqrt(5) - 10
*sqrt(-1/50*sqrt(5) - 1/10) - 1)^2 - 1/2*sqrt(5) - 5*sqrt(-1/50*sqrt(5) - 1
/10) - 5/2)*(sqrt(5) + 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)*(sqrt(5) - 10*sqr
t(-1/50*sqrt(5) - 1/10) - 1) - 1/64*((sqrt(5) + 10*sqrt(-1/50*sqrt(5) - 1/1
0) - 1)^2 + 4*sqrt(5) + 40*sqrt(-1/50*sqrt(5) - 1/10) - 4)*(sqrt(5) - 10*sq
rt(-1/50*sqrt(5) - 1/10) - 1) + 2*x) + 1/20*(sqrt(5) - 2*sqrt(-3/16*(sqrt(5
) + 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)^2 - 1/8*(sqrt(5) + 10*sqrt(-1/50*sqr
t(5) - 1/10) + 3)*(sqrt(5) - 10*sqrt(-1/50*sqrt(5) - 1/10) - 1) - 3/16*(sqr
t(5) - 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)^2 - 1/2*sqrt(5) - 5*sqrt(-1/50*sq
rt(5) - 1/10) - 5/2) + 1)*log(-1/64*(sqrt(5) + 10*sqrt(-1/50*sqrt(5) - 1/10
) - 1)*(sqrt(5) - 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)^2 - 1/16*sqrt(-3/16*(s
qrt(5) + 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)^2 - 1/8*(sqrt(5) + 10*sqrt(-1/5
0*sqrt(5) - 1/10) + 3)*(sqrt(5) - 10*sqrt(-1/50*sqrt(5) - 1/10) - 1) - 3/16
*(sqrt(5) - 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)^2 - 1/2*sqrt(5) - 5*sqrt(-1/
50*sqrt(5) - 1/10) - 5/2)*(sqrt(5) + 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)*(sq
rt(5) - 10*sqrt(-1/50*sqrt(5) - 1/10) - 1) - 1/64*((sqrt(5) + 10*sqrt(-1/50
*sqrt(5) - 1/10) - 1)^2 + 4*sqrt(5) + 40*sqrt(-1/50*sqrt(5) - 1/10) - 4)*(s
qrt(5) - 10*sqrt(-1/50*sqrt(5) - 1/10) - 1) + 2*x) - 1/5*log(x + 1)
```

giac [A] time = 0.18, size = 112, normalized size = 0.61

$$\frac{1}{20}\left(\sqrt{5}+1\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{5}+1\right)+1\right)-\frac{1}{20}\left(\sqrt{5}-1\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{5}-1\right)+1\right)+\frac{1}{10}\sqrt{2\sqrt{5}+10}\arctan\left(\frac{4x}{\sqrt{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^5+1),x, algorithm="giac")
```

```
[Out] 1/20*(sqrt(5) + 1)*log(x^2 - 1/2*x*(sqrt(5) + 1) + 1) - 1/20*(sqrt(5) - 1)*
log(x^2 + 1/2*x*(sqrt(5) - 1) + 1) + 1/10*sqrt(2*sqrt(5) + 10)*arctan((4*x
+ sqrt(5) - 1)/sqrt(2*sqrt(5) + 10)) + 1/10*sqrt(-2*sqrt(5) + 10)*arctan((4
*x - sqrt(5) - 1)/sqrt(-2*sqrt(5) + 10)) - 1/5*log(abs(x + 1))
```

maple [A] time = 0.02, size = 216, normalized size = 1.17

$$\frac{\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{10-2\sqrt{5}}}\right)}{\sqrt{10-2\sqrt{5}}}-\frac{\sqrt{5}\ \arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{10-2\sqrt{5}}}\right)}{5\sqrt{10-2\sqrt{5}}}+\frac{\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{10+2\sqrt{5}}}\right)}{\sqrt{10+2\sqrt{5}}}+\frac{\sqrt{5}\ \arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{10+2\sqrt{5}}}\right)}{5\sqrt{10+2\sqrt{5}}}-\frac{\ln(x+1)}{5}+\frac{\sqrt{5}\ \ln\left(2x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^5+1),x)
```

```
[Out] -1/20*5^(1/2)*ln(2*x^2+5^(1/2)*x-x+2)+1/20*ln(2*x^2+5^(1/2)*x-x+2)+1/(10+2*
5^(1/2))^(1/2)*arctan((4*x+5^(1/2)-1)/(10+2*5^(1/2))^(1/2))+1/5/(10+2*5^(1/
2))^(1/2)*5^(1/2)*arctan((4*x+5^(1/2)-1)/(10+2*5^(1/2))^(1/2))+1/20*5^(1/2)
*ln(2*x^2-5^(1/2)*x-x+2)+1/20*ln(2*x^2-5^(1/2)*x-x+2)+1/(10-2*5^(1/2))^(1/2
)*arctan((4*x-5^(1/2)-1)/(10-2*5^(1/2))^(1/2))-1/5/(10-2*5^(1/2))^(1/2)*5^(
1/2)*arctan((4*x-5^(1/2)-1)/(10-2*5^(1/2))^(1/2))-1/5*ln(x+1)
```

maxima [A] time = 2.28, size = 144, normalized size = 0.78

$$\frac{\sqrt{5}\left(\sqrt{5}+1\right)\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{2\sqrt{5}+10}}\right)}{5\sqrt{2\sqrt{5}+10}}+\frac{\sqrt{5}\left(\sqrt{5}-1\right)\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{-2\sqrt{5}+10}}\right)}{5\sqrt{-2\sqrt{5}+10}}+\frac{\left(\sqrt{5}+3\right)\log\left(2x^2-x\left(\sqrt{5}+1\right)+2\right)}{10\left(\sqrt{5}+1\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^5+1),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(5)*(sqrt(5) + 1)*arctan((4*x + sqrt(5) - 1)/sqrt(2*sqrt(5) + 10))/
sqrt(2*sqrt(5) + 10) + 1/5*sqrt(5)*(sqrt(5) - 1)*arctan((4*x - sqrt(5) - 1)
/sqrt(-2*sqrt(5) + 10))/sqrt(-2*sqrt(5) + 10) + 1/10*(sqrt(5) + 3)*log(2*x^
2 - x*(sqrt(5) + 1) + 2)/(sqrt(5) + 1) + 1/10*(sqrt(5) - 3)*log(2*x^2 + x*(
sqrt(5) - 1) + 2)/(sqrt(5) - 1) - 1/5*log(x + 1)
```

mupad [B] time = 1.27, size = 196, normalized size = 1.06

$$\ln\left(25x\left(\frac{\sqrt{2}\sqrt{-\sqrt{5}-5}}{20}-\frac{\sqrt{5}}{20}+\frac{1}{20}\right)-5\right)\left(\frac{\sqrt{2}\sqrt{-\sqrt{5}-5}}{20}-\frac{\sqrt{5}}{20}+\frac{1}{20}\right)-\frac{\ln(x+1)}{5}-\ln\left(25x\left(\frac{\sqrt{2}\sqrt{-\sqrt{5}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^5 + 1),x)
```

```
[Out] log(25*x*((2^(1/2)*(- 5^(1/2) - 5)^(1/2))/20 - 5^(1/2)/20 + 1/20) - 5)*((2^
(1/2)*(- 5^(1/2) - 5)^(1/2))/20 - 5^(1/2)/20 + 1/20) - log(x + 1)/5 - log(2
5*x*((2^(1/2)*(- 5^(1/2) - 5)^(1/2))/20 + 5^(1/2)/20 - 1/20) + 5)*((2^(1/2)
*(- 5^(1/2) - 5)^(1/2))/20 + 5^(1/2)/20 - 1/20) + log(25*x*(5^(1/2)/20 - (2
^(1/2)*(5^(1/2) - 5)^(1/2))/20 + 1/20) - 5)*(5^(1/2)/20 - (2^(1/2)*(5^(1/2)
 - 5)^(1/2))/20 + 1/20) + log(25*x*(5^(1/2)/20 + (2^(1/2)*(5^(1/2) - 5)^(1/
2))/20 + 1/20) - 5)*(5^(1/2)/20 + (2^(1/2)*(5^(1/2) - 5)^(1/2))/20 + 1/20)
```

sympy [A] time = 1.04, size = 36, normalized size = 0.19

$$-\frac{\log(x+1)}{5}+\operatorname{RootSum}\left(625t^4-125t^3+25t^2-5t+1,\left(t\mapsto t\log\left(625t^4+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**5+1),x)
```

```
[Out] -log(x + 1)/5 + RootSum(625*_t**4 - 125*_t**3 + 25*_t**2 - 5*_t + 1, Lambda
(_t, _t*log(625*_t**4 + x)))
```

3.1302 $\int \frac{x^2}{1+x^5}\,dx$

Optimal. Leaf size=185

$$-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1-\sqrt{5}\right)x+1\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1+\sqrt{5}\right)x+1\right)+\frac{1}{5}\log(x+1)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{}\right.}$$

```
[Out] 1/5*ln(1+x)-1/20*ln(1+x^2-1/2*x*(5^(1/2)+1))*(-5^(1/2)+1)-1/20*ln(1+x^2-1/2
*x*(-5^(1/2)+1))*(5^(1/2)+1)-1/10*arctan(1/5*(25-10*5^(1/2))^(1/2)+2*x*2^(1
/2)/(5+5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)+1/10*arctan(1/5*x*(50+10*5^(1/2
))^(1/2)-1/5*(25+10*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {293, 634, 618, 204, 628, 31}

$$-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1-\sqrt{5}\right)x+1\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1+\sqrt{5}\right)x+1\right)+\frac{1}{5}\log(x+1)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + x^5),x]
```

```
[Out] -(Sqrt[(5 - Sqrt[5])/2]*ArcTan[Sqrt[(5 - 2*Sqrt[5])/5] + 2*Sqrt[2/(5 + Sqrt
[5])]*x])/5 - (Sqrt[(5 + Sqrt[5])/2]*ArcTan[Sqrt[(5 + 2*Sqrt[5])/5] - Sqrt[
(2*(5 + Sqrt[5]))/5]*x])/5 + Log[1 + x]/5 - ((1 + Sqrt[5])*Log[1 - ((1 - Sq
rt[5])*x)/2 + x^2])/20 - ((1 - Sqrt[5])*Log[1 - ((1 + Sqrt[5])*x)/2 + x^2])
/20
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 293

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x]; -(((-r)^(m + 1)*Int[1/(r + s*x), x])/(a*n*s^m
)) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 1)/2}], x], x]] /; Fr
eeQ[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ
[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^2}{1+x^5}\,dx = \frac{2}{5}\int \frac{\frac{1}{4}\left(-1-\sqrt{5}\right)-\frac{1}{4}\left(1+\sqrt{5}\right)x}{1-\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx + \frac{2}{5}\int \frac{\frac{1}{4}\left(-1+\sqrt{5}\right)-\frac{1}{4}\left(1-\sqrt{5}\right)x}{1-\frac{1}{2}\left(1+\sqrt{5}\right)x+x^2}\,dx + \frac{1}{5}\int \frac{1}{1+x}\,dx$$

$$= \frac{1}{5}\log(1+x) + \frac{\int \frac{1}{1+\frac{1}{2}\left(-1-\sqrt{5}\right)x+x^2}\,dx}{2\sqrt{5}} - \frac{\int \frac{1}{1+\frac{1}{2}\left(-1+\sqrt{5}\right)x+x^2}\,dx}{2\sqrt{5}} + \frac{1}{20}\left(-1-\sqrt{5}\right)\int \frac{\frac{1}{2}\left(-1+\sqrt{5}\right.}{1+\frac{1}{2}\left(-1+\sqrt{5}\right.}$$

$$= \frac{1}{5}\log(1+x) - \frac{1}{20}\left(1-\sqrt{5}\right)\log\left(2-x-\sqrt{5}\,x+2x^2\right) - \frac{1}{20}\left(1+\sqrt{5}\right)\log\left(2-x+\sqrt{5}\,x+2x^2\right)$$

$$= \sqrt{\frac{2}{5\left(5+\sqrt{5}\right)}}\,\tan^{-1}\left(\frac{1-\sqrt{5}-4x}{\sqrt{2\left(5+\sqrt{5}\right)}}\right) - \frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\,\tan^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(1+\sqrt{5}-4\right.\right.$$

Mathematica [A] time = 0.10, size = 144, normalized size = 0.78

$$\frac{1}{20}\left(-\left(1+\sqrt{5}\right)\log\left(x^2+\frac{1}{2}\left(\sqrt{5}-1\right)x+1\right)+\left(\sqrt{5}-1\right)\log\left(x^2-\frac{1}{2}\left(1+\sqrt{5}\right)x+1\right)+4\log(x+1)-2\sqrt{2\left(5\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/(1 + x^5),x]
```

```
[Out] (-2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(1 + Sqrt[5] - 4*x)/Sqrt[10 - 2*Sqrt[5]]]
- 2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[(-1 + Sqrt[5] + 4*x)/Sqrt[2*(5 + Sqrt[5])]]
 + 4*Log[1 + x] - (1 + Sqrt[5])*Log[1 + ((-1 + Sqrt[5])*x)/2 + x^2] + (-1 +
 Sqrt[5])*Log[1 - ((1 + Sqrt[5])*x)/2 + x^2])/20
```

fricas [B] time = 2.88, size = 637, normalized size = 3.44

$$-\frac{1}{20}\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)\log\left(\frac{1}{16}\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^2+x\right)+\frac{1}{20}\left(\sqrt{5}+2\sqrt{-\frac{3}{16}\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{\right.}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^5+1),x, algorithm="fricas")
```

```
[Out] -1/20*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)*log(1/16*(2*sqrt(1/2)*s
qrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + x) + 1/20*(sqrt(5) + 2*sqrt(-3/16*(2*sq
rt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 1/8*(2*sqrt(1/2)*sqrt(sqrt(5)
- 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 3/16*(2
```

```
*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + sqrt(1/2)*sqrt(sqrt(5) - 5)
 + 1/2*sqrt(5) - 5/2) - 1)*log(-1/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(
5) + 1)^2 - 1/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 1/2*sqrt
(-3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 1/8*(2*sqrt(1/2)*s
qrt(sqrt(5) - 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) -
1) - 3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + sqrt(1/2)*sqrt(
sqrt(5) - 5) + 1/2*sqrt(5) - 5/2)*(sqrt(5) - 1) + 2*x - 1) + 1/20*(sqrt(5)
- 2*sqrt(-3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 1/8*(2*sqr
t(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sq
rt(5) - 1) - 3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + sqrt(1/
2)*sqrt(sqrt(5) - 5) + 1/2*sqrt(5) - 5/2) - 1)*log(-1/16*(2*sqrt(1/2)*sqrt(
sqrt(5) - 5) + sqrt(5) + 1)^2 - 1/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(
5) - 1)^2 - 1/2*sqrt(-3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2
+ 1/8*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt(
5) - 5) - sqrt(5) - 1) - 3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)
^2 + sqrt(1/2)*sqrt(sqrt(5) - 5) + 1/2*sqrt(5) - 5/2)*(sqrt(5) - 1) + 2*x -
 1) + 1/20*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)*log(1/16*(2*sqrt(1
/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + x) + 1/5*log(x + 1)
```

giac [A] time = 0.18, size = 112, normalized size = 0.61

$$\frac{1}{20}\left(\sqrt{5}-1\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{5}+1\right)+1\right)-\frac{1}{20}\left(\sqrt{5}+1\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{5}-1\right)+1\right)-\frac{1}{10}\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{4}{\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^5+1),x, algorithm="giac")
```

```
[Out] 1/20*(sqrt(5) - 1)*log(x^2 - 1/2*x*(sqrt(5) + 1) + 1) - 1/20*(sqrt(5) + 1)*
log(x^2 + 1/2*x*(sqrt(5) - 1) + 1) - 1/10*sqrt(-2*sqrt(5) + 10)*arctan((4*x
 + sqrt(5) - 1)/sqrt(2*sqrt(5) + 10)) + 1/10*sqrt(2*sqrt(5) + 10)*arctan((4
*x - sqrt(5) - 1)/sqrt(-2*sqrt(5) + 10)) + 1/5*log(abs(x + 1))
```

maple [A] time = 0.02, size = 156, normalized size = 0.84

$$\frac{2\sqrt{5}\ \arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{10-2\sqrt{5}}}\right)}{5\sqrt{10-2\sqrt{5}}}-\frac{2\sqrt{5}\ \arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{10+2\sqrt{5}}}\right)}{5\sqrt{10+2\sqrt{5}}}+\frac{\ln(x+1)}{5}+\frac{\sqrt{5}\ \ln\left(2x^2-\sqrt{5}\ x-x+2\right)}{20}-\frac{\ln\left(2x^2-\sqrt{5}\ x-x\right.}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^5+1),x)
```

```
[Out] -1/20*5^(1/2)*ln(2*x^2+5^(1/2)*x-x+2)-1/20*ln(2*x^2+5^(1/2)*x-x+2)-2/5/(10+
2*5^(1/2))^(1/2)*5^(1/2)*arctan((4*x+5^(1/2)-1)/(10+2*5^(1/2))^(1/2))+1/20*
5^(1/2)*ln(2*x^2-5^(1/2)*x-x+2)-1/20*ln(2*x^2-5^(1/2)*x-x+2)+2/5/(10-2*5^(1
/2))^(1/2)*5^(1/2)*arctan((4*x-5^(1/2)-1)/(10-2*5^(1/2))^(1/2))+1/5*ln(x+1)
```

maxima [A] time = 2.28, size = 124, normalized size = 0.67

$$-\frac{2\sqrt{5}\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{2\sqrt{5}+10}}\right)}{5\sqrt{2\sqrt{5}+10}}+\frac{2\sqrt{5}\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{-2\sqrt{5}+10}}\right)}{5\sqrt{-2\sqrt{5}+10}}+\frac{\log\left(2x^2-x\left(\sqrt{5}+1\right)+2\right)}{5\left(\sqrt{5}+1\right)}-\frac{\log\left(2x^2+x\left(\sqrt{5}-1\right)+2\right)}{5\left(\sqrt{5}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^5+1),x, algorithm="maxima")
```

```
[Out] -2/5*sqrt(5)*arctan((4*x + sqrt(5) - 1)/sqrt(2*sqrt(5) + 10))/sqrt(2*sqrt(5
) + 10) + 2/5*sqrt(5)*arctan((4*x - sqrt(5) - 1)/sqrt(-2*sqrt(5) + 10))/sqr
t(-2*sqrt(5) + 10) + 1/5*log(2*x^2 - x*(sqrt(5) + 1) + 2)/(sqrt(5) + 1) - 1
/5*log(2*x^2 + x*(sqrt(5) - 1) + 2)/(sqrt(5) - 1) + 1/5*log(x + 1)
```

mupad **[B]** time = 1.91, size = 197, normalized size = 1.06

$$\frac{\ln(x+1)}{5}-\ln\left(1-\frac{x\left(\sqrt{2}\,\sqrt{-\sqrt{5}-5}-\sqrt{5}+1\right)^3}{64}\right)\left(\frac{\sqrt{2}\,\sqrt{-\sqrt{5}-5}}{20}-\frac{\sqrt{5}}{20}+\frac{1}{20}\right)+\ln\left(\frac{x\left(\sqrt{2}\,\sqrt{-\sqrt{5}-5}+\sqrt{}\right.\right.}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^5 + 1),x)
```

```
[Out] log(x + 1)/5 - log(1 - (x*(2^(1/2)*(- 5^(1/2) - 5)^(1/2) - 5^(1/2) + 1)^3)/
64)*((2^(1/2)*(- 5^(1/2) - 5)^(1/2))/20 - 5^(1/2)/20 + 1/20) + log((x*(2^(1
/2)*(- 5^(1/2) - 5)^(1/2) + 5^(1/2) - 1)^3)/64 + 1)*((2^(1/2)*(- 5^(1/2) -
5)^(1/2))/20 + 5^(1/2)/20 - 1/20) - log(1 - (x*(5^(1/2) + 2^(1/2)*(5^(1/2)
- 5)^(1/2) + 1)^3)/64)*(5^(1/2)/20 + (2^(1/2)*(5^(1/2) - 5)^(1/2))/20 + 1/2
0) - log(1 - (x*(5^(1/2) - 2^(1/2)*(5^(1/2) - 5)^(1/2) + 1)^3)/64)*(5^(1/2)
/20 - (2^(1/2)*(5^(1/2) - 5)^(1/2))/20 + 1/20)
```

sympy **[A]** time = 2.25, size = 36, normalized size = 0.19

$$\frac{\log(x+1)}{5}+\operatorname{RootSum}\left(625t^4+125t^3+25t^2+5t+1,\left(t\mapsto t\log\left(25t^2+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**5+1),x)
```

```
[Out] log(x + 1)/5 + RootSum(625*_t**4 + 125*_t**3 + 25*_t**2 + 5*_t + 1, Lambda(
_t, _t*log(25*_t**2 + x)))
```

3.1303 $\int \frac{x}{1+x^5}\,dx$

Optimal. Leaf size=185

$$\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1-\sqrt{5}\right)x+1\right)+\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1+\sqrt{5}\right)x+1\right)-\frac{1}{5}\log(x+1)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right.}$$

```
[Out] -1/5*ln(1+x)+1/20*ln(1+x^2-1/2*x*(5^(1/2)+1))*(-5^(1/2)+1)+1/20*ln(1+x^2-1/
2*x*(-5^(1/2)+1))*(5^(1/2)+1)-1/10*arctan(1/5*(25-10*5^(1/2))^(1/2)+2*x*2^(
1/2)/(5+5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)+1/10*arctan(1/5*x*(50+10*5^(1/
2))^(1/2)-1/5*(25+10*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {293, 634, 618, 204, 628, 31}

$$\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1-\sqrt{5}\right)x+1\right)+\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1+\sqrt{5}\right)x+1\right)-\frac{1}{5}\log(x+1)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + x^5),x]
```

```
[Out] -(Sqrt[(5 - Sqrt[5])/2]*ArcTan[Sqrt[(5 - 2*Sqrt[5])/5] + 2*Sqrt[2/(5 + Sqrt
[5])]*x])/5 - (Sqrt[(5 + Sqrt[5])/2]*ArcTan[Sqrt[(5 + 2*Sqrt[5])/5] - Sqrt[
(2*(5 + Sqrt[5]))/5]*x])/5 - Log[1 + x]/5 + ((1 + Sqrt[5])*Log[1 - ((1 - Sq
rt[5])*x)/2 + x^2])/20 + ((1 - Sqrt[5])*Log[1 - ((1 + Sqrt[5])*x)/2 + x^2])
/20
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 293

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x]; -(((-r)^(m + 1)*Int[1/(r + s*x), x])/(a*n*s^m
)) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 1)/2}], x], x]] /; Fr
eeQ[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ
[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x}{1+x^5}\,dx = \frac{2}{5}\int \frac{\frac{1}{4}\left(1-\sqrt{5}\right)-\frac{1}{4}\left(-1-\sqrt{5}\right)x}{1-\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx + \frac{2}{5}\int \frac{\frac{1}{4}\left(1+\sqrt{5}\right)-\frac{1}{4}\left(-1+\sqrt{5}\right)x}{1-\frac{1}{2}\left(1+\sqrt{5}\right)x+x^2}\,dx - \frac{1}{5}\int \frac{1}{1+x}\,dx$$

$$= -\frac{1}{5}\log(1+x) + \frac{\int \frac{1}{1+\frac{1}{2}\left(-1-\sqrt{5}\right)x+x^2}\,dx}{2\sqrt{5}} - \frac{\int \frac{1}{1+\frac{1}{2}\left(-1+\sqrt{5}\right)x+x^2}\,dx}{2\sqrt{5}} + \frac{1}{20}\left(1-\sqrt{5}\right)\int \frac{\frac{1}{2}\left(-1-\sqrt{5}\right)}{1+\frac{1}{2}\left(-1-\sqrt{5}\right.}$$

$$= -\frac{1}{5}\log(1+x) + \frac{1}{20}\left(1-\sqrt{5}\right)\log\left(2-x-\sqrt{5}\,x+2x^2\right) + \frac{1}{20}\left(1+\sqrt{5}\right)\log\left(2-x+\sqrt{5}\,x+2x^2\right.$$

$$= \sqrt{\frac{2}{5\left(5+\sqrt{5}\right)}}\,\tan^{-1}\left(\frac{1-\sqrt{5}-4x}{\sqrt{2\left(5+\sqrt{5}\right)}}\right) - \frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\,\tan^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(1+\sqrt{5}-4x\right.\right.$$

Mathematica [A] time = 0.07, size = 144, normalized size = 0.78

$$\frac{1}{20}\left(\left(1+\sqrt{5}\right)\log\left(x^2+\frac{1}{2}\left(\sqrt{5}-1\right)x+1\right)-\left(\sqrt{5}-1\right)\log\left(x^2-\frac{1}{2}\left(1+\sqrt{5}\right)x+1\right)-4\log(x+1)-2\sqrt{2\left(5+\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(1 + x^5),x]
```

```
[Out] (-2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(1 + Sqrt[5] - 4*x)/Sqrt[10 - 2*Sqrt[5]]]
- 2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[(-1 + Sqrt[5] + 4*x)/Sqrt[2*(5 + Sqrt[5])]]
 - 4*Log[1 + x] + (1 + Sqrt[5])*Log[1 + ((-1 + Sqrt[5])*x)/2 + x^2] - (-1 +
 Sqrt[5])*Log[1 - ((1 + Sqrt[5])*x)/2 + x^2])/20
```

fricas [B] time = 2.70, size = 980, normalized size = 5.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^5+1),x, algorithm="fricas")
```

```
[Out] -1/20*(sqrt(5) + 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)*log(1/64*(sqrt(5) + 10*
sqrt(-1/50*sqrt(5) - 1/10) - 1)^3 + x) - 1/20*(sqrt(5) - 10*sqrt(-1/50*sqrt
(5) - 1/10) - 1)*log(-1/64*(sqrt(5) + 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)^3
- 1/64*(sqrt(5) + 10*sqrt(-1/50*sqrt(5) - 1/10) + 3)*(sqrt(5) - 10*sqrt(-1/
50*sqrt(5) - 1/10) - 1)^2 - 1/16*(sqrt(5) + 10*sqrt(-1/50*sqrt(5) - 1/10) -
 1)^2 - 1/64*((sqrt(5) + 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)^2 + 4*sqrt(5) +
 40*sqrt(-1/50*sqrt(5) - 1/10) + 12)*(sqrt(5) - 10*sqrt(-1/50*sqrt(5) - 1/1
0) - 1) + x - 1/4*sqrt(5) - 5/2*sqrt(-1/50*sqrt(5) - 1/10) - 3/4) + 1/20*(s
qrt(5) - 2*sqrt(-3/16*(sqrt(5) + 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)^2 - 1/8
```

```
*(sqrt(5) + 10*sqrt(-1/50*sqrt(5) - 1/10) + 3)*(sqrt(5) - 10*sqrt(-1/50*sqr
t(5) - 1/10) - 1) - 3/16*(sqrt(5) - 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)^2 -
1/2*sqrt(5) - 5*sqrt(-1/50*sqrt(5) - 1/10) - 5/2) + 1)*log(1/64*(sqrt(5) +
10*sqrt(-1/50*sqrt(5) - 1/10) + 3)*(sqrt(5) - 10*sqrt(-1/50*sqrt(5) - 1/10)
 - 1)^2 + 1/16*(sqrt(5) + 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)^2 + 1/64*((sqr
t(5) + 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)^2 + 4*sqrt(5) + 40*sqrt(-1/50*sqr
t(5) - 1/10) + 12)*(sqrt(5) - 10*sqrt(-1/50*sqrt(5) - 1/10) - 1) + 1/16*sqr
t(-3/16*(sqrt(5) + 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)^2 - 1/8*(sqrt(5) + 10
*sqrt(-1/50*sqrt(5) - 1/10) + 3)*(sqrt(5) - 10*sqrt(-1/50*sqrt(5) - 1/10) -
 1) - 3/16*(sqrt(5) - 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)^2 - 1/2*sqrt(5) -
5*sqrt(-1/50*sqrt(5) - 1/10) - 5/2)*((sqrt(5) + 10*sqrt(-1/50*sqrt(5) - 1/1
0) + 3)*(sqrt(5) - 10*sqrt(-1/50*sqrt(5) - 1/10) - 1) + 4*sqrt(5) + 40*sqrt
(-1/50*sqrt(5) - 1/10) - 4) + 2*x + 1/4*sqrt(5) + 5/2*sqrt(-1/50*sqrt(5) -
1/10) - 1/4) + 1/20*(sqrt(5) + 2*sqrt(-3/16*(sqrt(5) + 10*sqrt(-1/50*sqrt(5
) - 1/10) - 1)^2 - 1/8*(sqrt(5) + 10*sqrt(-1/50*sqrt(5) - 1/10) + 3)*(sqrt(
5) - 10*sqrt(-1/50*sqrt(5) - 1/10) - 1) - 3/16*(sqrt(5) - 10*sqrt(-1/50*sqr
t(5) - 1/10) - 1)^2 - 1/2*sqrt(5) - 5*sqrt(-1/50*sqrt(5) - 1/10) - 5/2) + 1
)*log(1/64*(sqrt(5) + 10*sqrt(-1/50*sqrt(5) - 1/10) + 3)*(sqrt(5) - 10*sqrt
(-1/50*sqrt(5) - 1/10) - 1)^2 + 1/16*(sqrt(5) + 10*sqrt(-1/50*sqrt(5) - 1/1
0) - 1)^2 + 1/64*((sqrt(5) + 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)^2 + 4*sqrt(
5) + 40*sqrt(-1/50*sqrt(5) - 1/10) + 12)*(sqrt(5) - 10*sqrt(-1/50*sqrt(5) -
 1/10) - 1) - 1/16*sqrt(-3/16*(sqrt(5) + 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)
^2 - 1/8*(sqrt(5) + 10*sqrt(-1/50*sqrt(5) - 1/10) + 3)*(sqrt(5) - 10*sqrt(-
1/50*sqrt(5) - 1/10) - 1) - 3/16*(sqrt(5) - 10*sqrt(-1/50*sqrt(5) - 1/10) -
 1)^2 - 1/2*sqrt(5) - 5*sqrt(-1/50*sqrt(5) - 1/10) - 5/2)*((sqrt(5) + 10*sq
rt(-1/50*sqrt(5) - 1/10) + 3)*(sqrt(5) - 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)
 + 4*sqrt(5) + 40*sqrt(-1/50*sqrt(5) - 1/10) - 4) + 2*x + 1/4*sqrt(5) + 5/2
*sqrt(-1/50*sqrt(5) - 1/10) - 1/4) - 1/5*log(x + 1)
```

giac [A] time = 0.18, size = 112, normalized size = 0.61

$$-\frac{1}{20}\left(\sqrt{5}-1\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{5}+1\right)+1\right)+\frac{1}{20}\left(\sqrt{5}+1\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{5}-1\right)+1\right)-\frac{1}{10}\sqrt{-2\sqrt{5}+10}\arctan\left($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^5+1),x, algorithm="giac")
```

```
[Out] -1/20*(sqrt(5) - 1)*log(x^2 - 1/2*x*(sqrt(5) + 1) + 1) + 1/20*(sqrt(5) + 1)
*log(x^2 + 1/2*x*(sqrt(5) - 1) + 1) - 1/10*sqrt(-2*sqrt(5) + 10)*arctan((4*
x + sqrt(5) - 1)/sqrt(2*sqrt(5) + 10)) + 1/10*sqrt(2*sqrt(5) + 10)*arctan((
4*x - sqrt(5) - 1)/sqrt(-2*sqrt(5) + 10)) - 1/5*log(abs(x + 1))
```

maple [A] time = 0.02, size = 156, normalized size = 0.84

$$\frac{2\sqrt{5}\ \arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{10-2\sqrt{5}}}\right)}{5\sqrt{10-2\sqrt{5}}}-\frac{2\sqrt{5}\ \arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{10+2\sqrt{5}}}\right)}{5\sqrt{10+2\sqrt{5}}}-\frac{\ln(x+1)}{5}-\frac{\sqrt{5}\ \ln\left(2x^2-\sqrt{5}\ x-x+2\right)}{20}+\frac{\ln\left(2x^2-\sqrt{5}\ x-x\right.}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^5+1),x)
```

```
[Out] 1/20*5^(1/2)*ln(2*x^2+5^(1/2)*x-x+2)+1/20*ln(2*x^2+5^(1/2)*x-x+2)-2/5/(10+2
*5^(1/2))^(1/2)*5^(1/2)*arctan((4*x+5^(1/2)-1)/(10+2*5^(1/2))^(1/2))-1/20*5
^(1/2)*ln(2*x^2-5^(1/2)*x-x+2)+1/20*ln(2*x^2-5^(1/2)*x-x+2)+2/5/(10-2*5^(1/
2))^(1/2)*5^(1/2)*arctan((4*x-5^(1/2)-1)/(10-2*5^(1/2))^(1/2))-1/5*ln(x+1)
```

maxima [A] time = 2.32, size = 124, normalized size = 0.67

$$-\frac{2\sqrt{5}\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{2\sqrt{5}+10}}\right)}{5\sqrt{2\sqrt{5}+10}}+\frac{2\sqrt{5}\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{-2\sqrt{5}+10}}\right)}{5\sqrt{-2\sqrt{5}+10}}-\frac{\log\left(2x^2-x\left(\sqrt{5}+1\right)+2\right)}{5\left(\sqrt{5}+1\right)}+\frac{\log\left(2x^2+x\left(\sqrt{5}-1\right)+\right.}{5\left(\sqrt{5}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^5+1),x, algorithm="maxima")
```

```
[Out] -2/5*sqrt(5)*arctan((4*x + sqrt(5) - 1)/sqrt(2*sqrt(5) + 10))/sqrt(2*sqrt(5
) + 10) + 2/5*sqrt(5)*arctan((4*x - sqrt(5) - 1)/sqrt(-2*sqrt(5) + 10))/sqr
t(-2*sqrt(5) + 10) - 1/5*log(2*x^2 - x*(sqrt(5) + 1) + 2)/(sqrt(5) + 1) + 1
/5*log(2*x^2 + x*(sqrt(5) - 1) + 2)/(sqrt(5) - 1) - 1/5*log(x + 1)
```

mupad [B] time = 0.26, size = 200, normalized size = 1.08

$$\ln\left(x-125\left(\frac{\sqrt{2}\,\sqrt{-\sqrt{5}-5}}{20}-\frac{\sqrt{5}}{20}+\frac{1}{20}\right)^3\right)\left(\frac{\sqrt{2}\,\sqrt{-\sqrt{5}-5}}{20}-\frac{\sqrt{5}}{20}+\frac{1}{20}\right)-\frac{\ln(x+1)}{5}-\ln\left(x+125\left(\frac{\sqrt{2}\,\sqrt{-}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^5 + 1),x)
```

```
[Out] log(x - 125*((2^(1/2)*(- 5^(1/2) - 5)^(1/2))/20 - 5^(1/2)/20 + 1/20)^3)*((2
^(1/2)*(- 5^(1/2) - 5)^(1/2))/20 - 5^(1/2)/20 + 1/20) - log(x + 1)/5 - log(
x + 125*((2^(1/2)*(- 5^(1/2) - 5)^(1/2))/20 + 5^(1/2)/20 - 1/20)^3)*((2^(1/
2)*(- 5^(1/2) - 5)^(1/2))/20 + 5^(1/2)/20 - 1/20) + log(x - 125*(5^(1/2)/20
 - (2^(1/2)*(5^(1/2) - 5)^(1/2))/20 + 1/20)^3)*(5^(1/2)/20 - (2^(1/2)*(5^(1
/2) - 5)^(1/2))/20 + 1/20) + log(x - 125*(5^(1/2)/20 + (2^(1/2)*(5^(1/2) -
5)^(1/2))/20 + 1/20)^3)*(5^(1/2)/20 + (2^(1/2)*(5^(1/2) - 5)^(1/2))/20 + 1/
20)
```

sympy [A] time = 2.13, size = 36, normalized size = 0.19

$$-\frac{\log(x+1)}{5}+\operatorname{RootSum}\left(625t^4-125t^3+25t^2-5t+1,\left(t\mapsto t\log\left(-125t^3+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**5+1),x)
```

```
[Out] -log(x + 1)/5 + RootSum(625*_t**4 - 125*_t**3 + 25*_t**2 - 5*_t + 1, Lambda
(_t, _t*log(-125*_t**3 + x)))
```

3.1304 $\int \frac{1}{1+x^5}\,dx$

Optimal. Leaf size=168

$$-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1-\sqrt{5}\right)x+1\right)-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1+\sqrt{5}\right)x+1\right)+\frac{1}{5}\log(x+1)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{}\right.}$$

```
[Out] 1/5*ln(1+x)-1/20*ln(1+x^2-1/2*x*(-5^(1/2)+1))*(-5^(1/2)+1)-1/20*ln(1+x^2-1/
2*x*(5^(1/2)+1))*(5^(1/2)+1)-1/10*arctan(1/20*(1-4*x+5^(1/2))*(50+10*5^(1/2
))^(1/2))*(10-2*5^(1/2))^(1/2)-1/10*arctan((1-4*x-5^(1/2))/(10+2*5^(1/2))^(
1/2))*(10+2*5^(1/2))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 185, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {201, 634, 618, 204, 628, 31}

$$-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1-\sqrt{5}\right)x+1\right)-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1+\sqrt{5}\right)x+1\right)+\frac{1}{5}\log(x+1)+\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(1 + x^5)^(-1),x]
```

```
[Out] (Sqrt[(5 + Sqrt[5])/2]*ArcTan[Sqrt[(5 - 2*Sqrt[5])/5] + 2*Sqrt[2/(5 + Sqrt[
5])]*x])/5 - (Sqrt[(5 - Sqrt[5])/2]*ArcTan[Sqrt[(5 + 2*Sqrt[5])/5] - Sqrt[(
2*(5 + Sqrt[5]))/5]*x])/5 + Log[1 + x]/5 - ((1 - Sqrt[5])*Log[1 - ((1 - Sqr
t[5])*x)/2 + x^2])/20 - ((1 + Sqrt[5])*Log[1 - ((1 + Sqrt[5])*x)/2 + x^2])/
20
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[((2*k -
 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]; (r*Int[1
/(r + s*x), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 1)/2}], x], x]
] /; FreeQ[{a, b}, x] && IGtQ[(n - 3)/2, 0] && PosQ[a/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{1+x^5}\,dx = \frac{2}{5}\int \frac{1-\frac{1}{4}\left(1-\sqrt{5}\right)x}{1-\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx + \frac{2}{5}\int \frac{1-\frac{1}{4}\left(1+\sqrt{5}\right)x}{1-\frac{1}{2}\left(1+\sqrt{5}\right)x+x^2}\,dx + \frac{1}{5}\int \frac{1}{1+x}\,dx$$

$$= \frac{1}{5}\log(1+x) + \frac{1}{20}\left(-1-\sqrt{5}\right)\int \frac{\frac{1}{2}\left(-1-\sqrt{5}\right)+2x}{1+\frac{1}{2}\left(-1-\sqrt{5}\right)x+x^2}\,dx + \frac{1}{20}\left(5-\sqrt{5}\right)\int \frac{1}{1+\frac{1}{2}\left(-1-\sqrt{5}\right.}$$

$$= \frac{1}{5}\log(1+x) - \frac{1}{20}\left(1+\sqrt{5}\right)\log\left(2-x-\sqrt{5}\,x+2x^2\right) - \frac{1}{20}\left(1-\sqrt{5}\right)\log\left(2-x+\sqrt{5}\,x+2x^2\right)$$

$$= -\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\,\tan^{-1}\left(\frac{1-\sqrt{5}-4x}{\sqrt{2\left(5+\sqrt{5}\right)}}\right) - \frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\,\tan^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(1+\sqrt{5}\right.\right.$$

Mathematica [A] time = 0.07, size = 144, normalized size = 0.86

$$\frac{1}{20}\left(\left(\sqrt{5}-1\right)\log\left(x^2+\frac{1}{2}\left(\sqrt{5}-1\right)x+1\right) - \left(1+\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1+\sqrt{5}\right)x+1\right) + 4\log(x+1) - 2\sqrt{10-2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^5)^(-1),x]
```

```
[Out] (-2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[(1 + Sqrt[5] - 4*x)/Sqrt[10 - 2*Sqrt[5]]] +
 2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(-1 + Sqrt[5] + 4*x)/Sqrt[2*(5 + Sqrt[5])]]
 + 4*Log[1 + x] + (-1 + Sqrt[5])*Log[1 + ((-1 + Sqrt[5])*x)/2 + x^2] - (1 +
 Sqrt[5])*Log[1 - ((1 + Sqrt[5])*x)/2 + x^2])/20
```

fricas [B] time = 2.88, size = 539, normalized size = 3.21

$$\frac{1}{20}\left(\sqrt{5}+2\sqrt{-\frac{3}{16}\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^2+\frac{1}{8}\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}+\sqrt{5}-3\right)\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^5+1),x, algorithm="fricas")
```

```
[Out] 1/20*(sqrt(5) + 2*sqrt(-3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^
2 + 1/8*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqr
t(5) - 5) - sqrt(5) - 1) - 3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) -
1)^2 + sqrt(1/2)*sqrt(sqrt(5) - 5) + 1/2*sqrt(5) - 5/2) - 1)*log(2*x + 1/2*
sqrt(5) + sqrt(-3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 1/8*
(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5
) - sqrt(5) - 1) - 3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + s
```

```
qrt(1/2)*sqrt(sqrt(5) - 5) + 1/2*sqrt(5) - 5/2) - 1/2) + 1/20*(sqrt(5) - 2*
sqrt(-3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 1/8*(2*sqrt(1/
2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5
) - 1) - 3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + sqrt(1/2)*s
qrt(sqrt(5) - 5) + 1/2*sqrt(5) - 5/2) - 1)*log(2*x + 1/2*sqrt(5) - sqrt(-3/
16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 1/8*(2*sqrt(1/2)*sqrt(
sqrt(5) - 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) -
 3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + sqrt(1/2)*sqrt(sqrt
(5) - 5) + 1/2*sqrt(5) - 5/2) - 1/2) + 1/20*(2*sqrt(1/2)*sqrt(sqrt(5) - 5)
- sqrt(5) - 1)*log(x + 1/2*sqrt(1/2)*sqrt(sqrt(5) - 5) - 1/4*sqrt(5) - 1/4)
 - 1/20*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)*log(x - 1/2*sqrt(1/2)
*sqrt(sqrt(5) - 5) - 1/4*sqrt(5) - 1/4) + 1/5*log(x + 1)
```

giac [A] time = 0.16, size = 112, normalized size = 0.67

$$-\frac{1}{20}\left(\sqrt{5}+1\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{5}+1\right)+1\right)+\frac{1}{20}\left(\sqrt{5}-1\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{5}-1\right)+1\right)+\frac{1}{10}\sqrt{2\sqrt{5}+10}\arctan\left($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^5+1),x, algorithm="giac")
```

```
[Out] -1/20*(sqrt(5) + 1)*log(x^2 - 1/2*x*(sqrt(5) + 1) + 1) + 1/20*(sqrt(5) - 1)
*log(x^2 + 1/2*x*(sqrt(5) - 1) + 1) + 1/10*sqrt(2*sqrt(5) + 10)*arctan((4*x
 + sqrt(5) - 1)/sqrt(2*sqrt(5) + 10)) + 1/10*sqrt(-2*sqrt(5) + 10)*arctan((
4*x - sqrt(5) - 1)/sqrt(-2*sqrt(5) + 10)) + 1/5*log(abs(x + 1))
```

maple [A] time = 0.02, size = 216, normalized size = 1.29

$$\frac{\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{10-2\sqrt{5}}}\right)}{\sqrt{10-2\sqrt{5}}}-\frac{\sqrt{5}\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{10-2\sqrt{5}}}\right)}{5\sqrt{10-2\sqrt{5}}}+\frac{\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{10+2\sqrt{5}}}\right)}{\sqrt{10+2\sqrt{5}}}+\frac{\sqrt{5}\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{10+2\sqrt{5}}}\right)}{5\sqrt{10+2\sqrt{5}}}+\frac{\ln(x+1)}{5}-\frac{\sqrt{5}\ln\left(2x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5+1),x)
```

```
[Out] 1/20*5^(1/2)*ln(2*x^2+5^(1/2)*x-x+2)-1/20*ln(2*x^2+5^(1/2)*x-x+2)+1/(10+2*5
^(1/2))^(1/2)*arctan((4*x+5^(1/2)-1)/(10+2*5^(1/2))^(1/2))+1/5/(10+2*5^(1/2
))^(1/2)*5^(1/2)*arctan((4*x+5^(1/2)-1)/(10+2*5^(1/2))^(1/2))-1/20*5^(1/2)*
ln(2*x^2-5^(1/2)*x-x+2)-1/20*ln(2*x^2-5^(1/2)*x-x+2)+1/(10-2*5^(1/2))^(1/2)
*arctan((4*x-5^(1/2)-1)/(10-2*5^(1/2))^(1/2))-1/5/(10-2*5^(1/2))^(1/2)*5^(1
/2)*arctan((4*x-5^(1/2)-1)/(10-2*5^(1/2))^(1/2))+1/5*ln(x+1)
```

maxima [A] time = 2.34, size = 144, normalized size = 0.86

$$\frac{\sqrt{5}\left(\sqrt{5}+1\right)\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{2\sqrt{5}+10}}\right)}{5\sqrt{2\sqrt{5}+10}}+\frac{\sqrt{5}\left(\sqrt{5}-1\right)\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{-2\sqrt{5}+10}}\right)}{5\sqrt{-2\sqrt{5}+10}}-\frac{\left(\sqrt{5}+3\right)\log\left(2x^2-x\left(\sqrt{5}+1\right)+2\right)}{10\left(\sqrt{5}+1\right)}-\frac{\left(\sqrt{5}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^5+1),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(5)*(sqrt(5) + 1)*arctan((4*x + sqrt(5) - 1)/sqrt(2*sqrt(5) + 10))/
sqrt(2*sqrt(5) + 10) + 1/5*sqrt(5)*(sqrt(5) - 1)*arctan((4*x - sqrt(5) - 1)
/sqrt(-2*sqrt(5) + 10))/sqrt(-2*sqrt(5) + 10) - 1/10*(sqrt(5) + 3)*log(2*x^
```

```
2 - x*(sqrt(5) + 1) + 2)/(sqrt(5) + 1) - 1/10*(sqrt(5) - 3)*log(2*x^2 + x*(
sqrt(5) - 1) + 2)/(sqrt(5) - 1) + 1/5*log(x + 1)
```

mupad [B] time = 1.17, size = 182, normalized size = 1.08

$$\frac{\ln(x+1)}{5}-\ln\left(x-\frac{\sqrt{5}}{4}-\frac{\sqrt{2}\,\sqrt{\sqrt{5}-5}}{4}-\frac{1}{4}\right)\left(\frac{\sqrt{5}}{20}+\frac{\sqrt{2}\,\sqrt{\sqrt{5}-5}}{20}+\frac{1}{20}\right)-\ln\left(x-\frac{\sqrt{5}}{4}+\frac{\sqrt{2}\,\sqrt{\sqrt{5}-5}}{4}-\frac{1}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5 + 1),x)
```

```
[Out] log(x + 1)/5 - log(x - 5^(1/2)/4 - (2^(1/2)*(5^(1/2) - 5)^(1/2))/4 - 1/4)*(
5^(1/2)/20 + (2^(1/2)*(5^(1/2) - 5)^(1/2))/20 + 1/20) - log(x - 5^(1/2)/4 +
 (2^(1/2)*(5^(1/2) - 5)^(1/2))/4 - 1/4)*(5^(1/2)/20 - (2^(1/2)*(5^(1/2) - 5
)^(1/2))/20 + 1/20) - log(x - (2^(1/2)*(- 5^(1/2) - 5)^(1/2))/4 + 5^(1/2)/4
 - 1/4)*((2^(1/2)*(- 5^(1/2) - 5)^(1/2))/20 - 5^(1/2)/20 + 1/20) + log(x +
(2^(1/2)*(- 5^(1/2) - 5)^(1/2))/4 + 5^(1/2)/4 - 1/4)*((2^(1/2)*(- 5^(1/2) -
 5)^(1/2))/20 + 5^(1/2)/20 - 1/20)
```

sympy [A] time = 1.93, size = 34, normalized size = 0.20

$$\frac{\log(x+1)}{5}+\operatorname{RootSum}\left(625t^4+125t^3+25t^2+5t+1,\left(t\mapsto t\log(5t+x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**5+1),x)
```

```
[Out] log(x + 1)/5 + RootSum(625*_t**4 + 125*_t**3 + 25*_t**2 + 5*_t + 1, Lambda(
_t, _t*log(5*_t + x)))
```

3.1305 $\int \frac{1}{x^2\left(1+x^5\right)}\,dx$

Optimal. Leaf size=190

$$-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1-\sqrt{5}\right)x+1\right)-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1+\sqrt{5}\right)x+1\right)-\frac{1}{x}+\frac{1}{5}\log(x+1)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\right.}$$

```
[Out] -1/x+1/5*ln(1+x)-1/20*ln(1+x^2-1/2*x*(-5^(1/2)+1))*(-5^(1/2)+1)-1/20*ln(1+x
^2-1/2*x*(5^(1/2)+1))*(5^(1/2)+1)-1/10*arctan(1/5*x*(50+10*5^(1/2))^(1/2)-1
/5*(25+10*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)-1/10*arctan(1/5*(25-10*5^(1/
2))^(1/2)+2*x*2^(1/2)/(5+5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {325, 293, 634, 618, 204, 628, 31}

$$-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1-\sqrt{5}\right)x+1\right)-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1+\sqrt{5}\right)x+1\right)-\frac{1}{x}+\frac{1}{5}\log(x+1)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 + x^5)),x]
```

```
[Out] -x^(-1) - (Sqrt[(5 + Sqrt[5])/2]*ArcTan[Sqrt[(5 - 2*Sqrt[5])/5] + 2*Sqrt[2/
(5 + Sqrt[5])]*x])/5 + (Sqrt[(5 - Sqrt[5])/2]*ArcTan[Sqrt[(5 + 2*Sqrt[5])/5
] - Sqrt[(2*(5 + Sqrt[5]))/5]*x])/5 + Log[1 + x]/5 - ((1 - Sqrt[5])*Log[1 -
 ((1 - Sqrt[5])*x)/2 + x^2])/20 - ((1 + Sqrt[5])*Log[1 - ((1 + Sqrt[5])*x)/
2 + x^2])/20
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 293

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x]; -(((-r)^(m + 1)*Int[1/(r + s*x), x])/(a*n*s^m
)) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 1)/2}], x], x]] /; Fr
eeQ[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ
[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^2\left(1+x^5\right)}\,dx = -\frac{1}{x} - \int \frac{x^3}{1+x^5}\,dx$$

$$= -\frac{1}{x} - \frac{2}{5}\int \frac{\frac{1}{4}\left(1+\sqrt{5}\right) - \frac{1}{4}\left(-1+\sqrt{5}\right)x}{1-\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx - \frac{2}{5}\int \frac{\frac{1}{4}\left(1-\sqrt{5}\right) - \frac{1}{4}\left(-1-\sqrt{5}\right)x}{1-\frac{1}{2}\left(1+\sqrt{5}\right)x+x^2}\,dx + \frac{1}{5}\int$$

$$= -\frac{1}{x} + \frac{1}{5}\log(1+x) - \frac{1}{20}\left(1-\sqrt{5}\right)\int \frac{\frac{1}{2}\left(-1+\sqrt{5}\right)+2x}{1+\frac{1}{2}\left(-1+\sqrt{5}\right)x+x^2}\,dx - \frac{1}{20}\left(5-\sqrt{5}\right)\int \frac{}{1+\frac{1}{2}\left($$

$$= -\frac{1}{x} + \frac{1}{5}\log(1+x) - \frac{1}{20}\left(1+\sqrt{5}\right)\log\left(2-x-\sqrt{5}\,x+2x^2\right) - \frac{1}{20}\left(1-\sqrt{5}\right)\log\left(2-x+\sqrt{\right.}$$

$$= -\frac{1}{x} + \frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\,\tan^{-1}\left(\frac{1-\sqrt{5}-4x}{\sqrt{2\left(5+\sqrt{5}\right)}}\right) + \frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\,\tan^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\right.$$

Mathematica [A] time = 0.10, size = 149, normalized size = 0.78

$$\frac{1}{20}\left(\left(\sqrt{5}-1\right)\log\left(x^2+\frac{1}{2}\left(\sqrt{5}-1\right)x+1\right) - \left(1+\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1+\sqrt{5}\right)x+1\right) - \frac{20}{x} + 4\log(x+1) + 2\sqrt{1}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 + x^5)),x]
```

```
[Out] (-20/x + 2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[(1 + Sqrt[5] - 4*x)/Sqrt[10 - 2*Sqrt
[5]]] - 2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(-1 + Sqrt[5] + 4*x)/Sqrt[2*(5 + Sqr
t[5])]] + 4*Log[1 + x] + (-1 + Sqrt[5])*Log[1 + ((-1 + Sqrt[5])*x)/2 + x^2]
 - (1 + Sqrt[5])*Log[1 - ((1 + Sqrt[5])*x)/2 + x^2])/20
```

fricas [B] time = 3.01, size = 1073, normalized size = 5.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^5+1),x, algorithm="fricas")
```

```
[Out] -1/40*(2*x*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)*log(1/64*(2*sqrt(1
/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^3 - 1/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5
) + sqrt(5) + 1)^2 + x + 1/2*sqrt(1/2)*sqrt(sqrt(5) - 5) + 1/4*sqrt(5) - 3/
4) - 2*x*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)*log(-1/64*(2*sqrt(1/
2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^3 - 1/64*(2*sqrt(1/2)*sqrt(sqrt(5) - 5)
 + sqrt(5) + 1)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 1/16*(2*s
qrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 1/64*((2*sqrt(1/2)*sqrt(sqrt(
5) - 5) + sqrt(5) + 1)^2 - 8*sqrt(1/2)*sqrt(sqrt(5) - 5) - 4*sqrt(5) - 4)*(
2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) + x - 1/2*sqrt(1/2)*sqrt(sqrt(
5) - 5) - 1/4*sqrt(5) - 1/4) - (x*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5)
+ 1) - x*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) + 4*sqrt(-3/16*(2*sq
rt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 1/8*(2*sqrt(1/2)*sqrt(sqrt(5)
 - 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 3/16*(2
*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + sqrt(1/2)*sqrt(sqrt(5) - 5)
 + 1/2*sqrt(5) - 5/2)*x - 4*x)*log(1/64*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sq
rt(5) + 1)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 1/16*sqrt(-3/1
6*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 1/8*(2*sqrt(1/2)*sqrt(s
qrt(5) - 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) -
3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + sqrt(1/2)*sqrt(sqrt(
5) - 5) + 1/2*sqrt(5) - 5/2)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)*
(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 1/64*((2*sqrt(1/2)*sqrt(sqr
t(5) - 5) + sqrt(5) + 1)^2 - 8*sqrt(1/2)*sqrt(sqrt(5) - 5) - 4*sqrt(5) - 4)
*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) + 2*x) - (x*(2*sqrt(1/2)*sqr
t(sqrt(5) - 5) + sqrt(5) + 1) - x*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5)
 - 1) - 4*sqrt(-3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 1/8*(
2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5)
 - sqrt(5) - 1) - 3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + sq
rt(1/2)*sqrt(sqrt(5) - 5) + 1/2*sqrt(5) - 5/2)*x - 4*x)*log(1/64*(2*sqrt(1/
2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5
) - 1)^2 - 1/16*sqrt(-3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2
+ 1/8*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt(
5) - 5) - sqrt(5) - 1) - 3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)
^2 + sqrt(1/2)*sqrt(sqrt(5) - 5) + 1/2*sqrt(5) - 5/2)*(2*sqrt(1/2)*sqrt(sqr
t(5) - 5) + sqrt(5) + 1)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 1/
64*((2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 - 8*sqrt(1/2)*sqrt(sqrt
(5) - 5) - 4*sqrt(5) - 4)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) + 2
*x) - 8*x*log(x + 1) + 40)/x
```

giac [A] time = 0.18, size = 132, normalized size = 0.69

$$-\frac{1}{10}\sqrt{2\sqrt{5}+10}\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{2\sqrt{5}+10}}\right)-\frac{1}{10}\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{-2\sqrt{5}+10}}\right)-\frac{1}{20}\sqrt{5}\log\left(x^2-\frac{1}{2}x\left(\sqrt{5}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^5+1),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(2*sqrt(5) + 10)*arctan((4*x + sqrt(5) - 1)/sqrt(2*sqrt(5) + 10))
 - 1/10*sqrt(-2*sqrt(5) + 10)*arctan((4*x - sqrt(5) - 1)/sqrt(-2*sqrt(5) +
10)) - 1/20*sqrt(5)*log(x^2 - 1/2*x*(sqrt(5) + 1) + 1) + 1/20*sqrt(5)*log(x
^2 + 1/2*x*(sqrt(5) - 1) + 1) - 1/x - 1/20*log(x^4 - x^3 + x^2 - x + 1) + 1
/5*log(abs(x + 1))
```

maple [A] time = 0.02, size = 223, normalized size = 1.17

$$-\frac{\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{10-2\sqrt{5}}}\right)}{\sqrt{10-2\sqrt{5}}}+\frac{\sqrt{5}\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{10-2\sqrt{5}}}\right)}{5\sqrt{10-2\sqrt{5}}}-\frac{\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{10+2\sqrt{5}}}\right)}{\sqrt{10+2\sqrt{5}}}-\frac{\sqrt{5}\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{10+2\sqrt{5}}}\right)}{5\sqrt{10+2\sqrt{5}}}+\frac{\ln(x+1)}{5}-\frac{\sqrt{5}\ln\left(2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^5+1),x)
```

```
[Out] -1/x+1/20*5^(1/2)*ln(2*x^2+5^(1/2)*x-x+2)-1/20*ln(2*x^2+5^(1/2)*x-x+2)-1/(1
0+2*5^(1/2))^(1/2)*arctan((4*x+5^(1/2)-1)/(10+2*5^(1/2))^(1/2))-1/5/(10+2*5
^(1/2))^(1/2)*5^(1/2)*arctan((4*x+5^(1/2)-1)/(10+2*5^(1/2))^(1/2))-1/20*5^(
1/2)*ln(2*x^2-5^(1/2)*x-x+2)-1/20*ln(2*x^2-5^(1/2)*x-x+2)-1/(10-2*5^(1/2))^
(1/2)*arctan((4*x-5^(1/2)-1)/(10-2*5^(1/2))^(1/2))+1/5/(10-2*5^(1/2))^(1/2)
*5^(1/2)*arctan((4*x-5^(1/2)-1)/(10-2*5^(1/2))^(1/2))+1/5*ln(x+1)
```

maxima [A] time = 2.41, size = 149, normalized size = 0.78

$$-\frac{\sqrt{5}\left(\sqrt{5}+1\right)\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{2\sqrt{5}+10}}\right)}{5\sqrt{2\sqrt{5}+10}}-\frac{\sqrt{5}\left(\sqrt{5}-1\right)\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{-2\sqrt{5}+10}}\right)}{5\sqrt{-2\sqrt{5}+10}}-\frac{\left(\sqrt{5}+3\right)\log\left(2x^2-x\left(\sqrt{5}+1\right)+2\right)}{10\left(\sqrt{5}+1\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^5+1),x, algorithm="maxima")
```

```
[Out] -1/5*sqrt(5)*(sqrt(5) + 1)*arctan((4*x + sqrt(5) - 1)/sqrt(2*sqrt(5) + 10))
/sqrt(2*sqrt(5) + 10) - 1/5*sqrt(5)*(sqrt(5) - 1)*arctan((4*x - sqrt(5) - 1
)/sqrt(-2*sqrt(5) + 10))/sqrt(-2*sqrt(5) + 10) - 1/10*(sqrt(5) + 3)*log(2*x
^2 - x*(sqrt(5) + 1) + 2)/(sqrt(5) + 1) - 1/10*(sqrt(5) - 3)*log(2*x^2 + x*
(sqrt(5) - 1) + 2)/(sqrt(5) - 1) - 1/x + 1/5*log(x + 1)
```

mupad [B] time = 1.25, size = 203, normalized size = 1.07

$$\frac{\ln(x+1)}{5}-\ln\left(25x\left(\frac{\sqrt{2}\sqrt{-\sqrt{5}-5}}{20}-\frac{\sqrt{5}}{20}+\frac{1}{20}\right)-5\right)\left(\frac{\sqrt{2}\sqrt{-\sqrt{5}-5}}{20}-\frac{\sqrt{5}}{20}+\frac{1}{20}\right)+\ln\left(25x\left(\frac{\sqrt{2}\sqrt{-\sqrt{5}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^5 + 1)),x)
```

```
[Out] log(x + 1)/5 - log(25*x*((2^(1/2)*(- 5^(1/2) - 5)^(1/2))/20 - 5^(1/2)/20 +
1/20) - 5)*((2^(1/2)*(- 5^(1/2) - 5)^(1/2))/20 - 5^(1/2)/20 + 1/20) + log(2
5*x*((2^(1/2)*(- 5^(1/2) - 5)^(1/2))/20 + 5^(1/2)/20 - 1/20) + 5)*((2^(1/2)
*(- 5^(1/2) - 5)^(1/2))/20 + 5^(1/2)/20 - 1/20) - 1/x - log(25*x*(5^(1/2)/2
0 - (2^(1/2)*(5^(1/2) - 5)^(1/2))/20 + 1/20) - 5)*(5^(1/2)/20 - (2^(1/2)*(5
^(1/2) - 5)^(1/2))/20 + 1/20) - log(25*x*(5^(1/2)/20 + (2^(1/2)*(5^(1/2) -
5)^(1/2))/20 + 1/20) - 5)*(5^(1/2)/20 + (2^(1/2)*(5^(1/2) - 5)^(1/2))/20 +
1/20)
```

sympy [A] time = 2.83, size = 39, normalized size = 0.21

$$\frac{\log(x+1)}{5}+\operatorname{RootSum}\left(625t^4+125t^3+25t^2+5t+1,\left(t\mapsto t\log\left(625t^4+x\right)\right)\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**5+1),x)
```

```
[Out] log(x + 1)/5 + RootSum(625*_t**4 + 125*_t**3 + 25*_t**2 + 5*_t + 1, Lambda(
_t, _t*log(625*_t**4 + x))) - 1/x
```

3.1306 $\int \frac{1}{x^3\left(1+x^5\right)}\,dx$

Optimal. Leaf size=192

$$-\frac{1}{2x^2}+\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1-\sqrt{5}\right)x+1\right)+\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1+\sqrt{5}\right)x+1\right)-\frac{1}{5}\log(x+1)+\frac{1}{5}\sqrt{\frac{1}{2}(}$$

```
[Out] -1/2/x^2-1/5*ln(1+x)+1/20*ln(1+x^2-1/2*x*(5^(1/2)+1))*(-5^(1/2)+1)+1/20*ln(
1+x^2-1/2*x*(-5^(1/2)+1))*(5^(1/2)+1)+1/10*arctan(1/5*(25-10*5^(1/2))^(1/2)
+2*x*2^(1/2)/(5+5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)-1/10*arctan(1/5*x*(50+
10*5^(1/2))^(1/2)-1/5*(25+10*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {325, 293, 634, 618, 204, 628, 31}

$$-\frac{1}{2x^2}+\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1-\sqrt{5}\right)x+1\right)+\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1+\sqrt{5}\right)x+1\right)-\frac{1}{5}\log(x+1)+\frac{1}{5}\sqrt{\frac{1}{2}(}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 + x^5)),x]
```

```
[Out] -1/(2*x^2) + (Sqrt[(5 - Sqrt[5])/2]*ArcTan[Sqrt[(5 - 2*Sqrt[5])/5] + 2*Sqrt
[2/(5 + Sqrt[5])]*x])/5 + (Sqrt[(5 + Sqrt[5])/2]*ArcTan[Sqrt[(5 + 2*Sqrt[5]
)/5] - Sqrt[(2*(5 + Sqrt[5]))/5]*x])/5 - Log[1 + x]/5 + ((1 + Sqrt[5])*Log[
1 - ((1 - Sqrt[5])*x)/2 + x^2])/20 + ((1 - Sqrt[5])*Log[1 - ((1 + Sqrt[5])*
x)/2 + x^2])/20
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 293

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x]; -(((-r)^(m + 1)*Int[1/(r + s*x), x])/(a*n*s^m
)) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 1)/2}], x], x]] /; Fr
eeQ[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ
[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^3\left(1+x^5\right)}\,dx = -\frac{1}{2x^2} - \int \frac{x^2}{1+x^5}\,dx$$

$$= -\frac{1}{2x^2} - \frac{2}{5}\int \frac{\frac{1}{4}\left(-1-\sqrt{5}\right) - \frac{1}{4}\left(1+\sqrt{5}\right)x}{1-\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx - \frac{2}{5}\int \frac{\frac{1}{4}\left(-1+\sqrt{5}\right) - \frac{1}{4}\left(1-\sqrt{5}\right)x}{1-\frac{1}{2}\left(1+\sqrt{5}\right)x+x^2}\,dx - \frac{1}{5}\int$$

$$= -\frac{1}{2x^2} - \frac{1}{5}\log(1+x) - \frac{\int \frac{1}{1+\frac{1}{2}\left(-1-\sqrt{5}\right)x+x^2}\,dx}{2\sqrt{5}} + \frac{\int \frac{1}{1+\frac{1}{2}\left(-1+\sqrt{5}\right)x+x^2}\,dx}{2\sqrt{5}} - \frac{1}{20}\left(-1-\sqrt{5}\right)\int \frac{}{1}$$

$$= -\frac{1}{2x^2} - \frac{1}{5}\log(1+x) + \frac{1}{20}\left(1-\sqrt{5}\right)\log\left(2-x-\sqrt{5}x+2x^2\right) + \frac{1}{20}\left(1+\sqrt{5}\right)\log\left(2-x+\right.$$

$$= -\frac{1}{2x^2} - \sqrt{\frac{2}{5\left(5+\sqrt{5}\right)}}\tan^{-1}\left(\frac{1-\sqrt{5}-4x}{\sqrt{2\left(5+\sqrt{5}\right)}}\right) + \frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\right.$$

Mathematica [A] time = 0.12, size = 149, normalized size = 0.78

$$\frac{1}{20}\left(-\frac{10}{x^2} + \left(1+\sqrt{5}\right)\log\left(x^2+\frac{1}{2}\left(\sqrt{5}-1\right)x+1\right) - \left(\sqrt{5}-1\right)\log\left(x^2-\frac{1}{2}\left(1+\sqrt{5}\right)x+1\right) - 4\log(x+1) + 2\sqrt{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(1 + x^5)),x]
```

```
[Out] (-10/x^2 + 2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(1 + Sqrt[5] - 4*x)/Sqrt[10 - 2*S
qrt[5]]] + 2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[(-1 + Sqrt[5] + 4*x)/Sqrt[2*(5 + S
qrt[5])]] - 4*Log[1 + x] + (1 + Sqrt[5])*Log[1 + ((-1 + Sqrt[5])*x)/2 + x^2
] - (-1 + Sqrt[5])*Log[1 - ((1 + Sqrt[5])*x)/2 + x^2])/20
```

fricas [B] time = 2.93, size = 692, normalized size = 3.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^5+1),x, algorithm="fricas")
```

```
[Out] -1/40*(2*x^2*(sqrt(5) + 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)*log(1/16*(sqrt(5
) + 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)^2 + x) + 2*x^2*(sqrt(5) - 10*sqrt(-1
/50*sqrt(5) - 1/10) - 1)*log(1/16*(sqrt(5) - 10*sqrt(-1/50*sqrt(5) - 1/10)
 - 1)^2 + x) + 8*x^2*log(x + 1) - (x^2*(sqrt(5) + 10*sqrt(-1/50*sqrt(5) - 1/
10) - 1) + x^2*(sqrt(5) - 10*sqrt(-1/50*sqrt(5) - 1/10) - 1) + 4*sqrt(-3/16
*(sqrt(5) + 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)^2 - 1/8*(sqrt(5) + 10*sqrt(-
1/50*sqrt(5) - 1/10) + 3)*(sqrt(5) - 10*sqrt(-1/50*sqrt(5) - 1/10) - 1) - 3
/16*(sqrt(5) - 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)^2 - 1/2*sqrt(5) - 5*sqrt(
-1/50*sqrt(5) - 1/10) - 5/2)*x^2 + 4*x^2)*log(-1/16*(sqrt(5) + 10*sqrt(-1/5
0*sqrt(5) - 1/10) - 1)^2 - 1/16*(sqrt(5) - 10*sqrt(-1/50*sqrt(5) - 1/10) -
 1)^2 + 1/2*sqrt(-3/16*(sqrt(5) + 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)^2 - 1/8
*(sqrt(5) + 10*sqrt(-1/50*sqrt(5) - 1/10) + 3)*(sqrt(5) - 10*sqrt(-1/50*sqr
t(5) - 1/10) - 1) - 3/16*(sqrt(5) - 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)^2 -
 1/2*sqrt(5) - 5*sqrt(-1/50*sqrt(5) - 1/10) - 5/2)*(sqrt(5) + 1) + 2*x - 1)
 - (x^2*(sqrt(5) + 10*sqrt(-1/50*sqrt(5) - 1/10) - 1) + x^2*(sqrt(5) - 10*sq
rt(-1/50*sqrt(5) - 1/10) - 1) - 4*sqrt(-3/16*(sqrt(5) + 10*sqrt(-1/50*sqrt(
5) - 1/10) - 1)^2 - 1/8*(sqrt(5) + 10*sqrt(-1/50*sqrt(5) - 1/10) + 3)*(sqrt
(5) - 10*sqrt(-1/50*sqrt(5) - 1/10) - 1) - 3/16*(sqrt(5) - 10*sqrt(-1/50*sq
rt(5) - 1/10) - 1)^2 - 1/2*sqrt(5) - 5*sqrt(-1/50*sqrt(5) - 1/10) - 5/2)*x^
2 + 4*x^2)*log(-1/16*(sqrt(5) + 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)^2 - 1/16
*(sqrt(5) - 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)^2 - 1/2*sqrt(-3/16*(sqrt(5)
 + 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)^2 - 1/8*(sqrt(5) + 10*sqrt(-1/50*sqrt(
5) - 1/10) + 3)*(sqrt(5) - 10*sqrt(-1/50*sqrt(5) - 1/10) - 1) - 3/16*(sqrt(
5) - 10*sqrt(-1/50*sqrt(5) - 1/10) - 1)^2 - 1/2*sqrt(5) - 5*sqrt(-1/50*sqrt
(5) - 1/10) - 5/2)*(sqrt(5) + 1) + 2*x - 1) + 20)/x^2
```

giac [A] time = 0.18, size = 132, normalized size = 0.69

$$\frac{1}{10}\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{2\sqrt{5}+10}}\right)-\frac{1}{10}\sqrt{2\sqrt{5}+10}\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{-2\sqrt{5}+10}}\right)-\frac{1}{20}\sqrt{5}\log\left(x^2-\frac{1}{2}x\left(\sqrt{5}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^5+1),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(-2*sqrt(5) + 10)*arctan((4*x + sqrt(5) - 1)/sqrt(2*sqrt(5) + 10))
 - 1/10*sqrt(2*sqrt(5) + 10)*arctan((4*x - sqrt(5) - 1)/sqrt(-2*sqrt(5) + 1
0)) - 1/20*sqrt(5)*log(x^2 - 1/2*x*(sqrt(5) + 1) + 1) + 1/20*sqrt(5)*log(x^
2 + 1/2*x*(sqrt(5) - 1) + 1) - 1/2/x^2 + 1/20*log(x^4 - x^3 + x^2 - x + 1)
 - 1/5*log(abs(x + 1))
```

maple [A] time = 0.02, size = 161, normalized size = 0.84

$$-\frac{2\sqrt{5}\ \arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{10-2\sqrt{5}}}\right)}{5\sqrt{10-2\sqrt{5}}}+\frac{2\sqrt{5}\ \arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{10+2\sqrt{5}}}\right)}{5\sqrt{10+2\sqrt{5}}}-\frac{\ln(x+1)}{5}-\frac{\sqrt{5}\ \ln\left(2x^2-\sqrt{5}\,x-x+2\right)}{20}+\frac{\ln\left(2x^2-\sqrt{5}\,x-\right.}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^5+1),x)
```

```
[Out] -1/2/x^2+1/20*5^(1/2)*ln(2*x^2+5^(1/2)*x-x+2)+1/20*ln(2*x^2+5^(1/2)*x-x+2)+
2/5/(10+2*5^(1/2))^(1/2)*5^(1/2)*arctan((4*x+5^(1/2)-1)/(10+2*5^(1/2))^(1/2
))-1/20*5^(1/2)*ln(2*x^2-5^(1/2)*x-x+2)+1/20*ln(2*x^2-5^(1/2)*x-x+2)-2/5/(1
0-2*5^(1/2))^(1/2)*5^(1/2)*arctan((4*x-5^(1/2)-1)/(10-2*5^(1/2))^(1/2))-1/5
*ln(x+1)
```

maxima [A] time = 2.27, size = 129, normalized size = 0.67

$$\frac{2\sqrt{5}\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{2\sqrt{5}+10}}\right)}{5\sqrt{2\sqrt{5}+10}}-\frac{2\sqrt{5}\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{-2\sqrt{5}+10}}\right)}{5\sqrt{-2\sqrt{5}+10}}-\frac{\log\left(2x^2-x\left(\sqrt{5}+1\right)+2\right)}{5\left(\sqrt{5}+1\right)}+\frac{\log\left(2x^2+x\left(\sqrt{5}-1\right)+2\right.}{5\left(\sqrt{5}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^5+1),x, algorithm="maxima")
```

```
[Out] 2/5*sqrt(5)*arctan((4*x + sqrt(5) - 1)/sqrt(2*sqrt(5) + 10))/sqrt(2*sqrt(5)
 + 10) - 2/5*sqrt(5)*arctan((4*x - sqrt(5) - 1)/sqrt(-2*sqrt(5) + 10))/sqrt
(-2*sqrt(5) + 10) - 1/5*log(2*x^2 - x*(sqrt(5) + 1) + 2)/(sqrt(5) + 1) + 1/
5*log(2*x^2 + x*(sqrt(5) - 1) + 2)/(sqrt(5) - 1) - 1/2/x^2 - 1/5*log(x + 1)
```

mupad [B] time = 1.70, size = 200, normalized size = 1.04

$$\ln\left(1-\frac{x\left(\sqrt{2}\,\sqrt{-\sqrt{5}-5}-\sqrt{5}+1\right)^3}{64}\right)\left(\frac{\sqrt{2}\,\sqrt{-\sqrt{5}-5}}{20}-\frac{\sqrt{5}}{20}+\frac{1}{20}\right)-\frac{\ln(x+1)}{5}-\ln\left(\frac{x\left(\sqrt{2}\,\sqrt{-\sqrt{5}-5}+\sqrt{\right.}\right.}{64}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x^5 + 1)),x)
```

```
[Out] log(1 - (x*(2^(1/2)*(- 5^(1/2) - 5)^(1/2) - 5^(1/2) + 1)^3)/64)*((2^(1/2)*(
- 5^(1/2) - 5)^(1/2))/20 - 5^(1/2)/20 + 1/20) - log(x + 1)/5 - log((x*(2^(1
/2)*(- 5^(1/2) - 5)^(1/2) + 5^(1/2) - 1)^3)/64 + 1)*((2^(1/2)*(- 5^(1/2) -
5)^(1/2))/20 + 5^(1/2)/20 - 1/20) - 1/(2*x^2) + log(1 - (x*(5^(1/2) + 2^(1/
2)*(5^(1/2) - 5)^(1/2) + 1)^3)/64)*(5^(1/2)/20 + (2^(1/2)*(5^(1/2) - 5)^(1/
2))/20 + 1/20) + log(1 - (x*(5^(1/2) - 2^(1/2)*(5^(1/2) - 5)^(1/2) + 1)^3)/
64)*(5^(1/2)/20 - (2^(1/2)*(5^(1/2) - 5)^(1/2))/20 + 1/20)
```

sympy [A] time = 2.00, size = 42, normalized size = 0.22

$$-\frac{\log(x+1)}{5}+\operatorname{RootSum}\left(625t^4-125t^3+25t^2-5t+1,\left(t\mapsto t\log\left(25t^2+x\right)\right)\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**5+1),x)
```

```
[Out] -log(x + 1)/5 + RootSum(625*_t**4 - 125*_t**3 + 25*_t**2 - 5*_t + 1, Lambda
(_t, _t*log(25*_t**2 + x))) - 1/(2*x**2)
```

3.1307 $\int \frac{1}{x^4\left(1+x^5\right)}\,dx$

Optimal. Leaf size=192

$$-\frac{1}{3x^3}-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1-\sqrt{5}\right)x+1\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1+\sqrt{5}\right)x+1\right)+\frac{1}{5}\log(x+1)+\frac{1}{5}\sqrt{\frac{1}{2}\left(\right.}$$

```
[Out] -1/3/x^3+1/5*ln(1+x)-1/20*ln(1+x^2-1/2*x*(5^(1/2)+1))*(-5^(1/2)+1)-1/20*ln(
1+x^2-1/2*x*(-5^(1/2)+1))*(5^(1/2)+1)+1/10*arctan(1/5*(25-10*5^(1/2))^(1/2)
+2*x*2^(1/2)/(5+5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)-1/10*arctan(1/5*x*(50+
10*5^(1/2))^(1/2)-1/5*(25+10*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {325, 293, 634, 618, 204, 628, 31}

$$-\frac{1}{3x^3}-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1-\sqrt{5}\right)x+1\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1+\sqrt{5}\right)x+1\right)+\frac{1}{5}\log(x+1)+\frac{1}{5}\sqrt{\frac{1}{2}\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 + x^5)),x]
```

```
[Out] -1/(3*x^3) + (Sqrt[(5 - Sqrt[5])/2]*ArcTan[Sqrt[(5 - 2*Sqrt[5])/5] + 2*Sqrt
[2/(5 + Sqrt[5])]*x])/5 + (Sqrt[(5 + Sqrt[5])/2]*ArcTan[Sqrt[(5 + 2*Sqrt[5]
)/5] - Sqrt[(2*(5 + Sqrt[5]))/5]*x])/5 + Log[1 + x]/5 - ((1 + Sqrt[5])*Log[
1 - ((1 - Sqrt[5])*x)/2 + x^2])/20 - ((1 - Sqrt[5])*Log[1 - ((1 + Sqrt[5])*
x)/2 + x^2])/20
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 293

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x]; -(((-r)^(m + 1)*Int[1/(r + s*x), x])/(a*n*s^m
)) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 1)/2}], x], x]] /; Fr
eeQ[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ
[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^4\left(1+x^5\right)}\,dx = -\frac{1}{3x^3} - \int \frac{x}{1+x^5}\,dx$$

$$= -\frac{1}{3x^3} - \frac{2}{5}\int \frac{\frac{1}{4}\left(1-\sqrt{5}\right) - \frac{1}{4}\left(-1-\sqrt{5}\right)x}{1-\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx - \frac{2}{5}\int \frac{\frac{1}{4}\left(1+\sqrt{5}\right) - \frac{1}{4}\left(-1+\sqrt{5}\right)x}{1-\frac{1}{2}\left(1+\sqrt{5}\right)x+x^2}\,dx + \frac{1}{5}$$

$$= -\frac{1}{3x^3} + \frac{1}{5}\log(1+x) - \frac{\int \frac{1}{1+\frac{1}{2}\left(-1-\sqrt{5}\right)x+x^2}\,dx}{2\sqrt{5}} + \frac{\int \frac{1}{1+\frac{1}{2}\left(-1+\sqrt{5}\right)x+x^2}\,dx}{2\sqrt{5}} - \frac{1}{20}\left(1-\sqrt{5}\right)\int \frac{1}{1+$$

$$= -\frac{1}{3x^3} + \frac{1}{5}\log(1+x) - \frac{1}{20}\left(1-\sqrt{5}\right)\log\left(2-x-\sqrt{5}\,x+2x^2\right) - \frac{1}{20}\left(1+\sqrt{5}\right)\log\left(2-x+\right.$$

$$= -\frac{1}{3x^3} - \sqrt{\frac{2}{5\left(5+\sqrt{5}\right)}}\,\tan^{-1}\left(\frac{1-\sqrt{5}-4x}{\sqrt{2\left(5+\sqrt{5}\right)}}\right) + \frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\,\tan^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\right.$$

Mathematica [A] time = 0.14, size = 150, normalized size = 0.78

$$\frac{1}{60}\left(-\frac{20}{x^3} - 3\left(1+\sqrt{5}\right)\log\left(x^2+\frac{1}{2}\left(\sqrt{5}-1\right)x+1\right) + 3\left(\sqrt{5}-1\right)\log\left(x^2-\frac{1}{2}\left(1+\sqrt{5}\right)x+1\right) + 12\log(x+1)\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^4*(1 + x^5)),x]
```

```
[Out] (-20/x^3 + 6*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(1 + Sqrt[5] - 4*x)/Sqrt[10 - 2*S
qrt[5]]] + 6*Sqrt[10 - 2*Sqrt[5]]*ArcTan[(-1 + Sqrt[5] + 4*x)/Sqrt[2*(5 + S
qrt[5])]] + 12*Log[1 + x] - 3*(1 + Sqrt[5])*Log[1 + ((-1 + Sqrt[5])*x)/2 +
x^2] + 3*(-1 + Sqrt[5])*Log[1 - ((1 + Sqrt[5])*x)/2 + x^2])/60
```

fricas [B] time = 2.65, size = 1174, normalized size = 6.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^5+1),x, algorithm="fricas")
```

```
[Out] 1/120*(6*x^3*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)*log(1/64*(2*sqrt
(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^3 + 1/64*(2*sqrt(1/2)*sqrt(sqrt(5) -
 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 - 1/16*(
2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 - 1/64*((2*sqrt(1/2)*sqrt(sq
rt(5) - 5) + sqrt(5) + 1)^2 - 8*sqrt(1/2)*sqrt(sqrt(5) - 5) - 4*sqrt(5) + 1
2)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) + x + 1/2*sqrt(1/2)*sqrt(s
qrt(5) - 5) + 1/4*sqrt(5) - 3/4) - 6*x^3*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + s
qrt(5) + 1)*log(-1/64*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^3 + x)
+ 24*x^3*log(x + 1) + 3*(x^3*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)
- x^3*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 4*sqrt(-3/16*(2*sqrt(
1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 1/8*(2*sqrt(1/2)*sqrt(sqrt(5) - 5
) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 3/16*(2*sq
rt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + sqrt(1/2)*sqrt(sqrt(5) - 5) +
1/2*sqrt(5) - 5/2)*x^3 - 4*x^3)*log(-1/64*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) +
sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 1/16*(2*sqrt
(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 1/64*((2*sqrt(1/2)*sqrt(sqrt(5)
- 5) + sqrt(5) + 1)^2 - 8*sqrt(1/2)*sqrt(sqrt(5) - 5) - 4*sqrt(5) + 12)*(2*
sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) + 1/16*sqrt(-3/16*(2*sqrt(1/2)*s
qrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 1/8*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sq
rt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 3/16*(2*sqrt(1/2
)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + sqrt(1/2)*sqrt(sqrt(5) - 5) + 1/2*sq
rt(5) - 5/2)*((2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sq
rt(sqrt(5) - 5) - sqrt(5) - 1) + 8*sqrt(1/2)*sqrt(sqrt(5) - 5) + 4*sqrt(5)
+ 4) + 2*x - 1/2*sqrt(1/2)*sqrt(sqrt(5) - 5) - 1/4*sqrt(5) - 1/4) + 3*(x^3*
(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1) - x^3*(2*sqrt(1/2)*sqrt(sqrt(
5) - 5) - sqrt(5) - 1) + 4*sqrt(-3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt
(5) + 1)^2 + 1/8*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(2*sqrt(1/2)
*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - s
qrt(5) - 1)^2 + sqrt(1/2)*sqrt(sqrt(5) - 5) + 1/2*sqrt(5) - 5/2)*x^3 - 4*x^
3)*log(-1/64*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqr
t(sqrt(5) - 5) - sqrt(5) - 1)^2 + 1/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqr
t(5) + 1)^2 + 1/64*((2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 - 8*sqr
t(1/2)*sqrt(sqrt(5) - 5) - 4*sqrt(5) + 12)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) -
 sqrt(5) - 1) - 1/16*sqrt(-3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) +
1)^2 + 1/8*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(
sqrt(5) - 5) - sqrt(5) - 1) - 3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5)
 - 1)^2 + sqrt(1/2)*sqrt(sqrt(5) - 5) + 1/2*sqrt(5) - 5/2)*((2*sqrt(1/2)*sq
rt(sqrt(5) - 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1
) + 8*sqrt(1/2)*sqrt(sqrt(5) - 5) + 4*sqrt(5) + 4) + 2*x - 1/2*sqrt(1/2)*sq
rt(sqrt(5) - 5) - 1/4*sqrt(5) - 1/4) - 40)/x^3
```

giac [A] time = 0.18, size = 132, normalized size = 0.69

$$\frac{1}{10}\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{2\sqrt{5}+10}}\right)-\frac{1}{10}\sqrt{2\sqrt{5}+10}\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{-2\sqrt{5}+10}}\right)+\frac{1}{20}\sqrt{5}\log\left(x^2-\frac{1}{2}x\left(\sqrt{5}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^5+1),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(-2*sqrt(5) + 10)*arctan((4*x + sqrt(5) - 1)/sqrt(2*sqrt(5) + 10))
 - 1/10*sqrt(2*sqrt(5) + 10)*arctan((4*x - sqrt(5) - 1)/sqrt(-2*sqrt(5) + 1
0)) + 1/20*sqrt(5)*log(x^2 - 1/2*x*(sqrt(5) + 1) + 1) - 1/20*sqrt(5)*log(x^
2 + 1/2*x*(sqrt(5) - 1) + 1) - 1/3/x^3 - 1/20*log(x^4 - x^3 + x^2 - x + 1)
+ 1/5*log(abs(x + 1))
```

maple [A] time = 0.02, size = 161, normalized size = 0.84

$$-\frac{2\sqrt{5}\ \arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{10-2\sqrt{5}}}\right)}{5\sqrt{10-2\sqrt{5}}}+\frac{2\sqrt{5}\ \arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{10+2\sqrt{5}}}\right)}{5\sqrt{10+2\sqrt{5}}}+\frac{\ln(x+1)}{5}+\frac{\sqrt{5}\ \ln\left(2x^2-\sqrt{5}\ x-x+2\right)}{20}-\frac{\ln\left(2x^2-\sqrt{5}\right.}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^5+1),x)
```

```
[Out] -1/3/x^3-1/20*5^(1/2)*ln(2*x^2+5^(1/2)*x-x+2)-1/20*ln(2*x^2+5^(1/2)*x-x+2)+
2/5/(10+2*5^(1/2))^(1/2)*5^(1/2)*arctan((4*x+5^(1/2)-1)/(10+2*5^(1/2))^(1/2
))+1/20*5^(1/2)*ln(2*x^2-5^(1/2)*x-x+2)-1/20*ln(2*x^2-5^(1/2)*x-x+2)-2/5/(1
0-2*5^(1/2))^(1/2)*5^(1/2)*arctan((4*x-5^(1/2)-1)/(10-2*5^(1/2))^(1/2))+1/5
*ln(x+1)
```

maxima [A] time = 2.29, size = 129, normalized size = 0.67

$$\frac{2\sqrt{5}\ \arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{2\sqrt{5}+10}}\right)}{5\sqrt{2\sqrt{5}+10}}-\frac{2\sqrt{5}\ \arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{-2\sqrt{5}+10}}\right)}{5\sqrt{-2\sqrt{5}+10}}+\frac{\log\left(2x^2-x\left(\sqrt{5}+1\right)+2\right)}{5\left(\sqrt{5}+1\right)}-\frac{\log\left(2x^2+x\left(\sqrt{5}-1\right)+2\right.}{5\left(\sqrt{5}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^5+1),x, algorithm="maxima")
```

```
[Out] 2/5*sqrt(5)*arctan((4*x + sqrt(5) - 1)/sqrt(2*sqrt(5) + 10))/sqrt(2*sqrt(5)
 + 10) - 2/5*sqrt(5)*arctan((4*x - sqrt(5) - 1)/sqrt(-2*sqrt(5) + 10))/sqrt
(-2*sqrt(5) + 10) + 1/5*log(2*x^2 - x*(sqrt(5) + 1) + 2)/(sqrt(5) + 1) - 1/
5*log(2*x^2 + x*(sqrt(5) - 1) + 2)/(sqrt(5) - 1) - 1/3/x^3 + 1/5*log(x + 1)
```

mupad [B] time = 0.26, size = 207, normalized size = 1.08

$$\frac{\ln(x+1)}{5}-\ln\left(x-125\left(\frac{\sqrt{2}\ \sqrt{-\sqrt{5}-5}}{20}-\frac{\sqrt{5}}{20}+\frac{1}{20}\right)^3\right)\left(\frac{\sqrt{2}\ \sqrt{-\sqrt{5}-5}}{20}-\frac{\sqrt{5}}{20}+\frac{1}{20}\right)+\ln\left(x+125\left(\frac{\sqrt{2}\ \sqrt{-}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x^5 + 1)),x)
```

```
[Out] log(x + 1)/5 - log(x - 125*((2^(1/2)*(- 5^(1/2) - 5)^(1/2))/20 - 5^(1/2)/20
 + 1/20)^3)*((2^(1/2)*(- 5^(1/2) - 5)^(1/2))/20 - 5^(1/2)/20 + 1/20) + log(
x + 125*((2^(1/2)*(- 5^(1/2) - 5)^(1/2))/20 + 5^(1/2)/20 - 1/20)^3)*((2^(1/
2)*(- 5^(1/2) - 5)^(1/2))/20 + 5^(1/2)/20 - 1/20) - log(x - 125*(5^(1/2)/20
 - (2^(1/2)*(5^(1/2) - 5)^(1/2))/20 + 1/20)^3)*(5^(1/2)/20 - (2^(1/2)*(5^(1
/2) - 5)^(1/2))/20 + 1/20) - log(x - 125*(5^(1/2)/20 + (2^(1/2)*(5^(1/2) -
5)^(1/2))/20 + 1/20)^3)*(5^(1/2)/20 + (2^(1/2)*(5^(1/2) - 5)^(1/2))/20 + 1/
20) - 1/(3*x^3)
```

sympy [A] time = 3.06, size = 42, normalized size = 0.22

$$\frac{\log(x+1)}{5}+\operatorname{RootSum}\left(625t^4+125t^3+25t^2+5t+1,\left(t\mapsto t\log\left(125t^3+x\right)\right)\right)-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**5+1),x)
```

```
[Out] log(x + 1)/5 + RootSum(625*_t**4 + 125*_t**3 + 25*_t**2 + 5*_t + 1, Lambda(
_t, _t*log(125*_t**3 + x))) - 1/(3*x**3)
```

3.1308 $\int \frac{x^{23/2}}{\sqrt{a+bx^5}}\,dx$

Optimal. Leaf size=83

$$\frac{3a^2 \tanh^{-1}\left(\frac{\sqrt{b}\,x^{5/2}}{\sqrt{a+bx^5}}\right)}{20b^{5/2}} - \frac{3ax^{5/2}\sqrt{a+bx^5}}{20b^2} + \frac{x^{15/2}\sqrt{a+bx^5}}{10b}$$

```
[Out] 3/20*a^2*arctanh(x^(5/2)*b^(1/2)/(b*x^5+a)^(1/2))/b^(5/2)-3/20*a*x^(5/2)*(b
*x^5+a)^(1/2)/b^2+1/10*x^(15/2)*(b*x^5+a)^(1/2)/b
```

Rubi [A] time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {321, 329, 275, 217, 206}

$$\frac{3a^2 \tanh^{-1}\left(\frac{\sqrt{b}\,x^{5/2}}{\sqrt{a+bx^5}}\right)}{20b^{5/2}} - \frac{3ax^{5/2}\sqrt{a+bx^5}}{20b^2} + \frac{x^{15/2}\sqrt{a+bx^5}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(23/2)/Sqrt[a + b*x^5],x]
```

```
[Out] (-3*a*x^(5/2)*Sqrt[a + b*x^5])/(20*b^2) + (x^(15/2)*Sqrt[a + b*x^5])/(10*b)
 + (3*a^2*ArcTanh[(Sqrt[b]*x^(5/2))/Sqrt[a + b*x^5]])/(20*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{23/2}}{\sqrt{a+bx^5}}\,dx &= \frac{x^{15/2}\sqrt{a+bx^5}}{10b} - \frac{(3a)\int \frac{x^{13/2}}{\sqrt{a+bx^5}}\,dx}{4b} \\
&= -\frac{3ax^{5/2}\sqrt{a+bx^5}}{20b^2} + \frac{x^{15/2}\sqrt{a+bx^5}}{10b} + \frac{\left(3a^2\right)\int \frac{x^{3/2}}{\sqrt{a+bx^5}}\,dx}{8b^2} \\
&= -\frac{3ax^{5/2}\sqrt{a+bx^5}}{20b^2} + \frac{x^{15/2}\sqrt{a+bx^5}}{10b} + \frac{\left(3a^2\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^{10}}}\,dx, x, \sqrt{x}\right)}{4b^2} \\
&= -\frac{3ax^{5/2}\sqrt{a+bx^5}}{20b^2} + \frac{x^{15/2}\sqrt{a+bx^5}}{10b} + \frac{\left(3a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, x^{5/2}\right)}{20b^2} \\
&= -\frac{3ax^{5/2}\sqrt{a+bx^5}}{20b^2} + \frac{x^{15/2}\sqrt{a+bx^5}}{10b} + \frac{\left(3a^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^{5/2}}{\sqrt{a+bx^5}}\right)}{20b^2} \\
&= -\frac{3ax^{5/2}\sqrt{a+bx^5}}{20b^2} + \frac{x^{15/2}\sqrt{a+bx^5}}{10b} + \frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x^{5/2}}{\sqrt{a+bx^5}}\right)}{20b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 70, normalized size = 0.84

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x^{5/2}}{\sqrt{a+bx^5}}\right) + \sqrt{b}\,x^{5/2}\sqrt{a+bx^5}\left(2bx^5-3a\right)}{20b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(23/2)/Sqrt[a + b*x^5],x]
```

```
[Out] (Sqrt[b]*x^(5/2)*Sqrt[a + b*x^5]*(-3*a + 2*b*x^5) + 3*a^2*ArcTanh[(Sqrt[b]*
x^(5/2))/Sqrt[a + b*x^5]])/(20*b^(5/2))
```

fricas [A] time = 1.77, size = 172, normalized size = 2.07

$$\left[\frac{3\,a^2\sqrt{b}\,\log\left(-8\,b^2x^{10}-8\,abx^5-4\left(2\,bx^7+ax^2\right)\sqrt{bx^5+a}\,\sqrt{b}\,\sqrt{x}\,-a^2\right)+4\left(2\,b^2x^7-3\,abx^2\right)\sqrt{bx^5+a}\,\sqrt{x}}{80\,b^3}, -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(23/2)/(b*x^5+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/80*(3*a^2*sqrt(b)*log(-8*b^2*x^10 - 8*a*b*x^5 - 4*(2*b*x^7 + a*x^2)*sqrt
(b*x^5 + a)*sqrt(b)*sqrt(x) - a^2) + 4*(2*b^2*x^7 - 3*a*b*x^2)*sqrt(b*x^5 +
 a)*sqrt(x))/b^3, -1/40*(3*a^2*sqrt(-b)*arctan(2*sqrt(b*x^5 + a)*sqrt(-b)*x
^(5/2)/(2*b*x^5 + a)) - 2*(2*b^2*x^7 - 3*a*b*x^2)*sqrt(b*x^5 + a)*sqrt(x))/
b^3]
```

giac [A] time = 0.30, size = 58, normalized size = 0.70

$$\frac{1}{20}\sqrt{bx^5+a}\left(\frac{2\,x^5}{b}-\frac{3\,a}{b^2}\right)x^{\frac{5}{2}} - \frac{3\,a^2\log\left(\left|-\sqrt{b}\,x^{\frac{5}{2}}+\sqrt{bx^5+a}\right|\right)}{20\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(23/2)/(b*x^5+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(b*x^5 + a)*(2*x^5/b - 3*a/b^2)*x^(5/2) - 3/20*a^2*log(abs(-sqrt(b
)*x^(5/2) + sqrt(b*x^5 + a)))/b^(5/2)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{23}{2}}}{\sqrt{b\,x^5+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(23/2)/(b*x^5+a)^(1/2),x)
```

```
[Out] int(x^(23/2)/(b*x^5+a)^(1/2),x)
```

maxima [B] time = 2.35, size = 124, normalized size = 1.49

$$-\frac{3\,a^2\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^5+a}}{x^{\frac{5}{2}}}}{\sqrt{b}+\frac{\sqrt{bx^5+a}}{x^{\frac{5}{2}}}}\right)}{40\,b^{\frac{5}{2}}}+\frac{\frac{5\sqrt{bx^5+a}\,a^2b}{x^{\frac{5}{2}}}-\frac{3\left(bx^5+a\right)^{\frac{3}{2}}a^2}{x^{\frac{15}{2}}}}{20\left(b^4-\frac{2\left(bx^5+a\right)b^3}{x^5}+\frac{\left(bx^5+a\right)^2b^2}{x^{10}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(23/2)/(b*x^5+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/40*a^2*log(-(sqrt(b) - sqrt(b*x^5 + a)/x^(5/2))/(sqrt(b) + sqrt(b*x^5 +
a)/x^(5/2)))/b^(5/2) + 1/20*(5*sqrt(b*x^5 + a)*a^2*b/x^(5/2) - 3*(b*x^5 + a
)^(3/2)*a^2/x^(15/2))/(b^4 - 2*(b*x^5 + a)*b^3/x^5 + (b*x^5 + a)^2*b^2/x^10
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{23/2}}{\sqrt{b\,x^5+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(23/2)/(a + b*x^5)^(1/2),x)
```

```
[Out] int(x^(23/2)/(a + b*x^5)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(23/2)/(b*x**5+a)**(1/2),x)
```

```
[Out] Timed out
```

3.1309 $\int \frac{x^{13/2}}{\sqrt{a+bx^5}}\,dx$

Optimal. Leaf size=57

$$\frac{x^{5/2}\sqrt{a+bx^5}}{5b} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^{5/2}}{\sqrt{a+bx^5}}\right)}{5b^{3/2}}$$

```
[Out] -1/5*a*arctanh(x^(5/2)*b^(1/2)/(b*x^5+a)^(1/2))/b^(3/2)+1/5*x^(5/2)*(b*x^5+
a)^(1/2)/b
```

Rubi [A] time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {321, 329, 275, 217, 206}

$$\frac{x^{5/2}\sqrt{a+bx^5}}{5b} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^{5/2}}{\sqrt{a+bx^5}}\right)}{5b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(13/2)/Sqrt[a + b*x^5],x]
```

```
[Out] (x^(5/2)*Sqrt[a + b*x^5])/(5*b) - (a*ArcTanh[(Sqrt[b]*x^(5/2))/Sqrt[a + b*x
^5]])/(5*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{13/2}}{\sqrt{a+bx^5}}\,dx &= \frac{x^{5/2}\sqrt{a+bx^5}}{5b} - \frac{a\int \frac{x^{3/2}}{\sqrt{a+bx^5}}\,dx}{2b} \\ &= \frac{x^{5/2}\sqrt{a+bx^5}}{5b} - \frac{a\,\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^{10}}}\,dx, x, \sqrt{x}\right)}{b} \\ &= \frac{x^{5/2}\sqrt{a+bx^5}}{5b} - \frac{a\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, x^{5/2}\right)}{5b} \\ &= \frac{x^{5/2}\sqrt{a+bx^5}}{5b} - \frac{a\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^{5/2}}{\sqrt{a+bx^5}}\right)}{5b} \\ &= \frac{x^{5/2}\sqrt{a+bx^5}}{5b} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^{5/2}}{\sqrt{a+bx^5}}\right)}{5b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 57, normalized size = 1.00

$$\frac{x^{5/2}\sqrt{a+bx^5}}{5b} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^{5/2}}{\sqrt{a+bx^5}}\right)}{5b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(13/2)/Sqrt[a + b*x^5],x]
```

```
[Out] (x^(5/2)*Sqrt[a + b*x^5])/(5*b) - (a*ArcTanh[(Sqrt[b]*x^(5/2))/Sqrt[a + b*x
^5]])/(5*b^(3/2))
```

fricas [A] time = 1.97, size = 136, normalized size = 2.39

$$\left[\frac{4\sqrt{bx^5+a}\,bx^{\frac{5}{2}} + a\sqrt{b}\log\left(-8b^2x^{10} - 8abx^5 + 4\left(2bx^7 + ax^2\right)\sqrt{bx^5+a}\sqrt{b}\sqrt{x} - a^2\right)}{20b^2}, \frac{2\sqrt{bx^5+a}\,bx^{\frac{5}{2}} + a\sqrt{-b}}{10}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(b*x^5+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/20*(4*sqrt(b*x^5 + a)*b*x^(5/2) + a*sqrt(b)*log(-8*b^2*x^10 - 8*a*b*x^5
+ 4*(2*b*x^7 + a*x^2)*sqrt(b*x^5 + a)*sqrt(b)*sqrt(x) - a^2))/b^2, 1/10*(2*
sqrt(b*x^5 + a)*b*x^(5/2) + a*sqrt(-b)*arctan(2*sqrt(b*x^5 + a)*sqrt(-b)*x^
(5/2)/(2*b*x^5 + a)))/b^2]
```

giac [A] time = 0.29, size = 44, normalized size = 0.77

$$\frac{\sqrt{bx^5+a}\,x^{\frac{5}{2}}}{5\,b} + \frac{a\log\left(\left|-\sqrt{b}\,x^{\frac{5}{2}} + \sqrt{bx^5+a}\right|\right)}{5\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(b*x^5+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/5*sqrt(b*x^5 + a)*x^(5/2)/b + 1/5*a*log(abs(-sqrt(b)*x^(5/2) + sqrt(b*x^5
 + a)))/b^(3/2)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{13}{2}}}{\sqrt{b\,x^5+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(b*x^5+a)^(1/2),x)
```

```
[Out] int(x^(13/2)/(b*x^5+a)^(1/2),x)
```

maxima [A] time = 2.35, size = 81, normalized size = 1.42

$$\frac{a\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^5+a}}{x^{\frac{5}{2}}}}{\sqrt{b}+\frac{\sqrt{bx^5+a}}{x^{\frac{5}{2}}}}\right)}{10\,b^{\frac{3}{2}}}-\frac{\sqrt{bx^5+a}\,a}{5\left(b^2-\frac{\left(bx^5+a\right)b}{x^5}\right)x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(b*x^5+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/10*a*log(-(sqrt(b) - sqrt(b*x^5 + a)/x^(5/2))/(sqrt(b) + sqrt(b*x^5 + a)/
x^(5/2)))/b^(3/2) - 1/5*sqrt(b*x^5 + a)*a/((b^2 - (b*x^5 + a)*b/x^5)*x^(5/2
))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{13/2}}{\sqrt{b\,x^5+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(a + b*x^5)^(1/2),x)
```

```
[Out] int(x^(13/2)/(a + b*x^5)^(1/2), x)
```

sympy [A] time = 165.09, size = 49, normalized size = 0.86

$$\frac{\sqrt{a}\,x^{\frac{5}{2}}\sqrt{1+\frac{bx^5}{a}}}{5b}-\frac{a\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{\frac{5}{2}}}{\sqrt{a}}\right)}{5b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)/(b*x**5+a)**(1/2),x)
```

```
[Out] sqrt(a)*x**(5/2)*sqrt(1 + b*x**5/a)/(5*b) - a*asinh(sqrt(b)*x**(5/2)/sqrt(a
))/(5*b**(3/2))
```

3.1310 $\int \frac{x^{3/2}}{\sqrt{a+bx^5}}\,dx$

Optimal. Leaf size=32

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x^{5/2}}{\sqrt{a+bx^5}}\right)}{5\sqrt{b}}$$

```
[Out] 2/5*arctanh(x^(5/2)*b^(1/2)/(b*x^5+a)^(1/2))/b^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {329, 275, 217, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x^{5/2}}{\sqrt{a+bx^5}}\right)}{5\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[a + b*x^5],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x^(5/2))/Sqrt[a + b*x^5]])/(5*Sqrt[b])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{3/2}}{\sqrt{a+bx^5}}\,dx &= 2\,\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^{10}}}\,dx, x, \sqrt{x}\right)\\ &= \frac{2}{5}\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, x^{5/2}\right)\\ &= \frac{2}{5}\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^{5/2}}{\sqrt{a+bx^5}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x^{5/2}}{\sqrt{a+bx^5}}\right)}{5\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x^{5/2}}{\sqrt{a+bx^5}}\right)}{5\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[a + b*x^5],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x^(5/2))/Sqrt[a + b*x^5]])/(5*Sqrt[b])
```

fricas [A] time = 1.89, size = 97, normalized size = 3.03

$$\left[\frac{\log\left(-8\,b^2x^{10}-8\,abx^5-4\left(2\,bx^7+ax^2\right)\sqrt{bx^5+a}\,\sqrt{b}\,\sqrt{x}\,-a^2\right)}{10\,\sqrt{b}}, -\frac{\sqrt{-b}\,\arctan\left(\frac{2\,\sqrt{bx^5+a}\,\sqrt{-b}\,x^{\frac{5}{2}}}{2\,bx^5+a}\right)}{5\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^5+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/10*log(-8*b^2*x^10 - 8*a*b*x^5 - 4*(2*b*x^7 + a*x^2)*sqrt(b*x^5 + a)*sqr
t(b)*sqrt(x) - a^2)/sqrt(b), -1/5*sqrt(-b)*arctan(2*sqrt(b*x^5 + a)*sqrt(-b
)*x^(5/2)/(2*b*x^5 + a))/b]
```

giac [A] time = 0.18, size = 25, normalized size = 0.78

$$-\frac{2\,\log\left(\left|-\sqrt{b}\,x^{\frac{5}{2}}+\sqrt{bx^5+a}\right|\right)}{5\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^5+a)^(1/2),x, algorithm="giac")
```

```
[Out] -2/5*log(abs(-sqrt(b)*x^(5/2) + sqrt(b*x^5 + a)))/sqrt(b)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{b\,x^5+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^5+a)^(1/2),x)
```

```
[Out] int(x^(3/2)/(b*x^5+a)^(1/2),x)
```

maxima [B] time = 2.27, size = 45, normalized size = 1.41

$$-\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^5+a}}{x^{\frac{5}{2}}}}{\sqrt{b}+\frac{\sqrt{bx^5+a}}{x^{\frac{5}{2}}}}\right)}{5\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^5+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/5*log(-(sqrt(b) - sqrt(b*x^5 + a)/x^(5/2))/(sqrt(b) + sqrt(b*x^5 + a)/x^
(5/2)))/sqrt(b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^{3/2}}{\sqrt{b\,x^5+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b*x^5)^(1/2),x)
```

```
[Out] int(x^(3/2)/(a + b*x^5)^(1/2), x)
```

sympy [A] time = 2.32, size = 24, normalized size = 0.75

$$\frac{2\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x^{\frac{5}{2}}}{\sqrt{a}}\right)}{5\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**5+a)**(1/2),x)
```

```
[Out] 2*asinh(sqrt(b)*x**(5/2)/sqrt(a))/(5*sqrt(b))
```

3.1311 $\int \frac{1}{x^{7/2}\sqrt{a+bx^5}}\,dx$

Optimal. Leaf size=23

$$-\frac{2\sqrt{a+bx^5}}{5ax^{5/2}}$$

```
[Out] -2/5*(b*x^5+a)^(1/2)/a/x^(5/2)
```

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {264}

$$-\frac{2\sqrt{a+bx^5}}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*Sqrt[a + b*x^5]),x]
```

```
[Out] (-2*Sqrt[a + b*x^5])/(5*a*x^(5/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\sqrt{a+bx^5}}\,dx = -\frac{2\sqrt{a+bx^5}}{5ax^{5/2}}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$-\frac{2\sqrt{a+bx^5}}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*Sqrt[a + b*x^5]),x]
```

```
[Out] (-2*Sqrt[a + b*x^5])/(5*a*x^(5/2))
```

fricas [A] time = 0.87, size = 17, normalized size = 0.74

$$-\frac{2\sqrt{bx^5+a}}{5\,ax^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^5+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/5*sqrt(b*x^5 + a)/(a*x^(5/2))
```

giac [A] time = 0.21, size = 23, normalized size = 1.00

$$-\frac{2\sqrt{b+\frac{a}{x^5}}}{5\,a}+\frac{2\sqrt{b}}{5\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^5+a)^(1/2),x, algorithm="giac")
[Out] -2/5*sqrt(b + a/x^5)/a + 2/5*sqrt(b)/a
```

maple [A] time = 0.00, size = 18, normalized size = 0.78

$$-\frac{2\sqrt{b\,x^5+a}}{5a\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x^5+a)^(1/2),x)
[Out] -2/5*(b*x^5+a)^(1/2)/a/x^(5/2)
```

maxima [A] time = 1.01, size = 17, normalized size = 0.74

$$-\frac{2\,\sqrt{bx^5+a}}{5\,ax^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^5+a)^(1/2),x, algorithm="maxima")
[Out] -2/5*sqrt(b*x^5 + a)/(a*x^(5/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^{7/2}\,\sqrt{b\,x^5+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a + b*x^5)^(1/2)),x)
[Out] int(1/(x^(7/2)*(a + b*x^5)^(1/2)), x)
```

sympy [A] time = 7.21, size = 22, normalized size = 0.96

$$-\frac{2\sqrt{b}\,\sqrt{\frac{a}{bx^5}+1}}{5a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**5+a)**(1/2),x)
[Out] -2*sqrt(b)*sqrt(a/(b*x**5) + 1)/(5*a)
```

3.1312 $\int \frac{1}{x^{17/2}\sqrt{a+bx^5}}\,dx$

Optimal. Leaf size=48

$$\frac{4b\sqrt{a+bx^5}}{15a^2x^{5/2}} - \frac{2\sqrt{a+bx^5}}{15ax^{15/2}}$$

```
[Out] -2/15*(b*x^5+a)^(1/2)/a/x^(15/2)+4/15*b*(b*x^5+a)^(1/2)/a^2/x^(5/2)
```

Rubi [A] time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {271, 264}

$$\frac{4b\sqrt{a+bx^5}}{15a^2x^{5/2}} - \frac{2\sqrt{a+bx^5}}{15ax^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(17/2)*Sqrt[a + b*x^5]),x]
```

```
[Out] (-2*Sqrt[a + b*x^5])/(15*a*x^(15/2)) + (4*b*Sqrt[a + b*x^5])/(15*a^2*x^(5/2
))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{17/2}\sqrt{a+bx^5}}\,dx &= -\frac{2\sqrt{a+bx^5}}{15ax^{15/2}} - \frac{(2b)\int \frac{1}{x^{7/2}\sqrt{a+bx^5}}\,dx}{3a} \\ &= -\frac{2\sqrt{a+bx^5}}{15ax^{15/2}} + \frac{4b\sqrt{a+bx^5}}{15a^2x^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.65

$$-\frac{2\left(a-2bx^5\right)\sqrt{a+bx^5}}{15a^2x^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(17/2)*Sqrt[a + b*x^5]),x]
```

```
[Out] (-2*(a - 2*b*x^5)*Sqrt[a + b*x^5])/(15*a^2*x^(15/2))
```

fricas [A] time = 1.07, size = 27, normalized size = 0.56

$$\frac{2\left(2\,bx^5-a\right)\sqrt{bx^5+a}}{15\,a^2x^{\frac{15}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(17/2)/(b*x^5+a)^(1/2),x, algorithm="fricas")
[Out] 2/15*(2*b*x^5 - a)*sqrt(b*x^5 + a)/(a^2*x^(15/2))
```

giac [A] time = 0.21, size = 38, normalized size = 0.79

$$-\frac{2\left(b+\frac{a}{x^5}\right)^{\frac{3}{2}}}{15\,a^2}+\frac{2\sqrt{b+\frac{a}{x^5}}\,b}{5\,a^2}-\frac{4\,b^{\frac{3}{2}}}{15\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(17/2)/(b*x^5+a)^(1/2),x, algorithm="giac")
[Out] -2/15*(b + a/x^5)^(3/2)/a^2 + 2/5*sqrt(b + a/x^5)*b/a^2 - 4/15*b^(3/2)/a^2
```

maple [A] time = 0.00, size = 26, normalized size = 0.54

$$-\frac{2\sqrt{b\,x^5+a}\,\left(-2b\,x^5+a\right)}{15a^2x^{\frac{15}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(17/2)/(b*x^5+a)^(1/2),x)
[Out] -2/15*(b*x^5+a)^(1/2)*(-2*b*x^5+a)/x^(15/2)/a^2
```

maxima [A] time = 1.10, size = 35, normalized size = 0.73

$$\frac{2\left(\frac{3\,\sqrt{bx^5+a}\,b}{x^{\frac{5}{2}}}-\frac{\left(bx^5+a\right)^{\frac{3}{2}}}{x^{\frac{15}{2}}}\right)}{15\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(17/2)/(b*x^5+a)^(1/2),x, algorithm="maxima")
[Out] 2/15*(3*sqrt(b*x^5 + a)*b/x^(5/2) - (b*x^5 + a)^(3/2)/x^(15/2))/a^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^{17/2}\,\sqrt{b\,x^5+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(17/2)*(a + b*x^5)^(1/2)),x)
[Out] int(1/(x^(17/2)*(a + b*x^5)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(17/2)/(b*x**5+a)**(1/2),x)
[Out] Timed out
```

3.1313 $\int \frac{x^{23/2}}{\sqrt{1+x^5}}\,dx$

Optimal. Leaf size=47

$$\frac{3}{20}\sinh^{-1}\left(x^{5/2}\right)+\frac{1}{10}\sqrt{x^5+1}\,x^{15/2}-\frac{3}{20}\sqrt{x^5+1}\,x^{5/2}$$

```
[Out] 3/20*arcsinh(x^(5/2))-3/20*x^(5/2)*(x^5+1)^(1/2)+1/10*x^(15/2)*(x^5+1)^(1/2
)
```

Rubi [A] time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {321, 329, 275, 215}

$$\frac{1}{10}\sqrt{x^5+1}\,x^{15/2}-\frac{3}{20}\sqrt{x^5+1}\,x^{5/2}+\frac{3}{20}\sinh^{-1}\left(x^{5/2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^(23/2)/Sqrt[1 + x^5],x]
```

```
[Out] (-3*x^(5/2)*Sqrt[1 + x^5])/20 + (x^(15/2)*Sqrt[1 + x^5])/10 + (3*ArcSinh[x^
(5/2)])/20
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{23/2}}{\sqrt{1+x^5}}\,dx &= \frac{1}{10}x^{15/2}\sqrt{1+x^5} - \frac{3}{4}\int \frac{x^{13/2}}{\sqrt{1+x^5}}\,dx \\ &= -\frac{3}{20}x^{5/2}\sqrt{1+x^5} + \frac{1}{10}x^{15/2}\sqrt{1+x^5} + \frac{3}{8}\int \frac{x^{3/2}}{\sqrt{1+x^5}}\,dx \\ &= -\frac{3}{20}x^{5/2}\sqrt{1+x^5} + \frac{1}{10}x^{15/2}\sqrt{1+x^5} + \frac{3}{4}\,\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^{10}}}\,dx, x, \sqrt{x}\right) \\ &= -\frac{3}{20}x^{5/2}\sqrt{1+x^5} + \frac{1}{10}x^{15/2}\sqrt{1+x^5} + \frac{3}{20}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, x^{5/2}\right) \\ &= -\frac{3}{20}x^{5/2}\sqrt{1+x^5} + \frac{1}{10}x^{15/2}\sqrt{1+x^5} + \frac{3}{20}\sinh^{-1}\left(x^{5/2}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.74

$$\frac{1}{20}\left(3\sinh^{-1}\left(x^{5/2}\right) + \sqrt{x^5+1}\,\left(2x^5-3\right)x^{5/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(23/2)/Sqrt[1 + x^5],x]
```

```
[Out] (x^(5/2)*Sqrt[1 + x^5]*(-3 + 2*x^5) + 3*ArcSinh[x^(5/2)])/20
```

fricas [A] time = 0.97, size = 46, normalized size = 0.98

$$\frac{1}{20}\left(2\,x^7 - 3\,x^2\right)\sqrt{x^5+1}\,\sqrt{x} + \frac{3}{40}\,\log\left(2\,x^5 + 2\,\sqrt{x^5+1}\,x^{\frac{5}{2}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(23/2)/(x^5+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/20*(2*x^7 - 3*x^2)*sqrt(x^5 + 1)*sqrt(x) + 3/40*log(2*x^5 + 2*sqrt(x^5 +
1)*x^(5/2) + 1)
```

giac [A] time = 0.22, size = 36, normalized size = 0.77

$$\frac{1}{20}\left(2\,x^5 - 3\right)\sqrt{x^5+1}\,x^{\frac{5}{2}} - \frac{3}{20}\,\log\left(-x^{\frac{5}{2}} + \sqrt{x^5+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(23/2)/(x^5+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/20*(2*x^5 - 3)*sqrt(x^5 + 1)*x^(5/2) - 3/20*log(-x^(5/2) + sqrt(x^5 + 1))
```

maple [A] time = 0.14, size = 46, normalized size = 0.98

$$\frac{\left(2x^5-3\right)\sqrt{x^5+1}\;x^{\frac{5}{2}}}{20} + \frac{3\sqrt{\left(x^5+1\right)x}\;\operatorname{arcsinh}\left(x^{\frac{5}{2}}\right)}{20\sqrt{x^5+1}\;\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(23/2)/(x^5+1)^(1/2),x)
```

```
[Out] 1/20*x^(5/2)*(2*x^5-3)*(x^5+1)^(1/2)+3/20*arcsinh(x^(5/2))*(x*(x^5+1))^(1/2
)/x^(1/2)/(x^5+1)^(1/2)
```

maxima [B] time = 1.12, size = 86, normalized size = 1.83

$$-\frac{\frac{5\sqrt{x^5+1}}{x^{\frac{5}{2}}}-\frac{3\left(x^5+1\right)^{\frac{3}{2}}}{x^{\frac{15}{2}}}}{20\left(\frac{2\left(x^5+1\right)}{x^5}-\frac{\left(x^5+1\right)^2}{x^{10}}-1\right)}+\frac{3}{40}\log\left(\frac{\sqrt{x^5+1}}{x^{\frac{5}{2}}}+1\right)-\frac{3}{40}\log\left(\frac{\sqrt{x^5+1}}{x^{\frac{5}{2}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(23/2)/(x^5+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/20*(5*sqrt(x^5 + 1)/x^(5/2) - 3*(x^5 + 1)^(3/2)/x^(15/2))/(2*(x^5 + 1)/x
^5 - (x^5 + 1)^2/x^10 - 1) + 3/40*log(sqrt(x^5 + 1)/x^(5/2) + 1) - 3/40*log
(sqrt(x^5 + 1)/x^(5/2) - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{23/2}}{\sqrt{x^5+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(23/2)/(x^5 + 1)^(1/2),x)
```

```
[Out] int(x^(23/2)/(x^5 + 1)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(23/2)/(x**5+1)**(1/2),x)
```

```
[Out] Timed out
```

3.1314 $\int \frac{x^{13/2}}{\sqrt{1+x^5}}\,dx$

Optimal. Leaf size=29

$$\frac{1}{5}x^{5/2}\sqrt{x^5+1}-\frac{1}{5}\sinh^{-1}\left(x^{5/2}\right)$$

```
[Out] -1/5*arcsinh(x^(5/2))+1/5*x^(5/2)*(x^5+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {321, 329, 275, 215}

$$\frac{1}{5}x^{5/2}\sqrt{x^5+1}-\frac{1}{5}\sinh^{-1}\left(x^{5/2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^(13/2)/Sqrt[1 + x^5],x]
```

```
[Out] (x^(5/2)*Sqrt[1 + x^5])/5 - ArcSinh[x^(5/2)]/5
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{13/2}}{\sqrt{1+x^5}}\,dx &= \frac{1}{5}x^{5/2}\sqrt{1+x^5}-\frac{1}{2}\int \frac{x^{3/2}}{\sqrt{1+x^5}}\,dx \\
&= \frac{1}{5}x^{5/2}\sqrt{1+x^5}-\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^{10}}}\,dx,x,\sqrt{x}\right) \\
&= \frac{1}{5}x^{5/2}\sqrt{1+x^5}-\frac{1}{5}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,x^{5/2}\right) \\
&= \frac{1}{5}x^{5/2}\sqrt{1+x^5}-\frac{1}{5}\sinh^{-1}\left(x^{5/2}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{1}{5}x^{5/2}\sqrt{x^5+1}\;-\frac{1}{5}\sinh^{-1}\left(x^{5/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(13/2)/Sqrt[1 + x^5],x]
[Out] (x^(5/2)*Sqrt[1 + x^5])/5 - ArcSinh[x^(5/2)]/5
```

fricas [A] time = 0.91, size = 35, normalized size = 1.21

$$\frac{1}{5}\sqrt{x^5+1}\,x^{\frac{5}{2}}+\frac{1}{10}\log\left(-2\,x^5+2\sqrt{x^5+1}\,x^{\frac{5}{2}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(x^5+1)^(1/2),x, algorithm="fricas")
[Out] 1/5*sqrt(x^5 + 1)*x^(5/2) + 1/10*log(-2*x^5 + 2*sqrt(x^5 + 1)*x^(5/2) - 1)
```

giac [A] time = 0.25, size = 29, normalized size = 1.00

$$\frac{1}{5}\sqrt{x^5+1}\,x^{\frac{5}{2}}+\frac{1}{5}\log\left(-x^{\frac{5}{2}}+\sqrt{x^5+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(x^5+1)^(1/2),x, algorithm="giac")
[Out] 1/5*sqrt(x^5 + 1)*x^(5/2) + 1/5*log(-x^(5/2) + sqrt(x^5 + 1))
```

maple [A] time = 0.13, size = 39, normalized size = 1.34

$$\frac{\sqrt{x^5+1}\;x^{\frac{5}{2}}}{5}-\frac{\sqrt{\left(x^5+1\right)x}\;\operatorname{arcsinh}\left(x^{\frac{5}{2}}\right)}{5\sqrt{x^5+1}\;\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(x^5+1)^(1/2),x)
[Out] 1/5*x^(5/2)*(x^5+1)^(1/2)-1/5*arcsinh(x^(5/2))*((x^5+1)*x)^(1/2)/x^(1/2)/(x
^5+1)^(1/2)
```

maxima [B] time = 1.05, size = 58, normalized size = 2.00

$$\frac{\sqrt{x^5+1}}{5\,x^{\frac{5}{2}}\left(\frac{x^5+1}{x^5}-1\right)}-\frac{1}{10}\log\left(\frac{\sqrt{x^5+1}}{x^{\frac{5}{2}}}+1\right)+\frac{1}{10}\log\left(\frac{\sqrt{x^5+1}}{x^{\frac{5}{2}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(x^5+1)^(1/2),x, algorithm="maxima")
[Out] 1/5*sqrt(x^5 + 1)/(x^(5/2)*((x^5 + 1)/x^5 - 1)) - 1/10*log(sqrt(x^5 + 1)/x^
(5/2) + 1) + 1/10*log(sqrt(x^5 + 1)/x^(5/2) - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^{13/2}}{\sqrt{x^5+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(x^5 + 1)^(1/2),x)
[Out] int(x^(13/2)/(x^5 + 1)^(1/2), x)
```

sympy [**A**] time = 145.06, size = 22, normalized size = 0.76

$$\frac{x^{\frac{5}{2}}\sqrt{x^5+1}}{5} - \frac{\operatorname{asinh}\left(x^{\frac{5}{2}}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)/(x**5+1)**(1/2),x)
[Out] x**(5/2)*sqrt(x**5 + 1)/5 - asinh(x**(5/2))/5
```

3.1315 $\int \frac{x^{3/2}}{\sqrt{1+x^5}}\,dx$

Optimal. Leaf size=10

$$\frac{2}{5}\sinh^{-1}\left(x^{5/2}\right)$$

```
[Out] 2/5*arcsinh(x^(5/2))
```

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {329, 275, 215}

$$\frac{2}{5}\sinh^{-1}\left(x^{5/2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[1 + x^5],x]
```

```
[Out] (2*ArcSinh[x^(5/2)])/5
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{3/2}}{\sqrt{1+x^5}}\,dx &= 2\,\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^{10}}}\,dx, x, \sqrt{x}\right)\\ &= \frac{2}{5}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, x^{5/2}\right)\\ &= \frac{2}{5}\sinh^{-1}\left(x^{5/2}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{2}{5}\sinh^{-1}\left(x^{5/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[1 + x^5],x]
```

```
[Out] (2*ArcSinh[x^(5/2)])/5
```

fricas [B] time = 0.88, size = 22, normalized size = 2.20

$$\frac{1}{5}\log\left(2x^5+2\sqrt{x^5+1}\,x^{\frac{5}{2}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(x^5+1)^(1/2),x, algorithm="fricas")
[Out] 1/5*log(2*x^5 + 2*sqrt(x^5 + 1)*x^(5/2) + 1)
```

giac [B] time = 0.17, size = 16, normalized size = 1.60

$$-\frac{2}{5}\log\left(-x^{\frac{5}{2}}+\sqrt{x^5+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(x^5+1)^(1/2),x, algorithm="giac")
[Out] -2/5*log(-x^(5/2) + sqrt(x^5 + 1))
```

maple [A] time = 0.13, size = 7, normalized size = 0.70

$$\frac{2\,\operatorname{arcsinh}\left(x^{\frac{5}{2}}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(x^5+1)^(1/2),x)
[Out] 2/5*arcsinh(x^(5/2))
```

maxima [B] time = 0.93, size = 33, normalized size = 3.30

$$\frac{1}{5}\log\left(\frac{\sqrt{x^5+1}}{x^{\frac{5}{2}}}+1\right)-\frac{1}{5}\log\left(\frac{\sqrt{x^5+1}}{x^{\frac{5}{2}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(x^5+1)^(1/2),x, algorithm="maxima")
[Out] 1/5*log(sqrt(x^5 + 1)/x^(5/2) + 1) - 1/5*log(sqrt(x^5 + 1)/x^(5/2) - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.10

$$\int \frac{x^{3/2}}{\sqrt{x^5+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(x^5 + 1)^(1/2),x)
[Out] int(x^(3/2)/(x^5 + 1)^(1/2), x)
```

sympy [A] time = 2.38, size = 8, normalized size = 0.80

$$\frac{2\,\operatorname{asinh}\left(x^{\frac{5}{2}}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(x**5+1)**(1/2),x)
[Out] 2*asinh(x**(5/2))/5
```

3.1316 $\int \frac{1}{x^{7/2}\sqrt{1+x^5}}\,dx$

Optimal. Leaf size=18

$$-\frac{2\sqrt{x^5+1}}{5x^{5/2}}$$

```
[Out] -2/5*(x^5+1)^(1/2)/x^(5/2)
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {264}

$$-\frac{2\sqrt{x^5+1}}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*Sqrt[1 + x^5]),x]
```

```
[Out] (-2*Sqrt[1 + x^5])/(5*x^(5/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\sqrt{1+x^5}}\,dx = -\frac{2\sqrt{1+x^5}}{5x^{5/2}}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$-\frac{2\sqrt{x^5+1}}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*Sqrt[1 + x^5]),x]
```

```
[Out] (-2*Sqrt[1 + x^5])/(5*x^(5/2))
```

fricas [A] time = 0.85, size = 12, normalized size = 0.67

$$-\frac{2\sqrt{x^5+1}}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(x^5+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/5*sqrt(x^5 + 1)/x^(5/2)
```

giac [A] time = 0.18, size = 11, normalized size = 0.61

$$-\frac{2}{5}\sqrt{\frac{1}{x^5}+1}+\frac{2}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(x^5+1)^(1/2),x, algorithm="giac")
[Out] -2/5*sqrt(1/x^5 + 1) + 2/5
```

maple [B] time = 0.01, size = 32, normalized size = 1.78

$$-\frac{2\,(x+1)\left(x^4-x^3+x^2-x+1\right)}{5\sqrt{x^5+1}\;x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(x^5+1)^(1/2),x)
[Out] -2/5/x^(5/2)*(x+1)*(x^4-x^3+x^2-x+1)/(x^5+1)^(1/2)
```

maxima [A] time = 1.05, size = 12, normalized size = 0.67

$$-\frac{2\sqrt{x^5+1}}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(x^5+1)^(1/2),x, algorithm="maxima")
[Out] -2/5*sqrt(x^5 + 1)/x^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{x^{7/2}\sqrt{x^5+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(x^5 + 1)^(1/2)),x)
[Out] int(1/(x^(7/2)*(x^5 + 1)^(1/2)), x)
```

sympy [A] time = 11.35, size = 14, normalized size = 0.78

$$-\frac{2\sqrt{1+\frac{1}{x^5}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(x**5+1)**(1/2),x)
[Out] -2*sqrt(1 + x**(-5))/5
```

3.1317 $\int \frac{1}{x^{17/2}\sqrt{1+x^5}}\,dx$

Optimal. Leaf size=37

$$\frac{4\sqrt{x^5+1}}{15x^{5/2}}-\frac{2\sqrt{x^5+1}}{15x^{15/2}}$$

```
[Out] -2/15*(x^5+1)^(1/2)/x^(15/2)+4/15*(x^5+1)^(1/2)/x^(5/2)
```

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$\frac{4\sqrt{x^5+1}}{15x^{5/2}}-\frac{2\sqrt{x^5+1}}{15x^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(17/2)*Sqrt[1 + x^5]),x]
```

```
[Out] (-2*Sqrt[1 + x^5])/(15*x^(15/2)) + (4*Sqrt[1 + x^5])/(15*x^(5/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{17/2}\sqrt{1+x^5}}\,dx &= -\frac{2\sqrt{1+x^5}}{15x^{15/2}}-\frac{2}{3}\int \frac{1}{x^{7/2}\sqrt{1+x^5}}\,dx\\ &= -\frac{2\sqrt{1+x^5}}{15x^{15/2}}+\frac{4\sqrt{1+x^5}}{15x^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.68

$$-\frac{2\left(1-2x^5\right)\sqrt{x^5+1}}{15x^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(17/2)*Sqrt[1 + x^5]),x]
```

```
[Out] (-2*(1 - 2*x^5)*Sqrt[1 + x^5])/(15*x^(15/2))
```

fricas [A] time = 0.85, size = 19, normalized size = 0.51

$$\frac{2\left(2x^5-1\right)\sqrt{x^5+1}}{15\,x^{\frac{15}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(17/2)/(x^5+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(2*x^5 - 1)*sqrt(x^5 + 1)/x^(15/2)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(17/2)/(x^5+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [abs(t_nostep)]sym2poly/r2sym(co
nst gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [**A**] time = 0.00, size = 39, normalized size = 1.05

$$\frac{2\,(x+1)\left(x^4-x^3+x^2-x+1\right)\left(2x^5-1\right)}{15\sqrt{x^5+1}\;x^{\frac{15}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(17/2)/(x^5+1)^(1/2),x)
```

```
[Out] 2/15*(x+1)*(x^4-x^3+x^2-x+1)*(2*x^5-1)/x^(15/2)/(x^5+1)^(1/2)
```

maxima [**A**] time = 0.95, size = 25, normalized size = 0.68

$$\frac{2\sqrt{x^5+1}}{5\,x^{\frac{5}{2}}}-\frac{2\left(x^5+1\right)^{\frac{3}{2}}}{15\,x^{\frac{15}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(17/2)/(x^5+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*sqrt(x^5 + 1)/x^(5/2) - 2/15*(x^5 + 1)^(3/2)/x^(15/2)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x^{17/2}\sqrt{x^5+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(17/2)*(x^5 + 1)^(1/2)),x)
```

```
[Out] int(1/(x^(17/2)*(x^5 + 1)^(1/2)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(17/2)/(x**5+1)**(1/2),x)
```

```
[Out] Timed out
```

3.1318 $\int \frac{x^8}{a+bx^6}\,dx$

Optimal. Leaf size=40

$$\frac{x^3}{3b} - \frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x^3}{\sqrt{a}}\right)}{3b^{3/2}}$$

```
[Out] 1/3*x^3/b-1/3*arctan(x^3*b^(1/2)/a^(1/2))*a^(1/2)/b^(3/2)
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {275, 321, 205}

$$\frac{x^3}{3b} - \frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x^3}{\sqrt{a}}\right)}{3b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^6),x]
```

```
[Out] x^3/(3*b) - (Sqrt[a]*ArcTan[(Sqrt[b]*x^3)/Sqrt[a]])/(3*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{a+bx^6}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{a+bx^2}\,dx, x, x^3\right) \\
&= \frac{x^3}{3b} - \frac{a\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^3\right)}{3b} \\
&= \frac{x^3}{3b} - \frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x^3}{\sqrt{a}}\right)}{3b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 1.00

$$\frac{x^3}{3b} - \frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x^3}{\sqrt{a}}\right)}{3b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^6),x]
```

```
[Out] x^3/(3*b) - (Sqrt[a]*ArcTan[(Sqrt[b]*x^3)/Sqrt[a]])/(3*b^(3/2))
```

fricas [A] time = 0.84, size = 89, normalized size = 2.22

$$\left[\frac{2\,x^3+\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^6-2\,bx^3\sqrt{-\frac{a}{b}}-a}{bx^6+a}\right)}{6\,b},\ \frac{x^3-\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx^3\sqrt{\frac{a}{b}}}{a}\right)}{3\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^6+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*x^3 + sqrt(-a/b)*log((b*x^6 - 2*b*x^3*sqrt(-a/b) - a)/(b*x^6 + a)))
/b, 1/3*(x^3 - sqrt(a/b)*arctan(b*x^3*sqrt(a/b)/a))/b]
```

giac [A] time = 0.15, size = 31, normalized size = 0.78

$$\frac{x^3}{3\,b}-\frac{a\arctan\left(\frac{bx^3}{\sqrt{ab}}\right)}{3\,\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^6+a),x, algorithm="giac")
```

```
[Out] 1/3*x^3/b - 1/3*a*arctan(b*x^3/sqrt(a*b))/(sqrt(a*b)*b)
```

maple [A] time = 0.00, size = 32, normalized size = 0.80

$$\frac{x^3}{3b}-\frac{a\arctan\left(\frac{b\,x^3}{\sqrt{ab}}\right)}{3\sqrt{ab}\ b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^6+a),x)
```

```
[Out] 1/3/b*x^3-1/3*a/b/(a*b)^(1/2)*arctan(b*x^3/(a*b)^(1/2))
```

maxima [A] time = 2.27, size = 31, normalized size = 0.78

$$\frac{x^3}{3\,b}-\frac{a\arctan\left(\frac{bx^3}{\sqrt{ab}}\right)}{3\,\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^6+a),x, algorithm="maxima")
```

```
[Out] 1/3*x^3/b - 1/3*a*arctan(b*x^3/sqrt(a*b))/(sqrt(a*b)*b)
```

mupad [B] time = 1.02, size = 28, normalized size = 0.70

$$\frac{x^3}{3\,b}-\frac{\sqrt{a}\ \mathrm{atan}\left(\frac{\sqrt{b}\,x^3}{\sqrt{a}}\right)}{3\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^6),x)
```

```
[Out] x^3/(3*b) - (a^(1/2)*atan((b^(1/2)*x^3)/a^(1/2)))/(3*b^(3/2))
```

sympy [**A**] time = 0.34, size = 63, normalized size = 1.58

$$\frac{\sqrt{-\frac{a}{b^3}}\,\log\left(-b\sqrt{-\frac{a}{b^3}}+x^3\right)}{6}-\frac{\sqrt{-\frac{a}{b^3}}\,\log\left(b\sqrt{-\frac{a}{b^3}}+x^3\right)}{6}+\frac{x^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**6+a),x)
```

```
[Out] sqrt(-a/b**3)*log(-b*sqrt(-a/b**3) + x**3)/6 - sqrt(-a/b**3)*log(b*sqrt(-a/
b**3) + x**3)/6 + x**3/(3*b)
```

3.1319 $\int \frac{x^7}{a+bx^6}\,dx$

Optimal. Leaf size=133

$$\frac{\sqrt[3]{a}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^2+b^{2/3}x^4\right)}{12b^{4/3}}-\frac{\sqrt[3]{a}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{6b^{4/3}}+\frac{\sqrt[3]{a}\,\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x^2}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2\sqrt{3}\,b^{4/3}}+\frac{x^2}{2b}$$

```
[Out] 1/2*x^2/b-1/6*a^(1/3)*ln(a^(1/3)+b^(1/3)*x^2)/b^(4/3)+1/12*a^(1/3)*ln(a^(2/
3)-a^(1/3)*b^(1/3)*x^2+b^(2/3)*x^4)/b^(4/3)+1/6*a^(1/3)*arctan(1/3*(a^(1/3)
-2*b^(1/3)*x^2)/a^(1/3)*3^(1/2))/b^(4/3)*3^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {275, 321, 200, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{a}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^2+b^{2/3}x^4\right)}{12b^{4/3}}-\frac{\sqrt[3]{a}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{6b^{4/3}}+\frac{\sqrt[3]{a}\,\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x^2}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2\sqrt{3}\,b^{4/3}}+\frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^6),x]
```

```
[Out] x^2/(2*b) + (a^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x^2)/(Sqrt[3]*a^(1/3))])/(
2*Sqrt[3]*b^(4/3)) - (a^(1/3)*Log[a^(1/3) + b^(1/3)*x^2])/(6*b^(4/3)) + (a^
(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^2 + b^(2/3)*x^4])/(12*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^7}{a+bx^6}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^3}{a+bx^3}\,dx, x, x^2\right)$$

$$= \frac{x^2}{2b} - \frac{a\operatorname{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, x^2\right)}{2b}$$

$$= \frac{x^2}{2b} - \frac{\sqrt[3]{a}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, x^2\right)}{6b} - \frac{\sqrt[3]{a}\operatorname{Subst}\left(\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, x^2\right)}{6b}$$

$$= \frac{x^2}{2b} - \frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{6b^{4/3}} + \frac{\sqrt[3]{a}\operatorname{Subst}\left(\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, x^2\right)}{12b^{4/3}} - \frac{a^{2/3}\operatorname{Subst}\left(\int \frac{}{a^{2/3}-\sqrt[3]{}}\right.}{}$$

$$= \frac{x^2}{2b} - \frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{6b^{4/3}} + \frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x^2+b^{2/3}x^4\right)}{12b^{4/3}} - \frac{\sqrt[3]{a}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x,\right.}{2b^{4/3}}$$

$$= \frac{x^2}{2b} + \frac{\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x^2}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}\,b^{4/3}} - \frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{6b^{4/3}} + \frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x^2+b^{2/3}x^4\right)}{12b^{4/3}}$$

Mathematica [A] time = 0.06, size = 186, normalized size = 1.40

$$\frac{-2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)+\sqrt[3]{a}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)+\sqrt[3]{a}\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)+2}{12b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^6),x]
```

```
[Out] (6*b^(1/3)*x^2 + 2*Sqrt[3]*a^(1/3)*ArcTan[Sqrt[3] - (2*b^(1/6)*x)/a^(1/6)]
+ 2*Sqrt[3]*a^(1/3)*ArcTan[Sqrt[3] + (2*b^(1/6)*x)/a^(1/6)] - 2*a^(1/3)*Log
[a^(1/3) + b^(1/3)*x^2] + a^(1/3)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*x +
 b^(1/3)*x^2] + a^(1/3)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*x + b^(1/3)*x
^2])/(12*b^(4/3))
```

fricas [A] time = 0.88, size = 114, normalized size = 0.86

$$\frac{6x^2 + 2\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx^2\left(-\frac{a}{b}\right)^{\frac{2}{3}} - \sqrt{3}a}{3a}\right) - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^4 + x^2\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right) + 2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2 - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^6+a),x, algorithm="fricas")
```

```
[Out] 1/12*(6*x^2 + 2*sqrt(3)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x^2*(-a/b)^(2/
3) - sqrt(3)*a)/a) - (-a/b)^(1/3)*log(x^4 + x^2*(-a/b)^(1/3) + (-a/b)^(2/3)
) + 2*(-a/b)^(1/3)*log(x^2 - (-a/b)^(1/3)))/b
```

giac [A] time = 0.16, size = 120, normalized size = 0.90

$$\frac{x^2}{2b} + \frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^2 - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{6b} - \frac{\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x^2 + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{6b^2} - \frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^4 + x^2\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{12b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^6+a),x, algorithm="giac")
```

```
[Out] 1/2*x^2/b + 1/6*(-a/b)^(1/3)*log(abs(x^2 - (-a/b)^(1/3)))/b - 1/6*sqrt(3)*(
-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^2 + (-a/b)^(1/3))/(-a/b)^(1/3))/b^2 -
 1/12*(-a*b^2)^(1/3)*log(x^4 + x^2*(-a/b)^(1/3) + (-a/b)^(2/3))/b^2
```

maple [A] time = 0.00, size = 108, normalized size = 0.81

$$\frac{x^2}{2b} - \frac{\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(\frac{2x^2}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2} - \frac{a\ln\left(x^2 + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2} + \frac{a\ln\left(x^4 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x^2 + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{12\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^6+a),x)
```

```
[Out] 1/2/b*x^2-1/6*a/b^2/(a/b)^(2/3)*ln(x^2+(a/b)^(1/3))+1/12*a/b^2/(a/b)^(2/3)*
ln(x^4-(a/b)^(1/3)*x^2+(a/b)^(2/3))-1/6*a/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/
3*3^(1/2)*(2/(a/b)^(1/3)*x^2-1))
```

maxima [A] time = 2.47, size = 115, normalized size = 0.86

$$\frac{x^2}{2b} - \frac{\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(2x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{6b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{a\log\left(x^4 - x^2\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{12b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{a\log\left(x^2 + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^6+a),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/b - 1/6*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x^2 - (a/b)^(1/3))/(a/b)^(1
/3))/(b^2*(a/b)^(2/3)) + 1/12*a*log(x^4 - x^2*(a/b)^(1/3) + (a/b)^(2/3))/(b
^2*(a/b)^(2/3)) - 1/6*a*log(x^2 + (a/b)^(1/3))/(b^2*(a/b)^(2/3))
```

mupad [B] time = 1.25, size = 130, normalized size = 0.98

$$\frac{x^2}{2\,b}+\frac{(-a)^{1/3}\,\ln\left((-a)^{10/3}+a^3\,b^{1/3}\,x^2\right)}{6\,b^{4/3}}+\frac{(-a)^{1/3}\,\ln\left(6\,a^3\,b\,x^2+6\,(-a)^{10/3}\,b^{2/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{6\,b^{4/3}}-\frac{(-a)^{1}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^6),x)
```

```
[Out] x^2/(2*b) + ((-a)^(1/3)*log((-a)^(10/3) + a^3*b^(1/3)*x^2))/(6*b^(4/3)) + (
(-a)^(1/3)*log(6*a^3*b*x^2 + 6*(-a)^(10/3)*b^(2/3)*((3^(1/2)*1i)/2 - 1/2))*
((3^(1/2)*1i)/2 - 1/2))/(6*b^(4/3)) - ((-a)^(1/3)*log(6*a^3*b*x^2 - 6*(-a)^
(10/3)*b^(2/3)*((3^(1/2)*1i)/2 + 1/2))*((3^(1/2)*1i)/2 + 1/2))/(6*b^(4/3))
```

sympy [A] time = 0.43, size = 27, normalized size = 0.20

$$\mathrm{RootSum}\left(216t^3b^4+a,\left(t\mapsto t\log\left(-6tb+x^2\right)\right)\right)+\frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**6+a),x)
```

```
[Out] RootSum(216*_t**3*b**4 + a, Lambda(_t, _t*log(-6*_t*b + x**2))) + x**2/(2*b
)
```

3.1320 $\int \frac{x^6}{a+bx^6}\,dx$

Optimal. Leaf size=220

$$\frac{\sqrt[6]{a}\,\log\left(-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{4\sqrt{3}\,b^{7/6}}-\frac{\sqrt[6]{a}\,\log\left(\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{4\sqrt{3}\,b^{7/6}}-\frac{\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{3b^{7/6}}+\frac{\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{}}{6b^{7/6}}\right.}$$

```
[Out] x/b-1/3*a^(1/6)*arctan(b^(1/6)*x/a^(1/6))/b^(7/6)+1/6*a^(1/6)*arctan((-2*b^
(1/6)*x+a^(1/6)*3^(1/2))/a^(1/6))/b^(7/6)-1/6*a^(1/6)*arctan((2*b^(1/6)*x+a
^(1/6)*3^(1/2))/a^(1/6))/b^(7/6)+1/12*a^(1/6)*ln(a^(1/3)+b^(1/3)*x^2-a^(1/6
)*b^(1/6)*x*3^(1/2))/b^(7/6)*3^(1/2)-1/12*a^(1/6)*ln(a^(1/3)+b^(1/3)*x^2+a^
(1/6)*b^(1/6)*x*3^(1/2))/b^(7/6)*3^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {321, 209, 634, 618, 204, 628, 205}

$$\frac{\sqrt[6]{a}\,\log\left(-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{4\sqrt{3}\,b^{7/6}}-\frac{\sqrt[6]{a}\,\log\left(\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{4\sqrt{3}\,b^{7/6}}-\frac{\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{3b^{7/6}}+\frac{\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{}}{6b^{7/6}}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^6),x]
```

```
[Out] x/b - (a^(1/6)*ArcTan[(b^(1/6)*x)/a^(1/6)])/(3*b^(7/6)) + (a^(1/6)*ArcTan[(
Sqrt[3]*a^(1/6) - 2*b^(1/6)*x)/a^(1/6)])/(6*b^(7/6)) - (a^(1/6)*ArcTan[(Sqr
t[3]*a^(1/6) + 2*b^(1/6)*x)/a^(1/6)])/(6*b^(7/6)) + (a^(1/6)*Log[a^(1/3) -
Sqrt[3]*a^(1/6)*b^(1/6)*x + b^(1/3)*x^2])/(4*Sqrt[3]*b^(7/6)) - (a^(1/6)*Lo
g[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*x + b^(1/3)*x^2])/(4*Sqrt[3]*b^(7/6))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^6}{a+bx^6}\,dx = \frac{x}{b} - \frac{a\int \frac{1}{a+bx^6}\,dx}{b}$$

$$= \frac{x}{b} - \frac{\sqrt[6]{a}\int \frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\,\sqrt[6]{b}\,x}{\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx}{3b} - \frac{\sqrt[6]{a}\int \frac{\sqrt[6]{a}+\frac{1}{2}\sqrt{3}\,\sqrt[6]{b}\,x}{\sqrt[3]{a}+\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx}{3b} - \frac{\sqrt[3]{a}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x^2}\,dx}{3b}$$

$$= \frac{x}{b} - \frac{\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{3b^{7/6}} + \frac{\sqrt[6]{a}\int \frac{-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}+2\sqrt[3]{b}\,x}{\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx}{4\sqrt{3}\,b^{7/6}} - \frac{\sqrt[6]{a}\int \frac{\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}+2\sqrt[3]{b}\,x}{\sqrt[3]{a}+\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx}{4\sqrt{3}\,b^{7/6}} - \frac{\sqrt[3]{a}\int$$

$$= \frac{x}{b} - \frac{\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{3b^{7/6}} + \frac{\sqrt[6]{a}\,\log\left(\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2\right)}{4\sqrt{3}\,b^{7/6}} - \frac{\sqrt[6]{a}\,\log\left(\sqrt[3]{a}+\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\right.}{4\sqrt{3}\,b^{7/6}}$$

$$= \frac{x}{b} - \frac{\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{3b^{7/6}} + \frac{\sqrt[6]{a}\,\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{6b^{7/6}} - \frac{\sqrt[6]{a}\,\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{6b^{7/6}} + \frac{\sqrt[6]{a}\,\log\left(\sqrt[3]{a}-\right.}{4}$$

Mathematica [A] time = 0.04, size = 182, normalized size = 0.83

$$\frac{\sqrt{3}\,\sqrt[6]{a}\,\log\left(-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)-\sqrt{3}\,\sqrt[6]{a}\,\log\left(\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)-4\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)+2}{12b^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^6),x]
```

```
[Out] (12*b^(1/6)*x - 4*a^(1/6)*ArcTan[(b^(1/6)*x)/a^(1/6)] + 2*a^(1/6)*ArcTan[Sq
rt[3] - (2*b^(1/6)*x)/a^(1/6)] - 2*a^(1/6)*ArcTan[Sqrt[3] + (2*b^(1/6)*x)/a
^(1/6)] + Sqrt[3]*a^(1/6)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*x + b^(1/3)
*x^2] - Sqrt[3]*a^(1/6)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*x + b^(1/3)*x
^2])/(12*b^(7/6))
```

fricas [B] time = 0.91, size = 314, normalized size = 1.43

$$-\frac{4\sqrt{3}\,b\left(-\frac{a}{b^7}\right)^{\frac{1}{6}}\arctan\left(-\frac{2\sqrt{3}\,b^6x\left(-\frac{a}{b^7}\right)^{\frac{5}{6}}-2\sqrt{3}\sqrt{b^2\left(-\frac{a}{b^7}\right)^{\frac{1}{3}}+bx\left(-\frac{a}{b^7}\right)^{\frac{1}{6}}+x^2}\,b^6\left(-\frac{a}{b^7}\right)^{\frac{5}{6}}-\sqrt{3}\,a}{3\,a}\right)+4\sqrt{3}\,b\left(-\frac{a}{b^7}\right)^{\frac{1}{6}}\arctan\left(-\frac{2\sqrt{3}\,b^6x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^6+a),x, algorithm="fricas")
```

```
[Out] -1/12*(4*sqrt(3)*b*(-a/b^7)^(1/6)*arctan(-1/3*(2*sqrt(3)*b^6*x*(-a/b^7)^(5/
6) - 2*sqrt(3)*sqrt(b^2*(-a/b^7)^(1/3) + b*x*(-a/b^7)^(1/6) + x^2)*b^6*(-a/
b^7)^(5/6) - sqrt(3)*a)/a) + 4*sqrt(3)*b*(-a/b^7)^(1/6)*arctan(-1/3*(2*sqrt
(3)*b^6*x*(-a/b^7)^(5/6) - 2*sqrt(3)*sqrt(b^2*(-a/b^7)^(1/3) - b*x*(-a/b^7)
^(1/6) + x^2)*b^6*(-a/b^7)^(5/6) + sqrt(3)*a)/a) + b*(-a/b^7)^(1/6)*log(b^2
*(-a/b^7)^(1/3) + b*x*(-a/b^7)^(1/6) + x^2) - b*(-a/b^7)^(1/6)*log(b^2*(-a/
b^7)^(1/3) - b*x*(-a/b^7)^(1/6) + x^2) + 2*b*(-a/b^7)^(1/6)*log(b*(-a/b^7)^
(1/6) + x) - 2*b*(-a/b^7)^(1/6)*log(-b*(-a/b^7)^(1/6) + x) - 12*x)/b
```

giac [A] time = 0.16, size = 180, normalized size = 0.82

$$\frac{x}{b}-\frac{\sqrt{3}\,\left(ab^5\right)^{\frac{1}{6}}\log\left(x^2+\sqrt{3}\,x\left(\frac{a}{b}\right)^{\frac{1}{6}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12\,b^2}+\frac{\sqrt{3}\,\left(ab^5\right)^{\frac{1}{6}}\log\left(x^2-\sqrt{3}\,x\left(\frac{a}{b}\right)^{\frac{1}{6}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12\,b^2}-\frac{\left(ab^5\right)^{\frac{1}{6}}\arctan\left(\frac{2\,x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right.}{6\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^6+a),x, algorithm="giac")
```

```
[Out] x/b - 1/12*sqrt(3)*(a*b^5)^(1/6)*log(x^2 + sqrt(3)*x*(a/b)^(1/6) + (a/b)^(1
/3))/b^2 + 1/12*sqrt(3)*(a*b^5)^(1/6)*log(x^2 - sqrt(3)*x*(a/b)^(1/6) + (a/
b)^(1/3))/b^2 - 1/6*(a*b^5)^(1/6)*arctan((2*x + sqrt(3)*(a/b)^(1/6))/(a/b)^
(1/6))/b^2 - 1/6*(a*b^5)^(1/6)*arctan((2*x - sqrt(3)*(a/b)^(1/6))/(a/b)^(1/
6))/b^2 - 1/3*(a*b^5)^(1/6)*arctan(x/(a/b)^(1/6))/b^2
```

maple [A] time = 0.10, size = 164, normalized size = 0.75

$$\frac{x}{b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6b}+\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x^2-\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}x+\left(\frac{a}{b}\right.\right.}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^6+a),x)
```

```
[Out] 1/b*x-1/3/b*(a/b)^(1/6)*arctan(x/(a/b)^(1/6))+1/12/b*3^(1/2)*(a/b)^(1/6)*ln
(x^2-3^(1/2)*(a/b)^(1/6)*x+(a/b)^(1/3))-1/6/b*(a/b)^(1/6)*arctan(2*x/(a/b)^
(1/6)-3^(1/2))-1/12/b*3^(1/2)*(a/b)^(1/6)*ln(x^2+3^(1/2)*(a/b)^(1/6)*x+(a/b
)^(1/3))-1/6/b*(a/b)^(1/6)*arctan(2*x/(a/b)^(1/6)+3^(1/2))
```

maxima [A] time = 2.43, size = 194, normalized size = 0.88

$$-\frac{\frac{\sqrt{3}\,a^{\frac{1}{6}}\log\left(b^{\frac{1}{3}}x^2+\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{b^{\frac{1}{6}}}-\frac{\sqrt{3}\,a^{\frac{1}{6}}\log\left(b^{\frac{1}{3}}x^2-\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{b^{\frac{1}{6}}}+\frac{4\,a^{\frac{1}{3}}\arctan\left(\frac{b^{\frac{1}{3}}x}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{2\,a^{\frac{1}{3}}\arctan\left(\frac{2\,b^{\frac{1}{3}}x+\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{2\,a^{\frac{1}{3}}\arcta}{}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^6+a),x, algorithm="maxima")
```

```
[Out] -1/12*(sqrt(3)*a^(1/6)*log(b^(1/3)*x^2 + sqrt(3)*a^(1/6)*b^(1/6)*x + a^(1/3
))/b^(1/6) - sqrt(3)*a^(1/6)*log(b^(1/3)*x^2 - sqrt(3)*a^(1/6)*b^(1/6)*x +
a^(1/3))/b^(1/6) + 4*a^(1/3)*arctan(b^(1/3)*x/sqrt(a^(1/3)*b^(1/3)))/sqrt(a
^(1/3)*b^(1/3)) + 2*a^(1/3)*arctan((2*b^(1/3)*x + sqrt(3)*a^(1/6)*b^(1/6))/
sqrt(a^(1/3)*b^(1/3)))/sqrt(a^(1/3)*b^(1/3)) + 2*a^(1/3)*arctan((2*b^(1/3)*
x - sqrt(3)*a^(1/6)*b^(1/6))/sqrt(a^(1/3)*b^(1/3)))/sqrt(a^(1/3)*b^(1/3)))/
b + x/b
```

mupad [B] time = 1.17, size = 227, normalized size = 1.03

$$\frac{x}{b}+\frac{(-a)^{1/6}\,\mathrm{atan}\left(\frac{b^{1/6}\,x\,1\mathrm{i}}{(-a)^{1/6}}\right)\,1\mathrm{i}}{3\,b^{7/6}}+\frac{(-a)^{1/6}\,\mathrm{atan}\left(\frac{(-a)^{25/6}\,x\,1\mathrm{i}}{b^{1/6}\left(\frac{(-a)^{13/3}}{b^{1/3}}+\frac{\sqrt{3}\,(-a)^{13/3}\,1\mathrm{i}}{b^{1/3}}\right)}+\frac{\sqrt{3}\,(-a)^{25/6}\,x}{b^{1/6}\left(\frac{(-a)^{13/3}}{b^{1/3}}+\frac{\sqrt{3}\,(-a)^{13/3}\,1\mathrm{i}}{b^{1/3}}\right)}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,1\mathrm{i}\quad($$
$$\frac{}{3\,b^{7/6}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^6),x)
```

```
[Out] x/b + ((-a)^(1/6)*atan((b^(1/6)*x*1i)/(-a)^(1/6))*1i)/(3*b^(7/6)) + ((-a)^(
1/6)*atan(((-a)^(25/6)*x*1i)/(b^(1/6)*((-a)^(13/3)/b^(1/3) + (3^(1/2)*(-a)^
(13/3)*1i)/b^(1/3))) + (3^(1/2)*(-a)^(25/6)*x)/(b^(1/6)*((-a)^(13/3)/b^(1/3
) + (3^(1/2)*(-a)^(13/3)*1i)/b^(1/3))))*((3^(1/2)*1i)/2 - 1/2)*1i)/(3*b^(7/
6)) - ((-a)^(1/6)*atan(((-a)^(25/6)*x*1i)/(b^(1/6)*((-a)^(13/3)/b^(1/3) - (
3^(1/2)*(-a)^(13/3)*1i)/b^(1/3))) - (3^(1/2)*(-a)^(25/6)*x)/(b^(1/6)*((-a)^
(13/3)/b^(1/3) - (3^(1/2)*(-a)^(13/3)*1i)/b^(1/3))))*((3^(1/2)*1i)/2 + 1/2)
*1i)/(3*b^(7/6))
```

sympy [A] time = 0.23, size = 22, normalized size = 0.10

$$\mathrm{RootSum}\left(46656t^6b^7+a,\left(t\mapsto t\log\left(-6tb+x\right)\right)\right)+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**6+a),x)
```

```
[Out] RootSum(46656*_t**6*b**7 + a, Lambda(_t, _t*log(-6*_t*b + x))) + x/b
```

3.1321 $\int \frac{x^5}{a+bx^6}\,dx$

Optimal. Leaf size=15

$$\frac{\log\left(a+bx^6\right)}{6b}$$

```
[Out] 1/6*ln(b*x^6+a)/b
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {260}

$$\frac{\log\left(a+bx^6\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^6),x]
```

```
[Out] Log[a + b*x^6]/(6*b)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^5}{a+bx^6}\,dx = \frac{\log\left(a+bx^6\right)}{6b}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log\left(a+bx^6\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^6),x]
```

```
[Out] Log[a + b*x^6]/(6*b)
```

fricas [A] time = 0.85, size = 13, normalized size = 0.87

$$\frac{\log\left(bx^6+a\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^6+a),x, algorithm="fricas")
```

```
[Out] 1/6*log(b*x^6 + a)/b
```

giac [A] time = 0.16, size = 14, normalized size = 0.93

$$\frac{\log\left(\left|bx^6+a\right|\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^6+a),x, algorithm="giac")
[Out] 1/6*log(abs(b*x^6 + a))/b
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{\ln\left(b\,x^6+a\right)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^6+a),x)
[Out] 1/6*ln(b*x^6+a)/b
```

maxima [A] time = 0.99, size = 13, normalized size = 0.87

$$\frac{\log\left(bx^6+a\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^6+a),x, algorithm="maxima")
[Out] 1/6*log(b*x^6 + a)/b
```

mupad [B] time = 1.03, size = 13, normalized size = 0.87

$$\frac{\ln\left(b\,x^6+a\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^6),x)
[Out] log(a + b*x^6)/(6*b)
```

sympy [A] time = 0.24, size = 10, normalized size = 0.67

$$\frac{\log\left(a+bx^6\right)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**6+a),x)
[Out] log(a + b*x**6)/(6*b)
```

3.1322 $\int \frac{x^4}{a+bx^6}\,dx$

Optimal. Leaf size=215

$$\frac{\log\left(-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{4\sqrt{3}\,\sqrt[6]{a}\,b^{5/6}}-\frac{\log\left(\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{4\sqrt{3}\,\sqrt[6]{a}\,b^{5/6}}+\frac{\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}\,b^{5/6}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}-2\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{6\sqrt[6]{a}\,b^{5/6}}+\frac{\tan^{-1}}{}$$

```
[Out] 1/3*arctan(b^(1/6)*x/a^(1/6))/a^(1/6)/b^(5/6)-1/6*arctan((-2*b^(1/6)*x+a^(1
/6)*3^(1/2))/a^(1/6))/a^(1/6)/b^(5/6)+1/6*arctan((2*b^(1/6)*x+a^(1/6)*3^(1/
2))/a^(1/6))/a^(1/6)/b^(5/6)+1/12*ln(a^(1/3)+b^(1/3)*x^2-a^(1/6)*b^(1/6)*x*
3^(1/2))/a^(1/6)/b^(5/6)*3^(1/2)-1/12*ln(a^(1/3)+b^(1/3)*x^2+a^(1/6)*b^(1/6
)*x*3^(1/2))/a^(1/6)/b^(5/6)*3^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {295, 634, 618, 204, 628, 205}

$$\frac{\log\left(-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{4\sqrt{3}\,\sqrt[6]{a}\,b^{5/6}}-\frac{\log\left(\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{4\sqrt{3}\,\sqrt[6]{a}\,b^{5/6}}+\frac{\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}\,b^{5/6}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}-2\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{6\sqrt[6]{a}\,b^{5/6}}+\frac{\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^6),x]
```

```
[Out] ArcTan[(b^(1/6)*x)/a^(1/6)]/(3*a^(1/6)*b^(5/6)) - ArcTan[(Sqrt[3]*a^(1/6) -
 2*b^(1/6)*x)/a^(1/6)]/(6*a^(1/6)*b^(5/6)) + ArcTan[(Sqrt[3]*a^(1/6) + 2*b^
(1/6)*x)/a^(1/6)]/(6*a^(1/6)*b^(5/6)) + Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/
6)*x + b^(1/3)*x^2]/(4*Sqrt[3]*a^(1/6)*b^(5/6)) - Log[a^(1/3) + Sqrt[3]*a^(
1/6)*b^(1/6)*x + b^(1/3)*x^2]/(4*Sqrt[3]*a^(1/6)*b^(5/6))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^4}{a+bx^6}\,dx = \frac{\int \frac{-\frac{\sqrt[6]{a}}{2}+\frac{1}{2}\sqrt{3}\,\sqrt[6]{b}\,x}{\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx}{3\sqrt[6]{a}\,b^{2/3}} + \frac{\int \frac{-\frac{\sqrt[6]{a}}{2}-\frac{1}{2}\sqrt{3}\,\sqrt[6]{b}\,x}{\sqrt[3]{a}+\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx}{3\sqrt[6]{a}\,b^{2/3}} + \frac{\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x^2}\,dx}{3b^{2/3}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}\,b^{5/6}} + \frac{\int \frac{-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}+2\sqrt[3]{b}\,x}{\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx}{4\sqrt{3}\,\sqrt[6]{a}\,b^{5/6}} - \frac{\int \frac{\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}+2\sqrt[3]{b}\,x}{\sqrt[3]{a}+\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx}{4\sqrt{3}\,\sqrt[6]{a}\,b^{5/6}} + \frac{\int \frac{1}{\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}}{12b^{2/3}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}\,b^{5/6}} + \frac{\log\left(\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2\right)}{4\sqrt{3}\,\sqrt[6]{a}\,b^{5/6}} - \frac{\log\left(\sqrt[3]{a}+\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2\right)}{4\sqrt{3}\,\sqrt[6]{a}\,b^{5/6}} + \frac{\text{Subst}}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}\,b^{5/6}} - \frac{\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{6\sqrt[6]{a}\,b^{5/6}} + \frac{\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{6\sqrt[6]{a}\,b^{5/6}} + \frac{\log\left(\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2\right.}{4\sqrt{3}\,\sqrt[6]{a}\,b^{5/6}}$$

Mathematica [A] time = 0.02, size = 154, normalized size = 0.72

$$\frac{\sqrt{3}\,\log\left(-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)-\sqrt{3}\,\log\left(\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)+4\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)-2\tan^{-1}\left(\sqrt{3}\right.}{12\sqrt[6]{a}\,b^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^6),x]
```

```
[Out] (4*ArcTan[(b^(1/6)*x)/a^(1/6)] - 2*ArcTan[Sqrt[3] - (2*b^(1/6)*x)/a^(1/6)]
+ 2*ArcTan[Sqrt[3] + (2*b^(1/6)*x)/a^(1/6)] + Sqrt[3]*Log[a^(1/3) - Sqrt[3]
*a^(1/6)*b^(1/6)*x + b^(1/3)*x^2] - Sqrt[3]*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b
^(1/6)*x + b^(1/3)*x^2])/(12*a^(1/6)*b^(5/6))
```

fricas [B] time = 0.93, size = 348, normalized size = 1.62

$$-\frac{1}{3}\sqrt{3}\left(-\frac{1}{ab^5}\right)^{\frac{1}{6}}\arctan\left(-\frac{2}{3}\sqrt{3}\,bx\left(-\frac{1}{ab^5}\right)^{\frac{1}{6}}+\frac{2}{3}\sqrt{3}\sqrt{ab^4x\left(-\frac{1}{ab^5}\right)^{\frac{5}{6}}-ab^3\left(-\frac{1}{ab^5}\right)^{\frac{2}{3}}+x^2}\,b\left(-\frac{1}{ab^5}\right)^{\frac{1}{6}}+\frac{1}{3}\sqrt{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^6+a),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*(-1/(a*b^5))^(1/6)*arctan(-2/3*sqrt(3)*b*x*(-1/(a*b^5))^(1/6)
+ 2/3*sqrt(3)*sqrt(a*b^4*x*(-1/(a*b^5))^(5/6) - a*b^3*(-1/(a*b^5))^(2/3) +
```

```
x^2)*b*(-1/(a*b^5))^(1/6) + 1/3*sqrt(3)) - 1/3*sqrt(3)*(-1/(a*b^5))^(1/6)*a
rctan(-2/3*sqrt(3)*b*x*(-1/(a*b^5))^(1/6) + 2/3*sqrt(3)*sqrt(-a*b^4*x*(-1/(
a*b^5))^(5/6) - a*b^3*(-1/(a*b^5))^(2/3) + x^2)*b*(-1/(a*b^5))^(1/6) - 1/3*
sqrt(3)) + 1/12*(-1/(a*b^5))^(1/6)*log(a*b^4*x*(-1/(a*b^5))^(5/6) - a*b^3*(
-1/(a*b^5))^(2/3) + x^2) - 1/12*(-1/(a*b^5))^(1/6)*log(-a*b^4*x*(-1/(a*b^5)
)^(5/6) - a*b^3*(-1/(a*b^5))^(2/3) + x^2) + 1/6*(-1/(a*b^5))^(1/6)*log(a*b^
4*(-1/(a*b^5))^(5/6) + x) - 1/6*(-1/(a*b^5))^(1/6)*log(-a*b^4*(-1/(a*b^5))^
(5/6) + x)
```

giac [A] time = 0.27, size = 187, normalized size = 0.87

$$\frac{\left(\frac{a}{b}\right)^{\frac{5}{6}}\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(ab^5\right)^{\frac{5}{6}}\log\left(x^2+\sqrt{3}\,x\left(\frac{a}{b}\right)^{\frac{1}{6}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12ab^5}+\frac{\sqrt{3}\left(ab^5\right)^{\frac{5}{6}}\log\left(x^2-\sqrt{3}\,x\left(\frac{a}{b}\right)^{\frac{1}{6}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12ab^5}+\frac{\left(ab^5\right)^{\frac{5}{6}}a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^6+a),x, algorithm="giac")
```

```
[Out] 1/3*(a/b)^(5/6)*arctan(x/(a/b)^(1/6))/a - 1/12*sqrt(3)*(a*b^5)^(5/6)*log(x^
2 + sqrt(3)*x*(a/b)^(1/6) + (a/b)^(1/3))/(a*b^5) + 1/12*sqrt(3)*(a*b^5)^(5/
6)*log(x^2 - sqrt(3)*x*(a/b)^(1/6) + (a/b)^(1/3))/(a*b^5) + 1/6*(a*b^5)^(5/
6)*arctan((2*x + sqrt(3)*(a/b)^(1/6))/(a/b)^(1/6))/(a*b^5) + 1/6*(a*b^5)^(5
/6)*arctan((2*x - sqrt(3)*(a/b)^(1/6))/(a/b)^(1/6))/(a*b^5)
```

maple [A] time = 0.06, size = 159, normalized size = 0.74

$$\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x^2-\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}-\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x^2+\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{6}}b}+\frac{\arctan\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{\right.}\right.}{6\left(\frac{a}{b}\right)^{\frac{1}{6}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^6+a),x)
```

```
[Out] 1/3/b/(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x)+1/12/a*3^(1/2)*(a/b)^(5/6)*ln(x^2
-3^(1/2)*(a/b)^(1/6)*x+(a/b)^(1/3))+1/6/b/(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*
x-3^(1/2))-1/12/a*3^(1/2)*(a/b)^(5/6)*ln(x^2+3^(1/2)*(a/b)^(1/6)*x+(a/b)^(1
/3))+1/6/b/(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x+3^(1/2))
```

maxima [A] time = 2.38, size = 184, normalized size = 0.86

$$-\frac{\sqrt{3}\,\log\left(b^{\frac{1}{3}}x^2+\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{12\,a^{\frac{1}{6}}b^{\frac{5}{6}}}+\frac{\sqrt{3}\,\log\left(b^{\frac{1}{3}}x^2-\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{12\,a^{\frac{1}{6}}b^{\frac{5}{6}}}+\frac{\arctan\left(\frac{b^{\frac{1}{3}}x}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{3\,b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{\arctan\left(\frac{2\,b^{\frac{1}{3}}x+\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^6+a),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*log(b^(1/3)*x^2 + sqrt(3)*a^(1/6)*b^(1/6)*x + a^(1/3))/(a^(1/
6)*b^(5/6)) + 1/12*sqrt(3)*log(b^(1/3)*x^2 - sqrt(3)*a^(1/6)*b^(1/6)*x + a^
(1/3))/(a^(1/6)*b^(5/6)) + 1/3*arctan(b^(1/3)*x/sqrt(a^(1/3)*b^(1/3)))/(b^(
2/3)*sqrt(a^(1/3)*b^(1/3))) + 1/6*arctan((2*b^(1/3)*x + sqrt(3)*a^(1/6)*b^(
1/6))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) + 1/6*arctan((
2*b^(1/3)*x - sqrt(3)*a^(1/6)*b^(1/6))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt
(a^(1/3)*b^(1/3)))
```

mupad [B] time = 1.14, size = 134, normalized size = 0.62

$$-\frac{\operatorname{atanh}\left(\frac{b^{1/6}x}{(-a)^{1/6}}\right)}{3(-a)^{1/6}b^{5/6}}-\frac{\operatorname{atanh}\left(\frac{2(-a)^{5/2}b^{3/2}x}{(-a)^{8/3}b^{4/3}-\sqrt{3}\,(-a)^{8/3}b^{4/3}\,1\mathrm{i}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6(-a)^{1/6}b^{5/6}}+\frac{\operatorname{atanh}\left(\frac{2(-a)^{5/2}b^{3/2}x}{(-a)^{8/3}b^{4/3}+\sqrt{3}\,(-a)^{8/3}b^{4/3}\,1\mathrm{i}}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6(-a)^{1/6}b^{5/6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^6),x)
```

```
[Out] (atanh((2*(-a)^(5/2)*b^(3/2)*x)/((-a)^(8/3)*b^(4/3) + 3^(1/2)*(-a)^(8/3)*b^
(4/3)*1i))*(3^(1/2)*1i - 1))/(6*(-a)^(1/6)*b^(5/6)) - (atanh((2*(-a)^(5/2)*
b^(3/2)*x)/((-a)^(8/3)*b^(4/3) - 3^(1/2)*(-a)^(8/3)*b^(4/3)*1i))*(3^(1/2)*1
i + 1))/(6*(-a)^(1/6)*b^(5/6)) - atanh((b^(1/6)*x)/(-a)^(1/6))/(3*(-a)^(1/6
)*b^(5/6))
```

sympy [A] time = 0.38, size = 26, normalized size = 0.12

$$\operatorname{RootSum}\left(46656t^6ab^5+1,\left(t\mapsto t\log\left(7776t^5ab^4+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**6+a),x)
```

```
[Out] RootSum(46656*_t**6*a*b**5 + 1, Lambda(_t, _t*log(7776*_t**5*a*b**4 + x)))
```

3.1323 $\int \frac{x^3}{a+bx^6}\,dx$

Optimal. Leaf size=123

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^2+b^{2/3}x^4\right)}{12\sqrt[3]{a}\,b^{2/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{6\sqrt[3]{a}\,b^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x^2}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2\sqrt{3}\,\sqrt[3]{a}\,b^{2/3}}$$

```
[Out] -1/6*ln(a^(1/3)+b^(1/3)*x^2)/a^(1/3)/b^(2/3)+1/12*ln(a^(2/3)-a^(1/3)*b^(1/3
)*x^2+b^(2/3)*x^4)/a^(1/3)/b^(2/3)-1/6*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^2)/a
^(1/3)*3^(1/2))/a^(1/3)/b^(2/3)*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {275, 292, 31, 634, 617, 204, 628}

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^2+b^{2/3}x^4\right)}{12\sqrt[3]{a}\,b^{2/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{6\sqrt[3]{a}\,b^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x^2}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2\sqrt{3}\,\sqrt[3]{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^6),x]
```

```
[Out] -ArcTan[(a^(1/3) - 2*b^(1/3)*x^2)/(Sqrt[3]*a^(1/3))]/(2*Sqrt[3]*a^(1/3)*b^(
2/3)) - Log[a^(1/3) + b^(1/3)*x^2]/(6*a^(1/3)*b^(2/3)) + Log[a^(2/3) - a^(1
/3)*b^(1/3)*x^2 + b^(2/3)*x^4]/(12*a^(1/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{a+bx^6}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{a+bx^3}\,dx, x, x^2\right) \\
&= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, x^2\right)}{6\sqrt[3]{a}\,\sqrt[3]{b}} + \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, x^2\right)}{6\sqrt[3]{a}\,\sqrt[3]{b}} \\
&= -\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{6\sqrt[3]{a}\,b^{2/3}} + \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, x^2\right)}{12\sqrt[3]{a}\,b^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x\right.}{4\sqrt[3]{b}} \\
&= -\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{6\sqrt[3]{a}\,b^{2/3}} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^2+b^{2/3}x^4\right)}{12\sqrt[3]{a}\,b^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}\,x^2}{\sqrt[3]{a}}\right)}{2\sqrt[3]{a}\,b^{2/3}} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x^2}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2\sqrt{3}\,\sqrt[3]{a}\,b^{2/3}} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{6\sqrt[3]{a}\,b^{2/3}} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^2+b^{2/3}x^4\right)}{12\sqrt[3]{a}\,b^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 154, normalized size = 1.25

$$\frac{-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)+\log\left(-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)+\log\left(\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)-2\sqrt{3}\,\tan^{-1}\left(\sqrt{}\right.}{12\sqrt[3]{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^6),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[Sqrt[3] - (2*b^(1/6)*x)/a^(1/6)] - 2*Sqrt[3]*ArcTan[Sqrt
[3] + (2*b^(1/6)*x)/a^(1/6)] - 2*Log[a^(1/3) + b^(1/3)*x^2] + Log[a^(1/3) -
 Sqrt[3]*a^(1/6)*b^(1/6)*x + b^(1/3)*x^2] + Log[a^(1/3) + Sqrt[3]*a^(1/6)*b
^(1/6)*x + b^(1/3)*x^2])/(12*a^(1/3)*b^(2/3))
```

fricas [A] time = 0.85, size = 318, normalized size = 2.59

$$\left[\frac{3\sqrt{\frac{1}{3}}\,ab\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\,\log\left(\frac{2b^2x^6-3\left(-ab^2\right)^{\frac{2}{3}}x^2-ab+3\sqrt{\frac{1}{3}}\left(2\left(-ab^2\right)^{\frac{2}{3}}x^4+abx^2+\left(-ab^2\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}}{bx^6+a}\right)+\left(-ab^2\right)^{\frac{2}{3}}\log\left(b^2x^4+\left(-ab\right.\right.\right.}{12\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^6+a),x, algorithm="fricas")
```

```
[Out] [1/12*(3*sqrt(1/3)*a*b*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^6 - 3*(-a*b^2)^(
2/3)*x^2 - a*b + 3*sqrt(1/3)*(2*(-a*b^2)^(2/3)*x^4 + a*b*x^2 + (-a*b^2)^(1/
3)*a)*sqrt((-a*b^2)^(1/3)/a))/(b*x^6 + a)) + (-a*b^2)^(2/3)*log(b^2*x^4 + (
-a*b^2)^(1/3)*b*x^2 + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*log(b*x^2 - (-a*b^
2)^(1/3)))/(a*b^2), 1/12*(6*sqrt(1/3)*a*b*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sq
rt(1/3)*(2*b*x^2 + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (-a*b^2)^(2
/3)*log(b^2*x^4 + (-a*b^2)^(1/3)*b*x^2 + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)
*log(b*x^2 - (-a*b^2)^(1/3)))/(a*b^2)]
```

giac [A] time = 0.17, size = 118, normalized size = 0.96

$$-\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^2-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{6a}-\frac{\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x^2+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{6ab^2}+\frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^4+x^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{12ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^6+a),x, algorithm="giac")
```

```
[Out] -1/6*(-a/b)^(2/3)*log(abs(x^2 - (-a/b)^(1/3)))/a - 1/6*sqrt(3)*(-a*b^2)^(2/
3)*arctan(1/3*sqrt(3)*(2*x^2 + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2) + 1/12*(
-a*b^2)^(2/3)*log(x^4 + x^2*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2)
```

maple [A] time = 0.00, size = 97, normalized size = 0.79

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^2}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b}-\frac{\ln\left(x^2+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b}+\frac{\ln\left(x^4-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^2+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{12\left(\frac{a}{b}\right)^{\frac{1}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^6+a),x)
```

```
[Out] -1/6/b/(a/b)^(1/3)*ln(x^2+(a/b)^(1/3))+1/12/b/(a/b)^(1/3)*ln(x^4-(a/b)^(1/3
)*x^2+(a/b)^(2/3))+1/6*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1
/3)*x^2-1))
```

maxima [A] time = 2.33, size = 104, normalized size = 0.85

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^4-x^2\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{12b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\log\left(x^2+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^6+a),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^
(1/3)) + 1/12*log(x^4 - x^2*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(1/3)) - 1/
6*log(x^2 + (a/b)^(1/3))/(b*(a/b)^(1/3))
```

mupad [B] time = 1.31, size = 118, normalized size = 0.96

$$\frac{\ln\left(a+(-a)^{2/3}\,b^{1/3}\,x^2\right)}{6\,(-a)^{1/3}\,b^{2/3}}+\frac{\ln\left(a\,b^2+\frac{(-a)^{2/3}\,b^{7/3}\,x^2\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12\,(-a)^{1/3}\,b^{2/3}}-\frac{\ln\left(a\,b^2-\frac{(-a)^{2/3}\,b^{7/3}\,x^2\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(1+\sqrt{}\right.}{12\,(-a)^{1/3}\,b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^6),x)
```

```
[Out] log(a + (-a)^(2/3)*b^(1/3)*x^2)/(6*(-a)^(1/3)*b^(2/3)) + (log(a*b^2 + ((-a)
^(2/3)*b^(7/3)*x^2*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(12*(-a)^(1/3)*b^
(2/3)) - (log(a*b^2 - ((-a)^(2/3)*b^(7/3)*x^2*(3^(1/2)*1i + 1))/2)*(3^(1/2)
*1i + 1))/(12*(-a)^(1/3)*b^(2/3))
```

sympy [A] time = 0.22, size = 26, normalized size = 0.21

$$\mathrm{RootSum}\left(216t^3ab^2+1,\left(t\mapsto t\log\left(36t^2ab+x^2\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**6+a),x)
```

```
[Out] RootSum(216*_t**3*a*b**2 + 1, Lambda(_t, _t*log(36*_t**2*a*b + x**2)))
```

3.1324 $\int \frac{x^2}{a+bx^6}\,dx$

Optimal. Leaf size=29

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x^3}{\sqrt{a}}\right)}{3\sqrt{a}\,\sqrt{b}}$$

```
[Out] 1/3*arctan(x^3*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {275, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x^3}{\sqrt{a}}\right)}{3\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^6),x]
```

```
[Out] ArcTan[(Sqrt[b]*x^3)/Sqrt[a]]/(3*Sqrt[a]*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{a+bx^6}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^3\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x^3}{\sqrt{a}}\right)}{3\sqrt{a}\,\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x^3}{\sqrt{a}}\right)}{3\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^6),x]
```

```
[Out] ArcTan[(Sqrt[b]*x^3)/Sqrt[a]]/(3*Sqrt[a]*Sqrt[b])
```

fricas [A] time = 0.89, size = 72, normalized size = 2.48

$$\left[-\frac{\sqrt{-ab}\,\log\left(\frac{bx^6-2\sqrt{-ab}\,x^3-a}{bx^6+a}\right)}{6\,ab}, \frac{\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x^3}{a}\right)}{3\,ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^6+a),x, algorithm="fricas")
```

```
[Out] [-1/6*sqrt(-a*b)*log((b*x^6 - 2*sqrt(-a*b)*x^3 - a)/(b*x^6 + a))/(a*b), 1/3
*sqrt(a*b)*arctan(sqrt(a*b)*x^3/a)/(a*b)]
```

giac [A] time = 0.16, size = 18, normalized size = 0.62

$$\frac{\arctan\left(\frac{bx^3}{\sqrt{ab}}\right)}{3\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^6+a),x, algorithm="giac")
```

```
[Out] 1/3*arctan(b*x^3/sqrt(a*b))/sqrt(a*b)
```

maple [A] time = 0.00, size = 19, normalized size = 0.66

$$\frac{\arctan\left(\frac{b\,x^3}{\sqrt{ab}}\right)}{3\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^6+a),x)
```

```
[Out] 1/3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^3)
```

maxima [A] time = 2.32, size = 18, normalized size = 0.62

$$\frac{\arctan\left(\frac{bx^3}{\sqrt{ab}}\right)}{3\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^6+a),x, algorithm="maxima")
```

```
[Out] 1/3*arctan(b*x^3/sqrt(a*b))/sqrt(a*b)
```

mupad [B] time = 0.05, size = 19, normalized size = 0.66

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x^3}{\sqrt{a}}\right)}{3\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^6),x)
```

```
[Out] atan((b^(1/2)*x^3)/a^(1/2))/(3*a^(1/2)*b^(1/2))
```

sympy [B] time = 0.26, size = 56, normalized size = 1.93

$$-\frac{\sqrt{-\frac{1}{ab}}\,\log\left(-a\sqrt{-\frac{1}{ab}}+x^3\right)}{6}+\frac{\sqrt{-\frac{1}{ab}}\,\log\left(a\sqrt{-\frac{1}{ab}}+x^3\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**6+a),x)
```

```
[Out] -sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x**3)/6 + sqrt(-1/(a*b))*log(a*sqrt
(-1/(a*b)) + x**3)/6
```

3.1325 $\int \frac{x}{a+bx^6}\,dx$

Optimal. Leaf size=123

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^2+b^{2/3}x^4\right)}{12a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{6a^{2/3}\sqrt[3]{b}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x^2}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2\sqrt{3}\,a^{2/3}\sqrt[3]{b}}$$

```
[Out] 1/6*ln(a^(1/3)+b^(1/3)*x^2)/a^(2/3)/b^(1/3)-1/12*ln(a^(2/3)-a^(1/3)*b^(1/3)
*x^2+b^(2/3)*x^4)/a^(2/3)/b^(1/3)-1/6*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^2)/a^
(1/3)*3^(1/2))/a^(2/3)/b^(1/3)*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {275, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^2+b^{2/3}x^4\right)}{12a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{6a^{2/3}\sqrt[3]{b}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x^2}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2\sqrt{3}\,a^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^6),x]
```

```
[Out] -ArcTan[(a^(1/3) - 2*b^(1/3)*x^2)/(Sqrt[3]*a^(1/3))]/(2*Sqrt[3]*a^(2/3)*b^(
1/3)) + Log[a^(1/3) + b^(1/3)*x^2]/(6*a^(2/3)*b^(1/3)) - Log[a^(2/3) - a^(1
/3)*b^(1/3)*x^2 + b^(2/3)*x^4]/(12*a^(2/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{a+bx^6}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, x^2\right) \\
&= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, x^2\right)}{6a^{2/3}} + \frac{\text{Subst}\left(\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, x^2\right)}{6a^{2/3}} \\
&= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{6a^{2/3}\sqrt[3]{b}} + \frac{\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, x^2\right)}{4\sqrt[3]{a}} - \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x,\right.}{12a^{2/3}\sqrt[3]{b}} \\
&= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{6a^{2/3}\sqrt[3]{b}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^2+b^{2/3}x^4\right)}{12a^{2/3}\sqrt[3]{b}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}\,x^2}{\sqrt[3]{a}}\right)}{2a^{2/3}\sqrt[3]{b}} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x^2}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2\sqrt{3}\,a^{2/3}\sqrt[3]{b}} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{6a^{2/3}\sqrt[3]{b}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^2+b^{2/3}x^4\right)}{12a^{2/3}\sqrt[3]{b}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 154, normalized size = 1.25

$$-\frac{-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)+\log\left(-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)+\log\left(\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)+2\sqrt{3}\,\tan^{-1}\left(\right.}{12a^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^6),x]
```

```
[Out] -1/12*(2*Sqrt[3]*ArcTan[Sqrt[3] - (2*b^(1/6)*x)/a^(1/6)] + 2*Sqrt[3]*ArcTan
[Sqrt[3] + (2*b^(1/6)*x)/a^(1/6)] - 2*Log[a^(1/3) + b^(1/3)*x^2] + Log[a^(1
/3) - Sqrt[3]*a^(1/6)*b^(1/6)*x + b^(1/3)*x^2] + Log[a^(1/3) + Sqrt[3]*a^(1
/6)*b^(1/6)*x + b^(1/3)*x^2])/(a^(2/3)*b^(1/3))
```

fricas [A] time = 0.99, size = 313, normalized size = 2.54

$$\left[\frac{3\sqrt{\frac{1}{3}}\,ab\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\,\log\left(\frac{2abx^6-3\left(a^2b\right)^{\frac{1}{3}}ax^2-a^2+3\sqrt{\frac{1}{3}}\left(2abx^4+\left(a^2b\right)^{\frac{2}{3}}x^2-\left(a^2b\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}}{bx^6+a}\right)-\left(a^2b\right)^{\frac{2}{3}}\log\left(abx^4-\left(a^2b\right)^{\frac{2}{3}}x\right.}{12\,a^2b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^6+a),x, algorithm="fricas")
```

```
[Out] [1/12*(3*sqrt(1/3)*a*b*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^6 - 3*(a^2*b)^(1
/3)*a*x^2 - a^2 + 3*sqrt(1/3)*(2*a*b*x^4 + (a^2*b)^(2/3)*x^2 - (a^2*b)^(1/3
)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^6 + a)) - (a^2*b)^(2/3)*log(a*b*x^4 - (a^
2*b)^(2/3)*x^2 + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*x^2 + (a^2*b)^(
2/3)))/(a^2*b), 1/12*(6*sqrt(1/3)*a*b*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3
)*(2*(a^2*b)^(2/3)*x^2 - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - (a^2
*b)^(2/3)*log(a*b*x^4 - (a^2*b)^(2/3)*x^2 + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2
/3)*log(a*b*x^2 + (a^2*b)^(2/3)))/(a^2*b)]
```

giac [A] time = 0.19, size = 118, normalized size = 0.96

$$-\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^2-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{6\,a}+\frac{\sqrt{3}\,\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^2+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{6\,ab}+\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^4+x^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{12\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^6+a),x, algorithm="giac")
```

```
[Out] -1/6*(-a/b)^(1/3)*log(abs(x^2 - (-a/b)^(1/3)))/a + 1/6*sqrt(3)*(-a*b^2)^(1/
3)*arctan(1/3*sqrt(3)*(2*x^2 + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b) + 1/12*(-a
*b^2)^(1/3)*log(x^4 + x^2*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b)
```

maple [A] time = 0.00, size = 97, normalized size = 0.79

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^2}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b}+\frac{\ln\left(x^2+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b}-\frac{\ln\left(x^4-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^2+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{12\left(\frac{a}{b}\right)^{\frac{2}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^6+a),x)
```

```
[Out] 1/6/b/(a/b)^(2/3)*ln(x^2+(a/b)^(1/3))-1/12/b/(a/b)^(2/3)*ln(x^4-(a/b)^(1/3)
*x^2+(a/b)^(2/3))+1/6/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/
3)*x^2-1))
```

maxima [A] time = 2.34, size = 104, normalized size = 0.85

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^4-x^2\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{12\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\log\left(x^2+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^6+a),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^
(2/3)) - 1/12*log(x^4 - x^2*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)) + 1/
6*log(x^2 + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

mupad [B] time = 0.24, size = 105, normalized size = 0.85

$$\frac{\ln\left(a^{1/3}+b^{1/3}x^2\right)}{6a^{2/3}b^{1/3}}+\frac{\ln\left(6b^4x^2+3a^{1/3}b^{11/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12a^{2/3}b^{1/3}}-\frac{\ln\left(6b^4x^2-3a^{1/3}b^{11/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)\right)}{12a^{2/3}b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^6),x)
```

```
[Out] log(a^(1/3) + b^(1/3)*x^2)/(6*a^(2/3)*b^(1/3)) + (log(6*b^4*x^2 + 3*a^(1/3)
*b^(11/3)*(3^(1/2)*1i - 1))*(3^(1/2)*1i - 1))/(12*a^(2/3)*b^(1/3)) - (log(6
*b^4*x^2 - 3*a^(1/3)*b^(11/3)*(3^(1/2)*1i + 1))*(3^(1/2)*1i + 1))/(12*a^(2/
3)*b^(1/3))
```

sympy [A] time = 0.24, size = 22, normalized size = 0.18

$$\mathrm{RootSum}\left(216t^3a^2b-1,\left(t\mapsto t\log\left(6ta+x^2\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**6+a),x)
```

```
[Out] RootSum(216*_t**3*a**2*b - 1, Lambda(_t, _t*log(6*_t*a + x**2)))
```

3.1326 $\int \frac{1}{a+bx^6}\,dx$

Optimal. Leaf size=215

$$-\frac{\log\left(-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{4\sqrt{3}\,a^{5/6}\sqrt[6]{b}}+\frac{\log\left(\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{4\sqrt{3}\,a^{5/6}\sqrt[6]{b}}+\frac{\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}-2\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{6a^{5/6}\sqrt[6]{b}}+\tan$$

```
[Out] 1/3*arctan(b^(1/6)*x/a^(1/6))/a^(5/6)/b^(1/6)-1/6*arctan((-2*b^(1/6)*x+a^(1
/6)*3^(1/2))/a^(1/6))/a^(5/6)/b^(1/6)+1/6*arctan((2*b^(1/6)*x+a^(1/6)*3^(1/
2))/a^(1/6))/a^(5/6)/b^(1/6)-1/12*ln(a^(1/3)+b^(1/3)*x^2-a^(1/6)*b^(1/6)*x*
3^(1/2))/a^(5/6)/b^(1/6)*3^(1/2)+1/12*ln(a^(1/3)+b^(1/3)*x^2+a^(1/6)*b^(1/6
)*x*3^(1/2))/a^(5/6)/b^(1/6)*3^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {209, 634, 618, 204, 628, 205}

$$-\frac{\log\left(-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{4\sqrt{3}\,a^{5/6}\sqrt[6]{b}}+\frac{\log\left(\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{4\sqrt{3}\,a^{5/6}\sqrt[6]{b}}+\frac{\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}-2\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{6a^{5/6}\sqrt[6]{b}}+\tan$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^6)^(-1),x]
```

```
[Out] ArcTan[(b^(1/6)*x)/a^(1/6)]/(3*a^(5/6)*b^(1/6)) - ArcTan[(Sqrt[3]*a^(1/6) -
 2*b^(1/6)*x)/a^(1/6)]/(6*a^(5/6)*b^(1/6)) + ArcTan[(Sqrt[3]*a^(1/6) + 2*b^
(1/6)*x)/a^(1/6)]/(6*a^(5/6)*b^(1/6)) - Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/
6)*x + b^(1/3)*x^2]/(4*Sqrt[3]*a^(5/6)*b^(1/6)) + Log[a^(1/3) + Sqrt[3]*a^(
1/6)*b^(1/6)*x + b^(1/3)*x^2]/(4*Sqrt[3]*a^(5/6)*b^(1/6))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{a+bx^6}\,dx &= \frac{\int \frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\,\sqrt[6]{b}\,x}{\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx}{3a^{5/6}} + \frac{\int \frac{\sqrt[6]{a}+\frac{1}{2}\sqrt{3}\,\sqrt[6]{b}\,x}{\sqrt[3]{a}+\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx}{3a^{5/6}} + \frac{\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x^2}\,dx}{3a^{2/3}} \\
&= \frac{\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}} + \frac{\int \frac{1}{\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx}{12a^{2/3}} + \frac{\int \frac{1}{\sqrt[3]{a}+\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx}{12a^{2/3}} - \frac{\int \frac{-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}+2\sqrt[3]{b}\,x}{\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx}{4\sqrt{3}\,a^{5/6}\sqrt[6]{b}} \\
&= \frac{\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}} - \frac{\log\left(\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2\right)}{4\sqrt{3}\,a^{5/6}\sqrt[6]{b}} + \frac{\log\left(\sqrt[3]{a}+\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2\right)}{4\sqrt{3}\,a^{5/6}\sqrt[6]{b}} + \frac{\text{Subst}}{} \\
&= \frac{\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}} - \frac{\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{6a^{5/6}\sqrt[6]{b}} + \frac{\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{6a^{5/6}\sqrt[6]{b}} - \frac{\log\left(\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2\right.}{4\sqrt{3}\,a^{5/6}\sqrt[6]{b}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 154, normalized size = 0.72

$$\frac{-\sqrt{3}\,\log\left(-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)+\sqrt{3}\,\log\left(\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)+4\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)-2\tan^{-1}\left(\sqrt{3}\right.}{12a^{5/6}\sqrt[6]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^6)^(-1),x]
```

```
[Out] (4*ArcTan[(b^(1/6)*x)/a^(1/6)] - 2*ArcTan[Sqrt[3] - (2*b^(1/6)*x)/a^(1/6)]
+ 2*ArcTan[Sqrt[3] + (2*b^(1/6)*x)/a^(1/6)] - Sqrt[3]*Log[a^(1/3) - Sqrt[3]
*a^(1/6)*b^(1/6)*x + b^(1/3)*x^2] + Sqrt[3]*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b
^(1/6)*x + b^(1/3)*x^2])/(12*a^(5/6)*b^(1/6))
```

fricas [B] time = 0.84, size = 334, normalized size = 1.55

$$\frac{1}{3}\sqrt{3}\left(-\frac{1}{a^5b}\right)^{\frac{1}{6}}\arctan\left(-\frac{2}{3}\sqrt{3}\,a^4bx\left(-\frac{1}{a^5b}\right)^{\frac{5}{6}}+\frac{2}{3}\sqrt{3}\sqrt{a^2\left(-\frac{1}{a^5b}\right)^{\frac{1}{3}}+ax\left(-\frac{1}{a^5b}\right)^{\frac{1}{6}}+x^2}\,a^4b\left(-\frac{1}{a^5b}\right)^{\frac{5}{6}}+\frac{1}{3}\sqrt{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*(-1/(a^5*b))^(1/6)*arctan(-2/3*sqrt(3)*a^4*b*x*(-1/(a^5*b))^(5/
6) + 2/3*sqrt(3)*sqrt(a^2*(-1/(a^5*b))^(1/3) + a*x*(-1/(a^5*b))^(1/6) + x^2
```

```
)*a^4*b*(-1/(a^5*b))^(5/6) + 1/3*sqrt(3)) + 1/3*sqrt(3)*(-1/(a^5*b))^(1/6)*
arctan(-2/3*sqrt(3)*a^4*b*x*(-1/(a^5*b))^(5/6) + 2/3*sqrt(3)*sqrt(a^2*(-1/(
a^5*b))^(1/3) - a*x*(-1/(a^5*b))^(1/6) + x^2)*a^4*b*(-1/(a^5*b))^(5/6) - 1/
3*sqrt(3)) + 1/12*(-1/(a^5*b))^(1/6)*log(a^2*(-1/(a^5*b))^(1/3) + a*x*(-1/(
a^5*b))^(1/6) + x^2) - 1/12*(-1/(a^5*b))^(1/6)*log(a^2*(-1/(a^5*b))^(1/3) -
 a*x*(-1/(a^5*b))^(1/6) + x^2) + 1/6*(-1/(a^5*b))^(1/6)*log(a*(-1/(a^5*b))^
(1/6) + x) - 1/6*(-1/(a^5*b))^(1/6)*log(-a*(-1/(a^5*b))^(1/6) + x)
```

giac [A] time = 0.17, size = 190, normalized size = 0.88

$$\frac{\sqrt{3}\,\left(ab^5\right)^{\frac{1}{6}}\log\left(x^2+\sqrt{3}\,x\left(\frac{a}{b}\right)^{\frac{1}{6}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12\,ab}-\frac{\sqrt{3}\,\left(ab^5\right)^{\frac{1}{6}}\log\left(x^2-\sqrt{3}\,x\left(\frac{a}{b}\right)^{\frac{1}{6}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12\,ab}+\frac{\left(ab^5\right)^{\frac{1}{6}}\arctan\left(\frac{2\,x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{6\,ab}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*(a*b^5)^(1/6)*log(x^2 + sqrt(3)*x*(a/b)^(1/6) + (a/b)^(1/3))/(
a*b) - 1/12*sqrt(3)*(a*b^5)^(1/6)*log(x^2 - sqrt(3)*x*(a/b)^(1/6) + (a/b)^(
1/3))/(a*b) + 1/6*(a*b^5)^(1/6)*arctan((2*x + sqrt(3)*(a/b)^(1/6))/(a/b)^(1
/6))/(a*b) + 1/6*(a*b^5)^(1/6)*arctan((2*x - sqrt(3)*(a/b)^(1/6))/(a/b)^(1/
6))/(a*b) + 1/3*(a*b^5)^(1/6)*arctan(x/(a/b)^(1/6))/(a*b)
```

maple [A] time = 0.05, size = 159, normalized size = 0.74

$$\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6a}-\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x^2-\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right.}{12a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^6+a),x)
```

```
[Out] 1/3/a*(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x)-1/12/a*3^(1/2)*(a/b)^(1/6)*ln(x^2
-3^(1/2)*(a/b)^(1/6)*x+(a/b)^(1/3))+1/6/a*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*
x-3^(1/2))+1/12/a*3^(1/2)*(a/b)^(1/6)*ln(x^2+3^(1/2)*(a/b)^(1/6)*x+(a/b)^(1
/3))+1/6/a*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x+3^(1/2))
```

maxima [A] time = 2.46, size = 184, normalized size = 0.86

$$\frac{\sqrt{3}\,\log\left(b^{\frac{1}{3}}x^2+\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{12\,a^{\frac{5}{6}}b^{\frac{1}{6}}}-\frac{\sqrt{3}\,\log\left(b^{\frac{1}{3}}x^2-\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{12\,a^{\frac{5}{6}}b^{\frac{1}{6}}}+\frac{\arctan\left(\frac{b^{\frac{1}{3}}x}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{3\,a^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{\arctan\left(\frac{2\,b^{\frac{1}{3}}x+\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{6\,a^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*log(b^(1/3)*x^2 + sqrt(3)*a^(1/6)*b^(1/6)*x + a^(1/3))/(a^(5/6
)*b^(1/6)) - 1/12*sqrt(3)*log(b^(1/3)*x^2 - sqrt(3)*a^(1/6)*b^(1/6)*x + a^(
1/3))/(a^(5/6)*b^(1/6)) + 1/3*arctan(b^(1/3)*x/sqrt(a^(1/3)*b^(1/3)))/(a^(2
/3)*sqrt(a^(1/3)*b^(1/3))) + 1/6*arctan((2*b^(1/3)*x + sqrt(3)*a^(1/6)*b^(1
/6))/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*sqrt(a^(1/3)*b^(1/3))) + 1/6*arctan((2
*b^(1/3)*x - sqrt(3)*a^(1/6)*b^(1/6))/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*sqrt(
a^(1/3)*b^(1/3)))
```

mupad [B] time = 1.12, size = 219, normalized size = 1.02

$$-\frac{\operatorname{atanh}\left(\frac{b^{1/6}x}{(-a)^{1/6}}\right)}{3(-a)^{5/6}b^{1/6}}+\frac{\operatorname{atan}\left(\frac{b^{29/6}x\,1\mathrm{i}}{(-a)^{5/6}\left(\frac{b^{14/3}}{(-a)^{2/3}}+\frac{\sqrt{3}\,b^{14/3}\,1\mathrm{i}}{(-a)^{2/3}}\right)}+\frac{\sqrt{3}\,b^{29/6}x}{(-a)^{5/6}\left(\frac{b^{14/3}}{(-a)^{2/3}}+\frac{\sqrt{3}\,b^{14/3}\,1\mathrm{i}}{(-a)^{2/3}}\right)}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)1\mathrm{i}}{6(-a)^{5/6}b^{1/6}}-\frac{\operatorname{atan}\left(\frac{b^{29/6}x}{(-a)^{5/6}\left(\frac{b^{14/3}}{(-a)^{2/3}}-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^6),x)
```

```
[Out] (atan((b^(29/6)*x*1i)/((-a)^(5/6)*(b^(14/3)/(-a)^(2/3) + (3^(1/2)*b^(14/3)*
1i)/(-a)^(2/3))) + (3^(1/2)*b^(29/6)*x)/((-a)^(5/6)*(b^(14/3)/(-a)^(2/3) +
(3^(1/2)*b^(14/3)*1i)/(-a)^(2/3))))*(3^(1/2)*1i - 1)*1i)/(6*(-a)^(5/6)*b^(1
/6)) - atanh((b^(1/6)*x)/(-a)^(1/6))/(3*(-a)^(5/6)*b^(1/6)) - (atan((b^(29/
6)*x*1i)/((-a)^(5/6)*(b^(14/3)/(-a)^(2/3) - (3^(1/2)*b^(14/3)*1i)/(-a)^(2/3
))) - (3^(1/2)*b^(29/6)*x)/((-a)^(5/6)*(b^(14/3)/(-a)^(2/3) - (3^(1/2)*b^(1
4/3)*1i)/(-a)^(2/3))))*(3^(1/2)*1i + 1)*1i)/(6*(-a)^(5/6)*b^(1/6))
```

sympy [A] time = 0.42, size = 20, normalized size = 0.09

$$\operatorname{RootSum}\left(46656t^6a^5b+1,\left(t\mapsto t\log\left(6ta+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**6+a),x)
```

```
[Out] RootSum(46656*_t**6*a**5*b + 1, Lambda(_t, _t*log(6*_t*a + x)))
```

3.1327 $\int \frac{1}{x\left(a+bx^6\right)}\,dx$

Optimal. Leaf size=22

$$\frac{\log(x)}{a}-\frac{\log\left(a+bx^6\right)}{6a}$$

```
[Out] ln(x)/a-1/6*ln(b*x^6+a)/a
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {266, 36, 29, 31}

$$\frac{\log(x)}{a}-\frac{\log\left(a+bx^6\right)}{6a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^6)),x]
```

```
[Out] Log[x]/a - Log[a + b*x^6]/(6*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a+bx^6\right)}\,dx &= \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{x(a+bx)}\,dx,x,x^6\right)\\ &= \frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,x^6\right)}{6a}-\frac{b\operatorname{Subst}\left(\int \frac{1}{a+bx}\,dx,x,x^6\right)}{6a}\\ &= \frac{\log(x)}{a}-\frac{\log\left(a+bx^6\right)}{6a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{\log(x)}{a}-\frac{\log\left(a+bx^6\right)}{6a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^6)),x]
[Out] Log[x]/a - Log[a + b*x^6]/(6*a)
```

fricas [A] time = 0.80, size = 18, normalized size = 0.82

$$-\frac{\log\left(bx^6+a\right)-6\,\log(x)}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^6+a),x, algorithm="fricas")
[Out] -1/6*(log(b*x^6 + a) - 6*log(x))/a
```

giac [A] time = 0.16, size = 24, normalized size = 1.09

$$\frac{\log\left(x^6\right)}{6\,a}-\frac{\log\left(\left|bx^6+a\right|\right)}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^6+a),x, algorithm="giac")
[Out] 1/6*log(x^6)/a - 1/6*log(abs(b*x^6 + a))/a
```

maple [A] time = 0.00, size = 21, normalized size = 0.95

$$\frac{\ln(x)}{a}-\frac{\ln\left(b\,x^6+a\right)}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^6+a),x)
[Out] 1/a*ln(x)-1/6*ln(b*x^6+a)/a
```

maxima [A] time = 1.07, size = 23, normalized size = 1.05

$$-\frac{\log\left(bx^6+a\right)}{6\,a}+\frac{\log\left(x^6\right)}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^6+a),x, algorithm="maxima")
[Out] -1/6*log(b*x^6 + a)/a + 1/6*log(x^6)/a
```

mupad [B] time = 1.07, size = 18, normalized size = 0.82

$$-\frac{\ln\left(b\,x^6+a\right)-6\,\ln(x)}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^6)),x)
[Out] -(log(a + b*x^6) - 6*log(x))/(6*a)
```

sympy [A] time = 0.61, size = 15, normalized size = 0.68

$$\frac{\log(x)}{a}-\frac{\log\left(\frac{a}{b}+x^6\right)}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**6+a),x)
[Out] log(x)/a - log(a/b + x**6)/(6*a)
```

3.1328 $\int \frac{1}{x^2\left(a+bx^6\right)}\,dx$

Optimal. Leaf size=223

$$-\frac{\sqrt[6]{b}\,\log\left(-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{4\sqrt{3}\,a^{7/6}}+\frac{\sqrt[6]{b}\,\log\left(\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{4\sqrt{3}\,a^{7/6}}-\frac{\sqrt[6]{b}\,\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{3a^{7/6}}+\frac{\sqrt[6]{b}\,\tan^{-1}\left($$

```
[Out] -1/a/x-1/3*b^(1/6)*arctan(b^(1/6)*x/a^(1/6))/a^(7/6)+1/6*b^(1/6)*arctan((-2
*b^(1/6)*x+a^(1/6)*3^(1/2))/a^(1/6))/a^(7/6)-1/6*b^(1/6)*arctan((2*b^(1/6)*
x+a^(1/6)*3^(1/2))/a^(1/6))/a^(7/6)-1/12*b^(1/6)*ln(a^(1/3)+b^(1/3)*x^2-a^(
1/6)*b^(1/6)*x*3^(1/2))/a^(7/6)*3^(1/2)+1/12*b^(1/6)*ln(a^(1/3)+b^(1/3)*x^2
+a^(1/6)*b^(1/6)*x*3^(1/2))/a^(7/6)*3^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {325, 295, 634, 618, 204, 628, 205}

$$-\frac{\sqrt[6]{b}\,\log\left(-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{4\sqrt{3}\,a^{7/6}}+\frac{\sqrt[6]{b}\,\log\left(\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{4\sqrt{3}\,a^{7/6}}-\frac{\sqrt[6]{b}\,\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{3a^{7/6}}+\frac{\sqrt[6]{b}\,\tan^{-1}\left($$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^6)),x]
```

```
[Out] -(1/(a*x)) - (b^(1/6)*ArcTan[(b^(1/6)*x)/a^(1/6)])/(3*a^(7/6)) + (b^(1/6)*A
rcTan[(Sqrt[3]*a^(1/6) - 2*b^(1/6)*x)/a^(1/6)])/(6*a^(7/6)) - (b^(1/6)*ArcT
an[(Sqrt[3]*a^(1/6) + 2*b^(1/6)*x)/a^(1/6)])/(6*a^(7/6)) - (b^(1/6)*Log[a^(
1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*x + b^(1/3)*x^2])/(4*Sqrt[3]*a^(7/6)) + (b^(
1/6)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*x + b^(1/3)*x^2])/(4*Sqrt[3]*a^(
7/6))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^6\right)}\,dx = -\frac{1}{ax} - \frac{b\int \frac{x^4}{a+bx^6}\,dx}{a}$$

$$= -\frac{1}{ax} - \frac{\sqrt[3]{b}\int \frac{-\frac{\sqrt[6]{a}}{2}+\frac{1}{2}\sqrt{3}\,\sqrt[6]{b}\,x}{\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx}{3a^{7/6}} - \frac{\sqrt[3]{b}\int \frac{-\frac{\sqrt[6]{a}}{2}-\frac{1}{2}\sqrt{3}\,\sqrt[6]{b}\,x}{\sqrt[3]{a}+\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx}{3a^{7/6}} - \frac{\sqrt[3]{b}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x^2}\,dx}{3a}$$

$$= -\frac{1}{ax} - \frac{\sqrt[6]{b}\,\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{3a^{7/6}} - \frac{\sqrt[6]{b}\int \frac{-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}+2\sqrt[3]{b}\,x}{\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx}{4\sqrt{3}\,a^{7/6}} + \frac{\sqrt[6]{b}\int \frac{\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}+2\sqrt[3]{b}\,x}{\sqrt[3]{a}+\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx}{4\sqrt{3}\,a^{7/6}} - \frac{\sqrt[3]{}}{}$$

$$= -\frac{1}{ax} - \frac{\sqrt[6]{b}\,\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{3a^{7/6}} - \frac{\sqrt[6]{b}\,\log\left(\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2\right)}{4\sqrt{3}\,a^{7/6}} + \frac{\sqrt[6]{b}\,\log\left(\sqrt[3]{a}+\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\right.}{4\sqrt{3}\,a^{7/6}}$$

$$= -\frac{1}{ax} - \frac{\sqrt[6]{b}\,\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{3a^{7/6}} + \frac{\sqrt[6]{b}\,\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{6a^{7/6}} - \frac{\sqrt[6]{b}\,\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{6a^{7/6}} - \frac{\sqrt[6]{b}\,\log\left(\sqrt[3]{}\right.}{}$$

Mathematica [A] time = 0.05, size = 189, normalized size = 0.85

$$-\frac{\sqrt{3}\,\sqrt[6]{b}\,x\log\left(-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)-\sqrt{3}\,\sqrt[6]{b}\,x\log\left(\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)+4\sqrt[6]{b}\,x\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)-}{12a^{7/6}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^6)),x]
```

```
[Out] -1/12*(12*a^(1/6) + 4*b^(1/6)*x*ArcTan[(b^(1/6)*x)/a^(1/6)] - 2*b^(1/6)*x*A
rcTan[Sqrt[3] - (2*b^(1/6)*x)/a^(1/6)] + 2*b^(1/6)*x*ArcTan[Sqrt[3] + (2*b^
(1/6)*x)/a^(1/6)] + Sqrt[3]*b^(1/6)*x*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)
```

```
*x + b^(1/3)*x^2] - Sqrt[3]*b^(1/6)*x*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)
*x + b^(1/3)*x^2])/(a^(7/6)*x)
```

fricas [B] time = 0.91, size = 338, normalized size = 1.52

$$\frac{4\sqrt{3}\,ax\left(-\frac{b}{a^7}\right)^{\frac{1}{6}}\arctan\left(-\frac{2}{3}\sqrt{3}\,ax\left(-\frac{b}{a^7}\right)^{\frac{1}{6}}+\frac{2}{3}\sqrt{3}\,a\sqrt{-\frac{a^6x\left(-\frac{b}{a^7}\right)^{\frac{5}{6}}+a^5\left(-\frac{b}{a^7}\right)^{\frac{2}{3}}-bx^2}{b}}\left(-\frac{b}{a^7}\right)^{\frac{1}{6}}-\frac{1}{3}\sqrt{3}\right)+4\sqrt{3}\,ax\left(-\frac{b}{a^7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^6+a),x, algorithm="fricas")
```

```
[Out] 1/12*(4*sqrt(3)*a*x*(-b/a^7)^(1/6)*arctan(-2/3*sqrt(3)*a*x*(-b/a^7)^(1/6) +
 2/3*sqrt(3)*a*sqrt(-(a^6*x*(-b/a^7)^(5/6) + a^5*(-b/a^7)^(2/3) - b*x^2)/b)
*(-b/a^7)^(1/6) - 1/3*sqrt(3)) + 4*sqrt(3)*a*x*(-b/a^7)^(1/6)*arctan(-2/3*s
qrt(3)*a*x*(-b/a^7)^(1/6) + 2/3*sqrt(3)*a*sqrt((a^6*x*(-b/a^7)^(5/6) - a^5*
(-b/a^7)^(2/3) + b*x^2)/b)*(-b/a^7)^(1/6) + 1/3*sqrt(3)) - a*x*(-b/a^7)^(1/
6)*log(a^6*x*(-b/a^7)^(5/6) - a^5*(-b/a^7)^(2/3) + b*x^2) + a*x*(-b/a^7)^(1
/6)*log(-a^6*x*(-b/a^7)^(5/6) - a^5*(-b/a^7)^(2/3) + b*x^2) - 2*a*x*(-b/a^7
)^(1/6)*log(a^6*(-b/a^7)^(5/6) + b*x) + 2*a*x*(-b/a^7)^(1/6)*log(-a^6*(-b/a
^7)^(5/6) + b*x) - 12)/(a*x)
```

giac [A] time = 0.26, size = 196, normalized size = 0.88

$$-\frac{b\left(\frac{a}{b}\right)^{\frac{5}{6}}\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a^2}-\frac{1}{ax}+\frac{\sqrt{3}\left(ab^5\right)^{\frac{5}{6}}\log\left(x^2+\sqrt{3}\,x\left(\frac{a}{b}\right)^{\frac{1}{6}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a^2b^4}-\frac{\sqrt{3}\left(ab^5\right)^{\frac{5}{6}}\log\left(x^2-\sqrt{3}\,x\left(\frac{a}{b}\right)^{\frac{1}{6}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^6+a),x, algorithm="giac")
```

```
[Out] -1/3*b*(a/b)^(5/6)*arctan(x/(a/b)^(1/6))/a^2 - 1/(a*x) + 1/12*sqrt(3)*(a*b^
5)^(5/6)*log(x^2 + sqrt(3)*x*(a/b)^(1/6) + (a/b)^(1/3))/(a^2*b^4) - 1/12*sq
rt(3)*(a*b^5)^(5/6)*log(x^2 - sqrt(3)*x*(a/b)^(1/6) + (a/b)^(1/3))/(a^2*b^4
) - 1/6*(a*b^5)^(5/6)*arctan((2*x + sqrt(3)*(a/b)^(1/6))/(a/b)^(1/6))/(a^2*
b^4) - 1/6*(a*b^5)^(5/6)*arctan((2*x - sqrt(3)*(a/b)^(1/6))/(a/b)^(1/6))/(a
^2*b^4)
```

maple [A] time = 0.05, size = 169, normalized size = 0.76

$$-\frac{\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{6}}a}-\frac{\arctan\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{6}}a}-\frac{\arctan\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{6}}a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}b\ln\left(x^2-\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a^2}+\frac{\sqrt{3}\left(\frac{a}{b}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^6+a),x)
```

```
[Out] -1/a/x-1/3/a/(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x)-1/12/a^2*b*3^(1/2)*(a/b)^(
5/6)*ln(x^2-3^(1/2)*(a/b)^(1/6)*x+(a/b)^(1/3))-1/6/a/(a/b)^(1/6)*arctan(2/(
a/b)^(1/6)*x-3^(1/2))+1/12/a^2*b*3^(1/2)*(a/b)^(5/6)*ln(x^2+3^(1/2)*(a/b)^(
1/6)*x+(a/b)^(1/3))-1/6/a/(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x+3^(1/2))
```

maxima [A] time = 2.48, size = 198, normalized size = 0.89

$$\frac{b\left(\frac{\sqrt{3}\log\left(b^{\frac{1}{3}}x^2+\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{\sqrt{3}\log\left(b^{\frac{1}{3}}x^2-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{4\arctan\left(\frac{b^{\frac{1}{3}}x}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-\frac{2\arctan\left(\frac{2b^{\frac{1}{3}}x+\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-\frac{2\arctan\left(\frac{2b^{\frac{1}{3}}x-\sqrt{3}a^{\frac{1}{6}}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^6+a),x, algorithm="maxima")
```

```
[Out] 1/12*b*(sqrt(3)*log(b^(1/3)*x^2 + sqrt(3)*a^(1/6)*b^(1/6)*x + a^(1/3))/(a^(
1/6)*b^(5/6)) - sqrt(3)*log(b^(1/3)*x^2 - sqrt(3)*a^(1/6)*b^(1/6)*x + a^(1/
3))/(a^(1/6)*b^(5/6)) - 4*arctan(b^(1/3)*x/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*
sqrt(a^(1/3)*b^(1/3))) - 2*arctan((2*b^(1/3)*x + sqrt(3)*a^(1/6)*b^(1/6))/s
qrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 2*arctan((2*b^(1/3)
*x - sqrt(3)*a^(1/6)*b^(1/6))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*
b^(1/3))))/a - 1/(a*x)
```

mupad [B] time = 0.18, size = 149, normalized size = 0.67

$$-\frac{1}{a\,x}-\frac{(-b)^{1/6}\,\mathrm{atan}\left(\frac{(-b)^{1/6}\,x\,1\mathrm{i}}{a^{1/6}}\right)\,1\mathrm{i}}{3\,a^{7/6}}-\frac{(-b)^{1/6}\,\mathrm{atan}\left(\frac{a^{13/2}\,(-b)^{13/2}\,x\,2\mathrm{i}}{a^{20/3}\,(-b)^{19/3}-\sqrt{3}\,a^{20/3}\,(-b)^{19/3}\,1\mathrm{i}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,1\mathrm{i}}{3\,a^{7/6}}+\frac{(-b)^{1/6}\,\mathrm{atan}\left(\frac{a}{a^{20/3}\,(-b)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^6)),x)
```

```
[Out] ((-b)^(1/6)*atan((a^(13/2)*(-b)^(13/2)*x*2i)/(a^(20/3)*(-b)^(19/3) + 3^(1/2
)*a^(20/3)*(-b)^(19/3)*1i))*((3^(1/2)*1i)/2 - 1/2)*1i)/(3*a^(7/6)) - ((-b)^
(1/6)*atan(((-b)^(1/6)*x*1i)/a^(1/6))*1i)/(3*a^(7/6)) - ((-b)^(1/6)*atan((a
^(13/2)*(-b)^(13/2)*x*2i)/(a^(20/3)*(-b)^(19/3) - 3^(1/2)*a^(20/3)*(-b)^(19
/3)*1i))*((3^(1/2)*1i)/2 + 1/2)*1i)/(3*a^(7/6)) - 1/(a*x)
```

sympy [A] time = 0.46, size = 29, normalized size = 0.13

$$\mathrm{RootSum}\left(46656t^6a^7+b,\left(t\mapsto t\log\left(-\frac{7776t^5a^6}{b}+x\right)\right)\right)-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**6+a),x)
```

```
[Out] RootSum(46656*_t**6*a**7 + b, Lambda(_t, _t*log(-7776*_t**5*a**6/b + x))) -
 1/(a*x)
```

3.1329 $\int \frac{1}{x^3\left(a+bx^6\right)}\,dx$

Optimal. Leaf size=133

$$-\frac{\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^2+b^{2/3}x^4\right)}{12a^{4/3}}+\frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{6a^{4/3}}+\frac{\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x^2}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2\sqrt{3}\,a^{4/3}}-\frac{1}{2ax^2}$$

```
[Out] -1/2/a/x^2+1/6*b^(1/3)*ln(a^(1/3)+b^(1/3)*x^2)/a^(4/3)-1/12*b^(1/3)*ln(a^(2
/3)-a^(1/3)*b^(1/3)*x^2+b^(2/3)*x^4)/a^(4/3)+1/6*b^(1/3)*arctan(1/3*(a^(1/3
)-2*b^(1/3)*x^2)/a^(1/3)*3^(1/2))/a^(4/3)*3^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {275, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^2+b^{2/3}x^4\right)}{12a^{4/3}}+\frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{6a^{4/3}}+\frac{\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x^2}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2\sqrt{3}\,a^{4/3}}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^6)),x]
```

```
[Out] -1/(2*a*x^2) + (b^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x^2)/(Sqrt[3]*a^(1/3))]
)/(2*Sqrt[3]*a^(4/3)) + (b^(1/3)*Log[a^(1/3) + b^(1/3)*x^2])/(6*a^(4/3)) -
(b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^2 + b^(2/3)*x^4])/(12*a^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(a+bx^6\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^3\right)}\,dx, x, x^2\right) \\
&= -\frac{1}{2ax^2} - \frac{b\,\text{Subst}\left(\int \frac{x}{a+bx^3}\,dx, x, x^2\right)}{2a} \\
&= -\frac{1}{2ax^2} + \frac{b^{2/3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, x^2\right)}{6a^{4/3}} - \frac{b^{2/3}\,\text{Subst}\left(\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, x^2\right)}{6a^{4/3}} \\
&= -\frac{1}{2ax^2} + \frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{6a^{4/3}} - \frac{\sqrt[3]{b}\,\text{Subst}\left(\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, x^2\right)}{12a^{4/3}} - \frac{b^{2/3}\,\text{Subst}\left(\int \right.}{} \\
&= -\frac{1}{2ax^2} + \frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{6a^{4/3}} - \frac{\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^2+b^{2/3}x^4\right)}{12a^{4/3}} - \frac{\sqrt[3]{b}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\right.}{2a^{4/} \\
&= -\frac{1}{2ax^2} + \frac{\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x^2}{\sqrt{3}\,\sqrt[3]{a}}\right)}{2\sqrt{3}\,a^{4/3}} + \frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{6a^{4/3}} - \frac{\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^2+b^{2/3}\right.}{12a^{4/3}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 203, normalized size = 1.53

$$\frac{2\sqrt[3]{b}\,x^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right) - \sqrt[3]{b}\,x^2\log\left(-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right) - \sqrt[3]{b}\,x^2\log\left(\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)+}{12a^{4/3}x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^6)),x]
```

```
[Out] (-6*a^(1/3) + 2*Sqrt[3]*b^(1/3)*x^2*ArcTan[Sqrt[3] - (2*b^(1/6)*x)/a^(1/6)]
 + 2*Sqrt[3]*b^(1/3)*x^2*ArcTan[Sqrt[3] + (2*b^(1/6)*x)/a^(1/6)] + 2*b^(1/3
)*x^2*Log[a^(1/3) + b^(1/3)*x^2] - b^(1/3)*x^2*Log[a^(1/3) - Sqrt[3]*a^(1/6
```

```
)*b^(1/6)*x + b^(1/3)*x^2] - b^(1/3)*x^2*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1
/6)*x + b^(1/3)*x^2])/(12*a^(4/3)*x^2)
```

fricas [A] time = 0.57, size = 115, normalized size = 0.86

$$-\frac{2\sqrt{3}\,x^2\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\,x^2\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+x^2\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^4-ax^2\left(\frac{b}{a}\right)^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-2\,x^2\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2+a\right.}{12\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^6+a),x, algorithm="fricas")
```

```
[Out] -1/12*(2*sqrt(3)*x^2*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x^2*(b/a)^(1/3) - 1/3*s
qrt(3)) + x^2*(b/a)^(1/3)*log(b*x^4 - a*x^2*(b/a)^(2/3) + a*(b/a)^(1/3)) -
2*x^2*(b/a)^(1/3)*log(b*x^2 + a*(b/a)^(2/3)) + 6)/(a*x^2)
```

giac [A] time = 0.18, size = 127, normalized size = 0.95

$$\frac{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^2-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{6\,a^2}+\frac{\sqrt{3}\,\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^2+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{6\,a^2b}-\frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^4+x^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{12\,a^2b}-\frac{1}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^6+a),x, algorithm="giac")
```

```
[Out] 1/6*b*(-a/b)^(2/3)*log(abs(x^2 - (-a/b)^(1/3)))/a^2 + 1/6*sqrt(3)*(-a*b^2)^
(2/3)*arctan(1/3*sqrt(3)*(2*x^2 + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b) - 1/1
2*(-a*b^2)^(2/3)*log(x^4 + x^2*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b) - 1/2/(
a*x^2)
```

maple [A] time = 0.00, size = 105, normalized size = 0.79

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^2}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}a}+\frac{\ln\left(x^2+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}a}-\frac{\ln\left(x^4-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^2+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{12\left(\frac{a}{b}\right)^{\frac{1}{3}}a}-\frac{1}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^6+a),x)
```

```
[Out] -1/2/a/x^2+1/6/a/(a/b)^(1/3)*ln(x^2+(a/b)^(1/3))-1/12/a/(a/b)^(1/3)*ln(x^4-
(a/b)^(1/3)*x^2+(a/b)^(2/3))-1/6/a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(
2/(a/b)^(1/3)*x^2-1))
```

maxima [A] time = 2.31, size = 112, normalized size = 0.84

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{6\,a\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\log\left(x^4-x^2\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{12\,a\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^2+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,a\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{1}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^6+a),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - (a/b)^(1/3))/(a/b)^(1/3))/(a*(a/b)
^(1/3)) - 1/12*log(x^4 - x^2*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(1/3)) + 1
/6*log(x^2 + (a/b)^(1/3))/(a*(a/b)^(1/3)) - 1/2/(a*x^2)
```

mupad [B] time = 1.17, size = 116, normalized size = 0.87

$$\frac{b^{1/3}\,\ln\left(a^{1/3}+b^{1/3}\,x^2\right)}{6\,a^{4/3}}-\frac{1}{2\,a\,x^2}+\frac{b^{1/3}\,\ln\left(a^4\,b^6+a^{11/3}\,b^{19/3}\,x^2\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{6\,a^{4/3}}-\frac{b^{1/3}\,\ln\left(a^4\,b^6-a^{11/3}\,b^{19}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^6)),x)
```

```
[Out] (b^(1/3)*log(a^(1/3) + b^(1/3)*x^2))/(6*a^(4/3)) - 1/(2*a*x^2) + (b^(1/3)*l
og(a^4*b^6 + a^(11/3)*b^(19/3)*x^2*((3^(1/2)*1i)/2 - 1/2))*((3^(1/2)*1i)/2
- 1/2))/(6*a^(4/3)) - (b^(1/3)*log(a^4*b^6 - a^(11/3)*b^(19/3)*x^2*((3^(1/2
)*1i)/2 + 1/2))*((3^(1/2)*1i)/2 + 1/2))/(6*a^(4/3))
```

sympy [A] time = 0.33, size = 34, normalized size = 0.26

$$\operatorname{RootSum}\left(216t^3a^4-b,\left(t\mapsto t\log\left(\frac{36t^2a^3}{b}+x^2\right)\right)\right)-\frac{1}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**6+a),x)
```

```
[Out] RootSum(216*_t**3*a**4 - b, Lambda(_t, _t*log(36*_t**2*a**3/b + x**2))) - 1
/(2*a*x**2)
```

3.1330 $\int \frac{1}{x^4\left(a+bx^6\right)}\,dx$

Optimal. Leaf size=40

$$-\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x^3}{\sqrt{a}}\right)}{3a^{3/2}}-\frac{1}{3ax^3}$$

```
[Out] -1/3/a/x^3-1/3*arctan(x^3*b^(1/2)/a^(1/2))*b^(1/2)/a^(3/2)
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {275, 325, 205}

$$-\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x^3}{\sqrt{a}}\right)}{3a^{3/2}}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^6)),x]
```

```
[Out] -1/(3*a*x^3) - (Sqrt[b]*ArcTan[(Sqrt[b]*x^3)/Sqrt[a]])/(3*a^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(a+bx^6\right)}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)}\,dx, x, x^3\right)\\ &= -\frac{1}{3ax^3}-\frac{b\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^3\right)}{3a}\\ &= -\frac{1}{3ax^3}-\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x^3}{\sqrt{a}}\right)}{3a^{3/2}}\end{aligned}$$

Mathematica [B] time = 0.02, size = 101, normalized size = 2.52

$$\frac{\sqrt{b}\,x^3\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)+\sqrt{b}\,x^3\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)-\sqrt{b}\,x^3\tan^{-1}\left(\frac{2\sqrt[6]{b}\,x}{\sqrt[6]{a}}+\sqrt{3}\right)-\sqrt{a}}{3a^{3/2}x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^6)),x]
```

```
[Out] (-Sqrt[a] + Sqrt[b]*x^3*ArcTan[(b^(1/6)*x)/a^(1/6)] + Sqrt[b]*x^3*ArcTan[Sq
rt[3] - (2*b^(1/6)*x)/a^(1/6)] - Sqrt[b]*x^3*ArcTan[Sqrt[3] + (2*b^(1/6)*x)
/a^(1/6)])/(3*a^(3/2)*x^3)
```

fricas [A] time = 0.84, size = 90, normalized size = 2.25

$$\left[\frac{x^3\sqrt{-\frac{b}{a}}\log\left(\frac{bx^6-2ax^3\sqrt{-\frac{b}{a}}-a}{bx^6+a}\right)-2}{6ax^3}, -\frac{x^3\sqrt{\frac{b}{a}}\arctan\left(x^3\sqrt{\frac{b}{a}}\right)+1}{3ax^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^6+a),x, algorithm="fricas")
```

```
[Out] [1/6*(x^3*sqrt(-b/a)*log((b*x^6 - 2*a*x^3*sqrt(-b/a) - a)/(b*x^6 + a)) - 2)
/(a*x^3), -1/3*(x^3*sqrt(b/a)*arctan(x^3*sqrt(b/a)) + 1)/(a*x^3)]
```

giac [A] time = 0.18, size = 31, normalized size = 0.78

$$-\frac{b\arctan\left(\frac{bx^3}{\sqrt{ab}}\right)}{3\sqrt{ab}\,a}-\frac{1}{3ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^6+a),x, algorithm="giac")
```

```
[Out] -1/3*b*arctan(b*x^3/sqrt(a*b))/(sqrt(a*b)*a) - 1/3/(a*x^3)
```

maple [A] time = 0.01, size = 32, normalized size = 0.80

$$-\frac{b\arctan\left(\frac{bx^3}{\sqrt{ab}}\right)}{3\sqrt{ab}\ a}-\frac{1}{3ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^6+a),x)
```

```
[Out] -1/3/a/x^3-1/3/a*b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^3)
```

maxima [A] time = 2.29, size = 31, normalized size = 0.78

$$-\frac{b\arctan\left(\frac{bx^3}{\sqrt{ab}}\right)}{3\sqrt{ab}\,a}-\frac{1}{3ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^6+a),x, algorithm="maxima")
```

```
[Out] -1/3*b*arctan(b*x^3/sqrt(a*b))/(sqrt(a*b)*a) - 1/3/(a*x^3)
```

mupad [B] time = 0.04, size = 28, normalized size = 0.70

$$-\frac{1}{3\,a\,x^3}-\frac{\sqrt{b}\ \mathrm{atan}\left(\frac{\sqrt{b}\,x^3}{\sqrt{a}}\right)}{3\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^6)),x)
```

```
[Out] - 1/(3*a*x^3) - (b^(1/2)*atan((b^(1/2)*x^3)/a^(1/2)))/(3*a^(3/2))
```

sympy [**A**] time = 0.56, size = 71, normalized size = 1.78

$$\frac{\sqrt{-\frac{b}{a^3}}\,\log\left(-\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x^3\right)}{6}-\frac{\sqrt{-\frac{b}{a^3}}\,\log\left(\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x^3\right)}{6}-\frac{1}{3ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**6+a),x)
```

```
[Out] sqrt(-b/a**3)*log(-a**2*sqrt(-b/a**3)/b + x**3)/6 - sqrt(-b/a**3)*log(a**2*
sqrt(-b/a**3)/b + x**3)/6 - 1/(3*a*x**3)
```

3.1331 $\int \frac{x^8}{\left(a+bx^6\right)^2}\,dx$

Optimal. Leaf size=49

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x^3}{\sqrt{a}}\right)}{6\sqrt{a}\,b^{3/2}}-\frac{x^3}{6b\left(a+bx^6\right)}$$

```
[Out] -1/6*x^3/b/(b*x^6+a)+1/6*arctan(x^3*b^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {275, 288, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x^3}{\sqrt{a}}\right)}{6\sqrt{a}\,b^{3/2}}-\frac{x^3}{6b\left(a+bx^6\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^6)^2,x]
```

```
[Out] -x^3/(6*b*(a + b*x^6)) + ArcTan[(Sqrt[b]*x^3)/Sqrt[a]]/(6*Sqrt[a]*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{\left(a+bx^6\right)^2}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^2\right)^2}\,dx, x, x^3\right)\\ &= -\frac{x^3}{6b\left(a+bx^6\right)}+\frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^3\right)}{6b}\\ &= -\frac{x^3}{6b\left(a+bx^6\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x^3}{\sqrt{a}}\right)}{6\sqrt{a}\,b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 49, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x^3}{\sqrt{a}}\right)}{6\sqrt{a}\,b^{3/2}} - \frac{x^3}{6b\left(a+bx^6\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^6)^2,x]
```

```
[Out] -1/6*x^3/(b*(a + b*x^6)) + ArcTan[(Sqrt[b]*x^3)/Sqrt[a]]/(6*Sqrt[a]*b^(3/2)
)
```

fricas [A] time = 0.73, size = 128, normalized size = 2.61

$$\left[-\frac{2\,abx^3+\left(bx^6+a\right)\sqrt{-ab}\,\log\left(\frac{bx^6-2\,\sqrt{-ab}\,x^3-a}{bx^6+a}\right)}{12\left(ab^3x^6+a^2b^2\right)}, -\frac{abx^3-\left(bx^6+a\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x^3}{a}\right)}{6\left(ab^3x^6+a^2b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^6+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(2*a*b*x^3 + (b*x^6 + a)*sqrt(-a*b)*log((b*x^6 - 2*sqrt(-a*b)*x^3 -
a)/(b*x^6 + a)))/(a*b^3*x^6 + a^2*b^2), -1/6*(a*b*x^3 - (b*x^6 + a)*sqrt(a*
b)*arctan(sqrt(a*b)*x^3/a))/(a*b^3*x^6 + a^2*b^2)]
```

giac [A] time = 0.17, size = 39, normalized size = 0.80

$$-\frac{x^3}{6\left(bx^6+a\right)b} + \frac{\arctan\left(\frac{bx^3}{\sqrt{ab}}\right)}{6\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^6+a)^2,x, algorithm="giac")
```

```
[Out] -1/6*x^3/((b*x^6 + a)*b) + 1/6*arctan(b*x^3/sqrt(a*b))/(sqrt(a*b)*b)
```

maple [A] time = 0.01, size = 40, normalized size = 0.82

$$-\frac{x^3}{6\left(b\,x^6+a\right)b} + \frac{\arctan\left(\frac{b\,x^3}{\sqrt{ab}}\right)}{6\sqrt{ab}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^6+a)^2,x)
```

```
[Out] -1/6*x^3/b/(b*x^6+a)+1/6/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^3)
```

maxima [A] time = 2.40, size = 40, normalized size = 0.82

$$-\frac{x^3}{6\left(b^2x^6+ab\right)} + \frac{\arctan\left(\frac{bx^3}{\sqrt{ab}}\right)}{6\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^6+a)^2,x, algorithm="maxima")
```

```
[Out] -1/6*x^3/(b^2*x^6 + a*b) + 1/6*arctan(b*x^3/sqrt(a*b))/(sqrt(a*b)*b)
```

mupad [B] time = 0.09, size = 37, normalized size = 0.76

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x^3}{\sqrt{a}}\right)}{6\sqrt{a}\,b^{3/2}}-\frac{x^3}{6\,b\left(b\,x^6+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^6)^2,x)
```

```
[Out] atan((b^(1/2)*x^3)/a^(1/2))/(6*a^(1/2)*b^(3/2)) - x^3/(6*b*(a + b*x^6))
```

sympy [B] time = 0.61, size = 83, normalized size = 1.69

$$-\frac{x^3}{6ab+6b^2x^6}-\frac{\sqrt{-\frac{1}{ab^3}}\,\log\left(-ab\sqrt{-\frac{1}{ab^3}}+x^3\right)}{12}+\frac{\sqrt{-\frac{1}{ab^3}}\,\log\left(ab\sqrt{-\frac{1}{ab^3}}+x^3\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**6+a)**2,x)
```

```
[Out] -x**3/(6*a*b + 6*b**2*x**6) - sqrt(-1/(a*b**3))*log(-a*b*sqrt(-1/(a*b**3))
+ x**3)/12 + sqrt(-1/(a*b**3))*log(a*b*sqrt(-1/(a*b**3)) + x**3)/12
```

3.1332 $\int \frac{x^7}{\left(a+bx^6\right)^2}\,dx$

Optimal. Leaf size=142

$$\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{18a^{2/3}b^{4/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x^2}{\sqrt{3}\,\sqrt[3]{a}}\right)}{6\sqrt{3}\,a^{2/3}b^{4/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^2+b^{2/3}x^4\right)}{36a^{2/3}b^{4/3}}-\frac{x^2}{6b\left(a+bx^6\right)}$$

```
[Out] -1/6*x^2/b/(b*x^6+a)+1/18*ln(a^(1/3)+b^(1/3)*x^2)/a^(2/3)/b^(4/3)-1/36*ln(a
^(2/3)-a^(1/3)*b^(1/3)*x^2+b^(2/3)*x^4)/a^(2/3)/b^(4/3)-1/18*arctan(1/3*(a^
(1/3)-2*b^(1/3)*x^2)/a^(1/3)*3^(1/2))/a^(2/3)/b^(4/3)*3^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {275, 288, 200, 31, 634, 617, 204, 628}

$$\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{18a^{2/3}b^{4/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^2+b^{2/3}x^4\right)}{36a^{2/3}b^{4/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x^2}{\sqrt{3}\,\sqrt[3]{a}}\right)}{6\sqrt{3}\,a^{2/3}b^{4/3}}-\frac{x^2}{6b\left(a+bx^6\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^6)^2,x]
```

```
[Out] -x^2/(6*b*(a + b*x^6)) - ArcTan[(a^(1/3) - 2*b^(1/3)*x^2)/(Sqrt[3]*a^(1/3))
]/(6*Sqrt[3]*a^(2/3)*b^(4/3)) + Log[a^(1/3) + b^(1/3)*x^2]/(18*a^(2/3)*b^(4
/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x^2 + b^(2/3)*x^4]/(36*a^(2/3)*b^(4/3)
)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^7}{\left(a+bx^6\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{\left(a+bx^3\right)^2}\,dx, x, x^2\right)$$

$$= -\frac{x^2}{6b\left(a+bx^6\right)} + \frac{\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, x^2\right)}{6b}$$

$$= -\frac{x^2}{6b\left(a+bx^6\right)} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, x^2\right)}{18a^{2/3}b} + \frac{\text{Subst}\left(\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, x^2\right)}{18a^{2/3}b}$$

$$= -\frac{x^2}{6b\left(a+bx^6\right)} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{18a^{2/3}b^{4/3}} - \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, x^2\right)}{36a^{2/3}b^{4/3}} + \frac{\text{Subst}\left(\int \frac{}{a^{2/3}-\sqrt[3]{}}\right.}{1}$$

$$= -\frac{x^2}{6b\left(a+bx^6\right)} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{18a^{2/3}b^{4/3}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^2+b^{2/3}x^4\right)}{36a^{2/3}b^{4/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x,\right.}{6a^{2/3}b^{4/3}}$$

$$= -\frac{x^2}{6b\left(a+bx^6\right)} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x^2}{\sqrt{3}\,\sqrt[3]{a}}\right)}{6\sqrt{3}\,a^{2/3}b^{4/3}} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{18a^{2/3}b^{4/3}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^2+b^{2/3}x^4\right)}{36a^{2/3}b^{4/3}}$$

Mathematica [A] time = 0.13, size = 197, normalized size = 1.39

$$\frac{\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{a^{2/3}} - \frac{\log\left(-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{a^{2/3}} - \frac{\log\left(\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{a^{2/3}} - \frac{2\sqrt{3}\,\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{a^{2/3}} - \frac{2\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[6]{b}\,x}{\sqrt[6]{a}}+\sqrt{3}\right)}{a^{2/3}} -}{36b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^6)^2,x]
```

```
[Out] ((-6*b^(1/3)*x^2)/(a + b*x^6) - (2*Sqrt[3]*ArcTan[Sqrt[3] - (2*b^(1/6)*x)/a
^(1/6)])/a^(2/3) - (2*Sqrt[3]*ArcTan[Sqrt[3] + (2*b^(1/6)*x)/a^(1/6)])/a^(2
/3) + (2*Log[a^(1/3) + b^(1/3)*x^2])/a^(2/3) - Log[a^(1/3) - Sqrt[3]*a^(1/6
)*b^(1/6)*x + b^(1/3)*x^2]/a^(2/3) - Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*
x + b^(1/3)*x^2]/a^(2/3))/(36*b^(4/3))
```

fricas [A] time = 0.91, size = 409, normalized size = 2.88

$$\left[-\frac{6\,a^2bx^2-3\sqrt{\frac{1}{3}}\left(ab^2x^6+a^2b\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\,\log\left(\frac{2\,abx^6-3\left(a^2b\right)^{\frac{1}{3}}ax^2-a^2+3\sqrt{\frac{1}{3}}\left(2\,abx^4+\left(a^2b\right)^{\frac{2}{3}}x^2-\left(a^2b\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}}{bx^6+a}\right)+\left(bx^6\right.}{36\left(a^2b^3x^6+a^3b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^6+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/36*(6*a^2*b*x^2 - 3*sqrt(1/3)*(a*b^2*x^6 + a^2*b)*sqrt(-(a^2*b)^(1/3)/b
)*log((2*a*b*x^6 - 3*(a^2*b)^(1/3)*a*x^2 - a^2 + 3*sqrt(1/3)*(2*a*b*x^4 + (
a^2*b)^(2/3)*x^2 - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^6 + a)) +
(b*x^6 + a)*(a^2*b)^(2/3)*log(a*b*x^4 - (a^2*b)^(2/3)*x^2 + (a^2*b)^(1/3)*a
) - 2*(b*x^6 + a)*(a^2*b)^(2/3)*log(a*b*x^2 + (a^2*b)^(2/3)))/(a^2*b^3*x^6
+ a^3*b^2), -1/36*(6*a^2*b*x^2 - 6*sqrt(1/3)*(a*b^2*x^6 + a^2*b)*sqrt((a^2*
b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x^2 - (a^2*b)^(1/3)*a)*sqrt((
a^2*b)^(1/3)/b)/a^2) + (b*x^6 + a)*(a^2*b)^(2/3)*log(a*b*x^4 - (a^2*b)^(2/3
)*x^2 + (a^2*b)^(1/3)*a) - 2*(b*x^6 + a)*(a^2*b)^(2/3)*log(a*b*x^2 + (a^2*b
)^(2/3)))/(a^2*b^3*x^6 + a^3*b^2)]
```

giac [A] time = 0.20, size = 138, normalized size = 0.97

$$-\frac{x^2}{6\left(bx^6+a\right)b}-\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^2-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{18\,ab}+\frac{\sqrt{3}\,\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^2+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{18\,ab^2}+\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^4+x^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right.}{36\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^6+a)^2,x, algorithm="giac")
```

```
[Out] -1/6*x^2/((b*x^6 + a)*b) - 1/18*(-a/b)^(1/3)*log(abs(x^2 - (-a/b)^(1/3)))/(
a*b) + 1/18*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^2 + (-a/b)^(1/3)
)/(-a/b)^(1/3))/(a*b^2) + 1/36*(-a*b^2)^(1/3)*log(x^4 + x^2*(-a/b)^(1/3) +
(-a/b)^(2/3))/(a*b^2)
```

maple [A] time = 0.01, size = 114, normalized size = 0.80

$$-\frac{x^2}{6\left(b\,x^6+a\right)b}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^2}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}+\frac{\ln\left(x^2+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}-\frac{\ln\left(x^4-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^2+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{36\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^6+a)^2,x)
```

```
[Out] -1/6*x^2/b/(b*x^6+a)+1/18/b^2/(a/b)^(2/3)*ln(x^2+(a/b)^(1/3))-1/36/b^2/(a/b
)^(2/3)*ln(x^4-(a/b)^(1/3)*x^2+(a/b)^(2/3))+1/18/b^2/(a/b)^(2/3)*3^(1/2)*ar
ctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^2-1))
```

maxima [A] time = 2.36, size = 122, normalized size = 0.86

$$-\frac{x^2}{6\left(b^2x^6+ab\right)}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{18\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^4-x^2\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{36\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\log\left(x^2+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{18\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^6+a)^2,x, algorithm="maxima")
```

```
[Out] -1/6*x^2/(b^2*x^6 + a*b) + 1/18*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - (a/b)^(
1/3))/(a/b)^(1/3))/(b^2*(a/b)^(2/3)) - 1/36*log(x^4 - x^2*(a/b)^(1/3) + (a/
b)^(2/3))/(b^2*(a/b)^(2/3)) + 1/18*log(x^2 + (a/b)^(1/3))/(b^2*(a/b)^(2/3))
```

mupad [B] time = 1.18, size = 118, normalized size = 0.83

$$\frac{\ln\left(a^{1/3}+b^{1/3}x^2\right)}{18\,a^{2/3}\,b^{4/3}}-\frac{x^2}{6\,b\left(b\,x^6+a\right)}+\frac{\ln\left(\frac{2\,b\,x^2}{9}+\frac{a^{1/3}\,b^{2/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{9}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{36\,a^{2/3}\,b^{4/3}}-\frac{\ln\left(\frac{2\,b\,x^2}{9}-\frac{a^{1/3}\,b^{2/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{9}\right)\left(1+\sqrt{}\right.}{36\,a^{2/3}\,b^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^6)^2,x)
```

```
[Out] log(a^(1/3) + b^(1/3)*x^2)/(18*a^(2/3)*b^(4/3)) - x^2/(6*b*(a + b*x^6)) + (
log((2*b*x^2)/9 + (a^(1/3)*b^(2/3)*(3^(1/2)*1i - 1))/9)*(3^(1/2)*1i - 1))/(
36*a^(2/3)*b^(4/3)) - (log((2*b*x^2)/9 - (a^(1/3)*b^(2/3)*(3^(1/2)*1i + 1))
/9)*(3^(1/2)*1i + 1))/(36*a^(2/3)*b^(4/3))
```

sympy [A] time = 0.78, size = 42, normalized size = 0.30

$$-\frac{x^2}{6ab+6b^2x^6}+\operatorname{RootSum}\left(5832t^3a^2b^4-1,\left(t\mapsto t\log\left(18tab+x^2\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**6+a)**2,x)
```

```
[Out] -x**2/(6*a*b + 6*b**2*x**6) + RootSum(5832*_t**3*a**2*b**4 - 1, Lambda(_t,
_t*log(18*_t*a*b + x**2)))
```

3.1333 $\int \frac{x^6}{\left(a+bx^6\right)^2}\,dx$

Optimal. Leaf size=232

$$-\frac{\log\left(-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{24\sqrt{3}\,a^{5/6}b^{7/6}}+\frac{\log\left(\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{24\sqrt{3}\,a^{5/6}b^{7/6}}+\frac{\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{18a^{5/6}b^{7/6}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}-2\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{36a^{5/6}b^{7/6}}+$$

```
[Out] -1/6*x/b/(b*x^6+a)+1/18*arctan(b^(1/6)*x/a^(1/6))/a^(5/6)/b^(7/6)-1/36*arct
an((-2*b^(1/6)*x+a^(1/6)*3^(1/2))/a^(1/6))/a^(5/6)/b^(7/6)+1/36*arctan((2*b
^(1/6)*x+a^(1/6)*3^(1/2))/a^(1/6))/a^(5/6)/b^(7/6)-1/72*ln(a^(1/3)+b^(1/3)*
x^2-a^(1/6)*b^(1/6)*x*3^(1/2))/a^(5/6)/b^(7/6)*3^(1/2)+1/72*ln(a^(1/3)+b^(1
/3)*x^2+a^(1/6)*b^(1/6)*x*3^(1/2))/a^(5/6)/b^(7/6)*3^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {288, 209, 634, 618, 204, 628, 205}

$$-\frac{\log\left(-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{24\sqrt{3}\,a^{5/6}b^{7/6}}+\frac{\log\left(\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{24\sqrt{3}\,a^{5/6}b^{7/6}}+\frac{\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{18a^{5/6}b^{7/6}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}-2\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{36a^{5/6}b^{7/6}}+$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^6)^2,x]
```

```
[Out] -x/(6*b*(a + b*x^6)) + ArcTan[(b^(1/6)*x)/a^(1/6)]/(18*a^(5/6)*b^(7/6)) - A
rcTan[(Sqrt[3]*a^(1/6) - 2*b^(1/6)*x)/a^(1/6)]/(36*a^(5/6)*b^(7/6)) + ArcTa
n[(Sqrt[3]*a^(1/6) + 2*b^(1/6)*x)/a^(1/6)]/(36*a^(5/6)*b^(7/6)) - Log[a^(1/
3) - Sqrt[3]*a^(1/6)*b^(1/6)*x + b^(1/3)*x^2]/(24*Sqrt[3]*a^(5/6)*b^(7/6))
 + Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*x + b^(1/3)*x^2]/(24*Sqrt[3]*a^(5/6
)*b^(7/6))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
```

```
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^6}{\left(a+bx^6\right)^2}\,dx = -\frac{x}{6b\left(a+bx^6\right)} + \frac{\int \frac{1}{a+bx^6}\,dx}{6b}$$

$$= -\frac{x}{6b\left(a+bx^6\right)} + \frac{\int \frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\,\sqrt[6]{b}\,x}{\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx}{18a^{5/6}b} + \frac{\int \frac{\sqrt[6]{a}+\frac{1}{2}\sqrt{3}\,\sqrt[6]{b}\,x}{\sqrt[3]{a}+\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx}{18a^{5/6}b} + \frac{\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x^2}\,dx}{18a^{2/3}b}$$

$$= -\frac{x}{6b\left(a+bx^6\right)} + \frac{\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{18a^{5/6}b^{7/6}} - \frac{\int \frac{-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}+2\sqrt[3]{b}\,x}{\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx}{24\sqrt{3}\,a^{5/6}b^{7/6}} + \frac{\int \frac{\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}+2\sqrt[3]{b}\,x}{\sqrt[3]{a}+\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx}{24\sqrt{3}\,a^{5/6}b^{7/6}} + \int$$

$$= -\frac{x}{6b\left(a+bx^6\right)} + \frac{\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{18a^{5/6}b^{7/6}} - \frac{\log\left(\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2\right)}{24\sqrt{3}\,a^{5/6}b^{7/6}} + \frac{\log\left(\sqrt[3]{a}+\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\right.}{24\sqrt{3}\,a^{5/6}b^{7/6}}$$

$$= -\frac{x}{6b\left(a+bx^6\right)} + \frac{\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{18a^{5/6}b^{7/6}} - \frac{\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{36a^{5/6}b^{7/6}} + \frac{\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{36a^{5/6}b^{7/6}} - \frac{\log\left(\sqrt[3]{a}-\sqrt{3}\right.}{24\sqrt{3}}$$

Mathematica [A] time = 0.12, size = 191, normalized size = 0.82

$$\frac{-\frac{\sqrt{3}\,\log\left(-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{a^{5/6}} + \frac{\sqrt{3}\,\log\left(\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{a^{5/6}} + \frac{4\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{a^{5/6}} - \frac{2\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{a^{5/6}} + \frac{2\tan^{-1}\left(\frac{2\sqrt[6]{b}\,x}{\sqrt[6]{a}}+\sqrt{3}\right)}{a^{5/6}} -}{72b^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^6)^2,x]
```

```
[Out] ((-12*b^(1/6)*x)/(a + b*x^6) + (4*ArcTan[(b^(1/6)*x)/a^(1/6)])/a^(5/6) - (2
*ArcTan[Sqrt[3] - (2*b^(1/6)*x)/a^(1/6)])/a^(5/6) + (2*ArcTan[Sqrt[3] + (2*
```

```
b^(1/6)*x)/a^(1/6)])/a^(5/6) - (Sqrt[3]*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/
6)*x + b^(1/3)*x^2])/a^(5/6) + (Sqrt[3]*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/
6)*x + b^(1/3)*x^2])/a^(5/6))/(72*b^(7/6))
```

fricas [B] time = 0.88, size = 443, normalized size = 1.91

$$\frac{4\sqrt{3}\left(b^2x^6+ab\right)\left(-\frac{1}{a^5b^7}\right)^{\frac{1}{6}}\arctan\left(-\frac{2}{3}\sqrt{3}\,a^4b^6x\left(-\frac{1}{a^5b^7}\right)^{\frac{5}{6}}+\frac{2}{3}\sqrt{3}\sqrt{a^2b^2\left(-\frac{1}{a^5b^7}\right)^{\frac{1}{3}}+abx\left(-\frac{1}{a^5b^7}\right)^{\frac{1}{6}}+x^2}\,a^4b^6\left(-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^6+a)^2,x, algorithm="fricas")
```

```
[Out] 1/72*(4*sqrt(3)*(b^2*x^6 + a*b)*(-1/(a^5*b^7))^(1/6)*arctan(-2/3*sqrt(3)*a^
4*b^6*x*(-1/(a^5*b^7))^(5/6) + 2/3*sqrt(3)*sqrt(a^2*b^2*(-1/(a^5*b^7))^(1/3
) + a*b*x*(-1/(a^5*b^7))^(1/6) + x^2)*a^4*b^6*(-1/(a^5*b^7))^(5/6) + 1/3*sq
rt(3)) + 4*sqrt(3)*(b^2*x^6 + a*b)*(-1/(a^5*b^7))^(1/6)*arctan(-2/3*sqrt(3)
*a^4*b^6*x*(-1/(a^5*b^7))^(5/6) + 2/3*sqrt(3)*sqrt(a^2*b^2*(-1/(a^5*b^7))^(
1/3) - a*b*x*(-1/(a^5*b^7))^(1/6) + x^2)*a^4*b^6*(-1/(a^5*b^7))^(5/6) - 1/3
*sqrt(3)) + (b^2*x^6 + a*b)*(-1/(a^5*b^7))^(1/6)*log(a^2*b^2*(-1/(a^5*b^7))
^(1/3) + a*b*x*(-1/(a^5*b^7))^(1/6) + x^2) - (b^2*x^6 + a*b)*(-1/(a^5*b^7))
^(1/6)*log(a^2*b^2*(-1/(a^5*b^7))^(1/3) - a*b*x*(-1/(a^5*b^7))^(1/6) + x^2)
 + 2*(b^2*x^6 + a*b)*(-1/(a^5*b^7))^(1/6)*log(a*b*(-1/(a^5*b^7))^(1/6) + x)
 - 2*(b^2*x^6 + a*b)*(-1/(a^5*b^7))^(1/6)*log(-a*b*(-1/(a^5*b^7))^(1/6) + x
) - 12*x)/(b^2*x^6 + a*b)
```

giac [A] time = 0.21, size = 205, normalized size = 0.88

$$-\frac{x}{6\left(bx^6+a\right)b}+\frac{\sqrt{3}\,\left(ab^5\right)^{\frac{1}{6}}\log\left(x^2+\sqrt{3}\,x\left(\frac{a}{b}\right)^{\frac{1}{6}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{72\,ab^2}-\frac{\sqrt{3}\,\left(ab^5\right)^{\frac{1}{6}}\log\left(x^2-\sqrt{3}\,x\left(\frac{a}{b}\right)^{\frac{1}{6}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{72\,ab^2}+\frac{\left(ab^5\right)^{\frac{1}{6}}\mathrm{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^6+a)^2,x, algorithm="giac")
```

```
[Out] -1/6*x/((b*x^6 + a)*b) + 1/72*sqrt(3)*(a*b^5)^(1/6)*log(x^2 + sqrt(3)*x*(a/
b)^(1/6) + (a/b)^(1/3))/(a*b^2) - 1/72*sqrt(3)*(a*b^5)^(1/6)*log(x^2 - sqrt
(3)*x*(a/b)^(1/6) + (a/b)^(1/3))/(a*b^2) + 1/36*(a*b^5)^(1/6)*arctan((2*x +
 sqrt(3)*(a/b)^(1/6))/(a/b)^(1/6))/(a*b^2) + 1/36*(a*b^5)^(1/6)*arctan((2*x
 - sqrt(3)*(a/b)^(1/6))/(a/b)^(1/6))/(a*b^2) + 1/18*(a*b^5)^(1/6)*arctan(x/
(a/b)^(1/6))/(a*b^2)
```

maple [A] time = 0.06, size = 189, normalized size = 0.81

$$-\frac{x}{6\left(b\,x^6+a\right)b}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18ab}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{36ab}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{36ab}-\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x^2-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^6+a)^2,x)
```

```
[Out] -1/6*x/b/(b*x^6+a)+1/18/b/a*(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x)-1/72/b/a*3^
(1/2)*(a/b)^(1/6)*ln(x^2-3^(1/2)*(a/b)^(1/6)*x+(a/b)^(1/3))+1/36/b/a*(a/b)^
(1/6)*arctan(2/(a/b)^(1/6)*x-3^(1/2))+1/72/b/a*3^(1/2)*(a/b)^(1/6)*ln(x^2+3
```

```
^(1/2)*(a/b)^(1/6)*x+(a/b)^(1/3))+1/36/b/a*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)
*x+3^(1/2))
```

maxima [A] time = 2.30, size = 205, normalized size = 0.88

$$-\frac{x}{6\left(b^2x^6+ab\right)}+\frac{\frac{\sqrt{3}\log\left(b^{\frac{1}{3}}x^2+\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}-\frac{\sqrt{3}\log\left(b^{\frac{1}{3}}x^2-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}+\frac{4\arctan\left(\frac{b^{\frac{1}{3}}x}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{2\arctan\left(\frac{2b^{\frac{1}{3}}x+\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{2}{}}{72\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^6+a)^2,x, algorithm="maxima")
```

```
[Out] -1/6*x/(b^2*x^6 + a*b) + 1/72*(sqrt(3)*log(b^(1/3)*x^2 + sqrt(3)*a^(1/6)*b^
(1/6)*x + a^(1/3))/(a^(5/6)*b^(1/6)) - sqrt(3)*log(b^(1/3)*x^2 - sqrt(3)*a^
(1/6)*b^(1/6)*x + a^(1/3))/(a^(5/6)*b^(1/6)) + 4*arctan(b^(1/3)*x/sqrt(a^(1
/3)*b^(1/3)))/(a^(2/3)*sqrt(a^(1/3)*b^(1/3))) + 2*arctan((2*b^(1/3)*x + sqr
t(3)*a^(1/6)*b^(1/6))/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*sqrt(a^(1/3)*b^(1/3))
) + 2*arctan((2*b^(1/3)*x - sqrt(3)*a^(1/6)*b^(1/6))/sqrt(a^(1/3)*b^(1/3)))
/(a^(2/3)*sqrt(a^(1/3)*b^(1/3))))/b
```

mupad [B] time = 1.15, size = 242, normalized size = 1.04

$$-\frac{x}{6\,b\,\left(b\,x^6+a\right)}+\frac{\operatorname{atan}\left(\frac{b^{1/6}\,x\,1\mathrm{i}}{(-a)^{1/6}}\right)\,1\mathrm{i}}{18\,(-a)^{5/6}\,b^{7/6}}-\frac{\operatorname{atan}\left(\frac{x\,1\mathrm{i}}{7776\,(-a)^{5/6}\,b^{1/6}\left(\frac{1}{7776\,(-a)^{2/3}\,b^{1/3}}-\frac{\sqrt{3}\,1\mathrm{i}}{7776\,(-a)^{2/3}\,b^{1/3}}\right)}-\frac{\sqrt{3}\,x}{7776\,(-a)^{5/6}\,b^{1/6}\left(\frac{1}{7776\,(-a)^{2/3}\,b^{1/3}}-\frac{}{7776\,(}\right.\right.}{36\,(-a)^{5/6}\,b^{7/6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^6)^2,x)
```

```
[Out] (atan((b^(1/6)*x*1i)/(-a)^(1/6))*1i)/(18*(-a)^(5/6)*b^(7/6)) - x/(6*b*(a +
b*x^6)) - (atan((x*1i)/(7776*(-a)^(5/6)*b^(1/6)*(1/(7776*(-a)^(2/3)*b^(1/3)
) - (3^(1/2)*1i)/(7776*(-a)^(2/3)*b^(1/3)))) - (3^(1/2)*x)/(7776*(-a)^(5/6)
*b^(1/6)*(1/(7776*(-a)^(2/3)*b^(1/3)) - (3^(1/2)*1i)/(7776*(-a)^(2/3)*b^(1/
3)))))*(3^(1/2)*1i + 1)*1i)/(36*(-a)^(5/6)*b^(7/6)) + (atan((x*1i)/(7776*(-
a)^(5/6)*b^(1/6)*(1/(7776*(-a)^(2/3)*b^(1/3)) + (3^(1/2)*1i)/(7776*(-a)^(2/
3)*b^(1/3)))) + (3^(1/2)*x)/(7776*(-a)^(5/6)*b^(1/6)*(1/(7776*(-a)^(2/3)*b^
(1/3)) + (3^(1/2)*1i)/(7776*(-a)^(2/3)*b^(1/3)))))*(3^(1/2)*1i - 1)*1i)/(36
*(-a)^(5/6)*b^(7/6))
```

sympy [A] time = 0.72, size = 39, normalized size = 0.17

$$-\frac{x}{6ab+6b^2x^6}+\operatorname{RootSum}\left(2176782336t^6a^5b^7+1,\left(t\mapsto t\log\left(36tab+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**6+a)**2,x)
```

```
[Out] -x/(6*a*b + 6*b**2*x**6) + RootSum(2176782336*_t**6*a**5*b**7 + 1, Lambda(_
t, _t*log(36*_t*a*b + x)))
```

3.1334 $\int \frac{x^5}{\left(a+bx^6\right)^2}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{6b\left(a+bx^6\right)}$$

```
[Out] -1/6/b/(b*x^6+a)
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$-\frac{1}{6b\left(a+bx^6\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^6)^2,x]
```

```
[Out] -1/(6*b*(a + b*x^6))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^5}{\left(a+bx^6\right)^2}\,dx = -\frac{1}{6b\left(a+bx^6\right)}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{6b\left(a+bx^6\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^6)^2,x]
```

```
[Out] -1/6*1/(b*(a + b*x^6))
```

fricas [A] time = 0.86, size = 15, normalized size = 0.94

$$-\frac{1}{6\left(b^2x^6+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^6+a)^2,x, algorithm="fricas")
```

```
[Out] -1/6/(b^2*x^6 + a*b)
```

giac [A] time = 0.18, size = 14, normalized size = 0.88

$$-\frac{1}{6\left(bx^6+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^6+a)^2,x, algorithm="giac")
[Out] -1/6/((b*x^6 + a)*b)
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$-\frac{1}{6\left(b\,x^6+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^6+a)^2,x)
[Out] -1/6/b/(b*x^6+a)
```

maxima [A] time = 1.01, size = 14, normalized size = 0.88

$$-\frac{1}{6\left(bx^6+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^6+a)^2,x, algorithm="maxima")
[Out] -1/6/((b*x^6 + a)*b)
```

mupad [B] time = 1.01, size = 14, normalized size = 0.88

$$-\frac{1}{6\,b\,\left(b\,x^6+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^6)^2,x)
[Out] -1/(6*b*(a + b*x^6))
```

sympy [A] time = 0.51, size = 15, normalized size = 0.94

$$-\frac{1}{6ab+6b^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**6+a)**2,x)
[Out] -1/(6*a*b + 6*b**2*x**6)
```

3.1335 $\int \frac{x^4}{\left(a+bx^6\right)^2}\,dx$

Optimal. Leaf size=234

$$\frac{\log\left(-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{24\sqrt{3}\,a^{7/6}b^{5/6}}-\frac{\log\left(\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{24\sqrt{3}\,a^{7/6}b^{5/6}}+\frac{\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{18a^{7/6}b^{5/6}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}-2\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{36a^{7/6}b^{5/6}}+\text{ta}$$

```
[Out] 1/6*x^5/a/(b*x^6+a)+1/18*arctan(b^(1/6)*x/a^(1/6))/a^(7/6)/b^(5/6)-1/36*arc
tan((-2*b^(1/6)*x+a^(1/6)*3^(1/2))/a^(1/6))/a^(7/6)/b^(5/6)+1/36*arctan((2*
b^(1/6)*x+a^(1/6)*3^(1/2))/a^(1/6))/a^(7/6)/b^(5/6)+1/72*ln(a^(1/3)+b^(1/3)
*x^2-a^(1/6)*b^(1/6)*x*3^(1/2))/a^(7/6)/b^(5/6)*3^(1/2)-1/72*ln(a^(1/3)+b^(
1/3)*x^2+a^(1/6)*b^(1/6)*x*3^(1/2))/a^(7/6)/b^(5/6)*3^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {290, 295, 634, 618, 204, 628, 205}

$$\frac{\log\left(-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{24\sqrt{3}\,a^{7/6}b^{5/6}}-\frac{\log\left(\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{24\sqrt{3}\,a^{7/6}b^{5/6}}+\frac{\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{18a^{7/6}b^{5/6}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}-2\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{36a^{7/6}b^{5/6}}+\text{ta}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^6)^2,x]
```

```
[Out] x^5/(6*a*(a + b*x^6)) + ArcTan[(b^(1/6)*x)/a^(1/6)]/(18*a^(7/6)*b^(5/6)) -
ArcTan[(Sqrt[3]*a^(1/6) - 2*b^(1/6)*x)/a^(1/6)]/(36*a^(7/6)*b^(5/6)) + ArcT
an[(Sqrt[3]*a^(1/6) + 2*b^(1/6)*x)/a^(1/6)]/(36*a^(7/6)*b^(5/6)) + Log[a^(1
/3) - Sqrt[3]*a^(1/6)*b^(1/6)*x + b^(1/3)*x^2]/(24*Sqrt[3]*a^(7/6)*b^(5/6))
 - Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*x + b^(1/3)*x^2]/(24*Sqrt[3]*a^(7/
6)*b^(5/6))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
```

```
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^6\right)^2}\,dx = \frac{x^5}{6a\left(a+bx^6\right)} + \frac{\int \frac{x^4}{a+bx^6}\,dx}{6a}$$

$$= \frac{x^5}{6a\left(a+bx^6\right)} + \frac{\int \frac{-\frac{\sqrt[6]{a}}{2}+\frac{1}{2}\sqrt{3}\,\sqrt[6]{b}\,x}{\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx}{18a^{7/6}b^{2/3}} + \frac{\int \frac{-\frac{\sqrt[6]{a}}{2}-\frac{1}{2}\sqrt{3}\,\sqrt[6]{b}\,x}{\sqrt[3]{a}+\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx}{18a^{7/6}b^{2/3}} + \frac{\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x^2}\,dx}{18ab^{2/3}}$$

$$= \frac{x^5}{6a\left(a+bx^6\right)} + \frac{\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{18a^{7/6}b^{5/6}} + \frac{\int \frac{-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}+2\sqrt[3]{b}\,x}{\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx}{24\sqrt{3}\,a^{7/6}b^{5/6}} - \frac{\int \frac{\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}+2\sqrt[3]{b}\,x}{\sqrt[3]{a}+\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx}{24\sqrt{3}\,a^{7/6}b^{5/6}} + \frac{\int \frac{}{\sqrt[3]{a}-}}{}$$

$$= \frac{x^5}{6a\left(a+bx^6\right)} + \frac{\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{18a^{7/6}b^{5/6}} + \frac{\log\left(\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2\right)}{24\sqrt{3}\,a^{7/6}b^{5/6}} - \frac{\log\left(\sqrt[3]{a}+\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\right.}{24\sqrt{3}\,a^{7/6}b^{5/6}}$$

$$= \frac{x^5}{6a\left(a+bx^6\right)} + \frac{\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{18a^{7/6}b^{5/6}} - \frac{\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{36a^{7/6}b^{5/6}} + \frac{\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{36a^{7/6}b^{5/6}} + \frac{\log\left(\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{}\right.}{24\sqrt{3}\,a}$$

Mathematica [A] time = 0.11, size = 193, normalized size = 0.82

$$\frac{\frac{\sqrt{3}\log\left(-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{b^{5/6}} - \frac{\sqrt{3}\log\left(\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{b^{5/6}} + \frac{4\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{b^{5/6}} - \frac{2\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{b^{5/6}} + \frac{2\tan^{-1}\left(\frac{2\sqrt[6]{b}\,x}{\sqrt[6]{a}}+\sqrt{3}\right)}{b^{5/6}} + \frac{12}{}}{72a^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^6)^2,x]
```

```
[Out] ((12*a^(1/6)*x^5)/(a + b*x^6) + (4*ArcTan[(b^(1/6)*x)/a^(1/6)])/b^(5/6) - (
2*ArcTan[Sqrt[3] - (2*b^(1/6)*x)/a^(1/6)])/b^(5/6) + (2*ArcTan[Sqrt[3] + (2
*b^(1/6)*x)/a^(1/6)])/b^(5/6) + (Sqrt[3]*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1
/6)*x + b^(1/3)*x^2])/b^(5/6) - (Sqrt[3]*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1
/6)*x + b^(1/3)*x^2])/b^(5/6))/(72*a^(7/6))
```

fricas [B] time = 0.69, size = 450, normalized size = 1.92

$$\frac{12\,x^5 - 4\sqrt{3}\left(abx^6 + a^2\right)\left(-\frac{1}{a^7b^5}\right)^{\frac{1}{6}}\arctan\left(-\frac{2}{3}\sqrt{3}\,abx\left(-\frac{1}{a^7b^5}\right)^{\frac{1}{6}} + \frac{2}{3}\sqrt{3}\sqrt{a^6b^4x\left(-\frac{1}{a^7b^5}\right)^{\frac{5}{6}} - a^5b^3\left(-\frac{1}{a^7b^5}\right)^{\frac{2}{3}} + x^2}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^6+a)^2,x, algorithm="fricas")
```

```
[Out] 1/72*(12*x^5 - 4*sqrt(3)*(a*b*x^6 + a^2)*(-1/(a^7*b^5))^(1/6)*arctan(-2/3*s
qrt(3)*a*b*x*(-1/(a^7*b^5))^(1/6) + 2/3*sqrt(3)*sqrt(a^6*b^4*x*(-1/(a^7*b^5
))^(5/6) - a^5*b^3*(-1/(a^7*b^5))^(2/3) + x^2)*a*b*(-1/(a^7*b^5))^(1/6) + 1
/3*sqrt(3)) - 4*sqrt(3)*(a*b*x^6 + a^2)*(-1/(a^7*b^5))^(1/6)*arctan(-2/3*sq
rt(3)*a*b*x*(-1/(a^7*b^5))^(1/6) + 2/3*sqrt(3)*sqrt(-a^6*b^4*x*(-1/(a^7*b^5
))^(5/6) - a^5*b^3*(-1/(a^7*b^5))^(2/3) + x^2)*a*b*(-1/(a^7*b^5))^(1/6) - 1
/3*sqrt(3)) + (a*b*x^6 + a^2)*(-1/(a^7*b^5))^(1/6)*log(a^6*b^4*x*(-1/(a^7*b
^5))^(5/6) - a^5*b^3*(-1/(a^7*b^5))^(2/3) + x^2) - (a*b*x^6 + a^2)*(-1/(a^7
*b^5))^(1/6)*log(-a^6*b^4*x*(-1/(a^7*b^5))^(5/6) - a^5*b^3*(-1/(a^7*b^5))^(
2/3) + x^2) + 2*(a*b*x^6 + a^2)*(-1/(a^7*b^5))^(1/6)*log(a^6*b^4*(-1/(a^7*b
^5))^(5/6) + x) - 2*(a*b*x^6 + a^2)*(-1/(a^7*b^5))^(1/6)*log(-a^6*b^4*(-1/(
a^7*b^5))^(5/6) + x))/(a*b*x^6 + a^2)
```

giac [A] time = 0.33, size = 204, normalized size = 0.87

$$\frac{x^5}{6\left(bx^6 + a\right)a} + \frac{\left(\frac{a}{b}\right)^{\frac{5}{6}}\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,a^2} - \frac{\sqrt{3}\,\left(ab^5\right)^{\frac{5}{6}}\log\left(x^2 + \sqrt{3}\,x\left(\frac{a}{b}\right)^{\frac{1}{6}} + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{72\,a^2b^5} + \frac{\sqrt{3}\,\left(ab^5\right)^{\frac{5}{6}}\log\left(x^2 - \sqrt{3}\,x\left(\frac{a}{b}\right)^{\frac{1}{6}} +\right.}{72\,a^2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^6+a)^2,x, algorithm="giac")
```

```
[Out] 1/6*x^5/((b*x^6 + a)*a) + 1/18*(a/b)^(5/6)*arctan(x/(a/b)^(1/6))/a^2 - 1/72
*sqrt(3)*(a*b^5)^(5/6)*log(x^2 + sqrt(3)*x*(a/b)^(1/6) + (a/b)^(1/3))/(a^2*
b^5) + 1/72*sqrt(3)*(a*b^5)^(5/6)*log(x^2 - sqrt(3)*x*(a/b)^(1/6) + (a/b)^(
1/3))/(a^2*b^5) + 1/36*(a*b^5)^(5/6)*arctan((2*x + sqrt(3)*(a/b)^(1/6))/(a/
b)^(1/6))/(a^2*b^5) + 1/36*(a*b^5)^(5/6)*arctan((2*x - sqrt(3)*(a/b)^(1/6))
/(a/b)^(1/6))/(a^2*b^5)
```

maple [B] time = 0.44, size = 346, normalized size = 1.48

$$\frac{x}{18\left(x^2 + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)ab} + \frac{x}{18\left(x^2 - \sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)ab} + \frac{x}{18\left(x^2 + \sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)ab} + \frac{\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{6}}ab} + \frac{\arctan}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^6+a)^2,x)
```

```
[Out] 1/18/b/a*x/(x^2+(a/b)^(1/3))+1/18/b/a/(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x)+1
/18/b/a/(x^2-3^(1/2)*(a/b)^(1/6)*x+(a/b)^(1/3))*x-1/36/b/a/(x^2-3^(1/2)*(a/
b)^(1/6)*x+(a/b)^(1/3))*(a/b)^(1/6)*3^(1/2)+1/72/a^2*(a/b)^(5/6)*3^(1/2)*ln
(x^2-3^(1/2)*(a/b)^(1/6)*x+(a/b)^(1/3))+1/36/b/a/(a/b)^(1/6)*arctan(2/(a/b)
^(1/6)*x-3^(1/2))+1/18/b/a/(x^2+3^(1/2)*(a/b)^(1/6)*x+(a/b)^(1/3))*x+1/36/b
/a/(x^2+3^(1/2)*(a/b)^(1/6)*x+(a/b)^(1/3))*(a/b)^(1/6)*3^(1/2)-1/72/a^2*(a/
b)^(5/6)*3^(1/2)*ln(x^2+3^(1/2)*(a/b)^(1/6)*x+(a/b)^(1/3))+1/36/b/a/(a/b)^(
1/6)*arctan(2/(a/b)^(1/6)*x+3^(1/2))
```

maxima [A] time = 2.46, size = 206, normalized size = 0.88

$$\frac{x^5}{6\left(abx^6+a^2\right)}-\frac{\frac{\sqrt{3}\log\left(b^{\frac{1}{3}}x^2+\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{\sqrt{3}\log\left(b^{\frac{1}{3}}x^2-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{4\arctan\left(\frac{b^{\frac{1}{3}}x}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-\frac{2\arctan\left(\frac{2b^{\frac{1}{3}}x+\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-\frac{2\ \mathrm{ar}}{}}{72\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^6+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*x^5/(a*b*x^6 + a^2) - 1/72*(sqrt(3)*log(b^(1/3)*x^2 + sqrt(3)*a^(1/6)*b
^(1/6)*x + a^(1/3))/(a^(1/6)*b^(5/6)) - sqrt(3)*log(b^(1/3)*x^2 - sqrt(3)*a
^(1/6)*b^(1/6)*x + a^(1/3))/(a^(1/6)*b^(5/6)) - 4*arctan(b^(1/3)*x/sqrt(a^(
1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 2*arctan((2*b^(1/3)*x + sq
rt(3)*a^(1/6)*b^(1/6))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3)
)) - 2*arctan((2*b^(1/3)*x - sqrt(3)*a^(1/6)*b^(1/6))/sqrt(a^(1/3)*b^(1/3))
)/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))))/a
```

mupad [B] time = 1.18, size = 160, normalized size = 0.68

$$\frac{x^5}{6\,a\,\left(b\,x^6+a\right)}-\frac{\mathrm{atan}\left(\frac{b^{1/6}\,x\,1\mathrm{i}}{(-a)^{1/6}}\right)\,1\mathrm{i}}{18\,(-a)^{7/6}\,b^{5/6}}-\frac{\mathrm{atan}\left(\frac{b^{3/2}\,x\,1\mathrm{i}}{3888\,(-a)^{5/2}\left(\frac{b^{4/3}}{7776\,(-a)^{7/3}}-\frac{\sqrt{3}\,b^{4/3}\,1\mathrm{i}}{7776\,(-a)^{7/3}}\right)}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)\,1\mathrm{i}}{36\,(-a)^{7/6}\,b^{5/6}}+\frac{\mathrm{atan}\left(\frac{b^{3/2}\,x\,1\mathrm{i}}{3888\,(-a)^{5/2}\left(\frac{b^{4/3}}{7776\,(-a)^{7/3}}\right.\right.}}{36\,(-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^6)^2,x)
```

```
[Out] x^5/(6*a*(a + b*x^6)) - (atan((b^(1/6)*x*1i)/(-a)^(1/6))*1i)/(18*(-a)^(7/6)
*b^(5/6)) - (atan((b^(3/2)*x*1i)/(3888*(-a)^(5/2)*(b^(4/3)/(7776*(-a)^(7/3)
) - (3^(1/2)*b^(4/3)*1i)/(7776*(-a)^(7/3)))))*(3^(1/2)*1i + 1)*1i)/(36*(-a)
^(7/6)*b^(5/6)) + (atan((b^(3/2)*x*1i)/(3888*(-a)^(5/2)*(b^(4/3)/(7776*(-a)
^(7/3)) + (3^(1/2)*b^(4/3)*1i)/(7776*(-a)^(7/3)))))*(3^(1/2)*1i - 1)*1i)/(3
6*(-a)^(7/6)*b^(5/6))
```

sympy [A] time = 0.66, size = 46, normalized size = 0.20

$$\frac{x^5}{6a^2+6abx^6}+\mathrm{RootSum}\left(2176782336t^6a^7b^5+1,\left(t\mapsto t\log\left(60466176t^5a^6b^4+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**6+a)**2,x)
```

```
[Out] x**5/(6*a**2 + 6*a*b*x**6) + RootSum(2176782336*_t**6*a**7*b**5 + 1, Lambda
(_t, _t*log(60466176*_t**5*a**6*b**4 + x)))
```

3.1336 $\int \frac{x^3}{\left(a+bx^6\right)^2}\,dx$

Optimal. Leaf size=142

$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{18a^{4/3}b^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x^2}{\sqrt{3}\,\sqrt[3]{a}}\right)}{6\sqrt{3}\,a^{4/3}b^{2/3}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^2+b^{2/3}x^4\right)}{36a^{4/3}b^{2/3}}+\frac{x^4}{6a\left(a+bx^6\right)}$$

```
[Out] 1/6*x^4/a/(b*x^6+a)-1/18*ln(a^(1/3)+b^(1/3)*x^2)/a^(4/3)/b^(2/3)+1/36*ln(a^
(2/3)-a^(1/3)*b^(1/3)*x^2+b^(2/3)*x^4)/a^(4/3)/b^(2/3)-1/18*arctan(1/3*(a^(
1/3)-2*b^(1/3)*x^2)/a^(1/3)*3^(1/2))/a^(4/3)/b^(2/3)*3^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {275, 290, 292, 31, 634, 617, 204, 628}

$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{18a^{4/3}b^{2/3}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^2+b^{2/3}x^4\right)}{36a^{4/3}b^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x^2}{\sqrt{3}\,\sqrt[3]{a}}\right)}{6\sqrt{3}\,a^{4/3}b^{2/3}}+\frac{x^4}{6a\left(a+bx^6\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^6)^2,x]
```

```
[Out] x^4/(6*a*(a + b*x^6)) - ArcTan[(a^(1/3) - 2*b^(1/3)*x^2)/(Sqrt[3]*a^(1/3))]
/(6*Sqrt[3]*a^(4/3)*b^(2/3)) - Log[a^(1/3) + b^(1/3)*x^2]/(18*a^(4/3)*b^(2/
3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x^2 + b^(2/3)*x^4]/(36*a^(4/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
```

```
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(a+bx^6\right)^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{\left(a+bx^3\right)^2}\,dx, x, x^2\right)\\
&= \frac{x^4}{6a\left(a+bx^6\right)} + \frac{\text{Subst}\left(\int \frac{x}{a+bx^3}\,dx, x, x^2\right)}{6a}\\
&= \frac{x^4}{6a\left(a+bx^6\right)} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, x^2\right)}{18a^{4/3}\sqrt[3]{b}} + \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, x^2\right)}{18a^{4/3}\sqrt[3]{b}}\\
&= \frac{x^4}{6a\left(a+bx^6\right)} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{18a^{4/3}b^{2/3}} + \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, x^2\right)}{36a^{4/3}b^{2/3}} + \frac{\text{Subst}\left(\int \frac{}{a^{2/3}-\sqrt[3]{a}}\right.}{12}\\
&= \frac{x^4}{6a\left(a+bx^6\right)} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{18a^{4/3}b^{2/3}} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^2+b^{2/3}x^4\right)}{36a^{4/3}b^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1\right.}{6a^{4/3}b^{2/3}}\\
&= \frac{x^4}{6a\left(a+bx^6\right)} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x^2}{\sqrt{3}\,\sqrt[3]{a}}\right)}{6\sqrt{3}\,a^{4/3}b^{2/3}} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{18a^{4/3}b^{2/3}} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^2+b^{2/3}x^4\right)}{36a^{4/3}b^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 195, normalized size = 1.37

$$\frac{-\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{b^{2/3}} + \frac{\log\left(-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{b^{2/3}} + \frac{\log\left(\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{b^{2/3}} - \frac{2\sqrt{3}\,\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{b^{2/3}} - \frac{2\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[6]{b}\,x}{\sqrt[6]{a}}+\sqrt{3}\right)}{b^{2/3}}}{36a^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^6)^2,x]
```

```
[Out] ((6*a^(1/3)*x^4)/(a + b*x^6) - (2*Sqrt[3]*ArcTan[Sqrt[3] - (2*b^(1/6)*x)/a^
(1/6)])/b^(2/3) - (2*Sqrt[3]*ArcTan[Sqrt[3] + (2*b^(1/6)*x)/a^(1/6)])/b^(2/
3) - (2*Log[a^(1/3) + b^(1/3)*x^2])/b^(2/3) + Log[a^(1/3) - Sqrt[3]*a^(1/6)
*b^(1/6)*x + b^(1/3)*x^2]/b^(2/3) + Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*x
 + b^(1/3)*x^2]/b^(2/3))/(36*a^(4/3))
```

fricas [A] time = 0.89, size = 416, normalized size = 2.93

$$\left[\frac{6\,ab^2x^4+3\sqrt{\frac{1}{3}}\left(ab^2x^6+a^2b\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\,\log\left(\frac{2\,b^2x^6-3\left(-ab^2\right)^{\frac{2}{3}}x^2-ab+3\sqrt{\frac{1}{3}}\left(2\left(-ab^2\right)^{\frac{2}{3}}x^4+abx^2+\left(-ab^2\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}}{bx^6+a}\right)+\left(bx^6\right.}{36\left(a^2b^3x^6+a^3b\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^6+a)^2,x, algorithm="fricas")
```

```
[Out] [1/36*(6*a*b^2*x^4 + 3*sqrt(1/3)*(a*b^2*x^6 + a^2*b)*sqrt((-a*b^2)^(1/3)/a)
*log((2*b^2*x^6 - 3*(-a*b^2)^(2/3)*x^2 - a*b + 3*sqrt(1/3)*(2*(-a*b^2)^(2/3
)*x^4 + a*b*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a))/(b*x^6 + a)) +
(b*x^6 + a)*(-a*b^2)^(2/3)*log(b^2*x^4 + (-a*b^2)^(1/3)*b*x^2 + (-a*b^2)^(2
/3)) - 2*(b*x^6 + a)*(-a*b^2)^(2/3)*log(b*x^2 - (-a*b^2)^(1/3)))/(a^2*b^3*x
^6 + a^3*b^2), 1/36*(6*a*b^2*x^4 + 6*sqrt(1/3)*(a*b^2*x^6 + a^2*b)*sqrt(-(-
a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x^2 + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^
(1/3)/a)/b) + (b*x^6 + a)*(-a*b^2)^(2/3)*log(b^2*x^4 + (-a*b^2)^(1/3)*b*x^2
 + (-a*b^2)^(2/3)) - 2*(b*x^6 + a)*(-a*b^2)^(2/3)*log(b*x^2 - (-a*b^2)^(1/3
)))/(a^2*b^3*x^6 + a^3*b^2)]
```

giac [A] time = 0.18, size = 135, normalized size = 0.95

$$\frac{x^4}{6\left(bx^6+a\right)a}-\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^2-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{18\,a^2}-\frac{\sqrt{3}\,\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^2+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{18\,a^2b^2}+\frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^4+x^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\right.}{36\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^6+a)^2,x, algorithm="giac")
```

```
[Out] 1/6*x^4/((b*x^6 + a)*a) - 1/18*(-a/b)^(2/3)*log(abs(x^2 - (-a/b)^(1/3)))/a^
2 - 1/18*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^2 + (-a/b)^(1/3))/(
-a/b)^(1/3))/(a^2*b^2) + 1/36*(-a*b^2)^(2/3)*log(x^4 + x^2*(-a/b)^(1/3) + (
-a/b)^(2/3))/(a^2*b^2)
```

maple [A] time = 0.01, size = 123, normalized size = 0.87

$$\frac{x^4}{6\left(b\,x^6+a\right)a}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^2}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}-\frac{\ln\left(x^2+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}+\frac{\ln\left(x^4-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^2+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{36\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^6+a)^2,x)
```

```
[Out] 1/6*x^4/a/(b*x^6+a)-1/18/a/b/(a/b)^(1/3)*ln(x^2+(a/b)^(1/3))+1/36/a/b/(a/b)
^(1/3)*ln(x^4-(a/b)^(1/3)*x^2+(a/b)^(2/3))+1/18/a*3^(1/2)/b/(a/b)^(1/3)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^2-1))
```

maxima [A] time = 2.39, size = 130, normalized size = 0.92

$$\frac{x^4}{6\left(abx^6+a^2\right)}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{18\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^4-x^2\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{36\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\log\left(x^2+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{18\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^6+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*x^4/(a*b*x^6 + a^2) + 1/18*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - (a/b)^(1
/3))/(a/b)^(1/3))/(a*b*(a/b)^(1/3)) + 1/36*log(x^4 - x^2*(a/b)^(1/3) + (a/b
)^(2/3))/(a*b*(a/b)^(1/3)) - 1/18*log(x^2 + (a/b)^(1/3))/(a*b*(a/b)^(1/3))
```

mupad [B] time = 1.34, size = 146, normalized size = 1.03

$$\frac{x^4}{6\,a\,\left(b\,x^6+a\right)}+\frac{(-1)^{1/3}\,\ln\left(\frac{b^2}{81\,a^3}-\frac{(-1)^{1/3}\,b^{7/3}\,x^2}{81\,a^{10/3}}\right)}{18\,a^{4/3}\,b^{2/3}}-\frac{(-1)^{1/3}\,\ln\left((-1)^{2/3}\,a^{1/3}-2\,b^{1/3}\,x^2+(-1)^{1/6}\,\sqrt{3}\,a^{1/3}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{18\,a^{4/3}\,b^{2/3}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^6)^2,x)
```

```
[Out] x^4/(6*a*(a + b*x^6)) + ((-1)^(1/3)*log(b^2/(81*a^3) - ((-1)^(1/3)*b^(7/3)*
x^2)/(81*a^(10/3))))/(18*a^(4/3)*b^(2/3)) - ((-1)^(1/3)*log((-1)^(2/3)*a^(1
/3) - 2*b^(1/3)*x^2 + (-1)^(1/6)*3^(1/2)*a^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(
18*a^(4/3)*b^(2/3)) + ((-1)^(1/3)*log(2*b^(1/3)*x^2 - (-1)^(2/3)*a^(1/3) +
(-1)^(1/6)*3^(1/2)*a^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(18*a^(4/3)*b^(2/3))
```

sympy [A] time = 0.67, size = 46, normalized size = 0.32

$$\frac{x^4}{6a^2+6abx^6}+\operatorname{RootSum}\left(5832t^3a^4b^2+1,\left(t\mapsto t\log\left(324t^2a^3b+x^2\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**6+a)**2,x)
```

```
[Out] x**4/(6*a**2 + 6*a*b*x**6) + RootSum(5832*_t**3*a**4*b**2 + 1, Lambda(_t, _
t*log(324*_t**2*a**3*b + x**2)))
```

3.1337 $\int \frac{x^2}{\left(a+bx^6\right)^2}\,dx$

Optimal. Leaf size=49

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x^3}{\sqrt{a}}\right)}{6a^{3/2}\sqrt{b}}+\frac{x^3}{6a\left(a+bx^6\right)}$$

```
[Out] 1/6*x^3/a/(b*x^6+a)+1/6*arctan(x^3*b^(1/2)/a^(1/2))/a^(3/2)/b^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {275, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x^3}{\sqrt{a}}\right)}{6a^{3/2}\sqrt{b}}+\frac{x^3}{6a\left(a+bx^6\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^6)^2,x]
```

```
[Out] x^3/(6*a*(a + b*x^6)) + ArcTan[(Sqrt[b]*x^3)/Sqrt[a]]/(6*a^(3/2)*Sqrt[b])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a+bx^6\right)^2}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^2}\,dx, x, x^3\right)\\ &= \frac{x^3}{6a\left(a+bx^6\right)}+\frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^3\right)}{6a}\\ &= \frac{x^3}{6a\left(a+bx^6\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x^3}{\sqrt{a}}\right)}{6a^{3/2}\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 49, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x^3}{\sqrt{a}}\right)}{6a^{3/2}\sqrt{b}} + \frac{x^3}{6a\left(a+bx^6\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^6)^2,x]
```

```
[Out] x^3/(6*a*(a + b*x^6)) + ArcTan[(Sqrt[b]*x^3)/Sqrt[a]]/(6*a^(3/2)*Sqrt[b])
```

fricas [A] time = 0.81, size = 128, normalized size = 2.61

$$\left[\frac{2\,abx^3 - \left(bx^6+a\right)\sqrt{-ab}\,\log\left(\frac{bx^6-2\,\sqrt{-ab}\,x^3-a}{bx^6+a}\right)}{12\left(a^2b^2x^6+a^3b\right)}, \frac{abx^3+\left(bx^6+a\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x^3}{a}\right)}{6\left(a^2b^2x^6+a^3b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^6+a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(2*a*b*x^3 - (b*x^6 + a)*sqrt(-a*b)*log((b*x^6 - 2*sqrt(-a*b)*x^3 - a
)/(b*x^6 + a)))/(a^2*b^2*x^6 + a^3*b), 1/6*(a*b*x^3 + (b*x^6 + a)*sqrt(a*b)
*arctan(sqrt(a*b)*x^3/a))/(a^2*b^2*x^6 + a^3*b)]
```

giac [A] time = 0.18, size = 39, normalized size = 0.80

$$\frac{x^3}{6\left(bx^6+a\right)a} + \frac{\arctan\left(\frac{bx^3}{\sqrt{ab}}\right)}{6\sqrt{ab}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^6+a)^2,x, algorithm="giac")
```

```
[Out] 1/6*x^3/((b*x^6 + a)*a) + 1/6*arctan(b*x^3/sqrt(a*b))/(sqrt(a*b)*a)
```

maple [A] time = 0.01, size = 40, normalized size = 0.82

$$\frac{x^3}{6\left(b\,x^6+a\right)a} + \frac{\arctan\left(\frac{b\,x^3}{\sqrt{ab}}\right)}{6\sqrt{ab}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^6+a)^2,x)
```

```
[Out] 1/6*x^3/a/(b*x^6+a)+1/6/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^3)
```

maxima [A] time = 2.38, size = 39, normalized size = 0.80

$$\frac{x^3}{6\left(abx^6+a^2\right)} + \frac{\arctan\left(\frac{bx^3}{\sqrt{ab}}\right)}{6\sqrt{ab}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^6+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*x^3/(a*b*x^6 + a^2) + 1/6*arctan(b*x^3/sqrt(a*b))/(sqrt(a*b)*a)
```

mupad [B] time = 1.06, size = 37, normalized size = 0.76

$$\frac{x^3}{6\,a\,\left(b\,x^6+a\right)}+\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x^3}{\sqrt{a}}\right)}{6\,a^{3/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^6)^2,x)
```

```
[Out] x^3/(6*a*(a + b*x^6)) + atan((b^(1/2)*x^3)/a^(1/2))/(6*a^(3/2)*b^(1/2))
```

sympy [B] time = 0.84, size = 83, normalized size = 1.69

$$\frac{x^3}{6a^2+6abx^6}-\frac{\sqrt{-\frac{1}{a^3b}}\,\log\left(-a^2\sqrt{-\frac{1}{a^3b}}+x^3\right)}{12}+\frac{\sqrt{-\frac{1}{a^3b}}\,\log\left(a^2\sqrt{-\frac{1}{a^3b}}+x^3\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**6+a)**2,x)
```

```
[Out] x**3/(6*a**2 + 6*a*b*x**6) - sqrt(-1/(a**3*b))*log(-a**2*sqrt(-1/(a**3*b))
+ x**3)/12 + sqrt(-1/(a**3*b))*log(a**2*sqrt(-1/(a**3*b)) + x**3)/12
```

3.1338 $\int \frac{x}{\left(a+bx^6\right)^2}\,dx$

Optimal. Leaf size=142

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^2+b^{2/3}x^4\right)}{18a^{5/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{9a^{5/3}\sqrt[3]{b}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x^2}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}\sqrt[3]{b}}+\frac{x^2}{6a\left(a+bx^6\right)}$$

```
[Out] 1/6*x^2/a/(b*x^6+a)+1/9*ln(a^(1/3)+b^(1/3)*x^2)/a^(5/3)/b^(1/3)-1/18*ln(a^(
2/3)-a^(1/3)*b^(1/3)*x^2+b^(2/3)*x^4)/a^(5/3)/b^(1/3)-1/9*arctan(1/3*(a^(1/
3)-2*b^(1/3)*x^2)/a^(1/3)*3^(1/2))/a^(5/3)/b^(1/3)*3^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {275, 199, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^2+b^{2/3}x^4\right)}{18a^{5/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{9a^{5/3}\sqrt[3]{b}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x^2}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}\sqrt[3]{b}}+\frac{x^2}{6a\left(a+bx^6\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^6)^2,x]
```

```
[Out] x^2/(6*a*(a + b*x^6)) - ArcTan[(a^(1/3) - 2*b^(1/3)*x^2)/(Sqrt[3]*a^(1/3))]
/(3*Sqrt[3]*a^(5/3)*b^(1/3)) + Log[a^(1/3) + b^(1/3)*x^2]/(9*a^(5/3)*b^(1/3
)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x^2 + b^(2/3)*x^4]/(18*a^(5/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^6\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\left(a+bx^3\right)^2}\,dx, x, x^2\right)$$

$$= \frac{x^2}{6a\left(a+bx^6\right)} + \frac{\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, x^2\right)}{3a}$$

$$= \frac{x^2}{6a\left(a+bx^6\right)} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, x^2\right)}{9a^{5/3}} + \frac{\text{Subst}\left(\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, x^2\right)}{9a^{5/3}}$$

$$= \frac{x^2}{6a\left(a+bx^6\right)} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{9a^{5/3}\sqrt[3]{b}} + \frac{\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, x^2\right)}{6a^{4/3}} - \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{}}{a^{2/3}_{-}}\right.}{1}$$

$$= \frac{x^2}{6a\left(a+bx^6\right)} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{9a^{5/3}\sqrt[3]{b}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^2+b^{2/3}x^4\right)}{18a^{5/3}\sqrt[3]{b}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x\right.}{3a^{5/3}\sqrt[3]{b}}$$

$$= \frac{x^2}{6a\left(a+bx^6\right)} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x^2}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}\sqrt[3]{b}} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{9a^{5/3}\sqrt[3]{b}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^2+b^{2/3}x^4\right)}{18a^{5/3}\sqrt[3]{b}}$$

Mathematica [A] time = 0.12, size = 197, normalized size = 1.39

$$\frac{\frac{3a^{2/3}x^2}{a+bx^6} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{\sqrt[3]{b}} - \frac{\log\left(-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{\sqrt[3]{b}} - \frac{\log\left(\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{\sqrt[3]{b}} - \frac{2\sqrt{3}\,\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{\sqrt[3]{b}} - \frac{2\sqrt{3}\,\tan^{-1}\left(\right.}{\sqrt[3]{}}}{18a^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^6)^2,x]
```

```
[Out] ((3*a^(2/3)*x^2)/(a + b*x^6) - (2*Sqrt[3]*ArcTan[Sqrt[3] - (2*b^(1/6)*x)/a^
(1/6)])/b^(1/3) - (2*Sqrt[3]*ArcTan[Sqrt[3] + (2*b^(1/6)*x)/a^(1/6)])/b^(1/
3) + (2*Log[a^(1/3) + b^(1/3)*x^2])/b^(1/3) - Log[a^(1/3) - Sqrt[3]*a^(1/6)
*b^(1/6)*x + b^(1/3)*x^2]/b^(1/3) - Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*x
 + b^(1/3)*x^2]/b^(1/3))/(18*a^(5/3))
```

fricas [A] time = 1.13, size = 407, normalized size = 2.87

$$\left[\frac{3\,a^2bx^2+3\,\sqrt{\frac{1}{3}}\left(ab^2x^6+a^2b\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\,\log\left(\frac{2\,abx^6-3\left(a^2b\right)^{\frac{1}{3}}ax^2-a^2+3\,\sqrt{\frac{1}{3}}\left(2\,abx^4+\left(a^2b\right)^{\frac{2}{3}}x^2-\left(a^2b\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}}{bx^6+a}\right)-\left(bx^6+a\right)}{18\left(a^3b^2x^6+a^4b\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^6+a)^2,x, algorithm="fricas")
```

```
[Out] [1/18*(3*a^2*b*x^2 + 3*sqrt(1/3)*(a*b^2*x^6 + a^2*b)*sqrt(-(a^2*b)^(1/3)/b)
*log((2*a*b*x^6 - 3*(a^2*b)^(1/3)*a*x^2 - a^2 + 3*sqrt(1/3)*(2*a*b*x^4 + (a
^2*b)^(2/3)*x^2 - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^6 + a)) - (
b*x^6 + a)*(a^2*b)^(2/3)*log(a*b*x^4 - (a^2*b)^(2/3)*x^2 + (a^2*b)^(1/3)*a)
 + 2*(b*x^6 + a)*(a^2*b)^(2/3)*log(a*b*x^2 + (a^2*b)^(2/3)))/(a^3*b^2*x^6 +
 a^4*b), 1/18*(3*a^2*b*x^2 + 6*sqrt(1/3)*(a*b^2*x^6 + a^2*b)*sqrt((a^2*b)^(
1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x^2 - (a^2*b)^(1/3)*a)*sqrt((a^2*
b)^(1/3)/b)/a^2) - (b*x^6 + a)*(a^2*b)^(2/3)*log(a*b*x^4 - (a^2*b)^(2/3)*x^
2 + (a^2*b)^(1/3)*a) + 2*(b*x^6 + a)*(a^2*b)^(2/3)*log(a*b*x^2 + (a^2*b)^(2
/3)))/(a^3*b^2*x^6 + a^4*b)]
```

giac [A] time = 0.20, size = 135, normalized size = 0.95

$$\frac{x^2}{6\left(bx^6+a\right)a}-\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^2-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2}+\frac{\sqrt{3}\,\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^2+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b}+\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^4+x^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\right.\right.}{18\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^6+a)^2,x, algorithm="giac")
```

```
[Out] 1/6*x^2/((b*x^6 + a)*a) - 1/9*(-a/b)^(1/3)*log(abs(x^2 - (-a/b)^(1/3)))/a^2
 + 1/9*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^2 + (-a/b)^(1/3))/(-a
/b)^(1/3))/(a^2*b) + 1/18*(-a*b^2)^(1/3)*log(x^4 + x^2*(-a/b)^(1/3) + (-a/b
)^(2/3))/(a^2*b)
```

maple [A] time = 0.01, size = 123, normalized size = 0.87

$$\frac{x^2}{6\left(b\,x^6+a\right)a}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^2}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}+\frac{\ln\left(x^2+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}-\frac{\ln\left(x^4-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^2+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^6+a)^2,x)
```

```
[Out] 1/6*x^2/a/(b*x^6+a)+1/9/a/b/(a/b)^(2/3)*ln(x^2+(a/b)^(1/3))-1/18/a/b/(a/b)^
(2/3)*ln(x^4-(a/b)^(1/3)*x^2+(a/b)^(2/3))+1/9/a/b/(a/b)^(2/3)*3^(1/2)*arcta
n(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^2-1))
```

maxima [A] time = 2.31, size = 130, normalized size = 0.92

$$\frac{x^2}{6\left(abx^6+a^2\right)}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^4-x^2\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\log\left(x^2+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^6+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*x^2/(a*b*x^6 + a^2) + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - (a/b)^(1/
3))/(a/b)^(1/3))/(a*b*(a/b)^(2/3)) - 1/18*log(x^4 - x^2*(a/b)^(1/3) + (a/b)
^(2/3))/(a*b*(a/b)^(2/3)) + 1/9*log(x^2 + (a/b)^(1/3))/(a*b*(a/b)^(2/3))
```

mupad [B] time = 1.19, size = 128, normalized size = 0.90

$$\frac{\ln\left(a^{1/3}+b^{1/3}x^2\right)}{9a^{5/3}b^{1/3}}+\frac{x^2}{6a\left(bx^6+a\right)}+\frac{\ln\left(\frac{16b^4x^2}{9a^3}+\frac{8b^{11/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{9a^{8/3}}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{18a^{5/3}b^{1/3}}-\frac{\ln\left(\frac{16b^4x^2}{9a^3}-\frac{8b^{11/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{9a^{8/3}}\right)\left(1\right.}{18a^{5/3}b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^6)^2,x)
```

```
[Out] log(a^(1/3) + b^(1/3)*x^2)/(9*a^(5/3)*b^(1/3)) + x^2/(6*a*(a + b*x^6)) + (l
og((16*b^4*x^2)/(9*a^3) + (8*b^(11/3)*(3^(1/2)*1i - 1))/(9*a^(8/3)))*(3^(1/
2)*1i - 1))/(18*a^(5/3)*b^(1/3)) - (log((16*b^4*x^2)/(9*a^3) - (8*b^(11/3)*
(3^(1/2)*1i + 1))/(9*a^(8/3)))*(3^(1/2)*1i + 1))/(18*a^(5/3)*b^(1/3))
```

sympy [A] time = 0.79, size = 41, normalized size = 0.29

$$\frac{x^2}{6a^2+6abx^6}+\operatorname{RootSum}\left(729t^3a^5b-1,\left(t\mapsto t\log\left(9ta^2+x^2\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**6+a)**2,x)
```

```
[Out] x**2/(6*a**2 + 6*a*b*x**6) + RootSum(729*_t**3*a**5*b - 1, Lambda(_t, _t*lo
g(9*_t*a**2 + x**2)))
```

3.1339 $\int \frac{1}{\left(a+bx^6\right)^2}\,dx$

Optimal. Leaf size=232

$$-\frac{5\log\left(-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{24\sqrt{3}\,a^{11/6}\sqrt[6]{b}}+\frac{5\log\left(\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{24\sqrt{3}\,a^{11/6}\sqrt[6]{b}}+\frac{5\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{18a^{11/6}\sqrt[6]{b}}-\frac{5\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}-2\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right.}{36a^{11/6}\sqrt[6]{b}}$$

```
[Out] 1/6*x/a/(b*x^6+a)+5/18*arctan(b^(1/6)*x/a^(1/6))/a^(11/6)/b^(1/6)-5/36*arct
an((-2*b^(1/6)*x+a^(1/6)*3^(1/2))/a^(1/6))/a^(11/6)/b^(1/6)+5/36*arctan((2*
b^(1/6)*x+a^(1/6)*3^(1/2))/a^(1/6))/a^(11/6)/b^(1/6)-5/72*ln(a^(1/3)+b^(1/3
)*x^2-a^(1/6)*b^(1/6)*x*3^(1/2))/a^(11/6)/b^(1/6)*3^(1/2)+5/72*ln(a^(1/3)+b
^(1/3)*x^2+a^(1/6)*b^(1/6)*x*3^(1/2))/a^(11/6)/b^(1/6)*3^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {199, 209, 634, 618, 204, 628, 205}

$$-\frac{5\log\left(-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{24\sqrt{3}\,a^{11/6}\sqrt[6]{b}}+\frac{5\log\left(\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{24\sqrt{3}\,a^{11/6}\sqrt[6]{b}}+\frac{5\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{18a^{11/6}\sqrt[6]{b}}-\frac{5\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}-2\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right.}{36a^{11/6}\sqrt[6]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^6)^(-2),x]
```

```
[Out] x/(6*a*(a + b*x^6)) + (5*ArcTan[(b^(1/6)*x)/a^(1/6)])/(18*a^(11/6)*b^(1/6))
 - (5*ArcTan[(Sqrt[3]*a^(1/6) - 2*b^(1/6)*x)/a^(1/6)])/(36*a^(11/6)*b^(1/6)
) + (5*ArcTan[(Sqrt[3]*a^(1/6) + 2*b^(1/6)*x)/a^(1/6)])/(36*a^(11/6)*b^(1/6
)) - (5*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*x + b^(1/3)*x^2])/(24*Sqrt[3]
*a^(11/6)*b^(1/6)) + (5*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*x + b^(1/3)*x
^2])/(24*Sqrt[3]*a^(11/6)*b^(1/6))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
```

```
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^6\right)^2}\,dx = \frac{x}{6a\left(a+bx^6\right)} + \frac{5\int \frac{1}{a+bx^6}\,dx}{6a}$$

$$= \frac{x}{6a\left(a+bx^6\right)} + \frac{5\int \frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\,\sqrt[6]{b}\,x}{\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx}{18a^{11/6}} + \frac{5\int \frac{\sqrt[6]{a}+\frac{1}{2}\sqrt{3}\,\sqrt[6]{b}\,x}{\sqrt[3]{a}+\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx}{18a^{11/6}} + \frac{5\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x^2}\,dx}{18a^{5/3}}$$

$$= \frac{x}{6a\left(a+bx^6\right)} + \frac{5\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{18a^{11/6}\sqrt[6]{b}} + \frac{5\int \frac{1}{\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx}{72a^{5/3}} + \frac{5\int \frac{1}{\sqrt[3]{a}+\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx}{72a^{5/3}} -$$

$$= \frac{x}{6a\left(a+bx^6\right)} + \frac{5\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{18a^{11/6}\sqrt[6]{b}} - \frac{5\log\left(\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2\right)}{24\sqrt{3}\,a^{11/6}\sqrt[6]{b}} + \frac{5\log\left(\sqrt[3]{a}+\sqrt{3}\,\sqrt[6]{a}\right.}{24\sqrt{3}\,a^{11/}}$$

$$= \frac{x}{6a\left(a+bx^6\right)} + \frac{5\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{18a^{11/6}\sqrt[6]{b}} - \frac{5\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{36a^{11/6}\sqrt[6]{b}} + \frac{5\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{36a^{11/6}\sqrt[6]{b}} - \frac{5\log\left(\sqrt[3]{}\right.}{}$$

Mathematica [A] time = 0.12, size = 192, normalized size = 0.83

$$\frac{\frac{12a^{5/6}x}{a+bx^6} - \frac{5\sqrt{3}\log\left(-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{\sqrt[6]{b}} + \frac{5\sqrt{3}\log\left(\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{\sqrt[6]{b}} + \frac{20\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{\sqrt[6]{b}} - \frac{10\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{\sqrt[6]{b}} + \frac{10\tan}{}}{72a^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^6)^(-2),x]
```

```
[Out] ((12*a^(5/6)*x)/(a + b*x^6) + (20*ArcTan[(b^(1/6)*x)/a^(1/6)])/b^(1/6) - (1
0*ArcTan[Sqrt[3] - (2*b^(1/6)*x)/a^(1/6)])/b^(1/6) + (10*ArcTan[Sqrt[3] + (
```

```
2*b^(1/6)*x)/a^(1/6)])/b^(1/6) - (5*Sqrt[3]*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b
^(1/6)*x + b^(1/3)*x^2])/b^(1/6) + (5*Sqrt[3]*Log[a^(1/3) + Sqrt[3]*a^(1/6)
*b^(1/6)*x + b^(1/3)*x^2])/b^(1/6))/(72*a^(11/6))
```

fricas [B] time = 0.87, size = 423, normalized size = 1.82

$$\frac{20\sqrt{3}\left(abx^6+a^2\right)\left(-\frac{1}{a^{11}b}\right)^{\frac{1}{6}}\arctan\left(-\frac{2}{3}\sqrt{3}\,a^9bx\left(-\frac{1}{a^{11}b}\right)^{\frac{5}{6}}+\frac{2}{3}\sqrt{3}\sqrt{a^4\left(-\frac{1}{a^{11}b}\right)^{\frac{1}{3}}+a^2x\left(-\frac{1}{a^{11}b}\right)^{\frac{1}{6}}+x^2}\,a^9b\left(-\frac{1}{a^{11}b}\right)^{\frac{5}{6}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a)^2,x, algorithm="fricas")
```

```
[Out] 1/72*(20*sqrt(3)*(a*b*x^6 + a^2)*(-1/(a^11*b))^(1/6)*arctan(-2/3*sqrt(3)*a^
9*b*x*(-1/(a^11*b))^(5/6) + 2/3*sqrt(3)*sqrt(a^4*(-1/(a^11*b))^(1/3) + a^2*
x*(-1/(a^11*b))^(1/6) + x^2)*a^9*b*(-1/(a^11*b))^(5/6) + 1/3*sqrt(3)) + 20*
sqrt(3)*(a*b*x^6 + a^2)*(-1/(a^11*b))^(1/6)*arctan(-2/3*sqrt(3)*a^9*b*x*(-1
/(a^11*b))^(5/6) + 2/3*sqrt(3)*sqrt(a^4*(-1/(a^11*b))^(1/3) - a^2*x*(-1/(a^
11*b))^(1/6) + x^2)*a^9*b*(-1/(a^11*b))^(5/6) - 1/3*sqrt(3)) + 5*(a*b*x^6 +
 a^2)*(-1/(a^11*b))^(1/6)*log(a^4*(-1/(a^11*b))^(1/3) + a^2*x*(-1/(a^11*b))
^(1/6) + x^2) - 5*(a*b*x^6 + a^2)*(-1/(a^11*b))^(1/6)*log(a^4*(-1/(a^11*b))
^(1/3) - a^2*x*(-1/(a^11*b))^(1/6) + x^2) + 10*(a*b*x^6 + a^2)*(-1/(a^11*b)
)^(1/6)*log(a^2*(-1/(a^11*b))^(1/6) + x) - 10*(a*b*x^6 + a^2)*(-1/(a^11*b))
^(1/6)*log(-a^2*(-1/(a^11*b))^(1/6) + x) + 12*x)/(a*b*x^6 + a^2)
```

giac [A] time = 0.16, size = 205, normalized size = 0.88

$$\frac{x}{6\left(bx^6+a\right)a}+\frac{5\sqrt{3}\,\left(ab^5\right)^{\frac{1}{6}}\log\left(x^2+\sqrt{3}\,x\left(\frac{a}{b}\right)^{\frac{1}{6}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{72\,a^2b}-\frac{5\sqrt{3}\,\left(ab^5\right)^{\frac{1}{6}}\log\left(x^2-\sqrt{3}\,x\left(\frac{a}{b}\right)^{\frac{1}{6}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{72\,a^2b}+\frac{5\,\left(ab^5\right)^{\frac{1}{6}}\mathrm{ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a)^2,x, algorithm="giac")
```

```
[Out] 1/6*x/((b*x^6 + a)*a) + 5/72*sqrt(3)*(a*b^5)^(1/6)*log(x^2 + sqrt(3)*x*(a/b
)^(1/6) + (a/b)^(1/3))/(a^2*b) - 5/72*sqrt(3)*(a*b^5)^(1/6)*log(x^2 - sqrt(
3)*x*(a/b)^(1/6) + (a/b)^(1/3))/(a^2*b) + 5/36*(a*b^5)^(1/6)*arctan((2*x +
sqrt(3)*(a/b)^(1/6))/(a/b)^(1/6))/(a^2*b) + 5/36*(a*b^5)^(1/6)*arctan((2*x
- sqrt(3)*(a/b)^(1/6))/(a/b)^(1/6))/(a^2*b) + 5/18*(a*b^5)^(1/6)*arctan(x/(
a/b)^(1/6))/(a^2*b)
```

maple [B] time = 0.45, size = 343, normalized size = 1.48

$$\frac{\left(\frac{a}{b}\right)^{\frac{1}{3}}x}{18\left(x^2+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)a^2}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{3}}x}{36\left(x^2-\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)a^2}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{3}}x}{36\left(x^2+\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)a^2}+\frac{5\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18a^2}+\frac{5\left(\frac{a}{b}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^6+a)^2,x)
```

```
[Out] 1/18*(a/b)^(1/3)/a^2*x/(x^2+(a/b)^(1/3))+5/18*(a/b)^(1/6)/a^2*arctan(1/(a/b
)^(1/6)*x)-1/36/a^2/(x^2-3^(1/2)*(a/b)^(1/6)*x+(a/b)^(1/3))*(a/b)^(1/3)*x+1
/36/a^2/(x^2-3^(1/2)*(a/b)^(1/6)*x+(a/b)^(1/3))*(a/b)^(1/2)*3^(1/2)-5/72/a^
```

```
2*3^(1/2)*(a/b)^(1/6)*ln(x^2-3^(1/2)*(a/b)^(1/6)*x+(a/b)^(1/3))+5/36/a^2*(a
/b)^(1/6)*arctan(2/(a/b)^(1/6)*x-3^(1/2))-1/36/a^2/(x^2+3^(1/2)*(a/b)^(1/6)
*x+(a/b)^(1/3))*(a/b)^(1/3)*x-1/36/a^2/(x^2+3^(1/2)*(a/b)^(1/6)*x+(a/b)^(1/
3))*(a/b)^(1/2)*3^(1/2)+5/72/a^2*3^(1/2)*(a/b)^(1/6)*ln(x^2+3^(1/2)*(a/b)^(
1/6)*x+(a/b)^(1/3))+5/36/a^2*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x+3^(1/2))
```

maxima [A] time = 2.33, size = 204, normalized size = 0.88

$$\frac{x}{6\left(abx^6+a^2\right)}+\frac{5\left(\frac{\sqrt{3}\log\left(b^{\frac{1}{3}}x^2+\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}-\frac{\sqrt{3}\log\left(b^{\frac{1}{3}}x^2-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}+\frac{4\arctan\left(\frac{b^{\frac{1}{3}}x}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{2\arctan\left(\frac{2b^{\frac{1}{3}}x+\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{72\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*x/(a*b*x^6 + a^2) + 5/72*(sqrt(3)*log(b^(1/3)*x^2 + sqrt(3)*a^(1/6)*b^(
1/6)*x + a^(1/3))/(a^(5/6)*b^(1/6)) - sqrt(3)*log(b^(1/3)*x^2 - sqrt(3)*a^(
1/6)*b^(1/6)*x + a^(1/3))/(a^(5/6)*b^(1/6)) + 4*arctan(b^(1/3)*x/sqrt(a^(1/
3)*b^(1/3)))/(a^(2/3)*sqrt(a^(1/3)*b^(1/3))) + 2*arctan((2*b^(1/3)*x + sqrt
(3)*a^(1/6)*b^(1/6))/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*sqrt(a^(1/3)*b^(1/3)))
 + 2*arctan((2*b^(1/3)*x - sqrt(3)*a^(1/6)*b^(1/6))/sqrt(a^(1/3)*b^(1/3)))/
(a^(2/3)*sqrt(a^(1/3)*b^(1/3))))/a
```

mupad [B] time = 1.14, size = 242, normalized size = 1.04

$$\frac{x}{6\,a\,\left(b\,x^6+a\right)}-\frac{\operatorname{atan}\left(\frac{b^{1/6}\,x\,1\mathrm{i}}{(-a)^{1/6}}\right)\,5\mathrm{i}}{18\,(-a)^{11/6}\,b^{1/6}}+\frac{\operatorname{atan}\left(\frac{b^{29/6}\,x\,3125\mathrm{i}}{7776\,(-a)^{35/6}\left(\frac{3125\,b^{14/3}}{7776\,(-a)^{17/3}}-\frac{\sqrt{3}\,b^{14/3}\,3125\mathrm{i}}{7776\,(-a)^{17/3}}\right)}-\frac{3125\,\sqrt{3}\,b^{29/6}\,x}{7776\,(-a)^{35/6}\left(\frac{3125\,b^{14/3}}{7776\,(-a)^{17/3}}-\frac{\sqrt{3}\,b^{14/3}\,3125\mathrm{i}}{7776\,(-a)^{17/3}}\right)}\right)}{36\,(-a)^{11/6}\,b^{1/6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^6)^2,x)
```

```
[Out] x/(6*a*(a + b*x^6)) - (atan((b^(1/6)*x*1i)/(-a)^(1/6))*5i)/(18*(-a)^(11/6)*
b^(1/6)) + (atan((b^(29/6)*x*3125i)/(7776*(-a)^(35/6)*((3125*b^(14/3))/(777
6*(-a)^(17/3)) - (3^(1/2)*b^(14/3)*3125i)/(7776*(-a)^(17/3)))) - (3125*3^(1
/2)*b^(29/6)*x)/(7776*(-a)^(35/6)*((3125*b^(14/3))/(7776*(-a)^(17/3)) - (3^
(1/2)*b^(14/3)*3125i)/(7776*(-a)^(17/3)))))*(3^(1/2)*5i + 5)*1i)/(36*(-a)^(
11/6)*b^(1/6)) - (atan((b^(29/6)*x*3125i)/(7776*(-a)^(35/6)*((3125*b^(14/3)
)/(7776*(-a)^(17/3)) + (3^(1/2)*b^(14/3)*3125i)/(7776*(-a)^(17/3)))) + (312
5*3^(1/2)*b^(29/6)*x)/(7776*(-a)^(35/6)*((3125*b^(14/3))/(7776*(-a)^(17/3))
 + (3^(1/2)*b^(14/3)*3125i)/(7776*(-a)^(17/3)))))*(3^(1/2)*5i - 5)*1i)/(36*
(-a)^(11/6)*b^(1/6))
```

sympy [A] time = 0.72, size = 39, normalized size = 0.17

$$\frac{x}{6a^2+6abx^6}+\operatorname{RootSum}\left(2176782336t^6a^{11}b+15625,\left(t\mapsto t\log\left(\frac{36ta^2}{5}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**6+a)**2,x)
```

```
[Out] x/(6*a**2 + 6*a*b*x**6) + RootSum(2176782336*_t**6*a**11*b + 15625, Lambda(
_t, _t*log(36*_t*a**2/5 + x)))
```

3.1340 $\int \frac{1}{x\left(a+bx^6\right)^2}\,dx$

Optimal. Leaf size=38

$$-\frac{\log\left(a+bx^6\right)}{6a^2}+\frac{\log(x)}{a^2}+\frac{1}{6a\left(a+bx^6\right)}$$

```
[Out] 1/6/a/(b*x^6+a)+ln(x)/a^2-1/6*ln(b*x^6+a)/a^2
```

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 44}

$$-\frac{\log\left(a+bx^6\right)}{6a^2}+\frac{\log(x)}{a^2}+\frac{1}{6a\left(a+bx^6\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^6)^2),x]
```

```
[Out] 1/(6*a*(a + b*x^6)) + Log[x]/a^2 - Log[a + b*x^6]/(6*a^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a+bx^6\right)^2}\,dx &= \frac{1}{6}\text{Subst}\left(\int \frac{1}{x(a+bx)^2}\,dx, x, x^6\right)\\ &= \frac{1}{6}\text{Subst}\left(\int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right)dx, x, x^6\right)\\ &= \frac{1}{6a\left(a+bx^6\right)}+\frac{\log(x)}{a^2}-\frac{\log\left(a+bx^6\right)}{6a^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.87

$$\frac{\frac{a}{a+bx^6}-\log\left(a+bx^6\right)+6\log(x)}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^6)^2),x]
```

```
[Out] (a/(a + b*x^6) + 6*Log[x] - Log[a + b*x^6])/(6*a^2)
```

fricas [A] time = 0.99, size = 47, normalized size = 1.24

$$-\frac{\left(bx^6+a\right)\log\left(bx^6+a\right)-6\left(bx^6+a\right)\log(x)-a}{6\left(a^2bx^6+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^6+a)^2,x, algorithm="fricas")
```

```
[Out] -1/6*((b*x^6 + a)*log(b*x^6 + a) - 6*(b*x^6 + a)*log(x) - a)/(a^2*b*x^6 + a
^3)
```

giac [A] time = 0.16, size = 47, normalized size = 1.24

$$\frac{\log\left(x^6\right)}{6\,a^2}-\frac{\log\left(\left|bx^6+a\right|\right)}{6\,a^2}+\frac{bx^6+2\,a}{6\left(bx^6+a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^6+a)^2,x, algorithm="giac")
```

```
[Out] 1/6*log(x^6)/a^2 - 1/6*log(abs(b*x^6 + a))/a^2 + 1/6*(b*x^6 + 2*a)/((b*x^6
+ a)*a^2)
```

maple [A] time = 0.01, size = 35, normalized size = 0.92

$$\frac{1}{6\left(b\,x^6+a\right)a}+\frac{\ln(x)}{a^2}-\frac{\ln\left(b\,x^6+a\right)}{6a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^6+a)^2,x)
```

```
[Out] 1/6/a/(b*x^6+a)+1/a^2*ln(x)-1/6*ln(b*x^6+a)/a^2
```

maxima [A] time = 1.09, size = 37, normalized size = 0.97

$$\frac{1}{6\left(abx^6+a^2\right)}-\frac{\log\left(bx^6+a\right)}{6\,a^2}+\frac{\log\left(x^6\right)}{6\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^6+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6/(a*b*x^6 + a^2) - 1/6*log(b*x^6 + a)/a^2 + 1/6*log(x^6)/a^2
```

mupad [B] time = 0.07, size = 34, normalized size = 0.89

$$\frac{\ln(x)}{a^2}+\frac{1}{6\,a\,\left(b\,x^6+a\right)}-\frac{\ln\left(b\,x^6+a\right)}{6\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^6)^2),x)
```

```
[Out] log(x)/a^2 + 1/(6*a*(a + b*x^6)) - log(a + b*x^6)/(6*a^2)
```

sympy [A] time = 0.60, size = 34, normalized size = 0.89

$$\frac{1}{6a^2+6abx^6}+\frac{\log(x)}{a^2}-\frac{\log\left(\frac{a}{b}+x^6\right)}{6a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**6+a)**2,x)
[Out] 1/(6*a**2 + 6*a*b*x**6) + log(x)/a**2 - log(a/b + x**6)/(6*a**2)
```

3.1341 $\int \frac{1}{x^2\left(a+bx^6\right)^2}\,dx$

Optimal. Leaf size=244

$$-\frac{7\sqrt[6]{b}\,\log\left(-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{24\sqrt{3}\,a^{13/6}}+\frac{7\sqrt[6]{b}\,\log\left(\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{24\sqrt{3}\,a^{13/6}}-\frac{7\sqrt[6]{b}\,\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{18a^{13/6}}+\frac{7\sqrt[6]{b}\,\text{ta}}{}$$

```
[Out] -7/6/a^2/x+1/6/a/x/(b*x^6+a)-7/18*b^(1/6)*arctan(b^(1/6)*x/a^(1/6))/a^(13/6
)+7/36*b^(1/6)*arctan((-2*b^(1/6)*x+a^(1/6)*3^(1/2))/a^(1/6))/a^(13/6)-7/36
*b^(1/6)*arctan((2*b^(1/6)*x+a^(1/6)*3^(1/2))/a^(1/6))/a^(13/6)-7/72*b^(1/6
)*ln(a^(1/3)+b^(1/3)*x^2-a^(1/6)*b^(1/6)*x*3^(1/2))/a^(13/6)*3^(1/2)+7/72*b
^(1/6)*ln(a^(1/3)+b^(1/3)*x^2+a^(1/6)*b^(1/6)*x*3^(1/2))/a^(13/6)*3^(1/2)
```

Rubi [A] time = 0.50, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {290, 325, 295, 634, 618, 204, 628, 205}

$$-\frac{7\sqrt[6]{b}\,\log\left(-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{24\sqrt{3}\,a^{13/6}}+\frac{7\sqrt[6]{b}\,\log\left(\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{24\sqrt{3}\,a^{13/6}}-\frac{7\sqrt[6]{b}\,\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{18a^{13/6}}+\frac{7\sqrt[6]{b}\,\text{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^6)^2),x]
```

```
[Out] -7/(6*a^2*x) + 1/(6*a*x*(a + b*x^6)) - (7*b^(1/6)*ArcTan[(b^(1/6)*x)/a^(1/6
)])/(18*a^(13/6)) + (7*b^(1/6)*ArcTan[(Sqrt[3]*a^(1/6) - 2*b^(1/6)*x)/a^(1/
6)])/(36*a^(13/6)) - (7*b^(1/6)*ArcTan[(Sqrt[3]*a^(1/6) + 2*b^(1/6)*x)/a^(1
/6)])/(36*a^(13/6)) - (7*b^(1/6)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*x +
b^(1/3)*x^2])/(24*Sqrt[3]*a^(13/6)) + (7*b^(1/6)*Log[a^(1/3) + Sqrt[3]*a^(1
/6)*b^(1/6)*x + b^(1/3)*x^2])/(24*Sqrt[3]*a^(13/6))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
```

```
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^6\right)^2}\,dx = \frac{1}{6ax\left(a+bx^6\right)} + \frac{7\int \frac{1}{x^2\left(a+bx^6\right)}\,dx}{6a}$$

$$= -\frac{7}{6a^2x} + \frac{1}{6ax\left(a+bx^6\right)} - \frac{(7b)\int \frac{x^4}{a+bx^6}\,dx}{6a^2}$$

$$= -\frac{7}{6a^2x} + \frac{1}{6ax\left(a+bx^6\right)} - \frac{\left(7\sqrt[3]{b}\right)\int \frac{-\frac{\sqrt[6]{a}}{2}+\frac{1}{2}\sqrt{3}\,\sqrt[6]{b}\,x}{\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx}{18a^{13/6}} - \frac{\left(7\sqrt[3]{b}\right)\int \frac{-\frac{\sqrt[6]{a}}{2}-\frac{1}{2}\sqrt{3}\,\sqrt[6]{b}\,x}{\sqrt[3]{a}+\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx}{18a^{13/6}}$$

$$= -\frac{7}{6a^2x} + \frac{1}{6ax\left(a+bx^6\right)} - \frac{7\sqrt[6]{b}\,\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{18a^{13/6}} - \frac{\left(7\sqrt[6]{b}\right)\int \frac{-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}+2\sqrt[3]{b}\,x}{\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx}{24\sqrt{3}\,a^{13/6}} + \frac{\left(7\sqrt[6]{b}\right)\int$$

$$= -\frac{7}{6a^2x} + \frac{1}{6ax\left(a+bx^6\right)} - \frac{7\sqrt[6]{b}\,\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{18a^{13/6}} - \frac{7\sqrt[6]{b}\,\log\left(\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2\right)}{24\sqrt{3}\,a^{13/6}} + \frac{7\sqrt[6]{}}{}$$

$$= -\frac{7}{6a^2x} + \frac{1}{6ax\left(a+bx^6\right)} - \frac{7\sqrt[6]{b}\,\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{18a^{13/6}} + \frac{7\sqrt[6]{b}\,\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right)}{36a^{13/6}} - \frac{7\sqrt[6]{b}\,\tan^{-1}\left(\sqrt{3}\right.}{36a^{13/6}}$$

Mathematica [A] time = 0.14, size = 205, normalized size = 0.84

$$\frac{-7\sqrt{3}\,\sqrt[6]{b}\,\log\left(-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)+7\sqrt{3}\,\sqrt[6]{b}\,\log\left(\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)-\frac{12\sqrt[6]{a}\,bx^5}{a+bx^6}-28\sqrt[6]{b}\,\text{ta}}{72a^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^6)^2),x]
```

```
[Out] ((-72*a^(1/6))/x - (12*a^(1/6)*b*x^5)/(a + b*x^6) - 28*b^(1/6)*ArcTan[(b^(1
/6)*x)/a^(1/6)] + 14*b^(1/6)*ArcTan[Sqrt[3] - (2*b^(1/6)*x)/a^(1/6)] - 14*b
^(1/6)*ArcTan[Sqrt[3] + (2*b^(1/6)*x)/a^(1/6)] - 7*Sqrt[3]*b^(1/6)*Log[a^(1
/3) - Sqrt[3]*a^(1/6)*b^(1/6)*x + b^(1/3)*x^2] + 7*Sqrt[3]*b^(1/6)*Log[a^(1
/3) + Sqrt[3]*a^(1/6)*b^(1/6)*x + b^(1/3)*x^2])/(72*a^(13/6))
```

fricas [B] time = 0.90, size = 443, normalized size = 1.82

$$-\frac{84\,bx^6-28\sqrt{3}\left(a^2bx^7+a^3x\right)\left(-\frac{b}{a^{13}}\right)^{\frac{1}{6}}\arctan\left(-\frac{2}{3}\sqrt{3}\,a^2x\left(-\frac{b}{a^{13}}\right)^{\frac{1}{6}}+\frac{2}{3}\sqrt{3}\,a^2\sqrt{-\frac{a^{11}x\left(-\frac{b}{a^{13}}\right)^{\frac{5}{6}}+a^9\left(-\frac{b}{a^{13}}\right)^{\frac{2}{3}}-bx^2}{b}}\left(-\frac{b}{a^{\cdots}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^6+a)^2,x, algorithm="fricas")
```

```
[Out] -1/72*(84*b*x^6 - 28*sqrt(3)*(a^2*b*x^7 + a^3*x)*(-b/a^13)^(1/6)*arctan(-2/
3*sqrt(3)*a^2*x*(-b/a^13)^(1/6) + 2/3*sqrt(3)*a^2*sqrt(-(a^11*x*(-b/a^13)^(
5/6) + a^9*(-b/a^13)^(2/3) - b*x^2)/b)*(-b/a^13)^(1/6) - 1/3*sqrt(3)) - 28*
sqrt(3)*(a^2*b*x^7 + a^3*x)*(-b/a^13)^(1/6)*arctan(-2/3*sqrt(3)*a^2*x*(-b/a
^13)^(1/6) + 2/3*sqrt(3)*a^2*sqrt((a^11*x*(-b/a^13)^(5/6) - a^9*(-b/a^13)^(
2/3) + b*x^2)/b)*(-b/a^13)^(1/6) + 1/3*sqrt(3)) + 7*(a^2*b*x^7 + a^3*x)*(-b
/a^13)^(1/6)*log(16807*a^11*x*(-b/a^13)^(5/6) - 16807*a^9*(-b/a^13)^(2/3) +
 16807*b*x^2) - 7*(a^2*b*x^7 + a^3*x)*(-b/a^13)^(1/6)*log(-16807*a^11*x*(-b
/a^13)^(5/6) - 16807*a^9*(-b/a^13)^(2/3) + 16807*b*x^2) + 14*(a^2*b*x^7 + a
^3*x)*(-b/a^13)^(1/6)*log(16807*a^11*(-b/a^13)^(5/6) + 16807*b*x) - 14*(a^2
*b*x^7 + a^3*x)*(-b/a^13)^(1/6)*log(-16807*a^11*(-b/a^13)^(5/6) + 16807*b*x
) + 72*a)/(a^2*b*x^7 + a^3*x)
```

giac [A] time = 0.25, size = 216, normalized size = 0.89

$$-\frac{7\,bx^6+6\,a}{6\left(bx^7+ax\right)a^2}+\frac{7\sqrt{3}\,\left(ab^5\right)^{\frac{5}{6}}\log\left(x^2+\sqrt{3}\,x\left(\frac{a}{b}\right)^{\frac{1}{6}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{72\,a^3b^4}-\frac{7\sqrt{3}\,\left(ab^5\right)^{\frac{5}{6}}\log\left(x^2-\sqrt{3}\,x\left(\frac{a}{b}\right)^{\frac{1}{6}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{72\,a^3b^4}-\frac{7\,\left(ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^6+a)^2,x, algorithm="giac")
```

```
[Out] -1/6*(7*b*x^6 + 6*a)/((b*x^7 + a*x)*a^2) + 7/72*sqrt(3)*(a*b^5)^(5/6)*log(x
^2 + sqrt(3)*x*(a/b)^(1/6) + (a/b)^(1/3))/(a^3*b^4) - 7/72*sqrt(3)*(a*b^5)^
(5/6)*log(x^2 - sqrt(3)*x*(a/b)^(1/6) + (a/b)^(1/3))/(a^3*b^4) - 7/36*(a*b^
5)^(5/6)*arctan((2*x + sqrt(3)*(a/b)^(1/6))/(a/b)^(1/6))/(a^3*b^4) - 7/36*(
a*b^5)^(5/6)*arctan((2*x - sqrt(3)*(a/b)^(1/6))/(a/b)^(1/6))/(a^3*b^4) - 7/
18*(a*b^5)^(5/6)*arctan(x/(a/b)^(1/6))/(a^3*b^4)
```

maple [A] time = 0.06, size = 187, normalized size = 0.77

$$-\frac{b\,x^5}{6\left(b\,x^6+a\right)a^2}-\frac{7\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{6}}a^2}-\frac{7\arctan\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{36\left(\frac{a}{b}\right)^{\frac{1}{6}}a^2}-\frac{7\arctan\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{36\left(\frac{a}{b}\right)^{\frac{1}{6}}a^2}-\frac{7\sqrt{3}\ \left(\frac{a}{b}\right)^{\frac{5}{6}}b\ln\left(x^2-\sqrt{3}\ \left(\frac{a}{b}\right)^{\frac{1}{6}}\right.}{72a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^6+a)^2,x)
```

```
[Out] -1/6/a^2*b*x^5/(b*x^6+a)-7/18/a^2/(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x)-7/72/
a^3*b*3^(1/2)*(a/b)^(5/6)*ln(x^2-3^(1/2)*(a/b)^(1/6)*x+(a/b)^(1/3))-7/36/a^
2/(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x-3^(1/2))+7/72/a^3*b*3^(1/2)*(a/b)^(5/6
)*ln(x^2+3^(1/2)*(a/b)^(1/6)*x+(a/b)^(1/3))-7/36/a^2/(a/b)^(1/6)*arctan(2/(
a/b)^(1/6)*x+3^(1/2))-1/a^2/x
```

maxima [A] time = 2.41, size = 218, normalized size = 0.89

$$-\frac{7\,bx^6+6\,a}{6\left(a^2bx^7+a^3x\right)}+\frac{7\,b\left(\frac{\sqrt{3}\,\log\left(b^{\frac{1}{3}}x^2+\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{\sqrt{3}\,\log\left(b^{\frac{1}{3}}x^2-\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{4\arctan\left(\frac{b^{\frac{1}{3}}x}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-\frac{2\arctan\left(\frac{2b^{\frac{1}{3}}x+\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right.}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right.}{72\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^6+a)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(7*b*x^6 + 6*a)/(a^2*b*x^7 + a^3*x) + 7/72*b*(sqrt(3)*log(b^(1/3)*x^2
+ sqrt(3)*a^(1/6)*b^(1/6)*x + a^(1/3))/(a^(1/6)*b^(5/6)) - sqrt(3)*log(b^(1
/3)*x^2 - sqrt(3)*a^(1/6)*b^(1/6)*x + a^(1/3))/(a^(1/6)*b^(5/6)) - 4*arctan
(b^(1/3)*x/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 2*arcta
n((2*b^(1/3)*x + sqrt(3)*a^(1/6)*b^(1/6))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*s
qrt(a^(1/3)*b^(1/3))) - 2*arctan((2*b^(1/3)*x - sqrt(3)*a^(1/6)*b^(1/6))/sq
rt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))))/a^2
```

mupad [B] time = 1.18, size = 169, normalized size = 0.69

$$-\frac{\frac{1}{a}+\frac{7\,b\,x^6}{6\,a^2}}{b\,x^7+a\,x}-\frac{(-b)^{1/6}\,\mathrm{atan}\left(\frac{(-b)^{1/6}\,x\,1\mathrm{i}}{a^{1/6}}\right)\,7\mathrm{i}}{18\,a^{13/6}}-\frac{(-b)^{1/6}\,\mathrm{atan}\left(\frac{a^{21/2}\,(-b)^{13/2}\,x\,43563744\mathrm{i}}{21781872\,a^{32/3}\,(-b)^{19/3}-\sqrt{3}\,a^{32/3}\,(-b)^{19/3}\,21781872\mathrm{i}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,7\mathrm{i}}{18\,a^{13/6}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^6)^2),x)
```

```
[Out] ((-b)^(1/6)*atan((a^(21/2)*(-b)^(13/2)*x*43563744i)/(21781872*a^(32/3)*(-b)
^(19/3) + 3^(1/2)*a^(32/3)*(-b)^(19/3)*21781872i))*((3^(1/2)*1i)/2 - 1/2)*7
i)/(18*a^(13/6)) - ((-b)^(1/6)*atan(((-b)^(1/6)*x*1i)/a^(1/6))*7i)/(18*a^(1
3/6)) - ((-b)^(1/6)*atan((a^(21/2)*(-b)^(13/2)*x*43563744i)/(21781872*a^(32
/3)*(-b)^(19/3) - 3^(1/2)*a^(32/3)*(-b)^(19/3)*21781872i))*((3^(1/2)*1i)/2
+ 1/2)*7i)/(18*a^(13/6)) - (1/a + (7*b*x^6)/(6*a^2))/(a*x + b*x^7)
```

sympy [A] time = 0.51, size = 56, normalized size = 0.23

$$\frac{-6a-7bx^6}{6a^3x+6a^2bx^7}+\mathrm{RootSum}\left(2176782336t^6a^{13}+117649b,\left(t\mapsto t\log\left(-\frac{60466176t^5a^{11}}{16807b}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**6+a)**2,x)

[Out] (-6*a - 7*b*x**6)/(6*a**3*x + 6*a**2*b*x**7) + RootSum(2176782336*_t**6*a**
13 + 117649*b, Lambda(_t, _t*log(-60466176*_t**5*a**11/(16807*b) + x)))
```

3.1342 $\int \frac{1}{x^3\left(a+bx^6\right)^2}\,dx$

Optimal. Leaf size=152

$$-\frac{\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^2+b^{2/3}x^4\right)}{9a^{7/3}}+\frac{2\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{9a^{7/3}}+\frac{2\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x^2}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}}-\frac{2}{3a^2x^2}+\frac{1}{6ax^2\left(a+bx^6\right)}$$

```
[Out] -2/3/a^2/x^2+1/6/a/x^2/(b*x^6+a)+2/9*b^(1/3)*ln(a^(1/3)+b^(1/3)*x^2)/a^(7/3
)-1/9*b^(1/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x^2+b^(2/3)*x^4)/a^(7/3)+2/9*b^(1/
3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^2)/a^(1/3)*3^(1/2))/a^(7/3)*3^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {275, 290, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^2+b^{2/3}x^4\right)}{9a^{7/3}}+\frac{2\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{9a^{7/3}}+\frac{2\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x^2}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}}-\frac{2}{3a^2x^2}+\frac{1}{6ax^2\left(a+bx^6\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^6)^2),x]
```

```
[Out] -2/(3*a^2*x^2) + 1/(6*a*x^2*(a + b*x^6)) + (2*b^(1/3)*ArcTan[(a^(1/3) - 2*b
^(1/3)*x^2)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)) + (2*b^(1/3)*Log[a^(1/3
) + b^(1/3)*x^2])/(9*a^(7/3)) - (b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^2
+ b^(2/3)*x^4])/(9*a^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
```

```
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^6\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^3\right)^2}\,dx, x, x^2\right)$$

$$= \frac{1}{6ax^2\left(a+bx^6\right)} + \frac{2\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^3\right)}\,dx, x, x^2\right)}{3a}$$

$$= -\frac{2}{3a^2x^2} + \frac{1}{6ax^2\left(a+bx^6\right)} - \frac{(2b)\,\text{Subst}\left(\int \frac{x}{a+bx^3}\,dx, x, x^2\right)}{3a^2}$$

$$= -\frac{2}{3a^2x^2} + \frac{1}{6ax^2\left(a+bx^6\right)} + \frac{\left(2b^{2/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, x^2\right)}{9a^{7/3}} - \frac{\left(2b^{2/3}\right)\text{Subst}\left(\int \frac{\sqrt[3]{a}}{a^{2/3}-\sqrt[3]{a}}\right.}{9a^{7/3}}$$

$$= -\frac{2}{3a^2x^2} + \frac{1}{6ax^2\left(a+bx^6\right)} + \frac{2\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{9a^{7/3}} - \frac{\sqrt[3]{b}\,\text{Subst}\left(\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x\right.}{9a^{7/3}}$$

$$= -\frac{2}{3a^2x^2} + \frac{1}{6ax^2\left(a+bx^6\right)} + \frac{2\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{9a^{7/3}} - \frac{\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^2+b^{2/3}x^4\right)}{9a^{7/3}} -$$

$$= -\frac{2}{3a^2x^2} + \frac{1}{6ax^2\left(a+bx^6\right)} + \frac{2\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x^2}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}} + \frac{2\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right)}{9a^{7/3}} - \frac{\sqrt[3]{b}\,\log\left(\right.}{}$$

Mathematica [A] time = 0.14, size = 208, normalized size = 1.37

$$\frac{4\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right) - 2\sqrt[3]{b}\,\log\left(-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right) - 2\sqrt[3]{b}\,\log\left(\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}+\sqrt[3]{b}\,x^2\right) - \frac{3\sqrt[3]{a}\,b}{a+bx}}{18a^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^6)^2),x]
```

```
[Out] ((-9*a^(1/3))/x^2 - (3*a^(1/3)*b*x^4)/(a + b*x^6) + 4*Sqrt[3]*b^(1/3)*ArcTa
n[Sqrt[3] - (2*b^(1/6)*x)/a^(1/6)] + 4*Sqrt[3]*b^(1/3)*ArcTan[Sqrt[3] + (2*
b^(1/6)*x)/a^(1/6)] + 4*b^(1/3)*Log[a^(1/3) + b^(1/3)*x^2] - 2*b^(1/3)*Log[
a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*x + b^(1/3)*x^2] - 2*b^(1/3)*Log[a^(1/3)
+ Sqrt[3]*a^(1/6)*b^(1/6)*x + b^(1/3)*x^2])/(18*a^(7/3))
```

fricas [A] time = 0.89, size = 160, normalized size = 1.05

$$-\frac{12\,bx^6 + 4\sqrt{3}\left(bx^8+ax^2\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\,x^2\left(\frac{b}{a}\right)^{\frac{1}{3}} - \frac{1}{3}\sqrt{3}\right) + 2\left(bx^8+ax^2\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^4 - ax^2\left(\frac{b}{a}\right)^{\frac{2}{3}} + a\left(\frac{b}{a}\right)\right.}{18\left(a^2bx^8+a^3x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^6+a)^2,x, algorithm="fricas")
```

```
[Out] -1/18*(12*b*x^6 + 4*sqrt(3)*(b*x^8 + a*x^2)*(b/a)^(1/3)*arctan(2/3*sqrt(3)*
x^2*(b/a)^(1/3) - 1/3*sqrt(3)) + 2*(b*x^8 + a*x^2)*(b/a)^(1/3)*log(b*x^4 -
a*x^2*(b/a)^(2/3) + a*(b/a)^(1/3)) - 4*(b*x^8 + a*x^2)*(b/a)^(1/3)*log(b*x^
2 + a*(b/a)^(2/3)) + 9*a)/(a^2*b*x^8 + a^3*x^2)
```

giac [A] time = 0.20, size = 147, normalized size = 0.97

$$\frac{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^2-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^3}+\frac{2\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x^2+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^3b}-\frac{4bx^6+3a}{6\left(bx^8+ax^2\right)a^2}-\frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^4+x^2\right.}{9a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^6+a)^2,x, algorithm="giac")
```

```
[Out] 2/9*b*(-a/b)^(2/3)*log(abs(x^2 - (-a/b)^(1/3)))/a^3 + 2/9*sqrt(3)*(-a*b^2)^
(2/3)*arctan(1/3*sqrt(3)*(2*x^2 + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b) - 1/6
*(4*b*x^6 + 3*a)/((b*x^8 + a*x^2)*a^2) - 1/9*(-a*b^2)^(2/3)*log(x^4 + x^2*(
-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b)
```

maple [A] time = 0.01, size = 123, normalized size = 0.81

$$-\frac{bx^4}{6\left(bx^6+a\right)a^2}-\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^2}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}+\frac{2\ln\left(x^2+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}-\frac{\ln\left(x^4-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^2+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}-\frac{1}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^6+a)^2,x)
```

```
[Out] -1/6/a^2*b*x^4/(b*x^6+a)+2/9/a^2/(a/b)^(1/3)*ln(x^2+(a/b)^(1/3))-1/9/a^2/(a
/b)^(1/3)*ln(x^4-(a/b)^(1/3)*x^2+(a/b)^(2/3))-2/9/a^2*3^(1/2)/(a/b)^(1/3)*a
rctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^2-1))-1/2/a^2/x^2
```

maxima [A] time = 2.32, size = 134, normalized size = 0.88

$$-\frac{4bx^6+3a}{6\left(a^2bx^8+a^3x^2\right)}-\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\log\left(x^4-x^2\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9a^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2\log\left(x^2+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9a^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^6+a)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(4*b*x^6 + 3*a)/(a^2*b*x^8 + a^3*x^2) - 2/9*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x^2 - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*(a/b)^(1/3)) - 1/9*log(x^4 - x^2*(
a/b)^(1/3) + (a/b)^(2/3))/(a^2*(a/b)^(1/3)) + 2/9*log(x^2 + (a/b)^(1/3))/(a
^2*(a/b)^(1/3))
```

mupad [B] time = 1.21, size = 140, normalized size = 0.92

$$\frac{2b^{1/3}\ln\left(a^{1/3}+b^{1/3}x^2\right)}{9a^{7/3}}-\frac{\frac{1}{2a}+\frac{2bx^6}{3a^2}}{bx^8+ax^2}-\frac{2b^{1/3}\ln\left(6912a^7b^6-6912a^{20/3}b^{19/3}x^2\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9a^{7/3}}+\frac{b^{1/3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^6)^2),x)
```

```
[Out] (2*b^(1/3)*log(a^(1/3) + b^(1/3)*x^2))/(9*a^(7/3)) - (1/(2*a) + (2*b*x^6)/(
3*a^2))/(a*x^2 + b*x^8) - (2*b^(1/3)*log(6912*a^7*b^6 - 6912*a^(20/3)*b^(19
/3)*x^2*((3^(1/2)*1i)/2 + 1/2))*((3^(1/2)*1i)/2 + 1/2))/(9*a^(7/3)) + (b^(1
/3)*log(6912*a^7*b^6 + 31104*a^(20/3)*b^(19/3)*x^2*((3^(1/2)*1i)/9 - 1/9))*
((3^(1/2)*1i)/9 - 1/9))/a^(7/3)
```

sympy **[A]** time = 0.96, size = 60, normalized size = 0.39

$$\frac{-3a - 4bx^6}{6a^3x^2 + 6a^2bx^8} + \operatorname{RootSum}\left(729t^3a^7 - 8b, \left(t \mapsto t\log\left(\frac{81t^2a^5}{4b} + x^2\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**6+a)**2,x)
```

```
[Out] (-3*a - 4*b*x**6)/(6*a**3*x**2 + 6*a**2*b*x**8) + RootSum(729*_t**3*a**7 -
8*b, Lambda(_t, _t*log(81*_t**2*a**5/(4*b) + x**2)))
```

3.1343 $\int \frac{1}{x^4\left(a+bx^6\right)^2}\,dx$

Optimal. Leaf size=59

$$-\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x^3}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{1}{2a^2x^3}+\frac{1}{6ax^3\left(a+bx^6\right)}$$

```
[Out] -1/2/a^2/x^3+1/6/a/x^3/(b*x^6+a)-1/2*arctan(x^3*b^(1/2)/a^(1/2))*b^(1/2)/a^
(5/2)
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {275, 290, 325, 205}

$$-\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x^3}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{1}{2a^2x^3}+\frac{1}{6ax^3\left(a+bx^6\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^6)^2),x]
```

```
[Out] -1/(2*a^2*x^3) + 1/(6*a*x^3*(a + b*x^6)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x^3)/Sq
rt[a]])/(2*a^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(a+bx^6\right)^2}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^2}\,dx, x, x^3\right)\\ &= \frac{1}{6ax^3\left(a+bx^6\right)} + \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)}\,dx, x, x^3\right)}{2a}\\ &= -\frac{1}{2a^2x^3} + \frac{1}{6ax^3\left(a+bx^6\right)} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^3\right)}{2a^2}\\ &= -\frac{1}{2a^2x^3} + \frac{1}{6ax^3\left(a+bx^6\right)} - \frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x^3}{\sqrt{a}}\right)}{2a^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.09, size = 114, normalized size = 1.93

$$\frac{-\frac{\sqrt{a}\,bx^3}{a+bx^6} + 3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right) + 3\sqrt{b}\,\tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[6]{b}\,x}{\sqrt[6]{a}}\right) - 3\sqrt{b}\,\tan^{-1}\left(\frac{2\sqrt[6]{b}\,x}{\sqrt[6]{a}} + \sqrt{3}\right) - \frac{2\sqrt{a}}{x^3}}{6a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^6)^2),x]
```

```
[Out] ((-2*Sqrt[a])/x^3 - (Sqrt[a]*b*x^3)/(a + b*x^6) + 3*Sqrt[b]*ArcTan[(b^(1/6)
*x)/a^(1/6)] + 3*Sqrt[b]*ArcTan[Sqrt[3] - (2*b^(1/6)*x)/a^(1/6)] - 3*Sqrt[b
]*ArcTan[Sqrt[3] + (2*b^(1/6)*x)/a^(1/6)])/(6*a^(5/2))
```

fricas [A] time = 0.95, size = 148, normalized size = 2.51

$$\left[-\frac{6\,bx^6 - 3\left(bx^9+ax^3\right)\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx^6-2\,ax^3\sqrt{-\frac{b}{a}}-a}{bx^6+a}\right) + 4\,a}{12\left(a^2bx^9+a^3x^3\right)}, -\frac{3\,bx^6 + 3\left(bx^9+ax^3\right)\sqrt{\frac{b}{a}}\,\arctan\left(x^3\sqrt{\frac{b}{a}}\right) + 2\,a}{6\left(a^2bx^9+a^3x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^6+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(6*b*x^6 - 3*(b*x^9 + a*x^3)*sqrt(-b/a)*log((b*x^6 - 2*a*x^3*sqrt(-b
/a) - a)/(b*x^6 + a)) + 4*a)/(a^2*b*x^9 + a^3*x^3), -1/6*(3*b*x^6 + 3*(b*x^
9 + a*x^3)*sqrt(b/a)*arctan(x^3*sqrt(b/a)) + 2*a)/(a^2*b*x^9 + a^3*x^3)]
```

giac [A] time = 0.16, size = 51, normalized size = 0.86

$$-\frac{b\arctan\left(\frac{bx^3}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a^2} - \frac{3\,bx^6+2\,a}{6\left(bx^9+ax^3\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^6+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*b*arctan(b*x^3/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/6*(3*b*x^6 + 2*a)/((b*x^
9 + a*x^3)*a^2)
```

maple [A] time = 0.01, size = 50, normalized size = 0.85

$$-\frac{b\,x^3}{6\left(b\,x^6+a\right)a^2}-\frac{b\arctan\left(\frac{b\,x^3}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^2}-\frac{1}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^6+a)^2,x)
```

```
[Out] -1/6/a^2*b*x^3/(b*x^6+a)-1/2/a^2*b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^3)-
1/3/a^2/x^3
```

maxima [A] time = 2.36, size = 53, normalized size = 0.90

$$-\frac{3\,bx^6+2\,a}{6\left(a^2bx^9+a^3x^3\right)}-\frac{b\arctan\left(\frac{bx^3}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^6+a)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*b*x^6 + 2*a)/(a^2*b*x^9 + a^3*x^3) - 1/2*b*arctan(b*x^3/sqrt(a*b))/
(sqrt(a*b)*a^2)
```

mupad [B] time = 0.09, size = 50, normalized size = 0.85

$$-\frac{\frac{1}{3\,a}+\frac{b\,x^6}{2\,a^2}}{b\,x^9+a\,x^3}-\frac{\sqrt{b}\,\operatorname{atan}\left(\frac{\sqrt{b}\,x^3}{\sqrt{a}}\right)}{2\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^6)^2),x)
```

```
[Out] - (1/(3*a) + (b*x^6)/(2*a^2))/(a*x^3 + b*x^9) - (b^(1/2)*atan((b^(1/2)*x^3)
/a^(1/2)))/(2*a^(5/2))
```

sympy [A] time = 0.97, size = 94, normalized size = 1.59

$$\frac{\sqrt{-\frac{b}{a^5}}\,\log\left(-\frac{a^3\sqrt{-\frac{b}{a^5}}}{b}+x^3\right)}{4}-\frac{\sqrt{-\frac{b}{a^5}}\,\log\left(\frac{a^3\sqrt{-\frac{b}{a^5}}}{b}+x^3\right)}{4}+\frac{-2a-3bx^6}{6a^3x^3+6a^2bx^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**6+a)**2,x)
```

```
[Out] sqrt(-b/a**5)*log(-a**3*sqrt(-b/a**5)/b + x**3)/4 - sqrt(-b/a**5)*log(a**3*
sqrt(-b/a**5)/b + x**3)/4 + (-2*a - 3*b*x**6)/(6*a**3*x**3 + 6*a**2*b*x**9)
```

3.1344 $\int \frac{x^8}{1-x^6}\,dx$

Optimal. Leaf size=16

$$\frac{1}{3}\tanh^{-1}\left(x^3\right)-\frac{x^3}{3}$$

```
[Out] -1/3*x^3+1/3*arctanh(x^3)
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {275, 321, 206}

$$\frac{1}{3}\tanh^{-1}\left(x^3\right)-\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(1 - x^6),x]
```

```
[Out] -x^3/3 + ArcTanh[x^3]/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{1-x^6}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, x^3\right)\\ &= -\frac{x^3}{3}+\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, x^3\right)\\ &= -\frac{x^3}{3}+\frac{1}{3}\tanh^{-1}\left(x^3\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.88

$$-\frac{x^3}{3}-\frac{1}{6}\log\left(1-x^3\right)+\frac{1}{6}\log\left(x^3+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(1 - x^6),x]
[Out] -1/3*x^3 - Log[1 - x^3]/6 + Log[1 + x^3]/6
```

fricas [A] time = 0.85, size = 22, normalized size = 1.38

$$-\frac{1}{3}x^3+\frac{1}{6}\log\left(x^3+1\right)-\frac{1}{6}\log\left(x^3-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^6+1),x, algorithm="fricas")
[Out] -1/3*x^3 + 1/6*log(x^3 + 1) - 1/6*log(x^3 - 1)
```

giac [A] time = 0.15, size = 24, normalized size = 1.50

$$-\frac{1}{3}x^3+\frac{1}{6}\log\left(\left|x^3+1\right|\right)-\frac{1}{6}\log\left(\left|x^3-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^6+1),x, algorithm="giac")
[Out] -1/3*x^3 + 1/6*log(abs(x^3 + 1)) - 1/6*log(abs(x^3 - 1))
```

maple [A] time = 0.00, size = 23, normalized size = 1.44

$$-\frac{x^3}{3}-\frac{\ln\left(x^3-1\right)}{6}+\frac{\ln\left(x^3+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-x^6+1),x)
[Out] -1/3*x^3-1/6*ln(x^3-1)+1/6*ln(x^3+1)
```

maxima [A] time = 0.98, size = 22, normalized size = 1.38

$$-\frac{1}{3}x^3+\frac{1}{6}\log\left(x^3+1\right)-\frac{1}{6}\log\left(x^3-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^6+1),x, algorithm="maxima")
[Out] -1/3*x^3 + 1/6*log(x^3 + 1) - 1/6*log(x^3 - 1)
```

mupad [B] time = 0.06, size = 12, normalized size = 0.75

$$\frac{\operatorname{atanh}\left(x^3\right)}{3}-\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^8/(x^6 - 1),x)
[Out] atanh(x^3)/3 - x^3/3
```

sympy [A] time = 0.13, size = 20, normalized size = 1.25

$$-\frac{x^3}{3}-\frac{\log\left(x^3-1\right)}{6}+\frac{\log\left(x^3+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-x**6+1),x)
[Out] -x**3/3 - log(x**3 - 1)/6 + log(x**3 + 1)/6
```

3.1345 $\int \frac{x^7}{1-x^6}\,dx$

Optimal. Leaf size=56

$$-\frac{x^2}{2}-\frac{1}{6}\log\left(1-x^2\right)+\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{12}\log\left(x^4+x^2+1\right)$$

```
[Out] -1/2*x^2-1/6*ln(-x^2+1)+1/12*ln(x^4+x^2+1)+1/6*arctan(1/3*(2*x^2+1)*3^(1/2)
)*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {275, 321, 200, 31, 634, 618, 204, 628}

$$-\frac{x^2}{2}-\frac{1}{6}\log\left(1-x^2\right)+\frac{1}{12}\log\left(x^4+x^2+1\right)+\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(1 - x^6),x]
```

```
[Out] -x^2/2 + ArcTan[(1 + 2*x^2)/Sqrt[3]]/(2*Sqrt[3]) - Log[1 - x^2]/6 + Log[1 +
 x^2 + x^4]/12
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{1-x^6}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{1-x^3}\,dx, x, x^2\right) \\
&= -\frac{x^2}{2} + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-x^3}\,dx, x, x^2\right) \\
&= -\frac{x^2}{2} + \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, x^2\right) + \frac{1}{6}\,\text{Subst}\left(\int \frac{2+x}{1+x+x^2}\,dx, x, x^2\right) \\
&= -\frac{x^2}{2} - \frac{1}{6}\log\left(1-x^2\right) + \frac{1}{12}\,\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, x^2\right) + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^2\right) \\
&= -\frac{x^2}{2} - \frac{1}{6}\log\left(1-x^2\right) + \frac{1}{12}\log\left(1+x^2+x^4\right) - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x^2\right) \\
&= -\frac{x^2}{2} + \frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{6}\log\left(1-x^2\right) + \frac{1}{12}\log\left(1+x^2+x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 78, normalized size = 1.39

$$\frac{1}{12}\left(-6x^2 + \log\left(x^2-x+1\right) + \log\left(x^2+x+1\right) - 2\log(1-x) - 2\log(x+1) + 2\sqrt{3}\,\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) - 2\sqrt{3}\,\tan\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(1 - x^6),x]
```

```
[Out] (-6*x^2 + 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)
/Sqrt[3]] - 2*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] + Log[1 + x + x^
2])/12
```

fricas [A] time = 0.90, size = 43, normalized size = 0.77

$$-\frac{1}{2}x^2 + \frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right) + \frac{1}{12}\log\left(x^4+x^2+1\right) - \frac{1}{6}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^6+1),x, algorithm="fricas")
```

```
[Out] -1/2*x^2 + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*log(x^4 + x^2
 + 1) - 1/6*log(x^2 - 1)
```

giac [A] time = 0.18, size = 44, normalized size = 0.79

$$-\frac{1}{2}x^2+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{12}\log\left(x^4+x^2+1\right)-\frac{1}{6}\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^6+1),x, algorithm="giac")
```

```
[Out] -1/2*x^2 + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*log(x^4 + x^2
 + 1) - 1/6*log(abs(x^2 - 1))
```

maple [A] time = 0.01, size = 71, normalized size = 1.27

$$-\frac{x^2}{2}+\frac{\sqrt{3}\ \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{\sqrt{3}\ \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{6}-\frac{\ln(x-1)}{6}-\frac{\ln(x+1)}{6}+\frac{\ln\left(x^2-x+1\right)}{12}+\frac{\ln\left(x^2+x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-x^6+1),x)
```

```
[Out] -1/2*x^2-1/6*ln(x-1)+1/12*ln(x^2+x+1)-1/6*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/
2)+1/12*ln(x^2-x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-1/6*ln(x+1)
```

maxima [A] time = 2.44, size = 43, normalized size = 0.77

$$-\frac{1}{2}x^2+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{12}\log\left(x^4+x^2+1\right)-\frac{1}{6}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^6+1),x, algorithm="maxima")
```

```
[Out] -1/2*x^2 + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*log(x^4 + x^2
 + 1) - 1/6*log(x^2 - 1)
```

mupad [B] time = 1.10, size = 57, normalized size = 1.02

$$-\frac{\ln\left(x^2-1\right)}{6}-\frac{x^2}{2}-\ln\left(x^2-\frac{\sqrt{3}\ 1\mathrm{i}}{2}+\frac{1}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\ 1\mathrm{i}}{12}\right)+\ln\left(x^2+\frac{\sqrt{3}\ 1\mathrm{i}}{2}+\frac{1}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\ 1\mathrm{i}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^7/(x^6 - 1),x)
```

```
[Out] log((3^(1/2)*1i)/2 + x^2 + 1/2)*((3^(1/2)*1i)/12 + 1/12) - x^2/2 - log(x^2
 - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/12 - 1/12) - log(x^2 - 1)/6
```

sympy [A] time = 0.18, size = 51, normalized size = 0.91

$$-\frac{x^2}{2}-\frac{\log\left(x^2-1\right)}{6}+\frac{\log\left(x^4+x^2+1\right)}{12}+\frac{\sqrt{3}\ \operatorname{atan}\left(\frac{2\sqrt{3}\,x^2}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-x**6+1),x)
```

```
[Out] -x**2/2 - log(x**2 - 1)/6 + log(x**4 + x**2 + 1)/12 + sqrt(3)*atan(2*sqrt(3
)*x**2/3 + sqrt(3)/3)/6
```

3.1346 $\int \frac{x^6}{1-x^6}\,dx$

Optimal. Leaf size=50

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\,x}{1-x^2}\right)}{2\sqrt{3}}+\frac{1}{6}\tanh^{-1}\left(\frac{x}{x^2+1}\right)-x+\frac{1}{3}\tanh^{-1}(x)$$

```
[Out] -x+1/3*arctanh(x)+1/6*arctanh(x/(x^2+1))+1/6*arctan(x*3^(1/2)/(-x^2+1))*3^(
1/2)
```

Rubi [A] time = 0.11, antiderivative size = 76, normalized size of antiderivative = 1.52, number of steps used = 11, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {321, 210, 634, 618, 204, 628, 206}

$$-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{12}\log\left(x^2+x+1\right)-x-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{3}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(1 - x^6),x]
```

```
[Out] -x - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(1 + 2*x)/Sqrt[3]]/(2*S
qrt[3]) + ArcTanh[x]/3 - Log[1 - x + x^2]/12 + Log[1 + x + x^2]/12
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{1-x^6}\,dx &= -x + \int \frac{1}{1-x^6}\,dx \\
&= -x + \frac{1}{3}\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx + \frac{1}{3}\int \frac{1+\frac{x}{2}}{1+x+x^2}\,dx + \frac{1}{3}\int \frac{1}{1-x^2}\,dx \\
&= -x + \frac{1}{3}\tanh^{-1}(x) - \frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx + \frac{1}{12}\int \frac{1+2x}{1+x+x^2}\,dx + \frac{1}{4}\int \frac{1}{1-x+x^2}\,dx + \frac{1}{4}\int \frac{1}{1+x} \\
&= -x + \frac{1}{3}\tanh^{-1}(x) - \frac{1}{12}\log\left(1-x+x^2\right) + \frac{1}{12}\log\left(1+x+x^2\right) - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\right) \\
&= -x - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{1}{3}\tanh^{-1}(x) - \frac{1}{12}\log\left(1-x+x^2\right) + \frac{1}{12}\log\left(1+x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 78, normalized size = 1.56

$$\frac{1}{12}\left(-\log\left(x^2-x+1\right) + \log\left(x^2+x+1\right) - 12x - 2\log(1-x) + 2\log(x+1) + 2\sqrt{3}\,\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + 2\sqrt{3}\,\tan^{-1}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(1 - x^6),x]
```

```
[Out] (-12*x + 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*Sqrt[3]*ArcTan[(1 + 2*x)/
Sqrt[3]] - 2*Log[1 - x] + 2*Log[1 + x] - Log[1 - x + x^2] + Log[1 + x + x^2
])/12
```

fricas [A] time = 0.74, size = 68, normalized size = 1.36

$$\frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right) + \frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right) - x + \frac{1}{12}\log\left(x^2+x+1\right) - \frac{1}{12}\log\left(x^2-x+1\right) + \frac{1}{6}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^6+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) - x + 1/12*log(x^2 + x + 1) - 1/12*log(x^2 - x + 1) + 1/6*log(x
+ 1) - 1/6*log(x - 1)
```

giac [A] time = 0.16, size = 70, normalized size = 1.40

$$\frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right) + \frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right) - x + \frac{1}{12}\log\left(x^2+x+1\right) - \frac{1}{12}\log\left(x^2-x+1\right) + \frac{1}{6}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^6+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) - x + 1/12*log(x^2 + x + 1) - 1/12*log(x^2 - x + 1) + 1/6*log(ab
s(x + 1)) - 1/6*log(abs(x - 1))
```

maple [A] time = 0.01, size = 69, normalized size = 1.38

$$-x+\frac{\sqrt{3}\ \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\sqrt{3}\ \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{6}-\frac{\ln(x-1)}{6}+\frac{\ln(x+1)}{6}-\frac{\ln\left(x^2-x+1\right)}{12}+\frac{\ln\left(x^2+x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-x^6+1),x)
```

```
[Out] -x-1/6*ln(x-1)+1/12*ln(x^2+x+1)+1/6*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))-1/1
2*ln(x^2-x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/6*ln(x+1)
```

maxima [A] time = 2.21, size = 68, normalized size = 1.36

$$\frac{1}{6}\sqrt{3}\ \arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+\frac{1}{6}\sqrt{3}\ \arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)-x+\frac{1}{12}\ \log\left(x^2+x+1\right)-\frac{1}{12}\ \log\left(x^2-x+1\right)+\frac{1}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^6+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) - x + 1/12*log(x^2 + x + 1) - 1/12*log(x^2 - x + 1) + 1/6*log(x
+ 1) - 1/6*log(x - 1)
```

mupad [B] time = 1.06, size = 93, normalized size = 1.86

$$-x-\frac{\operatorname{atan}(x\,1\mathrm{i})\ 1\mathrm{i}}{3}+\operatorname{atan}\left(\frac{x\,1\mathrm{i}}{1+\sqrt{3}\ 1\mathrm{i}}+\frac{\sqrt{3}\ x}{1+\sqrt{3}\ 1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{6}+\frac{1}{6}\mathrm{i}\right)+\operatorname{atan}\left(\frac{x\,1\mathrm{i}}{-1+\sqrt{3}\ 1\mathrm{i}}-\frac{\sqrt{3}\ x}{-1+\sqrt{3}\ 1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{6}-\frac{1}{6}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^6/(x^6 - 1),x)
```

```
[Out] atan((x*1i)/(3^(1/2)*1i + 1) + (3^(1/2)*x)/(3^(1/2)*1i + 1))*(3^(1/2)/6 + 1
i/6) - (atan(x*1i)*1i)/3 - x + atan((x*1i)/(3^(1/2)*1i - 1) - (3^(1/2)*x)/(
3^(1/2)*1i - 1))*(3^(1/2)/6 - 1i/6)
```

sympy [B] time = 0.52, size = 85, normalized size = 1.70

$$-x-\frac{\log(x-1)}{6}+\frac{\log(x+1)}{6}-\frac{\log\left(x^2-x+1\right)}{12}+\frac{\log\left(x^2+x+1\right)}{12}+\frac{\sqrt{3}\ \operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{6}+\frac{\sqrt{3}\ \operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-x**6+1),x)
```

```
[Out] -x - log(x - 1)/6 + log(x + 1)/6 - log(x**2 - x + 1)/12 + log(x**2 + x + 1)
/12 + sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/6 + sqrt(3)*atan(2*sqrt(3)*x/
3 + sqrt(3)/3)/6
```

3.1347 $\int \frac{x^5}{1-x^6}\,dx$

Optimal. Leaf size=12

$$-\frac{1}{6}\log\left(1-x^6\right)$$

```
[Out] -1/6*ln(-x^6+1)
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {260}

$$-\frac{1}{6}\log\left(1-x^6\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(1 - x^6),x]
```

```
[Out] -Log[1 - x^6]/6
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^5}{1-x^6}\,dx = -\frac{1}{6}\log\left(1-x^6\right)$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$-\frac{1}{6}\log\left(1-x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(1 - x^6),x]
```

```
[Out] -1/6*Log[1 - x^6]
```

fricas [A] time = 0.81, size = 8, normalized size = 0.67

$$-\frac{1}{6}\log\left(x^6-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^6+1),x, algorithm="fricas")
```

```
[Out] -1/6*log(x^6 - 1)
```

giac [A] time = 0.16, size = 9, normalized size = 0.75

$$-\frac{1}{6}\log\left(\left|x^6-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^6+1),x, algorithm="giac")
[Out] -1/6*log(abs(x^6 - 1))
```

maple [A] time = 0.00, size = 18, normalized size = 1.50

$$-\frac{\ln\left(x^3-1\right)}{6}-\frac{\ln\left(x^3+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-x^6+1),x)
[Out] -1/6*ln(x^3-1)-1/6*ln(x^3+1)
```

maxima [A] time = 1.11, size = 8, normalized size = 0.67

$$-\frac{1}{6}\log\left(x^6-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^6+1),x, algorithm="maxima")
[Out] -1/6*log(x^6 - 1)
```

mupad [B] time = 0.04, size = 8, normalized size = 0.67

$$-\frac{\ln\left(x^6-1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^5/(x^6 - 1),x)
[Out] -log(x^6 - 1)/6
```

sympy [A] time = 0.10, size = 8, normalized size = 0.67

$$-\frac{\log\left(x^6-1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-x**6+1),x)
[Out] -log(x**6 - 1)/6
```

3.1348 $\int \frac{x^4}{1-x^6}\,dx$

Optimal. Leaf size=47

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,x}{1-x^2}\right)}{2\sqrt{3}}+\frac{1}{6}\tanh^{-1}\left(\frac{x}{x^2+1}\right)+\frac{1}{3}\tanh^{-1}(x)$$

```
[Out] 1/3*arctanh(x)+1/6*arctanh(x/(x^2+1))-1/6*arctan(x*3^(1/2)/(-x^2+1))*3^(1/2
)
```

Rubi [A] time = 0.15, antiderivative size = 73, normalized size of antiderivative = 1.55, number of steps used = 10, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {296, 634, 618, 204, 628, 206}

$$-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{12}\log\left(x^2+x+1\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{3}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 - x^6),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(1 + 2*x)/Sqrt[3]]/(2*Sqrt[3
]) + ArcTanh[x]/3 - Log[1 - x + x^2]/12 + Log[1 + x + x^2]/12
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{1-x^6}\,dx &= \frac{1}{3}\int \frac{-\frac{1}{2}-\frac{x}{2}}{1-x+x^2}\,dx + \frac{1}{3}\int \frac{-\frac{1}{2}+\frac{x}{2}}{1+x+x^2}\,dx + \frac{1}{3}\int \frac{1}{1-x^2}\,dx \\ &= \frac{1}{3}\tanh^{-1}(x) - \frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx + \frac{1}{12}\int \frac{1+2x}{1+x+x^2}\,dx - \frac{1}{4}\int \frac{1}{1-x+x^2}\,dx - \frac{1}{4}\int \frac{1}{1+x+x} \\ &= \frac{1}{3}\tanh^{-1}(x) - \frac{1}{12}\log\left(1-x+x^2\right) + \frac{1}{12}\log\left(1+x+x^2\right) + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x\right) \\ &= \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{1}{3}\tanh^{-1}(x) - \frac{1}{12}\log\left(1-x+x^2\right) + \frac{1}{12}\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 75, normalized size = 1.60

$$\frac{1}{12}\left(-\log\left(x^2-x+1\right)+\log\left(x^2+x+1\right)-2\log(1-x)+2\log(x+1)-2\sqrt{3}\,\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)-2\sqrt{3}\,\tan^{-1}\left(\frac{2}{}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 - x^6),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]
] - 2*Log[1 - x] + 2*Log[1 + x] - Log[1 - x + x^2] + Log[1 + x + x^2])/12
```

fricas [A] time = 0.91, size = 65, normalized size = 1.38

$$-\frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)-\frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+\frac{1}{12}\,\log\left(x^2+x+1\right)-\frac{1}{12}\,\log\left(x^2-x+1\right)+\frac{1}{6}\,l$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^6+1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + 1/12*log(x^2 + x + 1) - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1
) - 1/6*log(x - 1)
```

giac [A] time = 0.19, size = 67, normalized size = 1.43

$$-\frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)-\frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+\frac{1}{12}\,\log\left(x^2+x+1\right)-\frac{1}{12}\,\log\left(x^2-x+1\right)+\frac{1}{6}\,l$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^6+1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + 1/12*log(x^2 + x + 1) - 1/12*log(x^2 - x + 1) + 1/6*log(abs(x
 + 1)) - 1/6*log(abs(x - 1))
```

maple [A] time = 0.01, size = 66, normalized size = 1.40

$$-\frac{\sqrt{3}\ \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{\sqrt{3}\ \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{6}-\frac{\ln(x-1)}{6}+\frac{\ln(x+1)}{6}-\frac{\ln\left(x^2-x+1\right)}{12}+\frac{\ln\left(x^2+x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^6+1),x)
```

```
[Out] -1/6*ln(x-1)+1/12*ln(x^2+x+1)-1/6*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))-1/12*
ln(x^2-x+1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/6*ln(x+1)
```

maxima [A] time = 2.31, size = 65, normalized size = 1.38

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{12}\log\left(x^2+x+1\right)-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^6+1),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + 1/12*log(x^2 + x + 1) - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1
) - 1/6*log(x - 1)
```

mupad [B] time = 0.06, size = 53, normalized size = 1.13

$$\frac{\operatorname{atanh}(x)}{3}-\operatorname{atanh}\left(\frac{2x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\operatorname{atanh}\left(\frac{2x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^4/(x^6 - 1),x)
```

```
[Out] atanh(x)/3 - atanh((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/6 + 1/6) - atanh((
2*x)/(3^(1/2)*1i + 1))*((3^(1/2)*1i)/6 - 1/6)
```

sympy [B] time = 0.52, size = 83, normalized size = 1.77

$$-\frac{\log(x-1)}{6}+\frac{\log(x+1)}{6}-\frac{\log\left(x^2-x+1\right)}{12}+\frac{\log\left(x^2+x+1\right)}{12}-\frac{\sqrt{3}\ \operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{6}-\frac{\sqrt{3}\ \operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**6+1),x)
```

```
[Out] -log(x - 1)/6 + log(x + 1)/6 - log(x**2 - x + 1)/12 + log(x**2 + x + 1)/12
- sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 +
sqrt(3)/3)/6
```

3.1349 $\int \frac{x^3}{1-x^6}\,dx$

Optimal. Leaf size=49

$$-\frac{1}{6}\log\left(1-x^2\right)-\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{12}\log\left(x^4+x^2+1\right)$$

```
[Out] -1/6*ln(-x^2+1)+1/12*ln(x^4+x^2+1)-1/6*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2
)
```

Rubi [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {275, 292, 31, 634, 618, 204, 628}

$$-\frac{1}{6}\log\left(1-x^2\right)+\frac{1}{12}\log\left(x^4+x^2+1\right)-\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 - x^6),x]
```

```
[Out] -ArcTan[(1 + 2*x^2)/Sqrt[3]]/(2*Sqrt[3]) - Log[1 - x^2]/6 + Log[1 + x^2 + x
^4]/12
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{1-x^6}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{1-x^3}\,dx, x, x^2\right) \\
&= \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, x^2\right) - \frac{1}{6}\,\text{Subst}\left(\int \frac{1-x}{1+x+x^2}\,dx, x, x^2\right) \\
&= -\frac{1}{6}\log\left(1-x^2\right) + \frac{1}{12}\,\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, x^2\right) - \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^2\right) \\
&= -\frac{1}{6}\log\left(1-x^2\right) + \frac{1}{12}\log\left(1+x^2+x^4\right) + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x^2\right) \\
&= -\frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{6}\log\left(1-x^2\right) + \frac{1}{12}\log\left(1+x^2+x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 73, normalized size = 1.49

$$\frac{1}{12}\left(\log\left(x^2-x+1\right) + \log\left(x^2+x+1\right) - 2\log(1-x) - 2\log(x+1) - 2\sqrt{3}\,\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + 2\sqrt{3}\,\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 - x^6),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]
] - 2*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] + Log[1 + x + x^2])/12
```

fricas [A] time = 0.87, size = 38, normalized size = 0.78

$$-\frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right) + \frac{1}{12}\log\left(x^4+x^2+1\right) - \frac{1}{6}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^6+1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*log(x^4 + x^2 + 1) - 1/
6*log(x^2 - 1)
```

giac [A] time = 0.16, size = 39, normalized size = 0.80

$$-\frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right) + \frac{1}{12}\log\left(x^4+x^2+1\right) - \frac{1}{6}\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^6+1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*log(x^4 + x^2 + 1) - 1/
6*log(abs(x^2 - 1))
```

maple [A] time = 0.01, size = 66, normalized size = 1.35

$$-\frac{\sqrt{3}\ \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\sqrt{3}\ \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{6}-\frac{\ln(x-1)}{6}-\frac{\ln(x+1)}{6}+\frac{\ln\left(x^2-x+1\right)}{12}+\frac{\ln\left(x^2+x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-x^6+1),x)
```

```
[Out] -1/6*ln(x-1)+1/12*ln(x^2+x+1)+1/6*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))+1/12*
ln(x^2-x+1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-1/6*ln(x+1)
```

maxima [A] time = 2.35, size = 38, normalized size = 0.78

$$-\frac{1}{6}\sqrt{3}\ \arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{12}\ \log\left(x^4+x^2+1\right)-\frac{1}{6}\ \log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^6+1),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*log(x^4 + x^2 + 1) - 1/
6*log(x^2 - 1)
```

mupad [B] time = 0.12, size = 52, normalized size = 1.06

$$-\frac{\ln\left(x^2-1\right)}{6}+\ln\left(x^2-\frac{\sqrt{3}\ 1\mathrm{i}}{2}+\frac{1}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\ 1\mathrm{i}}{12}\right)-\ln\left(x^2+\frac{\sqrt{3}\ 1\mathrm{i}}{2}+\frac{1}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\ 1\mathrm{i}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^3/(x^6 - 1),x)
```

```
[Out] log(x^2 - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/12 + 1/12) - log(x^2 - 1)/6 -
 log((3^(1/2)*1i)/2 + x^2 + 1/2)*((3^(1/2)*1i)/12 - 1/12)
```

sympy [A] time = 0.28, size = 46, normalized size = 0.94

$$-\frac{\log\left(x^2-1\right)}{6}+\frac{\log\left(x^4+x^2+1\right)}{12}-\frac{\sqrt{3}\ \operatorname{atan}\left(\frac{2\sqrt{3}\,x^2}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-x**6+1),x)
```

```
[Out] -log(x**2 - 1)/6 + log(x**4 + x**2 + 1)/12 - sqrt(3)*atan(2*sqrt(3)*x**2/3
+ sqrt(3)/3)/6
```

3.1350 $\int \frac{x^2}{1-x^6}\,dx$

Optimal. Leaf size=8

$$\frac{1}{3}\tanh^{-1}\left(x^3\right)$$

```
[Out] 1/3*arctanh(x^3)
```

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {275, 206}

$$\frac{1}{3}\tanh^{-1}\left(x^3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 - x^6),x]
```

```
[Out] ArcTanh[x^3]/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{1-x^6}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, x^3\right)\\ &= \frac{1}{3}\tanh^{-1}\left(x^3\right)\end{aligned}$$

Mathematica [B] time = 0.00, size = 23, normalized size = 2.88

$$\frac{1}{6}\log\left(x^3+1\right)-\frac{1}{6}\log\left(1-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 - x^6),x]
```

```
[Out] -1/6*Log[1 - x^3] + Log[1 + x^3]/6
```

fricas [B] time = 0.59, size = 17, normalized size = 2.12

$$\frac{1}{6}\log\left(x^3+1\right)-\frac{1}{6}\log\left(x^3-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^6+1),x, algorithm="fricas")
```

```
[Out] 1/6*log(x^3 + 1) - 1/6*log(x^3 - 1)
```

giac [B] time = 0.15, size = 19, normalized size = 2.38

$$\frac{1}{6}\log\left(\left|x^3+1\right|\right)-\frac{1}{6}\log\left(\left|x^3-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^6+1),x, algorithm="giac")
[Out] 1/6*log(abs(x^3 + 1)) - 1/6*log(abs(x^3 - 1))
```

maple [B] time = 0.00, size = 18, normalized size = 2.25

$$-\frac{\ln\left(x^3-1\right)}{6}+\frac{\ln\left(x^3+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^6+1),x)
[Out] -1/6*ln(x^3-1)+1/6*ln(x^3+1)
```

maxima [B] time = 1.01, size = 17, normalized size = 2.12

$$\frac{1}{6}\log\left(x^3+1\right)-\frac{1}{6}\log\left(x^3-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^6+1),x, algorithm="maxima")
[Out] 1/6*log(x^3 + 1) - 1/6*log(x^3 - 1)
```

mupad [B] time = 1.07, size = 6, normalized size = 0.75

$$\frac{\operatorname{atanh}\left(x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/(x^6 - 1),x)
[Out] atanh(x^3)/3
```

sympy [B] time = 0.12, size = 15, normalized size = 1.88

$$-\frac{\log\left(x^3-1\right)}{6}+\frac{\log\left(x^3+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**6+1),x)
[Out] -log(x**3 - 1)/6 + log(x**3 + 1)/6
```

3.1351 $\int \frac{x}{1-x^6}\,dx$

Optimal. Leaf size=49

$$-\frac{1}{6}\log\left(1-x^2\right)+\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{12}\log\left(x^4+x^2+1\right)$$

```
[Out] -1/6*ln(-x^2+1)+1/12*ln(x^4+x^2+1)+1/6*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2
)
```

Rubi [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {275, 200, 31, 634, 618, 204, 628}

$$-\frac{1}{6}\log\left(1-x^2\right)+\frac{1}{12}\log\left(x^4+x^2+1\right)+\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 - x^6),x]
```

```
[Out] ArcTan[(1 + 2*x^2)/Sqrt[3]]/(2*Sqrt[3]) - Log[1 - x^2]/6 + Log[1 + x^2 + x^
4]/12
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{1-x^6}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-x^3}\,dx, x, x^2\right) \\
&= \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, x^2\right) + \frac{1}{6}\,\text{Subst}\left(\int \frac{2+x}{1+x+x^2}\,dx, x, x^2\right) \\
&= -\frac{1}{6}\log\left(1-x^2\right) + \frac{1}{12}\,\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, x^2\right) + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^2\right) \\
&= -\frac{1}{6}\log\left(1-x^2\right) + \frac{1}{12}\log\left(1+x^2+x^4\right) - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x^2\right) \\
&= \frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{6}\log\left(1-x^2\right) + \frac{1}{12}\log\left(1+x^2+x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 73, normalized size = 1.49

$$\frac{1}{12}\left(\log\left(x^2-x+1\right) + \log\left(x^2+x+1\right) - 2\log(1-x) - 2\log(x+1) + 2\sqrt{3}\,\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) - 2\sqrt{3}\,\tan^{-1}\left(\frac{2x-}{\sqrt{}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 - x^6),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]]
 - 2*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] + Log[1 + x + x^2])/12
```

fricas [A] time = 0.85, size = 38, normalized size = 0.78

$$\frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right) + \frac{1}{12}\,\log\left(x^4+x^2+1\right) - \frac{1}{6}\,\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^6+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*log(x^4 + x^2 + 1) - 1/6
*log(x^2 - 1)
```

giac [A] time = 0.15, size = 39, normalized size = 0.80

$$\frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right) + \frac{1}{12}\,\log\left(x^4+x^2+1\right) - \frac{1}{6}\,\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^6+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*log(x^4 + x^2 + 1) - 1/6
*log(abs(x^2 - 1))
```

maple [A] time = 0.01, size = 66, normalized size = 1.35

$$\frac{\sqrt{3}\ \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{\sqrt{3}\ \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{6}-\frac{\ln(x-1)}{6}-\frac{\ln(x+1)}{6}+\frac{\ln\left(x^2-x+1\right)}{12}+\frac{\ln\left(x^2+x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^6+1),x)
```

```
[Out] -1/6*ln(x-1)+1/12*ln(x^2+x+1)-1/6*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))+1/12*
ln(x^2-x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-1/6*ln(x+1)
```

maxima [A] time = 2.34, size = 38, normalized size = 0.78

$$\frac{1}{6}\sqrt{3}\ \arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{12}\ \log\left(x^4+x^2+1\right)-\frac{1}{6}\ \log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^6+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*log(x^4 + x^2 + 1) - 1/6
*log(x^2 - 1)
```

mupad [B] time = 0.06, size = 52, normalized size = 1.06

$$-\frac{\ln\left(x^2-1\right)}{6}-\ln\left(x^2-\frac{\sqrt{3}\ 1\mathrm{i}}{2}+\frac{1}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\ 1\mathrm{i}}{12}\right)+\ln\left(x^2+\frac{\sqrt{3}\ 1\mathrm{i}}{2}+\frac{1}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\ 1\mathrm{i}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/(x^6 - 1),x)
```

```
[Out] log((3^(1/2)*1i)/2 + x^2 + 1/2)*((3^(1/2)*1i)/12 + 1/12) - log(x^2 - (3^(1/
2)*1i)/2 + 1/2)*((3^(1/2)*1i)/12 - 1/12) - log(x^2 - 1)/6
```

sympy [A] time = 0.26, size = 46, normalized size = 0.94

$$-\frac{\log\left(x^2-1\right)}{6}+\frac{\log\left(x^4+x^2+1\right)}{12}+\frac{\sqrt{3}\ \operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**6+1),x)
```

```
[Out] -log(x**2 - 1)/6 + log(x**4 + x**2 + 1)/12 + sqrt(3)*atan(2*sqrt(3)*x**2/3
+ sqrt(3)/3)/6
```

3.1352 $\int \frac{1}{1-x^6}\,dx$

Optimal. Leaf size=47

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\,x}{1-x^2}\right)}{2\sqrt{3}} + \frac{1}{6}\tanh^{-1}\left(\frac{x}{x^2+1}\right) + \frac{1}{3}\tanh^{-1}(x)$$

```
[Out] 1/3*arctanh(x)+1/6*arctanh(x/(x^2+1))+1/6*arctan(x*3^(1/2)/(-x^2+1))*3^(1/2
)
```

Rubi [A] time = 0.10, antiderivative size = 73, normalized size of antiderivative = 1.55, number of steps used = 10, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {210, 634, 618, 204, 628, 206}

$$-\frac{1}{12}\log\left(x^2-x+1\right) + \frac{1}{12}\log\left(x^2+x+1\right) - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{1}{3}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^6)^(-1),x]
```

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(1 + 2*x)/Sqrt[3]]/(2*Sqrt[
3]) + ArcTanh[x]/3 - Log[1 - x + x^2]/12 + Log[1 + x + x^2]/12
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1-x^6}\,dx &= \frac{1}{3}\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx + \frac{1}{3}\int \frac{1+\frac{x}{2}}{1+x+x^2}\,dx + \frac{1}{3}\int \frac{1}{1-x^2}\,dx \\ &= \frac{1}{3}\tanh^{-1}(x) - \frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx + \frac{1}{12}\int \frac{1+2x}{1+x+x^2}\,dx + \frac{1}{4}\int \frac{1}{1-x+x^2}\,dx + \frac{1}{4}\int \frac{1}{1+x+x^2}\,d \\ &= \frac{1}{3}\tanh^{-1}(x) - \frac{1}{12}\log\left(1-x+x^2\right) + \frac{1}{12}\log\left(1+x+x^2\right) - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x\right) - \\ &= -\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{1}{3}\tanh^{-1}(x) - \frac{1}{12}\log\left(1-x+x^2\right) + \frac{1}{12}\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 75, normalized size = 1.60

$$\frac{1}{12}\left(-\log\left(x^2-x+1\right) + \log\left(x^2+x+1\right) - 2\log(1-x) + 2\log(x+1) + 2\sqrt{3}\,\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + 2\sqrt{3}\,\tan^{-1}\left(\frac{2x+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^6)^(-1),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]]
 - 2*Log[1 - x] + 2*Log[1 + x] - Log[1 - x + x^2] + Log[1 + x + x^2])/12
```

fricas [A] time = 0.90, size = 65, normalized size = 1.38

$$\frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right) + \frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right) + \frac{1}{12}\log\left(x^2+x+1\right) - \frac{1}{12}\log\left(x^2-x+1\right) + \frac{1}{6}\log($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^6+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/12*log(x^2 + x + 1) - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
 - 1/6*log(x - 1)
```

giac [A] time = 0.21, size = 67, normalized size = 1.43

$$\frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right) + \frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right) + \frac{1}{12}\log\left(x^2+x+1\right) - \frac{1}{12}\log\left(x^2-x+1\right) + \frac{1}{6}\log($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^6+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/12*log(x^2 + x + 1) - 1/12*log(x^2 - x + 1) + 1/6*log(abs(x
+ 1)) - 1/6*log(abs(x - 1))
```

maple [A] time = 0.01, size = 66, normalized size = 1.40

$$\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6} + \frac{\sqrt{3}\,\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{6} - \frac{\ln(x-1)}{6} + \frac{\ln(x+1)}{6} - \frac{\ln\left(x^2-x+1\right)}{12} + \frac{\ln\left(x^2+x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^6+1),x)
```

```
[Out] -1/6*ln(x-1)+1/12*ln(x^2+x+1)+1/6*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))-1/12*
ln(x^2-x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/6*ln(x+1)
```

maxima [A] time = 2.36, size = 65, normalized size = 1.38

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+\frac{1}{12}\log\left(x^2+x+1\right)-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^6+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/12*log(x^2 + x + 1) - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
 - 1/6*log(x - 1)
```

mupad [B] time = 0.03, size = 86, normalized size = 1.83

$$\frac{\operatorname{atanh}(x)}{3}+\operatorname{atan}\left(\frac{x\,1\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}+\frac{\sqrt{3}\,x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{6}+\frac{1}{6}\mathrm{i}\right)+\operatorname{atan}\left(\frac{x\,1\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}-\frac{\sqrt{3}\,x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{6}-\frac{1}{6}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^6 - 1),x)
```

```
[Out] atanh(x)/3 + atan((x*1i)/(3^(1/2)*1i + 1) + (3^(1/2)*x)/(3^(1/2)*1i + 1))*(
3^(1/2)/6 + 1i/6) + atan((x*1i)/(3^(1/2)*1i - 1) - (3^(1/2)*x)/(3^(1/2)*1i
 - 1))*(3^(1/2)/6 - 1i/6)
```

sympy [B] time = 0.48, size = 83, normalized size = 1.77

$$-\frac{\log(x-1)}{6}+\frac{\log(x+1)}{6}-\frac{\log\left(x^2-x+1\right)}{12}+\frac{\log\left(x^2+x+1\right)}{12}+\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{6}+\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**6+1),x)
```

```
[Out] -log(x - 1)/6 + log(x + 1)/6 - log(x**2 - x + 1)/12 + log(x**2 + x + 1)/12
+ sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 +
sqrt(3)/3)/6
```

3.1353 $\int \frac{1}{x\left(1-x^6\right)}\,dx$

Optimal. Leaf size=15

$$\log(x)-\frac{1}{6}\log\left(1-x^6\right)$$

```
[Out] ln(x)-1/6*ln(-x^6+1)
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {266, 36, 31, 29}

$$\log(x)-\frac{1}{6}\log\left(1-x^6\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^6)),x]
```

```
[Out] Log[x] - Log[1 - x^6]/6
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(1-x^6\right)}\,dx &= \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{(1-x)x}\,dx, x, x^6\right)\\ &= \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1-x}\,dx, x, x^6\right)+\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, x^6\right)\\ &= \log(x)-\frac{1}{6}\log\left(1-x^6\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\log(x)-\frac{1}{6}\log\left(1-x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^6)),x]
[Out] Log[x] - Log[1 - x^6]/6
```

fricas [A] time = 0.85, size = 11, normalized size = 0.73

$$-\frac{1}{6}\log\left(x^6-1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^6+1),x, algorithm="fricas")
[Out] -1/6*log(x^6 - 1) + log(x)
```

giac [A] time = 0.15, size = 16, normalized size = 1.07

$$\frac{1}{6}\log\left(x^6\right)-\frac{1}{6}\log\left(\left|x^6-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^6+1),x, algorithm="giac")
[Out] 1/6*log(x^6) - 1/6*log(abs(x^6 - 1))
```

maple [B] time = 0.01, size = 36, normalized size = 2.40

$$\ln(x)-\frac{\ln(x-1)}{6}-\frac{\ln(x+1)}{6}-\frac{\ln\left(x^2-x+1\right)}{6}-\frac{\ln\left(x^2+x+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^6+1),x)
[Out] -1/6*ln(x-1)-1/6*ln(x^2-x+1)-1/6*ln(x+1)-1/6*ln(x^2+x+1)+ln(x)
```

maxima [A] time = 1.04, size = 15, normalized size = 1.00

$$-\frac{1}{6}\log\left(x^6-1\right)+\frac{1}{6}\log\left(x^6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^6+1),x, algorithm="maxima")
[Out] -1/6*log(x^6 - 1) + 1/6*log(x^6)
```

mupad [B] time = 1.05, size = 11, normalized size = 0.73

$$\ln(x)-\frac{\ln\left(x^6-1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x*(x^6 - 1)),x)
[Out] log(x) - log(x^6 - 1)/6
```

sympy [A] time = 0.18, size = 10, normalized size = 0.67

$$\log(x)-\frac{\log\left(x^6-1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**6+1),x)
[Out] log(x) - log(x**6 - 1)/6
```

3.1354 $\int \frac{1}{x^2\left(1-x^6\right)}\,dx$

Optimal. Leaf size=52

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,x}{1-x^2}\right)}{2\sqrt{3}}+\frac{1}{6}\tanh^{-1}\left(\frac{x}{x^2+1}\right)-\frac{1}{x}+\frac{1}{3}\tanh^{-1}(x)$$

```
[Out] -1/x+1/3*arctanh(x)+1/6*arctanh(x/(x^2+1))-1/6*arctan(x*3^(1/2)/(-x^2+1))*3
^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 78, normalized size of antiderivative = 1.50, number of steps used = 11, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {325, 296, 634, 618, 204, 628, 206}

$$-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{12}\log\left(x^2+x+1\right)-\frac{1}{x}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{3}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 - x^6)),x]
```

```
[Out] -x^(-1) + ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(1 + 2*x)/Sqrt[3]]
/(2*Sqrt[3]) + ArcTanh[x]/3 - Log[1 - x + x^2]/12 + Log[1 + x + x^2]/12
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^2\left(1-x^6\right)}\,dx = -\frac{1}{x} + \int \frac{x^4}{1-x^6}\,dx$$

$$= -\frac{1}{x} + \frac{1}{3}\int \frac{-\frac{1}{2}-\frac{x}{2}}{1-x+x^2}\,dx + \frac{1}{3}\int \frac{-\frac{1}{2}+\frac{x}{2}}{1+x+x^2}\,dx + \frac{1}{3}\int \frac{1}{1-x^2}\,dx$$

$$= -\frac{1}{x} + \frac{1}{3}\tanh^{-1}(x) - \frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx + \frac{1}{12}\int \frac{1+2x}{1+x+x^2}\,dx - \frac{1}{4}\int \frac{1}{1-x+x^2}\,dx - \frac{1}{4}$$

$$= -\frac{1}{x} + \frac{1}{3}\tanh^{-1}(x) - \frac{1}{12}\log\left(1-x+x^2\right) + \frac{1}{12}\log\left(1+x+x^2\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,$$

$$= -\frac{1}{x} + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{1}{3}\tanh^{-1}(x) - \frac{1}{12}\log\left(1-x+x^2\right) + \frac{1}{12}\log\left(1+x+\right.$$

Mathematica [A] time = 0.02, size = 86, normalized size = 1.65

$$-\frac{x\log\left(x^2-x+1\right) - x\log\left(x^2+x+1\right) + 2x\log(1-x) - 2x\log(x+1) + 2\sqrt{3}\,x\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + 2\sqrt{3}\,x\tan^{-1}\left($$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 - x^6)),x]
```

```
[Out] -1/12*(12 + 2*Sqrt[3]*x*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*Sqrt[3]*x*ArcTan[(1
+ 2*x)/Sqrt[3]] + 2*x*Log[1 - x] - 2*x*Log[1 + x] + x*Log[1 - x + x^2] - x*
Log[1 + x + x^2])/x
```

fricas [A] time = 0.68, size = 76, normalized size = 1.46

$$-\frac{2\sqrt{3}\,x\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right) + 2\sqrt{3}\,x\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right) - x\log\left(x^2+x+1\right) + x\log\left(x^2-x+1\right) - 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^6+1),x, algorithm="fricas")
```

```
[Out] -1/12*(2*sqrt(3)*x*arctan(1/3*sqrt(3)*(2*x + 1)) + 2*sqrt(3)*x*arctan(1/3*s
qrt(3)*(2*x - 1)) - x*log(x^2 + x + 1) + x*log(x^2 - x + 1) - 2*x*log(x + 1
) + 2*x*log(x - 1) + 12)/x
```

giac [A] time = 0.15, size = 72, normalized size = 1.38

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{x}+\frac{1}{12}\log\left(x^2+x+1\right)-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^6+1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) - 1/x + 1/12*log(x^2 + x + 1) - 1/12*log(x^2 - x + 1) + 1/6*log
(abs(x + 1)) - 1/6*log(abs(x - 1))
```

maple [A] time = 0.01, size = 71, normalized size = 1.37

$$-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{6}-\frac{\ln(x-1)}{6}+\frac{\ln(x+1)}{6}-\frac{\ln\left(x^2-x+1\right)}{12}+\frac{\ln\left(x^2+x+1\right)}{12}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-x^6+1),x)
```

```
[Out] -1/6*ln(x-1)-1/12*ln(x^2-x+1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-1/x+1
/6*ln(x+1)+1/12*ln(x^2+x+1)-1/6*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [A] time = 2.41, size = 70, normalized size = 1.35

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{x}+\frac{1}{12}\log\left(x^2+x+1\right)-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^6+1),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) - 1/x + 1/12*log(x^2 + x + 1) - 1/12*log(x^2 - x + 1) + 1/6*log
(x + 1) - 1/6*log(x - 1)
```

mupad [B] time = 0.06, size = 64, normalized size = 1.23

$$-\operatorname{atan}\left(\frac{x\,2\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{6}-\frac{1}{6}\mathrm{i}\right)-\operatorname{atan}\left(\frac{x\,2\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{6}+\frac{1}{6}\mathrm{i}\right)-\frac{1}{x}-\frac{\operatorname{atan}(x\,1\mathrm{i})\,1\mathrm{i}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^2*(x^6 - 1)),x)
```

```
[Out] - (atan(x*1i)*1i)/3 - atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/2)/6 - 1i/6) - at
an((x*2i)/(3^(1/2)*1i + 1))*(3^(1/2)/6 + 1i/6) - 1/x
```

sympy [B] time = 0.55, size = 87, normalized size = 1.67

$$-\frac{\log(x-1)}{6}+\frac{\log(x+1)}{6}-\frac{\log\left(x^2-x+1\right)}{12}+\frac{\log\left(x^2+x+1\right)}{12}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{6}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{6}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-x**6+1),x)
```

```
[Out] -log(x - 1)/6 + log(x + 1)/6 - log(x**2 - x + 1)/12 + log(x**2 + x + 1)/12
- sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 +
sqrt(3)/3)/6 - 1/x
```

3.1355 $\int \frac{1}{x^3\left(1-x^6\right)}\,dx$

Optimal. Leaf size=56

$$-\frac{1}{2x^2}-\frac{1}{6}\log\left(1-x^2\right)-\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{12}\log\left(x^4+x^2+1\right)$$

```
[Out] -1/2/x^2-1/6*ln(-x^2+1)+1/12*ln(x^4+x^2+1)-1/6*arctan(1/3*(2*x^2+1)*3^(1/2)
)*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {275, 325, 292, 31, 634, 618, 204, 628}

$$-\frac{1}{2x^2}-\frac{1}{6}\log\left(1-x^2\right)+\frac{1}{12}\log\left(x^4+x^2+1\right)-\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 - x^6)),x]
```

```
[Out] -1/(2*x^2) - ArcTan[(1 + 2*x^2)/Sqrt[3]]/(2*Sqrt[3]) - Log[1 - x^2]/6 + Log
[1 + x^2 + x^4]/12
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(1-x^6\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2\left(1-x^3\right)}\,dx, x, x^2\right) \\
&= -\frac{1}{2x^2} + \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{1-x^3}\,dx, x, x^2\right) \\
&= -\frac{1}{2x^2} + \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1-x}\,dx, x, x^2\right) - \frac{1}{6}\operatorname{Subst}\left(\int \frac{1-x}{1+x+x^2}\,dx, x, x^2\right) \\
&= -\frac{1}{2x^2} - \frac{1}{6}\log\left(1-x^2\right) + \frac{1}{12}\operatorname{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, x^2\right) - \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^2\right) \\
&= -\frac{1}{2x^2} - \frac{1}{6}\log\left(1-x^2\right) + \frac{1}{12}\log\left(1+x^2+x^4\right) + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x^2\right) \\
&= -\frac{1}{2x^2} - \frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{6}\log\left(1-x^2\right) + \frac{1}{12}\log\left(1+x^2+x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 78, normalized size = 1.39

$$\frac{1}{12}\left(-\frac{6}{x^2} + \log\left(x^2-x+1\right) + \log\left(x^2+x+1\right) - 2\log(1-x) - 2\log(x+1) - 2\sqrt{3}\,\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + 2\sqrt{3}\,\tan^{-1}\left($$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 - x^6)),x]
```

```
[Out] (-6/x^2 - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*Sqrt[3]*ArcTan[(1 + 2*x)
/Sqrt[3]] - 2*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] + Log[1 + x + x^
2])/12
```

fricas [A] time = 0.90, size = 53, normalized size = 0.95

$$-\frac{2\sqrt{3}\,x^2\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right) - x^2\log\left(x^4+x^2+1\right) + 2x^2\log\left(x^2-1\right) + 6}{12x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^6+1),x, algorithm="fricas")
```

```
[Out] -1/12*(2*sqrt(3)*x^2*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - x^2*log(x^4 + x^2 +
1) + 2*x^2*log(x^2 - 1) + 6)/x^2
```

giac [A] time = 0.15, size = 44, normalized size = 0.79

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)-\frac{1}{2x^2}+\frac{1}{12}\log\left(x^4+x^2+1\right)-\frac{1}{6}\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^6+1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/2/x^2 + 1/12*log(x^4 + x^2
 + 1) - 1/6*log(abs(x^2 - 1))
```

maple [A] time = 0.01, size = 71, normalized size = 1.27

$$-\frac{\sqrt{3}\ \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\sqrt{3}\ \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{6}-\frac{\ln(x-1)}{6}-\frac{\ln(x+1)}{6}+\frac{\ln\left(x^2-x+1\right)}{12}+\frac{\ln\left(x^2+x+1\right)}{12}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-x^6+1),x)
```

```
[Out] -1/6*ln(x-1)+1/12*ln(x^2-x+1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-1/2/x
^2-1/6*ln(x+1)+1/12*ln(x^2+x+1)+1/6*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [A] time = 2.35, size = 43, normalized size = 0.77

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)-\frac{1}{2x^2}+\frac{1}{12}\log\left(x^4+x^2+1\right)-\frac{1}{6}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^6+1),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/2/x^2 + 1/12*log(x^4 + x^2
 + 1) - 1/6*log(x^2 - 1)
```

mupad [B] time = 1.06, size = 57, normalized size = 1.02

$$-\frac{\ln\left(x^2-1\right)}{6}-\frac{1}{2x^2}+\ln\left(x^2-\frac{\sqrt{3}\,1\mathrm{i}}{2}+\frac{1}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\ln\left(x^2+\frac{\sqrt{3}\,1\mathrm{i}}{2}+\frac{1}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^3*(x^6 - 1)),x)
```

```
[Out] log(x^2 - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/12 + 1/12) - 1/(2*x^2) - log(
x^2 - 1)/6 - log((3^(1/2)*1i)/2 + x^2 + 1/2)*((3^(1/2)*1i)/12 - 1/12)
```

sympy [A] time = 0.20, size = 53, normalized size = 0.95

$$-\frac{\log\left(x^2-1\right)}{6}+\frac{\log\left(x^4+x^2+1\right)}{12}-\frac{\sqrt{3}\ \operatorname{atan}\left(\frac{2\sqrt{3}\,x^2}{3}+\frac{\sqrt{3}}{3}\right)}{6}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-x**6+1),x)
```

```
[Out] -log(x**2 - 1)/6 + log(x**4 + x**2 + 1)/12 - sqrt(3)*atan(2*sqrt(3)*x**2/3
+ sqrt(3)/3)/6 - 1/(2*x**2)
```

3.1356 $\int \frac{1}{x^4\left(1-x^6\right)}\,dx$

Optimal. Leaf size=16

$$\frac{1}{3}\tanh^{-1}\left(x^3\right)-\frac{1}{3x^3}$$

```
[Out] -1/3/x^3+1/3*arctanh(x^3)
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {275, 325, 206}

$$\frac{1}{3}\tanh^{-1}\left(x^3\right)-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 - x^6)),x]
```

```
[Out] -1/(3*x^3) + ArcTanh[x^3]/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(1-x^6\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x^2\left(1-x^2\right)}\,dx, x, x^3\right)\\ &= -\frac{1}{3x^3}+\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, x^3\right)\\ &= -\frac{1}{3x^3}+\frac{1}{3}\tanh^{-1}\left(x^3\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.88

$$-\frac{1}{3x^3}-\frac{1}{6}\log\left(1-x^3\right)+\frac{1}{6}\log\left(x^3+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 - x^6)),x]
[Out] -1/3*1/x^3 - Log[1 - x^3]/6 + Log[1 + x^3]/6
```

fricas [B] time = 0.77, size = 28, normalized size = 1.75

$$\frac{x^3 \log\left(x^3+1\right) - x^3 \log\left(x^3-1\right) - 2}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^6+1),x, algorithm="fricas")
[Out] 1/6*(x^3*log(x^3 + 1) - x^3*log(x^3 - 1) - 2)/x^3
```

giac [A] time = 0.16, size = 24, normalized size = 1.50

$$-\frac{1}{3\,x^3} + \frac{1}{6}\log\left(\left|x^3+1\right|\right) - \frac{1}{6}\log\left(\left|x^3-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^6+1),x, algorithm="giac")
[Out] -1/3/x^3 + 1/6*log(abs(x^3 + 1)) - 1/6*log(abs(x^3 - 1))
```

maple [B] time = 0.01, size = 39, normalized size = 2.44

$$-\frac{\ln(x-1)}{6} + \frac{\ln(x+1)}{6} + \frac{\ln\left(x^2-x+1\right)}{6} - \frac{\ln\left(x^2+x+1\right)}{6} - \frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-x^6+1),x)
[Out] -1/6*ln(x-1)+1/6*ln(x^2-x+1)-1/3/x^3+1/6*ln(x+1)-1/6*ln(x^2+x+1)
```

maxima [A] time = 1.13, size = 22, normalized size = 1.38

$$-\frac{1}{3\,x^3} + \frac{1}{6}\log\left(x^3+1\right) - \frac{1}{6}\log\left(x^3-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^6+1),x, algorithm="maxima")
[Out] -1/3/x^3 + 1/6*log(x^3 + 1) - 1/6*log(x^3 - 1)
```

mupad [B] time = 0.03, size = 12, normalized size = 0.75

$$\frac{\operatorname{atanh}\left(x^3\right)}{3} - \frac{1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^4*(x^6 - 1)),x)
[Out] atanh(x^3)/3 - 1/(3*x^3)
```

sympy [A] time = 0.24, size = 22, normalized size = 1.38

$$-\frac{\log\left(x^3-1\right)}{6} + \frac{\log\left(x^3+1\right)}{6} - \frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-x**6+1),x)
[Out] -log(x**3 - 1)/6 + log(x**3 + 1)/6 - 1/(3*x**3)
```

3.1357 $\int \frac{1}{x^5\left(1-x^6\right)}\,dx$

Optimal. Leaf size=56

$$-\frac{1}{4x^4}-\frac{1}{6}\log\left(1-x^2\right)+\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{12}\log\left(x^4+x^2+1\right)$$

```
[Out] -1/4/x^4-1/6*ln(-x^2+1)+1/12*ln(x^4+x^2+1)+1/6*arctan(1/3*(2*x^2+1)*3^(1/2)
)*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {275, 325, 200, 31, 634, 618, 204, 628}

$$-\frac{1}{4x^4}-\frac{1}{6}\log\left(1-x^2\right)+\frac{1}{12}\log\left(x^4+x^2+1\right)+\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 - x^6)),x]
```

```
[Out] -1/(4*x^4) + ArcTan[(1 + 2*x^2)/Sqrt[3]]/(2*Sqrt[3]) - Log[1 - x^2]/6 + Log
[1 + x^2 + x^4]/12
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\left(1-x^6\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3\left(1-x^3\right)}\,dx, x, x^2\right) \\
&= -\frac{1}{4x^4} + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-x^3}\,dx, x, x^2\right) \\
&= -\frac{1}{4x^4} + \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, x^2\right) + \frac{1}{6}\,\text{Subst}\left(\int \frac{2+x}{1+x+x^2}\,dx, x, x^2\right) \\
&= -\frac{1}{4x^4} - \frac{1}{6}\log\left(1-x^2\right) + \frac{1}{12}\,\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, x^2\right) + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x\right. \\
&= -\frac{1}{4x^4} - \frac{1}{6}\log\left(1-x^2\right) + \frac{1}{12}\log\left(1+x^2+x^4\right) - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x^2\right) \\
&= -\frac{1}{4x^4} + \frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{6}\log\left(1-x^2\right) + \frac{1}{12}\log\left(1+x^2+x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 78, normalized size = 1.39

$$\frac{1}{12}\left(-\frac{3}{x^4} + \log\left(x^2-x+1\right) + \log\left(x^2+x+1\right) - 2\log(1-x) - 2\log(x+1) + 2\sqrt{3}\,\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) - 2\sqrt{3}\,\tan^{-1}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 - x^6)),x]
```

```
[Out] (-3/x^4 + 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)
/Sqrt[3]] - 2*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] + Log[1 + x + x^
2])/12
```

fricas [A] time = 0.85, size = 52, normalized size = 0.93

$$\frac{2\sqrt{3}\,x^4\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right) + x^4\log\left(x^4+x^2+1\right) - 2x^4\log\left(x^2-1\right) - 3}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^6+1),x, algorithm="fricas")
```

```
[Out] 1/12*(2*sqrt(3)*x^4*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + x^4*log(x^4 + x^2 + 1
) - 2*x^4*log(x^2 - 1) - 3)/x^4
```

giac [A] time = 0.16, size = 44, normalized size = 0.79

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)-\frac{1}{4x^4}+\frac{1}{12}\log\left(x^4+x^2+1\right)-\frac{1}{6}\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^6+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/4/x^4 + 1/12*log(x^4 + x^2
 + 1) - 1/6*log(abs(x^2 - 1))
```

maple [A] time = 0.01, size = 71, normalized size = 1.27

$$\frac{\sqrt{3}\ \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{\sqrt{3}\ \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{6}-\frac{\ln(x-1)}{6}-\frac{\ln(x+1)}{6}+\frac{\ln\left(x^2-x+1\right)}{12}+\frac{\ln\left(x^2+x+1\right)}{12}-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-x^6+1),x)
```

```
[Out] -1/6*ln(x-1)-1/4/x^4+1/12*ln(x^2-x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2
))-1/6*ln(x+1)+1/12*ln(x^2+x+1)-1/6*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [A] time = 2.22, size = 43, normalized size = 0.77

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)-\frac{1}{4x^4}+\frac{1}{12}\log\left(x^4+x^2+1\right)-\frac{1}{6}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^6+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/4/x^4 + 1/12*log(x^4 + x^2
 + 1) - 1/6*log(x^2 - 1)
```

mupad [B] time = 0.08, size = 57, normalized size = 1.02

$$-\frac{\ln\left(x^2-1\right)}{6}-\frac{1}{4x^4}-\ln\left(x^2-\frac{\sqrt{3}\ 1\mathrm{i}}{2}+\frac{1}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\ 1\mathrm{i}}{12}\right)+\ln\left(x^2+\frac{\sqrt{3}\ 1\mathrm{i}}{2}+\frac{1}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\ 1\mathrm{i}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^5*(x^6 - 1)),x)
```

```
[Out] log((3^(1/2)*1i)/2 + x^2 + 1/2)*((3^(1/2)*1i)/12 + 1/12) - 1/(4*x^4) - log(
x^2 - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/12 - 1/12) - log(x^2 - 1)/6
```

sympy [A] time = 0.21, size = 53, normalized size = 0.95

$$-\frac{\log\left(x^2-1\right)}{6}+\frac{\log\left(x^4+x^2+1\right)}{12}+\frac{\sqrt{3}\ \operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{6}-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-x**6+1),x)
```

```
[Out] -log(x**2 - 1)/6 + log(x**4 + x**2 + 1)/12 + sqrt(3)*atan(2*sqrt(3)*x**2/3
 + sqrt(3)/3)/6 - 1/(4*x**4)
```

3.1358 $\int \frac{1}{x^6\left(1-x^6\right)}\,dx$

Optimal. Leaf size=80

$$-\frac{1}{5x^5}-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{12}\log\left(x^2+x+1\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{3}\tanh^{-1}(x)$$

```
[Out] -1/5/x^5+1/3*arctanh(x)-1/12*ln(x^2-x+1)+1/12*ln(x^2+x+1)-1/6*arctan(1/3*(1
-2*x)*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {325, 210, 634, 618, 204, 628, 206}

$$-\frac{1}{5x^5}-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{12}\log\left(x^2+x+1\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{3}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(1 - x^6)),x]
```

```
[Out] -1/(5*x^5) - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(1 + 2*x)/Sqrt[
3]]/(2*Sqrt[3]) + ArcTanh[x]/3 - Log[1 - x + x^2]/12 + Log[1 + x + x^2]/12
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^6\left(1-x^6\right)}\,dx = -\frac{1}{5x^5} + \int \frac{1}{1-x^6}\,dx$$

$$= -\frac{1}{5x^5} + \frac{1}{3}\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx + \frac{1}{3}\int \frac{1+\frac{x}{2}}{1+x+x^2}\,dx + \frac{1}{3}\int \frac{1}{1-x^2}\,dx$$

$$= -\frac{1}{5x^5} + \frac{1}{3}\tanh^{-1}(x) - \frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx + \frac{1}{12}\int \frac{1+2x}{1+x+x^2}\,dx + \frac{1}{4}\int \frac{1}{1-x+x^2}\,dx + \frac{1}{4}\int$$

$$= -\frac{1}{5x^5} + \frac{1}{3}\tanh^{-1}(x) - \frac{1}{12}\log\left(1-x+x^2\right) + \frac{1}{12}\log\left(1+x+x^2\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x\right.$$

$$= -\frac{1}{5x^5} - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{1}{3}\tanh^{-1}(x) - \frac{1}{12}\log\left(1-x+x^2\right) + \frac{1}{12}\log\left(1+x+\right.$$

Mathematica [A] time = 0.03, size = 82, normalized size = 1.02

$$\frac{1}{60}\left(-\frac{12}{x^5} - 5\log\left(x^2-x+1\right) + 5\log\left(x^2+x+1\right) - 10\log(1-x) + 10\log(x+1) + 10\sqrt{3}\,\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + 10\sqrt{3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(1 - x^6)),x]
```

```
[Out] (-12/x^5 + 10*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 10*Sqrt[3]*ArcTan[(1 + 2
*x)/Sqrt[3]] - 10*Log[1 - x] + 10*Log[1 + x] - 5*Log[1 - x + x^2] + 5*Log[1
 + x + x^2])/60
```

fricas [A] time = 0.58, size = 89, normalized size = 1.11

$$\frac{10\sqrt{3}\,x^5\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right) + 10\sqrt{3}\,x^5\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right) + 5\,x^5\log\left(x^2+x+1\right) - 5\,x^5\log\left(x^2-x+1\right.\right.}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^6+1),x, algorithm="fricas")
```

```
[Out] 1/60*(10*sqrt(3)*x^5*arctan(1/3*sqrt(3)*(2*x + 1)) + 10*sqrt(3)*x^5*arctan(
1/3*sqrt(3)*(2*x - 1)) + 5*x^5*log(x^2 + x + 1) - 5*x^5*log(x^2 - x + 1) +
10*x^5*log(x + 1) - 10*x^5*log(x - 1) - 12)/x^5
```

giac [A] time = 0.16, size = 72, normalized size = 0.90

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{5x^5}+\frac{1}{12}\log\left(x^2+x+1\right)-\frac{1}{12}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^6+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) - 1/5/x^5 + 1/12*log(x^2 + x + 1) - 1/12*log(x^2 - x + 1) + 1/6*
log(abs(x + 1)) - 1/6*log(abs(x - 1))
```

maple [A] time = 0.01, size = 71, normalized size = 0.89

$$\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{6}-\frac{\ln(x-1)}{6}+\frac{\ln(x+1)}{6}-\frac{\ln\left(x^2-x+1\right)}{12}+\frac{\ln\left(x^2+x+1\right)}{12}-\frac{1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-x^6+1),x)
```

```
[Out] -1/5/x^5-1/6*ln(x-1)-1/12*ln(x^2-x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2
))+1/6*ln(x+1)+1/12*ln(x^2+x+1)+1/6*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [A] time = 2.25, size = 70, normalized size = 0.88

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{5x^5}+\frac{1}{12}\log\left(x^2+x+1\right)-\frac{1}{12}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^6+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) - 1/5/x^5 + 1/12*log(x^2 + x + 1) - 1/12*log(x^2 - x + 1) + 1/6*
log(x + 1) - 1/6*log(x - 1)
```

mupad [B] time = 1.04, size = 95, normalized size = 1.19

$$\operatorname{atan}\left(\frac{x\,1\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}+\frac{\sqrt{3}\,x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{6}+\frac{1}{6}\mathrm{i}\right)+\operatorname{atan}\left(\frac{x\,1\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}-\frac{\sqrt{3}\,x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{6}-\frac{1}{6}\mathrm{i}\right)-\frac{1}{5x^5}-\frac{\operatorname{atan}(x\,1\mathrm{i})\,1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^6*(x^6 - 1)),x)
```

```
[Out] atan((x*1i)/(3^(1/2)*1i + 1) + (3^(1/2)*x)/(3^(1/2)*1i + 1))*(3^(1/2)/6 + 1
i/6) - (atan(x*1i)*1i)/3 + atan((x*1i)/(3^(1/2)*1i - 1) - (3^(1/2)*x)/(3^(1
/2)*1i - 1))*(3^(1/2)/6 - 1i/6) - 1/(5*x^5)
```

sympy [A] time = 0.63, size = 90, normalized size = 1.12

$$-\frac{\log(x-1)}{6}+\frac{\log(x+1)}{6}-\frac{\log\left(x^2-x+1\right)}{12}+\frac{\log\left(x^2+x+1\right)}{12}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{6}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-x**6+1),x)
```

```
[Out] -log(x - 1)/6 + log(x + 1)/6 - log(x**2 - x + 1)/12 + log(x**2 + x + 1)/12
+ sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 +
sqrt(3)/3)/6 - 1/(5*x**5)
```

3.1359 $\int \frac{1}{x^7\left(1-x^6\right)}\,dx$

Optimal. Leaf size=22

$$-\frac{1}{6x^6}-\frac{1}{6}\log\left(1-x^6\right)+\log(x)$$

```
[Out] -1/6/x^6+ln(x)-1/6*ln(-x^6+1)
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 44}

$$-\frac{1}{6x^6}-\frac{1}{6}\log\left(1-x^6\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(1 - x^6)),x]
```

```
[Out] -1/(6*x^6) + Log[x] - Log[1 - x^6]/6
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7\left(1-x^6\right)}\,dx &= \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{(1-x)x^2}\,dx, x, x^6\right)\\ &= \frac{1}{6}\operatorname{Subst}\left(\int \left(\frac{1}{1-x}+\frac{1}{x^2}+\frac{1}{x}\right)dx, x, x^6\right)\\ &= -\frac{1}{6x^6}+\log(x)-\frac{1}{6}\log\left(1-x^6\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$-\frac{1}{6x^6}-\frac{1}{6}\log\left(1-x^6\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(1 - x^6)),x]
```

```
[Out] -1/6*1/x^6 + Log[x] - Log[1 - x^6]/6
```

fricas [A] time = 0.76, size = 24, normalized size = 1.09

$$-\frac{x^6\log\left(x^6-1\right)-6\,x^6\log(x)+1}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-x^6+1),x, algorithm="fricas")
[Out] -1/6*(x^6*log(x^6 - 1) - 6*x^6*log(x) + 1)/x^6
```

giac [**A**] time = 0.17, size = 26, normalized size = 1.18

$$-\frac{x^6+1}{6\,x^6}+\frac{1}{6}\log\left(x^6\right)-\frac{1}{6}\log\left(\left|x^6-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-x^6+1),x, algorithm="giac")
[Out] -1/6*(x^6 + 1)/x^6 + 1/6*log(x^6) - 1/6*log(abs(x^6 - 1))
```

maple [**B**] time = 0.01, size = 41, normalized size = 1.86

$$\ln(x)-\frac{\ln(x-1)}{6}-\frac{\ln(x+1)}{6}-\frac{\ln\left(x^2-x+1\right)}{6}-\frac{\ln\left(x^2+x+1\right)}{6}-\frac{1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(-x^6+1),x)
[Out] -1/6*ln(x-1)-1/6*ln(x^2-x+1)-1/6*ln(x+1)-1/6*ln(x^2+x+1)-1/6/x^6+ln(x)
```

maxima [**A**] time = 1.08, size = 20, normalized size = 0.91

$$-\frac{1}{6\,x^6}-\frac{1}{6}\log\left(x^6-1\right)+\frac{1}{6}\log\left(x^6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-x^6+1),x, algorithm="maxima")
[Out] -1/6/x^6 - 1/6*log(x^6 - 1) + 1/6*log(x^6)
```

mupad [**B**] time = 1.02, size = 16, normalized size = 0.73

$$\ln(x)-\frac{\ln\left(x^6-1\right)}{6}-\frac{1}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^7*(x^6 - 1)),x)
[Out] log(x) - log(x^6 - 1)/6 - 1/(6*x^6)
```

sympy [**A**] time = 0.24, size = 17, normalized size = 0.77

$$\log(x)-\frac{\log\left(x^6-1\right)}{6}-\frac{1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(-x**6+1),x)
[Out] log(x) - log(x**6 - 1)/6 - 1/(6*x**6)
```

3.1360 $\int \frac{1}{x^8\left(1-x^6\right)}\,dx$

Optimal. Leaf size=85

$$-\frac{1}{7x^7}-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{12}\log\left(x^2+x+1\right)-\frac{1}{x}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{3}\tanh^{-1}(x)$$

```
[Out] -1/7/x^7-1/x+1/3*arctanh(x)-1/12*ln(x^2-x+1)+1/12*ln(x^2+x+1)+1/6*arctan(1/
3*(1-2*x)*3^(1/2))*3^(1/2)-1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {325, 296, 634, 618, 204, 628, 206}

$$-\frac{1}{7x^7}-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{12}\log\left(x^2+x+1\right)-\frac{1}{x}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{3}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(1 - x^6)),x]
```

```
[Out] -1/(7*x^7) - x^(-1) + ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(1 + 2
*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTanh[x]/3 - Log[1 - x + x^2]/12 + Log[1 + x +
 x^2]/12
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^8\left(1-x^6\right)}\,dx &= -\frac{1}{7x^7} + \int \frac{1}{x^2\left(1-x^6\right)}\,dx \\
&= -\frac{1}{7x^7} - \frac{1}{x} + \int \frac{x^4}{1-x^6}\,dx \\
&= -\frac{1}{7x^7} - \frac{1}{x} + \frac{1}{3}\int \frac{-\frac{1}{2}-\frac{x}{2}}{1-x+x^2}\,dx + \frac{1}{3}\int \frac{-\frac{1}{2}+\frac{x}{2}}{1+x+x^2}\,dx + \frac{1}{3}\int \frac{1}{1-x^2}\,dx \\
&= -\frac{1}{7x^7} - \frac{1}{x} + \frac{1}{3}\tanh^{-1}(x) - \frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx + \frac{1}{12}\int \frac{1+2x}{1+x+x^2}\,dx - \frac{1}{4}\int \frac{1}{1-x+x^2}\,dx \\
&= -\frac{1}{7x^7} - \frac{1}{x} + \frac{1}{3}\tanh^{-1}(x) - \frac{1}{12}\log\left(1-x+x^2\right) + \frac{1}{12}\log\left(1+x+x^2\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-}\right. \\
&= -\frac{1}{7x^7} - \frac{1}{x} + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{1}{3}\tanh^{-1}(x) - \frac{1}{12}\log\left(1-x+x^2\right) + \frac{1}{12}\log\left(1\right.
\end{aligned}$$

Mathematica [A] time = 0.03, size = 87, normalized size = 1.02

$$\frac{1}{84}\left(-\frac{12}{x^7} - 7\log\left(x^2-x+1\right) + 7\log\left(x^2+x+1\right) - \frac{84}{x} - 14\log(1-x) + 14\log(x+1) - 14\sqrt{3}\,\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(1 - x^6)),x]
```

```
[Out] (-12/x^7 - 84/x - 14*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 14*Sqrt[3]*ArcTan
[(1 + 2*x)/Sqrt[3]] - 14*Log[1 - x] + 14*Log[1 + x] - 7*Log[1 - x + x^2] +
7*Log[1 + x + x^2])/84
```

fricas [A] time = 0.99, size = 94, normalized size = 1.11

$$-\frac{14\sqrt{3}\,x^7\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right) + 14\sqrt{3}\,x^7\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right) - 7x^7\log\left(x^2+x+1\right) + 7x^7\log\left(x^2-x\right.}{84\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-x^6+1),x, algorithm="fricas")
```

```
[Out] -1/84*(14*sqrt(3)*x^7*arctan(1/3*sqrt(3)*(2*x + 1)) + 14*sqrt(3)*x^7*arctan
(1/3*sqrt(3)*(2*x - 1)) - 7*x^7*log(x^2 + x + 1) + 7*x^7*log(x^2 - x + 1) -
 14*x^7*log(x + 1) + 14*x^7*log(x - 1) + 84*x^6 + 12)/x^7
```

giac [A] time = 0.16, size = 79, normalized size = 0.93

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{7x^6+1}{7x^7}+\frac{1}{12}\log\left(x^2+x+1\right)-\frac{1}{12}\log\left(x^2-x+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-x^6+1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) - 1/7*(7*x^6 + 1)/x^7 + 1/12*log(x^2 + x + 1) - 1/12*log(x^2 -
x + 1) + 1/6*log(abs(x + 1)) - 1/6*log(abs(x - 1))
```

maple [A] time = 0.01, size = 76, normalized size = 0.89

$$-\frac{\sqrt{3}\ \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{\sqrt{3}\ \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{6}-\frac{\ln(x-1)}{6}+\frac{\ln(x+1)}{6}-\frac{\ln\left(x^2-x+1\right)}{12}+\frac{\ln\left(x^2+x+1\right)}{12}-\frac{1}{x}-\frac{1}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(-x^6+1),x)
```

```
[Out] -1/6*ln(x-1)-1/12*ln(x^2-x+1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/6*l
n(x+1)+1/12*ln(x^2+x+1)-1/6*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))-1/7/x^7-1/x
```

maxima [A] time = 2.25, size = 77, normalized size = 0.91

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{7x^6+1}{7x^7}+\frac{1}{12}\log\left(x^2+x+1\right)-\frac{1}{12}\log\left(x^2-x+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-x^6+1),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) - 1/7*(7*x^6 + 1)/x^7 + 1/12*log(x^2 + x + 1) - 1/12*log(x^2 -
x + 1) + 1/6*log(x + 1) - 1/6*log(x - 1)
```

mupad [B] time = 0.06, size = 69, normalized size = 0.81

$$-\operatorname{atan}\left(\frac{x\,2\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{6}-\frac{1}{6}\mathrm{i}\right)-\operatorname{atan}\left(\frac{x\,2\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{6}+\frac{1}{6}\mathrm{i}\right)-\frac{x^6+\frac{1}{7}}{x^7}-\frac{\operatorname{atan}(x\,1\mathrm{i})\,1\mathrm{i}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^8*(x^6 - 1)),x)
```

```
[Out] - (atan(x*1i)*1i)/3 - atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/2)/6 - 1i/6) - at
an((x*2i)/(3^(1/2)*1i + 1))*(3^(1/2)/6 + 1i/6) - (x^6 + 1/7)/x^7
```

sympy [A] time = 0.55, size = 95, normalized size = 1.12

$$-\frac{\log(x-1)}{6}+\frac{\log(x+1)}{6}-\frac{\log\left(x^2-x+1\right)}{12}+\frac{\log\left(x^2+x+1\right)}{12}-\frac{\sqrt{3}\ \operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{6}-\frac{\sqrt{3}\ \operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{6}-\frac{7}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(-x**6+1),x)
```

```
[Out] -log(x - 1)/6 + log(x + 1)/6 - log(x**2 - x + 1)/12 + log(x**2 + x + 1)/12
- sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 +
sqrt(3)/3)/6 - (7*x**6 + 1)/(7*x**7)
```

3.1361 $\int \frac{x^8}{1+x^6}\,dx$

Optimal. Leaf size=16

$$\frac{x^3}{3} - \frac{1}{3}\tan^{-1}\left(x^3\right)$$

```
[Out] 1/3*x^3-1/3*arctan(x^3)
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {275, 321, 203}

$$\frac{x^3}{3} - \frac{1}{3}\tan^{-1}\left(x^3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(1 + x^6),x]
```

```
[Out] x^3/3 - ArcTan[x^3]/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{1+x^6}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, x^3\right)\\ &= \frac{x^3}{3} - \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, x^3\right)\\ &= \frac{x^3}{3} - \frac{1}{3}\tan^{-1}\left(x^3\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{x^3}{3} - \frac{1}{3}\tan^{-1}\left(x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(1 + x^6),x]
[Out] x^3/3 - ArcTan[x^3]/3
```

fricas [A] time = 0.78, size = 12, normalized size = 0.75

$$\frac{1}{3}x^3 - \frac{1}{3}\arctan\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6+1),x, algorithm="fricas")
[Out] 1/3*x^3 - 1/3*arctan(x^3)
```

giac [A] time = 0.15, size = 12, normalized size = 0.75

$$\frac{1}{3}x^3 - \frac{1}{3}\arctan\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6+1),x, algorithm="giac")
[Out] 1/3*x^3 - 1/3*arctan(x^3)
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{x^3}{3} - \frac{\arctan\left(x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^6+1),x)
[Out] 1/3*x^3-1/3*arctan(x^3)
```

maxima [A] time = 2.46, size = 12, normalized size = 0.75

$$\frac{1}{3}x^3 - \frac{1}{3}\arctan\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6+1),x, algorithm="maxima")
[Out] 1/3*x^3 - 1/3*arctan(x^3)
```

mupad [B] time = 1.03, size = 12, normalized size = 0.75

$$\frac{x^3}{3} - \frac{\operatorname{atan}\left(x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^6 + 1),x)
[Out] x^3/3 - atan(x^3)/3
```

sympy [A] time = 0.21, size = 10, normalized size = 0.62

$$\frac{x^3}{3} - \frac{\operatorname{atan}\left(x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**6+1),x)
[Out] x**3/3 - atan(x**3)/3
```

3.1362 $\int \frac{x^7}{1+x^6}\,dx$

Optimal. Leaf size=56

$$\frac{x^2}{2}-\frac{1}{6}\log\left(x^2+1\right)+\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{12}\log\left(x^4-x^2+1\right)$$

```
[Out] 1/2*x^2-1/6*ln(x^2+1)+1/12*ln(x^4-x^2+1)+1/6*arctan(1/3*(-2*x^2+1)*3^(1/2))
*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {275, 321, 200, 31, 634, 618, 204, 628}

$$\frac{x^2}{2}-\frac{1}{6}\log\left(x^2+1\right)+\frac{1}{12}\log\left(x^4-x^2+1\right)+\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(1 + x^6),x]
```

```
[Out] x^2/2 + ArcTan[(1 - 2*x^2)/Sqrt[3]]/(2*Sqrt[3]) - Log[1 + x^2]/6 + Log[1 -
x^2 + x^4]/12
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{1+x^6}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{1+x^3}\,dx, x, x^2\right) \\
&= \frac{x^2}{2} - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+x^3}\,dx, x, x^2\right) \\
&= \frac{x^2}{2} - \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^2\right) - \frac{1}{6}\,\text{Subst}\left(\int \frac{2-x}{1-x+x^2}\,dx, x, x^2\right) \\
&= \frac{x^2}{2} - \frac{1}{6}\log\left(1+x^2\right) + \frac{1}{12}\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^2\right) - \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^2\right) \\
&= \frac{x^2}{2} - \frac{1}{6}\log\left(1+x^2\right) + \frac{1}{12}\log\left(1-x^2+x^4\right) + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^2\right) \\
&= \frac{x^2}{2} + \frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{6}\log\left(1+x^2\right) + \frac{1}{12}\log\left(1-x^2+x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 79, normalized size = 1.41

$$\frac{1}{12}\left(6x^2 - 2\log\left(x^2+1\right) + \log\left(x^2-\sqrt{3}\,x+1\right) + \log\left(x^2+\sqrt{3}\,x+1\right) + 2\sqrt{3}\,\tan^{-1}\left(\sqrt{3}\,-2x\right) + 2\sqrt{3}\,\tan^{-1}\left(2\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(1 + x^6),x]
```

```
[Out] (6*x^2 + 2*Sqrt[3]*ArcTan[Sqrt[3] - 2*x] + 2*Sqrt[3]*ArcTan[Sqrt[3] + 2*x]
- 2*Log[1 + x^2] + Log[1 - Sqrt[3]*x + x^2] + Log[1 + Sqrt[3]*x + x^2])/12
```

fricas [A] time = 0.84, size = 45, normalized size = 0.80

$$\frac{1}{2}x^2 - \frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right) + \frac{1}{12}\log\left(x^4-x^2+1\right) - \frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^6+1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) + 1/12*log(x^4 - x^2
+ 1) - 1/6*log(x^2 + 1)
```

giac [A] time = 0.15, size = 45, normalized size = 0.80

$$\frac{1}{2}x^2 - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right) + \frac{1}{12}\log\left(x^4-x^2+1\right) - \frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^6+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) + 1/12*log(x^4 - x^2
+ 1) - 1/6*log(x^2 + 1)
```

maple [A] time = 0.01, size = 46, normalized size = 0.82

$$\frac{x^2}{2} - \frac{\sqrt{3}\ \arctan\left(\frac{\left(2x^2-1\right)\sqrt{3}}{3}\right)}{6} - \frac{\ln\left(x^2+1\right)}{6} + \frac{\ln\left(x^4-x^2+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^6+1),x)
```

```
[Out] 1/2*x^2+1/12*ln(x^4-x^2+1)-1/6*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))-1/6*ln
(x^2+1)
```

maxima [A] time = 2.30, size = 45, normalized size = 0.80

$$\frac{1}{2}x^2 - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right) + \frac{1}{12}\log\left(x^4-x^2+1\right) - \frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^6+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) + 1/12*log(x^4 - x^2
+ 1) - 1/6*log(x^2 + 1)
```

mupad [B] time = 1.08, size = 57, normalized size = 1.02

$$\frac{x^2}{2} - \frac{\ln\left(x^2+1\right)}{6} + \ln\left(x^2 - \frac{\sqrt{3}\ 1\mathrm{i}}{2} - \frac{1}{2}\right)\left(\frac{1}{12} + \frac{\sqrt{3}\ 1\mathrm{i}}{12}\right) - \ln\left(x^2 + \frac{\sqrt{3}\ 1\mathrm{i}}{2} - \frac{1}{2}\right)\left(-\frac{1}{12} + \frac{\sqrt{3}\ 1\mathrm{i}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^6 + 1),x)
```

```
[Out] x^2/2 - log(x^2 + 1)/6 + log(x^2 - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12 +
 1/12) - log((3^(1/2)*1i)/2 + x^2 - 1/2)*((3^(1/2)*1i)/12 - 1/12)
```

sympy [A] time = 0.21, size = 51, normalized size = 0.91

$$\frac{x^2}{2} - \frac{\log\left(x^2+1\right)}{6} + \frac{\log\left(x^4-x^2+1\right)}{12} - \frac{\sqrt{3}\ \operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3} - \frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**6+1),x)
```

```
[Out] x**2/2 - log(x**2 + 1)/6 + log(x**4 - x**2 + 1)/12 - sqrt(3)*atan(2*sqrt(3)
*x**2/3 - sqrt(3)/3)/6
```

3.1363 $\int \frac{x^6}{1+x^6}\,dx$

Optimal. Leaf size=81

$$\frac{\log\left(x^2-\sqrt{3}\,x+1\right)}{4\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}\,x+1\right)}{4\sqrt{3}}+x+\frac{1}{6}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{1}{3}\tan^{-1}(x)-\frac{1}{6}\tan^{-1}\left(2x+\sqrt{3}\right)$$

```
[Out] x-1/3*arctan(x)-1/6*arctan(2*x-3^(1/2))-1/6*arctan(2*x+3^(1/2))+1/12*ln(1+x
^2-x*3^(1/2))*3^(1/2)-1/12*ln(1+x^2+x*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {321, 209, 634, 618, 204, 628, 203}

$$\frac{\log\left(x^2-\sqrt{3}\,x+1\right)}{4\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}\,x+1\right)}{4\sqrt{3}}+x+\frac{1}{6}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{1}{3}\tan^{-1}(x)-\frac{1}{6}\tan^{-1}\left(2x+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(1 + x^6),x]
```

```
[Out] x + ArcTan[Sqrt[3] - 2*x]/6 - ArcTan[x]/3 - ArcTan[Sqrt[3] + 2*x]/6 + Log[1
 - Sqrt[3]*x + x^2]/(4*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(4*Sqrt[3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{1+x^6}\,dx &= x - \int \frac{1}{1+x^6}\,dx \\
&= x - \frac{1}{3}\int \frac{1-\frac{\sqrt{3}\,x}{2}}{1-\sqrt{3}\,x+x^2}\,dx - \frac{1}{3}\int \frac{1+\frac{\sqrt{3}\,x}{2}}{1+\sqrt{3}\,x+x^2}\,dx - \frac{1}{3}\int \frac{1}{1+x^2}\,dx \\
&= x - \frac{1}{3}\tan^{-1}(x) - \frac{1}{12}\int \frac{1}{1-\sqrt{3}\,x+x^2}\,dx - \frac{1}{12}\int \frac{1}{1+\sqrt{3}\,x+x^2}\,dx + \frac{\int \frac{-\sqrt{3}+2x}{1-\sqrt{3}\,x+x^2}\,dx}{4\sqrt{3}} - \frac{\int \frac{\sqrt{3}+2x}{1+\sqrt{3}\,x+x^2}}{4\sqrt{3}} \\
&= x - \frac{1}{3}\tan^{-1}(x) + \frac{\log\left(1-\sqrt{3}\,x+x^2\right)}{4\sqrt{3}} - \frac{\log\left(1+\sqrt{3}\,x+x^2\right)}{4\sqrt{3}} + \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, -\sqrt{3}+ \right. \\
&= x + \frac{1}{6}\tan^{-1}\left(\sqrt{3}-2x\right) - \frac{1}{3}\tan^{-1}(x) - \frac{1}{6}\tan^{-1}\left(\sqrt{3}+2x\right) + \frac{\log\left(1-\sqrt{3}\,x+x^2\right)}{4\sqrt{3}} - \frac{\log\left(1+\sqrt{3}\,x \right.}{4\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 76, normalized size = 0.94

$$\frac{1}{12}\left(\sqrt{3}\log\left(x^2-\sqrt{3}\,x+1\right) - \sqrt{3}\log\left(x^2+\sqrt{3}\,x+1\right) + 12x + 2\tan^{-1}\left(\sqrt{3}-2x\right) - 4\tan^{-1}(x) - 2\tan^{-1}\left(2x+\sqrt{ \right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(1 + x^6),x]
```

```
[Out] (12*x + 2*ArcTan[Sqrt[3] - 2*x] - 4*ArcTan[x] - 2*ArcTan[Sqrt[3] + 2*x] + S
qrt[3]*Log[1 - Sqrt[3]*x + x^2] - Sqrt[3]*Log[1 + Sqrt[3]*x + x^2])/12
```

fricas [A] time = 0.89, size = 90, normalized size = 1.11

$$-\frac{1}{12}\sqrt{3}\log\left(x^2+\sqrt{3}\,x+1\right)+\frac{1}{12}\sqrt{3}\log\left(x^2-\sqrt{3}\,x+1\right)+x-\frac{1}{3}\arctan(x)+\frac{1}{3}\arctan\left(-2\,x+\sqrt{3}+2\sqrt{x^2-\sqrt{3}\,x \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6+1),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/12*sqrt(3)*log(x^2 - sqrt(3)*x +
 1) + x - 1/3*arctan(x) + 1/3*arctan(-2*x + sqrt(3) + 2*sqrt(x^2 - sqrt(3)*
x + 1)) + 1/3*arctan(-2*x - sqrt(3) + 2*sqrt(x^2 + sqrt(3)*x + 1))
```

giac [A] time = 0.16, size = 61, normalized size = 0.75

$$-\frac{1}{12}\sqrt{3}\log\left(x^2+\sqrt{3}\,x+1\right)+\frac{1}{12}\sqrt{3}\log\left(x^2-\sqrt{3}\,x+1\right)+x-\frac{1}{6}\arctan\left(2\,x+\sqrt{3}\right)-\frac{1}{6}\arctan\left(2\,x-\sqrt{3}\right)-\frac{1}{3}\arct$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/12*sqrt(3)*log(x^2 - sqrt(3)*x +
 1) + x - 1/6*arctan(2*x + sqrt(3)) - 1/6*arctan(2*x - sqrt(3)) - 1/3*arcta
n(x)
```

maple [A] time = 0.04, size = 62, normalized size = 0.77

$$x-\frac{\arctan(x)}{3}-\frac{\arctan\left(2x-\sqrt{3}\right)}{6}-\frac{\arctan\left(2x+\sqrt{3}\right)}{6}+\frac{\sqrt{3}\ \ln\left(x^2-\sqrt{3}\ x+1\right)}{12}-\frac{\sqrt{3}\ \ln\left(x^2+\sqrt{3}\ x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^6+1),x)
```

```
[Out] x-1/3*arctan(x)-1/6*arctan(2*x-3^(1/2))-1/6*arctan(2*x+3^(1/2))+1/12*ln(1+x
^2-3^(1/2)*x)*3^(1/2)-1/12*ln(1+x^2+3^(1/2)*x)*3^(1/2)
```

maxima [A] time = 2.38, size = 61, normalized size = 0.75

$$-\frac{1}{12}\sqrt{3}\ \log\left(x^2+\sqrt{3}\,x+1\right)+\frac{1}{12}\sqrt{3}\ \log\left(x^2-\sqrt{3}\,x+1\right)+x-\frac{1}{6}\ \arctan\left(2\,x+\sqrt{3}\right)-\frac{1}{6}\ \arctan\left(2\,x-\sqrt{3}\right)-\frac{1}{3}\ a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/12*sqrt(3)*log(x^2 - sqrt(3)*x +
 1) + x - 1/6*arctan(2*x + sqrt(3)) - 1/6*arctan(2*x - sqrt(3)) - 1/3*arcta
n(x)
```

mupad [B] time = 0.11, size = 88, normalized size = 1.09

$$x-\frac{\operatorname{atan}(x)}{3}-\operatorname{atan}\left(\frac{x}{-1+\sqrt{3}\ 1\mathrm{i}}+\frac{\sqrt{3}\ x\,1\mathrm{i}}{-1+\sqrt{3}\ 1\mathrm{i}}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\ 1\mathrm{i}}{6}\right)-\operatorname{atan}\left(\frac{x}{1+\sqrt{3}\ 1\mathrm{i}}-\frac{\sqrt{3}\ x\,1\mathrm{i}}{1+\sqrt{3}\ 1\mathrm{i}}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\ 1\mathrm{i}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^6 + 1),x)
```

```
[Out] x - atan(x)/3 - atan(x/(3^(1/2)*1i - 1) + (3^(1/2)*x*1i)/(3^(1/2)*1i - 1))*
((3^(1/2)*1i)/6 + 1/6) - atan(x/(3^(1/2)*1i + 1) - (3^(1/2)*x*1i)/(3^(1/2)*
1i + 1))*((3^(1/2)*1i)/6 - 1/6)
```

sympy [A] time = 0.42, size = 70, normalized size = 0.86

$$x+\frac{\sqrt{3}\ \log\left(x^2-\sqrt{3}\,x+1\right)}{12}-\frac{\sqrt{3}\ \log\left(x^2+\sqrt{3}\,x+1\right)}{12}-\frac{\operatorname{atan}\left(x\right)}{3}-\frac{\operatorname{atan}\left(2x-\sqrt{3}\right)}{6}-\frac{\operatorname{atan}\left(2x+\sqrt{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**6+1),x)
```

```
[Out] x + sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 - sqrt(3)*log(x**2 + sqrt(3)*x + 1
)/12 - atan(x)/3 - atan(2*x - sqrt(3))/6 - atan(2*x + sqrt(3))/6
```

3.1364 $\int \frac{x^5}{1+x^6}\,dx$

Optimal. Leaf size=10

$$\frac{1}{6}\log\left(x^6+1\right)$$

```
[Out] 1/6*ln(x^6+1)
```

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {260}

$$\frac{1}{6}\log\left(x^6+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(1 + x^6),x]
```

```
[Out] Log[1 + x^6]/6
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^5}{1+x^6}\,dx = \frac{1}{6}\log\left(1+x^6\right)$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{6}\log\left(x^6+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(1 + x^6),x]
```

```
[Out] Log[1 + x^6]/6
```

fricas [A] time = 0.84, size = 8, normalized size = 0.80

$$\frac{1}{6}\log\left(x^6+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+1),x, algorithm="fricas")
```

```
[Out] 1/6*log(x^6 + 1)
```

giac [A] time = 0.14, size = 8, normalized size = 0.80

$$\frac{1}{6}\log\left(x^6+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+1),x, algorithm="giac")
[Out] 1/6*log(x^6 + 1)
```

maple [A] time = 0.00, size = 9, normalized size = 0.90

$$\frac{\ln\left(x^6+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^6+1),x)
[Out] 1/6*ln(x^6+1)
```

maxima [A] time = 0.97, size = 8, normalized size = 0.80

$$\frac{1}{6}\log\left(x^6+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+1),x, algorithm="maxima")
[Out] 1/6*log(x^6 + 1)
```

mupad [B] time = 0.03, size = 8, normalized size = 0.80

$$\frac{\ln\left(x^6+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^6 + 1),x)
[Out] log(x^6 + 1)/6
```

sympy [A] time = 0.17, size = 7, normalized size = 0.70

$$\frac{\log\left(x^6+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**6+1),x)
[Out] log(x**6 + 1)/6
```

3.1365 $\int \frac{x^4}{1+x^6}\,dx$

Optimal. Leaf size=80

$$\frac{\log\left(x^2-\sqrt{3}\,x+1\right)}{4\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}\,x+1\right)}{4\sqrt{3}}-\frac{1}{6}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{1}{3}\tan^{-1}(x)+\frac{1}{6}\tan^{-1}\left(2x+\sqrt{3}\right)$$

```
[Out] 1/3*arctan(x)+1/6*arctan(2*x-3^(1/2))+1/6*arctan(2*x+3^(1/2))+1/12*ln(1+x^2
-x*3^(1/2))*3^(1/2)-1/12*ln(1+x^2+x*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {295, 634, 618, 204, 628, 203}

$$\frac{\log\left(x^2-\sqrt{3}\,x+1\right)}{4\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}\,x+1\right)}{4\sqrt{3}}-\frac{1}{6}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{1}{3}\tan^{-1}(x)+\frac{1}{6}\tan^{-1}\left(2x+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 + x^6),x]
```

```
[Out] -ArcTan[Sqrt[3] - 2*x]/6 + ArcTan[x]/3 + ArcTan[Sqrt[3] + 2*x]/6 + Log[1 -
Sqrt[3]*x + x^2]/(4*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(4*Sqrt[3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^4}{1+x^6}\,dx = \frac{1}{3}\int \frac{-\frac{1}{2}+\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx + \frac{1}{3}\int \frac{-\frac{1}{2}-\frac{\sqrt{3}x}{2}}{1+\sqrt{3}x+x^2}\,dx + \frac{1}{3}\int \frac{1}{1+x^2}\,dx$$

$$= \frac{1}{3}\tan^{-1}(x) + \frac{1}{12}\int \frac{1}{1-\sqrt{3}x+x^2}\,dx + \frac{1}{12}\int \frac{1}{1+\sqrt{3}x+x^2}\,dx + \frac{\int \frac{-\sqrt{3}+2x}{1-\sqrt{3}x+x^2}\,dx}{4\sqrt{3}} - \frac{\int \frac{\sqrt{3}+2x}{1+\sqrt{3}x+x^2}}{4\sqrt{3}}$$

$$= \frac{1}{3}\tan^{-1}(x) + \frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}} - \frac{\log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}} - \frac{1}{6}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, -\sqrt{3}+2\right.$$

$$= -\frac{1}{6}\tan^{-1}\left(\sqrt{3}-2x\right) + \frac{1}{3}\tan^{-1}(x) + \frac{1}{6}\tan^{-1}\left(\sqrt{3}+2x\right) + \frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}} - \frac{\log\left(1+\sqrt{3}x\right.}{4\sqrt{3}}$$

Mathematica [A] time = 0.04, size = 73, normalized size = 0.91

$$\frac{1}{12}\left(\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right) - \sqrt{3}\log\left(x^2+\sqrt{3}x+1\right) - 2\tan^{-1}\left(\sqrt{3}-2x\right) + 4\tan^{-1}(x) + 2\tan^{-1}\left(2x+\sqrt{3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 + x^6),x]
```

```
[Out] (-2*ArcTan[Sqrt[3] - 2*x] + 4*ArcTan[x] + 2*ArcTan[Sqrt[3] + 2*x] + Sqrt[3]
*Log[1 - Sqrt[3]*x + x^2] - Sqrt[3]*Log[1 + Sqrt[3]*x + x^2])/12
```

fricas [A] time = 0.66, size = 89, normalized size = 1.11

$$-\frac{1}{12}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)+\frac{1}{12}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)+\frac{1}{3}\arctan(x)-\frac{1}{3}\arctan\left(-2x+\sqrt{3}+2\sqrt{x^2-\sqrt{3}x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+1),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/12*sqrt(3)*log(x^2 - sqrt(3)*x +
 1) + 1/3*arctan(x) - 1/3*arctan(-2*x + sqrt(3) + 2*sqrt(x^2 - sqrt(3)*x +
1)) - 1/3*arctan(-2*x - sqrt(3) + 2*sqrt(x^2 + sqrt(3)*x + 1))
```

giac [A] time = 0.17, size = 60, normalized size = 0.75

$$-\frac{1}{12}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)+\frac{1}{12}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)+\frac{1}{6}\arctan\left(2x+\sqrt{3}\right)+\frac{1}{6}\arctan\left(2x-\sqrt{3}\right)+\frac{1}{3}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/12*sqrt(3)*log(x^2 - sqrt(3)*x +
 1) + 1/6*arctan(2*x + sqrt(3)) + 1/6*arctan(2*x - sqrt(3)) + 1/3*arctan(x)
```

maple [A] time = 0.02, size = 61, normalized size = 0.76

$$\frac{\arctan(x)}{3}+\frac{\arctan\left(2x-\sqrt{3}\right)}{6}+\frac{\arctan\left(2x+\sqrt{3}\right)}{6}+\frac{\sqrt{3}\ \ln\left(x^2-\sqrt{3}\ x+1\right)}{12}-\frac{\sqrt{3}\ \ln\left(x^2+\sqrt{3}\ x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^6+1),x)
```

```
[Out] 1/3*arctan(x)+1/6*arctan(2*x-3^(1/2))+1/6*arctan(2*x+3^(1/2))+1/12*3^(1/2)*
ln(x^2-3^(1/2)*x+1)-1/12*3^(1/2)*ln(x^2+3^(1/2)*x+1)
```

maxima [A] time = 2.48, size = 60, normalized size = 0.75

$$-\frac{1}{12}\sqrt{3}\log\left(x^2+\sqrt{3}\,x+1\right)+\frac{1}{12}\sqrt{3}\log\left(x^2-\sqrt{3}\,x+1\right)+\frac{1}{6}\arctan\left(2\,x+\sqrt{3}\right)+\frac{1}{6}\arctan\left(2\,x-\sqrt{3}\right)+\frac{1}{3}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/12*sqrt(3)*log(x^2 - sqrt(3)*x +
 1) + 1/6*arctan(2*x + sqrt(3)) + 1/6*arctan(2*x - sqrt(3)) + 1/3*arctan(x)
```

mupad [B] time = 1.05, size = 53, normalized size = 0.66

$$\frac{\operatorname{atan}(x)}{3}-\operatorname{atan}\left(\frac{2\,x}{-1+\sqrt{3}\ 1\mathrm{i}}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\ 1\mathrm{i}}{6}\right)-\operatorname{atan}\left(\frac{2\,x}{1+\sqrt{3}\ 1\mathrm{i}}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\ 1\mathrm{i}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^6 + 1),x)
```

```
[Out] atan(x)/3 - atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/6 + 1/6) - atan((2*x
)/(3^(1/2)*1i + 1))*((3^(1/2)*1i)/6 - 1/6)
```

sympy [A] time = 0.37, size = 68, normalized size = 0.85

$$\frac{\sqrt{3}\log\left(x^2-\sqrt{3}\,x+1\right)}{12}-\frac{\sqrt{3}\log\left(x^2+\sqrt{3}\,x+1\right)}{12}+\frac{\operatorname{atan}(x)}{3}+\frac{\operatorname{atan}\left(2x-\sqrt{3}\right)}{6}+\frac{\operatorname{atan}\left(2x+\sqrt{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**6+1),x)
```

```
[Out] sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 - sqrt(3)*log(x**2 + sqrt(3)*x + 1)/12
 + atan(x)/3 + atan(2*x - sqrt(3))/6 + atan(2*x + sqrt(3))/6
```

3.1366 $\int \frac{x^3}{1+x^6}\,dx$

Optimal. Leaf size=49

$$-\frac{1}{6}\log\left(x^2+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{12}\log\left(x^4-x^2+1\right)$$

```
[Out] -1/6*ln(x^2+1)+1/12*ln(x^4-x^2+1)-1/6*arctan(1/3*(-2*x^2+1)*3^(1/2))*3^(1/2
)
```

Rubi [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {275, 292, 31, 634, 618, 204, 628}

$$-\frac{1}{6}\log\left(x^2+1\right)+\frac{1}{12}\log\left(x^4-x^2+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 + x^6),x]
```

```
[Out] -ArcTan[(1 - 2*x^2)/Sqrt[3]]/(2*Sqrt[3]) - Log[1 + x^2]/6 + Log[1 - x^2 + x
^4]/12
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{1+x^6}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, x^2\right) \\
&= -\left(\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^2\right)\right) + \frac{1}{6}\,\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, x^2\right) \\
&= -\frac{1}{6}\log\left(1+x^2\right) + \frac{1}{12}\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^2\right) + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^2\right) \\
&= -\frac{1}{6}\log\left(1+x^2\right) + \frac{1}{12}\log\left(1-x^2+x^4\right) - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^2\right) \\
&= -\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{6}\log\left(1+x^2\right) + \frac{1}{12}\log\left(1-x^2+x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 74, normalized size = 1.51

$$\frac{1}{12}\left(-2\log\left(x^2+1\right) + \log\left(x^2-\sqrt{3}\,x+1\right) + \log\left(x^2+\sqrt{3}\,x+1\right) - 2\sqrt{3}\,\tan^{-1}\left(\sqrt{3}-2x\right) - 2\sqrt{3}\,\tan^{-1}\left(2x+\sqrt{3}\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 + x^6),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[Sqrt[3] - 2*x] - 2*Sqrt[3]*ArcTan[Sqrt[3] + 2*x] - 2*Log
[1 + x^2] + Log[1 - Sqrt[3]*x + x^2] + Log[1 + Sqrt[3]*x + x^2])/12
```

fricas [A] time = 0.87, size = 40, normalized size = 0.82

$$\frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^2-1\right)\right) + \frac{1}{12}\,\log\left(x^4-x^2+1\right) - \frac{1}{6}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) + 1/12*log(x^4 - x^2 + 1) - 1/6
*log(x^2 + 1)
```

giac [A] time = 0.15, size = 40, normalized size = 0.82

$$\frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^2-1\right)\right) + \frac{1}{12}\,\log\left(x^4-x^2+1\right) - \frac{1}{6}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) + 1/12*log(x^4 - x^2 + 1) - 1/6
*log(x^2 + 1)
```

maple [A] time = 0.01, size = 41, normalized size = 0.84

$$\frac{\sqrt{3}\ \arctan\left(\frac{\left(2x^2-1\right)\sqrt{3}}{3}\right)}{6}-\frac{\ln\left(x^2+1\right)}{6}+\frac{\ln\left(x^4-x^2+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^6+1),x)
```

```
[Out] 1/12*ln(x^4-x^2+1)+1/6*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))-1/6*ln(x^2+1)
```

maxima [A] time = 2.37, size = 40, normalized size = 0.82

$$\frac{1}{6}\sqrt{3}\ \arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)+\frac{1}{12}\ \log\left(x^4-x^2+1\right)-\frac{1}{6}\ \log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) + 1/12*log(x^4 - x^2 + 1) - 1/6
*log(x^2 + 1)
```

mupad [B] time = 0.12, size = 52, normalized size = 1.06

$$-\frac{\ln\left(x^2+1\right)}{6}-\ln\left(x^2-\frac{\sqrt{3}\ 1\mathrm{i}}{2}-\frac{1}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\ 1\mathrm{i}}{12}\right)+\ln\left(x^2+\frac{\sqrt{3}\ 1\mathrm{i}}{2}-\frac{1}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\ 1\mathrm{i}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^6 + 1),x)
```

```
[Out] log((3^(1/2)*1i)/2 + x^2 - 1/2)*((3^(1/2)*1i)/12 + 1/12) - log(x^2 - (3^(1/
2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12 - 1/12) - log(x^2 + 1)/6
```

sympy [A] time = 0.17, size = 46, normalized size = 0.94

$$-\frac{\log\left(x^2+1\right)}{6}+\frac{\log\left(x^4-x^2+1\right)}{12}+\frac{\sqrt{3}\ \operatorname{atan}\left(\frac{2\sqrt{3}\,x^2}{3}-\frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**6+1),x)
```

```
[Out] -log(x**2 + 1)/6 + log(x**4 - x**2 + 1)/12 + sqrt(3)*atan(2*sqrt(3)*x**2/3
- sqrt(3)/3)/6
```

3.1367 $\int \frac{x^2}{1+x^6}\,dx$

Optimal. Leaf size=8

$$\frac{1}{3}\tan^{-1}\left(x^3\right)$$

```
[Out] 1/3*arctan(x^3)
```

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {275, 203}

$$\frac{1}{3}\tan^{-1}\left(x^3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + x^6),x]
```

```
[Out] ArcTan[x^3]/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{1+x^6}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, x^3\right)\\ &= \frac{1}{3}\tan^{-1}\left(x^3\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\frac{1}{3}\tan^{-1}\left(x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + x^6),x]
```

```
[Out] ArcTan[x^3]/3
```

fricas [A] time = 0.97, size = 6, normalized size = 0.75

$$\frac{1}{3}\arctan\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+1),x, algorithm="fricas")
```

```
[Out] 1/3*arctan(x^3)
```

giac [A] time = 0.15, size = 6, normalized size = 0.75

$$\frac{1}{3}\arctan\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+1),x, algorithm="giac")
[Out] 1/3*arctan(x^3)
```

maple [A] time = 0.00, size = 7, normalized size = 0.88

$$\frac{\arctan\left(x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^6+1),x)
[Out] 1/3*arctan(x^3)
```

maxima [A] time = 2.34, size = 6, normalized size = 0.75

$$\frac{1}{3}\arctan\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+1),x, algorithm="maxima")
[Out] 1/3*arctan(x^3)
```

mupad [B] time = 0.04, size = 6, normalized size = 0.75

$$\frac{\operatorname{atan}\left(x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^6 + 1),x)
[Out] atan(x^3)/3
```

sympy [A] time = 0.15, size = 5, normalized size = 0.62

$$\frac{\operatorname{atan}\left(x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**6+1),x)
[Out] atan(x**3)/3
```

3.1368 $\int \frac{x}{1+x^6}\,dx$

Optimal. Leaf size=49

$$\frac{1}{6}\log\left(x^2+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{12}\log\left(x^4-x^2+1\right)$$

```
[Out] 1/6*ln(x^2+1)-1/12*ln(x^4-x^2+1)-1/6*arctan(1/3*(-2*x^2+1)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {275, 200, 31, 634, 618, 204, 628}

$$\frac{1}{6}\log\left(x^2+1\right)-\frac{1}{12}\log\left(x^4-x^2+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + x^6),x]
```

```
[Out] -ArcTan[(1 - 2*x^2)/Sqrt[3]]/(2*Sqrt[3]) + Log[1 + x^2]/6 - Log[1 - x^2 + x
^4]/12
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{1+x^6}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+x^3}\,dx, x, x^2\right) \\
&= \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^2\right) + \frac{1}{6}\,\text{Subst}\left(\int \frac{2-x}{1-x+x^2}\,dx, x, x^2\right) \\
&= \frac{1}{6}\log\left(1+x^2\right) - \frac{1}{12}\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^2\right) + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^2\right) \\
&= \frac{1}{6}\log\left(1+x^2\right) - \frac{1}{12}\log\left(1-x^2+x^4\right) - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^2\right) \\
&= -\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{1}{6}\log\left(1+x^2\right) - \frac{1}{12}\log\left(1-x^2+x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 78, normalized size = 1.59

$$\frac{1}{12}\left(2\log\left(x^2+1\right) - \log\left(x^2-\sqrt{3}\,x+1\right) - \log\left(x^2+\sqrt{3}\,x+1\right) - 2\sqrt{3}\,\tan^{-1}\left(\sqrt{3}\,-2x\right) - 2\sqrt{3}\,\tan^{-1}\left(2x+\sqrt{3}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + x^6),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[Sqrt[3] - 2*x] - 2*Sqrt[3]*ArcTan[Sqrt[3] + 2*x] + 2*Log
[1 + x^2] - Log[1 - Sqrt[3]*x + x^2] - Log[1 + Sqrt[3]*x + x^2])/12
```

fricas [A] time = 0.60, size = 40, normalized size = 0.82

$$\frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right) - \frac{1}{12}\,\log\left(x^4-x^2+1\right) + \frac{1}{6}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 1/12*log(x^4 - x^2 + 1) + 1/6
*log(x^2 + 1)
```

giac [A] time = 0.15, size = 40, normalized size = 0.82

$$\frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right) - \frac{1}{12}\,\log\left(x^4-x^2+1\right) + \frac{1}{6}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 1/12*log(x^4 - x^2 + 1) + 1/6
*log(x^2 + 1)
```

maple [A] time = 0.00, size = 41, normalized size = 0.84

$$\frac{\sqrt{3}\ \arctan\left(\frac{\left(2x^2-1\right)\sqrt{3}}{3}\right)}{6}+\frac{\ln\left(x^2+1\right)}{6}-\frac{\ln\left(x^4-x^2+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^6+1),x)
```

```
[Out] -1/12*ln(x^4-x^2+1)+1/6*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))+1/6*ln(x^2+1)
```

maxima [A] time = 2.42, size = 40, normalized size = 0.82

$$\frac{1}{6}\sqrt{3}\ \arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^2-1\right)\right)-\frac{1}{12}\ \log\left(x^4-x^2+1\right)+\frac{1}{6}\ \log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 1/12*log(x^4 - x^2 + 1) + 1/6
*log(x^2 + 1)
```

mupad [B] time = 1.06, size = 52, normalized size = 1.06

$$\frac{\ln\left(x^2+1\right)}{6}-\ln\left(x^2-\frac{\sqrt{3}\ 1\mathrm{i}}{2}-\frac{1}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\ 1\mathrm{i}}{12}\right)+\ln\left(x^2+\frac{\sqrt{3}\ 1\mathrm{i}}{2}-\frac{1}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\ 1\mathrm{i}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^6 + 1),x)
```

```
[Out] log(x^2 + 1)/6 - log(x^2 - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12 + 1/12) +
 log((3^(1/2)*1i)/2 + x^2 - 1/2)*((3^(1/2)*1i)/12 - 1/12)
```

sympy [A] time = 0.32, size = 46, normalized size = 0.94

$$\frac{\log\left(x^2+1\right)}{6}-\frac{\log\left(x^4-x^2+1\right)}{12}+\frac{\sqrt{3}\ \operatorname{atan}\left(\frac{2\sqrt{3}\,x^2}{3}-\frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**6+1),x)
```

```
[Out] log(x**2 + 1)/6 - log(x**4 - x**2 + 1)/12 + sqrt(3)*atan(2*sqrt(3)*x**2/3 -
 sqrt(3)/3)/6
```

3.1369 $\int \frac{1}{1+x^6}\,dx$

Optimal. Leaf size=80

$$-\frac{\log\left(x^2-\sqrt{3}\,x+1\right)}{4\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}\,x+1\right)}{4\sqrt{3}}-\frac{1}{6}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{1}{3}\tan^{-1}(x)+\frac{1}{6}\tan^{-1}\left(2x+\sqrt{3}\right)$$

```
[Out] 1/3*arctan(x)+1/6*arctan(2*x-3^(1/2))+1/6*arctan(2*x+3^(1/2))-1/12*ln(1+x^2
-x*3^(1/2))*3^(1/2)+1/12*ln(1+x^2+x*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {209, 634, 618, 204, 628, 203}

$$-\frac{\log\left(x^2-\sqrt{3}\,x+1\right)}{4\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}\,x+1\right)}{4\sqrt{3}}-\frac{1}{6}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{1}{3}\tan^{-1}(x)+\frac{1}{6}\tan^{-1}\left(2x+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^6)^(-1),x]
```

```
[Out] -ArcTan[Sqrt[3] - 2*x]/6 + ArcTan[x]/3 + ArcTan[Sqrt[3] + 2*x]/6 - Log[1 -
Sqrt[3]*x + x^2]/(4*Sqrt[3]) + Log[1 + Sqrt[3]*x + x^2]/(4*Sqrt[3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{1+x^6}\,dx = \frac{1}{3}\int \frac{1-\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx + \frac{1}{3}\int \frac{1+\frac{\sqrt{3}x}{2}}{1+\sqrt{3}x+x^2}\,dx + \frac{1}{3}\int \frac{1}{1+x^2}\,dx$$

$$= \frac{1}{3}\tan^{-1}(x) + \frac{1}{12}\int \frac{1}{1-\sqrt{3}x+x^2}\,dx + \frac{1}{12}\int \frac{1}{1+\sqrt{3}x+x^2}\,dx - \frac{\int \frac{-\sqrt{3}+2x}{1-\sqrt{3}x+x^2}\,dx}{4\sqrt{3}} + \frac{\int \frac{\sqrt{3}+2x}{1+\sqrt{3}x+x^2}\,dx}{4\sqrt{3}}$$

$$= \frac{1}{3}\tan^{-1}(x) - \frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}} + \frac{\log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}} - \frac{1}{6}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, -\sqrt{3}+2x\right)$$

$$= -\frac{1}{6}\tan^{-1}\left(\sqrt{3}-2x\right) + \frac{1}{3}\tan^{-1}(x) + \frac{1}{6}\tan^{-1}\left(\sqrt{3}+2x\right) - \frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}} + \frac{\log\left(1+\sqrt{3}x+\right.}{4\sqrt{3}}$$

Mathematica [A] time = 0.01, size = 73, normalized size = 0.91

$$\frac{1}{12}\left(-\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)+\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)-2\tan^{-1}\left(\sqrt{3}-2x\right)+4\tan^{-1}(x)+2\tan^{-1}\left(2x+\sqrt{3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^6)^(-1),x]
```

```
[Out] (-2*ArcTan[Sqrt[3] - 2*x] + 4*ArcTan[x] + 2*ArcTan[Sqrt[3] + 2*x] - Sqrt[3]
*Log[1 - Sqrt[3]*x + x^2] + Sqrt[3]*Log[1 + Sqrt[3]*x + x^2])/12
```

fricas [A] time = 0.61, size = 89, normalized size = 1.11

$$\frac{1}{12}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)-\frac{1}{12}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)+\frac{1}{3}\arctan(x)-\frac{1}{3}\arctan\left(-2x+\sqrt{3}+2\sqrt{x^2-\sqrt{3}x+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/12*sqrt(3)*log(x^2 - sqrt(3)*x +
1) + 1/3*arctan(x) - 1/3*arctan(-2*x + sqrt(3) + 2*sqrt(x^2 - sqrt(3)*x + 1
)) - 1/3*arctan(-2*x - sqrt(3) + 2*sqrt(x^2 + sqrt(3)*x + 1))
```

giac [A] time = 0.18, size = 60, normalized size = 0.75

$$\frac{1}{12}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)-\frac{1}{12}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)+\frac{1}{6}\arctan\left(2x+\sqrt{3}\right)+\frac{1}{6}\arctan\left(2x-\sqrt{3}\right)+\frac{1}{3}\arctan(x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/12*sqrt(3)*log(x^2 - sqrt(3)*x +
1) + 1/6*arctan(2*x + sqrt(3)) + 1/6*arctan(2*x - sqrt(3)) + 1/3*arctan(x)
```

maple [A] time = 0.02, size = 61, normalized size = 0.76

$$\frac{\arctan(x)}{3}+\frac{\arctan\left(2x-\sqrt{3}\right)}{6}+\frac{\arctan\left(2x+\sqrt{3}\right)}{6}-\frac{\sqrt{3}\ \ln\left(x^2-\sqrt{3}\ x+1\right)}{12}+\frac{\sqrt{3}\ \ln\left(x^2+\sqrt{3}\ x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6+1),x)
```

```
[Out] 1/3*arctan(x)+1/6*arctan(2*x-3^(1/2))+1/6*arctan(2*x+3^(1/2))-1/12*3^(1/2)*
ln(x^2-3^(1/2)*x+1)+1/12*3^(1/2)*ln(x^2+3^(1/2)*x+1)
```

maxima [A] time = 2.32, size = 60, normalized size = 0.75

$$\frac{1}{12}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)-\frac{1}{12}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)+\frac{1}{6}\arctan\left(2x+\sqrt{3}\right)+\frac{1}{6}\arctan\left(2x-\sqrt{3}\right)+\frac{1}{3}\text{ arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/12*sqrt(3)*log(x^2 - sqrt(3)*x +
1) + 1/6*arctan(2*x + sqrt(3)) + 1/6*arctan(2*x - sqrt(3)) + 1/3*arctan(x)
```

mupad [B] time = 0.04, size = 85, normalized size = 1.06

$$\frac{\text{atan}(x)}{3}+\text{atan}\left(\frac{x}{-1+\sqrt{3}\,1\text{i}}+\frac{\sqrt{3}\,x1\text{i}}{-1+\sqrt{3}\,1\text{i}}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\text{i}}{6}\right)+\text{atan}\left(\frac{x}{1+\sqrt{3}\,1\text{i}}-\frac{\sqrt{3}\,x1\text{i}}{1+\sqrt{3}\,1\text{i}}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\text{i}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6 + 1),x)
```

```
[Out] atan(x)/3 + atan(x/(3^(1/2)*1i - 1) + (3^(1/2)*x*1i)/(3^(1/2)*1i - 1))*((3^
(1/2)*1i)/6 + 1/6) + atan(x/(3^(1/2)*1i + 1) - (3^(1/2)*x*1i)/(3^(1/2)*1i +
 1))*((3^(1/2)*1i)/6 - 1/6)
```

sympy [A] time = 0.42, size = 68, normalized size = 0.85

$$-\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{12}+\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{12}+\frac{\text{atan}\left(x\right)}{3}+\frac{\text{atan}\left(2x-\sqrt{3}\right)}{6}+\frac{\text{atan}\left(2x+\sqrt{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6+1),x)
```

```
[Out] -sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 + sqrt(3)*log(x**2 + sqrt(3)*x + 1)/1
2 + atan(x)/3 + atan(2*x - sqrt(3))/6 + atan(2*x + sqrt(3))/6
```

3.1370 $\int \frac{1}{x\left(1+x^6\right)}\,dx$

Optimal. Leaf size=13

$$\log(x)-\frac{1}{6}\log\left(x^6+1\right)$$

```
[Out] ln(x)-1/6*ln(x^6+1)
```

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {266, 36, 29, 31}

$$\log(x)-\frac{1}{6}\log\left(x^6+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + x^6)),x]

[Out] Log[x] - Log[1 + x^6]/6
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(1+x^6\right)}\,dx &= \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{x(1+x)}\,dx,x,x^6\right)\\ &= \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,x^6\right)-\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx,x,x^6\right)\\ &= \log(x)-\frac{1}{6}\log\left(1+x^6\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\log(x)-\frac{1}{6}\log\left(x^6+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + x^6)),x]
[Out] Log[x] - Log[1 + x^6]/6
```

fricas [A] time = 0.75, size = 11, normalized size = 0.85

$$-\frac{1}{6}\log\left(x^6+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6+1),x, algorithm="fricas")
[Out] -1/6*log(x^6 + 1) + log(x)
```

giac [A] time = 0.22, size = 15, normalized size = 1.15

$$-\frac{1}{6}\log\left(x^6+1\right)+\frac{1}{6}\log\left(x^6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6+1),x, algorithm="giac")
[Out] -1/6*log(x^6 + 1) + 1/6*log(x^6)
```

maple [B] time = 0.01, size = 25, normalized size = 1.92

$$\ln(x)-\frac{\ln\left(x^2+1\right)}{6}-\frac{\ln\left(x^4-x^2+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^6+1),x)
[Out] -1/6*ln(x^2+1)-1/6*ln(x^4-x^2+1)+ln(x)
```

maxima [A] time = 1.03, size = 15, normalized size = 1.15

$$-\frac{1}{6}\log\left(x^6+1\right)+\frac{1}{6}\log\left(x^6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6+1),x, algorithm="maxima")
[Out] -1/6*log(x^6 + 1) + 1/6*log(x^6)
```

mupad [B] time = 0.04, size = 11, normalized size = 0.85

$$\ln(x)-\frac{\ln\left(x^6+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^6 + 1)),x)
[Out] log(x) - log(x^6 + 1)/6
```

sympy [A] time = 0.16, size = 10, normalized size = 0.77

$$\log(x)-\frac{\log\left(x^6+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**6+1),x)
[Out] log(x) - log(x**6 + 1)/6
```

3.1371 $\int \frac{1}{x^2\left(1+x^6\right)}\,dx$

Optimal. Leaf size=85

$$-\frac{\log\left(x^2-\sqrt{3}\,x+1\right)}{4\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}\,x+1\right)}{4\sqrt{3}}-\frac{1}{x}+\frac{1}{6}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{1}{3}\tan^{-1}(x)-\frac{1}{6}\tan^{-1}\left(2x+\sqrt{3}\right)$$

```
[Out] -1/x-1/3*arctan(x)-1/6*arctan(2*x-3^(1/2))-1/6*arctan(2*x+3^(1/2))-1/12*ln(
1+x^2-x*3^(1/2))*3^(1/2)+1/12*ln(1+x^2+x*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {325, 295, 634, 618, 204, 628, 203}

$$-\frac{\log\left(x^2-\sqrt{3}\,x+1\right)}{4\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}\,x+1\right)}{4\sqrt{3}}-\frac{1}{x}+\frac{1}{6}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{1}{3}\tan^{-1}(x)-\frac{1}{6}\tan^{-1}\left(2x+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 + x^6)),x]
```

```
[Out] -x^(-1) + ArcTan[Sqrt[3] - 2*x]/6 - ArcTan[x]/3 - ArcTan[Sqrt[3] + 2*x]/6 -
 Log[1 - Sqrt[3]*x + x^2]/(4*Sqrt[3]) + Log[1 + Sqrt[3]*x + x^2]/(4*Sqrt[3]
)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^2\left(1+x^6\right)}\,dx = -\frac{1}{x} - \int \frac{x^4}{1+x^6}\,dx$$

$$= -\frac{1}{x} - \frac{1}{3}\int \frac{-\frac{1}{2}+\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx - \frac{1}{3}\int \frac{-\frac{1}{2}-\frac{\sqrt{3}x}{2}}{1+\sqrt{3}x+x^2}\,dx - \frac{1}{3}\int \frac{1}{1+x^2}\,dx$$

$$= -\frac{1}{x} - \frac{1}{3}\tan^{-1}(x) - \frac{1}{12}\int \frac{1}{1-\sqrt{3}x+x^2}\,dx - \frac{1}{12}\int \frac{1}{1+\sqrt{3}x+x^2}\,dx - \frac{\int \frac{-\sqrt{3}+2x}{1-\sqrt{3}x+x^2}\,dx}{4\sqrt{3}} + \frac{\int$$

$$= -\frac{1}{x} - \frac{1}{3}\tan^{-1}(x) - \frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}} + \frac{\log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}} + \frac{1}{6}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,\right.$$

$$= -\frac{1}{x} + \frac{1}{6}\tan^{-1}\left(\sqrt{3}-2x\right) - \frac{1}{3}\tan^{-1}(x) - \frac{1}{6}\tan^{-1}\left(\sqrt{3}+2x\right) - \frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}} + \frac{\log($$

Mathematica [A] time = 0.04, size = 82, normalized size = 0.96

$$-\frac{\sqrt{3}x\log\left(x^2-\sqrt{3}x+1\right) - \sqrt{3}x\log\left(x^2+\sqrt{3}x+1\right) - 2x\tan^{-1}\left(\sqrt{3}-2x\right) + 4x\tan^{-1}(x) + 2x\tan^{-1}\left(2x+\sqrt{}\right.}{12x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 + x^6)),x]
```

```
[Out] -1/12*(12 - 2*x*ArcTan[Sqrt[3] - 2*x] + 4*x*ArcTan[x] + 2*x*ArcTan[Sqrt[3]
+ 2*x] + Sqrt[3]*x*Log[1 - Sqrt[3]*x + x^2] - Sqrt[3]*x*Log[1 + Sqrt[3]*x +
 x^2])/x
```

fricas [A] time = 0.90, size = 99, normalized size = 1.16

$$\frac{\sqrt{3}x\log\left(x^2+\sqrt{3}x+1\right) - \sqrt{3}x\log\left(x^2-\sqrt{3}x+1\right) - 4x\arctan(x) + 4x\arctan\left(-2x+\sqrt{3}+2\sqrt{x^2-\sqrt{3}x}\right.}{12x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6+1),x, algorithm="fricas")
```

```
[Out] 1/12*(sqrt(3)*x*log(x^2 + sqrt(3)*x + 1) - sqrt(3)*x*log(x^2 - sqrt(3)*x +
1) - 4*x*arctan(x) + 4*x*arctan(-2*x + sqrt(3) + 2*sqrt(x^2 - sqrt(3)*x + 1
)) + 4*x*arctan(-2*x - sqrt(3) + 2*sqrt(x^2 + sqrt(3)*x + 1)) - 12)/x
```

giac [A] time = 0.16, size = 65, normalized size = 0.76

$$\frac{1}{12}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)-\frac{1}{12}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)-\frac{1}{x}-\frac{1}{6}\arctan\left(2x+\sqrt{3}\right)-\frac{1}{6}\arctan\left(2x-\sqrt{3}\right)-\frac{1}{3}\text{ arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/12*sqrt(3)*log(x^2 - sqrt(3)*x +
1) - 1/x - 1/6*arctan(2*x + sqrt(3)) - 1/6*arctan(2*x - sqrt(3)) - 1/3*arct
an(x)
```

maple [A] time = 0.02, size = 66, normalized size = 0.78

$$-\frac{\arctan(x)}{3}-\frac{\arctan\left(2x-\sqrt{3}\right)}{6}-\frac{\arctan\left(2x+\sqrt{3}\right)}{6}-\frac{\sqrt{3}\,\ln\left(x^2-\sqrt{3}\,x+1\right)}{12}+\frac{\sqrt{3}\,\ln\left(x^2+\sqrt{3}\,x+1\right)}{12}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^6+1),x)
```

```
[Out] -1/x-1/3*arctan(x)-1/6*arctan(2*x-3^(1/2))-1/6*arctan(2*x+3^(1/2))-1/12*3^(
1/2)*ln(x^2-3^(1/2)*x+1)+1/12*3^(1/2)*ln(x^2+3^(1/2)*x+1)
```

maxima [A] time = 2.41, size = 65, normalized size = 0.76

$$\frac{1}{12}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)-\frac{1}{12}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)-\frac{1}{x}-\frac{1}{6}\arctan\left(2x+\sqrt{3}\right)-\frac{1}{6}\arctan\left(2x-\sqrt{3}\right)-\frac{1}{3}\text{ arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/12*sqrt(3)*log(x^2 - sqrt(3)*x +
1) - 1/x - 1/6*arctan(2*x + sqrt(3)) - 1/6*arctan(2*x - sqrt(3)) - 1/3*arct
an(x)
```

mupad [B] time = 0.06, size = 56, normalized size = 0.66

$$-\frac{\operatorname{atan}(x)}{3}+\operatorname{atan}\left(\frac{2x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\operatorname{atan}\left(\frac{2x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^6 + 1)),x)
```

```
[Out] atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/6 + 1/6) - atan(x)/3 + atan((2*x
)/(3^(1/2)*1i + 1))*((3^(1/2)*1i)/6 - 1/6) - 1/x
```

sympy [A] time = 0.50, size = 71, normalized size = 0.84

$$-\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{12}+\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{12}-\frac{\operatorname{atan}(x)}{3}-\frac{\operatorname{atan}\left(2x-\sqrt{3}\right)}{6}-\frac{\operatorname{atan}\left(2x+\sqrt{3}\right)}{6}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**6+1),x)
```

```
[Out] -sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 + sqrt(3)*log(x**2 + sqrt(3)*x + 1)/1
2 - atan(x)/3 - atan(2*x - sqrt(3))/6 - atan(2*x + sqrt(3))/6 - 1/x
```

3.1372 $\int \frac{1}{x^3\left(1+x^6\right)}\,dx$

Optimal. Leaf size=56

$$-\frac{1}{2x^2}+\frac{1}{6}\log\left(x^2+1\right)+\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{12}\log\left(x^4-x^2+1\right)$$

```
[Out] -1/2/x^2+1/6*ln(x^2+1)-1/12*ln(x^4-x^2+1)+1/6*arctan(1/3*(-2*x^2+1)*3^(1/2)
)*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {275, 325, 292, 31, 634, 618, 204, 628}

$$-\frac{1}{2x^2}+\frac{1}{6}\log\left(x^2+1\right)-\frac{1}{12}\log\left(x^4-x^2+1\right)+\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 + x^6)),x]
```

```
[Out] -1/(2*x^2) + ArcTan[(1 - 2*x^2)/Sqrt[3]]/(2*Sqrt[3]) + Log[1 + x^2]/6 - Log
[1 - x^2 + x^4]/12
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(1+x^6\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2\left(1+x^3\right)}\,dx, x, x^2\right) \\
&= -\frac{1}{2x^2} - \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, x^2\right) \\
&= -\frac{1}{2x^2} + \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^2\right) - \frac{1}{6}\operatorname{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, x^2\right) \\
&= -\frac{1}{2x^2} + \frac{1}{6}\log\left(1+x^2\right) - \frac{1}{12}\operatorname{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^2\right) - \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^2\right) \\
&= -\frac{1}{2x^2} + \frac{1}{6}\log\left(1+x^2\right) - \frac{1}{12}\log\left(1-x^2+x^4\right) + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^2\right) \\
&= -\frac{1}{2x^2} + \frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{1}{6}\log\left(1+x^2\right) - \frac{1}{12}\log\left(1-x^2+x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.08, size = 83, normalized size = 1.48

$$\frac{1}{12}\left(-\frac{6}{x^2} + 2\log\left(x^2+1\right) - \log\left(x^2-\sqrt{3}\,x+1\right) - \log\left(x^2+\sqrt{3}\,x+1\right) + 2\sqrt{3}\,\tan^{-1}\left(\sqrt{3}\,-2x\right) + 2\sqrt{3}\,\tan^{-1}\left(2x+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 + x^6)),x]
```

```
[Out] (-6/x^2 + 2*Sqrt[3]*ArcTan[Sqrt[3] - 2*x] + 2*Sqrt[3]*ArcTan[Sqrt[3] + 2*x]
 + 2*Log[1 + x^2] - Log[1 - Sqrt[3]*x + x^2] - Log[1 + Sqrt[3]*x + x^2])/12
```

fricas [A] time = 0.81, size = 54, normalized size = 0.96

$$-\frac{2\sqrt{3}\,x^2\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right) + x^2\log\left(x^4-x^2+1\right) - 2x^2\log\left(x^2+1\right) + 6}{12x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^6+1),x, algorithm="fricas")
```

```
[Out] -1/12*(2*sqrt(3)*x^2*arctan(1/3*sqrt(3)*(2*x^2 - 1)) + x^2*log(x^4 - x^2 +
1) - 2*x^2*log(x^2 + 1) + 6)/x^2
```

giac [A] time = 0.18, size = 45, normalized size = 0.80

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)-\frac{1}{2x^2}-\frac{1}{12}\log\left(x^4-x^2+1\right)+\frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^6+1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 1/2/x^2 - 1/12*log(x^4 - x^2
 + 1) + 1/6*log(x^2 + 1)
```

maple [A] time = 0.01, size = 46, normalized size = 0.82

$$-\frac{\sqrt{3}\ \arctan\left(\frac{\left(2x^2-1\right)\sqrt{3}}{3}\right)}{6}+\frac{\ln\left(x^2+1\right)}{6}-\frac{\ln\left(x^4-x^2+1\right)}{12}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^6+1),x)
```

```
[Out] 1/6*ln(x^2+1)-1/2/x^2-1/12*ln(x^4-x^2+1)-1/6*3^(1/2)*arctan(1/3*(2*x^2-1)*3
^(1/2))
```

maxima [A] time = 2.36, size = 45, normalized size = 0.80

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)-\frac{1}{2x^2}-\frac{1}{12}\log\left(x^4-x^2+1\right)+\frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^6+1),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 1/2/x^2 - 1/12*log(x^4 - x^2
 + 1) + 1/6*log(x^2 + 1)
```

mupad [B] time = 0.07, size = 57, normalized size = 1.02

$$\frac{\ln\left(x^2+1\right)}{6}-\frac{1}{2x^2}+\ln\left(x^2-\frac{\sqrt{3}\,1\mathrm{i}}{2}-\frac{1}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\ln\left(x^2+\frac{\sqrt{3}\,1\mathrm{i}}{2}-\frac{1}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x^6 + 1)),x)
```

```
[Out] log(x^2 + 1)/6 - 1/(2*x^2) + log(x^2 - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/
12 - 1/12) - log((3^(1/2)*1i)/2 + x^2 - 1/2)*((3^(1/2)*1i)/12 + 1/12)
```

sympy [A] time = 0.20, size = 53, normalized size = 0.95

$$\frac{\log\left(x^2+1\right)}{6}-\frac{\log\left(x^4-x^2+1\right)}{12}-\frac{\sqrt{3}\ \operatorname{atan}\left(\frac{2\sqrt{3}\,x^2}{3}-\frac{\sqrt{3}}{3}\right)}{6}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**6+1),x)
```

```
[Out] log(x**2 + 1)/6 - log(x**4 - x**2 + 1)/12 - sqrt(3)*atan(2*sqrt(3)*x**2/3 -
 sqrt(3)/3)/6 - 1/(2*x**2)
```

3.1373 $\int \frac{1}{x^4\left(1+x^6\right)}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{3x^3}-\frac{1}{3}\tan^{-1}\left(x^3\right)$$

```
[Out] -1/3/x^3-1/3*arctan(x^3)
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {275, 325, 203}

$$-\frac{1}{3x^3}-\frac{1}{3}\tan^{-1}\left(x^3\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 + x^6)),x]
```

```
[Out] -1/(3*x^3) - ArcTan[x^3]/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(1+x^6\right)}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^2\left(1+x^2\right)}\,dx, x, x^3\right)\\ &= -\frac{1}{3x^3}-\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, x^3\right)\\ &= -\frac{1}{3x^3}-\frac{1}{3}\tan^{-1}\left(x^3\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{1}{3}\tan^{-1}\left(\frac{1}{x^3}\right)-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 + x^6)),x]
[Out] -1/3*1/x^3 + ArcTan[x^(-3)]/3
```

fricas [A] time = 0.86, size = 15, normalized size = 0.94

$$-\frac{x^3 \arctan\left(x^3\right)+1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^6+1),x, algorithm="fricas")
[Out] -1/3*(x^3*arctan(x^3) + 1)/x^3
```

giac [A] time = 0.15, size = 12, normalized size = 0.75

$$-\frac{1}{3\,x^3}-\frac{1}{3}\arctan\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^6+1),x, algorithm="giac")
[Out] -1/3/x^3 - 1/3*arctan(x^3)
```

maple [B] time = 0.02, size = 33, normalized size = 2.06

$$\frac{\arctan(x)}{3}-\frac{\arctan\left(2x-\sqrt{3}\right)}{3}-\frac{\arctan\left(2x+\sqrt{3}\right)}{3}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^6+1),x)
[Out] -1/3/x^3+1/3*arctan(x)-1/3*arctan(2*x-3^(1/2))-1/3*arctan(2*x+3^(1/2))
```

maxima [A] time = 2.36, size = 12, normalized size = 0.75

$$-\frac{1}{3\,x^3}-\frac{1}{3}\arctan\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^6+1),x, algorithm="maxima")
[Out] -1/3/x^3 - 1/3*arctan(x^3)
```

mupad [B] time = 1.02, size = 12, normalized size = 0.75

$$-\frac{\operatorname{atan}\left(x^3\right)}{3}-\frac{1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x^6 + 1)),x)
[Out] - atan(x^3)/3 - 1/(3*x^3)
```

sympy [A] time = 0.28, size = 14, normalized size = 0.88

$$-\frac{\operatorname{atan}\left(x^3\right)}{3}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**6+1),x)
[Out] -atan(x**3)/3 - 1/(3*x**3)
```

3.1374 $\int \frac{1}{x^5\left(1+x^6\right)}\,dx$

Optimal. Leaf size=56

$$-\frac{1}{4x^4}-\frac{1}{6}\log\left(x^2+1\right)+\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{12}\log\left(x^4-x^2+1\right)$$

```
[Out] -1/4/x^4-1/6*ln(x^2+1)+1/12*ln(x^4-x^2+1)+1/6*arctan(1/3*(-2*x^2+1)*3^(1/2)
)*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {275, 325, 200, 31, 634, 618, 204, 628}

$$-\frac{1}{4x^4}-\frac{1}{6}\log\left(x^2+1\right)+\frac{1}{12}\log\left(x^4-x^2+1\right)+\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 + x^6)),x]
```

```
[Out] -1/(4*x^4) + ArcTan[(1 - 2*x^2)/Sqrt[3]]/(2*Sqrt[3]) - Log[1 + x^2]/6 + Log
[1 - x^2 + x^4]/12
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\left(1+x^6\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3\left(1+x^3\right)}\,dx, x, x^2\right)\\
&= -\frac{1}{4x^4} - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+x^3}\,dx, x, x^2\right)\\
&= -\frac{1}{4x^4} - \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^2\right) - \frac{1}{6}\,\text{Subst}\left(\int \frac{2-x}{1-x+x^2}\,dx, x, x^2\right)\\
&= -\frac{1}{4x^4} - \frac{1}{6}\log\left(1+x^2\right) + \frac{1}{12}\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^2\right) - \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x\right.\\
&= -\frac{1}{4x^4} - \frac{1}{6}\log\left(1+x^2\right) + \frac{1}{12}\log\left(1-x^2+x^4\right) + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^2\right)\\
&= -\frac{1}{4x^4} + \frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{6}\log\left(1+x^2\right) + \frac{1}{12}\log\left(1-x^2+x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 79, normalized size = 1.41

$$\frac{1}{12}\left(-\frac{3}{x^4} - 2\log\left(x^2+1\right) + \log\left(x^2-\sqrt{3}\,x+1\right) + \log\left(x^2+\sqrt{3}\,x+1\right) + 2\sqrt{3}\,\tan^{-1}\left(\sqrt{3}\,-2x\right) + 2\sqrt{3}\,\tan^{-1}\left(2\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 + x^6)),x]
```

```
[Out] (-3/x^4 + 2*Sqrt[3]*ArcTan[Sqrt[3] - 2*x] + 2*Sqrt[3]*ArcTan[Sqrt[3] + 2*x]
 - 2*Log[1 + x^2] + Log[1 - Sqrt[3]*x + x^2] + Log[1 + Sqrt[3]*x + x^2])/12
```

fricas [A] time = 0.87, size = 55, normalized size = 0.98

$$-\frac{2\sqrt{3}\,x^4\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right) - x^4\log\left(x^4-x^2+1\right) + 2x^4\log\left(x^2+1\right) + 3}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^6+1),x, algorithm="fricas")
```

```
[Out] -1/12*(2*sqrt(3)*x^4*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - x^4*log(x^4 - x^2 +
1) + 2*x^4*log(x^2 + 1) + 3)/x^4
```

giac [A] time = 0.17, size = 45, normalized size = 0.80

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)-\frac{1}{4x^4}+\frac{1}{12}\log\left(x^4-x^2+1\right)-\frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^6+1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 1/4/x^4 + 1/12*log(x^4 - x^2
 + 1) - 1/6*log(x^2 + 1)
```

maple [A] time = 0.01, size = 46, normalized size = 0.82

$$-\frac{\sqrt{3}\ \arctan\left(\frac{\left(2x^2-1\right)\sqrt{3}}{3}\right)}{6}-\frac{\ln\left(x^2+1\right)}{6}+\frac{\ln\left(x^4-x^2+1\right)}{12}-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^6+1),x)
```

```
[Out] -1/4/x^4-1/6*ln(x^2+1)+1/12*ln(x^4-x^2+1)-1/6*3^(1/2)*arctan(1/3*(2*x^2-1)*
3^(1/2))
```

maxima [A] time = 2.33, size = 45, normalized size = 0.80

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)-\frac{1}{4x^4}+\frac{1}{12}\log\left(x^4-x^2+1\right)-\frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^6+1),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 1/4/x^4 + 1/12*log(x^4 - x^2
 + 1) - 1/6*log(x^2 + 1)
```

mupad [B] time = 1.07, size = 57, normalized size = 1.02

$$-\frac{\ln\left(x^2+1\right)}{6}-\frac{1}{4x^4}+\ln\left(x^2-\frac{\sqrt{3}\,1\mathrm{i}}{2}-\frac{1}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\ln\left(x^2+\frac{\sqrt{3}\,1\mathrm{i}}{2}-\frac{1}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(x^6 + 1)),x)
```

```
[Out] log(x^2 - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12 + 1/12) - 1/(4*x^4) - log(
x^2 + 1)/6 - log((3^(1/2)*1i)/2 + x^2 - 1/2)*((3^(1/2)*1i)/12 - 1/12)
```

sympy [A] time = 0.20, size = 53, normalized size = 0.95

$$-\frac{\log\left(x^2+1\right)}{6}+\frac{\log\left(x^4-x^2+1\right)}{12}-\frac{\sqrt{3}\ \operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3}-\frac{\sqrt{3}}{3}\right)}{6}-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**6+1),x)
```

```
[Out] -log(x**2 + 1)/6 + log(x**4 - x**2 + 1)/12 - sqrt(3)*atan(2*sqrt(3)*x**2/3
 - sqrt(3)/3)/6 - 1/(4*x**4)
```

3.1375 $\int \frac{1}{x^6\left(1+x^6\right)}\,dx$

Optimal. Leaf size=87

$$-\frac{1}{5x^5}+\frac{\log\left(x^2-\sqrt{3}\,x+1\right)}{4\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}\,x+1\right)}{4\sqrt{3}}+\frac{1}{6}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{1}{3}\tan^{-1}(x)-\frac{1}{6}\tan^{-1}\left(2x+\sqrt{3}\right)$$

```
[Out] -1/5/x^5-1/3*arctan(x)-1/6*arctan(2*x-3^(1/2))-1/6*arctan(2*x+3^(1/2))+1/12
*ln(1+x^2-x*3^(1/2))*3^(1/2)-1/12*ln(1+x^2+x*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {325, 209, 634, 618, 204, 628, 203}

$$-\frac{1}{5x^5}+\frac{\log\left(x^2-\sqrt{3}\,x+1\right)}{4\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}\,x+1\right)}{4\sqrt{3}}+\frac{1}{6}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{1}{3}\tan^{-1}(x)-\frac{1}{6}\tan^{-1}\left(2x+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(1 + x^6)),x]
```

```
[Out] -1/(5*x^5) + ArcTan[Sqrt[3] - 2*x]/6 - ArcTan[x]/3 - ArcTan[Sqrt[3] + 2*x]/
6 + Log[1 - Sqrt[3]*x + x^2]/(4*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(4*Sqrt
[3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^6\left(1+x^6\right)}\,dx = -\frac{1}{5x^5} - \int \frac{1}{1+x^6}\,dx$$

$$= -\frac{1}{5x^5} - \frac{1}{3}\int \frac{1-\frac{\sqrt{3}\,x}{2}}{1-\sqrt{3}\,x+x^2}\,dx - \frac{1}{3}\int \frac{1+\frac{\sqrt{3}\,x}{2}}{1+\sqrt{3}\,x+x^2}\,dx - \frac{1}{3}\int \frac{1}{1+x^2}\,dx$$

$$= -\frac{1}{5x^5} - \frac{1}{3}\tan^{-1}(x) - \frac{1}{12}\int \frac{1}{1-\sqrt{3}\,x+x^2}\,dx - \frac{1}{12}\int \frac{1}{1+\sqrt{3}\,x+x^2}\,dx + \frac{\int \frac{-\sqrt{3}+2x}{1-\sqrt{3}\,x+x^2}\,dx}{4\sqrt{3}} - \frac{\int$$

$$= -\frac{1}{5x^5} - \frac{1}{3}\tan^{-1}(x) + \frac{\log\left(1-\sqrt{3}\,x+x^2\right)}{4\sqrt{3}} - \frac{\log\left(1+\sqrt{3}\,x+x^2\right)}{4\sqrt{3}} + \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x\right.$$

$$= -\frac{1}{5x^5} + \frac{1}{6}\tan^{-1}\left(\sqrt{3}-2x\right) - \frac{1}{3}\tan^{-1}(x) - \frac{1}{6}\tan^{-1}\left(\sqrt{3}+2x\right) + \frac{\log\left(1-\sqrt{3}\,x+x^2\right)}{4\sqrt{3}} - \frac{\log\left($$

Mathematica [A] time = 0.03, size = 79, normalized size = 0.91

$$\frac{1}{60}\left(-\frac{12}{x^5} + 5\sqrt{3}\,\log\left(x^2-\sqrt{3}\,x+1\right) - 5\sqrt{3}\,\log\left(x^2+\sqrt{3}\,x+1\right) + 10\tan^{-1}\left(\sqrt{3}-2x\right) - 20\tan^{-1}(x) - 10\tan^{-1}\left(2\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(1 + x^6)),x]
```

```
[Out] (-12/x^5 + 10*ArcTan[Sqrt[3] - 2*x] - 20*ArcTan[x] - 10*ArcTan[Sqrt[3] + 2*
x] + 5*Sqrt[3]*Log[1 - Sqrt[3]*x + x^2] - 5*Sqrt[3]*Log[1 + Sqrt[3]*x + x^2
])/60
```

fricas [A] time = 0.91, size = 110, normalized size = 1.26

$$-\frac{5\sqrt{3}\,x^5\log\left(x^2+\sqrt{3}\,x+1\right) - 5\sqrt{3}\,x^5\log\left(x^2-\sqrt{3}\,x+1\right) + 20\,x^5\arctan(x) - 20\,x^5\arctan\left(-2\,x+\sqrt{3}+2\sqrt{x^2}\right.}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^6+1),x, algorithm="fricas")
```

```
[Out] -1/60*(5*sqrt(3)*x^5*log(x^2 + sqrt(3)*x + 1) - 5*sqrt(3)*x^5*log(x^2 - sqr
t(3)*x + 1) + 20*x^5*arctan(x) - 20*x^5*arctan(-2*x + sqrt(3) + 2*sqrt(x^2
```

```
- sqrt(3)*x + 1)) - 20*x^5*arctan(-2*x - sqrt(3) + 2*sqrt(x^2 + sqrt(3)*x +
 1)) + 12)/x^5
```

giac [A] time = 0.21, size = 65, normalized size = 0.75

$$-\frac{1}{12}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)+\frac{1}{12}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)-\frac{1}{5x^5}-\frac{1}{6}\arctan\left(2x+\sqrt{3}\right)-\frac{1}{6}\arctan\left(2x-\sqrt{3}\right)-\frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^6+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/12*sqrt(3)*log(x^2 - sqrt(3)*x +
 1) - 1/5/x^5 - 1/6*arctan(2*x + sqrt(3)) - 1/6*arctan(2*x - sqrt(3)) - 1/3
*arctan(x)
```

maple [A] time = 0.02, size = 66, normalized size = 0.76

$$-\frac{\arctan(x)}{3}-\frac{\arctan\left(2x-\sqrt{3}\right)}{6}-\frac{\arctan\left(2x+\sqrt{3}\right)}{6}+\frac{\sqrt{3}\ \ln\left(x^2-\sqrt{3}\ x+1\right)}{12}-\frac{\sqrt{3}\ \ln\left(x^2+\sqrt{3}\ x+1\right)}{12}-\frac{1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^6+1),x)
```

```
[Out] -1/5/x^5-1/3*arctan(x)-1/6*arctan(2*x-3^(1/2))-1/6*arctan(2*x+3^(1/2))+1/12
*3^(1/2)*ln(x^2-3^(1/2)*x+1)-1/12*3^(1/2)*ln(x^2+3^(1/2)*x+1)
```

maxima [A] time = 2.35, size = 65, normalized size = 0.75

$$-\frac{1}{12}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)+\frac{1}{12}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)-\frac{1}{5x^5}-\frac{1}{6}\arctan\left(2x+\sqrt{3}\right)-\frac{1}{6}\arctan\left(2x-\sqrt{3}\right)-\frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^6+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/12*sqrt(3)*log(x^2 - sqrt(3)*x +
 1) - 1/5/x^5 - 1/6*arctan(2*x + sqrt(3)) - 1/6*arctan(2*x - sqrt(3)) - 1/3
*arctan(x)
```

mupad [B] time = 0.05, size = 92, normalized size = 1.06

$$-\frac{\operatorname{atan}(x)}{3}-\operatorname{atan}\left(\frac{x}{-1+\sqrt{3}\,1\mathrm{i}}+\frac{\sqrt{3}\,x\,1\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\operatorname{atan}\left(\frac{x}{1+\sqrt{3}\,1\mathrm{i}}-\frac{\sqrt{3}\,x\,1\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\frac{1}{5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(x^6 + 1)),x)
```

```
[Out] - atan(x)/3 - atan(x/(3^(1/2)*1i - 1) + (3^(1/2)*x*1i)/(3^(1/2)*1i - 1))*((
3^(1/2)*1i)/6 + 1/6) - atan(x/(3^(1/2)*1i + 1) - (3^(1/2)*x*1i)/(3^(1/2)*1i
 + 1))*((3^(1/2)*1i)/6 - 1/6) - 1/(5*x^5)
```

sympy [A] time = 0.48, size = 75, normalized size = 0.86

$$\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{12}-\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{12}-\frac{\operatorname{atan}(x)}{3}-\frac{\operatorname{atan}\left(2x-\sqrt{3}\right)}{6}-\frac{\operatorname{atan}\left(2x+\sqrt{3}\right)}{6}-\frac{1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**6+1),x)
```

```
[Out] sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 - sqrt(3)*log(x**2 + sqrt(3)*x + 1)/12
 - atan(x)/3 - atan(2*x - sqrt(3))/6 - atan(2*x + sqrt(3))/6 - 1/(5*x**5)
```

3.1376 $\int \frac{1}{x^7\left(1+x^6\right)}\,dx$

Optimal. Leaf size=22

$$-\frac{1}{6x^6}+\frac{1}{6}\log\left(x^6+1\right)-\log(x)$$

```
[Out] -1/6/x^6-ln(x)+1/6*ln(x^6+1)
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {266, 44}

$$-\frac{1}{6x^6}+\frac{1}{6}\log\left(x^6+1\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(1 + x^6)),x]
```

```
[Out] -1/(6*x^6) - Log[x] + Log[1 + x^6]/6
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7\left(1+x^6\right)}\,dx &= \frac{1}{6}\text{Subst}\left(\int \frac{1}{x^2(1+x)}\,dx, x, x^6\right)\\ &= \frac{1}{6}\text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{1}{x}+\frac{1}{1+x}\right)dx, x, x^6\right)\\ &= -\frac{1}{6x^6}-\log(x)+\frac{1}{6}\log\left(1+x^6\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$-\frac{1}{6x^6}+\frac{1}{6}\log\left(x^6+1\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(1 + x^6)),x]
```

```
[Out] -1/6*1/x^6 - Log[x] + Log[1 + x^6]/6
```

fricas [A] time = 0.87, size = 24, normalized size = 1.09

$$\frac{x^6\log\left(x^6+1\right)-6\,x^6\log(x)-1}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^6+1),x, algorithm="fricas")
[Out] 1/6*(x^6*log(x^6 + 1) - 6*x^6*log(x) - 1)/x^6
```

giac [A] time = 0.15, size = 25, normalized size = 1.14

$$\frac{x^6-1}{6\,x^6}+\frac{1}{6}\,\log\left(x^6+1\right)-\frac{1}{6}\,\log\left(x^6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^6+1),x, algorithm="giac")
[Out] 1/6*(x^6 - 1)/x^6 + 1/6*log(x^6 + 1) - 1/6*log(x^6)
```

maple [A] time = 0.01, size = 32, normalized size = 1.45

$$-\ln(x)+\frac{\ln\left(x^2+1\right)}{6}+\frac{\ln\left(x^4-x^2+1\right)}{6}-\frac{1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^6+1),x)
[Out] 1/6*ln(x^2+1)+1/6*ln(x^4-x^2+1)-1/6/x^6-ln(x)
```

maxima [A] time = 0.98, size = 20, normalized size = 0.91

$$-\frac{1}{6\,x^6}+\frac{1}{6}\,\log\left(x^6+1\right)-\frac{1}{6}\,\log\left(x^6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^6+1),x, algorithm="maxima")
[Out] -1/6/x^6 + 1/6*log(x^6 + 1) - 1/6*log(x^6)
```

mupad [B] time = 0.04, size = 18, normalized size = 0.82

$$\frac{\ln\left(x^6+1\right)}{6}-\ln(x)-\frac{1}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(x^6 + 1)),x)
[Out] log(x^6 + 1)/6 - log(x) - 1/(6*x^6)
```

sympy [A] time = 0.22, size = 17, normalized size = 0.77

$$-\log(x)+\frac{\log\left(x^6+1\right)}{6}-\frac{1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**6+1),x)
[Out] -log(x) + log(x**6 + 1)/6 - 1/(6*x**6)
```

3.1377 $\int \frac{1}{x^8\left(1+x^6\right)}\,dx$

Optimal. Leaf size=90

$$-\frac{1}{7x^7}+\frac{\log\left(x^2-\sqrt{3}\,x+1\right)}{4\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}\,x+1\right)}{4\sqrt{3}}+\frac{1}{x}-\frac{1}{6}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{1}{3}\tan^{-1}(x)+\frac{1}{6}\tan^{-1}\left(2x+\sqrt{3}\right)$$

```
[Out] -1/7/x^7+1/x+1/3*arctan(x)+1/6*arctan(2*x-3^(1/2))+1/6*arctan(2*x+3^(1/2))+
1/12*ln(1+x^2-x*3^(1/2))*3^(1/2)-1/12*ln(1+x^2+x*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {325, 295, 634, 618, 204, 628, 203}

$$-\frac{1}{7x^7}+\frac{\log\left(x^2-\sqrt{3}\,x+1\right)}{4\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}\,x+1\right)}{4\sqrt{3}}+\frac{1}{x}-\frac{1}{6}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{1}{3}\tan^{-1}(x)+\frac{1}{6}\tan^{-1}\left(2x+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(1 + x^6)),x]
```

```
[Out] -1/(7*x^7) + x^(-1) - ArcTan[Sqrt[3] - 2*x]/6 + ArcTan[x]/3 + ArcTan[Sqrt[3
] + 2*x]/6 + Log[1 - Sqrt[3]*x + x^2]/(4*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2
]/(4*Sqrt[3])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^8\left(1+x^6\right)}\,dx &= -\frac{1}{7x^7} - \int \frac{1}{x^2\left(1+x^6\right)}\,dx \\
&= -\frac{1}{7x^7} + \frac{1}{x} + \int \frac{x^4}{1+x^6}\,dx \\
&= -\frac{1}{7x^7} + \frac{1}{x} + \frac{1}{3}\int \frac{-\frac{1}{2}+\frac{\sqrt{3}x}{2}}{1-\sqrt{3}\,x+x^2}\,dx + \frac{1}{3}\int \frac{-\frac{1}{2}-\frac{\sqrt{3}x}{2}}{1+\sqrt{3}\,x+x^2}\,dx + \frac{1}{3}\int \frac{1}{1+x^2}\,dx \\
&= -\frac{1}{7x^7} + \frac{1}{x} + \frac{1}{3}\tan^{-1}(x) + \frac{1}{12}\int \frac{1}{1-\sqrt{3}\,x+x^2}\,dx + \frac{1}{12}\int \frac{1}{1+\sqrt{3}\,x+x^2}\,dx + \frac{\int \frac{-\sqrt{3}+2x}{1-\sqrt{3}\,x+x^2}}{4\sqrt{3}} \\
&= -\frac{1}{7x^7} + \frac{1}{x} + \frac{1}{3}\tan^{-1}(x) + \frac{\log\left(1-\sqrt{3}\,x+x^2\right)}{4\sqrt{3}} - \frac{\log\left(1+\sqrt{3}\,x+x^2\right)}{4\sqrt{3}} - \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{-1-} \right. \\
&= -\frac{1}{7x^7} + \frac{1}{x} - \frac{1}{6}\tan^{-1}\left(\sqrt{3}-2x\right) + \frac{1}{3}\tan^{-1}(x) + \frac{1}{6}\tan^{-1}\left(\sqrt{3}+2x\right) + \frac{\log\left(1-\sqrt{3}\,x+x^2\right)}{4\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 84, normalized size = 0.93

$$\frac{1}{84}\left(-\frac{12}{x^7} + 7\sqrt{3}\,\log\left(x^2-\sqrt{3}\,x+1\right) - 7\sqrt{3}\,\log\left(x^2+\sqrt{3}\,x+1\right) + \frac{84}{x} - 14\tan^{-1}\left(\sqrt{3}-2x\right) + 28\tan^{-1}(x) + 1\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(1 + x^6)),x]
```

```
[Out] (-12/x^7 + 84/x - 14*ArcTan[Sqrt[3] - 2*x] + 28*ArcTan[x] + 14*ArcTan[Sqrt[
3] + 2*x] + 7*Sqrt[3]*Log[1 - Sqrt[3]*x + x^2] - 7*Sqrt[3]*Log[1 + Sqrt[3]*
x + x^2])/84
```

fricas [A] time = 0.91, size = 115, normalized size = 1.28

$$-\frac{7\sqrt{3}\,x^7\log\left(x^2+\sqrt{3}\,x+1\right) - 7\sqrt{3}\,x^7\log\left(x^2-\sqrt{3}\,x+1\right) - 28\,x^7\arctan(x) + 28\,x^7\arctan\left(-2\,x+\sqrt{3}+2\right.}{84\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^6+1),x, algorithm="fricas")
```

```
[Out] -1/84*(7*sqrt(3)*x^7*log(x^2 + sqrt(3)*x + 1) - 7*sqrt(3)*x^7*log(x^2 - sqr
t(3)*x + 1) - 28*x^7*arctan(x) + 28*x^7*arctan(-2*x + sqrt(3) + 2*sqrt(x^2
- sqrt(3)*x + 1)) + 28*x^7*arctan(-2*x - sqrt(3) + 2*sqrt(x^2 + sqrt(3)*x +
 1)) - 84*x^6 + 12)/x^7
```

giac [A] time = 0.16, size = 39, normalized size = 0.43

$$\frac{7x^6-1}{7x^7}+\frac{1}{3}\arctan\left(2x+\sqrt{3}\right)+\frac{1}{3}\arctan\left(2x-\sqrt{3}\right)+\frac{1}{3}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^6+1),x, algorithm="giac")
```

```
[Out] 1/7*(7*x^6 - 1)/x^7 + 1/3*arctan(2*x + sqrt(3)) + 1/3*arctan(2*x - sqrt(3))
 + 1/3*arctan(x)
```

maple [A] time = 0.02, size = 69, normalized size = 0.77

$$\frac{\arctan(x)}{3}+\frac{\arctan\left(2x-\sqrt{3}\right)}{6}+\frac{\arctan\left(2x+\sqrt{3}\right)}{6}+\frac{\sqrt{3}\,\ln\left(x^2-\sqrt{3}\,x+1\right)}{12}-\frac{\sqrt{3}\,\ln\left(x^2+\sqrt{3}\,x+1\right)}{12}+\frac{1}{x}-\frac{1}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(x^6+1),x)
```

```
[Out] -1/7/x^7+1/x+1/3*arctan(x)+1/6*arctan(2*x-3^(1/2))+1/6*arctan(2*x+3^(1/2))+
1/12*3^(1/2)*ln(x^2-3^(1/2)*x+1)-1/12*3^(1/2)*ln(x^2+3^(1/2)*x+1)
```

maxima [A] time = 2.39, size = 72, normalized size = 0.80

$$-\frac{1}{12}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)+\frac{1}{12}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)+\frac{7x^6-1}{7x^7}+\frac{1}{6}\arctan\left(2x+\sqrt{3}\right)+\frac{1}{6}\arctan\left(2x-\sqrt{3}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^6+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/12*sqrt(3)*log(x^2 - sqrt(3)*x +
 1) + 1/7*(7*x^6 - 1)/x^7 + 1/6*arctan(2*x + sqrt(3)) + 1/6*arctan(2*x - sq
rt(3)) + 1/3*arctan(x)
```

mupad [B] time = 1.05, size = 62, normalized size = 0.69

$$\frac{\operatorname{atan}(x)}{3}-\operatorname{atan}\left(\frac{2x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\operatorname{atan}\left(\frac{2x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\frac{x^6-\frac{1}{7}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(x^6 + 1)),x)
```

```
[Out] atan(x)/3 - atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/6 + 1/6) - atan((2*x
)/(3^(1/2)*1i + 1))*((3^(1/2)*1i)/6 - 1/6) + (x^6 - 1/7)/x^7
```

sympy [A] time = 0.46, size = 80, normalized size = 0.89

$$\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{12}-\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{12}+\frac{\operatorname{atan}(x)}{3}+\frac{\operatorname{atan}\left(2x-\sqrt{3}\right)}{6}+\frac{\operatorname{atan}\left(2x+\sqrt{3}\right)}{6}+\frac{7x^6-1}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(x**6+1),x)
```

```
[Out] sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 - sqrt(3)*log(x**2 + sqrt(3)*x + 1)/12
 + atan(x)/3 + atan(2*x - sqrt(3))/6 + atan(2*x + sqrt(3))/6 + (7*x**6 - 1)
/(7*x**7)
```

3.1378 $\int \frac{1}{2-3x^6}\,dx$

Optimal. Leaf size=180

$$-\frac{\log\left(\sqrt[3]{3}\,x^2-\sqrt[6]{6}\,x+\sqrt[3]{2}\right)}{12\,2^{5/6}\sqrt[6]{3}}+\frac{\log\left(\sqrt[3]{3}\,x^2+\sqrt[6]{6}\,x+\sqrt[3]{2}\right)}{12\,2^{5/6}\sqrt[6]{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[6]{6}-2\sqrt[3]{3}\,x}{\sqrt[6]{2}\,3^{2/3}}\right)}{2\,2^{5/6}3^{2/3}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{3}\,x+\sqrt[6]{6}}{\sqrt[6]{2}\,3^{2/3}}\right)}{2\,2^{5/6}3^{2/3}}+\frac{\tanh^{-1}\left(\sqrt[6]{\frac{3}{2}}\,x\right)}{3\,2^{5/6}\sqrt[6]{3}}$$

```
[Out] 1/12*arctan(1/3*2^(5/6)*x*3^(2/3)-1/3*3^(1/2))*2^(1/6)*3^(1/3)+1/12*arctan(
1/3*2^(5/6)*x*3^(2/3)+1/3*3^(1/2))*2^(1/6)*3^(1/3)+1/18*arctanh(1/2*3^(1/6)
*2^(5/6)*x)*3^(5/6)*2^(1/6)-1/72*ln(2^(1/3)-6^(1/6)*x+3^(1/3)*x^2)*3^(5/6)*
2^(1/6)+1/72*ln(2^(1/3)+6^(1/6)*x+3^(1/3)*x^2)*3^(5/6)*2^(1/6)
```

Rubi [A] time = 0.28, antiderivative size = 167, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {210, 634, 618, 204, 628, 206}

$$-\frac{\log\left(\sqrt[3]{3}\,x^2-\sqrt[6]{6}\,x+\sqrt[3]{2}\right)}{12\,2^{5/6}\sqrt[6]{3}}+\frac{\log\left(\sqrt[3]{3}\,x^2+\sqrt[6]{6}\,x+\sqrt[3]{2}\right)}{12\,2^{5/6}\sqrt[6]{3}}-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{5/6}x}{\sqrt[3]{3}}\right)}{2\,2^{5/6}3^{2/3}}+\frac{\tan^{-1}\left(\frac{2^{5/6}x}{\sqrt[3]{3}}+\frac{1}{\sqrt{3}}\right)}{2\,2^{5/6}3^{2/3}}+\frac{\tanh^{-1}\left(\sqrt[6]{\frac{3}{2}}\,x\right)}{3\,2^{5/6}\sqrt[6]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 3*x^6)^(-1),x]
```

```
[Out] -ArcTan[1/Sqrt[3] - (2^(5/6)*x)/3^(1/3)]/(2*2^(5/6)*3^(2/3)) + ArcTan[1/Sqr
t[3] + (2^(5/6)*x)/3^(1/3)]/(2*2^(5/6)*3^(2/3)) + ArcTanh[(3/2)^(1/6)*x]/(3
*2^(5/6)*3^(1/6)) - Log[2^(1/3) - 6^(1/6)*x + 3^(1/3)*x^2]/(12*2^(5/6)*3^(1
/6)) + Log[2^(1/3) + 6^(1/6)*x + 3^(1/3)*x^2]/(12*2^(5/6)*3^(1/6))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{2-3x^6}\,dx = \frac{\int \frac{\sqrt[6]{2}-\frac{\sqrt[6]{3}x}{2}}{\sqrt[3]{2}-\sqrt[6]{6}x+\sqrt[3]{3}x^2}\,dx}{3\,2^{5/6}} + \frac{\int \frac{\sqrt[6]{2}+\frac{\sqrt[6]{3}x}{2}}{\sqrt[3]{2}+\sqrt[6]{6}x+\sqrt[3]{3}x^2}\,dx}{3\,2^{5/6}} + \frac{\int \frac{1}{\sqrt[3]{2}-\sqrt[3]{3}x^2}\,dx}{3\,2^{2/3}}$$

$$= \frac{\tanh^{-1}\left(\sqrt[6]{\frac{3}{2}}x\right)}{3\,2^{5/6}\sqrt[6]{3}} + \frac{\int \frac{1}{\sqrt[3]{2}-\sqrt[6]{6}x+\sqrt[3]{3}x^2}\,dx}{4\,2^{2/3}} + \frac{\int \frac{1}{\sqrt[3]{2}+\sqrt[6]{6}x+\sqrt[3]{3}x^2}\,dx}{4\,2^{2/3}} - \frac{\int \frac{-\sqrt[6]{6}+2\sqrt[3]{3}x}{\sqrt[3]{2}-\sqrt[6]{6}x+\sqrt[3]{3}x^2}\,dx}{12\,2^{5/6}\sqrt[6]{3}} + \frac{\int \frac{\sqrt[6]{6}}{\sqrt[3]{2}+\sqrt[6]{6}}}{12}$$

$$= \frac{\tanh^{-1}\left(\sqrt[6]{\frac{3}{2}}x\right)}{3\,2^{5/6}\sqrt[6]{3}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[6]{6}x+\sqrt[3]{3}x^2\right)}{12\,2^{5/6}\sqrt[6]{3}} + \frac{\log\left(\sqrt[3]{2}+\sqrt[6]{6}x+\sqrt[3]{3}x^2\right)}{12\,2^{5/6}\sqrt[6]{3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\right.}{2\,2^{5}}$$

$$= -\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{5/6}x}{\sqrt[3]{3}}\right)}{2\,2^{5/6}3^{2/3}} + \frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2^{5/6}x}{\sqrt[3]{3}}\right)}{2\,2^{5/6}3^{2/3}} + \frac{\tanh^{-1}\left(\sqrt[6]{\frac{3}{2}}x\right)}{3\,2^{5/6}\sqrt[6]{3}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[6]{6}x+\sqrt[3]{3}x^2\right)}{12\,2^{5/6}\sqrt[6]{3}} + -$$

Mathematica [A] time = 0.08, size = 162, normalized size = 0.90

$$\frac{\sqrt{3}\left(-\log\left(2^{2/3}\sqrt[3]{3}x^2-2^{5/6}\sqrt[6]{3}x+2\right)+\log\left(2^{2/3}\sqrt[3]{3}x^2+2^{5/6}\sqrt[6]{3}x+2\right)-2\log\left(2-2^{5/6}\sqrt[6]{3}x\right)+2\log\left(2^{5/6}\sqrt[6]{3}\right.\right.}{12\,2^{5/6}3^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 3*x^6)^(-1),x]
```

```
[Out] (6*ArcTan[1/Sqrt[3] + (2^(5/6)*x)/3^(1/3)] + 6*ArcTan[(-1 + 2^(5/6)*3^(1/6)
*x)/Sqrt[3]] + Sqrt[3]*(-2*Log[2 - 2^(5/6)*3^(1/6)*x] + 2*Log[2 + 2^(5/6)*3
^(1/6)*x] - Log[2 - 2^(5/6)*3^(1/6)*x + 2^(2/3)*3^(1/3)*x^2] + Log[2 + 2^(5
/6)*3^(1/6)*x + 2^(2/3)*3^(1/3)*x^2]))/(12*2^(5/6)*3^(2/3))
```

fricas [A] time = 0.96, size = 169, normalized size = 0.94

$$-\frac{1}{288}\cdot 96^{\frac{5}{6}}\sqrt{3}\,\arctan\left(-\frac{1}{3}\cdot 96^{\frac{1}{6}}\sqrt{3}x+\frac{1}{12}\cdot 96^{\frac{1}{6}}\sqrt{48x^2+96^{\frac{5}{6}}x+8\cdot 12^{\frac{2}{3}}}-\frac{1}{3}\sqrt{3}\right)-\frac{1}{288}\cdot 96^{\frac{5}{6}}\sqrt{3}\,\arctan\left(-\frac{1}{3}\cdot 9\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^6+2),x, algorithm="fricas")
```

```
[Out] -1/288*96^(5/6)*sqrt(3)*arctan(-1/3*96^(1/6)*sqrt(3)*x + 1/12*96^(1/6)*sqrt
(48*x^2 + 96^(5/6)*x + 8*12^(2/3)) - 1/3*sqrt(3)) - 1/288*96^(5/6)*sqrt(3)*
arctan(-1/3*96^(1/6)*sqrt(3)*x + 1/12*96^(1/6)*sqrt(48*x^2 - 96^(5/6)*x + 8
*12^(2/3)) + 1/3*sqrt(3)) + 1/1152*96^(5/6)*log(48*x^2 + 96^(5/6)*x + 8*12^
(2/3)) - 1/1152*96^(5/6)*log(48*x^2 - 96^(5/6)*x + 8*12^(2/3)) + 1/576*96^(
5/6)*log(48*x + 96^(5/6)) - 1/576*96^(5/6)*log(48*x - 96^(5/6))
```

giac [A] time = 0.17, size = 114, normalized size = 0.63

$$\frac{1}{12}\sqrt{3}\left(\frac{2}{3}\right)^{\frac{1}{6}}\arctan\left(\frac{1}{2}\sqrt{3}\left(\frac{2}{3}\right)^{\frac{5}{6}}\left(2x+\left(\frac{2}{3}\right)^{\frac{1}{6}}\right)\right)+\frac{1}{12}\sqrt{3}\left(\frac{2}{3}\right)^{\frac{1}{6}}\arctan\left(\frac{1}{2}\sqrt{3}\left(\frac{2}{3}\right)^{\frac{5}{6}}\left(2x-\left(\frac{2}{3}\right)^{\frac{1}{6}}\right)\right)+\frac{1}{72}\cdot 486^{\frac{1}{6}}\log\left(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^6+2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*(2/3)^(1/6)*arctan(1/2*sqrt(3)*(2/3)^(5/6)*(2*x + (2/3)^(1/6))
) + 1/12*sqrt(3)*(2/3)^(1/6)*arctan(1/2*sqrt(3)*(2/3)^(5/6)*(2*x - (2/3)^(1
/6))) + 1/72*486^(1/6)*log(x^2 + (2/3)^(1/6)*x + (2/3)^(1/3)) - 1/72*486^(1
/6)*log(x^2 - (2/3)^(1/6)*x + (2/3)^(1/3)) + 1/36*486^(1/6)*log(abs(x + (2/
3)^(1/6))) - 1/36*486^(1/6)*log(abs(x - (2/3)^(1/6)))
```

maple [A] time = 0.22, size = 228, normalized size = 1.27

$$-\frac{2^{\frac{1}{6}}3^{\frac{1}{3}}\arctan\left(\frac{\sqrt{2}\,12^{\frac{2}{3}}x}{6}-\frac{\sqrt{2}\,\sqrt{6}}{6}\right)}{36}+\frac{2^{\frac{5}{6}}3^{\frac{2}{3}}12^{\frac{2}{3}}\arctan\left(\frac{\sqrt{2}\,12^{\frac{2}{3}}x}{6}-\frac{\sqrt{2}\,\sqrt{6}}{6}\right)}{108}-\frac{2^{\frac{1}{6}}3^{\frac{1}{3}}\arctan\left(\frac{\sqrt{2}\,12^{\frac{2}{3}}x}{6}+\frac{\sqrt{2}\,\sqrt{6}}{6}\right)}{36}+\frac{2^{\frac{5}{6}}3^{\frac{2}{3}}1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^6+2),x)
```

```
[Out] -1/72*6^(1/2)*3^(1/3)*2^(2/3)*ln(-6^(1/2)*3^(1/3)*2^(2/3)+6*x)-1/144*2^(2/3
)*3^(1/3)*6^(1/2)*ln(-x*6^(1/2)*12^(1/3)+12^(2/3)+6*x^2)-1/36*2^(1/6)*3^(1/
3)*arctan(-1/6*2^(1/2)*6^(1/2)+1/6*2^(1/2)*12^(2/3)*x)+1/108*2^(5/6)*3^(2/3
)*12^(2/3)*arctan(-1/6*2^(1/2)*6^(1/2)+1/6*2^(1/2)*12^(2/3)*x)+1/72*6^(1/2)
*3^(1/3)*2^(2/3)*ln(6^(1/2)*3^(1/3)*2^(2/3)+6*x)+1/144*2^(2/3)*3^(1/3)*6^(1
/2)*ln(x*6^(1/2)*12^(1/3)+12^(2/3)+6*x^2)-1/36*2^(1/6)*3^(1/3)*arctan(1/6*2
^(1/2)*6^(1/2)+1/6*2^(1/2)*12^(2/3)*x)+1/108*2^(5/6)*3^(2/3)*12^(2/3)*arcta
n(1/6*2^(1/2)*6^(1/2)+1/6*2^(1/2)*12^(2/3)*x)
```

maxima [A] time = 2.34, size = 224, normalized size = 1.24

$$\frac{1}{12}\cdot 3^{\frac{2}{3}}2^{\frac{1}{6}}\left(\frac{1}{3}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{2}\cdot 3^{\frac{1}{3}}2^{\frac{5}{6}}\left(\frac{1}{3}\right)^{\frac{2}{3}}\left(2x+\left(\frac{1}{3}\right)^{\frac{1}{3}}\left(\sqrt{3}\,\sqrt{2}\right)^{\frac{1}{3}}\right)\right)+\frac{1}{12}\cdot 3^{\frac{2}{3}}2^{\frac{1}{6}}\left(\frac{1}{3}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{2}\cdot 3^{\frac{1}{3}}2^{\frac{5}{6}}\left(\frac{1}{3}\right)^{\frac{2}{3}}\left(2x-\left(\frac{1}{3}\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^6+2),x, algorithm="maxima")
```

```
[Out] 1/12*3^(2/3)*2^(1/6)*(1/3)^(1/3)*arctan(1/2*3^(1/3)*2^(5/6)*(1/3)^(2/3)*(2*
x + (1/3)^(1/3)*(sqrt(3)*sqrt(2))^(1/3))) + 1/12*3^(2/3)*2^(1/6)*(1/3)^(1/3
)*arctan(1/2*3^(1/3)*2^(5/6)*(1/3)^(2/3)*(2*x - (1/3)^(1/3)*(sqrt(3)*sqrt(2
))^(1/3))) + 1/24*3^(1/6)*2^(1/6)*(1/3)^(1/3)*log(x^2 + (1/3)^(1/3)*(sqrt(3
)*sqrt(2))^(1/3)*x + (1/3)^(2/3)*(sqrt(3)*sqrt(2))^(2/3)) - 1/24*3^(1/6)*2^
(1/6)*(1/3)^(1/3)*log(x^2 - (1/3)^(1/3)*(sqrt(3)*sqrt(2))^(1/3)*x + (1/3)^(
2/3)*(sqrt(3)*sqrt(2))^(2/3)) + 1/12*3^(1/6)*2^(1/6)*(1/3)^(1/3)*log(x + (1
/3)^(1/3)*(sqrt(3)*sqrt(2))^(1/3)) - 1/12*3^(1/6)*2^(1/6)*(1/3)^(1/3)*log(x
 - (1/3)^(1/3)*(sqrt(3)*sqrt(2))^(1/3))
```

mupad [B] time = 1.15, size = 164, normalized size = 0.91

$$\frac{486^{1/6}\operatorname{atanh}\left(\frac{486^{5/6}x}{162}\right)}{18}-\frac{2^{1/6}\operatorname{atanh}\left(\frac{2^{1/6}3^{1/3}x\,1\mathrm{i}}{162\left(\frac{2^{1/3}3^{2/3}}{486}-\frac{2^{1/3}3^{1/6}\,1\mathrm{i}}{162}\right)}+\frac{2^{1/6}3^{5/6}x}{486\left(\frac{2^{1/3}3^{2/3}}{486}-\frac{2^{1/3}3^{1/6}\,1\mathrm{i}}{162}\right)}\right)\left(3^{5/6}+3^{1/3}\,3\mathrm{i}\right)}{36}-\frac{2^{1/6}\operatorname{atan}\left(\frac{}{162\left(\frac{2}{}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(3*x^6 - 2),x)
```

```
[Out] (486^(1/6)*atanh((486^(5/6)*x)/162))/18 - (2^(1/6)*atanh((2^(1/6)*3^(1/3)*x
*1i)/(162*((2^(1/3)*3^(2/3))/486 - (2^(1/3)*3^(1/6)*1i)/162)) + (2^(1/6)*3^
(5/6)*x)/(486*((2^(1/3)*3^(2/3))/486 - (2^(1/3)*3^(1/6)*1i)/162)))*(3^(1/3)
*3i + 3^(5/6)))/36 - (2^(1/6)*atan((2^(1/6)*3^(1/3)*x)/(162*((2^(1/3)*3^(2/
3))/486 + (2^(1/3)*3^(1/6)*1i)/162)) + (2^(1/6)*3^(5/6)*x*1i)/(486*((2^(1/3
)*3^(2/3))/486 + (2^(1/3)*3^(1/6)*1i)/162)))*(3^(1/3)*3i - 3^(5/6))*1i)/36
```

sympy [**A**] time = 0.95, size = 15, normalized size = 0.08

$$-\operatorname{RootSum}\left(4478976t^6-1,\left(t\mapsto t\log\left(-12t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**6+2),x)
```

```
[Out] -RootSum(4478976*_t**6 - 1, Lambda(_t, _t*log(-12*_t + x)))
```

3.1379 $\int \frac{\sqrt[3]{x}}{1-x^6}\,dx$

Optimal. Leaf size=244

$$-\frac{1}{6}\log\left(1-x^{2/3}\right)+\frac{1}{12}\log\left(x^{4/3}+x^{2/3}+1\right)-\frac{\tan^{-1}\left(\frac{2x^{2/3}+1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{6}\cos\left(\frac{2\pi}{9}\right)\log\left(x^{4/3}+2x^{2/3}\cos\left(\frac{\pi}{9}\right)+1\right)+\frac{1}{6}\cos\left(\frac{\pi}{9}\right.$$

```
[Out] 1/3*arctan((x^(2/3)-cos(2/9*Pi))*csc(2/9*Pi))*cos(1/18*Pi)-1/6*ln(1-x^(2/3)
)+1/12*ln(1+x^(2/3)+x^(4/3))-1/6*cos(2/9*Pi)*ln(1+x^(4/3)+2*x^(2/3)*cos(1/9
*Pi))+1/6*cos(1/9*Pi)*ln(1+x^(4/3)-2*x^(2/3)*sin(1/18*Pi))-1/6*ln(1+x^(4/3)
-2*x^(2/3)*cos(2/9*Pi))*sin(1/18*Pi)+1/3*arctan(sec(1/18*Pi)*(x^(2/3)-sin(1
/18*Pi)))*sin(1/9*Pi)-1/3*arctan((x^(2/3)+cos(1/9*Pi))*csc(1/9*Pi))*sin(2/9
*Pi)-1/6*arctan(1/3*(1+2*x^(2/3))*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {329, 275, 294, 634, 618, 204, 628, 31}

$$-\frac{1}{6}\log\left(1-x^{2/3}\right)+\frac{1}{12}\log\left(x^{4/3}+x^{2/3}+1\right)-\frac{\tan^{-1}\left(\frac{2x^{2/3}+1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{6}\cos\left(\frac{2\pi}{9}\right)\log\left(x^{4/3}+2x^{2/3}\cos\left(\frac{\pi}{9}\right)+1\right)+\frac{1}{6}\cos\left(\frac{\pi}{9}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3)/(1 - x^6),x]
```

```
[Out] -ArcTan[(1 + 2*x^(2/3))/Sqrt[3]]/(2*Sqrt[3]) + (ArcTan[(x^(2/3) - Cos[(2*Pi
)/9])*Csc[(2*Pi)/9]]*Cos[Pi/18])/3 - Log[1 - x^(2/3)]/6 + Log[1 + x^(2/3) +
 x^(4/3)]/12 - (Cos[(2*Pi)/9]*Log[1 + x^(4/3) + 2*x^(2/3)*Cos[Pi/9]])/6 + (
Cos[Pi/9]*Log[1 + x^(4/3) - 2*x^(2/3)*Sin[Pi/18]])/6 - (Log[1 + x^(4/3) - 2
*x^(2/3)*Cos[(2*Pi)/9]]*Sin[Pi/18])/6 + (ArcTan[Sec[Pi/18]*(x^(2/3) - Sin[P
i/18])]*Sin[Pi/9])/3 - (ArcTan[(x^(2/3) + Cos[Pi/9])*Csc[Pi/9]]*Sin[(2*Pi)/
9])/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 294

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[
((2*k - 1)*Pi)/n]*x + s^2*x^2), x]; (r^(m + 1)*Int[1/(r - s*x), x])/(a*n*s^
m) - Dist[(2*(-r)^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 1)/2}], x], x]] /;
 FreeQ[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && N
```

```
egQ[a/b]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{x}}{1-x^6}\,dx &= 3\,\text{Subst}\left(\int \frac{x^3}{1-x^{18}}\,dx, x, \sqrt[3]{x}\right) \\
&= \frac{3}{2}\,\text{Subst}\left(\int \frac{x}{1-x^9}\,dx, x, x^{2/3}\right) \\
&= \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, x^{2/3}\right) - \frac{1}{3}\,\text{Subst}\left(\int \frac{\frac{1}{2}-\frac{x}{2}}{1+x+x^2}\,dx, x, x^{2/3}\right) - \frac{1}{3}\,\text{Subst}\left(\int \frac{\cos\left(\frac{\pi}{9}\right)+x\,c}{1+x^2+2x\,c}\right. \\
&= -\frac{1}{6}\log\left(1-x^{2/3}\right) + \frac{1}{12}\,\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, x^{2/3}\right) - \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^{2/3}\right) + \\
&= -\frac{1}{6}\log\left(1-x^{2/3}\right) + \frac{1}{12}\log\left(1+x^{2/3}+x^{4/3}\right) - \frac{1}{6}\cos\left(\frac{2\pi}{9}\right)\log\left(1+x^{4/3}+2x^{2/3}\cos\left(\frac{\pi}{9}\right)\right) + \frac{1}{6}\,c \\
&= -\frac{\tan^{-1}\left(\frac{1+2x^{2/3}}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{6}\log\left(1-x^{2/3}\right) + \frac{1}{12}\log\left(1+x^{2/3}+x^{4/3}\right) - \frac{1}{6}\cos\left(\frac{2\pi}{9}\right)\log\left(1+x^{4/3}+2x^{2/}\right.
\end{aligned}$$

Mathematica [C] time = 0.01, size = 20, normalized size = 0.08

$$\frac{3}{4}x^{4/3}\,{}_2F_1\left(\frac{2}{9},1;\frac{11}{9};x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3)/(1 - x^6),x]
```

```
[Out] (3*x^(4/3)*Hypergeometric2F1[2/9, 1, 11/9, x^6])/4
```

fricas [B] time = 1.06, size = 555, normalized size = 2.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(-x^6+1),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(3)*cos(1/9*pi) - sin(1/9*pi))*arctan(-(8*(2*cos(1/9*pi)^3 - cos(1
/9*pi))*sin(1/9*pi) + 2*(2*sqrt(3)*cos(1/9*pi)^2 + 2*cos(1/9*pi)*sin(1/9*pi
) - sqrt(3))*x^(2/3) - 2*(2*sqrt(3)*cos(1/9*pi)^2 + 2*cos(1/9*pi)*sin(1/9*p
i) - sqrt(3))*sqrt((2*sqrt(3)*cos(1/9*pi)*sin(1/9*pi) + 2*cos(1/9*pi)^2 - 1
)*x^(2/3) + x^(4/3) + 1) + sqrt(3))/(16*cos(1/9*pi)^4 - 16*cos(1/9*pi)^2 +
3)) - 1/3*(sqrt(3)*cos(1/9*pi) + sin(1/9*pi))*arctan(1/16*(16*cos(1/9*pi)^2
 - 8*x^(2/3) + sqrt(-128*(2*cos(1/9*pi)^2 - 1)*x^(2/3) + 64*x^(4/3) + 64) -
 8)/(cos(1/9*pi)*sin(1/9*pi))) - 1/12*(sqrt(3)*sin(1/9*pi) + cos(1/9*pi))*l
og(64*(2*sqrt(3)*cos(1/9*pi)*sin(1/9*pi) + 2*cos(1/9*pi)^2 - 1)*x^(2/3) + 6
4*x^(4/3) + 64) + 1/6*cos(1/9*pi)*log(-16*(2*sqrt(3)*cos(1/9*pi)*sin(1/9*pi
) - 2*cos(1/9*pi)^2 + 1)*x^(2/3) + 16*x^(4/3) + 16) + 1/12*(sqrt(3)*sin(1/9
*pi) - cos(1/9*pi))*log(-128*(2*cos(1/9*pi)^2 - 1)*x^(2/3) + 64*x^(4/3) + 6
4) + 2/3*arctan(-1/2*(16*(2*cos(1/9*pi)^3 - cos(1/9*pi))*sin(1/9*pi) - 4*(2
*sqrt(3)*cos(1/9*pi)^2 - 2*cos(1/9*pi)*sin(1/9*pi) - sqrt(3))*x^(2/3) + (2*
sqrt(3)*cos(1/9*pi)^2 - 2*cos(1/9*pi)*sin(1/9*pi) - sqrt(3))*sqrt(-16*(2*sq
rt(3)*cos(1/9*pi)*sin(1/9*pi) - 2*cos(1/9*pi)^2 + 1)*x^(2/3) + 16*x^(4/3) +
 16) - 2*sqrt(3))/(16*cos(1/9*pi)^4 - 16*cos(1/9*pi)^2 + 3))*sin(1/9*pi) -
1/6*sqrt(3)*arctan(2/3*sqrt(3)*x^(2/3) + 1/3*sqrt(3)) + 1/12*log(x^(4/3) +
x^(2/3) + 1) - 1/6*log(x^(2/3) - 1)
```

giac [A] time = 3.59, size = 217, normalized size = 0.89

$$\frac{2}{3}\arctan\left(\frac{x^{\frac{2}{3}}-\cos\left(\frac{4}{9}\pi\right)}{\sin\left(\frac{4}{9}\pi\right)}\right)\cos\left(\frac{4}{9}\pi\right)\sin\left(\frac{4}{9}\pi\right)+\frac{2}{3}\arctan\left(\frac{x^{\frac{2}{3}}-\cos\left(\frac{2}{9}\pi\right)}{\sin\left(\frac{2}{9}\pi\right)}\right)\cos\left(\frac{2}{9}\pi\right)\sin\left(\frac{2}{9}\pi\right)-\frac{2}{3}\arctan\left(\frac{x^{\frac{2}{3}}}{s}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(-x^6+1),x, algorithm="giac")
```

```
[Out] 2/3*arctan((x^(2/3) - cos(4/9*pi))/sin(4/9*pi))*cos(4/9*pi)*sin(4/9*pi) + 2
/3*arctan((x^(2/3) - cos(2/9*pi))/sin(2/9*pi))*cos(2/9*pi)*sin(2/9*pi) - 2/
3*arctan((x^(2/3) + cos(1/9*pi))/sin(1/9*pi))*cos(1/9*pi)*sin(1/9*pi) - 1/6
*(cos(4/9*pi)^2 - sin(4/9*pi)^2)*log(-2*x^(2/3)*cos(4/9*pi) + x^(4/3) + 1)
- 1/6*(cos(2/9*pi)^2 - sin(2/9*pi)^2)*log(-2*x^(2/3)*cos(2/9*pi) + x^(4/3)
+ 1) - 1/6*(cos(1/9*pi)^2 - sin(1/9*pi)^2)*log(2*x^(2/3)*cos(1/9*pi) + x^(4
/3) + 1) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(2/3) + 1)) + 1/12*log(x^(4/
3) + x^(2/3) + 1) - 1/6*log(abs(x^(2/3) - 1))
```

maple [C] time = 0.04, size = 162, normalized size = 0.66

$$-\frac{\sqrt{3}\arctan\left(\frac{\left(2x^{\frac{1}{3}}-1\right)\sqrt{3}}{3}\right)}{6}+\frac{\sqrt{3}\arctan\left(\frac{\left(2x^{\frac{1}{3}}+1\right)\sqrt{3}}{3}\right)}{6}-\frac{\ln\left(x^{\frac{1}{3}}+1\right)}{6}-\frac{\ln\left(x^{\frac{1}{3}}-1\right)}{6}+\frac{\ln\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}+1\right)}{12}+\frac{\ln\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}+\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)/(-x^6+1),x)
```

```
[Out] -1/6*ln(x^(1/3)-1)-1/6*sum((-_R^3+1)/(2*_R^5+_R^2)*ln(x^(1/3)-_R),_R=RootOf
(_Z^6+_Z^3+1))+1/12*ln(x^(2/3)+x^(1/3)+1)+1/6*3^(1/2)*arctan(1/3*(2*x^(1/3)
+1)*3^(1/2))+1/12*ln(x^(2/3)-x^(1/3)+1)-1/6*3^(1/2)*arctan(1/3*(2*x^(1/3)-1
)*3^(1/2))+1/6*sum((_R^3+1)/(2*_R^5-_R^2)*ln(x^(1/3)-_R),_R=RootOf(_Z^6-_Z^
3+1))-1/6*ln(1+x^(1/3))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{3}}+1\right)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{3}}-1\right)\right)+\int\frac{x^{\frac{4}{3}}+2x^{\frac{1}{3}}}{6\left(x^2+x+1\right)}\,dx-\int\frac{x^{\frac{4}{3}}-2x^{\frac{1}{3}}}{6\left(x^2-x+1\right)}\,dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(-x^6+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + 1)) - 1/6*sqrt(3)*arctan(1/3*sq
rt(3)*(2*x^(1/3) - 1)) + integrate(1/6*(x^(4/3) + 2*x^(1/3))/(x^2 + x + 1),
 x) - integrate(1/6*(x^(4/3) - 2*x^(1/3))/(x^2 - x + 1), x) + 1/12*log(x^(2
/3) + x^(1/3) + 1) + 1/12*log(x^(2/3) - x^(1/3) + 1) - 1/6*log(x^(1/3) + 1)
 - 1/6*log(x^(1/3) - 1)
```

mupad [B] time = 1.10, size = 186, normalized size = 0.76

$$-\frac{\ln\left(43046721\,x^{2/3}-43046721\right)}{6}+\frac{\ln\left(43046721\,x^{2/3}\,e^{\frac{\pi 4i}{9}}-43046721\right)e^{\frac{\pi 1i}{9}}}{6}+\frac{\ln\left(43046721\,x^{2/3}\,e^{\frac{\pi 2i}{9}}-43046\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^(1/3)/(x^6 - 1),x)
```

```
[Out] (log(43046721*x^(2/3)*exp((pi*4i)/9) - 43046721)*exp((pi*1i)/9))/6 - log(43
046721*x^(2/3) - 43046721)/6 + (log(43046721*x^(2/3)*exp((pi*2i)/9) - 43046
721)*exp((pi*5i)/9))/6 + (log(- 43046721*x^(2/3)*exp((pi*1i)/9) - 43046721)
*exp((pi*7i)/9))/6 - (log(43046721*x^(2/3)*exp((pi*8i)/9) - 43046721)*exp((
pi*2i)/9))/6 - (log(- 43046721*x^(2/3)*exp((pi*7i)/9) - 43046721)*exp((pi*4
i)/9))/6 - (log(- 43046721*x^(2/3)*exp((pi*5i)/9) - 43046721)*exp((pi*8i)/9
))/6 - log(55788550416*x^(2/3)*((3^(1/2)*1i)/12 - 1/12)^4 - 43046721)*((3^(
1/2)*1i)/12 - 1/12) + log(55788550416*x^(2/3)*((3^(1/2)*1i)/12 + 1/12)^4 -
43046721)*((3^(1/2)*1i)/12 + 1/12)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3)/(-x**6+1),x)
```

```
[Out] Timed out
```

3.1380 $\int x^8\sqrt{-1+4x^6}\,dx$

Optimal. Leaf size=58

$$\frac{1}{12}\sqrt{4x^6-1}\,x^9-\frac{1}{96}\sqrt{4x^6-1}\,x^3-\frac{1}{192}\tanh^{-1}\left(\frac{2x^3}{\sqrt{4x^6-1}}\right)$$

```
[Out] -1/192*arctanh(2*x^3/(4*x^6-1)^(1/2))-1/96*x^3*(4*x^6-1)^(1/2)+1/12*x^9*(4*
x^6-1)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {275, 279, 321, 217, 206}

$$\frac{1}{12}\sqrt{4x^6-1}\,x^9-\frac{1}{96}\sqrt{4x^6-1}\,x^3-\frac{1}{192}\tanh^{-1}\left(\frac{2x^3}{\sqrt{4x^6-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^8*Sqrt[-1 + 4*x^6],x]
```

```
[Out] -(x^3*Sqrt[-1 + 4*x^6])/96 + (x^9*Sqrt[-1 + 4*x^6])/12 - ArcTanh[(2*x^3)/Sq
rt[-1 + 4*x^6]]/192
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int x^8\sqrt{-1+4x^6}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int x^2\sqrt{-1+4x^2}\,dx, x, x^3\right)\\ &= \frac{1}{12}x^9\sqrt{-1+4x^6} - \frac{1}{12}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{-1+4x^2}}\,dx, x, x^3\right)\\ &= -\frac{1}{96}x^3\sqrt{-1+4x^6} + \frac{1}{12}x^9\sqrt{-1+4x^6} - \frac{1}{96}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+4x^2}}\,dx, x, x^3\right)\\ &= -\frac{1}{96}x^3\sqrt{-1+4x^6} + \frac{1}{12}x^9\sqrt{-1+4x^6} - \frac{1}{96}\,\text{Subst}\left(\int \frac{1}{1-4x^2}\,dx, x, \frac{x^3}{\sqrt{-1+4x^6}}\right)\\ &= -\frac{1}{96}x^3\sqrt{-1+4x^6} + \frac{1}{12}x^9\sqrt{-1+4x^6} - \frac{1}{192}\tanh^{-1}\left(\frac{2x^3}{\sqrt{-1+4x^6}}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 56, normalized size = 0.97

$$\frac{\left(4x^6-1\right)\left(\sin^{-1}\left(2x^3\right)+2\sqrt{1-4x^6}\left(8x^6-1\right)x^3\right)}{192\sqrt{-\left(1-4x^6\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*Sqrt[-1 + 4*x^6],x]
```

```
[Out] ((-1 + 4*x^6)*(2*x^3*Sqrt[1 - 4*x^6]*(-1 + 8*x^6) + ArcSin[2*x^3]))/(192*Sq
rt[-(1 - 4*x^6)^2])
```

fricas [A] time = 0.57, size = 41, normalized size = 0.71

$$\frac{1}{96}\left(8\,x^9 - x^3\right)\sqrt{4\,x^6-1} + \frac{1}{192}\,\log\left(-2\,x^3 + \sqrt{4\,x^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(4*x^6-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/96*(8*x^9 - x^3)*sqrt(4*x^6 - 1) + 1/192*log(-2*x^3 + sqrt(4*x^6 - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{4\,x^6-1}\,x^8\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(4*x^6-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(4*x^6 - 1)*x^8, x)
```

maple [C] time = 0.16, size = 53, normalized size = 0.91

$$\frac{\left(8x^6-1\right)\sqrt{4x^6-1}\,x^3}{96} - \frac{\sqrt{-\text{signum}\left(4x^6-1\right)}\,\arcsin\left(2x^3\right)}{192\sqrt{\text{signum}\left(4x^6-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(4*x^6-1)^(1/2),x)
```

```
[Out] 1/96*x^3*(8*x^6-1)*(4*x^6-1)^(1/2)-1/192/signum(4*x^6-1)^(1/2)*(-signum(4*x
^6-1))^(1/2)*arcsin(2*x^3)
```

maxima [B] time = 1.06, size = 97, normalized size = 1.67

$$-\frac{\frac{4\sqrt{4x^6-1}}{x^3}+\frac{\left(4x^6-1\right)^{\frac{3}{2}}}{x^9}}{96\left(\frac{8\left(4x^6-1\right)}{x^6}-\frac{\left(4x^6-1\right)^2}{x^{12}}-16\right)}-\frac{1}{384}\log\left(\frac{\sqrt{4x^6-1}}{x^3}+2\right)+\frac{1}{384}\log\left(\frac{\sqrt{4x^6-1}}{x^3}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(4*x^6-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/96*(4*sqrt(4*x^6 - 1)/x^3 + (4*x^6 - 1)^(3/2)/x^9)/(8*(4*x^6 - 1)/x^6 -
(4*x^6 - 1)^2/x^12 - 16) - 1/384*log(sqrt(4*x^6 - 1)/x^3 + 2) + 1/384*log(s
qrt(4*x^6 - 1)/x^3 - 2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^8\sqrt{4x^6-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(4*x^6 - 1)^(1/2),x)
```

```
[Out] int(x^8*(4*x^6 - 1)^(1/2), x)
```

sympy [A] time = 5.61, size = 119, normalized size = 2.05

$$\begin{cases}\frac{x^{15}}{3\sqrt{4x^6-1}}-\frac{x^9}{8\sqrt{4x^6-1}}+\frac{x^3}{96\sqrt{4x^6-1}}-\frac{\operatorname{acosh}\left(2x^3\right)}{192} & \text{for } 4\left|x^6\right|>1\\ -\frac{ix^{15}}{3\sqrt{1-4x^6}}+\frac{ix^9}{8\sqrt{1-4x^6}}-\frac{ix^3}{96\sqrt{1-4x^6}}+\frac{i\operatorname{asin}\left(2x^3\right)}{192} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(4*x**6-1)**(1/2),x)
```

```
[Out] Piecewise((x**15/(3*sqrt(4*x**6 - 1)) - x**9/(8*sqrt(4*x**6 - 1)) + x**3/(9
6*sqrt(4*x**6 - 1)) - acosh(2*x**3)/192, 4*Abs(x**6) > 1), (-I*x**15/(3*sqr
t(1 - 4*x**6)) + I*x**9/(8*sqrt(1 - 4*x**6)) - I*x**3/(96*sqrt(1 - 4*x**6))
 + I*asin(2*x**3)/192, True))
```

3.1381 $\int x^5\sqrt{a^6-x^6}\,dx$

Optimal. Leaf size=17

$$-\frac{1}{9}\left(a^6-x^6\right)^{3/2}$$

```
[Out] -1/9*(a^6-x^6)^(3/2)
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {261}

$$-\frac{1}{9}\left(a^6-x^6\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[a^6 - x^6],x]
```

```
[Out] -(a^6 - x^6)^(3/2)/9
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^5\sqrt{a^6-x^6}\,dx = -\frac{1}{9}\left(a^6-x^6\right)^{3/2}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$-\frac{1}{9}\left(a^6-x^6\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[a^6 - x^6],x]
```

```
[Out] -1/9*(a^6 - x^6)^(3/2)
```

fricas [A] time = 0.58, size = 13, normalized size = 0.76

$$-\frac{1}{9}\left(a^6-x^6\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a^6-x^6)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/9*(a^6 - x^6)^(3/2)
```

giac [A] time = 0.15, size = 13, normalized size = 0.76

$$-\frac{1}{9}\left(a^6-x^6\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a^6-x^6)^(1/2),x, algorithm="giac")
[Out] -1/9*(a^6 - x^6)^(3/2)
```

maple [B] time = 0.01, size = 43, normalized size = 2.53

$$-\frac{(a-x)(a+x)\left(a^2+ax+x^2\right)\left(a^2-ax+x^2\right)\sqrt{a^6-x^6}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a^6-x^6)^(1/2),x)
[Out] -1/9*(a-x)*(a+x)*(a^2+a*x+x^2)*(a^2-a*x+x^2)*(a^6-x^6)^(1/2)
```

maxima [A] time = 1.14, size = 13, normalized size = 0.76

$$-\frac{1}{9}\left(a^6-x^6\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a^6-x^6)^(1/2),x, algorithm="maxima")
[Out] -1/9*(a^6 - x^6)^(3/2)
```

mupad [B] time = 1.23, size = 13, normalized size = 0.76

$$-\frac{\left(a^6-x^6\right)^{3/2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a^6 - x^6)^(1/2),x)
[Out] -(a^6 - x^6)^(3/2)/9
```

sympy [B] time = 0.42, size = 29, normalized size = 1.71

$$-\frac{a^6\sqrt{a^6-x^6}}{9}+\frac{x^6\sqrt{a^6-x^6}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a**6-x**6)**(1/2),x)
[Out] -a**6*sqrt(a**6 - x**6)/9 + x**6*sqrt(a**6 - x**6)/9
```

3.1382 $\int x^2\sqrt{-2+x^6}\,dx$

Optimal. Leaf size=35

$$\frac{1}{6}x^3\sqrt{x^6-2}-\frac{1}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-2}}\right)$$

```
[Out] -1/3*arctanh(x^3/(x^6-2)^(1/2))+1/6*x^3*(x^6-2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {275, 195, 217, 206}

$$\frac{1}{6}x^3\sqrt{x^6-2}-\frac{1}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{x^6-2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[-2 + x^6],x]
```

```
[Out] (x^3*Sqrt[-2 + x^6])/6 - ArcTanh[x^3/Sqrt[-2 + x^6]]/3
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int x^2\sqrt{-2+x^6}\,dx &= \frac{1}{3}\text{Subst}\left(\int \sqrt{-2+x^2}\,dx, x, x^3\right)\\
&= \frac{1}{6}x^3\sqrt{-2+x^6}-\frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{-2+x^2}}\,dx, x, x^3\right)\\
&= \frac{1}{6}x^3\sqrt{-2+x^6}-\frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x^3}{\sqrt{-2+x^6}}\right)\\
&= \frac{1}{6}x^3\sqrt{-2+x^6}-\frac{1}{3}\tanh^{-1}\left(\frac{x^3}{\sqrt{-2+x^6}}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 1.43

$$\frac{\left(x^6-2\right)\left(2\sin^{-1}\left(\frac{x^3}{\sqrt{2}}\right)+\sqrt{2-x^6}\,x^3\right)}{6\sqrt{-\left(x^6-2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[-2 + x^6],x]
```

```
[Out] ((-2 + x^6)*(x^3*Sqrt[2 - x^6] + 2*ArcSin[x^3/Sqrt[2]]))/(6*Sqrt[-(-2 + x^6
)^2])
```

fricas [A] time = 0.91, size = 29, normalized size = 0.83

$$\frac{1}{6}\sqrt{x^6-2}\,x^3+\frac{1}{3}\log\left(-x^3+\sqrt{x^6-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^6-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(x^6 - 2)*x^3 + 1/3*log(-x^3 + sqrt(x^6 - 2))
```

giac [A] time = 0.18, size = 30, normalized size = 0.86

$$\frac{1}{6}\sqrt{x^6-2}\,x^3+\frac{1}{3}\log\left(\left|-x^3+\sqrt{x^6-2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^6-2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(x^6 - 2)*x^3 + 1/3*log(abs(-x^3 + sqrt(x^6 - 2)))
```

maple [C] time = 0.16, size = 47, normalized size = 1.34

$$\frac{\sqrt{x^6-2}\;x^3}{6}-\frac{\sqrt{-\operatorname{signum}\left(\frac{x^6}{2}-1\right)}\;\arcsin\left(\frac{\sqrt{2}\,x^3}{2}\right)}{3\sqrt{\operatorname{signum}\left(\frac{x^6}{2}-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^6-2)^(1/2),x)
```

```
[Out] 1/6*x^3*(x^6-2)^(1/2)-1/3/signum(-1+1/2*x^6)^(1/2)*(-signum(-1+1/2*x^6))^(1
/2)*arcsin(1/2*x^3*2^(1/2))
```

maxima [B] time = 0.99, size = 58, normalized size = 1.66

$$-\frac{\sqrt{x^6-2}}{3\,x^3\left(\frac{x^6-2}{x^6}-1\right)}-\frac{1}{6}\log\left(\frac{\sqrt{x^6-2}}{x^3}+1\right)+\frac{1}{6}\log\left(\frac{\sqrt{x^6-2}}{x^3}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^6-2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(x^6 - 2)/(x^3*((x^6 - 2)/x^6 - 1)) - 1/6*log(sqrt(x^6 - 2)/x^3 +
1) + 1/6*log(sqrt(x^6 - 2)/x^3 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x^2 \sqrt{x^6 - 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^6 - 2)^(1/2),x)
```

```
[Out] int(x^2*(x^6 - 2)^(1/2), x)
```

sympy [A] time = 3.14, size = 90, normalized size = 2.57

$$\begin{cases} \frac{x^9}{6\sqrt{x^6-2}} - \frac{x^3}{3\sqrt{x^6-2}} - \frac{\operatorname{acosh}\left(\frac{\sqrt{2}x^3}{2}\right)}{3} & \text{for } \frac{|x^6|}{2} > 1 \\ -\frac{ix^9}{6\sqrt{2-x^6}} + \frac{ix^3}{3\sqrt{2-x^6}} + \frac{i\operatorname{asin}\left(\frac{\sqrt{2}x^3}{2}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**6-2)**(1/2),x)
```

```
[Out] Piecewise((x**9/(6*sqrt(x**6 - 2)) - x**3/(3*sqrt(x**6 - 2)) - acosh(sqrt(2
)*x**3/2)/3, Abs(x**6)/2 > 1), (-I*x**9/(6*sqrt(2 - x**6)) + I*x**3/(3*sqrt
(2 - x**6)) + I*asin(sqrt(2)*x**3/2)/3, True))
```

3.1383 $\int \frac{x^{23}}{\sqrt{2+x^6}}\,dx$

Optimal. Leaf size=53

$$\frac{1}{21}\left(x^6+2\right)^{7/2}-\frac{2}{5}\left(x^6+2\right)^{5/2}+\frac{4}{3}\left(x^6+2\right)^{3/2}-\frac{8\sqrt{x^6+2}}{3}$$

```
[Out] 4/3*(x^6+2)^(3/2)-2/5*(x^6+2)^(5/2)+1/21*(x^6+2)^(7/2)-8/3*(x^6+2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{1}{21}\left(x^6+2\right)^{7/2}-\frac{2}{5}\left(x^6+2\right)^{5/2}+\frac{4}{3}\left(x^6+2\right)^{3/2}-\frac{8\sqrt{x^6+2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^23/Sqrt[2 + x^6],x]
```

```
[Out] (-8*Sqrt[2 + x^6])/3 + (4*(2 + x^6)^(3/2))/3 - (2*(2 + x^6)^(5/2))/5 + (2 +
 x^6)^(7/2)/21
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{23}}{\sqrt{2+x^6}}\,dx &= \frac{1}{6}\,\text{Subst}\left(\int \frac{x^3}{\sqrt{2+x}}\,dx, x, x^6\right)\\ &= \frac{1}{6}\,\text{Subst}\left(\int \left(-\frac{8}{\sqrt{2+x}}+12\sqrt{2+x}-6(2+x)^{3/2}+(2+x)^{5/2}\right)dx, x, x^6\right)\\ &= -\frac{8}{3}\sqrt{2+x^6}+\frac{4}{3}\left(2+x^6\right)^{3/2}-\frac{2}{5}\left(2+x^6\right)^{5/2}+\frac{1}{21}\left(2+x^6\right)^{7/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.57

$$\frac{1}{105}\sqrt{x^6+2}\left(5x^{18}-12x^{12}+32x^6-128\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^23/Sqrt[2 + x^6],x]
```

```
[Out] (Sqrt[2 + x^6]*(-128 + 32*x^6 - 12*x^12 + 5*x^18))/105
```

fricas [A] time = 0.55, size = 26, normalized size = 0.49

$$\frac{1}{105}\left(5\,x^{18}-12\,x^{12}+32\,x^{6}-128\right)\sqrt{x^{6}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(x^6+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(5*x^18 - 12*x^12 + 32*x^6 - 128)*sqrt(x^6 + 2)
```

giac [A] time = 0.15, size = 37, normalized size = 0.70

$$\frac{1}{21}\left(x^{6}+2\right)^{\frac{7}{2}}-\frac{2}{5}\left(x^{6}+2\right)^{\frac{5}{2}}+\frac{4}{3}\left(x^{6}+2\right)^{\frac{3}{2}}-\frac{8}{3}\sqrt{x^{6}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(x^6+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/21*(x^6 + 2)^(7/2) - 2/5*(x^6 + 2)^(5/2) + 4/3*(x^6 + 2)^(3/2) - 8/3*sqrt
(x^6 + 2)
```

maple [A] time = 0.01, size = 27, normalized size = 0.51

$$\frac{\sqrt{x^{6}+2}\,\left(5x^{18}-12x^{12}+32x^{6}-128\right)}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^23/(x^6+2)^(1/2),x)
```

```
[Out] 1/105*(x^6+2)^(1/2)*(5*x^18-12*x^12+32*x^6-128)
```

maxima [A] time = 1.01, size = 37, normalized size = 0.70

$$\frac{1}{21}\left(x^{6}+2\right)^{\frac{7}{2}}-\frac{2}{5}\left(x^{6}+2\right)^{\frac{5}{2}}+\frac{4}{3}\left(x^{6}+2\right)^{\frac{3}{2}}-\frac{8}{3}\sqrt{x^{6}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(x^6+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/21*(x^6 + 2)^(7/2) - 2/5*(x^6 + 2)^(5/2) + 4/3*(x^6 + 2)^(3/2) - 8/3*sqrt
(x^6 + 2)
```

mupad [B] time = 1.15, size = 25, normalized size = 0.47

$$\sqrt{x^{6}+2}\,\left(\frac{x^{18}}{21}-\frac{4\,x^{12}}{35}+\frac{32\,x^{6}}{105}-\frac{128}{105}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^23/(x^6 + 2)^(1/2),x)
```

```
[Out] (x^6 + 2)^(1/2)*((32*x^6)/105 - (4*x^12)/35 + x^18/21 - 128/105)
```

sympy [A] time = 24.03, size = 54, normalized size = 1.02

$$\frac{x^{18}\sqrt{x^{6}+2}}{21}-\frac{4x^{12}\sqrt{x^{6}+2}}{35}+\frac{32x^{6}\sqrt{x^{6}+2}}{105}-\frac{128\sqrt{x^{6}+2}}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**23/(x**6+2)**(1/2),x)
```

```
[Out] x**18*sqrt(x**6 + 2)/21 - 4*x**12*sqrt(x**6 + 2)/35 + 32*x**6*sqrt(x**6 + 2
)/105 - 128*sqrt(x**6 + 2)/105
```

3.1384 $\int \frac{x^{17}}{\sqrt{2+x^6}}\,dx$

Optimal. Leaf size=40

$$\frac{1}{15}\left(x^6+2\right)^{5/2}-\frac{4}{9}\left(x^6+2\right)^{3/2}+\frac{4\sqrt{x^6+2}}{3}$$

```
[Out] -4/9*(x^6+2)^(3/2)+1/15*(x^6+2)^(5/2)+4/3*(x^6+2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{1}{15}\left(x^6+2\right)^{5/2}-\frac{4}{9}\left(x^6+2\right)^{3/2}+\frac{4\sqrt{x^6+2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^17/Sqrt[2 + x^6],x]
```

```
[Out] (4*Sqrt[2 + x^6])/3 - (4*(2 + x^6)^(3/2))/9 + (2 + x^6)^(5/2)/15
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{17}}{\sqrt{2+x^6}}\,dx &= \frac{1}{6}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{2+x}}\,dx, x, x^6\right)\\ &= \frac{1}{6}\,\text{Subst}\left(\int \left(\frac{4}{\sqrt{2+x}}-4\sqrt{2+x}+(2+x)^{3/2}\right)dx, x, x^6\right)\\ &= \frac{4\sqrt{2+x^6}}{3}-\frac{4}{9}\left(2+x^6\right)^{3/2}+\frac{1}{15}\left(2+x^6\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.62

$$\frac{1}{45}\sqrt{x^6+2}\left(3x^{12}-8x^6+32\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^17/Sqrt[2 + x^6],x]
```

```
[Out] (Sqrt[2 + x^6]*(32 - 8*x^6 + 3*x^12))/45
```

fricas [A] time = 0.58, size = 21, normalized size = 0.52

$$\frac{1}{45}\left(3\,x^{12}-8\,x^6+32\right)\sqrt{x^6+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(x^6+2)^(1/2),x, algorithm="fricas")
[Out] 1/45*(3*x^12 - 8*x^6 + 32)*sqrt(x^6 + 2)
```

giac [A] time = 0.15, size = 28, normalized size = 0.70

$$\frac{1}{15}\left(x^6+2\right)^{\frac{5}{2}}-\frac{4}{9}\left(x^6+2\right)^{\frac{3}{2}}+\frac{4}{3}\sqrt{x^6+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(x^6+2)^(1/2),x, algorithm="giac")
[Out] 1/15*(x^6 + 2)^(5/2) - 4/9*(x^6 + 2)^(3/2) + 4/3*sqrt(x^6 + 2)
```

maple [A] time = 0.00, size = 22, normalized size = 0.55

$$\frac{\sqrt{x^6+2}\,\left(3x^{12}-8x^6+32\right)}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^17/(x^6+2)^(1/2),x)
[Out] 1/45*(x^6+2)^(1/2)*(3*x^12-8*x^6+32)
```

maxima [A] time = 0.98, size = 28, normalized size = 0.70

$$\frac{1}{15}\left(x^6+2\right)^{\frac{5}{2}}-\frac{4}{9}\left(x^6+2\right)^{\frac{3}{2}}+\frac{4}{3}\sqrt{x^6+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(x^6+2)^(1/2),x, algorithm="maxima")
[Out] 1/15*(x^6 + 2)^(5/2) - 4/9*(x^6 + 2)^(3/2) + 4/3*sqrt(x^6 + 2)
```

mupad [B] time = 1.15, size = 20, normalized size = 0.50

$$\sqrt{x^6+2}\,\left(\frac{x^{12}}{15}-\frac{8\,x^6}{45}+\frac{32}{45}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^17/(x^6 + 2)^(1/2),x)
[Out] (x^6 + 2)^(1/2)*(x^12/15 - (8*x^6)/45 + 32/45)
```

sympy [A] time = 9.14, size = 39, normalized size = 0.98

$$\frac{x^{12}\sqrt{x^6+2}}{15}-\frac{8x^6\sqrt{x^6+2}}{45}+\frac{32\sqrt{x^6+2}}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**17/(x**6+2)**(1/2),x)
[Out] x**12*sqrt(x**6 + 2)/15 - 8*x**6*sqrt(x**6 + 2)/45 + 32*sqrt(x**6 + 2)/45
```

3.1385 $\int \frac{x^{11}}{\sqrt{2+x^6}}\,dx$

Optimal. Leaf size=27

$$\frac{1}{9}\left(x^6+2\right)^{3/2}-\frac{2\sqrt{x^6+2}}{3}$$

```
[Out] 1/9*(x^6+2)^(3/2)-2/3*(x^6+2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{1}{9}\left(x^6+2\right)^{3/2}-\frac{2\sqrt{x^6+2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/Sqrt[2 + x^6],x]
```

```
[Out] (-2*Sqrt[2 + x^6])/3 + (2 + x^6)^(3/2)/9
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{\sqrt{2+x^6}}\,dx &= \frac{1}{6}\,\text{Subst}\left(\int \frac{x}{\sqrt{2+x}}\,dx, x, x^6\right)\\ &= \frac{1}{6}\,\text{Subst}\left(\int \left(-\frac{2}{\sqrt{2+x}}+\sqrt{2+x}\right)\,dx, x, x^6\right)\\ &= -\frac{2}{3}\sqrt{2+x^6}+\frac{1}{9}\left(2+x^6\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 0.67

$$\frac{1}{9}\left(x^6-4\right)\sqrt{x^6+2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/Sqrt[2 + x^6],x]
```

```
[Out] ((-4 + x^6)*Sqrt[2 + x^6])/9
```

fricas [A] time = 0.81, size = 14, normalized size = 0.52

$$\frac{1}{9}\sqrt{x^6+2}\left(x^6-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^6+2)^(1/2),x, algorithm="fricas")
[Out] 1/9*sqrt(x^6 + 2)*(x^6 - 4)
```

giac [A] time = 0.15, size = 19, normalized size = 0.70

$$\frac{1}{9}\left(x^6+2\right)^{\frac{3}{2}}-\frac{2}{3}\sqrt{x^6+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^6+2)^(1/2),x, algorithm="giac")
[Out] 1/9*(x^6 + 2)^(3/2) - 2/3*sqrt(x^6 + 2)
```

maple [A] time = 0.00, size = 15, normalized size = 0.56

$$\frac{\sqrt{x^6+2}\,\left(x^6-4\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^6+2)^(1/2),x)
[Out] 1/9*(x^6+2)^(1/2)*(x^6-4)
```

maxima [A] time = 0.99, size = 19, normalized size = 0.70

$$\frac{1}{9}\left(x^6+2\right)^{\frac{3}{2}}-\frac{2}{3}\sqrt{x^6+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^6+2)^(1/2),x, algorithm="maxima")
[Out] 1/9*(x^6 + 2)^(3/2) - 2/3*sqrt(x^6 + 2)
```

mupad [B] time = 1.16, size = 14, normalized size = 0.52

$$\frac{\sqrt{x^6+2}\,\left(x^6-4\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^6 + 2)^(1/2),x)
[Out] ((x^6 + 2)^(1/2)*(x^6 - 4))/9
```

sympy [A] time = 2.38, size = 24, normalized size = 0.89

$$\frac{x^6\sqrt{x^6+2}}{9}-\frac{4\sqrt{x^6+2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(x**6+2)**(1/2),x)
[Out] x**6*sqrt(x**6 + 2)/9 - 4*sqrt(x**6 + 2)/9
```

3.1386 $\int \frac{x^5}{\sqrt{2+x^6}}\,dx$

Optimal. Leaf size=13

$$\frac{\sqrt{x^6+2}}{3}$$

```
[Out] 1/3*(x^6+2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$\frac{\sqrt{x^6+2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[2 + x^6],x]
```

```
[Out] Sqrt[2 + x^6]/3
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^5}{\sqrt{2+x^6}}\,dx = \frac{\sqrt{2+x^6}}{3}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{\sqrt{x^6+2}}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[2 + x^6],x]
```

```
[Out] Sqrt[2 + x^6]/3
```

fricas [A] time = 0.69, size = 9, normalized size = 0.69

$$\frac{1}{3}\sqrt{x^6+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(x^6 + 2)
```

giac [A] time = 0.15, size = 9, normalized size = 0.69

$$\frac{1}{3}\sqrt{x^6+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+2)^(1/2),x, algorithm="giac")
[Out] 1/3*sqrt(x^6 + 2)
```

maple [A] time = 0.00, size = 10, normalized size = 0.77

$$\frac{\sqrt{x^6+2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^6+2)^(1/2),x)
[Out] 1/3*(x^6+2)^(1/2)
```

maxima [A] time = 1.11, size = 9, normalized size = 0.69

$$\frac{1}{3}\sqrt{x^6+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+2)^(1/2),x, algorithm="maxima")
[Out] 1/3*sqrt(x^6 + 2)
```

mupad [B] time = 1.15, size = 9, normalized size = 0.69

$$\frac{\sqrt{x^6+2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^6 + 2)^(1/2),x)
[Out] (x^6 + 2)^(1/2)/3
```

sympy [A] time = 0.53, size = 8, normalized size = 0.62

$$\frac{\sqrt{x^6+2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**6+2)**(1/2),x)
[Out] sqrt(x**6 + 2)/3
```

3.1387 $\int \frac{1}{x\sqrt{2+x^6}}\,dx$

Optimal. Leaf size=25

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x^6+2}}{\sqrt{2}}\right)}{3\sqrt{2}}$$

```
[Out] -1/6*arctanh(1/2*(x^6+2)^(1/2)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {266, 63, 207}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x^6+2}}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[2 + x^6]),x]
```

```
[Out] -ArcTanh[Sqrt[2 + x^6]/Sqrt[2]]/(3*Sqrt[2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{2+x^6}}\,dx &= \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{x\sqrt{2+x}}\,dx, x, x^6\right) \\
&= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{-2+x^2}\,dx, x, \sqrt{2+x^6}\right) \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{2+x^6}}{\sqrt{2}}\right)}{3\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x^6+2}}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[2 + x^6]),x]
[Out] -1/3*ArcTanh[Sqrt[2 + x^6]/Sqrt[2]]/Sqrt[2]
```

fricas [A] time = 0.84, size = 27, normalized size = 1.08

$$\frac{1}{12}\sqrt{2}\log\left(\frac{x^6 - 2\sqrt{2}\sqrt{x^6+2} + 4}{x^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6+2)^(1/2),x, algorithm="fricas")
[Out] 1/12*sqrt(2)*log((x^6 - 2*sqrt(2)*sqrt(x^6 + 2) + 4)/x^6)
```

giac [B] time = 0.18, size = 37, normalized size = 1.48

$$-\frac{1}{12}\sqrt{2}\log\left(\sqrt{2} + \sqrt{x^6+2}\right) + \frac{1}{12}\sqrt{2}\log\left(-\sqrt{2} + \sqrt{x^6+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6+2)^(1/2),x, algorithm="giac")
[Out] -1/12*sqrt(2)*log(sqrt(2) + sqrt(x^6 + 2)) + 1/12*sqrt(2)*log(-sqrt(2) + sq
rt(x^6 + 2))
```

maple [A] time = 0.01, size = 26, normalized size = 1.04

$$\frac{\sqrt{2}\,\ln\left(\frac{\sqrt{x^6+2}-\sqrt{2}}{\sqrt{x^6}}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^6+2)^(1/2),x)
[Out] 1/6*2^(1/2)*ln(((x^6+2)^(1/2)-2^(1/2))/(x^6)^(1/2))
```

maxima [A] time = 2.37, size = 34, normalized size = 1.36

$$\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2} - \sqrt{x^6+2}}{\sqrt{2} + \sqrt{x^6+2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6+2)^(1/2),x, algorithm="maxima")
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - sqrt(x^6 + 2))/(sqrt(2) + sqrt(x^6 + 2)))
```

mupad [B] time = 0.07, size = 18, normalized size = 0.72

$$-\frac{\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}\,\sqrt{x^6+2}}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^6 + 2)^(1/2)),x)
[Out] -(2^(1/2)*atanh((2^(1/2)*(x^6 + 2)^(1/2))/2))/6
```

sympy [**A**] time = 1.69, size = 17, normalized size = 0.68

$$-\frac{\sqrt{2}\,\operatorname{asinh}\left(\frac{\sqrt{2}}{x^3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**6+2)**(1/2),x)
[Out] -sqrt(2)*asinh(sqrt(2)/x**3)/6
```

3.1388 $\int \frac{1}{x^7\sqrt{2+x^6}}\,dx$

Optimal. Leaf size=42

$$\frac{\tanh^{-1}\left(\frac{\sqrt{x^6+2}}{\sqrt{2}}\right)}{12\sqrt{2}} - \frac{\sqrt{x^6+2}}{12x^6}$$

```
[Out] 1/24*arctanh(1/2*(x^6+2)^(1/2)*2^(1/2))*2^(1/2)-1/12*(x^6+2)^(1/2)/x^6
```

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {266, 51, 63, 207}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{x^6+2}}{\sqrt{2}}\right)}{12\sqrt{2}} - \frac{\sqrt{x^6+2}}{12x^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*Sqrt[2 + x^6]),x]
```

```
[Out] -Sqrt[2 + x^6]/(12*x^6) + ArcTanh[Sqrt[2 + x^6]/Sqrt[2]]/(12*Sqrt[2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7\sqrt{2+x^6}}\,dx &= \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{2+x}}\,dx, x, x^6\right)\\ &= -\frac{\sqrt{2+x^6}}{12x^6} - \frac{1}{24}\,\text{Subst}\left(\int \frac{1}{x\sqrt{2+x}}\,dx, x, x^6\right)\\ &= -\frac{\sqrt{2+x^6}}{12x^6} - \frac{1}{12}\,\text{Subst}\left(\int \frac{1}{-2+x^2}\,dx, x, \sqrt{2+x^6}\right)\\ &= -\frac{\sqrt{2+x^6}}{12x^6} + \frac{\tanh^{-1}\left(\frac{\sqrt{2+x^6}}{\sqrt{2}}\right)}{12\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.00

$$\frac{\sqrt{2}\,x^6\tanh^{-1}\left(\sqrt{\frac{x^6}{2}+1}\right) - 2\sqrt{x^6+2}}{24x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*Sqrt[2 + x^6]),x]
```

```
[Out] (-2*Sqrt[2 + x^6] + Sqrt[2]*x^6*ArcTanh[Sqrt[1 + x^6/2]])/(24*x^6)
```

fricas [A] time = 0.84, size = 44, normalized size = 1.05

$$\frac{\sqrt{2}\,x^6\log\left(\frac{x^6+2\sqrt{2}\sqrt{x^6+2}+4}{x^6}\right) - 4\sqrt{x^6+2}}{48\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^6+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/48*(sqrt(2)*x^6*log((x^6 + 2*sqrt(2)*sqrt(x^6 + 2) + 4)/x^6) - 4*sqrt(x^6
 + 2))/x^6
```

giac [A] time = 0.16, size = 47, normalized size = 1.12

$$-\frac{1}{48}\sqrt{2}\log\left(-\frac{\sqrt{2}-\sqrt{x^6+2}}{\sqrt{2}+\sqrt{x^6+2}}\right) - \frac{\sqrt{x^6+2}}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^6+2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/48*sqrt(2)*log(-(sqrt(2) - sqrt(x^6 + 2))/(sqrt(2) + sqrt(x^6 + 2))) - 1
/12*sqrt(x^6 + 2)/x^6
```

maple [A] time = 0.02, size = 39, normalized size = 0.93

$$-\frac{\sqrt{2}\,\ln\left(\frac{\sqrt{x^6+2}-\sqrt{2}}{\sqrt{x^6}}\right)}{24} - \frac{\sqrt{x^6+2}}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^6+2)^(1/2),x)
```

```
[Out] -1/12*(x^6+2)^(1/2)/x^6-1/24*2^(1/2)*ln(((x^6+2)^(1/2)-2^(1/2))/(x^6)^(1/2)
)
```

maxima [A] time = 2.44, size = 47, normalized size = 1.12

$$-\frac{1}{48}\sqrt{2}\log\left(-\frac{\sqrt{2}-\sqrt{x^6+2}}{\sqrt{2}+\sqrt{x^6+2}}\right)-\frac{\sqrt{x^6+2}}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^6+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/48*sqrt(2)*log(-(sqrt(2) - sqrt(x^6 + 2))/(sqrt(2) + sqrt(x^6 + 2))) - 1
/12*sqrt(x^6 + 2)/x^6
```

mupad [B] time = 1.26, size = 31, normalized size = 0.74

$$\frac{\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}\,\sqrt{x^6+2}}{2}\right)}{24}-\frac{\sqrt{x^6+2}}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(x^6 + 2)^(1/2)),x)
```

```
[Out] (2^(1/2)*atanh((2^(1/2)*(x^6 + 2)^(1/2))/2))/24 - (x^6 + 2)^(1/2)/(12*x^6)
```

sympy [A] time = 2.65, size = 31, normalized size = 0.74

$$\frac{\sqrt{2}\,\operatorname{asinh}\left(\frac{\sqrt{2}}{x^3}\right)}{24}-\frac{\sqrt{1+\frac{2}{x^6}}}{12x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**6+2)**(1/2),x)
```

```
[Out] sqrt(2)*asinh(sqrt(2)/x**3)/24 - sqrt(1 + 2/x**6)/(12*x**3)
```

3.1389 $\int \frac{1}{x^{13}\sqrt{2+x^6}}\,dx$

Optimal. Leaf size=58

$$\frac{\sqrt{x^6+2}}{32x^6} - \frac{\tanh^{-1}\left(\frac{\sqrt{x^6+2}}{\sqrt{2}}\right)}{32\sqrt{2}} - \frac{\sqrt{x^6+2}}{24x^{12}}$$

```
[Out] -1/64*arctanh(1/2*(x^6+2)^(1/2)*2^(1/2))*2^(1/2)-1/24*(x^6+2)^(1/2)/x^12+1/
32*(x^6+2)^(1/2)/x^6
```

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {266, 51, 63, 207}

$$\frac{\sqrt{x^6+2}}{32x^6} - \frac{\sqrt{x^6+2}}{24x^{12}} - \frac{\tanh^{-1}\left(\frac{\sqrt{x^6+2}}{\sqrt{2}}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^13*Sqrt[2 + x^6]),x]
```

```
[Out] -Sqrt[2 + x^6]/(24*x^12) + Sqrt[2 + x^6]/(32*x^6) - ArcTanh[Sqrt[2 + x^6]/S
qrt[2]]/(32*Sqrt[2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{13}\sqrt{2+x^6}}\,dx &= \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{2+x}}\,dx, x, x^6\right)\\ &= -\frac{\sqrt{2+x^6}}{24x^{12}} - \frac{1}{16}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{2+x}}\,dx, x, x^6\right)\\ &= -\frac{\sqrt{2+x^6}}{24x^{12}} + \frac{\sqrt{2+x^6}}{32x^6} + \frac{1}{64}\,\text{Subst}\left(\int \frac{1}{x\sqrt{2+x}}\,dx, x, x^6\right)\\ &= -\frac{\sqrt{2+x^6}}{24x^{12}} + \frac{\sqrt{2+x^6}}{32x^6} + \frac{1}{32}\,\text{Subst}\left(\int \frac{1}{-2+x^2}\,dx, x, \sqrt{2+x^6}\right)\\ &= -\frac{\sqrt{2+x^6}}{24x^{12}} + \frac{\sqrt{2+x^6}}{32x^6} - \frac{\tanh^{-1}\left(\frac{\sqrt{2+x^6}}{\sqrt{2}}\right)}{32\sqrt{2}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 30, normalized size = 0.52

$$-\frac{1}{24}\sqrt{x^6+2}\;{}_2F_1\left(\frac{1}{2}, 3; \frac{3}{2}; \frac{x^6}{2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^13*Sqrt[2 + x^6]),x]
```

```
[Out] -1/24*(Sqrt[2 + x^6]*Hypergeometric2F1[1/2, 3, 3/2, 1 + x^6/2])
```

fricas [A] time = 0.65, size = 52, normalized size = 0.90

$$\frac{3\sqrt{2}\,x^{12}\log\left(\frac{x^6-2\sqrt{2}\sqrt{x^6+2}+4}{x^6}\right) + 4\left(3x^6-4\right)\sqrt{x^6+2}}{384\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(x^6+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/384*(3*sqrt(2)*x^12*log((x^6 - 2*sqrt(2)*sqrt(x^6 + 2) + 4)/x^6) + 4*(3*x
^6 - 4)*sqrt(x^6 + 2))/x^12
```

giac [A] time = 0.16, size = 59, normalized size = 1.02

$$\frac{1}{128}\sqrt{2}\log\left(-\frac{\sqrt{2}-\sqrt{x^6+2}}{\sqrt{2}+\sqrt{x^6+2}}\right) + \frac{3\left(x^6+2\right)^{\frac{3}{2}} - 10\sqrt{x^6+2}}{96\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(x^6+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/128*sqrt(2)*log(-(sqrt(2) - sqrt(x^6 + 2))/(sqrt(2) + sqrt(x^6 + 2))) + 1
/96*(3*(x^6 + 2)^(3/2) - 10*sqrt(x^6 + 2))/x^12
```

maple [A] time = 0.02, size = 51, normalized size = 0.88

$$\frac{\sqrt{2}\;\ln\left(\frac{\sqrt{x^6+2}-\sqrt{2}}{\sqrt{x^6}}\right)}{64} + \frac{3x^{12}+2x^6-8}{96\sqrt{x^6+2}\;x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^13/(x^6+2)^(1/2),x)
```

```
[Out] 1/96*(3*x^12+2*x^6-8)/x^12/(x^6+2)^(1/2)+1/64*2^(1/2)*ln(((x^6+2)^(1/2)-2^(
1/2))/(x^6)^(1/2))
```

maxima [A] time = 2.30, size = 74, normalized size = 1.28

$$\frac{1}{128}\sqrt{2}\log\left(-\frac{\sqrt{2}-\sqrt{x^6+2}}{\sqrt{2}+\sqrt{x^6+2}}\right)-\frac{3\left(x^6+2\right)^{\frac{3}{2}}-10\sqrt{x^6+2}}{96\left(4x^6-\left(x^6+2\right)^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(x^6+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/128*sqrt(2)*log(-(sqrt(2) - sqrt(x^6 + 2))/(sqrt(2) + sqrt(x^6 + 2))) - 1
/96*(3*(x^6 + 2)^(3/2) - 10*sqrt(x^6 + 2))/(4*x^6 - (x^6 + 2)^2 + 4)
```

mupad [B] time = 1.28, size = 57, normalized size = 0.98

$$\frac{\frac{5\sqrt{x^6+2}}{48}-\frac{\left(x^6+2\right)^{3/2}}{32}}{4x^6-\left(x^6+2\right)^2+4}-\frac{\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}\,\sqrt{x^6+2}}{2}\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^13*(x^6 + 2)^(1/2)),x)
```

```
[Out] ((5*(x^6 + 2)^(1/2))/48 - (x^6 + 2)^(3/2)/32)/(4*x^6 - (x^6 + 2)^2 + 4) - (
2^(1/2)*atanh((2^(1/2)*(x^6 + 2)^(1/2))/2))/64
```

sympy [A] time = 4.15, size = 66, normalized size = 1.14

$$-\frac{\sqrt{2}\,\operatorname{asinh}\left(\frac{\sqrt{2}}{x^3}\right)}{64}+\frac{1}{32x^3\sqrt{1+\frac{2}{x^6}}}+\frac{1}{48x^9\sqrt{1+\frac{2}{x^6}}}-\frac{1}{12x^{15}\sqrt{1+\frac{2}{x^6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**13/(x**6+2)**(1/2),x)
```

```
[Out] -sqrt(2)*asinh(sqrt(2)/x**3)/64 + 1/(32*x**3*sqrt(1 + 2/x**6)) + 1/(48*x**9
*sqrt(1 + 2/x**6)) - 1/(12*x**15*sqrt(1 + 2/x**6))
```

3.1390 $\int \frac{x^{14}}{\sqrt{2+x^6}}\,dx$

Optimal. Leaf size=47

$$\frac{1}{2}\sinh^{-1}\left(\frac{x^3}{\sqrt{2}}\right)+\frac{1}{12}\sqrt{x^6+2}\,x^9-\frac{1}{4}\sqrt{x^6+2}\,x^3$$

```
[Out] 1/2*arcsinh(1/2*x^3*2^(1/2))-1/4*x^3*(x^6+2)^(1/2)+1/12*x^9*(x^6+2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {275, 321, 215}

$$\frac{1}{12}\sqrt{x^6+2}\,x^9-\frac{1}{4}\sqrt{x^6+2}\,x^3+\frac{1}{2}\sinh^{-1}\left(\frac{x^3}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^14/Sqrt[2 + x^6],x]
```

```
[Out] -(x^3*Sqrt[2 + x^6])/4 + (x^9*Sqrt[2 + x^6])/12 + ArcSinh[x^3/Sqrt[2]]/2
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{14}}{\sqrt{2+x^6}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^4}{\sqrt{2+x^2}}\,dx, x, x^3\right)\\
&= \frac{1}{12}x^9\sqrt{2+x^6}-\frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{2+x^2}}\,dx, x, x^3\right)\\
&= -\frac{1}{4}x^3\sqrt{2+x^6}+\frac{1}{12}x^9\sqrt{2+x^6}+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{2+x^2}}\,dx, x, x^3\right)\\
&= -\frac{1}{4}x^3\sqrt{2+x^6}+\frac{1}{12}x^9\sqrt{2+x^6}+\frac{1}{2}\sinh^{-1}\left(\frac{x^3}{\sqrt{2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.74

$$\frac{1}{12}\left(6\sinh^{-1}\left(\frac{x^3}{\sqrt{2}}\right)+\left(x^6-3\right)\sqrt{x^6+2}\,x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/Sqrt[2 + x^6],x]
[Out] (x^3*(-3 + x^6)*Sqrt[2 + x^6] + 6*ArcSinh[x^3/Sqrt[2]])/12
```

fricas [A] time = 0.71, size = 35, normalized size = 0.74

$$\frac{1}{12}\left(x^9 - 3\,x^3\right)\sqrt{x^6+2} - \frac{1}{2}\log\left(-x^3 + \sqrt{x^6+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(x^6+2)^(1/2),x, algorithm="fricas")
[Out] 1/12*(x^9 - 3*x^3)*sqrt(x^6 + 2) - 1/2*log(-x^3 + sqrt(x^6 + 2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\sqrt{x^6+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(x^6+2)^(1/2),x, algorithm="giac")
[Out] integrate(x^14/sqrt(x^6 + 2), x)
```

maple [A] time = 0.14, size = 30, normalized size = 0.64

$$\frac{\left(x^6-3\right)\sqrt{x^6+2}\,x^3}{12} + \frac{\operatorname{arcsinh}\left(\frac{\sqrt{2}\,x^3}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(x^6+2)^(1/2),x)
[Out] 1/12*x^3*(x^6-3)*(x^6+2)^(1/2)+1/2*arcsinh(1/2*2^(1/2)*x^3)
```

maxima [B] time = 1.00, size = 86, normalized size = 1.83

$$-\frac{\frac{5\sqrt{x^6+2}}{x^3} - \frac{3\left(x^6+2\right)^{\frac{3}{2}}}{x^9}}{6\left(\frac{2\left(x^6+2\right)}{x^6} - \frac{\left(x^6+2\right)^2}{x^{12}} - 1\right)} + \frac{1}{4}\log\left(\frac{\sqrt{x^6+2}}{x^3}+1\right) - \frac{1}{4}\log\left(\frac{\sqrt{x^6+2}}{x^3}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(x^6+2)^(1/2),x, algorithm="maxima")
[Out] -1/6*(5*sqrt(x^6 + 2)/x^3 - 3*(x^6 + 2)^(3/2)/x^9)/(2*(x^6 + 2)/x^6 - (x^6
+ 2)^2/x^12 - 1) + 1/4*log(sqrt(x^6 + 2)/x^3 + 1) - 1/4*log(sqrt(x^6 + 2)/x
^3 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{14}}{\sqrt{x^6+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(x^6 + 2)^(1/2),x)
```

```
[Out] int(x^14/(x^6 + 2)^(1/2), x)
```

sympy [**A**] time = 6.29, size = 53, normalized size = 1.13

$$\frac{x^{15}}{12\sqrt{x^6+2}} - \frac{x^9}{12\sqrt{x^6+2}} - \frac{x^3}{2\sqrt{x^6+2}} + \frac{\operatorname{asinh}\left(\frac{\sqrt{2}x^3}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(x**6+2)**(1/2),x)
```

```
[Out] x**15/(12*sqrt(x**6 + 2)) - x**9/(12*sqrt(x**6 + 2)) - x**3/(2*sqrt(x**6 +
2)) + asinh(sqrt(2)*x**3/2)/2
```

3.1391 $\int \frac{x^8}{\sqrt{2+x^6}}\,dx$

Optimal. Leaf size=31

$$\frac{1}{6}x^3\sqrt{x^6+2}-\frac{1}{3}\sinh^{-1}\left(\frac{x^3}{\sqrt{2}}\right)$$

```
[Out] -1/3*arcsinh(1/2*x^3*2^(1/2))+1/6*x^3*(x^6+2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {275, 321, 215}

$$\frac{1}{6}x^3\sqrt{x^6+2}-\frac{1}{3}\sinh^{-1}\left(\frac{x^3}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^8/Sqrt[2 + x^6],x]
```

```
[Out] (x^3*Sqrt[2 + x^6])/6 - ArcSinh[x^3/Sqrt[2]]/3
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{\sqrt{2+x^6}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{\sqrt{2+x^2}}\,dx, x, x^3\right)\\ &= \frac{1}{6}x^3\sqrt{2+x^6}-\frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{2+x^2}}\,dx, x, x^3\right)\\ &= \frac{1}{6}x^3\sqrt{2+x^6}-\frac{1}{3}\sinh^{-1}\left(\frac{x^3}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{1}{6}x^3\sqrt{x^6+2}-\frac{1}{3}\sinh^{-1}\left(\frac{x^3}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/Sqrt[2 + x^6],x]
[Out] (x^3*Sqrt[2 + x^6])/6 - ArcSinh[x^3/Sqrt[2]]/3
```

fricas [A] time = 0.86, size = 29, normalized size = 0.94

$$\frac{1}{6}\sqrt{x^6+2}\,x^3+\frac{1}{3}\log\left(-x^3+\sqrt{x^6+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6+2)^(1/2),x, algorithm="fricas")
[Out] 1/6*sqrt(x^6 + 2)*x^3 + 1/3*log(-x^3 + sqrt(x^6 + 2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\sqrt{x^6+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6+2)^(1/2),x, algorithm="giac")
[Out] integrate(x^8/sqrt(x^6 + 2), x)
```

maple [A] time = 0.14, size = 25, normalized size = 0.81

$$\frac{\sqrt{x^6+2}\,x^3}{6}-\frac{\operatorname{arcsinh}\left(\frac{\sqrt{2}\,x^3}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^6+2)^(1/2),x)
[Out] -1/3*arcsinh(1/2*2^(1/2)*x^3)+1/6*x^3*(x^6+2)^(1/2)
```

maxima [B] time = 0.99, size = 58, normalized size = 1.87

$$\frac{\sqrt{x^6+2}}{3\,x^3\left(\frac{x^6+2}{x^6}-1\right)}-\frac{1}{6}\log\left(\frac{\sqrt{x^6+2}}{x^3}+1\right)+\frac{1}{6}\log\left(\frac{\sqrt{x^6+2}}{x^3}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6+2)^(1/2),x, algorithm="maxima")
[Out] 1/3*sqrt(x^6 + 2)/(x^3*((x^6 + 2)/x^6 - 1)) - 1/6*log(sqrt(x^6 + 2)/x^3 + 1
) + 1/6*log(sqrt(x^6 + 2)/x^3 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^8}{\sqrt{x^6+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^6 + 2)^(1/2),x)
[Out] int(x^8/(x^6 + 2)^(1/2), x)
```

sympy [A] time = 3.32, size = 39, normalized size = 1.26

$$\frac{x^9}{6\sqrt{x^6+2}}+\frac{x^3}{3\sqrt{x^6+2}}-\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\,x^3}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**6+2)**(1/2),x)
[Out] x**9/(6*sqrt(x**6 + 2)) + x**3/(3*sqrt(x**6 + 2)) - asinh(sqrt(2)*x**3/2)/3
```

3.1392 $\int \frac{x^2}{\sqrt{2+x^6}}\,dx$

Optimal. Leaf size=14

$$\frac{1}{3}\sinh^{-1}\left(\frac{x^3}{\sqrt{2}}\right)$$

```
[Out] 1/3*arcsinh(1/2*x^3*2^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {275, 215}

$$\frac{1}{3}\sinh^{-1}\left(\frac{x^3}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[2 + x^6],x]
```

```
[Out] ArcSinh[x^3/Sqrt[2]]/3
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{2+x^6}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{2+x^2}}\,dx, x, x^3\right)\\ &= \frac{1}{3}\sinh^{-1}\left(\frac{x^3}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{1}{3}\sinh^{-1}\left(\frac{x^3}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[2 + x^6],x]
```

```
[Out] ArcSinh[x^3/Sqrt[2]]/3
```

fricas [A] time = 0.89, size = 16, normalized size = 1.14

$$-\frac{1}{3}\log\left(-x^3+\sqrt{x^6+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+2)^(1/2),x, algorithm="fricas")
[Out] -1/3*log(-x^3 + sqrt(x^6 + 2))
```

giac [A] time = 0.21, size = 16, normalized size = 1.14

$$-\frac{1}{3}\log\left(-x^3+\sqrt{x^6+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+2)^(1/2),x, algorithm="giac")
[Out] -1/3*log(-x^3 + sqrt(x^6 + 2))
```

maple [A] time = 0.14, size = 12, normalized size = 0.86

$$\frac{\operatorname{arcsinh}\left(\frac{\sqrt{2}\,x^3}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^6+2)^(1/2),x)
[Out] 1/3*arcsinh(1/2*2^(1/2)*x^3)
```

maxima [B] time = 1.05, size = 33, normalized size = 2.36

$$\frac{1}{6}\log\left(\frac{\sqrt{x^6+2}}{x^3}+1\right)-\frac{1}{6}\log\left(\frac{\sqrt{x^6+2}}{x^3}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+2)^(1/2),x, algorithm="maxima")
[Out] 1/6*log(sqrt(x^6 + 2)/x^3 + 1) - 1/6*log(sqrt(x^6 + 2)/x^3 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{x^2}{\sqrt{x^6+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^6 + 2)^(1/2),x)
[Out] int(x^2/(x^6 + 2)^(1/2), x)
```

sympy [A] time = 1.75, size = 12, normalized size = 0.86

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{2}x^3}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**6+2)**(1/2),x)
[Out] asinh(sqrt(2)*x**3/2)/3
```

3.1393 $\int \frac{1}{x^4\sqrt{2+x^6}}\,dx$

Optimal. Leaf size=16

$$-\frac{\sqrt{x^6+2}}{6x^3}$$

```
[Out] -1/6*(x^6+2)^(1/2)/x^3
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {264}

$$-\frac{\sqrt{x^6+2}}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[2 + x^6]),x]
```

```
[Out] -Sqrt[2 + x^6]/(6*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{2+x^6}}\,dx = -\frac{\sqrt{2+x^6}}{6x^3}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{\sqrt{x^6+2}}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[2 + x^6]),x]
```

```
[Out] -1/6*Sqrt[2 + x^6]/x^3
```

fricas [A] time = 0.88, size = 16, normalized size = 1.00

$$-\frac{x^3+\sqrt{x^6+2}}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^6+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(x^3 + sqrt(x^6 + 2))/x^3
```

giac [A] time = 0.20, size = 20, normalized size = 1.25

$$-\frac{\sqrt{\frac{2}{x^6}+1}}{6\,\mathrm{sgn}(x)}+\frac{1}{6}\,\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^6+2)^(1/2),x, algorithm="giac")
[Out] -1/6*sqrt(2/x^6 + 1)/sgn(x) + 1/6*sgn(x)
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$-\frac{\sqrt{x^6+2}}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^6+2)^(1/2),x)
[Out] -1/6*(x^6+2)^(1/2)/x^3
```

maxima [A] time = 0.92, size = 12, normalized size = 0.75

$$-\frac{\sqrt{x^6+2}}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^6+2)^(1/2),x, algorithm="maxima")
[Out] -1/6*sqrt(x^6 + 2)/x^3
```

mupad [B] time = 1.12, size = 12, normalized size = 0.75

$$-\frac{\sqrt{x^6+2}}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x^6 + 2)^(1/2)),x)
[Out] -(x^6 + 2)^(1/2)/(6*x^3)
```

sympy [A] time = 1.17, size = 12, normalized size = 0.75

$$-\frac{\sqrt{1+\frac{2}{x^6}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**6+2)**(1/2),x)
[Out] -sqrt(1 + 2/x**6)/6
```

3.1394 $\int \frac{1}{x^{10}\sqrt{2+x^6}}\,dx$

Optimal. Leaf size=33

$$\frac{\sqrt{x^6+2}}{18x^3}-\frac{\sqrt{x^6+2}}{18x^9}$$

```
[Out] -1/18*(x^6+2)^(1/2)/x^9+1/18*(x^6+2)^(1/2)/x^3
```

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {271, 264}

$$\frac{\sqrt{x^6+2}}{18x^3}-\frac{\sqrt{x^6+2}}{18x^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*Sqrt[2 + x^6]),x]
```

```
[Out] -Sqrt[2 + x^6]/(18*x^9) + Sqrt[2 + x^6]/(18*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{10}\sqrt{2+x^6}}\,dx &= -\frac{\sqrt{2+x^6}}{18x^9}-\frac{1}{3}\int \frac{1}{x^4\sqrt{2+x^6}}\,dx\\ &= -\frac{\sqrt{2+x^6}}{18x^9}+\frac{\sqrt{2+x^6}}{18x^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 0.64

$$\frac{\left(x^6-1\right)\sqrt{x^6+2}}{18x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^10*Sqrt[2 + x^6]),x]
```

```
[Out] ((-1 + x^6)*Sqrt[2 + x^6])/(18*x^9)
```

fricas [A] time = 0.83, size = 22, normalized size = 0.67

$$\frac{x^9+\sqrt{x^6+2}\left(x^6-1\right)}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(x^6+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/18*(x^9 + sqrt(x^6 + 2)*(x^6 - 1))/x^9
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(x^6+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [abs(x)]sym2poly/r2sym(const gen
 & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [**A**] time = 0.00, size = 18, normalized size = 0.55

$$\frac{\sqrt{x^6+2}\,\left(x^6-1\right)}{18x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(x^6+2)^(1/2),x)
```

```
[Out] 1/18*(x^6+2)^(1/2)*(x^6-1)/x^9
```

maxima [**A**] time = 0.99, size = 25, normalized size = 0.76

$$\frac{\sqrt{x^6+2}}{12\,x^3}-\frac{\left(x^6+2\right)^{\frac{3}{2}}}{36\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(x^6+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(x^6 + 2)/x^3 - 1/36*(x^6 + 2)^(3/2)/x^9
```

mupad [**B**] time = 1.17, size = 17, normalized size = 0.52

$$\frac{\left(x^6-1\right)\,\sqrt{x^6+2}}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^10*(x^6 + 2)^(1/2)),x)
```

```
[Out] ((x^6 - 1)*(x^6 + 2)^(1/2))/(18*x^9)
```

sympy [**A**] time = 1.98, size = 26, normalized size = 0.79

$$\frac{\sqrt{1+\frac{2}{x^6}}}{18}-\frac{\sqrt{1+\frac{2}{x^6}}}{18x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(x**6+2)**(1/2),x)
```

```
[Out] sqrt(1 + 2/x**6)/18 - sqrt(1 + 2/x**6)/(18*x**6)
```

3.1395 $\int \frac{1}{x^{16}\sqrt{2+x^6}}\,dx$

Optimal. Leaf size=49

$$-\frac{\sqrt{x^6+2}}{30x^{15}}+\frac{\sqrt{x^6+2}}{45x^9}-\frac{\sqrt{x^6+2}}{45x^3}$$

```
[Out] -1/30*(x^6+2)^(1/2)/x^15+1/45*(x^6+2)^(1/2)/x^9-1/45*(x^6+2)^(1/2)/x^3
```

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {271, 264}

$$-\frac{\sqrt{x^6+2}}{45x^3}+\frac{\sqrt{x^6+2}}{45x^9}-\frac{\sqrt{x^6+2}}{30x^{15}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^16*Sqrt[2 + x^6]),x]
```

```
[Out] -Sqrt[2 + x^6]/(30*x^15) + Sqrt[2 + x^6]/(45*x^9) - Sqrt[2 + x^6]/(45*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{16}\sqrt{2+x^6}}\,dx &= -\frac{\sqrt{2+x^6}}{30x^{15}}-\frac{2}{5}\int \frac{1}{x^{10}\sqrt{2+x^6}}\,dx\\ &= -\frac{\sqrt{2+x^6}}{30x^{15}}+\frac{\sqrt{2+x^6}}{45x^9}+\frac{2}{15}\int \frac{1}{x^4\sqrt{2+x^6}}\,dx\\ &= -\frac{\sqrt{2+x^6}}{30x^{15}}+\frac{\sqrt{2+x^6}}{45x^9}-\frac{\sqrt{2+x^6}}{45x^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.57

$$-\frac{\sqrt{x^6+2}\left(2x^{12}-2x^6+3\right)}{90x^{15}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^16*Sqrt[2 + x^6]),x]
```

```
[Out] -1/90*(Sqrt[2 + x^6]*(3 - 2*x^6 + 2*x^12))/x^15
```

fricas [A] time = 0.83, size = 31, normalized size = 0.63

$$-\frac{2\,x^{15}+\left(2\,x^{12}-2\,x^6+3\right)\sqrt{x^6+2}}{90\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^16/(x^6+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/90*(2*x^15 + (2*x^12 - 2*x^6 + 3)*sqrt(x^6 + 2))/x^15
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^16/(x^6+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [abs(x)]sym2poly/r2sym(const gen
 & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [**A**] time = 0.00, size = 25, normalized size = 0.51

$$-\frac{\sqrt{x^6+2}\,\left(2x^{12}-2x^6+3\right)}{90x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^16/(x^6+2)^(1/2),x)
```

```
[Out] -1/90*(x^6+2)^(1/2)*(2*x^12-2*x^6+3)/x^15
```

maxima [**A**] time = 1.03, size = 37, normalized size = 0.76

$$-\frac{\sqrt{x^6+2}}{24\,x^3}+\frac{\left(x^6+2\right)^{\frac{3}{2}}}{36\,x^9}-\frac{\left(x^6+2\right)^{\frac{5}{2}}}{120\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^16/(x^6+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/24*sqrt(x^6 + 2)/x^3 + 1/36*(x^6 + 2)^(3/2)/x^9 - 1/120*(x^6 + 2)^(5/2)/
x^15
```

mupad [**B**] time = 1.24, size = 24, normalized size = 0.49

$$-\frac{\sqrt{x^6+2}\,\left(2\,x^{12}-2\,x^6+3\right)}{90\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^16*(x^6 + 2)^(1/2)),x)
```

```
[Out] -((x^6 + 2)^(1/2)*(2*x^12 - 2*x^6 + 3))/(90*x^15)
```

sympy [**A**] time = 3.71, size = 41, normalized size = 0.84

$$-\frac{\sqrt{1+\frac{2}{x^6}}}{45}+\frac{\sqrt{1+\frac{2}{x^6}}}{45x^6}-\frac{\sqrt{1+\frac{2}{x^6}}}{30x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**16/(x**6+2)**(1/2),x)
```

```
[Out] -sqrt(1 + 2/x**6)/45 + sqrt(1 + 2/x**6)/(45*x**6) - sqrt(1 + 2/x**6)/(30*x*
*12)
```

3.1396 $\int \frac{x^7}{\sqrt{2+x^6}}\,dx$

Optimal. Leaf size=186

$$\frac{1}{5}x^2\sqrt{x^6+2}-\frac{2\,2^{5/6}\sqrt{2+\sqrt{3}}\,\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\,F\left(\sin^{-1}\left(\frac{x^2+\sqrt[3]{2}\left(1-\sqrt{3}\right)}{x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)}\right)|-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{x^2+\sqrt[3]{2}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\sqrt{x^6+2}}$$

```
[Out] 1/5*x^2*(x^6+2)^(1/2)-2/15*2^(5/6)*(2^(1/3)+x^2)*EllipticF((x^2+2^(1/3)*(1-
3^(1/2)))/(x^2+2^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2)
)*((2^(2/3)-2^(1/3)*x^2+x^4)/(x^2+2^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/(x^
6+2)^(1/2)/((2^(1/3)+x^2)/(x^2+2^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {275, 321, 218}

$$\frac{1}{5}x^2\sqrt{x^6+2}-\frac{2\,2^{5/6}\sqrt{2+\sqrt{3}}\,\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\,F\left(\sin^{-1}\left(\frac{x^2+\sqrt[3]{2}\left(1-\sqrt{3}\right)}{x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)}\right)|-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{x^2+\sqrt[3]{2}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/Sqrt[2 + x^6],x]
```

```
[Out] (x^2*Sqrt[2 + x^6])/5 - (2*2^(5/6)*Sqrt[2 + Sqrt[3]]*(2^(1/3) + x^2)*Sqrt[(
2^(2/3) - 2^(1/3)*x^2 + x^4)/(2^(1/3)*(1 + Sqrt[3]) + x^2)^2]*EllipticF[Arc
Sin[(2^(1/3)*(1 - Sqrt[3]) + x^2)/(2^(1/3)*(1 + Sqrt[3]) + x^2)], -7 - 4*Sq
rt[3]])/(5*3^(1/4)*Sqrt[(2^(1/3) + x^2)/(2^(1/3)*(1 + Sqrt[3]) + x^2)^2]*Sq
rt[2 + x^6])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^7}{\sqrt{2+x^6}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{\sqrt{2+x^3}}\,dx, x, x^2\right)$$
$$= \frac{1}{5}x^2\sqrt{2+x^6} - \frac{2}{5}\,\text{Subst}\left(\int \frac{1}{\sqrt{2+x^3}}\,dx, x, x^2\right)$$
$$= \frac{1}{5}x^2\sqrt{2+x^6} - \frac{2\,2^{5/6}\sqrt{2+\sqrt{3}}\,\left(\sqrt[3]{2}+x^2\right)\sqrt{\frac{2^{2/3}-\sqrt[3]{2}x^2+x^4}{\left(\sqrt[3]{2}\left(1+\sqrt{3}\right)+x^2\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{2}\left(1-\sqrt{3}\right)+x^2}{\sqrt[3]{2}\left(1+\sqrt{3}\right)+x^2}\right)|-7-4\sqrt{3}\right.}{5\sqrt[4]{3}\sqrt{\frac{\sqrt[3]{2}+x^2}{\left(\sqrt[3]{2}\left(1+\sqrt{3}\right)+x^2\right)^2}}\sqrt{2+x^6}}$$

Mathematica [C] time = 0.01, size = 41, normalized size = 0.22

$$\frac{1}{5}x^2\left(\sqrt{x^6+2} - \sqrt{2}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{x^6}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/Sqrt[2 + x^6],x]
```

```
[Out] (x^2*(Sqrt[2 + x^6] - Sqrt[2]*Hypergeometric2F1[1/3, 1/2, 4/3, -1/2*x^6]))/
5
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^7}{\sqrt{x^6+2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^6+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^7/sqrt(x^6 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\sqrt{x^6+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^6+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^7/sqrt(x^6 + 2), x)
```

maple [C] time = 0.14, size = 33, normalized size = 0.18

$$-\frac{\sqrt{2}\,x^2\,\text{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],-\frac{x^6}{2}\right)}{5} + \frac{\sqrt{x^6+2}\,x^2}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^6+2)^(1/2),x)
```

```
[Out] 1/5*x^2*(x^6+2)^(1/2)-1/5*2^(1/2)*x^2*hypergeom([1/3,1/2],[4/3],-1/2*x^6)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\sqrt{x^6+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^6+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^7/sqrt(x^6 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7}{\sqrt{x^6+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^6 + 2)^(1/2),x)
```

```
[Out] int(x^7/(x^6 + 2)^(1/2), x)
```

sympy [A] time = 0.93, size = 36, normalized size = 0.19

$$\frac{\sqrt{2}\,x^8\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\right|\frac{x^6e^{i\pi}}{2}\right)}{12\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**6+2)**(1/2),x)
```

```
[Out] sqrt(2)*x**8*gamma(4/3)*hyper((1/2, 4/3), (7/3,), x**6*exp_polar(I*pi)/2)/(
12*gamma(7/3))
```

3.1397 $\int \frac{x}{\sqrt{2+x^6}}\,dx$

Optimal. Leaf size=166

$$\frac{\sqrt{2+\sqrt{3}}\,\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\,F\left(\sin^{-1}\left(\frac{x^2+\sqrt[3]{2}\left(1-\sqrt{3}\right)}{x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)}\right)|-7-4\sqrt{3}\right)}{\sqrt[6]{2}\,\sqrt[4]{3}\,\sqrt{\frac{x^2+\sqrt[3]{2}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\,\sqrt{x^6+2}}$$

```
[Out] 1/6*2^(5/6)*(2^(1/3)+x^2)*EllipticF((x^2+2^(1/3)*(1-3^(1/2)))/(x^2+2^(1/3)*
(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((2^(2/3)-2^(1/3)*x^2
+x^4)/(x^2+2^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/(x^6+2)^(1/2)/((2^(1/3)+x^
2)/(x^2+2^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {275, 218}

$$\frac{\sqrt{2+\sqrt{3}}\,\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\,F\left(\sin^{-1}\left(\frac{x^2+\sqrt[3]{2}\left(1-\sqrt{3}\right)}{x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)}\right)|-7-4\sqrt{3}\right)}{\sqrt[6]{2}\,\sqrt[4]{3}\,\sqrt{\frac{x^2+\sqrt[3]{2}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\,\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[2 + x^6],x]
```

```
[Out] (Sqrt[2 + Sqrt[3]]*(2^(1/3) + x^2)*Sqrt[(2^(2/3) - 2^(1/3)*x^2 + x^4)/(2^(1
/3)*(1 + Sqrt[3]) + x^2)^2]*EllipticF[ArcSin[(2^(1/3)*(1 - Sqrt[3]) + x^2)/
(2^(1/3)*(1 + Sqrt[3]) + x^2)], -7 - 4*Sqrt[3]])/(2^(1/6)*3^(1/4)*Sqrt[(2^(
1/3) + x^2)/(2^(1/3)*(1 + Sqrt[3]) + x^2)^2]*Sqrt[2 + x^6])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x}{\sqrt{2+x^6}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{2+x^3}}\,dx, x, x^2\right)$$
$$= \frac{\sqrt{2+\sqrt{3}}\,\left(\sqrt[3]{2}+x^2\right)\sqrt{\frac{2^{2/3}-\sqrt[3]{2}\,x^2+x^4}{\left(\sqrt[3]{2}\left(1+\sqrt{3}\right)+x^2\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{2}\left(1-\sqrt{3}\right)+x^2}{\sqrt[3]{2}\left(1+\sqrt{3}\right)+x^2}\right)|-7-4\sqrt{3}\right)}{\sqrt[6]{2}\,\sqrt[4]{3}\,\sqrt{\frac{\sqrt[3]{2}+x^2}{\left(\sqrt[3]{2}\left(1+\sqrt{3}\right)+x^2\right)^2}}\,\sqrt{2+x^6}}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.17

$$\frac{x^2\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{x^6}{2}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[2 + x^6],x]
[Out] (x^2*Hypergeometric2F1[1/3, 1/2, 4/3, -1/2*x^6])/(2*Sqrt[2])
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{\sqrt{x^6+2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+2)^(1/2),x, algorithm="fricas")
[Out] integral(x/sqrt(x^6 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^6+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+2)^(1/2),x, algorithm="giac")
[Out] integrate(x/sqrt(x^6 + 2), x)
```

maple [C] time = 0.14, size = 20, normalized size = 0.12

$$\frac{\sqrt{2}\,x^2\,\text{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],-\frac{x^6}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^6+2)^(1/2),x)
[Out] 1/4*2^(1/2)*x^2*hypergeom([1/3,1/2],[4/3],-1/2*x^6)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^6+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(x^6 + 2), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{x^6+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^6 + 2)^(1/2),x)
```

```
[Out] int(x/(x^6 + 2)^(1/2), x)
```

sympy **[A]** time = 1.17, size = 36, normalized size = 0.22

$$\frac{\sqrt{2}\,x^2\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3}, \frac{1}{2} \\ \frac{4}{3}\end{matrix}\middle| \frac{x^6 e^{i\pi}}{2}\right)}{12\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**6+2)**(1/2),x)
```

```
[Out] sqrt(2)*x**2*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**6*exp_polar(I*pi)/2)/(
12*gamma(4/3))
```

3.1398 $\int \frac{1}{x^5\sqrt{2+x^6}}\,dx$

Optimal. Leaf size=186

$$-\frac{\sqrt{x^6+2}}{8x^4}-\frac{\sqrt{2+\sqrt{3}}\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}F\left(\sin^{-1}\left(\frac{x^2+\sqrt[3]{2}\left(1-\sqrt{3}\right)}{x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)}\right)|-7-4\sqrt{3}\right)}{8\sqrt[6]{2}\sqrt[4]{3}\sqrt{\frac{x^2+\sqrt[3]{2}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\sqrt{x^6+2}}$$

```
[Out] -1/8*(x^6+2)^(1/2)/x^4-1/48*2^(5/6)*(2^(1/3)+x^2)*EllipticF((x^2+2^(1/3)*(1
-3^(1/2)))/(x^2+2^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2
))*((2^(2/3)-2^(1/3)*x^2+x^4)/(x^2+2^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/(x
^6+2)^(1/2)/((2^(1/3)+x^2)/(x^2+2^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {275, 325, 218}

$$-\frac{\sqrt{x^6+2}}{8x^4}-\frac{\sqrt{2+\sqrt{3}}\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}F\left(\sin^{-1}\left(\frac{x^2+\sqrt[3]{2}\left(1-\sqrt{3}\right)}{x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)}\right)|-7-4\sqrt{3}\right)}{8\sqrt[6]{2}\sqrt[4]{3}\sqrt{\frac{x^2+\sqrt[3]{2}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[2 + x^6]),x]
```

```
[Out] -Sqrt[2 + x^6]/(8*x^4) - (Sqrt[2 + Sqrt[3]]*(2^(1/3) + x^2)*Sqrt[(2^(2/3) -
 2^(1/3)*x^2 + x^4)/(2^(1/3)*(1 + Sqrt[3]) + x^2)^2]*EllipticF[ArcSin[(2^(1
/3)*(1 - Sqrt[3]) + x^2)/(2^(1/3)*(1 + Sqrt[3]) + x^2)], -7 - 4*Sqrt[3]])/(
8*2^(1/6)*3^(1/4)*Sqrt[(2^(1/3) + x^2)/(2^(1/3)*(1 + Sqrt[3]) + x^2)^2]*Sqr
t[2 + x^6])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^5\sqrt{2+x^6}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{2+x^3}}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt{2+x^6}}{8x^4} - \frac{1}{16}\,\text{Subst}\left(\int \frac{1}{\sqrt{2+x^3}}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt{2+x^6}}{8x^4} - \frac{\sqrt{2+\sqrt{3}}\,\left(\sqrt[3]{2}+x^2\right)\sqrt{\frac{2^{2/3}-\sqrt[3]{2}\,x^2+x^4}{\left(\sqrt[3]{2}\left(1+\sqrt{3}\right)+x^2\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{2}\left(1-\sqrt{3}\right)+x^2}{\sqrt[3]{2}\left(1+\sqrt{3}\right)+x^2}\right)\middle|-7-4\sqrt{3}\right)}{8\sqrt[6]{2}\,\sqrt[4]{3}\,\sqrt{\frac{\sqrt[3]{2}+x^2}{\left(\sqrt[3]{2}\left(1+\sqrt{3}\right)+x^2\right)^2}}\,\sqrt{2+x^6}}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.16

$$-\frac{{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};-\frac{x^6}{2}\right)}{4\sqrt{2}\,x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[2 + x^6]),x]
[Out] -1/4*Hypergeometric2F1[-2/3, 1/2, 1/3, -1/2*x^6]/(Sqrt[2]*x^4)
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^6+2}}{x^{11}+2\,x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^6+2)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x^6 + 2)/(x^11 + 2*x^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^6+2}\,x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^6+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^6 + 2)*x^5), x)
```

maple [C] time = 0.14, size = 33, normalized size = 0.18

$$-\frac{\sqrt{2}\,x^2\,\text{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],-\frac{x^6}{2}\right)}{32} - \frac{\sqrt{x^6+2}}{8x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^6+2)^(1/2),x)
[Out] -1/8*(x^6+2)^(1/2)/x^4-1/32*2^(1/2)*x^2*hypergeom([1/3,1/2],[4/3],-1/2*x^6)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^6+2}\,x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^6+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^6 + 2)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^5 \sqrt{x^6+2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(x^6 + 2)^(1/2)),x)
```

```
[Out] int(1/(x^5*(x^6 + 2)^(1/2)), x)
```

sympy [A] time = 0.95, size = 39, normalized size = 0.21

$$\frac{\sqrt{2}\,\Gamma\left(-\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{2}{3}, \frac{1}{2} \\ \frac{1}{3}\end{matrix}\middle| \frac{x^6 e^{i\pi}}{2}\right)}{12 x^4 \Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**6+2)**(1/2),x)
```

```
[Out] sqrt(2)*gamma(-2/3)*hyper((-2/3, 1/2), (1/3,), x**6*exp_polar(I*pi)/2)/(12*
x**4*gamma(1/3))
```

3.1399 $\int \frac{x^6}{\sqrt{2+x^6}}\,dx$

Optimal. Leaf size=179

$$\frac{1}{4}x\sqrt{x^6+2}-\frac{x\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x^2+\sqrt[3]{2}}{\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[3]{2}\sqrt[4]{3}\sqrt{\frac{x^2\left(x^2+\sqrt[3]{2}\right)}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\sqrt{x^6+2}}$$

```
[Out] 1/4*x*(x^6+2)^(1/2)-1/24*x*(2^(1/3)+x^2)*((2^(1/3)+x^2*(1-3^(1/2)))^2/(2^(1
/3)+x^2*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)+x^2*(1-3^(1/2)))*(2^(1/3)+x^2*(1+3^(
1/2)))*EllipticF((1-(2^(1/3)+x^2*(1-3^(1/2)))^2/(2^(1/3)+x^2*(1+3^(1/2)))^2
)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((2^(2/3)-2^(1/3)*x^2+x^4)/(2^(1/3)+x^2*(1
+3^(1/2)))^2)^(1/2)*2^(2/3)*3^(3/4)/(x^6+2)^(1/2)/(x^2*(2^(1/3)+x^2)/(2^(1/
3)+x^2*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {321, 225}

$$\frac{1}{4}x\sqrt{x^6+2}-\frac{x\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x^2+\sqrt[3]{2}}{\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[3]{2}\sqrt[4]{3}\sqrt{\frac{x^2\left(x^2+\sqrt[3]{2}\right)}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/Sqrt[2 + x^6],x]
```

```
[Out] (x*Sqrt[2 + x^6])/4 - (x*(2^(1/3) + x^2)*Sqrt[(2^(2/3) - 2^(1/3)*x^2 + x^4)
/(2^(1/3) + (1 + Sqrt[3])*x^2)^2]*EllipticF[ArcCos[(2^(1/3) + (1 - Sqrt[3])
*x^2)/(2^(1/3) + (1 + Sqrt[3])*x^2)], (2 + Sqrt[3])/4])/(4*2^(1/3)*3^(1/4)*
Sqrt[(x^2*(2^(1/3) + x^2))/(2^(1/3) + (1 + Sqrt[3])*x^2)^2]*Sqrt[2 + x^6])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^6}{\sqrt{2+x^6}}\,dx = \frac{1}{4}x\sqrt{2+x^6} - \frac{1}{2}\int \frac{1}{\sqrt{2+x^6}}\,dx$$

$$= \frac{1}{4}x\sqrt{2+x^6} - \frac{x\left(\sqrt[3]{2}+x^2\right)\sqrt{\frac{2^{2/3}-\sqrt[3]{2}x^2+x^4}{\left(\sqrt[3]{2}+\left(1+\sqrt{3}\right)x^2\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{2}+\left(1-\sqrt{3}\right)x^2}{\sqrt[3]{2}+\left(1+\sqrt{3}\right)x^2}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[3]{2}\sqrt[4]{3}\sqrt{\frac{x^2\left(\sqrt[3]{2}+x^2\right)}{\left(\sqrt[3]{2}+\left(1+\sqrt{3}\right)x^2\right)^2}}\sqrt{2+x^6}}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.22

$$\frac{1}{4}x\left(\sqrt{x^6+2} - \sqrt{2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{x^6}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/Sqrt[2 + x^6],x]
[Out] (x*(Sqrt[2 + x^6] - Sqrt[2]*Hypergeometric2F1[1/6, 1/2, 7/6, -1/2*x^6]))/4
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^6}{\sqrt{x^6+2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6+2)^(1/2),x, algorithm="fricas")
[Out] integral(x^6/sqrt(x^6 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt{x^6+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6+2)^(1/2),x, algorithm="giac")
[Out] integrate(x^6/sqrt(x^6 + 2), x)
```

maple [C] time = 0.14, size = 29, normalized size = 0.16

$$-\frac{\sqrt{2}\,x\,\text{hypergeom}\left(\left[\frac{1}{6},\frac{1}{2}\right],\left[\frac{7}{6}\right],-\frac{x^6}{2}\right)}{4} + \frac{\sqrt{x^6+2}\,x}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^6+2)^(1/2),x)
[Out] 1/4*x*(x^6+2)^(1/2)-1/4*2^(1/2)*x*hypergeom([1/6,1/2],[7/6],-1/2*x^6)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt{x^6+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/sqrt(x^6 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{\sqrt{x^6+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^6 + 2)^(1/2),x)
```

```
[Out] int(x^6/(x^6 + 2)^(1/2), x)
```

sympy [C] time = 1.45, size = 36, normalized size = 0.20

$$\frac{\sqrt{2}\,x^7\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\right|\frac{x^6e^{i\pi}}{2}\right)}{12\Gamma\left(\frac{13}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**6+2)**(1/2),x)
```

```
[Out] sqrt(2)*x**7*gamma(7/6)*hyper((1/2, 7/6), (13/6,), x**6*exp_polar(I*pi)/2)/
(12*gamma(13/6))
```

3.1400 $\int \frac{1}{\sqrt{2+x^6}}\,dx$

Optimal. Leaf size=164

$$\frac{x\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x^2+\sqrt[3]{2}}{\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[3]{2}\,\sqrt[4]{3}\,\sqrt{\frac{x^2\left(x^2+\sqrt[3]{2}\right)}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\,\sqrt{x^6+2}}$$

```
[Out] 1/12*x*(2^(1/3)+x^2)*((2^(1/3)+x^2*(1-3^(1/2)))^2/(2^(1/3)+x^2*(1+3^(1/2)))
^2)^(1/2)/(2^(1/3)+x^2*(1-3^(1/2)))*(2^(1/3)+x^2*(1+3^(1/2)))*EllipticF((1-
(2^(1/3)+x^2*(1-3^(1/2)))^2/(2^(1/3)+x^2*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+
1/4*2^(1/2))*((2^(2/3)-2^(1/3)*x^2+x^4)/(2^(1/3)+x^2*(1+3^(1/2)))^2)^(1/2)*
2^(2/3)*3^(3/4)/(x^6+2)^(1/2)/(x^2*(2^(1/3)+x^2)/(2^(1/3)+x^2*(1+3^(1/2)))^
2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {225}

$$\frac{x\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x^2+\sqrt[3]{2}}{\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[3]{2}\,\sqrt[4]{3}\,\sqrt{\frac{x^2\left(x^2+\sqrt[3]{2}\right)}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\,\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + x^6],x]
```

```
[Out] (x*(2^(1/3) + x^2)*Sqrt[(2^(2/3) - 2^(1/3)*x^2 + x^4)/(2^(1/3) + (1 + Sqrt[
3])*x^2)^2]*EllipticF[ArcCos[(2^(1/3) + (1 - Sqrt[3])*x^2)/(2^(1/3) + (1 +
Sqrt[3])*x^2)], (2 + Sqrt[3])/4])/(2*2^(1/3)*3^(1/4)*Sqrt[(x^2*(2^(1/3) + x
^2))/(2^(1/3) + (1 + Sqrt[3])*x^2)^2]*Sqrt[2 + x^6])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+x^6}}\,dx = \frac{x\left(\sqrt[3]{2}+x^2\right)\sqrt{\frac{2^{2/3}-\sqrt[3]{2}x^2+x^4}{\left(\sqrt[3]{2}+\left(1+\sqrt{3}\right)x^2\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{2}+\left(1-\sqrt{3}\right)x^2}{\sqrt[3]{2}+\left(1+\sqrt{3}\right)x^2}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[3]{2}\,\sqrt[4]{3}\,\sqrt{\frac{x^2\left(\sqrt[3]{2}+x^2\right)}{\left(\sqrt[3]{2}+\left(1+\sqrt{3}\right)x^2\right)^2}}\,\sqrt{2+x^6}}$$

Mathematica [C] time = 0.00, size = 24, normalized size = 0.15

$$\frac{x\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{x^6}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + x^6],x]
[Out] (x*Hypergeometric2F1[1/6, 1/2, 7/6, -1/2*x^6])/Sqrt[2]
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{x^6+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+2)^(1/2),x, algorithm="fricas")
[Out] integral(1/sqrt(x^6 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^6+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(x^6 + 2), x)
```

maple [C] time = 0.13, size = 18, normalized size = 0.11

$$\frac{\sqrt{2}\,x\,\text{hypergeom}\left(\left[\frac{1}{6},\frac{1}{2}\right],\left[\frac{7}{6}\right],-\frac{x^6}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6+2)^(1/2),x)
[Out] 1/2*2^(1/2)*x*hypergeom([1/6,1/2],[7/6],-1/2*x^6)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^6+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(x^6 + 2), x)
```

mupad [B] time = 0.08, size = 16, normalized size = 0.10

$$\frac{\sqrt{2}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{6},\frac{1}{2};\ \frac{7}{6};\ -\frac{x^6}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6 + 2)^(1/2),x)
```

```
[Out] (2^(1/2)*x*hypergeom([1/6, 1/2], 7/6, -x^6/2))/2
```

sympy [C] time = 1.10, size = 34, normalized size = 0.21

$$\frac{\sqrt{2}\,x\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{6}, \frac{1}{2} \\ \frac{7}{6}\end{matrix}\right| \frac{x^6 e^{i\pi}}{2}\right)}{12\Gamma\left(\frac{7}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6+2)**(1/2),x)
```

```
[Out] sqrt(2)*x*gamma(1/6)*hyper((1/6, 1/2), (7/6,), x**6*exp_polar(I*pi)/2)/(12*
gamma(7/6))
```

3.1401 $\int \frac{1}{x^6\sqrt{2+x^6}}\,dx$

Optimal. Leaf size=181

$$-\frac{\sqrt{x^6+2}}{10x^5}-\frac{x\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x^2+\sqrt[3]{2}}{\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{10\sqrt[3]{2}\,\sqrt[4]{3}\,\sqrt{\frac{x^2\left(x^2+\sqrt[3]{2}\right)}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\,\sqrt{x^6+2}}$$

```
[Out] -1/10*(x^6+2)^(1/2)/x^5-1/60*x*(2^(1/3)+x^2)*((2^(1/3)+x^2*(1-3^(1/2)))^2/(
2^(1/3)+x^2*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)+x^2*(1-3^(1/2)))*(2^(1/3)+x^2*(1
+3^(1/2)))*EllipticF((1-(2^(1/3)+x^2*(1-3^(1/2)))^2/(2^(1/3)+x^2*(1+3^(1/2)
))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((2^(2/3)-2^(1/3)*x^2+x^4)/(2^(1/3)+x^
2*(1+3^(1/2)))^2)^(1/2)*2^(2/3)*3^(3/4)/(x^6+2)^(1/2)/(x^2*(2^(1/3)+x^2)/(2
^(1/3)+x^2*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {325, 225}

$$-\frac{\sqrt{x^6+2}}{10x^5}-\frac{x\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x^2+\sqrt[3]{2}}{\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{10\sqrt[3]{2}\,\sqrt[4]{3}\,\sqrt{\frac{x^2\left(x^2+\sqrt[3]{2}\right)}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\,\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*Sqrt[2 + x^6]),x]
```

```
[Out] -Sqrt[2 + x^6]/(10*x^5) - (x*(2^(1/3) + x^2)*Sqrt[(2^(2/3) - 2^(1/3)*x^2 +
x^4)/(2^(1/3) + (1 + Sqrt[3])*x^2)^2]*EllipticF[ArcCos[(2^(1/3) + (1 - Sqrt
[3])*x^2)/(2^(1/3) + (1 + Sqrt[3])*x^2)], (2 + Sqrt[3])/4])/(10*2^(1/3)*3^(
1/4)*Sqrt[(x^2*(2^(1/3) + x^2))/(2^(1/3) + (1 + Sqrt[3])*x^2)^2]*Sqrt[2 + x
^6])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^6\sqrt{2+x^6}}\,dx = -\frac{\sqrt{2+x^6}}{10x^5} - \frac{1}{5}\int \frac{1}{\sqrt{2+x^6}}\,dx$$

$$= -\frac{\sqrt{2+x^6}}{10x^5} - \frac{x\left(\sqrt[3]{2}+x^2\right)\sqrt{\frac{2^{2/3}-\sqrt[3]{2}x^2+x^4}{\left(\sqrt[3]{2}+\left(1+\sqrt{3}\right)x^2\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{2}+\left(1-\sqrt{3}\right)x^2}{\sqrt[3]{2}+\left(1+\sqrt{3}\right)x^2}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{10\sqrt[3]{2}\,\sqrt[4]{3}\,\sqrt{\frac{x^2\left(\sqrt[3]{2}+x^2\right)}{\left(\sqrt[3]{2}+\left(1+\sqrt{3}\right)x^2\right)^2}}\,\sqrt{2+x^6}}$$

Mathematica [C] time = 0.00, size = 29, normalized size = 0.16

$$-\frac{{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};-\frac{x^6}{2}\right)}{5\sqrt{2}\,x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*Sqrt[2 + x^6]),x]
[Out] -1/5*Hypergeometric2F1[-5/6, 1/2, 1/6, -1/2*x^6]/(Sqrt[2]*x^5)
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^6+2}}{x^{12}+2\,x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^6+2)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x^6 + 2)/(x^12 + 2*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^6+2}\,x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^6+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^6 + 2)*x^6), x)
```

maple [C] time = 0.13, size = 31, normalized size = 0.17

$$-\frac{\sqrt{2}\,x\,\text{hypergeom}\left(\left[\frac{1}{6},\frac{1}{2}\right],\left[\frac{7}{6}\right],-\frac{x^6}{2}\right)}{10} - \frac{\sqrt{x^6+2}}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^6+2)^(1/2),x)
[Out] -1/10*(x^6+2)^(1/2)/x^5-1/10*2^(1/2)*x*hypergeom([1/6,1/2],[7/6],-1/2*x^6)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^6+2}\,x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^6+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^6 + 2)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^6\sqrt{x^6+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(x^6 + 2)^(1/2)),x)
```

```
[Out] int(1/(x^6*(x^6 + 2)^(1/2)), x)
```

sympy [C] time = 1.70, size = 39, normalized size = 0.22

$$\frac{\sqrt{2}\,\Gamma\left(-\frac{5}{6}\right){}_2F_1\left(\left.\begin{matrix}-\frac{5}{6}, \frac{1}{2} \\ \frac{1}{6}\end{matrix}\right| \frac{x^6 e^{i\pi}}{2}\right)}{12x^5\Gamma\left(\frac{1}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**6+2)**(1/2),x)
```

```
[Out] sqrt(2)*gamma(-5/6)*hyper((-5/6, 1/2), (1/6,), x**6*exp_polar(I*pi)/2)/(12*
x**5*gamma(1/6))
```

3.1402 $\int \frac{x^9}{\sqrt{2+x^6}}\,dx$

Optimal. Leaf size=378

$$\frac{1}{7}\sqrt{x^6+2}\,x^4-\frac{8\sqrt{x^6+2}}{7\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)}-\frac{8\,2^{2/3}\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\,F\left(\sin^{-1}\left(\frac{x^2+\sqrt[3]{2}\left(1-\sqrt{3}\right)}{x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)}\right)|-7-4\sqrt{3}\right)}{7\sqrt[4]{3}\sqrt{\frac{x^2+\sqrt[3]{2}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\sqrt{x^6+2}}$$

```
[Out] 1/7*x^4*(x^6+2)^(1/2)-8/7*(x^6+2)^(1/2)/(x^2+2^(1/3)*(1+3^(1/2)))-8/21*2^(2
/3)*(2^(1/3)+x^2)*EllipticF((x^2+2^(1/3)*(1-3^(1/2)))/(x^2+2^(1/3)*(1+3^(1/
2))),I*3^(1/2)+2*I)*((2^(2/3)-2^(1/3)*x^2+x^4)/(x^2+2^(1/3)*(1+3^(1/2)))^2)
^(1/2)*3^(3/4)/(x^6+2)^(1/2)/((2^(1/3)+x^2)/(x^2+2^(1/3)*(1+3^(1/2)))^2)^(1
/2)+4/7*2^(1/6)*3^(1/4)*(2^(1/3)+x^2)*EllipticE((x^2+2^(1/3)*(1-3^(1/2)))/(
x^2+2^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((2^(2/3)
-2^(1/3)*x^2+x^4)/(x^2+2^(1/3)*(1+3^(1/2)))^2)^(1/2)/(x^6+2)^(1/2)/((2^(1/3
)+x^2)/(x^2+2^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {275, 321, 303, 218, 1877}

$$\frac{1}{7}\sqrt{x^6+2}\,x^4-\frac{8\sqrt{x^6+2}}{7\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)}-\frac{8\,2^{2/3}\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\,F\left(\sin^{-1}\left(\frac{x^2+\sqrt[3]{2}\left(1-\sqrt{3}\right)}{x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)}\right)|-7-4\sqrt{3}\right)}{7\sqrt[4]{3}\sqrt{\frac{x^2+\sqrt[3]{2}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/Sqrt[2 + x^6],x]
```

```
[Out] (x^4*Sqrt[2 + x^6])/7 - (8*Sqrt[2 + x^6])/(7*(2^(1/3)*(1 + Sqrt[3]) + x^2))
 + (4*2^(1/6)*3^(1/4)*Sqrt[2 - Sqrt[3]]*(2^(1/3) + x^2)*Sqrt[(2^(2/3) - 2^(
1/3)*x^2 + x^4)/(2^(1/3)*(1 + Sqrt[3]) + x^2)^2]*EllipticE[ArcSin[(2^(1/3)*
(1 - Sqrt[3]) + x^2)/(2^(1/3)*(1 + Sqrt[3]) + x^2)], -7 - 4*Sqrt[3]])/(7*Sq
rt[(2^(1/3) + x^2)/(2^(1/3)*(1 + Sqrt[3]) + x^2)^2]*Sqrt[2 + x^6]) - (8*2^(
2/3)*(2^(1/3) + x^2)*Sqrt[(2^(2/3) - 2^(1/3)*x^2 + x^4)/(2^(1/3)*(1 + Sqrt[
3]) + x^2)^2]*EllipticF[ArcSin[(2^(1/3)*(1 - Sqrt[3]) + x^2)/(2^(1/3)*(1 +
Sqrt[3]) + x^2)], -7 - 4*Sqrt[3]])/(7*3^(1/4)*Sqrt[(2^(1/3) + x^2)/(2^(1/3)
*(1 + Sqrt[3]) + x^2)^2]*Sqrt[2 + x^6])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9}{\sqrt{2+x^6}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{\sqrt{2+x^3}}\,dx, x, x^2\right) \\
&= \frac{1}{7}x^4\sqrt{2+x^6} - \frac{4}{7}\,\text{Subst}\left(\int \frac{x}{\sqrt{2+x^3}}\,dx, x, x^2\right) \\
&= \frac{1}{7}x^4\sqrt{2+x^6} - \frac{4}{7}\,\text{Subst}\left(\int \frac{\sqrt[3]{2}\left(1-\sqrt{3}\right)+x}{\sqrt{2+x^3}}\,dx, x, x^2\right) - \frac{\left(4\,2^{5/6}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2+x^3}}\,dx, x, x^2\right)}{7\sqrt{2+\sqrt{3}}} \\
&= \frac{1}{7}x^4\sqrt{2+x^6} - \frac{8\sqrt{2+x^6}}{7\left(\sqrt[3]{2}\left(1+\sqrt{3}\right)+x^2\right)} + \frac{4\sqrt[6]{2}\,\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\left(\sqrt[3]{2}+x^2\right)\sqrt{\frac{2^{2/3}-\sqrt[3]{2}x^2+x^4}{\left(\sqrt[3]{2}\left(1+\sqrt{3}\right)+x^2\right)^2}}\,E\left(\sin
\end{aligned}$$

$$\frac{4\sqrt[6]{2}\,\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\left(\sqrt[3]{2}+x^2\right)\sqrt{\frac{2^{2/3}-\sqrt[3]{2}x^2+x^4}{\left(\sqrt[3]{2}\left(1+\sqrt{3}\right)+x^2\right)^2}}\,E\left(\sin
}{7\sqrt{\frac{\sqrt[3]{2}+x^2}{\left(\sqrt[3]{2}\left(1+\sqrt{3}\right)+x^2\right)^2}}\,\sqrt{2+
}$$

Mathematica [C] time = 0.03, size = 41, normalized size = 0.11

$$\frac{1}{7}x^4\left(\sqrt{x^6+2} - \sqrt{2}\;{}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; -\frac{x^6}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/Sqrt[2 + x^6],x]
```

```
[Out] (x^4*(Sqrt[2 + x^6] - Sqrt[2]*Hypergeometric2F1[1/2, 2/3, 5/3, -1/2*x^6]))/
7
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^9}{\sqrt{x^6+2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^6+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^9/sqrt(x^6 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\sqrt{x^6+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^6+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^9/sqrt(x^6 + 2), x)
```

maple [C] time = 0.15, size = 33, normalized size = 0.09

$$-\frac{\sqrt{2}\,x^4\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{2}{3}\right],\left[\frac{5}{3}\right],-\frac{x^6}{2}\right)}{7}+\frac{\sqrt{x^6+2}\,x^4}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^6+2)^(1/2),x)
```

```
[Out] 1/7*x^4*(x^6+2)^(1/2)-1/7*2^(1/2)*x^4*hypergeom([1/2,2/3],[5/3],-1/2*x^6)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\sqrt{x^6+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^6+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^9/sqrt(x^6 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^9}{\sqrt{x^6+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^6 + 2)^(1/2),x)
```

```
[Out] int(x^9/(x^6 + 2)^(1/2), x)
```

sympy [A] time = 1.02, size = 36, normalized size = 0.10

$$\frac{\sqrt{2}\,x^{10}\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\right|\frac{x^6e^{i\pi}}{2}\right)}{12\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(x**6+2)**(1/2),x)
```

```
[Out] sqrt(2)*x**10*gamma(5/3)*hyper((1/2, 5/3), (8/3,), x**6*exp_polar(I*pi)/2)/
(12*gamma(8/3))
```

3.1403 $\int \frac{x^3}{\sqrt{2+x^6}}\,dx$

Optimal. Leaf size=354

$$\frac{\sqrt{x^6+2}}{x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)}+\frac{2^{2/3}\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}F\left(\sin^{-1}\left(\frac{x^2+\sqrt[3]{2}\left(1-\sqrt{3}\right)}{x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)}\right)|-7-4\sqrt{3}\right)\quad\sqrt[4]{3}\sqrt{2-\sqrt{3}}\ \left(x^2+\right.}{\sqrt[4]{3}\sqrt{\frac{x^2+\sqrt[3]{2}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\sqrt{x^6+2}}-$$

```
[Out] (x^6+2)^(1/2)/(x^2+2^(1/3)*(1+3^(1/2)))+1/3*2^(2/3)*(2^(1/3)+x^2)*EllipticF
((x^2+2^(1/3)*(1-3^(1/2)))/(x^2+2^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2^(2/
3)-2^(1/3)*x^2+x^4)/(x^2+2^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/(x^6+2)^(1/2
)/((2^(1/3)+x^2)/(x^2+2^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/2*2^(1/6)*3^(1/4)*(2^
(1/3)+x^2)*EllipticE((x^2+2^(1/3)*(1-3^(1/2)))/(x^2+2^(1/3)*(1+3^(1/2))),I*
3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((2^(2/3)-2^(1/3)*x^2+x^4)/(x^2+2^(1
/3)*(1+3^(1/2)))^2)^(1/2)/(x^6+2)^(1/2)/((2^(1/3)+x^2)/(x^2+2^(1/3)*(1+3^(1
/2)))^2)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {275, 303, 218, 1877}

$$\frac{\sqrt{x^6+2}}{x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)}+\frac{2^{2/3}\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}F\left(\sin^{-1}\left(\frac{x^2+\sqrt[3]{2}\left(1-\sqrt{3}\right)}{x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)}\right)|-7-4\sqrt{3}\right)\quad\sqrt[4]{3}\sqrt{2-\sqrt{3}}\ \left(x^2+\right.}{\sqrt[4]{3}\sqrt{\frac{x^2+\sqrt[3]{2}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\sqrt{x^6+2}}-$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[2 + x^6],x]
```

```
[Out] Sqrt[2 + x^6]/(2^(1/3)*(1 + Sqrt[3]) + x^2) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(2
^(1/3) + x^2)*Sqrt[(2^(2/3) - 2^(1/3)*x^2 + x^4)/(2^(1/3)*(1 + Sqrt[3]) + x
^2)^2]*EllipticE[ArcSin[(2^(1/3)*(1 - Sqrt[3]) + x^2)/(2^(1/3)*(1 + Sqrt[3]
) + x^2)], -7 - 4*Sqrt[3]])/(2^(5/6)*Sqrt[(2^(1/3) + x^2)/(2^(1/3)*(1 + Sqr
t[3]) + x^2)^2]*Sqrt[2 + x^6]) + (2^(2/3)*(2^(1/3) + x^2)*Sqrt[(2^(2/3) - 2
^(1/3)*x^2 + x^4)/(2^(1/3)*(1 + Sqrt[3]) + x^2)^2]*EllipticF[ArcSin[(2^(1/3
)*(1 - Sqrt[3]) + x^2)/(2^(1/3)*(1 + Sqrt[3]) + x^2)], -7 - 4*Sqrt[3]])/(3^
(1/4)*Sqrt[(2^(1/3) + x^2)/(2^(1/3)*(1 + Sqrt[3]) + x^2)^2]*Sqrt[2 + x^6])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{2+x^6}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{\sqrt{2+x^3}}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt[3]{2}\left(1-\sqrt{3}\right)+x}{\sqrt{2+x^3}}\,dx, x, x^2\right) + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2+x^3}}\,dx, x, x^2\right)}{\sqrt[6]{2}\sqrt{2+\sqrt{3}}}$$

$$= \frac{\sqrt{2+x^6}}{\sqrt[3]{2}\left(1+\sqrt{3}\right)+x^2} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\left(\sqrt[3]{2}+x^2\right)\sqrt{\frac{2^{2/3}-\sqrt[3]{2}x^2+x^4}{\left(\sqrt[3]{2}\left(1+\sqrt{3}\right)+x^2\right)^2}}\,E\left(\sin^{-1}\left(\frac{\sqrt[3]{2}\left(1-\sqrt{3}\right)+x^2}{\sqrt[3]{2}\left(1+\sqrt{3}\right)+x^2}\right)\right| -}{2^{5/6}\sqrt{\frac{\sqrt[3]{2}+x^2}{\left(\sqrt[3]{2}\left(1+\sqrt{3}\right)+x^2\right)^2}}\,\sqrt{2+x^6}}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.08

$$\frac{x^4\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{x^6}{2}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[2 + x^6],x]
```

```
[Out] (x^4*Hypergeometric2F1[1/2, 2/3, 5/3, -1/2*x^6])/(4*Sqrt[2])
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3}{\sqrt{x^6+2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^3/sqrt(x^6 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{x^6+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+2)^(1/2),x, algorithm="giac")
[Out] integrate(x^3/sqrt(x^6 + 2), x)
```

maple [C] time = 0.15, size = 20, normalized size = 0.06

$$\frac{\sqrt{2}\, x^4 \,\mathrm{hypergeom}\left(\left[\frac{1}{2}, \frac{2}{3}\right], \left[\frac{5}{3}\right], -\frac{x^6}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^6+2)^(1/2),x)
[Out] 1/8*2^(1/2)*x^4*hypergeom([1/2,2/3],[5/3],-1/2*x^6)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{x^6+2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+2)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/sqrt(x^6 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\sqrt{x^6+2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^6 + 2)^(1/2),x)
[Out] int(x^3/(x^6 + 2)^(1/2), x)
```

sympy [A] time = 0.72, size = 36, normalized size = 0.10

$$\frac{\sqrt{2}\, x^4 \Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{2}{3} \\ \frac{5}{3}\end{matrix}\right| \frac{x^6 e^{i\pi}}{2}\right)}{12\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**6+2)**(1/2),x)
[Out] sqrt(2)*x**4*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**6*exp_polar(I*pi)/2)/(
12*gamma(5/3))
```

3.1404 $\int \frac{1}{x^3\sqrt{2+x^6}}\,dx$

Optimal. Leaf size=378

$$\frac{\sqrt{x^6+2}}{4\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)}-\frac{\sqrt{x^6+2}}{4x^2}+\frac{\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}F\left(\sin^{-1}\left(\frac{x^2+\sqrt[3]{2}\left(1-\sqrt{3}\right)}{x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)}\right)|-7-4\sqrt{3}\right)\quad\sqrt[4]{3}\sqrt{2}}{2\sqrt[3]{2}\sqrt[4]{3}\sqrt{\frac{x^2+\sqrt[3]{2}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\sqrt{x^6+2}}-\frac{}{}$$

```
[Out] -1/4*(x^6+2)^(1/2)/x^2+1/4*(x^6+2)^(1/2)/(x^2+2^(1/3)*(1+3^(1/2)))+1/12*2^(
2/3)*(2^(1/3)+x^2)*EllipticF((x^2+2^(1/3)*(1-3^(1/2)))/(x^2+2^(1/3)*(1+3^(1
/2))),I*3^(1/2)+2*I)*((2^(2/3)-2^(1/3)*x^2+x^4)/(x^2+2^(1/3)*(1+3^(1/2)))^2
)^(1/2)*3^(3/4)/(x^6+2)^(1/2)/((2^(1/3)+x^2)/(x^2+2^(1/3)*(1+3^(1/2)))^2)^(
1/2)-1/8*2^(1/6)*3^(1/4)*(2^(1/3)+x^2)*EllipticE((x^2+2^(1/3)*(1-3^(1/2)))/
(x^2+2^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((2^(2/3
)-2^(1/3)*x^2+x^4)/(x^2+2^(1/3)*(1+3^(1/2)))^2)^(1/2)/(x^6+2)^(1/2)/((2^(1/
3)+x^2)/(x^2+2^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {275, 325, 303, 218, 1877}

$$\frac{\sqrt{x^6+2}}{4\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)}-\frac{\sqrt{x^6+2}}{4x^2}+\frac{\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}F\left(\sin^{-1}\left(\frac{x^2+\sqrt[3]{2}\left(1-\sqrt{3}\right)}{x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)}\right)|-7-4\sqrt{3}\right)\quad\sqrt[4]{3}\sqrt{2}}{2\sqrt[3]{2}\sqrt[4]{3}\sqrt{\frac{x^2+\sqrt[3]{2}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\sqrt{x^6+2}}-\frac{}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[2 + x^6]),x]
```

```
[Out] -Sqrt[2 + x^6]/(4*x^2) + Sqrt[2 + x^6]/(4*(2^(1/3)*(1 + Sqrt[3]) + x^2)) -
(3^(1/4)*Sqrt[2 - Sqrt[3]]*(2^(1/3) + x^2)*Sqrt[(2^(2/3) - 2^(1/3)*x^2 + x^
4)/(2^(1/3)*(1 + Sqrt[3]) + x^2)^2]*EllipticE[ArcSin[(2^(1/3)*(1 - Sqrt[3])
 + x^2)/(2^(1/3)*(1 + Sqrt[3]) + x^2)], -7 - 4*Sqrt[3]])/(4*2^(5/6)*Sqrt[(2
^(1/3) + x^2)/(2^(1/3)*(1 + Sqrt[3]) + x^2)^2]*Sqrt[2 + x^6]) + ((2^(1/3) +
 x^2)*Sqrt[(2^(2/3) - 2^(1/3)*x^2 + x^4)/(2^(1/3)*(1 + Sqrt[3]) + x^2)^2]*E
llipticF[ArcSin[(2^(1/3)*(1 - Sqrt[3]) + x^2)/(2^(1/3)*(1 + Sqrt[3]) + x^2)
], -7 - 4*Sqrt[3]])/(2*2^(1/3)*3^(1/4)*Sqrt[(2^(1/3) + x^2)/(2^(1/3)*(1 + S
qrt[3]) + x^2)^2]*Sqrt[2 + x^6])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
 + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
  b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
  x]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{2+x^6}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{2+x^3}}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt{2+x^6}}{4x^2} + \frac{1}{8}\,\text{Subst}\left(\int \frac{x}{\sqrt{2+x^3}}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt{2+x^6}}{4x^2} + \frac{1}{8}\,\text{Subst}\left(\int \frac{\sqrt[3]{2}\left(1-\sqrt{3}\right)+x}{\sqrt{2+x^3}}\,dx, x, x^2\right) + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2+x^3}}\,dx, x, x^2\right)}{4\sqrt[6]{2}\sqrt{2+\sqrt{3}}}$$

$$= -\frac{\sqrt{2+x^6}}{4x^2} + \frac{\sqrt{2+x^6}}{4\left(\sqrt[3]{2}\left(1+\sqrt{3}\right)+x^2\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\left(\sqrt[3]{2}+x^2\right)\sqrt{\frac{2^{2/3}-\sqrt[3]{2}x^2+x^4}{\left(\sqrt[3]{2}\left(1+\sqrt{3}\right)+x^2\right)^2}}\,E\left(\sin^{-1}\left(\right.\right.}{4\,2^{5/6}\sqrt{\frac{\sqrt[3]{2}+x^2}{\left(\sqrt[3]{2}\left(1+\sqrt{3}\right)+x^2\right)^2}}\,\sqrt{2+}}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.08

$$-\frac{{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};-\frac{x^6}{2}\right)}{2\sqrt{2}\,x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[2 + x^6]),x]
```

```
[Out] -1/2*Hypergeometric2F1[-1/3, 1/2, 2/3, -1/2*x^6]/(Sqrt[2]*x^2)
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^6+2}}{x^9+2\,x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^6+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^6 + 2)/(x^9 + 2*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^6+2}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^6+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^6 + 2)*x^3), x)
```

maple [C] time = 0.14, size = 33, normalized size = 0.09

$$\frac{\sqrt{2}\,x^4\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{2}{3}\right],\left[\frac{5}{3}\right],-\frac{x^6}{2}\right)}{32}-\frac{\sqrt{x^6+2}}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^6+2)^(1/2),x)
```

```
[Out] -1/4*(x^6+2)^(1/2)/x^2+1/32*2^(1/2)*x^4*hypergeom([1/2,2/3],[5/3],-1/2*x^6)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^6+2}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^6+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^6 + 2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,\sqrt{x^6+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x^6 + 2)^(1/2)),x)
```

```
[Out] int(1/(x^3*(x^6 + 2)^(1/2)), x)
```

sympy [A] time = 1.32, size = 39, normalized size = 0.10

$$\frac{\sqrt{2}\,\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},\frac{1}{2}\\ \frac{2}{3}\end{matrix}\right|\frac{x^6e^{i\pi}}{2}\right)}{12x^2\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**6+2)**(1/2),x)
```

```
[Out] sqrt(2)*gamma(-1/3)*hyper((-1/3, 1/2), (2/3,), x**6*exp_polar(I*pi)/2)/(12*
x**2*gamma(2/3))
```

3.1405 $\int \frac{x^{10}}{\sqrt{2+x^6}}\,dx$

Optimal. Leaf size=394

$$\frac{1}{8}\sqrt{x^6+2}\,x^5-\frac{5\left(1+\sqrt{3}\right)\sqrt{x^6+2}\,x}{8\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)}+\frac{5\left(1-\sqrt{3}\right)\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\,xF\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x^2+\sqrt[3]{2}}{\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}}\right)|\frac{1}{4}\left(2+\sqrt{\right.}\right.}{8\,2^{2/3}\sqrt[4]{3}\sqrt{\frac{x^2\left(x^2+\sqrt[3]{2}\right)}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\sqrt{x^6+2}}$$

```
[Out] 1/8*x^5*(x^6+2)^(1/2)-5/8*x*(1+3^(1/2))*(x^6+2)^(1/2)/(2^(1/3)+x^2*(1+3^(1/
2)))+5/8*3^(1/4)*x*(2^(1/3)+x^2)*((2^(1/3)+x^2*(1-3^(1/2)))^2/(2^(1/3)+x^2*
(1+3^(1/2)))^2)^(1/2)/(2^(1/3)+x^2*(1-3^(1/2)))*(2^(1/3)+x^2*(1+3^(1/2)))*E
llipticE((1-(2^(1/3)+x^2*(1-3^(1/2)))^2/(2^(1/3)+x^2*(1+3^(1/2)))^2)^(1/2),
1/4*6^(1/2)+1/4*2^(1/2))*((2^(2/3)-2^(1/3)*x^2+x^4)/(2^(1/3)+x^2*(1+3^(1/2)
))^2)^(1/2)*2^(1/3)/(x^6+2)^(1/2)/(x^2*(2^(1/3)+x^2)/(2^(1/3)+x^2*(1+3^(1/2
)))^2)^(1/2)+5/48*x*(2^(1/3)+x^2)*((2^(1/3)+x^2*(1-3^(1/2)))^2/(2^(1/3)+x^2
*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)+x^2*(1-3^(1/2)))*(2^(1/3)+x^2*(1+3^(1/2)))*
EllipticF((1-(2^(1/3)+x^2*(1-3^(1/2)))^2/(2^(1/3)+x^2*(1+3^(1/2)))^2)^(1/2)
,1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*((2^(2/3)-2^(1/3)*x^2+x^4)/(2^(1/3)+x
^2*(1+3^(1/2)))^2)^(1/2)*2^(1/3)*3^(3/4)/(x^6+2)^(1/2)/(x^2*(2^(1/3)+x^2)/(
2^(1/3)+x^2*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {321, 308, 225, 1881}

$$\frac{1}{8}\sqrt{x^6+2}\,x^5-\frac{5\left(1+\sqrt{3}\right)\sqrt{x^6+2}\,x}{8\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)}+\frac{5\left(1-\sqrt{3}\right)\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\,xF\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x^2+\sqrt[3]{2}}{\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}}\right)|\frac{1}{4}\left(2+\sqrt{\right.}\right.}{8\,2^{2/3}\sqrt[4]{3}\sqrt{\frac{x^2\left(x^2+\sqrt[3]{2}\right)}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/Sqrt[2 + x^6],x]
```

```
[Out] (x^5*Sqrt[2 + x^6])/8 - (5*(1 + Sqrt[3])*x*Sqrt[2 + x^6])/(8*(2^(1/3) + (1
+ Sqrt[3])*x^2)) + (5*3^(1/4)*x*(2^(1/3) + x^2)*Sqrt[(2^(2/3) - 2^(1/3)*x^2
 + x^4)/(2^(1/3) + (1 + Sqrt[3])*x^2)^2]*EllipticE[ArcCos[(2^(1/3) + (1 - S
qrt[3])*x^2)/(2^(1/3) + (1 + Sqrt[3])*x^2)], (2 + Sqrt[3])/4])/(4*2^(2/3)*S
qrt[(x^2*(2^(1/3) + x^2))/(2^(1/3) + (1 + Sqrt[3])*x^2)^2]*Sqrt[2 + x^6]) +
 (5*(1 - Sqrt[3])*x*(2^(1/3) + x^2)*Sqrt[(2^(2/3) - 2^(1/3)*x^2 + x^4)/(2^(
1/3) + (1 + Sqrt[3])*x^2)^2]*EllipticF[ArcCos[(2^(1/3) + (1 - Sqrt[3])*x^2)
/(2^(1/3) + (1 + Sqrt[3])*x^2)], (2 + Sqrt[3])/4])/(8*2^(2/3)*3^(1/4)*Sqrt[
(x^2*(2^(1/3) + x^2))/(2^(1/3) + (1 + Sqrt[3])*x^2)^2]*Sqrt[2 + x^6])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{x^{10}}{\sqrt{2+x^6}}\,dx = \frac{1}{8}x^5\sqrt{2+x^6} - \frac{5}{4}\int \frac{x^4}{\sqrt{2+x^6}}\,dx$$

$$= \frac{1}{8}x^5\sqrt{2+x^6} + \frac{5}{8}\int \frac{2^{2/3}\left(-1+\sqrt{3}\right)-2x^4}{\sqrt{2+x^6}}\,dx + \frac{\left(5\left(1-\sqrt{3}\right)\right)\int \frac{1}{\sqrt{2+x^6}}\,dx}{4\sqrt[3]{2}}$$

$$= \frac{1}{8}x^5\sqrt{2+x^6} - \frac{5\left(1+\sqrt{3}\right)x\sqrt{2+x^6}}{8\left(\sqrt[3]{2}+\left(1+\sqrt{3}\right)x^2\right)} + \frac{5\sqrt[4]{3}\,x\left(\sqrt[3]{2}+x^2\right)\sqrt{\frac{2^{2/3}-\sqrt[3]{2}x^2+x^4}{\left(\sqrt[3]{2}+\left(1+\sqrt{3}\right)x^2\right)^2}}\,E\left(\cos^{-1}\left(\frac{\sqrt[3]{2}+(}{\sqrt[3]{2}+(}\right.\right.}{4\,2^{2/3}\sqrt{\frac{x^2\left(\sqrt[3]{2}+x^2\right)}{\left(\sqrt[3]{2}+\left(1+\sqrt{3}\right)x^2\right)^2}}\sqrt{2+x}}$$

Mathematica [C] time = 0.02, size = 41, normalized size = 0.10

$$\frac{1}{8}x^5\left(\sqrt{x^6+2} - \sqrt{2}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};-\frac{x^6}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/Sqrt[2 + x^6],x]
```

```
[Out] (x^5*(Sqrt[2 + x^6] - Sqrt[2]*Hypergeometric2F1[1/2, 5/6, 11/6, -1/2*x^6]))
/8
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^{10}}{\sqrt{x^6+2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(x^6+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^10/sqrt(x^6 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\sqrt{x^6+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(x^6+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^10/sqrt(x^6 + 2), x)
```

maple [C] time = 0.14, size = 33, normalized size = 0.08

$$-\frac{\sqrt{2}\,x^5\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{5}{6}\right],\left[\frac{11}{6}\right],-\frac{x^6}{2}\right)}{8}+\frac{\sqrt{x^6+2}\,x^5}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(x^6+2)^(1/2),x)
```

```
[Out] 1/8*x^5*(x^6+2)^(1/2)-1/8*2^(1/2)*x^5*hypergeom([1/2,5/6],[11/6],-1/2*x^6)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\sqrt{x^6+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(x^6+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^10/sqrt(x^6 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{10}}{\sqrt{x^6+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(x^6 + 2)^(1/2),x)
```

```
[Out] int(x^10/(x^6 + 2)^(1/2), x)
```

sympy [C] time = 2.15, size = 36, normalized size = 0.09

$$\frac{\sqrt{2}\,x^{11}\Gamma\left(\frac{11}{6}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{11}{6}\\ \frac{17}{6}\end{matrix}\right|\frac{x^6e^{i\pi}}{2}\right)}{12\Gamma\left(\frac{17}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(x**6+2)**(1/2),x)
```

```
[Out] sqrt(2)*x**11*gamma(11/6)*hyper((1/2, 11/6), (17/6,), x**6*exp_polar(I*pi)/
2)/(12*gamma(17/6))
```

3.1406 $\int \frac{x^4}{\sqrt{2+x^6}}\,dx$

Optimal. Leaf size=376

$$\frac{\left(1+\sqrt{3}\right)\sqrt{x^6+2}\,x}{2\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)}-\frac{\left(1-\sqrt{3}\right)\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\,xF\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x^2+\sqrt[3]{2}}{\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)\quad\sqrt[4]{3}\,(x}{2\,2^{2/3}\sqrt[4]{3}\sqrt{\frac{x^2\left(x^2+\sqrt[3]{2}\right)}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\sqrt{x^6+2}}-$$

```
[Out] 1/2*x*(1+3^(1/2))*(x^6+2)^(1/2)/(2^(1/3)+x^2*(1+3^(1/2)))-1/2*3^(1/4)*x*(2^
(1/3)+x^2)*((2^(1/3)+x^2*(1-3^(1/2)))^2/(2^(1/3)+x^2*(1+3^(1/2)))^2)^(1/2)/
(2^(1/3)+x^2*(1-3^(1/2)))*(2^(1/3)+x^2*(1+3^(1/2)))*EllipticE((1-(2^(1/3)+x
^2*(1-3^(1/2)))^2/(2^(1/3)+x^2*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2
))*((2^(2/3)-2^(1/3)*x^2+x^4)/(2^(1/3)+x^2*(1+3^(1/2)))^2)^(1/2)*2^(1/3)/(x
^6+2)^(1/2)/(x^2*(2^(1/3)+x^2)/(2^(1/3)+x^2*(1+3^(1/2)))^2)^(1/2)-1/12*x*(2
^(1/3)+x^2)*((2^(1/3)+x^2*(1-3^(1/2)))^2/(2^(1/3)+x^2*(1+3^(1/2)))^2)^(1/2)
/(2^(1/3)+x^2*(1-3^(1/2)))*(2^(1/3)+x^2*(1+3^(1/2)))*EllipticF((1-(2^(1/3)+
x^2*(1-3^(1/2)))^2/(2^(1/3)+x^2*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/
2))*(1-3^(1/2))*((2^(2/3)-2^(1/3)*x^2+x^4)/(2^(1/3)+x^2*(1+3^(1/2)))^2)^(1/
2)*2^(1/3)*3^(3/4)/(x^6+2)^(1/2)/(x^2*(2^(1/3)+x^2)/(2^(1/3)+x^2*(1+3^(1/2)
))^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {308, 225, 1881}

$$\frac{\left(1+\sqrt{3}\right)\sqrt{x^6+2}\,x}{2\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)}-\frac{\left(1-\sqrt{3}\right)\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\,xF\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x^2+\sqrt[3]{2}}{\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)\quad\sqrt[4]{3}\,(x}{2\,2^{2/3}\sqrt[4]{3}\sqrt{\frac{x^2\left(x^2+\sqrt[3]{2}\right)}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\sqrt{x^6+2}}-$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[2 + x^6],x]
```

```
[Out] ((1 + Sqrt[3])*x*Sqrt[2 + x^6])/(2*(2^(1/3) + (1 + Sqrt[3])*x^2)) - (3^(1/4
)*x*(2^(1/3) + x^2)*Sqrt[(2^(2/3) - 2^(1/3)*x^2 + x^4)/(2^(1/3) + (1 + Sqrt
[3])*x^2)^2]*EllipticE[ArcCos[(2^(1/3) + (1 - Sqrt[3])*x^2)/(2^(1/3) + (1 +
 Sqrt[3])*x^2)], (2 + Sqrt[3])/4])/(2^(2/3)*Sqrt[(x^2*(2^(1/3) + x^2))/(2^(
1/3) + (1 + Sqrt[3])*x^2)^2]*Sqrt[2 + x^6]) - ((1 - Sqrt[3])*x*(2^(1/3) + x
^2)*Sqrt[(2^(2/3) - 2^(1/3)*x^2 + x^4)/(2^(1/3) + (1 + Sqrt[3])*x^2)^2]*Ell
ipticF[ArcCos[(2^(1/3) + (1 - Sqrt[3])*x^2)/(2^(1/3) + (1 + Sqrt[3])*x^2)],
 (2 + Sqrt[3])/4])/(2*2^(2/3)*3^(1/4)*Sqrt[(x^2*(2^(1/3) + x^2))/(2^(1/3) +
 (1 + Sqrt[3])*x^2)^2]*Sqrt[2 + x^6])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
 - Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{2+x^6}}\,dx = -\left(\frac{1}{2}\int \frac{2^{2/3}\left(-1+\sqrt{3}\right)-2x^4}{\sqrt{2+x^6}}\,dx\right) - \frac{\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{2+x^6}}\,dx}{\sqrt[3]{2}}$$

$$= \frac{\left(1+\sqrt{3}\right)x\sqrt{2+x^6}}{2\left(\sqrt[3]{2}+\left(1+\sqrt{3}\right)x^2\right)} - \frac{\sqrt[4]{3}\,x\left(\sqrt[3]{2}+x^2\right)\sqrt{\frac{2^{2/3}-\sqrt[3]{2}x^2+x^4}{\left(\sqrt[3]{2}+\left(1+\sqrt{3}\right)x^2\right)^2}}\,E\left(\cos^{-1}\left(\frac{\sqrt[3]{2}+\left(1-\sqrt{3}\right)x^2}{\sqrt[3]{2}+\left(1+\sqrt{3}\right)x^2}\right)|\frac{1}{4}\left(2+\sqrt{3}\right.\right.}{2^{2/3}\sqrt{\frac{x^2\left(\sqrt[3]{2}+x^2\right)}{\left(\sqrt[3]{2}+\left(1+\sqrt{3}\right)x^2\right)^2}}\sqrt{2+x^6}}$$

Mathematica [C] time = 0.00, size = 29, normalized size = 0.08

$$\frac{x^5\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};-\frac{x^6}{2}\right)}{5\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[2 + x^6],x]
```

```
[Out] (x^5*Hypergeometric2F1[1/2, 5/6, 11/6, -1/2*x^6])/(5*Sqrt[2])
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^4}{\sqrt{x^6+2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^4/sqrt(x^6 + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{x^6+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+2)^(1/2),x, algorithm="giac")
[Out] integrate(x^4/sqrt(x^6 + 2), x)
```

maple [C] time = 0.14, size = 20, normalized size = 0.05

$$\frac{\sqrt{2}\, x^5\, \text{hypergeom}\left(\left[\frac{1}{2}, \frac{5}{6}\right], \left[\frac{11}{6}\right], -\frac{x^6}{2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^6+2)^(1/2),x)
[Out] 1/10*2^(1/2)*x^5*hypergeom([1/2,5/6],[11/6],-1/2*x^6)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{x^6+2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+2)^(1/2),x, algorithm="maxima")
[Out] integrate(x^4/sqrt(x^6 + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\sqrt{x^6+2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^6 + 2)^(1/2),x)
[Out] int(x^4/(x^6 + 2)^(1/2), x)
```

sympy [C] time = 1.17, size = 36, normalized size = 0.10

$$\frac{\sqrt{2}\, x^5 \Gamma\left(\frac{5}{6}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{5}{6} \\ \frac{11}{6}\end{matrix}\right| \frac{x^6 e^{i\pi}}{2}\right)}{12\Gamma\left(\frac{11}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**6+2)**(1/2),x)
[Out] sqrt(2)*x**5*gamma(5/6)*hyper((1/2, 5/6), (11/6,), x**6*exp_polar(I*pi)/2)/
(12*gamma(11/6))
```

3.1407 $\int \frac{1}{x^2\sqrt{2+x^6}}\,dx$

Optimal. Leaf size=392

$$-\frac{\sqrt{x^6+2}}{2x}+\frac{\left(1+\sqrt{3}\right)\sqrt{x^6+2}\,x}{2\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)}-\frac{\left(1-\sqrt{3}\right)\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\,xF\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x^2+\sqrt[3]{2}}{\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right.}{2\,2^{2/3}\sqrt[4]{3}\sqrt{\frac{x^2\left(x^2+\sqrt[3]{2}\right)}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\sqrt{x^6+2}}$$

```
[Out] -1/2*(x^6+2)^(1/2)/x+1/2*x*(1+3^(1/2))*(x^6+2)^(1/2)/(2^(1/3)+x^2*(1+3^(1/2
))))-1/2*3^(1/4)*x*(2^(1/3)+x^2)*((2^(1/3)+x^2*(1-3^(1/2)))^2/(2^(1/3)+x^2*(
1+3^(1/2)))^2)^(1/2)/(2^(1/3)+x^2*(1-3^(1/2)))*(2^(1/3)+x^2*(1+3^(1/2)))*El
lipticE((1-(2^(1/3)+x^2*(1-3^(1/2)))^2/(2^(1/3)+x^2*(1+3^(1/2)))^2)^(1/2),1
/4*6^(1/2)+1/4*2^(1/2))*((2^(2/3)-2^(1/3)*x^2+x^4)/(2^(1/3)+x^2*(1+3^(1/2))
)^2)^(1/2)*2^(1/3)/(x^6+2)^(1/2)/(x^2*(2^(1/3)+x^2)/(2^(1/3)+x^2*(1+3^(1/2)
))^2)^(1/2)-1/12*x*(2^(1/3)+x^2)*((2^(1/3)+x^2*(1-3^(1/2)))^2/(2^(1/3)+x^2*
(1+3^(1/2)))^2)^(1/2)/(2^(1/3)+x^2*(1-3^(1/2)))*(2^(1/3)+x^2*(1+3^(1/2)))*E
llipticF((1-(2^(1/3)+x^2*(1-3^(1/2)))^2/(2^(1/3)+x^2*(1+3^(1/2)))^2)^(1/2),
1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*((2^(2/3)-2^(1/3)*x^2+x^4)/(2^(1/3)+x^
2*(1+3^(1/2)))^2)^(1/2)*2^(1/3)*3^(3/4)/(x^6+2)^(1/2)/(x^2*(2^(1/3)+x^2)/(2
^(1/3)+x^2*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 392, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {325, 308, 225, 1881}

$$\frac{\left(1+\sqrt{3}\right)\sqrt{x^6+2}\,x}{2\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)}-\frac{\sqrt{x^6+2}}{2x}-\frac{\left(1-\sqrt{3}\right)\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\,xF\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x^2+\sqrt[3]{2}}{\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\,2^{2/3}\sqrt[4]{3}\sqrt{\frac{x^2\left(x^2+\sqrt[3]{2}\right)}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[2 + x^6]),x]
```

```
[Out] -Sqrt[2 + x^6]/(2*x) + ((1 + Sqrt[3])*x*Sqrt[2 + x^6])/(2*(2^(1/3) + (1 + S
qrt[3])*x^2)) - (3^(1/4)*x*(2^(1/3) + x^2)*Sqrt[(2^(2/3) - 2^(1/3)*x^2 + x^
4)/(2^(1/3) + (1 + Sqrt[3])*x^2)^2]*EllipticE[ArcCos[(2^(1/3) + (1 - Sqrt[3
])*x^2)/(2^(1/3) + (1 + Sqrt[3])*x^2)], (2 + Sqrt[3])/4])/(2^(2/3)*Sqrt[(x^
2*(2^(1/3) + x^2))/(2^(1/3) + (1 + Sqrt[3])*x^2)^2]*Sqrt[2 + x^6]) - ((1 -
Sqrt[3])*x*(2^(1/3) + x^2)*Sqrt[(2^(2/3) - 2^(1/3)*x^2 + x^4)/(2^(1/3) + (1
 + Sqrt[3])*x^2)^2]*EllipticF[ArcCos[(2^(1/3) + (1 - Sqrt[3])*x^2)/(2^(1/3)
 + (1 + Sqrt[3])*x^2)], (2 + Sqrt[3])/4])/(2*2^(2/3)*3^(1/4)*Sqrt[(x^2*(2^(
1/3) + x^2))/(2^(1/3) + (1 + Sqrt[3])*x^2)^2]*Sqrt[2 + x^6])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{2+x^6}}\,dx &= -\frac{\sqrt{2+x^6}}{2x} + \int \frac{x^4}{\sqrt{2+x^6}}\,dx \\ &= -\frac{\sqrt{2+x^6}}{2x} - \frac{1}{2}\int \frac{2^{2/3}\left(-1+\sqrt{3}\right) - 2x^4}{\sqrt{2+x^6}}\,dx - \frac{\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{2+x^6}}\,dx}{\sqrt[3]{2}} \\ &= -\frac{\sqrt{2+x^6}}{2x} + \frac{\left(1+\sqrt{3}\right)x\sqrt{2+x^6}}{2\left(\sqrt[3]{2}+\left(1+\sqrt{3}\right)x^2\right)} - \frac{\sqrt[4]{3}\,x\left(\sqrt[3]{2}+x^2\right)\sqrt{\frac{2^{2/3}-\sqrt[3]{2}x^2+x^4}{\left(\sqrt[3]{2}+\left(1+\sqrt{3}\right)x^2\right)^2}}\,E\left(\cos^{-1}\left(\frac{\sqrt[3]{2}+\left(1-\right.\right.\right.}{\sqrt[3]{2}+\left(1-\right.}\right.}{2^{2/3}\sqrt{\frac{x^2\left(\sqrt[3]{2}+x^2\right)}{\left(\sqrt[3]{2}+\left(1+\sqrt{3}\right)x^2\right)^2}}\sqrt{2+x^6}}\end{aligned}$$

Mathematica [C] time = 0.00, size = 27, normalized size = 0.07

$$-\frac{2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};-\frac{x^6}{2}\right)}{\sqrt{2}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[2 + x^6]),x]
```

```
[Out] -(Hypergeometric2F1[-1/6, 1/2, 5/6, -1/2*x^6]/(Sqrt[2]*x))
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^6+2}}{x^8+2\,x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^6 + 2)/(x^8 + 2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^6+2}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^6 + 2)*x^2), x)
```

maple [C] time = 0.02, size = 33, normalized size = 0.08

$$\frac{\sqrt{2}\,x^5\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{5}{6}\right],\left[\frac{11}{6}\right],-\frac{x^6}{2}\right)}{10}-\frac{\sqrt{x^6+2}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^6+2)^(1/2),x)
```

```
[Out] -1/2*(x^6+2)^(1/2)/x+1/10*2^(1/2)*x^5*hypergeom([1/2,5/6],[11/6],-1/2*x^6)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^6+2}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^6 + 2)*x^2), x)
```

mupad [B] time = 1.21, size = 31, normalized size = 0.08

$$-\frac{\sqrt{\frac{2}{x^6}+1}\;{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{2}{3};\;\frac{5}{3};\;-\frac{2}{x^6}\right)}{4\,x\,\sqrt{x^6+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^6 + 2)^(1/2)),x)
```

```
[Out] -((2/x^6 + 1)^(1/2)*hypergeom([1/2, 2/3], 5/3, -2/x^6))/(4*x*(x^6 + 2)^(1/2
))
```

sympy [C] time = 1.32, size = 37, normalized size = 0.09

$$\frac{\sqrt{2}\,\Gamma\left(-\frac{1}{6}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{6},\frac{1}{2}\\ \frac{5}{6}\end{matrix}\right|\frac{x^6e^{i\pi}}{2}\right)}{12x\Gamma\left(\frac{5}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**6+2)**(1/2),x)
```

```
[Out] sqrt(2)*gamma(-1/6)*hyper((-1/6, 1/2), (5/6,), x**6*exp_polar(I*pi)/2)/(12*
x*gamma(5/6))
```

3.1408 $\int \frac{x^{23}}{\left(2+x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=51

$$\frac{1}{15}\left(x^6+2\right)^{5/2}-\frac{2}{3}\left(x^6+2\right)^{3/2}+4\sqrt{x^6+2}+\frac{8}{3\sqrt{x^6+2}}$$

```
[Out] -2/3*(x^6+2)^(3/2)+1/15*(x^6+2)^(5/2)+8/3/(x^6+2)^(1/2)+4*(x^6+2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{1}{15}\left(x^6+2\right)^{5/2}-\frac{2}{3}\left(x^6+2\right)^{3/2}+4\sqrt{x^6+2}+\frac{8}{3\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^23/(2 + x^6)^(3/2),x]
```

```
[Out] 8/(3*Sqrt[2 + x^6]) + 4*Sqrt[2 + x^6] - (2*(2 + x^6)^(3/2))/3 + (2 + x^6)^(
5/2)/15
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{23}}{\left(2+x^6\right)^{3/2}}\,dx &= \frac{1}{6}\,\text{Subst}\left(\int \frac{x^3}{(2+x)^{3/2}}\,dx, x, x^6\right)\\ &= \frac{1}{6}\,\text{Subst}\left(\int \left(-\frac{8}{(2+x)^{3/2}}+\frac{12}{\sqrt{2+x}}-6\sqrt{2+x}+(2+x)^{3/2}\right)dx, x, x^6\right)\\ &= \frac{8}{3\sqrt{2+x^6}}+4\sqrt{2+x^6}-\frac{2}{3}\left(2+x^6\right)^{3/2}+\frac{1}{15}\left(2+x^6\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.55

$$\frac{x^{18}-4x^{12}+32x^6+128}{15\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^23/(2 + x^6)^(3/2),x]
```

```
[Out] (128 + 32*x^6 - 4*x^12 + x^18)/(15*Sqrt[2 + x^6])
```

fricas [A] time = 0.57, size = 24, normalized size = 0.47

$$\frac{x^{18} - 4\,x^{12} + 32\,x^6 + 128}{15\,\sqrt{x^6 + 2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(x^6+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(x^18 - 4*x^12 + 32*x^6 + 128)/sqrt(x^6 + 2)
```

giac [A] time = 0.15, size = 37, normalized size = 0.73

$$\frac{1}{15}\left(x^6+2\right)^{\frac{5}{2}} - \frac{2}{3}\left(x^6+2\right)^{\frac{3}{2}} + 4\,\sqrt{x^6+2} + \frac{8}{3\,\sqrt{x^6+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(x^6+2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/15*(x^6 + 2)^(5/2) - 2/3*(x^6 + 2)^(3/2) + 4*sqrt(x^6 + 2) + 8/3/sqrt(x^6
 + 2)
```

maple [A] time = 0.01, size = 25, normalized size = 0.49

$$\frac{x^{18} - 4x^{12} + 32x^6 + 128}{15\sqrt{x^6 + 2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^23/(x^6+2)^(3/2),x)
```

```
[Out] 1/15*(x^18-4*x^12+32*x^6+128)/(x^6+2)^(1/2)
```

maxima [A] time = 1.02, size = 37, normalized size = 0.73

$$\frac{1}{15}\left(x^6+2\right)^{\frac{5}{2}} - \frac{2}{3}\left(x^6+2\right)^{\frac{3}{2}} + 4\,\sqrt{x^6+2} + \frac{8}{3\,\sqrt{x^6+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(x^6+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/15*(x^6 + 2)^(5/2) - 2/3*(x^6 + 2)^(3/2) + 4*sqrt(x^6 + 2) + 8/3/sqrt(x^6
 + 2)
```

mupad [B] time = 1.22, size = 32, normalized size = 0.63

$$\frac{\left(x^6+2\right)^3 - 10\left(x^6+2\right)^2 + 60\,x^6 + 160}{15\,\sqrt{x^6+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^23/(x^6 + 2)^(3/2),x)
```

```
[Out] ((x^6 + 2)^3 - 10*(x^6 + 2)^2 + 60*x^6 + 160)/(15*(x^6 + 2)^(1/2))
```

sympy [A] time = 19.15, size = 54, normalized size = 1.06

$$\frac{x^{18}}{15\sqrt{x^6+2}} - \frac{4x^{12}}{15\sqrt{x^6+2}} + \frac{32x^6}{15\sqrt{x^6+2}} + \frac{128}{15\sqrt{x^6+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**23/(x**6+2)**(3/2),x)
```

```
[Out] x**18/(15*sqrt(x**6 + 2)) - 4*x**12/(15*sqrt(x**6 + 2)) + 32*x**6/(15*sqrt(
x**6 + 2)) + 128/(15*sqrt(x**6 + 2))
```

3.1409 $\int \frac{x^{17}}{\left(2+x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=40

$$\frac{1}{9}\left(x^6+2\right)^{3/2}-\frac{4\sqrt{x^6+2}}{3}-\frac{4}{3\sqrt{x^6+2}}$$

```
[Out] 1/9*(x^6+2)^(3/2)-4/3/(x^6+2)^(1/2)-4/3*(x^6+2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{1}{9}\left(x^6+2\right)^{3/2}-\frac{4\sqrt{x^6+2}}{3}-\frac{4}{3\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^17/(2 + x^6)^(3/2),x]
```

```
[Out] -4/(3*Sqrt[2 + x^6]) - (4*Sqrt[2 + x^6])/3 + (2 + x^6)^(3/2)/9
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{17}}{\left(2+x^6\right)^{3/2}}\,dx &= \frac{1}{6}\,\text{Subst}\left(\int \frac{x^2}{(2+x)^{3/2}}\,dx, x, x^6\right)\\ &= \frac{1}{6}\,\text{Subst}\left(\int \left(\frac{4}{(2+x)^{3/2}}-\frac{4}{\sqrt{2+x}}+\sqrt{2+x}\right)dx, x, x^6\right)\\ &= -\frac{4}{3\sqrt{2+x^6}}-\frac{4\sqrt{2+x^6}}{3}+\frac{1}{9}\left(2+x^6\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.58

$$\frac{x^{12}-8x^6-32}{9\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^17/(2 + x^6)^(3/2),x]
```

```
[Out] (-32 - 8*x^6 + x^12)/(9*Sqrt[2 + x^6])
```

fricas [A] time = 0.56, size = 19, normalized size = 0.48

$$\frac{x^{12} - 8\,x^6 - 32}{9\,\sqrt{x^6 + 2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(x^6+2)^(3/2),x, algorithm="fricas")
[Out] 1/9*(x^12 - 8*x^6 - 32)/sqrt(x^6 + 2)
```

giac [A] time = 0.15, size = 28, normalized size = 0.70

$$\frac{1}{9}\left(x^6 + 2\right)^{\frac{3}{2}} - \frac{4}{3}\,\sqrt{x^6 + 2} - \frac{4}{3\,\sqrt{x^6 + 2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(x^6+2)^(3/2),x, algorithm="giac")
[Out] 1/9*(x^6 + 2)^(3/2) - 4/3*sqrt(x^6 + 2) - 4/3/sqrt(x^6 + 2)
```

maple [A] time = 0.01, size = 20, normalized size = 0.50

$$\frac{x^{12} - 8x^6 - 32}{9\sqrt{x^6 + 2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^17/(x^6+2)^(3/2),x)
[Out] 1/9*(x^12-8*x^6-32)/(x^6+2)^(1/2)
```

maxima [A] time = 0.98, size = 28, normalized size = 0.70

$$\frac{1}{9}\left(x^6 + 2\right)^{\frac{3}{2}} - \frac{4}{3}\,\sqrt{x^6 + 2} - \frac{4}{3\,\sqrt{x^6 + 2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(x^6+2)^(3/2),x, algorithm="maxima")
[Out] 1/9*(x^6 + 2)^(3/2) - 4/3*sqrt(x^6 + 2) - 4/3/sqrt(x^6 + 2)
```

mupad [B] time = 1.18, size = 25, normalized size = 0.62

$$-\frac{12\,x^6 - \left(x^6 + 2\right)^2 + 36}{9\,\sqrt{x^6 + 2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^17/(x^6 + 2)^(3/2),x)
[Out] -(12*x^6 - (x^6 + 2)^2 + 36)/(9*(x^6 + 2)^(1/2))
```

sympy [A] time = 8.23, size = 39, normalized size = 0.98

$$\frac{x^{12}}{9\sqrt{x^6 + 2}} - \frac{8x^6}{9\sqrt{x^6 + 2}} - \frac{32}{9\sqrt{x^6 + 2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**17/(x**6+2)**(3/2),x)
[Out] x**12/(9*sqrt(x**6 + 2)) - 8*x**6/(9*sqrt(x**6 + 2)) - 32/(9*sqrt(x**6 + 2)
)
```

3.1410 $\int \frac{x^{11}}{\left(2+x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=27

$$\frac{\sqrt{x^6+2}}{3}+\frac{2}{3\sqrt{x^6+2}}$$

```
[Out] 2/3/(x^6+2)^(1/2)+1/3*(x^6+2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{\sqrt{x^6+2}}{3}+\frac{2}{3\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(2 + x^6)^(3/2),x]
```

```
[Out] 2/(3*Sqrt[2 + x^6]) + Sqrt[2 + x^6]/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{\left(2+x^6\right)^{3/2}}\,dx &= \frac{1}{6}\,\text{Subst}\left(\int \frac{x}{(2+x)^{3/2}}\,dx, x, x^6\right)\\ &= \frac{1}{6}\,\text{Subst}\left(\int \left(-\frac{2}{(2+x)^{3/2}}+\frac{1}{\sqrt{2+x}}\right)dx, x, x^6\right)\\ &= \frac{2}{3\sqrt{2+x^6}}+\frac{\sqrt{2+x^6}}{3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 0.67

$$\frac{x^6+4}{3\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(2 + x^6)^(3/2),x]
```

```
[Out] (4 + x^6)/(3*Sqrt[2 + x^6])
```

fricas [A] time = 0.52, size = 14, normalized size = 0.52

$$\frac{x^6+4}{3\sqrt{x^6+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^6+2)^(3/2),x, algorithm="fricas")
[Out] 1/3*(x^6 + 4)/sqrt(x^6 + 2)
```

giac [A] time = 0.16, size = 19, normalized size = 0.70

$$\frac{1}{3}\sqrt{x^6+2}+\frac{2}{3\sqrt{x^6+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^6+2)^(3/2),x, algorithm="giac")
[Out] 1/3*sqrt(x^6 + 2) + 2/3/sqrt(x^6 + 2)
```

maple [A] time = 0.01, size = 15, normalized size = 0.56

$$\frac{x^6+4}{3\sqrt{x^6+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^6+2)^(3/2),x)
[Out] 1/3*(x^6+4)/(x^6+2)^(1/2)
```

maxima [A] time = 1.03, size = 19, normalized size = 0.70

$$\frac{1}{3}\sqrt{x^6+2}+\frac{2}{3\sqrt{x^6+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^6+2)^(3/2),x, algorithm="maxima")
[Out] 1/3*sqrt(x^6 + 2) + 2/3/sqrt(x^6 + 2)
```

mupad [B] time = 1.15, size = 14, normalized size = 0.52

$$\frac{x^6+4}{3\sqrt{x^6+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^6 + 2)^(3/2),x)
[Out] (x^6 + 4)/(3*(x^6 + 2)^(1/2))
```

sympy [A] time = 2.82, size = 24, normalized size = 0.89

$$\frac{x^6}{3\sqrt{x^6+2}}+\frac{4}{3\sqrt{x^6+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(x**6+2)**(3/2),x)
[Out] x**6/(3*sqrt(x**6 + 2)) + 4/(3*sqrt(x**6 + 2))
```

3.1411 $\int \frac{x^5}{\left(2+x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{3\sqrt{x^6+2}}$$

```
[Out] -1/3/(x^6+2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$-\frac{1}{3\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(2 + x^6)^(3/2),x]
```

```
[Out] -1/(3*Sqrt[2 + x^6])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^5}{\left(2+x^6\right)^{3/2}}\,dx = -\frac{1}{3\sqrt{2+x^6}}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$-\frac{1}{3\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(2 + x^6)^(3/2),x]
```

```
[Out] -1/3*1/Sqrt[2 + x^6]
```

fricas [A] time = 0.89, size = 9, normalized size = 0.69

$$-\frac{1}{3\sqrt{x^6+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3/sqrt(x^6 + 2)
```

giac [A] time = 0.17, size = 9, normalized size = 0.69

$$-\frac{1}{3\sqrt{x^6+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+2)^(3/2),x, algorithm="giac")
[Out] -1/3/sqrt(x^6 + 2)
```

maple [A] time = 0.00, size = 10, normalized size = 0.77

$$-\frac{1}{3\sqrt{x^6+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^6+2)^(3/2),x)
[Out] -1/3/(x^6+2)^(1/2)
```

maxima [A] time = 1.04, size = 9, normalized size = 0.69

$$-\frac{1}{3\sqrt{x^6+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+2)^(3/2),x, algorithm="maxima")
[Out] -1/3/sqrt(x^6 + 2)
```

mupad [B] time = 1.15, size = 9, normalized size = 0.69

$$-\frac{1}{3\sqrt{x^6+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^6 + 2)^(3/2),x)
[Out] -1/(3*(x^6 + 2)^(1/2))
```

sympy [A] time = 0.61, size = 12, normalized size = 0.92

$$-\frac{1}{3\sqrt{x^6+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**6+2)**(3/2),x)
[Out] -1/(3*sqrt(x**6 + 2))
```

3.1412 $\int \frac{1}{x\left(2+x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=39

$$\frac{1}{6\sqrt{x^6+2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^6+2}}{\sqrt{2}}\right)}{6\sqrt{2}}$$

```
[Out] -1/12*arctanh(1/2*(x^6+2)^(1/2)*2^(1/2))*2^(1/2)+1/6/(x^6+2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {266, 51, 63, 207}

$$\frac{1}{6\sqrt{x^6+2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^6+2}}{\sqrt{2}}\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(2 + x^6)^(3/2)),x]
```

```
[Out] 1/(6*Sqrt[2 + x^6]) - ArcTanh[Sqrt[2 + x^6]/Sqrt[2]]/(6*Sqrt[2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(2+x^6\right)^{3/2}}\,dx &= \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{x(2+x)^{3/2}}\,dx, x, x^6\right)\\ &= \frac{1}{6\sqrt{2+x^6}} + \frac{1}{12}\,\text{Subst}\left(\int \frac{1}{x\sqrt{2+x}}\,dx, x, x^6\right)\\ &= \frac{1}{6\sqrt{2+x^6}} + \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{-2+x^2}\,dx, x, \sqrt{2+x^6}\right)\\ &= \frac{1}{6\sqrt{2+x^6}} - \frac{\tanh^{-1}\left(\frac{\sqrt{2+x^6}}{\sqrt{2}}\right)}{6\sqrt{2}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 30, normalized size = 0.77

$$\frac{{}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{x^6}{2}+1\right)}{6\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(2 + x^6)^(3/2)),x]
```

```
[Out] Hypergeometric2F1[-1/2, 1, 1/2, 1 + x^6/2]/(6*Sqrt[2 + x^6])
```

fricas [A] time = 0.88, size = 50, normalized size = 1.28

$$\frac{\sqrt{2}\left(x^6+2\right)\log\left(\frac{x^6-2\sqrt{2}\sqrt{x^6+2}+4}{x^6}\right)+4\sqrt{x^6+2}}{24\left(x^6+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(sqrt(2)*(x^6 + 2)*log((x^6 - 2*sqrt(2)*sqrt(x^6 + 2) + 4)/x^6) + 4*sq
rt(x^6 + 2))/(x^6 + 2)
```

giac [A] time = 0.15, size = 44, normalized size = 1.13

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{\sqrt{2}-\sqrt{x^6+2}}{\sqrt{2}+\sqrt{x^6+2}}\right)+\frac{1}{6\sqrt{x^6+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6+2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(2)*log(-(sqrt(2) - sqrt(x^6 + 2))/(sqrt(2) + sqrt(x^6 + 2))) + 1/
6/sqrt(x^6 + 2)
```

maple [A] time = 0.02, size = 36, normalized size = 0.92

$$\frac{\sqrt{2}\,\ln\left(\frac{\sqrt{x^6+2}-\sqrt{2}}{\sqrt{x^6}}\right)}{12}+\frac{1}{6\sqrt{x^6+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^6+2)^(3/2),x)
```

```
[Out] 1/6/(x^6+2)^(1/2)+1/12*2^(1/2)*ln(((x^6+2)^(1/2)-2^(1/2))/(x^6)^(1/2))
```

maxima [A] time = 2.35, size = 44, normalized size = 1.13

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{\sqrt{2}-\sqrt{x^6+2}}{\sqrt{2}+\sqrt{x^6+2}}\right)+\frac{1}{6\sqrt{x^6+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/24*sqrt(2)*log(-(sqrt(2) - sqrt(x^6 + 2))/(sqrt(2) + sqrt(x^6 + 2))) + 1/
6/sqrt(x^6 + 2)
```

mupad [B] time = 1.22, size = 28, normalized size = 0.72

$$\frac{1}{6\sqrt{x^6+2}}-\frac{\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}\,\sqrt{x^6+2}}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^6 + 2)^(3/2)),x)
```

```
[Out] 1/(6*(x^6 + 2)^(1/2)) - (2^(1/2)*atanh((2^(1/2)*(x^6 + 2)^(1/2))/2))/12
```

sympy [B] time = 1.63, size = 194, normalized size = 4.97

$$\frac{x^6\log\left(x^6\right)}{12\sqrt{2}\,x^6+24\sqrt{2}}-\frac{2x^6\log\left(\sqrt{\frac{x^6}{2}+1}+1\right)}{12\sqrt{2}\,x^6+24\sqrt{2}}-\frac{x^6\log(2)}{12\sqrt{2}\,x^6+24\sqrt{2}}+\frac{2\sqrt{2}\,\sqrt{x^6+2}}{12\sqrt{2}\,x^6+24\sqrt{2}}+\frac{2\log\left(x^6\right)}{12\sqrt{2}\,x^6+24\sqrt{2}}-\frac{4\log\left(\sqrt{\frac{x^6}{2}}\right.}{12\sqrt{2}\,x^6+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**6+2)**(3/2),x)
```

```
[Out] x**6*log(x**6)/(12*sqrt(2)*x**6 + 24*sqrt(2)) - 2*x**6*log(sqrt(x**6/2 + 1)
 + 1)/(12*sqrt(2)*x**6 + 24*sqrt(2)) - x**6*log(2)/(12*sqrt(2)*x**6 + 24*sq
rt(2)) + 2*sqrt(2)*sqrt(x**6 + 2)/(12*sqrt(2)*x**6 + 24*sqrt(2)) + 2*log(x*
*6)/(12*sqrt(2)*x**6 + 24*sqrt(2)) - 4*log(sqrt(x**6/2 + 1) + 1)/(12*sqrt(2
)*x**6 + 24*sqrt(2)) - 2*log(2)/(12*sqrt(2)*x**6 + 24*sqrt(2))
```

3.1413 $\int \frac{1}{x^7\left(2+x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=55

$$-\frac{1}{12x^6\sqrt{x^6+2}}-\frac{1}{8\sqrt{x^6+2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{x^6+2}}{\sqrt{2}}\right)}{8\sqrt{2}}$$

```
[Out] 1/16*arctanh(1/2*(x^6+2)^(1/2)*2^(1/2))*2^(1/2)-1/8/(x^6+2)^(1/2)-1/12/x^6/
(x^6+2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {266, 51, 63, 207}

$$-\frac{\sqrt{x^6+2}}{8x^6}+\frac{1}{6x^6\sqrt{x^6+2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{x^6+2}}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(2 + x^6)^(3/2)),x]
```

```
[Out] 1/(6*x^6*Sqrt[2 + x^6]) - Sqrt[2 + x^6]/(8*x^6) + ArcTanh[Sqrt[2 + x^6]/Sqr
t[2]]/(8*Sqrt[2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7\left(2+x^6\right)^{3/2}}\,dx &= \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{x^2(2+x)^{3/2}}\,dx, x, x^6\right)\\ &= \frac{1}{6x^6\sqrt{2+x^6}} + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{2+x}}\,dx, x, x^6\right)\\ &= \frac{1}{6x^6\sqrt{2+x^6}} - \frac{\sqrt{2+x^6}}{8x^6} - \frac{1}{16}\,\text{Subst}\left(\int \frac{1}{x\sqrt{2+x}}\,dx, x, x^6\right)\\ &= \frac{1}{6x^6\sqrt{2+x^6}} - \frac{\sqrt{2+x^6}}{8x^6} - \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{-2+x^2}\,dx, x, \sqrt{2+x^6}\right)\\ &= \frac{1}{6x^6\sqrt{2+x^6}} - \frac{\sqrt{2+x^6}}{8x^6} + \frac{\tanh^{-1}\left(\frac{\sqrt{2+x^6}}{\sqrt{2}}\right)}{8\sqrt{2}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 30, normalized size = 0.55

$$-\frac{{}_2F_1\left(-\frac{1}{2}, 2; \frac{1}{2}; \frac{x^6}{2}+1\right)}{12\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(2 + x^6)^(3/2)),x]
```

```
[Out] -1/12*Hypergeometric2F1[-1/2, 2, 1/2, 1 + x^6/2]/Sqrt[2 + x^6]
```

fricas [A] time = 0.75, size = 66, normalized size = 1.20

$$\frac{3\sqrt{2}\left(x^{12}+2x^6\right)\log\left(\frac{x^6+2\sqrt{2}\sqrt{x^6+2}+4}{x^6}\right) - 4\left(3x^6+2\right)\sqrt{x^6+2}}{96\left(x^{12}+2x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^6+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/96*(3*sqrt(2)*(x^12 + 2*x^6)*log((x^6 + 2*sqrt(2)*sqrt(x^6 + 2) + 4)/x^6)
 - 4*(3*x^6 + 2)*sqrt(x^6 + 2))/(x^12 + 2*x^6)
```

giac [A] time = 0.17, size = 63, normalized size = 1.15

$$-\frac{1}{32}\sqrt{2}\log\left(-\frac{\sqrt{2}-\sqrt{x^6+2}}{\sqrt{2}+\sqrt{x^6+2}}\right) - \frac{3x^6+2}{24\left(\left(x^6+2\right)^{\frac{3}{2}} - 2\sqrt{x^6+2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^6+2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/32*sqrt(2)*log(-(sqrt(2) - sqrt(x^6 + 2))/(sqrt(2) + sqrt(x^6 + 2))) - 1
/24*(3*x^6 + 2)/((x^6 + 2)^(3/2) - 2*sqrt(x^6 + 2))
```

maple [A] time = 0.02, size = 46, normalized size = 0.84

$$-\frac{\sqrt{2}\,\ln\left(\frac{\sqrt{x^6+2}-\sqrt{2}}{\sqrt{x^6}}\right)}{16} - \frac{3x^6+2}{24\sqrt{x^6+2}\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^6+2)^(3/2),x)
```

```
[Out] -1/24*(3*x^6+2)/x^6/(x^6+2)^(1/2)-1/16*2^(1/2)*ln(((x^6+2)^(1/2)-2^(1/2))/(
x^6)^(1/2))
```

maxima [A] time = 2.39, size = 63, normalized size = 1.15

$$-\frac{1}{32}\sqrt{2}\log\left(-\frac{\sqrt{2}-\sqrt{x^6+2}}{\sqrt{2}+\sqrt{x^6+2}}\right)-\frac{3x^6+2}{24\left(\left(x^6+2\right)^{\frac{3}{2}}-2\sqrt{x^6+2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^6+2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/32*sqrt(2)*log(-(sqrt(2) - sqrt(x^6 + 2))/(sqrt(2) + sqrt(x^6 + 2))) - 1
/24*(3*x^6 + 2)/((x^6 + 2)^(3/2) - 2*sqrt(x^6 + 2))
```

mupad [B] time = 1.30, size = 40, normalized size = 0.73

$$\frac{\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}\,\sqrt{x^6+2}}{2}\right)}{16}-\frac{1}{8\sqrt{x^6+2}}-\frac{1}{12x^6\sqrt{x^6+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(x^6 + 2)^(3/2)),x)
```

```
[Out] (2^(1/2)*atanh((2^(1/2)*(x^6 + 2)^(1/2))/2))/16 - 1/(8*(x^6 + 2)^(1/2)) - 1
/(12*x^6*(x^6 + 2)^(1/2))
```

sympy [A] time = 2.84, size = 49, normalized size = 0.89

$$\frac{\sqrt{2}\,\operatorname{asinh}\left(\frac{\sqrt{2}}{x^3}\right)}{16}-\frac{1}{8x^3\sqrt{1+\frac{2}{x^6}}}-\frac{1}{12x^9\sqrt{1+\frac{2}{x^6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**6+2)**(3/2),x)
```

```
[Out] sqrt(2)*asinh(sqrt(2)/x**3)/16 - 1/(8*x**3*sqrt(1 + 2/x**6)) - 1/(12*x**9*s
qrt(1 + 2/x**6))
```

3.1414 $\int \frac{1}{x^{13}\left(2+x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=71

$$\frac{5}{96x^6\sqrt{x^6+2}}+\frac{5}{64\sqrt{x^6+2}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{x^6+2}}{\sqrt{2}}\right)}{64\sqrt{2}}-\frac{1}{24x^{12}\sqrt{x^6+2}}$$

```
[Out] -5/128*arctanh(1/2*(x^6+2)^(1/2)*2^(1/2))*2^(1/2)+5/64/(x^6+2)^(1/2)-1/24/x
^12/(x^6+2)^(1/2)+5/96/x^6/(x^6+2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {266, 51, 63, 207}

$$\frac{5\sqrt{x^6+2}}{64x^6}-\frac{5\sqrt{x^6+2}}{48x^{12}}+\frac{1}{6x^{12}\sqrt{x^6+2}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{x^6+2}}{\sqrt{2}}\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^13*(2 + x^6)^(3/2)),x]
```

```
[Out] 1/(6*x^12*Sqrt[2 + x^6]) - (5*Sqrt[2 + x^6])/(48*x^12) + (5*Sqrt[2 + x^6])/
(64*x^6) - (5*ArcTanh[Sqrt[2 + x^6]/Sqrt[2]])/(64*Sqrt[2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{13}\left(2+x^6\right)^{3/2}}\,dx &= \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{x^3(2+x)^{3/2}}\,dx, x, x^6\right)\\
&= \frac{1}{6x^{12}\sqrt{2+x^6}} + \frac{5}{12}\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{2+x}}\,dx, x, x^6\right)\\
&= \frac{1}{6x^{12}\sqrt{2+x^6}} - \frac{5\sqrt{2+x^6}}{48x^{12}} - \frac{5}{32}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{2+x}}\,dx, x, x^6\right)\\
&= \frac{1}{6x^{12}\sqrt{2+x^6}} - \frac{5\sqrt{2+x^6}}{48x^{12}} + \frac{5\sqrt{2+x^6}}{64x^6} + \frac{5}{128}\,\text{Subst}\left(\int \frac{1}{x\sqrt{2+x}}\,dx, x, x^6\right)\\
&= \frac{1}{6x^{12}\sqrt{2+x^6}} - \frac{5\sqrt{2+x^6}}{48x^{12}} + \frac{5\sqrt{2+x^6}}{64x^6} + \frac{5}{64}\,\text{Subst}\left(\int \frac{1}{-2+x^2}\,dx, x, \sqrt{2+x^6}\right)\\
&= \frac{1}{6x^{12}\sqrt{2+x^6}} - \frac{5\sqrt{2+x^6}}{48x^{12}} + \frac{5\sqrt{2+x^6}}{64x^6} - \frac{5\tanh^{-1}\left(\frac{\sqrt{2+x^6}}{\sqrt{2}}\right)}{64\sqrt{2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 30, normalized size = 0.42

$$\frac{{}_2F_1\left(-\frac{1}{2}, 3; \frac{1}{2}; \frac{x^6}{2}+1\right)}{24\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^13*(2 + x^6)^(3/2)),x]
```

```
[Out] Hypergeometric2F1[-1/2, 3, 1/2, 1 + x^6/2]/(24*Sqrt[2 + x^6])
```

fricas [A] time = 0.75, size = 71, normalized size = 1.00

$$\frac{15\sqrt{2}\left(x^{18}+2x^{12}\right)\log\left(\frac{x^6-2\sqrt{2}\sqrt{x^6+2}+4}{x^6}\right) + 4\left(15x^{12}+10x^6-8\right)\sqrt{x^6+2}}{768\left(x^{18}+2x^{12}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(x^6+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/768*(15*sqrt(2)*(x^18 + 2*x^12)*log((x^6 - 2*sqrt(2)*sqrt(x^6 + 2) + 4)/x
^6) + 4*(15*x^12 + 10*x^6 - 8)*sqrt(x^6 + 2))/(x^18 + 2*x^12)
```

giac [A] time = 0.15, size = 68, normalized size = 0.96

$$\frac{5}{256}\sqrt{2}\log\left(-\frac{\sqrt{2}-\sqrt{x^6+2}}{\sqrt{2}+\sqrt{x^6+2}}\right) + \frac{1}{24\sqrt{x^6+2}} + \frac{7\left(x^6+2\right)^{\frac{3}{2}} - 18\sqrt{x^6+2}}{192x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(x^6+2)^(3/2),x, algorithm="giac")
```

```
[Out] 5/256*sqrt(2)*log(-(sqrt(2) - sqrt(x^6 + 2))/(sqrt(2) + sqrt(x^6 + 2))) + 1
/24/sqrt(x^6 + 2) + 1/192*(7*(x^6 + 2)^(3/2) - 18*sqrt(x^6 + 2))/x^12
```

maple [A] time = 0.03, size = 51, normalized size = 0.72

$$\frac{5\sqrt{2}\,\ln\left(\frac{\sqrt{x^6+2}-\sqrt{2}}{\sqrt{x^6}}\right)}{128} + \frac{15x^{12}+10x^6-8}{192\sqrt{x^6+2}\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^13/(x^6+2)^(3/2),x)
```

```
[Out] 1/192*(15*x^12+10*x^6-8)/x^12/(x^6+2)^(1/2)+5/128*2^(1/2)*ln(((x^6+2)^(1/2)
-2^(1/2))/(x^6)^(1/2))
```

maxima [A] time = 2.27, size = 81, normalized size = 1.14

$$\frac{5}{256}\sqrt{2}\log\left(-\frac{\sqrt{2}-\sqrt{x^6+2}}{\sqrt{2}+\sqrt{x^6+2}}\right)-\frac{50x^6-15\left(x^6+2\right)^2+68}{192\left(\left(x^6+2\right)^{\frac{5}{2}}-4\left(x^6+2\right)^{\frac{3}{2}}+4\sqrt{x^6+2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(x^6+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 5/256*sqrt(2)*log(-(sqrt(2) - sqrt(x^6 + 2))/(sqrt(2) + sqrt(x^6 + 2))) - 1
/192*(50*x^6 - 15*(x^6 + 2)^2 + 68)/((x^6 + 2)^(5/2) - 4*(x^6 + 2)^(3/2) +
4*sqrt(x^6 + 2))
```

mupad [B] time = 1.53, size = 65, normalized size = 0.92

$$-\frac{\frac{25x^6}{96}-\frac{5\left(x^6+2\right)^2}{64}+\frac{17}{48}}{4\sqrt{x^6+2}-4\left(x^6+2\right)^{3/2}+\left(x^6+2\right)^{5/2}}-\frac{5\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}\,\sqrt{x^6+2}}{2}\right)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^13*(x^6 + 2)^(3/2)),x)
```

```
[Out] - ((25*x^6)/96 - (5*(x^6 + 2)^2)/64 + 17/48)/(4*(x^6 + 2)^(1/2) - 4*(x^6 +
2)^(3/2) + (x^6 + 2)^(5/2)) - (5*2^(1/2)*atanh((2^(1/2)*(x^6 + 2)^(1/2))/2)
)/128
```

sympy [A] time = 6.64, size = 68, normalized size = 0.96

$$-\frac{5\sqrt{2}\,\operatorname{asinh}\left(\frac{\sqrt{2}}{x^3}\right)}{128}+\frac{5}{64x^3\sqrt{1+\frac{2}{x^6}}}+\frac{5}{96x^9\sqrt{1+\frac{2}{x^6}}}-\frac{1}{24x^{15}\sqrt{1+\frac{2}{x^6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**13/(x**6+2)**(3/2),x)
```

```
[Out] -5*sqrt(2)*asinh(sqrt(2)/x**3)/128 + 5/(64*x**3*sqrt(1 + 2/x**6)) + 5/(96*x
**9*sqrt(1 + 2/x**6)) - 1/(24*x**15*sqrt(1 + 2/x**6))
```

3.1415 $\int \frac{x^{14}}{\left(2+x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=45

$$-\sinh^{-1}\left(\frac{x^3}{\sqrt{2}}\right)-\frac{x^9}{3\sqrt{x^6+2}}+\frac{1}{2}\sqrt{x^6+2}\,x^3$$

```
[Out] -arcsinh(1/2*x^3*2^(1/2))-1/3*x^9/(x^6+2)^(1/2)+1/2*x^3*(x^6+2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {275, 288, 321, 215}

$$-\frac{x^9}{3\sqrt{x^6+2}}+\frac{1}{2}\sqrt{x^6+2}\,x^3-\sinh^{-1}\left(\frac{x^3}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^14/(2 + x^6)^(3/2),x]
```

```
[Out] -x^9/(3*Sqrt[2 + x^6]) + (x^3*Sqrt[2 + x^6])/2 - ArcSinh[x^3/Sqrt[2]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{14}}{\left(2+x^6\right)^{3/2}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^4}{\left(2+x^2\right)^{3/2}}\,dx, x, x^3\right)\\ &= -\frac{x^9}{3\sqrt{2+x^6}} + \text{Subst}\left(\int \frac{x^2}{\sqrt{2+x^2}}\,dx, x, x^3\right)\\ &= -\frac{x^9}{3\sqrt{2+x^6}} + \frac{1}{2}x^3\sqrt{2+x^6} - \text{Subst}\left(\int \frac{1}{\sqrt{2+x^2}}\,dx, x, x^3\right)\\ &= -\frac{x^9}{3\sqrt{2+x^6}} + \frac{1}{2}x^3\sqrt{2+x^6} - \sinh^{-1}\left(\frac{x^3}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 0.96

$$\frac{x^9 + 6x^3 - 6\sqrt{x^6+2}\,\sinh^{-1}\left(\frac{x^3}{\sqrt{2}}\right)}{6\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/(2 + x^6)^(3/2),x]
```

```
[Out] (6*x^3 + x^9 - 6*Sqrt[2 + x^6]*ArcSinh[x^3/Sqrt[2]])/(6*Sqrt[2 + x^6])
```

fricas [A] time = 0.84, size = 54, normalized size = 1.20

$$\frac{4\,x^6 + 6\left(x^6+2\right)\log\left(-x^3+\sqrt{x^6+2}\right) + \left(x^9+6\,x^3\right)\sqrt{x^6+2} + 8}{6\left(x^6+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(x^6+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(4*x^6 + 6*(x^6 + 2)*log(-x^3 + sqrt(x^6 + 2)) + (x^9 + 6*x^3)*sqrt(x^6
 + 2) + 8)/(x^6 + 2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(x^6+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(x^6+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^14/(x^6 + 2)^(3/2), x)
```

maple [A] time = 0.14, size = 30, normalized size = 0.67

$$\frac{\left(x^6+6\right)x^3}{6\sqrt{x^6+2}} - \text{arcsinh}\left(\frac{\sqrt{2}\,x^3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(x^6+2)^(3/2),x)
```

```
[Out] 1/6*x^3*(x^6+6)/(x^6+2)^(1/2)-arcsinh(1/2*2^(1/2)*x^3)
```

maxima [B] time = 1.01, size = 73, normalized size = 1.62

$$-\frac{\frac{3\left(x^6+2\right)}{x^6}-2}{3\left(\frac{\sqrt{x^6+2}}{x^3}-\frac{\left(x^6+2\right)^{\frac{3}{2}}}{x^9}\right)}-\frac{1}{2}\log\left(\frac{\sqrt{x^6+2}}{x^3}+1\right)+\frac{1}{2}\log\left(\frac{\sqrt{x^6+2}}{x^3}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(x^6+2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*(x^6 + 2)/x^6 - 2)/(sqrt(x^6 + 2)/x^3 - (x^6 + 2)^(3/2)/x^9) - 1/2*
log(sqrt(x^6 + 2)/x^3 + 1) + 1/2*log(sqrt(x^6 + 2)/x^3 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{14}}{\left(x^6+2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(x^6 + 2)^(3/2),x)
```

```
[Out] int(x^14/(x^6 + 2)^(3/2), x)
```

sympy [A] time = 5.51, size = 36, normalized size = 0.80

$$\frac{x^9}{6\sqrt{x^6+2}}+\frac{x^3}{\sqrt{x^6+2}}-\operatorname{asinh}\left(\frac{\sqrt{2}\,x^3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(x**6+2)**(3/2),x)
```

```
[Out] x**9/(6*sqrt(x**6 + 2)) + x**3/sqrt(x**6 + 2) - asinh(sqrt(2)*x**3/2)
```

3.1416 $\int \frac{x^8}{\left(2+x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=31

$$\frac{1}{3}\sinh^{-1}\left(\frac{x^3}{\sqrt{2}}\right)-\frac{x^3}{3\sqrt{x^6+2}}$$

```
[Out] 1/3*arcsinh(1/2*x^3*2^(1/2))-1/3*x^3/(x^6+2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {275, 288, 215}

$$\frac{1}{3}\sinh^{-1}\left(\frac{x^3}{\sqrt{2}}\right)-\frac{x^3}{3\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(2 + x^6)^(3/2),x]
```

```
[Out] -x^3/(3*Sqrt[2 + x^6]) + ArcSinh[x^3/Sqrt[2]]/3
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{\left(2+x^6\right)^{3/2}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{\left(2+x^2\right)^{3/2}}\,dx, x, x^3\right)\\ &= -\frac{x^3}{3\sqrt{2+x^6}}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{2+x^2}}\,dx, x, x^3\right)\\ &= -\frac{x^3}{3\sqrt{2+x^6}}+\frac{1}{3}\sinh^{-1}\left(\frac{x^3}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 0.94

$$\frac{1}{3}\left(\sinh^{-1}\left(\frac{x^3}{\sqrt{2}}\right)-\frac{x^3}{\sqrt{x^6+2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(2 + x^6)^(3/2),x]
```

```
[Out] (-(x^3/Sqrt[2 + x^6]) + ArcSinh[x^3/Sqrt[2]])/3
```

fricas [A] time = 0.84, size = 45, normalized size = 1.45

$$-\frac{x^6+\sqrt{x^6+2}\,x^3+\left(x^6+2\right)\log\left(-x^3+\sqrt{x^6+2}\right)+2}{3\left(x^6+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(x^6 + sqrt(x^6 + 2)*x^3 + (x^6 + 2)*log(-x^3 + sqrt(x^6 + 2)) + 2)/(x
^6 + 2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(x^6+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^8/(x^6 + 2)^(3/2), x)
```

maple [A] time = 0.02, size = 25, normalized size = 0.81

$$-\frac{x^3}{3\sqrt{x^6+2}}+\frac{\operatorname{arcsinh}\left(\frac{\sqrt{2}\,x^3}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^6+2)^(3/2),x)
```

```
[Out] 1/3*arcsinh(1/2*2^(1/2)*x^3)-1/3*x^3/(x^6+2)^(1/2)
```

maxima [A] time = 1.00, size = 45, normalized size = 1.45

$$-\frac{x^3}{3\sqrt{x^6+2}}+\frac{1}{6}\log\left(\frac{\sqrt{x^6+2}}{x^3}+1\right)-\frac{1}{6}\log\left(\frac{\sqrt{x^6+2}}{x^3}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6+2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*x^3/sqrt(x^6 + 2) + 1/6*log(sqrt(x^6 + 2)/x^3 + 1) - 1/6*log(sqrt(x^6
+ 2)/x^3 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^8}{\left(x^6+2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^6 + 2)^(3/2),x)
```

```
[Out] int(x^8/(x^6 + 2)^(3/2), x)
```

sympy [**A**] time = 2.83, size = 26, normalized size = 0.84

$$-\frac{x^3}{3\sqrt{x^6+2}}+\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\,x^3}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**6+2)**(3/2),x)
[Out] -x**3/(3*sqrt(x**6 + 2)) + asinh(sqrt(2)*x**3/2)/3
```

3.1417 $\int \frac{x^2}{\left(2+x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=16

$$\frac{x^3}{6\sqrt{x^6+2}}$$

```
[Out] 1/6*x^3/(x^6+2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {264}

$$\frac{x^3}{6\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 + x^6)^(3/2),x]
```

```
[Out] x^3/(6*Sqrt[2 + x^6])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^2}{\left(2+x^6\right)^{3/2}}\,dx = \frac{x^3}{6\sqrt{2+x^6}}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{x^3}{6\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 + x^6)^(3/2),x]
```

```
[Out] x^3/(6*Sqrt[2 + x^6])
```

fricas [B] time = 0.68, size = 25, normalized size = 1.56

$$\frac{x^6+\sqrt{x^6+2}\,x^3+2}{6\left(x^6+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(x^6 + sqrt(x^6 + 2)*x^3 + 2)/(x^6 + 2)
```

giac [A] time = 0.19, size = 12, normalized size = 0.75

$$\frac{x^3}{6\sqrt{x^6+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+2)^(3/2),x, algorithm="giac")
[Out] 1/6*x^3/sqrt(x^6 + 2)
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{x^3}{6\sqrt{x^6+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^6+2)^(3/2),x)
[Out] 1/6/(x^6+2)^(1/2)*x^3
```

maxima [A] time = 1.08, size = 12, normalized size = 0.75

$$\frac{x^3}{6\sqrt{x^6+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+2)^(3/2),x, algorithm="maxima")
[Out] 1/6*x^3/sqrt(x^6 + 2)
```

mupad [B] time = 1.14, size = 12, normalized size = 0.75

$$\frac{x^3}{6\sqrt{x^6+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^6 + 2)^(3/2),x)
[Out] x^3/(6*(x^6 + 2)^(1/2))
```

sympy [A] time = 1.04, size = 12, normalized size = 0.75

$$\frac{x^3}{6\sqrt{x^6+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**6+2)**(3/2),x)
[Out] x**3/(6*sqrt(x**6 + 2))
```

3.1418 $\int \frac{1}{x^4\left(2+x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=33

$$-\frac{x^3}{6\sqrt{x^6+2}}-\frac{1}{6\sqrt{x^6+2}\,x^3}$$

```
[Out] -1/6/x^3/(x^6+2)^(1/2)-1/6*x^3/(x^6+2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {271, 264}

$$-\frac{x^3}{6\sqrt{x^6+2}}-\frac{1}{6\sqrt{x^6+2}\,x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(2 + x^6)^(3/2)),x]
```

```
[Out] -1/(6*x^3*Sqrt[2 + x^6]) - x^3/(6*Sqrt[2 + x^6])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(2+x^6\right)^{3/2}}\,dx &= -\frac{1}{6x^3\sqrt{2+x^6}}-\int \frac{x^2}{\left(2+x^6\right)^{3/2}}\,dx\\ &= -\frac{1}{6x^3\sqrt{2+x^6}}-\frac{x^3}{6\sqrt{2+x^6}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.70

$$\frac{-x^6-1}{6x^3\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(2 + x^6)^(3/2)),x]
```

```
[Out] (-1 - x^6)/(6*x^3*Sqrt[2 + x^6])
```

fricas [A] time = 0.56, size = 35, normalized size = 1.06

$$-\frac{x^9+2x^3+\sqrt{x^6+2}\left(x^6+1\right)}{6\left(x^9+2x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^6+2)^(3/2),x, algorithm="fricas")
[Out] -1/6*(x^9 + 2*x^3 + sqrt(x^6 + 2)*(x^6 + 1))/(x^9 + 2*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^6+2\right)^{\frac{3}{2}} x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^6+2)^(3/2),x, algorithm="giac")
[Out] integrate(1/((x^6 + 2)^(3/2)*x^4), x)
```

maple [A] time = 0.00, size = 18, normalized size = 0.55

$$-\frac{x^6+1}{6\sqrt{x^6+2}\, x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^6+2)^(3/2),x)
[Out] -1/6*(x^6+1)/x^3/(x^6+2)^(1/2)
```

maxima [A] time = 1.03, size = 25, normalized size = 0.76

$$-\frac{x^3}{12\sqrt{x^6+2}} - \frac{\sqrt{x^6+2}}{12\, x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^6+2)^(3/2),x, algorithm="maxima")
[Out] -1/12*x^3/sqrt(x^6 + 2) - 1/12*sqrt(x^6 + 2)/x^3
```

mupad [B] time = 1.15, size = 17, normalized size = 0.52

$$-\frac{x^6+1}{6\, x^3\sqrt{x^6+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x^6 + 2)^(3/2)),x)
[Out] -(x^6 + 1)/(6*x^3*(x^6 + 2)^(1/2))
```

sympy [A] time = 1.50, size = 31, normalized size = 0.94

$$-\frac{1}{6\sqrt{1+\frac{2}{x^6}}} - \frac{1}{6x^6\sqrt{1+\frac{2}{x^6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**6+2)**(3/2),x)
[Out] -1/(6*sqrt(1 + 2/x**6)) - 1/(6*x**6*sqrt(1 + 2/x**6))
```

3.1419 $\int \frac{1}{x^{10}\left(2+x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=49

$$-\frac{1}{18\sqrt{x^6+2}\,x^9}+\frac{x^3}{9\sqrt{x^6+2}}+\frac{1}{9\sqrt{x^6+2}\,x^3}$$

```
[Out] -1/18/x^9/(x^6+2)^(1/2)+1/9/x^3/(x^6+2)^(1/2)+1/9*x^3/(x^6+2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {271, 264}

$$\frac{x^3}{9\sqrt{x^6+2}}+\frac{1}{9\sqrt{x^6+2}\,x^3}-\frac{1}{18\sqrt{x^6+2}\,x^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*(2 + x^6)^(3/2)),x]
```

```
[Out] -1/(18*x^9*Sqrt[2 + x^6]) + 1/(9*x^3*Sqrt[2 + x^6]) + x^3/(9*Sqrt[2 + x^6])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{10}\left(2+x^6\right)^{3/2}}\,dx &= -\frac{1}{18x^9\sqrt{2+x^6}}-\frac{2}{3}\int \frac{1}{x^4\left(2+x^6\right)^{3/2}}\,dx\\ &= -\frac{1}{18x^9\sqrt{2+x^6}}+\frac{1}{9x^3\sqrt{2+x^6}}+\frac{2}{3}\int \frac{x^2}{\left(2+x^6\right)^{3/2}}\,dx\\ &= -\frac{1}{18x^9\sqrt{2+x^6}}+\frac{1}{9x^3\sqrt{2+x^6}}+\frac{x^3}{9\sqrt{2+x^6}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.57

$$\frac{2x^{12}+2x^6-1}{18x^9\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^10*(2 + x^6)^(3/2)),x]
```

```
[Out] (-1 + 2*x^6 + 2*x^12)/(18*x^9*Sqrt[2 + x^6])
```

fricas [A] time = 0.89, size = 44, normalized size = 0.90

$$\frac{2x^{15}+4x^9+\left(2x^{12}+2x^6-1\right)\sqrt{x^6+2}}{18\left(x^{15}+2x^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(x^6+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/18*(2*x^15 + 4*x^9 + (2*x^12 + 2*x^6 - 1)*sqrt(x^6 + 2))/(x^15 + 2*x^9)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(x^6+2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(t_nostep)]Warnin
g, integration of abs or sign assumes constant sign by intervals (correct i
f the argument is real):Check [abs(x)]sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.00, size = 25, normalized size = 0.51

$$\frac{2x^{12}+2x^6-1}{18\sqrt{x^6+2}\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(x^6+2)^(3/2),x)
```

```
[Out] 1/18*(2*x^12+2*x^6-1)/x^9/(x^6+2)^(1/2)
```

maxima [A] time = 1.11, size = 37, normalized size = 0.76

$$\frac{x^3}{24\sqrt{x^6+2}}+\frac{\sqrt{x^6+2}}{12x^3}-\frac{\left(x^6+2\right)^{\frac{3}{2}}}{72x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(x^6+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/24*x^3/sqrt(x^6 + 2) + 1/12*sqrt(x^6 + 2)/x^3 - 1/72*(x^6 + 2)^(3/2)/x^9
```

mupad [B] time = 1.24, size = 28, normalized size = 0.57

$$-\frac{24x^6-8\left(x^6+2\right)^2+36}{72x^9\sqrt{x^6+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^10*(x^6 + 2)^(3/2)),x)
```

```
[Out] -(24*x^6 - 8*(x^6 + 2)^2 + 36)/(72*x^9*(x^6 + 2)^(1/2))
```

sympy [A] time = 2.53, size = 70, normalized size = 1.43

$$\frac{2x^{12}\sqrt{1+\frac{2}{x^6}}}{18x^{12}+36x^6}+\frac{2x^6\sqrt{1+\frac{2}{x^6}}}{18x^{12}+36x^6}-\frac{\sqrt{1+\frac{2}{x^6}}}{18x^{12}+36x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(x**6+2)**(3/2),x)
```

```
[Out] 2*x**12*sqrt(1 + 2/x**6)/(18*x**12 + 36*x**6) + 2*x**6*sqrt(1 + 2/x**6)/(18
*x**12 + 36*x**6) - sqrt(1 + 2/x**6)/(18*x**12 + 36*x**6)
```

3.1420 $\int \frac{x^{13}}{\left(2+x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=202

$$-\frac{x^8}{3\sqrt{x^6+2}}+\frac{8}{15}\sqrt{x^6+2}\,x^2-\frac{16\,2^{5/6}\sqrt{2+\sqrt{3}}\,\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}\,x^2+2^{2/3}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\,F\left(\sin^{-1}\left(\frac{x^2+\sqrt[3]{2}\left(1-\sqrt{3}\right)}{x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)}\right)|-7-4\sqrt{3}\right)}{15\sqrt[4]{3}\,\sqrt{\frac{x^2+\sqrt[3]{2}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\,\sqrt{x^6+2}}$$

```
[Out] -1/3*x^8/(x^6+2)^(1/2)+8/15*x^2*(x^6+2)^(1/2)-16/45*2^(5/6)*(2^(1/3)+x^2)*E
llipticF((x^2+2^(1/3)*(1-3^(1/2)))/(x^2+2^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)
*(1/2*6^(1/2)+1/2*2^(1/2))*((2^(2/3)-2^(1/3)*x^2+x^4)/(x^2+2^(1/3)*(1+3^(1/
2)))^2)^(1/2)*3^(3/4)/(x^6+2)^(1/2)/((2^(1/3)+x^2)/(x^2+2^(1/3)*(1+3^(1/2))
)^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {275, 288, 321, 218}

$$-\frac{x^8}{3\sqrt{x^6+2}}+\frac{8}{15}\sqrt{x^6+2}\,x^2-\frac{16\,2^{5/6}\sqrt{2+\sqrt{3}}\,\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}\,x^2+2^{2/3}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\,F\left(\sin^{-1}\left(\frac{x^2+\sqrt[3]{2}\left(1-\sqrt{3}\right)}{x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)}\right)|-7-4\sqrt{3}\right)}{15\sqrt[4]{3}\,\sqrt{\frac{x^2+\sqrt[3]{2}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\,\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/(2 + x^6)^(3/2),x]
```

```
[Out] -x^8/(3*Sqrt[2 + x^6]) + (8*x^2*Sqrt[2 + x^6])/15 - (16*2^(5/6)*Sqrt[2 + Sq
rt[3]]*(2^(1/3) + x^2)*Sqrt[(2^(2/3) - 2^(1/3)*x^2 + x^4)/(2^(1/3)*(1 + Sqr
t[3]) + x^2)^2]*EllipticF[ArcSin[(2^(1/3)*(1 - Sqrt[3]) + x^2)/(2^(1/3)*(1
+ Sqrt[3]) + x^2)], -7 - 4*Sqrt[3]])/(15*3^(1/4)*Sqrt[(2^(1/3) + x^2)/(2^(1
/3)*(1 + Sqrt[3]) + x^2)^2]*Sqrt[2 + x^6])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{13}}{\left(2+x^6\right)^{3/2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^6}{\left(2+x^3\right)^{3/2}}\,dx, x, x^2\right) \\
&= -\frac{x^8}{3\sqrt{2+x^6}} + \frac{4}{3}\,\text{Subst}\left(\int \frac{x^3}{\sqrt{2+x^3}}\,dx, x, x^2\right) \\
&= -\frac{x^8}{3\sqrt{2+x^6}} + \frac{8}{15}x^2\sqrt{2+x^6} - \frac{16}{15}\,\text{Subst}\left(\int \frac{1}{\sqrt{2+x^3}}\,dx, x, x^2\right) \\
&= -\frac{x^8}{3\sqrt{2+x^6}} + \frac{8}{15}x^2\sqrt{2+x^6} - \frac{16\,2^{5/6}\sqrt{2+\sqrt{3}}\,\left(\sqrt[3]{2}+x^2\right)\sqrt{\frac{2^{2/3}-\sqrt[3]{2}x^2+x^4}{\left(\sqrt[3]{2}\left(1+\sqrt{3}\right)+x^2\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{2}}{\sqrt[3]{2}}\right.\right.}{15\sqrt[4]{3}\sqrt{\frac{\sqrt[3]{2}+x^2}{\left(\sqrt[3]{2}\left(1+\sqrt{3}\right)+x^2\right)^2}}\sqrt{2+x^6}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 56, normalized size = 0.28

$$\frac{x^2\left(-8\sqrt{2}\,\sqrt{x^6+2}\;{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{x^6}{2}\right)+3x^6+16\right)}{15\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13/(2 + x^6)^(3/2),x]
```

```
[Out] (x^2*(16 + 3*x^6 - 8*Sqrt[2]*Sqrt[2 + x^6]*Hypergeometric2F1[1/3, 1/2, 4/3,
 -1/2*x^6]))/(15*Sqrt[2 + x^6])
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^6+2}\,x^{13}}{x^{12}+4\,x^6+4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(x^6+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^6 + 2)*x^13/(x^12 + 4*x^6 + 4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{\left(x^6+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(x^6+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^13/(x^6 + 2)^(3/2), x)
```

maple [C] time = 0.15, size = 40, normalized size = 0.20

$$-\frac{8\sqrt{2}\,x^2\,\text{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],-\frac{x^6}{2}\right)}{15}+\frac{\left(3x^6+16\right)x^2}{15\sqrt{x^6+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(x^6+2)^(3/2),x)
```

```
[Out] 1/15*x^2*(3*x^6+16)/(x^6+2)^(1/2)-8/15*2^(1/2)*x^2*hypergeom([1/3,1/2],[4/3
],-1/2*x^6)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{\left(x^6+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(x^6+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^13/(x^6 + 2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{13}}{\left(x^6+2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(x^6 + 2)^(3/2),x)
```

```
[Out] int(x^13/(x^6 + 2)^(3/2), x)
```

sympy [A] time = 2.41, size = 36, normalized size = 0.18

$$\frac{\sqrt{2}\,x^{14}\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\right|\frac{x^6e^{i\pi}}{2}\right)}{24\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(x**6+2)**(3/2),x)
```

```
[Out] sqrt(2)*x**14*gamma(7/3)*hyper((3/2, 7/3), (10/3,), x**6*exp_polar(I*pi)/2)
/(24*gamma(10/3))
```

3.1421 $\int \frac{x^7}{\left(2+x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=186

$$\frac{2^{5/6}\sqrt{2+\sqrt{3}}\,\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}\,x^2+2^{2/3}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\,F\left(\sin^{-1}\left(\frac{x^2+\sqrt[3]{2}\left(1-\sqrt{3}\right)}{x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,\sqrt{\frac{x^2+\sqrt[3]{2}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\,\sqrt{x^6+2}}-\frac{x^2}{3\sqrt{x^6+2}}$$

```
[Out] -1/3*x^2/(x^6+2)^(1/2)+1/9*2^(5/6)*(2^(1/3)+x^2)*EllipticF((x^2+2^(1/3)*(1-
3^(1/2)))/(x^2+2^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2)
)*((2^(2/3)-2^(1/3)*x^2+x^4)/(x^2+2^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/(x^
6+2)^(1/2)/((2^(1/3)+x^2)/(x^2+2^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {275, 288, 218}

$$\frac{2^{5/6}\sqrt{2+\sqrt{3}}\,\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}\,x^2+2^{2/3}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\,F\left(\sin^{-1}\left(\frac{x^2+\sqrt[3]{2}\left(1-\sqrt{3}\right)}{x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,\sqrt{\frac{x^2+\sqrt[3]{2}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\,\sqrt{x^6+2}}-\frac{x^2}{3\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(2 + x^6)^(3/2),x]
```

```
[Out] -x^2/(3*Sqrt[2 + x^6]) + (2^(5/6)*Sqrt[2 + Sqrt[3]]*(2^(1/3) + x^2)*Sqrt[(2
^(2/3) - 2^(1/3)*x^2 + x^4)/(2^(1/3)*(1 + Sqrt[3]) + x^2)^2]*EllipticF[ArcS
in[(2^(1/3)*(1 - Sqrt[3]) + x^2)/(2^(1/3)*(1 + Sqrt[3]) + x^2)], -7 - 4*Sqr
t[3]])/(3*3^(1/4)*Sqrt[(2^(1/3) + x^2)/(2^(1/3)*(1 + Sqrt[3]) + x^2)^2]*Sqr
t[2 + x^6])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^7}{\left(2+x^6\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{\left(2+x^3\right)^{3/2}}\,dx, x, x^2\right)$$
$$= -\frac{x^2}{3\sqrt{2+x^6}} + \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{2+x^3}}\,dx, x, x^2\right)$$
$$= -\frac{x^2}{3\sqrt{2+x^6}} + \frac{2^{5/6}\sqrt{2+\sqrt{3}}\,\left(\sqrt[3]{2}+x^2\right)\sqrt{\frac{2^{2/3}-\sqrt[3]{2}\,x^2+x^4}{\left(\sqrt[3]{2}\left(1+\sqrt{3}\right)+x^2\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{2}\left(1-\sqrt{3}\right)+x^2}{\sqrt[3]{2}\left(1+\sqrt{3}\right)+x^2}\right)|-7-4\sqrt{3}\right.}{3\sqrt[4]{3}\,\sqrt{\frac{\sqrt[3]{2}+x^2}{\left(\sqrt[3]{2}\left(1+\sqrt{3}\right)+x^2\right)^2}}\,\sqrt{2+x^6}}$$

Mathematica [C] time = 0.01, size = 42, normalized size = 0.23

$$\frac{1}{6}x^2\left(\sqrt{2}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{x^6}{2}\right) - \frac{2}{\sqrt{x^6+2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(2 + x^6)^(3/2),x]
```

```
[Out] (x^2*(-2/Sqrt[2 + x^6] + Sqrt[2]*Hypergeometric2F1[1/3, 1/2, 4/3, -1/2*x^6]
))/6
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^6+2}\,x^7}{x^{12}+4\,x^6+4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^6+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^6 + 2)*x^7/(x^12 + 4*x^6 + 4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(x^6+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^6+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^7/(x^6 + 2)^(3/2), x)
```

maple [C] time = 0.13, size = 33, normalized size = 0.18

$$\frac{\sqrt{2}\,x^2\,\text{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],-\frac{x^6}{2}\right)}{6} - \frac{x^2}{3\sqrt{x^6+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^6+2)^(3/2),x)
```

```
[Out] -1/3*x^2/(x^6+2)^(1/2)+1/6*2^(1/2)*x^2*hypergeom([1/3,1/2],[4/3],-1/2*x^6)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(x^6+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^6+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^7/(x^6 + 2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7}{\left(x^6+2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^6 + 2)^(3/2),x)
```

```
[Out] int(x^7/(x^6 + 2)^(3/2), x)
```

sympy [A] time = 1.61, size = 36, normalized size = 0.19

$$\frac{\sqrt{2}\,x^8\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{4}{3}, \frac{3}{2}\\ \frac{7}{3}\end{matrix}\right| \frac{x^6 e^{i\pi}}{2}\right)}{24\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**6+2)**(3/2),x)
```

```
[Out] sqrt(2)*x**8*gamma(4/3)*hyper((4/3, 3/2), (7/3,), x**6*exp_polar(I*pi)/2)/(
24*gamma(7/3))
```

3.1422 $\int \frac{x}{\left(2+x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=186

$$\frac{x^2}{6\sqrt{x^6+2}}+\frac{\sqrt{2+\sqrt{3}}\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}F\left(\sin^{-1}\left(\frac{x^2+\sqrt[3]{2}\left(1-\sqrt{3}\right)}{x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)}\right)|-7-4\sqrt{3}\right)}{6\sqrt[6]{2}\,\sqrt[4]{3}\sqrt{\frac{x^2+\sqrt[3]{2}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\sqrt{x^6+2}}$$

```
[Out] 1/6*x^2/(x^6+2)^(1/2)+1/36*2^(5/6)*(2^(1/3)+x^2)*EllipticF((x^2+2^(1/3)*(1-
3^(1/2)))/(x^2+2^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2)
)*((2^(2/3)-2^(1/3)*x^2+x^4)/(x^2+2^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/(x^
6+2)^(1/2)/((2^(1/3)+x^2)/(x^2+2^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {275, 199, 218}

$$\frac{x^2}{6\sqrt{x^6+2}}+\frac{\sqrt{2+\sqrt{3}}\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}F\left(\sin^{-1}\left(\frac{x^2+\sqrt[3]{2}\left(1-\sqrt{3}\right)}{x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)}\right)|-7-4\sqrt{3}\right)}{6\sqrt[6]{2}\,\sqrt[4]{3}\sqrt{\frac{x^2+\sqrt[3]{2}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(2 + x^6)^(3/2),x]
```

```
[Out] x^2/(6*Sqrt[2 + x^6]) + (Sqrt[2 + Sqrt[3]]*(2^(1/3) + x^2)*Sqrt[(2^(2/3) -
2^(1/3)*x^2 + x^4)/(2^(1/3)*(1 + Sqrt[3]) + x^2)^2]*EllipticF[ArcSin[(2^(1/
3)*(1 - Sqrt[3]) + x^2)/(2^(1/3)*(1 + Sqrt[3]) + x^2)], -7 - 4*Sqrt[3]])/(6
*2^(1/6)*3^(1/4)*Sqrt[(2^(1/3) + x^2)/(2^(1/3)*(1 + Sqrt[3]) + x^2)^2]*Sqrt
[2 + x^6])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x}{\left(2+x^6\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\left(2+x^3\right)^{3/2}}\,dx, x, x^2\right)$$
$$= \frac{x^2}{6\sqrt{2+x^6}} + \frac{1}{12}\,\text{Subst}\left(\int \frac{1}{\sqrt{2+x^3}}\,dx, x, x^2\right)$$
$$= \frac{x^2}{6\sqrt{2+x^6}} + \frac{\sqrt{2+\sqrt{3}}\,\left(\sqrt[3]{2}+x^2\right)\sqrt{\frac{2^{2/3}-\sqrt[3]{2}\,x^2+x^4}{\left(\sqrt[3]{2}\left(1+\sqrt{3}\right)+x^2\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{2}\left(1-\sqrt{3}\right)+x^2}{\sqrt[3]{2}\left(1+\sqrt{3}\right)+x^2}\right)\middle|-7-4\sqrt{3}\right)}{6\sqrt[6]{2}\,\sqrt[4]{3}\,\sqrt{\frac{\sqrt[3]{2}+x^2}{\left(\sqrt[3]{2}\left(1+\sqrt{3}\right)+x^2\right)^2}}\,\sqrt{2+x^6}}$$

Mathematica [C] time = 0.01, size = 42, normalized size = 0.23

$$\frac{1}{24}x^2\left(\sqrt{2}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{x^6}{2}\right)+\frac{4}{\sqrt{x^6+2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(2 + x^6)^(3/2),x]
```

```
[Out] (x^2*(4/Sqrt[2 + x^6] + Sqrt[2]*Hypergeometric2F1[1/3, 1/2, 4/3, -1/2*x^6])
)/24
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^6+2}\,x}{x^{12}+4\,x^6+4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^6 + 2)*x/(x^12 + 4*x^6 + 4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^6+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/(x^6 + 2)^(3/2), x)
```

maple [C] time = 0.16, size = 33, normalized size = 0.18

$$\frac{\sqrt{2}\,x^2\,\text{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],-\frac{x^6}{2}\right)}{24}+\frac{x^2}{6\sqrt{x^6+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^6+2)^(3/2),x)
```

```
[Out] 1/6/(x^6+2)^(1/2)*x^2+1/24*2^(1/2)*x^2*hypergeom([1/3,1/2],[4/3],-1/2*x^6)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^6+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(x^6 + 2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\left(x^6+2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^6 + 2)^(3/2),x)
```

```
[Out] int(x/(x^6 + 2)^(3/2), x)
```

sympy [A] time = 1.42, size = 36, normalized size = 0.19

$$\frac{\sqrt{2}\,x^2\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3}, \frac{3}{2} \\ \frac{4}{3}\end{matrix}\right| \frac{x^6 e^{i\pi}}{2}\right)}{24\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**6+2)**(3/2),x)
```

```
[Out] sqrt(2)*x**2*gamma(1/3)*hyper((1/3, 3/2), (4/3,), x**6*exp_polar(I*pi)/2)/(
24*gamma(4/3))
```

3.1423 $\int \frac{1}{x^5\left(2+x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=202

$$-\frac{7\sqrt{x^6+2}}{48x^4}+\frac{1}{6x^4\sqrt{x^6+2}}-\frac{7\sqrt{2+\sqrt{3}}\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}F\left(\sin^{-1}\left(\frac{x^2+\sqrt[3]{2}\left(1-\sqrt{3}\right)}{x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)}\right)|-7-4\sqrt{3}\right)}{48\sqrt[6]{2}\,\sqrt[4]{3}\sqrt{\frac{x^2+\sqrt[3]{2}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\sqrt{x^6+2}}$$

```
[Out] 1/6/x^4/(x^6+2)^(1/2)-7/48*(x^6+2)^(1/2)/x^4-7/288*2^(5/6)*(2^(1/3)+x^2)*El
lipticF((x^2+2^(1/3)*(1-3^(1/2)))/(x^2+2^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*
(1/2*6^(1/2)+1/2*2^(1/2))*((2^(2/3)-2^(1/3)*x^2+x^4)/(x^2+2^(1/3)*(1+3^(1/2
)))^2)^(1/2)*3^(3/4)/(x^6+2)^(1/2)/((2^(1/3)+x^2)/(x^2+2^(1/3)*(1+3^(1/2)))
^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {275, 290, 325, 218}

$$-\frac{7\sqrt{x^6+2}}{48x^4}+\frac{1}{6x^4\sqrt{x^6+2}}-\frac{7\sqrt{2+\sqrt{3}}\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}F\left(\sin^{-1}\left(\frac{x^2+\sqrt[3]{2}\left(1-\sqrt{3}\right)}{x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)}\right)|-7-4\sqrt{3}\right)}{48\sqrt[6]{2}\,\sqrt[4]{3}\sqrt{\frac{x^2+\sqrt[3]{2}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(2 + x^6)^(3/2)),x]
```

```
[Out] 1/(6*x^4*Sqrt[2 + x^6]) - (7*Sqrt[2 + x^6])/(48*x^4) - (7*Sqrt[2 + Sqrt[3]]
*(2^(1/3) + x^2)*Sqrt[(2^(2/3) - 2^(1/3)*x^2 + x^4)/(2^(1/3)*(1 + Sqrt[3])
+ x^2)^2]*EllipticF[ArcSin[(2^(1/3)*(1 - Sqrt[3]) + x^2)/(2^(1/3)*(1 + Sqrt
[3]) + x^2)], -7 - 4*Sqrt[3]])/(48*2^(1/6)*3^(1/4)*Sqrt[(2^(1/3) + x^2)/(2^
(1/3)*(1 + Sqrt[3]) + x^2)^2]*Sqrt[2 + x^6])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\left(2+x^6\right)^{3/2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3\left(2+x^3\right)^{3/2}}\,dx, x, x^2\right)\\ &= \frac{1}{6x^4\sqrt{2+x^6}} + \frac{7}{12}\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{2+x^3}}\,dx, x, x^2\right)\\ &= \frac{1}{6x^4\sqrt{2+x^6}} - \frac{7\sqrt{2+x^6}}{48x^4} - \frac{7}{96}\,\text{Subst}\left(\int \frac{1}{\sqrt{2+x^3}}\,dx, x, x^2\right)\\ &= \frac{1}{6x^4\sqrt{2+x^6}} - \frac{7\sqrt{2+x^6}}{48x^4} - \frac{7\sqrt{2+\sqrt{3}}\,\left(\sqrt[3]{2}+x^2\right)\sqrt{\frac{2^{2/3}-\sqrt[3]{2}x^2+x^4}{\left(\sqrt[3]{2}\left(1+\sqrt{3}\right)+x^2\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{2}\left(1-\sqrt{3}\right)+x\right.\right.}{48\sqrt[6]{2}\,\sqrt[4]{3}\,\sqrt{\frac{\sqrt[3]{2}+x^2}{\left(\sqrt[3]{2}\left(1+\sqrt{3}\right)+x^2\right)^2}}\,\sqrt{2+x^6}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.14

$$-\frac{{}_2F_1\left(-\frac{2}{3},\frac{3}{2};\frac{1}{3};-\frac{x^6}{2}\right)}{8\sqrt{2}\,x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(2 + x^6)^(3/2)),x]
```

```
[Out] -1/8*Hypergeometric2F1[-2/3, 3/2, 1/3, -1/2*x^6]/(Sqrt[2]*x^4)
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^6+2}}{x^{17}+4\,x^{11}+4\,x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^6+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^6 + 2)/(x^17 + 4*x^11 + 4*x^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^6+2\right)^{\frac{3}{2}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^6+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^6 + 2)^(3/2)*x^5), x)
```

maple [C] time = 0.14, size = 40, normalized size = 0.20

$$-\frac{7\sqrt{2}\,x^2\,\text{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],-\frac{x^6}{2}\right)}{192}-\frac{7x^6+6}{48\sqrt{x^6+2}\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^6+2)^(3/2),x)
```

```
[Out] -1/48*(7*x^6+6)/x^4/(x^6+2)^(1/2)-7/192*2^(1/2)*x^2*hypergeom([1/3,1/2],[4/
3],-1/2*x^6)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^6+2\right)^{\frac{3}{2}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^6+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^6 + 2)^(3/2)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^5\left(x^6+2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(x^6 + 2)^(3/2)),x)
```

```
[Out] int(1/(x^5*(x^6 + 2)^(3/2)), x)
```

sympy [A] time = 1.97, size = 39, normalized size = 0.19

$$\frac{\sqrt{2}\,\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{3}{2}\\ \frac{1}{3}\end{matrix}\right|\frac{x^6e^{i\pi}}{2}\right)}{24x^4\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**6+2)**(3/2),x)
```

```
[Out] sqrt(2)*gamma(-2/3)*hyper((-2/3, 3/2), (1/3,), x**6*exp_polar(I*pi)/2)/(24*
x**4*gamma(1/3))
```

3.1424 $\int \frac{x^{12}}{\left(2+x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=195

$$\frac{7}{12}\sqrt{x^6+2}\,x-\frac{x^7}{3\sqrt{x^6+2}}-\frac{7\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\,xF\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x^2+\sqrt[3]{2}}{\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{12\sqrt[3]{2}\,\sqrt[4]{3}\,\sqrt{\frac{x^2\left(x^2+\sqrt[3]{2}\right)}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\,\sqrt{x^6+2}}$$

```
[Out] -1/3*x^7/(x^6+2)^(1/2)+7/12*x*(x^6+2)^(1/2)-7/72*x*(2^(1/3)+x^2)*((2^(1/3)+
x^2*(1-3^(1/2)))^2/(2^(1/3)+x^2*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)+x^2*(1-3^(1/
2)))*(2^(1/3)+x^2*(1+3^(1/2)))*EllipticF((1-(2^(1/3)+x^2*(1-3^(1/2)))^2/(2^
(1/3)+x^2*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((2^(2/3)-2^(1/3)*
x^2+x^4)/(2^(1/3)+x^2*(1+3^(1/2)))^2)^(1/2)*2^(2/3)*3^(3/4)/(x^6+2)^(1/2)/(
x^2*(2^(1/3)+x^2)/(2^(1/3)+x^2*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {288, 321, 225}

$$-\frac{x^7}{3\sqrt{x^6+2}}+\frac{7}{12}\sqrt{x^6+2}\,x-\frac{7\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\,xF\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x^2+\sqrt[3]{2}}{\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{12\sqrt[3]{2}\,\sqrt[4]{3}\,\sqrt{\frac{x^2\left(x^2+\sqrt[3]{2}\right)}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\,\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^12/(2 + x^6)^(3/2),x]
```

```
[Out] -x^7/(3*Sqrt[2 + x^6]) + (7*x*Sqrt[2 + x^6])/12 - (7*x*(2^(1/3) + x^2)*Sqrt
[(2^(2/3) - 2^(1/3)*x^2 + x^4)/(2^(1/3) + (1 + Sqrt[3])*x^2)^2]*EllipticF[A
rcCos[(2^(1/3) + (1 - Sqrt[3])*x^2)/(2^(1/3) + (1 + Sqrt[3])*x^2)], (2 + Sq
rt[3])/4])/(12*2^(1/3)*3^(1/4)*Sqrt[(x^2*(2^(1/3) + x^2))/(2^(1/3) + (1 + S
qrt[3])*x^2)^2]*Sqrt[2 + x^6])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^{12}}{\left(2+x^6\right)^{3/2}}\,dx = -\frac{x^7}{3\sqrt{2+x^6}} + \frac{7}{3}\int \frac{x^6}{\sqrt{2+x^6}}\,dx$$

$$= -\frac{x^7}{3\sqrt{2+x^6}} + \frac{7}{12}x\sqrt{2+x^6} - \frac{7}{6}\int \frac{1}{\sqrt{2+x^6}}\,dx$$

$$= -\frac{x^7}{3\sqrt{2+x^6}} + \frac{7}{12}x\sqrt{2+x^6} - \frac{7x\left(\sqrt[3]{2}+x^2\right)\sqrt{\frac{2^{2/3}-\sqrt[3]{2}x^2+x^4}{\left(\sqrt[3]{2}+\left(1+\sqrt{3}\right)x^2\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{2}+\left(1-\sqrt{3}\right)x^2}{\sqrt[3]{2}+\left(1+\sqrt{3}\right)x^2}\right)|\frac{1}{4}\left(\right.\right.}{12\sqrt[3]{2}\sqrt[4]{3}\sqrt{\frac{x^2\left(\sqrt[3]{2}+x^2\right)}{\left(\sqrt[3]{2}+\left(1+\sqrt{3}\right)x^2\right)^2}}\sqrt{2+x^6}}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.28

$$\frac{x\left(-7\sqrt{2}\sqrt{x^6+2}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{x^6}{2}\right)+3x^6+14\right)}{12\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12/(2 + x^6)^(3/2),x]
```

```
[Out] (x*(14 + 3*x^6 - 7*Sqrt[2]*Sqrt[2 + x^6]*Hypergeometric2F1[1/6, 1/2, 7/6, -
1/2*x^6]))/(12*Sqrt[2 + x^6])
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^6+2}\,x^{12}}{x^{12}+4\,x^6+4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(x^6+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^6 + 2)*x^12/(x^12 + 4*x^6 + 4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{12}}{\left(x^6+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(x^6+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^12/(x^6 + 2)^(3/2), x)
```

maple [C] time = 0.14, size = 36, normalized size = 0.18

$$-\frac{7\sqrt{2}\;x\,\text{hypergeom}\left(\left[\frac{1}{6},\frac{1}{2}\right],\left[\frac{7}{6}\right],-\frac{x^6}{2}\right)}{12}+\frac{\left(3x^6+14\right)x}{12\sqrt{x^6+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(x^6+2)^(3/2),x)
```

```
[Out] 1/12*x*(3*x^6+14)/(x^6+2)^(1/2)-7/12*2^(1/2)*x*hypergeom([1/6,1/2],[7/6],-1
/2*x^6)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{12}}{\left(x^6+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(x^6+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^12/(x^6 + 2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{12}}{\left(x^6+2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(x^6 + 2)^(3/2),x)
```

```
[Out] int(x^12/(x^6 + 2)^(3/2), x)
```

sympy [C] time = 2.35, size = 36, normalized size = 0.18

$$\frac{\sqrt{2}\,x^{13}\Gamma\left(\frac{13}{6}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2}, \frac{13}{6} \\ \frac{19}{6}\end{matrix}\right| \frac{x^6 e^{i\pi}}{2}\right)}{24\Gamma\left(\frac{19}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12/(x**6+2)**(3/2),x)
```

```
[Out] sqrt(2)*x**13*gamma(13/6)*hyper((3/2, 13/6), (19/6,), x**6*exp_polar(I*pi)/
2)/(24*gamma(19/6))
```

3.1425 $\int \frac{x^6}{\left(2+x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=179

$$\frac{x\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x^2+\sqrt[3]{2}}{\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{6\sqrt[3]{2}\,\sqrt[4]{3}\sqrt{\frac{x^2\left(x^2+\sqrt[3]{2}\right)}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\sqrt{x^6+2}}-\frac{x}{3\sqrt{x^6+2}}$$

```
[Out] -1/3*x/(x^6+2)^(1/2)+1/36*x*(2^(1/3)+x^2)*((2^(1/3)+x^2*(1-3^(1/2)))^2/(2^(
1/3)+x^2*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)+x^2*(1-3^(1/2)))*(2^(1/3)+x^2*(1+3^
(1/2)))*EllipticF((1-(2^(1/3)+x^2*(1-3^(1/2)))^2/(2^(1/3)+x^2*(1+3^(1/2)))^
2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((2^(2/3)-2^(1/3)*x^2+x^4)/(2^(1/3)+x^2*(
1+3^(1/2)))^2)^(1/2)*2^(2/3)*3^(3/4)/(x^6+2)^(1/2)/(x^2*(2^(1/3)+x^2)/(2^(1
/3)+x^2*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {288, 225}

$$\frac{x\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x^2+\sqrt[3]{2}}{\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{6\sqrt[3]{2}\,\sqrt[4]{3}\sqrt{\frac{x^2\left(x^2+\sqrt[3]{2}\right)}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\sqrt{x^6+2}}-\frac{x}{3\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(2 + x^6)^(3/2),x]
```

```
[Out] -x/(3*Sqrt[2 + x^6]) + (x*(2^(1/3) + x^2)*Sqrt[(2^(2/3) - 2^(1/3)*x^2 + x^4
)/(2^(1/3) + (1 + Sqrt[3])*x^2)^2]*EllipticF[ArcCos[(2^(1/3) + (1 - Sqrt[3]
)*x^2)/(2^(1/3) + (1 + Sqrt[3])*x^2)], (2 + Sqrt[3])/4])/(6*2^(1/3)*3^(1/4)
*Sqrt[(x^2*(2^(1/3) + x^2))/(2^(1/3) + (1 + Sqrt[3])*x^2)^2]*Sqrt[2 + x^6])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^6}{\left(2+x^6\right)^{3/2}}\,dx = -\frac{x}{3\sqrt{2+x^6}} + \frac{1}{3}\int \frac{1}{\sqrt{2+x^6}}\,dx$$

$$= -\frac{x}{3\sqrt{2+x^6}} + \frac{x\left(\sqrt[3]{2}+x^2\right)\sqrt{\frac{2^{2/3}-\sqrt[3]{2}x^2+x^4}{\left(\sqrt[3]{2}+\left(1+\sqrt{3}\right)x^2\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{2}+\left(1-\sqrt{3}\right)x^2}{\sqrt[3]{2}+\left(1+\sqrt{3}\right)x^2}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{6\sqrt[3]{2}\,\sqrt[4]{3}\,\sqrt{\frac{x^2\left(\sqrt[3]{2}+x^2\right)}{\left(\sqrt[3]{2}+\left(1+\sqrt{3}\right)x^2\right)^2}}\,\sqrt{2+x^6}}$$

Mathematica [C] time = 0.01, size = 42, normalized size = 0.23

$$\frac{x\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{x^6}{2}\right)}{3\sqrt{2}} - \frac{x}{3\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(2 + x^6)^(3/2),x]
```

```
[Out] -1/3*x/Sqrt[2 + x^6] + (x*Hypergeometric2F1[1/6, 1/2, 7/6, -1/2*x^6])/(3*Sq
rt[2])
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^6+2}\,x^6}{x^{12}+4\,x^6+4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^6 + 2)*x^6/(x^12 + 4*x^6 + 4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(x^6+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/(x^6 + 2)^(3/2), x)
```

maple [C] time = 0.14, size = 29, normalized size = 0.16

$$\frac{\sqrt{2}\,x\,\text{hypergeom}\left(\left[\frac{1}{6},\frac{1}{2}\right],\left[\frac{7}{6}\right],-\frac{x^6}{2}\right)}{6} - \frac{x}{3\sqrt{x^6+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^6+2)^(3/2),x)
```

```
[Out] -1/3*x/(x^6+2)^(1/2)+1/6*2^(1/2)*x*hypergeom([1/6,1/2],[7/6],-1/2*x^6)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(x^6+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(x^6 + 2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{\left(x^6+2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^6 + 2)^(3/2),x)
```

```
[Out] int(x^6/(x^6 + 2)^(3/2), x)
```

sympy [C] time = 1.56, size = 36, normalized size = 0.20

$$\frac{\sqrt{2}\,x^7\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\left.\begin{matrix}\frac{7}{6}, \frac{3}{2} \\ \frac{13}{6}\end{matrix}\right| \frac{x^6 e^{i\pi}}{2}\right)}{24\Gamma\left(\frac{13}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**6+2)**(3/2),x)
```

```
[Out] sqrt(2)*x**7*gamma(7/6)*hyper((7/6, 3/2), (13/6,), x**6*exp_polar(I*pi)/2)/
(24*gamma(13/6))
```

3.1426 $\int \frac{1}{\left(2+x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=179

$$\frac{x}{6\sqrt{x^6+2}}+\frac{\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\,xF\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x^2+\sqrt[3]{2}}{\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{6\sqrt[3]{2}\,\sqrt[4]{3}\sqrt{\frac{x^2\left(x^2+\sqrt[3]{2}\right)}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\sqrt{x^6+2}}$$

```
[Out] 1/6*x/(x^6+2)^(1/2)+1/36*x*(2^(1/3)+x^2)*((2^(1/3)+x^2*(1-3^(1/2)))^2/(2^(1
/3)+x^2*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)+x^2*(1-3^(1/2)))*(2^(1/3)+x^2*(1+3^(
1/2)))*EllipticF((1-(2^(1/3)+x^2*(1-3^(1/2)))^2/(2^(1/3)+x^2*(1+3^(1/2)))^2
)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((2^(2/3)-2^(1/3)*x^2+x^4)/(2^(1/3)+x^2*(1
+3^(1/2)))^2)^(1/2)*2^(2/3)*3^(3/4)/(x^6+2)^(1/2)/(x^2*(2^(1/3)+x^2)/(2^(1/
3)+x^2*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {199, 225}

$$\frac{x}{6\sqrt{x^6+2}}+\frac{\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\,xF\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x^2+\sqrt[3]{2}}{\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{6\sqrt[3]{2}\,\sqrt[4]{3}\sqrt{\frac{x^2\left(x^2+\sqrt[3]{2}\right)}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^6)^(-3/2),x]
```

```
[Out] x/(6*Sqrt[2 + x^6]) + (x*(2^(1/3) + x^2)*Sqrt[(2^(2/3) - 2^(1/3)*x^2 + x^4)
/(2^(1/3) + (1 + Sqrt[3])*x^2)^2]*EllipticF[ArcCos[(2^(1/3) + (1 - Sqrt[3])
*x^2)/(2^(1/3) + (1 + Sqrt[3])*x^2)], (2 + Sqrt[3])/4])/(6*2^(1/3)*3^(1/4)*
Sqrt[(x^2*(2^(1/3) + x^2))/(2^(1/3) + (1 + Sqrt[3])*x^2)^2]*Sqrt[2 + x^6])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{1}{\left(2+x^6\right)^{3/2}}\,dx = \frac{x}{6\sqrt{2+x^6}} + \frac{1}{3}\int \frac{1}{\sqrt{2+x^6}}\,dx$$

$$= \frac{x}{6\sqrt{2+x^6}} + \frac{x\left(\sqrt[3]{2}+x^2\right)\sqrt{\frac{2^{2/3}-\sqrt[3]{2}x^2+x^4}{\left(\sqrt[3]{2}+\left(1+\sqrt{3}\right)x^2\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{2}+\left(1-\sqrt{3}\right)x^2}{\sqrt[3]{2}+\left(1+\sqrt{3}\right)x^2}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{6\sqrt[3]{2}\,\sqrt[4]{3}\,\sqrt{\frac{x^2\left(\sqrt[3]{2}+x^2\right)}{\left(\sqrt[3]{2}+\left(1+\sqrt{3}\right)x^2\right)^2}}\,\sqrt{2+x^6}}$$

Mathematica [C] time = 0.02, size = 38, normalized size = 0.21

$$\frac{1}{6}x\left(\sqrt{2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{x^6}{2}\right)+\frac{1}{\sqrt{x^6+2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^6)^(-3/2),x]
```

```
[Out] (x*(1/Sqrt[2 + x^6] + Sqrt[2]*Hypergeometric2F1[1/6, 1/2, 7/6, -1/2*x^6]))/
6
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^6+2}}{x^{12}+4\,x^6+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^6 + 2)/(x^12 + 4*x^6 + 4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^6+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^6 + 2)^(-3/2), x)
```

maple [C] time = 0.14, size = 18, normalized size = 0.10

$$\frac{\sqrt{2}\,x\,\text{hypergeom}\left(\left[\frac{1}{6},\frac{3}{2}\right],\left[\frac{7}{6}\right],-\frac{x^6}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6+2)^(3/2),x)
```

```
[Out] 1/4*2^(1/2)*x*hypergeom([1/6,3/2],[7/6],-1/2*x^6)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^6+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^6 + 2)^(-3/2), x)
```

mupad [B] time = 0.08, size = 16, normalized size = 0.09

$$\frac{\sqrt{2}\, x\, {}_2\mathrm{F}_1\left(\frac{1}{6}, \frac{3}{2};\ \frac{7}{6};\ -\frac{x^6}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6 + 2)^(3/2),x)
```

```
[Out] (2^(1/2)*x*hypergeom([1/6, 3/2], 7/6, -x^6/2))/4
```

sympy [C] time = 0.74, size = 34, normalized size = 0.19

$$\frac{\sqrt{2}\, x \Gamma\left(\frac{1}{6}\right) {}_2F_1\left(\begin{matrix}\frac{1}{6}, \frac{3}{2} \\ \frac{7}{6}\end{matrix}\middle| \frac{x^6 e^{i\pi}}{2}\right)}{24\Gamma\left(\frac{7}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6+2)**(3/2),x)
```

```
[Out] sqrt(2)*x*gamma(1/6)*hyper((1/6, 3/2), (7/6,), x**6*exp_polar(I*pi)/2)/(24*
gamma(7/6))
```

3.1427 $\int \frac{1}{x^6\left(2+x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=197

$$-\frac{2\sqrt{x^6+2}}{15x^5}+\frac{1}{6x^5\sqrt{x^6+2}}-\frac{2^{2/3}x\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x^2+\sqrt[3]{2}}{\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{15\sqrt[4]{3}\sqrt{\frac{x^2\left(x^2+\sqrt[3]{2}\right)}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\sqrt{x^6+2}}$$

```
[Out] 1/6/x^5/(x^6+2)^(1/2)-2/15*(x^6+2)^(1/2)/x^5-1/45*x*(2^(1/3)+x^2)*((2^(1/3)
+x^2*(1-3^(1/2)))^2/(2^(1/3)+x^2*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)+x^2*(1-3^(1
/2)))*(2^(1/3)+x^2*(1+3^(1/2)))*EllipticF((1-(2^(1/3)+x^2*(1-3^(1/2)))^2/(2
^(1/3)+x^2*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((2^(2/3)-2^(1/3)
*x^2+x^4)/(2^(1/3)+x^2*(1+3^(1/2)))^2)^(1/2)*2^(2/3)*3^(3/4)/(x^6+2)^(1/2)/
(x^2*(2^(1/3)+x^2)/(2^(1/3)+x^2*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {290, 325, 225}

$$-\frac{2\sqrt{x^6+2}}{15x^5}+\frac{1}{6x^5\sqrt{x^6+2}}-\frac{2^{2/3}x\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x^2+\sqrt[3]{2}}{\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{15\sqrt[4]{3}\sqrt{\frac{x^2\left(x^2+\sqrt[3]{2}\right)}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(2 + x^6)^(3/2)),x]
```

```
[Out] 1/(6*x^5*Sqrt[2 + x^6]) - (2*Sqrt[2 + x^6])/(15*x^5) - (2^(2/3)*x*(2^(1/3)
+ x^2)*Sqrt[(2^(2/3) - 2^(1/3)*x^2 + x^4)/(2^(1/3) + (1 + Sqrt[3])*x^2)^2]*
EllipticF[ArcCos[(2^(1/3) + (1 - Sqrt[3])*x^2)/(2^(1/3) + (1 + Sqrt[3])*x^2
)], (2 + Sqrt[3])/4])/(15*3^(1/4)*Sqrt[(x^2*(2^(1/3) + x^2))/(2^(1/3) + (1
+ Sqrt[3])*x^2)^2]*Sqrt[2 + x^6])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^6\left(2+x^6\right)^{3/2}}\,dx &= \frac{1}{6x^5\sqrt{2+x^6}} + \frac{4}{3}\int \frac{1}{x^6\sqrt{2+x^6}}\,dx \\ &= \frac{1}{6x^5\sqrt{2+x^6}} - \frac{2\sqrt{2+x^6}}{15x^5} - \frac{4}{15}\int \frac{1}{\sqrt{2+x^6}}\,dx \\ &= \frac{1}{6x^5\sqrt{2+x^6}} - \frac{2\sqrt{2+x^6}}{15x^5} - \frac{2^{2/3}x\left(\sqrt[3]{2}+x^2\right)\sqrt{\frac{2^{2/3}-\sqrt[3]{2}x^2+x^4}{\left(\sqrt[3]{2}+\left(1+\sqrt{3}\right)x^2\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{2}+\left(1-\sqrt{3}\right)x^2}{\sqrt[3]{2}+\left(1+\sqrt{3}\right)x^2}\right)|\frac{1}{4}\right.}{15\sqrt[4]{3}\sqrt{\frac{x^2\left(\sqrt[3]{2}+x^2\right)}{\left(\sqrt[3]{2}+\left(1+\sqrt{3}\right)x^2\right)^2}}\sqrt{2+x^6}}\end{aligned}$$

Mathematica [C] time = 0.00, size = 29, normalized size = 0.15

$$-\frac{{}_2F_1\left(-\frac{5}{6},\frac{3}{2};\frac{1}{6};-\frac{x^6}{2}\right)}{10\sqrt{2}\,x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(2 + x^6)^(3/2)),x]
```

```
[Out] -1/10*Hypergeometric2F1[-5/6, 3/2, 1/6, -1/2*x^6]/(Sqrt[2]*x^5)
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^6+2}}{x^{18}+4\,x^{12}+4\,x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^6+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^6 + 2)/(x^18 + 4*x^12 + 4*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^6+2\right)^{\frac{3}{2}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^6+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^6 + 2)^(3/2)*x^6), x)
```

maple [C] time = 0.13, size = 38, normalized size = 0.19

$$-\frac{2\sqrt{2}\,x\,\text{hypergeom}\left(\left[\frac{1}{6},\frac{1}{2}\right],\left[\frac{7}{6}\right],-\frac{x^6}{2}\right)}{15} - \frac{4x^6+3}{30\sqrt{x^6+2}\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^6+2)^(3/2),x)
```

```
[Out] -1/30*(4*x^6+3)/x^5/(x^6+2)^(1/2)-2/15*2^(1/2)*x*hypergeom([1/6,1/2],[7/6],
-1/2*x^6)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^6+2\right)^{\frac{3}{2}} x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^6+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^6 + 2)^(3/2)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^6\left(x^6+2\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(x^6 + 2)^(3/2)),x)
```

```
[Out] int(1/(x^6*(x^6 + 2)^(3/2)), x)
```

sympy [C] time = 2.01, size = 39, normalized size = 0.20

$$\frac{\sqrt{2}\,\Gamma\left(-\frac{5}{6}\right){}_2F_1\left(\left.\begin{matrix}-\frac{5}{6}, \frac{3}{2} \\ \frac{1}{6}\end{matrix}\right| \frac{x^6 e^{i\pi}}{2}\right)}{24x^5\Gamma\left(\frac{1}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**6+2)**(3/2),x)
```

```
[Out] sqrt(2)*gamma(-5/6)*hyper((-5/6, 3/2), (1/6,), x**6*exp_polar(I*pi)/2)/(24*
x**5*gamma(1/6))
```

3.1428 $\int \frac{x^{15}}{\left(2+x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=394

$$-\frac{x^{10}}{3\sqrt{x^6+2}}+\frac{10}{21}\sqrt{x^6+2}\,x^4-\frac{80\sqrt{x^6+2}}{21\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)}-\frac{80\,2^{2/3}\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\,F\left(\sin^{-1}\left(\frac{x^2+\sqrt[3]{2}\left(1-\sqrt{3}\right)}{x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)}\right.\right.}{21\sqrt[4]{3}\sqrt{\frac{x^2+\sqrt[3]{2}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\sqrt{x^6+2}}$$

```
[Out] -1/3*x^10/(x^6+2)^(1/2)+10/21*x^4*(x^6+2)^(1/2)-80/21*(x^6+2)^(1/2)/(x^2+2^
(1/3)*(1+3^(1/2)))-80/63*2^(2/3)*(2^(1/3)+x^2)*EllipticF((x^2+2^(1/3)*(1-3^
(1/2)))/(x^2+2^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2^(2/3)-2^(1/3)*x^2+x^4)
/(x^2+2^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/(x^6+2)^(1/2)/((2^(1/3)+x^2)/(x
^2+2^(1/3)*(1+3^(1/2)))^2)^(1/2)+40/21*2^(1/6)*3^(1/4)*(2^(1/3)+x^2)*Ellipt
icE((x^2+2^(1/3)*(1-3^(1/2)))/(x^2+2^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2
*6^(1/2)-1/2*2^(1/2))*((2^(2/3)-2^(1/3)*x^2+x^4)/(x^2+2^(1/3)*(1+3^(1/2)))^
2)^(1/2)/(x^6+2)^(1/2)/((2^(1/3)+x^2)/(x^2+2^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {275, 288, 321, 303, 218, 1877}

$$-\frac{x^{10}}{3\sqrt{x^6+2}}+\frac{10}{21}\sqrt{x^6+2}\,x^4-\frac{80\sqrt{x^6+2}}{21\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)}-\frac{80\,2^{2/3}\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\,F\left(\sin^{-1}\left(\frac{x^2+\sqrt[3]{2}\left(1-\sqrt{3}\right)}{x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)}\right.\right.}{21\sqrt[4]{3}\sqrt{\frac{x^2+\sqrt[3]{2}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^15/(2 + x^6)^(3/2),x]
```

```
[Out] -x^10/(3*Sqrt[2 + x^6]) + (10*x^4*Sqrt[2 + x^6])/21 - (80*Sqrt[2 + x^6])/(2
1*(2^(1/3)*(1 + Sqrt[3]) + x^2)) + (40*2^(1/6)*Sqrt[2 - Sqrt[3]]*(2^(1/3) +
 x^2)*Sqrt[(2^(2/3) - 2^(1/3)*x^2 + x^4)/(2^(1/3)*(1 + Sqrt[3]) + x^2)^2]*E
llipticE[ArcSin[(2^(1/3)*(1 - Sqrt[3]) + x^2)/(2^(1/3)*(1 + Sqrt[3]) + x^2)
], -7 - 4*Sqrt[3]])/(7*3^(3/4)*Sqrt[(2^(1/3) + x^2)/(2^(1/3)*(1 + Sqrt[3])
 + x^2)^2]*Sqrt[2 + x^6]) - (80*2^(2/3)*(2^(1/3) + x^2)*Sqrt[(2^(2/3) - 2^(1
/3)*x^2 + x^4)/(2^(1/3)*(1 + Sqrt[3]) + x^2)^2]*EllipticF[ArcSin[(2^(1/3)*(
1 - Sqrt[3]) + x^2)/(2^(1/3)*(1 + Sqrt[3]) + x^2)], -7 - 4*Sqrt[3]])/(21*3^
(1/4)*Sqrt[(2^(1/3) + x^2)/(2^(1/3)*(1 + Sqrt[3]) + x^2)^2]*Sqrt[2 + x^6])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
 + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^{15}}{\left(2+x^6\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^7}{\left(2+x^3\right)^{3/2}}\,dx, x, x^2\right)$$

$$= -\frac{x^{10}}{3\sqrt{2+x^6}} + \frac{5}{3}\,\text{Subst}\left(\int \frac{x^4}{\sqrt{2+x^3}}\,dx, x, x^2\right)$$

$$= -\frac{x^{10}}{3\sqrt{2+x^6}} + \frac{10}{21}x^4\sqrt{2+x^6} - \frac{40}{21}\,\text{Subst}\left(\int \frac{x}{\sqrt{2+x^3}}\,dx, x, x^2\right)$$

$$= -\frac{x^{10}}{3\sqrt{2+x^6}} + \frac{10}{21}x^4\sqrt{2+x^6} - \frac{40}{21}\,\text{Subst}\left(\int \frac{\sqrt[3]{2}\left(1-\sqrt{3}\right)+x}{\sqrt{2+x^3}}\,dx, x, x^2\right) - \frac{\left(40\,2^{5/6}\right)\text{Subst}}{21\sqrt{}}$$

$$= -\frac{x^{10}}{3\sqrt{2+x^6}} + \frac{10}{21}x^4\sqrt{2+x^6} - \frac{80\sqrt{2+x^6}}{21\left(\sqrt[3]{2}\left(1+\sqrt{3}\right)+x^2\right)} + \frac{40\sqrt[6]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{2}+x^2\right)\sqrt{\frac{}{\left(\sqrt[3]{}\right.}}}{7\,3^{3/4}}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.14

$$\frac{x^4\left(10\sqrt{2}\,\sqrt{x^6+2}\;{}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{3};-\frac{x^6}{2}\right)+x^6-20\right)}{7\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15/(2 + x^6)^(3/2),x]
```

```
[Out] (x^4*(-20 + x^6 + 10*Sqrt[2]*Sqrt[2 + x^6]*Hypergeometric2F1[2/3, 3/2, 5/3,
 -1/2*x^6]))/(7*Sqrt[2 + x^6])
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^6+2}\,x^{15}}{x^{12}+4\,x^6+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(x^6+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^6 + 2)*x^15/(x^12 + 4*x^6 + 4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{15}}{\left(x^6+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(x^6+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^15/(x^6 + 2)^(3/2), x)
```

maple [C] time = 0.14, size = 40, normalized size = 0.10

$$-\frac{10\sqrt{2}\;x^4\,\text{hypergeom}\left(\left[\frac{1}{2},\frac{2}{3}\right],\left[\frac{5}{3}\right],-\frac{x^6}{2}\right)}{21}+\frac{\left(3x^6+20\right)x^4}{21\sqrt{x^6+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(x^6+2)^(3/2),x)
```

```
[Out] 1/21*x^4*(3*x^6+20)/(x^6+2)^(1/2)-10/21*2^(1/2)*x^4*hypergeom([1/2,2/3],[5/
3],-1/2*x^6)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{15}}{\left(x^6+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(x^6+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^15/(x^6 + 2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{15}}{\left(x^6+2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(x^6 + 2)^(3/2),x)
```

```
[Out] int(x^15/(x^6 + 2)^(3/2), x)
```

sympy [**A**] time = 2.82, size = 36, normalized size = 0.09

$$\frac{\sqrt{2}\,x^{16}\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2}, \frac{8}{3} \\ \frac{11}{3}\end{matrix}\middle| \frac{x^6 e^{i\pi}}{2}\right)}{24\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15/(x**6+2)**(3/2),x)
```

```
[Out] sqrt(2)*x**16*gamma(8/3)*hyper((3/2, 8/3), (11/3,), x**6*exp_polar(I*pi)/2)
/(24*gamma(11/3))
```

3.1429 $\int \frac{x^9}{\left(2+x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=376

$$-\frac{x^4}{3\sqrt{x^6+2}}+\frac{4\sqrt{x^6+2}}{3\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)}+\frac{4\,2^{2/3}\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\,F\left(\sin^{-1}\left(\frac{x^2+\sqrt[3]{2}\left(1-\sqrt{3}\right)}{x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)}\right)|-7-4\sqrt{3}\right)\quad 2}{3\sqrt[4]{3}\sqrt{\frac{x^2+\sqrt[3]{2}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\sqrt{x^6+2}}--$$

```
[Out] -1/3*x^4/(x^6+2)^(1/2)+4/3*(x^6+2)^(1/2)/(x^2+2^(1/3)*(1+3^(1/2)))+4/9*2^(2
/3)*(2^(1/3)+x^2)*EllipticF((x^2+2^(1/3)*(1-3^(1/2)))/(x^2+2^(1/3)*(1+3^(1/
2))),I*3^(1/2)+2*I)*((2^(2/3)-2^(1/3)*x^2+x^4)/(x^2+2^(1/3)*(1+3^(1/2)))^2)
^(1/2)*3^(3/4)/(x^6+2)^(1/2)/((2^(1/3)+x^2)/(x^2+2^(1/3)*(1+3^(1/2)))^2)^(1
/2)-2/3*2^(1/6)*3^(1/4)*(2^(1/3)+x^2)*EllipticE((x^2+2^(1/3)*(1-3^(1/2)))/(
x^2+2^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((2^(2/3)
-2^(1/3)*x^2+x^4)/(x^2+2^(1/3)*(1+3^(1/2)))^2)^(1/2)/(x^6+2)^(1/2)/((2^(1/3
)+x^2)/(x^2+2^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {275, 288, 303, 218, 1877}

$$-\frac{x^4}{3\sqrt{x^6+2}}+\frac{4\sqrt{x^6+2}}{3\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)}+\frac{4\,2^{2/3}\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\,F\left(\sin^{-1}\left(\frac{x^2+\sqrt[3]{2}\left(1-\sqrt{3}\right)}{x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)}\right)|-7-4\sqrt{3}\right)\quad 2}{3\sqrt[4]{3}\sqrt{\frac{x^2+\sqrt[3]{2}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\sqrt{x^6+2}}--$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(2 + x^6)^(3/2),x]
```

```
[Out] -x^4/(3*Sqrt[2 + x^6]) + (4*Sqrt[2 + x^6])/(3*(2^(1/3)*(1 + Sqrt[3]) + x^2)
) - (2*2^(1/6)*Sqrt[2 - Sqrt[3]]*(2^(1/3) + x^2)*Sqrt[(2^(2/3) - 2^(1/3)*x^
2 + x^4)/(2^(1/3)*(1 + Sqrt[3]) + x^2)^2]*EllipticE[ArcSin[(2^(1/3)*(1 - Sq
rt[3]) + x^2)/(2^(1/3)*(1 + Sqrt[3]) + x^2)], -7 - 4*Sqrt[3]])/(3^(3/4)*Sqr
t[(2^(1/3) + x^2)/(2^(1/3)*(1 + Sqrt[3]) + x^2)^2]*Sqrt[2 + x^6]) + (4*2^(2
/3)*(2^(1/3) + x^2)*Sqrt[(2^(2/3) - 2^(1/3)*x^2 + x^4)/(2^(1/3)*(1 + Sqrt[3
]) + x^2)^2]*EllipticF[ArcSin[(2^(1/3)*(1 - Sqrt[3]) + x^2)/(2^(1/3)*(1 + S
qrt[3]) + x^2)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(2^(1/3) + x^2)/(2^(1/3)*
(1 + Sqrt[3]) + x^2)^2]*Sqrt[2 + x^6])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^9}{\left(2+x^6\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{\left(2+x^3\right)^{3/2}}\,dx, x, x^2\right)$$

$$= -\frac{x^4}{3\sqrt{2+x^6}} + \frac{2}{3}\,\text{Subst}\left(\int \frac{x}{\sqrt{2+x^3}}\,dx, x, x^2\right)$$

$$= -\frac{x^4}{3\sqrt{2+x^6}} + \frac{2}{3}\,\text{Subst}\left(\int \frac{\sqrt[3]{2}\left(1-\sqrt{3}\right)+x}{\sqrt{2+x^3}}\,dx, x, x^2\right) + \frac{\left(2\,2^{5/6}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2+x^3}}\,dx, x, x^2\right.}{3\sqrt{2+\sqrt{3}}}$$

$$= -\frac{x^4}{3\sqrt{2+x^6}} + \frac{4\sqrt{2+x^6}}{3\left(\sqrt[3]{2}\left(1+\sqrt{3}\right)+x^2\right)} - \frac{2\sqrt[6]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{2}+x^2\right)\sqrt{\frac{2^{2/3}-\sqrt[3]{2}x^2+x^4}{\left(\sqrt[3]{2}\left(1+\sqrt{3}\right)+x^2\right)^2}}\,E\left(\text{si}\right.}{3^{3/4}\sqrt{\frac{\sqrt[3]{2}+x^2}{\left(\sqrt[3]{2}\left(1+\sqrt{3}\right)+x^2\right)^2}}\,\sqrt{2}}$$

Mathematica [C] time = 0.01, size = 41, normalized size = 0.11

$$\frac{x^4}{\sqrt{x^6+2}} - \frac{x^4\,{}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{3};-\frac{x^6}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(2 + x^6)^(3/2),x]
```

```
[Out] x^4/Sqrt[2 + x^6] - (x^4*Hypergeometric2F1[2/3, 3/2, 5/3, -1/2*x^6])/Sqrt[2
]
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^6+2}\,x^9}{x^{12}+4\,x^6+4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^6+2)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(x^6 + 2)*x^9/(x^12 + 4*x^6 + 4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(x^6+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^6+2)^(3/2),x, algorithm="giac")
[Out] integrate(x^9/(x^6 + 2)^(3/2), x)
```

maple [C] time = 0.14, size = 33, normalized size = 0.09

$$\frac{\sqrt{2}\,x^4\,\text{hypergeom}\left(\left[\frac{1}{2},\frac{2}{3}\right],\left[\frac{5}{3}\right],-\frac{x^6}{2}\right)}{6}-\frac{x^4}{3\sqrt{x^6+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^6+2)^(3/2),x)
[Out] -1/3*x^4/(x^6+2)^(1/2)+1/6*2^(1/2)*x^4*hypergeom([1/2,2/3],[5/3],-1/2*x^6)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(x^6+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^6+2)^(3/2),x, algorithm="maxima")
[Out] integrate(x^9/(x^6 + 2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^9}{\left(x^6+2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^6 + 2)^(3/2),x)
[Out] int(x^9/(x^6 + 2)^(3/2), x)
```

sympy [A] time = 1.75, size = 36, normalized size = 0.10

$$\frac{\sqrt{2}\,x^{10}\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\right|\frac{x^6e^{i\pi}}{2}\right)}{24\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(x**6+2)**(3/2),x)
```

```
[Out] sqrt(2)*x**10*gamma(5/3)*hyper((3/2, 5/3), (8/3,), x**6*exp_polar(I*pi)/2)/
(24*gamma(8/3))
```

3.1430 $\int \frac{x^3}{\left(2+x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=378

$$\frac{x^4}{6\sqrt{x^6+2}}-\frac{\sqrt{x^6+2}}{6\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)}-\frac{\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}F\left(\sin^{-1}\left(\frac{x^2+\sqrt[3]{2}\left(1-\sqrt{3}\right)}{x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)}\right)|-7-4\sqrt{3}\right)\sqrt{2-\sqrt{3}}}{3\sqrt[3]{2}\sqrt[4]{3}\sqrt{\frac{x^2+\sqrt[3]{2}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\sqrt{x^6+2}}+$$

```
[Out] 1/6*x^4/(x^6+2)^(1/2)-1/6*(x^6+2)^(1/2)/(x^2+2^(1/3)*(1+3^(1/2)))-1/18*2^(2
/3)*(2^(1/3)+x^2)*EllipticF((x^2+2^(1/3)*(1-3^(1/2)))/(x^2+2^(1/3)*(1+3^(1/
2))),I*3^(1/2)+2*I)*((2^(2/3)-2^(1/3)*x^2+x^4)/(x^2+2^(1/3)*(1+3^(1/2)))^2)
^(1/2)*3^(3/4)/(x^6+2)^(1/2)/((2^(1/3)+x^2)/(x^2+2^(1/3)*(1+3^(1/2)))^2)^(1
/2)+1/12*2^(1/6)*3^(1/4)*(2^(1/3)+x^2)*EllipticE((x^2+2^(1/3)*(1-3^(1/2)))/
(x^2+2^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((2^(2/3
)-2^(1/3)*x^2+x^4)/(x^2+2^(1/3)*(1+3^(1/2)))^2)^(1/2)/(x^6+2)^(1/2)/((2^(1/
3)+x^2)/(x^2+2^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {275, 290, 303, 218, 1877}

$$\frac{x^4}{6\sqrt{x^6+2}}-\frac{\sqrt{x^6+2}}{6\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)}-\frac{\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}F\left(\sin^{-1}\left(\frac{x^2+\sqrt[3]{2}\left(1-\sqrt{3}\right)}{x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)}\right)|-7-4\sqrt{3}\right)\sqrt{2-\sqrt{3}}}{3\sqrt[3]{2}\sqrt[4]{3}\sqrt{\frac{x^2+\sqrt[3]{2}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\sqrt{x^6+2}}+$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(2 + x^6)^(3/2),x]
```

```
[Out] x^4/(6*Sqrt[2 + x^6]) - Sqrt[2 + x^6]/(6*(2^(1/3)*(1 + Sqrt[3]) + x^2)) + (
Sqrt[2 - Sqrt[3]]*(2^(1/3) + x^2)*Sqrt[(2^(2/3) - 2^(1/3)*x^2 + x^4)/(2^(1/
3)*(1 + Sqrt[3]) + x^2)^2]*EllipticE[ArcSin[(2^(1/3)*(1 - Sqrt[3]) + x^2)/(
2^(1/3)*(1 + Sqrt[3]) + x^2)], -7 - 4*Sqrt[3]])/(2*2^(5/6)*3^(3/4)*Sqrt[(2^
(1/3) + x^2)/(2^(1/3)*(1 + Sqrt[3]) + x^2)^2]*Sqrt[2 + x^6]) - ((2^(1/3) +
x^2)*Sqrt[(2^(2/3) - 2^(1/3)*x^2 + x^4)/(2^(1/3)*(1 + Sqrt[3]) + x^2)^2]*El
lipticF[ArcSin[(2^(1/3)*(1 - Sqrt[3]) + x^2)/(2^(1/3)*(1 + Sqrt[3]) + x^2)]
, -7 - 4*Sqrt[3]])/(3*2^(1/3)*3^(1/4)*Sqrt[(2^(1/3) + x^2)/(2^(1/3)*(1 + Sq
rt[3]) + x^2)^2]*Sqrt[2 + x^6])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^3}{\left(2+x^6\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{\left(2+x^3\right)^{3/2}}\,dx, x, x^2\right)$$

$$= \frac{x^4}{6\sqrt{2+x^6}} - \frac{1}{12}\,\text{Subst}\left(\int \frac{x}{\sqrt{2+x^3}}\,dx, x, x^2\right)$$

$$= \frac{x^4}{6\sqrt{2+x^6}} - \frac{1}{12}\,\text{Subst}\left(\int \frac{\sqrt[3]{2}\left(1-\sqrt{3}\right)+x}{\sqrt{2+x^3}}\,dx, x, x^2\right) - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2+x^3}}\,dx, x, x^2\right)}{6\sqrt[6]{2}\sqrt{2+\sqrt{3}}}$$

$$= \frac{x^4}{6\sqrt{2+x^6}} - \frac{\sqrt{2+x^6}}{6\left(\sqrt[3]{2}\left(1+\sqrt{3}\right)+x^2\right)} + \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{2}+x^2\right)\sqrt{\frac{2^{2/3}-\sqrt[3]{2}x^2+x^4}{\left(\sqrt[3]{2}\left(1+\sqrt{3}\right)+x^2\right)^2}}\,E\left(\sin^{-1}\left(\frac{\sqrt[3]{}}{\sqrt[3]{}}\right.\right.}{2\,2^{5/6}3^{3/4}\sqrt{\frac{\sqrt[3]{2}+x^2}{\left(\sqrt[3]{2}\left(1+\sqrt{3}\right)+x^2\right)^2}}\sqrt{2}}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.08

$$\frac{x^4\,{}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{3};-\frac{x^6}{2}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(2 + x^6)^(3/2),x]
```

```
[Out] (x^4*Hypergeometric2F1[2/3, 3/2, 5/3, -1/2*x^6])/(8*Sqrt[2])
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^6+2}\,x^3}{x^{12}+4\,x^6+4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^6 + 2)*x^3/(x^12 + 4*x^6 + 4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(x^6+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(x^6 + 2)^(3/2), x)
```

maple [C] time = 0.14, size = 33, normalized size = 0.09

$$-\frac{\sqrt{2}\,x^4\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{2}{3}\right],\left[\frac{5}{3}\right],-\frac{x^6}{2}\right)}{48}+\frac{x^4}{6\sqrt{x^6+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^6+2)^(3/2),x)
```

```
[Out] 1/6/(x^6+2)^(1/2)*x^4-1/48*2^(1/2)*x^4*hypergeom([1/2,2/3],[5/3],-1/2*x^6)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(x^6+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(x^6 + 2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\left(x^6+2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^6 + 2)^(3/2),x)
```

```
[Out] int(x^3/(x^6 + 2)^(3/2), x)
```

sympy [A] time = 1.39, size = 36, normalized size = 0.10

$$\frac{\sqrt{2}\,x^4\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{2}{3},\frac{3}{2}\\ \frac{5}{3}\end{matrix}\right|\frac{x^6e^{i\pi}}{2}\right)}{24\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**6+2)**(3/2),x)
```

```
[Out] sqrt(2)*x**4*gamma(2/3)*hyper((2/3, 3/2), (5/3,), x**6*exp_polar(I*pi)/2)/(
24*gamma(5/3))
```

3.1431 $\int \frac{1}{x^3\left(2+x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=394

$$\frac{5\sqrt{x^6+2}}{24\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)}-\frac{5\sqrt{x^6+2}}{24x^2}+\frac{1}{6x^2\sqrt{x^6+2}}+\frac{5\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}F\left(\sin^{-1}\left(\frac{x^2+\sqrt[3]{2}\left(1-\sqrt{3}\right)}{x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)}\right)|-7\right.}{12\sqrt[3]{2}\sqrt[4]{3}\sqrt{\frac{x^2+\sqrt[3]{2}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\sqrt{x^6+2}}$$

```
[Out] 1/6/x^2/(x^6+2)^(1/2)-5/24*(x^6+2)^(1/2)/x^2+5/24*(x^6+2)^(1/2)/(x^2+2^(1/3
)*(1+3^(1/2)))+5/72*2^(2/3)*(2^(1/3)+x^2)*EllipticF((x^2+2^(1/3)*(1-3^(1/2)
))/(x^2+2^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2^(2/3)-2^(1/3)*x^2+x^4)/(x^2
+2^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/(x^6+2)^(1/2)/((2^(1/3)+x^2)/(x^2+2^
(1/3)*(1+3^(1/2)))^2)^(1/2)-5/48*2^(1/6)*3^(1/4)*(2^(1/3)+x^2)*EllipticE((x
^2+2^(1/3)*(1-3^(1/2)))/(x^2+2^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/
2)-1/2*2^(1/2))*((2^(2/3)-2^(1/3)*x^2+x^4)/(x^2+2^(1/3)*(1+3^(1/2)))^2)^(1/
2)/(x^6+2)^(1/2)/((2^(1/3)+x^2)/(x^2+2^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {275, 290, 325, 303, 218, 1877}

$$\frac{5\sqrt{x^6+2}}{24\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)}-\frac{5\sqrt{x^6+2}}{24x^2}+\frac{1}{6x^2\sqrt{x^6+2}}+\frac{5\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}F\left(\sin^{-1}\left(\frac{x^2+\sqrt[3]{2}\left(1-\sqrt{3}\right)}{x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)}\right)|-7\right.}{12\sqrt[3]{2}\sqrt[4]{3}\sqrt{\frac{x^2+\sqrt[3]{2}}{\left(x^2+\sqrt[3]{2}\left(1+\sqrt{3}\right)\right)^2}}\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(2 + x^6)^(3/2)),x]
```

```
[Out] 1/(6*x^2*Sqrt[2 + x^6]) - (5*Sqrt[2 + x^6])/(24*x^2) + (5*Sqrt[2 + x^6])/(2
4*(2^(1/3)*(1 + Sqrt[3]) + x^2)) - (5*Sqrt[2 - Sqrt[3]]*(2^(1/3) + x^2)*Sqr
t[(2^(2/3) - 2^(1/3)*x^2 + x^4)/(2^(1/3)*(1 + Sqrt[3]) + x^2)^2]*EllipticE[
ArcSin[(2^(1/3)*(1 - Sqrt[3]) + x^2)/(2^(1/3)*(1 + Sqrt[3]) + x^2)], -7 - 4
*Sqrt[3]])/(8*2^(5/6)*3^(3/4)*Sqrt[(2^(1/3) + x^2)/(2^(1/3)*(1 + Sqrt[3]) +
 x^2)^2]*Sqrt[2 + x^6]) + (5*(2^(1/3) + x^2)*Sqrt[(2^(2/3) - 2^(1/3)*x^2 +
x^4)/(2^(1/3)*(1 + Sqrt[3]) + x^2)^2]*EllipticF[ArcSin[(2^(1/3)*(1 - Sqrt[3
]) + x^2)/(2^(1/3)*(1 + Sqrt[3]) + x^2)], -7 - 4*Sqrt[3]])/(12*2^(1/3)*3^(1
/4)*Sqrt[(2^(1/3) + x^2)/(2^(1/3)*(1 + Sqrt[3]) + x^2)^2]*Sqrt[2 + x^6])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^3\left(2+x^6\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\left(2+x^3\right)^{3/2}}\,dx, x, x^2\right)$$

$$= \frac{1}{6x^2\sqrt{2+x^6}} + \frac{5}{12}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{2+x^3}}\,dx, x, x^2\right)$$

$$= \frac{1}{6x^2\sqrt{2+x^6}} - \frac{5\sqrt{2+x^6}}{24x^2} + \frac{5}{48}\,\text{Subst}\left(\int \frac{x}{\sqrt{2+x^3}}\,dx, x, x^2\right)$$

$$= \frac{1}{6x^2\sqrt{2+x^6}} - \frac{5\sqrt{2+x^6}}{24x^2} + \frac{5}{48}\,\text{Subst}\left(\int \frac{\sqrt[3]{2}\left(1-\sqrt{3}\right)+x}{\sqrt{2+x^3}}\,dx, x, x^2\right) + \frac{5\,\text{Subst}\left(\int \frac{1}{\sqrt{2+x^3}}\right.}{24\sqrt[6]{2}\sqrt{2+}}$$

$$= \frac{1}{6x^2\sqrt{2+x^6}} - \frac{5\sqrt{2+x^6}}{24x^2} + \frac{5\sqrt{2+x^6}}{24\left(\sqrt[3]{2}\left(1+\sqrt{3}\right)+x^2\right)} - \frac{5\sqrt{2-\sqrt{3}}\left(\sqrt[3]{2}+x^2\right)\sqrt{\frac{2^{2/3}-\sqrt[3]{2}x^2}{\left(\sqrt[3]{2}\left(1+\sqrt{3}\right)\right.}}}{8\,2^{5/6}3^{3/4}\sqrt{\frac{}{\left(\sqrt[3]{}\right.}}}$$

Mathematica [C] time = 0.03, size = 29, normalized size = 0.07

$$-\frac{{}_2F_1\left(-\frac{1}{3},\frac{3}{2};\frac{2}{3};-\frac{x^6}{2}\right)}{4\sqrt{2}\,x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(2 + x^6)^(3/2)),x]
[Out] -1/4*Hypergeometric2F1[-1/3, 3/2, 2/3, -1/2*x^6]/(Sqrt[2]*x^2)
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^6+2}}{x^{15}+4\,x^9+4\,x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^6+2)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(x^6 + 2)/(x^15 + 4*x^9 + 4*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^6+2\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^6+2)^(3/2),x, algorithm="giac")
[Out] integrate(1/((x^6 + 2)^(3/2)*x^3), x)
```

maple [C] time = 0.14, size = 40, normalized size = 0.10

$$\frac{5\sqrt{2}\,x^4\,\text{hypergeom}\left(\left[\frac{1}{2},\frac{2}{3}\right],\left[\frac{5}{3}\right],-\frac{x^6}{2}\right)}{192}-\frac{5x^6+6}{24\sqrt{x^6+2}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^6+2)^(3/2),x)
[Out] -1/24*(5*x^6+6)/x^2/(x^6+2)^(1/2)+5/192*2^(1/2)*x^4*hypergeom([1/2,2/3],[5/
3],-1/2*x^6)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^6+2\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^6+2)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((x^6 + 2)^(3/2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\left(x^6+2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x^6 + 2)^(3/2)),x)
```

```
[Out] int(1/(x^3*(x^6 + 2)^(3/2)), x)
```

sympy [**A**] time = 1.68, size = 39, normalized size = 0.10

$$\frac{\sqrt{2}\,\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3}, \frac{3}{2} \\ \frac{2}{3}\end{matrix}\right| \frac{x^6 e^{i\pi}}{2}\right)}{24x^2\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**6+2)**(3/2),x)
```

```
[Out] sqrt(2)*gamma(-1/3)*hyper((-1/3, 3/2), (2/3,), x**6*exp_polar(I*pi)/2)/(24*
x**2*gamma(2/3))
```

3.1432 $\int \frac{x^{10}}{\left(2+x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=392

$$-\frac{x^5}{3\sqrt{x^6+2}}+\frac{5\left(1+\sqrt{3}\right)\sqrt{x^6+2}\,x}{6\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)}-\frac{5\left(1-\sqrt{3}\right)\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\,xF\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x^2+\sqrt[3]{2}}{\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}}\right)|\frac{1}{4}\left(2+\right.\right.}{6\,2^{2/3}\sqrt[4]{3}\sqrt{\frac{x^2\left(x^2+\sqrt[3]{2}\right)}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\sqrt{x^6+2}}$$

```
[Out] -1/3*x^5/(x^6+2)^(1/2)+5/6*x*(1+3^(1/2))*(x^6+2)^(1/2)/(2^(1/3)+x^2*(1+3^(1
/2)))-5/6*3^(1/4)*x*(2^(1/3)+x^2)*((2^(1/3)+x^2*(1-3^(1/2)))^2/(2^(1/3)+x^2
*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)+x^2*(1-3^(1/2)))*(2^(1/3)+x^2*(1+3^(1/2)))*
EllipticE((1-(2^(1/3)+x^2*(1-3^(1/2)))^2/(2^(1/3)+x^2*(1+3^(1/2)))^2)^(1/2)
,1/4*6^(1/2)+1/4*2^(1/2))*((2^(2/3)-2^(1/3)*x^2+x^4)/(2^(1/3)+x^2*(1+3^(1/2
)))^2)^(1/2)*2^(1/3)/(x^6+2)^(1/2)/(x^2*(2^(1/3)+x^2)/(2^(1/3)+x^2*(1+3^(1/
2)))^2)^(1/2)-5/36*x*(2^(1/3)+x^2)*((2^(1/3)+x^2*(1-3^(1/2)))^2/(2^(1/3)+x^
2*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)+x^2*(1-3^(1/2)))*(2^(1/3)+x^2*(1+3^(1/2)))
*EllipticF((1-(2^(1/3)+x^2*(1-3^(1/2)))^2/(2^(1/3)+x^2*(1+3^(1/2)))^2)^(1/2
),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*((2^(2/3)-2^(1/3)*x^2+x^4)/(2^(1/3)+
x^2*(1+3^(1/2)))^2)^(1/2)*2^(1/3)*3^(3/4)/(x^6+2)^(1/2)/(x^2*(2^(1/3)+x^2)/
(2^(1/3)+x^2*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 392, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {288, 308, 225, 1881}

$$-\frac{x^5}{3\sqrt{x^6+2}}+\frac{5\left(1+\sqrt{3}\right)\sqrt{x^6+2}\,x}{6\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)}-\frac{5\left(1-\sqrt{3}\right)\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\,xF\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x^2+\sqrt[3]{2}}{\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}}\right)|\frac{1}{4}\left(2+\right.\right.}{6\,2^{2/3}\sqrt[4]{3}\sqrt{\frac{x^2\left(x^2+\sqrt[3]{2}\right)}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(2 + x^6)^(3/2),x]
```

```
[Out] -x^5/(3*Sqrt[2 + x^6]) + (5*(1 + Sqrt[3])*x*Sqrt[2 + x^6])/(6*(2^(1/3) + (1
 + Sqrt[3])*x^2)) - (5*x*(2^(1/3) + x^2)*Sqrt[(2^(2/3) - 2^(1/3)*x^2 + x^4)
/(2^(1/3) + (1 + Sqrt[3])*x^2)^2]*EllipticE[ArcCos[(2^(1/3) + (1 - Sqrt[3])
*x^2)/(2^(1/3) + (1 + Sqrt[3])*x^2)], (2 + Sqrt[3])/4])/(2^(2/3)*3^(3/4)*Sq
rt[(x^2*(2^(1/3) + x^2))/(2^(1/3) + (1 + Sqrt[3])*x^2)^2]*Sqrt[2 + x^6]) -
(5*(1 - Sqrt[3])*x*(2^(1/3) + x^2)*Sqrt[(2^(2/3) - 2^(1/3)*x^2 + x^4)/(2^(1
/3) + (1 + Sqrt[3])*x^2)^2]*EllipticF[ArcCos[(2^(1/3) + (1 - Sqrt[3])*x^2)/
(2^(1/3) + (1 + Sqrt[3])*x^2)], (2 + Sqrt[3])/4])/(6*2^(2/3)*3^(1/4)*Sqrt[(
x^2*(2^(1/3) + x^2))/(2^(1/3) + (1 + Sqrt[3])*x^2)^2]*Sqrt[2 + x^6])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{x^{10}}{\left(2+x^6\right)^{3/2}}\,dx = -\frac{x^5}{3\sqrt{2+x^6}} + \frac{5}{3}\int \frac{x^4}{\sqrt{2+x^6}}\,dx$$

$$= -\frac{x^5}{3\sqrt{2+x^6}} - \frac{5}{6}\int \frac{2^{2/3}\left(-1+\sqrt{3}\right)-2x^4}{\sqrt{2+x^6}}\,dx - \frac{\left(5\left(1-\sqrt{3}\right)\right)\int \frac{1}{\sqrt{2+x^6}}\,dx}{3\sqrt[3]{2}}$$

$$= -\frac{x^5}{3\sqrt{2+x^6}} + \frac{5\left(1+\sqrt{3}\right)x\sqrt{2+x^6}}{6\left(\sqrt[3]{2}+\left(1+\sqrt{3}\right)x^2\right)} - \frac{5x\left(\sqrt[3]{2}+x^2\right)\sqrt{\frac{2^{2/3}-\sqrt[3]{2}x^2+x^4}{\left(\sqrt[3]{2}+\left(1+\sqrt{3}\right)x^2\right)^2}}\,E\left(\cos^{-1}\left(\frac{\sqrt[3]{2}+\left(1-\sqrt{3}\right)x^2}{\sqrt[3]{2}+\left(1+\sqrt{3}\right)x^2}\right)\right)}{2^{2/3}3^{3/4}\sqrt{\frac{x^2\left(\sqrt[3]{2}+x^2\right)}{\left(\sqrt[3]{2}+\left(1+\sqrt{3}\right)x^2\right)^2}}\sqrt{2+x^6}}$$

Mathematica [C] time = 0.02, size = 43, normalized size = 0.11

$$\frac{1}{4}x^5\left(\frac{2}{\sqrt{x^6+2}} - \sqrt{2}\,{}_2F_1\left(\frac{5}{6},\frac{3}{2};\frac{11}{6};-\frac{x^6}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(2 + x^6)^(3/2),x]
```

```
[Out] (x^5*(2/Sqrt[2 + x^6] - Sqrt[2]*Hypergeometric2F1[5/6, 3/2, 11/6, -1/2*x^6]
))/4
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^6+2}\,x^{10}}{x^{12}+4\,x^6+4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(x^6+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^6 + 2)*x^10/(x^12 + 4*x^6 + 4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\left(x^6+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(x^6+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^10/(x^6 + 2)^(3/2), x)
```

maple [C] time = 0.13, size = 33, normalized size = 0.08

$$\frac{\sqrt{2}\,x^5\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{5}{6}\right],\left[\frac{11}{6}\right],-\frac{x^6}{2}\right)}{6}-\frac{x^5}{3\sqrt{x^6+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(x^6+2)^(3/2),x)
```

```
[Out] -1/3*x^5/(x^6+2)^(1/2)+1/6*2^(1/2)*x^5*hypergeom([1/2,5/6],[11/6],-1/2*x^6)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\left(x^6+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(x^6+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^10/(x^6 + 2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{10}}{\left(x^6+2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(x^6 + 2)^(3/2),x)
```

```
[Out] int(x^10/(x^6 + 2)^(3/2), x)
```

sympy [C] time = 1.98, size = 36, normalized size = 0.09

$$\frac{\sqrt{2}\,x^{11}\Gamma\left(\frac{11}{6}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{11}{6}\\ \frac{17}{6}\end{matrix}\right|\frac{x^6e^{i\pi}}{2}\right)}{24\Gamma\left(\frac{17}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(x**6+2)**(3/2),x)
```

```
[Out] sqrt(2)*x**11*gamma(11/6)*hyper((3/2, 11/6), (17/6,), x**6*exp_polar(I*pi)/
2)/(24*gamma(17/6))
```

3.1433 $\int \frac{x^4}{\left(2+x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=391

$$\frac{x^5}{6\sqrt{x^6+2}}-\frac{\left(1+\sqrt{3}\right)\sqrt{x^6+2}\,x}{6\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)}+\frac{\left(1-\sqrt{3}\right)\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\,xF\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x^2+\sqrt[3]{2}}{\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right.}{6\,2^{2/3}\sqrt[4]{3}\sqrt{\frac{x^2\left(x^2+\sqrt[3]{2}\right)}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\sqrt{x^6+2}}$$

```
[Out] 1/6*x^5/(x^6+2)^(1/2)-1/6*x*(1+3^(1/2))*(x^6+2)^(1/2)/(2^(1/3)+x^2*(1+3^(1/
2)))+1/6*3^(1/4)*x*(2^(1/3)+x^2)*((2^(1/3)+x^2*(1-3^(1/2)))^2/(2^(1/3)+x^2*
(1+3^(1/2)))^2)^(1/2)/(2^(1/3)+x^2*(1-3^(1/2)))*(2^(1/3)+x^2*(1+3^(1/2)))*E
llipticE((1-(2^(1/3)+x^2*(1-3^(1/2)))^2/(2^(1/3)+x^2*(1+3^(1/2)))^2)^(1/2),
1/4*6^(1/2)+1/4*2^(1/2))*((2^(2/3)-2^(1/3)*x^2+x^4)/(2^(1/3)+x^2*(1+3^(1/2)
))^2)^(1/2)*2^(1/3)/(x^6+2)^(1/2)/(x^2*(2^(1/3)+x^2)/(2^(1/3)+x^2*(1+3^(1/2
)))^2)^(1/2)+1/36*x*(2^(1/3)+x^2)*((2^(1/3)+x^2*(1-3^(1/2)))^2/(2^(1/3)+x^2
*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)+x^2*(1-3^(1/2)))*(2^(1/3)+x^2*(1+3^(1/2)))*
EllipticF((1-(2^(1/3)+x^2*(1-3^(1/2)))^2/(2^(1/3)+x^2*(1+3^(1/2)))^2)^(1/2)
,1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*((2^(2/3)-2^(1/3)*x^2+x^4)/(2^(1/3)+x
^2*(1+3^(1/2)))^2)^(1/2)*2^(1/3)*3^(3/4)/(x^6+2)^(1/2)/(x^2*(2^(1/3)+x^2)/(
2^(1/3)+x^2*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {290, 308, 225, 1881}

$$\frac{x^5}{6\sqrt{x^6+2}}-\frac{\left(1+\sqrt{3}\right)\sqrt{x^6+2}\,x}{6\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)}+\frac{\left(1-\sqrt{3}\right)\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\,xF\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x^2+\sqrt[3]{2}}{\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right.}{6\,2^{2/3}\sqrt[4]{3}\sqrt{\frac{x^2\left(x^2+\sqrt[3]{2}\right)}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(2 + x^6)^(3/2),x]
```

```
[Out] x^5/(6*Sqrt[2 + x^6]) - ((1 + Sqrt[3])*x*Sqrt[2 + x^6])/(6*(2^(1/3) + (1 +
Sqrt[3])*x^2)) + (x*(2^(1/3) + x^2)*Sqrt[(2^(2/3) - 2^(1/3)*x^2 + x^4)/(2^(
1/3) + (1 + Sqrt[3])*x^2)^2]*EllipticE[ArcCos[(2^(1/3) + (1 - Sqrt[3])*x^2)
/(2^(1/3) + (1 + Sqrt[3])*x^2)], (2 + Sqrt[3])/4])/(2^(2/3)*3^(3/4)*Sqrt[(x
^2*(2^(1/3) + x^2))/(2^(1/3) + (1 + Sqrt[3])*x^2)^2]*Sqrt[2 + x^6]) + ((1 -
 Sqrt[3])*x*(2^(1/3) + x^2)*Sqrt[(2^(2/3) - 2^(1/3)*x^2 + x^4)/(2^(1/3) + (
1 + Sqrt[3])*x^2)^2]*EllipticF[ArcCos[(2^(1/3) + (1 - Sqrt[3])*x^2)/(2^(1/3
) + (1 + Sqrt[3])*x^2)], (2 + Sqrt[3])/4])/(6*2^(2/3)*3^(1/4)*Sqrt[(x^2*(2^
(1/3) + x^2))/(2^(1/3) + (1 + Sqrt[3])*x^2)^2]*Sqrt[2 + x^6])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
 - Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{x^4}{\left(2+x^6\right)^{3/2}}\,dx = \frac{x^5}{6\sqrt{2+x^6}} - \frac{1}{3}\int \frac{x^4}{\sqrt{2+x^6}}\,dx$$

$$= \frac{x^5}{6\sqrt{2+x^6}} + \frac{1}{6}\int \frac{2^{2/3}\left(-1+\sqrt{3}\right) - 2x^4}{\sqrt{2+x^6}}\,dx + \frac{\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{2+x^6}}\,dx}{3\sqrt[3]{2}}$$

$$= \frac{x^5}{6\sqrt{2+x^6}} - \frac{\left(1+\sqrt{3}\right)x\sqrt{2+x^6}}{6\left(\sqrt[3]{2}+\left(1+\sqrt{3}\right)x^2\right)} + \frac{x\left(\sqrt[3]{2}+x^2\right)\sqrt{\frac{2^{2/3}-\sqrt[3]{2}x^2+x^4}{\left(\sqrt[3]{2}+\left(1+\sqrt{3}\right)x^2\right)^2}}\,E\left(\cos^{-1}\left(\frac{\sqrt[3]{2}+\left(1-\sqrt{3}\right)}{\sqrt[3]{2}+\left(1+\sqrt{3}\right)}\right.\right.}{2^{2/3}3^{3/4}\sqrt{\frac{x^2\left(\sqrt[3]{2}+x^2\right)}{\left(\sqrt[3]{2}+\left(1+\sqrt{3}\right)x^2\right)^2}}\sqrt{2+x^6}}$$

Mathematica [C] time = 0.00, size = 29, normalized size = 0.07

$$\frac{x^5\,{}_2F_1\left(\frac{5}{6},\frac{3}{2};\frac{11}{6};-\frac{x^6}{2}\right)}{10\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(2 + x^6)^(3/2),x]
```

```
[Out] (x^5*Hypergeometric2F1[5/6, 3/2, 11/6, -1/2*x^6])/(10*Sqrt[2])
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^6+2}\,x^4}{x^{12}+4\,x^6+4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^6 + 2)*x^4/(x^12 + 4*x^6 + 4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(x^6+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/(x^6 + 2)^(3/2), x)
```

maple [C] time = 0.14, size = 33, normalized size = 0.08

$$-\frac{\sqrt{2}\,x^5\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{5}{6}\right],\left[\frac{11}{6}\right],-\frac{x^6}{2}\right)}{30}+\frac{x^5}{6\sqrt{x^6+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^6+2)^(3/2),x)
```

```
[Out] 1/6/(x^6+2)^(1/2)*x^5-1/30*2^(1/2)*x^5*hypergeom([1/2,5/6],[11/6],-1/2*x^6)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(x^6+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(x^6 + 2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(x^6+2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^6 + 2)^(3/2),x)
```

```
[Out] int(x^4/(x^6 + 2)^(3/2), x)
```

sympy [C] time = 1.36, size = 36, normalized size = 0.09

$$\frac{\sqrt{2}\,x^5\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{6},\frac{3}{2}\\ \frac{11}{6}\end{matrix}\right|\frac{x^6e^{i\pi}}{2}\right)}{24\Gamma\left(\frac{11}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**6+2)**(3/2),x)
```

```
[Out] sqrt(2)*x**5*gamma(5/6)*hyper((5/6, 3/2), (11/6,), x**6*exp_polar(I*pi)/2)/
(24*gamma(11/6))
```

3.1434 $\int \frac{1}{x^2\left(2+x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=408

$$-\frac{\sqrt{x^6+2}}{3x}+\frac{1}{6\sqrt{x^6+2}\,x}+\frac{\left(1+\sqrt{3}\right)\sqrt{x^6+2}\,x}{3\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)}-\frac{\left(1-\sqrt{3}\right)\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\,xF\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x^2+}{\left(1+\sqrt{3}\right)x^2+}\right.\right.}{3\,2^{2/3}\sqrt[4]{3}\sqrt{\frac{x^2\left(x^2+\sqrt[3]{2}\right)}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\sqrt{x^6+2}}$$

```
[Out] 1/6/x/(x^6+2)^(1/2)-1/3*(x^6+2)^(1/2)/x+1/3*x*(1+3^(1/2))*(x^6+2)^(1/2)/(2^
(1/3)+x^2*(1+3^(1/2)))-1/3*3^(1/4)*x*(2^(1/3)+x^2)*((2^(1/3)+x^2*(1-3^(1/2)
))^2/(2^(1/3)+x^2*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)+x^2*(1-3^(1/2)))*(2^(1/3)+
x^2*(1+3^(1/2)))*EllipticE((1-(2^(1/3)+x^2*(1-3^(1/2)))^2/(2^(1/3)+x^2*(1+3
^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((2^(2/3)-2^(1/3)*x^2+x^4)/(2^(1
/3)+x^2*(1+3^(1/2)))^2)^(1/2)*2^(1/3)/(x^6+2)^(1/2)/(x^2*(2^(1/3)+x^2)/(2^(
1/3)+x^2*(1+3^(1/2)))^2)^(1/2)-1/18*x*(2^(1/3)+x^2)*((2^(1/3)+x^2*(1-3^(1/2
)))^2/(2^(1/3)+x^2*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)+x^2*(1-3^(1/2)))*(2^(1/3)
+x^2*(1+3^(1/2)))*EllipticF((1-(2^(1/3)+x^2*(1-3^(1/2)))^2/(2^(1/3)+x^2*(1+
3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*((2^(2/3)-2^(1/3)*x
^2+x^4)/(2^(1/3)+x^2*(1+3^(1/2)))^2)^(1/2)*2^(1/3)*3^(3/4)/(x^6+2)^(1/2)/(x
^2*(2^(1/3)+x^2)/(2^(1/3)+x^2*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {290, 325, 308, 225, 1881}

$$\frac{\left(1+\sqrt{3}\right)\sqrt{x^6+2}\,x}{3\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)}-\frac{\sqrt{x^6+2}}{3x}+\frac{1}{6\sqrt{x^6+2}\,x}-\frac{\left(1-\sqrt{3}\right)\left(x^2+\sqrt[3]{2}\right)\sqrt{\frac{x^4-\sqrt[3]{2}x^2+2^{2/3}}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\,xF\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)x^2+\sqrt[3]{2}}{\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}}\right.\right.}{3\,2^{2/3}\sqrt[4]{3}\sqrt{\frac{x^2\left(x^2+\sqrt[3]{2}\right)}{\left(\left(1+\sqrt{3}\right)x^2+\sqrt[3]{2}\right)^2}}\sqrt{x^6+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(2 + x^6)^(3/2)),x]
```

```
[Out] 1/(6*x*Sqrt[2 + x^6]) - Sqrt[2 + x^6]/(3*x) + ((1 + Sqrt[3])*x*Sqrt[2 + x^6
])/(3*(2^(1/3) + (1 + Sqrt[3])*x^2)) - (2^(1/3)*x*(2^(1/3) + x^2)*Sqrt[(2^(
2/3) - 2^(1/3)*x^2 + x^4)/(2^(1/3) + (1 + Sqrt[3])*x^2)^2]*EllipticE[ArcCos
[(2^(1/3) + (1 - Sqrt[3])*x^2)/(2^(1/3) + (1 + Sqrt[3])*x^2)], (2 + Sqrt[3]
)/4])/(3^(3/4)*Sqrt[(x^2*(2^(1/3) + x^2))/(2^(1/3) + (1 + Sqrt[3])*x^2)^2]*
Sqrt[2 + x^6]) - ((1 - Sqrt[3])*x*(2^(1/3) + x^2)*Sqrt[(2^(2/3) - 2^(1/3)*x
^2 + x^4)/(2^(1/3) + (1 + Sqrt[3])*x^2)^2]*EllipticF[ArcCos[(2^(1/3) + (1 -
 Sqrt[3])*x^2)/(2^(1/3) + (1 + Sqrt[3])*x^2)], (2 + Sqrt[3])/4])/(3*2^(2/3)
*3^(1/4)*Sqrt[(x^2*(2^(1/3) + x^2))/(2^(1/3) + (1 + Sqrt[3])*x^2)^2]*Sqrt[2
 + x^6])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
 - Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(2+x^6\right)^{3/2}}\,dx = \frac{1}{6x\sqrt{2+x^6}} + \frac{2}{3}\int \frac{1}{x^2\sqrt{2+x^6}}\,dx$$

$$= \frac{1}{6x\sqrt{2+x^6}} - \frac{\sqrt{2+x^6}}{3x} + \frac{2}{3}\int \frac{x^4}{\sqrt{2+x^6}}\,dx$$

$$= \frac{1}{6x\sqrt{2+x^6}} - \frac{\sqrt{2+x^6}}{3x} - \frac{1}{3}\int \frac{2^{2/3}\left(-1+\sqrt{3}\right)-2x^4}{\sqrt{2+x^6}}\,dx - \frac{1}{3}\left(2^{2/3}\left(1-\sqrt{3}\right)\right)\int \frac{1}{\sqrt{2+x^6}}\,dx$$

$$= \frac{1}{6x\sqrt{2+x^6}} - \frac{\sqrt{2+x^6}}{3x} + \frac{\left(1+\sqrt{3}\right)x\sqrt{2+x^6}}{3\left(\sqrt[3]{2}+\left(1+\sqrt{3}\right)x^2\right)} - \frac{\sqrt[3]{2}\,x\left(\sqrt[3]{2}+x^2\right)\sqrt{\frac{2^{2/3}-\sqrt[3]{2}x^2+x^4}{\left(\sqrt[3]{2}+\left(1+\sqrt{3}\right)x^2\right)^2}}\,E\left(\text{c}\right.}{3^{3/4}\sqrt{\frac{x^2\left(\sqrt[3]{2}+x^2\right.}{\left(\sqrt[3]{2}+\left(1+\sqrt{3}\right).\right.}}}$$

Mathematica [C] time = 0.00, size = 29, normalized size = 0.07

$$-\frac{{}_2F_1\left(-\frac{1}{6},\frac{3}{2};\frac{5}{6};-\frac{x^6}{2}\right)}{2\sqrt{2}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(2 + x^6)^(3/2)),x]
```

```
[Out] -1/2*Hypergeometric2F1[-1/6, 3/2, 5/6, -1/2*x^6]/(Sqrt[2]*x)
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^6+2}}{x^{14}+4\,x^8+4\,x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^6 + 2)/(x^14 + 4*x^8 + 4*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^6+2\right)^{\frac{3}{2}} x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^6 + 2)^(3/2)*x^2), x)
```

maple [C] time = 0.14, size = 40, normalized size = 0.10

$$\frac{\sqrt{2}\,x^5\,\text{hypergeom}\left(\left[\frac{1}{2},\frac{5}{6}\right],\left[\frac{11}{6}\right],-\frac{x^6}{2}\right)}{15}-\frac{2x^6+3}{6\sqrt{x^6+2}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^6+2)^(3/2),x)
```

```
[Out] -1/6*(2*x^6+3)/x/(x^6+2)^(1/2)+1/15*2^(1/2)*x^5*hypergeom([1/2,5/6],[11/6],
-1/2*x^6)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^6+2\right)^{\frac{3}{2}} x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^6 + 2)^(3/2)*x^2), x)
```

mupad [B] time = 1.26, size = 31, normalized size = 0.08

$$-\frac{\left(\frac{2}{x^6}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{5}{3};\ \frac{8}{3};\ -\frac{2}{x^6}\right)}{10\,x\left(x^6+2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^6 + 2)^(3/2)),x)
```

```
[Out] -((2/x^6 + 1)^(3/2)*hypergeom([3/2, 5/3], 8/3, -2/x^6))/(10*x*(x^6 + 2)^(3/
2))
```

sympy [C] time = 1.54, size = 37, normalized size = 0.09

$$\frac{\sqrt{2}\,\Gamma\left(-\frac{1}{6}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{6}, \frac{3}{2} \\ \frac{5}{6}\end{matrix}\right| \frac{x^6 e^{i\pi}}{2}\right)}{24x\Gamma\left(\frac{5}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**6+2)**(3/2),x)
```

```
[Out] sqrt(2)*gamma(-1/6)*hyper((-1/6, 3/2), (5/6,), x**6*exp_polar(I*pi)/2)/(24*
x*gamma(5/6))
```

3.1435 $\int x^m \left(a + bx^7\right) dx$

Optimal. Leaf size=25

$$\frac{ax^{m+1}}{m+1} + \frac{bx^{m+8}}{m+8}$$

```
[Out] a*x^(1+m)/(1+m)+b*x^(8+m)/(8+m)
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$\frac{ax^{m+1}}{m+1} + \frac{bx^{m+8}}{m+8}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^7),x]
```

```
[Out] (a*x^(1 + m))/(1 + m) + (b*x^(8 + m))/(8 + m)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^m \left(a + bx^7\right) dx &= \int \left(ax^m + bx^{7+m}\right) dx \\ &= \frac{ax^{1+m}}{1+m} + \frac{bx^{8+m}}{8+m}\end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 1.00

$$\frac{ax^{m+1}}{m+1} + \frac{bx^{m+8}}{m+8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^7),x]
```

```
[Out] (a*x^(1 + m))/(1 + m) + (b*x^(8 + m))/(8 + m)
```

fricas [A] time = 0.85, size = 33, normalized size = 1.32

$$\frac{\left((bm+b)x^8 + (am+8\,a)x\right)x^m}{m^2+9\,m+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^7+a),x, algorithm="fricas")
```

```
[Out] ((b*m + b)*x^8 + (a*m + 8*a)*x)*x^m/(m^2 + 9*m + 8)
```

giac [A] time = 0.16, size = 43, normalized size = 1.72

$$\frac{bmx^8x^m + bx^8x^m + amxx^m + 8\,axx^m}{m^2+9\,m+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^7+a),x, algorithm="giac")
```

```
[Out] (b*m*x^8*x^m + b*x^8*x^m + a*m*x*x^m + 8*a*x*x^m)/(m^2 + 9*m + 8)
```

maple [A] time = 0.00, size = 35, normalized size = 1.40

$$\frac{\left(bm\,x^7 + b\,x^7 + am + 8a\right)x^{m+1}}{(m+8)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^7+a),x)
```

```
[Out] x^(m+1)*(b*m*x^7+b*x^7+a*m+8*a)/(8+m)/(m+1)
```

maxima [A] time = 1.02, size = 25, normalized size = 1.00

$$\frac{bx^{m+8}}{m+8} + \frac{ax^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^7+a),x, algorithm="maxima")
```

```
[Out] b*x^(m + 8)/(m + 8) + a*x^(m + 1)/(m + 1)
```

mupad [B] time = 1.32, size = 34, normalized size = 1.36

$$\frac{x^{m+1}\left(8\,a + a\,m + b\,x^7 + b\,m\,x^7\right)}{m^2 + 9\,m + 8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^7),x)
```

```
[Out] (x^(m + 1)*(8*a + a*m + b*x^7 + b*m*x^7))/(9*m + m^2 + 8)
```

sympy [A] time = 2.97, size = 94, normalized size = 3.76

$$\begin{cases} -\frac{a}{7x^7} + b\log(x) & \text{for } m = -8 \\ a\log(x) + \frac{bx^7}{7} & \text{for } m = -1 \\ \frac{amxx^m}{m^2+9m+8} + \frac{8axx^m}{m^2+9m+8} + \frac{bmx^8x^m}{m^2+9m+8} + \frac{bx^8x^m}{m^2+9m+8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**7+a),x)
```

```
[Out] Piecewise((-a/(7*x**7) + b*log(x), Eq(m, -8)), (a*log(x) + b*x**7/7, Eq(m,
-1)), (a*m*x*x**m/(m**2 + 9*m + 8) + 8*a*x*x**m/(m**2 + 9*m + 8) + b*m*x**8
*x**m/(m**2 + 9*m + 8) + b*x**8*x**m/(m**2 + 9*m + 8), True))
```

3.1436 $\int x^8 \left(a + bx^7\right) dx$

Optimal. Leaf size=17

$$\frac{ax^9}{9} + \frac{bx^{16}}{16}$$

```
[Out] 1/9*a*x^9+1/16*b*x^16
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$\frac{ax^9}{9} + \frac{bx^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x^7),x]
```

```
[Out] (a*x^9)/9 + (b*x^16)/16
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^8 \left(a + bx^7\right) dx &= \int \left(ax^8 + bx^{15}\right) dx \\ &= \frac{ax^9}{9} + \frac{bx^{16}}{16}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^9}{9} + \frac{bx^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x^7),x]
```

```
[Out] (a*x^9)/9 + (b*x^16)/16
```

fricas [A] time = 0.47, size = 13, normalized size = 0.76

$$\frac{1}{16}x^{16}b + \frac{1}{9}x^9 a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^7+a),x, algorithm="fricas")
```

```
[Out] 1/16*x^16*b + 1/9*x^9*a
```

giac [A] time = 0.19, size = 13, normalized size = 0.76

$$\frac{1}{16}\, bx^{16} + \frac{1}{9}\, ax^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^7+a),x, algorithm="giac")
[Out] 1/16*b*x^16 + 1/9*a*x^9
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{16} b\, x^{16} + \frac{1}{9} a\, x^{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^7+a),x)
[Out] 1/9*a*x^9+1/16*b*x^16
```

maxima [A] time = 0.96, size = 13, normalized size = 0.76

$$\frac{1}{16} bx^{16} + \frac{1}{9} ax^{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^7+a),x, algorithm="maxima")
[Out] 1/16*b*x^16 + 1/9*a*x^9
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{b\, x^{16}}{16} + \frac{a\, x^{9}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*x^7),x)
[Out] (a*x^9)/9 + (b*x^16)/16
```

sympy [A] time = 0.07, size = 12, normalized size = 0.71

$$\frac{ax^{9}}{9} + \frac{bx^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**7+a),x)
[Out] a*x**9/9 + b*x**16/16
```

3.1437 $\int \frac{a+bx^7}{x^8}\,dx$

Optimal. Leaf size=13

$$b\log(x)-\frac{a}{7x^7}$$

```
[Out] -1/7*a/x^7+b*ln(x)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$b\log(x)-\frac{a}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^7)/x^8,x]
```

```
[Out] -a/(7*x^7) + b*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^7}{x^8}\,dx &= \int \left(\frac{a}{x^8}+\frac{b}{x}\right)dx\\ &= -\frac{a}{7x^7}+b\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$b\log(x)-\frac{a}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^7)/x^8,x]
```

```
[Out] -1/7*a/x^7 + b*Log[x]
```

fricas [A] time = 0.81, size = 17, normalized size = 1.31

$$\frac{7\,bx^7\log(x)-a}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^7+a)/x^8,x, algorithm="fricas")
```

```
[Out] 1/7*(7*b*x^7*log(x) - a)/x^7
```

giac [A] time = 0.15, size = 18, normalized size = 1.38

$$b\log\left(|x|\right)-\frac{bx^7+a}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^7+a)/x^8,x, algorithm="giac")
[Out] b*log(abs(x)) - 1/7*(b*x^7 + a)/x^7
```

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$b \ln(x) - \frac{a}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^7+a)/x^8,x)
[Out] -1/7*a/x^7+b*ln(x)
```

maxima [A] time = 1.02, size = 14, normalized size = 1.08

$$\frac{1}{7} b \log\left(x^7\right) - \frac{a}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^7+a)/x^8,x, algorithm="maxima")
[Out] 1/7*b*log(x^7) - 1/7*a/x^7
```

mupad [B] time = 1.03, size = 11, normalized size = 0.85

$$b\,\ln(x) - \frac{a}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^7)/x^8,x)
[Out] b*log(x) - a/(7*x^7)
```

sympy [A] time = 0.34, size = 10, normalized size = 0.77

$$-\frac{a}{7x^7} + b \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**7+a)/x**8,x)
[Out] -a/(7*x**7) + b*log(x)
```

3.1438 $\int x^m \left(a + bx^7\right)^2 \, dx$

Optimal. Leaf size=43

$$\frac{a^2 x^{m+1}}{m+1} + \frac{2abx^{m+8}}{m+8} + \frac{b^2 x^{m+15}}{m+15}$$

```
[Out] a^2*x^(1+m)/(1+m)+2*a*b*x^(8+m)/(8+m)+b^2*x^(15+m)/(15+m)
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{a^2 x^{m+1}}{m+1} + \frac{2abx^{m+8}}{m+8} + \frac{b^2 x^{m+15}}{m+15}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^7)^2,x]
```

```
[Out] (a^2*x^(1 + m))/(1 + m) + (2*a*b*x^(8 + m))/(8 + m) + (b^2*x^(15 + m))/(15
+ m)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^m \left(a + bx^7\right)^2 \, dx &= \int \left(a^2 x^m + 2abx^{7+m} + b^2 x^{14+m}\right) dx \\ &= \frac{a^2 x^{1+m}}{1+m} + \frac{2abx^{8+m}}{8+m} + \frac{b^2 x^{15+m}}{15+m}\end{aligned}$$

Mathematica [A] time = 0.03, size = 40, normalized size = 0.93

$$x^{m+1}\left(\frac{a^2}{m+1} + \frac{2abx^7}{m+8} + \frac{b^2 x^{14}}{m+15}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^7)^2,x]
```

```
[Out] x^(1 + m)*(a^2/(1 + m) + (2*a*b*x^7)/(8 + m) + (b^2*x^14)/(15 + m))
```

fricas [A] time = 0.92, size = 85, normalized size = 1.98

$$\frac{\left(\left(b^2 m^2 + 9\,b^2 m + 8\,b^2\right)x^{15} + 2\left(abm^2 + 16\,abm + 15\,ab\right)x^8 + \left(a^2 m^2 + 23\,a^2 m + 120\,a^2\right)x\right)x^m}{m^3 + 24\,m^2 + 143\,m + 120}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^7+a)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*m^2 + 9*b^2*m + 8*b^2)*x^15 + 2*(a*b*m^2 + 16*a*b*m + 15*a*b)*x^8 + (
a^2*m^2 + 23*a^2*m + 120*a^2)*x)*x^m/(m^3 + 24*m^2 + 143*m + 120)
```

giac [B] time = 0.17, size = 117, normalized size = 2.72

$$\frac{b^2m^2x^{15}x^m + 9\,b^2mx^{15}x^m + 8\,b^2x^{15}x^m + 2\,abm^2x^8x^m + 32\,abmx^8x^m + 30\,abx^8x^m + a^2m^2xx^m + 23\,a^2mxx^m + 120}{m^3 + 24\,m^2 + 143\,m + 120}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^7+a)^2,x, algorithm="giac")
```

```
[Out] (b^2*m^2*x^15*x^m + 9*b^2*m*x^15*x^m + 8*b^2*x^15*x^m + 2*a*b*m^2*x^8*x^m +
 32*a*b*m*x^8*x^m + 30*a*b*x^8*x^m + a^2*m^2*x*x^m + 23*a^2*m*x*x^m + 120*a
^2*x*x^m)/(m^3 + 24*m^2 + 143*m + 120)
```

maple [B] time = 0.01, size = 93, normalized size = 2.16

$$\frac{\left(b^2m^2x^{14} + 9b^2m\,x^{14} + 8b^2x^{14} + 2ab\,m^2x^7 + 32abm\,x^7 + 30ab\,x^7 + a^2m^2 + 23a^2m + 120a^2\right)x^{m+1}}{(m+1)\,(m+8)\,(m+15)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^7+a)^2,x)
```

```
[Out] x^(m+1)*(b^2*m^2*x^14+9*b^2*m*x^14+8*b^2*x^14+2*a*b*m^2*x^7+32*a*b*m*x^7+30
*a*b*x^7+a^2*m^2+23*a^2*m+120*a^2)/(m+1)/(m+8)/(15+m)
```

maxima [A] time = 1.12, size = 43, normalized size = 1.00

$$\frac{b^2x^{m+15}}{m+15} + \frac{2\,abx^{m+8}}{m+8} + \frac{a^2x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^7+a)^2,x, algorithm="maxima")
```

```
[Out] b^2*x^(m + 15)/(m + 15) + 2*a*b*x^(m + 8)/(m + 8) + a^2*x^(m + 1)/(m + 1)
```

mupad [B] time = 1.37, size = 93, normalized size = 2.16

$$x^m\left(\frac{a^2\,x\,\left(m^2+23\,m+120\right)}{m^3+24\,m^2+143\,m+120} + \frac{b^2\,x^{15}\,\left(m^2+9\,m+8\right)}{m^3+24\,m^2+143\,m+120} + \frac{2\,a\,b\,x^8\,\left(m^2+16\,m+15\right)}{m^3+24\,m^2+143\,m+120}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^7)^2,x)
```

```
[Out] x^m*((a^2*x*(23*m + m^2 + 120))/(143*m + 24*m^2 + m^3 + 120) + (b^2*x^15*(9
*m + m^2 + 8))/(143*m + 24*m^2 + m^3 + 120) + (2*a*b*x^8*(16*m + m^2 + 15))
/(143*m + 24*m^2 + m^3 + 120))
```

sympy [A] time = 13.85, size = 313, normalized size = 7.28

$$\begin{cases} -\frac{a^2}{14x^{14}} - \frac{2ab}{7x^7} + b^2\log(x) \\ -\frac{a^2}{7x^7} + 2ab\log(x) + \frac{b^2x^7}{7} \\ a^2\log(x) + \frac{2abx^7}{7} + \frac{b^2x^{14}}{14} \\ \frac{a^2m^2xx^m}{m^3+24m^2+143m+120} + \frac{23a^2mxx^m}{m^3+24m^2+143m+120} + \frac{120a^2xx^m}{m^3+24m^2+143m+120} + \frac{2abm^2x^8x^m}{m^3+24m^2+143m+120} + \frac{32abmx^8x^m}{m^3+24m^2+143m+120} + \frac{30abx^8x}{m^3+24m^2+14} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**7+a)**2,x)
```

```
[Out] Piecewise((-a**2/(14*x**14) - 2*a*b/(7*x**7) + b**2*log(x), Eq(m, -15)), (-
a**2/(7*x**7) + 2*a*b*log(x) + b**2*x**7/7, Eq(m, -8)), (a**2*log(x) + 2*a*
b*x**7/7 + b**2*x**14/14, Eq(m, -1)), (a**2*m**2*x*x**m/(m**3 + 24*m**2 + 1
43*m + 120) + 23*a**2*m*x*x**m/(m**3 + 24*m**2 + 143*m + 120) + 120*a**2*x*
x**m/(m**3 + 24*m**2 + 143*m + 120) + 2*a*b*m**2*x**8*x**m/(m**3 + 24*m**2
+ 143*m + 120) + 32*a*b*m*x**8*x**m/(m**3 + 24*m**2 + 143*m + 120) + 30*a*b
*x**8*x**m/(m**3 + 24*m**2 + 143*m + 120) + b**2*m**2*x**15*x**m/(m**3 + 24
*m**2 + 143*m + 120) + 9*b**2*m*x**15*x**m/(m**3 + 24*m**2 + 143*m + 120) +
 8*b**2*x**15*x**m/(m**3 + 24*m**2 + 143*m + 120), True))
```

3.1439 $\int x^8 \left(a + bx^7\right)^2 \, dx$

Optimal. Leaf size=30

$$\frac{a^2 x^9}{9} + \frac{1}{8} abx^{16} + \frac{b^2 x^{23}}{23}$$

```
[Out] 1/9*a^2*x^9+1/8*a*b*x^16+1/23*b^2*x^23
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{a^2 x^9}{9} + \frac{1}{8} abx^{16} + \frac{b^2 x^{23}}{23}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x^7)^2,x]
```

```
[Out] (a^2*x^9)/9 + (a*b*x^16)/8 + (b^2*x^23)/23
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^8 \left(a + bx^7\right)^2 \, dx &= \int \left(a^2 x^8 + 2abx^{15} + b^2 x^{22}\right) \, dx \\ &= \frac{a^2 x^9}{9} + \frac{1}{8} abx^{16} + \frac{b^2 x^{23}}{23}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2 x^9}{9} + \frac{1}{8} abx^{16} + \frac{b^2 x^{23}}{23}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x^7)^2,x]
```

```
[Out] (a^2*x^9)/9 + (a*b*x^16)/8 + (b^2*x^23)/23
```

fricas [A] time = 0.67, size = 24, normalized size = 0.80

$$\frac{1}{23} x^{23} b^2 + \frac{1}{8} x^{16} ba + \frac{1}{9} x^9 a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^7+a)^2,x, algorithm="fricas")
```

```
[Out] 1/23*x^23*b^2 + 1/8*x^16*b*a + 1/9*x^9*a^2
```

giac [A] time = 0.15, size = 24, normalized size = 0.80

$$\frac{1}{23} b^2 x^{23} + \frac{1}{8} abx^{16} + \frac{1}{9} a^2 x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^7+a)^2,x, algorithm="giac")
[Out] 1/23*b^2*x^23 + 1/8*a*b*x^16 + 1/9*a^2*x^9
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$\frac{1}{23}b^2x^{23} + \frac{1}{8}ab\,x^{16} + \frac{1}{9}a^2x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^7+a)^2,x)
[Out] 1/9*a^2*x^9+1/8*a*b*x^16+1/23*b^2*x^23
```

maxima [A] time = 0.94, size = 24, normalized size = 0.80

$$\frac{1}{23}\,b^2x^{23} + \frac{1}{8}\,abx^{16} + \frac{1}{9}\,a^2x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^7+a)^2,x, algorithm="maxima")
[Out] 1/23*b^2*x^23 + 1/8*a*b*x^16 + 1/9*a^2*x^9
```

mupad [B] time = 0.04, size = 24, normalized size = 0.80

$$\frac{a^2\,x^9}{9} + \frac{a\,b\,x^{16}}{8} + \frac{b^2\,x^{23}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*x^7)^2,x)
[Out] (a^2*x^9)/9 + (b^2*x^23)/23 + (a*b*x^16)/8
```

sympy [A] time = 0.07, size = 24, normalized size = 0.80

$$\frac{a^2x^9}{9} + \frac{abx^{16}}{8} + \frac{b^2x^{23}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**7+a)**2,x)
[Out] a**2*x**9/9 + a*b*x**16/8 + b**2*x**23/23
```

3.1440 $\int \frac{\left(a+bx^7\right)^2}{x^8}\,dx$

Optimal. Leaf size=27

$$-\frac{a^2}{7x^7}+2ab\log(x)+\frac{b^2x^7}{7}$$

```
[Out] -1/7*a^2/x^7+1/7*b^2*x^7+2*a*b*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$-\frac{a^2}{7x^7}+2ab\log(x)+\frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^7)^2/x^8,x]
```

```
[Out] -a^2/(7*x^7) + (b^2*x^7)/7 + 2*a*b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^7\right)^2}{x^8}\,dx &= \frac{1}{7}\operatorname{Subst}\left(\int \frac{(a+bx)^2}{x^2}\,dx, x, x^7\right)\\ &= \frac{1}{7}\operatorname{Subst}\left(\int \left(b^2+\frac{a^2}{x^2}+\frac{2ab}{x}\right)\,dx, x, x^7\right)\\ &= -\frac{a^2}{7x^7}+\frac{b^2x^7}{7}+2ab\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.00

$$-\frac{a^2}{7x^7}+2ab\log(x)+\frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^7)^2/x^8,x]
```

```
[Out] -1/7*a^2/x^7 + (b^2*x^7)/7 + 2*a*b*Log[x]
```

fricas [A] time = 0.79, size = 27, normalized size = 1.00

$$\frac{b^2x^{14}+14\,abx^7\log(x)-a^2}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^7+a)^2/x^8,x, algorithm="fricas")
[Out] 1/7*(b^2*x^14 + 14*a*b*x^7*log(x) - a^2)/x^7
```

giac [A] time = 0.15, size = 32, normalized size = 1.19

$$\frac{1}{7}\,b^2x^7 + 2\,ab\log\left(|x|\right) - \frac{2\,abx^7 + a^2}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^7+a)^2/x^8,x, algorithm="giac")
[Out] 1/7*b^2*x^7 + 2*a*b*log(abs(x)) - 1/7*(2*a*b*x^7 + a^2)/x^7
```

maple [A] time = 0.01, size = 24, normalized size = 0.89

$$\frac{b^2x^7}{7} + 2ab\ln(x) - \frac{a^2}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^7+a)^2/x^8,x)
[Out] -1/7*a^2/x^7+1/7*b^2*x^7+2*a*b*ln(x)
```

maxima [A] time = 1.06, size = 25, normalized size = 0.93

$$\frac{1}{7}\,b^2x^7 + \frac{2}{7}\,ab\log\left(x^7\right) - \frac{a^2}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^7+a)^2/x^8,x, algorithm="maxima")
[Out] 1/7*b^2*x^7 + 2/7*a*b*log(x^7) - 1/7*a^2/x^7
```

mupad [B] time = 1.02, size = 23, normalized size = 0.85

$$\frac{b^2\,x^7}{7} - \frac{a^2}{7\,x^7} + 2\,a\,b\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^7)^2/x^8,x)
[Out] (b^2*x^7)/7 - a^2/(7*x^7) + 2*a*b*log(x)
```

sympy [A] time = 0.28, size = 24, normalized size = 0.89

$$-\frac{a^2}{7x^7} + 2ab\log\left(x\right) + \frac{b^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**7+a)**2/x**8,x)
[Out] -a**2/(7*x**7) + 2*a*b*log(x) + b**2*x**7/7
```

3.1441 $\int \frac{x^m}{a+bx^7}\,dx$

Optimal. Leaf size=39

$$\frac{x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{7};\frac{m+8}{7};-\frac{bx^7}{a}\right)}{a(m+1)}$$

```
[Out] x^(1+m)*hypergeom([1, 1/7+1/7*m],[8/7+1/7*m],-b*x^7/a)/a/(1+m)
```

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {364}

$$\frac{x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{7};\frac{m+8}{7};-\frac{bx^7}{a}\right)}{a(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x^7),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1, (1 + m)/7, (8 + m)/7, -((b*x^7)/a)])/(a*(1
+ m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^m}{a+bx^7}\,dx = \frac{x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{7};\frac{8+m}{7};-\frac{bx^7}{a}\right)}{a(1+m)}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.05

$$\frac{x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{7};\frac{m+1}{7}+1;-\frac{bx^7}{a}\right)}{a(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x^7),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1, (1 + m)/7, 1 + (1 + m)/7, -((b*x^7)/a)])/(a
*(1 + m))
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m}{bx^7+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^7+a),x, algorithm="fricas")
```

```
[Out] integral(x^m/(b*x^7 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{bx^7 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^7+a),x, algorithm="giac")
[Out] integrate(x^m/(b*x^7 + a), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^m}{b\,x^7 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^7+a),x)
[Out] int(x^m/(b*x^7+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{bx^7 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^7+a),x, algorithm="maxima")
[Out] integrate(x^m/(b*x^7 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{b\,x^7 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^7),x)
[Out] int(x^m/(a + b*x^7), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x**7+a),x)
[Out] Timed out
```

3.1442 $\int \frac{x^6}{a+bx^7}\,dx$

Optimal. Leaf size=15

$$\frac{\log\left(a+bx^7\right)}{7b}$$

```
[Out] 1/7*ln(b*x^7+a)/b
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {260}

$$\frac{\log\left(a+bx^7\right)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^7),x]
```

```
[Out] Log[a + b*x^7]/(7*b)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^6}{a+bx^7}\,dx = \frac{\log\left(a+bx^7\right)}{7b}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log\left(a+bx^7\right)}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^7),x]
```

```
[Out] Log[a + b*x^7]/(7*b)
```

fricas [A] time = 0.77, size = 13, normalized size = 0.87

$$\frac{\log\left(bx^7+a\right)}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^7+a),x, algorithm="fricas")
```

```
[Out] 1/7*log(b*x^7 + a)/b
```

giac [A] time = 0.16, size = 14, normalized size = 0.93

$$\frac{\log\left(\left|bx^7+a\right|\right)}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^7+a),x, algorithm="giac")
[Out] 1/7*log(abs(b*x^7 + a))/b
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{\ln\left(b\,x^7+a\right)}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^7+a),x)
[Out] 1/7*ln(b*x^7+a)/b
```

maxima [A] time = 1.01, size = 13, normalized size = 0.87

$$\frac{\log\left(bx^7+a\right)}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^7+a),x, algorithm="maxima")
[Out] 1/7*log(b*x^7 + a)/b
```

mupad [B] time = 0.04, size = 13, normalized size = 0.87

$$\frac{\ln\left(b\,x^7+a\right)}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^7),x)
[Out] log(a + b*x^7)/(7*b)
```

sympy [A] time = 0.33, size = 10, normalized size = 0.67

$$\frac{\log\left(a+bx^7\right)}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**7+a),x)
[Out] log(a + b*x**7)/(7*b)
```

3.1443 $\int \frac{1}{a+bx^7}\,dx$

Optimal. Leaf size=335

$$\frac{\sin\left(\frac{3\pi}{14}\right)\log\left(a^{2/7}+2\sqrt[7]{a}\,\sqrt[7]{b}\,x\sin\left(\frac{3\pi}{14}\right)+b^{2/7}x^2\right)}{7a^{6/7}\sqrt[7]{b}}-\frac{\sin\left(\frac{\pi}{14}\right)\log\left(a^{2/7}-2\sqrt[7]{a}\,\sqrt[7]{b}\,x\sin\left(\frac{\pi}{14}\right)+b^{2/7}x^2\right)}{7a^{6/7}\sqrt[7]{b}}-\frac{\cos\left(\frac{\pi}{7}\right)\log\left(a\right.}{}$$

```
[Out] 2/7*arctan(b^(1/7)*x*sec(1/14*Pi)/a^(1/7)-tan(1/14*Pi))*cos(1/14*Pi)/a^(6/7
)/b^(1/7)+2/7*arctan(b^(1/7)*x*sec(3/14*Pi)/a^(1/7)+tan(3/14*Pi))*cos(3/14*
Pi)/a^(6/7)/b^(1/7)+1/7*ln(a^(1/7)+b^(1/7)*x)/a^(6/7)/b^(1/7)-1/7*cos(1/7*P
i)*ln(a^(2/7)+b^(2/7)*x^2-2*a^(1/7)*b^(1/7)*x*cos(1/7*Pi))/a^(6/7)/b^(1/7)-
1/7*ln(a^(2/7)+b^(2/7)*x^2-2*a^(1/7)*b^(1/7)*x*sin(1/14*Pi))*sin(1/14*Pi)/a
^(6/7)/b^(1/7)+2/7*arctan(-cot(1/7*Pi)+b^(1/7)*x*csc(1/7*Pi)/a^(1/7))*sin(1
/7*Pi)/a^(6/7)/b^(1/7)+1/7*ln(a^(2/7)+b^(2/7)*x^2+2*a^(1/7)*b^(1/7)*x*sin(3
/14*Pi))*sin(3/14*Pi)/a^(6/7)/b^(1/7)
```

Rubi [A] time = 0.54, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {201, 634, 618, 204, 628, 31}

$$\frac{\sin\left(\frac{3\pi}{14}\right)\log\left(a^{2/7}+2\sqrt[7]{a}\,\sqrt[7]{b}\,x\sin\left(\frac{3\pi}{14}\right)+b^{2/7}x^2\right)}{7a^{6/7}\sqrt[7]{b}}-\frac{\sin\left(\frac{\pi}{14}\right)\log\left(a^{2/7}-2\sqrt[7]{a}\,\sqrt[7]{b}\,x\sin\left(\frac{\pi}{14}\right)+b^{2/7}x^2\right)}{7a^{6/7}\sqrt[7]{b}}-\frac{\cos\left(\frac{\pi}{7}\right)\log\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^7)^(-1),x]
```

```
[Out] (2*ArcTan[(b^(1/7)*x*Sec[Pi/14])/a^(1/7) - Tan[Pi/14]]*Cos[Pi/14])/(7*a^(6/
7)*b^(1/7)) + (2*ArcTan[(b^(1/7)*x*Sec[(3*Pi)/14])/a^(1/7) + Tan[(3*Pi)/14]
]*Cos[(3*Pi)/14])/(7*a^(6/7)*b^(1/7)) + Log[a^(1/7) + b^(1/7)*x]/(7*a^(6/7)
*b^(1/7)) - (Cos[Pi/7]*Log[a^(2/7) + b^(2/7)*x^2 - 2*a^(1/7)*b^(1/7)*x*Cos[
Pi/7]])/(7*a^(6/7)*b^(1/7)) - (Log[a^(2/7) + b^(2/7)*x^2 - 2*a^(1/7)*b^(1/7
)*x*Sin[Pi/14]]*Sin[Pi/14])/(7*a^(6/7)*b^(1/7)) - (2*ArcTan[Cot[Pi/7] - (b^
(1/7)*x*Csc[Pi/7])/a^(1/7)]*Sin[Pi/7])/(7*a^(6/7)*b^(1/7)) + (Log[a^(2/7) +
 b^(2/7)*x^2 + 2*a^(1/7)*b^(1/7)*x*Sin[(3*Pi)/14]]*Sin[(3*Pi)/14])/(7*a^(6/
7)*b^(1/7))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[((2*k -
 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]; (r*Int[1
/(r + s*x), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 1)/2}], x], x]
] /; FreeQ[{a, b}, x] && IGtQ[(n - 3)/2, 0] && PosQ[a/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{a+bx^7}\,dx = \frac{2\int \frac{\sqrt[7]{a}-\sqrt[7]{b}\,x\cos\left(\frac{\pi}{7}\right)}{a^{2/7}+b^{2/7}x^2-2\sqrt[7]{a}\,\sqrt[7]{b}\,x\cos\left(\frac{\pi}{7}\right)}\,dx}{7a^{6/7}} + \frac{2\int \frac{\sqrt[7]{a}-\sqrt[7]{b}\,x\sin\left(\frac{\pi}{14}\right)}{a^{2/7}+b^{2/7}x^2-2\sqrt[7]{a}\,\sqrt[7]{b}\,x\sin\left(\frac{\pi}{14}\right)}\,dx}{7a^{6/7}} + \frac{2\int \frac{\sqrt[7]{a}+\sqrt[7]{b}\,x\sin\left(\frac{3\pi}{14}\right)}{a^{2/7}+b^{2/7}x^2+2\sqrt[7]{a}\,\sqrt[7]{b}\,x\sin\left(\frac{3}{1}\right.}}{7a^{6/7}}$$

$$= \frac{\log\left(\sqrt[7]{a}+\sqrt[7]{b}\,x\right)}{7a^{6/7}\sqrt[7]{b}} + \frac{\left(2\cos^2\left(\frac{\pi}{14}\right)\right)\int \frac{1}{a^{2/7}+b^{2/7}x^2-2\sqrt[7]{a}\,\sqrt[7]{b}\,x\sin\left(\frac{\pi}{14}\right)}\,dx}{7a^{5/7}} - \frac{\cos\left(\frac{\pi}{7}\right)\int \frac{2b^{2/7}x-2\sqrt[7]{a}\,\sqrt[7]{b}\,\cos}{a^{2/7}+b^{2/7}x^2-2\sqrt[7]{a}\,\sqrt[7]{b}\,x}}{7a^{6/7}\sqrt[7]{b}}$$

$$= \frac{\log\left(\sqrt[7]{a}+\sqrt[7]{b}\,x\right)}{7a^{6/7}\sqrt[7]{b}} - \frac{\cos\left(\frac{\pi}{7}\right)\log\left(a^{2/7}+b^{2/7}x^2-2\sqrt[7]{a}\,\sqrt[7]{b}\,x\cos\left(\frac{\pi}{7}\right)\right)}{7a^{6/7}\sqrt[7]{b}} - \frac{\log\left(a^{2/7}+b^{2/7}x^2-2\sqrt[7]{}\right.}{7a^{6/}}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt[7]{b}\,x\sec\left(\frac{\pi}{14}\right)}{\sqrt[7]{a}}-\tan\left(\frac{\pi}{14}\right)\right)\cos\left(\frac{\pi}{14}\right)}{7a^{6/7}\sqrt[7]{b}} + \frac{2\tan^{-1}\left(\frac{\sqrt[7]{b}\,x\sec\left(\frac{3\pi}{14}\right)}{\sqrt[7]{a}}+\tan\left(\frac{3\pi}{14}\right)\right)\cos\left(\frac{3\pi}{14}\right)}{7a^{6/7}\sqrt[7]{b}} + \frac{\log\left(\right.}{7}$$

Mathematica [A] time = 0.39, size = 262, normalized size = 0.78

$$\frac{\sin\left(\frac{3\pi}{14}\right)\log\left(a^{2/7}+2\sqrt[7]{a}\,\sqrt[7]{b}\,x\sin\left(\frac{3\pi}{14}\right)+b^{2/7}x^2\right)-\sin\left(\frac{\pi}{14}\right)\log\left(a^{2/7}-2\sqrt[7]{a}\,\sqrt[7]{b}\,x\sin\left(\frac{\pi}{14}\right)+b^{2/7}x^2\right)-\cos\left(\frac{\pi}{7}\right)\log}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^7)^(-1),x]
```

```
[Out] (2*ArcTan[(b^(1/7)*x*Sec[Pi/14])/a^(1/7) - Tan[Pi/14]]*Cos[Pi/14] + 2*ArcTa
n[(b^(1/7)*x*Sec[(3*Pi)/14])/a^(1/7) + Tan[(3*Pi)/14]]*Cos[(3*Pi)/14] + Log
[a^(1/7) + b^(1/7)*x] - Cos[Pi/7]*Log[a^(2/7) + b^(2/7)*x^2 - 2*a^(1/7)*b^(
1/7)*x*Cos[Pi/7]] - Log[a^(2/7) + b^(2/7)*x^2 - 2*a^(1/7)*b^(1/7)*x*Sin[Pi/
14]]*Sin[Pi/14] - 2*ArcTan[Cot[Pi/7] - (b^(1/7)*x*Csc[Pi/7])/a^(1/7)]*Sin[P
i/7] + Log[a^(2/7) + b^(2/7)*x^2 + 2*a^(1/7)*b^(1/7)*x*Sin[(3*Pi)/14]]*Sin[
(3*Pi)/14])/(7*a^(6/7)*b^(1/7))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^7+a),x, algorithm="fricas")
```

```
[Out] Exception raised: RuntimeError >> no explicit roots found
```

giac [A] time = 0.25, size = 310, normalized size = 0.93

$$\frac{\left(-\frac{a}{b}\right)^{\frac{1}{7}}\cos\left(\frac{3}{7}\pi\right)\log\left(2x\left(-\frac{a}{b}\right)^{\frac{1}{7}}\cos\left(\frac{3}{7}\pi\right)+x^2+\left(-\frac{a}{b}\right)^{\frac{2}{7}}\right)}{7a}-\frac{\left(-\frac{a}{b}\right)^{\frac{1}{7}}\cos\left(\frac{2}{7}\pi\right)\log\left(-2x\left(-\frac{a}{b}\right)^{\frac{1}{7}}\cos\left(\frac{2}{7}\pi\right)+x^2+\left(-\frac{a}{b}\right.\right.}{7a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^7+a),x, algorithm="giac")
```

```
[Out] 1/7*(-a/b)^(1/7)*cos(3/7*pi)*log(2*x*(-a/b)^(1/7)*cos(3/7*pi) + x^2 + (-a/b
)^(2/7))/a - 1/7*(-a/b)^(1/7)*cos(2/7*pi)*log(-2*x*(-a/b)^(1/7)*cos(2/7*pi)
 + x^2 + (-a/b)^(2/7))/a + 1/7*(-a/b)^(1/7)*cos(1/7*pi)*log(2*x*(-a/b)^(1/7
)*cos(1/7*pi) + x^2 + (-a/b)^(2/7))/a + 2/7*(-a/b)^(1/7)*arctan(((-a/b)^(1/
7)*cos(3/7*pi) + x)/((-a/b)^(1/7)*sin(3/7*pi)))*sin(3/7*pi)/a + 2/7*(-a/b)^
(1/7)*arctan(-((-a/b)^(1/7)*cos(2/7*pi) - x)/((-a/b)^(1/7)*sin(2/7*pi)))*si
n(2/7*pi)/a + 2/7*(-a/b)^(1/7)*arctan(((-a/b)^(1/7)*cos(1/7*pi) + x)/((-a/b
)^(1/7)*sin(1/7*pi)))*sin(1/7*pi)/a - 1/7*(-a/b)^(1/7)*log(abs(x - (-a/b)^(
1/7)))/a
```

maple [C] time = 0.44, size = 27, normalized size = 0.08

$$\frac{\ln\left(-\operatorname{RootOf}\left(b_Z^7+a\right)+x\right)}{7b\operatorname{RootOf}\left(b_Z^7+a\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^7+a),x)
```

```
[Out] 1/7/b*sum(1/_R^6*ln(x-_R),_R=RootOf(_Z^7*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{bx^7+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^7+a),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*x^7 + a), x)
```

mupad [B] time = 1.85, size = 196, normalized size = 0.59

$$\frac{\ln\left(b^{1/7}x+a^{1/7}\right)}{7a^{6/7}b^{1/7}}-\frac{e^{\frac{\pi 1i}{7}}\ln\left(b^{1/7}x-a^{1/7}e^{\frac{\pi 1i}{7}}\right)}{7a^{6/7}b^{1/7}}+\frac{e^{\frac{\pi 2i}{7}}\ln\left(a^{1/7}e^{\frac{\pi 2i}{7}}+b^{1/7}x\right)}{7a^{6/7}b^{1/7}}-\frac{e^{\frac{\pi 3i}{7}}\ln\left(b^{1/7}x-a^{1/7}e^{\frac{\pi 3i}{7}}\right)}{7a^{6/7}b^{1/7}}+\frac{e^{\frac{\pi 4i}{7}}\ln\left(a\right.}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^7),x)
```

```
[Out] log(b^(1/7)*x + a^(1/7))/(7*a^(6/7)*b^(1/7)) - (exp((pi*1i)/7)*log(b^(1/7)*
x - a^(1/7)*exp((pi*1i)/7)))/(7*a^(6/7)*b^(1/7)) + (exp((pi*2i)/7)*log(a^(1
/7)*exp((pi*2i)/7) + b^(1/7)*x))/(7*a^(6/7)*b^(1/7)) - (exp((pi*3i)/7)*log(
b^(1/7)*x - a^(1/7)*exp((pi*3i)/7)))/(7*a^(6/7)*b^(1/7)) + (exp((pi*4i)/7)*
```

```
log(a^(1/7)*exp((pi*4i)/7) + b^(1/7)*x))/(7*a^(6/7)*b^(1/7)) - (exp((pi*5i)
/7)*log(b^(1/7)*x - a^(1/7)*exp((pi*5i)/7)))/(7*a^(6/7)*b^(1/7)) + (exp((pi
*6i)/7)*log(a^(1/7)*exp((pi*6i)/7) + b^(1/7)*x))/(7*a^(6/7)*b^(1/7))
```

sympy [**A**] time = 0.27, size = 20, normalized size = 0.06

$$\text{RootSum}\left(823543t^7a^6b - 1, \left(t \mapsto t\log\left(7ta + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**7+a),x)
```

```
[Out] RootSum(823543*_t**7*a**6*b - 1, Lambda(_t, _t*log(7*_t*a + x)))
```

3.1444 $\int \frac{1}{x\left(a+bx^7\right)}\,dx$

Optimal. Leaf size=22

$$\frac{\log(x)}{a}-\frac{\log\left(a+bx^7\right)}{7a}$$

```
[Out] ln(x)/a-1/7*ln(b*x^7+a)/a
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {266, 36, 29, 31}

$$\frac{\log(x)}{a}-\frac{\log\left(a+bx^7\right)}{7a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^7)),x]
```

```
[Out] Log[x]/a - Log[a + b*x^7]/(7*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a+bx^7\right)}\,dx &= \frac{1}{7}\operatorname{Subst}\left(\int \frac{1}{x(a+bx)}\,dx, x, x^7\right)\\ &= \frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, x^7\right)}{7a}-\frac{b\operatorname{Subst}\left(\int \frac{1}{a+bx}\,dx, x, x^7\right)}{7a}\\ &= \frac{\log(x)}{a}-\frac{\log\left(a+bx^7\right)}{7a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{\log(x)}{a}-\frac{\log\left(a+bx^7\right)}{7a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^7)),x]
[Out] Log[x]/a - Log[a + b*x^7]/(7*a)
```

fricas [A] time = 0.58, size = 18, normalized size = 0.82

$$-\frac{\log\left(bx^7+a\right)-7\,\log(x)}{7\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^7+a),x, algorithm="fricas")
[Out] -1/7*(log(b*x^7 + a) - 7*log(x))/a
```

giac [A] time = 0.16, size = 22, normalized size = 1.00

$$-\frac{\log\left(\left|bx^7+a\right|\right)}{7\,a}+\frac{\log\left(|x|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^7+a),x, algorithm="giac")
[Out] -1/7*log(abs(b*x^7 + a))/a + log(abs(x))/a
```

maple [A] time = 0.00, size = 21, normalized size = 0.95

$$\frac{\ln(x)}{a}-\frac{\ln\left(b\,x^7+a\right)}{7a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^7+a),x)
[Out] 1/a*ln(x)-1/7*ln(b*x^7+a)/a
```

maxima [A] time = 1.07, size = 23, normalized size = 1.05

$$-\frac{\log\left(bx^7+a\right)}{7\,a}+\frac{\log\left(x^7\right)}{7\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^7+a),x, algorithm="maxima")
[Out] -1/7*log(b*x^7 + a)/a + 1/7*log(x^7)/a
```

mupad [B] time = 0.09, size = 18, normalized size = 0.82

$$-\frac{\ln\left(b\,x^7+a\right)-7\,\ln(x)}{7\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^7)),x)
[Out] -(log(a + b*x^7) - 7*log(x))/(7*a)
```

sympy [A] time = 0.52, size = 15, normalized size = 0.68

$$\frac{\log\left(x\right)}{a}-\frac{\log\left(\frac{a}{b}+x^7\right)}{7a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**7+a),x)
[Out] log(x)/a - log(a/b + x**7)/(7*a)
```

3.1445 $\int \frac{x^m}{a-bx^7}\,dx$

Optimal. Leaf size=38

$$\frac{x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{7};\frac{m+8}{7};\frac{bx^7}{a}\right)}{a(m+1)}$$

```
[Out] x^(1+m)*hypergeom([1, 1/7+1/7*m],[8/7+1/7*m],b*x^7/a)/a/(1+m)
```

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {364}

$$\frac{x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{7};\frac{m+8}{7};\frac{bx^7}{a}\right)}{a(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a - b*x^7),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1, (1 + m)/7, (8 + m)/7, (b*x^7)/a])/(a*(1 + m
))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^m}{a-bx^7}\,dx = \frac{x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{7};\frac{8+m}{7};\frac{bx^7}{a}\right)}{a(1+m)}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.05

$$\frac{x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{7};\frac{m+1}{7}+1;\frac{bx^7}{a}\right)}{a(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a - b*x^7),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1, (1 + m)/7, 1 + (1 + m)/7, (b*x^7)/a])/(a*(1
 + m))
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{x^m}{bx^7-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-b*x^7+a),x, algorithm="fricas")
```

```
[Out] integral(-x^m/(b*x^7 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^m}{bx^7 - a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-b*x^7+a),x, algorithm="giac")
[Out] integrate(-x^m/(b*x^7 - a), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^m}{-b\,x^7 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(-b*x^7+a),x)
[Out] int(x^m/(-b*x^7+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^m}{bx^7 - a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-b*x^7+a),x, algorithm="maxima")
[Out] -integrate(x^m/(b*x^7 - a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{a - b\,x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a - b*x^7),x)
[Out] int(x^m/(a - b*x^7), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(-b*x**7+a),x)
[Out] Timed out
```

3.1446 $\int \frac{x^6}{a-bx^7}\,dx$

Optimal. Leaf size=16

$$-\frac{\log\left(a-bx^7\right)}{7b}$$

```
[Out] -1/7*ln(-b*x^7+a)/b
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {260}

$$-\frac{\log\left(a-bx^7\right)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a - b*x^7),x]
```

```
[Out] -Log[a - b*x^7]/(7*b)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^6}{a-bx^7}\,dx = -\frac{\log\left(a-bx^7\right)}{7b}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{\log\left(a-bx^7\right)}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a - b*x^7),x]
```

```
[Out] -1/7*Log[a - b*x^7]/b
```

fricas [A] time = 0.79, size = 15, normalized size = 0.94

$$-\frac{\log\left(bx^7-a\right)}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-b*x^7+a),x, algorithm="fricas")
```

```
[Out] -1/7*log(b*x^7 - a)/b
```

giac [A] time = 0.16, size = 16, normalized size = 1.00

$$-\frac{\log\left(\left|bx^7-a\right|\right)}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-b*x^7+a),x, algorithm="giac")
[Out] -1/7*log(abs(b*x^7 - a))/b
```

maple [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{\ln\left(b\,x^7-a\right)}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-b*x^7+a),x)
[Out] -1/7/b*ln(b*x^7-a)
```

maxima [A] time = 0.98, size = 15, normalized size = 0.94

$$-\frac{\log\left(bx^7-a\right)}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-b*x^7+a),x, algorithm="maxima")
[Out] -1/7*log(b*x^7 - a)/b
```

mupad [B] time = 0.05, size = 15, normalized size = 0.94

$$-\frac{\ln\left(b\,x^7-a\right)}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a - b*x^7),x)
[Out] -log(b*x^7 - a)/(7*b)
```

sympy [A] time = 0.38, size = 12, normalized size = 0.75

$$-\frac{\log\left(-a+bx^7\right)}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-b*x**7+a),x)
[Out] -log(-a + b*x**7)/(7*b)
```

3.1447 $\int \frac{1}{a-bx^7}\,dx$

Optimal. Leaf size=335

$$-\frac{\sin\left(\frac{3\pi}{14}\right)\log\left(a^{2/7}-2\sqrt[7]{a}\,\sqrt[7]{b}\,x\sin\left(\frac{3\pi}{14}\right)+b^{2/7}x^2\right)}{7a^{6/7}\sqrt[7]{b}}+\frac{\sin\left(\frac{\pi}{14}\right)\log\left(a^{2/7}+2\sqrt[7]{a}\,\sqrt[7]{b}\,x\sin\left(\frac{\pi}{14}\right)+b^{2/7}x^2\right)}{7a^{6/7}\sqrt[7]{b}}+\frac{\cos\left(\frac{\pi}{7}\right)}$$

```
[Out] 2/7*arctan(b^(1/7)*x*sec(1/14*Pi)/a^(1/7)+tan(1/14*Pi))*cos(1/14*Pi)/a^(6/7
)/b^(1/7)+2/7*arctan(b^(1/7)*x*sec(3/14*Pi)/a^(1/7)-tan(3/14*Pi))*cos(3/14*
Pi)/a^(6/7)/b^(1/7)-1/7*ln(a^(1/7)-b^(1/7)*x)/a^(6/7)/b^(1/7)+1/7*cos(1/7*P
i)*ln(a^(2/7)+b^(2/7)*x^2+2*a^(1/7)*b^(1/7)*x*cos(1/7*Pi))/a^(6/7)/b^(1/7)+
1/7*ln(a^(2/7)+b^(2/7)*x^2+2*a^(1/7)*b^(1/7)*x*sin(1/14*Pi))*sin(1/14*Pi)/a
^(6/7)/b^(1/7)+2/7*arctan(cot(1/7*Pi)+b^(1/7)*x*csc(1/7*Pi)/a^(1/7))*sin(1/
7*Pi)/a^(6/7)/b^(1/7)-1/7*ln(a^(2/7)+b^(2/7)*x^2-2*a^(1/7)*b^(1/7)*x*sin(3/
14*Pi))*sin(3/14*Pi)/a^(6/7)/b^(1/7)
```

Rubi [A] time = 0.44, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {202, 634, 618, 204, 628, 31}

$$-\frac{\sin\left(\frac{3\pi}{14}\right)\log\left(a^{2/7}-2\sqrt[7]{a}\,\sqrt[7]{b}\,x\sin\left(\frac{3\pi}{14}\right)+b^{2/7}x^2\right)}{7a^{6/7}\sqrt[7]{b}}+\frac{\sin\left(\frac{\pi}{14}\right)\log\left(a^{2/7}+2\sqrt[7]{a}\,\sqrt[7]{b}\,x\sin\left(\frac{\pi}{14}\right)+b^{2/7}x^2\right)}{7a^{6/7}\sqrt[7]{b}}+\frac{\cos\left(\frac{\pi}{7}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^7)^(-1),x]
```

```
[Out] (2*ArcTan[(b^(1/7)*x*Sec[Pi/14])/a^(1/7) + Tan[Pi/14]]*Cos[Pi/14])/(7*a^(6/
7)*b^(1/7)) + (2*ArcTan[(b^(1/7)*x*Sec[(3*Pi)/14])/a^(1/7) - Tan[(3*Pi)/14]
]*Cos[(3*Pi)/14])/(7*a^(6/7)*b^(1/7)) - Log[a^(1/7) - b^(1/7)*x]/(7*a^(6/7)
*b^(1/7)) + (Cos[Pi/7]*Log[a^(2/7) + b^(2/7)*x^2 + 2*a^(1/7)*b^(1/7)*x*Cos[
Pi/7]])/(7*a^(6/7)*b^(1/7)) + (Log[a^(2/7) + b^(2/7)*x^2 + 2*a^(1/7)*b^(1/7
)*x*Sin[Pi/14]]*Sin[Pi/14])/(7*a^(6/7)*b^(1/7)) + (2*ArcTan[Cot[Pi/7] + (b^
(1/7)*x*Csc[Pi/7])/a^(1/7)]*Sin[Pi/7])/(7*a^(6/7)*b^(1/7)) - (Log[a^(2/7) +
 b^(2/7)*x^2 - 2*a^(1/7)*b^(1/7)*x*Sin[(3*Pi)/14]]*Sin[(3*Pi)/14])/(7*a^(6/
7)*b^(1/7))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r + s*Cos[(
(2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]; (r
*Int[1/(r - s*x), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 1)/2}],
x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 3)/2, 0] && NegQ[a/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{a-bx^7}\,dx = \frac{2\int \frac{\sqrt[7]{a}+\sqrt[7]{b}\,x\cos\left(\frac{\pi}{7}\right)}{a^{2/7}+b^{2/7}x^2+2\sqrt[7]{a}\,\sqrt[7]{b}\,x\cos\left(\frac{\pi}{7}\right)}\,dx}{7a^{6/7}} + \frac{2\int \frac{\sqrt[7]{a}+\sqrt[7]{b}\,x\sin\left(\frac{\pi}{14}\right)}{a^{2/7}+b^{2/7}x^2+2\sqrt[7]{a}\,\sqrt[7]{b}\,x\sin\left(\frac{\pi}{14}\right)}\,dx}{7a^{6/7}} + \frac{2\int \frac{\sqrt[7]{a}-\sqrt[7]{b}\,x\sin\left(\frac{3\pi}{14}\right)}{a^{2/7}+b^{2/7}x^2-2\sqrt[7]{a}\,\sqrt[7]{b}\,x\sin\left(\frac{3\pi}{14}\right)}}{7a^{6/7}}$$

$$= -\frac{\log\left(\sqrt[7]{a}-\sqrt[7]{b}\,x\right)}{7a^{6/7}\sqrt[7]{b}} + \frac{\left(2\cos^2\left(\frac{\pi}{14}\right)\right)\int \frac{1}{a^{2/7}+b^{2/7}x^2+2\sqrt[7]{a}\,\sqrt[7]{b}\,x\sin\left(\frac{\pi}{14}\right)}\,dx}{7a^{5/7}} + \frac{\cos\left(\frac{\pi}{7}\right)\int \frac{2b^{2/7}x+2\sqrt[7]{a}\,\sqrt[7]{b}\cos\left(}{a^{2/7}+b^{2/7}x^2+2\sqrt[7]{a}\,\sqrt[7]{b}\,x\,c}}{7a^{6/7}\sqrt[7]{b}}$$

$$= -\frac{\log\left(\sqrt[7]{a}-\sqrt[7]{b}\,x\right)}{7a^{6/7}\sqrt[7]{b}} + \frac{\cos\left(\frac{\pi}{7}\right)\log\left(a^{2/7}+b^{2/7}x^2+2\sqrt[7]{a}\,\sqrt[7]{b}\,x\cos\left(\frac{\pi}{7}\right)\right)}{7a^{6/7}\sqrt[7]{b}} + \frac{\log\left(a^{2/7}+b^{2/7}x^2+2\sqrt[7]{a}\right.}{7a^{6/7}}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt[7]{b}\,x\sec\left(\frac{\pi}{14}\right)}{\sqrt[7]{a}}+\tan\left(\frac{\pi}{14}\right)\right)\cos\left(\frac{\pi}{14}\right)}{7a^{6/7}\sqrt[7]{b}} + \frac{2\tan^{-1}\left(\frac{\sqrt[7]{b}\,x\sec\left(\frac{3\pi}{14}\right)}{\sqrt[7]{a}}-\tan\left(\frac{3\pi}{14}\right)\right)\cos\left(\frac{3\pi}{14}\right)}{7a^{6/7}\sqrt[7]{b}} - \frac{\log\left(\sqrt[7]{a}\right.}{7a^{6/7}}$$

Mathematica [A] time = 0.30, size = 263, normalized size = 0.79

$$\frac{-\sin\left(\frac{3\pi}{14}\right)\log\left(a^{2/7}-2\sqrt[7]{a}\,\sqrt[7]{b}\,x\sin\left(\frac{3\pi}{14}\right)+b^{2/7}x^2\right)+\sin\left(\frac{\pi}{14}\right)\log\left(a^{2/7}+2\sqrt[7]{a}\,\sqrt[7]{b}\,x\sin\left(\frac{\pi}{14}\right)+b^{2/7}x^2\right)+\cos\left(\frac{\pi}{7}\right)\log}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^7)^(-1),x]
```

```
[Out] (2*ArcTan[(b^(1/7)*x*Sec[Pi/14])/a^(1/7) + Tan[Pi/14]]*Cos[Pi/14] + 2*ArcTa
n[(b^(1/7)*x*Sec[(3*Pi)/14])/a^(1/7) - Tan[(3*Pi)/14]]*Cos[(3*Pi)/14] - Log
[a^(1/7) - b^(1/7)*x] + Cos[Pi/7]*Log[a^(2/7) + b^(2/7)*x^2 + 2*a^(1/7)*b^(
1/7)*x*Cos[Pi/7]] + Log[a^(2/7) + b^(2/7)*x^2 + 2*a^(1/7)*b^(1/7)*x*Sin[Pi/
14]]*Sin[Pi/14] + 2*ArcTan[Cot[Pi/7] + (b^(1/7)*x*Csc[Pi/7])/a^(1/7)]*Sin[P
i/7] - Log[a^(2/7) + b^(2/7)*x^2 - 2*a^(1/7)*b^(1/7)*x*Sin[(3*Pi)/14]]*Sin[
(3*Pi)/14])/(7*a^(6/7)*b^(1/7))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^7+a),x, algorithm="fricas")
```

```
[Out] Exception raised: RuntimeError >> no explicit roots found
```

giac [A] time = 0.18, size = 290, normalized size = 0.87

$$\frac{\left(\frac{a}{b}\right)^{\frac{1}{7}}\cos\left(\frac{3}{7}\pi\right)\log\left(2x\left(\frac{a}{b}\right)^{\frac{1}{7}}\cos\left(\frac{3}{7}\pi\right)+x^2+\left(\frac{a}{b}\right)^{\frac{2}{7}}\right)}{7a}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{7}}\cos\left(\frac{2}{7}\pi\right)\log\left(-2x\left(\frac{a}{b}\right)^{\frac{1}{7}}\cos\left(\frac{2}{7}\pi\right)+x^2+\left(\frac{a}{b}\right)^{\frac{2}{7}}\right)}{7a}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^7+a),x, algorithm="giac")
```

```
[Out] 1/7*(a/b)^(1/7)*cos(3/7*pi)*log(2*x*(a/b)^(1/7)*cos(3/7*pi) + x^2 + (a/b)^(
2/7))/a - 1/7*(a/b)^(1/7)*cos(2/7*pi)*log(-2*x*(a/b)^(1/7)*cos(2/7*pi) + x^
2 + (a/b)^(2/7))/a + 1/7*(a/b)^(1/7)*cos(1/7*pi)*log(2*x*(a/b)^(1/7)*cos(1/
7*pi) + x^2 + (a/b)^(2/7))/a + 2/7*(a/b)^(1/7)*arctan(((a/b)^(1/7)*cos(3/7*
pi) + x)/((a/b)^(1/7)*sin(3/7*pi)))*sin(3/7*pi)/a + 2/7*(a/b)^(1/7)*arctan(
-((a/b)^(1/7)*cos(2/7*pi) - x)/((a/b)^(1/7)*sin(2/7*pi)))*sin(2/7*pi)/a + 2
/7*(a/b)^(1/7)*arctan(((a/b)^(1/7)*cos(1/7*pi) + x)/((a/b)^(1/7)*sin(1/7*pi
)))*sin(1/7*pi)/a - 1/7*(a/b)^(1/7)*log(abs(x - (a/b)^(1/7)))/a
```

maple [C] time = 0.06, size = 29, normalized size = 0.09

$$-\frac{\ln\left(-\operatorname{RootOf}\left(b_Z^7-a\right)+x\right)}{7b\operatorname{RootOf}\left(b_Z^7-a\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^7+a),x)
```

```
[Out] -1/7/b*sum(1/_R^6*ln(-_R+x),_R=RootOf(_Z^7*b-a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{1}{bx^7-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^7+a),x, algorithm="maxima")
```

```
[Out] -integrate(1/(b*x^7 - a), x)
```

mupad [B] time = 1.89, size = 246, normalized size = 0.73

$$\frac{\ln\left(a^{1/7}(-b)^{41/7}+b^6x\right)}{7a^{6/7}(-b)^{1/7}}-\frac{e^{\frac{\pi 1i}{7}}\ln\left(b^6x-a^{1/7}(-b)^{41/7}e^{\frac{\pi 1i}{7}}\right)}{7a^{6/7}(-b)^{1/7}}+\frac{e^{\frac{\pi 2i}{7}}\ln\left(b^6x+a^{1/7}(-b)^{41/7}e^{\frac{\pi 2i}{7}}\right)}{7a^{6/7}(-b)^{1/7}}-\frac{e^{\frac{\pi 3i}{7}}\ln\left(b^6x-\right.}{7a^{6/7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*x^7),x)
```

```
[Out] log(a^(1/7)*(-b)^(41/7) + b^6*x)/(7*a^(6/7)*(-b)^(1/7)) - (exp((pi*1i)/7)*l
og(b^6*x - a^(1/7)*(-b)^(41/7)*exp((pi*1i)/7)))/(7*a^(6/7)*(-b)^(1/7)) + (e
xp((pi*2i)/7)*log(b^6*x + a^(1/7)*(-b)^(41/7)*exp((pi*2i)/7)))/(7*a^(6/7)*(
-b)^(1/7)) - (exp((pi*3i)/7)*log(b^6*x - a^(1/7)*(-b)^(41/7)*exp((pi*3i)/7)
```

```
))/(7*a^(6/7)*(-b)^(1/7)) + (exp((pi*4i)/7)*log(b^6*x + a^(1/7)*(-b)^(41/7)
*exp((pi*4i)/7)))/(7*a^(6/7)*(-b)^(1/7)) - (exp((pi*5i)/7)*log(b^6*x - a^(1
/7)*(-b)^(41/7)*exp((pi*5i)/7)))/(7*a^(6/7)*(-b)^(1/7)) + (exp((pi*6i)/7)*l
og(b^6*x + a^(1/7)*(-b)^(41/7)*exp((pi*6i)/7)))/(7*a^(6/7)*(-b)^(1/7))
```

sympy [**A**] time = 0.22, size = 22, normalized size = 0.07

$$-\operatorname{RootSum}\left(823543t^7a^6b-1,\left(t\mapsto t\log\left(-7ta+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**7+a),x)
```

```
[Out] -RootSum(823543*_t**7*a**6*b - 1, Lambda(_t, _t*log(-7*_t*a + x)))
```

3.1448 $\int \frac{1}{x\left(a-bx^7\right)}\,dx$

Optimal. Leaf size=23

$$\frac{\log(x)}{a}-\frac{\log\left(a-bx^7\right)}{7a}$$

```
[Out] ln(x)/a-1/7*ln(-b*x^7+a)/a
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {266, 36, 29, 31}

$$\frac{\log(x)}{a}-\frac{\log\left(a-bx^7\right)}{7a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a - b*x^7)),x]
```

```
[Out] Log[x]/a - Log[a - b*x^7]/(7*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a-bx^7\right)}\,dx &= \frac{1}{7}\operatorname{Subst}\left(\int \frac{1}{x(a-bx)}\,dx, x, x^7\right)\\ &= \frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, x^7\right)}{7a}+\frac{b\operatorname{Subst}\left(\int \frac{1}{a-bx}\,dx, x, x^7\right)}{7a}\\ &= \frac{\log(x)}{a}-\frac{\log\left(a-bx^7\right)}{7a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{\log(x)}{a}-\frac{\log\left(a-bx^7\right)}{7a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a - b*x^7)),x]
[Out] Log[x]/a - Log[a - b*x^7]/(7*a)
```

fricas [A] time = 0.48, size = 20, normalized size = 0.87

$$-\frac{\log\left(bx^7-a\right)-7\,\log(x)}{7\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^7+a),x, algorithm="fricas")
[Out] -1/7*(log(b*x^7 - a) - 7*log(x))/a
```

giac [A] time = 0.15, size = 24, normalized size = 1.04

$$-\frac{\log\left(\left|bx^7-a\right|\right)}{7\,a}+\frac{\log\left(|x|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^7+a),x, algorithm="giac")
[Out] -1/7*log(abs(b*x^7 - a))/a + log(abs(x))/a
```

maple [A] time = 0.00, size = 23, normalized size = 1.00

$$\frac{\ln(x)}{a}-\frac{\ln\left(b\,x^7-a\right)}{7a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-b*x^7+a),x)
[Out] -1/7/a*ln(b*x^7-a)+1/a*ln(x)
```

maxima [A] time = 1.01, size = 25, normalized size = 1.09

$$-\frac{\log\left(bx^7-a\right)}{7\,a}+\frac{\log\left(x^7\right)}{7\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x^7+a),x, algorithm="maxima")
[Out] -1/7*log(b*x^7 - a)/a + 1/7*log(x^7)/a
```

mupad [B] time = 0.10, size = 21, normalized size = 0.91

$$\frac{\ln(x)}{a}-\frac{\ln\left(a-b\,x^7\right)}{7\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a - b*x^7)),x)
[Out] log(x)/a - log(a - b*x^7)/(7*a)
```

sympy [A] time = 0.63, size = 15, normalized size = 0.65

$$\frac{\log(x)}{a}-\frac{\log\left(-\frac{a}{b}+x^7\right)}{7a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b*x**7+a),x)
[Out] log(x)/a - log(-a/b + x**7)/(7*a)
```

3.1449 $\int \frac{1}{1-x^7}\,dx$

Optimal. Leaf size=166

$$-\frac{1}{7}\sin\left(\frac{3\pi}{14}\right)\log\left(x^2-2x\sin\left(\frac{3\pi}{14}\right)+1\right)+\frac{1}{7}\sin\left(\frac{\pi}{14}\right)\log\left(x^2+2x\sin\left(\frac{\pi}{14}\right)+1\right)+\frac{1}{7}\cos\left(\frac{\pi}{7}\right)\log\left(x^2+2x\cos\left(\frac{\pi}{7}\right)\right.$$

```
[Out] 2/7*arctan(sec(1/14*Pi)*(x+sin(1/14*Pi)))*cos(1/14*Pi)+2/7*arctan(sec(3/14*
Pi)*(x-sin(3/14*Pi)))*cos(3/14*Pi)-1/7*ln(1-x)+1/7*cos(1/7*Pi)*ln(1+x^2+2*x
*cos(1/7*Pi))+1/7*ln(1+x^2+2*x*sin(1/14*Pi))*sin(1/14*Pi)+2/7*arctan((x+cos
(1/7*Pi))*csc(1/7*Pi))*sin(1/7*Pi)-1/7*ln(1+x^2-2*x*sin(3/14*Pi))*sin(3/14*
Pi)
```

Rubi [A] time = 0.14, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {202, 634, 618, 204, 628, 31}

$$-\frac{1}{7}\sin\left(\frac{3\pi}{14}\right)\log\left(x^2-2x\sin\left(\frac{3\pi}{14}\right)+1\right)+\frac{1}{7}\sin\left(\frac{\pi}{14}\right)\log\left(x^2+2x\sin\left(\frac{\pi}{14}\right)+1\right)+\frac{1}{7}\cos\left(\frac{\pi}{7}\right)\log\left(x^2+2x\cos\left(\frac{\pi}{7}\right)\right.$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^7)^(-1),x]
```

```
[Out] (2*ArcTan[Sec[Pi/14]*(x + Sin[Pi/14])]*Cos[Pi/14])/7 + (2*ArcTan[Sec[(3*Pi)
/14]*(x - Sin[(3*Pi)/14])]*Cos[(3*Pi)/14])/7 - Log[1 - x]/7 + (Cos[Pi/7]*Lo
g[1 + x^2 + 2*x*Cos[Pi/7]])/7 + (Log[1 + x^2 + 2*x*Sin[Pi/14]]*Sin[Pi/14])/
7 + (2*ArcTan[(x + Cos[Pi/7])*Csc[Pi/7]]*Sin[Pi/7])/7 - (Log[1 + x^2 - 2*x*
Sin[(3*Pi)/14]]*Sin[(3*Pi)/14])/7
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r + s*Cos[(
(2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]; (r
*Int[1/(r - s*x), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 1)/2}],
x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 3)/2, 0] && NegQ[a/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{1-x^7}\,dx = \frac{2}{7}\int \frac{1+x\cos\left(\frac{\pi}{7}\right)}{1+x^2+2x\cos\left(\frac{\pi}{7}\right)}\,dx + \frac{2}{7}\int \frac{1+x\sin\left(\frac{\pi}{14}\right)}{1+x^2+2x\sin\left(\frac{\pi}{14}\right)}\,dx + \frac{2}{7}\int \frac{1-x\sin\left(\frac{3\pi}{14}\right)}{1+x^2-2x\sin\left(\frac{3\pi}{14}\right)}\,dx + \frac{1}{7}$$

$$= -\frac{1}{7}\log(1-x) + \frac{1}{7}\left(2\cos^2\left(\frac{\pi}{14}\right)\right)\int \frac{1}{1+x^2+2x\sin\left(\frac{\pi}{14}\right)}\,dx + \frac{1}{7}\cos\left(\frac{\pi}{7}\right)\int \frac{2x+2\cos\left(\frac{\pi}{7}\right)}{1+x^2+2x\cos\left(\frac{\pi}{7}\right)}$$

$$= -\frac{1}{7}\log(1-x) + \frac{1}{7}\cos\left(\frac{\pi}{7}\right)\log\left(1+x^2+2x\cos\left(\frac{\pi}{7}\right)\right) + \frac{1}{7}\log\left(1+x^2+2x\sin\left(\frac{\pi}{14}\right)\right)\sin\left(\frac{\pi}{14}\right)$$

$$= \frac{2}{7}\tan^{-1}\left(\sec\left(\frac{\pi}{14}\right)\left(x+\sin\left(\frac{\pi}{14}\right)\right)\right)\cos\left(\frac{\pi}{14}\right) + \frac{2}{7}\tan^{-1}\left(\sec\left(\frac{3\pi}{14}\right)\left(x-\sin\left(\frac{3\pi}{14}\right)\right)\right)\cos\left(\frac{3\pi}{14}\right) -$$

Mathematica [A] time = 0.00, size = 166, normalized size = 1.00

$$-\frac{1}{7}\sin\left(\frac{3\pi}{14}\right)\log\left(x^2-2x\sin\left(\frac{3\pi}{14}\right)+1\right)+\frac{1}{7}\sin\left(\frac{\pi}{14}\right)\log\left(x^2+2x\sin\left(\frac{\pi}{14}\right)+1\right)+\frac{1}{7}\cos\left(\frac{\pi}{7}\right)\log\left(x^2+2x\cos\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^7)^(-1),x]
```

```
[Out] (2*ArcTan[Sec[Pi/14]*(x + Sin[Pi/14])]*Cos[Pi/14])/7 + (2*ArcTan[Sec[(3*Pi)
/14]*(x - Sin[(3*Pi)/14])]*Cos[(3*Pi)/14])/7 - Log[1 - x]/7 + (Cos[Pi/7]*Lo
g[1 + x^2 + 2*x*Cos[Pi/7]])/7 + (Log[1 + x^2 + 2*x*Sin[Pi/14]]*Sin[Pi/14])/
7 + (2*ArcTan[(x + Cos[Pi/7])*Csc[Pi/7]]*Sin[Pi/7])/7 - (Log[1 + x^2 - 2*x*
Sin[(3*Pi)/14]]*Sin[(3*Pi)/14])/7
```

fricas [C] time = 10.96, size = 65, normalized size = 0.39

$$\frac{1}{14}\left(\sqrt{-0.7530203962825330? + 0.?e - 36\sqrt{-1}} + 1.801937735804839? + 0.?e - 36\sqrt{-1}\right)\log\left(2\,x + \sqrt{-0.753}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^7+1),x, algorithm="fricas")
```

```
[Out] 1/14*(sqrt(-0.7530203962825330? + 0.?e-36*I) + 1.801937735804839? + 0.?e-36
*I)*log(2*x + sqrt(-0.7530203962825330? + 0.?e-36*I) + 1.801937735804839? +
 0.?e-36*I) - 1/14*(sqrt(-0.7530203962825330? + 0.?e-36*I) - 1.801937735804
839? + 0.?e-36*I)*log(2*x - sqrt(-0.7530203962825330? + 0.?e-36*I) + 1.8019
37735804839? + 0.?e-36*I) + (0.03178870485090206? + 0.1392754160259748?*I)*
log(x + 0.2225209339563144? + 0.9749279121818236?*I) + (0.03178870485090206
? - 0.1392754160259748?*I)*log(x + 0.2225209339563144? - 0.9749279121818236
?*I) - (0.08906997169410479? - 0.11169021178114711?*I)*log(x - 0.6234898018
587335? + 0.7818314824680299?*I) - (0.08906997169410479? + 0.11169021178114
711?*I)*log(x - 0.6234898018587335? - 0.7818314824680299?*I) - 1/7*log(x -
1)
```

giac [A] time = 0.16, size = 127, normalized size = 0.77

$$\frac{1}{7}\cos\left(\frac{3}{7}\pi\right)\log\left(x^2+2x\cos\left(\frac{3}{7}\pi\right)+1\right)-\frac{1}{7}\cos\left(\frac{2}{7}\pi\right)\log\left(x^2-2x\cos\left(\frac{2}{7}\pi\right)+1\right)+\frac{1}{7}\cos\left(\frac{1}{7}\pi\right)\log\left(x^2+2x\,c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^7+1),x, algorithm="giac")
```

```
[Out] 1/7*cos(3/7*pi)*log(x^2 + 2*x*cos(3/7*pi) + 1) - 1/7*cos(2/7*pi)*log(x^2 -
2*x*cos(2/7*pi) + 1) + 1/7*cos(1/7*pi)*log(x^2 + 2*x*cos(1/7*pi) + 1) + 2/7
*arctan((x + cos(3/7*pi))/sin(3/7*pi))*sin(3/7*pi) + 2/7*arctan((x - cos(2/
7*pi))/sin(2/7*pi))*sin(2/7*pi) + 2/7*arctan((x + cos(1/7*pi))/sin(1/7*pi))
*sin(1/7*pi) - 1/7*log(abs(x - 1))
```

maple [C] time = 0.01, size = 89, normalized size = 0.54

$$-\frac{\ln(x-1)}{7}+\frac{\left(\text{RootOf}\left(_Z^6+_Z^5+_Z^4+_Z^3+_Z^2+_Z+1\right)^5+2\,\text{RootOf}\left(_Z^6+_Z^5+_Z^4+_Z^3+_Z^2+\right.\right.}{42\,\text{RootOf}\left(_Z^6+_Z^5+_Z^4+_Z^3+_Z^2+_Z+1\right)^5+35\,\text{RootOf}\left(_Z\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^7+1),x)
```

```
[Out] -1/7*ln(x-1)+1/7*sum((_R^5+2*_R^4+3*_R^3+4*_R^2+5*_R+6)/(6*_R^5+5*_R^4+4*_R
^3+3*_R^2+2*_R+1)*ln(-_R+x),_R=RootOf(_Z^6+_Z^5+_Z^4+_Z^3+_Z^2+_Z+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{7}\int\frac{x^5+2x^4+3x^3+4x^2+5x+6}{x^6+x^5+x^4+x^3+x^2+x+1}\,dx-\frac{1}{7}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^7+1),x, algorithm="maxima")
```

```
[Out] 1/7*integrate((x^5 + 2*x^4 + 3*x^3 + 4*x^2 + 5*x + 6)/(x^6 + x^5 + x^4 + x^
3 + x^2 + x + 1), x) - 1/7*log(x - 1)
```

mupad [B] time = 1.37, size = 103, normalized size = 0.62

$$-\frac{\ln(x-1)}{7}-\frac{e^{\frac{\pi 2i}{7}}\ln\left(x-e^{\frac{\pi 2i}{7}}\right)}{7}-\frac{e^{\frac{\pi 4i}{7}}\ln\left(x-e^{\frac{\pi 4i}{7}}\right)}{7}-\frac{e^{\frac{\pi 6i}{7}}\ln\left(x-e^{\frac{\pi 6i}{7}}\right)}{7}+\frac{\ln\left(x+e^{\frac{\pi 1i}{7}}\right)e^{\frac{\pi 1i}{7}}}{7}+\frac{\ln\left(x+e^{\frac{\pi 3i}{7}}\right)e^{\frac{\pi 3i}{7}}}{7}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^7 - 1),x)
```

```
[Out] (log(x + exp((pi*1i)/7))*exp((pi*1i)/7))/7 - (exp((pi*2i)/7)*log(x - exp((p
i*2i)/7)))/7 - (exp((pi*4i)/7)*log(x - exp((pi*4i)/7)))/7 - (exp((pi*6i)/7)
*log(x - exp((pi*6i)/7)))/7 - log(x - 1)/7 + (log(x + exp((pi*3i)/7))*exp((
pi*3i)/7))/7 + (log(x + exp((pi*5i)/7))*exp((pi*5i)/7))/7
```

sympy [A] time = 0.35, size = 46, normalized size = 0.28

$$-\frac{\log(x-1)}{7}-\text{RootSum}\left(117649t^6+16807t^5+2401t^4+343t^3+49t^2+7t+1,\left(t\mapsto t\log(-7t+x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**7+1),x)
```

```
[Out] -log(x - 1)/7 - RootSum(117649*_t**6 + 16807*_t**5 + 2401*_t**4 + 343*_t**3
 + 49*_t**2 + 7*_t + 1, Lambda(_t, _t*log(-7*_t + x)))
```

3.1450 $\int \frac{1}{1+x^7}\,dx$

Optimal. Leaf size=165

$$\frac{1}{7}\sin\left(\frac{3\pi}{14}\right)\log\left(x^2+2x\sin\left(\frac{3\pi}{14}\right)+1\right)-\frac{1}{7}\sin\left(\frac{\pi}{14}\right)\log\left(x^2-2x\sin\left(\frac{\pi}{14}\right)+1\right)-\frac{1}{7}\cos\left(\frac{\pi}{7}\right)\log\left(x^2-2x\cos\left(\frac{\pi}{7}\right)\right.$$

```
[Out] 2/7*arctan(x*sec(1/14*Pi)-tan(1/14*Pi))*cos(1/14*Pi)+2/7*arctan(x*sec(3/14*
Pi)+tan(3/14*Pi))*cos(3/14*Pi)+1/7*ln(1+x)-1/7*cos(1/7*Pi)*ln(1+x^2-2*x*cos
(1/7*Pi))-1/7*ln(1+x^2-2*x*sin(1/14*Pi))*sin(1/14*Pi)+2/7*arctan(-cot(1/7*P
i)+x*csc(1/7*Pi))*sin(1/7*Pi)+1/7*ln(1+x^2+2*x*sin(3/14*Pi))*sin(3/14*Pi)
```

Rubi [A] time = 0.14, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {201, 634, 618, 204, 628, 31}

$$\frac{1}{7}\sin\left(\frac{3\pi}{14}\right)\log\left(x^2+2x\sin\left(\frac{3\pi}{14}\right)+1\right)-\frac{1}{7}\sin\left(\frac{\pi}{14}\right)\log\left(x^2-2x\sin\left(\frac{\pi}{14}\right)+1\right)-\frac{1}{7}\cos\left(\frac{\pi}{7}\right)\log\left(x^2-2x\cos\left(\frac{\pi}{7}\right)\right.$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^7)^(-1),x]
```

```
[Out] (2*ArcTan[x*Sec[Pi/14] - Tan[Pi/14]]*Cos[Pi/14])/7 + (2*ArcTan[x*Sec[(3*Pi)
/14] + Tan[(3*Pi)/14]]*Cos[(3*Pi)/14])/7 + Log[1 + x]/7 - (Cos[Pi/7]*Log[1
+ x^2 - 2*x*Cos[Pi/7]])/7 - (Log[1 + x^2 - 2*x*Sin[Pi/14]]*Sin[Pi/14])/7 -
(2*ArcTan[Cot[Pi/7] - x*Csc[Pi/7]]*Sin[Pi/7])/7 + (Log[1 + x^2 + 2*x*Sin[(3
*Pi)/14]]*Sin[(3*Pi)/14])/7
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[((2*k -
 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]; (r*Int[1
/(r + s*x), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 1)/2}], x], x]
] /; FreeQ[{a, b}, x] && IGtQ[(n - 3)/2, 0] && PosQ[a/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{1+x^7}\,dx = \frac{2}{7}\int \frac{1-x\cos\left(\frac{\pi}{7}\right)}{1+x^2-2x\cos\left(\frac{\pi}{7}\right)}\,dx + \frac{2}{7}\int \frac{1-x\sin\left(\frac{\pi}{14}\right)}{1+x^2-2x\sin\left(\frac{\pi}{14}\right)}\,dx + \frac{2}{7}\int \frac{1+x\sin\left(\frac{3\pi}{14}\right)}{1+x^2+2x\sin\left(\frac{3\pi}{14}\right)}\,dx + \frac{1}{7}\int$$

$$= \frac{1}{7}\log(1+x) + \frac{1}{7}\left(2\cos^2\left(\frac{\pi}{14}\right)\right)\int \frac{1}{1+x^2-2x\sin\left(\frac{\pi}{14}\right)}\,dx - \frac{1}{7}\cos\left(\frac{\pi}{7}\right)\int \frac{2x-2\cos\left(\frac{\pi}{7}\right)}{1+x^2-2x\cos\left(\frac{\pi}{7}\right)}\,dx$$

$$= \frac{1}{7}\log(1+x) - \frac{1}{7}\cos\left(\frac{\pi}{7}\right)\log\left(1+x^2-2x\cos\left(\frac{\pi}{7}\right)\right) - \frac{1}{7}\log\left(1+x^2-2x\sin\left(\frac{\pi}{14}\right)\right)\sin\left(\frac{\pi}{14}\right) + \frac{1}{7}$$

$$= \frac{2}{7}\tan^{-1}\left(\sec\left(\frac{\pi}{14}\right)\left(x-\sin\left(\frac{\pi}{14}\right)\right)\right)\cos\left(\frac{\pi}{14}\right) + \frac{2}{7}\tan^{-1}\left(\sec\left(\frac{3\pi}{14}\right)\left(x+\sin\left(\frac{3\pi}{14}\right)\right)\right)\cos\left(\frac{3\pi}{14}\right) + \frac{1}{7}$$

Mathematica [A] time = 0.00, size = 166, normalized size = 1.01

$$\frac{1}{7}\sin\left(\frac{3\pi}{14}\right)\log\left(x^2+2x\sin\left(\frac{3\pi}{14}\right)+1\right) - \frac{1}{7}\sin\left(\frac{\pi}{14}\right)\log\left(x^2-2x\sin\left(\frac{\pi}{14}\right)+1\right) - \frac{1}{7}\cos\left(\frac{\pi}{7}\right)\log\left(x^2-2x\cos\left(\frac{\pi}{7}\right)+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^7)^(-1),x]
```

```
[Out] (2*ArcTan[Sec[Pi/14]*(x - Sin[Pi/14])]*Cos[Pi/14])/7 + (2*ArcTan[Sec[(3*Pi)
/14]*(x + Sin[(3*Pi)/14])]*Cos[(3*Pi)/14])/7 + Log[1 + x]/7 - (Cos[Pi/7]*Lo
g[1 + x^2 - 2*x*Cos[Pi/7]])/7 - (Log[1 + x^2 - 2*x*Sin[Pi/14]]*Sin[Pi/14])/
7 + (2*ArcTan[(x - Cos[Pi/7])*Csc[Pi/7]]*Sin[Pi/7])/7 + (Log[1 + x^2 + 2*x*
Sin[(3*Pi)/14]]*Sin[(3*Pi)/14])/7
```

fricas [C] time = 11.36, size = 65, normalized size = 0.39

$$\frac{1}{14}\left(\sqrt{-2.445041867912629? + 0.?e - 37\sqrt{-1}}\ + 1.246979603717467? + 0.?e - 36\sqrt{-1}\right)\log\left(2x + \sqrt{-2.445041\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^7+1),x, algorithm="fricas")
```

```
[Out] 1/14*(sqrt(-2.445041867912629? + 0.?e-37*I) + 1.246979603717467? + 0.?e-36*
I)*log(2*x + sqrt(-2.445041867912629? + 0.?e-37*I) + 1.246979603717467? + 0
.?e-36*I) - 1/14*(sqrt(-2.445041867912629? + 0.?e-37*I) - 1.246979603717467
? + 0.?e-36*I)*log(2*x - sqrt(-2.445041867912629? + 0.?e-37*I) + 1.24697960
3717467? + 0.?e-36*I) + 1/7*log(x + 1) - (0.03178870485090206? - 0.13927541
60259748?*I)*log(x - 0.2225209339563144? + 0.9749279121818236?*I) - (0.0317
8870485090206? + 0.1392754160259748?*I)*log(x - 0.2225209339563144? - 0.974
9279121818236?*I) - (0.1287098382717742? - 0.0619833913025083 1?*I)*log(x -
0.9009688679024191? + 0.4338837391175581?*I) - (0.1287098382717742? + 0.061
98339130250831?*I)*log(x - 0.9009688679024191? - 0.4338837391175582?*I)
```

giac [A] time = 0.16, size = 129, normalized size = 0.78

$$-\frac{1}{7}\cos\left(\frac{3}{7}\pi\right)\log\left(x^2-2x\cos\left(\frac{3}{7}\pi\right)+1\right)+\frac{1}{7}\cos\left(\frac{2}{7}\pi\right)\log\left(x^2+2x\cos\left(\frac{2}{7}\pi\right)+1\right)-\frac{1}{7}\cos\left(\frac{1}{7}\pi\right)\log\left(x^2-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^7+1),x, algorithm="giac")
```

```
[Out] -1/7*cos(3/7*pi)*log(x^2 - 2*x*cos(3/7*pi) + 1) + 1/7*cos(2/7*pi)*log(x^2 +
 2*x*cos(2/7*pi) + 1) - 1/7*cos(1/7*pi)*log(x^2 - 2*x*cos(1/7*pi) + 1) + 2/
7*arctan((x - cos(3/7*pi))/sin(3/7*pi))*sin(3/7*pi) + 2/7*arctan((x + cos(2
/7*pi))/sin(2/7*pi))*sin(2/7*pi) + 2/7*arctan((x - cos(1/7*pi))/sin(1/7*pi)
)*sin(1/7*pi) + 1/7*log(abs(x + 1))
```

maple [C] time = 0.01, size = 97, normalized size = 0.59

$$\frac{\ln(x+1)}{7}+\frac{\left(-\operatorname{RootOf}\left(_Z^6-_Z^5+_Z^4-_Z^3+_Z^2-_Z+1\right)^5+2\operatorname{RootOf}\left(_Z^6-_Z^5+_Z^4-_Z^3+_Z^2\right.\right.}{42\operatorname{RootOf}\left(_Z^6-_Z^5+_Z^4-_Z^3+_Z^2-_Z+1\right)^5-35\operatorname{RootOf}\left(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7+1),x)
```

```
[Out] 1/7*sum((-_R^5+2*_R^4-3*_R^3+4*_R^2-5*_R+6)/(6*_R^5-5*_R^4+4*_R^3-3*_R^2+2*
_R-1)*ln(-_R+x),_R=RootOf(_Z^6-_Z^5+_Z^4-_Z^3+_Z^2-_Z+1))+1/7*ln(x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{7}\int\frac{x^5-2x^4+3x^3-4x^2+5x-6}{x^6-x^5+x^4-x^3+x^2-x+1}\,dx+\frac{1}{7}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^7+1),x, algorithm="maxima")
```

```
[Out] -1/7*integrate((x^5 - 2*x^4 + 3*x^3 - 4*x^2 + 5*x - 6)/(x^6 - x^5 + x^4 - x
^3 + x^2 - x + 1), x) + 1/7*log(x + 1)
```

mupad [B] time = 1.34, size = 103, normalized size = 0.62

$$\frac{\ln(x+1)}{7}-\frac{e^{\frac{\pi 1i}{7}}\ln\left(x-e^{\frac{\pi 1i}{7}}\right)}{7}-\frac{e^{\frac{\pi 3i}{7}}\ln\left(x-e^{\frac{\pi 3i}{7}}\right)}{7}-\frac{e^{\frac{\pi 5i}{7}}\ln\left(x-e^{\frac{\pi 5i}{7}}\right)}{7}+\frac{\ln\left(x+e^{\frac{\pi 2i}{7}}\right)e^{\frac{\pi 2i}{7}}}{7}+\frac{\ln\left(x+e^{\frac{\pi 4i}{7}}\right)e^{\frac{\pi 4i}{7}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7 + 1),x)
```

```
[Out] log(x + 1)/7 - (exp((pi*1i)/7)*log(x - exp((pi*1i)/7)))/7 - (exp((pi*3i)/7)
*log(x - exp((pi*3i)/7)))/7 - (exp((pi*5i)/7)*log(x - exp((pi*5i)/7)))/7 +
(log(x + exp((pi*2i)/7))*exp((pi*2i)/7))/7 + (log(x + exp((pi*4i)/7))*exp((
pi*4i)/7))/7 + (log(x + exp((pi*6i)/7))*exp((pi*6i)/7))/7
```

sympy [A] time = 0.35, size = 44, normalized size = 0.27

$$\frac{\log(x+1)}{7}+\operatorname{RootSum}\left(117649t^6+16807t^5+2401t^4+343t^3+49t^2+7t+1,\left(t\mapsto t\log(7t+x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**7+1),x)
```

```
[Out] log(x + 1)/7 + RootSum(117649*_t**6 + 16807*_t**5 + 2401*_t**4 + 343*_t**3
+ 49*_t**2 + 7*_t + 1, Lambda(_t, _t*log(7*_t + x)))
```

3.1451 $\int \frac{x^9}{a+bx^8}\,dx$

Optimal. Leaf size=203

$$\frac{\sqrt[4]{a}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x^2}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,b^{5/4}}-\frac{\sqrt[4]{a}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x^2}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,b^{5/4}}+\frac{\sqrt[4]{a}\,\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^2+\sqrt{a}+\sqrt{b}\,x^4\right)}{8\sqrt{2}\,b^{5/4}}-\frac{\sqrt[4]{a}\,\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^2+\sqrt{a}+\sqrt{b}\,x^4\right)}{8\sqrt{2}\,b^{5/4}}$$

```
[Out] 1/2*x^2/b-1/8*a^(1/4)*arctan(-1+b^(1/4)*x^2*2^(1/2)/a^(1/4))/b^(5/4)*2^(1/2
)-1/8*a^(1/4)*arctan(1+b^(1/4)*x^2*2^(1/2)/a^(1/4))/b^(5/4)*2^(1/2)+1/16*a^
(1/4)*ln(-a^(1/4)*b^(1/4)*x^2*2^(1/2)+a^(1/2)+x^4*b^(1/2))/b^(5/4)*2^(1/2)-
1/16*a^(1/4)*ln(a^(1/4)*b^(1/4)*x^2*2^(1/2)+a^(1/2)+x^4*b^(1/2))/b^(5/4)*2^
(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {275, 321, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt[4]{a}\,\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^2+\sqrt{a}+\sqrt{b}\,x^4\right)}{8\sqrt{2}\,b^{5/4}}-\frac{\sqrt[4]{a}\,\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^2+\sqrt{a}+\sqrt{b}\,x^4\right)}{8\sqrt{2}\,b^{5/4}}+\frac{\sqrt[4]{a}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x^2}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,b^{5/4}}-\frac{\sqrt[4]{a}\,\text{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x^8),x]
```

```
[Out] x^2/(2*b) + (a^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x^2)/a^(1/4)])/(4*Sqrt[2]*
b^(5/4)) - (a^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x^2)/a^(1/4)])/(4*Sqrt[2]*b
^(5/4)) + (a^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x^2 + Sqrt[b]*x^4]
)/(8*Sqrt[2]*b^(5/4)) - (a^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x^2
+ Sqrt[b]*x^4])/(8*Sqrt[2]*b^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9}{a+bx^8}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{a+bx^4}\,dx, x, x^2\right) \\
&= \frac{x^2}{2b} - \frac{a\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, x^2\right)}{2b} \\
&= \frac{x^2}{2b} - \frac{\sqrt{a}\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, x^2\right)}{4b} - \frac{\sqrt{a}\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, x^2\right)}{4b} \\
&= \frac{x^2}{2b} - \frac{\sqrt{a}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}} + x^2}\,dx, x, x^2\right)}{8b^{3/2}} - \frac{\sqrt{a}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}} + x^2}\,dx, x, x^2\right)}{8b^{3/2}} + \frac{\sqrt[4]{a}\,\text{Sub}}{} \\
&= \frac{x^2}{2b} + \frac{\sqrt[4]{a}\log\left(\sqrt{a} - \sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^2 + \sqrt{b}\,x^4\right)}{8\sqrt{2}\,b^{5/4}} - \frac{\sqrt[4]{a}\log\left(\sqrt{a} + \sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^2 + \sqrt{b}\,x^4\right)}{8\sqrt{2}\,b^{5/4}} - \frac{\sqrt[4]{a}\,\text{Su}}{} \\
&= \frac{x^2}{2b} + \frac{\sqrt[4]{a}\tan^{-1}\left(1 - \frac{\sqrt{2}\,\sqrt[4]{b}\,x^2}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,b^{5/4}} - \frac{\sqrt[4]{a}\tan^{-1}\left(1 + \frac{\sqrt{2}\,\sqrt[4]{b}\,x^2}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,b^{5/4}} + \frac{\sqrt[4]{a}\log\left(\sqrt{a} - \sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^2 + \sqrt{b}\right.}{8\sqrt{2}\,b^{5/4}}
\end{aligned}$$

Mathematica [A] time = 0.33, size = 361, normalized size = 1.78

$$\frac{-\sqrt{2}\,\sqrt[4]{a}\log\left(-2\sqrt[8]{a}\,\sqrt[8]{b}\,x\sin\left(\frac{\pi}{8}\right) + \sqrt[4]{a} + \sqrt[4]{b}\,x^2\right) - \sqrt{2}\,\sqrt[4]{a}\log\left(2\sqrt[8]{a}\,\sqrt[8]{b}\,x\sin\left(\frac{\pi}{8}\right) + \sqrt[4]{a} + \sqrt[4]{b}\,x^2\right) + \sqrt{2}\,\sqrt[4]{a}\log}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x^8),x]
```

```
[Out] (8*b^(1/4)*x^2 + 2*Sqrt[2]*a^(1/4)*ArcTan[Cot[Pi/8] - (b^(1/8)*x*Csc[Pi/8])
/a^(1/8)] + 2*Sqrt[2]*a^(1/4)*ArcTan[Cot[Pi/8] + (b^(1/8)*x*Csc[Pi/8])/a^(1
/8)] - 2*Sqrt[2]*a^(1/4)*ArcTan[(b^(1/8)*x*Sec[Pi/8])/a^(1/8) - Tan[Pi/8]]
+ 2*Sqrt[2]*a^(1/4)*ArcTan[(b^(1/8)*x*Sec[Pi/8])/a^(1/8) + Tan[Pi/8]] + Sqr
t[2]*a^(1/4)*Log[a^(1/4) + b^(1/4)*x^2 - 2*a^(1/8)*b^(1/8)*x*Cos[Pi/8]] + S
qrt[2]*a^(1/4)*Log[a^(1/4) + b^(1/4)*x^2 + 2*a^(1/8)*b^(1/8)*x*Cos[Pi/8]] -
 Sqrt[2]*a^(1/4)*Log[a^(1/4) + b^(1/4)*x^2 - 2*a^(1/8)*b^(1/8)*x*Sin[Pi/8]]
 - Sqrt[2]*a^(1/4)*Log[a^(1/4) + b^(1/4)*x^2 + 2*a^(1/8)*b^(1/8)*x*Sin[Pi/8
]])/(16*b^(5/4))
```

fricas [A] time = 0.82, size = 127, normalized size = 0.63

$$-\frac{4\,b\left(-\frac{a}{b^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^4x^2\left(-\frac{a}{b^5}\right)^{\frac{3}{4}}-\sqrt{x^4+b^2\sqrt{-\frac{a}{b^5}}}\,b^4\left(-\frac{a}{b^5}\right)^{\frac{3}{4}}}{a}\right)+b\left(-\frac{a}{b^5}\right)^{\frac{1}{4}}\log\left(x^2+b\left(-\frac{a}{b^5}\right)^{\frac{1}{4}}\right)-b\left(-\frac{a}{b^5}\right)^{\frac{1}{4}}\log\left(x^2-b\left(-\frac{a}{b^5}\right.\right.}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^8+a),x, algorithm="fricas")
```

```
[Out] -1/8*(4*b*(-a/b^5)^(1/4)*arctan(-(b^4*x^2*(-a/b^5)^(3/4) - sqrt(x^4 + b^2*s
qrt(-a/b^5))*b^4*(-a/b^5)^(3/4))/a) + b*(-a/b^5)^(1/4)*log(x^2 + b*(-a/b^5)
^(1/4)) - b*(-a/b^5)^(1/4)*log(x^2 - b*(-a/b^5)^(1/4)) - 4*x^2)/b
```

giac [A] time = 0.19, size = 183, normalized size = 0.90

$$\frac{x^2}{2\,b}-\frac{\sqrt{2}\,\left(ab^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x^2+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,b^2}-\frac{\sqrt{2}\,\left(ab^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x^2-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,b^2}-\frac{\sqrt{2}\,\left(ab^3\right)^{\frac{1}{4}}\log\left(x^4+\sqrt{2}\,x^2\left(\frac{a}{b}\right)^{\frac{1}{4}}\right.}{16\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^8+a),x, algorithm="giac")
```

```
[Out] 1/2*x^2/b - 1/8*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x^2 + sqrt(2)*(
a/b)^(1/4))/(a/b)^(1/4))/b^2 - 1/8*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)
*(2*x^2 - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/b^2 - 1/16*sqrt(2)*(a*b^3)^(1/4
)*log(x^4 + sqrt(2)*x^2*(a/b)^(1/4) + sqrt(a/b))/b^2 + 1/16*sqrt(2)*(a*b^3)
^(1/4)*log(x^4 - sqrt(2)*x^2*(a/b)^(1/4) + sqrt(a/b))/b^2
```

maple [A] time = 0.01, size = 144, normalized size = 0.71

$$\frac{x^2}{2b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x^2}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x^2}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x^4+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x^2+\sqrt{\frac{a}{b}}}{x^4-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x^2+\sqrt{\frac{a}{b}}}\right)}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^8+a),x)
```

```
[Out] 1/2/b*x^2-1/16/b*(a/b)^(1/4)*2^(1/2)*ln((x^4+(a/b)^(1/4)*x^2*2^(1/2)+(a/b)^
(1/2))/(x^4-(a/b)^(1/4)*x^2*2^(1/2)+(a/b)^(1/2)))-1/8/b*(a/b)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(a/b)^(1/4)*x^2+1)-1/8/b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)
/(a/b)^(1/4)*x^2-1)
```

maxima [A] time = 2.37, size = 190, normalized size = 0.94

$$\frac{x^2}{2b} - \frac{2\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}a^{\frac{1}{4}}\log\left(\sqrt{b}x^4+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x^2+\sqrt{a}\right)}{b^{\frac{1}{4}}} - \frac{\sqrt{2}a^{\frac{1}{4}}\log}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^8+a),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/b - 1/16*(2*sqrt(2)*sqrt(a)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x^2 + sqr
t(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 2*sqrt
(2)*sqrt(a)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4))/sq
rt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + sqrt(2)*a^(1/4)*log(sqrt(b)*x^
4 + sqrt(2)*a^(1/4)*b^(1/4)*x^2 + sqrt(a))/b^(1/4) - sqrt(2)*a^(1/4)*log(sq
rt(b)*x^4 - sqrt(2)*a^(1/4)*b^(1/4)*x^2 + sqrt(a))/b^(1/4))/b
```

mupad [B] time = 1.14, size = 55, normalized size = 0.27

$$\frac{x^2}{2b} - \frac{(-a)^{1/4}\operatorname{atan}\left(\frac{b^{1/4}x^2}{(-a)^{1/4}}\right)}{4b^{5/4}} - \frac{(-a)^{1/4}\operatorname{atanh}\left(\frac{b^{1/4}x^2}{(-a)^{1/4}}\right)}{4b^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + b*x^8),x)
```

```
[Out] x^2/(2*b) - ((-a)^(1/4)*atan((b^(1/4)*x^2)/(-a)^(1/4)))/(4*b^(5/4)) - ((-a)
^(1/4)*atanh((b^(1/4)*x^2)/(-a)^(1/4)))/(4*b^(5/4))
```

sympy [A] time = 0.38, size = 27, normalized size = 0.13

$$\operatorname{RootSum}\left(4096t^4b^5 + a, \left(t \mapsto t\log\left(-8tb + x^2\right)\right)\right) + \frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**8+a),x)
```

```
[Out] RootSum(4096*_t**4*b**5 + a, Lambda(_t, _t*log(-8*_t*b + x**2))) + x**2/(2*
b)
```

3.1452 $\int \frac{x^7}{a+bx^8}\,dx$

Optimal. Leaf size=15

$$\frac{\log\left(a+bx^8\right)}{8b}$$

```
[Out] 1/8*ln(b*x^8+a)/b
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {260}

$$\frac{\log\left(a+bx^8\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^8),x]
```

```
[Out] Log[a + b*x^8]/(8*b)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^7}{a+bx^8}\,dx = \frac{\log\left(a+bx^8\right)}{8b}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log\left(a+bx^8\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^8),x]
```

```
[Out] Log[a + b*x^8]/(8*b)
```

fricas [A] time = 0.55, size = 13, normalized size = 0.87

$$\frac{\log\left(bx^8+a\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^8+a),x, algorithm="fricas")
```

```
[Out] 1/8*log(b*x^8 + a)/b
```

giac [A] time = 0.18, size = 14, normalized size = 0.93

$$\frac{\log\left(\left|bx^8+a\right|\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^8+a),x, algorithm="giac")
[Out] 1/8*log(abs(b*x^8 + a))/b
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{\ln\left(b\,x^8+a\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^8+a),x)
[Out] 1/8*ln(b*x^8+a)/b
```

maxima [A] time = 1.00, size = 13, normalized size = 0.87

$$\frac{\log\left(bx^8+a\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^8+a),x, algorithm="maxima")
[Out] 1/8*log(b*x^8 + a)/b
```

mupad [B] time = 0.04, size = 13, normalized size = 0.87

$$\frac{\ln\left(b\,x^8+a\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^8),x)
[Out] log(a + b*x^8)/(8*b)
```

sympy [A] time = 0.40, size = 10, normalized size = 0.67

$$\frac{\log\left(a+bx^8\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**8+a),x)
[Out] log(a + b*x**8)/(8*b)
```

3.1453 $\int \frac{x^5}{a+bx^8}\,dx$

Optimal. Leaf size=193

$$-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x^2}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,\sqrt[4]{a}\,b^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x^2}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,\sqrt[4]{a}\,b^{3/4}}+\frac{\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^2+\sqrt{a}+\sqrt{b}\,x^4\right)}{8\sqrt{2}\,\sqrt[4]{a}\,b^{3/4}}-\frac{\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^2+\sqrt{a}+\sqrt{b}\right.}{8\sqrt{2}\,\sqrt[4]{a}\,b^{3/4}}$$

```
[Out] 1/8*arctan(-1+b^(1/4)*x^2*2^(1/2)/a^(1/4))/a^(1/4)/b^(3/4)*2^(1/2)+1/8*arct
an(1+b^(1/4)*x^2*2^(1/2)/a^(1/4))/a^(1/4)/b^(3/4)*2^(1/2)+1/16*ln(-a^(1/4)*
b^(1/4)*x^2*2^(1/2)+a^(1/2)+x^4*b^(1/2))/a^(1/4)/b^(3/4)*2^(1/2)-1/16*ln(a^
(1/4)*b^(1/4)*x^2*2^(1/2)+a^(1/2)+x^4*b^(1/2))/a^(1/4)/b^(3/4)*2^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {275, 297, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^2+\sqrt{a}+\sqrt{b}\,x^4\right)}{8\sqrt{2}\,\sqrt[4]{a}\,b^{3/4}}-\frac{\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^2+\sqrt{a}+\sqrt{b}\,x^4\right)}{8\sqrt{2}\,\sqrt[4]{a}\,b^{3/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x^2}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,\sqrt[4]{a}\,b^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x^2}{\sqrt[4]{a}}+1\right.}{4\sqrt{2}\,\sqrt[4]{a}\,b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^8),x]
```

```
[Out] -ArcTan[1 - (Sqrt[2]*b^(1/4)*x^2)/a^(1/4)]/(4*Sqrt[2]*a^(1/4)*b^(3/4)) + Ar
cTan[1 + (Sqrt[2]*b^(1/4)*x^2)/a^(1/4)]/(4*Sqrt[2]*a^(1/4)*b^(3/4)) + Log[S
qrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x^2 + Sqrt[b]*x^4]/(8*Sqrt[2]*a^(1/4)*b^(3
/4)) - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x^2 + Sqrt[b]*x^4]/(8*Sqrt[2]*
a^(1/4)*b^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^5}{a+bx^8}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, x^2\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, x^2\right)}{4\sqrt{b}} + \frac{\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, x^2\right)}{4\sqrt{b}}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, x^2\right)}{8b} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, x^2\right)}{8b} + \frac{\text{Subst}\left(\int \frac{\frac{\sqrt{2}\,\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}} \right.}{8\sqrt{2}\,\sqrt[4]{a}\,b^3}$$

$$= \frac{\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^2+\sqrt{b}\,x^4\right)}{8\sqrt{2}\,\sqrt[4]{a}\,b^{3/4}} - \frac{\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^2+\sqrt{b}\,x^4\right)}{8\sqrt{2}\,\sqrt[4]{a}\,b^{3/4}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,\right.}{4\sqrt{2}\,\sqrt[4]{a}}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x^2}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,\sqrt[4]{a}\,b^{3/4}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,x^2}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,\sqrt[4]{a}\,b^{3/4}} + \frac{\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^2+\sqrt{b}\,x^4\right)}{8\sqrt{2}\,\sqrt[4]{a}\,b^{3/4}} - \frac{\log\left(\sqrt{a}\right.}{}$$

Mathematica [A] time = 0.07, size = 279, normalized size = 1.45

$$-\frac{\log\left(-2\sqrt[8]{a}\,\sqrt[8]{b}\,x\sin\left(\frac{\pi}{8}\right)+\sqrt[4]{a}+\sqrt[4]{b}\,x^2\right)+\log\left(2\sqrt[8]{a}\,\sqrt[8]{b}\,x\sin\left(\frac{\pi}{8}\right)+\sqrt[4]{a}+\sqrt[4]{b}\,x^2\right)-\log\left(-2\sqrt[8]{a}\,\sqrt[8]{b}\,x\cos\left(\frac{\pi}{8}\right)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^8),x]
```

```
[Out] -1/8*(2*ArcTan[Cot[Pi/8] - (b^(1/8)*x*Csc[Pi/8])/a^(1/8)] + 2*ArcTan[Cot[Pi
/8] + (b^(1/8)*x*Csc[Pi/8])/a^(1/8)] - 2*ArcTan[(b^(1/8)*x*Sec[Pi/8])/a^(1/
8) - Tan[Pi/8]] + 2*ArcTan[(b^(1/8)*x*Sec[Pi/8])/a^(1/8) + Tan[Pi/8]] - Log
[a^(1/4) + b^(1/4)*x^2 - 2*a^(1/8)*b^(1/8)*x*Cos[Pi/8]] - Log[a^(1/4) + b^(
1/4)*x^2 + 2*a^(1/8)*b^(1/8)*x*Cos[Pi/8]] + Log[a^(1/4) + b^(1/4)*x^2 - 2*a
^(1/8)*b^(1/8)*x*Sin[Pi/8]] + Log[a^(1/4) + b^(1/4)*x^2 + 2*a^(1/8)*b^(1/8)
*x*Sin[Pi/8]])/(Sqrt[2]*a^(1/4)*b^(3/4))
```

fricas [A] time = 0.65, size = 128, normalized size = 0.66

$$-\frac{1}{2}\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}\arctan\left(-bx^2\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}+\sqrt{x^4-ab\sqrt{-\frac{1}{ab^3}}}\,b\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}\right)+\frac{1}{8}\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}\log\left(ab^2\left(-\frac{1}{ab^3}\right)^{\frac{3}{4}}+x^2\right)-\frac{1}{8}\left(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^8+a),x, algorithm="fricas")
```

```
[Out] -1/2*(-1/(a*b^3))^(1/4)*arctan(-b*x^2*(-1/(a*b^3))^(1/4) + sqrt(x^4 - a*b*s
qrt(-1/(a*b^3)))*b*(-1/(a*b^3))^(1/4)) + 1/8*(-1/(a*b^3))^(1/4)*log(a*b^2*(
-1/(a*b^3))^(3/4) + x^2) - 1/8*(-1/(a*b^3))^(1/4)*log(-a*b^2*(-1/(a*b^3))^(
3/4) + x^2)
```

giac [A] time = 0.21, size = 199, normalized size = 1.03

$$\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}x^4\arctan\left(\frac{\sqrt{2}\left(2x^2+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8ab}+\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}x^4\arctan\left(\frac{\sqrt{2}\left(2x^2-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8ab}+\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}x^4\log\left(x^4+\sqrt{2}\,x^2\left(\right.\right.}{16ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^8+a),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(a*b^3)^(1/4)*x^4*arctan(1/2*sqrt(2)*(2*x^2 + sqrt(2)*(a/b)^(1/
4))/(a/b)^(1/4))/(a*b) + 1/8*sqrt(2)*(a*b^3)^(1/4)*x^4*arctan(1/2*sqrt(2)*(
2*x^2 - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b) + 1/16*sqrt(2)*(a*b^3)^(1/4
)*x^4*log(x^4 + sqrt(2)*x^2*(a/b)^(1/4) + sqrt(a/b))/(a*b) - 1/16*sqrt(2)*(
a*b^3)^(1/4)*x^4*log(x^4 - sqrt(2)*x^2*(a/b)^(1/4) + sqrt(a/b))/(a*b)
```

maple [A] time = 0.00, size = 136, normalized size = 0.70

$$\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x^2}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}b}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x^2}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}b}+\frac{\sqrt{2}\,\ln\left(\frac{x^4-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x^2+\sqrt{\frac{a}{b}}}{x^4+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x^2+\sqrt{\frac{a}{b}}}\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^8+a),x)
```

```
[Out] 1/16/b/(a/b)^(1/4)*2^(1/2)*ln((x^4-(a/b)^(1/4)*2^(1/2)*x^2+(a/b)^(1/2))/(x^
4+(a/b)^(1/4)*2^(1/2)*x^2+(a/b)^(1/2)))+1/8/b/(a/b)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(a/b)^(1/4)*x^2+1)+1/8/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/
4)*x^2-1)
```

maxima [A] time = 2.28, size = 177, normalized size = 0.92

$$\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{8\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{8\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\,\log\left(\sqrt{b}\,x^4+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x^2+\sqrt{a}\right)}{16\,a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^8+a),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4))/sq
rt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 1/8*sqrt(2)*arctan(1
/2*sqrt(2)*(2*sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))
/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - 1/16*sqrt(2)*log(sqrt(b)*x^4 + sqrt(2)*a
^(1/4)*b^(1/4)*x^2 + sqrt(a))/(a^(1/4)*b^(3/4)) + 1/16*sqrt(2)*log(sqrt(b)*
x^4 - sqrt(2)*a^(1/4)*b^(1/4)*x^2 + sqrt(a))/(a^(1/4)*b^(3/4))
```

mupad **[B]** time = 1.11, size = 39, normalized size = 0.20

$$\frac{\operatorname{atan}\left(\frac{b^{1/4}x^2}{(-a)^{1/4}}\right)-\operatorname{atanh}\left(\frac{b^{1/4}x^2}{(-a)^{1/4}}\right)}{4\,(-a)^{1/4}\,b^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^8),x)
```

```
[Out] (atan((b^(1/4)*x^2)/(-a)^(1/4)) - atanh((b^(1/4)*x^2)/(-a)^(1/4)))/(4*(-a)^
(1/4)*b^(3/4))
```

sympy **[A]** time = 0.28, size = 27, normalized size = 0.14

$$\operatorname{RootSum}\left(4096t^4ab^3+1,\left(t\mapsto t\log\left(512t^3ab^2+x^2\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**8+a),x)
```

```
[Out] RootSum(4096*_t**4*a*b**3 + 1, Lambda(_t, _t*log(512*_t**3*a*b**2 + x**2)))
```

3.1454 $\int \frac{x^3}{a+bx^8}\,dx$

Optimal. Leaf size=29

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x^4}{\sqrt{a}}\right)}{4\sqrt{a}\,\sqrt{b}}$$

```
[Out] 1/4*arctan(x^4*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {275, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x^4}{\sqrt{a}}\right)}{4\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^8),x]
```

```
[Out] ArcTan[(Sqrt[b]*x^4)/Sqrt[a]]/(4*Sqrt[a]*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{a+bx^8}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^4\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x^4}{\sqrt{a}}\right)}{4\sqrt{a}\,\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x^4}{\sqrt{a}}\right)}{4\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^8),x]
```

```
[Out] ArcTan[(Sqrt[b]*x^4)/Sqrt[a]]/(4*Sqrt[a]*Sqrt[b])
```

fricas [A] time = 0.60, size = 72, normalized size = 2.48

$$\left[-\frac{\sqrt{-ab}\,\log\left(\frac{bx^8-2\sqrt{-ab}\,x^4-a}{bx^8+a}\right)}{8\,ab}, -\frac{\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}}{bx^4}\right)}{4\,ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^8+a),x, algorithm="fricas")
```

```
[Out] [-1/8*sqrt(-a*b)*log((b*x^8 - 2*sqrt(-a*b)*x^4 - a)/(b*x^8 + a))/(a*b), -1/
4*sqrt(a*b)*arctan(sqrt(a*b)/(b*x^4))/(a*b)]
```

giac [A] time = 0.19, size = 18, normalized size = 0.62

$$\frac{\arctan\left(\frac{bx^4}{\sqrt{ab}}\right)}{4\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^8+a),x, algorithm="giac")
```

```
[Out] 1/4*arctan(b*x^4/sqrt(a*b))/sqrt(a*b)
```

maple [A] time = 0.00, size = 19, normalized size = 0.66

$$\frac{\arctan\left(\frac{b\,x^4}{\sqrt{ab}}\right)}{4\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^8+a),x)
```

```
[Out] 1/4/(a*b)^(1/2)*arctan(b*x^4/(a*b)^(1/2))
```

maxima [A] time = 2.34, size = 18, normalized size = 0.62

$$\frac{\arctan\left(\frac{bx^4}{\sqrt{ab}}\right)}{4\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^8+a),x, algorithm="maxima")
```

```
[Out] 1/4*arctan(b*x^4/sqrt(a*b))/sqrt(a*b)
```

mupad [B] time = 0.06, size = 19, normalized size = 0.66

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x^4}{\sqrt{a}}\right)}{4\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^8),x)
```

```
[Out] atan((b^(1/2)*x^4)/a^(1/2))/(4*a^(1/2)*b^(1/2))
```

sympy [B] time = 0.42, size = 56, normalized size = 1.93

$$-\frac{\sqrt{-\frac{1}{ab}}\log\left(-a\sqrt{-\frac{1}{ab}}+x^4\right)}{8}+\frac{\sqrt{-\frac{1}{ab}}\log\left(a\sqrt{-\frac{1}{ab}}+x^4\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**8+a),x)
```

```
[Out] -sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x**4)/8 + sqrt(-1/(a*b))*log(a*sqrt
(-1/(a*b)) + x**4)/8
```

3.1455 $\int \frac{x}{a+bx^8}\,dx$

Optimal. Leaf size=193

$$-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x^2}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x^2}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^2+\sqrt{a}+\sqrt{b}\,x^4\right)}{8\sqrt{2}\,a^{3/4}\sqrt[4]{b}}+\frac{\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^2+\sqrt{a}+\sqrt{b}\right.}{8\sqrt{2}\,a^{3/4}\sqrt[4]{b}}$$

```
[Out] 1/8*arctan(-1+b^(1/4)*x^2*2^(1/2)/a^(1/4))/a^(3/4)/b^(1/4)*2^(1/2)+1/8*arct
an(1+b^(1/4)*x^2*2^(1/2)/a^(1/4))/a^(3/4)/b^(1/4)*2^(1/2)-1/16*ln(-a^(1/4)*
b^(1/4)*x^2*2^(1/2)+a^(1/2)+x^4*b^(1/2))/a^(3/4)/b^(1/4)*2^(1/2)+1/16*ln(a^
(1/4)*b^(1/4)*x^2*2^(1/2)+a^(1/2)+x^4*b^(1/2))/a^(3/4)/b^(1/4)*2^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {275, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^2+\sqrt{a}+\sqrt{b}\,x^4\right)}{8\sqrt{2}\,a^{3/4}\sqrt[4]{b}}+\frac{\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^2+\sqrt{a}+\sqrt{b}\,x^4\right)}{8\sqrt{2}\,a^{3/4}\sqrt[4]{b}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x^2}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x^2}{\sqrt[4]{a}}+\right.}{4\sqrt{2}\,a^{3/4}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^8),x]
```

```
[Out] -ArcTan[1 - (Sqrt[2]*b^(1/4)*x^2)/a^(1/4)]/(4*Sqrt[2]*a^(3/4)*b^(1/4)) + Ar
cTan[1 + (Sqrt[2]*b^(1/4)*x^2)/a^(1/4)]/(4*Sqrt[2]*a^(3/4)*b^(1/4)) - Log[S
qrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x^2 + Sqrt[b]*x^4]/(8*Sqrt[2]*a^(3/4)*b^(1
/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x^2 + Sqrt[b]*x^4]/(8*Sqrt[2]*
a^(3/4)*b^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x}{a+bx^8}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, x^2\right)}{4\sqrt{a}} + \frac{\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, x^2\right)}{4\sqrt{a}}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, x^2\right)}{8\sqrt{a}\,\sqrt{b}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, x^2\right)}{8\sqrt{a}\,\sqrt{b}} - \frac{\text{Subst}\left(\int \frac{\frac{\sqrt{2}\,\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}}\right.}{8\sqrt{2}\,a^{3/4}\sqrt[4]{}}$$

$$= -\frac{\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^2+\sqrt{b}\,x^4\right)}{8\sqrt{2}\,a^{3/4}\sqrt[4]{b}} + \frac{\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^2+\sqrt{b}\,x^4\right)}{8\sqrt{2}\,a^{3/4}\sqrt[4]{b}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx\right.}{4\sqrt{2}\,a^3}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x^2}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{b}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,x^2}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{b}} - \frac{\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^2+\sqrt{b}\,x^4\right)}{8\sqrt{2}\,a^{3/4}\sqrt[4]{b}} + \frac{\log\left(\sqrt{a}\right.}{}$$

Mathematica [A] time = 0.04, size = 279, normalized size = 1.45

$$-\frac{-\log\left(-2\sqrt[8]{a}\,\sqrt[8]{b}\,x\sin\left(\frac{\pi}{8}\right)+\sqrt[4]{a}+\sqrt[4]{b}\,x^2\right)-\log\left(2\sqrt[8]{a}\,\sqrt[8]{b}\,x\sin\left(\frac{\pi}{8}\right)+\sqrt[4]{a}+\sqrt[4]{b}\,x^2\right)+\log\left(-2\sqrt[8]{a}\,\sqrt[8]{b}\,x\cos\left(\frac{\pi}{8}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^8),x]
```

```
[Out] -1/8*(2*ArcTan[Cot[Pi/8] - (b^(1/8)*x*Csc[Pi/8])/a^(1/8)] + 2*ArcTan[Cot[Pi
/8] + (b^(1/8)*x*Csc[Pi/8])/a^(1/8)] - 2*ArcTan[(b^(1/8)*x*Sec[Pi/8])/a^(1/
8) - Tan[Pi/8]] + 2*ArcTan[(b^(1/8)*x*Sec[Pi/8])/a^(1/8) + Tan[Pi/8]] + Log
[a^(1/4) + b^(1/4)*x^2 - 2*a^(1/8)*b^(1/8)*x*Cos[Pi/8]] + Log[a^(1/4) + b^(
1/4)*x^2 + 2*a^(1/8)*b^(1/8)*x*Cos[Pi/8]] - Log[a^(1/4) + b^(1/4)*x^2 - 2*a
^(1/8)*b^(1/8)*x*Sin[Pi/8]] - Log[a^(1/4) + b^(1/4)*x^2 + 2*a^(1/8)*b^(1/8)
*x*Sin[Pi/8]])/(Sqrt[2]*a^(3/4)*b^(1/4))
```

fricas [A] time = 0.59, size = 128, normalized size = 0.66

$$\frac{1}{2}\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}\arctan\left(-a^2bx^2\left(-\frac{1}{a^3b}\right)^{\frac{3}{4}}+\sqrt{x^4+a^2\sqrt{-\frac{1}{a^3b}}}\,a^2b\left(-\frac{1}{a^3b}\right)^{\frac{3}{4}}\right)+\frac{1}{8}\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}\log\left(x^2+a\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}\right)-\frac{1}{8}\left(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^8+a),x, algorithm="fricas")
```

```
[Out] 1/2*(-1/(a^3*b))^(1/4)*arctan(-a^2*b*x^2*(-1/(a^3*b))^(3/4) + sqrt(x^4 + a^
2*sqrt(-1/(a^3*b)))*a^2*b*(-1/(a^3*b))^(3/4)) + 1/8*(-1/(a^3*b))^(1/4)*log(
x^2 + a*(-1/(a^3*b))^(1/4)) - 1/8*(-1/(a^3*b))^(1/4)*log(x^2 - a*(-1/(a^3*b
))^(1/4))
```

giac [A] time = 0.22, size = 187, normalized size = 0.97

$$\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x^2+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8ab}+\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x^2-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8ab}+\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\log\left(x^4+\sqrt{2}\,x^2\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{}\right.}{16ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^8+a),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x^2 + sqrt(2)*(a/b)^(1/4))/
(a/b)^(1/4))/(a*b) + 1/8*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x^2 -
sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b) + 1/16*sqrt(2)*(a*b^3)^(1/4)*log(x^
4 + sqrt(2)*x^2*(a/b)^(1/4) + sqrt(a/b))/(a*b) - 1/16*sqrt(2)*(a*b^3)^(1/4)
*log(x^4 - sqrt(2)*x^2*(a/b)^(1/4) + sqrt(a/b))/(a*b)
```

maple [A] time = 0.00, size = 136, normalized size = 0.70

$$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x^2}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x^2}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x^4+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x^2+\sqrt{\frac{a}{b}}}{x^4-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x^2+\sqrt{\frac{a}{b}}}\right)}{16a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^8+a),x)
```

```
[Out] 1/16*(a/b)^(1/4)/a*2^(1/2)*ln((x^4+(a/b)^(1/4)*2^(1/2)*x^2+(a/b)^(1/2))/(x^
4-(a/b)^(1/4)*2^(1/2)*x^2+(a/b)^(1/2)))+1/8*(a/b)^(1/4)/a*2^(1/2)*arctan(2^
(1/2)/(a/b)^(1/4)*x^2+1)+1/8*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/
4)*x^2-1)
```

maxima [A] time = 2.34, size = 177, normalized size = 0.92

$$\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{8\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{8\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,\log\left(\sqrt{b}\,x^4+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x^2+\sqrt{a}\right)}{16\,a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^8+a),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4))/sq
rt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 1/8*sqrt(2)*arctan(1
```

```
/2*sqrt(2)*(2*sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))
/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 1/16*sqrt(2)*log(sqrt(b)*x^4 + sqrt(2)*a
^(1/4)*b^(1/4)*x^2 + sqrt(a))/(a^(3/4)*b^(1/4)) - 1/16*sqrt(2)*log(sqrt(b)*
x^4 - sqrt(2)*a^(1/4)*b^(1/4)*x^2 + sqrt(a))/(a^(3/4)*b^(1/4))
```

mupad **[B]** time = 1.10, size = 37, normalized size = 0.19

$$-\frac{\operatorname{atan}\left(\frac{b^{1/4}x^2}{(-a)^{1/4}}\right)+\operatorname{atanh}\left(\frac{b^{1/4}x^2}{(-a)^{1/4}}\right)}{4\,(-a)^{3/4}\,b^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^8),x)
```

```
[Out] -(atan((b^(1/4)*x^2)/(-a)^(1/4)) + atanh((b^(1/4)*x^2)/(-a)^(1/4)))/(4*(-a)
^(3/4)*b^(1/4))
```

sympy **[A]** time = 0.22, size = 22, normalized size = 0.11

$$\operatorname{RootSum}\left(4096t^4a^3b+1,\left(t\mapsto t\log\left(8ta+x^2\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**8+a),x)
```

```
[Out] RootSum(4096*_t**4*a**3*b + 1, Lambda(_t, _t*log(8*_t*a + x**2)))
```

3.1456 $\int \frac{1}{x\left(a+bx^8\right)}\,dx$

Optimal. Leaf size=22

$$\frac{\log(x)}{a}-\frac{\log\left(a+bx^8\right)}{8a}$$

```
[Out] ln(x)/a-1/8*ln(b*x^8+a)/a
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {266, 36, 29, 31}

$$\frac{\log(x)}{a}-\frac{\log\left(a+bx^8\right)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^8)),x]
```

```
[Out] Log[x]/a - Log[a + b*x^8]/(8*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a+bx^8\right)}\,dx &= \frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{x(a+bx)}\,dx,x,x^8\right)\\ &= \frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,x^8\right)}{8a}-\frac{b\operatorname{Subst}\left(\int \frac{1}{a+bx}\,dx,x,x^8\right)}{8a}\\ &= \frac{\log(x)}{a}-\frac{\log\left(a+bx^8\right)}{8a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{\log(x)}{a}-\frac{\log\left(a+bx^8\right)}{8a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^8)),x]
[Out] Log[x]/a - Log[a + b*x^8]/(8*a)
```

fricas [A] time = 0.57, size = 18, normalized size = 0.82

$$-\frac{\log\left(bx^8+a\right)-8\,\log(x)}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^8+a),x, algorithm="fricas")
[Out] -1/8*(log(b*x^8 + a) - 8*log(x))/a
```

giac [A] time = 0.20, size = 24, normalized size = 1.09

$$\frac{\log\left(x^8\right)}{8\,a}-\frac{\log\left(\left|bx^8+a\right|\right)}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^8+a),x, algorithm="giac")
[Out] 1/8*log(x^8)/a - 1/8*log(abs(b*x^8 + a))/a
```

maple [A] time = 0.01, size = 21, normalized size = 0.95

$$\frac{\ln(x)}{a}-\frac{\ln\left(b\,x^8+a\right)}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^8+a),x)
[Out] 1/a*ln(x)-1/8*ln(b*x^8+a)/a
```

maxima [A] time = 1.06, size = 23, normalized size = 1.05

$$-\frac{\log\left(bx^8+a\right)}{8\,a}+\frac{\log\left(x^8\right)}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^8+a),x, algorithm="maxima")
[Out] -1/8*log(b*x^8 + a)/a + 1/8*log(x^8)/a
```

mupad [B] time = 1.10, size = 18, normalized size = 0.82

$$-\frac{\ln\left(b\,x^8+a\right)-8\,\ln(x)}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^8)),x)
[Out] -(log(a + b*x^8) - 8*log(x))/(8*a)
```

sympy [A] time = 0.65, size = 15, normalized size = 0.68

$$\frac{\log(x)}{a}-\frac{\log\left(\frac{a}{b}+x^8\right)}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**8+a),x)
[Out] log(x)/a - log(a/b + x**8)/(8*a)
```

3.1457 $\int \frac{1}{x^3\left(a+bx^8\right)}\,dx$

Optimal. Leaf size=203

$$\frac{\sqrt[4]{b}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x^2}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{5/4}}-\frac{\sqrt[4]{b}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x^2}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,a^{5/4}}-\frac{\sqrt[4]{b}\,\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^2+\sqrt{a}+\sqrt{b}\,x^4\right)}{8\sqrt{2}\,a^{5/4}}+\frac{\sqrt[4]{b}\,\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{}\right.}{8\sqrt{}}$$

```
[Out] -1/2/a/x^2-1/8*b^(1/4)*arctan(-1+b^(1/4)*x^2*2^(1/2)/a^(1/4))/a^(5/4)*2^(1/
2)-1/8*b^(1/4)*arctan(1+b^(1/4)*x^2*2^(1/2)/a^(1/4))/a^(5/4)*2^(1/2)-1/16*b
^(1/4)*ln(-a^(1/4)*b^(1/4)*x^2*2^(1/2)+a^(1/2)+x^4*b^(1/2))/a^(5/4)*2^(1/2)
+1/16*b^(1/4)*ln(a^(1/4)*b^(1/4)*x^2*2^(1/2)+a^(1/2)+x^4*b^(1/2))/a^(5/4)*2
^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {275, 325, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{b}\,\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^2+\sqrt{a}+\sqrt{b}\,x^4\right)}{8\sqrt{2}\,a^{5/4}}+\frac{\sqrt[4]{b}\,\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^2+\sqrt{a}+\sqrt{b}\,x^4\right)}{8\sqrt{2}\,a^{5/4}}+\frac{\sqrt[4]{b}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x^2}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{5/4}}-\frac{\sqrt[4]{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^8)),x]
```

```
[Out] -1/(2*a*x^2) + (b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x^2)/a^(1/4)])/(4*Sqrt[
2]*a^(5/4)) - (b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x^2)/a^(1/4)])/(4*Sqrt[2
]*a^(5/4)) - (b^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x^2 + Sqrt[b]*x
^4])/(8*Sqrt[2]*a^(5/4)) + (b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x
^2 + Sqrt[b]*x^4])/(8*Sqrt[2]*a^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(a+bx^8\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^4\right)}\,dx, x, x^2\right) \\
&= -\frac{1}{2ax^2} - \frac{b\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, x^2\right)}{2a} \\
&= -\frac{1}{2ax^2} + \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, x^2\right)}{4a} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, x^2\right)}{4a} \\
&= -\frac{1}{2ax^2} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, x^2\right)}{8a} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, x^2\right)}{8a} - \frac{\sqrt[4]{b}\,\text{Subst}}{} \\
&= -\frac{1}{2ax^2} - \frac{\sqrt[4]{b}\,\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^2+\sqrt{b}\,x^4\right)}{8\sqrt{2}\,a^{5/4}} + \frac{\sqrt[4]{b}\,\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^2+\sqrt{b}\,x^4\right)}{8\sqrt{2}\,a^{5/4}} - \\
&= -\frac{1}{2ax^2} + \frac{\sqrt[4]{b}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x^2}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{5/4}} - \frac{\sqrt[4]{b}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,x^2}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{5/4}} - \frac{\sqrt[4]{b}\,\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^2\right.}{8\sqrt{2}\,a^{5/4}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 385, normalized size = 1.90

$$\frac{\sqrt{2}\,\sqrt[4]{b}\,x^2\log\left(-2\sqrt[8]{a}\,\sqrt[8]{b}\,x\sin\left(\frac{\pi}{8}\right)+\sqrt[4]{a}+\sqrt[4]{b}\,x^2\right)+\sqrt{2}\,\sqrt[4]{b}\,x^2\log\left(2\sqrt[8]{a}\,\sqrt[8]{b}\,x\sin\left(\frac{\pi}{8}\right)+\sqrt[4]{a}+\sqrt[4]{b}\,x^2\right)-\sqrt{2}\,\sqrt[4]{b}\,x^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^8)),x]
```

```
[Out] (-8*a^(1/4) + 2*Sqrt[2]*b^(1/4)*x^2*ArcTan[Cot[Pi/8] - (b^(1/8)*x*Csc[Pi/8]
)/a^(1/8)] + 2*Sqrt[2]*b^(1/4)*x^2*ArcTan[Cot[Pi/8] + (b^(1/8)*x*Csc[Pi/8])
/a^(1/8)] - 2*Sqrt[2]*b^(1/4)*x^2*ArcTan[(b^(1/8)*x*Sec[Pi/8])/a^(1/8) - Ta
n[Pi/8]] + 2*Sqrt[2]*b^(1/4)*x^2*ArcTan[(b^(1/8)*x*Sec[Pi/8])/a^(1/8) + Tan
[Pi/8]] - Sqrt[2]*b^(1/4)*x^2*Log[a^(1/4) + b^(1/4)*x^2 - 2*a^(1/8)*b^(1/8)
*x*Cos[Pi/8]] - Sqrt[2]*b^(1/4)*x^2*Log[a^(1/4) + b^(1/4)*x^2 + 2*a^(1/8)*b
^(1/8)*x*Cos[Pi/8]] + Sqrt[2]*b^(1/4)*x^2*Log[a^(1/4) + b^(1/4)*x^2 - 2*a^(
1/8)*b^(1/8)*x*Sin[Pi/8]] + Sqrt[2]*b^(1/4)*x^2*Log[a^(1/4) + b^(1/4)*x^2 +
 2*a^(1/8)*b^(1/8)*x*Sin[Pi/8]])/(16*a^(5/4)*x^2)
```

fricas [A] time = 0.73, size = 141, normalized size = 0.69

$$\frac{4\,ax^2\left(-\frac{b}{a^5}\right)^{\frac{1}{4}}\arctan\left(-ax^2\left(-\frac{b}{a^5}\right)^{\frac{1}{4}}+a\sqrt{\frac{bx^4-a^3\sqrt{-\frac{b}{a^5}}}{b}}\left(-\frac{b}{a^5}\right)^{\frac{1}{4}}\right)-ax^2\left(-\frac{b}{a^5}\right)^{\frac{1}{4}}\log\left(a^4\left(-\frac{b}{a^5}\right)^{\frac{3}{4}}+bx^2\right)+ax^2\left(-\frac{b}{a^5}\right)^{}}{8\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^8+a),x, algorithm="fricas")
```

```
[Out] 1/8*(4*a*x^2*(-b/a^5)^(1/4)*arctan(-a*x^2*(-b/a^5)^(1/4) + a*sqrt((b*x^4 -
a^3*sqrt(-b/a^5))/b)*(-b/a^5)^(1/4)) - a*x^2*(-b/a^5)^(1/4)*log(a^4*(-b/a^5
)^(3/4) + b*x^2) + a*x^2*(-b/a^5)^(1/4)*log(-a^4*(-b/a^5)^(3/4) + b*x^2) -
4)/(a*x^2)
```

giac [A] time = 0.19, size = 195, normalized size = 0.96

$$-\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}x^4\arctan\left(\frac{\sqrt{2}\left(2x^2+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^2}-\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}x^4\arctan\left(\frac{\sqrt{2}\left(2x^2-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^2}-\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}x^4\log\left(x^4+\sqrt{2}\right.}{16\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^8+a),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*(a*b^3)^(1/4)*x^4*arctan(1/2*sqrt(2)*(2*x^2 + sqrt(2)*(a/b)^(1
/4))/(a/b)^(1/4))/a^2 - 1/8*sqrt(2)*(a*b^3)^(1/4)*x^4*arctan(1/2*sqrt(2)*(2
*x^2 - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/a^2 - 1/16*sqrt(2)*(a*b^3)^(1/4)*x
^4*log(x^4 + sqrt(2)*x^2*(a/b)^(1/4) + sqrt(a/b))/a^2 + 1/16*sqrt(2)*(a*b^3
)^(1/4)*x^4*log(x^4 - sqrt(2)*x^2*(a/b)^(1/4) + sqrt(a/b))/a^2 - 1/2/(a*x^2
)
```

maple [A] time = 0.01, size = 144, normalized size = 0.71

$$-\frac{\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\,x^2}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}a}-\frac{\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\,x^2}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}a}-\frac{\sqrt{2}\ \ln\left(\frac{x^4-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x^2+\sqrt{\frac{a}{b}}}{x^4+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x^2+\sqrt{\frac{a}{b}}}\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}a}-\frac{1}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^8+a),x)
```

```
[Out] -1/16/a/(a/b)^(1/4)*2^(1/2)*ln((x^4-(a/b)^(1/4)*2^(1/2)*x^2+(a/b)^(1/2))/(x
^4+(a/b)^(1/4)*2^(1/2)*x^2+(a/b)^(1/2)))-1/8/a/(a/b)^(1/4)*2^(1/2)*arctan(2
```

```
^(1/2)/(a/b)^(1/4)*x^2+1)-1/8/a/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1
/4)*x^2-1)-1/2/a/x^2
```

maxima [A] time = 2.43, size = 191, normalized size = 0.94

$$-\frac{b\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}x^4+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x^2+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}x^4-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x\right.}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right.}{16\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^8+a),x, algorithm="maxima")
```

```
[Out] -1/16*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1
/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arc
tan(1/2*sqrt(2)*(2*sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt
(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(b)*x^4 + sqrt(2)*a
^(1/4)*b^(1/4)*x^2 + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*x^4 -
 sqrt(2)*a^(1/4)*b^(1/4)*x^2 + sqrt(a))/(a^(1/4)*b^(3/4)))/a - 1/2/(a*x^2)
```

mupad [B] time = 1.11, size = 55, normalized size = 0.27

$$\frac{(-b)^{1/4}\,\operatorname{atanh}\left(\frac{(-b)^{1/4}\,x^2}{a^{1/4}}\right)}{4\,a^{5/4}}-\frac{(-b)^{1/4}\,\operatorname{atan}\left(\frac{(-b)^{1/4}\,x^2}{a^{1/4}}\right)}{4\,a^{5/4}}-\frac{1}{2\,a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^8)),x)
```

```
[Out] ((-b)^(1/4)*atanh(((-b)^(1/4)*x^2)/a^(1/4)))/(4*a^(5/4)) - ((-b)^(1/4)*atan
(((-b)^(1/4)*x^2)/a^(1/4)))/(4*a^(5/4)) - 1/(2*a*x^2)
```

sympy [A] time = 0.51, size = 34, normalized size = 0.17

$$\operatorname{RootSum}\left(4096t^4a^5+b,\left(t\mapsto t\log\left(-\frac{512t^3a^4}{b}+x^2\right)\right)\right)-\frac{1}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**8+a),x)
```

```
[Out] RootSum(4096*_t**4*a**5 + b, Lambda(_t, _t*log(-512*_t**3*a**4/b + x**2)))
 - 1/(2*a*x**2)
```

3.1458 $\int \frac{1}{x^5\left(a+bx^8\right)}\,dx$

Optimal. Leaf size=40

$$-\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x^4}{\sqrt{a}}\right)}{4a^{3/2}}-\frac{1}{4ax^4}$$

```
[Out] -1/4/a/x^4-1/4*arctan(x^4*b^(1/2)/a^(1/2))*b^(1/2)/a^(3/2)
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {275, 325, 205}

$$-\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x^4}{\sqrt{a}}\right)}{4a^{3/2}}-\frac{1}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^8)),x]
```

```
[Out] -1/(4*a*x^4) - (Sqrt[b]*ArcTan[(Sqrt[b]*x^4)/Sqrt[a]])/(4*a^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\left(a+bx^8\right)}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)}\,dx, x, x^4\right)\\ &= -\frac{1}{4ax^4}-\frac{b\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^4\right)}{4a}\\ &= -\frac{1}{4ax^4}-\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x^4}{\sqrt{a}}\right)}{4a^{3/2}}\end{aligned}$$

Mathematica [B] time = 0.05, size = 164, normalized size = 4.10

$$\frac{\sqrt{b}\,x^4\tan^{-1}\left(\frac{\sqrt[8]{b}\,x\sec\left(\frac{\pi}{8}\right)}{\sqrt[8]{a}}-\tan\left(\frac{\pi}{8}\right)\right)-\sqrt{b}\,x^4\tan^{-1}\left(\frac{\sqrt[8]{b}\,x\sec\left(\frac{\pi}{8}\right)}{\sqrt[8]{a}}+\tan\left(\frac{\pi}{8}\right)\right)+\sqrt{b}\,x^4\tan^{-1}\left(\cot\left(\frac{\pi}{8}\right)-\frac{\sqrt[8]{b}\,x\csc\left(\frac{\pi}{8}\right)}{\sqrt[8]{a}}\right)}{4a^{3/2}x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^8)),x]
```

```
[Out] (-Sqrt[a] + Sqrt[b]*x^4*ArcTan[Cot[Pi/8] - (b^(1/8)*x*Csc[Pi/8])/a^(1/8)] +
 Sqrt[b]*x^4*ArcTan[Cot[Pi/8] + (b^(1/8)*x*Csc[Pi/8])/a^(1/8)] + Sqrt[b]*x^
4*ArcTan[(b^(1/8)*x*Sec[Pi/8])/a^(1/8) - Tan[Pi/8]] - Sqrt[b]*x^4*ArcTan[(b
^(1/8)*x*Sec[Pi/8])/a^(1/8) + Tan[Pi/8]])/(4*a^(3/2)*x^4)
```

fricas [A] time = 0.54, size = 94, normalized size = 2.35

$$\left[\frac{x^4\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx^8-2\,ax^4\sqrt{-\frac{b}{a}}-a}{bx^8+a}\right)-2}{8\,ax^4},\ \frac{x^4\sqrt{\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{bx^4}\right)-1}{4\,ax^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^8+a),x, algorithm="fricas")
```

```
[Out] [1/8*(x^4*sqrt(-b/a)*log((b*x^8 - 2*a*x^4*sqrt(-b/a) - a)/(b*x^8 + a)) - 2)
/(a*x^4), 1/4*(x^4*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*x^4)) - 1)/(a*x^4)]
```

giac [A] time = 0.19, size = 31, normalized size = 0.78

$$-\frac{b\arctan\left(\frac{bx^4}{\sqrt{ab}}\right)}{4\sqrt{ab}\,a}-\frac{1}{4\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^8+a),x, algorithm="giac")
```

```
[Out] -1/4*b*arctan(b*x^4/sqrt(a*b))/(sqrt(a*b)*a) - 1/4/(a*x^4)
```

maple [A] time = 0.01, size = 32, normalized size = 0.80

$$-\frac{b\arctan\left(\frac{b\,x^4}{\sqrt{ab}}\right)}{4\sqrt{ab}\ a}-\frac{1}{4a\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^8+a),x)
```

```
[Out] -1/4/a/x^4-1/4/a*b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^4)
```

maxima [A] time = 2.21, size = 31, normalized size = 0.78

$$-\frac{b\arctan\left(\frac{bx^4}{\sqrt{ab}}\right)}{4\sqrt{ab}\,a}-\frac{1}{4\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^8+a),x, algorithm="maxima")
```

```
[Out] -1/4*b*arctan(b*x^4/sqrt(a*b))/(sqrt(a*b)*a) - 1/4/(a*x^4)
```

mupad [B] time = 0.05, size = 28, normalized size = 0.70

$$-\frac{1}{4\,a\,x^4}-\frac{\sqrt{b}\,\operatorname{atan}\left(\frac{\sqrt{b}\,x^4}{\sqrt{a}}\right)}{4\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^8)),x)
```

```
[Out] - 1/(4*a*x^4) - (b^(1/2)*atan((b^(1/2)*x^4)/a^(1/2)))/(4*a^(3/2))
```

sympy [**A**] time = 0.69, size = 71, normalized size = 1.78

$$\frac{\sqrt{-\frac{b}{a^3}}\,\log\left(-\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x^4\right)}{8}-\frac{\sqrt{-\frac{b}{a^3}}\,\log\left(\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x^4\right)}{8}-\frac{1}{4ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**8+a),x)
```

```
[Out] sqrt(-b/a**3)*log(-a**2*sqrt(-b/a**3)/b + x**4)/8 - sqrt(-b/a**3)*log(a**2*
sqrt(-b/a**3)/b + x**4)/8 - 1/(4*a*x**4)
```

3.1459 $\int \frac{1}{x^7\left(a+bx^8\right)}\,dx$

Optimal. Leaf size=203

$$\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x^2}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{7/4}}-\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x^2}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,a^{7/4}}+\frac{b^{3/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^2+\sqrt{a}+\sqrt{b}\,x^4\right)}{8\sqrt{2}\,a^{7/4}}-\frac{b^{3/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\right.}{8\sqrt{2}\,a}$$

```
[Out] -1/6/a/x^6-1/8*b^(3/4)*arctan(-1+b^(1/4)*x^2*2^(1/2)/a^(1/4))/a^(7/4)*2^(1/
2)-1/8*b^(3/4)*arctan(1+b^(1/4)*x^2*2^(1/2)/a^(1/4))/a^(7/4)*2^(1/2)+1/16*b
^(3/4)*ln(-a^(1/4)*b^(1/4)*x^2*2^(1/2)+a^(1/2)+x^4*b^(1/2))/a^(7/4)*2^(1/2)
-1/16*b^(3/4)*ln(a^(1/4)*b^(1/4)*x^2*2^(1/2)+a^(1/2)+x^4*b^(1/2))/a^(7/4)*2
^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {275, 325, 211, 1165, 628, 1162, 617, 204}

$$\frac{b^{3/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^2+\sqrt{a}+\sqrt{b}\,x^4\right)}{8\sqrt{2}\,a^{7/4}}-\frac{b^{3/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^2+\sqrt{a}+\sqrt{b}\,x^4\right)}{8\sqrt{2}\,a^{7/4}}+\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x^2}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{7/4}}-\frac{b^{3/4}\,\text{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^8)),x]
```

```
[Out] -1/(6*a*x^6) + (b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x^2)/a^(1/4)])/(4*Sqrt[
2]*a^(7/4)) - (b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x^2)/a^(1/4)])/(4*Sqrt[2
]*a^(7/4)) + (b^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x^2 + Sqrt[b]*x
^4])/(8*Sqrt[2]*a^(7/4)) - (b^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x
^2 + Sqrt[b]*x^4])/(8*Sqrt[2]*a^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^7\left(a+bx^8\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^4\right)}\,dx, x, x^2\right)$$

$$= -\frac{1}{6ax^6} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, x^2\right)}{2a}$$

$$= -\frac{1}{6ax^6} - \frac{b\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, x^2\right)}{4a^{3/2}} - \frac{b\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, x^2\right)}{4a^{3/2}}$$

$$= -\frac{1}{6ax^6} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}} + x^2}\,dx, x, x^2\right)}{8a^{3/2}} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}} + x^2}\,dx, x, x^2\right)}{8a^{3/2}} +$$

$$= -\frac{1}{6ax^6} + \frac{b^{3/4}\log\left(\sqrt{a} - \sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^2 + \sqrt{b}\,x^4\right)}{8\sqrt{2}\,a^{7/4}} - \frac{b^{3/4}\log\left(\sqrt{a} + \sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^2 + \sqrt{b}\,x^4\right)}{8\sqrt{2}\,a^{7/4}}$$

$$= -\frac{1}{6ax^6} + \frac{b^{3/4}\tan^{-1}\left(1 - \frac{\sqrt{2}\,\sqrt[4]{b}\,x^2}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{7/4}} - \frac{b^{3/4}\tan^{-1}\left(1 + \frac{\sqrt{2}\,\sqrt[4]{b}\,x^2}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{7/4}} + \frac{b^{3/4}\log\left(\sqrt{a} - \sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\right.}{8\sqrt{2}\,a^{7/4}}$$

Mathematica [A] time = 0.10, size = 387, normalized size = 1.91

$$\frac{-8a^{3/4} - 6\sqrt{2}\,b^{3/4}x^6\tan^{-1}\left(\frac{\sqrt[8]{b}\,x\sec\left(\frac{\pi}{8}\right)}{\sqrt[8]{a}} - \tan\left(\frac{\pi}{8}\right)\right) + 6\sqrt{2}\,b^{3/4}x^6\tan^{-1}\left(\frac{\sqrt[8]{b}\,x\sec\left(\frac{\pi}{8}\right)}{\sqrt[8]{a}} + \tan\left(\frac{\pi}{8}\right)\right) + 6\sqrt{2}\,b^{3/4}x^6\tan^{-}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^8)),x]
```

```
[Out] (-8*a^(3/4) + 6*Sqrt[2]*b^(3/4)*x^6*ArcTan[Cot[Pi/8] - (b^(1/8)*x*Csc[Pi/8]
)/a^(1/8)] + 6*Sqrt[2]*b^(3/4)*x^6*ArcTan[Cot[Pi/8] + (b^(1/8)*x*Csc[Pi/8])
/a^(1/8)] - 6*Sqrt[2]*b^(3/4)*x^6*ArcTan[(b^(1/8)*x*Sec[Pi/8])/a^(1/8) - Ta
n[Pi/8]] + 6*Sqrt[2]*b^(3/4)*x^6*ArcTan[(b^(1/8)*x*Sec[Pi/8])/a^(1/8) + Tan
[Pi/8]] + 3*Sqrt[2]*b^(3/4)*x^6*Log[a^(1/4) + b^(1/4)*x^2 - 2*a^(1/8)*b^(1/
8)*x*Cos[Pi/8]] + 3*Sqrt[2]*b^(3/4)*x^6*Log[a^(1/4) + b^(1/4)*x^2 + 2*a^(1/
8)*b^(1/8)*x*Cos[Pi/8]] - 3*Sqrt[2]*b^(3/4)*x^6*Log[a^(1/4) + b^(1/4)*x^2 -
 2*a^(1/8)*b^(1/8)*x*Sin[Pi/8]] - 3*Sqrt[2]*b^(3/4)*x^6*Log[a^(1/4) + b^(1/
4)*x^2 + 2*a^(1/8)*b^(1/8)*x*Sin[Pi/8]])/(48*a^(7/4)*x^6)
```

fricas [A] time = 0.73, size = 168, normalized size = 0.83

$$-\frac{12\,ax^6\left(-\frac{b^3}{a^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{a^5x^2\left(-\frac{b^3}{a^7}\right)^{\frac{3}{4}}-a^5\sqrt{\frac{b^2x^4+a^4\sqrt{-\frac{b^3}{a^7}}}{b^2}}\left(-\frac{b^3}{a^7}\right)^{\frac{3}{4}}}{b^2}\right)+3\,ax^6\left(-\frac{b^3}{a^7}\right)^{\frac{1}{4}}\log\left(bx^2+a^2\left(-\frac{b^3}{a^7}\right)^{\frac{1}{4}}\right)-3\,ax^6\left(-\frac{b^3}{a^7}\right)^{\frac{1}{4}}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^8+a),x, algorithm="fricas")
```

```
[Out] -1/24*(12*a*x^6*(-b^3/a^7)^(1/4)*arctan(-(a^5*x^2*(-b^3/a^7)^(3/4) - a^5*sq
rt((b^2*x^4 + a^4*sqrt(-b^3/a^7))/b^2)*(-b^3/a^7)^(3/4))/b^2) + 3*a*x^6*(-b
^3/a^7)^(1/4)*log(b*x^2 + a^2*(-b^3/a^7)^(1/4)) - 3*a*x^6*(-b^3/a^7)^(1/4)*
log(b*x^2 - a^2*(-b^3/a^7)^(1/4)) + 4)/(a*x^6)
```

giac [A] time = 0.23, size = 183, normalized size = 0.90

$$-\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x^2+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^2}-\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x^2-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^2}-\frac{\sqrt{2}\left(ab^3\right)^{\frac{1}{4}}\log\left(x^4+\sqrt{2}\,x^2\left(\frac{a}{b}\right)^{\frac{1}{4}}+\right.}{16a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^8+a),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x^2 + sqrt(2)*(a/b)^(1/4))
/(a/b)^(1/4))/a^2 - 1/8*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x^2 - s
qrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/a^2 - 1/16*sqrt(2)*(a*b^3)^(1/4)*log(x^4 +
 sqrt(2)*x^2*(a/b)^(1/4) + sqrt(a/b))/a^2 + 1/16*sqrt(2)*(a*b^3)^(1/4)*log(
x^4 - sqrt(2)*x^2*(a/b)^(1/4) + sqrt(a/b))/a^2 - 1/6/(a*x^6)
```

maple [A] time = 0.00, size = 147, normalized size = 0.72

$$-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,x^2}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8a^2}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,x^2}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8a^2}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\ln\left(\frac{x^4+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x^2+\sqrt{\frac{a}{b}}}{x^4-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x^2+\sqrt{\frac{a}{b}}}\right)}{16a^2}-\frac{1}{6a\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^8+a),x)
```

```
[Out] -1/16/a^2*b*(a/b)^(1/4)*2^(1/2)*ln((x^4+(a/b)^(1/4)*2^(1/2)*x^2+(a/b)^(1/2)
)/(x^4-(a/b)^(1/4)*2^(1/2)*x^2+(a/b)^(1/2)))-1/8/a^2*b*(a/b)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(a/b)^(1/4)*x^2+1)-1/8/a^2*b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(a/b)^(1/4)*x^2-1)-1/6/a/x^6
```

maxima [A] time = 2.49, size = 192, normalized size = 0.95

$$-\frac{\frac{2\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(\sqrt{b}\,x^4+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x^2+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(\sqrt{b}\,x^4-}{a^{\frac{3}{4}}}}{16\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^8+a),x, algorithm="maxima")
```

```
[Out] -1/16*(2*sqrt(2)*b*arctan(1/2*sqrt(2)*(2*sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1
/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b*a
rctan(1/2*sqrt(2)*(2*sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sq
rt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(3/4)*log(sqrt(b)*x^4 +
 sqrt(2)*a^(1/4)*b^(1/4)*x^2 + sqrt(a))/a^(3/4) - sqrt(2)*b^(3/4)*log(sqrt(
b)*x^4 - sqrt(2)*a^(1/4)*b^(1/4)*x^2 + sqrt(a))/a^(3/4))/a - 1/6/(a*x^6)
```

mupad [B] time = 1.13, size = 55, normalized size = 0.27

$$\frac{(-b)^{3/4}\operatorname{atan}\left(\frac{(-b)^{1/4}x^2}{a^{1/4}}\right)}{4\,a^{7/4}}-\frac{1}{6\,a\,x^6}+\frac{(-b)^{3/4}\operatorname{atanh}\left(\frac{(-b)^{1/4}x^2}{a^{1/4}}\right)}{4\,a^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^8)),x)
```

```
[Out] ((-b)^(3/4)*atan(((-b)^(1/4)*x^2)/a^(1/4)))/(4*a^(7/4)) - 1/(6*a*x^6) + ((-
b)^(3/4)*atanh(((-b)^(1/4)*x^2)/a^(1/4)))/(4*a^(7/4))
```

sympy [A] time = 0.39, size = 34, normalized size = 0.17

$$\operatorname{RootSum}\left(4096t^4a^7+b^3,\left(t\mapsto t\log\left(-\frac{8ta^2}{b}+x^2\right)\right)\right)-\frac{1}{6ax^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**8+a),x)
```

```
[Out] RootSum(4096*_t**4*a**7 + b**3, Lambda(_t, _t*log(-8*_t*a**2/b + x**2))) -
1/(6*a*x**6)
```

3.1460 $\int \frac{1}{x^9\left(a+bx^8\right)}\,dx$

Optimal. Leaf size=35

$$\frac{b\log\left(a+bx^8\right)}{8a^2}-\frac{b\log(x)}{a^2}-\frac{1}{8ax^8}$$

```
[Out] -1/8/a/x^8-b*ln(x)/a^2+1/8*b*ln(b*x^8+a)/a^2
```

Rubi [A] time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 44}

$$\frac{b\log\left(a+bx^8\right)}{8a^2}-\frac{b\log(x)}{a^2}-\frac{1}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(a + b*x^8)),x]
```

```
[Out] -1/(8*a*x^8) - (b*Log[x])/a^2 + (b*Log[a + b*x^8])/(8*a^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^9\left(a+bx^8\right)}\,dx &= \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)}\,dx, x, x^8\right)\\ &= \frac{1}{8}\,\text{Subst}\left(\int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx, x, x^8\right)\\ &= -\frac{1}{8ax^8}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^8\right)}{8a^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.00

$$\frac{b\log\left(a+bx^8\right)}{8a^2}-\frac{b\log(x)}{a^2}-\frac{1}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^9*(a + b*x^8)),x]
```

```
[Out] -1/8*1/(a*x^8) - (b*Log[x])/a^2 + (b*Log[a + b*x^8])/(8*a^2)
```

fricas [A] time = 0.60, size = 33, normalized size = 0.94

$$\frac{bx^8\log\left(bx^8+a\right)-8\,bx^8\log(x)-a}{8\,a^2x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^8+a),x, algorithm="fricas")
```

```
[Out] 1/8*(b*x^8*log(b*x^8 + a) - 8*b*x^8*log(x) - a)/(a^2*x^8)
```

giac [A] time = 0.21, size = 43, normalized size = 1.23

$$-\frac{b\log\left(x^8\right)}{8\,a^2}+\frac{b\log\left(\left|bx^8+a\right|\right)}{8\,a^2}+\frac{bx^8-a}{8\,a^2x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^8+a),x, algorithm="giac")
```

```
[Out] -1/8*b*log(x^8)/a^2 + 1/8*b*log(abs(b*x^8 + a))/a^2 + 1/8*(b*x^8 - a)/(a^2*
x^8)
```

maple [A] time = 0.01, size = 32, normalized size = 0.91

$$-\frac{b\ln(x)}{a^2}+\frac{b\ln\left(b\,x^8+a\right)}{8a^2}-\frac{1}{8a\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(b*x^8+a),x)
```

```
[Out] -1/8/a/x^8-1/a^2*b*ln(x)+1/8*b*ln(b*x^8+a)/a^2
```

maxima [A] time = 1.05, size = 33, normalized size = 0.94

$$\frac{b\log\left(bx^8+a\right)}{8\,a^2}-\frac{b\log\left(x^8\right)}{8\,a^2}-\frac{1}{8\,ax^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^8+a),x, algorithm="maxima")
```

```
[Out] 1/8*b*log(b*x^8 + a)/a^2 - 1/8*b*log(x^8)/a^2 - 1/8/(a*x^8)
```

mupad [B] time = 0.08, size = 31, normalized size = 0.89

$$\frac{b\,\ln\left(b\,x^8+a\right)}{8\,a^2}-\frac{1}{8\,a\,x^8}-\frac{b\,\ln(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(a + b*x^8)),x)
```

```
[Out] (b*log(a + b*x^8))/(8*a^2) - 1/(8*a*x^8) - (b*log(x))/a^2
```

sympy [A] time = 1.68, size = 31, normalized size = 0.89

$$-\frac{1}{8ax^8}-\frac{b\log(x)}{a^2}+\frac{b\log\left(\frac{a}{b}+x^8\right)}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(b*x**8+a),x)
```

```
[Out] -1/(8*a*x**8) - b*log(x)/a**2 + b*log(a/b + x**8)/(8*a**2)
```

3.1461 $\int \frac{x^8}{a+bx^8}\,dx$

Optimal. Leaf size=272

$$\frac{\sqrt[8]{-a}\,\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{-a}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,b^{9/8}}-\frac{\sqrt[8]{-a}\,\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{-a}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,b^{9/8}}-\frac{\sqrt[8]{-a}\,\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4b^{9/8}}+\frac{\sqrt[8]{-a}}{}$$

```
[Out] x/b-1/4*(-a)^(1/8)*arctan(b^(1/8)*x/(-a)^(1/8))/b^(9/8)-1/4*(-a)^(1/8)*arct
anh(b^(1/8)*x/(-a)^(1/8))/b^(9/8)-1/8*(-a)^(1/8)*arctan(-1+b^(1/8)*x*2^(1/2
)/(-a)^(1/8))/b^(9/8)*2^(1/2)-1/8*(-a)^(1/8)*arctan(1+b^(1/8)*x*2^(1/2)/(-a
)^(1/8))/b^(9/8)*2^(1/2)+1/16*(-a)^(1/8)*ln((-a)^(1/4)+b^(1/4)*x^2-(-a)^(1/
8)*b^(1/8)*x*2^(1/2))/b^(9/8)*2^(1/2)-1/16*(-a)^(1/8)*ln((-a)^(1/4)+b^(1/4)
*x^2+(-a)^(1/8)*b^(1/8)*x*2^(1/2))/b^(9/8)*2^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {321, 214, 212, 208, 205, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt[8]{-a}\,\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{-a}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,b^{9/8}}-\frac{\sqrt[8]{-a}\,\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{-a}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,b^{9/8}}-\frac{\sqrt[8]{-a}\,\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4b^{9/8}}+\frac{\sqrt[8]{-a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^8),x]
```

```
[Out] x/b - ((-a)^(1/8)*ArcTan[(b^(1/8)*x)/(-a)^(1/8)])/(4*b^(9/8)) + ((-a)^(1/8)
*ArcTan[1 - (Sqrt[2]*b^(1/8)*x)/(-a)^(1/8)])/(4*Sqrt[2]*b^(9/8)) - ((-a)^(1
/8)*ArcTan[1 + (Sqrt[2]*b^(1/8)*x)/(-a)^(1/8)])/(4*Sqrt[2]*b^(9/8)) - ((-a)
^(1/8)*ArcTanh[(b^(1/8)*x)/(-a)^(1/8)])/(4*b^(9/8)) + ((-a)^(1/8)*Log[(-a)^
(1/4) - Sqrt[2]*(-a)^(1/8)*b^(1/8)*x + b^(1/4)*x^2])/(8*Sqrt[2]*b^(9/8)) -
((-a)^(1/8)*Log[(-a)^(1/4) + Sqrt[2]*(-a)^(1/8)*b^(1/8)*x + b^(1/4)*x^2])/(
8*Sqrt[2]*b^(9/8))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{a+bx^8}\,dx &= \frac{x}{b} - \frac{a\int \frac{1}{a+bx^8}\,dx}{b} \\
&= \frac{x}{b} - \frac{\sqrt{-a}\int \frac{1}{\sqrt{-a}-\sqrt{b}\,x^4}\,dx}{2b} - \frac{\sqrt{-a}\int \frac{1}{\sqrt{-a}+\sqrt{b}\,x^4}\,dx}{2b} \\
&= \frac{x}{b} - \frac{\sqrt[4]{-a}\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{b}\,x^2}\,dx}{4b} - \frac{\sqrt[4]{-a}\int \frac{1}{\sqrt[4]{-a}+\sqrt[4]{b}\,x^2}\,dx}{4b} - \frac{\sqrt[4]{-a}\int \frac{\sqrt[4]{-a}-\sqrt[4]{b}\,x^2}{\sqrt{-a}+\sqrt{b}\,x^4}\,dx}{4b} - \frac{\sqrt[4]{-a}\int \frac{\sqrt[4]{-a}+\sqrt[4]{b}\,x^2}{\sqrt{-a}+\sqrt{b}\,x^4}\,dx}{4b} \\
&= \frac{x}{b} - \frac{\sqrt[8]{-a}\,\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4b^{9/8}} - \frac{\sqrt[8]{-a}\,\tanh^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4b^{9/8}} - \frac{\sqrt[4]{-a}\int \frac{1}{\frac{\sqrt[4]{-a}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[8]{-a}\,x}{\sqrt[8]{b}}+x^2}\,dx}{8b^{5/4}} - \frac{\sqrt[4]{-a}\int \frac{1}{\frac{\sqrt[4]{-a}}{\sqrt{b}}+\frac{\sqrt{2}\,\sqrt[8]{-a}}{\sqrt[8]{b}}}}{8b^{5/4}} \\
&= \frac{x}{b} - \frac{\sqrt[8]{-a}\,\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4b^{9/8}} - \frac{\sqrt[8]{-a}\,\tanh^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4b^{9/8}} + \frac{\sqrt[8]{-a}\,\log\left(\sqrt[4]{-a}-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,b^{9/8}} - \frac{\sqrt[8]{-}}{} \\
&= \frac{x}{b} - \frac{\sqrt[8]{-a}\,\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4b^{9/8}} + \frac{\sqrt[8]{-a}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4\sqrt{2}\,b^{9/8}} - \frac{\sqrt[8]{-a}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4\sqrt{2}\,b^{9/8}} - \frac{\sqrt[8]{-a}\,\tanh^{-1}\left(}{4b^{9/8}}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 367, normalized size = 1.35

$$\frac{\sqrt[8]{a}\,\sin\left(\frac{\pi}{8}\right)\log\left(-2\sqrt[8]{a}\,\sqrt[8]{b}\,x\sin\left(\frac{\pi}{8}\right)+\sqrt[4]{a}+\sqrt[4]{b}\,x^2\right) - \sqrt[8]{a}\,\sin\left(\frac{\pi}{8}\right)\log\left(2\sqrt[8]{a}\,\sqrt[8]{b}\,x\sin\left(\frac{\pi}{8}\right)+\sqrt[4]{a}+\sqrt[4]{b}\,x^2\right) + \sqrt[8]{a}\,\mathrm{co}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^8),x]
```

```
[Out] (8*b^(1/8)*x - 2*a^(1/8)*ArcTan[(b^(1/8)*x*Sec[Pi/8])/a^(1/8) - Tan[Pi/8]]*
Cos[Pi/8] - 2*a^(1/8)*ArcTan[(b^(1/8)*x*Sec[Pi/8])/a^(1/8) + Tan[Pi/8]]*Cos
[Pi/8] + a^(1/8)*Cos[Pi/8]*Log[a^(1/4) + b^(1/4)*x^2 - 2*a^(1/8)*b^(1/8)*x*
Cos[Pi/8]] - a^(1/8)*Cos[Pi/8]*Log[a^(1/4) + b^(1/4)*x^2 + 2*a^(1/8)*b^(1/8
)*x*Cos[Pi/8]] + 2*a^(1/8)*ArcTan[Cot[Pi/8] - (b^(1/8)*x*Csc[Pi/8])/a^(1/8)
]*Sin[Pi/8] - 2*a^(1/8)*ArcTan[Cot[Pi/8] + (b^(1/8)*x*Csc[Pi/8])/a^(1/8)]*S
in[Pi/8] + a^(1/8)*Log[a^(1/4) + b^(1/4)*x^2 - 2*a^(1/8)*b^(1/8)*x*Sin[Pi/8
]]*Sin[Pi/8] - a^(1/8)*Log[a^(1/4) + b^(1/4)*x^2 + 2*a^(1/8)*b^(1/8)*x*Sin[
Pi/8]]*Sin[Pi/8])/(8*b^(9/8))
```

fricas [B] time = 0.74, size = 385, normalized size = 1.42

$$-\frac{4\sqrt{2}\,b\left(-\frac{a}{b^9}\right)^{\frac{1}{8}}\arctan\left(-\frac{\sqrt{2}\,b^8x\left(-\frac{a}{b^9}\right)^{\frac{7}{8}}-\sqrt{2}\,\sqrt{\sqrt{2}\,bx\left(-\frac{a}{b^9}\right)^{\frac{1}{8}}+b^2\left(-\frac{a}{b^9}\right)^{\frac{1}{4}}+x^2}\,b^8\left(-\frac{a}{b^9}\right)^{\frac{7}{8}}-a}{a}\right)+4\sqrt{2}\,b\left(-\frac{a}{b^9}\right)^{\frac{1}{8}}\arctan\left(-\frac{\sqrt{2}\,b^8x\left(-\frac{a}{b^9}\right.\right.}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^8+a),x, algorithm="fricas")
```

```
[Out] -1/16*(4*sqrt(2)*b*(-a/b^9)^(1/8)*arctan(-(sqrt(2)*b^8*x*(-a/b^9)^(7/8) - s
qrt(2)*sqrt(sqrt(2)*b*x*(-a/b^9)^(1/8) + b^2*(-a/b^9)^(1/4) + x^2)*b^8*(-a/
b^9)^(7/8) - a)/a) + 4*sqrt(2)*b*(-a/b^9)^(1/8)*arctan(-(sqrt(2)*b^8*x*(-a/
b^9)^(7/8) - sqrt(2)*sqrt(-sqrt(2)*b*x*(-a/b^9)^(1/8) + b^2*(-a/b^9)^(1/4)
```

```
+ x^2)*b^8*(-a/b^9)^(7/8) + a)/a) + sqrt(2)*b*(-a/b^9)^(1/8)*log(sqrt(2)*b*
x*(-a/b^9)^(1/8) + b^2*(-a/b^9)^(1/4) + x^2) - sqrt(2)*b*(-a/b^9)^(1/8)*log
(-sqrt(2)*b*x*(-a/b^9)^(1/8) + b^2*(-a/b^9)^(1/4) + x^2) + 8*b*(-a/b^9)^(1/
8)*arctan(-(b^8*x*(-a/b^9)^(7/8) - sqrt(b^2*(-a/b^9)^(1/4) + x^2)*b^8*(-a/b
^9)^(7/8))/a) + 2*b*(-a/b^9)^(1/8)*log(b*(-a/b^9)^(1/8) + x) - 2*b*(-a/b^9)
^(1/8)*log(-b*(-a/b^9)^(1/8) + x) - 16*x)/b
```

giac [B] time = 0.23, size = 442, normalized size = 1.62

$$\frac{x}{b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{8}}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{4b\sqrt{-2\sqrt{2}+4}}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{8}}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{4b\sqrt{-2\sqrt{2}+4}}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{8}}\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{4b\sqrt{2\sqrt{2}+4}}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{8}}\arcta}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^8+a),x, algorithm="giac")
```

```
[Out] x/b - 1/4*(a/b)^(1/8)*arctan((2*x + sqrt(-sqrt(2) + 2)*(a/b)^(1/8))/(sqrt(s
qrt(2) + 2)*(a/b)^(1/8)))/(b*sqrt(-2*sqrt(2) + 4)) - 1/4*(a/b)^(1/8)*arctan
((2*x - sqrt(-sqrt(2) + 2)*(a/b)^(1/8))/(sqrt(sqrt(2) + 2)*(a/b)^(1/8)))/(b
*sqrt(-2*sqrt(2) + 4)) - 1/4*(a/b)^(1/8)*arctan((2*x + sqrt(sqrt(2) + 2)*(a
/b)^(1/8))/(sqrt(-sqrt(2) + 2)*(a/b)^(1/8)))/(b*sqrt(2*sqrt(2) + 4)) - 1/4*
(a/b)^(1/8)*arctan((2*x - sqrt(sqrt(2) + 2)*(a/b)^(1/8))/(sqrt(-sqrt(2) + 2
)*(a/b)^(1/8)))/(b*sqrt(2*sqrt(2) + 4)) - 1/8*(a/b)^(1/8)*log(x^2 + x*sqrt(
sqrt(2) + 2)*(a/b)^(1/8) + (a/b)^(1/4))/(b*sqrt(-2*sqrt(2) + 4)) + 1/8*(a/b
)^(1/8)*log(x^2 - x*sqrt(sqrt(2) + 2)*(a/b)^(1/8) + (a/b)^(1/4))/(b*sqrt(-2
*sqrt(2) + 4)) - 1/8*(a/b)^(1/8)*log(x^2 + x*sqrt(-sqrt(2) + 2)*(a/b)^(1/8)
 + (a/b)^(1/4))/(b*sqrt(2*sqrt(2) + 4)) + 1/8*(a/b)^(1/8)*log(x^2 - x*sqrt(
-sqrt(2) + 2)*(a/b)^(1/8) + (a/b)^(1/4))/(b*sqrt(2*sqrt(2) + 4))
```

maple [C] time = 0.02, size = 34, normalized size = 0.12

$$-\frac{a\ln\left(-\operatorname{RootOf}\left(b_Z^8+a\right)+x\right)}{8b^2\operatorname{RootOf}\left(b_Z^8+a\right)^7}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^8+a),x)
```

```
[Out] 1/b*x-1/8*a/b^2*sum(1/_R^7*ln(-_R+x),_R=RootOf(_Z^8*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8}a\left(\frac{2\left(\frac{a}{b}\right)^{\frac{1}{8}}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{a\sqrt{-2\sqrt{2}+4}}+\frac{2\left(\frac{a}{b}\right)^{\frac{1}{8}}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{a\sqrt{-2\sqrt{2}+4}}+\frac{2\left(\frac{a}{b}\right)^{\frac{1}{8}}\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{a\sqrt{2\sqrt{2}+4}}+\frac{2\left(\frac{a}{b}\right)^{\frac{1}{8}}\arctan\left(\frac{2x-\sqrt{}}{a\sqrt{2\sqrt{2}+4}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^8+a),x, algorithm="maxima")
```

```
[Out] -a*integrate(1/(b*x^8 + a), x)/b + x/b
```

mupad [B] time = 0.15, size = 115, normalized size = 0.42

$$\frac{x}{b}-\frac{(-a)^{1/8}\operatorname{atan}\left(\frac{b^{1/8}x}{(-a)^{1/8}}\right)}{4b^{9/8}}+\frac{(-a)^{1/8}\operatorname{atan}\left(\frac{b^{1/8}x\,1\mathrm{i}}{(-a)^{1/8}}\right)1\mathrm{i}}{4b^{9/8}}+\frac{\sqrt{2}\,(-a)^{1/8}\operatorname{atan}\left(\frac{\sqrt{2}\,b^{1/8}x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\left(-\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)}{b^{9/8}}+\frac{\sqrt{2}\,(-a)^{1/8}\operatorname{atan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^8),x)
```

```
[Out] x/b - ((-a)^(1/8)*atan((b^(1/8)*x)/(-a)^(1/8)))/(4*b^(9/8)) + ((-a)^(1/8)*a
tan((b^(1/8)*x*1i)/(-a)^(1/8))*1i)/(4*b^(9/8)) - (2^(1/2)*(-a)^(1/8)*atan((
2^(1/2)*b^(1/8)*x*(1/2 - 1i/2))/(-a)^(1/8))*(1/8 + 1i/8))/b^(9/8) - (2^(1/2
)*(-a)^(1/8)*atan((2^(1/2)*b^(1/8)*x*(1/2 + 1i/2))/(-a)^(1/8))*(1/8 - 1i/8)
)/b^(9/8)
```

sympy [A] time = 0.42, size = 22, normalized size = 0.08

$$\operatorname{RootSum}\left(16777216t^8b^9+a,\left(t\mapsto t\log\left(-8tb+x\right)\right)\right)+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**8+a),x)
```

```
[Out] RootSum(16777216*_t**8*b**9 + a, Lambda(_t, _t*log(-8*_t*b + x))) + x/b
```

3.1462 $\int \frac{x^6}{a+bx^8}\,dx$

Optimal. Leaf size=267

$$\frac{\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{-a}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,\sqrt[8]{-a}\,b^{7/8}}-\frac{\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{-a}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,\sqrt[8]{-a}\,b^{7/8}}+\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4\sqrt[8]{-a}\,b^{7/8}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4\sqrt{2}\,\sqrt[8]{-a}\,b^{7/8}}$$

```
[Out] 1/4*arctan(b^(1/8)*x/(-a)^(1/8))/(-a)^(1/8)/b^(7/8)-1/4*arctanh(b^(1/8)*x/(
-a)^(1/8))/(-a)^(1/8)/b^(7/8)+1/8*arctan(-1+b^(1/8)*x*2^(1/2)/(-a)^(1/8))/(
-a)^(1/8)/b^(7/8)*2^(1/2)+1/8*arctan(1+b^(1/8)*x*2^(1/2)/(-a)^(1/8))/(-a)^(
1/8)/b^(7/8)*2^(1/2)+1/16*ln((-a)^(1/4)+b^(1/4)*x^2-(-a)^(1/8)*b^(1/8)*x*2^
(1/2))/(-a)^(1/8)/b^(7/8)*2^(1/2)-1/16*ln((-a)^(1/4)+b^(1/4)*x^2+(-a)^(1/8)
*b^(1/8)*x*2^(1/2))/(-a)^(1/8)/b^(7/8)*2^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {301, 297, 1162, 617, 204, 1165, 628, 298, 205, 208}

$$\frac{\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{-a}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,\sqrt[8]{-a}\,b^{7/8}}-\frac{\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{-a}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,\sqrt[8]{-a}\,b^{7/8}}+\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4\sqrt[8]{-a}\,b^{7/8}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4\sqrt{2}\,\sqrt[8]{-a}\,b^{7/8}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^8),x]
```

```
[Out] ArcTan[(b^(1/8)*x)/(-a)^(1/8)]/(4*(-a)^(1/8)*b^(7/8)) - ArcTan[1 - (Sqrt[2]
*b^(1/8)*x)/(-a)^(1/8)]/(4*Sqrt[2]*(-a)^(1/8)*b^(7/8)) + ArcTan[1 + (Sqrt[2
]*b^(1/8)*x)/(-a)^(1/8)]/(4*Sqrt[2]*(-a)^(1/8)*b^(7/8)) - ArcTanh[(b^(1/8)*
x)/(-a)^(1/8)]/(4*(-a)^(1/8)*b^(7/8)) + Log[(-a)^(1/4) - Sqrt[2]*(-a)^(1/8)
*b^(1/8)*x + b^(1/4)*x^2]/(8*Sqrt[2]*(-a)^(1/8)*b^(7/8)) - Log[(-a)^(1/4) +
 Sqrt[2]*(-a)^(1/8)*b^(1/8)*x + b^(1/4)*x^2]/(8*Sqrt[2]*(-a)^(1/8)*b^(7/8))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 301

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[x^(m - n/2
)/(r + s*x^(n/2)), x], x] - Dist[s/(2*b), Int[x^(m - n/2)/(r - s*x^(n/2)),
x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] &&
 LtQ[m, n] &&  !GtQ[a/b, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^6}{a+bx^8}\,dx = -\frac{\int \frac{x^2}{\sqrt{-a}-\sqrt{b}\,x^4}\,dx}{2\sqrt{b}} + \frac{\int \frac{x^2}{\sqrt{-a}+\sqrt{b}\,x^4}\,dx}{2\sqrt{b}}$$

$$= -\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{b}\,x^2}\,dx}{4b^{3/4}} + \frac{\int \frac{1}{\sqrt[4]{-a}+\sqrt[4]{b}\,x^2}\,dx}{4b^{3/4}} - \frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{b}\,x^2}{\sqrt{-a}+\sqrt{b}\,x^4}\,dx}{4b^{3/4}} + \frac{\int \frac{\sqrt[4]{-a}+\sqrt[4]{b}\,x^2}{\sqrt{-a}+\sqrt{b}\,x^4}\,dx}{4b^{3/4}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4\sqrt[8]{-a}\,b^{7/8}} - \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4\sqrt[8]{-a}\,b^{7/8}} + \frac{\int \frac{1}{\frac{\sqrt[4]{-a}}{\sqrt[4]{b}}-\frac{\sqrt{2}\,\sqrt[8]{-a}\,x}{\sqrt[8]{b}}+x^2}\,dx}{8b} + \frac{\int \frac{1}{\frac{\sqrt[4]{-a}}{\sqrt[4]{b}}+\frac{\sqrt{2}\,\sqrt[8]{-a}\,x}{\sqrt[8]{b}}+x^2}\,dx}{8b} + \frac{\int \frac{\frac{\sqrt{2}\,\sqrt[8]{-a}}{\sqrt[8]{b}}}{-\frac{\sqrt[4]{-a}}{\sqrt[4]{b}}-\frac{\sqrt{2}\,\sqrt[8]{}}{\sqrt[8]{}}}}{8\sqrt{2}\,\sqrt[8]{-}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4\sqrt[8]{-a}\,b^{7/8}} - \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4\sqrt[8]{-a}\,b^{7/8}} + \frac{\log\left(\sqrt[4]{-a}-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,\sqrt[8]{-a}\,b^{7/8}} - \frac{\log\left(\sqrt[4]{-a}+\sqrt{2}\,\sqrt[8]{-a}\right.}{8\sqrt{2}\,\sqrt[8]{-a}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4\sqrt[8]{-a}\,b^{7/8}} - \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4\sqrt{2}\,\sqrt[8]{-a}\,b^{7/8}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4\sqrt{2}\,\sqrt[8]{-a}\,b^{7/8}} - \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4\sqrt[8]{-a}\,b^{7/8}} + \frac{\log\left(\sqrt[4]{-a}-\sqrt{2}\right.}{8\sqrt{2}}$$

Mathematica [A] time = 0.07, size = 324, normalized size = 1.21

$$\sin\left(\frac{\pi}{8}\right)\log\left(-2\sqrt[8]{a}\,\sqrt[8]{b}\,x\sin\left(\frac{\pi}{8}\right)+\sqrt[4]{a}+\sqrt[4]{b}\,x^2\right)-\sin\left(\frac{\pi}{8}\right)\log\left(2\sqrt[8]{a}\,\sqrt[8]{b}\,x\sin\left(\frac{\pi}{8}\right)+\sqrt[4]{a}+\sqrt[4]{b}\,x^2\right)+\cos\left(\frac{\pi}{8}\right)\log$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^8),x]
```

```
[Out] (2*ArcTan[(b^(1/8)*x*Sec[Pi/8])/a^(1/8) - Tan[Pi/8]]*Cos[Pi/8] + 2*ArcTan[(
b^(1/8)*x*Sec[Pi/8])/a^(1/8) + Tan[Pi/8]]*Cos[Pi/8] + Cos[Pi/8]*Log[a^(1/4)
 + b^(1/4)*x^2 - 2*a^(1/8)*b^(1/8)*x*Cos[Pi/8]] - Cos[Pi/8]*Log[a^(1/4) + b
^(1/4)*x^2 + 2*a^(1/8)*b^(1/8)*x*Cos[Pi/8]] - 2*ArcTan[Cot[Pi/8] - (b^(1/8)
*x*Csc[Pi/8])/a^(1/8)]*Sin[Pi/8] + 2*ArcTan[Cot[Pi/8] + (b^(1/8)*x*Csc[Pi/8
])/a^(1/8)]*Sin[Pi/8] + Log[a^(1/4) + b^(1/4)*x^2 - 2*a^(1/8)*b^(1/8)*x*Sin
[Pi/8]]*Sin[Pi/8] - Log[a^(1/4) + b^(1/4)*x^2 + 2*a^(1/8)*b^(1/8)*x*Sin[Pi/
8]]*Sin[Pi/8])/(8*a^(1/8)*b^(7/8))
```

fricas [B] time = 0.90, size = 418, normalized size = 1.57

$$-\frac{1}{4}\sqrt{2}\left(-\frac{1}{ab^7}\right)^{\frac{1}{8}}\arctan\left(-\sqrt{2}\,bx\left(-\frac{1}{ab^7}\right)^{\frac{1}{8}}+\sqrt{2}\sqrt{\sqrt{2}\,ab^6x\left(-\frac{1}{ab^7}\right)^{\frac{7}{8}}-ab^5\left(-\frac{1}{ab^7}\right)^{\frac{3}{4}}+x^2}\,b\left(-\frac{1}{ab^7}\right)^{\frac{1}{8}}+1\right)-\frac{1}{4}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^8+a),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*(-1/(a*b^7))^(1/8)*arctan(-sqrt(2)*b*x*(-1/(a*b^7))^(1/8) + sq
rt(2)*sqrt(sqrt(2)*a*b^6*x*(-1/(a*b^7))^(7/8) - a*b^5*(-1/(a*b^7))^(3/4) +
x^2)*b*(-1/(a*b^7))^(1/8) + 1) - 1/4*sqrt(2)*(-1/(a*b^7))^(1/8)*arctan(-sqr
t(2)*b*x*(-1/(a*b^7))^(1/8) + sqrt(2)*sqrt(-sqrt(2)*a*b^6*x*(-1/(a*b^7))^(7
/8) - a*b^5*(-1/(a*b^7))^(3/4) + x^2)*b*(-1/(a*b^7))^(1/8) - 1) + 1/16*sqrt
(2)*(-1/(a*b^7))^(1/8)*log(sqrt(2)*a*b^6*x*(-1/(a*b^7))^(7/8) - a*b^5*(-1/(
a*b^7))^(3/4) + x^2) - 1/16*sqrt(2)*(-1/(a*b^7))^(1/8)*log(-sqrt(2)*a*b^6*x
*(-1/(a*b^7))^(7/8) - a*b^5*(-1/(a*b^7))^(3/4) + x^2) - 1/2*(-1/(a*b^7))^(1
```

```
/8)*arctan(-b*x*(-1/(a*b^7))^(1/8) + sqrt(-a*b^5*(-1/(a*b^7))^(3/4) + x^2)*
b*(-1/(a*b^7))^(1/8)) + 1/8*(-1/(a*b^7))^(1/8)*log(a*b^6*(-1/(a*b^7))^(7/8)
 + x) - 1/8*(-1/(a*b^7))^(1/8)*log(-a*b^6*(-1/(a*b^7))^(7/8) + x)
```

giac [B] time = 0.40, size = 437, normalized size = 1.64

$$\frac{\left(\frac{a}{b}\right)^{\frac{7}{8}}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{4a\sqrt{-2\sqrt{2}+4}}+\frac{\left(\frac{a}{b}\right)^{\frac{7}{8}}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{4a\sqrt{-2\sqrt{2}+4}}+\frac{\left(\frac{a}{b}\right)^{\frac{7}{8}}\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{4a\sqrt{2\sqrt{2}+4}}+\frac{\left(\frac{a}{b}\right)^{\frac{7}{8}}\arctan\left(\frac{2x-}{\sqrt{}}\right.}{4a\sqrt{2\sqrt{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^8+a),x, algorithm="giac")
```

```
[Out] 1/4*(a/b)^(7/8)*arctan((2*x + sqrt(-sqrt(2) + 2)*(a/b)^(1/8))/(sqrt(sqrt(2)
 + 2)*(a/b)^(1/8)))/(a*sqrt(-2*sqrt(2) + 4)) + 1/4*(a/b)^(7/8)*arctan((2*x
- sqrt(-sqrt(2) + 2)*(a/b)^(1/8))/(sqrt(sqrt(2) + 2)*(a/b)^(1/8)))/(a*sqrt(
-2*sqrt(2) + 4)) + 1/4*(a/b)^(7/8)*arctan((2*x + sqrt(sqrt(2) + 2)*(a/b)^(1
/8))/(sqrt(-sqrt(2) + 2)*(a/b)^(1/8)))/(a*sqrt(2*sqrt(2) + 4)) + 1/4*(a/b)^
(7/8)*arctan((2*x - sqrt(sqrt(2) + 2)*(a/b)^(1/8))/(sqrt(-sqrt(2) + 2)*(a/b
)^(1/8)))/(a*sqrt(2*sqrt(2) + 4)) - 1/8*(a/b)^(7/8)*log(x^2 + x*sqrt(sqrt(2
) + 2)*(a/b)^(1/8) + (a/b)^(1/4))/(a*sqrt(-2*sqrt(2) + 4)) + 1/8*(a/b)^(7/8
)*log(x^2 - x*sqrt(sqrt(2) + 2)*(a/b)^(1/8) + (a/b)^(1/4))/(a*sqrt(-2*sqrt(
2) + 4)) - 1/8*(a/b)^(7/8)*log(x^2 + x*sqrt(-sqrt(2) + 2)*(a/b)^(1/8) + (a/
b)^(1/4))/(a*sqrt(2*sqrt(2) + 4)) + 1/8*(a/b)^(7/8)*log(x^2 - x*sqrt(-sqrt(
2) + 2)*(a/b)^(1/8) + (a/b)^(1/4))/(a*sqrt(2*sqrt(2) + 4))
```

maple [C] time = 0.01, size = 27, normalized size = 0.10

$$\frac{\ln\left(-\operatorname{RootOf}\left(b_Z^8+a\right)+x\right)}{8b\operatorname{RootOf}\left(b_Z^8+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^8+a),x)
```

```
[Out] 1/8/b*sum(1/_R*ln(-_R+x),_R=RootOf(_Z^8*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{bx^8+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^8+a),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(b*x^8 + a), x)
```

mupad [B] time = 1.17, size = 110, normalized size = 0.41

$$\frac{\operatorname{atan}\left(\frac{b^{1/8}x}{(-a)^{1/8}}\right)}{4(-a)^{1/8}b^{7/8}}+\frac{\operatorname{atan}\left(\frac{b^{1/8}x\,1\mathrm{i}}{(-a)^{1/8}}\right)1\mathrm{i}}{4(-a)^{1/8}b^{7/8}}+\frac{\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,b^{1/8}x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\left(\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)}{(-a)^{1/8}b^{7/8}}+\frac{\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,b^{1/8}x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\left(\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)}{(-a)^{1/8}b^{7/8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^8),x)
```

```
[Out] atan((b^(1/8)*x)/(-a)^(1/8))/(4*(-a)^(1/8)*b^(7/8)) + (atan((b^(1/8)*x*1i)/
(-a)^(1/8))*1i)/(4*(-a)^(1/8)*b^(7/8)) + (2^(1/2)*atan((2^(1/2)*b^(1/8)*x*(
1/2 - 1i/2))/(-a)^(1/8))*(1/8 - 1i/8))/((-a)^(1/8)*b^(7/8)) + (2^(1/2)*atan
((2^(1/2)*b^(1/8)*x*(1/2 + 1i/2))/(-a)^(1/8))*(1/8 + 1i/8))/((-a)^(1/8)*b^(
7/8))
```

sympy [**A**] time = 0.34, size = 26, normalized size = 0.10

$$\text{RootSum}\left(16777216t^8ab^7+1,\left(t\mapsto t\log\left(2097152t^7ab^6+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**8+a),x)
```

```
[Out] RootSum(16777216*_t**8*a*b**7 + 1, Lambda(_t, _t*log(2097152*_t**7*a*b**6 +
 x)))
```

3.1463 $\int \frac{x^4}{a+bx^8}\,dx$

Optimal. Leaf size=267

$$-\frac{\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{-a}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,(-a)^{3/8}b^{5/8}}+\frac{\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{-a}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,(-a)^{3/8}b^{5/8}}-\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{3/8}b^{5/8}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4\sqrt{2}\,(-a)^{3/8}b^{5/8}}$$

```
[Out] -1/4*arctan(b^(1/8)*x/(-a)^(1/8))/(-a)^(3/8)/b^(5/8)-1/4*arctanh(b^(1/8)*x/
(-a)^(1/8))/(-a)^(3/8)/b^(5/8)+1/8*arctan(-1+b^(1/8)*x*2^(1/2)/(-a)^(1/8))/
(-a)^(3/8)/b^(5/8)*2^(1/2)+1/8*arctan(1+b^(1/8)*x*2^(1/2)/(-a)^(1/8))/(-a)^
(3/8)/b^(5/8)*2^(1/2)-1/16*ln((-a)^(1/4)+b^(1/4)*x^2-(-a)^(1/8)*b^(1/8)*x*2
^(1/2))/(-a)^(3/8)/b^(5/8)*2^(1/2)+1/16*ln((-a)^(1/4)+b^(1/4)*x^2+(-a)^(1/8
)*b^(1/8)*x*2^(1/2))/(-a)^(3/8)/b^(5/8)*2^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {301, 211, 1165, 628, 1162, 617, 204, 212, 208, 205}

$$-\frac{\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{-a}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,(-a)^{3/8}b^{5/8}}+\frac{\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{-a}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,(-a)^{3/8}b^{5/8}}-\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{3/8}b^{5/8}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4\sqrt{2}\,(-a)^{3/8}b^{5/8}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^8),x]
```

```
[Out] -ArcTan[(b^(1/8)*x)/(-a)^(1/8)]/(4*(-a)^(3/8)*b^(5/8)) - ArcTan[1 - (Sqrt[2
]*b^(1/8)*x)/(-a)^(1/8)]/(4*Sqrt[2]*(-a)^(3/8)*b^(5/8)) + ArcTan[1 + (Sqrt[
2]*b^(1/8)*x)/(-a)^(1/8)]/(4*Sqrt[2]*(-a)^(3/8)*b^(5/8)) - ArcTanh[(b^(1/8)
*x)/(-a)^(1/8)]/(4*(-a)^(3/8)*b^(5/8)) - Log[(-a)^(1/4) - Sqrt[2]*(-a)^(1/8
)*b^(1/8)*x + b^(1/4)*x^2]/(8*Sqrt[2]*(-a)^(3/8)*b^(5/8)) + Log[(-a)^(1/4)
+ Sqrt[2]*(-a)^(1/8)*b^(1/8)*x + b^(1/4)*x^2]/(8*Sqrt[2]*(-a)^(3/8)*b^(5/8)
)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 301

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[x^(m - n/2
)/(r + s*x^(n/2)), x], x] - Dist[s/(2*b), Int[x^(m - n/2)/(r - s*x^(n/2)),
x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] &&
 LtQ[m, n] &&  !GtQ[a/b, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^4}{a+bx^8}\,dx = -\frac{\int \frac{1}{\sqrt{-a}-\sqrt{b}\,x^4}\,dx}{2\sqrt{b}} + \frac{\int \frac{1}{\sqrt{-a}+\sqrt{b}\,x^4}\,dx}{2\sqrt{b}}$$

$$= -\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{b}\,x^2}\,dx}{4\sqrt[4]{-a}\,\sqrt{b}} - \frac{\int \frac{1}{\sqrt[4]{-a}+\sqrt[4]{b}\,x^2}\,dx}{4\sqrt[4]{-a}\,\sqrt{b}} + \frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{b}\,x^2}{\sqrt{-a}+\sqrt{b}\,x^4}\,dx}{4\sqrt[4]{-a}\,\sqrt{b}} + \frac{\int \frac{\sqrt[4]{-a}+\sqrt[4]{b}\,x^2}{\sqrt{-a}+\sqrt{b}\,x^4}\,dx}{4\sqrt[4]{-a}\,\sqrt{b}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{3/8}b^{5/8}} - \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{3/8}b^{5/8}} + \frac{\int \frac{1}{\frac{\sqrt[4]{-a}}{\sqrt[4]{b}}-\frac{\sqrt{2}\,\sqrt[8]{-a}\,x}{\sqrt[8]{b}}+x^2}\,dx}{8\sqrt[4]{-a}\,b^{3/4}} + \frac{\int \frac{1}{\frac{\sqrt[4]{-a}}{\sqrt[4]{b}}+\frac{\sqrt{2}\,\sqrt[8]{-a}\,x}{\sqrt[8]{b}}+x^2}\,dx}{8\sqrt[4]{-a}\,b^{3/4}} - \frac{\int \frac{\frac{\sqrt{2}\,\sqrt[8]{-a}}{\sqrt[8]{b}}+2}{-\frac{\sqrt[4]{-a}}{\sqrt[4]{b}}-\frac{\sqrt{2}\,\sqrt[8]{-a}\,x}{\sqrt[8]{b}}-x^2}\,dx}{8\sqrt{2}\,(-a)^{3/8}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{3/8}b^{5/8}} - \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{3/8}b^{5/8}} - \frac{\log\left(\sqrt[4]{-a}-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,(-a)^{3/8}b^{5/8}} + \frac{\log\left(\sqrt[4]{-a}+\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{}\right.}{8\sqrt{2}\,(-a)^{3/8}b}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{3/8}b^{5/8}} - \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4\sqrt{2}\,(-a)^{3/8}b^{5/8}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4\sqrt{2}\,(-a)^{3/8}b^{5/8}} - \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{3/8}b^{5/8}} - \frac{\log\left(\sqrt[4]{-a}-\sqrt{2}\,\right.}{8\sqrt{2}\,(-}$$

Mathematica [A] time = 0.10, size = 324, normalized size = 1.21

$$-\frac{\cos\left(\frac{\pi}{8}\right)\log\left(-2\sqrt[8]{a}\,\sqrt[8]{b}\,x\sin\left(\frac{\pi}{8}\right)+\sqrt[4]{a}+\sqrt[4]{b}\,x^2\right)-\cos\left(\frac{\pi}{8}\right)\log\left(2\sqrt[8]{a}\,\sqrt[8]{b}\,x\sin\left(\frac{\pi}{8}\right)+\sqrt[4]{a}+\sqrt[4]{b}\,x^2\right)-\sin\left(\frac{\pi}{8}\right)\log($$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^8),x]
```

```
[Out] -1/8*(2*ArcTan[Cot[Pi/8] - (b^(1/8)*x*Csc[Pi/8])/a^(1/8)]*Cos[Pi/8] - 2*Arc
Tan[Cot[Pi/8] + (b^(1/8)*x*Csc[Pi/8])/a^(1/8)]*Cos[Pi/8] + Cos[Pi/8]*Log[a^
(1/4) + b^(1/4)*x^2 - 2*a^(1/8)*b^(1/8)*x*Sin[Pi/8]] - Cos[Pi/8]*Log[a^(1/4
) + b^(1/4)*x^2 + 2*a^(1/8)*b^(1/8)*x*Sin[Pi/8]] + 2*ArcTan[(b^(1/8)*x*Sec[
Pi/8])/a^(1/8) - Tan[Pi/8]]*Sin[Pi/8] + 2*ArcTan[(b^(1/8)*x*Sec[Pi/8])/a^(1
/8) + Tan[Pi/8]]*Sin[Pi/8] - Log[a^(1/4) + b^(1/4)*x^2 - 2*a^(1/8)*b^(1/8)*
x*Cos[Pi/8]]*Sin[Pi/8] + Log[a^(1/4) + b^(1/4)*x^2 + 2*a^(1/8)*b^(1/8)*x*Co
s[Pi/8]]*Sin[Pi/8])/(a^(3/8)*b^(5/8))
```

fricas [B] time = 0.70, size = 438, normalized size = 1.64

$$\frac{1}{4}\sqrt{2}\left(-\frac{1}{a^3b^5}\right)^{\frac{1}{8}}\arctan\left(-\sqrt{2}\,ab^2x\left(-\frac{1}{a^3b^5}\right)^{\frac{3}{8}}+\sqrt{2}\sqrt{\sqrt{2}\,a^2b^3x\left(-\frac{1}{a^3b^5}\right)^{\frac{5}{8}}-ab\left(-\frac{1}{a^3b^5}\right)^{\frac{1}{4}}+x^2}\,ab^2\left(-\frac{1}{a^3b^5}\right)^{\frac{3}{8}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^8+a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*(-1/(a^3*b^5))^(1/8)*arctan(-sqrt(2)*a*b^2*x*(-1/(a^3*b^5))^(3/
8) + sqrt(2)*sqrt(sqrt(2)*a^2*b^3*x*(-1/(a^3*b^5))^(5/8) - a*b*(-1/(a^3*b^5
))^(1/4) + x^2)*a*b^2*(-1/(a^3*b^5))^(3/8) + 1) + 1/4*sqrt(2)*(-1/(a^3*b^5)
)^(1/8)*arctan(-sqrt(2)*a*b^2*x*(-1/(a^3*b^5))^(3/8) + sqrt(2)*sqrt(-sqrt(2
)*a^2*b^3*x*(-1/(a^3*b^5))^(5/8) - a*b*(-1/(a^3*b^5))^(1/4) + x^2)*a*b^2*(-
1/(a^3*b^5))^(3/8) - 1) + 1/16*sqrt(2)*(-1/(a^3*b^5))^(1/8)*log(sqrt(2)*a^2
*b^3*x*(-1/(a^3*b^5))^(5/8) - a*b*(-1/(a^3*b^5))^(1/4) + x^2) - 1/16*sqrt(2
)*(-1/(a^3*b^5))^(1/8)*log(-sqrt(2)*a^2*b^3*x*(-1/(a^3*b^5))^(5/8) - a*b*(-
1/(a^3*b^5))^(1/4) + x^2) - 1/2*(-1/(a^3*b^5))^(1/8)*arctan(-a*b^2*x*(-1/(a
```

```
^3*b^5))^(3/8) + sqrt(-a*b*(-1/(a^3*b^5))^(1/4) + x^2)*a*b^2*(-1/(a^3*b^5))
^(3/8)) - 1/8*(-1/(a^3*b^5))^(1/8)*log(a^2*b^3*(-1/(a^3*b^5))^(5/8) + x) +
1/8*(-1/(a^3*b^5))^(1/8)*log(-a^2*b^3*(-1/(a^3*b^5))^(5/8) + x)
```

giac [B] time = 0.37, size = 437, normalized size = 1.64

$$-\frac{\left(\frac{a}{b}\right)^{\frac{5}{8}}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{4a\sqrt{2\sqrt{2}+4}}-\frac{\left(\frac{a}{b}\right)^{\frac{5}{8}}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{4a\sqrt{2\sqrt{2}+4}}+\frac{\left(\frac{a}{b}\right)^{\frac{5}{8}}\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{4a\sqrt{-2\sqrt{2}+4}}+\frac{\left(\frac{a}{b}\right)^{\frac{5}{8}}\arctan}{4a\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^8+a),x, algorithm="giac")
```

```
[Out] -1/4*(a/b)^(5/8)*arctan((2*x + sqrt(-sqrt(2) + 2)*(a/b)^(1/8))/(sqrt(sqrt(2
) + 2)*(a/b)^(1/8)))/(a*sqrt(2*sqrt(2) + 4)) - 1/4*(a/b)^(5/8)*arctan((2*x
- sqrt(-sqrt(2) + 2)*(a/b)^(1/8))/(sqrt(sqrt(2) + 2)*(a/b)^(1/8)))/(a*sqrt(
2*sqrt(2) + 4)) + 1/4*(a/b)^(5/8)*arctan((2*x + sqrt(sqrt(2) + 2)*(a/b)^(1/
8))/(sqrt(-sqrt(2) + 2)*(a/b)^(1/8)))/(a*sqrt(-2*sqrt(2) + 4)) + 1/4*(a/b)^
(5/8)*arctan((2*x - sqrt(sqrt(2) + 2)*(a/b)^(1/8))/(sqrt(-sqrt(2) + 2)*(a/b
)^(1/8)))/(a*sqrt(-2*sqrt(2) + 4)) - 1/8*(a/b)^(5/8)*log(x^2 + x*sqrt(sqrt(
2) + 2)*(a/b)^(1/8) + (a/b)^(1/4))/(a*sqrt(2*sqrt(2) + 4)) + 1/8*(a/b)^(5/8
)*log(x^2 - x*sqrt(sqrt(2) + 2)*(a/b)^(1/8) + (a/b)^(1/4))/(a*sqrt(2*sqrt(2
) + 4)) + 1/8*(a/b)^(5/8)*log(x^2 + x*sqrt(-sqrt(2) + 2)*(a/b)^(1/8) + (a/b
)^(1/4))/(a*sqrt(-2*sqrt(2) + 4)) - 1/8*(a/b)^(5/8)*log(x^2 - x*sqrt(-sqrt(
2) + 2)*(a/b)^(1/8) + (a/b)^(1/4))/(a*sqrt(-2*sqrt(2) + 4))
```

maple [C] time = 0.01, size = 27, normalized size = 0.10

$$\frac{\ln\left(-\operatorname{RootOf}\left(b_Z^8+a\right)+x\right)}{8b\operatorname{RootOf}\left(b_Z^8+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^8+a),x)
```

```
[Out] 1/8/b*sum(1/_R^3*ln(-_R+x),_R=RootOf(_Z^8*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{bx^8+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^8+a),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(b*x^8 + a), x)
```

mupad [B] time = 1.17, size = 110, normalized size = 0.41

$$-\frac{\operatorname{atan}\left(\frac{b^{1/8}x}{(-a)^{1/8}}\right)}{4(-a)^{3/8}b^{5/8}}+\frac{\operatorname{atan}\left(\frac{b^{1/8}x\,1\mathrm{i}}{(-a)^{1/8}}\right)1\mathrm{i}}{4(-a)^{3/8}b^{5/8}}+\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\,b^{1/8}x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\left(\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)}{(-a)^{3/8}b^{5/8}}+\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\,b^{1/8}x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\left(\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)}{(-a)^{3/8}b^{5/8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^8),x)
```

```
[Out] (atan((b^(1/8)*x*1i)/(-a)^(1/8))*1i)/(4*(-a)^(3/8)*b^(5/8)) - atan((b^(1/8)
*x)/(-a)^(1/8))/(4*(-a)^(3/8)*b^(5/8)) + (2^(1/2)*atan((2^(1/2)*b^(1/8)*x*(
1/2 - 1i/2))/(-a)^(1/8))*(1/8 + 1i/8))/((-a)^(3/8)*b^(5/8)) + (2^(1/2)*atan
((2^(1/2)*b^(1/8)*x*(1/2 + 1i/2))/(-a)^(1/8))*(1/8 - 1i/8))/((-a)^(3/8)*b^(
5/8))
```

sympy [**A**] time = 0.34, size = 29, normalized size = 0.11

$$\text{RootSum}\left(16777216t^8a^3b^5+1,\left(t\mapsto t\log\left(-32768t^5a^2b^3+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**8+a),x)
```

```
[Out] RootSum(16777216*_t**8*a**3*b**5 + 1, Lambda(_t, _t*log(-32768*_t**5*a**2*b
**3 + x)))
```

3.1464 $\int \frac{x^2}{a+bx^8}\,dx$

Optimal. Leaf size=267

$$-\frac{\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{-a}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,(-a)^{5/8}b^{3/8}}+\frac{\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{-a}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,(-a)^{5/8}b^{3/8}}+\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{5/8}b^{3/8}}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{b}}{\sqrt[8]{-a}}\right.}{4\sqrt{2}\,(-a)^{5/8}b^{3/}}$$

```
[Out] 1/4*arctan(b^(1/8)*x/(-a)^(1/8))/(-a)^(5/8)/b^(3/8)-1/4*arctanh(b^(1/8)*x/(
-a)^(1/8))/(-a)^(5/8)/b^(3/8)-1/8*arctan(-1+b^(1/8)*x*2^(1/2)/(-a)^(1/8))/(
-a)^(5/8)/b^(3/8)*2^(1/2)-1/8*arctan(1+b^(1/8)*x*2^(1/2)/(-a)^(1/8))/(-a)^(
5/8)/b^(3/8)*2^(1/2)-1/16*ln((-a)^(1/4)+b^(1/4)*x^2-(-a)^(1/8)*b^(1/8)*x*2^
(1/2))/(-a)^(5/8)/b^(3/8)*2^(1/2)+1/16*ln((-a)^(1/4)+b^(1/4)*x^2+(-a)^(1/8)
*b^(1/8)*x*2^(1/2))/(-a)^(5/8)/b^(3/8)*2^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {300, 297, 1162, 617, 204, 1165, 628, 298, 205, 208}

$$-\frac{\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{-a}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,(-a)^{5/8}b^{3/8}}+\frac{\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{-a}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,(-a)^{5/8}b^{3/8}}+\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{5/8}b^{3/8}}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{b}}{\sqrt[8]{-a}}\right.}{4\sqrt{2}\,(-a)^{5/8}b^{3/}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^8),x]
```

```
[Out] ArcTan[(b^(1/8)*x)/(-a)^(1/8)]/(4*(-a)^(5/8)*b^(3/8)) + ArcTan[1 - (Sqrt[2]
*b^(1/8)*x)/(-a)^(1/8)]/(4*Sqrt[2]*(-a)^(5/8)*b^(3/8)) - ArcTan[1 + (Sqrt[2
]*b^(1/8)*x)/(-a)^(1/8)]/(4*Sqrt[2]*(-a)^(5/8)*b^(3/8)) - ArcTanh[(b^(1/8)*
x)/(-a)^(1/8)]/(4*(-a)^(5/8)*b^(3/8)) - Log[(-a)^(1/4) - Sqrt[2]*(-a)^(1/8)
*b^(1/8)*x + b^(1/4)*x^2]/(8*Sqrt[2]*(-a)^(5/8)*b^(3/8)) + Log[(-a)^(1/4) +
 Sqrt[2]*(-a)^(1/8)*b^(1/8)*x + b^(1/4)*x^2]/(8*Sqrt[2]*(-a)^(5/8)*b^(3/8))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 300

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[x^m/(r + s
*x^(n/2)), x], x] + Dist[r/(2*a), Int[x^m/(r - s*x^(n/2)), x], x]] /; FreeQ
[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n/2] &&  !GtQ[a/b, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^2}{a+bx^8}\,dx = -\frac{\int \frac{x^2}{\sqrt{-a}-\sqrt{b}\,x^4}\,dx}{2\sqrt{-a}} - \frac{\int \frac{x^2}{\sqrt{-a}+\sqrt{b}\,x^4}\,dx}{2\sqrt{-a}}$$

$$= -\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{b}\,x^2}\,dx}{4\sqrt{-a}\,\sqrt[4]{b}} + \frac{\int \frac{1}{\sqrt[4]{-a}+\sqrt[4]{b}\,x^2}\,dx}{4\sqrt{-a}\,\sqrt[4]{b}} + \frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{b}\,x^2}{\sqrt{-a}+\sqrt{b}\,x^4}\,dx}{4\sqrt{-a}\,\sqrt[4]{b}} - \frac{\int \frac{\sqrt[4]{-a}+\sqrt[4]{b}\,x^2}{\sqrt{-a}+\sqrt{b}\,x^4}\,dx}{4\sqrt{-a}\,\sqrt[4]{b}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{5/8}b^{3/8}} - \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{5/8}b^{3/8}} - \frac{\int \frac{1}{\frac{\sqrt[4]{-a}}{\sqrt[4]{b}}-\frac{\sqrt{2}\,\sqrt[8]{-a}\,x}{\sqrt[8]{b}}+x^2}\,dx}{8\sqrt{-a}\,\sqrt{b}} - \frac{\int \frac{1}{\frac{\sqrt[4]{-a}}{\sqrt[4]{b}}+\frac{\sqrt{2}\,\sqrt[8]{-a}\,x}{\sqrt[8]{b}}+x^2}\,dx}{8\sqrt{-a}\,\sqrt{b}} - \frac{\int \frac{\frac{\sqrt{2}\,\sqrt[8]{-a}}{\sqrt[8]{b}}+}{-\frac{\sqrt[4]{-a}}{\sqrt[4]{b}}-\frac{\sqrt{2}\,\sqrt[8]{}}{\sqrt[8]{b}}}}{8\sqrt{2}\,(-a)^{5}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{5/8}b^{3/8}} - \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{5/8}b^{3/8}} - \frac{\log\left(\sqrt[4]{-a}-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,(-a)^{5/8}b^{3/8}} + \frac{\log\left(\sqrt[4]{-a}+\sqrt{2}\,\sqrt[8]{-a}\right.}{8\sqrt{2}\,(-a)^{5/}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{5/8}b^{3/8}} + \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4\sqrt{2}\,(-a)^{5/8}b^{3/8}} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4\sqrt{2}\,(-a)^{5/8}b^{3/8}} - \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{5/8}b^{3/8}} - \frac{\log\left(\sqrt[4]{-a}-\sqrt{2}\right.}{8\sqrt{2}\,(}$$

Mathematica [A] time = 0.09, size = 324, normalized size = 1.21

$$-\frac{-\cos\left(\frac{\pi}{8}\right)\log\left(-2\sqrt[8]{a}\,\sqrt[8]{b}\,x\sin\left(\frac{\pi}{8}\right)+\sqrt[4]{a}+\sqrt[4]{b}\,x^2\right)+\cos\left(\frac{\pi}{8}\right)\log\left(2\sqrt[8]{a}\,\sqrt[8]{b}\,x\sin\left(\frac{\pi}{8}\right)+\sqrt[4]{a}+\sqrt[4]{b}\,x^2\right)+\sin\left(\frac{\pi}{8}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^8),x]
```

```
[Out] -1/8*(2*ArcTan[Cot[Pi/8] - (b^(1/8)*x*Csc[Pi/8])/a^(1/8)]*Cos[Pi/8] - 2*Arc
Tan[Cot[Pi/8] + (b^(1/8)*x*Csc[Pi/8])/a^(1/8)]*Cos[Pi/8] - Cos[Pi/8]*Log[a^
(1/4) + b^(1/4)*x^2 - 2*a^(1/8)*b^(1/8)*x*Sin[Pi/8]] + Cos[Pi/8]*Log[a^(1/4
) + b^(1/4)*x^2 + 2*a^(1/8)*b^(1/8)*x*Sin[Pi/8]] + 2*ArcTan[(b^(1/8)*x*Sec[
Pi/8])/a^(1/8) - Tan[Pi/8]]*Sin[Pi/8] + 2*ArcTan[(b^(1/8)*x*Sec[Pi/8])/a^(1
/8) + Tan[Pi/8]]*Sin[Pi/8] + Log[a^(1/4) + b^(1/4)*x^2 - 2*a^(1/8)*b^(1/8)*
x*Cos[Pi/8]]*Sin[Pi/8] - Log[a^(1/4) + b^(1/4)*x^2 + 2*a^(1/8)*b^(1/8)*x*Co
s[Pi/8]]*Sin[Pi/8])/(a^(5/8)*b^(3/8))
```

fricas [B] time = 0.58, size = 453, normalized size = 1.70

$$-\frac{1}{4}\sqrt{2}\left(-\frac{1}{a^5b^3}\right)^{\frac{1}{8}}\arctan\left(-\sqrt{2}\,a^3b^2x\left(-\frac{1}{a^5b^3}\right)^{\frac{5}{8}}+\sqrt{2}\sqrt{a^4b^2\left(-\frac{1}{a^5b^3}\right)^{\frac{3}{4}}+\sqrt{2}\,a^2bx\left(-\frac{1}{a^5b^3}\right)^{\frac{3}{8}}+x^2}\,a^3b^2\left(-\frac{1}{a^5b^3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^8+a),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*(-1/(a^5*b^3))^(1/8)*arctan(-sqrt(2)*a^3*b^2*x*(-1/(a^5*b^3))^
(5/8) + sqrt(2)*sqrt(a^4*b^2*(-1/(a^5*b^3))^(3/4) + sqrt(2)*a^2*b*x*(-1/(a^
5*b^3))^(3/8) + x^2)*a^3*b^2*(-1/(a^5*b^3))^(5/8) + 1) - 1/4*sqrt(2)*(-1/(a
^5*b^3))^(1/8)*arctan(-sqrt(2)*a^3*b^2*x*(-1/(a^5*b^3))^(5/8) + sqrt(2)*sqr
t(a^4*b^2*(-1/(a^5*b^3))^(3/4) - sqrt(2)*a^2*b*x*(-1/(a^5*b^3))^(3/8) + x^2
)*a^3*b^2*(-1/(a^5*b^3))^(5/8) - 1) + 1/16*sqrt(2)*(-1/(a^5*b^3))^(1/8)*log
(a^4*b^2*(-1/(a^5*b^3))^(3/4) + sqrt(2)*a^2*b*x*(-1/(a^5*b^3))^(3/8) + x^2)
 - 1/16*sqrt(2)*(-1/(a^5*b^3))^(1/8)*log(a^4*b^2*(-1/(a^5*b^3))^(3/4) - sqr
```

```
t(2)*a^2*b*x*(-1/(a^5*b^3))^(3/8) + x^2) + 1/2*(-1/(a^5*b^3))^(1/8)*arctan(
-a^3*b^2*x*(-1/(a^5*b^3))^(5/8) + sqrt(a^4*b^2*(-1/(a^5*b^3))^(3/4) + x^2)*
a^3*b^2*(-1/(a^5*b^3))^(5/8)) - 1/8*(-1/(a^5*b^3))^(1/8)*log(a^2*b*(-1/(a^5
*b^3))^(3/8) + x) + 1/8*(-1/(a^5*b^3))^(1/8)*log(-a^2*b*(-1/(a^5*b^3))^(3/8
) + x)
```

giac [B] time = 0.35, size = 437, normalized size = 1.64

$$-\frac{\left(\frac{a}{b}\right)^{\frac{3}{8}}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{4a\sqrt{2\sqrt{2}+4}}-\frac{\left(\frac{a}{b}\right)^{\frac{3}{8}}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{4a\sqrt{2\sqrt{2}+4}}+\frac{\left(\frac{a}{b}\right)^{\frac{3}{8}}\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{4a\sqrt{-2\sqrt{2}+4}}+\frac{\left(\frac{a}{b}\right)^{\frac{3}{8}}\arctan\left(\frac{2}{4a\sqrt{-2}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^8+a),x, algorithm="giac")
```

```
[Out] -1/4*(a/b)^(3/8)*arctan((2*x + sqrt(-sqrt(2) + 2)*(a/b)^(1/8))/(sqrt(sqrt(2
) + 2)*(a/b)^(1/8)))/(a*sqrt(2*sqrt(2) + 4)) - 1/4*(a/b)^(3/8)*arctan((2*x
- sqrt(-sqrt(2) + 2)*(a/b)^(1/8))/(sqrt(sqrt(2) + 2)*(a/b)^(1/8)))/(a*sqrt(
2*sqrt(2) + 4)) + 1/4*(a/b)^(3/8)*arctan((2*x + sqrt(sqrt(2) + 2)*(a/b)^(1/
8))/(sqrt(-sqrt(2) + 2)*(a/b)^(1/8)))/(a*sqrt(-2*sqrt(2) + 4)) + 1/4*(a/b)^
(3/8)*arctan((2*x - sqrt(sqrt(2) + 2)*(a/b)^(1/8))/(sqrt(-sqrt(2) + 2)*(a/b
)^(1/8)))/(a*sqrt(-2*sqrt(2) + 4)) + 1/8*(a/b)^(3/8)*log(x^2 + x*sqrt(sqrt(
2) + 2)*(a/b)^(1/8) + (a/b)^(1/4))/(a*sqrt(2*sqrt(2) + 4)) - 1/8*(a/b)^(3/8
)*log(x^2 - x*sqrt(sqrt(2) + 2)*(a/b)^(1/8) + (a/b)^(1/4))/(a*sqrt(2*sqrt(2
) + 4)) - 1/8*(a/b)^(3/8)*log(x^2 + x*sqrt(-sqrt(2) + 2)*(a/b)^(1/8) + (a/b
)^(1/4))/(a*sqrt(-2*sqrt(2) + 4)) + 1/8*(a/b)^(3/8)*log(x^2 - x*sqrt(-sqrt(
2) + 2)*(a/b)^(1/8) + (a/b)^(1/4))/(a*sqrt(-2*sqrt(2) + 4))
```

maple [C] time = 0.01, size = 27, normalized size = 0.10

$$\frac{\ln\left(-\operatorname{RootOf}\left(b_Z^8+a\right)+x\right)}{8b\operatorname{RootOf}\left(b_Z^8+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^8+a),x)
```

```
[Out] 1/8/b*sum(1/_R^5*ln(-_R+x),_R=RootOf(_Z^8*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{bx^8+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^8+a),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*x^8 + a), x)
```

mupad [B] time = 1.17, size = 110, normalized size = 0.41

$$\frac{\operatorname{atan}\left(\frac{b^{1/8}x}{(-a)^{1/8}}\right)}{4(-a)^{5/8}b^{3/8}}+\frac{\operatorname{atan}\left(\frac{b^{1/8}x\,1\mathrm{i}}{(-a)^{1/8}}\right)1\mathrm{i}}{4(-a)^{5/8}b^{3/8}}+\frac{\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,b^{1/8}x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\left(-\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)}{(-a)^{5/8}b^{3/8}}+\frac{\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,b^{1/8}x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\left(-\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)}{(-a)^{5/8}b^{3/8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^8),x)
```

```
[Out] atan((b^(1/8)*x)/(-a)^(1/8))/(4*(-a)^(5/8)*b^(3/8)) + (atan((b^(1/8)*x*1i)/
(-a)^(1/8))*1i)/(4*(-a)^(5/8)*b^(3/8)) - (2^(1/2)*atan((2^(1/2)*b^(1/8)*x*(
1/2 - 1i/2))/(-a)^(1/8))*(1/8 - 1i/8))/((-a)^(5/8)*b^(3/8)) - (2^(1/2)*atan
((2^(1/2)*b^(1/8)*x*(1/2 + 1i/2))/(-a)^(1/8))*(1/8 + 1i/8))/((-a)^(5/8)*b^(
3/8))
```

sympy [**A**] time = 0.35, size = 27, normalized size = 0.10

$$\operatorname{RootSum}\left(16777216t^8a^5b^3+1,\left(t\mapsto t\log\left(-512t^3a^2b+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**8+a),x)
```

```
[Out] RootSum(16777216*_t**8*a**5*b**3 + 1, Lambda(_t, _t*log(-512*_t**3*a**2*b +
 x)))
```

3.1465 $\int \frac{1}{a+bx^8}\,dx$

Optimal. Leaf size=267

$$\frac{\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{-a}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,(-a)^{7/8}\sqrt[8]{b}}-\frac{\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{-a}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,(-a)^{7/8}\sqrt[8]{b}}-\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt[8]{b}}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4\sqrt{2}\,(-a)^{7/8}\sqrt[8]{b}}-$$

```
[Out] -1/4*arctan(b^(1/8)*x/(-a)^(1/8))/(-a)^(7/8)/b^(1/8)-1/4*arctanh(b^(1/8)*x/
(-a)^(1/8))/(-a)^(7/8)/b^(1/8)-1/8*arctan(-1+b^(1/8)*x*2^(1/2)/(-a)^(1/8))/
(-a)^(7/8)/b^(1/8)*2^(1/2)-1/8*arctan(1+b^(1/8)*x*2^(1/2)/(-a)^(1/8))/(-a)^
(7/8)/b^(1/8)*2^(1/2)+1/16*ln((-a)^(1/4)+b^(1/4)*x^2-(-a)^(1/8)*b^(1/8)*x*2
^(1/2))/(-a)^(7/8)/b^(1/8)*2^(1/2)-1/16*ln((-a)^(1/4)+b^(1/4)*x^2+(-a)^(1/8
)*b^(1/8)*x*2^(1/2))/(-a)^(7/8)/b^(1/8)*2^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 1.111$, Rules used = {214, 212, 208, 205, 211, 1165, 628, 1162, 617, 204}

$$\frac{\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{-a}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,(-a)^{7/8}\sqrt[8]{b}}-\frac{\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{-a}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,(-a)^{7/8}\sqrt[8]{b}}-\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt[8]{b}}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4\sqrt{2}\,(-a)^{7/8}\sqrt[8]{b}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^8)^(-1),x]
```

```
[Out] -ArcTan[(b^(1/8)*x)/(-a)^(1/8)]/(4*(-a)^(7/8)*b^(1/8)) + ArcTan[1 - (Sqrt[2
]*b^(1/8)*x)/(-a)^(1/8)]/(4*Sqrt[2]*(-a)^(7/8)*b^(1/8)) - ArcTan[1 + (Sqrt[
2]*b^(1/8)*x)/(-a)^(1/8)]/(4*Sqrt[2]*(-a)^(7/8)*b^(1/8)) - ArcTanh[(b^(1/8)
*x)/(-a)^(1/8)]/(4*(-a)^(7/8)*b^(1/8)) + Log[(-a)^(1/4) - Sqrt[2]*(-a)^(1/8
)*b^(1/8)*x + b^(1/4)*x^2]/(8*Sqrt[2]*(-a)^(7/8)*b^(1/8)) - Log[(-a)^(1/4)
+ Sqrt[2]*(-a)^(1/8)*b^(1/8)*x + b^(1/4)*x^2]/(8*Sqrt[2]*(-a)^(7/8)*b^(1/8)
)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{a+bx^8}\,dx = -\frac{\int \frac{1}{\sqrt{-a}-\sqrt{b}\,x^4}\,dx}{2\sqrt{-a}} - \frac{\int \frac{1}{\sqrt{-a}+\sqrt{b}\,x^4}\,dx}{2\sqrt{-a}}$$

$$= -\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{b}\,x^2}\,dx}{4(-a)^{3/4}} - \frac{\int \frac{1}{\sqrt[4]{-a}+\sqrt[4]{b}\,x^2}\,dx}{4(-a)^{3/4}} - \frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{b}\,x^2}{\sqrt{-a}+\sqrt{b}\,x^4}\,dx}{4(-a)^{3/4}} - \frac{\int \frac{\sqrt[4]{-a}+\sqrt[4]{b}\,x^2}{\sqrt{-a}+\sqrt{b}\,x^4}\,dx}{4(-a)^{3/4}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt[8]{b}} - \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt[8]{b}} - \frac{\int \frac{1}{\frac{\sqrt[4]{-a}}{\sqrt[4]{b}}-\frac{\sqrt{2}\,\sqrt[8]{-a}\,x}{\sqrt[8]{b}}+x^2}\,dx}{8(-a)^{3/4}\sqrt[4]{b}} - \frac{\int \frac{1}{\frac{\sqrt[4]{-a}}{\sqrt[4]{b}}+\frac{\sqrt{2}\,\sqrt[8]{-a}\,x}{\sqrt[8]{b}}+x^2}\,dx}{8(-a)^{3/4}\sqrt[4]{b}} + \frac{\int \frac{\frac{\sqrt{2}\,\sqrt[8]{-a}}{\sqrt[8]{b}}+2}{-\frac{\sqrt[4]{-a}}{\sqrt[4]{b}}-\frac{\sqrt{2}\,\sqrt[8]{-a}}{\sqrt[8]{b}}}}{8\sqrt{2}\,(-a)^{7/8}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt[8]{b}} - \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt[8]{b}} + \frac{\log\left(\sqrt[4]{-a}-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,(-a)^{7/8}\sqrt[8]{b}} - \frac{\log\left(\sqrt[4]{-a}+\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{}\right.}{8\sqrt{2}\,(-a)^{7/8}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt[8]{b}} + \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4\sqrt{2}\,(-a)^{7/8}\sqrt[8]{b}} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4\sqrt{2}\,(-a)^{7/8}\sqrt[8]{b}} - \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt[8]{b}} + \frac{\log\left(\sqrt[4]{-a}-\sqrt{2}\right.}{8\sqrt{2}\,(-}$$

Mathematica [A] time = 0.10, size = 324, normalized size = 1.21

$$\frac{-\sin\left(\frac{\pi}{8}\right)\log\left(-2\sqrt[8]{a}\,\sqrt[8]{b}\,x\sin\left(\frac{\pi}{8}\right)+\sqrt[4]{a}+\sqrt[4]{b}\,x^2\right)+\sin\left(\frac{\pi}{8}\right)\log\left(2\sqrt[8]{a}\,\sqrt[8]{b}\,x\sin\left(\frac{\pi}{8}\right)+\sqrt[4]{a}+\sqrt[4]{b}\,x^2\right)-\cos\left(\frac{\pi}{8}\right)\log}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^8)^(-1),x]
```

```
[Out] (2*ArcTan[(b^(1/8)*x*Sec[Pi/8])/a^(1/8) - Tan[Pi/8]]*Cos[Pi/8] + 2*ArcTan[(
b^(1/8)*x*Sec[Pi/8])/a^(1/8) + Tan[Pi/8]]*Cos[Pi/8] - Cos[Pi/8]*Log[a^(1/4)
 + b^(1/4)*x^2 - 2*a^(1/8)*b^(1/8)*x*Cos[Pi/8]] + Cos[Pi/8]*Log[a^(1/4) + b
^(1/4)*x^2 + 2*a^(1/8)*b^(1/8)*x*Cos[Pi/8]] - 2*ArcTan[Cot[Pi/8] - (b^(1/8)
*x*Csc[Pi/8])/a^(1/8)]*Sin[Pi/8] + 2*ArcTan[Cot[Pi/8] + (b^(1/8)*x*Csc[Pi/8
])/a^(1/8)]*Sin[Pi/8] - Log[a^(1/4) + b^(1/4)*x^2 - 2*a^(1/8)*b^(1/8)*x*Sin
[Pi/8]]*Sin[Pi/8] + Log[a^(1/4) + b^(1/4)*x^2 + 2*a^(1/8)*b^(1/8)*x*Sin[Pi/
8]]*Sin[Pi/8])/(8*a^(7/8)*b^(1/8))
```

fricas [B] time = 0.67, size = 408, normalized size = 1.53

$$\frac{1}{4}\sqrt{2}\left(-\frac{1}{a^7b}\right)^{\frac{1}{8}}\arctan\left(-\sqrt{2}\,a^6bx\left(-\frac{1}{a^7b}\right)^{\frac{7}{8}}+\sqrt{2}\sqrt{\sqrt{2}\,ax\left(-\frac{1}{a^7b}\right)^{\frac{1}{8}}+a^2\left(-\frac{1}{a^7b}\right)^{\frac{1}{4}}+x^2}\,a^6b\left(-\frac{1}{a^7b}\right)^{\frac{7}{8}}+1\right)+\frac{1}{4}\sqrt{2}\,\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^8+a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*(-1/(a^7*b))^(1/8)*arctan(-sqrt(2)*a^6*b*x*(-1/(a^7*b))^(7/8) +
 sqrt(2)*sqrt(sqrt(2)*a*x*(-1/(a^7*b))^(1/8) + a^2*(-1/(a^7*b))^(1/4) + x^2
)*a^6*b*(-1/(a^7*b))^(7/8) + 1) + 1/4*sqrt(2)*(-1/(a^7*b))^(1/8)*arctan(-sq
rt(2)*a^6*b*x*(-1/(a^7*b))^(7/8) + sqrt(2)*sqrt(-sqrt(2)*a*x*(-1/(a^7*b))^(
1/8) + a^2*(-1/(a^7*b))^(1/4) + x^2)*a^6*b*(-1/(a^7*b))^(7/8) - 1) + 1/16*s
qrt(2)*(-1/(a^7*b))^(1/8)*log(sqrt(2)*a*x*(-1/(a^7*b))^(1/8) + a^2*(-1/(a^7
*b))^(1/4) + x^2) - 1/16*sqrt(2)*(-1/(a^7*b))^(1/8)*log(-sqrt(2)*a*x*(-1/(a
^7*b))^(1/8) + a^2*(-1/(a^7*b))^(1/4) + x^2) + 1/2*(-1/(a^7*b))^(1/8)*arcta
```

```
n(-a^6*b*x*(-1/(a^7*b))^(7/8) + sqrt(a^2*(-1/(a^7*b))^(1/4) + x^2)*a^6*b*(-
1/(a^7*b))^(7/8)) + 1/8*(-1/(a^7*b))^(1/8)*log(a*(-1/(a^7*b))^(1/8) + x) -
1/8*(-1/(a^7*b))^(1/8)*log(-a*(-1/(a^7*b))^(1/8) + x)
```

giac [B] time = 0.23, size = 437, normalized size = 1.64

$$\frac{\left(\frac{a}{b}\right)^{\frac{1}{8}} \arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{4a\sqrt{-2\sqrt{2}+4}} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{8}} \arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{4a\sqrt{-2\sqrt{2}+4}} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{8}} \arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{4a\sqrt{2\sqrt{2}+4}} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{8}} \arctan\left($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^8+a),x, algorithm="giac")
```

```
[Out] 1/4*(a/b)^(1/8)*arctan((2*x + sqrt(-sqrt(2) + 2)*(a/b)^(1/8))/(sqrt(sqrt(2)
 + 2)*(a/b)^(1/8)))/(a*sqrt(-2*sqrt(2) + 4)) + 1/4*(a/b)^(1/8)*arctan((2*x
- sqrt(-sqrt(2) + 2)*(a/b)^(1/8))/(sqrt(sqrt(2) + 2)*(a/b)^(1/8)))/(a*sqrt(
-2*sqrt(2) + 4)) + 1/4*(a/b)^(1/8)*arctan((2*x + sqrt(sqrt(2) + 2)*(a/b)^(1
/8))/(sqrt(-sqrt(2) + 2)*(a/b)^(1/8)))/(a*sqrt(2*sqrt(2) + 4)) + 1/4*(a/b)^
(1/8)*arctan((2*x - sqrt(sqrt(2) + 2)*(a/b)^(1/8))/(sqrt(-sqrt(2) + 2)*(a/b
)^(1/8)))/(a*sqrt(2*sqrt(2) + 4)) + 1/8*(a/b)^(1/8)*log(x^2 + x*sqrt(sqrt(2
) + 2)*(a/b)^(1/8) + (a/b)^(1/4))/(a*sqrt(-2*sqrt(2) + 4)) - 1/8*(a/b)^(1/8
)*log(x^2 - x*sqrt(sqrt(2) + 2)*(a/b)^(1/8) + (a/b)^(1/4))/(a*sqrt(-2*sqrt(
2) + 4)) + 1/8*(a/b)^(1/8)*log(x^2 + x*sqrt(-sqrt(2) + 2)*(a/b)^(1/8) + (a/
b)^(1/4))/(a*sqrt(2*sqrt(2) + 4)) - 1/8*(a/b)^(1/8)*log(x^2 - x*sqrt(-sqrt(
2) + 2)*(a/b)^(1/8) + (a/b)^(1/4))/(a*sqrt(2*sqrt(2) + 4))
```

maple [C] time = 0.00, size = 27, normalized size = 0.10

$$\frac{\ln\left(-\operatorname{RootOf}\left(b_Z^8+a\right)+x\right)}{8b\operatorname{RootOf}\left(b_Z^8+a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^8+a),x)
```

```
[Out] 1/8/b*sum(1/_R^7*ln(-_R+x),_R=RootOf(_Z^8*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{bx^8+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^8+a),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*x^8 + a), x)
```

mupad [B] time = 0.15, size = 110, normalized size = 0.41

$$-\frac{\operatorname{atan}\left(\frac{b^{1/8}x}{(-a)^{1/8}}\right)}{4(-a)^{7/8}b^{1/8}} + \frac{\operatorname{atan}\left(\frac{b^{1/8}x\,1\mathrm{i}}{(-a)^{1/8}}\right)1\mathrm{i}}{4(-a)^{7/8}b^{1/8}} + \frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\,b^{1/8}x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\left(-\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)}{(-a)^{7/8}b^{1/8}} + \frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\,b^{1/8}x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\left(-\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)}{(-a)^{7/8}b^{1/8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^8),x)
```

```
[Out] (atan((b^(1/8)*x*1i)/(-a)^(1/8))*1i)/(4*(-a)^(7/8)*b^(1/8)) - atan((b^(1/8)
*x)/(-a)^(1/8))/(4*(-a)^(7/8)*b^(1/8)) - (2^(1/2)*atan((2^(1/2)*b^(1/8)*x*(
1/2 - 1i/2))/(-a)^(1/8))*(1/8 + 1i/8))/((-a)^(7/8)*b^(1/8)) - (2^(1/2)*atan
((2^(1/2)*b^(1/8)*x*(1/2 + 1i/2))/(-a)^(1/8))*(1/8 - 1i/8))/((-a)^(7/8)*b^(
1/8))
```

sympy [**A**] time = 0.39, size = 20, normalized size = 0.07

$$\text{RootSum}\left(16777216t^8a^7b+1,\left(t\mapsto t\log\left(8ta+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**8+a),x)
```

```
[Out] RootSum(16777216*_t**8*a**7*b + 1, Lambda(_t, _t*log(8*_t*a + x)))
```

3.1466 $\int \frac{1}{x^2\left(a+bx^8\right)}\,dx$

Optimal. Leaf size=275

$$\frac{\sqrt[8]{b}\,\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{-a}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,(-a)^{9/8}}-\frac{\sqrt[8]{b}\,\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{-a}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,(-a)^{9/8}}+\frac{\sqrt[8]{b}\,\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{9/8}}-\frac{\sqrt[8]{b}\,\text{ta}}{4}$$

```
[Out] -1/a/x+1/4*b^(1/8)*arctan(b^(1/8)*x/(-a)^(1/8))/(-a)^(9/8)-1/4*b^(1/8)*arct
anh(b^(1/8)*x/(-a)^(1/8))/(-a)^(9/8)+1/8*b^(1/8)*arctan(-1+b^(1/8)*x*2^(1/2
)/(-a)^(1/8))/(-a)^(9/8)*2^(1/2)+1/8*b^(1/8)*arctan(1+b^(1/8)*x*2^(1/2)/(-a
)^(1/8))/(-a)^(9/8)*2^(1/2)+1/16*b^(1/8)*ln((-a)^(1/4)+b^(1/4)*x^2-(-a)^(1/
8)*b^(1/8)*x*2^(1/2))/(-a)^(9/8)*2^(1/2)-1/16*b^(1/8)*ln((-a)^(1/4)+b^(1/4)
*x^2+(-a)^(1/8)*b^(1/8)*x*2^(1/2))/(-a)^(9/8)*2^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {325, 301, 297, 1162, 617, 204, 1165, 628, 298, 205, 208}

$$\frac{\sqrt[8]{b}\,\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{-a}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,(-a)^{9/8}}-\frac{\sqrt[8]{b}\,\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{-a}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,(-a)^{9/8}}+\frac{\sqrt[8]{b}\,\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{9/8}}-\frac{\sqrt[8]{b}\,\text{ta}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^8)),x]
```

```
[Out] -(1/(a*x)) + (b^(1/8)*ArcTan[(b^(1/8)*x)/(-a)^(1/8)])/(4*(-a)^(9/8)) - (b^(
1/8)*ArcTan[1 - (Sqrt[2]*b^(1/8)*x)/(-a)^(1/8)])/(4*Sqrt[2]*(-a)^(9/8)) + (
b^(1/8)*ArcTan[1 + (Sqrt[2]*b^(1/8)*x)/(-a)^(1/8)])/(4*Sqrt[2]*(-a)^(9/8))
 - (b^(1/8)*ArcTanh[(b^(1/8)*x)/(-a)^(1/8)])/(4*(-a)^(9/8)) + (b^(1/8)*Log[(
-a)^(1/4) - Sqrt[2]*(-a)^(1/8)*b^(1/8)*x + b^(1/4)*x^2])/(8*Sqrt[2]*(-a)^(9
/8)) - (b^(1/8)*Log[(-a)^(1/4) + Sqrt[2]*(-a)^(1/8)*b^(1/8)*x + b^(1/4)*x^2
])/(8*Sqrt[2]*(-a)^(9/8))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 301

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[x^(m - n/2
)/(r + s*x^(n/2)), x], x] - Dist[s/(2*b), Int[x^(m - n/2)/(r - s*x^(n/2)),
x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] &&
 LtQ[m, n] &&  !GtQ[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^8\right)}\,dx = -\frac{1}{ax} - \frac{b\int \frac{x^6}{a+bx^8}\,dx}{a}$$

$$= -\frac{1}{ax} + \frac{\sqrt{b}\int \frac{x^2}{\sqrt{-a}-\sqrt{b}\,x^4}\,dx}{2a} - \frac{\sqrt{b}\int \frac{x^2}{\sqrt{-a}+\sqrt{b}\,x^4}\,dx}{2a}$$

$$= -\frac{1}{ax} + \frac{\sqrt[4]{b}\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{b}\,x^2}\,dx}{4a} - \frac{\sqrt[4]{b}\int \frac{1}{\sqrt[4]{-a}+\sqrt[4]{b}\,x^2}\,dx}{4a} + \frac{\sqrt[4]{b}\int \frac{\sqrt[4]{-a}-\sqrt[4]{b}\,x^2}{\sqrt{-a}+\sqrt{b}\,x^4}\,dx}{4a} - \frac{\sqrt[4]{b}\int \frac{\sqrt[4]{-a}+\sqrt[4]{b}}{\sqrt{-a}+\sqrt{b}}}{4a}$$

$$= -\frac{1}{ax} + \frac{\sqrt[8]{b}\,\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{9/8}} - \frac{\sqrt[8]{b}\,\tanh^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{9/8}} - \frac{\int \frac{1}{\frac{\sqrt[4]{-a}}{\sqrt[4]{b}}-\frac{\sqrt{2}\,\sqrt[8]{-a}\,x}{\sqrt[8]{b}}+x^2}\,dx}{8a} - \frac{\int \frac{1}{\frac{\sqrt[4]{-a}}{\sqrt[4]{b}}+\frac{\sqrt{2}\,\sqrt[8]{-a}\,x}{\sqrt[8]{b}}+x}}{8a}$$

$$= -\frac{1}{ax} + \frac{\sqrt[8]{b}\,\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{9/8}} - \frac{\sqrt[8]{b}\,\tanh^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{9/8}} + \frac{\sqrt[8]{b}\,\log\left(\sqrt[4]{-a}-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,(-a)^{9/8}}$$

$$= -\frac{1}{ax} + \frac{\sqrt[8]{b}\,\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{9/8}} - \frac{\sqrt[8]{b}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4\sqrt{2}\,(-a)^{9/8}} + \frac{\sqrt[8]{b}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4\sqrt{2}\,(-a)^{9/8}} - \frac{\sqrt[8]{b}\,\tanh}{4(-}$$

Mathematica [A] time = 0.13, size = 377, normalized size = 1.37

$$-\frac{\sqrt[8]{b}\,x\sin\left(\frac{\pi}{8}\right)\log\left(-2\sqrt[8]{a}\,\sqrt[8]{b}\,x\sin\left(\frac{\pi}{8}\right)+\sqrt[4]{a}+\sqrt[4]{b}\,x^2\right) - \sqrt[8]{b}\,x\sin\left(\frac{\pi}{8}\right)\log\left(2\sqrt[8]{a}\,\sqrt[8]{b}\,x\sin\left(\frac{\pi}{8}\right)+\sqrt[4]{a}+\sqrt[4]{b}\,x^2\right)+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^8)),x]
```

```
[Out] -1/8*(8*a^(1/8) + 2*b^(1/8)*x*ArcTan[(b^(1/8)*x*Sec[Pi/8])/a^(1/8) - Tan[Pi
/8]]*Cos[Pi/8] + 2*b^(1/8)*x*ArcTan[(b^(1/8)*x*Sec[Pi/8])/a^(1/8) + Tan[Pi/
8]]*Cos[Pi/8] + b^(1/8)*x*Cos[Pi/8]*Log[a^(1/4) + b^(1/4)*x^2 - 2*a^(1/8)*b
^(1/8)*x*Cos[Pi/8]] - b^(1/8)*x*Cos[Pi/8]*Log[a^(1/4) + b^(1/4)*x^2 + 2*a^(
1/8)*b^(1/8)*x*Cos[Pi/8]] - 2*b^(1/8)*x*ArcTan[Cot[Pi/8] - (b^(1/8)*x*Csc[P
i/8])/a^(1/8)]*Sin[Pi/8] + 2*b^(1/8)*x*ArcTan[Cot[Pi/8] + (b^(1/8)*x*Csc[Pi
/8])/a^(1/8)]*Sin[Pi/8] + b^(1/8)*x*Log[a^(1/4) + b^(1/4)*x^2 - 2*a^(1/8)*b
^(1/8)*x*Sin[Pi/8]]*Sin[Pi/8] - b^(1/8)*x*Log[a^(1/4) + b^(1/4)*x^2 + 2*a^(
1/8)*b^(1/8)*x*Sin[Pi/8]]*Sin[Pi/8])/(a^(9/8)*x)
```

fricas [B] time = 0.74, size = 408, normalized size = 1.48

$$\frac{4\sqrt{2}\,ax\left(-\frac{b}{a^9}\right)^{\frac{1}{8}}\arctan\left(-\sqrt{2}\,ax\left(-\frac{b}{a^9}\right)^{\frac{1}{8}}+\sqrt{2}\,a\sqrt{-\frac{\sqrt{2}\,a^8x\left(-\frac{b}{a^9}\right)^{\frac{7}{8}}+a^7\left(-\frac{b}{a^9}\right)^{\frac{3}{4}}-bx^2}{b}}\left(-\frac{b}{a^9}\right)^{\frac{1}{8}}-1\right)+4\sqrt{2}\,ax\left(-\frac{b}{a^9}\right)^{\frac{1}{8}}\text{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^8+a),x, algorithm="fricas")
```

```
[Out] 1/16*(4*sqrt(2)*a*x*(-b/a^9)^(1/8)*arctan(-sqrt(2)*a*x*(-b/a^9)^(1/8) + sqr
t(2)*a*sqrt(-(sqrt(2)*a^8*x*(-b/a^9)^(7/8) + a^7*(-b/a^9)^(3/4) - b*x^2)/b)
*(-b/a^9)^(1/8) - 1) + 4*sqrt(2)*a*x*(-b/a^9)^(1/8)*arctan(-sqrt(2)*a*x*(-b
/a^9)^(1/8) + sqrt(2)*a*sqrt((sqrt(2)*a^8*x*(-b/a^9)^(7/8) - a^7*(-b/a^9)^(
```

```
3/4) + b*x^2)/b)*(-b/a^9)^(1/8) + 1) - sqrt(2)*a*x*(-b/a^9)^(1/8)*log(sqrt(
2)*a^8*x*(-b/a^9)^(7/8) - a^7*(-b/a^9)^(3/4) + b*x^2) + sqrt(2)*a*x*(-b/a^9
)^(1/8)*log(-sqrt(2)*a^8*x*(-b/a^9)^(7/8) - a^7*(-b/a^9)^(3/4) + b*x^2) + 8
*a*x*(-b/a^9)^(1/8)*arctan(-a*x*(-b/a^9)^(1/8) + a*sqrt(-(a^7*(-b/a^9)^(3/4
) - b*x^2)/b)*(-b/a^9)^(1/8)) - 2*a*x*(-b/a^9)^(1/8)*log(a^8*(-b/a^9)^(7/8)
 + b*x) + 2*a*x*(-b/a^9)^(1/8)*log(-a^8*(-b/a^9)^(7/8) + b*x) - 16)/(a*x)
```

giac [B] time = 0.36, size = 453, normalized size = 1.65

$$-\frac{b\left(\frac{a}{b}\right)^{\frac{7}{8}}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{4a^2\sqrt{-2\sqrt{2}+4}}-\frac{b\left(\frac{a}{b}\right)^{\frac{7}{8}}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{4a^2\sqrt{-2\sqrt{2}+4}}-\frac{b\left(\frac{a}{b}\right)^{\frac{7}{8}}\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{4a^2\sqrt{2\sqrt{2}+4}}-\frac{b\left(\frac{a}{b}\right)^{\frac{7}{8}}\arct}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^8+a),x, algorithm="giac")
```

```
[Out] -1/4*b*(a/b)^(7/8)*arctan((2*x + sqrt(-sqrt(2) + 2)*(a/b)^(1/8))/(sqrt(sqrt
(2) + 2)*(a/b)^(1/8)))/(a^2*sqrt(-2*sqrt(2) + 4)) - 1/4*b*(a/b)^(7/8)*arcta
n((2*x - sqrt(-sqrt(2) + 2)*(a/b)^(1/8))/(sqrt(sqrt(2) + 2)*(a/b)^(1/8)))/(
a^2*sqrt(-2*sqrt(2) + 4)) - 1/4*b*(a/b)^(7/8)*arctan((2*x + sqrt(sqrt(2) +
2)*(a/b)^(1/8))/(sqrt(-sqrt(2) + 2)*(a/b)^(1/8)))/(a^2*sqrt(2*sqrt(2) + 4))
 - 1/4*b*(a/b)^(7/8)*arctan((2*x - sqrt(sqrt(2) + 2)*(a/b)^(1/8))/(sqrt(-sq
rt(2) + 2)*(a/b)^(1/8)))/(a^2*sqrt(2*sqrt(2) + 4)) + 1/8*b*(a/b)^(7/8)*log(
x^2 + x*sqrt(sqrt(2) + 2)*(a/b)^(1/8) + (a/b)^(1/4))/(a^2*sqrt(-2*sqrt(2) +
 4)) - 1/8*b*(a/b)^(7/8)*log(x^2 - x*sqrt(sqrt(2) + 2)*(a/b)^(1/8) + (a/b)^
(1/4))/(a^2*sqrt(-2*sqrt(2) + 4)) + 1/8*b*(a/b)^(7/8)*log(x^2 + x*sqrt(-sqr
t(2) + 2)*(a/b)^(1/8) + (a/b)^(1/4))/(a^2*sqrt(2*sqrt(2) + 4)) - 1/8*b*(a/b
)^(7/8)*log(x^2 - x*sqrt(-sqrt(2) + 2)*(a/b)^(1/8) + (a/b)^(1/4))/(a^2*sqrt
(2*sqrt(2) + 4)) - 1/(a*x)
```

maple [C] time = 0.01, size = 36, normalized size = 0.13

$$-\frac{\ln\left(-\operatorname{RootOf}\left(b_Z^8+a\right)+x\right)}{8a\operatorname{RootOf}\left(b_Z^8+a\right)}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^8+a),x)
```

```
[Out] -1/8/a*sum(1/_R*ln(-_R+x),_R=RootOf(_Z^8*b+a))-1/a/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{8}b\left(\frac{2\left(\frac{a}{b}\right)^{\frac{7}{8}}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{a\sqrt{-2\sqrt{2}+4}}+\frac{2\left(\frac{a}{b}\right)^{\frac{7}{8}}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{a\sqrt{-2\sqrt{2}+4}}+\frac{2\left(\frac{a}{b}\right)^{\frac{7}{8}}\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{a\sqrt{2\sqrt{2}+4}}+\frac{2\left(\frac{a}{b}\right)^{\frac{7}{8}}\arctan\left(\frac{2x-\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{a\sqrt{2\sqrt{2}+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^8+a),x, algorithm="maxima")
```

```
[Out] -b*integrate(x^6/(b*x^8 + a), x)/a - 1/(a*x)
```

mupad [B] time = 0.15, size = 118, normalized size = 0.43

$$-\frac{1}{a\,x}-\frac{(-b)^{1/8}\,\mathrm{atan}\left(\frac{(-b)^{1/8}\,x}{a^{1/8}}\right)}{4\,a^{9/8}}-\frac{(-b)^{1/8}\,\mathrm{atan}\left(\frac{(-b)^{1/8}\,x\,1\mathrm{i}}{a^{1/8}}\right)\,1\mathrm{i}}{4\,a^{9/8}}+\frac{\sqrt{2}\,(-b)^{1/8}\,\mathrm{atan}\left(\frac{\sqrt{2}\,(-b)^{1/8}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{a^{1/8}}\right)\left(-\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)}{a^{9/8}}+\frac{\sqrt{2}\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^8)),x)
```

```
[Out] - 1/(a*x) - ((-b)^(1/8)*atan(((-b)^(1/8)*x)/a^(1/8)))/(4*a^(9/8)) - ((-b)^(
1/8)*atan(((-b)^(1/8)*x*1i)/a^(1/8))*1i)/(4*a^(9/8)) - (2^(1/2)*(-b)^(1/8)*
atan((2^(1/2)*(-b)^(1/8)*x*(1/2 - 1i/2))/a^(1/8))*(1/8 - 1i/8))/a^(9/8) - (
2^(1/2)*(-b)^(1/8)*atan((2^(1/2)*(-b)^(1/8)*x*(1/2 + 1i/2))/a^(1/8))*(1/8 +
 1i/8))/a^(9/8)
```

sympy [A] time = 0.56, size = 29, normalized size = 0.11

$$\mathrm{RootSum}\left(16777216t^8a^9+b,\left(t\mapsto t\log\left(-\frac{2097152t^7a^8}{b}+x\right)\right)\right)-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**8+a),x)
```

```
[Out] RootSum(16777216*_t**8*a**9 + b, Lambda(_t, _t*log(-2097152*_t**7*a**8/b +
x))) - 1/(a*x)
```

3.1467 $\int \frac{1}{x^4\left(a+bx^8\right)}\,dx$

Optimal. Leaf size=277

$$-\frac{b^{3/8}\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{-a}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,(-a)^{11/8}}+\frac{b^{3/8}\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{-a}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,(-a)^{11/8}}-\frac{b^{3/8}\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{11/8}}-\frac{b^{3/8}\tan$$

```
[Out] -1/3/a/x^3-1/4*b^(3/8)*arctan(b^(1/8)*x/(-a)^(1/8))/(-a)^(11/8)-1/4*b^(3/8)
*arctanh(b^(1/8)*x/(-a)^(1/8))/(-a)^(11/8)+1/8*b^(3/8)*arctan(-1+b^(1/8)*x*
2^(1/2)/(-a)^(1/8))/(-a)^(11/8)*2^(1/2)+1/8*b^(3/8)*arctan(1+b^(1/8)*x*2^(1
/2)/(-a)^(1/8))/(-a)^(11/8)*2^(1/2)-1/16*b^(3/8)*ln((-a)^(1/4)+b^(1/4)*x^2-
(-a)^(1/8)*b^(1/8)*x*2^(1/2))/(-a)^(11/8)*2^(1/2)+1/16*b^(3/8)*ln((-a)^(1/4
)+b^(1/4)*x^2+(-a)^(1/8)*b^(1/8)*x*2^(1/2))/(-a)^(11/8)*2^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {325, 301, 211, 1165, 628, 1162, 617, 204, 212, 208, 205}

$$-\frac{b^{3/8}\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{-a}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,(-a)^{11/8}}+\frac{b^{3/8}\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{-a}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,(-a)^{11/8}}-\frac{b^{3/8}\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{11/8}}-\frac{b^{3/8}\tan$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^8)),x]
```

```
[Out] -1/(3*a*x^3) - (b^(3/8)*ArcTan[(b^(1/8)*x)/(-a)^(1/8)])/(4*(-a)^(11/8)) - (
b^(3/8)*ArcTan[1 - (Sqrt[2]*b^(1/8)*x)/(-a)^(1/8)])/(4*Sqrt[2]*(-a)^(11/8))
 + (b^(3/8)*ArcTan[1 + (Sqrt[2]*b^(1/8)*x)/(-a)^(1/8)])/(4*Sqrt[2]*(-a)^(11
/8)) - (b^(3/8)*ArcTanh[(b^(1/8)*x)/(-a)^(1/8)])/(4*(-a)^(11/8)) - (b^(3/8)
*Log[(-a)^(1/4) - Sqrt[2]*(-a)^(1/8)*b^(1/8)*x + b^(1/4)*x^2])/(8*Sqrt[2]*(
-a)^(11/8)) + (b^(3/8)*Log[(-a)^(1/4) + Sqrt[2]*(-a)^(1/8)*b^(1/8)*x + b^(1
/4)*x^2])/(8*Sqrt[2]*(-a)^(11/8))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 301

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[x^(m - n/2
)/(r + s*x^(n/2)), x], x] - Dist[s/(2*b), Int[x^(m - n/2)/(r - s*x^(n/2)),
x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] &&
 LtQ[m, n] &&  !GtQ[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^8\right)}\,dx = -\frac{1}{3ax^3} - \frac{b\int \frac{x^4}{a+bx^8}\,dx}{a}$$

$$= -\frac{1}{3ax^3} + \frac{\sqrt{b}\int \frac{1}{\sqrt{-a}-\sqrt{b}\,x^4}\,dx}{2a} - \frac{\sqrt{b}\int \frac{1}{\sqrt{-a}+\sqrt{b}\,x^4}\,dx}{2a}$$

$$= -\frac{1}{3ax^3} - \frac{\sqrt{b}\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{b}\,x^2}\,dx}{4(-a)^{5/4}} - \frac{\sqrt{b}\int \frac{1}{\sqrt[4]{-a}+\sqrt[4]{b}\,x^2}\,dx}{4(-a)^{5/4}} + \frac{\sqrt{b}\int \frac{\sqrt[4]{-a}-\sqrt[4]{b}\,x^2}{\sqrt{-a}+\sqrt{b}\,x^4}\,dx}{4(-a)^{5/4}} + \frac{\sqrt{b}\int \frac{\sqrt[4]{-a}+\sqrt[4]{b}\,x}{\sqrt{-a}+\sqrt{b}\,x^4}}{4(-a)^{5/4}}$$

$$= -\frac{1}{3ax^3} - \frac{b^{3/8}\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{11/8}} - \frac{b^{3/8}\tanh^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{11/8}} + \frac{\sqrt[4]{b}\int \frac{1}{\frac{\sqrt[4]{-a}}{\sqrt[4]{b}}-\frac{\sqrt{2}\,\sqrt[8]{-a}\,x}{\sqrt[8]{b}}+x^2}\,dx}{8(-a)^{5/4}} + \frac{\sqrt[4]{b}\int \frac{}{\frac{\sqrt[4]{-a}}{\sqrt[4]{b}}+\frac{\sqrt{}}{}}}{8(-a}$$

$$= -\frac{1}{3ax^3} - \frac{b^{3/8}\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{11/8}} - \frac{b^{3/8}\tanh^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{11/8}} - \frac{b^{3/8}\log\left(\sqrt[4]{-a}-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,(-a)^{11/8}}$$

$$= -\frac{1}{3ax^3} - \frac{b^{3/8}\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{11/8}} - \frac{b^{3/8}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4\sqrt{2}\,(-a)^{11/8}} + \frac{b^{3/8}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4\sqrt{2}\,(-a)^{11/8}} - \frac{b^{3/8}\tanh^{-}}{4(-a)}$$

Mathematica [A] time = 0.09, size = 395, normalized size = 1.43

$$\frac{-8a^{3/8} + 6b^{3/8}x^3\sin\left(\frac{\pi}{8}\right)\tan^{-1}\left(\frac{\sqrt[8]{b}\,x\sec\left(\frac{\pi}{8}\right)}{\sqrt[8]{a}} - \tan\left(\frac{\pi}{8}\right)\right) + 6b^{3/8}x^3\sin\left(\frac{\pi}{8}\right)\tan^{-1}\left(\frac{\sqrt[8]{b}\,x\sec\left(\frac{\pi}{8}\right)}{\sqrt[8]{a}} + \tan\left(\frac{\pi}{8}\right)\right) + 6b^{3/8}x^3\cos}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^8)),x]
```

```
[Out] (-8*a^(3/8) + 6*b^(3/8)*x^3*ArcTan[Cot[Pi/8] - (b^(1/8)*x*Csc[Pi/8])/a^(1/8
)]*Cos[Pi/8] - 6*b^(3/8)*x^3*ArcTan[Cot[Pi/8] + (b^(1/8)*x*Csc[Pi/8])/a^(1/
8)]*Cos[Pi/8] + 3*b^(3/8)*x^3*Cos[Pi/8]*Log[a^(1/4) + b^(1/4)*x^2 - 2*a^(1/
8)*b^(1/8)*x*Sin[Pi/8]] - 3*b^(3/8)*x^3*Cos[Pi/8]*Log[a^(1/4) + b^(1/4)*x^2
 + 2*a^(1/8)*b^(1/8)*x*Sin[Pi/8]] + 6*b^(3/8)*x^3*ArcTan[(b^(1/8)*x*Sec[Pi/
8])/a^(1/8) - Tan[Pi/8]]*Sin[Pi/8] + 6*b^(3/8)*x^3*ArcTan[(b^(1/8)*x*Sec[Pi
/8])/a^(1/8) + Tan[Pi/8]]*Sin[Pi/8] - 3*b^(3/8)*x^3*Log[a^(1/4) + b^(1/4)*x
^2 - 2*a^(1/8)*b^(1/8)*x*Cos[Pi/8]]*Sin[Pi/8] + 3*b^(3/8)*x^3*Log[a^(1/4) +
 b^(1/4)*x^2 + 2*a^(1/8)*b^(1/8)*x*Cos[Pi/8]]*Sin[Pi/8])/(24*a^(11/8)*x^3)
```

fricas [B] time = 0.70, size = 516, normalized size = 1.86

$$\frac{12\sqrt{2}\,ax^3\left(-\frac{b^3}{a^{11}}\right)^{\frac{1}{8}}\arctan\left(-\frac{\sqrt{2}\,a^4x\left(-\frac{b^3}{a^{11}}\right)^{\frac{3}{8}} - \sqrt{2}\,a^4\sqrt{-\frac{\sqrt{2}\,a^7x\left(-\frac{b^3}{a^{11}}\right)^{\frac{5}{8}} + a^3b\left(-\frac{b^3}{a^{11}}\right)^{\frac{1}{4}} - b^2x^2}{b^2}}\left(-\frac{b^3}{a^{11}}\right)^{\frac{3}{8}} + b}{b}\right) + 12\sqrt{2}\,ax^3\left(-\frac{b^3}{a^{11}}\right)^{\frac{1}{8}}\text{arcta}}{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^8+a),x, algorithm="fricas")
```

```
[Out] -1/48*(12*sqrt(2)*a*x^3*(-b^3/a^11)^(1/8)*arctan(-(sqrt(2)*a^4*x*(-b^3/a^11
)^(3/8) - sqrt(2)*a^4*sqrt(-(sqrt(2)*a^7*x*(-b^3/a^11)^(5/8) + a^3*b*(-b^3/
```

```
a^11)^(1/4) - b^2*x^2)/b^2)*(-b^3/a^11)^(3/8) + b)/b) + 12*sqrt(2)*a*x^3*(-
b^3/a^11)^(1/8)*arctan(-(sqrt(2)*a^4*x*(-b^3/a^11)^(3/8) - sqrt(2)*a^4*sqrt
((sqrt(2)*a^7*x*(-b^3/a^11)^(5/8) - a^3*b*(-b^3/a^11)^(1/4) + b^2*x^2)/b^2)
*(-b^3/a^11)^(3/8) - b)/b) + 3*sqrt(2)*a*x^3*(-b^3/a^11)^(1/8)*log(sqrt(2)*
a^7*x*(-b^3/a^11)^(5/8) - a^3*b*(-b^3/a^11)^(1/4) + b^2*x^2) - 3*sqrt(2)*a*
x^3*(-b^3/a^11)^(1/8)*log(-sqrt(2)*a^7*x*(-b^3/a^11)^(5/8) - a^3*b*(-b^3/a^
11)^(1/4) + b^2*x^2) - 24*a*x^3*(-b^3/a^11)^(1/8)*arctan(-(a^4*x*(-b^3/a^11
)^(3/8) - a^4*sqrt(-(a^3*(-b^3/a^11)^(1/4) - b*x^2)/b)*(-b^3/a^11)^(3/8))/b
) - 6*a*x^3*(-b^3/a^11)^(1/8)*log(a^7*(-b^3/a^11)^(5/8) + b^2*x) + 6*a*x^3*
(-b^3/a^11)^(1/8)*log(-a^7*(-b^3/a^11)^(5/8) + b^2*x) + 16)/(a*x^3)
```

giac [B] time = 0.46, size = 453, normalized size = 1.64

$$\frac{b\left(\frac{a}{b}\right)^{\frac{5}{8}}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{4a^2\sqrt{2\sqrt{2}+4}}+\frac{b\left(\frac{a}{b}\right)^{\frac{5}{8}}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{4a^2\sqrt{2\sqrt{2}+4}}-\frac{b\left(\frac{a}{b}\right)^{\frac{5}{8}}\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{4a^2\sqrt{-2\sqrt{2}+4}}-\frac{b\left(\frac{a}{b}\right)^{\frac{5}{8}}\arctan\left(\frac{2x-\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{4a^2\sqrt{-2\sqrt{2}+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^8+a),x, algorithm="giac")
```

```
[Out] 1/4*b*(a/b)^(5/8)*arctan((2*x + sqrt(-sqrt(2) + 2)*(a/b)^(1/8))/(sqrt(sqrt(
2) + 2)*(a/b)^(1/8)))/(a^2*sqrt(2*sqrt(2) + 4)) + 1/4*b*(a/b)^(5/8)*arctan(
(2*x - sqrt(-sqrt(2) + 2)*(a/b)^(1/8))/(sqrt(sqrt(2) + 2)*(a/b)^(1/8)))/(a^
2*sqrt(2*sqrt(2) + 4)) - 1/4*b*(a/b)^(5/8)*arctan((2*x + sqrt(sqrt(2) + 2)*
(a/b)^(1/8))/(sqrt(-sqrt(2) + 2)*(a/b)^(1/8)))/(a^2*sqrt(-2*sqrt(2) + 4)) -
 1/4*b*(a/b)^(5/8)*arctan((2*x - sqrt(sqrt(2) + 2)*(a/b)^(1/8))/(sqrt(-sqrt
(2) + 2)*(a/b)^(1/8)))/(a^2*sqrt(-2*sqrt(2) + 4)) + 1/8*b*(a/b)^(5/8)*log(x
^2 + x*sqrt(sqrt(2) + 2)*(a/b)^(1/8) + (a/b)^(1/4))/(a^2*sqrt(2*sqrt(2) + 4
)) - 1/8*b*(a/b)^(5/8)*log(x^2 - x*sqrt(sqrt(2) + 2)*(a/b)^(1/8) + (a/b)^(1
/4))/(a^2*sqrt(2*sqrt(2) + 4)) - 1/8*b*(a/b)^(5/8)*log(x^2 + x*sqrt(-sqrt(2
) + 2)*(a/b)^(1/8) + (a/b)^(1/4))/(a^2*sqrt(-2*sqrt(2) + 4)) + 1/8*b*(a/b)^
(5/8)*log(x^2 - x*sqrt(-sqrt(2) + 2)*(a/b)^(1/8) + (a/b)^(1/4))/(a^2*sqrt(-
2*sqrt(2) + 4)) - 1/3/(a*x^3)
```

maple [C] time = 0.00, size = 36, normalized size = 0.13

$$-\frac{\ln\left(-\operatorname{RootOf}\left(b_Z^8+a\right)+x\right)}{8a\operatorname{RootOf}\left(b_Z^8+a\right)^3}-\frac{1}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^8+a),x)
```

```
[Out] -1/3/a/x^3-1/8/a*sum(1/_R^3*ln(-_R+x),_R=RootOf(_Z^8*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{8}b\left(\frac{2\left(\frac{a}{b}\right)^{\frac{5}{8}}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{a\sqrt{2\sqrt{2}+4}}+\frac{2\left(\frac{a}{b}\right)^{\frac{5}{8}}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{a\sqrt{2\sqrt{2}+4}}-\frac{2\left(\frac{a}{b}\right)^{\frac{5}{8}}\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{a\sqrt{-2\sqrt{2}+4}}-\frac{2\left(\frac{a}{b}\right)^{\frac{5}{8}}\arctan\left(\frac{2x-\sqrt{}}{\sqrt{-\sqrt{}}}\right.}{a\sqrt{-2\sqrt{2}+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^8+a),x, algorithm="maxima")
```

```
[Out] -b*integrate(x^4/(b*x^8 + a), x)/a - 1/3/(a*x^3)
```

mupad [B] time = 1.15, size = 118, normalized size = 0.43

$$\frac{(-b)^{3/8}\,\mathrm{atan}\left(\frac{(-b)^{1/8}\,x}{a^{1/8}}\right)}{4\,a^{11/8}}-\frac{1}{3\,a\,x^3}-\frac{(-b)^{3/8}\,\mathrm{atan}\left(\frac{(-b)^{1/8}\,x\,1\mathrm{i}}{a^{1/8}}\right)\,1\mathrm{i}}{4\,a^{11/8}}+\frac{\sqrt{2}\,(-b)^{3/8}\,\mathrm{atan}\left(\frac{\sqrt{2}\,(-b)^{1/8}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{a^{1/8}}\right)\left(-\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)}{a^{11/8}}+\frac{\sqrt{2}\,(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^8)),x)
```

```
[Out] ((-b)^(3/8)*atan(((-b)^(1/8)*x)/a^(1/8)))/(4*a^(11/8)) - 1/(3*a*x^3) - ((-b
)^(3/8)*atan(((-b)^(1/8)*x*1i)/a^(1/8))*1i)/(4*a^(11/8)) - (2^(1/2)*(-b)^(3
/8)*atan((2^(1/2)*(-b)^(1/8)*x*(1/2 - 1i/2))/a^(1/8))*(1/8 + 1i/8))/a^(11/8
) - (2^(1/2)*(-b)^(3/8)*atan((2^(1/2)*(-b)^(1/8)*x*(1/2 + 1i/2))/a^(1/8))*(
1/8 - 1i/8))/a^(11/8)
```

sympy [A] time = 0.74, size = 36, normalized size = 0.13

$$\mathrm{RootSum}\left(16777216t^8a^{11}+b^3,\left(t\mapsto t\log\left(\frac{32768t^5a^7}{b^2}+x\right)\right)\right)-\frac{1}{3ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**8+a),x)
```

```
[Out] RootSum(16777216*_t**8*a**11 + b**3, Lambda(_t, _t*log(32768*_t**5*a**7/b**
2 + x))) - 1/(3*a*x**3)
```

3.1468 $\int \frac{1}{x^6\left(a+bx^8\right)}\,dx$

Optimal. Leaf size=277

$$-\frac{b^{5/8}\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{-a}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,(-a)^{13/8}}+\frac{b^{5/8}\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{-a}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,(-a)^{13/8}}+\frac{b^{5/8}\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{13/8}}+\frac{b^{5/8}}{}$$

```
[Out] -1/5/a/x^5+1/4*b^(5/8)*arctan(b^(1/8)*x/(-a)^(1/8))/(-a)^(13/8)-1/4*b^(5/8)
*arctanh(b^(1/8)*x/(-a)^(1/8))/(-a)^(13/8)-1/8*b^(5/8)*arctan(-1+b^(1/8)*x*
2^(1/2)/(-a)^(1/8))/(-a)^(13/8)*2^(1/2)-1/8*b^(5/8)*arctan(1+b^(1/8)*x*2^(1
/2)/(-a)^(1/8))/(-a)^(13/8)*2^(1/2)-1/16*b^(5/8)*ln((-a)^(1/4)+b^(1/4)*x^2-
(-a)^(1/8)*b^(1/8)*x*2^(1/2))/(-a)^(13/8)*2^(1/2)+1/16*b^(5/8)*ln((-a)^(1/4
)+b^(1/4)*x^2+(-a)^(1/8)*b^(1/8)*x*2^(1/2))/(-a)^(13/8)*2^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {325, 300, 297, 1162, 617, 204, 1165, 628, 298, 205, 208}

$$-\frac{b^{5/8}\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{-a}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,(-a)^{13/8}}+\frac{b^{5/8}\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{-a}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,(-a)^{13/8}}+\frac{b^{5/8}\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{13/8}}+\frac{b^{5/8}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^8)),x]
```

```
[Out] -1/(5*a*x^5) + (b^(5/8)*ArcTan[(b^(1/8)*x)/(-a)^(1/8)])/(4*(-a)^(13/8)) + (
b^(5/8)*ArcTan[1 - (Sqrt[2]*b^(1/8)*x)/(-a)^(1/8)])/(4*Sqrt[2]*(-a)^(13/8))
 - (b^(5/8)*ArcTan[1 + (Sqrt[2]*b^(1/8)*x)/(-a)^(1/8)])/(4*Sqrt[2]*(-a)^(13
/8)) - (b^(5/8)*ArcTanh[(b^(1/8)*x)/(-a)^(1/8)])/(4*(-a)^(13/8)) - (b^(5/8)
*Log[(-a)^(1/4) - Sqrt[2]*(-a)^(1/8)*b^(1/8)*x + b^(1/4)*x^2])/(8*Sqrt[2]*(
-a)^(13/8)) + (b^(5/8)*Log[(-a)^(1/4) + Sqrt[2]*(-a)^(1/8)*b^(1/8)*x + b^(1
/4)*x^2])/(8*Sqrt[2]*(-a)^(13/8))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 300

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[x^m/(r + s
*x^(n/2)), x], x] + Dist[r/(2*a), Int[x^m/(r - s*x^(n/2)), x], x]] /; FreeQ
[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n/2] &&  !GtQ[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
 + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
  b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
  x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^6\left(a+bx^8\right)}\,dx = -\frac{1}{5ax^5} - \frac{b\int \frac{x^2}{a+bx^8}\,dx}{a}$$

$$= -\frac{1}{5ax^5} - \frac{b\int \frac{x^2}{\sqrt{-a}-\sqrt{b}\,x^4}\,dx}{2(-a)^{3/2}} - \frac{b\int \frac{x^2}{\sqrt{-a}+\sqrt{b}\,x^4}\,dx}{2(-a)^{3/2}}$$

$$= -\frac{1}{5ax^5} - \frac{b^{3/4}\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{b}\,x^2}\,dx}{4(-a)^{3/2}} + \frac{b^{3/4}\int \frac{1}{\sqrt[4]{-a}+\sqrt[4]{b}\,x^2}\,dx}{4(-a)^{3/2}} + \frac{b^{3/4}\int \frac{\sqrt[4]{-a}-\sqrt[4]{b}\,x^2}{\sqrt{-a}+\sqrt{b}\,x^4}\,dx}{4(-a)^{3/2}} - \frac{b^{3/4}\int \frac{\sqrt[4]{-a}+}{\sqrt{-a}+}}{4(-a)^3}$$

$$= -\frac{1}{5ax^5} + \frac{b^{5/8}\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{13/8}} - \frac{b^{5/8}\tanh^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{13/8}} - \frac{\sqrt{b}\int \frac{1}{\frac{\sqrt[4]{-a}}{\sqrt[4]{b}} - \frac{\sqrt{2}\,\sqrt[8]{-a}\,x}{\sqrt[8]{b}}+x^2}\,dx}{8(-a)^{3/2}} - \frac{\sqrt{b}\int \frac{}{\frac{\sqrt[4]{-a}}{\sqrt[4]{b}}}}{8(}$$

$$= -\frac{1}{5ax^5} + \frac{b^{5/8}\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{13/8}} - \frac{b^{5/8}\tanh^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{13/8}} - \frac{b^{5/8}\log\left(\sqrt[4]{-a} - \sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x + \sqrt[4]{b}\,x\right.}{8\sqrt{2}\,(-a)^{13/8}}$$

$$= -\frac{1}{5ax^5} + \frac{b^{5/8}\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{13/8}} + \frac{b^{5/8}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4\sqrt{2}\,(-a)^{13/8}} - \frac{b^{5/8}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4\sqrt{2}\,(-a)^{13/8}} - \frac{b^{5/8}\tan}{4(}$$

Mathematica [A] time = 0.10, size = 395, normalized size = 1.43

$$\frac{-8a^{5/8} + 10b^{5/8}x^5\sin\left(\frac{\pi}{8}\right)\tan^{-1}\left(\frac{\sqrt[8]{b}\,x\sec\left(\frac{\pi}{8}\right)}{\sqrt[8]{a}} - \tan\left(\frac{\pi}{8}\right)\right) + 10b^{5/8}x^5\sin\left(\frac{\pi}{8}\right)\tan^{-1}\left(\frac{\sqrt[8]{b}\,x\sec\left(\frac{\pi}{8}\right)}{\sqrt[8]{a}} + \tan\left(\frac{\pi}{8}\right)\right) + 10b^{5/8}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^8)),x]
```

```
[Out] (-8*a^(5/8) + 10*b^(5/8)*x^5*ArcTan[Cot[Pi/8] - (b^(1/8)*x*Csc[Pi/8])/a^(1/
8)]*Cos[Pi/8] - 10*b^(5/8)*x^5*ArcTan[Cot[Pi/8] + (b^(1/8)*x*Csc[Pi/8])/a^(
1/8)]*Cos[Pi/8] - 5*b^(5/8)*x^5*Cos[Pi/8]*Log[a^(1/4) + b^(1/4)*x^2 - 2*a^(
1/8)*b^(1/8)*x*Sin[Pi/8]] + 5*b^(5/8)*x^5*Cos[Pi/8]*Log[a^(1/4) + b^(1/4)*x
^2 + 2*a^(1/8)*b^(1/8)*x*Sin[Pi/8]] + 10*b^(5/8)*x^5*ArcTan[(b^(1/8)*x*Sec[
Pi/8])/a^(1/8) - Tan[Pi/8]]*Sin[Pi/8] + 10*b^(5/8)*x^5*ArcTan[(b^(1/8)*x*Se
c[Pi/8])/a^(1/8) + Tan[Pi/8]]*Sin[Pi/8] + 5*b^(5/8)*x^5*Log[a^(1/4) + b^(1/
4)*x^2 - 2*a^(1/8)*b^(1/8)*x*Cos[Pi/8]]*Sin[Pi/8] - 5*b^(5/8)*x^5*Log[a^(1/
4) + b^(1/4)*x^2 + 2*a^(1/8)*b^(1/8)*x*Cos[Pi/8]]*Sin[Pi/8])/(40*a^(13/8)*x
^5)
```

fricas [B] time = 0.74, size = 524, normalized size = 1.89

$$\frac{20\sqrt{2}\,ax^5\left(-\frac{b^5}{a^{13}}\right)^{\frac{1}{8}}\arctan\left(-\frac{\sqrt{2}\,a^8x\left(-\frac{b^5}{a^{13}}\right)^{\frac{5}{8}} - \sqrt{2}\,a^8\sqrt{\frac{a^{10}\left(-\frac{b^5}{a^{13}}\right)^{\frac{3}{4}} + \sqrt{2}\,a^5b^2x\left(-\frac{b^5}{a^{13}}\right)^{\frac{3}{8}} + b^4x^2}{b^4}}\left(-\frac{b^5}{a^{13}}\right)^{\frac{5}{8}} - b^3}{b^3}\right) + 20\sqrt{2}\,ax^5\left(-\frac{b^5}{a^{13}}\right)^{\frac{1}{8}}\text{ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^8+a),x, algorithm="fricas")
```

```
[Out] 1/80*(20*sqrt(2)*a*x^5*(-b^5/a^13)^(1/8)*arctan(-(sqrt(2)*a^8*x*(-b^5/a^13)
^(5/8) - sqrt(2)*a^8*sqrt((a^10*(-b^5/a^13)^(3/4) + sqrt(2)*a^5*b^2*x*(-b^5
/a^13)^(3/8) + b^4*x^2)/b^4)*(-b^5/a^13)^(5/8) - b^3)/b^3) + 20*sqrt(2)*a*x
^5*(-b^5/a^13)^(1/8)*arctan(-(sqrt(2)*a^8*x*(-b^5/a^13)^(5/8) - sqrt(2)*a^8
*sqrt((a^10*(-b^5/a^13)^(3/4) - sqrt(2)*a^5*b^2*x*(-b^5/a^13)^(3/8) + b^4*x
^2)/b^4)*(-b^5/a^13)^(5/8) + b^3)/b^3) - 5*sqrt(2)*a*x^5*(-b^5/a^13)^(1/8)*
log(a^10*(-b^5/a^13)^(3/4) + sqrt(2)*a^5*b^2*x*(-b^5/a^13)^(3/8) + b^4*x^2)
 + 5*sqrt(2)*a*x^5*(-b^5/a^13)^(1/8)*log(a^10*(-b^5/a^13)^(3/4) - sqrt(2)*a
^5*b^2*x*(-b^5/a^13)^(3/8) + b^4*x^2) - 40*a*x^5*(-b^5/a^13)^(1/8)*arctan(-
(a^8*x*(-b^5/a^13)^(5/8) - a^8*sqrt((a^10*(-b^5/a^13)^(3/4) + b^4*x^2)/b^4)
*(-b^5/a^13)^(5/8))/b^3) + 10*a*x^5*(-b^5/a^13)^(1/8)*log(a^5*(-b^5/a^13)^(
3/8) + b^2*x) - 10*a*x^5*(-b^5/a^13)^(1/8)*log(-a^5*(-b^5/a^13)^(3/8) + b^2
*x) - 16)/(a*x^5)
```

giac [B] time = 0.36, size = 453, normalized size = 1.64

$$\frac{b\left(\frac{a}{b}\right)^{\frac{3}{8}}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{4a^2\sqrt{2\sqrt{2}+4}}+\frac{b\left(\frac{a}{b}\right)^{\frac{3}{8}}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{4a^2\sqrt{2\sqrt{2}+4}}-\frac{b\left(\frac{a}{b}\right)^{\frac{3}{8}}\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{4a^2\sqrt{-2\sqrt{2}+4}}-\frac{b\left(\frac{a}{b}\right)^{\frac{3}{8}}\arcta}{4a^2\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^8+a),x, algorithm="giac")
```

```
[Out] 1/4*b*(a/b)^(3/8)*arctan((2*x + sqrt(-sqrt(2) + 2)*(a/b)^(1/8))/(sqrt(sqrt(
2) + 2)*(a/b)^(1/8)))/(a^2*sqrt(2*sqrt(2) + 4)) + 1/4*b*(a/b)^(3/8)*arctan(
(2*x - sqrt(-sqrt(2) + 2)*(a/b)^(1/8))/(sqrt(sqrt(2) + 2)*(a/b)^(1/8)))/(a^
2*sqrt(2*sqrt(2) + 4)) - 1/4*b*(a/b)^(3/8)*arctan((2*x + sqrt(sqrt(2) + 2)*
(a/b)^(1/8))/(sqrt(-sqrt(2) + 2)*(a/b)^(1/8)))/(a^2*sqrt(-2*sqrt(2) + 4)) -
 1/4*b*(a/b)^(3/8)*arctan((2*x - sqrt(sqrt(2) + 2)*(a/b)^(1/8))/(sqrt(-sqrt
(2) + 2)*(a/b)^(1/8)))/(a^2*sqrt(-2*sqrt(2) + 4)) - 1/8*b*(a/b)^(3/8)*log(x
^2 + x*sqrt(sqrt(2) + 2)*(a/b)^(1/8) + (a/b)^(1/4))/(a^2*sqrt(2*sqrt(2) + 4
)) + 1/8*b*(a/b)^(3/8)*log(x^2 - x*sqrt(sqrt(2) + 2)*(a/b)^(1/8) + (a/b)^(1
/4))/(a^2*sqrt(2*sqrt(2) + 4)) + 1/8*b*(a/b)^(3/8)*log(x^2 + x*sqrt(-sqrt(2
) + 2)*(a/b)^(1/8) + (a/b)^(1/4))/(a^2*sqrt(-2*sqrt(2) + 4)) - 1/8*b*(a/b)^
(3/8)*log(x^2 - x*sqrt(-sqrt(2) + 2)*(a/b)^(1/8) + (a/b)^(1/4))/(a^2*sqrt(-
2*sqrt(2) + 4)) - 1/5/(a*x^5)
```

maple [C] time = 0.01, size = 36, normalized size = 0.13

$$-\frac{\ln\left(-\operatorname{RootOf}\left(b_Z^8+a\right)+x\right)}{8a\operatorname{RootOf}\left(b_Z^8+a\right)^5}-\frac{1}{5ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^8+a),x)
```

```
[Out] -1/5/a/x^5-1/8/a*sum(1/_R^5*ln(-_R+x),_R=RootOf(_Z^8*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{8}b\left(\frac{2\left(\frac{a}{b}\right)^{\frac{3}{8}}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{a\sqrt{2\sqrt{2}+4}}+\frac{2\left(\frac{a}{b}\right)^{\frac{3}{8}}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{a\sqrt{2\sqrt{2}+4}}-\frac{2\left(\frac{a}{b}\right)^{\frac{3}{8}}\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{a\sqrt{-2\sqrt{2}+4}}-\frac{2\left(\frac{a}{b}\right)^{\frac{3}{8}}\arctan\left(\frac{2x-\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right.}{a\sqrt{-2\sqrt{2}+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^8+a),x, algorithm="maxima")
```

```
[Out] -b*integrate(x^2/(b*x^8 + a), x)/a - 1/5/(a*x^5)
```

mupad **[B]** time = 0.15, size = 118, normalized size = 0.43

$$-\frac{1}{5\,a\,x^5}-\frac{(-b)^{5/8}\,\mathrm{atan}\left(\frac{(-b)^{1/8}\,x}{a^{1/8}}\right)}{4\,a^{13/8}}-\frac{(-b)^{5/8}\,\mathrm{atan}\left(\frac{(-b)^{1/8}\,x\,1\mathrm{i}}{a^{1/8}}\right)\,1\mathrm{i}}{4\,a^{13/8}}+\frac{\sqrt{2}\,(-b)^{5/8}\,\mathrm{atan}\left(\frac{\sqrt{2}\,(-b)^{1/8}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{a^{1/8}}\right)\left(\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)}{a^{13/8}}+\frac{\sqrt{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^8)),x)
```

```
[Out] (2^(1/2)*(-b)^(5/8)*atan((2^(1/2)*(-b)^(1/8)*x*(1/2 - 1i/2))/a^(1/8))*(1/8
- 1i/8))/a^(13/8) - ((-b)^(5/8)*atan(((-b)^(1/8)*x)/a^(1/8)))/(4*a^(13/8))
- ((-b)^(5/8)*atan(((-b)^(1/8)*x*1i)/a^(1/8))*1i)/(4*a^(13/8)) - 1/(5*a*x^5
) + (2^(1/2)*(-b)^(5/8)*atan((2^(1/2)*(-b)^(1/8)*x*(1/2 + 1i/2))/a^(1/8))*(
1/8 + 1i/8))/a^(13/8)
```

sympy **[A]** time = 0.58, size = 36, normalized size = 0.13

$$\mathrm{RootSum}\left(16777216t^8a^{13}+b^5,\left(t\mapsto t\log\left(\frac{512t^3a^5}{b^2}+x\right)\right)\right)-\frac{1}{5ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**8+a),x)
```

```
[Out] RootSum(16777216*_t**8*a**13 + b**5, Lambda(_t, _t*log(512*_t**3*a**5/b**2
+ x))) - 1/(5*a*x**5)
```

3.1469 $\int \frac{1}{x^8\left(a+bx^8\right)}\,dx$

Optimal. Leaf size=277

$$\frac{b^{7/8}\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{-a}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,(-a)^{15/8}}-\frac{b^{7/8}\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{-a}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,(-a)^{15/8}}-\frac{b^{7/8}\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{15/8}}+\frac{b^{7/8}\tan^{-}}{4\sqrt{2}}$$

```
[Out] -1/7/a/x^7-1/4*b^(7/8)*arctan(b^(1/8)*x/(-a)^(1/8))/(-a)^(15/8)-1/4*b^(7/8)
*arctanh(b^(1/8)*x/(-a)^(1/8))/(-a)^(15/8)-1/8*b^(7/8)*arctan(-1+b^(1/8)*x*
2^(1/2)/(-a)^(1/8))/(-a)^(15/8)*2^(1/2)-1/8*b^(7/8)*arctan(1+b^(1/8)*x*2^(1
/2)/(-a)^(1/8))/(-a)^(15/8)*2^(1/2)+1/16*b^(7/8)*ln((-a)^(1/4)+b^(1/4)*x^2-
(-a)^(1/8)*b^(1/8)*x*2^(1/2))/(-a)^(15/8)*2^(1/2)-1/16*b^(7/8)*ln((-a)^(1/4
)+b^(1/4)*x^2+(-a)^(1/8)*b^(1/8)*x*2^(1/2))/(-a)^(15/8)*2^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {325, 214, 212, 208, 205, 211, 1165, 628, 1162, 617, 204}

$$\frac{b^{7/8}\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{-a}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,(-a)^{15/8}}-\frac{b^{7/8}\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{-a}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,(-a)^{15/8}}-\frac{b^{7/8}\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{15/8}}+\frac{b^{7/8}\tan^{-}}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(a + b*x^8)),x]
```

```
[Out] -1/(7*a*x^7) - (b^(7/8)*ArcTan[(b^(1/8)*x)/(-a)^(1/8)])/(4*(-a)^(15/8)) + (
b^(7/8)*ArcTan[1 - (Sqrt[2]*b^(1/8)*x)/(-a)^(1/8)])/(4*Sqrt[2]*(-a)^(15/8))
 - (b^(7/8)*ArcTan[1 + (Sqrt[2]*b^(1/8)*x)/(-a)^(1/8)])/(4*Sqrt[2]*(-a)^(15
/8)) - (b^(7/8)*ArcTanh[(b^(1/8)*x)/(-a)^(1/8)])/(4*(-a)^(15/8)) + (b^(7/8)
*Log[(-a)^(1/4) - Sqrt[2]*(-a)^(1/8)*b^(1/8)*x + b^(1/4)*x^2])/(8*Sqrt[2]*(
-a)^(15/8)) - (b^(7/8)*Log[(-a)^(1/4) + Sqrt[2]*(-a)^(1/8)*b^(1/8)*x + b^(1
/4)*x^2])/(8*Sqrt[2]*(-a)^(15/8))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^8\left(a+bx^8\right)}\,dx = -\frac{1}{7ax^7} - \frac{b\int \frac{1}{a+bx^8}\,dx}{a}$$

$$= -\frac{1}{7ax^7} - \frac{b\int \frac{1}{\sqrt{-a}-\sqrt{b}\,x^4}\,dx}{2(-a)^{3/2}} - \frac{b\int \frac{1}{\sqrt{-a}+\sqrt{b}\,x^4}\,dx}{2(-a)^{3/2}}$$

$$= -\frac{1}{7ax^7} - \frac{b\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{b}\,x^2}\,dx}{4(-a)^{7/4}} - \frac{b\int \frac{1}{\sqrt[4]{-a}+\sqrt[4]{b}\,x^2}\,dx}{4(-a)^{7/4}} - \frac{b\int \frac{\sqrt[4]{-a}-\sqrt[4]{b}\,x^2}{\sqrt{-a}+\sqrt{b}\,x^4}\,dx}{4(-a)^{7/4}} - \frac{b\int \frac{\sqrt[4]{-a}+\sqrt[4]{b}\,x^2}{\sqrt{-a}+\sqrt{b}\,x^4}\,dx}{4(-a)^{7/4}}$$

$$= -\frac{1}{7ax^7} - \frac{b^{7/8}\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{15/8}} - \frac{b^{7/8}\tanh^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{15/8}} - \frac{b^{3/4}\int \frac{1}{\frac{\sqrt{-a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[8]{-a}\,x}{\sqrt[8]{b}}+x^2}\,dx}{8(-a)^{7/4}} - \frac{b^{3/4}\int \frac{}{}}{8(-a)}$$

$$= -\frac{1}{7ax^7} - \frac{b^{7/8}\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{15/8}} - \frac{b^{7/8}\tanh^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{15/8}} + \frac{b^{7/8}\log\left(\sqrt[4]{-a}-\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}\,x+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,(-a)^{15/8}}$$

$$= -\frac{1}{7ax^7} - \frac{b^{7/8}\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4(-a)^{15/8}} + \frac{b^{7/8}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4\sqrt{2}\,(-a)^{15/8}} - \frac{b^{7/8}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[8]{b}\,x}{\sqrt[8]{-a}}\right)}{4\sqrt{2}\,(-a)^{15/8}} - \frac{b^{7/8}\tanh^{-1}}{4(-a)}$$

Mathematica [A] time = 0.21, size = 395, normalized size = 1.43

$$-\frac{8a^{7/8} + 14b^{7/8}x^7\cos\left(\frac{\pi}{8}\right)\tan^{-1}\left(\frac{\sqrt[8]{b}\,x\sec\left(\frac{\pi}{8}\right)}{\sqrt[8]{a}} - \tan\left(\frac{\pi}{8}\right)\right) + 14b^{7/8}x^7\cos\left(\frac{\pi}{8}\right)\tan^{-1}\left(\frac{\sqrt[8]{b}\,x\sec\left(\frac{\pi}{8}\right)}{\sqrt[8]{a}} + \tan\left(\frac{\pi}{8}\right)\right) - 14b^{7/8}x^7}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(a + b*x^8)),x]
```

```
[Out] -1/56*(8*a^(7/8) + 14*b^(7/8)*x^7*ArcTan[(b^(1/8)*x*Sec[Pi/8])/a^(1/8) - Ta
n[Pi/8]]*Cos[Pi/8] + 14*b^(7/8)*x^7*ArcTan[(b^(1/8)*x*Sec[Pi/8])/a^(1/8) +
Tan[Pi/8]]*Cos[Pi/8] - 7*b^(7/8)*x^7*Cos[Pi/8]*Log[a^(1/4) + b^(1/4)*x^2 -
2*a^(1/8)*b^(1/8)*x*Cos[Pi/8]] + 7*b^(7/8)*x^7*Cos[Pi/8]*Log[a^(1/4) + b^(1
/4)*x^2 + 2*a^(1/8)*b^(1/8)*x*Cos[Pi/8]] - 14*b^(7/8)*x^7*ArcTan[Cot[Pi/8]
- (b^(1/8)*x*Csc[Pi/8])/a^(1/8)]*Sin[Pi/8] + 14*b^(7/8)*x^7*ArcTan[Cot[Pi/8
] + (b^(1/8)*x*Csc[Pi/8])/a^(1/8)]*Sin[Pi/8] - 7*b^(7/8)*x^7*Log[a^(1/4) +
b^(1/4)*x^2 - 2*a^(1/8)*b^(1/8)*x*Sin[Pi/8]]*Sin[Pi/8] + 7*b^(7/8)*x^7*Log[
a^(1/4) + b^(1/4)*x^2 + 2*a^(1/8)*b^(1/8)*x*Sin[Pi/8]]*Sin[Pi/8])/(a^(15/8)
*x^7)
```

fricas [B] time = 0.86, size = 514, normalized size = 1.86

$$-\frac{28\sqrt{2}\,ax^7\left(-\frac{b^7}{a^{15}}\right)^{\frac{1}{8}}\arctan\left(-\frac{\sqrt{2}\,a^{13}x\left(-\frac{b^7}{a^{15}}\right)^{\frac{7}{8}} - \sqrt{2}\,a^{13}\sqrt{\frac{\sqrt{2}\,a^2bx\left(-\frac{b^7}{a^{15}}\right)^{\frac{1}{8}} + a^4\left(-\frac{b^7}{a^{15}}\right)^{\frac{1}{4}} + b^2x^2}{b^2}}\left(-\frac{b^7}{a^{15}}\right)^{\frac{7}{8}} - b^6}{b^6}\right) + 28\sqrt{2}\,ax^7\left(-\frac{b^7}{a^{15}}\right)^{\frac{1}{8}}\arctan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^8+a),x, algorithm="fricas")
```

```
[Out] -1/112*(28*sqrt(2)*a*x^7*(-b^7/a^15)^(1/8)*arctan(-(sqrt(2)*a^13*x*(-b^7/a^
15)^(7/8) - sqrt(2)*a^13*sqrt((sqrt(2)*a^2*b*x*(-b^7/a^15)^(1/8) + a^4*(-b^
```

```
7/a^15)^(1/4) + b^2*x^2)/b^2)*(-b^7/a^15)^(7/8) - b^6)/b^6) + 28*sqrt(2)*a*
x^7*(-b^7/a^15)^(1/8)*arctan(-(sqrt(2)*a^13*x*(-b^7/a^15)^(7/8) - sqrt(2)*a
^13*sqrt(-(sqrt(2)*a^2*b*x*(-b^7/a^15)^(1/8) - a^4*(-b^7/a^15)^(1/4) - b^2*
x^2)/b^2)*(-b^7/a^15)^(7/8) + b^6)/b^6) + 7*sqrt(2)*a*x^7*(-b^7/a^15)^(1/8)
*log(sqrt(2)*a^2*b*x*(-b^7/a^15)^(1/8) + a^4*(-b^7/a^15)^(1/4) + b^2*x^2) -
 7*sqrt(2)*a*x^7*(-b^7/a^15)^(1/8)*log(-sqrt(2)*a^2*b*x*(-b^7/a^15)^(1/8) +
 a^4*(-b^7/a^15)^(1/4) + b^2*x^2) + 56*a*x^7*(-b^7/a^15)^(1/8)*arctan(-(a^1
3*x*(-b^7/a^15)^(7/8) - a^13*sqrt((a^4*(-b^7/a^15)^(1/4) + b^2*x^2)/b^2)*(-
b^7/a^15)^(7/8))/b^6) + 14*a*x^7*(-b^7/a^15)^(1/8)*log(a^2*(-b^7/a^15)^(1/8
) + b*x) - 14*a*x^7*(-b^7/a^15)^(1/8)*log(-a^2*(-b^7/a^15)^(1/8) + b*x) + 1
6)/(a*x^7)
```

giac [B] time = 0.23, size = 453, normalized size = 1.64

$$-\frac{b\left(\frac{a}{b}\right)^{\frac{1}{8}}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{4a^2\sqrt{-2\sqrt{2}+4}}-\frac{b\left(\frac{a}{b}\right)^{\frac{1}{8}}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{4a^2\sqrt{-2\sqrt{2}+4}}-\frac{b\left(\frac{a}{b}\right)^{\frac{1}{8}}\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{4a^2\sqrt{2\sqrt{2}+4}}-\frac{b\left(\frac{a}{b}\right)^{\frac{1}{8}}a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^8+a),x, algorithm="giac")
```

```
[Out] -1/4*b*(a/b)^(1/8)*arctan((2*x + sqrt(-sqrt(2) + 2)*(a/b)^(1/8))/(sqrt(sqrt
(2) + 2)*(a/b)^(1/8)))/(a^2*sqrt(-2*sqrt(2) + 4)) - 1/4*b*(a/b)^(1/8)*arcta
n((2*x - sqrt(-sqrt(2) + 2)*(a/b)^(1/8))/(sqrt(sqrt(2) + 2)*(a/b)^(1/8)))/(
a^2*sqrt(-2*sqrt(2) + 4)) - 1/4*b*(a/b)^(1/8)*arctan((2*x + sqrt(sqrt(2) +
2)*(a/b)^(1/8))/(sqrt(-sqrt(2) + 2)*(a/b)^(1/8)))/(a^2*sqrt(2*sqrt(2) + 4))
 - 1/4*b*(a/b)^(1/8)*arctan((2*x - sqrt(sqrt(2) + 2)*(a/b)^(1/8))/(sqrt(-sq
rt(2) + 2)*(a/b)^(1/8)))/(a^2*sqrt(2*sqrt(2) + 4)) - 1/8*b*(a/b)^(1/8)*log(
x^2 + x*sqrt(sqrt(2) + 2)*(a/b)^(1/8) + (a/b)^(1/4))/(a^2*sqrt(-2*sqrt(2) +
 4)) + 1/8*b*(a/b)^(1/8)*log(x^2 - x*sqrt(sqrt(2) + 2)*(a/b)^(1/8) + (a/b)^
(1/4))/(a^2*sqrt(-2*sqrt(2) + 4)) - 1/8*b*(a/b)^(1/8)*log(x^2 + x*sqrt(-sqr
t(2) + 2)*(a/b)^(1/8) + (a/b)^(1/4))/(a^2*sqrt(2*sqrt(2) + 4)) + 1/8*b*(a/b
)^(1/8)*log(x^2 - x*sqrt(-sqrt(2) + 2)*(a/b)^(1/8) + (a/b)^(1/4))/(a^2*sqrt
(2*sqrt(2) + 4)) - 1/7/(a*x^7)
```

maple [C] time = 0.00, size = 36, normalized size = 0.13

$$-\frac{\ln\left(-\operatorname{RootOf}\left(b_Z^8+a\right)+x\right)}{8a\operatorname{RootOf}\left(b_Z^8+a\right)^7}-\frac{1}{7a\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(b*x^8+a),x)
```

```
[Out] -1/8/a*sum(1/_R^7*ln(-_R+x),_R=RootOf(_Z^8*b+a))-1/7/a/x^7
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8}b\left(-\frac{2\left(\frac{a}{b}\right)^{\frac{1}{8}}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{a\sqrt{-2\sqrt{2}+4}}+\frac{2\left(\frac{a}{b}\right)^{\frac{1}{8}}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{a\sqrt{-2\sqrt{2}+4}}+\frac{2\left(\frac{a}{b}\right)^{\frac{1}{8}}\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{a\sqrt{2\sqrt{2}+4}}+\frac{2\left(\frac{a}{b}\right)^{\frac{1}{8}}\arctan\left(\frac{2x-\sqrt{\sqrt{2}}}{\sqrt{-\sqrt{2}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^8+a),x, algorithm="maxima")
```

```
[Out] -b*integrate(1/(b*x^8 + a), x)/a - 1/7/(a*x^7)
```

mupad [B] time = 1.14, size = 118, normalized size = 0.43

$$\frac{(-b)^{7/8}\,\mathrm{atan}\left(\frac{(-b)^{1/8}\,x}{a^{1/8}}\right)}{4\,a^{15/8}}-\frac{1}{7\,a\,x^7}-\frac{(-b)^{7/8}\,\mathrm{atan}\left(\frac{(-b)^{1/8}\,x\,1\mathrm{i}}{a^{1/8}}\right)1\mathrm{i}}{4\,a^{15/8}}+\frac{\sqrt{2}\,(-b)^{7/8}\,\mathrm{atan}\left(\frac{\sqrt{2}\,(-b)^{1/8}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{a^{1/8}}\right)\left(\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)}{a^{15/8}}+\frac{\sqrt{2}\,(-b)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(a + b*x^8)),x)
```

```
[Out] ((-b)^(7/8)*atan(((-b)^(1/8)*x)/a^(1/8)))/(4*a^(15/8)) - 1/(7*a*x^7) - ((-b
)^(7/8)*atan(((-b)^(1/8)*x*1i)/a^(1/8))*1i)/(4*a^(15/8)) + (2^(1/2)*(-b)^(7
/8)*atan((2^(1/2)*(-b)^(1/8)*x*(1/2 - 1i/2))/a^(1/8))*(1/8 + 1i/8))/a^(15/8
) + (2^(1/2)*(-b)^(7/8)*atan((2^(1/2)*(-b)^(1/8)*x*(1/2 + 1i/2))/a^(1/8))*(
1/8 - 1i/8))/a^(15/8)
```

sympy [A] time = 0.40, size = 32, normalized size = 0.12

$$\mathrm{RootSum}\left(16777216t^8a^{15}+b^7,\left(t\mapsto t\log\left(-\frac{8ta^2}{b}+x\right)\right)\right)-\frac{1}{7ax^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(b*x**8+a),x)
```

```
[Out] RootSum(16777216*_t**8*a**15 + b**7, Lambda(_t, _t*log(-8*_t*a**2/b + x)))
- 1/(7*a*x**7)
```

3.1470 $\int \frac{1}{a-bx^8}\,dx$

Optimal. Leaf size=239

$$-\frac{\log\left(-\sqrt{2}\,\sqrt[8]{a}\,\sqrt[8]{b}\,x+\sqrt[4]{a}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,a^{7/8}\sqrt[8]{b}}+\frac{\log\left(\sqrt{2}\,\sqrt[8]{a}\,\sqrt[8]{b}\,x+\sqrt[4]{a}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,a^{7/8}\sqrt[8]{b}}+\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt[8]{b}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{b}\,x}{\sqrt[8]{a}}\right)}{4\sqrt{2}\,a^{7/8}\sqrt[8]{b}}+\text{ta}$$

```
[Out] 1/4*arctan(b^(1/8)*x/a^(1/8))/a^(7/8)/b^(1/8)+1/4*arctanh(b^(1/8)*x/a^(1/8)
)/a^(7/8)/b^(1/8)+1/8*arctan(-1+b^(1/8)*x*2^(1/2)/a^(1/8))/a^(7/8)/b^(1/8)*
2^(1/2)+1/8*arctan(1+b^(1/8)*x*2^(1/2)/a^(1/8))/a^(7/8)/b^(1/8)*2^(1/2)-1/1
6*ln(a^(1/4)+b^(1/4)*x^2-a^(1/8)*b^(1/8)*x*2^(1/2))/a^(7/8)/b^(1/8)*2^(1/2)
+1/16*ln(a^(1/4)+b^(1/4)*x^2+a^(1/8)*b^(1/8)*x*2^(1/2))/a^(7/8)/b^(1/8)*2^(
1/2)
```

Rubi [A] time = 0.17, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {214, 212, 208, 205, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(-\sqrt{2}\,\sqrt[8]{a}\,\sqrt[8]{b}\,x+\sqrt[4]{a}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,a^{7/8}\sqrt[8]{b}}+\frac{\log\left(\sqrt{2}\,\sqrt[8]{a}\,\sqrt[8]{b}\,x+\sqrt[4]{a}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,a^{7/8}\sqrt[8]{b}}+\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt[8]{b}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{b}\,x}{\sqrt[8]{a}}\right)}{4\sqrt{2}\,a^{7/8}\sqrt[8]{b}}+\text{ta}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^8)^(-1),x]
```

```
[Out] ArcTan[(b^(1/8)*x)/a^(1/8)]/(4*a^(7/8)*b^(1/8)) - ArcTan[1 - (Sqrt[2]*b^(1/
8)*x)/a^(1/8)]/(4*Sqrt[2]*a^(7/8)*b^(1/8)) + ArcTan[1 + (Sqrt[2]*b^(1/8)*x)
/a^(1/8)]/(4*Sqrt[2]*a^(7/8)*b^(1/8)) + ArcTanh[(b^(1/8)*x)/a^(1/8)]/(4*a^(
7/8)*b^(1/8)) - Log[a^(1/4) - Sqrt[2]*a^(1/8)*b^(1/8)*x + b^(1/4)*x^2]/(8*S
qrt[2]*a^(7/8)*b^(1/8)) + Log[a^(1/4) + Sqrt[2]*a^(1/8)*b^(1/8)*x + b^(1/4)
*x^2]/(8*Sqrt[2]*a^(7/8)*b^(1/8))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{a-bx^8}\,dx = \frac{\int \frac{1}{\sqrt{a}-\sqrt{b}\,x^4}\,dx}{2\sqrt{a}} + \frac{\int \frac{1}{\sqrt{a}+\sqrt{b}\,x^4}\,dx}{2\sqrt{a}}$$

$$= \frac{\int \frac{1}{\sqrt[4]{a}-\sqrt[4]{b}\,x^2}\,dx}{4a^{3/4}} + \frac{\int \frac{1}{\sqrt[4]{a}+\sqrt[4]{b}\,x^2}\,dx}{4a^{3/4}} + \frac{\int \frac{\sqrt[4]{a}-\sqrt[4]{b}\,x^2}{\sqrt{a}+\sqrt{b}\,x^4}\,dx}{4a^{3/4}} + \frac{\int \frac{\sqrt[4]{a}+\sqrt[4]{b}\,x^2}{\sqrt{a}+\sqrt{b}\,x^4}\,dx}{4a^{3/4}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt[8]{b}} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt[8]{b}} + \frac{\int \frac{1}{\frac{\sqrt[4]{a}}{\sqrt[4]{b}}-\frac{\sqrt{2}\,\sqrt[8]{a}\,x}{\sqrt[8]{b}}+x^2}\,dx}{8a^{3/4}\sqrt[4]{b}} + \frac{\int \frac{1}{\frac{\sqrt[4]{a}}{\sqrt[4]{b}}+\frac{\sqrt{2}\,\sqrt[8]{a}\,x}{\sqrt[8]{b}}+x^2}\,dx}{8a^{3/4}\sqrt[4]{b}} - \frac{\int \frac{\frac{\sqrt{2}\,\sqrt[8]{a}}{\sqrt[8]{b}}+2x}{-\frac{\sqrt[4]{a}}{\sqrt[4]{b}}-\frac{\sqrt{2}\,\sqrt[8]{a}\,x}{\sqrt[8]{b}}-x^2}}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt[8]{b}} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt[8]{b}} - \frac{\log\left(\sqrt[4]{a}-\sqrt{2}\,\sqrt[8]{a}\,\sqrt[8]{b}\,x+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,a^{7/8}\sqrt[8]{b}} + \frac{\log\left(\sqrt[4]{a}+\sqrt{2}\,\sqrt[8]{a}\,\sqrt[8]{b}\,x+\right.}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt[8]{b}} - \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{b}\,x}{\sqrt[8]{a}}\right)}{4\sqrt{2}\,a^{7/8}\sqrt[8]{b}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[8]{b}\,x}{\sqrt[8]{a}}\right)}{4\sqrt{2}\,a^{7/8}\sqrt[8]{b}} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt[8]{b}} - \frac{\log\left(\sqrt[4]{a}-\sqrt{2}\,\sqrt[8]{}\right.}{}$$

Mathematica [A] time = 0.08, size = 198, normalized size = 0.83

$$\frac{-\sqrt{2}\,\log\left(-\sqrt{2}\,\sqrt[8]{a}\,\sqrt[8]{b}\,x+\sqrt[4]{a}+\sqrt[4]{b}\,x^2\right)+\sqrt{2}\,\log\left(\sqrt{2}\,\sqrt[8]{a}\,\sqrt[8]{b}\,x+\sqrt[4]{a}+\sqrt[4]{b}\,x^2\right)-2\log\left(\sqrt[8]{a}-\sqrt[8]{b}\,x\right)+2\log\left(}{16a^{7/8}\sqrt[8]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^8)^(-1),x]
```

```
[Out] (4*ArcTan[(b^(1/8)*x)/a^(1/8)] - 2*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/8)*x)/a
^(1/8)] + 2*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/8)*x)/a^(1/8)] - 2*Log[a^(1/8)
 - b^(1/8)*x] + 2*Log[a^(1/8) + b^(1/8)*x] - Sqrt[2]*Log[a^(1/4) - Sqrt[2]*
a^(1/8)*b^(1/8)*x + b^(1/4)*x^2] + Sqrt[2]*Log[a^(1/4) + Sqrt[2]*a^(1/8)*b^
(1/8)*x + b^(1/4)*x^2])/(16*a^(7/8)*b^(1/8))
```

fricas [B] time = 0.75, size = 384, normalized size = 1.61

$$-\frac{1}{4}\sqrt{2}\left(\frac{1}{a^7b}\right)^{\frac{1}{8}}\arctan\left(-\sqrt{2}\,a^6bx\left(\frac{1}{a^7b}\right)^{\frac{7}{8}}+\sqrt{2}\sqrt{\sqrt{2}\,ax\left(\frac{1}{a^7b}\right)^{\frac{1}{8}}+a^2\left(\frac{1}{a^7b}\right)^{\frac{1}{4}}+x^2}\,a^6b\left(\frac{1}{a^7b}\right)^{\frac{7}{8}}-1\right)-\frac{1}{4}\sqrt{2}\left(\frac{1}{a^7b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^8+a),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*(1/(a^7*b))^(1/8)*arctan(-sqrt(2)*a^6*b*x*(1/(a^7*b))^(7/8) +
sqrt(2)*sqrt(sqrt(2)*a*x*(1/(a^7*b))^(1/8) + a^2*(1/(a^7*b))^(1/4) + x^2)*a
^6*b*(1/(a^7*b))^(7/8) - 1) - 1/4*sqrt(2)*(1/(a^7*b))^(1/8)*arctan(-sqrt(2)
*a^6*b*x*(1/(a^7*b))^(7/8) + sqrt(2)*sqrt(-sqrt(2)*a*x*(1/(a^7*b))^(1/8) +
a^2*(1/(a^7*b))^(1/4) + x^2)*a^6*b*(1/(a^7*b))^(7/8) + 1) + 1/16*sqrt(2)*(1
/(a^7*b))^(1/8)*log(sqrt(2)*a*x*(1/(a^7*b))^(1/8) + a^2*(1/(a^7*b))^(1/4) +
 x^2) - 1/16*sqrt(2)*(1/(a^7*b))^(1/8)*log(-sqrt(2)*a*x*(1/(a^7*b))^(1/8) +
 a^2*(1/(a^7*b))^(1/4) + x^2) - 1/2*(1/(a^7*b))^(1/8)*arctan(-a^6*b*x*(1/(a
^7*b))^(7/8) + sqrt(a^2*(1/(a^7*b))^(1/4) + x^2)*a^6*b*(1/(a^7*b))^(7/8)) +
 1/8*(1/(a^7*b))^(1/8)*log(a*(1/(a^7*b))^(1/8) + x) - 1/8*(1/(a^7*b))^(1/8)
*log(-a*(1/(a^7*b))^(1/8) + x)
```

giac [B] time = 0.21, size = 461, normalized size = 1.93

$$\frac{\left(-\frac{a}{b}\right)^{\frac{1}{8}}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}\left(-\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(-\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{4a\sqrt{-2\sqrt{2}+4}}+\frac{\left(-\frac{a}{b}\right)^{\frac{1}{8}}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}\left(-\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(-\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{4a\sqrt{-2\sqrt{2}+4}}+\frac{\left(-\frac{a}{b}\right)^{\frac{1}{8}}\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}\left(-\frac{a}{b}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(-\frac{a}{b}\right)^{\frac{1}{8}}}\right)}{4a\sqrt{2\sqrt{2}+4}}+\frac{\left(-\frac{a}{b}\right)^{\frac{1}{8}}\ar}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^8+a),x, algorithm="giac")
```

```
[Out] 1/4*(-a/b)^(1/8)*arctan((2*x + sqrt(-sqrt(2) + 2)*(-a/b)^(1/8))/(sqrt(sqrt(
2) + 2)*(-a/b)^(1/8)))/(a*sqrt(-2*sqrt(2) + 4)) + 1/4*(-a/b)^(1/8)*arctan((
2*x - sqrt(-sqrt(2) + 2)*(-a/b)^(1/8))/(sqrt(sqrt(2) + 2)*(-a/b)^(1/8)))/(a
*sqrt(-2*sqrt(2) + 4)) + 1/4*(-a/b)^(1/8)*arctan((2*x + sqrt(sqrt(2) + 2)*(
-a/b)^(1/8))/(sqrt(-sqrt(2) + 2)*(-a/b)^(1/8)))/(a*sqrt(2*sqrt(2) + 4)) + 1
/4*(-a/b)^(1/8)*arctan((2*x - sqrt(sqrt(2) + 2)*(-a/b)^(1/8))/(sqrt(-sqrt(2
) + 2)*(-a/b)^(1/8)))/(a*sqrt(2*sqrt(2) + 4)) + 1/8*(-a/b)^(1/8)*log(x^2 +
x*sqrt(sqrt(2) + 2)*(-a/b)^(1/8) + (-a/b)^(1/4))/(a*sqrt(-2*sqrt(2) + 4)) -
 1/8*(-a/b)^(1/8)*log(x^2 - x*sqrt(sqrt(2) + 2)*(-a/b)^(1/8) + (-a/b)^(1/4)
)/(a*sqrt(-2*sqrt(2) + 4)) + 1/8*(-a/b)^(1/8)*log(x^2 + x*sqrt(-sqrt(2) + 2
)*(-a/b)^(1/8) + (-a/b)^(1/4))/(a*sqrt(2*sqrt(2) + 4)) - 1/8*(-a/b)^(1/8)*l
og(x^2 - x*sqrt(-sqrt(2) + 2)*(-a/b)^(1/8) + (-a/b)^(1/4))/(a*sqrt(2*sqrt(2
) + 4))
```

maple [C] time = 0.01, size = 29, normalized size = 0.12

$$-\frac{\ln\left(-\operatorname{RootOf}\left(b_Z^8-a\right)+x\right)}{8b\operatorname{RootOf}\left(b_Z^8-a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^8+a),x)
```

```
[Out] -1/8/b*sum(1/_R^7*ln(-_R+x),_R=RootOf(_Z^8*b-a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{bx^8-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^8+a),x, algorithm="maxima")
```

```
[Out] -integrate(1/(b*x^8 - a), x)
```

mupad [B] time = 0.14, size = 94, normalized size = 0.39

$$\frac{\operatorname{atan}\left(\frac{b^{1/8}x}{a^{1/8}}\right)}{4a^{7/8}b^{1/8}}-\frac{\operatorname{atan}\left(\frac{b^{1/8}x\,1\mathrm{i}}{a^{1/8}}\right)1\mathrm{i}}{4a^{7/8}b^{1/8}}+\frac{\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,b^{1/8}x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{a^{1/8}}\right)\left(\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)}{a^{7/8}b^{1/8}}+\frac{\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,b^{1/8}x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)}{a^{1/8}}\right)\left(\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)}{a^{7/8}b^{1/8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*x^8),x)
```

```
[Out] atan((b^(1/8)*x)/a^(1/8))/(4*a^(7/8)*b^(1/8)) - (atan((b^(1/8)*x*1i)/a^(1/8
))*1i)/(4*a^(7/8)*b^(1/8)) + (2^(1/2)*atan((2^(1/2)*b^(1/8)*x*(1/2 - 1i/2))
/a^(1/8))*(1/8 + 1i/8))/(a^(7/8)*b^(1/8)) + (2^(1/2)*atan((2^(1/2)*b^(1/8)*
x*(1/2 + 1i/2))/a^(1/8))*(1/8 - 1i/8))/(a^(7/8)*b^(1/8))
```

sympy [**A**] time = 0.40, size = 22, normalized size = 0.09

$$-\operatorname{RootSum}\left(16777216t^8a^7b-1,\left(t\mapsto t\log\left(-8ta+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**8+a),x)
[Out] -RootSum(16777216*_t**8*a**7*b - 1, Lambda(_t, _t*log(-8*_t*a + x)))
```

3.1471 $\int \frac{x^9}{1-x^8}\,dx$

Optimal. Leaf size=24

$$-\frac{x^2}{2}+\frac{1}{4}\tan^{-1}\left(x^2\right)+\frac{1}{4}\tanh^{-1}\left(x^2\right)$$

```
[Out] -1/2*x^2+1/4*arctan(x^2)+1/4*arctanh(x^2)
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {275, 321, 212, 206, 203}

$$-\frac{x^2}{2}+\frac{1}{4}\tan^{-1}\left(x^2\right)+\frac{1}{4}\tanh^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(1 - x^8),x]
```

```
[Out] -x^2/2 + ArcTan[x^2]/4 + ArcTanh[x^2]/4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{1-x^8}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{1-x^4}\,dx, x, x^2\right)\\ &= -\frac{x^2}{2} + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, x^2\right)\\ &= -\frac{x^2}{2} + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, x^2\right) + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, x^2\right)\\ &= -\frac{x^2}{2} + \frac{1}{4}\tan^{-1}\left(x^2\right) + \frac{1}{4}\tanh^{-1}\left(x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.58

$$-\frac{x^2}{2} - \frac{1}{8}\log\left(1-x^2\right) + \frac{1}{8}\log\left(x^2+1\right) - \frac{1}{4}\tan^{-1}\left(\frac{1}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(1 - x^8),x]
[Out] -1/2*x^2 - ArcTan[x^(-2)]/4 - Log[1 - x^2]/8 + Log[1 + x^2]/8
```

fricas [A] time = 0.54, size = 28, normalized size = 1.17

$$-\frac{1}{2}\,x^2 + \frac{1}{4}\,\arctan\left(x^2\right) + \frac{1}{8}\,\log\left(x^2+1\right) - \frac{1}{8}\,\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(-x^8+1),x, algorithm="fricas")
[Out] -1/2*x^2 + 1/4*arctan(x^2) + 1/8*log(x^2 + 1) - 1/8*log(x^2 - 1)
```

giac [A] time = 0.15, size = 29, normalized size = 1.21

$$-\frac{1}{2}\,x^2 + \frac{1}{4}\,\arctan\left(x^2\right) + \frac{1}{8}\,\log\left(x^2+1\right) - \frac{1}{8}\,\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(-x^8+1),x, algorithm="giac")
[Out] -1/2*x^2 + 1/4*arctan(x^2) + 1/8*log(x^2 + 1) - 1/8*log(abs(x^2 - 1))
```

maple [A] time = 0.01, size = 33, normalized size = 1.38

$$-\frac{x^2}{2} + \frac{\arctan\left(x^2\right)}{4} - \frac{\ln\left(x-1\right)}{8} - \frac{\ln\left(x+1\right)}{8} + \frac{\ln\left(x^2+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(-x^8+1),x)
[Out] -1/2*x^2-1/8*ln(x-1)-1/8*ln(x+1)+1/8*ln(x^2+1)+1/4*arctan(x^2)
```

maxima [A] time = 2.49, size = 28, normalized size = 1.17

$$-\frac{1}{2}\,x^2 + \frac{1}{4}\,\arctan\left(x^2\right) + \frac{1}{8}\,\log\left(x^2+1\right) - \frac{1}{8}\,\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(-x^8+1),x, algorithm="maxima")
[Out] -1/2*x^2 + 1/4*arctan(x^2) + 1/8*log(x^2 + 1) - 1/8*log(x^2 - 1)
```

mupad [**B**] time = 0.06, size = 18, normalized size = 0.75

$$\frac{\operatorname{atan}\left(x^2\right)}{4}+\frac{\operatorname{atanh}\left(x^2\right)}{4}-\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^9/(x^8 - 1),x)
[Out] atan(x^2)/4 + atanh(x^2)/4 - x^2/2
```

sympy [**A**] time = 0.30, size = 27, normalized size = 1.12

$$-\frac{x^2}{2}-\frac{\log\left(x^2-1\right)}{8}+\frac{\log\left(x^2+1\right)}{8}+\frac{\operatorname{atan}\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(-x**8+1),x)
[Out] -x**2/2 - log(x**2 - 1)/8 + log(x**2 + 1)/8 + atan(x**2)/4
```

3.1472 $\int \frac{x^7}{1-x^8}\,dx$

Optimal. Leaf size=12

$$-\frac{1}{8}\log\left(1-x^8\right)$$

```
[Out] -1/8*ln(-x^8+1)
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {260}

$$-\frac{1}{8}\log\left(1-x^8\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(1 - x^8),x]
```

```
[Out] -Log[1 - x^8]/8
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^7}{1-x^8}\,dx = -\frac{1}{8}\log\left(1-x^8\right)$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$-\frac{1}{8}\log\left(1-x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(1 - x^8),x]
```

```
[Out] -1/8*Log[1 - x^8]
```

fricas [A] time = 0.51, size = 8, normalized size = 0.67

$$-\frac{1}{8}\log\left(x^8-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^8+1),x, algorithm="fricas")
```

```
[Out] -1/8*log(x^8 - 1)
```

giac [A] time = 0.16, size = 9, normalized size = 0.75

$$-\frac{1}{8}\log\left(\left|x^8-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^8+1),x, algorithm="giac")
[Out] -1/8*log(abs(x^8 - 1))
```

maple [B] time = 0.01, size = 30, normalized size = 2.50

$$-\frac{\ln(x-1)}{8}-\frac{\ln(x+1)}{8}-\frac{\ln\left(x^2+1\right)}{8}-\frac{\ln\left(x^4+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-x^8+1),x)
[Out] -1/8*ln(x-1)-1/8*ln(x+1)-1/8*ln(x^2+1)-1/8*ln(x^4+1)
```

maxima [A] time = 1.06, size = 8, normalized size = 0.67

$$-\frac{1}{8}\log\left(x^8-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^8+1),x, algorithm="maxima")
[Out] -1/8*log(x^8 - 1)
```

mupad [B] time = 0.04, size = 8, normalized size = 0.67

$$-\frac{\ln\left(x^8-1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^7/(x^8 - 1),x)
[Out] -log(x^8 - 1)/8
```

sympy [A] time = 0.29, size = 8, normalized size = 0.67

$$-\frac{\log\left(x^8-1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-x**8+1),x)
[Out] -log(x**8 - 1)/8
```

3.1473 $\int \frac{x^5}{1-x^8}\,dx$

Optimal. Leaf size=17

$$\frac{1}{4}\tanh^{-1}\left(x^2\right)-\frac{1}{4}\tan^{-1}\left(x^2\right)$$

```
[Out] -1/4*arctan(x^2)+1/4*arctanh(x^2)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {275, 298, 203, 206}

$$\frac{1}{4}\tanh^{-1}\left(x^2\right)-\frac{1}{4}\tan^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(1 - x^8),x]
```

```
[Out] -ArcTan[x^2]/4 + ArcTanh[x^2]/4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{1-x^8}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, x^2\right)\\ &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, x^2\right)-\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, x^2\right)\\ &= -\frac{1}{4}\tan^{-1}\left(x^2\right)+\frac{1}{4}\tanh^{-1}\left(x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.82

$$-\frac{1}{8}\log\left(1-x^2\right)+\frac{1}{8}\log\left(x^2+1\right)+\frac{1}{4}\tan^{-1}\left(\frac{1}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(1 - x^8),x]
[Out] ArcTan[x^(-2)]/4 - Log[1 - x^2]/8 + Log[1 + x^2]/8
```

fricas [A] time = 0.55, size = 23, normalized size = 1.35

$$-\frac{1}{4}\arctan\left(x^2\right)+\frac{1}{8}\log\left(x^2+1\right)-\frac{1}{8}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^8+1),x, algorithm="fricas")
[Out] -1/4*arctan(x^2) + 1/8*log(x^2 + 1) - 1/8*log(x^2 - 1)
```

giac [B] time = 0.16, size = 55, normalized size = 3.24

$$\frac{1}{4}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)-\frac{1}{4}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{8}\log\left(x^2+1\right)-\frac{1}{8}\log\left(|x+1|\right)-\frac{1}{8}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^8+1),x, algorithm="giac")
[Out] 1/4*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 1/4*arctan(1/2*sqrt(2)*(2*x - sqr
t(2))) + 1/8*log(x^2 + 1) - 1/8*log(abs(x + 1)) - 1/8*log(abs(x - 1))
```

maple [B] time = 0.00, size = 28, normalized size = 1.65

$$-\frac{\arctan\left(x^2\right)}{4}-\frac{\ln\left(x-1\right)}{8}-\frac{\ln\left(x+1\right)}{8}+\frac{\ln\left(x^2+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-x^8+1),x)
[Out] -1/8*ln(x-1)-1/8*ln(x+1)+1/8*ln(x^2+1)-1/4*arctan(x^2)
```

maxima [A] time = 2.41, size = 23, normalized size = 1.35

$$-\frac{1}{4}\arctan\left(x^2\right)+\frac{1}{8}\log\left(x^2+1\right)-\frac{1}{8}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^8+1),x, algorithm="maxima")
[Out] -1/4*arctan(x^2) + 1/8*log(x^2 + 1) - 1/8*log(x^2 - 1)
```

mupad [B] time = 1.04, size = 13, normalized size = 0.76

$$\frac{\operatorname{atanh}\left(x^2\right)}{4}-\frac{\operatorname{atan}\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^5/(x^8 - 1),x)
[Out] atanh(x^2)/4 - atan(x^2)/4
```

sympy [A] time = 0.42, size = 22, normalized size = 1.29

$$-\frac{\log\left(x^2-1\right)}{8}+\frac{\log\left(x^2+1\right)}{8}-\frac{\operatorname{atan}\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-x**8+1),x)
[Out] -log(x**2 - 1)/8 + log(x**2 + 1)/8 - atan(x**2)/4
```

3.1474 $\int \frac{x^3}{1-x^8}\,dx$

Optimal. Leaf size=8

$$\frac{1}{4}\tanh^{-1}\left(x^4\right)$$

```
[Out] 1/4*arctanh(x^4)
```

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {275, 206}

$$\frac{1}{4}\tanh^{-1}\left(x^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 - x^8),x]
```

```
[Out] ArcTanh[x^4]/4
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{1-x^8}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, x^4\right)\\ &= \frac{1}{4}\tanh^{-1}\left(x^4\right)\end{aligned}$$

Mathematica [B] time = 0.00, size = 23, normalized size = 2.88

$$\frac{1}{8}\log\left(x^4+1\right)-\frac{1}{8}\log\left(1-x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 - x^8),x]
```

```
[Out] -1/8*Log[1 - x^4] + Log[1 + x^4]/8
```

fricas [B] time = 0.52, size = 17, normalized size = 2.12

$$\frac{1}{8}\,\log\left(x^4+1\right)-\frac{1}{8}\,\log\left(x^4-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^8+1),x, algorithm="fricas")
```

```
[Out] 1/8*log(x^4 + 1) - 1/8*log(x^4 - 1)
```

giac [B] time = 0.15, size = 18, normalized size = 2.25

$$\frac{1}{8}\log\left(x^4+1\right)-\frac{1}{8}\log\left(\left|x^4-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^8+1),x, algorithm="giac")
[Out] 1/8*log(x^4 + 1) - 1/8*log(abs(x^4 - 1))
```

maple [B] time = 0.00, size = 30, normalized size = 3.75

$$-\frac{\ln(x-1)}{8}-\frac{\ln(x+1)}{8}-\frac{\ln\left(x^2+1\right)}{8}+\frac{\ln\left(x^4+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-x^8+1),x)
[Out] -1/8*ln(x-1)-1/8*ln(x+1)-1/8*ln(x^2+1)+1/8*ln(x^4+1)
```

maxima [B] time = 1.06, size = 17, normalized size = 2.12

$$\frac{1}{8}\log\left(x^4+1\right)-\frac{1}{8}\log\left(x^4-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^8+1),x, algorithm="maxima")
[Out] 1/8*log(x^4 + 1) - 1/8*log(x^4 - 1)
```

mupad [B] time = 0.07, size = 6, normalized size = 0.75

$$\frac{\operatorname{atanh}\left(x^4\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^3/(x^8 - 1),x)
[Out] atanh(x^4)/4
```

sympy [B] time = 0.34, size = 15, normalized size = 1.88

$$-\frac{\log\left(x^4-1\right)}{8}+\frac{\log\left(x^4+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-x**8+1),x)
[Out] -log(x**4 - 1)/8 + log(x**4 + 1)/8
```

3.1475 $\int \frac{x}{1-x^8}\,dx$

Optimal. Leaf size=17

$$\frac{1}{4}\tan^{-1}\left(x^2\right)+\frac{1}{4}\tanh^{-1}\left(x^2\right)$$

```
[Out] 1/4*arctan(x^2)+1/4*arctanh(x^2)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {275, 212, 206, 203}

$$\frac{1}{4}\tan^{-1}\left(x^2\right)+\frac{1}{4}\tanh^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 - x^8),x]
```

```
[Out] ArcTan[x^2]/4 + ArcTanh[x^2]/4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{1-x^8}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, x^2\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, x^2\right)+\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, x^2\right)\\ &= \frac{1}{4}\tan^{-1}\left(x^2\right)+\frac{1}{4}\tanh^{-1}\left(x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.82

$$-\frac{1}{8}\log\left(1-x^2\right)+\frac{1}{8}\log\left(x^2+1\right)-\frac{1}{4}\tan^{-1}\left(\frac{1}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 - x^8),x]
[Out] -1/4*ArcTan[x^(-2)] - Log[1 - x^2]/8 + Log[1 + x^2]/8
```

fricas [A] time = 0.60, size = 23, normalized size = 1.35

$$\frac{1}{4}\arctan\left(x^2\right)+\frac{1}{8}\log\left(x^2+1\right)-\frac{1}{8}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^8+1),x, algorithm="fricas")
[Out] 1/4*arctan(x^2) + 1/8*log(x^2 + 1) - 1/8*log(x^2 - 1)
```

giac [A] time = 0.15, size = 24, normalized size = 1.41

$$\frac{1}{4}\arctan\left(x^2\right)+\frac{1}{8}\log\left(x^2+1\right)-\frac{1}{8}\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^8+1),x, algorithm="giac")
[Out] 1/4*arctan(x^2) + 1/8*log(x^2 + 1) - 1/8*log(abs(x^2 - 1))
```

maple [B] time = 0.00, size = 28, normalized size = 1.65

$$\frac{\arctan\left(x^2\right)}{4}-\frac{\ln\left(x-1\right)}{8}-\frac{\ln\left(x+1\right)}{8}+\frac{\ln\left(x^2+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^8+1),x)
[Out] -1/8*ln(x-1)-1/8*ln(x+1)+1/8*ln(x^2+1)+1/4*arctan(x^2)
```

maxima [A] time = 2.42, size = 23, normalized size = 1.35

$$\frac{1}{4}\arctan\left(x^2\right)+\frac{1}{8}\log\left(x^2+1\right)-\frac{1}{8}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^8+1),x, algorithm="maxima")
[Out] 1/4*arctan(x^2) + 1/8*log(x^2 + 1) - 1/8*log(x^2 - 1)
```

mupad [B] time = 0.03, size = 13, normalized size = 0.76

$$\frac{\operatorname{atan}\left(x^2\right)}{4}+\frac{\operatorname{atanh}\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/(x^8 - 1),x)
[Out] atan(x^2)/4 + atanh(x^2)/4
```

sympy [A] time = 0.28, size = 22, normalized size = 1.29

$$-\frac{\log\left(x^2-1\right)}{8}+\frac{\log\left(x^2+1\right)}{8}+\frac{\operatorname{atan}\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**8+1),x)
[Out] -log(x**2 - 1)/8 + log(x**2 + 1)/8 + atan(x**2)/4
```

3.1476 $\int \frac{1}{x\left(1-x^8\right)}\,dx$

Optimal. Leaf size=15

$$\log(x)-\frac{1}{8}\log\left(1-x^8\right)$$

```
[Out] ln(x)-1/8*ln(-x^8+1)
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {266, 36, 31, 29}

$$\log(x)-\frac{1}{8}\log\left(1-x^8\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^8)),x]
```

```
[Out] Log[x] - Log[1 - x^8]/8
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(1-x^8\right)}\,dx &= \frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{(1-x)x}\,dx, x, x^8\right)\\ &= \frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{1-x}\,dx, x, x^8\right)+\frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, x^8\right)\\ &= \log(x)-\frac{1}{8}\log\left(1-x^8\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\log(x)-\frac{1}{8}\log\left(1-x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^8)),x]
[Out] Log[x] - Log[1 - x^8]/8
```

fricas [A] time = 0.72, size = 11, normalized size = 0.73

$$-\frac{1}{8}\log\left(x^8-1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^8+1),x, algorithm="fricas")
[Out] -1/8*log(x^8 - 1) + log(x)
```

giac [A] time = 0.15, size = 16, normalized size = 1.07

$$\frac{1}{8}\log\left(x^8\right)-\frac{1}{8}\log\left(\left|x^8-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^8+1),x, algorithm="giac")
[Out] 1/8*log(x^8) - 1/8*log(abs(x^8 - 1))
```

maple [B] time = 0.01, size = 32, normalized size = 2.13

$$\ln(x)-\frac{\ln(x-1)}{8}-\frac{\ln(x+1)}{8}-\frac{\ln\left(x^2+1\right)}{8}-\frac{\ln\left(x^4+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^8+1),x)
[Out] -1/8*ln(x-1)-1/8*ln(x^2+1)-1/8*ln(x^4+1)-1/8*ln(x+1)+ln(x)
```

maxima [A] time = 1.09, size = 15, normalized size = 1.00

$$-\frac{1}{8}\log\left(x^8-1\right)+\frac{1}{8}\log\left(x^8\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^8+1),x, algorithm="maxima")
[Out] -1/8*log(x^8 - 1) + 1/8*log(x^8)
```

mupad [B] time = 1.07, size = 11, normalized size = 0.73

$$\ln(x)-\frac{\ln\left(x^8-1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x*(x^8 - 1)),x)
[Out] log(x) - log(x^8 - 1)/8
```

sympy [A] time = 0.29, size = 10, normalized size = 0.67

$$\log(x)-\frac{\log\left(x^8-1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**8+1),x)
[Out] log(x) - log(x**8 - 1)/8
```

3.1477 $\int \frac{1}{x^3\left(1-x^8\right)}\,dx$

Optimal. Leaf size=24

$$-\frac{1}{2x^2}-\frac{1}{4}\tan^{-1}\left(x^2\right)+\frac{1}{4}\tanh^{-1}\left(x^2\right)$$

```
[Out] -1/2/x^2-1/4*arctan(x^2)+1/4*arctanh(x^2)
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {275, 325, 298, 203, 206}

$$-\frac{1}{2x^2}-\frac{1}{4}\tan^{-1}\left(x^2\right)+\frac{1}{4}\tanh^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 - x^8)),x]
```

```
[Out] -1/(2*x^2) - ArcTan[x^2]/4 + ArcTanh[x^2]/4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(1-x^8\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\left(1-x^4\right)}\,dx, x, x^2\right)\\ &= -\frac{1}{2x^2} + \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, x^2\right)\\ &= -\frac{1}{2x^2} + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, x^2\right) - \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, x^2\right)\\ &= -\frac{1}{2x^2} - \frac{1}{4}\tan^{-1}\left(x^2\right) + \frac{1}{4}\tanh^{-1}\left(x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.58

$$-\frac{1}{2x^2} - \frac{1}{8}\log\left(1-x^2\right) + \frac{1}{8}\log\left(x^2+1\right) + \frac{1}{4}\tan^{-1}\left(\frac{1}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 - x^8)),x]
[Out] -1/2*1/x^2 + ArcTan[x^(-2)]/4 - Log[1 - x^2]/8 + Log[1 + x^2]/8
```

fricas [B] time = 0.59, size = 37, normalized size = 1.54

$$-\frac{2\,x^2\arctan\left(x^2\right) - x^2\log\left(x^2+1\right) + x^2\log\left(x^2-1\right) + 4}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^8+1),x, algorithm="fricas")
[Out] -1/8*(2*x^2*arctan(x^2) - x^2*log(x^2 + 1) + x^2*log(x^2 - 1) + 4)/x^2
```

giac [A] time = 0.16, size = 29, normalized size = 1.21

$$-\frac{1}{2\,x^2} - \frac{1}{4}\,\arctan\left(x^2\right) + \frac{1}{8}\,\log\left(x^2+1\right) - \frac{1}{8}\,\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^8+1),x, algorithm="giac")
[Out] -1/2/x^2 - 1/4*arctan(x^2) + 1/8*log(x^2 + 1) - 1/8*log(abs(x^2 - 1))
```

maple [A] time = 0.01, size = 33, normalized size = 1.38

$$-\frac{\arctan\left(x^2\right)}{4} - \frac{\ln\left(x-1\right)}{8} - \frac{\ln\left(x+1\right)}{8} + \frac{\ln\left(x^2+1\right)}{8} - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-x^8+1),x)
[Out] -1/8*ln(x-1)+1/8*ln(x^2+1)-1/4*arctan(x^2)-1/2/x^2-1/8*ln(x+1)
```

maxima [A] time = 2.38, size = 28, normalized size = 1.17

$$-\frac{1}{2\,x^2} - \frac{1}{4}\,\arctan\left(x^2\right) + \frac{1}{8}\,\log\left(x^2+1\right) - \frac{1}{8}\,\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^8+1),x, algorithm="maxima")
[Out] -1/2/x^2 - 1/4*arctan(x^2) + 1/8*log(x^2 + 1) - 1/8*log(x^2 - 1)
```

mupad **[B]** time = 1.05, size = 18, normalized size = 0.75

$$\frac{\operatorname{atanh}\left(x^2\right)}{4}-\frac{\operatorname{atan}\left(x^2\right)}{4}-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^3*(x^8 - 1)),x)
[Out] atanh(x^2)/4 - atan(x^2)/4 - 1/(2*x^2)
```

sympy **[A]** time = 0.20, size = 29, normalized size = 1.21

$$-\frac{\log\left(x^2-1\right)}{8}+\frac{\log\left(x^2+1\right)}{8}-\frac{\operatorname{atan}\left(x^2\right)}{4}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-x**8+1),x)
[Out] -log(x**2 - 1)/8 + log(x**2 + 1)/8 - atan(x**2)/4 - 1/(2*x**2)
```

3.1478 $\int \frac{1}{x^5\left(1-x^8\right)}\,dx$

Optimal. Leaf size=16

$$\frac{1}{4}\tanh^{-1}\left(x^4\right)-\frac{1}{4x^4}$$

```
[Out] -1/4/x^4+1/4*arctanh(x^4)
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {275, 325, 206}

$$\frac{1}{4}\tanh^{-1}\left(x^4\right)-\frac{1}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 - x^8)),x]
```

```
[Out] -1/(4*x^4) + ArcTanh[x^4]/4
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\left(1-x^8\right)}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x^2\left(1-x^2\right)}\,dx, x, x^4\right)\\ &= -\frac{1}{4x^4}+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, x^4\right)\\ &= -\frac{1}{4x^4}+\frac{1}{4}\tanh^{-1}\left(x^4\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.88

$$-\frac{1}{4x^4}-\frac{1}{8}\log\left(1-x^4\right)+\frac{1}{8}\log\left(x^4+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 - x^8)),x]
[Out] -1/4*1/x^4 - Log[1 - x^4]/8 + Log[1 + x^4]/8
```

fricas [B] time = 0.76, size = 28, normalized size = 1.75

$$\frac{x^4 \log\left(x^4+1\right) - x^4 \log\left(x^4-1\right) - 2}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^8+1),x, algorithm="fricas")
[Out] 1/8*(x^4*log(x^4 + 1) - x^4*log(x^4 - 1) - 2)/x^4
```

giac [A] time = 0.15, size = 23, normalized size = 1.44

$$-\frac{1}{4\,x^4} + \frac{1}{8}\log\left(x^4+1\right) - \frac{1}{8}\log\left(\left|x^4-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^8+1),x, algorithm="giac")
[Out] -1/4/x^4 + 1/8*log(x^4 + 1) - 1/8*log(abs(x^4 - 1))
```

maple [B] time = 0.01, size = 35, normalized size = 2.19

$$-\frac{\ln(x-1)}{8} - \frac{\ln(x+1)}{8} - \frac{\ln\left(x^2+1\right)}{8} + \frac{\ln\left(x^4+1\right)}{8} - \frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-x^8+1),x)
[Out] -1/8*ln(x-1)-1/4/x^4-1/8*ln(x^2+1)+1/8*ln(x^4+1)-1/8*ln(x+1)
```

maxima [A] time = 1.01, size = 22, normalized size = 1.38

$$-\frac{1}{4\,x^4} + \frac{1}{8}\log\left(x^4+1\right) - \frac{1}{8}\log\left(x^4-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^8+1),x, algorithm="maxima")
[Out] -1/4/x^4 + 1/8*log(x^4 + 1) - 1/8*log(x^4 - 1)
```

mupad [B] time = 1.05, size = 12, normalized size = 0.75

$$\frac{\operatorname{atanh}\left(x^4\right)}{4} - \frac{1}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^5*(x^8 - 1)),x)
[Out] atanh(x^4)/4 - 1/(4*x^4)
```

sympy [A] time = 0.27, size = 22, normalized size = 1.38

$$-\frac{\log\left(x^4-1\right)}{8} + \frac{\log\left(x^4+1\right)}{8} - \frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-x**8+1),x)
[Out] -log(x**4 - 1)/8 + log(x**4 + 1)/8 - 1/(4*x**4)
```

3.1479 $\int \frac{1}{x^7\left(1-x^8\right)}\,dx$

Optimal. Leaf size=24

$$-\frac{1}{6x^6}+\frac{1}{4}\tan^{-1}\left(x^2\right)+\frac{1}{4}\tanh^{-1}\left(x^2\right)$$

```
[Out] -1/6/x^6+1/4*arctan(x^2)+1/4*arctanh(x^2)
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {275, 325, 212, 206, 203}

$$-\frac{1}{6x^6}+\frac{1}{4}\tan^{-1}\left(x^2\right)+\frac{1}{4}\tanh^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(1 - x^8)),x]
```

```
[Out] -1/(6*x^6) + ArcTan[x^2]/4 + ArcTanh[x^2]/4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7\left(1-x^8\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^4\left(1-x^4\right)}\,dx, x, x^2\right)\\ &= -\frac{1}{6x^6}+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, x^2\right)\\ &= -\frac{1}{6x^6}+\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, x^2\right)+\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, x^2\right)\\ &= -\frac{1}{6x^6}+\frac{1}{4}\tan^{-1}\left(x^2\right)+\frac{1}{4}\tanh^{-1}\left(x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.58

$$-\frac{1}{6x^6}-\frac{1}{8}\log\left(1-x^2\right)+\frac{1}{8}\log\left(x^2+1\right)-\frac{1}{4}\tan^{-1}\left(\frac{1}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(1 - x^8)),x]
[Out] -1/6*1/x^6 - ArcTan[x^(-2)]/4 - Log[1 - x^2]/8 + Log[1 + x^2]/8
```

fricas [B] time = 0.52, size = 38, normalized size = 1.58

$$\frac{6\,x^6\arctan\left(x^2\right)+3\,x^6\log\left(x^2+1\right)-3\,x^6\log\left(x^2-1\right)-4}{24\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-x^8+1),x, algorithm="fricas")
[Out] 1/24*(6*x^6*arctan(x^2) + 3*x^6*log(x^2 + 1) - 3*x^6*log(x^2 - 1) - 4)/x^6
```

giac [A] time = 0.32, size = 29, normalized size = 1.21

$$-\frac{1}{6\,x^6}+\frac{1}{4}\,\arctan\left(x^2\right)+\frac{1}{8}\,\log\left(x^2+1\right)-\frac{1}{8}\,\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-x^8+1),x, algorithm="giac")
[Out] -1/6/x^6 + 1/4*arctan(x^2) + 1/8*log(x^2 + 1) - 1/8*log(abs(x^2 - 1))
```

maple [A] time = 0.01, size = 33, normalized size = 1.38

$$\frac{\arctan\left(x^2\right)}{4}-\frac{\ln\left(x-1\right)}{8}-\frac{\ln\left(x+1\right)}{8}+\frac{\ln\left(x^2+1\right)}{8}-\frac{1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(-x^8+1),x)
[Out] -1/8*ln(x-1)+1/8*ln(x^2+1)+1/4*arctan(x^2)-1/6/x^6-1/8*ln(x+1)
```

maxima [A] time = 2.35, size = 28, normalized size = 1.17

$$-\frac{1}{6\,x^6}+\frac{1}{4}\,\arctan\left(x^2\right)+\frac{1}{8}\,\log\left(x^2+1\right)-\frac{1}{8}\,\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-x^8+1),x, algorithm="maxima")
```

```
[Out] -1/6/x^6 + 1/4*arctan(x^2) + 1/8*log(x^2 + 1) - 1/8*log(x^2 - 1)
```

mupad [**B**] time = 0.05, size = 18, normalized size = 0.75

$$\frac{\operatorname{atan}\left(x^2\right)}{4}+\frac{\operatorname{atanh}\left(x^2\right)}{4}-\frac{1}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^7*(x^8 - 1)),x)
```

```
[Out] atan(x^2)/4 + atanh(x^2)/4 - 1/(6*x^6)
```

sympy [**A**] time = 0.21, size = 29, normalized size = 1.21

$$-\frac{\log\left(x^2-1\right)}{8}+\frac{\log\left(x^2+1\right)}{8}+\frac{\operatorname{atan}\left(x^2\right)}{4}-\frac{1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(-x**8+1),x)
```

```
[Out] -log(x**2 - 1)/8 + log(x**2 + 1)/8 + atan(x**2)/4 - 1/(6*x**6)
```

3.1480 $\int \frac{1}{x^9\left(1-x^8\right)}\,dx$

Optimal. Leaf size=22

$$-\frac{1}{8x^8}-\frac{1}{8}\log\left(1-x^8\right)+\log(x)$$

```
[Out] -1/8/x^8+ln(x)-1/8*ln(-x^8+1)
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 44}

$$-\frac{1}{8x^8}-\frac{1}{8}\log\left(1-x^8\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(1 - x^8)),x]
```

```
[Out] -1/(8*x^8) + Log[x] - Log[1 - x^8]/8
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^9\left(1-x^8\right)}\,dx &= \frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{(1-x)x^2}\,dx, x, x^8\right)\\ &= \frac{1}{8}\operatorname{Subst}\left(\int \left(\frac{1}{1-x}+\frac{1}{x^2}+\frac{1}{x}\right)dx, x, x^8\right)\\ &= -\frac{1}{8x^8}+\log(x)-\frac{1}{8}\log\left(1-x^8\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$-\frac{1}{8x^8}-\frac{1}{8}\log\left(1-x^8\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^9*(1 - x^8)),x]
```

```
[Out] -1/8*1/x^8 + Log[x] - Log[1 - x^8]/8
```

fricas [A] time = 0.55, size = 24, normalized size = 1.09

$$-\frac{x^8\log\left(x^8-1\right)-8\,x^8\log(x)+1}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(-x^8+1),x, algorithm="fricas")
[Out] -1/8*(x^8*log(x^8 - 1) - 8*x^8*log(x) + 1)/x^8
```

giac [A] time = 0.16, size = 26, normalized size = 1.18

$$-\frac{x^8+1}{8\,x^8}+\frac{1}{8}\log\left(x^8\right)-\frac{1}{8}\log\left(\left|x^8-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(-x^8+1),x, algorithm="giac")
[Out] -1/8*(x^8 + 1)/x^8 + 1/8*log(x^8) - 1/8*log(abs(x^8 - 1))
```

maple [A] time = 0.01, size = 37, normalized size = 1.68

$$\ln(x)-\frac{\ln(x-1)}{8}-\frac{\ln(x+1)}{8}-\frac{\ln\left(x^2+1\right)}{8}-\frac{\ln\left(x^4+1\right)}{8}-\frac{1}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(-x^8+1),x)
[Out] -1/8*ln(x-1)-1/8*ln(x^2+1)-1/8*ln(x^4+1)-1/8*ln(x+1)-1/8/x^8+ln(x)
```

maxima [A] time = 1.07, size = 20, normalized size = 0.91

$$-\frac{1}{8\,x^8}-\frac{1}{8}\log\left(x^8-1\right)+\frac{1}{8}\log\left(x^8\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(-x^8+1),x, algorithm="maxima")
[Out] -1/8/x^8 - 1/8*log(x^8 - 1) + 1/8*log(x^8)
```

mupad [B] time = 0.04, size = 16, normalized size = 0.73

$$\ln(x)-\frac{\ln\left(x^8-1\right)}{8}-\frac{1}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^9*(x^8 - 1)),x)
[Out] log(x) - log(x^8 - 1)/8 - 1/(8*x^8)
```

sympy [A] time = 0.27, size = 17, normalized size = 0.77

$$\log(x)-\frac{\log\left(x^8-1\right)}{8}-\frac{1}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(-x**8+1),x)
[Out] log(x) - log(x**8 - 1)/8 - 1/(8*x**8)
```

3.1481 $\int \frac{x^8}{1-x^8}\,dx$

Optimal. Leaf size=100

$$-\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{8\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{8\sqrt{2}}-x+\frac{1}{4}\tan^{-1}(x)-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\,x+1\right)}{4\sqrt{2}}+\frac{1}{4}\tanh^{-1}(x)$$

```
[Out] -x+1/4*arctan(x)+1/4*arctanh(x)+1/8*arctan(-1+x*2^(1/2))*2^(1/2)+1/8*arctan
(1+x*2^(1/2))*2^(1/2)-1/16*ln(1+x^2-x*2^(1/2))*2^(1/2)+1/16*ln(1+x^2+x*2^(1
/2))*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {321, 214, 212, 206, 203, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{8\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{8\sqrt{2}}-x+\frac{1}{4}\tan^{-1}(x)-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\,x+1\right)}{4\sqrt{2}}+\frac{1}{4}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(1 - x^8),x]
```

```
[Out] -x + ArcTan[x]/4 - ArcTan[1 - Sqrt[2]*x]/(4*Sqrt[2]) + ArcTan[1 + Sqrt[2]*x
]/(4*Sqrt[2]) + ArcTanh[x]/4 - Log[1 - Sqrt[2]*x + x^2]/(8*Sqrt[2]) + Log[1
 + Sqrt[2]*x + x^2]/(8*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{1-x^8}\,dx &= -x + \int \frac{1}{1-x^8}\,dx \\
&= -x + \frac{1}{2}\int \frac{1}{1-x^4}\,dx + \frac{1}{2}\int \frac{1}{1+x^4}\,dx \\
&= -x + \frac{1}{4}\int \frac{1}{1-x^2}\,dx + \frac{1}{4}\int \frac{1}{1+x^2}\,dx + \frac{1}{4}\int \frac{1-x^2}{1+x^4}\,dx + \frac{1}{4}\int \frac{1+x^2}{1+x^4}\,dx \\
&= -x + \frac{1}{4}\tan^{-1}(x) + \frac{1}{4}\tanh^{-1}(x) + \frac{1}{8}\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx + \frac{1}{8}\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx - \frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}\,x-}}{8\sqrt{2}} \\
&= -x + \frac{1}{4}\tan^{-1}(x) + \frac{1}{4}\tanh^{-1}(x) - \frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{8\sqrt{2}} + \frac{\log\left(1+\sqrt{2}\,x+x^2\right)}{8\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\right.}{4} \\
&= -x + \frac{1}{4}\tan^{-1}(x) - \frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{4\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}\,x\right)}{4\sqrt{2}} + \frac{1}{4}\tanh^{-1}(x) - \frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{8\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 101, normalized size = 1.01

$$\frac{1}{16}\left(-\sqrt{2}\,\log\left(x^2-\sqrt{2}\,x+1\right)+\sqrt{2}\,\log\left(x^2+\sqrt{2}\,x+1\right)-16x-2\log(1-x)+2\log(x+1)+4\tan^{-1}(x)-2\sqrt{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(1 - x^8),x]
```

```
[Out] (-16*x + 4*ArcTan[x] - 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 2*Sqrt[2]*ArcTan[1
 + Sqrt[2]*x] - 2*Log[1 - x] + 2*Log[1 + x] - Sqrt[2]*Log[1 - Sqrt[2]*x + x
^2] + Sqrt[2]*Log[1 + Sqrt[2]*x + x^2])/16
```

fricas [A] time = 0.69, size = 114, normalized size = 1.14

$$-\frac{1}{4}\sqrt{2}\,\arctan\left(-\sqrt{2}\,x+\sqrt{2}\,\sqrt{x^2+\sqrt{2}\,x+1}\,-1\right)-\frac{1}{4}\sqrt{2}\,\arctan\left(-\sqrt{2}\,x+\sqrt{2}\,\sqrt{x^2-\sqrt{2}\,x+1}\,+1\right)+\frac{1}{16}\sqrt{2}\,\text{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^8+1),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x + 1) - 1) - 1
/4*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) + 1) + 1/1
6*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/16*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
- x + 1/4*arctan(x) + 1/8*log(x + 1) - 1/8*log(x - 1)
```

giac [A] time = 0.16, size = 93, normalized size = 0.93

$$\frac{1}{8}\sqrt{2}\,\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{8}\sqrt{2}\,\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{16}\sqrt{2}\,\log\left(x^2+\sqrt{2}\,x+1\right)-\frac{1}{16}\sqrt{2}\,\log\left(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^8+1),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/8*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) + 1/16*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/16*sqrt(
2)*log(x^2 - sqrt(2)*x + 1) - x + 1/4*arctan(x) + 1/8*log(abs(x + 1)) - 1/8
*log(abs(x - 1))
```

maple [A] time = 0.01, size = 77, normalized size = 0.77

$$-x+\frac{\arctan(x)}{4}+\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ x-1\right)}{8}+\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ x+1\right)}{8}+\frac{\sqrt{2}\ \ln\left(\frac{x^2+\sqrt{2}\ x+1}{x^2-\sqrt{2}\ x+1}\right)}{16}-\frac{\ln(x-1)}{8}+\frac{\ln(x+1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-x^8+1),x)
```

```
[Out] -x-1/8*ln(x-1)+1/8*ln(x+1)+1/4*arctan(x)+1/8*arctan(1+2^(1/2)*x)*2^(1/2)+1/
8*arctan(-1+2^(1/2)*x)*2^(1/2)+1/16*2^(1/2)*ln((1+x^2+2^(1/2)*x)/(1+x^2-2^(
1/2)*x))
```

maxima [A] time = 2.31, size = 91, normalized size = 0.91

$$\frac{1}{8}\sqrt{2}\ \arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right)+\frac{1}{8}\sqrt{2}\ \arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right)+\frac{1}{16}\sqrt{2}\ \log\left(x^2+\sqrt{2}\,x+1\right)-\frac{1}{16}\sqrt{2}\ \log\left(x^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^8+1),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/8*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) + 1/16*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/16*sqrt(
2)*log(x^2 - sqrt(2)*x + 1) - x + 1/4*arctan(x) + 1/8*log(x + 1) - 1/8*log(
x - 1)
```

mupad [B] time = 0.07, size = 48, normalized size = 0.48

$$\frac{\operatorname{atan}(x)}{4}-\frac{\operatorname{atan}(x\,1\mathrm{i})\ 1\mathrm{i}}{4}-x+\sqrt{2}\ \operatorname{atan}\left(\sqrt{2}\ x\ \left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)+\sqrt{2}\ \operatorname{atan}\left(\sqrt{2}\ x\ \left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^8/(x^8 - 1),x)
```

```
[Out] atan(x)/4 - (atan(x*1i)*1i)/4 - x + 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1
/8 + 1i/8) + 2^(1/2)*atan(2^(1/2)*x*(1/2 + 1i/2))*(1/8 - 1i/8)
```

sympy [C] time = 176.28, size = 46, normalized size = 0.46

$$-x-\frac{\log(x-1)}{8}+\frac{\log(x+1)}{8}-\frac{i\log(x-i)}{8}+\frac{i\log(x+i)}{8}-\operatorname{RootSum}\left(4096t^4+1,\left(t\mapsto t\log(-8t+x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-x**8+1),x)
```

```
[Out] -x - log(x - 1)/8 + log(x + 1)/8 - I*log(x - I)/8 + I*log(x + I)/8 - RootSu
m(4096*_t**4 + 1, Lambda(_t, _t*log(-8*_t + x)))
```

3.1482 $\int \frac{x^6}{1-x^8}\,dx$

Optimal. Leaf size=97

$$-\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{8\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{8\sqrt{2}}-\frac{1}{4}\tan^{-1}(x)+\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\,x+1\right)}{4\sqrt{2}}+\frac{1}{4}\tanh^{-1}(x)$$

```
[Out] -1/4*arctan(x)+1/4*arctanh(x)-1/8*arctan(-1+x*2^(1/2))*2^(1/2)-1/8*arctan(1
+x*2^(1/2))*2^(1/2)-1/16*ln(1+x^2-x*2^(1/2))*2^(1/2)+1/16*ln(1+x^2+x*2^(1/2
))*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {301, 297, 1162, 617, 204, 1165, 628, 298, 203, 206}

$$-\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{8\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{8\sqrt{2}}-\frac{1}{4}\tan^{-1}(x)+\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\,x+1\right)}{4\sqrt{2}}+\frac{1}{4}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(1 - x^8),x]
```

```
[Out] -ArcTan[x]/4 + ArcTan[1 - Sqrt[2]*x]/(4*Sqrt[2]) - ArcTan[1 + Sqrt[2]*x]/(4
*Sqrt[2]) + ArcTanh[x]/4 - Log[1 - Sqrt[2]*x + x^2]/(8*Sqrt[2]) + Log[1 + S
qrt[2]*x + x^2]/(8*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 301

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[x^(m - n/2
)/(r + s*x^(n/2)), x], x] - Dist[s/(2*b), Int[x^(m - n/2)/(r - s*x^(n/2)),
x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] &&
 LtQ[m, n] &&  !GtQ[a/b, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{1-x^8}\,dx &= \frac{1}{2}\int \frac{x^2}{1-x^4}\,dx - \frac{1}{2}\int \frac{x^2}{1+x^4}\,dx \\
&= \frac{1}{4}\int \frac{1}{1-x^2}\,dx - \frac{1}{4}\int \frac{1}{1+x^2}\,dx + \frac{1}{4}\int \frac{1-x^2}{1+x^4}\,dx - \frac{1}{4}\int \frac{1+x^2}{1+x^4}\,dx \\
&= -\frac{1}{4}\tan^{-1}(x) + \frac{1}{4}\tanh^{-1}(x) - \frac{1}{8}\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx - \frac{1}{8}\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx - \frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}\,x-x^2}\,dx}{8\sqrt{2}} - \\
&= -\frac{1}{4}\tan^{-1}(x) + \frac{1}{4}\tanh^{-1}(x) - \frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{8\sqrt{2}} + \frac{\log\left(1+\sqrt{2}\,x+x^2\right)}{8\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1\right.}{4\sqrt{2}} \\
&= -\frac{1}{4}\tan^{-1}(x) + \frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{4\sqrt{2}} - \frac{\tan^{-1}\left(1+\sqrt{2}\,x\right)}{4\sqrt{2}} + \frac{1}{4}\tanh^{-1}(x) - \frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{8\sqrt{2}} + \frac{\log\left(\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 98, normalized size = 1.01

$$\frac{1}{16}\left(-\sqrt{2}\,\log\left(x^2-\sqrt{2}\,x+1\right) + \sqrt{2}\,\log\left(x^2+\sqrt{2}\,x+1\right) - 2\log(1-x) + 2\log(x+1) - 4\tan^{-1}(x) + 2\sqrt{2}\,\tan^{-1}\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(1 - x^8),x]
```

```
[Out] (-4*ArcTan[x] + 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] - 2*Sqrt[2]*ArcTan[1 + Sqrt
[2]*x] - 2*Log[1 - x] + 2*Log[1 + x] - Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] + S
qrt[2]*Log[1 + Sqrt[2]*x + x^2])/16
```

fricas [A] time = 0.64, size = 111, normalized size = 1.14

$$\frac{1}{4}\sqrt{2}\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}-1\right)+\frac{1}{4}\sqrt{2}\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}+1\right)+\frac{1}{16}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^8+1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x + 1) - 1) + 1/
4*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) + 1) + 1/16
*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/16*sqrt(2)*log(x^2 - sqrt(2)*x + 1) -
 1/4*arctan(x) + 1/8*log(x + 1) - 1/8*log(x - 1)
```

giac [A] time = 0.15, size = 90, normalized size = 0.93

$$-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{16}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{16}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^8+1),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 1/8*sqrt(2)*arctan(1/2*s
qrt(2)*(2*x - sqrt(2))) + 1/16*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/16*sqrt
(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*arctan(x) + 1/8*log(abs(x + 1)) - 1/8*lo
g(abs(x - 1))
```

maple [A] time = 0.01, size = 74, normalized size = 0.76

$$-\frac{\arctan(x)}{4}-\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ x-1\right)}{8}-\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ x+1\right)}{8}-\frac{\sqrt{2}\ \ln\left(\frac{x^2-\sqrt{2}\ x+1}{x^2+\sqrt{2}\ x+1}\right)}{16}-\frac{\ln(x-1)}{8}+\frac{\ln(x+1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-x^8+1),x)
```

```
[Out] -1/8*ln(x-1)+1/8*ln(x+1)-1/4*arctan(x)-1/8*2^(1/2)*arctan(2^(1/2)*x+1)-1/8*
2^(1/2)*arctan(2^(1/2)*x-1)-1/16*2^(1/2)*ln((x^2-2^(1/2)*x+1)/(x^2+2^(1/2)*
x+1))
```

maxima [A] time = 2.32, size = 88, normalized size = 0.91

$$-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{16}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{16}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^8+1),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 1/8*sqrt(2)*arctan(1/2*s
qrt(2)*(2*x - sqrt(2))) + 1/16*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/16*sqrt
(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*arctan(x) + 1/8*log(x + 1) - 1/8*log(x -
 1)
```

mupad [B] time = 0.06, size = 45, normalized size = 0.46

$$-\frac{\operatorname{atan}(x\,1\mathrm{i})\,1\mathrm{i}}{4}-\frac{\operatorname{atan}(x)}{4}+\sqrt{2}\ \operatorname{atan}\left(\sqrt{2}\ x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)+\sqrt{2}\ \operatorname{atan}\left(\sqrt{2}\ x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^6/(x^8 - 1),x)
```

```
[Out] - (atan(x*1i)*1i)/4 - atan(x)/4 - 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/8
 - 1i/8) - 2^(1/2)*atan(2^(1/2)*x*(1/2 + 1i/2))*(1/8 + 1i/8)
```

sympy [C] time = 171.71, size = 46, normalized size = 0.47

$$-\frac{\log(x-1)}{8}+\frac{\log(x+1)}{8}+\frac{i\log(x-i)}{8}-\frac{i\log(x+i)}{8}-\text{RootSum}\left(4096t^4+1,\left(t\mapsto t\log\left(-2097152t^7+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-x**8+1),x)
```

```
[Out] -log(x - 1)/8 + log(x + 1)/8 + I*log(x - I)/8 - I*log(x + I)/8 - RootSum(40
96*_t**4 + 1, Lambda(_t, _t*log(-2097152*_t**7 + x)))
```

3.1483 $\int \frac{x^4}{1-x^8}\,dx$

Optimal. Leaf size=97

$$\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{8\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{8\sqrt{2}}+\frac{1}{4}\tan^{-1}(x)+\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\,x+1\right)}{4\sqrt{2}}+\frac{1}{4}\tanh^{-1}(x)$$

```
[Out] 1/4*arctan(x)+1/4*arctanh(x)-1/8*arctan(-1+x*2^(1/2))*2^(1/2)-1/8*arctan(1+
x*2^(1/2))*2^(1/2)+1/16*ln(1+x^2-x*2^(1/2))*2^(1/2)-1/16*ln(1+x^2+x*2^(1/2)
)*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {301, 211, 1165, 628, 1162, 617, 204, 212, 206, 203}

$$\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{8\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{8\sqrt{2}}+\frac{1}{4}\tan^{-1}(x)+\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\,x+1\right)}{4\sqrt{2}}+\frac{1}{4}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 - x^8),x]
```

```
[Out] ArcTan[x]/4 + ArcTan[1 - Sqrt[2]*x]/(4*Sqrt[2]) - ArcTan[1 + Sqrt[2]*x]/(4*
Sqrt[2]) + ArcTanh[x]/4 + Log[1 - Sqrt[2]*x + x^2]/(8*Sqrt[2]) - Log[1 + Sq
rt[2]*x + x^2]/(8*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 301

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[x^(m - n/2
)/(r + s*x^(n/2)), x], x] - Dist[s/(2*b), Int[x^(m - n/2)/(r - s*x^(n/2)),
x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] &&
 LtQ[m, n] &&  !GtQ[a/b, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{1-x^8}\,dx &= \frac{1}{2}\int \frac{1}{1-x^4}\,dx - \frac{1}{2}\int \frac{1}{1+x^4}\,dx \\
&= \frac{1}{4}\int \frac{1}{1-x^2}\,dx + \frac{1}{4}\int \frac{1}{1+x^2}\,dx - \frac{1}{4}\int \frac{1-x^2}{1+x^4}\,dx - \frac{1}{4}\int \frac{1+x^2}{1+x^4}\,dx \\
&= \frac{1}{4}\tan^{-1}(x) + \frac{1}{4}\tanh^{-1}(x) - \frac{1}{8}\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx - \frac{1}{8}\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx + \frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}\,x-x^2}\,dx}{8\sqrt{2}} + \frac{\int}{} \\
&= \frac{1}{4}\tan^{-1}(x) + \frac{1}{4}\tanh^{-1}(x) + \frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{8\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\,x+x^2\right)}{8\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1 -}{4\sqrt{2}} \\
&= \frac{1}{4}\tan^{-1}(x) + \frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{4\sqrt{2}} - \frac{\tan^{-1}\left(1+\sqrt{2}\,x\right)}{4\sqrt{2}} + \frac{1}{4}\tanh^{-1}(x) + \frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{8\sqrt{2}} - \frac{\log\left(1\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 98, normalized size = 1.01

$$\frac{1}{16}\left(\sqrt{2}\,\log\left(x^2-\sqrt{2}\,x+1\right) - \sqrt{2}\,\log\left(x^2+\sqrt{2}\,x+1\right) - 2\log(1-x) + 2\log(x+1) + 4\tan^{-1}(x) + 2\sqrt{2}\,\tan^{-1}\left(1\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 - x^8),x]
```

```
[Out] (4*ArcTan[x] + 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] - 2*Sqrt[2]*ArcTan[1 + Sqrt[
2]*x] - 2*Log[1 - x] + 2*Log[1 + x] + Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - Sq
rt[2]*Log[1 + Sqrt[2]*x + x^2])/16
```

fricas [A] time = 0.70, size = 111, normalized size = 1.14

$$\frac{1}{4}\sqrt{2}\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}-1\right)+\frac{1}{4}\sqrt{2}\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}+1\right)-\frac{1}{16}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^8+1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x + 1) - 1) + 1/
4*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) + 1) - 1/16
*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/16*sqrt(2)*log(x^2 - sqrt(2)*x + 1) +
 1/4*arctan(x) + 1/8*log(x + 1) - 1/8*log(x - 1)
```

giac [A] time = 0.17, size = 90, normalized size = 0.93

$$-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{1}{16}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+\frac{1}{16}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^8+1),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 1/8*sqrt(2)*arctan(1/2*s
qrt(2)*(2*x - sqrt(2))) - 1/16*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/16*sqrt
(2)*log(x^2 - sqrt(2)*x + 1) + 1/4*arctan(x) + 1/8*log(abs(x + 1)) - 1/8*lo
g(abs(x - 1))
```

maple [A] time = 0.00, size = 74, normalized size = 0.76

$$\frac{\arctan(x)}{4}-\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ x-1\right)}{8}-\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ x+1\right)}{8}-\frac{\sqrt{2}\ \ln\left(\frac{x^2+\sqrt{2}\ x+1}{x^2-\sqrt{2}\ x+1}\right)}{16}-\frac{\ln(x-1)}{8}+\frac{\ln(x+1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^8+1),x)
```

```
[Out] -1/8*ln(x-1)+1/8*ln(x+1)+1/4*arctan(x)-1/8*2^(1/2)*arctan(2^(1/2)*x-1)-1/16
*2^(1/2)*ln((x^2+2^(1/2)*x+1)/(x^2-2^(1/2)*x+1))-1/8*2^(1/2)*arctan(2^(1/2)
*x+1)
```

maxima [A] time = 2.32, size = 88, normalized size = 0.91

$$-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{1}{16}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+\frac{1}{16}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^8+1),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 1/8*sqrt(2)*arctan(1/2*s
qrt(2)*(2*x - sqrt(2))) - 1/16*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/16*sqrt
(2)*log(x^2 - sqrt(2)*x + 1) + 1/4*arctan(x) + 1/8*log(x + 1) - 1/8*log(x -
 1)
```

mupad [B] time = 0.06, size = 45, normalized size = 0.46

$$\frac{\operatorname{atan}(x)}{4}-\frac{\operatorname{atan}(x\,1\mathrm{i})\,1\mathrm{i}}{4}+\sqrt{2}\ \operatorname{atan}\left(\sqrt{2}\ x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)+\sqrt{2}\ \operatorname{atan}\left(\sqrt{2}\ x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^4/(x^8 - 1),x)
```

```
[Out] atan(x)/4 - (atan(x*1i)*1i)/4 - 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/8 +
 1i/8) - 2^(1/2)*atan(2^(1/2)*x*(1/2 + 1i/2))*(1/8 - 1i/8)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**8+1),x)
```

```
[Out] Timed out
```

3.1484 $\int \frac{x^2}{1-x^8}\,dx$

Optimal. Leaf size=97

$$\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{8\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{8\sqrt{2}}-\frac{1}{4}\tan^{-1}(x)-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\,x+1\right)}{4\sqrt{2}}+\frac{1}{4}\tanh^{-1}(x)$$

```
[Out] -1/4*arctan(x)+1/4*arctanh(x)+1/8*arctan(-1+x*2^(1/2))*2^(1/2)+1/8*arctan(1
+x*2^(1/2))*2^(1/2)+1/16*ln(1+x^2-x*2^(1/2))*2^(1/2)-1/16*ln(1+x^2+x*2^(1/2
))*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {300, 297, 1162, 617, 204, 1165, 628, 298, 203, 206}

$$\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{8\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{8\sqrt{2}}-\frac{1}{4}\tan^{-1}(x)-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\,x+1\right)}{4\sqrt{2}}+\frac{1}{4}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 - x^8),x]
```

```
[Out] -ArcTan[x]/4 - ArcTan[1 - Sqrt[2]*x]/(4*Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]/(4
*Sqrt[2]) + ArcTanh[x]/4 + Log[1 - Sqrt[2]*x + x^2]/(8*Sqrt[2]) - Log[1 + S
qrt[2]*x + x^2]/(8*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 300

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[x^m/(r + s
*x^(n/2)), x], x] + Dist[r/(2*a), Int[x^m/(r - s*x^(n/2)), x], x]] /; FreeQ
[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n/2] &&  !GtQ[a/b, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{1-x^8}\,dx &= \frac{1}{2}\int \frac{x^2}{1-x^4}\,dx + \frac{1}{2}\int \frac{x^2}{1+x^4}\,dx \\
&= \frac{1}{4}\int \frac{1}{1-x^2}\,dx - \frac{1}{4}\int \frac{1}{1+x^2}\,dx - \frac{1}{4}\int \frac{1-x^2}{1+x^4}\,dx + \frac{1}{4}\int \frac{1+x^2}{1+x^4}\,dx \\
&= -\frac{1}{4}\tan^{-1}(x) + \frac{1}{4}\tanh^{-1}(x) + \frac{1}{8}\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx + \frac{1}{8}\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx + \frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}\,x-x^2}\,dx}{8\sqrt{2}} + \\
&= -\frac{1}{4}\tan^{-1}(x) + \frac{1}{4}\tanh^{-1}(x) + \frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{8\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\,x+x^2\right)}{8\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1\right.}{4\sqrt{2}} \\
&= -\frac{1}{4}\tan^{-1}(x) - \frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{4\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}\,x\right)}{4\sqrt{2}} + \frac{1}{4}\tanh^{-1}(x) + \frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{8\sqrt{2}} - \frac{\log\left(\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 98, normalized size = 1.01

$$\frac{1}{16}\left(\sqrt{2}\,\log\left(x^2-\sqrt{2}\,x+1\right) - \sqrt{2}\,\log\left(x^2+\sqrt{2}\,x+1\right) - 2\log(1-x) + 2\log(x+1) - 4\tan^{-1}(x) - 2\sqrt{2}\,\tan^{-1}\left(1\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 - x^8),x]
```

```
[Out] (-4*ArcTan[x] - 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 2*Sqrt[2]*ArcTan[1 + Sqrt
[2]*x] - 2*Log[1 - x] + 2*Log[1 + x] + Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - S
qrt[2]*Log[1 + Sqrt[2]*x + x^2])/16
```

fricas [A] time = 0.62, size = 111, normalized size = 1.14

$$-\frac{1}{4}\sqrt{2}\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}-1\right)-\frac{1}{4}\sqrt{2}\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}+1\right)-\frac{1}{16}\sqrt{2}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^8+1),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x + 1) - 1) - 1
/4*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) + 1) - 1/1
6*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/16*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
- 1/4*arctan(x) + 1/8*log(x + 1) - 1/8*log(x - 1)
```

giac [A] time = 0.15, size = 90, normalized size = 0.93

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{1}{16}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+\frac{1}{16}\sqrt{2}\log\left(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^8+1),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/8*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) - 1/16*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/16*sqrt(
2)*log(x^2 - sqrt(2)*x + 1) - 1/4*arctan(x) + 1/8*log(abs(x + 1)) - 1/8*log
(abs(x - 1))
```

maple [A] time = 0.01, size = 74, normalized size = 0.76

$$-\frac{\arctan(x)}{4}+\frac{\sqrt{2}\,\arctan\left(\sqrt{2}\,x-1\right)}{8}+\frac{\sqrt{2}\,\arctan\left(\sqrt{2}\,x+1\right)}{8}+\frac{\sqrt{2}\,\ln\left(\frac{x^2-\sqrt{2}\,x+1}{x^2+\sqrt{2}\,x+1}\right)}{16}-\frac{\ln(x-1)}{8}+\frac{\ln(x+1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^8+1),x)
```

```
[Out] -1/8*ln(x-1)+1/8*ln(x+1)-1/4*arctan(x)+1/8*2^(1/2)*arctan(2^(1/2)*x-1)+1/16
*2^(1/2)*ln((x^2-2^(1/2)*x+1)/(x^2+2^(1/2)*x+1))+1/8*2^(1/2)*arctan(2^(1/2)
*x+1)
```

maxima [A] time = 2.34, size = 88, normalized size = 0.91

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{1}{16}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+\frac{1}{16}\sqrt{2}\log\left(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^8+1),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/8*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) - 1/16*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/16*sqrt(
2)*log(x^2 - sqrt(2)*x + 1) - 1/4*arctan(x) + 1/8*log(x + 1) - 1/8*log(x -
1)
```

mupad [B] time = 1.06, size = 45, normalized size = 0.46

$$-\frac{\mathrm{atan}\left(x\,1\mathrm{i}\right)\,1\mathrm{i}}{4}-\frac{\mathrm{atan}(x)}{4}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/(x^8 - 1),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/8 - 1i/8) - atan(x)/4 - (atan(x*1i)
*1i)/4 + 2^(1/2)*atan(2^(1/2)*x*(1/2 + 1i/2))*(1/8 + 1i/8)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**8+1),x)
```

```
[Out] Timed out
```

3.1485 $\int \frac{1}{1-x^8}\,dx$

Optimal. Leaf size=97

$$-\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{8\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{8\sqrt{2}}+\frac{1}{4}\tan^{-1}(x)-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\,x+1\right)}{4\sqrt{2}}+\frac{1}{4}\tanh^{-1}(x)$$

```
[Out] 1/4*arctan(x)+1/4*arctanh(x)+1/8*arctan(-1+x*2^(1/2))*2^(1/2)+1/8*arctan(1+
x*2^(1/2))*2^(1/2)-1/16*ln(1+x^2-x*2^(1/2))*2^(1/2)+1/16*ln(1+x^2+x*2^(1/2)
)*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 1.111$, Rules used = {214, 212, 206, 203, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{8\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{8\sqrt{2}}+\frac{1}{4}\tan^{-1}(x)-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\,x+1\right)}{4\sqrt{2}}+\frac{1}{4}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^8)^(-1),x]
```

```
[Out] ArcTan[x]/4 - ArcTan[1 - Sqrt[2]*x]/(4*Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]/(4*
Sqrt[2]) + ArcTanh[x]/4 - Log[1 - Sqrt[2]*x + x^2]/(8*Sqrt[2]) + Log[1 + Sq
rt[2]*x + x^2]/(8*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1-x^8}\,dx &= \frac{1}{2}\int \frac{1}{1-x^4}\,dx + \frac{1}{2}\int \frac{1}{1+x^4}\,dx \\
&= \frac{1}{4}\int \frac{1}{1-x^2}\,dx + \frac{1}{4}\int \frac{1}{1+x^2}\,dx + \frac{1}{4}\int \frac{1-x^2}{1+x^4}\,dx + \frac{1}{4}\int \frac{1+x^2}{1+x^4}\,dx \\
&= \frac{1}{4}\tan^{-1}(x) + \frac{1}{4}\tanh^{-1}(x) + \frac{1}{8}\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx + \frac{1}{8}\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx - \frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}\,x-x^2}\,dx}{8\sqrt{2}} - \frac{\int}{} \\
&= \frac{1}{4}\tan^{-1}(x) + \frac{1}{4}\tanh^{-1}(x) - \frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{8\sqrt{2}} + \frac{\log\left(1+\sqrt{2}\,x+x^2\right)}{8\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1 -}{4\sqrt{2}} \\
&= \frac{1}{4}\tan^{-1}(x) - \frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{4\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}\,x\right)}{4\sqrt{2}} + \frac{1}{4}\tanh^{-1}(x) - \frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{8\sqrt{2}} + \frac{\log\left(1\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 98, normalized size = 1.01

$$\frac{1}{16}\left(-\sqrt{2}\,\log\left(x^2-\sqrt{2}\,x+1\right) + \sqrt{2}\,\log\left(x^2+\sqrt{2}\,x+1\right) - 2\log(1-x) + 2\log(x+1) + 4\tan^{-1}(x) - 2\sqrt{2}\,\tan^{-1}\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^8)^(-1),x]
```

```
[Out] (4*ArcTan[x] - 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 2*Sqrt[2]*ArcTan[1 + Sqrt[
2]*x] - 2*Log[1 - x] + 2*Log[1 + x] - Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] + Sq
rt[2]*Log[1 + Sqrt[2]*x + x^2])/16
```

fricas [A] time = 0.58, size = 111, normalized size = 1.14

$$-\frac{1}{4}\sqrt{2}\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}-1\right)-\frac{1}{4}\sqrt{2}\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}+1\right)+\frac{1}{16}\sqrt{2}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^8+1),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x + 1) - 1) - 1
/4*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) + 1) + 1/1
6*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/16*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
+ 1/4*arctan(x) + 1/8*log(x + 1) - 1/8*log(x - 1)
```

giac [A] time = 0.15, size = 90, normalized size = 0.93

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{16}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{16}\sqrt{2}\log\left(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^8+1),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/8*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) + 1/16*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/16*sqrt(
2)*log(x^2 - sqrt(2)*x + 1) + 1/4*arctan(x) + 1/8*log(abs(x + 1)) - 1/8*log
(abs(x - 1))
```

maple [A] time = 0.00, size = 66, normalized size = 0.68

$$\frac{\operatorname{arctanh}(x)}{4}+\frac{\arctan(x)}{4}+\frac{\sqrt{2}\,\arctan\left(\sqrt{2}\,x-1\right)}{8}+\frac{\sqrt{2}\,\arctan\left(\sqrt{2}\,x+1\right)}{8}+\frac{\sqrt{2}\,\ln\left(\frac{x^2+\sqrt{2}\,x+1}{x^2-\sqrt{2}\,x+1}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^8+1),x)
```

```
[Out] 1/4*arctanh(x)+1/4*arctan(x)+1/8*2^(1/2)*arctan(2^(1/2)*x-1)+1/16*2^(1/2)*l
n((x^2+2^(1/2)*x+1)/(x^2-2^(1/2)*x+1))+1/8*2^(1/2)*arctan(2^(1/2)*x+1)
```

maxima [A] time = 2.27, size = 88, normalized size = 0.91

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{16}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{16}\sqrt{2}\log\left(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^8+1),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/8*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) + 1/16*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/16*sqrt(
2)*log(x^2 - sqrt(2)*x + 1) + 1/4*arctan(x) + 1/8*log(x + 1) - 1/8*log(x -
1)
```

mupad [B] time = 0.04, size = 45, normalized size = 0.46

$$\frac{\operatorname{atan}(x)}{4}-\frac{\operatorname{atan}(x\,1\mathrm{i})\,1\mathrm{i}}{4}+\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)+\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^8 - 1),x)
```

```
[Out] atan(x)/4 - (atan(x*1i)*1i)/4 + 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/8 +
 1i/8) + 2^(1/2)*atan(2^(1/2)*x*(1/2 + 1i/2))*(1/8 - 1i/8)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**8+1),x)
```

```
[Out] Timed out
```

3.1486 $\int \frac{1}{x^2\left(1-x^8\right)}\,dx$

Optimal. Leaf size=102

$$-\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{8\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{8\sqrt{2}}-\frac{1}{x}-\frac{1}{4}\tan^{-1}(x)+\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\,x+1\right)}{4\sqrt{2}}+\frac{1}{4}\tanh^{-1}(x)$$

```
[Out] -1/x-1/4*arctan(x)+1/4*arctanh(x)-1/8*arctan(-1+x*2^(1/2))*2^(1/2)-1/8*arct
an(1+x*2^(1/2))*2^(1/2)-1/16*ln(1+x^2-x*2^(1/2))*2^(1/2)+1/16*ln(1+x^2+x*2^
(1/2))*2^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {325, 301, 297, 1162, 617, 204, 1165, 628, 298, 203, 206}

$$-\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{8\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{8\sqrt{2}}-\frac{1}{x}-\frac{1}{4}\tan^{-1}(x)+\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\,x+1\right)}{4\sqrt{2}}+\frac{1}{4}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 - x^8)),x]
```

```
[Out] -x^(-1) - ArcTan[x]/4 + ArcTan[1 - Sqrt[2]*x]/(4*Sqrt[2]) - ArcTan[1 + Sqrt
[2]*x]/(4*Sqrt[2]) + ArcTanh[x]/4 - Log[1 - Sqrt[2]*x + x^2]/(8*Sqrt[2]) +
Log[1 + Sqrt[2]*x + x^2]/(8*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 301

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[x^(m - n/2
)/(r + s*x^(n/2)), x], x] - Dist[s/(2*b), Int[x^(m - n/2)/(r - s*x^(n/2)),
x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] &&
 LtQ[m, n] &&  !GtQ[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(1-x^8\right)}\,dx &= -\frac{1}{x}+\int \frac{x^6}{1-x^8}\,dx \\ &= -\frac{1}{x}+\frac{1}{2}\int \frac{x^2}{1-x^4}\,dx-\frac{1}{2}\int \frac{x^2}{1+x^4}\,dx \\ &= -\frac{1}{x}+\frac{1}{4}\int \frac{1}{1-x^2}\,dx-\frac{1}{4}\int \frac{1}{1+x^2}\,dx+\frac{1}{4}\int \frac{1-x^2}{1+x^4}\,dx-\frac{1}{4}\int \frac{1+x^2}{1+x^4}\,dx \\ &= -\frac{1}{x}-\frac{1}{4}\tan^{-1}(x)+\frac{1}{4}\tanh^{-1}(x)-\frac{1}{8}\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx-\frac{1}{8}\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx-\frac{\int \frac{\sqrt{}}{-1-}}{8} \\ &= -\frac{1}{x}-\frac{1}{4}\tan^{-1}(x)+\frac{1}{4}\tanh^{-1}(x)-\frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{8\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\,x+x^2\right)}{8\sqrt{2}}-\frac{\text{Subst}\left(\int \right.}{} \\ &= -\frac{1}{x}-\frac{1}{4}\tan^{-1}(x)+\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(1+\sqrt{2}\,x\right)}{4\sqrt{2}}+\frac{1}{4}\tanh^{-1}(x)-\frac{\log\left(1-\sqrt{2}\,x+\right.}{8\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 109, normalized size = 1.07

$$-\frac{\sqrt{2}\,x\log\left(x^2-\sqrt{2}\,x+1\right)-\sqrt{2}\,x\log\left(x^2+\sqrt{2}\,x+1\right)+2x\log(1-x)-2x\log(x+1)+4x\tan^{-1}(x)-2\sqrt{2}\,x\,t}{16x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 - x^8)),x]
```

```
[Out] -1/16*(16 + 4*x*ArcTan[x] - 2*Sqrt[2]*x*ArcTan[1 - Sqrt[2]*x] + 2*Sqrt[2]*x
*ArcTan[1 + Sqrt[2]*x] + 2*x*Log[1 - x] - 2*x*Log[1 + x] + Sqrt[2]*x*Log[1
- Sqrt[2]*x + x^2] - Sqrt[2]*x*Log[1 + Sqrt[2]*x + x^2])/x
```

fricas [A] time = 0.61, size = 123, normalized size = 1.21

$$\frac{4\sqrt{2}\,x\arctan\left(-\sqrt{2}\,x+\sqrt{2}\sqrt{x^2+\sqrt{2}\,x+1}-1\right)+4\sqrt{2}\,x\arctan\left(-\sqrt{2}\,x+\sqrt{2}\sqrt{x^2-\sqrt{2}\,x+1}+1\right)+\sqrt{2}\,x\,l}{16\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^8+1),x, algorithm="fricas")
```

```
[Out] 1/16*(4*sqrt(2)*x*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x + 1) - 1
) + 4*sqrt(2)*x*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) + 1)
+ sqrt(2)*x*log(x^2 + sqrt(2)*x + 1) - sqrt(2)*x*log(x^2 - sqrt(2)*x + 1) -
 4*x*arctan(x) + 2*x*log(x + 1) - 2*x*log(x - 1) - 16)/x
```

giac [A] time = 0.17, size = 95, normalized size = 0.93

$$-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{16}\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right)-\frac{1}{16}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^8+1),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 1/8*sqrt(2)*arctan(1/2*s
qrt(2)*(2*x - sqrt(2))) + 1/16*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/16*sqrt
(2)*log(x^2 - sqrt(2)*x + 1) - 1/x - 1/4*arctan(x) + 1/8*log(abs(x + 1)) -
1/8*log(abs(x - 1))
```

maple [A] time = 0.01, size = 79, normalized size = 0.77

$$-\frac{\arctan(x)}{4}-\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ x-1\right)}{8}-\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ x+1\right)}{8}-\frac{\sqrt{2}\ \ln\left(\frac{x^2-\sqrt{2}\ x+1}{x^2+\sqrt{2}\ x+1}\right)}{16}-\frac{\ln(x-1)}{8}+\frac{\ln(x+1)}{8}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-x^8+1),x)
```

```
[Out] -1/8*ln(x-1)-1/4*arctan(x)-1/8*2^(1/2)*arctan(2^(1/2)*x+1)-1/8*2^(1/2)*arct
an(2^(1/2)*x-1)-1/16*2^(1/2)*ln((x^2-2^(1/2)*x+1)/(x^2+2^(1/2)*x+1))-1/x+1/
8*ln(x+1)
```

maxima [A] time = 2.35, size = 93, normalized size = 0.91

$$-\frac{1}{8}\sqrt{2}\ \arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right)-\frac{1}{8}\sqrt{2}\ \arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right)+\frac{1}{16}\sqrt{2}\ \log\left(x^2+\sqrt{2}\,x+1\right)-\frac{1}{16}\sqrt{2}\ \log\left(x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^8+1),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 1/8*sqrt(2)*arctan(1/2*s
qrt(2)*(2*x - sqrt(2))) + 1/16*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/16*sqrt
(2)*log(x^2 - sqrt(2)*x + 1) - 1/x - 1/4*arctan(x) + 1/8*log(x + 1) - 1/8*l
og(x - 1)
```

mupad [B] time = 1.04, size = 50, normalized size = 0.49

$$-\frac{\operatorname{atan}(x)}{4}-\frac{1}{x}-\frac{\operatorname{atan}(x\,1\mathrm{i})\ 1\mathrm{i}}{4}+\sqrt{2}\ \operatorname{atan}\left(\sqrt{2}\ x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)+\sqrt{2}\ \operatorname{atan}\left(\sqrt{2}\ x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^2*(x^8 - 1)),x)
```

```
[Out] - (atan(x*1i)*1i)/4 - atan(x)/4 - 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/8
 - 1i/8) - 2^(1/2)*atan(2^(1/2)*x*(1/2 + 1i/2))*(1/8 + 1i/8) - 1/x
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-x**8+1),x)
```

```
[Out] Timed out
```

3.1487 $\int \frac{1}{x^4\left(1-x^8\right)}\,dx$

Optimal. Leaf size=104

$$-\frac{1}{3x^3}+\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{8\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{8\sqrt{2}}+\frac{1}{4}\tan^{-1}(x)+\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\,x+1\right)}{4\sqrt{2}}+\frac{1}{4}\tanh^{-1}($$

```
[Out] -1/3/x^3+1/4*arctan(x)+1/4*arctanh(x)-1/8*arctan(-1+x*2^(1/2))*2^(1/2)-1/8*
arctan(1+x*2^(1/2))*2^(1/2)+1/16*ln(1+x^2-x*2^(1/2))*2^(1/2)-1/16*ln(1+x^2+
x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {325, 301, 211, 1165, 628, 1162, 617, 204, 212, 206, 203}

$$-\frac{1}{3x^3}+\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{8\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{8\sqrt{2}}+\frac{1}{4}\tan^{-1}(x)+\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\,x+1\right)}{4\sqrt{2}}+\frac{1}{4}\tanh^{-1}($$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 - x^8)),x]
```

```
[Out] -1/(3*x^3) + ArcTan[x]/4 + ArcTan[1 - Sqrt[2]*x]/(4*Sqrt[2]) - ArcTan[1 + S
qrt[2]*x]/(4*Sqrt[2]) + ArcTanh[x]/4 + Log[1 - Sqrt[2]*x + x^2]/(8*Sqrt[2])
 - Log[1 + Sqrt[2]*x + x^2]/(8*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 301

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[x^(m - n/2
)/(r + s*x^(n/2)), x], x] - Dist[s/(2*b), Int[x^(m - n/2)/(r - s*x^(n/2)),
x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] &&
 LtQ[m, n] &&  !GtQ[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(1-x^8\right)}\,dx &= -\frac{1}{3x^3} + \int \frac{x^4}{1-x^8}\,dx \\
&= -\frac{1}{3x^3} + \frac{1}{2}\int \frac{1}{1-x^4}\,dx - \frac{1}{2}\int \frac{1}{1+x^4}\,dx \\
&= -\frac{1}{3x^3} + \frac{1}{4}\int \frac{1}{1-x^2}\,dx + \frac{1}{4}\int \frac{1}{1+x^2}\,dx - \frac{1}{4}\int \frac{1-x^2}{1+x^4}\,dx - \frac{1}{4}\int \frac{1+x^2}{1+x^4}\,dx \\
&= -\frac{1}{3x^3} + \frac{1}{4}\tan^{-1}(x) + \frac{1}{4}\tanh^{-1}(x) - \frac{1}{8}\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx - \frac{1}{8}\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx + \frac{\int -}{} \\
&= -\frac{1}{3x^3} + \frac{1}{4}\tan^{-1}(x) + \frac{1}{4}\tanh^{-1}(x) + \frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{8\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\,x+x^2\right)}{8\sqrt{2}} - \frac{\text{Subst}\left(}{} \\
&= -\frac{1}{3x^3} + \frac{1}{4}\tan^{-1}(x) + \frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{4\sqrt{2}} - \frac{\tan^{-1}\left(1+\sqrt{2}\,x\right)}{4\sqrt{2}} + \frac{1}{4}\tanh^{-1}(x) + \frac{\log\left(1-\sqrt{2}\,x\right.}{8\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 104, normalized size = 1.00

$$\frac{1}{48}\left(-\frac{16}{x^3} + 3\sqrt{2}\,\log\left(x^2-\sqrt{2}\,x+1\right) - 3\sqrt{2}\,\log\left(x^2+\sqrt{2}\,x+1\right) - 6\log(1-x) + 6\log(x+1) + 12\tan^{-1}(x) +\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 - x^8)),x]
```

```
[Out] (-16/x^3 + 12*ArcTan[x] + 6*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] - 6*Sqrt[2]*ArcTa
n[1 + Sqrt[2]*x] - 6*Log[1 - x] + 6*Log[1 + x] + 3*Sqrt[2]*Log[1 - Sqrt[2]*
x + x^2] - 3*Sqrt[2]*Log[1 + Sqrt[2]*x + x^2])/48
```

fricas [A] time = 0.71, size = 138, normalized size = 1.33

$$\frac{12\sqrt{2}\,x^3\arctan\left(-\sqrt{2}\,x+\sqrt{2}\sqrt{x^2+\sqrt{2}\,x+1}\,-1\right) + 12\sqrt{2}\,x^3\arctan\left(-\sqrt{2}\,x+\sqrt{2}\sqrt{x^2-\sqrt{2}\,x+1}\,+1\right) - 3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^8+1),x, algorithm="fricas")
```

```
[Out] 1/48*(12*sqrt(2)*x^3*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x + 1)
- 1) + 12*sqrt(2)*x^3*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1)
 + 1) - 3*sqrt(2)*x^3*log(x^2 + sqrt(2)*x + 1) + 3*sqrt(2)*x^3*log(x^2 - sq
rt(2)*x + 1) + 12*x^3*arctan(x) + 6*x^3*log(x + 1) - 6*x^3*log(x - 1) - 16)
/x^3
```

giac [A] time = 0.15, size = 95, normalized size = 0.91

$$-\frac{1}{8}\sqrt{2}\,\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right) - \frac{1}{8}\sqrt{2}\,\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right) - \frac{1}{16}\sqrt{2}\,\log\left(x^2+\sqrt{2}\,x+1\right) + \frac{1}{16}\sqrt{2}\,\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^8+1),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 1/8*sqrt(2)*arctan(1/2*s
qrt(2)*(2*x - sqrt(2))) - 1/16*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/16*sqrt
(2)*log(x^2 - sqrt(2)*x + 1) - 1/3/x^3 + 1/4*arctan(x) + 1/8*log(abs(x + 1)
) - 1/8*log(abs(x - 1))
```

maple [A] time = 0.01, size = 79, normalized size = 0.76

$$\frac{\arctan(x)}{4}-\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ x-1\right)}{8}-\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ x+1\right)}{8}-\frac{\sqrt{2}\ \ln\left(\frac{x^2+\sqrt{2}\ x+1}{x^2-\sqrt{2}\ x+1}\right)}{16}-\frac{\ln(x-1)}{8}+\frac{\ln(x+1)}{8}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-x^8+1),x)
```

```
[Out] -1/8*ln(x-1)-1/3/x^3+1/4*arctan(x)-1/16*2^(1/2)*ln((x^2+2^(1/2)*x+1)/(x^2-2
^(1/2)*x+1))-1/8*2^(1/2)*arctan(2^(1/2)*x+1)-1/8*2^(1/2)*arctan(2^(1/2)*x-1
)+1/8*ln(x+1)
```

maxima [A] time = 2.35, size = 93, normalized size = 0.89

$$-\frac{1}{8}\sqrt{2}\ \arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)-\frac{1}{8}\sqrt{2}\ \arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{1}{16}\sqrt{2}\ \log\left(x^2+\sqrt{2}\,x+1\right)+\frac{1}{16}\sqrt{2}\ \log\left(x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^8+1),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 1/8*sqrt(2)*arctan(1/2*s
qrt(2)*(2*x - sqrt(2))) - 1/16*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/16*sqrt
(2)*log(x^2 - sqrt(2)*x + 1) - 1/3/x^3 + 1/4*arctan(x) + 1/8*log(x + 1) - 1
/8*log(x - 1)
```

mupad [B] time = 1.04, size = 50, normalized size = 0.48

$$\frac{\operatorname{atan}(x)}{4}-\frac{1}{3x^3}-\frac{\operatorname{atan}(x\,1\mathrm{i})\,1\mathrm{i}}{4}+\sqrt{2}\ \operatorname{atan}\left(\sqrt{2}\ x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)+\sqrt{2}\ \operatorname{atan}\left(\sqrt{2}\ x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^4*(x^8 - 1)),x)
```

```
[Out] atan(x)/4 - (atan(x*1i)*1i)/4 - 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/8 +
 1i/8) - 2^(1/2)*atan(2^(1/2)*x*(1/2 + 1i/2))*(1/8 - 1i/8) - 1/(3*x^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-x**8+1),x)
```

```
[Out] Timed out
```

3.1488 $\int \frac{1}{x^6\left(1-x^8\right)}\,dx$

Optimal. Leaf size=104

$$-\frac{1}{5x^5}+\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{8\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{8\sqrt{2}}-\frac{1}{4}\tan^{-1}(x)-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\,x+1\right)}{4\sqrt{2}}+\frac{1}{4}\tanh^{-1}($$

```
[Out] -1/5/x^5-1/4*arctan(x)+1/4*arctanh(x)+1/8*arctan(-1+x*2^(1/2))*2^(1/2)+1/8*
arctan(1+x*2^(1/2))*2^(1/2)+1/16*ln(1+x^2-x*2^(1/2))*2^(1/2)-1/16*ln(1+x^2+
x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {325, 300, 297, 1162, 617, 204, 1165, 628, 298, 203, 206}

$$-\frac{1}{5x^5}+\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{8\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{8\sqrt{2}}-\frac{1}{4}\tan^{-1}(x)-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\,x+1\right)}{4\sqrt{2}}+\frac{1}{4}\tanh^{-1}($$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(1 - x^8)),x]
```

```
[Out] -1/(5*x^5) - ArcTan[x]/4 - ArcTan[1 - Sqrt[2]*x]/(4*Sqrt[2]) + ArcTan[1 + S
qrt[2]*x]/(4*Sqrt[2]) + ArcTanh[x]/4 + Log[1 - Sqrt[2]*x + x^2]/(8*Sqrt[2])
 - Log[1 + Sqrt[2]*x + x^2]/(8*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 300

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[x^m/(r + s
*x^(n/2)), x], x] + Dist[r/(2*a), Int[x^m/(r - s*x^(n/2)), x], x]] /; FreeQ
[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n/2] &&  !GtQ[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^6\left(1-x^8\right)}\,dx &= -\frac{1}{5x^5} + \int \frac{x^2}{1-x^8}\,dx \\
&= -\frac{1}{5x^5} + \frac{1}{2}\int \frac{x^2}{1-x^4}\,dx + \frac{1}{2}\int \frac{x^2}{1+x^4}\,dx \\
&= -\frac{1}{5x^5} + \frac{1}{4}\int \frac{1}{1-x^2}\,dx - \frac{1}{4}\int \frac{1}{1+x^2}\,dx - \frac{1}{4}\int \frac{1-x^2}{1+x^4}\,dx + \frac{1}{4}\int \frac{1+x^2}{1+x^4}\,dx \\
&= -\frac{1}{5x^5} - \frac{1}{4}\tan^{-1}(x) + \frac{1}{4}\tanh^{-1}(x) + \frac{1}{8}\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx + \frac{1}{8}\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx + \frac{\int -}{} \\
&= -\frac{1}{5x^5} - \frac{1}{4}\tan^{-1}(x) + \frac{1}{4}\tanh^{-1}(x) + \frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{8\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\,x+x^2\right)}{8\sqrt{2}} + \frac{\text{Subst}\left(}{} \\
&= -\frac{1}{5x^5} - \frac{1}{4}\tan^{-1}(x) - \frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{4\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}\,x\right)}{4\sqrt{2}} + \frac{1}{4}\tanh^{-1}(x) + \frac{\log\left(1-\sqrt{2}\,x\right.}{8\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 104, normalized size = 1.00

$$\frac{1}{80}\left(-\frac{16}{x^5} + 5\sqrt{2}\,\log\left(x^2-\sqrt{2}\,x+1\right) - 5\sqrt{2}\,\log\left(x^2+\sqrt{2}\,x+1\right) - 10\log(1-x) + 10\log(x+1) - 20\tan^{-1}(x)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(1 - x^8)),x]
```

```
[Out] (-16/x^5 - 20*ArcTan[x] - 10*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 10*Sqrt[2]*Arc
Tan[1 + Sqrt[2]*x] - 10*Log[1 - x] + 10*Log[1 + x] + 5*Sqrt[2]*Log[1 - Sqrt
[2]*x + x^2] - 5*Sqrt[2]*Log[1 + Sqrt[2]*x + x^2])/80
```

fricas [A] time = 0.60, size = 138, normalized size = 1.33

$$-\frac{20\sqrt{2}\,x^5\arctan\left(-\sqrt{2}\,x+\sqrt{2}\sqrt{x^2+\sqrt{2}\,x+1}-1\right)+20\sqrt{2}\,x^5\arctan\left(-\sqrt{2}\,x+\sqrt{2}\sqrt{x^2-\sqrt{2}\,x+1}+1\right)+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^8+1),x, algorithm="fricas")
```

```
[Out] -1/80*(20*sqrt(2)*x^5*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x + 1)
 - 1) + 20*sqrt(2)*x^5*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1
) + 1) + 5*sqrt(2)*x^5*log(x^2 + sqrt(2)*x + 1) - 5*sqrt(2)*x^5*log(x^2 - s
qrt(2)*x + 1) + 20*x^5*arctan(x) - 10*x^5*log(x + 1) + 10*x^5*log(x - 1) +
16)/x^5
```

giac [A] time = 0.18, size = 95, normalized size = 0.91

$$\frac{1}{8}\sqrt{2}\,\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right)+\frac{1}{8}\sqrt{2}\,\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right)-\frac{1}{16}\sqrt{2}\,\log\left(x^2+\sqrt{2}\,x+1\right)+\frac{1}{16}\sqrt{2}\,\log\left(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^8+1),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/8*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) - 1/16*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/16*sqrt(
2)*log(x^2 - sqrt(2)*x + 1) - 1/5/x^5 - 1/4*arctan(x) + 1/8*log(abs(x + 1))
 - 1/8*log(abs(x - 1))
```

maple [A] time = 0.01, size = 79, normalized size = 0.76

$$-\frac{\arctan(x)}{4}+\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ x-1\right)}{8}+\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ x+1\right)}{8}+\frac{\sqrt{2}\ \ln\left(\frac{x^2-\sqrt{2}\ x+1}{x^2+\sqrt{2}\ x+1}\right)}{16}-\frac{\ln(x-1)}{8}+\frac{\ln(x+1)}{8}-\frac{1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-x^8+1),x)
```

```
[Out] -1/5/x^5-1/8*ln(x-1)-1/4*arctan(x)+1/8*2^(1/2)*arctan(2^(1/2)*x+1)+1/8*2^(1
/2)*arctan(2^(1/2)*x-1)+1/16*2^(1/2)*ln((x^2-2^(1/2)*x+1)/(x^2+2^(1/2)*x+1)
)+1/8*ln(x+1)
```

maxima [A] time = 2.32, size = 93, normalized size = 0.89

$$\frac{1}{8}\sqrt{2}\ \arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right)+\frac{1}{8}\sqrt{2}\ \arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right)-\frac{1}{16}\sqrt{2}\ \log\left(x^2+\sqrt{2}\,x+1\right)+\frac{1}{16}\sqrt{2}\ \log\left(x^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^8+1),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/8*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) - 1/16*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/16*sqrt(
2)*log(x^2 - sqrt(2)*x + 1) - 1/5/x^5 - 1/4*arctan(x) + 1/8*log(x + 1) - 1/
8*log(x - 1)
```

mupad [B] time = 0.04, size = 50, normalized size = 0.48

$$-\frac{\operatorname{atan}(x)}{4}-\frac{1}{5\,x^5}-\frac{\operatorname{atan}(x\,1\mathrm{i})\ 1\mathrm{i}}{4}+\sqrt{2}\ \operatorname{atan}\left(\sqrt{2}\ x\ \left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)+\sqrt{2}\ \operatorname{atan}\left(\sqrt{2}\ x\ \left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^6*(x^8 - 1)),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/8 - 1i/8) - atan(x)/4 - (atan(x*1i)
*1i)/4 + 2^(1/2)*atan(2^(1/2)*x*(1/2 + 1i/2))*(1/8 + 1i/8) - 1/(5*x^5)
```

sympy [C] time = 171.55, size = 53, normalized size = 0.51

$$-\frac{\log(x-1)}{8}+\frac{\log(x+1)}{8}+\frac{i\log(x-i)}{8}-\frac{i\log(x+i)}{8}-\operatorname{RootSum}\left(4096t^4+1,\left(t\mapsto t\log\left(-512t^3+x\right)\right)\right)-\frac{1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-x**8+1),x)
```

```
[Out] -log(x - 1)/8 + log(x + 1)/8 + I*log(x - I)/8 - I*log(x + I)/8 - RootSum(40
96*_t**4 + 1, Lambda(_t, _t*log(-512*_t**3 + x))) - 1/(5*x**5)
```

3.1489 $\int \frac{1}{x^8\left(1-x^8\right)}\,dx$

Optimal. Leaf size=104

$$-\frac{1}{7x^7}-\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{8\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{8\sqrt{2}}+\frac{1}{4}\tan^{-1}(x)-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\,x+1\right)}{4\sqrt{2}}+\frac{1}{4}\tanh^{-1}($$

```
[Out] -1/7/x^7+1/4*arctan(x)+1/4*arctanh(x)+1/8*arctan(-1+x*2^(1/2))*2^(1/2)+1/8*
arctan(1+x*2^(1/2))*2^(1/2)-1/16*ln(1+x^2-x*2^(1/2))*2^(1/2)+1/16*ln(1+x^2+
x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {325, 214, 212, 206, 203, 211, 1165, 628, 1162, 617, 204}

$$-\frac{1}{7x^7}-\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{8\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{8\sqrt{2}}+\frac{1}{4}\tan^{-1}(x)-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\,x+1\right)}{4\sqrt{2}}+\frac{1}{4}\tanh^{-1}($$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(1 - x^8)),x]
```

```
[Out] -1/(7*x^7) + ArcTan[x]/4 - ArcTan[1 - Sqrt[2]*x]/(4*Sqrt[2]) + ArcTan[1 + S
qrt[2]*x]/(4*Sqrt[2]) + ArcTanh[x]/4 - Log[1 - Sqrt[2]*x + x^2]/(8*Sqrt[2])
 + Log[1 + Sqrt[2]*x + x^2]/(8*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
 + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^8\left(1-x^8\right)}\,dx = -\frac{1}{7x^7} + \int \frac{1}{1-x^8}\,dx$$
$$= -\frac{1}{7x^7} + \frac{1}{2}\int \frac{1}{1-x^4}\,dx + \frac{1}{2}\int \frac{1}{1+x^4}\,dx$$
$$= -\frac{1}{7x^7} + \frac{1}{4}\int \frac{1}{1-x^2}\,dx + \frac{1}{4}\int \frac{1}{1+x^2}\,dx + \frac{1}{4}\int \frac{1-x^2}{1+x^4}\,dx + \frac{1}{4}\int \frac{1+x^2}{1+x^4}\,dx$$
$$= -\frac{1}{7x^7} + \frac{1}{4}\tan^{-1}(x) + \frac{1}{4}\tanh^{-1}(x) + \frac{1}{8}\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx + \frac{1}{8}\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx - \frac{\int$$
$$= -\frac{1}{7x^7} + \frac{1}{4}\tan^{-1}(x) + \frac{1}{4}\tanh^{-1}(x) - \frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{8\sqrt{2}} + \frac{\log\left(1+\sqrt{2}\,x+x^2\right)}{8\sqrt{2}} + \frac{\text{Subst}\left($$
$$= -\frac{1}{7x^7} + \frac{1}{4}\tan^{-1}(x) - \frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{4\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}\,x\right)}{4\sqrt{2}} + \frac{1}{4}\tanh^{-1}(x) - \frac{\log\left(1-\sqrt{2}\,x\right.}{8\sqrt{2}}$$

Mathematica [A] time = 0.06, size = 104, normalized size = 1.00

$$\frac{1}{112}\left(-\frac{16}{x^7} - 7\sqrt{2}\,\log\left(x^2-\sqrt{2}\,x+1\right) + 7\sqrt{2}\,\log\left(x^2+\sqrt{2}\,x+1\right) - 14\log(1-x) + 14\log(x+1) + 28\tan^{-1}(x\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(1 - x^8)),x]
```

```
[Out] (-16/x^7 + 28*ArcTan[x] - 14*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 14*Sqrt[2]*Arc
Tan[1 + Sqrt[2]*x] - 14*Log[1 - x] + 14*Log[1 + x] - 7*Sqrt[2]*Log[1 - Sqrt
[2]*x + x^2] + 7*Sqrt[2]*Log[1 + Sqrt[2]*x + x^2])/112
```

fricas [A] time = 0.61, size = 138, normalized size = 1.33

$$-\frac{28\sqrt{2}\,x^7\arctan\left(-\sqrt{2}\,x+\sqrt{2}\sqrt{x^2+\sqrt{2}\,x+1}-1\right) + 28\sqrt{2}\,x^7\arctan\left(-\sqrt{2}\,x+\sqrt{2}\sqrt{x^2-\sqrt{2}\,x+1}+1\right) -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-x^8+1),x, algorithm="fricas")
```

```
[Out] -1/112*(28*sqrt(2)*x^7*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x + 1
) - 1) + 28*sqrt(2)*x^7*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x +
1) + 1) - 7*sqrt(2)*x^7*log(x^2 + sqrt(2)*x + 1) + 7*sqrt(2)*x^7*log(x^2 -
sqrt(2)*x + 1) - 28*x^7*arctan(x) - 14*x^7*log(x + 1) + 14*x^7*log(x - 1) +
 16)/x^7
```

giac [A] time = 0.15, size = 95, normalized size = 0.91

$$\frac{1}{8}\sqrt{2}\,\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) + \frac{1}{8}\sqrt{2}\,\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) + \frac{1}{16}\sqrt{2}\,\log\left(x^2+\sqrt{2}\,x+1\right) - \frac{1}{16}\sqrt{2}\,\log\left($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-x^8+1),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/8*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) + 1/16*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/16*sqrt(
2)*log(x^2 - sqrt(2)*x + 1) - 1/7/x^7 + 1/4*arctan(x) + 1/8*log(abs(x + 1))
 - 1/8*log(abs(x - 1))
```

maple [A] time = 0.01, size = 79, normalized size = 0.76

$$\frac{\arctan(x)}{4}+\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ x-1\right)}{8}+\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ x+1\right)}{8}+\frac{\sqrt{2}\ \ln\left(\frac{x^2+\sqrt{2}\ x+1}{x^2-\sqrt{2}\ x+1}\right)}{16}-\frac{\ln(x-1)}{8}+\frac{\ln(x+1)}{8}-\frac{1}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(-x^8+1),x)
```

```
[Out] -1/8*ln(x-1)+1/4*arctan(x)+1/8*2^(1/2)*arctan(2^(1/2)*x+1)+1/8*2^(1/2)*arct
an(2^(1/2)*x-1)+1/16*2^(1/2)*ln((x^2+2^(1/2)*x+1)/(x^2-2^(1/2)*x+1))-1/7/x^
7+1/8*ln(x+1)
```

maxima [A] time = 2.31, size = 93, normalized size = 0.89

$$\frac{1}{8}\sqrt{2}\ \arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right)+\frac{1}{8}\sqrt{2}\ \arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right)+\frac{1}{16}\sqrt{2}\ \log\left(x^2+\sqrt{2}\,x+1\right)-\frac{1}{16}\sqrt{2}\ \log\left(x^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-x^8+1),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/8*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) + 1/16*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/16*sqrt(
2)*log(x^2 - sqrt(2)*x + 1) - 1/7/x^7 + 1/4*arctan(x) + 1/8*log(x + 1) - 1/
8*log(x - 1)
```

mupad [B] time = 1.02, size = 50, normalized size = 0.48

$$\frac{\operatorname{atan}(x)}{4}-\frac{1}{7x^7}-\frac{\operatorname{atan}(x\,1\mathrm{i})\ 1\mathrm{i}}{4}+\sqrt{2}\ \operatorname{atan}\left(\sqrt{2}\ x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)+\sqrt{2}\ \operatorname{atan}\left(\sqrt{2}\ x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^8*(x^8 - 1)),x)
```

```
[Out] atan(x)/4 - (atan(x*1i)*1i)/4 + 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/8 +
 1i/8) + 2^(1/2)*atan(2^(1/2)*x*(1/2 + 1i/2))*(1/8 - 1i/8) - 1/(7*x^7)
```

sympy [C] time = 122.76, size = 51, normalized size = 0.49

$$-\frac{\log(x-1)}{8}+\frac{\log(x+1)}{8}-\frac{i\log(x-i)}{8}+\frac{i\log(x+i)}{8}-\operatorname{RootSum}\left(4096t^4+1,\left(t\mapsto t\log(-8t+x)\right)\right)-\frac{1}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(-x**8+1),x)
```

```
[Out] -log(x - 1)/8 + log(x + 1)/8 - I*log(x - I)/8 + I*log(x + I)/8 - RootSum(40
96*_t**4 + 1, Lambda(_t, _t*log(-8*_t + x))) - 1/(7*x**7)
```

3.1490 $\int \frac{x^9}{1+x^8}\,dx$

Optimal. Leaf size=100

$$\frac{x^2}{2}+\frac{\tan^{-1}\left(1-\sqrt{2}\,x^2\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\,x^2+1\right)}{4\sqrt{2}}+\frac{\log\left(x^4-\sqrt{2}\,x^2+1\right)}{8\sqrt{2}}-\frac{\log\left(x^4+\sqrt{2}\,x^2+1\right)}{8\sqrt{2}}$$

```
[Out] 1/2*x^2-1/8*arctan(-1+x^2*2^(1/2))*2^(1/2)-1/8*arctan(1+x^2*2^(1/2))*2^(1/2
)+1/16*ln(1+x^4-x^2*2^(1/2))*2^(1/2)-1/16*ln(1+x^4+x^2*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {275, 321, 211, 1165, 628, 1162, 617, 204}

$$\frac{x^2}{2}+\frac{\log\left(x^4-\sqrt{2}\,x^2+1\right)}{8\sqrt{2}}-\frac{\log\left(x^4+\sqrt{2}\,x^2+1\right)}{8\sqrt{2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\,x^2\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\,x^2+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(1 + x^8),x]
```

```
[Out] x^2/2 + ArcTan[1 - Sqrt[2]*x^2]/(4*Sqrt[2]) - ArcTan[1 + Sqrt[2]*x^2]/(4*Sq
rt[2]) + Log[1 - Sqrt[2]*x^2 + x^4]/(8*Sqrt[2]) - Log[1 + Sqrt[2]*x^2 + x^4
]/(8*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9}{1+x^8}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{1+x^4}\,dx, x, x^2\right) \\
&= \frac{x^2}{2} - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, x^2\right) \\
&= \frac{x^2}{2} - \frac{1}{4}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, x^2\right) - \frac{1}{4}\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, x^2\right) \\
&= \frac{x^2}{2} - \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, x^2\right) - \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x, x^2\right) + \frac{\text{Subst}\left(\int \frac{\sqrt{}}{-1-}\right.}{8} \\
&= \frac{x^2}{2} + \frac{\log\left(1-\sqrt{2}\,x^2+x^4\right)}{8\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\,x^2+x^4\right)}{8\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\,x^2\right)}{4\sqrt{2}} + \frac{\text{Subst}\left(\int\right.}{} \\
&= \frac{x^2}{2} + \frac{\tan^{-1}\left(1-\sqrt{2}\,x^2\right)}{4\sqrt{2}} - \frac{\tan^{-1}\left(1+\sqrt{2}\,x^2\right)}{4\sqrt{2}} + \frac{\log\left(1-\sqrt{2}\,x^2+x^4\right)}{8\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\,x^2+x^4\right)}{8\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 191, normalized size = 1.91

$$\frac{1}{16}\left(8x^2 - \sqrt{2}\,\log\left(x^2 - 2x\sin\left(\frac{\pi}{8}\right)+1\right) - \sqrt{2}\,\log\left(x^2 + 2x\sin\left(\frac{\pi}{8}\right)+1\right) + \sqrt{2}\,\log\left(x^2 - 2x\cos\left(\frac{\pi}{8}\right)+1\right) + \sqrt{2}\,\text{l}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(1 + x^8),x]
```

```
[Out] (8*x^2 + 2*Sqrt[2]*ArcTan[(x + Cos[Pi/8])*Csc[Pi/8]] + 2*Sqrt[2]*ArcTan[Cot
[Pi/8] - x*Csc[Pi/8]] + 2*Sqrt[2]*ArcTan[Sec[Pi/8]*(x + Sin[Pi/8])] - 2*Sqr
t[2]*ArcTan[x*Sec[Pi/8] - Tan[Pi/8]] + Sqrt[2]*Log[1 + x^2 - 2*x*Cos[Pi/8]]
 + Sqrt[2]*Log[1 + x^2 + 2*x*Cos[Pi/8]] - Sqrt[2]*Log[1 + x^2 - 2*x*Sin[Pi/
8]] - Sqrt[2]*Log[1 + x^2 + 2*x*Sin[Pi/8]])/16
```

fricas [A] time = 0.68, size = 112, normalized size = 1.12

$$\frac{1}{2}\,x^2 + \frac{1}{4}\,\sqrt{2}\,\arctan\left(-\sqrt{2}\,x^2 + \sqrt{2}\,\sqrt{x^4+\sqrt{2}\,x^2+1} - 1\right) + \frac{1}{4}\,\sqrt{2}\,\arctan\left(-\sqrt{2}\,x^2 + \sqrt{2}\,\sqrt{x^4-\sqrt{2}\,x^2+1} + 1\right) - \frac{1}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 1/4*sqrt(2)*arctan(-sqrt(2)*x^2 + sqrt(2)*sqrt(x^4 + sqrt(2)*x^2
+ 1) - 1) + 1/4*sqrt(2)*arctan(-sqrt(2)*x^2 + sqrt(2)*sqrt(x^4 - sqrt(2)*x^
2 + 1) + 1) - 1/16*sqrt(2)*log(x^4 + sqrt(2)*x^2 + 1) + 1/16*sqrt(2)*log(x^
4 - sqrt(2)*x^2 + 1)
```

giac [A] time = 0.17, size = 85, normalized size = 0.85

$$\frac{1}{2}x^2-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x^2+\sqrt{2}\right)\right)-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x^2-\sqrt{2}\right)\right)-\frac{1}{16}\sqrt{2}\log\left(x^4+\sqrt{2}x^2+1\right)+\frac{1}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x^2 + sqrt(2))) - 1/8*sqrt(2)*a
rctan(1/2*sqrt(2)*(2*x^2 - sqrt(2))) - 1/16*sqrt(2)*log(x^4 + sqrt(2)*x^2 +
 1) + 1/16*sqrt(2)*log(x^4 - sqrt(2)*x^2 + 1)
```

maple [A] time = 0.00, size = 71, normalized size = 0.71

$$\frac{x^2}{2}-\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ x^2-1\right)}{8}-\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ x^2+1\right)}{8}-\frac{\sqrt{2}\ \ln\left(\frac{x^4+\sqrt{2}\ x^2+1}{x^4-\sqrt{2}\ x^2+1}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^8+1),x)
```

```
[Out] 1/2*x^2-1/8*arctan(1+x^2*2^(1/2))*2^(1/2)-1/8*arctan(-1+x^2*2^(1/2))*2^(1/2
)-1/16*2^(1/2)*ln((1+x^4+x^2*2^(1/2))/(1+x^4-x^2*2^(1/2)))
```

maxima [A] time = 2.27, size = 85, normalized size = 0.85

$$\frac{1}{2}x^2-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x^2+\sqrt{2}\right)\right)-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x^2-\sqrt{2}\right)\right)-\frac{1}{16}\sqrt{2}\log\left(x^4+\sqrt{2}x^2+1\right)+\frac{1}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x^2 + sqrt(2))) - 1/8*sqrt(2)*a
rctan(1/2*sqrt(2)*(2*x^2 - sqrt(2))) - 1/16*sqrt(2)*log(x^4 + sqrt(2)*x^2 +
 1) + 1/16*sqrt(2)*log(x^4 - sqrt(2)*x^2 + 1)
```

mupad [B] time = 0.07, size = 42, normalized size = 0.42

$$\frac{x^2}{2}+\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}\ x^2\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)+\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}\ x^2\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^8 + 1),x)
```

```
[Out] x^2/2 - 2^(1/2)*atan(2^(1/2)*x^2*(1/2 + 1i/2))*(1/8 - 1i/8) - 2^(1/2)*atan(
2^(1/2)*x^2*(1/2 - 1i/2))*(1/8 + 1i/8)
```

sympy [A] time = 0.18, size = 85, normalized size = 0.85

$$\frac{x^2}{2}+\frac{\sqrt{2}\log\left(x^4-\sqrt{2}x^2+1\right)}{16}-\frac{\sqrt{2}\log\left(x^4+\sqrt{2}x^2+1\right)}{16}-\frac{\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}x^2-1\right)}{8}-\frac{\sqrt{2}\ \mathrm{atan}\left(\sqrt{2}x^2+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(x**8+1),x)
```

```
[Out] x**2/2 + sqrt(2)*log(x**4 - sqrt(2)*x**2 + 1)/16 - sqrt(2)*log(x**4 + sqrt(
2)*x**2 + 1)/16 - sqrt(2)*atan(sqrt(2)*x**2 - 1)/8 - sqrt(2)*atan(sqrt(2)*x
**2 + 1)/8
```

3.1491 $\int \frac{x^7}{1+x^8}\,dx$

Optimal. Leaf size=10

$$\frac{1}{8}\log\left(x^8+1\right)$$

```
[Out] 1/8*ln(x^8+1)
```

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {260}

$$\frac{1}{8}\log\left(x^8+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(1 + x^8),x]
```

```
[Out] Log[1 + x^8]/8
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^7}{1+x^8}\,dx = \frac{1}{8}\log\left(1+x^8\right)$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{8}\log\left(x^8+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(1 + x^8),x]
```

```
[Out] Log[1 + x^8]/8
```

fricas [A] time = 0.61, size = 8, normalized size = 0.80

$$\frac{1}{8}\log\left(x^8+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+1),x, algorithm="fricas")
```

```
[Out] 1/8*log(x^8 + 1)
```

giac [A] time = 0.17, size = 8, normalized size = 0.80

$$\frac{1}{8}\log\left(x^8+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+1),x, algorithm="giac")
[Out] 1/8*log(x^8 + 1)
```

maple [A] time = 0.00, size = 9, normalized size = 0.90

$$\frac{\ln\left(x^8+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^8+1),x)
[Out] 1/8*ln(x^8+1)
```

maxima [A] time = 1.00, size = 8, normalized size = 0.80

$$\frac{1}{8}\log\left(x^8+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+1),x, algorithm="maxima")
[Out] 1/8*log(x^8 + 1)
```

mupad [B] time = 1.01, size = 8, normalized size = 0.80

$$\frac{\ln\left(x^8+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^8 + 1),x)
[Out] log(x^8 + 1)/8
```

sympy [A] time = 0.11, size = 7, normalized size = 0.70

$$\frac{\log\left(x^8+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**8+1),x)
[Out] log(x**8 + 1)/8
```

3.1492 $\int \frac{x^5}{1+x^8}\,dx$

Optimal. Leaf size=93

$$-\frac{\tan^{-1}\left(1-\sqrt{2}\,x^2\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\,x^2+1\right)}{4\sqrt{2}}+\frac{\log\left(x^4-\sqrt{2}\,x^2+1\right)}{8\sqrt{2}}-\frac{\log\left(x^4+\sqrt{2}\,x^2+1\right)}{8\sqrt{2}}$$

```
[Out] 1/8*arctan(-1+x^2*2^(1/2))*2^(1/2)+1/8*arctan(1+x^2*2^(1/2))*2^(1/2)+1/16*l
n(1+x^4-x^2*2^(1/2))*2^(1/2)-1/16*ln(1+x^4+x^2*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {275, 297, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(x^4-\sqrt{2}\,x^2+1\right)}{8\sqrt{2}}-\frac{\log\left(x^4+\sqrt{2}\,x^2+1\right)}{8\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\,x^2\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\,x^2+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(1 + x^8),x]
```

```
[Out] -ArcTan[1 - Sqrt[2]*x^2]/(4*Sqrt[2]) + ArcTan[1 + Sqrt[2]*x^2]/(4*Sqrt[2])
+ Log[1 - Sqrt[2]*x^2 + x^4]/(8*Sqrt[2]) - Log[1 + Sqrt[2]*x^2 + x^4]/(8*Sq
rt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{1+x^8}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, x^2\right) \\
&= -\left(\frac{1}{4}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, x^2\right)\right) + \frac{1}{4}\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, x^2\right) \\
&= \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, x^2\right) + \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x, x^2\right) + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}\,x-x}\right.}{8\sqrt{2}} \\
&= \frac{\log\left(1-\sqrt{2}\,x^2+x^4\right)}{8\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\,x^2+x^4\right)}{8\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\,x^2\right)}{4\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\right.}{4} \\
&= -\frac{\tan^{-1}\left(1-\sqrt{2}\,x^2\right)}{4\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}\,x^2\right)}{4\sqrt{2}} + \frac{\log\left(1-\sqrt{2}\,x^2+x^4\right)}{8\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\,x^2+x^4\right)}{8\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 149, normalized size = 1.60

$$-\frac{\log\left(x^2-2x\sin\left(\frac{\pi}{8}\right)+1\right)+\log\left(x^2+2x\sin\left(\frac{\pi}{8}\right)+1\right)-\log\left(x^2-2x\cos\left(\frac{\pi}{8}\right)+1\right)-\log\left(x^2+2x\cos\left(\frac{\pi}{8}\right)+1\right)-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(1 + x^8),x]
```

```
[Out] -1/8*(2*ArcTan[(x + Cos[Pi/8])*Csc[Pi/8]] + 2*ArcTan[Cot[Pi/8] - x*Csc[Pi/8
]] + 2*ArcTan[Sec[Pi/8]*(x + Sin[Pi/8])] - 2*ArcTan[x*Sec[Pi/8] - Tan[Pi/8]
] - Log[1 + x^2 - 2*x*Cos[Pi/8]] - Log[1 + x^2 + 2*x*Cos[Pi/8]] + Log[1 + x
^2 - 2*x*Sin[Pi/8]] + Log[1 + x^2 + 2*x*Sin[Pi/8]])/Sqrt[2]
```

fricas [A] time = 0.59, size = 107, normalized size = 1.15

$$-\frac{1}{4}\sqrt{2}\arctan\left(-\sqrt{2}\,x^2+\sqrt{2}\sqrt{x^4+\sqrt{2}\,x^2+1}-1\right)-\frac{1}{4}\sqrt{2}\arctan\left(-\sqrt{2}\,x^2+\sqrt{2}\sqrt{x^4-\sqrt{2}\,x^2+1}+1\right)-\frac{1}{16}\sqrt{2}\,\text{I}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8+1),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan(-sqrt(2)*x^2 + sqrt(2)*sqrt(x^4 + sqrt(2)*x^2 + 1) - 1)
 - 1/4*sqrt(2)*arctan(-sqrt(2)*x^2 + sqrt(2)*sqrt(x^4 - sqrt(2)*x^2 + 1) +
1) - 1/16*sqrt(2)*log(x^4 + sqrt(2)*x^2 + 1) + 1/16*sqrt(2)*log(x^4 - sqrt(
2)*x^2 + 1)
```

giac [B] time = 0.31, size = 199, normalized size = 2.14

$$-\frac{1}{8}\sqrt{2}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)+\frac{1}{8}\sqrt{2}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)-\frac{1}{8}\sqrt{2}\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right)+\frac{1}{8}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8+1),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*arctan((2*x + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)) + 1/8*sqr
t(2)*arctan((2*x - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)) - 1/8*sqrt(2)*arc
tan((2*x + sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) + 1/8*sqrt(2)*arctan((2*x
 - sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) + 1/16*sqrt(2)*log(x^2 + x*sqrt(s
qrt(2) + 2) + 1) + 1/16*sqrt(2)*log(x^2 - x*sqrt(sqrt(2) + 2) + 1) - 1/16*s
qrt(2)*log(x^2 + x*sqrt(-sqrt(2) + 2) + 1) - 1/16*sqrt(2)*log(x^2 - x*sqrt(
-sqrt(2) + 2) + 1)
```

maple [A] time = 0.00, size = 66, normalized size = 0.71

$$\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\,x^2-1\right)}{8}+\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\,x^2+1\right)}{8}+\frac{\sqrt{2}\ \ln\left(\frac{x^4-\sqrt{2}\,x^2+1}{x^4+\sqrt{2}\,x^2+1}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^8+1),x)
```

```
[Out] 1/8*2^(1/2)*arctan(2^(1/2)*x^2+1)+1/8*2^(1/2)*arctan(2^(1/2)*x^2-1)+1/16*2^
(1/2)*ln((x^4-2^(1/2)*x^2+1)/(x^4+2^(1/2)*x^2+1))
```

maxima [A] time = 2.32, size = 80, normalized size = 0.86

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x^2+\sqrt{2}\right)\right)+\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x^2-\sqrt{2}\right)\right)-\frac{1}{16}\sqrt{2}\log\left(x^4+\sqrt{2}\,x^2+1\right)+\frac{1}{16}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8+1),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x^2 + sqrt(2))) + 1/8*sqrt(2)*arctan(1/2*
sqrt(2)*(2*x^2 - sqrt(2))) - 1/16*sqrt(2)*log(x^4 + sqrt(2)*x^2 + 1) + 1/16
*sqrt(2)*log(x^4 - sqrt(2)*x^2 + 1)
```

mupad [B] time = 0.05, size = 37, normalized size = 0.40

$$\sqrt{2}\ \operatorname{atan}\left(\sqrt{2}\,x^2\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)+\sqrt{2}\ \operatorname{atan}\left(\sqrt{2}\,x^2\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^8 + 1),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^2*(1/2 - 1i/2))*(1/8 - 1i/8) + 2^(1/2)*atan(2^(1/2)*
x^2*(1/2 + 1i/2))*(1/8 + 1i/8)
```

sympy [A] time = 0.18, size = 80, normalized size = 0.86

$$\frac{\sqrt{2}\log\left(x^4-\sqrt{2}\,x^2+1\right)}{16}-\frac{\sqrt{2}\log\left(x^4+\sqrt{2}\,x^2+1\right)}{16}+\frac{\sqrt{2}\ \operatorname{atan}\left(\sqrt{2}\,x^2-1\right)}{8}+\frac{\sqrt{2}\ \operatorname{atan}\left(\sqrt{2}\,x^2+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**8+1),x)

[Out] sqrt(2)*log(x**4 - sqrt(2)*x**2 + 1)/16 - sqrt(2)*log(x**4 + sqrt(2)*x**2 +
 1)/16 + sqrt(2)*atan(sqrt(2)*x**2 - 1)/8 + sqrt(2)*atan(sqrt(2)*x**2 + 1)/
8
```

3.1493 $\int \frac{x^3}{1+x^8}\,dx$

Optimal. Leaf size=8

$$\frac{1}{4}\tan^{-1}\left(x^4\right)$$

```
[Out] 1/4*arctan(x^4)
```

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {275, 203}

$$\frac{1}{4}\tan^{-1}\left(x^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 + x^8),x]
```

```
[Out] ArcTan[x^4]/4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{1+x^8}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, x^4\right)\\ &= \frac{1}{4}\tan^{-1}\left(x^4\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\frac{1}{4}\tan^{-1}\left(x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 + x^8),x]
```

```
[Out] ArcTan[x^4]/4
```

fricas [A] time = 0.65, size = 6, normalized size = 0.75

$$\frac{1}{4}\arctan\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+1),x, algorithm="fricas")
```

```
[Out] 1/4*arctan(x^4)
```

giac [A] time = 0.17, size = 6, normalized size = 0.75

$$\frac{1}{4}\arctan\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+1),x, algorithm="giac")
[Out] 1/4*arctan(x^4)
```

maple [A] time = 0.00, size = 7, normalized size = 0.88

$$\frac{\arctan\left(x^4\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8+1),x)
[Out] 1/4*arctan(x^4)
```

maxima [A] time = 2.22, size = 6, normalized size = 0.75

$$\frac{1}{4}\arctan\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+1),x, algorithm="maxima")
[Out] 1/4*arctan(x^4)
```

mupad [B] time = 1.04, size = 6, normalized size = 0.75

$$\frac{\operatorname{atan}\left(x^4\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8 + 1),x)
[Out] atan(x^4)/4
```

sympy [A] time = 0.12, size = 5, normalized size = 0.62

$$\frac{\operatorname{atan}\left(x^4\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**8+1),x)
[Out] atan(x**4)/4
```

3.1494 $\int \frac{x}{1+x^8}\,dx$

Optimal. Leaf size=93

$$-\frac{\tan^{-1}\left(1-\sqrt{2}\,x^2\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\,x^2+1\right)}{4\sqrt{2}}-\frac{\log\left(x^4-\sqrt{2}\,x^2+1\right)}{8\sqrt{2}}+\frac{\log\left(x^4+\sqrt{2}\,x^2+1\right)}{8\sqrt{2}}$$

```
[Out] 1/8*arctan(-1+x^2*2^(1/2))*2^(1/2)+1/8*arctan(1+x^2*2^(1/2))*2^(1/2)-1/16*l
n(1+x^4-x^2*2^(1/2))*2^(1/2)+1/16*ln(1+x^4+x^2*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {275, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(x^4-\sqrt{2}\,x^2+1\right)}{8\sqrt{2}}+\frac{\log\left(x^4+\sqrt{2}\,x^2+1\right)}{8\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}\,x^2\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\,x^2+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + x^8),x]
```

```
[Out] -ArcTan[1 - Sqrt[2]*x^2]/(4*Sqrt[2]) + ArcTan[1 + Sqrt[2]*x^2]/(4*Sqrt[2])
- Log[1 - Sqrt[2]*x^2 + x^4]/(8*Sqrt[2]) + Log[1 + Sqrt[2]*x^2 + x^4]/(8*Sq
rt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{1+x^8}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, x^2\right) \\
&= \frac{1}{4}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, x^2\right) + \frac{1}{4}\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, x^2\right) \\
&= \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, x^2\right) + \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x, x^2\right) - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}\,x-x^2}\right.}{8\sqrt{2}} \\
&= -\frac{\log\left(1-\sqrt{2}\,x^2+x^4\right)}{8\sqrt{2}} + \frac{\log\left(1+\sqrt{2}\,x^2+x^4\right)}{8\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\,x^2\right)}{4\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-}\right.}{} \\
&= -\frac{\tan^{-1}\left(1-\sqrt{2}\,x^2\right)}{4\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}\,x^2\right)}{4\sqrt{2}} - \frac{\log\left(1-\sqrt{2}\,x^2+x^4\right)}{8\sqrt{2}} + \frac{\log\left(1+\sqrt{2}\,x^2+x^4\right)}{8\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 149, normalized size = 1.60

$$-\frac{-\log\left(x^2-2x\sin\left(\frac{\pi}{8}\right)+1\right)-\log\left(x^2+2x\sin\left(\frac{\pi}{8}\right)+1\right)+\log\left(x^2-2x\cos\left(\frac{\pi}{8}\right)+1\right)+\log\left(x^2+2x\cos\left(\frac{\pi}{8}\right)+1\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + x^8),x]
```

```
[Out] -1/8*(2*ArcTan[(x + Cos[Pi/8])*Csc[Pi/8]] + 2*ArcTan[Cot[Pi/8] - x*Csc[Pi/8
]] + 2*ArcTan[Sec[Pi/8]*(x + Sin[Pi/8])] - 2*ArcTan[x*Sec[Pi/8] - Tan[Pi/8]
] + Log[1 + x^2 - 2*x*Cos[Pi/8]] + Log[1 + x^2 + 2*x*Cos[Pi/8]] - Log[1 + x
^2 - 2*x*Sin[Pi/8]] - Log[1 + x^2 + 2*x*Sin[Pi/8]])/Sqrt[2]
```

fricas [A] time = 0.64, size = 107, normalized size = 1.15

$$-\frac{1}{4}\sqrt{2}\,\arctan\left(-\sqrt{2}\,x^2+\sqrt{2}\,\sqrt{x^4+\sqrt{2}\,x^2+1}\,-1\right)-\frac{1}{4}\sqrt{2}\,\arctan\left(-\sqrt{2}\,x^2+\sqrt{2}\,\sqrt{x^4-\sqrt{2}\,x^2+1}\,+1\right)+\frac{1}{16}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+1),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan(-sqrt(2)*x^2 + sqrt(2)*sqrt(x^4 + sqrt(2)*x^2 + 1) - 1)
 - 1/4*sqrt(2)*arctan(-sqrt(2)*x^2 + sqrt(2)*sqrt(x^4 - sqrt(2)*x^2 + 1) +
1) + 1/16*sqrt(2)*log(x^4 + sqrt(2)*x^2 + 1) - 1/16*sqrt(2)*log(x^4 - sqrt(
2)*x^2 + 1)
```

giac [A] time = 0.17, size = 80, normalized size = 0.86

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x^2+\sqrt{2}\right)\right)+\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x^2-\sqrt{2}\right)\right)+\frac{1}{16}\sqrt{2}\log\left(x^4+\sqrt{2}x^2+1\right)-\frac{1}{16}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+1),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x^2 + sqrt(2))) + 1/8*sqrt(2)*arctan(1/2*
sqrt(2)*(2*x^2 - sqrt(2))) + 1/16*sqrt(2)*log(x^4 + sqrt(2)*x^2 + 1) - 1/16
*sqrt(2)*log(x^4 - sqrt(2)*x^2 + 1)
```

maple [A] time = 0.00, size = 66, normalized size = 0.71

$$\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ x^2-1\right)}{8}+\frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ x^2+1\right)}{8}+\frac{\sqrt{2}\ \ln\left(\frac{x^4+\sqrt{2}\ x^2+1}{x^4-\sqrt{2}\ x^2+1}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^8+1),x)
```

```
[Out] 1/16*2^(1/2)*ln((x^4+2^(1/2)*x^2+1)/(x^4-2^(1/2)*x^2+1))+1/8*2^(1/2)*arctan
(2^(1/2)*x^2+1)+1/8*2^(1/2)*arctan(2^(1/2)*x^2-1)
```

maxima [A] time = 2.24, size = 80, normalized size = 0.86

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x^2+\sqrt{2}\right)\right)+\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x^2-\sqrt{2}\right)\right)+\frac{1}{16}\sqrt{2}\log\left(x^4+\sqrt{2}x^2+1\right)-\frac{1}{16}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+1),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x^2 + sqrt(2))) + 1/8*sqrt(2)*arctan(1/2*
sqrt(2)*(2*x^2 - sqrt(2))) + 1/16*sqrt(2)*log(x^4 + sqrt(2)*x^2 + 1) - 1/16
*sqrt(2)*log(x^4 - sqrt(2)*x^2 + 1)
```

mupad [B] time = 0.04, size = 37, normalized size = 0.40

$$\sqrt{2}\ \operatorname{atan}\left(\sqrt{2}\ x^2\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)+\sqrt{2}\ \operatorname{atan}\left(\sqrt{2}\ x^2\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^8 + 1),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^2*(1/2 - 1i/2))*(1/8 + 1i/8) + 2^(1/2)*atan(2^(1/2)*
x^2*(1/2 + 1i/2))*(1/8 - 1i/8)
```

sympy [A] time = 0.18, size = 80, normalized size = 0.86

$$-\frac{\sqrt{2}\log\left(x^4-\sqrt{2}\,x^2+1\right)}{16}+\frac{\sqrt{2}\log\left(x^4+\sqrt{2}\,x^2+1\right)}{16}+\frac{\sqrt{2}\arctan\left(\sqrt{2}\,x^2-1\right)}{8}+\frac{\sqrt{2}\arctan\left(\sqrt{2}\,x^2+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**8+1),x)
```

```
[Out] -sqrt(2)*log(x**4 - sqrt(2)*x**2 + 1)/16 + sqrt(2)*log(x**4 + sqrt(2)*x**2
+ 1)/16 + sqrt(2)*atan(sqrt(2)*x**2 - 1)/8 + sqrt(2)*atan(sqrt(2)*x**2 + 1)
/8
```

3.1495 $\int \frac{1}{x\left(1+x^8\right)}\,dx$

Optimal. Leaf size=13

$$\log(x)-\frac{1}{8}\log\left(x^8+1\right)$$

```
[Out] ln(x)-1/8*ln(x^8+1)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {266, 36, 29, 31}

$$\log(x)-\frac{1}{8}\log\left(x^8+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + x^8)),x]
```

```
[Out] Log[x] - Log[1 + x^8]/8
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(1+x^8\right)}\,dx &= \frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, x^8\right)\\ &= \frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, x^8\right)-\frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^8\right)\\ &= \log(x)-\frac{1}{8}\log\left(1+x^8\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\log(x)-\frac{1}{8}\log\left(x^8+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + x^8)),x]
[Out] Log[x] - Log[1 + x^8]/8
```

fricas [A] time = 0.76, size = 11, normalized size = 0.85

$$-\frac{1}{8}\log\left(x^8+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+1),x, algorithm="fricas")
[Out] -1/8*log(x^8 + 1) + log(x)
```

giac [A] time = 0.17, size = 15, normalized size = 1.15

$$-\frac{1}{8}\log\left(x^8+1\right)+\frac{1}{8}\log\left(x^8\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+1),x, algorithm="giac")
[Out] -1/8*log(x^8 + 1) + 1/8*log(x^8)
```

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$\ln(x)-\frac{\ln\left(x^8+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^8+1),x)
[Out] ln(x)-1/8*ln(x^8+1)
```

maxima [A] time = 0.90, size = 15, normalized size = 1.15

$$-\frac{1}{8}\log\left(x^8+1\right)+\frac{1}{8}\log\left(x^8\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+1),x, algorithm="maxima")
[Out] -1/8*log(x^8 + 1) + 1/8*log(x^8)
```

mupad [B] time = 0.04, size = 11, normalized size = 0.85

$$\ln(x)-\frac{\ln\left(x^8+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^8 + 1)),x)
[Out] log(x) - log(x^8 + 1)/8
```

sympy [A] time = 0.14, size = 10, normalized size = 0.77

$$\log(x)-\frac{\log\left(x^8+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**8+1),x)
[Out] log(x) - log(x**8 + 1)/8
```

3.1496 $\int \frac{1}{x^3\left(1+x^8\right)}\,dx$

Optimal. Leaf size=100

$$-\frac{1}{2x^2}+\frac{\tan^{-1}\left(1-\sqrt{2}\,x^2\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\,x^2+1\right)}{4\sqrt{2}}-\frac{\log\left(x^4-\sqrt{2}\,x^2+1\right)}{8\sqrt{2}}+\frac{\log\left(x^4+\sqrt{2}\,x^2+1\right)}{8\sqrt{2}}$$

```
[Out] -1/2/x^2-1/8*arctan(-1+x^2*2^(1/2))*2^(1/2)-1/8*arctan(1+x^2*2^(1/2))*2^(1/
2)-1/16*ln(1+x^4-x^2*2^(1/2))*2^(1/2)+1/16*ln(1+x^4+x^2*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {275, 325, 297, 1162, 617, 204, 1165, 628}

$$-\frac{1}{2x^2}-\frac{\log\left(x^4-\sqrt{2}\,x^2+1\right)}{8\sqrt{2}}+\frac{\log\left(x^4+\sqrt{2}\,x^2+1\right)}{8\sqrt{2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\,x^2\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\,x^2+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 + x^8)),x]
```

```
[Out] -1/(2*x^2) + ArcTan[1 - Sqrt[2]*x^2]/(4*Sqrt[2]) - ArcTan[1 + Sqrt[2]*x^2]/
(4*Sqrt[2]) - Log[1 - Sqrt[2]*x^2 + x^4]/(8*Sqrt[2]) + Log[1 + Sqrt[2]*x^2
+ x^4]/(8*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(1+x^8\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\left(1+x^4\right)}\,dx, x, x^2\right)\\
&= -\frac{1}{2x^2} - \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, x^2\right)\\
&= -\frac{1}{2x^2} + \frac{1}{4}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, x^2\right) - \frac{1}{4}\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, x^2\right)\\
&= -\frac{1}{2x^2} - \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, x^2\right) - \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x, x^2\right) - \frac{\text{Sub}}{}\\
&= -\frac{1}{2x^2} - \frac{\log\left(1-\sqrt{2}\,x^2+x^4\right)}{8\sqrt{2}} + \frac{\log\left(1+\sqrt{2}\,x^2+x^4\right)}{8\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\,x^2\right)}{4\sqrt{2}} +\\
&= -\frac{1}{2x^2} + \frac{\tan^{-1}\left(1-\sqrt{2}\,x^2\right)}{4\sqrt{2}} - \frac{\tan^{-1}\left(1+\sqrt{2}\,x^2\right)}{4\sqrt{2}} - \frac{\log\left(1-\sqrt{2}\,x^2+x^4\right)}{8\sqrt{2}} + \frac{\log\left(1+\sqrt{2}\,x^2-\right.}{8\sqrt{2}}
\end{aligned}$$

Mathematica [B] time = 0.04, size = 208, normalized size = 2.08

$$-\frac{1}{2x^2}+\frac{\log\left(x^2-2x\sin\left(\frac{\pi}{8}\right)+1\right)}{8\sqrt{2}}+\frac{\log\left(x^2+2x\sin\left(\frac{\pi}{8}\right)+1\right)}{8\sqrt{2}}-\frac{\log\left(x^2-2x\cos\left(\frac{\pi}{8}\right)+1\right)}{8\sqrt{2}}-\frac{\log\left(x^2+2x\cos\left(\frac{\pi}{8}\right)\right.}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 + x^8)),x]
```

```
[Out] -1/2*1/x^2 - ArcTan[(x - Cos[Pi/8])*Csc[Pi/8]]/(4*Sqrt[2]) + ArcTan[(x + Co
s[Pi/8])*Csc[Pi/8]]/(4*Sqrt[2]) - ArcTan[Sec[Pi/8]*(x - Sin[Pi/8])]/(4*Sqrt
[2]) + ArcTan[Sec[Pi/8]*(x + Sin[Pi/8])]/(4*Sqrt[2]) - Log[1 + x^2 - 2*x*Co
s[Pi/8]]/(8*Sqrt[2]) - Log[1 + x^2 + 2*x*Cos[Pi/8]]/(8*Sqrt[2]) + Log[1 + x
^2 - 2*x*Sin[Pi/8]]/(8*Sqrt[2]) + Log[1 + x^2 + 2*x*Sin[Pi/8]]/(8*Sqrt[2])
```

fricas [A] time = 0.87, size = 124, normalized size = 1.24

$$\frac{4\sqrt{2}\,x^2\arctan\left(-\sqrt{2}\,x^2+\sqrt{2}\sqrt{x^4+\sqrt{2}\,x^2+1}\,-1\right)+4\sqrt{2}\,x^2\arctan\left(-\sqrt{2}\,x^2+\sqrt{2}\sqrt{x^4-\sqrt{2}\,x^2+1}\,+1\right)+\sqrt{2}}{16\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+1),x, algorithm="fricas")
```

```
[Out] 1/16*(4*sqrt(2)*x^2*arctan(-sqrt(2)*x^2 + sqrt(2)*sqrt(x^4 + sqrt(2)*x^2 +
1) - 1) + 4*sqrt(2)*x^2*arctan(-sqrt(2)*x^2 + sqrt(2)*sqrt(x^4 - sqrt(2)*x^
2 + 1) + 1) + sqrt(2)*x^2*log(x^4 + sqrt(2)*x^2 + 1) - sqrt(2)*x^2*log(x^4
- sqrt(2)*x^2 + 1) - 8)/x^2
```

giac [A] time = 0.24, size = 97, normalized size = 0.97

$$-\frac{1}{8}\sqrt{2}\,x^4\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x^2+\sqrt{2}\right)\right)-\frac{1}{8}\sqrt{2}\,x^4\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x^2-\sqrt{2}\right)\right)-\frac{1}{16}\sqrt{2}\,x^4\log\left(x^4+\sqrt{2}\,x^2+1\right)+\frac{1}{16}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+1),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*x^4*arctan(1/2*sqrt(2)*(2*x^2 + sqrt(2))) - 1/8*sqrt(2)*x^4*ar
ctan(1/2*sqrt(2)*(2*x^2 - sqrt(2))) - 1/16*sqrt(2)*x^4*log(x^4 + sqrt(2)*x^
2 + 1) + 1/16*sqrt(2)*x^4*log(x^4 - sqrt(2)*x^2 + 1) - 1/2/x^2
```

maple [A] time = 0.00, size = 71, normalized size = 0.71

$$-\frac{\sqrt{2}\,\arctan\left(\sqrt{2}\,x^2-1\right)}{8}-\frac{\sqrt{2}\,\arctan\left(\sqrt{2}\,x^2+1\right)}{8}-\frac{\sqrt{2}\,\ln\left(\frac{x^4-\sqrt{2}\,x^2+1}{x^4+\sqrt{2}\,x^2+1}\right)}{16}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^8+1),x)
```

```
[Out] -1/8*2^(1/2)*arctan(2^(1/2)*x^2-1)-1/16*2^(1/2)*ln((x^4-2^(1/2)*x^2+1)/(x^4
+2^(1/2)*x^2+1))-1/8*2^(1/2)*arctan(2^(1/2)*x^2+1)-1/2/x^2
```

maxima [A] time = 2.33, size = 85, normalized size = 0.85

$$-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x^2+\sqrt{2}\right)\right)-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x^2-\sqrt{2}\right)\right)+\frac{1}{16}\sqrt{2}\log\left(x^4+\sqrt{2}\,x^2+1\right)-\frac{1}{16}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+1),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x^2 + sqrt(2))) - 1/8*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x^2 - sqrt(2))) + 1/16*sqrt(2)*log(x^4 + sqrt(2)*x^2 + 1) - 1/1
6*sqrt(2)*log(x^4 - sqrt(2)*x^2 + 1) - 1/2/x^2
```

mupad [B] time = 1.05, size = 42, normalized size = 0.42

$$-\frac{1}{2\,x^2}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x^2\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x^2\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x^8 + 1)),x)
```

```
[Out] - 2^(1/2)*atan(2^(1/2)*x^2*(1/2 - 1i/2))*(1/8 - 1i/8) - 2^(1/2)*atan(2^(1/2
)*x^2*(1/2 + 1i/2))*(1/8 + 1i/8) - 1/(2*x^2)
```

sympy [A] time = 0.22, size = 87, normalized size = 0.87

$$-\frac{\sqrt{2}\,\log\left(x^4-\sqrt{2}\,x^2+1\right)}{16}+\frac{\sqrt{2}\,\log\left(x^4+\sqrt{2}\,x^2+1\right)}{16}-\frac{\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\,x^2-1\right)}{8}-\frac{\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\,x^2+1\right)}{8}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**8+1),x)
```

```
[Out] -sqrt(2)*log(x**4 - sqrt(2)*x**2 + 1)/16 + sqrt(2)*log(x**4 + sqrt(2)*x**2
+ 1)/16 - sqrt(2)*atan(sqrt(2)*x**2 - 1)/8 - sqrt(2)*atan(sqrt(2)*x**2 + 1)
/8 - 1/(2*x**2)
```

3.1497 $\int \frac{1}{x^5\left(1+x^8\right)}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{4x^4}-\frac{1}{4}\tan^{-1}\left(x^4\right)$$

```
[Out] -1/4/x^4-1/4*arctan(x^4)
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {275, 325, 203}

$$-\frac{1}{4x^4}-\frac{1}{4}\tan^{-1}\left(x^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 + x^8)),x]
```

```
[Out] -1/(4*x^4) - ArcTan[x^4]/4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\left(1+x^8\right)}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x^2\left(1+x^2\right)}\,dx, x, x^4\right)\\ &= -\frac{1}{4x^4}-\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, x^4\right)\\ &= -\frac{1}{4x^4}-\frac{1}{4}\tan^{-1}\left(x^4\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{1}{4}\tan^{-1}\left(\frac{1}{x^4}\right)-\frac{1}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 + x^8)),x]
[Out] -1/4*1/x^4 + ArcTan[x^(-4)]/4
```

fricas [A] time = 0.81, size = 15, normalized size = 0.94

$$-\frac{x^4 \arctan\left(x^4\right)+1}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+1),x, algorithm="fricas")
[Out] -1/4*(x^4*arctan(x^4) + 1)/x^4
```

giac [A] time = 0.17, size = 12, normalized size = 0.75

$$-\frac{1}{4\,x^4}-\frac{1}{4}\arctan\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+1),x, algorithm="giac")
[Out] -1/4/x^4 - 1/4*arctan(x^4)
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$-\frac{\arctan\left(x^4\right)}{4}-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^8+1),x)
[Out] -1/4/x^4-1/4*arctan(x^4)
```

maxima [A] time = 2.40, size = 12, normalized size = 0.75

$$-\frac{1}{4\,x^4}-\frac{1}{4}\arctan\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+1),x, algorithm="maxima")
[Out] -1/4/x^4 - 1/4*arctan(x^4)
```

mupad [B] time = 0.02, size = 12, normalized size = 0.75

$$-\frac{\operatorname{atan}\left(x^4\right)}{4}-\frac{1}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(x^8 + 1)),x)
[Out] - atan(x^4)/4 - 1/(4*x^4)
```

sympy [A] time = 0.15, size = 14, normalized size = 0.88

$$-\frac{\operatorname{atan}\left(x^4\right)}{4}-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**8+1),x)
[Out] -atan(x**4)/4 - 1/(4*x**4)
```

3.1498 $\int \frac{1}{x^7\left(1+x^8\right)}\,dx$

Optimal. Leaf size=100

$$-\frac{1}{6x^6}+\frac{\tan^{-1}\left(1-\sqrt{2}\,x^2\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\,x^2+1\right)}{4\sqrt{2}}+\frac{\log\left(x^4-\sqrt{2}\,x^2+1\right)}{8\sqrt{2}}-\frac{\log\left(x^4+\sqrt{2}\,x^2+1\right)}{8\sqrt{2}}$$

```
[Out] -1/6/x^6-1/8*arctan(-1+x^2*2^(1/2))*2^(1/2)-1/8*arctan(1+x^2*2^(1/2))*2^(1/
2)+1/16*ln(1+x^4-x^2*2^(1/2))*2^(1/2)-1/16*ln(1+x^4+x^2*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {275, 325, 211, 1165, 628, 1162, 617, 204}

$$-\frac{1}{6x^6}+\frac{\log\left(x^4-\sqrt{2}\,x^2+1\right)}{8\sqrt{2}}-\frac{\log\left(x^4+\sqrt{2}\,x^2+1\right)}{8\sqrt{2}}+\frac{\tan^{-1}\left(1-\sqrt{2}\,x^2\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\,x^2+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(1 + x^8)),x]
```

```
[Out] -1/(6*x^6) + ArcTan[1 - Sqrt[2]*x^2]/(4*Sqrt[2]) - ArcTan[1 + Sqrt[2]*x^2]/
(4*Sqrt[2]) + Log[1 - Sqrt[2]*x^2 + x^4]/(8*Sqrt[2]) - Log[1 + Sqrt[2]*x^2
 + x^4]/(8*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
 + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^7\left(1+x^8\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^4\left(1+x^4\right)}\,dx, x, x^2\right)\\
&= -\frac{1}{6x^6} - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, x^2\right)\\
&= -\frac{1}{6x^6} - \frac{1}{4}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, x^2\right) - \frac{1}{4}\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, x^2\right)\\
&= -\frac{1}{6x^6} - \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, x^2\right) - \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x, x^2\right) + \frac{\text{Su}}{}\\
&= -\frac{1}{6x^6} + \frac{\log\left(1-\sqrt{2}\,x^2+x^4\right)}{8\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\,x^2+x^4\right)}{8\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\,x^2\right)}{4\sqrt{2}} +\\
&= -\frac{1}{6x^6} + \frac{\tan^{-1}\left(1-\sqrt{2}\,x^2\right)}{4\sqrt{2}} - \frac{\tan^{-1}\left(1+\sqrt{2}\,x^2\right)}{4\sqrt{2}} + \frac{\log\left(1-\sqrt{2}\,x^2+x^4\right)}{8\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\,x^2 -\right.}{8\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 193, normalized size = 1.93

$$\frac{1}{48}\left(-\frac{8}{x^6} - 3\sqrt{2}\,\log\left(x^2 - 2x\sin\left(\frac{\pi}{8}\right)+1\right) - 3\sqrt{2}\,\log\left(x^2 + 2x\sin\left(\frac{\pi}{8}\right)+1\right) + 3\sqrt{2}\,\log\left(x^2 - 2x\cos\left(\frac{\pi}{8}\right)+1\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(1 + x^8)),x]
```

```
[Out] (-8/x^6 + 6*Sqrt[2]*ArcTan[(x + Cos[Pi/8])*Csc[Pi/8]] + 6*Sqrt[2]*ArcTan[Co
t[Pi/8] - x*Csc[Pi/8]] + 6*Sqrt[2]*ArcTan[Sec[Pi/8]*(x + Sin[Pi/8])] - 6*Sq
rt[2]*ArcTan[x*Sec[Pi/8] - Tan[Pi/8]] + 3*Sqrt[2]*Log[1 + x^2 - 2*x*Cos[Pi/
8]] + 3*Sqrt[2]*Log[1 + x^2 + 2*x*Cos[Pi/8]] - 3*Sqrt[2]*Log[1 + x^2 - 2*x*
Sin[Pi/8]] - 3*Sqrt[2]*Log[1 + x^2 + 2*x*Sin[Pi/8]])/48
```

fricas [A] time = 0.91, size = 125, normalized size = 1.25

$$\frac{12\sqrt{2}\,x^6\arctan\left(-\sqrt{2}\,x^2+\sqrt{2}\sqrt{x^4+\sqrt{2}\,x^2+1}\,-1\right)+12\sqrt{2}\,x^6\arctan\left(-\sqrt{2}\,x^2+\sqrt{2}\sqrt{x^4-\sqrt{2}\,x^2+1}\,+1\right)-3}{48\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+1),x, algorithm="fricas")
```

```
[Out] 1/48*(12*sqrt(2)*x^6*arctan(-sqrt(2)*x^2 + sqrt(2)*sqrt(x^4 + sqrt(2)*x^2 +
 1) - 1) + 12*sqrt(2)*x^6*arctan(-sqrt(2)*x^2 + sqrt(2)*sqrt(x^4 - sqrt(2)*
x^2 + 1) + 1) - 3*sqrt(2)*x^6*log(x^4 + sqrt(2)*x^2 + 1) + 3*sqrt(2)*x^6*lo
g(x^4 - sqrt(2)*x^2 + 1) - 8)/x^6
```

giac [A] time = 0.17, size = 85, normalized size = 0.85

$$-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x^2+\sqrt{2}\right)\right)-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x^2-\sqrt{2}\right)\right)-\frac{1}{16}\sqrt{2}\log\left(x^4+\sqrt{2}\,x^2+1\right)+\frac{1}{16}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+1),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x^2 + sqrt(2))) - 1/8*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x^2 - sqrt(2))) - 1/16*sqrt(2)*log(x^4 + sqrt(2)*x^2 + 1) + 1/1
6*sqrt(2)*log(x^4 - sqrt(2)*x^2 + 1) - 1/6/x^6
```

maple [A] time = 0.00, size = 71, normalized size = 0.71

$$-\frac{\sqrt{2}\,\arctan\left(\sqrt{2}\,x^2-1\right)}{8}-\frac{\sqrt{2}\,\arctan\left(\sqrt{2}\,x^2+1\right)}{8}-\frac{\sqrt{2}\,\ln\left(\frac{x^4+\sqrt{2}\,x^2+1}{x^4-\sqrt{2}\,x^2+1}\right)}{16}-\frac{1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^8+1),x)
```

```
[Out] -1/16*2^(1/2)*ln((x^4+2^(1/2)*x^2+1)/(x^4-2^(1/2)*x^2+1))-1/8*2^(1/2)*arcta
n(2^(1/2)*x^2+1)-1/8*2^(1/2)*arctan(2^(1/2)*x^2-1)-1/6/x^6
```

maxima [A] time = 2.40, size = 85, normalized size = 0.85

$$-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x^2+\sqrt{2}\right)\right)-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x^2-\sqrt{2}\right)\right)-\frac{1}{16}\sqrt{2}\log\left(x^4+\sqrt{2}\,x^2+1\right)+\frac{1}{16}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+1),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x^2 + sqrt(2))) - 1/8*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x^2 - sqrt(2))) - 1/16*sqrt(2)*log(x^4 + sqrt(2)*x^2 + 1) + 1/1
6*sqrt(2)*log(x^4 - sqrt(2)*x^2 + 1) - 1/6/x^6
```

mupad [B] time = 1.05, size = 42, normalized size = 0.42

$$-\frac{1}{6\,x^6}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x^2\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x^2\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(x^8 + 1)),x)
```

```
[Out] - 2^(1/2)*atan(2^(1/2)*x^2*(1/2 - 1i/2))*(1/8 + 1i/8) - 2^(1/2)*atan(2^(1/2
)*x^2*(1/2 + 1i/2))*(1/8 - 1i/8) - 1/(6*x^6)
```

sympy [A] time = 0.22, size = 87, normalized size = 0.87

$$\frac{\sqrt{2}\,\log\left(x^4-\sqrt{2}\,x^2+1\right)}{16}-\frac{\sqrt{2}\,\log\left(x^4+\sqrt{2}\,x^2+1\right)}{16}-\frac{\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\,x^2-1\right)}{8}-\frac{\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\,x^2+1\right)}{8}-\frac{1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**8+1),x)
```

```
[Out] sqrt(2)*log(x**4 - sqrt(2)*x**2 + 1)/16 - sqrt(2)*log(x**4 + sqrt(2)*x**2 +
 1)/16 - sqrt(2)*atan(sqrt(2)*x**2 - 1)/8 - sqrt(2)*atan(sqrt(2)*x**2 + 1)/
8 - 1/(6*x**6)
```

3.1499 $\int \frac{1}{x^9\left(1+x^8\right)}\,dx$

Optimal. Leaf size=22

$$-\frac{1}{8x^8}+\frac{1}{8}\log\left(x^8+1\right)-\log(x)$$

```
[Out] -1/8/x^8-ln(x)+1/8*ln(x^8+1)
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {266, 44}

$$-\frac{1}{8x^8}+\frac{1}{8}\log\left(x^8+1\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(1 + x^8)),x]
```

```
[Out] -1/(8*x^8) - Log[x] + Log[1 + x^8]/8
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^9\left(1+x^8\right)}\,dx &= \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{x^2(1+x)}\,dx, x, x^8\right)\\ &= \frac{1}{8}\,\text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{1}{x}+\frac{1}{1+x}\right)dx, x, x^8\right)\\ &= -\frac{1}{8x^8}-\log(x)+\frac{1}{8}\log\left(1+x^8\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$-\frac{1}{8x^8}+\frac{1}{8}\log\left(x^8+1\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^9*(1 + x^8)),x]
```

```
[Out] -1/8*1/x^8 - Log[x] + Log[1 + x^8]/8
```

fricas [A] time = 0.84, size = 24, normalized size = 1.09

$$\frac{x^8\log\left(x^8+1\right)-8\,x^8\log(x)-1}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(x^8+1),x, algorithm="fricas")
[Out] 1/8*(x^8*log(x^8 + 1) - 8*x^8*log(x) - 1)/x^8
```

giac [A] time = 0.17, size = 25, normalized size = 1.14

$$\frac{x^8-1}{8\,x^8}+\frac{1}{8}\,\log\left(x^8+1\right)-\frac{1}{8}\,\log\left(x^8\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(x^8+1),x, algorithm="giac")
[Out] 1/8*(x^8 - 1)/x^8 + 1/8*log(x^8 + 1) - 1/8*log(x^8)
```

maple [A] time = 0.01, size = 19, normalized size = 0.86

$$-\ln(x)+\frac{\ln\left(x^8+1\right)}{8}-\frac{1}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(x^8+1),x)
[Out] -1/8/x^8-ln(x)+1/8*ln(x^8+1)
```

maxima [A] time = 1.01, size = 20, normalized size = 0.91

$$-\frac{1}{8\,x^8}+\frac{1}{8}\,\log\left(x^8+1\right)-\frac{1}{8}\,\log\left(x^8\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(x^8+1),x, algorithm="maxima")
[Out] -1/8/x^8 + 1/8*log(x^8 + 1) - 1/8*log(x^8)
```

mupad [B] time = 0.04, size = 18, normalized size = 0.82

$$\frac{\ln\left(x^8+1\right)}{8}-\ln(x)-\frac{1}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(x^8 + 1)),x)
[Out] log(x^8 + 1)/8 - log(x) - 1/(8*x^8)
```

sympy [A] time = 0.17, size = 17, normalized size = 0.77

$$-\log(x)+\frac{\log\left(x^8+1\right)}{8}-\frac{1}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(x**8+1),x)
[Out] -log(x) + log(x**8 + 1)/8 - 1/(8*x**8)
```

3.1500 $\int \frac{x^8}{1+x^8}\,dx$

Optimal. Leaf size=340

$$\frac{1}{16}\sqrt{2-\sqrt{2}}\,\log\left(x^2-\sqrt{2-\sqrt{2}}\,x+1\right)-\frac{1}{16}\sqrt{2-\sqrt{2}}\,\log\left(x^2+\sqrt{2-\sqrt{2}}\,x+1\right)+\frac{1}{16}\sqrt{2+\sqrt{2}}\,\log\left(x^2-\sqrt{2+\sqrt{}}\right.$$

```
[Out] x+1/16*ln(1+x^2-x*(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)-1/16*ln(1+x^2+x*(2-2
^(1/2))^(1/2))*(2-2^(1/2))^(1/2)+1/4*arctan((-2*x+(2-2^(1/2))^(1/2))/(2+2^(
1/2))^(1/2))/(4-2*2^(1/2))^(1/2)-1/4*arctan((2*x+(2-2^(1/2))^(1/2))/(2+2^(1
/2))^(1/2))/(4-2*2^(1/2))^(1/2)+1/16*ln(1+x^2-x*(2+2^(1/2))^(1/2))*(2+2^(1/
2))^(1/2)-1/16*ln(1+x^2+x*(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)+1/4*arctan((
-2*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)-1/4*arctan((
2*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {321, 213, 1169, 634, 618, 204, 628}

$$\frac{1}{16}\sqrt{2-\sqrt{2}}\,\log\left(x^2-\sqrt{2-\sqrt{2}}\,x+1\right)-\frac{1}{16}\sqrt{2-\sqrt{2}}\,\log\left(x^2+\sqrt{2-\sqrt{2}}\,x+1\right)+\frac{1}{16}\sqrt{2+\sqrt{2}}\,\log\left(x^2-\sqrt{2+\sqrt{}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(1 + x^8),x]
```

```
[Out] x + ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 + Sqrt[2]]]/(4*Sqrt[2*(2 - Sqrt
[2])]) + ArcTan[(Sqrt[2 + Sqrt[2]] - 2*x)/Sqrt[2 - Sqrt[2]]]/(4*Sqrt[2*(2 +
 Sqrt[2])]) - ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2 + Sqrt[2]]]/(4*Sqrt[2
*(2 - Sqrt[2])]) - ArcTan[(Sqrt[2 + Sqrt[2]] + 2*x)/Sqrt[2 - Sqrt[2]]]/(4*S
qrt[2*(2 + Sqrt[2])]) + (Sqrt[2 - Sqrt[2]]*Log[1 - Sqrt[2 - Sqrt[2]]*x + x^
2])/16 - (Sqrt[2 - Sqrt[2]]*Log[1 + Sqrt[2 - Sqrt[2]]*x + x^2])/16 + (Sqrt[
2 + Sqrt[2]]*Log[1 - Sqrt[2 + Sqrt[2]]*x + x^2])/16 - (Sqrt[2 + Sqrt[2]]*Lo
g[1 + Sqrt[2 + Sqrt[2]]*x + x^2])/16
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b,
 4]], s = Denominator[Rt[a/b, 4]]}, Dist[r/(2*Sqrt[2]*a), Int[(Sqrt[2]*r -
s*x^(n/4))/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] + Dist[r/(2*Sq
rt[2]*a), Int[(Sqrt[2]*r + s*x^(n/4))/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n
/2)), x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 1] && GtQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^8}{1+x^8}\,dx = x - \int \frac{1}{1+x^8}\,dx$$

$$= x - \frac{\int \frac{\sqrt{2}-x^2}{1-\sqrt{2}\,x^2+x^4}\,dx}{2\sqrt{2}} - \frac{\int \frac{\sqrt{2}+x^2}{1+\sqrt{2}\,x^2+x^4}\,dx}{2\sqrt{2}}$$

$$= x - \frac{\int \frac{\sqrt{2\left(2-\sqrt{2}\right)}-\left(-1+\sqrt{2}\right)x}{1-\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2\left(2-\sqrt{2}\right)}} - \frac{\int \frac{\sqrt{2\left(2-\sqrt{2}\right)}+\left(-1+\sqrt{2}\right)x}{1+\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2\left(2-\sqrt{2}\right)}} - \frac{\int \frac{\sqrt{2\left(2+\sqrt{2}\right)}-\left(1+\sqrt{2}\right)x}{1-\sqrt{2+\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2\left(2+\sqrt{2}\right)}} - \frac{\int \frac{\sqrt{2(2+\sqrt{}}}{1+\sqrt{2}}}{4\sqrt{2(}}$$

$$= x - \frac{1}{8}\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\int \frac{1}{1-\sqrt{2+\sqrt{2}}\,x+x^2}\,dx - \frac{1}{8}\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\int \frac{1}{1+\sqrt{2+\sqrt{2}}\,x+x^2}\,d$$

$$= x + \frac{1}{16}\sqrt{2-\sqrt{2}}\,\log\left(1-\sqrt{2-\sqrt{2}}\,x+x^2\right) - \frac{1}{16}\sqrt{2-\sqrt{2}}\,\log\left(1+\sqrt{2-\sqrt{2}}\,x+x^2\right) + \frac{1}{16}\sqrt{2}$$

$$= x + \frac{1}{8}\sqrt{2+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right) + \frac{1}{8}\sqrt{2-\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right) - \frac{1}{8}\sqrt{2+\sqrt{2}}\,t$$

Mathematica [A] time = 0.01, size = 210, normalized size = 0.62

$$\frac{1}{8}\sin\left(\frac{\pi}{8}\right)\log\left(x^2-2x\sin\left(\frac{\pi}{8}\right)+1\right) - \frac{1}{8}\sin\left(\frac{\pi}{8}\right)\log\left(x^2+2x\sin\left(\frac{\pi}{8}\right)+1\right) + \frac{1}{8}\cos\left(\frac{\pi}{8}\right)\log\left(x^2-2x\cos\left(\frac{\pi}{8}\right)+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(1 + x^8),x]
```

```
[Out] x - (ArcTan[Sec[Pi/8]*(x - Sin[Pi/8])]*Cos[Pi/8])/4 - (ArcTan[Sec[Pi/8]*(x
+ Sin[Pi/8])]*Cos[Pi/8])/4 + (Cos[Pi/8]*Log[1 + x^2 - 2*x*Cos[Pi/8]])/8 - (
Cos[Pi/8]*Log[1 + x^2 + 2*x*Cos[Pi/8]])/8 - (ArcTan[(x - Cos[Pi/8])*Csc[Pi/
8]]*Sin[Pi/8])/4 - (ArcTan[(x + Cos[Pi/8])*Csc[Pi/8]]*Sin[Pi/8])/4 + (Log[1
 + x^2 - 2*x*Sin[Pi/8]]*Sin[Pi/8])/8 - (Log[1 + x^2 + 2*x*Sin[Pi/8]]*Sin[Pi
/8])/8
```

fricas **[B]** time = 0.93, size = 1028, normalized size = 3.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+1),x, algorithm="fricas")
```

```
[Out] 1/16*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*arctan(-(2*sq
rt(2)*x - 2*sqrt(2)*sqrt(x^2 + 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) - 1/2*sqrt(2
)*x*sqrt(-sqrt(2) + 2) + 1) + sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(sqrt
(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))) + 1/16*(sqrt(2)*sqrt(sqrt(2) + 2) + sq
rt(2)*sqrt(-sqrt(2) + 2))*arctan(-(2*sqrt(2)*x - 2*sqrt(2)*sqrt(x^2 - 1/2*s
qrt(2)*x*sqrt(sqrt(2) + 2) + 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) - sqrt(s
qrt(2) + 2) + sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2)))
 - 1/16*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*arctan((2*
sqrt(2)*x - 2*sqrt(2)*sqrt(x^2 + 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) + 1/2*sqrt
(2)*x*sqrt(-sqrt(2) + 2) + 1) + sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))/(sq
rt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))) - 1/16*(sqrt(2)*sqrt(sqrt(2) + 2) -
sqrt(2)*sqrt(-sqrt(2) + 2))*arctan((2*sqrt(2)*x - 2*sqrt(2)*sqrt(x^2 - 1/2*
sqrt(2)*x*sqrt(sqrt(2) + 2) - 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) - sqrt(
sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))
) - 1/64*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*log(x^2 +
 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) + 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) -
1/64*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*log(x^2 + 1/2
*sqrt(2)*x*sqrt(sqrt(2) + 2) - 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) + 1/64
*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*log(x^2 - 1/2*sqr
t(2)*x*sqrt(sqrt(2) + 2) + 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) + 1/64*(sq
rt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*log(x^2 - 1/2*sqrt(2)
*x*sqrt(sqrt(2) + 2) - 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) + 1/8*sqrt(sqr
t(2) + 2)*arctan(-(2*x - 2*sqrt(x^2 + x*sqrt(-sqrt(2) + 2) + 1) + sqrt(-sqr
t(2) + 2))/sqrt(sqrt(2) + 2)) + 1/8*sqrt(sqrt(2) + 2)*arctan(-(2*x - 2*sqrt
(x^2 - x*sqrt(-sqrt(2) + 2) + 1) - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)) +
 1/8*sqrt(-sqrt(2) + 2)*arctan(-(2*x - 2*sqrt(x^2 + x*sqrt(sqrt(2) + 2) + 1
) + sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) + 1/8*sqrt(-sqrt(2) + 2)*arctan(
-(2*x - 2*sqrt(x^2 - x*sqrt(sqrt(2) + 2) + 1) - sqrt(sqrt(2) + 2))/sqrt(-sq
rt(2) + 2)) - 1/32*sqrt(sqrt(2) + 2)*log(x^2 + x*sqrt(sqrt(2) + 2) + 1) + 1
/32*sqrt(sqrt(2) + 2)*log(x^2 - x*sqrt(sqrt(2) + 2) + 1) - 1/32*sqrt(-sqrt(
2) + 2)*log(x^2 + x*sqrt(-sqrt(2) + 2) + 1) + 1/32*sqrt(-sqrt(2) + 2)*log(x
^2 - x*sqrt(-sqrt(2) + 2) + 1) + x
```

giac **[A]** time = 0.20, size = 240, normalized size = 0.71

$$-\frac{1}{8}\sqrt{\sqrt{2}+2}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)-\frac{1}{8}\sqrt{\sqrt{2}+2}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)-\frac{1}{8}\sqrt{-\sqrt{2}+2}\arctan\left(\frac{2x+}{\sqrt{-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+1),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(sqrt(2) + 2)*arctan((2*x + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2))
 - 1/8*sqrt(sqrt(2) + 2)*arctan((2*x - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2
)) - 1/8*sqrt(-sqrt(2) + 2)*arctan((2*x + sqrt(sqrt(2) + 2))/sqrt(-sqrt(2)
 + 2)) - 1/8*sqrt(-sqrt(2) + 2)*arctan((2*x - sqrt(sqrt(2) + 2))/sqrt(-sqrt(
2) + 2)) - 1/16*sqrt(sqrt(2) + 2)*log(x^2 + x*sqrt(sqrt(2) + 2) + 1) + 1/16
*sqrt(sqrt(2) + 2)*log(x^2 - x*sqrt(sqrt(2) + 2) + 1) - 1/16*sqrt(-sqrt(2)
 + 2)*log(x^2 + x*sqrt(-sqrt(2) + 2) + 1) + 1/16*sqrt(-sqrt(2) + 2)*log(x^2
 - x*sqrt(-sqrt(2) + 2) + 1) + x
```

maple [C] time = 0.01, size = 24, normalized size = 0.07

$$x - \frac{\ln\left(-\text{RootOf}\left(_Z^8+1\right)+x\right)}{8\,\text{RootOf}\left(_Z^8+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^8+1),x)
```

```
[Out] x-1/8*sum(1/_R^7*ln(-_R+x),_R=RootOf(_Z^8+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x - \int \frac{1}{x^8+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+1),x, algorithm="maxima")
```

```
[Out] x - integrate(1/(x^8 + 1), x)
```

mupad [B] time = 1.25, size = 289, normalized size = 0.85

$$x-\text{atan}\left(\frac{x\sqrt{-\sqrt{2}-2}\,1\text{i}}{\sqrt{2-\sqrt{2}}\,\sqrt{-\sqrt{2}-2}+\sqrt{2}}-\frac{x\sqrt{2-\sqrt{2}}\,1\text{i}}{\sqrt{2-\sqrt{2}}\,\sqrt{-\sqrt{2}-2}+\sqrt{2}}\right)\left(\frac{\sqrt{-\sqrt{2}-2}\,1\text{i}}{8}-\frac{\sqrt{2-\sqrt{2}}\,1\text{i}}{8}\right)+\text{atan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^8 + 1),x)
```

```
[Out] x - atan((x*(- 2^(1/2) - 2)^(1/2)*1i)/((2 - 2^(1/2))^(1/2)*(- 2^(1/2) - 2)^
(1/2) + 2^(1/2)) - (x*(2 - 2^(1/2))^(1/2)*1i)/((2 - 2^(1/2))^(1/2)*(- 2^(1/
2) - 2)^(1/2) + 2^(1/2)))*(((- 2^(1/2) - 2)^(1/2)*1i)/8 - ((2 - 2^(1/2))^(1
/2)*1i)/8) + atan((x*(2^(1/2) - 2)^(1/2)*1i)/(2^(1/2) + (2^(1/2) - 2)^(1/2)
*(2^(1/2) + 2)^(1/2)) + (x*(2^(1/2) + 2)^(1/2)*1i)/(2^(1/2) + (2^(1/2) - 2)
^(1/2)*(2^(1/2) + 2)^(1/2)))*(((2^(1/2) - 2)^(1/2)*1i)/8 + ((2^(1/2) + 2)^(
1/2)*1i)/8) - atan(x*(2^(1/2) + 2)^(1/2)*(1/2 + 1i/2) - (2^(1/2)*x*(2^(1/2)
 + 2)^(1/2))/2)*((2^(1/2)*1i)/16 - (1/16 + 1i/16))*(2^(1/2) + 2)^(1/2)*2i +
 atan(x*(2^(1/2) + 2)^(1/2)*(1/2 - 1i/2) + (2^(1/2)*x*(2^(1/2) + 2)^(1/2)*1
i)/2)*(2^(1/2)/16 - (1/16 - 1i/16))*(2^(1/2) + 2)^(1/2)*2i
```

sympy [A] time = 3.15, size = 15, normalized size = 0.04

$$x + \text{RootSum}\left(16777216t^8+1, \left(t \mapsto t\log\left(-8t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**8+1),x)
```

```
[Out] x + RootSum(16777216*_t**8 + 1, Lambda(_t, _t*log(-8*_t + x)))
```

3.1501 $\int \frac{x^6}{1+x^8}\,dx$

Optimal. Leaf size=339

$$\frac{1}{16}\sqrt{2-\sqrt{2}}\,\log\left(x^2-\sqrt{2-\sqrt{2}}\,x+1\right)-\frac{1}{16}\sqrt{2-\sqrt{2}}\,\log\left(x^2+\sqrt{2-\sqrt{2}}\,x+1\right)+\frac{1}{16}\sqrt{2+\sqrt{2}}\,\log\left(x^2-\sqrt{2+\sqrt{2}}\right.$$

```
[Out] 1/16*ln(1+x^2-x*(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)-1/16*ln(1+x^2+x*(2-2^(
1/2))^(1/2))*(2-2^(1/2))^(1/2)-1/4*arctan((-2*x+(2-2^(1/2))^(1/2))/(2+2^(1/
2))^(1/2))/(4-2*2^(1/2))^(1/2)+1/4*arctan((2*x+(2-2^(1/2))^(1/2))/(2+2^(1/2
))^(1/2))/(4-2*2^(1/2))^(1/2)+1/16*ln(1+x^2-x*(2+2^(1/2))^(1/2))*(2+2^(1/2)
)^(1/2)-1/16*ln(1+x^2+x*(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)-1/4*arctan((-2
*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)+1/4*arctan((2*
x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {299, 1122, 1169, 634, 618, 204, 628}

$$\frac{1}{16}\sqrt{2-\sqrt{2}}\,\log\left(x^2-\sqrt{2-\sqrt{2}}\,x+1\right)-\frac{1}{16}\sqrt{2-\sqrt{2}}\,\log\left(x^2+\sqrt{2-\sqrt{2}}\,x+1\right)+\frac{1}{16}\sqrt{2+\sqrt{2}}\,\log\left(x^2-\sqrt{2+\sqrt{2}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(1 + x^8),x]
```

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 + Sqrt[2]]]/(4*Sqrt[2*(2 - Sqrt[2]
)]) - ArcTan[(Sqrt[2 + Sqrt[2]] - 2*x)/Sqrt[2 - Sqrt[2]]]/(4*Sqrt[2*(2 + Sq
rt[2])]) + ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2 + Sqrt[2]]]/(4*Sqrt[2*(2
 - Sqrt[2])]) + ArcTan[(Sqrt[2 + Sqrt[2]] + 2*x)/Sqrt[2 - Sqrt[2]]]/(4*Sqrt
[2*(2 + Sqrt[2])]) + (Sqrt[2 - Sqrt[2]]*Log[1 - Sqrt[2 - Sqrt[2]]*x + x^2])
/16 - (Sqrt[2 - Sqrt[2]]*Log[1 + Sqrt[2 - Sqrt[2]]*x + x^2])/16 + (Sqrt[2 +
 Sqrt[2]]*Log[1 - Sqrt[2 + Sqrt[2]]*x + x^2])/16 - (Sqrt[2 + Sqrt[2]]*Log[1
 + Sqrt[2 + Sqrt[2]]*x + x^2])/16
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 299

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[s^3/(2*Sqrt[2]*b*r), Int[x^(
m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Dist[s^3/(2*Sq
rt[2]*b*r), Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && G
tQ[a/b, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{1+x^8}\,dx &= \frac{\int \frac{x^4}{1-\sqrt{2}\,x^2+x^4}\,dx}{2\sqrt{2}} - \frac{\int \frac{x^4}{1+\sqrt{2}\,x^2+x^4}\,dx}{2\sqrt{2}} \\
&= -\frac{\int \frac{1-\sqrt{2}\,x^2}{1-\sqrt{2}\,x^2+x^4}\,dx}{2\sqrt{2}} + \frac{\int \frac{1+\sqrt{2}\,x^2}{1+\sqrt{2}\,x^2+x^4}\,dx}{2\sqrt{2}} \\
&= \frac{\int \frac{\sqrt{2-\sqrt{2}}-\left(1-\sqrt{2}\right)x}{1-\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2\left(2-\sqrt{2}\right)}} + \frac{\int \frac{\sqrt{2-\sqrt{2}}+\left(1-\sqrt{2}\right)x}{1+\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2\left(2-\sqrt{2}\right)}} - \frac{\int \frac{\sqrt{2+\sqrt{2}}-\left(1+\sqrt{2}\right)x}{1-\sqrt{2+\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2\left(2+\sqrt{2}\right)}} - \frac{\int \frac{\sqrt{2+\sqrt{2}}+\left(1+\sqrt{2}\right)x}{1+\sqrt{2+\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2\left(2+\sqrt{2}\right)}} \\
&= \frac{1}{8}\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\int \frac{1}{1-\sqrt{2+\sqrt{2}}\,x+x^2}\,dx + \frac{1}{8}\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\int \frac{1}{1+\sqrt{2+\sqrt{2}}\,x+x^2}\,dx + \\
&= \frac{1}{16}\sqrt{2-\sqrt{2}}\,\log\left(1-\sqrt{2-\sqrt{2}}\,x+x^2\right) - \frac{1}{16}\sqrt{2-\sqrt{2}}\,\log\left(1+\sqrt{2-\sqrt{2}}\,x+x^2\right) + \frac{1}{16}\sqrt{2+\sqrt{} \\
&= -\frac{1}{8}\sqrt{2+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right) - \frac{1}{8}\sqrt{2-\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right) + \frac{1}{8}\sqrt{2+\sqrt{2}}\,\tan
\end{aligned}$$

Mathematica [A] time = 0.01, size = 209, normalized size = 0.62

$$\frac{1}{8}\sin\left(\frac{\pi}{8}\right)\log\left(x^2-2x\sin\left(\frac{\pi}{8}\right)+1\right) - \frac{1}{8}\sin\left(\frac{\pi}{8}\right)\log\left(x^2+2x\sin\left(\frac{\pi}{8}\right)+1\right) + \frac{1}{8}\cos\left(\frac{\pi}{8}\right)\log\left(x^2-2x\cos\left(\frac{\pi}{8}\right)+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(1 + x^8),x]
```

```
[Out] (ArcTan[Sec[Pi/8]*(x - Sin[Pi/8])]*Cos[Pi/8])/4 + (ArcTan[Sec[Pi/8]*(x + Si
n[Pi/8])]*Cos[Pi/8])/4 + (Cos[Pi/8]*Log[1 + x^2 - 2*x*Cos[Pi/8]])/8 - (Cos[
Pi/8]*Log[1 + x^2 + 2*x*Cos[Pi/8]])/8 + (ArcTan[(x - Cos[Pi/8])*Csc[Pi/8]]*
Sin[Pi/8])/4 + (ArcTan[(x + Cos[Pi/8])*Csc[Pi/8]]*Sin[Pi/8])/4 + (Log[1 + x
^2 - 2*x*Sin[Pi/8]]*Sin[Pi/8])/8 - (Log[1 + x^2 + 2*x*Sin[Pi/8]]*Sin[Pi/8])
/8
```

fricas **[B]** time = 0.97, size = 2513, normalized size = 7.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+1),x, algorithm="fricas")
```

```
[Out] 1/16*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*arctan((8*sqr
t(2)*x + (sqrt(2) + 2)^(3/2) + 3*sqrt(sqrt(2) + 2)*(sqrt(2) - 2) - 3*(sqrt(
2) + 2)*sqrt(-sqrt(2) + 2) + (-sqrt(2) + 2)^(3/2) - sqrt(2)*sqrt(8*sqrt(2)*
x*(sqrt(2) + 2)^(3/2) + (sqrt(2) + 2)^3 + 3*(sqrt(2) + 2)*(sqrt(2) - 2)^2 -
 (sqrt(2) - 2)^3 - 24*sqrt(2)*x*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) + 8*sqrt(2
)*x*(-sqrt(2) + 2)^(3/2) + 64*x^2 + 3*(8*sqrt(2)*x*sqrt(sqrt(2) + 2) - (sqr
t(2) + 2)^2)*(sqrt(2) - 2)))/((sqrt(2) + 2)^(3/2) + 3*sqrt(sqrt(2) + 2)*(sq
rt(2) - 2) + 3*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) - (-sqrt(2) + 2)^(3/2))) +
1/16*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*arctan((8*sqr
t(2)*x - (sqrt(2) + 2)^(3/2) - 3*sqrt(sqrt(2) + 2)*(sqrt(2) - 2) + 3*(sqrt(
2) + 2)*sqrt(-sqrt(2) + 2) - (-sqrt(2) + 2)^(3/2) - 8*sqrt(2)*sqrt(-1/8*sqr
t(2)*x*(sqrt(2) + 2)^(3/2) + 1/64*(sqrt(2) + 2)^3 + 3/64*(sqrt(2) + 2)*(sqr
t(2) - 2)^2 - 1/64*(sqrt(2) - 2)^3 + 3/8*sqrt(2)*x*(sqrt(2) + 2)*sqrt(-sqrt
(2) + 2) - 1/8*sqrt(2)*x*(-sqrt(2) + 2)^(3/2) + x^2 - 3/64*(8*sqrt(2)*x*sqr
t(sqrt(2) + 2) + (sqrt(2) + 2)^2)*(sqrt(2) - 2)))/((sqrt(2) + 2)^(3/2) + 3*
sqrt(sqrt(2) + 2)*(sqrt(2) - 2) + 3*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) - (-sq
rt(2) + 2)^(3/2))) + 1/16*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-sqrt(2
) + 2))*arctan(-(8*sqrt(2)*x + (sqrt(2) + 2)^(3/2) + 3*sqrt(sqrt(2) + 2)*(s
qrt(2) - 2) + 3*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) - (-sqrt(2) + 2)^(3/2) - s
qrt(2)*sqrt(8*sqrt(2)*x*(sqrt(2) + 2)^(3/2) + (sqrt(2) + 2)^3 + 3*(sqrt(2)
+ 2)*(sqrt(2) - 2)^2 - (sqrt(2) - 2)^3 + 24*sqrt(2)*x*(sqrt(2) + 2)*sqrt(-s
qrt(2) + 2) - 8*sqrt(2)*x*(-sqrt(2) + 2)^(3/2) + 64*x^2 + 3*(8*sqrt(2)*x*sq
rt(sqrt(2) + 2) - (sqrt(2) + 2)^2)*(sqrt(2) - 2)))/((sqrt(2) + 2)^(3/2) + 3
*sqrt(sqrt(2) + 2)*(sqrt(2) - 2) - 3*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) + (-s
qrt(2) + 2)^(3/2))) + 1/16*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-sqrt(
2) + 2))*arctan(-(8*sqrt(2)*x - (sqrt(2) + 2)^(3/2) - 3*sqrt(sqrt(2) + 2)*(
sqrt(2) - 2) - 3*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) + (-sqrt(2) + 2)^(3/2) -
8*sqrt(2)*sqrt(-1/8*sqrt(2)*x*(sqrt(2) + 2)^(3/2) + 1/64*(sqrt(2) + 2)^3 +
3/64*(sqrt(2) + 2)*(sqrt(2) - 2)^2 - 1/64*(sqrt(2) - 2)^3 - 3/8*sqrt(2)*x*(
sqrt(2) + 2)*sqrt(-sqrt(2) + 2) + 1/8*sqrt(2)*x*(-sqrt(2) + 2)^(3/2) + x^2
- 3/64*(8*sqrt(2)*x*sqrt(sqrt(2) + 2) + (sqrt(2) + 2)^2)*(sqrt(2) - 2)))/((
sqrt(2) + 2)^(3/2) + 3*sqrt(sqrt(2) + 2)*(sqrt(2) - 2) - 3*(sqrt(2) + 2)*sq
rt(-sqrt(2) + 2) + (-sqrt(2) + 2)^(3/2))) - 1/64*(sqrt(2)*sqrt(sqrt(2) + 2)
 + sqrt(2)*sqrt(-sqrt(2) + 2))*log(1/8*sqrt(2)*x*(sqrt(2) + 2)^(3/2) + 1/64
*(sqrt(2) + 2)^3 + 3/64*(sqrt(2) + 2)*(sqrt(2) - 2)^2 - 1/64*(sqrt(2) - 2)^
3 + 3/8*sqrt(2)*x*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) - 1/8*sqrt(2)*x*(-sqrt(2
) + 2)^(3/2) + x^2 + 3/64*(8*sqrt(2)*x*sqrt(sqrt(2) + 2) - (sqrt(2) + 2)^2)
*(sqrt(2) - 2)) + 1/64*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-sqrt(2) +
 2))*log(1/8*sqrt(2)*x*(sqrt(2) + 2)^(3/2) + 1/64*(sqrt(2) + 2)^3 + 3/64*(s
qrt(2) + 2)*(sqrt(2) - 2)^2 - 1/64*(sqrt(2) - 2)^3 - 3/8*sqrt(2)*x*(sqrt(2)
 + 2)*sqrt(-sqrt(2) + 2) + 1/8*sqrt(2)*x*(-sqrt(2) + 2)^(3/2) + x^2 + 3/64*
(8*sqrt(2)*x*sqrt(sqrt(2) + 2) - (sqrt(2) + 2)^2)*(sqrt(2) - 2)) - 1/64*(sq
```

```
rt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*log(-1/8*sqrt(2)*x*(s
qrt(2) + 2)^(3/2) + 1/64*(sqrt(2) + 2)^3 + 3/64*(sqrt(2) + 2)*(sqrt(2) - 2)
^2 - 1/64*(sqrt(2) - 2)^3 + 3/8*sqrt(2)*x*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2)
- 1/8*sqrt(2)*x*(-sqrt(2) + 2)^(3/2) + x^2 - 3/64*(8*sqrt(2)*x*sqrt(sqrt(2)
 + 2) + (sqrt(2) + 2)^2)*(sqrt(2) - 2)) + 1/64*(sqrt(2)*sqrt(sqrt(2) + 2) +
 sqrt(2)*sqrt(-sqrt(2) + 2))*log(-1/8*sqrt(2)*x*(sqrt(2) + 2)^(3/2) + 1/64*
(sqrt(2) + 2)^3 + 3/64*(sqrt(2) + 2)*(sqrt(2) - 2)^2 - 1/64*(sqrt(2) - 2)^3
 - 3/8*sqrt(2)*x*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) + 1/8*sqrt(2)*x*(-sqrt(2)
 + 2)^(3/2) + x^2 - 3/64*(8*sqrt(2)*x*sqrt(sqrt(2) + 2) + (sqrt(2) + 2)^2)*
(sqrt(2) - 2)) + 1/8*sqrt(-sqrt(2) + 2)*arctan((3*(sqrt(2) + 2)*sqrt(-sqrt(
2) + 2) - (-sqrt(2) + 2)^(3/2) + 8*x - sqrt((sqrt(2) + 2)^3 - 3*(sqrt(2) +
2)^2*(sqrt(2) - 2) + 3*(sqrt(2) + 2)*(sqrt(2) - 2)^2 - (sqrt(2) - 2)^3 + 48
*x*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) - 16*x*(-sqrt(2) + 2)^(3/2) + 64*x^2))/
((sqrt(2) + 2)^(3/2) + 3*sqrt(sqrt(2) + 2)*(sqrt(2) - 2))) + 1/8*sqrt(-sqrt
(2) + 2)*arctan(-(3*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) - (-sqrt(2) + 2)^(3/2)
 - 8*x + sqrt((sqrt(2) + 2)^3 - 3*(sqrt(2) + 2)^2*(sqrt(2) - 2) + 3*(sqrt(2
) + 2)*(sqrt(2) - 2)^2 - (sqrt(2) - 2)^3 - 48*x*(sqrt(2) + 2)*sqrt(-sqrt(2)
 + 2) + 16*x*(-sqrt(2) + 2)^(3/2) + 64*x^2))/((sqrt(2) + 2)^(3/2) + 3*sqrt(
sqrt(2) + 2)*(sqrt(2) - 2))) + 1/8*sqrt(sqrt(2) + 2)*arctan(((sqrt(2) + 2)^
(3/2) + 3*sqrt(sqrt(2) + 2)*(sqrt(2) - 2) + 8*x - sqrt((sqrt(2) + 2)^3 + 3*
(sqrt(2) + 2)*(sqrt(2) - 2)^2 - (sqrt(2) - 2)^3 + 16*x*(sqrt(2) + 2)^(3/2)
+ 64*x^2 - 3*((sqrt(2) + 2)^2 - 16*x*sqrt(sqrt(2) + 2))*(sqrt(2) - 2)))/(3*
(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) - (-sqrt(2) + 2)^(3/2))) + 1/8*sqrt(sqrt(2
) + 2)*arctan(-((sqrt(2) + 2)^(3/2) + 3*sqrt(sqrt(2) + 2)*(sqrt(2) - 2) - 8
*x + sqrt((sqrt(2) + 2)^3 + 3*(sqrt(2) + 2)*(sqrt(2) - 2)^2 - (sqrt(2) - 2)
^3 - 16*x*(sqrt(2) + 2)^(3/2) + 64*x^2 - 3*((sqrt(2) + 2)^2 + 16*x*sqrt(sqr
t(2) + 2))*(sqrt(2) - 2)))/(3*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) - (-sqrt(2)
+ 2)^(3/2))) - 1/32*sqrt(sqrt(2) + 2)*log(1/64*(sqrt(2) + 2)^3 - 3/64*(sqrt
(2) + 2)^2*(sqrt(2) - 2) + 3/64*(sqrt(2) + 2)*(sqrt(2) - 2)^2 - 1/64*(sqrt(
2) - 2)^3 + 3/4*x*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) - 1/4*x*(-sqrt(2) + 2)^(
3/2) + x^2) + 1/32*sqrt(sqrt(2) + 2)*log(1/64*(sqrt(2) + 2)^3 - 3/64*(sqrt(
2) + 2)^2*(sqrt(2) - 2) + 3/64*(sqrt(2) + 2)*(sqrt(2) - 2)^2 - 1/64*(sqrt(2
) - 2)^3 - 3/4*x*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) + 1/4*x*(-sqrt(2) + 2)^(3
/2) + x^2) - 1/32*sqrt(-sqrt(2) + 2)*log(1/64*(sqrt(2) + 2)^3 + 3/64*(sqrt(
2) + 2)*(sqrt(2) - 2)^2 - 1/64*(sqrt(2) - 2)^3 + 1/4*x*(sqrt(2) + 2)^(3/2)
+ x^2 - 3/64*((sqrt(2) + 2)^2 - 16*x*sqrt(sqrt(2) + 2))*(sqrt(2) - 2)) + 1/
32*sqrt(-sqrt(2) + 2)*log(1/64*(sqrt(2) + 2)^3 + 3/64*(sqrt(2) + 2)*(sqrt(2
) - 2)^2 - 1/64*(sqrt(2) - 2)^3 - 1/4*x*(sqrt(2) + 2)^(3/2) + x^2 - 3/64*((
sqrt(2) + 2)^2 + 16*x*sqrt(sqrt(2) + 2))*(sqrt(2) - 2))
```

giac [A] time = 0.31, size = 239, normalized size = 0.71

$$\frac{1}{8}\sqrt{\sqrt{2}+2}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)+\frac{1}{8}\sqrt{\sqrt{2}+2}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)+\frac{1}{8}\sqrt{-\sqrt{2}+2}\arctan\left(\frac{2x-}{\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+1),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(sqrt(2) + 2)*arctan((2*x + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2))
+ 1/8*sqrt(sqrt(2) + 2)*arctan((2*x - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)
) + 1/8*sqrt(-sqrt(2) + 2)*arctan((2*x + sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) +
 2)) + 1/8*sqrt(-sqrt(2) + 2)*arctan((2*x - sqrt(sqrt(2) + 2))/sqrt(-sqrt(2
) + 2)) - 1/16*sqrt(sqrt(2) + 2)*log(x^2 + x*sqrt(sqrt(2) + 2) + 1) + 1/16*
sqrt(sqrt(2) + 2)*log(x^2 - x*sqrt(sqrt(2) + 2) + 1) - 1/16*sqrt(-sqrt(2) +
 2)*log(x^2 + x*sqrt(-sqrt(2) + 2) + 1) + 1/16*sqrt(-sqrt(2) + 2)*log(x^2 -
 x*sqrt(-sqrt(2) + 2) + 1)
```

maple [C] time = 0.01, size = 22, normalized size = 0.06

$$\frac{\ln\left(-\text{RootOf}\left(_Z^8+1\right)+x\right)}{8\,\text{RootOf}\left(_Z^8+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^8+1),x)
```

```
[Out] 1/8*sum(1/_R*ln(-_R+x),_R=RootOf(_Z^8+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{x^8+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+1),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(x^8 + 1), x)
```

mupad [B] time = 1.08, size = 256, normalized size = 0.76

$$\text{atan}\left(-\frac{x\,1\text{i}}{\sqrt{\sqrt{2}-2}}+\frac{x\,1\text{i}}{\sqrt{\sqrt{2}+2}}+\frac{\sqrt{2}\,x\,1\text{i}}{2\sqrt{\sqrt{2}-2}}+\frac{\sqrt{2}\,x\,1\text{i}}{2\sqrt{\sqrt{2}+2}}\right)\left(\frac{\sqrt{\sqrt{2}-2}\,1\text{i}}{8}+\frac{\sqrt{\sqrt{2}+2}\,1\text{i}}{8}\right)-\text{atan}\left(\frac{x\,1\text{i}}{\sqrt{-\sqrt{2}-2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^8 + 1),x)
```

```
[Out] atan((x*1i)/(2^(1/2) + 2)^(1/2) - (x*1i)/(2^(1/2) - 2)^(1/2) + (2^(1/2)*x*1
i)/(2*(2^(1/2) - 2)^(1/2)) + (2^(1/2)*x*1i)/(2*(2^(1/2) + 2)^(1/2)))*(((2^(
1/2) - 2)^(1/2)*1i)/8 + ((2^(1/2) + 2)^(1/2)*1i)/8) - atan((x*1i)/(- 2^(1/2
) - 2)^(1/2) + (x*1i)/(2 - 2^(1/2))^(1/2) + (2^(1/2)*x*1i)/(2*(- 2^(1/2) -
2)^(1/2)) - (2^(1/2)*x*1i)/(2*(2 - 2^(1/2))^(1/2)))*(((- 2^(1/2) - 2)^(1/2)
*1i)/8 - ((2 - 2^(1/2))^(1/2)*1i)/8) + atan(x*(2^(1/2) + 2)^(1/2)*(1/2 - 1i
/2) - (2^(1/2)*x*(2^(1/2) + 2)^(1/2))/2)*((2^(1/2)*1i)/16 - (1/16 + 1i/16))
*(2^(1/2) + 2)^(1/2)*2i - atan(x*(2^(1/2) + 2)^(1/2)*(1/2 + 1i/2) - (2^(1/2
)*x*(2^(1/2) + 2)^(1/2)*1i)/2)*(2^(1/2)/16 - (1/16 - 1i/16))*(2^(1/2) + 2)^
(1/2)*2i
```

sympy [A] time = 2.96, size = 15, normalized size = 0.04

$$\text{RootSum}\left(16777216t^8+1,\left(t\mapsto t\log\left(2097152t^7+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**8+1),x)
```

```
[Out] RootSum(16777216*_t**8 + 1, Lambda(_t, _t*log(2097152*_t**7 + x)))
```

3.1502 $\int \frac{x^4}{1+x^8}\,dx$

Optimal. Leaf size=347

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{2}}\,x+1\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{2}}\,x+1\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}}+\frac{\log\left(x^2-\sqrt{2+\sqrt{2}}\,x+1\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}}-\frac{\log\left(x^2+\sqrt{2+\sqrt{2}}\,x+\right.}{8\sqrt{2\left(2+\sqrt{2}\right)}}$$

```
[Out] -1/4*arctan((-2*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(4-2*2^(1/2))^(1/2)
+1/4*arctan((2*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(4-2*2^(1/2))^(1/2)-
1/8*ln(1+x^2-x*(2-2^(1/2))^(1/2))/(4-2*2^(1/2))^(1/2)+1/8*ln(1+x^2+x*(2-2^(
1/2))^(1/2))/(4-2*2^(1/2))^(1/2)+1/4*arctan((-2*x+(2-2^(1/2))^(1/2))/(2+2^(
1/2))^(1/2))/(4+2*2^(1/2))^(1/2)-1/4*arctan((2*x+(2-2^(1/2))^(1/2))/(2+2^(1
/2))^(1/2))/(4+2*2^(1/2))^(1/2)+1/8*ln(1+x^2-x*(2+2^(1/2))^(1/2))/(4+2*2^(1
/2))^(1/2)-1/8*ln(1+x^2+x*(2+2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {299, 1127, 1161, 618, 204, 1164, 628}

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{2}}\,x+1\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{2}}\,x+1\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}}+\frac{\log\left(x^2-\sqrt{2+\sqrt{2}}\,x+1\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}}-\frac{\log\left(x^2+\sqrt{2+\sqrt{2}}\,x+\right.}{8\sqrt{2\left(2+\sqrt{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 + x^8),x]
```

```
[Out] ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 + Sqrt[2]]]/(4*Sqrt[2*(2 + Sqrt[2])
]) - ArcTan[(Sqrt[2 + Sqrt[2]] - 2*x)/Sqrt[2 - Sqrt[2]]]/(4*Sqrt[2*(2 - Sqr
t[2])]) - ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2 + Sqrt[2]]]/(4*Sqrt[2*(2
 + Sqrt[2])]) + ArcTan[(Sqrt[2 + Sqrt[2]] + 2*x)/Sqrt[2 - Sqrt[2]]]/(4*Sqrt[
2*(2 - Sqrt[2])]) - Log[1 - Sqrt[2 - Sqrt[2]]*x + x^2]/(8*Sqrt[2*(2 - Sqrt[
2])]) + Log[1 + Sqrt[2 - Sqrt[2]]*x + x^2]/(8*Sqrt[2*(2 - Sqrt[2])]) + Log[
1 - Sqrt[2 + Sqrt[2]]*x + x^2]/(8*Sqrt[2*(2 + Sqrt[2])]) - Log[1 + Sqrt[2 +
 Sqrt[2]]*x + x^2]/(8*Sqrt[2*(2 + Sqrt[2])])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 299

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[s^3/(2*Sqrt[2]*b*r), Int[x^(
m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Dist[s^3/(2*Sq
rt[2]*b*r), Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && G
tQ[a/b, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^4}{1+x^8}\,dx = \frac{\int \frac{x^2}{1-\sqrt{2}\,x^2+x^4}\,dx}{2\sqrt{2}} - \frac{\int \frac{x^2}{1+\sqrt{2}\,x^2+x^4}\,dx}{2\sqrt{2}}$$

$$= -\frac{\int \frac{1-x^2}{1-\sqrt{2}\,x^2+x^4}\,dx}{4\sqrt{2}} + \frac{\int \frac{1+x^2}{1-\sqrt{2}\,x^2+x^4}\,dx}{4\sqrt{2}} + \frac{\int \frac{1-x^2}{1+\sqrt{2}\,x^2+x^4}\,dx}{4\sqrt{2}} - \frac{\int \frac{1+x^2}{1+\sqrt{2}\,x^2+x^4}\,dx}{4\sqrt{2}}$$

$$= -\frac{\int \frac{1}{1-\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{8\sqrt{2}} - \frac{\int \frac{1}{1+\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{8\sqrt{2}} + \frac{\int \frac{1}{1-\sqrt{2+\sqrt{2}}\,x+x^2}\,dx}{8\sqrt{2}} + \frac{\int \frac{1}{1+\sqrt{2+\sqrt{2}}\,x+x^2}\,dx}{8\sqrt{2}} - \frac{\int \frac{\sqrt{2-\sqrt{}}}{-1-\sqrt{2-}}}{8\sqrt{2\left(2\right.}}$$

$$= -\frac{\log\left(1-\sqrt{2-\sqrt{2}}\,x+x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}} + \frac{\log\left(1+\sqrt{2-\sqrt{2}}\,x+x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}} + \frac{\log\left(1-\sqrt{2+\sqrt{2}}\,x+x^2\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}} - \frac{\log\left(1-\right.}{8}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2\left(2+\sqrt{2}\right)}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2\left(2-\sqrt{2}\right)}} - \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}+2x}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2\left(2+\sqrt{2}\right)}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}+2x}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2\left(2-\sqrt{2}\right)}} - \frac{\log\left(1-\sqrt{\right.}}{8\sqrt{2}}$$

Mathematica [A] time = 0.01, size = 209, normalized size = 0.60

$$-\frac{1}{8}\cos\left(\frac{\pi}{8}\right)\log\left(x^2-2x\sin\left(\frac{\pi}{8}\right)+1\right)+\frac{1}{8}\cos\left(\frac{\pi}{8}\right)\log\left(x^2+2x\sin\left(\frac{\pi}{8}\right)+1\right)+\frac{1}{8}\sin\left(\frac{\pi}{8}\right)\log\left(x^2-2x\cos\left(\frac{\pi}{8}\right)+1\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 + x^8),x]
```

```
[Out] (ArcTan[(x - Cos[Pi/8])*Csc[Pi/8]]*Cos[Pi/8])/4 + (ArcTan[(x + Cos[Pi/8])*C
sc[Pi/8]]*Cos[Pi/8])/4 - (Cos[Pi/8]*Log[1 + x^2 - 2*x*Sin[Pi/8]])/8 + (Cos[
Pi/8]*Log[1 + x^2 + 2*x*Sin[Pi/8]])/8 - (ArcTan[Sec[Pi/8]*(x - Sin[Pi/8])]*
Sin[Pi/8])/4 - (ArcTan[Sec[Pi/8]*(x + Sin[Pi/8])]*Sin[Pi/8])/4 + (Log[1 + x
^2 - 2*x*Cos[Pi/8]]*Sin[Pi/8])/8 - (Log[1 + x^2 + 2*x*Cos[Pi/8]]*Sin[Pi/8])
/8
```

fricas **[B]** time = 0.88, size = 1027, normalized size = 2.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+1),x, algorithm="fricas")
```

```
[Out] 1/16*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*arctan(-(2*sq
rt(2)*x - 2*sqrt(2)*sqrt(x^2 + 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) - 1/2*sqrt(2
)*x*sqrt(-sqrt(2) + 2) + 1) + sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(sqrt
(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))) + 1/16*(sqrt(2)*sqrt(sqrt(2) + 2) - sq
rt(2)*sqrt(-sqrt(2) + 2))*arctan(-(2*sqrt(2)*x - 2*sqrt(2)*sqrt(x^2 - 1/2*s
qrt(2)*x*sqrt(sqrt(2) + 2) + 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) - sqrt(s
qrt(2) + 2) + sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2)))
 + 1/16*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*arctan((2*
sqrt(2)*x - 2*sqrt(2)*sqrt(x^2 + 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) + 1/2*sqrt
(2)*x*sqrt(-sqrt(2) + 2) + 1) + sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))/(sq
rt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))) + 1/16*(sqrt(2)*sqrt(sqrt(2) + 2) +
sqrt(2)*sqrt(-sqrt(2) + 2))*arctan((2*sqrt(2)*x - 2*sqrt(2)*sqrt(x^2 - 1/2*
sqrt(2)*x*sqrt(sqrt(2) + 2) - 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) - sqrt(
sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))
) - 1/64*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*log(x^2 +
 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) + 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) +
1/64*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*log(x^2 + 1/2
*sqrt(2)*x*sqrt(sqrt(2) + 2) - 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) - 1/64
*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*log(x^2 - 1/2*sqr
t(2)*x*sqrt(sqrt(2) + 2) + 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) + 1/64*(sq
rt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*log(x^2 - 1/2*sqrt(2)
*x*sqrt(sqrt(2) + 2) - 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) + 1/8*sqrt(-sq
rt(2) + 2)*arctan(-(2*x - 2*sqrt(x^2 + x*sqrt(-sqrt(2) + 2) + 1) + sqrt(-sq
rt(2) + 2))/sqrt(sqrt(2) + 2)) + 1/8*sqrt(-sqrt(2) + 2)*arctan(-(2*x - 2*sq
rt(x^2 - x*sqrt(-sqrt(2) + 2) + 1) - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2))
 - 1/8*sqrt(sqrt(2) + 2)*arctan(-(2*x - 2*sqrt(x^2 + x*sqrt(sqrt(2) + 2) +
1) + sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) - 1/8*sqrt(sqrt(2) + 2)*arctan(
-(2*x - 2*sqrt(x^2 - x*sqrt(sqrt(2) + 2) + 1) - sqrt(sqrt(2) + 2))/sqrt(-sq
rt(2) + 2)) - 1/32*sqrt(-sqrt(2) + 2)*log(x^2 + x*sqrt(sqrt(2) + 2) + 1) +
1/32*sqrt(-sqrt(2) + 2)*log(x^2 - x*sqrt(sqrt(2) + 2) + 1) + 1/32*sqrt(sqrt
(2) + 2)*log(x^2 + x*sqrt(-sqrt(2) + 2) + 1) - 1/32*sqrt(sqrt(2) + 2)*log(x
^2 - x*sqrt(-sqrt(2) + 2) + 1)
```

giac **[A]** time = 0.31, size = 239, normalized size = 0.69

$$-\frac{1}{8}\sqrt{-\sqrt{2}+2}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)-\frac{1}{8}\sqrt{-\sqrt{2}+2}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)+\frac{1}{8}\sqrt{\sqrt{2}+2}\arctan\left($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+1),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(-sqrt(2) + 2)*arctan((2*x + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)
) - 1/8*sqrt(-sqrt(2) + 2)*arctan((2*x - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) +
 2)) + 1/8*sqrt(sqrt(2) + 2)*arctan((2*x + sqrt(sqrt(2) + 2))/sqrt(-sqrt(2)
 + 2)) + 1/8*sqrt(sqrt(2) + 2)*arctan((2*x - sqrt(sqrt(2) + 2))/sqrt(-sqrt(
2) + 2)) - 1/16*sqrt(-sqrt(2) + 2)*log(x^2 + x*sqrt(sqrt(2) + 2) + 1) + 1/1
6*sqrt(-sqrt(2) + 2)*log(x^2 - x*sqrt(sqrt(2) + 2) + 1) + 1/16*sqrt(sqrt(2)
 + 2)*log(x^2 + x*sqrt(-sqrt(2) + 2) + 1) - 1/16*sqrt(sqrt(2) + 2)*log(x^2
- x*sqrt(-sqrt(2) + 2) + 1)
```

maple [C] time = 0.00, size = 22, normalized size = 0.06

$$\frac{\ln\left(-\operatorname{RootOf}\left(_Z^8+1\right)+x\right)}{8\operatorname{RootOf}\left(_Z^8+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^8+1),x)
```

```
[Out] 1/8*sum(1/_R^3*ln(-_R+x),_R=RootOf(_Z^8+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{x^8+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+1),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(x^8 + 1), x)
```

mupad [B] time = 1.08, size = 295, normalized size = 0.85

$$-\operatorname{atan}\left(\frac{x\sqrt{-\sqrt{2}-2}\,1\mathrm{i}}{\sqrt{2-\sqrt{2}}\,\sqrt{-\sqrt{2}-2}-\sqrt{2}}-\frac{x\sqrt{2-\sqrt{2}}\,1\mathrm{i}}{\sqrt{2-\sqrt{2}}\,\sqrt{-\sqrt{2}-2}-\sqrt{2}}\right)\left(\frac{\sqrt{-\sqrt{2}-2}\,1\mathrm{i}}{8}-\frac{\sqrt{2-\sqrt{2}}\,1\mathrm{i}}{8}\right)-\operatorname{atan}\left(\frac{}{\sqrt{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^8 + 1),x)
```

```
[Out] atan(x*(2^(1/2) + 2)^(1/2)*(1/2 + 1i/2) - (2^(1/2)*x*(2^(1/2) + 2)^(1/2))/2
)*(2^(1/2)/16 - (1/16 - 1i/16))*(2^(1/2) + 2)^(1/2)*2i - atan((x*(2^(1/2) -
 2)^(1/2)*1i)/(2^(1/2) - (2^(1/2) - 2)^(1/2)*(2^(1/2) + 2)^(1/2)) + (x*(2^(
1/2) + 2)^(1/2)*1i)/(2^(1/2) - (2^(1/2) - 2)^(1/2)*(2^(1/2) + 2)^(1/2)))*((
(2^(1/2) - 2)^(1/2)*1i)/8 + ((2^(1/2) + 2)^(1/2)*1i)/8) - atan((x*(- 2^(1/2
) - 2)^(1/2)*1i)/((2 - 2^(1/2))^(1/2)*(- 2^(1/2) - 2)^(1/2) - 2^(1/2)) - (x
*(2 - 2^(1/2))^(1/2)*1i)/((2 - 2^(1/2))^(1/2)*(- 2^(1/2) - 2)^(1/2) - 2^(1/
2)))*(((- 2^(1/2) - 2)^(1/2)*1i)/8 - ((2 - 2^(1/2))^(1/2)*1i)/8) + atan(x*(
2^(1/2) + 2)^(1/2)*(1/2 - 1i/2) + (2^(1/2)*x*(2^(1/2) + 2)^(1/2)*1i)/2)*((2
^(1/2)*1i)/16 - (1/16 + 1i/16))*(2^(1/2) + 2)^(1/2)*2i
```

sympy [A] time = 2.99, size = 15, normalized size = 0.04

$$\operatorname{RootSum}\left(16777216t^8+1,\left(t\mapsto t\log\left(-32768t^5+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**8+1),x)
```

```
[Out] RootSum(16777216*_t**8 + 1, Lambda(_t, _t*log(-32768*_t**5 + x)))
```

3.1503 $\int \frac{x^2}{1+x^8}\,dx$

Optimal. Leaf size=339

$$\frac{\log\left(x^2-\sqrt{2-\sqrt{2}}\,x+1\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\log\left(x^2+\sqrt{2-\sqrt{2}}\,x+1\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{2}}\,x+1\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{2}}\,x+1\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}}$$

```
[Out] 1/8*arctan((-2*x+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)-1/
8*arctan((2*x+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)+1/8*l
n(1+x^2-x*(2-2^(1/2))^(1/2))/(4-2*2^(1/2))^(1/2)-1/8*ln(1+x^2+x*(2-2^(1/2))
^(1/2))/(4-2*2^(1/2))^(1/2)-1/8*arctan((-2*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))
^(1/2))*(2+2^(1/2))^(1/2)+1/8*arctan((2*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1
/2))*(2+2^(1/2))^(1/2)-1/8*ln(1+x^2-x*(2+2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2
)+1/8*ln(1+x^2+x*(2+2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {299, 1094, 634, 618, 204, 628}

$$\frac{\log\left(x^2-\sqrt{2-\sqrt{2}}\,x+1\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\log\left(x^2+\sqrt{2-\sqrt{2}}\,x+1\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{2}}\,x+1\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{2}}\,x+1\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + x^8),x]
```

```
[Out] (Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 + Sqrt[2]]])/8 -
 (Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] - 2*x)/Sqrt[2 - Sqrt[2]]])/8
 - (Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2 + Sqrt[2]]])/8
 + (Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] + 2*x)/Sqrt[2 - Sqrt[2]]])/
8 + Log[1 - Sqrt[2 - Sqrt[2]]*x + x^2]/(8*Sqrt[2*(2 - Sqrt[2])]) - Log[1 +
Sqrt[2 - Sqrt[2]]*x + x^2]/(8*Sqrt[2*(2 - Sqrt[2])]) - Log[1 - Sqrt[2 + Sqr
t[2]]*x + x^2]/(8*Sqrt[2*(2 + Sqrt[2])]) + Log[1 + Sqrt[2 + Sqrt[2]]*x + x^
2]/(8*Sqrt[2*(2 + Sqrt[2])])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 299

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[s^3/(2*Sqrt[2]*b*r), Int[x^(
m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Dist[s^3/(2*Sq
rt[2]*b*r), Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && G
tQ[a/b, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^2}{1+x^8}\,dx = \frac{\int \frac{1}{1-\sqrt{2}x^2+x^4}\,dx}{2\sqrt{2}} - \frac{\int \frac{1}{1+\sqrt{2}x^2+x^4}\,dx}{2\sqrt{2}}$$

$$= -\frac{\int \frac{\sqrt{2-\sqrt{2}}-x}{1-\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2\left(2-\sqrt{2}\right)}} - \frac{\int \frac{\sqrt{2-\sqrt{2}}+x}{1+\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2\left(2-\sqrt{2}\right)}} + \frac{\int \frac{\sqrt{2+\sqrt{2}}-x}{1-\sqrt{2+\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2\left(2+\sqrt{2}\right)}} + \frac{\int \frac{\sqrt{2+\sqrt{2}}+x}{1+\sqrt{2+\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2\left(2+\sqrt{2}\right)}}$$

$$= -\frac{\int \frac{1}{1-\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{8\sqrt{2}} - \frac{\int \frac{1}{1+\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{8\sqrt{2}} + \frac{\int \frac{1}{1-\sqrt{2+\sqrt{2}}\,x+x^2}\,dx}{8\sqrt{2}} + \frac{\int \frac{1}{1+\sqrt{2+\sqrt{2}}\,x+x^2}\,dx}{8\sqrt{2}} + \frac{\int \frac{-\sqrt{2-\sqrt{}}}{1-\sqrt{2-\sqrt{}}}}{8\sqrt{2\left(2\right.}}$$

$$= \frac{\log\left(1-\sqrt{2-\sqrt{2}}\,x+x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}} - \frac{\log\left(1+\sqrt{2-\sqrt{2}}\,x+x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}} - \frac{\log\left(1-\sqrt{2+\sqrt{2}}\,x+x^2\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}} + \frac{\log\left(1+\right.}{8\sqrt{}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2\left(2+\sqrt{2}\right)}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2\left(2-\sqrt{2}\right)}} - \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}+2x}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2\left(2+\sqrt{2}\right)}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}+2x}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2\left(2-\sqrt{2}\right)}} + \frac{\log\left(1-\sqrt{}\right.}{8\sqrt{2}}$$

Mathematica [A] time = 0.01, size = 209, normalized size = 0.62

$$\frac{1}{8}\cos\left(\frac{\pi}{8}\right)\log\left(x^2-2x\sin\left(\frac{\pi}{8}\right)+1\right)-\frac{1}{8}\cos\left(\frac{\pi}{8}\right)\log\left(x^2+2x\sin\left(\frac{\pi}{8}\right)+1\right)-\frac{1}{8}\sin\left(\frac{\pi}{8}\right)\log\left(x^2-2x\cos\left(\frac{\pi}{8}\right)+1\right)-$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + x^8),x]
```

```
[Out] (ArcTan[(x - Cos[Pi/8])*Csc[Pi/8]]*Cos[Pi/8])/4 + (ArcTan[(x + Cos[Pi/8])*C
sc[Pi/8]]*Cos[Pi/8])/4 + (Cos[Pi/8]*Log[1 + x^2 - 2*x*Sin[Pi/8]])/8 - (Cos[
Pi/8]*Log[1 + x^2 + 2*x*Sin[Pi/8]])/8 - (ArcTan[Sec[Pi/8]*(x - Sin[Pi/8])]*
Sin[Pi/8])/4 - (ArcTan[Sec[Pi/8]*(x + Sin[Pi/8])]*Sin[Pi/8])/4 - (Log[1 + x
```

```
^2 - 2*x*Cos[Pi/8]]*Sin[Pi/8])/8 + (Log[1 + x^2 + 2*x*Cos[Pi/8]]*Sin[Pi/8])
/8
```

fricas [**B**] time = 0.56, size = 2513, normalized size = 7.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+1),x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*arctan((8*sq
rt(2)*x + (sqrt(2) + 2)^(3/2) + 3*sqrt(sqrt(2) + 2)*(sqrt(2) - 2) - 3*(sqrt
(2) + 2)*sqrt(-sqrt(2) + 2) + (-sqrt(2) + 2)^(3/2) - sqrt(2)*sqrt(8*sqrt(2)
*x*(sqrt(2) + 2)^(3/2) + (sqrt(2) + 2)^3 + 3*(sqrt(2) + 2)*(sqrt(2) - 2)^2
 - (sqrt(2) - 2)^3 - 24*sqrt(2)*x*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) + 8*sqrt(
2)*x*(-sqrt(2) + 2)^(3/2) + 64*x^2 + 3*(8*sqrt(2)*x*sqrt(sqrt(2) + 2) - (sq
rt(2) + 2)^2)*(sqrt(2) - 2)))/((sqrt(2) + 2)^(3/2) + 3*sqrt(sqrt(2) + 2)*(s
qrt(2) - 2) + 3*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) - (-sqrt(2) + 2)^(3/2))) -
 1/16*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*arctan((8*sq
rt(2)*x - (sqrt(2) + 2)^(3/2) - 3*sqrt(sqrt(2) + 2)*(sqrt(2) - 2) + 3*(sqrt
(2) + 2)*sqrt(-sqrt(2) + 2) - (-sqrt(2) + 2)^(3/2) - 8*sqrt(2)*sqrt(-1/8*sq
rt(2)*x*(sqrt(2) + 2)^(3/2) + 1/64*(sqrt(2) + 2)^3 + 3/64*(sqrt(2) + 2)*(sq
rt(2) - 2)^2 - 1/64*(sqrt(2) - 2)^3 + 3/8*sqrt(2)*x*(sqrt(2) + 2)*sqrt(-sqr
t(2) + 2) - 1/8*sqrt(2)*x*(-sqrt(2) + 2)^(3/2) + x^2 - 3/64*(8*sqrt(2)*x*sq
rt(sqrt(2) + 2) + (sqrt(2) + 2)^2)*(sqrt(2) - 2)))/((sqrt(2) + 2)^(3/2) + 3
*sqrt(sqrt(2) + 2)*(sqrt(2) - 2) + 3*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) - (-s
qrt(2) + 2)^(3/2))) + 1/16*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(
2) + 2))*arctan(-(8*sqrt(2)*x + (sqrt(2) + 2)^(3/2) + 3*sqrt(sqrt(2) + 2)*(
sqrt(2) - 2) + 3*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) - (-sqrt(2) + 2)^(3/2) -
sqrt(2)*sqrt(8*sqrt(2)*x*(sqrt(2) + 2)^(3/2) + (sqrt(2) + 2)^3 + 3*(sqrt(2)
 + 2)*(sqrt(2) - 2)^2 - (sqrt(2) - 2)^3 + 24*sqrt(2)*x*(sqrt(2) + 2)*sqrt(-
sqrt(2) + 2) - 8*sqrt(2)*x*(-sqrt(2) + 2)^(3/2) + 64*x^2 + 3*(8*sqrt(2)*x*s
qrt(sqrt(2) + 2) - (sqrt(2) + 2)^2)*(sqrt(2) - 2)))/((sqrt(2) + 2)^(3/2) +
3*sqrt(sqrt(2) + 2)*(sqrt(2) - 2) - 3*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) + (-
sqrt(2) + 2)^(3/2))) + 1/16*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt
(2) + 2))*arctan(-(8*sqrt(2)*x - (sqrt(2) + 2)^(3/2) - 3*sqrt(sqrt(2) + 2)*
(sqrt(2) - 2) - 3*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) + (-sqrt(2) + 2)^(3/2) -
 8*sqrt(2)*sqrt(-1/8*sqrt(2)*x*(sqrt(2) + 2)^(3/2) + 1/64*(sqrt(2) + 2)^3 +
 3/64*(sqrt(2) + 2)*(sqrt(2) - 2)^2 - 1/64*(sqrt(2) - 2)^3 - 3/8*sqrt(2)*x*
(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) + 1/8*sqrt(2)*x*(-sqrt(2) + 2)^(3/2) + x^2
 - 3/64*(8*sqrt(2)*x*sqrt(sqrt(2) + 2) + (sqrt(2) + 2)^2)*(sqrt(2) - 2)))/(
(sqrt(2) + 2)^(3/2) + 3*sqrt(sqrt(2) + 2)*(sqrt(2) - 2) - 3*(sqrt(2) + 2)*s
qrt(-sqrt(2) + 2) + (-sqrt(2) + 2)^(3/2))) + 1/64*(sqrt(2)*sqrt(sqrt(2) + 2
) - sqrt(2)*sqrt(-sqrt(2) + 2))*log(1/8*sqrt(2)*x*(sqrt(2) + 2)^(3/2) + 1/6
4*(sqrt(2) + 2)^3 + 3/64*(sqrt(2) + 2)*(sqrt(2) - 2)^2 - 1/64*(sqrt(2) - 2)
^3 + 3/8*sqrt(2)*x*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) - 1/8*sqrt(2)*x*(-sqrt(
2) + 2)^(3/2) + x^2 + 3/64*(8*sqrt(2)*x*sqrt(sqrt(2) + 2) - (sqrt(2) + 2)^2
)*(sqrt(2) - 2)) + 1/64*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2)
+ 2))*log(1/8*sqrt(2)*x*(sqrt(2) + 2)^(3/2) + 1/64*(sqrt(2) + 2)^3 + 3/64*(
sqrt(2) + 2)*(sqrt(2) - 2)^2 - 1/64*(sqrt(2) - 2)^3 - 3/8*sqrt(2)*x*(sqrt(2
) + 2)*sqrt(-sqrt(2) + 2) + 1/8*sqrt(2)*x*(-sqrt(2) + 2)^(3/2) + x^2 + 3/64
*(8*sqrt(2)*x*sqrt(sqrt(2) + 2) - (sqrt(2) + 2)^2)*(sqrt(2) - 2)) - 1/64*(s
qrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*log(-1/8*sqrt(2)*x*(
sqrt(2) + 2)^(3/2) + 1/64*(sqrt(2) + 2)^3 + 3/64*(sqrt(2) + 2)*(sqrt(2) - 2
)^2 - 1/64*(sqrt(2) - 2)^3 + 3/8*sqrt(2)*x*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2)
 - 1/8*sqrt(2)*x*(-sqrt(2) + 2)^(3/2) + x^2 - 3/64*(8*sqrt(2)*x*sqrt(sqrt(2
) + 2) + (sqrt(2) + 2)^2)*(sqrt(2) - 2)) - 1/64*(sqrt(2)*sqrt(sqrt(2) + 2)
- sqrt(2)*sqrt(-sqrt(2) + 2))*log(-1/8*sqrt(2)*x*(sqrt(2) + 2)^(3/2) + 1/64
*(sqrt(2) + 2)^3 + 3/64*(sqrt(2) + 2)*(sqrt(2) - 2)^2 - 1/64*(sqrt(2) - 2)^
3 - 3/8*sqrt(2)*x*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) + 1/8*sqrt(2)*x*(-sqrt(2
```

```
) + 2)^(3/2) + x^2 - 3/64*(8*sqrt(2)*x*sqrt(sqrt(2) + 2) + (sqrt(2) + 2)^2)
*(sqrt(2) - 2)) + 1/8*sqrt(sqrt(2) + 2)*arctan((3*(sqrt(2) + 2)*sqrt(-sqrt(
2) + 2) - (-sqrt(2) + 2)^(3/2) + 8*x - sqrt((sqrt(2) + 2)^3 - 3*(sqrt(2) +
2)^2*(sqrt(2) - 2) + 3*(sqrt(2) + 2)*(sqrt(2) - 2)^2 - (sqrt(2) - 2)^3 + 48
*x*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) - 16*x*(-sqrt(2) + 2)^(3/2) + 64*x^2))/
((sqrt(2) + 2)^(3/2) + 3*sqrt(sqrt(2) + 2)*(sqrt(2) - 2))) + 1/8*sqrt(sqrt(
2) + 2)*arctan(-(3*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) - (-sqrt(2) + 2)^(3/2)
- 8*x + sqrt((sqrt(2) + 2)^3 - 3*(sqrt(2) + 2)^2*(sqrt(2) - 2) + 3*(sqrt(2)
 + 2)*(sqrt(2) - 2)^2 - (sqrt(2) - 2)^3 - 48*x*(sqrt(2) + 2)*sqrt(-sqrt(2)
+ 2) + 16*x*(-sqrt(2) + 2)^(3/2) + 64*x^2))/((sqrt(2) + 2)^(3/2) + 3*sqrt(s
qrt(2) + 2)*(sqrt(2) - 2))) - 1/8*sqrt(-sqrt(2) + 2)*arctan(((sqrt(2) + 2)^
(3/2) + 3*sqrt(sqrt(2) + 2)*(sqrt(2) - 2) + 8*x - sqrt((sqrt(2) + 2)^3 + 3*
(sqrt(2) + 2)*(sqrt(2) - 2)^2 - (sqrt(2) - 2)^3 + 16*x*(sqrt(2) + 2)^(3/2)
+ 64*x^2 - 3*((sqrt(2) + 2)^2 - 16*x*sqrt(sqrt(2) + 2))*(sqrt(2) - 2)))/(3*
(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) - (-sqrt(2) + 2)^(3/2))) - 1/8*sqrt(-sqrt(
2) + 2)*arctan(-((sqrt(2) + 2)^(3/2) + 3*sqrt(sqrt(2) + 2)*(sqrt(2) - 2) -
8*x + sqrt((sqrt(2) + 2)^3 + 3*(sqrt(2) + 2)*(sqrt(2) - 2)^2 - (sqrt(2) - 2
)^3 - 16*x*(sqrt(2) + 2)^(3/2) + 64*x^2 - 3*((sqrt(2) + 2)^2 + 16*x*sqrt(sq
rt(2) + 2))*(sqrt(2) - 2)))/(3*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) - (-sqrt(2)
 + 2)^(3/2))) + 1/32*sqrt(-sqrt(2) + 2)*log(1/64*(sqrt(2) + 2)^3 - 3/64*(sq
rt(2) + 2)^2*(sqrt(2) - 2) + 3/64*(sqrt(2) + 2)*(sqrt(2) - 2)^2 - 1/64*(sqr
t(2) - 2)^3 + 3/4*x*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) - 1/4*x*(-sqrt(2) + 2)
^(3/2) + x^2) - 1/32*sqrt(-sqrt(2) + 2)*log(1/64*(sqrt(2) + 2)^3 - 3/64*(sq
rt(2) + 2)^2*(sqrt(2) - 2) + 3/64*(sqrt(2) + 2)*(sqrt(2) - 2)^2 - 1/64*(sqr
t(2) - 2)^3 - 3/4*x*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) + 1/4*x*(-sqrt(2) + 2)
^(3/2) + x^2) - 1/32*sqrt(sqrt(2) + 2)*log(1/64*(sqrt(2) + 2)^3 + 3/64*(sqr
t(2) + 2)*(sqrt(2) - 2)^2 - 1/64*(sqrt(2) - 2)^3 + 1/4*x*(sqrt(2) + 2)^(3/2
) + x^2 - 3/64*((sqrt(2) + 2)^2 - 16*x*sqrt(sqrt(2) + 2))*(sqrt(2) - 2)) +
1/32*sqrt(sqrt(2) + 2)*log(1/64*(sqrt(2) + 2)^3 + 3/64*(sqrt(2) + 2)*(sqrt(
2) - 2)^2 - 1/64*(sqrt(2) - 2)^3 - 1/4*x*(sqrt(2) + 2)^(3/2) + x^2 - 3/64*(
(sqrt(2) + 2)^2 + 16*x*sqrt(sqrt(2) + 2))*(sqrt(2) - 2))
```

giac [A] time = 0.33, size = 239, normalized size = 0.71

$$-\frac{1}{8}\sqrt{-\sqrt{2}+2}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)-\frac{1}{8}\sqrt{-\sqrt{2}+2}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)+\frac{1}{8}\sqrt{\sqrt{2}+2}\arctan\left(\frac{2x-}{\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+1),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(-sqrt(2) + 2)*arctan((2*x + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)
) - 1/8*sqrt(-sqrt(2) + 2)*arctan((2*x - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) +
 2)) + 1/8*sqrt(sqrt(2) + 2)*arctan((2*x + sqrt(sqrt(2) + 2))/sqrt(-sqrt(2)
 + 2)) + 1/8*sqrt(sqrt(2) + 2)*arctan((2*x - sqrt(sqrt(2) + 2))/sqrt(-sqrt(
2) + 2)) + 1/16*sqrt(-sqrt(2) + 2)*log(x^2 + x*sqrt(sqrt(2) + 2) + 1) - 1/1
6*sqrt(-sqrt(2) + 2)*log(x^2 - x*sqrt(sqrt(2) + 2) + 1) - 1/16*sqrt(sqrt(2)
 + 2)*log(x^2 + x*sqrt(-sqrt(2) + 2) + 1) + 1/16*sqrt(sqrt(2) + 2)*log(x^2
- x*sqrt(-sqrt(2) + 2) + 1)
```

maple [C] time = 0.01, size = 22, normalized size = 0.06

$$\frac{\ln\left(-\operatorname{RootOf}\left(_Z^8+1\right)+x\right)}{8\operatorname{RootOf}\left(_Z^8+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^8+1),x)
```

```
[Out] 1/8*sum(1/_R^5*ln(-_R+x),_R=RootOf(_Z^8+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{x^8+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+1),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(x^8 + 1), x)
```

mupad [B] time = 0.09, size = 193, normalized size = 0.57

$$-\operatorname{atan}\left(x\left(\frac{\sqrt{-\sqrt{2}-2}}{16}-\frac{\sqrt{2-\sqrt{2}}}{16}\right)^5 32768\mathrm{i}\right)\left(\frac{\sqrt{-\sqrt{2}-2}\,1\mathrm{i}}{8}-\frac{\sqrt{2-\sqrt{2}}\,1\mathrm{i}}{8}\right)-\operatorname{atan}\left(x\left(\frac{\sqrt{\sqrt{2}-2}}{16}+\frac{\sqrt{\sqrt{2}+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^8 + 1),x)
```

```
[Out] - atan(x*((- 2^(1/2) - 2)^(1/2)/16 - (2 - 2^(1/2))^(1/2)/16)^5*32768i)*(((-
 2^(1/2) - 2)^(1/2)*1i)/8 - ((2 - 2^(1/2))^(1/2)*1i)/8) - atan(x*((2^(1/2)
- 2)^(1/2)/16 + (2^(1/2) + 2)^(1/2)/16)^5*32768i)*(((2^(1/2) - 2)^(1/2)*1i)
/8 + ((2^(1/2) + 2)^(1/2)*1i)/8) - atan(x*(2^(1/2)/16 - (1/16 - 1i/16))^5*(
2^(1/2) + 2)^(5/2)*32768i)*(2^(1/2)/16 - (1/16 - 1i/16))*(2^(1/2) + 2)^(1/2
)*2i - atan(x*((2^(1/2)*1i)/16 - (1/16 + 1i/16))^5*(2^(1/2) + 2)^(5/2)*3276
8i)*((2^(1/2)*1i)/16 - (1/16 + 1i/16))*(2^(1/2) + 2)^(1/2)*2i
```

sympy [A] time = 3.13, size = 15, normalized size = 0.04

$$\operatorname{RootSum}\left(16777216t^8+1,\left(t\mapsto t\log\left(-512t^3+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**8+1),x)
```

```
[Out] RootSum(16777216*_t**8 + 1, Lambda(_t, _t*log(-512*_t**3 + x)))
```

3.1504 $\int \frac{1}{1+x^8}\,dx$

Optimal. Leaf size=339

$$-\frac{1}{16}\sqrt{2-\sqrt{2}}\,\log\left(x^2-\sqrt{2-\sqrt{2}}\,x+1\right)+\frac{1}{16}\sqrt{2-\sqrt{2}}\,\log\left(x^2+\sqrt{2-\sqrt{2}}\,x+1\right)-\frac{1}{16}\sqrt{2+\sqrt{2}}\,\log\left(x^2-\sqrt{2+\right.$$

```
[Out] -1/16*ln(1+x^2-x*(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)+1/16*ln(1+x^2+x*(2-2^
(1/2))^(1/2))*(2-2^(1/2))^(1/2)-1/4*arctan((-2*x+(2-2^(1/2))^(1/2))/(2+2^(1
/2))^(1/2))/(4-2*2^(1/2))^(1/2)+1/4*arctan((2*x+(2-2^(1/2))^(1/2))/(2+2^(1/
2))^(1/2))/(4-2*2^(1/2))^(1/2)-1/16*ln(1+x^2-x*(2+2^(1/2))^(1/2))*(2+2^(1/2
))^(1/2)+1/16*ln(1+x^2+x*(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)-1/4*arctan((-
2*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)+1/4*arctan((2
*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {213, 1169, 634, 618, 204, 628}

$$-\frac{1}{16}\sqrt{2-\sqrt{2}}\,\log\left(x^2-\sqrt{2-\sqrt{2}}\,x+1\right)+\frac{1}{16}\sqrt{2-\sqrt{2}}\,\log\left(x^2+\sqrt{2-\sqrt{2}}\,x+1\right)-\frac{1}{16}\sqrt{2+\sqrt{2}}\,\log\left(x^2-\sqrt{2+\right.$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^8)^(-1),x]
```

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 + Sqrt[2]]]/(4*Sqrt[2*(2 - Sqrt[2]
)]) - ArcTan[(Sqrt[2 + Sqrt[2]] - 2*x)/Sqrt[2 - Sqrt[2]]]/(4*Sqrt[2*(2 + Sq
rt[2])]) + ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2 + Sqrt[2]]]/(4*Sqrt[2*(2
 - Sqrt[2])]) + ArcTan[(Sqrt[2 + Sqrt[2]] + 2*x)/Sqrt[2 - Sqrt[2]]]/(4*Sqrt
[2*(2 + Sqrt[2])]) - (Sqrt[2 - Sqrt[2]]*Log[1 - Sqrt[2 - Sqrt[2]]*x + x^2])
/16 + (Sqrt[2 - Sqrt[2]]*Log[1 + Sqrt[2 - Sqrt[2]]*x + x^2])/16 - (Sqrt[2 +
 Sqrt[2]]*Log[1 - Sqrt[2 + Sqrt[2]]*x + x^2])/16 + (Sqrt[2 + Sqrt[2]]*Log[1
 + Sqrt[2 + Sqrt[2]]*x + x^2])/16
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b,
 4]], s = Denominator[Rt[a/b, 4]]}, Dist[r/(2*Sqrt[2]*a), Int[(Sqrt[2]*r -
s*x^(n/4))/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] + Dist[r/(2*Sq
rt[2]*a), Int[(Sqrt[2]*r + s*x^(n/4))/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n
/2)), x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 1] && GtQ[a/b, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{1+x^8}\,dx = \frac{\int \frac{\sqrt{2}-x^2}{1-\sqrt{2}\,x^2+x^4}\,dx}{2\sqrt{2}} + \frac{\int \frac{\sqrt{2}+x^2}{1+\sqrt{2}\,x^2+x^4}\,dx}{2\sqrt{2}}$$

$$= \frac{\int \frac{\sqrt{2\left(2-\sqrt{2}\right)}-\left(-1+\sqrt{2}\right)x}{1-\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2\left(2-\sqrt{2}\right)}} + \frac{\int \frac{\sqrt{2\left(2-\sqrt{2}\right)}+\left(-1+\sqrt{2}\right)x}{1+\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2\left(2-\sqrt{2}\right)}} + \frac{\int \frac{\sqrt{2\left(2+\sqrt{2}\right)}-\left(1+\sqrt{2}\right)x}{1-\sqrt{2+\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2\left(2+\sqrt{2}\right)}} + \frac{\int \frac{\sqrt{2\left(2+\sqrt{2}\right)}+}{1+\sqrt{2+\sqrt{2}}}}{4\sqrt{2\left(2+\right.}}$$

$$= \frac{1}{8}\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\int \frac{1}{1-\sqrt{2+\sqrt{2}}\,x+x^2}\,dx + \frac{1}{8}\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\int \frac{1}{1+\sqrt{2+\sqrt{2}}\,x+x^2}\,dx -$$

$$= -\frac{1}{16}\sqrt{2-\sqrt{2}}\,\log\left(1-\sqrt{2-\sqrt{2}}\,x+x^2\right) + \frac{1}{16}\sqrt{2-\sqrt{2}}\,\log\left(1+\sqrt{2-\sqrt{2}}\,x+x^2\right) - \frac{1}{16}\sqrt{2+}$$

$$= -\frac{1}{8}\sqrt{2+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right) - \frac{1}{8}\sqrt{2-\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right) + \frac{1}{8}\sqrt{2+\sqrt{2}}\,\tan$$

Mathematica [A] time = 0.01, size = 209, normalized size = 0.62

$$-\frac{1}{8}\sin\left(\frac{\pi}{8}\right)\log\left(x^2-2x\sin\left(\frac{\pi}{8}\right)+1\right)+\frac{1}{8}\sin\left(\frac{\pi}{8}\right)\log\left(x^2+2x\sin\left(\frac{\pi}{8}\right)+1\right)-\frac{1}{8}\cos\left(\frac{\pi}{8}\right)\log\left(x^2-2x\cos\left(\frac{\pi}{8}\right)+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^8)^(-1),x]
```

```
[Out] (ArcTan[Sec[Pi/8]*(x - Sin[Pi/8])]*Cos[Pi/8])/4 + (ArcTan[Sec[Pi/8]*(x + Si
n[Pi/8])]*Cos[Pi/8])/4 - (Cos[Pi/8]*Log[1 + x^2 - 2*x*Cos[Pi/8]])/8 + (Cos[
Pi/8]*Log[1 + x^2 + 2*x*Cos[Pi/8]])/8 + (ArcTan[(x - Cos[Pi/8])*Csc[Pi/8]]*
Sin[Pi/8])/4 + (ArcTan[(x + Cos[Pi/8])*Csc[Pi/8]]*Sin[Pi/8])/4 - (Log[1 + x
^2 - 2*x*Sin[Pi/8]]*Sin[Pi/8])/8 + (Log[1 + x^2 + 2*x*Sin[Pi/8]]*Sin[Pi/8])
/8
```

fricas [B] time = 1.18, size = 1027, normalized size = 3.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+1),x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*arctan(-(2*s
qrt(2)*x - 2*sqrt(2)*sqrt(x^2 + 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) - 1/2*sqrt(
2)*x*sqrt(-sqrt(2) + 2) + 1) + sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(sqr
t(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))) - 1/16*(sqrt(2)*sqrt(sqrt(2) + 2) + s
qrt(2)*sqrt(-sqrt(2) + 2))*arctan(-(2*sqrt(2)*x - 2*sqrt(2)*sqrt(x^2 - 1/2*
sqrt(2)*x*sqrt(sqrt(2) + 2) + 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) - sqrt(
sqrt(2) + 2) + sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))
) + 1/16*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*arctan((2
*sqrt(2)*x - 2*sqrt(2)*sqrt(x^2 + 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) + 1/2*sqr
t(2)*x*sqrt(-sqrt(2) + 2) + 1) + sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))/(s
qrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))) + 1/16*(sqrt(2)*sqrt(sqrt(2) + 2) -
 sqrt(2)*sqrt(-sqrt(2) + 2))*arctan((2*sqrt(2)*x - 2*sqrt(2)*sqrt(x^2 - 1/2
*sqrt(2)*x*sqrt(sqrt(2) + 2) - 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) - sqrt
(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2)
)) + 1/64*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*log(x^2
+ 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) + 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) +
 1/64*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*log(x^2 + 1/
2*sqrt(2)*x*sqrt(sqrt(2) + 2) - 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) - 1/6
4*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*log(x^2 - 1/2*sq
rt(2)*x*sqrt(sqrt(2) + 2) + 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) - 1/64*(s
qrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*log(x^2 - 1/2*sqrt(2
)*x*sqrt(sqrt(2) + 2) - 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) - 1/8*sqrt(sq
rt(2) + 2)*arctan(-(2*x - 2*sqrt(x^2 + x*sqrt(-sqrt(2) + 2) + 1) + sqrt(-sq
rt(2) + 2))/sqrt(sqrt(2) + 2)) - 1/8*sqrt(sqrt(2) + 2)*arctan(-(2*x - 2*sqr
t(x^2 - x*sqrt(-sqrt(2) + 2) + 1) - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2))
- 1/8*sqrt(-sqrt(2) + 2)*arctan(-(2*x - 2*sqrt(x^2 + x*sqrt(sqrt(2) + 2) +
1) + sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) - 1/8*sqrt(-sqrt(2) + 2)*arctan
(-(2*x - 2*sqrt(x^2 - x*sqrt(sqrt(2) + 2) + 1) - sqrt(sqrt(2) + 2))/sqrt(-s
qrt(2) + 2)) + 1/32*sqrt(sqrt(2) + 2)*log(x^2 + x*sqrt(sqrt(2) + 2) + 1) -
1/32*sqrt(sqrt(2) + 2)*log(x^2 - x*sqrt(sqrt(2) + 2) + 1) + 1/32*sqrt(-sqrt
(2) + 2)*log(x^2 + x*sqrt(-sqrt(2) + 2) + 1) - 1/32*sqrt(-sqrt(2) + 2)*log(
x^2 - x*sqrt(-sqrt(2) + 2) + 1)
```

giac [A] time = 0.17, size = 239, normalized size = 0.71

$$\frac{1}{8}\sqrt{\sqrt{2}+2}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)+\frac{1}{8}\sqrt{\sqrt{2}+2}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)+\frac{1}{8}\sqrt{-\sqrt{2}+2}\arctan\left(\frac{2x+\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+1),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(sqrt(2) + 2)*arctan((2*x + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2))
+ 1/8*sqrt(sqrt(2) + 2)*arctan((2*x - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)
) + 1/8*sqrt(-sqrt(2) + 2)*arctan((2*x + sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) +
 2)) + 1/8*sqrt(-sqrt(2) + 2)*arctan((2*x - sqrt(sqrt(2) + 2))/sqrt(-sqrt(2
) + 2)) + 1/16*sqrt(sqrt(2) + 2)*log(x^2 + x*sqrt(sqrt(2) + 2) + 1) - 1/16*
sqrt(sqrt(2) + 2)*log(x^2 - x*sqrt(sqrt(2) + 2) + 1) + 1/16*sqrt(-sqrt(2) +
 2)*log(x^2 + x*sqrt(-sqrt(2) + 2) + 1) - 1/16*sqrt(-sqrt(2) + 2)*log(x^2 -
 x*sqrt(-sqrt(2) + 2) + 1)
```

maple [C] time = 0.00, size = 22, normalized size = 0.06

$$\frac{\ln\left(-\operatorname{RootOf}\left(_Z^8+1\right)+x\right)}{8\operatorname{RootOf}\left(_Z^8+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8+1),x)
```

```
[Out] 1/8*sum(1/_R^7*ln(-_R+x),_R=RootOf(_Z^8+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^8+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^8 + 1), x)
```

mupad [B] time = 0.07, size = 288, normalized size = 0.85

$$\operatorname{atan}\left(\frac{x\sqrt{-\sqrt{2}-2}\,1\mathrm{i}}{\sqrt{2-\sqrt{2}}\,\sqrt{-\sqrt{2}-2}+\sqrt{2}}-\frac{x\sqrt{2-\sqrt{2}}\,1\mathrm{i}}{\sqrt{2-\sqrt{2}}\,\sqrt{-\sqrt{2}-2}+\sqrt{2}}\right)\left(\frac{\sqrt{-\sqrt{2}-2}\,1\mathrm{i}}{8}-\frac{\sqrt{2-\sqrt{2}}\,1\mathrm{i}}{8}\right)-\operatorname{atan}\left(\frac{-}{\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8 + 1),x)
```

```
[Out] atan((x*(- 2^(1/2) - 2)^(1/2)*1i)/((2 - 2^(1/2))^(1/2)*(- 2^(1/2) - 2)^(1/2
) + 2^(1/2)) - (x*(2 - 2^(1/2))^(1/2)*1i)/((2 - 2^(1/2))^(1/2)*(- 2^(1/2) -
 2)^(1/2) + 2^(1/2)))*(((- 2^(1/2) - 2)^(1/2)*1i)/8 - ((2 - 2^(1/2))^(1/2)*
1i)/8) - atan((x*(2^(1/2) - 2)^(1/2)*1i)/(2^(1/2) + (2^(1/2) - 2)^(1/2)*(2^
(1/2) + 2)^(1/2)) + (x*(2^(1/2) + 2)^(1/2)*1i)/(2^(1/2) + (2^(1/2) - 2)^(1/
2)*(2^(1/2) + 2)^(1/2)))*(((2^(1/2) - 2)^(1/2)*1i)/8 + ((2^(1/2) + 2)^(1/2)
*1i)/8) + atan(x*(2^(1/2) + 2)^(1/2)*(1/2 + 1i/2) - (2^(1/2)*x*(2^(1/2) + 2
)^(1/2))/2)*((2^(1/2)*1i)/16 - (1/16 + 1i/16))*(2^(1/2) + 2)^(1/2)*2i - ata
n(x*(2^(1/2) + 2)^(1/2)*(1/2 - 1i/2) + (2^(1/2)*x*(2^(1/2) + 2)^(1/2)*1i)/2
)*(2^(1/2)/16 - (1/16 - 1i/16))*(2^(1/2) + 2)^(1/2)*2i
```

sympy [A] time = 2.97, size = 14, normalized size = 0.04

$$\operatorname{RootSum}\left(16777216t^8+1,\left(t\mapsto t\log\left(8t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**8+1),x)
```

```
[Out] RootSum(16777216*_t**8 + 1, Lambda(_t, _t*log(8*_t + x)))
```

3.1505 $\int \frac{1}{x^2\left(1+x^8\right)}\,dx$

Optimal. Leaf size=344

$$-\frac{1}{16}\sqrt{2-\sqrt{2}}\,\log\left(x^2-\sqrt{2-\sqrt{2}}\,x+1\right)+\frac{1}{16}\sqrt{2-\sqrt{2}}\,\log\left(x^2+\sqrt{2-\sqrt{2}}\,x+1\right)-\frac{1}{16}\sqrt{2+\sqrt{2}}\,\log\left(x^2-\sqrt{2+\right.$$

```
[Out] -1/x-1/16*ln(1+x^2-x*(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)+1/16*ln(1+x^2+x*(
2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)+1/4*arctan((-2*x+(2-2^(1/2))^(1/2))/(2+
2^(1/2))^(1/2))/(4-2*2^(1/2))^(1/2)-1/4*arctan((2*x+(2-2^(1/2))^(1/2))/(2+2
^(1/2))^(1/2))/(4-2*2^(1/2))^(1/2)-1/16*ln(1+x^2-x*(2+2^(1/2))^(1/2))*(2+2^
(1/2))^(1/2)+1/16*ln(1+x^2+x*(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)+1/4*arcta
n((-2*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)-1/4*arcta
n((2*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {325, 299, 1122, 1169, 634, 618, 204, 628}

$$-\frac{1}{16}\sqrt{2-\sqrt{2}}\,\log\left(x^2-\sqrt{2-\sqrt{2}}\,x+1\right)+\frac{1}{16}\sqrt{2-\sqrt{2}}\,\log\left(x^2+\sqrt{2-\sqrt{2}}\,x+1\right)-\frac{1}{16}\sqrt{2+\sqrt{2}}\,\log\left(x^2-\sqrt{2+\right.$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 + x^8)),x]
```

```
[Out] -x^(-1) + ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 + Sqrt[2]]]/(4*Sqrt[2*(2
- Sqrt[2])]) + ArcTan[(Sqrt[2 + Sqrt[2]] - 2*x)/Sqrt[2 - Sqrt[2]]]/(4*Sqrt[
2*(2 + Sqrt[2])]) - ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2 + Sqrt[2]]]/(4*
Sqrt[2*(2 - Sqrt[2])]) - ArcTan[(Sqrt[2 + Sqrt[2]] + 2*x)/Sqrt[2 - Sqrt[2]]
]/(4*Sqrt[2*(2 + Sqrt[2])]) - (Sqrt[2 - Sqrt[2]]*Log[1 - Sqrt[2 - Sqrt[2]]*
x + x^2])/16 + (Sqrt[2 - Sqrt[2]]*Log[1 + Sqrt[2 - Sqrt[2]]*x + x^2])/16 -
(Sqrt[2 + Sqrt[2]]*Log[1 - Sqrt[2 + Sqrt[2]]*x + x^2])/16 + (Sqrt[2 + Sqrt[
2]]*Log[1 + Sqrt[2 + Sqrt[2]]*x + x^2])/16
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 299

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[s^3/(2*Sqrt[2]*b*r), Int[x^(
m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Dist[s^3/(2*Sq
rt[2]*b*r), Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && G
tQ[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^2\left(1+x^8\right)}\,dx = -\frac{1}{x} - \int \frac{x^6}{1+x^8}\,dx$$

$$= -\frac{1}{x} - \frac{\int \frac{x^4}{1-\sqrt{2}\,x^2+x^4}\,dx}{2\sqrt{2}} + \frac{\int \frac{x^4}{1+\sqrt{2}\,x^2+x^4}\,dx}{2\sqrt{2}}$$

$$= -\frac{1}{x} + \frac{\int \frac{1-\sqrt{2}\,x^2}{1-\sqrt{2}\,x^2+x^4}\,dx}{2\sqrt{2}} - \frac{\int \frac{1+\sqrt{2}\,x^2}{1+\sqrt{2}\,x^2+x^4}\,dx}{2\sqrt{2}}$$

$$= -\frac{1}{x} - \frac{\int \frac{\sqrt{2-\sqrt{2}}-\left(1-\sqrt{2}\right)x}{1-\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2\left(2-\sqrt{2}\right)}} - \frac{\int \frac{\sqrt{2-\sqrt{2}}+\left(1-\sqrt{2}\right)x}{1+\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2\left(2-\sqrt{2}\right)}} + \frac{\int \frac{\sqrt{2+\sqrt{2}}-\left(1+\sqrt{2}\right)x}{1-\sqrt{2+\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2\left(2+\sqrt{2}\right)}} + \frac{\int \frac{\sqrt{2+\sqrt{2}}+\left(1+\sqrt{2}\right)x}{1+\sqrt{2+\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2\left(2+\sqrt{2}\right)}}$$

$$= -\frac{1}{x} - \frac{1}{8}\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\int \frac{1}{1-\sqrt{2+\sqrt{2}}\,x+x^2}\,dx - \frac{1}{8}\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\int \frac{1}{1+\sqrt{2+\sqrt{2}}\,x+x^2}\,dx$$

$$= -\frac{1}{x} - \frac{1}{16}\sqrt{2-\sqrt{2}}\,\log\left(1-\sqrt{2-\sqrt{2}}\,x+x^2\right) + \frac{1}{16}\sqrt{2-\sqrt{2}}\,\log\left(1+\sqrt{2-\sqrt{2}}\,x+x^2\right) - \frac{1}{16}$$

$$= -\frac{1}{x} + \frac{1}{8}\sqrt{2+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right) + \frac{1}{8}\sqrt{2-\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right) - \frac{1}{8}\sqrt{2+\sqrt{}}$$

Mathematica [A] time = 0.01, size = 214, normalized size = 0.62

$$-\frac{1}{8}\sin\left(\frac{\pi}{8}\right)\log\left(x^2-2x\sin\left(\frac{\pi}{8}\right)+1\right)+\frac{1}{8}\sin\left(\frac{\pi}{8}\right)\log\left(x^2+2x\sin\left(\frac{\pi}{8}\right)+1\right)-\frac{1}{8}\cos\left(\frac{\pi}{8}\right)\log\left(x^2-2x\cos\left(\frac{\pi}{8}\right)+1\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 + x^8)),x]
```

```
[Out] -x^(-1) - (ArcTan[Sec[Pi/8]*(x - Sin[Pi/8])]*Cos[Pi/8])/4 - (ArcTan[Sec[Pi/
8]*(x + Sin[Pi/8])]*Cos[Pi/8])/4 - (Cos[Pi/8]*Log[1 + x^2 - 2*x*Cos[Pi/8]])
/8 + (Cos[Pi/8]*Log[1 + x^2 + 2*x*Cos[Pi/8]])/8 - (ArcTan[(x - Cos[Pi/8])*C
sc[Pi/8]]*Sin[Pi/8])/4 - (ArcTan[(x + Cos[Pi/8])*Csc[Pi/8]]*Sin[Pi/8])/4 -
(Log[1 + x^2 - 2*x*Sin[Pi/8]]*Sin[Pi/8])/8 + (Log[1 + x^2 + 2*x*Sin[Pi/8]]*
Sin[Pi/8])/8
```

fricas [B] time = 0.70, size = 2541, normalized size = 7.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+1),x, algorithm="fricas")
```

```
[Out] -1/64*(8*x*sqrt(-sqrt(2) + 2)*arctan((3*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) -
(-sqrt(2) + 2)^(3/2) + 8*x - sqrt((sqrt(2) + 2)^3 - 3*(sqrt(2) + 2)^2*(sqrt
(2) - 2) + 3*(sqrt(2) + 2)*(sqrt(2) - 2)^2 - (sqrt(2) - 2)^3 + 48*x*(sqrt(2
) + 2)*sqrt(-sqrt(2) + 2) - 16*x*(-sqrt(2) + 2)^(3/2) + 64*x^2))/((sqrt(2)
+ 2)^(3/2) + 3*sqrt(sqrt(2) + 2)*(sqrt(2) - 2))) + 8*x*sqrt(-sqrt(2) + 2)*a
rctan(-(3*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) - (-sqrt(2) + 2)^(3/2) - 8*x + s
qrt((sqrt(2) + 2)^3 - 3*(sqrt(2) + 2)^2*(sqrt(2) - 2) + 3*(sqrt(2) + 2)*(sq
rt(2) - 2)^2 - (sqrt(2) - 2)^3 - 48*x*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) + 16
*x*(-sqrt(2) + 2)^(3/2) + 64*x^2))/((sqrt(2) + 2)^(3/2) + 3*sqrt(sqrt(2) +
```

```
2)*(sqrt(2) - 2))) + 8*x*sqrt(sqrt(2) + 2)*arctan(((sqrt(2) + 2)^(3/2) + 3*
sqrt(sqrt(2) + 2)*(sqrt(2) - 2) + 8*x - sqrt((sqrt(2) + 2)^3 + 3*(sqrt(2) +
 2)*(sqrt(2) - 2)^2 - (sqrt(2) - 2)^3 + 16*x*(sqrt(2) + 2)^(3/2) + 64*x^2 -
 3*((sqrt(2) + 2)^2 - 16*x*sqrt(sqrt(2) + 2))*(sqrt(2) - 2)))/(3*(sqrt(2) +
 2)*sqrt(-sqrt(2) + 2) - (-sqrt(2) + 2)^(3/2))) + 8*x*sqrt(sqrt(2) + 2)*arc
tan(-((sqrt(2) + 2)^(3/2) + 3*sqrt(sqrt(2) + 2)*(sqrt(2) - 2) - 8*x + sqrt(
(sqrt(2) + 2)^3 + 3*(sqrt(2) + 2)*(sqrt(2) - 2)^2 - (sqrt(2) - 2)^3 - 16*x*
(sqrt(2) + 2)^(3/2) + 64*x^2 - 3*((sqrt(2) + 2)^2 + 16*x*sqrt(sqrt(2) + 2))
*(sqrt(2) - 2)))/(3*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) - (-sqrt(2) + 2)^(3/2)
)) - 2*x*sqrt(sqrt(2) + 2)*log(1/64*(sqrt(2) + 2)^3 - 3/64*(sqrt(2) + 2)^2*
(sqrt(2) - 2) + 3/64*(sqrt(2) + 2)*(sqrt(2) - 2)^2 - 1/64*(sqrt(2) - 2)^3 +
 3/4*x*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) - 1/4*x*(-sqrt(2) + 2)^(3/2) + x^2)
 + 2*x*sqrt(sqrt(2) + 2)*log(1/64*(sqrt(2) + 2)^3 - 3/64*(sqrt(2) + 2)^2*(s
qrt(2) - 2) + 3/64*(sqrt(2) + 2)*(sqrt(2) - 2)^2 - 1/64*(sqrt(2) - 2)^3 - 3
/4*x*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) + 1/4*x*(-sqrt(2) + 2)^(3/2) + x^2) -
 2*x*sqrt(-sqrt(2) + 2)*log(1/64*(sqrt(2) + 2)^3 + 3/64*(sqrt(2) + 2)*(sqrt
(2) - 2)^2 - 1/64*(sqrt(2) - 2)^3 + 1/4*x*(sqrt(2) + 2)^(3/2) + x^2 - 3/64*
((sqrt(2) + 2)^2 - 16*x*sqrt(sqrt(2) + 2))*(sqrt(2) - 2)) + 2*x*sqrt(-sqrt(
2) + 2)*log(1/64*(sqrt(2) + 2)^3 + 3/64*(sqrt(2) + 2)*(sqrt(2) - 2)^2 - 1/6
4*(sqrt(2) - 2)^3 - 1/4*x*(sqrt(2) + 2)^(3/2) + x^2 - 3/64*((sqrt(2) + 2)^2
 + 16*x*sqrt(sqrt(2) + 2))*(sqrt(2) - 2)) + 4*(sqrt(2)*x*sqrt(sqrt(2) + 2)
+ sqrt(2)*x*sqrt(-sqrt(2) + 2))*arctan((8*sqrt(2)*x + (sqrt(2) + 2)^(3/2) +
 3*sqrt(sqrt(2) + 2)*(sqrt(2) - 2) - 3*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) + (
-sqrt(2) + 2)^(3/2) - sqrt(2)*sqrt(8*sqrt(2)*x*(sqrt(2) + 2)^(3/2) + (sqrt(
2) + 2)^3 + 3*(sqrt(2) + 2)*(sqrt(2) - 2)^2 - (sqrt(2) - 2)^3 - 24*sqrt(2)*
x*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) + 8*sqrt(2)*x*(-sqrt(2) + 2)^(3/2) + 64*
x^2 + 3*(8*sqrt(2)*x*sqrt(sqrt(2) + 2) - (sqrt(2) + 2)^2)*(sqrt(2) - 2)))/(
(sqrt(2) + 2)^(3/2) + 3*sqrt(sqrt(2) + 2)*(sqrt(2) - 2) + 3*(sqrt(2) + 2)*s
qrt(-sqrt(2) + 2) - (-sqrt(2) + 2)^(3/2))) + 4*(sqrt(2)*x*sqrt(sqrt(2) + 2)
 + sqrt(2)*x*sqrt(-sqrt(2) + 2))*arctan((8*sqrt(2)*x - (sqrt(2) + 2)^(3/2)
- 3*sqrt(sqrt(2) + 2)*(sqrt(2) - 2) + 3*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) -
(-sqrt(2) + 2)^(3/2) - 8*sqrt(2)*sqrt(-1/8*sqrt(2)*x*(sqrt(2) + 2)^(3/2) +
1/64*(sqrt(2) + 2)^3 + 3/64*(sqrt(2) + 2)*(sqrt(2) - 2)^2 - 1/64*(sqrt(2) -
 2)^3 + 3/8*sqrt(2)*x*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) - 1/8*sqrt(2)*x*(-sq
rt(2) + 2)^(3/2) + x^2 - 3/64*(8*sqrt(2)*x*sqrt(sqrt(2) + 2) + (sqrt(2) + 2
)^2)*(sqrt(2) - 2)))/((sqrt(2) + 2)^(3/2) + 3*sqrt(sqrt(2) + 2)*(sqrt(2) -
2) + 3*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) - (-sqrt(2) + 2)^(3/2))) + 4*(sqrt(
2)*x*sqrt(sqrt(2) + 2) - sqrt(2)*x*sqrt(-sqrt(2) + 2))*arctan(-(8*sqrt(2)*x
 + (sqrt(2) + 2)^(3/2) + 3*sqrt(sqrt(2) + 2)*(sqrt(2) - 2) + 3*(sqrt(2) + 2
)*sqrt(-sqrt(2) + 2) - (-sqrt(2) + 2)^(3/2) - sqrt(2)*sqrt(8*sqrt(2)*x*(sqr
t(2) + 2)^(3/2) + (sqrt(2) + 2)^3 + 3*(sqrt(2) + 2)*(sqrt(2) - 2)^2 - (sqrt
(2) - 2)^3 + 24*sqrt(2)*x*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) - 8*sqrt(2)*x*(-
sqrt(2) + 2)^(3/2) + 64*x^2 + 3*(8*sqrt(2)*x*sqrt(sqrt(2) + 2) - (sqrt(2) +
 2)^2)*(sqrt(2) - 2)))/((sqrt(2) + 2)^(3/2) + 3*sqrt(sqrt(2) + 2)*(sqrt(2)
- 2) - 3*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) + (-sqrt(2) + 2)^(3/2))) + 4*(sqr
t(2)*x*sqrt(sqrt(2) + 2) - sqrt(2)*x*sqrt(-sqrt(2) + 2))*arctan(-(8*sqrt(2)
*x - (sqrt(2) + 2)^(3/2) - 3*sqrt(sqrt(2) + 2)*(sqrt(2) - 2) - 3*(sqrt(2) +
 2)*sqrt(-sqrt(2) + 2) + (-sqrt(2) + 2)^(3/2) - 8*sqrt(2)*sqrt(-1/8*sqrt(2)
*x*(sqrt(2) + 2)^(3/2) + 1/64*(sqrt(2) + 2)^3 + 3/64*(sqrt(2) + 2)*(sqrt(2)
 - 2)^2 - 1/64*(sqrt(2) - 2)^3 - 3/8*sqrt(2)*x*(sqrt(2) + 2)*sqrt(-sqrt(2)
+ 2) + 1/8*sqrt(2)*x*(-sqrt(2) + 2)^(3/2) + x^2 - 3/64*(8*sqrt(2)*x*sqrt(sq
rt(2) + 2) + (sqrt(2) + 2)^2)*(sqrt(2) - 2)))/((sqrt(2) + 2)^(3/2) + 3*sqrt
(sqrt(2) + 2)*(sqrt(2) - 2) - 3*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) + (-sqrt(2
) + 2)^(3/2))) - (sqrt(2)*x*sqrt(sqrt(2) + 2) + sqrt(2)*x*sqrt(-sqrt(2) + 2
))*log(1/8*sqrt(2)*x*(sqrt(2) + 2)^(3/2) + 1/64*(sqrt(2) + 2)^3 + 3/64*(sqr
t(2) + 2)*(sqrt(2) - 2)^2 - 1/64*(sqrt(2) - 2)^3 + 3/8*sqrt(2)*x*(sqrt(2) +
 2)*sqrt(-sqrt(2) + 2) - 1/8*sqrt(2)*x*(-sqrt(2) + 2)^(3/2) + x^2 + 3/64*(8
*sqrt(2)*x*sqrt(sqrt(2) + 2) - (sqrt(2) + 2)^2)*(sqrt(2) - 2)) + (sqrt(2)*x
*sqrt(sqrt(2) + 2) - sqrt(2)*x*sqrt(-sqrt(2) + 2))*log(1/8*sqrt(2)*x*(sqrt(
```

```
2) + 2)^(3/2) + 1/64*(sqrt(2) + 2)^3 + 3/64*(sqrt(2) + 2)*(sqrt(2) - 2)^2 -
 1/64*(sqrt(2) - 2)^3 - 3/8*sqrt(2)*x*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) + 1/
8*sqrt(2)*x*(-sqrt(2) + 2)^(3/2) + x^2 + 3/64*(8*sqrt(2)*x*sqrt(sqrt(2) + 2
) - (sqrt(2) + 2)^2)*(sqrt(2) - 2)) - (sqrt(2)*x*sqrt(sqrt(2) + 2) - sqrt(2
)*x*sqrt(-sqrt(2) + 2))*log(-1/8*sqrt(2)*x*(sqrt(2) + 2)^(3/2) + 1/64*(sqrt
(2) + 2)^3 + 3/64*(sqrt(2) + 2)*(sqrt(2) - 2)^2 - 1/64*(sqrt(2) - 2)^3 + 3/
8*sqrt(2)*x*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) - 1/8*sqrt(2)*x*(-sqrt(2) + 2)
^(3/2) + x^2 - 3/64*(8*sqrt(2)*x*sqrt(sqrt(2) + 2) + (sqrt(2) + 2)^2)*(sqrt
(2) - 2)) + (sqrt(2)*x*sqrt(sqrt(2) + 2) + sqrt(2)*x*sqrt(-sqrt(2) + 2))*lo
g(-1/8*sqrt(2)*x*(sqrt(2) + 2)^(3/2) + 1/64*(sqrt(2) + 2)^3 + 3/64*(sqrt(2)
 + 2)*(sqrt(2) - 2)^2 - 1/64*(sqrt(2) - 2)^3 - 3/8*sqrt(2)*x*(sqrt(2) + 2)*
sqrt(-sqrt(2) + 2) + 1/8*sqrt(2)*x*(-sqrt(2) + 2)^(3/2) + x^2 - 3/64*(8*sqr
t(2)*x*sqrt(sqrt(2) + 2) + (sqrt(2) + 2)^2)*(sqrt(2) - 2)) + 64)/x
```

giac [A] time = 0.33, size = 244, normalized size = 0.71

$$-\frac{1}{8}\sqrt{\sqrt{2}+2}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)-\frac{1}{8}\sqrt{\sqrt{2}+2}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)-\frac{1}{8}\sqrt{-\sqrt{2}+2}\arctan\left(\frac{2x+}{\sqrt{-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+1),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(sqrt(2) + 2)*arctan((2*x + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2))
 - 1/8*sqrt(sqrt(2) + 2)*arctan((2*x - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2
)) - 1/8*sqrt(-sqrt(2) + 2)*arctan((2*x + sqrt(sqrt(2) + 2))/sqrt(-sqrt(2)
+ 2)) - 1/8*sqrt(-sqrt(2) + 2)*arctan((2*x - sqrt(sqrt(2) + 2))/sqrt(-sqrt(
2) + 2)) + 1/16*sqrt(sqrt(2) + 2)*log(x^2 + x*sqrt(sqrt(2) + 2) + 1) - 1/16
*sqrt(sqrt(2) + 2)*log(x^2 - x*sqrt(sqrt(2) + 2) + 1) + 1/16*sqrt(-sqrt(2)
+ 2)*log(x^2 + x*sqrt(-sqrt(2) + 2) + 1) - 1/16*sqrt(-sqrt(2) + 2)*log(x^2
- x*sqrt(-sqrt(2) + 2) + 1) - 1/x
```

maple [C] time = 0.01, size = 28, normalized size = 0.08

$$-\frac{\ln\left(-\operatorname{RootOf}\left(_Z^8+1\right)+x\right)}{8\operatorname{RootOf}\left(_Z^8+1\right)}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^8+1),x)
```

```
[Out] -1/8*sum(1/_R*ln(-_R+x),_R=RootOf(_Z^8+1))-1/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{x}-\int\frac{x^6}{x^8+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+1),x, algorithm="maxima")
```

```
[Out] -1/x - integrate(x^6/(x^8 + 1), x)
```

mupad [B] time = 0.06, size = 261, normalized size = 0.76

$$-\operatorname{atan}\left(-\frac{x\,1\mathrm{i}}{\sqrt{\sqrt{2}-2}}+\frac{x\,1\mathrm{i}}{\sqrt{\sqrt{2}+2}}+\frac{\sqrt{2}\,x\,1\mathrm{i}}{2\sqrt{\sqrt{2}-2}}+\frac{\sqrt{2}\,x\,1\mathrm{i}}{2\sqrt{\sqrt{2}+2}}\right)\left(\frac{\sqrt{\sqrt{2}-2}\,1\mathrm{i}}{8}+\frac{\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{8}\right)+\operatorname{atan}\left(\frac{x\,1\mathrm{i}}{\sqrt{-\sqrt{2}-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^8 + 1)),x)
```

```
[Out] atan((x*1i)/(- 2^(1/2) - 2)^(1/2) + (x*1i)/(2 - 2^(1/2))^(1/2) + (2^(1/2)*x
*1i)/(2*(- 2^(1/2) - 2)^(1/2)) - (2^(1/2)*x*1i)/(2*(2 - 2^(1/2))^(1/2)))*((
(- 2^(1/2) - 2)^(1/2)*1i)/8 - ((2 - 2^(1/2))^(1/2)*1i)/8) - atan((x*1i)/(2^
(1/2) + 2)^(1/2) - (x*1i)/(2^(1/2) - 2)^(1/2) + (2^(1/2)*x*1i)/(2*(2^(1/2)
- 2)^(1/2)) + (2^(1/2)*x*1i)/(2*(2^(1/2) + 2)^(1/2)))*(((2^(1/2) - 2)^(1/2)
*1i)/8 + ((2^(1/2) + 2)^(1/2)*1i)/8) - 1/x - atan(x*(2^(1/2) + 2)^(1/2)*(1/
2 - 1i/2) - (2^(1/2)*x*(2^(1/2) + 2)^(1/2))/2)*((2^(1/2)*1i)/16 - (1/16 + 1
i/16))*(2^(1/2) + 2)^(1/2)*2i + atan(x*(2^(1/2) + 2)^(1/2)*(1/2 + 1i/2) - (
2^(1/2)*x*(2^(1/2) + 2)^(1/2)*1i)/2)*(2^(1/2)/16 - (1/16 - 1i/16))*(2^(1/2)
 + 2)^(1/2)*2i
```

sympy **[A]** time = 3.18, size = 19, normalized size = 0.06

$$\operatorname{RootSum}\left(16777216t^8+1,\left(t\mapsto t\log\left(-2097152t^7+x\right)\right)\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**8+1),x)
```

```
[Out] RootSum(16777216*_t**8 + 1, Lambda(_t, _t*log(-2097152*_t**7 + x))) - 1/x
```

3.1506 $\int \frac{1}{x^4\left(1+x^8\right)}\,dx$

Optimal. Leaf size=354

$$-\frac{1}{3x^3}+\frac{\log\left(x^2-\sqrt{2-\sqrt{2}}\,x+1\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\log\left(x^2+\sqrt{2-\sqrt{2}}\,x+1\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{2}}\,x+1\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{2}}\,x+1\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}}$$

```
[Out] -1/3/x^3+1/4*arctan((-2*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(4-2*2^(1/2
))^(1/2)-1/4*arctan((2*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(4-2*2^(1/2)
)^(1/2)+1/8*ln(1+x^2-x*(2-2^(1/2))^(1/2))/(4-2*2^(1/2))^(1/2)-1/8*ln(1+x^2+
x*(2-2^(1/2))^(1/2))/(4-2*2^(1/2))^(1/2)-1/4*arctan((-2*x+(2-2^(1/2))^(1/2)
)/(2+2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)+1/4*arctan((2*x+(2-2^(1/2))^(1/2))
/(2+2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)-1/8*ln(1+x^2-x*(2+2^(1/2))^(1/2))/(
4+2*2^(1/2))^(1/2)+1/8*ln(1+x^2+x*(2+2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {325, 299, 1127, 1161, 618, 204, 1164, 628}

$$-\frac{1}{3x^3}+\frac{\log\left(x^2-\sqrt{2-\sqrt{2}}\,x+1\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\log\left(x^2+\sqrt{2-\sqrt{2}}\,x+1\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{2}}\,x+1\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{2}}\,x+1\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 + x^8)),x]
```

```
[Out] -1/(3*x^3) - ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 + Sqrt[2]]]/(4*Sqrt[2*
(2 + Sqrt[2])]) + ArcTan[(Sqrt[2 + Sqrt[2]] - 2*x)/Sqrt[2 - Sqrt[2]]]/(4*Sq
rt[2*(2 - Sqrt[2])]) + ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2 + Sqrt[2]]]/
(4*Sqrt[2*(2 + Sqrt[2])]) - ArcTan[(Sqrt[2 + Sqrt[2]] + 2*x)/Sqrt[2 - Sqrt[
2]]]/(4*Sqrt[2*(2 - Sqrt[2])]) + Log[1 - Sqrt[2 - Sqrt[2]]*x + x^2]/(8*Sqrt
[2*(2 - Sqrt[2])]) - Log[1 + Sqrt[2 - Sqrt[2]]*x + x^2]/(8*Sqrt[2*(2 - Sqrt
[2])]) - Log[1 - Sqrt[2 + Sqrt[2]]*x + x^2]/(8*Sqrt[2*(2 + Sqrt[2])]) + Log
[1 + Sqrt[2 + Sqrt[2]]*x + x^2]/(8*Sqrt[2*(2 + Sqrt[2])])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 299

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[s^3/(2*Sqrt[2]*b*r), Int[x^(
m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Dist[s^3/(2*Sq
rt[2]*b*r), Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && G
tQ[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^4\left(1+x^8\right)}\,dx = -\frac{1}{3x^3} - \int \frac{x^4}{1+x^8}\,dx$$

$$= -\frac{1}{3x^3} - \frac{\int \frac{x^2}{1-\sqrt{2}\,x^2+x^4}\,dx}{2\sqrt{2}} + \frac{\int \frac{x^2}{1+\sqrt{2}\,x^2+x^4}\,dx}{2\sqrt{2}}$$

$$= -\frac{1}{3x^3} + \frac{\int \frac{1-x^2}{1-\sqrt{2}\,x^2+x^4}\,dx}{4\sqrt{2}} - \frac{\int \frac{1+x^2}{1-\sqrt{2}\,x^2+x^4}\,dx}{4\sqrt{2}} - \frac{\int \frac{1-x^2}{1+\sqrt{2}\,x^2+x^4}\,dx}{4\sqrt{2}} + \frac{\int \frac{1+x^2}{1+\sqrt{2}\,x^2+x^4}\,dx}{4\sqrt{2}}$$

$$= -\frac{1}{3x^3} + \frac{\int \frac{1}{1-\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{8\sqrt{2}} + \frac{\int \frac{1}{1+\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{8\sqrt{2}} - \frac{\int \frac{1}{1-\sqrt{2+\sqrt{2}}\,x+x^2}\,dx}{8\sqrt{2}} - \frac{\int \frac{1}{1+\sqrt{2+\sqrt{2}}\,x+x^2}\,dx}{8\sqrt{2}} -$$

$$= -\frac{1}{3x^3} + \frac{\log\left(1-\sqrt{2-\sqrt{2}}\,x+x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}} - \frac{\log\left(1+\sqrt{2-\sqrt{2}}\,x+x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}} - \frac{\log\left(1-\sqrt{2+\sqrt{2}}\,x+x^2\right.}{8\sqrt{2\left(2+\sqrt{2}\right)}}$$

$$= -\frac{1}{3x^3} - \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2\left(2+\sqrt{2}\right)}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2\left(2-\sqrt{2}\right)}} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}+2x}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2\left(2+\sqrt{2}\right)}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}+2x}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2\left(2-\sqrt{2}\right)}}$$

Mathematica [A] time = 0.01, size = 216, normalized size = 0.61

$$-\frac{1}{3x^3}+\frac{1}{8}\cos\left(\frac{\pi}{8}\right)\log\left(x^2-2x\sin\left(\frac{\pi}{8}\right)+1\right)-\frac{1}{8}\cos\left(\frac{\pi}{8}\right)\log\left(x^2+2x\sin\left(\frac{\pi}{8}\right)+1\right)-\frac{1}{8}\sin\left(\frac{\pi}{8}\right)\log\left(x^2-2x\cos\left(\frac{\pi}{8}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 + x^8)),x]
```

```
[Out] -1/3*1/x^3 - (ArcTan[(x - Cos[Pi/8])*Csc[Pi/8]]*Cos[Pi/8])/4 - (ArcTan[(x +
 Cos[Pi/8])*Csc[Pi/8]]*Cos[Pi/8])/4 + (Cos[Pi/8]*Log[1 + x^2 - 2*x*Sin[Pi/8
]])/8 - (Cos[Pi/8]*Log[1 + x^2 + 2*x*Sin[Pi/8]])/8 + (ArcTan[Sec[Pi/8]*(x -
 Sin[Pi/8])]*Sin[Pi/8])/4 + (ArcTan[Sec[Pi/8]*(x + Sin[Pi/8])]*Sin[Pi/8])/4
 - (Log[1 + x^2 - 2*x*Cos[Pi/8]]*Sin[Pi/8])/8 + (Log[1 + x^2 + 2*x*Cos[Pi/8
]]*Sin[Pi/8])/8
```

fricas [B] time = 0.89, size = 1105, normalized size = 3.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+1),x, algorithm="fricas")
```

```
[Out] -1/192*(24*x^3*sqrt(-sqrt(2) + 2)*arctan(-(2*x - 2*sqrt(x^2 + x*sqrt(-sqrt(
2) + 2) + 1) + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)) + 24*x^3*sqrt(-sqrt(2
) + 2)*arctan(-(2*x - 2*sqrt(x^2 - x*sqrt(-sqrt(2) + 2) + 1) - sqrt(-sqrt(2
) + 2))/sqrt(sqrt(2) + 2)) - 24*x^3*sqrt(sqrt(2) + 2)*arctan(-(2*x - 2*sqrt
(x^2 + x*sqrt(sqrt(2) + 2) + 1) + sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) -
24*x^3*sqrt(sqrt(2) + 2)*arctan(-(2*x - 2*sqrt(x^2 - x*sqrt(sqrt(2) + 2) +
1) - sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) - 6*x^3*sqrt(-sqrt(2) + 2)*log(
x^2 + x*sqrt(sqrt(2) + 2) + 1) + 6*x^3*sqrt(-sqrt(2) + 2)*log(x^2 - x*sqrt(
sqrt(2) + 2) + 1) + 6*x^3*sqrt(sqrt(2) + 2)*log(x^2 + x*sqrt(-sqrt(2) + 2)
+ 1) - 6*x^3*sqrt(sqrt(2) + 2)*log(x^2 - x*sqrt(-sqrt(2) + 2) + 1) + 12*(sq
rt(2)*x^3*sqrt(sqrt(2) + 2) - sqrt(2)*x^3*sqrt(-sqrt(2) + 2))*arctan(-(2*sq
```

```
rt(2)*x - 2*sqrt(2)*sqrt(x^2 + 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) - 1/2*sqrt(2
)*x*sqrt(-sqrt(2) + 2) + 1) + sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(sqrt
(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))) + 12*(sqrt(2)*x^3*sqrt(sqrt(2) + 2) -
sqrt(2)*x^3*sqrt(-sqrt(2) + 2))*arctan(-(2*sqrt(2)*x - 2*sqrt(2)*sqrt(x^2 -
 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) + 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) -
sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2)
+ 2))) + 12*(sqrt(2)*x^3*sqrt(sqrt(2) + 2) + sqrt(2)*x^3*sqrt(-sqrt(2) + 2)
)*arctan((2*sqrt(2)*x - 2*sqrt(2)*sqrt(x^2 + 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2
) + 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) + sqrt(sqrt(2) + 2) + sqrt(-sqrt(
2) + 2))/(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))) + 12*(sqrt(2)*x^3*sqrt(s
qrt(2) + 2) + sqrt(2)*x^3*sqrt(-sqrt(2) + 2))*arctan((2*sqrt(2)*x - 2*sqrt(
2)*sqrt(x^2 - 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) - 1/2*sqrt(2)*x*sqrt(-sqrt(2)
 + 2) + 1) - sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) - s
qrt(-sqrt(2) + 2))) - 3*(sqrt(2)*x^3*sqrt(sqrt(2) + 2) - sqrt(2)*x^3*sqrt(-
sqrt(2) + 2))*log(x^2 + 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) + 1/2*sqrt(2)*x*sqr
t(-sqrt(2) + 2) + 1) + 3*(sqrt(2)*x^3*sqrt(sqrt(2) + 2) + sqrt(2)*x^3*sqrt(
-sqrt(2) + 2))*log(x^2 + 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) - 1/2*sqrt(2)*x*sq
rt(-sqrt(2) + 2) + 1) - 3*(sqrt(2)*x^3*sqrt(sqrt(2) + 2) + sqrt(2)*x^3*sqrt
(-sqrt(2) + 2))*log(x^2 - 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) + 1/2*sqrt(2)*x*s
qrt(-sqrt(2) + 2) + 1) + 3*(sqrt(2)*x^3*sqrt(sqrt(2) + 2) - sqrt(2)*x^3*sqr
t(-sqrt(2) + 2))*log(x^2 - 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) - 1/2*sqrt(2)*x*
sqrt(-sqrt(2) + 2) + 1) + 64)/x^3
```

giac [A] time = 0.31, size = 244, normalized size = 0.69

$$\frac{1}{8}\sqrt{-\sqrt{2}+2}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)+\frac{1}{8}\sqrt{-\sqrt{2}+2}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)-\frac{1}{8}\sqrt{\sqrt{2}+2}\arctan\left(\frac{2x}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+1),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(-sqrt(2) + 2)*arctan((2*x + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2))
 + 1/8*sqrt(-sqrt(2) + 2)*arctan((2*x - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) +
2)) - 1/8*sqrt(sqrt(2) + 2)*arctan((2*x + sqrt(sqrt(2) + 2))/sqrt(-sqrt(2)
+ 2)) - 1/8*sqrt(sqrt(2) + 2)*arctan((2*x - sqrt(sqrt(2) + 2))/sqrt(-sqrt(2
) + 2)) + 1/16*sqrt(-sqrt(2) + 2)*log(x^2 + x*sqrt(sqrt(2) + 2) + 1) - 1/16
*sqrt(-sqrt(2) + 2)*log(x^2 - x*sqrt(sqrt(2) + 2) + 1) - 1/16*sqrt(sqrt(2)
+ 2)*log(x^2 + x*sqrt(-sqrt(2) + 2) + 1) + 1/16*sqrt(sqrt(2) + 2)*log(x^2 -
 x*sqrt(-sqrt(2) + 2) + 1) - 1/3/x^3
```

maple [C] time = 0.01, size = 28, normalized size = 0.08

$$-\frac{\ln\left(-\operatorname{RootOf}\left(_Z^8+1\right)+x\right)}{8\operatorname{RootOf}\left(_Z^8+1\right)^3}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^8+1),x)
```

```
[Out] -1/3/x^3-1/8*sum(1/_R^3*ln(-_R+x),_R=RootOf(_Z^8+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3x^3}-\int\frac{x^4}{x^8+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+1),x, algorithm="maxima")
```

```
[Out] -1/3/x^3 - integrate(x^4/(x^8 + 1), x)
```

mupad [B] time = 0.06, size = 298, normalized size = 0.84

$$\operatorname{atan}\left(\frac{x\sqrt{-\sqrt{2}-2}\,1\mathrm{i}}{\sqrt{2-\sqrt{2}}\,\sqrt{-\sqrt{2}-2}-\sqrt{2}}-\frac{x\sqrt{2-\sqrt{2}}\,1\mathrm{i}}{\sqrt{2-\sqrt{2}}\,\sqrt{-\sqrt{2}-2}-\sqrt{2}}\right)\left(\frac{\sqrt{-\sqrt{2}-2}\,1\mathrm{i}}{8}-\frac{\sqrt{2-\sqrt{2}}\,1\mathrm{i}}{8}\right)-\frac{1}{3\,x^3}+\operatorname{atan}($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x^8 + 1)),x)
```

```
[Out] atan((x*(- 2^(1/2) - 2)^(1/2)*1i)/((2 - 2^(1/2))^(1/2)*(- 2^(1/2) - 2)^(1/2
) - 2^(1/2)) - (x*(2 - 2^(1/2))^(1/2)*1i)/((2 - 2^(1/2))^(1/2)*(- 2^(1/2) -
 2)^(1/2) - 2^(1/2)))*(((- 2^(1/2) - 2)^(1/2)*1i)/8 - ((2 - 2^(1/2))^(1/2)*
1i)/8) - 1/(3*x^3) + atan((x*(2^(1/2) - 2)^(1/2)*1i)/(2^(1/2) - (2^(1/2) -
2)^(1/2)*(2^(1/2) + 2)^(1/2)) + (x*(2^(1/2) + 2)^(1/2)*1i)/(2^(1/2) - (2^(1
/2) - 2)^(1/2)*(2^(1/2) + 2)^(1/2)))*(((2^(1/2) - 2)^(1/2)*1i)/8 + ((2^(1/2
) + 2)^(1/2)*1i)/8) - atan(x*(2^(1/2) + 2)^(1/2)*(1/2 + 1i/2) - (2^(1/2)*x*
(2^(1/2) + 2)^(1/2))/2)*(2^(1/2)/16 - (1/16 - 1i/16))*(2^(1/2) + 2)^(1/2)*2
i - atan(x*(2^(1/2) + 2)^(1/2)*(1/2 - 1i/2) + (2^(1/2)*x*(2^(1/2) + 2)^(1/2
)*1i)/2)*((2^(1/2)*1i)/16 - (1/16 + 1i/16))*(2^(1/2) + 2)^(1/2)*2i
```

sympy [A] time = 3.17, size = 22, normalized size = 0.06

$$\operatorname{RootSum}\left(16777216t^8+1,\left(t\mapsto t\log\left(32768t^5+x\right)\right)\right)-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**8+1),x)
```

```
[Out] RootSum(16777216*_t**8 + 1, Lambda(_t, _t*log(32768*_t**5 + x))) - 1/(3*x**
3)
```

3.1507 $\int \frac{1}{x^6\left(1+x^8\right)}\,dx$

Optimal. Leaf size=346

$$-\frac{1}{5x^5}-\frac{\log\left(x^2-\sqrt{2-\sqrt{2}}\,x+1\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{2}}\,x+1\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}}+\frac{\log\left(x^2-\sqrt{2+\sqrt{2}}\,x+1\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}}-\frac{\log\left(x^2+\sqrt{2+\sqrt{}}\right.}{8\sqrt{2\left(2+\sqrt{}\right.}}$$

```
[Out] -1/5/x^5-1/8*arctan((-2*x+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*(2-2^(1/2))
^(1/2)+1/8*arctan((2*x+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*(2-2^(1/2))^(1
/2)-1/8*ln(1+x^2-x*(2-2^(1/2))^(1/2))/(4-2*2^(1/2))^(1/2)+1/8*ln(1+x^2+x*(2
-2^(1/2))^(1/2))/(4-2*2^(1/2))^(1/2)+1/8*arctan((-2*x+(2+2^(1/2))^(1/2))/(2
-2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)-1/8*arctan((2*x+(2+2^(1/2))^(1/2))/(2-2^
(1/2))^(1/2))*(2+2^(1/2))^(1/2)+1/8*ln(1+x^2-x*(2+2^(1/2))^(1/2))/(4+2*2^(1
/2))^(1/2)-1/8*ln(1+x^2+x*(2+2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {325, 299, 1094, 634, 618, 204, 628}

$$-\frac{1}{5x^5}-\frac{\log\left(x^2-\sqrt{2-\sqrt{2}}\,x+1\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{2}}\,x+1\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}}+\frac{\log\left(x^2-\sqrt{2+\sqrt{2}}\,x+1\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}}-\frac{\log\left(x^2+\sqrt{2+\sqrt{}}\right.}{8\sqrt{2\left(2+\sqrt{}\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(1 + x^8)),x]
```

```
[Out] -1/(5*x^5) - (Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 + S
qrt[2]]])/8 + (Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] - 2*x)/Sqrt[2 -
Sqrt[2]]])/8 + (Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2 +
 Sqrt[2]]])/8 - (Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] + 2*x)/Sqrt[2
- Sqrt[2]]])/8 - Log[1 - Sqrt[2 - Sqrt[2]]*x + x^2]/(8*Sqrt[2*(2 - Sqrt[2])
]) + Log[1 + Sqrt[2 - Sqrt[2]]*x + x^2]/(8*Sqrt[2*(2 - Sqrt[2])]) + Log[1 -
 Sqrt[2 + Sqrt[2]]*x + x^2]/(8*Sqrt[2*(2 + Sqrt[2])]) - Log[1 + Sqrt[2 + Sq
rt[2]]*x + x^2]/(8*Sqrt[2*(2 + Sqrt[2])])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 299

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[s^3/(2*Sqrt[2]*b*r), Int[x^(
m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Dist[s^3/(2*Sq
rt[2]*b*r), Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && G
tQ[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^6\left(1+x^8\right)}\,dx = -\frac{1}{5x^5} - \int \frac{x^2}{1+x^8}\,dx$$

$$= -\frac{1}{5x^5} - \frac{\int \frac{1}{1-\sqrt{2}\,x^2+x^4}\,dx}{2\sqrt{2}} + \frac{\int \frac{1}{1+\sqrt{2}\,x^2+x^4}\,dx}{2\sqrt{2}}$$

$$= -\frac{1}{5x^5} + \frac{\int \frac{\sqrt{2-\sqrt{2}}-x}{1-\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2\left(2-\sqrt{2}\right)}} + \frac{\int \frac{\sqrt{2-\sqrt{2}}+x}{1+\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2\left(2-\sqrt{2}\right)}} - \frac{\int \frac{\sqrt{2+\sqrt{2}}-x}{1-\sqrt{2+\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2\left(2+\sqrt{2}\right)}} - \frac{\int \frac{\sqrt{2+\sqrt{2}}+x}{1+\sqrt{2+\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2\left(2+\sqrt{2}\right)}}$$

$$= -\frac{1}{5x^5} + \frac{\int \frac{1}{1-\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{8\sqrt{2}} + \frac{\int \frac{1}{1+\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{8\sqrt{2}} - \frac{\int \frac{1}{1-\sqrt{2+\sqrt{2}}\,x+x^2}\,dx}{8\sqrt{2}} - \frac{\int \frac{1}{1+\sqrt{2+\sqrt{2}}\,x+x^2}\,dx}{8\sqrt{2}}$$

$$= -\frac{1}{5x^5} - \frac{\log\left(1-\sqrt{2-\sqrt{2}}\,x+x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}} + \frac{\log\left(1+\sqrt{2-\sqrt{2}}\,x+x^2\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}} + \frac{\log\left(1-\sqrt{2+\sqrt{2}}\,x+x^2\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}}$$

$$= -\frac{1}{5x^5} - \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2\left(2+\sqrt{2}\right)}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2\left(2-\sqrt{2}\right)}} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}+2x}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2\left(2+\sqrt{2}\right)}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}+2x}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2\left(2-\sqrt{2}\right)}}$$

Mathematica [A] time = 0.01, size = 216, normalized size = 0.62

$$-\frac{1}{5x^5}-\frac{1}{8}\cos\left(\frac{\pi}{8}\right)\log\left(x^2-2x\sin\left(\frac{\pi}{8}\right)+1\right)+\frac{1}{8}\cos\left(\frac{\pi}{8}\right)\log\left(x^2+2x\sin\left(\frac{\pi}{8}\right)+1\right)+\frac{1}{8}\sin\left(\frac{\pi}{8}\right)\log\left(x^2-2x\cos\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(1 + x^8)),x]
```

```
[Out] -1/5*1/x^5 - (ArcTan[(x - Cos[Pi/8])*Csc[Pi/8]]*Cos[Pi/8])/4 - (ArcTan[(x +
 Cos[Pi/8])*Csc[Pi/8]]*Cos[Pi/8])/4 - (Cos[Pi/8]*Log[1 + x^2 - 2*x*Sin[Pi/8
]])/8 + (Cos[Pi/8]*Log[1 + x^2 + 2*x*Sin[Pi/8]])/8 + (ArcTan[Sec[Pi/8]*(x -
 Sin[Pi/8])]*Sin[Pi/8])/4 + (ArcTan[Sec[Pi/8]*(x + Sin[Pi/8])]*Sin[Pi/8])/4
 + (Log[1 + x^2 - 2*x*Cos[Pi/8]]*Sin[Pi/8])/8 - (Log[1 + x^2 + 2*x*Cos[Pi/8
]]*Sin[Pi/8])/8
```

fricas [B] time = 0.99, size = 2591, normalized size = 7.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8+1),x, algorithm="fricas")
```

```
[Out] -1/320*(40*x^5*sqrt(sqrt(2) + 2)*arctan((3*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2)
 - (-sqrt(2) + 2)^(3/2) + 8*x - sqrt((sqrt(2) + 2)^3 - 3*(sqrt(2) + 2)^2*(s
qrt(2) - 2) + 3*(sqrt(2) + 2)*(sqrt(2) - 2)^2 - (sqrt(2) - 2)^3 + 48*x*(sqr
t(2) + 2)*sqrt(-sqrt(2) + 2) - 16*x*(-sqrt(2) + 2)^(3/2) + 64*x^2))/((sqrt(
2) + 2)^(3/2) + 3*sqrt(sqrt(2) + 2)*(sqrt(2) - 2))) + 40*x^5*sqrt(sqrt(2) +
 2)*arctan(-(3*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) - (-sqrt(2) + 2)^(3/2) - 8*
x + sqrt((sqrt(2) + 2)^3 - 3*(sqrt(2) + 2)^2*(sqrt(2) - 2) + 3*(sqrt(2) + 2
)*(sqrt(2) - 2)^2 - (sqrt(2) - 2)^3 - 48*x*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2)
 + 16*x*(-sqrt(2) + 2)^(3/2) + 64*x^2))/((sqrt(2) + 2)^(3/2) + 3*sqrt(sqrt(
2) + 2)*(sqrt(2) - 2))) - 40*x^5*sqrt(-sqrt(2) + 2)*arctan(((sqrt(2) + 2)^(
3/2) + 3*sqrt(sqrt(2) + 2)*(sqrt(2) - 2) + 8*x - sqrt((sqrt(2) + 2)^3 + 3*(
sqrt(2) + 2)*(sqrt(2) - 2)^2 - (sqrt(2) - 2)^3 + 16*x*(sqrt(2) + 2)^(3/2) +
 64*x^2 - 3*((sqrt(2) + 2)^2 - 16*x*sqrt(sqrt(2) + 2))*(sqrt(2) - 2)))/(3*(
sqrt(2) + 2)*sqrt(-sqrt(2) + 2) - (-sqrt(2) + 2)^(3/2))) - 40*x^5*sqrt(-sqr
t(2) + 2)*arctan(-((sqrt(2) + 2)^(3/2) + 3*sqrt(sqrt(2) + 2)*(sqrt(2) - 2)
- 8*x + sqrt((sqrt(2) + 2)^3 + 3*(sqrt(2) + 2)*(sqrt(2) - 2)^2 - (sqrt(2) -
 2)^3 - 16*x*(sqrt(2) + 2)^(3/2) + 64*x^2 - 3*((sqrt(2) + 2)^2 + 16*x*sqrt(
sqrt(2) + 2))*(sqrt(2) - 2)))/(3*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) - (-sqrt(
2) + 2)^(3/2))) + 10*x^5*sqrt(-sqrt(2) + 2)*log(1/64*(sqrt(2) + 2)^3 - 3/64
*(sqrt(2) + 2)^2*(sqrt(2) - 2) + 3/64*(sqrt(2) + 2)*(sqrt(2) - 2)^2 - 1/64*
(sqrt(2) - 2)^3 + 3/4*x*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) - 1/4*x*(-sqrt(2)
+ 2)^(3/2) + x^2) - 10*x^5*sqrt(-sqrt(2) + 2)*log(1/64*(sqrt(2) + 2)^3 - 3/
64*(sqrt(2) + 2)^2*(sqrt(2) - 2) + 3/64*(sqrt(2) + 2)*(sqrt(2) - 2)^2 - 1/6
4*(sqrt(2) - 2)^3 - 3/4*x*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) + 1/4*x*(-sqrt(2
) + 2)^(3/2) + x^2) - 10*x^5*sqrt(sqrt(2) + 2)*log(1/64*(sqrt(2) + 2)^3 + 3
/64*(sqrt(2) + 2)*(sqrt(2) - 2)^2 - 1/64*(sqrt(2) - 2)^3 + 1/4*x*(sqrt(2) +
 2)^(3/2) + x^2 - 3/64*((sqrt(2) + 2)^2 - 16*x*sqrt(sqrt(2) + 2))*(sqrt(2)
- 2)) + 10*x^5*sqrt(sqrt(2) + 2)*log(1/64*(sqrt(2) + 2)^3 + 3/64*(sqrt(2) +
 2)*(sqrt(2) - 2)^2 - 1/64*(sqrt(2) - 2)^3 - 1/4*x*(sqrt(2) + 2)^(3/2) + x^
2 - 3/64*((sqrt(2) + 2)^2 + 16*x*sqrt(sqrt(2) + 2))*(sqrt(2) - 2)) - 20*(sq
rt(2)*x^5*sqrt(sqrt(2) + 2) - sqrt(2)*x^5*sqrt(-sqrt(2) + 2))*arctan((8*sqr
t(2)*x + (sqrt(2) + 2)^(3/2) + 3*sqrt(sqrt(2) + 2)*(sqrt(2) - 2) - 3*(sqrt(
2) + 2)*sqrt(-sqrt(2) + 2) + (-sqrt(2) + 2)^(3/2) - sqrt(2)*sqrt(8*sqrt(2)*
x*(sqrt(2) + 2)^(3/2) + (sqrt(2) + 2)^3 + 3*(sqrt(2) + 2)*(sqrt(2) - 2)^2 -
 (sqrt(2) - 2)^3 - 24*sqrt(2)*x*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) + 8*sqrt(2
)*x*(-sqrt(2) + 2)^(3/2) + 64*x^2 + 3*(8*sqrt(2)*x*sqrt(sqrt(2) + 2) - (sqr
t(2) + 2)^2)*(sqrt(2) - 2)))/((sqrt(2) + 2)^(3/2) + 3*sqrt(sqrt(2) + 2)*(sq
```

```
rt(2) - 2) + 3*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) - (-sqrt(2) + 2)^(3/2))) -
20*(sqrt(2)*x^5*sqrt(sqrt(2) + 2) - sqrt(2)*x^5*sqrt(-sqrt(2) + 2))*arctan(
(8*sqrt(2)*x - (sqrt(2) + 2)^(3/2) - 3*sqrt(sqrt(2) + 2)*(sqrt(2) - 2) + 3*
(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) - (-sqrt(2) + 2)^(3/2) - 8*sqrt(2)*sqrt(-1
/8*sqrt(2)*x*(sqrt(2) + 2)^(3/2) + 1/64*(sqrt(2) + 2)^3 + 3/64*(sqrt(2) + 2
)*(sqrt(2) - 2)^2 - 1/64*(sqrt(2) - 2)^3 + 3/8*sqrt(2)*x*(sqrt(2) + 2)*sqrt
(-sqrt(2) + 2) - 1/8*sqrt(2)*x*(-sqrt(2) + 2)^(3/2) + x^2 - 3/64*(8*sqrt(2)
*x*sqrt(sqrt(2) + 2) + (sqrt(2) + 2)^2)*(sqrt(2) - 2)))/((sqrt(2) + 2)^(3/2
) + 3*sqrt(sqrt(2) + 2)*(sqrt(2) - 2) + 3*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2)
- (-sqrt(2) + 2)^(3/2))) + 20*(sqrt(2)*x^5*sqrt(sqrt(2) + 2) + sqrt(2)*x^5*
sqrt(-sqrt(2) + 2))*arctan(-(8*sqrt(2)*x + (sqrt(2) + 2)^(3/2) + 3*sqrt(sqr
t(2) + 2)*(sqrt(2) - 2) + 3*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) - (-sqrt(2) +
2)^(3/2) - sqrt(2)*sqrt(8*sqrt(2)*x*(sqrt(2) + 2)^(3/2) + (sqrt(2) + 2)^3 +
 3*(sqrt(2) + 2)*(sqrt(2) - 2)^2 - (sqrt(2) - 2)^3 + 24*sqrt(2)*x*(sqrt(2)
+ 2)*sqrt(-sqrt(2) + 2) - 8*sqrt(2)*x*(-sqrt(2) + 2)^(3/2) + 64*x^2 + 3*(8*
sqrt(2)*x*sqrt(sqrt(2) + 2) - (sqrt(2) + 2)^2)*(sqrt(2) - 2)))/((sqrt(2) +
2)^(3/2) + 3*sqrt(sqrt(2) + 2)*(sqrt(2) - 2) - 3*(sqrt(2) + 2)*sqrt(-sqrt(2
) + 2) + (-sqrt(2) + 2)^(3/2))) + 20*(sqrt(2)*x^5*sqrt(sqrt(2) + 2) + sqrt(
2)*x^5*sqrt(-sqrt(2) + 2))*arctan(-(8*sqrt(2)*x - (sqrt(2) + 2)^(3/2) - 3*s
qrt(sqrt(2) + 2)*(sqrt(2) - 2) - 3*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) + (-sqr
t(2) + 2)^(3/2) - 8*sqrt(2)*sqrt(-1/8*sqrt(2)*x*(sqrt(2) + 2)^(3/2) + 1/64*
(sqrt(2) + 2)^3 + 3/64*(sqrt(2) + 2)*(sqrt(2) - 2)^2 - 1/64*(sqrt(2) - 2)^3
 - 3/8*sqrt(2)*x*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) + 1/8*sqrt(2)*x*(-sqrt(2)
 + 2)^(3/2) + x^2 - 3/64*(8*sqrt(2)*x*sqrt(sqrt(2) + 2) + (sqrt(2) + 2)^2)*
(sqrt(2) - 2)))/((sqrt(2) + 2)^(3/2) + 3*sqrt(sqrt(2) + 2)*(sqrt(2) - 2) -
3*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) + (-sqrt(2) + 2)^(3/2))) + 5*(sqrt(2)*x^
5*sqrt(sqrt(2) + 2) - sqrt(2)*x^5*sqrt(-sqrt(2) + 2))*log(1/8*sqrt(2)*x*(sq
rt(2) + 2)^(3/2) + 1/64*(sqrt(2) + 2)^3 + 3/64*(sqrt(2) + 2)*(sqrt(2) - 2)^
2 - 1/64*(sqrt(2) - 2)^3 + 3/8*sqrt(2)*x*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) -
 1/8*sqrt(2)*x*(-sqrt(2) + 2)^(3/2) + x^2 + 3/64*(8*sqrt(2)*x*sqrt(sqrt(2)
+ 2) - (sqrt(2) + 2)^2)*(sqrt(2) - 2)) + 5*(sqrt(2)*x^5*sqrt(sqrt(2) + 2) +
 sqrt(2)*x^5*sqrt(-sqrt(2) + 2))*log(1/8*sqrt(2)*x*(sqrt(2) + 2)^(3/2) + 1/
64*(sqrt(2) + 2)^3 + 3/64*(sqrt(2) + 2)*(sqrt(2) - 2)^2 - 1/64*(sqrt(2) - 2
)^3 - 3/8*sqrt(2)*x*(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) + 1/8*sqrt(2)*x*(-sqrt
(2) + 2)^(3/2) + x^2 + 3/64*(8*sqrt(2)*x*sqrt(sqrt(2) + 2) - (sqrt(2) + 2)^
2)*(sqrt(2) - 2)) - 5*(sqrt(2)*x^5*sqrt(sqrt(2) + 2) + sqrt(2)*x^5*sqrt(-sq
rt(2) + 2))*log(-1/8*sqrt(2)*x*(sqrt(2) + 2)^(3/2) + 1/64*(sqrt(2) + 2)^3 +
 3/64*(sqrt(2) + 2)*(sqrt(2) - 2)^2 - 1/64*(sqrt(2) - 2)^3 + 3/8*sqrt(2)*x*
(sqrt(2) + 2)*sqrt(-sqrt(2) + 2) - 1/8*sqrt(2)*x*(-sqrt(2) + 2)^(3/2) + x^2
 - 3/64*(8*sqrt(2)*x*sqrt(sqrt(2) + 2) + (sqrt(2) + 2)^2)*(sqrt(2) - 2)) -
5*(sqrt(2)*x^5*sqrt(sqrt(2) + 2) - sqrt(2)*x^5*sqrt(-sqrt(2) + 2))*log(-1/8
*sqrt(2)*x*(sqrt(2) + 2)^(3/2) + 1/64*(sqrt(2) + 2)^3 + 3/64*(sqrt(2) + 2)*
(sqrt(2) - 2)^2 - 1/64*(sqrt(2) - 2)^3 - 3/8*sqrt(2)*x*(sqrt(2) + 2)*sqrt(-
sqrt(2) + 2) + 1/8*sqrt(2)*x*(-sqrt(2) + 2)^(3/2) + x^2 - 3/64*(8*sqrt(2)*x
*sqrt(sqrt(2) + 2) + (sqrt(2) + 2)^2)*(sqrt(2) - 2)) + 64)/x^5
```

giac [A] time = 0.34, size = 244, normalized size = 0.71

$$\frac{1}{8}\sqrt{-\sqrt{2}+2}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)+\frac{1}{8}\sqrt{-\sqrt{2}+2}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)-\frac{1}{8}\sqrt{\sqrt{2}+2}\arctan\left(\frac{2x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8+1),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(-sqrt(2) + 2)*arctan((2*x + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2))
 + 1/8*sqrt(-sqrt(2) + 2)*arctan((2*x - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) +
2)) - 1/8*sqrt(sqrt(2) + 2)*arctan((2*x + sqrt(sqrt(2) + 2))/sqrt(-sqrt(2)
+ 2)) - 1/8*sqrt(sqrt(2) + 2)*arctan((2*x - sqrt(sqrt(2) + 2))/sqrt(-sqrt(2
```

```
) + 2)) - 1/16*sqrt(-sqrt(2) + 2)*log(x^2 + x*sqrt(sqrt(2) + 2) + 1) + 1/16
*sqrt(-sqrt(2) + 2)*log(x^2 - x*sqrt(sqrt(2) + 2) + 1) + 1/16*sqrt(sqrt(2)
+ 2)*log(x^2 + x*sqrt(-sqrt(2) + 2) + 1) - 1/16*sqrt(sqrt(2) + 2)*log(x^2 -
 x*sqrt(-sqrt(2) + 2) + 1) - 1/5/x^5
```

maple [C] time = 0.01, size = 28, normalized size = 0.08

$$-\frac{\ln\left(-\operatorname{RootOf}\left(_Z^8+1\right)+x\right)}{8\operatorname{RootOf}\left(_Z^8+1\right)^5}-\frac{1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^8+1),x)
```

```
[Out] -1/5/x^5-1/8*sum(1/_R^5*ln(-_R+x),_R=RootOf(_Z^8+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{5\,x^5}-\int\frac{x^2}{x^8+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8+1),x, algorithm="maxima")
```

```
[Out] -1/5/x^5 - integrate(x^2/(x^8 + 1), x)
```

mupad [B] time = 1.06, size = 196, normalized size = 0.57

$$\operatorname{atan}\left(x\left(\frac{\sqrt{-\sqrt{2}-2}}{16}-\frac{\sqrt{2-\sqrt{2}}}{16}\right)^5 32768\mathrm{i}\right)\left(\frac{\sqrt{-\sqrt{2}-2}\,1\mathrm{i}}{8}-\frac{\sqrt{2-\sqrt{2}}\,1\mathrm{i}}{8}\right)+\operatorname{atan}\left(x\left(\frac{\sqrt{\sqrt{2}-2}}{16}+\frac{\sqrt{\sqrt{2}+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(x^8 + 1)),x)
```

```
[Out] atan(x*((- 2^(1/2) - 2)^(1/2)/16 - (2 - 2^(1/2))^(1/2)/16)^5*32768i)*(((- 2
^(1/2) - 2)^(1/2)*1i)/8 - ((2 - 2^(1/2))^(1/2)*1i)/8) + atan(x*((2^(1/2) -
2)^(1/2)/16 + (2^(1/2) + 2)^(1/2)/16)^5*32768i)*(((2^(1/2) - 2)^(1/2)*1i)/8
 + ((2^(1/2) + 2)^(1/2)*1i)/8) - 1/(5*x^5) + atan(x*(2^(1/2)/16 - (1/16 - 1
i/16))^5*(2^(1/2) + 2)^(5/2)*32768i)*(2^(1/2)/16 - (1/16 - 1i/16))*(2^(1/2)
 + 2)^(1/2)*2i + atan(x*((2^(1/2)*1i)/16 - (1/16 + 1i/16))^5*(2^(1/2) + 2)^
(5/2)*32768i)*((2^(1/2)*1i)/16 - (1/16 + 1i/16))*(2^(1/2) + 2)^(1/2)*2i
```

sympy [A] time = 3.18, size = 22, normalized size = 0.06

$$\operatorname{RootSum}\left(16777216t^8+1,\left(t\mapsto t\log\left(512t^3+x\right)\right)\right)-\frac{1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**8+1),x)
```

```
[Out] RootSum(16777216*_t**8 + 1, Lambda(_t, _t*log(512*_t**3 + x))) - 1/(5*x**5)
```

3.1508 $\int \frac{1}{x^8\left(1+x^8\right)}\,dx$

Optimal. Leaf size=346

$$-\frac{1}{7x^7}+\frac{1}{16}\sqrt{2-\sqrt{2}}\,\log\left(x^2-\sqrt{2-\sqrt{2}}\,x+1\right)-\frac{1}{16}\sqrt{2-\sqrt{2}}\,\log\left(x^2+\sqrt{2-\sqrt{2}}\,x+1\right)+\frac{1}{16}\sqrt{2+\sqrt{2}}\,\log\left(x^2-$$

```
[Out] -1/7/x^7+1/16*ln(1+x^2-x*(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)-1/16*ln(1+x^2
+x*(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)+1/4*arctan((-2*x+(2-2^(1/2))^(1/2))
/(2+2^(1/2))^(1/2))/(4-2*2^(1/2))^(1/2)-1/4*arctan((2*x+(2-2^(1/2))^(1/2))/
(2+2^(1/2))^(1/2))/(4-2*2^(1/2))^(1/2)+1/16*ln(1+x^2-x*(2+2^(1/2))^(1/2))*(
2+2^(1/2))^(1/2)-1/16*ln(1+x^2+x*(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)+1/4*a
rctan((-2*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)-1/4*a
rctan((2*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {325, 213, 1169, 634, 618, 204, 628}

$$-\frac{1}{7x^7}+\frac{1}{16}\sqrt{2-\sqrt{2}}\,\log\left(x^2-\sqrt{2-\sqrt{2}}\,x+1\right)-\frac{1}{16}\sqrt{2-\sqrt{2}}\,\log\left(x^2+\sqrt{2-\sqrt{2}}\,x+1\right)+\frac{1}{16}\sqrt{2+\sqrt{2}}\,\log\left(x^2-$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(1 + x^8)),x]
```

```
[Out] -1/(7*x^7) + ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 + Sqrt[2]]]/(4*Sqrt[2*
(2 - Sqrt[2])]) + ArcTan[(Sqrt[2 + Sqrt[2]] - 2*x)/Sqrt[2 - Sqrt[2]]]/(4*Sq
rt[2*(2 + Sqrt[2])]) - ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2 + Sqrt[2]]]/
(4*Sqrt[2*(2 - Sqrt[2])]) - ArcTan[(Sqrt[2 + Sqrt[2]] + 2*x)/Sqrt[2 - Sqrt[
2]]]/(4*Sqrt[2*(2 + Sqrt[2])]) + (Sqrt[2 - Sqrt[2]]*Log[1 - Sqrt[2 - Sqrt[2
]]*x + x^2])/16 - (Sqrt[2 - Sqrt[2]]*Log[1 + Sqrt[2 - Sqrt[2]]*x + x^2])/16
 + (Sqrt[2 + Sqrt[2]]*Log[1 - Sqrt[2 + Sqrt[2]]*x + x^2])/16 - (Sqrt[2 + Sq
rt[2]]*Log[1 + Sqrt[2 + Sqrt[2]]*x + x^2])/16
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b,
 4]], s = Denominator[Rt[a/b, 4]]}, Dist[r/(2*Sqrt[2]*a), Int[(Sqrt[2]*r -
s*x^(n/4))/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] + Dist[r/(2*Sq
rt[2]*a), Int[(Sqrt[2]*r + s*x^(n/4))/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n
/2)), x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 1] && GtQ[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^8\left(1+x^8\right)}\,dx = -\frac{1}{7x^7} - \int \frac{1}{1+x^8}\,dx$$

$$= -\frac{1}{7x^7} - \frac{\int \frac{\sqrt{2}-x^2}{1-\sqrt{2}\,x^2+x^4}\,dx}{2\sqrt{2}} - \frac{\int \frac{\sqrt{2}+x^2}{1+\sqrt{2}\,x^2+x^4}\,dx}{2\sqrt{2}}$$

$$= -\frac{1}{7x^7} - \frac{\int \frac{\sqrt{2\left(2-\sqrt{2}\right)}-\left(-1+\sqrt{2}\right)x}{1-\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2\left(2-\sqrt{2}\right)}} - \frac{\int \frac{\sqrt{2\left(2-\sqrt{2}\right)}+\left(-1+\sqrt{2}\right)x}{1+\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2\left(2-\sqrt{2}\right)}} - \frac{\int \frac{\sqrt{2\left(2+\sqrt{2}\right)}-\left(1+\sqrt{2}\right)x}{1-\sqrt{2+\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2\left(2+\sqrt{2}\right)}} - -$$

$$= -\frac{1}{7x^7} - \frac{1}{8}\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\int \frac{1}{1-\sqrt{2+\sqrt{2}}\,x+x^2}\,dx - \frac{1}{8}\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\int \frac{1}{1+\sqrt{2+\sqrt{2}}}$$

$$= -\frac{1}{7x^7} + \frac{1}{16}\sqrt{2-\sqrt{2}}\,\log\left(1-\sqrt{2-\sqrt{2}}\,x+x^2\right) - \frac{1}{16}\sqrt{2-\sqrt{2}}\,\log\left(1+\sqrt{2-\sqrt{2}}\,x+x^2\right)$$

$$= -\frac{1}{7x^7} + \frac{1}{8}\sqrt{2+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right) + \frac{1}{8}\sqrt{2-\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right) - \frac{1}{8}\sqrt{}$$

Mathematica [A] time = 0.01, size = 216, normalized size = 0.62

$$-\frac{1}{7x^7}+\frac{1}{8}\sin\left(\frac{\pi}{8}\right)\log\left(x^2-2x\sin\left(\frac{\pi}{8}\right)+1\right)-\frac{1}{8}\sin\left(\frac{\pi}{8}\right)\log\left(x^2+2x\sin\left(\frac{\pi}{8}\right)+1\right)+\frac{1}{8}\cos\left(\frac{\pi}{8}\right)\log\left(x^2-2x\cos\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(1 + x^8)),x]
```

```
[Out] -1/7*1/x^7 - (ArcTan[Sec[Pi/8]*(x - Sin[Pi/8])]*Cos[Pi/8])/4 - (ArcTan[Sec[
Pi/8]*(x + Sin[Pi/8])]*Cos[Pi/8])/4 + (Cos[Pi/8]*Log[1 + x^2 - 2*x*Cos[Pi/8
]])/8 - (Cos[Pi/8]*Log[1 + x^2 + 2*x*Cos[Pi/8]])/8 - (ArcTan[(x - Cos[Pi/8]
)*Csc[Pi/8]]*Sin[Pi/8])/4 - (ArcTan[(x + Cos[Pi/8])*Csc[Pi/8]]*Sin[Pi/8])/4
 + (Log[1 + x^2 - 2*x*Sin[Pi/8]]*Sin[Pi/8])/8 - (Log[1 + x^2 + 2*x*Sin[Pi/8
]]*Sin[Pi/8])/8
```

fricas **[B]** time = 0.85, size = 1105, normalized size = 3.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8+1),x, algorithm="fricas")
```

```
[Out] 1/448*(56*x^7*sqrt(sqrt(2) + 2)*arctan(-(2*x - 2*sqrt(x^2 + x*sqrt(-sqrt(2)
 + 2) + 1) + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)) + 56*x^7*sqrt(sqrt(2) +
 2)*arctan(-(2*x - 2*sqrt(x^2 - x*sqrt(-sqrt(2) + 2) + 1) - sqrt(-sqrt(2) +
 2))/sqrt(sqrt(2) + 2)) + 56*x^7*sqrt(-sqrt(2) + 2)*arctan(-(2*x - 2*sqrt(x
^2 + x*sqrt(sqrt(2) + 2) + 1) + sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) + 56
*x^7*sqrt(-sqrt(2) + 2)*arctan(-(2*x - 2*sqrt(x^2 - x*sqrt(sqrt(2) + 2) + 1
) - sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) - 14*x^7*sqrt(sqrt(2) + 2)*log(x
^2 + x*sqrt(sqrt(2) + 2) + 1) + 14*x^7*sqrt(sqrt(2) + 2)*log(x^2 - x*sqrt(s
qrt(2) + 2) + 1) - 14*x^7*sqrt(-sqrt(2) + 2)*log(x^2 + x*sqrt(-sqrt(2) + 2)
 + 1) + 14*x^7*sqrt(-sqrt(2) + 2)*log(x^2 - x*sqrt(-sqrt(2) + 2) + 1) + 28*
(sqrt(2)*x^7*sqrt(sqrt(2) + 2) + sqrt(2)*x^7*sqrt(-sqrt(2) + 2))*arctan(-(2
*sqrt(2)*x - 2*sqrt(2)*sqrt(x^2 + 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) - 1/2*sqr
t(2)*x*sqrt(-sqrt(2) + 2) + 1) + sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(s
qrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))) + 28*(sqrt(2)*x^7*sqrt(sqrt(2) + 2)
 + sqrt(2)*x^7*sqrt(-sqrt(2) + 2))*arctan(-(2*sqrt(2)*x - 2*sqrt(2)*sqrt(x^
2 - 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) + 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1)
 - sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) + sqrt(-sqrt(
2) + 2))) - 28*(sqrt(2)*x^7*sqrt(sqrt(2) + 2) - sqrt(2)*x^7*sqrt(-sqrt(2) +
 2))*arctan((2*sqrt(2)*x - 2*sqrt(2)*sqrt(x^2 + 1/2*sqrt(2)*x*sqrt(sqrt(2)
+ 2) + 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) + sqrt(sqrt(2) + 2) + sqrt(-sq
rt(2) + 2))/(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))) - 28*(sqrt(2)*x^7*sqr
t(sqrt(2) + 2) - sqrt(2)*x^7*sqrt(-sqrt(2) + 2))*arctan((2*sqrt(2)*x - 2*sq
rt(2)*sqrt(x^2 - 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) - 1/2*sqrt(2)*x*sqrt(-sqrt
(2) + 2) + 1) - sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2)
- sqrt(-sqrt(2) + 2))) - 7*(sqrt(2)*x^7*sqrt(sqrt(2) + 2) + sqrt(2)*x^7*sqr
t(-sqrt(2) + 2))*log(x^2 + 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) + 1/2*sqrt(2)*x*
sqrt(-sqrt(2) + 2) + 1) - 7*(sqrt(2)*x^7*sqrt(sqrt(2) + 2) - sqrt(2)*x^7*sq
rt(-sqrt(2) + 2))*log(x^2 + 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) - 1/2*sqrt(2)*x
*sqrt(-sqrt(2) + 2) + 1) + 7*(sqrt(2)*x^7*sqrt(sqrt(2) + 2) - sqrt(2)*x^7*s
qrt(-sqrt(2) + 2))*log(x^2 - 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) + 1/2*sqrt(2)*
x*sqrt(-sqrt(2) + 2) + 1) + 7*(sqrt(2)*x^7*sqrt(sqrt(2) + 2) + sqrt(2)*x^7*
sqrt(-sqrt(2) + 2))*log(x^2 - 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) - 1/2*sqrt(2)
*x*sqrt(-sqrt(2) + 2) + 1) - 64)/x^7
```

giac **[A]** time = 0.19, size = 244, normalized size = 0.71

$$-\frac{1}{8}\sqrt{\sqrt{2}+2}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)-\frac{1}{8}\sqrt{\sqrt{2}+2}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)-\frac{1}{8}\sqrt{-\sqrt{2}+2}\arctan\left(\frac{2x+}{\sqrt{-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8+1),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(sqrt(2) + 2)*arctan((2*x + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2))
 - 1/8*sqrt(sqrt(2) + 2)*arctan((2*x - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2
)) - 1/8*sqrt(-sqrt(2) + 2)*arctan((2*x + sqrt(sqrt(2) + 2))/sqrt(-sqrt(2)
 + 2)) - 1/8*sqrt(-sqrt(2) + 2)*arctan((2*x - sqrt(sqrt(2) + 2))/sqrt(-sqrt(
2) + 2)) - 1/16*sqrt(sqrt(2) + 2)*log(x^2 + x*sqrt(sqrt(2) + 2) + 1) + 1/16
*sqrt(sqrt(2) + 2)*log(x^2 - x*sqrt(sqrt(2) + 2) + 1) - 1/16*sqrt(-sqrt(2)
 + 2)*log(x^2 + x*sqrt(-sqrt(2) + 2) + 1) + 1/16*sqrt(-sqrt(2) + 2)*log(x^2
 - x*sqrt(-sqrt(2) + 2) + 1) - 1/7/x^7
```

maple [C] time = 0.01, size = 28, normalized size = 0.08

$$-\frac{\ln\left(-\operatorname{RootOf}\left(_Z^8+1\right)+x\right)}{8\operatorname{RootOf}\left(_Z^8+1\right)^7}-\frac{1}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(x^8+1),x)
```

```
[Out] -1/8*sum(1/_R^7*ln(-_R+x),_R=RootOf(_Z^8+1))-1/7/x^7
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{7x^7}-\int\frac{1}{x^8+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8+1),x, algorithm="maxima")
```

```
[Out] -1/7/x^7 - integrate(1/(x^8 + 1), x)
```

mupad [B] time = 0.04, size = 293, normalized size = 0.85

$$-\operatorname{atan}\left(\frac{x\sqrt{-\sqrt{2}-2}\,1\mathrm{i}}{\sqrt{2-\sqrt{2}}\,\sqrt{-\sqrt{2}-2}+\sqrt{2}}-\frac{x\sqrt{2-\sqrt{2}}\,1\mathrm{i}}{\sqrt{2-\sqrt{2}}\,\sqrt{-\sqrt{2}-2}+\sqrt{2}}\right)\left(\frac{\sqrt{-\sqrt{2}-2}\,1\mathrm{i}}{8}-\frac{\sqrt{2-\sqrt{2}}\,1\mathrm{i}}{8}\right)-\frac{1}{7x^7}+a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(x^8 + 1)),x)
```

```
[Out] atan((x*(2^(1/2) - 2)^(1/2)*1i)/(2^(1/2) + (2^(1/2) - 2)^(1/2)*(2^(1/2) + 2
)^(1/2)) + (x*(2^(1/2) + 2)^(1/2)*1i)/(2^(1/2) + (2^(1/2) - 2)^(1/2)*(2^(1/
2) + 2)^(1/2)))*(((2^(1/2) - 2)^(1/2)*1i)/8 + ((2^(1/2) + 2)^(1/2)*1i)/8) -
 1/(7*x^7) - atan((x*(- 2^(1/2) - 2)^(1/2)*1i)/((2 - 2^(1/2))^(1/2)*(- 2^(1
/2) - 2)^(1/2) + 2^(1/2)) - (x*(2 - 2^(1/2))^(1/2)*1i)/((2 - 2^(1/2))^(1/2)
*(- 2^(1/2) - 2)^(1/2) + 2^(1/2)))*(((- 2^(1/2) - 2)^(1/2)*1i)/8 - ((2 - 2^
(1/2))^(1/2)*1i)/8) - atan(x*(2^(1/2) + 2)^(1/2)*(1/2 + 1i/2) - (2^(1/2)*x*
(2^(1/2) + 2)^(1/2))/2)*((2^(1/2)*1i)/16 - (1/16 + 1i/16))*(2^(1/2) + 2)^(1
/2)*2i + atan(x*(2^(1/2) + 2)^(1/2)*(1/2 - 1i/2) + (2^(1/2)*x*(2^(1/2) + 2)
^(1/2)*1i)/2)*(2^(1/2)/16 - (1/16 - 1i/16))*(2^(1/2) + 2)^(1/2)*2i
```

sympy [A] time = 3.24, size = 20, normalized size = 0.06

$$\operatorname{RootSum}\left(16777216t^8+1,\left(t\mapsto t\log\left(-8t+x\right)\right)\right)-\frac{1}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(x**8+1),x)
```

```
[Out] RootSum(16777216*_t**8 + 1, Lambda(_t, _t*log(-8*_t + x))) - 1/(7*x**7)
```

3.1509 $\int x^3\sqrt{1+x^8}\,dx$

Optimal. Leaf size=25

$$\frac{1}{8}\sinh^{-1}\left(x^4\right)+\frac{1}{8}\sqrt{x^8+1}\,x^4$$

```
[Out] 1/8*arcsinh(x^4)+1/8*x^4*(x^8+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {275, 195, 215}

$$\frac{1}{8}\sqrt{x^8+1}\,x^4+\frac{1}{8}\sinh^{-1}\left(x^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[1 + x^8],x]
```

```
[Out] (x^4*Sqrt[1 + x^8])/8 + ArcSinh[x^4]/8
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{1+x^8}\,dx &= \frac{1}{4}\text{Subst}\left(\int \sqrt{1+x^2}\,dx, x, x^4\right)\\ &= \frac{1}{8}x^4\sqrt{1+x^8}+\frac{1}{8}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, x^4\right)\\ &= \frac{1}{8}x^4\sqrt{1+x^8}+\frac{1}{8}\sinh^{-1}\left(x^4\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.88

$$\frac{1}{8}\left(\sinh^{-1}\left(x^4\right)+\sqrt{x^8+1}\,x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[1 + x^8],x]
```

```
[Out] (x^4*Sqrt[1 + x^8] + ArcSinh[x^4])/8
```

fricas [A] time = 0.85, size = 29, normalized size = 1.16

$$\frac{1}{8}\sqrt{x^8+1}\,x^4-\frac{1}{8}\log\left(-x^4+\sqrt{x^8+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^8+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(x^8 + 1)*x^4 - 1/8*log(-x^4 + sqrt(x^8 + 1))
```

giac [A] time = 0.16, size = 29, normalized size = 1.16

$$\frac{1}{8}\sqrt{x^8+1}\,x^4-\frac{1}{8}\log\left(-x^4+\sqrt{x^8+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^8+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(x^8 + 1)*x^4 - 1/8*log(-x^4 + sqrt(x^8 + 1))
```

maple [A] time = 0.15, size = 20, normalized size = 0.80

$$\frac{\sqrt{x^8+1}\;x^4}{8}+\frac{\operatorname{arcsinh}\left(x^4\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^8+1)^(1/2),x)
```

```
[Out] 1/8*arcsinh(x^4)+1/8*x^4*(x^8+1)^(1/2)
```

maxima [B] time = 1.10, size = 58, normalized size = 2.32

$$\frac{\sqrt{x^8+1}}{8\,x^4\left(\frac{x^8+1}{x^8}-1\right)}+\frac{1}{16}\log\left(\frac{\sqrt{x^8+1}}{x^4}+1\right)-\frac{1}{16}\log\left(\frac{\sqrt{x^8+1}}{x^4}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^8+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(x^8 + 1)/(x^4*((x^8 + 1)/x^8 - 1)) + 1/16*log(sqrt(x^8 + 1)/x^4 +
1) - 1/16*log(sqrt(x^8 + 1)/x^4 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int x^3\sqrt{x^8+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^8 + 1)^(1/2),x)
```

```
[Out] int(x^3*(x^8 + 1)^(1/2), x)
```

sympy [A] time = 1.57, size = 19, normalized size = 0.76

$$\frac{x^4\sqrt{x^8+1}}{8}+\frac{\operatorname{asinh}\left(x^4\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(x**8+1)**(1/2),x)
```

```
[Out] x**4*sqrt(x**8 + 1)/8 + asinh(x**4)/8
```

3.1510 $\int x\sqrt{1+x^8}\,dx$

Optimal. Leaf size=62

$$\frac{1}{6}\sqrt{x^8+1}\,x^2+\frac{\left(x^4+1\right)\sqrt{\frac{x^8+1}{\left(x^4+1\right)^2}}\,F\left(2\tan^{-1}\left(x^2\right)|\frac{1}{2}\right)}{6\sqrt{x^8+1}}$$

```
[Out] 1/6*x^2*(x^8+1)^(1/2)+1/6*(x^4+1)*(cos(2*arctan(x^2))^2)^(1/2)/cos(2*arctan
(x^2))*EllipticF(sin(2*arctan(x^2)),1/2*2^(1/2))*((x^8+1)/(x^4+1)^2)^(1/2)/
(x^8+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {275, 195, 220}

$$\frac{1}{6}\sqrt{x^8+1}\,x^2+\frac{\left(x^4+1\right)\sqrt{\frac{x^8+1}{\left(x^4+1\right)^2}}\,F\left(2\tan^{-1}\left(x^2\right)|\frac{1}{2}\right)}{6\sqrt{x^8+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[1 + x^8],x]
```

```
[Out] (x^2*Sqrt[1 + x^8])/6 + ((1 + x^4)*Sqrt[(1 + x^8)/(1 + x^4)^2]*EllipticF[2*
ArcTan[x^2], 1/2])/(6*Sqrt[1 + x^8])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{1+x^8}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \sqrt{1+x^4}\,dx, x, x^2\right)\\ &= \frac{1}{6}x^2\sqrt{1+x^8}+\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx, x, x^2\right)\\ &= \frac{1}{6}x^2\sqrt{1+x^8}+\frac{\left(1+x^4\right)\sqrt{\frac{1+x^8}{\left(1+x^4\right)^2}}\,F\left(2\tan^{-1}\left(x^2\right)|\frac{1}{2}\right)}{6\sqrt{1+x^8}}\end{aligned}$$

Mathematica [C] time = 0.00, size = 22, normalized size = 0.35

$$\frac{1}{2}x^2\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[1 + x^8],x]
[Out] (x^2*Hypergeometric2F1[-1/2, 1/4, 5/4, -x^8])/2
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{x^8+1}\,x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^8+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x^8 + 1)*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^8+1}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^8+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^8 + 1)*x, x)
```

maple [C] time = 0.15, size = 30, normalized size = 0.48

$$\frac{x^2\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],-x^8\right)}{3}+\frac{\sqrt{x^8+1}\,x^2}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^8+1)^(1/2),x)
[Out] 1/6*x^2*(x^8+1)^(1/2)+1/3*x^2*hypergeom([1/4,1/2],[5/4],-x^8)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^8+1}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^8+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^8 + 1)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\sqrt{x^8+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^8 + 1)^(1/2),x)
[Out] int(x*(x^8 + 1)^(1/2), x)
```

sympy [C] time = 0.76, size = 31, normalized size = 0.50

$$\frac{x^2\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2}, \frac{1}{4} \\ \frac{5}{4}\end{matrix}\right| x^8 e^{i\pi}\right)}{8\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**8+1)**(1/2),x)
```

```
[Out] x**2*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), x**8*exp_polar(I*pi))/(8*gamma(5
/4))
```

3.1511 $\int \frac{\sqrt{1+x^8}}{x}\,dx$

Optimal. Leaf size=28

$$\frac{\sqrt{x^8+1}}{4}-\frac{1}{4}\tanh^{-1}\left(\sqrt{x^8+1}\right)$$

```
[Out] -1/4*arctanh((x^8+1)^(1/2))+1/4*(x^8+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {266, 50, 63, 207}

$$\frac{\sqrt{x^8+1}}{4}-\frac{1}{4}\tanh^{-1}\left(\sqrt{x^8+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^8]/x,x]
```

```
[Out] Sqrt[1 + x^8]/4 - ArcTanh[Sqrt[1 + x^8]]/4
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+x^8}}{x}\,dx &= \frac{1}{8}\,\text{Subst}\left(\int \frac{\sqrt{1+x}}{x}\,dx, x, x^8\right)\\ &= \frac{\sqrt{1+x^8}}{4} + \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^8\right)\\ &= \frac{\sqrt{1+x^8}}{4} + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^8}\right)\\ &= \frac{\sqrt{1+x^8}}{4} - \frac{1}{4}\tanh^{-1}\left(\sqrt{1+x^8}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 28, normalized size = 1.00

$$\frac{\sqrt{x^8+1}}{4} - \frac{1}{4}\tanh^{-1}\left(\sqrt{x^8+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^8]/x,x]
[Out] Sqrt[1 + x^8]/4 - ArcTanh[Sqrt[1 + x^8]]/4
```

fricas [A] time = 0.78, size = 34, normalized size = 1.21

$$\frac{1}{4}\sqrt{x^8+1} - \frac{1}{8}\log\left(\sqrt{x^8+1}+1\right) + \frac{1}{8}\log\left(\sqrt{x^8+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8+1)^(1/2)/x,x, algorithm="fricas")
[Out] 1/4*sqrt(x^8 + 1) - 1/8*log(sqrt(x^8 + 1) + 1) + 1/8*log(sqrt(x^8 + 1) - 1)
```

giac [A] time = 0.15, size = 34, normalized size = 1.21

$$\frac{1}{4}\sqrt{x^8+1} - \frac{1}{8}\log\left(\sqrt{x^8+1}+1\right) + \frac{1}{8}\log\left(\sqrt{x^8+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8+1)^(1/2)/x,x, algorithm="giac")
[Out] 1/4*sqrt(x^8 + 1) - 1/8*log(sqrt(x^8 + 1) + 1) + 1/8*log(sqrt(x^8 + 1) - 1)
```

maple [B] time = 0.17, size = 56, normalized size = 2.00

$$-\frac{4\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^8+1}}{2}\right) + 4\sqrt{\pi} - 4\sqrt{\pi}\,\sqrt{x^8+1} - 2\left(8\ln(x)+2-2\ln(2)\right)\sqrt{\pi}}{16\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8+1)^(1/2)/x,x)
[Out] -1/16/Pi^(1/2)*(4*Pi^(1/2)-4*Pi^(1/2)*(x^8+1)^(1/2)+4*Pi^(1/2)*ln(1/2+1/2*(
x^8+1)^(1/2))-2*(2-2*ln(2)+8*ln(x))*Pi^(1/2))
```

maxima [A] time = 1.05, size = 34, normalized size = 1.21

$$\frac{1}{4}\sqrt{x^8+1} - \frac{1}{8}\log\left(\sqrt{x^8+1}+1\right) + \frac{1}{8}\log\left(\sqrt{x^8+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8+1)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(x^8 + 1) - 1/8*log(sqrt(x^8 + 1) + 1) + 1/8*log(sqrt(x^8 + 1) - 1)
```

mupad [B] time = 1.14, size = 20, normalized size = 0.71

$$\frac{\sqrt{x^8+1}}{4} - \frac{\operatorname{atanh}\left(\sqrt{x^8+1}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8 + 1)^(1/2)/x,x)
```

```
[Out] (x^8 + 1)^(1/2)/4 - atanh((x^8 + 1)^(1/2))/4
```

sympy [A] time = 1.30, size = 39, normalized size = 1.39

$$\frac{x^4}{4\sqrt{1+\frac{1}{x^8}}} - \frac{\operatorname{asinh}\left(\frac{1}{x^4}\right)}{4} + \frac{1}{4x^4\sqrt{1+\frac{1}{x^8}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**8+1)**(1/2)/x,x)
```

```
[Out] x**4/(4*sqrt(1 + x**(-8))) - asinh(x**(-4))/4 + 1/(4*x**4*sqrt(1 + x**(-8))
)
```

3.1512 $\int \frac{\sqrt{1+x^8}}{x^3}\,dx$

Optimal. Leaf size=125

$$-\frac{\sqrt{x^8+1}}{2x^2}+\frac{\sqrt{x^8+1}\,x^2}{x^4+1}+\frac{\left(x^4+1\right)\sqrt{\frac{x^8+1}{\left(x^4+1\right)^2}}\,F\left(2\tan^{-1}\left(x^2\right)|\frac{1}{2}\right)}{2\sqrt{x^8+1}}-\frac{\left(x^4+1\right)\sqrt{\frac{x^8+1}{\left(x^4+1\right)^2}}\,E\left(2\tan^{-1}\left(x^2\right)|\frac{1}{2}\right)}{\sqrt{x^8+1}}$$

```
[Out] -1/2*(x^8+1)^(1/2)/x^2+x^2*(x^8+1)^(1/2)/(x^4+1)-(x^4+1)*(cos(2*arctan(x^2)
)^2)^(1/2)/cos(2*arctan(x^2))*EllipticE(sin(2*arctan(x^2)),1/2*2^(1/2))*((x
^8+1)/(x^4+1)^2)^(1/2)/(x^8+1)^(1/2)+1/2*(x^4+1)*(cos(2*arctan(x^2))^2)^(1/
2)/cos(2*arctan(x^2))*EllipticF(sin(2*arctan(x^2)),1/2*2^(1/2))*((x^8+1)/(x
^4+1)^2)^(1/2)/(x^8+1)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {275, 277, 305, 220, 1196}

$$\frac{\sqrt{x^8+1}\,x^2}{x^4+1}-\frac{\sqrt{x^8+1}}{2x^2}+\frac{\left(x^4+1\right)\sqrt{\frac{x^8+1}{\left(x^4+1\right)^2}}\,F\left(2\tan^{-1}\left(x^2\right)|\frac{1}{2}\right)}{2\sqrt{x^8+1}}-\frac{\left(x^4+1\right)\sqrt{\frac{x^8+1}{\left(x^4+1\right)^2}}\,E\left(2\tan^{-1}\left(x^2\right)|\frac{1}{2}\right)}{\sqrt{x^8+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^8]/x^3,x]
```

```
[Out] -Sqrt[1 + x^8]/(2*x^2) + (x^2*Sqrt[1 + x^8])/(1 + x^4) - ((1 + x^4)*Sqrt[(1
 + x^8)/(1 + x^4)^2]*EllipticE[2*ArcTan[x^2], 1/2])/Sqrt[1 + x^8] + ((1 + x
^4)*Sqrt[(1 + x^8)/(1 + x^4)^2]*EllipticF[2*ArcTan[x^2], 1/2])/(2*Sqrt[1 +
x^8])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+x^8}}{x^3}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{1+x^4}}{x^2}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{1+x^8}}{2x^2} + \text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^4}}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{1+x^8}}{2x^2} + \text{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx, x, x^2\right) - \text{Subst}\left(\int \frac{1-x^2}{\sqrt{1+x^4}}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{1+x^8}}{2x^2} + \frac{x^2\sqrt{1+x^8}}{1+x^4} - \frac{\left(1+x^4\right)\sqrt{\frac{1+x^8}{\left(1+x^4\right)^2}}\,E\left(2\tan^{-1}\left(x^2\right)|\frac{1}{2}\right)}{\sqrt{1+x^8}} + \frac{\left(1+x^4\right)\sqrt{\frac{1+x^8}{\left(1+x^4\right)^2}}\,F\left(2\tan\right.}{2\sqrt{1+x^8}}\end{aligned}$$

Mathematica [C] time = 0.00, size = 22, normalized size = 0.18

$$-\frac{{}_2F_1\left(-\frac{1}{2}, -\frac{1}{4}; \frac{3}{4}; -x^8\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^8]/x^3,x]
```

```
[Out] -1/2*Hypergeometric2F1[-1/2, -1/4, 3/4, -x^8]/x^2
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^8+1}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8+1)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^8 + 1)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^8+1}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8+1)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^8 + 1)/x^3, x)
```

maple [C] time = 0.14, size = 30, normalized size = 0.24

$$\frac{x^6\,\text{hypergeom}\left(\left[\frac{1}{2}, \frac{3}{4}\right], \left[\frac{7}{4}\right], -x^8\right)}{3} - \frac{\sqrt{x^8+1}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8+1)^(1/2)/x^3,x)
```

```
[Out] -1/2*(x^8+1)^(1/2)/x^2+1/3*x^6*hypergeom([1/2,3/4],[7/4],-x^8)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^8+1}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^8+1)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^8 + 1)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x^8+1}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8 + 1)^(1/2)/x^3,x)
```

```
[Out] int((x^8 + 1)^(1/2)/x^3, x)
```

sympy [C] time = 0.76, size = 34, normalized size = 0.27

$$\frac{\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2}, -\frac{1}{4} \\ \frac{3}{4}\end{matrix}\middle| x^8 e^{i\pi}\right)}{8x^2\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**8+1)**(1/2)/x**3,x)
```

```
[Out] gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), x**8*exp_polar(I*pi))/(8*x**2*gamma
(3/4))
```

3.1513 $\int x^3\sqrt{-2+x^8}\,dx$

Optimal. Leaf size=35

$$\frac{1}{8}x^4\sqrt{x^8-2}-\frac{1}{4}\tanh^{-1}\left(\frac{x^4}{\sqrt{x^8-2}}\right)$$

```
[Out] -1/4*arctanh(x^4/(x^8-2)^(1/2))+1/8*x^4*(x^8-2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {275, 195, 217, 206}

$$\frac{1}{8}x^4\sqrt{x^8-2}-\frac{1}{4}\tanh^{-1}\left(\frac{x^4}{\sqrt{x^8-2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[-2 + x^8],x]
```

```
[Out] (x^4*Sqrt[-2 + x^8])/8 - ArcTanh[x^4/Sqrt[-2 + x^8]]/4
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int x^3\sqrt{-2+x^8}\,dx &= \frac{1}{4}\text{Subst}\left(\int \sqrt{-2+x^2}\,dx, x, x^4\right)\\
&= \frac{1}{8}x^4\sqrt{-2+x^8}-\frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt{-2+x^2}}\,dx, x, x^4\right)\\
&= \frac{1}{8}x^4\sqrt{-2+x^8}-\frac{1}{4}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x^4}{\sqrt{-2+x^8}}\right)\\
&= \frac{1}{8}x^4\sqrt{-2+x^8}-\frac{1}{4}\tanh^{-1}\left(\frac{x^4}{\sqrt{-2+x^8}}\right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 50, normalized size = 1.43

$$\frac{\left(x^8-2\right)\left(2\sin^{-1}\left(\frac{x^4}{\sqrt{2}}\right)+\sqrt{2-x^8}\,x^4\right)}{8\sqrt{-\left(x^8-2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[-2 + x^8],x]
```

```
[Out] ((-2 + x^8)*(x^4*Sqrt[2 - x^8] + 2*ArcSin[x^4/Sqrt[2]]))/(8*Sqrt[-(-2 + x^8
)^2])
```

fricas [A] time = 0.88, size = 29, normalized size = 0.83

$$\frac{1}{8}\sqrt{x^8-2}\,x^4+\frac{1}{4}\log\left(-x^4+\sqrt{x^8-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^8-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(x^8 - 2)*x^4 + 1/4*log(-x^4 + sqrt(x^8 - 2))
```

giac [A] time = 0.16, size = 29, normalized size = 0.83

$$\frac{1}{8}\sqrt{x^8-2}\,x^4+\frac{1}{4}\log\left(x^4-\sqrt{x^8-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^8-2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(x^8 - 2)*x^4 + 1/4*log(x^4 - sqrt(x^8 - 2))
```

maple [C] time = 0.16, size = 47, normalized size = 1.34

$$\frac{\sqrt{x^8-2}\;x^4}{8}-\frac{\sqrt{-\operatorname{signum}\left(\frac{x^8}{2}-1\right)}\;\arcsin\left(\frac{\sqrt{2}\,x^4}{2}\right)}{4\sqrt{\operatorname{signum}\left(\frac{x^8}{2}-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^8-2)^(1/2),x)
```

```
[Out] 1/8*x^4*(x^8-2)^(1/2)-1/4/signum(-1+1/2*x^8)^(1/2)*(-signum(-1+1/2*x^8))^(1
/2)*arcsin(1/2*x^4*2^(1/2))
```

maxima [B] time = 1.08, size = 58, normalized size = 1.66

$$-\frac{\sqrt{x^8-2}}{4\,x^4\left(\frac{x^8-2}{x^8}-1\right)}-\frac{1}{8}\log\left(\frac{\sqrt{x^8-2}}{x^4}+1\right)+\frac{1}{8}\log\left(\frac{\sqrt{x^8-2}}{x^4}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^8-2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(x^8 - 2)/(x^4*((x^8 - 2)/x^8 - 1)) - 1/8*log(sqrt(x^8 - 2)/x^4 +
1) + 1/8*log(sqrt(x^8 - 2)/x^4 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x^3 \sqrt{x^8 - 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^8 - 2)^(1/2),x)
```

```
[Out] int(x^3*(x^8 - 2)^(1/2), x)
```

sympy [A] time = 1.77, size = 90, normalized size = 2.57

$$\begin{cases} \frac{x^{12}}{8\sqrt{x^8-2}} - \frac{x^4}{4\sqrt{x^8-2}} - \frac{\operatorname{acosh}\left(\frac{\sqrt{2}x^4}{2}\right)}{4} & \text{for } \frac{|x^8|}{2} > 1 \\ -\frac{ix^{12}}{8\sqrt{2-x^8}} + \frac{ix^4}{4\sqrt{2-x^8}} + \frac{i\operatorname{asin}\left(\frac{\sqrt{2}x^4}{2}\right)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(x**8-2)**(1/2),x)
```

```
[Out] Piecewise((x**12/(8*sqrt(x**8 - 2)) - x**4/(4*sqrt(x**8 - 2)) - acosh(sqrt(
2)*x**4/2)/4, Abs(x**8)/2 > 1), (-I*x**12/(8*sqrt(2 - x**8)) + I*x**4/(4*sq
rt(2 - x**8)) + I*asin(sqrt(2)*x**4/2)/4, True))
```

3.1514 $\int \frac{x^{19}}{\sqrt{1+x^8}}\,dx$

Optimal. Leaf size=41

$$\frac{3}{32}\sinh^{-1}\left(x^4\right)+\frac{1}{16}\sqrt{x^8+1}\,x^{12}-\frac{3}{32}\sqrt{x^8+1}\,x^4$$

```
[Out] 3/32*arcsinh(x^4)-3/32*x^4*(x^8+1)^(1/2)+1/16*x^12*(x^8+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {275, 321, 215}

$$\frac{1}{16}\sqrt{x^8+1}\,x^{12}-\frac{3}{32}\sqrt{x^8+1}\,x^4+\frac{3}{32}\sinh^{-1}\left(x^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^19/Sqrt[1 + x^8],x]
```

```
[Out] (-3*x^4*Sqrt[1 + x^8])/32 + (x^12*Sqrt[1 + x^8])/16 + (3*ArcSinh[x^4])/32
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{19}}{\sqrt{1+x^8}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^2}}\,dx, x, x^4\right)\\
&= \frac{1}{16}x^{12}\sqrt{1+x^8}-\frac{3}{16}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}}\,dx, x, x^4\right)\\
&= -\frac{3}{32}x^4\sqrt{1+x^8}+\frac{1}{16}x^{12}\sqrt{1+x^8}+\frac{3}{32}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, x^4\right)\\
&= -\frac{3}{32}x^4\sqrt{1+x^8}+\frac{1}{16}x^{12}\sqrt{1+x^8}+\frac{3}{32}\sinh^{-1}\left(x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.76

$$\frac{1}{32}\left(3\sinh^{-1}\left(x^4\right)+\sqrt{x^8+1}\,\left(2x^8-3\right)x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^19/Sqrt[1 + x^8],x]
[Out] (x^4*Sqrt[1 + x^8]*(-3 + 2*x^8) + 3*ArcSinh[x^4])/32
```

fricas [A] time = 0.96, size = 37, normalized size = 0.90

$$\frac{1}{32}\left(2x^{12}-3x^4\right)\sqrt{x^8+1}-\frac{3}{32}\log\left(-x^4+\sqrt{x^8+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(x^8+1)^(1/2),x, algorithm="fricas")
[Out] 1/32*(2*x^12 - 3*x^4)*sqrt(x^8 + 1) - 3/32*log(-x^4 + sqrt(x^8 + 1))
```

giac [A] time = 0.17, size = 36, normalized size = 0.88

$$\frac{1}{32}\left(2x^8-3\right)\sqrt{x^8+1}\,x^4-\frac{3}{32}\log\left(-x^4+\sqrt{x^8+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(x^8+1)^(1/2),x, algorithm="giac")
[Out] 1/32*(2*x^8 - 3)*sqrt(x^8 + 1)*x^4 - 3/32*log(-x^4 + sqrt(x^8 + 1))
```

maple [A] time = 0.14, size = 27, normalized size = 0.66

$$\frac{\left(2x^8-3\right)\sqrt{x^8+1}\;x^4}{32}+\frac{3\,\mathrm{arcsinh}\left(x^4\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19/(x^8+1)^(1/2),x)
[Out] 1/32*x^4*(2*x^8-3)*(x^8+1)^(1/2)+3/32*arcsinh(x^4)
```

maxima [B] time = 1.06, size = 86, normalized size = 2.10

$$-\frac{\frac{5\sqrt{x^8+1}}{x^4}-\frac{3\left(x^8+1\right)^{\frac{3}{2}}}{x^{12}}}{32\left(\frac{2\left(x^8+1\right)}{x^8}-\frac{\left(x^8+1\right)^2}{x^{16}}-1\right)}+\frac{3}{64}\log\left(\frac{\sqrt{x^8+1}}{x^4}+1\right)-\frac{3}{64}\log\left(\frac{\sqrt{x^8+1}}{x^4}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(x^8+1)^(1/2),x, algorithm="maxima")
[Out] -1/32*(5*sqrt(x^8 + 1)/x^4 - 3*(x^8 + 1)^(3/2)/x^12)/(2*(x^8 + 1)/x^8 - (x^
8 + 1)^2/x^16 - 1) + 3/64*log(sqrt(x^8 + 1)/x^4 + 1) - 3/64*log(sqrt(x^8 +
1)/x^4 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{19}}{\sqrt{x^8+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19/(x^8 + 1)^(1/2),x)
[Out] int(x^19/(x^8 + 1)^(1/2), x)
```

sympy [**A**] time = 4.30, size = 49, normalized size = 1.20

$$\frac{x^{20}}{16\sqrt{x^8+1}} - \frac{x^{12}}{32\sqrt{x^8+1}} - \frac{3x^4}{32\sqrt{x^8+1}} + \frac{3\,\mathrm{asinh}\left(x^4\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**19/(x**8+1)**(1/2),x)
```

```
[Out] x**20/(16*sqrt(x**8 + 1)) - x**12/(32*sqrt(x**8 + 1)) - 3*x**4/(32*sqrt(x**
8 + 1)) + 3*asinh(x**4)/32
```

3.1515 $\int \frac{x^{15}}{\sqrt{1+x^8}}\,dx$

Optimal. Leaf size=27

$$\frac{1}{12}\left(x^8+1\right)^{3/2}-\frac{\sqrt{x^8+1}}{4}$$

```
[Out] 1/12*(x^8+1)^(3/2)-1/4*(x^8+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{1}{12}\left(x^8+1\right)^{3/2}-\frac{\sqrt{x^8+1}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^15/Sqrt[1 + x^8],x]
```

```
[Out] -Sqrt[1 + x^8]/4 + (1 + x^8)^(3/2)/12
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{15}}{\sqrt{1+x^8}}\,dx &= \frac{1}{8}\,\text{Subst}\left(\int \frac{x}{\sqrt{1+x}}\,dx, x, x^8\right)\\ &= \frac{1}{8}\,\text{Subst}\left(\int \left(-\frac{1}{\sqrt{1+x}}+\sqrt{1+x}\right)\,dx, x, x^8\right)\\ &= -\frac{1}{4}\sqrt{1+x^8}+\frac{1}{12}\left(1+x^8\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 0.67

$$\frac{1}{12}\left(x^8-2\right)\sqrt{x^8+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15/Sqrt[1 + x^8],x]
```

```
[Out] ((-2 + x^8)*Sqrt[1 + x^8])/12
```

fricas [A] time = 0.79, size = 14, normalized size = 0.52

$$\frac{1}{12}\sqrt{x^8+1}\left(x^8-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(x^8+1)^(1/2),x, algorithm="fricas")
[Out] 1/12*sqrt(x^8 + 1)*(x^8 - 2)
```

giac [A] time = 0.15, size = 19, normalized size = 0.70

$$\frac{1}{12}\left(x^8+1\right)^{\frac{3}{2}}-\frac{1}{4}\sqrt{x^8+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(x^8+1)^(1/2),x, algorithm="giac")
[Out] 1/12*(x^8 + 1)^(3/2) - 1/4*sqrt(x^8 + 1)
```

maple [A] time = 0.00, size = 15, normalized size = 0.56

$$\frac{\sqrt{x^8+1}\,\left(x^8-2\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(x^8+1)^(1/2),x)
[Out] 1/12*(x^8+1)^(1/2)*(x^8-2)
```

maxima [A] time = 1.02, size = 19, normalized size = 0.70

$$\frac{1}{12}\left(x^8+1\right)^{\frac{3}{2}}-\frac{1}{4}\sqrt{x^8+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(x^8+1)^(1/2),x, algorithm="maxima")
[Out] 1/12*(x^8 + 1)^(3/2) - 1/4*sqrt(x^8 + 1)
```

mupad [B] time = 1.16, size = 14, normalized size = 0.52

$$\frac{\sqrt{x^8+1}\,\left(x^8-2\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(x^8 + 1)^(1/2),x)
[Out] ((x^8 + 1)^(1/2)*(x^8 - 2))/12
```

sympy [A] time = 2.83, size = 22, normalized size = 0.81

$$\frac{x^8\sqrt{x^8+1}}{12}-\frac{\sqrt{x^8+1}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15/(x**8+1)**(1/2),x)
[Out] x**8*sqrt(x**8 + 1)/12 - sqrt(x**8 + 1)/6
```

3.1516 $\int \frac{x^{11}}{\sqrt{1+x^8}}\,dx$

Optimal. Leaf size=25

$$\frac{1}{8}x^4\sqrt{x^8+1}-\frac{1}{8}\sinh^{-1}\left(x^4\right)$$

```
[Out] -1/8*arcsinh(x^4)+1/8*x^4*(x^8+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {275, 321, 215}

$$\frac{1}{8}x^4\sqrt{x^8+1}-\frac{1}{8}\sinh^{-1}\left(x^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^11/Sqrt[1 + x^8],x]
```

```
[Out] (x^4*Sqrt[1 + x^8])/8 - ArcSinh[x^4]/8
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{\sqrt{1+x^8}}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}}\,dx, x, x^4\right)\\ &= \frac{1}{8}x^4\sqrt{1+x^8}-\frac{1}{8}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, x^4\right)\\ &= \frac{1}{8}x^4\sqrt{1+x^8}-\frac{1}{8}\sinh^{-1}\left(x^4\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{1}{8}x^4\sqrt{x^8+1}-\frac{1}{8}\sinh^{-1}\left(x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/Sqrt[1 + x^8],x]
```

```
[Out] (x^4*Sqrt[1 + x^8])/8 - ArcSinh[x^4]/8
```

fricas [A] time = 0.78, size = 29, normalized size = 1.16

$$\frac{1}{8}\sqrt{x^8+1}\,x^4+\frac{1}{8}\log\left(-x^4+\sqrt{x^8+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(x^8 + 1)*x^4 + 1/8*log(-x^4 + sqrt(x^8 + 1))
```

giac [A] time = 0.16, size = 29, normalized size = 1.16

$$\frac{1}{8}\sqrt{x^8+1}\,x^4+\frac{1}{8}\log\left(-x^4+\sqrt{x^8+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(x^8 + 1)*x^4 + 1/8*log(-x^4 + sqrt(x^8 + 1))
```

maple [A] time = 0.14, size = 20, normalized size = 0.80

$$\frac{\sqrt{x^8+1}\;x^4}{8}-\frac{\operatorname{arcsinh}\left(x^4\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^8+1)^(1/2),x)
```

```
[Out] -1/8*arcsinh(x^4)+1/8*(x^8+1)^(1/2)*x^4
```

maxima [B] time = 1.07, size = 58, normalized size = 2.32

$$\frac{\sqrt{x^8+1}}{8\,x^4\left(\frac{x^8+1}{x^8}-1\right)}-\frac{1}{16}\log\left(\frac{\sqrt{x^8+1}}{x^4}+1\right)+\frac{1}{16}\log\left(\frac{\sqrt{x^8+1}}{x^4}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(x^8 + 1)/(x^4*((x^8 + 1)/x^8 - 1)) - 1/16*log(sqrt(x^8 + 1)/x^4 +
1) + 1/16*log(sqrt(x^8 + 1)/x^4 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^{11}}{\sqrt{x^8+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^8 + 1)^(1/2),x)
```

```
[Out] int(x^11/(x^8 + 1)^(1/2), x)
```

sympy [A] time = 2.28, size = 19, normalized size = 0.76

$$\frac{x^4\sqrt{x^8+1}}{8}-\frac{\operatorname{asinh}\left(x^4\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(x**8+1)**(1/2),x)
```

```
[Out] x**4*sqrt(x**8 + 1)/8 - asinh(x**4)/8
```

3.1517 $\int \frac{x^7}{\sqrt{1+x^8}}\,dx$

Optimal. Leaf size=13

$$\frac{\sqrt{x^8+1}}{4}$$

```
[Out] 1/4*(x^8+1)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$\frac{\sqrt{x^8+1}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/Sqrt[1 + x^8],x]
```

```
[Out] Sqrt[1 + x^8]/4
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^7}{\sqrt{1+x^8}}\,dx = \frac{\sqrt{1+x^8}}{4}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{\sqrt{x^8+1}}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/Sqrt[1 + x^8],x]
```

```
[Out] Sqrt[1 + x^8]/4
```

fricas [A] time = 0.86, size = 9, normalized size = 0.69

$$\frac{1}{4}\sqrt{x^8+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(x^8 + 1)
```

giac [A] time = 0.15, size = 9, normalized size = 0.69

$$\frac{1}{4}\sqrt{x^8+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+1)^(1/2),x, algorithm="giac")
[Out] 1/4*sqrt(x^8 + 1)
```

maple [A] time = 0.00, size = 10, normalized size = 0.77

$$\frac{\sqrt{x^8+1}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^8+1)^(1/2),x)
[Out] 1/4*(x^8+1)^(1/2)
```

maxima [A] time = 0.97, size = 9, normalized size = 0.69

$$\frac{1}{4}\sqrt{x^8+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+1)^(1/2),x, algorithm="maxima")
[Out] 1/4*sqrt(x^8 + 1)
```

mupad [B] time = 0.09, size = 9, normalized size = 0.69

$$\frac{\sqrt{x^8+1}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^8 + 1)^(1/2),x)
[Out] (x^8 + 1)^(1/2)/4
```

sympy [A] time = 0.49, size = 8, normalized size = 0.62

$$\frac{\sqrt{x^8+1}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**8+1)**(1/2),x)
[Out] sqrt(x**8 + 1)/4
```

3.1518 $\int \frac{x^3}{\sqrt{1+x^8}}\,dx$

Optimal. Leaf size=8

$$\frac{1}{4}\sinh^{-1}\left(x^4\right)$$

```
[Out] 1/4*arcsinh(x^4)
```

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {275, 215}

$$\frac{1}{4}\sinh^{-1}\left(x^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[1 + x^8],x]
```

```
[Out] ArcSinh[x^4]/4
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{1+x^8}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, x^4\right)\\ &= \frac{1}{4}\sinh^{-1}\left(x^4\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\frac{1}{4}\sinh^{-1}\left(x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[1 + x^8],x]
```

```
[Out] ArcSinh[x^4]/4
```

fricas [B] time = 0.80, size = 16, normalized size = 2.00

$$-\frac{1}{4}\log\left(-x^4+\sqrt{x^8+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*log(-x^4 + sqrt(x^8 + 1))
```

giac [B] time = 0.16, size = 16, normalized size = 2.00

$$-\frac{1}{4}\log\left(-x^4+\sqrt{x^8+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+1)^(1/2),x, algorithm="giac")
[Out] -1/4*log(-x^4 + sqrt(x^8 + 1))
```

maple [A] time = 0.14, size = 7, normalized size = 0.88

$$\frac{\operatorname{arcsinh}\left(x^4\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8+1)^(1/2),x)
[Out] 1/4*arcsinh(x^4)
```

maxima [B] time = 1.06, size = 33, normalized size = 4.12

$$\frac{1}{8}\log\left(\frac{\sqrt{x^8+1}}{x^4}+1\right)-\frac{1}{8}\log\left(\frac{\sqrt{x^8+1}}{x^4}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+1)^(1/2),x, algorithm="maxima")
[Out] 1/8*log(sqrt(x^8 + 1)/x^4 + 1) - 1/8*log(sqrt(x^8 + 1)/x^4 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.12

$$\int \frac{x^3}{\sqrt{x^8+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8 + 1)^(1/2),x)
[Out] int(x^3/(x^8 + 1)^(1/2), x)
```

sympy [A] time = 1.06, size = 5, normalized size = 0.62

$$\frac{\operatorname{asinh}\left(x^4\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**8+1)**(1/2),x)
[Out] asinh(x**4)/4
```

3.1519 $\int \frac{1}{x\sqrt{1+x^8}}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{4}\tanh^{-1}\left(\sqrt{x^8+1}\right)$$

```
[Out] -1/4*arctanh((x^8+1)^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {266, 63, 207}

$$-\frac{1}{4}\tanh^{-1}\left(\sqrt{x^8+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[1 + x^8]),x]
```

```
[Out] -ArcTanh[Sqrt[1 + x^8]]/4
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{1+x^8}}\,dx &= \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^8\right)\\ &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^8}\right)\\ &= -\frac{1}{4}\tanh^{-1}\left(\sqrt{1+x^8}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{4}\tanh^{-1}\left(\sqrt{x^8+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[1 + x^8]),x]
[Out] -1/4*ArcTanh[Sqrt[1 + x^8]]
```

fricas [B] time = 0.64, size = 25, normalized size = 1.79

$$-\frac{1}{8}\log\left(\sqrt{x^8+1}+1\right)+\frac{1}{8}\log\left(\sqrt{x^8+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+1)^(1/2),x, algorithm="fricas")
[Out] -1/8*log(sqrt(x^8 + 1) + 1) + 1/8*log(sqrt(x^8 + 1) - 1)
```

giac [B] time = 0.21, size = 25, normalized size = 1.79

$$-\frac{1}{8}\log\left(\sqrt{x^8+1}+1\right)+\frac{1}{8}\log\left(\sqrt{x^8+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+1)^(1/2),x, algorithm="giac")
[Out] -1/8*log(sqrt(x^8 + 1) + 1) + 1/8*log(sqrt(x^8 + 1) - 1)
```

maple [A] time = 0.01, size = 19, normalized size = 1.36

$$\frac{\ln\left(\frac{\sqrt{x^8+1}-1}{\sqrt{x^8}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^8+1)^(1/2),x)
[Out] 1/4*ln(((x^8+1)^(1/2)-1)/(x^8)^(1/2))
```

maxima [B] time = 1.01, size = 25, normalized size = 1.79

$$-\frac{1}{8}\log\left(\sqrt{x^8+1}+1\right)+\frac{1}{8}\log\left(\sqrt{x^8+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+1)^(1/2),x, algorithm="maxima")
[Out] -1/8*log(sqrt(x^8 + 1) + 1) + 1/8*log(sqrt(x^8 + 1) - 1)
```

mupad [B] time = 0.05, size = 10, normalized size = 0.71

$$-\frac{\operatorname{atanh}\left(\sqrt{x^8+1}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^8 + 1)^(1/2)),x)
[Out] -atanh((x^8 + 1)^(1/2))/4
```

sympy [A] time = 1.02, size = 8, normalized size = 0.57

$$-\frac{\operatorname{asinh}\left(\frac{1}{x^4}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**8+1)**(1/2),x)
[Out] -asinh(x**(-4))/4
```

3.1520 $\int \frac{1}{x^5\sqrt{1+x^8}}\,dx$

Optimal. Leaf size=16

$$-\frac{\sqrt{x^8+1}}{4x^4}$$

```
[Out] -1/4*(x^8+1)^(1/2)/x^4
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {264}

$$-\frac{\sqrt{x^8+1}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[1 + x^8]),x]
```

```
[Out] -Sqrt[1 + x^8]/(4*x^4)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^5\sqrt{1+x^8}}\,dx = -\frac{\sqrt{1+x^8}}{4x^4}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{\sqrt{x^8+1}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[1 + x^8]),x]
```

```
[Out] -1/4*Sqrt[1 + x^8]/x^4
```

fricas [A] time = 0.83, size = 16, normalized size = 1.00

$$-\frac{x^4+\sqrt{x^8+1}}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(x^4 + sqrt(x^8 + 1))/x^4
```

giac [A] time = 0.17, size = 21, normalized size = 1.31

$$\frac{1}{2\left(\left(x^4-\sqrt{x^8+1}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+1)^(1/2),x, algorithm="giac")
[Out] 1/2/((x^4 - sqrt(x^8 + 1))^2 - 1)
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$-\frac{\sqrt{x^8+1}}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^8+1)^(1/2),x)
[Out] -1/4*(x^8+1)^(1/2)/x^4
```

maxima [A] time = 1.11, size = 12, normalized size = 0.75

$$-\frac{\sqrt{x^8+1}}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+1)^(1/2),x, algorithm="maxima")
[Out] -1/4*sqrt(x^8 + 1)/x^4
```

mupad [B] time = 1.13, size = 12, normalized size = 0.75

$$-\frac{\sqrt{x^8+1}}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(x^8 + 1)^(1/2)),x)
[Out] -(x^8 + 1)^(1/2)/(4*x^4)
```

sympy [A] time = 0.74, size = 12, normalized size = 0.75

$$-\frac{\sqrt{1+\frac{1}{x^8}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**8+1)**(1/2),x)
[Out] -sqrt(1 + x**(-8))/4
```

3.1521 $\int \frac{1}{x^9\sqrt{1+x^8}}\,dx$

Optimal. Leaf size=31

$$\frac{1}{8}\tanh^{-1}\left(\sqrt{x^8+1}\right)-\frac{\sqrt{x^8+1}}{8x^8}$$

```
[Out] 1/8*arctanh((x^8+1)^(1/2))-1/8*(x^8+1)^(1/2)/x^8
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {266, 51, 63, 207}

$$\frac{1}{8}\tanh^{-1}\left(\sqrt{x^8+1}\right)-\frac{\sqrt{x^8+1}}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*Sqrt[1 + x^8]),x]
```

```
[Out] -Sqrt[1 + x^8]/(8*x^8) + ArcTanh[Sqrt[1 + x^8]]/8
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^9\sqrt{1+x^8}}\,dx &= \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x}}\,dx, x, x^8\right)\\ &= -\frac{\sqrt{1+x^8}}{8x^8} - \frac{1}{16}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^8\right)\\ &= -\frac{\sqrt{1+x^8}}{8x^8} - \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^8}\right)\\ &= -\frac{\sqrt{1+x^8}}{8x^8} + \frac{1}{8}\tanh^{-1}\left(\sqrt{1+x^8}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 1.00

$$\frac{1}{8}\tanh^{-1}\left(\sqrt{x^8+1}\right) - \frac{\sqrt{x^8+1}}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^9*Sqrt[1 + x^8]),x]
```

```
[Out] -1/8*Sqrt[1 + x^8]/x^8 + ArcTanh[Sqrt[1 + x^8]]/8
```

fricas [A] time = 0.72, size = 44, normalized size = 1.42

$$\frac{x^8\log\left(\sqrt{x^8+1}+1\right) - x^8\log\left(\sqrt{x^8+1}-1\right) - 2\sqrt{x^8+1}}{16\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(x^8+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(x^8*log(sqrt(x^8 + 1) + 1) - x^8*log(sqrt(x^8 + 1) - 1) - 2*sqrt(x^8
+ 1))/x^8
```

giac [A] time = 0.15, size = 37, normalized size = 1.19

$$-\frac{\sqrt{x^8+1}}{8\,x^8} + \frac{1}{16}\log\left(\sqrt{x^8+1}+1\right) - \frac{1}{16}\log\left(\sqrt{x^8+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(x^8+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(x^8 + 1)/x^8 + 1/16*log(sqrt(x^8 + 1) + 1) - 1/16*log(sqrt(x^8 +
1) - 1)
```

maple [A] time = 0.02, size = 32, normalized size = 1.03

$$-\frac{\ln\left(\frac{\sqrt{x^8+1}-1}{\sqrt{x^8}}\right)}{8} - \frac{\sqrt{x^8+1}}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(x^8+1)^(1/2),x)
```

```
[Out] -1/8*(x^8+1)^(1/2)/x^8-1/8*ln(((x^8+1)^(1/2)-1)/(x^8)^(1/2))
```

maxima [A] time = 1.05, size = 37, normalized size = 1.19

$$-\frac{\sqrt{x^8+1}}{8\,x^8}+\frac{1}{16}\log\left(\sqrt{x^8+1}+1\right)-\frac{1}{16}\log\left(\sqrt{x^8+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(x^8+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(x^8 + 1)/x^8 + 1/16*log(sqrt(x^8 + 1) + 1) - 1/16*log(sqrt(x^8 +
1) - 1)
```

mupad [B] time = 1.41, size = 23, normalized size = 0.74

$$\frac{\operatorname{atanh}\left(\sqrt{x^8+1}\right)}{8}-\frac{\sqrt{x^8+1}}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(x^8 + 1)^(1/2)),x)
```

```
[Out] atanh((x^8 + 1)^(1/2))/8 - (x^8 + 1)^(1/2)/(8*x^8)
```

sympy [A] time = 2.29, size = 22, normalized size = 0.71

$$\frac{\operatorname{asinh}\left(\frac{1}{x^4}\right)}{8}-\frac{\sqrt{1+\frac{1}{x^8}}}{8x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(x**8+1)**(1/2),x)
```

```
[Out] asinh(x**(-4))/8 - sqrt(1 + x**(-8))/(8*x**4)
```

3.1522 $\int \frac{1}{x^{13}\sqrt{1+x^8}}\,dx$

Optimal. Leaf size=33

$$\frac{\sqrt{x^8+1}}{6x^4}-\frac{\sqrt{x^8+1}}{12x^{12}}$$

```
[Out] -1/12*(x^8+1)^(1/2)/x^12+1/6*(x^8+1)^(1/2)/x^4
```

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {271, 264}

$$\frac{\sqrt{x^8+1}}{6x^4}-\frac{\sqrt{x^8+1}}{12x^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^13*Sqrt[1 + x^8]),x]
```

```
[Out] -Sqrt[1 + x^8]/(12*x^12) + Sqrt[1 + x^8]/(6*x^4)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{13}\sqrt{1+x^8}}\,dx &= -\frac{\sqrt{1+x^8}}{12x^{12}}-\frac{2}{3}\int \frac{1}{x^5\sqrt{1+x^8}}\,dx\\ &= -\frac{\sqrt{1+x^8}}{12x^{12}}+\frac{\sqrt{1+x^8}}{6x^4}\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 0.70

$$-\frac{\left(1-2x^8\right)\sqrt{x^8+1}}{12x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^13*Sqrt[1 + x^8]),x]
```

```
[Out] -1/12*((1 - 2*x^8)*Sqrt[1 + x^8])/x^12
```

fricas [A] time = 0.80, size = 26, normalized size = 0.79

$$\frac{2\,x^{12}+\left(2\,x^8-1\right)\sqrt{x^8+1}}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(x^8+1)^(1/2),x, algorithm="fricas")
[Out] 1/12*(2*x^12 + (2*x^8 - 1)*sqrt(x^8 + 1))/x^12
```

giac [A] time = 0.17, size = 40, normalized size = 1.21

$$\frac{3\left(x^4-\sqrt{x^8+1}\right)^2-1}{3\left(\left(x^4-\sqrt{x^8+1}\right)^2-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(x^8+1)^(1/2),x, algorithm="giac")
[Out] 1/3*(3*(x^4 - sqrt(x^8 + 1))^2 - 1)/((x^4 - sqrt(x^8 + 1))^2 - 1)^3
```

maple [A] time = 0.00, size = 20, normalized size = 0.61

$$\frac{\sqrt{x^8+1}\,\left(2x^8-1\right)}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^13/(x^8+1)^(1/2),x)
[Out] 1/12*(x^8+1)^(1/2)*(2*x^8-1)/x^12
```

maxima [A] time = 1.08, size = 25, normalized size = 0.76

$$\frac{\sqrt{x^8+1}}{4\,x^4}-\frac{\left(x^8+1\right)^{\frac{3}{2}}}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(x^8+1)^(1/2),x, algorithm="maxima")
[Out] 1/4*sqrt(x^8 + 1)/x^4 - 1/12*(x^8 + 1)^(3/2)/x^12
```

mupad [B] time = 1.19, size = 19, normalized size = 0.58

$$\frac{\sqrt{x^8+1}\,\left(2\,x^8-1\right)}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^13*(x^8 + 1)^(1/2)),x)
[Out] ((x^8 + 1)^(1/2)*(2*x^8 - 1))/(12*x^12)
```

sympy [A] time = 1.57, size = 26, normalized size = 0.79

$$\frac{\sqrt{1+\frac{1}{x^8}}}{6}-\frac{\sqrt{1+\frac{1}{x^8}}}{12x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**13/(x**8+1)**(1/2),x)
[Out] sqrt(1 + x**(-8))/6 - sqrt(1 + x**(-8))/(12*x**8)
```

3.1523 $\int \frac{1}{x^{17}\sqrt{1+x^8}}\,dx$

Optimal. Leaf size=47

$$\frac{3\sqrt{x^8+1}}{32x^8}-\frac{3}{32}\tanh^{-1}\left(\sqrt{x^8+1}\right)-\frac{\sqrt{x^8+1}}{16x^{16}}$$

```
[Out] -3/32*arctanh((x^8+1)^(1/2))-1/16*(x^8+1)^(1/2)/x^16+3/32*(x^8+1)^(1/2)/x^8
```

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {266, 51, 63, 207}

$$\frac{3\sqrt{x^8+1}}{32x^8}-\frac{\sqrt{x^8+1}}{16x^{16}}-\frac{3}{32}\tanh^{-1}\left(\sqrt{x^8+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^17*Sqrt[1 + x^8]),x]
```

```
[Out] -Sqrt[1 + x^8]/(16*x^16) + (3*Sqrt[1 + x^8])/(32*x^8) - (3*ArcTanh[Sqrt[1 +
 x^8]])/32
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{17}\sqrt{1+x^8}}\,dx &= \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{1+x}}\,dx, x, x^8\right)\\ &= -\frac{\sqrt{1+x^8}}{16x^{16}} - \frac{3}{32}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x}}\,dx, x, x^8\right)\\ &= -\frac{\sqrt{1+x^8}}{16x^{16}} + \frac{3\sqrt{1+x^8}}{32x^8} + \frac{3}{64}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^8\right)\\ &= -\frac{\sqrt{1+x^8}}{16x^{16}} + \frac{3\sqrt{1+x^8}}{32x^8} + \frac{3}{32}\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^8}\right)\\ &= -\frac{\sqrt{1+x^8}}{16x^{16}} + \frac{3\sqrt{1+x^8}}{32x^8} - \frac{3}{32}\tanh^{-1}\left(\sqrt{1+x^8}\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 26, normalized size = 0.55

$$-\frac{1}{4}\sqrt{x^8+1}\;{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};x^8+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^17*Sqrt[1 + x^8]),x]
```

```
[Out] -1/4*(Sqrt[1 + x^8]*Hypergeometric2F1[1/2, 3, 3/2, 1 + x^8])
```

fricas [A] time = 0.74, size = 52, normalized size = 1.11

$$-\frac{3\,x^{16}\log\left(\sqrt{x^8+1}+1\right) - 3\,x^{16}\log\left(\sqrt{x^8+1}-1\right) - 2\left(3\,x^8-2\right)\sqrt{x^8+1}}{64\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^17/(x^8+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/64*(3*x^16*log(sqrt(x^8 + 1) + 1) - 3*x^16*log(sqrt(x^8 + 1) - 1) - 2*(3
*x^8 - 2)*sqrt(x^8 + 1))/x^16
```

giac [A] time = 0.15, size = 49, normalized size = 1.04

$$\frac{3\left(x^8+1\right)^{\frac{3}{2}} - 5\sqrt{x^8+1}}{32\,x^{16}} - \frac{3}{64}\log\left(\sqrt{x^8+1}+1\right) + \frac{3}{64}\log\left(\sqrt{x^8+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^17/(x^8+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/32*(3*(x^8 + 1)^(3/2) - 5*sqrt(x^8 + 1))/x^16 - 3/64*log(sqrt(x^8 + 1) +
1) + 3/64*log(sqrt(x^8 + 1) - 1)
```

maple [A] time = 0.03, size = 42, normalized size = 0.89

$$\frac{3\ln\left(\frac{\sqrt{x^8+1}-1}{\sqrt{x^8}}\right)}{32} + \frac{3x^{16}+x^8-2}{32\sqrt{x^8+1}\;x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^17/(x^8+1)^(1/2),x)
```

```
[Out] 1/32*(3*x^16+x^8-2)/x^16/(x^8+1)^(1/2)+3/32*ln(((x^8+1)^(1/2)-1)/(x^8)^(1/2
))
```

maxima [A] time = 0.99, size = 64, normalized size = 1.36

$$-\frac{3\left(x^8+1\right)^{\frac{3}{2}}-5\sqrt{x^8+1}}{32\left(2x^8-\left(x^8+1\right)^2+1\right)}-\frac{3}{64}\log\left(\sqrt{x^8+1}+1\right)+\frac{3}{64}\log\left(\sqrt{x^8+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^17/(x^8+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/32*(3*(x^8 + 1)^(3/2) - 5*sqrt(x^8 + 1))/(2*x^8 - (x^8 + 1)^2 + 1) - 3/6
4*log(sqrt(x^8 + 1) + 1) + 3/64*log(sqrt(x^8 + 1) - 1)
```

mupad [B] time = 1.46, size = 49, normalized size = 1.04

$$\frac{\frac{5\sqrt{x^8+1}}{32}-\frac{3\left(x^8+1\right)^{3/2}}{32}}{2x^8-\left(x^8+1\right)^2+1}-\frac{3\,\mathrm{atanh}\left(\sqrt{x^8+1}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^17*(x^8 + 1)^(1/2)),x)
```

```
[Out] ((5*(x^8 + 1)^(1/2))/32 - (3*(x^8 + 1)^(3/2))/32)/(2*x^8 - (x^8 + 1)^2 + 1)
 - (3*atanh((x^8 + 1)^(1/2)))/32
```

sympy [A] time = 4.66, size = 60, normalized size = 1.28

$$-\frac{3\,\mathrm{asinh}\left(\frac{1}{x^4}\right)}{32}+\frac{3}{32x^4\sqrt{1+\frac{1}{x^8}}}+\frac{1}{32x^{12}\sqrt{1+\frac{1}{x^8}}}-\frac{1}{16x^{20}\sqrt{1+\frac{1}{x^8}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**17/(x**8+1)**(1/2),x)
```

```
[Out] -3*asinh(x**(-4))/32 + 3/(32*x**4*sqrt(1 + x**(-8))) + 1/(32*x**12*sqrt(1 +
 x**(-8))) - 1/(16*x**20*sqrt(1 + x**(-8)))
```

3.1524 $\int \frac{x^{13}}{\sqrt{1+x^8}}\,dx$

Optimal. Leaf size=130

$$\frac{1}{10}\sqrt{x^8+1}\,x^6-\frac{3\sqrt{x^8+1}\,x^2}{10\left(x^4+1\right)}-\frac{3\left(x^4+1\right)\sqrt{\frac{x^8+1}{\left(x^4+1\right)^2}}\,F\left(2\tan^{-1}\left(x^2\right)|\frac{1}{2}\right)}{20\sqrt{x^8+1}}+\frac{3\left(x^4+1\right)\sqrt{\frac{x^8+1}{\left(x^4+1\right)^2}}\,E\left(2\tan^{-1}\left(x^2\right)|\frac{1}{2}\right)}{10\sqrt{x^8+1}}$$

```
[Out] 1/10*x^6*(x^8+1)^(1/2)-3/10*x^2*(x^8+1)^(1/2)/(x^4+1)+3/10*(x^4+1)*(cos(2*a
rctan(x^2))^2)^(1/2)/cos(2*arctan(x^2))*EllipticE(sin(2*arctan(x^2)),1/2*2^
(1/2))*((x^8+1)/(x^4+1)^2)^(1/2)/(x^8+1)^(1/2)-3/20*(x^4+1)*(cos(2*arctan(x
^2))^2)^(1/2)/cos(2*arctan(x^2))*EllipticF(sin(2*arctan(x^2)),1/2*2^(1/2))*
((x^8+1)/(x^4+1)^2)^(1/2)/(x^8+1)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {275, 321, 305, 220, 1196}

$$\frac{1}{10}\sqrt{x^8+1}\,x^6-\frac{3\sqrt{x^8+1}\,x^2}{10\left(x^4+1\right)}-\frac{3\left(x^4+1\right)\sqrt{\frac{x^8+1}{\left(x^4+1\right)^2}}\,F\left(2\tan^{-1}\left(x^2\right)|\frac{1}{2}\right)}{20\sqrt{x^8+1}}+\frac{3\left(x^4+1\right)\sqrt{\frac{x^8+1}{\left(x^4+1\right)^2}}\,E\left(2\tan^{-1}\left(x^2\right)|\frac{1}{2}\right)}{10\sqrt{x^8+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/Sqrt[1 + x^8],x]
```

```
[Out] (x^6*Sqrt[1 + x^8])/10 - (3*x^2*Sqrt[1 + x^8])/(10*(1 + x^4)) + (3*(1 + x^4
)*Sqrt[(1 + x^8)/(1 + x^4)^2]*EllipticE[2*ArcTan[x^2], 1/2])/(10*Sqrt[1 + x
^8]) - (3*(1 + x^4)*Sqrt[(1 + x^8)/(1 + x^4)^2]*EllipticF[2*ArcTan[x^2], 1/
2])/(20*Sqrt[1 + x^8])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{13}}{\sqrt{1+x^8}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^6}{\sqrt{1+x^4}}\,dx, x, x^2\right)\\ &= \frac{1}{10}x^6\sqrt{1+x^8} - \frac{3}{10}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^4}}\,dx, x, x^2\right)\\ &= \frac{1}{10}x^6\sqrt{1+x^8} - \frac{3}{10}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx, x, x^2\right) + \frac{3}{10}\,\text{Subst}\left(\int \frac{1-x^2}{\sqrt{1+x^4}}\,dx, x, x^2\right)\\ &= \frac{1}{10}x^6\sqrt{1+x^8} - \frac{3x^2\sqrt{1+x^8}}{10\left(1+x^4\right)} + \frac{3\left(1+x^4\right)\sqrt{\frac{1+x^8}{\left(1+x^4\right)^2}}\,E\left(2\tan^{-1}\left(x^2\right)|\frac{1}{2}\right)}{10\sqrt{1+x^8}} - \frac{3\left(1+x^4\right)\sqrt{\frac{1+x^8}{\left(1+x^4\right)^2}}}{20\sqrt{1}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.26

$$\frac{1}{10}x^6\left(\sqrt{x^8+1} - {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -x^8\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13/Sqrt[1 + x^8],x]
```

```
[Out] (x^6*(Sqrt[1 + x^8] - Hypergeometric2F1[1/2, 3/4, 7/4, -x^8]))/10
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^{13}}{\sqrt{x^8+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(x^8+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^13/sqrt(x^8 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{\sqrt{x^8+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(x^8+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^13/sqrt(x^8 + 1), x)
```

maple [C] time = 0.15, size = 30, normalized size = 0.23

$$-\frac{x^6\,\text{hypergeom}\left(\left[\frac{1}{2}, \frac{3}{4}\right], \left[\frac{7}{4}\right], -x^8\right)}{10} + \frac{\sqrt{x^8+1}\,x^6}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(x^8+1)^(1/2),x)
```

```
[Out] 1/10*x^6*(x^8+1)^(1/2)-1/10*x^6*hypergeom([1/2,3/4],[7/4],-x^8)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{\sqrt{x^8+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(x^8+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^13/sqrt(x^8 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{13}}{\sqrt{x^8+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(x^8 + 1)^(1/2),x)
```

```
[Out] int(x^13/(x^8 + 1)^(1/2), x)
```

sympy [C] time = 1.29, size = 29, normalized size = 0.22

$$\frac{x^{14}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{7}{4} \\ \frac{11}{4}\end{matrix}\right| x^8 e^{i\pi}\right)}{8\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(x**8+1)**(1/2),x)
```

```
[Out] x**14*gamma(7/4)*hyper((1/2, 7/4), (11/4,), x**8*exp_polar(I*pi))/(8*gamma(
11/4))
```

3.1525 $\int \frac{x^9}{\sqrt{1+x^8}}\,dx$

Optimal. Leaf size=62

$$\frac{1}{6}x^2\sqrt{x^8+1} - \frac{\left(x^4+1\right)\sqrt{\frac{x^8+1}{\left(x^4+1\right)^2}}\,F\left(2\tan^{-1}\left(x^2\right)|\frac{1}{2}\right)}{12\sqrt{x^8+1}}$$

```
[Out] 1/6*x^2*(x^8+1)^(1/2)-1/12*(x^4+1)*(cos(2*arctan(x^2))^2)^(1/2)/cos(2*arcta
n(x^2))*EllipticF(sin(2*arctan(x^2)),1/2*2^(1/2))*((x^8+1)/(x^4+1)^2)^(1/2)
/(x^8+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {275, 321, 220}

$$\frac{1}{6}x^2\sqrt{x^8+1} - \frac{\left(x^4+1\right)\sqrt{\frac{x^8+1}{\left(x^4+1\right)^2}}\,F\left(2\tan^{-1}\left(x^2\right)|\frac{1}{2}\right)}{12\sqrt{x^8+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/Sqrt[1 + x^8],x]
```

```
[Out] (x^2*Sqrt[1 + x^8])/6 - ((1 + x^4)*Sqrt[(1 + x^8)/(1 + x^4)^2]*EllipticF[2*
ArcTan[x^2], 1/2])/(12*Sqrt[1 + x^8])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{\sqrt{1+x^8}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^4}}\,dx, x, x^2\right)\\ &= \frac{1}{6}x^2\sqrt{1+x^8} - \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx, x, x^2\right)\\ &= \frac{1}{6}x^2\sqrt{1+x^8} - \frac{\left(1+x^4\right)\sqrt{\frac{1+x^8}{\left(1+x^4\right)^2}}\,F\left(2\tan^{-1}\left(x^2\right)|\frac{1}{2}\right)}{12\sqrt{1+x^8}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.55

$$\frac{1}{6}x^2\left(\sqrt{x^8+1} - {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -x^8\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/Sqrt[1 + x^8],x]
[Out] (x^2*(Sqrt[1 + x^8] - Hypergeometric2F1[1/4, 1/2, 5/4, -x^8]))/6
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^9}{\sqrt{x^8+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+1)^(1/2),x, algorithm="fricas")
[Out] integral(x^9/sqrt(x^8 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\sqrt{x^8+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^9/sqrt(x^8 + 1), x)
```

maple [C] time = 0.14, size = 30, normalized size = 0.48

$$-\frac{x^2\,\text{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right], \left[\frac{5}{4}\right], -x^8\right)}{6} + \frac{\sqrt{x^8+1}\,x^2}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^8+1)^(1/2),x)
[Out] 1/6*(x^8+1)^(1/2)*x^2-1/6*x^2*hypergeom([1/4,1/2],[5/4],-x^8)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\sqrt{x^8+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^9/sqrt(x^8 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^9}{\sqrt{x^8+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^8 + 1)^(1/2),x)
```

```
[Out] int(x^9/(x^8 + 1)^(1/2), x)
```

sympy [C] time = 0.97, size = 29, normalized size = 0.47

$$\frac{x^{10}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle| x^8 e^{i\pi}\right)}{8\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(x**8+1)**(1/2),x)
```

```
[Out] x**10*gamma(5/4)*hyper((1/2, 5/4), (9/4,), x**8*exp_polar(I*pi))/(8*gamma(9
/4))
```

3.1526 $\int \frac{x^5}{\sqrt{1+x^8}}\,dx$

Optimal. Leaf size=114

$$\frac{\sqrt{x^8+1}\,x^2}{2\left(x^4+1\right)}+\frac{\left(x^4+1\right)\sqrt{\frac{x^8+1}{\left(x^4+1\right)^2}}\,F\left(2\tan^{-1}\left(x^2\right)|\frac{1}{2}\right)}{4\sqrt{x^8+1}}-\frac{\left(x^4+1\right)\sqrt{\frac{x^8+1}{\left(x^4+1\right)^2}}\,E\left(2\tan^{-1}\left(x^2\right)|\frac{1}{2}\right)}{2\sqrt{x^8+1}}$$

```
[Out] 1/2*x^2*(x^8+1)^(1/2)/(x^4+1)-1/2*(x^4+1)*(cos(2*arctan(x^2))^2)^(1/2)/cos(
2*arctan(x^2))*EllipticE(sin(2*arctan(x^2)),1/2*2^(1/2))*((x^8+1)/(x^4+1)^2
)^(1/2)/(x^8+1)^(1/2)+1/4*(x^4+1)*(cos(2*arctan(x^2))^2)^(1/2)/cos(2*arctan
(x^2))*EllipticF(sin(2*arctan(x^2)),1/2*2^(1/2))*((x^8+1)/(x^4+1)^2)^(1/2)/
(x^8+1)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {275, 305, 220, 1196}

$$\frac{\sqrt{x^8+1}\,x^2}{2\left(x^4+1\right)}+\frac{\left(x^4+1\right)\sqrt{\frac{x^8+1}{\left(x^4+1\right)^2}}\,F\left(2\tan^{-1}\left(x^2\right)|\frac{1}{2}\right)}{4\sqrt{x^8+1}}-\frac{\left(x^4+1\right)\sqrt{\frac{x^8+1}{\left(x^4+1\right)^2}}\,E\left(2\tan^{-1}\left(x^2\right)|\frac{1}{2}\right)}{2\sqrt{x^8+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[1 + x^8],x]
```

```
[Out] (x^2*Sqrt[1 + x^8])/(2*(1 + x^4)) - ((1 + x^4)*Sqrt[(1 + x^8)/(1 + x^4)^2]*
EllipticE[2*ArcTan[x^2], 1/2])/(2*Sqrt[1 + x^8]) + ((1 + x^4)*Sqrt[(1 + x^8
)/(1 + x^4)^2]*EllipticF[2*ArcTan[x^2], 1/2])/(4*Sqrt[1 + x^8])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\sqrt{1+x^8}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^4}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx, x, x^2\right) - \frac{1}{2}\,\text{Subst}\left(\int \frac{1-x^2}{\sqrt{1+x^4}}\,dx, x, x^2\right)\\ &= \frac{x^2\sqrt{1+x^8}}{2\left(1+x^4\right)} - \frac{\left(1+x^4\right)\sqrt{\frac{1+x^8}{\left(1+x^4\right)^2}}\,E\left(2\tan^{-1}\left(x^2\right)|\frac{1}{2}\right)}{2\sqrt{1+x^8}} + \frac{\left(1+x^4\right)\sqrt{\frac{1+x^8}{\left(1+x^4\right)^2}}\,F\left(2\tan^{-1}\left(x^2\right)|\frac{1}{2}\right)}{4\sqrt{1+x^8}}\end{aligned}$$

Mathematica [C] time = 0.00, size = 22, normalized size = 0.19

$$\frac{1}{6}x^6\,{}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[1 + x^8],x]
[Out] (x^6*Hypergeometric2F1[1/2, 3/4, 7/4, -x^8])/6
```

fricas [F] time = 1.13, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^5}{\sqrt{x^8+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8+1)^(1/2),x, algorithm="fricas")
[Out] integral(x^5/sqrt(x^8 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{x^8+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^5/sqrt(x^8 + 1), x)
```

maple [C] time = 0.15, size = 17, normalized size = 0.15

$$\frac{x^6\,\text{hypergeom}\left(\left[\frac{1}{2}, \frac{3}{4}\right], \left[\frac{7}{4}\right], -x^8\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^8+1)^(1/2),x)
[Out] 1/6*x^6*hypergeom([1/2,3/4],[7/4],-x^8)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{x^8+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^5/sqrt(x^8 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\sqrt{x^8+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^8 + 1)^(1/2),x)
```

```
[Out] int(x^5/(x^8 + 1)^(1/2), x)
```

sympy [C] time = 0.77, size = 29, normalized size = 0.25

$$\frac{x^6\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle| x^8 e^{i\pi}\right)}{8\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**8+1)**(1/2),x)
```

```
[Out] x**6*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**8*exp_polar(I*pi))/(8*gamma(7/
4))
```

3.1527 $\int \frac{x}{\sqrt{1+x^8}}\,dx$

Optimal. Leaf size=45

$$\frac{\left(x^4+1\right)\sqrt{\frac{x^8+1}{\left(x^4+1\right)^2}}\,F\left(2\tan^{-1}\left(x^2\right)|\frac{1}{2}\right)}{4\sqrt{x^8+1}}$$

```
[Out] 1/4*(x^4+1)*(cos(2*arctan(x^2))^2)^(1/2)/cos(2*arctan(x^2))*EllipticF(sin(2
*arctan(x^2)),1/2*2^(1/2))*((x^8+1)/(x^4+1)^2)^(1/2)/(x^8+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {275, 220}

$$\frac{\left(x^4+1\right)\sqrt{\frac{x^8+1}{\left(x^4+1\right)^2}}\,F\left(2\tan^{-1}\left(x^2\right)|\frac{1}{2}\right)}{4\sqrt{x^8+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[1 + x^8],x]
```

```
[Out] ((1 + x^4)*Sqrt[(1 + x^8)/(1 + x^4)^2]*EllipticF[2*ArcTan[x^2], 1/2])/(4*Sq
rt[1 + x^8])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{1+x^8}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx, x, x^2\right)\\ &= \frac{\left(1+x^4\right)\sqrt{\frac{1+x^8}{\left(1+x^4\right)^2}}\,F\left(2\tan^{-1}\left(x^2\right)|\frac{1}{2}\right)}{4\sqrt{1+x^8}}\end{aligned}$$

Mathematica [C] time = 0.00, size = 22, normalized size = 0.49

$$\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[1 + x^8],x]
```

```
[Out] (x^2*Hypergeometric2F1[1/4, 1/2, 5/4, -x^8])/2
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{\sqrt{x^8+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+1)^(1/2),x, algorithm="fricas")
[Out] integral(x/sqrt(x^8 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^8+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+1)^(1/2),x, algorithm="giac")
[Out] integrate(x/sqrt(x^8 + 1), x)
```

maple [C] time = 0.14, size = 17, normalized size = 0.38

$$\frac{x^2\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],-x^8\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^8+1)^(1/2),x)
[Out] 1/2*x^2*hypergeom([1/4,1/2],[5/4],-x^8)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^8+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x/sqrt(x^8 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\sqrt{x^8+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^8 + 1)^(1/2),x)
[Out] int(x/(x^8 + 1)^(1/2), x)
```

sympy [C] time = 0.64, size = 29, normalized size = 0.64

$$\frac{x^2\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right| x^8 e^{i\pi}\right)}{8\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**8+1)**(1/2),x)
```

```
[Out] x**2*gamma(1/4)*hyper((1/4, 1/2), (5/4,), x**8*exp_polar(I*pi))/(8*gamma(5/
4))
```

3.1528 $\int \frac{1}{x^3\sqrt{1+x^8}}\,dx$

Optimal. Leaf size=130

$$-\frac{\sqrt{x^8+1}}{2x^2}+\frac{\sqrt{x^8+1}\,x^2}{2\left(x^4+1\right)}+\frac{\left(x^4+1\right)\sqrt{\frac{x^8+1}{\left(x^4+1\right)^2}}F\left(2\tan^{-1}\left(x^2\right)|\frac{1}{2}\right)}{4\sqrt{x^8+1}}-\frac{\left(x^4+1\right)\sqrt{\frac{x^8+1}{\left(x^4+1\right)^2}}E\left(2\tan^{-1}\left(x^2\right)|\frac{1}{2}\right)}{2\sqrt{x^8+1}}$$

```
[Out] -1/2*(x^8+1)^(1/2)/x^2+1/2*x^2*(x^8+1)^(1/2)/(x^4+1)-1/2*(x^4+1)*(cos(2*arc
tan(x^2))^2)^(1/2)/cos(2*arctan(x^2))*EllipticE(sin(2*arctan(x^2)),1/2*2^(1
/2))*((x^8+1)/(x^4+1)^2)^(1/2)/(x^8+1)^(1/2)+1/4*(x^4+1)*(cos(2*arctan(x^2)
)^2)^(1/2)/cos(2*arctan(x^2))*EllipticF(sin(2*arctan(x^2)),1/2*2^(1/2))*((x
^8+1)/(x^4+1)^2)^(1/2)/(x^8+1)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {275, 325, 305, 220, 1196}

$$\frac{\sqrt{x^8+1}\,x^2}{2\left(x^4+1\right)}-\frac{\sqrt{x^8+1}}{2x^2}+\frac{\left(x^4+1\right)\sqrt{\frac{x^8+1}{\left(x^4+1\right)^2}}F\left(2\tan^{-1}\left(x^2\right)|\frac{1}{2}\right)}{4\sqrt{x^8+1}}-\frac{\left(x^4+1\right)\sqrt{\frac{x^8+1}{\left(x^4+1\right)^2}}E\left(2\tan^{-1}\left(x^2\right)|\frac{1}{2}\right)}{2\sqrt{x^8+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[1 + x^8]),x]
```

```
[Out] -Sqrt[1 + x^8]/(2*x^2) + (x^2*Sqrt[1 + x^8])/(2*(1 + x^4)) - ((1 + x^4)*Sqr
t[(1 + x^8)/(1 + x^4)^2]*EllipticE[2*ArcTan[x^2], 1/2])/(2*Sqrt[1 + x^8]) +
 ((1 + x^4)*Sqrt[(1 + x^8)/(1 + x^4)^2]*EllipticF[2*ArcTan[x^2], 1/2])/(4*S
qrt[1 + x^8])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{1+x^8}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x^4}}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{1+x^8}}{2x^2} + \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^4}}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{1+x^8}}{2x^2} + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx, x, x^2\right) - \frac{1}{2}\,\text{Subst}\left(\int \frac{1-x^2}{\sqrt{1+x^4}}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{1+x^8}}{2x^2} + \frac{x^2\sqrt{1+x^8}}{2\left(1+x^4\right)} - \frac{\left(1+x^4\right)\sqrt{\frac{1+x^8}{\left(1+x^4\right)^2}}\,E\left(2\tan^{-1}\left(x^2\right)|\frac{1}{2}\right)}{2\sqrt{1+x^8}} + \frac{\left(1+x^4\right)\sqrt{\frac{1+x^8}{\left(1+x^4\right)^2}}\,F\left(2\right.}{4\sqrt{1+x^8}}\end{aligned}$$

Mathematica [C] time = 0.00, size = 22, normalized size = 0.17

$$-\frac{{}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; -x^8\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[1 + x^8]),x]
```

```
[Out] -1/2*Hypergeometric2F1[-1/4, 1/2, 3/4, -x^8]/x^2
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^8+1}}{x^{11}+x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^8 + 1)/(x^11 + x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^8+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^8 + 1)*x^3), x)
```

maple [C] time = 0.02, size = 30, normalized size = 0.23

$$\frac{x^6\,\text{hypergeom}\left(\left[\frac{1}{2}, \frac{3}{4}\right], \left[\frac{7}{4}\right], -x^8\right)}{6} - \frac{\sqrt{x^8+1}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^8+1)^(1/2),x)
```

```
[Out] -1/2*(x^8+1)^(1/2)/x^2+1/6*x^6*hypergeom([1/2,3/4],[7/4],-x^8)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^8+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^8 + 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\sqrt{x^8+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x^8 + 1)^(1/2)),x)
```

```
[Out] int(1/(x^3*(x^8 + 1)^(1/2)), x)
```

sympy [C] time = 0.80, size = 32, normalized size = 0.25

$$\frac{\Gamma\left(-\frac{1}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{4}, \frac{1}{2} \\ \frac{3}{4}\end{matrix}\right| x^8 e^{i\pi}\right)}{8x^2\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**8+1)**(1/2),x)
```

```
[Out] gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), x**8*exp_polar(I*pi))/(8*x**2*gamma(
3/4))
```

3.1529 $\int \frac{1}{x^7\sqrt{1+x^8}}\,dx$

Optimal. Leaf size=62

$$-\frac{\sqrt{x^8+1}}{6x^6}-\frac{\left(x^4+1\right)\sqrt{\frac{x^8+1}{\left(x^4+1\right)^2}}F\left(2\tan^{-1}\left(x^2\right)|\frac{1}{2}\right)}{12\sqrt{x^8+1}}$$

```
[Out] -1/6*(x^8+1)^(1/2)/x^6-1/12*(x^4+1)*(cos(2*arctan(x^2))^2)^(1/2)/cos(2*arct
an(x^2))*EllipticF(sin(2*arctan(x^2)),1/2*2^(1/2))*((x^8+1)/(x^4+1)^2)^(1/2
)/(x^8+1)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {275, 325, 220}

$$-\frac{\sqrt{x^8+1}}{6x^6}-\frac{\left(x^4+1\right)\sqrt{\frac{x^8+1}{\left(x^4+1\right)^2}}F\left(2\tan^{-1}\left(x^2\right)|\frac{1}{2}\right)}{12\sqrt{x^8+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*Sqrt[1 + x^8]),x]
```

```
[Out] -Sqrt[1 + x^8]/(6*x^6) - ((1 + x^4)*Sqrt[(1 + x^8)/(1 + x^4)^2]*EllipticF[2
*ArcTan[x^2], 1/2])/(12*Sqrt[1 + x^8])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7\sqrt{1+x^8}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^4\sqrt{1+x^4}}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{1+x^8}}{6x^6} - \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{1+x^8}}{6x^6} - \frac{\left(1+x^4\right)\sqrt{\frac{1+x^8}{\left(1+x^4\right)^2}}\,F\left(2\tan^{-1}\left(x^2\right)|\frac{1}{2}\right)}{12\sqrt{1+x^8}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 22, normalized size = 0.35

$$-\frac{{}_2F_1\left(-\frac{3}{4}, \frac{1}{2}; \frac{1}{4}; -x^8\right)}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*Sqrt[1 + x^8]),x]
[Out] -1/6*Hypergeometric2F1[-3/4, 1/2, 1/4, -x^8]/x^6
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^8+1}}{x^{15}+x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x^8 + 1)/(x^15 + x^7), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^8+1}\,x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^8 + 1)*x^7), x)
```

maple [C] time = 0.02, size = 30, normalized size = 0.48

$$-\frac{x^2\,\text{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right], \left[\frac{5}{4}\right], -x^8\right)}{6} - \frac{\sqrt{x^8+1}}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^8+1)^(1/2),x)
[Out] -1/6*(x^8+1)^(1/2)/x^6-1/6*x^2*hypergeom([1/4,1/2],[5/4],-x^8)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^8+1}\,x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^8 + 1)*x^7), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^7 \sqrt{x^8+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(x^8 + 1)^(1/2)),x)
```

```
[Out] int(1/(x^7*(x^8 + 1)^(1/2)), x)
```

sympy [C] time = 0.99, size = 32, normalized size = 0.52

$$\frac{\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{1}{2} \\ \frac{1}{4} \end{matrix} \middle| x^8 e^{i\pi}\right)}{8x^6 \Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**8+1)**(1/2),x)
```

```
[Out] gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), x**8*exp_polar(I*pi))/(8*x**6*gamma(
1/4))
```

3.1530 $\int \frac{x^{10}}{\sqrt{1+x^8}}\,dx$

Optimal. Leaf size=239

$$\frac{1}{7}\sqrt{x^8+1}\,x^3+\frac{3\sqrt{\frac{\left(x^2+1\right)^2}{x^2}}\sqrt{-\frac{x^8+1}{x^4}}\,x^3F\left(\sin^{-1}\left(\frac{1}{2}\sqrt{-\frac{\sqrt{2}\,x^4-2x^2+\sqrt{2}}{x^2}}\right)|-2\left(1-\sqrt{2}\right)\right)}{14\sqrt{2+\sqrt{2}}\,\left(x^2+1\right)\sqrt{x^8+1}}+\frac{3\sqrt{-\frac{\left(1-x^2\right)^2}{x^2}}\sqrt{-\frac{x^8+1}{x^4}}\,x^3F\left(\sin^{-}\right.}{14\sqrt{2+\sqrt{2}}}$$

```
[Out] 1/7*x^3*(x^8+1)^(1/2)+3/14*x^3*EllipticF(1/2*((2*x^2+2^(1/2)+x^4*2^(1/2))/x
^2)^(1/2),(-2+2*2^(1/2))^(1/2))*(-(-x^2+1)^2/x^2)^(1/2)*((-x^8-1)/x^4)^(1/2
)/(-x^2+1)/(x^8+1)^(1/2)/(2+2^(1/2))^(1/2)+3/14*x^3*EllipticF(1/2*((2*x^2-2
^(1/2)-x^4*2^(1/2))/x^2)^(1/2),(-2+2*2^(1/2))^(1/2))*((x^2+1)^2/x^2)^(1/2)*
((-x^8-1)/x^4)^(1/2)/(x^2+1)/(x^8+1)^(1/2)/(2+2^(1/2))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {321, 309, 1883}

$$\frac{1}{7}\sqrt{x^8+1}\,x^3+\frac{3\sqrt{\frac{\left(x^2+1\right)^2}{x^2}}\sqrt{-\frac{x^8+1}{x^4}}\,x^3F\left(\sin^{-1}\left(\frac{1}{2}\sqrt{-\frac{\sqrt{2}\,x^4-2x^2+\sqrt{2}}{x^2}}\right)|-2\left(1-\sqrt{2}\right)\right)}{14\sqrt{2+\sqrt{2}}\,\left(x^2+1\right)\sqrt{x^8+1}}+\frac{3\sqrt{-\frac{\left(1-x^2\right)^2}{x^2}}\sqrt{-\frac{x^8+1}{x^4}}\,x^3F\left(\sin^{-}\right.}{14\sqrt{2+\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/Sqrt[1 + x^8],x]
```

```
[Out] (x^3*Sqrt[1 + x^8])/7 + (3*x^3*Sqrt[(1 + x^2)^2/x^2]*Sqrt[-((1 + x^8)/x^4)]
*EllipticF[ArcSin[Sqrt[-((Sqrt[2] - 2*x^2 + Sqrt[2]*x^4)/x^2)]/2], -2*(1 -
Sqrt[2])])/(14*Sqrt[2 + Sqrt[2]]*(1 + x^2)*Sqrt[1 + x^8]) + (3*x^3*Sqrt[-((
1 - x^2)^2/x^2)]*Sqrt[-((1 + x^8)/x^4)]*EllipticF[ArcSin[Sqrt[(Sqrt[2] + 2*
x^2 + Sqrt[2]*x^4)/x^2]/2], -2*(1 - Sqrt[2])])/(14*Sqrt[2 + Sqrt[2]]*(1 - x
^2)*Sqrt[1 + x^8])
```

Rule 309

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> Dist[1/(2*Rt[b/a, 4]), I
nt[(1 + Rt[b/a, 4]*x^2)/Sqrt[a + b*x^8], x], x] - Dist[1/(2*Rt[b/a, 4]), In
t[(1 - Rt[b/a, 4]*x^2)/Sqrt[a + b*x^8], x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1883

```
Int[((c_) + (d_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> -Simp[(c*
d*x^3*Sqrt[-((c - d*x^2)^2/(c*d*x^2))]*Sqrt[-((d^2*(a + b*x^8))/(b*c^2*x^4)
)]*EllipticF[ArcSin[(1*Sqrt[(Sqrt[2]*c^2 + 2*c*d*x^2 + Sqrt[2]*d^2*x^4)/(c*
d*x^2)])/2], -2*(1 - Sqrt[2])])/(Sqrt[2 + Sqrt[2]]*(c - d*x^2)*Sqrt[a + b*x
^8]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^4 - a*d^4, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{10}}{\sqrt{1+x^8}}\,dx &= \frac{1}{7}x^3\sqrt{1+x^8} - \frac{3}{7}\int \frac{x^2}{\sqrt{1+x^8}}\,dx \\ &= \frac{1}{7}x^3\sqrt{1+x^8} + \frac{3}{14}\int \frac{1-x^2}{\sqrt{1+x^8}}\,dx - \frac{3}{14}\int \frac{1+x^2}{\sqrt{1+x^8}}\,dx \\ &= \frac{1}{7}x^3\sqrt{1+x^8} + \frac{3x^3\sqrt{\frac{\left(1+x^2\right)^2}{x^2}}\sqrt{-\frac{1+x^8}{x^4}}\,F\left(\sin^{-1}\left(\frac{1}{2}\sqrt{-\frac{\sqrt{2}-2x^2+\sqrt{2}\,x^4}{x^2}}\right)\middle|-2\left(1-\sqrt{2}\right)\right)}{14\sqrt{2+\sqrt{2}}\,\left(1+x^2\right)\sqrt{1+x^8}} + \frac{3x^3\sqrt{-}}{}\end{aligned}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.14

$$\frac{1}{7}x^3\left(\sqrt{x^8+1} - {}_2F_1\left(\frac{3}{8},\frac{1}{2};\frac{11}{8};-x^8\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/Sqrt[1 + x^8],x]
[Out] (x^3*(Sqrt[1 + x^8] - Hypergeometric2F1[3/8, 1/2, 11/8, -x^8]))/7
```

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^{10}}{\sqrt{x^8+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(x^8+1)^(1/2),x, algorithm="fricas")
[Out] integral(x^10/sqrt(x^8 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\sqrt{x^8+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(x^8+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^10/sqrt(x^8 + 1), x)
```

maple [C] time = 0.15, size = 30, normalized size = 0.13

$$-\frac{x^3\,\text{hypergeom}\left(\left[\frac{3}{8},\frac{1}{2}\right],\left[\frac{11}{8}\right],-x^8\right)}{7} + \frac{\sqrt{x^8+1}\,x^3}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(x^8+1)^(1/2),x)
[Out] 1/7*x^3*(x^8+1)^(1/2)-1/7*x^3*hypergeom([3/8,1/2],[11/8],-x^8)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\sqrt{x^8+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(x^8+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^10/sqrt(x^8 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{10}}{\sqrt{x^8+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(x^8 + 1)^(1/2),x)
```

```
[Out] int(x^10/(x^8 + 1)^(1/2), x)
```

sympy [C] time = 1.05, size = 29, normalized size = 0.12

$$\frac{x^{11}\Gamma\left(\frac{11}{8}\right){}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{11}{8} \\ \frac{19}{8}\end{matrix}\middle| x^8 e^{i\pi}\right)}{8\Gamma\left(\frac{19}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(x**8+1)**(1/2),x)
```

```
[Out] x**11*gamma(11/8)*hyper((1/2, 11/8), (19/8,), x**8*exp_polar(I*pi))/(8*gamm
a(19/8))
```

3.1531 $\int \frac{x^8}{\sqrt{1+x^8}}\,dx$

Optimal. Leaf size=237

$$\frac{1}{5}\sqrt{x^8+1}\,x-\frac{\sqrt{\frac{\left(x^2+1\right)^2}{x^2}}\sqrt{-\frac{x^8+1}{x^4}}\,x^3F\left(\sin^{-1}\left(\frac{1}{2}\sqrt{-\frac{\sqrt{2}\,x^4-2x^2+\sqrt{2}}{x^2}}\right)|-2\left(1-\sqrt{2}\right)\right)}{10\sqrt{2+\sqrt{2}}\,\left(x^2+1\right)\sqrt{x^8+1}}+\frac{\sqrt{-\frac{\left(1-x^2\right)^2}{x^2}}\sqrt{-\frac{x^8+1}{x^4}}\,x^3F\left(\sin^{-1}\right.}{10\sqrt{2+\sqrt{2}}}$$

```
[Out] 1/5*x*(x^8+1)^(1/2)+1/10*x^3*EllipticF(1/2*((2*x^2+2^(1/2)+x^4*2^(1/2))/x^2
)^(1/2),(-2+2*2^(1/2))^(1/2))*(-(-x^2+1)^2/x^2)^(1/2)*((-x^8-1)/x^4)^(1/2)/
(-x^2+1)/(x^8+1)^(1/2)/(2+2^(1/2))^(1/2)-1/10*x^3*EllipticF(1/2*((2*x^2-2^(
1/2)-x^4*2^(1/2))/x^2)^(1/2),(-2+2*2^(1/2))^(1/2))*((x^2+1)^2/x^2)^(1/2)*((
-x^8-1)/x^4)^(1/2)/(x^2+1)/(x^8+1)^(1/2)/(2+2^(1/2))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {321, 226, 1883}

$$\frac{1}{5}\sqrt{x^8+1}\,x-\frac{\sqrt{\frac{\left(x^2+1\right)^2}{x^2}}\sqrt{-\frac{x^8+1}{x^4}}\,x^3F\left(\sin^{-1}\left(\frac{1}{2}\sqrt{-\frac{\sqrt{2}\,x^4-2x^2+\sqrt{2}}{x^2}}\right)|-2\left(1-\sqrt{2}\right)\right)}{10\sqrt{2+\sqrt{2}}\,\left(x^2+1\right)\sqrt{x^8+1}}+\frac{\sqrt{-\frac{\left(1-x^2\right)^2}{x^2}}\sqrt{-\frac{x^8+1}{x^4}}\,x^3F\left(\sin^{-1}\right.}{10\sqrt{2+\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/Sqrt[1 + x^8],x]
```

```
[Out] (x*Sqrt[1 + x^8])/5 - (x^3*Sqrt[(1 + x^2)^2/x^2]*Sqrt[-((1 + x^8)/x^4)]*Ell
ipticF[ArcSin[Sqrt[-((Sqrt[2] - 2*x^2 + Sqrt[2]*x^4)/x^2)]/2], -2*(1 - Sqrt
[2])])/(10*Sqrt[2 + Sqrt[2]]*(1 + x^2)*Sqrt[1 + x^8]) + (x^3*Sqrt[-((1 - x^
2)^2/x^2)]*Sqrt[-((1 + x^8)/x^4)]*EllipticF[ArcSin[Sqrt[(Sqrt[2] + 2*x^2 +
Sqrt[2]*x^4)/x^2]/2], -2*(1 - Sqrt[2])])/(10*Sqrt[2 + Sqrt[2]]*(1 - x^2)*Sq
rt[1 + x^8])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> Dist[1/2, Int[(1 - Rt[b/a, 4]
*x^2)/Sqrt[a + b*x^8], x], x] + Dist[1/2, Int[(1 + Rt[b/a, 4]*x^2)/Sqrt[a +
 b*x^8], x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1883

```
Int[((c_) + (d_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> -Simp[(c*
d*x^3*Sqrt[-((c - d*x^2)^2/(c*d*x^2))]*Sqrt[-((d^2*(a + b*x^8))/(b*c^2*x^4)
)]*EllipticF[ArcSin[(1*Sqrt[(Sqrt[2]*c^2 + 2*c*d*x^2 + Sqrt[2]*d^2*x^4)/(c*
d*x^2)])/2], -2*(1 - Sqrt[2])])/(Sqrt[2 + Sqrt[2]]*(c - d*x^2)*Sqrt[a + b*x
^8]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^4 - a*d^4, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{\sqrt{1+x^8}}\,dx &= \frac{1}{5}x\sqrt{1+x^8} - \frac{1}{5}\int \frac{1}{\sqrt{1+x^8}}\,dx \\ &= \frac{1}{5}x\sqrt{1+x^8} - \frac{1}{10}\int \frac{1-x^2}{\sqrt{1+x^8}}\,dx - \frac{1}{10}\int \frac{1+x^2}{\sqrt{1+x^8}}\,dx \\ &= \frac{1}{5}x\sqrt{1+x^8} - \frac{x^3\sqrt{\frac{\left(1+x^2\right)^2}{x^2}}\sqrt{-\frac{1+x^8}{x^4}}\,F\left(\sin^{-1}\left(\frac{1}{2}\sqrt{-\frac{\sqrt{2}-2x^2+\sqrt{2}\,x^4}{x^2}}\right)\middle|\, -2\left(1-\sqrt{2}\right)\right)}{10\sqrt{2+\sqrt{2}}\,\left(1+x^2\right)\sqrt{1+x^8}} + \frac{x^3\sqrt{-\frac{\left(1-x^2\right)^2}{x^2}}}{}\end{aligned}$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.14

$$\frac{1}{5}x\left(\sqrt{x^8+1} - {}_2F_1\left(\frac{1}{8},\frac{1}{2};\frac{9}{8};-x^8\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/Sqrt[1 + x^8],x]
[Out] (x*(Sqrt[1 + x^8] - Hypergeometric2F1[1/8, 1/2, 9/8, -x^8]))/5
```

fricas [F] time = 1.07, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^8}{\sqrt{x^8+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+1)^(1/2),x, algorithm="fricas")
[Out] integral(x^8/sqrt(x^8 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\sqrt{x^8+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^8/sqrt(x^8 + 1), x)
```

maple [C] time = 0.15, size = 26, normalized size = 0.11

$$-\frac{x\,\text{hypergeom}\left(\left[\frac{1}{8},\frac{1}{2}\right],\left[\frac{9}{8}\right],-x^8\right)}{5} + \frac{\sqrt{x^8+1}\,x}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^8+1)^(1/2),x)
[Out] 1/5*x*(x^8+1)^(1/2)-1/5*x*hypergeom([1/8,1/2],[9/8],-x^8)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\sqrt{x^8+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^8/sqrt(x^8 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^8}{\sqrt{x^8+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^8 + 1)^(1/2),x)
```

```
[Out] int(x^8/(x^8 + 1)^(1/2), x)
```

sympy [C] time = 0.87, size = 29, normalized size = 0.12

$$\frac{x^9\Gamma\left(\frac{9}{8}\right){}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{9}{8}\\ \frac{17}{8}\end{matrix}\middle| x^8 e^{i\pi}\right)}{8\Gamma\left(\frac{17}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**8+1)**(1/2),x)
```

```
[Out] x**9*gamma(9/8)*hyper((1/2, 9/8), (17/8,), x**8*exp_polar(I*pi))/(8*gamma(1
7/8))
```

3.1532 $\int \frac{x^6}{\sqrt{1+x^8}}\,dx$

Optimal. Leaf size=22

$$\frac{1}{7}x^7\,{}_2F_1\left(\frac{1}{2},\frac{7}{8};\frac{15}{8};-x^8\right)$$

```
[Out] 1/7*x^7*hypergeom([1/2, 7/8],[15/8],-x^8)
```

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {364}

$$\frac{1}{7}x^7\,{}_2F_1\left(\frac{1}{2},\frac{7}{8};\frac{15}{8};-x^8\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^6/Sqrt[1 + x^8],x]
```

```
[Out] (x^7*Hypergeometric2F1[1/2, 7/8, 15/8, -x^8])/7
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^6}{\sqrt{1+x^8}}\,dx = \frac{1}{7}x^7\,{}_2F_1\left(\frac{1}{2},\frac{7}{8};\frac{15}{8};-x^8\right)$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$\frac{1}{7}x^7\,{}_2F_1\left(\frac{1}{2},\frac{7}{8};\frac{15}{8};-x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/Sqrt[1 + x^8],x]
```

```
[Out] (x^7*Hypergeometric2F1[1/2, 7/8, 15/8, -x^8])/7
```

fricas [F] time = 1.05, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^6}{\sqrt{x^8+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^6/sqrt(x^8 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt{x^8+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/sqrt(x^8 + 1), x)
```

maple [A] time = 0.15, size = 17, normalized size = 0.77

$$\frac{x^7\,\text{hypergeom}\left(\left[\frac{1}{2},\frac{7}{8}\right],\left[\frac{15}{8}\right],-x^8\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^8+1)^(1/2),x)
```

```
[Out] 1/7*x^7*hypergeom([1/2,7/8],[15/8],-x^8)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt{x^8+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/sqrt(x^8 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^6}{\sqrt{x^8+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^8 + 1)^(1/2),x)
```

```
[Out] int(x^6/(x^8 + 1)^(1/2), x)
```

sympy [C] time = 0.86, size = 29, normalized size = 1.32

$$\frac{x^7\Gamma\left(\frac{7}{8}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{7}{8}\\ \frac{15}{8}\end{matrix}\right|x^8e^{i\pi}\right)}{8\Gamma\left(\frac{15}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**8+1)**(1/2),x)
```

```
[Out] x**7*gamma(7/8)*hyper((1/2, 7/8), (15/8,), x**8*exp_polar(I*pi))/(8*gamma(1
5/8))
```

3.1533 $\int \frac{x^4}{\sqrt{1+x^8}}\,dx$

Optimal. Leaf size=22

$$\frac{1}{5}x^5\,{}_2F_1\left(\frac{1}{2},\frac{5}{8};\frac{13}{8};-x^8\right)$$

```
[Out] 1/5*x^5*hypergeom([1/2, 5/8],[13/8],-x^8)
```

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {364}

$$\frac{1}{5}x^5\,{}_2F_1\left(\frac{1}{2},\frac{5}{8};\frac{13}{8};-x^8\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[1 + x^8],x]
```

```
[Out] (x^5*Hypergeometric2F1[1/2, 5/8, 13/8, -x^8])/5
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^4}{\sqrt{1+x^8}}\,dx = \frac{1}{5}x^5\,{}_2F_1\left(\frac{1}{2},\frac{5}{8};\frac{13}{8};-x^8\right)$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$\frac{1}{5}x^5\,{}_2F_1\left(\frac{1}{2},\frac{5}{8};\frac{13}{8};-x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[1 + x^8],x]
```

```
[Out] (x^5*Hypergeometric2F1[1/2, 5/8, 13/8, -x^8])/5
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^4}{\sqrt{x^8+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^4/sqrt(x^8 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{x^8+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(x^8 + 1), x)
```

maple [A] time = 0.15, size = 17, normalized size = 0.77

$$\frac{x^5\,\text{hypergeom}\left(\left[\frac{1}{2},\frac{5}{8}\right],\left[\frac{13}{8}\right],-x^8\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^8+1)^(1/2),x)
```

```
[Out] 1/5*x^5*hypergeom([1/2,5/8],[13/8],-x^8)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{x^8+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(x^8 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^4}{\sqrt{x^8+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^8 + 1)^(1/2),x)
```

```
[Out] int(x^4/(x^8 + 1)^(1/2), x)
```

sympy [C] time = 0.74, size = 29, normalized size = 1.32

$$\frac{x^5\Gamma\left(\frac{5}{8}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{5}{8}\\ \frac{13}{8}\end{matrix}\right|x^8e^{i\pi}\right)}{8\Gamma\left(\frac{13}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**8+1)**(1/2),x)
```

```
[Out] x**5*gamma(5/8)*hyper((1/2, 5/8), (13/8,), x**8*exp_polar(I*pi))/(8*gamma(1
3/8))
```

3.1534 $\int \frac{x^2}{\sqrt{1+x^8}}\,dx$

Optimal. Leaf size=223

$$-\frac{\sqrt{\frac{\left(x^2+1\right)^2}{x^2}}\sqrt{-\frac{x^8+1}{x^4}}\,x^3F\left(\sin^{-1}\left(\frac{1}{2}\sqrt{-\frac{\sqrt{2}\,x^4-2x^2+\sqrt{2}}{x^2}}\right)|-2\left(1-\sqrt{2}\right)\right)}{2\sqrt{2+\sqrt{2}}\,\left(x^2+1\right)\sqrt{x^8+1}}-\frac{\sqrt{-\frac{\left(1-x^2\right)^2}{x^2}}\sqrt{-\frac{x^8+1}{x^4}}\,x^3F\left(\sin^{-1}\left(\frac{1}{2}\sqrt{\frac{\sqrt{2}\,x^4+2x^2}{x^2}}\right.\right.}{2\sqrt{2+\sqrt{2}}\,\left(1-x^2\right)\sqrt{x^8+}}$$

```
[Out] -1/2*x^3*EllipticF(1/2*((2*x^2+2^(1/2)+x^4*2^(1/2))/x^2)^(1/2),(-2+2*2^(1/2
))^(1/2))*(-(-x^2+1)^2/x^2)^(1/2)*((-x^8-1)/x^4)^(1/2)/(-x^2+1)/(x^8+1)^(1/
2)/(2+2^(1/2))^(1/2)-1/2*x^3*EllipticF(1/2*((2*x^2-2^(1/2)-x^4*2^(1/2))/x^2
)^(1/2),(-2+2*2^(1/2))^(1/2))*((x^2+1)^2/x^2)^(1/2)*((-x^8-1)/x^4)^(1/2)/(x
^2+1)/(x^8+1)^(1/2)/(2+2^(1/2))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {309, 1883}

$$-\frac{\sqrt{\frac{\left(x^2+1\right)^2}{x^2}}\sqrt{-\frac{x^8+1}{x^4}}\,x^3F\left(\sin^{-1}\left(\frac{1}{2}\sqrt{-\frac{\sqrt{2}\,x^4-2x^2+\sqrt{2}}{x^2}}\right)|-2\left(1-\sqrt{2}\right)\right)}{2\sqrt{2+\sqrt{2}}\,\left(x^2+1\right)\sqrt{x^8+1}}-\frac{\sqrt{-\frac{\left(1-x^2\right)^2}{x^2}}\sqrt{-\frac{x^8+1}{x^4}}\,x^3F\left(\sin^{-1}\left(\frac{1}{2}\sqrt{\frac{\sqrt{2}\,x^4+2x^2}{x^2}}\right.\right.}{2\sqrt{2+\sqrt{2}}\,\left(1-x^2\right)\sqrt{x^8+}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[1 + x^8],x]
```

```
[Out] -(x^3*Sqrt[(1 + x^2)^2/x^2]*Sqrt[-((1 + x^8)/x^4)]*EllipticF[ArcSin[Sqrt[-(
(Sqrt[2] - 2*x^2 + Sqrt[2]*x^4)/x^2)]/2], -2*(1 - Sqrt[2])])/(2*Sqrt[2 + Sq
rt[2]]*(1 + x^2)*Sqrt[1 + x^8]) - (x^3*Sqrt[-((1 - x^2)^2/x^2)]*Sqrt[-((1 +
 x^8)/x^4)]*EllipticF[ArcSin[Sqrt[(Sqrt[2] + 2*x^2 + Sqrt[2]*x^4)/x^2]/2],
-2*(1 - Sqrt[2])])/(2*Sqrt[2 + Sqrt[2]]*(1 - x^2)*Sqrt[1 + x^8])
```

Rule 309

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> Dist[1/(2*Rt[b/a, 4]), I
nt[(1 + Rt[b/a, 4]*x^2)/Sqrt[a + b*x^8], x], x] - Dist[1/(2*Rt[b/a, 4]), In
t[(1 - Rt[b/a, 4]*x^2)/Sqrt[a + b*x^8], x], x] /; FreeQ[{a, b}, x]
```

Rule 1883

```
Int[((c_) + (d_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> -Simp[(c*
d*x^3*Sqrt[-((c - d*x^2)^2/(c*d*x^2))]*Sqrt[-((d^2*(a + b*x^8))/(b*c^2*x^4)
)]*EllipticF[ArcSin[(1*Sqrt[(Sqrt[2]*c^2 + 2*c*d*x^2 + Sqrt[2]*d^2*x^4)/(c*
d*x^2)])/2], -2*(1 - Sqrt[2])])/(Sqrt[2 + Sqrt[2]]*(c - d*x^2)*Sqrt[a + b*x
^8]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^4 - a*d^4, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{1+x^8}}\,dx = -\left(\frac{1}{2}\int \frac{1-x^2}{\sqrt{1+x^8}}\,dx\right)+\frac{1}{2}\int \frac{1+x^2}{\sqrt{1+x^8}}\,dx$$

$$= -\frac{x^3\sqrt{\frac{\left(1+x^2\right)^2}{x^2}}\sqrt{-\frac{1+x^8}{x^4}}\,F\left(\sin^{-1}\left(\frac{1}{2}\sqrt{-\frac{\sqrt{2}-2x^2+\sqrt{2}\,x^4}{x^2}}\right)|-2\left(1-\sqrt{2}\right)\right)}{2\sqrt{2+\sqrt{2}}\,\left(1+x^2\right)\sqrt{1+x^8}}-\frac{x^3\sqrt{-\frac{\left(1-x^2\right)^2}{x^2}}\sqrt{-\frac{1+x^8}{x^4}}\,F\left(s\right.}{2\sqrt{2+}}$$

Mathematica [C] time = 0.00, size = 22, normalized size = 0.10

$$\frac{1}{3}x^3\,{}_2F_1\left(\frac{3}{8},\frac{1}{2};\frac{11}{8};-x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[1 + x^8],x]
[Out] (x^3*Hypergeometric2F1[3/8, 1/2, 11/8, -x^8])/3
```

fricas [F] time = 1.15, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2}{\sqrt{x^8+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+1)^(1/2),x, algorithm="fricas")
[Out] integral(x^2/sqrt(x^8 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{x^8+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/sqrt(x^8 + 1), x)
```

maple [C] time = 0.14, size = 17, normalized size = 0.08

$$\frac{x^3\,\text{hypergeom}\left(\left[\frac{3}{8},\frac{1}{2}\right],\left[\frac{11}{8}\right],-x^8\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^8+1)^(1/2),x)
[Out] 1/3*x^3*hypergeom([3/8,1/2],[11/8],-x^8)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{x^8+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/sqrt(x^8 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{x^8+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^8 + 1)^(1/2),x)
```

```
[Out] int(x^2/(x^8 + 1)^(1/2), x)
```

sympy [C] time = 0.68, size = 29, normalized size = 0.13

$$\frac{x^3\Gamma\left(\frac{3}{8}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{8}, \frac{1}{2} \\ \frac{11}{8}\end{matrix}\right| x^8 e^{i\pi}\right)}{8\Gamma\left(\frac{11}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**8+1)**(1/2),x)
```

```
[Out] x**3*gamma(3/8)*hyper((3/8, 1/2), (11/8,), x**8*exp_polar(I*pi))/(8*gamma(1
1/8))
```

3.1535 $\int \frac{1}{\sqrt{1+x^8}}\,dx$

Optimal. Leaf size=223

$$\frac{x^3\sqrt{\frac{\left(x^2+1\right)^2}{x^2}}\sqrt{-\frac{x^8+1}{x^4}}\,F\left(\sin^{-1}\left(\frac{1}{2}\sqrt{-\frac{\sqrt{2}\,x^4-2x^2+\sqrt{2}}{x^2}}\right)|-2\left(1-\sqrt{2}\right)\right)}{2\sqrt{2+\sqrt{2}}\,\left(x^2+1\right)\sqrt{x^8+1}} - \frac{x^3\sqrt{-\frac{\left(1-x^2\right)^2}{x^2}}\sqrt{-\frac{x^8+1}{x^4}}\,F\left(\sin^{-1}\left(\frac{1}{2}\sqrt{\frac{\sqrt{2}\,x^4+2x}{x^2}}\right)\right.}{2\sqrt{2+\sqrt{2}}\,\left(1-x^2\right)\sqrt{x^8}}$$

```
[Out] -1/2*x^3*EllipticF(1/2*((2*x^2+2^(1/2)+x^4*2^(1/2))/x^2)^(1/2),(-2+2*2^(1/2
))^(1/2))*(-(-x^2+1)^2/x^2)^(1/2)*((-x^8-1)/x^4)^(1/2)/(-x^2+1)/(x^8+1)^(1/
2)/(2+2^(1/2))^(1/2)+1/2*x^3*EllipticF(1/2*((2*x^2-2^(1/2)-x^4*2^(1/2))/x^2
)^(1/2),(-2+2*2^(1/2))^(1/2))*((x^2+1)^2/x^2)^(1/2)*((-x^8-1)/x^4)^(1/2)/(x
^2+1)/(x^8+1)^(1/2)/(2+2^(1/2))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {226, 1883}

$$\frac{x^3\sqrt{\frac{\left(x^2+1\right)^2}{x^2}}\sqrt{-\frac{x^8+1}{x^4}}\,F\left(\sin^{-1}\left(\frac{1}{2}\sqrt{-\frac{\sqrt{2}\,x^4-2x^2+\sqrt{2}}{x^2}}\right)|-2\left(1-\sqrt{2}\right)\right)}{2\sqrt{2+\sqrt{2}}\,\left(x^2+1\right)\sqrt{x^8+1}} - \frac{x^3\sqrt{-\frac{\left(1-x^2\right)^2}{x^2}}\sqrt{-\frac{x^8+1}{x^4}}\,F\left(\sin^{-1}\left(\frac{1}{2}\sqrt{\frac{\sqrt{2}\,x^4+2x}{x^2}}\right)\right.}{2\sqrt{2+\sqrt{2}}\,\left(1-x^2\right)\sqrt{x^8}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + x^8],x]
```

```
[Out] (x^3*Sqrt[(1 + x^2)^2/x^2]*Sqrt[-((1 + x^8)/x^4)]*EllipticF[ArcSin[Sqrt[-((
Sqrt[2] - 2*x^2 + Sqrt[2]*x^4)/x^2)]/2], -2*(1 - Sqrt[2])])/(2*Sqrt[2 + Sqr
t[2]]*(1 + x^2)*Sqrt[1 + x^8]) - (x^3*Sqrt[-((1 - x^2)^2/x^2)]*Sqrt[-((1 +
x^8)/x^4)]*EllipticF[ArcSin[Sqrt[(Sqrt[2] + 2*x^2 + Sqrt[2]*x^4)/x^2]/2], -
2*(1 - Sqrt[2])])/(2*Sqrt[2 + Sqrt[2]]*(1 - x^2)*Sqrt[1 + x^8])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> Dist[1/2, Int[(1 - Rt[b/a, 4]
*x^2)/Sqrt[a + b*x^8], x], x] + Dist[1/2, Int[(1 + Rt[b/a, 4]*x^2)/Sqrt[a +
 b*x^8], x], x] /; FreeQ[{a, b}, x]
```

Rule 1883

```
Int[((c_) + (d_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> -Simp[(c*
d*x^3*Sqrt[-((c - d*x^2)^2/(c*d*x^2))]*Sqrt[-((d^2*(a + b*x^8))/(b*c^2*x^4)
)]*EllipticF[ArcSin[(1*Sqrt[(Sqrt[2]*c^2 + 2*c*d*x^2 + Sqrt[2]*d^2*x^4)/(c*
d*x^2)])/2], -2*(1 - Sqrt[2])])/(Sqrt[2 + Sqrt[2]]*(c - d*x^2)*Sqrt[a + b*x
^8]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^4 - a*d^4, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+x^8}}\,dx = \frac{1}{2}\int \frac{1-x^2}{\sqrt{1+x^8}}\,dx + \frac{1}{2}\int \frac{1+x^2}{\sqrt{1+x^8}}\,dx$$

$$= \frac{x^3\sqrt{\frac{\left(1+x^2\right)^2}{x^2}}\sqrt{-\frac{1+x^8}{x^4}}\,F\left(\sin^{-1}\left(\frac{1}{2}\sqrt{-\frac{\sqrt{2}-2x^2+\sqrt{2}\,x^4}{x^2}}\right)|-2\left(1-\sqrt{2}\right)\right)}{2\sqrt{2+\sqrt{2}}\,\left(1+x^2\right)\sqrt{1+x^8}} - \frac{x^3\sqrt{-\frac{\left(1-x^2\right)^2}{x^2}}\sqrt{-\frac{1+x^8}{x^4}}\,F\left(\right.}{2\sqrt{2}}$$

Mathematica [C] time = 0.00, size = 17, normalized size = 0.08

$$x\,{}_2F_1\left(\frac{1}{8},\frac{1}{2};\frac{9}{8};-x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + x^8],x]
[Out] x*Hypergeometric2F1[1/8, 1/2, 9/8, -x^8]
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{x^8+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+1)^(1/2),x, algorithm="fricas")
[Out] integral(1/sqrt(x^8 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^8+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(x^8 + 1), x)
```

maple [C] time = 0.15, size = 14, normalized size = 0.06

$$x\,\text{hypergeom}\left(\left[\frac{1}{8},\frac{1}{2}\right],\left[\frac{9}{8}\right],-x^8\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8+1)^(1/2),x)
[Out] x*hypergeom([1/8,1/2],[9/8],-x^8)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^8+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(x^8 + 1), x)
```

mupad [B] time = 0.07, size = 12, normalized size = 0.05

$$x\,{}_2\mathrm{F}_1\left(\frac{1}{8},\frac{1}{2};\ \frac{9}{8};\ -x^8\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8 + 1)^(1/2),x)
```

```
[Out] x*hypergeom([1/8, 1/2], 9/8, -x^8)
```

sympy [C] time = 0.67, size = 27, normalized size = 0.12

$$\frac{x\Gamma\left(\frac{1}{8}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{8}, \frac{1}{2} \\ \frac{9}{8}\end{matrix}\right| x^8 e^{i\pi}\right)}{8\Gamma\left(\frac{9}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**8+1)**(1/2),x)
[Out] x*gamma(1/8)*hyper((1/8, 1/2), (9/8,), x**8*exp_polar(I*pi))/(8*gamma(9/8))
```

3.1536 $\int \frac{1}{x^2\sqrt{1+x^8}}\,dx$

Optimal. Leaf size=37

$$\frac{3}{7}x^7\,{}_2F_1\left(\frac{1}{2},\frac{7}{8};\frac{15}{8};-x^8\right)-\frac{\sqrt{x^8+1}}{x}$$

```
[Out] 3/7*x^7*hypergeom([1/2, 7/8],[15/8],-x^8)-(x^8+1)^(1/2)/x
```

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {325, 364}

$$\frac{3}{7}x^7\,{}_2F_1\left(\frac{1}{2},\frac{7}{8};\frac{15}{8};-x^8\right)-\frac{\sqrt{x^8+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[1 + x^8]),x]
```

```
[Out] -(Sqrt[1 + x^8]/x) + (3*x^7*Hypergeometric2F1[1/2, 7/8, 15/8, -x^8])/7
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{1+x^8}}\,dx &= -\frac{\sqrt{1+x^8}}{x}+3\int \frac{x^6}{\sqrt{1+x^8}}\,dx\\ &= -\frac{\sqrt{1+x^8}}{x}+\frac{3}{7}x^7\,{}_2F_1\left(\frac{1}{2},\frac{7}{8};\frac{15}{8};-x^8\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 0.54

$$-\frac{{}_2F_1\left(-\frac{1}{8},\frac{1}{2};\frac{7}{8};-x^8\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[1 + x^8]),x]
```

```
[Out] -(Hypergeometric2F1[-1/8, 1/2, 7/8, -x^8]/x)
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^8+1}}{x^{10}+x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x^8 + 1)/(x^10 + x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^8+1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^8 + 1)*x^2), x)
```

maple [A] time = 0.14, size = 30, normalized size = 0.81

$$\frac{3x^7\,\text{hypergeom}\left(\left[\frac{1}{2},\frac{7}{8}\right],\left[\frac{15}{8}\right],-x^8\right)}{7}-\frac{\sqrt{x^8+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^8+1)^(1/2),x)
[Out] 3/7*x^7*hypergeom([1/2,7/8],[15/8],-x^8)-(x^8+1)^(1/2)/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^8+1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^8 + 1)*x^2), x)
```

mupad [B] time = 1.12, size = 15, normalized size = 0.41

$$-\frac{{}_2\mathrm{F}_1\left(-\frac{1}{8},\frac{1}{2};\ \frac{7}{8};\ -x^8\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^8 + 1)^(1/2)),x)
[Out] -hypergeom([-1/8, 1/2], 7/8, -x^8)/x
```

sympy [C] time = 0.81, size = 31, normalized size = 0.84

$$\frac{\Gamma\left(-\frac{1}{8}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{8},\frac{1}{2}\\ \frac{7}{8}\end{matrix}\right| x^8 e^{i\pi}\right)}{8x\Gamma\left(\frac{7}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**8+1)**(1/2),x)
```

```
[Out] gamma(-1/8)*hyper((-1/8, 1/2), (7/8,), x**8*exp_polar(I*pi))/(8*x*gamma(7/8
))
```

3.1537 $\int \frac{1}{x^4\sqrt{1+x^8}}\,dx$

Optimal. Leaf size=39

$$\frac{1}{15}x^5\,{}_2F_1\left(\frac{1}{2},\frac{5}{8};\frac{13}{8};-x^8\right)-\frac{\sqrt{x^8+1}}{3x^3}$$

```
[Out] 1/15*x^5*hypergeom([1/2, 5/8],[13/8],-x^8)-1/3*(x^8+1)^(1/2)/x^3
```

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {325, 364}

$$\frac{1}{15}x^5\,{}_2F_1\left(\frac{1}{2},\frac{5}{8};\frac{13}{8};-x^8\right)-\frac{\sqrt{x^8+1}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[1 + x^8]),x]
```

```
[Out] -Sqrt[1 + x^8]/(3*x^3) + (x^5*Hypergeometric2F1[1/2, 5/8, 13/8, -x^8])/15
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{1+x^8}}\,dx &= -\frac{\sqrt{1+x^8}}{3x^3}+\frac{1}{3}\int \frac{x^4}{\sqrt{1+x^8}}\,dx\\ &= -\frac{\sqrt{1+x^8}}{3x^3}+\frac{1}{15}x^5\,{}_2F_1\left(\frac{1}{2},\frac{5}{8};\frac{13}{8};-x^8\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 0.56

$$-\frac{{}_2F_1\left(-\frac{3}{8},\frac{1}{2};\frac{5}{8};-x^8\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[1 + x^8]),x]
```

```
[Out] -1/3*Hypergeometric2F1[-3/8, 1/2, 5/8, -x^8]/x^3
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^8+1}}{x^{12}+x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x^8 + 1)/(x^12 + x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^8+1}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^8 + 1)*x^4), x)
```

maple [A] time = 0.15, size = 30, normalized size = 0.77

$$\frac{x^5\,\text{hypergeom}\left(\left[\frac{1}{2},\frac{5}{8}\right],\left[\frac{13}{8}\right],-x^8\right)}{15}-\frac{\sqrt{x^8+1}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^8+1)^(1/2),x)
[Out] 1/15*x^5*hypergeom([1/2,5/8],[13/8],-x^8)-1/3*(x^8+1)^(1/2)/x^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^8+1}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^8 + 1)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x^4\,\sqrt{x^8+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x^8 + 1)^(1/2)),x)
[Out] int(1/(x^4*(x^8 + 1)^(1/2)), x)
```

sympy [C] time = 0.84, size = 32, normalized size = 0.82

$$\frac{\Gamma\left(-\frac{3}{8}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{8},\frac{1}{2}\\ \frac{5}{8}\end{matrix}\right|x^8e^{i\pi}\right)}{8x^3\Gamma\left(\frac{5}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**8+1)**(1/2),x)
```

```
[Out] gamma(-3/8)*hyper((-3/8, 1/2), (5/8,), x**8*exp_polar(I*pi))/(8*x**3*gamma(
5/8))
```

3.1538 $\int \frac{1}{x^6\sqrt{1+x^8}}\,dx$

Optimal. Leaf size=239

$$-\frac{\sqrt{x^8+1}}{5x^5}+\frac{\sqrt{\frac{\left(x^2+1\right)^2}{x^2}}\sqrt{-\frac{x^8+1}{x^4}}\,x^3F\left(\sin^{-1}\left(\frac{1}{2}\sqrt{-\frac{\sqrt{2}\,x^4-2x^2+\sqrt{2}}{x^2}}\right)|-2\left(1-\sqrt{2}\right)\right)}{10\sqrt{2+\sqrt{2}}\,\left(x^2+1\right)\sqrt{x^8+1}}+\frac{\sqrt{-\frac{\left(1-x^2\right)^2}{x^2}}\sqrt{-\frac{x^8+1}{x^4}}\,x^3F\left(\sin^{-1}\left(\frac{1}{2}\right.\right.}{10\sqrt{2+\sqrt{2}}\,\left(1\right.}$$

```
[Out] -1/5*(x^8+1)^(1/2)/x^5+1/10*x^3*EllipticF(1/2*((2*x^2+2^(1/2)+x^4*2^(1/2))/
x^2)^(1/2),(-2+2*2^(1/2))^(1/2))*(-(-x^2+1)^2/x^2)^(1/2)*((-x^8-1)/x^4)^(1/
2)/(-x^2+1)/(x^8+1)^(1/2)/(2+2^(1/2))^(1/2)+1/10*x^3*EllipticF(1/2*((2*x^2-
2^(1/2)-x^4*2^(1/2))/x^2)^(1/2),(-2+2*2^(1/2))^(1/2))*((x^2+1)^2/x^2)^(1/2)
*((-x^8-1)/x^4)^(1/2)/(x^2+1)/(x^8+1)^(1/2)/(2+2^(1/2))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {325, 309, 1883}

$$-\frac{\sqrt{x^8+1}}{5x^5}+\frac{\sqrt{\frac{\left(x^2+1\right)^2}{x^2}}\sqrt{-\frac{x^8+1}{x^4}}\,x^3F\left(\sin^{-1}\left(\frac{1}{2}\sqrt{-\frac{\sqrt{2}\,x^4-2x^2+\sqrt{2}}{x^2}}\right)|-2\left(1-\sqrt{2}\right)\right)}{10\sqrt{2+\sqrt{2}}\,\left(x^2+1\right)\sqrt{x^8+1}}+\frac{\sqrt{-\frac{\left(1-x^2\right)^2}{x^2}}\sqrt{-\frac{x^8+1}{x^4}}\,x^3F\left(\sin^{-1}\left(\frac{1}{2}\right.\right.}{10\sqrt{2+\sqrt{2}}\,\left(1\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*Sqrt[1 + x^8]),x]
```

```
[Out] -Sqrt[1 + x^8]/(5*x^5) + (x^3*Sqrt[(1 + x^2)^2/x^2]*Sqrt[-((1 + x^8)/x^4)]*
EllipticF[ArcSin[Sqrt[-((Sqrt[2] - 2*x^2 + Sqrt[2]*x^4)/x^2)]/2], -2*(1 - S
qrt[2])])/(10*Sqrt[2 + Sqrt[2]]*(1 + x^2)*Sqrt[1 + x^8]) + (x^3*Sqrt[-((1 -
 x^2)^2/x^2)]*Sqrt[-((1 + x^8)/x^4)]*EllipticF[ArcSin[Sqrt[(Sqrt[2] + 2*x^2
 + Sqrt[2]*x^4)/x^2]/2], -2*(1 - Sqrt[2])])/(10*Sqrt[2 + Sqrt[2]]*(1 - x^2)
*Sqrt[1 + x^8])
```

Rule 309

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> Dist[1/(2*Rt[b/a, 4]), I
nt[(1 + Rt[b/a, 4]*x^2)/Sqrt[a + b*x^8], x], x] - Dist[1/(2*Rt[b/a, 4]), In
t[(1 - Rt[b/a, 4]*x^2)/Sqrt[a + b*x^8], x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1883

```
Int[((c_) + (d_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> -Simp[(c*
d*x^3*Sqrt[-((c - d*x^2)^2/(c*d*x^2))]*Sqrt[-((d^2*(a + b*x^8))/(b*c^2*x^4)
)]*EllipticF[ArcSin[(1*Sqrt[(Sqrt[2]*c^2 + 2*c*d*x^2 + Sqrt[2]*d^2*x^4)/(c*
d*x^2)])/2], -2*(1 - Sqrt[2])])/(Sqrt[2 + Sqrt[2]]*(c - d*x^2)*Sqrt[a + b*x
^8]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^4 - a*d^4, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^6\sqrt{1+x^8}}\,dx &= -\frac{\sqrt{1+x^8}}{5x^5} - \frac{1}{5}\int \frac{x^2}{\sqrt{1+x^8}}\,dx \\ &= -\frac{\sqrt{1+x^8}}{5x^5} + \frac{1}{10}\int \frac{1-x^2}{\sqrt{1+x^8}}\,dx - \frac{1}{10}\int \frac{1+x^2}{\sqrt{1+x^8}}\,dx \\ &= -\frac{\sqrt{1+x^8}}{5x^5} + \frac{x^3\sqrt{\frac{\left(1+x^2\right)^2}{x^2}}\sqrt{-\frac{1+x^8}{x^4}}\,F\left(\sin^{-1}\left(\frac{1}{2}\sqrt{-\frac{\sqrt{2}-2x^2+\sqrt{2}\,x^4}{x^2}}\right)\middle|-2\left(1-\sqrt{2}\right)\right)}{10\sqrt{2+\sqrt{2}}\,\left(1+x^2\right)\sqrt{1+x^8}} + \frac{x^3\sqrt{-\frac{(1}{}}}{}\end{aligned}$$

Mathematica [C] time = 0.00, size = 22, normalized size = 0.09

$$-\frac{{}_2F_1\left(-\frac{5}{8},\frac{1}{2};\frac{3}{8};-x^8\right)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*Sqrt[1 + x^8]),x]
[Out] -1/5*Hypergeometric2F1[-5/8, 1/2, 3/8, -x^8]/x^5
```

fricas [F] time = 1.09, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^8+1}}{x^{14}+x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x^8 + 1)/(x^14 + x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^8+1}\,x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^8 + 1)*x^6), x)
```

maple [C] time = 0.14, size = 30, normalized size = 0.13

$$-\frac{x^3\,\text{hypergeom}\left(\left[\frac{3}{8},\frac{1}{2}\right],\left[\frac{11}{8}\right],-x^8\right)}{15} - \frac{\sqrt{x^8+1}}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^8+1)^(1/2),x)
[Out] -1/5*(x^8+1)^(1/2)/x^5-1/15*x^3*hypergeom([3/8,1/2],[11/8],-x^8)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^8+1}\,x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^8 + 1)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^6\sqrt{x^8+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(x^8 + 1)^(1/2)),x)
```

```
[Out] int(1/(x^6*(x^8 + 1)^(1/2)), x)
```

sympy [C] time = 0.95, size = 32, normalized size = 0.13

$$\frac{\Gamma\left(-\frac{5}{8}\right){}_2F_1\left(\left.\begin{matrix}-\frac{5}{8}, \frac{1}{2} \\ \frac{3}{8}\end{matrix}\right| x^8 e^{i\pi}\right)}{8x^5\Gamma\left(\frac{3}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**8+1)**(1/2),x)
```

```
[Out] gamma(-5/8)*hyper((-5/8, 1/2), (3/8,), x**8*exp_polar(I*pi))/(8*x**5*gamma(
3/8))
```

3.1539 $\int \frac{1}{x^8\sqrt{1+x^8}}\,dx$

Optimal. Leaf size=239

$$-\frac{\sqrt{x^8+1}}{7x^7}-\frac{3\sqrt{\left(x^2+1\right)^2}\,\sqrt{-\frac{x^8+1}{x^4}}\,x^3F\left(\sin^{-1}\left(\frac{1}{2}\sqrt{-\frac{\sqrt{2}\,x^4-2x^2+\sqrt{2}}{x^2}}\right)|-2\left(1-\sqrt{2}\right)\right)}{14\sqrt{2+\sqrt{2}}\,\left(x^2+1\right)\sqrt{x^8+1}}+\frac{3\sqrt{-\frac{\left(1-x^2\right)^2}{x^2}}\,\sqrt{-\frac{x^8+1}{x^4}}\,x^3F\left(\sin\right.}{14\sqrt{2+\sqrt{2}}}$$

```
[Out] -1/7*(x^8+1)^(1/2)/x^7+3/14*x^3*EllipticF(1/2*((2*x^2+2^(1/2)+x^4*2^(1/2))/
x^2)^(1/2),(-2+2*2^(1/2))^(1/2))*(-(-x^2+1)^2/x^2)^(1/2)*((-x^8-1)/x^4)^(1/
2)/(-x^2+1)/(x^8+1)^(1/2)/(2+2^(1/2))^(1/2)-3/14*x^3*EllipticF(1/2*((2*x^2-
2^(1/2)-x^4*2^(1/2))/x^2)^(1/2),(-2+2*2^(1/2))^(1/2))*((x^2+1)^2/x^2)^(1/2)
*((-x^8-1)/x^4)^(1/2)/(x^2+1)/(x^8+1)^(1/2)/(2+2^(1/2))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {325, 226, 1883}

$$-\frac{\sqrt{x^8+1}}{7x^7}-\frac{3\sqrt{\left(x^2+1\right)^2}\,\sqrt{-\frac{x^8+1}{x^4}}\,x^3F\left(\sin^{-1}\left(\frac{1}{2}\sqrt{-\frac{\sqrt{2}\,x^4-2x^2+\sqrt{2}}{x^2}}\right)|-2\left(1-\sqrt{2}\right)\right)}{14\sqrt{2+\sqrt{2}}\,\left(x^2+1\right)\sqrt{x^8+1}}+\frac{3\sqrt{-\frac{\left(1-x^2\right)^2}{x^2}}\,\sqrt{-\frac{x^8+1}{x^4}}\,x^3F\left(\sin\right.}{14\sqrt{2+\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*Sqrt[1 + x^8]),x]
```

```
[Out] -Sqrt[1 + x^8]/(7*x^7) - (3*x^3*Sqrt[(1 + x^2)^2/x^2]*Sqrt[-((1 + x^8)/x^4)
]*EllipticF[ArcSin[Sqrt[-((Sqrt[2] - 2*x^2 + Sqrt[2]*x^4)/x^2)]/2], -2*(1 -
 Sqrt[2])])/(14*Sqrt[2 + Sqrt[2]]*(1 + x^2)*Sqrt[1 + x^8]) + (3*x^3*Sqrt[-(
(1 - x^2)^2/x^2)]*Sqrt[-((1 + x^8)/x^4)]*EllipticF[ArcSin[Sqrt[(Sqrt[2] + 2
*x^2 + Sqrt[2]*x^4)/x^2]/2], -2*(1 - Sqrt[2])])/(14*Sqrt[2 + Sqrt[2]]*(1 -
x^2)*Sqrt[1 + x^8])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> Dist[1/2, Int[(1 - Rt[b/a, 4]
*x^2)/Sqrt[a + b*x^8], x], x] + Dist[1/2, Int[(1 + Rt[b/a, 4]*x^2)/Sqrt[a +
 b*x^8], x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1883

```
Int[((c_) + (d_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> -Simp[(c*
d*x^3*Sqrt[-((c - d*x^2)^2/(c*d*x^2))]*Sqrt[-((d^2*(a + b*x^8))/(b*c^2*x^4)
)]*EllipticF[ArcSin[(1*Sqrt[(Sqrt[2]*c^2 + 2*c*d*x^2 + Sqrt[2]*d^2*x^4)/(c*
d*x^2)])/2], -2*(1 - Sqrt[2])])/(Sqrt[2 + Sqrt[2]]*(c - d*x^2)*Sqrt[a + b*x
^8]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^4 - a*d^4, 0]
```

Rubi steps

$$\int \frac{1}{x^8\sqrt{1+x^8}}\,dx = -\frac{\sqrt{1+x^8}}{7x^7} - \frac{3}{7}\int \frac{1}{\sqrt{1+x^8}}\,dx$$
$$= -\frac{\sqrt{1+x^8}}{7x^7} - \frac{3}{14}\int \frac{1-x^2}{\sqrt{1+x^8}}\,dx - \frac{3}{14}\int \frac{1+x^2}{\sqrt{1+x^8}}\,dx$$
$$= -\frac{\sqrt{1+x^8}}{7x^7} - \frac{3x^3\sqrt{\frac{\left(1+x^2\right)^2}{x^2}}\sqrt{-\frac{1+x^8}{x^4}}\,F\left(\sin^{-1}\left(\frac{1}{2}\sqrt{-\frac{\sqrt{2}-2x^2+\sqrt{2}\,x^4}{x^2}}\right)|-2\left(1-\sqrt{2}\right)\right)}{14\sqrt{2+\sqrt{2}}\,\left(1+x^2\right)\sqrt{1+x^8}} + \frac{3x^3\sqrt{-\frac{\left(1-}{}}$$

Mathematica [C] time = 0.00, size = 22, normalized size = 0.09

$$-\frac{{}_2F_1\left(-\frac{7}{8},\frac{1}{2};\frac{1}{8};-x^8\right)}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*Sqrt[1 + x^8]),x]
[Out] -1/7*Hypergeometric2F1[-7/8, 1/2, 1/8, -x^8]/x^7
```

fricas [F] time = 1.06, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x^8+1}}{x^{16}+x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x^8 + 1)/(x^16 + x^8), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^8+1}\,x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^8 + 1)*x^8), x)
```

maple [C] time = 0.14, size = 28, normalized size = 0.12

$$-\frac{3x\,\text{hypergeom}\left(\left[\frac{1}{8},\frac{1}{2}\right],\left[\frac{9}{8}\right],-x^8\right)}{7} - \frac{\sqrt{x^8+1}}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(x^8+1)^(1/2),x)
[Out] -1/7*(x^8+1)^(1/2)/x^7-3/7*x*hypergeom([1/8,1/2],[9/8],-x^8)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^8+1}\,x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^8 + 1)*x^8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^8 \sqrt{x^8+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(x^8 + 1)^(1/2)),x)
```

```
[Out] int(1/(x^8*(x^8 + 1)^(1/2)), x)
```

sympy [C] time = 1.05, size = 32, normalized size = 0.13

$$\frac{\Gamma\left(-\frac{7}{8}\right) {}_2F_1\left(\left.\begin{matrix} -\frac{7}{8}, \frac{1}{2} \\ \frac{1}{8} \end{matrix}\right| x^8 e^{i\pi}\right)}{8x^7 \Gamma\left(\frac{1}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(x**8+1)**(1/2),x)
```

```
[Out] gamma(-7/8)*hyper((-7/8, 1/2), (1/8,), x**8*exp_polar(I*pi))/(8*x**7*gamma(
1/8))
```

3.1540 $\int \frac{1}{1-x^{10}}\,dx$

Optimal. Leaf size=163

$$\frac{1}{20}\sqrt{10-2\sqrt{5}}\,\tan^{-1}\left(\frac{\sqrt{10-2\sqrt{5}}\,x}{2\left(1-x^2\right)}\right)+\frac{1}{20}\sqrt{10+2\sqrt{5}}\,\tan^{-1}\left(\frac{\sqrt{10+2\sqrt{5}}\,x}{2\left(1-x^2\right)}\right)+\frac{1}{20}\left(1-\sqrt{5}\right)\tanh^{-1}\left(\frac{\left(1-\sqrt{5}\right)x}{2\left(x^2+1\right)}\right.$$

```
[Out] 1/5*arctanh(x)+1/20*arctanh(1/2*x*(-5^(1/2)+1)/(x^2+1))*(-5^(1/2)+1)+1/20*a
rctanh(1/2*x*(5^(1/2)+1)/(x^2+1))*(5^(1/2)+1)+1/20*arctan(1/2*x*(10-2*5^(1/
2))^(1/2)/(-x^2+1))*(10-2*5^(1/2))^(1/2)+1/20*arctan(1/2*x*(10+2*5^(1/2))^(
1/2)/(-x^2+1))*(10+2*5^(1/2))^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 325, normalized size of antiderivative = 1.99, number of steps used = 10, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {210, 634, 618, 204, 628, 206}

$$-\frac{1}{40}\left(1-\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1-\sqrt{5}\right)x+1\right)+\frac{1}{40}\left(1-\sqrt{5}\right)\log\left(x^2+\frac{1}{2}\left(1-\sqrt{5}\right)x+1\right)-\frac{1}{40}\left(1+\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^10)^(-1),x]
```

```
[Out] -(Sqrt[(5 + Sqrt[5])/2]*ArcTan[(1 - Sqrt[5] - 4*x)/Sqrt[2*(5 + Sqrt[5])]])/
10 - (Sqrt[(5 - Sqrt[5])/2]*ArcTan[(Sqrt[(5 + Sqrt[5])/10]*(1 + Sqrt[5] - 4
*x))/2])/10 + (Sqrt[(5 + Sqrt[5])/2]*ArcTan[(1 - Sqrt[5] + 4*x)/Sqrt[2*(5 +
 Sqrt[5])]])/10 + (Sqrt[(5 - Sqrt[5])/2]*ArcTan[(Sqrt[(5 + Sqrt[5])/10]*(1
+ Sqrt[5] + 4*x))/2])/10 + ArcTanh[x]/5 - ((1 - Sqrt[5])*Log[1 - ((1 - Sqrt
[5])*x)/2 + x^2])/40 + ((1 - Sqrt[5])*Log[1 + ((1 - Sqrt[5])*x)/2 + x^2])/4
0 - ((1 + Sqrt[5])*Log[1 - ((1 + Sqrt[5])*x)/2 + x^2])/40 + ((1 + Sqrt[5])*
Log[1 + ((1 + Sqrt[5])*x)/2 + x^2])/40
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{1-x^{10}}\,dx = \frac{1}{5}\int \frac{1-\frac{1}{4}\left(-1+\sqrt{5}\right)x}{1-\frac{1}{2}\left(-1+\sqrt{5}\right)x+x^2}\,dx + \frac{1}{5}\int \frac{1+\frac{1}{4}\left(-1+\sqrt{5}\right)x}{1+\frac{1}{2}\left(-1+\sqrt{5}\right)x+x^2}\,dx + \frac{1}{5}\int \frac{1-\frac{1}{4}\left(1+\sqrt{5}\right)x}{1-\frac{1}{2}\left(1+\sqrt{5}\right)x+}$$

$$= \frac{1}{5}\tanh^{-1}(x) + \frac{1}{40}\left(-1-\sqrt{5}\right)\int \frac{\frac{1}{2}\left(-1-\sqrt{5}\right)+2x}{1+\frac{1}{2}\left(-1-\sqrt{5}\right)x+x^2}\,dx + \frac{1}{40}\left(1-\sqrt{5}\right)\int \frac{\frac{1}{2}\left(1-\sqrt{5}\right)+}{1+\frac{1}{2}\left(1-\sqrt{5}\right)}$$

$$= \frac{1}{5}\tanh^{-1}(x) - \frac{1}{40}\left(1+\sqrt{5}\right)\log\left(2-x-\sqrt{5}\,x+2x^2\right) + \frac{1}{40}\left(1-\sqrt{5}\right)\log\left(2+x-\sqrt{5}\,x+2x^2\right)$$

$$= -\frac{1}{10}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\,\tan^{-1}\left(\frac{1-\sqrt{5}-4x}{\sqrt{2\left(5+\sqrt{5}\right)}}\right) - \frac{1}{10}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\,\tan^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\,\left(1+\sqrt{\right.}\right.$$

Mathematica [A] time = 0.26, size = 289, normalized size = 1.77

$$\frac{1}{40}\left(-\left(\sqrt{5}-1\right)\log\left(x^2-\frac{1}{2}\left(\sqrt{5}-1\right)x+1\right)+\left(\sqrt{5}-1\right)\log\left(x^2+\frac{1}{2}\left(\sqrt{5}-1\right)x+1\right)-\left(1+\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^10)^(-1),x]
```

```
[Out] (-2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[(1 + Sqrt[5] - 4*x)/Sqrt[10 - 2*Sqrt[5]]] +
 2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(1 - Sqrt[5] + 4*x)/Sqrt[2*(5 + Sqrt[5])]]
+ 2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(-1 + Sqrt[5] + 4*x)/Sqrt[2*(5 + Sqrt[5])]
] + 2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[(1 + Sqrt[5] + 4*x)/Sqrt[10 - 2*Sqrt[5]]]
 - 4*Log[1 - x] + 4*Log[1 + x] - (-1 + Sqrt[5])*Log[1 - ((-1 + Sqrt[5])*x)/
2 + x^2] + (-1 + Sqrt[5])*Log[1 + ((-1 + Sqrt[5])*x)/2 + x^2] - (1 + Sqrt[5
])*Log[1 - ((1 + Sqrt[5])*x)/2 + x^2] + (1 + Sqrt[5])*Log[(2 + x + Sqrt[5]*
x + 2*x^2)/2])/40
```

fricas [B] time = 4.32, size = 1039, normalized size = 6.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^10+1),x, algorithm="fricas")
```

```
[Out] 1/40*(sqrt(5) + 2*sqrt(-3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^
2 + 1/8*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqr
t(5) - 5) - sqrt(5) - 1) - 3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) -
1)^2 + sqrt(1/2)*sqrt(sqrt(5) - 5) + 1/2*sqrt(5) - 5/2) - 1)*log(2*x + 1/2*
sqrt(5) + sqrt(-3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 1/8*
(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5
) - sqrt(5) - 1) - 3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + s
qrt(1/2)*sqrt(sqrt(5) - 5) + 1/2*sqrt(5) - 5/2) - 1/2) + 1/40*(sqrt(5) - 2*
sqrt(-3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 1/8*(2*sqrt(1/
2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5
) - 1) - 3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + sqrt(1/2)*s
qrt(sqrt(5) - 5) + 1/2*sqrt(5) - 5/2) - 1)*log(2*x + 1/2*sqrt(5) - sqrt(-3/
16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 1/8*(2*sqrt(1/2)*sqrt(
sqrt(5) - 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) -
 3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + sqrt(1/2)*sqrt(sqrt
(5) - 5) + 1/2*sqrt(5) - 5/2) - 1/2) + 1/40*(sqrt(5) + 2*sqrt(-3/16*(sqrt(5
) + 20*sqrt(-1/200*sqrt(5) - 1/40) - 1)^2 - 1/8*(sqrt(5) + 20*sqrt(-1/200*s
qrt(5) - 1/40) + 3)*(sqrt(5) - 20*sqrt(-1/200*sqrt(5) - 1/40) - 1) - 3/16*(
sqrt(5) - 20*sqrt(-1/200*sqrt(5) - 1/40) - 1)^2 - 1/2*sqrt(5) - 10*sqrt(-1/
200*sqrt(5) - 1/40) - 5/2) + 1)*log(2*x + 1/2*sqrt(5) + sqrt(-3/16*(sqrt(5)
 + 20*sqrt(-1/200*sqrt(5) - 1/40) - 1)^2 - 1/8*(sqrt(5) + 20*sqrt(-1/200*sq
rt(5) - 1/40) + 3)*(sqrt(5) - 20*sqrt(-1/200*sqrt(5) - 1/40) - 1) - 3/16*(s
qrt(5) - 20*sqrt(-1/200*sqrt(5) - 1/40) - 1)^2 - 1/2*sqrt(5) - 10*sqrt(-1/2
00*sqrt(5) - 1/40) - 5/2) + 1/2) + 1/40*(sqrt(5) - 2*sqrt(-3/16*(sqrt(5) +
20*sqrt(-1/200*sqrt(5) - 1/40) - 1)^2 - 1/8*(sqrt(5) + 20*sqrt(-1/200*sqrt(
5) - 1/40) + 3)*(sqrt(5) - 20*sqrt(-1/200*sqrt(5) - 1/40) - 1) - 3/16*(sqrt
(5) - 20*sqrt(-1/200*sqrt(5) - 1/40) - 1)^2 - 1/2*sqrt(5) - 10*sqrt(-1/200*
sqrt(5) - 1/40) - 5/2) + 1)*log(2*x + 1/2*sqrt(5) - sqrt(-3/16*(sqrt(5) + 2
0*sqrt(-1/200*sqrt(5) - 1/40) - 1)^2 - 1/8*(sqrt(5) + 20*sqrt(-1/200*sqrt(5
) - 1/40) + 3)*(sqrt(5) - 20*sqrt(-1/200*sqrt(5) - 1/40) - 1) - 3/16*(sqrt(
5) - 20*sqrt(-1/200*sqrt(5) - 1/40) - 1)^2 - 1/2*sqrt(5) - 10*sqrt(-1/200*s
qrt(5) - 1/40) - 5/2) + 1/2) + 1/40*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5
) - 1)*log(x + 1/2*sqrt(1/2)*sqrt(sqrt(5) - 5) - 1/4*sqrt(5) - 1/4) - 1/40*
(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)*log(x - 1/2*sqrt(1/2)*sqrt(sq
rt(5) - 5) - 1/4*sqrt(5) - 1/4) - 1/40*(sqrt(5) - 20*sqrt(-1/200*sqrt(5) -
1/40) - 1)*log(x - 1/4*sqrt(5) + 5*sqrt(-1/200*sqrt(5) - 1/40) + 1/4) - 1/4
0*(sqrt(5) + 20*sqrt(-1/200*sqrt(5) - 1/40) - 1)*log(x - 1/4*sqrt(5) - 5*sq
rt(-1/200*sqrt(5) - 1/40) + 1/4) + 1/10*log(x + 1) - 1/10*log(x - 1)
```

giac [A] time = 0.20, size = 223, normalized size = 1.37

$$\frac{1}{40}\left(\sqrt{5}+1\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{5}+1\right)+1\right)-\frac{1}{40}\left(\sqrt{5}+1\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{5}+1\right)+1\right)+\frac{1}{40}\left(\sqrt{5}-1\right)\log\left(x^2+\frac{1}{2}x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^10+1),x, algorithm="giac")
```

```
[Out] 1/40*(sqrt(5) + 1)*log(x^2 + 1/2*x*(sqrt(5) + 1) + 1) - 1/40*(sqrt(5) + 1)*
log(x^2 - 1/2*x*(sqrt(5) + 1) + 1) + 1/40*(sqrt(5) - 1)*log(x^2 + 1/2*x*(sq
rt(5) - 1) + 1) - 1/40*(sqrt(5) - 1)*log(x^2 - 1/2*x*(sqrt(5) - 1) + 1) + 1
/20*sqrt(2*sqrt(5) + 10)*arctan((4*x + sqrt(5) - 1)/sqrt(2*sqrt(5) + 10)) +
 1/20*sqrt(2*sqrt(5) + 10)*arctan((4*x - sqrt(5) + 1)/sqrt(2*sqrt(5) + 10))
 + 1/20*sqrt(-2*sqrt(5) + 10)*arctan((4*x + sqrt(5) + 1)/sqrt(-2*sqrt(5) +
10)) + 1/20*sqrt(-2*sqrt(5) + 10)*arctan((4*x - sqrt(5) - 1)/sqrt(-2*sqrt(5
) + 10)) + 1/10*log(abs(x + 1)) - 1/10*log(abs(x - 1))
```

maple [B] time = 0.04, size = 426, normalized size = 2.61

$$\frac{\arctan\left(\frac{4x+1-\sqrt{5}}{\sqrt{10+2\sqrt{5}}}\right)}{2\sqrt{10+2\sqrt{5}}}+\frac{\sqrt{5}\ \arctan\left(\frac{4x+1-\sqrt{5}}{\sqrt{10+2\sqrt{5}}}\right)}{10\sqrt{10+2\sqrt{5}}}+\frac{\arctan\left(\frac{4x+1+\sqrt{5}}{\sqrt{10-2\sqrt{5}}}\right)}{2\sqrt{10-2\sqrt{5}}}-\frac{\sqrt{5}\ \arctan\left(\frac{4x+1+\sqrt{5}}{\sqrt{10-2\sqrt{5}}}\right)}{10\sqrt{10-2\sqrt{5}}}+\frac{\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{10-2\sqrt{5}}}\right)}{2\sqrt{10-2\sqrt{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^10+1),x)
```

```
[Out] -1/40*ln(-5^(1/2)*x+2*x^2+x+2)*5^(1/2)+1/40*ln(-5^(1/2)*x+2*x^2+x+2)+1/2/(1
0+2*5^(1/2))^(1/2)*arctan((1+4*x-5^(1/2))/(10+2*5^(1/2))^(1/2))+1/10/(10+2*
5^(1/2))^(1/2)*arctan((1+4*x-5^(1/2))/(10+2*5^(1/2))^(1/2))*5^(1/2)+1/40*ln
(5^(1/2)*x+2*x^2+x+2)*5^(1/2)+1/40*ln(5^(1/2)*x+2*x^2+x+2)+1/2/(10-2*5^(1/2
))^(1/2)*arctan((1+4*x+5^(1/2))/(10-2*5^(1/2))^(1/2))-1/10/(10-2*5^(1/2))^(
1/2)*arctan((1+4*x+5^(1/2))/(10-2*5^(1/2))^(1/2))*5^(1/2)-1/10*ln(x-1)+1/40
*5^(1/2)*ln(2*x^2+5^(1/2)*x-x+2)-1/40*ln(2*x^2+5^(1/2)*x-x+2)+1/2/(10+2*5^(
1/2))^(1/2)*arctan((4*x+5^(1/2)-1)/(10+2*5^(1/2))^(1/2))+1/10/(10+2*5^(1/2)
)^(1/2)*5^(1/2)*arctan((4*x+5^(1/2)-1)/(10+2*5^(1/2))^(1/2))-1/40*5^(1/2)*l
n(2*x^2-5^(1/2)*x-x+2)-1/40*ln(2*x^2-5^(1/2)*x-x+2)+1/2/(10-2*5^(1/2))^(1/2
)*arctan((4*x-5^(1/2)-1)/(10-2*5^(1/2))^(1/2))-1/10/(10-2*5^(1/2))^(1/2)*5^
(1/2)*arctan((4*x-5^(1/2)-1)/(10-2*5^(1/2))^(1/2))+1/10*ln(x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{10}\int\frac{x^3+2x^2+3x+4}{x^4+x^3+x^2+x+1}\,dx-\frac{1}{10}\int\frac{x^3-2x^2+3x-4}{x^4-x^3+x^2-x+1}\,dx+\frac{1}{10}\log(x+1)-\frac{1}{10}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^10+1),x, algorithm="maxima")
```

```
[Out] 1/10*integrate((x^3 + 2*x^2 + 3*x + 4)/(x^4 + x^3 + x^2 + x + 1), x) - 1/10
*integrate((x^3 - 2*x^2 + 3*x - 4)/(x^4 - x^3 + x^2 - x + 1), x) + 1/10*log
(x + 1) - 1/10*log(x - 1)
```

mupad [B] time = 1.29, size = 852, normalized size = 5.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^10 - 1),x)
```

```
[Out] atan((((2^(1/2)*(- 5^(1/2) - 5)^(1/2))/40 - 5^(1/2)/40 + 1/40)*(10*x - (5*2
^(1/2)*(- 5^(1/2) - 5)^(1/2))/2 + (5*5^(1/2))/2 - 5/2)*1i + ((2^(1/2)*(- 5^
(1/2) - 5)^(1/2))/40 - 5^(1/2)/40 + 1/40)*(10*x + (5*2^(1/2)*(- 5^(1/2) - 5
)^(1/2))/2 - (5*5^(1/2))/2 + 5/2)*1i)/(((2^(1/2)*(- 5^(1/2) - 5)^(1/2))/40
 - 5^(1/2)/40 + 1/40)*(10*x - (5*2^(1/2)*(- 5^(1/2) - 5)^(1/2))/2 + (5*5^(1/
2))/2 - 5/2) - ((2^(1/2)*(- 5^(1/2) - 5)^(1/2))/40 - 5^(1/2)/40 + 1/40)*(10
*x + (5*2^(1/2)*(- 5^(1/2) - 5)^(1/2))/2 - (5*5^(1/2))/2 + 5/2)))*((2^(1/2)
*(- 5^(1/2) - 5)^(1/2)*1i)/20 - (5^(1/2)*1i)/20 + 1i/20) - (atan(x*1i)*1i)/
5 + atan((((2^(1/2)*(- 5^(1/2) - 5)^(1/2))/40 + 5^(1/2)/40 - 1/40)*(10*x -
(5*2^(1/2)*(- 5^(1/2) - 5)^(1/2))/2 - (5*5^(1/2))/2 + 5/2)*1i + ((2^(1/2)*(
- 5^(1/2) - 5)^(1/2))/40 + 5^(1/2)/40 - 1/40)*(10*x + (5*2^(1/2)*(- 5^(1/2)
 - 5)^(1/2))/2 + (5*5^(1/2))/2 - 5/2)*1i)/(((2^(1/2)*(- 5^(1/2) - 5)^(1/2))
/40 + 5^(1/2)/40 - 1/40)*(10*x - (5*2^(1/2)*(- 5^(1/2) - 5)^(1/2))/2 - (5*5
^(1/2))/2 + 5/2) - ((2^(1/2)*(- 5^(1/2) - 5)^(1/2))/40 + 5^(1/2)/40 - 1/40)
*(10*x + (5*2^(1/2)*(- 5^(1/2) - 5)^(1/2))/2 + (5*5^(1/2))/2 - 5/2)))*((2^(
1/2)*(- 5^(1/2) - 5)^(1/2)*1i)/20 + (5^(1/2)*1i)/20 - 1i/20) + atan(((5^(1/
```

```
2)/40 + (2^(1/2)*(5^(1/2) - 5)^(1/2))/40 + 1/40)*((5*5^(1/2))/2 - 10*x + (5
*2^(1/2)*(5^(1/2) - 5)^(1/2))/2 + 5/2)*1i - (5^(1/2)/40 + (2^(1/2)*(5^(1/2)
 - 5)^(1/2))/40 + 1/40)*(10*x + (5*5^(1/2))/2 + (5*2^(1/2)*(5^(1/2) - 5)^(1
/2))/2 + 5/2)*1i)/((5^(1/2)/40 + (2^(1/2)*(5^(1/2) - 5)^(1/2))/40 + 1/40)*(
(5*5^(1/2))/2 - 10*x + (5*2^(1/2)*(5^(1/2) - 5)^(1/2))/2 + 5/2) + (5^(1/2)/
40 + (2^(1/2)*(5^(1/2) - 5)^(1/2))/40 + 1/40)*(10*x + (5*5^(1/2))/2 + (5*2^
(1/2)*(5^(1/2) - 5)^(1/2))/2 + 5/2)))*((5^(1/2)*1i)/20 + (2^(1/2)*(5^(1/2)
- 5)^(1/2)*1i)/20 + 1i/20) + atan(((5^(1/2)/40 - (2^(1/2)*(5^(1/2) - 5)^(1/
2))/40 + 1/40)*(10*x - (5*5^(1/2))/2 + (5*2^(1/2)*(5^(1/2) - 5)^(1/2))/2 -
5/2)*1i + (5^(1/2)/40 - (2^(1/2)*(5^(1/2) - 5)^(1/2))/40 + 1/40)*(10*x + (5
*5^(1/2))/2 - (5*2^(1/2)*(5^(1/2) - 5)^(1/2))/2 + 5/2)*1i)/((5^(1/2)/40 - (
2^(1/2)*(5^(1/2) - 5)^(1/2))/40 + 1/40)*(10*x - (5*5^(1/2))/2 + (5*2^(1/2)*
(5^(1/2) - 5)^(1/2))/2 - 5/2) - (5^(1/2)/40 - (2^(1/2)*(5^(1/2) - 5)^(1/2))
/40 + 1/40)*(10*x + (5*5^(1/2))/2 - (5*2^(1/2)*(5^(1/2) - 5)^(1/2))/2 + 5/2
)))*((5^(1/2)*1i)/20 - (2^(1/2)*(5^(1/2) - 5)^(1/2)*1i)/20 + 1i/20)
```

sympy [**A**] time = 7.28, size = 70, normalized size = 0.43

$$-\frac{\log(x-1)}{10}+\frac{\log(x+1)}{10}-\text{RootSum}\left(10000t^4-1000t^3+100t^2-10t+1,\left(t\mapsto t\log(-10t+x)\right)\right)-\text{RootSum}\left(10$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**10+1),x)
```

```
[Out] -log(x - 1)/10 + log(x + 1)/10 - RootSum(10000*_t**4 - 1000*_t**3 + 100*_t*
*2 - 10*_t + 1, Lambda(_t, _t*log(-10*_t + x))) - RootSum(10000*_t**4 + 100
0*_t**3 + 100*_t**2 + 10*_t + 1, Lambda(_t, _t*log(-10*_t + x)))
```

3.1541 $\int \frac{x^4}{\sqrt{1-x^{10}}}\,dx$

Optimal. Leaf size=8

$$\frac{1}{5}\sin^{-1}\left(x^5\right)$$

```
[Out] 1/5*arcsin(x^5)
```

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {275, 216}

$$\frac{1}{5}\sin^{-1}\left(x^5\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[1 - x^10],x]
```

```
[Out] ArcSin[x^5]/5
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt{1-x^{10}}}\,dx &= \frac{1}{5}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, x^5\right)\\ &= \frac{1}{5}\sin^{-1}\left(x^5\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\frac{1}{5}\sin^{-1}\left(x^5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[1 - x^10],x]
```

```
[Out] ArcSin[x^5]/5
```

fricas [B] time = 0.97, size = 18, normalized size = 2.25

$$-\frac{2}{5}\arctan\left(\frac{\sqrt{-x^{10}+1}-1}{x^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^10+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/5*arctan((sqrt(-x^10 + 1) - 1)/x^5)
```

giac [A] time = 0.20, size = 6, normalized size = 0.75

$$\frac{1}{5}\arcsin\left(x^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^10+1)^(1/2),x, algorithm="giac")
[Out] 1/5*arcsin(x^5)
```

maple [A] time = 0.15, size = 7, normalized size = 0.88

$$\frac{\arcsin\left(x^5\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^10+1)^(1/2),x)
[Out] 1/5*arcsin(x^5)
```

maxima [B] time = 2.23, size = 16, normalized size = 2.00

$$-\frac{1}{5}\arctan\left(\frac{\sqrt{-x^{10}+1}}{x^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^10+1)^(1/2),x, algorithm="maxima")
[Out] -1/5*arctan(sqrt(-x^10 + 1)/x^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.12

$$\int \frac{x^4}{\sqrt{1-x^{10}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(1 - x^10)^(1/2),x)
[Out] int(x^4/(1 - x^10)^(1/2), x)
```

sympy [A] time = 1.12, size = 19, normalized size = 2.38

$$\begin{cases} -\frac{i\,\operatorname{acosh}\left(x^5\right)}{5} & \text{for } \left|x^{10}\right| > 1 \\ \frac{\operatorname{asin}\left(x^5\right)}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**10+1)**(1/2),x)
[Out] Piecewise((-I*acosh(x**5)/5, Abs(x**10) > 1), (asin(x**5)/5, True))
```

3.1542 $\int \frac{x^4}{\sqrt{-2+x^{10}}}\,dx$

Optimal. Leaf size=18

$$\frac{1}{5}\tanh^{-1}\left(\frac{x^5}{\sqrt{x^{10}-2}}\right)$$

```
[Out] 1/5*arctanh(x^5/(x^10-2)^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {275, 217, 206}

$$\frac{1}{5}\tanh^{-1}\left(\frac{x^5}{\sqrt{x^{10}-2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[-2 + x^10],x]
```

```
[Out] ArcTanh[x^5/Sqrt[-2 + x^10]]/5
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt{-2+x^{10}}}\,dx &= \frac{1}{5}\text{Subst}\left(\int \frac{1}{\sqrt{-2+x^2}}\,dx, x, x^5\right)\\ &= \frac{1}{5}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x^5}{\sqrt{-2+x^{10}}}\right)\\ &= \frac{1}{5}\tanh^{-1}\left(\frac{x^5}{\sqrt{-2+x^{10}}}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{1}{5}\tanh^{-1}\left(\frac{x^5}{\sqrt{x^{10}-2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[-2 + x^10],x]
```

```
[Out] ArcTanh[x^5/Sqrt[-2 + x^10]]/5
```

fricas [A] time = 0.86, size = 16, normalized size = 0.89

$$-\frac{1}{5}\log\left(-x^5+\sqrt{x^{10}-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^10-2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/5*log(-x^5 + sqrt(x^10 - 2))
```

giac [A] time = 0.20, size = 17, normalized size = 0.94

$$-\frac{1}{5}\log\left(\left|-x^5+\sqrt{x^{10}-2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^10-2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/5*log(abs(-x^5 + sqrt(x^10 - 2)))
```

maple [C] time = 0.17, size = 34, normalized size = 1.89

$$\frac{\sqrt{-\operatorname{signum}\left(\frac{x^{10}}{2}-1\right)}\,\arcsin\left(\frac{\sqrt{2}\,x^5}{2}\right)}{5\sqrt{\operatorname{signum}\left(\frac{x^{10}}{2}-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^10-2)^(1/2),x)
```

```
[Out] 1/5/signum(-1+1/2*x^10)^(1/2)*(-signum(-1+1/2*x^10))^(1/2)*arcsin(1/2*x^5*2
^(1/2))
```

maxima [B] time = 1.09, size = 33, normalized size = 1.83

$$\frac{1}{10}\log\left(\frac{\sqrt{x^{10}-2}}{x^5}+1\right)-\frac{1}{10}\log\left(\frac{\sqrt{x^{10}-2}}{x^5}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^10-2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/10*log(sqrt(x^10 - 2)/x^5 + 1) - 1/10*log(sqrt(x^10 - 2)/x^5 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{x^4}{\sqrt{x^{10}-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^10 - 2)^(1/2),x)
```

```
[Out] int(x^4/(x^10 - 2)^(1/2), x)
```

sympy [**A**] time = 1.17, size = 34, normalized size = 1.89

$$\begin{cases} \frac{\operatorname{acosh}\left(\frac{\sqrt{2}x^5}{2}\right)}{5} & \text{for } \frac{|x^{10}|}{2} > 1 \\ -\frac{i \operatorname{asin}\left(\frac{\sqrt{2}x^5}{2}\right)}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**10-2)**(1/2),x)
```

```
[Out] Piecewise((acosh(sqrt(2)*x**5/2)/5, Abs(x**10)/2 > 1), (-I*asin(sqrt(2)*x**
5/2)/5, True))
```

3.1543 $\int \frac{x^5}{9+x^{12}}\,dx$

Optimal. Leaf size=12

$$\frac{1}{18}\tan^{-1}\left(\frac{x^6}{3}\right)$$

```
[Out] 1/18*arctan(1/3*x^6)
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {275, 203}

$$\frac{1}{18}\tan^{-1}\left(\frac{x^6}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(9 + x^12),x]
```

```
[Out] ArcTan[x^6/3]/18
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{9+x^{12}}\,dx &= \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{9+x^2}\,dx, x, x^6\right)\\ &= \frac{1}{18}\tan^{-1}\left(\frac{x^6}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$\frac{1}{18}\tan^{-1}\left(\frac{x^6}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(9 + x^12),x]
```

```
[Out] ArcTan[x^6/3]/18
```

fricas [A] time = 1.08, size = 8, normalized size = 0.67

$$\frac{1}{18}\,\arctan\left(\frac{1}{3}\,x^6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^12+9),x, algorithm="fricas")
[Out] 1/18*arctan(1/3*x^6)
```

giac [A] time = 0.17, size = 8, normalized size = 0.67

$$\frac{1}{18}\arctan\left(\frac{1}{3}x^6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^12+9),x, algorithm="giac")
[Out] 1/18*arctan(1/3*x^6)
```

maple [A] time = 0.00, size = 9, normalized size = 0.75

$$\frac{\arctan\left(\frac{x^6}{3}\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^12+9),x)
[Out] 1/18*arctan(1/3*x^6)
```

maxima [A] time = 2.30, size = 8, normalized size = 0.67

$$\frac{1}{18}\arctan\left(\frac{1}{3}x^6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^12+9),x, algorithm="maxima")
[Out] 1/18*arctan(1/3*x^6)
```

mupad [B] time = 0.06, size = 8, normalized size = 0.67

$$\frac{\operatorname{atan}\left(\frac{x^6}{3}\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^12 + 9),x)
[Out] atan(x^6/3)/18
```

sympy [A] time = 0.14, size = 7, normalized size = 0.58

$$\frac{\operatorname{atan}\left(\frac{x^6}{3}\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**12+9),x)
[Out] atan(x**6/3)/18
```

3.1544 $\int \frac{x^5}{9-x^{12}}\,dx$

Optimal. Leaf size=12

$$\frac{1}{18}\tanh^{-1}\left(\frac{x^6}{3}\right)$$

```
[Out] 1/18*arctanh(1/3*x^6)
```

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {275, 206}

$$\frac{1}{18}\tanh^{-1}\left(\frac{x^6}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(9 - x^12),x]
```

```
[Out] ArcTanh[x^6/3]/18
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{9-x^{12}}\,dx &= \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{9-x^2}\,dx, x, x^6\right)\\ &= \frac{1}{18}\tanh^{-1}\left(\frac{x^6}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.92

$$\frac{1}{36}\log\left(x^6+3\right) - \frac{1}{36}\log\left(3-x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(9 - x^12),x]
```

```
[Out] -1/36*Log[3 - x^6] + Log[3 + x^6]/36
```

fricas [B] time = 0.89, size = 17, normalized size = 1.42

$$\frac{1}{36}\,\log\left(x^6+3\right) - \frac{1}{36}\,\log\left(x^6-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^12+9),x, algorithm="fricas")
[Out] 1/36*log(x^6 + 3) - 1/36*log(x^6 - 3)
```

giac [B] time = 0.20, size = 18, normalized size = 1.50

$$\frac{1}{36}\log\left(x^6+3\right)-\frac{1}{36}\log\left(\left|x^6-3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^12+9),x, algorithm="giac")
[Out] 1/36*log(x^6 + 3) - 1/36*log(abs(x^6 - 3))
```

maple [B] time = 0.01, size = 18, normalized size = 1.50

$$-\frac{\ln\left(x^6-3\right)}{36}+\frac{\ln\left(x^6+3\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-x^12+9),x)
[Out] -1/36*ln(x^6-3)+1/36*ln(x^6+3)
```

maxima [B] time = 1.01, size = 17, normalized size = 1.42

$$\frac{1}{36}\log\left(x^6+3\right)-\frac{1}{36}\log\left(x^6-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^12+9),x, algorithm="maxima")
[Out] 1/36*log(x^6 + 3) - 1/36*log(x^6 - 3)
```

mupad [B] time = 1.08, size = 8, normalized size = 0.67

$$\frac{\operatorname{atanh}\left(\frac{x^6}{3}\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^5/(x^12 - 9),x)
[Out] atanh(x^6/3)/18
```

sympy [B] time = 0.14, size = 15, normalized size = 1.25

$$-\frac{\log\left(x^6-3\right)}{36}+\frac{\log\left(x^6+3\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-x**12+9),x)
[Out] -log(x**6 - 3)/36 + log(x**6 + 3)/36
```

3.1545 $\int x^5\sqrt{9+x^{12}}\,dx$

Optimal. Leaf size=29

$$\frac{3}{4}\sinh^{-1}\left(\frac{x^6}{3}\right)+\frac{1}{12}\sqrt{x^{12}+9}\,x^6$$

```
[Out] 3/4*arcsinh(1/3*x^6)+1/12*x^6*(x^12+9)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {275, 195, 215}

$$\frac{1}{12}\sqrt{x^{12}+9}\,x^6+\frac{3}{4}\sinh^{-1}\left(\frac{x^6}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[9 + x^12],x]
```

```
[Out] (x^6*Sqrt[9 + x^12])/12 + (3*ArcSinh[x^6/3])/4
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int x^5\sqrt{9+x^{12}}\,dx &= \frac{1}{6}\text{Subst}\left(\int \sqrt{9+x^2}\,dx, x, x^6\right)\\ &= \frac{1}{12}x^6\sqrt{9+x^{12}}+\frac{3}{4}\text{Subst}\left(\int \frac{1}{\sqrt{9+x^2}}\,dx, x, x^6\right)\\ &= \frac{1}{12}x^6\sqrt{9+x^{12}}+\frac{3}{4}\sinh^{-1}\left(\frac{x^6}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.97

$$\frac{1}{12}\left(9\sinh^{-1}\left(\frac{x^6}{3}\right)+\sqrt{x^{12}+9}\,x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[9 + x^12],x]
```

```
[Out] (x^6*Sqrt[9 + x^12] + 9*ArcSinh[x^6/3])/12
```

fricas [A] time = 0.97, size = 29, normalized size = 1.00

$$\frac{1}{12}\sqrt{x^{12}+9}\,x^6 - \frac{3}{4}\log\left(-x^6+\sqrt{x^{12}+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^12+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(x^12 + 9)*x^6 - 3/4*log(-x^6 + sqrt(x^12 + 9))
```

giac [A] time = 0.19, size = 29, normalized size = 1.00

$$\frac{1}{12}\sqrt{x^{12}+9}\,x^6 - \frac{3}{4}\log\left(-x^6+\sqrt{x^{12}+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^12+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(x^12 + 9)*x^6 - 3/4*log(-x^6 + sqrt(x^12 + 9))
```

maple [A] time = 0.15, size = 22, normalized size = 0.76

$$\frac{\sqrt{x^{12}+9}\;x^6}{12} + \frac{3\,\mathrm{arcsinh}\left(\frac{x^6}{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^12+9)^(1/2),x)
```

```
[Out] 3/4*arcsinh(1/3*x^6)+1/12*x^6*(x^12+9)^(1/2)
```

maxima [B] time = 0.97, size = 58, normalized size = 2.00

$$\frac{3\sqrt{x^{12}+9}}{4\,x^6\left(\frac{x^{12}+9}{x^{12}}-1\right)} + \frac{3}{8}\log\left(\frac{\sqrt{x^{12}+9}}{x^6}+1\right) - \frac{3}{8}\log\left(\frac{\sqrt{x^{12}+9}}{x^6}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^12+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/4*sqrt(x^12 + 9)/(x^6*((x^12 + 9)/x^12 - 1)) + 3/8*log(sqrt(x^12 + 9)/x^6
 + 1) - 3/8*log(sqrt(x^12 + 9)/x^6 - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x^5\sqrt{x^{12}+9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^12 + 9)^(1/2),x)
```

```
[Out] int(x^5*(x^12 + 9)^(1/2), x)
```

sympy [A] time = 1.64, size = 37, normalized size = 1.28

$$\frac{x^{18}}{12\sqrt{x^{12}+9}} + \frac{3x^6}{4\sqrt{x^{12}+9}} + \frac{3\,\mathrm{asinh}\left(\frac{x^6}{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(x**12+9)**(1/2),x)
```

```
[Out] x**18/(12*sqrt(x**12 + 9)) + 3*x**6/(4*sqrt(x**12 + 9)) + 3*asinh(x**6/3)/4
```

3.1546 $\int \left(a + \frac{b}{x}\right) x^6 \, dx$

Optimal. Leaf size=17

$$\frac{ax^7}{7} + \frac{bx^6}{6}$$

```
[Out] 1/6*b*x^6+1/7*a*x^7
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$\frac{ax^7}{7} + \frac{bx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)*x^6,x]
```

```
[Out] (b*x^6)/6 + (a*x^7)/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \left(a + \frac{b}{x}\right) x^6 \, dx &= \int \left(bx^5 + ax^6\right) dx \\ &= \frac{bx^6}{6} + \frac{ax^7}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^7}{7} + \frac{bx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)*x^6,x]
```

```
[Out] (b*x^6)/6 + (a*x^7)/7
```

fricas [A] time = 0.92, size = 13, normalized size = 0.76

$$\frac{1}{7} ax^7 + \frac{1}{6} bx^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)*x^6,x, algorithm="fricas")
```

```
[Out] 1/7*a*x^7 + 1/6*b*x^6
```

giac [A] time = 0.16, size = 13, normalized size = 0.76

$$\frac{1}{7} ax^7 + \frac{1}{6} bx^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)*x^6,x, algorithm="giac")
[Out] 1/7*a*x^7 + 1/6*b*x^6
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{7}a\,x^7+\frac{1}{6}b\,x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)*x^6,x)
[Out] 1/6*b*x^6+1/7*a*x^7
```

maxima [A] time = 1.01, size = 13, normalized size = 0.76

$$\frac{1}{7}\,ax^7+\frac{1}{6}\,bx^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)*x^6,x, algorithm="maxima")
[Out] 1/7*a*x^7 + 1/6*b*x^6
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{x^6\,(7\,b+6\,a\,x)}{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b/x),x)
[Out] (x^6*(7*b + 6*a*x))/42
```

sympy [A] time = 0.06, size = 12, normalized size = 0.71

$$\frac{ax^7}{7}+\frac{bx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)*x**6,x)
[Out] a*x**7/7 + b*x**6/6
```

3.1547 $\int \left(a + \frac{b}{x}\right) x^5 \, dx$

Optimal. Leaf size=17

$$\frac{ax^6}{6} + \frac{bx^5}{5}$$

```
[Out] 1/5*b*x^5+1/6*a*x^6
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$\frac{ax^6}{6} + \frac{bx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)*x^5,x]
```

```
[Out] (b*x^5)/5 + (a*x^6)/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \left(a + \frac{b}{x}\right) x^5 \, dx &= \int \left(bx^4 + ax^5\right) \, dx \\ &= \frac{bx^5}{5} + \frac{ax^6}{6}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^6}{6} + \frac{bx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)*x^5,x]
```

```
[Out] (b*x^5)/5 + (a*x^6)/6
```

fricas [A] time = 0.94, size = 13, normalized size = 0.76

$$\frac{1}{6} ax^6 + \frac{1}{5} bx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)*x^5,x, algorithm="fricas")
```

```
[Out] 1/6*a*x^6 + 1/5*b*x^5
```

giac [A] time = 0.19, size = 13, normalized size = 0.76

$$\frac{1}{6} ax^6 + \frac{1}{5} bx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)*x^5,x, algorithm="giac")
[Out] 1/6*a*x^6 + 1/5*b*x^5
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{6}a\,x^6+\frac{1}{5}b\,x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)*x^5,x)
[Out] 1/5*b*x^5+1/6*a*x^6
```

maxima [A] time = 1.06, size = 13, normalized size = 0.76

$$\frac{1}{6}\,ax^6+\frac{1}{5}\,bx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)*x^5,x, algorithm="maxima")
[Out] 1/6*a*x^6 + 1/5*b*x^5
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{x^5\,(6\,b+5\,a\,x)}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b/x),x)
[Out] (x^5*(6*b + 5*a*x))/30
```

sympy [A] time = 0.06, size = 12, normalized size = 0.71

$$\frac{ax^6}{6}+\frac{bx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)*x**5,x)
[Out] a*x**6/6 + b*x**5/5
```

3.1548 $\int \left(a + \frac{b}{x}\right) x^4 \, dx$

Optimal. Leaf size=17

$$\frac{ax^5}{5} + \frac{bx^4}{4}$$

```
[Out] 1/4*b*x^4+1/5*a*x^5
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$\frac{ax^5}{5} + \frac{bx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)*x^4,x]
```

```
[Out] (b*x^4)/4 + (a*x^5)/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \left(a + \frac{b}{x}\right) x^4 \, dx &= \int \left(bx^3 + ax^4\right) \, dx \\ &= \frac{bx^4}{4} + \frac{ax^5}{5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^5}{5} + \frac{bx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)*x^4,x]
```

```
[Out] (b*x^4)/4 + (a*x^5)/5
```

fricas [A] time = 0.89, size = 13, normalized size = 0.76

$$\frac{1}{5} ax^5 + \frac{1}{4} bx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)*x^4,x, algorithm="fricas")
```

```
[Out] 1/5*a*x^5 + 1/4*b*x^4
```

giac [A] time = 0.15, size = 13, normalized size = 0.76

$$\frac{1}{5} ax^5 + \frac{1}{4} bx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)*x^4,x, algorithm="giac")
[Out] 1/5*a*x^5 + 1/4*b*x^4
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{5}a\,x^5+\frac{1}{4}b\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)*x^4,x)
[Out] 1/4*b*x^4+1/5*a*x^5
```

maxima [A] time = 1.02, size = 13, normalized size = 0.76

$$\frac{1}{5}\,ax^5+\frac{1}{4}\,bx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)*x^4,x, algorithm="maxima")
[Out] 1/5*a*x^5 + 1/4*b*x^4
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{x^4\,(5\,b+4\,a\,x)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b/x),x)
[Out] (x^4*(5*b + 4*a*x))/20
```

sympy [A] time = 0.06, size = 12, normalized size = 0.71

$$\frac{ax^5}{5}+\frac{bx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)*x**4,x)
[Out] a*x**5/5 + b*x**4/4
```

3.1549 $\int \left(a + \frac{b}{x}\right) x^3 \, dx$

Optimal. Leaf size=17

$$\frac{ax^4}{4} + \frac{bx^3}{3}$$

```
[Out] 1/3*b*x^3+1/4*a*x^4
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$\frac{ax^4}{4} + \frac{bx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)*x^3,x]
```

```
[Out] (b*x^3)/3 + (a*x^4)/4
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \left(a + \frac{b}{x}\right) x^3 \, dx &= \int \left(bx^2 + ax^3\right) \, dx \\ &= \frac{bx^3}{3} + \frac{ax^4}{4}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^4}{4} + \frac{bx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)*x^3,x]
```

```
[Out] (b*x^3)/3 + (a*x^4)/4
```

fricas [A] time = 0.97, size = 13, normalized size = 0.76

$$\frac{1}{4}\, ax^4 + \frac{1}{3}\, bx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)*x^3,x, algorithm="fricas")
```

```
[Out] 1/4*a*x^4 + 1/3*b*x^3
```

giac [A] time = 0.16, size = 13, normalized size = 0.76

$$\frac{1}{4}\, ax^4 + \frac{1}{3}\, bx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)*x^3,x, algorithm="giac")
[Out] 1/4*a*x^4 + 1/3*b*x^3
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{4}a\,x^4+\frac{1}{3}b\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)*x^3,x)
[Out] 1/3*b*x^3+1/4*a*x^4
```

maxima [A] time = 0.99, size = 13, normalized size = 0.76

$$\frac{1}{4}\,ax^4+\frac{1}{3}\,bx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)*x^3,x, algorithm="maxima")
[Out] 1/4*a*x^4 + 1/3*b*x^3
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{x^3\,(4\,b+3\,a\,x)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b/x),x)
[Out] (x^3*(4*b + 3*a*x))/12
```

sympy [A] time = 0.06, size = 12, normalized size = 0.71

$$\frac{ax^4}{4}+\frac{bx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)*x**3,x)
[Out] a*x**4/4 + b*x**3/3
```

3.1550 $\int \left(a + \frac{b}{x}\right) x^2 \, dx$

Optimal. Leaf size=17

$$\frac{ax^3}{3} + \frac{bx^2}{2}$$

```
[Out] 1/2*b*x^2+1/3*a*x^3
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$\frac{ax^3}{3} + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)*x^2,x]
```

```
[Out] (b*x^2)/2 + (a*x^3)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \left(a + \frac{b}{x}\right) x^2 \, dx &= \int \left(bx + ax^2\right) \, dx \\ &= \frac{bx^2}{2} + \frac{ax^3}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^3}{3} + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)*x^2,x]
```

```
[Out] (b*x^2)/2 + (a*x^3)/3
```

fricas [A] time = 0.77, size = 13, normalized size = 0.76

$$\frac{1}{3} ax^3 + \frac{1}{2} bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)*x^2,x, algorithm="fricas")
```

```
[Out] 1/3*a*x^3 + 1/2*b*x^2
```

giac [A] time = 0.15, size = 13, normalized size = 0.76

$$\frac{1}{3} ax^3 + \frac{1}{2} bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)*x^2,x, algorithm="giac")
[Out] 1/3*a*x^3 + 1/2*b*x^2
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{3}a\,x^3+\frac{1}{2}b\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)*x^2,x)
[Out] 1/2*b*x^2+1/3*a*x^3
```

maxima [A] time = 1.02, size = 13, normalized size = 0.76

$$\frac{1}{3}\,ax^3+\frac{1}{2}\,bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)*x^2,x, algorithm="maxima")
[Out] 1/3*a*x^3 + 1/2*b*x^2
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{x^2\,(3\,b+2\,a\,x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b/x),x)
[Out] (x^2*(3*b + 2*a*x))/6
```

sympy [A] time = 0.06, size = 12, normalized size = 0.71

$$\frac{ax^3}{3}+\frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)*x**2,x)
[Out] a*x**3/3 + b*x**2/2
```

3.1551 $\int \left(a + \frac{b}{x}\right) x \, dx$

Optimal. Leaf size=12

$$\frac{ax^2}{2} + bx$$

```
[Out] b*x+1/2*a*x^2
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {14}

$$\frac{ax^2}{2} + bx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)*x,x]
```

```
[Out] b*x + (a*x^2)/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \left(a + \frac{b}{x}\right) x \, dx &= \int (b + ax) \, dx \\ &= bx + \frac{ax^2}{2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\frac{ax^2}{2} + bx$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)*x,x]
```

```
[Out] b*x + (a*x^2)/2
```

fricas [A] time = 0.97, size = 10, normalized size = 0.83

$$\frac{1}{2} ax^2 + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)*x,x, algorithm="fricas")
```

```
[Out] 1/2*a*x^2 + b*x
```

giac [A] time = 0.16, size = 10, normalized size = 0.83

$$\frac{1}{2} ax^2 + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)*x,x, algorithm="giac")
[Out] 1/2*a*x^2 + b*x
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$\frac{1}{2}a\,x^2 + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)*x,x)
[Out] b*x+1/2*a*x^2
```

maxima [A] time = 1.00, size = 10, normalized size = 0.83

$$\frac{1}{2}\,ax^2 + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)*x,x, algorithm="maxima")
[Out] 1/2*a*x^2 + b*x
```

mupad [B] time = 0.02, size = 10, normalized size = 0.83

$$\frac{a\,x^2}{2} + b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b/x),x)
[Out] b*x + (a*x^2)/2
```

sympy [A] time = 0.06, size = 8, normalized size = 0.67

$$\frac{ax^2}{2} + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)*x,x)
[Out] a*x**2/2 + b*x
```

3.1552 $\int \left(a + \frac{b}{x}\right) dx$

Optimal. Leaf size=8

$$ax + b\log(x)$$

```
[Out] a*x+b*ln(x)
```

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$ax + b\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[a + b/x,x]
```

```
[Out] a*x + b*Log[x]
```

Rubi steps

$$\int \left(a + \frac{b}{x}\right) dx = ax + b\log(x)$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$ax + b\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b/x,x]
```

```
[Out] a*x + b*Log[x]
```

fricas [A] time = 0.85, size = 8, normalized size = 1.00

$$ax + b\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b/x,x, algorithm="fricas")
```

```
[Out] a*x + b*log(x)
```

giac [A] time = 0.17, size = 9, normalized size = 1.12

$$ax + b\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b/x,x, algorithm="giac")
```

```
[Out] a*x + b*log(abs(x))
```

maple [A] time = 0.00, size = 9, normalized size = 1.12

$$ax + b\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b/x,x)
```

```
[Out] a*x+b*ln(x)
```

maxima [A] time = 0.96, size = 8, normalized size = 1.00

$$ax + b\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b/x,x, algorithm="maxima")
[Out] a*x + b*log(x)
```

mupad [B] time = 0.02, size = 8, normalized size = 1.00

$$a\,x + b\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b/x,x)
[Out] a*x + b*log(x)
```

sympy [A] time = 0.09, size = 7, normalized size = 0.88

$$ax + b\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b/x,x)
[Out] a*x + b*log(x)
```

3.1553 $\int \frac{a+\frac{b}{x}}{x}\,dx$

Optimal. Leaf size=11

$$a\log(x)-\frac{b}{x}$$

```
[Out] -b/x+a*ln(x)
```

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$a\log(x)-\frac{b}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)/x,x]
```

```
[Out] -(b/x) + a*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+\frac{b}{x}}{x}\,dx &= \int \left(\frac{b}{x^2}+\frac{a}{x}\right)dx\\ &= -\frac{b}{x}+a\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$a\log(x)-\frac{b}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)/x,x]
```

```
[Out] -(b/x) + a*Log[x]
```

fricas [A] time = 0.99, size = 13, normalized size = 1.18

$$\frac{ax\log(x)-b}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)/x,x, algorithm="fricas")
```

```
[Out] (a*x*log(x) - b)/x
```

giac [A] time = 0.18, size = 12, normalized size = 1.09

$$a\log\left(|x|\right)-\frac{b}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)/x,x, algorithm="giac")
[Out] a*log(abs(x)) - b/x
```

maple [A] time = 0.01, size = 12, normalized size = 1.09

$$a \ln(x) - \frac{b}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)/x,x)
[Out] -b/x+a*ln(x)
```

maxima [A] time = 1.03, size = 11, normalized size = 1.00

$$a \log(x) - \frac{b}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)/x,x, algorithm="maxima")
[Out] a*log(x) - b/x
```

mupad [B] time = 0.03, size = 11, normalized size = 1.00

$$a\ \ln(x) - \frac{b}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)/x,x)
[Out] a*log(x) - b/x
```

sympy [A] time = 0.11, size = 7, normalized size = 0.64

$$a \log(x) - \frac{b}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)/x,x)
[Out] a*log(x) - b/x
```

3.1554 $\int \frac{a+\frac{b}{x}}{x^2}\,dx$

Optimal. Leaf size=15

$$-\frac{a}{x}-\frac{b}{2x^2}$$

```
[Out] -1/2*b/x^2-a/x
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$-\frac{a}{x}-\frac{b}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)/x^2,x]
```

```
[Out] -b/(2*x^2) - a/x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+\frac{b}{x}}{x^2}\,dx &= \int \left(\frac{b}{x^3}+\frac{a}{x^2}\right)dx\\ &= -\frac{b}{2x^2}-\frac{a}{x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$-\frac{a}{x}-\frac{b}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)/x^2,x]
```

```
[Out] -1/2*b/x^2 - a/x
```

fricas [A] time = 0.85, size = 11, normalized size = 0.73

$$-\frac{2\,ax+b}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*x + b)/x^2
```

giac [A] time = 0.15, size = 13, normalized size = 0.87

$$-\frac{a}{x}-\frac{b}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)/x^2,x, algorithm="giac")
[Out] -a/x - 1/2*b/x^2
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$-\frac{a}{x}-\frac{b}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)/x^2,x)
[Out] -1/2*b/x^2-a/x
```

maxima [A] time = 1.11, size = 14, normalized size = 0.93

$$-\frac{\left(a+\frac{b}{x}\right)^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)/x^2,x, algorithm="maxima")
[Out] -1/2*(a + b/x)^2/b
```

mupad [B] time = 0.02, size = 11, normalized size = 0.73

$$-\frac{b+2\,a\,x}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)/x^2,x)
[Out] -(b + 2*a*x)/(2*x^2)
```

sympy [A] time = 0.12, size = 12, normalized size = 0.80

$$\frac{-2ax-b}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)/x**2,x)
[Out] (-2*a*x - b)/(2*x**2)
```

3.1555 $\int \frac{a+\frac{b}{x}}{x^3}\,dx$

Optimal. Leaf size=17

$$-\frac{a}{2x^2}-\frac{b}{3x^3}$$

```
[Out] -1/3*b/x^3-1/2*a/x^2
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$-\frac{a}{2x^2}-\frac{b}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)/x^3,x]
```

```
[Out] -b/(3*x^3) - a/(2*x^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+\frac{b}{x}}{x^3}\,dx &= \int \left(\frac{b}{x^4}+\frac{a}{x^3}\right)dx\\ &= -\frac{b}{3x^3}-\frac{a}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$-\frac{a}{2x^2}-\frac{b}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)/x^3,x]
```

```
[Out] -1/3*b/x^3 - a/(2*x^2)
```

fricas [A] time = 0.54, size = 13, normalized size = 0.76

$$-\frac{3\,ax+2\,b}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)/x^3,x, algorithm="fricas")
```

```
[Out] -1/6*(3*a*x + 2*b)/x^3
```

giac [A] time = 0.15, size = 13, normalized size = 0.76

$$-\frac{3\,ax+2\,b}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)/x^3,x, algorithm="giac")
[Out] -1/6*(3*a*x + 2*b)/x^3
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$-\frac{a}{2x^2}-\frac{b}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)/x^3,x)
[Out] -1/3*b/x^3-1/2*a/x^2
```

maxima [A] time = 1.00, size = 13, normalized size = 0.76

$$-\frac{3\,ax+2\,b}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)/x^3,x, algorithm="maxima")
[Out] -1/6*(3*a*x + 2*b)/x^3
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$-\frac{2\,b+3\,a\,x}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)/x^3,x)
[Out] -(2*b + 3*a*x)/(6*x^3)
```

sympy [A] time = 0.12, size = 14, normalized size = 0.82

$$\frac{-3ax-2b}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)/x**3,x)
[Out] (-3*a*x - 2*b)/(6*x**3)
```

3.1556 $\int \frac{a+\frac{b}{x}}{x^4}\,dx$

Optimal. Leaf size=17

$$-\frac{a}{3x^3}-\frac{b}{4x^4}$$

```
[Out] -1/4*b/x^4-1/3*a/x^3
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$-\frac{a}{3x^3}-\frac{b}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)/x^4,x]
```

```
[Out] -b/(4*x^4) - a/(3*x^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+\frac{b}{x}}{x^4}\,dx &= \int \left(\frac{b}{x^5}+\frac{a}{x^4}\right)dx\\ &= -\frac{b}{4x^4}-\frac{a}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$-\frac{a}{3x^3}-\frac{b}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)/x^4,x]
```

```
[Out] -1/4*b/x^4 - a/(3*x^3)
```

fricas [A] time = 0.90, size = 13, normalized size = 0.76

$$-\frac{4\,ax+3\,b}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(4*a*x + 3*b)/x^4
```

giac [A] time = 0.15, size = 13, normalized size = 0.76

$$-\frac{4\,ax+3\,b}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)/x^4,x, algorithm="giac")
[Out] -1/12*(4*a*x + 3*b)/x^4
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$-\frac{a}{3x^3}-\frac{b}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)/x^4,x)
[Out] -1/4*b/x^4-1/3*a/x^3
```

maxima [A] time = 1.05, size = 13, normalized size = 0.76

$$-\frac{4\,ax+3\,b}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)/x^4,x, algorithm="maxima")
[Out] -1/12*(4*a*x + 3*b)/x^4
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$-\frac{3\,b+4\,a\,x}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)/x^4,x)
[Out] -(3*b + 4*a*x)/(12*x^4)
```

sympy [A] time = 0.13, size = 14, normalized size = 0.82

$$\frac{-4ax-3b}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)/x**4,x)
[Out] (-4*a*x - 3*b)/(12*x**4)
```

3.1557 $\int \frac{a+\frac{b}{x}}{x^5}\,dx$

Optimal. Leaf size=17

$$-\frac{a}{4x^4}-\frac{b}{5x^5}$$

```
[Out] -1/5*b/x^5-1/4*a/x^4
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$-\frac{a}{4x^4}-\frac{b}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)/x^5,x]
```

```
[Out] -b/(5*x^5) - a/(4*x^4)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+\frac{b}{x}}{x^5}\,dx &= \int \left(\frac{b}{x^6}+\frac{a}{x^5}\right)dx\\ &= -\frac{b}{5x^5}-\frac{a}{4x^4}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$-\frac{a}{4x^4}-\frac{b}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)/x^5,x]
```

```
[Out] -1/5*b/x^5 - a/(4*x^4)
```

fricas [A] time = 0.97, size = 13, normalized size = 0.76

$$-\frac{5\,ax+4\,b}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)/x^5,x, algorithm="fricas")
```

```
[Out] -1/20*(5*a*x + 4*b)/x^5
```

giac [A] time = 0.17, size = 13, normalized size = 0.76

$$-\frac{5\,ax+4\,b}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)/x^5,x, algorithm="giac")
[Out] -1/20*(5*a*x + 4*b)/x^5
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$-\frac{a}{4x^4}-\frac{b}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)/x^5,x)
[Out] -1/5*b/x^5-1/4*a/x^4
```

maxima [A] time = 1.10, size = 13, normalized size = 0.76

$$-\frac{5\,ax+4\,b}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)/x^5,x, algorithm="maxima")
[Out] -1/20*(5*a*x + 4*b)/x^5
```

mupad [B] time = 0.03, size = 13, normalized size = 0.76

$$-\frac{4\,b+5\,a\,x}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)/x^5,x)
[Out] -(4*b + 5*a*x)/(20*x^5)
```

sympy [A] time = 0.15, size = 14, normalized size = 0.82

$$\frac{-5ax-4b}{20x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)/x**5,x)
[Out] (-5*a*x - 4*b)/(20*x**5)
```

3.1558 $\int \frac{a+\frac{b}{x}}{x^6}\,dx$

Optimal. Leaf size=17

$$-\frac{a}{5x^5}-\frac{b}{6x^6}$$

```
[Out] -1/6*b/x^6-1/5*a/x^5
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$-\frac{a}{5x^5}-\frac{b}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)/x^6,x]
```

```
[Out] -b/(6*x^6) - a/(5*x^5)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+\frac{b}{x}}{x^6}\,dx &= \int \left(\frac{b}{x^7}+\frac{a}{x^6}\right)dx\\ &= -\frac{b}{6x^6}-\frac{a}{5x^5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$-\frac{a}{5x^5}-\frac{b}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)/x^6,x]
```

```
[Out] -1/6*b/x^6 - a/(5*x^5)
```

fricas [A] time = 0.88, size = 13, normalized size = 0.76

$$-\frac{6\,ax+5\,b}{30\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)/x^6,x, algorithm="fricas")
```

```
[Out] -1/30*(6*a*x + 5*b)/x^6
```

giac [A] time = 0.16, size = 13, normalized size = 0.76

$$-\frac{6\,ax+5\,b}{30\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)/x^6,x, algorithm="giac")
[Out] -1/30*(6*a*x + 5*b)/x^6
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$-\frac{a}{5x^5}-\frac{b}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)/x^6,x)
[Out] -1/6*b/x^6-1/5*a/x^5
```

maxima [A] time = 0.97, size = 13, normalized size = 0.76

$$-\frac{6\,ax+5\,b}{30\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)/x^6,x, algorithm="maxima")
[Out] -1/30*(6*a*x + 5*b)/x^6
```

mupad [B] time = 0.03, size = 13, normalized size = 0.76

$$-\frac{5\,b+6\,a\,x}{30\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)/x^6,x)
[Out] -(5*b + 6*a*x)/(30*x^6)
```

sympy [A] time = 0.15, size = 14, normalized size = 0.82

$$\frac{-6ax-5b}{30x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)/x**6,x)
[Out] (-6*a*x - 5*b)/(30*x**6)
```

3.1559 $\int \left(a+\frac{b}{x}\right)^2 x^5\,dx$

Optimal. Leaf size=30

$$\frac{a^2x^6}{6}+\frac{2}{5}abx^5+\frac{b^2x^4}{4}$$

```
[Out] 1/4*b^2*x^4+2/5*a*b*x^5+1/6*a^2*x^6
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$\frac{a^2x^6}{6}+\frac{2}{5}abx^5+\frac{b^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^2*x^5,x]
```

```
[Out] (b^2*x^4)/4 + (2*a*b*x^5)/5 + (a^2*x^6)/6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^2 x^5\,dx &= \int x^3(b+ax)^2\,dx\\ &= \int \left(b^2x^3+2abx^4+a^2x^5\right)\,dx\\ &= \frac{b^2x^4}{4}+\frac{2}{5}abx^5+\frac{a^2x^6}{6}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^6}{6}+\frac{2}{5}abx^5+\frac{b^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^2*x^5,x]
```

```
[Out] (b^2*x^4)/4 + (2*a*b*x^5)/5 + (a^2*x^6)/6
```

fricas [A] time = 1.00, size = 24, normalized size = 0.80

$$\frac{1}{6}\,a^2x^6+\frac{2}{5}\,abx^5+\frac{1}{4}\,b^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2*x^5,x, algorithm="fricas")
[Out] 1/6*a^2*x^6 + 2/5*a*b*x^5 + 1/4*b^2*x^4
```

giac [A] time = 0.16, size = 24, normalized size = 0.80

$$\frac{1}{6}a^2x^6 + \frac{2}{5}abx^5 + \frac{1}{4}b^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2*x^5,x, algorithm="giac")
[Out] 1/6*a^2*x^6 + 2/5*a*b*x^5 + 1/4*b^2*x^4
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$\frac{1}{6}a^2x^6 + \frac{2}{5}ab\,x^5 + \frac{1}{4}b^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^2*x^5,x)
[Out] 1/4*b^2*x^4+2/5*a*b*x^5+1/6*a^2*x^6
```

maxima [A] time = 1.01, size = 24, normalized size = 0.80

$$\frac{1}{6}a^2x^6 + \frac{2}{5}abx^5 + \frac{1}{4}b^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2*x^5,x, algorithm="maxima")
[Out] 1/6*a^2*x^6 + 2/5*a*b*x^5 + 1/4*b^2*x^4
```

mupad [B] time = 0.04, size = 24, normalized size = 0.80

$$\frac{a^2\,x^6}{6} + \frac{2\,a\,b\,x^5}{5} + \frac{b^2\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b/x)^2,x)
[Out] (a^2*x^6)/6 + (b^2*x^4)/4 + (2*a*b*x^5)/5
```

sympy [A] time = 0.08, size = 26, normalized size = 0.87

$$\frac{a^2x^6}{6} + \frac{2abx^5}{5} + \frac{b^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**2*x**5,x)
[Out] a**2*x**6/6 + 2*a*b*x**5/5 + b**2*x**4/4
```

3.1560 $\int \left(a+\frac{b}{x}\right)^2 x^4\,dx$

Optimal. Leaf size=30

$$\frac{a^2x^5}{5}+\frac{1}{2}abx^4+\frac{b^2x^3}{3}$$

```
[Out] 1/3*b^2*x^3+1/2*a*b*x^4+1/5*a^2*x^5
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$\frac{a^2x^5}{5}+\frac{1}{2}abx^4+\frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^2*x^4,x]
```

```
[Out] (b^2*x^3)/3 + (a*b*x^4)/2 + (a^2*x^5)/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^2 x^4\,dx &= \int x^2(b+ax)^2\,dx\\ &= \int \left(b^2x^2+2abx^3+a^2x^4\right)\,dx\\ &= \frac{b^2x^3}{3}+\frac{1}{2}abx^4+\frac{a^2x^5}{5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^5}{5}+\frac{1}{2}abx^4+\frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^2*x^4,x]
```

```
[Out] (b^2*x^3)/3 + (a*b*x^4)/2 + (a^2*x^5)/5
```

fricas [A] time = 0.89, size = 24, normalized size = 0.80

$$\frac{1}{5}\,a^2x^5+\frac{1}{2}\,abx^4+\frac{1}{3}\,b^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2*x^4,x, algorithm="fricas")
[Out] 1/5*a^2*x^5 + 1/2*a*b*x^4 + 1/3*b^2*x^3
```

giac [A] time = 0.15, size = 24, normalized size = 0.80

$$\frac{1}{5}a^2x^5 + \frac{1}{2}abx^4 + \frac{1}{3}b^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2*x^4,x, algorithm="giac")
[Out] 1/5*a^2*x^5 + 1/2*a*b*x^4 + 1/3*b^2*x^3
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$\frac{1}{5}a^2x^5 + \frac{1}{2}ab\,x^4 + \frac{1}{3}b^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^2*x^4,x)
[Out] 1/3*b^2*x^3+1/2*a*b*x^4+1/5*a^2*x^5
```

maxima [A] time = 1.05, size = 24, normalized size = 0.80

$$\frac{1}{5}a^2x^5 + \frac{1}{2}abx^4 + \frac{1}{3}b^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2*x^4,x, algorithm="maxima")
[Out] 1/5*a^2*x^5 + 1/2*a*b*x^4 + 1/3*b^2*x^3
```

mupad [B] time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^5}{5} + \frac{a\,b\,x^4}{2} + \frac{b^2\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b/x)^2,x)
[Out] (a^2*x^5)/5 + (b^2*x^3)/3 + (a*b*x^4)/2
```

sympy [A] time = 0.07, size = 24, normalized size = 0.80

$$\frac{a^2x^5}{5} + \frac{abx^4}{2} + \frac{b^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**2*x**4,x)
[Out] a**2*x**5/5 + a*b*x**4/2 + b**2*x**3/3
```

3.1561 $\int \left(a+\frac{b}{x}\right)^2 x^3\,dx$

Optimal. Leaf size=30

$$\frac{a^2x^4}{4}+\frac{2}{3}abx^3+\frac{b^2x^2}{2}$$

```
[Out] 1/2*b^2*x^2+2/3*a*b*x^3+1/4*a^2*x^4
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$\frac{a^2x^4}{4}+\frac{2}{3}abx^3+\frac{b^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^2*x^3,x]
```

```
[Out] (b^2*x^2)/2 + (2*a*b*x^3)/3 + (a^2*x^4)/4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^2 x^3\,dx &= \int x(b+ax)^2\,dx\\ &= \int \left(b^2x+2abx^2+a^2x^3\right)\,dx\\ &= \frac{b^2x^2}{2}+\frac{2}{3}abx^3+\frac{a^2x^4}{4}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^4}{4}+\frac{2}{3}abx^3+\frac{b^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^2*x^3,x]
```

```
[Out] (b^2*x^2)/2 + (2*a*b*x^3)/3 + (a^2*x^4)/4
```

fricas [A] time = 1.02, size = 24, normalized size = 0.80

$$\frac{1}{4}\,a^2x^4+\frac{2}{3}\,abx^3+\frac{1}{2}\,b^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2*x^3,x, algorithm="fricas")
[Out] 1/4*a^2*x^4 + 2/3*a*b*x^3 + 1/2*b^2*x^2
```

giac [A] time = 0.18, size = 24, normalized size = 0.80

$$\frac{1}{4}a^2x^4 + \frac{2}{3}abx^3 + \frac{1}{2}b^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2*x^3,x, algorithm="giac")
[Out] 1/4*a^2*x^4 + 2/3*a*b*x^3 + 1/2*b^2*x^2
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$\frac{1}{4}a^2x^4 + \frac{2}{3}ab\,x^3 + \frac{1}{2}b^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^2*x^3,x)
[Out] 1/2*b^2*x^2+2/3*a*b*x^3+1/4*a^2*x^4
```

maxima [A] time = 1.04, size = 24, normalized size = 0.80

$$\frac{1}{4}a^2x^4 + \frac{2}{3}abx^3 + \frac{1}{2}b^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2*x^3,x, algorithm="maxima")
[Out] 1/4*a^2*x^4 + 2/3*a*b*x^3 + 1/2*b^2*x^2
```

mupad [B] time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^4}{4} + \frac{2\,a\,b\,x^3}{3} + \frac{b^2\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b/x)^2,x)
[Out] (a^2*x^4)/4 + (b^2*x^2)/2 + (2*a*b*x^3)/3
```

sympy [A] time = 0.07, size = 26, normalized size = 0.87

$$\frac{a^2x^4}{4} + \frac{2abx^3}{3} + \frac{b^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**2*x**3,x)
[Out] a**2*x**4/4 + 2*a*b*x**3/3 + b**2*x**2/2
```

3.1562 $\int \left(a + \frac{b}{x}\right)^2 x^2\, dx$

Optimal. Leaf size=14

$$\frac{(ax+b)^3}{3a}$$

```
[Out] 1/3*(a*x+b)^3/a
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 32}

$$\frac{(ax+b)^3}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^2*x^2,x]
```

```
[Out] (b + a*x)^3/(3*a)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(a + \frac{b}{x}\right)^2 x^2\, dx &= \int (b+ax)^2\, dx \\ &= \frac{(b+ax)^3}{3a}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{(ax+b)^3}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^2*x^2,x]
```

```
[Out] (b + a*x)^3/(3*a)
```

fricas [A] time = 0.93, size = 20, normalized size = 1.43

$$\frac{1}{3}a^2x^3 + abx^2 + b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2*x^2,x, algorithm="fricas")
```

```
[Out] 1/3*a^2*x^3 + a*b*x^2 + b^2*x
```

giac [A] time = 0.15, size = 20, normalized size = 1.43

$$\frac{1}{3}a^2x^3 + abx^2 + b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2*x^2,x, algorithm="giac")
[Out] 1/3*a^2*x^3 + a*b*x^2 + b^2*x
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$\frac{(ax+b)^3}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^2*x^2,x)
[Out] 1/3*(a*x+b)^3/a
```

maxima [A] time = 1.00, size = 20, normalized size = 1.43

$$\frac{1}{3}a^2x^3 + abx^2 + b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2*x^2,x, algorithm="maxima")
[Out] 1/3*a^2*x^3 + a*b*x^2 + b^2*x
```

mupad [B] time = 0.03, size = 20, normalized size = 1.43

$$\frac{a^2\,x^3}{3} + a\,b\,x^2 + b^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b/x)^2,x)
[Out] b^2*x + (a^2*x^3)/3 + a*b*x^2
```

sympy [B] time = 0.07, size = 19, normalized size = 1.36

$$\frac{a^2x^3}{3} + abx^2 + b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**2*x**2,x)
[Out] a**2*x**3/3 + a*b*x**2 + b**2*x
```

3.1563 $\int \left(a+\frac{b}{x}\right)^2 x\,dx$

Optimal. Leaf size=22

$$\frac{a^2x^2}{2}+2abx+b^2\log(x)$$

```
[Out] 2*a*b*x+1/2*a^2*x^2+b^2*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {263, 43}

$$\frac{a^2x^2}{2}+2abx+b^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^2*x,x]
```

```
[Out] 2*a*b*x + (a^2*x^2)/2 + b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^2 x\,dx &= \int \frac{(b+ax)^2}{x}\,dx\\ &= \int \left(2ab+\frac{b^2}{x}+a^2x\right)dx\\ &= 2abx+\frac{a^2x^2}{2}+b^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$\frac{a^2x^2}{2}+2abx+b^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^2*x,x]
```

```
[Out] 2*a*b*x + (a^2*x^2)/2 + b^2*Log[x]
```

fricas [A] time = 0.90, size = 20, normalized size = 0.91

$$\frac{1}{2}\,a^2x^2+2\,abx+b^2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2*x,x, algorithm="fricas")
[Out] 1/2*a^2*x^2 + 2*a*b*x + b^2*log(x)
```

giac [A] time = 0.16, size = 21, normalized size = 0.95

$$\frac{1}{2}\,a^2x^2 + 2\,abx + b^2\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2*x,x, algorithm="giac")
[Out] 1/2*a^2*x^2 + 2*a*b*x + b^2*log(abs(x))
```

maple [A] time = 0.00, size = 21, normalized size = 0.95

$$\frac{a^2x^2}{2} + 2abx + b^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^2*x,x)
[Out] 2*a*b*x+1/2*a^2*x^2+b^2*ln(x)
```

maxima [A] time = 1.05, size = 20, normalized size = 0.91

$$\frac{1}{2}\,a^2x^2 + 2\,abx + b^2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2*x,x, algorithm="maxima")
[Out] 1/2*a^2*x^2 + 2*a*b*x + b^2*log(x)
```

mupad [B] time = 0.03, size = 20, normalized size = 0.91

$$b^2\,\ln(x) + \frac{a^2\,x^2}{2} + 2\,a\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b/x)^2,x)
[Out] b^2*log(x) + (a^2*x^2)/2 + 2*a*b*x
```

sympy [A] time = 0.11, size = 20, normalized size = 0.91

$$\frac{a^2x^2}{2} + 2abx + b^2\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**2*x,x)
[Out] a**2*x**2/2 + 2*a*b*x + b**2*log(x)
```

3.1564 $\int \left(a+\frac{b}{x}\right)^2 dx$

Optimal. Leaf size=20

$$a^2x + 2ab\log(x) - \frac{b^2}{x}$$

```
[Out] -b^2/x+a^2*x+2*a*b*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {193, 43}

$$a^2x + 2ab\log(x) - \frac{b^2}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^2,x]
```

```
[Out] -(b^2/x) + a^2*x + 2*a*b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^2 dx &= \int \frac{(b+ax)^2}{x^2}\,dx \\ &= \int \left(a^2 + \frac{b^2}{x^2} + \frac{2ab}{x}\right) dx \\ &= -\frac{b^2}{x} + a^2x + 2ab\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$a^2x + 2ab\log(x) - \frac{b^2}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^2,x]
```

```
[Out] -(b^2/x) + a^2*x + 2*a*b*Log[x]
```

fricas [A] time = 0.75, size = 24, normalized size = 1.20

$$\frac{a^2x^2 + 2\,abx\log(x) - b^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2,x, algorithm="fricas")
[Out] (a^2*x^2 + 2*a*b*x*log(x) - b^2)/x
```

giac [**A**] time = 0.17, size = 21, normalized size = 1.05

$$a^2 x + 2\,ab \log(|x|) - \frac{b^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2,x, algorithm="giac")
[Out] a^2*x + 2*a*b*log(abs(x)) - b^2/x
```

maple [**A**] time = 0.00, size = 21, normalized size = 1.05

$$a^2 x + 2ab \ln(x) - \frac{b^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^2,x)
[Out] -b^2/x+a^2*x+2*a*b*ln(x)
```

maxima [**A**] time = 1.05, size = 20, normalized size = 1.00

$$a^2 x + 2\,ab \log(x) - \frac{b^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2,x, algorithm="maxima")
[Out] a^2*x + 2*a*b*log(x) - b^2/x
```

mupad [**B**] time = 0.03, size = 20, normalized size = 1.00

$$a^2\,x - \frac{b^2}{x} + 2\,a\,b\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^2,x)
[Out] a^2*x - b^2/x + 2*a*b*log(x)
```

sympy [**A**] time = 0.12, size = 17, normalized size = 0.85

$$a^2 x + 2ab \log(x) - \frac{b^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**2,x)
[Out] a**2*x + 2*a*b*log(x) - b**2/x
```

3.1565 $\int \frac{\left(a+\frac{b}{x}\right)^2}{x}\,dx$

Optimal. Leaf size=24

$$a^2\log(x)-\frac{2ab}{x}-\frac{b^2}{2x^2}$$

```
[Out] -1/2*b^2/x^2-2*a*b/x+a^2*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$a^2\log(x)-\frac{2ab}{x}-\frac{b^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^2/x,x]
```

```
[Out] -b^2/(2*x^2) - (2*a*b)/x + a^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^2}{x}\,dx &= \int \frac{(b+ax)^2}{x^3}\,dx\\ &= \int \left(\frac{b^2}{x^3}+\frac{2ab}{x^2}+\frac{a^2}{x}\right)dx\\ &= -\frac{b^2}{2x^2}-\frac{2ab}{x}+a^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$a^2\log(x)-\frac{2ab}{x}-\frac{b^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^2/x,x]
```

```
[Out] -1/2*b^2/x^2 - (2*a*b)/x + a^2*Log[x]
```

fricas [A] time = 0.84, size = 26, normalized size = 1.08

$$\frac{2\,a^2x^2\log(x)-4\,abx-b^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2/x,x, algorithm="fricas")
[Out] 1/2*(2*a^2*x^2*log(x) - 4*a*b*x - b^2)/x^2
```

giac [**A**] time = 0.15, size = 22, normalized size = 0.92

$$a^2 \log(|x|) - \frac{4\,abx + b^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2/x,x, algorithm="giac")
[Out] a^2*log(abs(x)) - 1/2*(4*a*b*x + b^2)/x^2
```

maple [**A**] time = 0.01, size = 23, normalized size = 0.96

$$a^2 \ln(x) - \frac{2ab}{x} - \frac{b^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^2/x,x)
[Out] -1/2*b^2/x^2-2*a*b/x+a^2*ln(x)
```

maxima [**A**] time = 1.02, size = 21, normalized size = 0.88

$$a^2 \log(x) - \frac{4\,abx + b^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2/x,x, algorithm="maxima")
[Out] a^2*log(x) - 1/2*(4*a*b*x + b^2)/x^2
```

mupad [**B**] time = 0.04, size = 23, normalized size = 0.96

$$a^2\ \ln(x) - \frac{\frac{b^2}{2} + 2\,a\,x\,b}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^2/x,x)
[Out] a^2*log(x) - (b^2/2 + 2*a*b*x)/x^2
```

sympy [**A**] time = 0.16, size = 22, normalized size = 0.92

$$a^2 \log(x) + \frac{-4abx - b^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**2/x,x)
[Out] a**2*log(x) + (-4*a*b*x - b**2)/(2*x**2)
```

3.1566 $\int \frac{\left(a+\frac{b}{x}\right)^2}{x^2}\,dx$

Optimal. Leaf size=16

$$-\frac{\left(a+\frac{b}{x}\right)^3}{3b}$$

```
[Out] -1/3*(a+b/x)^3/b
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$-\frac{\left(a+\frac{b}{x}\right)^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^2/x^2,x]
```

```
[Out] -(a + b/x)^3/(3*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^2}\,dx = -\frac{\left(a+\frac{b}{x}\right)^3}{3b}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.62

$$-\frac{a^2}{x}-\frac{ab}{x^2}-\frac{b^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^2/x^2,x]
```

```
[Out] -1/3*b^2/x^3 - (a*b)/x^2 - a^2/x
```

fricas [A] time = 0.93, size = 22, normalized size = 1.38

$$-\frac{3\,a^2x^2+3\,abx+b^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2/x^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^2*x^2 + 3*a*b*x + b^2)/x^3
```

giac [A] time = 0.15, size = 14, normalized size = 0.88

$$-\frac{\left(a+\frac{b}{x}\right)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2/x^2,x, algorithm="giac")
[Out] -1/3*(a + b/x)^3/b
```

maple [A] time = 0.01, size = 25, normalized size = 1.56

$$-\frac{a^2}{x}-\frac{ab}{x^2}-\frac{b^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^2/x^2,x)
[Out] -1/3*b^2/x^3-a*b/x^2-a^2/x
```

maxima [A] time = 1.00, size = 14, normalized size = 0.88

$$-\frac{\left(a+\frac{b}{x}\right)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2/x^2,x, algorithm="maxima")
[Out] -1/3*(a + b/x)^3/b
```

mupad [B] time = 0.04, size = 22, normalized size = 1.38

$$-\frac{a^2\,x^2+a\,b\,x+\frac{b^2}{3}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^2/x^2,x)
[Out] -(b^2/3 + a^2*x^2 + a*b*x)/x^3
```

sympy [B] time = 0.17, size = 24, normalized size = 1.50

$$\frac{-3a^2x^2-3abx-b^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**2/x**2,x)
[Out] (-3*a**2*x**2 - 3*a*b*x - b**2)/(3*x**3)
```

3.1567 $\int \frac{\left(a+\frac{b}{x}\right)^2}{x^3}\,dx$

Optimal. Leaf size=30

$$-\frac{a^2}{2x^2}-\frac{2ab}{3x^3}-\frac{b^2}{4x^4}$$

```
[Out] -1/4*b^2/x^4-2/3*a*b/x^3-1/2*a^2/x^2
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$-\frac{a^2}{2x^2}-\frac{2ab}{3x^3}-\frac{b^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^2/x^3,x]
```

```
[Out] -b^2/(4*x^4) - (2*a*b)/(3*x^3) - a^2/(2*x^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^2}{x^3}\,dx &= \int \frac{(b+ax)^2}{x^5}\,dx\\ &= \int \left(\frac{b^2}{x^5}+\frac{2ab}{x^4}+\frac{a^2}{x^3}\right)dx\\ &= -\frac{b^2}{4x^4}-\frac{2ab}{3x^3}-\frac{a^2}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$-\frac{a^2}{2x^2}-\frac{2ab}{3x^3}-\frac{b^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^2/x^3,x]
```

```
[Out] -1/4*b^2/x^4 - (2*a*b)/(3*x^3) - a^2/(2*x^2)
```

fricas [A] time = 1.11, size = 24, normalized size = 0.80

$$-\frac{6\,a^2x^2+8\,abx+3\,b^2}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2/x^3,x, algorithm="fricas")
[Out] -1/12*(6*a^2*x^2 + 8*a*b*x + 3*b^2)/x^4
```

giac [A] time = 0.15, size = 24, normalized size = 0.80

$$-\frac{6\,a^2x^2+8\,abx+3\,b^2}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2/x^3,x, algorithm="giac")
[Out] -1/12*(6*a^2*x^2 + 8*a*b*x + 3*b^2)/x^4
```

maple [A] time = 0.01, size = 25, normalized size = 0.83

$$-\frac{a^2}{2x^2}-\frac{2ab}{3x^3}-\frac{b^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^2/x^3,x)
[Out] -1/4*b^2/x^4-2/3*a*b/x^3-1/2*a^2/x^2
```

maxima [A] time = 1.02, size = 24, normalized size = 0.80

$$-\frac{6\,a^2x^2+8\,abx+3\,b^2}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2/x^3,x, algorithm="maxima")
[Out] -1/12*(6*a^2*x^2 + 8*a*b*x + 3*b^2)/x^4
```

mupad [B] time = 0.04, size = 24, normalized size = 0.80

$$-\frac{\frac{a^2x^2}{2}+\frac{2\,a\,b\,x}{3}+\frac{b^2}{4}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^2/x^3,x)
[Out] -(b^2/4 + (a^2*x^2)/2 + (2*a*b*x)/3)/x^4
```

sympy [A] time = 0.18, size = 26, normalized size = 0.87

$$\frac{-6a^2x^2-8abx-3b^2}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**2/x**3,x)
[Out] (-6*a**2*x**2 - 8*a*b*x - 3*b**2)/(12*x**4)
```

3.1568 $\int \frac{\left(a+\frac{b}{x}\right)^2}{x^4}\,dx$

Optimal. Leaf size=30

$$-\frac{a^2}{3x^3}-\frac{ab}{2x^4}-\frac{b^2}{5x^5}$$

```
[Out] -1/5*b^2/x^5-1/2*a*b/x^4-1/3*a^2/x^3
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$-\frac{a^2}{3x^3}-\frac{ab}{2x^4}-\frac{b^2}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^2/x^4,x]
```

```
[Out] -b^2/(5*x^5) - (a*b)/(2*x^4) - a^2/(3*x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^2}{x^4}\,dx &= \int \frac{(b+ax)^2}{x^6}\,dx\\ &= \int \left(\frac{b^2}{x^6}+\frac{2ab}{x^5}+\frac{a^2}{x^4}\right)dx\\ &= -\frac{b^2}{5x^5}-\frac{ab}{2x^4}-\frac{a^2}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$-\frac{a^2}{3x^3}-\frac{ab}{2x^4}-\frac{b^2}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^2/x^4,x]
```

```
[Out] -1/5*b^2/x^5 - (a*b)/(2*x^4) - a^2/(3*x^3)
```

fricas [A] time = 0.89, size = 24, normalized size = 0.80

$$-\frac{10\,a^2x^2+15\,abx+6\,b^2}{30\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2/x^4,x, algorithm="fricas")
[Out] -1/30*(10*a^2*x^2 + 15*a*b*x + 6*b^2)/x^5
```

giac [A] time = 0.17, size = 24, normalized size = 0.80

$$-\frac{10\,a^2x^2+15\,abx+6\,b^2}{30\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2/x^4,x, algorithm="giac")
[Out] -1/30*(10*a^2*x^2 + 15*a*b*x + 6*b^2)/x^5
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$-\frac{a^2}{3x^3}-\frac{ab}{2x^4}-\frac{b^2}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^2/x^4,x)
[Out] -1/5*b^2/x^5-1/2*a*b/x^4-1/3*a^2/x^3
```

maxima [A] time = 1.00, size = 24, normalized size = 0.80

$$-\frac{10\,a^2x^2+15\,abx+6\,b^2}{30\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2/x^4,x, algorithm="maxima")
[Out] -1/30*(10*a^2*x^2 + 15*a*b*x + 6*b^2)/x^5
```

mupad [B] time = 0.03, size = 24, normalized size = 0.80

$$-\frac{\frac{a^2x^2}{3}+\frac{a\,b\,x}{2}+\frac{b^2}{5}}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^2/x^4,x)
[Out] -(b^2/5 + (a^2*x^2)/3 + (a*b*x)/2)/x^5
```

sympy [A] time = 0.19, size = 26, normalized size = 0.87

$$\frac{-10a^2x^2-15abx-6b^2}{30x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**2/x**4,x)
[Out] (-10*a**2*x**2 - 15*a*b*x - 6*b**2)/(30*x**5)
```

3.1569 $\int \frac{\left(a+\frac{b}{x}\right)^2}{x^5}\,dx$

Optimal. Leaf size=30

$$-\frac{a^2}{4x^4}-\frac{2ab}{5x^5}-\frac{b^2}{6x^6}$$

```
[Out] -1/6*b^2/x^6-2/5*a*b/x^5-1/4*a^2/x^4
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$-\frac{a^2}{4x^4}-\frac{2ab}{5x^5}-\frac{b^2}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^2/x^5,x]
```

```
[Out] -b^2/(6*x^6) - (2*a*b)/(5*x^5) - a^2/(4*x^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^2}{x^5}\,dx &= \int \frac{(b+ax)^2}{x^7}\,dx\\ &= \int \left(\frac{b^2}{x^7}+\frac{2ab}{x^6}+\frac{a^2}{x^5}\right)dx\\ &= -\frac{b^2}{6x^6}-\frac{2ab}{5x^5}-\frac{a^2}{4x^4}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$-\frac{a^2}{4x^4}-\frac{2ab}{5x^5}-\frac{b^2}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^2/x^5,x]
```

```
[Out] -1/6*b^2/x^6 - (2*a*b)/(5*x^5) - a^2/(4*x^4)
```

fricas [A] time = 0.89, size = 24, normalized size = 0.80

$$-\frac{15\,a^2x^2+24\,abx+10\,b^2}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2/x^5,x, algorithm="fricas")
[Out] -1/60*(15*a^2*x^2 + 24*a*b*x + 10*b^2)/x^6
```

giac [A] time = 0.15, size = 24, normalized size = 0.80

$$-\frac{15\,a^2x^2+24\,abx+10\,b^2}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2/x^5,x, algorithm="giac")
[Out] -1/60*(15*a^2*x^2 + 24*a*b*x + 10*b^2)/x^6
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$-\frac{a^2}{4x^4}-\frac{2ab}{5x^5}-\frac{b^2}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^2/x^5,x)
[Out] -1/6*b^2/x^6-2/5*a*b/x^5-1/4*a^2/x^4
```

maxima [A] time = 0.96, size = 24, normalized size = 0.80

$$-\frac{15\,a^2x^2+24\,abx+10\,b^2}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2/x^5,x, algorithm="maxima")
[Out] -1/60*(15*a^2*x^2 + 24*a*b*x + 10*b^2)/x^6
```

mupad [B] time = 0.04, size = 24, normalized size = 0.80

$$-\frac{\frac{a^2x^2}{4}+\frac{2\,a\,b\,x}{5}+\frac{b^2}{6}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^2/x^5,x)
[Out] -(b^2/6 + (a^2*x^2)/4 + (2*a*b*x)/5)/x^6
```

sympy [A] time = 0.22, size = 26, normalized size = 0.87

$$\frac{-15a^2x^2-24abx-10b^2}{60x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**2/x**5,x)
[Out] (-15*a**2*x**2 - 24*a*b*x - 10*b**2)/(60*x**6)
```

3.1570 $\int \left(a+\frac{b}{x}\right)^3 x^6\,dx$

Optimal. Leaf size=43

$$\frac{a^3x^7}{7}+\frac{1}{2}a^2bx^6+\frac{3}{5}ab^2x^5+\frac{b^3x^4}{4}$$

```
[Out] 1/4*b^3*x^4+3/5*a*b^2*x^5+1/2*a^2*b*x^6+1/7*a^3*x^7
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$\frac{1}{2}a^2bx^6+\frac{a^3x^7}{7}+\frac{3}{5}ab^2x^5+\frac{b^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^3*x^6,x]
```

```
[Out] (b^3*x^4)/4 + (3*a*b^2*x^5)/5 + (a^2*b*x^6)/2 + (a^3*x^7)/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^3 x^6\,dx &= \int x^3(b+ax)^3\,dx\\ &= \int \left(b^3x^3+3ab^2x^4+3a^2bx^5+a^3x^6\right)dx\\ &= \frac{b^3x^4}{4}+\frac{3}{5}ab^2x^5+\frac{1}{2}a^2bx^6+\frac{a^3x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^7}{7}+\frac{1}{2}a^2bx^6+\frac{3}{5}ab^2x^5+\frac{b^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^3*x^6,x]
```

```
[Out] (b^3*x^4)/4 + (3*a*b^2*x^5)/5 + (a^2*b*x^6)/2 + (a^3*x^7)/7
```

fricas [A] time = 0.97, size = 35, normalized size = 0.81

$$\frac{1}{7}\,a^3x^7+\frac{1}{2}\,a^2bx^6+\frac{3}{5}\,ab^2x^5+\frac{1}{4}\,b^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3*x^6,x, algorithm="fricas")
[Out] 1/7*a^3*x^7 + 1/2*a^2*b*x^6 + 3/5*a*b^2*x^5 + 1/4*b^3*x^4
```

giac [A] time = 0.16, size = 35, normalized size = 0.81

$$\frac{1}{7}a^3x^7 + \frac{1}{2}a^2bx^6 + \frac{3}{5}ab^2x^5 + \frac{1}{4}b^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3*x^6,x, algorithm="giac")
[Out] 1/7*a^3*x^7 + 1/2*a^2*b*x^6 + 3/5*a*b^2*x^5 + 1/4*b^3*x^4
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$\frac{1}{7}a^3x^7 + \frac{1}{2}a^2b\,x^6 + \frac{3}{5}a\,b^2x^5 + \frac{1}{4}b^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^3*x^6,x)
[Out] 1/4*b^3*x^4+3/5*a*b^2*x^5+1/2*a^2*b*x^6+1/7*a^3*x^7
```

maxima [A] time = 1.11, size = 35, normalized size = 0.81

$$\frac{1}{7}a^3x^7 + \frac{1}{2}a^2bx^6 + \frac{3}{5}ab^2x^5 + \frac{1}{4}b^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3*x^6,x, algorithm="maxima")
[Out] 1/7*a^3*x^7 + 1/2*a^2*b*x^6 + 3/5*a*b^2*x^5 + 1/4*b^3*x^4
```

mupad [B] time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3\,x^7}{7} + \frac{a^2\,b\,x^6}{2} + \frac{3\,a\,b^2\,x^5}{5} + \frac{b^3\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b/x)^3,x)
[Out] (a^3*x^7)/7 + (b^3*x^4)/4 + (3*a*b^2*x^5)/5 + (a^2*b*x^6)/2
```

sympy [A] time = 0.08, size = 37, normalized size = 0.86

$$\frac{a^3x^7}{7} + \frac{a^2bx^6}{2} + \frac{3ab^2x^5}{5} + \frac{b^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**3*x**6,x)
[Out] a**3*x**7/7 + a**2*b*x**6/2 + 3*a*b**2*x**5/5 + b**3*x**4/4
```

3.1571 $\int \left(a+\frac{b}{x}\right)^3 x^5\,dx$

Optimal. Leaf size=43

$$\frac{a^3x^6}{6}+\frac{3}{5}a^2bx^5+\frac{3}{4}ab^2x^4+\frac{b^3x^3}{3}$$

```
[Out] 1/3*b^3*x^3+3/4*a*b^2*x^4+3/5*a^2*b*x^5+1/6*a^3*x^6
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$\frac{3}{5}a^2bx^5+\frac{a^3x^6}{6}+\frac{3}{4}ab^2x^4+\frac{b^3x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^3*x^5,x]
```

```
[Out] (b^3*x^3)/3 + (3*a*b^2*x^4)/4 + (3*a^2*b*x^5)/5 + (a^3*x^6)/6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^3 x^5\,dx &= \int x^2(b+ax)^3\,dx\\ &= \int \left(b^3x^2+3ab^2x^3+3a^2bx^4+a^3x^5\right)dx\\ &= \frac{b^3x^3}{3}+\frac{3}{4}ab^2x^4+\frac{3}{5}a^2bx^5+\frac{a^3x^6}{6}\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^6}{6}+\frac{3}{5}a^2bx^5+\frac{3}{4}ab^2x^4+\frac{b^3x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^3*x^5,x]
```

```
[Out] (b^3*x^3)/3 + (3*a*b^2*x^4)/4 + (3*a^2*b*x^5)/5 + (a^3*x^6)/6
```

fricas [A] time = 0.97, size = 35, normalized size = 0.81

$$\frac{1}{6}\,a^3x^6+\frac{3}{5}\,a^2bx^5+\frac{3}{4}\,ab^2x^4+\frac{1}{3}\,b^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3*x^5,x, algorithm="fricas")
[Out] 1/6*a^3*x^6 + 3/5*a^2*b*x^5 + 3/4*a*b^2*x^4 + 1/3*b^3*x^3
```

giac [A] time = 0.17, size = 35, normalized size = 0.81

$$\frac{1}{6}a^3x^6 + \frac{3}{5}a^2bx^5 + \frac{3}{4}ab^2x^4 + \frac{1}{3}b^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3*x^5,x, algorithm="giac")
[Out] 1/6*a^3*x^6 + 3/5*a^2*b*x^5 + 3/4*a*b^2*x^4 + 1/3*b^3*x^3
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$\frac{1}{6}a^3x^6 + \frac{3}{5}a^2b\,x^5 + \frac{3}{4}a\,b^2x^4 + \frac{1}{3}b^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^3*x^5,x)
[Out] 1/3*b^3*x^3+3/4*a*b^2*x^4+3/5*a^2*b*x^5+1/6*a^3*x^6
```

maxima [A] time = 1.11, size = 35, normalized size = 0.81

$$\frac{1}{6}a^3x^6 + \frac{3}{5}a^2bx^5 + \frac{3}{4}ab^2x^4 + \frac{1}{3}b^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3*x^5,x, algorithm="maxima")
[Out] 1/6*a^3*x^6 + 3/5*a^2*b*x^5 + 3/4*a*b^2*x^4 + 1/3*b^3*x^3
```

mupad [B] time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3\,x^6}{6} + \frac{3\,a^2\,b\,x^5}{5} + \frac{3\,a\,b^2\,x^4}{4} + \frac{b^3\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b/x)^3,x)
[Out] (a^3*x^6)/6 + (b^3*x^3)/3 + (3*a*b^2*x^4)/4 + (3*a^2*b*x^5)/5
```

sympy [A] time = 0.07, size = 39, normalized size = 0.91

$$\frac{a^3x^6}{6} + \frac{3a^2bx^5}{5} + \frac{3ab^2x^4}{4} + \frac{b^3x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**3*x**5,x)
[Out] a**3*x**6/6 + 3*a**2*b*x**5/5 + 3*a*b**2*x**4/4 + b**3*x**3/3
```

3.1572 $\int \left(a+\frac{b}{x}\right)^3 x^4\,dx$

Optimal. Leaf size=30

$$\frac{(ax+b)^5}{5a^2}-\frac{b(ax+b)^4}{4a^2}$$

```
[Out] -1/4*b*(a*x+b)^4/a^2+1/5*(a*x+b)^5/a^2
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$\frac{(ax+b)^5}{5a^2}-\frac{b(ax+b)^4}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^3*x^4,x]
```

```
[Out] -(b*(b + a*x)^4)/(4*a^2) + (b + a*x)^5/(5*a^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^3 x^4\,dx &= \int x(b+ax)^3\,dx\\ &= \int \left(-\frac{b(b+ax)^3}{a}+\frac{(b+ax)^4}{a}\right)dx\\ &= -\frac{b(b+ax)^4}{4a^2}+\frac{(b+ax)^5}{5a^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 40, normalized size = 1.33

$$\frac{a^3x^5}{5}+\frac{3}{4}a^2bx^4+ab^2x^3+\frac{b^3x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^3*x^4,x]
```

```
[Out] (b^3*x^2)/2 + a*b^2*x^3 + (3*a^2*b*x^4)/4 + (a^3*x^5)/5
```

fricas [A] time = 0.85, size = 34, normalized size = 1.13

$$\frac{1}{5}\,a^3x^5+\frac{3}{4}\,a^2bx^4+ab^2x^3+\frac{1}{2}\,b^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3*x^4,x, algorithm="fricas")
[Out] 1/5*a^3*x^5 + 3/4*a^2*b*x^4 + a*b^2*x^3 + 1/2*b^3*x^2
```

giac [A] time = 0.15, size = 34, normalized size = 1.13

$$\frac{1}{5}a^3x^5 + \frac{3}{4}a^2bx^4 + ab^2x^3 + \frac{1}{2}b^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3*x^4,x, algorithm="giac")
[Out] 1/5*a^3*x^5 + 3/4*a^2*b*x^4 + a*b^2*x^3 + 1/2*b^3*x^2
```

maple [A] time = 0.00, size = 35, normalized size = 1.17

$$\frac{1}{5}a^3x^5 + \frac{3}{4}a^2b\,x^4 + a\,b^2x^3 + \frac{1}{2}b^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^3*x^4,x)
[Out] 1/5*a^3*x^5+3/4*a^2*b*x^4+a*b^2*x^3+1/2*b^3*x^2
```

maxima [A] time = 1.15, size = 34, normalized size = 1.13

$$\frac{1}{5}a^3x^5 + \frac{3}{4}a^2bx^4 + ab^2x^3 + \frac{1}{2}b^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3*x^4,x, algorithm="maxima")
[Out] 1/5*a^3*x^5 + 3/4*a^2*b*x^4 + a*b^2*x^3 + 1/2*b^3*x^2
```

mupad [B] time = 0.04, size = 34, normalized size = 1.13

$$\frac{a^3\,x^5}{5} + \frac{3\,a^2\,b\,x^4}{4} + a\,b^2\,x^3 + \frac{b^3\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b/x)^3,x)
[Out] (a^3*x^5)/5 + (b^3*x^2)/2 + a*b^2*x^3 + (3*a^2*b*x^4)/4
```

sympy [A] time = 0.08, size = 36, normalized size = 1.20

$$\frac{a^3x^5}{5} + \frac{3a^2bx^4}{4} + ab^2x^3 + \frac{b^3x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**3*x**4,x)
[Out] a**3*x**5/5 + 3*a**2*b*x**4/4 + a*b**2*x**3 + b**3*x**2/2
```

3.1573 $\int \left(a + \frac{b}{x}\right)^3 x^3\, dx$

Optimal. Leaf size=14

$$\frac{(ax+b)^4}{4a}$$

```
[Out] 1/4*(a*x+b)^4/a
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 32}

$$\frac{(ax+b)^4}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^3*x^3,x]
```

```
[Out] (b + a*x)^4/(4*a)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(a + \frac{b}{x}\right)^3 x^3\, dx &= \int (b+ax)^3\, dx \\ &= \frac{(b+ax)^4}{4a}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{(ax+b)^4}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^3*x^3,x]
```

```
[Out] (b + a*x)^4/(4*a)
```

fricas [B] time = 1.13, size = 31, normalized size = 2.21

$$\frac{1}{4}a^3x^4 + a^2bx^3 + \frac{3}{2}ab^2x^2 + b^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3*x^3,x, algorithm="fricas")
```

```
[Out] 1/4*a^3*x^4 + a^2*b*x^3 + 3/2*a*b^2*x^2 + b^3*x
```

giac [**B**] time = 0.15, size = 31, normalized size = 2.21

$$\frac{1}{4}a^3x^4 + a^2bx^3 + \frac{3}{2}ab^2x^2 + b^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3*x^3,x, algorithm="giac")
[Out] 1/4*a^3*x^4 + a^2*b*x^3 + 3/2*a*b^2*x^2 + b^3*x
```

maple [**A**] time = 0.00, size = 13, normalized size = 0.93

$$\frac{(ax+b)^4}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^3*x^3,x)
[Out] 1/4*(a*x+b)^4/a
```

maxima [**B**] time = 1.10, size = 31, normalized size = 2.21

$$\frac{1}{4}a^3x^4 + a^2bx^3 + \frac{3}{2}ab^2x^2 + b^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3*x^3,x, algorithm="maxima")
[Out] 1/4*a^3*x^4 + a^2*b*x^3 + 3/2*a*b^2*x^2 + b^3*x
```

mupad [**B**] time = 0.04, size = 31, normalized size = 2.21

$$\frac{a^3\,x^4}{4} + a^2\,b\,x^3 + \frac{3\,a\,b^2\,x^2}{2} + b^3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b/x)^3,x)
[Out] b^3*x + (a^3*x^4)/4 + (3*a*b^2*x^2)/2 + a^2*b*x^3
```

sympy [**B**] time = 0.07, size = 32, normalized size = 2.29

$$\frac{a^3x^4}{4} + a^2bx^3 + \frac{3ab^2x^2}{2} + b^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**3*x**3,x)
[Out] a**3*x**4/4 + a**2*b*x**3 + 3*a*b**2*x**2/2 + b**3*x
```

3.1574 $\int \left(a+\frac{b}{x}\right)^3 x^2\,dx$

Optimal. Leaf size=35

$$\frac{a^3x^3}{3}+\frac{3}{2}a^2bx^2+3ab^2x+b^3\log(x)$$

```
[Out] 3*a*b^2*x+3/2*a^2*b*x^2+1/3*a^3*x^3+b^3*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$\frac{3}{2}a^2bx^2+\frac{a^3x^3}{3}+3ab^2x+b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^3*x^2,x]
```

```
[Out] 3*a*b^2*x + (3*a^2*b*x^2)/2 + (a^3*x^3)/3 + b^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^3 x^2\,dx &= \int \frac{(b+ax)^3}{x}\,dx\\ &= \int \left(3ab^2+\frac{b^3}{x}+3a^2bx+a^3x^2\right)dx\\ &= 3ab^2x+\frac{3}{2}a^2bx^2+\frac{a^3x^3}{3}+b^3\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 35, normalized size = 1.00

$$\frac{a^3x^3}{3}+\frac{3}{2}a^2bx^2+3ab^2x+b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^3*x^2,x]
```

```
[Out] 3*a*b^2*x + (3*a^2*b*x^2)/2 + (a^3*x^3)/3 + b^3*Log[x]
```

fricas [A] time = 1.03, size = 31, normalized size = 0.89

$$\frac{1}{3}\,a^3x^3+\frac{3}{2}\,a^2bx^2+3\,ab^2x+b^3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3*x^2,x, algorithm="fricas")
[Out] 1/3*a^3*x^3 + 3/2*a^2*b*x^2 + 3*a*b^2*x + b^3*log(x)
```

giac [A] time = 0.15, size = 32, normalized size = 0.91

$$\frac{1}{3}\,a^3x^3 + \frac{3}{2}\,a^2bx^2 + 3\,ab^2x + b^3\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3*x^2,x, algorithm="giac")
[Out] 1/3*a^3*x^3 + 3/2*a^2*b*x^2 + 3*a*b^2*x + b^3*log(abs(x))
```

maple [A] time = 0.00, size = 32, normalized size = 0.91

$$\frac{a^3x^3}{3} + \frac{3a^2b\,x^2}{2} + 3a\,b^2x + b^3\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^3*x^2,x)
[Out] 3*a*b^2*x+3/2*a^2*b*x^2+1/3*a^3*x^3+b^3*ln(x)
```

maxima [A] time = 1.00, size = 31, normalized size = 0.89

$$\frac{1}{3}\,a^3x^3 + \frac{3}{2}\,a^2bx^2 + 3\,ab^2x + b^3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3*x^2,x, algorithm="maxima")
[Out] 1/3*a^3*x^3 + 3/2*a^2*b*x^2 + 3*a*b^2*x + b^3*log(x)
```

mupad [B] time = 0.04, size = 31, normalized size = 0.89

$$b^3\,\ln(x) + \frac{a^3\,x^3}{3} + \frac{3\,a^2\,b\,x^2}{2} + 3\,a\,b^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b/x)^3,x)
[Out] b^3*log(x) + (a^3*x^3)/3 + (3*a^2*b*x^2)/2 + 3*a*b^2*x
```

sympy [A] time = 0.12, size = 34, normalized size = 0.97

$$\frac{a^3x^3}{3} + \frac{3a^2bx^2}{2} + 3ab^2x + b^3\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**3*x**2,x)
[Out] a**3*x**3/3 + 3*a**2*b*x**2/2 + 3*a*b**2*x + b**3*log(x)
```

3.1575 $\int \left(a+\frac{b}{x}\right)^3 x\,dx$

Optimal. Leaf size=34

$$\frac{a^3x^2}{2}+3a^2bx+3ab^2\log(x)-\frac{b^3}{x}$$

```
[Out] -b^3/x+3*a^2*b*x+1/2*a^3*x^2+3*a*b^2*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {263, 43}

$$3a^2bx+\frac{a^3x^2}{2}+3ab^2\log(x)-\frac{b^3}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^3*x,x]
```

```
[Out] -(b^3/x) + 3*a^2*b*x + (a^3*x^2)/2 + 3*a*b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^3 x\,dx &= \int \frac{(b+ax)^3}{x^2}\,dx\\ &= \int \left(3a^2b+\frac{b^3}{x^2}+\frac{3ab^2}{x}+a^3x\right)dx\\ &= -\frac{b^3}{x}+3a^2bx+\frac{a^3x^2}{2}+3ab^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 34, normalized size = 1.00

$$\frac{a^3x^2}{2}+3a^2bx+3ab^2\log(x)-\frac{b^3}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^3*x,x]
```

```
[Out] -(b^3/x) + 3*a^2*b*x + (a^3*x^2)/2 + 3*a*b^2*Log[x]
```

fricas [A] time = 0.89, size = 36, normalized size = 1.06

$$\frac{a^3x^3+6\,a^2bx^2+6\,ab^2x\log(x)-2\,b^3}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3*x,x, algorithm="fricas")
[Out] 1/2*(a^3*x^3 + 6*a^2*b*x^2 + 6*a*b^2*x*log(x) - 2*b^3)/x
```

giac [A] time = 0.15, size = 33, normalized size = 0.97

$$\frac{1}{2}\,a^3x^2 + 3\,a^2bx + 3\,ab^2\log\left(|x|\right) - \frac{b^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3*x,x, algorithm="giac")
[Out] 1/2*a^3*x^2 + 3*a^2*b*x + 3*a*b^2*log(abs(x)) - b^3/x
```

maple [A] time = 0.01, size = 33, normalized size = 0.97

$$\frac{a^3x^2}{2} + 3a^2bx + 3a\,b^2\ln(x) - \frac{b^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^3*x,x)
[Out] -b^3/x+3*a^2*b*x+1/2*a^3*x^2+3*a*b^2*ln(x)
```

maxima [A] time = 1.04, size = 32, normalized size = 0.94

$$\frac{1}{2}\,a^3x^2 + 3\,a^2bx + 3\,ab^2\log(x) - \frac{b^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3*x,x, algorithm="maxima")
[Out] 1/2*a^3*x^2 + 3*a^2*b*x + 3*a*b^2*log(x) - b^3/x
```

mupad [B] time = 0.04, size = 32, normalized size = 0.94

$$\frac{a^3\,x^2}{2} - \frac{b^3}{x} + 3\,a\,b^2\,\ln(x) + 3\,a^2\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b/x)^3,x)
[Out] (a^3*x^2)/2 - b^3/x + 3*a*b^2*log(x) + 3*a^2*b*x
```

sympy [A] time = 0.14, size = 31, normalized size = 0.91

$$\frac{a^3x^2}{2} + 3a^2bx + 3ab^2\log\left(x\right) - \frac{b^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**3*x,x)
[Out] a**3*x**2/2 + 3*a**2*b*x + 3*a*b**2*log(x) - b**3/x
```

3.1576 $\int \left(a+\frac{b}{x}\right)^3 dx$

Optimal. Leaf size=33

$$a^3x+3a^2b\log(x)-\frac{3ab^2}{x}-\frac{b^3}{2x^2}$$

```
[Out] -1/2*b^3/x^2-3*a*b^2/x+a^3*x+3*a^2*b*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {193, 43}

$$3a^2b\log(x)+a^3x-\frac{3ab^2}{x}-\frac{b^3}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^3,x]
```

```
[Out] -b^3/(2*x^2) - (3*a*b^2)/x + a^3*x + 3*a^2*b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^3 dx &= \int \frac{(b+ax)^3}{x^3}\,dx\\ &= \int \left(a^3+\frac{b^3}{x^3}+\frac{3ab^2}{x^2}+\frac{3a^2b}{x}\right)dx\\ &= -\frac{b^3}{2x^2}-\frac{3ab^2}{x}+a^3x+3a^2b\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 33, normalized size = 1.00

$$a^3x+3a^2b\log(x)-\frac{3ab^2}{x}-\frac{b^3}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^3,x]
```

```
[Out] -1/2*b^3/x^2 - (3*a*b^2)/x + a^3*x + 3*a^2*b*Log[x]
```

fricas [A] time = 0.91, size = 37, normalized size = 1.12

$$\frac{2\,a^3x^3+6\,a^2bx^2\log(x)-6\,ab^2x-b^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3,x, algorithm="fricas")
[Out] 1/2*(2*a^3*x^3 + 6*a^2*b*x^2*log(x) - 6*a*b^2*x - b^3)/x^2
```

giac [**A**] time = 0.15, size = 31, normalized size = 0.94

$$a^3x + 3\,a^2b\log\left(|x|\right) - \frac{6\,ab^2x + b^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3,x, algorithm="giac")
[Out] a^3*x + 3*a^2*b*log(abs(x)) - 1/2*(6*a*b^2*x + b^3)/x^2
```

maple [**A**] time = 0.00, size = 32, normalized size = 0.97

$$a^3x + 3a^2b\ln(x) - \frac{3a\,b^2}{x} - \frac{b^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^3,x)
[Out] -1/2*b^3/x^2-3*a*b^2/x+a^3*x+3*a^2*b*ln(x)
```

maxima [**A**] time = 1.03, size = 31, normalized size = 0.94

$$a^3x + 3\,a^2b\log(x) - \frac{3\,ab^2}{x} - \frac{b^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3,x, algorithm="maxima")
[Out] a^3*x + 3*a^2*b*log(x) - 3*a*b^2/x - 1/2*b^3/x^2
```

mupad [**B**] time = 0.03, size = 32, normalized size = 0.97

$$a^3\,x - \frac{\frac{b^3}{2} + 3\,a\,x\,b^2}{x^2} + 3\,a^2\,b\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^3,x)
[Out] a^3*x - (b^3/2 + 3*a*b^2*x)/x^2 + 3*a^2*b*log(x)
```

sympy [**A**] time = 0.18, size = 32, normalized size = 0.97

$$a^3x + 3a^2b\log\left(x\right) + \frac{-6ab^2x - b^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**3,x)
[Out] a**3*x + 3*a**2*b*log(x) + (-6*a*b**2*x - b**3)/(2*x**2)
```

3.1577 $\int \frac{\left(a+\frac{b}{x}\right)^3}{x}\,dx$

Optimal. Leaf size=37

$$a^3\log(x)-\frac{3a^2b}{x}-\frac{3ab^2}{2x^2}-\frac{b^3}{3x^3}$$

```
[Out] -1/3*b^3/x^3-3/2*a*b^2/x^2-3*a^2*b/x+a^3*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$-\frac{3a^2b}{x}+a^3\log(x)-\frac{3ab^2}{2x^2}-\frac{b^3}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^3/x,x]
```

```
[Out] -b^3/(3*x^3) - (3*a*b^2)/(2*x^2) - (3*a^2*b)/x + a^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^3}{x}\,dx &= \int \frac{(b+ax)^3}{x^4}\,dx\\ &= \int\left(\frac{b^3}{x^4}+\frac{3ab^2}{x^3}+\frac{3a^2b}{x^2}+\frac{a^3}{x}\right)dx\\ &= -\frac{b^3}{3x^3}-\frac{3ab^2}{2x^2}-\frac{3a^2b}{x}+a^3\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 37, normalized size = 1.00

$$a^3\log(x)-\frac{3a^2b}{x}-\frac{3ab^2}{2x^2}-\frac{b^3}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^3/x,x]
```

```
[Out] -1/3*b^3/x^3 - (3*a*b^2)/(2*x^2) - (3*a^2*b)/x + a^3*Log[x]
```

fricas [A] time = 0.94, size = 37, normalized size = 1.00

$$\frac{6\,a^3x^3\log(x)-18\,a^2bx^2-9\,ab^2x-2\,b^3}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3/x,x, algorithm="fricas")
[Out] 1/6*(6*a^3*x^3*log(x) - 18*a^2*b*x^2 - 9*a*b^2*x - 2*b^3)/x^3
```

giac [A] time = 0.19, size = 35, normalized size = 0.95

$$a^3 \log(|x|) - \frac{18\,a^2bx^2 + 9\,ab^2x + 2\,b^3}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3/x,x, algorithm="giac")
[Out] a^3*log(abs(x)) - 1/6*(18*a^2*b*x^2 + 9*a*b^2*x + 2*b^3)/x^3
```

maple [A] time = 0.01, size = 34, normalized size = 0.92

$$a^3 \ln(x) - \frac{3a^2b}{x} - \frac{3a\,b^2}{2x^2} - \frac{b^3}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^3/x,x)
[Out] -1/3*b^3/x^3-3/2*a*b^2/x^2-3*a^2*b/x+a^3*ln(x)
```

maxima [A] time = 1.01, size = 34, normalized size = 0.92

$$a^3 \log(x) - \frac{18\,a^2bx^2 + 9\,ab^2x + 2\,b^3}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3/x,x, algorithm="maxima")
[Out] a^3*log(x) - 1/6*(18*a^2*b*x^2 + 9*a*b^2*x + 2*b^3)/x^3
```

mupad [B] time = 0.04, size = 34, normalized size = 0.92

$$a^3\,\ln(x) - \frac{3\,a^2\,b\,x^2 + \frac{3\,a\,b^2\,x}{2} + \frac{b^3}{3}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^3/x,x)
[Out] a^3*log(x) - (b^3/3 + 3*a^2*b*x^2 + (3*a*b^2*x)/2)/x^3
```

sympy [A] time = 0.21, size = 36, normalized size = 0.97

$$a^3 \log(x) + \frac{-18a^2bx^2 - 9ab^2x - 2b^3}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**3/x,x)
[Out] a**3*log(x) + (-18*a**2*b*x**2 - 9*a*b**2*x - 2*b**3)/(6*x**3)
```

3.1578 $\int \frac{\left(a+\frac{b}{x}\right)^3}{x^2}\,dx$

Optimal. Leaf size=16

$$-\frac{\left(a+\frac{b}{x}\right)^4}{4b}$$

```
[Out] -1/4*(a+b/x)^4/b
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$-\frac{\left(a+\frac{b}{x}\right)^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^3/x^2,x]
```

```
[Out] -(a + b/x)^4/(4*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^2}\,dx = -\frac{\left(a+\frac{b}{x}\right)^4}{4b}$$

Mathematica [B] time = 0.01, size = 39, normalized size = 2.44

$$-\frac{a^3}{x} - \frac{3a^2b}{2x^2} - \frac{ab^2}{x^3} - \frac{b^3}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^3/x^2,x]
```

```
[Out] -1/4*b^3/x^4 - (a*b^2)/x^3 - (3*a^2*b)/(2*x^2) - a^3/x
```

fricas [B] time = 1.07, size = 33, normalized size = 2.06

$$-\frac{4\,a^3x^3 + 6\,a^2bx^2 + 4\,ab^2x + b^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*a^3*x^3 + 6*a^2*b*x^2 + 4*a*b^2*x + b^3)/x^4
```

giac [A] time = 0.15, size = 14, normalized size = 0.88

$$-\frac{\left(a+\frac{b}{x}\right)^4}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3/x^2,x, algorithm="giac")
[Out] -1/4*(a + b/x)^4/b
```

maple [B] time = 0.00, size = 36, normalized size = 2.25

$$-\frac{a^3}{x}-\frac{3a^2b}{2x^2}-\frac{a\,b^2}{x^3}-\frac{b^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^3/x^2,x)
[Out] -1/4*b^3/x^4-a*b^2/x^3-3/2*a^2*b/x^2-a^3/x
```

maxima [A] time = 1.02, size = 14, normalized size = 0.88

$$-\frac{\left(a+\frac{b}{x}\right)^4}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3/x^2,x, algorithm="maxima")
[Out] -1/4*(a + b/x)^4/b
```

mupad [B] time = 0.03, size = 33, normalized size = 2.06

$$-\frac{a^3\,x^3+\frac{3\,a^2\,b\,x^2}{2}+a\,b^2\,x+\frac{b^3}{4}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^3/x^2,x)
[Out] -(b^3/4 + a^3*x^3 + (3*a^2*b*x^2)/2 + a*b^2*x)/x^4
```

sympy [B] time = 0.22, size = 36, normalized size = 2.25

$$\frac{-4a^3x^3-6a^2bx^2-4ab^2x-b^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**3/x**2,x)
[Out] (-4*a**3*x**3 - 6*a**2*b*x**2 - 4*a*b**2*x - b**3)/(4*x**4)
```

3.1579 $\int \frac{\left(a+\frac{b}{x}\right)^3}{x^3}\,dx$

Optimal. Leaf size=36

$$\frac{a(ax+b)^4}{20b^2x^4}-\frac{(ax+b)^4}{5bx^5}$$

```
[Out] -1/5*(a*x+b)^4/b/x^5+1/20*a*(a*x+b)^4/b^2/x^4
```

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 45, 37}

$$\frac{a(ax+b)^4}{20b^2x^4}-\frac{(ax+b)^4}{5bx^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^3/x^3,x]
```

```
[Out] -(b + a*x)^4/(5*b*x^5) + (a*(b + a*x)^4)/(20*b^2*x^4)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^3}{x^3}\,dx &= \int \frac{(b+ax)^3}{x^6}\,dx\\ &= -\frac{(b+ax)^4}{5bx^5}-\frac{a\int \frac{(b+ax)^3}{x^5}\,dx}{5b}\\ &= -\frac{(b+ax)^4}{5bx^5}+\frac{a(b+ax)^4}{20b^2x^4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.14

$$-\frac{a^3}{2x^2}-\frac{a^2b}{x^3}-\frac{3ab^2}{4x^4}-\frac{b^3}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^3/x^3,x]
[Out] -1/5*b^3/x^5 - (3*a*b^2)/(4*x^4) - (a^2*b)/x^3 - a^3/(2*x^2)
```

fricas [A] time = 1.14, size = 35, normalized size = 0.97

$$-\frac{10\,a^3x^3 + 20\,a^2bx^2 + 15\,ab^2x + 4\,b^3}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3/x^3,x, algorithm="fricas")
[Out] -1/20*(10*a^3*x^3 + 20*a^2*b*x^2 + 15*a*b^2*x + 4*b^3)/x^5
```

giac [A] time = 0.15, size = 35, normalized size = 0.97

$$-\frac{10\,a^3x^3 + 20\,a^2bx^2 + 15\,ab^2x + 4\,b^3}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3/x^3,x, algorithm="giac")
[Out] -1/20*(10*a^3*x^3 + 20*a^2*b*x^2 + 15*a*b^2*x + 4*b^3)/x^5
```

maple [A] time = 0.00, size = 36, normalized size = 1.00

$$-\frac{a^3}{2x^2} - \frac{a^2b}{x^3} - \frac{3a\,b^2}{4x^4} - \frac{b^3}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^3/x^3,x)
[Out] -1/5*b^3/x^5-3/4*a*b^2/x^4-a^2*b/x^3-1/2*a^3/x^2
```

maxima [A] time = 1.04, size = 35, normalized size = 0.97

$$-\frac{10\,a^3x^3 + 20\,a^2bx^2 + 15\,ab^2x + 4\,b^3}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3/x^3,x, algorithm="maxima")
[Out] -1/20*(10*a^3*x^3 + 20*a^2*b*x^2 + 15*a*b^2*x + 4*b^3)/x^5
```

mupad [B] time = 0.03, size = 34, normalized size = 0.94

$$-\frac{\frac{a^3x^3}{2} + a^2\,b\,x^2 + \frac{3\,a\,b^2\,x}{4} + \frac{b^3}{5}}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^3/x^3,x)
[Out] -(b^3/5 + (a^3*x^3)/2 + a^2*b*x^2 + (3*a*b^2*x)/4)/x^5
```

sympy [A] time = 0.24, size = 37, normalized size = 1.03

$$\frac{-10a^3x^3 - 20a^2bx^2 - 15ab^2x - 4b^3}{20x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**3/x**3,x)
[Out] (-10*a**3*x**3 - 20*a**2*b*x**2 - 15*a*b**2*x - 4*b**3)/(20*x**5)
```

3.1580 $\int \frac{\left(a+\frac{b}{x}\right)^3}{x^4}\,dx$

Optimal. Leaf size=43

$$-\frac{a^3}{3x^3}-\frac{3a^2b}{4x^4}-\frac{3ab^2}{5x^5}-\frac{b^3}{6x^6}$$

```
[Out] -1/6*b^3/x^6-3/5*a*b^2/x^5-3/4*a^2*b/x^4-1/3*a^3/x^3
```

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$-\frac{3a^2b}{4x^4}-\frac{a^3}{3x^3}-\frac{3ab^2}{5x^5}-\frac{b^3}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^3/x^4,x]
```

```
[Out] -b^3/(6*x^6) - (3*a*b^2)/(5*x^5) - (3*a^2*b)/(4*x^4) - a^3/(3*x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^3}{x^4}\,dx &= \int \frac{(b+ax)^3}{x^7}\,dx\\ &= \int \left(\frac{b^3}{x^7}+\frac{3ab^2}{x^6}+\frac{3a^2b}{x^5}+\frac{a^3}{x^4}\right)dx\\ &= -\frac{b^3}{6x^6}-\frac{3ab^2}{5x^5}-\frac{3a^2b}{4x^4}-\frac{a^3}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$-\frac{a^3}{3x^3}-\frac{3a^2b}{4x^4}-\frac{3ab^2}{5x^5}-\frac{b^3}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^3/x^4,x]
```

```
[Out] -1/6*b^3/x^6 - (3*a*b^2)/(5*x^5) - (3*a^2*b)/(4*x^4) - a^3/(3*x^3)
```

fricas [A] time = 1.00, size = 35, normalized size = 0.81

$$-\frac{20\,a^3x^3+45\,a^2bx^2+36\,ab^2x+10\,b^3}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3/x^4,x, algorithm="fricas")
[Out] -1/60*(20*a^3*x^3 + 45*a^2*b*x^2 + 36*a*b^2*x + 10*b^3)/x^6
```

giac [A] time = 0.15, size = 35, normalized size = 0.81

$$-\frac{20\,a^3x^3+45\,a^2bx^2+36\,ab^2x+10\,b^3}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3/x^4,x, algorithm="giac")
[Out] -1/60*(20*a^3*x^3 + 45*a^2*b*x^2 + 36*a*b^2*x + 10*b^3)/x^6
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$-\frac{a^3}{3x^3}-\frac{3a^2b}{4x^4}-\frac{3a\,b^2}{5x^5}-\frac{b^3}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^3/x^4,x)
[Out] -1/6*b^3/x^6-3/5*a*b^2/x^5-3/4*a^2*b/x^4-1/3*a^3/x^3
```

maxima [A] time = 0.98, size = 35, normalized size = 0.81

$$-\frac{20\,a^3x^3+45\,a^2bx^2+36\,ab^2x+10\,b^3}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3/x^4,x, algorithm="maxima")
[Out] -1/60*(20*a^3*x^3 + 45*a^2*b*x^2 + 36*a*b^2*x + 10*b^3)/x^6
```

mupad [B] time = 0.03, size = 35, normalized size = 0.81

$$-\frac{\frac{a^3x^3}{3}+\frac{3\,a^2\,b\,x^2}{4}+\frac{3\,a\,b^2\,x}{5}+\frac{b^3}{6}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^3/x^4,x)
[Out] -(b^3/6 + (a^3*x^3)/3 + (3*a^2*b*x^2)/4 + (3*a*b^2*x)/5)/x^6
```

sympy [A] time = 0.26, size = 37, normalized size = 0.86

$$\frac{-20a^3x^3-45a^2bx^2-36ab^2x-10b^3}{60x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**3/x**4,x)
[Out] (-20*a**3*x**3 - 45*a**2*b*x**2 - 36*a*b**2*x - 10*b**3)/(60*x**6)
```

3.1581 $\int \frac{\left(a+\frac{b}{x}\right)^3}{x^5}\,dx$

Optimal. Leaf size=43

$$-\frac{a^3}{4x^4}-\frac{3a^2b}{5x^5}-\frac{ab^2}{2x^6}-\frac{b^3}{7x^7}$$

```
[Out] -1/7*b^3/x^7-1/2*a*b^2/x^6-3/5*a^2*b/x^5-1/4*a^3/x^4
```

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$-\frac{3a^2b}{5x^5}-\frac{a^3}{4x^4}-\frac{ab^2}{2x^6}-\frac{b^3}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^3/x^5,x]
```

```
[Out] -b^3/(7*x^7) - (a*b^2)/(2*x^6) - (3*a^2*b)/(5*x^5) - a^3/(4*x^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^3}{x^5}\,dx &= \int \frac{(b+ax)^3}{x^8}\,dx\\ &= \int \left(\frac{b^3}{x^8}+\frac{3ab^2}{x^7}+\frac{3a^2b}{x^6}+\frac{a^3}{x^5}\right)dx\\ &= -\frac{b^3}{7x^7}-\frac{ab^2}{2x^6}-\frac{3a^2b}{5x^5}-\frac{a^3}{4x^4}\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$-\frac{a^3}{4x^4}-\frac{3a^2b}{5x^5}-\frac{ab^2}{2x^6}-\frac{b^3}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^3/x^5,x]
```

```
[Out] -1/7*b^3/x^7 - (a*b^2)/(2*x^6) - (3*a^2*b)/(5*x^5) - a^3/(4*x^4)
```

fricas [A] time = 0.62, size = 35, normalized size = 0.81

$$-\frac{35\,a^3x^3+84\,a^2bx^2+70\,ab^2x+20\,b^3}{140\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3/x^5,x, algorithm="fricas")
[Out] -1/140*(35*a^3*x^3 + 84*a^2*b*x^2 + 70*a*b^2*x + 20*b^3)/x^7
```

giac [A] time = 0.15, size = 35, normalized size = 0.81

$$-\frac{35\,a^3x^3+84\,a^2bx^2+70\,ab^2x+20\,b^3}{140\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3/x^5,x, algorithm="giac")
[Out] -1/140*(35*a^3*x^3 + 84*a^2*b*x^2 + 70*a*b^2*x + 20*b^3)/x^7
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$-\frac{a^3}{4x^4}-\frac{3a^2b}{5x^5}-\frac{a\,b^2}{2x^6}-\frac{b^3}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^3/x^5,x)
[Out] -1/7*b^3/x^7-1/2*a*b^2/x^6-3/5*a^2*b/x^5-1/4*a^3/x^4
```

maxima [A] time = 1.03, size = 35, normalized size = 0.81

$$-\frac{35\,a^3x^3+84\,a^2bx^2+70\,ab^2x+20\,b^3}{140\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3/x^5,x, algorithm="maxima")
[Out] -1/140*(35*a^3*x^3 + 84*a^2*b*x^2 + 70*a*b^2*x + 20*b^3)/x^7
```

mupad [B] time = 0.03, size = 35, normalized size = 0.81

$$-\frac{\frac{a^3x^3}{4}+\frac{3a^2bx^2}{5}+\frac{ab^2x}{2}+\frac{b^3}{7}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^3/x^5,x)
[Out] -(b^3/7 + (a^3*x^3)/4 + (3*a^2*b*x^2)/5 + (a*b^2*x)/2)/x^7
```

sympy [A] time = 0.28, size = 37, normalized size = 0.86

$$\frac{-35a^3x^3-84a^2bx^2-70ab^2x-20b^3}{140x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**3/x**5,x)
[Out] (-35*a**3*x**3 - 84*a**2*b*x**2 - 70*a*b**2*x - 20*b**3)/(140*x**7)
```

3.1582 $\int \frac{\left(a+\frac{b}{x}\right)^3}{x^6}\,dx$

Optimal. Leaf size=43

$$-\frac{a^3}{5x^5}-\frac{a^2b}{2x^6}-\frac{3ab^2}{7x^7}-\frac{b^3}{8x^8}$$

```
[Out] -1/8*b^3/x^8-3/7*a*b^2/x^7-1/2*a^2*b/x^6-1/5*a^3/x^5
```

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$-\frac{a^2b}{2x^6}-\frac{a^3}{5x^5}-\frac{3ab^2}{7x^7}-\frac{b^3}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^3/x^6,x]
```

```
[Out] -b^3/(8*x^8) - (3*a*b^2)/(7*x^7) - (a^2*b)/(2*x^6) - a^3/(5*x^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^3}{x^6}\,dx &= \int \frac{(b+ax)^3}{x^9}\,dx\\ &= \int \left(\frac{b^3}{x^9}+\frac{3ab^2}{x^8}+\frac{3a^2b}{x^7}+\frac{a^3}{x^6}\right)dx\\ &= -\frac{b^3}{8x^8}-\frac{3ab^2}{7x^7}-\frac{a^2b}{2x^6}-\frac{a^3}{5x^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 1.00

$$-\frac{a^3}{5x^5}-\frac{a^2b}{2x^6}-\frac{3ab^2}{7x^7}-\frac{b^3}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^3/x^6,x]
```

```
[Out] -1/8*b^3/x^8 - (3*a*b^2)/(7*x^7) - (a^2*b)/(2*x^6) - a^3/(5*x^5)
```

fricas [A] time = 1.01, size = 35, normalized size = 0.81

$$-\frac{56\,a^3x^3+140\,a^2bx^2+120\,ab^2x+35\,b^3}{280\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3/x^6,x, algorithm="fricas")
[Out] -1/280*(56*a^3*x^3 + 140*a^2*b*x^2 + 120*a*b^2*x + 35*b^3)/x^8
```

giac [A] time = 0.15, size = 35, normalized size = 0.81

$$-\frac{56\,a^3x^3+140\,a^2bx^2+120\,ab^2x+35\,b^3}{280\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3/x^6,x, algorithm="giac")
[Out] -1/280*(56*a^3*x^3 + 140*a^2*b*x^2 + 120*a*b^2*x + 35*b^3)/x^8
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$-\frac{a^3}{5x^5}-\frac{a^2b}{2x^6}-\frac{3a\,b^2}{7x^7}-\frac{b^3}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^3/x^6,x)
[Out] -1/8*b^3/x^8-3/7*a*b^2/x^7-1/2*a^2*b/x^6-1/5*a^3/x^5
```

maxima [A] time = 1.03, size = 35, normalized size = 0.81

$$-\frac{56\,a^3x^3+140\,a^2bx^2+120\,ab^2x+35\,b^3}{280\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3/x^6,x, algorithm="maxima")
[Out] -1/280*(56*a^3*x^3 + 140*a^2*b*x^2 + 120*a*b^2*x + 35*b^3)/x^8
```

mupad [B] time = 0.03, size = 35, normalized size = 0.81

$$-\frac{\frac{a^3x^3}{5}+\frac{a^2bx^2}{2}+\frac{3ab^2x}{7}+\frac{b^3}{8}}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^3/x^6,x)
[Out] -(b^3/8 + (a^3*x^3)/5 + (a^2*b*x^2)/2 + (3*a*b^2*x)/7)/x^8
```

sympy [A] time = 0.32, size = 37, normalized size = 0.86

$$\frac{-56a^3x^3-140a^2bx^2-120ab^2x-35b^3}{280x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**3/x**6,x)
[Out] (-56*a**3*x**3 - 140*a**2*b*x**2 - 120*a*b**2*x - 35*b**3)/(280*x**8)
```

3.1583 $\int \left(a+\frac{b}{x}\right)^8 x^{16}\,dx$

Optimal. Leaf size=106

$$\frac{a^8x^{17}}{17}+\frac{1}{2}a^7bx^{16}+\frac{28}{15}a^6b^2x^{15}+4a^5b^3x^{14}+\frac{70}{13}a^4b^4x^{13}+\frac{14}{3}a^3b^5x^{12}+\frac{28}{11}a^2b^6x^{11}+\frac{4}{5}ab^7x^{10}+\frac{b^8x^9}{9}$$

```
[Out] 1/9*b^8*x^9+4/5*a*b^7*x^10+28/11*a^2*b^6*x^11+14/3*a^3*b^5*x^12+70/13*a^4*b
^4*x^13+4*a^5*b^3*x^14+28/15*a^6*b^2*x^15+1/2*a^7*b*x^16+1/17*a^8*x^17
```

Rubi [A] time = 0.06, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$\frac{28}{15}a^6b^2x^{15}+4a^5b^3x^{14}+\frac{70}{13}a^4b^4x^{13}+\frac{14}{3}a^3b^5x^{12}+\frac{28}{11}a^2b^6x^{11}+\frac{1}{2}a^7bx^{16}+\frac{a^8x^{17}}{17}+\frac{4}{5}ab^7x^{10}+\frac{b^8x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^8*x^16,x]
```

```
[Out] (b^8*x^9)/9 + (4*a*b^7*x^10)/5 + (28*a^2*b^6*x^11)/11 + (14*a^3*b^5*x^12)/3
 + (70*a^4*b^4*x^13)/13 + 4*a^5*b^3*x^14 + (28*a^6*b^2*x^15)/15 + (a^7*b*x^
16)/2 + (a^8*x^17)/17
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^8 x^{16}\,dx &= \int x^8(b+ax)^8\,dx\\ &= \int \left(b^8x^8+8ab^7x^9+28a^2b^6x^{10}+56a^3b^5x^{11}+70a^4b^4x^{12}+56a^5b^3x^{13}+28a^6b^2x^{14}+8a^7\right.\\ &= \frac{b^8x^9}{9}+\frac{4}{5}ab^7x^{10}+\frac{28}{11}a^2b^6x^{11}+\frac{14}{3}a^3b^5x^{12}+\frac{70}{13}a^4b^4x^{13}+4a^5b^3x^{14}+\frac{28}{15}a^6b^2x^{15}+\frac{1}{2}a^7b\end{aligned}$$

Mathematica [A] time = 0.00, size = 106, normalized size = 1.00

$$\frac{a^8x^{17}}{17}+\frac{1}{2}a^7bx^{16}+\frac{28}{15}a^6b^2x^{15}+4a^5b^3x^{14}+\frac{70}{13}a^4b^4x^{13}+\frac{14}{3}a^3b^5x^{12}+\frac{28}{11}a^2b^6x^{11}+\frac{4}{5}ab^7x^{10}+\frac{b^8x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^8*x^16,x]
```

```
[Out] (b^8*x^9)/9 + (4*a*b^7*x^10)/5 + (28*a^2*b^6*x^11)/11 + (14*a^3*b^5*x^12)/3
 + (70*a^4*b^4*x^13)/13 + 4*a^5*b^3*x^14 + (28*a^6*b^2*x^15)/15 + (a^7*b*x^
16)/2 + (a^8*x^17)/17
```

fricas [A] time = 0.67, size = 90, normalized size = 0.85

$$\frac{1}{17}a^8x^{17}+\frac{1}{2}a^7bx^{16}+\frac{28}{15}a^6b^2x^{15}+4a^5b^3x^{14}+\frac{70}{13}a^4b^4x^{13}+\frac{14}{3}a^3b^5x^{12}+\frac{28}{11}a^2b^6x^{11}+\frac{4}{5}ab^7x^{10}+\frac{1}{9}b^8x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^16,x, algorithm="fricas")
```

```
[Out] 1/17*a^8*x^17 + 1/2*a^7*b*x^16 + 28/15*a^6*b^2*x^15 + 4*a^5*b^3*x^14 + 70/1
3*a^4*b^4*x^13 + 14/3*a^3*b^5*x^12 + 28/11*a^2*b^6*x^11 + 4/5*a*b^7*x^10 +
1/9*b^8*x^9
```

giac [A] time = 0.15, size = 90, normalized size = 0.85

$$\frac{1}{17}a^8x^{17}+\frac{1}{2}a^7bx^{16}+\frac{28}{15}a^6b^2x^{15}+4a^5b^3x^{14}+\frac{70}{13}a^4b^4x^{13}+\frac{14}{3}a^3b^5x^{12}+\frac{28}{11}a^2b^6x^{11}+\frac{4}{5}ab^7x^{10}+\frac{1}{9}b^8x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^16,x, algorithm="giac")
```

```
[Out] 1/17*a^8*x^17 + 1/2*a^7*b*x^16 + 28/15*a^6*b^2*x^15 + 4*a^5*b^3*x^14 + 70/1
3*a^4*b^4*x^13 + 14/3*a^3*b^5*x^12 + 28/11*a^2*b^6*x^11 + 4/5*a*b^7*x^10 +
1/9*b^8*x^9
```

maple [A] time = 0.00, size = 91, normalized size = 0.86

$$\frac{1}{17}a^8x^{17}+\frac{1}{2}a^7b\,x^{16}+\frac{28}{15}a^6b^2x^{15}+4a^5b^3x^{14}+\frac{70}{13}a^4b^4x^{13}+\frac{14}{3}a^3b^5x^{12}+\frac{28}{11}a^2b^6x^{11}+\frac{4}{5}a\,b^7x^{10}+\frac{1}{9}b^8x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^8*x^16,x)
```

```
[Out] 1/9*b^8*x^9+4/5*a*b^7*x^10+28/11*a^2*b^6*x^11+14/3*a^3*b^5*x^12+70/13*a^4*b
^4*x^13+4*a^5*b^3*x^14+28/15*a^6*b^2*x^15+1/2*a^7*b*x^16+1/17*a^8*x^17
```

maxima [A] time = 0.95, size = 90, normalized size = 0.85

$$\frac{1}{17}a^8x^{17}+\frac{1}{2}a^7bx^{16}+\frac{28}{15}a^6b^2x^{15}+4a^5b^3x^{14}+\frac{70}{13}a^4b^4x^{13}+\frac{14}{3}a^3b^5x^{12}+\frac{28}{11}a^2b^6x^{11}+\frac{4}{5}ab^7x^{10}+\frac{1}{9}b^8x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^16,x, algorithm="maxima")
```

```
[Out] 1/17*a^8*x^17 + 1/2*a^7*b*x^16 + 28/15*a^6*b^2*x^15 + 4*a^5*b^3*x^14 + 70/1
3*a^4*b^4*x^13 + 14/3*a^3*b^5*x^12 + 28/11*a^2*b^6*x^11 + 4/5*a*b^7*x^10 +
1/9*b^8*x^9
```

mupad [B] time = 0.06, size = 90, normalized size = 0.85

$$\frac{a^8\,x^{17}}{17}+\frac{a^7\,b\,x^{16}}{2}+\frac{28\,a^6\,b^2\,x^{15}}{15}+4\,a^5\,b^3\,x^{14}+\frac{70\,a^4\,b^4\,x^{13}}{13}+\frac{14\,a^3\,b^5\,x^{12}}{3}+\frac{28\,a^2\,b^6\,x^{11}}{11}+\frac{4\,a\,b^7\,x^{10}}{5}+\frac{b^8\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^16*(a + b/x)^8,x)
```

```
[Out] (a^8*x^17)/17 + (b^8*x^9)/9 + (4*a*b^7*x^10)/5 + (a^7*b*x^16)/2 + (28*a^2*b
^6*x^11)/11 + (14*a^3*b^5*x^12)/3 + (70*a^4*b^4*x^13)/13 + 4*a^5*b^3*x^14 +
 (28*a^6*b^2*x^15)/15
```

sympy [**A**] time = 0.09, size = 104, normalized size = 0.98

$$\frac{a^8x^{17}}{17}+\frac{a^7bx^{16}}{2}+\frac{28a^6b^2x^{15}}{15}+4a^5b^3x^{14}+\frac{70a^4b^4x^{13}}{13}+\frac{14a^3b^5x^{12}}{3}+\frac{28a^2b^6x^{11}}{11}+\frac{4ab^7x^{10}}{5}+\frac{b^8x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**8*x**16,x)
```

```
[Out] a**8*x**17/17 + a**7*b*x**16/2 + 28*a**6*b**2*x**15/15 + 4*a**5*b**3*x**14
+ 70*a**4*b**4*x**13/13 + 14*a**3*b**5*x**12/3 + 28*a**2*b**6*x**11/11 + 4*
a*b**7*x**10/5 + b**8*x**9/9
```

3.1584 $\int \left(a+\frac{b}{x}\right)^8 x^{15}\,dx$

Optimal. Leaf size=106

$$\frac{a^8x^{16}}{16}+\frac{8}{15}a^7bx^{15}+2a^6b^2x^{14}+\frac{56}{13}a^5b^3x^{13}+\frac{35}{6}a^4b^4x^{12}+\frac{56}{11}a^3b^5x^{11}+\frac{14}{5}a^2b^6x^{10}+\frac{8}{9}ab^7x^9+\frac{b^8x^8}{8}$$

```
[Out] 1/8*b^8*x^8+8/9*a*b^7*x^9+14/5*a^2*b^6*x^10+56/11*a^3*b^5*x^11+35/6*a^4*b^4
*x^12+56/13*a^5*b^3*x^13+2*a^6*b^2*x^14+8/15*a^7*b*x^15+1/16*a^8*x^16
```

Rubi [A] time = 0.05, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$2a^6b^2x^{14}+\frac{56}{13}a^5b^3x^{13}+\frac{35}{6}a^4b^4x^{12}+\frac{56}{11}a^3b^5x^{11}+\frac{14}{5}a^2b^6x^{10}+\frac{8}{15}a^7bx^{15}+\frac{a^8x^{16}}{16}+\frac{8}{9}ab^7x^9+\frac{b^8x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^8*x^15,x]
```

```
[Out] (b^8*x^8)/8 + (8*a*b^7*x^9)/9 + (14*a^2*b^6*x^10)/5 + (56*a^3*b^5*x^11)/11
+ (35*a^4*b^4*x^12)/6 + (56*a^5*b^3*x^13)/13 + 2*a^6*b^2*x^14 + (8*a^7*b*x^
15)/15 + (a^8*x^16)/16
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^8 x^{15}\,dx &= \int x^7(b+ax)^8\,dx\\ &= \int \left(b^8x^7+8ab^7x^8+28a^2b^6x^9+56a^3b^5x^{10}+70a^4b^4x^{11}+56a^5b^3x^{12}+28a^6b^2x^{13}+8a^7bx^{14}\right.\\ &= \frac{b^8x^8}{8}+\frac{8}{9}ab^7x^9+\frac{14}{5}a^2b^6x^{10}+\frac{56}{11}a^3b^5x^{11}+\frac{35}{6}a^4b^4x^{12}+\frac{56}{13}a^5b^3x^{13}+2a^6b^2x^{14}+\frac{8}{15}a^7bx^{15}\end{aligned}$$

Mathematica [A] time = 0.00, size = 106, normalized size = 1.00

$$\frac{a^8x^{16}}{16}+\frac{8}{15}a^7bx^{15}+2a^6b^2x^{14}+\frac{56}{13}a^5b^3x^{13}+\frac{35}{6}a^4b^4x^{12}+\frac{56}{11}a^3b^5x^{11}+\frac{14}{5}a^2b^6x^{10}+\frac{8}{9}ab^7x^9+\frac{b^8x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^8*x^15,x]
```

```
[Out] (b^8*x^8)/8 + (8*a*b^7*x^9)/9 + (14*a^2*b^6*x^10)/5 + (56*a^3*b^5*x^11)/11
+ (35*a^4*b^4*x^12)/6 + (56*a^5*b^3*x^13)/13 + 2*a^6*b^2*x^14 + (8*a^7*b*x^
15)/15 + (a^8*x^16)/16
```

fricas [A] time = 0.95, size = 90, normalized size = 0.85

$$\frac{1}{16}a^8x^{16}+\frac{8}{15}a^7bx^{15}+2a^6b^2x^{14}+\frac{56}{13}a^5b^3x^{13}+\frac{35}{6}a^4b^4x^{12}+\frac{56}{11}a^3b^5x^{11}+\frac{14}{5}a^2b^6x^{10}+\frac{8}{9}ab^7x^9+\frac{1}{8}b^8x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^15,x, algorithm="fricas")
```

```
[Out] 1/16*a^8*x^16 + 8/15*a^7*b*x^15 + 2*a^6*b^2*x^14 + 56/13*a^5*b^3*x^13 + 35/
6*a^4*b^4*x^12 + 56/11*a^3*b^5*x^11 + 14/5*a^2*b^6*x^10 + 8/9*a*b^7*x^9 + 1
/8*b^8*x^8
```

giac [A] time = 0.16, size = 90, normalized size = 0.85

$$\frac{1}{16}a^8x^{16}+\frac{8}{15}a^7bx^{15}+2a^6b^2x^{14}+\frac{56}{13}a^5b^3x^{13}+\frac{35}{6}a^4b^4x^{12}+\frac{56}{11}a^3b^5x^{11}+\frac{14}{5}a^2b^6x^{10}+\frac{8}{9}ab^7x^9+\frac{1}{8}b^8x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^15,x, algorithm="giac")
```

```
[Out] 1/16*a^8*x^16 + 8/15*a^7*b*x^15 + 2*a^6*b^2*x^14 + 56/13*a^5*b^3*x^13 + 35/
6*a^4*b^4*x^12 + 56/11*a^3*b^5*x^11 + 14/5*a^2*b^6*x^10 + 8/9*a*b^7*x^9 + 1
/8*b^8*x^8
```

maple [A] time = 0.00, size = 91, normalized size = 0.86

$$\frac{1}{16}a^8x^{16}+\frac{8}{15}a^7b\,x^{15}+2a^6b^2x^{14}+\frac{56}{13}a^5b^3x^{13}+\frac{35}{6}a^4b^4x^{12}+\frac{56}{11}a^3b^5x^{11}+\frac{14}{5}a^2b^6x^{10}+\frac{8}{9}a\,b^7x^9+\frac{1}{8}b^8x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^8*x^15,x)
```

```
[Out] 1/8*b^8*x^8+8/9*a*b^7*x^9+14/5*a^2*b^6*x^10+56/11*a^3*b^5*x^11+35/6*a^4*b^4
*x^12+56/13*a^5*b^3*x^13+2*a^6*b^2*x^14+8/15*a^7*b*x^15+1/16*a^8*x^16
```

maxima [A] time = 0.98, size = 90, normalized size = 0.85

$$\frac{1}{16}a^8x^{16}+\frac{8}{15}a^7bx^{15}+2a^6b^2x^{14}+\frac{56}{13}a^5b^3x^{13}+\frac{35}{6}a^4b^4x^{12}+\frac{56}{11}a^3b^5x^{11}+\frac{14}{5}a^2b^6x^{10}+\frac{8}{9}ab^7x^9+\frac{1}{8}b^8x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^15,x, algorithm="maxima")
```

```
[Out] 1/16*a^8*x^16 + 8/15*a^7*b*x^15 + 2*a^6*b^2*x^14 + 56/13*a^5*b^3*x^13 + 35/
6*a^4*b^4*x^12 + 56/11*a^3*b^5*x^11 + 14/5*a^2*b^6*x^10 + 8/9*a*b^7*x^9 + 1
/8*b^8*x^8
```

mupad [B] time = 1.07, size = 90, normalized size = 0.85

$$\frac{a^8\,x^{16}}{16}+\frac{8\,a^7\,b\,x^{15}}{15}+2\,a^6\,b^2\,x^{14}+\frac{56\,a^5\,b^3\,x^{13}}{13}+\frac{35\,a^4\,b^4\,x^{12}}{6}+\frac{56\,a^3\,b^5\,x^{11}}{11}+\frac{14\,a^2\,b^6\,x^{10}}{5}+\frac{8\,a\,b^7\,x^9}{9}+\frac{b^8\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15*(a + b/x)^8,x)
```

```
[Out] (a^8*x^16)/16 + (b^8*x^8)/8 + (8*a*b^7*x^9)/9 + (8*a^7*b*x^15)/15 + (14*a^2
*b^6*x^10)/5 + (56*a^3*b^5*x^11)/11 + (35*a^4*b^4*x^12)/6 + (56*a^5*b^3*x^1
3)/13 + 2*a^6*b^2*x^14
```

sympy [**A**] time = 0.09, size = 105, normalized size = 0.99

$$\frac{a^8x^{16}}{16}+\frac{8a^7bx^{15}}{15}+2a^6b^2x^{14}+\frac{56a^5b^3x^{13}}{13}+\frac{35a^4b^4x^{12}}{6}+\frac{56a^3b^5x^{11}}{11}+\frac{14a^2b^6x^{10}}{5}+\frac{8ab^7x^9}{9}+\frac{b^8x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**8*x**15,x)
```

```
[Out] a**8*x**16/16 + 8*a**7*b*x**15/15 + 2*a**6*b**2*x**14 + 56*a**5*b**3*x**13/
13 + 35*a**4*b**4*x**12/6 + 56*a**3*b**5*x**11/11 + 14*a**2*b**6*x**10/5 +
8*a*b**7*x**9/9 + b**8*x**8/8
```

3.1585 $\int \left(a+\frac{b}{x}\right)^8 x^{13}\,dx$

Optimal. Leaf size=98

$$-\frac{b^5(ax+b)^9}{9a^6}+\frac{b^4(ax+b)^{10}}{2a^6}-\frac{10b^3(ax+b)^{11}}{11a^6}+\frac{5b^2(ax+b)^{12}}{6a^6}+\frac{(ax+b)^{14}}{14a^6}-\frac{5b(ax+b)^{13}}{13a^6}$$

```
[Out] -1/9*b^5*(a*x+b)^9/a^6+1/2*b^4*(a*x+b)^10/a^6-10/11*b^3*(a*x+b)^11/a^6+5/6*
b^2*(a*x+b)^12/a^6-5/13*b*(a*x+b)^13/a^6+1/14*(a*x+b)^14/a^6
```

Rubi [A] time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$\frac{5b^2(ax+b)^{12}}{6a^6}-\frac{10b^3(ax+b)^{11}}{11a^6}+\frac{b^4(ax+b)^{10}}{2a^6}-\frac{b^5(ax+b)^9}{9a^6}+\frac{(ax+b)^{14}}{14a^6}-\frac{5b(ax+b)^{13}}{13a^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^8*x^13,x]
```

```
[Out] -(b^5*(b + a*x)^9)/(9*a^6) + (b^4*(b + a*x)^10)/(2*a^6) - (10*b^3*(b + a*x)
^11)/(11*a^6) + (5*b^2*(b + a*x)^12)/(6*a^6) - (5*b*(b + a*x)^13)/(13*a^6)
+ (b + a*x)^14/(14*a^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^8 x^{13}\,dx &= \int x^5(b+ax)^8\,dx\\ &= \int\left(-\frac{b^5(b+ax)^8}{a^5}+\frac{5b^4(b+ax)^9}{a^5}-\frac{10b^3(b+ax)^{10}}{a^5}+\frac{10b^2(b+ax)^{11}}{a^5}-\frac{5b(b+ax)^{12}}{a^5}+\frac{(b\\ &= -\frac{b^5(b+ax)^9}{9a^6}+\frac{b^4(b+ax)^{10}}{2a^6}-\frac{10b^3(b+ax)^{11}}{11a^6}+\frac{5b^2(b+ax)^{12}}{6a^6}-\frac{5b(b+ax)^{13}}{13a^6}+\frac{(b+ax}{14a^6}\end{aligned}$$

Mathematica [A] time = 0.00, size = 106, normalized size = 1.08

$$\frac{a^8x^{14}}{14}+\frac{8}{13}a^7bx^{13}+\frac{7}{3}a^6b^2x^{12}+\frac{56}{11}a^5b^3x^{11}+7a^4b^4x^{10}+\frac{56}{9}a^3b^5x^9+\frac{7}{2}a^2b^6x^8+\frac{8}{7}ab^7x^7+\frac{b^8x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^8*x^13,x]
```

```
[Out] (b^8*x^6)/6 + (8*a*b^7*x^7)/7 + (7*a^2*b^6*x^8)/2 + (56*a^3*b^5*x^9)/9 + 7*
a^4*b^4*x^10 + (56*a^5*b^3*x^11)/11 + (7*a^6*b^2*x^12)/3 + (8*a^7*b*x^13)/1
3 + (a^8*x^14)/14
```

fricas [A] time = 1.03, size = 90, normalized size = 0.92

$$\frac{1}{14}a^8x^{14}+\frac{8}{13}a^7bx^{13}+\frac{7}{3}a^6b^2x^{12}+\frac{56}{11}a^5b^3x^{11}+7a^4b^4x^{10}+\frac{56}{9}a^3b^5x^9+\frac{7}{2}a^2b^6x^8+\frac{8}{7}ab^7x^7+\frac{1}{6}b^8x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^13,x, algorithm="fricas")
```

```
[Out] 1/14*a^8*x^14 + 8/13*a^7*b*x^13 + 7/3*a^6*b^2*x^12 + 56/11*a^5*b^3*x^11 + 7
*a^4*b^4*x^10 + 56/9*a^3*b^5*x^9 + 7/2*a^2*b^6*x^8 + 8/7*a*b^7*x^7 + 1/6*b^
8*x^6
```

giac [A] time = 0.15, size = 90, normalized size = 0.92

$$\frac{1}{14}a^8x^{14}+\frac{8}{13}a^7bx^{13}+\frac{7}{3}a^6b^2x^{12}+\frac{56}{11}a^5b^3x^{11}+7a^4b^4x^{10}+\frac{56}{9}a^3b^5x^9+\frac{7}{2}a^2b^6x^8+\frac{8}{7}ab^7x^7+\frac{1}{6}b^8x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^13,x, algorithm="giac")
```

```
[Out] 1/14*a^8*x^14 + 8/13*a^7*b*x^13 + 7/3*a^6*b^2*x^12 + 56/11*a^5*b^3*x^11 + 7
*a^4*b^4*x^10 + 56/9*a^3*b^5*x^9 + 7/2*a^2*b^6*x^8 + 8/7*a*b^7*x^7 + 1/6*b^
8*x^6
```

maple [A] time = 0.00, size = 91, normalized size = 0.93

$$\frac{1}{14}a^8x^{14}+\frac{8}{13}a^7b\,x^{13}+\frac{7}{3}a^6b^2x^{12}+\frac{56}{11}a^5b^3x^{11}+7a^4b^4x^{10}+\frac{56}{9}a^3b^5x^9+\frac{7}{2}a^2b^6x^8+\frac{8}{7}a\,b^7x^7+\frac{1}{6}b^8x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^8*x^13,x)
```

```
[Out] 1/14*a^8*x^14+8/13*a^7*b*x^13+7/3*b^2*a^6*x^12+56/11*a^5*b^3*x^11+7*a^4*b^4
*x^10+56/9*a^3*b^5*x^9+7/2*b^6*a^2*x^8+8/7*a*b^7*x^7+1/6*b^8*x^6
```

maxima [A] time = 1.03, size = 90, normalized size = 0.92

$$\frac{1}{14}a^8x^{14}+\frac{8}{13}a^7bx^{13}+\frac{7}{3}a^6b^2x^{12}+\frac{56}{11}a^5b^3x^{11}+7a^4b^4x^{10}+\frac{56}{9}a^3b^5x^9+\frac{7}{2}a^2b^6x^8+\frac{8}{7}ab^7x^7+\frac{1}{6}b^8x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^13,x, algorithm="maxima")
```

```
[Out] 1/14*a^8*x^14 + 8/13*a^7*b*x^13 + 7/3*a^6*b^2*x^12 + 56/11*a^5*b^3*x^11 + 7
*a^4*b^4*x^10 + 56/9*a^3*b^5*x^9 + 7/2*a^2*b^6*x^8 + 8/7*a*b^7*x^7 + 1/6*b^
8*x^6
```

mupad [B] time = 0.05, size = 90, normalized size = 0.92

$$\frac{a^8\,x^{14}}{14}+\frac{8\,a^7\,b\,x^{13}}{13}+\frac{7\,a^6\,b^2\,x^{12}}{3}+\frac{56\,a^5\,b^3\,x^{11}}{11}+7\,a^4\,b^4\,x^{10}+\frac{56\,a^3\,b^5\,x^9}{9}+\frac{7\,a^2\,b^6\,x^8}{2}+\frac{8\,a\,b^7\,x^7}{7}+\frac{b^8\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(a + b/x)^8,x)
```

```
[Out] (a^8*x^14)/14 + (b^8*x^6)/6 + (8*a*b^7*x^7)/7 + (8*a^7*b*x^13)/13 + (7*a^2*
b^6*x^8)/2 + (56*a^3*b^5*x^9)/9 + 7*a^4*b^4*x^10 + (56*a^5*b^3*x^11)/11 + (
7*a^6*b^2*x^12)/3
```

sympy [**A**] time = 0.09, size = 105, normalized size = 1.07

$$\frac{a^8x^{14}}{14}+\frac{8a^7bx^{13}}{13}+\frac{7a^6b^2x^{12}}{3}+\frac{56a^5b^3x^{11}}{11}+7a^4b^4x^{10}+\frac{56a^3b^5x^9}{9}+\frac{7a^2b^6x^8}{2}+\frac{8ab^7x^7}{7}+\frac{b^8x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**8*x**13,x)
```

```
[Out] a**8*x**14/14 + 8*a**7*b*x**13/13 + 7*a**6*b**2*x**12/3 + 56*a**5*b**3*x**1
1/11 + 7*a**4*b**4*x**10 + 56*a**3*b**5*x**9/9 + 7*a**2*b**6*x**8/2 + 8*a*b
**7*x**7/7 + b**8*x**6/6
```

3.1586 $\int \left(a+\frac{b}{x}\right)^8 x^{12}\,dx$

Optimal. Leaf size=81

$$\frac{b^4(ax+b)^9}{9a^5}-\frac{2b^3(ax+b)^{10}}{5a^5}+\frac{6b^2(ax+b)^{11}}{11a^5}+\frac{(ax+b)^{13}}{13a^5}-\frac{b(ax+b)^{12}}{3a^5}$$

```
[Out] 1/9*b^4*(a*x+b)^9/a^5-2/5*b^3*(a*x+b)^10/a^5+6/11*b^2*(a*x+b)^11/a^5-1/3*b*
(a*x+b)^12/a^5+1/13*(a*x+b)^13/a^5
```

Rubi [A] time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$\frac{6b^2(ax+b)^{11}}{11a^5}-\frac{2b^3(ax+b)^{10}}{5a^5}+\frac{b^4(ax+b)^9}{9a^5}+\frac{(ax+b)^{13}}{13a^5}-\frac{b(ax+b)^{12}}{3a^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^8*x^12,x]
```

```
[Out] (b^4*(b + a*x)^9)/(9*a^5) - (2*b^3*(b + a*x)^10)/(5*a^5) + (6*b^2*(b + a*x)
^11)/(11*a^5) - (b*(b + a*x)^12)/(3*a^5) + (b + a*x)^13/(13*a^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^8 x^{12}\,dx &= \int x^4(b+ax)^8\,dx\\ &= \int \left(\frac{b^4(b+ax)^8}{a^4}-\frac{4b^3(b+ax)^9}{a^4}+\frac{6b^2(b+ax)^{10}}{a^4}-\frac{4b(b+ax)^{11}}{a^4}+\frac{(b+ax)^{12}}{a^4}\right)dx\\ &= \frac{b^4(b+ax)^9}{9a^5}-\frac{2b^3(b+ax)^{10}}{5a^5}+\frac{6b^2(b+ax)^{11}}{11a^5}-\frac{b(b+ax)^{12}}{3a^5}+\frac{(b+ax)^{13}}{13a^5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 104, normalized size = 1.28

$$\frac{a^8x^{13}}{13}+\frac{2}{3}a^7bx^{12}+\frac{28}{11}a^6b^2x^{11}+\frac{28}{5}a^5b^3x^{10}+\frac{70}{9}a^4b^4x^9+7a^3b^5x^8+4a^2b^6x^7+\frac{4}{3}ab^7x^6+\frac{b^8x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^8*x^12,x]
```

```
[Out] (b^8*x^5)/5 + (4*a*b^7*x^6)/3 + 4*a^2*b^6*x^7 + 7*a^3*b^5*x^8 + (70*a^4*b^4
*x^9)/9 + (28*a^5*b^3*x^10)/5 + (28*a^6*b^2*x^11)/11 + (2*a^7*b*x^12)/3 + (
a^8*x^13)/13
```

fricas [A] time = 0.93, size = 90, normalized size = 1.11

$$\frac{1}{13}a^8x^{13}+\frac{2}{3}a^7bx^{12}+\frac{28}{11}a^6b^2x^{11}+\frac{28}{5}a^5b^3x^{10}+\frac{70}{9}a^4b^4x^9+7a^3b^5x^8+4a^2b^6x^7+\frac{4}{3}ab^7x^6+\frac{1}{5}b^8x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^12,x, algorithm="fricas")
```

```
[Out] 1/13*a^8*x^13 + 2/3*a^7*b*x^12 + 28/11*a^6*b^2*x^11 + 28/5*a^5*b^3*x^10 + 7
0/9*a^4*b^4*x^9 + 7*a^3*b^5*x^8 + 4*a^2*b^6*x^7 + 4/3*a*b^7*x^6 + 1/5*b^8*x
^5
```

giac [A] time = 0.16, size = 90, normalized size = 1.11

$$\frac{1}{13}a^8x^{13}+\frac{2}{3}a^7bx^{12}+\frac{28}{11}a^6b^2x^{11}+\frac{28}{5}a^5b^3x^{10}+\frac{70}{9}a^4b^4x^9+7a^3b^5x^8+4a^2b^6x^7+\frac{4}{3}ab^7x^6+\frac{1}{5}b^8x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^12,x, algorithm="giac")
```

```
[Out] 1/13*a^8*x^13 + 2/3*a^7*b*x^12 + 28/11*a^6*b^2*x^11 + 28/5*a^5*b^3*x^10 + 7
0/9*a^4*b^4*x^9 + 7*a^3*b^5*x^8 + 4*a^2*b^6*x^7 + 4/3*a*b^7*x^6 + 1/5*b^8*x
^5
```

maple [A] time = 0.00, size = 91, normalized size = 1.12

$$\frac{1}{13}a^8x^{13}+\frac{2}{3}a^7b\,x^{12}+\frac{28}{11}a^6b^2x^{11}+\frac{28}{5}a^5b^3x^{10}+\frac{70}{9}a^4b^4x^9+7a^3b^5x^8+4a^2b^6x^7+\frac{4}{3}a\,b^7x^6+\frac{1}{5}b^8x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^8*x^12,x)
```

```
[Out] 1/13*a^8*x^13+2/3*a^7*b*x^12+28/11*a^6*b^2*x^11+28/5*a^5*b^3*x^10+70/9*a^4*
b^4*x^9+7*a^3*b^5*x^8+4*b^6*a^2*x^7+4/3*a*b^7*x^6+1/5*b^8*x^5
```

maxima [A] time = 0.99, size = 90, normalized size = 1.11

$$\frac{1}{13}a^8x^{13}+\frac{2}{3}a^7bx^{12}+\frac{28}{11}a^6b^2x^{11}+\frac{28}{5}a^5b^3x^{10}+\frac{70}{9}a^4b^4x^9+7a^3b^5x^8+4a^2b^6x^7+\frac{4}{3}ab^7x^6+\frac{1}{5}b^8x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^12,x, algorithm="maxima")
```

```
[Out] 1/13*a^8*x^13 + 2/3*a^7*b*x^12 + 28/11*a^6*b^2*x^11 + 28/5*a^5*b^3*x^10 + 7
0/9*a^4*b^4*x^9 + 7*a^3*b^5*x^8 + 4*a^2*b^6*x^7 + 4/3*a*b^7*x^6 + 1/5*b^8*x
^5
```

mupad [B] time = 1.06, size = 90, normalized size = 1.11

$$\frac{a^8\,x^{13}}{13}+\frac{2\,a^7\,b\,x^{12}}{3}+\frac{28\,a^6\,b^2\,x^{11}}{11}+\frac{28\,a^5\,b^3\,x^{10}}{5}+\frac{70\,a^4\,b^4\,x^9}{9}+7\,a^3\,b^5\,x^8+4\,a^2\,b^6\,x^7+\frac{4\,a\,b^7\,x^6}{3}+\frac{b^8\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(a + b/x)^8,x)
```

```
[Out] (a^8*x^13)/13 + (b^8*x^5)/5 + (4*a*b^7*x^6)/3 + (2*a^7*b*x^12)/3 + 4*a^2*b^
6*x^7 + 7*a^3*b^5*x^8 + (70*a^4*b^4*x^9)/9 + (28*a^5*b^3*x^10)/5 + (28*a^6*
b^2*x^11)/11
```

sympy [**A**] time = 0.09, size = 104, normalized size = 1.28

$$\frac{a^8x^{13}}{13}+\frac{2a^7bx^{12}}{3}+\frac{28a^6b^2x^{11}}{11}+\frac{28a^5b^3x^{10}}{5}+\frac{70a^4b^4x^9}{9}+7a^3b^5x^8+4a^2b^6x^7+\frac{4ab^7x^6}{3}+\frac{b^8x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**8*x**12,x)
```

```
[Out] a**8*x**13/13 + 2*a**7*b*x**12/3 + 28*a**6*b**2*x**11/11 + 28*a**5*b**3*x**
10/5 + 70*a**4*b**4*x**9/9 + 7*a**3*b**5*x**8 + 4*a**2*b**6*x**7 + 4*a*b**7
*x**6/3 + b**8*x**5/5
```

3.1587 $\int \left(a+\frac{b}{x}\right)^8 x^{11}\,dx$

Optimal. Leaf size=64

$$-\frac{b^3(ax+b)^9}{9a^4}+\frac{3b^2(ax+b)^{10}}{10a^4}+\frac{(ax+b)^{12}}{12a^4}-\frac{3b(ax+b)^{11}}{11a^4}$$

```
[Out] -1/9*b^3*(a*x+b)^9/a^4+3/10*b^2*(a*x+b)^10/a^4-3/11*b*(a*x+b)^11/a^4+1/12*(
a*x+b)^12/a^4
```

Rubi [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$\frac{3b^2(ax+b)^{10}}{10a^4}-\frac{b^3(ax+b)^9}{9a^4}+\frac{(ax+b)^{12}}{12a^4}-\frac{3b(ax+b)^{11}}{11a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^8*x^11,x]
```

```
[Out] -(b^3*(b + a*x)^9)/(9*a^4) + (3*b^2*(b + a*x)^10)/(10*a^4) - (3*b*(b + a*x)
^11)/(11*a^4) + (b + a*x)^12/(12*a^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^8 x^{11}\,dx &= \int x^3(b+ax)^8\,dx\\ &= \int\left(-\frac{b^3(b+ax)^8}{a^3}+\frac{3b^2(b+ax)^9}{a^3}-\frac{3b(b+ax)^{10}}{a^3}+\frac{(b+ax)^{11}}{a^3}\right)dx\\ &= -\frac{b^3(b+ax)^9}{9a^4}+\frac{3b^2(b+ax)^{10}}{10a^4}-\frac{3b(b+ax)^{11}}{11a^4}+\frac{(b+ax)^{12}}{12a^4}\end{aligned}$$

Mathematica [A] time = 0.00, size = 106, normalized size = 1.66

$$\frac{a^8x^{12}}{12}+\frac{8}{11}a^7bx^{11}+\frac{14}{5}a^6b^2x^{10}+\frac{56}{9}a^5b^3x^9+\frac{35}{4}a^4b^4x^8+8a^3b^5x^7+\frac{14}{3}a^2b^6x^6+\frac{8}{5}ab^7x^5+\frac{b^8x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^8*x^11,x]
```

```
[Out] (b^8*x^4)/4 + (8*a*b^7*x^5)/5 + (14*a^2*b^6*x^6)/3 + 8*a^3*b^5*x^7 + (35*a^
4*b^4*x^8)/4 + (56*a^5*b^3*x^9)/9 + (14*a^6*b^2*x^10)/5 + (8*a^7*b*x^11)/11
 + (a^8*x^12)/12
```

fricas [A] time = 1.05, size = 90, normalized size = 1.41

$$\frac{1}{12}a^8x^{12}+\frac{8}{11}a^7bx^{11}+\frac{14}{5}a^6b^2x^{10}+\frac{56}{9}a^5b^3x^9+\frac{35}{4}a^4b^4x^8+8a^3b^5x^7+\frac{14}{3}a^2b^6x^6+\frac{8}{5}ab^7x^5+\frac{1}{4}b^8x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^11,x, algorithm="fricas")
```

```
[Out] 1/12*a^8*x^12 + 8/11*a^7*b*x^11 + 14/5*a^6*b^2*x^10 + 56/9*a^5*b^3*x^9 + 35
/4*a^4*b^4*x^8 + 8*a^3*b^5*x^7 + 14/3*a^2*b^6*x^6 + 8/5*a*b^7*x^5 + 1/4*b^8
*x^4
```

giac [A] time = 0.15, size = 90, normalized size = 1.41

$$\frac{1}{12}a^8x^{12}+\frac{8}{11}a^7bx^{11}+\frac{14}{5}a^6b^2x^{10}+\frac{56}{9}a^5b^3x^9+\frac{35}{4}a^4b^4x^8+8a^3b^5x^7+\frac{14}{3}a^2b^6x^6+\frac{8}{5}ab^7x^5+\frac{1}{4}b^8x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^11,x, algorithm="giac")
```

```
[Out] 1/12*a^8*x^12 + 8/11*a^7*b*x^11 + 14/5*a^6*b^2*x^10 + 56/9*a^5*b^3*x^9 + 35
/4*a^4*b^4*x^8 + 8*a^3*b^5*x^7 + 14/3*a^2*b^6*x^6 + 8/5*a*b^7*x^5 + 1/4*b^8
*x^4
```

maple [A] time = 0.00, size = 91, normalized size = 1.42

$$\frac{1}{12}a^8x^{12}+\frac{8}{11}a^7b\,x^{11}+\frac{14}{5}a^6b^2x^{10}+\frac{56}{9}a^5b^3x^9+\frac{35}{4}a^4b^4x^8+8a^3b^5x^7+\frac{14}{3}a^2b^6x^6+\frac{8}{5}a\,b^7x^5+\frac{1}{4}b^8x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^8*x^11,x)
```

```
[Out] 1/12*a^8*x^12+8/11*a^7*b*x^11+14/5*a^6*b^2*x^10+56/9*a^5*b^3*x^9+35/4*a^4*b
^4*x^8+8*a^3*b^5*x^7+14/3*a^2*b^6*x^6+8/5*a*b^7*x^5+1/4*b^8*x^4
```

maxima [A] time = 1.06, size = 90, normalized size = 1.41

$$\frac{1}{12}a^8x^{12}+\frac{8}{11}a^7bx^{11}+\frac{14}{5}a^6b^2x^{10}+\frac{56}{9}a^5b^3x^9+\frac{35}{4}a^4b^4x^8+8a^3b^5x^7+\frac{14}{3}a^2b^6x^6+\frac{8}{5}ab^7x^5+\frac{1}{4}b^8x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^11,x, algorithm="maxima")
```

```
[Out] 1/12*a^8*x^12 + 8/11*a^7*b*x^11 + 14/5*a^6*b^2*x^10 + 56/9*a^5*b^3*x^9 + 35
/4*a^4*b^4*x^8 + 8*a^3*b^5*x^7 + 14/3*a^2*b^6*x^6 + 8/5*a*b^7*x^5 + 1/4*b^8
*x^4
```

mupad [B] time = 0.05, size = 90, normalized size = 1.41

$$\frac{a^8\,x^{12}}{12}+\frac{8\,a^7\,b\,x^{11}}{11}+\frac{14\,a^6\,b^2\,x^{10}}{5}+\frac{56\,a^5\,b^3\,x^9}{9}+\frac{35\,a^4\,b^4\,x^8}{4}+8\,a^3\,b^5\,x^7+\frac{14\,a^2\,b^6\,x^6}{3}+\frac{8\,a\,b^7\,x^5}{5}+\frac{b^8\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a + b/x)^8,x)
```

```
[Out] (a^8*x^12)/12 + (b^8*x^4)/4 + (8*a*b^7*x^5)/5 + (8*a^7*b*x^11)/11 + (14*a^2
*b^6*x^6)/3 + 8*a^3*b^5*x^7 + (35*a^4*b^4*x^8)/4 + (56*a^5*b^3*x^9)/9 + (14
*a^6*b^2*x^10)/5
```

sympy [**A**] time = 0.10, size = 105, normalized size = 1.64

$$\frac{a^8x^{12}}{12}+\frac{8a^7bx^{11}}{11}+\frac{14a^6b^2x^{10}}{5}+\frac{56a^5b^3x^9}{9}+\frac{35a^4b^4x^8}{4}+8a^3b^5x^7+\frac{14a^2b^6x^6}{3}+\frac{8ab^7x^5}{5}+\frac{b^8x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**8*x**11,x)
```

```
[Out] a**8*x**12/12 + 8*a**7*b*x**11/11 + 14*a**6*b**2*x**10/5 + 56*a**5*b**3*x**
9/9 + 35*a**4*b**4*x**8/4 + 8*a**3*b**5*x**7 + 14*a**2*b**6*x**6/3 + 8*a*b*
*7*x**5/5 + b**8*x**4/4
```

3.1588 $\int \left(a+\frac{b}{x}\right)^8 x^{10}\,dx$

Optimal. Leaf size=47

$$\frac{b^2(ax+b)^9}{9a^3}+\frac{(ax+b)^{11}}{11a^3}-\frac{b(ax+b)^{10}}{5a^3}$$

```
[Out] 1/9*b^2*(a*x+b)^9/a^3-1/5*b*(a*x+b)^10/a^3+1/11*(a*x+b)^11/a^3
```

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$\frac{b^2(ax+b)^9}{9a^3}+\frac{(ax+b)^{11}}{11a^3}-\frac{b(ax+b)^{10}}{5a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^8*x^10,x]
```

```
[Out] (b^2*(b + a*x)^9)/(9*a^3) - (b*(b + a*x)^10)/(5*a^3) + (b + a*x)^11/(11*a^3
)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^8 x^{10}\,dx &= \int x^2(b+ax)^8\,dx\\ &= \int \left(\frac{b^2(b+ax)^8}{a^2}-\frac{2b(b+ax)^9}{a^2}+\frac{(b+ax)^{10}}{a^2}\right)dx\\ &= \frac{b^2(b+ax)^9}{9a^3}-\frac{b(b+ax)^{10}}{5a^3}+\frac{(b+ax)^{11}}{11a^3}\end{aligned}$$

Mathematica [B] time = 0.00, size = 102, normalized size = 2.17

$$\frac{a^8x^{11}}{11}+\frac{4}{5}a^7bx^{10}+\frac{28}{9}a^6b^2x^9+7a^5b^3x^8+10a^4b^4x^7+\frac{28}{3}a^3b^5x^6+\frac{28}{5}a^2b^6x^5+2ab^7x^4+\frac{b^8x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^8*x^10,x]
```

```
[Out] (b^8*x^3)/3 + 2*a*b^7*x^4 + (28*a^2*b^6*x^5)/5 + (28*a^3*b^5*x^6)/3 + 10*a^
4*b^4*x^7 + 7*a^5*b^3*x^8 + (28*a^6*b^2*x^9)/9 + (4*a^7*b*x^10)/5 + (a^8*x^
11)/11
```

fricas [B] time = 0.90, size = 90, normalized size = 1.91

$$\frac{1}{11}a^8x^{11}+\frac{4}{5}a^7bx^{10}+\frac{28}{9}a^6b^2x^9+7a^5b^3x^8+10a^4b^4x^7+\frac{28}{3}a^3b^5x^6+\frac{28}{5}a^2b^6x^5+2ab^7x^4+\frac{1}{3}b^8x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^10,x, algorithm="fricas")
```

```
[Out] 1/11*a^8*x^11 + 4/5*a^7*b*x^10 + 28/9*a^6*b^2*x^9 + 7*a^5*b^3*x^8 + 10*a^4*
b^4*x^7 + 28/3*a^3*b^5*x^6 + 28/5*a^2*b^6*x^5 + 2*a*b^7*x^4 + 1/3*b^8*x^3
```

giac [B] time = 0.16, size = 90, normalized size = 1.91

$$\frac{1}{11}a^8x^{11}+\frac{4}{5}a^7bx^{10}+\frac{28}{9}a^6b^2x^9+7a^5b^3x^8+10a^4b^4x^7+\frac{28}{3}a^3b^5x^6+\frac{28}{5}a^2b^6x^5+2ab^7x^4+\frac{1}{3}b^8x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^10,x, algorithm="giac")
```

```
[Out] 1/11*a^8*x^11 + 4/5*a^7*b*x^10 + 28/9*a^6*b^2*x^9 + 7*a^5*b^3*x^8 + 10*a^4*
b^4*x^7 + 28/3*a^3*b^5*x^6 + 28/5*a^2*b^6*x^5 + 2*a*b^7*x^4 + 1/3*b^8*x^3
```

maple [B] time = 0.00, size = 91, normalized size = 1.94

$$\frac{1}{11}a^8x^{11}+\frac{4}{5}a^7b\,x^{10}+\frac{28}{9}a^6b^2x^9+7a^5b^3x^8+10a^4b^4x^7+\frac{28}{3}a^3b^5x^6+\frac{28}{5}a^2b^6x^5+2a\,b^7x^4+\frac{1}{3}b^8x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^8*x^10,x)
```

```
[Out] 1/11*a^8*x^11+4/5*a^7*b*x^10+28/9*b^2*a^6*x^9+7*a^5*b^3*x^8+10*a^4*b^4*x^7+
28/3*a^3*b^5*x^6+28/5*b^6*a^2*x^5+2*a*b^7*x^4+1/3*b^8*x^3
```

maxima [B] time = 0.96, size = 90, normalized size = 1.91

$$\frac{1}{11}a^8x^{11}+\frac{4}{5}a^7bx^{10}+\frac{28}{9}a^6b^2x^9+7a^5b^3x^8+10a^4b^4x^7+\frac{28}{3}a^3b^5x^6+\frac{28}{5}a^2b^6x^5+2ab^7x^4+\frac{1}{3}b^8x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^10,x, algorithm="maxima")
```

```
[Out] 1/11*a^8*x^11 + 4/5*a^7*b*x^10 + 28/9*a^6*b^2*x^9 + 7*a^5*b^3*x^8 + 10*a^4*
b^4*x^7 + 28/3*a^3*b^5*x^6 + 28/5*a^2*b^6*x^5 + 2*a*b^7*x^4 + 1/3*b^8*x^3
```

mupad [B] time = 1.06, size = 90, normalized size = 1.91

$$\frac{a^8\,x^{11}}{11}+\frac{4\,a^7\,b\,x^{10}}{5}+\frac{28\,a^6\,b^2\,x^9}{9}+7\,a^5\,b^3\,x^8+10\,a^4\,b^4\,x^7+\frac{28\,a^3\,b^5\,x^6}{3}+\frac{28\,a^2\,b^6\,x^5}{5}+2\,a\,b^7\,x^4+\frac{b^8\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(a + b/x)^8,x)
```

```
[Out] (a^8*x^11)/11 + (b^8*x^3)/3 + 2*a*b^7*x^4 + (4*a^7*b*x^10)/5 + (28*a^2*b^6*
x^5)/5 + (28*a^3*b^5*x^6)/3 + 10*a^4*b^4*x^7 + 7*a^5*b^3*x^8 + (28*a^6*b^2*
x^9)/9
```

sympy [B] time = 0.09, size = 102, normalized size = 2.17

$$\frac{a^8x^{11}}{11}+\frac{4a^7bx^{10}}{5}+\frac{28a^6b^2x^9}{9}+7a^5b^3x^8+10a^4b^4x^7+\frac{28a^3b^5x^6}{3}+\frac{28a^2b^6x^5}{5}+2ab^7x^4+\frac{b^8x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**8*x**10,x)
```

```
[Out] a**8*x**11/11 + 4*a**7*b*x**10/5 + 28*a**6*b**2*x**9/9 + 7*a**5*b**3*x**8 +
 10*a**4*b**4*x**7 + 28*a**3*b**5*x**6/3 + 28*a**2*b**6*x**5/5 + 2*a*b**7*x
**4 + b**8*x**3/3
```

3.1589 $\int \left(a+\frac{b}{x}\right)^8 x^9\,dx$

Optimal. Leaf size=30

$$\frac{(ax+b)^{10}}{10a^2}-\frac{b(ax+b)^9}{9a^2}$$

```
[Out] -1/9*b*(a*x+b)^9/a^2+1/10*(a*x+b)^10/a^2
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$\frac{(ax+b)^{10}}{10a^2}-\frac{b(ax+b)^9}{9a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^8*x^9,x]
```

```
[Out] -(b*(b + a*x)^9)/(9*a^2) + (b + a*x)^10/(10*a^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^8 x^9\,dx &= \int x(b+ax)^8\,dx\\ &= \int \left(-\frac{b(b+ax)^8}{a}+\frac{(b+ax)^9}{a}\right)dx\\ &= -\frac{b(b+ax)^9}{9a^2}+\frac{(b+ax)^{10}}{10a^2}\end{aligned}$$

Mathematica [B] time = 0.00, size = 104, normalized size = 3.47

$$\frac{a^8x^{10}}{10}+\frac{8}{9}a^7bx^9+\frac{7}{2}a^6b^2x^8+8a^5b^3x^7+\frac{35}{3}a^4b^4x^6+\frac{56}{5}a^3b^5x^5+7a^2b^6x^4+\frac{8}{3}ab^7x^3+\frac{b^8x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^8*x^9,x]
```

```
[Out] (b^8*x^2)/2 + (8*a*b^7*x^3)/3 + 7*a^2*b^6*x^4 + (56*a^3*b^5*x^5)/5 + (35*a^
4*b^4*x^6)/3 + 8*a^5*b^3*x^7 + (7*a^6*b^2*x^8)/2 + (8*a^7*b*x^9)/9 + (a^8*x
^10)/10
```

fricas [B] time = 0.96, size = 90, normalized size = 3.00

$$\frac{1}{10}a^8x^{10}+\frac{8}{9}a^7bx^9+\frac{7}{2}a^6b^2x^8+8a^5b^3x^7+\frac{35}{3}a^4b^4x^6+\frac{56}{5}a^3b^5x^5+7a^2b^6x^4+\frac{8}{3}ab^7x^3+\frac{1}{2}b^8x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^9,x, algorithm="fricas")
```

```
[Out] 1/10*a^8*x^10 + 8/9*a^7*b*x^9 + 7/2*a^6*b^2*x^8 + 8*a^5*b^3*x^7 + 35/3*a^4*
b^4*x^6 + 56/5*a^3*b^5*x^5 + 7*a^2*b^6*x^4 + 8/3*a*b^7*x^3 + 1/2*b^8*x^2
```

giac [B] time = 0.17, size = 90, normalized size = 3.00

$$\frac{1}{10}a^8x^{10}+\frac{8}{9}a^7bx^9+\frac{7}{2}a^6b^2x^8+8a^5b^3x^7+\frac{35}{3}a^4b^4x^6+\frac{56}{5}a^3b^5x^5+7a^2b^6x^4+\frac{8}{3}ab^7x^3+\frac{1}{2}b^8x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^9,x, algorithm="giac")
```

```
[Out] 1/10*a^8*x^10 + 8/9*a^7*b*x^9 + 7/2*a^6*b^2*x^8 + 8*a^5*b^3*x^7 + 35/3*a^4*
b^4*x^6 + 56/5*a^3*b^5*x^5 + 7*a^2*b^6*x^4 + 8/3*a*b^7*x^3 + 1/2*b^8*x^2
```

maple [B] time = 0.00, size = 91, normalized size = 3.03

$$\frac{1}{10}a^8x^{10}+\frac{8}{9}a^7b\,x^9+\frac{7}{2}a^6b^2x^8+8a^5b^3x^7+\frac{35}{3}a^4b^4x^6+\frac{56}{5}a^3b^5x^5+7a^2b^6x^4+\frac{8}{3}a\,b^7x^3+\frac{1}{2}b^8x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^8*x^9,x)
```

```
[Out] 1/10*a^8*x^10+8/9*a^7*b*x^9+7/2*a^6*b^2*x^8+8*a^5*b^3*x^7+35/3*a^4*b^4*x^6+
56/5*b^5*a^3*x^5+7*b^6*a^2*x^4+8/3*a*b^7*x^3+1/2*b^8*x^2
```

maxima [B] time = 1.06, size = 90, normalized size = 3.00

$$\frac{1}{10}a^8x^{10}+\frac{8}{9}a^7bx^9+\frac{7}{2}a^6b^2x^8+8a^5b^3x^7+\frac{35}{3}a^4b^4x^6+\frac{56}{5}a^3b^5x^5+7a^2b^6x^4+\frac{8}{3}ab^7x^3+\frac{1}{2}b^8x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^9,x, algorithm="maxima")
```

```
[Out] 1/10*a^8*x^10 + 8/9*a^7*b*x^9 + 7/2*a^6*b^2*x^8 + 8*a^5*b^3*x^7 + 35/3*a^4*
b^4*x^6 + 56/5*a^3*b^5*x^5 + 7*a^2*b^6*x^4 + 8/3*a*b^7*x^3 + 1/2*b^8*x^2
```

mupad [B] time = 1.06, size = 90, normalized size = 3.00

$$\frac{a^8\,x^{10}}{10}+\frac{8\,a^7\,b\,x^9}{9}+\frac{7\,a^6\,b^2\,x^8}{2}+8\,a^5\,b^3\,x^7+\frac{35\,a^4\,b^4\,x^6}{3}+\frac{56\,a^3\,b^5\,x^5}{5}+7\,a^2\,b^6\,x^4+\frac{8\,a\,b^7\,x^3}{3}+\frac{b^8\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(a + b/x)^8,x)
```

```
[Out] (a^8*x^10)/10 + (b^8*x^2)/2 + (8*a*b^7*x^3)/3 + (8*a^7*b*x^9)/9 + 7*a^2*b^6
*x^4 + (56*a^3*b^5*x^5)/5 + (35*a^4*b^4*x^6)/3 + 8*a^5*b^3*x^7 + (7*a^6*b^2
*x^8)/2
```

sympy [B] time = 0.11, size = 104, normalized size = 3.47

$$\frac{a^8x^{10}}{10}+\frac{8a^7bx^9}{9}+\frac{7a^6b^2x^8}{2}+8a^5b^3x^7+\frac{35a^4b^4x^6}{3}+\frac{56a^3b^5x^5}{5}+7a^2b^6x^4+\frac{8ab^7x^3}{3}+\frac{b^8x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**8*x**9,x)
```

```
[Out] a**8*x**10/10 + 8*a**7*b*x**9/9 + 7*a**6*b**2*x**8/2 + 8*a**5*b**3*x**7 + 3
5*a**4*b**4*x**6/3 + 56*a**3*b**5*x**5/5 + 7*a**2*b**6*x**4 + 8*a*b**7*x**3
/3 + b**8*x**2/2
```

3.1590 $\int \left(a + \frac{b}{x}\right)^8 x^8 \, dx$

Optimal. Leaf size=14

$$\frac{(ax+b)^9}{9a}$$

```
[Out] 1/9*(a*x+b)^9/a
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 32}

$$\frac{(ax+b)^9}{9a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^8*x^8,x]
```

```
[Out] (b + a*x)^9/(9*a)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(a + \frac{b}{x}\right)^8 x^8 \, dx &= \int (b+ax)^8 \, dx \\ &= \frac{(b+ax)^9}{9a}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{(ax+b)^9}{9a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^8*x^8,x]
```

```
[Out] (b + a*x)^9/(9*a)
```

fricas [B] time = 0.86, size = 86, normalized size = 6.14

$$\frac{1}{9}a^8x^9 + a^7bx^8 + 4\,a^6b^2x^7 + \frac{28}{3}a^5b^3x^6 + 14\,a^4b^4x^5 + 14\,a^3b^5x^4 + \frac{28}{3}a^2b^6x^3 + 4\,ab^7x^2 + b^8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^8,x, algorithm="fricas")
```

```
[Out] 1/9*a^8*x^9 + a^7*b*x^8 + 4*a^6*b^2*x^7 + 28/3*a^5*b^3*x^6 + 14*a^4*b^4*x^5
 + 14*a^3*b^5*x^4 + 28/3*a^2*b^6*x^3 + 4*a*b^7*x^2 + b^8*x
```

giac [**B**] time = 0.15, size = 86, normalized size = 6.14

$$\frac{1}{9}a^8x^9 + a^7bx^8 + 4\,a^6b^2x^7 + \frac{28}{3}a^5b^3x^6 + 14\,a^4b^4x^5 + 14\,a^3b^5x^4 + \frac{28}{3}a^2b^6x^3 + 4\,ab^7x^2 + b^8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^8,x, algorithm="giac")
```

```
[Out] 1/9*a^8*x^9 + a^7*b*x^8 + 4*a^6*b^2*x^7 + 28/3*a^5*b^3*x^6 + 14*a^4*b^4*x^5
 + 14*a^3*b^5*x^4 + 28/3*a^2*b^6*x^3 + 4*a*b^7*x^2 + b^8*x
```

maple [**A**] time = 0.00, size = 13, normalized size = 0.93

$$\frac{(ax+b)^9}{9a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^8*x^8,x)
```

```
[Out] 1/9*(a*x+b)^9/a
```

maxima [**B**] time = 0.97, size = 86, normalized size = 6.14

$$\frac{1}{9}a^8x^9 + a^7bx^8 + 4\,a^6b^2x^7 + \frac{28}{3}a^5b^3x^6 + 14\,a^4b^4x^5 + 14\,a^3b^5x^4 + \frac{28}{3}a^2b^6x^3 + 4\,ab^7x^2 + b^8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^8,x, algorithm="maxima")
```

```
[Out] 1/9*a^8*x^9 + a^7*b*x^8 + 4*a^6*b^2*x^7 + 28/3*a^5*b^3*x^6 + 14*a^4*b^4*x^5
 + 14*a^3*b^5*x^4 + 28/3*a^2*b^6*x^3 + 4*a*b^7*x^2 + b^8*x
```

mupad [**B**] time = 0.05, size = 86, normalized size = 6.14

$$\frac{a^8\,x^9}{9} + a^7\,b\,x^8 + 4\,a^6\,b^2\,x^7 + \frac{28\,a^5\,b^3\,x^6}{3} + 14\,a^4\,b^4\,x^5 + 14\,a^3\,b^5\,x^4 + \frac{28\,a^2\,b^6\,x^3}{3} + 4\,a\,b^7\,x^2 + b^8\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b/x)^8,x)
```

```
[Out] b^8*x + (a^8*x^9)/9 + 4*a*b^7*x^2 + a^7*b*x^8 + (28*a^2*b^6*x^3)/3 + 14*a^3
*b^5*x^4 + 14*a^4*b^4*x^5 + (28*a^5*b^3*x^6)/3 + 4*a^6*b^2*x^7
```

sympy [**B**] time = 0.09, size = 94, normalized size = 6.71

$$\frac{a^8x^9}{9} + a^7bx^8 + 4a^6b^2x^7 + \frac{28a^5b^3x^6}{3} + 14a^4b^4x^5 + 14a^3b^5x^4 + \frac{28a^2b^6x^3}{3} + 4ab^7x^2 + b^8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**8*x**8,x)
```

```
[Out] a**8*x**9/9 + a**7*b*x**8 + 4*a**6*b**2*x**7 + 28*a**5*b**3*x**6/3 + 14*a**
4*b**4*x**5 + 14*a**3*b**5*x**4 + 28*a**2*b**6*x**3/3 + 4*a*b**7*x**2 + b**
8*x
```

3.1591 $\int \left(a+\frac{b}{x}\right)^8 x^7\,dx$

Optimal. Leaf size=98

$$\frac{a^8x^8}{8}+\frac{8}{7}a^7bx^7+\frac{14}{3}a^6b^2x^6+\frac{56}{5}a^5b^3x^5+\frac{35}{2}a^4b^4x^4+\frac{56}{3}a^3b^5x^3+14a^2b^6x^2+8ab^7x+b^8\log(x)$$

```
[Out] 8*a*b^7*x+14*a^2*b^6*x^2+56/3*a^3*b^5*x^3+35/2*a^4*b^4*x^4+56/5*a^5*b^3*x^5
+14/3*a^6*b^2*x^6+8/7*a^7*b*x^7+1/8*a^8*x^8+b^8*ln(x)
```

Rubi [A] time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$14a^2b^6x^2+\frac{56}{3}a^3b^5x^3+\frac{35}{2}a^4b^4x^4+\frac{56}{5}a^5b^3x^5+\frac{14}{3}a^6b^2x^6+\frac{8}{7}a^7bx^7+\frac{a^8x^8}{8}+8ab^7x+b^8\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^8*x^7,x]
```

```
[Out] 8*a*b^7*x + 14*a^2*b^6*x^2 + (56*a^3*b^5*x^3)/3 + (35*a^4*b^4*x^4)/2 + (56*
a^5*b^3*x^5)/5 + (14*a^6*b^2*x^6)/3 + (8*a^7*b*x^7)/7 + (a^8*x^8)/8 + b^8*L
og[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^8 x^7\,dx &= \int \frac{(b+ax)^8}{x}\,dx\\ &= \int \left(8ab^7+\frac{b^8}{x}+28a^2b^6x+56a^3b^5x^2+70a^4b^4x^3+56a^5b^3x^4+28a^6b^2x^5+8a^7bx^6+a^8x^7\right.\\ &= 8ab^7x+14a^2b^6x^2+\frac{56}{3}a^3b^5x^3+\frac{35}{2}a^4b^4x^4+\frac{56}{5}a^5b^3x^5+\frac{14}{3}a^6b^2x^6+\frac{8}{7}a^7bx^7+\frac{a^8x^8}{8}+b^8\end{aligned}$$

Mathematica [A] time = 0.01, size = 98, normalized size = 1.00

$$\frac{a^8x^8}{8}+\frac{8}{7}a^7bx^7+\frac{14}{3}a^6b^2x^6+\frac{56}{5}a^5b^3x^5+\frac{35}{2}a^4b^4x^4+\frac{56}{3}a^3b^5x^3+14a^2b^6x^2+8ab^7x+b^8\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^8*x^7,x]
```

```
[Out] 8*a*b^7*x + 14*a^2*b^6*x^2 + (56*a^3*b^5*x^3)/3 + (35*a^4*b^4*x^4)/2 + (56*
a^5*b^3*x^5)/5 + (14*a^6*b^2*x^6)/3 + (8*a^7*b*x^7)/7 + (a^8*x^8)/8 + b^8*L
og[x]
```

fricas [A] time = 1.09, size = 86, normalized size = 0.88

$$\frac{1}{8}a^8x^8+\frac{8}{7}a^7bx^7+\frac{14}{3}a^6b^2x^6+\frac{56}{5}a^5b^3x^5+\frac{35}{2}a^4b^4x^4+\frac{56}{3}a^3b^5x^3+14a^2b^6x^2+8ab^7x+b^8\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^7,x, algorithm="fricas")
```

```
[Out] 1/8*a^8*x^8 + 8/7*a^7*b*x^7 + 14/3*a^6*b^2*x^6 + 56/5*a^5*b^3*x^5 + 35/2*a^
4*b^4*x^4 + 56/3*a^3*b^5*x^3 + 14*a^2*b^6*x^2 + 8*a*b^7*x + b^8*log(x)
```

giac [A] time = 0.18, size = 87, normalized size = 0.89

$$\frac{1}{8}a^8x^8+\frac{8}{7}a^7bx^7+\frac{14}{3}a^6b^2x^6+\frac{56}{5}a^5b^3x^5+\frac{35}{2}a^4b^4x^4+\frac{56}{3}a^3b^5x^3+14a^2b^6x^2+8ab^7x+b^8\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^7,x, algorithm="giac")
```

```
[Out] 1/8*a^8*x^8 + 8/7*a^7*b*x^7 + 14/3*a^6*b^2*x^6 + 56/5*a^5*b^3*x^5 + 35/2*a^
4*b^4*x^4 + 56/3*a^3*b^5*x^3 + 14*a^2*b^6*x^2 + 8*a*b^7*x + b^8*log(abs(x))
```

maple [A] time = 0.00, size = 87, normalized size = 0.89

$$\frac{a^8x^8}{8}+\frac{8a^7b\,x^7}{7}+\frac{14a^6b^2x^6}{3}+\frac{56a^5b^3x^5}{5}+\frac{35a^4b^4x^4}{2}+\frac{56a^3b^5x^3}{3}+14a^2b^6x^2+8a\,b^7x+b^8\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^8*x^7,x)
```

```
[Out] 8*a*b^7*x+14*a^2*b^6*x^2+56/3*a^3*b^5*x^3+35/2*a^4*b^4*x^4+56/5*a^5*b^3*x^5
+14/3*a^6*b^2*x^6+8/7*a^7*b*x^7+1/8*a^8*x^8+b^8*ln(x)
```

maxima [A] time = 0.97, size = 86, normalized size = 0.88

$$\frac{1}{8}a^8x^8+\frac{8}{7}a^7bx^7+\frac{14}{3}a^6b^2x^6+\frac{56}{5}a^5b^3x^5+\frac{35}{2}a^4b^4x^4+\frac{56}{3}a^3b^5x^3+14a^2b^6x^2+8ab^7x+b^8\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^7,x, algorithm="maxima")
```

```
[Out] 1/8*a^8*x^8 + 8/7*a^7*b*x^7 + 14/3*a^6*b^2*x^6 + 56/5*a^5*b^3*x^5 + 35/2*a^
4*b^4*x^4 + 56/3*a^3*b^5*x^3 + 14*a^2*b^6*x^2 + 8*a*b^7*x + b^8*log(x)
```

mupad [B] time = 1.06, size = 86, normalized size = 0.88

$$b^8\,\ln(x)+\frac{a^8\,x^8}{8}+\frac{8\,a^7\,b\,x^7}{7}+14\,a^2\,b^6\,x^2+\frac{56\,a^3\,b^5\,x^3}{3}+\frac{35\,a^4\,b^4\,x^4}{2}+\frac{56\,a^5\,b^3\,x^5}{5}+\frac{14\,a^6\,b^2\,x^6}{3}+8\,a\,b^7\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b/x)^8,x)
```

```
[Out] b^8*log(x) + (a^8*x^8)/8 + (8*a^7*b*x^7)/7 + 14*a^2*b^6*x^2 + (56*a^3*b^5*x
^3)/3 + (35*a^4*b^4*x^4)/2 + (56*a^5*b^3*x^5)/5 + (14*a^6*b^2*x^6)/3 + 8*a*
b^7*x
```

sympy [**A**] time = 0.21, size = 100, normalized size = 1.02

$$\frac{a^8x^8}{8}+\frac{8a^7bx^7}{7}+\frac{14a^6b^2x^6}{3}+\frac{56a^5b^3x^5}{5}+\frac{35a^4b^4x^4}{2}+\frac{56a^3b^5x^3}{3}+14a^2b^6x^2+8ab^7x+b^8\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**8*x**7,x)
```

```
[Out] a**8*x**8/8 + 8*a**7*b*x**7/7 + 14*a**6*b**2*x**6/3 + 56*a**5*b**3*x**5/5 +
 35*a**4*b**4*x**4/2 + 56*a**3*b**5*x**3/3 + 14*a**2*b**6*x**2 + 8*a*b**7*x
 + b**8*log(x)
```

3.1592 $\int \left(a+\frac{b}{x}\right)^8 x^6\,dx$

Optimal. Leaf size=95

$$\frac{a^8x^7}{7}+\frac{4}{3}a^7bx^6+\frac{28}{5}a^6b^2x^5+14a^5b^3x^4+\frac{70}{3}a^4b^4x^3+28a^3b^5x^2+28a^2b^6x+8ab^7\log(x)-\frac{b^8}{x}$$

```
[Out] -b^8/x+28*a^2*b^6*x+28*a^3*b^5*x^2+70/3*a^4*b^4*x^3+14*a^5*b^3*x^4+28/5*a^6
*b^2*x^5+4/3*a^7*b*x^6+1/7*a^8*x^7+8*a*b^7*ln(x)
```

Rubi [A] time = 0.04, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$\frac{28}{5}a^6b^2x^5+14a^5b^3x^4+\frac{70}{3}a^4b^4x^3+28a^3b^5x^2+28a^2b^6x+\frac{4}{3}a^7bx^6+\frac{a^8x^7}{7}+8ab^7\log(x)-\frac{b^8}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^8*x^6,x]
```

```
[Out] -(b^8/x) + 28*a^2*b^6*x + 28*a^3*b^5*x^2 + (70*a^4*b^4*x^3)/3 + 14*a^5*b^3*
x^4 + (28*a^6*b^2*x^5)/5 + (4*a^7*b*x^6)/3 + (a^8*x^7)/7 + 8*a*b^7*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^8 x^6\,dx &= \int \frac{(b+ax)^8}{x^2}\,dx\\ &= \int \left(28a^2b^6+\frac{b^8}{x^2}+\frac{8ab^7}{x}+56a^3b^5x+70a^4b^4x^2+56a^5b^3x^3+28a^6b^2x^4+8a^7bx^5+a^8x^6\right)dx\\ &= -\frac{b^8}{x}+28a^2b^6x+28a^3b^5x^2+\frac{70}{3}a^4b^4x^3+14a^5b^3x^4+\frac{28}{5}a^6b^2x^5+\frac{4}{3}a^7bx^6+\frac{a^8x^7}{7}+8ab^7\log\end{aligned}$$

Mathematica [A] time = 0.01, size = 95, normalized size = 1.00

$$\frac{a^8x^7}{7}+\frac{4}{3}a^7bx^6+\frac{28}{5}a^6b^2x^5+14a^5b^3x^4+\frac{70}{3}a^4b^4x^3+28a^3b^5x^2+28a^2b^6x+8ab^7\log(x)-\frac{b^8}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^8*x^6,x]
```

```
[Out] -(b^8/x) + 28*a^2*b^6*x + 28*a^3*b^5*x^2 + (70*a^4*b^4*x^3)/3 + 14*a^5*b^3*
x^4 + (28*a^6*b^2*x^5)/5 + (4*a^7*b*x^6)/3 + (a^8*x^7)/7 + 8*a*b^7*Log[x]
```

fricas [A] time = 0.85, size = 92, normalized size = 0.97

$$\frac{15\,a^8x^8+140\,a^7bx^7+588\,a^6b^2x^6+1470\,a^5b^3x^5+2450\,a^4b^4x^4+2940\,a^3b^5x^3+2940\,a^2b^6x^2+840\,ab^7x\log(x}{105\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^6,x, algorithm="fricas")
```

```
[Out] 1/105*(15*a^8*x^8 + 140*a^7*b*x^7 + 588*a^6*b^2*x^6 + 1470*a^5*b^3*x^5 + 24
50*a^4*b^4*x^4 + 2940*a^3*b^5*x^3 + 2940*a^2*b^6*x^2 + 840*a*b^7*x*log(x) -
 105*b^8)/x
```

giac [A] time = 0.15, size = 88, normalized size = 0.93

$$\frac{1}{7}\,a^8x^7+\frac{4}{3}\,a^7bx^6+\frac{28}{5}\,a^6b^2x^5+14\,a^5b^3x^4+\frac{70}{3}\,a^4b^4x^3+28\,a^3b^5x^2+28\,a^2b^6x+8\,ab^7\log\left(|x|\right)-\frac{b^8}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^6,x, algorithm="giac")
```

```
[Out] 1/7*a^8*x^7 + 4/3*a^7*b*x^6 + 28/5*a^6*b^2*x^5 + 14*a^5*b^3*x^4 + 70/3*a^4*
b^4*x^3 + 28*a^3*b^5*x^2 + 28*a^2*b^6*x + 8*a*b^7*log(abs(x)) - b^8/x
```

maple [A] time = 0.00, size = 88, normalized size = 0.93

$$\frac{a^8x^7}{7}+\frac{4a^7b\,x^6}{3}+\frac{28a^6b^2x^5}{5}+14a^5b^3x^4+\frac{70a^4b^4x^3}{3}+28a^3b^5x^2+28a^2b^6x+8a\,b^7\ln(x)-\frac{b^8}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^8*x^6,x)
```

```
[Out] -b^8/x+28*a^2*b^6*x+28*a^3*b^5*x^2+70/3*a^4*b^4*x^3+14*a^5*b^3*x^4+28/5*a^6
*b^2*x^5+4/3*a^7*b*x^6+1/7*a^8*x^7+8*a*b^7*ln(x)
```

maxima [A] time = 1.01, size = 87, normalized size = 0.92

$$\frac{1}{7}\,a^8x^7+\frac{4}{3}\,a^7bx^6+\frac{28}{5}\,a^6b^2x^5+14\,a^5b^3x^4+\frac{70}{3}\,a^4b^4x^3+28\,a^3b^5x^2+28\,a^2b^6x+8\,ab^7\log(x)-\frac{b^8}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^6,x, algorithm="maxima")
```

```
[Out] 1/7*a^8*x^7 + 4/3*a^7*b*x^6 + 28/5*a^6*b^2*x^5 + 14*a^5*b^3*x^4 + 70/3*a^4*
b^4*x^3 + 28*a^3*b^5*x^2 + 28*a^2*b^6*x + 8*a*b^7*log(x) - b^8/x
```

mupad [B] time = 0.06, size = 87, normalized size = 0.92

$$\frac{a^8\,x^7}{7}-\frac{b^8}{x}+28\,a^2\,b^6\,x+\frac{4\,a^7\,b\,x^6}{3}+8\,a\,b^7\,\ln(x)+28\,a^3\,b^5\,x^2+\frac{70\,a^4\,b^4\,x^3}{3}+14\,a^5\,b^3\,x^4+\frac{28\,a^6\,b^2\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b/x)^8,x)
```

```
[Out] (a^8*x^7)/7 - b^8/x + 28*a^2*b^6*x + (4*a^7*b*x^6)/3 + 8*a*b^7*log(x) + 28*
a^3*b^5*x^2 + (70*a^4*b^4*x^3)/3 + 14*a^5*b^3*x^4 + (28*a^6*b^2*x^5)/5
```

sympy [A] time = 0.22, size = 95, normalized size = 1.00

$$\frac{a^8x^7}{7}+\frac{4a^7bx^6}{3}+\frac{28a^6b^2x^5}{5}+14a^5b^3x^4+\frac{70a^4b^4x^3}{3}+28a^3b^5x^2+28a^2b^6x+8ab^7\log\left(x\right)-\frac{b^8}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**8*x**6,x)
```

```
[Out] a**8*x**7/7 + 4*a**7*b*x**6/3 + 28*a**6*b**2*x**5/5 + 14*a**5*b**3*x**4 + 7
0*a**4*b**4*x**3/3 + 28*a**3*b**5*x**2 + 28*a**2*b**6*x + 8*a*b**7*log(x) -
 b**8/x
```

3.1593 $\int \left(a+\frac{b}{x}\right)^8 x^5\,dx$

Optimal. Leaf size=95

$$\frac{a^8x^6}{6}+\frac{8}{5}a^7bx^5+7a^6b^2x^4+\frac{56}{3}a^5b^3x^3+35a^4b^4x^2+56a^3b^5x+28a^2b^6\log(x)-\frac{8ab^7}{x}-\frac{b^8}{2x^2}$$

```
[Out] -1/2*b^8/x^2-8*a*b^7/x+56*a^3*b^5*x+35*a^4*b^4*x^2+56/3*a^5*b^3*x^3+7*a^6*b
^2*x^4+8/5*a^7*b*x^5+1/6*a^8*x^6+28*a^2*b^6*ln(x)
```

Rubi [A] time = 0.04, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$7a^6b^2x^4+\frac{56}{3}a^5b^3x^3+35a^4b^4x^2+56a^3b^5x+28a^2b^6\log(x)+\frac{8}{5}a^7bx^5+\frac{a^8x^6}{6}-\frac{8ab^7}{x}-\frac{b^8}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^8*x^5,x]
```

```
[Out] -b^8/(2*x^2) - (8*a*b^7)/x + 56*a^3*b^5*x + 35*a^4*b^4*x^2 + (56*a^5*b^3*x^
3)/3 + 7*a^6*b^2*x^4 + (8*a^7*b*x^5)/5 + (a^8*x^6)/6 + 28*a^2*b^6*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^8 x^5\,dx &= \int \frac{(b+ax)^8}{x^3}\,dx\\ &= \int \left(56a^3b^5+\frac{b^8}{x^3}+\frac{8ab^7}{x^2}+\frac{28a^2b^6}{x}+70a^4b^4x+56a^5b^3x^2+28a^6b^2x^3+8a^7bx^4+a^8x^5\right)dx\\ &= -\frac{b^8}{2x^2}-\frac{8ab^7}{x}+56a^3b^5x+35a^4b^4x^2+\frac{56}{3}a^5b^3x^3+7a^6b^2x^4+\frac{8}{5}a^7bx^5+\frac{a^8x^6}{6}+28a^2b^6\log\end{aligned}$$

Mathematica [A] time = 0.01, size = 95, normalized size = 1.00

$$\frac{a^8x^6}{6}+\frac{8}{5}a^7bx^5+7a^6b^2x^4+\frac{56}{3}a^5b^3x^3+35a^4b^4x^2+56a^3b^5x+28a^2b^6\log(x)-\frac{8ab^7}{x}-\frac{b^8}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^8*x^5,x]
```

```
[Out] -1/2*b^8/x^2 - (8*a*b^7)/x + 56*a^3*b^5*x + 35*a^4*b^4*x^2 + (56*a^5*b^3*x^
3)/3 + 7*a^6*b^2*x^4 + (8*a^7*b*x^5)/5 + (a^8*x^6)/6 + 28*a^2*b^6*Log[x]
```

fricas [A] time = 0.63, size = 92, normalized size = 0.97

$$\frac{5\,a^8x^8+48\,a^7bx^7+210\,a^6b^2x^6+560\,a^5b^3x^5+1050\,a^4b^4x^4+1680\,a^3b^5x^3+840\,a^2b^6x^2\log(x)-240\,ab^7x-15\,b^8}{30\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^5,x, algorithm="fricas")
```

```
[Out] 1/30*(5*a^8*x^8 + 48*a^7*b*x^7 + 210*a^6*b^2*x^6 + 560*a^5*b^3*x^5 + 1050*a
^4*b^4*x^4 + 1680*a^3*b^5*x^3 + 840*a^2*b^6*x^2*log(x) - 240*a*b^7*x - 15*b
^8)/x^2
```

giac [A] time = 0.15, size = 87, normalized size = 0.92

$$\frac{1}{6}\,a^8x^6+\frac{8}{5}\,a^7bx^5+7\,a^6b^2x^4+\frac{56}{3}\,a^5b^3x^3+35\,a^4b^4x^2+56\,a^3b^5x+28\,a^2b^6\log\left(|x|\right)-\frac{16\,ab^7x+b^8}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^5,x, algorithm="giac")
```

```
[Out] 1/6*a^8*x^6 + 8/5*a^7*b*x^5 + 7*a^6*b^2*x^4 + 56/3*a^5*b^3*x^3 + 35*a^4*b^4
*x^2 + 56*a^3*b^5*x + 28*a^2*b^6*log(abs(x)) - 1/2*(16*a*b^7*x + b^8)/x^2
```

maple [A] time = 0.01, size = 88, normalized size = 0.93

$$\frac{a^8x^6}{6}+\frac{8a^7b\,x^5}{5}+7a^6b^2x^4+\frac{56a^5b^3x^3}{3}+35a^4b^4x^2+56a^3b^5x+28a^2b^6\ln(x)-\frac{8a\,b^7}{x}-\frac{b^8}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^8*x^5,x)
```

```
[Out] -1/2*b^8/x^2-8*a*b^7/x+56*a^3*b^5*x+35*a^4*b^4*x^2+56/3*a^5*b^3*x^3+7*a^6*b
^2*x^4+8/5*a^7*b*x^5+1/6*a^8*x^6+28*a^2*b^6*ln(x)
```

maxima [A] time = 0.99, size = 86, normalized size = 0.91

$$\frac{1}{6}\,a^8x^6+\frac{8}{5}\,a^7bx^5+7\,a^6b^2x^4+\frac{56}{3}\,a^5b^3x^3+35\,a^4b^4x^2+56\,a^3b^5x+28\,a^2b^6\log(x)-\frac{16\,ab^7x+b^8}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^5,x, algorithm="maxima")
```

```
[Out] 1/6*a^8*x^6 + 8/5*a^7*b*x^5 + 7*a^6*b^2*x^4 + 56/3*a^5*b^3*x^3 + 35*a^4*b^4
*x^2 + 56*a^3*b^5*x + 28*a^2*b^6*log(x) - 1/2*(16*a*b^7*x + b^8)/x^2
```

mupad [B] time = 0.05, size = 88, normalized size = 0.93

$$\frac{a^8\,x^6}{6}-\frac{\frac{b^8}{2}+8\,a\,x\,b^7}{x^2}+56\,a^3\,b^5\,x+\frac{8\,a^7\,b\,x^5}{5}+35\,a^4\,b^4\,x^2+\frac{56\,a^5\,b^3\,x^3}{3}+7\,a^6\,b^2\,x^4+28\,a^2\,b^6\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b/x)^8,x)
```

```
[Out] (a^8*x^6)/6 - (b^8/2 + 8*a*b^7*x)/x^2 + 56*a^3*b^5*x + (8*a^7*b*x^5)/5 + 35
*a^4*b^4*x^2 + (56*a^5*b^3*x^3)/3 + 7*a^6*b^2*x^4 + 28*a^2*b^6*log(x)
```

sympy [A] time = 0.26, size = 97, normalized size = 1.02

$$\frac{a^8x^6}{6}+\frac{8a^7bx^5}{5}+7a^6b^2x^4+\frac{56a^5b^3x^3}{3}+35a^4b^4x^2+56a^3b^5x+28a^2b^6\log(x)+\frac{-16ab^7x-b^8}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**8*x**5,x)
```

```
[Out] a**8*x**6/6 + 8*a**7*b*x**5/5 + 7*a**6*b**2*x**4 + 56*a**5*b**3*x**3/3 + 35
*a**4*b**4*x**2 + 56*a**3*b**5*x + 28*a**2*b**6*log(x) + (-16*a*b**7*x - b*
*8)/(2*x**2)
```

3.1594 $\int \left(a+\frac{b}{x}\right)^8 x^4\,dx$

Optimal. Leaf size=93

$$\frac{a^8x^5}{5}+2a^7bx^4+\frac{28}{3}a^6b^2x^3+28a^5b^3x^2+70a^4b^4x+56a^3b^5\log(x)-\frac{28a^2b^6}{x}-\frac{4ab^7}{x^2}-\frac{b^8}{3x^3}$$

```
[Out] -1/3*b^8/x^3-4*a*b^7/x^2-28*a^2*b^6/x+70*a^4*b^4*x+28*a^5*b^3*x^2+28/3*a^6*
b^2*x^3+2*a^7*b*x^4+1/5*a^8*x^5+56*a^3*b^5*ln(x)
```

Rubi [A] time = 0.04, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$\frac{28}{3}a^6b^2x^3+28a^5b^3x^2+70a^4b^4x-\frac{28a^2b^6}{x}+56a^3b^5\log(x)+2a^7bx^4+\frac{a^8x^5}{5}-\frac{4ab^7}{x^2}-\frac{b^8}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^8*x^4,x]
```

```
[Out] -b^8/(3*x^3) - (4*a*b^7)/x^2 - (28*a^2*b^6)/x + 70*a^4*b^4*x + 28*a^5*b^3*x
^2 + (28*a^6*b^2*x^3)/3 + 2*a^7*b*x^4 + (a^8*x^5)/5 + 56*a^3*b^5*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^8 x^4\,dx &= \int \frac{(b+ax)^8}{x^4}\,dx\\ &= \int \left(70a^4b^4+\frac{b^8}{x^4}+\frac{8ab^7}{x^3}+\frac{28a^2b^6}{x^2}+\frac{56a^3b^5}{x}+56a^5b^3x+28a^6b^2x^2+8a^7bx^3+a^8x^4\right)dx\\ &= -\frac{b^8}{3x^3}-\frac{4ab^7}{x^2}-\frac{28a^2b^6}{x}+70a^4b^4x+28a^5b^3x^2+\frac{28}{3}a^6b^2x^3+2a^7bx^4+\frac{a^8x^5}{5}+56a^3b^5\log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 93, normalized size = 1.00

$$\frac{a^8x^5}{5}+2a^7bx^4+\frac{28}{3}a^6b^2x^3+28a^5b^3x^2+70a^4b^4x+56a^3b^5\log(x)-\frac{28a^2b^6}{x}-\frac{4ab^7}{x^2}-\frac{b^8}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^8*x^4,x]
```

```
[Out] -1/3*b^8/x^3 - (4*a*b^7)/x^2 - (28*a^2*b^6)/x + 70*a^4*b^4*x + 28*a^5*b^3*x
^2 + (28*a^6*b^2*x^3)/3 + 2*a^7*b*x^4 + (a^8*x^5)/5 + 56*a^3*b^5*Log[x]
```

fricas [A] time = 1.03, size = 92, normalized size = 0.99

$$\frac{3\,a^8x^8+30\,a^7bx^7+140\,a^6b^2x^6+420\,a^5b^3x^5+1050\,a^4b^4x^4+840\,a^3b^5x^3\log(x)-420\,a^2b^6x^2-60\,ab^7x-5\,b^8}{15\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^4,x, algorithm="fricas")
```

```
[Out] 1/15*(3*a^8*x^8 + 30*a^7*b*x^7 + 140*a^6*b^2*x^6 + 420*a^5*b^3*x^5 + 1050*a
^4*b^4*x^4 + 840*a^3*b^5*x^3*log(x) - 420*a^2*b^6*x^2 - 60*a*b^7*x - 5*b^8)
/x^3
```

giac [A] time = 0.15, size = 87, normalized size = 0.94

$$\frac{1}{5}\,a^8x^5+2\,a^7bx^4+\frac{28}{3}\,a^6b^2x^3+28\,a^5b^3x^2+70\,a^4b^4x+56\,a^3b^5\log\left(|x|\right)-\frac{84\,a^2b^6x^2+12\,ab^7x+b^8}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^4,x, algorithm="giac")
```

```
[Out] 1/5*a^8*x^5 + 2*a^7*b*x^4 + 28/3*a^6*b^2*x^3 + 28*a^5*b^3*x^2 + 70*a^4*b^4*
x + 56*a^3*b^5*log(abs(x)) - 1/3*(84*a^2*b^6*x^2 + 12*a*b^7*x + b^8)/x^3
```

maple [A] time = 0.01, size = 88, normalized size = 0.95

$$\frac{a^8x^5}{5}+2a^7b\,x^4+\frac{28a^6b^2x^3}{3}+28a^5b^3x^2+70a^4b^4x+56a^3b^5\ln(x)-\frac{28a^2b^6}{x}-\frac{4a\,b^7}{x^2}-\frac{b^8}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^8*x^4,x)
```

```
[Out] -1/3*b^8/x^3-4*a*b^7/x^2-28*a^2*b^6/x+70*a^4*b^4*x+28*a^5*b^3*x^2+28/3*a^6*
b^2*x^3+2*a^7*b*x^4+1/5*a^8*x^5+56*a^3*b^5*ln(x)
```

maxima [A] time = 0.94, size = 86, normalized size = 0.92

$$\frac{1}{5}\,a^8x^5+2\,a^7bx^4+\frac{28}{3}\,a^6b^2x^3+28\,a^5b^3x^2+70\,a^4b^4x+56\,a^3b^5\log(x)-\frac{84\,a^2b^6x^2+12\,ab^7x+b^8}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^4,x, algorithm="maxima")
```

```
[Out] 1/5*a^8*x^5 + 2*a^7*b*x^4 + 28/3*a^6*b^2*x^3 + 28*a^5*b^3*x^2 + 70*a^4*b^4*
x + 56*a^3*b^5*log(x) - 1/3*(84*a^2*b^6*x^2 + 12*a*b^7*x + b^8)/x^3
```

mupad [B] time = 0.05, size = 88, normalized size = 0.95

$$\frac{a^8\,x^5}{5}-\frac{28\,a^2\,b^6\,x^2+4\,a\,b^7\,x+\frac{b^8}{3}}{x^3}+70\,a^4\,b^4\,x+2\,a^7\,b\,x^4+28\,a^5\,b^3\,x^2+\frac{28\,a^6\,b^2\,x^3}{3}+56\,a^3\,b^5\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b/x)^8,x)
```

```
[Out] (a^8*x^5)/5 - (b^8/3 + 28*a^2*b^6*x^2 + 4*a*b^7*x)/x^3 + 70*a^4*b^4*x + 2*a
^7*b*x^4 + 28*a^5*b^3*x^2 + (28*a^6*b^2*x^3)/3 + 56*a^3*b^5*log(x)
```

sympy [A] time = 0.32, size = 95, normalized size = 1.02

$$\frac{a^8x^5}{5}+2a^7bx^4+\frac{28a^6b^2x^3}{3}+28a^5b^3x^2+70a^4b^4x+56a^3b^5\log(x)+\frac{-84a^2b^6x^2-12ab^7x-b^8}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**8*x**4,x)
```

```
[Out] a**8*x**5/5 + 2*a**7*b*x**4 + 28*a**6*b**2*x**3/3 + 28*a**5*b**3*x**2 + 70*
a**4*b**4*x + 56*a**3*b**5*log(x) + (-84*a**2*b**6*x**2 - 12*a*b**7*x - b**
8)/(3*x**3)
```

3.1595 $\int \left(a+\frac{b}{x}\right)^8 x^3\,dx$

Optimal. Leaf size=95

$$\frac{a^8x^4}{4}+\frac{8}{3}a^7bx^3+14a^6b^2x^2+56a^5b^3x+70a^4b^4\log(x)-\frac{56a^3b^5}{x}-\frac{14a^2b^6}{x^2}-\frac{8ab^7}{3x^3}-\frac{b^8}{4x^4}$$

```
[Out] -1/4*b^8/x^4-8/3*a*b^7/x^3-14*a^2*b^6/x^2-56*a^3*b^5/x+56*a^5*b^3*x+14*a^6*
b^2*x^2+8/3*a^7*b*x^3+1/4*a^8*x^4+70*a^4*b^4*ln(x)
```

Rubi [A] time = 0.04, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$14a^6b^2x^2-\frac{14a^2b^6}{x^2}+56a^5b^3x-\frac{56a^3b^5}{x}+70a^4b^4\log(x)+\frac{8}{3}a^7bx^3+\frac{a^8x^4}{4}-\frac{8ab^7}{3x^3}-\frac{b^8}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^8*x^3,x]
```

```
[Out] -b^8/(4*x^4) - (8*a*b^7)/(3*x^3) - (14*a^2*b^6)/x^2 - (56*a^3*b^5)/x + 56*a
^5*b^3*x + 14*a^6*b^2*x^2 + (8*a^7*b*x^3)/3 + (a^8*x^4)/4 + 70*a^4*b^4*Log[
x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^8 x^3\,dx &= \int \frac{(b+ax)^8}{x^5}\,dx\\ &= \int \left(56a^5b^3+\frac{b^8}{x^5}+\frac{8ab^7}{x^4}+\frac{28a^2b^6}{x^3}+\frac{56a^3b^5}{x^2}+\frac{70a^4b^4}{x}+28a^6b^2x+8a^7bx^2+a^8x^3\right)dx\\ &= -\frac{b^8}{4x^4}-\frac{8ab^7}{3x^3}-\frac{14a^2b^6}{x^2}-\frac{56a^3b^5}{x}+56a^5b^3x+14a^6b^2x^2+\frac{8}{3}a^7bx^3+\frac{a^8x^4}{4}+70a^4b^4\log(x\end{aligned}$$

Mathematica [A] time = 0.00, size = 95, normalized size = 1.00

$$\frac{a^8x^4}{4}+\frac{8}{3}a^7bx^3+14a^6b^2x^2+56a^5b^3x+70a^4b^4\log(x)-\frac{56a^3b^5}{x}-\frac{14a^2b^6}{x^2}-\frac{8ab^7}{3x^3}-\frac{b^8}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^8*x^3,x]
```

```
[Out] -1/4*b^8/x^4 - (8*a*b^7)/(3*x^3) - (14*a^2*b^6)/x^2 - (56*a^3*b^5)/x + 56*a
^5*b^3*x + 14*a^6*b^2*x^2 + (8*a^7*b*x^3)/3 + (a^8*x^4)/4 + 70*a^4*b^4*Log[
x]
```

fricas [A] time = 0.66, size = 92, normalized size = 0.97

$$\frac{3\,a^8x^8+32\,a^7bx^7+168\,a^6b^2x^6+672\,a^5b^3x^5+840\,a^4b^4x^4\log(x)-672\,a^3b^5x^3-168\,a^2b^6x^2-32\,ab^7x-3\,b^8}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a^8*x^8 + 32*a^7*b*x^7 + 168*a^6*b^2*x^6 + 672*a^5*b^3*x^5 + 840*a^
4*b^4*x^4*log(x) - 672*a^3*b^5*x^3 - 168*a^2*b^6*x^2 - 32*a*b^7*x - 3*b^8)/
x^4
```

giac [A] time = 0.17, size = 89, normalized size = 0.94

$$\frac{1}{4}\,a^8x^4+\frac{8}{3}\,a^7bx^3+14\,a^6b^2x^2+56\,a^5b^3x+70\,a^4b^4\log\left(|x|\right)-\frac{672\,a^3b^5x^3+168\,a^2b^6x^2+32\,ab^7x+3\,b^8}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^3,x, algorithm="giac")
```

```
[Out] 1/4*a^8*x^4 + 8/3*a^7*b*x^3 + 14*a^6*b^2*x^2 + 56*a^5*b^3*x + 70*a^4*b^4*lo
g(abs(x)) - 1/12*(672*a^3*b^5*x^3 + 168*a^2*b^6*x^2 + 32*a*b^7*x + 3*b^8)/x
^4
```

maple [A] time = 0.01, size = 88, normalized size = 0.93

$$\frac{a^8x^4}{4}+\frac{8a^7b\,x^3}{3}+14a^6b^2x^2+56a^5b^3x+70a^4b^4\ln(x)-\frac{56a^3b^5}{x}-\frac{14a^2b^6}{x^2}-\frac{8a\,b^7}{3x^3}-\frac{b^8}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^8*x^3,x)
```

```
[Out] -1/4*b^8/x^4-8/3*a*b^7/x^3-14*a^2*b^6/x^2-56*a^3*b^5/x+56*a^5*b^3*x+14*a^6*
b^2*x^2+8/3*a^7*b*x^3+1/4*a^8*x^4+70*a^4*b^4*ln(x)
```

maxima [A] time = 1.06, size = 88, normalized size = 0.93

$$\frac{1}{4}\,a^8x^4+\frac{8}{3}\,a^7bx^3+14\,a^6b^2x^2+56\,a^5b^3x+70\,a^4b^4\log(x)-\frac{672\,a^3b^5x^3+168\,a^2b^6x^2+32\,ab^7x+3\,b^8}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^3,x, algorithm="maxima")
```

```
[Out] 1/4*a^8*x^4 + 8/3*a^7*b*x^3 + 14*a^6*b^2*x^2 + 56*a^5*b^3*x + 70*a^4*b^4*lo
g(x) - 1/12*(672*a^3*b^5*x^3 + 168*a^2*b^6*x^2 + 32*a*b^7*x + 3*b^8)/x^4
```

mupad [B] time = 0.05, size = 88, normalized size = 0.93

$$\frac{a^8\,x^4}{4}-\frac{56\,a^3\,b^5\,x^3+14\,a^2\,b^6\,x^2+\frac{8\,a\,b^7\,x}{3}+\frac{b^8}{4}}{x^4}+56\,a^5\,b^3\,x+\frac{8\,a^7\,b\,x^3}{3}+14\,a^6\,b^2\,x^2+70\,a^4\,b^4\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b/x)^8,x)
```

```
[Out] (a^8*x^4)/4 - (b^8/4 + 14*a^2*b^6*x^2 + 56*a^3*b^5*x^3 + (8*a*b^7*x)/3)/x^4
 + 56*a^5*b^3*x + (8*a^7*b*x^3)/3 + 14*a^6*b^2*x^2 + 70*a^4*b^4*log(x)
```

sympy [A] time = 0.37, size = 97, normalized size = 1.02

$$\frac{a^8x^4}{4}+\frac{8a^7bx^3}{3}+14a^6b^2x^2+56a^5b^3x+70a^4b^4\log(x)+\frac{-672a^3b^5x^3-168a^2b^6x^2-32ab^7x-3b^8}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**8*x**3,x)
```

```
[Out] a**8*x**4/4 + 8*a**7*b*x**3/3 + 14*a**6*b**2*x**2 + 56*a**5*b**3*x + 70*a**
4*b**4*log(x) + (-672*a**3*b**5*x**3 - 168*a**2*b**6*x**2 - 32*a*b**7*x - 3
*b**8)/(12*x**4)
```

3.1596 $\int \left(a+\frac{b}{x}\right)^8 x^2\,dx$

Optimal. Leaf size=93

$$\frac{a^8x^3}{3}+4a^7bx^2+28a^6b^2x+56a^5b^3\log(x)-\frac{70a^4b^4}{x}-\frac{28a^3b^5}{x^2}-\frac{28a^2b^6}{3x^3}-\frac{2ab^7}{x^4}-\frac{b^8}{5x^5}$$

```
[Out] -1/5*b^8/x^5-2*a*b^7/x^4-28/3*a^2*b^6/x^3-28*a^3*b^5/x^2-70*a^4*b^4/x+28*a^
6*b^2*x+4*a^7*b*x^2+1/3*a^8*x^3+56*a^5*b^3*ln(x)
```

Rubi [A] time = 0.04, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$-\frac{28a^3b^5}{x^2}-\frac{28a^2b^6}{3x^3}+28a^6b^2x-\frac{70a^4b^4}{x}+56a^5b^3\log(x)+4a^7bx^2+\frac{a^8x^3}{3}-\frac{2ab^7}{x^4}-\frac{b^8}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^8*x^2,x]
```

```
[Out] -b^8/(5*x^5) - (2*a*b^7)/x^4 - (28*a^2*b^6)/(3*x^3) - (28*a^3*b^5)/x^2 - (7
0*a^4*b^4)/x + 28*a^6*b^2*x + 4*a^7*b*x^2 + (a^8*x^3)/3 + 56*a^5*b^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^8 x^2\,dx &= \int \frac{(b+ax)^8}{x^6}\,dx\\ &= \int \left(28a^6b^2+\frac{b^8}{x^6}+\frac{8ab^7}{x^5}+\frac{28a^2b^6}{x^4}+\frac{56a^3b^5}{x^3}+\frac{70a^4b^4}{x^2}+\frac{56a^5b^3}{x}+8a^7bx+a^8x^2\right)dx\\ &= -\frac{b^8}{5x^5}-\frac{2ab^7}{x^4}-\frac{28a^2b^6}{3x^3}-\frac{28a^3b^5}{x^2}-\frac{70a^4b^4}{x}+28a^6b^2x+4a^7bx^2+\frac{a^8x^3}{3}+56a^5b^3\log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 93, normalized size = 1.00

$$\frac{a^8x^3}{3}+4a^7bx^2+28a^6b^2x+56a^5b^3\log(x)-\frac{70a^4b^4}{x}-\frac{28a^3b^5}{x^2}-\frac{28a^2b^6}{3x^3}-\frac{2ab^7}{x^4}-\frac{b^8}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^8*x^2,x]
```

```
[Out] -1/5*b^8/x^5 - (2*a*b^7)/x^4 - (28*a^2*b^6)/(3*x^3) - (28*a^3*b^5)/x^2 - (7
0*a^4*b^4)/x + 28*a^6*b^2*x + 4*a^7*b*x^2 + (a^8*x^3)/3 + 56*a^5*b^3*Log[x]
```

fricas [A] time = 1.02, size = 92, normalized size = 0.99

$$\frac{5\,a^8x^8+60\,a^7bx^7+420\,a^6b^2x^6+840\,a^5b^3x^5\log(x)-1050\,a^4b^4x^4-420\,a^3b^5x^3-140\,a^2b^6x^2-30\,ab^7x-3\,b^8}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^2,x, algorithm="fricas")
```

```
[Out] 1/15*(5*a^8*x^8 + 60*a^7*b*x^7 + 420*a^6*b^2*x^6 + 840*a^5*b^3*x^5*log(x) -
 1050*a^4*b^4*x^4 - 420*a^3*b^5*x^3 - 140*a^2*b^6*x^2 - 30*a*b^7*x - 3*b^8)
/x^5
```

giac [A] time = 0.19, size = 89, normalized size = 0.96

$$\frac{1}{3}\,a^8x^3+4\,a^7bx^2+28\,a^6b^2x+56\,a^5b^3\log\left(|x|\right)-\frac{1050\,a^4b^4x^4+420\,a^3b^5x^3+140\,a^2b^6x^2+30\,ab^7x+3\,b^8}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^2,x, algorithm="giac")
```

```
[Out] 1/3*a^8*x^3 + 4*a^7*b*x^2 + 28*a^6*b^2*x + 56*a^5*b^3*log(abs(x)) - 1/15*(1
050*a^4*b^4*x^4 + 420*a^3*b^5*x^3 + 140*a^2*b^6*x^2 + 30*a*b^7*x + 3*b^8)/x
^5
```

maple [A] time = 0.01, size = 88, normalized size = 0.95

$$\frac{a^8x^3}{3}+4a^7b\,x^2+28a^6b^2x+56a^5b^3\ln(x)-\frac{70a^4b^4}{x}-\frac{28a^3b^5}{x^2}-\frac{28a^2b^6}{3x^3}-\frac{2a\,b^7}{x^4}-\frac{b^8}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^8*x^2,x)
```

```
[Out] -1/5*b^8/x^5-2*a*b^7/x^4-28/3*a^2*b^6/x^3-28*a^3*b^5/x^2-70*a^4*b^4/x+28*a^
6*b^2*x+4*a^7*b*x^2+1/3*a^8*x^3+56*a^5*b^3*ln(x)
```

maxima [A] time = 0.95, size = 88, normalized size = 0.95

$$\frac{1}{3}\,a^8x^3+4\,a^7bx^2+28\,a^6b^2x+56\,a^5b^3\log(x)-\frac{1050\,a^4b^4x^4+420\,a^3b^5x^3+140\,a^2b^6x^2+30\,ab^7x+3\,b^8}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^8*x^3 + 4*a^7*b*x^2 + 28*a^6*b^2*x + 56*a^5*b^3*log(x) - 1/15*(1050*a
^4*b^4*x^4 + 420*a^3*b^5*x^3 + 140*a^2*b^6*x^2 + 30*a*b^7*x + 3*b^8)/x^5
```

mupad [B] time = 1.05, size = 88, normalized size = 0.95

$$\frac{a^8\,x^3}{3}-\frac{70\,a^4\,b^4\,x^4+28\,a^3\,b^5\,x^3+\frac{28\,a^2\,b^6\,x^2}{3}+2\,a\,b^7\,x+\frac{b^8}{5}}{x^5}+28\,a^6\,b^2\,x+4\,a^7\,b\,x^2+56\,a^5\,b^3\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b/x)^8,x)
```

```
[Out] (a^8*x^3)/3 - (b^8/5 + (28*a^2*b^6*x^2)/3 + 28*a^3*b^5*x^3 + 70*a^4*b^4*x^4
 + 2*a*b^7*x)/x^5 + 28*a^6*b^2*x + 4*a^7*b*x^2 + 56*a^5*b^3*log(x)
```

sympy [**A**] time = 0.45, size = 95, normalized size = 1.02

$$\frac{a^8x^3}{3}+4a^7bx^2+28a^6b^2x+56a^5b^3\log(x)+\frac{-1050a^4b^4x^4-420a^3b^5x^3-140a^2b^6x^2-30ab^7x-3b^8}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**8*x**2,x)
```

```
[Out] a**8*x**3/3 + 4*a**7*b*x**2 + 28*a**6*b**2*x + 56*a**5*b**3*log(x) + (-1050
*a**4*b**4*x**4 - 420*a**3*b**5*x**3 - 140*a**2*b**6*x**2 - 30*a*b**7*x - 3
*b**8)/(15*x**5)
```

3.1597 $\int \left(a+\frac{b}{x}\right)^8 x\,dx$

Optimal. Leaf size=95

$$\frac{a^8x^2}{2}+8a^7bx+28a^6b^2\log(x)-\frac{56a^5b^3}{x}-\frac{35a^4b^4}{x^2}-\frac{56a^3b^5}{3x^3}-\frac{7a^2b^6}{x^4}-\frac{8ab^7}{5x^5}-\frac{b^8}{6x^6}$$

```
[Out] -1/6*b^8/x^6-8/5*a*b^7/x^5-7*a^2*b^6/x^4-56/3*a^3*b^5/x^3-35*a^4*b^4/x^2-56
*a^5*b^3/x+8*a^7*b*x+1/2*a^8*x^2+28*a^6*b^2*ln(x)
```

Rubi [A] time = 0.04, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {263, 43}

$$-\frac{35a^4b^4}{x^2}-\frac{56a^3b^5}{3x^3}-\frac{7a^2b^6}{x^4}-\frac{56a^5b^3}{x}+28a^6b^2\log(x)+8a^7bx+\frac{a^8x^2}{2}-\frac{8ab^7}{5x^5}-\frac{b^8}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^8*x,x]
```

```
[Out] -b^8/(6*x^6) - (8*a*b^7)/(5*x^5) - (7*a^2*b^6)/x^4 - (56*a^3*b^5)/(3*x^3) -
 (35*a^4*b^4)/x^2 - (56*a^5*b^3)/x + 8*a^7*b*x + (a^8*x^2)/2 + 28*a^6*b^2*L
og[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^8 x\,dx &= \int \frac{(b+ax)^8}{x^7}\,dx\\ &= \int \left(8a^7b+\frac{b^8}{x^7}+\frac{8ab^7}{x^6}+\frac{28a^2b^6}{x^5}+\frac{56a^3b^5}{x^4}+\frac{70a^4b^4}{x^3}+\frac{56a^5b^3}{x^2}+\frac{28a^6b^2}{x}+a^8x\right)dx\\ &= -\frac{b^8}{6x^6}-\frac{8ab^7}{5x^5}-\frac{7a^2b^6}{x^4}-\frac{56a^3b^5}{3x^3}-\frac{35a^4b^4}{x^2}-\frac{56a^5b^3}{x}+8a^7bx+\frac{a^8x^2}{2}+28a^6b^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 95, normalized size = 1.00

$$\frac{a^8x^2}{2}+8a^7bx+28a^6b^2\log(x)-\frac{56a^5b^3}{x}-\frac{35a^4b^4}{x^2}-\frac{56a^3b^5}{3x^3}-\frac{7a^2b^6}{x^4}-\frac{8ab^7}{5x^5}-\frac{b^8}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^8*x,x]
```

```
[Out] -1/6*b^8/x^6 - (8*a*b^7)/(5*x^5) - (7*a^2*b^6)/x^4 - (56*a^3*b^5)/(3*x^3) -
 (35*a^4*b^4)/x^2 - (56*a^5*b^3)/x + 8*a^7*b*x + (a^8*x^2)/2 + 28*a^6*b^2*L
og[x]
```

fricas [A] time = 0.87, size = 92, normalized size = 0.97

$$\frac{15\,a^8x^8 + 240\,a^7bx^7 + 840\,a^6b^2x^6\log(x) - 1680\,a^5b^3x^5 - 1050\,a^4b^4x^4 - 560\,a^3b^5x^3 - 210\,a^2b^6x^2 - 48\,ab^7x - 5\,b^8}{30\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x,x, algorithm="fricas")
```

```
[Out] 1/30*(15*a^8*x^8 + 240*a^7*b*x^7 + 840*a^6*b^2*x^6*log(x) - 1680*a^5*b^3*x^
5 - 1050*a^4*b^4*x^4 - 560*a^3*b^5*x^3 - 210*a^2*b^6*x^2 - 48*a*b^7*x - 5*b
^8)/x^6
```

giac [A] time = 0.15, size = 89, normalized size = 0.94

$$\frac{1}{2}\,a^8x^2+8\,a^7bx+28\,a^6b^2\log\left(|x|\right)-\frac{1680\,a^5b^3x^5 + 1050\,a^4b^4x^4 + 560\,a^3b^5x^3 + 210\,a^2b^6x^2 + 48\,ab^7x + 5\,b^8}{30\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x,x, algorithm="giac")
```

```
[Out] 1/2*a^8*x^2 + 8*a^7*b*x + 28*a^6*b^2*log(abs(x)) - 1/30*(1680*a^5*b^3*x^5 +
 1050*a^4*b^4*x^4 + 560*a^3*b^5*x^3 + 210*a^2*b^6*x^2 + 48*a*b^7*x + 5*b^8)
/x^6
```

maple [A] time = 0.01, size = 88, normalized size = 0.93

$$\frac{a^8x^2}{2} + 8a^7bx + 28a^6b^2\ln(x) - \frac{56a^5b^3}{x} - \frac{35a^4b^4}{x^2} - \frac{56a^3b^5}{3x^3} - \frac{7a^2b^6}{x^4} - \frac{8a\,b^7}{5x^5} - \frac{b^8}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^8*x,x)
```

```
[Out] -1/6*b^8/x^6-8/5*a*b^7/x^5-7*a^2*b^6/x^4-56/3*a^3*b^5/x^3-35*a^4*b^4/x^2-56
*a^5*b^3/x+8*a^7*b*x+1/2*a^8*x^2+28*a^6*b^2*ln(x)
```

maxima [A] time = 0.92, size = 88, normalized size = 0.93

$$\frac{1}{2}\,a^8x^2+8\,a^7bx+28\,a^6b^2\log(x)-\frac{1680\,a^5b^3x^5 + 1050\,a^4b^4x^4 + 560\,a^3b^5x^3 + 210\,a^2b^6x^2 + 48\,ab^7x + 5\,b^8}{30\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8*x,x, algorithm="maxima")
```

```
[Out] 1/2*a^8*x^2 + 8*a^7*b*x + 28*a^6*b^2*log(x) - 1/30*(1680*a^5*b^3*x^5 + 1050
*a^4*b^4*x^4 + 560*a^3*b^5*x^3 + 210*a^2*b^6*x^2 + 48*a*b^7*x + 5*b^8)/x^6
```

mupad [B] time = 1.06, size = 88, normalized size = 0.93

$$\frac{a^8\,x^2}{2} - \frac{56\,a^5\,b^3\,x^5 + 35\,a^4\,b^4\,x^4 + \frac{56\,a^3\,b^5\,x^3}{3} + 7\,a^2\,b^6\,x^2 + \frac{8\,a\,b^7\,x}{5} + \frac{b^8}{6}}{x^6} + 28\,a^6\,b^2\,\ln(x) + 8\,a^7\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b/x)^8,x)
```

```
[Out] (a^8*x^2)/2 - (b^8/6 + 7*a^2*b^6*x^2 + (56*a^3*b^5*x^3)/3 + 35*a^4*b^4*x^4
+ 56*a^5*b^3*x^5 + (8*a*b^7*x)/5)/x^6 + 28*a^6*b^2*log(x) + 8*a^7*b*x
```

sympy [**A**] time = 0.52, size = 95, normalized size = 1.00

$$\frac{a^8x^2}{2}+8a^7bx+28a^6b^2\log(x)+\frac{-1680a^5b^3x^5-1050a^4b^4x^4-560a^3b^5x^3-210a^2b^6x^2-48ab^7x-5b^8}{30x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**8*x,x)
```

```
[Out] a**8*x**2/2 + 8*a**7*b*x + 28*a**6*b**2*log(x) + (-1680*a**5*b**3*x**5 - 10
50*a**4*b**4*x**4 - 560*a**3*b**5*x**3 - 210*a**2*b**6*x**2 - 48*a*b**7*x -
 5*b**8)/(30*x**6)
```

3.1598 $\int \left(a+\frac{b}{x}\right)^8 dx$

Optimal. Leaf size=94

$$a^8x+8a^7b\log(x)-\frac{28a^6b^2}{x}-\frac{28a^5b^3}{x^2}-\frac{70a^4b^4}{3x^3}-\frac{14a^3b^5}{x^4}-\frac{28a^2b^6}{5x^5}-\frac{4ab^7}{3x^6}-\frac{b^8}{7x^7}$$

```
[Out] -1/7*b^8/x^7-4/3*a*b^7/x^6-28/5*a^2*b^6/x^5-14*a^3*b^5/x^4-70/3*a^4*b^4/x^3
-28*a^5*b^3/x^2-28*a^6*b^2/x+a^8*x+8*a^7*b*ln(x)
```

Rubi [A] time = 0.04, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {193, 43}

$$-\frac{28a^5b^3}{x^2}-\frac{70a^4b^4}{3x^3}-\frac{14a^3b^5}{x^4}-\frac{28a^2b^6}{5x^5}-\frac{28a^6b^2}{x}+8a^7b\log(x)+a^8x-\frac{4ab^7}{3x^6}-\frac{b^8}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^8,x]
```

```
[Out] -b^8/(7*x^7) - (4*a*b^7)/(3*x^6) - (28*a^2*b^6)/(5*x^5) - (14*a^3*b^5)/x^4
- (70*a^4*b^4)/(3*x^3) - (28*a^5*b^3)/x^2 - (28*a^6*b^2)/x + a^8*x + 8*a^7*
b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^8 dx &= \int \frac{(b+ax)^8}{x^8}\,dx\\ &= \int \left(a^8+\frac{b^8}{x^8}+\frac{8ab^7}{x^7}+\frac{28a^2b^6}{x^6}+\frac{56a^3b^5}{x^5}+\frac{70a^4b^4}{x^4}+\frac{56a^5b^3}{x^3}+\frac{28a^6b^2}{x^2}+\frac{8a^7b}{x}\right)dx\\ &= -\frac{b^8}{7x^7}-\frac{4ab^7}{3x^6}-\frac{28a^2b^6}{5x^5}-\frac{14a^3b^5}{x^4}-\frac{70a^4b^4}{3x^3}-\frac{28a^5b^3}{x^2}-\frac{28a^6b^2}{x}+a^8x+8a^7b\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 94, normalized size = 1.00

$$a^8x+8a^7b\log(x)-\frac{28a^6b^2}{x}-\frac{28a^5b^3}{x^2}-\frac{70a^4b^4}{3x^3}-\frac{14a^3b^5}{x^4}-\frac{28a^2b^6}{5x^5}-\frac{4ab^7}{3x^6}-\frac{b^8}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^8,x]
```

```
[Out] -1/7*b^8/x^7 - (4*a*b^7)/(3*x^6) - (28*a^2*b^6)/(5*x^5) - (14*a^3*b^5)/x^4
- (70*a^4*b^4)/(3*x^3) - (28*a^5*b^3)/x^2 - (28*a^6*b^2)/x + a^8*x + 8*a^7*
b*Log[x]
```

fricas [A] time = 0.86, size = 92, normalized size = 0.98

$$\frac{105\,a^8x^8 + 840\,a^7bx^7\log(x) - 2940\,a^6b^2x^6 - 2940\,a^5b^3x^5 - 2450\,a^4b^4x^4 - 1470\,a^3b^5x^3 - 588\,a^2b^6x^2 - 140\,ab^7}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8,x, algorithm="fricas")
```

```
[Out] 1/105*(105*a^8*x^8 + 840*a^7*b*x^7*log(x) - 2940*a^6*b^2*x^6 - 2940*a^5*b^3
*x^5 - 2450*a^4*b^4*x^4 - 1470*a^3*b^5*x^3 - 588*a^2*b^6*x^2 - 140*a*b^7*x
- 15*b^8)/x^7
```

giac [A] time = 0.17, size = 88, normalized size = 0.94

$$a^8x+8\,a^7b\log\left(|x|\right)-\frac{2940\,a^6b^2x^6 + 2940\,a^5b^3x^5 + 2450\,a^4b^4x^4 + 1470\,a^3b^5x^3 + 588\,a^2b^6x^2 + 140\,ab^7x + 15\,b^8}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8,x, algorithm="giac")
```

```
[Out] a^8*x + 8*a^7*b*log(abs(x)) - 1/105*(2940*a^6*b^2*x^6 + 2940*a^5*b^3*x^5 +
2450*a^4*b^4*x^4 + 1470*a^3*b^5*x^3 + 588*a^2*b^6*x^2 + 140*a*b^7*x + 15*b^
8)/x^7
```

maple [A] time = 0.01, size = 87, normalized size = 0.93

$$a^8x + 8a^7b\ln(x) - \frac{28a^6b^2}{x} - \frac{28a^5b^3}{x^2} - \frac{70a^4b^4}{3x^3} - \frac{14a^3b^5}{x^4} - \frac{28a^2b^6}{5x^5} - \frac{4a\,b^7}{3x^6} - \frac{b^8}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^8,x)
```

```
[Out] -1/7*b^8/x^7-4/3*a*b^7/x^6-28/5*a^2*b^6/x^5-14*a^3*b^5/x^4-70/3*a^4*b^4/x^3
-28*a^5*b^3/x^2-28*a^6*b^2/x+a^8*x+8*a^7*b*ln(x)
```

maxima [A] time = 1.04, size = 87, normalized size = 0.93

$$a^8x+8\,a^7b\log(x)-\frac{2940\,a^6b^2x^6 + 2940\,a^5b^3x^5 + 2450\,a^4b^4x^4 + 1470\,a^3b^5x^3 + 588\,a^2b^6x^2 + 140\,ab^7x + 15\,b^8}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8,x, algorithm="maxima")
```

```
[Out] a^8*x + 8*a^7*b*log(x) - 1/105*(2940*a^6*b^2*x^6 + 2940*a^5*b^3*x^5 + 2450*
a^4*b^4*x^4 + 1470*a^3*b^5*x^3 + 588*a^2*b^6*x^2 + 140*a*b^7*x + 15*b^8)/x^
7
```

mupad [B] time = 1.07, size = 92, normalized size = 0.98

$$-\frac{\frac{b^8}{7} - a^8\,x^8 + \frac{28\,a^2\,b^6\,x^2}{5} + 14\,a^3\,b^5\,x^3 + \frac{70\,a^4\,b^4\,x^4}{3} + 28\,a^5\,b^3\,x^5 + 28\,a^6\,b^2\,x^6 + \frac{4\,a\,b^7\,x}{3} - 8\,a^7\,b\,x^7\,\ln(x)}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^8,x)
```

```
[Out] -(b^8/7 - a^8*x^8 + (28*a^2*b^6*x^2)/5 + 14*a^3*b^5*x^3 + (70*a^4*b^4*x^4)/
3 + 28*a^5*b^3*x^5 + 28*a^6*b^2*x^6 + (4*a*b^7*x)/3 - 8*a^7*b*x^7*log(x))/x
^7
```

sympy [**A**] time = 0.56, size = 94, normalized size = 1.00

$$a^8x+8a^7b\log(x)+\frac{-2940a^6b^2x^6-2940a^5b^3x^5-2450a^4b^4x^4-1470a^3b^5x^3-588a^2b^6x^2-140ab^7x-15b^8}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**8,x)
```

```
[Out] a**8*x + 8*a**7*b*log(x) + (-2940*a**6*b**2*x**6 - 2940*a**5*b**3*x**5 - 24
50*a**4*b**4*x**4 - 1470*a**3*b**5*x**3 - 588*a**2*b**6*x**2 - 140*a*b**7*x
 - 15*b**8)/(105*x**7)
```

3.1599 $\int \frac{\left(a+\frac{b}{x}\right)^8}{x}\,dx$

Optimal. Leaf size=100

$$a^8\log(x)-\frac{8a^7b}{x}-\frac{14a^6b^2}{x^2}-\frac{56a^5b^3}{3x^3}-\frac{35a^4b^4}{2x^4}-\frac{56a^3b^5}{5x^5}-\frac{14a^2b^6}{3x^6}-\frac{8ab^7}{7x^7}-\frac{b^8}{8x^8}$$

```
[Out] -1/8*b^8/x^8-8/7*a*b^7/x^7-14/3*a^2*b^6/x^6-56/5*a^3*b^5/x^5-35/2*a^4*b^4/x
^4-56/3*a^5*b^3/x^3-14*a^6*b^2/x^2-8*a^7*b/x+a^8*ln(x)
```

Rubi [A] time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$-\frac{14a^6b^2}{x^2}-\frac{56a^5b^3}{3x^3}-\frac{35a^4b^4}{2x^4}-\frac{56a^3b^5}{5x^5}-\frac{14a^2b^6}{3x^6}-\frac{8a^7b}{x}+a^8\log(x)-\frac{8ab^7}{7x^7}-\frac{b^8}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^8/x,x]
```

```
[Out] -b^8/(8*x^8) - (8*a*b^7)/(7*x^7) - (14*a^2*b^6)/(3*x^6) - (56*a^3*b^5)/(5*x
^5) - (35*a^4*b^4)/(2*x^4) - (56*a^5*b^3)/(3*x^3) - (14*a^6*b^2)/x^2 - (8*a
^7*b)/x + a^8*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^8}{x}\,dx &= \int \frac{(b+ax)^8}{x^9}\,dx\\ &= \int\left(\frac{b^8}{x^9}+\frac{8ab^7}{x^8}+\frac{28a^2b^6}{x^7}+\frac{56a^3b^5}{x^6}+\frac{70a^4b^4}{x^5}+\frac{56a^5b^3}{x^4}+\frac{28a^6b^2}{x^3}+\frac{8a^7b}{x^2}+\frac{a^8}{x}\right)dx\\ &= -\frac{b^8}{8x^8}-\frac{8ab^7}{7x^7}-\frac{14a^2b^6}{3x^6}-\frac{56a^3b^5}{5x^5}-\frac{35a^4b^4}{2x^4}-\frac{56a^5b^3}{3x^3}-\frac{14a^6b^2}{x^2}-\frac{8a^7b}{x}+a^8\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 100, normalized size = 1.00

$$a^8\log(x)-\frac{8a^7b}{x}-\frac{14a^6b^2}{x^2}-\frac{56a^5b^3}{3x^3}-\frac{35a^4b^4}{2x^4}-\frac{56a^3b^5}{5x^5}-\frac{14a^2b^6}{3x^6}-\frac{8ab^7}{7x^7}-\frac{b^8}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^8/x,x]
```

```
[Out] -1/8*b^8/x^8 - (8*a*b^7)/(7*x^7) - (14*a^2*b^6)/(3*x^6) - (56*a^3*b^5)/(5*x
^5) - (35*a^4*b^4)/(2*x^4) - (56*a^5*b^3)/(3*x^3) - (14*a^6*b^2)/x^2 - (8*a
^7*b)/x + a^8*Log[x]
```

fricas [A] time = 1.06, size = 92, normalized size = 0.92

$$\frac{840\,a^8x^8\log(x) - 6720\,a^7bx^7 - 11760\,a^6b^2x^6 - 15680\,a^5b^3x^5 - 14700\,a^4b^4x^4 - 9408\,a^3b^5x^3 - 3920\,a^2b^6x^2 - 960\,a}{840\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8/x,x, algorithm="fricas")
```

```
[Out] 1/840*(840*a^8*x^8*log(x) - 6720*a^7*b*x^7 - 11760*a^6*b^2*x^6 - 15680*a^5*
b^3*x^5 - 14700*a^4*b^4*x^4 - 9408*a^3*b^5*x^3 - 3920*a^2*b^6*x^2 - 960*a*b
^7*x - 105*b^8)/x^8
```

giac [A] time = 0.15, size = 90, normalized size = 0.90

$$a^8\log\left(|x|\right) - \frac{6720\,a^7bx^7 + 11760\,a^6b^2x^6 + 15680\,a^5b^3x^5 + 14700\,a^4b^4x^4 + 9408\,a^3b^5x^3 + 3920\,a^2b^6x^2 + 960\,ab^7x}{840\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8/x,x, algorithm="giac")
```

```
[Out] a^8*log(abs(x)) - 1/840*(6720*a^7*b*x^7 + 11760*a^6*b^2*x^6 + 15680*a^5*b^3
*x^5 + 14700*a^4*b^4*x^4 + 9408*a^3*b^5*x^3 + 3920*a^2*b^6*x^2 + 960*a*b^7*
x + 105*b^8)/x^8
```

maple [A] time = 0.01, size = 89, normalized size = 0.89

$$a^8\ln(x) - \frac{8a^7b}{x} - \frac{14a^6b^2}{x^2} - \frac{56a^5b^3}{3x^3} - \frac{35a^4b^4}{2x^4} - \frac{56a^3b^5}{5x^5} - \frac{14a^2b^6}{3x^6} - \frac{8a\,b^7}{7x^7} - \frac{b^8}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^8/x,x)
```

```
[Out] -1/8*b^8/x^8-8/7*a*b^7/x^7-14/3*a^2*b^6/x^6-56/5*a^3*b^5/x^5-35/2*a^4*b^4/x
^4-56/3*a^5*b^3/x^3-14*a^6*b^2/x^2-8*a^7*b/x+a^8*ln(x)
```

maxima [A] time = 1.07, size = 89, normalized size = 0.89

$$a^8\log(x) - \frac{6720\,a^7bx^7 + 11760\,a^6b^2x^6 + 15680\,a^5b^3x^5 + 14700\,a^4b^4x^4 + 9408\,a^3b^5x^3 + 3920\,a^2b^6x^2 + 960\,ab^7x +}{840\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8/x,x, algorithm="maxima")
```

```
[Out] a^8*log(x) - 1/840*(6720*a^7*b*x^7 + 11760*a^6*b^2*x^6 + 15680*a^5*b^3*x^5
+ 14700*a^4*b^4*x^4 + 9408*a^3*b^5*x^3 + 3920*a^2*b^6*x^2 + 960*a*b^7*x + 1
05*b^8)/x^8
```

mupad [B] time = 0.07, size = 89, normalized size = 0.89

$$a^8\,\ln(x) - \frac{8\,a^7\,b\,x^7 + 14\,a^6\,b^2\,x^6 + \frac{56\,a^5\,b^3\,x^5}{3} + \frac{35\,a^4\,b^4\,x^4}{2} + \frac{56\,a^3\,b^5\,x^3}{5} + \frac{14\,a^2\,b^6\,x^2}{3} + \frac{8\,a\,b^7\,x}{7} + \frac{b^8}{8}}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^8/x,x)
```

```
[Out] a^8*log(x) - (b^8/8 + 8*a^7*b*x^7 + (14*a^2*b^6*x^2)/3 + (56*a^3*b^5*x^3)/5
 + (35*a^4*b^4*x^4)/2 + (56*a^5*b^3*x^5)/3 + 14*a^6*b^2*x^6 + (8*a*b^7*x)/7
)/x^8
```

sympy [**A**] time = 0.64, size = 95, normalized size = 0.95

$$a^8 \log(x) + \frac{-6720a^7bx^7 - 11760a^6b^2x^6 - 15680a^5b^3x^5 - 14700a^4b^4x^4 - 9408a^3b^5x^3 - 3920a^2b^6x^2 - 960ab^7x -}{840x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**8/x,x)
```

```
[Out] a**8*log(x) + (-6720*a**7*b*x**7 - 11760*a**6*b**2*x**6 - 15680*a**5*b**3*x
**5 - 14700*a**4*b**4*x**4 - 9408*a**3*b**5*x**3 - 3920*a**2*b**6*x**2 - 96
0*a*b**7*x - 105*b**8)/(840*x**8)
```

3.1600 $\int \frac{\left(a+\frac{b}{x}\right)^8}{x^2}\,dx$

Optimal. Leaf size=16

$$-\frac{\left(a+\frac{b}{x}\right)^9}{9b}$$

```
[Out] -1/9*(a+b/x)^9/b
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$-\frac{\left(a+\frac{b}{x}\right)^9}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^8/x^2,x]
```

```
[Out] -(a + b/x)^9/(9*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^2}\,dx = -\frac{\left(a+\frac{b}{x}\right)^9}{9b}$$

Mathematica [B] time = 0.01, size = 96, normalized size = 6.00

$$-\frac{a^8}{x}-\frac{4a^7b}{x^2}-\frac{28a^6b^2}{3x^3}-\frac{14a^5b^3}{x^4}-\frac{14a^4b^4}{x^5}-\frac{28a^3b^5}{3x^6}-\frac{4a^2b^6}{x^7}-\frac{ab^7}{x^8}-\frac{b^8}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^8/x^2,x]
```

```
[Out] -1/9*b^8/x^9 - (a*b^7)/x^8 - (4*a^2*b^6)/x^7 - (28*a^3*b^5)/(3*x^6) - (14*a
^4*b^4)/x^5 - (14*a^5*b^3)/x^4 - (28*a^6*b^2)/(3*x^3) - (4*a^7*b)/x^2 - a^8
/x
```

fricas [B] time = 0.99, size = 88, normalized size = 5.50

$$-\frac{9\,a^8x^8+36\,a^7bx^7+84\,a^6b^2x^6+126\,a^5b^3x^5+126\,a^4b^4x^4+84\,a^3b^5x^3+36\,a^2b^6x^2+9\,ab^7x+b^8}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8/x^2,x, algorithm="fricas")
```

```
[Out] -1/9*(9*a^8*x^8 + 36*a^7*b*x^7 + 84*a^6*b^2*x^6 + 126*a^5*b^3*x^5 + 126*a^4
*b^4*x^4 + 84*a^3*b^5*x^3 + 36*a^2*b^6*x^2 + 9*a*b^7*x + b^8)/x^9
```

giac [A] time = 0.15, size = 14, normalized size = 0.88

$$-\frac{\left(a+\frac{b}{x}\right)^9}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8/x^2,x, algorithm="giac")
```

```
[Out] -1/9*(a + b/x)^9/b
```

maple [B] time = 0.01, size = 91, normalized size = 5.69

$$-\frac{a^8}{x}-\frac{4a^7b}{x^2}-\frac{28a^6b^2}{3x^3}-\frac{14a^5b^3}{x^4}-\frac{14a^4b^4}{x^5}-\frac{28a^3b^5}{3x^6}-\frac{4a^2b^6}{x^7}-\frac{a\,b^7}{x^8}-\frac{b^8}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^8/x^2,x)
```

```
[Out] -14*a^4*b^4/x^5-14*a^5*b^3/x^4-28/3*a^6*b^2/x^3-a*b^7/x^8-4*a^7*b/x^2-1/9*b
^8/x^9-4*a^2*b^6/x^7-28/3*a^3*b^5/x^6-a^8/x
```

maxima [A] time = 1.10, size = 14, normalized size = 0.88

$$-\frac{\left(a+\frac{b}{x}\right)^9}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8/x^2,x, algorithm="maxima")
```

```
[Out] -1/9*(a + b/x)^9/b
```

mupad [B] time = 1.10, size = 88, normalized size = 5.50

$$-\frac{a^8\,x^8+4\,a^7\,b\,x^7+\frac{28\,a^6\,b^2\,x^6}{3}+14\,a^5\,b^3\,x^5+14\,a^4\,b^4\,x^4+\frac{28\,a^3\,b^5\,x^3}{3}+4\,a^2\,b^6\,x^2+a\,b^7\,x+\frac{b^8}{9}}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^8/x^2,x)
```

```
[Out] -(b^8/9 + a^8*x^8 + 4*a^7*b*x^7 + 4*a^2*b^6*x^2 + (28*a^3*b^5*x^3)/3 + 14*a
^4*b^4*x^4 + 14*a^5*b^3*x^5 + (28*a^6*b^2*x^6)/3 + a*b^7*x)/x^9
```

sympy [B] time = 0.69, size = 95, normalized size = 5.94

$$\frac{-9a^8x^8-36a^7bx^7-84a^6b^2x^6-126a^5b^3x^5-126a^4b^4x^4-84a^3b^5x^3-36a^2b^6x^2-9ab^7x-b^8}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**8/x**2,x)
```

```
[Out] (-9*a**8*x**8 - 36*a**7*b*x**7 - 84*a**6*b**2*x**6 - 126*a**5*b**3*x**5 - 1
26*a**4*b**4*x**4 - 84*a**3*b**5*x**3 - 36*a**2*b**6*x**2 - 9*a*b**7*x - b*
*8)/(9*x**9)
```

3.1601 $\int \frac{\left(a+\frac{b}{x}\right)^8}{x^3}\,dx$

Optimal. Leaf size=36

$$\frac{a(ax+b)^9}{90b^2x^9}-\frac{(ax+b)^9}{10bx^{10}}$$

```
[Out] -1/10*(a*x+b)^9/b/x^10+1/90*a*(a*x+b)^9/b^2/x^9
```

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 45, 37}

$$\frac{a(ax+b)^9}{90b^2x^9}-\frac{(ax+b)^9}{10bx^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^8/x^3,x]
```

```
[Out] -(b + a*x)^9/(10*b*x^10) + (a*(b + a*x)^9)/(90*b^2*x^9)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^8}{x^3}\,dx &= \int \frac{(b+ax)^8}{x^{11}}\,dx\\ &= -\frac{(b+ax)^9}{10bx^{10}}-\frac{a\int \frac{(b+ax)^8}{x^{10}}\,dx}{10b}\\ &= -\frac{(b+ax)^9}{10bx^{10}}+\frac{a(b+ax)^9}{90b^2x^9}\end{aligned}$$

Mathematica [B] time = 0.00, size = 104, normalized size = 2.89

$$-\frac{a^8}{2x^2}-\frac{8a^7b}{3x^3}-\frac{7a^6b^2}{x^4}-\frac{56a^5b^3}{5x^5}-\frac{35a^4b^4}{3x^6}-\frac{8a^3b^5}{x^7}-\frac{7a^2b^6}{2x^8}-\frac{8ab^7}{9x^9}-\frac{b^8}{10x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^8/x^3,x]
```

```
[Out] -1/10*b^8/x^10 - (8*a*b^7)/(9*x^9) - (7*a^2*b^6)/(2*x^8) - (8*a^3*b^5)/x^7
- (35*a^4*b^4)/(3*x^6) - (56*a^5*b^3)/(5*x^5) - (7*a^6*b^2)/x^4 - (8*a^7*b)
/(3*x^3) - a^8/(2*x^2)
```

fricas [B] time = 1.00, size = 90, normalized size = 2.50

$$-\frac{45\,a^8x^8 + 240\,a^7bx^7 + 630\,a^6b^2x^6 + 1008\,a^5b^3x^5 + 1050\,a^4b^4x^4 + 720\,a^3b^5x^3 + 315\,a^2b^6x^2 + 80\,ab^7x + 9\,b^8}{90\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8/x^3,x, algorithm="fricas")
```

```
[Out] -1/90*(45*a^8*x^8 + 240*a^7*b*x^7 + 630*a^6*b^2*x^6 + 1008*a^5*b^3*x^5 + 10
50*a^4*b^4*x^4 + 720*a^3*b^5*x^3 + 315*a^2*b^6*x^2 + 80*a*b^7*x + 9*b^8)/x^
10
```

giac [B] time = 0.21, size = 90, normalized size = 2.50

$$-\frac{45\,a^8x^8 + 240\,a^7bx^7 + 630\,a^6b^2x^6 + 1008\,a^5b^3x^5 + 1050\,a^4b^4x^4 + 720\,a^3b^5x^3 + 315\,a^2b^6x^2 + 80\,ab^7x + 9\,b^8}{90\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8/x^3,x, algorithm="giac")
```

```
[Out] -1/90*(45*a^8*x^8 + 240*a^7*b*x^7 + 630*a^6*b^2*x^6 + 1008*a^5*b^3*x^5 + 10
50*a^4*b^4*x^4 + 720*a^3*b^5*x^3 + 315*a^2*b^6*x^2 + 80*a*b^7*x + 9*b^8)/x^
10
```

maple [B] time = 0.00, size = 91, normalized size = 2.53

$$-\frac{a^8}{2x^2} - \frac{8a^7b}{3x^3} - \frac{7a^6b^2}{x^4} - \frac{56a^5b^3}{5x^5} - \frac{35a^4b^4}{3x^6} - \frac{8a^3b^5}{x^7} - \frac{7a^2b^6}{2x^8} - \frac{8a\,b^7}{9x^9} - \frac{b^8}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^8/x^3,x)
```

```
[Out] -56/5*a^5*b^3/x^5-7*a^6*b^2/x^4-8/3*a^7*b/x^3-7/2*a^2*b^6/x^8-1/2*a^8/x^2-8
/9*a*b^7/x^9-8*a^3*b^5/x^7-35/3*a^4*b^4/x^6-1/10*b^8/x^10
```

maxima [B] time = 0.98, size = 90, normalized size = 2.50

$$-\frac{45\,a^8x^8 + 240\,a^7bx^7 + 630\,a^6b^2x^6 + 1008\,a^5b^3x^5 + 1050\,a^4b^4x^4 + 720\,a^3b^5x^3 + 315\,a^2b^6x^2 + 80\,ab^7x + 9\,b^8}{90\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8/x^3,x, algorithm="maxima")
```

```
[Out] -1/90*(45*a^8*x^8 + 240*a^7*b*x^7 + 630*a^6*b^2*x^6 + 1008*a^5*b^3*x^5 + 10
50*a^4*b^4*x^4 + 720*a^3*b^5*x^3 + 315*a^2*b^6*x^2 + 80*a*b^7*x + 9*b^8)/x^
10
```

mupad [B] time = 0.07, size = 90, normalized size = 2.50

$$-\frac{\frac{a^8x^8}{2} + \frac{8a^7bx^7}{3} + 7a^6b^2x^6 + \frac{56a^5b^3x^5}{5} + \frac{35a^4b^4x^4}{3} + 8a^3b^5x^3 + \frac{7a^2b^6x^2}{2} + \frac{8ab^7x}{9} + \frac{b^8}{10}}{x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^8/x^3,x)
```

```
[Out] -(b^8/10 + (a^8*x^8)/2 + (8*a^7*b*x^7)/3 + (7*a^2*b^6*x^2)/2 + 8*a^3*b^5*x^
3 + (35*a^4*b^4*x^4)/3 + (56*a^5*b^3*x^5)/5 + 7*a^6*b^2*x^6 + (8*a*b^7*x)/9
)/x^10
```

sympy [**B**] time = 0.71, size = 97, normalized size = 2.69

$$\frac{-45a^8x^8 - 240a^7bx^7 - 630a^6b^2x^6 - 1008a^5b^3x^5 - 1050a^4b^4x^4 - 720a^3b^5x^3 - 315a^2b^6x^2 - 80ab^7x - 9b^8}{90x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**8/x**3,x)
```

```
[Out] (-45*a**8*x**8 - 240*a**7*b*x**7 - 630*a**6*b**2*x**6 - 1008*a**5*b**3*x**5
 - 1050*a**4*b**4*x**4 - 720*a**3*b**5*x**3 - 315*a**2*b**6*x**2 - 80*a*b**
7*x - 9*b**8)/(90*x**10)
```

3.1602 $\int \frac{\left(a+\frac{b}{x}\right)^8}{x^4}\,dx$

Optimal. Leaf size=56

$$-\frac{a^2(ax+b)^9}{495b^3x^9}+\frac{a(ax+b)^9}{55b^2x^{10}}-\frac{(ax+b)^9}{11bx^{11}}$$

```
[Out] -1/11*(a*x+b)^9/b/x^11+1/55*a*(a*x+b)^9/b^2/x^10-1/495*a^2*(a*x+b)^9/b^3/x^
9
```

Rubi [A] time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 45, 37}

$$-\frac{a^2(ax+b)^9}{495b^3x^9}+\frac{a(ax+b)^9}{55b^2x^{10}}-\frac{(ax+b)^9}{11bx^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^8/x^4,x]
```

```
[Out] -(b + a*x)^9/(11*b*x^11) + (a*(b + a*x)^9)/(55*b^2*x^10) - (a^2*(b + a*x)^9
)/(495*b^3*x^9)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^8}{x^4}\,dx &= \int \frac{(b+ax)^8}{x^{12}}\,dx \\ &= -\frac{(b+ax)^9}{11bx^{11}} - \frac{(2a)\int \frac{(b+ax)^8}{x^{11}}\,dx}{11b} \\ &= -\frac{(b+ax)^9}{11bx^{11}} + \frac{a(b+ax)^9}{55b^2x^{10}} + \frac{a^2\int \frac{(b+ax)^8}{x^{10}}\,dx}{55b^2} \\ &= -\frac{(b+ax)^9}{11bx^{11}} + \frac{a(b+ax)^9}{55b^2x^{10}} - \frac{a^2(b+ax)^9}{495b^3x^9}\end{aligned}$$

Mathematica [A] time = 0.01, size = 102, normalized size = 1.82

$$-\frac{a^8}{3x^3} - \frac{2a^7b}{x^4} - \frac{28a^6b^2}{5x^5} - \frac{28a^5b^3}{3x^6} - \frac{10a^4b^4}{x^7} - \frac{7a^3b^5}{x^8} - \frac{28a^2b^6}{9x^9} - \frac{4ab^7}{5x^{10}} - \frac{b^8}{11x^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^8/x^4,x]
```

```
[Out] -1/11*b^8/x^11 - (4*a*b^7)/(5*x^10) - (28*a^2*b^6)/(9*x^9) - (7*a^3*b^5)/x^
8 - (10*a^4*b^4)/x^7 - (28*a^5*b^3)/(3*x^6) - (28*a^6*b^2)/(5*x^5) - (2*a^7
*b)/x^4 - a^8/(3*x^3)
```

fricas [A] time = 1.24, size = 90, normalized size = 1.61

$$-\frac{165\,a^8x^8 + 990\,a^7bx^7 + 2772\,a^6b^2x^6 + 4620\,a^5b^3x^5 + 4950\,a^4b^4x^4 + 3465\,a^3b^5x^3 + 1540\,a^2b^6x^2 + 396\,ab^7x + 45}{495\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8/x^4,x, algorithm="fricas")
```

```
[Out] -1/495*(165*a^8*x^8 + 990*a^7*b*x^7 + 2772*a^6*b^2*x^6 + 4620*a^5*b^3*x^5 +
 4950*a^4*b^4*x^4 + 3465*a^3*b^5*x^3 + 1540*a^2*b^6*x^2 + 396*a*b^7*x + 45*
b^8)/x^11
```

giac [A] time = 0.17, size = 90, normalized size = 1.61

$$-\frac{165\,a^8x^8 + 990\,a^7bx^7 + 2772\,a^6b^2x^6 + 4620\,a^5b^3x^5 + 4950\,a^4b^4x^4 + 3465\,a^3b^5x^3 + 1540\,a^2b^6x^2 + 396\,ab^7x + 45}{495\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8/x^4,x, algorithm="giac")
```

```
[Out] -1/495*(165*a^8*x^8 + 990*a^7*b*x^7 + 2772*a^6*b^2*x^6 + 4620*a^5*b^3*x^5 +
 4950*a^4*b^4*x^4 + 3465*a^3*b^5*x^3 + 1540*a^2*b^6*x^2 + 396*a*b^7*x + 45*
b^8)/x^11
```

maple [A] time = 0.01, size = 91, normalized size = 1.62

$$-\frac{a^8}{3x^3} - \frac{2a^7b}{x^4} - \frac{28a^6b^2}{5x^5} - \frac{28a^5b^3}{3x^6} - \frac{10a^4b^4}{x^7} - \frac{7a^3b^5}{x^8} - \frac{28a^2b^6}{9x^9} - \frac{4a\,b^7}{5x^{10}} - \frac{b^8}{11x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^8/x^4,x)
```

```
[Out] -28/5*a^6*b^2/x^5-2*a^7*b/x^4-1/3*a^8/x^3-7*a^3*b^5/x^8-28/9*a^2*b^6/x^9-10
*a^4*b^4/x^7-28/3*a^5*b^3/x^6-4/5*a*b^7/x^10-1/11*b^8/x^11
```

maxima [A] time = 1.05, size = 90, normalized size = 1.61

$$-\frac{165\,a^8x^8+990\,a^7bx^7+2772\,a^6b^2x^6+4620\,a^5b^3x^5+4950\,a^4b^4x^4+3465\,a^3b^5x^3+1540\,a^2b^6x^2+396\,ab^7x+}{495\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8/x^4,x, algorithm="maxima")
```

```
[Out] -1/495*(165*a^8*x^8 + 990*a^7*b*x^7 + 2772*a^6*b^2*x^6 + 4620*a^5*b^3*x^5 +
 4950*a^4*b^4*x^4 + 3465*a^3*b^5*x^3 + 1540*a^2*b^6*x^2 + 396*a*b^7*x + 45*
b^8)/x^11
```

mupad [B] time = 1.08, size = 90, normalized size = 1.61

$$-\frac{\frac{a^8x^8}{3}+2\,a^7\,b\,x^7+\frac{28\,a^6\,b^2\,x^6}{5}+\frac{28\,a^5\,b^3\,x^5}{3}+10\,a^4\,b^4\,x^4+7\,a^3\,b^5\,x^3+\frac{28\,a^2\,b^6\,x^2}{9}+\frac{4\,a\,b^7\,x}{5}+\frac{b^8}{11}}{x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^8/x^4,x)
```

```
[Out] -(b^8/11 + (a^8*x^8)/3 + 2*a^7*b*x^7 + (28*a^2*b^6*x^2)/9 + 7*a^3*b^5*x^3 +
 10*a^4*b^4*x^4 + (28*a^5*b^3*x^5)/3 + (28*a^6*b^2*x^6)/5 + (4*a*b^7*x)/5)/
x^11
```

sympy [B] time = 0.76, size = 97, normalized size = 1.73

$$\frac{-165a^8x^8-990a^7bx^7-2772a^6b^2x^6-4620a^5b^3x^5-4950a^4b^4x^4-3465a^3b^5x^3-1540a^2b^6x^2-396ab^7x-45b}{495x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**8/x**4,x)
```

```
[Out] (-165*a**8*x**8 - 990*a**7*b*x**7 - 2772*a**6*b**2*x**6 - 4620*a**5*b**3*x*
*5 - 4950*a**4*b**4*x**4 - 3465*a**3*b**5*x**3 - 1540*a**2*b**6*x**2 - 396*
a*b**7*x - 45*b**8)/(495*x**11)
```

3.1603 $\int \frac{\left(a+\frac{b}{x}\right)^8}{x^5}\,dx$

Optimal. Leaf size=76

$$\frac{a^3(ax+b)^9}{1980b^4x^9}-\frac{a^2(ax+b)^9}{220b^3x^{10}}+\frac{a(ax+b)^9}{44b^2x^{11}}-\frac{(ax+b)^9}{12bx^{12}}$$

```
[Out] -1/12*(a*x+b)^9/b/x^12+1/44*a*(a*x+b)^9/b^2/x^11-1/220*a^2*(a*x+b)^9/b^3/x^
10+1/1980*a^3*(a*x+b)^9/b^4/x^9
```

Rubi [A] time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 45, 37}

$$\frac{a^3(ax+b)^9}{1980b^4x^9}-\frac{a^2(ax+b)^9}{220b^3x^{10}}+\frac{a(ax+b)^9}{44b^2x^{11}}-\frac{(ax+b)^9}{12bx^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^8/x^5,x]
```

```
[Out] -(b + a*x)^9/(12*b*x^12) + (a*(b + a*x)^9)/(44*b^2*x^11) - (a^2*(b + a*x)^9
)/(220*b^3*x^10) + (a^3*(b + a*x)^9)/(1980*b^4*x^9)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^8}{x^5}\,dx &= \int \frac{(b+ax)^8}{x^{13}}\,dx \\ &= -\frac{(b+ax)^9}{12bx^{12}} - \frac{a\int \frac{(b+ax)^8}{x^{12}}\,dx}{4b} \\ &= -\frac{(b+ax)^9}{12bx^{12}} + \frac{a(b+ax)^9}{44b^2x^{11}} + \frac{a^2\int \frac{(b+ax)^8}{x^{11}}\,dx}{22b^2} \\ &= -\frac{(b+ax)^9}{12bx^{12}} + \frac{a(b+ax)^9}{44b^2x^{11}} - \frac{a^2(b+ax)^9}{220b^3x^{10}} - \frac{a^3\int \frac{(b+ax)^8}{x^{10}}\,dx}{220b^3} \\ &= -\frac{(b+ax)^9}{12bx^{12}} + \frac{a(b+ax)^9}{44b^2x^{11}} - \frac{a^2(b+ax)^9}{220b^3x^{10}} + \frac{a^3(b+ax)^9}{1980b^4x^9}\end{aligned}$$

Mathematica [A] time = 0.00, size = 106, normalized size = 1.39

$$-\frac{a^8}{4x^4} - \frac{8a^7b}{5x^5} - \frac{14a^6b^2}{3x^6} - \frac{8a^5b^3}{x^7} - \frac{35a^4b^4}{4x^8} - \frac{56a^3b^5}{9x^9} - \frac{14a^2b^6}{5x^{10}} - \frac{8ab^7}{11x^{11}} - \frac{b^8}{12x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^8/x^5,x]
```

```
[Out] -1/12*b^8/x^12 - (8*a*b^7)/(11*x^11) - (14*a^2*b^6)/(5*x^10) - (56*a^3*b^5)
/(9*x^9) - (35*a^4*b^4)/(4*x^8) - (8*a^5*b^3)/x^7 - (14*a^6*b^2)/(3*x^6) -
(8*a^7*b)/(5*x^5) - a^8/(4*x^4)
```

fricas [A] time = 0.86, size = 90, normalized size = 1.18

$$-\frac{495\,a^8x^8 + 3168\,a^7bx^7 + 9240\,a^6b^2x^6 + 15840\,a^5b^3x^5 + 17325\,a^4b^4x^4 + 12320\,a^3b^5x^3 + 5544\,a^2b^6x^2 + 1440\,a}{1980\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8/x^5,x, algorithm="fricas")
```

```
[Out] -1/1980*(495*a^8*x^8 + 3168*a^7*b*x^7 + 9240*a^6*b^2*x^6 + 15840*a^5*b^3*x^
5 + 17325*a^4*b^4*x^4 + 12320*a^3*b^5*x^3 + 5544*a^2*b^6*x^2 + 1440*a*b^7*x
 + 165*b^8)/x^12
```

giac [A] time = 0.17, size = 90, normalized size = 1.18

$$-\frac{495\,a^8x^8 + 3168\,a^7bx^7 + 9240\,a^6b^2x^6 + 15840\,a^5b^3x^5 + 17325\,a^4b^4x^4 + 12320\,a^3b^5x^3 + 5544\,a^2b^6x^2 + 1440\,a}{1980\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8/x^5,x, algorithm="giac")
```

```
[Out] -1/1980*(495*a^8*x^8 + 3168*a^7*b*x^7 + 9240*a^6*b^2*x^6 + 15840*a^5*b^3*x^
5 + 17325*a^4*b^4*x^4 + 12320*a^3*b^5*x^3 + 5544*a^2*b^6*x^2 + 1440*a*b^7*x
 + 165*b^8)/x^12
```

maple [A] time = 0.01, size = 91, normalized size = 1.20

$$-\frac{a^8}{4x^4} - \frac{8a^7b}{5x^5} - \frac{14a^6b^2}{3x^6} - \frac{8a^5b^3}{x^7} - \frac{35a^4b^4}{4x^8} - \frac{56a^3b^5}{9x^9} - \frac{14a^2b^6}{5x^{10}} - \frac{8a\,b^7}{11x^{11}} - \frac{b^8}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^8/x^5,x)
```

```
[Out] -8/5*a^7*b/x^5-1/4*a^8/x^4-35/4*a^4*b^4/x^8-56/9*a^3*b^5/x^9-8*a^5*b^3/x^7-
14/3*a^6*b^2/x^6-1/12*b^8/x^12-14/5*a^2*b^6/x^10-8/11*a*b^7/x^11
```

maxima [A] time = 1.16, size = 90, normalized size = 1.18

$$-\frac{495\,a^8x^8+3168\,a^7bx^7+9240\,a^6b^2x^6+15840\,a^5b^3x^5+17325\,a^4b^4x^4+12320\,a^3b^5x^3+5544\,a^2b^6x^2+1440\,ab^7}{1980\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8/x^5,x, algorithm="maxima")
```

```
[Out] -1/1980*(495*a^8*x^8 + 3168*a^7*b*x^7 + 9240*a^6*b^2*x^6 + 15840*a^5*b^3*x^
5 + 17325*a^4*b^4*x^4 + 12320*a^3*b^5*x^3 + 5544*a^2*b^6*x^2 + 1440*a*b^7*x
 + 165*b^8)/x^12
```

mupad [B] time = 0.07, size = 90, normalized size = 1.18

$$-\frac{\frac{a^8x^8}{4}+\frac{8a^7bx^7}{5}+\frac{14a^6b^2x^6}{3}+8a^5b^3x^5+\frac{35a^4b^4x^4}{4}+\frac{56a^3b^5x^3}{9}+\frac{14a^2b^6x^2}{5}+\frac{8ab^7x}{11}+\frac{b^8}{12}}{x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^8/x^5,x)
```

```
[Out] -(b^8/12 + (a^8*x^8)/4 + (8*a^7*b*x^7)/5 + (14*a^2*b^6*x^2)/5 + (56*a^3*b^5
*x^3)/9 + (35*a^4*b^4*x^4)/4 + 8*a^5*b^3*x^5 + (14*a^6*b^2*x^6)/3 + (8*a*b^
7*x)/11)/x^12
```

sympy [A] time = 0.85, size = 97, normalized size = 1.28

$$\frac{-495a^8x^8-3168a^7bx^7-9240a^6b^2x^6-15840a^5b^3x^5-17325a^4b^4x^4-12320a^3b^5x^3-5544a^2b^6x^2-1440ab^7x-1}{1980x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**8/x**5,x)
```

```
[Out] (-495*a**8*x**8 - 3168*a**7*b*x**7 - 9240*a**6*b**2*x**6 - 15840*a**5*b**3*
x**5 - 17325*a**4*b**4*x**4 - 12320*a**3*b**5*x**3 - 5544*a**2*b**6*x**2 -
1440*a*b**7*x - 165*b**8)/(1980*x**12)
```

3.1604 $\int \frac{\left(a+\frac{b}{x}\right)^8}{x^6}\,dx$

Optimal. Leaf size=96

$$-\frac{a^4(ax+b)^9}{6435b^5x^9}+\frac{a^3(ax+b)^9}{715b^4x^{10}}-\frac{a^2(ax+b)^9}{143b^3x^{11}}+\frac{a(ax+b)^9}{39b^2x^{12}}-\frac{(ax+b)^9}{13bx^{13}}$$

```
[Out] -1/13*(a*x+b)^9/b/x^13+1/39*a*(a*x+b)^9/b^2/x^12-1/143*a^2*(a*x+b)^9/b^3/x^
11+1/715*a^3*(a*x+b)^9/b^4/x^10-1/6435*a^4*(a*x+b)^9/b^5/x^9
```

Rubi [A] time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 45, 37}

$$-\frac{a^4(ax+b)^9}{6435b^5x^9}+\frac{a^3(ax+b)^9}{715b^4x^{10}}-\frac{a^2(ax+b)^9}{143b^3x^{11}}+\frac{a(ax+b)^9}{39b^2x^{12}}-\frac{(ax+b)^9}{13bx^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^8/x^6,x]
```

```
[Out] -(b + a*x)^9/(13*b*x^13) + (a*(b + a*x)^9)/(39*b^2*x^12) - (a^2*(b + a*x)^9
)/(143*b^3*x^11) + (a^3*(b + a*x)^9)/(715*b^4*x^10) - (a^4*(b + a*x)^9)/(64
35*b^5*x^9)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^8}{x^6}\,dx &= \int \frac{(b+ax)^8}{x^{14}}\,dx \\ &= -\frac{(b+ax)^9}{13bx^{13}} - \frac{(4a)\int \frac{(b+ax)^8}{x^{13}}\,dx}{13b} \\ &= -\frac{(b+ax)^9}{13bx^{13}} + \frac{a(b+ax)^9}{39b^2x^{12}} + \frac{a^2\int \frac{(b+ax)^8}{x^{12}}\,dx}{13b^2} \\ &= -\frac{(b+ax)^9}{13bx^{13}} + \frac{a(b+ax)^9}{39b^2x^{12}} - \frac{a^2(b+ax)^9}{143b^3x^{11}} - \frac{\left(2a^3\right)\int \frac{(b+ax)^8}{x^{11}}\,dx}{143b^3} \\ &= -\frac{(b+ax)^9}{13bx^{13}} + \frac{a(b+ax)^9}{39b^2x^{12}} - \frac{a^2(b+ax)^9}{143b^3x^{11}} + \frac{a^3(b+ax)^9}{715b^4x^{10}} + \frac{a^4\int \frac{(b+ax)^8}{x^{10}}\,dx}{715b^4} \\ &= -\frac{(b+ax)^9}{13bx^{13}} + \frac{a(b+ax)^9}{39b^2x^{12}} - \frac{a^2(b+ax)^9}{143b^3x^{11}} + \frac{a^3(b+ax)^9}{715b^4x^{10}} - \frac{a^4(b+ax)^9}{6435b^5x^9}\end{aligned}$$

Mathematica [A] time = 0.02, size = 104, normalized size = 1.08

$$-\frac{a^8}{5x^5} - \frac{4a^7b}{3x^6} - \frac{4a^6b^2}{x^7} - \frac{7a^5b^3}{x^8} - \frac{70a^4b^4}{9x^9} - \frac{28a^3b^5}{5x^{10}} - \frac{28a^2b^6}{11x^{11}} - \frac{2ab^7}{3x^{12}} - \frac{b^8}{13x^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^8/x^6,x]
```

```
[Out] -1/13*b^8/x^13 - (2*a*b^7)/(3*x^12) - (28*a^2*b^6)/(11*x^11) - (28*a^3*b^5)
/(5*x^10) - (70*a^4*b^4)/(9*x^9) - (7*a^5*b^3)/x^8 - (4*a^6*b^2)/x^7 - (4*a
^7*b)/(3*x^6) - a^8/(5*x^5)
```

fricas [A] time = 1.03, size = 90, normalized size = 0.94

$$-\frac{1287\,a^8x^8 + 8580\,a^7bx^7 + 25740\,a^6b^2x^6 + 45045\,a^5b^3x^5 + 50050\,a^4b^4x^4 + 36036\,a^3b^5x^3 + 16380\,a^2b^6x^2 + 4290}{6435\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8/x^6,x, algorithm="fricas")
```

```
[Out] -1/6435*(1287*a^8*x^8 + 8580*a^7*b*x^7 + 25740*a^6*b^2*x^6 + 45045*a^5*b^3*
x^5 + 50050*a^4*b^4*x^4 + 36036*a^3*b^5*x^3 + 16380*a^2*b^6*x^2 + 4290*a*b^
7*x + 495*b^8)/x^13
```

giac [A] time = 0.15, size = 90, normalized size = 0.94

$$-\frac{1287\,a^8x^8 + 8580\,a^7bx^7 + 25740\,a^6b^2x^6 + 45045\,a^5b^3x^5 + 50050\,a^4b^4x^4 + 36036\,a^3b^5x^3 + 16380\,a^2b^6x^2 + 4290}{6435\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8/x^6,x, algorithm="giac")
```

```
[Out] -1/6435*(1287*a^8*x^8 + 8580*a^7*b*x^7 + 25740*a^6*b^2*x^6 + 45045*a^5*b^3*
x^5 + 50050*a^4*b^4*x^4 + 36036*a^3*b^5*x^3 + 16380*a^2*b^6*x^2 + 4290*a*b^
7*x + 495*b^8)/x^13
```

maple [A] time = 0.01, size = 91, normalized size = 0.95

$$-\frac{a^8}{5x^5} - \frac{4a^7b}{3x^6} - \frac{4a^6b^2}{x^7} - \frac{7a^5b^3}{x^8} - \frac{70a^4b^4}{9x^9} - \frac{28a^3b^5}{5x^{10}} - \frac{28a^2b^6}{11x^{11}} - \frac{2a\,b^7}{3x^{12}} - \frac{b^8}{13x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^8/x^6,x)
```

```
[Out] -1/5*a^8/x^5-7*a^5*b^3/x^8-1/13*b^8/x^13-70/9*a^4*b^4/x^9-4*a^6*b^2/x^7-4/3
*a^7*b/x^6-2/3*a*b^7/x^12-28/5*a^3*b^5/x^10-28/11*a^2*b^6/x^11
```

maxima [A] time = 1.01, size = 90, normalized size = 0.94

$$-\frac{1287\,a^8x^8+8580\,a^7bx^7+25740\,a^6b^2x^6+45045\,a^5b^3x^5+50050\,a^4b^4x^4+36036\,a^3b^5x^3+16380\,a^2b^6x^2+42}{6435\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8/x^6,x, algorithm="maxima")
```

```
[Out] -1/6435*(1287*a^8*x^8 + 8580*a^7*b*x^7 + 25740*a^6*b^2*x^6 + 45045*a^5*b^3*
x^5 + 50050*a^4*b^4*x^4 + 36036*a^3*b^5*x^3 + 16380*a^2*b^6*x^2 + 4290*a*b^
7*x + 495*b^8)/x^13
```

mupad [B] time = 1.08, size = 90, normalized size = 0.94

$$-\frac{\frac{a^8x^8}{5}+\frac{4a^7bx^7}{3}+4a^6b^2x^6+7a^5b^3x^5+\frac{70a^4b^4x^4}{9}+\frac{28a^3b^5x^3}{5}+\frac{28a^2b^6x^2}{11}+\frac{2ab^7x}{3}+\frac{b^8}{13}}{x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^8/x^6,x)
```

```
[Out] -(b^8/13 + (a^8*x^8)/5 + (4*a^7*b*x^7)/3 + (28*a^2*b^6*x^2)/11 + (28*a^3*b^
5*x^3)/5 + (70*a^4*b^4*x^4)/9 + 7*a^5*b^3*x^5 + 4*a^6*b^2*x^6 + (2*a*b^7*x)
/3)/x^13
```

sympy [A] time = 0.89, size = 97, normalized size = 1.01

$$\frac{-1287a^8x^8-8580a^7bx^7-25740a^6b^2x^6-45045a^5b^3x^5-50050a^4b^4x^4-36036a^3b^5x^3-16380a^2b^6x^2-4290a}{6435x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**8/x**6,x)
```

```
[Out] (-1287*a**8*x**8 - 8580*a**7*b*x**7 - 25740*a**6*b**2*x**6 - 45045*a**5*b**
3*x**5 - 50050*a**4*b**4*x**4 - 36036*a**3*b**5*x**3 - 16380*a**2*b**6*x**2
 - 4290*a*b**7*x - 495*b**8)/(6435*x**13)
```

3.1605 $\int \frac{\left(a+\frac{b}{x}\right)^8}{x^7}\,dx$

Optimal. Leaf size=106

$$-\frac{a^8}{6x^6}-\frac{8a^7b}{7x^7}-\frac{7a^6b^2}{2x^8}-\frac{56a^5b^3}{9x^9}-\frac{7a^4b^4}{x^{10}}-\frac{56a^3b^5}{11x^{11}}-\frac{7a^2b^6}{3x^{12}}-\frac{8ab^7}{13x^{13}}-\frac{b^8}{14x^{14}}$$

```
[Out] -1/14*b^8/x^14-8/13*a*b^7/x^13-7/3*a^2*b^6/x^12-56/11*a^3*b^5/x^11-7*a^4*b^
4/x^10-56/9*a^5*b^3/x^9-7/2*a^6*b^2/x^8-8/7*a^7*b/x^7-1/6*a^8/x^6
```

Rubi [A] time = 0.04, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$-\frac{7a^6b^2}{2x^8}-\frac{56a^5b^3}{9x^9}-\frac{7a^4b^4}{x^{10}}-\frac{56a^3b^5}{11x^{11}}-\frac{7a^2b^6}{3x^{12}}-\frac{8a^7b}{7x^7}-\frac{a^8}{6x^6}-\frac{8ab^7}{13x^{13}}-\frac{b^8}{14x^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^8/x^7,x]
```

```
[Out] -b^8/(14*x^14) - (8*a*b^7)/(13*x^13) - (7*a^2*b^6)/(3*x^12) - (56*a^3*b^5)/
(11*x^11) - (7*a^4*b^4)/x^10 - (56*a^5*b^3)/(9*x^9) - (7*a^6*b^2)/(2*x^8) -
 (8*a^7*b)/(7*x^7) - a^8/(6*x^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^8}{x^7}\,dx &= \int \frac{(b+ax)^8}{x^{15}}\,dx\\ &= \int \left(\frac{b^8}{x^{15}}+\frac{8ab^7}{x^{14}}+\frac{28a^2b^6}{x^{13}}+\frac{56a^3b^5}{x^{12}}+\frac{70a^4b^4}{x^{11}}+\frac{56a^5b^3}{x^{10}}+\frac{28a^6b^2}{x^9}+\frac{8a^7b}{x^8}+\frac{a^8}{x^7}\right)dx\\ &= -\frac{b^8}{14x^{14}}-\frac{8ab^7}{13x^{13}}-\frac{7a^2b^6}{3x^{12}}-\frac{56a^3b^5}{11x^{11}}-\frac{7a^4b^4}{x^{10}}-\frac{56a^5b^3}{9x^9}-\frac{7a^6b^2}{2x^8}-\frac{8a^7b}{7x^7}-\frac{a^8}{6x^6}\end{aligned}$$

Mathematica [A] time = 0.01, size = 106, normalized size = 1.00

$$-\frac{a^8}{6x^6}-\frac{8a^7b}{7x^7}-\frac{7a^6b^2}{2x^8}-\frac{56a^5b^3}{9x^9}-\frac{7a^4b^4}{x^{10}}-\frac{56a^3b^5}{11x^{11}}-\frac{7a^2b^6}{3x^{12}}-\frac{8ab^7}{13x^{13}}-\frac{b^8}{14x^{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^8/x^7,x]
```

```
[Out] -1/14*b^8/x^14 - (8*a*b^7)/(13*x^13) - (7*a^2*b^6)/(3*x^12) - (56*a^3*b^5)/
(11*x^11) - (7*a^4*b^4)/x^10 - (56*a^5*b^3)/(9*x^9) - (7*a^6*b^2)/(2*x^8) -
 (8*a^7*b)/(7*x^7) - a^8/(6*x^6)
```

fricas [A] time = 0.84, size = 90, normalized size = 0.85

$$-\frac{3003\,a^8x^8 + 20592\,a^7bx^7 + 63063\,a^6b^2x^6 + 112112\,a^5b^3x^5 + 126126\,a^4b^4x^4 + 91728\,a^3b^5x^3 + 42042\,a^2b^6x^2 +}{18018\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8/x^7,x, algorithm="fricas")
```

```
[Out] -1/18018*(3003*a^8*x^8 + 20592*a^7*b*x^7 + 63063*a^6*b^2*x^6 + 112112*a^5*b
^3*x^5 + 126126*a^4*b^4*x^4 + 91728*a^3*b^5*x^3 + 42042*a^2*b^6*x^2 + 11088
*a*b^7*x + 1287*b^8)/x^14
```

giac [A] time = 0.17, size = 90, normalized size = 0.85

$$-\frac{3003\,a^8x^8 + 20592\,a^7bx^7 + 63063\,a^6b^2x^6 + 112112\,a^5b^3x^5 + 126126\,a^4b^4x^4 + 91728\,a^3b^5x^3 + 42042\,a^2b^6x^2 +}{18018\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8/x^7,x, algorithm="giac")
```

```
[Out] -1/18018*(3003*a^8*x^8 + 20592*a^7*b*x^7 + 63063*a^6*b^2*x^6 + 112112*a^5*b
^3*x^5 + 126126*a^4*b^4*x^4 + 91728*a^3*b^5*x^3 + 42042*a^2*b^6*x^2 + 11088
*a*b^7*x + 1287*b^8)/x^14
```

maple [A] time = 0.01, size = 91, normalized size = 0.86

$$-\frac{a^8}{6x^6} - \frac{8a^7b}{7x^7} - \frac{7a^6b^2}{2x^8} - \frac{56a^5b^3}{9x^9} - \frac{7a^4b^4}{x^{10}} - \frac{56a^3b^5}{11x^{11}} - \frac{7a^2b^6}{3x^{12}} - \frac{8a\,b^7}{13x^{13}} - \frac{b^8}{14x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^8/x^7,x)
```

```
[Out] -1/14*b^8/x^14-8/13*a*b^7/x^13-7/3*a^2*b^6/x^12-56/11*a^3*b^5/x^11-7*a^4*b^
4/x^10-56/9*a^5*b^3/x^9-7/2*a^6*b^2/x^8-8/7*a^7*b/x^7-1/6*a^8/x^6
```

maxima [A] time = 1.08, size = 90, normalized size = 0.85

$$-\frac{3003\,a^8x^8 + 20592\,a^7bx^7 + 63063\,a^6b^2x^6 + 112112\,a^5b^3x^5 + 126126\,a^4b^4x^4 + 91728\,a^3b^5x^3 + 42042\,a^2b^6x^2 +}{18018\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8/x^7,x, algorithm="maxima")
```

```
[Out] -1/18018*(3003*a^8*x^8 + 20592*a^7*b*x^7 + 63063*a^6*b^2*x^6 + 112112*a^5*b
^3*x^5 + 126126*a^4*b^4*x^4 + 91728*a^3*b^5*x^3 + 42042*a^2*b^6*x^2 + 11088
*a*b^7*x + 1287*b^8)/x^14
```

mupad [B] time = 0.07, size = 90, normalized size = 0.85

$$-\frac{\frac{a^8x^8}{6} + \frac{8a^7bx^7}{7} + \frac{7a^6b^2x^6}{2} + \frac{56a^5b^3x^5}{9} + 7a^4b^4x^4 + \frac{56a^3b^5x^3}{11} + \frac{7a^2b^6x^2}{3} + \frac{8ab^7x}{13} + \frac{b^8}{14}}{x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^8/x^7,x)
```

```
[Out] -(b^8/14 + (a^8*x^8)/6 + (8*a^7*b*x^7)/7 + (7*a^2*b^6*x^2)/3 + (56*a^3*b^5*
x^3)/11 + 7*a^4*b^4*x^4 + (56*a^5*b^3*x^5)/9 + (7*a^6*b^2*x^6)/2 + (8*a*b^7
*x)/13)/x^14
```

sympy [**A**] time = 0.93, size = 97, normalized size = 0.92

$$\frac{-3003a^8x^8 - 20592a^7bx^7 - 63063a^6b^2x^6 - 112112a^5b^3x^5 - 126126a^4b^4x^4 - 91728a^3b^5x^3 - 42042a^2b^6x^2 - 11088}{18018x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**8/x**7,x)
```

```
[Out] (-3003*a**8*x**8 - 20592*a**7*b*x**7 - 63063*a**6*b**2*x**6 - 112112*a**5*b
**3*x**5 - 126126*a**4*b**4*x**4 - 91728*a**3*b**5*x**3 - 42042*a**2*b**6*x
**2 - 11088*a*b**7*x - 1287*b**8)/(18018*x**14)
```

3.1606 $\int \frac{\left(a+\frac{b}{x}\right)^8}{x^8}\,dx$

Optimal. Leaf size=106

$$-\frac{a^8}{7x^7}-\frac{a^7b}{x^8}-\frac{28a^6b^2}{9x^9}-\frac{28a^5b^3}{5x^{10}}-\frac{70a^4b^4}{11x^{11}}-\frac{14a^3b^5}{3x^{12}}-\frac{28a^2b^6}{13x^{13}}-\frac{4ab^7}{7x^{14}}-\frac{b^8}{15x^{15}}$$

```
[Out] -1/15*b^8/x^15-4/7*a*b^7/x^14-28/13*a^2*b^6/x^13-14/3*a^3*b^5/x^12-70/11*a^
4*b^4/x^11-28/5*a^5*b^3/x^10-28/9*a^6*b^2/x^9-a^7*b/x^8-1/7*a^8/x^7
```

Rubi [A] time = 0.04, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$-\frac{28a^6b^2}{9x^9}-\frac{28a^5b^3}{5x^{10}}-\frac{70a^4b^4}{11x^{11}}-\frac{14a^3b^5}{3x^{12}}-\frac{28a^2b^6}{13x^{13}}-\frac{a^7b}{x^8}-\frac{a^8}{7x^7}-\frac{4ab^7}{7x^{14}}-\frac{b^8}{15x^{15}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^8/x^8,x]
```

```
[Out] -b^8/(15*x^15) - (4*a*b^7)/(7*x^14) - (28*a^2*b^6)/(13*x^13) - (14*a^3*b^5)
/(3*x^12) - (70*a^4*b^4)/(11*x^11) - (28*a^5*b^3)/(5*x^10) - (28*a^6*b^2)/(
9*x^9) - (a^7*b)/x^8 - a^8/(7*x^7)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^8}{x^8}\,dx &= \int \frac{(b+ax)^8}{x^{16}}\,dx \\ &= \int \left(\frac{b^8}{x^{16}}+\frac{8ab^7}{x^{15}}+\frac{28a^2b^6}{x^{14}}+\frac{56a^3b^5}{x^{13}}+\frac{70a^4b^4}{x^{12}}+\frac{56a^5b^3}{x^{11}}+\frac{28a^6b^2}{x^{10}}+\frac{8a^7b}{x^9}+\frac{a^8}{x^8}\right)dx \\ &= -\frac{b^8}{15x^{15}}-\frac{4ab^7}{7x^{14}}-\frac{28a^2b^6}{13x^{13}}-\frac{14a^3b^5}{3x^{12}}-\frac{70a^4b^4}{11x^{11}}-\frac{28a^5b^3}{5x^{10}}-\frac{28a^6b^2}{9x^9}-\frac{a^7b}{x^8}-\frac{a^8}{7x^7}\end{aligned}$$

Mathematica [A] time = 0.01, size = 106, normalized size = 1.00

$$-\frac{a^8}{7x^7}-\frac{a^7b}{x^8}-\frac{28a^6b^2}{9x^9}-\frac{28a^5b^3}{5x^{10}}-\frac{70a^4b^4}{11x^{11}}-\frac{14a^3b^5}{3x^{12}}-\frac{28a^2b^6}{13x^{13}}-\frac{4ab^7}{7x^{14}}-\frac{b^8}{15x^{15}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^8/x^8,x]
```

```
[Out] -1/15*b^8/x^15 - (4*a*b^7)/(7*x^14) - (28*a^2*b^6)/(13*x^13) - (14*a^3*b^5)
/(3*x^12) - (70*a^4*b^4)/(11*x^11) - (28*a^5*b^3)/(5*x^10) - (28*a^6*b^2)/(
9*x^9) - (a^7*b)/x^8 - a^8/(7*x^7)
```

fricas [A] time = 0.98, size = 90, normalized size = 0.85

$$-\frac{6435\,a^8x^8+45045\,a^7bx^7+140140\,a^6b^2x^6+252252\,a^5b^3x^5+286650\,a^4b^4x^4+210210\,a^3b^5x^3+97020\,a^2b^6x^2+}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8/x^8,x, algorithm="fricas")
```

```
[Out] -1/45045*(6435*a^8*x^8 + 45045*a^7*b*x^7 + 140140*a^6*b^2*x^6 + 252252*a^5*
b^3*x^5 + 286650*a^4*b^4*x^4 + 210210*a^3*b^5*x^3 + 97020*a^2*b^6*x^2 + 257
40*a*b^7*x + 3003*b^8)/x^15
```

giac [A] time = 0.15, size = 90, normalized size = 0.85

$$-\frac{6435\,a^8x^8+45045\,a^7bx^7+140140\,a^6b^2x^6+252252\,a^5b^3x^5+286650\,a^4b^4x^4+210210\,a^3b^5x^3+97020\,a^2b^6x^2+}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8/x^8,x, algorithm="giac")
```

```
[Out] -1/45045*(6435*a^8*x^8 + 45045*a^7*b*x^7 + 140140*a^6*b^2*x^6 + 252252*a^5*
b^3*x^5 + 286650*a^4*b^4*x^4 + 210210*a^3*b^5*x^3 + 97020*a^2*b^6*x^2 + 257
40*a*b^7*x + 3003*b^8)/x^15
```

maple [A] time = 0.01, size = 91, normalized size = 0.86

$$-\frac{a^8}{7x^7}-\frac{a^7b}{x^8}-\frac{28a^6b^2}{9x^9}-\frac{28a^5b^3}{5x^{10}}-\frac{70a^4b^4}{11x^{11}}-\frac{14a^3b^5}{3x^{12}}-\frac{28a^2b^6}{13x^{13}}-\frac{4a\,b^7}{7x^{14}}-\frac{b^8}{15x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^8/x^8,x)
```

```
[Out] -1/15*b^8/x^15-4/7*a*b^7/x^14-28/13*a^2*b^6/x^13-14/3*a^3*b^5/x^12-70/11*a^
4*b^4/x^11-28/5*a^5*b^3/x^10-28/9*a^6*b^2/x^9-a^7*b/x^8-1/7*a^8/x^7
```

maxima [A] time = 1.01, size = 90, normalized size = 0.85

$$-\frac{6435\,a^8x^8+45045\,a^7bx^7+140140\,a^6b^2x^6+252252\,a^5b^3x^5+286650\,a^4b^4x^4+210210\,a^3b^5x^3+97020\,a^2b^6x^2+}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^8/x^8,x, algorithm="maxima")
```

```
[Out] -1/45045*(6435*a^8*x^8 + 45045*a^7*b*x^7 + 140140*a^6*b^2*x^6 + 252252*a^5*
b^3*x^5 + 286650*a^4*b^4*x^4 + 210210*a^3*b^5*x^3 + 97020*a^2*b^6*x^2 + 257
40*a*b^7*x + 3003*b^8)/x^15
```

mupad [B] time = 0.07, size = 89, normalized size = 0.84

$$-\frac{\frac{a^8\,x^8}{7}+a^7\,b\,x^7+\frac{28\,a^6\,b^2\,x^6}{9}+\frac{28\,a^5\,b^3\,x^5}{5}+\frac{70\,a^4\,b^4\,x^4}{11}+\frac{14\,a^3\,b^5\,x^3}{3}+\frac{28\,a^2\,b^6\,x^2}{13}+\frac{4\,a\,b^7\,x}{7}+\frac{b^8}{15}}{x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^8/x^8,x)
```

```
[Out] -(b^8/15 + (a^8*x^8)/7 + a^7*b*x^7 + (28*a^2*b^6*x^2)/13 + (14*a^3*b^5*x^3)
/3 + (70*a^4*b^4*x^4)/11 + (28*a^5*b^3*x^5)/5 + (28*a^6*b^2*x^6)/9 + (4*a*b
^7*x)/7)/x^15
```

sympy [**A**] time = 0.92, size = 97, normalized size = 0.92

$$\frac{-6435a^8x^8 - 45045a^7bx^7 - 140140a^6b^2x^6 - 252252a^5b^3x^5 - 286650a^4b^4x^4 - 210210a^3b^5x^3 - 97020a^2b^6x^2 -}{45045x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**8/x**8,x)
```

```
[Out] (-6435*a**8*x**8 - 45045*a**7*b*x**7 - 140140*a**6*b**2*x**6 - 252252*a**5*
b**3*x**5 - 286650*a**4*b**4*x**4 - 210210*a**3*b**5*x**3 - 97020*a**2*b**6
*x**2 - 25740*a*b**7*x - 3003*b**8)/(45045*x**15)
```

3.1607 $\int \frac{x^4}{a+\frac{b}{x}}\,dx$

Optimal. Leaf size=70

$$-\frac{b^5\log(ax+b)}{a^6}+\frac{b^4x}{a^5}-\frac{b^3x^2}{2a^4}+\frac{b^2x^3}{3a^3}-\frac{bx^4}{4a^2}+\frac{x^5}{5a}$$

```
[Out] b^4*x/a^5-1/2*b^3*x^2/a^4+1/3*b^2*x^3/a^3-1/4*b*x^4/a^2+1/5*x^5/a-b^5*ln(a*
x+b)/a^6
```

Rubi [A] time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$-\frac{b^3x^2}{2a^4}+\frac{b^2x^3}{3a^3}+\frac{b^4x}{a^5}-\frac{b^5\log(ax+b)}{a^6}-\frac{bx^4}{4a^2}+\frac{x^5}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b/x),x]
```

```
[Out] (b^4*x)/a^5 - (b^3*x^2)/(2*a^4) + (b^2*x^3)/(3*a^3) - (b*x^4)/(4*a^2) + x^5
/(5*a) - (b^5*Log[b + a*x])/a^6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{a+\frac{b}{x}}\,dx &= \int \frac{x^5}{b+ax}\,dx\\ &= \int\left(\frac{b^4}{a^5}-\frac{b^3x}{a^4}+\frac{b^2x^2}{a^3}-\frac{bx^3}{a^2}+\frac{x^4}{a}-\frac{b^5}{a^5(b+ax)}\right)dx\\ &= \frac{b^4x}{a^5}-\frac{b^3x^2}{2a^4}+\frac{b^2x^3}{3a^3}-\frac{bx^4}{4a^2}+\frac{x^5}{5a}-\frac{b^5\log(b+ax)}{a^6}\end{aligned}$$

Mathematica [A] time = 0.01, size = 70, normalized size = 1.00

$$-\frac{b^5\log(ax+b)}{a^6}+\frac{b^4x}{a^5}-\frac{b^3x^2}{2a^4}+\frac{b^2x^3}{3a^3}-\frac{bx^4}{4a^2}+\frac{x^5}{5a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b/x),x]
```

```
[Out] (b^4*x)/a^5 - (b^3*x^2)/(2*a^4) + (b^2*x^3)/(3*a^3) - (b*x^4)/(4*a^2) + x^5
/(5*a) - (b^5*Log[b + a*x])/a^6
```

fricas [A] time = 0.96, size = 63, normalized size = 0.90

$$\frac{12\,a^5x^5 - 15\,a^4bx^4 + 20\,a^3b^2x^3 - 30\,a^2b^3x^2 + 60\,ab^4x - 60\,b^5\log(ax+b)}{60\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/x),x, algorithm="fricas")
```

```
[Out] 1/60*(12*a^5*x^5 - 15*a^4*b*x^4 + 20*a^3*b^2*x^3 - 30*a^2*b^3*x^2 + 60*a*b^
4*x - 60*b^5*log(a*x + b))/a^6
```

giac [A] time = 0.17, size = 65, normalized size = 0.93

$$-\frac{b^5\log(|ax+b|)}{a^6} + \frac{12\,a^4x^5 - 15\,a^3bx^4 + 20\,a^2b^2x^3 - 30\,ab^3x^2 + 60\,b^4x}{60\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/x),x, algorithm="giac")
```

```
[Out] -b^5*log(abs(a*x + b))/a^6 + 1/60*(12*a^4*x^5 - 15*a^3*b*x^4 + 20*a^2*b^2*x
^3 - 30*a*b^3*x^2 + 60*b^4*x)/a^5
```

maple [A] time = 0.00, size = 63, normalized size = 0.90

$$\frac{x^5}{5a} - \frac{b\,x^4}{4a^2} + \frac{b^2x^3}{3a^3} - \frac{b^3x^2}{2a^4} + \frac{b^4x}{a^5} - \frac{b^5\ln(ax+b)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a+b/x),x)
```

```
[Out] b^4*x/a^5-1/2*b^3*x^2/a^4+1/3*b^2*x^3/a^3-1/4*b*x^4/a^2+1/5/a*x^5-b^5*ln(a*
x+b)/a^6
```

maxima [A] time = 1.09, size = 64, normalized size = 0.91

$$-\frac{b^5\log(ax+b)}{a^6} + \frac{12\,a^4x^5 - 15\,a^3bx^4 + 20\,a^2b^2x^3 - 30\,ab^3x^2 + 60\,b^4x}{60\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/x),x, algorithm="maxima")
```

```
[Out] -b^5*log(a*x + b)/a^6 + 1/60*(12*a^4*x^5 - 15*a^3*b*x^4 + 20*a^2*b^2*x^3 -
30*a*b^3*x^2 + 60*b^4*x)/a^5
```

mupad [B] time = 1.04, size = 62, normalized size = 0.89

$$\frac{x^5}{5\,a} - \frac{b^5\,\ln(b+a\,x)}{a^6} - \frac{b\,x^4}{4\,a^2} + \frac{b^4\,x}{a^5} + \frac{b^2\,x^3}{3\,a^3} - \frac{b^3\,x^2}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b/x),x)
```

```
[Out] x^5/(5*a) - (b^5*log(b + a*x))/a^6 - (b*x^4)/(4*a^2) + (b^4*x)/a^5 + (b^2*x
^3)/(3*a^3) - (b^3*x^2)/(2*a^4)
```

sympy [A] time = 0.15, size = 61, normalized size = 0.87

$$\frac{x^5}{5a} - \frac{bx^4}{4a^2} + \frac{b^2x^3}{3a^3} - \frac{b^3x^2}{2a^4} + \frac{b^4x}{a^5} - \frac{b^5\log(ax+b)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a+b/x),x)
```

```
[Out] x**5/(5*a) - b*x**4/(4*a**2) + b**2*x**3/(3*a**3) - b**3*x**2/(2*a**4) + b*
**4*x/a**5 - b**5*log(a*x + b)/a**6
```

3.1608 $\int \frac{x^3}{a+\frac{b}{x}}\,dx$

Optimal. Leaf size=57

$$\frac{b^4\log(ax+b)}{a^5}-\frac{b^3x}{a^4}+\frac{b^2x^2}{2a^3}-\frac{bx^3}{3a^2}+\frac{x^4}{4a}$$

```
[Out] -b^3*x/a^4+1/2*b^2*x^2/a^3-1/3*b*x^3/a^2+1/4*x^4/a+b^4*ln(a*x+b)/a^5
```

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$\frac{b^2x^2}{2a^3}-\frac{b^3x}{a^4}+\frac{b^4\log(ax+b)}{a^5}-\frac{bx^3}{3a^2}+\frac{x^4}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b/x),x]
```

```
[Out] -((b^3*x)/a^4) + (b^2*x^2)/(2*a^3) - (b*x^3)/(3*a^2) + x^4/(4*a) + (b^4*Log
[b + a*x])/a^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{a+\frac{b}{x}}\,dx &= \int \frac{x^4}{b+ax}\,dx\\ &= \int\left(-\frac{b^3}{a^4}+\frac{b^2x}{a^3}-\frac{bx^2}{a^2}+\frac{x^3}{a}+\frac{b^4}{a^4(b+ax)}\right)dx\\ &= -\frac{b^3x}{a^4}+\frac{b^2x^2}{2a^3}-\frac{bx^3}{3a^2}+\frac{x^4}{4a}+\frac{b^4\log(b+ax)}{a^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 57, normalized size = 1.00

$$\frac{b^4\log(ax+b)}{a^5}-\frac{b^3x}{a^4}+\frac{b^2x^2}{2a^3}-\frac{bx^3}{3a^2}+\frac{x^4}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b/x),x]
```

```
[Out] -((b^3*x)/a^4) + (b^2*x^2)/(2*a^3) - (b*x^3)/(3*a^2) + x^4/(4*a) + (b^4*Log
[b + a*x])/a^5
```

fricas [A] time = 1.19, size = 52, normalized size = 0.91

$$\frac{3\,a^4x^4 - 4\,a^3bx^3 + 6\,a^2b^2x^2 - 12\,ab^3x + 12\,b^4\log(ax+b)}{12\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a^4*x^4 - 4*a^3*b*x^3 + 6*a^2*b^2*x^2 - 12*a*b^3*x + 12*b^4*log(a*x
 + b))/a^5
```

giac [A] time = 0.17, size = 53, normalized size = 0.93

$$\frac{b^4\log(|ax+b|)}{a^5} + \frac{3\,a^3x^4 - 4\,a^2bx^3 + 6\,ab^2x^2 - 12\,b^3x}{12\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x),x, algorithm="giac")
```

```
[Out] b^4*log(abs(a*x + b))/a^5 + 1/12*(3*a^3*x^4 - 4*a^2*b*x^3 + 6*a*b^2*x^2 - 1
2*b^3*x)/a^4
```

maple [A] time = 0.00, size = 52, normalized size = 0.91

$$\frac{x^4}{4a} - \frac{b\,x^3}{3a^2} + \frac{b^2x^2}{2a^3} - \frac{b^3x}{a^4} + \frac{b^4\ln(ax+b)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b/x),x)
```

```
[Out] -b^3*x/a^4+1/2*b^2*x^2/a^3-1/3*b*x^3/a^2+1/4/a*x^4+b^4*ln(a*x+b)/a^5
```

maxima [A] time = 1.08, size = 52, normalized size = 0.91

$$\frac{b^4\log(ax+b)}{a^5} + \frac{3\,a^3x^4 - 4\,a^2bx^3 + 6\,ab^2x^2 - 12\,b^3x}{12\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x),x, algorithm="maxima")
```

```
[Out] b^4*log(a*x + b)/a^5 + 1/12*(3*a^3*x^4 - 4*a^2*b*x^3 + 6*a*b^2*x^2 - 12*b^3
*x)/a^4
```

mupad [B] time = 0.03, size = 51, normalized size = 0.89

$$\frac{x^4}{4\,a} + \frac{b^4\,\ln(b+a\,x)}{a^5} - \frac{b\,x^3}{3\,a^2} - \frac{b^3\,x}{a^4} + \frac{b^2\,x^2}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b/x),x)
```

```
[Out] x^4/(4*a) + (b^4*log(b + a*x))/a^5 - (b*x^3)/(3*a^2) - (b^3*x)/a^4 + (b^2*x
^2)/(2*a^3)
```

sympy [A] time = 0.14, size = 49, normalized size = 0.86

$$\frac{x^4}{4a} - \frac{bx^3}{3a^2} + \frac{b^2x^2}{2a^3} - \frac{b^3x}{a^4} + \frac{b^4\log(ax+b)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b/x),x)

[Out] x**4/(4*a) - b*x**3/(3*a**2) + b**2*x**2/(2*a**3) - b**3*x/a**4 + b**4*log(
a*x + b)/a**5
```

3.1609 $\int \frac{x^2}{a+\frac{b}{x}}\,dx$

Optimal. Leaf size=44

$$-\frac{b^3\log(ax+b)}{a^4}+\frac{b^2x}{a^3}-\frac{bx^2}{2a^2}+\frac{x^3}{3a}$$

```
[Out] b^2*x/a^3-1/2*b*x^2/a^2+1/3*x^3/a-b^3*ln(a*x+b)/a^4
```

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$\frac{b^2x}{a^3}-\frac{b^3\log(ax+b)}{a^4}-\frac{bx^2}{2a^2}+\frac{x^3}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b/x),x]
```

```
[Out] (b^2*x)/a^3 - (b*x^2)/(2*a^2) + x^3/(3*a) - (b^3*Log[b + a*x])/a^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{a+\frac{b}{x}}\,dx &= \int \frac{x^3}{b+ax}\,dx\\ &= \int\left(\frac{b^2}{a^3}-\frac{bx}{a^2}+\frac{x^2}{a}-\frac{b^3}{a^3(b+ax)}\right)dx\\ &= \frac{b^2x}{a^3}-\frac{bx^2}{2a^2}+\frac{x^3}{3a}-\frac{b^3\log(b+ax)}{a^4}\end{aligned}$$

Mathematica [A] time = 0.00, size = 44, normalized size = 1.00

$$-\frac{b^3\log(ax+b)}{a^4}+\frac{b^2x}{a^3}-\frac{bx^2}{2a^2}+\frac{x^3}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b/x),x]
```

```
[Out] (b^2*x)/a^3 - (b*x^2)/(2*a^2) + x^3/(3*a) - (b^3*Log[b + a*x])/a^4
```

fricas [A] time = 0.72, size = 41, normalized size = 0.93

$$\frac{2\,a^3x^3-3\,a^2bx^2+6\,ab^2x-6\,b^3\log(ax+b)}{6\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x),x, algorithm="fricas")
[Out] 1/6*(2*a^3*x^3 - 3*a^2*b*x^2 + 6*a*b^2*x - 6*b^3*log(a*x + b))/a^4
```

giac [A] time = 0.15, size = 43, normalized size = 0.98

$$-\frac{b^3 \log(|ax+b|)}{a^4} + \frac{2a^2x^3 - 3abx^2 + 6b^2x}{6a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x),x, algorithm="giac")
[Out] -b^3*log(abs(a*x + b))/a^4 + 1/6*(2*a^2*x^3 - 3*a*b*x^2 + 6*b^2*x)/a^3
```

maple [A] time = 0.00, size = 41, normalized size = 0.93

$$\frac{x^3}{3a} - \frac{b\,x^2}{2a^2} + \frac{b^2x}{a^3} - \frac{b^3 \ln(ax+b)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b/x),x)
[Out] b^2*x/a^3-1/2*b*x^2/a^2+1/3*x^3/a-b^3*ln(a*x+b)/a^4
```

maxima [A] time = 1.00, size = 42, normalized size = 0.95

$$-\frac{b^3 \log(ax+b)}{a^4} + \frac{2a^2x^3 - 3abx^2 + 6b^2x}{6a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x),x, algorithm="maxima")
[Out] -b^3*log(a*x + b)/a^4 + 1/6*(2*a^2*x^3 - 3*a*b*x^2 + 6*b^2*x)/a^3
```

mupad [B] time = 0.04, size = 40, normalized size = 0.91

$$\frac{x^3}{3\,a} - \frac{b^3\,\ln(b+a\,x)}{a^4} - \frac{b\,x^2}{2\,a^2} + \frac{b^2\,x}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b/x),x)
[Out] x^3/(3*a) - (b^3*log(b + a*x))/a^4 - (b*x^2)/(2*a^2) + (b^2*x)/a^3
```

sympy [A] time = 0.14, size = 37, normalized size = 0.84

$$\frac{x^3}{3a} - \frac{bx^2}{2a^2} + \frac{b^2x}{a^3} - \frac{b^3 \log(ax+b)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b/x),x)
[Out] x**3/(3*a) - b*x**2/(2*a**2) + b**2*x/a**3 - b**3*log(a*x + b)/a**4
```

3.1610 $\int \frac{x}{a+\frac{b}{x}}\,dx$

Optimal. Leaf size=31

$$\frac{b^2\log(ax+b)}{a^3}-\frac{bx}{a^2}+\frac{x^2}{2a}$$

```
[Out] -b*x/a^2+1/2*x^2/a+b^2*ln(a*x+b)/a^3
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {263, 43}

$$\frac{b^2\log(ax+b)}{a^3}-\frac{bx}{a^2}+\frac{x^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b/x),x]
```

```
[Out] -((b*x)/a^2) + x^2/(2*a) + (b^2*Log[b + a*x])/a^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{a+\frac{b}{x}}\,dx &= \int \frac{x^2}{b+ax}\,dx\\ &= \int\left(-\frac{b}{a^2}+\frac{x}{a}+\frac{b^2}{a^2(b+ax)}\right)dx\\ &= -\frac{bx}{a^2}+\frac{x^2}{2a}+\frac{b^2\log(b+ax)}{a^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 31, normalized size = 1.00

$$\frac{b^2\log(ax+b)}{a^3}-\frac{bx}{a^2}+\frac{x^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b/x),x]
```

```
[Out] -((b*x)/a^2) + x^2/(2*a) + (b^2*Log[b + a*x])/a^3
```

fricas [A] time = 0.68, size = 29, normalized size = 0.94

$$\frac{a^2x^2-2\,abx+2\,b^2\log(ax+b)}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x),x, algorithm="fricas")
[Out] 1/2*(a^2*x^2 - 2*a*b*x + 2*b^2*log(a*x + b))/a^3
```

giac [A] time = 0.15, size = 30, normalized size = 0.97

$$\frac{b^2 \log(|ax+b|)}{a^3} + \frac{ax^2 - 2bx}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x),x, algorithm="giac")
[Out] b^2*log(abs(a*x + b))/a^3 + 1/2*(a*x^2 - 2*b*x)/a^2
```

maple [A] time = 0.00, size = 30, normalized size = 0.97

$$\frac{x^2}{2a} - \frac{bx}{a^2} + \frac{b^2 \ln(ax+b)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b/x),x)
[Out] -b*x/a^2+1/2*x^2/a+b^2*ln(a*x+b)/a^3
```

maxima [A] time = 1.10, size = 29, normalized size = 0.94

$$\frac{b^2 \log(ax+b)}{a^3} + \frac{ax^2 - 2bx}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x),x, algorithm="maxima")
[Out] b^2*log(a*x + b)/a^3 + 1/2*(a*x^2 - 2*b*x)/a^2
```

mupad [B] time = 0.04, size = 29, normalized size = 0.94

$$\frac{2b^2 \ln(b+ax) + a^2x^2 - 2abx}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b/x),x)
[Out] (2*b^2*log(b + a*x) + a^2*x^2 - 2*a*b*x)/(2*a^3)
```

sympy [A] time = 0.12, size = 26, normalized size = 0.84

$$\frac{x^2}{2a} - \frac{bx}{a^2} + \frac{b^2 \log(ax+b)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x),x)
[Out] x**2/(2*a) - b*x/a**2 + b**2*log(a*x + b)/a**3
```

3.1611 $\int \frac{1}{a+\frac{b}{x}}\,dx$

Optimal. Leaf size=18

$$\frac{x}{a}-\frac{b\log(ax+b)}{a^2}$$

```
[Out] x/a-b*ln(a*x+b)/a^2
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {193, 43}

$$\frac{x}{a}-\frac{b\log(ax+b)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(-1),x]
```

```
[Out] x/a - (b*Log[b + a*x])/a^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+\frac{b}{x}}\,dx &= \int \frac{x}{b+ax}\,dx\\ &= \int\left(\frac{1}{a}-\frac{b}{a(b+ax)}\right)dx\\ &= \frac{x}{a}-\frac{b\log(b+ax)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{x}{a}-\frac{b\log(ax+b)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(-1),x]
```

```
[Out] x/a - (b*Log[b + a*x])/a^2
```

fricas [A] time = 1.05, size = 17, normalized size = 0.94

$$\frac{ax-b\log(ax+b)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x),x, algorithm="fricas")
[Out] (a*x - b*log(a*x + b))/a^2
```

giac [A] time = 0.18, size = 19, normalized size = 1.06

$$\frac{x}{a} - \frac{b \log(|ax+b|)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x),x, algorithm="giac")
[Out] x/a - b*log(abs(a*x + b))/a^2
```

maple [A] time = 0.00, size = 19, normalized size = 1.06

$$\frac{x}{a} - \frac{b \ln(ax+b)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x),x)
[Out] x/a-b*ln(a*x+b)/a^2
```

maxima [A] time = 0.95, size = 18, normalized size = 1.00

$$\frac{x}{a} - \frac{b \log(ax+b)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x),x, algorithm="maxima")
[Out] x/a - b*log(a*x + b)/a^2
```

mupad [B] time = 1.05, size = 18, normalized size = 1.00

$$-\frac{b \ln(b + a\,x) - a\,x}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x),x)
[Out] -(b*log(b + a*x) - a*x)/a^2
```

sympy [A] time = 0.12, size = 14, normalized size = 0.78

$$\frac{x}{a} - \frac{b \log(ax+b)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x),x)
[Out] x/a - b*log(a*x + b)/a**2
```

3.1612 $\int \frac{1}{\left(a+\frac{b}{x}\right)x}\,dx$

Optimal. Leaf size=10

$$\frac{\log(ax+b)}{a}$$

```
[Out] ln(a*x+b)/a
```

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 31}

$$\frac{\log(ax+b)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)*x),x]
```

```
[Out] Log[b + a*x]/a
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x}\right)x}\,dx &= \int \frac{1}{b+ax}\,dx \\ &= \frac{\log(b+ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{\log(ax+b)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)*x),x]
```

```
[Out] Log[b + a*x]/a
```

fricas [A] time = 0.72, size = 10, normalized size = 1.00

$$\frac{\log(ax+b)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x,x, algorithm="fricas")
```

```
[Out] log(a*x + b)/a
```

giac [A] time = 0.15, size = 11, normalized size = 1.10

$$\frac{\log(|ax+b|)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x,x, algorithm="giac")
[Out] log(abs(a*x + b))/a
```

maple [A] time = 0.00, size = 11, normalized size = 1.10

$$\frac{\ln(ax+b)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)/x,x)
[Out] ln(a*x+b)/a
```

maxima [A] time = 0.99, size = 10, normalized size = 1.00

$$\frac{\log(ax+b)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x,x, algorithm="maxima")
[Out] log(a*x + b)/a
```

mupad [B] time = 0.02, size = 10, normalized size = 1.00

$$\frac{\ln(b+a\,x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b/x)),x)
[Out] log(b + a*x)/a
```

sympy [A] time = 0.07, size = 7, normalized size = 0.70

$$\frac{\log(ax+b)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x,x)
[Out] log(a*x + b)/a
```

3.1613 $\int \frac{1}{\left(a+\frac{b}{x}\right)x^2}\,dx$

Optimal. Leaf size=13

$$-\frac{\log\left(a+\frac{b}{x}\right)}{b}$$

```
[Out] -ln(a+b/x)/b
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {260}

$$-\frac{\log\left(a+\frac{b}{x}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)*x^2),x]
```

```
[Out] -(Log[a + b/x]/b)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^2}\,dx = -\frac{\log\left(a+\frac{b}{x}\right)}{b}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.38

$$\frac{\log(x)}{b} - \frac{\log(ax+b)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)*x^2),x]
```

```
[Out] Log[x]/b - Log[b + a*x]/b
```

fricas [A] time = 0.96, size = 16, normalized size = 1.23

$$-\frac{\log(ax+b)-\log(x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x^2,x, algorithm="fricas")
```

```
[Out] -(log(a*x + b) - log(x))/b
```

giac [A] time = 0.23, size = 14, normalized size = 1.08

$$-\frac{\log\left(\left|a+\frac{b}{x}\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x^2,x, algorithm="giac")
[Out] -log(abs(a + b/x))/b
```

maple [A] time = 0.00, size = 19, normalized size = 1.46

$$\frac{\ln(x)}{b} - \frac{\ln(ax+b)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)/x^2,x)
[Out] -1/b*ln(a*x+b)+1/b*ln(x)
```

maxima [A] time = 1.10, size = 13, normalized size = 1.00

$$-\frac{\log\left(a+\frac{b}{x}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x^2,x, algorithm="maxima")
[Out] -log(a + b/x)/b
```

mupad [B] time = 0.04, size = 15, normalized size = 1.15

$$-\frac{2\,\operatorname{atanh}\left(\frac{2\,a\,x}{b}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b/x)),x)
[Out] -(2*atanh((2*a*x)/b + 1))/b
```

sympy [A] time = 0.15, size = 10, normalized size = 0.77

$$\frac{\log(x) - \log\left(x+\frac{b}{a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x**2,x)
[Out] (log(x) - log(x + b/a))/b
```

3.1614 $\int \frac{1}{\left(a+\frac{b}{x}\right)x^3}\,dx$

Optimal. Leaf size=28

$$-\frac{a\log(x)}{b^2}+\frac{a\log(ax+b)}{b^2}-\frac{1}{bx}$$

```
[Out] -1/b/x-a*ln(x)/b^2+a*ln(a*x+b)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 44}

$$-\frac{a\log(x)}{b^2}+\frac{a\log(ax+b)}{b^2}-\frac{1}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)*x^3),x]
```

```
[Out] -(1/(b*x)) - (a*Log[x])/b^2 + (a*Log[b + a*x])/b^2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x}\right)x^3}\,dx &= \int \frac{1}{x^2(b+ax)}\,dx\\ &= \int\left(\frac{1}{bx^2}-\frac{a}{b^2x}+\frac{a^2}{b^2(b+ax)}\right)dx\\ &= -\frac{1}{bx}-\frac{a\log(x)}{b^2}+\frac{a\log(b+ax)}{b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$-\frac{a\log(x)}{b^2}+\frac{a\log(ax+b)}{b^2}-\frac{1}{bx}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)*x^3),x]
```

```
[Out] -(1/(b*x)) - (a*Log[x])/b^2 + (a*Log[b + a*x])/b^2
```

fricas [A] time = 0.90, size = 26, normalized size = 0.93

$$\frac{ax\log(ax+b)-ax\log(x)-b}{b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x^3,x, algorithm="fricas")
[Out] (a*x*log(a*x + b) - a*x*log(x) - b)/(b^2*x)
```

giac [**A**] time = 0.17, size = 30, normalized size = 1.07

$$\frac{a\log(|ax+b|)}{b^2} - \frac{a\log(|x|)}{b^2} - \frac{1}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x^3,x, algorithm="giac")
[Out] a*log(abs(a*x + b))/b^2 - a*log(abs(x))/b^2 - 1/(b*x)
```

maple [**A**] time = 0.01, size = 29, normalized size = 1.04

$$-\frac{a\ln(x)}{b^2} + \frac{a\ln(ax+b)}{b^2} - \frac{1}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)/x^3,x)
[Out] -1/b/x-a*ln(x)/b^2+a*ln(a*x+b)/b^2
```

maxima [**A**] time = 1.03, size = 28, normalized size = 1.00

$$\frac{a\log(ax+b)}{b^2} - \frac{a\log(x)}{b^2} - \frac{1}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x^3,x, algorithm="maxima")
[Out] a*log(a*x + b)/b^2 - a*log(x)/b^2 - 1/(b*x)
```

mupad [**B**] time = 0.06, size = 25, normalized size = 0.89

$$\frac{2\,a\,\operatorname{atanh}\left(\frac{2\,a\,x}{b}+1\right)}{b^2} - \frac{1}{b\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b/x)),x)
[Out] (2*a*atanh((2*a*x)/b + 1))/b^2 - 1/(b*x)
```

sympy [**A**] time = 0.20, size = 19, normalized size = 0.68

$$\frac{a\left(-\log(x) + \log\left(x + \frac{b}{a}\right)\right)}{b^2} - \frac{1}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x**3,x)
[Out] a*(-log(x) + log(x + b/a))/b**2 - 1/(b*x)
```

3.1615 $\int \frac{1}{\left(a+\frac{b}{x}\right)x^4}\,dx$

Optimal. Leaf size=42

$$\frac{a^2\log(x)}{b^3}-\frac{a^2\log(ax+b)}{b^3}+\frac{a}{b^2x}-\frac{1}{2bx^2}$$

```
[Out] -1/2/b/x^2+a/b^2/x+a^2*ln(x)/b^3-a^2*ln(a*x+b)/b^3
```

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 44}

$$\frac{a^2\log(x)}{b^3}-\frac{a^2\log(ax+b)}{b^3}+\frac{a}{b^2x}-\frac{1}{2bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)*x^4),x]
```

```
[Out] -1/(2*b*x^2) + a/(b^2*x) + (a^2*Log[x])/b^3 - (a^2*Log[b + a*x])/b^3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x}\right)x^4}\,dx &= \int \frac{1}{x^3(b+ax)}\,dx\\ &= \int\left(\frac{1}{bx^3}-\frac{a}{b^2x^2}+\frac{a^2}{b^3x}-\frac{a^3}{b^3(b+ax)}\right)dx\\ &= -\frac{1}{2bx^2}+\frac{a}{b^2x}+\frac{a^2\log(x)}{b^3}-\frac{a^2\log(b+ax)}{b^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 42, normalized size = 1.00

$$\frac{a^2\log(x)}{b^3}-\frac{a^2\log(ax+b)}{b^3}+\frac{a}{b^2x}-\frac{1}{2bx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)*x^4),x]
```

```
[Out] -1/2*1/(b*x^2) + a/(b^2*x) + (a^2*Log[x])/b^3 - (a^2*Log[b + a*x])/b^3
```

fricas [A] time = 0.89, size = 41, normalized size = 0.98

$$-\frac{2\,a^2x^2\log(ax+b)-2\,a^2x^2\log(x)-2\,abx+b^2}{2\,b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x^4,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*x^2*log(a*x + b) - 2*a^2*x^2*log(x) - 2*a*b*x + b^2)/(b^3*x^2)
```

giac [A] time = 0.17, size = 45, normalized size = 1.07

$$-\frac{a^2\log\left(|ax+b|\right)}{b^3}+\frac{a^2\log\left(|x|\right)}{b^3}+\frac{2\,abx-b^2}{2\,b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x^4,x, algorithm="giac")
```

```
[Out] -a^2*log(abs(a*x + b))/b^3 + a^2*log(abs(x))/b^3 + 1/2*(2*a*b*x - b^2)/(b^3
*x^2)
```

maple [A] time = 0.01, size = 41, normalized size = 0.98

$$\frac{a^2\ln(x)}{b^3}-\frac{a^2\ln\left(ax+b\right)}{b^3}+\frac{a}{b^2x}-\frac{1}{2b\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)/x^4,x)
```

```
[Out] -1/2/b/x^2+a/b^2/x+a^2*ln(x)/b^3-a^2*ln(a*x+b)/b^3
```

maxima [A] time = 1.12, size = 40, normalized size = 0.95

$$-\frac{a^2\log\left(ax+b\right)}{b^3}+\frac{a^2\log(x)}{b^3}+\frac{2\,ax-b}{2\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x^4,x, algorithm="maxima")
```

```
[Out] -a^2*log(a*x + b)/b^3 + a^2*log(x)/b^3 + 1/2*(2*a*x - b)/(b^2*x^2)
```

mupad [B] time = 0.06, size = 38, normalized size = 0.90

$$-\frac{\frac{b^2}{2}-a\,b\,x}{b^3\,x^2}-\frac{2\,a^2\,\mathrm{atanh}\left(\frac{2\,a\,x}{b}+1\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b/x)),x)
```

```
[Out] - (b^2/2 - a*b*x)/(b^3*x^2) - (2*a^2*atanh((2*a*x)/b + 1))/b^3
```

sympy [A] time = 0.23, size = 31, normalized size = 0.74

$$\frac{a^2\left(\log\left(x\right)-\log\left(x+\frac{b}{a}\right)\right)}{b^3}+\frac{2ax-b}{2b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x**4,x)
```

```
[Out] a**2*(log(x) - log(x + b/a))/b**3 + (2*a*x - b)/(2*b**2*x**2)
```

3.1616 $\int \frac{1}{\left(a+\frac{b}{x}\right)x^5}\,dx$

Optimal. Leaf size=56

$$-\frac{a^3\log(x)}{b^4}+\frac{a^3\log(ax+b)}{b^4}-\frac{a^2}{b^3x}+\frac{a}{2b^2x^2}-\frac{1}{3bx^3}$$

```
[Out] -1/3/b/x^3+1/2*a/b^2/x^2-a^2/b^3/x-a^3*ln(x)/b^4+a^3*ln(a*x+b)/b^4
```

Rubi [A] time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 44}

$$-\frac{a^2}{b^3x}-\frac{a^3\log(x)}{b^4}+\frac{a^3\log(ax+b)}{b^4}+\frac{a}{2b^2x^2}-\frac{1}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)*x^5),x]
```

```
[Out] -1/(3*b*x^3) + a/(2*b^2*x^2) - a^2/(b^3*x) - (a^3*Log[x])/b^4 + (a^3*Log[b
+ a*x])/b^4
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x}\right)x^5}\,dx &= \int \frac{1}{x^4(b+ax)}\,dx\\ &= \int\left(\frac{1}{bx^4}-\frac{a}{b^2x^3}+\frac{a^2}{b^3x^2}-\frac{a^3}{b^4x}+\frac{a^4}{b^4(b+ax)}\right)dx\\ &= -\frac{1}{3bx^3}+\frac{a}{2b^2x^2}-\frac{a^2}{b^3x}-\frac{a^3\log(x)}{b^4}+\frac{a^3\log(b+ax)}{b^4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 56, normalized size = 1.00

$$-\frac{a^3\log(x)}{b^4}+\frac{a^3\log(ax+b)}{b^4}-\frac{a^2}{b^3x}+\frac{a}{2b^2x^2}-\frac{1}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)*x^5),x]
```

```
[Out] -1/3*1/(b*x^3) + a/(2*b^2*x^2) - a^2/(b^3*x) - (a^3*Log[x])/b^4 + (a^3*Log[
b + a*x])/b^4
```

fricas [A] time = 0.98, size = 54, normalized size = 0.96

$$\frac{6\,a^3x^3\log(ax+b)-6\,a^3x^3\log(x)-6\,a^2bx^2+3\,ab^2x-2\,b^3}{6\,b^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x^5,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^3*x^3*log(a*x + b) - 6*a^3*x^3*log(x) - 6*a^2*b*x^2 + 3*a*b^2*x -
2*b^3)/(b^4*x^3)
```

giac [A] time = 0.15, size = 56, normalized size = 1.00

$$\frac{a^3\log(|ax+b|)}{b^4}-\frac{a^3\log(|x|)}{b^4}-\frac{6\,a^2bx^2-3\,ab^2x+2\,b^3}{6\,b^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x^5,x, algorithm="giac")
```

```
[Out] a^3*log(abs(a*x + b))/b^4 - a^3*log(abs(x))/b^4 - 1/6*(6*a^2*b*x^2 - 3*a*b^
2*x + 2*b^3)/(b^4*x^3)
```

maple [A] time = 0.01, size = 53, normalized size = 0.95

$$-\frac{a^3\ln(x)}{b^4}+\frac{a^3\ln(ax+b)}{b^4}-\frac{a^2}{b^3x}+\frac{a}{2b^2x^2}-\frac{1}{3b\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)/x^5,x)
```

```
[Out] -1/3/b/x^3+1/2*a/b^2/x^2-a^2/b^3/x-a^3*ln(x)/b^4+a^3*ln(a*x+b)/b^4
```

maxima [A] time = 1.03, size = 51, normalized size = 0.91

$$\frac{a^3\log(ax+b)}{b^4}-\frac{a^3\log(x)}{b^4}-\frac{6\,a^2x^2-3\,abx+2\,b^2}{6\,b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x^5,x, algorithm="maxima")
```

```
[Out] a^3*log(a*x + b)/b^4 - a^3*log(x)/b^4 - 1/6*(6*a^2*x^2 - 3*a*b*x + 2*b^2)/(
b^3*x^3)
```

mupad [B] time = 1.08, size = 48, normalized size = 0.86

$$\frac{2\,a^3\,\mathrm{atanh}\left(\frac{2\,a\,x}{b}+1\right)}{b^4}-\frac{a^2\,b\,x^2-\frac{a\,b^2\,x}{2}+\frac{b^3}{3}}{b^4\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b/x)),x)
```

```
[Out] (2*a^3*atanh((2*a*x)/b + 1))/b^4 - (b^3/3 + a^2*b*x^2 - (a*b^2*x)/2)/(b^4*x
^3)
```

sympy [A] time = 0.24, size = 44, normalized size = 0.79

$$\frac{a^3\left(-\log(x)+\log\left(x+\frac{b}{a}\right)\right)}{b^4}+\frac{-6a^2x^2+3abx-2b^2}{6b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x**5,x)
```

```
[Out] a**3*(-log(x) + log(x + b/a))/b**4 + (-6*a**2*x**2 + 3*a*b*x - 2*b**2)/(6*b
**3*x**3)
```

3.1617 $\int \frac{1}{\left(a+\frac{b}{x}\right)x^6}\,dx$

Optimal. Leaf size=68

$$\frac{a^4\log(x)}{b^5}-\frac{a^4\log(ax+b)}{b^5}+\frac{a^3}{b^4x}-\frac{a^2}{2b^3x^2}+\frac{a}{3b^2x^3}-\frac{1}{4bx^4}$$

```
[Out] -1/4/b/x^4+1/3*a/b^2/x^3-1/2*a^2/b^3/x^2+a^3/b^4/x+a^4*ln(x)/b^5-a^4*ln(a*x
+b)/b^5
```

Rubi [A] time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 44}

$$-\frac{a^2}{2b^3x^2}+\frac{a^3}{b^4x}+\frac{a^4\log(x)}{b^5}-\frac{a^4\log(ax+b)}{b^5}+\frac{a}{3b^2x^3}-\frac{1}{4bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)*x^6),x]
```

```
[Out] -1/(4*b*x^4) + a/(3*b^2*x^3) - a^2/(2*b^3*x^2) + a^3/(b^4*x) + (a^4*Log[x])
/b^5 - (a^4*Log[b + a*x])/b^5
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x}\right)x^6}\,dx &= \int \frac{1}{x^5(b+ax)}\,dx\\ &= \int\left(\frac{1}{bx^5}-\frac{a}{b^2x^4}+\frac{a^2}{b^3x^3}-\frac{a^3}{b^4x^2}+\frac{a^4}{b^5x}-\frac{a^5}{b^5(b+ax)}\right)dx\\ &= -\frac{1}{4bx^4}+\frac{a}{3b^2x^3}-\frac{a^2}{2b^3x^2}+\frac{a^3}{b^4x}+\frac{a^4\log(x)}{b^5}-\frac{a^4\log(b+ax)}{b^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 68, normalized size = 1.00

$$\frac{a^4\log(x)}{b^5}-\frac{a^4\log(ax+b)}{b^5}+\frac{a^3}{b^4x}-\frac{a^2}{2b^3x^2}+\frac{a}{3b^2x^3}-\frac{1}{4bx^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)*x^6),x]
```

```
[Out] -1/4*1/(b*x^4) + a/(3*b^2*x^3) - a^2/(2*b^3*x^2) + a^3/(b^4*x) + (a^4*Log[x
])/b^5 - (a^4*Log[b + a*x])/b^5
```

fricas [A] time = 0.69, size = 65, normalized size = 0.96

$$-\frac{12\,a^4x^4\log(ax+b)-12\,a^4x^4\log(x)-12\,a^3bx^3+6\,a^2b^2x^2-4\,ab^3x+3\,b^4}{12\,b^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x^6,x, algorithm="fricas")
```

```
[Out] -1/12*(12*a^4*x^4*log(a*x + b) - 12*a^4*x^4*log(x) - 12*a^3*b*x^3 + 6*a^2*b
^2*x^2 - 4*a*b^3*x + 3*b^4)/(b^5*x^4)
```

giac [A] time = 0.21, size = 67, normalized size = 0.99

$$-\frac{a^4\log(|ax+b|)}{b^5}+\frac{a^4\log(|x|)}{b^5}+\frac{12\,a^3bx^3-6\,a^2b^2x^2+4\,ab^3x-3\,b^4}{12\,b^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x^6,x, algorithm="giac")
```

```
[Out] -a^4*log(abs(a*x + b))/b^5 + a^4*log(abs(x))/b^5 + 1/12*(12*a^3*b*x^3 - 6*a
^2*b^2*x^2 + 4*a*b^3*x - 3*b^4)/(b^5*x^4)
```

maple [A] time = 0.01, size = 63, normalized size = 0.93

$$\frac{a^4\ln(x)}{b^5}-\frac{a^4\ln(ax+b)}{b^5}+\frac{a^3}{b^4x}-\frac{a^2}{2b^3x^2}+\frac{a}{3b^2x^3}-\frac{1}{4b\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)/x^6,x)
```

```
[Out] -1/4/b/x^4+1/3*a/b^2/x^3-1/2*a^2/b^3/x^2+a^3/b^4/x+a^4*ln(x)/b^5-a^4*ln(a*x
+b)/b^5
```

maxima [A] time = 0.94, size = 62, normalized size = 0.91

$$-\frac{a^4\log(ax+b)}{b^5}+\frac{a^4\log(x)}{b^5}+\frac{12\,a^3x^3-6\,a^2bx^2+4\,ab^2x-3\,b^3}{12\,b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x^6,x, algorithm="maxima")
```

```
[Out] -a^4*log(a*x + b)/b^5 + a^4*log(x)/b^5 + 1/12*(12*a^3*x^3 - 6*a^2*b*x^2 + 4
*a*b^2*x - 3*b^3)/(b^4*x^4)
```

mupad [B] time = 1.09, size = 60, normalized size = 0.88

$$-\frac{-a^3\,b\,x^3+\frac{a^2\,b^2\,x^2}{2}-\frac{a\,b^3\,x}{3}+\frac{b^4}{4}}{b^5\,x^4}-\frac{2\,a^4\,\mathrm{atanh}\left(\frac{2\,a\,x}{b}+1\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b/x)),x)
```

```
[Out] - (b^4/4 - a^3*b*x^3 + (a^2*b^2*x^2)/2 - (a*b^3*x)/3)/(b^5*x^4) - (2*a^4*at
anh((2*a*x)/b + 1))/b^5
```

sympy [A] time = 0.26, size = 56, normalized size = 0.82

$$\frac{a^4\left(\log(x)-\log\left(x+\frac{b}{a}\right)\right)}{b^5}+\frac{12a^3x^3-6a^2bx^2+4ab^2x-3b^3}{12b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x**6,x)
```

```
[Out] a**4*(log(x) - log(x + b/a))/b**5 + (12*a**3*x**3 - 6*a**2*b*x**2 + 4*a*b**
2*x - 3*b**3)/(12*b**4*x**4)
```

3.1618 $\int \frac{1}{\left(a+\frac{b}{x}\right)x^7}\,dx$

Optimal. Leaf size=82

$$-\frac{a^5\log(x)}{b^6}+\frac{a^5\log(ax+b)}{b^6}-\frac{a^4}{b^5x}+\frac{a^3}{2b^4x^2}-\frac{a^2}{3b^3x^3}+\frac{a}{4b^2x^4}-\frac{1}{5bx^5}$$

```
[Out] -1/5/b/x^5+1/4*a/b^2/x^4-1/3*a^2/b^3/x^3+1/2*a^3/b^4/x^2-a^4/b^5/x-a^5*ln(x
)/b^6+a^5*ln(a*x+b)/b^6
```

Rubi [A] time = 0.03, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 44}

$$\frac{a^3}{2b^4x^2}-\frac{a^2}{3b^3x^3}-\frac{a^4}{b^5x}-\frac{a^5\log(x)}{b^6}+\frac{a^5\log(ax+b)}{b^6}+\frac{a}{4b^2x^4}-\frac{1}{5bx^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)*x^7),x]
```

```
[Out] -1/(5*b*x^5) + a/(4*b^2*x^4) - a^2/(3*b^3*x^3) + a^3/(2*b^4*x^2) - a^4/(b^5
*x) - (a^5*Log[x])/b^6 + (a^5*Log[b + a*x])/b^6
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x}\right)x^7}\,dx &= \int \frac{1}{x^6(b+ax)}\,dx\\ &= \int\left(\frac{1}{bx^6}-\frac{a}{b^2x^5}+\frac{a^2}{b^3x^4}-\frac{a^3}{b^4x^3}+\frac{a^4}{b^5x^2}-\frac{a^5}{b^6x}+\frac{a^6}{b^6(b+ax)}\right)dx\\ &= -\frac{1}{5bx^5}+\frac{a}{4b^2x^4}-\frac{a^2}{3b^3x^3}+\frac{a^3}{2b^4x^2}-\frac{a^4}{b^5x}-\frac{a^5\log(x)}{b^6}+\frac{a^5\log(b+ax)}{b^6}\end{aligned}$$

Mathematica [A] time = 0.01, size = 82, normalized size = 1.00

$$-\frac{a^5\log(x)}{b^6}+\frac{a^5\log(ax+b)}{b^6}-\frac{a^4}{b^5x}+\frac{a^3}{2b^4x^2}-\frac{a^2}{3b^3x^3}+\frac{a}{4b^2x^4}-\frac{1}{5bx^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)*x^7),x]
```

```
[Out] -1/5*1/(b*x^5) + a/(4*b^2*x^4) - a^2/(3*b^3*x^3) + a^3/(2*b^4*x^2) - a^4/(b
^5*x) - (a^5*Log[x])/b^6 + (a^5*Log[b + a*x])/b^6
```

fricas [A] time = 1.13, size = 76, normalized size = 0.93

$$\frac{60\,a^5x^5\log(ax+b)-60\,a^5x^5\log(x)-60\,a^4bx^4+30\,a^3b^2x^3-20\,a^2b^3x^2+15\,ab^4x-12\,b^5}{60\,b^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x^7,x, algorithm="fricas")
```

```
[Out] 1/60*(60*a^5*x^5*log(a*x + b) - 60*a^5*x^5*log(x) - 60*a^4*b*x^4 + 30*a^3*b
^2*x^3 - 20*a^2*b^3*x^2 + 15*a*b^4*x - 12*b^5)/(b^6*x^5)
```

giac [A] time = 0.17, size = 78, normalized size = 0.95

$$\frac{a^5\log(|ax+b|)}{b^6}-\frac{a^5\log(|x|)}{b^6}-\frac{60\,a^4bx^4-30\,a^3b^2x^3+20\,a^2b^3x^2-15\,ab^4x+12\,b^5}{60\,b^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x^7,x, algorithm="giac")
```

```
[Out] a^5*log(abs(a*x + b))/b^6 - a^5*log(abs(x))/b^6 - 1/60*(60*a^4*b*x^4 - 30*a
^3*b^2*x^3 + 20*a^2*b^3*x^2 - 15*a*b^4*x + 12*b^5)/(b^6*x^5)
```

maple [A] time = 0.01, size = 75, normalized size = 0.91

$$-\frac{a^5\ln(x)}{b^6}+\frac{a^5\ln(ax+b)}{b^6}-\frac{a^4}{b^5x}+\frac{a^3}{2b^4x^2}-\frac{a^2}{3b^3x^3}+\frac{a}{4b^2x^4}-\frac{1}{5b\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)/x^7,x)
```

```
[Out] -1/5/b/x^5+1/4*a/b^2/x^4-1/3*a^2/b^3/x^3+1/2*a^3/b^4/x^2-a^4/b^5/x-a^5*ln(x
)/b^6+a^5*ln(a*x+b)/b^6
```

maxima [A] time = 1.07, size = 73, normalized size = 0.89

$$\frac{a^5\log(ax+b)}{b^6}-\frac{a^5\log(x)}{b^6}-\frac{60\,a^4x^4-30\,a^3bx^3+20\,a^2b^2x^2-15\,ab^3x+12\,b^4}{60\,b^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x^7,x, algorithm="maxima")
```

```
[Out] a^5*log(a*x + b)/b^6 - a^5*log(x)/b^6 - 1/60*(60*a^4*x^4 - 30*a^3*b*x^3 + 2
0*a^2*b^2*x^2 - 15*a*b^3*x + 12*b^4)/(b^5*x^5)
```

mupad [B] time = 1.10, size = 70, normalized size = 0.85

$$\frac{2\,a^5\,\mathrm{atanh}\left(\frac{2\,a\,x}{b}+1\right)}{b^6}-\frac{a^4\,b\,x^4-\frac{a^3\,b^2\,x^3}{2}+\frac{a^2\,b^3\,x^2}{3}-\frac{a\,b^4\,x}{4}+\frac{b^5}{5}}{b^6\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b/x)),x)
```

```
[Out] (2*a^5*atanh((2*a*x)/b + 1))/b^6 - (b^5/5 + a^4*b*x^4 + (a^2*b^3*x^2)/3 - (
a^3*b^2*x^3)/2 - (a*b^4*x)/4)/(b^6*x^5)
```

sympy [A] time = 0.30, size = 68, normalized size = 0.83

$$\frac{a^5\left(-\log(x)+\log\left(x+\frac{b}{a}\right)\right)}{b^6}+\frac{-60a^4x^4+30a^3bx^3-20a^2b^2x^2+15ab^3x-12b^4}{60b^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x**7,x)
```

```
[Out] a**5*(-log(x) + log(x + b/a))/b**6 + (-60*a**4*x**4 + 30*a**3*b*x**3 - 20*a
**2*b**2*x**2 + 15*a*b**3*x - 12*b**4)/(60*b**5*x**5)
```

3.1619 $\int \frac{x^5}{\left(a+\frac{b}{x}\right)^2}\,dx$

Optimal. Leaf size=98

$$\frac{b^7}{a^8(ax+b)}+\frac{7b^6\log(ax+b)}{a^8}-\frac{6b^5x}{a^7}+\frac{5b^4x^2}{2a^6}-\frac{4b^3x^3}{3a^5}+\frac{3b^2x^4}{4a^4}-\frac{2bx^5}{5a^3}+\frac{x^6}{6a^2}$$

```
[Out] -6*b^5*x/a^7+5/2*b^4*x^2/a^6-4/3*b^3*x^3/a^5+3/4*b^2*x^4/a^4-2/5*b*x^5/a^3+
1/6*x^6/a^2+b^7/a^8/(a*x+b)+7*b^6*ln(a*x+b)/a^8
```

Rubi [A] time = 0.07, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$\frac{5b^4x^2}{2a^6}-\frac{4b^3x^3}{3a^5}+\frac{3b^2x^4}{4a^4}+\frac{b^7}{a^8(ax+b)}-\frac{6b^5x}{a^7}+\frac{7b^6\log(ax+b)}{a^8}-\frac{2bx^5}{5a^3}+\frac{x^6}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b/x)^2,x]
```

```
[Out] (-6*b^5*x)/a^7 + (5*b^4*x^2)/(2*a^6) - (4*b^3*x^3)/(3*a^5) + (3*b^2*x^4)/(4
*a^4) - (2*b*x^5)/(5*a^3) + x^6/(6*a^2) + b^7/(a^8*(b + a*x)) + (7*b^6*Log[
b + a*x])/a^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\left(a+\frac{b}{x}\right)^2}\,dx &= \int \frac{x^7}{(b+ax)^2}\,dx\\ &= \int\left(-\frac{6b^5}{a^7}+\frac{5b^4x}{a^6}-\frac{4b^3x^2}{a^5}+\frac{3b^2x^3}{a^4}-\frac{2bx^4}{a^3}+\frac{x^5}{a^2}-\frac{b^7}{a^7(b+ax)^2}+\frac{7b^6}{a^7(b+ax)}\right)dx\\ &= -\frac{6b^5x}{a^7}+\frac{5b^4x^2}{2a^6}-\frac{4b^3x^3}{3a^5}+\frac{3b^2x^4}{4a^4}-\frac{2bx^5}{5a^3}+\frac{x^6}{6a^2}+\frac{b^7}{a^8(b+ax)}+\frac{7b^6\log(b+ax)}{a^8}\end{aligned}$$

Mathematica [A] time = 0.02, size = 88, normalized size = 0.90

$$\frac{10a^6x^6-24a^5bx^5+45a^4b^2x^4-80a^3b^3x^3+150a^2b^4x^2+\frac{60b^7}{ax+b}+420b^6\log(ax+b)-360ab^5x}{60a^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b/x)^2,x]
```

```
[Out] (-360*a*b^5*x + 150*a^2*b^4*x^2 - 80*a^3*b^3*x^3 + 45*a^4*b^2*x^4 - 24*a^5*
b*x^5 + 10*a^6*x^6 + (60*b^7)/(b + a*x) + 420*b^6*Log[b + a*x])/(60*a^8)
```

fricas [A] time = 0.84, size = 107, normalized size = 1.09

$$\frac{10\,a^7x^7 - 14\,a^6bx^6 + 21\,a^5b^2x^5 - 35\,a^4b^3x^4 + 70\,a^3b^4x^3 - 210\,a^2b^5x^2 - 360\,ab^6x + 60\,b^7 + 420\left(ab^6x + b^7\right)\log(a}{60\left(a^9x + a^8b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/x)^2,x, algorithm="fricas")
```

```
[Out] 1/60*(10*a^7*x^7 - 14*a^6*b*x^6 + 21*a^5*b^2*x^5 - 35*a^4*b^3*x^4 + 70*a^3*
b^4*x^3 - 210*a^2*b^5*x^2 - 360*a*b^6*x + 60*b^7 + 420*(a*b^6*x + b^7)*log(
a*x + b))/(a^9*x + a^8*b)
```

giac [A] time = 0.16, size = 95, normalized size = 0.97

$$\frac{7\,b^6\log(|ax+b|)}{a^8} + \frac{b^7}{(ax+b)a^8} + \frac{10\,a^{10}x^6 - 24\,a^9bx^5 + 45\,a^8b^2x^4 - 80\,a^7b^3x^3 + 150\,a^6b^4x^2 - 360\,a^5b^5x}{60\,a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/x)^2,x, algorithm="giac")
```

```
[Out] 7*b^6*log(abs(a*x + b))/a^8 + b^7/((a*x + b)*a^8) + 1/60*(10*a^10*x^6 - 24*
a^9*b*x^5 + 45*a^8*b^2*x^4 - 80*a^7*b^3*x^3 + 150*a^6*b^4*x^2 - 360*a^5*b^5
*x)/a^12
```

maple [A] time = 0.01, size = 89, normalized size = 0.91

$$\frac{x^6}{6a^2} - \frac{2b\,x^5}{5a^3} + \frac{3b^2x^4}{4a^4} - \frac{4b^3x^3}{3a^5} + \frac{5b^4x^2}{2a^6} - \frac{6b^5x}{a^7} + \frac{b^7}{(ax+b)\,a^8} + \frac{7b^6\ln(ax+b)}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a+b/x)^2,x)
```

```
[Out] -6*b^5*x/a^7+5/2*b^4*x^2/a^6-4/3*b^3*x^3/a^5+3/4*b^2*x^4/a^4-2/5*b*x^5/a^3+
1/6*x^6/a^2+b^7/a^8/(a*x+b)+7*b^6*ln(a*x+b)/a^8
```

maxima [A] time = 1.06, size = 92, normalized size = 0.94

$$\frac{b^7}{a^9x + a^8b} + \frac{7\,b^6\log(ax+b)}{a^8} + \frac{10\,a^5x^6 - 24\,a^4bx^5 + 45\,a^3b^2x^4 - 80\,a^2b^3x^3 + 150\,ab^4x^2 - 360\,b^5x}{60\,a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/x)^2,x, algorithm="maxima")
```

```
[Out] b^7/(a^9*x + a^8*b) + 7*b^6*log(a*x + b)/a^8 + 1/60*(10*a^5*x^6 - 24*a^4*b*
x^5 + 45*a^3*b^2*x^4 - 80*a^2*b^3*x^3 + 150*a*b^4*x^2 - 360*b^5*x)/a^7
```

mupad [B] time = 0.06, size = 94, normalized size = 0.96

$$\frac{x^6}{6\,a^2} + \frac{7\,b^6\,\ln(b + a\,x)}{a^8} - \frac{2\,b\,x^5}{5\,a^3} - \frac{6\,b^5\,x}{a^7} + \frac{3\,b^2\,x^4}{4\,a^4} - \frac{4\,b^3\,x^3}{3\,a^5} + \frac{5\,b^4\,x^2}{2\,a^6} + \frac{b^7}{a\left(x\,a^8 + b\,a^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b/x)^2,x)
```

```
[Out] x^6/(6*a^2) + (7*b^6*log(b + a*x))/a^8 - (2*b*x^5)/(5*a^3) - (6*b^5*x)/a^7
+ (3*b^2*x^4)/(4*a^4) - (4*b^3*x^3)/(3*a^5) + (5*b^4*x^2)/(2*a^6) + b^7/(a*
(a^7*b + a^8*x))
```

sympy [**A**] time = 0.29, size = 99, normalized size = 1.01

$$\frac{b^7}{a^9x + a^8b} + \frac{x^6}{6a^2} - \frac{2bx^5}{5a^3} + \frac{3b^2x^4}{4a^4} - \frac{4b^3x^3}{3a^5} + \frac{5b^4x^2}{2a^6} - \frac{6b^5x}{a^7} + \frac{7b^6\log(ax+b)}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a+b/x)**2,x)
```

```
[Out] b**7/(a**9*x + a**8*b) + x**6/(6*a**2) - 2*b*x**5/(5*a**3) + 3*b**2*x**4/(4
*a**4) - 4*b**3*x**3/(3*a**5) + 5*b**4*x**2/(2*a**6) - 6*b**5*x/a**7 + 7*b*
*6*log(a*x + b)/a**8
```

3.1620 $\int \frac{x^4}{\left(a+\frac{b}{x}\right)^2}\,dx$

Optimal. Leaf size=81

$$-\frac{b^6}{a^7(ax+b)}-\frac{6b^5\log(ax+b)}{a^7}+\frac{5b^4x}{a^6}-\frac{2b^3x^2}{a^5}+\frac{b^2x^3}{a^4}-\frac{bx^4}{2a^3}+\frac{x^5}{5a^2}$$

```
[Out] 5*b^4*x/a^6-2*b^3*x^2/a^5+b^2*x^3/a^4-1/2*b*x^4/a^3+1/5*x^5/a^2-b^6/a^7/(a*
x+b)-6*b^5*ln(a*x+b)/a^7
```

Rubi [A] time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$-\frac{2b^3x^2}{a^5}+\frac{b^2x^3}{a^4}-\frac{b^6}{a^7(ax+b)}+\frac{5b^4x}{a^6}-\frac{6b^5\log(ax+b)}{a^7}-\frac{bx^4}{2a^3}+\frac{x^5}{5a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b/x)^2,x]
```

```
[Out] (5*b^4*x)/a^6 - (2*b^3*x^2)/a^5 + (b^2*x^3)/a^4 - (b*x^4)/(2*a^3) + x^5/(5*
a^2) - b^6/(a^7*(b + a*x)) - (6*b^5*Log[b + a*x])/a^7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\left(a+\frac{b}{x}\right)^2}\,dx &= \int \frac{x^6}{(b+ax)^2}\,dx\\ &= \int\left(\frac{5b^4}{a^6}-\frac{4b^3x}{a^5}+\frac{3b^2x^2}{a^4}-\frac{2bx^3}{a^3}+\frac{x^4}{a^2}+\frac{b^6}{a^6(b+ax)^2}-\frac{6b^5}{a^6(b+ax)}\right)dx\\ &= \frac{5b^4x}{a^6}-\frac{2b^3x^2}{a^5}+\frac{b^2x^3}{a^4}-\frac{bx^4}{2a^3}+\frac{x^5}{5a^2}-\frac{b^6}{a^7(b+ax)}-\frac{6b^5\log(b+ax)}{a^7}\end{aligned}$$

Mathematica [A] time = 0.03, size = 77, normalized size = 0.95

$$\frac{2a^5x^5-5a^4bx^4+10a^3b^2x^3-20a^2b^3x^2-\frac{10b^6}{ax+b}-60b^5\log(ax+b)+50ab^4x}{10a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b/x)^2,x]
```

```
[Out] (50*a*b^4*x - 20*a^2*b^3*x^2 + 10*a^3*b^2*x^3 - 5*a^4*b*x^4 + 2*a^5*x^5 - (
10*b^6)/(b + a*x) - 60*b^5*Log[b + a*x])/(10*a^7)
```

fricas [A] time = 1.14, size = 96, normalized size = 1.19

$$\frac{2\,a^6x^6 - 3\,a^5bx^5 + 5\,a^4b^2x^4 - 10\,a^3b^3x^3 + 30\,a^2b^4x^2 + 50\,ab^5x - 10\,b^6 - 60\left(ab^5x + b^6\right)\log\left(ax + b\right)}{10\left(a^8x + a^7b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/x)^2,x, algorithm="fricas")
```

```
[Out] 1/10*(2*a^6*x^6 - 3*a^5*b*x^5 + 5*a^4*b^2*x^4 - 10*a^3*b^3*x^3 + 30*a^2*b^4
*x^2 + 50*a*b^5*x - 10*b^6 - 60*(a*b^5*x + b^6)*log(a*x + b))/(a^8*x + a^7*
b)
```

giac [A] time = 0.15, size = 85, normalized size = 1.05

$$-\frac{6\,b^5\log\left(\left|ax + b\right|\right)}{a^7} - \frac{b^6}{(ax + b)a^7} + \frac{2\,a^8x^5 - 5\,a^7bx^4 + 10\,a^6b^2x^3 - 20\,a^5b^3x^2 + 50\,a^4b^4x}{10\,a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/x)^2,x, algorithm="giac")
```

```
[Out] -6*b^5*log(abs(a*x + b))/a^7 - b^6/((a*x + b)*a^7) + 1/10*(2*a^8*x^5 - 5*a^
7*b*x^4 + 10*a^6*b^2*x^3 - 20*a^5*b^3*x^2 + 50*a^4*b^4*x)/a^10
```

maple [A] time = 0.01, size = 78, normalized size = 0.96

$$\frac{x^5}{5a^2} - \frac{b\,x^4}{2a^3} + \frac{b^2x^3}{a^4} - \frac{2b^3x^2}{a^5} + \frac{5b^4x}{a^6} - \frac{b^6}{(ax + b)\,a^7} - \frac{6b^5\ln\left(ax + b\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a+b/x)^2,x)
```

```
[Out] 5*b^4*x/a^6-2*b^3*x^2/a^5+b^2*x^3/a^4-1/2*b*x^4/a^3+1/5*x^5/a^2-b^6/a^7/(a*
x+b)-6*b^5*ln(a*x+b)/a^7
```

maxima [A] time = 1.18, size = 82, normalized size = 1.01

$$-\frac{b^6}{a^8x + a^7b} - \frac{6\,b^5\log\left(ax + b\right)}{a^7} + \frac{2\,a^4x^5 - 5\,a^3bx^4 + 10\,a^2b^2x^3 - 20\,ab^3x^2 + 50\,b^4x}{10\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/x)^2,x, algorithm="maxima")
```

```
[Out] -b^6/(a^8*x + a^7*b) - 6*b^5*log(a*x + b)/a^7 + 1/10*(2*a^4*x^5 - 5*a^3*b*x
^4 + 10*a^2*b^2*x^3 - 20*a*b^3*x^2 + 50*b^4*x)/a^6
```

mupad [B] time = 0.05, size = 83, normalized size = 1.02

$$\frac{x^5}{5\,a^2} - \frac{6\,b^5\,\ln\left(b + a\,x\right)}{a^7} - \frac{b\,x^4}{2\,a^3} + \frac{5\,b^4\,x}{a^6} + \frac{b^2\,x^3}{a^4} - \frac{2\,b^3\,x^2}{a^5} - \frac{b^6}{a\left(x\,a^7 + b\,a^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b/x)^2,x)
```

```
[Out] x^5/(5*a^2) - (6*b^5*log(b + a*x))/a^7 - (b*x^4)/(2*a^3) + (5*b^4*x)/a^6 +
(b^2*x^3)/a^4 - (2*b^3*x^2)/a^5 - b^6/(a*(a^6*b + a^7*x))
```

sympy [**A**] time = 0.26, size = 78, normalized size = 0.96

$$-\frac{b^6}{a^8x+a^7b}+\frac{x^5}{5a^2}-\frac{bx^4}{2a^3}+\frac{b^2x^3}{a^4}-\frac{2b^3x^2}{a^5}+\frac{5b^4x}{a^6}-\frac{6b^5\log(ax+b)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a+b/x)**2,x)
```

```
[Out] -b**6/(a**8*x + a**7*b) + x**5/(5*a**2) - b*x**4/(2*a**3) + b**2*x**3/a**4
- 2*b**3*x**2/a**5 + 5*b**4*x/a**6 - 6*b**5*log(a*x + b)/a**7
```

3.1621 $\int \frac{x^3}{\left(a+\frac{b}{x}\right)^2}\,dx$

Optimal. Leaf size=72

$$\frac{b^5}{a^6(ax+b)}+\frac{5b^4\log(ax+b)}{a^6}-\frac{4b^3x}{a^5}+\frac{3b^2x^2}{2a^4}-\frac{2bx^3}{3a^3}+\frac{x^4}{4a^2}$$

```
[Out] -4*b^3*x/a^5+3/2*b^2*x^2/a^4-2/3*b*x^3/a^3+1/4*x^4/a^2+b^5/a^6/(a*x+b)+5*b^
4*ln(a*x+b)/a^6
```

Rubi [A] time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$\frac{3b^2x^2}{2a^4}+\frac{b^5}{a^6(ax+b)}-\frac{4b^3x}{a^5}+\frac{5b^4\log(ax+b)}{a^6}-\frac{2bx^3}{3a^3}+\frac{x^4}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b/x)^2,x]
```

```
[Out] (-4*b^3*x)/a^5 + (3*b^2*x^2)/(2*a^4) - (2*b*x^3)/(3*a^3) + x^4/(4*a^2) + b^
5/(a^6*(b + a*x)) + (5*b^4*Log[b + a*x])/a^6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(a+\frac{b}{x}\right)^2}\,dx &= \int \frac{x^5}{(b+ax)^2}\,dx\\ &= \int\left(-\frac{4b^3}{a^5}+\frac{3b^2x}{a^4}-\frac{2bx^2}{a^3}+\frac{x^3}{a^2}-\frac{b^5}{a^5(b+ax)^2}+\frac{5b^4}{a^5(b+ax)}\right)dx\\ &= -\frac{4b^3x}{a^5}+\frac{3b^2x^2}{2a^4}-\frac{2bx^3}{3a^3}+\frac{x^4}{4a^2}+\frac{b^5}{a^6(b+ax)}+\frac{5b^4\log(b+ax)}{a^6}\end{aligned}$$

Mathematica [A] time = 0.02, size = 66, normalized size = 0.92

$$\frac{3a^4x^4-8a^3bx^3+18a^2b^2x^2+\frac{12b^5}{ax+b}+60b^4\log(ax+b)-48ab^3x}{12a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b/x)^2,x]
```

```
[Out] (-48*a*b^3*x + 18*a^2*b^2*x^2 - 8*a^3*b*x^3 + 3*a^4*x^4 + (12*b^5)/(b + a*x
) + 60*b^4*Log[b + a*x])/(12*a^6)
```

fricas [A] time = 0.97, size = 85, normalized size = 1.18

$$\frac{3\,a^5x^5 - 5\,a^4bx^4 + 10\,a^3b^2x^3 - 30\,a^2b^3x^2 - 48\,ab^4x + 12\,b^5 + 60\left(ab^4x + b^5\right)\log\left(ax + b\right)}{12\left(a^7x + a^6b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a^5*x^5 - 5*a^4*b*x^4 + 10*a^3*b^2*x^3 - 30*a^2*b^3*x^2 - 48*a*b^4*
x + 12*b^5 + 60*(a*b^4*x + b^5)*log(a*x + b))/(a^7*x + a^6*b)
```

giac [A] time = 0.15, size = 73, normalized size = 1.01

$$\frac{5\,b^4\log\left(|ax + b|\right)}{a^6} + \frac{b^5}{(ax + b)a^6} + \frac{3\,a^6x^4 - 8\,a^5bx^3 + 18\,a^4b^2x^2 - 48\,a^3b^3x}{12\,a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x)^2,x, algorithm="giac")
```

```
[Out] 5*b^4*log(abs(a*x + b))/a^6 + b^5/((a*x + b)*a^6) + 1/12*(3*a^6*x^4 - 8*a^5
*b*x^3 + 18*a^4*b^2*x^2 - 48*a^3*b^3*x)/a^8
```

maple [A] time = 0.01, size = 67, normalized size = 0.93

$$\frac{x^4}{4a^2} - \frac{2b\,x^3}{3a^3} + \frac{3b^2x^2}{2a^4} - \frac{4b^3x}{a^5} + \frac{b^5}{(ax + b)\,a^6} + \frac{5b^4\ln\left(ax + b\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b/x)^2,x)
```

```
[Out] -4*b^3*x/a^5+3/2*b^2*x^2/a^4-2/3*b*x^3/a^3+1/4*x^4/a^2+b^5/a^6/(a*x+b)+5*b^
4*ln(a*x+b)/a^6
```

maxima [A] time = 1.09, size = 70, normalized size = 0.97

$$\frac{b^5}{a^7x + a^6b} + \frac{5\,b^4\log\left(ax + b\right)}{a^6} + \frac{3\,a^3x^4 - 8\,a^2bx^3 + 18\,ab^2x^2 - 48\,b^3x}{12\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x)^2,x, algorithm="maxima")
```

```
[Out] b^5/(a^7*x + a^6*b) + 5*b^4*log(a*x + b)/a^6 + 1/12*(3*a^3*x^4 - 8*a^2*b*x^
3 + 18*a*b^2*x^2 - 48*b^3*x)/a^5
```

mupad [B] time = 0.05, size = 72, normalized size = 1.00

$$\frac{x^4}{4\,a^2} + \frac{5\,b^4\,\ln\left(b + a\,x\right)}{a^6} - \frac{2\,b\,x^3}{3\,a^3} - \frac{4\,b^3\,x}{a^5} + \frac{3\,b^2\,x^2}{2\,a^4} + \frac{b^5}{a\,\left(x\,a^6 + b\,a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b/x)^2,x)
```

```
[Out] x^4/(4*a^2) + (5*b^4*log(b + a*x))/a^6 - (2*b*x^3)/(3*a^3) - (4*b^3*x)/a^5
+ (3*b^2*x^2)/(2*a^4) + b^5/(a*(a^5*b + a^6*x))
```

sympy [**A**] time = 0.26, size = 71, normalized size = 0.99

$$\frac{b^5}{a^7x + a^6b} + \frac{x^4}{4a^2} - \frac{2bx^3}{3a^3} + \frac{3b^2x^2}{2a^4} - \frac{4b^3x}{a^5} + \frac{5b^4\log(ax+b)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b/x)**2,x)
```

```
[Out] b**5/(a**7*x + a**6*b) + x**4/(4*a**2) - 2*b*x**3/(3*a**3) + 3*b**2*x**2/(2
*a**4) - 4*b**3*x/a**5 + 5*b**4*log(a*x + b)/a**6
```

3.1622 $\int \frac{x^2}{\left(a+\frac{b}{x}\right)^2}\,dx$

Optimal. Leaf size=58

$$-\frac{b^4}{a^5(ax+b)}-\frac{4b^3\log(ax+b)}{a^5}+\frac{3b^2x}{a^4}-\frac{bx^2}{a^3}+\frac{x^3}{3a^2}$$

```
[Out] 3*b^2*x/a^4-b*x^2/a^3+1/3*x^3/a^2-b^4/a^5/(a*x+b)-4*b^3*ln(a*x+b)/a^5
```

Rubi [A] time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$-\frac{b^4}{a^5(ax+b)}+\frac{3b^2x}{a^4}-\frac{4b^3\log(ax+b)}{a^5}-\frac{bx^2}{a^3}+\frac{x^3}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b/x)^2,x]
```

```
[Out] (3*b^2*x)/a^4 - (b*x^2)/a^3 + x^3/(3*a^2) - b^4/(a^5*(b + a*x)) - (4*b^3*Lo
g[b + a*x])/a^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a+\frac{b}{x}\right)^2}\,dx &= \int \frac{x^4}{(b+ax)^2}\,dx\\ &= \int\left(\frac{3b^2}{a^4}-\frac{2bx}{a^3}+\frac{x^2}{a^2}+\frac{b^4}{a^4(b+ax)^2}-\frac{4b^3}{a^4(b+ax)}\right)dx\\ &= \frac{3b^2x}{a^4}-\frac{bx^2}{a^3}+\frac{x^3}{3a^2}-\frac{b^4}{a^5(b+ax)}-\frac{4b^3\log(b+ax)}{a^5}\end{aligned}$$

Mathematica [A] time = 0.03, size = 54, normalized size = 0.93

$$\frac{a^3x^3-3a^2bx^2-\frac{3b^4}{ax+b}-12b^3\log(ax+b)+9ab^2x}{3a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b/x)^2,x]
```

```
[Out] (9*a*b^2*x - 3*a^2*b*x^2 + a^3*x^3 - (3*b^4)/(b + a*x) - 12*b^3*Log[b + a*x
])/(3*a^5)
```

fricas [A] time = 0.89, size = 73, normalized size = 1.26

$$\frac{a^4x^4 - 2\,a^3bx^3 + 6\,a^2b^2x^2 + 9\,ab^3x - 3\,b^4 - 12\left(ab^3x + b^4\right)\log\left(ax + b\right)}{3\left(a^6x + a^5b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(a^4*x^4 - 2*a^3*b*x^3 + 6*a^2*b^2*x^2 + 9*a*b^3*x - 3*b^4 - 12*(a*b^3*
x + b^4)*log(a*x + b))/(a^6*x + a^5*b)
```

giac [A] time = 0.18, size = 62, normalized size = 1.07

$$-\frac{4\,b^3\log\left(|ax + b|\right)}{a^5} - \frac{b^4}{(ax + b)a^5} + \frac{a^4x^3 - 3\,a^3bx^2 + 9\,a^2b^2x}{3\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x)^2,x, algorithm="giac")
```

```
[Out] -4*b^3*log(abs(a*x + b))/a^5 - b^4/((a*x + b)*a^5) + 1/3*(a^4*x^3 - 3*a^3*b
*x^2 + 9*a^2*b^2*x)/a^6
```

maple [A] time = 0.01, size = 57, normalized size = 0.98

$$\frac{x^3}{3a^2} - \frac{b\,x^2}{a^3} + \frac{3b^2x}{a^4} - \frac{b^4}{(ax + b)\,a^5} - \frac{4b^3\ln\left(ax + b\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b/x)^2,x)
```

```
[Out] 3*b^2*x/a^4-b*x^2/a^3+1/3*x^3/a^2-b^4/a^5/(a*x+b)-4*b^3*ln(a*x+b)/a^5
```

maxima [A] time = 1.05, size = 59, normalized size = 1.02

$$-\frac{b^4}{a^6x + a^5b} - \frac{4\,b^3\log\left(ax + b\right)}{a^5} + \frac{a^2x^3 - 3\,abx^2 + 9\,b^2x}{3\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x)^2,x, algorithm="maxima")
```

```
[Out] -b^4/(a^6*x + a^5*b) - 4*b^3*log(a*x + b)/a^5 + 1/3*(a^2*x^3 - 3*a*b*x^2 +
9*b^2*x)/a^4
```

mupad [B] time = 1.05, size = 62, normalized size = 1.07

$$\frac{x^3}{3\,a^2} - \frac{4\,b^3\,\ln\left(b + a\,x\right)}{a^5} - \frac{b\,x^2}{a^3} + \frac{3\,b^2\,x}{a^4} - \frac{b^4}{a\,\left(x\,a^5 + b\,a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b/x)^2,x)
```

```
[Out] x^3/(3*a^2) - (4*b^3*log(b + a*x))/a^5 - (b*x^2)/a^3 + (3*b^2*x)/a^4 - b^4/
(a*(a^4*b + a^5*x))
```

sympy [**A**] time = 0.22, size = 54, normalized size = 0.93

$$-\frac{b^4}{a^6x + a^5b} + \frac{x^3}{3a^2} - \frac{bx^2}{a^3} + \frac{3b^2x}{a^4} - \frac{4b^3\log(ax+b)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b/x)**2,x)
```

```
[Out] -b**4/(a**6*x + a**5*b) + x**3/(3*a**2) - b*x**2/a**3 + 3*b**2*x/a**4 - 4*b
**3*log(a*x + b)/a**5
```

3.1623 $\int \frac{x}{\left(a+\frac{b}{x}\right)^2}\,dx$

Optimal. Leaf size=46

$$\frac{b^3}{a^4(ax+b)}+\frac{3b^2\log(ax+b)}{a^4}-\frac{2bx}{a^3}+\frac{x^2}{2a^2}$$

```
[Out] -2*b*x/a^3+1/2*x^2/a^2+b^3/a^4/(a*x+b)+3*b^2*ln(a*x+b)/a^4
```

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {263, 43}

$$\frac{b^3}{a^4(ax+b)}+\frac{3b^2\log(ax+b)}{a^4}-\frac{2bx}{a^3}+\frac{x^2}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b/x)^2,x]
```

```
[Out] (-2*b*x)/a^3 + x^2/(2*a^2) + b^3/(a^4*(b + a*x)) + (3*b^2*Log[b + a*x])/a^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a+\frac{b}{x}\right)^2}\,dx &= \int \frac{x^3}{(b+ax)^2}\,dx\\ &= \int\left(-\frac{2b}{a^3}+\frac{x}{a^2}-\frac{b^3}{a^3(b+ax)^2}+\frac{3b^2}{a^3(b+ax)}\right)dx\\ &= -\frac{2bx}{a^3}+\frac{x^2}{2a^2}+\frac{b^3}{a^4(b+ax)}+\frac{3b^2\log(b+ax)}{a^4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 0.93

$$\frac{a^2x^2+\frac{2b^3}{ax+b}+6b^2\log(ax+b)-4abx}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b/x)^2,x]
```

```
[Out] (-4*a*b*x + a^2*x^2 + (2*b^3)/(b + a*x) + 6*b^2*Log[b + a*x])/(2*a^4)
```

fricas [A] time = 1.06, size = 62, normalized size = 1.35

$$\frac{a^3x^3 - 3\,a^2bx^2 - 4\,ab^2x + 2\,b^3 + 6\left(ab^2x + b^3\right)\log\left(ax + b\right)}{2\left(a^5x + a^4b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a^3*x^3 - 3*a^2*b*x^2 - 4*a*b^2*x + 2*b^3 + 6*(a*b^2*x + b^3)*log(a*x
+ b))/(a^5*x + a^4*b)
```

giac [A] time = 0.15, size = 48, normalized size = 1.04

$$\frac{3\,b^2\log\left(|ax + b|\right)}{a^4} + \frac{b^3}{(ax + b)a^4} + \frac{a^2x^2 - 4\,abx}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x)^2,x, algorithm="giac")
```

```
[Out] 3*b^2*log(abs(a*x + b))/a^4 + b^3/((a*x + b)*a^4) + 1/2*(a^2*x^2 - 4*a*b*x)
/a^4
```

maple [A] time = 0.01, size = 45, normalized size = 0.98

$$\frac{x^2}{2a^2} - \frac{2bx}{a^3} + \frac{b^3}{(ax + b)\,a^4} + \frac{3b^2\ln\left(ax + b\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b/x)^2,x)
```

```
[Out] -2*b*x/a^3+1/2*x^2/a^2+b^3/a^4/(a*x+b)+3*b^2*ln(a*x+b)/a^4
```

maxima [A] time = 1.09, size = 47, normalized size = 1.02

$$\frac{b^3}{a^5x + a^4b} + \frac{3\,b^2\log\left(ax + b\right)}{a^4} + \frac{ax^2 - 4\,bx}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x)^2,x, algorithm="maxima")
```

```
[Out] b^3/(a^5*x + a^4*b) + 3*b^2*log(a*x + b)/a^4 + 1/2*(a*x^2 - 4*b*x)/a^3
```

mupad [B] time = 0.04, size = 50, normalized size = 1.09

$$\frac{x^2}{2\,a^2} + \frac{3\,b^2\,\ln\left(b + a\,x\right)}{a^4} + \frac{b^3}{a\,\left(x\,a^4 + b\,a^3\right)} - \frac{2\,b\,x}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b/x)^2,x)
```

```
[Out] x^2/(2*a^2) + (3*b^2*log(b + a*x))/a^4 + b^3/(a*(a^3*b + a^4*x)) - (2*b*x)/
a^3
```

sympy [A] time = 0.20, size = 44, normalized size = 0.96

$$\frac{b^3}{a^5x + a^4b} + \frac{x^2}{2a^2} - \frac{2bx}{a^3} + \frac{3b^2\log\left(ax + b\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x)**2,x)
```

```
[Out] b**3/(a**5*x + a**4*b) + x**2/(2*a**2) - 2*b*x/a**3 + 3*b**2*log(a*x + b)/a
**4
```

3.1624 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2}\,dx$

Optimal. Leaf size=34

$$-\frac{2b\log(ax+b)}{a^3}+\frac{2x}{a^2}-\frac{x}{a\left(a+\frac{b}{x}\right)}$$

```
[Out] 2*x/a^2-x/a/(a+b/x)-2*b*ln(a*x+b)/a^3
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {192, 193, 43}

$$-\frac{2b\log(ax+b)}{a^3}+\frac{2x}{a^2}-\frac{x}{a\left(a+\frac{b}{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(-2),x]
```

```
[Out] (2*x)/a^2 - x/(a*(a + b/x)) - (2*b*Log[b + a*x])/a^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x}\right)^2}\,dx &= -\frac{x}{a\left(a+\frac{b}{x}\right)} + \frac{2\int \frac{1}{a+\frac{b}{x}}\,dx}{a} \\ &= -\frac{x}{a\left(a+\frac{b}{x}\right)} + \frac{2\int \frac{x}{b+ax}\,dx}{a} \\ &= -\frac{x}{a\left(a+\frac{b}{x}\right)} + \frac{2\int \left(\frac{1}{a}-\frac{b}{a(b+ax)}\right)\,dx}{a} \\ &= \frac{2x}{a^2} - \frac{x}{a\left(a+\frac{b}{x}\right)} - \frac{2b\log(b+ax)}{a^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.85

$$\frac{-\frac{b^2}{ax+b} - 2b\log(ax+b) + ax}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(-2),x]
[Out] (a*x - b^2/(b + a*x) - 2*b*Log[b + a*x])/a^3
```

fricas [A] time = 1.07, size = 47, normalized size = 1.38

$$\frac{a^2x^2 + abx - b^2 - 2\left(abx+b^2\right)\log\left(ax+b\right)}{a^4x + a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2,x, algorithm="fricas")
[Out] (a^2*x^2 + a*b*x - b^2 - 2*(a*b*x + b^2)*log(a*x + b))/(a^4*x + a^3*b)
```

giac [A] time = 0.15, size = 34, normalized size = 1.00

$$\frac{x}{a^2} - \frac{2\,b\log\left(|ax+b|\right)}{a^3} - \frac{b^2}{(ax+b)a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2,x, algorithm="giac")
[Out] x/a^2 - 2*b*log(abs(a*x + b))/a^3 - b^2/((a*x + b)*a^3)
```

maple [A] time = 0.00, size = 34, normalized size = 1.00

$$\frac{x}{a^2} - \frac{b^2}{(ax+b)\,a^3} - \frac{2b\ln\left(ax+b\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^2,x)
[Out] x/a^2-2*b*ln(a*x+b)/a^3-1/a^3*b^2/(a*x+b)
```

maxima [A] time = 1.25, size = 36, normalized size = 1.06

$$-\frac{b^2}{a^4x + a^3b} + \frac{x}{a^2} - \frac{2\,b\log\left(ax + b\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2,x, algorithm="maxima")
```

```
[Out] -b^2/(a^4*x + a^3*b) + x/a^2 - 2*b*log(a*x + b)/a^3
```

mupad [B] time = 1.06, size = 36, normalized size = 1.06

$$\frac{x}{a^2} - \frac{b^2}{x\,a^4 + b\,a^3} - \frac{2\,b\,\ln\left(b + a\,x\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x)^2,x)
```

```
[Out] x/a^2 - b^2/(a^3*b + a^4*x) - (2*b*log(b + a*x))/a^3
```

sympy [A] time = 0.17, size = 31, normalized size = 0.91

$$-\frac{b^2}{a^4x + a^3b} + \frac{x}{a^2} - \frac{2b\log\left(ax + b\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**2,x)
```

```
[Out] -b**2/(a**4*x + a**3*b) + x/a**2 - 2*b*log(a*x + b)/a**3
```

3.1625 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x}\,dx$

Optimal. Leaf size=23

$$\frac{b}{a^2(ax+b)}+\frac{\log(ax+b)}{a^2}$$

```
[Out] b/a^2/(a*x+b)+ln(a*x+b)/a^2
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$\frac{b}{a^2(ax+b)}+\frac{\log(ax+b)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^2*x),x]
```

```
[Out] b/(a^2*(b + a*x)) + Log[b + a*x]/a^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x}\,dx &= \int \frac{x}{(b+ax)^2}\,dx\\ &= \int \left(-\frac{b}{a(b+ax)^2}+\frac{1}{a(b+ax)}\right)dx\\ &= \frac{b}{a^2(b+ax)}+\frac{\log(b+ax)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.87

$$\frac{\frac{b}{ax+b}+\log(ax+b)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^2*x),x]
```

```
[Out] (b/(b + a*x) + Log[b + a*x])/a^2
```

fricas [A] time = 0.85, size = 28, normalized size = 1.22

$$\frac{(ax+b)\log(ax+b)+b}{a^3x+a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x,x, algorithm="fricas")
[Out] ((a*x + b)*log(a*x + b) + b)/(a^3*x + a^2*b)
```

giac [A] time = 0.19, size = 24, normalized size = 1.04

$$\frac{\log(|ax+b|)}{a^2} + \frac{b}{(ax+b)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x,x, algorithm="giac")
[Out] log(abs(a*x + b))/a^2 + b/((a*x + b)*a^2)
```

maple [A] time = 0.01, size = 24, normalized size = 1.04

$$\frac{b}{(ax+b)\,a^2} + \frac{\ln(ax+b)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^2/x,x)
[Out] b/a^2/(a*x+b)+ln(a*x+b)/a^2
```

maxima [A] time = 1.04, size = 26, normalized size = 1.13

$$\frac{b}{a^3x + a^2b} + \frac{\log(ax+b)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x,x, algorithm="maxima")
[Out] b/(a^3*x + a^2*b) + log(a*x + b)/a^2
```

mupad [B] time = 1.06, size = 23, normalized size = 1.00

$$\frac{\ln(b + a\,x)}{a^2} + \frac{b}{a^2\,(b + a\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b/x)^2),x)
[Out] log(b + a*x)/a^2 + b/(a^2*(b + a*x))
```

sympy [A] time = 0.16, size = 20, normalized size = 0.87

$$\frac{b}{a^3x + a^2b} + \frac{\log(ax+b)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**2/x,x)
[Out] b/(a**3*x + a**2*b) + log(a*x + b)/a**2
```

3.1626 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^2}\,dx$

Optimal. Leaf size=13

$$\frac{1}{b\left(a+\frac{b}{x}\right)}$$

```
[Out] 1/b/(a+b/x)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$\frac{1}{b\left(a+\frac{b}{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^2*x^2),x]
```

```
[Out] 1/(b*(a + b/x))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^2}\,dx = \frac{1}{b\left(a+\frac{b}{x}\right)}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 0.92

$$-\frac{1}{a(ax+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^2*x^2),x]
```

```
[Out] -(1/(a*(b + a*x)))
```

fricas [A] time = 0.80, size = 13, normalized size = 1.00

$$-\frac{1}{a^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x^2,x, algorithm="fricas")
```

```
[Out] -1/(a^2*x + a*b)
```

giac [**A**] time = 0.15, size = 13, normalized size = 1.00

$$\frac{1}{\left(a+\frac{b}{x}\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x^2,x, algorithm="giac")
[Out] 1/((a + b/x)*b)
```

maple [**A**] time = 0.00, size = 13, normalized size = 1.00

$$-\frac{1}{(ax+b)\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^2/x^2,x)
[Out] -1/a/(a*x+b)
```

maxima [**A**] time = 1.05, size = 13, normalized size = 1.00

$$\frac{1}{\left(a+\frac{b}{x}\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x^2,x, algorithm="maxima")
[Out] 1/((a + b/x)*b)
```

mupad [**B**] time = 0.03, size = 12, normalized size = 0.92

$$-\frac{1}{a\ (b+a\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b/x)^2),x)
[Out] -1/(a*(b + a*x))
```

sympy [**A**] time = 0.14, size = 10, normalized size = 0.77

$$-\frac{1}{a^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**2/x**2,x)
[Out] -1/(a**2*x + a*b)
```

3.1627 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^3}\,dx$

Optimal. Leaf size=29

$$-\frac{\log(ax+b)}{b^2}+\frac{1}{b(ax+b)}+\frac{\log(x)}{b^2}$$

```
[Out] 1/b/(a*x+b)+ln(x)/b^2-ln(a*x+b)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 44}

$$-\frac{\log(ax+b)}{b^2}+\frac{1}{b(ax+b)}+\frac{\log(x)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^2*x^3),x]
```

```
[Out] 1/(b*(b + a*x)) + Log[x]/b^2 - Log[b + a*x]/b^2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^3}\,dx &= \int \frac{1}{x(b+ax)^2}\,dx\\ &= \int \left(\frac{1}{b^2 x}-\frac{a}{b(b+ax)^2}-\frac{a}{b^2(b+ax)}\right)dx\\ &= \frac{1}{b(b+ax)}+\frac{\log(x)}{b^2}-\frac{\log(b+ax)}{b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.83

$$\frac{\frac{b}{ax+b}-\log(ax+b)+\log(x)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^2*x^3),x]
```

```
[Out] (b/(b + a*x) + Log[x] - Log[b + a*x])/b^2
```

fricas [A] time = 1.03, size = 39, normalized size = 1.34

$$-\frac{(ax+b)\log(ax+b)-(ax+b)\log(x)-b}{ab^2x+b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x^3,x, algorithm="fricas")
[Out] -((a*x + b)*log(a*x + b) - (a*x + b)*log(x) - b)/(a*b^2*x + b^3)
```

giac [A] time = 0.15, size = 31, normalized size = 1.07

$$-\frac{\log(|ax+b|)}{b^2}+\frac{\log(|x|)}{b^2}+\frac{1}{(ax+b)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x^3,x, algorithm="giac")
[Out] -log(abs(a*x + b))/b^2 + log(abs(x))/b^2 + 1/((a*x + b)*b)
```

maple [A] time = 0.01, size = 30, normalized size = 1.03

$$\frac{1}{(ax+b)\,b}+\frac{\ln(x)}{b^2}-\frac{\ln(ax+b)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^2/x^3,x)
[Out] 1/b/(a*x+b)+ln(x)/b^2-ln(a*x+b)/b^2
```

maxima [A] time = 1.08, size = 28, normalized size = 0.97

$$\frac{1}{abx+b^2}-\frac{\log(ax+b)}{b^2}+\frac{\log(x)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x^3,x, algorithm="maxima")
[Out] 1/(a*b*x + b^2) - log(a*x + b)/b^2 + log(x)/b^2
```

mupad [B] time = 0.04, size = 26, normalized size = 0.90

$$\frac{1}{b^2+a\,x\,b}-\frac{2\,\mathrm{atanh}\left(\frac{2\,a\,x}{b}+1\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b/x)^2),x)
[Out] 1/(b^2 + a*b*x) - (2*atanh((2*a*x)/b + 1))/b^2
```

sympy [A] time = 0.22, size = 22, normalized size = 0.76

$$\frac{1}{abx+b^2}+\frac{\log(x)-\log\left(x+\frac{b}{a}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**2/x**3,x)
[Out] 1/(a*b*x + b**2) + (log(x) - log(x + b/a))/b**2
```

3.1628 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^4}\,dx$

Optimal. Leaf size=42

$$-\frac{2a\log(x)}{b^3}+\frac{2a\log(ax+b)}{b^3}-\frac{a}{b^2(ax+b)}-\frac{1}{b^2x}$$

```
[Out] -1/b^2/x-a/b^2/(a*x+b)-2*a*ln(x)/b^3+2*a*ln(a*x+b)/b^3
```

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 44}

$$-\frac{a}{b^2(ax+b)}-\frac{2a\log(x)}{b^3}+\frac{2a\log(ax+b)}{b^3}-\frac{1}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^2*x^4),x]
```

```
[Out] -(1/(b^2*x)) - a/(b^2*(b + a*x)) - (2*a*Log[x])/b^3 + (2*a*Log[b + a*x])/b^
3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^4}\,dx &= \int \frac{1}{x^2(b+ax)^2}\,dx\\ &= \int\left(\frac{1}{b^2x^2}-\frac{2a}{b^3x}+\frac{a^2}{b^2(b+ax)^2}+\frac{2a^2}{b^3(b+ax)}\right)dx\\ &= -\frac{1}{b^2x}-\frac{a}{b^2(b+ax)}-\frac{2a\log(x)}{b^3}+\frac{2a\log(b+ax)}{b^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 35, normalized size = 0.83

$$-\frac{b\left(\frac{a}{ax+b}+\frac{1}{x}\right)-2a\log(ax+b)+2a\log(x)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^2*x^4),x]
```

```
[Out] -((b*(x^(-1) + a/(b + a*x)) + 2*a*Log[x] - 2*a*Log[b + a*x])/b^3)
```

fricas [A] time = 0.90, size = 63, normalized size = 1.50

$$-\frac{2\,abx + b^2 - 2\left(a^2x^2 + abx\right)\log\left(ax + b\right) + 2\left(a^2x^2 + abx\right)\log(x)}{ab^3x^2 + b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x^4,x, algorithm="fricas")
```

```
[Out] -(2*a*b*x + b^2 - 2*(a^2*x^2 + a*b*x)*log(a*x + b) + 2*(a^2*x^2 + a*b*x)*lo
g(x))/(a*b^3*x^2 + b^4*x)
```

giac [A] time = 0.15, size = 45, normalized size = 1.07

$$\frac{2\,a\log\left(|ax + b|\right)}{b^3} - \frac{2\,a\log\left(|x|\right)}{b^3} - \frac{2\,ax + b}{\left(ax^2 + bx\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x^4,x, algorithm="giac")
```

```
[Out] 2*a*log(abs(a*x + b))/b^3 - 2*a*log(abs(x))/b^3 - (2*a*x + b)/((a*x^2 + b*x
)*b^2)
```

maple [A] time = 0.01, size = 43, normalized size = 1.02

$$-\frac{a}{(ax + b)\,b^2} - \frac{2a\ln(x)}{b^3} + \frac{2a\ln\left(ax + b\right)}{b^3} - \frac{1}{b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^2/x^4,x)
```

```
[Out] -1/b^2/x-a/b^2/(a*x+b)-2*a*ln(x)/b^3+2*a*ln(a*x+b)/b^3
```

maxima [A] time = 1.14, size = 45, normalized size = 1.07

$$-\frac{2\,ax + b}{ab^2x^2 + b^3x} + \frac{2\,a\log\left(ax + b\right)}{b^3} - \frac{2\,a\log(x)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x^4,x, algorithm="maxima")
```

```
[Out] -(2*a*x + b)/(a*b^2*x^2 + b^3*x) + 2*a*log(a*x + b)/b^3 - 2*a*log(x)/b^3
```

mupad [B] time = 1.07, size = 41, normalized size = 0.98

$$\frac{4\,a\,\operatorname{atanh}\left(\frac{2\,a\,x}{b} + 1\right)}{b^3} - \frac{\frac{1}{b} + \frac{2\,a\,x}{b^2}}{a\,x^2 + b\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b/x)^2),x)
```

```
[Out] (4*a*atanh((2*a*x)/b + 1))/b^3 - (1/b + (2*a*x)/b^2)/(b*x + a*x^2)
```

sympy [A] time = 0.28, size = 37, normalized size = 0.88

$$\frac{2a\left(-\log\left(x\right) + \log\left(x + \frac{b}{a}\right)\right)}{b^3} + \frac{-2ax - b}{ab^2x^2 + b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**2/x**4,x)
```

```
[Out] 2*a*(-log(x) + log(x + b/a))/b**3 + (-2*a*x - b)/(a*b**2*x**2 + b**3*x)
```

3.1629 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^5}\,dx$

Optimal. Leaf size=58

$$\frac{3a^2\log(x)}{b^4}-\frac{3a^2\log(ax+b)}{b^4}+\frac{a^2}{b^3(ax+b)}+\frac{2a}{b^3x}-\frac{1}{2b^2x^2}$$

```
[Out] -1/2/b^2/x^2+2*a/b^3/x+a^2/b^3/(a*x+b)+3*a^2*ln(x)/b^4-3*a^2*ln(a*x+b)/b^4
```

Rubi [A] time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 44}

$$\frac{a^2}{b^3(ax+b)}+\frac{3a^2\log(x)}{b^4}-\frac{3a^2\log(ax+b)}{b^4}+\frac{2a}{b^3x}-\frac{1}{2b^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^2*x^5),x]
```

```
[Out] -1/(2*b^2*x^2) + (2*a)/(b^3*x) + a^2/(b^3*(b + a*x)) + (3*a^2*Log[x])/b^4 -
 (3*a^2*Log[b + a*x])/b^4
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^5}\,dx &= \int \frac{1}{x^3(b+ax)^2}\,dx\\ &= \int\left(\frac{1}{b^2x^3}-\frac{2a}{b^3x^2}+\frac{3a^2}{b^4x}-\frac{a^3}{b^3(b+ax)^2}-\frac{3a^3}{b^4(b+ax)}\right)dx\\ &= -\frac{1}{2b^2x^2}+\frac{2a}{b^3x}+\frac{a^2}{b^3(b+ax)}+\frac{3a^2\log(x)}{b^4}-\frac{3a^2\log(b+ax)}{b^4}\end{aligned}$$

Mathematica [A] time = 0.06, size = 53, normalized size = 0.91

$$\frac{b\left(\frac{2a^2}{ax+b}+\frac{4a}{x}-\frac{b}{x^2}\right)-6a^2\log(ax+b)+6a^2\log(x)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^2*x^5),x]
```

```
[Out] (b*(-(b/x^2) + (4*a)/x + (2*a^2)/(b + a*x)) + 6*a^2*Log[x] - 6*a^2*Log[b +
a*x])/(2*b^4)
```

fricas [A] time = 0.91, size = 86, normalized size = 1.48

$$\frac{6\,a^2bx^2 + 3\,ab^2x - b^3 - 6\left(a^3x^3 + a^2bx^2\right)\log\left(ax+b\right) + 6\left(a^3x^3 + a^2bx^2\right)\log(x)}{2\left(ab^4x^3 + b^5x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a^2*b*x^2 + 3*a*b^2*x - b^3 - 6*(a^3*x^3 + a^2*b*x^2)*log(a*x + b) +
 6*(a^3*x^3 + a^2*b*x^2)*log(x))/(a*b^4*x^3 + b^5*x^2)
```

giac [A] time = 0.15, size = 64, normalized size = 1.10

$$-\frac{3\,a^2\log\left(\left|ax+b\right|\right)}{b^4} + \frac{3\,a^2\log\left(\left|x\right|\right)}{b^4} + \frac{6\,a^2bx^2 + 3\,ab^2x - b^3}{2\,(ax+b)b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x^5,x, algorithm="giac")
```

```
[Out] -3*a^2*log(abs(a*x + b))/b^4 + 3*a^2*log(abs(x))/b^4 + 1/2*(6*a^2*b*x^2 + 3
*a*b^2*x - b^3)/((a*x + b)*b^4*x^2)
```

maple [A] time = 0.01, size = 57, normalized size = 0.98

$$\frac{a^2}{(ax+b)\,b^3} + \frac{3a^2\ln(x)}{b^4} - \frac{3a^2\ln\left(ax+b\right)}{b^4} + \frac{2a}{b^3x} - \frac{1}{2b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^2/x^5,x)
```

```
[Out] -1/2/b^2/x^2+2*a/b^3/x+a^2/b^3/(a*x+b)+3*a^2*ln(x)/b^4-3*a^2*ln(a*x+b)/b^4
```

maxima [A] time = 1.03, size = 64, normalized size = 1.10

$$\frac{6\,a^2x^2 + 3\,abx - b^2}{2\left(ab^3x^3 + b^4x^2\right)} - \frac{3\,a^2\log\left(ax+b\right)}{b^4} + \frac{3\,a^2\log(x)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*(6*a^2*x^2 + 3*a*b*x - b^2)/(a*b^3*x^3 + b^4*x^2) - 3*a^2*log(a*x + b)/
b^4 + 3*a^2*log(x)/b^4
```

mupad [B] time = 1.08, size = 57, normalized size = 0.98

$$\frac{\frac{3\,a^2\,x^2}{b^3} - \frac{1}{2\,b} + \frac{3\,a\,x}{2\,b^2}}{a\,x^3 + b\,x^2} - \frac{6\,a^2\,\mathrm{atanh}\left(\frac{2\,a\,x}{b} + 1\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b/x)^2),x)
```

```
[Out] ((3*a^2*x^2)/b^3 - 1/(2*b) + (3*a*x)/(2*b^2))/(a*x^3 + b*x^2) - (6*a^2*atan
h((2*a*x)/b + 1))/b^4
```

sympy [**A**] time = 0.32, size = 54, normalized size = 0.93

$$\frac{3a^2\left(\log(x)-\log\left(x+\frac{b}{a}\right)\right)}{b^4}+\frac{6a^2x^2+3abx-b^2}{2ab^3x^3+2b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**2/x**5,x)
```

```
[Out] 3*a**2*(log(x) - log(x + b/a))/b**4 + (6*a**2*x**2 + 3*a*b*x - b**2)/(2*a*b
**3*x**3 + 2*b**4*x**2)
```

3.1630 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^6}\,dx$

Optimal. Leaf size=69

$$-\frac{4a^3\log(x)}{b^5}+\frac{4a^3\log(ax+b)}{b^5}-\frac{a^3}{b^4(ax+b)}-\frac{3a^2}{b^4x}+\frac{a}{b^3x^2}-\frac{1}{3b^2x^3}$$

```
[Out] -1/3/b^2/x^3+a/b^3/x^2-3*a^2/b^4/x-a^3/b^4/(a*x+b)-4*a^3*ln(x)/b^5+4*a^3*ln
(a*x+b)/b^5
```

Rubi [A] time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 44}

$$-\frac{a^3}{b^4(ax+b)}-\frac{3a^2}{b^4x}-\frac{4a^3\log(x)}{b^5}+\frac{4a^3\log(ax+b)}{b^5}+\frac{a}{b^3x^2}-\frac{1}{3b^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^2*x^6),x]
```

```
[Out] -1/(3*b^2*x^3) + a/(b^3*x^2) - (3*a^2)/(b^4*x) - a^3/(b^4*(b + a*x)) - (4*a
^3*Log[x])/b^5 + (4*a^3*Log[b + a*x])/b^5
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^6}\,dx &= \int \frac{1}{x^4(b+ax)^2}\,dx\\ &= \int\left(\frac{1}{b^2x^4}-\frac{2a}{b^3x^3}+\frac{3a^2}{b^4x^2}-\frac{4a^3}{b^5x}+\frac{a^4}{b^4(b+ax)^2}+\frac{4a^4}{b^5(b+ax)}\right)dx\\ &= -\frac{1}{3b^2x^3}+\frac{a}{b^3x^2}-\frac{3a^2}{b^4x}-\frac{a^3}{b^4(b+ax)}-\frac{4a^3\log(x)}{b^5}+\frac{4a^3\log(b+ax)}{b^5}\end{aligned}$$

Mathematica [A] time = 0.06, size = 66, normalized size = 0.96

$$-\frac{-12a^3\log(ax+b)+12a^3\log(x)+\frac{b\left(12a^3x^3+6a^2bx^2-2ab^2x+b^3\right)}{x^3(ax+b)}}{3b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^2*x^6),x]
```

```
[Out] -1/3*((b*(b^3 - 2*a*b^2*x + 6*a^2*b*x^2 + 12*a^3*x^3))/(x^3*(b + a*x)) + 12
*a^3*Log[x] - 12*a^3*Log[b + a*x])/b^5
```

fricas [A] time = 0.82, size = 95, normalized size = 1.38

$$-\frac{12\,a^3bx^3+6\,a^2b^2x^2-2\,ab^3x+b^4-12\left(a^4x^4+a^3bx^3\right)\log\left(ax+b\right)+12\left(a^4x^4+a^3bx^3\right)\log(x)}{3\left(ab^5x^4+b^6x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x^6,x, algorithm="fricas")
```

```
[Out] -1/3*(12*a^3*b*x^3 + 6*a^2*b^2*x^2 - 2*a*b^3*x + b^4 - 12*(a^4*x^4 + a^3*b*
x^3)*log(a*x + b) + 12*(a^4*x^4 + a^3*b*x^3)*log(x))/(a*b^5*x^4 + b^6*x^3)
```

giac [A] time = 0.16, size = 73, normalized size = 1.06

$$\frac{4\,a^3\log\left(\left|ax+b\right|\right)}{b^5}-\frac{4\,a^3\log\left(\left|x\right|\right)}{b^5}-\frac{12\,a^3bx^3+6\,a^2b^2x^2-2\,ab^3x+b^4}{3\left(ax+b\right)b^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x^6,x, algorithm="giac")
```

```
[Out] 4*a^3*log(abs(a*x + b))/b^5 - 4*a^3*log(abs(x))/b^5 - 1/3*(12*a^3*b*x^3 + 6
*a^2*b^2*x^2 - 2*a*b^3*x + b^4)/((a*x + b)*b^5*x^3)
```

maple [A] time = 0.01, size = 68, normalized size = 0.99

$$-\frac{a^3}{\left(ax+b\right)b^4}-\frac{4a^3\ln(x)}{b^5}+\frac{4a^3\ln\left(ax+b\right)}{b^5}-\frac{3a^2}{b^4x}+\frac{a}{b^3x^2}-\frac{1}{3b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^2/x^6,x)
```

```
[Out] -1/3/b^2/x^3+a/b^3/x^2-3*a^2/b^4/x-a^3/b^4/(a*x+b)-4*a^3*ln(x)/b^5+4*a^3*ln
(a*x+b)/b^5
```

maxima [A] time = 1.06, size = 73, normalized size = 1.06

$$-\frac{12\,a^3x^3+6\,a^2bx^2-2\,ab^2x+b^3}{3\left(ab^4x^4+b^5x^3\right)}+\frac{4\,a^3\log\left(ax+b\right)}{b^5}-\frac{4\,a^3\log(x)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x^6,x, algorithm="maxima")
```

```
[Out] -1/3*(12*a^3*x^3 + 6*a^2*b*x^2 - 2*a*b^2*x + b^3)/(a*b^4*x^4 + b^5*x^3) + 4
*a^3*log(a*x + b)/b^5 - 4*a^3*log(x)/b^5
```

mupad [B] time = 0.08, size = 69, normalized size = 1.00

$$\frac{8\,a^3\,\mathrm{atanh}\left(\frac{2\,a\,x}{b}+1\right)}{b^5}-\frac{\frac{1}{3\,b}+\frac{2\,a^2\,x^2}{b^3}+\frac{4\,a^3\,x^3}{b^4}-\frac{2\,a\,x}{3\,b^2}}{a\,x^4+b\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b/x)^2),x)
```

```
[Out] (8*a^3*atanh((2*a*x)/b + 1))/b^5 - (1/(3*b) + (2*a^2*x^2)/b^3 + (4*a^3*x^3)
/b^4 - (2*a*x)/(3*b^2))/(a*x^4 + b*x^3)
```

sympy [**A**] time = 0.39, size = 66, normalized size = 0.96

$$\frac{4a^3\left(-\log(x)+\log\left(x+\frac{b}{a}\right)\right)}{b^5}+\frac{-12a^3x^3-6a^2bx^2+2ab^2x-b^3}{3ab^4x^4+3b^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**2/x**6,x)
```

```
[Out] 4*a**3*(-log(x) + log(x + b/a))/b**5 + (-12*a**3*x**3 - 6*a**2*b*x**2 + 2*a
*b**2*x - b**3)/(3*a*b**4*x**4 + 3*b**5*x**3)
```

3.1631 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^7}\,dx$

Optimal. Leaf size=84

$$\frac{5a^4\log(x)}{b^6}-\frac{5a^4\log(ax+b)}{b^6}+\frac{a^4}{b^5(ax+b)}+\frac{4a^3}{b^5x}-\frac{3a^2}{2b^4x^2}+\frac{2a}{3b^3x^3}-\frac{1}{4b^2x^4}$$

```
[Out] -1/4/b^2/x^4+2/3*a/b^3/x^3-3/2*a^2/b^4/x^2+4*a^3/b^5/x+a^4/b^5/(a*x+b)+5*a^
4*ln(x)/b^6-5*a^4*ln(a*x+b)/b^6
```

Rubi [A] time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 44}

$$-\frac{3a^2}{2b^4x^2}+\frac{a^4}{b^5(ax+b)}+\frac{4a^3}{b^5x}+\frac{5a^4\log(x)}{b^6}-\frac{5a^4\log(ax+b)}{b^6}+\frac{2a}{3b^3x^3}-\frac{1}{4b^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^2*x^7),x]
```

```
[Out] -1/(4*b^2*x^4) + (2*a)/(3*b^3*x^3) - (3*a^2)/(2*b^4*x^2) + (4*a^3)/(b^5*x)
+ a^4/(b^5*(b + a*x)) + (5*a^4*Log[x])/b^6 - (5*a^4*Log[b + a*x])/b^6
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^7}\,dx &= \int \frac{1}{x^5(b+ax)^2}\,dx\\ &= \int\left(\frac{1}{b^2x^5}-\frac{2a}{b^3x^4}+\frac{3a^2}{b^4x^3}-\frac{4a^3}{b^5x^2}+\frac{5a^4}{b^6x}-\frac{a^5}{b^5(b+ax)^2}-\frac{5a^5}{b^6(b+ax)}\right)dx\\ &= -\frac{1}{4b^2x^4}+\frac{2a}{3b^3x^3}-\frac{3a^2}{2b^4x^2}+\frac{4a^3}{b^5x}+\frac{a^4}{b^5(b+ax)}+\frac{5a^4\log(x)}{b^6}-\frac{5a^4\log(b+ax)}{b^6}\end{aligned}$$

Mathematica [A] time = 0.04, size = 79, normalized size = 0.94

$$\frac{-60a^4\log(ax+b)+60a^4\log(x)+\frac{b\left(60a^4x^4+30a^3bx^3-10a^2b^2x^2+5ab^3x-3b^4\right)}{x^4(ax+b)}}{12b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^2*x^7),x]
```

```
[Out] ((b*(-3*b^4 + 5*a*b^3*x - 10*a^2*b^2*x^2 + 30*a^3*b*x^3 + 60*a^4*x^4))/(x^4
*(b + a*x)) + 60*a^4*Log[x] - 60*a^4*Log[b + a*x])/(12*b^6)
```

fricas [A] time = 1.01, size = 108, normalized size = 1.29

$$\frac{60\,a^4bx^4 + 30\,a^3b^2x^3 - 10\,a^2b^3x^2 + 5\,ab^4x - 3\,b^5 - 60\left(a^5x^5 + a^4bx^4\right)\log\left(ax + b\right) + 60\left(a^5x^5 + a^4bx^4\right)\log(x)}{12\left(ab^6x^5 + b^7x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(60*a^4*b*x^4 + 30*a^3*b^2*x^3 - 10*a^2*b^3*x^2 + 5*a*b^4*x - 3*b^5 -
60*(a^5*x^5 + a^4*b*x^4)*log(a*x + b) + 60*(a^5*x^5 + a^4*b*x^4)*log(x))/(a
*b^6*x^5 + b^7*x^4)
```

giac [A] time = 0.21, size = 86, normalized size = 1.02

$$-\frac{5\,a^4\log\left(\left|ax + b\right|\right)}{b^6} + \frac{5\,a^4\log\left(\left|x\right|\right)}{b^6} + \frac{60\,a^4bx^4 + 30\,a^3b^2x^3 - 10\,a^2b^3x^2 + 5\,ab^4x - 3\,b^5}{12\left(ax + b\right)b^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x^7,x, algorithm="giac")
```

```
[Out] -5*a^4*log(abs(a*x + b))/b^6 + 5*a^4*log(abs(x))/b^6 + 1/12*(60*a^4*b*x^4 +
 30*a^3*b^2*x^3 - 10*a^2*b^3*x^2 + 5*a*b^4*x - 3*b^5)/((a*x + b)*b^6*x^4)
```

maple [A] time = 0.01, size = 79, normalized size = 0.94

$$\frac{a^4}{\left(ax + b\right)b^5} + \frac{5a^4\ln(x)}{b^6} - \frac{5a^4\ln\left(ax + b\right)}{b^6} + \frac{4a^3}{b^5x} - \frac{3a^2}{2b^4x^2} + \frac{2a}{3b^3x^3} - \frac{1}{4b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^2/x^7,x)
```

```
[Out] -1/4/b^2/x^4+2/3*a/b^3/x^3-3/2*a^2/b^4/x^2+4*a^3/b^5/x+a^4/b^5/(a*x+b)+5*a^
4*ln(x)/b^6-5*a^4*ln(a*x+b)/b^6
```

maxima [A] time = 1.01, size = 86, normalized size = 1.02

$$\frac{60\,a^4x^4 + 30\,a^3bx^3 - 10\,a^2b^2x^2 + 5\,ab^3x - 3\,b^4}{12\left(ab^5x^5 + b^6x^4\right)} - \frac{5\,a^4\log\left(ax + b\right)}{b^6} + \frac{5\,a^4\log(x)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x^7,x, algorithm="maxima")
```

```
[Out] 1/12*(60*a^4*x^4 + 30*a^3*b*x^3 - 10*a^2*b^2*x^2 + 5*a*b^3*x - 3*b^4)/(a*b^
5*x^5 + b^6*x^4) - 5*a^4*log(a*x + b)/b^6 + 5*a^4*log(x)/b^6
```

mupad [B] time = 1.10, size = 79, normalized size = 0.94

$$\frac{\frac{5\,a^3x^3}{2\,b^4} - \frac{5\,a^2x^2}{6\,b^3} - \frac{1}{4\,b} + \frac{5\,a^4x^4}{b^5} + \frac{5\,ax}{12\,b^2}}{a\,x^5 + b\,x^4} - \frac{10\,a^4\,\mathrm{atanh}\left(\frac{2\,ax}{b} + 1\right)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b/x)^2),x)
```

```
[Out] ((5*a^3*x^3)/(2*b^4) - (5*a^2*x^2)/(6*b^3) - 1/(4*b) + (5*a^4*x^4)/b^5 + (5
*a*x)/(12*b^2))/(a*x^5 + b*x^4) - (10*a^4*atanh((2*a*x)/b + 1))/b^6
```

sympy [A] time = 0.37, size = 80, normalized size = 0.95

$$\frac{5a^4\left(\log(x)-\log\left(x+\frac{b}{a}\right)\right)}{b^6}+\frac{60a^4x^4+30a^3bx^3-10a^2b^2x^2+5ab^3x-3b^4}{12ab^5x^5+12b^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**2/x**7,x)
```

```
[Out] 5*a**4*(log(x) - log(x + b/a))/b**6 + (60*a**4*x**4 + 30*a**3*b*x**3 - 10*a
**2*b**2*x**2 + 5*a*b**3*x - 3*b**4)/(12*a*b**5*x**5 + 12*b**6*x**4)
```

3.1632 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^8}\,dx$

Optimal. Leaf size=94

$$-\frac{6a^5\log(x)}{b^7}+\frac{6a^5\log(ax+b)}{b^7}-\frac{a^5}{b^6(ax+b)}-\frac{5a^4}{b^6x}+\frac{2a^3}{b^5x^2}-\frac{a^2}{b^4x^3}+\frac{a}{2b^3x^4}-\frac{1}{5b^2x^5}$$

```
[Out] -1/5/b^2/x^5+1/2*a/b^3/x^4-a^2/b^4/x^3+2*a^3/b^5/x^2-5*a^4/b^6/x-a^5/b^6/(a
*x+b)-6*a^5*ln(x)/b^7+6*a^5*ln(a*x+b)/b^7
```

Rubi [A] time = 0.05, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 44}

$$\frac{2a^3}{b^5x^2}-\frac{a^2}{b^4x^3}-\frac{a^5}{b^6(ax+b)}-\frac{5a^4}{b^6x}-\frac{6a^5\log(x)}{b^7}+\frac{6a^5\log(ax+b)}{b^7}+\frac{a}{2b^3x^4}-\frac{1}{5b^2x^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^2*x^8),x]
```

```
[Out] -1/(5*b^2*x^5) + a/(2*b^3*x^4) - a^2/(b^4*x^3) + (2*a^3)/(b^5*x^2) - (5*a^4
)/(b^6*x) - a^5/(b^6*(b + a*x)) - (6*a^5*Log[x])/b^7 + (6*a^5*Log[b + a*x])
/b^7
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^8}\,dx &= \int \frac{1}{x^6(b+ax)^2}\,dx\\ &= \int\left(\frac{1}{b^2x^6}-\frac{2a}{b^3x^5}+\frac{3a^2}{b^4x^4}-\frac{4a^3}{b^5x^3}+\frac{5a^4}{b^6x^2}-\frac{6a^5}{b^7x}+\frac{a^6}{b^6(b+ax)^2}+\frac{6a^6}{b^7(b+ax)}\right)dx\\ &= -\frac{1}{5b^2x^5}+\frac{a}{2b^3x^4}-\frac{a^2}{b^4x^3}+\frac{2a^3}{b^5x^2}-\frac{5a^4}{b^6x}-\frac{a^5}{b^6(b+ax)}-\frac{6a^5\log(x)}{b^7}+\frac{6a^5\log(b+ax)}{b^7}\end{aligned}$$

Mathematica [A] time = 0.07, size = 90, normalized size = 0.96

$$-\frac{-60a^5\log(ax+b)+60a^5\log(x)+\frac{b\left(60a^5x^5+30a^4bx^4-10a^3b^2x^3+5a^2b^3x^2-3ab^4x+2b^5\right)}{x^5(ax+b)}}{10b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^2*x^8),x]
```

```
[Out] -1/10*((b*(2*b^5 - 3*a*b^4*x + 5*a^2*b^3*x^2 - 10*a^3*b^2*x^3 + 30*a^4*b*x^
4 + 60*a^5*x^5))/(x^5*(b + a*x)) + 60*a^5*Log[x] - 60*a^5*Log[b + a*x])/b^7
```

fricas [A] time = 0.98, size = 119, normalized size = 1.27

$$-\frac{60\,a^5bx^5+30\,a^4b^2x^4-10\,a^3b^3x^3+5\,a^2b^4x^2-3\,ab^5x+2\,b^6-60\left(a^6x^6+a^5bx^5\right)\log\left(ax+b\right)+60\left(a^6x^6+a^5\right.}{10\left(ab^7x^6+b^8x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x^8,x, algorithm="fricas")
```

```
[Out] -1/10*(60*a^5*b*x^5 + 30*a^4*b^2*x^4 - 10*a^3*b^3*x^3 + 5*a^2*b^4*x^2 - 3*a
*b^5*x + 2*b^6 - 60*(a^6*x^6 + a^5*b*x^5)*log(a*x + b) + 60*(a^6*x^6 + a^5*
b*x^5)*log(x))/(a*b^7*x^6 + b^8*x^5)
```

giac [A] time = 0.15, size = 97, normalized size = 1.03

$$\frac{6\,a^5\log\left(\left|ax+b\right|\right)}{b^7}-\frac{6\,a^5\log\left(\left|x\right|\right)}{b^7}-\frac{60\,a^5bx^5+30\,a^4b^2x^4-10\,a^3b^3x^3+5\,a^2b^4x^2-3\,ab^5x+2\,b^6}{10\left(ax+b\right)b^7x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x^8,x, algorithm="giac")
```

```
[Out] 6*a^5*log(abs(a*x + b))/b^7 - 6*a^5*log(abs(x))/b^7 - 1/10*(60*a^5*b*x^5 +
30*a^4*b^2*x^4 - 10*a^3*b^3*x^3 + 5*a^2*b^4*x^2 - 3*a*b^5*x + 2*b^6)/((a*x
+ b)*b^7*x^5)
```

maple [A] time = 0.01, size = 91, normalized size = 0.97

$$-\frac{a^5}{\left(ax+b\right)b^6}-\frac{6a^5\ln(x)}{b^7}+\frac{6a^5\ln\left(ax+b\right)}{b^7}-\frac{5a^4}{b^6x}+\frac{2a^3}{b^5x^2}-\frac{a^2}{b^4x^3}+\frac{a}{2b^3x^4}-\frac{1}{5b^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^2/x^8,x)
```

```
[Out] -1/5/b^2/x^5+1/2*a/b^3/x^4-a^2/b^4/x^3+2*a^3/b^5/x^2-5*a^4/b^6/x-a^5/b^6/(a
*x+b)-6*a^5*ln(x)/b^7+6*a^5*ln(a*x+b)/b^7
```

maxima [A] time = 1.09, size = 97, normalized size = 1.03

$$-\frac{60\,a^5x^5+30\,a^4bx^4-10\,a^3b^2x^3+5\,a^2b^3x^2-3\,ab^4x+2\,b^5}{10\left(ab^6x^6+b^7x^5\right)}+\frac{6\,a^5\log\left(ax+b\right)}{b^7}-\frac{6\,a^5\log(x)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x^8,x, algorithm="maxima")
```

```
[Out] -1/10*(60*a^5*x^5 + 30*a^4*b*x^4 - 10*a^3*b^2*x^3 + 5*a^2*b^3*x^2 - 3*a*b^4
*x + 2*b^5)/(a*b^6*x^6 + b^7*x^5) + 6*a^5*log(a*x + b)/b^7 - 6*a^5*log(x)/b
^7
```

mupad [B] time = 1.10, size = 91, normalized size = 0.97

$$\frac{12\,a^5\,\mathrm{atanh}\left(\frac{2\,a\,x}{b}+1\right)}{b^7}-\frac{\frac{1}{5\,b}+\frac{a^2\,x^2}{2\,b^3}-\frac{a^3\,x^3}{b^4}+\frac{3\,a^4\,x^4}{b^5}+\frac{6\,a^5\,x^5}{b^6}-\frac{3\,a\,x}{10\,b^2}}{a\,x^6+b\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(a + b/x)^2),x)
```

```
[Out] (12*a^5*atanh((2*a*x)/b + 1))/b^7 - (1/(5*b) + (a^2*x^2)/(2*b^3) - (a^3*x^3
)/b^4 + (3*a^4*x^4)/b^5 + (6*a^5*x^5)/b^6 - (3*a*x)/(10*b^2))/(a*x^6 + b*x^
5)
```

sympy [**A**] time = 0.43, size = 92, normalized size = 0.98

$$\frac{6a^5\left(-\log(x)+\log\left(x+\frac{b}{a}\right)\right)}{b^7}+\frac{-60a^5x^5-30a^4bx^4+10a^3b^2x^3-5a^2b^3x^2+3ab^4x-2b^5}{10ab^6x^6+10b^7x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**2/x**8,x)
```

```
[Out] 6*a**5*(-log(x) + log(x + b/a))/b**7 + (-60*a**5*x**5 - 30*a**4*b*x**4 + 10
*a**3*b**2*x**3 - 5*a**2*b**3*x**2 + 3*a*b**4*x - 2*b**5)/(10*a*b**6*x**6 +
 10*b**7*x**5)
```

3.1633 $\int \frac{x^4}{\left(a+\frac{b}{x}\right)^3}\,dx$

Optimal. Leaf size=99

$$\frac{b^7}{2a^8(ax+b)^2}-\frac{7b^6}{a^8(ax+b)}-\frac{21b^5\log(ax+b)}{a^8}+\frac{15b^4x}{a^7}-\frac{5b^3x^2}{a^6}+\frac{2b^2x^3}{a^5}-\frac{3bx^4}{4a^4}+\frac{x^5}{5a^3}$$

```
[Out] 15*b^4*x/a^7-5*b^3*x^2/a^6+2*b^2*x^3/a^5-3/4*b*x^4/a^4+1/5*x^5/a^3+1/2*b^7/
a^8/(a*x+b)^2-7*b^6/a^8/(a*x+b)-21*b^5*ln(a*x+b)/a^8
```

Rubi [A] time = 0.07, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$-\frac{5b^3x^2}{a^6}+\frac{2b^2x^3}{a^5}+\frac{b^7}{2a^8(ax+b)^2}-\frac{7b^6}{a^8(ax+b)}+\frac{15b^4x}{a^7}-\frac{21b^5\log(ax+b)}{a^8}-\frac{3bx^4}{4a^4}+\frac{x^5}{5a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b/x)^3,x]
```

```
[Out] (15*b^4*x)/a^7 - (5*b^3*x^2)/a^6 + (2*b^2*x^3)/a^5 - (3*b*x^4)/(4*a^4) + x^
5/(5*a^3) + b^7/(2*a^8*(b + a*x)^2) - (7*b^6)/(a^8*(b + a*x)) - (21*b^5*Log
[b + a*x])/a^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\left(a+\frac{b}{x}\right)^3}\,dx &= \int \frac{x^7}{(b+ax)^3}\,dx\\ &= \int\left(\frac{15b^4}{a^7}-\frac{10b^3x}{a^6}+\frac{6b^2x^2}{a^5}-\frac{3bx^3}{a^4}+\frac{x^4}{a^3}-\frac{b^7}{a^7(b+ax)^3}+\frac{7b^6}{a^7(b+ax)^2}-\frac{21b^5}{a^7(b+ax)}\right)dx\\ &= \frac{15b^4x}{a^7}-\frac{5b^3x^2}{a^6}+\frac{2b^2x^3}{a^5}-\frac{3bx^4}{4a^4}+\frac{x^5}{5a^3}+\frac{b^7}{2a^8(b+ax)^2}-\frac{7b^6}{a^8(b+ax)}-\frac{21b^5\log(b+ax)}{a^8}\end{aligned}$$

Mathematica [A] time = 0.06, size = 85, normalized size = 0.86

$$\frac{4a^5x^5-15a^4bx^4+40a^3b^2x^3-100a^2b^3x^2-\frac{10b^6(14ax+13b)}{(ax+b)^2}-420b^5\log(ax+b)+300ab^4x}{20a^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b/x)^3,x]
```

```
[Out] (300*a*b^4*x - 100*a^2*b^3*x^2 + 40*a^3*b^2*x^3 - 15*a^4*b*x^4 + 4*a^5*x^5
- (10*b^6*(13*b + 14*a*x))/(b + a*x)^2 - 420*b^5*Log[b + a*x])/(20*a^8)
```

fricas [A] time = 0.87, size = 129, normalized size = 1.30

$$\frac{4\,a^7x^7 - 7\,a^6bx^6 + 14\,a^5b^2x^5 - 35\,a^4b^3x^4 + 140\,a^3b^4x^3 + 500\,a^2b^5x^2 + 160\,ab^6x - 130\,b^7 - 420\left(a^2b^5x^2 + 2\,ab^6x +\right.}{20\left(a^{10}x^2 + 2\,a^9bx + a^8b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/x)^3,x, algorithm="fricas")
```

```
[Out] 1/20*(4*a^7*x^7 - 7*a^6*b*x^6 + 14*a^5*b^2*x^5 - 35*a^4*b^3*x^4 + 140*a^3*b
^4*x^3 + 500*a^2*b^5*x^2 + 160*a*b^6*x - 130*b^7 - 420*(a^2*b^5*x^2 + 2*a*b
^6*x + b^7)*log(a*x + b))/(a^10*x^2 + 2*a^9*b*x + a^8*b^2)
```

giac [A] time = 0.15, size = 95, normalized size = 0.96

$$-\frac{21\,b^5\log\left(|ax+b|\right)}{a^8} - \frac{14\,ab^6x + 13\,b^7}{2\,(ax+b)^2a^8} + \frac{4\,a^{12}x^5 - 15\,a^{11}bx^4 + 40\,a^{10}b^2x^3 - 100\,a^9b^3x^2 + 300\,a^8b^4x}{20\,a^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/x)^3,x, algorithm="giac")
```

```
[Out] -21*b^5*log(abs(a*x + b))/a^8 - 1/2*(14*a*b^6*x + 13*b^7)/((a*x + b)^2*a^8)
 + 1/20*(4*a^12*x^5 - 15*a^11*b*x^4 + 40*a^10*b^2*x^3 - 100*a^9*b^3*x^2 + 3
00*a^8*b^4*x)/a^15
```

maple [A] time = 0.01, size = 94, normalized size = 0.95

$$\frac{x^5}{5a^3} - \frac{3b\,x^4}{4a^4} + \frac{2b^2x^3}{a^5} - \frac{5b^3x^2}{a^6} + \frac{b^7}{2\,(ax+b)^2\,a^8} + \frac{15b^4x}{a^7} - \frac{7b^6}{(ax+b)\,a^8} - \frac{21b^5\ln\left(ax+b\right)}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a+b/x)^3,x)
```

```
[Out] 15*b^4*x/a^7-5*b^3*x^2/a^6+2*b^2*x^3/a^5-3/4*b*x^4/a^4+1/5*x^5/a^3+1/2*b^7/
a^8/(a*x+b)^2-7*b^6/a^8/(a*x+b)-21*b^5*ln(a*x+b)/a^8
```

maxima [A] time = 1.02, size = 103, normalized size = 1.04

$$-\frac{14\,ab^6x + 13\,b^7}{2\left(a^{10}x^2 + 2\,a^9bx + a^8b^2\right)} - \frac{21\,b^5\log\left(ax+b\right)}{a^8} + \frac{4\,a^4x^5 - 15\,a^3bx^4 + 40\,a^2b^2x^3 - 100\,ab^3x^2 + 300\,b^4x}{20\,a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(14*a*b^6*x + 13*b^7)/(a^10*x^2 + 2*a^9*b*x + a^8*b^2) - 21*b^5*log(a*
x + b)/a^8 + 1/20*(4*a^4*x^5 - 15*a^3*b*x^4 + 40*a^2*b^2*x^3 - 100*a*b^3*x^
2 + 300*b^4*x)/a^7
```

mupad [B] time = 1.09, size = 104, normalized size = 1.05

$$\frac{x^5}{5\,a^3} - \frac{7\,b^6\,x + \frac{13\,b^7}{2\,a}}{a^9\,x^2 + 2\,a^8\,b\,x + a^7\,b^2} - \frac{21\,b^5\,\ln\left(b + a\,x\right)}{a^8} - \frac{3\,b\,x^4}{4\,a^4} + \frac{15\,b^4\,x}{a^7} + \frac{2\,b^2\,x^3}{a^5} - \frac{5\,b^3\,x^2}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b/x)^3,x)
```

```
[Out] x^5/(5*a^3) - (7*b^6*x + (13*b^7)/(2*a))/(a^7*b^2 + a^9*x^2 + 2*a^8*b*x) -
(21*b^5*log(b + a*x))/a^8 - (3*b*x^4)/(4*a^4) + (15*b^4*x)/a^7 + (2*b^2*x^3
)/a^5 - (5*b^3*x^2)/a^6
```

sympy [**A**] time = 0.40, size = 109, normalized size = 1.10

$$\frac{-14ab^6x - 13b^7}{2a^{10}x^2 + 4a^9bx + 2a^8b^2} + \frac{x^5}{5a^3} - \frac{3bx^4}{4a^4} + \frac{2b^2x^3}{a^5} - \frac{5b^3x^2}{a^6} + \frac{15b^4x}{a^7} - \frac{21b^5\log(ax+b)}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a+b/x)**3,x)
```

```
[Out] (-14*a*b**6*x - 13*b**7)/(2*a**10*x**2 + 4*a**9*b*x + 2*a**8*b**2) + x**5/(
5*a**3) - 3*b*x**4/(4*a**4) + 2*b**2*x**3/a**5 - 5*b**3*x**2/a**6 + 15*b**4
*x/a**7 - 21*b**5*log(a*x + b)/a**8
```

3.1634 $\int \frac{x^3}{\left(a+\frac{b}{x}\right)^3}\,dx$

Optimal. Leaf size=86

$$-\frac{b^6}{2a^7(ax+b)^2}+\frac{6b^5}{a^7(ax+b)}+\frac{15b^4\log(ax+b)}{a^7}-\frac{10b^3x}{a^6}+\frac{3b^2x^2}{a^5}-\frac{bx^3}{a^4}+\frac{x^4}{4a^3}$$

```
[Out] -10*b^3*x/a^6+3*b^2*x^2/a^5-b*x^3/a^4+1/4*x^4/a^3-1/2*b^6/a^7/(a*x+b)^2+6*b
^5/a^7/(a*x+b)+15*b^4*ln(a*x+b)/a^7
```

Rubi [A] time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$\frac{3b^2x^2}{a^5}-\frac{b^6}{2a^7(ax+b)^2}+\frac{6b^5}{a^7(ax+b)}-\frac{10b^3x}{a^6}+\frac{15b^4\log(ax+b)}{a^7}-\frac{bx^3}{a^4}+\frac{x^4}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b/x)^3,x]
```

```
[Out] (-10*b^3*x)/a^6 + (3*b^2*x^2)/a^5 - (b*x^3)/a^4 + x^4/(4*a^3) - b^6/(2*a^7*
(b + a*x)^2) + (6*b^5)/(a^7*(b + a*x)) + (15*b^4*Log[b + a*x])/a^7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(a+\frac{b}{x}\right)^3}\,dx &= \int \frac{x^6}{(b+ax)^3}\,dx\\ &= \int\left(-\frac{10b^3}{a^6}+\frac{6b^2x}{a^5}-\frac{3bx^2}{a^4}+\frac{x^3}{a^3}+\frac{b^6}{a^6(b+ax)^3}-\frac{6b^5}{a^6(b+ax)^2}+\frac{15b^4}{a^6(b+ax)}\right)dx\\ &= -\frac{10b^3x}{a^6}+\frac{3b^2x^2}{a^5}-\frac{bx^3}{a^4}+\frac{x^4}{4a^3}-\frac{b^6}{2a^7(b+ax)^2}+\frac{6b^5}{a^7(b+ax)}+\frac{15b^4\log(b+ax)}{a^7}\end{aligned}$$

Mathematica [A] time = 0.06, size = 73, normalized size = 0.85

$$\frac{a^4x^4-4a^3bx^3+12a^2b^2x^2+\frac{2b^5(12ax+11b)}{(ax+b)^2}+60b^4\log(ax+b)-40ab^3x}{4a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b/x)^3,x]
```

```
[Out] (-40*a*b^3*x + 12*a^2*b^2*x^2 - 4*a^3*b*x^3 + a^4*x^4 + (2*b^5*(11*b + 12*a
*x))/(b + a*x)^2 + 60*b^4*Log[b + a*x])/(4*a^7)
```

fricas [A] time = 0.76, size = 117, normalized size = 1.36

$$\frac{a^6x^6 - 2\,a^5bx^5 + 5\,a^4b^2x^4 - 20\,a^3b^3x^3 - 68\,a^2b^4x^2 - 16\,ab^5x + 22\,b^6 + 60\left(a^2b^4x^2 + 2\,ab^5x + b^6\right)\log\left(ax + b\right)}{4\left(a^9x^2 + 2\,a^8bx + a^7b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(a^6*x^6 - 2*a^5*b*x^5 + 5*a^4*b^2*x^4 - 20*a^3*b^3*x^3 - 68*a^2*b^4*x^
2 - 16*a*b^5*x + 22*b^6 + 60*(a^2*b^4*x^2 + 2*a*b^5*x + b^6)*log(a*x + b))/
(a^9*x^2 + 2*a^8*b*x + a^7*b^2)
```

giac [A] time = 0.18, size = 83, normalized size = 0.97

$$\frac{15\,b^4\log\left(\left|ax + b\right|\right)}{a^7} + \frac{12\,ab^5x + 11\,b^6}{2\left(ax + b\right)^2a^7} + \frac{a^9x^4 - 4\,a^8bx^3 + 12\,a^7b^2x^2 - 40\,a^6b^3x}{4\,a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x)^3,x, algorithm="giac")
```

```
[Out] 15*b^4*log(abs(a*x + b))/a^7 + 1/2*(12*a*b^5*x + 11*b^6)/((a*x + b)^2*a^7)
+ 1/4*(a^9*x^4 - 4*a^8*b*x^3 + 12*a^7*b^2*x^2 - 40*a^6*b^3*x)/a^12
```

maple [A] time = 0.01, size = 83, normalized size = 0.97

$$\frac{x^4}{4a^3} - \frac{b\,x^3}{a^4} + \frac{3b^2x^2}{a^5} - \frac{b^6}{2\left(ax + b\right)^2a^7} - \frac{10b^3x}{a^6} + \frac{6b^5}{\left(ax + b\right)a^7} + \frac{15b^4\ln\left(ax + b\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b/x)^3,x)
```

```
[Out] -10*b^3*x/a^6+3*b^2*x^2/a^5-b*x^3/a^4+1/4*x^4/a^3-1/2*b^6/a^7/(a*x+b)^2+6*b
^5/a^7/(a*x+b)+15*b^4*ln(a*x+b)/a^7
```

maxima [A] time = 1.08, size = 91, normalized size = 1.06

$$\frac{12\,ab^5x + 11\,b^6}{2\left(a^9x^2 + 2\,a^8bx + a^7b^2\right)} + \frac{15\,b^4\log\left(ax + b\right)}{a^7} + \frac{a^3x^4 - 4\,a^2bx^3 + 12\,ab^2x^2 - 40\,b^3x}{4\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(12*a*b^5*x + 11*b^6)/(a^9*x^2 + 2*a^8*b*x + a^7*b^2) + 15*b^4*log(a*x
+ b)/a^7 + 1/4*(a^3*x^4 - 4*a^2*b*x^3 + 12*a*b^2*x^2 - 40*b^3*x)/a^6
```

mupad [B] time = 0.06, size = 92, normalized size = 1.07

$$\frac{6\,b^5\,x + \frac{11\,b^6}{2\,a}}{a^8\,x^2 + 2\,a^7\,b\,x + a^6\,b^2} + \frac{x^4}{4\,a^3} + \frac{15\,b^4\,\ln\left(b + a\,x\right)}{a^7} - \frac{b\,x^3}{a^4} - \frac{10\,b^3\,x}{a^6} + \frac{3\,b^2\,x^2}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b/x)^3,x)
```

```
[Out] (6*b^5*x + (11*b^6)/(2*a))/(a^6*b^2 + a^8*x^2 + 2*a^7*b*x) + x^4/(4*a^3) +
(15*b^4*log(b + a*x))/a^7 - (b*x^3)/a^4 - (10*b^3*x)/a^6 + (3*b^2*x^2)/a^5
```

sympy [**A**] time = 0.35, size = 92, normalized size = 1.07

$$\frac{12ab^5x + 11b^6}{2a^9x^2 + 4a^8bx + 2a^7b^2} + \frac{x^4}{4a^3} - \frac{bx^3}{a^4} + \frac{3b^2x^2}{a^5} - \frac{10b^3x}{a^6} + \frac{15b^4\log(ax+b)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b/x)**3,x)
```

```
[Out] (12*a*b**5*x + 11*b**6)/(2*a**9*x**2 + 4*a**8*b*x + 2*a**7*b**2) + x**4/(4*
a**3) - b*x**3/a**4 + 3*b**2*x**2/a**5 - 10*b**3*x/a**6 + 15*b**4*log(a*x +
 b)/a**7
```

3.1635 $\int \frac{x^2}{\left(a+\frac{b}{x}\right)^3}\,dx$

Optimal. Leaf size=77

$$\frac{b^5}{2a^6(ax+b)^2}-\frac{5b^4}{a^6(ax+b)}-\frac{10b^3\log(ax+b)}{a^6}+\frac{6b^2x}{a^5}-\frac{3bx^2}{2a^4}+\frac{x^3}{3a^3}$$

```
[Out] 6*b^2*x/a^5-3/2*b*x^2/a^4+1/3*x^3/a^3+1/2*b^5/a^6/(a*x+b)^2-5*b^4/a^6/(a*x+
b)-10*b^3*ln(a*x+b)/a^6
```

Rubi [A] time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$\frac{b^5}{2a^6(ax+b)^2}-\frac{5b^4}{a^6(ax+b)}+\frac{6b^2x}{a^5}-\frac{10b^3\log(ax+b)}{a^6}-\frac{3bx^2}{2a^4}+\frac{x^3}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b/x)^3,x]
```

```
[Out] (6*b^2*x)/a^5 - (3*b*x^2)/(2*a^4) + x^3/(3*a^3) + b^5/(2*a^6*(b + a*x)^2) -
 (5*b^4)/(a^6*(b + a*x)) - (10*b^3*Log[b + a*x])/a^6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a+\frac{b}{x}\right)^3}\,dx &= \int \frac{x^5}{(b+ax)^3}\,dx\\ &= \int\left(\frac{6b^2}{a^5}-\frac{3bx}{a^4}+\frac{x^2}{a^3}-\frac{b^5}{a^5(b+ax)^3}+\frac{5b^4}{a^5(b+ax)^2}-\frac{10b^3}{a^5(b+ax)}\right)dx\\ &= \frac{6b^2x}{a^5}-\frac{3bx^2}{2a^4}+\frac{x^3}{3a^3}+\frac{b^5}{2a^6(b+ax)^2}-\frac{5b^4}{a^6(b+ax)}-\frac{10b^3\log(b+ax)}{a^6}\end{aligned}$$

Mathematica [A] time = 0.06, size = 63, normalized size = 0.82

$$\frac{2a^3x^3-9a^2bx^2-\frac{3b^4(10ax+9b)}{(ax+b)^2}-60b^3\log(ax+b)+36ab^2x}{6a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b/x)^3,x]
```

```
[Out] (36*a*b^2*x - 9*a^2*b*x^2 + 2*a^3*x^3 - (3*b^4*(9*b + 10*a*x))/(b + a*x)^2
- 60*b^3*Log[b + a*x])/(6*a^6)
```

fricas [A] time = 0.74, size = 107, normalized size = 1.39

$$\frac{2\,a^5x^5 - 5\,a^4bx^4 + 20\,a^3b^2x^3 + 63\,a^2b^3x^2 + 6\,ab^4x - 27\,b^5 - 60\left(a^2b^3x^2 + 2\,ab^4x + b^5\right)\log\left(ax+b\right)}{6\left(a^8x^2 + 2\,a^7bx + a^6b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a^5*x^5 - 5*a^4*b*x^4 + 20*a^3*b^2*x^3 + 63*a^2*b^3*x^2 + 6*a*b^4*x
- 27*b^5 - 60*(a^2*b^3*x^2 + 2*a*b^4*x + b^5)*log(a*x + b))/(a^8*x^2 + 2*a^
7*b*x + a^6*b^2)
```

giac [A] time = 0.15, size = 73, normalized size = 0.95

$$-\frac{10\,b^3\log\left(\left|ax+b\right|\right)}{a^6} - \frac{10\,ab^4x + 9\,b^5}{2\left(ax+b\right)^2a^6} + \frac{2\,a^6x^3 - 9\,a^5bx^2 + 36\,a^4b^2x}{6\,a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x)^3,x, algorithm="giac")
```

```
[Out] -10*b^3*log(abs(a*x + b))/a^6 - 1/2*(10*a*b^4*x + 9*b^5)/((a*x + b)^2*a^6)
+ 1/6*(2*a^6*x^3 - 9*a^5*b*x^2 + 36*a^4*b^2*x)/a^9
```

maple [A] time = 0.01, size = 72, normalized size = 0.94

$$\frac{x^3}{3a^3} - \frac{3b\,x^2}{2a^4} + \frac{b^5}{2\left(ax+b\right)^2a^6} + \frac{6b^2x}{a^5} - \frac{5b^4}{\left(ax+b\right)a^6} - \frac{10b^3\ln\left(ax+b\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b/x)^3,x)
```

```
[Out] 6*b^2*x/a^5-3/2*b*x^2/a^4+1/3*x^3/a^3+1/2*b^5/a^6/(a*x+b)^2-5*b^4/a^6/(a*x+
b)-10*b^3*ln(a*x+b)/a^6
```

maxima [A] time = 1.12, size = 81, normalized size = 1.05

$$-\frac{10\,ab^4x + 9\,b^5}{2\left(a^8x^2 + 2\,a^7bx + a^6b^2\right)} - \frac{10\,b^3\log\left(ax+b\right)}{a^6} + \frac{2\,a^2x^3 - 9\,abx^2 + 36\,b^2x}{6\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(10*a*b^4*x + 9*b^5)/(a^8*x^2 + 2*a^7*b*x + a^6*b^2) - 10*b^3*log(a*x
+ b)/a^6 + 1/6*(2*a^2*x^3 - 9*a*b*x^2 + 36*b^2*x)/a^5
```

mupad [B] time = 0.05, size = 82, normalized size = 1.06

$$\frac{x^3}{3\,a^3} - \frac{5\,b^4\,x + \frac{9\,b^5}{2\,a}}{a^7\,x^2 + 2\,a^6\,b\,x + a^5\,b^2} - \frac{10\,b^3\,\ln\left(b + a\,x\right)}{a^6} - \frac{3\,b\,x^2}{2\,a^4} + \frac{6\,b^2\,x}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b/x)^3,x)
```

```
[Out] x^3/(3*a^3) - (5*b^4*x + (9*b^5)/(2*a))/(a^5*b^2 + a^7*x^2 + 2*a^6*b*x) - (
10*b^3*log(b + a*x))/a^6 - (3*b*x^2)/(2*a^4) + (6*b^2*x)/a^5
```

sympy [A] time = 0.33, size = 85, normalized size = 1.10

$$\frac{-10ab^4x - 9b^5}{2a^8x^2 + 4a^7bx + 2a^6b^2} + \frac{x^3}{3a^3} - \frac{3bx^2}{2a^4} + \frac{6b^2x}{a^5} - \frac{10b^3\log(ax+b)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b/x)**3,x)
```

```
[Out] (-10*a*b**4*x - 9*b**5)/(2*a**8*x**2 + 4*a**7*b*x + 2*a**6*b**2) + x**3/(3*
a**3) - 3*b*x**2/(2*a**4) + 6*b**2*x/a**5 - 10*b**3*log(a*x + b)/a**6
```

3.1636 $\int \frac{x}{\left(a+\frac{b}{x}\right)^3}\,dx$

Optimal. Leaf size=64

$$-\frac{b^4}{2a^5(ax+b)^2}+\frac{4b^3}{a^5(ax+b)}+\frac{6b^2\log(ax+b)}{a^5}-\frac{3bx}{a^4}+\frac{x^2}{2a^3}$$

```
[Out] -3*b*x/a^4+1/2*x^2/a^3-1/2*b^4/a^5/(a*x+b)^2+4*b^3/a^5/(a*x+b)+6*b^2*ln(a*x
+b)/a^5
```

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {263, 43}

$$-\frac{b^4}{2a^5(ax+b)^2}+\frac{4b^3}{a^5(ax+b)}+\frac{6b^2\log(ax+b)}{a^5}-\frac{3bx}{a^4}+\frac{x^2}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b/x)^3,x]
```

```
[Out] (-3*b*x)/a^4 + x^2/(2*a^3) - b^4/(2*a^5*(b + a*x)^2) + (4*b^3)/(a^5*(b + a*
x)) + (6*b^2*Log[b + a*x])/a^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a+\frac{b}{x}\right)^3}\,dx &= \int \frac{x^4}{(b+ax)^3}\,dx\\ &= \int\left(-\frac{3b}{a^4}+\frac{x}{a^3}+\frac{b^4}{a^4(b+ax)^3}-\frac{4b^3}{a^4(b+ax)^2}+\frac{6b^2}{a^4(b+ax)}\right)dx\\ &= -\frac{3bx}{a^4}+\frac{x^2}{2a^3}-\frac{b^4}{2a^5(b+ax)^2}+\frac{4b^3}{a^5(b+ax)}+\frac{6b^2\log(b+ax)}{a^5}\end{aligned}$$

Mathematica [A] time = 0.04, size = 50, normalized size = 0.78

$$\frac{a^2x^2+\frac{b^3(8ax+7b)}{(ax+b)^2}+12b^2\log(ax+b)-6abx}{2a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b/x)^3,x]
```

```
[Out] (-6*a*b*x + a^2*x^2 + (b^3*(7*b + 8*a*x))/(b + a*x)^2 + 12*b^2*Log[b + a*x]
)/(2*a^5)
```

fricas [A] time = 1.06, size = 95, normalized size = 1.48

$$\frac{a^4x^4 - 4\,a^3bx^3 - 11\,a^2b^2x^2 + 2\,ab^3x + 7\,b^4 + 12\left(a^2b^2x^2 + 2\,ab^3x + b^4\right)\log\left(ax+b\right)}{2\left(a^7x^2 + 2\,a^6bx + a^5b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a^4*x^4 - 4*a^3*b*x^3 - 11*a^2*b^2*x^2 + 2*a*b^3*x + 7*b^4 + 12*(a^2*b
^2*x^2 + 2*a*b^3*x + b^4)*log(a*x + b))/(a^7*x^2 + 2*a^6*b*x + a^5*b^2)
```

giac [A] time = 0.15, size = 61, normalized size = 0.95

$$\frac{6\,b^2\log\left(\left|ax+b\right|\right)}{a^5} + \frac{a^3x^2 - 6\,a^2bx}{2\,a^6} + \frac{8\,ab^3x + 7\,b^4}{2\left(ax+b\right)^2a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x)^3,x, algorithm="giac")
```

```
[Out] 6*b^2*log(abs(a*x + b))/a^5 + 1/2*(a^3*x^2 - 6*a^2*b*x)/a^6 + 1/2*(8*a*b^3*
x + 7*b^4)/((a*x + b)^2*a^5)
```

maple [A] time = 0.01, size = 61, normalized size = 0.95

$$\frac{x^2}{2a^3} - \frac{b^4}{2\left(ax+b\right)^2a^5} - \frac{3bx}{a^4} + \frac{4b^3}{\left(ax+b\right)a^5} + \frac{6b^2\ln\left(ax+b\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b/x)^3,x)
```

```
[Out] -3*b*x/a^4+1/2*x^2/a^3-1/2*b^4/a^5/(a*x+b)^2+4*b^3/a^5/(a*x+b)+6*b^2*ln(a*x
+b)/a^5
```

maxima [A] time = 1.07, size = 69, normalized size = 1.08

$$\frac{8\,ab^3x + 7\,b^4}{2\left(a^7x^2 + 2\,a^6bx + a^5b^2\right)} + \frac{6\,b^2\log\left(ax+b\right)}{a^5} + \frac{ax^2 - 6\,bx}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(8*a*b^3*x + 7*b^4)/(a^7*x^2 + 2*a^6*b*x + a^5*b^2) + 6*b^2*log(a*x + b
)/a^5 + 1/2*(a*x^2 - 6*b*x)/a^4
```

mupad [B] time = 1.07, size = 70, normalized size = 1.09

$$\frac{4\,b^3\,x + \frac{7\,b^4}{2\,a}}{a^6\,x^2 + 2\,a^5\,b\,x + a^4\,b^2} + \frac{x^2}{2\,a^3} + \frac{6\,b^2\,\ln\left(b + a\,x\right)}{a^5} - \frac{3\,b\,x}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b/x)^3,x)
```

```
[Out] (4*b^3*x + (7*b^4)/(2*a))/(a^4*b^2 + a^6*x^2 + 2*a^5*b*x) + x^2/(2*a^3) + (
6*b^2*log(b + a*x))/a^5 - (3*b*x)/a^4
```

sympy [**A**] time = 0.30, size = 70, normalized size = 1.09

$$\frac{8ab^3x + 7b^4}{2a^7x^2 + 4a^6bx + 2a^5b^2} + \frac{x^2}{2a^3} - \frac{3bx}{a^4} + \frac{6b^2\log(ax+b)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x)**3,x)
```

```
[Out] (8*a*b**3*x + 7*b**4)/(2*a**7*x**2 + 4*a**6*b*x + 2*a**5*b**2) + x**2/(2*a*
**3) - 3*b*x/a**4 + 6*b**2*log(a*x + b)/a**5
```

3.1637 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3}\,dx$

Optimal. Leaf size=53

$$-\frac{3b\log(ax+b)}{a^4}+\frac{3x}{a^3}-\frac{3x}{2a^2\left(a+\frac{b}{x}\right)}-\frac{x}{2a\left(a+\frac{b}{x}\right)^2}$$

```
[Out] 3*x/a^3-1/2*x/a/(a+b/x)^2-3/2*x/a^2/(a+b/x)-3*b*ln(a*x+b)/a^4
```

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {192, 193, 43}

$$-\frac{3x}{2a^2\left(a+\frac{b}{x}\right)}-\frac{3b\log(ax+b)}{a^4}+\frac{3x}{a^3}-\frac{x}{2a\left(a+\frac{b}{x}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(-3),x]
```

```
[Out] (3*x)/a^3 - x/(2*a*(a + b/x)^2) - (3*x)/(2*a^2*(a + b/x)) - (3*b*Log[b + a*
x])/a^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3}\,dx = -\frac{x}{2a\left(a+\frac{b}{x}\right)^2} + \frac{3\int \frac{1}{\left(a+\frac{b}{x}\right)^2}\,dx}{2a}$$

$$= -\frac{x}{2a\left(a+\frac{b}{x}\right)^2} - \frac{3x}{2a^2\left(a+\frac{b}{x}\right)} + \frac{3\int \frac{1}{a+\frac{b}{x}}\,dx}{a^2}$$

$$= -\frac{x}{2a\left(a+\frac{b}{x}\right)^2} - \frac{3x}{2a^2\left(a+\frac{b}{x}\right)} + \frac{3\int \frac{x}{b+ax}\,dx}{a^2}$$

$$= -\frac{x}{2a\left(a+\frac{b}{x}\right)^2} - \frac{3x}{2a^2\left(a+\frac{b}{x}\right)} + \frac{3\int \left(\frac{1}{a} - \frac{b}{a(b+ax)}\right)\,dx}{a^2}$$

$$= \frac{3x}{a^3} - \frac{x}{2a\left(a+\frac{b}{x}\right)^2} - \frac{3x}{2a^2\left(a+\frac{b}{x}\right)} - \frac{3b\log(b+ax)}{a^4}$$

Mathematica [A] time = 0.04, size = 40, normalized size = 0.75

$$-\frac{\frac{b^2(6ax+5b)}{(ax+b)^2} + 6b\log(ax+b) - 2ax}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(-3),x]
```

```
[Out] -1/2*(-2*a*x + (b^2*(5*b + 6*a*x))/(b + a*x)^2 + 6*b*Log[b + a*x])/a^4
```

fricas [A] time = 0.74, size = 83, normalized size = 1.57

$$\frac{2\,a^3x^3 + 4\,a^2bx^2 - 4\,ab^2x - 5\,b^3 - 6\left(a^2bx^2 + 2\,ab^2x + b^3\right)\log\left(ax+b\right)}{2\left(a^6x^2 + 2\,a^5bx + a^4b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^3*x^3 + 4*a^2*b*x^2 - 4*a*b^2*x - 5*b^3 - 6*(a^2*b*x^2 + 2*a*b^2*x
 + b^3)*log(a*x + b))/(a^6*x^2 + 2*a^5*b*x + a^4*b^2)
```

giac [A] time = 0.15, size = 44, normalized size = 0.83

$$\frac{x}{a^3} - \frac{3\,b\log\left(|ax+b|\right)}{a^4} - \frac{6\,ab^2x + 5\,b^3}{2\left(ax+b\right)^2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3,x, algorithm="giac")
```

```
[Out] x/a^3 - 3*b*log(abs(a*x + b))/a^4 - 1/2*(6*a*b^2*x + 5*b^3)/((a*x + b)^2*a^
4)
```

maple [A] time = 0.01, size = 49, normalized size = 0.92

$$\frac{b^3}{2\left(ax+b\right)^2a^4} + \frac{x}{a^3} - \frac{3b^2}{\left(ax+b\right)a^4} - \frac{3b\ln\left(ax+b\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^3,x)
```

```
[Out] x/a^3-3*b*ln(a*x+b)/a^4+1/2*b^3/a^4/(a*x+b)^2-3/a^4*b^2/(a*x+b)
```

maxima [A] time = 1.04, size = 57, normalized size = 1.08

$$-\frac{6\,ab^2x+5\,b^3}{2\left(a^6x^2+2\,a^5bx+a^4b^2\right)}+\frac{x}{a^3}-\frac{3\,b\log\left(ax+b\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(6*a*b^2*x + 5*b^3)/(a^6*x^2 + 2*a^5*b*x + a^4*b^2) + x/a^3 - 3*b*log(
a*x + b)/a^4
```

mupad [B] time = 0.06, size = 59, normalized size = 1.11

$$\frac{x}{a^3}-\frac{3\,b^2\,x+\frac{5\,b^3}{2\,a}}{a^5\,x^2+2\,a^4\,b\,x+a^3\,b^2}-\frac{3\,b\,\ln\left(b+a\,x\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x)^3,x)
```

```
[Out] x/a^3 - (3*b^2*x + (5*b^3)/(2*a))/(a^3*b^2 + a^5*x^2 + 2*a^4*b*x) - (3*b*lo
g(b + a*x))/a^4
```

sympy [A] time = 0.27, size = 58, normalized size = 1.09

$$\frac{-6ab^2x-5b^3}{2a^6x^2+4a^5bx+2a^4b^2}+\frac{x}{a^3}-\frac{3b\log\left(ax+b\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**3,x)
```

```
[Out] (-6*a*b**2*x - 5*b**3)/(2*a**6*x**2 + 4*a**5*b*x + 2*a**4*b**2) + x/a**3 -
3*b*log(a*x + b)/a**4
```

3.1638 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x}\,dx$

Optimal. Leaf size=41

$$-\frac{b^2}{2a^3(ax+b)^2}+\frac{2b}{a^3(ax+b)}+\frac{\log(ax+b)}{a^3}$$

```
[Out] -1/2*b^2/a^3/(a*x+b)^2+2*b/a^3/(a*x+b)+ln(a*x+b)/a^3
```

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 43}

$$-\frac{b^2}{2a^3(ax+b)^2}+\frac{2b}{a^3(ax+b)}+\frac{\log(ax+b)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^3*x),x]
```

```
[Out] -b^2/(2*a^3*(b + a*x)^2) + (2*b)/(a^3*(b + a*x)) + Log[b + a*x]/a^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x}\,dx &= \int \frac{x^2}{(b+ax)^3}\,dx \\ &= \int \left(\frac{b^2}{a^2(b+ax)^3}-\frac{2b}{a^2(b+ax)^2}+\frac{1}{a^2(b+ax)}\right)dx \\ &= -\frac{b^2}{2a^3(b+ax)^2}+\frac{2b}{a^3(b+ax)}+\frac{\log(b+ax)}{a^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.80

$$\frac{\frac{b(4ax+3b)}{(ax+b)^2}+2\log(ax+b)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^3*x),x]
```

```
[Out] ((b*(3*b + 4*a*x))/(b + a*x)^2 + 2*Log[b + a*x])/(2*a^3)
```

fricas [A] time = 0.82, size = 61, normalized size = 1.49

$$\frac{4\,abx + 3\,b^2 + 2\left(a^2x^2 + 2\,abx + b^2\right)\log\left(ax + b\right)}{2\left(a^5x^2 + 2\,a^4bx + a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*b*x + 3*b^2 + 2*(a^2*x^2 + 2*a*b*x + b^2)*log(a*x + b))/(a^5*x^2 +
 2*a^4*b*x + a^3*b^2)
```

giac [A] time = 0.15, size = 37, normalized size = 0.90

$$\frac{\log\left(|ax + b|\right)}{a^3} + \frac{4\,bx + \frac{3\,b^2}{a}}{2\left(ax + b\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x,x, algorithm="giac")
```

```
[Out] log(abs(a*x + b))/a^3 + 1/2*(4*b*x + 3*b^2/a)/((a*x + b)^2*a^2)
```

maple [A] time = 0.01, size = 40, normalized size = 0.98

$$-\frac{b^2}{2\left(ax + b\right)^2a^3} + \frac{2b}{\left(ax + b\right)a^3} + \frac{\ln\left(ax + b\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^3/x,x)
```

```
[Out] -1/2*b^2/a^3/(a*x+b)^2+2*b/a^3/(a*x+b)+ln(a*x+b)/a^3
```

maxima [A] time = 0.98, size = 48, normalized size = 1.17

$$\frac{4\,abx + 3\,b^2}{2\left(a^5x^2 + 2\,a^4bx + a^3b^2\right)} + \frac{\log\left(ax + b\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x,x, algorithm="maxima")
```

```
[Out] 1/2*(4*a*b*x + 3*b^2)/(a^5*x^2 + 2*a^4*b*x + a^3*b^2) + log(a*x + b)/a^3
```

mupad [B] time = 1.07, size = 46, normalized size = 1.12

$$\frac{\ln\left(b + a\,x\right)}{a^3} + \frac{\frac{3\,b^2}{2\,a^3} + \frac{2\,b\,x}{a^2}}{a^2\,x^2 + 2\,a\,b\,x + b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b/x)^3),x)
```

```
[Out] log(b + a*x)/a^3 + ((3*b^2)/(2*a^3) + (2*b*x)/a^2)/(b^2 + a^2*x^2 + 2*a*b*x
)
```

sympy [A] time = 0.22, size = 46, normalized size = 1.12

$$\frac{4abx + 3b^2}{2a^5x^2 + 4a^4bx + 2a^3b^2} + \frac{\log\left(ax + b\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**3/x,x)
```

```
[Out] (4*a*b*x + 3*b**2)/(2*a**5*x**2 + 4*a**4*b*x + 2*a**3*b**2) + log(a*x + b)/
a**3
```

3.1639 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^2}\,dx$

Optimal. Leaf size=16

$$\frac{1}{2b\left(a+\frac{b}{x}\right)^2}$$

```
[Out] 1/2/b/(a+b/x)^2
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$\frac{1}{2b\left(a+\frac{b}{x}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^3*x^2),x]
```

```
[Out] 1/(2*b*(a + b/x)^2)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^2}\,dx = \frac{1}{2b\left(a+\frac{b}{x}\right)^2}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.25

$$-\frac{2ax+b}{2a^2(ax+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^3*x^2),x]
```

```
[Out] -1/2*(b + 2*a*x)/(a^2*(b + a*x)^2)
```

fricas [B] time = 1.01, size = 32, normalized size = 2.00

$$-\frac{2\,ax+b}{2\left(a^4x^2+2\,a^3bx+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*x + b)/(a^4*x^2 + 2*a^3*b*x + a^2*b^2)
```

giac [A] time = 0.15, size = 14, normalized size = 0.88

$$\frac{1}{2\left(a+\frac{b}{x}\right)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^2,x, algorithm="giac")
[Out] 1/2/((a + b/x)^2*b)
```

maple [A] time = 0.00, size = 27, normalized size = 1.69

$$\frac{b}{2\left(ax+b\right)^2 a^2}-\frac{1}{\left(ax+b\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^3/x^2,x)
[Out] 1/2/a^2*b/(a*x+b)^2-1/a^2/(a*x+b)
```

maxima [A] time = 1.02, size = 14, normalized size = 0.88

$$\frac{1}{2\left(a+\frac{b}{x}\right)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^2,x, algorithm="maxima")
[Out] 1/2/((a + b/x)^2*b)
```

mupad [B] time = 1.06, size = 32, normalized size = 2.00

$$-\frac{\frac{b}{2a^2}+\frac{x}{a}}{a^2x^2+2abx+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b/x)^3),x)
[Out] -(b/(2*a^2) + x/a)/(b^2 + a^2*x^2 + 2*a*b*x)
```

sympy [B] time = 0.21, size = 32, normalized size = 2.00

$$\frac{-2ax-b}{2a^4x^2+4a^3bx+2a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**3/x**2,x)
[Out] (-2*a*x - b)/(2*a**4*x**2 + 4*a**3*b*x + 2*a**2*b**2)
```

3.1640 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^3}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{2a(ax+b)^2}$$

```
[Out] -1/2/a/(a*x+b)^2
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 32}

$$-\frac{1}{2a(ax+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^3*x^3),x]
```

```
[Out] -1/(2*a*(b + a*x)^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^3}\,dx &= \int \frac{1}{(b+ax)^3}\,dx \\ &= -\frac{1}{2a(b+ax)^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{2a(ax+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^3*x^3),x]
```

```
[Out] -1/2*1/(a*(b + a*x)^2)
```

fricas [A] time = 1.10, size = 24, normalized size = 1.71

$$-\frac{1}{2\left(a^3x^2+2\,a^2bx+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^3,x, algorithm="fricas")
[Out] -1/2/(a^3*x^2 + 2*a^2*b*x + a*b^2)
```

giac [A] time = 0.15, size = 12, normalized size = 0.86

$$-\frac{1}{2\left(ax+b\right)^2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^3,x, algorithm="giac")
[Out] -1/2/((a*x + b)^2*a)
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$-\frac{1}{2\left(ax+b\right)^2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^3/x^3,x)
[Out] -1/2/a/(a*x+b)^2
```

maxima [A] time = 1.02, size = 24, normalized size = 1.71

$$-\frac{1}{2\left(a^3x^2+2\,a^2bx+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^3,x, algorithm="maxima")
[Out] -1/2/(a^3*x^2 + 2*a^2*b*x + a*b^2)
```

mupad [B] time = 1.05, size = 26, normalized size = 1.86

$$-\frac{1}{2\,a^3\,x^2+4\,a^2\,b\,x+2\,a\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b/x)^3),x)
[Out] -1/(2*a*b^2 + 2*a^3*x^2 + 4*a^2*b*x)
```

sympy [B] time = 0.21, size = 26, normalized size = 1.86

$$-\frac{1}{2a^3x^2+4a^2bx+2ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**3/x**3,x)
[Out] -1/(2*a**3*x**2 + 4*a**2*b*x + 2*a*b**2)
```

3.1641 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^4}\,dx$

Optimal. Leaf size=43

$$-\frac{\log(ax+b)}{b^3}+\frac{1}{b^2(ax+b)}+\frac{1}{2b(ax+b)^2}+\frac{\log(x)}{b^3}$$

```
[Out] 1/2/b/(a*x+b)^2+1/b^2/(a*x+b)+ln(x)/b^3-ln(a*x+b)/b^3
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 44}

$$\frac{1}{b^2(ax+b)}-\frac{\log(ax+b)}{b^3}+\frac{1}{2b(ax+b)^2}+\frac{\log(x)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^3*x^4),x]
```

```
[Out] 1/(2*b*(b + a*x)^2) + 1/(b^2*(b + a*x)) + Log[x]/b^3 - Log[b + a*x]/b^3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^4}\,dx &= \int \frac{1}{x(b+ax)^3}\,dx\\ &= \int\left(\frac{1}{b^3x}-\frac{a}{b(b+ax)^3}-\frac{a}{b^2(b+ax)^2}-\frac{a}{b^3(b+ax)}\right)dx\\ &= \frac{1}{2b(b+ax)^2}+\frac{1}{b^2(b+ax)}+\frac{\log(x)}{b^3}-\frac{\log(b+ax)}{b^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 37, normalized size = 0.86

$$\frac{\frac{b(2ax+3b)}{(ax+b)^2}-2\log(ax+b)+2\log(x)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^3*x^4),x]
```

```
[Out] ((b*(3*b + 2*a*x))/(b + a*x)^2 + 2*Log[x] - 2*Log[b + a*x])/(2*b^3)
```

fricas [A] time = 0.80, size = 80, normalized size = 1.86

$$\frac{2\,abx + 3\,b^2 - 2\left(a^2x^2 + 2\,abx + b^2\right)\log\left(ax + b\right) + 2\left(a^2x^2 + 2\,abx + b^2\right)\log(x)}{2\left(a^2b^3x^2 + 2\,ab^4x + b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*x + 3*b^2 - 2*(a^2*x^2 + 2*a*b*x + b^2)*log(a*x + b) + 2*(a^2*x^
2 + 2*a*b*x + b^2)*log(x))/(a^2*b^3*x^2 + 2*a*b^4*x + b^5)
```

giac [A] time = 0.15, size = 43, normalized size = 1.00

$$-\frac{\log\left(|ax + b|\right)}{b^3} + \frac{\log\left(|x|\right)}{b^3} + \frac{2\,abx + 3\,b^2}{2\left(ax + b\right)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^4,x, algorithm="giac")
```

```
[Out] -log(abs(a*x + b))/b^3 + log(abs(x))/b^3 + 1/2*(2*a*b*x + 3*b^2)/((a*x + b)
^2*b^3)
```

maple [A] time = 0.01, size = 42, normalized size = 0.98

$$\frac{1}{2\left(ax + b\right)^2b} + \frac{1}{\left(ax + b\right)b^2} + \frac{\ln(x)}{b^3} - \frac{\ln\left(ax + b\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^3/x^4,x)
```

```
[Out] 1/2/b/(a*x+b)^2+1/b^2/(a*x+b)+ln(x)/b^3-ln(a*x+b)/b^3
```

maxima [A] time = 1.02, size = 51, normalized size = 1.19

$$\frac{2\,ax + 3\,b}{2\left(a^2b^2x^2 + 2\,ab^3x + b^4\right)} - \frac{\log\left(ax + b\right)}{b^3} + \frac{\log(x)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*x + 3*b)/(a^2*b^2*x^2 + 2*a*b^3*x + b^4) - log(a*x + b)/b^3 + log(
x)/b^3
```

mupad [B] time = 1.08, size = 47, normalized size = 1.09

$$\frac{\frac{3}{2\,b} + \frac{a\,x}{b^2}}{a^2\,x^2 + 2\,a\,b\,x + b^2} - \frac{2\,\mathrm{atanh}\left(\frac{2\,a\,x}{b} + 1\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b/x)^3),x)
```

```
[Out] (3/(2*b) + (a*x)/b^2)/(b^2 + a^2*x^2 + 2*a*b*x) - (2*atanh((2*a*x)/b + 1))/
b^3
```

sympy [A] time = 0.31, size = 46, normalized size = 1.07

$$\frac{2ax + 3b}{2a^2b^2x^2 + 4ab^3x + 2b^4} + \frac{\log(x) - \log\left(x + \frac{b}{a}\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**3/x**4,x)
```

```
[Out] (2*a*x + 3*b)/(2*a**2*b**2*x**2 + 4*a*b**3*x + 2*b**4) + (log(x) - log(x +
b/a))/b**3
```

3.1642 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^5}\,dx$

Optimal. Leaf size=57

$$-\frac{3a\log(x)}{b^4}+\frac{3a\log(ax+b)}{b^4}-\frac{2a}{b^3(ax+b)}-\frac{a}{2b^2(ax+b)^2}-\frac{1}{b^3x}$$

```
[Out] -1/b^3/x-1/2*a/b^2/(a*x+b)^2-2*a/b^3/(a*x+b)-3*a*ln(x)/b^4+3*a*ln(a*x+b)/b^
4
```

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 44}

$$-\frac{2a}{b^3(ax+b)}-\frac{a}{2b^2(ax+b)^2}-\frac{3a\log(x)}{b^4}+\frac{3a\log(ax+b)}{b^4}-\frac{1}{b^3x}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^3*x^5),x]
```

```
[Out] -(1/(b^3*x)) - a/(2*b^2*(b + a*x)^2) - (2*a)/(b^3*(b + a*x)) - (3*a*Log[x])
/b^4 + (3*a*Log[b + a*x])/b^4
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^5}\,dx &= \int \frac{1}{x^2(b+ax)^3}\,dx\\ &= \int\left(\frac{1}{b^3x^2}-\frac{3a}{b^4x}+\frac{a^2}{b^2(b+ax)^3}+\frac{2a^2}{b^3(b+ax)^2}+\frac{3a^2}{b^4(b+ax)}\right)dx\\ &= -\frac{1}{b^3x}-\frac{a}{2b^2(b+ax)^2}-\frac{2a}{b^3(b+ax)}-\frac{3a\log(x)}{b^4}+\frac{3a\log(b+ax)}{b^4}\end{aligned}$$

Mathematica [A] time = 0.05, size = 53, normalized size = 0.93

$$-\frac{\frac{b\left(6a^2x^2+9abx+2b^2\right)}{x(ax+b)^2}-6a\log(ax+b)+6a\log(x)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^3*x^5),x]
```

```
[Out] -1/2*((b*(2*b^2 + 9*a*b*x + 6*a^2*x^2))/(x*(b + a*x)^2) + 6*a*Log[x] - 6*a*
Log[b + a*x])/b^4
```

fricas [A] time = 1.02, size = 109, normalized size = 1.91

$$-\frac{6\,a^2bx^2+9\,ab^2x+2\,b^3-6\left(a^3x^3+2\,a^2bx^2+ab^2x\right)\log\left(ax+b\right)+6\left(a^3x^3+2\,a^2bx^2+ab^2x\right)\log(x)}{2\left(a^2b^4x^3+2\,ab^5x^2+b^6x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^5,x, algorithm="fricas")
```

```
[Out] -1/2*(6*a^2*b*x^2 + 9*a*b^2*x + 2*b^3 - 6*(a^3*x^3 + 2*a^2*b*x^2 + a*b^2*x)
*log(a*x + b) + 6*(a^3*x^3 + 2*a^2*b*x^2 + a*b^2*x)*log(x))/(a^2*b^4*x^3 +
2*a*b^5*x^2 + b^6*x)
```

giac [A] time = 0.15, size = 60, normalized size = 1.05

$$\frac{3\,a\log\left(\left|ax+b\right|\right)}{b^4}-\frac{3\,a\log\left(\left|x\right|\right)}{b^4}-\frac{6\,a^2bx^2+9\,ab^2x+2\,b^3}{2\left(ax+b\right)^2b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^5,x, algorithm="giac")
```

```
[Out] 3*a*log(abs(a*x + b))/b^4 - 3*a*log(abs(x))/b^4 - 1/2*(6*a^2*b*x^2 + 9*a*b^
2*x + 2*b^3)/((a*x + b)^2*b^4*x)
```

maple [A] time = 0.01, size = 56, normalized size = 0.98

$$-\frac{a}{2\left(ax+b\right)^2b^2}-\frac{2a}{\left(ax+b\right)b^3}-\frac{3a\ln(x)}{b^4}+\frac{3a\ln\left(ax+b\right)}{b^4}-\frac{1}{b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^3/x^5,x)
```

```
[Out] -1/b^3/x-1/2*a/b^2/(a*x+b)^2-2*a/b^3/(a*x+b)-3*a*ln(x)/b^4+3*a*ln(a*x+b)/b^
4
```

maxima [A] time = 1.02, size = 69, normalized size = 1.21

$$-\frac{6\,a^2x^2+9\,abx+2\,b^2}{2\left(a^2b^3x^3+2\,ab^4x^2+b^5x\right)}+\frac{3\,a\log\left(ax+b\right)}{b^4}-\frac{3\,a\log(x)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^5,x, algorithm="maxima")
```

```
[Out] -1/2*(6*a^2*x^2 + 9*a*b*x + 2*b^2)/(a^2*b^3*x^3 + 2*a*b^4*x^2 + b^5*x) + 3*
a*log(a*x + b)/b^4 - 3*a*log(x)/b^4
```

mupad [B] time = 0.08, size = 63, normalized size = 1.11

$$\frac{6\,a\,\mathrm{atanh}\left(\frac{2\,a\,x}{b}+1\right)}{b^4}-\frac{\frac{1}{b}+\frac{3\,a^2\,x^2}{b^3}+\frac{9\,a\,x}{2\,b^2}}{a^2\,x^3+2\,a\,b\,x^2+b^2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b/x)^3),x)
```

```
[Out] (6*a*atanh((2*a*x)/b + 1))/b^4 - (1/b + (3*a^2*x^2)/b^3 + (9*a*x)/(2*b^2))/
(b^2*x + a^2*x^3 + 2*a*b*x^2)
```

sympy [**A**] time = 0.37, size = 66, normalized size = 1.16

$$\frac{3a\left(-\log(x)+\log\left(x+\frac{b}{a}\right)\right)}{b^4}+\frac{-6a^2x^2-9abx-2b^2}{2a^2b^3x^3+4ab^4x^2+2b^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**3/x**5,x)
```

```
[Out] 3*a*(-log(x) + log(x + b/a))/b**4 + (-6*a**2*x**2 - 9*a*b*x - 2*b**2)/(2*a*
**2*b**3*x**3 + 4*a*b**4*x**2 + 2*b**5*x)
```

3.1643 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^6}\,dx$

Optimal. Leaf size=76

$$\frac{6a^2\log(x)}{b^5}-\frac{6a^2\log(ax+b)}{b^5}+\frac{3a^2}{b^4(ax+b)}+\frac{a^2}{2b^3(ax+b)^2}+\frac{3a}{b^4x}-\frac{1}{2b^3x^2}$$

```
[Out] -1/2/b^3/x^2+3*a/b^4/x+1/2*a^2/b^3/(a*x+b)^2+3*a^2/b^4/(a*x+b)+6*a^2*ln(x)/
b^5-6*a^2*ln(a*x+b)/b^5
```

Rubi [A] time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 44}

$$\frac{3a^2}{b^4(ax+b)}+\frac{a^2}{2b^3(ax+b)^2}+\frac{6a^2\log(x)}{b^5}-\frac{6a^2\log(ax+b)}{b^5}+\frac{3a}{b^4x}-\frac{1}{2b^3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^3*x^6),x]
```

```
[Out] -1/(2*b^3*x^2) + (3*a)/(b^4*x) + a^2/(2*b^3*(b + a*x)^2) + (3*a^2)/(b^4*(b
+ a*x)) + (6*a^2*Log[x])/b^5 - (6*a^2*Log[b + a*x])/b^5
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^6}\,dx &= \int \frac{1}{x^3(b+ax)^3}\,dx\\ &= \int\left(\frac{1}{b^3x^3}-\frac{3a}{b^4x^2}+\frac{6a^2}{b^5x}-\frac{a^3}{b^3(b+ax)^3}-\frac{3a^3}{b^4(b+ax)^2}-\frac{6a^3}{b^5(b+ax)}\right)dx\\ &= -\frac{1}{2b^3x^2}+\frac{3a}{b^4x}+\frac{a^2}{2b^3(b+ax)^2}+\frac{3a^2}{b^4(b+ax)}+\frac{6a^2\log(x)}{b^5}-\frac{6a^2\log(b+ax)}{b^5}\end{aligned}$$

Mathematica [A] time = 0.05, size = 68, normalized size = 0.89

$$\frac{-12a^2\log(ax+b)+12a^2\log(x)+\frac{b\left(12a^3x^3+18a^2bx^2+4ab^2x-b^3\right)}{x^2(ax+b)^2}}{2b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^3*x^6),x]
```

```
[Out] ((b*(-b^3 + 4*a*b^2*x + 18*a^2*b*x^2 + 12*a^3*x^3))/(x^2*(b + a*x)^2) + 12*
a^2*Log[x] - 12*a^2*Log[b + a*x])/(2*b^5)
```

fricas [A] time = 0.90, size = 130, normalized size = 1.71

$$\frac{12\,a^3bx^3+18\,a^2b^2x^2+4\,ab^3x-b^4-12\left(a^4x^4+2\,a^3bx^3+a^2b^2x^2\right)\log\left(ax+b\right)+12\left(a^4x^4+2\,a^3bx^3+a^2b^2x^2\right)\log}{2\left(a^2b^5x^4+2\,ab^6x^3+b^7x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^6,x, algorithm="fricas")
```

```
[Out] 1/2*(12*a^3*b*x^3 + 18*a^2*b^2*x^2 + 4*a*b^3*x - b^4 - 12*(a^4*x^4 + 2*a^3*
b*x^3 + a^2*b^2*x^2)*log(a*x + b) + 12*(a^4*x^4 + 2*a^3*b*x^3 + a^2*b^2*x^2
)*log(x))/(a^2*b^5*x^4 + 2*a*b^6*x^3 + b^7*x^2)
```

giac [A] time = 0.16, size = 73, normalized size = 0.96

$$-\frac{6\,a^2\log\left(|ax+b|\right)}{b^5}+\frac{6\,a^2\log\left(|x|\right)}{b^5}+\frac{12\,a^3x^3+18\,a^2bx^2+4\,ab^2x-b^3}{2\left(ax^2+bx\right)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^6,x, algorithm="giac")
```

```
[Out] -6*a^2*log(abs(a*x + b))/b^5 + 6*a^2*log(abs(x))/b^5 + 1/2*(12*a^3*x^3 + 18
*a^2*b*x^2 + 4*a*b^2*x - b^3)/((a*x^2 + b*x)^2*b^4)
```

maple [A] time = 0.01, size = 73, normalized size = 0.96

$$\frac{a^2}{2\left(ax+b\right)^2b^3}+\frac{3a^2}{\left(ax+b\right)b^4}+\frac{6a^2\ln(x)}{b^5}-\frac{6a^2\ln\left(ax+b\right)}{b^5}+\frac{3a}{b^4x}-\frac{1}{2b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^3/x^6,x)
```

```
[Out] -1/2/b^3/x^2+3*a/b^4/x+1/2*a^2/b^3/(a*x+b)^2+3*a^2/b^4/(a*x+b)+6*a^2*ln(x)/
b^5-6*a^2*ln(a*x+b)/b^5
```

maxima [A] time = 1.10, size = 86, normalized size = 1.13

$$\frac{12\,a^3x^3+18\,a^2bx^2+4\,ab^2x-b^3}{2\left(a^2b^4x^4+2\,ab^5x^3+b^6x^2\right)}-\frac{6\,a^2\log\left(ax+b\right)}{b^5}+\frac{6\,a^2\log(x)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^6,x, algorithm="maxima")
```

```
[Out] 1/2*(12*a^3*x^3 + 18*a^2*b*x^2 + 4*a*b^2*x - b^3)/(a^2*b^4*x^4 + 2*a*b^5*x^
3 + b^6*x^2) - 6*a^2*log(a*x + b)/b^5 + 6*a^2*log(x)/b^5
```

mupad [B] time = 0.09, size = 79, normalized size = 1.04

$$\frac{\frac{9\,a^2\,x^2}{b^3}-\frac{1}{2\,b}+\frac{6\,a^3\,x^3}{b^4}+\frac{2\,a\,x}{b^2}}{a^2\,x^4+2\,a\,b\,x^3+b^2\,x^2}-\frac{12\,a^2\,\mathrm{atanh}\left(\frac{2\,a\,x}{b}+1\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b/x)^3),x)
```

```
[Out] ((9*a^2*x^2)/b^3 - 1/(2*b) + (6*a^3*x^3)/b^4 + (2*a*x)/b^2)/(a^2*x^4 + b^2*
x^2 + 2*a*b*x^3) - (12*a^2*atanh((2*a*x)/b + 1))/b^5
```

sympy [A] time = 0.42, size = 78, normalized size = 1.03

$$\frac{6a^2\left(\log(x) - \log\left(x + \frac{b}{a}\right)\right)}{b^5} + \frac{12a^3x^3 + 18a^2bx^2 + 4ab^2x - b^3}{2a^2b^4x^4 + 4ab^5x^3 + 2b^6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**3/x**6,x)
```

```
[Out] 6*a**2*(log(x) - log(x + b/a))/b**5 + (12*a**3*x**3 + 18*a**2*b*x**2 + 4*a*
b**2*x - b**3)/(2*a**2*b**4*x**4 + 4*a*b**5*x**3 + 2*b**6*x**2)
```

3.1644 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^7}\,dx$

Optimal. Leaf size=89

$$-\frac{10a^3\log(x)}{b^6}+\frac{10a^3\log(ax+b)}{b^6}-\frac{4a^3}{b^5(ax+b)}-\frac{a^3}{2b^4(ax+b)^2}-\frac{6a^2}{b^5x}+\frac{3a}{2b^4x^2}-\frac{1}{3b^3x^3}$$

```
[Out] -1/3/b^3/x^3+3/2*a/b^4/x^2-6*a^2/b^5/x-1/2*a^3/b^4/(a*x+b)^2-4*a^3/b^5/(a*x
+b)-10*a^3*ln(x)/b^6+10*a^3*ln(a*x+b)/b^6
```

Rubi [A] time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 44}

$$-\frac{4a^3}{b^5(ax+b)}-\frac{a^3}{2b^4(ax+b)^2}-\frac{6a^2}{b^5x}-\frac{10a^3\log(x)}{b^6}+\frac{10a^3\log(ax+b)}{b^6}+\frac{3a}{2b^4x^2}-\frac{1}{3b^3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^3*x^7),x]
```

```
[Out] -1/(3*b^3*x^3) + (3*a)/(2*b^4*x^2) - (6*a^2)/(b^5*x) - a^3/(2*b^4*(b + a*x)
^2) - (4*a^3)/(b^5*(b + a*x)) - (10*a^3*Log[x])/b^6 + (10*a^3*Log[b + a*x])
/b^6
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^7}\,dx &= \int \frac{1}{x^4(b+ax)^3}\,dx\\ &= \int\left(\frac{1}{b^3x^4}-\frac{3a}{b^4x^3}+\frac{6a^2}{b^5x^2}-\frac{10a^3}{b^6x}+\frac{a^4}{b^4(b+ax)^3}+\frac{4a^4}{b^5(b+ax)^2}+\frac{10a^4}{b^6(b+ax)}\right)dx\\ &= -\frac{1}{3b^3x^3}+\frac{3a}{2b^4x^2}-\frac{6a^2}{b^5x}-\frac{a^3}{2b^4(b+ax)^2}-\frac{4a^3}{b^5(b+ax)}-\frac{10a^3\log(x)}{b^6}+\frac{10a^3\log(b+ax)}{b^6}\end{aligned}$$

Mathematica [A] time = 0.08, size = 79, normalized size = 0.89

$$-\frac{-60a^3\log(ax+b)+60a^3\log(x)+\frac{b\left(60a^4x^4+90a^3bx^3+20a^2b^2x^2-5ab^3x+2b^4\right)}{x^3(ax+b)^2}}{6b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^3*x^7),x]
```

```
[Out] -1/6*((b*(2*b^4 - 5*a*b^3*x + 20*a^2*b^2*x^2 + 90*a^3*b*x^3 + 60*a^4*x^4))/
(x^3*(b + a*x)^2) + 60*a^3*Log[x] - 60*a^3*Log[b + a*x])/b^6
```

fricas [A] time = 1.01, size = 141, normalized size = 1.58

$$-\frac{60\,a^4bx^4+90\,a^3b^2x^3+20\,a^2b^3x^2-5\,ab^4x+2\,b^5-60\left(a^5x^5+2\,a^4bx^4+a^3b^2x^3\right)\log\left(ax+b\right)+60\left(a^5x^5+2\right.}{6\left(a^2b^6x^5+2\,ab^7x^4+b^8x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^7,x, algorithm="fricas")
```

```
[Out] -1/6*(60*a^4*b*x^4 + 90*a^3*b^2*x^3 + 20*a^2*b^3*x^2 - 5*a*b^4*x + 2*b^5 -
60*(a^5*x^5 + 2*a^4*b*x^4 + a^3*b^2*x^3)*log(a*x + b) + 60*(a^5*x^5 + 2*a^4
*b*x^4 + a^3*b^2*x^3)*log(x))/(a^2*b^6*x^5 + 2*a*b^7*x^4 + b^8*x^3)
```

giac [A] time = 0.17, size = 86, normalized size = 0.97

$$\frac{10\,a^3\log\left(\left|ax+b\right|\right)}{b^6}-\frac{10\,a^3\log\left(\left|x\right|\right)}{b^6}-\frac{60\,a^4bx^4+90\,a^3b^2x^3+20\,a^2b^3x^2-5\,ab^4x+2\,b^5}{6\left(ax+b\right)^2b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^7,x, algorithm="giac")
```

```
[Out] 10*a^3*log(abs(a*x + b))/b^6 - 10*a^3*log(abs(x))/b^6 - 1/6*(60*a^4*b*x^4 +
 90*a^3*b^2*x^3 + 20*a^2*b^3*x^2 - 5*a*b^4*x + 2*b^5)/((a*x + b)^2*b^6*x^3)
```

maple [A] time = 0.01, size = 84, normalized size = 0.94

$$-\frac{a^3}{2\left(ax+b\right)^2b^4}-\frac{4a^3}{\left(ax+b\right)b^5}-\frac{10a^3\ln(x)}{b^6}+\frac{10a^3\ln\left(ax+b\right)}{b^6}-\frac{6a^2}{b^5x}+\frac{3a}{2b^4x^2}-\frac{1}{3b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^3/x^7,x)
```

```
[Out] -1/3/b^3/x^3+3/2*a/b^4/x^2-6*a^2/b^5/x-1/2*a^3/b^4/(a*x+b)^2-4*a^3/b^5/(a*x
+b)-10*a^3*ln(x)/b^6+10*a^3*ln(a*x+b)/b^6
```

maxima [A] time = 1.07, size = 97, normalized size = 1.09

$$-\frac{60\,a^4x^4+90\,a^3bx^3+20\,a^2b^2x^2-5\,ab^3x+2\,b^4}{6\left(a^2b^5x^5+2\,ab^6x^4+b^7x^3\right)}+\frac{10\,a^3\log\left(ax+b\right)}{b^6}-\frac{10\,a^3\log(x)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^7,x, algorithm="maxima")
```

```
[Out] -1/6*(60*a^4*x^4 + 90*a^3*b*x^3 + 20*a^2*b^2*x^2 - 5*a*b^3*x + 2*b^4)/(a^2*
b^5*x^5 + 2*a*b^6*x^4 + b^7*x^3) + 10*a^3*log(a*x + b)/b^6 - 10*a^3*log(x)/
b^6
```

mupad [B] time = 0.09, size = 91, normalized size = 1.02

$$\frac{20\,a^3\,\mathrm{atanh}\left(\frac{2\,a\,x}{b}+1\right)}{b^6}-\frac{\frac{1}{3\,b}+\frac{10\,a^2\,x^2}{3\,b^3}+\frac{15\,a^3\,x^3}{b^4}+\frac{10\,a^4\,x^4}{b^5}-\frac{5\,a\,x}{6\,b^2}}{a^2\,x^5+2\,a\,b\,x^4+b^2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b/x)^3),x)
```

```
[Out] (20*a^3*atanh((2*a*x)/b + 1))/b^6 - (1/(3*b) + (10*a^2*x^2)/(3*b^3) + (15*a
^3*x^3)/b^4 + (10*a^4*x^4)/b^5 - (5*a*x)/(6*b^2))/(a^2*x^5 + b^2*x^3 + 2*a*
b*x^4)
```

sympy [**A**] time = 0.44, size = 92, normalized size = 1.03

$$\frac{10a^3\left(-\log(x)+\log\left(x+\frac{b}{a}\right)\right)}{b^6}+\frac{-60a^4x^4-90a^3bx^3-20a^2b^2x^2+5ab^3x-2b^4}{6a^2b^5x^5+12ab^6x^4+6b^7x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**3/x**7,x)
```

```
[Out] 10*a**3*(-log(x) + log(x + b/a))/b**6 + (-60*a**4*x**4 - 90*a**3*b*x**3 - 2
0*a**2*b**2*x**2 + 5*a*b**3*x - 2*b**4)/(6*a**2*b**5*x**5 + 12*a*b**6*x**4
+ 6*b**7*x**3)
```

3.1645 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^8}\,dx$

Optimal. Leaf size=97

$$\frac{15a^4\log(x)}{b^7}-\frac{15a^4\log(ax+b)}{b^7}+\frac{5a^4}{b^6(ax+b)}+\frac{a^4}{2b^5(ax+b)^2}+\frac{10a^3}{b^6x}-\frac{3a^2}{b^5x^2}+\frac{a}{b^4x^3}-\frac{1}{4b^3x^4}$$

```
[Out] -1/4/b^3/x^4+a/b^4/x^3-3*a^2/b^5/x^2+10*a^3/b^6/x+1/2*a^4/b^5/(a*x+b)^2+5*a
^4/b^6/(a*x+b)+15*a^4*ln(x)/b^7-15*a^4*ln(a*x+b)/b^7
```

Rubi [A] time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 44}

$$-\frac{3a^2}{b^5x^2}+\frac{5a^4}{b^6(ax+b)}+\frac{a^4}{2b^5(ax+b)^2}+\frac{10a^3}{b^6x}+\frac{15a^4\log(x)}{b^7}-\frac{15a^4\log(ax+b)}{b^7}+\frac{a}{b^4x^3}-\frac{1}{4b^3x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^3*x^8),x]
```

```
[Out] -1/(4*b^3*x^4) + a/(b^4*x^3) - (3*a^2)/(b^5*x^2) + (10*a^3)/(b^6*x) + a^4/(
2*b^5*(b + a*x)^2) + (5*a^4)/(b^6*(b + a*x)) + (15*a^4*Log[x])/b^7 - (15*a^
4*Log[b + a*x])/b^7
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^8}\,dx &= \int \frac{1}{x^5(b+ax)^3}\,dx\\ &= \int\left(\frac{1}{b^3x^5}-\frac{3a}{b^4x^4}+\frac{6a^2}{b^5x^3}-\frac{10a^3}{b^6x^2}+\frac{15a^4}{b^7x}-\frac{a^5}{b^5(b+ax)^3}-\frac{5a^5}{b^6(b+ax)^2}-\frac{15a^5}{b^7(b+ax)}\right)dx\\ &= -\frac{1}{4b^3x^4}+\frac{a}{b^4x^3}-\frac{3a^2}{b^5x^2}+\frac{10a^3}{b^6x}+\frac{a^4}{2b^5(b+ax)^2}+\frac{5a^4}{b^6(b+ax)}+\frac{15a^4\log(x)}{b^7}-\frac{15a^4\log(b+\end{aligned}$$

Mathematica [A] time = 0.06, size = 90, normalized size = 0.93

$$\frac{-60a^4\log(ax+b)+60a^4\log(x)+\frac{b\left(60a^5x^5+90a^4bx^4+20a^3b^2x^3-5a^2b^3x^2+2ab^4x-b^5\right)}{x^4(ax+b)^2}}{4b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^3*x^8),x]
```

```
[Out] ((b*(-b^5 + 2*a*b^4*x - 5*a^2*b^3*x^2 + 20*a^3*b^2*x^3 + 90*a^4*b*x^4 + 60*
a^5*x^5))/(x^4*(b + a*x)^2) + 60*a^4*Log[x] - 60*a^4*Log[b + a*x])/(4*b^7)
```

fricas [A] time = 0.99, size = 152, normalized size = 1.57

$$\frac{60\,a^5bx^5+90\,a^4b^2x^4+20\,a^3b^3x^3-5\,a^2b^4x^2+2\,ab^5x-b^6-60\left(a^6x^6+2\,a^5bx^5+a^4b^2x^4\right)\log\left(ax+b\right)+60\left(a^6x^6\right.}{4\left(a^2b^7x^6+2\,ab^8x^5+b^9x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^8,x, algorithm="fricas")
```

```
[Out] 1/4*(60*a^5*b*x^5 + 90*a^4*b^2*x^4 + 20*a^3*b^3*x^3 - 5*a^2*b^4*x^2 + 2*a*b
^5*x - b^6 - 60*(a^6*x^6 + 2*a^5*b*x^5 + a^4*b^2*x^4)*log(a*x + b) + 60*(a^
6*x^6 + 2*a^5*b*x^5 + a^4*b^2*x^4)*log(x))/(a^2*b^7*x^6 + 2*a*b^8*x^5 + b^9
*x^4)
```

giac [A] time = 0.15, size = 97, normalized size = 1.00

$$-\frac{15\,a^4\log\left(\left|ax+b\right|\right)}{b^7}+\frac{15\,a^4\log\left(\left|x\right|\right)}{b^7}+\frac{60\,a^5bx^5+90\,a^4b^2x^4+20\,a^3b^3x^3-5\,a^2b^4x^2+2\,ab^5x-b^6}{4\left(ax+b\right)^2b^7x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^8,x, algorithm="giac")
```

```
[Out] -15*a^4*log(abs(a*x + b))/b^7 + 15*a^4*log(abs(x))/b^7 + 1/4*(60*a^5*b*x^5
+ 90*a^4*b^2*x^4 + 20*a^3*b^3*x^3 - 5*a^2*b^4*x^2 + 2*a*b^5*x - b^6)/((a*x
+ b)^2*b^7*x^4)
```

maple [A] time = 0.01, size = 94, normalized size = 0.97

$$\frac{a^4}{2\left(ax+b\right)^2b^5}+\frac{5a^4}{\left(ax+b\right)b^6}+\frac{15a^4\ln(x)}{b^7}-\frac{15a^4\ln\left(ax+b\right)}{b^7}+\frac{10a^3}{b^6x}-\frac{3a^2}{b^5x^2}+\frac{a}{b^4x^3}-\frac{1}{4b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^3/x^8,x)
```

```
[Out] -1/4/b^3/x^4+a/b^4/x^3-3*a^2/b^5/x^2+10*a^3/b^6/x+1/2*a^4/b^5/(a*x+b)^2+5*a
^4/b^6/(a*x+b)+15*a^4*ln(x)/b^7-15*a^4*ln(a*x+b)/b^7
```

maxima [A] time = 1.14, size = 108, normalized size = 1.11

$$\frac{60\,a^5x^5+90\,a^4bx^4+20\,a^3b^2x^3-5\,a^2b^3x^2+2\,ab^4x-b^5}{4\left(a^2b^6x^6+2\,ab^7x^5+b^8x^4\right)}-\frac{15\,a^4\log\left(ax+b\right)}{b^7}+\frac{15\,a^4\log(x)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^8,x, algorithm="maxima")
```

```
[Out] 1/4*(60*a^5*x^5 + 90*a^4*b*x^4 + 20*a^3*b^2*x^3 - 5*a^2*b^3*x^2 + 2*a*b^4*x
 - b^5)/(a^2*b^6*x^6 + 2*a*b^7*x^5 + b^8*x^4) - 15*a^4*log(a*x + b)/b^7 + 1
5*a^4*log(x)/b^7
```

mupad [B] time = 1.13, size = 101, normalized size = 1.04

$$\frac{\frac{5\,a^3\,x^3}{b^4}-\frac{5\,a^2\,x^2}{4\,b^3}-\frac{1}{4\,b}+\frac{45\,a^4\,x^4}{2\,b^5}+\frac{15\,a^5\,x^5}{b^6}+\frac{a\,x}{2\,b^2}}{a^2\,x^6+2\,a\,b\,x^5+b^2\,x^4}-\frac{30\,a^4\,\operatorname{atanh}\left(\frac{2\,a\,x}{b}+1\right)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(a + b/x)^3),x)
```

```
[Out] ((5*a^3*x^3)/b^4 - (5*a^2*x^2)/(4*b^3) - 1/(4*b) + (45*a^4*x^4)/(2*b^5) + (
15*a^5*x^5)/b^6 + (a*x)/(2*b^2))/(a^2*x^6 + b^2*x^4 + 2*a*b*x^5) - (30*a^4*
atanh((2*a*x)/b + 1))/b^7
```

sympy [**A**] time = 0.51, size = 102, normalized size = 1.05

$$\frac{15a^4\left(\log(x) - \log\left(x + \frac{b}{a}\right)\right)}{b^7} + \frac{60a^5x^5 + 90a^4bx^4 + 20a^3b^2x^3 - 5a^2b^3x^2 + 2ab^4x - b^5}{4a^2b^6x^6 + 8ab^7x^5 + 4b^8x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**3/x**8,x)
```

```
[Out] 15*a**4*(log(x) - log(x + b/a))/b**7 + (60*a**5*x**5 + 90*a**4*b*x**4 + 20*
a**3*b**2*x**3 - 5*a**2*b**3*x**2 + 2*a*b**4*x - b**5)/(4*a**2*b**6*x**6 +
8*a*b**7*x**5 + 4*b**8*x**4)
```

3.1646 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^9}\,dx$

Optimal. Leaf size=111

$$-\frac{21a^5\log(x)}{b^8}+\frac{21a^5\log(ax+b)}{b^8}-\frac{6a^5}{b^7(ax+b)}-\frac{a^5}{2b^6(ax+b)^2}-\frac{15a^4}{b^7x}+\frac{5a^3}{b^6x^2}-\frac{2a^2}{b^5x^3}+\frac{3a}{4b^4x^4}-\frac{1}{5b^3x^5}$$

```
[Out] -1/5/b^3/x^5+3/4*a/b^4/x^4-2*a^2/b^5/x^3+5*a^3/b^6/x^2-15*a^4/b^7/x-1/2*a^5
/b^6/(a*x+b)^2-6*a^5/b^7/(a*x+b)-21*a^5*ln(x)/b^8+21*a^5*ln(a*x+b)/b^8
```

Rubi [A] time = 0.07, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 44}

$$\frac{5a^3}{b^6x^2}-\frac{2a^2}{b^5x^3}-\frac{6a^5}{b^7(ax+b)}-\frac{a^5}{2b^6(ax+b)^2}-\frac{15a^4}{b^7x}-\frac{21a^5\log(x)}{b^8}+\frac{21a^5\log(ax+b)}{b^8}+\frac{3a}{4b^4x^4}-\frac{1}{5b^3x^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^3*x^9),x]
```

```
[Out] -1/(5*b^3*x^5) + (3*a)/(4*b^4*x^4) - (2*a^2)/(b^5*x^3) + (5*a^3)/(b^6*x^2)
- (15*a^4)/(b^7*x) - a^5/(2*b^6*(b + a*x)^2) - (6*a^5)/(b^7*(b + a*x)) - (2
1*a^5*Log[x])/b^8 + (21*a^5*Log[b + a*x])/b^8
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^9}\,dx = \int \frac{1}{x^6(b+ax)^3}\,dx$$

$$= \int \left(\frac{1}{b^3x^6}-\frac{3a}{b^4x^5}+\frac{6a^2}{b^5x^4}-\frac{10a^3}{b^6x^3}+\frac{15a^4}{b^7x^2}-\frac{21a^5}{b^8x}+\frac{a^6}{b^6(b+ax)^3}+\frac{6a^6}{b^7(b+ax)^2}+\frac{21a^6}{b^8(b+ax)}\right) a$$

$$= -\frac{1}{5b^3x^5}+\frac{3a}{4b^4x^4}-\frac{2a^2}{b^5x^3}+\frac{5a^3}{b^6x^2}-\frac{15a^4}{b^7x}-\frac{a^5}{2b^6(b+ax)^2}-\frac{6a^5}{b^7(b+ax)}-\frac{21a^5\log(x)}{b^8}+\frac{21a^5 \text{lo}}{}$$

Mathematica [A] time = 0.10, size = 101, normalized size = 0.91

$$-\frac{-420a^5\log(ax+b)+420a^5\log(x)+\frac{b\left(420a^6x^6+630a^5bx^5+140a^4b^2x^4-35a^3b^3x^3+14a^2b^4x^2-7ab^5x+4b^6\right)}{x^5(ax+b)^2}}{20b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^3*x^9),x]
```

```
[Out] -1/20*((b*(4*b^6 - 7*a*b^5*x + 14*a^2*b^4*x^2 - 35*a^3*b^3*x^3 + 140*a^4*b^
2*x^4 + 630*a^5*b*x^5 + 420*a^6*x^6))/(x^5*(b + a*x)^2) + 420*a^5*Log[x] -
420*a^5*Log[b + a*x])/b^8
```

fricas [A] time = 0.95, size = 163, normalized size = 1.47

$$-\frac{420\,a^6bx^6+630\,a^5b^2x^5+140\,a^4b^3x^4-35\,a^3b^4x^3+14\,a^2b^5x^2-7\,ab^6x+4\,b^7-420\left(a^7x^7+2\,a^6bx^6+a^5b^2x^5\right.}{20\left(a^2b^8x^7+2\,ab^9x^6+b^{10}x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^9,x, algorithm="fricas")
```

```
[Out] -1/20*(420*a^6*b*x^6 + 630*a^5*b^2*x^5 + 140*a^4*b^3*x^4 - 35*a^3*b^4*x^3 +
 14*a^2*b^5*x^2 - 7*a*b^6*x + 4*b^7 - 420*(a^7*x^7 + 2*a^6*b*x^6 + a^5*b^2*
x^5)*log(a*x + b) + 420*(a^7*x^7 + 2*a^6*b*x^6 + a^5*b^2*x^5)*log(x))/(a^2*
b^8*x^7 + 2*a*b^9*x^6 + b^10*x^5)
```

giac [A] time = 0.15, size = 108, normalized size = 0.97

$$\frac{21\,a^5\log\left(|ax+b|\right)}{b^8}-\frac{21\,a^5\log\left(|x|\right)}{b^8}-\frac{420\,a^6bx^6+630\,a^5b^2x^5+140\,a^4b^3x^4-35\,a^3b^4x^3+14\,a^2b^5x^2-7\,ab^6x+4}{20\,(ax+b)^2b^8x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^9,x, algorithm="giac")
```

```
[Out] 21*a^5*log(abs(a*x + b))/b^8 - 21*a^5*log(abs(x))/b^8 - 1/20*(420*a^6*b*x^6
 + 630*a^5*b^2*x^5 + 140*a^4*b^3*x^4 - 35*a^3*b^4*x^3 + 14*a^2*b^5*x^2 - 7*
a*b^6*x + 4*b^7)/((a*x + b)^2*b^8*x^5)
```

maple [A] time = 0.01, size = 106, normalized size = 0.95

$$-\frac{a^5}{2\,(ax+b)^2\,b^6}-\frac{6a^5}{(ax+b)\,b^7}-\frac{21a^5\ln(x)}{b^8}+\frac{21a^5\ln\left(ax+b\right)}{b^8}-\frac{15a^4}{b^7x}+\frac{5a^3}{b^6x^2}-\frac{2a^2}{b^5x^3}+\frac{3a}{4b^4x^4}-\frac{1}{5b^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^3/x^9,x)
```

```
[Out] -1/5/b^3/x^5+3/4*a/b^4/x^4-2*a^2/b^5/x^3+5*a^3/b^6/x^2-15*a^4/b^7/x-1/2*a^5
/b^6/(a*x+b)^2-6*a^5/b^7/(a*x+b)-21*a^5*ln(x)/b^8+21*a^5*ln(a*x+b)/b^8
```

maxima [A] time = 1.01, size = 119, normalized size = 1.07

$$-\frac{420\,a^6x^6+630\,a^5bx^5+140\,a^4b^2x^4-35\,a^3b^3x^3+14\,a^2b^4x^2-7\,ab^5x+4\,b^6}{20\left(a^2b^7x^7+2\,ab^8x^6+b^9x^5\right)}+\frac{21\,a^5\log\left(ax+b\right)}{b^8}-\frac{21\,a^5\log(x)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^9,x, algorithm="maxima")
```

```
[Out] -1/20*(420*a^6*x^6 + 630*a^5*b*x^5 + 140*a^4*b^2*x^4 - 35*a^3*b^3*x^3 + 14*
a^2*b^4*x^2 - 7*a*b^5*x + 4*b^6)/(a^2*b^7*x^7 + 2*a*b^8*x^6 + b^9*x^5) + 21
*a^5*log(a*x + b)/b^8 - 21*a^5*log(x)/b^8
```

mupad [B] time = 1.18, size = 113, normalized size = 1.02

$$\frac{42\,a^5\,\mathrm{atanh}\left(\frac{2\,a\,x}{b}+1\right)}{b^8}-\frac{\frac{1}{5\,b}+\frac{7\,a^2\,x^2}{10\,b^3}-\frac{7\,a^3\,x^3}{4\,b^4}+\frac{7\,a^4\,x^4}{b^5}+\frac{63\,a^5\,x^5}{2\,b^6}+\frac{21\,a^6\,x^6}{b^7}-\frac{7\,a\,x}{20\,b^2}}{a^2\,x^7+2\,a\,b\,x^6+b^2\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(a + b/x)^3),x)
```

```
[Out] (42*a^5*atanh((2*a*x)/b + 1))/b^8 - (1/(5*b) + (7*a^2*x^2)/(10*b^3) - (7*a^
3*x^3)/(4*b^4) + (7*a^4*x^4)/b^5 + (63*a^5*x^5)/(2*b^6) + (21*a^6*x^6)/b^7
- (7*a*x)/(20*b^2))/(a^2*x^7 + b^2*x^5 + 2*a*b*x^6)
```

sympy [**A**] time = 0.51, size = 116, normalized size = 1.05

$$\frac{21a^5\left(-\log(x)+\log\left(x+\frac{b}{a}\right)\right)}{b^8}+\frac{-420a^6x^6-630a^5bx^5-140a^4b^2x^4+35a^3b^3x^3-14a^2b^4x^2+7ab^5x-4b^6}{20a^2b^7x^7+40ab^8x^6+20b^9x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**3/x**9,x)
```

```
[Out] 21*a**5*(-log(x) + log(x + b/a))/b**8 + (-420*a**6*x**6 - 630*a**5*b*x**5 -
 140*a**4*b**2*x**4 + 35*a**3*b**3*x**3 - 14*a**2*b**4*x**2 + 7*a*b**5*x -
4*b**6)/(20*a**2*b**7*x**7 + 40*a*b**8*x**6 + 20*b**9*x**5)
```

3.1647 $\int \left(a + \frac{b}{x}\right) x^{5/2}\, dx$

Optimal. Leaf size=21

$$\frac{2}{7}ax^{7/2} + \frac{2}{5}bx^{5/2}$$

```
[Out] 2/5*b*x^(5/2)+2/7*a*x^(7/2)
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$\frac{2}{7}ax^{7/2} + \frac{2}{5}bx^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)*x^(5/2),x]
```

```
[Out] (2*b*x^(5/2))/5 + (2*a*x^(7/2))/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \left(a + \frac{b}{x}\right) x^{5/2}\, dx &= \int \left(bx^{3/2} + ax^{5/2}\right) dx \\ &= \frac{2}{5}bx^{5/2} + \frac{2}{7}ax^{7/2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 0.81

$$\frac{2}{35}x^{5/2}(5ax + 7b)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)*x^(5/2),x]
```

```
[Out] (2*x^(5/2)*(7*b + 5*a*x))/35
```

fricas [A] time = 1.16, size = 18, normalized size = 0.86

$$\frac{2}{35}\left(5\,ax^3 + 7\,bx^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)*x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*a*x^3 + 7*b*x^2)*sqrt(x)
```

giac [A] time = 0.15, size = 13, normalized size = 0.62

$$\frac{2}{7}\,ax^{\frac{7}{2}} + \frac{2}{5}\,bx^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)*x^(5/2),x, algorithm="giac")
[Out] 2/7*a*x^(7/2) + 2/5*b*x^(5/2)
```

maple [A] time = 0.00, size = 14, normalized size = 0.67

$$\frac{2\,(5ax+7b)\,x^{\frac{5}{2}}}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)*x^(5/2),x)
[Out] 2/35*(5*a*x+7*b)*x^(5/2)
```

maxima [A] time = 1.01, size = 15, normalized size = 0.71

$$\frac{2}{35}\left(5\,a+\frac{7\,b}{x}\right)x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)*x^(5/2),x, algorithm="maxima")
[Out] 2/35*(5*a + 7*b/x)*x^(7/2)
```

mupad [B] time = 1.07, size = 13, normalized size = 0.62

$$\frac{2\,x^{5/2}\,(7\,b+5\,a\,x)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b/x),x)
[Out] (2*x^(5/2)*(7*b + 5*a*x))/35
```

sympy [A] time = 1.77, size = 19, normalized size = 0.90

$$\frac{2ax^{\frac{7}{2}}}{7}+\frac{2bx^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)*x**(5/2),x)
[Out] 2*a*x**(7/2)/7 + 2*b*x**(5/2)/5
```

3.1648 $\int \left(a + \frac{b}{x}\right) x^{3/2}\, dx$

Optimal. Leaf size=21

$$\frac{2}{5}ax^{5/2} + \frac{2}{3}bx^{3/2}$$

```
[Out] 2/3*b*x^(3/2)+2/5*a*x^(5/2)
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$\frac{2}{5}ax^{5/2} + \frac{2}{3}bx^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)*x^(3/2),x]
```

```
[Out] (2*b*x^(3/2))/3 + (2*a*x^(5/2))/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \left(a + \frac{b}{x}\right) x^{3/2}\, dx &= \int \left(b\sqrt{x} + ax^{3/2}\right) dx \\ &= \frac{2}{3}bx^{3/2} + \frac{2}{5}ax^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 0.81

$$\frac{2}{15}x^{3/2}(3ax + 5b)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)*x^(3/2),x]
```

```
[Out] (2*x^(3/2)*(5*b + 3*a*x))/15
```

fricas [A] time = 1.23, size = 16, normalized size = 0.76

$$\frac{2}{15}\left(3\,ax^2 + 5\,bx\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)*x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*a*x^2 + 5*b*x)*sqrt(x)
```

giac [A] time = 0.16, size = 13, normalized size = 0.62

$$\frac{2}{5}\,ax^{\frac{5}{2}} + \frac{2}{3}\,bx^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)*x^(3/2),x, algorithm="giac")
[Out] 2/5*a*x^(5/2) + 2/3*b*x^(3/2)
```

maple [A] time = 0.00, size = 14, normalized size = 0.67

$$\frac{2\,(3ax+5b)\,x^{\frac{3}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)*x^(3/2),x)
[Out] 2/15*(3*a*x+5*b)*x^(3/2)
```

maxima [A] time = 1.11, size = 15, normalized size = 0.71

$$\frac{2}{15}\left(3\,a+\frac{5\,b}{x}\right)x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)*x^(3/2),x, algorithm="maxima")
[Out] 2/15*(3*a + 5*b/x)*x^(5/2)
```

mupad [B] time = 0.03, size = 13, normalized size = 0.62

$$\frac{2\,x^{3/2}\,(5\,b+3\,a\,x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b/x),x)
[Out] (2*x^(3/2)*(5*b + 3*a*x))/15
```

sympy [A] time = 0.72, size = 19, normalized size = 0.90

$$\frac{2ax^{\frac{5}{2}}}{5}+\frac{2bx^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)*x**(3/2),x)
[Out] 2*a*x**(5/2)/5 + 2*b*x**(3/2)/3
```

3.1649 $\int \left(a + \frac{b}{x}\right) \sqrt{x}\, dx$

Optimal. Leaf size=19

$$\frac{2}{3} a x^{3/2} + 2b\sqrt{x}$$

```
[Out] 2/3*a*x^(3/2)+2*b*x^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$\frac{2}{3} a x^{3/2} + 2b\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)*Sqrt[x],x]
```

```
[Out] 2*b*Sqrt[x] + (2*a*x^(3/2))/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \left(a + \frac{b}{x}\right) \sqrt{x}\, dx &= \int \left(\frac{b}{\sqrt{x}} + a\sqrt{x}\right) dx \\ &= 2b\sqrt{x} + \frac{2}{3} a x^{3/2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 0.84

$$\frac{2}{3} \sqrt{x}\, (ax + 3b)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)*Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(3*b + a*x))/3
```

fricas [A] time = 1.00, size = 12, normalized size = 0.63

$$\frac{2}{3} (ax + 3\, b)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(a*x + 3*b)*sqrt(x)
```

giac [A] time = 0.17, size = 13, normalized size = 0.68

$$\frac{2}{3} a x^{\frac{3}{2}} + 2\, b\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)*x^(1/2),x, algorithm="giac")
[Out] 2/3*a*x^(3/2) + 2*b*sqrt(x)
```

maple [A] time = 0.00, size = 13, normalized size = 0.68

$$\frac{2\,(ax+3b)\,\sqrt{x}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)*x^(1/2),x)
[Out] 2/3*(a*x+3*b)*x^(1/2)
```

maxima [A] time = 0.99, size = 13, normalized size = 0.68

$$\frac{2}{3}\left(a+\frac{3\,b}{x}\right)x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)*x^(1/2),x, algorithm="maxima")
[Out] 2/3*(a + 3*b/x)*x^(3/2)
```

mupad [B] time = 0.03, size = 12, normalized size = 0.63

$$\frac{2\,\sqrt{x}\;\;(3\,b+a\,x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b/x),x)
[Out] (2*x^(1/2)*(3*b + a*x))/3
```

sympy [A] time = 0.22, size = 17, normalized size = 0.89

$$\frac{2ax^{\frac{3}{2}}}{3}+2b\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)*x**(1/2),x)
[Out] 2*a*x**(3/2)/3 + 2*b*sqrt(x)
```

3.1650 $\int \frac{a+\frac{b}{x}}{\sqrt{x}}\,dx$

Optimal. Leaf size=17

$$2a\sqrt{x}-\frac{2b}{\sqrt{x}}$$

```
[Out] -2*b/x^(1/2)+2*a*x^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$2a\sqrt{x}-\frac{2b}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)/Sqrt[x],x]
```

```
[Out] (-2*b)/Sqrt[x] + 2*a*Sqrt[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+\frac{b}{x}}{\sqrt{x}}\,dx &= \int \left(\frac{b}{x^{3/2}}+\frac{a}{\sqrt{x}}\right)dx\\ &= -\frac{2b}{\sqrt{x}}+2a\sqrt{x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 0.82

$$\frac{2(ax-b)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)/Sqrt[x],x]
```

```
[Out] (2*(-b + a*x))/Sqrt[x]
```

fricas [A] time = 0.65, size = 12, normalized size = 0.71

$$\frac{2\,(ax-b)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(a*x - b)/sqrt(x)
```

giac [A] time = 0.15, size = 13, normalized size = 0.76

$$2\,a\sqrt{x} - \frac{2\,b}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)/x^(1/2),x, algorithm="giac")
[Out] 2*a*sqrt(x) - 2*b/sqrt(x)
```

maple [A] time = 0.00, size = 13, normalized size = 0.76

$$\frac{2ax - 2b}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)/x^(1/2),x)
[Out] 2*(a*x-b)/x^(1/2)
```

maxima [A] time = 1.07, size = 13, normalized size = 0.76

$$2\,a\sqrt{x} - \frac{2\,b}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)/x^(1/2),x, algorithm="maxima")
[Out] 2*a*sqrt(x) - 2*b/sqrt(x)
```

mupad [B] time = 0.03, size = 11, normalized size = 0.65

$$-\frac{2\,(b - a\,x)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)/x^(1/2),x)
[Out] -(2*(b - a*x))/x^(1/2)
```

sympy [A] time = 0.36, size = 15, normalized size = 0.88

$$2a\sqrt{x} - \frac{2b}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)/x**(1/2),x)
[Out] 2*a*sqrt(x) - 2*b/sqrt(x)
```

3.1651 $\int \frac{a+\frac{b}{x}}{x^{3/2}}\,dx$

Optimal. Leaf size=19

$$-\frac{2a}{\sqrt{x}}-\frac{2b}{3x^{3/2}}$$

```
[Out] -2/3*b/x^(3/2)-2*a/x^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$-\frac{2a}{\sqrt{x}}-\frac{2b}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)/x^(3/2),x]
```

```
[Out] (-2*b)/(3*x^(3/2)) - (2*a)/Sqrt[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+\frac{b}{x}}{x^{3/2}}\,dx &= \int \left(\frac{b}{x^{5/2}}+\frac{a}{x^{3/2}}\right)dx\\ &= -\frac{2b}{3x^{3/2}}-\frac{2a}{\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 0.79

$$-\frac{2(3ax+b)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)/x^(3/2),x]
```

```
[Out] (-2*(b + 3*a*x))/(3*x^(3/2))
```

fricas [A] time = 0.94, size = 11, normalized size = 0.58

$$-\frac{2\,(3\,ax+b)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)/x^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*(3*a*x + b)/x^(3/2)
```

giac [A] time = 0.15, size = 11, normalized size = 0.58

$$-\frac{2\,(3\,ax+b)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)/x^(3/2),x, algorithm="giac")
[Out] -2/3*(3*a*x + b)/x^(3/2)
```

maple [A] time = 0.00, size = 12, normalized size = 0.63

$$-\frac{2\,(3ax+b)}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)/x^(3/2),x)
[Out] -2/3*(3*a*x+b)/x^(3/2)
```

maxima [A] time = 1.07, size = 13, normalized size = 0.68

$$-\frac{2\,a}{\sqrt{x}}-\frac{2\,b}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)/x^(3/2),x, algorithm="maxima")
[Out] -2*a/sqrt(x) - 2/3*b/x^(3/2)
```

mupad [B] time = 0.03, size = 13, normalized size = 0.68

$$-\frac{2\,b+6\,a\,x}{3\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)/x^(3/2),x)
[Out] -(2*b + 6*a*x)/(3*x^(3/2))
```

sympy [A] time = 0.50, size = 19, normalized size = 1.00

$$-\frac{2a}{\sqrt{x}}-\frac{2b}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)/x**(3/2),x)
[Out] -2*a/sqrt(x) - 2*b/(3*x**(3/2))
```

3.1652 $\int \frac{a+\frac{b}{x}}{x^{5/2}}\,dx$

Optimal. Leaf size=21

$$-\frac{2a}{3x^{3/2}}-\frac{2b}{5x^{5/2}}$$

```
[Out] -2/5*b/x^(5/2)-2/3*a/x^(3/2)
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$-\frac{2a}{3x^{3/2}}-\frac{2b}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)/x^(5/2),x]
```

```
[Out] (-2*b)/(5*x^(5/2)) - (2*a)/(3*x^(3/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+\frac{b}{x}}{x^{5/2}}\,dx &= \int \left(\frac{b}{x^{7/2}}+\frac{a}{x^{5/2}}\right)dx\\ &= -\frac{2b}{5x^{5/2}}-\frac{2a}{3x^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 0.81

$$-\frac{2(5ax+3b)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)/x^(5/2),x]
```

```
[Out] (-2*(3*b + 5*a*x))/(15*x^(5/2))
```

fricas [A] time = 0.98, size = 13, normalized size = 0.62

$$-\frac{2\,(5\,ax+3\,b)}{15\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)/x^(5/2),x, algorithm="fricas")
```

```
[Out] -2/15*(5*a*x + 3*b)/x^(5/2)
```

giac [A] time = 0.15, size = 13, normalized size = 0.62

$$-\frac{2\,(5\,ax+3\,b)}{15\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)/x^(5/2),x, algorithm="giac")
[Out] -2/15*(5*a*x + 3*b)/x^(5/2)
```

maple [A] time = 0.00, size = 14, normalized size = 0.67

$$-\frac{2\,(5ax+3b)}{15x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)/x^(5/2),x)
[Out] -2/15*(5*a*x+3*b)/x^(5/2)
```

maxima [A] time = 1.02, size = 13, normalized size = 0.62

$$-\frac{2\,a}{3\,x^{\frac{3}{2}}}-\frac{2\,b}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)/x^(5/2),x, algorithm="maxima")
[Out] -2/3*a/x^(3/2) - 2/5*b/x^(5/2)
```

mupad [B] time = 0.03, size = 13, normalized size = 0.62

$$-\frac{6\,b+10\,a\,x}{15\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)/x^(5/2),x)
[Out] -(6*b + 10*a*x)/(15*x^(5/2))
```

sympy [A] time = 0.94, size = 20, normalized size = 0.95

$$-\frac{2a}{3x^{\frac{3}{2}}}-\frac{2b}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)/x**(5/2),x)
[Out] -2*a/(3*x**(3/2)) - 2*b/(5*x**(5/2))
```

3.1653 $\int \left(a+\frac{b}{x}\right)^2 x^{5/2}\,dx$

Optimal. Leaf size=36

$$\frac{2}{7}a^2x^{7/2}+\frac{4}{5}abx^{5/2}+\frac{2}{3}b^2x^{3/2}$$

```
[Out] 2/3*b^2*x^(3/2)+4/5*a*b*x^(5/2)+2/7*a^2*x^(7/2)
```

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {263, 43}

$$\frac{2}{7}a^2x^{7/2}+\frac{4}{5}abx^{5/2}+\frac{2}{3}b^2x^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^2*x^(5/2),x]
```

```
[Out] (2*b^2*x^(3/2))/3 + (4*a*b*x^(5/2))/5 + (2*a^2*x^(7/2))/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^2 x^{5/2}\,dx &= \int \sqrt{x}\,(b+ax)^2\,dx\\ &= \int \left(b^2\sqrt{x}+2abx^{3/2}+a^2x^{5/2}\right)dx\\ &= \frac{2}{3}b^2x^{3/2}+\frac{4}{5}abx^{5/2}+\frac{2}{7}a^2x^{7/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.78

$$\frac{2}{105}x^{3/2}\left(15a^2x^2+42abx+35b^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^2*x^(5/2),x]
```

```
[Out] (2*x^(3/2)*(35*b^2 + 42*a*b*x + 15*a^2*x^2))/105
```

fricas [A] time = 1.06, size = 27, normalized size = 0.75

$$\frac{2}{105}\left(15\,a^2x^3+42\,abx^2+35\,b^2x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2*x^(5/2),x, algorithm="fricas")
[Out] 2/105*(15*a^2*x^3 + 42*a*b*x^2 + 35*b^2*x)*sqrt(x)
```

giac [A] time = 0.15, size = 24, normalized size = 0.67

$$\frac{2}{7}a^2x^{\frac{7}{2}} + \frac{4}{5}abx^{\frac{5}{2}} + \frac{2}{3}b^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2*x^(5/2),x, algorithm="giac")
[Out] 2/7*a^2*x^(7/2) + 4/5*a*b*x^(5/2) + 2/3*b^2*x^(3/2)
```

maple [A] time = 0.00, size = 25, normalized size = 0.69

$$\frac{2\left(15a^2x^2 + 42abx + 35b^2\right)x^{\frac{3}{2}}}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^2*x^(5/2),x)
[Out] 2/105*(15*a^2*x^2+42*a*b*x+35*b^2)*x^(3/2)
```

maxima [A] time = 1.05, size = 26, normalized size = 0.72

$$\frac{2}{105}\left(15\,a^2 + \frac{42\,ab}{x} + \frac{35\,b^2}{x^2}\right)x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2*x^(5/2),x, algorithm="maxima")
[Out] 2/105*(15*a^2 + 42*a*b/x + 35*b^2/x^2)*x^(7/2)
```

mupad [B] time = 0.05, size = 24, normalized size = 0.67

$$\frac{2\,x^{3/2}\,\left(15\,a^2\,x^2 + 42\,a\,b\,x + 35\,b^2\right)}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b/x)^2,x)
[Out] (2*x^(3/2)*(35*b^2 + 15*a^2*x^2 + 42*a*b*x))/105
```

sympy [A] time = 2.56, size = 34, normalized size = 0.94

$$\frac{2a^2x^{\frac{7}{2}}}{7} + \frac{4abx^{\frac{5}{2}}}{5} + \frac{2b^2x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**2*x**(5/2),x)
[Out] 2*a**2*x**(7/2)/7 + 4*a*b*x**(5/2)/5 + 2*b**2*x**(3/2)/3
```

3.1654 $\int \left(a+\frac{b}{x}\right)^2 x^{3/2}\,dx$

Optimal. Leaf size=34

$$\frac{2}{5}a^2x^{5/2}+\frac{4}{3}abx^{3/2}+2b^2\sqrt{x}$$

```
[Out] 4/3*a*b*x^(3/2)+2/5*a^2*x^(5/2)+2*b^2*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {263, 43}

$$\frac{2}{5}a^2x^{5/2}+\frac{4}{3}abx^{3/2}+2b^2\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^2*x^(3/2),x]
```

```
[Out] 2*b^2*Sqrt[x] + (4*a*b*x^(3/2))/3 + (2*a^2*x^(5/2))/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^2 x^{3/2}\,dx &= \int \frac{(b+ax)^2}{\sqrt{x}}\,dx\\ &= \int \left(\frac{b^2}{\sqrt{x}}+2ab\sqrt{x}+a^2x^{3/2}\right)dx\\ &= 2b^2\sqrt{x}+\frac{4}{3}abx^{3/2}+\frac{2}{5}a^2x^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.82

$$\frac{2}{15}\sqrt{x}\left(3a^2x^2+10abx+15b^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^2*x^(3/2),x]
```

```
[Out] (2*Sqrt[x]*(15*b^2 + 10*a*b*x + 3*a^2*x^2))/15
```

fricas [A] time = 1.01, size = 24, normalized size = 0.71

$$\frac{2}{15}\left(3\,a^2x^2+10\,abx+15\,b^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2*x^(3/2),x, algorithm="fricas")
[Out] 2/15*(3*a^2*x^2 + 10*a*b*x + 15*b^2)*sqrt(x)
```

giac [A] time = 0.15, size = 24, normalized size = 0.71

$$\frac{2}{5}a^2x^{\frac{5}{2}} + \frac{4}{3}abx^{\frac{3}{2}} + 2b^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2*x^(3/2),x, algorithm="giac")
[Out] 2/5*a^2*x^(5/2) + 4/3*a*b*x^(3/2) + 2*b^2*sqrt(x)
```

maple [A] time = 0.00, size = 25, normalized size = 0.74

$$\frac{2\left(3a^2x^2 + 10abx + 15b^2\right)\sqrt{x}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^2*x^(3/2),x)
[Out] 2/15*(3*a^2*x^2+10*a*b*x+15*b^2)*x^(1/2)
```

maxima [A] time = 1.02, size = 26, normalized size = 0.76

$$\frac{2}{15}\left(3\,a^2 + \frac{10\,ab}{x} + \frac{15\,b^2}{x^2}\right)x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2*x^(3/2),x, algorithm="maxima")
[Out] 2/15*(3*a^2 + 10*a*b/x + 15*b^2/x^2)*x^(5/2)
```

mupad [B] time = 0.04, size = 24, normalized size = 0.71

$$\frac{2\sqrt{x}\,\left(3\,a^2\,x^2 + 10\,a\,b\,x + 15\,b^2\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b/x)^2,x)
[Out] (2*x^(1/2)*(15*b^2 + 3*a^2*x^2 + 10*a*b*x))/15
```

sympy [A] time = 1.16, size = 32, normalized size = 0.94

$$\frac{2a^2x^{\frac{5}{2}}}{5} + \frac{4abx^{\frac{3}{2}}}{3} + 2b^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**2*x**(3/2),x)
[Out] 2*a**2*x**(5/2)/5 + 4*a*b*x**(3/2)/3 + 2*b**2*sqrt(x)
```

3.1655 $\int \left(a+\frac{b}{x}\right)^2 \sqrt{x}\, dx$

Optimal. Leaf size=32

$$\frac{2}{3}a^2x^{3/2}+4ab\sqrt{x}-\frac{2b^2}{\sqrt{x}}$$

```
[Out] 2/3*a^2*x^(3/2)-2*b^2/x^(1/2)+4*a*b*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {263, 43}

$$\frac{2}{3}a^2x^{3/2}+4ab\sqrt{x}-\frac{2b^2}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^2*Sqrt[x],x]
```

```
[Out] (-2*b^2)/Sqrt[x] + 4*a*b*Sqrt[x] + (2*a^2*x^(3/2))/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^2 \sqrt{x}\, dx &= \int \frac{(b+ax)^2}{x^{3/2}}\, dx \\ &= \int \left(\frac{b^2}{x^{3/2}}+\frac{2ab}{\sqrt{x}}+a^2\sqrt{x}\right) dx \\ &= -\frac{2b^2}{\sqrt{x}}+4ab\sqrt{x}+\frac{2}{3}a^2x^{3/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.84

$$\frac{2\left(a^2x^2+6abx-3b^2\right)}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^2*Sqrt[x],x]
```

```
[Out] (2*(-3*b^2 + 6*a*b*x + a^2*x^2))/(3*Sqrt[x])
```

fricas [A] time = 1.08, size = 23, normalized size = 0.72

$$\frac{2\left(a^2x^2+6\,abx-3\,b^2\right)}{3\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2*x^(1/2),x, algorithm="fricas")
[Out] 2/3*(a^2*x^2 + 6*a*b*x - 3*b^2)/sqrt(x)
```

giac [A] time = 0.15, size = 24, normalized size = 0.75

$$\frac{2}{3}\,a^2 x^{\frac{3}{2}} + 4\,ab\sqrt{x} - \frac{2\,b^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2*x^(1/2),x, algorithm="giac")
[Out] 2/3*a^2*x^(3/2) + 4*a*b*sqrt(x) - 2*b^2/sqrt(x)
```

maple [A] time = 0.00, size = 24, normalized size = 0.75

$$\frac{\frac{2}{3}a^2x^2 + 4abx - 2b^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^2*x^(1/2),x)
[Out] 2/3*(a^2*x^2+6*a*b*x-3*b^2)/x^(1/2)
```

maxima [A] time = 0.98, size = 25, normalized size = 0.78

$$\frac{2}{3}\left(a^2 + \frac{6\,ab}{x}\right)x^{\frac{3}{2}} - \frac{2\,b^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2*x^(1/2),x, algorithm="maxima")
[Out] 2/3*(a^2 + 6*a*b/x)*x^(3/2) - 2*b^2/sqrt(x)
```

mupad [B] time = 0.04, size = 24, normalized size = 0.75

$$\frac{2\,a^2\,x^2 + 12\,a\,b\,x - 6\,b^2}{3\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b/x)^2,x)
[Out] (2*a^2*x^2 - 6*b^2 + 12*a*b*x)/(3*x^(1/2))
```

sympy [A] time = 0.50, size = 31, normalized size = 0.97

$$\frac{2a^2x^{\frac{3}{2}}}{3} + 4ab\sqrt{x} - \frac{2b^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**2*x**(1/2),x)
[Out] 2*a**2*x**(3/2)/3 + 4*a*b*sqrt(x) - 2*b**2/sqrt(x)
```

3.1656 $\int \frac{\left(a+\frac{b}{x}\right)^2}{\sqrt{x}}\,dx$

Optimal. Leaf size=32

$$2a^2\sqrt{x}-\frac{4ab}{\sqrt{x}}-\frac{2b^2}{3x^{3/2}}$$

```
[Out] -2/3*b^2/x^(3/2)-4*a*b/x^(1/2)+2*a^2*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {263, 43}

$$2a^2\sqrt{x}-\frac{4ab}{\sqrt{x}}-\frac{2b^2}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^2/Sqrt[x],x]
```

```
[Out] (-2*b^2)/(3*x^(3/2)) - (4*a*b)/Sqrt[x] + 2*a^2*Sqrt[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^2}{\sqrt{x}}\,dx &= \int \frac{(b+ax)^2}{x^{5/2}}\,dx\\ &= \int \left(\frac{b^2}{x^{5/2}}+\frac{2ab}{x^{3/2}}+\frac{a^2}{\sqrt{x}}\right)dx\\ &= -\frac{2b^2}{3x^{3/2}}-\frac{4ab}{\sqrt{x}}+2a^2\sqrt{x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.81

$$-\frac{2\left(-3a^2x^2+6abx+b^2\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^2/Sqrt[x],x]
```

```
[Out] (-2*(b^2 + 6*a*b*x - 3*a^2*x^2))/(3*x^(3/2))
```

fricas [A] time = 1.02, size = 24, normalized size = 0.75

$$\frac{2\left(3\,a^2x^2-6\,abx-b^2\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2/x^(1/2),x, algorithm="fricas")
[Out] 2/3*(3*a^2*x^2 - 6*a*b*x - b^2)/x^(3/2)
```

giac [A] time = 0.15, size = 23, normalized size = 0.72

$$2\,a^2\sqrt{x}-\frac{2\left(6\,abx+b^2\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2/x^(1/2),x, algorithm="giac")
[Out] 2*a^2*sqrt(x) - 2/3*(6*a*b*x + b^2)/x^(3/2)
```

maple [A] time = 0.00, size = 25, normalized size = 0.78

$$\frac{2a^2x^2-4abx-\frac{2}{3}b^2}{x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^2/x^(1/2),x)
[Out] 2/3*(3*a^2*x^2-6*a*b*x-b^2)/x^(3/2)
```

maxima [A] time = 1.04, size = 24, normalized size = 0.75

$$2\,a^2\sqrt{x}-\frac{4\,ab}{\sqrt{x}}-\frac{2\,b^2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2/x^(1/2),x, algorithm="maxima")
[Out] 2*a^2*sqrt(x) - 4*a*b/sqrt(x) - 2/3*b^2/x^(3/2)
```

mupad [B] time = 1.08, size = 24, normalized size = 0.75

$$-\frac{-6\,a^2\,x^2+12\,a\,b\,x+2\,b^2}{3\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^2/x^(1/2),x)
[Out] -(2*b^2 - 6*a^2*x^2 + 12*a*b*x)/(3*x^(3/2))
```

sympy [A] time = 0.54, size = 31, normalized size = 0.97

$$2a^2\sqrt{x}-\frac{4ab}{\sqrt{x}}-\frac{2b^2}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**2/x**(1/2),x)
[Out] 2*a**2*sqrt(x) - 4*a*b/sqrt(x) - 2*b**2/(3*x**(3/2))
```

3.1657 $\int \frac{\left(a+\frac{b}{x}\right)^2}{x^{3/2}}\,dx$

Optimal. Leaf size=34

$$-\frac{2a^2}{\sqrt{x}}-\frac{4ab}{3x^{3/2}}-\frac{2b^2}{5x^{5/2}}$$

```
[Out] -2/5*b^2/x^(5/2)-4/3*a*b/x^(3/2)-2*a^2/x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {263, 43}

$$-\frac{2a^2}{\sqrt{x}}-\frac{4ab}{3x^{3/2}}-\frac{2b^2}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^2/x^(3/2),x]
```

```
[Out] (-2*b^2)/(5*x^(5/2)) - (4*a*b)/(3*x^(3/2)) - (2*a^2)/Sqrt[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^2}{x^{3/2}}\,dx &= \int \frac{(b+ax)^2}{x^{7/2}}\,dx\\ &= \int \left(\frac{b^2}{x^{7/2}}+\frac{2ab}{x^{5/2}}+\frac{a^2}{x^{3/2}}\right)dx\\ &= -\frac{2b^2}{5x^{5/2}}-\frac{4ab}{3x^{3/2}}-\frac{2a^2}{\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.82

$$-\frac{2\left(15a^2x^2+10abx+3b^2\right)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^2/x^(3/2),x]
```

```
[Out] (-2*(3*b^2 + 10*a*b*x + 15*a^2*x^2))/(15*x^(5/2))
```

fricas [A] time = 0.90, size = 24, normalized size = 0.71

$$-\frac{2\left(15\,a^2x^2+10\,abx+3\,b^2\right)}{15\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2/x^(3/2),x, algorithm="fricas")
[Out] -2/15*(15*a^2*x^2 + 10*a*b*x + 3*b^2)/x^(5/2)
```

giac [A] time = 0.15, size = 24, normalized size = 0.71

$$-\frac{2\left(15\,a^2x^2+10\,abx+3\,b^2\right)}{15\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2/x^(3/2),x, algorithm="giac")
[Out] -2/15*(15*a^2*x^2 + 10*a*b*x + 3*b^2)/x^(5/2)
```

maple [A] time = 0.00, size = 25, normalized size = 0.74

$$-\frac{2\left(15a^2x^2+10abx+3b^2\right)}{15x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^2/x^(3/2),x)
[Out] -2/15*(15*a^2*x^2+10*a*b*x+3*b^2)/x^(5/2)
```

maxima [A] time = 1.07, size = 24, normalized size = 0.71

$$-\frac{2\,a^2}{\sqrt{x}}-\frac{4\,ab}{3\,x^{\frac{3}{2}}}-\frac{2\,b^2}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2/x^(3/2),x, algorithm="maxima")
[Out] -2*a^2/sqrt(x) - 4/3*a*b/x^(3/2) - 2/5*b^2/x^(5/2)
```

mupad [B] time = 0.03, size = 24, normalized size = 0.71

$$-\frac{30\,a^2\,x^2+20\,a\,b\,x+6\,b^2}{15\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^2/x^(3/2),x)
[Out] -(6*b^2 + 30*a^2*x^2 + 20*a*b*x)/(15*x^(5/2))
```

sympy [A] time = 0.85, size = 34, normalized size = 1.00

$$-\frac{2a^2}{\sqrt{x}}-\frac{4ab}{3x^{\frac{3}{2}}}-\frac{2b^2}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**2/x**(3/2),x)
[Out] -2*a**2/sqrt(x) - 4*a*b/(3*x**(3/2)) - 2*b**2/(5*x**(5/2))
```

3.1658 $\int \frac{\left(a+\frac{b}{x}\right)^2}{x^{5/2}}\,dx$

Optimal. Leaf size=36

$$-\frac{2a^2}{3x^{3/2}}-\frac{4ab}{5x^{5/2}}-\frac{2b^2}{7x^{7/2}}$$

```
[Out] -2/7*b^2/x^(7/2)-4/5*a*b/x^(5/2)-2/3*a^2/x^(3/2)
```

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {263, 43}

$$-\frac{2a^2}{3x^{3/2}}-\frac{4ab}{5x^{5/2}}-\frac{2b^2}{7x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^2/x^(5/2),x]
```

```
[Out] (-2*b^2)/(7*x^(7/2)) - (4*a*b)/(5*x^(5/2)) - (2*a^2)/(3*x^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^2}{x^{5/2}}\,dx &= \int \frac{(b+ax)^2}{x^{9/2}}\,dx\\ &= \int \left(\frac{b^2}{x^{9/2}}+\frac{2ab}{x^{7/2}}+\frac{a^2}{x^{5/2}}\right)dx\\ &= -\frac{2b^2}{7x^{7/2}}-\frac{4ab}{5x^{5/2}}-\frac{2a^2}{3x^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.78

$$-\frac{2\left(35a^2x^2+42abx+15b^2\right)}{105x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^2/x^(5/2),x]
```

```
[Out] (-2*(15*b^2 + 42*a*b*x + 35*a^2*x^2))/(105*x^(7/2))
```

fricas [A] time = 1.05, size = 24, normalized size = 0.67

$$-\frac{2\left(35\,a^2x^2+42\,abx+15\,b^2\right)}{105\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2/x^(5/2),x, algorithm="fricas")
[Out] -2/105*(35*a^2*x^2 + 42*a*b*x + 15*b^2)/x^(7/2)
```

giac [A] time = 0.15, size = 24, normalized size = 0.67

$$-\frac{2\left(35\,a^2x^2+42\,abx+15\,b^2\right)}{105\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2/x^(5/2),x, algorithm="giac")
[Out] -2/105*(35*a^2*x^2 + 42*a*b*x + 15*b^2)/x^(7/2)
```

maple [A] time = 0.01, size = 25, normalized size = 0.69

$$-\frac{2\left(35a^2x^2+42abx+15b^2\right)}{105x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^2/x^(5/2),x)
[Out] -2/105*(35*a^2*x^2+42*a*b*x+15*b^2)/x^(7/2)
```

maxima [A] time = 1.17, size = 24, normalized size = 0.67

$$-\frac{2\,a^2}{3\,x^{\frac{3}{2}}}-\frac{4\,ab}{5\,x^{\frac{5}{2}}}-\frac{2\,b^2}{7\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2/x^(5/2),x, algorithm="maxima")
[Out] -2/3*a^2/x^(3/2) - 4/5*a*b/x^(5/2) - 2/7*b^2/x^(7/2)
```

mupad [B] time = 0.03, size = 24, normalized size = 0.67

$$-\frac{70\,a^2\,x^2+84\,a\,b\,x+30\,b^2}{105\,x^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^2/x^(5/2),x)
[Out] -(30*b^2 + 70*a^2*x^2 + 84*a*b*x)/(105*x^(7/2))
```

sympy [A] time = 1.48, size = 36, normalized size = 1.00

$$-\frac{2a^2}{3x^{\frac{3}{2}}}-\frac{4ab}{5x^{\frac{5}{2}}}-\frac{2b^2}{7x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**2/x**(5/2),x)
[Out] -2*a**2/(3*x**(3/2)) - 4*a*b/(5*x**(5/2)) - 2*b**2/(7*x**(7/2))
```

3.1659 $\int \left(a+\frac{b}{x}\right)^3 x^{5/2}\,dx$

Optimal. Leaf size=47

$$\frac{2}{7}a^3x^{7/2}+\frac{6}{5}a^2bx^{5/2}+2ab^2x^{3/2}+2b^3\sqrt{x}$$

```
[Out] 2*a*b^2*x^(3/2)+6/5*a^2*b*x^(5/2)+2/7*a^3*x^(7/2)+2*b^3*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {263, 43}

$$\frac{6}{5}a^2bx^{5/2}+\frac{2}{7}a^3x^{7/2}+2ab^2x^{3/2}+2b^3\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^3*x^(5/2),x]
```

```
[Out] 2*b^3*Sqrt[x] + 2*a*b^2*x^(3/2) + (6*a^2*b*x^(5/2))/5 + (2*a^3*x^(7/2))/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^3 x^{5/2}\,dx &= \int \frac{(b+ax)^3}{\sqrt{x}}\,dx\\ &= \int \left(\frac{b^3}{\sqrt{x}}+3ab^2\sqrt{x}+3a^2bx^{3/2}+a^3x^{5/2}\right)dx\\ &= 2b^3\sqrt{x}+2ab^2x^{3/2}+\frac{6}{5}a^2bx^{5/2}+\frac{2}{7}a^3x^{7/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.83

$$\frac{2}{35}\sqrt{x}\left(5a^3x^3+21a^2bx^2+35ab^2x+35b^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^3*x^(5/2),x]
```

```
[Out] (2*Sqrt[x]*(35*b^3 + 35*a*b^2*x + 21*a^2*b*x^2 + 5*a^3*x^3))/35
```

fricas [A] time = 0.91, size = 35, normalized size = 0.74

$$\frac{2}{35}\left(5\,a^3x^3+21\,a^2bx^2+35\,ab^2x+35\,b^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3*x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*a^3*x^3 + 21*a^2*b*x^2 + 35*a*b^2*x + 35*b^3)*sqrt(x)
```

giac [A] time = 0.15, size = 35, normalized size = 0.74

$$\frac{2}{7}a^3x^{\frac{7}{2}} + \frac{6}{5}a^2bx^{\frac{5}{2}} + 2\,ab^2x^{\frac{3}{2}} + 2\,b^3\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3*x^(5/2),x, algorithm="giac")
```

```
[Out] 2/7*a^3*x^(7/2) + 6/5*a^2*b*x^(5/2) + 2*a*b^2*x^(3/2) + 2*b^3*sqrt(x)
```

maple [A] time = 0.00, size = 36, normalized size = 0.77

$$\frac{2\left(5a^3x^3 + 21a^2b\,x^2 + 35a\,b^2x + 35b^3\right)\sqrt{x}}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^3*x^(5/2),x)
```

```
[Out] 2/35*(5*a^3*x^3+21*a^2*b*x^2+35*a*b^2*x+35*b^3)*x^(1/2)
```

maxima [A] time = 1.06, size = 37, normalized size = 0.79

$$\frac{2}{35}\left(5\,a^3 + \frac{21\,a^2b}{x} + \frac{35\,ab^2}{x^2} + \frac{35\,b^3}{x^3}\right)x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3*x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/35*(5*a^3 + 21*a^2*b/x + 35*a*b^2/x^2 + 35*b^3/x^3)*x^(7/2)
```

mupad [B] time = 0.05, size = 35, normalized size = 0.74

$$\frac{2\,a^3\,x^{7/2}}{7} + 2\,b^3\,\sqrt{x} + 2\,a\,b^2\,x^{3/2} + \frac{6\,a^2\,b\,x^{5/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b/x)^3,x)
```

```
[Out] (2*a^3*x^(7/2))/7 + 2*b^3*x^(1/2) + 2*a*b^2*x^(3/2) + (6*a^2*b*x^(5/2))/5
```

sympy [A] time = 3.88, size = 46, normalized size = 0.98

$$\frac{2a^3x^{\frac{7}{2}}}{7} + \frac{6a^2bx^{\frac{5}{2}}}{5} + 2ab^2x^{\frac{3}{2}} + 2b^3\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**3*x**(5/2),x)
```

```
[Out] 2*a**3*x**(7/2)/7 + 6*a**2*b*x**(5/2)/5 + 2*a*b**2*x**(3/2) + 2*b**3*sqrt(x
)
```

3.1660 $\int \left(a+\frac{b}{x}\right)^3 x^{3/2}\,dx$

Optimal. Leaf size=45

$$\frac{2}{5}a^3x^{5/2}+2a^2bx^{3/2}+6ab^2\sqrt{x}-\frac{2b^3}{\sqrt{x}}$$

```
[Out] 2*a^2*b*x^(3/2)+2/5*a^3*x^(5/2)-2*b^3/x^(1/2)+6*a*b^2*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {263, 43}

$$2a^2bx^{3/2}+\frac{2}{5}a^3x^{5/2}+6ab^2\sqrt{x}-\frac{2b^3}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^3*x^(3/2),x]
```

```
[Out] (-2*b^3)/Sqrt[x] + 6*a*b^2*Sqrt[x] + 2*a^2*b*x^(3/2) + (2*a^3*x^(5/2))/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^3 x^{3/2}\,dx &= \int \frac{(b+ax)^3}{x^{3/2}}\,dx\\ &= \int \left(\frac{b^3}{x^{3/2}}+\frac{3ab^2}{\sqrt{x}}+3a^2b\sqrt{x}+a^3x^{3/2}\right)dx\\ &= -\frac{2b^3}{\sqrt{x}}+6ab^2\sqrt{x}+2a^2bx^{3/2}+\frac{2}{5}a^3x^{5/2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 0.84

$$\frac{2\left(a^3x^3+5a^2bx^2+15ab^2x-5b^3\right)}{5\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^3*x^(3/2),x]
```

```
[Out] (2*(-5*b^3 + 15*a*b^2*x + 5*a^2*b*x^2 + a^3*x^3))/(5*Sqrt[x])
```

fricas [A] time = 0.79, size = 34, normalized size = 0.76

$$\frac{2\left(a^3x^3+5\,a^2bx^2+15\,ab^2x-5\,b^3\right)}{5\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3*x^(3/2),x, algorithm="fricas")
[Out] 2/5*(a^3*x^3 + 5*a^2*b*x^2 + 15*a*b^2*x - 5*b^3)/sqrt(x)
```

giac [A] time = 0.15, size = 35, normalized size = 0.78

$$\frac{2}{5}\,a^3x^{\frac{5}{2}}+2\,a^2bx^{\frac{3}{2}}+6\,ab^2\sqrt{x}-\frac{2\,b^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3*x^(3/2),x, algorithm="giac")
[Out] 2/5*a^3*x^(5/2) + 2*a^2*b*x^(3/2) + 6*a*b^2*sqrt(x) - 2*b^3/sqrt(x)
```

maple [A] time = 0.01, size = 35, normalized size = 0.78

$$\frac{\frac{2}{5}a^3x^3+2a^2b\,x^2+6a\,b^2x-2b^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^3*x^(3/2),x)
[Out] 2/5*(a^3*x^3+5*a^2*b*x^2+15*a*b^2*x-5*b^3)/x^(1/2)
```

maxima [A] time = 1.02, size = 36, normalized size = 0.80

$$\frac{2}{5}\left(a^3+\frac{5\,a^2b}{x}+\frac{15\,ab^2}{x^2}\right)x^{\frac{5}{2}}-\frac{2\,b^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3*x^(3/2),x, algorithm="maxima")
[Out] 2/5*(a^3 + 5*a^2*b/x + 15*a*b^2/x^2)*x^(5/2) - 2*b^3/sqrt(x)
```

mupad [B] time = 0.05, size = 35, normalized size = 0.78

$$\frac{2\,a^3\,x^{5/2}}{5}-\frac{2\,b^3}{\sqrt{x}}+6\,a\,b^2\,\sqrt{x}+2\,a^2\,b\,x^{3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b/x)^3,x)
[Out] (2*a^3*x^(5/2))/5 - (2*b^3)/x^(1/2) + 6*a*b^2*x^(1/2) + 2*a^2*b*x^(3/2)
```

sympy [A] time = 1.81, size = 44, normalized size = 0.98

$$\frac{2a^3x^{\frac{5}{2}}}{5}+2a^2bx^{\frac{3}{2}}+6ab^2\sqrt{x}-\frac{2b^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**3*x**(3/2),x)
[Out] 2*a**3*x**(5/2)/5 + 2*a**2*b*x**(3/2) + 6*a*b**2*sqrt(x) - 2*b**3/sqrt(x)
```

3.1661 $\int \left(a+\frac{b}{x}\right)^3 \sqrt{x}\, dx$

Optimal. Leaf size=47

$$\frac{2}{3}a^3x^{3/2}+6a^2b\sqrt{x}-\frac{6ab^2}{\sqrt{x}}-\frac{2b^3}{3x^{3/2}}$$

```
[Out] -2/3*b^3/x^(3/2)+2/3*a^3*x^(3/2)-6*a*b^2/x^(1/2)+6*a^2*b*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {263, 43}

$$6a^2b\sqrt{x}+\frac{2}{3}a^3x^{3/2}-\frac{6ab^2}{\sqrt{x}}-\frac{2b^3}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^3*Sqrt[x],x]
```

```
[Out] (-2*b^3)/(3*x^(3/2)) - (6*a*b^2)/Sqrt[x] + 6*a^2*b*Sqrt[x] + (2*a^3*x^(3/2)
)/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^3 \sqrt{x}\, dx &= \int \frac{(b+ax)^3}{x^{5/2}}\, dx \\ &= \int \left(\frac{b^3}{x^{5/2}}+\frac{3ab^2}{x^{3/2}}+\frac{3a^2b}{\sqrt{x}}+a^3\sqrt{x}\right) dx \\ &= -\frac{2b^3}{3x^{3/2}}-\frac{6ab^2}{\sqrt{x}}+6a^2b\sqrt{x}+\frac{2}{3}a^3x^{3/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 0.81

$$\frac{2\left(a^3x^3+9a^2bx^2-9ab^2x-b^3\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^3*Sqrt[x],x]
```

```
[Out] (2*(-b^3 - 9*a*b^2*x + 9*a^2*b*x^2 + a^3*x^3))/(3*x^(3/2))
```

fricas [A] time = 0.86, size = 34, normalized size = 0.72

$$\frac{2\left(a^3x^3+9\,a^2bx^2-9\,ab^2x-b^3\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(a^3*x^3 + 9*a^2*b*x^2 - 9*a*b^2*x - b^3)/x^(3/2)
```

giac [A] time = 0.18, size = 34, normalized size = 0.72

$$\frac{2}{3}\,a^3x^{\frac{3}{2}}+6\,a^2b\sqrt{x}-\frac{2\left(9\,ab^2x+b^3\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*a^3*x^(3/2) + 6*a^2*b*sqrt(x) - 2/3*(9*a*b^2*x + b^3)/x^(3/2)
```

maple [A] time = 0.00, size = 35, normalized size = 0.74

$$\frac{\frac{2}{3}a^3x^3+6a^2b\,x^2-6a\,b^2x-\frac{2}{3}b^3}{x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^3*x^(1/2),x)
```

```
[Out] 2/3*(a^3*x^3+9*a^2*b*x^2-9*a*b^2*x-b^3)/x^(3/2)
```

maxima [A] time = 0.86, size = 36, normalized size = 0.77

$$-\frac{6\,ab^2}{\sqrt{x}}+\frac{2}{3}\left(a^3+\frac{9\,a^2b}{x}\right)x^{\frac{3}{2}}-\frac{2\,b^3}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3*x^(1/2),x, algorithm="maxima")
```

```
[Out] -6*a*b^2/sqrt(x) + 2/3*(a^3 + 9*a^2*b/x)*x^(3/2) - 2/3*b^3/x^(3/2)
```

mupad [B] time = 1.08, size = 35, normalized size = 0.74

$$-\frac{-2\,a^3\,x^3-18\,a^2\,b\,x^2+18\,a\,b^2\,x+2\,b^3}{3\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b/x)^3,x)
```

```
[Out] -(2*b^3 - 2*a^3*x^3 - 18*a^2*b*x^2 + 18*a*b^2*x)/(3*x^(3/2))
```

sympy [A] time = 0.80, size = 46, normalized size = 0.98

$$\frac{2a^3x^{\frac{3}{2}}}{3}+6a^2b\sqrt{x}-\frac{6ab^2}{\sqrt{x}}-\frac{2b^3}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**3*x**(1/2),x)
```

```
[Out] 2*a**3*x**(3/2)/3 + 6*a**2*b*sqrt(x) - 6*a*b**2/sqrt(x) - 2*b**3/(3*x**(3/2
))
```

3.1662 $\int \frac{\left(a+\frac{b}{x}\right)^3}{\sqrt{x}}\,dx$

Optimal. Leaf size=45

$$2a^3\sqrt{x}-\frac{6a^2b}{\sqrt{x}}-\frac{2ab^2}{x^{3/2}}-\frac{2b^3}{5x^{5/2}}$$

```
[Out] -2/5*b^3/x^(5/2)-2*a*b^2/x^(3/2)-6*a^2*b/x^(1/2)+2*a^3*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {263, 43}

$$-\frac{6a^2b}{\sqrt{x}}+2a^3\sqrt{x}-\frac{2ab^2}{x^{3/2}}-\frac{2b^3}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^3/Sqrt[x],x]
```

```
[Out] (-2*b^3)/(5*x^(5/2)) - (2*a*b^2)/x^(3/2) - (6*a^2*b)/Sqrt[x] + 2*a^3*Sqrt[x
]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^3}{\sqrt{x}}\,dx &= \int \frac{(b+ax)^3}{x^{7/2}}\,dx\\ &= \int \left(\frac{b^3}{x^{7/2}}+\frac{3ab^2}{x^{5/2}}+\frac{3a^2b}{x^{3/2}}+\frac{a^3}{\sqrt{x}}\right)dx\\ &= -\frac{2b^3}{5x^{5/2}}-\frac{2ab^2}{x^{3/2}}-\frac{6a^2b}{\sqrt{x}}+2a^3\sqrt{x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.87

$$\frac{2\left(5a^3x^3-15a^2bx^2-5ab^2x-b^3\right)}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^3/Sqrt[x],x]
```

```
[Out] (2*(-b^3 - 5*a*b^2*x - 15*a^2*b*x^2 + 5*a^3*x^3))/(5*x^(5/2))
```

fricas [A] time = 1.03, size = 35, normalized size = 0.78

$$\frac{2\left(5\,a^3x^3-15\,a^2bx^2-5\,ab^2x-b^3\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3/x^(1/2),x, algorithm="fricas")
[Out] 2/5*(5*a^3*x^3 - 15*a^2*b*x^2 - 5*a*b^2*x - b^3)/x^(5/2)
```

giac [A] time = 0.16, size = 34, normalized size = 0.76

$$2\,a^3\sqrt{x}-\frac{2\left(15\,a^2bx^2+5\,ab^2x+b^3\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3/x^(1/2),x, algorithm="giac")
[Out] 2*a^3*sqrt(x) - 2/5*(15*a^2*b*x^2 + 5*a*b^2*x + b^3)/x^(5/2)
```

maple [A] time = 0.01, size = 36, normalized size = 0.80

$$\frac{2a^3x^3-6a^2b\,x^2-2a\,b^2x-\frac{2}{5}b^3}{x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^3/x^(1/2),x)
[Out] 2/5*(5*a^3*x^3-15*a^2*b*x^2-5*a*b^2*x-b^3)/x^(5/2)
```

maxima [A] time = 1.05, size = 35, normalized size = 0.78

$$2\,a^3\sqrt{x}-\frac{6\,a^2b}{\sqrt{x}}-\frac{2\,ab^2}{x^{\frac{3}{2}}}-\frac{2\,b^3}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3/x^(1/2),x, algorithm="maxima")
[Out] 2*a^3*sqrt(x) - 6*a^2*b/sqrt(x) - 2*a*b^2/x^(3/2) - 2/5*b^3/x^(5/2)
```

mupad [B] time = 1.07, size = 35, normalized size = 0.78

$$-\frac{-10\,a^3\,x^3+30\,a^2\,b\,x^2+10\,a\,b^2\,x+2\,b^3}{5\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^3/x^(1/2),x)
[Out] -(2*b^3 - 10*a^3*x^3 + 30*a^2*b*x^2 + 10*a*b^2*x)/(5*x^(5/2))
```

sympy [A] time = 0.85, size = 44, normalized size = 0.98

$$2a^3\sqrt{x}-\frac{6a^2b}{\sqrt{x}}-\frac{2ab^2}{x^{\frac{3}{2}}}-\frac{2b^3}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**3/x**(1/2),x)
[Out] 2*a**3*sqrt(x) - 6*a**2*b/sqrt(x) - 2*a*b**2/x**(3/2) - 2*b**3/(5*x**(5/2))
```

3.1663 $\int \frac{\left(a+\frac{b}{x}\right)^3}{x^{3/2}}\,dx$

Optimal. Leaf size=47

$$-\frac{2a^3}{\sqrt{x}}-\frac{2a^2b}{x^{3/2}}-\frac{6ab^2}{5x^{5/2}}-\frac{2b^3}{7x^{7/2}}$$

```
[Out] -2/7*b^3/x^(7/2)-6/5*a*b^2/x^(5/2)-2*a^2*b/x^(3/2)-2*a^3/x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {263, 43}

$$-\frac{2a^2b}{x^{3/2}}-\frac{2a^3}{\sqrt{x}}-\frac{6ab^2}{5x^{5/2}}-\frac{2b^3}{7x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^3/x^(3/2),x]
```

```
[Out] (-2*b^3)/(7*x^(7/2)) - (6*a*b^2)/(5*x^(5/2)) - (2*a^2*b)/x^(3/2) - (2*a^3)/
Sqrt[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^3}{x^{3/2}}\,dx &= \int \frac{(b+ax)^3}{x^{9/2}}\,dx\\ &= \int \left(\frac{b^3}{x^{9/2}}+\frac{3ab^2}{x^{7/2}}+\frac{3a^2b}{x^{5/2}}+\frac{a^3}{x^{3/2}}\right)dx\\ &= -\frac{2b^3}{7x^{7/2}}-\frac{6ab^2}{5x^{5/2}}-\frac{2a^2b}{x^{3/2}}-\frac{2a^3}{\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.83

$$-\frac{2\left(35a^3x^3+35a^2bx^2+21ab^2x+5b^3\right)}{35x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^3/x^(3/2),x]
```

```
[Out] (-2*(5*b^3 + 21*a*b^2*x + 35*a^2*b*x^2 + 35*a^3*x^3))/(35*x^(7/2))
```

fricas [A] time = 0.77, size = 35, normalized size = 0.74

$$-\frac{2\left(35\,a^3x^3+35\,a^2bx^2+21\,ab^2x+5\,b^3\right)}{35\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3/x^(3/2),x, algorithm="fricas")
[Out] -2/35*(35*a^3*x^3 + 35*a^2*b*x^2 + 21*a*b^2*x + 5*b^3)/x^(7/2)
```

giac [A] time = 0.17, size = 35, normalized size = 0.74

$$-\frac{2\left(35\,a^3x^3+35\,a^2bx^2+21\,ab^2x+5\,b^3\right)}{35\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3/x^(3/2),x, algorithm="giac")
[Out] -2/35*(35*a^3*x^3 + 35*a^2*b*x^2 + 21*a*b^2*x + 5*b^3)/x^(7/2)
```

maple [A] time = 0.00, size = 36, normalized size = 0.77

$$-\frac{2\left(35a^3x^3+35a^2b\,x^2+21a\,b^2x+5b^3\right)}{35x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^3/x^(3/2),x)
[Out] -2/35*(35*a^3*x^3+35*a^2*b*x^2+21*a*b^2*x+5*b^3)/x^(7/2)
```

maxima [A] time = 1.07, size = 35, normalized size = 0.74

$$-\frac{2\,a^3}{\sqrt{x}}-\frac{2\,a^2b}{x^{\frac{3}{2}}}-\frac{6\,ab^2}{5\,x^{\frac{5}{2}}}-\frac{2\,b^3}{7\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3/x^(3/2),x, algorithm="maxima")
[Out] -2*a^3/sqrt(x) - 2*a^2*b/x^(3/2) - 6/5*a*b^2/x^(5/2) - 2/7*b^3/x^(7/2)
```

mupad [B] time = 0.03, size = 35, normalized size = 0.74

$$-\frac{70\,a^3\,x^3+70\,a^2\,b\,x^2+42\,a\,b^2\,x+10\,b^3}{35\,x^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^3/x^(3/2),x)
[Out] -(10*b^3 + 70*a^3*x^3 + 70*a^2*b*x^2 + 42*a*b^2*x)/(35*x^(7/2))
```

sympy [A] time = 1.22, size = 48, normalized size = 1.02

$$-\frac{2a^3}{\sqrt{x}}-\frac{2a^2b}{x^{\frac{3}{2}}}-\frac{6ab^2}{5x^{\frac{5}{2}}}-\frac{2b^3}{7x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**3/x**(3/2),x)
[Out] -2*a**3/sqrt(x) - 2*a**2*b/x**(3/2) - 6*a*b**2/(5*x**(5/2)) - 2*b**3/(7*x**
(7/2))
```

3.1664 $\int \frac{\left(a+\frac{b}{x}\right)^3}{x^{5/2}}\,dx$

Optimal. Leaf size=51

$$-\frac{2a^3}{3x^{3/2}}-\frac{6a^2b}{5x^{5/2}}-\frac{6ab^2}{7x^{7/2}}-\frac{2b^3}{9x^{9/2}}$$

```
[Out] -2/9*b^3/x^(9/2)-6/7*a*b^2/x^(7/2)-6/5*a^2*b/x^(5/2)-2/3*a^3/x^(3/2)
```

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {263, 43}

$$-\frac{6a^2b}{5x^{5/2}}-\frac{2a^3}{3x^{3/2}}-\frac{6ab^2}{7x^{7/2}}-\frac{2b^3}{9x^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^3/x^(5/2),x]
```

```
[Out] (-2*b^3)/(9*x^(9/2)) - (6*a*b^2)/(7*x^(7/2)) - (6*a^2*b)/(5*x^(5/2)) - (2*a
^3)/(3*x^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^3}{x^{5/2}}\,dx &= \int \frac{(b+ax)^3}{x^{11/2}}\,dx\\ &= \int \left(\frac{b^3}{x^{11/2}}+\frac{3ab^2}{x^{9/2}}+\frac{3a^2b}{x^{7/2}}+\frac{a^3}{x^{5/2}}\right)dx\\ &= -\frac{2b^3}{9x^{9/2}}-\frac{6ab^2}{7x^{7/2}}-\frac{6a^2b}{5x^{5/2}}-\frac{2a^3}{3x^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.76

$$-\frac{2\left(105a^3x^3+189a^2bx^2+135ab^2x+35b^3\right)}{315x^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^3/x^(5/2),x]
```

```
[Out] (-2*(35*b^3 + 135*a*b^2*x + 189*a^2*b*x^2 + 105*a^3*x^3))/(315*x^(9/2))
```

fricas [A] time = 0.78, size = 35, normalized size = 0.69

$$-\frac{2\left(105\,a^3x^3+189\,a^2bx^2+135\,ab^2x+35\,b^3\right)}{315\,x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3/x^(5/2),x, algorithm="fricas")
```

```
[Out] -2/315*(105*a^3*x^3 + 189*a^2*b*x^2 + 135*a*b^2*x + 35*b^3)/x^(9/2)
```

giac [A] time = 0.16, size = 35, normalized size = 0.69

$$-\frac{2\left(105\,a^3x^3+189\,a^2bx^2+135\,ab^2x+35\,b^3\right)}{315\,x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3/x^(5/2),x, algorithm="giac")
```

```
[Out] -2/315*(105*a^3*x^3 + 189*a^2*b*x^2 + 135*a*b^2*x + 35*b^3)/x^(9/2)
```

maple [A] time = 0.00, size = 36, normalized size = 0.71

$$-\frac{2\left(105a^3x^3+189a^2b\,x^2+135a\,b^2x+35b^3\right)}{315x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^3/x^(5/2),x)
```

```
[Out] -2/315*(105*a^3*x^3+189*a^2*b*x^2+135*a*b^2*x+35*b^3)/x^(9/2)
```

maxima [A] time = 1.03, size = 35, normalized size = 0.69

$$-\frac{2\,a^3}{3\,x^{\frac{3}{2}}}-\frac{6\,a^2b}{5\,x^{\frac{5}{2}}}-\frac{6\,ab^2}{7\,x^{\frac{7}{2}}}-\frac{2\,b^3}{9\,x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^3/x^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*a^3/x^(3/2) - 6/5*a^2*b/x^(5/2) - 6/7*a*b^2/x^(7/2) - 2/9*b^3/x^(9/2)
```

mupad [B] time = 0.04, size = 35, normalized size = 0.69

$$-\frac{210\,a^3\,x^3+378\,a^2\,b\,x^2+270\,a\,b^2\,x+70\,b^3}{315\,x^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^3/x^(5/2),x)
```

```
[Out] -(70*b^3 + 210*a^3*x^3 + 378*a^2*b*x^2 + 270*a*b^2*x)/(315*x^(9/2))
```

sympy [A] time = 2.04, size = 51, normalized size = 1.00

$$-\frac{2a^3}{3x^{\frac{3}{2}}}-\frac{6a^2b}{5x^{\frac{5}{2}}}-\frac{6ab^2}{7x^{\frac{7}{2}}}-\frac{2b^3}{9x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**3/x**(5/2),x)
```

```
[Out] -2*a**3/(3*x**(3/2)) - 6*a**2*b/(5*x**(5/2)) - 6*a*b**2/(7*x**(7/2)) - 2*b*
**3/(9*x**(9/2))
```

3.1665 $\int \frac{x^{5/2}}{a+\frac{b}{x}}\,dx$

Optimal. Leaf size=83

$$\frac{2b^{7/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{9/2}}-\frac{2b^3\sqrt{x}}{a^4}+\frac{2b^2x^{3/2}}{3a^3}-\frac{2bx^{5/2}}{5a^2}+\frac{2x^{7/2}}{7a}$$

```
[Out] 2/3*b^2*x^(3/2)/a^3-2/5*b*x^(5/2)/a^2+2/7*x^(7/2)/a+2*b^(7/2)*arctan(a^(1/2
)*x^(1/2)/b^(1/2))/a^(9/2)-2*b^3*x^(1/2)/a^4
```

Rubi [A] time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {263, 50, 63, 205}

$$\frac{2b^2x^{3/2}}{3a^3}-\frac{2b^3\sqrt{x}}{a^4}+\frac{2b^{7/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{9/2}}-\frac{2bx^{5/2}}{5a^2}+\frac{2x^{7/2}}{7a}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b/x),x]
```

```
[Out] (-2*b^3*Sqrt[x])/a^4 + (2*b^2*x^(3/2))/(3*a^3) - (2*b*x^(5/2))/(5*a^2) + (2
*x^(7/2))/(7*a) + (2*b^(7/2)*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/a^(9/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\int \frac{x^{5/2}}{a+\frac{b}{x}}\,dx = \int \frac{x^{7/2}}{b+ax}\,dx$$

$$= \frac{2x^{7/2}}{7a} - \frac{b\int \frac{x^{5/2}}{b+ax}\,dx}{a}$$

$$= -\frac{2bx^{5/2}}{5a^2} + \frac{2x^{7/2}}{7a} + \frac{b^2\int \frac{x^{3/2}}{b+ax}\,dx}{a^2}$$

$$= \frac{2b^2x^{3/2}}{3a^3} - \frac{2bx^{5/2}}{5a^2} + \frac{2x^{7/2}}{7a} - \frac{b^3\int \frac{\sqrt{x}}{b+ax}\,dx}{a^3}$$

$$= -\frac{2b^3\sqrt{x}}{a^4} + \frac{2b^2x^{3/2}}{3a^3} - \frac{2bx^{5/2}}{5a^2} + \frac{2x^{7/2}}{7a} + \frac{b^4\int \frac{1}{\sqrt{x}\,(b+ax)}\,dx}{a^4}$$

$$= -\frac{2b^3\sqrt{x}}{a^4} + \frac{2b^2x^{3/2}}{3a^3} - \frac{2bx^{5/2}}{5a^2} + \frac{2x^{7/2}}{7a} + \frac{\left(2b^4\right)\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \sqrt{x}\right)}{a^4}$$

$$= -\frac{2b^3\sqrt{x}}{a^4} + \frac{2b^2x^{3/2}}{3a^3} - \frac{2bx^{5/2}}{5a^2} + \frac{2x^{7/2}}{7a} + \frac{2b^{7/2}\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{9/2}}$$

Mathematica [A] time = 0.04, size = 72, normalized size = 0.87

$$\frac{2b^{7/2}\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{9/2}} + \frac{2\sqrt{x}\left(15a^3x^3 - 21a^2bx^2 + 35ab^2x - 105b^3\right)}{105a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b/x),x]
```

```
[Out] (2*Sqrt[x]*(-105*b^3 + 35*a*b^2*x - 21*a^2*b*x^2 + 15*a^3*x^3))/(105*a^4) +
 (2*b^(7/2)*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/a^(9/2)
```

fricas [A] time = 1.06, size = 153, normalized size = 1.84

$$\left[\frac{105\,b^3\sqrt{-\frac{b}{a}}\,\log\left(\frac{ax+2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-b}{ax+b}\right) + 2\left(15\,a^3x^3 - 21\,a^2bx^2 + 35\,ab^2x - 105\,b^3\right)\sqrt{x}}{105\,a^4}, \frac{2\left(105\,b^3\sqrt{\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{x}\sqrt{\frac{b}{a}}}{b}\right)\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(a+b/x),x, algorithm="fricas")
```

```
[Out] [1/105*(105*b^3*sqrt(-b/a)*log((a*x + 2*a*sqrt(x)*sqrt(-b/a) - b)/(a*x + b)
) + 2*(15*a^3*x^3 - 21*a^2*b*x^2 + 35*a*b^2*x - 105*b^3)*sqrt(x))/a^4, 2/10
5*(105*b^3*sqrt(b/a)*arctan(a*sqrt(x)*sqrt(b/a)/b) + (15*a^3*x^3 - 21*a^2*b
*x^2 + 35*a*b^2*x - 105*b^3)*sqrt(x))/a^4]
```

giac [A] time = 0.15, size = 70, normalized size = 0.84

$$\frac{2\,b^4\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^4} + \frac{2\left(15\,a^6x^{\frac{7}{2}} - 21\,a^5bx^{\frac{5}{2}} + 35\,a^4b^2x^{\frac{3}{2}} - 105\,a^3b^3\sqrt{x}\right)}{105\,a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(a+b/x),x, algorithm="giac")
```

```
[Out] 2*b^4*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^4) + 2/105*(15*a^6*x^(7/2) -
 21*a^5*b*x^(5/2) + 35*a^4*b^2*x^(3/2) - 105*a^3*b^3*sqrt(x))/a^7
```

maple [A] time = 0.01, size = 65, normalized size = 0.78

$$\frac{2x^{\frac{7}{2}}}{7a} - \frac{2b\,x^{\frac{5}{2}}}{5a^2} + \frac{2b^4 \arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^4} + \frac{2b^2x^{\frac{3}{2}}}{3a^3} - \frac{2b^3\sqrt{x}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a+b/x),x)
```

```
[Out] 2/7/a*x^(7/2)-2/5*b*x^(5/2)/a^2+2/3*b^2*x^(3/2)/a^3-2*b^3*x^(1/2)/a^4+2*b^4
/a^4/(a*b)^(1/2)*arctan(x^(1/2)*a/(a*b)^(1/2))
```

maxima [A] time = 2.29, size = 65, normalized size = 0.78

$$\frac{2\left(15\,a^3 - \frac{21\,a^2b}{x} + \frac{35\,ab^2}{x^2} - \frac{105\,b^3}{x^3}\right)x^{\frac{7}{2}}}{105\,a^4} - \frac{2\,b^4 \arctan\left(\frac{b}{\sqrt{ab}\,\sqrt{x}}\right)}{\sqrt{ab}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(a+b/x),x, algorithm="maxima")
```

```
[Out] 2/105*(15*a^3 - 21*a^2*b/x + 35*a*b^2/x^2 - 105*b^3/x^3)*x^(7/2)/a^4 - 2*b^
4*arctan(b/(sqrt(a*b)*sqrt(x)))/(sqrt(a*b)*a^4)
```

mupad [B] time = 0.04, size = 59, normalized size = 0.71

$$\frac{2\,x^{7/2}}{7\,a} - \frac{2\,b\,x^{5/2}}{5\,a^2} + \frac{2\,b^2\,x^{3/2}}{3\,a^3} - \frac{2\,b^3\,\sqrt{x}}{a^4} + \frac{2\,b^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b/x),x)
```

```
[Out] (2*x^(7/2))/(7*a) - (2*b*x^(5/2))/(5*a^2) + (2*b^2*x^(3/2))/(3*a^3) - (2*b^
3*x^(1/2))/a^4 + (2*b^(7/2)*atan((a^(1/2)*x^(1/2))/b^(1/2)))/a^(9/2)
```

sympy [A] time = 22.58, size = 136, normalized size = 1.64

$$\begin{cases} \frac{2x^{\frac{7}{2}}}{7a} - \frac{2bx^{\frac{5}{2}}}{5a^2} + \frac{2b^2x^{\frac{3}{2}}}{3a^3} - \frac{2b^3\sqrt{x}}{a^4} - \frac{ib^{\frac{7}{2}}\log\left(-i\sqrt{b}\sqrt{\frac{1}{a}}+\sqrt{x}\right)}{a^5\sqrt{\frac{1}{a}}} + \frac{ib^{\frac{7}{2}}\log\left(i\sqrt{b}\sqrt{\frac{1}{a}}+\sqrt{x}\right)}{a^5\sqrt{\frac{1}{a}}} & \text{for } a \neq 0 \\ \frac{2x^{\frac{9}{2}}}{9b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(a+b/x),x)
```

```
[Out] Piecewise((2*x**(7/2)/(7*a) - 2*b*x**(5/2)/(5*a**2) + 2*b**2*x**(3/2)/(3*a*
*3) - 2*b**3*sqrt(x)/a**4 - I*b**(7/2)*log(-I*sqrt(b)*sqrt(1/a) + sqrt(x))/
(a**5*sqrt(1/a)) + I*b**(7/2)*log(I*sqrt(b)*sqrt(1/a) + sqrt(x))/(a**5*sqrt
(1/a)), Ne(a, 0)), (2*x**(9/2)/(9*b), True))
```

3.1666 $\int \frac{x^{3/2}}{a+\frac{b}{x}}\,dx$

Optimal. Leaf size=68

$$-\frac{2b^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{7/2}}+\frac{2b^2\sqrt{x}}{a^3}-\frac{2bx^{3/2}}{3a^2}+\frac{2x^{5/2}}{5a}$$

```
[Out] -2/3*b*x^(3/2)/a^2+2/5*x^(5/2)/a-2*b^(5/2)*arctan(a^(1/2)*x^(1/2)/b^(1/2))/
a^(7/2)+2*b^2*x^(1/2)/a^3
```

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {263, 50, 63, 205}

$$\frac{2b^2\sqrt{x}}{a^3}-\frac{2b^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{7/2}}-\frac{2bx^{3/2}}{3a^2}+\frac{2x^{5/2}}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b/x),x]
```

```
[Out] (2*b^2*Sqrt[x])/a^3 - (2*b*x^(3/2))/(3*a^2) + (2*x^(5/2))/(5*a) - (2*b^(5/2
)*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/a^(7/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{3/2}}{a+\frac{b}{x}}\,dx &= \int \frac{x^{5/2}}{b+ax}\,dx \\ &= \frac{2x^{5/2}}{5a} - \frac{b\int \frac{x^{3/2}}{b+ax}\,dx}{a} \\ &= -\frac{2bx^{3/2}}{3a^2} + \frac{2x^{5/2}}{5a} + \frac{b^2\int \frac{\sqrt{x}}{b+ax}\,dx}{a^2} \\ &= \frac{2b^2\sqrt{x}}{a^3} - \frac{2bx^{3/2}}{3a^2} + \frac{2x^{5/2}}{5a} - \frac{b^3\int \frac{1}{\sqrt{x}\,(b+ax)}\,dx}{a^3} \\ &= \frac{2b^2\sqrt{x}}{a^3} - \frac{2bx^{3/2}}{3a^2} + \frac{2x^{5/2}}{5a} - \frac{\left(2b^3\right)\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \sqrt{x}\right)}{a^3} \\ &= \frac{2b^2\sqrt{x}}{a^3} - \frac{2bx^{3/2}}{3a^2} + \frac{2x^{5/2}}{5a} - \frac{2b^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 61, normalized size = 0.90

$$\frac{2\sqrt{x}\left(3a^2x^2 - 5abx + 15b^2\right)}{15a^3} - \frac{2b^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b/x),x]
```

```
[Out] (2*Sqrt[x]*(15*b^2 - 5*a*b*x + 3*a^2*x^2))/(15*a^3) - (2*b^(5/2)*ArcTan[(Sq
rt[a]*Sqrt[x])/Sqrt[b]])/a^(7/2)
```

fricas [A] time = 0.72, size = 132, normalized size = 1.94

$$\left[\frac{15\,b^2\sqrt{-\frac{b}{a}}\,\log\left(\frac{ax-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-b}{ax+b}\right) + 2\left(3\,a^2x^2 - 5\,abx + 15\,b^2\right)\sqrt{x}}{15\,a^3}, -\frac{2\left(15\,b^2\sqrt{\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{x}\sqrt{\frac{b}{a}}}{b}\right) - \left(3\,a^2x^2 - 5\right.\right.}{15\,a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(a+b/x),x, algorithm="fricas")
```

```
[Out] [1/15*(15*b^2*sqrt(-b/a)*log((a*x - 2*a*sqrt(x)*sqrt(-b/a) - b)/(a*x + b))
+ 2*(3*a^2*x^2 - 5*a*b*x + 15*b^2)*sqrt(x))/a^3, -2/15*(15*b^2*sqrt(b/a)*ar
ctan(a*sqrt(x)*sqrt(b/a)/b) - (3*a^2*x^2 - 5*a*b*x + 15*b^2)*sqrt(x))/a^3]
```

giac [A] time = 0.21, size = 59, normalized size = 0.87

$$-\frac{2\,b^3\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^3} + \frac{2\left(3\,a^4x^{\frac{5}{2}} - 5\,a^3bx^{\frac{3}{2}} + 15\,a^2b^2\sqrt{x}\right)}{15\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(a+b/x),x, algorithm="giac")
```

```
[Out] -2*b^3*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) + 2/15*(3*a^4*x^(5/2) -
5*a^3*b*x^(3/2) + 15*a^2*b^2*sqrt(x))/a^5
```

maple [A] time = 0.01, size = 54, normalized size = 0.79

$$\frac{2x^{\frac{5}{2}}}{5a} - \frac{2b^3 \arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\ a^3} - \frac{2b\,x^{\frac{3}{2}}}{3a^2} + \frac{2b^2\sqrt{x}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a+b/x),x)
```

```
[Out] 2/5/a*x^(5/2)-2/3*b*x^(3/2)/a^2+2*b^2*x^(1/2)/a^3-2*b^3/a^3/(a*b)^(1/2)*arc
tan(1/(a*b)^(1/2)*a*x^(1/2))
```

maxima [A] time = 2.37, size = 54, normalized size = 0.79

$$\frac{2\left(3a^2 - \frac{5ab}{x} + \frac{15b^2}{x^2}\right)x^{\frac{5}{2}}}{15a^3} + \frac{2b^3 \arctan\left(\frac{b}{\sqrt{ab}\sqrt{x}}\right)}{\sqrt{ab}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(a+b/x),x, algorithm="maxima")
```

```
[Out] 2/15*(3*a^2 - 5*a*b/x + 15*b^2/x^2)*x^(5/2)/a^3 + 2*b^3*arctan(b/(sqrt(a*b)
*sqrt(x)))/(sqrt(a*b)*a^3)
```

mupad [B] time = 0.05, size = 48, normalized size = 0.71

$$\frac{2\,x^{5/2}}{5\,a} - \frac{2\,b\,x^{3/2}}{3\,a^2} + \frac{2\,b^2\,\sqrt{x}}{a^3} - \frac{2\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b/x),x)
```

```
[Out] (2*x^(5/2))/(5*a) - (2*b*x^(3/2))/(3*a^2) + (2*b^2*x^(1/2))/a^3 - (2*b^(5/2
)*atan((a^(1/2)*x^(1/2))/b^(1/2)))/a^(7/2)
```

sympy [A] time = 5.75, size = 121, normalized size = 1.78

$$\begin{cases} \frac{2x^{\frac{5}{2}}}{5a} - \frac{2bx^{\frac{3}{2}}}{3a^2} + \frac{2b^2\sqrt{x}}{a^3} + \frac{ib^{\frac{5}{2}}\log\left(-i\sqrt{b}\sqrt{\frac{1}{a}}+\sqrt{x}\right)}{a^4\sqrt{\frac{1}{a}}} - \frac{ib^{\frac{5}{2}}\log\left(i\sqrt{b}\sqrt{\frac{1}{a}}+\sqrt{x}\right)}{a^4\sqrt{\frac{1}{a}}} & \text{for } a \neq 0 \\ \frac{2x^{\frac{7}{2}}}{7b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(a+b/x),x)
```

```
[Out] Piecewise((2*x**(5/2)/(5*a) - 2*b*x**(3/2)/(3*a**2) + 2*b**2*sqrt(x)/a**3 +
 I*b**(5/2)*log(-I*sqrt(b)*sqrt(1/a) + sqrt(x))/(a**4*sqrt(1/a)) - I*b**(5/
2)*log(I*sqrt(b)*sqrt(1/a) + sqrt(x))/(a**4*sqrt(1/a)), Ne(a, 0)), (2*x**(7
/2)/(7*b), True))
```

3.1667 $\int \frac{\sqrt{x}}{a+\frac{b}{x}}\,dx$

Optimal. Leaf size=53

$$\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{5/2}}-\frac{2b\sqrt{x}}{a^2}+\frac{2x^{3/2}}{3a}$$

```
[Out] 2/3*x^(3/2)/a+2*b^(3/2)*arctan(a^(1/2)*x^(1/2)/b^(1/2))/a^(5/2)-2*b*x^(1/2)
/a^2
```

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {263, 50, 63, 205}

$$\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{5/2}}-\frac{2b\sqrt{x}}{a^2}+\frac{2x^{3/2}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b/x),x]
```

```
[Out] (-2*b*Sqrt[x])/a^2 + (2*x^(3/2))/(3*a) + (2*b^(3/2)*ArcTan[(Sqrt[a]*Sqrt[x]
)/Sqrt[b]])/a^(5/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\int \frac{\sqrt{x}}{a+\frac{b}{x}}\,dx = \int \frac{x^{3/2}}{b+ax}\,dx$$
$$= \frac{2x^{3/2}}{3a} - \frac{b\int \frac{\sqrt{x}}{b+ax}\,dx}{a}$$
$$= -\frac{2b\sqrt{x}}{a^2} + \frac{2x^{3/2}}{3a} + \frac{b^2\int \frac{1}{\sqrt{x}\,(b+ax)}\,dx}{a^2}$$
$$= -\frac{2b\sqrt{x}}{a^2} + \frac{2x^{3/2}}{3a} + \frac{\left(2b^2\right)\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \sqrt{x}\right)}{a^2}$$
$$= -\frac{2b\sqrt{x}}{a^2} + \frac{2x^{3/2}}{3a} + \frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{5/2}}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.92

$$\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{5/2}} + \frac{2\sqrt{x}\,(ax-3b)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b/x),x]
```

```
[Out] (2*Sqrt[x]*(-3*b + a*x))/(3*a^2) + (2*b^(3/2)*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt
[b]])/a^(5/2)
```

fricas [A] time = 0.86, size = 103, normalized size = 1.94

$$\left[\frac{3\,b\sqrt{-\frac{b}{a}}\,\log\left(\frac{ax+2\,a\sqrt{x}\,\sqrt{-\frac{b}{a}}-b}{ax+b}\right)+2\,(ax-3\,b)\sqrt{x}}{3\,a^2}, \frac{2\left(3\,b\sqrt{\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{x}\,\sqrt{\frac{b}{a}}}{b}\right)+(ax-3\,b)\sqrt{x}\right)}{3\,a^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(a+b/x),x, algorithm="fricas")
```

```
[Out] [1/3*(3*b*sqrt(-b/a)*log((a*x + 2*a*sqrt(x)*sqrt(-b/a) - b)/(a*x + b)) + 2*
(a*x - 3*b)*sqrt(x))/a^2, 2/3*(3*b*sqrt(b/a)*arctan(a*sqrt(x)*sqrt(b/a)/b)
+ (a*x - 3*b)*sqrt(x))/a^2]
```

giac [A] time = 0.17, size = 45, normalized size = 0.85

$$\frac{2\,b^2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2} + \frac{2\left(a^2x^{\frac{3}{2}}-3\,ab\sqrt{x}\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(a+b/x),x, algorithm="giac")
```

```
[Out] 2*b^2*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) + 2/3*(a^2*x^(3/2) - 3*a*
b*sqrt(x))/a^3
```

maple [A] time = 0.01, size = 43, normalized size = 0.81

$$\frac{2b^2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^2} + \frac{2x^{\frac{3}{2}}}{3a} - \frac{2b\sqrt{x}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a+b/x),x)
```

```
[Out] 2/3/a*x^(3/2)-2*b*x^(1/2)/a^2+2/a^2*b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*a*
x^(1/2))
```

maxima [A] time = 2.33, size = 41, normalized size = 0.77

$$\frac{2\left(a-\frac{3b}{x}\right)x^{\frac{3}{2}}}{3a^2}-\frac{2b^2\arctan\left(\frac{b}{\sqrt{ab}\sqrt{x}}\right)}{\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(a+b/x),x, algorithm="maxima")
```

```
[Out] 2/3*(a - 3*b/x)*x^(3/2)/a^2 - 2*b^2*arctan(b/(sqrt(a*b)*sqrt(x)))/(sqrt(a*b
)*a^2)
```

mupad [B] time = 0.05, size = 37, normalized size = 0.70

$$\frac{2\,x^{3/2}}{3\,a}-\frac{2\,b\,\sqrt{x}}{a^2}+\frac{2\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b/x),x)
```

```
[Out] (2*x^(3/2))/(3*a) - (2*b*x^(1/2))/a^2 + (2*b^(3/2)*atan((a^(1/2)*x^(1/2))/b
^(1/2)))/a^(5/2)
```

sympy [A] time = 1.37, size = 105, normalized size = 1.98

$$\begin{cases}\frac{2x^{\frac{3}{2}}}{3a}-\frac{2b\sqrt{x}}{a^2}-\frac{ib^{\frac{3}{2}}\log\left(-i\sqrt{b}\sqrt{\frac{1}{a}}+\sqrt{x}\right)}{a^3\sqrt{\frac{1}{a}}}+\frac{ib^{\frac{3}{2}}\log\left(i\sqrt{b}\sqrt{\frac{1}{a}}+\sqrt{x}\right)}{a^3\sqrt{\frac{1}{a}}} & \text{for } a\neq 0\\ \frac{2x^{\frac{5}{2}}}{5b} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(a+b/x),x)
```

```
[Out] Piecewise((2*x**(3/2)/(3*a) - 2*b*sqrt(x)/a**2 - I*b**(3/2)*log(-I*sqrt(b)*
sqrt(1/a) + sqrt(x))/(a**3*sqrt(1/a)) + I*b**(3/2)*log(I*sqrt(b)*sqrt(1/a)
+ sqrt(x))/(a**3*sqrt(1/a)), Ne(a, 0)), (2*x**(5/2)/(5*b), True))
```

3.1668 $\int \frac{1}{\left(a+\frac{b}{x}\right)\sqrt{x}}\,dx$

Optimal. Leaf size=40

$$\frac{2\sqrt{x}}{a}-\frac{2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{3/2}}$$

```
[Out] -2*arctan(a^(1/2)*x^(1/2)/b^(1/2))*b^(1/2)/a^(3/2)+2*x^(1/2)/a
```

Rubi [A] time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {263, 50, 63, 205}

$$\frac{2\sqrt{x}}{a}-\frac{2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)*Sqrt[x]),x]
```

```
[Out] (2*Sqrt[x])/a - (2*Sqrt[b]*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/a^(3/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)\sqrt{x}}\,dx = \int \frac{\sqrt{x}}{b+ax}\,dx$$
$$= \frac{2\sqrt{x}}{a} - \frac{b\int \frac{1}{\sqrt{x}\,(b+ax)}\,dx}{a}$$
$$= \frac{2\sqrt{x}}{a} - \frac{(2b)\,\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \sqrt{x}\right)}{a}$$
$$= \frac{2\sqrt{x}}{a} - \frac{2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{3/2}}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.00

$$\frac{2\sqrt{x}}{a} - \frac{2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)*Sqrt[x]),x]
```

```
[Out] (2*Sqrt[x])/a - (2*Sqrt[b]*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/a^(3/2)
```

fricas [A] time = 0.94, size = 85, normalized size = 2.12

$$\left[\frac{\sqrt{-\frac{b}{a}}\,\log\left(\frac{ax-2\,a\sqrt{x}\,\sqrt{-\frac{b}{a}}-b}{ax+b}\right)+2\sqrt{x}}{a}, -\frac{2\left(\sqrt{\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{x}\,\sqrt{\frac{b}{a}}}{b}\right)-\sqrt{x}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(-b/a)*log((a*x - 2*a*sqrt(x)*sqrt(-b/a) - b)/(a*x + b)) + 2*sqrt(x))
/a, -2*(sqrt(b/a)*arctan(a*sqrt(x)*sqrt(b/a)/b) - sqrt(x))/a]
```

giac [A] time = 0.15, size = 31, normalized size = 0.78

$$-\frac{2\,b\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a} + \frac{2\sqrt{x}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x^(1/2),x, algorithm="giac")
```

```
[Out] -2*b*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a) + 2*sqrt(x)/a
```

maple [A] time = 0.01, size = 32, normalized size = 0.80

$$-\frac{2b\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\;a} + \frac{2\sqrt{x}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)/x^(1/2),x)
```

```
[Out] 2/a*x^(1/2)-2/a*b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*a*x^(1/2))
```

maxima [A] time = 2.65, size = 31, normalized size = 0.78

$$\frac{2\,b\arctan\left(\frac{b}{\sqrt{ab}\,\sqrt{x}}\right)}{\sqrt{ab}\,a}+\frac{2\,\sqrt{x}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*b*arctan(b/(sqrt(a*b)*sqrt(x)))/(sqrt(a*b)*a) + 2*sqrt(x)/a
```

mupad [B] time = 0.04, size = 28, normalized size = 0.70

$$\frac{2\,\sqrt{x}}{a}-\frac{2\,\sqrt{b}\,\operatorname{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b/x)),x)
```

```
[Out] (2*x^(1/2))/a - (2*b^(1/2)*atan((a^(1/2)*x^(1/2))/b^(1/2)))/a^(3/2)
```

sympy [A] time = 1.35, size = 92, normalized size = 2.30

$$\begin{cases}\frac{2\sqrt{x}}{a}+\frac{i\sqrt{b}\,\log\left(-i\sqrt{b}\,\sqrt{\frac{1}{a}}+\sqrt{x}\right)}{a^2\sqrt{\frac{1}{a}}}-\frac{i\sqrt{b}\,\log\left(i\sqrt{b}\,\sqrt{\frac{1}{a}}+\sqrt{x}\right)}{a^2\sqrt{\frac{1}{a}}} & \text{for } a\neq 0\\ \frac{2x^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x**(1/2),x)
```

```
[Out] Piecewise((2*sqrt(x)/a + I*sqrt(b)*log(-I*sqrt(b)*sqrt(1/a) + sqrt(x))/(a**
2*sqrt(1/a)) - I*sqrt(b)*log(I*sqrt(b)*sqrt(1/a) + sqrt(x))/(a**2*sqrt(1/a)
), Ne(a, 0)), (2*x**(3/2)/(3*b), True))
```

3.1669 $\int \frac{1}{\left(a+\frac{b}{x}\right)x^{3/2}}\,dx$

Optimal. Leaf size=29

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}$$

```
[Out] 2*arctan(a^(1/2)*x^(1/2)/b^(1/2))/a^(1/2)/b^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {263, 63, 205}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)*x^(3/2)),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/(Sqrt[a]*Sqrt[b])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x}\right)x^{3/2}}\,dx &= \int \frac{1}{\sqrt{x}\,(b+ax)}\,dx \\ &= 2\,\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \sqrt{x}\right) \\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)*x^(3/2)),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/(Sqrt[a]*Sqrt[b])
```

fricas [A] time = 1.04, size = 68, normalized size = 2.34

$$\left[-\frac{\sqrt{-ab}\,\log\left(\frac{ax-b-2\sqrt{-ab}\,\sqrt{x}}{ax+b}\right)}{ab}, -\frac{2\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}}{a\sqrt{x}}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-a*b)*log((a*x - b - 2*sqrt(-a*b)*sqrt(x))/(a*x + b))/(a*b), -2*sqrt
(a*b)*arctan(sqrt(a*b)/(a*sqrt(x)))/(a*b)]
```

giac [A] time = 0.15, size = 18, normalized size = 0.62

$$\frac{2\,\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x^(3/2),x, algorithm="giac")
```

```
[Out] 2*arctan(a*sqrt(x)/sqrt(a*b))/sqrt(a*b)
```

maple [A] time = 0.01, size = 19, normalized size = 0.66

$$\frac{2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)/x^(3/2),x)
```

```
[Out] 2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*a*x^(1/2))
```

maxima [A] time = 2.33, size = 18, normalized size = 0.62

$$-\frac{2\,\arctan\left(\frac{b}{\sqrt{ab}\,\sqrt{x}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x^(3/2),x, algorithm="maxima")
```

```
[Out] -2*arctan(b/(sqrt(a*b)*sqrt(x)))/sqrt(a*b)
```

mupad [B] time = 1.06, size = 19, normalized size = 0.66

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b/x)),x)
```

```
[Out] (2*atan((a^(1/2)*x^(1/2))/b^(1/2)))/(a^(1/2)*b^(1/2))
```

sympy [**A**] time = 3.29, size = 94, normalized size = 3.24

$$\begin{cases} \tilde{\infty}\sqrt{x} & \text{for } a = 0 \wedge b = 0 \\ \frac{2\sqrt{x}}{b} & \text{for } a = 0 \\ -\frac{2}{a\sqrt{x}} & \text{for } b = 0 \\ -\frac{i\log\left(-i\sqrt{b}\sqrt{\frac{1}{a}}+\sqrt{x}\right)}{a\sqrt{b}\sqrt{\frac{1}{a}}} + \frac{i\log\left(i\sqrt{b}\sqrt{\frac{1}{a}}+\sqrt{x}\right)}{a\sqrt{b}\sqrt{\frac{1}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x**(3/2),x)
```

```
[Out] Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/b, Eq(a, 0)), (-2/
(a*sqrt(x)), Eq(b, 0)), (-I*log(-I*sqrt(b)*sqrt(1/a) + sqrt(x))/(a*sqrt(b)*
sqrt(1/a)) + I*log(I*sqrt(b)*sqrt(1/a) + sqrt(x))/(a*sqrt(b)*sqrt(1/a)), Tr
ue))
```

3.1670 $\int \frac{1}{\left(a+\frac{b}{x}\right)x^{5/2}}\,dx$

Optimal. Leaf size=40

$$-\frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{b^{3/2}}-\frac{2}{b\sqrt{x}}$$

```
[Out] -2*arctan(a^(1/2)*x^(1/2)/b^(1/2))*a^(1/2)/b^(3/2)-2/b/x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {263, 51, 63, 205}

$$-\frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{b^{3/2}}-\frac{2}{b\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)*x^(5/2)),x]
```

```
[Out] -2/(b*Sqrt[x]) - (2*Sqrt[a]*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/b^(3/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^{5/2}}\,dx = \int \frac{1}{x^{3/2}(b+ax)}\,dx$$

$$= -\frac{2}{b\sqrt{x}} - \frac{a\int \frac{1}{\sqrt{x}\,(b+ax)}\,dx}{b}$$

$$= -\frac{2}{b\sqrt{x}} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \sqrt{x}\right)}{b}$$

$$= -\frac{2}{b\sqrt{x}} - \frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{b^{3/2}}$$

Mathematica [C] time = 0.01, size = 25, normalized size = 0.62

$$-\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{ax}{b}\right)}{b\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)*x^(5/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/2, 1, 1/2, -((a*x)/b)])/(b*Sqrt[x])
```

fricas [A] time = 1.02, size = 93, normalized size = 2.32

$$\left[\frac{x\sqrt{-\frac{a}{b}}\,\log\left(\frac{ax-2\,b\sqrt{x}\,\sqrt{-\frac{a}{b}}-b}{ax+b}\right)-2\sqrt{x}}{bx}, \frac{2\left(x\sqrt{\frac{a}{b}}\,\arctan\left(\frac{b\sqrt{\frac{a}{b}}}{a\sqrt{x}}\right)-\sqrt{x}\right)}{bx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [(x*sqrt(-a/b)*log((a*x - 2*b*sqrt(x)*sqrt(-a/b) - b)/(a*x + b)) - 2*sqrt(x
))/(b*x), 2*(x*sqrt(a/b)*arctan(b*sqrt(a/b)/(a*sqrt(x))) - sqrt(x))/(b*x)]
```

giac [A] time = 0.17, size = 31, normalized size = 0.78

$$-\frac{2\,a\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b} - \frac{2}{b\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x^(5/2),x, algorithm="giac")
```

```
[Out] -2*a*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b) - 2/(b*sqrt(x))
```

maple [A] time = 0.01, size = 32, normalized size = 0.80

$$-\frac{2a\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\;b} - \frac{2}{b\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)/x^(5/2),x)
```

```
[Out] -2/b/x^(1/2)-2*a/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*a*x^(1/2))
```

maxima [A] time = 2.48, size = 31, normalized size = 0.78

$$\frac{2\,a\arctan\left(\frac{b}{\sqrt{ab}\,\sqrt{x}}\right)}{\sqrt{ab}\,b}-\frac{2}{b\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2*a*arctan(b/(sqrt(a*b)*sqrt(x)))/(sqrt(a*b)*b) - 2/(b*sqrt(x))
```

mupad [B] time = 0.06, size = 28, normalized size = 0.70

$$-\frac{2}{b\,\sqrt{x}}-\frac{2\,\sqrt{a}\,\operatorname{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b/x)),x)
```

```
[Out] - 2/(b*x^(1/2)) - (2*a^(1/2)*atan((a^(1/2)*x^(1/2))/b^(1/2)))/b^(3/2)
```

sympy [A] time = 12.59, size = 102, normalized size = 2.55

$$\begin{cases}\frac{\tilde{\infty}}{\sqrt{x}} & \text{for } a=0\wedge b=0\\ -\frac{2}{3ax^{\frac{3}{2}}} & \text{for } b=0\\ -\frac{2}{b\sqrt{x}} & \text{for } a=0\\ -\frac{2}{b\sqrt{x}}+\frac{i\log\left(-i\sqrt{b}\,\sqrt{\frac{1}{a}}+\sqrt{x}\right)}{b^{\frac{3}{2}}\sqrt{\frac{1}{a}}}-\frac{i\log\left(i\sqrt{b}\,\sqrt{\frac{1}{a}}+\sqrt{x}\right)}{b^{\frac{3}{2}}\sqrt{\frac{1}{a}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x**(5/2),x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (-2/(3*a*x**(3/2)), Eq(b, 0))
, (-2/(b*sqrt(x)), Eq(a, 0)), (-2/(b*sqrt(x)) + I*log(-I*sqrt(b)*sqrt(1/a)
+ sqrt(x))/(b**(3/2)*sqrt(1/a)) - I*log(I*sqrt(b)*sqrt(1/a) + sqrt(x))/(b**
(3/2)*sqrt(1/a)), True))
```

3.1671 $\int \frac{1}{\left(a+\frac{b}{x}\right)x^{7/2}}\,dx$

Optimal. Leaf size=53

$$\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{b^{5/2}}+\frac{2a}{b^2\sqrt{x}}-\frac{2}{3bx^{3/2}}$$

```
[Out] -2/3/b/x^(3/2)+2*a^(3/2)*arctan(a^(1/2)*x^(1/2)/b^(1/2))/b^(5/2)+2*a/b^2/x^
(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {263, 51, 63, 205}

$$\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{b^{5/2}}+\frac{2a}{b^2\sqrt{x}}-\frac{2}{3bx^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)*x^(7/2)),x]
```

```
[Out] -2/(3*b*x^(3/2)) + (2*a)/(b^2*Sqrt[x]) + (2*a^(3/2)*ArcTan[(Sqrt[a]*Sqrt[x]
)/Sqrt[b]])/b^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^{7/2}}\,dx = \int \frac{1}{x^{5/2}(b+ax)}\,dx$$

$$= -\frac{2}{3bx^{3/2}} - \frac{a\int \frac{1}{x^{3/2}(b+ax)}\,dx}{b}$$

$$= -\frac{2}{3bx^{3/2}} + \frac{2a}{b^2\sqrt{x}} + \frac{a^2\int \frac{1}{\sqrt{x}\,(b+ax)}\,dx}{b^2}$$

$$= -\frac{2}{3bx^{3/2}} + \frac{2a}{b^2\sqrt{x}} + \frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \sqrt{x}\right)}{b^2}$$

$$= -\frac{2}{3bx^{3/2}} + \frac{2a}{b^2\sqrt{x}} + \frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{b^{5/2}}$$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.51

$$-\frac{2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\frac{ax}{b}\right)}{3bx^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)*x^(7/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/2, 1, -1/2, -((a*x)/b)])/(3*b*x^(3/2))
```

fricas [A] time = 0.89, size = 118, normalized size = 2.23

$$\left[\frac{3\,ax^2\sqrt{-\frac{a}{b}}\,\log\left(\frac{ax+2\,b\sqrt{x}\,\sqrt{-\frac{a}{b}}-b}{ax+b}\right)+2\,(3\,ax-b)\sqrt{x}}{3\,b^2x^2}, -\frac{2\left(3\,ax^2\sqrt{\frac{a}{b}}\,\arctan\left(\frac{b\sqrt{\frac{a}{b}}}{a\sqrt{x}}\right)-(3\,ax-b)\sqrt{x}\right)}{3\,b^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x^(7/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*a*x^2*sqrt(-a/b)*log((a*x + 2*b*sqrt(x)*sqrt(-a/b) - b)/(a*x + b))
+ 2*(3*a*x - b)*sqrt(x))/(b^2*x^2), -2/3*(3*a*x^2*sqrt(a/b)*arctan(b*sqrt(a
/b)/(a*sqrt(x))) - (3*a*x - b)*sqrt(x))/(b^2*x^2)]
```

giac [A] time = 0.18, size = 41, normalized size = 0.77

$$\frac{2\,a^2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2} + \frac{2\,(3\,ax-b)}{3\,b^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x^(7/2),x, algorithm="giac")
```

```
[Out] 2*a^2*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + 2/3*(3*a*x - b)/(b^2*x^
(3/2))
```

maple [A] time = 0.01, size = 43, normalized size = 0.81

$$\frac{2a^2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^2} + \frac{2a}{b^2\sqrt{x}} - \frac{2}{3b\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)/x^(7/2),x)
```

```
[Out] 2*a^2/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*a*x^(1/2))-2/3/b/x^(3/2)+2*a/b^2
/x^(1/2)
```

maxima [A] time = 2.31, size = 43, normalized size = 0.81

$$-\frac{2\,a^2\arctan\left(\frac{b}{\sqrt{ab}\,\sqrt{x}}\right)}{\sqrt{ab}\,b^2}+\frac{2\left(\frac{3\,a}{\sqrt{x}}-\frac{b}{x^{\frac{3}{2}}}\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x^(7/2),x, algorithm="maxima")
```

```
[Out] -2*a^2*arctan(b/(sqrt(a*b)*sqrt(x)))/(sqrt(a*b)*b^2) + 2/3*(3*a/sqrt(x) - b
/x^(3/2))/b^2
```

mupad [B] time = 0.06, size = 38, normalized size = 0.72

$$\frac{2\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{b^{5/2}}-\frac{\frac{2}{3\,b}-\frac{2\,a\,x}{b^2}}{x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a + b/x)),x)
```

```
[Out] (2*a^(3/2)*atan((a^(1/2)*x^(1/2))/b^(1/2)))/b^(5/2) - (2/(3*b) - (2*a*x)/b^
2)/x^(3/2)
```

sympy [A] time = 57.64, size = 121, normalized size = 2.28

$$\begin{cases}\frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for } a=0\wedge b=0\\ -\frac{2}{5ax^{\frac{5}{2}}} & \text{for } b=0\\ -\frac{2}{3bx^{\frac{3}{2}}} & \text{for } a=0\\ \frac{2a}{b^2\sqrt{x}}-\frac{ia\log\left(-i\sqrt{b}\,\sqrt{\frac{1}{a}}+\sqrt{x}\right)}{b^{\frac{5}{2}}\sqrt{\frac{1}{a}}}+\frac{ia\log\left(i\sqrt{b}\,\sqrt{\frac{1}{a}}+\sqrt{x}\right)}{b^{\frac{5}{2}}\sqrt{\frac{1}{a}}}-\frac{2}{3bx^{\frac{3}{2}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x**(7/2),x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (-2/(5*a*x**(5/2)), Eq(b, 0)
), (-2/(3*b*x**(3/2)), Eq(a, 0)), (2*a/(b**2*sqrt(x)) - I*a*log(-I*sqrt(b)*
sqrt(1/a) + sqrt(x))/(b**(5/2)*sqrt(1/a)) + I*a*log(I*sqrt(b)*sqrt(1/a) + s
qrt(x))/(b**(5/2)*sqrt(1/a)) - 2/(3*b*x**(3/2)), True))
```

3.1672 $\int \frac{1}{\left(a+\frac{b}{x}\right)x^{9/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{b^{7/2}}-\frac{2a^2}{b^3\sqrt{x}}+\frac{2a}{3b^2x^{3/2}}-\frac{2}{5bx^{5/2}}$$

```
[Out] -2/5/b/x^(5/2)+2/3*a/b^2/x^(3/2)-2*a^(5/2)*arctan(a^(1/2)*x^(1/2)/b^(1/2))/
b^(7/2)-2*a^2/b^3/x^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {263, 51, 63, 205}

$$-\frac{2a^2}{b^3\sqrt{x}}-\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{b^{7/2}}+\frac{2a}{3b^2x^{3/2}}-\frac{2}{5bx^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)*x^(9/2)),x]
```

```
[Out] -2/(5*b*x^(5/2)) + (2*a)/(3*b^2*x^(3/2)) - (2*a^2)/(b^3*Sqrt[x]) - (2*a^(5/
2)*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/b^(7/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^{9/2}}\,dx = \int \frac{1}{x^{7/2}(b+ax)}\,dx$$

$$= -\frac{2}{5bx^{5/2}} - \frac{a\int \frac{1}{x^{5/2}(b+ax)}\,dx}{b}$$

$$= -\frac{2}{5bx^{5/2}} + \frac{2a}{3b^2x^{3/2}} + \frac{a^2\int \frac{1}{x^{3/2}(b+ax)}\,dx}{b^2}$$

$$= -\frac{2}{5bx^{5/2}} + \frac{2a}{3b^2x^{3/2}} - \frac{2a^2}{b^3\sqrt{x}} - \frac{a^3\int \frac{1}{\sqrt{x}\,(b+ax)}\,dx}{b^3}$$

$$= -\frac{2}{5bx^{5/2}} + \frac{2a}{3b^2x^{3/2}} - \frac{2a^2}{b^3\sqrt{x}} - \frac{\left(2a^3\right)\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \sqrt{x}\right)}{b^3}$$

$$= -\frac{2}{5bx^{5/2}} + \frac{2a}{3b^2x^{3/2}} - \frac{2a^2}{b^3\sqrt{x}} - \frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{b^{7/2}}$$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.40

$$-\frac{2\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\frac{ax}{b}\right)}{5bx^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)*x^(9/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-5/2, 1, -3/2, -((a*x)/b)])/(5*b*x^(5/2))
```

fricas [A] time = 0.79, size = 144, normalized size = 2.12

$$\left[\frac{15\,a^2x^3\sqrt{-\frac{a}{b}}\,\log\left(\frac{ax-2\,b\sqrt{x}\,\sqrt{-\frac{a}{b}}-b}{ax+b}\right) - 2\left(15\,a^2x^2 - 5\,abx + 3\,b^2\right)\sqrt{x}}{15\,b^3x^3}, \frac{2\left(15\,a^2x^3\sqrt{\frac{a}{b}}\,\arctan\left(\frac{b\sqrt{\frac{a}{b}}}{a\sqrt{x}}\right) - \left(15\,a^2x^2 - 5\right.\right.}{15\,b^3x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x^(9/2),x, algorithm="fricas")
```

```
[Out] [1/15*(15*a^2*x^3*sqrt(-a/b)*log((a*x - 2*b*sqrt(x)*sqrt(-a/b) - b)/(a*x +
b)) - 2*(15*a^2*x^2 - 5*a*b*x + 3*b^2)*sqrt(x))/(b^3*x^3), 2/15*(15*a^2*x^3
*sqrt(a/b)*arctan(b*sqrt(a/b)/(a*sqrt(x))) - (15*a^2*x^2 - 5*a*b*x + 3*b^2)
*sqrt(x))/(b^3*x^3)]
```

giac [A] time = 0.16, size = 52, normalized size = 0.76

$$-\frac{2\,a^3\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3} - \frac{2\left(15\,a^2x^2 - 5\,abx + 3\,b^2\right)}{15\,b^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x^(9/2),x, algorithm="giac")
```

```
[Out] -2*a^3*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) - 2/15*(15*a^2*x^2 - 5*a
*b*x + 3*b^2)/(b^3*x^(5/2))
```

maple [A] time = 0.01, size = 54, normalized size = 0.79

$$-\frac{2a^3 \arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\, b^3} - \frac{2a^2}{b^3\sqrt{x}} + \frac{2a}{3b^2 x^{\frac{3}{2}}} - \frac{2}{5b\, x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)/x^(9/2),x)
```

```
[Out] -2*a^3/b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*a*x^(1/2))-2/5/b/x^(5/2)-2*a^2/
b^3/x^(1/2)+2/3*a/b^2/x^(3/2)
```

maxima [A] time = 2.29, size = 54, normalized size = 0.79

$$\frac{2\, a^3 \arctan\left(\frac{b}{\sqrt{ab}\,\sqrt{x}}\right)}{\sqrt{ab}\, b^3} - \frac{2\left(\frac{15\, a^2}{\sqrt{x}} - \frac{5\, ab}{x^{\frac{3}{2}}} + \frac{3\, b^2}{x^{\frac{5}{2}}}\right)}{15\, b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x^(9/2),x, algorithm="maxima")
```

```
[Out] 2*a^3*arctan(b/(sqrt(a*b)*sqrt(x)))/(sqrt(a*b)*b^3) - 2/15*(15*a^2/sqrt(x)
- 5*a*b/x^(3/2) + 3*b^2/x^(5/2))/b^3
```

mupad [B] time = 0.06, size = 49, normalized size = 0.72

$$-\frac{\frac{2}{5\, b} + \frac{2\, a^2 x^2}{b^3} - \frac{2\, a\, x}{3\, b^2}}{x^{5/2}} - \frac{2\, a^{5/2} \operatorname{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(9/2)*(a + b/x)),x)
```

```
[Out] - (2/(5*b) + (2*a^2*x^2)/b^3 - (2*a*x)/(3*b^2))/x^(5/2) - (2*a^(5/2)*atan((
a^(1/2)*x^(1/2))/b^(1/2)))/b^(7/2)
```

sympy [A] time = 149.24, size = 139, normalized size = 2.04

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2}{7ax^{\frac{7}{2}}} & \text{for } b = 0 \\ -\frac{2}{5bx^{\frac{5}{2}}} & \text{for } a = 0 \\ -\frac{2a^2}{b^3\sqrt{x}} + \frac{ia^2 \log\left(-i\sqrt{b}\,\sqrt{\frac{1}{a}} + \sqrt{x}\right)}{b^{\frac{7}{2}}\sqrt{\frac{1}{a}}} - \frac{ia^2 \log\left(i\sqrt{b}\,\sqrt{\frac{1}{a}} + \sqrt{x}\right)}{b^{\frac{7}{2}}\sqrt{\frac{1}{a}}} + \frac{2a}{3b^2 x^{\frac{3}{2}}} - \frac{2}{5bx^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)/x**(9/2),x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(7*a*x**(7/2)), Eq(b, 0)
), (-2/(5*b*x**(5/2)), Eq(a, 0)), (-2*a**2/(b**3*sqrt(x)) + I*a**2*log(-I*s
qrt(b)*sqrt(1/a) + sqrt(x))/(b**(7/2)*sqrt(1/a)) - I*a**2*log(I*sqrt(b)*sqr
t(1/a) + sqrt(x))/(b**(7/2)*sqrt(1/a)) + 2*a/(3*b**2*x**(3/2)) - 2/(5*b*x**
(5/2)), True))
```

3.1673 $\int \frac{x^{5/2}}{\left(a+\frac{b}{x}\right)^2}\,dx$

Optimal. Leaf size=98

$$\frac{9b^{7/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{11/2}}-\frac{9b^3\sqrt{x}}{a^5}+\frac{3b^2x^{3/2}}{a^4}-\frac{9bx^{5/2}}{5a^3}+\frac{9x^{7/2}}{7a^2}-\frac{x^{9/2}}{a(ax+b)}$$

```
[Out] 3*b^2*x^(3/2)/a^4-9/5*b*x^(5/2)/a^3+9/7*x^(7/2)/a^2-x^(9/2)/a/(a*x+b)+9*b^(
7/2)*arctan(a^(1/2)*x^(1/2)/b^(1/2))/a^(11/2)-9*b^3*x^(1/2)/a^5
```

Rubi [A] time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {263, 47, 50, 63, 205}

$$\frac{3b^2x^{3/2}}{a^4}-\frac{9b^3\sqrt{x}}{a^5}+\frac{9b^{7/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{11/2}}-\frac{9bx^{5/2}}{5a^3}+\frac{9x^{7/2}}{7a^2}-\frac{x^{9/2}}{a(ax+b)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b/x)^2,x]
```

```
[Out] (-9*b^3*Sqrt[x])/a^5 + (3*b^2*x^(3/2))/a^4 - (9*b*x^(5/2))/(5*a^3) + (9*x^(
7/2))/(7*a^2) - x^(9/2)/(a*(b + a*x)) + (9*b^(7/2)*ArcTan[(Sqrt[a]*Sqrt[x])
/Sqrt[b]])/a^(11/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\int \frac{x^{5/2}}{\left(a+\frac{b}{x}\right)^2}\,dx = \int \frac{x^{9/2}}{(b+ax)^2}\,dx$$

$$= -\frac{x^{9/2}}{a(b+ax)} + \frac{9\int \frac{x^{7/2}}{b+ax}\,dx}{2a}$$

$$= \frac{9x^{7/2}}{7a^2} - \frac{x^{9/2}}{a(b+ax)} - \frac{(9b)\int \frac{x^{5/2}}{b+ax}\,dx}{2a^2}$$

$$= -\frac{9bx^{5/2}}{5a^3} + \frac{9x^{7/2}}{7a^2} - \frac{x^{9/2}}{a(b+ax)} + \frac{\left(9b^2\right)\int \frac{x^{3/2}}{b+ax}\,dx}{2a^3}$$

$$= \frac{3b^2x^{3/2}}{a^4} - \frac{9bx^{5/2}}{5a^3} + \frac{9x^{7/2}}{7a^2} - \frac{x^{9/2}}{a(b+ax)} - \frac{\left(9b^3\right)\int \frac{\sqrt{x}}{b+ax}\,dx}{2a^4}$$

$$= -\frac{9b^3\sqrt{x}}{a^5} + \frac{3b^2x^{3/2}}{a^4} - \frac{9bx^{5/2}}{5a^3} + \frac{9x^{7/2}}{7a^2} - \frac{x^{9/2}}{a(b+ax)} + \frac{\left(9b^4\right)\int \frac{1}{\sqrt{x}\,(b+ax)}\,dx}{2a^5}$$

$$= -\frac{9b^3\sqrt{x}}{a^5} + \frac{3b^2x^{3/2}}{a^4} - \frac{9bx^{5/2}}{5a^3} + \frac{9x^{7/2}}{7a^2} - \frac{x^{9/2}}{a(b+ax)} + \frac{\left(9b^4\right)\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \sqrt{x}\right)}{a^5}$$

$$= -\frac{9b^3\sqrt{x}}{a^5} + \frac{3b^2x^{3/2}}{a^4} - \frac{9bx^{5/2}}{5a^3} + \frac{9x^{7/2}}{7a^2} - \frac{x^{9/2}}{a(b+ax)} + \frac{9b^{7/2}\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{11/2}}$$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.28

$$\frac{2x^{11/2}\,{}_2F_1\left(2, \frac{11}{2}; \frac{13}{2}; -\frac{ax}{b}\right)}{11b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b/x)^2,x]
```

```
[Out] (2*x^(11/2)*Hypergeometric2F1[2, 11/2, 13/2, -((a*x)/b)])/(11*b^2)
```

fricas [A] time = 1.01, size = 209, normalized size = 2.13

$$\left[\frac{315\left(ab^3x+b^4\right)\sqrt{-\frac{b}{a}}\,\log\left(\frac{ax+2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-b}{ax+b}\right)+2\left(10\,a^4x^4-18\,a^3bx^3+42\,a^2b^2x^2-210\,ab^3x-315\,b^4\right)\sqrt{x}}{70\left(a^6x+a^5b\right)}, \frac{315\left(ab^{3}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(a+b/x)^2,x, algorithm="fricas")
```

```
[Out] [1/70*(315*(a*b^3*x + b^4)*sqrt(-b/a)*log((a*x + 2*a*sqrt(x)*sqrt(-b/a) - b
)/(a*x + b)) + 2*(10*a^4*x^4 - 18*a^3*b*x^3 + 42*a^2*b^2*x^2 - 210*a*b^3*x
- 315*b^4)*sqrt(x))/(a^6*x + a^5*b), 1/35*(315*(a*b^3*x + b^4)*sqrt(b/a)*ar
ctan(a*sqrt(x)*sqrt(b/a)/b) + (10*a^4*x^4 - 18*a^3*b*x^3 + 42*a^2*b^2*x^2 -
 210*a*b^3*x - 315*b^4)*sqrt(x))/(a^6*x + a^5*b)]
```

giac [A] time = 0.15, size = 88, normalized size = 0.90

$$\frac{9\,b^4 \arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^5} - \frac{b^4\sqrt{x}}{(ax+b)a^5} + \frac{2\left(5\,a^{12}x^{\frac{7}{2}} - 14\,a^{11}bx^{\frac{5}{2}} + 35\,a^{10}b^2x^{\frac{3}{2}} - 140\,a^9b^3\sqrt{x}\right)}{35\,a^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(a+b/x)^2,x, algorithm="giac")
```

```
[Out] 9*b^4*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^5) - b^4*sqrt(x)/((a*x + b)*
a^5) + 2/35*(5*a^12*x^(7/2) - 14*a^11*b*x^(5/2) + 35*a^10*b^2*x^(3/2) - 140
*a^9*b^3*sqrt(x))/a^14
```

maple [A] time = 0.01, size = 83, normalized size = 0.85

$$\frac{2x^{\frac{7}{2}}}{7a^2} - \frac{4b\,x^{\frac{5}{2}}}{5a^3} + \frac{9b^4 \arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^5} + \frac{2b^2x^{\frac{3}{2}}}{a^4} - \frac{b^4\sqrt{x}}{(ax+b)\,a^5} - \frac{8b^3\sqrt{x}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a+b/x)^2,x)
```

```
[Out] 2/7*x^(7/2)/a^2-4/5*b*x^(5/2)/a^3+2*b^2*x^(3/2)/a^4-8*b^3*x^(1/2)/a^5-1/a^5
*b^4*x^(1/2)/(a*x+b)+9/a^5*b^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*a*x^(1/2))
```

maxima [A] time = 2.39, size = 88, normalized size = 0.90

$$\frac{10\,a^4 - \frac{18\,a^3b}{x} + \frac{42\,a^2b^2}{x^2} - \frac{210\,ab^3}{x^3} - \frac{315\,b^4}{x^4}}{35\left(\frac{a^6}{x^{\frac{7}{2}}} + \frac{a^5b}{x^{\frac{9}{2}}}\right)} - \frac{9\,b^4 \arctan\left(\frac{b}{\sqrt{ab}\,\sqrt{x}}\right)}{\sqrt{ab}\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(a+b/x)^2,x, algorithm="maxima")
```

```
[Out] 1/35*(10*a^4 - 18*a^3*b/x + 42*a^2*b^2/x^2 - 210*a*b^3/x^3 - 315*b^4/x^4)/(
a^6/x^(7/2) + a^5*b/x^(9/2)) - 9*b^4*arctan(b/(sqrt(a*b)*sqrt(x)))/(sqrt(a*
b)*a^5)
```

mupad [B] time = 1.08, size = 80, normalized size = 0.82

$$\frac{2\,x^{7/2}}{7\,a^2} - \frac{4\,b\,x^{5/2}}{5\,a^3} + \frac{2\,b^2\,x^{3/2}}{a^4} - \frac{8\,b^3\,\sqrt{x}}{a^5} - \frac{b^4\,\sqrt{x}}{x\,a^6 + b\,a^5} + \frac{9\,b^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b/x)^2,x)
```

```
[Out] (2*x^(7/2))/(7*a^2) - (4*b*x^(5/2))/(5*a^3) + (2*b^2*x^(3/2))/a^4 - (8*b^3*
x^(1/2))/a^5 - (b^4*x^(1/2))/(a^5*b + a^6*x) + (9*b^(7/2)*atan((a^(1/2)*x^(
1/2))/b^(1/2)))/a^(11/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(a+b/x)**2,x)
```

```
[Out] Timed out
```

3.1674 $\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^2}\,dx$

Optimal. Leaf size=85

$$-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{9/2}}+\frac{7b^2\sqrt{x}}{a^4}-\frac{7bx^{3/2}}{3a^3}+\frac{7x^{5/2}}{5a^2}-\frac{x^{7/2}}{a(ax+b)}$$

```
[Out] -7/3*b*x^(3/2)/a^3+7/5*x^(5/2)/a^2-x^(7/2)/a/(a*x+b)-7*b^(5/2)*arctan(a^(1/
2)*x^(1/2)/b^(1/2))/a^(9/2)+7*b^2*x^(1/2)/a^4
```

Rubi [A] time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {263, 47, 50, 63, 205}

$$\frac{7b^2\sqrt{x}}{a^4}-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{9/2}}-\frac{7bx^{3/2}}{3a^3}+\frac{7x^{5/2}}{5a^2}-\frac{x^{7/2}}{a(ax+b)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b/x)^2,x]
```

```
[Out] (7*b^2*Sqrt[x])/a^4 - (7*b*x^(3/2))/(3*a^3) + (7*x^(5/2))/(5*a^2) - x^(7/2)
/(a*(b + a*x)) - (7*b^(5/2)*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/a^(9/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^2}\,dx = \int \frac{x^{7/2}}{(b+ax)^2}\,dx$$

$$= -\frac{x^{7/2}}{a(b+ax)} + \frac{7\int \frac{x^{5/2}}{b+ax}\,dx}{2a}$$

$$= \frac{7x^{5/2}}{5a^2} - \frac{x^{7/2}}{a(b+ax)} - \frac{(7b)\int \frac{x^{3/2}}{b+ax}\,dx}{2a^2}$$

$$= -\frac{7bx^{3/2}}{3a^3} + \frac{7x^{5/2}}{5a^2} - \frac{x^{7/2}}{a(b+ax)} + \frac{\left(7b^2\right)\int \frac{\sqrt{x}}{b+ax}\,dx}{2a^3}$$

$$= \frac{7b^2\sqrt{x}}{a^4} - \frac{7bx^{3/2}}{3a^3} + \frac{7x^{5/2}}{5a^2} - \frac{x^{7/2}}{a(b+ax)} - \frac{\left(7b^3\right)\int \frac{1}{\sqrt{x}\,(b+ax)}\,dx}{2a^4}$$

$$= \frac{7b^2\sqrt{x}}{a^4} - \frac{7bx^{3/2}}{3a^3} + \frac{7x^{5/2}}{5a^2} - \frac{x^{7/2}}{a(b+ax)} - \frac{\left(7b^3\right)\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \sqrt{x}\right)}{a^4}$$

$$= \frac{7b^2\sqrt{x}}{a^4} - \frac{7bx^{3/2}}{3a^3} + \frac{7x^{5/2}}{5a^2} - \frac{x^{7/2}}{a(b+ax)} - \frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{9/2}}$$

Mathematica [C] time = 0.00, size = 27, normalized size = 0.32

$$\frac{2x^{9/2}\,{}_2F_1\left(2, \frac{9}{2}; \frac{11}{2}; -\frac{ax}{b}\right)}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b/x)^2,x]
```

```
[Out] (2*x^(9/2)*Hypergeometric2F1[2, 9/2, 11/2, -((a*x)/b)])/(9*b^2)
```

fricas [A] time = 0.63, size = 188, normalized size = 2.21

$$\left[\frac{105\left(ab^2x+b^3\right)\sqrt{-\frac{b}{a}}\log\left(\frac{ax-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-b}{ax+b}\right)+2\left(6\,a^3x^3-14\,a^2bx^2+70\,ab^2x+105\,b^3\right)\sqrt{x}}{30\left(a^5x+a^4b\right)}, -\frac{105\left(ab^2x+b^3\right)\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(a+b/x)^2,x, algorithm="fricas")
```

```
[Out] [1/30*(105*(a*b^2*x + b^3)*sqrt(-b/a)*log((a*x - 2*a*sqrt(x)*sqrt(-b/a) - b
)/(a*x + b)) + 2*(6*a^3*x^3 - 14*a^2*b*x^2 + 70*a*b^2*x + 105*b^3)*sqrt(x))
/(a^5*x + a^4*b), -1/15*(105*(a*b^2*x + b^3)*sqrt(b/a)*arctan(a*sqrt(x)*sqr
t(b/a)/b) - (6*a^3*x^3 - 14*a^2*b*x^2 + 70*a*b^2*x + 105*b^3)*sqrt(x))/(a^5
*x + a^4*b)]
```

giac [A] time = 0.16, size = 76, normalized size = 0.89

$$-\frac{7\,b^3\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^4}+\frac{b^3\sqrt{x}}{(ax+b)a^4}+\frac{2\left(3\,a^8x^{\frac{5}{2}}-10\,a^7bx^{\frac{3}{2}}+45\,a^6b^2\sqrt{x}\right)}{15\,a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(a+b/x)^2,x, algorithm="giac")
```

```
[Out] -7*b^3*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^4) + b^3*sqrt(x)/((a*x + b)
*a^4) + 2/15*(3*a^8*x^(5/2) - 10*a^7*b*x^(3/2) + 45*a^6*b^2*sqrt(x))/a^10
```

maple [A] time = 0.01, size = 71, normalized size = 0.84

$$\frac{2x^{\frac{5}{2}}}{5a^2}-\frac{7b^3\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^4}-\frac{4b\,x^{\frac{3}{2}}}{3a^3}+\frac{b^3\sqrt{x}}{(ax+b)\,a^4}+\frac{6b^2\sqrt{x}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a+b/x)^2,x)
```

```
[Out] 2/5*x^(5/2)/a^2-4/3*b*x^(3/2)/a^3+6*b^2*x^(1/2)/a^4+1/a^4*b^3*x^(1/2)/(a*x+
b)-7/a^4*b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*a*x^(1/2))
```

maxima [A] time = 2.56, size = 77, normalized size = 0.91

$$\frac{6\,a^3-\frac{14\,a^2b}{x}+\frac{70\,ab^2}{x^2}+\frac{105\,b^3}{x^3}}{15\left(\frac{a^5}{x^{\frac{5}{2}}}+\frac{a^4b}{x^{\frac{7}{2}}}\right)}+\frac{7\,b^3\arctan\left(\frac{b}{\sqrt{ab}\,\sqrt{x}}\right)}{\sqrt{ab}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(a+b/x)^2,x, algorithm="maxima")
```

```
[Out] 1/15*(6*a^3 - 14*a^2*b/x + 70*a*b^2/x^2 + 105*b^3/x^3)/(a^5/x^(5/2) + a^4*b
/x^(7/2)) + 7*b^3*arctan(b/(sqrt(a*b)*sqrt(x)))/(sqrt(a*b)*a^4)
```

mupad [B] time = 0.04, size = 68, normalized size = 0.80

$$\frac{2\,x^{5/2}}{5\,a^2}-\frac{4\,b\,x^{3/2}}{3\,a^3}+\frac{6\,b^2\,\sqrt{x}}{a^4}+\frac{b^3\,\sqrt{x}}{x\,a^5+b\,a^4}-\frac{7\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\;\sqrt{x}}{\sqrt{b}}\right)}{a^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b/x)^2,x)
```

```
[Out] (2*x^(5/2))/(5*a^2) - (4*b*x^(3/2))/(3*a^3) + (6*b^2*x^(1/2))/a^4 + (b^3*x^
(1/2))/(a^4*b + a^5*x) - (7*b^(5/2)*atan((a^(1/2)*x^(1/2))/b^(1/2)))/a^(9/2
)
```

sympy [A] time = 57.53, size = 542, normalized size = 6.38

$$\begin{cases}\tilde{\infty}x^{\frac{9}{2}} & \\ \frac{2x^{\frac{9}{2}}}{9b^2} & \\ \frac{2x^{\frac{5}{2}}}{5a^2} & \\ \frac{12ia^4\sqrt{b}\,x^{\frac{7}{2}}\sqrt{\frac{1}{a}}}{30ia^6\sqrt{b}\,x\sqrt{\frac{1}{a}}+30ia^5b^{\frac{3}{2}}\sqrt{\frac{1}{a}}}-\frac{28ia^3b^{\frac{3}{2}}x^{\frac{5}{2}}\sqrt{\frac{1}{a}}}{30ia^6\sqrt{b}\,x\sqrt{\frac{1}{a}}+30ia^5b^{\frac{3}{2}}\sqrt{\frac{1}{a}}}+\frac{140ia^2b^{\frac{5}{2}}x^{\frac{3}{2}}\sqrt{\frac{1}{a}}}{30ia^6\sqrt{b}\,x\sqrt{\frac{1}{a}}+30ia^5b^{\frac{3}{2}}\sqrt{\frac{1}{a}}}+\frac{210iab^{\frac{7}{2}}\sqrt{x}\sqrt{\frac{1}{a}}}{30ia^6\sqrt{b}\,x\sqrt{\frac{1}{a}}+30ia^5b^{\frac{3}{2}}\sqrt{\frac{1}{a}}}-\frac{105ab^3x\log\left(-i\sqrt{\;}\right.}{30ia^6\sqrt{b}\,x\sqrt{\frac{1}{a}}+} & \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(a+b/x)**2,x)
```

```
[Out] Piecewise((zoo*x**(9/2), Eq(a, 0) & Eq(b, 0)), (2*x**(9/2)/(9*b**2), Eq(a,
0)), (2*x**(5/2)/(5*a**2), Eq(b, 0)), (12*I*a**4*sqrt(b)*x**(7/2)*sqrt(1/a)
/(30*I*a**6*sqrt(b)*x*sqrt(1/a) + 30*I*a**5*b**(3/2)*sqrt(1/a)) - 28*I*a**3
*b**(3/2)*x**(5/2)*sqrt(1/a)/(30*I*a**6*sqrt(b)*x*sqrt(1/a) + 30*I*a**5*b**
(3/2)*sqrt(1/a)) + 140*I*a**2*b**(5/2)*x**(3/2)*sqrt(1/a)/(30*I*a**6*sqrt(b
)*x*sqrt(1/a) + 30*I*a**5*b**(3/2)*sqrt(1/a)) + 210*I*a*b**(7/2)*sqrt(x)*sq
rt(1/a)/(30*I*a**6*sqrt(b)*x*sqrt(1/a) + 30*I*a**5*b**(3/2)*sqrt(1/a)) - 10
5*a*b**3*x*log(-I*sqrt(b)*sqrt(1/a) + sqrt(x))/(30*I*a**6*sqrt(b)*x*sqrt(1/
a) + 30*I*a**5*b**(3/2)*sqrt(1/a)) + 105*a*b**3*x*log(I*sqrt(b)*sqrt(1/a) +
 sqrt(x))/(30*I*a**6*sqrt(b)*x*sqrt(1/a) + 30*I*a**5*b**(3/2)*sqrt(1/a)) -
105*b**4*log(-I*sqrt(b)*sqrt(1/a) + sqrt(x))/(30*I*a**6*sqrt(b)*x*sqrt(1/a)
 + 30*I*a**5*b**(3/2)*sqrt(1/a)) + 105*b**4*log(I*sqrt(b)*sqrt(1/a) + sqrt(
x))/(30*I*a**6*sqrt(b)*x*sqrt(1/a) + 30*I*a**5*b**(3/2)*sqrt(1/a)), True))
```

3.1675 $\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^2}\,dx$

Optimal. Leaf size=70

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{7/2}}-\frac{5b\sqrt{x}}{a^3}+\frac{5x^{3/2}}{3a^2}-\frac{x^{5/2}}{a(ax+b)}$$

```
[Out] 5/3*x^(3/2)/a^2-x^(5/2)/a/(a*x+b)+5*b^(3/2)*arctan(a^(1/2)*x^(1/2)/b^(1/2))
/a^(7/2)-5*b*x^(1/2)/a^3
```

Rubi [A] time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {263, 47, 50, 63, 205}

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{7/2}}-\frac{5b\sqrt{x}}{a^3}+\frac{5x^{3/2}}{3a^2}-\frac{x^{5/2}}{a(ax+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b/x)^2,x]
```

```
[Out] (-5*b*Sqrt[x])/a^3 + (5*x^(3/2))/(3*a^2) - x^(5/2)/(a*(b + a*x)) + (5*b^(3/
2)*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/a^(7/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
 + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^2}\,dx &= \int \frac{x^{5/2}}{(b+ax)^2}\,dx \\
&= -\frac{x^{5/2}}{a(b+ax)} + \frac{5\int \frac{x^{3/2}}{b+ax}\,dx}{2a} \\
&= \frac{5x^{3/2}}{3a^2} - \frac{x^{5/2}}{a(b+ax)} - \frac{(5b)\int \frac{\sqrt{x}}{b+ax}\,dx}{2a^2} \\
&= -\frac{5b\sqrt{x}}{a^3} + \frac{5x^{3/2}}{3a^2} - \frac{x^{5/2}}{a(b+ax)} + \frac{\left(5b^2\right)\int \frac{1}{\sqrt{x}\,(b+ax)}\,dx}{2a^3} \\
&= -\frac{5b\sqrt{x}}{a^3} + \frac{5x^{3/2}}{3a^2} - \frac{x^{5/2}}{a(b+ax)} + \frac{\left(5b^2\right)\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \sqrt{x}\right)}{a^3} \\
&= -\frac{5b\sqrt{x}}{a^3} + \frac{5x^{3/2}}{3a^2} - \frac{x^{5/2}}{a(b+ax)} + \frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{7/2}}
\end{aligned}$$

Mathematica [C] time = 0.00, size = 27, normalized size = 0.39

$$\frac{2x^{7/2}\,{}_2F_1\left(2, \frac{7}{2}; \frac{9}{2}; -\frac{ax}{b}\right)}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b/x)^2,x]
```

```
[Out] (2*x^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, -((a*x)/b)])/(7*b^2)
```

fricas [A] time = 0.92, size = 161, normalized size = 2.30

$$\left[\frac{15\left(abx+b^2\right)\sqrt{-\frac{b}{a}}\,\log\left(\frac{ax+2\,a\sqrt{x}\,\sqrt{-\frac{b}{a}}-b}{ax+b}\right)+2\left(2\,a^2x^2-10\,abx-15\,b^2\right)\sqrt{x}}{6\left(a^4x+a^3b\right)}, \frac{15\left(abx+b^2\right)\sqrt{\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{x}\,\sqrt{\frac{b}{a}}}{b}\right)}{3\left(a^4x+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(a+b/x)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(15*(a*b*x + b^2)*sqrt(-b/a)*log((a*x + 2*a*sqrt(x)*sqrt(-b/a) - b)/(a
*x + b)) + 2*(2*a^2*x^2 - 10*a*b*x - 15*b^2)*sqrt(x))/(a^4*x + a^3*b), 1/3*
(15*(a*b*x + b^2)*sqrt(b/a)*arctan(a*sqrt(x)*sqrt(b/a)/b) + (2*a^2*x^2 - 10
*a*b*x - 15*b^2)*sqrt(x))/(a^4*x + a^3*b)]
```

giac [A] time = 0.18, size = 65, normalized size = 0.93

$$\frac{5\,b^2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^3} - \frac{b^2\sqrt{x}}{(ax+b)a^3} + \frac{2\left(a^4x^{\frac{3}{2}} - 6\,a^3b\sqrt{x}\right)}{3\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(a+b/x)^2,x, algorithm="giac")
```

```
[Out] 5*b^2*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) - b^2*sqrt(x)/((a*x + b)*
a^3) + 2/3*(a^4*x^(3/2) - 6*a^3*b*sqrt(x))/a^6
```

maple [A] time = 0.01, size = 61, normalized size = 0.87

$$\frac{5b^2 \arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\ a^3} + \frac{2x^{\frac{3}{2}}}{3a^2} - \frac{b^2\sqrt{x}}{(ax+b)\,a^3} - \frac{4b\sqrt{x}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a+b/x)^2,x)
```

```
[Out] 2/3*x^(3/2)/a^2-4*b*x^(1/2)/a^3-1/a^3*b^2*x^(1/2)/(a*x+b)+5/a^3*b^2/(a*b)^(
1/2)*arctan(1/(a*b)^(1/2)*a*x^(1/2))
```

maxima [A] time = 2.35, size = 66, normalized size = 0.94

$$\frac{2\,a^2 - \frac{10\,ab}{x} - \frac{15\,b^2}{x^2}}{3\left(\frac{a^4}{x^{\frac{3}{2}}} + \frac{a^3 b}{x^{\frac{5}{2}}}\right)} - \frac{5\,b^2 \arctan\left(\frac{b}{\sqrt{ab}\,\sqrt{x}}\right)}{\sqrt{ab}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(a+b/x)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(2*a^2 - 10*a*b/x - 15*b^2/x^2)/(a^4/x^(3/2) + a^3*b/x^(5/2)) - 5*b^2*a
rctan(b/(sqrt(a*b)*sqrt(x)))/(sqrt(a*b)*a^3)
```

mupad [B] time = 0.06, size = 58, normalized size = 0.83

$$\frac{2\,x^{3/2}}{3\,a^2} - \frac{4\,b\,\sqrt{x}}{a^3} - \frac{b^2\,\sqrt{x}}{x\,a^4 + b\,a^3} + \frac{5\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b/x)^2,x)
```

```
[Out] (2*x^(3/2))/(3*a^2) - (4*b*x^(1/2))/a^3 - (b^2*x^(1/2))/(a^3*b + a^4*x) + (
5*b^(3/2)*atan((a^(1/2)*x^(1/2))/b^(1/2)))/a^(7/2)
```

sympy [A] time = 9.64, size = 479, normalized size = 6.84

$$\begin{cases} \tilde{\infty} x^{\frac{7}{2}} \\ \frac{2x^{\frac{7}{2}}}{7b^2} \\ \frac{2x^{\frac{3}{2}}}{3a^2} \\ \frac{4ia^3\sqrt{b}\,x^{\frac{5}{2}}\sqrt{\frac{1}{a}}}{6ia^5\sqrt{b}\,x\sqrt{\frac{1}{a}}+6ia^4b^{\frac{3}{2}}\sqrt{\frac{1}{a}}} - \frac{20ia^2b^{\frac{3}{2}}x^{\frac{3}{2}}\sqrt{\frac{1}{a}}}{6ia^5\sqrt{b}\,x\sqrt{\frac{1}{a}}+6ia^4b^{\frac{3}{2}}\sqrt{\frac{1}{a}}} - \frac{30iab^{\frac{5}{2}}\sqrt{x}\sqrt{\frac{1}{a}}}{6ia^5\sqrt{b}\,x\sqrt{\frac{1}{a}}+6ia^4b^{\frac{3}{2}}\sqrt{\frac{1}{a}}} + \frac{15ab^2x\log\left(-i\sqrt{b}\sqrt{\frac{1}{a}}+\sqrt{x}\right)}{6ia^5\sqrt{b}\,x\sqrt{\frac{1}{a}}+6ia^4b^{\frac{3}{2}}\sqrt{\frac{1}{a}}} - \frac{15ab^2x\log\left(i\sqrt{b}\sqrt{\frac{1}{a}}+\sqrt{}\right.}{6ia^5\sqrt{b}\,x\sqrt{\frac{1}{a}}+6ia^4b^{\frac{3}{2}}\sqrt{} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(a+b/x)**2,x)
```

```
[Out] Piecewise((zoo*x**(7/2), Eq(a, 0) & Eq(b, 0)), (2*x**(7/2)/(7*b**2), Eq(a,
0)), (2*x**(3/2)/(3*a**2), Eq(b, 0)), (4*I*a**3*sqrt(b)*x**(5/2)*sqrt(1/a)/
(6*I*a**5*sqrt(b)*x*sqrt(1/a) + 6*I*a**4*b**(3/2)*sqrt(1/a)) - 20*I*a**2*b*
*(3/2)*x**(3/2)*sqrt(1/a)/(6*I*a**5*sqrt(b)*x*sqrt(1/a) + 6*I*a**4*b**(3/2)
*sqrt(1/a)) - 30*I*a*b**(5/2)*sqrt(x)*sqrt(1/a)/(6*I*a**5*sqrt(b)*x*sqrt(1/
a) + 6*I*a**4*b**(3/2)*sqrt(1/a)) + 15*a*b**2*x*log(-I*sqrt(b)*sqrt(1/a) +
sqrt(x))/(6*I*a**5*sqrt(b)*x*sqrt(1/a) + 6*I*a**4*b**(3/2)*sqrt(1/a)) - 15*
a*b**2*x*log(I*sqrt(b)*sqrt(1/a) + sqrt(x))/(6*I*a**5*sqrt(b)*x*sqrt(1/a) +
 6*I*a**4*b**(3/2)*sqrt(1/a)) + 15*b**3*log(-I*sqrt(b)*sqrt(1/a) + sqrt(x))
/(6*I*a**5*sqrt(b)*x*sqrt(1/a) + 6*I*a**4*b**(3/2)*sqrt(1/a)) - 15*b**3*log
(I*sqrt(b)*sqrt(1/a) + sqrt(x))/(6*I*a**5*sqrt(b)*x*sqrt(1/a) + 6*I*a**4*b*
*(3/2)*sqrt(1/a)), True))
```

3.1676 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2 \sqrt{x}}\,dx$

Optimal. Leaf size=57

$$-\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{5/2}}+\frac{3\sqrt{x}}{a^2}-\frac{x^{3/2}}{a(ax+b)}$$

```
[Out] -x^(3/2)/a/(a*x+b)-3*arctan(a^(1/2)*x^(1/2)/b^(1/2))*b^(1/2)/a^(5/2)+3*x^(1
/2)/a^2
```

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {263, 47, 50, 63, 205}

$$-\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{5/2}}+\frac{3\sqrt{x}}{a^2}-\frac{x^{3/2}}{a(ax+b)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^2*Sqrt[x]),x]
```

```
[Out] (3*Sqrt[x])/a^2 - x^(3/2)/(a*(b + a*x)) - (3*Sqrt[b]*ArcTan[(Sqrt[a]*Sqrt[x
])/Sqrt[b]])/a^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x}\right)^2\sqrt{x}}\,dx &= \int \frac{x^{3/2}}{(b+ax)^2}\,dx \\ &= -\frac{x^{3/2}}{a(b+ax)} + \frac{3\int \frac{\sqrt{x}}{b+ax}\,dx}{2a} \\ &= \frac{3\sqrt{x}}{a^2} - \frac{x^{3/2}}{a(b+ax)} - \frac{(3b)\int \frac{1}{\sqrt{x}\,(b+ax)}\,dx}{2a^2} \\ &= \frac{3\sqrt{x}}{a^2} - \frac{x^{3/2}}{a(b+ax)} - \frac{(3b)\,\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \sqrt{x}\right)}{a^2} \\ &= \frac{3\sqrt{x}}{a^2} - \frac{x^{3/2}}{a(b+ax)} - \frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{5/2}}\end{aligned}$$

Mathematica [C] time = 0.00, size = 27, normalized size = 0.47

$$\frac{2x^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};-\frac{ax}{b}\right)}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^2*Sqrt[x]),x]
```

```
[Out] (2*x^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, -((a*x)/b)])/(5*b^2)
```

fricas [A] time = 0.91, size = 134, normalized size = 2.35

$$\left[\frac{3\,(ax+b)\sqrt{-\frac{b}{a}}\,\log\left(\frac{ax-2\,a\sqrt{x}\,\sqrt{-\frac{b}{a}}-b}{ax+b}\right)+2\,(2\,ax+3\,b)\sqrt{x}}{2\left(a^3x+a^2b\right)}, -\frac{3\,(ax+b)\sqrt{\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{x}\,\sqrt{\frac{b}{a}}}{b}\right)-(2\,ax+3\,b)\sqrt{x}}{a^3x+a^2b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*(a*x + b)*sqrt(-b/a)*log((a*x - 2*a*sqrt(x)*sqrt(-b/a) - b)/(a*x +
b)) + 2*(2*a*x + 3*b)*sqrt(x))/(a^3*x + a^2*b), -(3*(a*x + b)*sqrt(b/a)*arc
tan(a*sqrt(x)*sqrt(b/a)/b) - (2*a*x + 3*b)*sqrt(x))/(a^3*x + a^2*b)]
```

giac [A] time = 0.16, size = 46, normalized size = 0.81

$$-\frac{3\,b\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}+\frac{2\sqrt{x}}{a^2}+\frac{b\sqrt{x}}{(ax+b)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] -3*b*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) + 2*sqrt(x)/a^2 + b*sqrt(x
)/((a*x + b)*a^2)
```

maple [A] time = 0.01, size = 47, normalized size = 0.82

$$-\frac{3b\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}+\frac{b\sqrt{x}}{(ax+b)\,a^2}+\frac{2\sqrt{x}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^2/x^(1/2),x)
```

```
[Out] 2*x^(1/2)/a^2+1/a^2*b*x^(1/2)/(a*x+b)-3/a^2*b/(a*b)^(1/2)*arctan(1/(a*b)^(1
/2)*a*x^(1/2))
```

maxima [A] time = 2.35, size = 52, normalized size = 0.91

$$\frac{2\,a+\frac{3\,b}{x}}{\frac{a^3}{\sqrt{x}}+\frac{a^2b}{x^{\frac{3}{2}}}}+\frac{3\,b\arctan\left(\frac{b}{\sqrt{ab}\,\sqrt{x}}\right)}{\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] (2*a + 3*b/x)/(a^3/sqrt(x) + a^2*b/x^(3/2)) + 3*b*arctan(b/(sqrt(a*b)*sqrt(
x)))/(sqrt(a*b)*a^2)
```

mupad [B] time = 1.10, size = 46, normalized size = 0.81

$$\frac{2\,\sqrt{x}}{a^2}+\frac{b\,\sqrt{x}}{x\,a^3+b\,a^2}-\frac{3\,\sqrt{b}\,\operatorname{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b/x)^2),x)
```

```
[Out] (2*x^(1/2))/a^2 + (b*x^(1/2))/(a^2*b + a^3*x) - (3*b^(1/2)*atan((a^(1/2)*x^
(1/2))/b^(1/2)))/a^(5/2)
```

sympy [A] time = 12.34, size = 411, normalized size = 7.21

$$\begin{cases}\tilde{\infty}x^{\frac{5}{2}} & \\ \frac{2\sqrt{x}}{a^2} & \\ \frac{2x^{\frac{5}{2}}}{5b^2} & \\ \frac{4ia^2\sqrt{b}\,x^{\frac{3}{2}}\sqrt{\frac{1}{a}}}{2ia^4\sqrt{b}\,x\sqrt{\frac{1}{a}}+2ia^3b^{\frac{3}{2}}\sqrt{\frac{1}{a}}}+\frac{6iab^{\frac{3}{2}}\sqrt{x}\sqrt{\frac{1}{a}}}{2ia^4\sqrt{b}\,x\sqrt{\frac{1}{a}}+2ia^3b^{\frac{3}{2}}\sqrt{\frac{1}{a}}}-\frac{3abx\log\left(-i\sqrt{b}\sqrt{\frac{1}{a}}+\sqrt{x}\right)}{2ia^4\sqrt{b}\,x\sqrt{\frac{1}{a}}+2ia^3b^{\frac{3}{2}}\sqrt{\frac{1}{a}}}+\frac{3abx\log\left(i\sqrt{b}\sqrt{\frac{1}{a}}+\sqrt{x}\right)}{2ia^4\sqrt{b}\,x\sqrt{\frac{1}{a}}+2ia^3b^{\frac{3}{2}}\sqrt{\frac{1}{a}}}-\frac{3b^2\log\left(-i\sqrt{b}\sqrt{\frac{1}{a}}+\sqrt{x}\right)}{2ia^4\sqrt{b}\,x\sqrt{\frac{1}{a}}+2ia^3b^{\frac{3}{2}}\sqrt{\frac{1}{a}}} & \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**2/x**(1/2),x)
```

```
[Out] Piecewise((zoo*x**(5/2), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/a**2, Eq(b, 0)),
(2*x**(5/2)/(5*b**2), Eq(a, 0)), (4*I*a**2*sqrt(b)*x**(3/2)*sqrt(1/a)/(2*I*
a**4*sqrt(b)*x*sqrt(1/a) + 2*I*a**3*b**(3/2)*sqrt(1/a)) + 6*I*a*b**(3/2)*sq
```

```
rt(x)*sqrt(1/a)/(2*I*a**4*sqrt(b)*x*sqrt(1/a) + 2*I*a**3*b**(3/2)*sqrt(1/a)
) - 3*a*b*x*log(-I*sqrt(b)*sqrt(1/a) + sqrt(x))/(2*I*a**4*sqrt(b)*x*sqrt(1/
a) + 2*I*a**3*b**(3/2)*sqrt(1/a)) + 3*a*b*x*log(I*sqrt(b)*sqrt(1/a) + sqrt(
x))/(2*I*a**4*sqrt(b)*x*sqrt(1/a) + 2*I*a**3*b**(3/2)*sqrt(1/a)) - 3*b**2*l
og(-I*sqrt(b)*sqrt(1/a) + sqrt(x))/(2*I*a**4*sqrt(b)*x*sqrt(1/a) + 2*I*a**3
*b**(3/2)*sqrt(1/a)) + 3*b**2*log(I*sqrt(b)*sqrt(1/a) + sqrt(x))/(2*I*a**4*
sqrt(b)*x*sqrt(1/a) + 2*I*a**3*b**(3/2)*sqrt(1/a)), True))
```

3.1677 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{3/2}}\,dx$

Optimal. Leaf size=46

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{3/2}\sqrt{b}} - \frac{\sqrt{x}}{a(ax+b)}$$

```
[Out] arctan(a^(1/2)*x^(1/2)/b^(1/2))/a^(3/2)/b^(1/2)-x^(1/2)/a/(a*x+b)
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {263, 47, 63, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{3/2}\sqrt{b}} - \frac{\sqrt{x}}{a(ax+b)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^2*x^(3/2)),x]
```

```
[Out] -(Sqrt[x]/(a*(b + a*x))) + ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/(a^(3/2)*Sqrt[
b])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{3/2}}\,dx = \int \frac{\sqrt{x}}{(b+ax)^2}\,dx$$

$$= -\frac{\sqrt{x}}{a(b+ax)} + \frac{\int \frac{1}{\sqrt{x}\,(b+ax)}\,dx}{2a}$$

$$= -\frac{\sqrt{x}}{a(b+ax)} + \frac{\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \sqrt{x}\right)}{a}$$

$$= -\frac{\sqrt{x}}{a(b+ax)} + \frac{\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{3/2}\sqrt{b}}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{3/2}\sqrt{b}} - \frac{\sqrt{x}}{a(ax+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^2*x^(3/2)),x]
```

```
[Out] -(Sqrt[x]/(a*(b + a*x))) + ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/(a^(3/2)*Sqrt[
b])
```

fricas [A] time = 1.12, size = 115, normalized size = 2.50

$$\left[-\frac{2\,ab\sqrt{x} + \sqrt{-ab}\,(ax+b)\log\left(\frac{ax-b-2\sqrt{-ab}\,\sqrt{x}}{ax+b}\right)}{2\left(a^3bx+a^2b^2\right)}, -\frac{ab\sqrt{x} + \sqrt{ab}\,(ax+b)\arctan\left(\frac{\sqrt{ab}}{a\sqrt{x}}\right)}{a^3bx+a^2b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*b*sqrt(x) + sqrt(-a*b)*(a*x + b)*log((a*x - b - 2*sqrt(-a*b)*sqr
t(x))/(a*x + b)))/(a^3*b*x + a^2*b^2), -(a*b*sqrt(x) + sqrt(a*b)*(a*x + b)*
arctan(sqrt(a*b)/(a*sqrt(x))))/(a^3*b*x + a^2*b^2)]
```

giac [A] time = 0.15, size = 36, normalized size = 0.78

$$\frac{\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a} - \frac{\sqrt{x}}{(ax+b)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x^(3/2),x, algorithm="giac")
```

```
[Out] arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a) - sqrt(x)/((a*x + b)*a)
```

maple [A] time = 0.01, size = 37, normalized size = 0.80

$$\frac{\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\;a} - \frac{\sqrt{x}}{(ax+b)\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^2/x^(3/2),x)
[Out] -x^(1/2)/a/(a*x+b)+1/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*a*x^(1/2))
```

maxima [A] time = 2.35, size = 39, normalized size = 0.85

$$-\frac{\arctan\left(\frac{b}{\sqrt{ab}\,\sqrt{x}}\right)}{\sqrt{ab}\,a}-\frac{1}{\left(a^2+\frac{ab}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x^(3/2),x, algorithm="maxima")
[Out] -arctan(b/(sqrt(a*b)*sqrt(x)))/(sqrt(a*b)*a) - 1/((a^2 + a*b/x)*sqrt(x))
```

mupad [B] time = 0.04, size = 34, normalized size = 0.74

$$\frac{\operatorname{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{3/2}\,\sqrt{b}}-\frac{\sqrt{x}}{a\,(b+a\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b/x)^2),x)
[Out] atan((a^(1/2)*x^(1/2))/b^(1/2))/(a^(3/2)*b^(1/2)) - x^(1/2)/(a*(b + a*x))
```

sympy [A] time = 28.92, size = 337, normalized size = 7.33

$$\begin{cases}\tilde{\infty}x^{\frac{3}{2}} & \\ -\frac{2}{a^2\sqrt{x}} & \\ \frac{2x^{\frac{3}{2}}}{3b^2} & \\ -\frac{2ia\sqrt{b}\,\sqrt{x}\,\sqrt{\frac{1}{a}}}{2ia^3\sqrt{b}\,x\sqrt{\frac{1}{a}}+2ia^2b^{\frac{3}{2}}\sqrt{\frac{1}{a}}}+\frac{ax\log\left(-i\sqrt{b}\,\sqrt{\frac{1}{a}}+\sqrt{x}\right)}{2ia^3\sqrt{b}\,x\sqrt{\frac{1}{a}}+2ia^2b^{\frac{3}{2}}\sqrt{\frac{1}{a}}}-\frac{ax\log\left(i\sqrt{b}\,\sqrt{\frac{1}{a}}+\sqrt{x}\right)}{2ia^3\sqrt{b}\,x\sqrt{\frac{1}{a}}+2ia^2b^{\frac{3}{2}}\sqrt{\frac{1}{a}}}+\frac{b\log\left(-i\sqrt{b}\,\sqrt{\frac{1}{a}}+\sqrt{x}\right)}{2ia^3\sqrt{b}\,x\sqrt{\frac{1}{a}}+2ia^2b^{\frac{3}{2}}\sqrt{\frac{1}{a}}}-\frac{b\log\left(i\sqrt{b}\,\sqrt{\frac{1}{a}}+\sqrt{x}\right)}{2ia^3\sqrt{b}\,x\sqrt{\frac{1}{a}}+2ia^2b^{\frac{3}{2}}\sqrt{} & \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**2/x**(3/2),x)
[Out] Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (-2/(a**2*sqrt(x)), Eq(b, 0
), (2*x**(3/2)/(3*b**2), Eq(a, 0)), (-2*I*a*sqrt(b)*sqrt(x)*sqrt(1/a)/(2*I*
a**3*sqrt(b)*x*sqrt(1/a) + 2*I*a**2*b**(3/2)*sqrt(1/a)) + a*x*log(-I*sqrt(b
)*sqrt(1/a) + sqrt(x))/(2*I*a**3*sqrt(b)*x*sqrt(1/a) + 2*I*a**2*b**(3/2)*sq
rt(1/a)) - a*x*log(I*sqrt(b)*sqrt(1/a) + sqrt(x))/(2*I*a**3*sqrt(b)*x*sqrt(
1/a) + 2*I*a**2*b**(3/2)*sqrt(1/a)) + b*log(-I*sqrt(b)*sqrt(1/a) + sqrt(x))
/(2*I*a**3*sqrt(b)*x*sqrt(1/a) + 2*I*a**2*b**(3/2)*sqrt(1/a)) - b*log(I*sqr
t(b)*sqrt(1/a) + sqrt(x))/(2*I*a**3*sqrt(b)*x*sqrt(1/a) + 2*I*a**2*b**(3/2)
*sqrt(1/a)), True))
```

3.1678 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{5/2}}\,dx$

Optimal. Leaf size=45

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\,b^{3/2}}+\frac{\sqrt{x}}{b(ax+b)}$$

```
[Out] arctan(a^(1/2)*x^(1/2)/b^(1/2))/b^(3/2)/a^(1/2)+x^(1/2)/b/(a*x+b)
```

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {263, 51, 63, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\,b^{3/2}}+\frac{\sqrt{x}}{b(ax+b)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^2*x^(5/2)),x]
```

```
[Out] Sqrt[x]/(b*(b + a*x)) + ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/(Sqrt[a]*b^(3/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{5/2}}\,dx = \int \frac{1}{\sqrt{x}\,(b+ax)^2}\,dx$$

$$= \frac{\sqrt{x}}{b(b+ax)} + \frac{\int \frac{1}{\sqrt{x}\,(b+ax)}\,dx}{2b}$$

$$= \frac{\sqrt{x}}{b(b+ax)} + \frac{\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \sqrt{x}\right)}{b}$$

$$= \frac{\sqrt{x}}{b(b+ax)} + \frac{\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\,b^{3/2}}$$

Mathematica [A] time = 0.03, size = 45, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\,b^{3/2}} + \frac{\sqrt{x}}{abx+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^2*x^(5/2)),x]
```

```
[Out] Sqrt[x]/(b^2 + a*b*x) + ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/(Sqrt[a]*b^(3/2))
```

fricas [A] time = 0.91, size = 116, normalized size = 2.58

$$\left[\frac{2\,ab\sqrt{x} - \sqrt{-ab}\,(ax+b)\log\left(\frac{ax-b-2\sqrt{-ab}\,\sqrt{x}}{ax+b}\right)}{2\left(a^2b^2x+ab^3\right)}, \frac{ab\sqrt{x} - \sqrt{ab}\,(ax+b)\arctan\left(\frac{\sqrt{ab}}{a\sqrt{x}}\right)}{a^2b^2x+ab^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b*sqrt(x) - sqrt(-a*b)*(a*x + b)*log((a*x - b - 2*sqrt(-a*b)*sqrt
(x))/(a*x + b)))/(a^2*b^2*x + a*b^3), (a*b*sqrt(x) - sqrt(a*b)*(a*x + b)*ar
ctan(sqrt(a*b)/(a*sqrt(x))))/(a^2*b^2*x + a*b^3)]
```

giac [A] time = 0.16, size = 35, normalized size = 0.78

$$\frac{\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b} + \frac{\sqrt{x}}{(ax+b)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x^(5/2),x, algorithm="giac")
```

```
[Out] arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b) + sqrt(x)/((a*x + b)*b)
```

maple [A] time = 0.01, size = 36, normalized size = 0.80

$$\frac{\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\;b} + \frac{\sqrt{x}}{(ax+b)\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^2/x^(5/2),x)
[Out] x^(1/2)/b/(a*x+b)+1/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*a*x^(1/2))
```

maxima [A] time = 2.33, size = 39, normalized size = 0.87

$$-\frac{\arctan\left(\frac{b}{\sqrt{ab}\,\sqrt{x}}\right)}{\sqrt{ab}\,b}+\frac{1}{\left(ab+\frac{b^2}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x^(5/2),x, algorithm="maxima")
[Out] -arctan(b/(sqrt(a*b)*sqrt(x)))/(sqrt(a*b)*b) + 1/((a*b + b^2/x)*sqrt(x))
```

mupad [B] time = 1.10, size = 33, normalized size = 0.73

$$\frac{\sqrt{x}}{b\,(b+a\,x)}+\frac{\operatorname{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\;b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b/x)^2),x)
[Out] x^(1/2)/(b*(b + a*x)) + atan((a^(1/2)*x^(1/2))/b^(1/2))/(a^(1/2)*b^(3/2))
```

sympy [A] time = 102.66, size = 328, normalized size = 7.29

$$\begin{cases}\tilde{\infty}\sqrt{x} & \text{f}\\ \frac{2\sqrt{x}}{b^2} & \text{f}\\ -\frac{2}{3a^2x^{\frac{3}{2}}} & \text{f}\\ \frac{2ia\sqrt{b}\,\sqrt{x}\,\sqrt{\frac{1}{a}}}{2ia^2b^{\frac{3}{2}}x\sqrt{\frac{1}{a}}+2iab^{\frac{5}{2}}\sqrt{\frac{1}{a}}}+\frac{ax\log\left(-i\sqrt{b}\,\sqrt{\frac{1}{a}}+\sqrt{x}\right)}{2ia^2b^{\frac{3}{2}}x\sqrt{\frac{1}{a}}+2iab^{\frac{5}{2}}\sqrt{\frac{1}{a}}}-\frac{ax\log\left(i\sqrt{b}\,\sqrt{\frac{1}{a}}+\sqrt{x}\right)}{2ia^2b^{\frac{3}{2}}x\sqrt{\frac{1}{a}}+2iab^{\frac{5}{2}}\sqrt{\frac{1}{a}}}+\frac{b\log\left(-i\sqrt{b}\,\sqrt{\frac{1}{a}}+\sqrt{x}\right)}{2ia^2b^{\frac{3}{2}}x\sqrt{\frac{1}{a}}+2iab^{\frac{5}{2}}\sqrt{\frac{1}{a}}}-\frac{b\log\left(i\sqrt{b}\,\sqrt{\frac{1}{a}}+\sqrt{x}\right)}{2ia^2b^{\frac{3}{2}}x\sqrt{\frac{1}{a}}+2iab^{\frac{5}{2}}\sqrt{\frac{1}{a}}} & \text{c}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**2/x**(5/2),x)
[Out] Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/b**2, Eq(a, 0)), (
-2/(3*a**2*x**(3/2)), Eq(b, 0)), (2*I*a*sqrt(b)*sqrt(x)*sqrt(1/a)/(2*I*a**2
*b**(3/2)*x*sqrt(1/a) + 2*I*a*b**(5/2)*sqrt(1/a)) + a*x*log(-I*sqrt(b)*sqrt
(1/a) + sqrt(x))/(2*I*a**2*b**(3/2)*x*sqrt(1/a) + 2*I*a*b**(5/2)*sqrt(1/a))
 - a*x*log(I*sqrt(b)*sqrt(1/a) + sqrt(x))/(2*I*a**2*b**(3/2)*x*sqrt(1/a) +
2*I*a*b**(5/2)*sqrt(1/a)) + b*log(-I*sqrt(b)*sqrt(1/a) + sqrt(x))/(2*I*a**2
*b**(3/2)*x*sqrt(1/a) + 2*I*a*b**(5/2)*sqrt(1/a)) - b*log(I*sqrt(b)*sqrt(1/
a) + sqrt(x))/(2*I*a**2*b**(3/2)*x*sqrt(1/a) + 2*I*a*b**(5/2)*sqrt(1/a)), T
rue))
```

3.1679 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{7/2}}\,dx$

Optimal. Leaf size=56

$$-\frac{3\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{b^{5/2}}+\frac{1}{b\sqrt{x}\,(ax+b)}-\frac{3}{b^2\sqrt{x}}$$

```
[Out] -3*arctan(a^(1/2)*x^(1/2)/b^(1/2))*a^(1/2)/b^(5/2)-3/b^2/x^(1/2)+1/b/(a*x+b
)/x^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {263, 51, 63, 205}

$$-\frac{3\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{b^{5/2}}+\frac{1}{b\sqrt{x}\,(ax+b)}-\frac{3}{b^2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^2*x^(7/2)),x]
```

```
[Out] -3/(b^2*Sqrt[x]) + 1/(b*Sqrt[x]*(b + a*x)) - (3*Sqrt[a]*ArcTan[(Sqrt[a]*Sqr
t[x])/Sqrt[b]])/b^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{7/2}}\,dx = \int \frac{1}{x^{3/2}(b+ax)^2}\,dx$$

$$= \frac{1}{b\sqrt{x}\,(b+ax)} + \frac{3\int \frac{1}{x^{3/2}(b+ax)}\,dx}{2b}$$

$$= -\frac{3}{b^2\sqrt{x}} + \frac{1}{b\sqrt{x}\,(b+ax)} - \frac{(3a)\int \frac{1}{\sqrt{x}\,(b+ax)}\,dx}{2b^2}$$

$$= -\frac{3}{b^2\sqrt{x}} + \frac{1}{b\sqrt{x}\,(b+ax)} - \frac{(3a)\,\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \sqrt{x}\right)}{b^2}$$

$$= -\frac{3}{b^2\sqrt{x}} + \frac{1}{b\sqrt{x}\,(b+ax)} - \frac{3\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{b^{5/2}}$$

Mathematica [C] time = 0.00, size = 25, normalized size = 0.45

$$-\frac{2\,{}_2F_1\left(-\frac{1}{2}, 2; \frac{1}{2}; -\frac{ax}{b}\right)}{b^2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^2*x^(7/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/2, 2, 1/2, -((a*x)/b)])/(b^2*Sqrt[x])
```

fricas [A] time = 0.72, size = 147, normalized size = 2.62

$$\left[\frac{3\left(ax^2+bx\right)\sqrt{-\frac{a}{b}}\,\log\left(\frac{ax-2\,b\sqrt{x}\,\sqrt{-\frac{a}{b}}-b}{ax+b}\right) - 2\,(3\,ax+2\,b)\sqrt{x}}{2\left(ab^2x^2+b^3x\right)}, \frac{3\left(ax^2+bx\right)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{b\sqrt{\frac{a}{b}}}{a\sqrt{x}}\right) - (3\,ax+2\,b)\sqrt{x}}{ab^2x^2+b^3x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x^(7/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*(a*x^2 + b*x)*sqrt(-a/b)*log((a*x - 2*b*sqrt(x)*sqrt(-a/b) - b)/(a*
x + b)) - 2*(3*a*x + 2*b)*sqrt(x))/(a*b^2*x^2 + b^3*x), (3*(a*x^2 + b*x)*sq
rt(a/b)*arctan(b*sqrt(a/b)/(a*sqrt(x))) - (3*a*x + 2*b)*sqrt(x))/(a*b^2*x^2
 + b^3*x)]
```

giac [A] time = 0.16, size = 49, normalized size = 0.88

$$-\frac{3\,a\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2} - \frac{3\,ax+2\,b}{\left(ax^{\frac{3}{2}}+b\sqrt{x}\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x^(7/2),x, algorithm="giac")
```

```
[Out] -3*a*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) - (3*a*x + 2*b)/((a*x^(3/2
) + b*sqrt(x))*b^2)
```

maple [A] time = 0.01, size = 48, normalized size = 0.86

$$-\frac{3a\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}-\frac{a\sqrt{x}}{(ax+b)\,b^2}-\frac{2}{b^2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^2/x^(7/2),x)
```

```
[Out] -a/b^2*x^(1/2)/(a*x+b)-3*a/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*a*x^(1/2))-
2/b^2/x^(1/2)
```

maxima [A] time = 2.35, size = 52, normalized size = 0.93

$$-\frac{a}{\left(ab^2+\frac{b^3}{x}\right)\sqrt{x}}+\frac{3\,a\arctan\left(\frac{b}{\sqrt{ab}\,\sqrt{x}}\right)}{\sqrt{ab}\,b^2}-\frac{2}{b^2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x^(7/2),x, algorithm="maxima")
```

```
[Out] -a/((a*b^2 + b^3/x)*sqrt(x)) + 3*a*arctan(b/(sqrt(a*b)*sqrt(x)))/(sqrt(a*b)
*b^2) - 2/(b^2*sqrt(x))
```

mupad [B] time = 1.13, size = 48, normalized size = 0.86

$$-\frac{\frac{2}{b}+\frac{3\,a\,x}{b^2}}{a\,x^{3/2}+b\,\sqrt{x}}-\frac{3\,\sqrt{a}\,\operatorname{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a + b/x)^2),x)
```

```
[Out] - (2/b + (3*a*x)/b^2)/(a*x^(3/2) + b*x^(1/2)) - (3*a^(1/2)*atan((a^(1/2)*x^
(1/2))/b^(1/2)))/b^(5/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**2/x**(7/2),x)
```

```
[Out] Timed out
```

3.1680 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{9/2}}\,dx$

Optimal. Leaf size=69

$$\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{b^{7/2}}+\frac{5a}{b^3\sqrt{x}}+\frac{1}{bx^{3/2}(ax+b)}-\frac{5}{3b^2x^{3/2}}$$

```
[Out] -5/3/b^2/x^(3/2)+1/b/x^(3/2)/(a*x+b)+5*a^(3/2)*arctan(a^(1/2)*x^(1/2)/b^(1/
2))/b^(7/2)+5*a/b^3/x^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {263, 51, 63, 205}

$$\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{b^{7/2}}+\frac{5a}{b^3\sqrt{x}}+\frac{1}{bx^{3/2}(ax+b)}-\frac{5}{3b^2x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^2*x^(9/2)),x]
```

```
[Out] -5/(3*b^2*x^(3/2)) + (5*a)/(b^3*Sqrt[x]) + 1/(b*x^(3/2)*(b + a*x)) + (5*a^(
3/2)*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/b^(7/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{9/2}}\,dx = \int \frac{1}{x^{5/2}(b+ax)^2}\,dx$$

$$= \frac{1}{bx^{3/2}(b+ax)} + \frac{5\int \frac{1}{x^{5/2}(b+ax)}\,dx}{2b}$$

$$= -\frac{5}{3b^2x^{3/2}} + \frac{1}{bx^{3/2}(b+ax)} - \frac{(5a)\int \frac{1}{x^{3/2}(b+ax)}\,dx}{2b^2}$$

$$= -\frac{5}{3b^2x^{3/2}} + \frac{5a}{b^3\sqrt{x}} + \frac{1}{bx^{3/2}(b+ax)} + \frac{\left(5a^2\right)\int \frac{1}{\sqrt{x}\,(b+ax)}\,dx}{2b^3}$$

$$= -\frac{5}{3b^2x^{3/2}} + \frac{5a}{b^3\sqrt{x}} + \frac{1}{bx^{3/2}(b+ax)} + \frac{\left(5a^2\right)\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \sqrt{x}\right)}{b^3}$$

$$= -\frac{5}{3b^2x^{3/2}} + \frac{5a}{b^3\sqrt{x}} + \frac{1}{bx^{3/2}(b+ax)} + \frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{b^{7/2}}$$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.39

$$-\frac{2\,{}_2F_1\left(-\frac{3}{2}, 2; -\frac{1}{2}; -\frac{ax}{b}\right)}{3b^2x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^2*x^(9/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/2, 2, -1/2, -((a*x)/b)])/(3*b^2*x^(3/2))
```

fricas [A] time = 0.78, size = 184, normalized size = 2.67

$$\left[\frac{15\left(a^2x^3+abx^2\right)\sqrt{-\frac{a}{b}}\,\log\left(\frac{ax+2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-b}{ax+b}\right)+2\left(15\,a^2x^2+10\,abx-2\,b^2\right)\sqrt{x}}{6\left(ab^3x^3+b^4x^2\right)}, -\frac{15\left(a^2x^3+abx^2\right)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{b\sqrt{\frac{a}{b}}}{a\sqrt{x}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x^(9/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*(a^2*x^3 + a*b*x^2)*sqrt(-a/b)*log((a*x + 2*b*sqrt(x)*sqrt(-a/b) -
 b)/(a*x + b)) + 2*(15*a^2*x^2 + 10*a*b*x - 2*b^2)*sqrt(x))/(a*b^3*x^3 + b^
4*x^2), -1/3*(15*(a^2*x^3 + a*b*x^2)*sqrt(a/b)*arctan(b*sqrt(a/b)/(a*sqrt(x
))) - (15*a^2*x^2 + 10*a*b*x - 2*b^2)*sqrt(x))/(a*b^3*x^3 + b^4*x^2)]
```

giac [A] time = 0.17, size = 58, normalized size = 0.84

$$\frac{5\,a^2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3} + \frac{a^2\sqrt{x}}{(ax+b)b^3} + \frac{2\,(6\,ax-b)}{3\,b^3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x^(9/2),x, algorithm="giac")
```

```
[Out] 5*a^2*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + a^2*sqrt(x)/((a*x + b)*
b^3) + 2/3*(6*a*x - b)/(b^3*x^(3/2))
```

maple [A] time = 0.02, size = 60, normalized size = 0.87

$$\frac{5a^2 \arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\ b^3} + \frac{a^2\sqrt{x}}{(ax+b)\,b^3} + \frac{4a}{b^3\sqrt{x}} - \frac{2}{3b^2 x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^2/x^(9/2),x)
```

```
[Out] a^2/b^3*x^(1/2)/(a*x+b)+5*a^2/b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*a*x^(1/2
))-2/3/b^2/x^(3/2)+4*a/b^3/x^(1/2)
```

maxima [A] time = 2.24, size = 65, normalized size = 0.94

$$\frac{a^2}{\left(ab^3 + \frac{b^4}{x}\right)\sqrt{x}} - \frac{5\,a^2 \arctan\left(\frac{b}{\sqrt{ab}\,\sqrt{x}}\right)}{\sqrt{ab}\,b^3} + \frac{2\left(\frac{6\,a}{\sqrt{x}} - \frac{b}{x^{\frac{3}{2}}}\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x^(9/2),x, algorithm="maxima")
```

```
[Out] a^2/((a*b^3 + b^4/x)*sqrt(x)) - 5*a^2*arctan(b/(sqrt(a*b)*sqrt(x)))/(sqrt(a
*b)*b^3) + 2/3*(6*a/sqrt(x) - b/x^(3/2))/b^3
```

mupad [B] time = 1.14, size = 58, normalized size = 0.84

$$\frac{\frac{5\,a^2\,x^2}{b^3} - \frac{2}{3\,b} + \frac{10\,a\,x}{3\,b^2}}{a\,x^{5/2} + b\,x^{3/2}} + \frac{5\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(9/2)*(a + b/x)^2),x)
```

```
[Out] ((5*a^2*x^2)/b^3 - 2/(3*b) + (10*a*x)/(3*b^2))/(a*x^(5/2) + b*x^(3/2)) + (5
*a^(3/2)*atan((a^(1/2)*x^(1/2))/b^(1/2)))/b^(7/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**2/x**(9/2),x)
```

```
[Out] Timed out
```

3.1681 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{11/2}}\,dx$

Optimal. Leaf size=84

$$-\frac{7a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{b^{9/2}}-\frac{7a^2}{b^4\sqrt{x}}+\frac{7a}{3b^3x^{3/2}}+\frac{1}{bx^{5/2}(ax+b)}-\frac{7}{5b^2x^{5/2}}$$

```
[Out] -7/5/b^2/x^(5/2)+7/3*a/b^3/x^(3/2)+1/b/x^(5/2)/(a*x+b)-7*a^(5/2)*arctan(a^(
1/2)*x^(1/2)/b^(1/2))/b^(9/2)-7*a^2/b^4/x^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {263, 51, 63, 205}

$$-\frac{7a^2}{b^4\sqrt{x}}-\frac{7a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{b^{9/2}}+\frac{7a}{3b^3x^{3/2}}+\frac{1}{bx^{5/2}(ax+b)}-\frac{7}{5b^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^2*x^(11/2)),x]
```

```
[Out] -7/(5*b^2*x^(5/2)) + (7*a)/(3*b^3*x^(3/2)) - (7*a^2)/(b^4*Sqrt[x]) + 1/(b*x
^(5/2)*(b + a*x)) - (7*a^(5/2)*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/b^(9/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{11/2}}\,dx = \int \frac{1}{x^{7/2}(b+ax)^2}\,dx$$

$$= \frac{1}{bx^{5/2}(b+ax)} + \frac{7\int \frac{1}{x^{7/2}(b+ax)}\,dx}{2b}$$

$$= -\frac{7}{5b^2x^{5/2}} + \frac{1}{bx^{5/2}(b+ax)} - \frac{(7a)\int \frac{1}{x^{5/2}(b+ax)}\,dx}{2b^2}$$

$$= -\frac{7}{5b^2x^{5/2}} + \frac{7a}{3b^3x^{3/2}} + \frac{1}{bx^{5/2}(b+ax)} + \frac{\left(7a^2\right)\int \frac{1}{x^{3/2}(b+ax)}\,dx}{2b^3}$$

$$= -\frac{7}{5b^2x^{5/2}} + \frac{7a}{3b^3x^{3/2}} - \frac{7a^2}{b^4\sqrt{x}} + \frac{1}{bx^{5/2}(b+ax)} - \frac{\left(7a^3\right)\int \frac{1}{\sqrt{x}\,(b+ax)}\,dx}{2b^4}$$

$$= -\frac{7}{5b^2x^{5/2}} + \frac{7a}{3b^3x^{3/2}} - \frac{7a^2}{b^4\sqrt{x}} + \frac{1}{bx^{5/2}(b+ax)} - \frac{\left(7a^3\right)\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \sqrt{x}\right)}{b^4}$$

$$= -\frac{7}{5b^2x^{5/2}} + \frac{7a}{3b^3x^{3/2}} - \frac{7a^2}{b^4\sqrt{x}} + \frac{1}{bx^{5/2}(b+ax)} - \frac{7a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{b^{9/2}}$$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.32

$$-\frac{2\,{}_2F_1\left(-\frac{5}{2}, 2; -\frac{3}{2}; -\frac{ax}{b}\right)}{5b^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^2*x^(11/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-5/2, 2, -3/2, -((a*x)/b)])/(5*b^2*x^(5/2))
```

fricas [A] time = 0.98, size = 210, normalized size = 2.50

$$\left[\frac{105\left(a^3x^4+a^2bx^3\right)\sqrt{-\frac{a}{b}}\log\left(\frac{ax-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-b}{ax+b}\right) - 2\left(105\,a^3x^3+70\,a^2bx^2-14\,ab^2x+6\,b^3\right)\sqrt{x}}{30\left(ab^4x^4+b^5x^3\right)}, \frac{105\left(a^3x^4+a^2b\right.}{\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x^(11/2),x, algorithm="fricas")
```

```
[Out] [1/30*(105*(a^3*x^4 + a^2*b*x^3)*sqrt(-a/b)*log((a*x - 2*b*sqrt(x)*sqrt(-a/
b) - b)/(a*x + b)) - 2*(105*a^3*x^3 + 70*a^2*b*x^2 - 14*a*b^2*x + 6*b^3)*sq
rt(x))/(a*b^4*x^4 + b^5*x^3), 1/15*(105*(a^3*x^4 + a^2*b*x^3)*sqrt(a/b)*arc
tan(b*sqrt(a/b)/(a*sqrt(x))) - (105*a^3*x^3 + 70*a^2*b*x^2 - 14*a*b^2*x + 6
*b^3)*sqrt(x))/(a*b^4*x^4 + b^5*x^3)]
```

giac [A] time = 0.16, size = 70, normalized size = 0.83

$$-\frac{7\,a^3\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^4} - \frac{a^3\sqrt{x}}{(ax+b)b^4} - \frac{2\left(45\,a^2x^2-10\,abx+3\,b^2\right)}{15\,b^4x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x^(11/2),x, algorithm="giac")
```

```
[Out] -7*a^3*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4) - a^3*sqrt(x)/((a*x + b)
*b^4) - 2/15*(45*a^2*x^2 - 10*a*b*x + 3*b^2)/(b^4*x^(5/2))
```

maple [A] time = 0.02, size = 72, normalized size = 0.86

$$-\frac{7a^3 \arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\, b^4} - \frac{a^3\sqrt{x}}{(ax+b)\, b^4} - \frac{6a^2}{b^4\sqrt{x}} + \frac{4a}{3b^3 x^{\frac{3}{2}}} - \frac{2}{5b^2 x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^2/x^(11/2),x)
```

```
[Out] -a^3/b^4*x^(1/2)/(a*x+b)-7*a^3/b^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*a*x^(1/
2))-2/5/b^2/x^(5/2)-6*a^2/b^4/x^(1/2)+4/3*a/b^3/x^(3/2)
```

maxima [A] time = 2.30, size = 77, normalized size = 0.92

$$-\frac{a^3}{\left(ab^4 + \frac{b^5}{x}\right)\sqrt{x}} + \frac{7\, a^3 \arctan\left(\frac{b}{\sqrt{ab}\,\sqrt{x}}\right)}{\sqrt{ab}\, b^4} - \frac{2\left(\frac{45\, a^2}{\sqrt{x}} - \frac{10\, ab}{x^{\frac{3}{2}}} + \frac{3\, b^2}{x^{\frac{5}{2}}}\right)}{15\, b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^2/x^(11/2),x, algorithm="maxima")
```

```
[Out] -a^3/((a*b^4 + b^5/x)*sqrt(x)) + 7*a^3*arctan(b/(sqrt(a*b)*sqrt(x)))/(sqrt(
a*b)*b^4) - 2/15*(45*a^2/sqrt(x) - 10*a*b/x^(3/2) + 3*b^2/x^(5/2))/b^4
```

mupad [B] time = 1.15, size = 70, normalized size = 0.83

$$-\frac{\frac{2}{5\, b} + \frac{14\, a^2 x^2}{3\, b^3} + \frac{7\, a^3 x^3}{b^4} - \frac{14\, a\, x}{15\, b^2}}{a\, x^{7/2} + b\, x^{5/2}} - \frac{7\, a^{5/2} \operatorname{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(11/2)*(a + b/x)^2),x)
```

```
[Out] - (2/(5*b) + (14*a^2*x^2)/(3*b^3) + (7*a^3*x^3)/b^4 - (14*a*x)/(15*b^2))/(a
*x^(7/2) + b*x^(5/2)) - (7*a^(5/2)*atan((a^(1/2)*x^(1/2))/b^(1/2)))/b^(9/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**2/x**(11/2),x)
```

```
[Out] Timed out
```

3.1682 $\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^3}\,dx$

Optimal. Leaf size=110

$$-\frac{63b^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{4a^{11/2}}+\frac{63b^2\sqrt{x}}{4a^5}-\frac{21bx^{3/2}}{4a^4}+\frac{63x^{5/2}}{20a^3}-\frac{9x^{7/2}}{4a^2(ax+b)}-\frac{x^{9/2}}{2a(ax+b)^2}$$

```
[Out] -21/4*b*x^(3/2)/a^4+63/20*x^(5/2)/a^3-1/2*x^(9/2)/a/(a*x+b)^2-9/4*x^(7/2)/a
^2/(a*x+b)-63/4*b^(5/2)*arctan(a^(1/2)*x^(1/2)/b^(1/2))/a^(11/2)+63/4*b^2*x
^(1/2)/a^5
```

Rubi [A] time = 0.04, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {263, 47, 50, 63, 205}

$$\frac{63b^2\sqrt{x}}{4a^5}-\frac{63b^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{4a^{11/2}}-\frac{9x^{7/2}}{4a^2(ax+b)}-\frac{21bx^{3/2}}{4a^4}+\frac{63x^{5/2}}{20a^3}-\frac{x^{9/2}}{2a(ax+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b/x)^3,x]
```

```
[Out] (63*b^2*Sqrt[x])/(4*a^5) - (21*b*x^(3/2))/(4*a^4) + (63*x^(5/2))/(20*a^3) -
 x^(9/2)/(2*a*(b + a*x)^2) - (9*x^(7/2))/(4*a^2*(b + a*x)) - (63*b^(5/2)*Ar
cTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/(4*a^(11/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^3}\,dx = \int \frac{x^{9/2}}{(b+ax)^3}\,dx$$

$$= -\frac{x^{9/2}}{2a(b+ax)^2} + \frac{9\int \frac{x^{7/2}}{(b+ax)^2}\,dx}{4a}$$

$$= -\frac{x^{9/2}}{2a(b+ax)^2} - \frac{9x^{7/2}}{4a^2(b+ax)} + \frac{63\int \frac{x^{5/2}}{b+ax}\,dx}{8a^2}$$

$$= \frac{63x^{5/2}}{20a^3} - \frac{x^{9/2}}{2a(b+ax)^2} - \frac{9x^{7/2}}{4a^2(b+ax)} - \frac{(63b)\int \frac{x^{3/2}}{b+ax}\,dx}{8a^3}$$

$$= -\frac{21bx^{3/2}}{4a^4} + \frac{63x^{5/2}}{20a^3} - \frac{x^{9/2}}{2a(b+ax)^2} - \frac{9x^{7/2}}{4a^2(b+ax)} + \frac{\left(63b^2\right)\int \frac{\sqrt{x}}{b+ax}\,dx}{8a^4}$$

$$= \frac{63b^2\sqrt{x}}{4a^5} - \frac{21bx^{3/2}}{4a^4} + \frac{63x^{5/2}}{20a^3} - \frac{x^{9/2}}{2a(b+ax)^2} - \frac{9x^{7/2}}{4a^2(b+ax)} - \frac{\left(63b^3\right)\int \frac{1}{\sqrt{x}\,(b+ax)}\,dx}{8a^5}$$

$$= \frac{63b^2\sqrt{x}}{4a^5} - \frac{21bx^{3/2}}{4a^4} + \frac{63x^{5/2}}{20a^3} - \frac{x^{9/2}}{2a(b+ax)^2} - \frac{9x^{7/2}}{4a^2(b+ax)} - \frac{\left(63b^3\right)\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \sqrt{x}\right)}{4a^5}$$

$$= \frac{63b^2\sqrt{x}}{4a^5} - \frac{21bx^{3/2}}{4a^4} + \frac{63x^{5/2}}{20a^3} - \frac{x^{9/2}}{2a(b+ax)^2} - \frac{9x^{7/2}}{4a^2(b+ax)} - \frac{63b^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{4a^{11/2}}$$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.25

$$\frac{2x^{11/2}\,{}_2F_1\left(3, \frac{11}{2}; \frac{13}{2}; -\frac{ax}{b}\right)}{11b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b/x)^3,x]
```

```
[Out] (2*x^(11/2)*Hypergeometric2F1[3, 11/2, 13/2, -((a*x)/b)])/(11*b^3)
```

fricas [A] time = 0.78, size = 254, normalized size = 2.31

$$\left[\frac{315\left(a^2b^2x^2 + 2ab^3x + b^4\right)\sqrt{-\frac{b}{a}}\log\left(\frac{ax - 2a\sqrt{x}\sqrt{-\frac{b}{a}} - b}{ax+b}\right) + 2\left(8a^4x^4 - 24a^3bx^3 + 168a^2b^2x^2 + 525ab^3x + 315b^4\right)\sqrt{}}{40\left(a^7x^2 + 2a^6bx + a^5b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(a+b/x)^3,x, algorithm="fricas")
```

```
[Out] [1/40*(315*(a^2*b^2*x^2 + 2*a*b^3*x + b^4)*sqrt(-b/a)*log((a*x - 2*a*sqrt(x
)*sqrt(-b/a) - b)/(a*x + b)) + 2*(8*a^4*x^4 - 24*a^3*b*x^3 + 168*a^2*b^2*x^
2 + 525*a*b^3*x + 315*b^4)*sqrt(x))/(a^7*x^2 + 2*a^6*b*x + a^5*b^2), -1/20*
```

```
(315*(a^2*b^2*x^2 + 2*a*b^3*x + b^4)*sqrt(b/a)*arctan(a*sqrt(x)*sqrt(b/a)/b
) - (8*a^4*x^4 - 24*a^3*b*x^3 + 168*a^2*b^2*x^2 + 525*a*b^3*x + 315*b^4)*sq
rt(x))/(a^7*x^2 + 2*a^6*b*x + a^5*b^2)]
```

giac [A] time = 0.20, size = 88, normalized size = 0.80

$$-\frac{63\,b^3\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\,a^5}+\frac{17\,ab^3x^{\frac{3}{2}}+15\,b^4\sqrt{x}}{4\,(ax+b)^2a^5}+\frac{2\left(a^{12}x^{\frac{5}{2}}-5\,a^{11}bx^{\frac{3}{2}}+30\,a^{10}b^2\sqrt{x}\right)}{5\,a^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(a+b/x)^3,x, algorithm="giac")
```

```
[Out] -63/4*b^3*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^5) + 1/4*(17*a*b^3*x^(3/
2) + 15*b^4*sqrt(x))/((a*x + b)^2*a^5) + 2/5*(a^12*x^(5/2) - 5*a^11*b*x^(3/
2) + 30*a^10*b^2*sqrt(x))/a^15
```

maple [A] time = 0.01, size = 90, normalized size = 0.82

$$\frac{17b^3x^{\frac{3}{2}}}{4\,(ax+b)^2\,a^4}+\frac{2x^{\frac{5}{2}}}{5a^3}+\frac{15b^4\sqrt{x}}{4\,(ax+b)^2\,a^5}-\frac{63b^3\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\;a^5}-\frac{2b\,x^{\frac{3}{2}}}{a^4}+\frac{12b^2\sqrt{x}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a+b/x)^3,x)
```

```
[Out] 2/5*x^(5/2)/a^3-2*b*x^(3/2)/a^4+12*b^2*x^(1/2)/a^5+17/4/a^4*b^3/(a*x+b)^2*x
^(3/2)+15/4/a^5*b^4/(a*x+b)^2*x^(1/2)-63/4/a^5*b^3/(a*b)^(1/2)*arctan(1/(a*
b)^(1/2)*a*x^(1/2))
```

maxima [A] time = 2.36, size = 99, normalized size = 0.90

$$\frac{8\,a^4-\frac{24\,a^3b}{x}+\frac{168\,a^2b^2}{x^2}+\frac{525\,ab^3}{x^3}+\frac{315\,b^4}{x^4}}{20\left(\frac{a^7}{x^{\frac{5}{2}}}+\frac{2\,a^6b}{x^{\frac{7}{2}}}+\frac{a^5b^2}{x^{\frac{9}{2}}}\right)}+\frac{63\,b^3\arctan\left(\frac{b}{\sqrt{ab}\,\sqrt{x}}\right)}{4\,\sqrt{ab}\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(a+b/x)^3,x, algorithm="maxima")
```

```
[Out] 1/20*(8*a^4 - 24*a^3*b/x + 168*a^2*b^2/x^2 + 525*a*b^3/x^3 + 315*b^4/x^4)/(
a^7/x^(5/2) + 2*a^6*b/x^(7/2) + a^5*b^2/x^(9/2)) + 63/4*b^3*arctan(b/(sqrt(
a*b)*sqrt(x)))/(sqrt(a*b)*a^5)
```

mupad [B] time = 1.08, size = 91, normalized size = 0.83

$$\frac{\frac{15\,b^4\,\sqrt{x}}{4}+\frac{17\,a\,b^3\,x^{3/2}}{4}}{a^7\,x^2+2\,a^6\,b\,x+a^5\,b^2}+\frac{2\,x^{5/2}}{5\,a^3}-\frac{2\,b\,x^{3/2}}{a^4}+\frac{12\,b^2\,\sqrt{x}}{a^5}-\frac{63\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{4\,a^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b/x)^3,x)
```

```
[Out] ((15*b^4*x^(1/2))/4 + (17*a*b^3*x^(3/2))/4)/(a^5*b^2 + a^7*x^2 + 2*a^6*b*x)
 + (2*x^(5/2))/(5*a^3) - (2*b*x^(3/2))/a^4 + (12*b^2*x^(1/2))/a^5 - (63*b^(
5/2)*atan((a^(1/2)*x^(1/2))/b^(1/2)))/(4*a^(11/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(a+b/x)**3,x)
```

```
[Out] Timed out
```

3.1683 $\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^3}\,dx$

Optimal. Leaf size=95

$$\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{4a^{9/2}}-\frac{35b\sqrt{x}}{4a^4}+\frac{35x^{3/2}}{12a^3}-\frac{7x^{5/2}}{4a^2(ax+b)}-\frac{x^{7/2}}{2a(ax+b)^2}$$

```
[Out] 35/12*x^(3/2)/a^3-1/2*x^(7/2)/a/(a*x+b)^2-7/4*x^(5/2)/a^2/(a*x+b)+35/4*b^(3
/2)*arctan(a^(1/2)*x^(1/2)/b^(1/2))/a^(9/2)-35/4*b*x^(1/2)/a^4
```

Rubi [A] time = 0.03, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {263, 47, 50, 63, 205}

$$\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{4a^{9/2}}-\frac{7x^{5/2}}{4a^2(ax+b)}-\frac{35b\sqrt{x}}{4a^4}+\frac{35x^{3/2}}{12a^3}-\frac{x^{7/2}}{2a(ax+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b/x)^3,x]
```

```
[Out] (-35*b*Sqrt[x])/(4*a^4) + (35*x^(3/2))/(12*a^3) - x^(7/2)/(2*a*(b + a*x)^2)
 - (7*x^(5/2))/(4*a^2*(b + a*x)) + (35*b^(3/2)*ArcTan[(Sqrt[a]*Sqrt[x])/Sqr
t[b]])/(4*a^(9/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^3}\,dx = \int \frac{x^{7/2}}{(b+ax)^3}\,dx$$

$$= -\frac{x^{7/2}}{2a(b+ax)^2} + \frac{7\int \frac{x^{5/2}}{(b+ax)^2}\,dx}{4a}$$

$$= -\frac{x^{7/2}}{2a(b+ax)^2} - \frac{7x^{5/2}}{4a^2(b+ax)} + \frac{35\int \frac{x^{3/2}}{b+ax}\,dx}{8a^2}$$

$$= \frac{35x^{3/2}}{12a^3} - \frac{x^{7/2}}{2a(b+ax)^2} - \frac{7x^{5/2}}{4a^2(b+ax)} - \frac{(35b)\int \frac{\sqrt{x}}{b+ax}\,dx}{8a^3}$$

$$= -\frac{35b\sqrt{x}}{4a^4} + \frac{35x^{3/2}}{12a^3} - \frac{x^{7/2}}{2a(b+ax)^2} - \frac{7x^{5/2}}{4a^2(b+ax)} + \frac{\left(35b^2\right)\int \frac{1}{\sqrt{x}\,(b+ax)}\,dx}{8a^4}$$

$$= -\frac{35b\sqrt{x}}{4a^4} + \frac{35x^{3/2}}{12a^3} - \frac{x^{7/2}}{2a(b+ax)^2} - \frac{7x^{5/2}}{4a^2(b+ax)} + \frac{\left(35b^2\right)\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \sqrt{x}\right)}{4a^4}$$

$$= -\frac{35b\sqrt{x}}{4a^4} + \frac{35x^{3/2}}{12a^3} - \frac{x^{7/2}}{2a(b+ax)^2} - \frac{7x^{5/2}}{4a^2(b+ax)} + \frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{4a^{9/2}}$$

Mathematica [C] time = 0.00, size = 27, normalized size = 0.28

$$\frac{2x^{9/2}\,{}_2F_1\left(3, \frac{9}{2}; \frac{11}{2}; -\frac{ax}{b}\right)}{9b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b/x)^3,x]
```

```
[Out] (2*x^(9/2)*Hypergeometric2F1[3, 9/2, 11/2, -((a*x)/b)])/(9*b^3)
```

fricas [A] time = 1.16, size = 227, normalized size = 2.39

$$\left[\frac{105\left(a^2bx^2+2\,ab^2x+b^3\right)\sqrt{-\frac{b}{a}}\log\left(\frac{ax+2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-b}{ax+b}\right)+2\left(8\,a^3x^3-56\,a^2bx^2-175\,ab^2x-105\,b^3\right)\sqrt{x}}{24\left(a^6x^2+2\,a^5bx+a^4b^2\right)}, \frac{105\left(a^2bx^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(a+b/x)^3,x, algorithm="fricas")
```

```
[Out] [1/24*(105*(a^2*b*x^2 + 2*a*b^2*x + b^3)*sqrt(-b/a)*log((a*x + 2*a*sqrt(x)*
sqrt(-b/a) - b)/(a*x + b)) + 2*(8*a^3*x^3 - 56*a^2*b*x^2 - 175*a*b^2*x - 10
5*b^3)*sqrt(x))/(a^6*x^2 + 2*a^5*b*x + a^4*b^2), 1/12*(105*(a^2*b*x^2 + 2*a
*b^2*x + b^3)*sqrt(b/a)*arctan(a*sqrt(x)*sqrt(b/a)/b) + (8*a^3*x^3 - 56*a^2
*b*x^2 - 175*a*b^2*x - 105*b^3)*sqrt(x))/(a^6*x^2 + 2*a^5*b*x + a^4*b^2)]
```

giac [A] time = 0.19, size = 77, normalized size = 0.81

$$\frac{35\,b^2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\,a^4}-\frac{13\,ab^2x^{\frac{3}{2}}+11\,b^3\sqrt{x}}{4\,(ax+b)^2a^4}+\frac{2\left(a^6x^{\frac{3}{2}}-9\,a^5b\sqrt{x}\right)}{3\,a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(a+b/x)^3,x, algorithm="giac")
```

```
[Out] 35/4*b^2*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^4) - 1/4*(13*a*b^2*x^(3/2
) + 11*b^3*sqrt(x))/((a*x + b)^2*a^4) + 2/3*(a^6*x^(3/2) - 9*a^5*b*sqrt(x))
/a^9
```

maple [A] time = 0.02, size = 79, normalized size = 0.83

$$-\frac{13b^2x^{\frac{3}{2}}}{4\,(ax+b)^2\,a^3}-\frac{11b^3\sqrt{x}}{4\,(ax+b)^2\,a^4}+\frac{35b^2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\;a^4}+\frac{2x^{\frac{3}{2}}}{3a^3}-\frac{6b\sqrt{x}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a+b/x)^3,x)
```

```
[Out] 2/3*x^(3/2)/a^3-6*b*x^(1/2)/a^4-13/4/a^3*b^2/(a*x+b)^2*x^(3/2)-11/4/a^4*b^3
/(a*x+b)^2*x^(1/2)+35/4/a^4*b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*a*x^(1/2))
```

maxima [A] time = 2.37, size = 88, normalized size = 0.93

$$\frac{8\,a^3-\frac{56\,a^2b}{x}-\frac{175\,ab^2}{x^2}-\frac{105\,b^3}{x^3}}{12\left(\frac{a^6}{x^{\frac{3}{2}}}+\frac{2\,a^5b}{x^{\frac{5}{2}}}+\frac{a^4b^2}{x^{\frac{7}{2}}}\right)}-\frac{35\,b^2\arctan\left(\frac{b}{\sqrt{ab}\,\sqrt{x}}\right)}{4\sqrt{ab}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(a+b/x)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(8*a^3 - 56*a^2*b/x - 175*a*b^2/x^2 - 105*b^3/x^3)/(a^6/x^(3/2) + 2*a^
5*b/x^(5/2) + a^4*b^2/x^(7/2)) - 35/4*b^2*arctan(b/(sqrt(a*b)*sqrt(x)))/(sq
rt(a*b)*a^4)
```

mupad [B] time = 0.07, size = 81, normalized size = 0.85

$$\frac{2\,x^{3/2}}{3\,a^3}-\frac{\frac{11\,b^3\,\sqrt{x}}{4}+\frac{13\,a\,b^2\,x^{3/2}}{4}}{a^6\,x^2+2\,a^5\,b\,x+a^4\,b^2}-\frac{6\,b\,\sqrt{x}}{a^4}+\frac{35\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{4\,a^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b/x)^3,x)
```

```
[Out] (2*x^(3/2))/(3*a^3) - ((11*b^3*x^(1/2))/4 + (13*a*b^2*x^(3/2))/4)/(a^4*b^2
+ a^6*x^2 + 2*a^5*b*x) - (6*b*x^(1/2))/a^4 + (35*b^(3/2)*atan((a^(1/2)*x^(1
/2))/b^(1/2)))/(4*a^(9/2))
```

sympy [A] time = 43.54, size = 906, normalized size = 9.54

$$\begin{cases} \tilde{\infty} x^{\frac{9}{2}} \\ \frac{2x^{\frac{9}{2}}}{9b^3} \\ \frac{2x^{\frac{3}{2}}}{3a^3} \\ \frac{16ia^4\sqrt{b}\,x^{\frac{7}{2}}\sqrt{\frac{1}{a}}}{24ia^7\sqrt{b}\,x^2\sqrt{\frac{1}{a}}+48ia^6b^{\frac{3}{2}}x\sqrt{\frac{1}{a}}+24ia^5b^{\frac{5}{2}}\sqrt{\frac{1}{a}}} - \frac{112ia^3b^{\frac{3}{2}}x^{\frac{5}{2}}\sqrt{\frac{1}{a}}}{24ia^7\sqrt{b}\,x^2\sqrt{\frac{1}{a}}+48ia^6b^{\frac{3}{2}}x\sqrt{\frac{1}{a}}+24ia^5b^{\frac{5}{2}}\sqrt{\frac{1}{a}}} - \frac{350ia^2b^{\frac{5}{2}}x^{\frac{3}{2}}\sqrt{\frac{1}{a}}}{24ia^7\sqrt{b}\,x^2\sqrt{\frac{1}{a}}+48ia^6b^{\frac{3}{2}}x\sqrt{\frac{1}{a}}+24ia^5b^{\frac{5}{2}}\sqrt{\frac{1}{a}}} + \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(a+b/x)**3,x)
```

```
[Out] Piecewise((zoo*x**(9/2), Eq(a, 0) & Eq(b, 0)), (2*x**(9/2)/(9*b**3), Eq(a,
0)), (2*x**(3/2)/(3*a**3), Eq(b, 0)), (16*I*a**4*sqrt(b)*x**(7/2)*sqrt(1/a)
/(24*I*a**7*sqrt(b)*x**2*sqrt(1/a) + 48*I*a**6*b**(3/2)*x*sqrt(1/a) + 24*I*
a**5*b**(5/2)*sqrt(1/a)) - 112*I*a**3*b**(3/2)*x**(5/2)*sqrt(1/a)/(24*I*a**
7*sqrt(b)*x**2*sqrt(1/a) + 48*I*a**6*b**(3/2)*x*sqrt(1/a) + 24*I*a**5*b**(5
/2)*sqrt(1/a)) - 350*I*a**2*b**(5/2)*x**(3/2)*sqrt(1/a)/(24*I*a**7*sqrt(b)*
x**2*sqrt(1/a) + 48*I*a**6*b**(3/2)*x*sqrt(1/a) + 24*I*a**5*b**(5/2)*sqrt(1
/a)) + 105*a**2*b**2*x**2*log(-I*sqrt(b)*sqrt(1/a) + sqrt(x))/(24*I*a**7*sq
rt(b)*x**2*sqrt(1/a) + 48*I*a**6*b**(3/2)*x*sqrt(1/a) + 24*I*a**5*b**(5/2)*
sqrt(1/a)) - 105*a**2*b**2*x**2*log(I*sqrt(b)*sqrt(1/a) + sqrt(x))/(24*I*a*
*7*sqrt(b)*x**2*sqrt(1/a) + 48*I*a**6*b**(3/2)*x*sqrt(1/a) + 24*I*a**5*b**(
5/2)*sqrt(1/a)) - 210*I*a*b**(7/2)*sqrt(x)*sqrt(1/a)/(24*I*a**7*sqrt(b)*x**
2*sqrt(1/a) + 48*I*a**6*b**(3/2)*x*sqrt(1/a) + 24*I*a**5*b**(5/2)*sqrt(1/a)
) + 210*a*b**3*x*log(-I*sqrt(b)*sqrt(1/a) + sqrt(x))/(24*I*a**7*sqrt(b)*x**
2*sqrt(1/a) + 48*I*a**6*b**(3/2)*x*sqrt(1/a) + 24*I*a**5*b**(5/2)*sqrt(1/a)
) - 210*a*b**3*x*log(I*sqrt(b)*sqrt(1/a) + sqrt(x))/(24*I*a**7*sqrt(b)*x**2
*sqrt(1/a) + 48*I*a**6*b**(3/2)*x*sqrt(1/a) + 24*I*a**5*b**(5/2)*sqrt(1/a))
 + 105*b**4*log(-I*sqrt(b)*sqrt(1/a) + sqrt(x))/(24*I*a**7*sqrt(b)*x**2*sqr
t(1/a) + 48*I*a**6*b**(3/2)*x*sqrt(1/a) + 24*I*a**5*b**(5/2)*sqrt(1/a)) - 1
05*b**4*log(I*sqrt(b)*sqrt(1/a) + sqrt(x))/(24*I*a**7*sqrt(b)*x**2*sqrt(1/a
) + 48*I*a**6*b**(3/2)*x*sqrt(1/a) + 24*I*a**5*b**(5/2)*sqrt(1/a)), True))
```

3.1684 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 \sqrt{x}}\,dx$

Optimal. Leaf size=82

$$-\frac{15\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{4a^{7/2}}+\frac{15\sqrt{x}}{4a^3}-\frac{5x^{3/2}}{4a^2(ax+b)}-\frac{x^{5/2}}{2a(ax+b)^2}$$

```
[Out] -1/2*x^(5/2)/a/(a*x+b)^2-5/4*x^(3/2)/a^2/(a*x+b)-15/4*arctan(a^(1/2)*x^(1/2
)/b^(1/2))*b^(1/2)/a^(7/2)+15/4*x^(1/2)/a^3
```

Rubi [A] time = 0.03, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {263, 47, 50, 63, 205}

$$-\frac{5x^{3/2}}{4a^2(ax+b)}-\frac{15\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{4a^{7/2}}+\frac{15\sqrt{x}}{4a^3}-\frac{x^{5/2}}{2a(ax+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^3*Sqrt[x]),x]
```

```
[Out] (15*Sqrt[x])/(4*a^3) - x^(5/2)/(2*a*(b + a*x)^2) - (5*x^(3/2))/(4*a^2*(b +
a*x)) - (15*Sqrt[b]*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/(4*a^(7/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 \sqrt{x}}\,dx = \int \frac{x^{5/2}}{(b+ax)^3}\,dx$$

$$= -\frac{x^{5/2}}{2a(b+ax)^2} + \frac{5\int \frac{x^{3/2}}{(b+ax)^2}\,dx}{4a}$$

$$= -\frac{x^{5/2}}{2a(b+ax)^2} - \frac{5x^{3/2}}{4a^2(b+ax)} + \frac{15\int \frac{\sqrt{x}}{b+ax}\,dx}{8a^2}$$

$$= \frac{15\sqrt{x}}{4a^3} - \frac{x^{5/2}}{2a(b+ax)^2} - \frac{5x^{3/2}}{4a^2(b+ax)} - \frac{(15b)\int \frac{1}{\sqrt{x}\,(b+ax)}\,dx}{8a^3}$$

$$= \frac{15\sqrt{x}}{4a^3} - \frac{x^{5/2}}{2a(b+ax)^2} - \frac{5x^{3/2}}{4a^2(b+ax)} - \frac{(15b)\,\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \sqrt{x}\right)}{4a^3}$$

$$= \frac{15\sqrt{x}}{4a^3} - \frac{x^{5/2}}{2a(b+ax)^2} - \frac{5x^{3/2}}{4a^2(b+ax)} - \frac{15\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{4a^{7/2}}$$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.33

$$\frac{2x^{7/2}\,{}_2F_1\left(3, \frac{7}{2}; \frac{9}{2}; -\frac{ax}{b}\right)}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^3*Sqrt[x]),x]
```

```
[Out] (2*x^(7/2)*Hypergeometric2F1[3, 7/2, 9/2, -((a*x)/b)])/(7*b^3)
```

fricas [A] time = 1.03, size = 200, normalized size = 2.44

$$\left[\frac{15\left(a^2x^2+2\,abx+b^2\right)\sqrt{-\frac{b}{a}}\,\log\left(\frac{ax-2\,a\sqrt{x}\,\sqrt{-\frac{b}{a}}-b}{ax+b}\right)+2\left(8\,a^2x^2+25\,abx+15\,b^2\right)\sqrt{x}}{8\left(a^5x^2+2\,a^4bx+a^3b^2\right)}, -\frac{15\left(a^2x^2+2\,abx+b^2\right)\sqrt{\frac{b}{a}}\,a}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(15*(a^2*x^2 + 2*a*b*x + b^2)*sqrt(-b/a)*log((a*x - 2*a*sqrt(x)*sqrt(-
b/a) - b)/(a*x + b)) + 2*(8*a^2*x^2 + 25*a*b*x + 15*b^2)*sqrt(x))/(a^5*x^2
+ 2*a^4*b*x + a^3*b^2), -1/4*(15*(a^2*x^2 + 2*a*b*x + b^2)*sqrt(b/a)*arctan
(a*sqrt(x)*sqrt(b/a)/b) - (8*a^2*x^2 + 25*a*b*x + 15*b^2)*sqrt(x))/(a^5*x^2
 + 2*a^4*b*x + a^3*b^2)]
```

giac [A] time = 0.16, size = 59, normalized size = 0.72

$$-\frac{15\,b\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\,a^3} + \frac{2\sqrt{x}}{a^3} + \frac{9\,abx^{\frac{3}{2}}+7\,b^2\sqrt{x}}{4\,(ax+b)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] -15/4*b*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) + 2*sqrt(x)/a^3 + 1/4*(
9*a*b*x^(3/2) + 7*b^2*sqrt(x))/((a*x + b)^2*a^3)
```

maple [A] time = 0.02, size = 66, normalized size = 0.80

$$\frac{9b\,x^{\frac{3}{2}}}{4\,(ax+b)^2\,a^2}+\frac{7b^2\sqrt{x}}{4\,(ax+b)^2\,a^3}-\frac{15b\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\;a^3}+\frac{2\sqrt{x}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^3/x^(1/2),x)
```

```
[Out] 2*x^(1/2)/a^3+9/4/a^2*b/(a*x+b)^2*x^(3/2)+7/4/a^3*b^2/(a*x+b)^2*x^(1/2)-15/
4/a^3*b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*a*x^(1/2))
```

maxima [A] time = 2.41, size = 75, normalized size = 0.91

$$\frac{8\,a^2+\frac{25\,ab}{x}+\frac{15\,b^2}{x^2}}{4\left(\frac{a^5}{\sqrt{x}}+\frac{2\,a^4b}{x^{\frac{3}{2}}}+\frac{a^3b^2}{x^{\frac{5}{2}}}\right)}+\frac{15\,b\arctan\left(\frac{b}{\sqrt{ab}\,\sqrt{x}}\right)}{4\,\sqrt{ab}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(8*a^2 + 25*a*b/x + 15*b^2/x^2)/(a^5/sqrt(x) + 2*a^4*b/x^(3/2) + a^3*b^
2/x^(5/2)) + 15/4*b*arctan(b/(sqrt(a*b)*sqrt(x)))/(sqrt(a*b)*a^3)
```

mupad [B] time = 0.08, size = 69, normalized size = 0.84

$$\frac{\frac{7\,b^2\,\sqrt{x}}{4}+\frac{9\,a\,b\,x^{3/2}}{4}}{a^5\,x^2+2\,a^4\,b\,x+a^3\,b^2}+\frac{2\,\sqrt{x}}{a^3}-\frac{15\,\sqrt{b}\;\mathrm{atan}\left(\frac{\sqrt{a}\;\sqrt{x}}{\sqrt{b}}\right)}{4\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b/x)^3),x)
```

```
[Out] ((7*b^2*x^(1/2))/4 + (9*a*b*x^(3/2))/4)/(a^3*b^2 + a^5*x^2 + 2*a^4*b*x) + (
2*x^(1/2))/a^3 - (15*b^(1/2)*atan((a^(1/2)*x^(1/2))/b^(1/2)))/(4*a^(7/2))
```

sympy [A] time = 39.81, size = 816, normalized size = 9.95

$$\begin{cases}\tilde{\infty}x^{\frac{7}{2}} & \\ \frac{2\sqrt{x}}{a^3} & \\ \frac{2x^{\frac{7}{2}}}{7b^3} & \\ \frac{16ia^3\sqrt{b}\,x^{\frac{5}{2}}\sqrt{\frac{1}{a}}}{8ia^6\sqrt{b}\,x^2\sqrt{\frac{1}{a}}+16ia^5b^{\frac{3}{2}}x\sqrt{\frac{1}{a}}+8ia^4b^{\frac{5}{2}}\sqrt{\frac{1}{a}}}+\frac{50ia^2b^{\frac{3}{2}}x^{\frac{3}{2}}\sqrt{\frac{1}{a}}}{8ia^6\sqrt{b}\,x^2\sqrt{\frac{1}{a}}+16ia^5b^{\frac{3}{2}}x\sqrt{\frac{1}{a}}+8ia^4b^{\frac{5}{2}}\sqrt{\frac{1}{a}}}-\frac{15a^2bx^2\log\left(-i\sqrt{b}\,\sqrt{\frac{1}{a}}+\sqrt{x}\right)}{8ia^6\sqrt{b}\,x^2\sqrt{\frac{1}{a}}+16ia^5b^{\frac{3}{2}}x\sqrt{\frac{1}{a}}+8ia^4b^{\frac{5}{2}}\sqrt{\frac{1}{a}}}+\frac{}{8ia} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**3/x**(1/2),x)
```

```
[Out] Piecewise((zoo*x**(7/2), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/a**3, Eq(b, 0)),
(2*x**(7/2)/(7*b**3), Eq(a, 0)), (16*I*a**3*sqrt(b)*x**(5/2)*sqrt(1/a)/(8*I
*a**6*sqrt(b)*x**2*sqrt(1/a) + 16*I*a**5*b**(3/2)*x*sqrt(1/a) + 8*I*a**4*b*
*(5/2)*sqrt(1/a)) + 50*I*a**2*b**(3/2)*x**(3/2)*sqrt(1/a)/(8*I*a**6*sqrt(b)
*x**2*sqrt(1/a) + 16*I*a**5*b**(3/2)*x*sqrt(1/a) + 8*I*a**4*b**(5/2)*sqrt(1
/a)) - 15*a**2*b*x**2*log(-I*sqrt(b)*sqrt(1/a) + sqrt(x))/(8*I*a**6*sqrt(b)
*x**2*sqrt(1/a) + 16*I*a**5*b**(3/2)*x*sqrt(1/a) + 8*I*a**4*b**(5/2)*sqrt(1
/a)) + 15*a**2*b*x**2*log(I*sqrt(b)*sqrt(1/a) + sqrt(x))/(8*I*a**6*sqrt(b)*
x**2*sqrt(1/a) + 16*I*a**5*b**(3/2)*x*sqrt(1/a) + 8*I*a**4*b**(5/2)*sqrt(1/
a)) + 30*I*a*b**(5/2)*sqrt(x)*sqrt(1/a)/(8*I*a**6*sqrt(b)*x**2*sqrt(1/a) +
16*I*a**5*b**(3/2)*x*sqrt(1/a) + 8*I*a**4*b**(5/2)*sqrt(1/a)) - 30*a*b**2*x
*log(-I*sqrt(b)*sqrt(1/a) + sqrt(x))/(8*I*a**6*sqrt(b)*x**2*sqrt(1/a) + 16*
I*a**5*b**(3/2)*x*sqrt(1/a) + 8*I*a**4*b**(5/2)*sqrt(1/a)) + 30*a*b**2*x*lo
g(I*sqrt(b)*sqrt(1/a) + sqrt(x))/(8*I*a**6*sqrt(b)*x**2*sqrt(1/a) + 16*I*a*
*5*b**(3/2)*x*sqrt(1/a) + 8*I*a**4*b**(5/2)*sqrt(1/a)) - 15*b**3*log(-I*sqr
t(b)*sqrt(1/a) + sqrt(x))/(8*I*a**6*sqrt(b)*x**2*sqrt(1/a) + 16*I*a**5*b**(
3/2)*x*sqrt(1/a) + 8*I*a**4*b**(5/2)*sqrt(1/a)) + 15*b**3*log(I*sqrt(b)*sqr
t(1/a) + sqrt(x))/(8*I*a**6*sqrt(b)*x**2*sqrt(1/a) + 16*I*a**5*b**(3/2)*x*s
qrt(1/a) + 8*I*a**4*b**(5/2)*sqrt(1/a)), True))
```

3.1685 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{3/2}}\,dx$

Optimal. Leaf size=70

$$\frac{3\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{4a^{5/2}\sqrt{b}} - \frac{3\sqrt{x}}{4a^2(ax+b)} - \frac{x^{3/2}}{2a(ax+b)^2}$$

```
[Out] -1/2*x^(3/2)/a/(a*x+b)^2+3/4*arctan(a^(1/2)*x^(1/2)/b^(1/2))/a^(5/2)/b^(1/2
)-3/4*x^(1/2)/a^2/(a*x+b)
```

Rubi [A] time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {263, 47, 63, 205}

$$-\frac{3\sqrt{x}}{4a^2(ax+b)} + \frac{3\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{4a^{5/2}\sqrt{b}} - \frac{x^{3/2}}{2a(ax+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^3*x^(3/2)),x]
```

```
[Out] -x^(3/2)/(2*a*(b + a*x)^2) - (3*Sqrt[x])/(4*a^2*(b + a*x)) + (3*ArcTan[(Sqr
t[a]*Sqrt[x])/Sqrt[b]])/(4*a^(5/2)*Sqrt[b])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{3/2}}\,dx = \int \frac{x^{3/2}}{(b+ax)^3}\,dx$$

$$= -\frac{x^{3/2}}{2a(b+ax)^2} + \frac{3\int \frac{\sqrt{x}}{(b+ax)^2}\,dx}{4a}$$

$$= -\frac{x^{3/2}}{2a(b+ax)^2} - \frac{3\sqrt{x}}{4a^2(b+ax)} + \frac{3\int \frac{1}{\sqrt{x}\,(b+ax)}\,dx}{8a^2}$$

$$= -\frac{x^{3/2}}{2a(b+ax)^2} - \frac{3\sqrt{x}}{4a^2(b+ax)} + \frac{3\,\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \sqrt{x}\right)}{4a^2}$$

$$= -\frac{x^{3/2}}{2a(b+ax)^2} - \frac{3\sqrt{x}}{4a^2(b+ax)} + \frac{3\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{4a^{5/2}\sqrt{b}}$$

Mathematica [A] time = 0.05, size = 59, normalized size = 0.84

$$\frac{3\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{4a^{5/2}\sqrt{b}} - \frac{\sqrt{x}\,(5ax+3b)}{4a^2(ax+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^3*x^(3/2)),x]
```

```
[Out] -1/4*(Sqrt[x]*(3*b + 5*a*x))/(a^2*(b + a*x)^2) + (3*ArcTan[(Sqrt[a]*Sqrt[x]
)/Sqrt[b]])/(4*a^(5/2)*Sqrt[b])
```

fricas [A] time = 1.01, size = 185, normalized size = 2.64

$$\left[-\frac{3\left(a^2x^2+2\,abx+b^2\right)\sqrt{-ab}\,\log\left(\frac{ax-b-2\,\sqrt{-ab}\,\sqrt{x}}{ax+b}\right)+2\left(5\,a^2bx+3\,ab^2\right)\sqrt{x}}{8\left(a^5bx^2+2\,a^4b^2x+a^3b^3\right)}, -\frac{3\left(a^2x^2+2\,abx+b^2\right)\sqrt{ab}\,\arctan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*(a^2*x^2 + 2*a*b*x + b^2)*sqrt(-a*b)*log((a*x - b - 2*sqrt(-a*b)*s
qrt(x))/(a*x + b)) + 2*(5*a^2*b*x + 3*a*b^2)*sqrt(x))/(a^5*b*x^2 + 2*a^4*b^
2*x + a^3*b^3), -1/4*(3*(a^2*x^2 + 2*a*b*x + b^2)*sqrt(a*b)*arctan(sqrt(a*b
)/(a*sqrt(x))) + (5*a^2*b*x + 3*a*b^2)*sqrt(x))/(a^5*b*x^2 + 2*a^4*b^2*x +
a^3*b^3)]
```

giac [A] time = 0.16, size = 47, normalized size = 0.67

$$\frac{3\,\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,a^2} - \frac{5\,ax^{\frac{3}{2}}+3\,b\sqrt{x}}{4\,(ax+b)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^(3/2),x, algorithm="giac")
```

```
[Out] 3/4*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/4*(5*a*x^(3/2) + 3*b*sq
rt(x))/((a*x + b)^2*a^2)
```

maple [A] time = 0.01, size = 50, normalized size = 0.71

$$\frac{3\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\,a^2}+\frac{-\frac{5x^{\frac{3}{2}}}{4a}-\frac{3b\sqrt{x}}{4a^2}}{(ax+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^3/x^(3/2),x)
```

```
[Out] 2*(-5/8/a*x^(3/2)-3/8/a^2*b*x^(1/2))/(a*x+b)^2+3/4/a^2/(a*b)^(1/2)*arctan(1
/(a*b)^(1/2)*a*x^(1/2))
```

maxima [A] time = 2.33, size = 62, normalized size = 0.89

$$-\frac{\frac{5a}{\sqrt{x}}+\frac{3b}{x^{\frac{3}{2}}}}{4\left(a^4+\frac{2a^3b}{x}+\frac{a^2b^2}{x^2}\right)}-\frac{3\arctan\left(\frac{b}{\sqrt{ab}\sqrt{x}}\right)}{4\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(5*a/sqrt(x) + 3*b/x^(3/2))/(a^4 + 2*a^3*b/x + a^2*b^2/x^2) - 3/4*arct
an(b/(sqrt(a*b)*sqrt(x)))/(sqrt(a*b)*a^2)
```

mupad [B] time = 1.12, size = 58, normalized size = 0.83

$$\frac{3\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{4\,a^{5/2}\,\sqrt{b}}-\frac{\frac{5\,x^{3/2}}{4\,a}+\frac{3\,b\,\sqrt{x}}{4\,a^2}}{a^2\,x^2+2\,a\,b\,x+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b/x)^3),x)
```

```
[Out] (3*atan((a^(1/2)*x^(1/2))/b^(1/2)))/(4*a^(5/2)*b^(1/2)) - ((5*x^(3/2))/(4*a
) + (3*b*x^(1/2))/(4*a^2))/(b^2 + a^2*x^2 + 2*a*b*x)
```

sympy [A] time = 110.84, size = 726, normalized size = 10.37

$$\begin{cases}\tilde{\infty}x^{\frac{5}{2}} \\ \frac{2x^{\frac{5}{2}}}{5b^3} \\ -\frac{2}{a^3\sqrt{x}} \\ -\frac{10ia^2\sqrt{b}\,x^{\frac{3}{2}}\sqrt{\frac{1}{a}}}{8ia^5\sqrt{b}\,x^2\sqrt{\frac{1}{a}}+16ia^4b^{\frac{3}{2}}x\sqrt{\frac{1}{a}}+8ia^3b^{\frac{5}{2}}\sqrt{\frac{1}{a}}}+\frac{3a^2x^2\log\left(-i\sqrt{b}\sqrt{\frac{1}{a}}+\sqrt{x}\right)}{8ia^5\sqrt{b}\,x^2\sqrt{\frac{1}{a}}+16ia^4b^{\frac{3}{2}}x\sqrt{\frac{1}{a}}+8ia^3b^{\frac{5}{2}}\sqrt{\frac{1}{a}}}-\frac{3a^2x^2\log\left(i\sqrt{b}\sqrt{\frac{1}{a}}+\sqrt{x}\right)}{8ia^5\sqrt{b}\,x^2\sqrt{\frac{1}{a}}+16ia^4b^{\frac{3}{2}}x\sqrt{\frac{1}{a}}+8ia^3b^{\frac{5}{2}}\sqrt{\frac{1}{a}}}- \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**3/x**(3/2),x)
```

```
[Out] Piecewise((zoo*x**(5/2), Eq(a, 0) & Eq(b, 0)), (2*x**(5/2)/(5*b**3), Eq(a,
0)), (-2/(a**3*sqrt(x)), Eq(b, 0)), (-10*I*a**2*sqrt(b)*x**(3/2)*sqrt(1/a)/
(8*I*a**5*sqrt(b)*x**2*sqrt(1/a) + 16*I*a**4*b**(3/2)*x*sqrt(1/a) + 8*I*a**
3*b**(5/2)*sqrt(1/a)) + 3*a**2*x**2*log(-I*sqrt(b)*sqrt(1/a) + sqrt(x))/(8*
I*a**5*sqrt(b)*x**2*sqrt(1/a) + 16*I*a**4*b**(3/2)*x*sqrt(1/a) + 8*I*a**3*b
```

```
**(5/2)*sqrt(1/a)) - 3*a**2*x**2*log(I*sqrt(b)*sqrt(1/a) + sqrt(x))/(8*I*a*
*5*sqrt(b)*x**2*sqrt(1/a) + 16*I*a**4*b**(3/2)*x*sqrt(1/a) + 8*I*a**3*b**(5
/2)*sqrt(1/a)) - 6*I*a*b**(3/2)*sqrt(x)*sqrt(1/a)/(8*I*a**5*sqrt(b)*x**2*sq
rt(1/a) + 16*I*a**4*b**(3/2)*x*sqrt(1/a) + 8*I*a**3*b**(5/2)*sqrt(1/a)) + 6
*a*b*x*log(-I*sqrt(b)*sqrt(1/a) + sqrt(x))/(8*I*a**5*sqrt(b)*x**2*sqrt(1/a)
 + 16*I*a**4*b**(3/2)*x*sqrt(1/a) + 8*I*a**3*b**(5/2)*sqrt(1/a)) - 6*a*b*x*
log(I*sqrt(b)*sqrt(1/a) + sqrt(x))/(8*I*a**5*sqrt(b)*x**2*sqrt(1/a) + 16*I*
a**4*b**(3/2)*x*sqrt(1/a) + 8*I*a**3*b**(5/2)*sqrt(1/a)) + 3*b**2*log(-I*sq
rt(b)*sqrt(1/a) + sqrt(x))/(8*I*a**5*sqrt(b)*x**2*sqrt(1/a) + 16*I*a**4*b**
(3/2)*x*sqrt(1/a) + 8*I*a**3*b**(5/2)*sqrt(1/a)) - 3*b**2*log(I*sqrt(b)*sqr
t(1/a) + sqrt(x))/(8*I*a**5*sqrt(b)*x**2*sqrt(1/a) + 16*I*a**4*b**(3/2)*x*s
qrt(1/a) + 8*I*a**3*b**(5/2)*sqrt(1/a)), True))
```

3.1686 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{5/2}}\,dx$

Optimal. Leaf size=73

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{4a^{3/2}b^{3/2}}+\frac{\sqrt{x}}{4ab(ax+b)}-\frac{\sqrt{x}}{2a(ax+b)^2}$$

```
[Out] 1/4*arctan(a^(1/2)*x^(1/2)/b^(1/2))/a^(3/2)/b^(3/2)-1/2*x^(1/2)/a/(a*x+b)^2
+1/4*x^(1/2)/a/b/(a*x+b)
```

Rubi [A] time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {263, 47, 51, 63, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{4a^{3/2}b^{3/2}}+\frac{\sqrt{x}}{4ab(ax+b)}-\frac{\sqrt{x}}{2a(ax+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^3*x^(5/2)),x]
```

```
[Out] -Sqrt[x]/(2*a*(b + a*x)^2) + Sqrt[x]/(4*a*b*(b + a*x)) + ArcTan[(Sqrt[a]*Sq
rt[x])/Sqrt[b]]/(4*a^(3/2)*b^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{5/2}}\,dx &= \int \frac{\sqrt{x}}{(b+ax)^3}\,dx \\ &= -\frac{\sqrt{x}}{2a(b+ax)^2} + \frac{\int \frac{1}{\sqrt{x}\,(b+ax)^2}\,dx}{4a} \\ &= -\frac{\sqrt{x}}{2a(b+ax)^2} + \frac{\sqrt{x}}{4ab(b+ax)} + \frac{\int \frac{1}{\sqrt{x}\,(b+ax)}\,dx}{8ab} \\ &= -\frac{\sqrt{x}}{2a(b+ax)^2} + \frac{\sqrt{x}}{4ab(b+ax)} + \frac{\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \sqrt{x}\right)}{4ab} \\ &= -\frac{\sqrt{x}}{2a(b+ax)^2} + \frac{\sqrt{x}}{4ab(b+ax)} + \frac{\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{4a^{3/2}b^{3/2}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.37

$$\frac{2x^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};-\frac{ax}{b}\right)}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^3*x^(5/2)),x]
```

```
[Out] (2*x^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, -((a*x)/b)])/(3*b^3)
```

fricas [A] time = 0.83, size = 186, normalized size = 2.55

$$\left[-\frac{\left(a^2x^2+2\,abx+b^2\right)\sqrt{-ab}\,\log\left(\frac{ax-b-2\,\sqrt{-ab}\,\sqrt{x}}{ax+b}\right)-2\left(a^2bx-ab^2\right)\sqrt{x}}{8\left(a^4b^2x^2+2\,a^3b^3x+a^2b^4\right)}, -\frac{\left(a^2x^2+2\,abx+b^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}}{a\sqrt{x}}\right)-}{4\left(a^4b^2x^2+2\,a^3b^3x+a^2b^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((a^2*x^2 + 2*a*b*x + b^2)*sqrt(-a*b)*log((a*x - b - 2*sqrt(-a*b)*sqr
t(x))/(a*x + b)) - 2*(a^2*b*x - a*b^2)*sqrt(x))/(a^4*b^2*x^2 + 2*a^3*b^3*x
+ a^2*b^4), -1/4*((a^2*x^2 + 2*a*b*x + b^2)*sqrt(a*b)*arctan(sqrt(a*b)/(a*s
qrt(x))) - (a^2*b*x - a*b^2)*sqrt(x))/(a^4*b^2*x^2 + 2*a^3*b^3*x + a^2*b^4)
]
```

giac [A] time = 0.16, size = 52, normalized size = 0.71

$$\frac{\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\,ab} + \frac{ax^{\frac{3}{2}} - b\sqrt{x}}{4\,(ax+b)^2 ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^(5/2),x, algorithm="giac")
```

```
[Out] 1/4*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a*b) + 1/4*(a*x^(3/2) - b*sqrt(x
))/((a*x + b)^2*a*b)
```

maple [A] time = 0.01, size = 52, normalized size = 0.71

$$\frac{\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\,ab} + \frac{\frac{x^{\frac{3}{2}}}{4b} - \frac{\sqrt{x}}{4a}}{(ax+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^3/x^(5/2),x)
```

```
[Out] 2*(1/8*x^(3/2)/b-1/8/a*x^(1/2))/(a*x+b)^2+1/4/b/a/(a*b)^(1/2)*arctan(1/(a*b
)^(1/2)*a*x^(1/2))
```

maxima [A] time = 2.20, size = 66, normalized size = 0.90

$$\frac{\frac{a}{\sqrt{x}} - \frac{b}{x^{\frac{3}{2}}}}{4\left(a^3 b + \frac{2\,a^2 b^2}{x} + \frac{a b^3}{x^2}\right)} - \frac{\arctan\left(\frac{b}{\sqrt{ab}\,\sqrt{x}}\right)}{4\sqrt{ab}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(a/sqrt(x) - b/x^(3/2))/(a^3*b + 2*a^2*b^2/x + a*b^3/x^2) - 1/4*arctan(
b/(sqrt(a*b)*sqrt(x)))/(sqrt(a*b)*a*b)
```

mupad [B] time = 1.13, size = 57, normalized size = 0.78

$$\frac{\operatorname{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{4\,a^{3/2}\,b^{3/2}} - \frac{\frac{\sqrt{x}}{4\,a} - \frac{x^{3/2}}{4\,b}}{a^2\,x^2 + 2\,a\,b\,x + b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b/x)^3),x)
```

```
[Out] atan((a^(1/2)*x^(1/2))/b^(1/2))/(4*a^(3/2)*b^(3/2)) - (x^(1/2)/(4*a) - x^(3
/2)/(4*b))/(b^2 + a^2*x^2 + 2*a*b*x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**3/x**(5/2),x)
```

```
[Out] Timed out
```

3.1687 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{7/2}}\,dx$

Optimal. Leaf size=70

$$\frac{3\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{4\sqrt{a}\,b^{5/2}}+\frac{3\sqrt{x}}{4b^2(ax+b)}+\frac{\sqrt{x}}{2b(ax+b)^2}$$

```
[Out] 3/4*arctan(a^(1/2)*x^(1/2)/b^(1/2))/b^(5/2)/a^(1/2)+1/2*x^(1/2)/b/(a*x+b)^2
+3/4*x^(1/2)/b^2/(a*x+b)
```

Rubi [A] time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {263, 51, 63, 205}

$$\frac{3\sqrt{x}}{4b^2(ax+b)}+\frac{3\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{4\sqrt{a}\,b^{5/2}}+\frac{\sqrt{x}}{2b(ax+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^3*x^(7/2)),x]
```

```
[Out] Sqrt[x]/(2*b*(b + a*x)^2) + (3*Sqrt[x])/(4*b^2*(b + a*x)) + (3*ArcTan[(Sqrt
[a]*Sqrt[x])/Sqrt[b]])/(4*Sqrt[a]*b^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{7/2}}\,dx = \int \frac{1}{\sqrt{x}\,(b+ax)^3}\,dx$$

$$= \frac{\sqrt{x}}{2b(b+ax)^2} + \frac{3\int \frac{1}{\sqrt{x}\,(b+ax)^2}\,dx}{4b}$$

$$= \frac{\sqrt{x}}{2b(b+ax)^2} + \frac{3\sqrt{x}}{4b^2(b+ax)} + \frac{3\int \frac{1}{\sqrt{x}\,(b+ax)}\,dx}{8b^2}$$

$$= \frac{\sqrt{x}}{2b(b+ax)^2} + \frac{3\sqrt{x}}{4b^2(b+ax)} + \frac{3\,\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \sqrt{x}\right)}{4b^2}$$

$$= \frac{\sqrt{x}}{2b(b+ax)^2} + \frac{3\sqrt{x}}{4b^2(b+ax)} + \frac{3\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{4\sqrt{a}\,b^{5/2}}$$

Mathematica [C] time = 0.00, size = 25, normalized size = 0.36

$$\frac{2\sqrt{x}\;{}_2F_1\left(\frac{1}{2}, 3; \frac{3}{2}; -\frac{ax}{b}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^3*x^(7/2)),x]
```

```
[Out] (2*Sqrt[x]*Hypergeometric2F1[1/2, 3, 3/2, -((a*x)/b)])/b^3
```

fricas [A] time = 1.03, size = 186, normalized size = 2.66

$$\left[-\frac{3\left(a^2x^2+2\,abx+b^2\right)\sqrt{-ab}\,\log\left(\frac{ax-b-2\,\sqrt{-ab}\,\sqrt{x}}{ax+b}\right)-2\left(3\,a^2bx+5\,ab^2\right)\sqrt{x}}{8\left(a^3b^3x^2+2\,a^2b^4x+ab^5\right)}, -\frac{3\left(a^2x^2+2\,abx+b^2\right)\sqrt{ab}\,\arctan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*(a^2*x^2 + 2*a*b*x + b^2)*sqrt(-a*b)*log((a*x - b - 2*sqrt(-a*b)*s
qrt(x))/(a*x + b)) - 2*(3*a^2*b*x + 5*a*b^2)*sqrt(x))/(a^3*b^3*x^2 + 2*a^2*
b^4*x + a*b^5), -1/4*(3*(a^2*x^2 + 2*a*b*x + b^2)*sqrt(a*b)*arctan(sqrt(a*b
)/(a*sqrt(x))) - (3*a^2*b*x + 5*a*b^2)*sqrt(x))/(a^3*b^3*x^2 + 2*a^2*b^4*x
+ a*b^5)]
```

giac [A] time = 0.15, size = 47, normalized size = 0.67

$$\frac{3\,\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\,b^2} + \frac{3\,ax^{\frac{3}{2}}+5\,b\sqrt{x}}{4\,(ax+b)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^(7/2),x, algorithm="giac")
```

```
[Out] 3/4*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/4*(3*a*x^(3/2) + 5*b*sq
rt(x))/((a*x + b)^2*b^2)
```

maple [A] time = 0.01, size = 53, normalized size = 0.76

$$\frac{\sqrt{x}}{2\,(ax+b)^2\,b}+\frac{3\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\;b^2}+\frac{3\sqrt{x}}{4\,(ax+b)\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^3/x^(7/2),x)
```

```
[Out] 1/2*x^(1/2)/b/(a*x+b)^2+3/4*x^(1/2)/b^2/(a*x+b)+3/4/b^2/(a*b)^(1/2)*arctan(
1/(a*b)^(1/2)*a*x^(1/2))
```

maxima [A] time = 2.37, size = 63, normalized size = 0.90

$$\frac{\frac{3\,a}{\sqrt{x}}+\frac{5\,b}{x^{\frac{3}{2}}}}{4\left(a^2b^2+\frac{2\,ab^3}{x}+\frac{b^4}{x^2}\right)}-\frac{3\,\arctan\left(\frac{b}{\sqrt{ab}\,\sqrt{x}}\right)}{4\,\sqrt{ab}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^(7/2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*a/sqrt(x) + 5*b/x^(3/2))/(a^2*b^2 + 2*a*b^3/x + b^4/x^2) - 3/4*arcta
n(b/(sqrt(a*b)*sqrt(x)))/(sqrt(a*b)*b^2)
```

mupad [B] time = 1.10, size = 57, normalized size = 0.81

$$\frac{\frac{5\,\sqrt{x}}{4\,b}+\frac{3\,a\,x^{3/2}}{4\,b^2}}{a^2\,x^2+2\,a\,b\,x+b^2}+\frac{3\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{4\,\sqrt{a}\;b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a + b/x)^3),x)
```

```
[Out] ((5*x^(1/2))/(4*b) + (3*a*x^(3/2))/(4*b^2))/(b^2 + a^2*x^2 + 2*a*b*x) + (3*
atan((a^(1/2)*x^(1/2))/b^(1/2)))/(4*a^(1/2)*b^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**3/x**(7/2),x)
```

```
[Out] Timed out
```

3.1688 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{9/2}}\,dx$

Optimal. Leaf size=82

$$-\frac{15\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{4b^{7/2}}+\frac{5}{4b^2\sqrt{x}\,(ax+b)}+\frac{1}{2b\sqrt{x}\,(ax+b)^2}-\frac{15}{4b^3\sqrt{x}}$$

```
[Out] -15/4*arctan(a^(1/2)*x^(1/2)/b^(1/2))*a^(1/2)/b^(7/2)-15/4/b^3/x^(1/2)+1/2/
b/(a*x+b)^2/x^(1/2)+5/4/b^2/(a*x+b)/x^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {263, 51, 63, 205}

$$\frac{5}{4b^2\sqrt{x}\,(ax+b)}-\frac{15\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{4b^{7/2}}+\frac{1}{2b\sqrt{x}\,(ax+b)^2}-\frac{15}{4b^3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^3*x^(9/2)),x]
```

```
[Out] -15/(4*b^3*Sqrt[x]) + 1/(2*b*Sqrt[x]*(b + a*x)^2) + 5/(4*b^2*Sqrt[x]*(b + a
*x)) - (15*Sqrt[a]*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/(4*b^(7/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{9/2}}\,dx = \int \frac{1}{x^{3/2}(b+ax)^3}\,dx$$

$$= \frac{1}{2b\sqrt{x}\,(b+ax)^2} + \frac{5\int \frac{1}{x^{3/2}(b+ax)^2}\,dx}{4b}$$

$$= \frac{1}{2b\sqrt{x}\,(b+ax)^2} + \frac{5}{4b^2\sqrt{x}\,(b+ax)} + \frac{15\int \frac{1}{x^{3/2}(b+ax)}\,dx}{8b^2}$$

$$= -\frac{15}{4b^3\sqrt{x}} + \frac{1}{2b\sqrt{x}\,(b+ax)^2} + \frac{5}{4b^2\sqrt{x}\,(b+ax)} - \frac{(15a)\int \frac{1}{\sqrt{x}\,(b+ax)}\,dx}{8b^3}$$

$$= -\frac{15}{4b^3\sqrt{x}} + \frac{1}{2b\sqrt{x}\,(b+ax)^2} + \frac{5}{4b^2\sqrt{x}\,(b+ax)} - \frac{(15a)\,\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \sqrt{x}\right)}{4b^3}$$

$$= -\frac{15}{4b^3\sqrt{x}} + \frac{1}{2b\sqrt{x}\,(b+ax)^2} + \frac{5}{4b^2\sqrt{x}\,(b+ax)} - \frac{15\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{4b^{7/2}}$$

Mathematica [C] time = 0.00, size = 25, normalized size = 0.30

$$-\frac{2\,{}_2F_1\left(-\frac{1}{2}, 3; \frac{1}{2}; -\frac{ax}{b}\right)}{b^3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^3*x^(9/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/2, 3, 1/2, -((a*x)/b)])/(b^3*Sqrt[x])
```

fricas [A] time = 1.09, size = 214, normalized size = 2.61

$$\left[\frac{15\left(a^2x^3+2abx^2+b^2x\right)\sqrt{-\frac{a}{b}}\log\left(\frac{ax-2b\sqrt{x}\sqrt{-\frac{a}{b}}-b}{ax+b}\right)-2\left(15a^2x^2+25abx+8b^2\right)\sqrt{x}}{8\left(a^2b^3x^3+2ab^4x^2+b^5x\right)}, \frac{15\left(a^2x^3+2abx^2+b^2x\right)\sqrt{}}{4\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^(9/2),x, algorithm="fricas")
```

```
[Out] [1/8*(15*(a^2*x^3 + 2*a*b*x^2 + b^2*x)*sqrt(-a/b)*log((a*x - 2*b*sqrt(x)*sq
rt(-a/b) - b)/(a*x + b)) - 2*(15*a^2*x^2 + 25*a*b*x + 8*b^2)*sqrt(x))/(a^2*
b^3*x^3 + 2*a*b^4*x^2 + b^5*x), 1/4*(15*(a^2*x^3 + 2*a*b*x^2 + b^2*x)*sqrt(
a/b)*arctan(b*sqrt(a/b)/(a*sqrt(x))) - (15*a^2*x^2 + 25*a*b*x + 8*b^2)*sqrt
(x))/(a^2*b^3*x^3 + 2*a*b^4*x^2 + b^5*x)]
```

giac [A] time = 0.16, size = 59, normalized size = 0.72

$$-\frac{15\,a\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\,b^3} - \frac{2}{b^3\sqrt{x}} - \frac{7\,a^2x^{\frac{3}{2}}+9\,ab\sqrt{x}}{4\,(ax+b)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^(9/2),x, algorithm="giac")
```

```
[Out] -15/4*a*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) - 2/(b^3*sqrt(x)) - 1/4
*(7*a^2*x^(3/2) + 9*a*b*sqrt(x))/((a*x + b)^2*b^3)
```

maple [A] time = 0.02, size = 66, normalized size = 0.80

$$-\frac{7a^2x^{\frac{3}{2}}}{4\left(ax+b\right)^2b^3}-\frac{9a\sqrt{x}}{4\left(ax+b\right)^2b^2}-\frac{15a\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\,b^3}-\frac{2}{b^3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^3/x^(9/2),x)
```

```
[Out] -7/4*a^2/b^3/(a*x+b)^2*x^(3/2)-9/4*a/b^2/(a*x+b)^2*x^(1/2)-15/4*a/b^3/(a*b)
^(1/2)*arctan(1/(a*b)^(1/2)*a*x^(1/2))-2/b^3/x^(1/2)
```

maxima [A] time = 2.42, size = 75, normalized size = 0.91

$$-\frac{\frac{7a^2}{\sqrt{x}}+\frac{9ab}{x^{\frac{3}{2}}}}{4\left(a^2b^3+\frac{2ab^4}{x}+\frac{b^5}{x^2}\right)}+\frac{15\,a\arctan\left(\frac{b}{\sqrt{ab}\,\sqrt{x}}\right)}{4\sqrt{ab}\,b^3}-\frac{2}{b^3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^(9/2),x, algorithm="maxima")
```

```
[Out] -1/4*(7*a^2/sqrt(x) + 9*a*b/x^(3/2))/(a^2*b^3 + 2*a*b^4/x + b^5/x^2) + 15/4
*a*arctan(b/(sqrt(a*b)*sqrt(x)))/(sqrt(a*b)*b^3) - 2/(b^3*sqrt(x))
```

mupad [B] time = 1.13, size = 70, normalized size = 0.85

$$-\frac{\frac{2}{b}+\frac{15\,a^2\,x^2}{4\,b^3}+\frac{25\,a\,x}{4\,b^2}}{a^2\,x^{5/2}+b^2\,\sqrt{x}+2\,a\,b\,x^{3/2}}-\frac{15\,\sqrt{a}\,\operatorname{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{4\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(9/2)*(a + b/x)^3),x)
```

```
[Out] - (2/b + (15*a^2*x^2)/(4*b^3) + (25*a*x)/(4*b^2))/(a^2*x^(5/2) + b^2*x^(1/2
) + 2*a*b*x^(3/2)) - (15*a^(1/2)*atan((a^(1/2)*x^(1/2))/b^(1/2)))/(4*b^(7/2
))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**3/x**(9/2),x)
```

```
[Out] Timed out
```

3.1689 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{11/2}}\,dx$

Optimal. Leaf size=95

$$\frac{35a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{4b^{9/2}}+\frac{35a}{4b^4\sqrt{x}}+\frac{7}{4b^2x^{3/2}(ax+b)}+\frac{1}{2bx^{3/2}(ax+b)^2}-\frac{35}{12b^3x^{3/2}}$$

```
[Out] -35/12/b^3/x^(3/2)+1/2/b/x^(3/2)/(a*x+b)^2+7/4/b^2/x^(3/2)/(a*x+b)+35/4*a^(
3/2)*arctan(a^(1/2)*x^(1/2)/b^(1/2))/b^(9/2)+35/4*a/b^4/x^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {263, 51, 63, 205}

$$\frac{35a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{4b^{9/2}}+\frac{7}{4b^2x^{3/2}(ax+b)}+\frac{35a}{4b^4\sqrt{x}}+\frac{1}{2bx^{3/2}(ax+b)^2}-\frac{35}{12b^3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^3*x^(11/2)),x]
```

```
[Out] -35/(12*b^3*x^(3/2)) + (35*a)/(4*b^4*Sqrt[x]) + 1/(2*b*x^(3/2)*(b + a*x)^2)
 + 7/(4*b^2*x^(3/2)*(b + a*x)) + (35*a^(3/2)*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[
b]])/(4*b^(9/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{11/2}}\,dx = \int \frac{1}{x^{5/2}(b+ax)^3}\,dx$$

$$= \frac{1}{2bx^{3/2}(b+ax)^2} + \frac{7\int \frac{1}{x^{5/2}(b+ax)^2}\,dx}{4b}$$

$$= \frac{1}{2bx^{3/2}(b+ax)^2} + \frac{7}{4b^2x^{3/2}(b+ax)} + \frac{35\int \frac{1}{x^{5/2}(b+ax)}\,dx}{8b^2}$$

$$= -\frac{35}{12b^3x^{3/2}} + \frac{1}{2bx^{3/2}(b+ax)^2} + \frac{7}{4b^2x^{3/2}(b+ax)} - \frac{(35a)\int \frac{1}{x^{3/2}(b+ax)}\,dx}{8b^3}$$

$$= -\frac{35}{12b^3x^{3/2}} + \frac{35a}{4b^4\sqrt{x}} + \frac{1}{2bx^{3/2}(b+ax)^2} + \frac{7}{4b^2x^{3/2}(b+ax)} + \frac{\left(35a^2\right)\int \frac{1}{\sqrt{x}\,(b+ax)}\,dx}{8b^4}$$

$$= -\frac{35}{12b^3x^{3/2}} + \frac{35a}{4b^4\sqrt{x}} + \frac{1}{2bx^{3/2}(b+ax)^2} + \frac{7}{4b^2x^{3/2}(b+ax)} + \frac{\left(35a^2\right)\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x\right.}{4b^4}$$

$$= -\frac{35}{12b^3x^{3/2}} + \frac{35a}{4b^4\sqrt{x}} + \frac{1}{2bx^{3/2}(b+ax)^2} + \frac{7}{4b^2x^{3/2}(b+ax)} + \frac{35a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{4b^{9/2}}$$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.28

$$-\frac{2\,{}_2F_1\left(-\frac{3}{2},3;-\frac{1}{2};-\frac{ax}{b}\right)}{3b^3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^3*x^(11/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/2, 3, -1/2, -((a*x)/b)])/(3*b^3*x^(3/2))
```

fricas [A] time = 1.09, size = 250, normalized size = 2.63

$$\left[\frac{105\left(a^3x^4+2\,a^2bx^3+ab^2x^2\right)\sqrt{-\frac{a}{b}}\,\log\left(\frac{ax+2\,b\sqrt{x}\,\sqrt{-\frac{a}{b}}-b}{ax+b}\right)+2\left(105\,a^3x^3+175\,a^2bx^2+56\,ab^2x-8\,b^3\right)\sqrt{x}}{24\left(a^2b^4x^4+2\,ab^5x^3+b^6x^2\right)}, -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^(11/2),x, algorithm="fricas")
```

```
[Out] [1/24*(105*(a^3*x^4 + 2*a^2*b*x^3 + a*b^2*x^2)*sqrt(-a/b)*log((a*x + 2*b*sq
rt(x)*sqrt(-a/b) - b)/(a*x + b)) + 2*(105*a^3*x^3 + 175*a^2*b*x^2 + 56*a*b^
2*x - 8*b^3)*sqrt(x))/(a^2*b^4*x^4 + 2*a*b^5*x^3 + b^6*x^2), -1/12*(105*(a^
3*x^4 + 2*a^2*b*x^3 + a*b^2*x^2)*sqrt(a/b)*arctan(b*sqrt(a/b)/(a*sqrt(x)))
 - (105*a^3*x^3 + 175*a^2*b*x^2 + 56*a*b^2*x - 8*b^3)*sqrt(x))/(a^2*b^4*x^4
 + 2*a*b^5*x^3 + b^6*x^2)]
```

giac [A] time = 0.15, size = 71, normalized size = 0.75

$$\frac{35\,a^2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\,b^4} + \frac{2\,(9\,ax-b)}{3\,b^4x^{\frac{3}{2}}} + \frac{11\,a^3x^{\frac{3}{2}}+13\,a^2b\sqrt{x}}{4\,(ax+b)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^(11/2),x, algorithm="giac")
```

```
[Out] 35/4*a^2*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4) + 2/3*(9*a*x - b)/(b^4
*x^(3/2)) + 1/4*(11*a^3*x^(3/2) + 13*a^2*b*sqrt(x))/((a*x + b)^2*b^4)
```

maple **[A]** time = 0.02, size = 79, normalized size = 0.83

$$\frac{11a^3x^{\frac{3}{2}}}{4\left(ax+b\right)^2b^4}+\frac{13a^2\sqrt{x}}{4\left(ax+b\right)^2b^3}+\frac{35a^2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\ b^4}+\frac{6a}{b^4\sqrt{x}}-\frac{2}{3b^3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^3/x^(11/2),x)
```

```
[Out] 11/4*a^3/b^4/(a*x+b)^2*x^(3/2)+13/4*a^2/b^3/(a*x+b)^2*x^(1/2)+35/4*a^2/b^4/
(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*a*x^(1/2))-2/3/b^3/x^(3/2)+6*a/b^4/x^(1/2)
```

maxima **[A]** time = 2.36, size = 89, normalized size = 0.94

$$\frac{\frac{11\,a^3}{\sqrt{x}}+\frac{13\,a^2b}{x^{\frac{3}{2}}}}{4\left(a^2b^4+\frac{2\,ab^5}{x}+\frac{b^6}{x^2}\right)}-\frac{35\,a^2\arctan\left(\frac{b}{\sqrt{ab}\,\sqrt{x}}\right)}{4\,\sqrt{ab}\,b^4}+\frac{2\left(\frac{9\,a}{\sqrt{x}}-\frac{b}{x^{\frac{3}{2}}}\right)}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^(11/2),x, algorithm="maxima")
```

```
[Out] 1/4*(11*a^3/sqrt(x) + 13*a^2*b/x^(3/2))/(a^2*b^4 + 2*a*b^5/x + b^6/x^2) - 3
5/4*a^2*arctan(b/(sqrt(a*b)*sqrt(x)))/(sqrt(a*b)*b^4) + 2/3*(9*a/sqrt(x) -
b/x^(3/2))/b^4
```

mupad **[B]** time = 0.10, size = 80, normalized size = 0.84

$$\frac{\frac{175\,a^2x^2}{12\,b^3}-\frac{2}{3\,b}+\frac{35\,a^3x^3}{4\,b^4}+\frac{14\,a\,x}{3\,b^2}}{a^2\,x^{7/2}+b^2\,x^{3/2}+2\,a\,b\,x^{5/2}}+\frac{35\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{4\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(11/2)*(a + b/x)^3),x)
```

```
[Out] ((175*a^2*x^2)/(12*b^3) - 2/(3*b) + (35*a^3*x^3)/(4*b^4) + (14*a*x)/(3*b^2)
)/(a^2*x^(7/2) + b^2*x^(3/2) + 2*a*b*x^(5/2)) + (35*a^(3/2)*atan((a^(1/2)*x
^(1/2))/b^(1/2)))/(4*b^(9/2))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**3/x**(11/2),x)
```

```
[Out] Timed out
```

3.1690 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{13/2}}\,dx$

Optimal. Leaf size=110

$$-\frac{63a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{4b^{11/2}}-\frac{63a^2}{4b^5\sqrt{x}}+\frac{21a}{4b^4x^{3/2}}+\frac{9}{4b^2x^{5/2}(ax+b)}+\frac{1}{2bx^{5/2}(ax+b)^2}-\frac{63}{20b^3x^{5/2}}$$

```
[Out] -63/20/b^3/x^(5/2)+21/4*a/b^4/x^(3/2)+1/2/b/x^(5/2)/(a*x+b)^2+9/4/b^2/x^(5/
2)/(a*x+b)-63/4*a^(5/2)*arctan(a^(1/2)*x^(1/2)/b^(1/2))/b^(11/2)-63/4*a^2/b
^5/x^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {263, 51, 63, 205}

$$-\frac{63a^2}{4b^5\sqrt{x}}-\frac{63a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{4b^{11/2}}+\frac{21a}{4b^4x^{3/2}}+\frac{9}{4b^2x^{5/2}(ax+b)}+\frac{1}{2bx^{5/2}(ax+b)^2}-\frac{63}{20b^3x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^3*x^(13/2)),x]
```

```
[Out] -63/(20*b^3*x^(5/2)) + (21*a)/(4*b^4*x^(3/2)) - (63*a^2)/(4*b^5*Sqrt[x]) +
1/(2*b*x^(5/2)*(b + a*x)^2) + 9/(4*b^2*x^(5/2)*(b + a*x)) - (63*a^(5/2)*Arc
Tan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/(4*b^(11/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{13/2}}\,dx = \int \frac{1}{x^{7/2}(b+ax)^3}\,dx$$

$$= \frac{1}{2bx^{5/2}(b+ax)^2} + \frac{9\int \frac{1}{x^{7/2}(b+ax)^2}\,dx}{4b}$$

$$= \frac{1}{2bx^{5/2}(b+ax)^2} + \frac{9}{4b^2x^{5/2}(b+ax)} + \frac{63\int \frac{1}{x^{7/2}(b+ax)}\,dx}{8b^2}$$

$$= -\frac{63}{20b^3x^{5/2}} + \frac{1}{2bx^{5/2}(b+ax)^2} + \frac{9}{4b^2x^{5/2}(b+ax)} - \frac{(63a)\int \frac{1}{x^{5/2}(b+ax)}\,dx}{8b^3}$$

$$= -\frac{63}{20b^3x^{5/2}} + \frac{21a}{4b^4x^{3/2}} + \frac{1}{2bx^{5/2}(b+ax)^2} + \frac{9}{4b^2x^{5/2}(b+ax)} + \frac{\left(63a^2\right)\int \frac{1}{x^{3/2}(b+ax)}\,dx}{8b^4}$$

$$= -\frac{63}{20b^3x^{5/2}} + \frac{21a}{4b^4x^{3/2}} - \frac{63a^2}{4b^5\sqrt{x}} + \frac{1}{2bx^{5/2}(b+ax)^2} + \frac{9}{4b^2x^{5/2}(b+ax)} - \frac{\left(63a^3\right)\int \frac{1}{\sqrt{x}\,(b+ax)}\,d}{8b^5}$$

$$= -\frac{63}{20b^3x^{5/2}} + \frac{21a}{4b^4x^{3/2}} - \frac{63a^2}{4b^5\sqrt{x}} + \frac{1}{2bx^{5/2}(b+ax)^2} + \frac{9}{4b^2x^{5/2}(b+ax)} - \frac{\left(63a^3\right)\text{Subst}\left(\int \frac{1}{b+a}\right.}{4b^5}$$

$$= -\frac{63}{20b^3x^{5/2}} + \frac{21a}{4b^4x^{3/2}} - \frac{63a^2}{4b^5\sqrt{x}} + \frac{1}{2bx^{5/2}(b+ax)^2} + \frac{9}{4b^2x^{5/2}(b+ax)} - \frac{63a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right.}{4b^{11/2}}$$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.25

$$-\frac{2\,{}_2F_1\left(-\frac{5}{2},3;-\frac{3}{2};-\frac{ax}{b}\right)}{5b^3x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^3*x^(13/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-5/2, 3, -3/2, -((a*x)/b)])/(5*b^3*x^(5/2))
```

fricas [A] time = 1.02, size = 276, normalized size = 2.51

$$\left[\frac{315\left(a^4x^5+2\,a^3bx^4+a^2b^2x^3\right)\sqrt{-\frac{a}{b}}\,\log\left(\frac{ax-2\,b\sqrt{x}\,\sqrt{-\frac{a}{b}}-b}{ax+b}\right)-2\left(315\,a^4x^4+525\,a^3bx^3+168\,a^2b^2x^2-24\,ab^3x+8\,b\right.}{40\left(a^2b^5x^5+2\,ab^6x^4+b^7x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^(13/2),x, algorithm="fricas")
```

```
[Out] [1/40*(315*(a^4*x^5 + 2*a^3*b*x^4 + a^2*b^2*x^3)*sqrt(-a/b)*log((a*x - 2*b*
sqrt(x)*sqrt(-a/b) - b)/(a*x + b)) - 2*(315*a^4*x^4 + 525*a^3*b*x^3 + 168*a
^2*b^2*x^2 - 24*a*b^3*x + 8*b^4)*sqrt(x))/(a^2*b^5*x^5 + 2*a*b^6*x^4 + b^7*
x^3), 1/20*(315*(a^4*x^5 + 2*a^3*b*x^4 + a^2*b^2*x^3)*sqrt(a/b)*arctan(b*sq
rt(a/b)/(a*sqrt(x))) - (315*a^4*x^4 + 525*a^3*b*x^3 + 168*a^2*b^2*x^2 - 24*
a*b^3*x + 8*b^4)*sqrt(x))/(a^2*b^5*x^5 + 2*a*b^6*x^4 + b^7*x^3)]
```

giac [A] time = 0.16, size = 80, normalized size = 0.73

$$-\frac{63\,a^3\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\,b^5} - \frac{15\,a^4x^{\frac{3}{2}}+17\,a^3b\sqrt{x}}{4\,(ax+b)^2b^5} - \frac{2\left(30\,a^2x^2-5\,abx+b^2\right)}{5\,b^5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^(13/2),x, algorithm="giac")
```

```
[Out] -63/4*a^3*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^5) - 1/4*(15*a^4*x^(3/2)
 + 17*a^3*b*sqrt(x))/((a*x + b)^2*b^5) - 2/5*(30*a^2*x^2 - 5*a*b*x + b^2)/(
b^5*x^(5/2))
```

maple [A] time = 0.02, size = 90, normalized size = 0.82

$$-\frac{15a^4x^{\frac{3}{2}}}{4\left(ax+b\right)^2b^5}-\frac{17a^3\sqrt{x}}{4\left(ax+b\right)^2b^4}-\frac{63a^3\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\,b^5}-\frac{12a^2}{b^5\sqrt{x}}+\frac{2a}{b^4x^{\frac{3}{2}}}-\frac{2}{5b^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^3/x^(13/2),x)
```

```
[Out] -15/4*a^4/b^5/(a*x+b)^2*x^(3/2)-17/4*a^3/b^4/(a*x+b)^2*x^(1/2)-63/4*a^3/b^5
/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*a*x^(1/2))-2/5/b^3/x^(5/2)-12*a^2/b^5/x^(
1/2)+2*a/b^4/x^(3/2)
```

maxima [A] time = 2.31, size = 99, normalized size = 0.90

$$-\frac{\frac{15\,a^4}{\sqrt{x}}+\frac{17\,a^3b}{x^{\frac{3}{2}}}}{4\left(a^2b^5+\frac{2\,ab^6}{x}+\frac{b^7}{x^2}\right)}+\frac{63\,a^3\arctan\left(\frac{b}{\sqrt{ab}\,\sqrt{x}}\right)}{4\,\sqrt{ab}\,b^5}-\frac{2\left(\frac{30\,a^2}{\sqrt{x}}-\frac{5\,ab}{x^{\frac{3}{2}}}+\frac{b^2}{x^{\frac{5}{2}}}\right)}{5\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^3/x^(13/2),x, algorithm="maxima")
```

```
[Out] -1/4*(15*a^4/sqrt(x) + 17*a^3*b/x^(3/2))/(a^2*b^5 + 2*a*b^6/x + b^7/x^2) +
63/4*a^3*arctan(b/(sqrt(a*b)*sqrt(x)))/(sqrt(a*b)*b^5) - 2/5*(30*a^2/sqrt(x
) - 5*a*b/x^(3/2) + b^2/x^(5/2))/b^5
```

mupad [B] time = 1.18, size = 92, normalized size = 0.84

$$-\frac{\frac{2}{5\,b}+\frac{42\,a^2\,x^2}{5\,b^3}+\frac{105\,a^3\,x^3}{4\,b^4}+\frac{63\,a^4\,x^4}{4\,b^5}-\frac{6\,a\,x}{5\,b^2}}{a^2\,x^{9/2}+b^2\,x^{5/2}+2\,a\,b\,x^{7/2}}-\frac{63\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{4\,b^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(13/2)*(a + b/x)^3),x)
```

```
[Out] - (2/(5*b) + (42*a^2*x^2)/(5*b^3) + (105*a^3*x^3)/(4*b^4) + (63*a^4*x^4)/(4
*b^5) - (6*a*x)/(5*b^2))/(a^2*x^(9/2) + b^2*x^(5/2) + 2*a*b*x^(7/2)) - (63*
a^(5/2)*atan((a^(1/2)*x^(1/2))/b^(1/2)))/(4*b^(11/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**3/x**(13/2),x)
```

```
[Out] Timed out
```

3.1691 $\int \sqrt{a+\frac{b}{x}}\, x^3\, dx$

Optimal. Leaf size=117

$$-\frac{5b^4 \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{64a^{7/2}}+\frac{5b^3x\sqrt{a+\frac{b}{x}}}{64a^3}-\frac{5b^2x^2\sqrt{a+\frac{b}{x}}}{96a^2}+\frac{1}{4}x^4\sqrt{a+\frac{b}{x}}+\frac{bx^3\sqrt{a+\frac{b}{x}}}{24a}$$

```
[Out] -5/64*b^4*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(7/2)+5/64*b^3*x*(a+b/x)^(1/2)/a
^3-5/96*b^2*x^2*(a+b/x)^(1/2)/a^2+1/24*b*x^3*(a+b/x)^(1/2)/a+1/4*x^4*(a+b/x
)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {266, 47, 51, 63, 208}

$$-\frac{5b^2x^2\sqrt{a+\frac{b}{x}}}{96a^2}+\frac{5b^3x\sqrt{a+\frac{b}{x}}}{64a^3}-\frac{5b^4 \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{64a^{7/2}}+\frac{bx^3\sqrt{a+\frac{b}{x}}}{24a}+\frac{1}{4}x^4\sqrt{a+\frac{b}{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]*x^3,x]
```

```
[Out] (5*b^3*Sqrt[a + b/x]*x)/(64*a^3) - (5*b^2*Sqrt[a + b/x]*x^2)/(96*a^2) + (b*
Sqrt[a + b/x]*x^3)/(24*a) + (Sqrt[a + b/x]*x^4)/4 - (5*b^4*ArcTanh[Sqrt[a +
 b/x]/Sqrt[a]])/(64*a^(7/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a+\frac{b}{x}}\, x^3\, dx &= -\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^5}\, dx, x, \frac{1}{x}\right) \\
&= \frac{1}{4}\sqrt{a+\frac{b}{x}}\, x^4 - \frac{1}{8}b\,\text{Subst}\left(\int \frac{1}{x^4\sqrt{a+bx}}\, dx, x, \frac{1}{x}\right) \\
&= \frac{b\sqrt{a+\frac{b}{x}}\, x^3}{24a} + \frac{1}{4}\sqrt{a+\frac{b}{x}}\, x^4 + \frac{\left(5b^2\right)\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\, dx, x, \frac{1}{x}\right)}{48a} \\
&= -\frac{5b^2\sqrt{a+\frac{b}{x}}\, x^2}{96a^2} + \frac{b\sqrt{a+\frac{b}{x}}\, x^3}{24a} + \frac{1}{4}\sqrt{a+\frac{b}{x}}\, x^4 - \frac{\left(5b^3\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\, dx, x, \frac{1}{x}\right)}{64a^2} \\
&= \frac{5b^3\sqrt{a+\frac{b}{x}}\, x}{64a^3} - \frac{5b^2\sqrt{a+\frac{b}{x}}\, x^2}{96a^2} + \frac{b\sqrt{a+\frac{b}{x}}\, x^3}{24a} + \frac{1}{4}\sqrt{a+\frac{b}{x}}\, x^4 + \frac{\left(5b^4\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\, dx, x\right.}{128a^3} \\
&= \frac{5b^3\sqrt{a+\frac{b}{x}}\, x}{64a^3} - \frac{5b^2\sqrt{a+\frac{b}{x}}\, x^2}{96a^2} + \frac{b\sqrt{a+\frac{b}{x}}\, x^3}{24a} + \frac{1}{4}\sqrt{a+\frac{b}{x}}\, x^4 + \frac{\left(5b^3\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\, dx, x,\right.}{64a^3} \\
&= \frac{5b^3\sqrt{a+\frac{b}{x}}\, x}{64a^3} - \frac{5b^2\sqrt{a+\frac{b}{x}}\, x^2}{96a^2} + \frac{b\sqrt{a+\frac{b}{x}}\, x^3}{24a} + \frac{1}{4}\sqrt{a+\frac{b}{x}}\, x^4 - \frac{5b^4\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{64a^{7/2}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 39, normalized size = 0.33

$$\frac{2b^4\left(a+\frac{b}{x}\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},5;\frac{5}{2};\frac{b}{ax}+1\right)}{3a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]*x^3,x]
```

```
[Out] (2*b^4*(a + b/x)^(3/2)*Hypergeometric2F1[3/2, 5, 5/2, 1 + b/(a*x)])/(3*a^5)
```

fricas [A] time = 1.02, size = 173, normalized size = 1.48

$$\left[\frac{15\sqrt{a}\,b^4\log\left(2\,ax-2\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(48\,a^4x^4+8\,a^3bx^3-10\,a^2b^2x^2+15\,ab^3x\right)\sqrt{\frac{ax+b}{x}}}{384\,a^4},\ \frac{15\sqrt{-a}\,b^4\arcta\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/384*(15*sqrt(a)*b^4*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(
48*a^4*x^4 + 8*a^3*b*x^3 - 10*a^2*b^2*x^2 + 15*a*b^3*x)*sqrt((a*x + b)/x))/
a^4, 1/192*(15*sqrt(-a)*b^4*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (48*a^4*
x^4 + 8*a^3*b*x^3 - 10*a^2*b^2*x^2 + 15*a*b^3*x)*sqrt((a*x + b)/x))/a^4]
```

giac [A] time = 0.22, size = 108, normalized size = 0.92

$$\frac{5\,b^4\log\left(\left|-2\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)\sqrt{a}-b\right|\right)\operatorname{sgn}(x)}{128\,a^{\frac{7}{2}}}-\frac{5\,b^4\log\left(|b|\right)\operatorname{sgn}(x)}{128\,a^{\frac{7}{2}}}+\frac{1}{192}\sqrt{ax^2+bx}\left(2\left(4\left(6\,x\operatorname{sgn}(x)+\frac{b\operatorname{sgn}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] 5/128*b^4*log(abs(-2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) - b))*sgn(x)/a
^(7/2) - 5/128*b^4*log(abs(b))*sgn(x)/a^(7/2) + 1/192*sqrt(a*x^2 + b*x)*(2*
(4*(6*x*sgn(x) + b*sgn(x)/a)*x - 5*b^2*sgn(x)/a^2)*x + 15*b^3*sgn(x)/a^3)
```

maple [A] time = 0.02, size = 135, normalized size = 1.15

$$-\frac{\sqrt{\frac{ax+b}{x}}\left(15a\,b^4\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)-60\sqrt{a\,x^2+bx}\;a^{\frac{5}{2}}b^2x-96\left(a\,x^2+bx\right)^{\frac{3}{2}}a^{\frac{7}{2}}x-30\sqrt{a\,x^2+bx}\;a^{\frac{3}{2}}b^3+80\right.}{384\sqrt{(ax+b)\,x}\;a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b/x)^(1/2),x)
```

```
[Out] -1/384*((a*x+b)/x)^(1/2)*x*(-96*x*(a*x^2+b*x)^(3/2)*a^(7/2)+80*a^(5/2)*(a*x
^2+b*x)^(3/2)*b-60*a^(5/2)*(a*x^2+b*x)^(1/2)*x*b^2-30*a^(3/2)*(a*x^2+b*x)^(
1/2)*b^3+15*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*b^4)/((
a*x+b)*x)^(1/2)/a^(9/2)
```

maxima [A] time = 2.37, size = 166, normalized size = 1.42

$$\frac{5\,b^4\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{128\,a^{\frac{7}{2}}}+\frac{15\left(a+\frac{b}{x}\right)^{\frac{7}{2}}b^4-55\left(a+\frac{b}{x}\right)^{\frac{5}{2}}ab^4+73\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2b^4+15\sqrt{a+\frac{b}{x}}\,a^3b^4}{192\left(\left(a+\frac{b}{x}\right)^4a^3-4\left(a+\frac{b}{x}\right)^3a^4+6\left(a+\frac{b}{x}\right)^2a^5-4\left(a+\frac{b}{x}\right)a^6+a^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] 5/128*b^4*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(7/2)
+ 1/192*(15*(a + b/x)^(7/2)*b^4 - 55*(a + b/x)^(5/2)*a*b^4 + 73*(a + b/x)^(
3/2)*a^2*b^4 + 15*sqrt(a + b/x)*a^3*b^4)/((a + b/x)^4*a^3 - 4*(a + b/x)^3*a
^4 + 6*(a + b/x)^2*a^5 - 4*(a + b/x)*a^6 + a^7)
```

mupad [B] time = 1.71, size = 91, normalized size = 0.78

$$\frac{5\,x^4\sqrt{a+\frac{b}{x}}}{64}+\frac{73\,x^4\left(a+\frac{b}{x}\right)^{3/2}}{192\,a}-\frac{55\,x^4\left(a+\frac{b}{x}\right)^{5/2}}{192\,a^2}+\frac{5\,x^4\left(a+\frac{b}{x}\right)^{7/2}}{64\,a^3}+\frac{b^4\operatorname{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1\mathrm{i}}{\sqrt{a}}\right)5\mathrm{i}}{64\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b/x)^(1/2),x)
```

```
[Out] (5*x^4*(a + b/x)^(1/2))/64 + (b^4*atan(((a + b/x)^(1/2)*1i)/a^(1/2))*5i)/(6
4*a^(7/2)) + (73*x^4*(a + b/x)^(3/2))/(192*a) - (55*x^4*(a + b/x)^(5/2))/(1
92*a^2) + (5*x^4*(a + b/x)^(7/2))/(64*a^3)
```

sympy [**A**] time = 10.57, size = 153, normalized size = 1.31

$$\frac{a x^{\frac{9}{2}}}{4\sqrt{b}\sqrt{\frac{ax}{b}+1}}+\frac{7\sqrt{b}\,x^{\frac{7}{2}}}{24\sqrt{\frac{ax}{b}+1}}-\frac{b^{\frac{3}{2}}x^{\frac{5}{2}}}{96a\sqrt{\frac{ax}{b}+1}}+\frac{5b^{\frac{5}{2}}x^{\frac{3}{2}}}{192a^2\sqrt{\frac{ax}{b}+1}}+\frac{5b^{\frac{7}{2}}\sqrt{x}}{64a^3\sqrt{\frac{ax}{b}+1}}-\frac{5b^4\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{64a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b/x)**(1/2),x)
```

```
[Out] a*x**(9/2)/(4*sqrt(b)*sqrt(a*x/b + 1)) + 7*sqrt(b)*x**(7/2)/(24*sqrt(a*x/b
+ 1)) - b**(3/2)*x**(5/2)/(96*a*sqrt(a*x/b + 1)) + 5*b**(5/2)*x**(3/2)/(192
*a**2*sqrt(a*x/b + 1)) + 5*b**(7/2)*sqrt(x)/(64*a**3*sqrt(a*x/b + 1)) - 5*b
**4*asinh(sqrt(a)*sqrt(x)/sqrt(b))/(64*a**(7/2))
```

3.1692 $\int \sqrt{a+\frac{b}{x}}\, x^2\, dx$

Optimal. Leaf size=93

$$\frac{b^3 \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{8a^{5/2}} - \frac{b^2 x\sqrt{a+\frac{b}{x}}}{8a^2} + \frac{1}{3}x^3\sqrt{a+\frac{b}{x}} + \frac{bx^2\sqrt{a+\frac{b}{x}}}{12a}$$

```
[Out] 1/8*b^3*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(5/2)-1/8*b^2*x*(a+b/x)^(1/2)/a^2+
1/12*b*x^2*(a+b/x)^(1/2)/a+1/3*x^3*(a+b/x)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {266, 47, 51, 63, 208}

$$-\frac{b^2 x\sqrt{a+\frac{b}{x}}}{8a^2} + \frac{b^3 \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{8a^{5/2}} + \frac{bx^2\sqrt{a+\frac{b}{x}}}{12a} + \frac{1}{3}x^3\sqrt{a+\frac{b}{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]*x^2,x]
```

```
[Out] -(b^2*Sqrt[a + b/x]*x)/(8*a^2) + (b*Sqrt[a + b/x]*x^2)/(12*a) + (Sqrt[a + b
/x]*x^3)/3 + (b^3*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(8*a^(5/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a+\frac{b}{x}}\, x^2\, dx &= -\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^4}\, dx, x, \frac{1}{x}\right) \\
&= \frac{1}{3}\sqrt{a+\frac{b}{x}}\, x^3 - \frac{1}{6}b\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\, dx, x, \frac{1}{x}\right) \\
&= \frac{b\sqrt{a+\frac{b}{x}}\, x^2}{12a} + \frac{1}{3}\sqrt{a+\frac{b}{x}}\, x^3 + \frac{b^2\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\, dx, x, \frac{1}{x}\right)}{8a} \\
&= -\frac{b^2\sqrt{a+\frac{b}{x}}\, x}{8a^2} + \frac{b\sqrt{a+\frac{b}{x}}\, x^2}{12a} + \frac{1}{3}\sqrt{a+\frac{b}{x}}\, x^3 - \frac{b^3\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\, dx, x, \frac{1}{x}\right)}{16a^2} \\
&= -\frac{b^2\sqrt{a+\frac{b}{x}}\, x}{8a^2} + \frac{b\sqrt{a+\frac{b}{x}}\, x^2}{12a} + \frac{1}{3}\sqrt{a+\frac{b}{x}}\, x^3 - \frac{b^2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\, dx, x, \sqrt{a+\frac{b}{x}}\right)}{8a^2} \\
&= -\frac{b^2\sqrt{a+\frac{b}{x}}\, x}{8a^2} + \frac{b\sqrt{a+\frac{b}{x}}\, x^2}{12a} + \frac{1}{3}\sqrt{a+\frac{b}{x}}\, x^3 + \frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{8a^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.42

$$-\frac{2b^3\left(a+\frac{b}{x}\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},4;\frac{5}{2};\frac{b}{ax}+1\right)}{3a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]*x^2,x]
```

```
[Out] (-2*b^3*(a + b/x)^(3/2)*Hypergeometric2F1[3/2, 4, 5/2, 1 + b/(a*x)])/(3*a^4
)
```

fricas [A] time = 1.09, size = 152, normalized size = 1.63

$$\left[\frac{3\sqrt{a}\,b^3\log\left(2ax+2\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(8a^3x^3+2a^2bx^2-3ab^2x\right)\sqrt{\frac{ax+b}{x}}}{48a^3}, -\frac{3\sqrt{-a}\,b^3\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*sqrt(a)*b^3*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(8*
a^3*x^3 + 2*a^2*b*x^2 - 3*a*b^2*x)*sqrt((a*x + b)/x))/a^3, -1/24*(3*sqrt(-a
)*b^3*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (8*a^3*x^3 + 2*a^2*b*x^2 - 3*a
*b^2*x)*sqrt((a*x + b)/x))/a^3]
```

giac [A] time = 0.30, size = 94, normalized size = 1.01

$$-\frac{b^3\log\left(\left|-2\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)\sqrt{a}-b\right|\right)\text{sgn}(x)}{16\,a^{\frac{5}{2}}}+\frac{b^3\log\left(|b|\right)\text{sgn}(x)}{16\,a^{\frac{5}{2}}}+\frac{1}{24}\sqrt{ax^2+bx}\left(2\left(4\,x\text{sgn}(x)+\frac{b\text{sgn}(x)}{a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/16*b^3*log(abs(-2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) - b))*sgn(x)/a
^(5/2) + 1/16*b^3*log(abs(b))*sgn(x)/a^(5/2) + 1/24*sqrt(a*x^2 + b*x)*(2*(4
*x*sgn(x) + b*sgn(x)/a)*x - 3*b^2*sgn(x)/a^2)
```

maple [A] time = 0.01, size = 115, normalized size = 1.24

$$\frac{\sqrt{\frac{ax+b}{x}}\left(3a\,b^3\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)-12\sqrt{a\,x^2+bx}\;a^{\frac{5}{2}}bx-6\sqrt{a\,x^2+bx}\;a^{\frac{3}{2}}b^2+16\left(a\,x^2+bx\right)^{\frac{3}{2}}a^{\frac{5}{2}}\right)x}{48\sqrt{(ax+b)\,x}\;a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b/x)^(1/2),x)
```

```
[Out] 1/48*((a*x+b)/x)^(1/2)*x*(16*(a*x^2+b*x)^(3/2)*a^(5/2)-12*(a*x^2+b*x)^(1/2)
*a^(5/2)*x*b-6*(a*x^2+b*x)^(1/2)*a^(3/2)*b^2+3*ln(1/2*(2*a*x+b+2*(a*x^2+b*x
)^(1/2)*a^(1/2))/a^(1/2))*a*b^3)/((a*x+b)*x)^(1/2)/a^(7/2)
```

maxima [A] time = 2.35, size = 137, normalized size = 1.47

$$-\frac{b^3\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{16\,a^{\frac{5}{2}}}-\frac{3\left(a+\frac{b}{x}\right)^{\frac{5}{2}}b^3-8\left(a+\frac{b}{x}\right)^{\frac{3}{2}}ab^3-3\sqrt{a+\frac{b}{x}}\,a^2b^3}{24\left(\left(a+\frac{b}{x}\right)^3a^2-3\left(a+\frac{b}{x}\right)^2a^3+3\left(a+\frac{b}{x}\right)a^4-a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*b^3*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(5/2)
- 1/24*(3*(a + b/x)^(5/2)*b^3 - 8*(a + b/x)^(3/2)*a*b^3 - 3*sqrt(a + b/x)*a
^2*b^3)/((a + b/x)^3*a^2 - 3*(a + b/x)^2*a^3 + 3*(a + b/x)*a^4 - a^5)
```

mupad [B] time = 1.35, size = 74, normalized size = 0.80

$$\frac{x^3\sqrt{a+\frac{b}{x}}}{8}+\frac{x^3\left(a+\frac{b}{x}\right)^{3/2}}{3\,a}-\frac{x^3\left(a+\frac{b}{x}\right)^{5/2}}{8\,a^2}-\frac{b^3\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1\mathrm{i}}{\sqrt{a}}\right)1\mathrm{i}}{8\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b/x)^(1/2),x)
```

```
[Out] (x^3*(a + b/x)^(1/2))/8 - (b^3*atan(((a + b/x)^(1/2)*1i)/a^(1/2))*1i)/(8*a^
(5/2)) + (x^3*(a + b/x)^(3/2))/(3*a) - (x^3*(a + b/x)^(5/2))/(8*a^2)
```

sympy [A] time = 6.17, size = 122, normalized size = 1.31

$$\frac{ax^{\frac{7}{2}}}{3\sqrt{b}\sqrt{\frac{ax}{b}+1}}+\frac{5\sqrt{b}\,x^{\frac{5}{2}}}{12\sqrt{\frac{ax}{b}+1}}-\frac{b^{\frac{3}{2}}x^{\frac{3}{2}}}{24a\sqrt{\frac{ax}{b}+1}}-\frac{b^{\frac{5}{2}}\sqrt{x}}{8a^2\sqrt{\frac{ax}{b}+1}}+\frac{b^3\,\mathrm{asinh}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{8a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b/x)**(1/2),x)
```

```
[Out] a*x**(7/2)/(3*sqrt(b)*sqrt(a*x/b + 1)) + 5*sqrt(b)*x**(5/2)/(12*sqrt(a*x/b
+ 1)) - b**(3/2)*x**(3/2)/(24*a*sqrt(a*x/b + 1)) - b**(5/2)*sqrt(x)/(8*a**2
*sqrt(a*x/b + 1)) + b**3*asinh(sqrt(a)*sqrt(x)/sqrt(b))/(8*a**(5/2))
```

3.1693 $\int \sqrt{a+\frac{b}{x}}\,x\,dx$

Optimal. Leaf size=69

$$-\frac{b^2 \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4a^{3/2}}+\frac{1}{2}x^2\sqrt{a+\frac{b}{x}}+\frac{bx\sqrt{a+\frac{b}{x}}}{4a}$$

```
[Out] -1/4*b^2*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(3/2)+1/4*b*x*(a+b/x)^(1/2)/a+1/2
*x^2*(a+b/x)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {266, 47, 51, 63, 208}

$$-\frac{b^2 \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4a^{3/2}}+\frac{1}{2}x^2\sqrt{a+\frac{b}{x}}+\frac{bx\sqrt{a+\frac{b}{x}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]*x,x]
```

```
[Out] (b*Sqrt[a + b/x]*x)/(4*a) + (Sqrt[a + b/x]*x^2)/2 - (b^2*ArcTanh[Sqrt[a + b
/x]/Sqrt[a]])/(4*a^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a+\frac{b}{x}}\,x\,dx &= -\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^3}\,dx, x, \frac{1}{x}\right) \\
&= \frac{1}{2}\sqrt{a+\frac{b}{x}}\,x^2 - \frac{1}{4}b\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right) \\
&= \frac{b\sqrt{a+\frac{b}{x}}\,x}{4a} + \frac{1}{2}\sqrt{a+\frac{b}{x}}\,x^2 + \frac{b^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{8a} \\
&= \frac{b\sqrt{a+\frac{b}{x}}\,x}{4a} + \frac{1}{2}\sqrt{a+\frac{b}{x}}\,x^2 + \frac{b\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{4a} \\
&= \frac{b\sqrt{a+\frac{b}{x}}\,x}{4a} + \frac{1}{2}\sqrt{a+\frac{b}{x}}\,x^2 - \frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4a^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.57

$$\frac{2b^2\left(a+\frac{b}{x}\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{b}{ax}+1\right)}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]*x,x]
```

```
[Out] (2*b^2*(a + b/x)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + b/(a*x)])/(3*a^3)
```

fricas [A] time = 1.06, size = 125, normalized size = 1.81

$$\left[\frac{\sqrt{a}\,b^2\log\left(2\,ax - 2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}} + b\right) + 2\left(2\,a^2x^2 + abx\right)\sqrt{\frac{ax+b}{x}}}{8\,a^2}, \frac{\sqrt{-a}\,b^2\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right) + \left(2\,a^2x^2 + abx\right)\sqrt{\frac{ax}{}}}{4\,a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a)*b^2*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(2*a^2
*x^2 + a*b*x)*sqrt((a*x + b)/x))/a^2, 1/4*(sqrt(-a)*b^2*arctan(sqrt(-a)*sqr
t((a*x + b)/x)/a) + (2*a^2*x^2 + a*b*x)*sqrt((a*x + b)/x))/a^2]
```

giac [A] time = 0.21, size = 78, normalized size = 1.13

$$-\frac{b^2\log\left(|b|\right)\operatorname{sgn}(x)}{8\,a^{\frac{3}{2}}} + \frac{1}{8}\left(2\sqrt{ax^2+bx}\left(2\,x+\frac{b}{a}\right) + \frac{b^2\log\left(\left|-2\left(\sqrt{a}\,x - \sqrt{ax^2+bx}\right)\sqrt{a} - b\right|\right)}{a^{\frac{3}{2}}}\right)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*b^2*log(abs(b))*sgn(x)/a^(3/2) + 1/8*(2*sqrt(a*x^2 + b*x)*(2*x + b/a)
+ b^2*log(abs(-2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) - b))/a^(3/2))*sgn
(x)
```

maple [A] time = 0.01, size = 96, normalized size = 1.39

$$\frac{\sqrt{\frac{ax+b}{x}}\left(-a\,b^2\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)+4\sqrt{a\,x^2+bx}\;a^{\frac{5}{2}}x+2\sqrt{a\,x^2+bx}\;a^{\frac{3}{2}}b\right)x}{8\sqrt{(ax+b)\,x}\;a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b/x)^(1/2),x)
```

```
[Out] 1/8*((a*x+b)/x)^(1/2)*x*(4*(a*x^2+b*x)^(1/2)*a^(5/2)*x+2*(a*x^2+b*x)^(1/2)*
a^(3/2)*b-b^2*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*a)/((a*
x+b)*x)^(1/2)/a^(5/2)
```

maxima [A] time = 2.39, size = 100, normalized size = 1.45

$$\frac{b^2\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{8\,a^{\frac{3}{2}}}+\frac{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^2+\sqrt{a+\frac{b}{x}}\,ab^2}{4\left(\left(a+\frac{b}{x}\right)^2a-2\left(a+\frac{b}{x}\right)a^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*b^2*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2) +
1/4*((a + b/x)^(3/2)*b^2 + sqrt(a + b/x)*a*b^2)/((a + b/x)^2*a - 2*(a + b/x
)*a^2 + a^3)
```

mupad [B] time = 1.20, size = 54, normalized size = 0.78

$$\frac{x^2\sqrt{a+\frac{b}{x}}}{4}-\frac{b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4\,a^{3/2}}+\frac{x^2\left(a+\frac{b}{x}\right)^{3/2}}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b/x)^(1/2),x)
```

```
[Out] (x^2*(a + b/x)^(1/2))/4 - (b^2*atanh((a + b/x)^(1/2)/a^(1/2)))/(4*a^(3/2))
+ (x^2*(a + b/x)^(3/2))/(4*a)
```

sympy [A] time = 4.18, size = 97, normalized size = 1.41

$$\frac{ax^{\frac{5}{2}}}{2\sqrt{b}\sqrt{\frac{ax}{b}+1}}+\frac{3\sqrt{b}\,x^{\frac{3}{2}}}{4\sqrt{\frac{ax}{b}+1}}+\frac{b^{\frac{3}{2}}\sqrt{x}}{4a\sqrt{\frac{ax}{b}+1}}-\frac{b^2\,\mathrm{asinh}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{4a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/x)**(1/2),x)
```

```
[Out] a*x**(5/2)/(2*sqrt(b)*sqrt(a*x/b + 1)) + 3*sqrt(b)*x**(3/2)/(4*sqrt(a*x/b +
 1)) + b**(3/2)*sqrt(x)/(4*a*sqrt(a*x/b + 1)) - b**2*asinh(sqrt(a)*sqrt(x)/
sqrt(b))/(4*a**(3/2))
```

3.1694 $\int \sqrt{a+\frac{b}{x}}\,dx$

Optimal. Leaf size=39

$$x\sqrt{a+\frac{b}{x}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] b*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(1/2)+x*(a+b/x)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {242, 47, 63, 208}

$$x\sqrt{a+\frac{b}{x}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x],x]
```

```
[Out] Sqrt[a + b/x]*x + (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+\frac{b}{x}}\,dx &= -\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx, x, \frac{1}{x}\right)\\ &= \sqrt{a+\frac{b}{x}}\,x - \frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)\\ &= \sqrt{a+\frac{b}{x}}\,x - \text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)\\ &= \sqrt{a+\frac{b}{x}}\,x + \frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 1.00

$$x\sqrt{a+\frac{b}{x}} + \frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x],x]
```

```
[Out] Sqrt[a + b/x]*x + (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

fricas [A] time = 1.05, size = 99, normalized size = 2.54

$$\left[\frac{2\,ax\sqrt{\frac{ax+b}{x}} + \sqrt{a}\,b\log\left(2\,ax + 2\sqrt{a}\,x\sqrt{\frac{ax+b}{x}} + b\right)}{2\,a}, \frac{ax\sqrt{\frac{ax+b}{x}} - \sqrt{-a}\,b\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a*x + b)/x) + sqrt(a)*b*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x
 + b)/x) + b))/a, (a*x*sqrt((a*x + b)/x) - sqrt(-a)*b*arctan(sqrt(-a)*sqrt(
(a*x + b)/x)/a))/a]
```

giac [B] time = 0.20, size = 64, normalized size = 1.64

$$-\frac{b\log\left(\left|-2\left(\sqrt{a}\,x - \sqrt{ax^2+bx}\right)\sqrt{a} - b\right|\right)\text{sgn}(x)}{2\sqrt{a}} + \frac{b\log\left(|b|\right)\text{sgn}(x)}{2\sqrt{a}} + \sqrt{ax^2+bx}\,\text{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*b*log(abs(-2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) - b))*sgn(x)/sqrt
(a) + 1/2*b*log(abs(b))*sgn(x)/sqrt(a) + sqrt(a*x^2 + b*x)*sgn(x)
```

maple [B] time = 0.01, size = 74, normalized size = 1.90

$$\frac{\sqrt{\frac{ax+b}{x}}\,\left(b\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right) + 2\sqrt{a\,x^2+bx}\,\sqrt{a}\right)x}{2\sqrt{(ax+b)\,x}\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(1/2),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+b*ln(1/2*(2*a*x+b+2*(a
*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2)))/((a*x+b)*x)^(1/2)/a^(1/2)
```

maxima [A] time = 2.38, size = 50, normalized size = 1.28

$$\sqrt{a+\frac{b}{x}}\,x-\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a + b/x)*x - 1/2*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt
(a)))/sqrt(a)
```

mupad [B] time = 0.09, size = 58, normalized size = 1.49

$$x\sqrt{a\,x^2+b\,x}\,\sqrt{\frac{1}{x^2}}+\frac{b\,x\,\ln\left(\frac{\frac{b}{2}+a\,x+\sqrt{a}\,\sqrt{a\,x^2+b\,x}}{\sqrt{a}}\right)\sqrt{\frac{1}{x^2}}}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(1/2),x)
```

```
[Out] x*(b*x + a*x^2)^(1/2)*(1/x^2)^(1/2) + (b*x*log((b/2 + a*x + a^(1/2)*(b*x +
a*x^2)^(1/2))/a^(1/2))*(1/x^2)^(1/2))/(2*a^(1/2))
```

sympy [A] time = 2.01, size = 42, normalized size = 1.08

$$\sqrt{b}\,\sqrt{x}\,\sqrt{\frac{a x}{b}+1}+\frac{b\,\operatorname{asinh}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2),x)
```

```
[Out] sqrt(b)*sqrt(x)*sqrt(a*x/b + 1) + b*asinh(sqrt(a)*sqrt(x)/sqrt(b))/sqrt(a)
```

3.1695 $\int \frac{\sqrt{a+\frac{b}{x}}}{x}\,dx$

Optimal. Leaf size=39

$$2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-2\sqrt{a+\frac{b}{x}}$$

```
[Out] 2*arctanh((a+b/x)^(1/2)/a^(1/2))*a^(1/2)-2*(a+b/x)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 50, 63, 208}

$$2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-2\sqrt{a+\frac{b}{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]/x,x]
```

```
[Out] -2*Sqrt[a + b/x] + 2*Sqrt[a]*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+\frac{b}{x}}}{x}\,dx &= -\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x}\right)\\ &= -2\sqrt{a+\frac{b}{x}} - a\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)\\ &= -2\sqrt{a+\frac{b}{x}} - \frac{(2a)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{b}\\ &= -2\sqrt{a+\frac{b}{x}} + 2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 1.00

$$2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right) - 2\sqrt{a+\frac{b}{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]/x,x]
```

```
[Out] -2*Sqrt[a + b/x] + 2*Sqrt[a]*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

fricas [A] time = 1.01, size = 85, normalized size = 2.18

$$\left[\sqrt{a}\,\log\left(2\,ax + 2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}} + b\right) - 2\sqrt{\frac{ax+b}{x}}, -2\,\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}\,\sqrt{\frac{ax+b}{x}}}{a}\right) - 2\sqrt{\frac{ax+b}{x}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*sqrt((a*x + b)/
x), -2*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - 2*sqrt((a*x + b)/x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%%}+%%%{-2
,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%%%}+%%%{2
,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters values [86
,-97,-82]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%
%},0,%%%{1,[0,2,2]%%%}] at parameters values [82.1195442914,26,-89]Warning,
 choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[0,2,2
]%%%}] at parameters values [85.3561567818,-64,-30]Warning, choosing root o
f [1,0,%%%{-2,[1,0,1]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,2]%%%}] at parame
```

```
ters values [42,43.9628838282,-9]Sign error (%%%{-b,0%%%}+%%%{2*sqrt(a)*sqr
t(b),1/2%%%}+%%%{-2*a,1%%%}+%%%{a*sqrt(a)*sqrt(b)/b,3/2%%%}+%%%{-a^2*sqrt(a
)*sqrt(b)/(4*b^2),5/2%%%}+%%%{undef,7/2%%%})Limit: Max order reached or una
ble to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.01, size = 103, normalized size = 2.64

$$-\frac{\sqrt{\frac{ax+b}{x}}\left(-ab\,x^2\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)-2\sqrt{a\,x^2+bx}\;a^{\frac{3}{2}}x^2+2\left(a\,x^2+bx\right)^{\frac{3}{2}}\sqrt{a}\right)}{\sqrt{(ax+b)\,x}\;\sqrt{a}\;bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(1/2)/x,x)
```

```
[Out] -((a*x+b)/x)^(1/2)/x*(-2*(a*x^2+b*x)^(1/2)*a^(3/2)*x^2-ln(1/2*(2*a*x+b+2*(a
*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x^2*a*b+2*(a*x^2+b*x)^(3/2)*a^(1/2))/((a*
x+b)*x)^(1/2)/b/a^(1/2)
```

maxima [A] time = 2.39, size = 49, normalized size = 1.26

$$-\sqrt{a}\,\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-2\sqrt{a+\frac{b}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -sqrt(a)*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) - 2*sqrt(
a + b/x)
```

mupad [B] time = 1.50, size = 31, normalized size = 0.79

$$2\sqrt{a}\,\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-2\sqrt{a+\frac{b}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(1/2)/x,x)
```

```
[Out] 2*a^(1/2)*atanh((a + b/x)^(1/2)/a^(1/2)) - 2*(a + b/x)^(1/2)
```

sympy [B] time = 1.82, size = 68, normalized size = 1.74

$$2\sqrt{a}\,\operatorname{asinh}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)-\frac{2a\sqrt{x}}{\sqrt{b}\,\sqrt{\frac{ax}{b}+1}}-\frac{2\sqrt{b}}{\sqrt{x}\,\sqrt{\frac{ax}{b}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2)/x,x)
```

```
[Out] 2*sqrt(a)*asinh(sqrt(a)*sqrt(x)/sqrt(b)) - 2*a*sqrt(x)/(sqrt(b)*sqrt(a*x/b
+ 1)) - 2*sqrt(b)/(sqrt(x)*sqrt(a*x/b + 1))
```

3.1696 $\int \frac{\sqrt{a+\frac{b}{x}}}{x^2}\,dx$

Optimal. Leaf size=18

$$-\frac{2\left(a+\frac{b}{x}\right)^{3/2}}{3b}$$

```
[Out] -2/3*(a+b/x)^(3/2)/b
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$-\frac{2\left(a+\frac{b}{x}\right)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]/x^2,x]
```

```
[Out] (-2*(a + b/x)^(3/2))/(3*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^2}\,dx = -\frac{2\left(a+\frac{b}{x}\right)^{3/2}}{3b}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$-\frac{2\left(a+\frac{b}{x}\right)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]/x^2,x]
```

```
[Out] (-2*(a + b/x)^(3/2))/(3*b)
```

fricas [A] time = 1.19, size = 24, normalized size = 1.33

$$-\frac{2\,(ax+b)\sqrt{\frac{ax+b}{x}}}{3\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] -2/3*(a*x + b)*sqrt((a*x + b)/x)/(b*x)
```

giac [B] time = 0.22, size = 83, normalized size = 4.61

$$\frac{2\left(3\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)^2 a\,\mathrm{sgn}(x)+3\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)\sqrt{a}\,b\,\mathrm{sgn}(x)+b^2\mathrm{sgn}(x)\right)}{3\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 2/3*(3*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a*sgn(x) + 3*(sqrt(a)*x - sqrt(a*x
^2 + b*x))*sqrt(a)*b*sgn(x) + b^2*sgn(x))/(sqrt(a)*x - sqrt(a*x^2 + b*x))^3
```

maple [A] time = 0.00, size = 25, normalized size = 1.39

$$-\frac{2\,(ax+b)\sqrt{\frac{ax+b}{x}}}{3bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(1/2)/x^2,x)
```

```
[Out] -2/3*(a*x+b)/x/b*((a*x+b)/x)^(1/2)
```

maxima [A] time = 1.07, size = 14, normalized size = 0.78

$$-\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -2/3*(a + b/x)^(3/2)/b
```

mupad [B] time = 1.37, size = 22, normalized size = 1.22

$$-\frac{2\sqrt{a+\frac{b}{x}}\;(b+a\,x)}{3\,b\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(1/2)/x^2,x)
```

```
[Out] -(2*(a + b/x)^(1/2)*(b + a*x))/(3*b*x)
```

sympy [B] time = 0.94, size = 41, normalized size = 2.28

$$-\frac{2a^{\frac{3}{2}}\sqrt{1+\frac{b}{ax}}}{3b}-\frac{2\sqrt{a}\sqrt{1+\frac{b}{ax}}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2)/x**2,x)
```

```
[Out] -2*a**(3/2)*sqrt(1 + b/(a*x))/(3*b) - 2*sqrt(a)*sqrt(1 + b/(a*x))/(3*x)
```

3.1697 $\int \frac{\sqrt{a+\frac{b}{x}}}{x^3}\,dx$

Optimal. Leaf size=38

$$\frac{2a\left(a+\frac{b}{x}\right)^{3/2}}{3b^2}-\frac{2\left(a+\frac{b}{x}\right)^{5/2}}{5b^2}$$

```
[Out] 2/3*a*(a+b/x)^(3/2)/b^2-2/5*(a+b/x)^(5/2)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{2a\left(a+\frac{b}{x}\right)^{3/2}}{3b^2}-\frac{2\left(a+\frac{b}{x}\right)^{5/2}}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]/x^3,x]
```

```
[Out] (2*a*(a + b/x)^(3/2))/(3*b^2) - (2*(a + b/x)^(5/2))/(5*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+\frac{b}{x}}}{x^3}\,dx &= -\text{Subst}\left(\int x\sqrt{a+bx}\,dx, x, \frac{1}{x}\right)\\ &= -\text{Subst}\left(\int\left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx, x, \frac{1}{x}\right)\\ &= \frac{2a\left(a+\frac{b}{x}\right)^{3/2}}{3b^2}-\frac{2\left(a+\frac{b}{x}\right)^{5/2}}{5b^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 0.89

$$\frac{2\sqrt{a+\frac{b}{x}}\,(ax+b)(2ax-3b)}{15b^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]/x^3,x]
```

```
[Out] (2*Sqrt[a + b/x]*(b + a*x)*(-3*b + 2*a*x))/(15*b^2*x^2)
```

fricas [A] time = 0.90, size = 38, normalized size = 1.00

$$\frac{2\left(2\,a^2x^2 - abx - 3\,b^2\right)\sqrt{\frac{ax+b}{x}}}{15\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 2/15*(2*a^2*x^2 - a*b*x - 3*b^2)*sqrt((a*x + b)/x)/(b^2*x^2)
```

giac [B] time = 0.23, size = 115, normalized size = 3.03

$$\frac{2\left(15\left(\sqrt{a}\,x - \sqrt{ax^2+bx}\right)^3 a^{\frac{3}{2}}\mathrm{sgn}(x) + 25\left(\sqrt{a}\,x - \sqrt{ax^2+bx}\right)^2 ab\,\mathrm{sgn}(x) + 15\left(\sqrt{a}\,x - \sqrt{ax^2+bx}\right)\sqrt{a}\,b^2\mathrm{sgn}(x\right.}{15\left(\sqrt{a}\,x - \sqrt{ax^2+bx}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 2/15*(15*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a^(3/2)*sgn(x) + 25*(sqrt(a)*x -
 sqrt(a*x^2 + b*x))^2*a*b*sgn(x) + 15*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(
a)*b^2*sgn(x) + 3*b^3*sgn(x))/(sqrt(a)*x - sqrt(a*x^2 + b*x))^5
```

maple [A] time = 0.00, size = 33, normalized size = 0.87

$$\frac{2\,(ax+b)\,(2ax-3b)\,\sqrt{\frac{ax+b}{x}}}{15b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(1/2)/x^3,x)
```

```
[Out] 2/15*(a*x+b)*(2*a*x-3*b)*((a*x+b)/x)^(1/2)/b^2/x^2
```

maxima [A] time = 0.96, size = 30, normalized size = 0.79

$$-\frac{2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{5\,b^2} + \frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -2/5*(a + b/x)^(5/2)/b^2 + 2/3*(a + b/x)^(3/2)*a/b^2
```

mupad [B] time = 1.46, size = 35, normalized size = 0.92

$$-\frac{2\sqrt{a+\frac{b}{x}}\,\left(-2\,a^2\,x^2 + a\,b\,x + 3\,b^2\right)}{15\,b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(1/2)/x^3,x)
```

```
[Out] -(2*(a + b/x)^(1/2)*(3*b^2 - 2*a^2*x^2 + a*b*x))/(15*b^2*x^2)
```

sympy [**B**] time = 1.43, size = 304, normalized size = 8.00

$$\frac{4a^{\frac{11}{2}}b^{\frac{3}{2}}x^3\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}+\frac{2a^{\frac{9}{2}}b^{\frac{5}{2}}x^2\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{8a^{\frac{7}{2}}b^{\frac{7}{2}}x\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{6a^{\frac{5}{2}}b^{\frac{9}{2}}\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{4a^6bx^{\frac{7}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2)/x**3,x)
```

```
[Out] 4*a**(11/2)*b**(3/2)*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a
**(5/2)*b**4*x**(5/2)) + 2*a**(9/2)*b**(5/2)*x**2*sqrt(a*x/b + 1)/(15*a**(7
/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a**(7/2)*b**(7/2)*x*sqrt
(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 6*a**
(5/2)*b**(9/2)*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**
4*x**(5/2)) - 4*a**6*b*x**(7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b*
*4*x**(5/2)) - 4*a**5*b**2*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2
)*b**4*x**(5/2))
```

3.1698 $\int \frac{\sqrt{a+\frac{b}{x}}}{x^4}\,dx$

Optimal. Leaf size=59

$$-\frac{2a^2\left(a+\frac{b}{x}\right)^{3/2}}{3b^3}-\frac{2\left(a+\frac{b}{x}\right)^{7/2}}{7b^3}+\frac{4a\left(a+\frac{b}{x}\right)^{5/2}}{5b^3}$$

```
[Out] -2/3*a^2*(a+b/x)^(3/2)/b^3+4/5*a*(a+b/x)^(5/2)/b^3-2/7*(a+b/x)^(7/2)/b^3
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{2a^2\left(a+\frac{b}{x}\right)^{3/2}}{3b^3}-\frac{2\left(a+\frac{b}{x}\right)^{7/2}}{7b^3}+\frac{4a\left(a+\frac{b}{x}\right)^{5/2}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]/x^4,x]
```

```
[Out] (-2*a^2*(a + b/x)^(3/2))/(3*b^3) + (4*a*(a + b/x)^(5/2))/(5*b^3) - (2*(a +
b/x)^(7/2))/(7*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+\frac{b}{x}}}{x^4}\,dx &= -\text{Subst}\left(\int x^2\sqrt{a+bx}\,dx, x, \frac{1}{x}\right)\\ &= -\text{Subst}\left(\int\left(\frac{a^2\sqrt{a+bx}}{b^2}-\frac{2a(a+bx)^{3/2}}{b^2}+\frac{(a+bx)^{5/2}}{b^2}\right)dx, x, \frac{1}{x}\right)\\ &= -\frac{2a^2\left(a+\frac{b}{x}\right)^{3/2}}{3b^3}+\frac{4a\left(a+\frac{b}{x}\right)^{5/2}}{5b^3}-\frac{2\left(a+\frac{b}{x}\right)^{7/2}}{7b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 0.76

$$-\frac{2\sqrt{a+\frac{b}{x}}\,(ax+b)\left(8a^2x^2-12abx+15b^2\right)}{105b^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]/x^4,x]
```

```
[Out] (-2*Sqrt[a + b/x]*(b + a*x)*(15*b^2 - 12*a*b*x + 8*a^2*x^2))/(105*b^3*x^3)
```

fricas [A] time = 1.06, size = 49, normalized size = 0.83

$$-\frac{2\left(8\,a^3x^3-4\,a^2bx^2+3\,ab^2x+15\,b^3\right)\sqrt{\frac{ax+b}{x}}}{105\,b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -2/105*(8*a^3*x^3 - 4*a^2*b*x^2 + 3*a*b^2*x + 15*b^3)*sqrt((a*x + b)/x)/(b^
3*x^3)
```

giac [B] time = 0.25, size = 146, normalized size = 2.47

$$\frac{2\left(140\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)^4a^2\mathrm{sgn}(x)+315\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)^3a^{\frac{3}{2}}b\mathrm{sgn}(x)+273\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)^2ab^2\mathrm{sgn}(\right.}{105\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 2/105*(140*(sqrt(a)*x - sqrt(a*x^2 + b*x))^4*a^2*sgn(x) + 315*(sqrt(a)*x -
sqrt(a*x^2 + b*x))^3*a^(3/2)*b*sgn(x) + 273*(sqrt(a)*x - sqrt(a*x^2 + b*x))
^2*a*b^2*sgn(x) + 105*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*b^3*sgn(x) +
15*b^4*sgn(x))/(sqrt(a)*x - sqrt(a*x^2 + b*x))^7
```

maple [A] time = 0.00, size = 44, normalized size = 0.75

$$-\frac{2\,(ax+b)\left(8a^2x^2-12abx+15b^2\right)\sqrt{\frac{ax+b}{x}}}{105b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(1/2)/x^4,x)
```

```
[Out] -2/105*(a*x+b)*(8*a^2*x^2-12*a*b*x+15*b^2)*((a*x+b)/x)^(1/2)/b^3/x^3
```

maxima [A] time = 1.06, size = 47, normalized size = 0.80

$$-\frac{2\left(a+\frac{b}{x}\right)^{\frac{7}{2}}}{7\,b^3}+\frac{4\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a}{5\,b^3}-\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] -2/7*(a + b/x)^(7/2)/b^3 + 4/5*(a + b/x)^(5/2)*a/b^3 - 2/3*(a + b/x)^(3/2)*
a^2/b^3
```

mupad [B] time = 1.53, size = 70, normalized size = 1.19

$$\frac{8\,a^2\sqrt{a+\frac{b}{x}}}{105\,b^2\,x}-\frac{16\,a^3\sqrt{a+\frac{b}{x}}}{105\,b^3}-\frac{2\,a\sqrt{a+\frac{b}{x}}}{35\,b\,x^2}-\frac{2\sqrt{a+\frac{b}{x}}}{7\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(1/2)/x^4,x)
```

```
[Out] (8*a^2*(a + b/x)^(1/2))/(105*b^2*x) - (16*a^3*(a + b/x)^(1/2))/(105*b^3) -
(2*a*(a + b/x)^(1/2))/(35*b*x^2) - (2*(a + b/x)^(1/2))/(7*x^3)
```

sympy [**B**] time = 2.19, size = 899, normalized size = 15.24

$$-\frac{16a^{\frac{19}{2}}b^{\frac{9}{2}}x^6\sqrt{\frac{ax}{b}+1}}{105a^{\frac{13}{2}}b^7x^{\frac{13}{2}}+315a^{\frac{11}{2}}b^8x^{\frac{11}{2}}+315a^{\frac{9}{2}}b^9x^{\frac{9}{2}}+105a^{\frac{7}{2}}b^{10}x^{\frac{7}{2}}}-\frac{40a^{\frac{17}{2}}b^{\frac{11}{2}}x^5\sqrt{\frac{ax}{b}+1}}{105a^{\frac{13}{2}}b^7x^{\frac{13}{2}}+315a^{\frac{11}{2}}b^8x^{\frac{11}{2}}+315a^{\frac{9}{2}}b^9x^{\frac{9}{2}}+105a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2)/x**4,x)
```

```
[Out] -16*a**(19/2)*b**(9/2)*x**6*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) +
 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b
**10*x**(7/2)) - 40*a**(17/2)*b**(11/2)*x**5*sqrt(a*x/b + 1)/(105*a**(13/2)
*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2)
 + 105*a**(7/2)*b**10*x**(7/2)) - 30*a**(15/2)*b**(13/2)*x**4*sqrt(a*x/b +
1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9
/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 40*a**(13/2)*b**(15/2)*x
**3*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(
11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 100*a**
(11/2)*b**(17/2)*x**2*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a
**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x
**(7/2)) - 96*a**(9/2)*b**(19/2)*x*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(
13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**
(7/2)*b**10*x**(7/2)) - 30*a**(7/2)*b**(21/2)*sqrt(a*x/b + 1)/(105*a**(13/2
)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2
) + 105*a**(7/2)*b**10*x**(7/2)) + 16*a**10*b**4*x**(13/2)/(105*a**(13/2)*b
**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) +
 105*a**(7/2)*b**10*x**(7/2)) + 48*a**9*b**5*x**(11/2)/(105*a**(13/2)*b**7*
x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105
*a**(7/2)*b**10*x**(7/2)) + 48*a**8*b**6*x**(9/2)/(105*a**(13/2)*b**7*x**(1
3/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(
7/2)*b**10*x**(7/2)) + 16*a**7*b**7*x**(7/2)/(105*a**(13/2)*b**7*x**(13/2)
+ 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*
b**10*x**(7/2))
```

3.1699 $\int \frac{\sqrt{a+\frac{b}{x}}}{x^5}\,dx$

Optimal. Leaf size=80

$$\frac{2a^3\left(a+\frac{b}{x}\right)^{3/2}}{3b^4}-\frac{6a^2\left(a+\frac{b}{x}\right)^{5/2}}{5b^4}-\frac{2\left(a+\frac{b}{x}\right)^{9/2}}{9b^4}+\frac{6a\left(a+\frac{b}{x}\right)^{7/2}}{7b^4}$$

```
[Out] 2/3*a^3*(a+b/x)^(3/2)/b^4-6/5*a^2*(a+b/x)^(5/2)/b^4+6/7*a*(a+b/x)^(7/2)/b^4
-2/9*(a+b/x)^(9/2)/b^4
```

Rubi [A] time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{6a^2\left(a+\frac{b}{x}\right)^{5/2}}{5b^4}+\frac{2a^3\left(a+\frac{b}{x}\right)^{3/2}}{3b^4}-\frac{2\left(a+\frac{b}{x}\right)^{9/2}}{9b^4}+\frac{6a\left(a+\frac{b}{x}\right)^{7/2}}{7b^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]/x^5,x]
```

```
[Out] (2*a^3*(a + b/x)^(3/2))/(3*b^4) - (6*a^2*(a + b/x)^(5/2))/(5*b^4) + (6*a*(a
 + b/x)^(7/2))/(7*b^4) - (2*(a + b/x)^(9/2))/(9*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+\frac{b}{x}}}{x^5}\,dx &= -\text{Subst}\left(\int x^3\sqrt{a+bx}\,dx, x, \frac{1}{x}\right)\\ &= -\text{Subst}\left(\int\left(-\frac{a^3\sqrt{a+bx}}{b^3}+\frac{3a^2(a+bx)^{3/2}}{b^3}-\frac{3a(a+bx)^{5/2}}{b^3}+\frac{(a+bx)^{7/2}}{b^3}\right)dx, x, \frac{1}{x}\right)\\ &= \frac{2a^3\left(a+\frac{b}{x}\right)^{3/2}}{3b^4}-\frac{6a^2\left(a+\frac{b}{x}\right)^{5/2}}{5b^4}+\frac{6a\left(a+\frac{b}{x}\right)^{7/2}}{7b^4}-\frac{2\left(a+\frac{b}{x}\right)^{9/2}}{9b^4}\end{aligned}$$

Mathematica [A] time = 0.03, size = 56, normalized size = 0.70

$$\frac{2\sqrt{a+\frac{b}{x}}\,(ax+b)\left(16a^3x^3-24a^2bx^2+30ab^2x-35b^3\right)}{315b^4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]/x^5,x]
```

```
[Out] (2*Sqrt[a + b/x]*(b + a*x)*(-35*b^3 + 30*a*b^2*x - 24*a^2*b*x^2 + 16*a^3*x^
3))/(315*b^4*x^4)
```

fricas [A] time = 1.03, size = 60, normalized size = 0.75

$$\frac{2\left(16\,a^4x^4-8\,a^3bx^3+6\,a^2b^2x^2-5\,ab^3x-35\,b^4\right)\sqrt{\frac{ax+b}{x}}}{315\,b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 2/315*(16*a^4*x^4 - 8*a^3*b*x^3 + 6*a^2*b^2*x^2 - 5*a*b^3*x - 35*b^4)*sqrt(
(a*x + b)/x)/(b^4*x^4)
```

giac [B] time = 0.25, size = 177, normalized size = 2.21

$$\frac{2\left(630\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)^5a^{\frac{5}{2}}\mathrm{sgn}(x)+1764\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)^4a^2b\mathrm{sgn}(x)+1995\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)^3a^{\frac{3}{2}}\right.}{315\left(\sqrt{a}\,x-\sqrt{a}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 2/315*(630*(sqrt(a)*x - sqrt(a*x^2 + b*x))^5*a^(5/2)*sgn(x) + 1764*(sqrt(a)
*x - sqrt(a*x^2 + b*x))^4*a^2*b*sgn(x) + 1995*(sqrt(a)*x - sqrt(a*x^2 + b*x
))^3*a^(3/2)*b^2*sgn(x) + 1125*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a*b^3*sgn(
x) + 315*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*b^4*sgn(x) + 35*b^5*sgn(x)
)/(sqrt(a)*x - sqrt(a*x^2 + b*x))^9
```

maple [A] time = 0.01, size = 55, normalized size = 0.69

$$\frac{2\,(ax+b)\left(16a^3x^3-24a^2b\,x^2+30a\,b^2x-35b^3\right)\sqrt{\frac{ax+b}{x}}}{315b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(1/2)/x^5,x)
```

```
[Out] 2/315*(a*x+b)*(16*a^3*x^3-24*a^2*b*x^2+30*a*b^2*x-35*b^3)*((a*x+b)/x)^(1/2)
/x^4/b^4
```

maxima [A] time = 0.95, size = 64, normalized size = 0.80

$$-\frac{2\left(a+\frac{b}{x}\right)^{\frac{9}{2}}}{9\,b^4}+\frac{6\left(a+\frac{b}{x}\right)^{\frac{7}{2}}a}{7\,b^4}-\frac{6\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^2}{5\,b^4}+\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^3}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -2/9*(a + b/x)^(9/2)/b^4 + 6/7*(a + b/x)^(7/2)*a/b^4 - 6/5*(a + b/x)^(5/2)*
a^2/b^4 + 2/3*(a + b/x)^(3/2)*a^3/b^4
```

mupad [B] time = 1.61, size = 90, normalized size = 1.12

$$\frac{32\,a^4\sqrt{a+\frac{b}{x}}}{315\,b^4}-\frac{2\sqrt{a+\frac{b}{x}}}{9\,x^4}-\frac{2\,a\sqrt{a+\frac{b}{x}}}{63\,b\,x^3}+\frac{4\,a^2\sqrt{a+\frac{b}{x}}}{105\,b^2\,x^2}-\frac{16\,a^3\sqrt{a+\frac{b}{x}}}{315\,b^3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(1/2)/x^5,x)
```

```
[Out] (32*a^4*(a + b/x)^(1/2))/(315*b^4) - (2*(a + b/x)^(1/2))/(9*x^4) - (2*a*(a
+ b/x)^(1/2))/(63*b*x^3) + (4*a^2*(a + b/x)^(1/2))/(105*b^2*x^2) - (16*a^3*
(a + b/x)^(1/2))/(315*b^3*x)
```

sympy [**B**] time = 3.01, size = 2297, normalized size = 28.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2)/x**5,x)
```

```
[Out] 32*a**(29/2)*b**(23/2)*x**10*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2)
 + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a
**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*
b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) + 176*a**(27/2)*b**(25/2)*x*
*9*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x*
*(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) +
 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(
9/2)*b**21*x**(9/2)) + 396*a**(25/2)*b**(27/2)*x**8*sqrt(a*x/b + 1)/(315*a*
*(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b
**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(
13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) + 462
*a**(23/2)*b**(29/2)*x**7*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) +
1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(
15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**
20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) + 210*a**(21/2)*b**(31/2)*x**6*
sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(1
9/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 47
25*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2
)*b**21*x**(9/2)) - 378*a**(19/2)*b**(33/2)*x**5*sqrt(a*x/b + 1)/(315*a**(2
1/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**1
7*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/
2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) - 1134*a
**(17/2)*b**(35/2)*x**4*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 18
90*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15
/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20
*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) - 1494*a**(15/2)*b**(37/2)*x**3*s
qrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19
/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 472
5*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)
*b**21*x**(9/2)) - 1098*a**(13/2)*b**(39/2)*x**2*sqrt(a*x/b + 1)/(315*a**(2
1/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**1
7*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/
2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) - 430*a*
*(11/2)*b**(41/2)*x*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a
**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*
b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**
(11/2) + 315*a**(9/2)*b**21*x**(9/2)) - 70*a**(9/2)*b**(43/2)*sqrt(a*x/b +
1)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a
**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*
b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9
/2)) - 32*a**15*b**11*x**(21/2)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(1
9/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**1
8*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/
2) + 315*a**(9/2)*b**21*x**(9/2)) - 192*a**14*b**12*x**(19/2)/(315*a**(21/2
)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x
```

```
**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2)
+ 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) - 480*a**13
*b**13*x**(17/2)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(
19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4
725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/
2)*b**21*x**(9/2)) - 640*a**12*b**14*x**(15/2)/(315*a**(21/2)*b**15*x**(21/
2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300
*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2
)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) - 480*a**11*b**15*x**(13/2
)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a*
*(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b
**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/
2)) - 192*a**10*b**16*x**(11/2)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(1
9/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**1
8*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/
2) + 315*a**(9/2)*b**21*x**(9/2)) - 32*a**9*b**17*x**(9/2)/(315*a**(21/2)*b
**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(
17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1
890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2))
```

3.1700 $\int \frac{\sqrt{a+\frac{b}{x}}}{x^6}\,dx$

Optimal. Leaf size=101

$$-\frac{2a^4\left(a+\frac{b}{x}\right)^{3/2}}{3b^5}+\frac{8a^3\left(a+\frac{b}{x}\right)^{5/2}}{5b^5}-\frac{12a^2\left(a+\frac{b}{x}\right)^{7/2}}{7b^5}-\frac{2\left(a+\frac{b}{x}\right)^{11/2}}{11b^5}+\frac{8a\left(a+\frac{b}{x}\right)^{9/2}}{9b^5}$$

```
[Out] -2/3*a^4*(a+b/x)^(3/2)/b^5+8/5*a^3*(a+b/x)^(5/2)/b^5-12/7*a^2*(a+b/x)^(7/2)
/b^5+8/9*a*(a+b/x)^(9/2)/b^5-2/11*(a+b/x)^(11/2)/b^5
```

Rubi [A] time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{12a^2\left(a+\frac{b}{x}\right)^{7/2}}{7b^5}+\frac{8a^3\left(a+\frac{b}{x}\right)^{5/2}}{5b^5}-\frac{2a^4\left(a+\frac{b}{x}\right)^{3/2}}{3b^5}-\frac{2\left(a+\frac{b}{x}\right)^{11/2}}{11b^5}+\frac{8a\left(a+\frac{b}{x}\right)^{9/2}}{9b^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]/x^6,x]
```

```
[Out] (-2*a^4*(a + b/x)^(3/2))/(3*b^5) + (8*a^3*(a + b/x)^(5/2))/(5*b^5) - (12*a^
2*(a + b/x)^(7/2))/(7*b^5) + (8*a*(a + b/x)^(9/2))/(9*b^5) - (2*(a + b/x)^(
11/2))/(11*b^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+\frac{b}{x}}}{x^6}\,dx &= -\text{Subst}\left(\int x^4\sqrt{a+bx}\,dx, x, \frac{1}{x}\right)\\ &= -\text{Subst}\left(\int\left(\frac{a^4\sqrt{a+bx}}{b^4}-\frac{4a^3(a+bx)^{3/2}}{b^4}+\frac{6a^2(a+bx)^{5/2}}{b^4}-\frac{4a(a+bx)^{7/2}}{b^4}+\frac{(a+bx)^{9/2}}{b^4}\right)dx, x\right)\\ &= -\frac{2a^4\left(a+\frac{b}{x}\right)^{3/2}}{3b^5}+\frac{8a^3\left(a+\frac{b}{x}\right)^{5/2}}{5b^5}-\frac{12a^2\left(a+\frac{b}{x}\right)^{7/2}}{7b^5}+\frac{8a\left(a+\frac{b}{x}\right)^{9/2}}{9b^5}-\frac{2\left(a+\frac{b}{x}\right)^{11/2}}{11b^5}\end{aligned}$$

Mathematica [A] time = 0.06, size = 67, normalized size = 0.66

$$-\frac{2\sqrt{a+\frac{b}{x}}\,(ax+b)\left(128a^4x^4-192a^3bx^3+240a^2b^2x^2-280ab^3x+315b^4\right)}{3465b^5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]/x^6,x]
```

```
[Out] (-2*Sqrt[a + b/x]*(b + a*x)*(315*b^4 - 280*a*b^3*x + 240*a^2*b^2*x^2 - 192*
a^3*b*x^3 + 128*a^4*x^4))/(3465*b^5*x^5)
```

fricas [A] time = 0.84, size = 71, normalized size = 0.70

$$-\frac{2\left(128\,a^5x^5-64\,a^4bx^4+48\,a^3b^2x^3-40\,a^2b^3x^2+35\,ab^4x+315\,b^5\right)\sqrt{\frac{ax+b}{x}}}{3465\,b^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] -2/3465*(128*a^5*x^5 - 64*a^4*b*x^4 + 48*a^3*b^2*x^3 - 40*a^2*b^3*x^2 + 35*
a*b^4*x + 315*b^5)*sqrt((a*x + b)/x)/(b^5*x^5)
```

giac [B] time = 0.34, size = 208, normalized size = 2.06

$$\frac{2\left(11088\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)^6a^3\mathrm{sgn}(x)+36960\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)^5a^{\frac{5}{2}}b\mathrm{sgn}(x)+51480\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] 2/3465*(11088*(sqrt(a)*x - sqrt(a*x^2 + b*x))^6*a^3*sgn(x) + 36960*(sqrt(a)
*x - sqrt(a*x^2 + b*x))^5*a^(5/2)*b*sgn(x) + 51480*(sqrt(a)*x - sqrt(a*x^2
+ b*x))^4*a^2*b^2*sgn(x) + 38115*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a^(3/2)*
b^3*sgn(x) + 15785*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a*b^4*sgn(x) + 3465*(s
qrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*b^5*sgn(x) + 315*b^6*sgn(x))/(sqrt(a)
*x - sqrt(a*x^2 + b*x))^11
```

maple [A] time = 0.01, size = 66, normalized size = 0.65

$$-\frac{2\,(ax+b)\left(128a^4x^4-192a^3x^3b+240a^2x^2b^2-280ax\,b^3+315b^4\right)\sqrt{\frac{ax+b}{x}}}{3465b^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(1/2)/x^6,x)
```

```
[Out] -2/3465*(a*x+b)*(128*a^4*x^4-192*a^3*b*x^3+240*a^2*b^2*x^2-280*a*b^3*x+315*
b^4)*((a*x+b)/x)^(1/2)/x^5/b^5
```

maxima [A] time = 1.04, size = 81, normalized size = 0.80

$$-\frac{2\left(a+\frac{b}{x}\right)^{\frac{11}{2}}}{11\,b^5}+\frac{8\left(a+\frac{b}{x}\right)^{\frac{9}{2}}a}{9\,b^5}-\frac{12\left(a+\frac{b}{x}\right)^{\frac{7}{2}}a^2}{7\,b^5}+\frac{8\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^3}{5\,b^5}-\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^4}{3\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] -2/11*(a + b/x)^(11/2)/b^5 + 8/9*(a + b/x)^(9/2)*a/b^5 - 12/7*(a + b/x)^(7/
2)*a^2/b^5 + 8/5*(a + b/x)^(5/2)*a^3/b^5 - 2/3*(a + b/x)^(3/2)*a^4/b^5
```

mupad [B] time = 1.70, size = 110, normalized size = 1.09

$$\frac{16\,a^2\sqrt{a+\frac{b}{x}}}{693\,b^2\,x^3}-\frac{256\,a^5\sqrt{a+\frac{b}{x}}}{3465\,b^5}-\frac{2\,a\sqrt{a+\frac{b}{x}}}{99\,b\,x^4}-\frac{2\sqrt{a+\frac{b}{x}}}{11\,x^5}-\frac{32\,a^3\sqrt{a+\frac{b}{x}}}{1155\,b^3\,x^2}+\frac{128\,a^4\sqrt{a+\frac{b}{x}}}{3465\,b^4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(1/2)/x^6,x)
```

```
[Out] (16*a^2*(a + b/x)^(1/2))/(693*b^2*x^3) - (256*a^5*(a + b/x)^(1/2))/(3465*b^
5) - (2*a*(a + b/x)^(1/2))/(99*b*x^4) - (2*(a + b/x)^(1/2))/(11*x^5) - (32*
a^3*(a + b/x)^(1/2))/(1155*b^3*x^2) + (128*a^4*(a + b/x)^(1/2))/(3465*b^4*x
)
```

sympy [B] time = 5.62, size = 5095, normalized size = 50.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2)/x**6,x)
```

```
[Out] -256*a**(41/2)*b**(49/2)*x**15*sqrt(a*x/b + 1)/(3465*a**(31/2)*b**29*x**(31
/2) + 34650*a**(29/2)*b**30*x**(29/2) + 155925*a**(27/2)*b**31*x**(27/2) +
415800*a**(25/2)*b**32*x**(25/2) + 727650*a**(23/2)*b**33*x**(23/2) + 87318
0*a**(21/2)*b**34*x**(21/2) + 727650*a**(19/2)*b**35*x**(19/2) + 415800*a**
(17/2)*b**36*x**(17/2) + 155925*a**(15/2)*b**37*x**(15/2) + 34650*a**(13/2)
*b**38*x**(13/2) + 3465*a**(11/2)*b**39*x**(11/2)) - 2432*a**(39/2)*b**(51/
2)*x**14*sqrt(a*x/b + 1)/(3465*a**(31/2)*b**29*x**(31/2) + 34650*a**(29/2)*
b**30*x**(29/2) + 155925*a**(27/2)*b**31*x**(27/2) + 415800*a**(25/2)*b**32
*x**(25/2) + 727650*a**(23/2)*b**33*x**(23/2) + 873180*a**(21/2)*b**34*x**(
21/2) + 727650*a**(19/2)*b**35*x**(19/2) + 415800*a**(17/2)*b**36*x**(17/2)
 + 155925*a**(15/2)*b**37*x**(15/2) + 34650*a**(13/2)*b**38*x**(13/2) + 346
5*a**(11/2)*b**39*x**(11/2)) - 10336*a**(37/2)*b**(53/2)*x**13*sqrt(a*x/b +
 1)/(3465*a**(31/2)*b**29*x**(31/2) + 34650*a**(29/2)*b**30*x**(29/2) + 155
925*a**(27/2)*b**31*x**(27/2) + 415800*a**(25/2)*b**32*x**(25/2) + 727650*a
**(23/2)*b**33*x**(23/2) + 873180*a**(21/2)*b**34*x**(21/2) + 727650*a**(19
/2)*b**35*x**(19/2) + 415800*a**(17/2)*b**36*x**(17/2) + 155925*a**(15/2)*b
**37*x**(15/2) + 34650*a**(13/2)*b**38*x**(13/2) + 3465*a**(11/2)*b**39*x**
(11/2)) - 25840*a**(35/2)*b**(55/2)*x**12*sqrt(a*x/b + 1)/(3465*a**(31/2)*b
**29*x**(31/2) + 34650*a**(29/2)*b**30*x**(29/2) + 155925*a**(27/2)*b**31*x
**(27/2) + 415800*a**(25/2)*b**32*x**(25/2) + 727650*a**(23/2)*b**33*x**(23
/2) + 873180*a**(21/2)*b**34*x**(21/2) + 727650*a**(19/2)*b**35*x**(19/2) +
 415800*a**(17/2)*b**36*x**(17/2) + 155925*a**(15/2)*b**37*x**(15/2) + 3465
0*a**(13/2)*b**38*x**(13/2) + 3465*a**(11/2)*b**39*x**(11/2)) - 41990*a**(3
3/2)*b**(57/2)*x**11*sqrt(a*x/b + 1)/(3465*a**(31/2)*b**29*x**(31/2) + 3465
0*a**(29/2)*b**30*x**(29/2) + 155925*a**(27/2)*b**31*x**(27/2) + 415800*a**
(25/2)*b**32*x**(25/2) + 727650*a**(23/2)*b**33*x**(23/2) + 873180*a**(21/2
)*b**34*x**(21/2) + 727650*a**(19/2)*b**35*x**(19/2) + 415800*a**(17/2)*b**
36*x**(17/2) + 155925*a**(15/2)*b**37*x**(15/2) + 34650*a**(13/2)*b**38*x**
(13/2) + 3465*a**(11/2)*b**39*x**(11/2)) - 46882*a**(31/2)*b**(59/2)*x**10*
sqrt(a*x/b + 1)/(3465*a**(31/2)*b**29*x**(31/2) + 34650*a**(29/2)*b**30*x**
(29/2) + 155925*a**(27/2)*b**31*x**(27/2) + 415800*a**(25/2)*b**32*x**(25/2
) + 727650*a**(23/2)*b**33*x**(23/2) + 873180*a**(21/2)*b**34*x**(21/2) + 7
27650*a**(19/2)*b**35*x**(19/2) + 415800*a**(17/2)*b**36*x**(17/2) + 155925
*a**(15/2)*b**37*x**(15/2) + 34650*a**(13/2)*b**38*x**(13/2) + 3465*a**(11/
2)*b**39*x**(11/2)) - 41514*a**(29/2)*b**(61/2)*x**9*sqrt(a*x/b + 1)/(3465*
a**(31/2)*b**29*x**(31/2) + 34650*a**(29/2)*b**30*x**(29/2) + 155925*a**(27
/2)*b**31*x**(27/2) + 415800*a**(25/2)*b**32*x**(25/2) + 727650*a**(23/2)*b
**33*x**(23/2) + 873180*a**(21/2)*b**34*x**(21/2) + 727650*a**(19/2)*b**35*
```

```
x**(19/2) + 415800*a**(17/2)*b**36*x**(17/2) + 155925*a**(15/2)*b**37*x**(1
5/2) + 34650*a**(13/2)*b**38*x**(13/2) + 3465*a**(11/2)*b**39*x**(11/2)) -
47982*a**(27/2)*b**(63/2)*x**8*sqrt(a*x/b + 1)/(3465*a**(31/2)*b**29*x**(31
/2) + 34650*a**(29/2)*b**30*x**(29/2) + 155925*a**(27/2)*b**31*x**(27/2) +
415800*a**(25/2)*b**32*x**(25/2) + 727650*a**(23/2)*b**33*x**(23/2) + 87318
0*a**(21/2)*b**34*x**(21/2) + 727650*a**(19/2)*b**35*x**(19/2) + 415800*a**
(17/2)*b**36*x**(17/2) + 155925*a**(15/2)*b**37*x**(15/2) + 34650*a**(13/2)
*b**38*x**(13/2) + 3465*a**(11/2)*b**39*x**(11/2)) - 86460*a**(25/2)*b**(65
/2)*x**7*sqrt(a*x/b + 1)/(3465*a**(31/2)*b**29*x**(31/2) + 34650*a**(29/2)*
b**30*x**(29/2) + 155925*a**(27/2)*b**31*x**(27/2) + 415800*a**(25/2)*b**32
*x**(25/2) + 727650*a**(23/2)*b**33*x**(23/2) + 873180*a**(21/2)*b**34*x**(
21/2) + 727650*a**(19/2)*b**35*x**(19/2) + 415800*a**(17/2)*b**36*x**(17/2)
 + 155925*a**(15/2)*b**37*x**(15/2) + 34650*a**(13/2)*b**38*x**(13/2) + 346
5*a**(11/2)*b**39*x**(11/2)) - 141460*a**(23/2)*b**(67/2)*x**6*sqrt(a*x/b +
 1)/(3465*a**(31/2)*b**29*x**(31/2) + 34650*a**(29/2)*b**30*x**(29/2) + 155
925*a**(27/2)*b**31*x**(27/2) + 415800*a**(25/2)*b**32*x**(25/2) + 727650*a
**(23/2)*b**33*x**(23/2) + 873180*a**(21/2)*b**34*x**(21/2) + 727650*a**(19
/2)*b**35*x**(19/2) + 415800*a**(17/2)*b**36*x**(17/2) + 155925*a**(15/2)*b
**37*x**(15/2) + 34650*a**(13/2)*b**38*x**(13/2) + 3465*a**(11/2)*b**39*x**
(11/2)) - 167156*a**(21/2)*b**(69/2)*x**5*sqrt(a*x/b + 1)/(3465*a**(31/2)*b
**29*x**(31/2) + 34650*a**(29/2)*b**30*x**(29/2) + 155925*a**(27/2)*b**31*x
**(27/2) + 415800*a**(25/2)*b**32*x**(25/2) + 727650*a**(23/2)*b**33*x**(23
/2) + 873180*a**(21/2)*b**34*x**(21/2) + 727650*a**(19/2)*b**35*x**(19/2) +
 415800*a**(17/2)*b**36*x**(17/2) + 155925*a**(15/2)*b**37*x**(15/2) + 3465
0*a**(13/2)*b**38*x**(13/2) + 3465*a**(11/2)*b**39*x**(11/2)) - 137932*a**(
19/2)*b**(71/2)*x**4*sqrt(a*x/b + 1)/(3465*a**(31/2)*b**29*x**(31/2) + 3465
0*a**(29/2)*b**30*x**(29/2) + 155925*a**(27/2)*b**31*x**(27/2) + 415800*a**
(25/2)*b**32*x**(25/2) + 727650*a**(23/2)*b**33*x**(23/2) + 873180*a**(21/2
)*b**34*x**(21/2) + 727650*a**(19/2)*b**35*x**(19/2) + 415800*a**(17/2)*b**
36*x**(17/2) + 155925*a**(15/2)*b**37*x**(15/2) + 34650*a**(13/2)*b**38*x**
(13/2) + 3465*a**(11/2)*b**39*x**(11/2)) - 78046*a**(17/2)*b**(73/2)*x**3*s
qrt(a*x/b + 1)/(3465*a**(31/2)*b**29*x**(31/2) + 34650*a**(29/2)*b**30*x**(
29/2) + 155925*a**(27/2)*b**31*x**(27/2) + 415800*a**(25/2)*b**32*x**(25/2)
 + 727650*a**(23/2)*b**33*x**(23/2) + 873180*a**(21/2)*b**34*x**(21/2) + 72
7650*a**(19/2)*b**35*x**(19/2) + 415800*a**(17/2)*b**36*x**(17/2) + 155925*
a**(15/2)*b**37*x**(15/2) + 34650*a**(13/2)*b**38*x**(13/2) + 3465*a**(11/2
)*b**39*x**(11/2)) - 28970*a**(15/2)*b**(75/2)*x**2*sqrt(a*x/b + 1)/(3465*a
**(31/2)*b**29*x**(31/2) + 34650*a**(29/2)*b**30*x**(29/2) + 155925*a**(27/
2)*b**31*x**(27/2) + 415800*a**(25/2)*b**32*x**(25/2) + 727650*a**(23/2)*b*
*33*x**(23/2) + 873180*a**(21/2)*b**34*x**(21/2) + 727650*a**(19/2)*b**35*x
**(19/2) + 415800*a**(17/2)*b**36*x**(17/2) + 155925*a**(15/2)*b**37*x**(15
/2) + 34650*a**(13/2)*b**38*x**(13/2) + 3465*a**(11/2)*b**39*x**(11/2)) - 6
370*a**(13/2)*b**(77/2)*x*sqrt(a*x/b + 1)/(3465*a**(31/2)*b**29*x**(31/2) +
 34650*a**(29/2)*b**30*x**(29/2) + 155925*a**(27/2)*b**31*x**(27/2) + 41580
0*a**(25/2)*b**32*x**(25/2) + 727650*a**(23/2)*b**33*x**(23/2) + 873180*a**
(21/2)*b**34*x**(21/2) + 727650*a**(19/2)*b**35*x**(19/2) + 415800*a**(17/2
)*b**36*x**(17/2) + 155925*a**(15/2)*b**37*x**(15/2) + 34650*a**(13/2)*b**3
8*x**(13/2) + 3465*a**(11/2)*b**39*x**(11/2)) - 630*a**(11/2)*b**(79/2)*sqr
t(a*x/b + 1)/(3465*a**(31/2)*b**29*x**(31/2) + 34650*a**(29/2)*b**30*x**(29
/2) + 155925*a**(27/2)*b**31*x**(27/2) + 415800*a**(25/2)*b**32*x**(25/2) +
 727650*a**(23/2)*b**33*x**(23/2) + 873180*a**(21/2)*b**34*x**(21/2) + 7276
50*a**(19/2)*b**35*x**(19/2) + 415800*a**(17/2)*b**36*x**(17/2) + 155925*a*
*(15/2)*b**37*x**(15/2) + 34650*a**(13/2)*b**38*x**(13/2) + 3465*a**(11/2)*
b**39*x**(11/2)) + 256*a**21*b**24*x**(31/2)/(3465*a**(31/2)*b**29*x**(31/2
) + 34650*a**(29/2)*b**30*x**(29/2) + 155925*a**(27/2)*b**31*x**(27/2) + 41
5800*a**(25/2)*b**32*x**(25/2) + 727650*a**(23/2)*b**33*x**(23/2) + 873180*
a**(21/2)*b**34*x**(21/2) + 727650*a**(19/2)*b**35*x**(19/2) + 415800*a**(1
7/2)*b**36*x**(17/2) + 155925*a**(15/2)*b**37*x**(15/2) + 34650*a**(13/2)*b
**38*x**(13/2) + 3465*a**(11/2)*b**39*x**(11/2)) + 2560*a**20*b**25*x**(29/
```

```
2)/(3465*a**(31/2)*b**29*x**(31/2) + 34650*a**(29/2)*b**30*x**(29/2) + 1559
25*a**(27/2)*b**31*x**(27/2) + 415800*a**(25/2)*b**32*x**(25/2) + 727650*a*
*(23/2)*b**33*x**(23/2) + 873180*a**(21/2)*b**34*x**(21/2) + 727650*a**(19/
2)*b**35*x**(19/2) + 415800*a**(17/2)*b**36*x**(17/2) + 155925*a**(15/2)*b*
*37*x**(15/2) + 34650*a**(13/2)*b**38*x**(13/2) + 3465*a**(11/2)*b**39*x**(
11/2)) + 11520*a**19*b**26*x**(27/2)/(3465*a**(31/2)*b**29*x**(31/2) + 3465
0*a**(29/2)*b**30*x**(29/2) + 155925*a**(27/2)*b**31*x**(27/2) + 415800*a**
(25/2)*b**32*x**(25/2) + 727650*a**(23/2)*b**33*x**(23/2) + 873180*a**(21/2
)*b**34*x**(21/2) + 727650*a**(19/2)*b**35*x**(19/2) + 415800*a**(17/2)*b**
36*x**(17/2) + 155925*a**(15/2)*b**37*x**(15/2) + 34650*a**(13/2)*b**38*x**
(13/2) + 3465*a**(11/2)*b**39*x**(11/2)) + 30720*a**18*b**27*x**(25/2)/(346
5*a**(31/2)*b**29*x**(31/2) + 34650*a**(29/2)*b**30*x**(29/2) + 155925*a**(
27/2)*b**31*x**(27/2) + 415800*a**(25/2)*b**32*x**(25/2) + 727650*a**(23/2)
*b**33*x**(23/2) + 873180*a**(21/2)*b**34*x**(21/2) + 727650*a**(19/2)*b**3
5*x**(19/2) + 415800*a**(17/2)*b**36*x**(17/2) + 155925*a**(15/2)*b**37*x**
(15/2) + 34650*a**(13/2)*b**38*x**(13/2) + 3465*a**(11/2)*b**39*x**(11/2))
+ 53760*a**17*b**28*x**(23/2)/(3465*a**(31/2)*b**29*x**(31/2) + 34650*a**(2
9/2)*b**30*x**(29/2) + 155925*a**(27/2)*b**31*x**(27/2) + 415800*a**(25/2)*
b**32*x**(25/2) + 727650*a**(23/2)*b**33*x**(23/2) + 873180*a**(21/2)*b**34
*x**(21/2) + 727650*a**(19/2)*b**35*x**(19/2) + 415800*a**(17/2)*b**36*x**(
17/2) + 155925*a**(15/2)*b**37*x**(15/2) + 34650*a**(13/2)*b**38*x**(13/2)
+ 3465*a**(11/2)*b**39*x**(11/2)) + 64512*a**16*b**29*x**(21/2)/(3465*a**(3
1/2)*b**29*x**(31/2) + 34650*a**(29/2)*b**30*x**(29/2) + 155925*a**(27/2)*b
**31*x**(27/2) + 415800*a**(25/2)*b**32*x**(25/2) + 727650*a**(23/2)*b**33*
x**(23/2) + 873180*a**(21/2)*b**34*x**(21/2) + 727650*a**(19/2)*b**35*x**(1
9/2) + 415800*a**(17/2)*b**36*x**(17/2) + 155925*a**(15/2)*b**37*x**(15/2)
+ 34650*a**(13/2)*b**38*x**(13/2) + 3465*a**(11/2)*b**39*x**(11/2)) + 53760
*a**15*b**30*x**(19/2)/(3465*a**(31/2)*b**29*x**(31/2) + 34650*a**(29/2)*b*
*30*x**(29/2) + 155925*a**(27/2)*b**31*x**(27/2) + 415800*a**(25/2)*b**32*x
**(25/2) + 727650*a**(23/2)*b**33*x**(23/2) + 873180*a**(21/2)*b**34*x**(21
/2) + 727650*a**(19/2)*b**35*x**(19/2) + 415800*a**(17/2)*b**36*x**(17/2) +
 155925*a**(15/2)*b**37*x**(15/2) + 34650*a**(13/2)*b**38*x**(13/2) + 3465*
a**(11/2)*b**39*x**(11/2)) + 30720*a**14*b**31*x**(17/2)/(3465*a**(31/2)*b*
*29*x**(31/2) + 34650*a**(29/2)*b**30*x**(29/2) + 155925*a**(27/2)*b**31*x*
*(27/2) + 415800*a**(25/2)*b**32*x**(25/2) + 727650*a**(23/2)*b**33*x**(23/
2) + 873180*a**(21/2)*b**34*x**(21/2) + 727650*a**(19/2)*b**35*x**(19/2) +
415800*a**(17/2)*b**36*x**(17/2) + 155925*a**(15/2)*b**37*x**(15/2) + 34650
*a**(13/2)*b**38*x**(13/2) + 3465*a**(11/2)*b**39*x**(11/2)) + 11520*a**13*
b**32*x**(15/2)/(3465*a**(31/2)*b**29*x**(31/2) + 34650*a**(29/2)*b**30*x**
(29/2) + 155925*a**(27/2)*b**31*x**(27/2) + 415800*a**(25/2)*b**32*x**(25/2
) + 727650*a**(23/2)*b**33*x**(23/2) + 873180*a**(21/2)*b**34*x**(21/2) + 7
27650*a**(19/2)*b**35*x**(19/2) + 415800*a**(17/2)*b**36*x**(17/2) + 155925
*a**(15/2)*b**37*x**(15/2) + 34650*a**(13/2)*b**38*x**(13/2) + 3465*a**(11/
2)*b**39*x**(11/2)) + 2560*a**12*b**33*x**(13/2)/(3465*a**(31/2)*b**29*x**(
31/2) + 34650*a**(29/2)*b**30*x**(29/2) + 155925*a**(27/2)*b**31*x**(27/2)
+ 415800*a**(25/2)*b**32*x**(25/2) + 727650*a**(23/2)*b**33*x**(23/2) + 873
180*a**(21/2)*b**34*x**(21/2) + 727650*a**(19/2)*b**35*x**(19/2) + 415800*a
**(17/2)*b**36*x**(17/2) + 155925*a**(15/2)*b**37*x**(15/2) + 34650*a**(13/
2)*b**38*x**(13/2) + 3465*a**(11/2)*b**39*x**(11/2)) + 256*a**11*b**34*x**(
11/2)/(3465*a**(31/2)*b**29*x**(31/2) + 34650*a**(29/2)*b**30*x**(29/2) + 1
55925*a**(27/2)*b**31*x**(27/2) + 415800*a**(25/2)*b**32*x**(25/2) + 727650
*a**(23/2)*b**33*x**(23/2) + 873180*a**(21/2)*b**34*x**(21/2) + 727650*a**(
19/2)*b**35*x**(19/2) + 415800*a**(17/2)*b**36*x**(17/2) + 155925*a**(15/2)
*b**37*x**(15/2) + 34650*a**(13/2)*b**38*x**(13/2) + 3465*a**(11/2)*b**39*x
**(11/2))
```

3.1701 $\int \left(a+\frac{b}{x}\right)^{3/2} x^3\,dx$

Optimal. Leaf size=114

$$\frac{3b^4 \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{64a^{5/2}} - \frac{3b^3 x\sqrt{a+\frac{b}{x}}}{64a^2} + \frac{b^2x^2\sqrt{a+\frac{b}{x}}}{32a} + \frac{1}{4}x^4\left(a+\frac{b}{x}\right)^{3/2} + \frac{1}{8}bx^3\sqrt{a+\frac{b}{x}}$$

```
[Out] 1/4*(a+b/x)^(3/2)*x^4+3/64*b^4*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(5/2)-3/64*
b^3*x*(a+b/x)^(1/2)/a^2+1/32*b^2*x^2*(a+b/x)^(1/2)/a+1/8*b*x^3*(a+b/x)^(1/2
)
```

Rubi [A] time = 0.05, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {266, 47, 51, 63, 208}

$$-\frac{3b^3 x\sqrt{a+\frac{b}{x}}}{64a^2} + \frac{3b^4 \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{64a^{5/2}} + \frac{b^2x^2\sqrt{a+\frac{b}{x}}}{32a} + \frac{1}{8}bx^3\sqrt{a+\frac{b}{x}} + \frac{1}{4}x^4\left(a+\frac{b}{x}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)*x^3,x]
```

```
[Out] (-3*b^3*Sqrt[a + b/x]*x)/(64*a^2) + (b^2*Sqrt[a + b/x]*x^2)/(32*a) + (b*Sqr
t[a + b/x]*x^3)/8 + ((a + b/x)^(3/2)*x^4)/4 + (3*b^4*ArcTanh[Sqrt[a + b/x]/
Sqrt[a]])/(64*a^(5/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a+\frac{b}{x}\right)^{3/2} x^3\,dx &= -\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^5}\,dx, x, \frac{1}{x}\right) \\
&= \frac{1}{4}\left(a+\frac{b}{x}\right)^{3/2} x^4 - \frac{1}{8}(3b)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^4}\,dx, x, \frac{1}{x}\right) \\
&= \frac{1}{8}b\sqrt{a+\frac{b}{x}}\,x^3 + \frac{1}{4}\left(a+\frac{b}{x}\right)^{3/2} x^4 - \frac{1}{16}b^2\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right) \\
&= \frac{b^2\sqrt{a+\frac{b}{x}}\,x^2}{32a} + \frac{1}{8}b\sqrt{a+\frac{b}{x}}\,x^3 + \frac{1}{4}\left(a+\frac{b}{x}\right)^{3/2} x^4 + \frac{\left(3b^3\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{64a} \\
&= -\frac{3b^3\sqrt{a+\frac{b}{x}}\,x}{64a^2} + \frac{b^2\sqrt{a+\frac{b}{x}}\,x^2}{32a} + \frac{1}{8}b\sqrt{a+\frac{b}{x}}\,x^3 + \frac{1}{4}\left(a+\frac{b}{x}\right)^{3/2} x^4 - \frac{\left(3b^4\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\right.}{128a^2} \\
&= -\frac{3b^3\sqrt{a+\frac{b}{x}}\,x}{64a^2} + \frac{b^2\sqrt{a+\frac{b}{x}}\,x^2}{32a} + \frac{1}{8}b\sqrt{a+\frac{b}{x}}\,x^3 + \frac{1}{4}\left(a+\frac{b}{x}\right)^{3/2} x^4 - \frac{\left(3b^3\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\right.}{64a^2} \\
&= -\frac{3b^3\sqrt{a+\frac{b}{x}}\,x}{64a^2} + \frac{b^2\sqrt{a+\frac{b}{x}}\,x^2}{32a} + \frac{1}{8}b\sqrt{a+\frac{b}{x}}\,x^3 + \frac{1}{4}\left(a+\frac{b}{x}\right)^{3/2} x^4 + \frac{3b^4\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{64a^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 39, normalized size = 0.34

$$\frac{2b^4\left(a+\frac{b}{x}\right)^{5/2}\,{}_2F_1\left(\frac{5}{2},5;\frac{7}{2};\frac{b}{ax}+1\right)}{5a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)*x^3,x]
```

```
[Out] (2*b^4*(a + b/x)^(5/2)*Hypergeometric2F1[5/2, 5, 7/2, 1 + b/(a*x)])/(5*a^5)
```

fricas [A] time = 0.89, size = 174, normalized size = 1.53

$$\left[\frac{3\sqrt{a}\,b^4\log\left(2\,ax+2\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(16\,a^4x^4+24\,a^3bx^3+2\,a^2b^2x^2-3\,ab^3x\right)\sqrt{\frac{ax+b}{x}}}{128\,a^3}, -\frac{3\sqrt{-a}\,b^4\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*x^3,x, algorithm="fricas")
```

```
[Out] [1/128*(3*sqrt(a)*b^4*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(1
6*a^4*x^4 + 24*a^3*b*x^3 + 2*a^2*b^2*x^2 - 3*a*b^3*x)*sqrt((a*x + b)/x))/a^
3, -1/64*(3*sqrt(-a)*b^4*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (16*a^4*x^4
 + 24*a^3*b*x^3 + 2*a^2*b^2*x^2 - 3*a*b^3*x)*sqrt((a*x + b)/x))/a^3]
```

giac [A] time = 0.26, size = 106, normalized size = 0.93

$$-\frac{3\,b^4\log\left(\left|-2\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)\sqrt{a}-b\right|\right)\operatorname{sgn}(x)}{128\,a^{\frac{5}{2}}}+\frac{3\,b^4\log\left(|b|\right)\operatorname{sgn}(x)}{128\,a^{\frac{5}{2}}}+\frac{1}{64}\sqrt{ax^2+bx}\left(2\left(4\left(2\,ax\operatorname{sgn}(x)+3\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*x^3,x, algorithm="giac")
```

```
[Out] -3/128*b^4*log(abs(-2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) - b))*sgn(x)/
a^(5/2) + 3/128*b^4*log(abs(b))*sgn(x)/a^(5/2) + 1/64*sqrt(a*x^2 + b*x)*(2*
(4*(2*a*x*sgn(x) + 3*b*sgn(x))*x + b^2*sgn(x)/a)*x - 3*b^3*sgn(x)/a^2)
```

maple [A] time = 0.01, size = 135, normalized size = 1.18

$$\frac{\sqrt{\frac{ax+b}{x}}\left(3a\,b^4\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)-12\sqrt{a\,x^2+bx}\;a^{\frac{5}{2}}b^2x+32\left(a\,x^2+bx\right)^{\frac{3}{2}}a^{\frac{7}{2}}x-6\sqrt{a\,x^2+bx}\;a^{\frac{3}{2}}b^3+16\left(a\right.\right.}{128\sqrt{(ax+b)\,x}\;a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(3/2)*x^3,x)
```

```
[Out] 1/128*((a*x+b)/x)^(1/2)*x/a^(7/2)*(32*(a*x^2+b*x)^(3/2)*a^(7/2)*x+16*(a*x^2
+b*x)^(3/2)*a^(5/2)*b-12*(a*x^2+b*x)^(1/2)*a^(5/2)*b^2*x-6*(a*x^2+b*x)^(1/2
)*a^(3/2)*b^3+3*a*b^4*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))
)/((a*x+b)*x)^(1/2)
```

maxima [A] time = 2.28, size = 166, normalized size = 1.46

$$-\frac{3\,b^4\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{128\,a^{\frac{5}{2}}}-\frac{3\left(a+\frac{b}{x}\right)^{\frac{7}{2}}b^4-11\left(a+\frac{b}{x}\right)^{\frac{5}{2}}ab^4-11\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2b^4+3\sqrt{a+\frac{b}{x}}\,a^3b^4}{64\left(\left(a+\frac{b}{x}\right)^4a^2-4\left(a+\frac{b}{x}\right)^3a^3+6\left(a+\frac{b}{x}\right)^2a^4-4\left(a+\frac{b}{x}\right)a^5+a^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*x^3,x, algorithm="maxima")
```

```
[Out] -3/128*b^4*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(5/2)
 - 1/64*(3*(a + b/x)^(7/2)*b^4 - 11*(a + b/x)^(5/2)*a*b^4 - 11*(a + b/x)^(3
/2)*a^2*b^4 + 3*sqrt(a + b/x)*a^3*b^4)/((a + b/x)^4*a^2 - 4*(a + b/x)^3*a^3
 + 6*(a + b/x)^2*a^4 - 4*(a + b/x)*a^5 + a^6)
```

mupad [B] time = 1.36, size = 89, normalized size = 0.78

$$\frac{11\,x^4\left(a+\frac{b}{x}\right)^{3/2}}{64}-\frac{3\,a\,x^4\sqrt{a+\frac{b}{x}}}{64}+\frac{11\,x^4\left(a+\frac{b}{x}\right)^{5/2}}{64\,a}-\frac{3\,x^4\left(a+\frac{b}{x}\right)^{7/2}}{64\,a^2}-\frac{b^4\operatorname{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1\mathrm{i}}{\sqrt{a}}\right)3\mathrm{i}}{64\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b/x)^(3/2),x)
```

```
[Out] (11*x^4*(a + b/x)^(3/2))/64 - (b^4*atan(((a + b/x)^(1/2)*1i)/a^(1/2))*3i)/(
64*a^(5/2)) - (3*a*x^4*(a + b/x)^(1/2))/64 + (11*x^4*(a + b/x)^(5/2))/(64*a
) - (3*x^4*(a + b/x)^(7/2))/(64*a^2)
```

sympy [**A**] time = 8.94, size = 153, normalized size = 1.34

$$\frac{a^2 x^{\frac{9}{2}}}{4\sqrt{b}\sqrt{\frac{ax}{b}+1}} + \frac{5a\sqrt{b}\,x^{\frac{7}{2}}}{8\sqrt{\frac{ax}{b}+1}} + \frac{13b^{\frac{3}{2}}x^{\frac{5}{2}}}{32\sqrt{\frac{ax}{b}+1}} - \frac{b^{\frac{5}{2}}x^{\frac{3}{2}}}{64a\sqrt{\frac{ax}{b}+1}} - \frac{3b^{\frac{7}{2}}\sqrt{x}}{64a^2\sqrt{\frac{ax}{b}+1}} + \frac{3b^4 \operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{64a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)*x**3,x)
```

```
[Out] a**2*x**(9/2)/(4*sqrt(b)*sqrt(a*x/b + 1)) + 5*a*sqrt(b)*x**(7/2)/(8*sqrt(a*
x/b + 1)) + 13*b**(3/2)*x**(5/2)/(32*sqrt(a*x/b + 1)) - b**(5/2)*x**(3/2)/(
64*a*sqrt(a*x/b + 1)) - 3*b**(7/2)*sqrt(x)/(64*a**2*sqrt(a*x/b + 1)) + 3*b*
*4*asinh(sqrt(a)*sqrt(x)/sqrt(b))/(64*a**(5/2))
```

3.1702 $\int \left(a+\frac{b}{x}\right)^{3/2} x^2\,dx$

Optimal. Leaf size=90

$$-\frac{b^3 \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{8a^{3/2}}+\frac{b^2 x\sqrt{a+\frac{b}{x}}}{8a}+\frac{1}{3}x^3\left(a+\frac{b}{x}\right)^{3/2}+\frac{1}{4}bx^2\sqrt{a+\frac{b}{x}}$$

```
[Out] 1/3*(a+b/x)^(3/2)*x^3-1/8*b^3*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(3/2)+1/8*b^
2*x*(a+b/x)^(1/2)/a+1/4*b*x^2*(a+b/x)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {266, 47, 51, 63, 208}

$$-\frac{b^3 \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{8a^{3/2}}+\frac{b^2 x\sqrt{a+\frac{b}{x}}}{8a}+\frac{1}{4}bx^2\sqrt{a+\frac{b}{x}}+\frac{1}{3}x^3\left(a+\frac{b}{x}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)*x^2,x]
```

```
[Out] (b^2*Sqrt[a + b/x]*x)/(8*a) + (b*Sqrt[a + b/x]*x^2)/4 + ((a + b/x)^(3/2)*x^
3)/3 - (b^3*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(8*a^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a+\frac{b}{x}\right)^{3/2} x^2\,dx &= -\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^4}\,dx, x, \frac{1}{x}\right) \\
&= \frac{1}{3}\left(a+\frac{b}{x}\right)^{3/2} x^3 - \frac{1}{2}b\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x^3}\,dx, x, \frac{1}{x}\right) \\
&= \frac{1}{4}b\sqrt{a+\frac{b}{x}}\,x^2 + \frac{1}{3}\left(a+\frac{b}{x}\right)^{3/2} x^3 - \frac{1}{8}b^2\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right) \\
&= \frac{b^2\sqrt{a+\frac{b}{x}}\,x}{8a} + \frac{1}{4}b\sqrt{a+\frac{b}{x}}\,x^2 + \frac{1}{3}\left(a+\frac{b}{x}\right)^{3/2} x^3 + \frac{b^3\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{16a} \\
&= \frac{b^2\sqrt{a+\frac{b}{x}}\,x}{8a} + \frac{1}{4}b\sqrt{a+\frac{b}{x}}\,x^2 + \frac{1}{3}\left(a+\frac{b}{x}\right)^{3/2} x^3 + \frac{b^2\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{8a} \\
&= \frac{b^2\sqrt{a+\frac{b}{x}}\,x}{8a} + \frac{1}{4}b\sqrt{a+\frac{b}{x}}\,x^2 + \frac{1}{3}\left(a+\frac{b}{x}\right)^{3/2} x^3 - \frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{8a^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.43

$$-\frac{2b^3\left(a+\frac{b}{x}\right)^{5/2}\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};\frac{b}{ax}+1\right)}{5a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)*x^2,x]
```

```
[Out] (-2*b^3*(a + b/x)^(5/2)*Hypergeometric2F1[5/2, 4, 7/2, 1 + b/(a*x)])/(5*a^4
)
```

fricas [A] time = 1.17, size = 151, normalized size = 1.68

$$\left[\frac{3\sqrt{a}\,b^3\log\left(2\,ax-2\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(8\,a^3x^3+14\,a^2bx^2+3\,ab^2x\right)\sqrt{\frac{ax+b}{x}}}{48\,a^2}, \frac{3\sqrt{-a}\,b^3\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)+\left(8\right.}{24}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*x^2,x, algorithm="fricas")
```

```
[Out] [1/48*(3*sqrt(a)*b^3*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(8*
a^3*x^3 + 14*a^2*b*x^2 + 3*a*b^2*x)*sqrt((a*x + b)/x))/a^2, 1/24*(3*sqrt(-a
)*b^3*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (8*a^3*x^3 + 14*a^2*b*x^2 + 3*
a*b^2*x)*sqrt((a*x + b)/x))/a^2]
```

giac [A] time = 0.22, size = 93, normalized size = 1.03

$$\frac{b^3\log\left(\left|-2\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)\sqrt{a}-b\right|\right)\operatorname{sgn}(x)}{16\,a^{\frac{3}{2}}}-\frac{b^3\log\left(|b|\right)\operatorname{sgn}(x)}{16\,a^{\frac{3}{2}}}+\frac{1}{24}\sqrt{ax^2+bx}\left(2\left(4\,ax\operatorname{sgn}(x)+7\,b\operatorname{sgn}(x)\right)x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*x^2,x, algorithm="giac")
```

```
[Out] 1/16*b^3*log(abs(-2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) - b))*sgn(x)/a^
(3/2) - 1/16*b^3*log(abs(b))*sgn(x)/a^(3/2) + 1/24*sqrt(a*x^2 + b*x)*(2*(4*
a*x*sgn(x) + 7*b*sgn(x))*x + 3*b^2*sgn(x)/a)
```

maple [A] time = 0.01, size = 115, normalized size = 1.28

$$\frac{\sqrt{\frac{ax+b}{x}}\left(-3ab^3\ln\left(\frac{2ax+b+2\sqrt{ax^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)+12\sqrt{ax^2+bx}\,a^{\frac{5}{2}}bx+6\sqrt{ax^2+bx}\,a^{\frac{3}{2}}b^2+16\left(ax^2+bx\right)^{\frac{3}{2}}a^{\frac{5}{2}}\right)x}{48\sqrt{(ax+b)x}\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(3/2)*x^2,x)
```

```
[Out] 1/48*((a*x+b)/x)^(1/2)*x/a^(5/2)*(16*(a*x^2+b*x)^(3/2)*a^(5/2)+12*(a*x^2+b*
x)^(1/2)*a^(5/2)*b*x+6*(a*x^2+b*x)^(1/2)*a^(3/2)*b^2-3*a*b^3*ln(1/2*(2*a*x+
b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2)))/((a*x+b)*x)^(1/2)
```

maxima [A] time = 2.34, size = 135, normalized size = 1.50

$$\frac{b^3\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{16\,a^{\frac{3}{2}}}+\frac{3\left(a+\frac{b}{x}\right)^{\frac{5}{2}}b^3+8\left(a+\frac{b}{x}\right)^{\frac{3}{2}}ab^3-3\sqrt{a+\frac{b}{x}}\,a^2b^3}{24\left(\left(a+\frac{b}{x}\right)^3a-3\left(a+\frac{b}{x}\right)^2a^2+3\left(a+\frac{b}{x}\right)a^3-a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*x^2,x, algorithm="maxima")
```

```
[Out] 1/16*b^3*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2) +
 1/24*(3*(a + b/x)^(5/2)*b^3 + 8*(a + b/x)^(3/2)*a*b^3 - 3*sqrt(a + b/x)*a^
2*b^3)/((a + b/x)^3*a - 3*(a + b/x)^2*a^2 + 3*(a + b/x)*a^3 - a^4)
```

mupad [B] time = 1.36, size = 72, normalized size = 0.80

$$\frac{x^3\left(a+\frac{b}{x}\right)^{3/2}}{3}-\frac{a\,x^3\sqrt{a+\frac{b}{x}}}{8}+\frac{x^3\left(a+\frac{b}{x}\right)^{5/2}}{8\,a}+\frac{b^3\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1\mathrm{i}}{\sqrt{a}}\right)1\mathrm{i}}{8\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b/x)^(3/2),x)
```

```
[Out] (x^3*(a + b/x)^(3/2))/3 + (b^3*atan(((a + b/x)^(1/2)*1i)/a^(1/2))*1i)/(8*a^
(3/2)) - (a*x^3*(a + b/x)^(1/2))/8 + (x^3*(a + b/x)^(5/2))/(8*a)
```

sympy [A] time = 9.20, size = 124, normalized size = 1.38

$$\frac{a^2x^{\frac{7}{2}}}{3\sqrt{b}\sqrt{\frac{ax}{b}+1}}+\frac{11a\sqrt{b}\,x^{\frac{5}{2}}}{12\sqrt{\frac{ax}{b}+1}}+\frac{17b^{\frac{3}{2}}x^{\frac{3}{2}}}{24\sqrt{\frac{ax}{b}+1}}+\frac{b^{\frac{5}{2}}\sqrt{x}}{8a\sqrt{\frac{ax}{b}+1}}-\frac{b^3\,\mathrm{asinh}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{8a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)*x**2,x)
```

```
[Out] a**2*x**(7/2)/(3*sqrt(b)*sqrt(a*x/b + 1)) + 11*a*sqrt(b)*x**(5/2)/(12*sqrt(
a*x/b + 1)) + 17*b**(3/2)*x**(3/2)/(24*sqrt(a*x/b + 1)) + b**(5/2)*sqrt(x)/
(8*a*sqrt(a*x/b + 1)) - b**3*asinh(sqrt(a)*sqrt(x)/sqrt(b))/(8*a**(3/2))
```

3.1703 $\int \left(a+\frac{b}{x}\right)^{3/2} x\,dx$

Optimal. Leaf size=66

$$\frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4\sqrt{a}} + \frac{1}{2}x^2\left(a+\frac{b}{x}\right)^{3/2} + \frac{3}{4}bx\sqrt{a+\frac{b}{x}}$$

```
[Out] 1/2*(a+b/x)^(3/2)*x^2+3/4*b^2*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(1/2)+3/4*b*
x*(a+b/x)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {266, 47, 63, 208}

$$\frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4\sqrt{a}} + \frac{1}{2}x^2\left(a+\frac{b}{x}\right)^{3/2} + \frac{3}{4}bx\sqrt{a+\frac{b}{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)*x,x]
```

```
[Out] (3*b*Sqrt[a + b/x]*x)/4 + ((a + b/x)^(3/2)*x^2)/2 + (3*b^2*ArcTanh[Sqrt[a +
 b/x]/Sqrt[a]])/(4*Sqrt[a])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^{3/2} x\,dx &= -\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^3}\,dx, x, \frac{1}{x}\right)\\ &= \frac{1}{2}\left(a+\frac{b}{x}\right)^{3/2}x^2 - \frac{1}{4}(3b)\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx, x, \frac{1}{x}\right)\\ &= \frac{3}{4}b\sqrt{a+\frac{b}{x}}\,x + \frac{1}{2}\left(a+\frac{b}{x}\right)^{3/2}x^2 - \frac{1}{8}\left(3b^2\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)\\ &= \frac{3}{4}b\sqrt{a+\frac{b}{x}}\,x + \frac{1}{2}\left(a+\frac{b}{x}\right)^{3/2}x^2 - \frac{1}{4}(3b)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)\\ &= \frac{3}{4}b\sqrt{a+\frac{b}{x}}\,x + \frac{1}{2}\left(a+\frac{b}{x}\right)^{3/2}x^2 + \frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 76, normalized size = 1.15

$$\frac{x\sqrt{a+\frac{b}{x}}\left(2a^2x^2+3b^2\sqrt{\frac{b}{ax}+1}\,\tanh^{-1}\left(\sqrt{\frac{b}{ax}+1}\right)+7abx+5b^2\right)}{4(ax+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)*x,x]
```

```
[Out] (Sqrt[a + b/x]*x*(5*b^2 + 7*a*b*x + 2*a^2*x^2 + 3*b^2*Sqrt[1 + b/(a*x)]*Arc
Tanh[Sqrt[1 + b/(a*x)]]))/(4*(b + a*x))
```

fricas [A] time = 1.04, size = 130, normalized size = 1.97

$$\left[\frac{3\sqrt{a}\,b^2\log\left(2\,ax+2\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(2\,a^2x^2+5\,abx\right)\sqrt{\frac{ax+b}{x}}}{8\,a}, -\frac{3\sqrt{-a}\,b^2\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-\left(2\,a^2x^2+\right.}{4\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*x,x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(a)*b^2*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(2*a
^2*x^2 + 5*a*b*x)*sqrt((a*x + b)/x))/a, -1/4*(3*sqrt(-a)*b^2*arctan(sqrt(-a
)*sqrt((a*x + b)/x)/a) - (2*a^2*x^2 + 5*a*b*x)*sqrt((a*x + b)/x))/a]
```

giac [A] time = 0.22, size = 79, normalized size = 1.20

$$-\frac{3\,b^2\log\left(\left|-2\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)\sqrt{a}-b\right|\right)\operatorname{sgn}(x)}{8\sqrt{a}}+\frac{3\,b^2\log\left(|b|\right)\operatorname{sgn}(x)}{8\sqrt{a}}+\frac{1}{4}\sqrt{ax^2+bx}\left(2\,ax\operatorname{sgn}(x)+5\,b\operatorname{sgn}(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*x,x, algorithm="giac")
```

```
[Out] -3/8*b^2*log(abs(-2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) - b))*sgn(x)/sq
rt(a) + 3/8*b^2*log(abs(b))*sgn(x)/sqrt(a) + 1/4*sqrt(a*x^2 + b*x)*(2*a*x*s
gn(x) + 5*b*sgn(x))
```

maple [A] time = 0.01, size = 96, normalized size = 1.45

$$\frac{\sqrt{\frac{ax+b}{x}}\left(3a\,b^2\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)+4\sqrt{a\,x^2+bx}\,a^{\frac{5}{2}}x+10\sqrt{a\,x^2+bx}\,a^{\frac{3}{2}}b\right)x}{8\sqrt{(ax+b)\,x}\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(3/2)*x,x)
```

```
[Out] 1/8*((a*x+b)/x)^(1/2)*x*(4*(a*x^2+b*x)^(1/2)*a^(5/2)*x+10*(a*x^2+b*x)^(1/2)
*a^(3/2)*b+3*a*b^2*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2)))/(
(a*x+b)*x)^(1/2)/a^(3/2)
```

maxima [A] time = 2.23, size = 98, normalized size = 1.48

$$-\frac{3\,b^2\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{8\sqrt{a}}+\frac{5\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^2-3\sqrt{a+\frac{b}{x}}\,ab^2}{4\left(\left(a+\frac{b}{x}\right)^2-2\left(a+\frac{b}{x}\right)a+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*x,x, algorithm="maxima")
```

```
[Out] -3/8*b^2*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/sqrt(a) +
 1/4*(5*(a + b/x)^(3/2)*b^2 - 3*sqrt(a + b/x)*a*b^2)/((a + b/x)^2 - 2*(a +
b/x)*a + a^2)
```

mupad [B] time = 1.17, size = 52, normalized size = 0.79

$$\frac{5\,x^2\left(a+\frac{b}{x}\right)^{3/2}}{4}+\frac{3\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4\sqrt{a}}-\frac{3\,a\,x^2\sqrt{a+\frac{b}{x}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b/x)^(3/2),x)
```

```
[Out] (5*x^2*(a + b/x)^(3/2))/4 + (3*b^2*atanh((a + b/x)^(1/2)/a^(1/2)))/(4*a^(1/
2)) - (3*a*x^2*(a + b/x)^(1/2))/4
```

sympy [A] time = 4.11, size = 75, normalized size = 1.14

$$\frac{a\sqrt{b}\,x^{\frac{3}{2}}\sqrt{\frac{ax}{b}+1}}{2}+\frac{5b^{\frac{3}{2}}\sqrt{x}\sqrt{\frac{ax}{b}+1}}{4}+\frac{3b^2\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)*x,x)
```

```
[Out] a*sqrt(b)*x**(3/2)*sqrt(a*x/b + 1)/2 + 5*b**(3/2)*sqrt(x)*sqrt(a*x/b + 1)/4
 + 3*b**2*asinh(sqrt(a)*sqrt(x)/sqrt(b))/(4*sqrt(a))
```

3.1704 $\int \left(a+\frac{b}{x}\right)^{3/2} dx$

Optimal. Leaf size=54

$$x\left(a+\frac{b}{x}\right)^{3/2}-3b\sqrt{a+\frac{b}{x}}+3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

```
[Out] (a+b/x)^(3/2)*x+3*b*arctanh((a+b/x)^(1/2)/a^(1/2))*a^(1/2)-3*b*(a+b/x)^(1/2
)
```

Rubi [A] time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {242, 47, 50, 63, 208}

$$x\left(a+\frac{b}{x}\right)^{3/2}-3b\sqrt{a+\frac{b}{x}}+3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2),x]
```

```
[Out] -3*b*Sqrt[a + b/x] + (a + b/x)^(3/2)*x + 3*Sqrt[a]*b*ArcTanh[Sqrt[a + b/x]/
Sqrt[a]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^{3/2} dx &= -\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2}\, dx, x, \frac{1}{x}\right)\\ &= \left(a+\frac{b}{x}\right)^{3/2} x - \frac{1}{2}(3b)\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\, dx, x, \frac{1}{x}\right)\\ &= -3b\sqrt{a+\frac{b}{x}} + \left(a+\frac{b}{x}\right)^{3/2} x - \frac{1}{2}(3ab)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\, dx, x, \frac{1}{x}\right)\\ &= -3b\sqrt{a+\frac{b}{x}} + \left(a+\frac{b}{x}\right)^{3/2} x - (3a)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\, dx, x, \sqrt{a+\frac{b}{x}}\right)\\ &= -3b\sqrt{a+\frac{b}{x}} + \left(a+\frac{b}{x}\right)^{3/2} x + 3\sqrt{a}\, b \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 0.85

$$\sqrt{a+\frac{b}{x}}\,(ax-2b) + 3\sqrt{a}\, b \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2),x]
```

```
[Out] Sqrt[a + b/x]*(-2*b + a*x) + 3*Sqrt[a]*b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

fricas [A] time = 1.07, size = 100, normalized size = 1.85

$$\left[\frac{3}{2}\sqrt{a}\, b \log\left(2\,ax + 2\,\sqrt{a}\, x\sqrt{\frac{ax+b}{x}} + b\right) + (ax-2\,b)\sqrt{\frac{ax+b}{x}}\,, -3\,\sqrt{-a}\, b \arctan\left(\frac{\sqrt{-a}\,\sqrt{\frac{ax+b}{x}}}{a}\right) + (ax-2\,b)\sqrt{\frac{ax}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [3/2*sqrt(a)*b*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (a*x - 2*b)
*sqrt((a*x + b)/x), -3*sqrt(-a)*b*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a
*x - 2*b)*sqrt((a*x + b)/x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
```

```
Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%%}+%%%{-2
,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%%%}+%%%{2
,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters values [86
,-97,-82]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%
%},0,%%%{1,[0,2,2]%%%}] at parameters values [82.1195442914,26,-89]Warning,
 choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[0,2,2
]%%%}] at parameters values [85.3561567818,-64,-30]Warning, choosing root o
f [1,0,%%%{-2,[1,0,1]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,2]%%%}] at parame
ters values [42,43.9628838282,-9]Sign error (%%%{-b,0%%%}+%%%{2*sqrt(a)*sqr
t(b),1/2%%%}+%%%{-2*a,1%%%}+%%%{a*sqrt(a)*sqrt(b)/b,3/2%%%}+%%%{-a^2*sqrt(a
)*sqrt(b)/(4*b^2),5/2%%%}+%%%{undef,7/2%%%})Limit: Max order reached or una
ble to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.01, size = 100, normalized size = 1.85

$$-\frac{\sqrt{\frac{ax+b}{x}}\left(-3ab\,x^2\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)-6\sqrt{a\,x^2+bx}\;a^{\frac{3}{2}}x^2+4\left(a\,x^2+bx\right)^{\frac{3}{2}}\sqrt{a}\right)}{2\sqrt{(ax+b)\,x}\;\sqrt{a}\;x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(3/2),x)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)/x*(-6*(a*x^2+b*x)^(1/2)*a^(3/2)*x^2-3*a*b*x^2*ln(1/2
*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))+4*(a*x^2+b*x)^(3/2)*a^(1/2)
)/((a*x+b)*x)^(1/2)/a^(1/2)
```

maxima [A] time = 2.36, size = 63, normalized size = 1.17

$$\sqrt{a+\frac{b}{x}}\,ax-\frac{3}{2}\sqrt{a}\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-2\sqrt{a+\frac{b}{x}}\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2),x, algorithm="maxima")
```

```
[Out] sqrt(a + b/x)*a*x - 3/2*sqrt(a)*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b
/x) + sqrt(a))) - 2*sqrt(a + b/x)*b
```

mupad [B] time = 1.25, size = 34, normalized size = 0.63

$$-\frac{2\,x\left(a+\frac{b}{x}\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{a\,x}{b}\right)}{\left(\frac{a\,x}{b}+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(3/2),x)
```

```
[Out] -(2*x*(a + b/x)^(3/2)*hypergeom([-3/2, -1/2], 1/2, -(a*x)/b))/((a*x)/b + 1)
^(3/2)
```

sympy [B] time = 3.75, size = 92, normalized size = 1.70

$$3\sqrt{a}\,b\,\operatorname{asinh}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)+\frac{a^2x^{\frac{3}{2}}}{\sqrt{b}\,\sqrt{\frac{ax}{b}+1}}-\frac{a\sqrt{b}\,\sqrt{x}}{\sqrt{\frac{ax}{b}+1}}-\frac{2b^{\frac{3}{2}}}{\sqrt{x}\,\sqrt{\frac{ax}{b}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2),x)
```

```
[Out] 3*sqrt(a)*b*asinh(sqrt(a)*sqrt(x)/sqrt(b)) + a**2*x**(3/2)/(sqrt(b)*sqrt(a*
x/b + 1)) - a*sqrt(b)*sqrt(x)/sqrt(a*x/b + 1) - 2*b**(3/2)/(sqrt(x)*sqrt(a*
x/b + 1))
```

3.1705 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=55

$$2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-2a\sqrt{a+\frac{b}{x}}-\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}$$

```
[Out] -2/3*(a+b/x)^(3/2)+2*a^(3/2)*arctanh((a+b/x)^(1/2)/a^(1/2))-2*a*(a+b/x)^(1/
2)
```

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 50, 63, 208}

$$2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-2a\sqrt{a+\frac{b}{x}}-\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)/x,x]
```

```
[Out] -2*a*Sqrt[a + b/x] - (2*(a + b/x)^(3/2))/3 + 2*a^(3/2)*ArcTanh[Sqrt[a + b/x
]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x}\,dx &= -\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, \frac{1}{x}\right) \\
&= -\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2} - a\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x}\right) \\
&= -2a\sqrt{a+\frac{b}{x}} - \frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2} - a^2\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right) \\
&= -2a\sqrt{a+\frac{b}{x}} - \frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2} - \frac{\left(2a^2\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{b} \\
&= -2a\sqrt{a+\frac{b}{x}} - \frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2} + 2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [A] time = 0.06, size = 50, normalized size = 0.91

$$2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right) - \frac{2\sqrt{a+\frac{b}{x}}\,(4ax+b)}{3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)/x,x]
```

```
[Out] (-2*Sqrt[a + b/x]*(b + 4*a*x))/(3*x) + 2*a^(3/2)*ArcTanh[Sqrt[a + b/x]/Sqrt
[a]]
```

fricas [A] time = 0.75, size = 110, normalized size = 2.00

$$\left[\frac{3\,a^{\frac{3}{2}}x\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)-2\,(4\,ax+b)\sqrt{\frac{ax+b}{x}}}{3\,x}, -\frac{2\left(3\,\sqrt{-a}\,ax\arctan\left(\frac{\sqrt{-a}\,\sqrt{\frac{ax+b}{x}}}{a}\right)+(4\,ax+b)\sqrt{\frac{ax+b}{x}}\right)}{3\,x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/3*(3*a^(3/2)*x*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(4*a*x
 + b)*sqrt((a*x + b)/x))/x, -2/3*(3*sqrt(-a)*a*x*arctan(sqrt(-a)*sqrt((a*x
+ b)/x)/a) + (4*a*x + b)*sqrt((a*x + b)/x))/x]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%%}+%%%{-2
```

```
,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%%%}+%%%{2
,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters values [86
,-97,-82]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%
%},0,%%%{1,[0,2,2]%%%}] at parameters values [82.1195442914,26,-89]Warning,
 choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[0,2,2
]%%%}] at parameters values [85.3561567818,-64,-30]Warning, choosing root o
f [1,0,%%%{-2,[1,0,1]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,2]%%%}] at parame
ters values [42,43.9628838282,-9]Sign error (%%%{-b,0%%%}+%%%{2*sqrt(a)*sqr
t(b),1/2%%%}+%%%{-2*a,1%%%}+%%%{a*sqrt(a)*sqrt(b)/b,3/2%%%}+%%%{-a^2*sqrt(a
)*sqrt(b)/(4*b^2),5/2%%%}+%%%{undef,7/2%%%})Limit: Max order reached or una
ble to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.01, size = 123, normalized size = 2.24

$$-\frac{\sqrt{\frac{ax+b}{x}}\left(-3a^2bx^3\ln\left(\frac{2ax+b+2\sqrt{ax^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)-6\sqrt{ax^2+bx}\,a^{\frac{5}{2}}x^3+6\left(ax^2+bx\right)^{\frac{3}{2}}a^{\frac{3}{2}}x+2\left(ax^2+bx\right)^{\frac{3}{2}}\sqrt{a}\,b\right)}{3\sqrt{(ax+b)x}\,\sqrt{a}\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(3/2)/x,x)
```

```
[Out] -1/3*((a*x+b)/x)^(1/2)/x^2*(-6*(a*x^2+b*x)^(1/2)*a^(5/2)*x^3-3*ln(1/2*(2*a*
x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x^3*a^2*b+6*(a*x^2+b*x)^(3/2)*a^(
3/2)*x+2*(a*x^2+b*x)^(3/2)*b*a^(1/2))/((a*x+b)*x)^(1/2)/b/a^(1/2)
```

maxima [A] time = 2.31, size = 61, normalized size = 1.11

$$-a^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-\frac{2}{3}\left(a+\frac{b}{x}\right)^{\frac{3}{2}}-2\sqrt{a+\frac{b}{x}}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] -a^(3/2)*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) - 2/3*(a
+ b/x)^(3/2) - 2*sqrt(a + b/x)*a
```

mupad [B] time = 1.51, size = 43, normalized size = 0.78

$$2\,a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-2\,a\,\sqrt{a+\frac{b}{x}}-\frac{2\left(a+\frac{b}{x}\right)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(3/2)/x,x)
```

```
[Out] 2*a^(3/2)*atanh((a + b/x)^(1/2)/a^(1/2)) - 2*a*(a + b/x)^(1/2) - (2*(a + b/
x)^(3/2))/3
```

sympy [A] time = 2.31, size = 71, normalized size = 1.29

$$-\frac{8a^{\frac{3}{2}}\sqrt{1+\frac{b}{ax}}}{3}-a^{\frac{3}{2}}\log\left(\frac{b}{ax}\right)+2a^{\frac{3}{2}}\log\left(\sqrt{1+\frac{b}{ax}}+1\right)-\frac{2\sqrt{a}\,b\sqrt{1+\frac{b}{ax}}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)/x,x)
```

```
[Out] -8*a**(3/2)*sqrt(1 + b/(a*x))/3 - a**(3/2)*log(b/(a*x)) + 2*a**(3/2)*log(sq
rt(1 + b/(a*x)) + 1) - 2*sqrt(a)*b*sqrt(1 + b/(a*x))/(3*x)
```

3.1706 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=18

$$-\frac{2\left(a+\frac{b}{x}\right)^{5/2}}{5b}$$

```
[Out] -2/5*(a+b/x)^(5/2)/b
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$-\frac{2\left(a+\frac{b}{x}\right)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)/x^2,x]
```

```
[Out] (-2*(a + b/x)^(5/2))/(5*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^2}\,dx = -\frac{2\left(a+\frac{b}{x}\right)^{5/2}}{5b}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$-\frac{2\left(a+\frac{b}{x}\right)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)/x^2,x]
```

```
[Out] (-2*(a + b/x)^(5/2))/(5*b)
```

fricas [B] time = 1.03, size = 35, normalized size = 1.94

$$-\frac{2\left(a^2x^2+2\,abx+b^2\right)\sqrt{\frac{ax+b}{x}}}{5\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] -2/5*(a^2*x^2 + 2*a*b*x + b^2)*sqrt((a*x + b)/x)/(b*x^2)
```

giac [B] time = 0.25, size = 145, normalized size = 8.06

$$\frac{2\left(5\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)^4 a^2\mathrm{sgn}(x)+10\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)^3 a^{\frac{3}{2}}b\mathrm{sgn}(x)+10\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)^2 ab^2\mathrm{sgn}(x)\right.}{5\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 2/5*(5*(sqrt(a)*x - sqrt(a*x^2 + b*x))^4*a^2*sgn(x) + 10*(sqrt(a)*x - sqrt(
a*x^2 + b*x))^3*a^(3/2)*b*sgn(x) + 10*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a*b
^2*sgn(x) + 5*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*b^3*sgn(x) + b^4*sgn(
x))/(sqrt(a)*x - sqrt(a*x^2 + b*x))^5
```

maple [A] time = 0.00, size = 25, normalized size = 1.39

$$-\frac{2\,(ax+b)\left(\frac{ax+b}{x}\right)^{\frac{3}{2}}}{5bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(3/2)/x^2,x)
```

```
[Out] -2/5*(a*x+b)/x/b*((a*x+b)/x)^(3/2)
```

maxima [A] time = 0.97, size = 14, normalized size = 0.78

$$-\frac{2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] -2/5*(a + b/x)^(5/2)/b
```

mupad [B] time = 1.59, size = 24, normalized size = 1.33

$$-\frac{2\sqrt{a+\frac{b}{x}}\,(b+a\,x)^2}{5\,b\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(3/2)/x^2,x)
```

```
[Out] -(2*(a + b/x)^(1/2)*(b + a*x)^2)/(5*b*x^2)
```

sympy [B] time = 1.36, size = 65, normalized size = 3.61

$$-\frac{2a^{\frac{5}{2}}\sqrt{1+\frac{b}{ax}}}{5b}-\frac{4a^{\frac{3}{2}}\sqrt{1+\frac{b}{ax}}}{5x}-\frac{2\sqrt{a}\,b\sqrt{1+\frac{b}{ax}}}{5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)/x**2,x)
```

```
[Out] -2*a**(5/2)*sqrt(1 + b/(a*x))/(5*b) - 4*a**(3/2)*sqrt(1 + b/(a*x))/(5*x) -
2*sqrt(a)*b*sqrt(1 + b/(a*x))/(5*x**2)
```

3.1707 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=38

$$\frac{2a\left(a+\frac{b}{x}\right)^{5/2}}{5b^2}-\frac{2\left(a+\frac{b}{x}\right)^{7/2}}{7b^2}$$

```
[Out] 2/5*a*(a+b/x)^(5/2)/b^2-2/7*(a+b/x)^(7/2)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{2a\left(a+\frac{b}{x}\right)^{5/2}}{5b^2}-\frac{2\left(a+\frac{b}{x}\right)^{7/2}}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)/x^3,x]
```

```
[Out] (2*a*(a + b/x)^(5/2))/(5*b^2) - (2*(a + b/x)^(7/2))/(7*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^3}\,dx &= -\text{Subst}\left(\int x(a+bx)^{3/2}\,dx, x, \frac{1}{x}\right)\\ &= -\text{Subst}\left(\int\left(-\frac{a(a+bx)^{3/2}}{b}+\frac{(a+bx)^{5/2}}{b}\right)dx, x, \frac{1}{x}\right)\\ &= \frac{2a\left(a+\frac{b}{x}\right)^{5/2}}{5b^2}-\frac{2\left(a+\frac{b}{x}\right)^{7/2}}{7b^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 38, normalized size = 1.00

$$\frac{2a\left(a+\frac{b}{x}\right)^{5/2}}{5b^2}-\frac{2\left(a+\frac{b}{x}\right)^{7/2}}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)/x^3,x]
```

```
[Out] (2*a*(a + b/x)^(5/2))/(5*b^2) - (2*(a + b/x)^(7/2))/(7*b^2)
```

fricas [A] time = 0.70, size = 49, normalized size = 1.29

$$\frac{2\left(2\,a^3x^3 - a^2bx^2 - 8\,ab^2x - 5\,b^3\right)\sqrt{\frac{ax+b}{x}}}{35\,b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 2/35*(2*a^3*x^3 - a^2*b*x^2 - 8*a*b^2*x - 5*b^3)*sqrt((a*x + b)/x)/(b^2*x^3
)
```

giac [B] time = 0.25, size = 177, normalized size = 4.66

$$\frac{2\left(35\left(\sqrt{a}\,x - \sqrt{ax^2+bx}\right)^5 a^{\frac{5}{2}}\mathrm{sgn}(x) + 105\left(\sqrt{a}\,x - \sqrt{ax^2+bx}\right)^4 a^2b\,\mathrm{sgn}(x) + 140\left(\sqrt{a}\,x - \sqrt{ax^2+bx}\right)^3 a^{\frac{3}{2}}b^2\mathrm{sg}\right.}{35\left(\sqrt{a}\,x - \sqrt{ax^2+}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 2/35*(35*(sqrt(a)*x - sqrt(a*x^2 + b*x))^5*a^(5/2)*sgn(x) + 105*(sqrt(a)*x
 - sqrt(a*x^2 + b*x))^4*a^2*b*sgn(x) + 140*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3
*a^(3/2)*b^2*sgn(x) + 98*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a*b^3*sgn(x) + 3
5*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*b^4*sgn(x) + 5*b^5*sgn(x))/(sqrt(
a)*x - sqrt(a*x^2 + b*x))^7
```

maple [A] time = 0.00, size = 33, normalized size = 0.87

$$\frac{2\,(ax+b)\,(2ax-5b)\left(\frac{ax+b}{x}\right)^{\frac{3}{2}}}{35b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(3/2)/x^3,x)
```

```
[Out] 2/35*(a*x+b)*(2*a*x-5*b)*((a*x+b)/x)^(3/2)/b^2/x^2
```

maxima [A] time = 1.05, size = 30, normalized size = 0.79

$$-\frac{2\left(a+\frac{b}{x}\right)^{\frac{7}{2}}}{7\,b^2} + \frac{2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a}{5\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] -2/7*(a + b/x)^(7/2)/b^2 + 2/5*(a + b/x)^(5/2)*a/b^2
```

mupad [B] time = 1.73, size = 68, normalized size = 1.79

$$\frac{4\,a^3\sqrt{a+\frac{b}{x}}}{35\,b^2} - \frac{2\,b\sqrt{a+\frac{b}{x}}}{7\,x^3} - \frac{16\,a\sqrt{a+\frac{b}{x}}}{35\,x^2} - \frac{2\,a^2\sqrt{a+\frac{b}{x}}}{35\,b\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(3/2)/x^3,x)
```

```
[Out] (4*a^3*(a + b/x)^(1/2))/(35*b^2) - (2*b*(a + b/x)^(1/2))/(7*x^3) - (16*a*(a
 + b/x)^(1/2))/(35*x^2) - (2*a^2*(a + b/x)^(1/2))/(35*b*x)
```

sympy [B] time = 1.47, size = 360, normalized size = 9.47

$$\frac{4a^{\frac{15}{2}}b^{\frac{3}{2}}x^4\sqrt{\frac{ax}{b}+1}}{35a^{\frac{9}{2}}b^3x^{\frac{9}{2}}+35a^{\frac{7}{2}}b^4x^{\frac{7}{2}}}+\frac{2a^{\frac{13}{2}}b^{\frac{5}{2}}x^3\sqrt{\frac{ax}{b}+1}}{35a^{\frac{9}{2}}b^3x^{\frac{9}{2}}+35a^{\frac{7}{2}}b^4x^{\frac{7}{2}}}-\frac{18a^{\frac{11}{2}}b^{\frac{7}{2}}x^2\sqrt{\frac{ax}{b}+1}}{35a^{\frac{9}{2}}b^3x^{\frac{9}{2}}+35a^{\frac{7}{2}}b^4x^{\frac{7}{2}}}-\frac{26a^{\frac{9}{2}}b^{\frac{9}{2}}x\sqrt{\frac{ax}{b}+1}}{35a^{\frac{9}{2}}b^3x^{\frac{9}{2}}+35a^{\frac{7}{2}}b^4x^{\frac{7}{2}}}-\frac{10a^{\frac{7}{2}}b^{\frac{11}{2}}\sqrt{\frac{ax}{b}}}{35a^{\frac{9}{2}}b^3x^{\frac{9}{2}}+35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)/x**3,x)
```

```
[Out] 4*a**(15/2)*b**(3/2)*x**4*sqrt(a*x/b + 1)/(35*a**(9/2)*b**3*x**(9/2) + 35*a
**(7/2)*b**4*x**(7/2)) + 2*a**(13/2)*b**(5/2)*x**3*sqrt(a*x/b + 1)/(35*a**(
9/2)*b**3*x**(9/2) + 35*a**(7/2)*b**4*x**(7/2)) - 18*a**(11/2)*b**(7/2)*x**
2*sqrt(a*x/b + 1)/(35*a**(9/2)*b**3*x**(9/2) + 35*a**(7/2)*b**4*x**(7/2)) -
 26*a**(9/2)*b**(9/2)*x*sqrt(a*x/b + 1)/(35*a**(9/2)*b**3*x**(9/2) + 35*a**
(7/2)*b**4*x**(7/2)) - 10*a**(7/2)*b**(11/2)*sqrt(a*x/b + 1)/(35*a**(9/2)*b
**3*x**(9/2) + 35*a**(7/2)*b**4*x**(7/2)) - 4*a**8*b*x**(9/2)/(35*a**(9/2)*
b**3*x**(9/2) + 35*a**(7/2)*b**4*x**(7/2)) - 4*a**7*b**2*x**(7/2)/(35*a**(9
/2)*b**3*x**(9/2) + 35*a**(7/2)*b**4*x**(7/2))
```

3.1708 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=59

$$-\frac{2a^2\left(a+\frac{b}{x}\right)^{5/2}}{5b^3}-\frac{2\left(a+\frac{b}{x}\right)^{9/2}}{9b^3}+\frac{4a\left(a+\frac{b}{x}\right)^{7/2}}{7b^3}$$

```
[Out] -2/5*a^2*(a+b/x)^(5/2)/b^3+4/7*a*(a+b/x)^(7/2)/b^3-2/9*(a+b/x)^(9/2)/b^3
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{2a^2\left(a+\frac{b}{x}\right)^{5/2}}{5b^3}-\frac{2\left(a+\frac{b}{x}\right)^{9/2}}{9b^3}+\frac{4a\left(a+\frac{b}{x}\right)^{7/2}}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)/x^4,x]
```

```
[Out] (-2*a^2*(a + b/x)^(5/2))/(5*b^3) + (4*a*(a + b/x)^(7/2))/(7*b^3) - (2*(a +
b/x)^(9/2))/(9*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^4}\,dx &= -\text{Subst}\left(\int x^2(a+bx)^{3/2}\,dx, x, \frac{1}{x}\right)\\ &= -\text{Subst}\left(\int\left(\frac{a^2(a+bx)^{3/2}}{b^2}-\frac{2a(a+bx)^{5/2}}{b^2}+\frac{(a+bx)^{7/2}}{b^2}\right)dx, x, \frac{1}{x}\right)\\ &= -\frac{2a^2\left(a+\frac{b}{x}\right)^{5/2}}{5b^3}+\frac{4a\left(a+\frac{b}{x}\right)^{7/2}}{7b^3}-\frac{2\left(a+\frac{b}{x}\right)^{9/2}}{9b^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 47, normalized size = 0.80

$$-\frac{2\sqrt{a+\frac{b}{x}}\,(ax+b)^2\left(8a^2x^2-20abx+35b^2\right)}{315b^3x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)/x^4,x]
```

```
[Out] (-2*Sqrt[a + b/x]*(b + a*x)^2*(35*b^2 - 20*a*b*x + 8*a^2*x^2))/(315*b^3*x^4
)
```

fricas [A] time = 1.38, size = 60, normalized size = 1.02

$$-\frac{2\left(8\,a^4x^4-4\,a^3bx^3+3\,a^2b^2x^2+50\,ab^3x+35\,b^4\right)\sqrt{\frac{ax+b}{x}}}{315\,b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] -2/315*(8*a^4*x^4 - 4*a^3*b*x^3 + 3*a^2*b^2*x^2 + 50*a*b^3*x + 35*b^4)*sqrt
((a*x + b)/x)/(b^3*x^4)
```

giac [B] time = 0.32, size = 208, normalized size = 3.53

$$2\left(420\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)^6a^3\mathrm{sgn}(x)+1575\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)^5a^{\frac{5}{2}}b\mathrm{sgn}(x)+2583\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)^4a^2b^2\mathrm{s}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] 2/315*(420*(sqrt(a)*x - sqrt(a*x^2 + b*x))^6*a^3*sgn(x) + 1575*(sqrt(a)*x -
 sqrt(a*x^2 + b*x))^5*a^(5/2)*b*sgn(x) + 2583*(sqrt(a)*x - sqrt(a*x^2 + b*x
))^4*a^2*b^2*sgn(x) + 2310*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a^(3/2)*b^3*sg
n(x) + 1170*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a*b^4*sgn(x) + 315*(sqrt(a)*x
 - sqrt(a*x^2 + b*x))*sqrt(a)*b^5*sgn(x) + 35*b^6*sgn(x))/(sqrt(a)*x - sqrt
(a*x^2 + b*x))^9
```

maple [A] time = 0.01, size = 44, normalized size = 0.75

$$-\frac{2\,(ax+b)\left(8a^2x^2-20abx+35b^2\right)\left(\frac{ax+b}{x}\right)^{\frac{3}{2}}}{315b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(3/2)/x^4,x)
```

```
[Out] -2/315*(a*x+b)*(8*a^2*x^2-20*a*b*x+35*b^2)*((a*x+b)/x)^(3/2)/b^3/x^3
```

maxima [A] time = 0.99, size = 47, normalized size = 0.80

$$-\frac{2\left(a+\frac{b}{x}\right)^{\frac{9}{2}}}{9\,b^3}+\frac{4\left(a+\frac{b}{x}\right)^{\frac{7}{2}}a}{7\,b^3}-\frac{2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^2}{5\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] -2/9*(a + b/x)^(9/2)/b^3 + 4/7*(a + b/x)^(7/2)*a/b^3 - 2/5*(a + b/x)^(5/2)*
a^2/b^3
```

mupad [B] time = 1.89, size = 88, normalized size = 1.49

$$\frac{8\,a^3\sqrt{a+\frac{b}{x}}}{315\,b^2\,x}-\frac{2\,b\sqrt{a+\frac{b}{x}}}{9\,x^4}-\frac{16\,a^4\sqrt{a+\frac{b}{x}}}{315\,b^3}-\frac{2\,a^2\sqrt{a+\frac{b}{x}}}{105\,b\,x^2}-\frac{20\,a\sqrt{a+\frac{b}{x}}}{63\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(3/2)/x^4,x)
```

```
[Out] (8*a^3*(a + b/x)^(1/2))/(315*b^2*x) - (2*b*(a + b/x)^(1/2))/(9*x^4) - (16*a
^4*(a + b/x)^(1/2))/(315*b^3) - (2*a^2*(a + b/x)^(1/2))/(105*b*x^2) - (20*a
*(a + b/x)^(1/2))/(63*x^3)
```

sympy [B] time = 2.40, size = 986, normalized size = 16.71

$$-\frac{16a^{\frac{23}{2}}b^{\frac{9}{2}}x^7\sqrt{\frac{ax}{b}+1}}{315a^{\frac{15}{2}}b^7x^{\frac{15}{2}}+945a^{\frac{13}{2}}b^8x^{\frac{13}{2}}+945a^{\frac{11}{2}}b^9x^{\frac{11}{2}}+315a^{\frac{9}{2}}b^{10}x^{\frac{9}{2}}}-\frac{40a^{\frac{21}{2}}b^{\frac{11}{2}}x^6\sqrt{\frac{ax}{b}+1}}{315a^{\frac{15}{2}}b^7x^{\frac{15}{2}}+945a^{\frac{13}{2}}b^8x^{\frac{13}{2}}+945a^{\frac{11}{2}}b^9x^{\frac{11}{2}}+315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)/x**4,x)
```

```
[Out] -16*a**(23/2)*b**(9/2)*x**7*sqrt(a*x/b + 1)/(315*a**(15/2)*b**7*x**(15/2) +
 945*a**(13/2)*b**8*x**(13/2) + 945*a**(11/2)*b**9*x**(11/2) + 315*a**(9/2)
*b**10*x**(9/2)) - 40*a**(21/2)*b**(11/2)*x**6*sqrt(a*x/b + 1)/(315*a**(15/
2)*b**7*x**(15/2) + 945*a**(13/2)*b**8*x**(13/2) + 945*a**(11/2)*b**9*x**(1
1/2) + 315*a**(9/2)*b**10*x**(9/2)) - 30*a**(19/2)*b**(13/2)*x**5*sqrt(a*x/
b + 1)/(315*a**(15/2)*b**7*x**(15/2) + 945*a**(13/2)*b**8*x**(13/2) + 945*a
**(11/2)*b**9*x**(11/2) + 315*a**(9/2)*b**10*x**(9/2)) - 110*a**(17/2)*b**(
15/2)*x**4*sqrt(a*x/b + 1)/(315*a**(15/2)*b**7*x**(15/2) + 945*a**(13/2)*b*
*8*x**(13/2) + 945*a**(11/2)*b**9*x**(11/2) + 315*a**(9/2)*b**10*x**(9/2))
 - 380*a**(15/2)*b**(17/2)*x**3*sqrt(a*x/b + 1)/(315*a**(15/2)*b**7*x**(15/2
) + 945*a**(13/2)*b**8*x**(13/2) + 945*a**(11/2)*b**9*x**(11/2) + 315*a**(9
/2)*b**10*x**(9/2)) - 516*a**(13/2)*b**(19/2)*x**2*sqrt(a*x/b + 1)/(315*a**
(15/2)*b**7*x**(15/2) + 945*a**(13/2)*b**8*x**(13/2) + 945*a**(11/2)*b**9*x
**(11/2) + 315*a**(9/2)*b**10*x**(9/2)) - 310*a**(11/2)*b**(21/2)*x*sqrt(a*
x/b + 1)/(315*a**(15/2)*b**7*x**(15/2) + 945*a**(13/2)*b**8*x**(13/2) + 945
*a**(11/2)*b**9*x**(11/2) + 315*a**(9/2)*b**10*x**(9/2)) - 70*a**(9/2)*b**(
23/2)*sqrt(a*x/b + 1)/(315*a**(15/2)*b**7*x**(15/2) + 945*a**(13/2)*b**8*x*
*(13/2) + 945*a**(11/2)*b**9*x**(11/2) + 315*a**(9/2)*b**10*x**(9/2)) + 16*
a**12*b**4*x**(15/2)/(315*a**(15/2)*b**7*x**(15/2) + 945*a**(13/2)*b**8*x**
(13/2) + 945*a**(11/2)*b**9*x**(11/2) + 315*a**(9/2)*b**10*x**(9/2)) + 48*a
**11*b**5*x**(13/2)/(315*a**(15/2)*b**7*x**(15/2) + 945*a**(13/2)*b**8*x**(
13/2) + 945*a**(11/2)*b**9*x**(11/2) + 315*a**(9/2)*b**10*x**(9/2)) + 48*a*
*10*b**6*x**(11/2)/(315*a**(15/2)*b**7*x**(15/2) + 945*a**(13/2)*b**8*x**(1
3/2) + 945*a**(11/2)*b**9*x**(11/2) + 315*a**(9/2)*b**10*x**(9/2)) + 16*a**
9*b**7*x**(9/2)/(315*a**(15/2)*b**7*x**(15/2) + 945*a**(13/2)*b**8*x**(13/2
) + 945*a**(11/2)*b**9*x**(11/2) + 315*a**(9/2)*b**10*x**(9/2))
```

3.1709 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=80

$$\frac{2a^3\left(a+\frac{b}{x}\right)^{5/2}}{5b^4}-\frac{6a^2\left(a+\frac{b}{x}\right)^{7/2}}{7b^4}-\frac{2\left(a+\frac{b}{x}\right)^{11/2}}{11b^4}+\frac{2a\left(a+\frac{b}{x}\right)^{9/2}}{3b^4}$$

```
[Out] 2/5*a^3*(a+b/x)^(5/2)/b^4-6/7*a^2*(a+b/x)^(7/2)/b^4+2/3*a*(a+b/x)^(9/2)/b^4
-2/11*(a+b/x)^(11/2)/b^4
```

Rubi [A] time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{6a^2\left(a+\frac{b}{x}\right)^{7/2}}{7b^4}+\frac{2a^3\left(a+\frac{b}{x}\right)^{5/2}}{5b^4}-\frac{2\left(a+\frac{b}{x}\right)^{11/2}}{11b^4}+\frac{2a\left(a+\frac{b}{x}\right)^{9/2}}{3b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)/x^5,x]
```

```
[Out] (2*a^3*(a + b/x)^(5/2))/(5*b^4) - (6*a^2*(a + b/x)^(7/2))/(7*b^4) + (2*a*(a
 + b/x)^(9/2))/(3*b^4) - (2*(a + b/x)^(11/2))/(11*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^5}\,dx &= -\text{Subst}\left(\int x^3(a+bx)^{3/2}\,dx, x, \frac{1}{x}\right)\\ &= -\text{Subst}\left(\int\left(-\frac{a^3(a+bx)^{3/2}}{b^3}+\frac{3a^2(a+bx)^{5/2}}{b^3}-\frac{3a(a+bx)^{7/2}}{b^3}+\frac{(a+bx)^{9/2}}{b^3}\right)dx, x, \frac{1}{x}\right)\\ &= \frac{2a^3\left(a+\frac{b}{x}\right)^{5/2}}{5b^4}-\frac{6a^2\left(a+\frac{b}{x}\right)^{7/2}}{7b^4}+\frac{2a\left(a+\frac{b}{x}\right)^{9/2}}{3b^4}-\frac{2\left(a+\frac{b}{x}\right)^{11/2}}{11b^4}\end{aligned}$$

Mathematica [A] time = 0.03, size = 58, normalized size = 0.72

$$\frac{2\sqrt{a+\frac{b}{x}}\,(ax+b)^2\left(16a^3x^3-40a^2bx^2+70ab^2x-105b^3\right)}{1155b^4x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)/x^5,x]
```

```
[Out] (2*Sqrt[a + b/x]*(b + a*x)^2*(-105*b^3 + 70*a*b^2*x - 40*a^2*b*x^2 + 16*a^3
*x^3))/(1155*b^4*x^5)
```

fricas [A] time = 0.91, size = 71, normalized size = 0.89

$$\frac{2\left(16\,a^5x^5-8\,a^4bx^4+6\,a^3b^2x^3-5\,a^2b^3x^2-140\,ab^4x-105\,b^5\right)\sqrt{\frac{ax+b}{x}}}{1155\,b^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] 2/1155*(16*a^5*x^5 - 8*a^4*b*x^4 + 6*a^3*b^2*x^3 - 5*a^2*b^3*x^2 - 140*a*b^
4*x - 105*b^5)*sqrt((a*x + b)/x)/(b^4*x^5)
```

giac [B] time = 0.28, size = 239, normalized size = 2.99

$$2\left(2310\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)^7a^{\frac{7}{2}}\mathrm{sgn}(x)+10164\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)^6a^3b\mathrm{sgn}(x)+19635\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 2/1155*(2310*(sqrt(a)*x - sqrt(a*x^2 + b*x))^7*a^(7/2)*sgn(x) + 10164*(sqrt
(a)*x - sqrt(a*x^2 + b*x))^6*a^3*b*sgn(x) + 19635*(sqrt(a)*x - sqrt(a*x^2 +
 b*x))^5*a^(5/2)*b^2*sgn(x) + 21285*(sqrt(a)*x - sqrt(a*x^2 + b*x))^4*a^2*b
^3*sgn(x) + 13860*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a^(3/2)*b^4*sgn(x) + 53
90*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a*b^5*sgn(x) + 1155*(sqrt(a)*x - sqrt(
a*x^2 + b*x))*sqrt(a)*b^6*sgn(x) + 105*b^7*sgn(x))/(sqrt(a)*x - sqrt(a*x^2
 + b*x))^11
```

maple [A] time = 0.00, size = 55, normalized size = 0.69

$$\frac{2\,(ax+b)\left(16a^3x^3-40a^2b\,x^2+70a\,b^2x-105b^3\right)\left(\frac{ax+b}{x}\right)^{\frac{3}{2}}}{1155b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(3/2)/x^5,x)
```

```
[Out] 2/1155*(a*x+b)*(16*a^3*x^3-40*a^2*b*x^2+70*a*b^2*x-105*b^3)*((a*x+b)/x)^(3/
2)/x^4/b^4
```

maxima [A] time = 1.05, size = 64, normalized size = 0.80

$$-\frac{2\left(a+\frac{b}{x}\right)^{\frac{11}{2}}}{11\,b^4}+\frac{2\left(a+\frac{b}{x}\right)^{\frac{9}{2}}a}{3\,b^4}-\frac{6\left(a+\frac{b}{x}\right)^{\frac{7}{2}}a^2}{7\,b^4}+\frac{2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^3}{5\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] -2/11*(a + b/x)^(11/2)/b^4 + 2/3*(a + b/x)^(9/2)*a/b^4 - 6/7*(a + b/x)^(7/2
)*a^2/b^4 + 2/5*(a + b/x)^(5/2)*a^3/b^4
```

mupad [B] time = 2.09, size = 108, normalized size = 1.35

$$\frac{32\,a^5\sqrt{a+\frac{b}{x}}}{1155\,b^4}-\frac{2\,b\sqrt{a+\frac{b}{x}}}{11\,x^5}-\frac{8\,a\sqrt{a+\frac{b}{x}}}{33\,x^4}-\frac{2\,a^2\sqrt{a+\frac{b}{x}}}{231\,b\,x^3}+\frac{4\,a^3\sqrt{a+\frac{b}{x}}}{385\,b^2\,x^2}-\frac{16\,a^4\sqrt{a+\frac{b}{x}}}{1155\,b^3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(3/2)/x^5,x)
```

```
[Out] (32*a^5*(a + b/x)^(1/2))/(1155*b^4) - (2*b*(a + b/x)^(1/2))/(11*x^5) - (8*a
*(a + b/x)^(1/2))/(33*x^4) - (2*a^2*(a + b/x)^(1/2))/(231*b*x^3) + (4*a^3*(
a + b/x)^(1/2))/(385*b^2*x^2) - (16*a^4*(a + b/x)^(1/2))/(1155*b^3*x)
```

sympy [B] time = 3.51, size = 2297, normalized size = 28.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)/x**5,x)
```

```
[Out] 32*a**(33/2)*b**(23/2)*x**11*sqrt(a*x/b + 1)/(1155*a**(23/2)*b**15*x**(23/2
) + 6930*a**(21/2)*b**16*x**(21/2) + 17325*a**(19/2)*b**17*x**(19/2) + 2310
0*a**(17/2)*b**18*x**(17/2) + 17325*a**(15/2)*b**19*x**(15/2) + 6930*a**(13
/2)*b**20*x**(13/2) + 1155*a**(11/2)*b**21*x**(11/2)) + 176*a**(31/2)*b**(2
5/2)*x**10*sqrt(a*x/b + 1)/(1155*a**(23/2)*b**15*x**(23/2) + 6930*a**(21/2)
*b**16*x**(21/2) + 17325*a**(19/2)*b**17*x**(19/2) + 23100*a**(17/2)*b**18*
x**(17/2) + 17325*a**(15/2)*b**19*x**(15/2) + 6930*a**(13/2)*b**20*x**(13/2
) + 1155*a**(11/2)*b**21*x**(11/2)) + 396*a**(29/2)*b**(27/2)*x**9*sqrt(a*x
/b + 1)/(1155*a**(23/2)*b**15*x**(23/2) + 6930*a**(21/2)*b**16*x**(21/2) +
17325*a**(19/2)*b**17*x**(19/2) + 23100*a**(17/2)*b**18*x**(17/2) + 17325*a
**(15/2)*b**19*x**(15/2) + 6930*a**(13/2)*b**20*x**(13/2) + 1155*a**(11/2)*
b**21*x**(11/2)) + 462*a**(27/2)*b**(29/2)*x**8*sqrt(a*x/b + 1)/(1155*a**(2
3/2)*b**15*x**(23/2) + 6930*a**(21/2)*b**16*x**(21/2) + 17325*a**(19/2)*b**
17*x**(19/2) + 23100*a**(17/2)*b**18*x**(17/2) + 17325*a**(15/2)*b**19*x**(
15/2) + 6930*a**(13/2)*b**20*x**(13/2) + 1155*a**(11/2)*b**21*x**(11/2)) -
1848*a**(23/2)*b**(33/2)*x**6*sqrt(a*x/b + 1)/(1155*a**(23/2)*b**15*x**(23/
2) + 6930*a**(21/2)*b**16*x**(21/2) + 17325*a**(19/2)*b**17*x**(19/2) + 231
00*a**(17/2)*b**18*x**(17/2) + 17325*a**(15/2)*b**19*x**(15/2) + 6930*a**(1
3/2)*b**20*x**(13/2) + 1155*a**(11/2)*b**21*x**(11/2)) - 5544*a**(21/2)*b**
(35/2)*x**5*sqrt(a*x/b + 1)/(1155*a**(23/2)*b**15*x**(23/2) + 6930*a**(21/2
)*b**16*x**(21/2) + 17325*a**(19/2)*b**17*x**(19/2) + 23100*a**(17/2)*b**18
*x**(17/2) + 17325*a**(15/2)*b**19*x**(15/2) + 6930*a**(13/2)*b**20*x**(13/
2) + 1155*a**(11/2)*b**21*x**(11/2)) - 8844*a**(19/2)*b**(37/2)*x**4*sqrt(a
*x/b + 1)/(1155*a**(23/2)*b**15*x**(23/2) + 6930*a**(21/2)*b**16*x**(21/2)
+ 17325*a**(19/2)*b**17*x**(19/2) + 23100*a**(17/2)*b**18*x**(17/2) + 17325
*a**(15/2)*b**19*x**(15/2) + 6930*a**(13/2)*b**20*x**(13/2) + 1155*a**(11/2
)*b**21*x**(11/2)) - 8448*a**(17/2)*b**(39/2)*x**3*sqrt(a*x/b + 1)/(1155*a*
*(23/2)*b**15*x**(23/2) + 6930*a**(21/2)*b**16*x**(21/2) + 17325*a**(19/2)*
b**17*x**(19/2) + 23100*a**(17/2)*b**18*x**(17/2) + 17325*a**(15/2)*b**19*x
**(15/2) + 6930*a**(13/2)*b**20*x**(13/2) + 1155*a**(11/2)*b**21*x**(11/2))
 - 4840*a**(15/2)*b**(41/2)*x**2*sqrt(a*x/b + 1)/(1155*a**(23/2)*b**15*x**(
23/2) + 6930*a**(21/2)*b**16*x**(21/2) + 17325*a**(19/2)*b**17*x**(19/2) +
23100*a**(17/2)*b**18*x**(17/2) + 17325*a**(15/2)*b**19*x**(15/2) + 6930*a*
*(13/2)*b**20*x**(13/2) + 1155*a**(11/2)*b**21*x**(11/2)) - 1540*a**(13/2)*
b**(43/2)*x*sqrt(a*x/b + 1)/(1155*a**(23/2)*b**15*x**(23/2) + 6930*a**(21/2
)*b**16*x**(21/2) + 17325*a**(19/2)*b**17*x**(19/2) + 23100*a**(17/2)*b**18
*x**(17/2) + 17325*a**(15/2)*b**19*x**(15/2) + 6930*a**(13/2)*b**20*x**(13/
2) + 1155*a**(11/2)*b**21*x**(11/2)) - 210*a**(11/2)*b**(45/2)*sqrt(a*x/b +
 1)/(1155*a**(23/2)*b**15*x**(23/2) + 6930*a**(21/2)*b**16*x**(21/2) + 1732
```

```
5*a**(19/2)*b**17*x**(19/2) + 23100*a**(17/2)*b**18*x**(17/2) + 17325*a**(1
5/2)*b**19*x**(15/2) + 6930*a**(13/2)*b**20*x**(13/2) + 1155*a**(11/2)*b**2
1*x**(11/2)) - 32*a**17*b**11*x**(23/2)/(1155*a**(23/2)*b**15*x**(23/2) + 6
930*a**(21/2)*b**16*x**(21/2) + 17325*a**(19/2)*b**17*x**(19/2) + 23100*a**
(17/2)*b**18*x**(17/2) + 17325*a**(15/2)*b**19*x**(15/2) + 6930*a**(13/2)*b
**20*x**(13/2) + 1155*a**(11/2)*b**21*x**(11/2)) - 192*a**16*b**12*x**(21/2
)/(1155*a**(23/2)*b**15*x**(23/2) + 6930*a**(21/2)*b**16*x**(21/2) + 17325*
a**(19/2)*b**17*x**(19/2) + 23100*a**(17/2)*b**18*x**(17/2) + 17325*a**(15/
2)*b**19*x**(15/2) + 6930*a**(13/2)*b**20*x**(13/2) + 1155*a**(11/2)*b**21*
x**(11/2)) - 480*a**15*b**13*x**(19/2)/(1155*a**(23/2)*b**15*x**(23/2) + 69
30*a**(21/2)*b**16*x**(21/2) + 17325*a**(19/2)*b**17*x**(19/2) + 23100*a**(
17/2)*b**18*x**(17/2) + 17325*a**(15/2)*b**19*x**(15/2) + 6930*a**(13/2)*b*
*20*x**(13/2) + 1155*a**(11/2)*b**21*x**(11/2)) - 640*a**14*b**14*x**(17/2)
/(1155*a**(23/2)*b**15*x**(23/2) + 6930*a**(21/2)*b**16*x**(21/2) + 17325*a
**(19/2)*b**17*x**(19/2) + 23100*a**(17/2)*b**18*x**(17/2) + 17325*a**(15/2
)*b**19*x**(15/2) + 6930*a**(13/2)*b**20*x**(13/2) + 1155*a**(11/2)*b**21*x
**(11/2)) - 480*a**13*b**15*x**(15/2)/(1155*a**(23/2)*b**15*x**(23/2) + 693
0*a**(21/2)*b**16*x**(21/2) + 17325*a**(19/2)*b**17*x**(19/2) + 23100*a**(1
7/2)*b**18*x**(17/2) + 17325*a**(15/2)*b**19*x**(15/2) + 6930*a**(13/2)*b**
20*x**(13/2) + 1155*a**(11/2)*b**21*x**(11/2)) - 192*a**12*b**16*x**(13/2)/
(1155*a**(23/2)*b**15*x**(23/2) + 6930*a**(21/2)*b**16*x**(21/2) + 17325*a*
*(19/2)*b**17*x**(19/2) + 23100*a**(17/2)*b**18*x**(17/2) + 17325*a**(15/2)
*b**19*x**(15/2) + 6930*a**(13/2)*b**20*x**(13/2) + 1155*a**(11/2)*b**21*x*
*(11/2)) - 32*a**11*b**17*x**(11/2)/(1155*a**(23/2)*b**15*x**(23/2) + 6930*
a**(21/2)*b**16*x**(21/2) + 17325*a**(19/2)*b**17*x**(19/2) + 23100*a**(17/
2)*b**18*x**(17/2) + 17325*a**(15/2)*b**19*x**(15/2) + 6930*a**(13/2)*b**20
*x**(13/2) + 1155*a**(11/2)*b**21*x**(11/2))
```

3.1710 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=101

$$-\frac{2a^4\left(a+\frac{b}{x}\right)^{5/2}}{5b^5}+\frac{8a^3\left(a+\frac{b}{x}\right)^{7/2}}{7b^5}-\frac{4a^2\left(a+\frac{b}{x}\right)^{9/2}}{3b^5}-\frac{2\left(a+\frac{b}{x}\right)^{13/2}}{13b^5}+\frac{8a\left(a+\frac{b}{x}\right)^{11/2}}{11b^5}$$

```
[Out] -2/5*a^4*(a+b/x)^(5/2)/b^5+8/7*a^3*(a+b/x)^(7/2)/b^5-4/3*a^2*(a+b/x)^(9/2)/
b^5+8/11*a*(a+b/x)^(11/2)/b^5-2/13*(a+b/x)^(13/2)/b^5
```

Rubi [A] time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{4a^2\left(a+\frac{b}{x}\right)^{9/2}}{3b^5}+\frac{8a^3\left(a+\frac{b}{x}\right)^{7/2}}{7b^5}-\frac{2a^4\left(a+\frac{b}{x}\right)^{5/2}}{5b^5}-\frac{2\left(a+\frac{b}{x}\right)^{13/2}}{13b^5}+\frac{8a\left(a+\frac{b}{x}\right)^{11/2}}{11b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)/x^6,x]
```

```
[Out] (-2*a^4*(a + b/x)^(5/2))/(5*b^5) + (8*a^3*(a + b/x)^(7/2))/(7*b^5) - (4*a^2
*(a + b/x)^(9/2))/(3*b^5) + (8*a*(a + b/x)^(11/2))/(11*b^5) - (2*(a + b/x)^
(13/2))/(13*b^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^6}\,dx &= -\text{Subst}\left(\int x^4(a+bx)^{3/2}\,dx, x, \frac{1}{x}\right)\\ &= -\text{Subst}\left(\int\left(\frac{a^4(a+bx)^{3/2}}{b^4}-\frac{4a^3(a+bx)^{5/2}}{b^4}+\frac{6a^2(a+bx)^{7/2}}{b^4}-\frac{4a(a+bx)^{9/2}}{b^4}+\frac{(a+bx)^{11/2}}{b^4}\right)\right)\\ &= -\frac{2a^4\left(a+\frac{b}{x}\right)^{5/2}}{5b^5}+\frac{8a^3\left(a+\frac{b}{x}\right)^{7/2}}{7b^5}-\frac{4a^2\left(a+\frac{b}{x}\right)^{9/2}}{3b^5}+\frac{8a\left(a+\frac{b}{x}\right)^{11/2}}{11b^5}-\frac{2\left(a+\frac{b}{x}\right)^{13/2}}{13b^5}\end{aligned}$$

Mathematica [A] time = 0.05, size = 69, normalized size = 0.68

$$-\frac{2\sqrt{a+\frac{b}{x}}\,(ax+b)^2\left(128a^4x^4-320a^3bx^3+560a^2b^2x^2-840ab^3x+1155b^4\right)}{15015b^5x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)/x^6,x]
```

```
[Out] (-2*Sqrt[a + b/x]*(b + a*x)^2*(1155*b^4 - 840*a*b^3*x + 560*a^2*b^2*x^2 - 3
20*a^3*b*x^3 + 128*a^4*x^4))/(15015*b^5*x^6)
```

fricas [A] time = 1.09, size = 82, normalized size = 0.81

$$-\frac{2\left(128\,a^6x^6-64\,a^5bx^5+48\,a^4b^2x^4-40\,a^3b^3x^3+35\,a^2b^4x^2+1470\,ab^5x+1155\,b^6\right)\sqrt{\frac{ax+b}{x}}}{15015\,b^5x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] -2/15015*(128*a^6*x^6 - 64*a^5*b*x^5 + 48*a^4*b^2*x^4 - 40*a^3*b^3*x^3 + 35
*a^2*b^4*x^2 + 1470*a*b^5*x + 1155*b^6)*sqrt((a*x + b)/x)/(b^5*x^6)
```

giac [B] time = 0.33, size = 270, normalized size = 2.67

$$2\left(48048\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)^8a^4\mathrm{sgn}(x)+240240\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)^7a^{\frac{7}{2}}b\mathrm{sgn}(x)+531960\left(\sqrt{a}\,x-\sqrt{ax^2+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] 2/15015*(48048*(sqrt(a)*x - sqrt(a*x^2 + b*x))^8*a^4*sgn(x) + 240240*(sqrt(
a)*x - sqrt(a*x^2 + b*x))^7*a^(7/2)*b*sgn(x) + 531960*(sqrt(a)*x - sqrt(a*x
^2 + b*x))^6*a^3*b^2*sgn(x) + 675675*(sqrt(a)*x - sqrt(a*x^2 + b*x))^5*a^(5
/2)*b^3*sgn(x) + 535535*(sqrt(a)*x - sqrt(a*x^2 + b*x))^4*a^2*b^4*sgn(x) +
270270*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a^(3/2)*b^5*sgn(x) + 84630*(sqrt(a
)*x - sqrt(a*x^2 + b*x))^2*a*b^6*sgn(x) + 15015*(sqrt(a)*x - sqrt(a*x^2 + b
*x))*sqrt(a)*b^7*sgn(x) + 1155*b^8*sgn(x))/(sqrt(a)*x - sqrt(a*x^2 + b*x))^
13
```

maple [A] time = 0.01, size = 66, normalized size = 0.65

$$-\frac{2\,(ax+b)\left(128a^4x^4-320a^3x^3b+560a^2x^2b^2-840ax\,b^3+1155b^4\right)\left(\frac{ax+b}{x}\right)^{\frac{3}{2}}}{15015b^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(3/2)/x^6,x)
```

```
[Out] -2/15015*(a*x+b)*(128*a^4*x^4-320*a^3*b*x^3+560*a^2*b^2*x^2-840*a*b^3*x+115
5*b^4)*((a*x+b)/x)^(3/2)/x^5/b^5
```

maxima [A] time = 1.06, size = 81, normalized size = 0.80

$$-\frac{2\left(a+\frac{b}{x}\right)^{\frac{13}{2}}}{13\,b^5}+\frac{8\left(a+\frac{b}{x}\right)^{\frac{11}{2}}a}{11\,b^5}-\frac{4\left(a+\frac{b}{x}\right)^{\frac{9}{2}}a^2}{3\,b^5}+\frac{8\left(a+\frac{b}{x}\right)^{\frac{7}{2}}a^3}{7\,b^5}-\frac{2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^4}{5\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] -2/13*(a + b/x)^(13/2)/b^5 + 8/11*(a + b/x)^(11/2)*a/b^5 - 4/3*(a + b/x)^(9
/2)*a^2/b^5 + 8/7*(a + b/x)^(7/2)*a^3/b^5 - 2/5*(a + b/x)^(5/2)*a^4/b^5
```

mupad [B] time = 2.31, size = 128, normalized size = 1.27

$$\frac{16\,a^3\sqrt{a+\frac{b}{x}}}{3003\,b^2\,x^3}-\frac{2\,b\sqrt{a+\frac{b}{x}}}{13\,x^6}-\frac{256\,a^6\sqrt{a+\frac{b}{x}}}{15015\,b^5}-\frac{2\,a^2\sqrt{a+\frac{b}{x}}}{429\,b\,x^4}-\frac{28\,a\sqrt{a+\frac{b}{x}}}{143\,x^5}-\frac{32\,a^4\sqrt{a+\frac{b}{x}}}{5005\,b^3\,x^2}+\frac{128\,a^5\sqrt{a+\frac{b}{x}}}{15015\,b^4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(3/2)/x^6,x)
```

```
[Out] (16*a^3*(a + b/x)^(1/2))/(3003*b^2*x^3) - (2*b*(a + b/x)^(1/2))/(13*x^6) -
(256*a^6*(a + b/x)^(1/2))/(15015*b^5) - (2*a^2*(a + b/x)^(1/2))/(429*b*x^4)
 - (28*a*(a + b/x)^(1/2))/(143*x^5) - (32*a^4*(a + b/x)^(1/2))/(5005*b^3*x^
2) + (128*a^5*(a + b/x)^(1/2))/(15015*b^4*x)
```

sympy [B] time = 6.15, size = 5289, normalized size = 52.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)/x**6,x)
```

```
[Out] -256*a**(45/2)*b**(49/2)*x**16*sqrt(a*x/b + 1)/(15015*a**(33/2)*b**29*x**(3
3/2) + 150150*a**(31/2)*b**30*x**(31/2) + 675675*a**(29/2)*b**31*x**(29/2)
+ 1801800*a**(27/2)*b**32*x**(27/2) + 3153150*a**(25/2)*b**33*x**(25/2) + 3
783780*a**(23/2)*b**34*x**(23/2) + 3153150*a**(21/2)*b**35*x**(21/2) + 1801
800*a**(19/2)*b**36*x**(19/2) + 675675*a**(17/2)*b**37*x**(17/2) + 150150*a
**(15/2)*b**38*x**(15/2) + 15015*a**(13/2)*b**39*x**(13/2)) - 2432*a**(43/2
)*b**(51/2)*x**15*sqrt(a*x/b + 1)/(15015*a**(33/2)*b**29*x**(33/2) + 150150
*a**(31/2)*b**30*x**(31/2) + 675675*a**(29/2)*b**31*x**(29/2) + 1801800*a**
(27/2)*b**32*x**(27/2) + 3153150*a**(25/2)*b**33*x**(25/2) + 3783780*a**(23
/2)*b**34*x**(23/2) + 3153150*a**(21/2)*b**35*x**(21/2) + 1801800*a**(19/2)
*b**36*x**(19/2) + 675675*a**(17/2)*b**37*x**(17/2) + 150150*a**(15/2)*b**3
8*x**(15/2) + 15015*a**(13/2)*b**39*x**(13/2)) - 10336*a**(41/2)*b**(53/2)*
x**14*sqrt(a*x/b + 1)/(15015*a**(33/2)*b**29*x**(33/2) + 150150*a**(31/2)*b
**30*x**(31/2) + 675675*a**(29/2)*b**31*x**(29/2) + 1801800*a**(27/2)*b**32
*x**(27/2) + 3153150*a**(25/2)*b**33*x**(25/2) + 3783780*a**(23/2)*b**34*x*
*(23/2) + 3153150*a**(21/2)*b**35*x**(21/2) + 1801800*a**(19/2)*b**36*x**(1
9/2) + 675675*a**(17/2)*b**37*x**(17/2) + 150150*a**(15/2)*b**38*x**(15/2)
+ 15015*a**(13/2)*b**39*x**(13/2)) - 25840*a**(39/2)*b**(55/2)*x**13*sqrt(a
*x/b + 1)/(15015*a**(33/2)*b**29*x**(33/2) + 150150*a**(31/2)*b**30*x**(31/
2) + 675675*a**(29/2)*b**31*x**(29/2) + 1801800*a**(27/2)*b**32*x**(27/2) +
 3153150*a**(25/2)*b**33*x**(25/2) + 3783780*a**(23/2)*b**34*x**(23/2) + 31
53150*a**(21/2)*b**35*x**(21/2) + 1801800*a**(19/2)*b**36*x**(19/2) + 67567
5*a**(17/2)*b**37*x**(17/2) + 150150*a**(15/2)*b**38*x**(15/2) + 15015*a**(
13/2)*b**39*x**(13/2)) - 41990*a**(37/2)*b**(57/2)*x**12*sqrt(a*x/b + 1)/(1
5015*a**(33/2)*b**29*x**(33/2) + 150150*a**(31/2)*b**30*x**(31/2) + 675675*
a**(29/2)*b**31*x**(29/2) + 1801800*a**(27/2)*b**32*x**(27/2) + 3153150*a**
(25/2)*b**33*x**(25/2) + 3783780*a**(23/2)*b**34*x**(23/2) + 3153150*a**(21
/2)*b**35*x**(21/2) + 1801800*a**(19/2)*b**36*x**(19/2) + 675675*a**(17/2)*
b**37*x**(17/2) + 150150*a**(15/2)*b**38*x**(15/2) + 15015*a**(13/2)*b**39*
x**(13/2)) - 49192*a**(35/2)*b**(59/2)*x**11*sqrt(a*x/b + 1)/(15015*a**(33/
2)*b**29*x**(33/2) + 150150*a**(31/2)*b**30*x**(31/2) + 675675*a**(29/2)*b*
*31*x**(29/2) + 1801800*a**(27/2)*b**32*x**(27/2) + 3153150*a**(25/2)*b**33
*x**(25/2) + 3783780*a**(23/2)*b**34*x**(23/2) + 3153150*a**(21/2)*b**35*x*
*(21/2) + 1801800*a**(19/2)*b**36*x**(19/2) + 675675*a**(17/2)*b**37*x**(17
/2) + 150150*a**(15/2)*b**38*x**(15/2) + 15015*a**(13/2)*b**39*x**(13/2)) -
 66924*a**(33/2)*b**(61/2)*x**10*sqrt(a*x/b + 1)/(15015*a**(33/2)*b**29*x**
```

```
(33/2) + 150150*a**(31/2)*b**30*x**(31/2) + 675675*a**(29/2)*b**31*x**(29/2
) + 1801800*a**(27/2)*b**32*x**(27/2) + 3153150*a**(25/2)*b**33*x**(25/2) +
 3783780*a**(23/2)*b**34*x**(23/2) + 3153150*a**(21/2)*b**35*x**(21/2) + 18
01800*a**(19/2)*b**36*x**(19/2) + 675675*a**(17/2)*b**37*x**(17/2) + 150150
*a**(15/2)*b**38*x**(15/2) + 15015*a**(13/2)*b**39*x**(13/2)) - 175032*a**(
31/2)*b**(63/2)*x**9*sqrt(a*x/b + 1)/(15015*a**(33/2)*b**29*x**(33/2) + 150
150*a**(31/2)*b**30*x**(31/2) + 675675*a**(29/2)*b**31*x**(29/2) + 1801800*
a**(27/2)*b**32*x**(27/2) + 3153150*a**(25/2)*b**33*x**(25/2) + 3783780*a**
(23/2)*b**34*x**(23/2) + 3153150*a**(21/2)*b**35*x**(21/2) + 1801800*a**(19
/2)*b**36*x**(19/2) + 675675*a**(17/2)*b**37*x**(17/2) + 150150*a**(15/2)*b
**38*x**(15/2) + 15015*a**(13/2)*b**39*x**(13/2)) - 467610*a**(29/2)*b**(65
/2)*x**8*sqrt(a*x/b + 1)/(15015*a**(33/2)*b**29*x**(33/2) + 150150*a**(31/2
)*b**30*x**(31/2) + 675675*a**(29/2)*b**31*x**(29/2) + 1801800*a**(27/2)*b*
*32*x**(27/2) + 3153150*a**(25/2)*b**33*x**(25/2) + 3783780*a**(23/2)*b**34
*x**(23/2) + 3153150*a**(21/2)*b**35*x**(21/2) + 1801800*a**(19/2)*b**36*x*
*(19/2) + 675675*a**(17/2)*b**37*x**(17/2) + 150150*a**(15/2)*b**38*x**(15/
2) + 15015*a**(13/2)*b**39*x**(13/2)) - 903760*a**(27/2)*b**(67/2)*x**7*sqr
t(a*x/b + 1)/(15015*a**(33/2)*b**29*x**(33/2) + 150150*a**(31/2)*b**30*x**(
31/2) + 675675*a**(29/2)*b**31*x**(29/2) + 1801800*a**(27/2)*b**32*x**(27/2
) + 3153150*a**(25/2)*b**33*x**(25/2) + 3783780*a**(23/2)*b**34*x**(23/2) +
 3153150*a**(21/2)*b**35*x**(21/2) + 1801800*a**(19/2)*b**36*x**(19/2) + 67
5675*a**(17/2)*b**37*x**(17/2) + 150150*a**(15/2)*b**38*x**(15/2) + 15015*a
**(13/2)*b**39*x**(13/2)) - 1234376*a**(25/2)*b**(69/2)*x**6*sqrt(a*x/b + 1
)/(15015*a**(33/2)*b**29*x**(33/2) + 150150*a**(31/2)*b**30*x**(31/2) + 675
675*a**(29/2)*b**31*x**(29/2) + 1801800*a**(27/2)*b**32*x**(27/2) + 3153150
*a**(25/2)*b**33*x**(25/2) + 3783780*a**(23/2)*b**34*x**(23/2) + 3153150*a*
*(21/2)*b**35*x**(21/2) + 1801800*a**(19/2)*b**36*x**(19/2) + 675675*a**(17
/2)*b**37*x**(17/2) + 150150*a**(15/2)*b**38*x**(15/2) + 15015*a**(13/2)*b*
*39*x**(13/2)) - 1205152*a**(23/2)*b**(71/2)*x**5*sqrt(a*x/b + 1)/(15015*a*
*(33/2)*b**29*x**(33/2) + 150150*a**(31/2)*b**30*x**(31/2) + 675675*a**(29/
2)*b**31*x**(29/2) + 1801800*a**(27/2)*b**32*x**(27/2) + 3153150*a**(25/2)*
b**33*x**(25/2) + 3783780*a**(23/2)*b**34*x**(23/2) + 3153150*a**(21/2)*b**
35*x**(21/2) + 1801800*a**(19/2)*b**36*x**(19/2) + 675675*a**(17/2)*b**37*x
**(17/2) + 150150*a**(15/2)*b**38*x**(15/2) + 15015*a**(13/2)*b**39*x**(13/
2)) - 840346*a**(21/2)*b**(73/2)*x**4*sqrt(a*x/b + 1)/(15015*a**(33/2)*b**2
9*x**(33/2) + 150150*a**(31/2)*b**30*x**(31/2) + 675675*a**(29/2)*b**31*x**
(29/2) + 1801800*a**(27/2)*b**32*x**(27/2) + 3153150*a**(25/2)*b**33*x**(25
/2) + 3783780*a**(23/2)*b**34*x**(23/2) + 3153150*a**(21/2)*b**35*x**(21/2)
 + 1801800*a**(19/2)*b**36*x**(19/2) + 675675*a**(17/2)*b**37*x**(17/2) + 1
50150*a**(15/2)*b**38*x**(15/2) + 15015*a**(13/2)*b**39*x**(13/2)) - 410120
*a**(19/2)*b**(75/2)*x**3*sqrt(a*x/b + 1)/(15015*a**(33/2)*b**29*x**(33/2)
 + 150150*a**(31/2)*b**30*x**(31/2) + 675675*a**(29/2)*b**31*x**(29/2) + 180
1800*a**(27/2)*b**32*x**(27/2) + 3153150*a**(25/2)*b**33*x**(25/2) + 378378
0*a**(23/2)*b**34*x**(23/2) + 3153150*a**(21/2)*b**35*x**(21/2) + 1801800*a
**(19/2)*b**36*x**(19/2) + 675675*a**(17/2)*b**37*x**(17/2) + 150150*a**(15
/2)*b**38*x**(15/2) + 15015*a**(13/2)*b**39*x**(13/2)) - 133420*a**(17/2)*b
**(77/2)*x**2*sqrt(a*x/b + 1)/(15015*a**(33/2)*b**29*x**(33/2) + 150150*a**
(31/2)*b**30*x**(31/2) + 675675*a**(29/2)*b**31*x**(29/2) + 1801800*a**(27/
2)*b**32*x**(27/2) + 3153150*a**(25/2)*b**33*x**(25/2) + 3783780*a**(23/2)*
b**34*x**(23/2) + 3153150*a**(21/2)*b**35*x**(21/2) + 1801800*a**(19/2)*b**
36*x**(19/2) + 675675*a**(17/2)*b**37*x**(17/2) + 150150*a**(15/2)*b**38*x*
*(15/2) + 15015*a**(13/2)*b**39*x**(13/2)) - 26040*a**(15/2)*b**(79/2)*x*sq
rt(a*x/b + 1)/(15015*a**(33/2)*b**29*x**(33/2) + 150150*a**(31/2)*b**30*x**
(31/2) + 675675*a**(29/2)*b**31*x**(29/2) + 1801800*a**(27/2)*b**32*x**(27/
2) + 3153150*a**(25/2)*b**33*x**(25/2) + 3783780*a**(23/2)*b**34*x**(23/2)
+ 3153150*a**(21/2)*b**35*x**(21/2) + 1801800*a**(19/2)*b**36*x**(19/2) + 6
75675*a**(17/2)*b**37*x**(17/2) + 150150*a**(15/2)*b**38*x**(15/2) + 15015*
a**(13/2)*b**39*x**(13/2)) - 2310*a**(13/2)*b**(81/2)*sqrt(a*x/b + 1)/(1501
5*a**(33/2)*b**29*x**(33/2) + 150150*a**(31/2)*b**30*x**(31/2) + 675675*a**
```

```
(29/2)*b**31*x**(29/2) + 1801800*a**(27/2)*b**32*x**(27/2) + 3153150*a**(25
/2)*b**33*x**(25/2) + 3783780*a**(23/2)*b**34*x**(23/2) + 3153150*a**(21/2)
*b**35*x**(21/2) + 1801800*a**(19/2)*b**36*x**(19/2) + 675675*a**(17/2)*b**
37*x**(17/2) + 150150*a**(15/2)*b**38*x**(15/2) + 15015*a**(13/2)*b**39*x**
(13/2)) + 256*a**23*b**24*x**(33/2)/(15015*a**(33/2)*b**29*x**(33/2) + 1501
50*a**(31/2)*b**30*x**(31/2) + 675675*a**(29/2)*b**31*x**(29/2) + 1801800*a
**(27/2)*b**32*x**(27/2) + 3153150*a**(25/2)*b**33*x**(25/2) + 3783780*a**(
23/2)*b**34*x**(23/2) + 3153150*a**(21/2)*b**35*x**(21/2) + 1801800*a**(19/
2)*b**36*x**(19/2) + 675675*a**(17/2)*b**37*x**(17/2) + 150150*a**(15/2)*b*
*38*x**(15/2) + 15015*a**(13/2)*b**39*x**(13/2)) + 2560*a**22*b**25*x**(31/
2)/(15015*a**(33/2)*b**29*x**(33/2) + 150150*a**(31/2)*b**30*x**(31/2) + 67
5675*a**(29/2)*b**31*x**(29/2) + 1801800*a**(27/2)*b**32*x**(27/2) + 315315
0*a**(25/2)*b**33*x**(25/2) + 3783780*a**(23/2)*b**34*x**(23/2) + 3153150*a
**(21/2)*b**35*x**(21/2) + 1801800*a**(19/2)*b**36*x**(19/2) + 675675*a**(1
7/2)*b**37*x**(17/2) + 150150*a**(15/2)*b**38*x**(15/2) + 15015*a**(13/2)*b
**39*x**(13/2)) + 11520*a**21*b**26*x**(29/2)/(15015*a**(33/2)*b**29*x**(33
/2) + 150150*a**(31/2)*b**30*x**(31/2) + 675675*a**(29/2)*b**31*x**(29/2) +
 1801800*a**(27/2)*b**32*x**(27/2) + 3153150*a**(25/2)*b**33*x**(25/2) + 37
83780*a**(23/2)*b**34*x**(23/2) + 3153150*a**(21/2)*b**35*x**(21/2) + 18018
00*a**(19/2)*b**36*x**(19/2) + 675675*a**(17/2)*b**37*x**(17/2) + 150150*a*
*(15/2)*b**38*x**(15/2) + 15015*a**(13/2)*b**39*x**(13/2)) + 30720*a**20*b*
*27*x**(27/2)/(15015*a**(33/2)*b**29*x**(33/2) + 150150*a**(31/2)*b**30*x**
(31/2) + 675675*a**(29/2)*b**31*x**(29/2) + 1801800*a**(27/2)*b**32*x**(27/
2) + 3153150*a**(25/2)*b**33*x**(25/2) + 3783780*a**(23/2)*b**34*x**(23/2)
+ 3153150*a**(21/2)*b**35*x**(21/2) + 1801800*a**(19/2)*b**36*x**(19/2) + 6
75675*a**(17/2)*b**37*x**(17/2) + 150150*a**(15/2)*b**38*x**(15/2) + 15015*
a**(13/2)*b**39*x**(13/2)) + 53760*a**19*b**28*x**(25/2)/(15015*a**(33/2)*b
**29*x**(33/2) + 150150*a**(31/2)*b**30*x**(31/2) + 675675*a**(29/2)*b**31*
x**(29/2) + 1801800*a**(27/2)*b**32*x**(27/2) + 3153150*a**(25/2)*b**33*x**
(25/2) + 3783780*a**(23/2)*b**34*x**(23/2) + 3153150*a**(21/2)*b**35*x**(21
/2) + 1801800*a**(19/2)*b**36*x**(19/2) + 675675*a**(17/2)*b**37*x**(17/2)
+ 150150*a**(15/2)*b**38*x**(15/2) + 15015*a**(13/2)*b**39*x**(13/2)) + 645
12*a**18*b**29*x**(23/2)/(15015*a**(33/2)*b**29*x**(33/2) + 150150*a**(31/2
)*b**30*x**(31/2) + 675675*a**(29/2)*b**31*x**(29/2) + 1801800*a**(27/2)*b*
*32*x**(27/2) + 3153150*a**(25/2)*b**33*x**(25/2) + 3783780*a**(23/2)*b**34
*x**(23/2) + 3153150*a**(21/2)*b**35*x**(21/2) + 1801800*a**(19/2)*b**36*x*
*(19/2) + 675675*a**(17/2)*b**37*x**(17/2) + 150150*a**(15/2)*b**38*x**(15/
2) + 15015*a**(13/2)*b**39*x**(13/2)) + 53760*a**17*b**30*x**(21/2)/(15015*
a**(33/2)*b**29*x**(33/2) + 150150*a**(31/2)*b**30*x**(31/2) + 675675*a**(2
9/2)*b**31*x**(29/2) + 1801800*a**(27/2)*b**32*x**(27/2) + 3153150*a**(25/2
)*b**33*x**(25/2) + 3783780*a**(23/2)*b**34*x**(23/2) + 3153150*a**(21/2)*b
**35*x**(21/2) + 1801800*a**(19/2)*b**36*x**(19/2) + 675675*a**(17/2)*b**37
*x**(17/2) + 150150*a**(15/2)*b**38*x**(15/2) + 15015*a**(13/2)*b**39*x**(1
3/2)) + 30720*a**16*b**31*x**(19/2)/(15015*a**(33/2)*b**29*x**(33/2) + 1501
50*a**(31/2)*b**30*x**(31/2) + 675675*a**(29/2)*b**31*x**(29/2) + 1801800*a
**(27/2)*b**32*x**(27/2) + 3153150*a**(25/2)*b**33*x**(25/2) + 3783780*a**(
23/2)*b**34*x**(23/2) + 3153150*a**(21/2)*b**35*x**(21/2) + 1801800*a**(19/
2)*b**36*x**(19/2) + 675675*a**(17/2)*b**37*x**(17/2) + 150150*a**(15/2)*b*
*38*x**(15/2) + 15015*a**(13/2)*b**39*x**(13/2)) + 11520*a**15*b**32*x**(17
/2)/(15015*a**(33/2)*b**29*x**(33/2) + 150150*a**(31/2)*b**30*x**(31/2) + 6
75675*a**(29/2)*b**31*x**(29/2) + 1801800*a**(27/2)*b**32*x**(27/2) + 31531
50*a**(25/2)*b**33*x**(25/2) + 3783780*a**(23/2)*b**34*x**(23/2) + 3153150*
a**(21/2)*b**35*x**(21/2) + 1801800*a**(19/2)*b**36*x**(19/2) + 675675*a**(
17/2)*b**37*x**(17/2) + 150150*a**(15/2)*b**38*x**(15/2) + 15015*a**(13/2)*
b**39*x**(13/2)) + 2560*a**14*b**33*x**(15/2)/(15015*a**(33/2)*b**29*x**(33
/2) + 150150*a**(31/2)*b**30*x**(31/2) + 675675*a**(29/2)*b**31*x**(29/2) +
 1801800*a**(27/2)*b**32*x**(27/2) + 3153150*a**(25/2)*b**33*x**(25/2) + 37
83780*a**(23/2)*b**34*x**(23/2) + 3153150*a**(21/2)*b**35*x**(21/2) + 18018
00*a**(19/2)*b**36*x**(19/2) + 675675*a**(17/2)*b**37*x**(17/2) + 150150*a*
```

```
*(15/2)*b**38*x**(15/2) + 15015*a**(13/2)*b**39*x**(13/2)) + 256*a**13*b**3
4*x**(13/2)/(15015*a**(33/2)*b**29*x**(33/2) + 150150*a**(31/2)*b**30*x**(3
1/2) + 675675*a**(29/2)*b**31*x**(29/2) + 1801800*a**(27/2)*b**32*x**(27/2)
 + 3153150*a**(25/2)*b**33*x**(25/2) + 3783780*a**(23/2)*b**34*x**(23/2) +
3153150*a**(21/2)*b**35*x**(21/2) + 1801800*a**(19/2)*b**36*x**(19/2) + 675
675*a**(17/2)*b**37*x**(17/2) + 150150*a**(15/2)*b**38*x**(15/2) + 15015*a*
*(13/2)*b**39*x**(13/2))
```

3.1711 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=122

$$\frac{2a^5\left(a+\frac{b}{x}\right)^{5/2}}{5b^6}-\frac{10a^4\left(a+\frac{b}{x}\right)^{7/2}}{7b^6}+\frac{20a^3\left(a+\frac{b}{x}\right)^{9/2}}{9b^6}-\frac{20a^2\left(a+\frac{b}{x}\right)^{11/2}}{11b^6}-\frac{2\left(a+\frac{b}{x}\right)^{15/2}}{15b^6}+\frac{10a\left(a+\frac{b}{x}\right)^{13/2}}{13b^6}$$

```
[Out] 2/5*a^5*(a+b/x)^(5/2)/b^6-10/7*a^4*(a+b/x)^(7/2)/b^6+20/9*a^3*(a+b/x)^(9/2)
/b^6-20/11*a^2*(a+b/x)^(11/2)/b^6+10/13*a*(a+b/x)^(13/2)/b^6-2/15*(a+b/x)^(
15/2)/b^6
```

Rubi [A] time = 0.05, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{20a^2\left(a+\frac{b}{x}\right)^{11/2}}{11b^6}+\frac{20a^3\left(a+\frac{b}{x}\right)^{9/2}}{9b^6}-\frac{10a^4\left(a+\frac{b}{x}\right)^{7/2}}{7b^6}+\frac{2a^5\left(a+\frac{b}{x}\right)^{5/2}}{5b^6}-\frac{2\left(a+\frac{b}{x}\right)^{15/2}}{15b^6}+\frac{10a\left(a+\frac{b}{x}\right)^{13/2}}{13b^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)/x^7,x]
```

```
[Out] (2*a^5*(a + b/x)^(5/2))/(5*b^6) - (10*a^4*(a + b/x)^(7/2))/(7*b^6) + (20*a^
3*(a + b/x)^(9/2))/(9*b^6) - (20*a^2*(a + b/x)^(11/2))/(11*b^6) + (10*a*(a
+ b/x)^(13/2))/(13*b^6) - (2*(a + b/x)^(15/2))/(15*b^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^7}\,dx &= -\text{Subst}\left(\int x^5(a+bx)^{3/2}\,dx, x, \frac{1}{x}\right)\\ &= -\text{Subst}\left(\int\left(-\frac{a^5(a+bx)^{3/2}}{b^5}+\frac{5a^4(a+bx)^{5/2}}{b^5}-\frac{10a^3(a+bx)^{7/2}}{b^5}+\frac{10a^2(a+bx)^{9/2}}{b^5}-\frac{5a(a+b\right.\right.\\ &= \frac{2a^5\left(a+\frac{b}{x}\right)^{5/2}}{5b^6}-\frac{10a^4\left(a+\frac{b}{x}\right)^{7/2}}{7b^6}+\frac{20a^3\left(a+\frac{b}{x}\right)^{9/2}}{9b^6}-\frac{20a^2\left(a+\frac{b}{x}\right)^{11/2}}{11b^6}+\frac{10a\left(a+\frac{b}{x}\right)^{13/2}}{13b^6}-\frac{2\left(\right.}{}\end{aligned}$$

Mathematica [A] time = 0.05, size = 80, normalized size = 0.66

$$\frac{2\sqrt{a+\frac{b}{x}}\,(ax+b)^2\left(256a^5x^5-640a^4bx^4+1120a^3b^2x^3-1680a^2b^3x^2+2310ab^4x-3003b^5\right)}{45045b^6x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)/x^7,x]
```

```
[Out] (2*Sqrt[a + b/x]*(b + a*x)^2*(-3003*b^5 + 2310*a*b^4*x - 1680*a^2*b^3*x^2 +
 1120*a^3*b^2*x^3 - 640*a^4*b*x^4 + 256*a^5*x^5))/(45045*b^6*x^7)
```

fricas [A] time = 0.95, size = 93, normalized size = 0.76

$$\frac{2\left(256\,a^7x^7-128\,a^6bx^6+96\,a^5b^2x^5-80\,a^4b^3x^4+70\,a^3b^4x^3-63\,a^2b^5x^2-3696\,ab^6x-3003\,b^7\right)\sqrt{\frac{ax+b}{x}}}{45045\,b^6x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] 2/45045*(256*a^7*x^7 - 128*a^6*b*x^6 + 96*a^5*b^2*x^5 - 80*a^4*b^3*x^4 + 70
*a^3*b^4*x^3 - 63*a^2*b^5*x^2 - 3696*a*b^6*x - 3003*b^7)*sqrt((a*x + b)/x)/
(b^6*x^7)
```

giac [B] time = 0.34, size = 301, normalized size = 2.47

$$\frac{2\left(240240\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)^9a^{\frac{9}{2}}\mathrm{sgn}(x)+1338480\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)^8a^4b\mathrm{sgn}(x)+3333330\left(\sqrt{a}\,x-\sqrt{ax\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] 2/45045*(240240*(sqrt(a)*x - sqrt(a*x^2 + b*x))^9*a^(9/2)*sgn(x) + 1338480*
(sqrt(a)*x - sqrt(a*x^2 + b*x))^8*a^4*b*sgn(x) + 3333330*(sqrt(a)*x - sqrt(
a*x^2 + b*x))^7*a^(7/2)*b^2*sgn(x) + 4844840*(sqrt(a)*x - sqrt(a*x^2 + b*x)
)^6*a^3*b^3*sgn(x) + 4513509*(sqrt(a)*x - sqrt(a*x^2 + b*x))^5*a^(5/2)*b^4*
sgn(x) + 2788695*(sqrt(a)*x - sqrt(a*x^2 + b*x))^4*a^2*b^5*sgn(x) + 1141140
*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a^(3/2)*b^6*sgn(x) + 297990*(sqrt(a)*x -
 sqrt(a*x^2 + b*x))^2*a*b^7*sgn(x) + 45045*(sqrt(a)*x - sqrt(a*x^2 + b*x))*
sqrt(a)*b^8*sgn(x) + 3003*b^9*sgn(x))/(sqrt(a)*x - sqrt(a*x^2 + b*x))^15
```

maple [A] time = 0.01, size = 77, normalized size = 0.63

$$\frac{2\,(ax+b)\left(256a^5x^5-640a^4b\,x^4+1120a^3b^2x^3-1680a^2b^3x^2+2310a\,b^4x-3003b^5\right)\left(\frac{ax+b}{x}\right)^{\frac{3}{2}}}{45045b^6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(3/2)/x^7,x)
```

```
[Out] 2/45045*(a*x+b)*(256*a^5*x^5-640*a^4*b*x^4+1120*a^3*b^2*x^3-1680*a^2*b^3*x^
2+2310*a*b^4*x-3003*b^5)*((a*x+b)/x)^(3/2)/x^6/b^6
```

maxima [A] time = 0.97, size = 98, normalized size = 0.80

$$-\frac{2\left(a+\frac{b}{x}\right)^{\frac{15}{2}}}{15\,b^6}+\frac{10\left(a+\frac{b}{x}\right)^{\frac{13}{2}}a}{13\,b^6}-\frac{20\left(a+\frac{b}{x}\right)^{\frac{11}{2}}a^2}{11\,b^6}+\frac{20\left(a+\frac{b}{x}\right)^{\frac{9}{2}}a^3}{9\,b^6}-\frac{10\left(a+\frac{b}{x}\right)^{\frac{7}{2}}a^4}{7\,b^6}+\frac{2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^5}{5\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] -2/15*(a + b/x)^(15/2)/b^6 + 10/13*(a + b/x)^(13/2)*a/b^6 - 20/11*(a + b/x)
^(11/2)*a^2/b^6 + 20/9*(a + b/x)^(9/2)*a^3/b^6 - 10/7*(a + b/x)^(7/2)*a^4/b
^6 + 2/5*(a + b/x)^(5/2)*a^5/b^6
```

mupad [B] time = 2.52, size = 148, normalized size = 1.21

$$\frac{512\,a^7\sqrt{a+\frac{b}{x}}}{45045\,b^6}-\frac{2\,b\sqrt{a+\frac{b}{x}}}{15\,x^7}-\frac{32\,a\sqrt{a+\frac{b}{x}}}{195\,x^6}-\frac{2\,a^2\sqrt{a+\frac{b}{x}}}{715\,b\,x^5}+\frac{4\,a^3\sqrt{a+\frac{b}{x}}}{1287\,b^2\,x^4}-\frac{32\,a^4\sqrt{a+\frac{b}{x}}}{9009\,b^3\,x^3}+\frac{64\,a^5\sqrt{a+\frac{b}{x}}}{15015\,b^4\,x^2}-\frac{256\,a^6\sqrt{}}{45045}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(3/2)/x^7,x)
```

```
[Out] (512*a^7*(a + b/x)^(1/2))/(45045*b^6) - (2*b*(a + b/x)^(1/2))/(15*x^7) - (3
2*a*(a + b/x)^(1/2))/(195*x^6) - (2*a^2*(a + b/x)^(1/2))/(715*b*x^5) + (4*a
^3*(a + b/x)^(1/2))/(1287*b^2*x^4) - (32*a^4*(a + b/x)^(1/2))/(9009*b^3*x^3
) + (64*a^5*(a + b/x)^(1/2))/(15015*b^4*x^2) - (256*a^6*(a + b/x)^(1/2))/(4
5045*b^5*x)
```

sympy [B] time = 11.85, size = 10344, normalized size = 84.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)/x**7,x)
```

```
[Out] 512*a**(59/2)*b**(91/2)*x**22*sqrt(a*x/b + 1)/(45045*a**(45/2)*b**51*x**(45
/2) + 675675*a**(43/2)*b**52*x**(43/2) + 4729725*a**(41/2)*b**53*x**(41/2)
+ 20495475*a**(39/2)*b**54*x**(39/2) + 61486425*a**(37/2)*b**55*x**(37/2) +
 135270135*a**(35/2)*b**56*x**(35/2) + 225450225*a**(33/2)*b**57*x**(33/2)
+ 289864575*a**(31/2)*b**58*x**(31/2) + 289864575*a**(29/2)*b**59*x**(29/2)
 + 225450225*a**(27/2)*b**60*x**(27/2) + 135270135*a**(25/2)*b**61*x**(25/2
) + 61486425*a**(23/2)*b**62*x**(23/2) + 20495475*a**(21/2)*b**63*x**(21/2)
 + 4729725*a**(19/2)*b**64*x**(19/2) + 675675*a**(17/2)*b**65*x**(17/2) + 4
5045*a**(15/2)*b**66*x**(15/2)) + 7424*a**(57/2)*b**(93/2)*x**21*sqrt(a*x/b
 + 1)/(45045*a**(45/2)*b**51*x**(45/2) + 675675*a**(43/2)*b**52*x**(43/2) +
 4729725*a**(41/2)*b**53*x**(41/2) + 20495475*a**(39/2)*b**54*x**(39/2) + 6
1486425*a**(37/2)*b**55*x**(37/2) + 135270135*a**(35/2)*b**56*x**(35/2) + 2
25450225*a**(33/2)*b**57*x**(33/2) + 289864575*a**(31/2)*b**58*x**(31/2) +
289864575*a**(29/2)*b**59*x**(29/2) + 225450225*a**(27/2)*b**60*x**(27/2) +
 135270135*a**(25/2)*b**61*x**(25/2) + 61486425*a**(23/2)*b**62*x**(23/2) +
 20495475*a**(21/2)*b**63*x**(21/2) + 4729725*a**(19/2)*b**64*x**(19/2) + 6
75675*a**(17/2)*b**65*x**(17/2) + 45045*a**(15/2)*b**66*x**(15/2)) + 50112*
a**(55/2)*b**(95/2)*x**20*sqrt(a*x/b + 1)/(45045*a**(45/2)*b**51*x**(45/2)
+ 675675*a**(43/2)*b**52*x**(43/2) + 4729725*a**(41/2)*b**53*x**(41/2) + 20
495475*a**(39/2)*b**54*x**(39/2) + 61486425*a**(37/2)*b**55*x**(37/2) + 135
270135*a**(35/2)*b**56*x**(35/2) + 225450225*a**(33/2)*b**57*x**(33/2) + 28
9864575*a**(31/2)*b**58*x**(31/2) + 289864575*a**(29/2)*b**59*x**(29/2) + 2
25450225*a**(27/2)*b**60*x**(27/2) + 135270135*a**(25/2)*b**61*x**(25/2) +
61486425*a**(23/2)*b**62*x**(23/2) + 20495475*a**(21/2)*b**63*x**(21/2) + 4
729725*a**(19/2)*b**64*x**(19/2) + 675675*a**(17/2)*b**65*x**(17/2) + 45045
*a**(15/2)*b**66*x**(15/2)) + 208800*a**(53/2)*b**(97/2)*x**19*sqrt(a*x/b +
 1)/(45045*a**(45/2)*b**51*x**(45/2) + 675675*a**(43/2)*b**52*x**(43/2) + 4
729725*a**(41/2)*b**53*x**(41/2) + 20495475*a**(39/2)*b**54*x**(39/2) + 614
86425*a**(37/2)*b**55*x**(37/2) + 135270135*a**(35/2)*b**56*x**(35/2) + 225
450225*a**(33/2)*b**57*x**(33/2) + 289864575*a**(31/2)*b**58*x**(31/2) + 28
9864575*a**(29/2)*b**59*x**(29/2) + 225450225*a**(27/2)*b**60*x**(27/2) + 1
35270135*a**(25/2)*b**61*x**(25/2) + 61486425*a**(23/2)*b**62*x**(23/2) + 2
0495475*a**(21/2)*b**63*x**(21/2) + 4729725*a**(19/2)*b**64*x**(19/2) + 675
675*a**(17/2)*b**65*x**(17/2) + 45045*a**(15/2)*b**66*x**(15/2)) + 600300*a
```

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**(51/2)*b**(99/2)*x**18*sqrt(a*x/b + 1)/(45045*a**(45/2)*b**51*x**(45/2) +
 675675*a**(43/2)*b**52*x**(43/2) + 4729725*a**(41/2)*b**53*x**(41/2) + 204
95475*a**(39/2)*b**54*x**(39/2) + 61486425*a**(37/2)*b**55*x**(37/2) + 1352
70135*a**(35/2)*b**56*x**(35/2) + 225450225*a**(33/2)*b**57*x**(33/2) + 289
864575*a**(31/2)*b**58*x**(31/2) + 289864575*a**(29/2)*b**59*x**(29/2) + 22
5450225*a**(27/2)*b**60*x**(27/2) + 135270135*a**(25/2)*b**61*x**(25/2) + 6
1486425*a**(23/2)*b**62*x**(23/2) + 20495475*a**(21/2)*b**63*x**(21/2) + 47
29725*a**(19/2)*b**64*x**(19/2) + 675675*a**(17/2)*b**65*x**(17/2) + 45045*
a**(15/2)*b**66*x**(15/2)) + 1260630*a**(49/2)*b**(101/2)*x**17*sqrt(a*x/b
+ 1)/(45045*a**(45/2)*b**51*x**(45/2) + 675675*a**(43/2)*b**52*x**(43/2) +
4729725*a**(41/2)*b**53*x**(41/2) + 20495475*a**(39/2)*b**54*x**(39/2) + 61
486425*a**(37/2)*b**55*x**(37/2) + 135270135*a**(35/2)*b**56*x**(35/2) + 22
5450225*a**(33/2)*b**57*x**(33/2) + 289864575*a**(31/2)*b**58*x**(31/2) + 2
89864575*a**(29/2)*b**59*x**(29/2) + 225450225*a**(27/2)*b**60*x**(27/2) +
135270135*a**(25/2)*b**61*x**(25/2) + 61486425*a**(23/2)*b**62*x**(23/2) +
20495475*a**(21/2)*b**63*x**(21/2) + 4729725*a**(19/2)*b**64*x**(19/2) + 67
5675*a**(17/2)*b**65*x**(17/2) + 45045*a**(15/2)*b**66*x**(15/2)) + 1988490
*a**(47/2)*b**(103/2)*x**16*sqrt(a*x/b + 1)/(45045*a**(45/2)*b**51*x**(45/2
) + 675675*a**(43/2)*b**52*x**(43/2) + 4729725*a**(41/2)*b**53*x**(41/2) +
20495475*a**(39/2)*b**54*x**(39/2) + 61486425*a**(37/2)*b**55*x**(37/2) + 1
35270135*a**(35/2)*b**56*x**(35/2) + 225450225*a**(33/2)*b**57*x**(33/2) +
289864575*a**(31/2)*b**58*x**(31/2) + 289864575*a**(29/2)*b**59*x**(29/2) +
 225450225*a**(27/2)*b**60*x**(27/2) + 135270135*a**(25/2)*b**61*x**(25/2)
+ 61486425*a**(23/2)*b**62*x**(23/2) + 20495475*a**(21/2)*b**63*x**(21/2) +
 4729725*a**(19/2)*b**64*x**(19/2) + 675675*a**(17/2)*b**65*x**(17/2) + 450
45*a**(15/2)*b**66*x**(15/2)) + 2305200*a**(45/2)*b**(105/2)*x**15*sqrt(a*x
/b + 1)/(45045*a**(45/2)*b**51*x**(45/2) + 675675*a**(43/2)*b**52*x**(43/2)
 + 4729725*a**(41/2)*b**53*x**(41/2) + 20495475*a**(39/2)*b**54*x**(39/2) +
 61486425*a**(37/2)*b**55*x**(37/2) + 135270135*a**(35/2)*b**56*x**(35/2) +
 225450225*a**(33/2)*b**57*x**(33/2) + 289864575*a**(31/2)*b**58*x**(31/2)
+ 289864575*a**(29/2)*b**59*x**(29/2) + 225450225*a**(27/2)*b**60*x**(27/2)
 + 135270135*a**(25/2)*b**61*x**(25/2) + 61486425*a**(23/2)*b**62*x**(23/2)
 + 20495475*a**(21/2)*b**63*x**(21/2) + 4729725*a**(19/2)*b**64*x**(19/2) +
 675675*a**(17/2)*b**65*x**(17/2) + 45045*a**(15/2)*b**66*x**(15/2)) + 1395
360*a**(43/2)*b**(107/2)*x**14*sqrt(a*x/b + 1)/(45045*a**(45/2)*b**51*x**(4
5/2) + 675675*a**(43/2)*b**52*x**(43/2) + 4729725*a**(41/2)*b**53*x**(41/2)
 + 20495475*a**(39/2)*b**54*x**(39/2) + 61486425*a**(37/2)*b**55*x**(37/2)
+ 135270135*a**(35/2)*b**56*x**(35/2) + 225450225*a**(33/2)*b**57*x**(33/2)
 + 289864575*a**(31/2)*b**58*x**(31/2) + 289864575*a**(29/2)*b**59*x**(29/2
) + 225450225*a**(27/2)*b**60*x**(27/2) + 135270135*a**(25/2)*b**61*x**(25/
2) + 61486425*a**(23/2)*b**62*x**(23/2) + 20495475*a**(21/2)*b**63*x**(21/2
) + 4729725*a**(19/2)*b**64*x**(19/2) + 675675*a**(17/2)*b**65*x**(17/2) +
45045*a**(15/2)*b**66*x**(15/2)) - 2395640*a**(41/2)*b**(109/2)*x**13*sqrt(
a*x/b + 1)/(45045*a**(45/2)*b**51*x**(45/2) + 675675*a**(43/2)*b**52*x**(43
/2) + 4729725*a**(41/2)*b**53*x**(41/2) + 20495475*a**(39/2)*b**54*x**(39/2
) + 61486425*a**(37/2)*b**55*x**(37/2) + 135270135*a**(35/2)*b**56*x**(35/2
) + 225450225*a**(33/2)*b**57*x**(33/2) + 289864575*a**(31/2)*b**58*x**(31/
2) + 289864575*a**(29/2)*b**59*x**(29/2) + 225450225*a**(27/2)*b**60*x**(27
/2) + 135270135*a**(25/2)*b**61*x**(25/2) + 61486425*a**(23/2)*b**62*x**(23
/2) + 20495475*a**(21/2)*b**63*x**(21/2) + 4729725*a**(19/2)*b**64*x**(19/2
) + 675675*a**(17/2)*b**65*x**(17/2) + 45045*a**(15/2)*b**66*x**(15/2)) - 1
2038312*a**(39/2)*b**(111/2)*x**12*sqrt(a*x/b + 1)/(45045*a**(45/2)*b**51*x
**(45/2) + 675675*a**(43/2)*b**52*x**(43/2) + 4729725*a**(41/2)*b**53*x**(4
1/2) + 20495475*a**(39/2)*b**54*x**(39/2) + 61486425*a**(37/2)*b**55*x**(37
/2) + 135270135*a**(35/2)*b**56*x**(35/2) + 225450225*a**(33/2)*b**57*x**(3
3/2) + 289864575*a**(31/2)*b**58*x**(31/2) + 289864575*a**(29/2)*b**59*x**(
29/2) + 225450225*a**(27/2)*b**60*x**(27/2) + 135270135*a**(25/2)*b**61*x**
(25/2) + 61486425*a**(23/2)*b**62*x**(23/2) + 20495475*a**(21/2)*b**63*x**(
21/2) + 4729725*a**(19/2)*b**64*x**(19/2) + 675675*a**(17/2)*b**65*x**(17/2
```

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) + 45045*a**(15/2)*b**66*x**(15/2)) - 30264624*a**(37/2)*b**(113/2)*x**11*
sqrt(a*x/b + 1)/(45045*a**(45/2)*b**51*x**(45/2) + 675675*a**(43/2)*b**52*x
**(43/2) + 4729725*a**(41/2)*b**53*x**(41/2) + 20495475*a**(39/2)*b**54*x**
(39/2) + 61486425*a**(37/2)*b**55*x**(37/2) + 135270135*a**(35/2)*b**56*x**
(35/2) + 225450225*a**(33/2)*b**57*x**(33/2) + 289864575*a**(31/2)*b**58*x*
*(31/2) + 289864575*a**(29/2)*b**59*x**(29/2) + 225450225*a**(27/2)*b**60*x
**(27/2) + 135270135*a**(25/2)*b**61*x**(25/2) + 61486425*a**(23/2)*b**62*x
**(23/2) + 20495475*a**(21/2)*b**63*x**(21/2) + 4729725*a**(19/2)*b**64*x**
(19/2) + 675675*a**(17/2)*b**65*x**(17/2) + 45045*a**(15/2)*b**66*x**(15/2)
) - 55283592*a**(35/2)*b**(115/2)*x**10*sqrt(a*x/b + 1)/(45045*a**(45/2)*b*
*51*x**(45/2) + 675675*a**(43/2)*b**52*x**(43/2) + 4729725*a**(41/2)*b**53*
x**(41/2) + 20495475*a**(39/2)*b**54*x**(39/2) + 61486425*a**(37/2)*b**55*x
**(37/2) + 135270135*a**(35/2)*b**56*x**(35/2) + 225450225*a**(33/2)*b**57*
x**(33/2) + 289864575*a**(31/2)*b**58*x**(31/2) + 289864575*a**(29/2)*b**59
*x**(29/2) + 225450225*a**(27/2)*b**60*x**(27/2) + 135270135*a**(25/2)*b**6
1*x**(25/2) + 61486425*a**(23/2)*b**62*x**(23/2) + 20495475*a**(21/2)*b**63
*x**(21/2) + 4729725*a**(19/2)*b**64*x**(19/2) + 675675*a**(17/2)*b**65*x**
(17/2) + 45045*a**(15/2)*b**66*x**(15/2)) - 78018780*a**(33/2)*b**(117/2)*x
**9*sqrt(a*x/b + 1)/(45045*a**(45/2)*b**51*x**(45/2) + 675675*a**(43/2)*b**
52*x**(43/2) + 4729725*a**(41/2)*b**53*x**(41/2) + 20495475*a**(39/2)*b**54
*x**(39/2) + 61486425*a**(37/2)*b**55*x**(37/2) + 135270135*a**(35/2)*b**56
*x**(35/2) + 225450225*a**(33/2)*b**57*x**(33/2) + 289864575*a**(31/2)*b**5
8*x**(31/2) + 289864575*a**(29/2)*b**59*x**(29/2) + 225450225*a**(27/2)*b**
60*x**(27/2) + 135270135*a**(25/2)*b**61*x**(25/2) + 61486425*a**(23/2)*b**
62*x**(23/2) + 20495475*a**(21/2)*b**63*x**(21/2) + 4729725*a**(19/2)*b**64
*x**(19/2) + 675675*a**(17/2)*b**65*x**(17/2) + 45045*a**(15/2)*b**66*x**(1
5/2)) - 86576580*a**(31/2)*b**(119/2)*x**8*sqrt(a*x/b + 1)/(45045*a**(45/2)
*b**51*x**(45/2) + 675675*a**(43/2)*b**52*x**(43/2) + 4729725*a**(41/2)*b**
53*x**(41/2) + 20495475*a**(39/2)*b**54*x**(39/2) + 61486425*a**(37/2)*b**5
5*x**(37/2) + 135270135*a**(35/2)*b**56*x**(35/2) + 225450225*a**(33/2)*b**
57*x**(33/2) + 289864575*a**(31/2)*b**58*x**(31/2) + 289864575*a**(29/2)*b*
*59*x**(29/2) + 225450225*a**(27/2)*b**60*x**(27/2) + 135270135*a**(25/2)*b
**61*x**(25/2) + 61486425*a**(23/2)*b**62*x**(23/2) + 20495475*a**(21/2)*b*
*63*x**(21/2) + 4729725*a**(19/2)*b**64*x**(19/2) + 675675*a**(17/2)*b**65*
x**(17/2) + 45045*a**(15/2)*b**66*x**(15/2)) - 75888816*a**(29/2)*b**(121/2
)*x**7*sqrt(a*x/b + 1)/(45045*a**(45/2)*b**51*x**(45/2) + 675675*a**(43/2)*
b**52*x**(43/2) + 4729725*a**(41/2)*b**53*x**(41/2) + 20495475*a**(39/2)*b*
*54*x**(39/2) + 61486425*a**(37/2)*b**55*x**(37/2) + 135270135*a**(35/2)*b*
*56*x**(35/2) + 225450225*a**(33/2)*b**57*x**(33/2) + 289864575*a**(31/2)*b
**58*x**(31/2) + 289864575*a**(29/2)*b**59*x**(29/2) + 225450225*a**(27/2)*
b**60*x**(27/2) + 135270135*a**(25/2)*b**61*x**(25/2) + 61486425*a**(23/2)*
b**62*x**(23/2) + 20495475*a**(21/2)*b**63*x**(21/2) + 4729725*a**(19/2)*b*
*64*x**(19/2) + 675675*a**(17/2)*b**65*x**(17/2) + 45045*a**(15/2)*b**66*x*
*(15/2)) - 52380672*a**(27/2)*b**(123/2)*x**6*sqrt(a*x/b + 1)/(45045*a**(45
/2)*b**51*x**(45/2) + 675675*a**(43/2)*b**52*x**(43/2) + 4729725*a**(41/2)*
b**53*x**(41/2) + 20495475*a**(39/2)*b**54*x**(39/2) + 61486425*a**(37/2)*b
**55*x**(37/2) + 135270135*a**(35/2)*b**56*x**(35/2) + 225450225*a**(33/2)*
b**57*x**(33/2) + 289864575*a**(31/2)*b**58*x**(31/2) + 289864575*a**(29/2)
*b**59*x**(29/2) + 225450225*a**(27/2)*b**60*x**(27/2) + 135270135*a**(25/2
)*b**61*x**(25/2) + 61486425*a**(23/2)*b**62*x**(23/2) + 20495475*a**(21/2)
*b**63*x**(21/2) + 4729725*a**(19/2)*b**64*x**(19/2) + 675675*a**(17/2)*b**
65*x**(17/2) + 45045*a**(15/2)*b**66*x**(15/2)) - 28170936*a**(25/2)*b**(12
5/2)*x**5*sqrt(a*x/b + 1)/(45045*a**(45/2)*b**51*x**(45/2) + 675675*a**(43/
2)*b**52*x**(43/2) + 4729725*a**(41/2)*b**53*x**(41/2) + 20495475*a**(39/2)
*b**54*x**(39/2) + 61486425*a**(37/2)*b**55*x**(37/2) + 135270135*a**(35/2)
*b**56*x**(35/2) + 225450225*a**(33/2)*b**57*x**(33/2) + 289864575*a**(31/2
)*b**58*x**(31/2) + 289864575*a**(29/2)*b**59*x**(29/2) + 225450225*a**(27/
2)*b**60*x**(27/2) + 135270135*a**(25/2)*b**61*x**(25/2) + 61486425*a**(23/
2)*b**62*x**(23/2) + 20495475*a**(21/2)*b**63*x**(21/2) + 4729725*a**(19/2)
```

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*b**64*x**(19/2) + 675675*a**(17/2)*b**65*x**(17/2) + 45045*a**(15/2)*b**66
*x**(15/2)) - 11572840*a**(23/2)*b**(127/2)*x**4*sqrt(a*x/b + 1)/(45045*a**
(45/2)*b**51*x**(45/2) + 675675*a**(43/2)*b**52*x**(43/2) + 4729725*a**(41/
2)*b**53*x**(41/2) + 20495475*a**(39/2)*b**54*x**(39/2) + 61486425*a**(37/2
)*b**55*x**(37/2) + 135270135*a**(35/2)*b**56*x**(35/2) + 225450225*a**(33/
2)*b**57*x**(33/2) + 289864575*a**(31/2)*b**58*x**(31/2) + 289864575*a**(29
/2)*b**59*x**(29/2) + 225450225*a**(27/2)*b**60*x**(27/2) + 135270135*a**(2
5/2)*b**61*x**(25/2) + 61486425*a**(23/2)*b**62*x**(23/2) + 20495475*a**(21
/2)*b**63*x**(21/2) + 4729725*a**(19/2)*b**64*x**(19/2) + 675675*a**(17/2)*
b**65*x**(17/2) + 45045*a**(15/2)*b**66*x**(15/2)) - 3510640*a**(21/2)*b**(
129/2)*x**3*sqrt(a*x/b + 1)/(45045*a**(45/2)*b**51*x**(45/2) + 675675*a**(4
3/2)*b**52*x**(43/2) + 4729725*a**(41/2)*b**53*x**(41/2) + 20495475*a**(39/
2)*b**54*x**(39/2) + 61486425*a**(37/2)*b**55*x**(37/2) + 135270135*a**(35/
2)*b**56*x**(35/2) + 225450225*a**(33/2)*b**57*x**(33/2) + 289864575*a**(31
/2)*b**58*x**(31/2) + 289864575*a**(29/2)*b**59*x**(29/2) + 225450225*a**(2
7/2)*b**60*x**(27/2) + 135270135*a**(25/2)*b**61*x**(25/2) + 61486425*a**(2
3/2)*b**62*x**(23/2) + 20495475*a**(21/2)*b**63*x**(21/2) + 4729725*a**(19/
2)*b**64*x**(19/2) + 675675*a**(17/2)*b**65*x**(17/2) + 45045*a**(15/2)*b**
66*x**(15/2)) - 741636*a**(19/2)*b**(131/2)*x**2*sqrt(a*x/b + 1)/(45045*a**
(45/2)*b**51*x**(45/2) + 675675*a**(43/2)*b**52*x**(43/2) + 4729725*a**(41/
2)*b**53*x**(41/2) + 20495475*a**(39/2)*b**54*x**(39/2) + 61486425*a**(37/2
)*b**55*x**(37/2) + 135270135*a**(35/2)*b**56*x**(35/2) + 225450225*a**(33/
2)*b**57*x**(33/2) + 289864575*a**(31/2)*b**58*x**(31/2) + 289864575*a**(29
/2)*b**59*x**(29/2) + 225450225*a**(27/2)*b**60*x**(27/2) + 135270135*a**(2
5/2)*b**61*x**(25/2) + 61486425*a**(23/2)*b**62*x**(23/2) + 20495475*a**(21
/2)*b**63*x**(21/2) + 4729725*a**(19/2)*b**64*x**(19/2) + 675675*a**(17/2)*
b**65*x**(17/2) + 45045*a**(15/2)*b**66*x**(15/2)) - 97482*a**(17/2)*b**(13
3/2)*x*sqrt(a*x/b + 1)/(45045*a**(45/2)*b**51*x**(45/2) + 675675*a**(43/2)*
b**52*x**(43/2) + 4729725*a**(41/2)*b**53*x**(41/2) + 20495475*a**(39/2)*b*
*54*x**(39/2) + 61486425*a**(37/2)*b**55*x**(37/2) + 135270135*a**(35/2)*b*
*56*x**(35/2) + 225450225*a**(33/2)*b**57*x**(33/2) + 289864575*a**(31/2)*b
**58*x**(31/2) + 289864575*a**(29/2)*b**59*x**(29/2) + 225450225*a**(27/2)*
b**60*x**(27/2) + 135270135*a**(25/2)*b**61*x**(25/2) + 61486425*a**(23/2)*
b**62*x**(23/2) + 20495475*a**(21/2)*b**63*x**(21/2) + 4729725*a**(19/2)*b*
*64*x**(19/2) + 675675*a**(17/2)*b**65*x**(17/2) + 45045*a**(15/2)*b**66*x*
*(15/2)) - 6006*a**(15/2)*b**(135/2)*sqrt(a*x/b + 1)/(45045*a**(45/2)*b**51
*x**(45/2) + 675675*a**(43/2)*b**52*x**(43/2) + 4729725*a**(41/2)*b**53*x**
(41/2) + 20495475*a**(39/2)*b**54*x**(39/2) + 61486425*a**(37/2)*b**55*x**(
37/2) + 135270135*a**(35/2)*b**56*x**(35/2) + 225450225*a**(33/2)*b**57*x**
(33/2) + 289864575*a**(31/2)*b**58*x**(31/2) + 289864575*a**(29/2)*b**59*x*
*(29/2) + 225450225*a**(27/2)*b**60*x**(27/2) + 135270135*a**(25/2)*b**61*x
**(25/2) + 61486425*a**(23/2)*b**62*x**(23/2) + 20495475*a**(21/2)*b**63*x*
*(21/2) + 4729725*a**(19/2)*b**64*x**(19/2) + 675675*a**(17/2)*b**65*x**(17
/2) + 45045*a**(15/2)*b**66*x**(15/2)) - 512*a**30*b**45*x**(45/2)/(45045*a
**(45/2)*b**51*x**(45/2) + 675675*a**(43/2)*b**52*x**(43/2) + 4729725*a**(4
1/2)*b**53*x**(41/2) + 20495475*a**(39/2)*b**54*x**(39/2) + 61486425*a**(37
/2)*b**55*x**(37/2) + 135270135*a**(35/2)*b**56*x**(35/2) + 225450225*a**(3
3/2)*b**57*x**(33/2) + 289864575*a**(31/2)*b**58*x**(31/2) + 289864575*a**(
29/2)*b**59*x**(29/2) + 225450225*a**(27/2)*b**60*x**(27/2) + 135270135*a**
(25/2)*b**61*x**(25/2) + 61486425*a**(23/2)*b**62*x**(23/2) + 20495475*a**(
21/2)*b**63*x**(21/2) + 4729725*a**(19/2)*b**64*x**(19/2) + 675675*a**(17/2
)*b**65*x**(17/2) + 45045*a**(15/2)*b**66*x**(15/2)) - 7680*a**29*b**46*x**
(43/2)/(45045*a**(45/2)*b**51*x**(45/2) + 675675*a**(43/2)*b**52*x**(43/2)
+ 4729725*a**(41/2)*b**53*x**(41/2) + 20495475*a**(39/2)*b**54*x**(39/2) +
61486425*a**(37/2)*b**55*x**(37/2) + 135270135*a**(35/2)*b**56*x**(35/2) +
225450225*a**(33/2)*b**57*x**(33/2) + 289864575*a**(31/2)*b**58*x**(31/2) +
 289864575*a**(29/2)*b**59*x**(29/2) + 225450225*a**(27/2)*b**60*x**(27/2)
+ 135270135*a**(25/2)*b**61*x**(25/2) + 61486425*a**(23/2)*b**62*x**(23/2)
+ 20495475*a**(21/2)*b**63*x**(21/2) + 4729725*a**(19/2)*b**64*x**(19/2) +
```

```
675675*a**(17/2)*b**65*x**(17/2) + 45045*a**(15/2)*b**66*x**(15/2)) - 53760
*a**28*b**47*x**(41/2)/(45045*a**(45/2)*b**51*x**(45/2) + 675675*a**(43/2)*
b**52*x**(43/2) + 4729725*a**(41/2)*b**53*x**(41/2) + 20495475*a**(39/2)*b*
*54*x**(39/2) + 61486425*a**(37/2)*b**55*x**(37/2) + 135270135*a**(35/2)*b*
*56*x**(35/2) + 225450225*a**(33/2)*b**57*x**(33/2) + 289864575*a**(31/2)*b
**58*x**(31/2) + 289864575*a**(29/2)*b**59*x**(29/2) + 225450225*a**(27/2)*
b**60*x**(27/2) + 135270135*a**(25/2)*b**61*x**(25/2) + 61486425*a**(23/2)*
b**62*x**(23/2) + 20495475*a**(21/2)*b**63*x**(21/2) + 4729725*a**(19/2)*b*
*64*x**(19/2) + 675675*a**(17/2)*b**65*x**(17/2) + 45045*a**(15/2)*b**66*x*
*(15/2)) - 232960*a**27*b**48*x**(39/2)/(45045*a**(45/2)*b**51*x**(45/2) +
675675*a**(43/2)*b**52*x**(43/2) + 4729725*a**(41/2)*b**53*x**(41/2) + 2049
5475*a**(39/2)*b**54*x**(39/2) + 61486425*a**(37/2)*b**55*x**(37/2) + 13527
0135*a**(35/2)*b**56*x**(35/2) + 225450225*a**(33/2)*b**57*x**(33/2) + 2898
64575*a**(31/2)*b**58*x**(31/2) + 289864575*a**(29/2)*b**59*x**(29/2) + 225
450225*a**(27/2)*b**60*x**(27/2) + 135270135*a**(25/2)*b**61*x**(25/2) + 61
486425*a**(23/2)*b**62*x**(23/2) + 20495475*a**(21/2)*b**63*x**(21/2) + 472
9725*a**(19/2)*b**64*x**(19/2) + 675675*a**(17/2)*b**65*x**(17/2) + 45045*a
**(15/2)*b**66*x**(15/2)) - 698880*a**26*b**49*x**(37/2)/(45045*a**(45/2)*b
**51*x**(45/2) + 675675*a**(43/2)*b**52*x**(43/2) + 4729725*a**(41/2)*b**53
*x**(41/2) + 20495475*a**(39/2)*b**54*x**(39/2) + 61486425*a**(37/2)*b**55*
x**(37/2) + 135270135*a**(35/2)*b**56*x**(35/2) + 225450225*a**(33/2)*b**57
*x**(33/2) + 289864575*a**(31/2)*b**58*x**(31/2) + 289864575*a**(29/2)*b**5
9*x**(29/2) + 225450225*a**(27/2)*b**60*x**(27/2) + 135270135*a**(25/2)*b**
61*x**(25/2) + 61486425*a**(23/2)*b**62*x**(23/2) + 20495475*a**(21/2)*b**6
3*x**(21/2) + 4729725*a**(19/2)*b**64*x**(19/2) + 675675*a**(17/2)*b**65*x*
*(17/2) + 45045*a**(15/2)*b**66*x**(15/2)) - 1537536*a**25*b**50*x**(35/2)/
(45045*a**(45/2)*b**51*x**(45/2) + 675675*a**(43/2)*b**52*x**(43/2) + 47297
25*a**(41/2)*b**53*x**(41/2) + 20495475*a**(39/2)*b**54*x**(39/2) + 6148642
5*a**(37/2)*b**55*x**(37/2) + 135270135*a**(35/2)*b**56*x**(35/2) + 2254502
25*a**(33/2)*b**57*x**(33/2) + 289864575*a**(31/2)*b**58*x**(31/2) + 289864
575*a**(29/2)*b**59*x**(29/2) + 225450225*a**(27/2)*b**60*x**(27/2) + 13527
0135*a**(25/2)*b**61*x**(25/2) + 61486425*a**(23/2)*b**62*x**(23/2) + 20495
475*a**(21/2)*b**63*x**(21/2) + 4729725*a**(19/2)*b**64*x**(19/2) + 675675*
a**(17/2)*b**65*x**(17/2) + 45045*a**(15/2)*b**66*x**(15/2)) - 2562560*a**2
4*b**51*x**(33/2)/(45045*a**(45/2)*b**51*x**(45/2) + 675675*a**(43/2)*b**52
*x**(43/2) + 4729725*a**(41/2)*b**53*x**(41/2) + 20495475*a**(39/2)*b**54*x
**(39/2) + 61486425*a**(37/2)*b**55*x**(37/2) + 135270135*a**(35/2)*b**56*x
**(35/2) + 225450225*a**(33/2)*b**57*x**(33/2) + 289864575*a**(31/2)*b**58*
x**(31/2) + 289864575*a**(29/2)*b**59*x**(29/2) + 225450225*a**(27/2)*b**60
*x**(27/2) + 135270135*a**(25/2)*b**61*x**(25/2) + 61486425*a**(23/2)*b**62
*x**(23/2) + 20495475*a**(21/2)*b**63*x**(21/2) + 4729725*a**(19/2)*b**64*x
**(19/2) + 675675*a**(17/2)*b**65*x**(17/2) + 45045*a**(15/2)*b**66*x**(15/
2)) - 3294720*a**23*b**52*x**(31/2)/(45045*a**(45/2)*b**51*x**(45/2) + 6756
75*a**(43/2)*b**52*x**(43/2) + 4729725*a**(41/2)*b**53*x**(41/2) + 20495475
*a**(39/2)*b**54*x**(39/2) + 61486425*a**(37/2)*b**55*x**(37/2) + 135270135
*a**(35/2)*b**56*x**(35/2) + 225450225*a**(33/2)*b**57*x**(33/2) + 28986457
5*a**(31/2)*b**58*x**(31/2) + 289864575*a**(29/2)*b**59*x**(29/2) + 2254502
25*a**(27/2)*b**60*x**(27/2) + 135270135*a**(25/2)*b**61*x**(25/2) + 614864
25*a**(23/2)*b**62*x**(23/2) + 20495475*a**(21/2)*b**63*x**(21/2) + 4729725
*a**(19/2)*b**64*x**(19/2) + 675675*a**(17/2)*b**65*x**(17/2) + 45045*a**(1
5/2)*b**66*x**(15/2)) - 3294720*a**22*b**53*x**(29/2)/(45045*a**(45/2)*b**5
1*x**(45/2) + 675675*a**(43/2)*b**52*x**(43/2) + 4729725*a**(41/2)*b**53*x*
*(41/2) + 20495475*a**(39/2)*b**54*x**(39/2) + 61486425*a**(37/2)*b**55*x**
(37/2) + 135270135*a**(35/2)*b**56*x**(35/2) + 225450225*a**(33/2)*b**57*x*
*(33/2) + 289864575*a**(31/2)*b**58*x**(31/2) + 289864575*a**(29/2)*b**59*x
**(29/2) + 225450225*a**(27/2)*b**60*x**(27/2) + 135270135*a**(25/2)*b**61*
x**(25/2) + 61486425*a**(23/2)*b**62*x**(23/2) + 20495475*a**(21/2)*b**63*x
**(21/2) + 4729725*a**(19/2)*b**64*x**(19/2) + 675675*a**(17/2)*b**65*x**(1
7/2) + 45045*a**(15/2)*b**66*x**(15/2)) - 2562560*a**21*b**54*x**(27/2)/(45
```

```
045*a**(45/2)*b**51*x**(45/2) + 675675*a**(43/2)*b**52*x**(43/2) + 4729725*
a**(41/2)*b**53*x**(41/2) + 20495475*a**(39/2)*b**54*x**(39/2) + 61486425*a
**(37/2)*b**55*x**(37/2) + 135270135*a**(35/2)*b**56*x**(35/2) + 225450225*
a**(33/2)*b**57*x**(33/2) + 289864575*a**(31/2)*b**58*x**(31/2) + 289864575
*a**(29/2)*b**59*x**(29/2) + 225450225*a**(27/2)*b**60*x**(27/2) + 13527013
5*a**(25/2)*b**61*x**(25/2) + 61486425*a**(23/2)*b**62*x**(23/2) + 20495475
*a**(21/2)*b**63*x**(21/2) + 4729725*a**(19/2)*b**64*x**(19/2) + 675675*a**
(17/2)*b**65*x**(17/2) + 45045*a**(15/2)*b**66*x**(15/2)) - 1537536*a**20*b
**55*x**(25/2)/(45045*a**(45/2)*b**51*x**(45/2) + 675675*a**(43/2)*b**52*x*
*(43/2) + 4729725*a**(41/2)*b**53*x**(41/2) + 20495475*a**(39/2)*b**54*x**(
39/2) + 61486425*a**(37/2)*b**55*x**(37/2) + 135270135*a**(35/2)*b**56*x**(
35/2) + 225450225*a**(33/2)*b**57*x**(33/2) + 289864575*a**(31/2)*b**58*x**
(31/2) + 289864575*a**(29/2)*b**59*x**(29/2) + 225450225*a**(27/2)*b**60*x*
*(27/2) + 135270135*a**(25/2)*b**61*x**(25/2) + 61486425*a**(23/2)*b**62*x*
*(23/2) + 20495475*a**(21/2)*b**63*x**(21/2) + 4729725*a**(19/2)*b**64*x**(
19/2) + 675675*a**(17/2)*b**65*x**(17/2) + 45045*a**(15/2)*b**66*x**(15/2))
 - 698880*a**19*b**56*x**(23/2)/(45045*a**(45/2)*b**51*x**(45/2) + 675675*a
**(43/2)*b**52*x**(43/2) + 4729725*a**(41/2)*b**53*x**(41/2) + 20495475*a**
(39/2)*b**54*x**(39/2) + 61486425*a**(37/2)*b**55*x**(37/2) + 135270135*a**
(35/2)*b**56*x**(35/2) + 225450225*a**(33/2)*b**57*x**(33/2) + 289864575*a*
*(31/2)*b**58*x**(31/2) + 289864575*a**(29/2)*b**59*x**(29/2) + 225450225*a
**(27/2)*b**60*x**(27/2) + 135270135*a**(25/2)*b**61*x**(25/2) + 61486425*a
**(23/2)*b**62*x**(23/2) + 20495475*a**(21/2)*b**63*x**(21/2) + 4729725*a**
(19/2)*b**64*x**(19/2) + 675675*a**(17/2)*b**65*x**(17/2) + 45045*a**(15/2)
*b**66*x**(15/2)) - 232960*a**18*b**57*x**(21/2)/(45045*a**(45/2)*b**51*x**
(45/2) + 675675*a**(43/2)*b**52*x**(43/2) + 4729725*a**(41/2)*b**53*x**(41/
2) + 20495475*a**(39/2)*b**54*x**(39/2) + 61486425*a**(37/2)*b**55*x**(37/2
) + 135270135*a**(35/2)*b**56*x**(35/2) + 225450225*a**(33/2)*b**57*x**(33/
2) + 289864575*a**(31/2)*b**58*x**(31/2) + 289864575*a**(29/2)*b**59*x**(29
/2) + 225450225*a**(27/2)*b**60*x**(27/2) + 135270135*a**(25/2)*b**61*x**(2
5/2) + 61486425*a**(23/2)*b**62*x**(23/2) + 20495475*a**(21/2)*b**63*x**(21
/2) + 4729725*a**(19/2)*b**64*x**(19/2) + 675675*a**(17/2)*b**65*x**(17/2)
+ 45045*a**(15/2)*b**66*x**(15/2)) - 53760*a**17*b**58*x**(19/2)/(45045*a**
(45/2)*b**51*x**(45/2) + 675675*a**(43/2)*b**52*x**(43/2) + 4729725*a**(41/
2)*b**53*x**(41/2) + 20495475*a**(39/2)*b**54*x**(39/2) + 61486425*a**(37/2
)*b**55*x**(37/2) + 135270135*a**(35/2)*b**56*x**(35/2) + 225450225*a**(33/
2)*b**57*x**(33/2) + 289864575*a**(31/2)*b**58*x**(31/2) + 289864575*a**(29
/2)*b**59*x**(29/2) + 225450225*a**(27/2)*b**60*x**(27/2) + 135270135*a**(2
5/2)*b**61*x**(25/2) + 61486425*a**(23/2)*b**62*x**(23/2) + 20495475*a**(21
/2)*b**63*x**(21/2) + 4729725*a**(19/2)*b**64*x**(19/2) + 675675*a**(17/2)*
b**65*x**(17/2) + 45045*a**(15/2)*b**66*x**(15/2)) - 7680*a**16*b**59*x**(1
7/2)/(45045*a**(45/2)*b**51*x**(45/2) + 675675*a**(43/2)*b**52*x**(43/2) +
4729725*a**(41/2)*b**53*x**(41/2) + 20495475*a**(39/2)*b**54*x**(39/2) + 61
486425*a**(37/2)*b**55*x**(37/2) + 135270135*a**(35/2)*b**56*x**(35/2) + 22
5450225*a**(33/2)*b**57*x**(33/2) + 289864575*a**(31/2)*b**58*x**(31/2) + 2
89864575*a**(29/2)*b**59*x**(29/2) + 225450225*a**(27/2)*b**60*x**(27/2) +
135270135*a**(25/2)*b**61*x**(25/2) + 61486425*a**(23/2)*b**62*x**(23/2) +
20495475*a**(21/2)*b**63*x**(21/2) + 4729725*a**(19/2)*b**64*x**(19/2) + 67
5675*a**(17/2)*b**65*x**(17/2) + 45045*a**(15/2)*b**66*x**(15/2)) - 512*a**
15*b**60*x**(15/2)/(45045*a**(45/2)*b**51*x**(45/2) + 675675*a**(43/2)*b**5
2*x**(43/2) + 4729725*a**(41/2)*b**53*x**(41/2) + 20495475*a**(39/2)*b**54*
x**(39/2) + 61486425*a**(37/2)*b**55*x**(37/2) + 135270135*a**(35/2)*b**56*
x**(35/2) + 225450225*a**(33/2)*b**57*x**(33/2) + 289864575*a**(31/2)*b**58
*x**(31/2) + 289864575*a**(29/2)*b**59*x**(29/2) + 225450225*a**(27/2)*b**6
0*x**(27/2) + 135270135*a**(25/2)*b**61*x**(25/2) + 61486425*a**(23/2)*b**6
2*x**(23/2) + 20495475*a**(21/2)*b**63*x**(21/2) + 4729725*a**(19/2)*b**64*
x**(19/2) + 675675*a**(17/2)*b**65*x**(17/2) + 45045*a**(15/2)*b**66*x**(15
/2))
```

3.1712 $\int \left(a+\frac{b}{x}\right)^{5/2} x^3\,dx$

Optimal. Leaf size=111

$$-\frac{5b^4 \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{64a^{3/2}}+\frac{5b^3 x\sqrt{a+\frac{b}{x}}}{64a}+\frac{5}{32}b^2x^2\sqrt{a+\frac{b}{x}}+\frac{1}{4}x^4\left(a+\frac{b}{x}\right)^{5/2}+\frac{5}{24}bx^3\left(a+\frac{b}{x}\right)^{3/2}$$

```
[Out] 5/24*b*(a+b/x)^(3/2)*x^3+1/4*(a+b/x)^(5/2)*x^4-5/64*b^4*arctanh((a+b/x)^(1/
2)/a^(1/2))/a^(3/2)+5/64*b^3*x*(a+b/x)^(1/2)/a+5/32*b^2*x^2*(a+b/x)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {266, 47, 51, 63, 208}

$$-\frac{5b^4 \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{64a^{3/2}}+\frac{5}{32}b^2x^2\sqrt{a+\frac{b}{x}}+\frac{5b^3 x\sqrt{a+\frac{b}{x}}}{64a}+\frac{5}{24}bx^3\left(a+\frac{b}{x}\right)^{3/2}+\frac{1}{4}x^4\left(a+\frac{b}{x}\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)*x^3,x]
```

```
[Out] (5*b^3*Sqrt[a + b/x]*x)/(64*a) + (5*b^2*Sqrt[a + b/x]*x^2)/32 + (5*b*(a + b
/x)^(3/2)*x^3)/24 + ((a + b/x)^(5/2)*x^4)/4 - (5*b^4*ArcTanh[Sqrt[a + b/x]/
Sqrt[a]])/(64*a^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a+\frac{b}{x}\right)^{5/2} x^3\,dx &= -\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^5}\,dx, x, \frac{1}{x}\right) \\
&= \frac{1}{4}\left(a+\frac{b}{x}\right)^{5/2} x^4 - \frac{1}{8}(5b)\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^4}\,dx, x, \frac{1}{x}\right) \\
&= \frac{5}{24}b\left(a+\frac{b}{x}\right)^{3/2} x^3 + \frac{1}{4}\left(a+\frac{b}{x}\right)^{5/2} x^4 - \frac{1}{16}\left(5b^2\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^3}\,dx, x, \frac{1}{x}\right) \\
&= \frac{5}{32}b^2\sqrt{a+\frac{b}{x}}\,x^2 + \frac{5}{24}b\left(a+\frac{b}{x}\right)^{3/2} x^3 + \frac{1}{4}\left(a+\frac{b}{x}\right)^{5/2} x^4 - \frac{1}{64}\left(5b^3\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\right. \\
&= \frac{5b^3\sqrt{a+\frac{b}{x}}\,x}{64a} + \frac{5}{32}b^2\sqrt{a+\frac{b}{x}}\,x^2 + \frac{5}{24}b\left(a+\frac{b}{x}\right)^{3/2} x^3 + \frac{1}{4}\left(a+\frac{b}{x}\right)^{5/2} x^4 + \frac{\left(5b^4\right)\text{Subst}\left(\int\right.}{1} \\
&= \frac{5b^3\sqrt{a+\frac{b}{x}}\,x}{64a} + \frac{5}{32}b^2\sqrt{a+\frac{b}{x}}\,x^2 + \frac{5}{24}b\left(a+\frac{b}{x}\right)^{3/2} x^3 + \frac{1}{4}\left(a+\frac{b}{x}\right)^{5/2} x^4 + \frac{\left(5b^3\right)\text{Subst}\left(\int\right.}{} \\
&= \frac{5b^3\sqrt{a+\frac{b}{x}}\,x}{64a} + \frac{5}{32}b^2\sqrt{a+\frac{b}{x}}\,x^2 + \frac{5}{24}b\left(a+\frac{b}{x}\right)^{3/2} x^3 + \frac{1}{4}\left(a+\frac{b}{x}\right)^{5/2} x^4 - \frac{5b^4\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{64a^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 39, normalized size = 0.35

$$\frac{2b^4\left(a+\frac{b}{x}\right)^{7/2}\,{}_2F_1\left(\frac{7}{2},5;\frac{9}{2};\frac{b}{ax}+1\right)}{7a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)*x^3,x]
```

```
[Out] (2*b^4*(a + b/x)^(7/2)*Hypergeometric2F1[7/2, 5, 9/2, 1 + b/(a*x)])/(7*a^5)
```

fricas [A] time = 0.96, size = 173, normalized size = 1.56

$$\left[\frac{15\sqrt{a}\,b^4\log\left(2\,ax - 2\sqrt{a}\,x\sqrt{\frac{ax+b}{x}} + b\right) + 2\left(48\,a^4x^4 + 136\,a^3bx^3 + 118\,a^2b^2x^2 + 15\,ab^3x\right)\sqrt{\frac{ax+b}{x}}}{384\,a^2}, \frac{15\sqrt{-a}\,b^4\,\text{a}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*x^3,x, algorithm="fricas")
```

```
[Out] [1/384*(15*sqrt(a)*b^4*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(
48*a^4*x^4 + 136*a^3*b*x^3 + 118*a^2*b^2*x^2 + 15*a*b^3*x)*sqrt((a*x + b)/x
))/a^2, 1/192*(15*sqrt(-a)*b^4*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (48*a
```

```
^4*x^4 + 136*a^3*b*x^3 + 118*a^2*b^2*x^2 + 15*a*b^3*x)*sqrt((a*x + b)/x))/a
^2]
```

giac [A] time = 0.23, size = 107, normalized size = 0.96

$$\frac{5\,b^4\log\left(\left|-2\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)\sqrt{a}-b\right|\right)\operatorname{sgn}(x)}{128\,a^{\frac{3}{2}}}-\frac{5\,b^4\log\left(|b|\right)\operatorname{sgn}(x)}{128\,a^{\frac{3}{2}}}+\frac{1}{192}\sqrt{ax^2+bx}\left(\frac{15\,b^3\operatorname{sgn}(x)}{a}+2\left(59\,b^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*x^3,x, algorithm="giac")
```

```
[Out] 5/128*b^4*log(abs(-2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) - b))*sgn(x)/a
^(3/2) - 5/128*b^4*log(abs(b))*sgn(x)/a^(3/2) + 1/192*sqrt(a*x^2 + b*x)*(15
*b^3*sgn(x)/a + 2*(59*b^2*sgn(x) + 4*(6*a^2*x*sgn(x) + 17*a*b*sgn(x))*x)*x)
```

maple [A] time = 0.01, size = 135, normalized size = 1.22

$$\frac{\sqrt{\frac{ax+b}{x}}\left(-15a\,b^4\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)+60\sqrt{a\,x^2+bx}\;a^{\frac{5}{2}}b^2x+96\left(a\,x^2+bx\right)^{\frac{3}{2}}a^{\frac{7}{2}}x+30\sqrt{a\,x^2+bx}\;a^{\frac{3}{2}}b^3+176\right.}{384\sqrt{(ax+b)\,x}\;a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2)*x^3,x)
```

```
[Out] 1/384*((a*x+b)/x)^(1/2)*x/a^(5/2)*(96*(a*x^2+b*x)^(3/2)*a^(7/2)*x+176*(a*x^
2+b*x)^(3/2)*a^(5/2)*b+60*(a*x^2+b*x)^(1/2)*a^(5/2)*b^2*x+30*(a*x^2+b*x)^(1
/2)*a^(3/2)*b^3-15*a*b^4*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/
2)))/((a*x+b)*x)^(1/2)
```

maxima [A] time = 2.29, size = 164, normalized size = 1.48

$$\frac{5\,b^4\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{128\,a^{\frac{3}{2}}}+\frac{15\left(a+\frac{b}{x}\right)^{\frac{7}{2}}b^4+73\left(a+\frac{b}{x}\right)^{\frac{5}{2}}ab^4-55\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2b^4+15\sqrt{a+\frac{b}{x}}\,a^3b^4}{192\left(\left(a+\frac{b}{x}\right)^4a-4\left(a+\frac{b}{x}\right)^3a^2+6\left(a+\frac{b}{x}\right)^2a^3-4\left(a+\frac{b}{x}\right)a^4+a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*x^3,x, algorithm="maxima")
```

```
[Out] 5/128*b^4*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2)
+ 1/192*(15*(a + b/x)^(7/2)*b^4 + 73*(a + b/x)^(5/2)*a*b^4 - 55*(a + b/x)^(
3/2)*a^2*b^4 + 15*sqrt(a + b/x)*a^3*b^4)/((a + b/x)^4*a - 4*(a + b/x)^3*a^2
 + 6*(a + b/x)^2*a^3 - 4*(a + b/x)*a^4 + a^5)
```

mupad [B] time = 1.37, size = 89, normalized size = 0.80

$$\frac{73\,x^4\left(a+\frac{b}{x}\right)^{5/2}}{192}-\frac{55\,a\,x^4\left(a+\frac{b}{x}\right)^{3/2}}{192}+\frac{5\,a^2\,x^4\sqrt{a+\frac{b}{x}}}{64}+\frac{5\,x^4\left(a+\frac{b}{x}\right)^{7/2}}{64\,a}+\frac{b^4\operatorname{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1\mathrm{i}}{\sqrt{a}}\right)5\mathrm{i}}{64\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b/x)^(5/2),x)
```

```
[Out] (73*x^4*(a + b/x)^(5/2))/192 + (b^4*atan(((a + b/x)^(1/2)*1i)/a^(1/2))*5i)/
(64*a^(3/2)) - (55*a*x^4*(a + b/x)^(3/2))/192 + (5*a^2*x^4*(a + b/x)^(1/2))
/64 + (5*x^4*(a + b/x)^(7/2))/(64*a)
```

sympy [**A**] time = 8.59, size = 155, normalized size = 1.40

$$\frac{a^3 x^{\frac{9}{2}}}{4\sqrt{b}\sqrt{\frac{ax}{b}+1}} + \frac{23a^2\sqrt{b}\,x^{\frac{7}{2}}}{24\sqrt{\frac{ax}{b}+1}} + \frac{127ab^{\frac{3}{2}}x^{\frac{5}{2}}}{96\sqrt{\frac{ax}{b}+1}} + \frac{133b^{\frac{5}{2}}x^{\frac{3}{2}}}{192\sqrt{\frac{ax}{b}+1}} + \frac{5b^{\frac{7}{2}}\sqrt{x}}{64a\sqrt{\frac{ax}{b}+1}} - \frac{5b^4\,\mathrm{asinh}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{64a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)*x**3,x)
```

```
[Out] a**3*x**(9/2)/(4*sqrt(b)*sqrt(a*x/b + 1)) + 23*a**2*sqrt(b)*x**(7/2)/(24*sq
rt(a*x/b + 1)) + 127*a*b**(3/2)*x**(5/2)/(96*sqrt(a*x/b + 1)) + 133*b**(5/2
)*x**(3/2)/(192*sqrt(a*x/b + 1)) + 5*b**(7/2)*sqrt(x)/(64*a*sqrt(a*x/b + 1)
) - 5*b**4*asinh(sqrt(a)*sqrt(x)/sqrt(b))/(64*a**(3/2))
```

3.1713 $\int \left(a+\frac{b}{x}\right)^{5/2} x^2\,dx$

Optimal. Leaf size=87

$$\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{8\sqrt{a}}+\frac{5}{8}b^2x\sqrt{a+\frac{b}{x}}+\frac{1}{3}x^3\left(a+\frac{b}{x}\right)^{5/2}+\frac{5}{12}bx^2\left(a+\frac{b}{x}\right)^{3/2}$$

```
[Out] 5/12*b*(a+b/x)^(3/2)*x^2+1/3*(a+b/x)^(5/2)*x^3+5/8*b^3*arctanh((a+b/x)^(1/2
)/a^(1/2))/a^(1/2)+5/8*b^2*x*(a+b/x)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 47, 63, 208}

$$\frac{5}{8}b^2x\sqrt{a+\frac{b}{x}}+\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{8\sqrt{a}}+\frac{5}{12}bx^2\left(a+\frac{b}{x}\right)^{3/2}+\frac{1}{3}x^3\left(a+\frac{b}{x}\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)*x^2,x]
```

```
[Out] (5*b^2*Sqrt[a + b/x]*x)/8 + (5*b*(a + b/x)^(3/2)*x^2)/12 + ((a + b/x)^(5/2)
*x^3)/3 + (5*b^3*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(8*Sqrt[a])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^2\,dx = -\operatorname{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^4}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{3}\left(a+\frac{b}{x}\right)^{5/2} x^3 - \frac{1}{6}(5b)\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^3}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{5}{12}b\left(a+\frac{b}{x}\right)^{3/2} x^2 + \frac{1}{3}\left(a+\frac{b}{x}\right)^{5/2} x^3 - \frac{1}{8}\left(5b^2\right)\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{5}{8}b^2\sqrt{a+\frac{b}{x}}\,x + \frac{5}{12}b\left(a+\frac{b}{x}\right)^{3/2} x^2 + \frac{1}{3}\left(a+\frac{b}{x}\right)^{5/2} x^3 - \frac{1}{16}\left(5b^3\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,$$
$$= \frac{5}{8}b^2\sqrt{a+\frac{b}{x}}\,x + \frac{5}{12}b\left(a+\frac{b}{x}\right)^{3/2} x^2 + \frac{1}{3}\left(a+\frac{b}{x}\right)^{5/2} x^3 - \frac{1}{8}\left(5b^2\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x,$$
$$= \frac{5}{8}b^2\sqrt{a+\frac{b}{x}}\,x + \frac{5}{12}b\left(a+\frac{b}{x}\right)^{3/2} x^2 + \frac{1}{3}\left(a+\frac{b}{x}\right)^{5/2} x^3 + \frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{8\sqrt{a}}$$

Mathematica [A] time = 0.04, size = 87, normalized size = 1.00

$$\frac{x\sqrt{a+\frac{b}{x}}\left(8a^3x^3+34a^2bx^2+15b^3\sqrt{\frac{b}{ax}+1}\,\tanh^{-1}\left(\sqrt{\frac{b}{ax}+1}\right)+59ab^2x+33b^3\right)}{24(ax+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)*x^2,x]
```

```
[Out] (Sqrt[a + b/x]*x*(33*b^3 + 59*a*b^2*x + 34*a^2*b*x^2 + 8*a^3*x^3 + 15*b^3*S
qrt[1 + b/(a*x)]*ArcTanh[Sqrt[1 + b/(a*x)]]))/(24*(b + a*x))
```

fricas [A] time = 1.21, size = 152, normalized size = 1.75

$$\left[\frac{15\sqrt{a}\,b^3\log\left(2ax+2\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(8a^3x^3+26a^2bx^2+33ab^2x\right)\sqrt{\frac{ax+b}{x}}}{48a}, -\frac{15\sqrt{-a}\,b^3\arctan\left(\frac{\sqrt{-a}\sqrt{a}}{a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*x^2,x, algorithm="fricas")
```

```
[Out] [1/48*(15*sqrt(a)*b^3*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(8
*a^3*x^3 + 26*a^2*b*x^2 + 33*a*b^2*x)*sqrt((a*x + b)/x))/a, -1/24*(15*sqrt(
-a)*b^3*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (8*a^3*x^3 + 26*a^2*b*x^2 +
33*a*b^2*x)*sqrt((a*x + b)/x))/a]
```

giac [A] time = 0.25, size = 93, normalized size = 1.07

$$-\frac{5\,b^3\log\left(\left|-2\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)\sqrt{a}-b\right|\right)\operatorname{sgn}(x)}{16\sqrt{a}}+\frac{5\,b^3\log\left(|b|\right)\operatorname{sgn}(x)}{16\sqrt{a}}+\frac{1}{24}\sqrt{ax^2+bx}\left(33\,b^2\operatorname{sgn}(x)+2\left(4\,a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*x^2,x, algorithm="giac")
```

```
[Out] -5/16*b^3*log(abs(-2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) - b))*sgn(x)/s
qrt(a) + 5/16*b^3*log(abs(b))*sgn(x)/sqrt(a) + 1/24*sqrt(a*x^2 + b*x)*(33*b
^2*sgn(x) + 2*(4*a^2*x*sgn(x) + 13*a*b*sgn(x))*x)
```

maple [A] time = 0.01, size = 115, normalized size = 1.32

$$\frac{\sqrt{\frac{ax+b}{x}}\left(15a\,b^3\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)+36\sqrt{a\,x^2+bx}\;a^{\frac{5}{2}}bx+66\sqrt{a\,x^2+bx}\;a^{\frac{3}{2}}b^2+16\left(a\,x^2+bx\right)^{\frac{3}{2}}a^{\frac{5}{2}}\right)x}{48\sqrt{(ax+b)\,x}\;a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2)*x^2,x)
```

```
[Out] 1/48*((a*x+b)/x)^(1/2)*x*(16*(a*x^2+b*x)^(3/2)*a^(5/2)+36*(a*x^2+b*x)^(1/2)
*a^(5/2)*b*x+66*(a*x^2+b*x)^(1/2)*a^(3/2)*b^2+15*a*b^3*ln(1/2*(2*a*x+b+2*(a
*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2)))/((a*x+b)*x)^(1/2)/a^(3/2)
```

maxima [A] time = 2.31, size = 131, normalized size = 1.51

$$-\frac{5\,b^3\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{16\sqrt{a}}+\frac{33\left(a+\frac{b}{x}\right)^{\frac{5}{2}}b^3-40\left(a+\frac{b}{x}\right)^{\frac{3}{2}}ab^3+15\sqrt{a+\frac{b}{x}}\,a^2b^3}{24\left(\left(a+\frac{b}{x}\right)^3-3\left(a+\frac{b}{x}\right)^2a+3\left(a+\frac{b}{x}\right)a^2-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*x^2,x, algorithm="maxima")
```

```
[Out] -5/16*b^3*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/sqrt(a)
+ 1/24*(33*(a + b/x)^(5/2)*b^3 - 40*(a + b/x)^(3/2)*a*b^3 + 15*sqrt(a + b/x
)*a^2*b^3)/((a + b/x)^3 - 3*(a + b/x)^2*a + 3*(a + b/x)*a^2 - a^3)
```

mupad [B] time = 1.35, size = 72, normalized size = 0.83

$$\frac{11\,x^3\left(a+\frac{b}{x}\right)^{5/2}}{8}-\frac{5\,a\,x^3\left(a+\frac{b}{x}\right)^{3/2}}{3}+\frac{5\,a^2\,x^3\sqrt{a+\frac{b}{x}}}{8}-\frac{b^3\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1\mathrm{i}}{\sqrt{a}}\right)5\mathrm{i}}{8\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b/x)^(5/2),x)
```

```
[Out] (11*x^3*(a + b/x)^(5/2))/8 - (b^3*atan(((a + b/x)^(1/2)*1i)/a^(1/2))*5i)/(8
*a^(1/2)) - (5*a*x^3*(a + b/x)^(3/2))/3 + (5*a^2*x^3*(a + b/x)^(1/2))/8
```

sympy [A] time = 5.58, size = 102, normalized size = 1.17

$$\frac{a^2\sqrt{b}\,x^{\frac{5}{2}}\sqrt{\frac{ax}{b}+1}}{3}+\frac{13ab^{\frac{3}{2}}x^{\frac{3}{2}}\sqrt{\frac{ax}{b}+1}}{12}+\frac{11b^{\frac{5}{2}}\sqrt{x}\sqrt{\frac{ax}{b}+1}}{8}+\frac{5b^3\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{8\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)*x**2,x)
```

```
[Out] a**2*sqrt(b)*x**(5/2)*sqrt(a*x/b + 1)/3 + 13*a*b**(3/2)*x**(3/2)*sqrt(a*x/b
 + 1)/12 + 11*b**(5/2)*sqrt(x)*sqrt(a*x/b + 1)/8 + 5*b**3*asinh(sqrt(a)*sqr
t(x)/sqrt(b))/(8*sqrt(a))
```

3.1714 $\int \left(a+\frac{b}{x}\right)^{5/2} x\,dx$

Optimal. Leaf size=84

$$-\frac{15}{4}b^2\sqrt{a+\frac{b}{x}}+\frac{15}{4}\sqrt{a}\,b^2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+\frac{1}{2}x^2\left(a+\frac{b}{x}\right)^{5/2}+\frac{5}{4}bx\left(a+\frac{b}{x}\right)^{3/2}$$

```
[Out] 5/4*b*(a+b/x)^(3/2)*x+1/2*(a+b/x)^(5/2)*x^2+15/4*b^2*arctanh((a+b/x)^(1/2)/
a^(1/2))*a^(1/2)-15/4*b^2*(a+b/x)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {266, 47, 50, 63, 208}

$$-\frac{15}{4}b^2\sqrt{a+\frac{b}{x}}+\frac{15}{4}\sqrt{a}\,b^2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+\frac{1}{2}x^2\left(a+\frac{b}{x}\right)^{5/2}+\frac{5}{4}bx\left(a+\frac{b}{x}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)*x,x]
```

```
[Out] (-15*b^2*Sqrt[a + b/x])/4 + (5*b*(a + b/x)^(3/2)*x)/4 + ((a + b/x)^(5/2)*x^
2)/2 + (15*Sqrt[a]*b^2*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/4
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a+\frac{b}{x}\right)^{5/2} x\,dx &= -\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^3}\,dx, x, \frac{1}{x}\right) \\
&= \frac{1}{2}\left(a+\frac{b}{x}\right)^{5/2} x^2 - \frac{1}{4}(5b)\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2}\,dx, x, \frac{1}{x}\right) \\
&= \frac{5}{4}b\left(a+\frac{b}{x}\right)^{3/2} x + \frac{1}{2}\left(a+\frac{b}{x}\right)^{5/2} x^2 - \frac{1}{8}\left(15b^2\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x}\right) \\
&= -\frac{15}{4}b^2\sqrt{a+\frac{b}{x}} + \frac{5}{4}b\left(a+\frac{b}{x}\right)^{3/2} x + \frac{1}{2}\left(a+\frac{b}{x}\right)^{5/2} x^2 - \frac{1}{8}\left(15ab^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \right. \\
&= -\frac{15}{4}b^2\sqrt{a+\frac{b}{x}} + \frac{5}{4}b\left(a+\frac{b}{x}\right)^{3/2} x + \frac{1}{2}\left(a+\frac{b}{x}\right)^{5/2} x^2 - \frac{1}{4}(15ab)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\right. \\
&= -\frac{15}{4}b^2\sqrt{a+\frac{b}{x}} + \frac{5}{4}b\left(a+\frac{b}{x}\right)^{3/2} x + \frac{1}{2}\left(a+\frac{b}{x}\right)^{5/2} x^2 + \frac{15}{4}\sqrt{a}\,b^2 \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [C] time = 0.04, size = 39, normalized size = 0.46

$$\frac{2b^2\left(a+\frac{b}{x}\right)^{7/2}\,{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};\frac{b}{ax}+1\right)}{7a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)*x,x]
```

```
[Out] (2*b^2*(a + b/x)^(7/2)*Hypergeometric2F1[3, 7/2, 9/2, 1 + b/(a*x)])/(7*a^3)
```

fricas [A] time = 0.79, size = 130, normalized size = 1.55

$$\left[\frac{15}{8}\sqrt{a}\,b^2\log\left(2\,ax+2\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+\frac{1}{4}\left(2\,a^2x^2+9\,abx-8\,b^2\right)\sqrt{\frac{ax+b}{x}}, -\frac{15}{4}\sqrt{-a}\,b^2\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+}{x}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*x,x, algorithm="fricas")
```

```
[Out] [15/8*sqrt(a)*b^2*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 1/4*(2*a
^2*x^2 + 9*a*b*x - 8*b^2)*sqrt((a*x + b)/x), -15/4*sqrt(-a)*b^2*arctan(sqrt
(-a)*sqrt((a*x + b)/x)/a) + 1/4*(2*a^2*x^2 + 9*a*b*x - 8*b^2)*sqrt((a*x + b
)/x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%%}+%%%{-2
,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%%%}+%%%{2
,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters values [86
,-97,-82]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%
%},0,%%%{1,[0,2,2]%%%}] at parameters values [82.1195442914,26,-89]Warning,
 choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[0,2,2
]%%%}] at parameters values [85.3561567818,-64,-30]Warning, choosing root o
f [1,0,%%%{-2,[1,0,1]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,2]%%%}] at parame
ters values [42,43.9628838282,-9]Sign error (%%%{-b,0%%%}+%%%{2*sqrt(a)*sqr
t(b),1/2%%%}+%%%{-2*a,1%%%}+%%%{a*sqrt(a)*sqrt(b)/b,3/2%%%}+%%%{-a^2*sqrt(a
)*sqrt(b)/(4*b^2),5/2%%%}+%%%{undef,7/2%%%})Limit: Max order reached or una
ble to make series expansion Error: Bad Argument Value
```

maple [A] time = 0.01, size = 125, normalized size = 1.49

$$-\frac{\sqrt{\frac{ax+b}{x}}\left(-15a^2b^2x^2\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)-4\sqrt{a\,x^2+bx}\;a^{\frac{7}{2}}x^3-34\sqrt{a\,x^2+bx}\;a^{\frac{5}{2}}b\,x^2+16\left(a\,x^2+bx\right)^{\frac{3}{2}}a^{\frac{3}{2}}b\right.}{8\sqrt{(ax+b)\,x}\;a^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2)*x,x)
```

```
[Out] -1/8*((a*x+b)/x)^(1/2)/x*(-4*(a*x^2+b*x)^(1/2)*a^(7/2)*x^3-34*(a*x^2+b*x)^(
1/2)*a^(5/2)*x^2*b-15*a^2*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1
/2))*x^2*b^2+16*(a*x^2+b*x)^(3/2)*a^(3/2)*b)/((a*x+b)*x)^(1/2)/a^(3/2)
```

maxima [A] time = 2.53, size = 115, normalized size = 1.37

$$-\frac{15}{8}\sqrt{a}\,b^2\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-2\sqrt{a+\frac{b}{x}}\,b^2+\frac{9\left(a+\frac{b}{x}\right)^{\frac{3}{2}}ab^2-7\sqrt{a+\frac{b}{x}}\,a^2b^2}{4\left(\left(a+\frac{b}{x}\right)^2-2\left(a+\frac{b}{x}\right)a+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*x,x, algorithm="maxima")
```

```
[Out] -15/8*sqrt(a)*b^2*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))
- 2*sqrt(a + b/x)*b^2 + 1/4*(9*(a + b/x)^(3/2)*a*b^2 - 7*sqrt(a + b/x)*a^2*
b^2)/((a + b/x)^2 - 2*(a + b/x)*a + a^2)
```

mupad [B] time = 1.32, size = 72, normalized size = 0.86

$$\frac{9\,a\,x^2\left(a+\frac{b}{x}\right)^{3/2}}{4}-2\,b^2\sqrt{a+\frac{b}{x}}-\frac{7\,a^2\,x^2\sqrt{a+\frac{b}{x}}}{4}-\frac{\sqrt{a}\;b^2\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1\mathrm{i}}{\sqrt{a}}\right)15\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b/x)^(5/2),x)
```

```
[Out] (9*a*x^2*(a + b/x)^(3/2))/4 - (a^(1/2)*b^2*atan(((a + b/x)^(1/2)*1i)/a^(1/2
))*15i)/4 - 2*b^2*(a + b/x)^(1/2) - (7*a^2*x^2*(a + b/x)^(1/2))/4
```

sympy [A] time = 4.20, size = 126, normalized size = 1.50

$$\frac{15\sqrt{a}\,b^2\,\mathrm{asinh}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{4}+\frac{a^3x^{\frac{5}{2}}}{2\sqrt{b}\,\sqrt{\frac{ax}{b}+1}}+\frac{11a^2\sqrt{b}\,x^{\frac{3}{2}}}{4\sqrt{\frac{ax}{b}+1}}+\frac{ab^{\frac{3}{2}}\sqrt{x}}{4\sqrt{\frac{ax}{b}+1}}-\frac{2b^{\frac{5}{2}}}{\sqrt{x}\,\sqrt{\frac{ax}{b}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)*x,x)
```

```
[Out] 15*sqrt(a)*b**2*asinh(sqrt(a)*sqrt(x)/sqrt(b))/4 + a**3*x**(5/2)/(2*sqrt(b)
*sqrt(a*x/b + 1)) + 11*a**2*sqrt(b)*x**(3/2)/(4*sqrt(a*x/b + 1)) + a*b**(3/
2)*sqrt(x)/(4*sqrt(a*x/b + 1)) - 2*b**(5/2)/(sqrt(x)*sqrt(a*x/b + 1))
```

3.1715 $\int \left(a+\frac{b}{x}\right)^{5/2} dx$

Optimal. Leaf size=71

$$5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+x\left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{3}b\left(a+\frac{b}{x}\right)^{3/2}-5ab\sqrt{a+\frac{b}{x}}$$

```
[Out] -5/3*b*(a+b/x)^(3/2)+(a+b/x)^(5/2)*x+5*a^(3/2)*b*arctanh((a+b/x)^(1/2)/a^(1
/2))-5*a*b*(a+b/x)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {242, 47, 50, 63, 208}

$$5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+x\left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{3}b\left(a+\frac{b}{x}\right)^{3/2}-5ab\sqrt{a+\frac{b}{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2),x]
```

```
[Out] -5*a*b*Sqrt[a + b/x] - (5*b*(a + b/x)^(3/2))/3 + (a + b/x)^(5/2)*x + 5*a^(3
/2)*b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(a+\frac{b}{x}\right)^{5/2} dx &= -\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^2}\,dx, x, \frac{1}{x}\right) \\
&= \left(a+\frac{b}{x}\right)^{5/2} x - \frac{1}{2}(5b)\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, \frac{1}{x}\right) \\
&= -\frac{5}{3}b\left(a+\frac{b}{x}\right)^{3/2} + \left(a+\frac{b}{x}\right)^{5/2} x - \frac{1}{2}(5ab)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x}\right) \\
&= -5ab\sqrt{a+\frac{b}{x}} - \frac{5}{3}b\left(a+\frac{b}{x}\right)^{3/2} + \left(a+\frac{b}{x}\right)^{5/2} x - \frac{1}{2}\left(5a^2b\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right) \\
&= -5ab\sqrt{a+\frac{b}{x}} - \frac{5}{3}b\left(a+\frac{b}{x}\right)^{3/2} + \left(a+\frac{b}{x}\right)^{5/2} x - \left(5a^2\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right) \\
&= -5ab\sqrt{a+\frac{b}{x}} - \frac{5}{3}b\left(a+\frac{b}{x}\right)^{3/2} + \left(a+\frac{b}{x}\right)^{5/2} x + 5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [A] time = 0.07, size = 64, normalized size = 0.90

$$5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right) + \frac{\sqrt{a+\frac{b}{x}}\left(3a^2x^2-14abx-2b^2\right)}{3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2),x]
```

```
[Out] (Sqrt[a + b/x]*(-2*b^2 - 14*a*b*x + 3*a^2*x^2))/(3*x) + 5*a^(3/2)*b*ArcTanh
[Sqrt[a + b/x]/Sqrt[a]]
```

fricas [A] time = 0.77, size = 139, normalized size = 1.96

$$\left[\frac{15\,a^{\frac{3}{2}}bx\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(3\,a^2x^2-14\,abx-2\,b^2\right)\sqrt{\frac{ax+b}{x}}}{6\,x}, -\frac{15\,\sqrt{-a}\,abx\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-\left(3\right.}{3\,x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*a^(3/2)*b*x*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(3*
a^2*x^2 - 14*a*b*x - 2*b^2)*sqrt((a*x + b)/x))/x, -1/3*(15*sqrt(-a)*a*b*x*a
rctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (3*a^2*x^2 - 14*a*b*x - 2*b^2)*sqrt((
a*x + b)/x))/x]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%%}+%%%{-2
,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%%%}+%%%{2
,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters values [86
,-97,-82]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%
%},0,%%%{1,[0,2,2]%%%}] at parameters values [82.1195442914,26,-89]Warning,
 choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[0,2,2
]%%%}] at parameters values [85.3561567818,-64,-30]Warning, choosing root o
f [1,0,%%%{-2,[1,0,1]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,2]%%%}] at parame
ters values [42,43.9628838282,-9]Sign error (%%%{-b,0%%%}+%%%{2*sqrt(a)*sqr
t(b),1/2%%%}+%%%{-2*a,1%%%}+%%%{a*sqrt(a)*sqrt(b)/b,3/2%%%}+%%%{-a^2*sqrt(a
)*sqrt(b)/(4*b^2),5/2%%%}+%%%{undef,7/2%%%})Limit: Max order reached or una
ble to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.01, size = 120, normalized size = 1.69

$$-\frac{\sqrt{\frac{ax+b}{x}}\left(-15a^2bx^3\ln\left(\frac{2ax+b+2\sqrt{ax^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)-30\sqrt{ax^2+bx}\,a^{\frac{5}{2}}x^3+24\left(ax^2+bx\right)^{\frac{3}{2}}a^{\frac{3}{2}}x+4\left(ax^2+bx\right)^{\frac{3}{2}}\sqrt{a}\,b\right.}{6\sqrt{(ax+b)x}\,\sqrt{a}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2),x)
```

```
[Out] -1/6*((a*x+b)/x)^(1/2)/x^2*(-30*(a*x^2+b*x)^(1/2)*a^(5/2)*x^3-15*a^2*b*x^3*
ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))+24*(a*x^2+b*x)^(3/2)*
a^(3/2)*x+4*(a*x^2+b*x)^(3/2)*a^(1/2)*b)/((a*x+b)*x)^(1/2)/a^(1/2)
```

maxima [A] time = 2.43, size = 78, normalized size = 1.10

$$\sqrt{a+\frac{b}{x}}\,a^2x-\frac{5}{2}a^{\frac{3}{2}}b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-\frac{2}{3}\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b-4\sqrt{a+\frac{b}{x}}\,ab$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2),x, algorithm="maxima")
```

```
[Out] sqrt(a + b/x)*a^2*x - 5/2*a^(3/2)*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a +
 b/x) + sqrt(a))) - 2/3*(a + b/x)^(3/2)*b - 4*sqrt(a + b/x)*a*b
```

mupad [B] time = 1.39, size = 34, normalized size = 0.48

$$-\frac{2x\left(a+\frac{b}{x}\right)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},-\frac{3}{2};\,-\frac{1}{2};\,-\frac{ax}{b}\right)}{3\left(\frac{ax}{b}+1\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(5/2),x)
```

```
[Out] -(2*x*(a + b/x)^(5/2)*hypergeom([-5/2, -3/2], -1/2, -(a*x)/b))/(3*((a*x)/b
+ 1)^(5/2))
```

sympy [A] time = 3.90, size = 99, normalized size = 1.39

$$a^{\frac{5}{2}}x\sqrt{1+\frac{b}{ax}}-\frac{14a^{\frac{3}{2}}b\sqrt{1+\frac{b}{ax}}}{3}-\frac{5a^{\frac{3}{2}}b\log\left(\frac{b}{ax}\right)}{2}+5a^{\frac{3}{2}}b\log\left(\sqrt{1+\frac{b}{ax}}+1\right)-\frac{2\sqrt{a}\,b^2\sqrt{1+\frac{b}{ax}}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2),x)
```

```
[Out] a**(5/2)*x*sqrt(1 + b/(a*x)) - 14*a**(3/2)*b*sqrt(1 + b/(a*x))/3 - 5*a**(3/
2)*b*log(b/(a*x))/2 + 5*a**(3/2)*b*log(sqrt(1 + b/(a*x)) + 1) - 2*sqrt(a)*b
**2*sqrt(1 + b/(a*x))/(3*x)
```

3.1716 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x}\,dx$

Optimal. Leaf size=73

$$2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-2a^2\sqrt{a+\frac{b}{x}}-\frac{2}{3}a\left(a+\frac{b}{x}\right)^{3/2}-\frac{2}{5}\left(a+\frac{b}{x}\right)^{5/2}$$

```
[Out] -2/3*a*(a+b/x)^(3/2)-2/5*(a+b/x)^(5/2)+2*a^(5/2)*arctanh((a+b/x)^(1/2)/a^(1
/2))-2*a^2*(a+b/x)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 50, 63, 208}

$$-2a^2\sqrt{a+\frac{b}{x}}+2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-\frac{2}{3}a\left(a+\frac{b}{x}\right)^{3/2}-\frac{2}{5}\left(a+\frac{b}{x}\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)/x,x]
```

```
[Out] -2*a^2*Sqrt[a + b/x] - (2*a*(a + b/x)^(3/2))/3 - (2*(a + b/x)^(5/2))/5 + 2*
a^(5/2)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x}\,dx &= -\operatorname{Subst}\left(\int \frac{(a+bx)^{5/2}}{x}\,dx, x, \frac{1}{x}\right) \\
&= -\frac{2}{5}\left(a+\frac{b}{x}\right)^{5/2} - a\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, \frac{1}{x}\right) \\
&= -\frac{2}{3}a\left(a+\frac{b}{x}\right)^{3/2} - \frac{2}{5}\left(a+\frac{b}{x}\right)^{5/2} - a^2\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x}\right) \\
&= -2a^2\sqrt{a+\frac{b}{x}} - \frac{2}{3}a\left(a+\frac{b}{x}\right)^{3/2} - \frac{2}{5}\left(a+\frac{b}{x}\right)^{5/2} - a^3\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right) \\
&= -2a^2\sqrt{a+\frac{b}{x}} - \frac{2}{3}a\left(a+\frac{b}{x}\right)^{3/2} - \frac{2}{5}\left(a+\frac{b}{x}\right)^{5/2} - \frac{\left(2a^3\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{b} \\
&= -2a^2\sqrt{a+\frac{b}{x}} - \frac{2}{3}a\left(a+\frac{b}{x}\right)^{3/2} - \frac{2}{5}\left(a+\frac{b}{x}\right)^{5/2} + 2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [A] time = 0.08, size = 63, normalized size = 0.86

$$2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right) - \frac{2\sqrt{a+\frac{b}{x}}\left(23a^2x^2+11abx+3b^2\right)}{15x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)/x,x]
```

```
[Out] (-2*Sqrt[a + b/x]*(3*b^2 + 11*a*b*x + 23*a^2*x^2))/(15*x^2) + 2*a^(5/2)*Arc
Tanh[Sqrt[a + b/x]/Sqrt[a]]
```

fricas [A] time = 1.10, size = 142, normalized size = 1.95

$$\left[\frac{15\,a^{\frac{5}{2}}x^2\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)-2\left(23\,a^2x^2+11\,abx+3\,b^2\right)\sqrt{\frac{ax+b}{x}}}{15\,x^2}, -\frac{2\left(15\,\sqrt{-a}\,a^2x^2\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right.\right.}{15}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [1/15*(15*a^(5/2)*x^2*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(2
3*a^2*x^2 + 11*a*b*x + 3*b^2)*sqrt((a*x + b)/x))/x^2, -2/15*(15*sqrt(-a)*a^
2*x^2*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (23*a^2*x^2 + 11*a*b*x + 3*b^2
)*sqrt((a*x + b)/x))/x^2]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%%}+%%%{-2
,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%%%}+%%%{2
,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters values [86
,-97,-82]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%
%},0,%%%{1,[0,2,2]%%%}] at parameters values [82.1195442914,26,-89]Warning,
 choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[0,2,2
]%%%}] at parameters values [85.3561567818,-64,-30]Warning, choosing root o
f [1,0,%%%{-2,[1,0,1]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,2]%%%}] at parame
ters values [42,43.9628838282,-9]Sign error (%%%{-b,0%%%}+%%%{2*sqrt(a)*sqr
t(b),1/2%%%}+%%%{-2*a,1%%%}+%%%{a*sqrt(a)*sqrt(b)/b,3/2%%%}+%%%{-a^2*sqrt(a
)*sqrt(b)/(4*b^2),5/2%%%}+%%%{undef,7/2%%%})Limit: Max order reached or una
ble to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.01, size = 145, normalized size = 1.99

$$-\frac{\sqrt{\frac{ax+b}{x}}\left(-15a^3bx^4\ln\left(\frac{2ax+b+2\sqrt{ax^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)-30\sqrt{ax^2+bx}\,a^{\frac{7}{2}}x^4+30\left(ax^2+bx\right)^{\frac{3}{2}}a^{\frac{5}{2}}x^2+16\left(ax^2+bx\right)^{\frac{3}{2}}a^{\frac{3}{2}}\right.}{15\sqrt{(ax+b)x}\,\sqrt{a}\,bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2)/x,x)
```

```
[Out] -1/15*((a*x+b)/x)^(1/2)/x^3/b*(-30*(a*x^2+b*x)^(1/2)*a^(7/2)*x^4-15*ln(1/2*
(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x^4*a^3*b+30*(a*x^2+b*x)^(3/
2)*a^(5/2)*x^2+16*a^(3/2)*(a*x^2+b*x)^(3/2)*b*x+6*(a*x^2+b*x)^(3/2)*a^(1/2)
*b^2)/((a*x+b)*x)^(1/2)/a^(1/2)
```

maxima [A] time = 2.43, size = 75, normalized size = 1.03

$$-a^{\frac{5}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-\frac{2}{5}\left(a+\frac{b}{x}\right)^{\frac{5}{2}}-\frac{2}{3}\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a-2\sqrt{a+\frac{b}{x}}\,a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] -a^(5/2)*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) - 2/5*(a
+ b/x)^(5/2) - 2/3*(a + b/x)^(3/2)*a - 2*sqrt(a + b/x)*a^2
```

mupad [B] time = 1.75, size = 60, normalized size = 0.82

$$-\frac{2a\left(a+\frac{b}{x}\right)^{3/2}}{3}-\frac{2\left(a+\frac{b}{x}\right)^{5/2}}{5}-2a^2\sqrt{a+\frac{b}{x}}-a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1\mathrm{i}}{\sqrt{a}}\right)2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(5/2)/x,x)
```

```
[Out] - a^(5/2)*atan(((a + b/x)^(1/2)*1i)/a^(1/2))*2i - (2*a*(a + b/x)^(3/2))/3 -
 (2*(a + b/x)^(5/2))/5 - 2*a^2*(a + b/x)^(1/2)
```

sympy [A] time = 4.42, size = 97, normalized size = 1.33

$$-\frac{46a^{\frac{5}{2}}\sqrt{1+\frac{b}{ax}}}{15}-a^{\frac{5}{2}}\log\left(\frac{b}{ax}\right)+2a^{\frac{5}{2}}\log\left(\sqrt{1+\frac{b}{ax}}+1\right)-\frac{22a^{\frac{3}{2}}b\sqrt{1+\frac{b}{ax}}}{15x}-\frac{2\sqrt{a}\,b^2\sqrt{1+\frac{b}{ax}}}{5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)/x,x)
```

```
[Out] -46*a**(5/2)*sqrt(1 + b/(a*x))/15 - a**(5/2)*log(b/(a*x)) + 2*a**(5/2)*log(
sqrt(1 + b/(a*x)) + 1) - 22*a**(3/2)*b*sqrt(1 + b/(a*x))/(15*x) - 2*sqrt(a)
*b**2*sqrt(1 + b/(a*x))/(5*x**2)
```

3.1717 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^2}\,dx$

Optimal. Leaf size=18

$$-\frac{2\left(a+\frac{b}{x}\right)^{7/2}}{7b}$$

```
[Out] -2/7*(a+b/x)^(7/2)/b
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$-\frac{2\left(a+\frac{b}{x}\right)^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)/x^2,x]
```

```
[Out] (-2*(a + b/x)^(7/2))/(7*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^2}\,dx = -\frac{2\left(a+\frac{b}{x}\right)^{7/2}}{7b}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$-\frac{2\left(a+\frac{b}{x}\right)^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)/x^2,x]
```

```
[Out] (-2*(a + b/x)^(7/2))/(7*b)
```

fricas [B] time = 0.83, size = 46, normalized size = 2.56

$$-\frac{2\left(a^3x^3+3\,a^2bx^2+3\,ab^2x+b^3\right)\sqrt{\frac{ax+b}{x}}}{7\,bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] -2/7*(a^3*x^3 + 3*a^2*b*x^2 + 3*a*b^2*x + b^3)*sqrt((a*x + b)/x)/(b*x^3)
```

giac [B] time = 0.27, size = 207, normalized size = 11.50

$$\frac{2\left(7\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)^6 a^3\mathrm{sgn}(x)+21\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)^5 a^{\frac{5}{2}} b\mathrm{sgn}(x)+35\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)^4 a^2b^2\mathrm{sgn}(x)+\right.}{7\left(\sqrt{a}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] 2/7*(7*(sqrt(a)*x - sqrt(a*x^2 + b*x))^6*a^3*sgn(x) + 21*(sqrt(a)*x - sqrt(
a*x^2 + b*x))^5*a^(5/2)*b*sgn(x) + 35*(sqrt(a)*x - sqrt(a*x^2 + b*x))^4*a^2
*b^2*sgn(x) + 35*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a^(3/2)*b^3*sgn(x) + 21*
(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a*b^4*sgn(x) + 7*(sqrt(a)*x - sqrt(a*x^2
+ b*x))*sqrt(a)*b^5*sgn(x) + b^6*sgn(x))/(sqrt(a)*x - sqrt(a*x^2 + b*x))^7
```

maple [A] time = 0.00, size = 25, normalized size = 1.39

$$-\frac{2\,(ax+b)\left(\frac{ax+b}{x}\right)^{\frac{5}{2}}}{7bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2)/x^2,x)
```

```
[Out] -2/7*(a*x+b)/x/b*((a*x+b)/x)^(5/2)
```

maxima [A] time = 1.08, size = 14, normalized size = 0.78

$$-\frac{2\left(a+\frac{b}{x}\right)^{\frac{7}{2}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] -2/7*(a + b/x)^(7/2)/b
```

mupad [B] time = 1.88, size = 68, normalized size = 3.78

$$-\frac{2\,a^3\sqrt{a+\frac{b}{x}}}{7\,b}-\frac{6\,a^2\sqrt{a+\frac{b}{x}}}{7\,x}-\frac{2\,b^2\sqrt{a+\frac{b}{x}}}{7\,x^3}-\frac{6\,a\,b\sqrt{a+\frac{b}{x}}}{7\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(5/2)/x^2,x)
```

```
[Out] - (2*a^3*(a + b/x)^(1/2))/(7*b) - (6*a^2*(a + b/x)^(1/2))/(7*x) - (2*b^2*(a
 + b/x)^(1/2))/(7*x^3) - (6*a*b*(a + b/x)^(1/2))/(7*x^2)
```

sympy [A] time = 3.53, size = 80, normalized size = 4.44

$$\begin{cases} -\frac{2a^3\sqrt{a+\frac{b}{x}}}{7b}-\frac{6a^2\sqrt{a+\frac{b}{x}}}{7x}-\frac{6ab\sqrt{a+\frac{b}{x}}}{7x^2}-\frac{2b^2\sqrt{a+\frac{b}{x}}}{7x^3} & \text{for } b\neq 0 \\ -\frac{a^{\frac{5}{2}}}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)/x**2,x)
```

```
[Out] Piecewise((-2*a**3*sqrt(a + b/x)/(7*b) - 6*a**2*sqrt(a + b/x)/(7*x) - 6*a*b
*sqrt(a + b/x)/(7*x**2) - 2*b**2*sqrt(a + b/x)/(7*x**3), Ne(b, 0)), (-a**(5
/2)/x, True))
```

3.1718 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^3}\,dx$

Optimal. Leaf size=38

$$\frac{2a\left(a+\frac{b}{x}\right)^{7/2}}{7b^2}-\frac{2\left(a+\frac{b}{x}\right)^{9/2}}{9b^2}$$

```
[Out] 2/7*a*(a+b/x)^(7/2)/b^2-2/9*(a+b/x)^(9/2)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{2a\left(a+\frac{b}{x}\right)^{7/2}}{7b^2}-\frac{2\left(a+\frac{b}{x}\right)^{9/2}}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)/x^3,x]
```

```
[Out] (2*a*(a + b/x)^(7/2))/(7*b^2) - (2*(a + b/x)^(9/2))/(9*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^3}\,dx &= -\text{Subst}\left(\int x(a+bx)^{5/2}\,dx, x, \frac{1}{x}\right)\\ &= -\text{Subst}\left(\int\left(-\frac{a(a+bx)^{5/2}}{b}+\frac{(a+bx)^{7/2}}{b}\right)dx, x, \frac{1}{x}\right)\\ &= \frac{2a\left(a+\frac{b}{x}\right)^{7/2}}{7b^2}-\frac{2\left(a+\frac{b}{x}\right)^{9/2}}{9b^2}\end{aligned}$$

Mathematica [A] time = 0.04, size = 38, normalized size = 1.00

$$\frac{2a\left(a+\frac{b}{x}\right)^{7/2}}{7b^2}-\frac{2\left(a+\frac{b}{x}\right)^{9/2}}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)/x^3,x]
```

```
[Out] (2*a*(a + b/x)^(7/2))/(7*b^2) - (2*(a + b/x)^(9/2))/(9*b^2)
```

fricas [A] time = 0.96, size = 60, normalized size = 1.58

$$\frac{2\left(2\,a^4x^4 - a^3bx^3 - 15\,a^2b^2x^2 - 19\,ab^3x - 7\,b^4\right)\sqrt{\frac{ax+b}{x}}}{63\,b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] 2/63*(2*a^4*x^4 - a^3*b*x^3 - 15*a^2*b^2*x^2 - 19*a*b^3*x - 7*b^4)*sqrt((a*
x + b)/x)/(b^2*x^4)
```

giac [B] time = 0.32, size = 239, normalized size = 6.29

$$2\left(63\left(\sqrt{a}\,x - \sqrt{ax^2+bx}\right)^7 a^{\frac{7}{2}}\mathrm{sgn}(x) + 273\left(\sqrt{a}\,x - \sqrt{ax^2+bx}\right)^6 a^3b\mathrm{sgn}(x) + 567\left(\sqrt{a}\,x - \sqrt{ax^2+bx}\right)^5 a^{\frac{5}{2}}b^2\mathrm{sg}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] 2/63*(63*(sqrt(a)*x - sqrt(a*x^2 + b*x))^7*a^(7/2)*sgn(x) + 273*(sqrt(a)*x
- sqrt(a*x^2 + b*x))^6*a^3*b*sgn(x) + 567*(sqrt(a)*x - sqrt(a*x^2 + b*x))^5
*a^(5/2)*b^2*sgn(x) + 693*(sqrt(a)*x - sqrt(a*x^2 + b*x))^4*a^2*b^3*sgn(x)
+ 525*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a^(3/2)*b^4*sgn(x) + 243*(sqrt(a)*x
 - sqrt(a*x^2 + b*x))^2*a*b^5*sgn(x) + 63*(sqrt(a)*x - sqrt(a*x^2 + b*x))*s
qrt(a)*b^6*sgn(x) + 7*b^7*sgn(x))/(sqrt(a)*x - sqrt(a*x^2 + b*x))^9
```

maple [A] time = 0.01, size = 33, normalized size = 0.87

$$\frac{2\,(ax+b)\,(2ax-7b)\left(\frac{ax+b}{x}\right)^{\frac{5}{2}}}{63b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2)/x^3,x)
```

```
[Out] 2/63*(a*x+b)*(2*a*x-7*b)*((a*x+b)/x)^(5/2)/b^2/x^2
```

maxima [A] time = 0.99, size = 30, normalized size = 0.79

$$-\frac{2\left(a+\frac{b}{x}\right)^{\frac{9}{2}}}{9\,b^2} + \frac{2\left(a+\frac{b}{x}\right)^{\frac{7}{2}}a}{7\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] -2/9*(a + b/x)^(9/2)/b^2 + 2/7*(a + b/x)^(7/2)*a/b^2
```

mupad [B] time = 2.12, size = 88, normalized size = 2.32

$$\frac{4\,a^4\sqrt{a+\frac{b}{x}}}{63\,b^2} - \frac{10\,a^2\sqrt{a+\frac{b}{x}}}{21\,x^2} - \frac{2\,b^2\sqrt{a+\frac{b}{x}}}{9\,x^4} - \frac{2\,a^3\sqrt{a+\frac{b}{x}}}{63\,b\,x} - \frac{38\,a\,b\sqrt{a+\frac{b}{x}}}{63\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(5/2)/x^3,x)
```

```
[Out] (4*a^4*(a + b/x)^(1/2))/(63*b^2) - (10*a^2*(a + b/x)^(1/2))/(21*x^2) - (2*b
^2*(a + b/x)^(1/2))/(9*x^4) - (2*a^3*(a + b/x)^(1/2))/(63*b*x) - (38*a*b*(a
 + b/x)^(1/2))/(63*x^3)
```

sympy [B] time = 1.76, size = 416, normalized size = 10.95

$$\frac{4a^{\frac{19}{2}}b^{\frac{3}{2}}x^5\sqrt{\frac{ax}{b}+1}}{63a^{\frac{11}{2}}b^3x^{\frac{11}{2}}+63a^{\frac{9}{2}}b^4x^{\frac{9}{2}}}+\frac{2a^{\frac{17}{2}}b^{\frac{5}{2}}x^4\sqrt{\frac{ax}{b}+1}}{63a^{\frac{11}{2}}b^3x^{\frac{11}{2}}+63a^{\frac{9}{2}}b^4x^{\frac{9}{2}}}-\frac{32a^{\frac{15}{2}}b^{\frac{7}{2}}x^3\sqrt{\frac{ax}{b}+1}}{63a^{\frac{11}{2}}b^3x^{\frac{11}{2}}+63a^{\frac{9}{2}}b^4x^{\frac{9}{2}}}-\frac{68a^{\frac{13}{2}}b^{\frac{9}{2}}x^2\sqrt{\frac{ax}{b}+1}}{63a^{\frac{11}{2}}b^3x^{\frac{11}{2}}+63a^{\frac{9}{2}}b^4x^{\frac{9}{2}}}-\frac{52a^{\frac{11}{2}}b}{63a^{\frac{11}{2}}b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)/x**3,x)
```

```
[Out] 4*a**(19/2)*b**(3/2)*x**5*sqrt(a*x/b + 1)/(63*a**(11/2)*b**3*x**(11/2) + 63
*a**(9/2)*b**4*x**(9/2)) + 2*a**(17/2)*b**(5/2)*x**4*sqrt(a*x/b + 1)/(63*a*
*(11/2)*b**3*x**(11/2) + 63*a**(9/2)*b**4*x**(9/2)) - 32*a**(15/2)*b**(7/2)
*x**3*sqrt(a*x/b + 1)/(63*a**(11/2)*b**3*x**(11/2) + 63*a**(9/2)*b**4*x**(9
/2)) - 68*a**(13/2)*b**(9/2)*x**2*sqrt(a*x/b + 1)/(63*a**(11/2)*b**3*x**(11
/2) + 63*a**(9/2)*b**4*x**(9/2)) - 52*a**(11/2)*b**(11/2)*x*sqrt(a*x/b + 1)
/(63*a**(11/2)*b**3*x**(11/2) + 63*a**(9/2)*b**4*x**(9/2)) - 14*a**(9/2)*b*
*(13/2)*sqrt(a*x/b + 1)/(63*a**(11/2)*b**3*x**(11/2) + 63*a**(9/2)*b**4*x**
(9/2)) - 4*a**10*b*x**(11/2)/(63*a**(11/2)*b**3*x**(11/2) + 63*a**(9/2)*b**
4*x**(9/2)) - 4*a**9*b**2*x**(9/2)/(63*a**(11/2)*b**3*x**(11/2) + 63*a**(9/
2)*b**4*x**(9/2))
```

3.1719 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^4}\,dx$

Optimal. Leaf size=59

$$-\frac{2a^2\left(a+\frac{b}{x}\right)^{7/2}}{7b^3}-\frac{2\left(a+\frac{b}{x}\right)^{11/2}}{11b^3}+\frac{4a\left(a+\frac{b}{x}\right)^{9/2}}{9b^3}$$

```
[Out] -2/7*a^2*(a+b/x)^(7/2)/b^3+4/9*a*(a+b/x)^(9/2)/b^3-2/11*(a+b/x)^(11/2)/b^3
```

Rubi [A] time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{2a^2\left(a+\frac{b}{x}\right)^{7/2}}{7b^3}-\frac{2\left(a+\frac{b}{x}\right)^{11/2}}{11b^3}+\frac{4a\left(a+\frac{b}{x}\right)^{9/2}}{9b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)/x^4,x]
```

```
[Out] (-2*a^2*(a + b/x)^(7/2))/(7*b^3) + (4*a*(a + b/x)^(9/2))/(9*b^3) - (2*(a +
b/x)^(11/2))/(11*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^4}\,dx &= -\text{Subst}\left(\int x^2(a+bx)^{5/2}\,dx, x, \frac{1}{x}\right)\\ &= -\text{Subst}\left(\int\left(\frac{a^2(a+bx)^{5/2}}{b^2}-\frac{2a(a+bx)^{7/2}}{b^2}+\frac{(a+bx)^{9/2}}{b^2}\right)dx, x, \frac{1}{x}\right)\\ &= -\frac{2a^2\left(a+\frac{b}{x}\right)^{7/2}}{7b^3}+\frac{4a\left(a+\frac{b}{x}\right)^{9/2}}{9b^3}-\frac{2\left(a+\frac{b}{x}\right)^{11/2}}{11b^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 47, normalized size = 0.80

$$-\frac{2\sqrt{a+\frac{b}{x}}\,(ax+b)^3\left(8a^2x^2-28abx+63b^2\right)}{693b^3x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)/x^4,x]
```

```
[Out] (-2*Sqrt[a + b/x]*(b + a*x)^3*(63*b^2 - 28*a*b*x + 8*a^2*x^2))/(693*b^3*x^5
)
```

fricas [A] time = 0.91, size = 71, normalized size = 1.20

$$-\frac{2\left(8\,a^5x^5-4\,a^4bx^4+3\,a^3b^2x^3+113\,a^2b^3x^2+161\,ab^4x+63\,b^5\right)\sqrt{\frac{ax+b}{x}}}{693\,b^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] -2/693*(8*a^5*x^5 - 4*a^4*b*x^4 + 3*a^3*b^2*x^3 + 113*a^2*b^3*x^2 + 161*a*b
^4*x + 63*b^5)*sqrt((a*x + b)/x)/(b^3*x^5)
```

giac [B] time = 0.29, size = 270, normalized size = 4.58

$$2\left(924\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)^8a^4\mathrm{sgn}(x)+4851\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)^7a^{\frac{7}{2}}b\mathrm{sgn}(x)+11781\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)^6a^3b^2s$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] 2/693*(924*(sqrt(a)*x - sqrt(a*x^2 + b*x))^8*a^4*sgn(x) + 4851*(sqrt(a)*x -
 sqrt(a*x^2 + b*x))^7*a^(7/2)*b*sgn(x) + 11781*(sqrt(a)*x - sqrt(a*x^2 + b*
x))^6*a^3*b^2*sgn(x) + 16863*(sqrt(a)*x - sqrt(a*x^2 + b*x))^5*a^(5/2)*b^3*
sgn(x) + 15345*(sqrt(a)*x - sqrt(a*x^2 + b*x))^4*a^2*b^4*sgn(x) + 9009*(sqr
t(a)*x - sqrt(a*x^2 + b*x))^3*a^(3/2)*b^5*sgn(x) + 3311*(sqrt(a)*x - sqrt(a
*x^2 + b*x))^2*a*b^6*sgn(x) + 693*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*b
^7*sgn(x) + 63*b^8*sgn(x))/(sqrt(a)*x - sqrt(a*x^2 + b*x))^11
```

maple [A] time = 0.00, size = 44, normalized size = 0.75

$$-\frac{2\,(ax+b)\left(8a^2x^2-28abx+63b^2\right)\left(\frac{ax+b}{x}\right)^{\frac{5}{2}}}{693b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2)/x^4,x)
```

```
[Out] -2/693*(a*x+b)*(8*a^2*x^2-28*a*b*x+63*b^2)*((a*x+b)/x)^(5/2)/b^3/x^3
```

maxima [A] time = 0.99, size = 47, normalized size = 0.80

$$-\frac{2\left(a+\frac{b}{x}\right)^{\frac{11}{2}}}{11\,b^3}+\frac{4\left(a+\frac{b}{x}\right)^{\frac{9}{2}}a}{9\,b^3}-\frac{2\left(a+\frac{b}{x}\right)^{\frac{7}{2}}a^2}{7\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] -2/11*(a + b/x)^(11/2)/b^3 + 4/9*(a + b/x)^(9/2)*a/b^3 - 2/7*(a + b/x)^(7/2
)*a^2/b^3
```

mupad [B] time = 2.41, size = 108, normalized size = 1.83

$$\frac{8\,a^4\sqrt{a+\frac{b}{x}}}{693\,b^2\,x}-\frac{226\,a^2\sqrt{a+\frac{b}{x}}}{693\,x^3}-\frac{2\,b^2\sqrt{a+\frac{b}{x}}}{11\,x^5}-\frac{2\,a^3\sqrt{a+\frac{b}{x}}}{231\,b\,x^2}-\frac{16\,a^5\sqrt{a+\frac{b}{x}}}{693\,b^3}-\frac{46\,a\,b\sqrt{a+\frac{b}{x}}}{99\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(5/2)/x^4,x)
```

```
[Out] (8*a^4*(a + b/x)^(1/2))/(693*b^2*x) - (226*a^2*(a + b/x)^(1/2))/(693*x^3) -
 (2*b^2*(a + b/x)^(1/2))/(11*x^5) - (2*a^3*(a + b/x)^(1/2))/(231*b*x^2) - (
16*a^5*(a + b/x)^(1/2))/(693*b^3) - (46*a*b*(a + b/x)^(1/2))/(99*x^4)
```

sympy [B] time = 2.76, size = 1073, normalized size = 18.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)/x**4,x)
```

```
[Out] -16*a**(27/2)*b**(9/2)*x**8*sqrt(a*x/b + 1)/(693*a**(17/2)*b**7*x**(17/2) +
 2079*a**(15/2)*b**8*x**(15/2) + 2079*a**(13/2)*b**9*x**(13/2) + 693*a**(11
/2)*b**10*x**(11/2)) - 40*a**(25/2)*b**(11/2)*x**7*sqrt(a*x/b + 1)/(693*a**
(17/2)*b**7*x**(17/2) + 2079*a**(15/2)*b**8*x**(15/2) + 2079*a**(13/2)*b**9
*x**(13/2) + 693*a**(11/2)*b**10*x**(11/2)) - 30*a**(23/2)*b**(13/2)*x**6*s
qrt(a*x/b + 1)/(693*a**(17/2)*b**7*x**(17/2) + 2079*a**(15/2)*b**8*x**(15/2
) + 2079*a**(13/2)*b**9*x**(13/2) + 693*a**(11/2)*b**10*x**(11/2)) - 236*a*
*(21/2)*b**(15/2)*x**5*sqrt(a*x/b + 1)/(693*a**(17/2)*b**7*x**(17/2) + 2079
*a**(15/2)*b**8*x**(15/2) + 2079*a**(13/2)*b**9*x**(13/2) + 693*a**(11/2)*b
**10*x**(11/2)) - 1010*a**(19/2)*b**(17/2)*x**4*sqrt(a*x/b + 1)/(693*a**(17
/2)*b**7*x**(17/2) + 2079*a**(15/2)*b**8*x**(15/2) + 2079*a**(13/2)*b**9*x*
*(13/2) + 693*a**(11/2)*b**10*x**(11/2)) - 1776*a**(17/2)*b**(19/2)*x**3*sq
rt(a*x/b + 1)/(693*a**(17/2)*b**7*x**(17/2) + 2079*a**(15/2)*b**8*x**(15/2)
 + 2079*a**(13/2)*b**9*x**(13/2) + 693*a**(11/2)*b**10*x**(11/2)) - 1570*a*
*(15/2)*b**(21/2)*x**2*sqrt(a*x/b + 1)/(693*a**(17/2)*b**7*x**(17/2) + 2079
*a**(15/2)*b**8*x**(15/2) + 2079*a**(13/2)*b**9*x**(13/2) + 693*a**(11/2)*b
**10*x**(11/2)) - 700*a**(13/2)*b**(23/2)*x*sqrt(a*x/b + 1)/(693*a**(17/2)*
b**7*x**(17/2) + 2079*a**(15/2)*b**8*x**(15/2) + 2079*a**(13/2)*b**9*x**(13
/2) + 693*a**(11/2)*b**10*x**(11/2)) - 126*a**(11/2)*b**(25/2)*sqrt(a*x/b +
 1)/(693*a**(17/2)*b**7*x**(17/2) + 2079*a**(15/2)*b**8*x**(15/2) + 2079*a*
*(13/2)*b**9*x**(13/2) + 693*a**(11/2)*b**10*x**(11/2)) + 16*a**14*b**4*x**
(17/2)/(693*a**(17/2)*b**7*x**(17/2) + 2079*a**(15/2)*b**8*x**(15/2) + 2079
*a**(13/2)*b**9*x**(13/2) + 693*a**(11/2)*b**10*x**(11/2)) + 48*a**13*b**5*
x**(15/2)/(693*a**(17/2)*b**7*x**(17/2) + 2079*a**(15/2)*b**8*x**(15/2) + 2
079*a**(13/2)*b**9*x**(13/2) + 693*a**(11/2)*b**10*x**(11/2)) + 48*a**12*b*
*6*x**(13/2)/(693*a**(17/2)*b**7*x**(17/2) + 2079*a**(15/2)*b**8*x**(15/2)
+ 2079*a**(13/2)*b**9*x**(13/2) + 693*a**(11/2)*b**10*x**(11/2)) + 16*a**11
*b**7*x**(11/2)/(693*a**(17/2)*b**7*x**(17/2) + 2079*a**(15/2)*b**8*x**(15/
2) + 2079*a**(13/2)*b**9*x**(13/2) + 693*a**(11/2)*b**10*x**(11/2))
```

3.1720 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^5}\,dx$

Optimal. Leaf size=80

$$\frac{2a^3\left(a+\frac{b}{x}\right)^{7/2}}{7b^4}-\frac{2a^2\left(a+\frac{b}{x}\right)^{9/2}}{3b^4}-\frac{2\left(a+\frac{b}{x}\right)^{13/2}}{13b^4}+\frac{6a\left(a+\frac{b}{x}\right)^{11/2}}{11b^4}$$

```
[Out] 2/7*a^3*(a+b/x)^(7/2)/b^4-2/3*a^2*(a+b/x)^(9/2)/b^4+6/11*a*(a+b/x)^(11/2)/b
^4-2/13*(a+b/x)^(13/2)/b^4
```

Rubi [A] time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{2a^2\left(a+\frac{b}{x}\right)^{9/2}}{3b^4}+\frac{2a^3\left(a+\frac{b}{x}\right)^{7/2}}{7b^4}-\frac{2\left(a+\frac{b}{x}\right)^{13/2}}{13b^4}+\frac{6a\left(a+\frac{b}{x}\right)^{11/2}}{11b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)/x^5,x]
```

```
[Out] (2*a^3*(a + b/x)^(7/2))/(7*b^4) - (2*a^2*(a + b/x)^(9/2))/(3*b^4) + (6*a*(a
 + b/x)^(11/2))/(11*b^4) - (2*(a + b/x)^(13/2))/(13*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^5}\,dx &= -\text{Subst}\left(\int x^3(a+bx)^{5/2}\,dx, x, \frac{1}{x}\right)\\ &= -\text{Subst}\left(\int\left(-\frac{a^3(a+bx)^{5/2}}{b^3}+\frac{3a^2(a+bx)^{7/2}}{b^3}-\frac{3a(a+bx)^{9/2}}{b^3}+\frac{(a+bx)^{11/2}}{b^3}\right)dx, x, \frac{1}{x}\right)\\ &= \frac{2a^3\left(a+\frac{b}{x}\right)^{7/2}}{7b^4}-\frac{2a^2\left(a+\frac{b}{x}\right)^{9/2}}{3b^4}+\frac{6a\left(a+\frac{b}{x}\right)^{11/2}}{11b^4}-\frac{2\left(a+\frac{b}{x}\right)^{13/2}}{13b^4}\end{aligned}$$

Mathematica [A] time = 0.04, size = 58, normalized size = 0.72

$$\frac{2\sqrt{a+\frac{b}{x}}\,(ax+b)^3\left(16a^3x^3-56a^2bx^2+126ab^2x-231b^3\right)}{3003b^4x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)/x^5,x]
```

```
[Out] (2*Sqrt[a + b/x]*(b + a*x)^3*(-231*b^3 + 126*a*b^2*x - 56*a^2*b*x^2 + 16*a^
3*x^3))/(3003*b^4*x^6)
```

fricas [A] time = 0.56, size = 82, normalized size = 1.02

$$\frac{2\left(16\,a^6x^6-8\,a^5bx^5+6\,a^4b^2x^4-5\,a^3b^3x^3-371\,a^2b^4x^2-567\,ab^5x-231\,b^6\right)\sqrt{\frac{ax+b}{x}}}{3003\,b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] 2/3003*(16*a^6*x^6 - 8*a^5*b*x^5 + 6*a^4*b^2*x^4 - 5*a^3*b^3*x^3 - 371*a^2*
b^4*x^2 - 567*a*b^5*x - 231*b^6)*sqrt((a*x + b)/x)/(b^4*x^6)
```

giac [B] time = 0.47, size = 301, normalized size = 3.76

$$\frac{2\left(6006\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)^9a^{\frac{9}{2}}\mathrm{sgn}(x)+36036\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)^8a^4b\mathrm{sgn}(x)+99099\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] 2/3003*(6006*(sqrt(a)*x - sqrt(a*x^2 + b*x))^9*a^(9/2)*sgn(x) + 36036*(sqrt
(a)*x - sqrt(a*x^2 + b*x))^8*a^4*b*sgn(x) + 99099*(sqrt(a)*x - sqrt(a*x^2 +
 b*x))^7*a^(7/2)*b^2*sgn(x) + 161733*(sqrt(a)*x - sqrt(a*x^2 + b*x))^6*a^3*
b^3*sgn(x) + 171171*(sqrt(a)*x - sqrt(a*x^2 + b*x))^5*a^(5/2)*b^4*sgn(x) +
121121*(sqrt(a)*x - sqrt(a*x^2 + b*x))^4*a^2*b^5*sgn(x) + 57057*(sqrt(a)*x
- sqrt(a*x^2 + b*x))^3*a^(3/2)*b^6*sgn(x) + 17199*(sqrt(a)*x - sqrt(a*x^2 +
 b*x))^2*a*b^7*sgn(x) + 3003*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*b^8*sg
n(x) + 231*b^9*sgn(x))/(sqrt(a)*x - sqrt(a*x^2 + b*x))^13
```

maple [A] time = 0.01, size = 55, normalized size = 0.69

$$\frac{2\,(ax+b)\left(16a^3x^3-56a^2b\,x^2+126a\,b^2x-231b^3\right)\left(\frac{ax+b}{x}\right)^{\frac{5}{2}}}{3003b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2)/x^5,x)
```

```
[Out] 2/3003*(a*x+b)*(16*a^3*x^3-56*a^2*b*x^2+126*a*b^2*x-231*b^3)*((a*x+b)/x)^(5
/2)/x^4/b^4
```

maxima [A] time = 1.07, size = 64, normalized size = 0.80

$$-\frac{2\left(a+\frac{b}{x}\right)^{\frac{13}{2}}}{13\,b^4}+\frac{6\left(a+\frac{b}{x}\right)^{\frac{11}{2}}a}{11\,b^4}-\frac{2\left(a+\frac{b}{x}\right)^{\frac{9}{2}}a^2}{3\,b^4}+\frac{2\left(a+\frac{b}{x}\right)^{\frac{7}{2}}a^3}{7\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] -2/13*(a + b/x)^(13/2)/b^4 + 6/11*(a + b/x)^(11/2)*a/b^4 - 2/3*(a + b/x)^(9
/2)*a^2/b^4 + 2/7*(a + b/x)^(7/2)*a^3/b^4
```

mupad [B] time = 2.71, size = 128, normalized size = 1.60

$$\frac{32\,a^6\,\sqrt{a+\frac{b}{x}}}{3003\,b^4}-\frac{106\,a^2\,\sqrt{a+\frac{b}{x}}}{429\,x^4}-\frac{2\,b^2\,\sqrt{a+\frac{b}{x}}}{13\,x^6}-\frac{10\,a^3\,\sqrt{a+\frac{b}{x}}}{3003\,b\,x^3}+\frac{4\,a^4\,\sqrt{a+\frac{b}{x}}}{1001\,b^2\,x^2}-\frac{16\,a^5\,\sqrt{a+\frac{b}{x}}}{3003\,b^3\,x}-\frac{54\,a\,b\,\sqrt{a+\frac{b}{x}}}{143\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(5/2)/x^5,x)
```

```
[Out] (32*a^6*(a + b/x)^(1/2))/(3003*b^4) - (106*a^2*(a + b/x)^(1/2))/(429*x^4) -
 (2*b^2*(a + b/x)^(1/2))/(13*x^6) - (10*a^3*(a + b/x)^(1/2))/(3003*b*x^3) +
 (4*a^4*(a + b/x)^(1/2))/(1001*b^2*x^2) - (16*a^5*(a + b/x)^(1/2))/(3003*b^
3*x) - (54*a*b*(a + b/x)^(1/2))/(143*x^5)
```

sympy [B] time = 3.36, size = 2562, normalized size = 32.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)/x**5,x)
```

```
[Out] 32*a**(37/2)*b**(23/2)*x**12*sqrt(a*x/b + 1)/(3003*a**(25/2)*b**15*x**(25/2
) + 18018*a**(23/2)*b**16*x**(23/2) + 45045*a**(21/2)*b**17*x**(21/2) + 600
60*a**(19/2)*b**18*x**(19/2) + 45045*a**(17/2)*b**19*x**(17/2) + 18018*a**(
15/2)*b**20*x**(15/2) + 3003*a**(13/2)*b**21*x**(13/2)) + 176*a**(35/2)*b**
(25/2)*x**11*sqrt(a*x/b + 1)/(3003*a**(25/2)*b**15*x**(25/2) + 18018*a**(23
/2)*b**16*x**(23/2) + 45045*a**(21/2)*b**17*x**(21/2) + 60060*a**(19/2)*b**
18*x**(19/2) + 45045*a**(17/2)*b**19*x**(17/2) + 18018*a**(15/2)*b**20*x**(
15/2) + 3003*a**(13/2)*b**21*x**(13/2)) + 396*a**(33/2)*b**(27/2)*x**10*sqr
t(a*x/b + 1)/(3003*a**(25/2)*b**15*x**(25/2) + 18018*a**(23/2)*b**16*x**(23
/2) + 45045*a**(21/2)*b**17*x**(21/2) + 60060*a**(19/2)*b**18*x**(19/2) + 4
5045*a**(17/2)*b**19*x**(17/2) + 18018*a**(15/2)*b**20*x**(15/2) + 3003*a**
(13/2)*b**21*x**(13/2)) + 462*a**(31/2)*b**(29/2)*x**9*sqrt(a*x/b + 1)/(300
3*a**(25/2)*b**15*x**(25/2) + 18018*a**(23/2)*b**16*x**(23/2) + 45045*a**(2
1/2)*b**17*x**(21/2) + 60060*a**(19/2)*b**18*x**(19/2) + 45045*a**(17/2)*b*
*19*x**(17/2) + 18018*a**(15/2)*b**20*x**(15/2) + 3003*a**(13/2)*b**21*x**(
13/2)) - 462*a**(29/2)*b**(31/2)*x**8*sqrt(a*x/b + 1)/(3003*a**(25/2)*b**15
*x**(25/2) + 18018*a**(23/2)*b**16*x**(23/2) + 45045*a**(21/2)*b**17*x**(21
/2) + 60060*a**(19/2)*b**18*x**(19/2) + 45045*a**(17/2)*b**19*x**(17/2) + 1
8018*a**(15/2)*b**20*x**(15/2) + 3003*a**(13/2)*b**21*x**(13/2)) - 5544*a**
(27/2)*b**(33/2)*x**7*sqrt(a*x/b + 1)/(3003*a**(25/2)*b**15*x**(25/2) + 180
18*a**(23/2)*b**16*x**(23/2) + 45045*a**(21/2)*b**17*x**(21/2) + 60060*a**(
19/2)*b**18*x**(19/2) + 45045*a**(17/2)*b**19*x**(17/2) + 18018*a**(15/2)*b
**20*x**(15/2) + 3003*a**(13/2)*b**21*x**(13/2)) - 18480*a**(25/2)*b**(35/2
)*x**6*sqrt(a*x/b + 1)/(3003*a**(25/2)*b**15*x**(25/2) + 18018*a**(23/2)*b*
*16*x**(23/2) + 45045*a**(21/2)*b**17*x**(21/2) + 60060*a**(19/2)*b**18*x**
(19/2) + 45045*a**(17/2)*b**19*x**(17/2) + 18018*a**(15/2)*b**20*x**(15/2)
+ 3003*a**(13/2)*b**21*x**(13/2)) - 34716*a**(23/2)*b**(37/2)*x**5*sqrt(a*x
/b + 1)/(3003*a**(25/2)*b**15*x**(25/2) + 18018*a**(23/2)*b**16*x**(23/2) +
 45045*a**(21/2)*b**17*x**(21/2) + 60060*a**(19/2)*b**18*x**(19/2) + 45045*
a**(17/2)*b**19*x**(17/2) + 18018*a**(15/2)*b**20*x**(15/2) + 3003*a**(13/2
)*b**21*x**(13/2)) - 40788*a**(21/2)*b**(39/2)*x**4*sqrt(a*x/b + 1)/(3003*a
**(25/2)*b**15*x**(25/2) + 18018*a**(23/2)*b**16*x**(23/2) + 45045*a**(21/2
)*b**17*x**(21/2) + 60060*a**(19/2)*b**18*x**(19/2) + 45045*a**(17/2)*b**19
*x**(17/2) + 18018*a**(15/2)*b**20*x**(15/2) + 3003*a**(13/2)*b**21*x**(13/
2)) - 30712*a**(19/2)*b**(41/2)*x**3*sqrt(a*x/b + 1)/(3003*a**(25/2)*b**15*
x**(25/2) + 18018*a**(23/2)*b**16*x**(23/2) + 45045*a**(21/2)*b**17*x**(21/
```

```
2) + 60060*a**(19/2)*b**18*x**(19/2) + 45045*a**(17/2)*b**19*x**(17/2) + 18
018*a**(15/2)*b**20*x**(15/2) + 3003*a**(13/2)*b**21*x**(13/2)) - 14476*a**
(17/2)*b**(43/2)*x**2*sqrt(a*x/b + 1)/(3003*a**(25/2)*b**15*x**(25/2) + 180
18*a**(23/2)*b**16*x**(23/2) + 45045*a**(21/2)*b**17*x**(21/2) + 60060*a**(
19/2)*b**18*x**(19/2) + 45045*a**(17/2)*b**19*x**(17/2) + 18018*a**(15/2)*b
**20*x**(15/2) + 3003*a**(13/2)*b**21*x**(13/2)) - 3906*a**(15/2)*b**(45/2)
*x*sqrt(a*x/b + 1)/(3003*a**(25/2)*b**15*x**(25/2) + 18018*a**(23/2)*b**16*
x**(23/2) + 45045*a**(21/2)*b**17*x**(21/2) + 60060*a**(19/2)*b**18*x**(19/
2) + 45045*a**(17/2)*b**19*x**(17/2) + 18018*a**(15/2)*b**20*x**(15/2) + 30
03*a**(13/2)*b**21*x**(13/2)) - 462*a**(13/2)*b**(47/2)*sqrt(a*x/b + 1)/(30
03*a**(25/2)*b**15*x**(25/2) + 18018*a**(23/2)*b**16*x**(23/2) + 45045*a**(
21/2)*b**17*x**(21/2) + 60060*a**(19/2)*b**18*x**(19/2) + 45045*a**(17/2)*b
**19*x**(17/2) + 18018*a**(15/2)*b**20*x**(15/2) + 3003*a**(13/2)*b**21*x**
(13/2)) - 32*a**19*b**11*x**(25/2)/(3003*a**(25/2)*b**15*x**(25/2) + 18018*
a**(23/2)*b**16*x**(23/2) + 45045*a**(21/2)*b**17*x**(21/2) + 60060*a**(19/
2)*b**18*x**(19/2) + 45045*a**(17/2)*b**19*x**(17/2) + 18018*a**(15/2)*b**2
0*x**(15/2) + 3003*a**(13/2)*b**21*x**(13/2)) - 192*a**18*b**12*x**(23/2)/(
3003*a**(25/2)*b**15*x**(25/2) + 18018*a**(23/2)*b**16*x**(23/2) + 45045*a*
*(21/2)*b**17*x**(21/2) + 60060*a**(19/2)*b**18*x**(19/2) + 45045*a**(17/2)
*b**19*x**(17/2) + 18018*a**(15/2)*b**20*x**(15/2) + 3003*a**(13/2)*b**21*x
**(13/2)) - 480*a**17*b**13*x**(21/2)/(3003*a**(25/2)*b**15*x**(25/2) + 180
18*a**(23/2)*b**16*x**(23/2) + 45045*a**(21/2)*b**17*x**(21/2) + 60060*a**(
19/2)*b**18*x**(19/2) + 45045*a**(17/2)*b**19*x**(17/2) + 18018*a**(15/2)*b
**20*x**(15/2) + 3003*a**(13/2)*b**21*x**(13/2)) - 640*a**16*b**14*x**(19/2
)/(3003*a**(25/2)*b**15*x**(25/2) + 18018*a**(23/2)*b**16*x**(23/2) + 45045
*a**(21/2)*b**17*x**(21/2) + 60060*a**(19/2)*b**18*x**(19/2) + 45045*a**(17
/2)*b**19*x**(17/2) + 18018*a**(15/2)*b**20*x**(15/2) + 3003*a**(13/2)*b**2
1*x**(13/2)) - 480*a**15*b**15*x**(17/2)/(3003*a**(25/2)*b**15*x**(25/2) +
18018*a**(23/2)*b**16*x**(23/2) + 45045*a**(21/2)*b**17*x**(21/2) + 60060*a
**(19/2)*b**18*x**(19/2) + 45045*a**(17/2)*b**19*x**(17/2) + 18018*a**(15/2
)*b**20*x**(15/2) + 3003*a**(13/2)*b**21*x**(13/2)) - 192*a**14*b**16*x**(1
5/2)/(3003*a**(25/2)*b**15*x**(25/2) + 18018*a**(23/2)*b**16*x**(23/2) + 45
045*a**(21/2)*b**17*x**(21/2) + 60060*a**(19/2)*b**18*x**(19/2) + 45045*a**
(17/2)*b**19*x**(17/2) + 18018*a**(15/2)*b**20*x**(15/2) + 3003*a**(13/2)*b
**21*x**(13/2)) - 32*a**13*b**17*x**(13/2)/(3003*a**(25/2)*b**15*x**(25/2)
+ 18018*a**(23/2)*b**16*x**(23/2) + 45045*a**(21/2)*b**17*x**(21/2) + 60060
*a**(19/2)*b**18*x**(19/2) + 45045*a**(17/2)*b**19*x**(17/2) + 18018*a**(15
/2)*b**20*x**(15/2) + 3003*a**(13/2)*b**21*x**(13/2))
```

3.1721 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^6}\,dx$

Optimal. Leaf size=101

$$-\frac{2a^4\left(a+\frac{b}{x}\right)^{7/2}}{7b^5}+\frac{8a^3\left(a+\frac{b}{x}\right)^{9/2}}{9b^5}-\frac{12a^2\left(a+\frac{b}{x}\right)^{11/2}}{11b^5}-\frac{2\left(a+\frac{b}{x}\right)^{15/2}}{15b^5}+\frac{8a\left(a+\frac{b}{x}\right)^{13/2}}{13b^5}$$

```
[Out] -2/7*a^4*(a+b/x)^(7/2)/b^5+8/9*a^3*(a+b/x)^(9/2)/b^5-12/11*a^2*(a+b/x)^(11/
2)/b^5+8/13*a*(a+b/x)^(13/2)/b^5-2/15*(a+b/x)^(15/2)/b^5
```

Rubi [A] time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{12a^2\left(a+\frac{b}{x}\right)^{11/2}}{11b^5}+\frac{8a^3\left(a+\frac{b}{x}\right)^{9/2}}{9b^5}-\frac{2a^4\left(a+\frac{b}{x}\right)^{7/2}}{7b^5}-\frac{2\left(a+\frac{b}{x}\right)^{15/2}}{15b^5}+\frac{8a\left(a+\frac{b}{x}\right)^{13/2}}{13b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)/x^6,x]
```

```
[Out] (-2*a^4*(a + b/x)^(7/2))/(7*b^5) + (8*a^3*(a + b/x)^(9/2))/(9*b^5) - (12*a^
2*(a + b/x)^(11/2))/(11*b^5) + (8*a*(a + b/x)^(13/2))/(13*b^5) - (2*(a + b/
x)^(15/2))/(15*b^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^6}\,dx &= -\text{Subst}\left(\int x^4(a+bx)^{5/2}\,dx, x, \frac{1}{x}\right)\\ &= -\text{Subst}\left(\int\left(\frac{a^4(a+bx)^{5/2}}{b^4}-\frac{4a^3(a+bx)^{7/2}}{b^4}+\frac{6a^2(a+bx)^{9/2}}{b^4}-\frac{4a(a+bx)^{11/2}}{b^4}+\frac{(a+bx)^{13/2}}{b^4}\right)\right)\\ &= -\frac{2a^4\left(a+\frac{b}{x}\right)^{7/2}}{7b^5}+\frac{8a^3\left(a+\frac{b}{x}\right)^{9/2}}{9b^5}-\frac{12a^2\left(a+\frac{b}{x}\right)^{11/2}}{11b^5}+\frac{8a\left(a+\frac{b}{x}\right)^{13/2}}{13b^5}-\frac{2\left(a+\frac{b}{x}\right)^{15/2}}{15b^5}\end{aligned}$$

Mathematica [A] time = 0.04, size = 69, normalized size = 0.68

$$-\frac{2\sqrt{a+\frac{b}{x}}\,(ax+b)^3\left(128a^4x^4-448a^3bx^3+1008a^2b^2x^2-1848ab^3x+3003b^4\right)}{45045b^5x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)/x^6,x]
```

```
[Out] (-2*Sqrt[a + b/x]*(b + a*x)^3*(3003*b^4 - 1848*a*b^3*x + 1008*a^2*b^2*x^2 -
 448*a^3*b*x^3 + 128*a^4*x^4))/(45045*b^5*x^7)
```

fricas [A] time = 0.99, size = 93, normalized size = 0.92

$$-\frac{2\left(128\,a^7x^7-64\,a^6bx^6+48\,a^5b^2x^5-40\,a^4b^3x^4+35\,a^3b^4x^3+4473\,a^2b^5x^2+7161\,ab^6x+3003\,b^7\right)\sqrt{\frac{ax+b}{x}}}{45045\,b^5x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/x^6,x, algorithm="fricas")
```

```
[Out] -2/45045*(128*a^7*x^7 - 64*a^6*b*x^6 + 48*a^5*b^2*x^5 - 40*a^4*b^3*x^4 + 35
*a^3*b^4*x^3 + 4473*a^2*b^5*x^2 + 7161*a*b^6*x + 3003*b^7)*sqrt((a*x + b)/x
)/(b^5*x^7)
```

giac [B] time = 0.38, size = 332, normalized size = 3.29

$$2\left(144144\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)^{10}a^5\mathrm{sgn}(x)+960960\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)^{9}a^{\frac{9}{2}}b\mathrm{sgn}(x)+2934360\left(\sqrt{a}\,x-\sqrt{ax^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/x^6,x, algorithm="giac")
```

```
[Out] 2/45045*(144144*(sqrt(a)*x - sqrt(a*x^2 + b*x))^10*a^5*sgn(x) + 960960*(sqr
t(a)*x - sqrt(a*x^2 + b*x))^9*a^(9/2)*b*sgn(x) + 2934360*(sqrt(a)*x - sqrt(
a*x^2 + b*x))^8*a^4*b^2*sgn(x) + 5360355*(sqrt(a)*x - sqrt(a*x^2 + b*x))^7*
a^(7/2)*b^3*sgn(x) + 6451445*(sqrt(a)*x - sqrt(a*x^2 + b*x))^6*a^3*b^4*sgn(
x) + 5324319*(sqrt(a)*x - sqrt(a*x^2 + b*x))^5*a^(5/2)*b^5*sgn(x) + 3042585
*(sqrt(a)*x - sqrt(a*x^2 + b*x))^4*a^2*b^6*sgn(x) + 1186185*(sqrt(a)*x - sq
rt(a*x^2 + b*x))^3*a^(3/2)*b^7*sgn(x) + 301455*(sqrt(a)*x - sqrt(a*x^2 + b*
x))^2*a*b^8*sgn(x) + 45045*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*b^9*sgn(
x) + 3003*b^10*sgn(x))/(sqrt(a)*x - sqrt(a*x^2 + b*x))^15
```

maple [A] time = 0.01, size = 66, normalized size = 0.65

$$-\frac{2\,(ax+b)\left(128a^4x^4-448a^3x^3b+1008a^2x^2b^2-1848ax\,b^3+3003b^4\right)\left(\frac{ax+b}{x}\right)^{\frac{5}{2}}}{45045b^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2)/x^6,x)
```

```
[Out] -2/45045*(a*x+b)*(128*a^4*x^4-448*a^3*b*x^3+1008*a^2*b^2*x^2-1848*a*b^3*x+3
003*b^4)*((a*x+b)/x)^(5/2)/x^5/b^5
```

maxima [A] time = 1.05, size = 81, normalized size = 0.80

$$-\frac{2\left(a+\frac{b}{x}\right)^{\frac{15}{2}}}{15\,b^5}+\frac{8\left(a+\frac{b}{x}\right)^{\frac{13}{2}}a}{13\,b^5}-\frac{12\left(a+\frac{b}{x}\right)^{\frac{11}{2}}a^2}{11\,b^5}+\frac{8\left(a+\frac{b}{x}\right)^{\frac{9}{2}}a^3}{9\,b^5}-\frac{2\left(a+\frac{b}{x}\right)^{\frac{7}{2}}a^4}{7\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/x^6,x, algorithm="maxima")
```

```
[Out] -2/15*(a + b/x)^(15/2)/b^5 + 8/13*(a + b/x)^(13/2)*a/b^5 - 12/11*(a + b/x)^
(11/2)*a^2/b^5 + 8/9*(a + b/x)^(9/2)*a^3/b^5 - 2/7*(a + b/x)^(7/2)*a^4/b^5
```

mupad [B] time = 3.01, size = 148, normalized size = 1.47

$$\frac{16\,a^4\sqrt{a+\frac{b}{x}}}{9009\,b^2\,x^3}-\frac{142\,a^2\sqrt{a+\frac{b}{x}}}{715\,x^5}-\frac{2\,b^2\sqrt{a+\frac{b}{x}}}{15\,x^7}-\frac{2\,a^3\sqrt{a+\frac{b}{x}}}{1287\,b\,x^4}-\frac{256\,a^7\sqrt{a+\frac{b}{x}}}{45045\,b^5}-\frac{32\,a^5\sqrt{a+\frac{b}{x}}}{15015\,b^3\,x^2}+\frac{128\,a^6\sqrt{a+\frac{b}{x}}}{45045\,b^4\,x}-\frac{62\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(5/2)/x^6,x)
```

```
[Out] (16*a^4*(a + b/x)^(1/2))/(9009*b^2*x^3) - (142*a^2*(a + b/x)^(1/2))/(715*x^
5) - (2*b^2*(a + b/x)^(1/2))/(15*x^7) - (2*a^3*(a + b/x)^(1/2))/(1287*b*x^4
) - (256*a^7*(a + b/x)^(1/2))/(45045*b^5) - (32*a^5*(a + b/x)^(1/2))/(15015
*b^3*x^2) + (128*a^6*(a + b/x)^(1/2))/(45045*b^4*x) - (62*a*b*(a + b/x)^(1/
2))/(195*x^6)
```

sympy [B] time = 5.71, size = 5482, normalized size = 54.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)/x**6,x)
```

```
[Out] -256*a**(49/2)*b**(49/2)*x**17*sqrt(a*x/b + 1)/(45045*a**(35/2)*b**29*x**(3
5/2) + 450450*a**(33/2)*b**30*x**(33/2) + 2027025*a**(31/2)*b**31*x**(31/2)
 + 5405400*a**(29/2)*b**32*x**(29/2) + 9459450*a**(27/2)*b**33*x**(27/2) +
11351340*a**(25/2)*b**34*x**(25/2) + 9459450*a**(23/2)*b**35*x**(23/2) + 54
05400*a**(21/2)*b**36*x**(21/2) + 2027025*a**(19/2)*b**37*x**(19/2) + 45045
0*a**(17/2)*b**38*x**(17/2) + 45045*a**(15/2)*b**39*x**(15/2)) - 2432*a**(4
7/2)*b**(51/2)*x**16*sqrt(a*x/b + 1)/(45045*a**(35/2)*b**29*x**(35/2) + 450
450*a**(33/2)*b**30*x**(33/2) + 2027025*a**(31/2)*b**31*x**(31/2) + 5405400
*a**(29/2)*b**32*x**(29/2) + 9459450*a**(27/2)*b**33*x**(27/2) + 11351340*a
**(25/2)*b**34*x**(25/2) + 9459450*a**(23/2)*b**35*x**(23/2) + 5405400*a**(
21/2)*b**36*x**(21/2) + 2027025*a**(19/2)*b**37*x**(19/2) + 450450*a**(17/2
)*b**38*x**(17/2) + 45045*a**(15/2)*b**39*x**(15/2)) - 10336*a**(45/2)*b**(
53/2)*x**15*sqrt(a*x/b + 1)/(45045*a**(35/2)*b**29*x**(35/2) + 450450*a**(3
3/2)*b**30*x**(33/2) + 2027025*a**(31/2)*b**31*x**(31/2) + 5405400*a**(29/2
)*b**32*x**(29/2) + 9459450*a**(27/2)*b**33*x**(27/2) + 11351340*a**(25/2)*
b**34*x**(25/2) + 9459450*a**(23/2)*b**35*x**(23/2) + 5405400*a**(21/2)*b**
36*x**(21/2) + 2027025*a**(19/2)*b**37*x**(19/2) + 450450*a**(17/2)*b**38*x
**(17/2) + 45045*a**(15/2)*b**39*x**(15/2)) - 25840*a**(43/2)*b**(55/2)*x**
14*sqrt(a*x/b + 1)/(45045*a**(35/2)*b**29*x**(35/2) + 450450*a**(33/2)*b**3
0*x**(33/2) + 2027025*a**(31/2)*b**31*x**(31/2) + 5405400*a**(29/2)*b**32*x
**(29/2) + 9459450*a**(27/2)*b**33*x**(27/2) + 11351340*a**(25/2)*b**34*x**
(25/2) + 9459450*a**(23/2)*b**35*x**(23/2) + 5405400*a**(21/2)*b**36*x**(21
/2) + 2027025*a**(19/2)*b**37*x**(19/2) + 450450*a**(17/2)*b**38*x**(17/2)
+ 45045*a**(15/2)*b**39*x**(15/2)) - 41990*a**(41/2)*b**(57/2)*x**13*sqrt(a
*x/b + 1)/(45045*a**(35/2)*b**29*x**(35/2) + 450450*a**(33/2)*b**30*x**(33/
2) + 2027025*a**(31/2)*b**31*x**(31/2) + 5405400*a**(29/2)*b**32*x**(29/2)
+ 9459450*a**(27/2)*b**33*x**(27/2) + 11351340*a**(25/2)*b**34*x**(25/2) +
9459450*a**(23/2)*b**35*x**(23/2) + 5405400*a**(21/2)*b**36*x**(21/2) + 202
7025*a**(19/2)*b**37*x**(19/2) + 450450*a**(17/2)*b**38*x**(17/2) + 45045*a
**(15/2)*b**39*x**(15/2)) - 55198*a**(39/2)*b**(59/2)*x**12*sqrt(a*x/b + 1)
/(45045*a**(35/2)*b**29*x**(35/2) + 450450*a**(33/2)*b**30*x**(33/2) + 2027
025*a**(31/2)*b**31*x**(31/2) + 5405400*a**(29/2)*b**32*x**(29/2) + 9459450
*a**(27/2)*b**33*x**(27/2) + 11351340*a**(25/2)*b**34*x**(25/2) + 9459450*a
```

```
**(23/2)*b**35*x**(23/2) + 5405400*a**(21/2)*b**36*x**(21/2) + 2027025*a**(
19/2)*b**37*x**(19/2) + 450450*a**(17/2)*b**38*x**(17/2) + 45045*a**(15/2)*
b**39*x**(15/2)) - 138996*a**(37/2)*b**(61/2)*x**11*sqrt(a*x/b + 1)/(45045*
a**(35/2)*b**29*x**(35/2) + 450450*a**(33/2)*b**30*x**(33/2) + 2027025*a**(
31/2)*b**31*x**(31/2) + 5405400*a**(29/2)*b**32*x**(29/2) + 9459450*a**(27/
2)*b**33*x**(27/2) + 11351340*a**(25/2)*b**34*x**(25/2) + 9459450*a**(23/2)
*b**35*x**(23/2) + 5405400*a**(21/2)*b**36*x**(21/2) + 2027025*a**(19/2)*b*
*37*x**(19/2) + 450450*a**(17/2)*b**38*x**(17/2) + 45045*a**(15/2)*b**39*x*
*(15/2)) - 571428*a**(35/2)*b**(63/2)*x**10*sqrt(a*x/b + 1)/(45045*a**(35/2
)*b**29*x**(35/2) + 450450*a**(33/2)*b**30*x**(33/2) + 2027025*a**(31/2)*b*
*31*x**(31/2) + 5405400*a**(29/2)*b**32*x**(29/2) + 9459450*a**(27/2)*b**33
*x**(27/2) + 11351340*a**(25/2)*b**34*x**(25/2) + 9459450*a**(23/2)*b**35*x
**(23/2) + 5405400*a**(21/2)*b**36*x**(21/2) + 2027025*a**(19/2)*b**37*x**(
19/2) + 450450*a**(17/2)*b**38*x**(17/2) + 45045*a**(15/2)*b**39*x**(15/2))
 - 1788930*a**(33/2)*b**(65/2)*x**9*sqrt(a*x/b + 1)/(45045*a**(35/2)*b**29*
x**(35/2) + 450450*a**(33/2)*b**30*x**(33/2) + 2027025*a**(31/2)*b**31*x**(
31/2) + 5405400*a**(29/2)*b**32*x**(29/2) + 9459450*a**(27/2)*b**33*x**(27/
2) + 11351340*a**(25/2)*b**34*x**(25/2) + 9459450*a**(23/2)*b**35*x**(23/2)
 + 5405400*a**(21/2)*b**36*x**(21/2) + 2027025*a**(19/2)*b**37*x**(19/2) +
450450*a**(17/2)*b**38*x**(17/2) + 45045*a**(15/2)*b**39*x**(15/2)) - 38767
30*a**(31/2)*b**(67/2)*x**8*sqrt(a*x/b + 1)/(45045*a**(35/2)*b**29*x**(35/2
) + 450450*a**(33/2)*b**30*x**(33/2) + 2027025*a**(31/2)*b**31*x**(31/2) +
5405400*a**(29/2)*b**32*x**(29/2) + 9459450*a**(27/2)*b**33*x**(27/2) + 113
51340*a**(25/2)*b**34*x**(25/2) + 9459450*a**(23/2)*b**35*x**(23/2) + 54054
00*a**(21/2)*b**36*x**(21/2) + 2027025*a**(19/2)*b**37*x**(19/2) + 450450*a
**(17/2)*b**38*x**(17/2) + 45045*a**(15/2)*b**39*x**(15/2)) - 5991128*a**(2
9/2)*b**(69/2)*x**7*sqrt(a*x/b + 1)/(45045*a**(35/2)*b**29*x**(35/2) + 4504
50*a**(33/2)*b**30*x**(33/2) + 2027025*a**(31/2)*b**31*x**(31/2) + 5405400*
a**(29/2)*b**32*x**(29/2) + 9459450*a**(27/2)*b**33*x**(27/2) + 11351340*a*
*(25/2)*b**34*x**(25/2) + 9459450*a**(23/2)*b**35*x**(23/2) + 5405400*a**(2
1/2)*b**36*x**(21/2) + 2027025*a**(19/2)*b**37*x**(19/2) + 450450*a**(17/2)
*b**38*x**(17/2) + 45045*a**(15/2)*b**39*x**(15/2)) - 6754696*a**(27/2)*b**
(71/2)*x**6*sqrt(a*x/b + 1)/(45045*a**(35/2)*b**29*x**(35/2) + 450450*a**(3
3/2)*b**30*x**(33/2) + 2027025*a**(31/2)*b**31*x**(31/2) + 5405400*a**(29/2
)*b**32*x**(29/2) + 9459450*a**(27/2)*b**33*x**(27/2) + 11351340*a**(25/2)*
b**34*x**(25/2) + 9459450*a**(23/2)*b**35*x**(23/2) + 5405400*a**(21/2)*b**
36*x**(21/2) + 2027025*a**(19/2)*b**37*x**(19/2) + 450450*a**(17/2)*b**38*x
**(17/2) + 45045*a**(15/2)*b**39*x**(15/2)) - 5597098*a**(25/2)*b**(73/2)*x
**5*sqrt(a*x/b + 1)/(45045*a**(35/2)*b**29*x**(35/2) + 450450*a**(33/2)*b**
30*x**(33/2) + 2027025*a**(31/2)*b**31*x**(31/2) + 5405400*a**(29/2)*b**32*
x**(29/2) + 9459450*a**(27/2)*b**33*x**(27/2) + 11351340*a**(25/2)*b**34*x*
*(25/2) + 9459450*a**(23/2)*b**35*x**(23/2) + 5405400*a**(21/2)*b**36*x**(2
1/2) + 2027025*a**(19/2)*b**37*x**(19/2) + 450450*a**(17/2)*b**38*x**(17/2)
 + 45045*a**(15/2)*b**39*x**(15/2)) - 3383090*a**(23/2)*b**(75/2)*x**4*sqrt
(a*x/b + 1)/(45045*a**(35/2)*b**29*x**(35/2) + 450450*a**(33/2)*b**30*x**(3
3/2) + 2027025*a**(31/2)*b**31*x**(31/2) + 5405400*a**(29/2)*b**32*x**(29/2
) + 9459450*a**(27/2)*b**33*x**(27/2) + 11351340*a**(25/2)*b**34*x**(25/2)
+ 9459450*a**(23/2)*b**35*x**(23/2) + 5405400*a**(21/2)*b**36*x**(21/2) + 2
027025*a**(19/2)*b**37*x**(19/2) + 450450*a**(17/2)*b**38*x**(17/2) + 45045
*a**(15/2)*b**39*x**(15/2)) - 1454740*a**(21/2)*b**(77/2)*x**3*sqrt(a*x/b +
 1)/(45045*a**(35/2)*b**29*x**(35/2) + 450450*a**(33/2)*b**30*x**(33/2) + 2
027025*a**(31/2)*b**31*x**(31/2) + 5405400*a**(29/2)*b**32*x**(29/2) + 9459
450*a**(27/2)*b**33*x**(27/2) + 11351340*a**(25/2)*b**34*x**(25/2) + 945945
0*a**(23/2)*b**35*x**(23/2) + 5405400*a**(21/2)*b**36*x**(21/2) + 2027025*a
**(19/2)*b**37*x**(19/2) + 450450*a**(17/2)*b**38*x**(17/2) + 45045*a**(15/
2)*b**39*x**(15/2)) - 422436*a**(19/2)*b**(79/2)*x**2*sqrt(a*x/b + 1)/(4504
5*a**(35/2)*b**29*x**(35/2) + 450450*a**(33/2)*b**30*x**(33/2) + 2027025*a*
*(31/2)*b**31*x**(31/2) + 5405400*a**(29/2)*b**32*x**(29/2) + 9459450*a**(2
7/2)*b**33*x**(27/2) + 11351340*a**(25/2)*b**34*x**(25/2) + 9459450*a**(23/
```

```
2)*b**35*x**(23/2) + 5405400*a**(21/2)*b**36*x**(21/2) + 2027025*a**(19/2)*
b**37*x**(19/2) + 450450*a**(17/2)*b**38*x**(17/2) + 45045*a**(15/2)*b**39*
x**(15/2)) - 74382*a**(17/2)*b**(81/2)*x*sqrt(a*x/b + 1)/(45045*a**(35/2)*b
**29*x**(35/2) + 450450*a**(33/2)*b**30*x**(33/2) + 2027025*a**(31/2)*b**31
*x**(31/2) + 5405400*a**(29/2)*b**32*x**(29/2) + 9459450*a**(27/2)*b**33*x*
*(27/2) + 11351340*a**(25/2)*b**34*x**(25/2) + 9459450*a**(23/2)*b**35*x**(
23/2) + 5405400*a**(21/2)*b**36*x**(21/2) + 2027025*a**(19/2)*b**37*x**(19/
2) + 450450*a**(17/2)*b**38*x**(17/2) + 45045*a**(15/2)*b**39*x**(15/2)) -
6006*a**(15/2)*b**(83/2)*sqrt(a*x/b + 1)/(45045*a**(35/2)*b**29*x**(35/2) +
 450450*a**(33/2)*b**30*x**(33/2) + 2027025*a**(31/2)*b**31*x**(31/2) + 540
5400*a**(29/2)*b**32*x**(29/2) + 9459450*a**(27/2)*b**33*x**(27/2) + 113513
40*a**(25/2)*b**34*x**(25/2) + 9459450*a**(23/2)*b**35*x**(23/2) + 5405400*
a**(21/2)*b**36*x**(21/2) + 2027025*a**(19/2)*b**37*x**(19/2) + 450450*a**(
17/2)*b**38*x**(17/2) + 45045*a**(15/2)*b**39*x**(15/2)) + 256*a**25*b**24*
x**(35/2)/(45045*a**(35/2)*b**29*x**(35/2) + 450450*a**(33/2)*b**30*x**(33/
2) + 2027025*a**(31/2)*b**31*x**(31/2) + 5405400*a**(29/2)*b**32*x**(29/2)
 + 9459450*a**(27/2)*b**33*x**(27/2) + 11351340*a**(25/2)*b**34*x**(25/2) +
 9459450*a**(23/2)*b**35*x**(23/2) + 5405400*a**(21/2)*b**36*x**(21/2) + 202
7025*a**(19/2)*b**37*x**(19/2) + 450450*a**(17/2)*b**38*x**(17/2) + 45045*a
**(15/2)*b**39*x**(15/2)) + 2560*a**24*b**25*x**(33/2)/(45045*a**(35/2)*b**
29*x**(35/2) + 450450*a**(33/2)*b**30*x**(33/2) + 2027025*a**(31/2)*b**31*x
**(31/2) + 5405400*a**(29/2)*b**32*x**(29/2) + 9459450*a**(27/2)*b**33*x**(
27/2) + 11351340*a**(25/2)*b**34*x**(25/2) + 9459450*a**(23/2)*b**35*x**(23
/2) + 5405400*a**(21/2)*b**36*x**(21/2) + 2027025*a**(19/2)*b**37*x**(19/2)
 + 450450*a**(17/2)*b**38*x**(17/2) + 45045*a**(15/2)*b**39*x**(15/2)) + 11
520*a**23*b**26*x**(31/2)/(45045*a**(35/2)*b**29*x**(35/2) + 450450*a**(33/
2)*b**30*x**(33/2) + 2027025*a**(31/2)*b**31*x**(31/2) + 5405400*a**(29/2)*
b**32*x**(29/2) + 9459450*a**(27/2)*b**33*x**(27/2) + 11351340*a**(25/2)*b*
*34*x**(25/2) + 9459450*a**(23/2)*b**35*x**(23/2) + 5405400*a**(21/2)*b**36
*x**(21/2) + 2027025*a**(19/2)*b**37*x**(19/2) + 450450*a**(17/2)*b**38*x**
(17/2) + 45045*a**(15/2)*b**39*x**(15/2)) + 30720*a**22*b**27*x**(29/2)/(45
045*a**(35/2)*b**29*x**(35/2) + 450450*a**(33/2)*b**30*x**(33/2) + 2027025*
a**(31/2)*b**31*x**(31/2) + 5405400*a**(29/2)*b**32*x**(29/2) + 9459450*a**
(27/2)*b**33*x**(27/2) + 11351340*a**(25/2)*b**34*x**(25/2) + 9459450*a**(2
3/2)*b**35*x**(23/2) + 5405400*a**(21/2)*b**36*x**(21/2) + 2027025*a**(19/2
)*b**37*x**(19/2) + 450450*a**(17/2)*b**38*x**(17/2) + 45045*a**(15/2)*b**3
9*x**(15/2)) + 53760*a**21*b**28*x**(27/2)/(45045*a**(35/2)*b**29*x**(35/2)
 + 450450*a**(33/2)*b**30*x**(33/2) + 2027025*a**(31/2)*b**31*x**(31/2) + 5
405400*a**(29/2)*b**32*x**(29/2) + 9459450*a**(27/2)*b**33*x**(27/2) + 1135
1340*a**(25/2)*b**34*x**(25/2) + 9459450*a**(23/2)*b**35*x**(23/2) + 540540
0*a**(21/2)*b**36*x**(21/2) + 2027025*a**(19/2)*b**37*x**(19/2) + 450450*a*
*(17/2)*b**38*x**(17/2) + 45045*a**(15/2)*b**39*x**(15/2)) + 64512*a**20*b*
*29*x**(25/2)/(45045*a**(35/2)*b**29*x**(35/2) + 450450*a**(33/2)*b**30*x**
(33/2) + 2027025*a**(31/2)*b**31*x**(31/2) + 5405400*a**(29/2)*b**32*x**(29
/2) + 9459450*a**(27/2)*b**33*x**(27/2) + 11351340*a**(25/2)*b**34*x**(25/2
) + 9459450*a**(23/2)*b**35*x**(23/2) + 5405400*a**(21/2)*b**36*x**(21/2) +
 2027025*a**(19/2)*b**37*x**(19/2) + 450450*a**(17/2)*b**38*x**(17/2) + 450
45*a**(15/2)*b**39*x**(15/2)) + 53760*a**19*b**30*x**(23/2)/(45045*a**(35/2
)*b**29*x**(35/2) + 450450*a**(33/2)*b**30*x**(33/2) + 2027025*a**(31/2)*b*
*31*x**(31/2) + 5405400*a**(29/2)*b**32*x**(29/2) + 9459450*a**(27/2)*b**33
*x**(27/2) + 11351340*a**(25/2)*b**34*x**(25/2) + 9459450*a**(23/2)*b**35*x
**(23/2) + 5405400*a**(21/2)*b**36*x**(21/2) + 2027025*a**(19/2)*b**37*x**(
19/2) + 450450*a**(17/2)*b**38*x**(17/2) + 45045*a**(15/2)*b**39*x**(15/2))
 + 30720*a**18*b**31*x**(21/2)/(45045*a**(35/2)*b**29*x**(35/2) + 450450*a*
*(33/2)*b**30*x**(33/2) + 2027025*a**(31/2)*b**31*x**(31/2) + 5405400*a**(2
9/2)*b**32*x**(29/2) + 9459450*a**(27/2)*b**33*x**(27/2) + 11351340*a**(25/
2)*b**34*x**(25/2) + 9459450*a**(23/2)*b**35*x**(23/2) + 5405400*a**(21/2)*
b**36*x**(21/2) + 2027025*a**(19/2)*b**37*x**(19/2) + 450450*a**(17/2)*b**3
8*x**(17/2) + 45045*a**(15/2)*b**39*x**(15/2)) + 11520*a**17*b**32*x**(19/2
```

```
)/(45045*a**(35/2)*b**29*x**(35/2) + 450450*a**(33/2)*b**30*x**(33/2) + 202
7025*a**(31/2)*b**31*x**(31/2) + 5405400*a**(29/2)*b**32*x**(29/2) + 945945
0*a**(27/2)*b**33*x**(27/2) + 11351340*a**(25/2)*b**34*x**(25/2) + 9459450*
a**(23/2)*b**35*x**(23/2) + 5405400*a**(21/2)*b**36*x**(21/2) + 2027025*a**
(19/2)*b**37*x**(19/2) + 450450*a**(17/2)*b**38*x**(17/2) + 45045*a**(15/2)
*b**39*x**(15/2)) + 2560*a**16*b**33*x**(17/2)/(45045*a**(35/2)*b**29*x**(3
5/2) + 450450*a**(33/2)*b**30*x**(33/2) + 2027025*a**(31/2)*b**31*x**(31/2)
 + 5405400*a**(29/2)*b**32*x**(29/2) + 9459450*a**(27/2)*b**33*x**(27/2) +
11351340*a**(25/2)*b**34*x**(25/2) + 9459450*a**(23/2)*b**35*x**(23/2) + 54
05400*a**(21/2)*b**36*x**(21/2) + 2027025*a**(19/2)*b**37*x**(19/2) + 45045
0*a**(17/2)*b**38*x**(17/2) + 45045*a**(15/2)*b**39*x**(15/2)) + 256*a**15*
b**34*x**(15/2)/(45045*a**(35/2)*b**29*x**(35/2) + 450450*a**(33/2)*b**30*x
**(33/2) + 2027025*a**(31/2)*b**31*x**(31/2) + 5405400*a**(29/2)*b**32*x**(
29/2) + 9459450*a**(27/2)*b**33*x**(27/2) + 11351340*a**(25/2)*b**34*x**(25
/2) + 9459450*a**(23/2)*b**35*x**(23/2) + 5405400*a**(21/2)*b**36*x**(21/2)
 + 2027025*a**(19/2)*b**37*x**(19/2) + 450450*a**(17/2)*b**38*x**(17/2) + 4
5045*a**(15/2)*b**39*x**(15/2))
```

3.1722 $\int \frac{x^3}{\sqrt{a+\frac{b}{x}}}\,dx$

Optimal. Leaf size=120

$$\frac{35b^4\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{64a^{9/2}}-\frac{35b^3x\sqrt{a+\frac{b}{x}}}{64a^4}+\frac{35b^2x^2\sqrt{a+\frac{b}{x}}}{96a^3}-\frac{7bx^3\sqrt{a+\frac{b}{x}}}{24a^2}+\frac{x^4\sqrt{a+\frac{b}{x}}}{4a}$$

```
[Out] 35/64*b^4*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(9/2)-35/64*b^3*x*(a+b/x)^(1/2)/
a^4+35/96*b^2*x^2*(a+b/x)^(1/2)/a^3-7/24*b*x^3*(a+b/x)^(1/2)/a^2+1/4*x^4*(a
+b/x)^(1/2)/a
```

Rubi [A] time = 0.05, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 51, 63, 208}

$$\frac{35b^2x^2\sqrt{a+\frac{b}{x}}}{96a^3}-\frac{35b^3x\sqrt{a+\frac{b}{x}}}{64a^4}+\frac{35b^4\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{64a^{9/2}}-\frac{7bx^3\sqrt{a+\frac{b}{x}}}{24a^2}+\frac{x^4\sqrt{a+\frac{b}{x}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b/x],x]
```

```
[Out] (-35*b^3*Sqrt[a + b/x]*x)/(64*a^4) + (35*b^2*Sqrt[a + b/x]*x^2)/(96*a^3) -
(7*b*Sqrt[a + b/x]*x^3)/(24*a^2) + (Sqrt[a + b/x]*x^4)/(4*a) + (35*b^4*ArcT
anh[Sqrt[a + b/x]/Sqrt[a]])/(64*a^(9/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x}}}\,dx = -\text{Subst}\left(\int \frac{1}{x^5\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{\sqrt{a+\frac{b}{x}}\,x^4}{4a} + \frac{(7b)\,\text{Subst}\left(\int \frac{1}{x^4\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{8a}$$
$$= -\frac{7b\sqrt{a+\frac{b}{x}}\,x^3}{24a^2} + \frac{\sqrt{a+\frac{b}{x}}\,x^4}{4a} - \frac{\left(35b^2\right)\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{48a^2}$$
$$= \frac{35b^2\sqrt{a+\frac{b}{x}}\,x^2}{96a^3} - \frac{7b\sqrt{a+\frac{b}{x}}\,x^3}{24a^2} + \frac{\sqrt{a+\frac{b}{x}}\,x^4}{4a} + \frac{\left(35b^3\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{64a^3}$$
$$= -\frac{35b^3\sqrt{a+\frac{b}{x}}\,x}{64a^4} + \frac{35b^2\sqrt{a+\frac{b}{x}}\,x^2}{96a^3} - \frac{7b\sqrt{a+\frac{b}{x}}\,x^3}{24a^2} + \frac{\sqrt{a+\frac{b}{x}}\,x^4}{4a} - \frac{\left(35b^4\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx\right.}{128a^4}$$
$$= -\frac{35b^3\sqrt{a+\frac{b}{x}}\,x}{64a^4} + \frac{35b^2\sqrt{a+\frac{b}{x}}\,x^2}{96a^3} - \frac{7b\sqrt{a+\frac{b}{x}}\,x^3}{24a^2} + \frac{\sqrt{a+\frac{b}{x}}\,x^4}{4a} - \frac{\left(35b^3\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,\right.}{64a^4}$$
$$= -\frac{35b^3\sqrt{a+\frac{b}{x}}\,x}{64a^4} + \frac{35b^2\sqrt{a+\frac{b}{x}}\,x^2}{96a^3} - \frac{7b\sqrt{a+\frac{b}{x}}\,x^3}{24a^2} + \frac{\sqrt{a+\frac{b}{x}}\,x^4}{4a} + \frac{35b^4\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{64a^{9/2}}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.31

$$\frac{2b^4\sqrt{a+\frac{b}{x}}\;{}_2F_1\left(\frac{1}{2},5;\frac{3}{2};\frac{b}{ax}+1\right)}{a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + b/x],x]
```

```
[Out] (2*b^4*Sqrt[a + b/x]*Hypergeometric2F1[1/2, 5, 3/2, 1 + b/(a*x)])/a^5
```

fricas [A] time = 1.08, size = 174, normalized size = 1.45

$$\left[\frac{105\,\sqrt{a}\,b^4\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(48\,a^4x^4-56\,a^3bx^3+70\,a^2b^2x^2-105\,ab^3x\right)\sqrt{\frac{ax+b}{x}}}{384\,a^5}, -\frac{105\,\sqrt{-a}\,b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/384*(105*sqrt(a)*b^4*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*
(48*a^4*x^4 - 56*a^3*b*x^3 + 70*a^2*b^2*x^2 - 105*a*b^3*x)*sqrt((a*x + b)/x
))/a^5, -1/192*(105*sqrt(-a)*b^4*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (48
*a^4*x^4 - 56*a^3*b*x^3 + 70*a^2*b^2*x^2 - 105*a*b^3*x)*sqrt((a*x + b)/x))/
a^5]
```

giac [A] time = 0.18, size = 141, normalized size = 1.18

$$-\frac{1}{192}\,b^4\left(\frac{105\,\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^4} - \frac{279\,a^3\sqrt{\frac{ax+b}{x}} - \frac{511\,(ax+b)a^2\sqrt{\frac{ax+b}{x}}}{x} + \frac{385\,(ax+b)^2a\sqrt{\frac{ax+b}{x}}}{x^2} - \frac{105\,(ax+b)^3\sqrt{\frac{ax+b}{x}}}{x^3}}{\left(a-\frac{ax+b}{x}\right)^4a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/192*b^4*(105*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^4) - (279*a^
3*sqrt((a*x + b)/x) - 511*(a*x + b)*a^2*sqrt((a*x + b)/x)/x + 385*(a*x + b)
^2*a*sqrt((a*x + b)/x)/x^2 - 105*(a*x + b)^3*sqrt((a*x + b)/x)/x^3)/((a - (
a*x + b)/x)^4*a^4))
```

maple [A] time = 0.01, size = 184, normalized size = 1.53

$$\frac{\sqrt{\frac{ax+b}{x}}\left(192ab^4\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)-87ab^4\ln\left(\frac{2ax+b+2\sqrt{ax^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)+348\sqrt{ax^2+bx}\,a^{\frac{5}{2}}b^2x+96\left(ax^2+bx\right)\right.}{384\sqrt{(ax+b)x}\,a^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b/x)^(1/2),x)
```

```
[Out] 1/384*((a*x+b)/x)^(1/2)*x*(96*(a*x^2+b*x)^(3/2)*a^(7/2)*x-208*(a*x^2+b*x)^(
3/2)*a^(5/2)*b+348*(a*x^2+b*x)^(1/2)*a^(5/2)*b^2*x+174*(a*x^2+b*x)^(1/2)*a^
(3/2)*b^3+192*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*b^4-3
84*a^(3/2)*((a*x+b)*x)^(1/2)*b^3-87*a*b^4*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/
2)*a^(1/2))/a^(1/2)))/((a*x+b)*x)^(1/2)/a^(11/2)
```

maxima [A] time = 2.42, size = 166, normalized size = 1.38

$$-\frac{35b^4\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{128\,a^{\frac{9}{2}}}-\frac{105\left(a+\frac{b}{x}\right)^{\frac{7}{2}}b^4-385\left(a+\frac{b}{x}\right)^{\frac{5}{2}}ab^4+511\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2b^4-279\sqrt{a+\frac{b}{x}}\,a^3b^4}{192\left(\left(a+\frac{b}{x}\right)^4a^4-4\left(a+\frac{b}{x}\right)^3a^5+6\left(a+\frac{b}{x}\right)^2a^6-4\left(a+\frac{b}{x}\right)a^7+a^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] -35/128*b^4*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(9/2
) - 1/192*(105*(a + b/x)^(7/2)*b^4 - 385*(a + b/x)^(5/2)*a*b^4 + 511*(a + b
/x)^(3/2)*a^2*b^4 - 279*sqrt(a + b/x)*a^3*b^4)/((a + b/x)^4*a^4 - 4*(a + b/
x)^3*a^5 + 6*(a + b/x)^2*a^6 - 4*(a + b/x)*a^7 + a^8)
```

mupad [B] time = 1.43, size = 94, normalized size = 0.78

$$\frac{93x^4\sqrt{a+\frac{b}{x}}}{64\,a}-\frac{511x^4\left(a+\frac{b}{x}\right)^{3/2}}{192\,a^2}+\frac{385x^4\left(a+\frac{b}{x}\right)^{5/2}}{192\,a^3}-\frac{35x^4\left(a+\frac{b}{x}\right)^{7/2}}{64\,a^4}-\frac{b^4\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1\mathrm{i}}{\sqrt{a}}\right)35\mathrm{i}}{64\,a^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b/x)^(1/2),x)
```

```
[Out] (93*x^4*(a + b/x)^(1/2))/(64*a) - (b^4*atan(((a + b/x)^(1/2)*1i)/a^(1/2))*3
5i)/(64*a^(9/2)) - (511*x^4*(a + b/x)^(3/2))/(192*a^2) + (385*x^4*(a + b/x)
^(5/2))/(192*a^3) - (35*x^4*(a + b/x)^(7/2))/(64*a^4)
```

sympy [A] time = 11.84, size = 155, normalized size = 1.29

$$\frac{x^{\frac{9}{2}}}{4\sqrt{b}\sqrt{\frac{ax}{b}+1}}-\frac{\sqrt{b}\,x^{\frac{7}{2}}}{24a\sqrt{\frac{ax}{b}+1}}+\frac{7b^{\frac{3}{2}}x^{\frac{5}{2}}}{96a^2\sqrt{\frac{ax}{b}+1}}-\frac{35b^{\frac{5}{2}}x^{\frac{3}{2}}}{192a^3\sqrt{\frac{ax}{b}+1}}-\frac{35b^{\frac{7}{2}}\sqrt{x}}{64a^4\sqrt{\frac{ax}{b}+1}}+\frac{35b^4\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{64a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b/x)**(1/2),x)
```

```
[Out] x**(9/2)/(4*sqrt(b)*sqrt(a*x/b + 1)) - sqrt(b)*x**(7/2)/(24*a*sqrt(a*x/b +
1)) + 7*b**(3/2)*x**(5/2)/(96*a**2*sqrt(a*x/b + 1)) - 35*b**(5/2)*x**(3/2)/
(192*a**3*sqrt(a*x/b + 1)) - 35*b**(7/2)*sqrt(x)/(64*a**4*sqrt(a*x/b + 1))
+ 35*b**4*asinh(sqrt(a)*sqrt(x)/sqrt(b))/(64*a**(9/2))
```

3.1723 $\int \frac{x^2}{\sqrt{a+\frac{b}{x}}}\,dx$

Optimal. Leaf size=96

$$-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{8a^{7/2}}+\frac{5b^2x\sqrt{a+\frac{b}{x}}}{8a^3}-\frac{5bx^2\sqrt{a+\frac{b}{x}}}{12a^2}+\frac{x^3\sqrt{a+\frac{b}{x}}}{3a}$$

```
[Out] -5/8*b^3*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(7/2)+5/8*b^2*x*(a+b/x)^(1/2)/a^3
-5/12*b*x^2*(a+b/x)^(1/2)/a^2+1/3*x^3*(a+b/x)^(1/2)/a
```

Rubi [A] time = 0.04, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 51, 63, 208}

$$\frac{5b^2x\sqrt{a+\frac{b}{x}}}{8a^3}-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{8a^{7/2}}-\frac{5bx^2\sqrt{a+\frac{b}{x}}}{12a^2}+\frac{x^3\sqrt{a+\frac{b}{x}}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b/x],x]
```

```
[Out] (5*b^2*Sqrt[a + b/x]*x)/(8*a^3) - (5*b*Sqrt[a + b/x]*x^2)/(12*a^2) + (Sqrt[
a + b/x]*x^3)/(3*a) - (5*b^3*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(8*a^(7/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x}}}\,dx = -\text{Subst}\left(\int \frac{1}{x^4\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{\sqrt{a+\frac{b}{x}}\,x^3}{3a} + \frac{(5b)\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{6a}$$
$$= -\frac{5b\sqrt{a+\frac{b}{x}}\,x^2}{12a^2} + \frac{\sqrt{a+\frac{b}{x}}\,x^3}{3a} - \frac{\left(5b^2\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{8a^2}$$
$$= \frac{5b^2\sqrt{a+\frac{b}{x}}\,x}{8a^3} - \frac{5b\sqrt{a+\frac{b}{x}}\,x^2}{12a^2} + \frac{\sqrt{a+\frac{b}{x}}\,x^3}{3a} + \frac{\left(5b^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{16a^3}$$
$$= \frac{5b^2\sqrt{a+\frac{b}{x}}\,x}{8a^3} - \frac{5b\sqrt{a+\frac{b}{x}}\,x^2}{12a^2} + \frac{\sqrt{a+\frac{b}{x}}\,x^3}{3a} + \frac{\left(5b^2\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{8a^3}$$
$$= \frac{5b^2\sqrt{a+\frac{b}{x}}\,x}{8a^3} - \frac{5b\sqrt{a+\frac{b}{x}}\,x^2}{12a^2} + \frac{\sqrt{a+\frac{b}{x}}\,x^3}{3a} - \frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{8a^{7/2}}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.39

$$-\frac{2b^3\sqrt{a+\frac{b}{x}}\;{}_2F_1\left(\frac{1}{2}, 4; \frac{3}{2}; \frac{b}{ax}+1\right)}{a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b/x],x]
```

```
[Out] (-2*b^3*Sqrt[a + b/x]*Hypergeometric2F1[1/2, 4, 3/2, 1 + b/(a*x)])/a^4
```

fricas [A] time = 0.75, size = 151, normalized size = 1.57

$$\left[\frac{15\sqrt{a}\,b^3\log\left(2ax - 2\sqrt{a}\,x\sqrt{\frac{ax+b}{x}} + b\right) + 2\left(8a^3x^3 - 10a^2bx^2 + 15ab^2x\right)\sqrt{\frac{ax+b}{x}}}{48a^4}, \frac{15\sqrt{-a}\,b^3\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*sqrt(a)*b^3*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(8
*a^3*x^3 - 10*a^2*b*x^2 + 15*a*b^2*x)*sqrt((a*x + b)/x))/a^4, 1/24*(15*sqrt
(-a)*b^3*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (8*a^3*x^3 - 10*a^2*b*x^2 +
 15*a*b^2*x)*sqrt((a*x + b)/x))/a^4]
```

giac [A] time = 0.17, size = 115, normalized size = 1.20

$$\frac{1}{24}b^3\left(\frac{15\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^3} - \frac{33a^2\sqrt{\frac{ax+b}{x}} - \frac{40(ax+b)a\sqrt{\frac{ax+b}{x}}}{x} + \frac{15(ax+b)^2\sqrt{\frac{ax+b}{x}}}{x^2}}{\left(a-\frac{ax+b}{x}\right)^3a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*b^3*(15*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^3) - (33*a^2*sq
rt((a*x + b)/x) - 40*(a*x + b)*a*sqrt((a*x + b)/x)/x + 15*(a*x + b)^2*sqrt(
(a*x + b)/x)/x^2)/((a - (a*x + b)/x)^3*a^3))
```

maple [B] time = 0.01, size = 164, normalized size = 1.71

$$\frac{\sqrt{\frac{ax+b}{x}}\left(-24a\,b^3\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)+9a\,b^3\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)-36\sqrt{a\,x^2+bx}\;a^{\frac{5}{2}}bx-18\sqrt{a\,x^2+bx}\;a^{\frac{3}{2}}b\right.}{48\sqrt{(ax+b)\,x}\;a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b/x)^(1/2),x)
```

```
[Out] 1/48*((a*x+b)/x)^(1/2)*x/a^(9/2)*(16*(a*x^2+b*x)^(3/2)*a^(5/2)-36*(a*x^2+b*
x)^(1/2)*a^(5/2)*b*x-18*(a*x^2+b*x)^(1/2)*a^(3/2)*b^2+48*a^(3/2)*((a*x+b)*x
)^(1/2)*b^2-24*a*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*b^3+
9*a*b^3*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2)))/((a*x+b)*x)^
(1/2)
```

maxima [A] time = 2.35, size = 137, normalized size = 1.43

$$\frac{5\,b^3\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{16\,a^{\frac{7}{2}}}+\frac{15\left(a+\frac{b}{x}\right)^{\frac{5}{2}}b^3-40\left(a+\frac{b}{x}\right)^{\frac{3}{2}}ab^3+33\sqrt{a+\frac{b}{x}}\,a^2b^3}{24\left(\left(a+\frac{b}{x}\right)^3a^3-3\left(a+\frac{b}{x}\right)^2a^4+3\left(a+\frac{b}{x}\right)a^5-a^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] 5/16*b^3*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(7/2) +
 1/24*(15*(a + b/x)^(5/2)*b^3 - 40*(a + b/x)^(3/2)*a*b^3 + 33*sqrt(a + b/x)
*a^2*b^3)/((a + b/x)^3*a^3 - 3*(a + b/x)^2*a^4 + 3*(a + b/x)*a^5 - a^6)
```

mupad [B] time = 1.40, size = 77, normalized size = 0.80

$$\frac{11\,x^3\sqrt{a+\frac{b}{x}}}{8\,a}-\frac{5\,x^3\left(a+\frac{b}{x}\right)^{3/2}}{3\,a^2}+\frac{5\,x^3\left(a+\frac{b}{x}\right)^{5/2}}{8\,a^3}+\frac{b^3\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1\mathrm{i}}{\sqrt{a}}\right)5\mathrm{i}}{8\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b/x)^(1/2),x)
```

```
[Out] (b^3*atan(((a + b/x)^(1/2)*1i)/a^(1/2))*5i)/(8*a^(7/2)) + (11*x^3*(a + b/x)
^(1/2))/(8*a) - (5*x^3*(a + b/x)^(3/2))/(3*a^2) + (5*x^3*(a + b/x)^(5/2))/(
8*a^3)
```

sympy [A] time = 7.92, size = 128, normalized size = 1.33

$$\frac{x^{\frac{7}{2}}}{3\sqrt{b}\sqrt{\frac{ax}{b}+1}}-\frac{\sqrt{b}\,x^{\frac{5}{2}}}{12a\sqrt{\frac{ax}{b}+1}}+\frac{5b^{\frac{3}{2}}x^{\frac{3}{2}}}{24a^2\sqrt{\frac{ax}{b}+1}}+\frac{5b^{\frac{5}{2}}\sqrt{x}}{8a^3\sqrt{\frac{ax}{b}+1}}-\frac{5b^3\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{8a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b/x)**(1/2),x)
```

```
[Out] x**(7/2)/(3*sqrt(b)*sqrt(a*x/b + 1)) - sqrt(b)*x**(5/2)/(12*a*sqrt(a*x/b +
1)) + 5*b**(3/2)*x**(3/2)/(24*a**2*sqrt(a*x/b + 1)) + 5*b**(5/2)*sqrt(x)/(8
*a**3*sqrt(a*x/b + 1)) - 5*b**3*asinh(sqrt(a)*sqrt(x)/sqrt(b))/(8*a**(7/2))
```

3.1724 $\int \frac{x}{\sqrt{a+\frac{b}{x}}}\,dx$

Optimal. Leaf size=72

$$\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4a^{5/2}}-\frac{3bx\sqrt{a+\frac{b}{x}}}{4a^2}+\frac{x^2\sqrt{a+\frac{b}{x}}}{2a}$$

```
[Out] 3/4*b^2*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(5/2)-3/4*b*x*(a+b/x)^(1/2)/a^2+1/
2*x^2*(a+b/x)^(1/2)/a
```

Rubi [A] time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {266, 51, 63, 208}

$$\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4a^{5/2}}-\frac{3bx\sqrt{a+\frac{b}{x}}}{4a^2}+\frac{x^2\sqrt{a+\frac{b}{x}}}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b/x],x]
```

```
[Out] (-3*b*Sqrt[a + b/x]*x)/(4*a^2) + (Sqrt[a + b/x]*x^2)/(2*a) + (3*b^2*ArcTanh
[Sqrt[a + b/x]/Sqrt[a]])/(4*a^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{a+\frac{b}{x}}}\,dx &= -\operatorname{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)\\ &= \frac{\sqrt{a+\frac{b}{x}}\,x^2}{2a} + \frac{(3b)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{4a}\\ &= -\frac{3b\sqrt{a+\frac{b}{x}}\,x}{4a^2} + \frac{\sqrt{a+\frac{b}{x}}\,x^2}{2a} - \frac{\left(3b^2\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{8a^2}\\ &= -\frac{3b\sqrt{a+\frac{b}{x}}\,x}{4a^2} + \frac{\sqrt{a+\frac{b}{x}}\,x^2}{2a} - \frac{(3b)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{4a^2}\\ &= -\frac{3b\sqrt{a+\frac{b}{x}}\,x}{4a^2} + \frac{\sqrt{a+\frac{b}{x}}\,x^2}{2a} + \frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4a^{5/2}}\end{aligned}$$

Mathematica [C] time = 0.04, size = 37, normalized size = 0.51

$$\frac{2b^2\sqrt{a+\frac{b}{x}}\;{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{b}{ax}+1\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b/x],x]
```

```
[Out] (2*b^2*Sqrt[a + b/x]*Hypergeometric2F1[1/2, 3, 3/2, 1 + b/(a*x)])/a^3
```

fricas [A] time = 1.05, size = 130, normalized size = 1.81

$$\left[\frac{3\sqrt{a}\,b^2\log\left(2\,ax+2\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(2\,a^2x^2-3\,abx\right)\sqrt{\frac{ax+b}{x}}}{8\,a^3}, -\frac{3\sqrt{-a}\,b^2\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-\left(2\,a^2x^2-3\,a\right.}{4\,a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(a)*b^2*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(2*a
^2*x^2 - 3*a*b*x)*sqrt((a*x + b)/x))/a^3, -1/4*(3*sqrt(-a)*b^2*arctan(sqrt(
-a)*sqrt((a*x + b)/x)/a) - (2*a^2*x^2 - 3*a*b*x)*sqrt((a*x + b)/x))/a^3]
```

giac [A] time = 0.16, size = 89, normalized size = 1.24

$$-\frac{1}{4}\,b^2\left(\frac{3\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2} - \frac{5\,a\sqrt{\frac{ax+b}{x}} - \frac{3\,(ax+b)\sqrt{\frac{ax+b}{x}}}{x}}{\left(a-\frac{ax+b}{x}\right)^2a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*b^2*(3*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^2) - (5*a*sqrt((
a*x + b)/x) - 3*(a*x + b)*sqrt((a*x + b)/x)/x)/((a - (a*x + b)/x)^2*a^2))
```

maple [B] time = 0.01, size = 142, normalized size = 1.97

$$-\frac{\sqrt{\frac{ax+b}{x}}\left(-4a\,b^2\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)+a\,b^2\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)-4\sqrt{a\,x^2+bx}\;a^{\frac{5}{2}}x+8\sqrt{(ax+b)\,x}\;a^{\frac{3}{2}}b\right.}{8\sqrt{(ax+b)\,x}\;a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b/x)^(1/2),x)
```

```
[Out] -1/8*((a*x+b)/x)^(1/2)*x*(-4*(a*x^2+b*x)^(1/2)*a^(5/2)*x+8*((a*x+b)*x)^(1/2
)*a^(3/2)*b-2*(a*x^2+b*x)^(1/2)*a^(3/2)*b-4*a*ln(1/2*(2*a*x+b+2*((a*x+b)*x)
^(1/2)*a^(1/2))/a^(1/2))*b^2+a*b^2*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1
/2))/a^(1/2)))/((a*x+b)*x)^(1/2)/a^(7/2)
```

maxima [A] time = 2.34, size = 104, normalized size = 1.44

$$-\frac{3\,b^2\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{8\,a^{\frac{5}{2}}}-\frac{3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^2-5\sqrt{a+\frac{b}{x}}\,ab^2}{4\left(\left(a+\frac{b}{x}\right)^2a^2-2\left(a+\frac{b}{x}\right)a^3+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/8*b^2*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(5/2) -
 1/4*(3*(a + b/x)^(3/2)*b^2 - 5*sqrt(a + b/x)*a*b^2)/((a + b/x)^2*a^2 - 2*(
a + b/x)*a^3 + a^4)
```

mupad [B] time = 1.21, size = 57, normalized size = 0.79

$$\frac{3\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4\,a^{5/2}}+\frac{5\,x^2\sqrt{a+\frac{b}{x}}}{4\,a}-\frac{3\,x^2\left(a+\frac{b}{x}\right)^{3/2}}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b/x)^(1/2),x)
```

```
[Out] (3*b^2*atanh((a + b/x)^(1/2)/a^(1/2)))/(4*a^(5/2)) + (5*x^2*(a + b/x)^(1/2)
)/(4*a) - (3*x^2*(a + b/x)^(3/2))/(4*a^2)
```

sympy [A] time = 4.48, size = 100, normalized size = 1.39

$$\frac{x^{\frac{5}{2}}}{2\sqrt{b}\,\sqrt{\frac{ax}{b}+1}}-\frac{\sqrt{b}\,x^{\frac{3}{2}}}{4a\sqrt{\frac{ax}{b}+1}}-\frac{3b^{\frac{3}{2}}\sqrt{x}}{4a^2\sqrt{\frac{ax}{b}+1}}+\frac{3b^2\,\mathrm{asinh}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{4a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x)**(1/2),x)
```

```
[Out] x**(5/2)/(2*sqrt(b)*sqrt(a*x/b + 1)) - sqrt(b)*x**(3/2)/(4*a*sqrt(a*x/b + 1
)) - 3*b**(3/2)*sqrt(x)/(4*a**2*sqrt(a*x/b + 1)) + 3*b**2*asinh(sqrt(a)*sqr
t(x)/sqrt(b))/(4*a**(5/2))
```

3.1725 $\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx$

Optimal. Leaf size=43

$$\frac{x\sqrt{a+\frac{b}{x}}}{a}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] -b*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(3/2)+x*(a+b/x)^(1/2)/a
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {242, 51, 63, 208}

$$\frac{x\sqrt{a+\frac{b}{x}}}{a}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b/x],x]
```

```
[Out] (Sqrt[a + b/x]*x)/a - (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx = -\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{\sqrt{a+\frac{b}{x}}\,x}{a} + \frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a}$$
$$= \frac{\sqrt{a+\frac{b}{x}}\,x}{a} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{a}$$
$$= \frac{\sqrt{a+\frac{b}{x}}\,x}{a} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 1.00

$$\frac{x\sqrt{a+\frac{b}{x}}}{a} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b/x],x]
```

```
[Out] (Sqrt[a + b/x]*x)/a - (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

fricas [A] time = 0.98, size = 98, normalized size = 2.28

$$\left[\frac{2\,ax\sqrt{\frac{ax+b}{x}} + \sqrt{a}\,b\log\left(2\,ax - 2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}} + b\right)}{2\,a^2}, \frac{ax\sqrt{\frac{ax+b}{x}} + \sqrt{-a}\,b\arctan\left(\frac{\sqrt{-a}\,\sqrt{\frac{ax+b}{x}}}{a}\right)}{a^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a*x + b)/x) + sqrt(a)*b*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x
 + b)/x) + b))/a^2, (a*x*sqrt((a*x + b)/x) + sqrt(-a)*b*arctan(sqrt(-a)*sqr
t((a*x + b)/x)/a))/a^2]
```

giac [B] time = 0.23, size = 71, normalized size = 1.65

$$-\frac{b\log\left(|b|\right)\text{sgn}(x)}{2\,a^{\frac{3}{2}}} + \frac{b\log\left(\left|-2\left(\sqrt{a}\,x - \sqrt{ax^2+bx}\right)\sqrt{a} - b\right|\right)}{2\,a^{\frac{3}{2}}\text{sgn}(x)} + \frac{\sqrt{ax^2+bx}}{a\text{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*b*log(abs(b))*sgn(x)/a^(3/2) + 1/2*b*log(abs(-2*(sqrt(a)*x - sqrt(a*x^
2 + b*x))*sqrt(a) - b))/(a^(3/2)*sgn(x)) + sqrt(a*x^2 + b*x)/(a*sgn(x))
```

maple [A] time = 0.00, size = 70, normalized size = 1.63

$$-\frac{\sqrt{\frac{ax+b}{x}}\,\left(b\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right) - 2\sqrt{(ax+b)\,x}\,\sqrt{a}\right)x}{2\sqrt{(ax+b)\,x}\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(1/2),x)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)*x*(b*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^
(1/2))-2*((a*x+b)*x)^(1/2)*a^(1/2))/((a*x+b)*x)^(1/2)/a^(3/2)
```

maxima [A] time = 2.23, size = 67, normalized size = 1.56

$$\frac{\sqrt{a+\frac{b}{x}}\,b}{\left(a+\frac{b}{x}\right)a-a^2}+\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{2\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a + b/x)*b/((a + b/x)*a - a^2) + 1/2*b*log((sqrt(a + b/x) - sqrt(a))/(
sqrt(a + b/x) + sqrt(a)))/a^(3/2)
```

mupad [B] time = 1.20, size = 66, normalized size = 1.53

$$\frac{2\,x\left(\frac{3\,\sqrt{b}\,\sqrt{b+a\,x}}{2\,a\,x}+\frac{b^{3/2}\,\mathrm{asin}\left(\frac{\sqrt{a}\,\sqrt{x}\,1\mathrm{i}}{\sqrt{b}}\right)3\mathrm{i}}{2\,a^{3/2}\,x^{3/2}}\right)\sqrt{\frac{a\,x}{b}+1}}{3\sqrt{a+\frac{b}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x)^(1/2),x)
```

```
[Out] (2*x*((3*b^(1/2)*(b + a*x)^(1/2))/(2*a*x) + (b^(3/2)*asin((a^(1/2)*x^(1/2)*
1i)/b^(1/2))*3i)/(2*a^(3/2)*x^(3/2)))*((a*x)/b + 1)^(1/2))/(3*(a + b/x)^(1/
2))
```

sympy [A] time = 2.63, size = 44, normalized size = 1.02

$$\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{\frac{ax}{b}+1}}{a}-\frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(1/2),x)
```

```
[Out] sqrt(b)*sqrt(x)*sqrt(a*x/b + 1)/a - b*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a**(3/
2)
```

3.1726 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\,x}\,dx$

Optimal. Leaf size=25

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] 2*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {266, 63, 208}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x]*x),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+\frac{b}{x}}\,x}\,dx &= -\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)\\ &= -\frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{b}\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x]*x),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

fricas [A] time = 1.11, size = 60, normalized size = 2.40

$$\left[\frac{\log\left(2\,ax + 2\sqrt{a}\,x\sqrt{\frac{ax+b}{x}} + b\right)}{\sqrt{a}}, -\frac{2\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b)/sqrt(a), -2*sqrt(-a)*arctan
(sqrt(-a)*sqrt((a*x + b)/x)/a)/a]
```

giac [A] time = 0.18, size = 25, normalized size = 1.00

$$-\frac{2\,\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] -2*arctan(sqrt((a*x + b)/x)/sqrt(-a))/sqrt(-a)
```

maple [B] time = 0.01, size = 121, normalized size = 4.84

$$-\frac{\sqrt{\frac{ax+b}{x}}\left(-b\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right) - b\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right) + 2\sqrt{(ax+b)\,x}\,\sqrt{a} - 2\sqrt{a\,x^2+bx}\,\sqrt{a}\right)x}{2\sqrt{(ax+b)\,x}\,\sqrt{a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b/x)^(1/2),x)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)*x*(2*((a*x+b)*x)^(1/2)*a^(1/2)-b*ln(1/2*(2*a*x+b+2*(
(a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))-2*(a*x^2+b*x)^(1/2)*a^(1/2)-b*ln(1/2*(2*
a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2)))/((a*x+b)*x)^(1/2)/b/a^(1/2)
```

maxima [A] time = 2.34, size = 37, normalized size = 1.48

$$-\frac{\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/x)^(1/2),x, algorithm="maxima")
[Out] -log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/sqrt(a)
```

mupad [B] time = 1.41, size = 19, normalized size = 0.76

$$\frac{2\,\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b/x)^(1/2)),x)
[Out] (2*atanh((a + b/x)^(1/2)/a^(1/2)))/a^(1/2)
```

sympy [A] time = 1.20, size = 22, normalized size = 0.88

$$\frac{2\,\operatorname{asinh}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/x)**(1/2),x)
[Out] 2*asinh(sqrt(a)*sqrt(x)/sqrt(b))/sqrt(a)
```

3.1727 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\,x^2}\,dx$

Optimal. Leaf size=16

$$-\frac{2\sqrt{a+\frac{b}{x}}}{b}$$

```
[Out] -2*(a+b/x)^(1/2)/b
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$-\frac{2\sqrt{a+\frac{b}{x}}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x]*x^2),x]
```

```
[Out] (-2*Sqrt[a + b/x])/b
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\,x^2}\,dx = -\frac{2\sqrt{a+\frac{b}{x}}}{b}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$-\frac{2\sqrt{a+\frac{b}{x}}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x]*x^2),x]
```

```
[Out] (-2*Sqrt[a + b/x])/b
```

fricas [A] time = 1.02, size = 16, normalized size = 1.00

$$-\frac{2\sqrt{\frac{ax+b}{x}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt((a*x + b)/x)/b
```

giac [A] time = 0.15, size = 16, normalized size = 1.00

$$-\frac{2\sqrt{\frac{ax+b}{x}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b/x)^(1/2),x, algorithm="giac")
[Out] -2*sqrt((a*x + b)/x)/b
```

maple [A] time = 0.00, size = 25, normalized size = 1.56

$$-\frac{2\,(ax+b)}{\sqrt{\frac{ax+b}{x}}\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b/x)^(1/2),x)
[Out] -2*(a*x+b)/x/b/((a*x+b)/x)^(1/2)
```

maxima [A] time = 1.14, size = 14, normalized size = 0.88

$$-\frac{2\sqrt{a+\frac{b}{x}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b/x)^(1/2),x, algorithm="maxima")
[Out] -2*sqrt(a + b/x)/b
```

mupad [B] time = 1.32, size = 14, normalized size = 0.88

$$-\frac{2\sqrt{a+\frac{b}{x}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b/x)^(1/2)),x)
[Out] -(2*(a + b/x)^(1/2))/b
```

sympy [A] time = 0.96, size = 22, normalized size = 1.38

$$\begin{cases} -\frac{2\sqrt{a+\frac{b}{x}}}{b} & \text{for } b \neq 0 \\ -\frac{1}{\sqrt{a}\,x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b/x)**(1/2),x)
[Out] Piecewise((-2*sqrt(a + b/x)/b, Ne(b, 0)), (-1/(sqrt(a)*x), True))
```

3.1728 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\,x^3}\,dx$

Optimal. Leaf size=36

$$\frac{2a\sqrt{a+\frac{b}{x}}}{b^2}-\frac{2\left(a+\frac{b}{x}\right)^{3/2}}{3b^2}$$

```
[Out] -2/3*(a+b/x)^(3/2)/b^2+2*a*(a+b/x)^(1/2)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{2a\sqrt{a+\frac{b}{x}}}{b^2}-\frac{2\left(a+\frac{b}{x}\right)^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x]*x^3),x]
```

```
[Out] (2*a*Sqrt[a + b/x])/b^2 - (2*(a + b/x)^(3/2))/(3*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+\frac{b}{x}}\,x^3}\,dx &= -\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)\\ &= -\text{Subst}\left(\int\left(-\frac{a}{b\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b}\right)dx, x, \frac{1}{x}\right)\\ &= \frac{2a\sqrt{a+\frac{b}{x}}}{b^2}-\frac{2\left(a+\frac{b}{x}\right)^{3/2}}{3b^2}\end{aligned}$$

Mathematica [A] time = 0.04, size = 29, normalized size = 0.81

$$\frac{2\sqrt{a+\frac{b}{x}}\,(2ax-b)}{3b^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x]*x^3),x]
```

```
[Out] (2*Sqrt[a + b/x]*(-b + 2*a*x))/(3*b^2*x)
```

fricas [A] time = 0.86, size = 27, normalized size = 0.75

$$\frac{2\,(2\,ax-b)\sqrt{\frac{ax+b}{x}}}{3\,b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b/x)^(1/2),x, algorithm="fricas")
[Out] 2/3*(2*a*x - b)*sqrt((a*x + b)/x)/(b^2*x)
```

giac [A] time = 0.19, size = 41, normalized size = 1.14

$$\frac{2\left(3\,a\sqrt{\frac{ax+b}{x}}-\frac{(ax+b)\sqrt{\frac{ax+b}{x}}}{x}\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b/x)^(1/2),x, algorithm="giac")
[Out] 2/3*(3*a*sqrt((a*x + b)/x) - (a*x + b)*sqrt((a*x + b)/x)/x)/b^2
```

maple [A] time = 0.00, size = 33, normalized size = 0.92

$$\frac{2\,(ax+b)\,(2ax-b)}{3\sqrt{\frac{ax+b}{x}}\;b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b/x)^(1/2),x)
[Out] 2/3*(a*x+b)*(2*a*x-b)/x^2/b^2/((a*x+b)/x)^(1/2)
```

maxima [A] time = 0.90, size = 30, normalized size = 0.83

$$-\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3\,b^2}+\frac{2\sqrt{a+\frac{b}{x}}\,a}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b/x)^(1/2),x, algorithm="maxima")
[Out] -2/3*(a + b/x)^(3/2)/b^2 + 2*sqrt(a + b/x)*a/b^2
```

mupad [B] time = 1.30, size = 23, normalized size = 0.64

$$-\frac{2\sqrt{a+\frac{b}{x}}\;(b-2\,a\,x)}{3\,b^2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b/x)^(1/2)),x)
[Out] -(2*(a + b/x)^(1/2)*(b - 2*a*x))/(3*b^2*x)
```

sympy [B] time = 1.45, size = 248, normalized size = 6.89

$$\frac{4a^{\frac{7}{2}}b^{\frac{3}{2}}x^2\sqrt{\frac{ax}{b}+1}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}}+\frac{2a^{\frac{5}{2}}b^{\frac{5}{2}}x\sqrt{\frac{ax}{b}+1}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}}-\frac{2a^{\frac{3}{2}}b^{\frac{7}{2}}\sqrt{\frac{ax}{b}+1}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}}-\frac{4a^4bx^{\frac{5}{2}}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}}-\frac{4a^3b^2x^{\frac{3}{2}}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b/x)**(1/2),x)
```

```
[Out] 4*a**(7/2)*b**(3/2)*x**2*sqrt(a*x/b + 1)/(3*a**(5/2)*b**3*x**(5/2) + 3*a**(
3/2)*b**4*x**(3/2)) + 2*a**(5/2)*b**(5/2)*x*sqrt(a*x/b + 1)/(3*a**(5/2)*b**
3*x**(5/2) + 3*a**(3/2)*b**4*x**(3/2)) - 2*a**(3/2)*b**(7/2)*sqrt(a*x/b + 1
)/(3*a**(5/2)*b**3*x**(5/2) + 3*a**(3/2)*b**4*x**(3/2)) - 4*a**4*b*x**(5/2)
/(3*a**(5/2)*b**3*x**(5/2) + 3*a**(3/2)*b**4*x**(3/2)) - 4*a**3*b**2*x**(3/
2)/(3*a**(5/2)*b**3*x**(5/2) + 3*a**(3/2)*b**4*x**(3/2))
```

3.1729 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\,x^4}\,dx$

Optimal. Leaf size=57

$$-\frac{2a^2\sqrt{a+\frac{b}{x}}}{b^3}-\frac{2\left(a+\frac{b}{x}\right)^{5/2}}{5b^3}+\frac{4a\left(a+\frac{b}{x}\right)^{3/2}}{3b^3}$$

```
[Out] 4/3*a*(a+b/x)^(3/2)/b^3-2/5*(a+b/x)^(5/2)/b^3-2*a^2*(a+b/x)^(1/2)/b^3
```

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{2a^2\sqrt{a+\frac{b}{x}}}{b^3}-\frac{2\left(a+\frac{b}{x}\right)^{5/2}}{5b^3}+\frac{4a\left(a+\frac{b}{x}\right)^{3/2}}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x]*x^4),x]
```

```
[Out] (-2*a^2*Sqrt[a + b/x])/b^3 + (4*a*(a + b/x)^(3/2))/(3*b^3) - (2*(a + b/x)^(
5/2))/(5*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+\frac{b}{x}}\,x^4}\,dx &= -\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)\\ &= -\text{Subst}\left(\int\left(\frac{a^2}{b^2\sqrt{a+bx}}-\frac{2a\sqrt{a+bx}}{b^2}+\frac{(a+bx)^{3/2}}{b^2}\right)dx, x, \frac{1}{x}\right)\\ &= -\frac{2a^2\sqrt{a+\frac{b}{x}}}{b^3}+\frac{4a\left(a+\frac{b}{x}\right)^{3/2}}{3b^3}-\frac{2\left(a+\frac{b}{x}\right)^{5/2}}{5b^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 40, normalized size = 0.70

$$-\frac{2\sqrt{a+\frac{b}{x}}\left(8a^2x^2-4abx+3b^2\right)}{15b^3x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x]*x^4),x]
```

```
[Out] (-2*Sqrt[a + b/x]*(3*b^2 - 4*a*b*x + 8*a^2*x^2))/(15*b^3*x^2)
```

fricas [A] time = 0.98, size = 38, normalized size = 0.67

$$-\frac{2\left(8\,a^2x^2-4\,abx+3\,b^2\right)\sqrt{\frac{ax+b}{x}}}{15\,b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/15*(8*a^2*x^2 - 4*a*b*x + 3*b^2)*sqrt((a*x + b)/x)/(b^3*x^2)
```

giac [A] time = 0.17, size = 67, normalized size = 1.18

$$-\frac{2\left(15\,a^2\sqrt{\frac{ax+b}{x}}-\frac{10\,(ax+b)a\sqrt{\frac{ax+b}{x}}}{x}+\frac{3\,(ax+b)^2\sqrt{\frac{ax+b}{x}}}{x^2}\right)}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] -2/15*(15*a^2*sqrt((a*x + b)/x) - 10*(a*x + b)*a*sqrt((a*x + b)/x)/x + 3*(a
*x + b)^2*sqrt((a*x + b)/x)/x^2)/b^3
```

maple [A] time = 0.00, size = 44, normalized size = 0.77

$$-\frac{2\,(ax+b)\left(8a^2x^2-4abx+3b^2\right)}{15\sqrt{\frac{ax+b}{x}}\;b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a+b/x)^(1/2),x)
```

```
[Out] -2/15*(a*x+b)*(8*a^2*x^2-4*a*b*x+3*b^2)/x^3/b^3/((a*x+b)/x)^(1/2)
```

maxima [A] time = 1.05, size = 47, normalized size = 0.82

$$-\frac{2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{5\,b^3}+\frac{4\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a}{3\,b^3}-\frac{2\sqrt{a+\frac{b}{x}}\,a^2}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/5*(a + b/x)^(5/2)/b^3 + 4/3*(a + b/x)^(3/2)*a/b^3 - 2*sqrt(a + b/x)*a^2/
b^3
```

mupad [B] time = 1.30, size = 54, normalized size = 0.95

$$-\frac{6\,b^2\sqrt{a+\frac{b}{x}}+16\,a^2\,x^2\sqrt{a+\frac{b}{x}}-8\,a\,b\,x\sqrt{a+\frac{b}{x}}}{15\,b^3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b/x)^(1/2)),x)
```

```
[Out] -(6*b^2*(a + b/x)^(1/2) + 16*a^2*x^2*(a + b/x)^(1/2) - 8*a*b*x*(a + b/x)^(1
/2))/(15*b^3*x^2)
```

sympy [B] time = 2.25, size = 813, normalized size = 14.26

$$-\frac{16a^{\frac{15}{2}}b^{\frac{9}{2}}x^5\sqrt{\frac{ax}{b}+1}}{15a^{\frac{11}{2}}b^7x^{\frac{11}{2}}+45a^{\frac{9}{2}}b^8x^{\frac{9}{2}}+45a^{\frac{7}{2}}b^9x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^{10}x^{\frac{5}{2}}}-\frac{40a^{\frac{13}{2}}b^{\frac{11}{2}}x^4\sqrt{\frac{ax}{b}+1}}{15a^{\frac{11}{2}}b^7x^{\frac{11}{2}}+45a^{\frac{9}{2}}b^8x^{\frac{9}{2}}+45a^{\frac{7}{2}}b^9x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^{10}x^{\frac{5}{2}}}-\frac{}{15a^{\frac{1}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a+b/x)**(1/2),x)
```

```
[Out] -16*a**(15/2)*b**(9/2)*x**5*sqrt(a*x/b + 1)/(15*a**(11/2)*b**7*x**(11/2) +
45*a**(9/2)*b**8*x**(9/2) + 45*a**(7/2)*b**9*x**(7/2) + 15*a**(5/2)*b**10*x
**(5/2)) - 40*a**(13/2)*b**(11/2)*x**4*sqrt(a*x/b + 1)/(15*a**(11/2)*b**7*x
**(11/2) + 45*a**(9/2)*b**8*x**(9/2) + 45*a**(7/2)*b**9*x**(7/2) + 15*a**(5
/2)*b**10*x**(5/2)) - 30*a**(11/2)*b**(13/2)*x**3*sqrt(a*x/b + 1)/(15*a**(1
1/2)*b**7*x**(11/2) + 45*a**(9/2)*b**8*x**(9/2) + 45*a**(7/2)*b**9*x**(7/2)
 + 15*a**(5/2)*b**10*x**(5/2)) - 10*a**(9/2)*b**(15/2)*x**2*sqrt(a*x/b + 1)
/(15*a**(11/2)*b**7*x**(11/2) + 45*a**(9/2)*b**8*x**(9/2) + 45*a**(7/2)*b**
9*x**(7/2) + 15*a**(5/2)*b**10*x**(5/2)) - 10*a**(7/2)*b**(17/2)*x*sqrt(a*x
/b + 1)/(15*a**(11/2)*b**7*x**(11/2) + 45*a**(9/2)*b**8*x**(9/2) + 45*a**(7
/2)*b**9*x**(7/2) + 15*a**(5/2)*b**10*x**(5/2)) - 6*a**(5/2)*b**(19/2)*sqrt
(a*x/b + 1)/(15*a**(11/2)*b**7*x**(11/2) + 45*a**(9/2)*b**8*x**(9/2) + 45*a
**(7/2)*b**9*x**(7/2) + 15*a**(5/2)*b**10*x**(5/2)) + 16*a**8*b**4*x**(11/2
)/(15*a**(11/2)*b**7*x**(11/2) + 45*a**(9/2)*b**8*x**(9/2) + 45*a**(7/2)*b*
*9*x**(7/2) + 15*a**(5/2)*b**10*x**(5/2)) + 48*a**7*b**5*x**(9/2)/(15*a**(1
1/2)*b**7*x**(11/2) + 45*a**(9/2)*b**8*x**(9/2) + 45*a**(7/2)*b**9*x**(7/2)
 + 15*a**(5/2)*b**10*x**(5/2)) + 48*a**6*b**6*x**(7/2)/(15*a**(11/2)*b**7*x
**(11/2) + 45*a**(9/2)*b**8*x**(9/2) + 45*a**(7/2)*b**9*x**(7/2) + 15*a**(5
/2)*b**10*x**(5/2)) + 16*a**5*b**7*x**(5/2)/(15*a**(11/2)*b**7*x**(11/2) +
45*a**(9/2)*b**8*x**(9/2) + 45*a**(7/2)*b**9*x**(7/2) + 15*a**(5/2)*b**10*x
**(5/2))
```

3.1730 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\,x^5}\,dx$

Optimal. Leaf size=76

$$\frac{2a^3\sqrt{a+\frac{b}{x}}}{b^4}-\frac{2a^2\left(a+\frac{b}{x}\right)^{3/2}}{b^4}-\frac{2\left(a+\frac{b}{x}\right)^{7/2}}{7b^4}+\frac{6a\left(a+\frac{b}{x}\right)^{5/2}}{5b^4}$$

```
[Out] -2*a^2*(a+b/x)^(3/2)/b^4+6/5*a*(a+b/x)^(5/2)/b^4-2/7*(a+b/x)^(7/2)/b^4+2*a^
3*(a+b/x)^(1/2)/b^4
```

Rubi [A] time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{2a^2\left(a+\frac{b}{x}\right)^{3/2}}{b^4}+\frac{2a^3\sqrt{a+\frac{b}{x}}}{b^4}-\frac{2\left(a+\frac{b}{x}\right)^{7/2}}{7b^4}+\frac{6a\left(a+\frac{b}{x}\right)^{5/2}}{5b^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x]*x^5),x]
```

```
[Out] (2*a^3*Sqrt[a + b/x])/b^4 - (2*a^2*(a + b/x)^(3/2))/b^4 + (6*a*(a + b/x)^(5
/2))/(5*b^4) - (2*(a + b/x)^(7/2))/(7*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+\frac{b}{x}}\,x^5}\,dx &= -\text{Subst}\left(\int \frac{x^3}{\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)\\ &= -\text{Subst}\left(\int\left(-\frac{a^3}{b^3\sqrt{a+bx}}+\frac{3a^2\sqrt{a+bx}}{b^3}-\frac{3a(a+bx)^{3/2}}{b^3}+\frac{(a+bx)^{5/2}}{b^3}\right)dx, x, \frac{1}{x}\right)\\ &= \frac{2a^3\sqrt{a+\frac{b}{x}}}{b^4}-\frac{2a^2\left(a+\frac{b}{x}\right)^{3/2}}{b^4}+\frac{6a\left(a+\frac{b}{x}\right)^{5/2}}{5b^4}-\frac{2\left(a+\frac{b}{x}\right)^{7/2}}{7b^4}\end{aligned}$$

Mathematica [A] time = 0.04, size = 51, normalized size = 0.67

$$\frac{2\sqrt{a+\frac{b}{x}}\left(16a^3x^3-8a^2bx^2+6ab^2x-5b^3\right)}{35b^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x]*x^5),x]
```

```
[Out] (2*Sqrt[a + b/x]*(-5*b^3 + 6*a*b^2*x - 8*a^2*b*x^2 + 16*a^3*x^3))/(35*b^4*x
^3)
```

fricas [A] time = 0.57, size = 49, normalized size = 0.64

$$\frac{2\left(16\,a^3x^3-8\,a^2bx^2+6\,ab^2x-5\,b^3\right)\sqrt{\frac{ax+b}{x}}}{35\,b^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(16*a^3*x^3 - 8*a^2*b*x^2 + 6*a*b^2*x - 5*b^3)*sqrt((a*x + b)/x)/(b^4*
x^3)
```

giac [A] time = 0.20, size = 93, normalized size = 1.22

$$\frac{2\left(35\,a^3\sqrt{\frac{ax+b}{x}}-\frac{35\,(ax+b)a^2\sqrt{\frac{ax+b}{x}}}{x}+\frac{21\,(ax+b)^2a\sqrt{\frac{ax+b}{x}}}{x^2}-\frac{5\,(ax+b)^3\sqrt{\frac{ax+b}{x}}}{x^3}\right)}{35\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/35*(35*a^3*sqrt((a*x + b)/x) - 35*(a*x + b)*a^2*sqrt((a*x + b)/x)/x + 21*
(a*x + b)^2*a*sqrt((a*x + b)/x)/x^2 - 5*(a*x + b)^3*sqrt((a*x + b)/x)/x^3)/
b^4
```

maple [A] time = 0.00, size = 55, normalized size = 0.72

$$\frac{2\,(ax+b)\left(16a^3x^3-8a^2b\,x^2+6a\,b^2x-5b^3\right)}{35\sqrt{\frac{ax+b}{x}}\;b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(a+b/x)^(1/2),x)
```

```
[Out] 2/35*(a*x+b)*(16*a^3*x^3-8*a^2*b*x^2+6*a*b^2*x-5*b^3)/x^4/b^4/((a*x+b)/x)^(
1/2)
```

maxima [A] time = 0.98, size = 64, normalized size = 0.84

$$-\frac{2\left(a+\frac{b}{x}\right)^{\frac{7}{2}}}{7\,b^4}+\frac{6\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a}{5\,b^4}-\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2}{b^4}+\frac{2\sqrt{a+\frac{b}{x}}\,a^3}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/7*(a + b/x)^(7/2)/b^4 + 6/5*(a + b/x)^(5/2)*a/b^4 - 2*(a + b/x)^(3/2)*a^
2/b^4 + 2*sqrt(a + b/x)*a^3/b^4
```

mupad [B] time = 1.28, size = 73, normalized size = 0.96

$$\frac{32\,a^3\sqrt{a+\frac{b}{x}}}{35\,b^4}-\frac{2\sqrt{a+\frac{b}{x}}}{7\,b\,x^3}+\frac{12\,a\sqrt{a+\frac{b}{x}}}{35\,b^2\,x^2}-\frac{16\,a^2\sqrt{a+\frac{b}{x}}}{35\,b^3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b/x)^(1/2)),x)
```

```
[Out] (32*a^3*(a + b/x)^(1/2))/(35*b^4) - (2*(a + b/x)^(1/2))/(7*b*x^3) + (12*a*(
a + b/x)^(1/2))/(35*b^2*x^2) - (16*a^2*(a + b/x)^(1/2))/(35*b^3*x)
```

sympy [**B**] time = 3.95, size = 2164, normalized size = 28.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(a+b/x)**(1/2),x)
```

```
[Out] 32*a**(25/2)*b**(23/2)*x**9*sqrt(a*x/b + 1)/(35*a**(19/2)*b**15*x**(19/2) +
 210*a**(17/2)*b**16*x**(17/2) + 525*a**(15/2)*b**17*x**(15/2) + 700*a**(13
/2)*b**18*x**(13/2) + 525*a**(11/2)*b**19*x**(11/2) + 210*a**(9/2)*b**20*x*
*(9/2) + 35*a**(7/2)*b**21*x**(7/2)) + 176*a**(23/2)*b**(25/2)*x**8*sqrt(a*
x/b + 1)/(35*a**(19/2)*b**15*x**(19/2) + 210*a**(17/2)*b**16*x**(17/2) + 52
5*a**(15/2)*b**17*x**(15/2) + 700*a**(13/2)*b**18*x**(13/2) + 525*a**(11/2)
*b**19*x**(11/2) + 210*a**(9/2)*b**20*x**(9/2) + 35*a**(7/2)*b**21*x**(7/2)
) + 396*a**(21/2)*b**(27/2)*x**7*sqrt(a*x/b + 1)/(35*a**(19/2)*b**15*x**(19
/2) + 210*a**(17/2)*b**16*x**(17/2) + 525*a**(15/2)*b**17*x**(15/2) + 700*a
**(13/2)*b**18*x**(13/2) + 525*a**(11/2)*b**19*x**(11/2) + 210*a**(9/2)*b**
20*x**(9/2) + 35*a**(7/2)*b**21*x**(7/2)) + 462*a**(19/2)*b**(29/2)*x**6*sq
rt(a*x/b + 1)/(35*a**(19/2)*b**15*x**(19/2) + 210*a**(17/2)*b**16*x**(17/2)
 + 525*a**(15/2)*b**17*x**(15/2) + 700*a**(13/2)*b**18*x**(13/2) + 525*a**(
11/2)*b**19*x**(11/2) + 210*a**(9/2)*b**20*x**(9/2) + 35*a**(7/2)*b**21*x**
(7/2)) + 280*a**(17/2)*b**(31/2)*x**5*sqrt(a*x/b + 1)/(35*a**(19/2)*b**15*x
**(19/2) + 210*a**(17/2)*b**16*x**(17/2) + 525*a**(15/2)*b**17*x**(15/2) +
700*a**(13/2)*b**18*x**(13/2) + 525*a**(11/2)*b**19*x**(11/2) + 210*a**(9/2
)*b**20*x**(9/2) + 35*a**(7/2)*b**21*x**(7/2)) + 42*a**(15/2)*b**(33/2)*x**
4*sqrt(a*x/b + 1)/(35*a**(19/2)*b**15*x**(19/2) + 210*a**(17/2)*b**16*x**(1
7/2) + 525*a**(15/2)*b**17*x**(15/2) + 700*a**(13/2)*b**18*x**(13/2) + 525*
a**(11/2)*b**19*x**(11/2) + 210*a**(9/2)*b**20*x**(9/2) + 35*a**(7/2)*b**21
*x**(7/2)) - 84*a**(13/2)*b**(35/2)*x**3*sqrt(a*x/b + 1)/(35*a**(19/2)*b**1
5*x**(19/2) + 210*a**(17/2)*b**16*x**(17/2) + 525*a**(15/2)*b**17*x**(15/2)
 + 700*a**(13/2)*b**18*x**(13/2) + 525*a**(11/2)*b**19*x**(11/2) + 210*a**(
9/2)*b**20*x**(9/2) + 35*a**(7/2)*b**21*x**(7/2)) - 94*a**(11/2)*b**(37/2)*
x**2*sqrt(a*x/b + 1)/(35*a**(19/2)*b**15*x**(19/2) + 210*a**(17/2)*b**16*x*
*(17/2) + 525*a**(15/2)*b**17*x**(15/2) + 700*a**(13/2)*b**18*x**(13/2) + 5
25*a**(11/2)*b**19*x**(11/2) + 210*a**(9/2)*b**20*x**(9/2) + 35*a**(7/2)*b*
*21*x**(7/2)) - 48*a**(9/2)*b**(39/2)*x*sqrt(a*x/b + 1)/(35*a**(19/2)*b**15
*x**(19/2) + 210*a**(17/2)*b**16*x**(17/2) + 525*a**(15/2)*b**17*x**(15/2)
+ 700*a**(13/2)*b**18*x**(13/2) + 525*a**(11/2)*b**19*x**(11/2) + 210*a**(9
/2)*b**20*x**(9/2) + 35*a**(7/2)*b**21*x**(7/2)) - 10*a**(7/2)*b**(41/2)*sq
rt(a*x/b + 1)/(35*a**(19/2)*b**15*x**(19/2) + 210*a**(17/2)*b**16*x**(17/2)
 + 525*a**(15/2)*b**17*x**(15/2) + 700*a**(13/2)*b**18*x**(13/2) + 525*a**(
11/2)*b**19*x**(11/2) + 210*a**(9/2)*b**20*x**(9/2) + 35*a**(7/2)*b**21*x**
(7/2)) - 32*a**13*b**11*x**(19/2)/(35*a**(19/2)*b**15*x**(19/2) + 210*a**(1
7/2)*b**16*x**(17/2) + 525*a**(15/2)*b**17*x**(15/2) + 700*a**(13/2)*b**18*
x**(13/2) + 525*a**(11/2)*b**19*x**(11/2) + 210*a**(9/2)*b**20*x**(9/2) + 3
5*a**(7/2)*b**21*x**(7/2)) - 192*a**12*b**12*x**(17/2)/(35*a**(19/2)*b**15*
x**(19/2) + 210*a**(17/2)*b**16*x**(17/2) + 525*a**(15/2)*b**17*x**(15/2) +
 700*a**(13/2)*b**18*x**(13/2) + 525*a**(11/2)*b**19*x**(11/2) + 210*a**(9/
2)*b**20*x**(9/2) + 35*a**(7/2)*b**21*x**(7/2)) - 480*a**11*b**13*x**(15/2)
/(35*a**(19/2)*b**15*x**(19/2) + 210*a**(17/2)*b**16*x**(17/2) + 525*a**(15
/2)*b**17*x**(15/2) + 700*a**(13/2)*b**18*x**(13/2) + 525*a**(11/2)*b**19*x
**(11/2) + 210*a**(9/2)*b**20*x**(9/2) + 35*a**(7/2)*b**21*x**(7/2)) - 640*
a**10*b**14*x**(13/2)/(35*a**(19/2)*b**15*x**(19/2) + 210*a**(17/2)*b**16*x
```

```
**(17/2) + 525*a**(15/2)*b**17*x**(15/2) + 700*a**(13/2)*b**18*x**(13/2) +
525*a**(11/2)*b**19*x**(11/2) + 210*a**(9/2)*b**20*x**(9/2) + 35*a**(7/2)*b
**21*x**(7/2)) - 480*a**9*b**15*x**(11/2)/(35*a**(19/2)*b**15*x**(19/2) + 2
10*a**(17/2)*b**16*x**(17/2) + 525*a**(15/2)*b**17*x**(15/2) + 700*a**(13/2
)*b**18*x**(13/2) + 525*a**(11/2)*b**19*x**(11/2) + 210*a**(9/2)*b**20*x**(
9/2) + 35*a**(7/2)*b**21*x**(7/2)) - 192*a**8*b**16*x**(9/2)/(35*a**(19/2)*
b**15*x**(19/2) + 210*a**(17/2)*b**16*x**(17/2) + 525*a**(15/2)*b**17*x**(1
5/2) + 700*a**(13/2)*b**18*x**(13/2) + 525*a**(11/2)*b**19*x**(11/2) + 210*
a**(9/2)*b**20*x**(9/2) + 35*a**(7/2)*b**21*x**(7/2)) - 32*a**7*b**17*x**(7
/2)/(35*a**(19/2)*b**15*x**(19/2) + 210*a**(17/2)*b**16*x**(17/2) + 525*a**
(15/2)*b**17*x**(15/2) + 700*a**(13/2)*b**18*x**(13/2) + 525*a**(11/2)*b**1
9*x**(11/2) + 210*a**(9/2)*b**20*x**(9/2) + 35*a**(7/2)*b**21*x**(7/2))
```

3.1731 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\,x^6}\,dx$

Optimal. Leaf size=99

$$-\frac{2a^4\sqrt{a+\frac{b}{x}}}{b^5}+\frac{8a^3\left(a+\frac{b}{x}\right)^{3/2}}{3b^5}-\frac{12a^2\left(a+\frac{b}{x}\right)^{5/2}}{5b^5}-\frac{2\left(a+\frac{b}{x}\right)^{9/2}}{9b^5}+\frac{8a\left(a+\frac{b}{x}\right)^{7/2}}{7b^5}$$

```
[Out] 8/3*a^3*(a+b/x)^(3/2)/b^5-12/5*a^2*(a+b/x)^(5/2)/b^5+8/7*a*(a+b/x)^(7/2)/b^
5-2/9*(a+b/x)^(9/2)/b^5-2*a^4*(a+b/x)^(1/2)/b^5
```

Rubi [A] time = 0.04, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{12a^2\left(a+\frac{b}{x}\right)^{5/2}}{5b^5}+\frac{8a^3\left(a+\frac{b}{x}\right)^{3/2}}{3b^5}-\frac{2a^4\sqrt{a+\frac{b}{x}}}{b^5}-\frac{2\left(a+\frac{b}{x}\right)^{9/2}}{9b^5}+\frac{8a\left(a+\frac{b}{x}\right)^{7/2}}{7b^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x]*x^6),x]
```

```
[Out] (-2*a^4*Sqrt[a + b/x])/b^5 + (8*a^3*(a + b/x)^(3/2))/(3*b^5) - (12*a^2*(a +
 b/x)^(5/2))/(5*b^5) + (8*a*(a + b/x)^(7/2))/(7*b^5) - (2*(a + b/x)^(9/2))/
(9*b^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+\frac{b}{x}}\,x^6}\,dx &= -\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)\\ &= -\text{Subst}\left(\int\left(\frac{a^4}{b^4\sqrt{a+bx}}-\frac{4a^3\sqrt{a+bx}}{b^4}+\frac{6a^2(a+bx)^{3/2}}{b^4}-\frac{4a(a+bx)^{5/2}}{b^4}+\frac{(a+bx)^{7/2}}{b^4}\right)dx, x\right)\\ &= -\frac{2a^4\sqrt{a+\frac{b}{x}}}{b^5}+\frac{8a^3\left(a+\frac{b}{x}\right)^{3/2}}{3b^5}-\frac{12a^2\left(a+\frac{b}{x}\right)^{5/2}}{5b^5}+\frac{8a\left(a+\frac{b}{x}\right)^{7/2}}{7b^5}-\frac{2\left(a+\frac{b}{x}\right)^{9/2}}{9b^5}\end{aligned}$$

Mathematica [A] time = 0.05, size = 62, normalized size = 0.63

$$-\frac{2\sqrt{a+\frac{b}{x}}\left(128a^4x^4-64a^3bx^3+48a^2b^2x^2-40ab^3x+35b^4\right)}{315b^5x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x]*x^6),x]
```

```
[Out] (-2*Sqrt[a + b/x]*(35*b^4 - 40*a*b^3*x + 48*a^2*b^2*x^2 - 64*a^3*b*x^3 + 12
8*a^4*x^4))/(315*b^5*x^4)
```

fricas [A] time = 1.28, size = 60, normalized size = 0.61

$$-\frac{2\left(128\,a^4x^4-64\,a^3bx^3+48\,a^2b^2x^2-40\,ab^3x+35\,b^4\right)\sqrt{\frac{ax+b}{x}}}{315\,b^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/315*(128*a^4*x^4 - 64*a^3*b*x^3 + 48*a^2*b^2*x^2 - 40*a*b^3*x + 35*b^4)*
sqrt((a*x + b)/x)/(b^5*x^4)
```

giac [A] time = 0.17, size = 119, normalized size = 1.20

$$-\frac{2\left(315\,a^4\sqrt{\frac{ax+b}{x}}-\frac{420\,(ax+b)a^3\sqrt{\frac{ax+b}{x}}}{x}+\frac{378\,(ax+b)^2a^2\sqrt{\frac{ax+b}{x}}}{x^2}-\frac{180\,(ax+b)^3a\sqrt{\frac{ax+b}{x}}}{x^3}+\frac{35\,(ax+b)^4\sqrt{\frac{ax+b}{x}}}{x^4}\right)}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] -2/315*(315*a^4*sqrt((a*x + b)/x) - 420*(a*x + b)*a^3*sqrt((a*x + b)/x)/x +
 378*(a*x + b)^2*a^2*sqrt((a*x + b)/x)/x^2 - 180*(a*x + b)^3*a*sqrt((a*x +
b)/x)/x^3 + 35*(a*x + b)^4*sqrt((a*x + b)/x)/x^4)/b^5
```

maple [A] time = 0.00, size = 66, normalized size = 0.67

$$-\frac{2\,(ax+b)\left(128a^4x^4-64a^3x^3b+48a^2x^2b^2-40ax\,b^3+35b^4\right)}{315\sqrt{\frac{ax+b}{x}}\;b^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(a+b/x)^(1/2),x)
```

```
[Out] -2/315*(a*x+b)*(128*a^4*x^4-64*a^3*b*x^3+48*a^2*b^2*x^2-40*a*b^3*x+35*b^4)/
x^5/b^5/((a*x+b)/x)^(1/2)
```

maxima [A] time = 0.96, size = 81, normalized size = 0.82

$$-\frac{2\left(a+\frac{b}{x}\right)^{\frac{9}{2}}}{9\,b^5}+\frac{8\left(a+\frac{b}{x}\right)^{\frac{7}{2}}a}{7\,b^5}-\frac{12\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^2}{5\,b^5}+\frac{8\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^3}{3\,b^5}-\frac{2\sqrt{a+\frac{b}{x}}\,a^4}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/9*(a + b/x)^(9/2)/b^5 + 8/7*(a + b/x)^(7/2)*a/b^5 - 12/5*(a + b/x)^(5/2)
*a^2/b^5 + 8/3*(a + b/x)^(3/2)*a^3/b^5 - 2*sqrt(a + b/x)*a^4/b^5
```

mupad [B] time = 1.28, size = 93, normalized size = 0.94

$$\frac{16\,a\sqrt{a+\frac{b}{x}}}{63\,b^2\,x^3}-\frac{2\sqrt{a+\frac{b}{x}}}{9\,b\,x^4}-\frac{256\,a^4\sqrt{a+\frac{b}{x}}}{315\,b^5}-\frac{32\,a^2\sqrt{a+\frac{b}{x}}}{105\,b^3\,x^2}+\frac{128\,a^3\sqrt{a+\frac{b}{x}}}{315\,b^4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b/x)^(1/2)),x)
```

```
[Out] (16*a*(a + b/x)^(1/2))/(63*b^2*x^3) - (2*(a + b/x)^(1/2))/(9*b*x^4) - (256*
a^4*(a + b/x)^(1/2))/(315*b^5) - (32*a^2*(a + b/x)^(1/2))/(105*b^3*x^2) + (
128*a^3*(a + b/x)^(1/2))/(315*b^4*x)
```

sympy [**B**] time = 5.83, size = 4901, normalized size = 49.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(a+b/x)**(1/2),x)
```

```
[Out] -256*a**(37/2)*b**(49/2)*x**14*sqrt(a*x/b + 1)/(315*a**(29/2)*b**29*x**(29/
2) + 3150*a**(27/2)*b**30*x**(27/2) + 14175*a**(25/2)*b**31*x**(25/2) + 378
00*a**(23/2)*b**32*x**(23/2) + 66150*a**(21/2)*b**33*x**(21/2) + 79380*a**(
19/2)*b**34*x**(19/2) + 66150*a**(17/2)*b**35*x**(17/2) + 37800*a**(15/2)*b
**36*x**(15/2) + 14175*a**(13/2)*b**37*x**(13/2) + 3150*a**(11/2)*b**38*x**
(11/2) + 315*a**(9/2)*b**39*x**(9/2)) - 2432*a**(35/2)*b**(51/2)*x**13*sqrt
(a*x/b + 1)/(315*a**(29/2)*b**29*x**(29/2) + 3150*a**(27/2)*b**30*x**(27/2)
 + 14175*a**(25/2)*b**31*x**(25/2) + 37800*a**(23/2)*b**32*x**(23/2) + 6615
0*a**(21/2)*b**33*x**(21/2) + 79380*a**(19/2)*b**34*x**(19/2) + 66150*a**(1
7/2)*b**35*x**(17/2) + 37800*a**(15/2)*b**36*x**(15/2) + 14175*a**(13/2)*b*
*37*x**(13/2) + 3150*a**(11/2)*b**38*x**(11/2) + 315*a**(9/2)*b**39*x**(9/2
)) - 10336*a**(33/2)*b**(53/2)*x**12*sqrt(a*x/b + 1)/(315*a**(29/2)*b**29*x
**(29/2) + 3150*a**(27/2)*b**30*x**(27/2) + 14175*a**(25/2)*b**31*x**(25/2)
 + 37800*a**(23/2)*b**32*x**(23/2) + 66150*a**(21/2)*b**33*x**(21/2) + 7938
0*a**(19/2)*b**34*x**(19/2) + 66150*a**(17/2)*b**35*x**(17/2) + 37800*a**(1
5/2)*b**36*x**(15/2) + 14175*a**(13/2)*b**37*x**(13/2) + 3150*a**(11/2)*b**
38*x**(11/2) + 315*a**(9/2)*b**39*x**(9/2)) - 25840*a**(31/2)*b**(55/2)*x**
11*sqrt(a*x/b + 1)/(315*a**(29/2)*b**29*x**(29/2) + 3150*a**(27/2)*b**30*x*
*(27/2) + 14175*a**(25/2)*b**31*x**(25/2) + 37800*a**(23/2)*b**32*x**(23/2)
 + 66150*a**(21/2)*b**33*x**(21/2) + 79380*a**(19/2)*b**34*x**(19/2) + 6615
0*a**(17/2)*b**35*x**(17/2) + 37800*a**(15/2)*b**36*x**(15/2) + 14175*a**(1
3/2)*b**37*x**(13/2) + 3150*a**(11/2)*b**38*x**(11/2) + 315*a**(9/2)*b**39*
x**(9/2)) - 41990*a**(29/2)*b**(57/2)*x**10*sqrt(a*x/b + 1)/(315*a**(29/2)*
b**29*x**(29/2) + 3150*a**(27/2)*b**30*x**(27/2) + 14175*a**(25/2)*b**31*x*
*(25/2) + 37800*a**(23/2)*b**32*x**(23/2) + 66150*a**(21/2)*b**33*x**(21/2)
 + 79380*a**(19/2)*b**34*x**(19/2) + 66150*a**(17/2)*b**35*x**(17/2) + 3780
0*a**(15/2)*b**36*x**(15/2) + 14175*a**(13/2)*b**37*x**(13/2) + 3150*a**(11
/2)*b**38*x**(11/2) + 315*a**(9/2)*b**39*x**(9/2)) - 46252*a**(27/2)*b**(59
/2)*x**9*sqrt(a*x/b + 1)/(315*a**(29/2)*b**29*x**(29/2) + 3150*a**(27/2)*b*
*30*x**(27/2) + 14175*a**(25/2)*b**31*x**(25/2) + 37800*a**(23/2)*b**32*x**
(23/2) + 66150*a**(21/2)*b**33*x**(21/2) + 79380*a**(19/2)*b**34*x**(19/2)
+ 66150*a**(17/2)*b**35*x**(17/2) + 37800*a**(15/2)*b**36*x**(15/2) + 14175
*a**(13/2)*b**37*x**(13/2) + 3150*a**(11/2)*b**38*x**(11/2) + 315*a**(9/2)*
b**39*x**(9/2)) - 35214*a**(25/2)*b**(61/2)*x**8*sqrt(a*x/b + 1)/(315*a**(2
9/2)*b**29*x**(29/2) + 3150*a**(27/2)*b**30*x**(27/2) + 14175*a**(25/2)*b**
31*x**(25/2) + 37800*a**(23/2)*b**32*x**(23/2) + 66150*a**(21/2)*b**33*x**(
21/2) + 79380*a**(19/2)*b**34*x**(19/2) + 66150*a**(17/2)*b**35*x**(17/2) +
 37800*a**(15/2)*b**36*x**(15/2) + 14175*a**(13/2)*b**37*x**(13/2) + 3150*a
**(11/2)*b**38*x**(11/2) + 315*a**(9/2)*b**39*x**(9/2)) - 19632*a**(23/2)*b
**(63/2)*x**7*sqrt(a*x/b + 1)/(315*a**(29/2)*b**29*x**(29/2) + 3150*a**(27/
2)*b**30*x**(27/2) + 14175*a**(25/2)*b**31*x**(25/2) + 37800*a**(23/2)*b**3
2*x**(23/2) + 66150*a**(21/2)*b**33*x**(21/2) + 79380*a**(19/2)*b**34*x**(1
9/2) + 66150*a**(17/2)*b**35*x**(17/2) + 37800*a**(15/2)*b**36*x**(15/2) +
14175*a**(13/2)*b**37*x**(13/2) + 3150*a**(11/2)*b**38*x**(11/2) + 315*a**(
9/2)*b**39*x**(9/2)) - 10860*a**(21/2)*b**(65/2)*x**6*sqrt(a*x/b + 1)/(315*
```

```
a**(29/2)*b**29*x**(29/2) + 3150*a**(27/2)*b**30*x**(27/2) + 14175*a**(25/2
)*b**31*x**(25/2) + 37800*a**(23/2)*b**32*x**(23/2) + 66150*a**(21/2)*b**33
*x**(21/2) + 79380*a**(19/2)*b**34*x**(19/2) + 66150*a**(17/2)*b**35*x**(17
/2) + 37800*a**(15/2)*b**36*x**(15/2) + 14175*a**(13/2)*b**37*x**(13/2) + 3
150*a**(11/2)*b**38*x**(11/2) + 315*a**(9/2)*b**39*x**(9/2)) - 9160*a**(19/
2)*b**(67/2)*x**5*sqrt(a*x/b + 1)/(315*a**(29/2)*b**29*x**(29/2) + 3150*a**
(27/2)*b**30*x**(27/2) + 14175*a**(25/2)*b**31*x**(25/2) + 37800*a**(23/2)*
b**32*x**(23/2) + 66150*a**(21/2)*b**33*x**(21/2) + 79380*a**(19/2)*b**34*x
**(19/2) + 66150*a**(17/2)*b**35*x**(17/2) + 37800*a**(15/2)*b**36*x**(15/2
) + 14175*a**(13/2)*b**37*x**(13/2) + 3150*a**(11/2)*b**38*x**(11/2) + 315*
a**(9/2)*b**39*x**(9/2)) - 8396*a**(17/2)*b**(69/2)*x**4*sqrt(a*x/b + 1)/(3
15*a**(29/2)*b**29*x**(29/2) + 3150*a**(27/2)*b**30*x**(27/2) + 14175*a**(2
5/2)*b**31*x**(25/2) + 37800*a**(23/2)*b**32*x**(23/2) + 66150*a**(21/2)*b*
*33*x**(21/2) + 79380*a**(19/2)*b**34*x**(19/2) + 66150*a**(17/2)*b**35*x**
(17/2) + 37800*a**(15/2)*b**36*x**(15/2) + 14175*a**(13/2)*b**37*x**(13/2)
+ 3150*a**(11/2)*b**38*x**(11/2) + 315*a**(9/2)*b**39*x**(9/2)) - 5632*a**(
15/2)*b**(71/2)*x**3*sqrt(a*x/b + 1)/(315*a**(29/2)*b**29*x**(29/2) + 3150*
a**(27/2)*b**30*x**(27/2) + 14175*a**(25/2)*b**31*x**(25/2) + 37800*a**(23/
2)*b**32*x**(23/2) + 66150*a**(21/2)*b**33*x**(21/2) + 79380*a**(19/2)*b**3
4*x**(19/2) + 66150*a**(17/2)*b**35*x**(17/2) + 37800*a**(15/2)*b**36*x**(1
5/2) + 14175*a**(13/2)*b**37*x**(13/2) + 3150*a**(11/2)*b**38*x**(11/2) + 3
15*a**(9/2)*b**39*x**(9/2)) - 2446*a**(13/2)*b**(73/2)*x**2*sqrt(a*x/b + 1)
/(315*a**(29/2)*b**29*x**(29/2) + 3150*a**(27/2)*b**30*x**(27/2) + 14175*a*
*(25/2)*b**31*x**(25/2) + 37800*a**(23/2)*b**32*x**(23/2) + 66150*a**(21/2)
*b**33*x**(21/2) + 79380*a**(19/2)*b**34*x**(19/2) + 66150*a**(17/2)*b**35*
x**(17/2) + 37800*a**(15/2)*b**36*x**(15/2) + 14175*a**(13/2)*b**37*x**(13/
2) + 3150*a**(11/2)*b**38*x**(11/2) + 315*a**(9/2)*b**39*x**(9/2)) - 620*a*
*(11/2)*b**(75/2)*x*sqrt(a*x/b + 1)/(315*a**(29/2)*b**29*x**(29/2) + 3150*a
**(27/2)*b**30*x**(27/2) + 14175*a**(25/2)*b**31*x**(25/2) + 37800*a**(23/2
)*b**32*x**(23/2) + 66150*a**(21/2)*b**33*x**(21/2) + 79380*a**(19/2)*b**34
*x**(19/2) + 66150*a**(17/2)*b**35*x**(17/2) + 37800*a**(15/2)*b**36*x**(15
/2) + 14175*a**(13/2)*b**37*x**(13/2) + 3150*a**(11/2)*b**38*x**(11/2) + 31
5*a**(9/2)*b**39*x**(9/2)) - 70*a**(9/2)*b**(77/2)*sqrt(a*x/b + 1)/(315*a**
(29/2)*b**29*x**(29/2) + 3150*a**(27/2)*b**30*x**(27/2) + 14175*a**(25/2)*b
**31*x**(25/2) + 37800*a**(23/2)*b**32*x**(23/2) + 66150*a**(21/2)*b**33*x*
*(21/2) + 79380*a**(19/2)*b**34*x**(19/2) + 66150*a**(17/2)*b**35*x**(17/2)
 + 37800*a**(15/2)*b**36*x**(15/2) + 14175*a**(13/2)*b**37*x**(13/2) + 3150
*a**(11/2)*b**38*x**(11/2) + 315*a**(9/2)*b**39*x**(9/2)) + 256*a**19*b**24
*x**(29/2)/(315*a**(29/2)*b**29*x**(29/2) + 3150*a**(27/2)*b**30*x**(27/2)
+ 14175*a**(25/2)*b**31*x**(25/2) + 37800*a**(23/2)*b**32*x**(23/2) + 66150
*a**(21/2)*b**33*x**(21/2) + 79380*a**(19/2)*b**34*x**(19/2) + 66150*a**(17
/2)*b**35*x**(17/2) + 37800*a**(15/2)*b**36*x**(15/2) + 14175*a**(13/2)*b**
37*x**(13/2) + 3150*a**(11/2)*b**38*x**(11/2) + 315*a**(9/2)*b**39*x**(9/2)
) + 2560*a**18*b**25*x**(27/2)/(315*a**(29/2)*b**29*x**(29/2) + 3150*a**(27
/2)*b**30*x**(27/2) + 14175*a**(25/2)*b**31*x**(25/2) + 37800*a**(23/2)*b**
32*x**(23/2) + 66150*a**(21/2)*b**33*x**(21/2) + 79380*a**(19/2)*b**34*x**(
19/2) + 66150*a**(17/2)*b**35*x**(17/2) + 37800*a**(15/2)*b**36*x**(15/2) +
 14175*a**(13/2)*b**37*x**(13/2) + 3150*a**(11/2)*b**38*x**(11/2) + 315*a**
(9/2)*b**39*x**(9/2)) + 11520*a**17*b**26*x**(25/2)/(315*a**(29/2)*b**29*x*
*(29/2) + 3150*a**(27/2)*b**30*x**(27/2) + 14175*a**(25/2)*b**31*x**(25/2)
+ 37800*a**(23/2)*b**32*x**(23/2) + 66150*a**(21/2)*b**33*x**(21/2) + 79380
*a**(19/2)*b**34*x**(19/2) + 66150*a**(17/2)*b**35*x**(17/2) + 37800*a**(15
/2)*b**36*x**(15/2) + 14175*a**(13/2)*b**37*x**(13/2) + 3150*a**(11/2)*b**3
8*x**(11/2) + 315*a**(9/2)*b**39*x**(9/2)) + 30720*a**16*b**27*x**(23/2)/(3
15*a**(29/2)*b**29*x**(29/2) + 3150*a**(27/2)*b**30*x**(27/2) + 14175*a**(2
5/2)*b**31*x**(25/2) + 37800*a**(23/2)*b**32*x**(23/2) + 66150*a**(21/2)*b*
*33*x**(21/2) + 79380*a**(19/2)*b**34*x**(19/2) + 66150*a**(17/2)*b**35*x**
(17/2) + 37800*a**(15/2)*b**36*x**(15/2) + 14175*a**(13/2)*b**37*x**(13/2)
+ 3150*a**(11/2)*b**38*x**(11/2) + 315*a**(9/2)*b**39*x**(9/2)) + 53760*a**
```

```
15*b**28*x**(21/2)/(315*a**(29/2)*b**29*x**(29/2) + 3150*a**(27/2)*b**30*x*
*(27/2) + 14175*a**(25/2)*b**31*x**(25/2) + 37800*a**(23/2)*b**32*x**(23/2)
 + 66150*a**(21/2)*b**33*x**(21/2) + 79380*a**(19/2)*b**34*x**(19/2) + 6615
0*a**(17/2)*b**35*x**(17/2) + 37800*a**(15/2)*b**36*x**(15/2) + 14175*a**(1
3/2)*b**37*x**(13/2) + 3150*a**(11/2)*b**38*x**(11/2) + 315*a**(9/2)*b**39*
x**(9/2)) + 64512*a**14*b**29*x**(19/2)/(315*a**(29/2)*b**29*x**(29/2) + 31
50*a**(27/2)*b**30*x**(27/2) + 14175*a**(25/2)*b**31*x**(25/2) + 37800*a**(
23/2)*b**32*x**(23/2) + 66150*a**(21/2)*b**33*x**(21/2) + 79380*a**(19/2)*b
**34*x**(19/2) + 66150*a**(17/2)*b**35*x**(17/2) + 37800*a**(15/2)*b**36*x*
*(15/2) + 14175*a**(13/2)*b**37*x**(13/2) + 3150*a**(11/2)*b**38*x**(11/2)
+ 315*a**(9/2)*b**39*x**(9/2)) + 53760*a**13*b**30*x**(17/2)/(315*a**(29/2)
*b**29*x**(29/2) + 3150*a**(27/2)*b**30*x**(27/2) + 14175*a**(25/2)*b**31*x
**(25/2) + 37800*a**(23/2)*b**32*x**(23/2) + 66150*a**(21/2)*b**33*x**(21/2
) + 79380*a**(19/2)*b**34*x**(19/2) + 66150*a**(17/2)*b**35*x**(17/2) + 378
00*a**(15/2)*b**36*x**(15/2) + 14175*a**(13/2)*b**37*x**(13/2) + 3150*a**(1
1/2)*b**38*x**(11/2) + 315*a**(9/2)*b**39*x**(9/2)) + 30720*a**12*b**31*x**
(15/2)/(315*a**(29/2)*b**29*x**(29/2) + 3150*a**(27/2)*b**30*x**(27/2) + 14
175*a**(25/2)*b**31*x**(25/2) + 37800*a**(23/2)*b**32*x**(23/2) + 66150*a**
(21/2)*b**33*x**(21/2) + 79380*a**(19/2)*b**34*x**(19/2) + 66150*a**(17/2)*
b**35*x**(17/2) + 37800*a**(15/2)*b**36*x**(15/2) + 14175*a**(13/2)*b**37*x
**(13/2) + 3150*a**(11/2)*b**38*x**(11/2) + 315*a**(9/2)*b**39*x**(9/2)) +
11520*a**11*b**32*x**(13/2)/(315*a**(29/2)*b**29*x**(29/2) + 3150*a**(27/2)
*b**30*x**(27/2) + 14175*a**(25/2)*b**31*x**(25/2) + 37800*a**(23/2)*b**32*
x**(23/2) + 66150*a**(21/2)*b**33*x**(21/2) + 79380*a**(19/2)*b**34*x**(19/
2) + 66150*a**(17/2)*b**35*x**(17/2) + 37800*a**(15/2)*b**36*x**(15/2) + 14
175*a**(13/2)*b**37*x**(13/2) + 3150*a**(11/2)*b**38*x**(11/2) + 315*a**(9/
2)*b**39*x**(9/2)) + 2560*a**10*b**33*x**(11/2)/(315*a**(29/2)*b**29*x**(29
/2) + 3150*a**(27/2)*b**30*x**(27/2) + 14175*a**(25/2)*b**31*x**(25/2) + 37
800*a**(23/2)*b**32*x**(23/2) + 66150*a**(21/2)*b**33*x**(21/2) + 79380*a**
(19/2)*b**34*x**(19/2) + 66150*a**(17/2)*b**35*x**(17/2) + 37800*a**(15/2)*
b**36*x**(15/2) + 14175*a**(13/2)*b**37*x**(13/2) + 3150*a**(11/2)*b**38*x*
*(11/2) + 315*a**(9/2)*b**39*x**(9/2)) + 256*a**9*b**34*x**(9/2)/(315*a**(2
9/2)*b**29*x**(29/2) + 3150*a**(27/2)*b**30*x**(27/2) + 14175*a**(25/2)*b**
31*x**(25/2) + 37800*a**(23/2)*b**32*x**(23/2) + 66150*a**(21/2)*b**33*x**(
21/2) + 79380*a**(19/2)*b**34*x**(19/2) + 66150*a**(17/2)*b**35*x**(17/2) +
 37800*a**(15/2)*b**36*x**(15/2) + 14175*a**(13/2)*b**37*x**(13/2) + 3150*a
**(11/2)*b**38*x**(11/2) + 315*a**(9/2)*b**39*x**(9/2))
```

3.1732 $\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$

Optimal. Leaf size=117

$$-\frac{35b^3\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{8a^{9/2}}+\frac{35b^3}{8a^4\sqrt{a+\frac{b}{x}}}+\frac{35b^2x}{24a^3\sqrt{a+\frac{b}{x}}}-\frac{7bx^2}{12a^2\sqrt{a+\frac{b}{x}}}+\frac{x^3}{3a\sqrt{a+\frac{b}{x}}}$$

```
[Out] -35/8*b^3*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(9/2)+35/8*b^3/a^4/(a+b/x)^(1/2)
+35/24*b^2*x/a^3/(a+b/x)^(1/2)-7/12*b*x^2/a^2/(a+b/x)^(1/2)+1/3*x^3/a/(a+b/
x)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 115, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 51, 63, 208}

$$\frac{35b^2x\sqrt{a+\frac{b}{x}}}{8a^4}-\frac{35b^3\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{8a^{9/2}}-\frac{35bx^2\sqrt{a+\frac{b}{x}}}{12a^3}+\frac{7x^3\sqrt{a+\frac{b}{x}}}{3a^2}-\frac{2x^3}{a\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b/x)^(3/2),x]
```

```
[Out] (35*b^2*Sqrt[a + b/x]*x)/(8*a^4) - (35*b*Sqrt[a + b/x]*x^2)/(12*a^3) - (2*x
^3)/(a*Sqrt[a + b/x]) + (7*Sqrt[a + b/x]*x^3)/(3*a^2) - (35*b^3*ArcTanh[Sqr
t[a + b/x]/Sqrt[a]])/(8*a^(9/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = -\operatorname{Subst}\left(\int \frac{1}{x^4(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2x^3}{a\sqrt{a+\frac{b}{x}}} - \frac{7\operatorname{Subst}\left(\int \frac{1}{x^4\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{2x^3}{a\sqrt{a+\frac{b}{x}}} + \frac{7\sqrt{a+\frac{b}{x}}\,x^3}{3a^2} + \frac{(35b)\operatorname{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{6a^2}$$

$$= -\frac{35b\sqrt{a+\frac{b}{x}}\,x^2}{12a^3} - \frac{2x^3}{a\sqrt{a+\frac{b}{x}}} + \frac{7\sqrt{a+\frac{b}{x}}\,x^3}{3a^2} - \frac{\left(35b^2\right)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{8a^3}$$

$$= \frac{35b^2\sqrt{a+\frac{b}{x}}\,x}{8a^4} - \frac{35b\sqrt{a+\frac{b}{x}}\,x^2}{12a^3} - \frac{2x^3}{a\sqrt{a+\frac{b}{x}}} + \frac{7\sqrt{a+\frac{b}{x}}\,x^3}{3a^2} + \frac{\left(35b^3\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{16a^4}$$

$$= \frac{35b^2\sqrt{a+\frac{b}{x}}\,x}{8a^4} - \frac{35b\sqrt{a+\frac{b}{x}}\,x^2}{12a^3} - \frac{2x^3}{a\sqrt{a+\frac{b}{x}}} + \frac{7\sqrt{a+\frac{b}{x}}\,x^3}{3a^2} + \frac{\left(35b^2\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a}\right.}{8a^4}$$

$$= \frac{35b^2\sqrt{a+\frac{b}{x}}\,x}{8a^4} - \frac{35b\sqrt{a+\frac{b}{x}}\,x^2}{12a^3} - \frac{2x^3}{a\sqrt{a+\frac{b}{x}}} + \frac{7\sqrt{a+\frac{b}{x}}\,x^3}{3a^2} - \frac{35b^3\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{8a^{9/2}}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.32

$$\frac{2b^3\,{}_2F_1\left(-\frac{1}{2}, 4; \frac{1}{2}; \frac{b}{ax}+1\right)}{a^4\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b/x)^(3/2),x]
```

```
[Out] (2*b^3*Hypergeometric2F1[-1/2, 4, 1/2, 1 + b/(a*x)])/(a^4*Sqrt[a + b/x])
```

fricas [A] time = 1.06, size = 207, normalized size = 1.77

$$\left[\frac{105\left(ab^3x+b^4\right)\sqrt{a}\,\log\left(2\,ax-2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(8\,a^4x^4-14\,a^3bx^3+35\,a^2b^2x^2+105\,ab^3x\right)\sqrt{\frac{ax+b}{x}}}{48\left(a^6x+a^5b\right)}, \frac{105\left(a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(105*(a*b^3*x + b^4)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x
) + b) + 2*(8*a^4*x^4 - 14*a^3*b*x^3 + 35*a^2*b^2*x^2 + 105*a*b^3*x)*sqrt((
a*x + b)/x))/(a^6*x + a^5*b), 1/24*(105*(a*b^3*x + b^4)*sqrt(-a)*arctan(sqr
```

```
t(-a)*sqrt((a*x + b)/x)/a) + (8*a^4*x^4 - 14*a^3*b*x^3 + 35*a^2*b^2*x^2 + 1
05*a*b^3*x)*sqrt((a*x + b)/x))/(a^6*x + a^5*b)]
```

giac [A] time = 0.19, size = 131, normalized size = 1.12

$$\frac{1}{24}\,b^3\left(\frac{105\,\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^4}+\frac{48}{a^4\sqrt{\frac{ax+b}{x}}}-\frac{87\,a^2\sqrt{\frac{ax+b}{x}}-\frac{136\,(ax+b)a\sqrt{\frac{ax+b}{x}}}{x}+\frac{57\,(ax+b)^2\sqrt{\frac{ax+b}{x}}}{x^2}}{\left(a-\frac{ax+b}{x}\right)^3a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/24*b^3*(105*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^4) + 48/(a^4*s
qrt((a*x + b)/x)) - (87*a^2*sqrt((a*x + b)/x) - 136*(a*x + b)*a*sqrt((a*x +
 b)/x)/x + 57*(a*x + b)^2*sqrt((a*x + b)/x)/x^2)/((a - (a*x + b)/x)^3*a^4))
```

maple [B] time = 0.02, size = 458, normalized size = 3.91

$$\frac{\sqrt{\frac{ax+b}{x}}\left(-120a^3b^3x^2\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)+15a^3b^3x^2\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)-60\sqrt{a\,x^2+bx}\;a^{\frac{9}{2}}b\,x^3-240a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b/x)^(3/2),x)
```

```
[Out] 1/48*((a*x+b)/x)^(1/2)*x/a^(11/2)*(16*(a*x^2+b*x)^(3/2)*a^(9/2)*x^2-60*(a*x
^2+b*x)^(1/2)*a^(9/2)*x^3*b+32*(a*x^2+b*x)^(3/2)*a^(7/2)*x*b-150*(a*x^2+b*x
)^(1/2)*a^(7/2)*x^2*b^2+240*a^(7/2)*((a*x+b)*x)^(1/2)*x^2*b^2-120*a^3*ln(1/
2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^2*b^3+16*(a*x^2+b*x)^(3/
2)*a^(5/2)*b^2-120*(a*x^2+b*x)^(1/2)*a^(5/2)*x*b^3-96*a^(5/2)*((a*x+b)*x)^(
3/2)*b^2+480*a^(5/2)*((a*x+b)*x)^(1/2)*x*b^3-240*a^2*ln(1/2*(2*a*x+b+2*((a*
x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x*b^4+15*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2
)*a^(1/2))/a^(1/2))*x^2*a^3*b^3-30*(a*x^2+b*x)^(1/2)*a^(3/2)*b^4+240*a^(3/2
)*((a*x+b)*x)^(1/2)*b^4-120*a*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/
a^(1/2))*b^5+30*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x*a^2
*b^4+15*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*a*b^5)/((a*x+
b)*x)^(1/2)/(a*x+b)^2
```

maxima [A] time = 2.44, size = 154, normalized size = 1.32

$$\frac{105\left(a+\frac{b}{x}\right)^3b^3-280\left(a+\frac{b}{x}\right)^2ab^3+231\left(a+\frac{b}{x}\right)a^2b^3-48\,a^3b^3}{24\left(\left(a+\frac{b}{x}\right)^{\frac{7}{2}}a^4-3\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^5+3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^6-\sqrt{a+\frac{b}{x}}\,a^7\right)}+\frac{35\,b^3\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{16\,a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/24*(105*(a + b/x)^3*b^3 - 280*(a + b/x)^2*a*b^3 + 231*(a + b/x)*a^2*b^3 -
 48*a^3*b^3)/((a + b/x)^(7/2)*a^4 - 3*(a + b/x)^(5/2)*a^5 + 3*(a + b/x)^(3/
2)*a^6 - sqrt(a + b/x)*a^7) + 35/16*b^3*log((sqrt(a + b/x) - sqrt(a))/(sqrt
(a + b/x) + sqrt(a)))/a^(9/2)
```

mupad [B] time = 1.57, size = 93, normalized size = 0.79

$$\frac{35\,b^3}{8\,a^4\sqrt{a+\frac{b}{x}}}-\frac{35\,b^3\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{8\,a^{9/2}}+\frac{x^3}{3\,a\sqrt{a+\frac{b}{x}}}-\frac{7\,b\,x^2}{12\,a^2\sqrt{a+\frac{b}{x}}}+\frac{35\,b^2\,x}{24\,a^3\sqrt{a+\frac{b}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b/x)^(3/2),x)
```

```
[Out] (35*b^3)/(8*a^4*(a + b/x)^(1/2)) - (35*b^3*atanh((a + b/x)^(1/2)/a^(1/2)))/
(8*a^(9/2)) + x^3/(3*a*(a + b/x)^(1/2)) - (7*b*x^2)/(12*a^2*(a + b/x)^(1/2)
) + (35*b^2*x)/(24*a^3*(a + b/x)^(1/2))
```

sympy [A] time = 9.82, size = 133, normalized size = 1.14

$$\frac{x^{\frac{7}{2}}}{3a\sqrt{b}\sqrt{\frac{ax}{b}+1}}-\frac{7\sqrt{b}\,x^{\frac{5}{2}}}{12a^2\sqrt{\frac{ax}{b}+1}}+\frac{35b^{\frac{3}{2}}x^{\frac{3}{2}}}{24a^3\sqrt{\frac{ax}{b}+1}}+\frac{35b^{\frac{5}{2}}\sqrt{x}}{8a^4\sqrt{\frac{ax}{b}+1}}-\frac{35b^3\,\mathrm{asinh}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{8a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b/x)**(3/2),x)
```

```
[Out] x**(7/2)/(3*a*sqrt(b)*sqrt(a*x/b + 1)) - 7*sqrt(b)*x**(5/2)/(12*a**2*sqrt(a
*x/b + 1)) + 35*b**(3/2)*x**(3/2)/(24*a**3*sqrt(a*x/b + 1)) + 35*b**(5/2)*s
qrt(x)/(8*a**4*sqrt(a*x/b + 1)) - 35*b**3*asinh(sqrt(a)*sqrt(x)/sqrt(b))/(8
*a**(9/2))
```

3.1733 $\int \frac{x}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$

Optimal. Leaf size=93

$$\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4a^{7/2}}-\frac{15b^2}{4a^3\sqrt{a+\frac{b}{x}}}-\frac{5bx}{4a^2\sqrt{a+\frac{b}{x}}}+\frac{x^2}{2a\sqrt{a+\frac{b}{x}}}$$

```
[Out] 15/4*b^2*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(7/2)-15/4*b^2/a^3/(a+b/x)^(1/2)-
5/4*b*x/a^2/(a+b/x)^(1/2)+1/2*x^2/a/(a+b/x)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 91, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {266, 51, 63, 208}

$$\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4a^{7/2}}+\frac{5x^2\sqrt{a+\frac{b}{x}}}{2a^2}-\frac{15bx\sqrt{a+\frac{b}{x}}}{4a^3}-\frac{2x^2}{a\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b/x)^(3/2),x]
```

```
[Out] (-15*b*Sqrt[a + b/x]*x)/(4*a^3) - (2*x^2)/(a*Sqrt[a + b/x]) + (5*Sqrt[a + b
/x]*x^2)/(2*a^2) + (15*b^2*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(4*a^(7/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = -\text{Subst}\left(\int \frac{1}{x^3(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2x^2}{a\sqrt{a+\frac{b}{x}}} - \frac{5\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{2x^2}{a\sqrt{a+\frac{b}{x}}} + \frac{5\sqrt{a+\frac{b}{x}}\,x^2}{2a^2} + \frac{(15b)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{4a^2}$$

$$= -\frac{15b\sqrt{a+\frac{b}{x}}\,x}{4a^3} - \frac{2x^2}{a\sqrt{a+\frac{b}{x}}} + \frac{5\sqrt{a+\frac{b}{x}}\,x^2}{2a^2} - \frac{\left(15b^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{8a^3}$$

$$= -\frac{15b\sqrt{a+\frac{b}{x}}\,x}{4a^3} - \frac{2x^2}{a\sqrt{a+\frac{b}{x}}} + \frac{5\sqrt{a+\frac{b}{x}}\,x^2}{2a^2} - \frac{(15b)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{4a^3}$$

$$= -\frac{15b\sqrt{a+\frac{b}{x}}\,x}{4a^3} - \frac{2x^2}{a\sqrt{a+\frac{b}{x}}} + \frac{5\sqrt{a+\frac{b}{x}}\,x^2}{2a^2} + \frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4a^{7/2}}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.40

$$-\frac{2b^2\,{}_2F_1\left(-\frac{1}{2}, 3; \frac{1}{2}; \frac{b}{ax}+1\right)}{a^3\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b/x)^(3/2),x]
```

```
[Out] (-2*b^2*Hypergeometric2F1[-1/2, 3, 1/2, 1 + b/(a*x)])/(a^3*Sqrt[a + b/x])
```

fricas [A] time = 0.96, size = 186, normalized size = 2.00

$$\left[\frac{15\left(ab^2x+b^3\right)\sqrt{a}\,\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(2\,a^3x^3-5\,a^2bx^2-15\,ab^2x\right)\sqrt{\frac{ax+b}{x}}}{8\left(a^5x+a^4b\right)}, -\frac{15\left(ab^2x+b^3\right)\sqrt{-a}\,\text{a}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(15*(a*b^2*x + b^3)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x)
+ b) + 2*(2*a^3*x^3 - 5*a^2*b*x^2 - 15*a*b^2*x)*sqrt((a*x + b)/x))/(a^5*x +
 a^4*b), -1/4*(15*(a*b^2*x + b^3)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x
)/a) - (2*a^3*x^3 - 5*a^2*b*x^2 - 15*a*b^2*x)*sqrt((a*x + b)/x))/(a^5*x + a
^4*b)]
```

giac [A] time = 0.17, size = 105, normalized size = 1.13

$$-\frac{1}{4}b^2\left(\frac{15\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^3}+\frac{8}{a^3\sqrt{\frac{ax+b}{x}}}-\frac{9\,a\sqrt{\frac{ax+b}{x}}-\frac{7\,(ax+b)\sqrt{\frac{ax+b}{x}}}{x}}{\left(a-\frac{ax+b}{x}\right)^2a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x)^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*b^2*(15*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^3) + 8/(a^3*sqr
t((a*x + b)/x)) - (9*a*sqrt((a*x + b)/x) - 7*(a*x + b)*sqrt((a*x + b)/x)/x)
/((a - (a*x + b)/x)^2*a^3))
```

maple [B] time = 0.01, size = 395, normalized size = 4.25

$$\frac{\sqrt{\frac{ax+b}{x}}\left(16a^3b^2x^2\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)-a^3b^2x^2\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)+4\sqrt{a\,x^2+bx}\;a^{\frac{9}{2}}x^3+32a^2b^3x\ln\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b/x)^(3/2),x)
```

```
[Out] 1/8*((a*x+b)/x)^(1/2)*x/a^(9/2)*(4*(a*x^2+b*x)^(1/2)*a^(9/2)*x^3+10*(a*x^2+
b*x)^(1/2)*a^(7/2)*x^2*b-32*a^(7/2)*((a*x+b)*x)^(1/2)*x^2*b+16*a^3*ln(1/2*(
2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^2*b^2+8*(a*x^2+b*x)^(1/2)*a
^(5/2)*b^2*x+16*a^(5/2)*((a*x+b)*x)^(3/2)*b-64*a^(5/2)*((a*x+b)*x)^(1/2)*x*
b^2+32*a^2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x*b^3-ln(1
/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x^2*a^3*b^2+2*(a*x^2+b*x)
^(1/2)*a^(3/2)*b^3-32*((a*x+b)*x)^(1/2)*a^(3/2)*b^3+16*a*b^4*ln(1/2*(2*a*x+
b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))-2*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/
2)*a^(1/2))/a^(1/2))*x*a^2*b^3-a*b^4*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^
(1/2))/a^(1/2)))/((a*x+b)*x)^(1/2)/(a*x+b)^2
```

maxima [A] time = 2.37, size = 122, normalized size = 1.31

$$-\frac{15\left(a+\frac{b}{x}\right)^2b^2-25\left(a+\frac{b}{x}\right)ab^2+8\,a^2b^2}{4\left(\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^3-2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^4+\sqrt{a+\frac{b}{x}}\,a^5\right)}-\frac{15\,b^2\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{8\,a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(15*(a + b/x)^2*b^2 - 25*(a + b/x)*a*b^2 + 8*a^2*b^2)/((a + b/x)^(5/2)
*a^3 - 2*(a + b/x)^(3/2)*a^4 + sqrt(a + b/x)*a^5) - 15/8*b^2*log((sqrt(a +
b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(7/2)
```

mupad [B] time = 1.35, size = 73, normalized size = 0.78

$$\frac{15\,b^2\,\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4\,a^{7/2}}-\frac{15\,b^2}{4\,a^3\sqrt{a+\frac{b}{x}}}+\frac{x^2}{2\,a\sqrt{a+\frac{b}{x}}}-\frac{5\,b\,x}{4\,a^2\sqrt{a+\frac{b}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b/x)^(3/2),x)
```

```
[Out] (15*b^2*atanh((a + b/x)^(1/2)/a^(1/2)))/(4*a^(7/2)) - (15*b^2)/(4*a^3*(a +
b/x)^(1/2)) + x^2/(2*a*(a + b/x)^(1/2)) - (5*b*x)/(4*a^2*(a + b/x)^(1/2))
```

sympy [**A**] time = 5.97, size = 105, normalized size = 1.13

$$\frac{x^{\frac{5}{2}}}{2a\sqrt{b}\sqrt{\frac{ax}{b}+1}} - \frac{5\sqrt{b}\,x^{\frac{3}{2}}}{4a^2\sqrt{\frac{ax}{b}+1}} - \frac{15b^{\frac{3}{2}}\sqrt{x}}{4a^3\sqrt{\frac{ax}{b}+1}} + \frac{15b^2\,\operatorname{asinh}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{4a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x)**(3/2),x)
```

```
[Out] x**(5/2)/(2*a*sqrt(b)*sqrt(a*x/b + 1)) - 5*sqrt(b)*x**(3/2)/(4*a**2*sqrt(a*
x/b + 1)) - 15*b**(3/2)*sqrt(x)/(4*a**3*sqrt(a*x/b + 1)) + 15*b**2*asinh(sq
rt(a)*sqrt(x)/sqrt(b))/(4*a**(7/2))
```

3.1734 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$

Optimal. Leaf size=60

$$-\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{3b}{a^2\sqrt{a+\frac{b}{x}}}+\frac{x}{a\sqrt{a+\frac{b}{x}}}$$

```
[Out] -3*b*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(5/2)+3*b/a^2/(a+b/x)^(1/2)+x/a/(a+b/
x)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {242, 51, 63, 208}

$$\frac{3x\sqrt{a+\frac{b}{x}}}{a^2}-\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{2x}{a\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(-3/2),x]
```

```
[Out] (-2*x)/(a*Sqrt[a + b/x]) + (3*Sqrt[a + b/x]*x)/a^2 - (3*b*ArcTanh[Sqrt[a +
b/x]/Sqrt[a]])/a^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = -\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2x}{a\sqrt{a+\frac{b}{x}}} - \frac{3\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{2x}{a\sqrt{a+\frac{b}{x}}} + \frac{3\sqrt{a+\frac{b}{x}}\,x}{a^2} + \frac{(3b)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a^2}$$

$$= -\frac{2x}{a\sqrt{a+\frac{b}{x}}} + \frac{3\sqrt{a+\frac{b}{x}}\,x}{a^2} + \frac{3\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{a^2}$$

$$= -\frac{2x}{a\sqrt{a+\frac{b}{x}}} + \frac{3\sqrt{a+\frac{b}{x}}\,x}{a^2} - \frac{3b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

Mathematica [C] time = 0.04, size = 36, normalized size = 0.60

$$\frac{2b\,{}_2F_1\left(-\frac{1}{2}, 2; \frac{1}{2}; \frac{a+\frac{b}{x}}{a}\right)}{a^2\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(-3/2),x]
```

```
[Out] (2*b*Hypergeometric2F1[-1/2, 2, 1/2, (a + b/x)/a])/(a^2*Sqrt[a + b/x])
```

fricas [A] time = 1.13, size = 156, normalized size = 2.60

$$\left[\frac{3\left(abx+b^2\right)\sqrt{a}\,\log\left(2\,ax-2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(a^2x^2+3\,abx\right)\sqrt{\frac{ax+b}{x}}}{2\left(a^4x+a^3b\right)}, \frac{3\left(abx+b^2\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-}{a^4x+a^3b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*(a*b*x + b^2)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b
) + 2*(a^2*x^2 + 3*a*b*x)*sqrt((a*x + b)/x))/(a^4*x + a^3*b), (3*(a*b*x + b
^2)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a^2*x^2 + 3*a*b*x)*sqr
t((a*x + b)/x))/(a^4*x + a^3*b)]
```

giac [A] time = 0.16, size = 86, normalized size = 1.43

$$b\left(\frac{3\,\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2} + \frac{2\,a-\frac{3\,(ax+b)}{x}}{\left(a\sqrt{\frac{ax+b}{x}}-\frac{(ax+b)\sqrt{\frac{ax+b}{x}}}{x}\right)a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2),x, algorithm="giac")
```

```
[Out] b*(3*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^2) + (2*a - 3*(a*x + b)
/x)/((a*sqrt((a*x + b)/x) - (a*x + b)*sqrt((a*x + b)/x)/x)*a^2))
```

maple [B] time = 0.01, size = 198, normalized size = 3.30

$$-\frac{\sqrt{\frac{ax+b}{x}}\left(3a^2b\,x^2\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)+6a\,b^2x\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)-6\sqrt{(ax+b)\,x}\;a^{\frac{5}{2}}x^2+3b^3\ln\left(\frac{2ax+b}{}\right.\right.}{2\sqrt{(ax+b)\,x}\;(ax+b)^2\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(3/2),x)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)*x/a^(5/2)*(-6*((a*x+b)*x)^(1/2)*a^(5/2)*x^2+3*ln(1/2
*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^2*a^2*b+4*a^(3/2)*((a*x+b
)*x)^(3/2)-12*((a*x+b)*x)^(1/2)*a^(3/2)*x*b+6*ln(1/2*(2*a*x+b+2*((a*x+b)*x)
^(1/2)*a^(1/2))/a^(1/2))*x*a*b^2-6*((a*x+b)*x)^(1/2)*a^(1/2)*b^2+3*ln(1/2*(
2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*b^3)/((a*x+b)*x)^(1/2)/(a*x+b
)^2
```

maxima [A] time = 2.31, size = 85, normalized size = 1.42

$$\frac{3\left(a+\frac{b}{x}\right)b-2\,ab}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2-\sqrt{a+\frac{b}{x}}\,a^3}+\frac{3\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{2\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2),x, algorithm="maxima")
```

```
[Out] (3*(a + b/x)*b - 2*a*b)/((a + b/x)^(3/2)*a^2 - sqrt(a + b/x)*a^3) + 3/2*b*l
og((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(5/2)
```

mupad [B] time = 1.28, size = 34, normalized size = 0.57

$$\frac{2\,x\left(\frac{a\,x}{b}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{5}{2};\ \frac{7}{2};\ -\frac{a\,x}{b}\right)}{5\left(a+\frac{b}{x}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x)^(3/2),x)
```

```
[Out] (2*x*((a*x)/b + 1)^(3/2)*hypergeom([3/2, 5/2], 7/2, -(a*x)/b))/(5*(a + b/x)
^(3/2))
```

sympy [A] time = 5.49, size = 71, normalized size = 1.18

$$\frac{x^{\frac{3}{2}}}{a\sqrt{b}\,\sqrt{\frac{ax}{b}+1}}+\frac{3\sqrt{b}\,\sqrt{x}}{a^2\sqrt{\frac{ax}{b}+1}}-\frac{3b\,\operatorname{asinh}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(3/2),x)
```

```
[Out] x**(3/2)/(a*sqrt(b)*sqrt(a*x/b + 1)) + 3*sqrt(b)*sqrt(x)/(a**2*sqrt(a*x/b +
 1)) - 3*b*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a**(5/2)
```

3.1735 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x}\,dx$

Optimal. Leaf size=42

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{a+\frac{b}{x}}}$$

```
[Out] 2*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(3/2)-2/a/(a+b/x)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 51, 63, 208}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(3/2)*x),x]
```

```
[Out] -2/(a*Sqrt[a + b/x]) + (2*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x}\,dx = -\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2}{a\sqrt{a+\frac{b}{x}}} - \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{2}{a\sqrt{a+\frac{b}{x}}} - \frac{2\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{ab}$$

$$= -\frac{2}{a\sqrt{a+\frac{b}{x}}} + \frac{2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

Mathematica [C] time = 0.02, size = 34, normalized size = 0.81

$$-\frac{2\,{}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{b}{ax}+1\right)}{a\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(3/2)*x),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + b/(a*x)])/(a*Sqrt[a + b/x])
```

fricas [A] time = 1.09, size = 128, normalized size = 3.05

$$\left[-\frac{2\,ax\sqrt{\frac{ax+b}{x}} - (ax+b)\sqrt{a}\,\log\left(2\,ax + 2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}} + b\right)}{a^3x + a^2b}, -\frac{2\left(ax\sqrt{\frac{ax+b}{x}} + (ax+b)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)\right)}{a^3x + a^2b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [-(2*a*x*sqrt((a*x + b)/x) - (a*x + b)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqrt
((a*x + b)/x) + b))/(a^3*x + a^2*b), -2*(a*x*sqrt((a*x + b)/x) + (a*x + b)*
sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a))/(a^3*x + a^2*b)]
```

giac [A] time = 0.20, size = 45, normalized size = 1.07

$$-\frac{2\,\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a} - \frac{2}{a\sqrt{\frac{ax+b}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/x,x, algorithm="giac")
```

```
[Out] -2*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a) - 2/(a*sqrt((a*x + b)/x)
)
```

maple [B] time = 0.01, size = 198, normalized size = 4.71

$$\frac{\sqrt{\frac{ax+b}{x}}\left(a^2 b\,x^2\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)+2a\,b^2 x\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)-2\sqrt{(ax+b)\,x}\;a^{\frac{5}{2}}x^2+b^3\ln\left(\frac{2ax+b+2\sqrt{(ax+}\right.\right.}{2\sqrt{a}}}{\sqrt{(ax+b)\,x}\;(ax+b)^2\,a^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(3/2)/x,x)
```

```
[Out] ((a*x+b)/x)^(1/2)*x/a^(3/2)*(a^2*b*x^2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*
a^(1/2))/a^(1/2))-2*((a*x+b)*x)^(1/2)*a^(5/2)*x^2+2*a*b^2*x*ln(1/2*(2*a*x+b
+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))+2*((a*x+b)*x)^(3/2)*a^(3/2)-4*((a*x+
b)*x)^(1/2)*a^(3/2)*b*x+b^3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^
(1/2))-2*((a*x+b)*x)^(1/2)*a^(1/2)*b^2)/((a*x+b)*x)^(1/2)/b/(a*x+b)^2
```

maxima [A] time = 2.44, size = 52, normalized size = 1.24

$$-\frac{\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}-\frac{2}{\sqrt{a+\frac{b}{x}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] -log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2) - 2/(sqrt
(a + b/x)*a)
```

mupad [B] time = 1.42, size = 34, normalized size = 0.81

$$\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{a+\frac{b}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b/x)^(3/2)),x)
```

```
[Out] (2*atanh((a + b/x)^(1/2)/a^(1/2)))/a^(3/2) - 2/(a*(a + b/x)^(1/2))
```

sympy [B] time = 3.82, size = 148, normalized size = 3.52

$$-\frac{2a^3x\sqrt{1+\frac{b}{ax}}}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b}-\frac{a^3x\log\left(\frac{b}{ax}\right)}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b}+\frac{2a^3x\log\left(\sqrt{1+\frac{b}{ax}}+1\right)}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b}-\frac{a^2b\log\left(\frac{b}{ax}\right)}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b}+\frac{2a^2b\log\left(\sqrt{1+\frac{b}{ax}}+1\right)}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(3/2)/x,x)
```

```
[Out] -2*a**3*x*sqrt(1 + b/(a*x))/(a**(9/2)*x + a**(7/2)*b) - a**3*x*log(b/(a*x))
/(a**(9/2)*x + a**(7/2)*b) + 2*a**3*x*log(sqrt(1 + b/(a*x)) + 1)/(a**(9/2)*
x + a**(7/2)*b) - a**2*b*log(b/(a*x))/(a**(9/2)*x + a**(7/2)*b) + 2*a**2*b*
log(sqrt(1 + b/(a*x)) + 1)/(a**(9/2)*x + a**(7/2)*b)
```

3.1736 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^2}\,dx$

Optimal. Leaf size=16

$$\frac{2}{b\sqrt{a+\frac{b}{x}}}$$

```
[Out] 2/b/(a+b/x)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$\frac{2}{b\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(3/2)*x^2),x]
```

```
[Out] 2/(b*Sqrt[a + b/x])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^2}\,dx = \frac{2}{b\sqrt{a+\frac{b}{x}}}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{2}{b\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(3/2)*x^2),x]
```

```
[Out] 2/(b*Sqrt[a + b/x])
```

fricas [A] time = 1.01, size = 24, normalized size = 1.50

$$\frac{2\,x\sqrt{\frac{ax+b}{x}}}{abx+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 2*x*sqrt((a*x + b)/x)/(a*b*x + b^2)
```

giac [A] time = 0.16, size = 16, normalized size = 1.00

$$\frac{2}{b\sqrt{\frac{ax+b}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/x^2,x, algorithm="giac")
[Out] 2/(b*sqrt((a*x + b)/x))
```

maple [A] time = 0.00, size = 25, normalized size = 1.56

$$\frac{2ax+2b}{\left(\frac{ax+b}{x}\right)^{\frac{3}{2}} bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(3/2)/x^2,x)
[Out] 2*(a*x+b)/x/b/((a*x+b)/x)^(3/2)
```

maxima [A] time = 0.99, size = 14, normalized size = 0.88

$$\frac{2}{\sqrt{a+\frac{b}{x}}\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/x^2,x, algorithm="maxima")
[Out] 2/(sqrt(a + b/x)*b)
```

mupad [B] time = 1.31, size = 14, normalized size = 0.88

$$\frac{2}{b\sqrt{a+\frac{b}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b/x)^(3/2)),x)
[Out] 2/(b*(a + b/x)^(1/2))
```

sympy [A] time = 1.80, size = 20, normalized size = 1.25

$$\begin{cases} \frac{2}{b\sqrt{a+\frac{b}{x}}} & \text{for } b \neq 0 \\ -\frac{1}{a^{\frac{3}{2}} x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(3/2)/x**2,x)
[Out] Piecewise((2/(b*sqrt(a + b/x)), Ne(b, 0)), (-1/(a**(3/2)*x), True))
```

3.1737 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^3}\,dx$

Optimal. Leaf size=34

$$-\frac{2a}{b^2\sqrt{a+\frac{b}{x}}}-\frac{2\sqrt{a+\frac{b}{x}}}{b^2}$$

```
[Out] -2*a/b^2/(a+b/x)^(1/2)-2*(a+b/x)^(1/2)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{2a}{b^2\sqrt{a+\frac{b}{x}}}-\frac{2\sqrt{a+\frac{b}{x}}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(3/2)*x^3),x]
```

```
[Out] (-2*a)/(b^2*Sqrt[a + b/x]) - (2*Sqrt[a + b/x])/b^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^3}\,dx &= -\text{Subst}\left(\int \frac{x}{(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)\\ &= -\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^{3/2}}+\frac{1}{b\sqrt{a+bx}}\right)dx, x, \frac{1}{x}\right)\\ &= -\frac{2a}{b^2\sqrt{a+\frac{b}{x}}}-\frac{2\sqrt{a+\frac{b}{x}}}{b^2}\end{aligned}$$

Mathematica [A] time = 0.04, size = 25, normalized size = 0.74

$$-\frac{2(2ax+b)}{b^2x\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(3/2)*x^3),x]
[Out] (-2*(b + 2*a*x))/(b^2*Sqrt[a + b/x]*x)
```

fricas [A] time = 0.62, size = 31, normalized size = 0.91

$$-\frac{2\,(2\,ax+b)\sqrt{\frac{ax+b}{x}}}{ab^2x+b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/x^3,x, algorithm="fricas")
[Out] -2*(2*a*x + b)*sqrt((a*x + b)/x)/(a*b^2*x + b^3)
```

giac [A] time = 0.16, size = 30, normalized size = 0.88

$$-\frac{2\left(\frac{a}{\sqrt{\frac{ax+b}{x}}}+\sqrt{\frac{ax+b}{x}}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/x^3,x, algorithm="giac")
[Out] -2*(a/sqrt((a*x + b)/x) + sqrt((a*x + b)/x))/b^2
```

maple [A] time = 0.00, size = 31, normalized size = 0.91

$$-\frac{2\,(ax+b)\,(2ax+b)}{\left(\frac{ax+b}{x}\right)^{\frac{3}{2}}b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(3/2)/x^3,x)
[Out] -2*(a*x+b)*(2*a*x+b)/x^2/b^2/((a*x+b)/x)^(3/2)
```

maxima [A] time = 0.98, size = 30, normalized size = 0.88

$$-\frac{2\sqrt{a+\frac{b}{x}}}{b^2}-\frac{2\,a}{\sqrt{a+\frac{b}{x}}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/x^3,x, algorithm="maxima")
[Out] -2*sqrt(a + b/x)/b^2 - 2*a/(sqrt(a + b/x)*b^2)
```

mupad [B] time = 1.26, size = 36, normalized size = 1.06

$$-\frac{x\sqrt{a+\frac{b}{x}}\,\left(\frac{2}{b}+\frac{4\,a\,x}{b^2}\right)}{a\,x^2+b\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b/x)^(3/2)),x)
[Out] -(x*(a + b/x)^(1/2)*(2/b + (4*a*x)/b^2))/(b*x + a*x^2)
```

sympy [**A**] time = 3.37, size = 42, normalized size = 1.24

$$\begin{cases} -\frac{4a}{b^2\sqrt{a+\frac{b}{x}}} - \frac{2}{bx\sqrt{a+\frac{b}{x}}} & \text{for } b \neq 0 \\ -\frac{1}{2a^{\frac{3}{2}}x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(3/2)/x**3,x)
```

```
[Out] Piecewise((-4*a/(b**2*sqrt(a + b/x)) - 2/(b*x*sqrt(a + b/x)), Ne(b, 0)), (-
1/(2*a**(3/2)*x**2), True))
```

3.1738 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^4}\,dx$

Optimal. Leaf size=55

$$\frac{2a^2}{b^3\sqrt{a+\frac{b}{x}}}+\frac{4a\sqrt{a+\frac{b}{x}}}{b^3}-\frac{2\left(a+\frac{b}{x}\right)^{3/2}}{3b^3}$$

```
[Out] -2/3*(a+b/x)^(3/2)/b^3+2*a^2/b^3/(a+b/x)^(1/2)+4*a*(a+b/x)^(1/2)/b^3
```

Rubi [A] time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{2a^2}{b^3\sqrt{a+\frac{b}{x}}}+\frac{4a\sqrt{a+\frac{b}{x}}}{b^3}-\frac{2\left(a+\frac{b}{x}\right)^{3/2}}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(3/2)*x^4),x]
```

```
[Out] (2*a^2)/(b^3*Sqrt[a + b/x]) + (4*a*Sqrt[a + b/x])/b^3 - (2*(a + b/x)^(3/2))
/(3*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^4}\,dx &= -\text{Subst}\left(\int \frac{x^2}{(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)\\ &= -\text{Subst}\left(\int\left(\frac{a^2}{b^2(a+bx)^{3/2}}-\frac{2a}{b^2\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b^2}\right)dx, x, \frac{1}{x}\right)\\ &= \frac{2a^2}{b^3\sqrt{a+\frac{b}{x}}}+\frac{4a\sqrt{a+\frac{b}{x}}}{b^3}-\frac{2\left(a+\frac{b}{x}\right)^{3/2}}{3b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 0.73

$$\frac{16a^2x^2+8abx-2b^2}{3b^3x^2\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(3/2)*x^4),x]
```

```
[Out] (-2*b^2 + 8*a*b*x + 16*a^2*x^2)/(3*b^3*Sqrt[a + b/x]*x^2)
```

fricas [A] time = 0.96, size = 48, normalized size = 0.87

$$\frac{2\left(8\,a^2x^2+4\,abx-b^2\right)\sqrt{\frac{ax+b}{x}}}{3\left(ab^3x^2+b^4x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] 2/3*(8*a^2*x^2 + 4*a*b*x - b^2)*sqrt((a*x + b)/x)/(a*b^3*x^2 + b^4*x)
```

giac [A] time = 0.17, size = 57, normalized size = 1.04

$$\frac{2\left(\frac{3\,a^2}{\sqrt{\frac{ax+b}{x}}}+6\,a\sqrt{\frac{ax+b}{x}}-\frac{(ax+b)\sqrt{\frac{ax+b}{x}}}{x}\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] 2/3*(3*a^2/sqrt((a*x + b)/x) + 6*a*sqrt((a*x + b)/x) - (a*x + b)*sqrt((a*x
+ b)/x)/x)/b^3
```

maple [A] time = 0.01, size = 44, normalized size = 0.80

$$\frac{2\,(ax+b)\left(8a^2x^2+4abx-b^2\right)}{3\left(\frac{ax+b}{x}\right)^{\frac{3}{2}}b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(3/2)/x^4,x)
```

```
[Out] 2/3*(a*x+b)*(8*a^2*x^2+4*a*b*x-b^2)/x^3/b^3/((a*x+b)/x)^(3/2)
```

maxima [A] time = 0.97, size = 47, normalized size = 0.85

$$-\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3\,b^3}+\frac{4\sqrt{a+\frac{b}{x}}\,a}{b^3}+\frac{2\,a^2}{\sqrt{a+\frac{b}{x}}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] -2/3*(a + b/x)^(3/2)/b^3 + 4*sqrt(a + b/x)*a/b^3 + 2*a^2/(sqrt(a + b/x)*b^3
)
```

mupad [B] time = 1.32, size = 43, normalized size = 0.78

$$\frac{2\sqrt{a+\frac{b}{x}}\,\left(8\,a^2\,x^2+4\,a\,b\,x-b^2\right)}{3\,b^3\,x\,\left(b+a\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b/x)^(3/2)),x)
```

```
[Out] (2*(a + b/x)^(1/2)*(8*a^2*x^2 - b^2 + 4*a*b*x))/(3*b^3*x*(b + a*x))
```

sympy [**B**] time = 2.39, size = 457, normalized size = 8.31

$$\frac{16a^{\frac{9}{2}}b^{\frac{7}{2}}x^3\sqrt{\frac{ax}{b}+1}}{3a^{\frac{7}{2}}b^6x^{\frac{7}{2}}+6a^{\frac{5}{2}}b^7x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^8x^{\frac{3}{2}}}+\frac{24a^{\frac{7}{2}}b^{\frac{9}{2}}x^2\sqrt{\frac{ax}{b}+1}}{3a^{\frac{7}{2}}b^6x^{\frac{7}{2}}+6a^{\frac{5}{2}}b^7x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^8x^{\frac{3}{2}}}+\frac{6a^{\frac{5}{2}}b^{\frac{11}{2}}x\sqrt{\frac{ax}{b}+1}}{3a^{\frac{7}{2}}b^6x^{\frac{7}{2}}+6a^{\frac{5}{2}}b^7x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^8x^{\frac{3}{2}}}-\frac{2a^{\frac{3}{2}}b}{3a^{\frac{7}{2}}b^6x^{\frac{7}{2}}+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(3/2)/x**4,x)
```

```
[Out] 16*a**(9/2)*b**(7/2)*x**3*sqrt(a*x/b + 1)/(3*a**(7/2)*b**6*x**(7/2) + 6*a**
(5/2)*b**7*x**(5/2) + 3*a**(3/2)*b**8*x**(3/2)) + 24*a**(7/2)*b**(9/2)*x**2
*sqrt(a*x/b + 1)/(3*a**(7/2)*b**6*x**(7/2) + 6*a**(5/2)*b**7*x**(5/2) + 3*a
**(3/2)*b**8*x**(3/2)) + 6*a**(5/2)*b**(11/2)*x*sqrt(a*x/b + 1)/(3*a**(7/2)
*b**6*x**(7/2) + 6*a**(5/2)*b**7*x**(5/2) + 3*a**(3/2)*b**8*x**(3/2)) - 2*a
**(3/2)*b**(13/2)*sqrt(a*x/b + 1)/(3*a**(7/2)*b**6*x**(7/2) + 6*a**(5/2)*b*
*7*x**(5/2) + 3*a**(3/2)*b**8*x**(3/2)) - 16*a**5*b**3*x**(7/2)/(3*a**(7/2)
*b**6*x**(7/2) + 6*a**(5/2)*b**7*x**(5/2) + 3*a**(3/2)*b**8*x**(3/2)) - 32*
a**4*b**4*x**(5/2)/(3*a**(7/2)*b**6*x**(7/2) + 6*a**(5/2)*b**7*x**(5/2) + 3
*a**(3/2)*b**8*x**(3/2)) - 16*a**3*b**5*x**(3/2)/(3*a**(7/2)*b**6*x**(7/2)
+ 6*a**(5/2)*b**7*x**(5/2) + 3*a**(3/2)*b**8*x**(3/2))
```

3.1739 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^5}\,dx$

Optimal. Leaf size=74

$$-\frac{2a^3}{b^4\sqrt{a+\frac{b}{x}}}-\frac{6a^2\sqrt{a+\frac{b}{x}}}{b^4}+\frac{2a\left(a+\frac{b}{x}\right)^{3/2}}{b^4}-\frac{2\left(a+\frac{b}{x}\right)^{5/2}}{5b^4}$$

```
[Out] 2*a*(a+b/x)^(3/2)/b^4-2/5*(a+b/x)^(5/2)/b^4-2*a^3/b^4/(a+b/x)^(1/2)-6*a^2*(
a+b/x)^(1/2)/b^4
```

Rubi [A] time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{2a^3}{b^4\sqrt{a+\frac{b}{x}}}-\frac{6a^2\sqrt{a+\frac{b}{x}}}{b^4}+\frac{2a\left(a+\frac{b}{x}\right)^{3/2}}{b^4}-\frac{2\left(a+\frac{b}{x}\right)^{5/2}}{5b^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(3/2)*x^5),x]
```

```
[Out] (-2*a^3)/(b^4*Sqrt[a + b/x]) - (6*a^2*Sqrt[a + b/x])/b^4 + (2*a*(a + b/x)^(
3/2))/b^4 - (2*(a + b/x)^(5/2))/(5*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^5}\,dx &= -\text{Subst}\left(\int \frac{x^3}{(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)\\ &= -\text{Subst}\left(\int\left(-\frac{a^3}{b^3(a+bx)^{3/2}}+\frac{3a^2}{b^3\sqrt{a+bx}}-\frac{3a\sqrt{a+bx}}{b^3}+\frac{(a+bx)^{3/2}}{b^3}\right)dx, x, \frac{1}{x}\right)\\ &= -\frac{2a^3}{b^4\sqrt{a+\frac{b}{x}}}-\frac{6a^2\sqrt{a+\frac{b}{x}}}{b^4}+\frac{2a\left(a+\frac{b}{x}\right)^{3/2}}{b^4}-\frac{2\left(a+\frac{b}{x}\right)^{5/2}}{5b^4}\end{aligned}$$

Mathematica [A] time = 0.04, size = 49, normalized size = 0.66

$$-\frac{2\left(16a^3x^3+8a^2bx^2-2ab^2x+b^3\right)}{5b^4x^3\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(3/2)*x^5),x]
```

```
[Out] (-2*(b^3 - 2*a*b^2*x + 8*a^2*b*x^2 + 16*a^3*x^3))/(5*b^4*Sqrt[a + b/x]*x^3)
```

fricas [A] time = 1.06, size = 59, normalized size = 0.80

$$-\frac{2\left(16\,a^3x^3+8\,a^2bx^2-2\,ab^2x+b^3\right)\sqrt{\frac{ax+b}{x}}}{5\left(ab^4x^3+b^5x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] -2/5*(16*a^3*x^3 + 8*a^2*b*x^2 - 2*a*b^2*x + b^3)*sqrt((a*x + b)/x)/(a*b^4*
x^3 + b^5*x^2)
```

giac [A] time = 0.21, size = 82, normalized size = 1.11

$$-\frac{2\left(\frac{5\,a^3}{\sqrt{\frac{ax+b}{x}}}+15\,a^2\sqrt{\frac{ax+b}{x}}-\frac{5\,(ax+b)a\sqrt{\frac{ax+b}{x}}}{x}+\frac{(ax+b)^2\sqrt{\frac{ax+b}{x}}}{x^2}\right)}{5\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] -2/5*(5*a^3/sqrt((a*x + b)/x) + 15*a^2*sqrt((a*x + b)/x) - 5*(a*x + b)*a*sq
rt((a*x + b)/x)/x + (a*x + b)^2*sqrt((a*x + b)/x)/x^2)/b^4
```

maple [A] time = 0.01, size = 53, normalized size = 0.72

$$-\frac{2\,(ax+b)\left(16a^3x^3+8a^2b\,x^2-2a\,b^2x+b^3\right)}{5\left(\frac{ax+b}{x}\right)^{\frac{3}{2}}b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(3/2)/x^5,x)
```

```
[Out] -2/5*(a*x+b)*(16*a^3*x^3+8*a^2*b*x^2-2*a*b^2*x+b^3)/x^4/b^4/((a*x+b)/x)^(3/
2)
```

maxima [A] time = 1.05, size = 64, normalized size = 0.86

$$-\frac{2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{5\,b^4}+\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a}{b^4}-\frac{6\sqrt{a+\frac{b}{x}}\,a^2}{b^4}-\frac{2\,a^3}{\sqrt{a+\frac{b}{x}}\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] -2/5*(a + b/x)^(5/2)/b^4 + 2*(a + b/x)^(3/2)*a/b^4 - 6*sqrt(a + b/x)*a^2/b^
4 - 2*a^3/(sqrt(a + b/x)*b^4)
```

mupad [B] time = 1.37, size = 52, normalized size = 0.70

$$-\frac{2\sqrt{a+\frac{b}{x}}\,\left(16\,a^3\,x^3+8\,a^2\,b\,x^2-2\,a\,b^2\,x+b^3\right)}{5\,b^4\,x^2\,\left(b+a\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b/x)^(3/2)),x)
```

```
[Out] -(2*(a + b/x)^(1/2)*(b^3 + 16*a^3*x^3 + 8*a^2*b*x^2 - 2*a*b^2*x))/(5*b^4*x^
2*(b + a*x))
```

sympy [**B**] time = 3.61, size = 2032, normalized size = 27.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(3/2)/x**5,x)
```

```
[Out] -32*a**(21/2)*b**(23/2)*x**8*sqrt(a*x/b + 1)/(5*a**(17/2)*b**15*x**(17/2) +
 30*a**(15/2)*b**16*x**(15/2) + 75*a**(13/2)*b**17*x**(13/2) + 100*a**(11/2
)*b**18*x**(11/2) + 75*a**(9/2)*b**19*x**(9/2) + 30*a**(7/2)*b**20*x**(7/2)
 + 5*a**(5/2)*b**21*x**(5/2)) - 176*a**(19/2)*b**(25/2)*x**7*sqrt(a*x/b + 1
)/(5*a**(17/2)*b**15*x**(17/2) + 30*a**(15/2)*b**16*x**(15/2) + 75*a**(13/2
)*b**17*x**(13/2) + 100*a**(11/2)*b**18*x**(11/2) + 75*a**(9/2)*b**19*x**(9
/2) + 30*a**(7/2)*b**20*x**(7/2) + 5*a**(5/2)*b**21*x**(5/2)) - 396*a**(17/
2)*b**(27/2)*x**6*sqrt(a*x/b + 1)/(5*a**(17/2)*b**15*x**(17/2) + 30*a**(15/
2)*b**16*x**(15/2) + 75*a**(13/2)*b**17*x**(13/2) + 100*a**(11/2)*b**18*x**
(11/2) + 75*a**(9/2)*b**19*x**(9/2) + 30*a**(7/2)*b**20*x**(7/2) + 5*a**(5/
2)*b**21*x**(5/2)) - 462*a**(15/2)*b**(29/2)*x**5*sqrt(a*x/b + 1)/(5*a**(17
/2)*b**15*x**(17/2) + 30*a**(15/2)*b**16*x**(15/2) + 75*a**(13/2)*b**17*x**
(13/2) + 100*a**(11/2)*b**18*x**(11/2) + 75*a**(9/2)*b**19*x**(9/2) + 30*a*
*(7/2)*b**20*x**(7/2) + 5*a**(5/2)*b**21*x**(5/2)) - 290*a**(13/2)*b**(31/2
)*x**4*sqrt(a*x/b + 1)/(5*a**(17/2)*b**15*x**(17/2) + 30*a**(15/2)*b**16*x*
*(15/2) + 75*a**(13/2)*b**17*x**(13/2) + 100*a**(11/2)*b**18*x**(11/2) + 75
*a**(9/2)*b**19*x**(9/2) + 30*a**(7/2)*b**20*x**(7/2) + 5*a**(5/2)*b**21*x*
*(5/2)) - 92*a**(11/2)*b**(33/2)*x**3*sqrt(a*x/b + 1)/(5*a**(17/2)*b**15*x*
*(17/2) + 30*a**(15/2)*b**16*x**(15/2) + 75*a**(13/2)*b**17*x**(13/2) + 100
*a**(11/2)*b**18*x**(11/2) + 75*a**(9/2)*b**19*x**(9/2) + 30*a**(7/2)*b**20
*x**(7/2) + 5*a**(5/2)*b**21*x**(5/2)) - 16*a**(9/2)*b**(35/2)*x**2*sqrt(a*
x/b + 1)/(5*a**(17/2)*b**15*x**(17/2) + 30*a**(15/2)*b**16*x**(15/2) + 75*a
**(13/2)*b**17*x**(13/2) + 100*a**(11/2)*b**18*x**(11/2) + 75*a**(9/2)*b**1
9*x**(9/2) + 30*a**(7/2)*b**20*x**(7/2) + 5*a**(5/2)*b**21*x**(5/2)) - 6*a*
*(7/2)*b**(37/2)*x*sqrt(a*x/b + 1)/(5*a**(17/2)*b**15*x**(17/2) + 30*a**(15
/2)*b**16*x**(15/2) + 75*a**(13/2)*b**17*x**(13/2) + 100*a**(11/2)*b**18*x*
*(11/2) + 75*a**(9/2)*b**19*x**(9/2) + 30*a**(7/2)*b**20*x**(7/2) + 5*a**(5
/2)*b**21*x**(5/2)) - 2*a**(5/2)*b**(39/2)*sqrt(a*x/b + 1)/(5*a**(17/2)*b**
15*x**(17/2) + 30*a**(15/2)*b**16*x**(15/2) + 75*a**(13/2)*b**17*x**(13/2)
+ 100*a**(11/2)*b**18*x**(11/2) + 75*a**(9/2)*b**19*x**(9/2) + 30*a**(7/2)*
b**20*x**(7/2) + 5*a**(5/2)*b**21*x**(5/2)) + 32*a**11*b**11*x**(17/2)/(5*a
**(17/2)*b**15*x**(17/2) + 30*a**(15/2)*b**16*x**(15/2) + 75*a**(13/2)*b**1
7*x**(13/2) + 100*a**(11/2)*b**18*x**(11/2) + 75*a**(9/2)*b**19*x**(9/2) +
30*a**(7/2)*b**20*x**(7/2) + 5*a**(5/2)*b**21*x**(5/2)) + 192*a**10*b**12*x
**(15/2)/(5*a**(17/2)*b**15*x**(17/2) + 30*a**(15/2)*b**16*x**(15/2) + 75*a
**(13/2)*b**17*x**(13/2) + 100*a**(11/2)*b**18*x**(11/2) + 75*a**(9/2)*b**1
9*x**(9/2) + 30*a**(7/2)*b**20*x**(7/2) + 5*a**(5/2)*b**21*x**(5/2)) + 480*
a**9*b**13*x**(13/2)/(5*a**(17/2)*b**15*x**(17/2) + 30*a**(15/2)*b**16*x**(
15/2) + 75*a**(13/2)*b**17*x**(13/2) + 100*a**(11/2)*b**18*x**(11/2) + 75*a
**(9/2)*b**19*x**(9/2) + 30*a**(7/2)*b**20*x**(7/2) + 5*a**(5/2)*b**21*x**(
5/2)) + 640*a**8*b**14*x**(11/2)/(5*a**(17/2)*b**15*x**(17/2) + 30*a**(15/2
)*b**16*x**(15/2) + 75*a**(13/2)*b**17*x**(13/2) + 100*a**(11/2)*b**18*x**(
11/2) + 75*a**(9/2)*b**19*x**(9/2) + 30*a**(7/2)*b**20*x**(7/2) + 5*a**(5/2
)*b**21*x**(5/2)) + 480*a**7*b**15*x**(9/2)/(5*a**(17/2)*b**15*x**(17/2) +
30*a**(15/2)*b**16*x**(15/2) + 75*a**(13/2)*b**17*x**(13/2) + 100*a**(11/2)
*b**18*x**(11/2) + 75*a**(9/2)*b**19*x**(9/2) + 30*a**(7/2)*b**20*x**(7/2)
```

```
+ 5*a**(5/2)*b**21*x**(5/2)) + 192*a**6*b**16*x**(7/2)/(5*a**(17/2)*b**15*x
**(17/2) + 30*a**(15/2)*b**16*x**(15/2) + 75*a**(13/2)*b**17*x**(13/2) + 10
0*a**(11/2)*b**18*x**(11/2) + 75*a**(9/2)*b**19*x**(9/2) + 30*a**(7/2)*b**2
0*x**(7/2) + 5*a**(5/2)*b**21*x**(5/2)) + 32*a**5*b**17*x**(5/2)/(5*a**(17/
2)*b**15*x**(17/2) + 30*a**(15/2)*b**16*x**(15/2) + 75*a**(13/2)*b**17*x**(
13/2) + 100*a**(11/2)*b**18*x**(11/2) + 75*a**(9/2)*b**19*x**(9/2) + 30*a**
(7/2)*b**20*x**(7/2) + 5*a**(5/2)*b**21*x**(5/2))
```

3.1740 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^6}\,dx$

Optimal. Leaf size=95

$$\frac{2a^4}{b^5\sqrt{a+\frac{b}{x}}}+\frac{8a^3\sqrt{a+\frac{b}{x}}}{b^5}-\frac{4a^2\left(a+\frac{b}{x}\right)^{3/2}}{b^5}+\frac{8a\left(a+\frac{b}{x}\right)^{5/2}}{5b^5}-\frac{2\left(a+\frac{b}{x}\right)^{7/2}}{7b^5}$$

```
[Out] -4*a^2*(a+b/x)^(3/2)/b^5+8/5*a*(a+b/x)^(5/2)/b^5-2/7*(a+b/x)^(7/2)/b^5+2*a^
4/b^5/(a+b/x)^(1/2)+8*a^3*(a+b/x)^(1/2)/b^5
```

Rubi [A] time = 0.04, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{2a^4}{b^5\sqrt{a+\frac{b}{x}}}+\frac{8a^3\sqrt{a+\frac{b}{x}}}{b^5}-\frac{4a^2\left(a+\frac{b}{x}\right)^{3/2}}{b^5}+\frac{8a\left(a+\frac{b}{x}\right)^{5/2}}{5b^5}-\frac{2\left(a+\frac{b}{x}\right)^{7/2}}{7b^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(3/2)*x^6),x]
```

```
[Out] (2*a^4)/(b^5*Sqrt[a + b/x]) + (8*a^3*Sqrt[a + b/x])/b^5 - (4*a^2*(a + b/x)^
(3/2))/b^5 + (8*a*(a + b/x)^(5/2))/(5*b^5) - (2*(a + b/x)^(7/2))/(7*b^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}x^6}\,dx &= -\text{Subst}\left(\int \frac{x^4}{(a+bx)^{3/2}}\,dx,x,\frac{1}{x}\right)\\ &= -\text{Subst}\left(\int\left(\frac{a^4}{b^4(a+bx)^{3/2}}-\frac{4a^3}{b^4\sqrt{a+bx}}+\frac{6a^2\sqrt{a+bx}}{b^4}-\frac{4a(a+bx)^{3/2}}{b^4}+\frac{(a+bx)^{5/2}}{b^4}\right)\right.\\ &= \frac{2a^4}{b^5\sqrt{a+\frac{b}{x}}}+\frac{8a^3\sqrt{a+\frac{b}{x}}}{b^5}-\frac{4a^2\left(a+\frac{b}{x}\right)^{3/2}}{b^5}+\frac{8a\left(a+\frac{b}{x}\right)^{5/2}}{5b^5}-\frac{2\left(a+\frac{b}{x}\right)^{7/2}}{7b^5}\end{aligned}$$

Mathematica [A] time = 0.04, size = 62, normalized size = 0.65

$$\frac{2\left(128a^4x^4+64a^3bx^3-16a^2b^2x^2+8ab^3x-5b^4\right)}{35b^5x^4\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(3/2)*x^6),x]
```

```
[Out] (2*(-5*b^4 + 8*a*b^3*x - 16*a^2*b^2*x^2 + 64*a^3*b*x^3 + 128*a^4*x^4))/(35*
b^5*Sqrt[a + b/x]*x^4)
```

fricas [A] time = 1.15, size = 72, normalized size = 0.76

$$\frac{2\left(128\,a^4x^4+64\,a^3bx^3-16\,a^2b^2x^2+8\,ab^3x-5\,b^4\right)\sqrt{\frac{ax+b}{x}}}{35\left(ab^5x^4+b^6x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] 2/35*(128*a^4*x^4 + 64*a^3*b*x^3 - 16*a^2*b^2*x^2 + 8*a*b^3*x - 5*b^4)*sqrt
((a*x + b)/x)/(a*b^5*x^4 + b^6*x^3)
```

giac [A] time = 0.19, size = 109, normalized size = 1.15

$$\frac{2\left(\frac{35\,a^4}{\sqrt{\frac{ax+b}{x}}}+140\,a^3\sqrt{\frac{ax+b}{x}}-\frac{70\,(ax+b)a^2\sqrt{\frac{ax+b}{x}}}{x}+\frac{28\,(ax+b)^2a\sqrt{\frac{ax+b}{x}}}{x^2}-\frac{5\,(ax+b)^3\sqrt{\frac{ax+b}{x}}}{x^3}\right)}{35\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] 2/35*(35*a^4/sqrt((a*x + b)/x) + 140*a^3*sqrt((a*x + b)/x) - 70*(a*x + b)*a
^2*sqrt((a*x + b)/x)/x + 28*(a*x + b)^2*a*sqrt((a*x + b)/x)/x^2 - 5*(a*x +
b)^3*sqrt((a*x + b)/x)/x^3)/b^5
```

maple [A] time = 0.01, size = 66, normalized size = 0.69

$$\frac{2\,(ax+b)\left(128a^4x^4+64a^3x^3b-16a^2x^2b^2+8ax\,b^3-5b^4\right)}{35\left(\frac{ax+b}{x}\right)^{\frac{3}{2}}b^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(3/2)/x^6,x)
```

```
[Out] 2/35*(a*x+b)*(128*a^4*x^4+64*a^3*b*x^3-16*a^2*b^2*x^2+8*a*b^3*x-5*b^4)/x^5/
b^5/((a*x+b)/x)^(3/2)
```

maxima [A] time = 0.95, size = 81, normalized size = 0.85

$$-\frac{2\left(a+\frac{b}{x}\right)^{\frac{7}{2}}}{7\,b^5}+\frac{8\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a}{5\,b^5}-\frac{4\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2}{b^5}+\frac{8\sqrt{a+\frac{b}{x}}\,a^3}{b^5}+\frac{2\,a^4}{\sqrt{a+\frac{b}{x}}\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] -2/7*(a + b/x)^(7/2)/b^5 + 8/5*(a + b/x)^(5/2)*a/b^5 - 4*(a + b/x)^(3/2)*a^
2/b^5 + 8*sqrt(a + b/x)*a^3/b^5 + 2*a^4/(sqrt(a + b/x)*b^5)
```

mupad [B] time = 1.45, size = 91, normalized size = 0.96

$$\frac{\sqrt{a+\frac{b}{x}}\left(\frac{186\,a^3}{35\,b^4}+\frac{256\,a^4\,x}{35\,b^5}\right)}{b+a\,x}-\frac{2\sqrt{a+\frac{b}{x}}}{7\,b^2\,x^3}+\frac{26\,a\sqrt{a+\frac{b}{x}}}{35\,b^3\,x^2}-\frac{58\,a^2\sqrt{a+\frac{b}{x}}}{35\,b^4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b/x)^(3/2)),x)
```

```
[Out] ((a + b/x)^(1/2)*((186*a^3)/(35*b^4) + (256*a^4*x)/(35*b^5)))/(b + a*x) - (
2*(a + b/x)^(1/2))/(7*b^2*x^3) + (26*a*(a + b/x)^(1/2))/(35*b^3*x^2) - (58*
a^2*(a + b/x)^(1/2))/(35*b^4*x)
```

sympy [B] time = 6.29, size = 4707, normalized size = 49.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(3/2)/x**6,x)
```

```
[Out] 256*a**(33/2)*b**(49/2)*x**13*sqrt(a*x/b + 1)/(35*a**(27/2)*b**29*x**(27/2)
 + 350*a**(25/2)*b**30*x**(25/2) + 1575*a**(23/2)*b**31*x**(23/2) + 4200*a*
*(21/2)*b**32*x**(21/2) + 7350*a**(19/2)*b**33*x**(19/2) + 8820*a**(17/2)*b
**34*x**(17/2) + 7350*a**(15/2)*b**35*x**(15/2) + 4200*a**(13/2)*b**36*x**(
13/2) + 1575*a**(11/2)*b**37*x**(11/2) + 350*a**(9/2)*b**38*x**(9/2) + 35*a
**(7/2)*b**39*x**(7/2)) + 2432*a**(31/2)*b**(51/2)*x**12*sqrt(a*x/b + 1)/(3
5*a**(27/2)*b**29*x**(27/2) + 350*a**(25/2)*b**30*x**(25/2) + 1575*a**(23/2
)*b**31*x**(23/2) + 4200*a**(21/2)*b**32*x**(21/2) + 7350*a**(19/2)*b**33*x
**(19/2) + 8820*a**(17/2)*b**34*x**(17/2) + 7350*a**(15/2)*b**35*x**(15/2)
+ 4200*a**(13/2)*b**36*x**(13/2) + 1575*a**(11/2)*b**37*x**(11/2) + 350*a**
(9/2)*b**38*x**(9/2) + 35*a**(7/2)*b**39*x**(7/2)) + 10336*a**(29/2)*b**(53
/2)*x**11*sqrt(a*x/b + 1)/(35*a**(27/2)*b**29*x**(27/2) + 350*a**(25/2)*b**
30*x**(25/2) + 1575*a**(23/2)*b**31*x**(23/2) + 4200*a**(21/2)*b**32*x**(21
/2) + 7350*a**(19/2)*b**33*x**(19/2) + 8820*a**(17/2)*b**34*x**(17/2) + 735
0*a**(15/2)*b**35*x**(15/2) + 4200*a**(13/2)*b**36*x**(13/2) + 1575*a**(11/
2)*b**37*x**(11/2) + 350*a**(9/2)*b**38*x**(9/2) + 35*a**(7/2)*b**39*x**(7/
2)) + 25840*a**(27/2)*b**(55/2)*x**10*sqrt(a*x/b + 1)/(35*a**(27/2)*b**29*x
**(27/2) + 350*a**(25/2)*b**30*x**(25/2) + 1575*a**(23/2)*b**31*x**(23/2) +
 4200*a**(21/2)*b**32*x**(21/2) + 7350*a**(19/2)*b**33*x**(19/2) + 8820*a**
(17/2)*b**34*x**(17/2) + 7350*a**(15/2)*b**35*x**(15/2) + 4200*a**(13/2)*b*
*36*x**(13/2) + 1575*a**(11/2)*b**37*x**(11/2) + 350*a**(9/2)*b**38*x**(9/2
) + 35*a**(7/2)*b**39*x**(7/2)) + 41990*a**(25/2)*b**(57/2)*x**9*sqrt(a*x/b
 + 1)/(35*a**(27/2)*b**29*x**(27/2) + 350*a**(25/2)*b**30*x**(25/2) + 1575*
a**(23/2)*b**31*x**(23/2) + 4200*a**(21/2)*b**32*x**(21/2) + 7350*a**(19/2)
*b**33*x**(19/2) + 8820*a**(17/2)*b**34*x**(17/2) + 7350*a**(15/2)*b**35*x*
*(15/2) + 4200*a**(13/2)*b**36*x**(13/2) + 1575*a**(11/2)*b**37*x**(11/2) +
 350*a**(9/2)*b**38*x**(9/2) + 35*a**(7/2)*b**39*x**(7/2)) + 46182*a**(23/2
)*b**(59/2)*x**8*sqrt(a*x/b + 1)/(35*a**(27/2)*b**29*x**(27/2) + 350*a**(25
/2)*b**30*x**(25/2) + 1575*a**(23/2)*b**31*x**(23/2) + 4200*a**(21/2)*b**32
*x**(21/2) + 7350*a**(19/2)*b**33*x**(19/2) + 8820*a**(17/2)*b**34*x**(17/2
) + 7350*a**(15/2)*b**35*x**(15/2) + 4200*a**(13/2)*b**36*x**(13/2) + 1575*
a**(11/2)*b**37*x**(11/2) + 350*a**(9/2)*b**38*x**(9/2) + 35*a**(7/2)*b**39
*x**(7/2)) + 34584*a**(21/2)*b**(61/2)*x**7*sqrt(a*x/b + 1)/(35*a**(27/2)*b
**29*x**(27/2) + 350*a**(25/2)*b**30*x**(25/2) + 1575*a**(23/2)*b**31*x**(2
3/2) + 4200*a**(21/2)*b**32*x**(21/2) + 7350*a**(19/2)*b**33*x**(19/2) + 88
20*a**(17/2)*b**34*x**(17/2) + 7350*a**(15/2)*b**35*x**(15/2) + 4200*a**(13
/2)*b**36*x**(13/2) + 1575*a**(11/2)*b**37*x**(11/2) + 350*a**(9/2)*b**38*x
**(9/2) + 35*a**(7/2)*b**39*x**(7/2)) + 17112*a**(19/2)*b**(63/2)*x**6*sqrt
(a*x/b + 1)/(35*a**(27/2)*b**29*x**(27/2) + 350*a**(25/2)*b**30*x**(25/2) +
```

```
 1575*a**(23/2)*b**31*x**(23/2) + 4200*a**(21/2)*b**32*x**(21/2) + 7350*a**
(19/2)*b**33*x**(19/2) + 8820*a**(17/2)*b**34*x**(17/2) + 7350*a**(15/2)*b*
*35*x**(15/2) + 4200*a**(13/2)*b**36*x**(13/2) + 1575*a**(11/2)*b**37*x**(1
1/2) + 350*a**(9/2)*b**38*x**(9/2) + 35*a**(7/2)*b**39*x**(7/2)) + 4980*a**
(17/2)*b**(65/2)*x**5*sqrt(a*x/b + 1)/(35*a**(27/2)*b**29*x**(27/2) + 350*a
**(25/2)*b**30*x**(25/2) + 1575*a**(23/2)*b**31*x**(23/2) + 4200*a**(21/2)*
b**32*x**(21/2) + 7350*a**(19/2)*b**33*x**(19/2) + 8820*a**(17/2)*b**34*x**
(17/2) + 7350*a**(15/2)*b**35*x**(15/2) + 4200*a**(13/2)*b**36*x**(13/2) +
1575*a**(11/2)*b**37*x**(11/2) + 350*a**(9/2)*b**38*x**(9/2) + 35*a**(7/2)*
b**39*x**(7/2)) + 340*a**(15/2)*b**(67/2)*x**4*sqrt(a*x/b + 1)/(35*a**(27/2
)*b**29*x**(27/2) + 350*a**(25/2)*b**30*x**(25/2) + 1575*a**(23/2)*b**31*x*
*(23/2) + 4200*a**(21/2)*b**32*x**(21/2) + 7350*a**(19/2)*b**33*x**(19/2) +
 8820*a**(17/2)*b**34*x**(17/2) + 7350*a**(15/2)*b**35*x**(15/2) + 4200*a**
(13/2)*b**36*x**(13/2) + 1575*a**(11/2)*b**37*x**(11/2) + 350*a**(9/2)*b**3
8*x**(9/2) + 35*a**(7/2)*b**39*x**(7/2)) - 424*a**(13/2)*b**(69/2)*x**3*sqr
t(a*x/b + 1)/(35*a**(27/2)*b**29*x**(27/2) + 350*a**(25/2)*b**30*x**(25/2)
+ 1575*a**(23/2)*b**31*x**(23/2) + 4200*a**(21/2)*b**32*x**(21/2) + 7350*a*
*(19/2)*b**33*x**(19/2) + 8820*a**(17/2)*b**34*x**(17/2) + 7350*a**(15/2)*b
**35*x**(15/2) + 4200*a**(13/2)*b**36*x**(13/2) + 1575*a**(11/2)*b**37*x**(
11/2) + 350*a**(9/2)*b**38*x**(9/2) + 35*a**(7/2)*b**39*x**(7/2)) - 248*a**
(11/2)*b**(71/2)*x**2*sqrt(a*x/b + 1)/(35*a**(27/2)*b**29*x**(27/2) + 350*a
**(25/2)*b**30*x**(25/2) + 1575*a**(23/2)*b**31*x**(23/2) + 4200*a**(21/2)*
b**32*x**(21/2) + 7350*a**(19/2)*b**33*x**(19/2) + 8820*a**(17/2)*b**34*x**
(17/2) + 7350*a**(15/2)*b**35*x**(15/2) + 4200*a**(13/2)*b**36*x**(13/2) +
1575*a**(11/2)*b**37*x**(11/2) + 350*a**(9/2)*b**38*x**(9/2) + 35*a**(7/2)*
b**39*x**(7/2)) - 74*a**(9/2)*b**(73/2)*x*sqrt(a*x/b + 1)/(35*a**(27/2)*b**
29*x**(27/2) + 350*a**(25/2)*b**30*x**(25/2) + 1575*a**(23/2)*b**31*x**(23/
2) + 4200*a**(21/2)*b**32*x**(21/2) + 7350*a**(19/2)*b**33*x**(19/2) + 8820
*a**(17/2)*b**34*x**(17/2) + 7350*a**(15/2)*b**35*x**(15/2) + 4200*a**(13/2
)*b**36*x**(13/2) + 1575*a**(11/2)*b**37*x**(11/2) + 350*a**(9/2)*b**38*x**
(9/2) + 35*a**(7/2)*b**39*x**(7/2)) - 10*a**(7/2)*b**(75/2)*sqrt(a*x/b + 1)
/(35*a**(27/2)*b**29*x**(27/2) + 350*a**(25/2)*b**30*x**(25/2) + 1575*a**(2
3/2)*b**31*x**(23/2) + 4200*a**(21/2)*b**32*x**(21/2) + 7350*a**(19/2)*b**3
3*x**(19/2) + 8820*a**(17/2)*b**34*x**(17/2) + 7350*a**(15/2)*b**35*x**(15/
2) + 4200*a**(13/2)*b**36*x**(13/2) + 1575*a**(11/2)*b**37*x**(11/2) + 350*
a**(9/2)*b**38*x**(9/2) + 35*a**(7/2)*b**39*x**(7/2)) - 256*a**17*b**24*x**
(27/2)/(35*a**(27/2)*b**29*x**(27/2) + 350*a**(25/2)*b**30*x**(25/2) + 1575
*a**(23/2)*b**31*x**(23/2) + 4200*a**(21/2)*b**32*x**(21/2) + 7350*a**(19/2
)*b**33*x**(19/2) + 8820*a**(17/2)*b**34*x**(17/2) + 7350*a**(15/2)*b**35*x
**(15/2) + 4200*a**(13/2)*b**36*x**(13/2) + 1575*a**(11/2)*b**37*x**(11/2)
+ 350*a**(9/2)*b**38*x**(9/2) + 35*a**(7/2)*b**39*x**(7/2)) - 2560*a**16*b*
*25*x**(25/2)/(35*a**(27/2)*b**29*x**(27/2) + 350*a**(25/2)*b**30*x**(25/2)
 + 1575*a**(23/2)*b**31*x**(23/2) + 4200*a**(21/2)*b**32*x**(21/2) + 7350*a
**(19/2)*b**33*x**(19/2) + 8820*a**(17/2)*b**34*x**(17/2) + 7350*a**(15/2)*
b**35*x**(15/2) + 4200*a**(13/2)*b**36*x**(13/2) + 1575*a**(11/2)*b**37*x**
(11/2) + 350*a**(9/2)*b**38*x**(9/2) + 35*a**(7/2)*b**39*x**(7/2)) - 11520*
a**15*b**26*x**(23/2)/(35*a**(27/2)*b**29*x**(27/2) + 350*a**(25/2)*b**30*x
**(25/2) + 1575*a**(23/2)*b**31*x**(23/2) + 4200*a**(21/2)*b**32*x**(21/2)
+ 7350*a**(19/2)*b**33*x**(19/2) + 8820*a**(17/2)*b**34*x**(17/2) + 7350*a*
*(15/2)*b**35*x**(15/2) + 4200*a**(13/2)*b**36*x**(13/2) + 1575*a**(11/2)*b
**37*x**(11/2) + 350*a**(9/2)*b**38*x**(9/2) + 35*a**(7/2)*b**39*x**(7/2))
- 30720*a**14*b**27*x**(21/2)/(35*a**(27/2)*b**29*x**(27/2) + 350*a**(25/2)
*b**30*x**(25/2) + 1575*a**(23/2)*b**31*x**(23/2) + 4200*a**(21/2)*b**32*x*
*(21/2) + 7350*a**(19/2)*b**33*x**(19/2) + 8820*a**(17/2)*b**34*x**(17/2) +
 7350*a**(15/2)*b**35*x**(15/2) + 4200*a**(13/2)*b**36*x**(13/2) + 1575*a**
(11/2)*b**37*x**(11/2) + 350*a**(9/2)*b**38*x**(9/2) + 35*a**(7/2)*b**39*x*
*(7/2)) - 53760*a**13*b**28*x**(19/2)/(35*a**(27/2)*b**29*x**(27/2) + 350*a
**(25/2)*b**30*x**(25/2) + 1575*a**(23/2)*b**31*x**(23/2) + 4200*a**(21/2)*
b**32*x**(21/2) + 7350*a**(19/2)*b**33*x**(19/2) + 8820*a**(17/2)*b**34*x**
```

```
(17/2) + 7350*a**(15/2)*b**35*x**(15/2) + 4200*a**(13/2)*b**36*x**(13/2) +
1575*a**(11/2)*b**37*x**(11/2) + 350*a**(9/2)*b**38*x**(9/2) + 35*a**(7/2)*
b**39*x**(7/2)) - 64512*a**12*b**29*x**(17/2)/(35*a**(27/2)*b**29*x**(27/2)
 + 350*a**(25/2)*b**30*x**(25/2) + 1575*a**(23/2)*b**31*x**(23/2) + 4200*a*
*(21/2)*b**32*x**(21/2) + 7350*a**(19/2)*b**33*x**(19/2) + 8820*a**(17/2)*b
**34*x**(17/2) + 7350*a**(15/2)*b**35*x**(15/2) + 4200*a**(13/2)*b**36*x**(
13/2) + 1575*a**(11/2)*b**37*x**(11/2) + 350*a**(9/2)*b**38*x**(9/2) + 35*a
**(7/2)*b**39*x**(7/2)) - 53760*a**11*b**30*x**(15/2)/(35*a**(27/2)*b**29*x
**(27/2) + 350*a**(25/2)*b**30*x**(25/2) + 1575*a**(23/2)*b**31*x**(23/2) +
 4200*a**(21/2)*b**32*x**(21/2) + 7350*a**(19/2)*b**33*x**(19/2) + 8820*a**
(17/2)*b**34*x**(17/2) + 7350*a**(15/2)*b**35*x**(15/2) + 4200*a**(13/2)*b*
*36*x**(13/2) + 1575*a**(11/2)*b**37*x**(11/2) + 350*a**(9/2)*b**38*x**(9/2
) + 35*a**(7/2)*b**39*x**(7/2)) - 30720*a**10*b**31*x**(13/2)/(35*a**(27/2)
*b**29*x**(27/2) + 350*a**(25/2)*b**30*x**(25/2) + 1575*a**(23/2)*b**31*x**
(23/2) + 4200*a**(21/2)*b**32*x**(21/2) + 7350*a**(19/2)*b**33*x**(19/2) +
8820*a**(17/2)*b**34*x**(17/2) + 7350*a**(15/2)*b**35*x**(15/2) + 4200*a**(
13/2)*b**36*x**(13/2) + 1575*a**(11/2)*b**37*x**(11/2) + 350*a**(9/2)*b**38
*x**(9/2) + 35*a**(7/2)*b**39*x**(7/2)) - 11520*a**9*b**32*x**(11/2)/(35*a*
*(27/2)*b**29*x**(27/2) + 350*a**(25/2)*b**30*x**(25/2) + 1575*a**(23/2)*b*
*31*x**(23/2) + 4200*a**(21/2)*b**32*x**(21/2) + 7350*a**(19/2)*b**33*x**(1
9/2) + 8820*a**(17/2)*b**34*x**(17/2) + 7350*a**(15/2)*b**35*x**(15/2) + 42
00*a**(13/2)*b**36*x**(13/2) + 1575*a**(11/2)*b**37*x**(11/2) + 350*a**(9/2
)*b**38*x**(9/2) + 35*a**(7/2)*b**39*x**(7/2)) - 2560*a**8*b**33*x**(9/2)/(
35*a**(27/2)*b**29*x**(27/2) + 350*a**(25/2)*b**30*x**(25/2) + 1575*a**(23/
2)*b**31*x**(23/2) + 4200*a**(21/2)*b**32*x**(21/2) + 7350*a**(19/2)*b**33*
x**(19/2) + 8820*a**(17/2)*b**34*x**(17/2) + 7350*a**(15/2)*b**35*x**(15/2)
 + 4200*a**(13/2)*b**36*x**(13/2) + 1575*a**(11/2)*b**37*x**(11/2) + 350*a*
*(9/2)*b**38*x**(9/2) + 35*a**(7/2)*b**39*x**(7/2)) - 256*a**7*b**34*x**(7/
2)/(35*a**(27/2)*b**29*x**(27/2) + 350*a**(25/2)*b**30*x**(25/2) + 1575*a**
(23/2)*b**31*x**(23/2) + 4200*a**(21/2)*b**32*x**(21/2) + 7350*a**(19/2)*b*
*33*x**(19/2) + 8820*a**(17/2)*b**34*x**(17/2) + 7350*a**(15/2)*b**35*x**(1
5/2) + 4200*a**(13/2)*b**36*x**(13/2) + 1575*a**(11/2)*b**37*x**(11/2) + 35
0*a**(9/2)*b**38*x**(9/2) + 35*a**(7/2)*b**39*x**(7/2))
```

3.1741 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^7}\,dx$

Optimal. Leaf size=116

$$-\frac{2a^5}{b^6\sqrt{a+\frac{b}{x}}}-\frac{10a^4\sqrt{a+\frac{b}{x}}}{b^6}+\frac{20a^3\left(a+\frac{b}{x}\right)^{3/2}}{3b^6}-\frac{4a^2\left(a+\frac{b}{x}\right)^{5/2}}{b^6}+\frac{10a\left(a+\frac{b}{x}\right)^{7/2}}{7b^6}-\frac{2\left(a+\frac{b}{x}\right)^{9/2}}{9b^6}$$

```
[Out] 20/3*a^3*(a+b/x)^(3/2)/b^6-4*a^2*(a+b/x)^(5/2)/b^6+10/7*a*(a+b/x)^(7/2)/b^6
-2/9*(a+b/x)^(9/2)/b^6-2*a^5/b^6/(a+b/x)^(1/2)-10*a^4*(a+b/x)^(1/2)/b^6
```

Rubi [A] time = 0.05, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{2a^5}{b^6\sqrt{a+\frac{b}{x}}}-\frac{10a^4\sqrt{a+\frac{b}{x}}}{b^6}+\frac{20a^3\left(a+\frac{b}{x}\right)^{3/2}}{3b^6}-\frac{4a^2\left(a+\frac{b}{x}\right)^{5/2}}{b^6}+\frac{10a\left(a+\frac{b}{x}\right)^{7/2}}{7b^6}-\frac{2\left(a+\frac{b}{x}\right)^{9/2}}{9b^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(3/2)*x^7),x]
```

```
[Out] (-2*a^5)/(b^6*Sqrt[a + b/x]) - (10*a^4*Sqrt[a + b/x])/b^6 + (20*a^3*(a + b/
x)^(3/2))/(3*b^6) - (4*a^2*(a + b/x)^(5/2))/b^6 + (10*a*(a + b/x)^(7/2))/(7
*b^6) - (2*(a + b/x)^(9/2))/(9*b^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^7}\,dx = -\text{Subst}\left(\int \frac{x^5}{(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(-\frac{a^5}{b^5(a+bx)^{3/2}}+\frac{5a^4}{b^5\sqrt{a+bx}}-\frac{10a^3\sqrt{a+bx}}{b^5}+\frac{10a^2(a+bx)^{3/2}}{b^5}-\frac{5a(a+bx)^5}{b^5}\right.\right.$$

$$= -\frac{2a^5}{b^6\sqrt{a+\frac{b}{x}}}-\frac{10a^4\sqrt{a+\frac{b}{x}}}{b^6}+\frac{20a^3\left(a+\frac{b}{x}\right)^{3/2}}{3b^6}-\frac{4a^2\left(a+\frac{b}{x}\right)^{5/2}}{b^6}+\frac{10a\left(a+\frac{b}{x}\right)^{7/2}}{7b^6}-\frac{2\left(a+\frac{b}{x}\right)^{9/2}}{9b^6}$$

Mathematica [A] time = 0.08, size = 73, normalized size = 0.63

$$-\frac{2\left(256a^5x^5+128a^4bx^4-32a^3b^2x^3+16a^2b^3x^2-10ab^4x+7b^5\right)}{63b^6x^5\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(3/2)*x^7),x]
```

```
[Out] (-2*(7*b^5 - 10*a*b^4*x + 16*a^2*b^3*x^2 - 32*a^3*b^2*x^3 + 128*a^4*b*x^4 +
 256*a^5*x^5))/(63*b^6*Sqrt[a + b/x]*x^5)
```

fricas [A] time = 0.70, size = 83, normalized size = 0.72

$$-\frac{2\left(256\,a^5x^5+128\,a^4bx^4-32\,a^3b^2x^3+16\,a^2b^3x^2-10\,ab^4x+7\,b^5\right)\sqrt{\frac{ax+b}{x}}}{63\left(ab^6x^5+b^7x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] -2/63*(256*a^5*x^5 + 128*a^4*b*x^4 - 32*a^3*b^2*x^3 + 16*a^2*b^3*x^2 - 10*a
*b^4*x + 7*b^5)*sqrt((a*x + b)/x)/(a*b^6*x^5 + b^7*x^4)
```

giac [A] time = 0.18, size = 135, normalized size = 1.16

$$-\frac{2\left(\frac{63\,a^5}{\sqrt{\frac{ax+b}{x}}}+315\,a^4\sqrt{\frac{ax+b}{x}}-\frac{210\,(ax+b)a^3\sqrt{\frac{ax+b}{x}}}{x}+\frac{126\,(ax+b)^2a^2\sqrt{\frac{ax+b}{x}}}{x^2}-\frac{45\,(ax+b)^3a\sqrt{\frac{ax+b}{x}}}{x^3}+\frac{7\,(ax+b)^4\sqrt{\frac{ax+b}{x}}}{x^4}\right)}{63\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] -2/63*(63*a^5/sqrt((a*x + b)/x) + 315*a^4*sqrt((a*x + b)/x) - 210*(a*x + b)
*a^3*sqrt((a*x + b)/x)/x + 126*(a*x + b)^2*a^2*sqrt((a*x + b)/x)/x^2 - 45*(
a*x + b)^3*a*sqrt((a*x + b)/x)/x^3 + 7*(a*x + b)^4*sqrt((a*x + b)/x)/x^4)/b
^6
```

maple [A] time = 0.00, size = 77, normalized size = 0.66

$$-\frac{2\,(ax+b)\left(256a^5x^5+128a^4b\,x^4-32a^3b^2x^3+16a^2b^3x^2-10a\,b^4x+7b^5\right)}{63\left(\frac{ax+b}{x}\right)^{\frac{3}{2}}b^6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(3/2)/x^7,x)
```

```
[Out] -2/63*(a*x+b)*(256*a^5*x^5+128*a^4*b*x^4-32*a^3*b^2*x^3+16*a^2*b^3*x^2-10*a
*b^4*x+7*b^5)/x^6/b^6/((a*x+b)/x)^(3/2)
```

maxima [A] time = 1.00, size = 98, normalized size = 0.84

$$-\frac{2\left(a+\frac{b}{x}\right)^{\frac{9}{2}}}{9\,b^6}+\frac{10\left(a+\frac{b}{x}\right)^{\frac{7}{2}}a}{7\,b^6}-\frac{4\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^2}{b^6}+\frac{20\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^3}{3\,b^6}-\frac{10\sqrt{a+\frac{b}{x}}\,a^4}{b^6}-\frac{2\,a^5}{\sqrt{a+\frac{b}{x}}\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] -2/9*(a + b/x)^(9/2)/b^6 + 10/7*(a + b/x)^(7/2)*a/b^6 - 4*(a + b/x)^(5/2)*a
^2/b^6 + 20/3*(a + b/x)^(3/2)*a^3/b^6 - 10*sqrt(a + b/x)*a^4/b^6 - 2*a^5/(s
qrt(a + b/x)*b^6)
```

mupad [B] time = 1.44, size = 112, normalized size = 0.97

$$\frac{34\,a\,\sqrt{a+\frac{b}{x}}}{63\,b^3\,x^3}-\frac{2\,\sqrt{a+\frac{b}{x}}}{9\,b^2\,x^4}-\frac{\sqrt{a+\frac{b}{x}}\,\left(\frac{386\,a^4}{63\,b^5}+\frac{512\,a^5\,x}{63\,b^6}\right)}{b+a\,x}-\frac{22\,a^2\,\sqrt{a+\frac{b}{x}}}{21\,b^4\,x^2}+\frac{130\,a^3\,\sqrt{a+\frac{b}{x}}}{63\,b^5\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b/x)^(3/2)),x)
```

```
[Out] (34*a*(a + b/x)^(1/2))/(63*b^3*x^3) - (2*(a + b/x)^(1/2))/(9*b^2*x^4) - ((a
 + b/x)^(1/2)*((386*a^4)/(63*b^5) + (512*a^5*x)/(63*b^6)))/(b + a*x) - (22*
a^2*(a + b/x)^(1/2))/(21*b^4*x^2) + (130*a^3*(a + b/x)^(1/2))/(63*b^5*x)
```

sympy [B] time = 11.38, size = 9534, normalized size = 82.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(3/2)/x**7,x)
```

```
[Out] -512*a**(47/2)*b**(91/2)*x**19*sqrt(a*x/b + 1)/(63*a**(39/2)*b**51*x**(39/2
) + 945*a**(37/2)*b**52*x**(37/2) + 6615*a**(35/2)*b**53*x**(35/2) + 28665*
a**(33/2)*b**54*x**(33/2) + 85995*a**(31/2)*b**55*x**(31/2) + 189189*a**(29
/2)*b**56*x**(29/2) + 315315*a**(27/2)*b**57*x**(27/2) + 405405*a**(25/2)*b
**58*x**(25/2) + 405405*a**(23/2)*b**59*x**(23/2) + 315315*a**(21/2)*b**60*
x**(21/2) + 189189*a**(19/2)*b**61*x**(19/2) + 85995*a**(17/2)*b**62*x**(17
/2) + 28665*a**(15/2)*b**63*x**(15/2) + 6615*a**(13/2)*b**64*x**(13/2) + 94
5*a**(11/2)*b**65*x**(11/2) + 63*a**(9/2)*b**66*x**(9/2)) - 7424*a**(45/2)*
b**(93/2)*x**18*sqrt(a*x/b + 1)/(63*a**(39/2)*b**51*x**(39/2) + 945*a**(37/
2)*b**52*x**(37/2) + 6615*a**(35/2)*b**53*x**(35/2) + 28665*a**(33/2)*b**54
*x**(33/2) + 85995*a**(31/2)*b**55*x**(31/2) + 189189*a**(29/2)*b**56*x**(2
9/2) + 315315*a**(27/2)*b**57*x**(27/2) + 405405*a**(25/2)*b**58*x**(25/2)
 + 405405*a**(23/2)*b**59*x**(23/2) + 315315*a**(21/2)*b**60*x**(21/2) + 189
189*a**(19/2)*b**61*x**(19/2) + 85995*a**(17/2)*b**62*x**(17/2) + 28665*a**
(15/2)*b**63*x**(15/2) + 6615*a**(13/2)*b**64*x**(13/2) + 945*a**(11/2)*b**
65*x**(11/2) + 63*a**(9/2)*b**66*x**(9/2)) - 50112*a**(43/2)*b**(95/2)*x**1
7*sqrt(a*x/b + 1)/(63*a**(39/2)*b**51*x**(39/2) + 945*a**(37/2)*b**52*x**(3
7/2) + 6615*a**(35/2)*b**53*x**(35/2) + 28665*a**(33/2)*b**54*x**(33/2) + 8
5995*a**(31/2)*b**55*x**(31/2) + 189189*a**(29/2)*b**56*x**(29/2) + 315315*
a**(27/2)*b**57*x**(27/2) + 405405*a**(25/2)*b**58*x**(25/2) + 405405*a**(2
3/2)*b**59*x**(23/2) + 315315*a**(21/2)*b**60*x**(21/2) + 189189*a**(19/2)*
b**61*x**(19/2) + 85995*a**(17/2)*b**62*x**(17/2) + 28665*a**(15/2)*b**63*x
**(15/2) + 6615*a**(13/2)*b**64*x**(13/2) + 945*a**(11/2)*b**65*x**(11/2) +
 63*a**(9/2)*b**66*x**(9/2)) - 208800*a**(41/2)*b**(97/2)*x**16*sqrt(a*x/b
+ 1)/(63*a**(39/2)*b**51*x**(39/2) + 945*a**(37/2)*b**52*x**(37/2) + 6615*a
**(35/2)*b**53*x**(35/2) + 28665*a**(33/2)*b**54*x**(33/2) + 85995*a**(31/2
)*b**55*x**(31/2) + 189189*a**(29/2)*b**56*x**(29/2) + 315315*a**(27/2)*b**
57*x**(27/2) + 405405*a**(25/2)*b**58*x**(25/2) + 405405*a**(23/2)*b**59*x*
*(23/2) + 315315*a**(21/2)*b**60*x**(21/2) + 189189*a**(19/2)*b**61*x**(19/
2) + 85995*a**(17/2)*b**62*x**(17/2) + 28665*a**(15/2)*b**63*x**(15/2) + 66
15*a**(13/2)*b**64*x**(13/2) + 945*a**(11/2)*b**65*x**(11/2) + 63*a**(9/2)*
b**66*x**(9/2)) - 600300*a**(39/2)*b**(99/2)*x**15*sqrt(a*x/b + 1)/(63*a**(
39/2)*b**51*x**(39/2) + 945*a**(37/2)*b**52*x**(37/2) + 6615*a**(35/2)*b**5
3*x**(35/2) + 28665*a**(33/2)*b**54*x**(33/2) + 85995*a**(31/2)*b**55*x**(3
```

```
1/2) + 189189*a**(29/2)*b**56*x**(29/2) + 315315*a**(27/2)*b**57*x**(27/2)
+ 405405*a**(25/2)*b**58*x**(25/2) + 405405*a**(23/2)*b**59*x**(23/2) + 315
315*a**(21/2)*b**60*x**(21/2) + 189189*a**(19/2)*b**61*x**(19/2) + 85995*a*
*(17/2)*b**62*x**(17/2) + 28665*a**(15/2)*b**63*x**(15/2) + 6615*a**(13/2)*
b**64*x**(13/2) + 945*a**(11/2)*b**65*x**(11/2) + 63*a**(9/2)*b**66*x**(9/2
)) - 1260630*a**(37/2)*b**(101/2)*x**14*sqrt(a*x/b + 1)/(63*a**(39/2)*b**51
*x**(39/2) + 945*a**(37/2)*b**52*x**(37/2) + 6615*a**(35/2)*b**53*x**(35/2)
 + 28665*a**(33/2)*b**54*x**(33/2) + 85995*a**(31/2)*b**55*x**(31/2) + 1891
89*a**(29/2)*b**56*x**(29/2) + 315315*a**(27/2)*b**57*x**(27/2) + 405405*a*
*(25/2)*b**58*x**(25/2) + 405405*a**(23/2)*b**59*x**(23/2) + 315315*a**(21/
2)*b**60*x**(21/2) + 189189*a**(19/2)*b**61*x**(19/2) + 85995*a**(17/2)*b**
62*x**(17/2) + 28665*a**(15/2)*b**63*x**(15/2) + 6615*a**(13/2)*b**64*x**(1
3/2) + 945*a**(11/2)*b**65*x**(11/2) + 63*a**(9/2)*b**66*x**(9/2)) - 199600
8*a**(35/2)*b**(103/2)*x**13*sqrt(a*x/b + 1)/(63*a**(39/2)*b**51*x**(39/2)
+ 945*a**(37/2)*b**52*x**(37/2) + 6615*a**(35/2)*b**53*x**(35/2) + 28665*a*
*(33/2)*b**54*x**(33/2) + 85995*a**(31/2)*b**55*x**(31/2) + 189189*a**(29/2
)*b**56*x**(29/2) + 315315*a**(27/2)*b**57*x**(27/2) + 405405*a**(25/2)*b**
58*x**(25/2) + 405405*a**(23/2)*b**59*x**(23/2) + 315315*a**(21/2)*b**60*x*
*(21/2) + 189189*a**(19/2)*b**61*x**(19/2) + 85995*a**(17/2)*b**62*x**(17/2
) + 28665*a**(15/2)*b**63*x**(15/2) + 6615*a**(13/2)*b**64*x**(13/2) + 945*
a**(11/2)*b**65*x**(11/2) + 63*a**(9/2)*b**66*x**(9/2)) - 2423850*a**(33/2)
*b**(105/2)*x**12*sqrt(a*x/b + 1)/(63*a**(39/2)*b**51*x**(39/2) + 945*a**(3
7/2)*b**52*x**(37/2) + 6615*a**(35/2)*b**53*x**(35/2) + 28665*a**(33/2)*b**
54*x**(33/2) + 85995*a**(31/2)*b**55*x**(31/2) + 189189*a**(29/2)*b**56*x**
(29/2) + 315315*a**(27/2)*b**57*x**(27/2) + 405405*a**(25/2)*b**58*x**(25/2
) + 405405*a**(23/2)*b**59*x**(23/2) + 315315*a**(21/2)*b**60*x**(21/2) + 1
89189*a**(19/2)*b**61*x**(19/2) + 85995*a**(17/2)*b**62*x**(17/2) + 28665*a
**(15/2)*b**63*x**(15/2) + 6615*a**(13/2)*b**64*x**(13/2) + 945*a**(11/2)*b
**65*x**(11/2) + 63*a**(9/2)*b**66*x**(9/2)) - 2273076*a**(31/2)*b**(107/2)
*x**11*sqrt(a*x/b + 1)/(63*a**(39/2)*b**51*x**(39/2) + 945*a**(37/2)*b**52*
x**(37/2) + 6615*a**(35/2)*b**53*x**(35/2) + 28665*a**(33/2)*b**54*x**(33/2
) + 85995*a**(31/2)*b**55*x**(31/2) + 189189*a**(29/2)*b**56*x**(29/2) + 31
5315*a**(27/2)*b**57*x**(27/2) + 405405*a**(25/2)*b**58*x**(25/2) + 405405*
a**(23/2)*b**59*x**(23/2) + 315315*a**(21/2)*b**60*x**(21/2) + 189189*a**(1
9/2)*b**61*x**(19/2) + 85995*a**(17/2)*b**62*x**(17/2) + 28665*a**(15/2)*b*
*63*x**(15/2) + 6615*a**(13/2)*b**64*x**(13/2) + 945*a**(11/2)*b**65*x**(11
/2) + 63*a**(9/2)*b**66*x**(9/2)) - 1644214*a**(29/2)*b**(109/2)*x**10*sqrt
(a*x/b + 1)/(63*a**(39/2)*b**51*x**(39/2) + 945*a**(37/2)*b**52*x**(37/2) +
 6615*a**(35/2)*b**53*x**(35/2) + 28665*a**(33/2)*b**54*x**(33/2) + 85995*a
**(31/2)*b**55*x**(31/2) + 189189*a**(29/2)*b**56*x**(29/2) + 315315*a**(27
/2)*b**57*x**(27/2) + 405405*a**(25/2)*b**58*x**(25/2) + 405405*a**(23/2)*b
**59*x**(23/2) + 315315*a**(21/2)*b**60*x**(21/2) + 189189*a**(19/2)*b**61*
x**(19/2) + 85995*a**(17/2)*b**62*x**(17/2) + 28665*a**(15/2)*b**63*x**(15/
2) + 6615*a**(13/2)*b**64*x**(13/2) + 945*a**(11/2)*b**65*x**(11/2) + 63*a*
*(9/2)*b**66*x**(9/2)) - 910624*a**(27/2)*b**(111/2)*x**9*sqrt(a*x/b + 1)/(
63*a**(39/2)*b**51*x**(39/2) + 945*a**(37/2)*b**52*x**(37/2) + 6615*a**(35/
2)*b**53*x**(35/2) + 28665*a**(33/2)*b**54*x**(33/2) + 85995*a**(31/2)*b**5
5*x**(31/2) + 189189*a**(29/2)*b**56*x**(29/2) + 315315*a**(27/2)*b**57*x**
(27/2) + 405405*a**(25/2)*b**58*x**(25/2) + 405405*a**(23/2)*b**59*x**(23/2
) + 315315*a**(21/2)*b**60*x**(21/2) + 189189*a**(19/2)*b**61*x**(19/2) + 8
5995*a**(17/2)*b**62*x**(17/2) + 28665*a**(15/2)*b**63*x**(15/2) + 6615*a**
(13/2)*b**64*x**(13/2) + 945*a**(11/2)*b**65*x**(11/2) + 63*a**(9/2)*b**66*
x**(9/2)) - 383994*a**(25/2)*b**(113/2)*x**8*sqrt(a*x/b + 1)/(63*a**(39/2)*
b**51*x**(39/2) + 945*a**(37/2)*b**52*x**(37/2) + 6615*a**(35/2)*b**53*x**(
35/2) + 28665*a**(33/2)*b**54*x**(33/2) + 85995*a**(31/2)*b**55*x**(31/2) +
 189189*a**(29/2)*b**56*x**(29/2) + 315315*a**(27/2)*b**57*x**(27/2) + 4054
05*a**(25/2)*b**58*x**(25/2) + 405405*a**(23/2)*b**59*x**(23/2) + 315315*a*
*(21/2)*b**60*x**(21/2) + 189189*a**(19/2)*b**61*x**(19/2) + 85995*a**(17/2
)*b**62*x**(17/2) + 28665*a**(15/2)*b**63*x**(15/2) + 6615*a**(13/2)*b**64*
```

```
x**(13/2) + 945*a**(11/2)*b**65*x**(11/2) + 63*a**(9/2)*b**66*x**(9/2)) - 1
27764*a**(23/2)*b**(115/2)*x**7*sqrt(a*x/b + 1)/(63*a**(39/2)*b**51*x**(39/
2) + 945*a**(37/2)*b**52*x**(37/2) + 6615*a**(35/2)*b**53*x**(35/2) + 28665
*a**(33/2)*b**54*x**(33/2) + 85995*a**(31/2)*b**55*x**(31/2) + 189189*a**(2
9/2)*b**56*x**(29/2) + 315315*a**(27/2)*b**57*x**(27/2) + 405405*a**(25/2)*
b**58*x**(25/2) + 405405*a**(23/2)*b**59*x**(23/2) + 315315*a**(21/2)*b**60
*x**(21/2) + 189189*a**(19/2)*b**61*x**(19/2) + 85995*a**(17/2)*b**62*x**(1
7/2) + 28665*a**(15/2)*b**63*x**(15/2) + 6615*a**(13/2)*b**64*x**(13/2) + 9
45*a**(11/2)*b**65*x**(11/2) + 63*a**(9/2)*b**66*x**(9/2)) - 41202*a**(21/2
)*b**(117/2)*x**6*sqrt(a*x/b + 1)/(63*a**(39/2)*b**51*x**(39/2) + 945*a**(3
7/2)*b**52*x**(37/2) + 6615*a**(35/2)*b**53*x**(35/2) + 28665*a**(33/2)*b**
54*x**(33/2) + 85995*a**(31/2)*b**55*x**(31/2) + 189189*a**(29/2)*b**56*x**
(29/2) + 315315*a**(27/2)*b**57*x**(27/2) + 405405*a**(25/2)*b**58*x**(25/2
) + 405405*a**(23/2)*b**59*x**(23/2) + 315315*a**(21/2)*b**60*x**(21/2) + 1
89189*a**(19/2)*b**61*x**(19/2) + 85995*a**(17/2)*b**62*x**(17/2) + 28665*a
**(15/2)*b**63*x**(15/2) + 6615*a**(13/2)*b**64*x**(13/2) + 945*a**(11/2)*b
**65*x**(11/2) + 63*a**(9/2)*b**66*x**(9/2)) - 17928*a**(19/2)*b**(119/2)*x
**5*sqrt(a*x/b + 1)/(63*a**(39/2)*b**51*x**(39/2) + 945*a**(37/2)*b**52*x**
(37/2) + 6615*a**(35/2)*b**53*x**(35/2) + 28665*a**(33/2)*b**54*x**(33/2) +
 85995*a**(31/2)*b**55*x**(31/2) + 189189*a**(29/2)*b**56*x**(29/2) + 31531
5*a**(27/2)*b**57*x**(27/2) + 405405*a**(25/2)*b**58*x**(25/2) + 405405*a**
(23/2)*b**59*x**(23/2) + 315315*a**(21/2)*b**60*x**(21/2) + 189189*a**(19/2
)*b**61*x**(19/2) + 85995*a**(17/2)*b**62*x**(17/2) + 28665*a**(15/2)*b**63
*x**(15/2) + 6615*a**(13/2)*b**64*x**(13/2) + 945*a**(11/2)*b**65*x**(11/2)
 + 63*a**(9/2)*b**66*x**(9/2)) - 9006*a**(17/2)*b**(121/2)*x**4*sqrt(a*x/b
+ 1)/(63*a**(39/2)*b**51*x**(39/2) + 945*a**(37/2)*b**52*x**(37/2) + 6615*a
**(35/2)*b**53*x**(35/2) + 28665*a**(33/2)*b**54*x**(33/2) + 85995*a**(31/2
)*b**55*x**(31/2) + 189189*a**(29/2)*b**56*x**(29/2) + 315315*a**(27/2)*b**
57*x**(27/2) + 405405*a**(25/2)*b**58*x**(25/2) + 405405*a**(23/2)*b**59*x*
*(23/2) + 315315*a**(21/2)*b**60*x**(21/2) + 189189*a**(19/2)*b**61*x**(19/
2) + 85995*a**(17/2)*b**62*x**(17/2) + 28665*a**(15/2)*b**63*x**(15/2) + 66
15*a**(13/2)*b**64*x**(13/2) + 945*a**(11/2)*b**65*x**(11/2) + 63*a**(9/2)*
b**66*x**(9/2)) - 3660*a**(15/2)*b**(123/2)*x**3*sqrt(a*x/b + 1)/(63*a**(39
/2)*b**51*x**(39/2) + 945*a**(37/2)*b**52*x**(37/2) + 6615*a**(35/2)*b**53*
x**(35/2) + 28665*a**(33/2)*b**54*x**(33/2) + 85995*a**(31/2)*b**55*x**(31/
2) + 189189*a**(29/2)*b**56*x**(29/2) + 315315*a**(27/2)*b**57*x**(27/2) +
405405*a**(25/2)*b**58*x**(25/2) + 405405*a**(23/2)*b**59*x**(23/2) + 31531
5*a**(21/2)*b**60*x**(21/2) + 189189*a**(19/2)*b**61*x**(19/2) + 85995*a**(
17/2)*b**62*x**(17/2) + 28665*a**(15/2)*b**63*x**(15/2) + 6615*a**(13/2)*b*
*64*x**(13/2) + 945*a**(11/2)*b**65*x**(11/2) + 63*a**(9/2)*b**66*x**(9/2))
 - 1026*a**(13/2)*b**(125/2)*x**2*sqrt(a*x/b + 1)/(63*a**(39/2)*b**51*x**(3
9/2) + 945*a**(37/2)*b**52*x**(37/2) + 6615*a**(35/2)*b**53*x**(35/2) + 286
65*a**(33/2)*b**54*x**(33/2) + 85995*a**(31/2)*b**55*x**(31/2) + 189189*a**
(29/2)*b**56*x**(29/2) + 315315*a**(27/2)*b**57*x**(27/2) + 405405*a**(25/2
)*b**58*x**(25/2) + 405405*a**(23/2)*b**59*x**(23/2) + 315315*a**(21/2)*b**
60*x**(21/2) + 189189*a**(19/2)*b**61*x**(19/2) + 85995*a**(17/2)*b**62*x**
(17/2) + 28665*a**(15/2)*b**63*x**(15/2) + 6615*a**(13/2)*b**64*x**(13/2) +
 945*a**(11/2)*b**65*x**(11/2) + 63*a**(9/2)*b**66*x**(9/2)) - 176*a**(11/2
)*b**(127/2)*x*sqrt(a*x/b + 1)/(63*a**(39/2)*b**51*x**(39/2) + 945*a**(37/2
)*b**52*x**(37/2) + 6615*a**(35/2)*b**53*x**(35/2) + 28665*a**(33/2)*b**54*
x**(33/2) + 85995*a**(31/2)*b**55*x**(31/2) + 189189*a**(29/2)*b**56*x**(29
/2) + 315315*a**(27/2)*b**57*x**(27/2) + 405405*a**(25/2)*b**58*x**(25/2) +
 405405*a**(23/2)*b**59*x**(23/2) + 315315*a**(21/2)*b**60*x**(21/2) + 1891
89*a**(19/2)*b**61*x**(19/2) + 85995*a**(17/2)*b**62*x**(17/2) + 28665*a**(
15/2)*b**63*x**(15/2) + 6615*a**(13/2)*b**64*x**(13/2) + 945*a**(11/2)*b**6
5*x**(11/2) + 63*a**(9/2)*b**66*x**(9/2)) - 14*a**(9/2)*b**(129/2)*sqrt(a*x
/b + 1)/(63*a**(39/2)*b**51*x**(39/2) + 945*a**(37/2)*b**52*x**(37/2) + 661
5*a**(35/2)*b**53*x**(35/2) + 28665*a**(33/2)*b**54*x**(33/2) + 85995*a**(3
1/2)*b**55*x**(31/2) + 189189*a**(29/2)*b**56*x**(29/2) + 315315*a**(27/2)*
```

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b**57*x**(27/2) + 405405*a**(25/2)*b**58*x**(25/2) + 405405*a**(23/2)*b**59
*x**(23/2) + 315315*a**(21/2)*b**60*x**(21/2) + 189189*a**(19/2)*b**61*x**(
19/2) + 85995*a**(17/2)*b**62*x**(17/2) + 28665*a**(15/2)*b**63*x**(15/2) +
 6615*a**(13/2)*b**64*x**(13/2) + 945*a**(11/2)*b**65*x**(11/2) + 63*a**(9/
2)*b**66*x**(9/2)) + 512*a**24*b**45*x**(39/2)/(63*a**(39/2)*b**51*x**(39/2
) + 945*a**(37/2)*b**52*x**(37/2) + 6615*a**(35/2)*b**53*x**(35/2) + 28665*
a**(33/2)*b**54*x**(33/2) + 85995*a**(31/2)*b**55*x**(31/2) + 189189*a**(29
/2)*b**56*x**(29/2) + 315315*a**(27/2)*b**57*x**(27/2) + 405405*a**(25/2)*b
**58*x**(25/2) + 405405*a**(23/2)*b**59*x**(23/2) + 315315*a**(21/2)*b**60*
x**(21/2) + 189189*a**(19/2)*b**61*x**(19/2) + 85995*a**(17/2)*b**62*x**(17
/2) + 28665*a**(15/2)*b**63*x**(15/2) + 6615*a**(13/2)*b**64*x**(13/2) + 94
5*a**(11/2)*b**65*x**(11/2) + 63*a**(9/2)*b**66*x**(9/2)) + 7680*a**23*b**4
6*x**(37/2)/(63*a**(39/2)*b**51*x**(39/2) + 945*a**(37/2)*b**52*x**(37/2) +
 6615*a**(35/2)*b**53*x**(35/2) + 28665*a**(33/2)*b**54*x**(33/2) + 85995*a
**(31/2)*b**55*x**(31/2) + 189189*a**(29/2)*b**56*x**(29/2) + 315315*a**(27
/2)*b**57*x**(27/2) + 405405*a**(25/2)*b**58*x**(25/2) + 405405*a**(23/2)*b
**59*x**(23/2) + 315315*a**(21/2)*b**60*x**(21/2) + 189189*a**(19/2)*b**61*
x**(19/2) + 85995*a**(17/2)*b**62*x**(17/2) + 28665*a**(15/2)*b**63*x**(15/
2) + 6615*a**(13/2)*b**64*x**(13/2) + 945*a**(11/2)*b**65*x**(11/2) + 63*a*
*(9/2)*b**66*x**(9/2)) + 53760*a**22*b**47*x**(35/2)/(63*a**(39/2)*b**51*x*
*(39/2) + 945*a**(37/2)*b**52*x**(37/2) + 6615*a**(35/2)*b**53*x**(35/2) +
28665*a**(33/2)*b**54*x**(33/2) + 85995*a**(31/2)*b**55*x**(31/2) + 189189*
a**(29/2)*b**56*x**(29/2) + 315315*a**(27/2)*b**57*x**(27/2) + 405405*a**(2
5/2)*b**58*x**(25/2) + 405405*a**(23/2)*b**59*x**(23/2) + 315315*a**(21/2)*
b**60*x**(21/2) + 189189*a**(19/2)*b**61*x**(19/2) + 85995*a**(17/2)*b**62*
x**(17/2) + 28665*a**(15/2)*b**63*x**(15/2) + 6615*a**(13/2)*b**64*x**(13/2
) + 945*a**(11/2)*b**65*x**(11/2) + 63*a**(9/2)*b**66*x**(9/2)) + 232960*a*
*21*b**48*x**(33/2)/(63*a**(39/2)*b**51*x**(39/2) + 945*a**(37/2)*b**52*x**
(37/2) + 6615*a**(35/2)*b**53*x**(35/2) + 28665*a**(33/2)*b**54*x**(33/2) +
 85995*a**(31/2)*b**55*x**(31/2) + 189189*a**(29/2)*b**56*x**(29/2) + 31531
5*a**(27/2)*b**57*x**(27/2) + 405405*a**(25/2)*b**58*x**(25/2) + 405405*a**
(23/2)*b**59*x**(23/2) + 315315*a**(21/2)*b**60*x**(21/2) + 189189*a**(19/2
)*b**61*x**(19/2) + 85995*a**(17/2)*b**62*x**(17/2) + 28665*a**(15/2)*b**63
*x**(15/2) + 6615*a**(13/2)*b**64*x**(13/2) + 945*a**(11/2)*b**65*x**(11/2)
 + 63*a**(9/2)*b**66*x**(9/2)) + 698880*a**20*b**49*x**(31/2)/(63*a**(39/2)
*b**51*x**(39/2) + 945*a**(37/2)*b**52*x**(37/2) + 6615*a**(35/2)*b**53*x**
(35/2) + 28665*a**(33/2)*b**54*x**(33/2) + 85995*a**(31/2)*b**55*x**(31/2)
+ 189189*a**(29/2)*b**56*x**(29/2) + 315315*a**(27/2)*b**57*x**(27/2) + 405
405*a**(25/2)*b**58*x**(25/2) + 405405*a**(23/2)*b**59*x**(23/2) + 315315*a
**(21/2)*b**60*x**(21/2) + 189189*a**(19/2)*b**61*x**(19/2) + 85995*a**(17/
2)*b**62*x**(17/2) + 28665*a**(15/2)*b**63*x**(15/2) + 6615*a**(13/2)*b**64
*x**(13/2) + 945*a**(11/2)*b**65*x**(11/2) + 63*a**(9/2)*b**66*x**(9/2)) +
1537536*a**19*b**50*x**(29/2)/(63*a**(39/2)*b**51*x**(39/2) + 945*a**(37/2)
*b**52*x**(37/2) + 6615*a**(35/2)*b**53*x**(35/2) + 28665*a**(33/2)*b**54*x
**(33/2) + 85995*a**(31/2)*b**55*x**(31/2) + 189189*a**(29/2)*b**56*x**(29/
2) + 315315*a**(27/2)*b**57*x**(27/2) + 405405*a**(25/2)*b**58*x**(25/2) +
405405*a**(23/2)*b**59*x**(23/2) + 315315*a**(21/2)*b**60*x**(21/2) + 18918
9*a**(19/2)*b**61*x**(19/2) + 85995*a**(17/2)*b**62*x**(17/2) + 28665*a**(1
5/2)*b**63*x**(15/2) + 6615*a**(13/2)*b**64*x**(13/2) + 945*a**(11/2)*b**65
*x**(11/2) + 63*a**(9/2)*b**66*x**(9/2)) + 2562560*a**18*b**51*x**(27/2)/(6
3*a**(39/2)*b**51*x**(39/2) + 945*a**(37/2)*b**52*x**(37/2) + 6615*a**(35/2
)*b**53*x**(35/2) + 28665*a**(33/2)*b**54*x**(33/2) + 85995*a**(31/2)*b**55
*x**(31/2) + 189189*a**(29/2)*b**56*x**(29/2) + 315315*a**(27/2)*b**57*x**(
27/2) + 405405*a**(25/2)*b**58*x**(25/2) + 405405*a**(23/2)*b**59*x**(23/2)
 + 315315*a**(21/2)*b**60*x**(21/2) + 189189*a**(19/2)*b**61*x**(19/2) + 85
995*a**(17/2)*b**62*x**(17/2) + 28665*a**(15/2)*b**63*x**(15/2) + 6615*a**(
13/2)*b**64*x**(13/2) + 945*a**(11/2)*b**65*x**(11/2) + 63*a**(9/2)*b**66*x
**(9/2)) + 3294720*a**17*b**52*x**(25/2)/(63*a**(39/2)*b**51*x**(39/2) + 94
5*a**(37/2)*b**52*x**(37/2) + 6615*a**(35/2)*b**53*x**(35/2) + 28665*a**(33
```

```
/2)*b**54*x**(33/2) + 85995*a**(31/2)*b**55*x**(31/2) + 189189*a**(29/2)*b*
*56*x**(29/2) + 315315*a**(27/2)*b**57*x**(27/2) + 405405*a**(25/2)*b**58*x
**(25/2) + 405405*a**(23/2)*b**59*x**(23/2) + 315315*a**(21/2)*b**60*x**(21
/2) + 189189*a**(19/2)*b**61*x**(19/2) + 85995*a**(17/2)*b**62*x**(17/2) +
28665*a**(15/2)*b**63*x**(15/2) + 6615*a**(13/2)*b**64*x**(13/2) + 945*a**(
11/2)*b**65*x**(11/2) + 63*a**(9/2)*b**66*x**(9/2)) + 3294720*a**16*b**53*x
**(23/2)/(63*a**(39/2)*b**51*x**(39/2) + 945*a**(37/2)*b**52*x**(37/2) + 66
15*a**(35/2)*b**53*x**(35/2) + 28665*a**(33/2)*b**54*x**(33/2) + 85995*a**(
31/2)*b**55*x**(31/2) + 189189*a**(29/2)*b**56*x**(29/2) + 315315*a**(27/2)
*b**57*x**(27/2) + 405405*a**(25/2)*b**58*x**(25/2) + 405405*a**(23/2)*b**5
9*x**(23/2) + 315315*a**(21/2)*b**60*x**(21/2) + 189189*a**(19/2)*b**61*x**
(19/2) + 85995*a**(17/2)*b**62*x**(17/2) + 28665*a**(15/2)*b**63*x**(15/2)
+ 6615*a**(13/2)*b**64*x**(13/2) + 945*a**(11/2)*b**65*x**(11/2) + 63*a**(9
/2)*b**66*x**(9/2)) + 2562560*a**15*b**54*x**(21/2)/(63*a**(39/2)*b**51*x**
(39/2) + 945*a**(37/2)*b**52*x**(37/2) + 6615*a**(35/2)*b**53*x**(35/2) + 2
8665*a**(33/2)*b**54*x**(33/2) + 85995*a**(31/2)*b**55*x**(31/2) + 189189*a
**(29/2)*b**56*x**(29/2) + 315315*a**(27/2)*b**57*x**(27/2) + 405405*a**(25
/2)*b**58*x**(25/2) + 405405*a**(23/2)*b**59*x**(23/2) + 315315*a**(21/2)*b
**60*x**(21/2) + 189189*a**(19/2)*b**61*x**(19/2) + 85995*a**(17/2)*b**62*x
**(17/2) + 28665*a**(15/2)*b**63*x**(15/2) + 6615*a**(13/2)*b**64*x**(13/2)
 + 945*a**(11/2)*b**65*x**(11/2) + 63*a**(9/2)*b**66*x**(9/2)) + 1537536*a*
*14*b**55*x**(19/2)/(63*a**(39/2)*b**51*x**(39/2) + 945*a**(37/2)*b**52*x**
(37/2) + 6615*a**(35/2)*b**53*x**(35/2) + 28665*a**(33/2)*b**54*x**(33/2) +
 85995*a**(31/2)*b**55*x**(31/2) + 189189*a**(29/2)*b**56*x**(29/2) + 31531
5*a**(27/2)*b**57*x**(27/2) + 405405*a**(25/2)*b**58*x**(25/2) + 405405*a**
(23/2)*b**59*x**(23/2) + 315315*a**(21/2)*b**60*x**(21/2) + 189189*a**(19/2
)*b**61*x**(19/2) + 85995*a**(17/2)*b**62*x**(17/2) + 28665*a**(15/2)*b**63
*x**(15/2) + 6615*a**(13/2)*b**64*x**(13/2) + 945*a**(11/2)*b**65*x**(11/2)
 + 63*a**(9/2)*b**66*x**(9/2)) + 698880*a**13*b**56*x**(17/2)/(63*a**(39/2)
*b**51*x**(39/2) + 945*a**(37/2)*b**52*x**(37/2) + 6615*a**(35/2)*b**53*x**
(35/2) + 28665*a**(33/2)*b**54*x**(33/2) + 85995*a**(31/2)*b**55*x**(31/2)
+ 189189*a**(29/2)*b**56*x**(29/2) + 315315*a**(27/2)*b**57*x**(27/2) + 405
405*a**(25/2)*b**58*x**(25/2) + 405405*a**(23/2)*b**59*x**(23/2) + 315315*a
**(21/2)*b**60*x**(21/2) + 189189*a**(19/2)*b**61*x**(19/2) + 85995*a**(17/
2)*b**62*x**(17/2) + 28665*a**(15/2)*b**63*x**(15/2) + 6615*a**(13/2)*b**64
*x**(13/2) + 945*a**(11/2)*b**65*x**(11/2) + 63*a**(9/2)*b**66*x**(9/2)) +
232960*a**12*b**57*x**(15/2)/(63*a**(39/2)*b**51*x**(39/2) + 945*a**(37/2)*
b**52*x**(37/2) + 6615*a**(35/2)*b**53*x**(35/2) + 28665*a**(33/2)*b**54*x*
*(33/2) + 85995*a**(31/2)*b**55*x**(31/2) + 189189*a**(29/2)*b**56*x**(29/2
) + 315315*a**(27/2)*b**57*x**(27/2) + 405405*a**(25/2)*b**58*x**(25/2) + 4
05405*a**(23/2)*b**59*x**(23/2) + 315315*a**(21/2)*b**60*x**(21/2) + 189189
*a**(19/2)*b**61*x**(19/2) + 85995*a**(17/2)*b**62*x**(17/2) + 28665*a**(15
/2)*b**63*x**(15/2) + 6615*a**(13/2)*b**64*x**(13/2) + 945*a**(11/2)*b**65*
x**(11/2) + 63*a**(9/2)*b**66*x**(9/2)) + 53760*a**11*b**58*x**(13/2)/(63*a
**(39/2)*b**51*x**(39/2) + 945*a**(37/2)*b**52*x**(37/2) + 6615*a**(35/2)*b
**53*x**(35/2) + 28665*a**(33/2)*b**54*x**(33/2) + 85995*a**(31/2)*b**55*x*
*(31/2) + 189189*a**(29/2)*b**56*x**(29/2) + 315315*a**(27/2)*b**57*x**(27/
2) + 405405*a**(25/2)*b**58*x**(25/2) + 405405*a**(23/2)*b**59*x**(23/2) +
315315*a**(21/2)*b**60*x**(21/2) + 189189*a**(19/2)*b**61*x**(19/2) + 85995
*a**(17/2)*b**62*x**(17/2) + 28665*a**(15/2)*b**63*x**(15/2) + 6615*a**(13/
2)*b**64*x**(13/2) + 945*a**(11/2)*b**65*x**(11/2) + 63*a**(9/2)*b**66*x**(
9/2)) + 7680*a**10*b**59*x**(11/2)/(63*a**(39/2)*b**51*x**(39/2) + 945*a**(
37/2)*b**52*x**(37/2) + 6615*a**(35/2)*b**53*x**(35/2) + 28665*a**(33/2)*b*
*54*x**(33/2) + 85995*a**(31/2)*b**55*x**(31/2) + 189189*a**(29/2)*b**56*x*
*(29/2) + 315315*a**(27/2)*b**57*x**(27/2) + 405405*a**(25/2)*b**58*x**(25/
2) + 405405*a**(23/2)*b**59*x**(23/2) + 315315*a**(21/2)*b**60*x**(21/2) +
189189*a**(19/2)*b**61*x**(19/2) + 85995*a**(17/2)*b**62*x**(17/2) + 28665*
a**(15/2)*b**63*x**(15/2) + 6615*a**(13/2)*b**64*x**(13/2) + 945*a**(11/2)*
b**65*x**(11/2) + 63*a**(9/2)*b**66*x**(9/2)) + 512*a**9*b**60*x**(9/2)/(63
```

```
*a**(39/2)*b**51*x**(39/2) + 945*a**(37/2)*b**52*x**(37/2) + 6615*a**(35/2)
*b**53*x**(35/2) + 28665*a**(33/2)*b**54*x**(33/2) + 85995*a**(31/2)*b**55*
x**(31/2) + 189189*a**(29/2)*b**56*x**(29/2) + 315315*a**(27/2)*b**57*x**(2
7/2) + 405405*a**(25/2)*b**58*x**(25/2) + 405405*a**(23/2)*b**59*x**(23/2)
+ 315315*a**(21/2)*b**60*x**(21/2) + 189189*a**(19/2)*b**61*x**(19/2) + 859
95*a**(17/2)*b**62*x**(17/2) + 28665*a**(15/2)*b**63*x**(15/2) + 6615*a**(1
3/2)*b**64*x**(13/2) + 945*a**(11/2)*b**65*x**(11/2) + 63*a**(9/2)*b**66*x*
*(9/2))
```

3.1742 $\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$

Optimal. Leaf size=138

$$-\frac{105b^3\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{8a^{11/2}}+\frac{105b^3}{8a^5\sqrt{a+\frac{b}{x}}}+\frac{35b^3}{8a^4\left(a+\frac{b}{x}\right)^{3/2}}+\frac{21b^2x}{8a^3\left(a+\frac{b}{x}\right)^{3/2}}-\frac{3bx^2}{4a^2\left(a+\frac{b}{x}\right)^{3/2}}+\frac{x^3}{3a\left(a+\frac{b}{x}\right)^{3/2}}$$

```
[Out] 35/8*b^3/a^4/(a+b/x)^(3/2)+21/8*b^2*x/a^3/(a+b/x)^(3/2)-3/4*b*x^2/a^2/(a+b/
x)^(3/2)+1/3*x^3/a/(a+b/x)^(3/2)-105/8*b^3*arctanh((a+b/x)^(1/2)/a^(1/2))/a
^(11/2)+105/8*b^3/a^5/(a+b/x)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 134, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 51, 63, 208}

$$\frac{105b^2x\sqrt{a+\frac{b}{x}}}{8a^5}-\frac{105b^3\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{8a^{11/2}}-\frac{35bx^2\sqrt{a+\frac{b}{x}}}{4a^4}+\frac{7x^3\sqrt{a+\frac{b}{x}}}{a^3}-\frac{6x^3}{a^2\sqrt{a+\frac{b}{x}}}-\frac{2x^3}{3a\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b/x)^(5/2),x]
```

```
[Out] (105*b^2*Sqrt[a + b/x]*x)/(8*a^5) - (35*b*Sqrt[a + b/x]*x^2)/(4*a^4) - (2*x
^3)/(3*a*(a + b/x)^(3/2)) - (6*x^3)/(a^2*Sqrt[a + b/x]) + (7*Sqrt[a + b/x]*
x^3)/a^3 - (105*b^3*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(8*a^(11/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = -\operatorname{Subst}\left(\int \frac{1}{x^4(a+bx)^{5/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2x^3}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{3\operatorname{Subst}\left(\int \frac{1}{x^4(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{2x^3}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{6x^3}{a^2\sqrt{a+\frac{b}{x}}} - \frac{21\operatorname{Subst}\left(\int \frac{1}{x^4\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{a^2}$$

$$= -\frac{2x^3}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{6x^3}{a^2\sqrt{a+\frac{b}{x}}} + \frac{7\sqrt{a+\frac{b}{x}}\,x^3}{a^3} + \frac{(35b)\operatorname{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a^3}$$

$$= -\frac{35b\sqrt{a+\frac{b}{x}}\,x^2}{4a^4} - \frac{2x^3}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{6x^3}{a^2\sqrt{a+\frac{b}{x}}} + \frac{7\sqrt{a+\frac{b}{x}}\,x^3}{a^3} - \frac{\left(105b^2\right)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx\right.}{8a^4}$$

$$= \frac{105b^2\sqrt{a+\frac{b}{x}}\,x}{8a^5} - \frac{35b\sqrt{a+\frac{b}{x}}\,x^2}{4a^4} - \frac{2x^3}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{6x^3}{a^2\sqrt{a+\frac{b}{x}}} + \frac{7\sqrt{a+\frac{b}{x}}\,x^3}{a^3} + \frac{\left(105b^3\right)\mathrm{Su}}{}$$

$$= \frac{105b^2\sqrt{a+\frac{b}{x}}\,x}{8a^5} - \frac{35b\sqrt{a+\frac{b}{x}}\,x^2}{4a^4} - \frac{2x^3}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{6x^3}{a^2\sqrt{a+\frac{b}{x}}} + \frac{7\sqrt{a+\frac{b}{x}}\,x^3}{a^3} + \frac{\left(105b^2\right)\mathrm{Su}}{}$$

$$= \frac{105b^2\sqrt{a+\frac{b}{x}}\,x}{8a^5} - \frac{35b\sqrt{a+\frac{b}{x}}\,x^2}{4a^4} - \frac{2x^3}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{6x^3}{a^2\sqrt{a+\frac{b}{x}}} + \frac{7\sqrt{a+\frac{b}{x}}\,x^3}{a^3} - \frac{105b^3\tanh}{8a^1}$$

Mathematica [C] time = 0.02, size = 39, normalized size = 0.28

$$\frac{2b^3\,{}_2F_1\left(-\frac{3}{2}, 4; -\frac{1}{2}; \frac{b}{ax}+1\right)}{3a^4\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b/x)^(5/2),x]
```

```
[Out] (2*b^3*Hypergeometric2F1[-3/2, 4, -1/2, 1 + b/(a*x)])/(3*a^4*(a + b/x)^(3/2
))
```

fricas [A] time = 1.10, size = 273, normalized size = 1.98

$$\left[\frac{315\left(a^2b^3x^2+2ab^4x+b^5\right)\sqrt{a}\,\log\left(2ax-2\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(8a^5x^5-18a^4bx^4+63a^3b^2x^3+420a^2b^3x^2\right.}{48\left(a^8x^2+2a^7bx+a^6b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(315*(a^2*b^3*x^2 + 2*a*b^4*x + b^5)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*
sqrt((a*x + b)/x) + b) + 2*(8*a^5*x^5 - 18*a^4*b*x^4 + 63*a^3*b^2*x^3 + 420
*a^2*b^3*x^2 + 315*a*b^4*x)*sqrt((a*x + b)/x))/(a^8*x^2 + 2*a^7*b*x + a^6*b
^2), 1/24*(315*(a^2*b^3*x^2 + 2*a*b^4*x + b^5)*sqrt(-a)*arctan(sqrt(-a)*sqr
t((a*x + b)/x)/a) + (8*a^5*x^5 - 18*a^4*b*x^4 + 63*a^3*b^2*x^3 + 420*a^2*b^
3*x^2 + 315*a*b^4*x)*sqrt((a*x + b)/x))/(a^8*x^2 + 2*a^7*b*x + a^6*b^2)]
```

giac [A] time = 0.18, size = 134, normalized size = 0.97

$$\frac{1}{24}\,b^3\left(\frac{315\,\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^5}+\frac{16\,a^4+\frac{144\,(ax+b)a^3}{x}-\frac{693\,(ax+b)^2a^2}{x^2}+\frac{840\,(ax+b)^3a}{x^3}-\frac{315\,(ax+b)^4}{x^4}}{\left(a\sqrt{\frac{ax+b}{x}}-\frac{(ax+b)\sqrt{\frac{ax+b}{x}}}{x}\right)^3a^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*b^3*(315*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^5) + (16*a^4 +
 144*(a*x + b)*a^3/x - 693*(a*x + b)^2*a^2/x^2 + 840*(a*x + b)^3*a/x^3 - 31
5*(a*x + b)^4/x^4)/((a*sqrt((a*x + b)/x) - (a*x + b)*sqrt((a*x + b)/x)/x)^3
*a^5))
```

maple [B] time = 0.02, size = 616, normalized size = 4.46

$$\frac{\sqrt{\frac{ax+b}{x}}\left(-336a^4b^3x^3\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)+21a^4b^3x^3\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)-84\sqrt{a\,x^2+bx}\;a^{\frac{11}{2}}b\,x^4-1008a^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b/x)^(5/2),x)
```

```
[Out] 1/48*((a*x+b)/x)^(1/2)*x/a^(13/2)*(16*(a*x^2+b*x)^(3/2)*a^(11/2)*x^3-84*(a*
x^2+b*x)^(1/2)*a^(11/2)*x^4*b+48*(a*x^2+b*x)^(3/2)*a^(9/2)*x^2*b-294*(a*x^2
+b*x)^(1/2)*a^(9/2)*x^3*b^2+672*a^(9/2)*((a*x+b)*x)^(1/2)*x^3*b^2-336*a^4*l
n(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^3*b^3+48*(a*x^2+b*x)
^(3/2)*a^(7/2)*x*b^2-378*(a*x^2+b*x)^(1/2)*a^(7/2)*x^2*b^3-384*a^(7/2)*((a*
x+b)*x)^(3/2)*x*b^2+2016*a^(7/2)*((a*x+b)*x)^(1/2)*x^2*b^3-1008*a^3*ln(1/2*
(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^2*b^4+21*ln(1/2*(2*a*x+b+2
*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x^3*a^4*b^3+16*(a*x^2+b*x)^(3/2)*a^(5/
2)*b^3-210*(a*x^2+b*x)^(1/2)*a^(5/2)*x*b^4-352*b^3*a^(5/2)*((a*x+b)*x)^(3/2
)+2016*a^(5/2)*((a*x+b)*x)^(1/2)*x*b^4-1008*a^2*ln(1/2*(2*a*x+b+2*((a*x+b)*
x)^(1/2)*a^(1/2))/a^(1/2))*x*b^5+63*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(
1/2))/a^(1/2))*x^2*a^3*b^4-42*(a*x^2+b*x)^(1/2)*a^(3/2)*b^5+672*a^(3/2)*((a
*x+b)*x)^(1/2)*b^5-336*a*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/
2))*b^6+63*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x*a^2*b^5+
21*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*a*b^6)/((a*x+b)*x)
^(1/2)/(a*x+b)^3
```

maxima [A] time = 2.33, size = 171, normalized size = 1.24

$$\frac{315\left(a+\frac{b}{x}\right)^4b^3-840\left(a+\frac{b}{x}\right)^3ab^3+693\left(a+\frac{b}{x}\right)^2a^2b^3-144\left(a+\frac{b}{x}\right)a^3b^3-16\,a^4b^3}{24\left(\left(a+\frac{b}{x}\right)^{\frac{9}{2}}a^5-3\left(a+\frac{b}{x}\right)^{\frac{7}{2}}a^6+3\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^7-\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^8\right)}+\frac{105\,b^3\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{16\,a^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/24*(315*(a + b/x)^4*b^3 - 840*(a + b/x)^3*a*b^3 + 693*(a + b/x)^2*a^2*b^3
 - 144*(a + b/x)*a^3*b^3 - 16*a^4*b^3)/((a + b/x)^(9/2)*a^5 - 3*(a + b/x)^(
7/2)*a^6 + 3*(a + b/x)^(5/2)*a^7 - (a + b/x)^(3/2)*a^8) + 105/16*b^3*log((s
qrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(11/2)
```

mupad [B] time = 1.79, size = 113, normalized size = 0.82

$$\frac{35\,b^3}{2\,a^4\left(a+\frac{b}{x}\right)^{3/2}}-\frac{105\,b^3\,\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{8\,a^{11/2}}+\frac{x^3}{3\,a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{3\,b\,x^2}{4\,a^2\left(a+\frac{b}{x}\right)^{3/2}}+\frac{21\,b^2\,x}{8\,a^3\left(a+\frac{b}{x}\right)^{3/2}}+\frac{105\,b^4}{8\,a^5\,x\left(a+\frac{b}{x}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b/x)^(5/2),x)
```

```
[Out] (35*b^3)/(2*a^4*(a + b/x)^(3/2)) - (105*b^3*atanh((a + b/x)^(1/2)/a^(1/2)))
/(8*a^(11/2)) + x^3/(3*a*(a + b/x)^(3/2)) - (3*b*x^2)/(4*a^2*(a + b/x)^(3/2
)) + (21*b^2*x)/(8*a^3*(a + b/x)^(3/2)) + (105*b^4)/(8*a^5*x*(a + b/x)^(3/2
))
```

sympy [B] time = 17.64, size = 532, normalized size = 3.86

$$\frac{8a^{\frac{133}{2}}b^{128}x^{72}}{24a^{\frac{137}{2}}b^{\frac{257}{2}}x^{\frac{137}{2}}\sqrt{\frac{ax}{b}+1}+24a^{\frac{135}{2}}b^{\frac{259}{2}}x^{\frac{135}{2}}\sqrt{\frac{ax}{b}+1}}-\frac{18a^{\frac{131}{2}}b^{129}x^{71}}{24a^{\frac{137}{2}}b^{\frac{257}{2}}x^{\frac{137}{2}}\sqrt{\frac{ax}{b}+1}+24a^{\frac{135}{2}}b^{\frac{259}{2}}x^{\frac{135}{2}}\sqrt{\frac{ax}{b}+1}}+\frac{}{24a^{\frac{137}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b/x)**(5/2),x)
```

```
[Out] 8*a**(133/2)*b**128*x**72/(24*a**(137/2)*b**(257/2)*x**(137/2)*sqrt(a*x/b +
 1) + 24*a**(135/2)*b**(259/2)*x**(135/2)*sqrt(a*x/b + 1)) - 18*a**(131/2)*
b**129*x**71/(24*a**(137/2)*b**(257/2)*x**(137/2)*sqrt(a*x/b + 1) + 24*a**(
135/2)*b**(259/2)*x**(135/2)*sqrt(a*x/b + 1)) + 63*a**(129/2)*b**130*x**70/
(24*a**(137/2)*b**(257/2)*x**(137/2)*sqrt(a*x/b + 1) + 24*a**(135/2)*b**(25
9/2)*x**(135/2)*sqrt(a*x/b + 1)) + 420*a**(127/2)*b**131*x**69/(24*a**(137/
2)*b**(257/2)*x**(137/2)*sqrt(a*x/b + 1) + 24*a**(135/2)*b**(259/2)*x**(135
/2)*sqrt(a*x/b + 1)) + 315*a**(125/2)*b**132*x**68/(24*a**(137/2)*b**(257/2
)*x**(137/2)*sqrt(a*x/b + 1) + 24*a**(135/2)*b**(259/2)*x**(135/2)*sqrt(a*x
/b + 1)) - 315*a**63*b**(263/2)*x**(137/2)*sqrt(a*x/b + 1)*asinh(sqrt(a)*sq
rt(x)/sqrt(b))/(24*a**(137/2)*b**(257/2)*x**(137/2)*sqrt(a*x/b + 1) + 24*a*
*(135/2)*b**(259/2)*x**(135/2)*sqrt(a*x/b + 1)) - 315*a**62*b**(265/2)*x**(
135/2)*sqrt(a*x/b + 1)*asinh(sqrt(a)*sqrt(x)/sqrt(b))/(24*a**(137/2)*b**(25
7/2)*x**(137/2)*sqrt(a*x/b + 1) + 24*a**(135/2)*b**(259/2)*x**(135/2)*sqrt(
a*x/b + 1))
```

3.1743 $\int \frac{x}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$

Optimal. Leaf size=114

$$\frac{35b^2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4a^{9/2}}-\frac{35b^2}{4a^4\sqrt{a+\frac{b}{x}}}-\frac{35b^2}{12a^3\left(a+\frac{b}{x}\right)^{3/2}}-\frac{7bx}{4a^2\left(a+\frac{b}{x}\right)^{3/2}}+\frac{x^2}{2a\left(a+\frac{b}{x}\right)^{3/2}}$$

```
[Out] -35/12*b^2/a^3/(a+b/x)^(3/2)-7/4*b*x/a^2/(a+b/x)^(3/2)+1/2*x^2/a/(a+b/x)^(3
/2)+35/4*b^2*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(9/2)-35/4*b^2/a^4/(a+b/x)^(1
/2)
```

Rubi [A] time = 0.05, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {266, 51, 63, 208}

$$\frac{35b^2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4a^{9/2}}+\frac{35x^2\sqrt{a+\frac{b}{x}}}{6a^3}-\frac{14x^2}{3a^2\sqrt{a+\frac{b}{x}}}-\frac{35bx\sqrt{a+\frac{b}{x}}}{4a^4}-\frac{2x^2}{3a\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b/x)^(5/2),x]
```

```
[Out] (-35*b*Sqrt[a + b/x]*x)/(4*a^4) - (2*x^2)/(3*a*(a + b/x)^(3/2)) - (14*x^2)/
(3*a^2*Sqrt[a + b/x]) + (35*Sqrt[a + b/x]*x^2)/(6*a^3) + (35*b^2*ArcTanh[Sq
rt[a + b/x]/Sqrt[a]])/(4*a^(9/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = -\text{Subst}\left(\int \frac{1}{x^3(a+bx)^{5/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2x^2}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{7\,\text{Subst}\left(\int \frac{1}{x^3(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3a}$$

$$= -\frac{2x^2}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{14x^2}{3a^2\sqrt{a+\frac{b}{x}}} - \frac{35\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{3a^2}$$

$$= -\frac{2x^2}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{14x^2}{3a^2\sqrt{a+\frac{b}{x}}} + \frac{35\sqrt{a+\frac{b}{x}}\,x^2}{6a^3} + \frac{(35b)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{4a^3}$$

$$= -\frac{35b\sqrt{a+\frac{b}{x}}\,x}{4a^4} - \frac{2x^2}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{14x^2}{3a^2\sqrt{a+\frac{b}{x}}} + \frac{35\sqrt{a+\frac{b}{x}}\,x^2}{6a^3} - \frac{\left(35b^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx\right.}{8a^4}$$

$$= -\frac{35b\sqrt{a+\frac{b}{x}}\,x}{4a^4} - \frac{2x^2}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{14x^2}{3a^2\sqrt{a+\frac{b}{x}}} + \frac{35\sqrt{a+\frac{b}{x}}\,x^2}{6a^3} - \frac{(35b)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x\right.}{4a^4}$$

$$= -\frac{35b\sqrt{a+\frac{b}{x}}\,x}{4a^4} - \frac{2x^2}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{14x^2}{3a^2\sqrt{a+\frac{b}{x}}} + \frac{35\sqrt{a+\frac{b}{x}}\,x^2}{6a^3} + \frac{35b^2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4a^{9/2}}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.34

$$-\frac{2b^2\,{}_2F_1\left(-\frac{3}{2}, 3; -\frac{1}{2}; \frac{b}{ax}+1\right)}{3a^3\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b/x)^(5/2),x]
```

```
[Out] (-2*b^2*Hypergeometric2F1[-3/2, 3, -1/2, 1 + b/(a*x)])/(3*a^3*(a + b/x)^(3/
2))
```

fricas [A] time = 1.00, size = 252, normalized size = 2.21

$$\left[\frac{105\left(a^2b^2x^2+2\,ab^3x+b^4\right)\sqrt{a}\,\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(6\,a^4x^4-21\,a^3bx^3-140\,a^2b^2x^2-105\,ab^3x\right)}{24\left(a^7x^2+2\,a^6bx+a^5b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(105*(a^2*b^2*x^2 + 2*a*b^3*x + b^4)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*
sqrt((a*x + b)/x) + b) + 2*(6*a^4*x^4 - 21*a^3*b*x^3 - 140*a^2*b^2*x^2 - 10
5*a*b^3*x)*sqrt((a*x + b)/x))/(a^7*x^2 + 2*a^6*b*x + a^5*b^2), -1/12*(105*(
a^2*b^2*x^2 + 2*a*b^3*x + b^4)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a
) - (6*a^4*x^4 - 21*a^3*b*x^3 - 140*a^2*b^2*x^2 - 105*a*b^3*x)*sqrt((a*x +
b)/x))/(a^7*x^2 + 2*a^6*b*x + a^5*b^2)]
```

giac [A] time = 0.19, size = 125, normalized size = 1.10

$$-\frac{1}{12}\,b^2\left(\frac{8\left(a+\frac{9\,(ax+b)}{x}\right)x}{(ax+b)a^4\sqrt{\frac{ax+b}{x}}}+\frac{105\,\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^4}-\frac{3\left(13\,a\sqrt{\frac{ax+b}{x}}-\frac{11\,(ax+b)\sqrt{\frac{ax+b}{x}}}{x}\right)}{\left(a-\frac{ax+b}{x}\right)^2a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/12*b^2*(8*(a + 9*(a*x + b)/x)*x/((a*x + b)*a^4*sqrt((a*x + b)/x)) + 105*
arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^4) - 3*(13*a*sqrt((a*x + b)/
x) - 11*(a*x + b)*sqrt((a*x + b)/x)/x)/((a - (a*x + b)/x)^2*a^4))
```

maple [B] time = 0.01, size = 531, normalized size = 4.66

$$\frac{\sqrt{\frac{ax+b}{x}}\left(108a^4b^2x^3\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)-3a^4b^2x^3\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)+12\sqrt{a\,x^2+bx}\;a^{\frac{11}{2}}x^4+324a^3b^3x^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b/x)^(5/2),x)
```

```
[Out] 1/24*((a*x+b)/x)^(1/2)*x/a^(11/2)*(12*(a*x^2+b*x)^(1/2)*a^(11/2)*x^4+42*(a*
x^2+b*x)^(1/2)*a^(9/2)*b*x^3-216*a^(9/2)*((a*x+b)*x)^(1/2)*x^3*b+108*a^4*ln
(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^3*b^2+54*(a*x^2+b*x)^
(1/2)*a^(7/2)*b^2*x^2+144*a^(7/2)*((a*x+b)*x)^(3/2)*x*b-648*((a*x+b)*x)^(1/
2)*a^(7/2)*b^2*x^2+324*a^3*b^3*x^2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1
/2))/a^(1/2))-3*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x^3*a
^4*b^2+30*(a*x^2+b*x)^(1/2)*a^(5/2)*b^3*x+128*((a*x+b)*x)^(3/2)*a^(5/2)*b^2
-648*((a*x+b)*x)^(1/2)*a^(5/2)*b^3*x+324*a^2*b^4*x*ln(1/2*(2*a*x+b+2*((a*x+
b)*x)^(1/2)*a^(1/2))/a^(1/2))-9*a^3*b^3*x^2*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(
1/2)*a^(1/2))/a^(1/2))+6*(a*x^2+b*x)^(1/2)*a^(3/2)*b^4-216*((a*x+b)*x)^(1/2
)*a^(3/2)*b^4+108*a*b^5*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2
))-9*a^2*b^4*x*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))-3*a*b^
5*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2)))/((a*x+b)*x)^(1/2)/
(a*x+b)^3
```

maxima [A] time = 2.54, size = 139, normalized size = 1.22

$$-\frac{105\left(a+\frac{b}{x}\right)^3b^2-175\left(a+\frac{b}{x}\right)^2ab^2+56\left(a+\frac{b}{x}\right)a^2b^2+8\,a^3b^2}{12\left(\left(a+\frac{b}{x}\right)^{\frac{7}{2}}a^4-2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^5+\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^6\right)}-\frac{35\,b^2\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{8\,a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/12*(105*(a + b/x)^3*b^2 - 175*(a + b/x)^2*a*b^2 + 56*(a + b/x)*a^2*b^2 +
 8*a^3*b^2)/((a + b/x)^(7/2)*a^4 - 2*(a + b/x)^(5/2)*a^5 + (a + b/x)^(3/2)*
a^6) - 35/8*b^2*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^
(9/2)
```

mupad **[B]** time = 1.54, size = 93, normalized size = 0.82

$$\frac{35\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4\,a^{9/2}}-\frac{35\,b^2}{3\,a^3\left(a+\frac{b}{x}\right)^{3/2}}+\frac{x^2}{2\,a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{35\,b^3}{4\,a^4\,x\left(a+\frac{b}{x}\right)^{3/2}}-\frac{7\,b\,x}{4\,a^2\left(a+\frac{b}{x}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b/x)^(5/2),x)
```

```
[Out] (35*b^2*atanh((a + b/x)^(1/2)/a^(1/2)))/(4*a^(9/2)) - (35*b^2)/(3*a^3*(a +
b/x)^(3/2)) + x^2/(2*a*(a + b/x)^(3/2)) - (35*b^3)/(4*a^4*x*(a + b/x)^(3/2)
) - (7*b*x)/(4*a^2*(a + b/x)^(3/2))
```

sympy **[B]** time = 10.57, size = 464, normalized size = 4.07

$$\frac{6a^{\frac{89}{2}}b^{75}x^{49}}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{93}{2}}\sqrt{\frac{ax}{b}+1}+12a^{\frac{91}{2}}b^{\frac{153}{2}}x^{\frac{91}{2}}\sqrt{\frac{ax}{b}+1}}-\frac{21a^{\frac{87}{2}}b^{76}x^{48}}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{93}{2}}\sqrt{\frac{ax}{b}+1}+12a^{\frac{91}{2}}b^{\frac{153}{2}}x^{\frac{91}{2}}\sqrt{\frac{ax}{b}+1}}-\frac{}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{93}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x)**(5/2),x)
```

```
[Out] 6*a**(89/2)*b**75*x**49/(12*a**(93/2)*b**(151/2)*x**(93/2)*sqrt(a*x/b + 1)
+ 12*a**(91/2)*b**(153/2)*x**(91/2)*sqrt(a*x/b + 1)) - 21*a**(87/2)*b**76*x
**48/(12*a**(93/2)*b**(151/2)*x**(93/2)*sqrt(a*x/b + 1) + 12*a**(91/2)*b**(
153/2)*x**(91/2)*sqrt(a*x/b + 1)) - 140*a**(85/2)*b**77*x**47/(12*a**(93/2)
*b**(151/2)*x**(93/2)*sqrt(a*x/b + 1) + 12*a**(91/2)*b**(153/2)*x**(91/2)*s
qrt(a*x/b + 1)) - 105*a**(83/2)*b**78*x**46/(12*a**(93/2)*b**(151/2)*x**(93
/2)*sqrt(a*x/b + 1) + 12*a**(91/2)*b**(153/2)*x**(91/2)*sqrt(a*x/b + 1)) +
105*a**42*b**(155/2)*x**(93/2)*sqrt(a*x/b + 1)*asinh(sqrt(a)*sqrt(x)/sqrt(b
))/(12*a**(93/2)*b**(151/2)*x**(93/2)*sqrt(a*x/b + 1) + 12*a**(91/2)*b**(15
3/2)*x**(91/2)*sqrt(a*x/b + 1)) + 105*a**41*b**(157/2)*x**(91/2)*sqrt(a*x/b
 + 1)*asinh(sqrt(a)*sqrt(x)/sqrt(b))/(12*a**(93/2)*b**(151/2)*x**(93/2)*sqr
t(a*x/b + 1) + 12*a**(91/2)*b**(153/2)*x**(91/2)*sqrt(a*x/b + 1))
```

3.1744 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$

Optimal. Leaf size=79

$$-\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{5b}{a^3\sqrt{a+\frac{b}{x}}}+\frac{5b}{3a^2\left(a+\frac{b}{x}\right)^{3/2}}+\frac{x}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

```
[Out] 5/3*b/a^2/(a+b/x)^(3/2)+x/a/(a+b/x)^(3/2)-5*b*arctanh((a+b/x)^(1/2)/a^(1/2)
)/a^(7/2)+5*b/a^3/(a+b/x)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {242, 51, 63, 208}

$$\frac{5x\sqrt{a+\frac{b}{x}}}{a^3}-\frac{10x}{3a^2\sqrt{a+\frac{b}{x}}}-\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}-\frac{2x}{3a\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(-5/2),x]
```

```
[Out] (-2*x)/(3*a*(a + b/x)^(3/2)) - (10*x)/(3*a^2*Sqrt[a + b/x]) + (5*Sqrt[a + b
/x]*x)/a^3 - (5*b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = -\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)^{5/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2x}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{5\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3a}$$

$$= -\frac{2x}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{10x}{3a^2\sqrt{a+\frac{b}{x}}} - \frac{5\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{a^2}$$

$$= -\frac{2x}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{10x}{3a^2\sqrt{a+\frac{b}{x}}} + \frac{5\sqrt{a+\frac{b}{x}}\,x}{a^3} + \frac{(5b)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a^3}$$

$$= -\frac{2x}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{10x}{3a^2\sqrt{a+\frac{b}{x}}} + \frac{5\sqrt{a+\frac{b}{x}}\,x}{a^3} + \frac{5\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{a^3}$$

$$= -\frac{2x}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{10x}{3a^2\sqrt{a+\frac{b}{x}}} + \frac{5\sqrt{a+\frac{b}{x}}\,x}{a^3} - \frac{5b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

Mathematica [C] time = 0.02, size = 38, normalized size = 0.48

$$\frac{2b\,{}_2F_1\left(-\frac{3}{2}, 2; -\frac{1}{2}; \frac{a+\frac{b}{x}}{a}\right)}{3a^2\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(-5/2),x]
```

```
[Out] (2*b*Hypergeometric2F1[-3/2, 2, -1/2, (a + b/x)/a])/(3*a^2*(a + b/x)^(3/2))
```

fricas [A] time = 0.97, size = 225, normalized size = 2.85

$$\left[\frac{15\left(a^2bx^2+2\,ab^2x+b^3\right)\sqrt{a}\,\log\left(2\,ax-2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(3\,a^3x^3+20\,a^2bx^2+15\,ab^2x\right)\sqrt{\frac{ax+b}{x}}}{6\left(a^6x^2+2\,a^5bx+a^4b^2\right)}, \frac{15\left(a^2bx\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*(a^2*b*x^2 + 2*a*b^2*x + b^3)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt
((a*x + b)/x) + b) + 2*(3*a^3*x^3 + 20*a^2*b*x^2 + 15*a*b^2*x)*sqrt((a*x +
b)/x))/(a^6*x^2 + 2*a^5*b*x + a^4*b^2), 1/3*(15*(a^2*b*x^2 + 2*a*b^2*x + b^
3)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (3*a^3*x^3 + 20*a^2*b*x^
2 + 15*a*b^2*x)*sqrt((a*x + b)/x))/(a^6*x^2 + 2*a^5*b*x + a^4*b^2)]
```

giac [A] time = 0.17, size = 98, normalized size = 1.24

$$\frac{1}{3}b\left(\frac{2\left(a+\frac{6(ax+b)}{x}\right)x}{(ax+b)a^3\sqrt{\frac{ax+b}{x}}}+\frac{15\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^3}-\frac{3\sqrt{\frac{ax+b}{x}}}{\left(a-\frac{ax+b}{x}\right)a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*b*(2*(a + 6*(a*x + b)/x)*x/((a*x + b)*a^3*sqrt((a*x + b)/x)) + 15*arcta
n(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^3) - 3*sqrt((a*x + b)/x)/((a - (a
*x + b)/x)*a^3))
```

maple [B] time = 0.01, size = 271, normalized size = 3.43

$$-\frac{\sqrt{\frac{ax+b}{x}}\left(15a^3bx^3\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)+45a^2b^2x^2\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)-30\sqrt{(ax+b)x}\,a^{\frac{7}{2}}x^3+45ab^3x\ln\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(5/2),x)
```

```
[Out] -1/6*((a*x+b)/x)^(1/2)*x/a^(7/2)*(-30*((a*x+b)*x)^(1/2)*a^(7/2)*x^3+15*ln(1
/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^3*a^3*b+24*((a*x+b)*x)^
(3/2)*a^(5/2)*x-90*((a*x+b)*x)^(1/2)*a^(5/2)*x^2*b+45*ln(1/2*(2*a*x+b+2*((a
*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^2*a^2*b^2+20*b*a^(3/2)*((a*x+b)*x)^(3/2)
-90*((a*x+b)*x)^(1/2)*a^(3/2)*x*b^2+45*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*
a^(1/2))/a^(1/2))*x*a*b^3-30*((a*x+b)*x)^(1/2)*a^(1/2)*b^3+15*ln(1/2*(2*a*x
+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*b^4)/((a*x+b)*x)^(1/2)/(a*x+b)^3
```

maxima [A] time = 2.32, size = 101, normalized size = 1.28

$$\frac{15\left(a+\frac{b}{x}\right)^2b-10\left(a+\frac{b}{x}\right)ab-2a^2b}{3\left(\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^3-\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^4\right)}+\frac{5b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{2a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(15*(a + b/x)^2*b - 10*(a + b/x)*a*b - 2*a^2*b)/((a + b/x)^(5/2)*a^3 -
(a + b/x)^(3/2)*a^4) + 5/2*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) +
 sqrt(a)))/a^(7/2)
```

mupad [B] time = 1.44, size = 34, normalized size = 0.43

$$\frac{2x\left(\frac{ax}{b}+1\right)^{5/2}{}_2\mathrm{F}_1\left(\frac{5}{2},\frac{7}{2};\ \frac{9}{2};\ -\frac{ax}{b}\right)}{7\left(a+\frac{b}{x}\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x)^(5/2),x)
```

```
[Out] (2*x*((a*x)/b + 1)^(5/2)*hypergeom([5/2, 7/2], 9/2, -(a*x)/b))/(7*(a + b/x)
^(5/2))
```

sympy [B] time = 6.57, size = 774, normalized size = 9.80

$$\frac{6a^{17}x^4\sqrt{1+\frac{b}{ax}}}{6a^{\frac{39}{2}}x^3+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x+6a^{\frac{33}{2}}b^3}+\frac{46a^{16}bx^3\sqrt{1+\frac{b}{ax}}}{6a^{\frac{39}{2}}x^3+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x+6a^{\frac{33}{2}}b^3}+\frac{15a^{16}bx^3\log\left($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(5/2),x)
```

```
[Out] 6*a**17*x**4*sqrt(1 + b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18
*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 46*a**16*b*x**3*sqrt(1 + b/(a*x))/(
6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*
b**3) + 15*a**16*b*x**3*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**
2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) - 30*a**16*b*x**3*log(sqrt(1 +
b/(a*x)) + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x
 + 6*a**(33/2)*b**3) + 70*a**15*b**2*x**2*sqrt(1 + b/(a*x))/(6*a**(39/2)*x*
*3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 45*a**
15*b**2*x**2*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(
35/2)*b**2*x + 6*a**(33/2)*b**3) - 90*a**15*b**2*x**2*log(sqrt(1 + b/(a*x))
 + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**
(33/2)*b**3) + 30*a**14*b**3*x*sqrt(1 + b/(a*x))/(6*a**(39/2)*x**3 + 18*a**
(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 45*a**14*b**3*x*l
og(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x +
 6*a**(33/2)*b**3) - 90*a**14*b**3*x*log(sqrt(1 + b/(a*x)) + 1)/(6*a**(39/2
)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 15
*a**13*b**4*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(3
5/2)*b**2*x + 6*a**(33/2)*b**3) - 30*a**13*b**4*log(sqrt(1 + b/(a*x)) + 1)/
(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)
*b**3)
```

3.1745 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x}\,dx$

Optimal. Leaf size=60

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{2}{a^2\sqrt{a+\frac{b}{x}}} - \frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2}}$$

```
[Out] -2/3/a/(a+b/x)^(3/2)+2*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(5/2)-2/a^2/(a+b/x)
^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 51, 63, 208}

$$-\frac{2}{a^2\sqrt{a+\frac{b}{x}}} + \frac{2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(5/2)*x),x]
```

```
[Out] -2/(3*a*(a + b/x)^(3/2)) - 2/(a^2*Sqrt[a + b/x]) + (2*ArcTanh[Sqrt[a + b/x]
/Sqrt[a]])/a^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x}\,dx = -\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{2}{a^2\sqrt{a+\frac{b}{x}}} - \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{a^2}$$

$$= -\frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{2}{a^2\sqrt{a+\frac{b}{x}}} - \frac{2\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{a^2 b}$$

$$= -\frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{2}{a^2\sqrt{a+\frac{b}{x}}} + \frac{2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

Mathematica [C] time = 0.03, size = 36, normalized size = 0.60

$$-\frac{2\,{}_2F_1\left(-\frac{3}{2}, 1; -\frac{1}{2}; \frac{b}{ax}+1\right)}{3a\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(5/2)*x),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/2, 1, -1/2, 1 + b/(a*x)])/(3*a*(a + b/x)^(3/2))
```

fricas [B] time = 1.04, size = 197, normalized size = 3.28

$$\left[\frac{3\left(a^2x^2+2\,abx+b^2\right)\sqrt{a}\,\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)-2\left(4\,a^2x^2+3\,abx\right)\sqrt{\frac{ax+b}{x}}}{3\left(a^5x^2+2\,a^4bx+a^3b^2\right)}, -\frac{2\left(3\left(a^2x^2+2\,abx+b^2\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [1/3*(3*(a^2*x^2 + 2*a*b*x + b^2)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x
 + b)/x) + b) - 2*(4*a^2*x^2 + 3*a*b*x)*sqrt((a*x + b)/x))/(a^5*x^2 + 2*a^4
*b*x + a^3*b^2), -2/3*(3*(a^2*x^2 + 2*a*b*x + b^2)*sqrt(-a)*arctan(sqrt(-a)
*sqrt((a*x + b)/x)/a) + (4*a^2*x^2 + 3*a*b*x)*sqrt((a*x + b)/x))/(a^5*x^2 +
 2*a^4*b*x + a^3*b^2)]
```

giac [A] time = 0.17, size = 65, normalized size = 1.08

$$-\frac{2\left(a+\frac{3\,(ax+b)}{x}\right)x}{3\,(ax+b)a^2\sqrt{\frac{ax+b}{x}}} - \frac{2\,\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/x,x, algorithm="giac")
```

```
[Out] -2/3*(a + 3*(a*x + b)/x)*x/((a*x + b)*a^2*sqrt((a*x + b)/x)) - 2*arctan(sqr
t((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^2)
```

maple [B] time = 0.01, size = 274, normalized size = 4.57

$$\frac{\sqrt{\frac{ax+b}{x}}\left(3a^3bx^3\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)+9a^2b^2x^2\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)-6\sqrt{(ax+b)\,x}\;a^{\frac{7}{2}}x^3+9ab^3x\ln\left(\frac{2ax+b}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(5/2)/x,x)
```

```
[Out] 1/3*((a*x+b)/x)^(1/2)*x/a^(5/2)*(3*a^3*b*x^3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^
(1/2)*a^(1/2))/a^(1/2))-6*((a*x+b)*x)^(1/2)*a^(7/2)*x^3+9*a^2*b^2*x^2*ln(1/
2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))+6*((a*x+b)*x)^(3/2)*a^(5/2
)*x-18*((a*x+b)*x)^(1/2)*a^(5/2)*b*x^2+9*a*b^3*x*ln(1/2*(2*a*x+b+2*((a*x+b)
*x)^(1/2)*a^(1/2))/a^(1/2))+4*((a*x+b)*x)^(3/2)*a^(3/2)*b-18*((a*x+b)*x)^(1
/2)*a^(3/2)*b^2*x+3*b^4*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2
))-6*((a*x+b)*x)^(1/2)*a^(1/2)*b^3)/((a*x+b)*x)^(1/2)/(a*x+b)^3/b
```

maxima [A] time = 2.21, size = 62, normalized size = 1.03

$$-\frac{\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}-\frac{2\left(4a+\frac{3b}{x}\right)}{3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] -log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(5/2) - 2/3*(4*
a + 3*b/x)/((a + b/x)^(3/2)*a^2)
```

mupad [B] time = 1.58, size = 49, normalized size = 0.82

$$\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{\frac{2\left(a+\frac{b}{x}\right)}{a^2}+\frac{2}{3a}}{\left(a+\frac{b}{x}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b/x)^(5/2)),x)
```

```
[Out] (2*atanh((a + b/x)^(1/2)/a^(1/2)))/a^(5/2) - ((2*(a + b/x))/a^2 + 2/(3*a))/
(a + b/x)^(3/2)
```

sympy [B] time = 3.56, size = 700, normalized size = 11.67

$$-\frac{8a^7x^3\sqrt{1+\frac{b}{ax}}}{3a^{\frac{19}{2}}x^3+9a^{\frac{17}{2}}bx^2+9a^{\frac{15}{2}}b^2x+3a^{\frac{13}{2}}b^3}-\frac{3a^7x^3\log\left(\frac{b}{ax}\right)}{3a^{\frac{19}{2}}x^3+9a^{\frac{17}{2}}bx^2+9a^{\frac{15}{2}}b^2x+3a^{\frac{13}{2}}b^3}+\frac{6a^7x^3\log\left(\sqrt{1+\frac{b}{ax}}+1\right)}{3a^{\frac{19}{2}}x^3+9a^{\frac{17}{2}}bx^2+9a^{\frac{15}{2}}b^2x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(5/2)/x,x)
```

```
[Out] -8*a**7*x**3*sqrt(1 + b/(a*x))/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a
**(15/2)*b**2*x + 3*a**(13/2)*b**3) - 3*a**7*x**3*log(b/(a*x))/(3*a**(19/2)
*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3) + 6*a**
7*x**3*log(sqrt(1 + b/(a*x)) + 1)/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 +
9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3) - 14*a**6*b*x**2*sqrt(1 + b/(a*x))/(
3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b*
*3) - 9*a**6*b*x**2*log(b/(a*x))/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9
*a**(15/2)*b**2*x + 3*a**(13/2)*b**3) + 18*a**6*b*x**2*log(sqrt(1 + b/(a*x)
) + 1)/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(
13/2)*b**3) - 6*a**5*b**2*x*sqrt(1 + b/(a*x))/(3*a**(19/2)*x**3 + 9*a**(17/
2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3) - 9*a**5*b**2*x*log(b/(a
*x))/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13
/2)*b**3) + 18*a**5*b**2*x*log(sqrt(1 + b/(a*x)) + 1)/(3*a**(19/2)*x**3 + 9
*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3) - 3*a**4*b**3*lo
g(b/(a*x))/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*
a**(13/2)*b**3) + 6*a**4*b**3*log(sqrt(1 + b/(a*x)) + 1)/(3*a**(19/2)*x**3
+ 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3)
```

3.1746 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^2}\,dx$

Optimal. Leaf size=18

$$\frac{2}{3b\left(a+\frac{b}{x}\right)^{3/2}}$$

```
[Out] 2/3/b/(a+b/x)^(3/2)
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$\frac{2}{3b\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(5/2)*x^2),x]
```

```
[Out] 2/(3*b*(a + b/x)^(3/2))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^2}\,dx = \frac{2}{3b\left(a+\frac{b}{x}\right)^{3/2}}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{2}{3b\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(5/2)*x^2),x]
```

```
[Out] 2/(3*b*(a + b/x)^(3/2))
```

fricas [B] time = 1.15, size = 37, normalized size = 2.06

$$\frac{2\,x^2\sqrt{\frac{ax+b}{x}}}{3\left(a^2bx^2+2\,ab^2x+b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] 2/3*x^2*sqrt((a*x + b)/x)/(a^2*b*x^2 + 2*a*b^2*x + b^3)
```

giac [A] time = 0.16, size = 24, normalized size = 1.33

$$\frac{2\,x}{3\,(ax+b)b\sqrt{\frac{ax+b}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] 2/3*x/((a*x + b)*b*sqrt((a*x + b)/x))
```

maple [A] time = 0.00, size = 25, normalized size = 1.39

$$\frac{\frac{2ax}{3}+\frac{2b}{3}}{\left(\frac{ax+b}{x}\right)^{\frac{5}{2}}bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(5/2)/x^2,x)
```

```
[Out] 2/3*(a*x+b)/x/b/((a*x+b)/x)^(5/2)
```

maxima [A] time = 1.04, size = 14, normalized size = 0.78

$$\frac{2}{3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] 2/3/((a + b/x)^(3/2)*b)
```

mupad [B] time = 1.40, size = 14, normalized size = 0.78

$$\frac{2}{3\,b\left(a+\frac{b}{x}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b/x)^(5/2)),x)
```

```
[Out] 2/(3*b*(a + b/x)^(3/2))
```

sympy [A] time = 2.88, size = 39, normalized size = 2.17

$$\begin{cases} \frac{2}{3ab\sqrt{a+\frac{b}{x}}+\frac{3b^2\sqrt{a+\frac{b}{x}}}{x}} & \text{for } b \neq 0 \\ -\frac{1}{a^{\frac{5}{2}}x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(5/2)/x**2,x)
```

```
[Out] Piecewise((2/(3*a*b*sqrt(a + b/x) + 3*b**2*sqrt(a + b/x)/x), Ne(b, 0)), (-1
/(a**(5/2)*x), True))
```

3.1747 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^3}\,dx$

Optimal. Leaf size=36

$$\frac{2}{b^2\sqrt{a+\frac{b}{x}}}-\frac{2a}{3b^2\left(a+\frac{b}{x}\right)^{3/2}}$$

```
[Out] -2/3*a/b^2/(a+b/x)^(3/2)+2/b^2/(a+b/x)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{2}{b^2\sqrt{a+\frac{b}{x}}}-\frac{2a}{3b^2\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(5/2)*x^3),x]
```

```
[Out] (-2*a)/(3*b^2*(a + b/x)^(3/2)) + 2/(b^2*Sqrt[a + b/x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^3}\,dx &= -\text{Subst}\left(\int \frac{x}{(a+bx)^{5/2}}\,dx, x, \frac{1}{x}\right)\\ &= -\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^{5/2}}+\frac{1}{b(a+bx)^{3/2}}\right)dx, x, \frac{1}{x}\right)\\ &= -\frac{2a}{3b^2\left(a+\frac{b}{x}\right)^{3/2}}+\frac{2}{b^2\sqrt{a+\frac{b}{x}}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 33, normalized size = 0.92

$$\frac{4ax+6b}{3b^2\sqrt{a+\frac{b}{x}}\,(ax+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(5/2)*x^3),x]
```

```
[Out] (6*b + 4*a*x)/(3*b^2*Sqrt[a + b/x]*(b + a*x))
```

fricas [A] time = 1.21, size = 47, normalized size = 1.31

$$\frac{2\left(2\,ax^2+3\,bx\right)\sqrt{\frac{ax+b}{x}}}{3\left(a^2b^2x^2+2\,ab^3x+b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] 2/3*(2*a*x^2 + 3*b*x)*sqrt((a*x + b)/x)/(a^2*b^2*x^2 + 2*a*b^3*x + b^4)
```

giac [A] time = 0.16, size = 36, normalized size = 1.00

$$-\frac{2\left(a-\frac{3\,(ax+b)}{x}\right)x}{3\,(ax+b)b^2\sqrt{\frac{ax+b}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] -2/3*(a - 3*(a*x + b)/x)*x/((a*x + b)*b^2*sqrt((a*x + b)/x))
```

maple [A] time = 0.00, size = 33, normalized size = 0.92

$$\frac{2\,(ax+b)\,(2ax+3b)}{3\left(\frac{ax+b}{x}\right)^{\frac{5}{2}}b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(5/2)/x^3,x)
```

```
[Out] 2/3*(a*x+b)*(2*a*x+3*b)/x^2/b^2/((a*x+b)/x)^(5/2)
```

maxima [A] time = 1.09, size = 30, normalized size = 0.83

$$\frac{2}{\sqrt{a+\frac{b}{x}}\,b^2}-\frac{2\,a}{3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] 2/(sqrt(a + b/x)*b^2) - 2/3*a/((a + b/x)^(3/2)*b^2)
```

mupad [B] time = 1.30, size = 25, normalized size = 0.69

$$\frac{6\,b+4\,a\,x}{3\,b^2\,x\left(a+\frac{b}{x}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b/x)^(5/2)),x)
```

```
[Out] (6*b + 4*a*x)/(3*b^2*x*(a + b/x)^(3/2))
```

sympy [**A**] time = 4.04, size = 82, normalized size = 2.28

$$\begin{cases} \frac{4ax}{3ab^2x\sqrt{a+\frac{b}{x}}+3b^3\sqrt{a+\frac{b}{x}}} + \frac{6b}{3ab^2x\sqrt{a+\frac{b}{x}}+3b^3\sqrt{a+\frac{b}{x}}} & \text{for } b \neq 0 \\ -\frac{1}{2a^{\frac{5}{2}}x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(5/2)/x**3,x)
```

```
[Out] Piecewise((4*a*x/(3*a*b**2*x*sqrt(a + b/x) + 3*b**3*sqrt(a + b/x)) + 6*b/(3
*a*b**2*x*sqrt(a + b/x) + 3*b**3*sqrt(a + b/x)), Ne(b, 0)), (-1/(2*a**(5/2)
*x**2), True))
```

3.1748 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^4}\,dx$

Optimal. Leaf size=55

$$\frac{2a^2}{3b^3\left(a+\frac{b}{x}\right)^{3/2}}-\frac{4a}{b^3\sqrt{a+\frac{b}{x}}}-\frac{2\sqrt{a+\frac{b}{x}}}{b^3}$$

```
[Out] 2/3*a^2/b^3/(a+b/x)^(3/2)-4*a/b^3/(a+b/x)^(1/2)-2*(a+b/x)^(1/2)/b^3
```

Rubi [A] time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{2a^2}{3b^3\left(a+\frac{b}{x}\right)^{3/2}}-\frac{4a}{b^3\sqrt{a+\frac{b}{x}}}-\frac{2\sqrt{a+\frac{b}{x}}}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(5/2)*x^4),x]
```

```
[Out] (2*a^2)/(3*b^3*(a + b/x)^(3/2)) - (4*a)/(b^3*Sqrt[a + b/x]) - (2*Sqrt[a + b
/x])/b^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^4}\,dx &= -\text{Subst}\left(\int \frac{x^2}{(a+bx)^{5/2}}\,dx, x, \frac{1}{x}\right)\\ &= -\text{Subst}\left(\int \left(\frac{a^2}{b^2(a+bx)^{5/2}}-\frac{2a}{b^2(a+bx)^{3/2}}+\frac{1}{b^2\sqrt{a+bx}}\right)dx, x, \frac{1}{x}\right)\\ &= \frac{2a^2}{3b^3\left(a+\frac{b}{x}\right)^{3/2}}-\frac{4a}{b^3\sqrt{a+\frac{b}{x}}}-\frac{2\sqrt{a+\frac{b}{x}}}{b^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 44, normalized size = 0.80

$$-\frac{2\sqrt{a+\frac{b}{x}}\left(8a^2x^2+12abx+3b^2\right)}{3b^3(ax+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(5/2)*x^4),x]
```

```
[Out] (-2*Sqrt[a + b/x]*(3*b^2 + 12*a*b*x + 8*a^2*x^2))/(3*b^3*(b + a*x)^2)
```

fricas [A] time = 0.94, size = 55, normalized size = 1.00

$$-\frac{2\left(8\,a^2x^2+12\,abx+3\,b^2\right)\sqrt{\frac{ax+b}{x}}}{3\left(a^2b^3x^2+2\,ab^4x+b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] -2/3*(8*a^2*x^2 + 12*a*b*x + 3*b^2)*sqrt((a*x + b)/x)/(a^2*b^3*x^2 + 2*a*b^
4*x + b^5)
```

giac [A] time = 0.17, size = 54, normalized size = 0.98

$$\frac{2\left(\frac{\left(a^2-\frac{6\,(ax+b)a}{x}\right)x}{(ax+b)\sqrt{\frac{ax+b}{x}}}-3\sqrt{\frac{ax+b}{x}}\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] 2/3*((a^2 - 6*(a*x + b)*a/x)*x/((a*x + b)*sqrt((a*x + b)/x)) - 3*sqrt((a*x
+ b)/x))/b^3
```

maple [A] time = 0.01, size = 44, normalized size = 0.80

$$-\frac{2\,(ax+b)\left(8a^2x^2+12abx+3b^2\right)}{3\left(\frac{ax+b}{x}\right)^{\frac{5}{2}}b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(5/2)/x^4,x)
```

```
[Out] -2/3*(a*x+b)*(8*a^2*x^2+12*a*b*x+3*b^2)/x^3/b^3/((a*x+b)/x)^(5/2)
```

maxima [A] time = 1.00, size = 47, normalized size = 0.85

$$-\frac{2\sqrt{a+\frac{b}{x}}}{b^3}-\frac{4\,a}{\sqrt{a+\frac{b}{x}}\,b^3}+\frac{2\,a^2}{3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] -2*sqrt(a + b/x)/b^3 - 4*a/(sqrt(a + b/x)*b^3) + 2/3*a^2/((a + b/x)^(3/2)*b
^3)
```

mupad [B] time = 1.35, size = 40, normalized size = 0.73

$$-\frac{2\sqrt{a+\frac{b}{x}}\,\left(8\,a^2\,x^2+12\,a\,b\,x+3\,b^2\right)}{3\,b^3\,(b+a\,x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b/x)^(5/2)),x)
```

```
[Out] -(2*(a + b/x)^(1/2)*(3*b^2 + 8*a^2*x^2 + 12*a*b*x))/(3*b^3*(b + a*x)^2)
```

sympy [**A**] time = 4.27, size = 136, normalized size = 2.47

$$\begin{cases} -\frac{16a^2x^2}{3ab^3x^2\sqrt{a+\frac{b}{x}}+3b^4x\sqrt{a+\frac{b}{x}}} - \frac{24abx}{3ab^3x^2\sqrt{a+\frac{b}{x}}+3b^4x\sqrt{a+\frac{b}{x}}} - \frac{6b^2}{3ab^3x^2\sqrt{a+\frac{b}{x}}+3b^4x\sqrt{a+\frac{b}{x}}} & \text{for } b \neq 0 \\ -\frac{1}{3a^{\frac{5}{2}}x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(5/2)/x**4,x)
```

```
[Out] Piecewise((-16*a**2*x**2/(3*a*b**3*x**2*sqrt(a + b/x) + 3*b**4*x*sqrt(a + b
/x)) - 24*a*b*x/(3*a*b**3*x**2*sqrt(a + b/x) + 3*b**4*x*sqrt(a + b/x)) - 6*
b**2/(3*a*b**3*x**2*sqrt(a + b/x) + 3*b**4*x*sqrt(a + b/x)), Ne(b, 0)), (-1
/(3*a**(5/2)*x**3), True))
```

3.1749 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^5}\,dx$

Optimal. Leaf size=76

$$-\frac{2a^3}{3b^4\left(a+\frac{b}{x}\right)^{3/2}}+\frac{6a^2}{b^4\sqrt{a+\frac{b}{x}}}+\frac{6a\sqrt{a+\frac{b}{x}}}{b^4}-\frac{2\left(a+\frac{b}{x}\right)^{3/2}}{3b^4}$$

```
[Out] -2/3*a^3/b^4/(a+b/x)^(3/2)-2/3*(a+b/x)^(3/2)/b^4+6*a^2/b^4/(a+b/x)^(1/2)+6*
a*(a+b/x)^(1/2)/b^4
```

Rubi [A] time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{2a^3}{3b^4\left(a+\frac{b}{x}\right)^{3/2}}+\frac{6a^2}{b^4\sqrt{a+\frac{b}{x}}}+\frac{6a\sqrt{a+\frac{b}{x}}}{b^4}-\frac{2\left(a+\frac{b}{x}\right)^{3/2}}{3b^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(5/2)*x^5),x]
```

```
[Out] (-2*a^3)/(3*b^4*(a + b/x)^(3/2)) + (6*a^2)/(b^4*Sqrt[a + b/x]) + (6*a*Sqrt[
a + b/x])/b^4 - (2*(a + b/x)^(3/2))/(3*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^5}\,dx &= -\text{Subst}\left(\int \frac{x^3}{(a+bx)^{5/2}}\,dx, x, \frac{1}{x}\right)\\ &= -\text{Subst}\left(\int\left(-\frac{a^3}{b^3(a+bx)^{5/2}}+\frac{3a^2}{b^3(a+bx)^{3/2}}-\frac{3a}{b^3\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b^3}\right)dx, x, \frac{1}{x}\right)\\ &= -\frac{2a^3}{3b^4\left(a+\frac{b}{x}\right)^{3/2}}+\frac{6a^2}{b^4\sqrt{a+\frac{b}{x}}}+\frac{6a\sqrt{a+\frac{b}{x}}}{b^4}-\frac{2\left(a+\frac{b}{x}\right)^{3/2}}{3b^4}\end{aligned}$$

Mathematica [A] time = 0.04, size = 58, normalized size = 0.76

$$\frac{32a^3x^3+48a^2bx^2+12ab^2x-2b^3}{3b^4x^2\sqrt{a+\frac{b}{x}}\,(ax+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(5/2)*x^5),x]
```

```
[Out] (-2*b^3 + 12*a*b^2*x + 48*a^2*b*x^2 + 32*a^3*x^3)/(3*b^4*Sqrt[a + b/x]*x^2*
(b + a*x))
```

fricas [A] time = 0.92, size = 70, normalized size = 0.92

$$\frac{2\left(16\,a^3x^3+24\,a^2bx^2+6\,ab^2x-b^3\right)\sqrt{\frac{ax+b}{x}}}{3\left(a^2b^4x^3+2\,ab^5x^2+b^6x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] 2/3*(16*a^3*x^3 + 24*a^2*b*x^2 + 6*a*b^2*x - b^3)*sqrt((a*x + b)/x)/(a^2*b^
4*x^3 + 2*a*b^5*x^2 + b^6*x)
```

giac [A] time = 0.24, size = 79, normalized size = 1.04

$$\frac{2\left(9\,a\sqrt{\frac{ax+b}{x}}-\frac{\left(a^3-\frac{9\,(ax+b)a^2}{x}\right)x}{(ax+b)\sqrt{\frac{ax+b}{x}}}-\frac{(ax+b)\sqrt{\frac{ax+b}{x}}}{x}\right)}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] 2/3*(9*a*sqrt((a*x + b)/x) - (a^3 - 9*(a*x + b)*a^2/x)*x/((a*x + b)*sqrt((a
*x + b)/x)) - (a*x + b)*sqrt((a*x + b)/x)/x)/b^4
```

maple [A] time = 0.01, size = 55, normalized size = 0.72

$$\frac{2\,(ax+b)\left(16a^3x^3+24a^2b\,x^2+6a\,b^2x-b^3\right)}{3\left(\frac{ax+b}{x}\right)^{\frac{5}{2}}b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(5/2)/x^5,x)
```

```
[Out] 2/3*(a*x+b)*(16*a^3*x^3+24*a^2*b*x^2+6*a*b^2*x-b^3)/x^4/b^4/((a*x+b)/x)^(5/
2)
```

maxima [A] time = 1.05, size = 64, normalized size = 0.84

$$-\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3\,b^4}+\frac{6\sqrt{a+\frac{b}{x}}\,a}{b^4}+\frac{6\,a^2}{\sqrt{a+\frac{b}{x}}\,b^4}-\frac{2\,a^3}{3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] -2/3*(a + b/x)^(3/2)/b^4 + 6*sqrt(a + b/x)*a/b^4 + 6*a^2/(sqrt(a + b/x)*b^4
) - 2/3*a^3/((a + b/x)^(3/2)*b^4)
```

mupad [B] time = 1.32, size = 54, normalized size = 0.71

$$\frac{2\sqrt{a+\frac{b}{x}}\left(16a^3x^3+24a^2bx^2+6ab^2x-b^3\right)}{3b^4x(b+ax)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b/x)^(5/2)),x)
```

```
[Out] (2*(a + b/x)^(1/2)*(16*a^3*x^3 - b^3 + 24*a^2*b*x^2 + 6*a*b^2*x))/(3*b^4*x*
(b + a*x)^2)
```

sympy [A] time = 5.25, size = 187, normalized size = 2.46

$$\begin{cases} \frac{32a^3x^3}{3ab^4x^3\sqrt{a+\frac{b}{x}}+3b^5x^2\sqrt{a+\frac{b}{x}}}+\frac{48a^2bx^2}{3ab^4x^3\sqrt{a+\frac{b}{x}}+3b^5x^2\sqrt{a+\frac{b}{x}}}+\frac{12ab^2x}{3ab^4x^3\sqrt{a+\frac{b}{x}}+3b^5x^2\sqrt{a+\frac{b}{x}}}-\frac{2b^3}{3ab^4x^3\sqrt{a+\frac{b}{x}}+3b^5x^2\sqrt{a+\frac{b}{x}}} & \text{for } b\neq 0 \\ -\frac{1}{4a^{\frac{5}{2}}x^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(5/2)/x**5,x)
```

```
[Out] Piecewise((32*a**3*x**3/(3*a*b**4*x**3*sqrt(a + b/x) + 3*b**5*x**2*sqrt(a +
 b/x)) + 48*a**2*b*x**2/(3*a*b**4*x**3*sqrt(a + b/x) + 3*b**5*x**2*sqrt(a +
 b/x)) + 12*a*b**2*x/(3*a*b**4*x**3*sqrt(a + b/x) + 3*b**5*x**2*sqrt(a + b/
x)) - 2*b**3/(3*a*b**4*x**3*sqrt(a + b/x) + 3*b**5*x**2*sqrt(a + b/x)), Ne(
b, 0)), (-1/(4*a**(5/2)*x**4), True))
```

3.1750 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^6}\,dx$

Optimal. Leaf size=97

$$\frac{2a^4}{3b^5\left(a+\frac{b}{x}\right)^{3/2}}-\frac{8a^3}{b^5\sqrt{a+\frac{b}{x}}}-\frac{12a^2\sqrt{a+\frac{b}{x}}}{b^5}+\frac{8a\left(a+\frac{b}{x}\right)^{3/2}}{3b^5}-\frac{2\left(a+\frac{b}{x}\right)^{5/2}}{5b^5}$$

```
[Out] 2/3*a^4/b^5/(a+b/x)^(3/2)+8/3*a*(a+b/x)^(3/2)/b^5-2/5*(a+b/x)^(5/2)/b^5-8*a
^3/b^5/(a+b/x)^(1/2)-12*a^2*(a+b/x)^(1/2)/b^5
```

Rubi [A] time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{2a^4}{3b^5\left(a+\frac{b}{x}\right)^{3/2}}-\frac{8a^3}{b^5\sqrt{a+\frac{b}{x}}}-\frac{12a^2\sqrt{a+\frac{b}{x}}}{b^5}+\frac{8a\left(a+\frac{b}{x}\right)^{3/2}}{3b^5}-\frac{2\left(a+\frac{b}{x}\right)^{5/2}}{5b^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(5/2)*x^6),x]
```

```
[Out] (2*a^4)/(3*b^5*(a + b/x)^(3/2)) - (8*a^3)/(b^5*Sqrt[a + b/x]) - (12*a^2*Sqr
t[a + b/x])/b^5 + (8*a*(a + b/x)^(3/2))/(3*b^5) - (2*(a + b/x)^(5/2))/(5*b^
5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^6}\,dx = -\text{Subst}\left(\int \frac{x^4}{(a+bx)^{5/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a^4}{b^4(a+bx)^{5/2}}-\frac{4a^3}{b^4(a+bx)^{3/2}}+\frac{6a^2}{b^4\sqrt{a+bx}}-\frac{4a\sqrt{a+bx}}{b^4}+\frac{(a+bx)^{3/2}}{b^4}\right) d\right.$$

$$= \frac{2a^4}{3b^5\left(a+\frac{b}{x}\right)^{3/2}}-\frac{8a^3}{b^5\sqrt{a+\frac{b}{x}}}-\frac{12a^2\sqrt{a+\frac{b}{x}}}{b^5}+\frac{8a\left(a+\frac{b}{x}\right)^{3/2}}{3b^5}-\frac{2\left(a+\frac{b}{x}\right)^{5/2}}{5b^5}$$

Mathematica [A] time = 0.06, size = 69, normalized size = 0.71

$$-\frac{2\left(128a^4x^4+192a^3bx^3+48a^2b^2x^2-8ab^3x+3b^4\right)}{15b^5x^3\sqrt{a+\frac{b}{x}}\,(ax+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(5/2)*x^6),x]
```

```
[Out] (-2*(3*b^4 - 8*a*b^3*x + 48*a^2*b^2*x^2 + 192*a^3*b*x^3 + 128*a^4*x^4))/(15
*b^5*Sqrt[a + b/x]*x^3*(b + a*x))
```

fricas [A] time = 0.88, size = 83, normalized size = 0.86

$$-\frac{2\left(128\,a^4x^4+192\,a^3bx^3+48\,a^2b^2x^2-8\,ab^3x+3\,b^4\right)\sqrt{\frac{ax+b}{x}}}{15\left(a^2b^5x^4+2\,ab^6x^3+b^7x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/x^6,x, algorithm="fricas")
```

```
[Out] -2/15*(128*a^4*x^4 + 192*a^3*b*x^3 + 48*a^2*b^2*x^2 - 8*a*b^3*x + 3*b^4)*sq
rt((a*x + b)/x)/(a^2*b^5*x^4 + 2*a*b^6*x^3 + b^7*x^2)
```

giac [A] time = 0.17, size = 105, normalized size = 1.08

$$-\frac{2\left(90\,a^2\sqrt{\frac{ax+b}{x}}-\frac{20\,(ax+b)a\sqrt{\frac{ax+b}{x}}}{x}-\frac{5\left(a^4-\frac{12\,(ax+b)a^3}{x}\right)x}{(ax+b)\sqrt{\frac{ax+b}{x}}}+\frac{3\,(ax+b)^2\sqrt{\frac{ax+b}{x}}}{x^2}\right)}{15\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/x^6,x, algorithm="giac")
```

```
[Out] -2/15*(90*a^2*sqrt((a*x + b)/x) - 20*(a*x + b)*a*sqrt((a*x + b)/x)/x - 5*(a
^4 - 12*(a*x + b)*a^3/x)*x/((a*x + b)*sqrt((a*x + b)/x)) + 3*(a*x + b)^2*sq
rt((a*x + b)/x)/x^2)/b^5
```

maple [A] time = 0.01, size = 66, normalized size = 0.68

$$-\frac{2\,(ax+b)\left(128a^4x^4+192a^3x^3b+48a^2x^2b^2-8ax\,b^3+3b^4\right)}{15\left(\frac{ax+b}{x}\right)^{\frac{5}{2}}b^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(5/2)/x^6,x)
```

```
[Out] -2/15*(a*x+b)*(128*a^4*x^4+192*a^3*b*x^3+48*a^2*b^2*x^2-8*a*b^3*x+3*b^4)/x^
5/b^5/((a*x+b)/x)^(5/2)
```

maxima [A] time = 1.03, size = 81, normalized size = 0.84

$$-\frac{2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{5\,b^5}+\frac{8\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a}{3\,b^5}-\frac{12\sqrt{a+\frac{b}{x}}\,a^2}{b^5}-\frac{8\,a^3}{\sqrt{a+\frac{b}{x}}\,b^5}+\frac{2\,a^4}{3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/x^6,x, algorithm="maxima")
```

```
[Out] -2/5*(a + b/x)^(5/2)/b^5 + 8/3*(a + b/x)^(3/2)*a/b^5 - 12*sqrt(a + b/x)*a^2
/b^5 - 8*a^3/(sqrt(a + b/x)*b^5) + 2/3*a^4/((a + b/x)^(3/2)*b^5)
```

mupad [B] time = 1.38, size = 65, normalized size = 0.67

$$-\frac{2\sqrt{a+\frac{b}{x}}\,\left(128\,a^4\,x^4+192\,a^3\,b\,x^3+48\,a^2\,b^2\,x^2-8\,a\,b^3\,x+3\,b^4\right)}{15\,b^5\,x^2\,{(b+a\,x)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b/x)^(5/2)),x)
```

```
[Out] -(2*(a + b/x)^(1/2)*(3*b^4 + 128*a^4*x^4 + 192*a^3*b*x^3 + 48*a^2*b^2*x^2 -
 8*a*b^3*x))/(15*b^5*x^2*(b + a*x)^2)
```

sympy [B] time = 6.36, size = 2032, normalized size = 20.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(5/2)/x**6,x)
```

```
[Out] -256*a**(21/2)*b**(33/2)*x**8*sqrt(a*x/b + 1)/(15*a**(17/2)*b**21*x**(17/2)
 + 90*a**(15/2)*b**22*x**(15/2) + 225*a**(13/2)*b**23*x**(13/2) + 300*a**(1
1/2)*b**24*x**(11/2) + 225*a**(9/2)*b**25*x**(9/2) + 90*a**(7/2)*b**26*x**(
7/2) + 15*a**(5/2)*b**27*x**(5/2)) - 1408*a**(19/2)*b**(35/2)*x**7*sqrt(a*x
/b + 1)/(15*a**(17/2)*b**21*x**(17/2) + 90*a**(15/2)*b**22*x**(15/2) + 225*
a**(13/2)*b**23*x**(13/2) + 300*a**(11/2)*b**24*x**(11/2) + 225*a**(9/2)*b*
*25*x**(9/2) + 90*a**(7/2)*b**26*x**(7/2) + 15*a**(5/2)*b**27*x**(5/2)) - 3
168*a**(17/2)*b**(37/2)*x**6*sqrt(a*x/b + 1)/(15*a**(17/2)*b**21*x**(17/2)
+ 90*a**(15/2)*b**22*x**(15/2) + 225*a**(13/2)*b**23*x**(13/2) + 300*a**(11
/2)*b**24*x**(11/2) + 225*a**(9/2)*b**25*x**(9/2) + 90*a**(7/2)*b**26*x**(7
/2) + 15*a**(5/2)*b**27*x**(5/2)) - 3696*a**(15/2)*b**(39/2)*x**5*sqrt(a*x/
b + 1)/(15*a**(17/2)*b**21*x**(17/2) + 90*a**(15/2)*b**22*x**(15/2) + 225*a
**(13/2)*b**23*x**(13/2) + 300*a**(11/2)*b**24*x**(11/2) + 225*a**(9/2)*b**
25*x**(9/2) + 90*a**(7/2)*b**26*x**(7/2) + 15*a**(5/2)*b**27*x**(5/2)) - 23
10*a**(13/2)*b**(41/2)*x**4*sqrt(a*x/b + 1)/(15*a**(17/2)*b**21*x**(17/2) +
 90*a**(15/2)*b**22*x**(15/2) + 225*a**(13/2)*b**23*x**(13/2) + 300*a**(11/
2)*b**24*x**(11/2) + 225*a**(9/2)*b**25*x**(9/2) + 90*a**(7/2)*b**26*x**(7/
2) + 15*a**(5/2)*b**27*x**(5/2)) - 696*a**(11/2)*b**(43/2)*x**3*sqrt(a*x/b
 + 1)/(15*a**(17/2)*b**21*x**(17/2) + 90*a**(15/2)*b**22*x**(15/2) + 225*a**
(13/2)*b**23*x**(13/2) + 300*a**(11/2)*b**24*x**(11/2) + 225*a**(9/2)*b**25
*x**(9/2) + 90*a**(7/2)*b**26*x**(7/2) + 15*a**(5/2)*b**27*x**(5/2)) - 68*a
**(9/2)*b**(45/2)*x**2*sqrt(a*x/b + 1)/(15*a**(17/2)*b**21*x**(17/2) + 90*a
**(15/2)*b**22*x**(15/2) + 225*a**(13/2)*b**23*x**(13/2) + 300*a**(11/2)*b*
*24*x**(11/2) + 225*a**(9/2)*b**25*x**(9/2) + 90*a**(7/2)*b**26*x**(7/2) +
15*a**(5/2)*b**27*x**(5/2)) - 8*a**(7/2)*b**(47/2)*x*sqrt(a*x/b + 1)/(15*a*
*(17/2)*b**21*x**(17/2) + 90*a**(15/2)*b**22*x**(15/2) + 225*a**(13/2)*b**2
3*x**(13/2) + 300*a**(11/2)*b**24*x**(11/2) + 225*a**(9/2)*b**25*x**(9/2) +
 90*a**(7/2)*b**26*x**(7/2) + 15*a**(5/2)*b**27*x**(5/2)) - 6*a**(5/2)*b**(
49/2)*sqrt(a*x/b + 1)/(15*a**(17/2)*b**21*x**(17/2) + 90*a**(15/2)*b**22*x*
*(15/2) + 225*a**(13/2)*b**23*x**(13/2) + 300*a**(11/2)*b**24*x**(11/2) + 2
25*a**(9/2)*b**25*x**(9/2) + 90*a**(7/2)*b**26*x**(7/2) + 15*a**(5/2)*b**27
*x**(5/2)) + 256*a**11*b**16*x**(17/2)/(15*a**(17/2)*b**21*x**(17/2) + 90*a
**(15/2)*b**22*x**(15/2) + 225*a**(13/2)*b**23*x**(13/2) + 300*a**(11/2)*b*
*24*x**(11/2) + 225*a**(9/2)*b**25*x**(9/2) + 90*a**(7/2)*b**26*x**(7/2) +
15*a**(5/2)*b**27*x**(5/2)) + 1536*a**10*b**17*x**(15/2)/(15*a**(17/2)*b**2
1*x**(17/2) + 90*a**(15/2)*b**22*x**(15/2) + 225*a**(13/2)*b**23*x**(13/2)
```

```
+ 300*a**(11/2)*b**24*x**(11/2) + 225*a**(9/2)*b**25*x**(9/2) + 90*a**(7/2)
*b**26*x**(7/2) + 15*a**(5/2)*b**27*x**(5/2)) + 3840*a**9*b**18*x**(13/2)/(
15*a**(17/2)*b**21*x**(17/2) + 90*a**(15/2)*b**22*x**(15/2) + 225*a**(13/2)
*b**23*x**(13/2) + 300*a**(11/2)*b**24*x**(11/2) + 225*a**(9/2)*b**25*x**(9
/2) + 90*a**(7/2)*b**26*x**(7/2) + 15*a**(5/2)*b**27*x**(5/2)) + 5120*a**8*
b**19*x**(11/2)/(15*a**(17/2)*b**21*x**(17/2) + 90*a**(15/2)*b**22*x**(15/2
) + 225*a**(13/2)*b**23*x**(13/2) + 300*a**(11/2)*b**24*x**(11/2) + 225*a**
(9/2)*b**25*x**(9/2) + 90*a**(7/2)*b**26*x**(7/2) + 15*a**(5/2)*b**27*x**(5
/2)) + 3840*a**7*b**20*x**(9/2)/(15*a**(17/2)*b**21*x**(17/2) + 90*a**(15/2
)*b**22*x**(15/2) + 225*a**(13/2)*b**23*x**(13/2) + 300*a**(11/2)*b**24*x**
(11/2) + 225*a**(9/2)*b**25*x**(9/2) + 90*a**(7/2)*b**26*x**(7/2) + 15*a**(
5/2)*b**27*x**(5/2)) + 1536*a**6*b**21*x**(7/2)/(15*a**(17/2)*b**21*x**(17/
2) + 90*a**(15/2)*b**22*x**(15/2) + 225*a**(13/2)*b**23*x**(13/2) + 300*a**
(11/2)*b**24*x**(11/2) + 225*a**(9/2)*b**25*x**(9/2) + 90*a**(7/2)*b**26*x*
*(7/2) + 15*a**(5/2)*b**27*x**(5/2)) + 256*a**5*b**22*x**(5/2)/(15*a**(17/2
)*b**21*x**(17/2) + 90*a**(15/2)*b**22*x**(15/2) + 225*a**(13/2)*b**23*x**(
13/2) + 300*a**(11/2)*b**24*x**(11/2) + 225*a**(9/2)*b**25*x**(9/2) + 90*a*
*(7/2)*b**26*x**(7/2) + 15*a**(5/2)*b**27*x**(5/2))
```

3.1751 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^7}\,dx$

Optimal. Leaf size=116

$$-\frac{2a^5}{3b^6\left(a+\frac{b}{x}\right)^{3/2}}+\frac{10a^4}{b^6\sqrt{a+\frac{b}{x}}}+\frac{20a^3\sqrt{a+\frac{b}{x}}}{b^6}-\frac{20a^2\left(a+\frac{b}{x}\right)^{3/2}}{3b^6}+\frac{2a\left(a+\frac{b}{x}\right)^{5/2}}{b^6}-\frac{2\left(a+\frac{b}{x}\right)^{7/2}}{7b^6}$$

```
[Out] -2/3*a^5/b^6/(a+b/x)^(3/2)-20/3*a^2*(a+b/x)^(3/2)/b^6+2*a*(a+b/x)^(5/2)/b^6
-2/7*(a+b/x)^(7/2)/b^6+10*a^4/b^6/(a+b/x)^(1/2)+20*a^3*(a+b/x)^(1/2)/b^6
```

Rubi [A] time = 0.05, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{2a^5}{3b^6\left(a+\frac{b}{x}\right)^{3/2}}+\frac{10a^4}{b^6\sqrt{a+\frac{b}{x}}}+\frac{20a^3\sqrt{a+\frac{b}{x}}}{b^6}-\frac{20a^2\left(a+\frac{b}{x}\right)^{3/2}}{3b^6}+\frac{2a\left(a+\frac{b}{x}\right)^{5/2}}{b^6}-\frac{2\left(a+\frac{b}{x}\right)^{7/2}}{7b^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(5/2)*x^7),x]
```

```
[Out] (-2*a^5)/(3*b^6*(a + b/x)^(3/2)) + (10*a^4)/(b^6*Sqrt[a + b/x]) + (20*a^3*S
qrt[a + b/x])/b^6 - (20*a^2*(a + b/x)^(3/2))/(3*b^6) + (2*a*(a + b/x)^(5/2)
)/b^6 - (2*(a + b/x)^(7/2))/(7*b^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^7}\,dx = -\text{Subst}\left(\int \frac{x^5}{(a+bx)^{5/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(-\frac{a^5}{b^5(a+bx)^{5/2}}+\frac{5a^4}{b^5(a+bx)^{3/2}}-\frac{10a^3}{b^5\sqrt{a+bx}}+\frac{10a^2\sqrt{a+bx}}{b^5}-\frac{5a(a+bx)}{b^5}\right.\right.$$

$$= -\frac{2a^5}{3b^6\left(a+\frac{b}{x}\right)^{3/2}}+\frac{10a^4}{b^6\sqrt{a+\frac{b}{x}}}+\frac{20a^3\sqrt{a+\frac{b}{x}}}{b^6}-\frac{20a^2\left(a+\frac{b}{x}\right)^{3/2}}{3b^6}+\frac{2a\left(a+\frac{b}{x}\right)^{5/2}}{b^6}-\frac{2\left(a+\frac{b}{x}\right)^{7/2}}{7}$$

Mathematica [A] time = 0.08, size = 80, normalized size = 0.69

$$\frac{2\left(256a^5x^5+384a^4bx^4+96a^3b^2x^3-16a^2b^3x^2+6ab^4x-3b^5\right)}{21b^6x^4\sqrt{a+\frac{b}{x}}\,(ax+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(5/2)*x^7),x]
```

```
[Out] (2*(-3*b^5 + 6*a*b^4*x - 16*a^2*b^3*x^2 + 96*a^3*b^2*x^3 + 384*a^4*b*x^4 +
256*a^5*x^5))/(21*b^6*Sqrt[a + b/x]*x^4*(b + a*x))
```

fricas [A] time = 0.83, size = 94, normalized size = 0.81

$$\frac{2\left(256\,a^5x^5+384\,a^4bx^4+96\,a^3b^2x^3-16\,a^2b^3x^2+6\,ab^4x-3\,b^5\right)\sqrt{\frac{ax+b}{x}}}{21\left(a^2b^6x^5+2\,ab^7x^4+b^8x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/x^7,x, algorithm="fricas")
```

```
[Out] 2/21*(256*a^5*x^5 + 384*a^4*b*x^4 + 96*a^3*b^2*x^3 - 16*a^2*b^3*x^2 + 6*a*b
^4*x - 3*b^5)*sqrt((a*x + b)/x)/(a^2*b^6*x^5 + 2*a*b^7*x^4 + b^8*x^3)
```

giac [A] time = 0.17, size = 131, normalized size = 1.13

$$\frac{2\left(210\,a^3\sqrt{\frac{ax+b}{x}}-\frac{70\,(ax+b)a^2\sqrt{\frac{ax+b}{x}}}{x}+\frac{21\,(ax+b)^2a\sqrt{\frac{ax+b}{x}}}{x^2}-\frac{7\left(a^5-\frac{15\,(ax+b)a^4}{x}\right)x}{(ax+b)\sqrt{\frac{ax+b}{x}}}-\frac{3\,(ax+b)^3\sqrt{\frac{ax+b}{x}}}{x^3}\right)}{21\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/x^7,x, algorithm="giac")
```

```
[Out] 2/21*(210*a^3*sqrt((a*x + b)/x) - 70*(a*x + b)*a^2*sqrt((a*x + b)/x)/x + 21
*(a*x + b)^2*a*sqrt((a*x + b)/x)/x^2 - 7*(a^5 - 15*(a*x + b)*a^4/x)*x/((a*x
 + b)*sqrt((a*x + b)/x)) - 3*(a*x + b)^3*sqrt((a*x + b)/x)/x^3)/b^6
```

maple [A] time = 0.01, size = 77, normalized size = 0.66

$$\frac{2\,(ax+b)\left(256a^5x^5+384a^4b\,x^4+96a^3b^2x^3-16a^2b^3x^2+6a\,b^4x-3b^5\right)}{21\left(\frac{ax+b}{x}\right)^{\frac{5}{2}}b^6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(5/2)/x^7,x)
```

```
[Out] 2/21*(a*x+b)*(256*a^5*x^5+384*a^4*b*x^4+96*a^3*b^2*x^3-16*a^2*b^3*x^2+6*a*b
^4*x-3*b^5)/x^6/b^6/((a*x+b)/x)^(5/2)
```

maxima [A] time = 0.96, size = 98, normalized size = 0.84

$$-\frac{2\left(a+\frac{b}{x}\right)^{\frac{7}{2}}}{7\,b^6}+\frac{2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a}{b^6}-\frac{20\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2}{3\,b^6}+\frac{20\sqrt{a+\frac{b}{x}}\,a^3}{b^6}+\frac{10\,a^4}{\sqrt{a+\frac{b}{x}}\,b^6}-\frac{2\,a^5}{3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/x^7,x, algorithm="maxima")
```

```
[Out] -2/7*(a + b/x)^(7/2)/b^6 + 2*(a + b/x)^(5/2)*a/b^6 - 20/3*(a + b/x)^(3/2)*a
^2/b^6 + 20*sqrt(a + b/x)*a^3/b^6 + 10*a^4/(sqrt(a + b/x)*b^6) - 2/3*a^5/((
a + b/x)^(3/2)*b^6)
```

mupad **[B]** time = 1.42, size = 110, normalized size = 0.95

$$\frac{\sqrt{a+\frac{b}{x}}\left(\frac{256\,a^3}{21\,b^5}+\frac{512\,a^4\,x}{21\,b^6}\right)}{b+a\,x}-\frac{2\sqrt{a+\frac{b}{x}}}{7\,b^3\,x^3}+\frac{8\,a\sqrt{a+\frac{b}{x}}}{7\,b^4\,x^2}-\frac{\sqrt{a+\frac{b}{x}}\left(\frac{74\,a^2}{21\,b^3}+\frac{88\,a^3\,x}{21\,b^4}\right)}{x\,(b+a\,x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b/x)^(5/2)),x)
```

```
[Out] ((a + b/x)^(1/2)*((256*a^3)/(21*b^5) + (512*a^4*x)/(21*b^6)))/(b + a*x) - (
2*(a + b/x)^(1/2))/(7*b^3*x^3) + (8*a*(a + b/x)^(1/2))/(7*b^4*x^2) - ((a +
b/x)^(1/2)*((74*a^2)/(21*b^3) + (88*a^3*x)/(21*b^4)))/(x*(b + a*x)^2)
```

sympy **[B]** time = 11.99, size = 9263, normalized size = 79.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(5/2)/x**7,x)
```

```
[Out] 512*a**(43/2)*b**(91/2)*x**18*sqrt(a*x/b + 1)/(21*a**(37/2)*b**51*x**(37/2)
 + 315*a**(35/2)*b**52*x**(35/2) + 2205*a**(33/2)*b**53*x**(33/2) + 9555*a*
*(31/2)*b**54*x**(31/2) + 28665*a**(29/2)*b**55*x**(29/2) + 63063*a**(27/2)
*b**56*x**(27/2) + 105105*a**(25/2)*b**57*x**(25/2) + 135135*a**(23/2)*b**5
8*x**(23/2) + 135135*a**(21/2)*b**59*x**(21/2) + 105105*a**(19/2)*b**60*x**
(19/2) + 63063*a**(17/2)*b**61*x**(17/2) + 28665*a**(15/2)*b**62*x**(15/2)
+ 9555*a**(13/2)*b**63*x**(13/2) + 2205*a**(11/2)*b**64*x**(11/2) + 315*a**
(9/2)*b**65*x**(9/2) + 21*a**(7/2)*b**66*x**(7/2)) + 7424*a**(41/2)*b**(93/
2)*x**17*sqrt(a*x/b + 1)/(21*a**(37/2)*b**51*x**(37/2) + 315*a**(35/2)*b**5
2*x**(35/2) + 2205*a**(33/2)*b**53*x**(33/2) + 9555*a**(31/2)*b**54*x**(31/
2) + 28665*a**(29/2)*b**55*x**(29/2) + 63063*a**(27/2)*b**56*x**(27/2) + 10
5105*a**(25/2)*b**57*x**(25/2) + 135135*a**(23/2)*b**58*x**(23/2) + 135135*
a**(21/2)*b**59*x**(21/2) + 105105*a**(19/2)*b**60*x**(19/2) + 63063*a**(17
/2)*b**61*x**(17/2) + 28665*a**(15/2)*b**62*x**(15/2) + 9555*a**(13/2)*b**6
3*x**(13/2) + 2205*a**(11/2)*b**64*x**(11/2) + 315*a**(9/2)*b**65*x**(9/2)
+ 21*a**(7/2)*b**66*x**(7/2)) + 50112*a**(39/2)*b**(95/2)*x**16*sqrt(a*x/b
+ 1)/(21*a**(37/2)*b**51*x**(37/2) + 315*a**(35/2)*b**52*x**(35/2) + 2205*a
**(33/2)*b**53*x**(33/2) + 9555*a**(31/2)*b**54*x**(31/2) + 28665*a**(29/2)
*b**55*x**(29/2) + 63063*a**(27/2)*b**56*x**(27/2) + 105105*a**(25/2)*b**57
*x**(25/2) + 135135*a**(23/2)*b**58*x**(23/2) + 135135*a**(21/2)*b**59*x**(
21/2) + 105105*a**(19/2)*b**60*x**(19/2) + 63063*a**(17/2)*b**61*x**(17/2)
+ 28665*a**(15/2)*b**62*x**(15/2) + 9555*a**(13/2)*b**63*x**(13/2) + 2205*a
**(11/2)*b**64*x**(11/2) + 315*a**(9/2)*b**65*x**(9/2) + 21*a**(7/2)*b**66*
x**(7/2)) + 208800*a**(37/2)*b**(97/2)*x**15*sqrt(a*x/b + 1)/(21*a**(37/2)*
b**51*x**(37/2) + 315*a**(35/2)*b**52*x**(35/2) + 2205*a**(33/2)*b**53*x**(
33/2) + 9555*a**(31/2)*b**54*x**(31/2) + 28665*a**(29/2)*b**55*x**(29/2) +
63063*a**(27/2)*b**56*x**(27/2) + 105105*a**(25/2)*b**57*x**(25/2) + 135135
*a**(23/2)*b**58*x**(23/2) + 135135*a**(21/2)*b**59*x**(21/2) + 105105*a**(
19/2)*b**60*x**(19/2) + 63063*a**(17/2)*b**61*x**(17/2) + 28665*a**(15/2)*b
**62*x**(15/2) + 9555*a**(13/2)*b**63*x**(13/2) + 2205*a**(11/2)*b**64*x**(
11/2) + 315*a**(9/2)*b**65*x**(9/2) + 21*a**(7/2)*b**66*x**(7/2)) + 600300*
a**(35/2)*b**(99/2)*x**14*sqrt(a*x/b + 1)/(21*a**(37/2)*b**51*x**(37/2) + 3
15*a**(35/2)*b**52*x**(35/2) + 2205*a**(33/2)*b**53*x**(33/2) + 9555*a**(31
/2)*b**54*x**(31/2) + 28665*a**(29/2)*b**55*x**(29/2) + 63063*a**(27/2)*b**
```

```
56*x**(27/2) + 105105*a**(25/2)*b**57*x**(25/2) + 135135*a**(23/2)*b**58*x*
*(23/2) + 135135*a**(21/2)*b**59*x**(21/2) + 105105*a**(19/2)*b**60*x**(19/
2) + 63063*a**(17/2)*b**61*x**(17/2) + 28665*a**(15/2)*b**62*x**(15/2) + 95
55*a**(13/2)*b**63*x**(13/2) + 2205*a**(11/2)*b**64*x**(11/2) + 315*a**(9/2
)*b**65*x**(9/2) + 21*a**(7/2)*b**66*x**(7/2)) + 1260630*a**(33/2)*b**(101/
2)*x**13*sqrt(a*x/b + 1)/(21*a**(37/2)*b**51*x**(37/2) + 315*a**(35/2)*b**5
2*x**(35/2) + 2205*a**(33/2)*b**53*x**(33/2) + 9555*a**(31/2)*b**54*x**(31/
2) + 28665*a**(29/2)*b**55*x**(29/2) + 63063*a**(27/2)*b**56*x**(27/2) + 10
5105*a**(25/2)*b**57*x**(25/2) + 135135*a**(23/2)*b**58*x**(23/2) + 135135*
a**(21/2)*b**59*x**(21/2) + 105105*a**(19/2)*b**60*x**(19/2) + 63063*a**(17
/2)*b**61*x**(17/2) + 28665*a**(15/2)*b**62*x**(15/2) + 9555*a**(13/2)*b**6
3*x**(13/2) + 2205*a**(11/2)*b**64*x**(11/2) + 315*a**(9/2)*b**65*x**(9/2)
+ 21*a**(7/2)*b**66*x**(7/2)) + 1995994*a**(31/2)*b**(103/2)*x**12*sqrt(a*x
/b + 1)/(21*a**(37/2)*b**51*x**(37/2) + 315*a**(35/2)*b**52*x**(35/2) + 220
5*a**(33/2)*b**53*x**(33/2) + 9555*a**(31/2)*b**54*x**(31/2) + 28665*a**(29
/2)*b**55*x**(29/2) + 63063*a**(27/2)*b**56*x**(27/2) + 105105*a**(25/2)*b*
*57*x**(25/2) + 135135*a**(23/2)*b**58*x**(23/2) + 135135*a**(21/2)*b**59*x
**(21/2) + 105105*a**(19/2)*b**60*x**(19/2) + 63063*a**(17/2)*b**61*x**(17/
2) + 28665*a**(15/2)*b**62*x**(15/2) + 9555*a**(13/2)*b**63*x**(13/2) + 220
5*a**(11/2)*b**64*x**(11/2) + 315*a**(9/2)*b**65*x**(9/2) + 21*a**(7/2)*b**
66*x**(7/2)) + 2423668*a**(29/2)*b**(105/2)*x**11*sqrt(a*x/b + 1)/(21*a**(3
7/2)*b**51*x**(37/2) + 315*a**(35/2)*b**52*x**(35/2) + 2205*a**(33/2)*b**53
*x**(33/2) + 9555*a**(31/2)*b**54*x**(31/2) + 28665*a**(29/2)*b**55*x**(29/
2) + 63063*a**(27/2)*b**56*x**(27/2) + 105105*a**(25/2)*b**57*x**(25/2) + 1
35135*a**(23/2)*b**58*x**(23/2) + 135135*a**(21/2)*b**59*x**(21/2) + 105105
*a**(19/2)*b**60*x**(19/2) + 63063*a**(17/2)*b**61*x**(17/2) + 28665*a**(15
/2)*b**62*x**(15/2) + 9555*a**(13/2)*b**63*x**(13/2) + 2205*a**(11/2)*b**64
*x**(11/2) + 315*a**(9/2)*b**65*x**(9/2) + 21*a**(7/2)*b**66*x**(7/2)) + 22
71984*a**(27/2)*b**(107/2)*x**10*sqrt(a*x/b + 1)/(21*a**(37/2)*b**51*x**(37
/2) + 315*a**(35/2)*b**52*x**(35/2) + 2205*a**(33/2)*b**53*x**(33/2) + 9555
*a**(31/2)*b**54*x**(31/2) + 28665*a**(29/2)*b**55*x**(29/2) + 63063*a**(27
/2)*b**56*x**(27/2) + 105105*a**(25/2)*b**57*x**(25/2) + 135135*a**(23/2)*b
**58*x**(23/2) + 135135*a**(21/2)*b**59*x**(21/2) + 105105*a**(19/2)*b**60*
x**(19/2) + 63063*a**(17/2)*b**61*x**(17/2) + 28665*a**(15/2)*b**62*x**(15/
2) + 9555*a**(13/2)*b**63*x**(13/2) + 2205*a**(11/2)*b**64*x**(11/2) + 315*
a**(9/2)*b**65*x**(9/2) + 21*a**(7/2)*b**66*x**(7/2)) + 1640210*a**(25/2)*b
**(109/2)*x**9*sqrt(a*x/b + 1)/(21*a**(37/2)*b**51*x**(37/2) + 315*a**(35/2
)*b**52*x**(35/2) + 2205*a**(33/2)*b**53*x**(33/2) + 9555*a**(31/2)*b**54*x
**(31/2) + 28665*a**(29/2)*b**55*x**(29/2) + 63063*a**(27/2)*b**56*x**(27/2
) + 105105*a**(25/2)*b**57*x**(25/2) + 135135*a**(23/2)*b**58*x**(23/2) + 1
35135*a**(21/2)*b**59*x**(21/2) + 105105*a**(19/2)*b**60*x**(19/2) + 63063*
a**(17/2)*b**61*x**(17/2) + 28665*a**(15/2)*b**62*x**(15/2) + 9555*a**(13/2
)*b**63*x**(13/2) + 2205*a**(11/2)*b**64*x**(11/2) + 315*a**(9/2)*b**65*x**
(9/2) + 21*a**(7/2)*b**66*x**(7/2)) + 900614*a**(23/2)*b**(111/2)*x**8*sqrt
(a*x/b + 1)/(21*a**(37/2)*b**51*x**(37/2) + 315*a**(35/2)*b**52*x**(35/2) +
 2205*a**(33/2)*b**53*x**(33/2) + 9555*a**(31/2)*b**54*x**(31/2) + 28665*a*
*(29/2)*b**55*x**(29/2) + 63063*a**(27/2)*b**56*x**(27/2) + 105105*a**(25/2
)*b**57*x**(25/2) + 135135*a**(23/2)*b**58*x**(23/2) + 135135*a**(21/2)*b**
59*x**(21/2) + 105105*a**(19/2)*b**60*x**(19/2) + 63063*a**(17/2)*b**61*x**
(17/2) + 28665*a**(15/2)*b**62*x**(15/2) + 9555*a**(13/2)*b**63*x**(13/2) +
 2205*a**(11/2)*b**64*x**(11/2) + 315*a**(9/2)*b**65*x**(9/2) + 21*a**(7/2)
*b**66*x**(7/2)) + 365976*a**(21/2)*b**(113/2)*x**7*sqrt(a*x/b + 1)/(21*a**
(37/2)*b**51*x**(37/2) + 315*a**(35/2)*b**52*x**(35/2) + 2205*a**(33/2)*b**
53*x**(33/2) + 9555*a**(31/2)*b**54*x**(31/2) + 28665*a**(29/2)*b**55*x**(2
9/2) + 63063*a**(27/2)*b**56*x**(27/2) + 105105*a**(25/2)*b**57*x**(25/2) +
 135135*a**(23/2)*b**58*x**(23/2) + 135135*a**(21/2)*b**59*x**(21/2) + 1051
05*a**(19/2)*b**60*x**(19/2) + 63063*a**(17/2)*b**61*x**(17/2) + 28665*a**(
15/2)*b**62*x**(15/2) + 9555*a**(13/2)*b**63*x**(13/2) + 2205*a**(11/2)*b**
64*x**(11/2) + 315*a**(9/2)*b**65*x**(9/2) + 21*a**(7/2)*b**66*x**(7/2)) +
```

```
103740*a**(19/2)*b**(115/2)*x**6*sqrt(a*x/b + 1)/(21*a**(37/2)*b**51*x**(37
/2) + 315*a**(35/2)*b**52*x**(35/2) + 2205*a**(33/2)*b**53*x**(33/2) + 9555
*a**(31/2)*b**54*x**(31/2) + 28665*a**(29/2)*b**55*x**(29/2) + 63063*a**(27
/2)*b**56*x**(27/2) + 105105*a**(25/2)*b**57*x**(25/2) + 135135*a**(23/2)*b
**58*x**(23/2) + 135135*a**(21/2)*b**59*x**(21/2) + 105105*a**(19/2)*b**60*
x**(19/2) + 63063*a**(17/2)*b**61*x**(17/2) + 28665*a**(15/2)*b**62*x**(15/
2) + 9555*a**(13/2)*b**63*x**(13/2) + 2205*a**(11/2)*b**64*x**(11/2) + 315*
a**(9/2)*b**65*x**(9/2) + 21*a**(7/2)*b**66*x**(7/2)) + 17178*a**(17/2)*b**
(117/2)*x**5*sqrt(a*x/b + 1)/(21*a**(37/2)*b**51*x**(37/2) + 315*a**(35/2)*
b**52*x**(35/2) + 2205*a**(33/2)*b**53*x**(33/2) + 9555*a**(31/2)*b**54*x**
(31/2) + 28665*a**(29/2)*b**55*x**(29/2) + 63063*a**(27/2)*b**56*x**(27/2)
+ 105105*a**(25/2)*b**57*x**(25/2) + 135135*a**(23/2)*b**58*x**(23/2) + 135
135*a**(21/2)*b**59*x**(21/2) + 105105*a**(19/2)*b**60*x**(19/2) + 63063*a*
*(17/2)*b**61*x**(17/2) + 28665*a**(15/2)*b**62*x**(15/2) + 9555*a**(13/2)*
b**63*x**(13/2) + 2205*a**(11/2)*b**64*x**(11/2) + 315*a**(9/2)*b**65*x**(9
/2) + 21*a**(7/2)*b**66*x**(7/2)) - 90*a**(15/2)*b**(119/2)*x**4*sqrt(a*x/b
 + 1)/(21*a**(37/2)*b**51*x**(37/2) + 315*a**(35/2)*b**52*x**(35/2) + 2205*
a**(33/2)*b**53*x**(33/2) + 9555*a**(31/2)*b**54*x**(31/2) + 28665*a**(29/2
)*b**55*x**(29/2) + 63063*a**(27/2)*b**56*x**(27/2) + 105105*a**(25/2)*b**5
7*x**(25/2) + 135135*a**(23/2)*b**58*x**(23/2) + 135135*a**(21/2)*b**59*x**
(21/2) + 105105*a**(19/2)*b**60*x**(19/2) + 63063*a**(17/2)*b**61*x**(17/2)
 + 28665*a**(15/2)*b**62*x**(15/2) + 9555*a**(13/2)*b**63*x**(13/2) + 2205*
a**(11/2)*b**64*x**(11/2) + 315*a**(9/2)*b**65*x**(9/2) + 21*a**(7/2)*b**66
*x**(7/2)) - 1004*a**(13/2)*b**(121/2)*x**3*sqrt(a*x/b + 1)/(21*a**(37/2)*b
**51*x**(37/2) + 315*a**(35/2)*b**52*x**(35/2) + 2205*a**(33/2)*b**53*x**(3
3/2) + 9555*a**(31/2)*b**54*x**(31/2) + 28665*a**(29/2)*b**55*x**(29/2) + 6
3063*a**(27/2)*b**56*x**(27/2) + 105105*a**(25/2)*b**57*x**(25/2) + 135135*
a**(23/2)*b**58*x**(23/2) + 135135*a**(21/2)*b**59*x**(21/2) + 105105*a**(1
9/2)*b**60*x**(19/2) + 63063*a**(17/2)*b**61*x**(17/2) + 28665*a**(15/2)*b*
*62*x**(15/2) + 9555*a**(13/2)*b**63*x**(13/2) + 2205*a**(11/2)*b**64*x**(1
1/2) + 315*a**(9/2)*b**65*x**(9/2) + 21*a**(7/2)*b**66*x**(7/2)) - 344*a**(
11/2)*b**(123/2)*x**2*sqrt(a*x/b + 1)/(21*a**(37/2)*b**51*x**(37/2) + 315*a
**(35/2)*b**52*x**(35/2) + 2205*a**(33/2)*b**53*x**(33/2) + 9555*a**(31/2)*
b**54*x**(31/2) + 28665*a**(29/2)*b**55*x**(29/2) + 63063*a**(27/2)*b**56*x
**(27/2) + 105105*a**(25/2)*b**57*x**(25/2) + 135135*a**(23/2)*b**58*x**(23
/2) + 135135*a**(21/2)*b**59*x**(21/2) + 105105*a**(19/2)*b**60*x**(19/2) +
 63063*a**(17/2)*b**61*x**(17/2) + 28665*a**(15/2)*b**62*x**(15/2) + 9555*a
**(13/2)*b**63*x**(13/2) + 2205*a**(11/2)*b**64*x**(11/2) + 315*a**(9/2)*b*
*65*x**(9/2) + 21*a**(7/2)*b**66*x**(7/2)) - 66*a**(9/2)*b**(125/2)*x*sqrt(
a*x/b + 1)/(21*a**(37/2)*b**51*x**(37/2) + 315*a**(35/2)*b**52*x**(35/2) +
2205*a**(33/2)*b**53*x**(33/2) + 9555*a**(31/2)*b**54*x**(31/2) + 28665*a**
(29/2)*b**55*x**(29/2) + 63063*a**(27/2)*b**56*x**(27/2) + 105105*a**(25/2)
*b**57*x**(25/2) + 135135*a**(23/2)*b**58*x**(23/2) + 135135*a**(21/2)*b**5
9*x**(21/2) + 105105*a**(19/2)*b**60*x**(19/2) + 63063*a**(17/2)*b**61*x**(
17/2) + 28665*a**(15/2)*b**62*x**(15/2) + 9555*a**(13/2)*b**63*x**(13/2) +
2205*a**(11/2)*b**64*x**(11/2) + 315*a**(9/2)*b**65*x**(9/2) + 21*a**(7/2)*
b**66*x**(7/2)) - 6*a**(7/2)*b**(127/2)*sqrt(a*x/b + 1)/(21*a**(37/2)*b**51
*x**(37/2) + 315*a**(35/2)*b**52*x**(35/2) + 2205*a**(33/2)*b**53*x**(33/2)
 + 9555*a**(31/2)*b**54*x**(31/2) + 28665*a**(29/2)*b**55*x**(29/2) + 63063
*a**(27/2)*b**56*x**(27/2) + 105105*a**(25/2)*b**57*x**(25/2) + 135135*a**(
23/2)*b**58*x**(23/2) + 135135*a**(21/2)*b**59*x**(21/2) + 105105*a**(19/2)
*b**60*x**(19/2) + 63063*a**(17/2)*b**61*x**(17/2) + 28665*a**(15/2)*b**62*
x**(15/2) + 9555*a**(13/2)*b**63*x**(13/2) + 2205*a**(11/2)*b**64*x**(11/2)
 + 315*a**(9/2)*b**65*x**(9/2) + 21*a**(7/2)*b**66*x**(7/2)) - 512*a**22*b*
*45*x**(37/2)/(21*a**(37/2)*b**51*x**(37/2) + 315*a**(35/2)*b**52*x**(35/2)
 + 2205*a**(33/2)*b**53*x**(33/2) + 9555*a**(31/2)*b**54*x**(31/2) + 28665*
a**(29/2)*b**55*x**(29/2) + 63063*a**(27/2)*b**56*x**(27/2) + 105105*a**(25
/2)*b**57*x**(25/2) + 135135*a**(23/2)*b**58*x**(23/2) + 135135*a**(21/2)*b
**59*x**(21/2) + 105105*a**(19/2)*b**60*x**(19/2) + 63063*a**(17/2)*b**61*x
```

```
**(17/2) + 28665*a**(15/2)*b**62*x**(15/2) + 9555*a**(13/2)*b**63*x**(13/2)
 + 2205*a**(11/2)*b**64*x**(11/2) + 315*a**(9/2)*b**65*x**(9/2) + 21*a**(7/
2)*b**66*x**(7/2)) - 7680*a**21*b**46*x**(35/2)/(21*a**(37/2)*b**51*x**(37/
2) + 315*a**(35/2)*b**52*x**(35/2) + 2205*a**(33/2)*b**53*x**(33/2) + 9555*
a**(31/2)*b**54*x**(31/2) + 28665*a**(29/2)*b**55*x**(29/2) + 63063*a**(27/
2)*b**56*x**(27/2) + 105105*a**(25/2)*b**57*x**(25/2) + 135135*a**(23/2)*b*
*58*x**(23/2) + 135135*a**(21/2)*b**59*x**(21/2) + 105105*a**(19/2)*b**60*x
**(19/2) + 63063*a**(17/2)*b**61*x**(17/2) + 28665*a**(15/2)*b**62*x**(15/2
) + 9555*a**(13/2)*b**63*x**(13/2) + 2205*a**(11/2)*b**64*x**(11/2) + 315*a
**(9/2)*b**65*x**(9/2) + 21*a**(7/2)*b**66*x**(7/2)) - 53760*a**20*b**47*x*
*(33/2)/(21*a**(37/2)*b**51*x**(37/2) + 315*a**(35/2)*b**52*x**(35/2) + 220
5*a**(33/2)*b**53*x**(33/2) + 9555*a**(31/2)*b**54*x**(31/2) + 28665*a**(29
/2)*b**55*x**(29/2) + 63063*a**(27/2)*b**56*x**(27/2) + 105105*a**(25/2)*b*
*57*x**(25/2) + 135135*a**(23/2)*b**58*x**(23/2) + 135135*a**(21/2)*b**59*x
**(21/2) + 105105*a**(19/2)*b**60*x**(19/2) + 63063*a**(17/2)*b**61*x**(17/
2) + 28665*a**(15/2)*b**62*x**(15/2) + 9555*a**(13/2)*b**63*x**(13/2) + 220
5*a**(11/2)*b**64*x**(11/2) + 315*a**(9/2)*b**65*x**(9/2) + 21*a**(7/2)*b**
66*x**(7/2)) - 232960*a**19*b**48*x**(31/2)/(21*a**(37/2)*b**51*x**(37/2) +
 315*a**(35/2)*b**52*x**(35/2) + 2205*a**(33/2)*b**53*x**(33/2) + 9555*a**(
31/2)*b**54*x**(31/2) + 28665*a**(29/2)*b**55*x**(29/2) + 63063*a**(27/2)*b
**56*x**(27/2) + 105105*a**(25/2)*b**57*x**(25/2) + 135135*a**(23/2)*b**58*
x**(23/2) + 135135*a**(21/2)*b**59*x**(21/2) + 105105*a**(19/2)*b**60*x**(1
9/2) + 63063*a**(17/2)*b**61*x**(17/2) + 28665*a**(15/2)*b**62*x**(15/2) +
 9555*a**(13/2)*b**63*x**(13/2) + 2205*a**(11/2)*b**64*x**(11/2) + 315*a**(9
/2)*b**65*x**(9/2) + 21*a**(7/2)*b**66*x**(7/2)) - 698880*a**18*b**49*x**(2
9/2)/(21*a**(37/2)*b**51*x**(37/2) + 315*a**(35/2)*b**52*x**(35/2) + 2205*a
**(33/2)*b**53*x**(33/2) + 9555*a**(31/2)*b**54*x**(31/2) + 28665*a**(29/2)
*b**55*x**(29/2) + 63063*a**(27/2)*b**56*x**(27/2) + 105105*a**(25/2)*b**57
*x**(25/2) + 135135*a**(23/2)*b**58*x**(23/2) + 135135*a**(21/2)*b**59*x**(
21/2) + 105105*a**(19/2)*b**60*x**(19/2) + 63063*a**(17/2)*b**61*x**(17/2)
+ 28665*a**(15/2)*b**62*x**(15/2) + 9555*a**(13/2)*b**63*x**(13/2) + 2205*a
**(11/2)*b**64*x**(11/2) + 315*a**(9/2)*b**65*x**(9/2) + 21*a**(7/2)*b**66*
x**(7/2)) - 1537536*a**17*b**50*x**(27/2)/(21*a**(37/2)*b**51*x**(37/2) + 3
15*a**(35/2)*b**52*x**(35/2) + 2205*a**(33/2)*b**53*x**(33/2) + 9555*a**(31
/2)*b**54*x**(31/2) + 28665*a**(29/2)*b**55*x**(29/2) + 63063*a**(27/2)*b**
56*x**(27/2) + 105105*a**(25/2)*b**57*x**(25/2) + 135135*a**(23/2)*b**58*x*
*(23/2) + 135135*a**(21/2)*b**59*x**(21/2) + 105105*a**(19/2)*b**60*x**(19/
2) + 63063*a**(17/2)*b**61*x**(17/2) + 28665*a**(15/2)*b**62*x**(15/2) + 95
55*a**(13/2)*b**63*x**(13/2) + 2205*a**(11/2)*b**64*x**(11/2) + 315*a**(9/2
)*b**65*x**(9/2) + 21*a**(7/2)*b**66*x**(7/2)) - 2562560*a**16*b**51*x**(25
/2)/(21*a**(37/2)*b**51*x**(37/2) + 315*a**(35/2)*b**52*x**(35/2) + 2205*a*
*(33/2)*b**53*x**(33/2) + 9555*a**(31/2)*b**54*x**(31/2) + 28665*a**(29/2)*
b**55*x**(29/2) + 63063*a**(27/2)*b**56*x**(27/2) + 105105*a**(25/2)*b**57*
x**(25/2) + 135135*a**(23/2)*b**58*x**(23/2) + 135135*a**(21/2)*b**59*x**(2
1/2) + 105105*a**(19/2)*b**60*x**(19/2) + 63063*a**(17/2)*b**61*x**(17/2) +
 28665*a**(15/2)*b**62*x**(15/2) + 9555*a**(13/2)*b**63*x**(13/2) + 2205*a*
*(11/2)*b**64*x**(11/2) + 315*a**(9/2)*b**65*x**(9/2) + 21*a**(7/2)*b**66*x
**(7/2)) - 3294720*a**15*b**52*x**(23/2)/(21*a**(37/2)*b**51*x**(37/2) + 31
5*a**(35/2)*b**52*x**(35/2) + 2205*a**(33/2)*b**53*x**(33/2) + 9555*a**(31/
2)*b**54*x**(31/2) + 28665*a**(29/2)*b**55*x**(29/2) + 63063*a**(27/2)*b**5
6*x**(27/2) + 105105*a**(25/2)*b**57*x**(25/2) + 135135*a**(23/2)*b**58*x**
(23/2) + 135135*a**(21/2)*b**59*x**(21/2) + 105105*a**(19/2)*b**60*x**(19/2
) + 63063*a**(17/2)*b**61*x**(17/2) + 28665*a**(15/2)*b**62*x**(15/2) + 955
5*a**(13/2)*b**63*x**(13/2) + 2205*a**(11/2)*b**64*x**(11/2) + 315*a**(9/2)
*b**65*x**(9/2) + 21*a**(7/2)*b**66*x**(7/2)) - 3294720*a**14*b**53*x**(21/
2)/(21*a**(37/2)*b**51*x**(37/2) + 315*a**(35/2)*b**52*x**(35/2) + 2205*a**
(33/2)*b**53*x**(33/2) + 9555*a**(31/2)*b**54*x**(31/2) + 28665*a**(29/2)*b
**55*x**(29/2) + 63063*a**(27/2)*b**56*x**(27/2) + 105105*a**(25/2)*b**57*x
**(25/2) + 135135*a**(23/2)*b**58*x**(23/2) + 135135*a**(21/2)*b**59*x**(21
```

```
/2) + 105105*a**(19/2)*b**60*x**(19/2) + 63063*a**(17/2)*b**61*x**(17/2) +
28665*a**(15/2)*b**62*x**(15/2) + 9555*a**(13/2)*b**63*x**(13/2) + 2205*a**
(11/2)*b**64*x**(11/2) + 315*a**(9/2)*b**65*x**(9/2) + 21*a**(7/2)*b**66*x*
*(7/2)) - 2562560*a**13*b**54*x**(19/2)/(21*a**(37/2)*b**51*x**(37/2) + 315
*a**(35/2)*b**52*x**(35/2) + 2205*a**(33/2)*b**53*x**(33/2) + 9555*a**(31/2
)*b**54*x**(31/2) + 28665*a**(29/2)*b**55*x**(29/2) + 63063*a**(27/2)*b**56
*x**(27/2) + 105105*a**(25/2)*b**57*x**(25/2) + 135135*a**(23/2)*b**58*x**(
23/2) + 135135*a**(21/2)*b**59*x**(21/2) + 105105*a**(19/2)*b**60*x**(19/2)
 + 63063*a**(17/2)*b**61*x**(17/2) + 28665*a**(15/2)*b**62*x**(15/2) + 9555
*a**(13/2)*b**63*x**(13/2) + 2205*a**(11/2)*b**64*x**(11/2) + 315*a**(9/2)*
b**65*x**(9/2) + 21*a**(7/2)*b**66*x**(7/2)) - 1537536*a**12*b**55*x**(17/2
)/(21*a**(37/2)*b**51*x**(37/2) + 315*a**(35/2)*b**52*x**(35/2) + 2205*a**(
33/2)*b**53*x**(33/2) + 9555*a**(31/2)*b**54*x**(31/2) + 28665*a**(29/2)*b*
*55*x**(29/2) + 63063*a**(27/2)*b**56*x**(27/2) + 105105*a**(25/2)*b**57*x*
*(25/2) + 135135*a**(23/2)*b**58*x**(23/2) + 135135*a**(21/2)*b**59*x**(21/
2) + 105105*a**(19/2)*b**60*x**(19/2) + 63063*a**(17/2)*b**61*x**(17/2) + 2
8665*a**(15/2)*b**62*x**(15/2) + 9555*a**(13/2)*b**63*x**(13/2) + 2205*a**(
11/2)*b**64*x**(11/2) + 315*a**(9/2)*b**65*x**(9/2) + 21*a**(7/2)*b**66*x**
(7/2)) - 698880*a**11*b**56*x**(15/2)/(21*a**(37/2)*b**51*x**(37/2) + 315*a
**(35/2)*b**52*x**(35/2) + 2205*a**(33/2)*b**53*x**(33/2) + 9555*a**(31/2)*
b**54*x**(31/2) + 28665*a**(29/2)*b**55*x**(29/2) + 63063*a**(27/2)*b**56*x
**(27/2) + 105105*a**(25/2)*b**57*x**(25/2) + 135135*a**(23/2)*b**58*x**(23
/2) + 135135*a**(21/2)*b**59*x**(21/2) + 105105*a**(19/2)*b**60*x**(19/2) +
 63063*a**(17/2)*b**61*x**(17/2) + 28665*a**(15/2)*b**62*x**(15/2) + 9555*a
**(13/2)*b**63*x**(13/2) + 2205*a**(11/2)*b**64*x**(11/2) + 315*a**(9/2)*b*
*65*x**(9/2) + 21*a**(7/2)*b**66*x**(7/2)) - 232960*a**10*b**57*x**(13/2)/(
21*a**(37/2)*b**51*x**(37/2) + 315*a**(35/2)*b**52*x**(35/2) + 2205*a**(33/
2)*b**53*x**(33/2) + 9555*a**(31/2)*b**54*x**(31/2) + 28665*a**(29/2)*b**55
*x**(29/2) + 63063*a**(27/2)*b**56*x**(27/2) + 105105*a**(25/2)*b**57*x**(2
5/2) + 135135*a**(23/2)*b**58*x**(23/2) + 135135*a**(21/2)*b**59*x**(21/2)
+ 105105*a**(19/2)*b**60*x**(19/2) + 63063*a**(17/2)*b**61*x**(17/2) + 2866
5*a**(15/2)*b**62*x**(15/2) + 9555*a**(13/2)*b**63*x**(13/2) + 2205*a**(11/
2)*b**64*x**(11/2) + 315*a**(9/2)*b**65*x**(9/2) + 21*a**(7/2)*b**66*x**(7/
2)) - 53760*a**9*b**58*x**(11/2)/(21*a**(37/2)*b**51*x**(37/2) + 315*a**(35
/2)*b**52*x**(35/2) + 2205*a**(33/2)*b**53*x**(33/2) + 9555*a**(31/2)*b**54
*x**(31/2) + 28665*a**(29/2)*b**55*x**(29/2) + 63063*a**(27/2)*b**56*x**(27
/2) + 105105*a**(25/2)*b**57*x**(25/2) + 135135*a**(23/2)*b**58*x**(23/2) +
 135135*a**(21/2)*b**59*x**(21/2) + 105105*a**(19/2)*b**60*x**(19/2) + 6306
3*a**(17/2)*b**61*x**(17/2) + 28665*a**(15/2)*b**62*x**(15/2) + 9555*a**(13
/2)*b**63*x**(13/2) + 2205*a**(11/2)*b**64*x**(11/2) + 315*a**(9/2)*b**65*x
**(9/2) + 21*a**(7/2)*b**66*x**(7/2)) - 7680*a**8*b**59*x**(9/2)/(21*a**(37
/2)*b**51*x**(37/2) + 315*a**(35/2)*b**52*x**(35/2) + 2205*a**(33/2)*b**53*
x**(33/2) + 9555*a**(31/2)*b**54*x**(31/2) + 28665*a**(29/2)*b**55*x**(29/2
) + 63063*a**(27/2)*b**56*x**(27/2) + 105105*a**(25/2)*b**57*x**(25/2) + 13
5135*a**(23/2)*b**58*x**(23/2) + 135135*a**(21/2)*b**59*x**(21/2) + 105105*
a**(19/2)*b**60*x**(19/2) + 63063*a**(17/2)*b**61*x**(17/2) + 28665*a**(15/
2)*b**62*x**(15/2) + 9555*a**(13/2)*b**63*x**(13/2) + 2205*a**(11/2)*b**64*
x**(11/2) + 315*a**(9/2)*b**65*x**(9/2) + 21*a**(7/2)*b**66*x**(7/2)) - 512
*a**7*b**60*x**(7/2)/(21*a**(37/2)*b**51*x**(37/2) + 315*a**(35/2)*b**52*x*
*(35/2) + 2205*a**(33/2)*b**53*x**(33/2) + 9555*a**(31/2)*b**54*x**(31/2) +
 28665*a**(29/2)*b**55*x**(29/2) + 63063*a**(27/2)*b**56*x**(27/2) + 105105
*a**(25/2)*b**57*x**(25/2) + 135135*a**(23/2)*b**58*x**(23/2) + 135135*a**(
21/2)*b**59*x**(21/2) + 105105*a**(19/2)*b**60*x**(19/2) + 63063*a**(17/2)*
b**61*x**(17/2) + 28665*a**(15/2)*b**62*x**(15/2) + 9555*a**(13/2)*b**63*x*
*(13/2) + 2205*a**(11/2)*b**64*x**(11/2) + 315*a**(9/2)*b**65*x**(9/2) + 21
*a**(7/2)*b**66*x**(7/2))
```

3.1752 $\int \sqrt{a+\frac{b}{x}}\, x^{7/2}\, dx$

Optimal. Leaf size=100

$$-\frac{32b^3x^{3/2}\left(a+\frac{b}{x}\right)^{3/2}}{315a^4}+\frac{16b^2x^{5/2}\left(a+\frac{b}{x}\right)^{3/2}}{105a^3}-\frac{4bx^{7/2}\left(a+\frac{b}{x}\right)^{3/2}}{21a^2}+\frac{2x^{9/2}\left(a+\frac{b}{x}\right)^{3/2}}{9a}$$

```
[Out] -32/315*b^3*(a+b/x)^(3/2)*x^(3/2)/a^4+16/105*b^2*(a+b/x)^(3/2)*x^(5/2)/a^3-
4/21*b*(a+b/x)^(3/2)*x^(7/2)/a^2+2/9*(a+b/x)^(3/2)*x^(9/2)/a
```

Rubi [A] time = 0.03, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {271, 264}

$$\frac{16b^2x^{5/2}\left(a+\frac{b}{x}\right)^{3/2}}{105a^3}-\frac{32b^3x^{3/2}\left(a+\frac{b}{x}\right)^{3/2}}{315a^4}-\frac{4bx^{7/2}\left(a+\frac{b}{x}\right)^{3/2}}{21a^2}+\frac{2x^{9/2}\left(a+\frac{b}{x}\right)^{3/2}}{9a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]*x^(7/2),x]
```

```
[Out] (-32*b^3*(a + b/x)^(3/2)*x^(3/2))/(315*a^4) + (16*b^2*(a + b/x)^(3/2)*x^(5/
2))/(105*a^3) - (4*b*(a + b/x)^(3/2)*x^(7/2))/(21*a^2) + (2*(a + b/x)^(3/2)
*x^(9/2))/(9*a)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+\frac{b}{x}}\, x^{7/2}\, dx &= \frac{2\left(a+\frac{b}{x}\right)^{3/2}x^{9/2}}{9a}-\frac{(2b)\int \sqrt{a+\frac{b}{x}}\, x^{5/2}\, dx}{3a}\\ &= -\frac{4b\left(a+\frac{b}{x}\right)^{3/2}x^{7/2}}{21a^2}+\frac{2\left(a+\frac{b}{x}\right)^{3/2}x^{9/2}}{9a}+\frac{\left(8b^2\right)\int \sqrt{a+\frac{b}{x}}\, x^{3/2}\, dx}{21a^2}\\ &= \frac{16b^2\left(a+\frac{b}{x}\right)^{3/2}x^{5/2}}{105a^3}-\frac{4b\left(a+\frac{b}{x}\right)^{3/2}x^{7/2}}{21a^2}+\frac{2\left(a+\frac{b}{x}\right)^{3/2}x^{9/2}}{9a}-\frac{\left(16b^3\right)\int \sqrt{a+\frac{b}{x}}\, \sqrt{x}\, dx}{105a^3}\\ &= -\frac{32b^3\left(a+\frac{b}{x}\right)^{3/2}x^{3/2}}{315a^4}+\frac{16b^2\left(a+\frac{b}{x}\right)^{3/2}x^{5/2}}{105a^3}-\frac{4b\left(a+\frac{b}{x}\right)^{3/2}x^{7/2}}{21a^2}+\frac{2\left(a+\frac{b}{x}\right)^{3/2}x^{9/2}}{9a}\end{aligned}$$

Mathematica [A] time = 0.02, size = 58, normalized size = 0.58

$$\frac{2\sqrt{x}\sqrt{a+\frac{b}{x}}\,(ax+b)\left(35a^3x^3-30a^2bx^2+24ab^2x-16b^3\right)}{315a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]*x^(7/2),x]
```

```
[Out] (2*Sqrt[a + b/x]*Sqrt[x]*(b + a*x)*(-16*b^3 + 24*a*b^2*x - 30*a^2*b*x^2 + 3
5*a^3*x^3))/(315*a^4)
```

fricas [A] time = 0.86, size = 60, normalized size = 0.60

$$\frac{2\left(35\,a^4x^4+5\,a^3bx^3-6\,a^2b^2x^2+8\,ab^3x-16\,b^4\right)\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}{315\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)*x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*a^4*x^4 + 5*a^3*b*x^3 - 6*a^2*b^2*x^2 + 8*a*b^3*x - 16*b^4)*sqrt(
x)*sqrt((a*x + b)/x)/a^4
```

giac [A] time = 0.17, size = 61, normalized size = 0.61

$$\frac{2}{315}\left(\frac{16\,b^{\frac{9}{2}}}{a^4}+\frac{35\,(ax+b)^{\frac{9}{2}}-135\,(ax+b)^{\frac{7}{2}}b+189\,(ax+b)^{\frac{5}{2}}b^2-105\,(ax+b)^{\frac{3}{2}}b^3}{a^4}\right)\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)*x^(7/2),x, algorithm="giac")
```

```
[Out] 2/315*(16*b^(9/2)/a^4 + (35*(a*x + b)^(9/2) - 135*(a*x + b)^(7/2)*b + 189*(
a*x + b)^(5/2)*b^2 - 105*(a*x + b)^(3/2)*b^3)/a^4)*sgn(x)
```

maple [A] time = 0.01, size = 55, normalized size = 0.55

$$\frac{2\,(ax+b)\left(35a^3x^3-30a^2b\,x^2+24a\,b^2x-16b^3\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x}}{315a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(1/2)*x^(7/2),x)
```

```
[Out] 2/315*(a*x+b)*(35*a^3*x^3-30*a^2*b*x^2+24*a*b^2*x-16*b^3)*x^(1/2)*((a*x+b)/
x)^(1/2)/a^4
```

maxima [A] time = 1.09, size = 69, normalized size = 0.69

$$\frac{2\left(35\left(a+\frac{b}{x}\right)^{\frac{9}{2}}x^{\frac{9}{2}}-135\left(a+\frac{b}{x}\right)^{\frac{7}{2}}bx^{\frac{7}{2}}+189\left(a+\frac{b}{x}\right)^{\frac{5}{2}}b^2x^{\frac{5}{2}}-105\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^3x^{\frac{3}{2}}\right)}{315\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)*x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*(a + b/x)^(9/2)*x^(9/2) - 135*(a + b/x)^(7/2)*b*x^(7/2) + 189*(a
+ b/x)^(5/2)*b^2*x^(5/2) - 105*(a + b/x)^(3/2)*b^3*x^(3/2))/a^4
```

mupad [B] time = 1.67, size = 58, normalized size = 0.58

$$\sqrt{a+\frac{b}{x}}\left(\frac{2\,x^{9/2}}{9}+\frac{2\,b\,x^{7/2}}{63\,a}-\frac{4\,b^2\,x^{5/2}}{105\,a^2}+\frac{16\,b^3\,x^{3/2}}{315\,a^3}-\frac{32\,b^4\,\sqrt{x}}{315\,a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(a + b/x)^(1/2),x)
```

```
[Out] (a + b/x)^(1/2)*((2*x^(9/2))/9 + (2*b*x^(7/2))/(63*a) - (4*b^2*x^(5/2))/(10
5*a^2) + (16*b^3*x^(3/2))/(315*a^3) - (32*b^4*x^(1/2))/(315*a^4))
```

sympy [**B**] time = 38.79, size = 518, normalized size = 5.18

$$\frac{70a^7b^{\frac{19}{2}}x^7\sqrt{\frac{ax}{b}+1}}{315a^7b^9x^3+945a^6b^{10}x^2+945a^5b^{11}x+315a^4b^{12}}+\frac{220a^6b^{\frac{21}{2}}x^6\sqrt{\frac{ax}{b}+1}}{315a^7b^9x^3+945a^6b^{10}x^2+945a^5b^{11}x+315a^4b^{12}}+\frac{}{315a^7b^9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2)*x**(7/2),x)
```

```
[Out] 70*a**7*b**(19/2)*x**7*sqrt(a*x/b + 1)/(315*a**7*b**9*x**3 + 945*a**6*b**10
*x**2 + 945*a**5*b**11*x + 315*a**4*b**12) + 220*a**6*b**(21/2)*x**6*sqrt(a
*x/b + 1)/(315*a**7*b**9*x**3 + 945*a**6*b**10*x**2 + 945*a**5*b**11*x + 31
5*a**4*b**12) + 228*a**5*b**(23/2)*x**5*sqrt(a*x/b + 1)/(315*a**7*b**9*x**3
 + 945*a**6*b**10*x**2 + 945*a**5*b**11*x + 315*a**4*b**12) + 80*a**4*b**(2
5/2)*x**4*sqrt(a*x/b + 1)/(315*a**7*b**9*x**3 + 945*a**6*b**10*x**2 + 945*a
**5*b**11*x + 315*a**4*b**12) - 10*a**3*b**(27/2)*x**3*sqrt(a*x/b + 1)/(315
*a**7*b**9*x**3 + 945*a**6*b**10*x**2 + 945*a**5*b**11*x + 315*a**4*b**12)
- 60*a**2*b**(29/2)*x**2*sqrt(a*x/b + 1)/(315*a**7*b**9*x**3 + 945*a**6*b**
10*x**2 + 945*a**5*b**11*x + 315*a**4*b**12) - 80*a*b**(31/2)*x*sqrt(a*x/b
+ 1)/(315*a**7*b**9*x**3 + 945*a**6*b**10*x**2 + 945*a**5*b**11*x + 315*a**
4*b**12) - 32*b**(33/2)*sqrt(a*x/b + 1)/(315*a**7*b**9*x**3 + 945*a**6*b**1
0*x**2 + 945*a**5*b**11*x + 315*a**4*b**12)
```

3.1753 $\int \sqrt{a+\frac{b}{x}}\, x^{5/2}\, dx$

Optimal. Leaf size=74

$$\frac{16b^2x^{3/2}\left(a+\frac{b}{x}\right)^{3/2}}{105a^3}-\frac{8bx^{5/2}\left(a+\frac{b}{x}\right)^{3/2}}{35a^2}+\frac{2x^{7/2}\left(a+\frac{b}{x}\right)^{3/2}}{7a}$$

```
[Out] 16/105*b^2*(a+b/x)^(3/2)*x^(3/2)/a^3-8/35*b*(a+b/x)^(3/2)*x^(5/2)/a^2+2/7*(
a+b/x)^(3/2)*x^(7/2)/a
```

Rubi [A] time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {271, 264}

$$\frac{16b^2x^{3/2}\left(a+\frac{b}{x}\right)^{3/2}}{105a^3}-\frac{8bx^{5/2}\left(a+\frac{b}{x}\right)^{3/2}}{35a^2}+\frac{2x^{7/2}\left(a+\frac{b}{x}\right)^{3/2}}{7a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]*x^(5/2),x]
```

```
[Out] (16*b^2*(a + b/x)^(3/2)*x^(3/2))/(105*a^3) - (8*b*(a + b/x)^(3/2)*x^(5/2))/
(35*a^2) + (2*(a + b/x)^(3/2)*x^(7/2))/(7*a)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+\frac{b}{x}}\, x^{5/2}\, dx &= \frac{2\left(a+\frac{b}{x}\right)^{3/2}x^{7/2}}{7a}-\frac{(4b)\int \sqrt{a+\frac{b}{x}}\, x^{3/2}\, dx}{7a}\\ &= -\frac{8b\left(a+\frac{b}{x}\right)^{3/2}x^{5/2}}{35a^2}+\frac{2\left(a+\frac{b}{x}\right)^{3/2}x^{7/2}}{7a}+\frac{\left(8b^2\right)\int \sqrt{a+\frac{b}{x}}\,\sqrt{x}\, dx}{35a^2}\\ &= \frac{16b^2\left(a+\frac{b}{x}\right)^{3/2}x^{3/2}}{105a^3}-\frac{8b\left(a+\frac{b}{x}\right)^{3/2}x^{5/2}}{35a^2}+\frac{2\left(a+\frac{b}{x}\right)^{3/2}x^{7/2}}{7a}\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 0.64

$$\frac{2\sqrt{x}\,\sqrt{a+\frac{b}{x}}\,(ax+b)\left(15a^2x^2-12abx+8b^2\right)}{105a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]*x^(5/2),x]
```

```
[Out] (2*Sqrt[a + b/x]*Sqrt[x]*(b + a*x)*(8*b^2 - 12*a*b*x + 15*a^2*x^2))/(105*a^
3)
```

fricas [A] time = 0.78, size = 49, normalized size = 0.66

$$\frac{2\left(15\,a^3x^3+3\,a^2bx^2-4\,ab^2x+8\,b^3\right)\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}{105\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)*x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*a^3*x^3 + 3*a^2*b*x^2 - 4*a*b^2*x + 8*b^3)*sqrt(x)*sqrt((a*x + b)
/x)/a^3
```

giac [A] time = 0.16, size = 50, normalized size = 0.68

$$-\frac{2}{105}\left(\frac{8\,b^{\frac{7}{2}}}{a^3}-\frac{15\,(ax+b)^{\frac{7}{2}}-42\,(ax+b)^{\frac{5}{2}}b+35\,(ax+b)^{\frac{3}{2}}b^2}{a^3}\right)\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)*x^(5/2),x, algorithm="giac")
```

```
[Out] -2/105*(8*b^(7/2)/a^3 - (15*(a*x + b)^(7/2) - 42*(a*x + b)^(5/2)*b + 35*(a*
x + b)^(3/2)*b^2)/a^3)*sgn(x)
```

maple [A] time = 0.00, size = 44, normalized size = 0.59

$$\frac{2\,(ax+b)\left(15a^2x^2-12abx+8b^2\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x}}{105a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(1/2)*x^(5/2),x)
```

```
[Out] 2/105*(a*x+b)*(15*a^2*x^2-12*a*b*x+8*b^2)*x^(1/2)*((a*x+b)/x)^(1/2)/a^3
```

maxima [A] time = 1.11, size = 52, normalized size = 0.70

$$\frac{2\left(15\left(a+\frac{b}{x}\right)^{\frac{7}{2}}x^{\frac{7}{2}}-42\left(a+\frac{b}{x}\right)^{\frac{5}{2}}bx^{\frac{5}{2}}+35\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^2x^{\frac{3}{2}}\right)}{105\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)*x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/105*(15*(a + b/x)^(7/2)*x^(7/2) - 42*(a + b/x)^(5/2)*b*x^(5/2) + 35*(a +
b/x)^(3/2)*b^2*x^(3/2))/a^3
```

mupad [B] time = 1.40, size = 47, normalized size = 0.64

$$\sqrt{a+\frac{b}{x}}\,\left(\frac{2\,x^{7/2}}{7}+\frac{2\,b\,x^{5/2}}{35\,a}-\frac{8\,b^2\,x^{3/2}}{105\,a^2}+\frac{16\,b^3\,\sqrt{x}}{105\,a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b/x)^(1/2),x)
```

```
[Out] (a + b/x)^(1/2)*((2*x^(7/2))/7 + (2*b*x^(5/2))/(35*a) - (8*b^2*x^(3/2))/(10
5*a^2) + (16*b^3*x^(1/2))/(105*a^3))
```

sympy [B] time = 16.44, size = 314, normalized size = 4.24

$$\frac{30a^5b^{\frac{9}{2}}x^5\sqrt{\frac{ax}{b}+1}}{105a^5b^4x^2+210a^4b^5x+105a^3b^6}+\frac{66a^4b^{\frac{11}{2}}x^4\sqrt{\frac{ax}{b}+1}}{105a^5b^4x^2+210a^4b^5x+105a^3b^6}+\frac{34a^3b^{\frac{13}{2}}x^3\sqrt{\frac{ax}{b}+1}}{105a^5b^4x^2+210a^4b^5x+105a^3b^6}+\frac{}{105a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2)*x**(5/2),x)
```

```
[Out] 30*a**5*b**(9/2)*x**5*sqrt(a*x/b + 1)/(105*a**5*b**4*x**2 + 210*a**4*b**5*x
 + 105*a**3*b**6) + 66*a**4*b**(11/2)*x**4*sqrt(a*x/b + 1)/(105*a**5*b**4*x
**2 + 210*a**4*b**5*x + 105*a**3*b**6) + 34*a**3*b**(13/2)*x**3*sqrt(a*x/b
+ 1)/(105*a**5*b**4*x**2 + 210*a**4*b**5*x + 105*a**3*b**6) + 6*a**2*b**(15
/2)*x**2*sqrt(a*x/b + 1)/(105*a**5*b**4*x**2 + 210*a**4*b**5*x + 105*a**3*b
**6) + 24*a*b**(17/2)*x*sqrt(a*x/b + 1)/(105*a**5*b**4*x**2 + 210*a**4*b**5
*x + 105*a**3*b**6) + 16*b**(19/2)*sqrt(a*x/b + 1)/(105*a**5*b**4*x**2 + 21
0*a**4*b**5*x + 105*a**3*b**6)
```

3.1754 $\int \sqrt{a+\frac{b}{x}}\, x^{3/2}\, dx$

Optimal. Leaf size=48

$$\frac{2x^{5/2}\left(a+\frac{b}{x}\right)^{3/2}}{5a}-\frac{4bx^{3/2}\left(a+\frac{b}{x}\right)^{3/2}}{15a^2}$$

```
[Out] -4/15*b*(a+b/x)^(3/2)*x^(3/2)/a^2+2/5*(a+b/x)^(3/2)*x^(5/2)/a
```

Rubi [A] time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {271, 264}

$$\frac{2x^{5/2}\left(a+\frac{b}{x}\right)^{3/2}}{5a}-\frac{4bx^{3/2}\left(a+\frac{b}{x}\right)^{3/2}}{15a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]*x^(3/2),x]
```

```
[Out] (-4*b*(a + b/x)^(3/2)*x^(3/2))/(15*a^2) + (2*(a + b/x)^(3/2)*x^(5/2))/(5*a)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+\frac{b}{x}}\, x^{3/2}\, dx &= \frac{2\left(a+\frac{b}{x}\right)^{3/2}x^{5/2}}{5a}-\frac{(2b)\int \sqrt{a+\frac{b}{x}}\,\sqrt{x}\, dx}{5a}\\ &= -\frac{4b\left(a+\frac{b}{x}\right)^{3/2}x^{3/2}}{15a^2}+\frac{2\left(a+\frac{b}{x}\right)^{3/2}x^{5/2}}{5a}\end{aligned}$$

Mathematica [A] time = 0.04, size = 36, normalized size = 0.75

$$\frac{2\sqrt{x}\,\sqrt{a+\frac{b}{x}}\,(ax+b)(3ax-2b)}{15a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]*x^(3/2),x]
```

```
[Out] (2*Sqrt[a + b/x]*Sqrt[x]*(b + a*x)*(-2*b + 3*a*x))/(15*a^2)
```

fricas [A] time = 0.70, size = 37, normalized size = 0.77

$$\frac{2\left(3a^2x^2+abx-2b^2\right)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{15a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)*x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*a^2*x^2 + a*b*x - 2*b^2)*sqrt(x)*sqrt((a*x + b)/x)/a^2
```

giac [A] time = 0.16, size = 37, normalized size = 0.77

$$\frac{2}{15}\left(\frac{2b^{\frac{5}{2}}}{a^2}+\frac{3(ax+b)^{\frac{5}{2}}-5(ax+b)^{\frac{3}{2}}b}{a^2}\right)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)*x^(3/2),x, algorithm="giac")
```

```
[Out] 2/15*(2*b^(5/2)/a^2 + (3*(a*x + b)^(5/2) - 5*(a*x + b)^(3/2)*b)/a^2)*sgn(x)
```

maple [A] time = 0.00, size = 33, normalized size = 0.69

$$\frac{2(ax+b)(3ax-2b)\sqrt{\frac{ax+b}{x}}\sqrt{x}}{15a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(1/2)*x^(3/2),x)
```

```
[Out] 2/15*(a*x+b)*(3*a*x-2*b)*x^(1/2)*((a*x+b)/x)^(1/2)/a^2
```

maxima [A] time = 1.17, size = 35, normalized size = 0.73

$$\frac{2\left(3\left(a+\frac{b}{x}\right)^{\frac{5}{2}}x^{\frac{5}{2}}-5\left(a+\frac{b}{x}\right)^{\frac{3}{2}}bx^{\frac{3}{2}}\right)}{15a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)*x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*(a + b/x)^(5/2)*x^(5/2) - 5*(a + b/x)^(3/2)*b*x^(3/2))/a^2
```

mupad [B] time = 1.40, size = 36, normalized size = 0.75

$$\sqrt{a+\frac{b}{x}}\left(\frac{2x^{5/2}}{5}+\frac{2bx^{3/2}}{15a}-\frac{4b^2\sqrt{x}}{15a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b/x)^(1/2),x)
```

```
[Out] (a + b/x)^(1/2)*((2*x^(5/2))/5 + (2*b*x^(3/2))/(15*a) - (4*b^2*x^(1/2))/(15
*a^2))
```

sympy [A] time = 5.24, size = 65, normalized size = 1.35

$$\frac{2\sqrt{b}x^2\sqrt{\frac{ax}{b}+1}}{5}+\frac{2b^{\frac{3}{2}}x\sqrt{\frac{ax}{b}+1}}{15a}-\frac{4b^{\frac{5}{2}}\sqrt{\frac{ax}{b}+1}}{15a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2)*x**(3/2),x)
```

```
[Out] 2*sqrt(b)*x**2*sqrt(a*x/b + 1)/5 + 2*b**(3/2)*x*sqrt(a*x/b + 1)/(15*a) - 4*
b**(5/2)*sqrt(a*x/b + 1)/(15*a**2)
```

3.1755 $\int \sqrt{a+\frac{b}{x}}\,\sqrt{x}\,dx$

Optimal. Leaf size=23

$$\frac{2x^{3/2}\left(a+\frac{b}{x}\right)^{3/2}}{3a}$$

```
[Out] 2/3*(a+b/x)^(3/2)*x^(3/2)/a
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {264}

$$\frac{2x^{3/2}\left(a+\frac{b}{x}\right)^{3/2}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]*Sqrt[x],x]
```

```
[Out] (2*(a + b/x)^(3/2)*x^(3/2))/(3*a)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{a+\frac{b}{x}}\,\sqrt{x}\,dx = \frac{2\left(a+\frac{b}{x}\right)^{3/2}x^{3/2}}{3a}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{2x^{3/2}\left(a+\frac{b}{x}\right)^{3/2}}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]*Sqrt[x],x]
```

```
[Out] (2*(a + b/x)^(3/2)*x^(3/2))/(3*a)
```

fricas [A] time = 0.85, size = 24, normalized size = 1.04

$$\frac{2\,(ax+b)\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(a*x + b)*sqrt(x)*sqrt((a*x + b)/x)/a
```

giac [A] time = 0.19, size = 24, normalized size = 1.04

$$\frac{2}{3}\left(\frac{(ax+b)^{\frac{3}{2}}}{a}-\frac{b^{\frac{3}{2}}}{a}\right)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)*x^(1/2),x, algorithm="giac")
[Out] 2/3*((a*x + b)^(3/2)/a - b^(3/2)/a)*sgn(x)
```

maple [A] time = 0.00, size = 25, normalized size = 1.09

$$\frac{2\,(ax+b)\,\sqrt{\frac{ax+b}{x}}\,\sqrt{x}}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(1/2)*x^(1/2),x)
[Out] 2/3*(a*x+b)*((a*x+b)/x)^(1/2)*x^(1/2)/a
```

maxima [A] time = 1.01, size = 17, normalized size = 0.74

$$\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}x^{\frac{3}{2}}}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)*x^(1/2),x, algorithm="maxima")
[Out] 2/3*(a + b/x)^(3/2)*x^(3/2)/a
```

mupad [B] time = 1.31, size = 22, normalized size = 0.96

$$\frac{2\,\sqrt{x}\,\sqrt{a+\frac{b}{x}}\,(b+a\,x)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b/x)^(1/2),x)
[Out] (2*x^(1/2)*(a + b/x)^(1/2)*(b + a*x))/(3*a)
```

sympy [B] time = 2.06, size = 39, normalized size = 1.70

$$\frac{2\sqrt{b}\,x\sqrt{\frac{ax}{b}+1}}{3}+\frac{2b^{\frac{3}{2}}\sqrt{\frac{ax}{b}+1}}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2)*x**(1/2),x)
[Out] 2*sqrt(b)*x*sqrt(a*x/b + 1)/3 + 2*b**(3/2)*sqrt(a*x/b + 1)/(3*a)
```

3.1756 $\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{x}}\,dx$

Optimal. Leaf size=49

$$2\sqrt{x}\sqrt{a+\frac{b}{x}}-2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)$$

```
[Out] -2*arctanh(b^(1/2)/(a+b/x)^(1/2)/x^(1/2))*b^(1/2)+2*(a+b/x)^(1/2)*x^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {337, 277, 217, 206}

$$2\sqrt{x}\sqrt{a+\frac{b}{x}}-2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]/Sqrt[x],x]
```

```
[Out] 2*Sqrt[a + b/x]*Sqrt[x] - 2*Sqrt[b]*ArcTanh[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x])
]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 337

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, -Dist[k/c, Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1)
 + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0] &
& FractionQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{x}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)\\ &= 2\sqrt{a+\frac{b}{x}}\,\sqrt{x} - (2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{\sqrt{x}}\right)\\ &= 2\sqrt{a+\frac{b}{x}}\,\sqrt{x} - (2b)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{b}{x}}\,\sqrt{x}}\right)\\ &= 2\sqrt{a+\frac{b}{x}}\,\sqrt{x} - 2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\,\sqrt{x}}\right)\end{aligned}$$

Mathematica [A] time = 0.12, size = 78, normalized size = 1.59

$$\frac{2\sqrt{x}\,\sqrt{a+\frac{b}{x}}\left(-\sqrt{a}\,\sqrt{b}\,\sqrt{x}\,\sqrt{\frac{b}{ax}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}\,\sqrt{x}}\right)+ax+b\right)}{ax+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]/Sqrt[x],x]
```

```
[Out] (2*Sqrt[a + b/x]*Sqrt[x]*(b + a*x - Sqrt[a]*Sqrt[b]*Sqrt[1 + b/(a*x)]*Sqrt[
x]*ArcSinh[Sqrt[b]/(Sqrt[a]*Sqrt[x])]))/(b + a*x)
```

fricas [A] time = 0.89, size = 101, normalized size = 2.06

$$\left[\sqrt{b}\,\log\left(\frac{ax-2\,\sqrt{b}\,\sqrt{x}\,\sqrt{\frac{ax+b}{x}}+2\,b}{x}\right)+2\,\sqrt{x}\,\sqrt{\frac{ax+b}{x}}\,,2\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}{b}\right)+2\,\sqrt{x}\,\sqrt{\frac{ax+b}{x}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(b)*log((a*x - 2*sqrt(b)*sqrt(x)*sqrt((a*x + b)/x) + 2*b)/x) + 2*sqrt(
x)*sqrt((a*x + b)/x), 2*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)*sqrt((a*x + b)/x)/
b) + 2*sqrt(x)*sqrt((a*x + b)/x)]
```

giac [A] time = 0.18, size = 62, normalized size = 1.27

$$2\left(\frac{b\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}+\sqrt{ax+b}-\frac{b\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+\sqrt{-b}\,\sqrt{b}}{\sqrt{-b}}\right)\text{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*(b*arctan(sqrt(a*x + b)/sqrt(-b))/sqrt(-b) + sqrt(a*x + b) - (b*arctan(sq
rt(b)/sqrt(-b)) + sqrt(-b)*sqrt(b))/sqrt(-b))*sgn(x)
```

maple [A] time = 0.01, size = 50, normalized size = 1.02

$$-\frac{2\sqrt{\frac{ax+b}{x}}\,\left(\sqrt{b}\,\,\text{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)-\sqrt{ax+b}\right)\sqrt{x}}{\sqrt{ax+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(1/2)/x^(1/2),x)
```

```
[Out] -2*((a*x+b)/x)^(1/2)*x^(1/2)*(b^(1/2)*arctanh((a*x+b)^(1/2)/b^(1/2))-(a*x+b
)^(1/2))/(a*x+b)^(1/2)
```

maxima [A] time = 2.42, size = 59, normalized size = 1.20

$$\sqrt{b}\,\log\left(\frac{\sqrt{a+\frac{b}{x}}\,\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\,\sqrt{x}+\sqrt{b}}\right)+2\sqrt{a+\frac{b}{x}}\,\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(b)*log((sqrt(a + b/x)*sqrt(x) - sqrt(b))/(sqrt(a + b/x)*sqrt(x) + sqrt
(b))) + 2*sqrt(a + b/x)*sqrt(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(1/2)/x^(1/2),x)
```

```
[Out] int((a + b/x)^(1/2)/x^(1/2), x)
```

sympy [A] time = 2.41, size = 68, normalized size = 1.39

$$\frac{2\sqrt{a}\,\sqrt{x}}{\sqrt{1+\frac{b}{ax}}}-2\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\,\sqrt{x}}\right)+\frac{2b}{\sqrt{a}\,\sqrt{x}\,\sqrt{1+\frac{b}{ax}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2)/x**(1/2),x)
```

```
[Out] 2*sqrt(a)*sqrt(x)/sqrt(1 + b/(a*x)) - 2*sqrt(b)*asinh(sqrt(b)/(sqrt(a)*sqrt
(x))) + 2*b/(sqrt(a)*sqrt(x)*sqrt(1 + b/(a*x)))
```

3.1757 $\int \frac{\sqrt{a+\frac{b}{x}}}{x^{3/2}}\,dx$

Optimal. Leaf size=50

$$-\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{x}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{\sqrt{b}}$$

```
[Out] -a*arctanh(b^(1/2)/(a+b/x)^(1/2)/x^(1/2))/b^(1/2)-(a+b/x)^(1/2)/x^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {337, 195, 217, 206}

$$-\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{x}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]/x^(3/2),x]
```

```
[Out] -(Sqrt[a + b/x]/Sqrt[x]) - (a*ArcTanh[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x])])/Sqr
t[b]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 337

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, -Dist[k/c, Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1)
 + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0] &
& FractionQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+\frac{b}{x}}}{x^{3/2}}\,dx &= -\left(2\,\text{Subst}\left(\int \sqrt{a+bx^2}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)\\ &= -\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{x}} - a\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{\sqrt{x}}\right)\\ &= -\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{x}} - a\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)\\ &= -\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{x}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.23, size = 74, normalized size = 1.48

$$\frac{\sqrt{a+\frac{b}{x}}\left(-\frac{a^{3/2}x^{3/2}\sqrt{\frac{b}{ax}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right)}{\sqrt{b}\,(ax+b)}-1\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]/x^(3/2),x]
```

```
[Out] (Sqrt[a + b/x]*(-1 - (a^(3/2)*Sqrt[1 + b/(a*x)]*x^(3/2)*ArcSinh[Sqrt[b]/(Sq
rt[a]*Sqrt[x])])/(Sqrt[b]*(b + a*x))))/Sqrt[x]
```

fricas [A] time = 0.95, size = 121, normalized size = 2.42

$$\left[\frac{a\sqrt{b}\,x\log\left(\frac{ax-2\sqrt{b}\sqrt{x}\sqrt{\frac{ax+b}{x}}+2b}{x}\right)-2b\sqrt{x}\sqrt{\frac{ax+b}{x}}}{2bx}, \frac{a\sqrt{-b}\,x\arctan\left(\frac{\sqrt{-b}\sqrt{x}\sqrt{\frac{ax+b}{x}}}{b}\right)-b\sqrt{x}\sqrt{\frac{ax+b}{x}}}{bx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(a*sqrt(b)*x*log((a*x - 2*sqrt(b)*sqrt(x)*sqrt((a*x + b)/x) + 2*b)/x)
- 2*b*sqrt(x)*sqrt((a*x + b)/x))/(b*x), (a*sqrt(-b)*x*arctan(sqrt(-b)*sqrt(
x)*sqrt((a*x + b)/x)/b) - b*sqrt(x)*sqrt((a*x + b)/x))/(b*x)]
```

giac [A] time = 0.26, size = 43, normalized size = 0.86

$$\frac{\left(\frac{a^2\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}-\frac{\sqrt{ax+b}\,a}{x}\right)\text{sgn}(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] (a^2*arctan(sqrt(a*x + b)/sqrt(-b))/sqrt(-b) - sqrt(a*x + b)*a/x)*sgn(x)/a
```

maple [A] time = 0.02, size = 54, normalized size = 1.08

$$-\frac{\sqrt{\frac{ax+b}{x}}\ \left(ax\,\text{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)+\sqrt{ax+b}\ \sqrt{b}\right)}{\sqrt{ax+b}\ \sqrt{b}\ \sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(1/2)/x^(3/2),x)
```

```
[Out] -((a*x+b)/x)^(1/2)*(arctanh((a*x+b)^(1/2)/b^(1/2))*x*a+(a*x+b)^(1/2)*b^(1/2
))/x^(1/2)/(a*x+b)^(1/2)/b^(1/2)
```

maxima [B] time = 2.34, size = 77, normalized size = 1.54

$$\frac{a\log\left(\frac{\sqrt{a+\frac{b}{x}}\,\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\,\sqrt{x}+\sqrt{b}}\right)}{2\sqrt{b}}-\frac{\sqrt{a+\frac{b}{x}}\,a\sqrt{x}}{\left(a+\frac{b}{x}\right)x-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*a*log((sqrt(a + b/x)*sqrt(x) - sqrt(b))/(sqrt(a + b/x)*sqrt(x) + sqrt(b
)))/sqrt(b) - sqrt(a + b/x)*a*sqrt(x)/((a + b/x)*x - b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(1/2)/x^(3/2),x)
```

```
[Out] int((a + b/x)^(1/2)/x^(3/2), x)
```

sympy [A] time = 3.48, size = 44, normalized size = 0.88

$$-\frac{\sqrt{a}\,\sqrt{1+\frac{b}{ax}}}{\sqrt{x}}-\frac{a\,\text{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\,\sqrt{x}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2)/x**(3/2),x)
```

```
[Out] -sqrt(a)*sqrt(1 + b/(a*x))/sqrt(x) - a*asinh(sqrt(b)/(sqrt(a)*sqrt(x)))/sqr
t(b)
```

3.1758 $\int \frac{\sqrt{a+\frac{b}{x}}}{x^{5/2}}\,dx$

Optimal. Leaf size=80

$$\frac{a^2 \tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{4b^{3/2}} - \frac{\sqrt{a+\frac{b}{x}}}{2x^{3/2}} - \frac{a\sqrt{a+\frac{b}{x}}}{4b\sqrt{x}}$$

```
[Out] 1/4*a^2*arctanh(b^(1/2)/(a+b/x)^(1/2)/x^(1/2))/b^(3/2)-1/2*(a+b/x)^(1/2)/x^
(3/2)-1/4*a*(a+b/x)^(1/2)/b/x^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {337, 279, 321, 217, 206}

$$\frac{a^2 \tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{4b^{3/2}} - \frac{\sqrt{a+\frac{b}{x}}}{2x^{3/2}} - \frac{a\sqrt{a+\frac{b}{x}}}{4b\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]/x^(5/2),x]
```

```
[Out] -Sqrt[a + b/x]/(2*x^(3/2)) - (a*Sqrt[a + b/x])/(4*b*Sqrt[x]) + (a^2*ArcTanh
[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x])])/(4*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 337

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, -Dist[k/c, Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1)
 + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0] &
```

```
& FractionQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+\frac{b}{x}}}{x^{5/2}}\,dx &= -\left(2\,\text{Subst}\left(\int x^2\sqrt{a+bx^2}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)\\
&= -\frac{\sqrt{a+\frac{b}{x}}}{2x^{3/2}} - \frac{1}{2}a\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{\sqrt{x}}\right)\\
&= -\frac{\sqrt{a+\frac{b}{x}}}{2x^{3/2}} - \frac{a\sqrt{a+\frac{b}{x}}}{4b\sqrt{x}} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{\sqrt{x}}\right)}{4b}\\
&= -\frac{\sqrt{a+\frac{b}{x}}}{2x^{3/2}} - \frac{a\sqrt{a+\frac{b}{x}}}{4b\sqrt{x}} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)}{4b}\\
&= -\frac{\sqrt{a+\frac{b}{x}}}{2x^{3/2}} - \frac{a\sqrt{a+\frac{b}{x}}}{4b\sqrt{x}} + \frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)}{4b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 77, normalized size = 0.96

$$\frac{\sqrt{a+\frac{b}{x}}\left(\frac{a^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right)}{\sqrt{\frac{b}{ax}+1}} - \frac{\sqrt{b}(ax+2b)}{x^{3/2}}\right)}{4b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]/x^(5/2),x]
```

```
[Out] (Sqrt[a + b/x]*(-((Sqrt[b]*(2*b + a*x))/x^(3/2)) + (a^(3/2)*ArcSinh[Sqrt[b]
/(Sqrt[a]*Sqrt[x])])/Sqrt[1 + b/(a*x)]))/(4*b^(3/2))
```

fricas [A] time = 0.88, size = 147, normalized size = 1.84

$$\left[\frac{a^2\sqrt{b}\,x^2\log\left(\frac{ax+2\sqrt{b}\sqrt{x}\sqrt{\frac{ax+b}{x}}+2b}{x}\right) - 2\left(abx+2b^2\right)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{8b^2x^2}, -\frac{a^2\sqrt{-b}\,x^2\arctan\left(\frac{\sqrt{-b}\sqrt{x}\sqrt{\frac{ax+b}{x}}}{b}\right) + \left(abx+2b^2\right)\sqrt{}}{4b^2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/8*(a^2*sqrt(b)*x^2*log((a*x + 2*sqrt(b)*sqrt(x)*sqrt((a*x + b)/x) + 2*b)
/x) - 2*(a*b*x + 2*b^2)*sqrt(x)*sqrt((a*x + b)/x))/(b^2*x^2), -1/4*(a^2*sqr
t(-b)*x^2*arctan(sqrt(-b)*sqrt(x)*sqrt((a*x + b)/x)/b) + (a*b*x + 2*b^2)*sq
rt(x)*sqrt((a*x + b)/x))/(b^2*x^2)]
```

giac [A] time = 0.29, size = 68, normalized size = 0.85

$$-\frac{\left(\frac{a^3\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b} + \frac{(ax+b)^{\frac{3}{2}}a^3+\sqrt{ax+b}\,a^3b}{a^2bx^2}\right)\text{sgn}(x)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] -1/4*(a^3*arctan(sqrt(a*x + b)/sqrt(-b))/(sqrt(-b)*b) + ((a*x + b)^(3/2)*a^
3 + sqrt(a*x + b)*a^3*b)/(a^2*b*x^2))*sgn(x)/a
```

maple [A] time = 0.02, size = 73, normalized size = 0.91

$$-\frac{\sqrt{\frac{ax+b}{x}}\left(-a^2x^2\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)+\sqrt{ax+b}\,a\sqrt{b}\,x+2\sqrt{ax+b}\,b^{\frac{3}{2}}\right)}{4\sqrt{ax+b}\,b^{\frac{3}{2}}x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(1/2)/x^(5/2),x)
```

```
[Out] -1/4*((a*x+b)/x)^(1/2)*(-arctanh((a*x+b)^(1/2)/b^(1/2))*a^2*x^2+2*b^(3/2)*(
a*x+b)^(1/2)+x*a*(a*x+b)^(1/2)*b^(1/2))/x^(3/2)/b^(3/2)/(a*x+b)^(1/2)
```

maxima [B] time = 2.30, size = 118, normalized size = 1.48

$$-\frac{a^2\log\left(\frac{\sqrt{a+\frac{b}{x}}\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}+\sqrt{b}}\right)}{8\,b^{\frac{3}{2}}}-\frac{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2x^{\frac{3}{2}}+\sqrt{a+\frac{b}{x}}\,a^2b\sqrt{x}}{4\left(\left(a+\frac{b}{x}\right)^2bx^2-2\left(a+\frac{b}{x}\right)b^2x+b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -1/8*a^2*log((sqrt(a + b/x)*sqrt(x) - sqrt(b))/(sqrt(a + b/x)*sqrt(x) + sqr
t(b)))/b^(3/2) - 1/4*((a + b/x)^(3/2)*a^2*x^(3/2) + sqrt(a + b/x)*a^2*b*sqr
t(x))/((a + b/x)^2*b*x^2 - 2*(a + b/x)*b^2*x + b^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(1/2)/x^(5/2),x)
```

```
[Out] int((a + b/x)^(1/2)/x^(5/2), x)
```

sympy [A] time = 8.94, size = 97, normalized size = 1.21

$$-\frac{a^{\frac{3}{2}}}{4b\sqrt{x}\sqrt{1+\frac{b}{ax}}}-\frac{3\sqrt{a}}{4x^{\frac{3}{2}}\sqrt{1+\frac{b}{ax}}}+\frac{a^2\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right)}{4b^{\frac{3}{2}}}-\frac{b}{2\sqrt{a}\,x^{\frac{5}{2}}\sqrt{1+\frac{b}{ax}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2)/x**(5/2),x)
```

```
[Out] -a**(3/2)/(4*b*sqrt(x)*sqrt(1 + b/(a*x))) - 3*sqrt(a)/(4*x**(3/2)*sqrt(1 +
b/(a*x))) + a**2*asinh(sqrt(b)/(sqrt(a)*sqrt(x)))/(4*b**(3/2)) - b/(2*sqrt(
a)*x**(5/2)*sqrt(1 + b/(a*x)))
```

3.1759 $\int \frac{\sqrt{a+\frac{b}{x}}}{x^{7/2}}\,dx$

Optimal. Leaf size=106

$$-\frac{a^3 \tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{8b^{5/2}}+\frac{a^2\sqrt{a+\frac{b}{x}}}{8b^2\sqrt{x}}-\frac{a\sqrt{a+\frac{b}{x}}}{12bx^{3/2}}-\frac{\sqrt{a+\frac{b}{x}}}{3x^{5/2}}$$

```
[Out] -1/8*a^3*arctanh(b^(1/2)/(a+b/x)^(1/2)/x^(1/2))/b^(5/2)-1/3*(a+b/x)^(1/2)/x
^(5/2)-1/12*a*(a+b/x)^(1/2)/b/x^(3/2)+1/8*a^2*(a+b/x)^(1/2)/b^2/x^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {337, 279, 321, 217, 206}

$$\frac{a^2\sqrt{a+\frac{b}{x}}}{8b^2\sqrt{x}}-\frac{a^3 \tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{8b^{5/2}}-\frac{a\sqrt{a+\frac{b}{x}}}{12bx^{3/2}}-\frac{\sqrt{a+\frac{b}{x}}}{3x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]/x^(7/2),x]
```

```
[Out] -Sqrt[a + b/x]/(3*x^(5/2)) - (a*Sqrt[a + b/x])/(12*b*x^(3/2)) + (a^2*Sqrt[a
 + b/x])/(8*b^2*Sqrt[x]) - (a^3*ArcTanh[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x])])/(
8*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 337

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, -Dist[k/c, Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1)
```

```
 + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0] &
& FractionQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+\frac{b}{x}}}{x^{7/2}}\,dx &= -\left(2\,\text{Subst}\left(\int x^4\sqrt{a+bx^2}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)\\
&= -\frac{\sqrt{a+\frac{b}{x}}}{3x^{5/2}} - \frac{1}{3}a\,\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{\sqrt{x}}\right)\\
&= -\frac{\sqrt{a+\frac{b}{x}}}{3x^{5/2}} - \frac{a\sqrt{a+\frac{b}{x}}}{12bx^{3/2}} + \frac{a^2\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{\sqrt{x}}\right)}{4b}\\
&= -\frac{\sqrt{a+\frac{b}{x}}}{3x^{5/2}} - \frac{a\sqrt{a+\frac{b}{x}}}{12bx^{3/2}} + \frac{a^2\sqrt{a+\frac{b}{x}}}{8b^2\sqrt{x}} - \frac{a^3\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{\sqrt{x}}\right)}{8b^2}\\
&= -\frac{\sqrt{a+\frac{b}{x}}}{3x^{5/2}} - \frac{a\sqrt{a+\frac{b}{x}}}{12bx^{3/2}} + \frac{a^2\sqrt{a+\frac{b}{x}}}{8b^2\sqrt{x}} - \frac{a^3\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{b}{x}}\,\sqrt{x}}\right)}{8b^2}\\
&= -\frac{\sqrt{a+\frac{b}{x}}}{3x^{5/2}} - \frac{a\sqrt{a+\frac{b}{x}}}{12bx^{3/2}} + \frac{a^2\sqrt{a+\frac{b}{x}}}{8b^2\sqrt{x}} - \frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\,\sqrt{x}}\right)}{8b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.27, size = 89, normalized size = 0.84

$$\frac{\sqrt{a+\frac{b}{x}}\left(\frac{\sqrt{b}\left(3a^2x^2-2abx-8b^2\right)}{x^{5/2}} - \frac{3a^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}\,\sqrt{x}}\right)}{\sqrt{\frac{b}{ax}+1}}\right)}{24b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]/x^(7/2),x]
```

```
[Out] (Sqrt[a + b/x]*((Sqrt[b]*(-8*b^2 - 2*a*b*x + 3*a^2*x^2))/x^(5/2) - (3*a^(5/
2)*ArcSinh[Sqrt[b]/(Sqrt[a]*Sqrt[x])])/Sqrt[1 + b/(a*x)]))/(24*b^(5/2))
```

fricas [A] time = 0.73, size = 173, normalized size = 1.63

$$\left[\frac{3\,a^3\sqrt{b}\,x^3\log\left(\frac{ax-2\,\sqrt{b}\,\sqrt{x}\,\sqrt{\frac{ax+b}{x}}+2\,b}{x}\right) + 2\left(3\,a^2bx^2 - 2\,ab^2x - 8\,b^3\right)\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}{48\,b^3x^3},\ \frac{3\,a^3\sqrt{-b}\,x^3\arctan\left(\frac{\sqrt{-b}\,\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}{b}\right.}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*a^3*sqrt(b)*x^3*log((a*x - 2*sqrt(b)*sqrt(x)*sqrt((a*x + b)/x) + 2
*b)/x) + 2*(3*a^2*b*x^2 - 2*a*b^2*x - 8*b^3)*sqrt(x)*sqrt((a*x + b)/x))/(b^
3*x^3), 1/24*(3*a^3*sqrt(-b)*x^3*arctan(sqrt(-b)*sqrt(x)*sqrt((a*x + b)/x)/
b) + (3*a^2*b*x^2 - 2*a*b^2*x - 8*b^3)*sqrt(x)*sqrt((a*x + b)/x))/(b^3*x^3)
]
```

giac [A] time = 0.32, size = 86, normalized size = 0.81

$$\frac{\left(\frac{3\,a^4\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b^2}+\frac{3\,(ax+b)^{\frac{5}{2}}a^4-8\,(ax+b)^{\frac{3}{2}}a^4b-3\,\sqrt{ax+b}\,a^4b^2}{a^3b^2x^3}\right)\mathrm{sgn}(x)}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] 1/24*(3*a^4*arctan(sqrt(a*x + b)/sqrt(-b))/(sqrt(-b)*b^2) + (3*(a*x + b)^(5
/2)*a^4 - 8*(a*x + b)^(3/2)*a^4*b - 3*sqrt(a*x + b)*a^4*b^2)/(a^3*b^2*x^3))
*sgn(x)/a
```

maple [A] time = 0.02, size = 92, normalized size = 0.87

$$-\frac{\sqrt{\frac{ax+b}{x}}\,\left(3a^3x^3\,\mathrm{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)-3\sqrt{ax+b}\;a^2\sqrt{b}\;x^2+2\sqrt{ax+b}\;a\,b^{\frac{3}{2}}x+8\sqrt{ax+b}\;b^{\frac{5}{2}}\right)}{24\sqrt{ax+b}\;b^{\frac{5}{2}}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(1/2)/x^(7/2),x)
```

```
[Out] -1/24*((a*x+b)/x)^(1/2)/x^(5/2)/b^(5/2)*(3*arctanh((a*x+b)^(1/2)/b^(1/2))*x
^3*a^3-3*x^2*a^2*b^(1/2)*(a*x+b)^(1/2)+2*x*a*b^(3/2)*(a*x+b)^(1/2)+8*b^(5/2
)*(a*x+b)^(1/2))/(a*x+b)^(1/2)
```

maxima [B] time = 2.35, size = 161, normalized size = 1.52

$$\frac{a^3\log\left(\frac{\sqrt{a+\frac{b}{x}}\,\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\,\sqrt{x}+\sqrt{b}}\right)}{16\,b^{\frac{5}{2}}}+\frac{3\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^3x^{\frac{5}{2}}-8\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^3bx^{\frac{3}{2}}-3\,\sqrt{a+\frac{b}{x}}\,a^3b^2\sqrt{x}}{24\left(\left(a+\frac{b}{x}\right)^3b^2x^3-3\left(a+\frac{b}{x}\right)^2b^3x^2+3\left(a+\frac{b}{x}\right)b^4x-b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/x^(7/2),x, algorithm="maxima")
```

```
[Out] 1/16*a^3*log((sqrt(a + b/x)*sqrt(x) - sqrt(b))/(sqrt(a + b/x)*sqrt(x) + sqr
t(b)))/b^(5/2) + 1/24*(3*(a + b/x)^(5/2)*a^3*x^(5/2) - 8*(a + b/x)^(3/2)*a^
3*b*x^(3/2) - 3*sqrt(a + b/x)*a^3*b^2*sqrt(x))/((a + b/x)^3*b^2*x^3 - 3*(a
+ b/x)^2*b^3*x^2 + 3*(a + b/x)*b^4*x - b^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(1/2)/x^(7/2),x)
```

```
[Out] int((a + b/x)^(1/2)/x^(7/2), x)
```

sympy [A] time = 20.97, size = 122, normalized size = 1.15

$$\frac{a^{\frac{5}{2}}}{8b^2\sqrt{x}\,\sqrt{1+\frac{b}{ax}}}+\frac{a^{\frac{3}{2}}}{24bx^{\frac{3}{2}}\sqrt{1+\frac{b}{ax}}}-\frac{5\sqrt{a}}{12x^{\frac{5}{2}}\sqrt{1+\frac{b}{ax}}}-\frac{a^3\,\mathrm{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\,\sqrt{x}}\right)}{8b^{\frac{5}{2}}}-\frac{b}{3\sqrt{a}\,x^{\frac{7}{2}}\sqrt{1+\frac{b}{ax}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2)/x**(7/2),x)
```

```
[Out] a**(5/2)/(8*b**2*sqrt(x)*sqrt(1 + b/(a*x))) + a**(3/2)/(24*b*x**(3/2)*sqrt(
1 + b/(a*x))) - 5*sqrt(a)/(12*x**(5/2)*sqrt(1 + b/(a*x))) - a**3*asinh(sqrt
(b)/(sqrt(a)*sqrt(x)))/(8*b**(5/2)) - b/(3*sqrt(a)*x**(7/2)*sqrt(1 + b/(a*x
)))
```

3.1760 $\int \left(a+\frac{b}{x}\right)^{3/2} x^{9/2}\,dx$

Optimal. Leaf size=100

$$-\frac{32b^3x^{5/2}\left(a+\frac{b}{x}\right)^{5/2}}{1155a^4}+\frac{16b^2x^{7/2}\left(a+\frac{b}{x}\right)^{5/2}}{231a^3}-\frac{4bx^{9/2}\left(a+\frac{b}{x}\right)^{5/2}}{33a^2}+\frac{2x^{11/2}\left(a+\frac{b}{x}\right)^{5/2}}{11a}$$

```
[Out] -32/1155*b^3*(a+b/x)^(5/2)*x^(5/2)/a^4+16/231*b^2*(a+b/x)^(5/2)*x^(7/2)/a^3
-4/33*b*(a+b/x)^(5/2)*x^(9/2)/a^2+2/11*(a+b/x)^(5/2)*x^(11/2)/a
```

Rubi [A] time = 0.03, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {271, 264}

$$\frac{16b^2x^{7/2}\left(a+\frac{b}{x}\right)^{5/2}}{231a^3}-\frac{32b^3x^{5/2}\left(a+\frac{b}{x}\right)^{5/2}}{1155a^4}-\frac{4bx^{9/2}\left(a+\frac{b}{x}\right)^{5/2}}{33a^2}+\frac{2x^{11/2}\left(a+\frac{b}{x}\right)^{5/2}}{11a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)*x^(9/2),x]
```

```
[Out] (-32*b^3*(a + b/x)^(5/2)*x^(5/2))/(1155*a^4) + (16*b^2*(a + b/x)^(5/2)*x^(7
/2))/(231*a^3) - (4*b*(a + b/x)^(5/2)*x^(9/2))/(33*a^2) + (2*(a + b/x)^(5/2
)*x^(11/2))/(11*a)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \left(a+\frac{b}{x}\right)^{3/2} x^{9/2}\,dx &= \frac{2\left(a+\frac{b}{x}\right)^{5/2}x^{11/2}}{11a}-\frac{(6b)\int \left(a+\frac{b}{x}\right)^{3/2}x^{7/2}\,dx}{11a}\\
&= -\frac{4b\left(a+\frac{b}{x}\right)^{5/2}x^{9/2}}{33a^2}+\frac{2\left(a+\frac{b}{x}\right)^{5/2}x^{11/2}}{11a}+\frac{\left(8b^2\right)\int \left(a+\frac{b}{x}\right)^{3/2}x^{5/2}\,dx}{33a^2}\\
&= \frac{16b^2\left(a+\frac{b}{x}\right)^{5/2}x^{7/2}}{231a^3}-\frac{4b\left(a+\frac{b}{x}\right)^{5/2}x^{9/2}}{33a^2}+\frac{2\left(a+\frac{b}{x}\right)^{5/2}x^{11/2}}{11a}-\frac{\left(16b^3\right)\int \left(a+\frac{b}{x}\right)^{3/2}x^{3/2}\,dx}{231a^3}\\
&= -\frac{32b^3\left(a+\frac{b}{x}\right)^{5/2}x^{5/2}}{1155a^4}+\frac{16b^2\left(a+\frac{b}{x}\right)^{5/2}x^{7/2}}{231a^3}-\frac{4b\left(a+\frac{b}{x}\right)^{5/2}x^{9/2}}{33a^2}+\frac{2\left(a+\frac{b}{x}\right)^{5/2}x^{11/2}}{11a}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 60, normalized size = 0.60

$$\frac{2\sqrt{x}\sqrt{a+\frac{b}{x}}(ax+b)^2\left(105a^3x^3-70a^2bx^2+40ab^2x-16b^3\right)}{1155a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)*x^(9/2),x]
```

```
[Out] (2*Sqrt[a + b/x]*Sqrt[x]*(b + a*x)^2*(-16*b^3 + 40*a*b^2*x - 70*a^2*b*x^2 +
 105*a^3*x^3))/(1155*a^4)
```

fricas [A] time = 0.68, size = 71, normalized size = 0.71

$$\frac{2\left(105\,a^5x^5+140\,a^4bx^4+5\,a^3b^2x^3-6\,a^2b^3x^2+8\,ab^4x-16\,b^5\right)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{1155\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*x^(9/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(105*a^5*x^5 + 140*a^4*b*x^4 + 5*a^3*b^2*x^3 - 6*a^2*b^3*x^2 + 8*a*b
^4*x - 16*b^5)*sqrt(x)*sqrt((a*x + b)/x)/a^4
```

giac [A] time = 0.20, size = 138, normalized size = 1.38

$$\frac{2}{315}b\left(\frac{16\,b^{\frac{9}{2}}}{a^4}+\frac{35\,(ax+b)^{\frac{9}{2}}-135\,(ax+b)^{\frac{7}{2}}b+189\,(ax+b)^{\frac{5}{2}}b^2-105\,(ax+b)^{\frac{3}{2}}b^3}{a^4}\right)\mathrm{sgn}(x)-\frac{2}{3465}a\left(\frac{128\,b^{\frac{11}{2}}}{a^5}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*x^(9/2),x, algorithm="giac")
```

```
[Out] 2/315*b*(16*b^(9/2)/a^4 + (35*(a*x + b)^(9/2) - 135*(a*x + b)^(7/2)*b + 189
*(a*x + b)^(5/2)*b^2 - 105*(a*x + b)^(3/2)*b^3)/a^4)*sgn(x) - 2/3465*a*(128
*b^(11/2)/a^5 - (315*(a*x + b)^(11/2) - 1540*(a*x + b)^(9/2)*b + 2970*(a*x
+ b)^(7/2)*b^2 - 2772*(a*x + b)^(5/2)*b^3 + 1155*(a*x + b)^(3/2)*b^4)/a^5)*
sgn(x)
```

maple [A] time = 0.00, size = 55, normalized size = 0.55

$$\frac{2\,(ax+b)\left(105a^3x^3-70a^2b\,x^2+40a\,b^2x-16b^3\right)\left(\frac{ax+b}{x}\right)^{\frac{3}{2}}x^{\frac{3}{2}}}{1155a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(3/2)*x^(9/2),x)
```

```
[Out] 2/1155*(a*x+b)*(105*a^3*x^3-70*a^2*b*x^2+40*a*b^2*x-16*b^3)*x^(3/2)*((a*x+b
)/x)^(3/2)/a^4
```

maxima [A] time = 0.99, size = 69, normalized size = 0.69

$$\frac{2\left(105\left(a+\frac{b}{x}\right)^{\frac{11}{2}}x^{\frac{11}{2}}-385\left(a+\frac{b}{x}\right)^{\frac{9}{2}}bx^{\frac{9}{2}}+495\left(a+\frac{b}{x}\right)^{\frac{7}{2}}b^2x^{\frac{7}{2}}-231\left(a+\frac{b}{x}\right)^{\frac{5}{2}}b^3x^{\frac{5}{2}}\right)}{1155\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*x^(9/2),x, algorithm="maxima")
```

```
[Out] 2/1155*(105*(a + b/x)^(11/2)*x^(11/2) - 385*(a + b/x)^(9/2)*b*x^(9/2) + 495
*(a + b/x)^(7/2)*b^2*x^(7/2) - 231*(a + b/x)^(5/2)*b^3*x^(5/2))/a^4
```

mupad [B] time = 1.46, size = 67, normalized size = 0.67

$$\sqrt{a+\frac{b}{x}}\left(\frac{2\,a\,x^{11/2}}{11}+\frac{8\,b\,x^{9/2}}{33}+\frac{2\,b^2\,x^{7/2}}{231\,a}-\frac{4\,b^3\,x^{5/2}}{385\,a^2}+\frac{16\,b^4\,x^{3/2}}{1155\,a^3}-\frac{32\,b^5\,\sqrt{x}}{1155\,a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)*(a + b/x)^(3/2),x)
```

```
[Out] (a + b/x)^(1/2)*((2*a*x^(11/2))/11 + (8*b*x^(9/2))/33 + (2*b^2*x^(7/2))/(23
1*a) - (4*b^3*x^(5/2))/(385*a^2) + (16*b^4*x^(3/2))/(1155*a^3) - (32*b^5*x^
(1/2))/(1155*a^4))
```

sympy [B] time = 171.29, size = 585, normalized size = 5.85

$$\frac{210a^8b^{\frac{19}{2}}x^8\sqrt{\frac{ax}{b}+1}}{1155a^7b^9x^3+3465a^6b^{10}x^2+3465a^5b^{11}x+1155a^4b^{12}}+\frac{910a^7b^{\frac{21}{2}}x^7\sqrt{\frac{ax}{b}+1}}{1155a^7b^9x^3+3465a^6b^{10}x^2+3465a^5b^{11}x+1155a^4b^{12}}+\frac{}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)*x**(9/2),x)
```

```
[Out] 210*a**8*b**(19/2)*x**8*sqrt(a*x/b + 1)/(1155*a**7*b**9*x**3 + 3465*a**6*b*
*10*x**2 + 3465*a**5*b**11*x + 1155*a**4*b**12) + 910*a**7*b**(21/2)*x**7*s
qrt(a*x/b + 1)/(1155*a**7*b**9*x**3 + 3465*a**6*b**10*x**2 + 3465*a**5*b**1
1*x + 1155*a**4*b**12) + 1480*a**6*b**(23/2)*x**6*sqrt(a*x/b + 1)/(1155*a**
7*b**9*x**3 + 3465*a**6*b**10*x**2 + 3465*a**5*b**11*x + 1155*a**4*b**12) +
 1068*a**5*b**(25/2)*x**5*sqrt(a*x/b + 1)/(1155*a**7*b**9*x**3 + 3465*a**6*
b**10*x**2 + 3465*a**5*b**11*x + 1155*a**4*b**12) + 290*a**4*b**(27/2)*x**4
*sqrt(a*x/b + 1)/(1155*a**7*b**9*x**3 + 3465*a**6*b**10*x**2 + 3465*a**5*b*
*11*x + 1155*a**4*b**12) - 10*a**3*b**(29/2)*x**3*sqrt(a*x/b + 1)/(1155*a**
7*b**9*x**3 + 3465*a**6*b**10*x**2 + 3465*a**5*b**11*x + 1155*a**4*b**12) -
 60*a**2*b**(31/2)*x**2*sqrt(a*x/b + 1)/(1155*a**7*b**9*x**3 + 3465*a**6*b*
*10*x**2 + 3465*a**5*b**11*x + 1155*a**4*b**12) - 80*a*b**(33/2)*x*sqrt(a*x
/b + 1)/(1155*a**7*b**9*x**3 + 3465*a**6*b**10*x**2 + 3465*a**5*b**11*x + 1
155*a**4*b**12) - 32*b**(35/2)*sqrt(a*x/b + 1)/(1155*a**7*b**9*x**3 + 3465*
a**6*b**10*x**2 + 3465*a**5*b**11*x + 1155*a**4*b**12)
```

3.1761 $\int \left(a+\frac{b}{x}\right)^{3/2} x^{7/2}\,dx$

Optimal. Leaf size=74

$$\frac{16b^2x^{5/2}\left(a+\frac{b}{x}\right)^{5/2}}{315a^3}-\frac{8bx^{7/2}\left(a+\frac{b}{x}\right)^{5/2}}{63a^2}+\frac{2x^{9/2}\left(a+\frac{b}{x}\right)^{5/2}}{9a}$$

```
[Out] 16/315*b^2*(a+b/x)^(5/2)*x^(5/2)/a^3-8/63*b*(a+b/x)^(5/2)*x^(7/2)/a^2+2/9*(
a+b/x)^(5/2)*x^(9/2)/a
```

Rubi [A] time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {271, 264}

$$\frac{16b^2x^{5/2}\left(a+\frac{b}{x}\right)^{5/2}}{315a^3}-\frac{8bx^{7/2}\left(a+\frac{b}{x}\right)^{5/2}}{63a^2}+\frac{2x^{9/2}\left(a+\frac{b}{x}\right)^{5/2}}{9a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)*x^(7/2),x]
```

```
[Out] (16*b^2*(a + b/x)^(5/2)*x^(5/2))/(315*a^3) - (8*b*(a + b/x)^(5/2)*x^(7/2))/
(63*a^2) + (2*(a + b/x)^(5/2)*x^(9/2))/(9*a)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^{3/2} x^{7/2}\,dx &= \frac{2\left(a+\frac{b}{x}\right)^{5/2}x^{9/2}}{9a}-\frac{(4b)\int \left(a+\frac{b}{x}\right)^{3/2}x^{5/2}\,dx}{9a}\\ &= -\frac{8b\left(a+\frac{b}{x}\right)^{5/2}x^{7/2}}{63a^2}+\frac{2\left(a+\frac{b}{x}\right)^{5/2}x^{9/2}}{9a}+\frac{\left(8b^2\right)\int \left(a+\frac{b}{x}\right)^{3/2}x^{3/2}\,dx}{63a^2}\\ &= \frac{16b^2\left(a+\frac{b}{x}\right)^{5/2}x^{5/2}}{315a^3}-\frac{8b\left(a+\frac{b}{x}\right)^{5/2}x^{7/2}}{63a^2}+\frac{2\left(a+\frac{b}{x}\right)^{5/2}x^{9/2}}{9a}\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.66

$$\frac{2\sqrt{x}\sqrt{a+\frac{b}{x}}(ax+b)^2\left(35a^2x^2-20abx+8b^2\right)}{315a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)*x^(7/2),x]
```

```
[Out] (2*Sqrt[a + b/x]*Sqrt[x]*(b + a*x)^2*(8*b^2 - 20*a*b*x + 35*a^2*x^2))/(315*
a^3)
```

fricas [A] time = 0.87, size = 60, normalized size = 0.81

$$\frac{2\left(35\,a^4x^4+50\,a^3bx^3+3\,a^2b^2x^2-4\,ab^3x+8\,b^4\right)\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}{315\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*a^4*x^4 + 50*a^3*b*x^3 + 3*a^2*b^2*x^2 - 4*a*b^3*x + 8*b^4)*sqrt(
x)*sqrt((a*x + b)/x)/a^3
```

giac [B] time = 0.23, size = 114, normalized size = 1.54

$$-\frac{2}{105}\,b\left(\frac{8\,b^{\frac{7}{2}}}{a^3}-\frac{15\,(ax+b)^{\frac{7}{2}}-42\,(ax+b)^{\frac{5}{2}}b+35\,(ax+b)^{\frac{3}{2}}b^2}{a^3}\right)\mathrm{sgn}(x)+\frac{2}{315}\,a\left(\frac{16\,b^{\frac{9}{2}}}{a^4}+\frac{35\,(ax+b)^{\frac{9}{2}}-135\,(ax+b)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*x^(7/2),x, algorithm="giac")
```

```
[Out] -2/105*b*(8*b^(7/2)/a^3 - (15*(a*x + b)^(7/2) - 42*(a*x + b)^(5/2)*b + 35*(
a*x + b)^(3/2)*b^2)/a^3)*sgn(x) + 2/315*a*(16*b^(9/2)/a^4 + (35*(a*x + b)^(
9/2) - 135*(a*x + b)^(7/2)*b + 189*(a*x + b)^(5/2)*b^2 - 105*(a*x + b)^(3/2
)*b^3)/a^4)*sgn(x)
```

maple [A] time = 0.00, size = 44, normalized size = 0.59

$$\frac{2\,(ax+b)\left(35a^2x^2-20abx+8b^2\right)\left(\frac{ax+b}{x}\right)^{\frac{3}{2}}x^{\frac{3}{2}}}{315a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(3/2)*x^(7/2),x)
```

```
[Out] 2/315*(a*x+b)*(35*a^2*x^2-20*a*b*x+8*b^2)*x^(3/2)*((a*x+b)/x)^(3/2)/a^3
```

maxima [A] time = 1.08, size = 52, normalized size = 0.70

$$\frac{2\left(35\left(a+\frac{b}{x}\right)^{\frac{9}{2}}x^{\frac{9}{2}}-90\left(a+\frac{b}{x}\right)^{\frac{7}{2}}bx^{\frac{7}{2}}+63\left(a+\frac{b}{x}\right)^{\frac{5}{2}}b^2x^{\frac{5}{2}}\right)}{315\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*(a + b/x)^(9/2)*x^(9/2) - 90*(a + b/x)^(7/2)*b*x^(7/2) + 63*(a +
b/x)^(5/2)*b^2*x^(5/2))/a^3
```

mupad [B] time = 1.47, size = 56, normalized size = 0.76

$$\sqrt{a+\frac{b}{x}}\,\left(\frac{2\,a\,x^{9/2}}{9}+\frac{20\,b\,x^{7/2}}{63}+\frac{2\,b^2\,x^{5/2}}{105\,a}-\frac{8\,b^3\,x^{3/2}}{315\,a^2}+\frac{16\,b^4\,\sqrt{x}}{315\,a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(a + b/x)^(3/2),x)
```

```
[Out] (a + b/x)^(1/2)*((2*a*x^(9/2))/9 + (20*b*x^(7/2))/63 + (2*b^2*x^(5/2))/(105
*a) - (8*b^3*x^(3/2))/(315*a^2) + (16*b^4*x^(1/2))/(315*a^3))
```

sympy [**B**] time = 64.43, size = 369, normalized size = 4.99

$$\frac{70a^6b^{\frac{9}{2}}x^6\sqrt{\frac{ax}{b}+1}}{315a^5b^4x^2+630a^4b^5x+315a^3b^6}+\frac{240a^5b^{\frac{11}{2}}x^5\sqrt{\frac{ax}{b}+1}}{315a^5b^4x^2+630a^4b^5x+315a^3b^6}+\frac{276a^4b^{\frac{13}{2}}x^4\sqrt{\frac{ax}{b}+1}}{315a^5b^4x^2+630a^4b^5x+315a^3b^6}+\frac{315}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)*x**(7/2),x)
```

```
[Out] 70*a**6*b**(9/2)*x**6*sqrt(a*x/b + 1)/(315*a**5*b**4*x**2 + 630*a**4*b**5*x
 + 315*a**3*b**6) + 240*a**5*b**(11/2)*x**5*sqrt(a*x/b + 1)/(315*a**5*b**4*
x**2 + 630*a**4*b**5*x + 315*a**3*b**6) + 276*a**4*b**(13/2)*x**4*sqrt(a*x/
b + 1)/(315*a**5*b**4*x**2 + 630*a**4*b**5*x + 315*a**3*b**6) + 104*a**3*b*
*(15/2)*x**3*sqrt(a*x/b + 1)/(315*a**5*b**4*x**2 + 630*a**4*b**5*x + 315*a*
*3*b**6) + 6*a**2*b**(17/2)*x**2*sqrt(a*x/b + 1)/(315*a**5*b**4*x**2 + 630*
a**4*b**5*x + 315*a**3*b**6) + 24*a*b**(19/2)*x*sqrt(a*x/b + 1)/(315*a**5*b
**4*x**2 + 630*a**4*b**5*x + 315*a**3*b**6) + 16*b**(21/2)*sqrt(a*x/b + 1)/
(315*a**5*b**4*x**2 + 630*a**4*b**5*x + 315*a**3*b**6)
```

3.1762 $\int \left(a+\frac{b}{x}\right)^{3/2} x^{5/2}\,dx$

Optimal. Leaf size=48

$$\frac{2x^{7/2}\left(a+\frac{b}{x}\right)^{5/2}}{7a}-\frac{4bx^{5/2}\left(a+\frac{b}{x}\right)^{5/2}}{35a^2}$$

```
[Out] -4/35*b*(a+b/x)^(5/2)*x^(5/2)/a^2+2/7*(a+b/x)^(5/2)*x^(7/2)/a
```

Rubi [A] time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {271, 264}

$$\frac{2x^{7/2}\left(a+\frac{b}{x}\right)^{5/2}}{7a}-\frac{4bx^{5/2}\left(a+\frac{b}{x}\right)^{5/2}}{35a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)*x^(5/2),x]
```

```
[Out] (-4*b*(a + b/x)^(5/2)*x^(5/2))/(35*a^2) + (2*(a + b/x)^(5/2)*x^(7/2))/(7*a)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^{3/2} x^{5/2}\,dx &= \frac{2\left(a+\frac{b}{x}\right)^{5/2}x^{7/2}}{7a}-\frac{(2b)\int \left(a+\frac{b}{x}\right)^{3/2}x^{3/2}\,dx}{7a}\\ &= -\frac{4b\left(a+\frac{b}{x}\right)^{5/2}x^{5/2}}{35a^2}+\frac{2\left(a+\frac{b}{x}\right)^{5/2}x^{7/2}}{7a}\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 0.83

$$\frac{2}{35}x^{5/2}\sqrt{a+\frac{b}{x}}\left(\frac{b}{ax}+1\right)^2(5ax-2b)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)*x^(5/2),x]
```

```
[Out] (2*Sqrt[a + b/x]*(1 + b/(a*x))^2*x^(5/2)*(-2*b + 5*a*x))/35
```

fricas [A] time = 0.81, size = 48, normalized size = 1.00

$$\frac{2\left(5\,a^3x^3+8\,a^2bx^2+ab^2x-2\,b^3\right)\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}{35\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*a^3*x^3 + 8*a^2*b*x^2 + a*b^2*x - 2*b^3)*sqrt(x)*sqrt((a*x + b)/x)/
a^2
```

giac [B] time = 0.18, size = 90, normalized size = 1.88

$$\frac{2}{15}\,b\left(\frac{2\,b^{\frac{5}{2}}}{a^2}+\frac{3\,(ax+b)^{\frac{5}{2}}-5\,(ax+b)^{\frac{3}{2}}b}{a^2}\right)\mathrm{sgn}(x)-\frac{2}{105}\,a\left(\frac{8\,b^{\frac{7}{2}}}{a^3}-\frac{15\,(ax+b)^{\frac{7}{2}}-42\,(ax+b)^{\frac{5}{2}}b+35\,(ax+b)^{\frac{3}{2}}b^2}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*x^(5/2),x, algorithm="giac")
```

```
[Out] 2/15*b*(2*b^(5/2)/a^2 + (3*(a*x + b)^(5/2) - 5*(a*x + b)^(3/2)*b)/a^2)*sgn(
x) - 2/105*a*(8*b^(7/2)/a^3 - (15*(a*x + b)^(7/2) - 42*(a*x + b)^(5/2)*b +
35*(a*x + b)^(3/2)*b^2)/a^3)*sgn(x)
```

maple [A] time = 0.00, size = 33, normalized size = 0.69

$$\frac{2\,(ax+b)\,(5ax-2b)\left(\frac{ax+b}{x}\right)^{\frac{3}{2}}x^{\frac{3}{2}}}{35a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(3/2)*x^(5/2),x)
```

```
[Out] 2/35*(a*x+b)*(5*a*x-2*b)*x^(3/2)*((a*x+b)/x)^(3/2)/a^2
```

maxima [A] time = 1.05, size = 35, normalized size = 0.73

$$\frac{2\left(5\left(a+\frac{b}{x}\right)^{\frac{7}{2}}x^{\frac{7}{2}}-7\left(a+\frac{b}{x}\right)^{\frac{5}{2}}bx^{\frac{5}{2}}\right)}{35\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/35*(5*(a + b/x)^(7/2)*x^(7/2) - 7*(a + b/x)^(5/2)*b*x^(5/2))/a^2
```

mupad [B] time = 1.43, size = 45, normalized size = 0.94

$$\sqrt{a+\frac{b}{x}}\left(\frac{2\,a\,x^{7/2}}{7}+\frac{16\,b\,x^{5/2}}{35}+\frac{2\,b^2\,x^{3/2}}{35\,a}-\frac{4\,b^3\,\sqrt{x}}{35\,a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b/x)^(3/2),x)
```

```
[Out] (a + b/x)^(1/2)*((2*a*x^(7/2))/7 + (16*b*x^(5/2))/35 + (2*b^2*x^(3/2))/(35*
a) - (4*b^3*x^(1/2))/(35*a^2))
```

sympy [**B**] time = 27.92, size = 88, normalized size = 1.83

$$\frac{2a\sqrt{b}\,x^3\sqrt{\frac{ax}{b}+1}}{7}+\frac{16b^{\frac{3}{2}}x^2\sqrt{\frac{ax}{b}+1}}{35}+\frac{2b^{\frac{5}{2}}x\sqrt{\frac{ax}{b}+1}}{35a}-\frac{4b^{\frac{7}{2}}\sqrt{\frac{ax}{b}+1}}{35a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)*x**(5/2),x)
```

```
[Out] 2*a*sqrt(b)*x**3*sqrt(a*x/b + 1)/7 + 16*b**(3/2)*x**2*sqrt(a*x/b + 1)/35 +
2*b**(5/2)*x*sqrt(a*x/b + 1)/(35*a) - 4*b**(7/2)*sqrt(a*x/b + 1)/(35*a**2)
```

3.1763 $\int \left(a+\frac{b}{x}\right)^{3/2} x^{3/2}\,dx$

Optimal. Leaf size=23

$$\frac{2x^{5/2}\left(a+\frac{b}{x}\right)^{5/2}}{5a}$$

```
[Out] 2/5*(a+b/x)^(5/2)*x^(5/2)/a
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {264}

$$\frac{2x^{5/2}\left(a+\frac{b}{x}\right)^{5/2}}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)*x^(3/2),x]
```

```
[Out] (2*(a + b/x)^(5/2)*x^(5/2))/(5*a)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^{3/2}\,dx = \frac{2\left(a+\frac{b}{x}\right)^{5/2} x^{5/2}}{5a}$$

Mathematica [A] time = 0.02, size = 23, normalized size = 1.00

$$\frac{2x^{5/2}\left(a+\frac{b}{x}\right)^{5/2}}{5a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)*x^(3/2),x]
```

```
[Out] (2*(a + b/x)^(5/2)*x^(5/2))/(5*a)
```

fricas [B] time = 0.79, size = 35, normalized size = 1.52

$$\frac{2\left(a^2x^2+2\,abx+b^2\right)\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}{5\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(a^2*x^2 + 2*a*b*x + b^2)*sqrt(x)*sqrt((a*x + b)/x)/a
```

giac [B] time = 0.17, size = 64, normalized size = 2.78

$$\frac{2}{3}b\left(\frac{(ax+b)^{\frac{3}{2}}}{a}-\frac{b^{\frac{3}{2}}}{a}\right)\operatorname{sgn}(x)+\frac{2}{15}a\left(\frac{2\,b^{\frac{5}{2}}}{a^2}+\frac{3\,(ax+b)^{\frac{5}{2}}-5\,(ax+b)^{\frac{3}{2}}b}{a^2}\right)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*x^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*b*((a*x + b)^(3/2)/a - b^(3/2)/a)*sgn(x) + 2/15*a*(2*b^(5/2)/a^2 + (3*(
a*x + b)^(5/2) - 5*(a*x + b)^(3/2)*b)/a^2)*sgn(x)
```

maple [A] time = 0.00, size = 25, normalized size = 1.09

$$\frac{2\,(ax+b)\left(\frac{ax+b}{x}\right)^{\frac{3}{2}}x^{\frac{3}{2}}}{5a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(3/2)*x^(3/2),x)
```

```
[Out] 2/5*(a*x+b)*((a*x+b)/x)^(3/2)*x^(3/2)/a
```

maxima [A] time = 1.08, size = 17, normalized size = 0.74

$$\frac{2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}x^{\frac{5}{2}}}{5\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*(a + b/x)^(5/2)*x^(5/2)/a
```

mupad [B] time = 1.38, size = 34, normalized size = 1.48

$$\sqrt{a+\frac{b}{x}}\left(\frac{2\,a\,x^{5/2}}{5}+\frac{4\,b\,x^{3/2}}{5}+\frac{2\,b^2\,\sqrt{x}}{5\,a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b/x)^(3/2),x)
```

```
[Out] (a + b/x)^(1/2)*((2*a*x^(5/2))/5 + (4*b*x^(3/2))/5 + (2*b^2*x^(1/2))/(5*a))
```

sympy [B] time = 9.46, size = 63, normalized size = 2.74

$$\frac{2a\sqrt{b}\,x^2\sqrt{\frac{ax}{b}+1}}{5}+\frac{4b^{\frac{3}{2}}x\sqrt{\frac{ax}{b}+1}}{5}+\frac{2b^{\frac{5}{2}}\sqrt{\frac{ax}{b}+1}}{5a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)*x**(3/2),x)
```

```
[Out] 2*a*sqrt(b)*x**2*sqrt(a*x/b + 1)/5 + 4*b**(3/2)*x*sqrt(a*x/b + 1)/5 + 2*b**
(5/2)*sqrt(a*x/b + 1)/(5*a)
```

3.1764 $\int \left(a+\frac{b}{x}\right)^{3/2}\sqrt{x}\,dx$

Optimal. Leaf size=70

$$-2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)+\frac{2}{3}x^{3/2}\left(a+\frac{b}{x}\right)^{3/2}+2b\sqrt{x}\sqrt{a+\frac{b}{x}}$$

```
[Out] 2/3*(a+b/x)^(3/2)*x^(3/2)-2*b^(3/2)*arctanh(b^(1/2)/(a+b/x)^(1/2)/x^(1/2))+
2*b*(a+b/x)^(1/2)*x^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {337, 277, 217, 206}

$$-2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)+\frac{2}{3}x^{3/2}\left(a+\frac{b}{x}\right)^{3/2}+2b\sqrt{x}\sqrt{a+\frac{b}{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)*Sqrt[x],x]
```

```
[Out] 2*b*Sqrt[a + b/x]*Sqrt[x] + (2*(a + b/x)^(3/2)*x^(3/2))/3 - 2*b^(3/2)*ArcTa
nh[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x])]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 337

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, -Dist[k/c, Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1)
 + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0] &
& FractionQ[m]
```

Rubi steps

$$\int \left(a+\frac{b}{x}\right)^{3/2}\sqrt{x}\,dx = -\left(2\,\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x^4}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$
$$= \frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}x^{3/2} - (2b)\,\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2}\,dx, x, \frac{1}{\sqrt{x}}\right)$$
$$= 2b\sqrt{a+\frac{b}{x}}\sqrt{x} + \frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}x^{3/2} - \left(2b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{\sqrt{x}}\right)$$
$$= 2b\sqrt{a+\frac{b}{x}}\sqrt{x} + \frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}x^{3/2} - \left(2b^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)$$
$$= 2b\sqrt{a+\frac{b}{x}}\sqrt{x} + \frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}x^{3/2} - 2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.77

$$\frac{2ax^{3/2}\sqrt{a+\frac{b}{x}}\;{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};-\frac{b}{ax}\right)}{3\sqrt{\frac{b}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)*Sqrt[x],x]
```

```
[Out] (2*a*Sqrt[a + b/x]*x^(3/2)*Hypergeometric2F1[-3/2, -3/2, -1/2, -(b/(a*x))])
/(3*Sqrt[1 + b/(a*x)])
```

fricas [A] time = 0.84, size = 116, normalized size = 1.66

$$\left[b^{\frac{3}{2}}\log\left(\frac{ax-2\sqrt{b}\sqrt{x}\sqrt{\frac{ax+b}{x}}+2b}{x}\right)+\frac{2}{3}(ax+4b)\sqrt{x}\sqrt{\frac{ax+b}{x}}, 2\sqrt{-b}\,b\arctan\left(\frac{\sqrt{-b}\sqrt{x}\sqrt{\frac{ax+b}{x}}}{b}\right)+\frac{2}{3}(ax+4b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] [b^(3/2)*log((a*x - 2*sqrt(b)*sqrt(x)*sqrt((a*x + b)/x) + 2*b)/x) + 2/3*(a*
x + 4*b)*sqrt(x)*sqrt((a*x + b)/x), 2*sqrt(-b)*b*arctan(sqrt(-b)*sqrt(x)*sq
rt((a*x + b)/x)/b) + 2/3*(a*x + 4*b)*sqrt(x)*sqrt((a*x + b)/x)]
```

giac [A] time = 0.21, size = 44, normalized size = 0.63

$$\frac{2\,b^2\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}+\frac{2}{3}(ax+b)^{\frac{3}{2}}+2\sqrt{ax+b}\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*x^(1/2),x, algorithm="giac")
```

```
[Out] 2*b^2*arctan(sqrt(a*x + b)/sqrt(-b))/sqrt(-b) + 2/3*(a*x + b)^(3/2) + 2*sqr
t(a*x + b)*b
```

maple [A] time = 0.01, size = 63, normalized size = 0.90

$$-\frac{2\sqrt{\frac{ax+b}{x}}\left(3b^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)-\sqrt{ax+b}\,ax-4\sqrt{ax+b}\,b\right)\sqrt{x}}{3\sqrt{ax+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(3/2)*x^(1/2),x)
```

```
[Out] -2/3*((a*x+b)/x)^(1/2)*x^(1/2)*(3*b^(3/2)*arctanh((a*x+b)^(1/2)/b^(1/2))-x*
a*(a*x+b)^(1/2)-4*(a*x+b)^(1/2)*b)/(a*x+b)^(1/2)
```

maxima [A] time = 2.29, size = 74, normalized size = 1.06

$$\frac{2}{3}\left(a+\frac{b}{x}\right)^{\frac{3}{2}}x^{\frac{3}{2}}+b^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}+\sqrt{b}}\right)+2\sqrt{a+\frac{b}{x}}\,b\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(a + b/x)^(3/2)*x^(3/2) + b^(3/2)*log((sqrt(a + b/x)*sqrt(x) - sqrt(b))
/(sqrt(a + b/x)*sqrt(x) + sqrt(b))) + 2*sqrt(a + b/x)*b*sqrt(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\left(a+\frac{b}{x}\right)^{3/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b/x)^(3/2),x)
```

```
[Out] int(x^(1/2)*(a + b/x)^(3/2), x)
```

sympy [A] time = 6.74, size = 71, normalized size = 1.01

$$\frac{2a\sqrt{b}\,x\sqrt{\frac{ax}{b}+1}}{3}+\frac{8b^{\frac{3}{2}}\sqrt{\frac{ax}{b}+1}}{3}+b^{\frac{3}{2}}\log\left(\frac{ax}{b}\right)-2b^{\frac{3}{2}}\log\left(\sqrt{\frac{ax}{b}+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)*x**(1/2),x)
```

```
[Out] 2*a*sqrt(b)*x*sqrt(a*x/b + 1)/3 + 8*b**(3/2)*sqrt(a*x/b + 1)/3 + b**(3/2)*l
og(a*x/b) - 2*b**(3/2)*log(sqrt(a*x/b + 1) + 1)
```

3.1765 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\sqrt{x}}\,dx$

Optimal. Leaf size=69

$$2\sqrt{x}\left(a+\frac{b}{x}\right)^{3/2}-\frac{3b\sqrt{a+\frac{b}{x}}}{\sqrt{x}}-3a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)$$

```
[Out] -3*a*arctanh(b^(1/2)/(a+b/x)^(1/2)/x^(1/2))*b^(1/2)-3*b*(a+b/x)^(1/2)/x^(1/
2)+2*(a+b/x)^(3/2)*x^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {337, 277, 195, 217, 206}

$$2\sqrt{x}\left(a+\frac{b}{x}\right)^{3/2}-\frac{3b\sqrt{a+\frac{b}{x}}}{\sqrt{x}}-3a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)/Sqrt[x],x]
```

```
[Out] (-3*b*Sqrt[a + b/x])/Sqrt[x] + 2*(a + b/x)^(3/2)*Sqrt[x] - 3*a*Sqrt[b]*ArcT
anh[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x])]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 337

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, -Dist[k/c, Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1)
 + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0] &
```

```
& FractionQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\sqrt{x}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x^2}\,dx, x, \frac{1}{\sqrt{x}}\right)\right) \\
&= 2\left(a+\frac{b}{x}\right)^{3/2}\sqrt{x} - (6b)\,\text{Subst}\left(\int \sqrt{a+bx^2}\,dx, x, \frac{1}{\sqrt{x}}\right) \\
&= -\frac{3b\sqrt{a+\frac{b}{x}}}{\sqrt{x}} + 2\left(a+\frac{b}{x}\right)^{3/2}\sqrt{x} - (3ab)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{\sqrt{x}}\right) \\
&= -\frac{3b\sqrt{a+\frac{b}{x}}}{\sqrt{x}} + 2\left(a+\frac{b}{x}\right)^{3/2}\sqrt{x} - (3ab)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right) \\
&= -\frac{3b\sqrt{a+\frac{b}{x}}}{\sqrt{x}} + 2\left(a+\frac{b}{x}\right)^{3/2}\sqrt{x} - 3a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.75

$$\frac{2a\sqrt{x}\sqrt{a+\frac{b}{x}}\;{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{b}{ax}\right)}{\sqrt{\frac{b}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)/Sqrt[x],x]
```

```
[Out] (2*a*Sqrt[a + b/x]*Sqrt[x]*Hypergeometric2F1[-3/2, -1/2, 1/2, -(b/(a*x))])/
Sqrt[1 + b/(a*x)]
```

fricas [A] time = 0.93, size = 130, normalized size = 1.88

$$\left[\frac{3\,a\sqrt{b}\,x\log\left(\frac{ax-2\sqrt{b}\sqrt{x}\sqrt{\frac{ax+b}{x}}+2b}{x}\right)+2\,(2\,ax-b)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{2\,x},\ \frac{3\,a\sqrt{-b}\,x\arctan\left(\frac{\sqrt{-b}\sqrt{x}\sqrt{\frac{ax+b}{x}}}{b}\right)+(2\,ax-b)\sqrt{x}\sqrt{}}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*a*sqrt(b)*x*log((a*x - 2*sqrt(b)*sqrt(x)*sqrt((a*x + b)/x) + 2*b)/x
) + 2*(2*a*x - b)*sqrt(x)*sqrt((a*x + b)/x))/x, (3*a*sqrt(-b)*x*arctan(sqrt
(-b)*sqrt(x)*sqrt((a*x + b)/x)/b) + (2*a*x - b)*sqrt(x)*sqrt((a*x + b)/x))/
x]
```

giac [A] time = 0.27, size = 56, normalized size = 0.81

$$\frac{\frac{3\,a^2b\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}+2\sqrt{ax+b}\,a^2-\frac{\sqrt{ax+b}\,ab}{x}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] (3*a^2*b*arctan(sqrt(a*x + b)/sqrt(-b))/sqrt(-b) + 2*sqrt(a*x + b)*a^2 - sq
rt(a*x + b)*a*b/x)/a
```

maple [A] time = 0.02, size = 70, normalized size = 1.01

$$-\frac{\sqrt{\frac{ax+b}{x}}\left(3abx\,\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)-2\sqrt{ax+b}\;a\sqrt{b}\;x+\sqrt{ax+b}\;b^{\frac{3}{2}}\right)}{\sqrt{ax+b}\;\sqrt{b}\;\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(3/2)/x^(1/2),x)
```

```
[Out] -((a*x+b)/x)^(1/2)*((a*x+b)^(1/2)*b^(3/2)-2*(a*x+b)^(1/2)*a*b^(1/2)*x+3*arc
tanh((a*x+b)^(1/2)/b^(1/2))*x*a*b)/x^(1/2)/(a*x+b)^(1/2)/b^(1/2)
```

maxima [A] time = 2.33, size = 93, normalized size = 1.35

$$\frac{3}{2}\,a\sqrt{b}\,\log\left(\frac{\sqrt{a+\frac{b}{x}}\,\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\,\sqrt{x}+\sqrt{b}}\right)+2\sqrt{a+\frac{b}{x}}\,a\sqrt{x}-\frac{\sqrt{a+\frac{b}{x}}\,ab\sqrt{x}}{\left(a+\frac{b}{x}\right)x-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 3/2*a*sqrt(b)*log((sqrt(a + b/x)*sqrt(x) - sqrt(b))/(sqrt(a + b/x)*sqrt(x)
+ sqrt(b))) + 2*sqrt(a + b/x)*a*sqrt(x) - sqrt(a + b/x)*a*b*sqrt(x)/((a + b
/x)*x - b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(3/2)/x^(1/2),x)
```

```
[Out] int((a + b/x)^(3/2)/x^(1/2), x)
```

sympy [A] time = 9.08, size = 92, normalized size = 1.33

$$\frac{2a^{\frac{3}{2}}\sqrt{x}}{\sqrt{1+\frac{b}{ax}}}+\frac{\sqrt{a}\,b}{\sqrt{x}\,\sqrt{1+\frac{b}{ax}}}-3a\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\,\sqrt{x}}\right)-\frac{b^2}{\sqrt{a}\,x^{\frac{3}{2}}\sqrt{1+\frac{b}{ax}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)/x**(1/2),x)
```

```
[Out] 2*a**(3/2)*sqrt(x)/sqrt(1 + b/(a*x)) + sqrt(a)*b/(sqrt(x)*sqrt(1 + b/(a*x))
) - 3*a*sqrt(b)*asinh(sqrt(b)/(sqrt(a)*sqrt(x))) - b**2/(sqrt(a)*x**(3/2)*s
qrt(1 + b/(a*x)))
```

3.1766 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^{3/2}}\,dx$

Optimal. Leaf size=77

$$-\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{4\sqrt{b}}-\frac{3a\sqrt{a+\frac{b}{x}}}{4\sqrt{x}}-\frac{\left(a+\frac{b}{x}\right)^{3/2}}{2\sqrt{x}}$$

```
[Out] -3/4*a^2*arctanh(b^(1/2)/(a+b/x)^(1/2)/x^(1/2))/b^(1/2)-1/2*(a+b/x)^(3/2)/x
^(1/2)-3/4*a*(a+b/x)^(1/2)/x^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {337, 195, 217, 206}

$$-\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{4\sqrt{b}}-\frac{3a\sqrt{a+\frac{b}{x}}}{4\sqrt{x}}-\frac{\left(a+\frac{b}{x}\right)^{3/2}}{2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)/x^(3/2),x]
```

```
[Out] (-3*a*Sqrt[a + b/x])/(4*Sqrt[x]) - (a + b/x)^(3/2)/(2*Sqrt[x]) - (3*a^2*Arc
Tanh[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x])])/(4*Sqrt[b])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 337

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, -Dist[k/c, Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1)
 + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0] &
& FractionQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^{3/2}}\,dx &= -\left(2\,\text{Subst}\left(\int \left(a+bx^2\right)^{3/2}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)\\ &= -\frac{\left(a+\frac{b}{x}\right)^{3/2}}{2\sqrt{x}} - \frac{1}{2}(3a)\,\text{Subst}\left(\int \sqrt{a+bx^2}\,dx, x, \frac{1}{\sqrt{x}}\right)\\ &= -\frac{3a\sqrt{a+\frac{b}{x}}}{4\sqrt{x}} - \frac{\left(a+\frac{b}{x}\right)^{3/2}}{2\sqrt{x}} - \frac{1}{4}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{\sqrt{x}}\right)\\ &= -\frac{3a\sqrt{a+\frac{b}{x}}}{4\sqrt{x}} - \frac{\left(a+\frac{b}{x}\right)^{3/2}}{2\sqrt{x}} - \frac{1}{4}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)\\ &= -\frac{3a\sqrt{a+\frac{b}{x}}}{4\sqrt{x}} - \frac{\left(a+\frac{b}{x}\right)^{3/2}}{2\sqrt{x}} - \frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)}{4\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.19, size = 73, normalized size = 0.95

$$\frac{1}{4}\sqrt{a+\frac{b}{x}}\left(\frac{-5ax-2b}{x^{3/2}} - \frac{3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right)}{\sqrt{b}\sqrt{\frac{b}{ax}+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)/x^(3/2),x]
```

```
[Out] (Sqrt[a + b/x]*((-2*b - 5*a*x)/x^(3/2) - (3*a^(3/2)*ArcSinh[Sqrt[b]/(Sqrt[a
]*Sqrt[x])])/(Sqrt[b]*Sqrt[1 + b/(a*x)])))/4
```

fricas [A] time = 0.95, size = 152, normalized size = 1.97

$$\left[\frac{3\,a^2\sqrt{b}\,x^2\log\left(\frac{ax-2\sqrt{b}\sqrt{x}\sqrt{\frac{ax+b}{x}}+2\,b}{x}\right) - 2\left(5\,abx+2\,b^2\right)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{8\,bx^2}, \frac{3\,a^2\sqrt{-b}\,x^2\arctan\left(\frac{\sqrt{-b}\sqrt{x}\sqrt{\frac{ax+b}{x}}}{b}\right) - \left(5\,abx+2\right.}{4\,bx^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*a^2*sqrt(b)*x^2*log((a*x - 2*sqrt(b)*sqrt(x)*sqrt((a*x + b)/x) + 2*
b)/x) - 2*(5*a*b*x + 2*b^2)*sqrt(x)*sqrt((a*x + b)/x))/(b*x^2), 1/4*(3*a^2*
sqrt(-b)*x^2*arctan(sqrt(-b)*sqrt(x)*sqrt((a*x + b)/x)/b) - (5*a*b*x + 2*b^
2)*sqrt(x)*sqrt((a*x + b)/x))/(b*x^2)]
```

giac [A] time = 0.34, size = 64, normalized size = 0.83

$$\frac{\frac{3\,a^3\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{\sqrt{-b}} - \frac{5\,(ax+b)^{\frac{3}{2}}a^3 - 3\sqrt{ax+b}\,a^3 b}{a^2x^2}}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*a^3*arctan(sqrt(a*x + b)/sqrt(-b))/sqrt(-b) - (5*(a*x + b)^(3/2)*a^3
 - 3*sqrt(a*x + b)*a^3*b)/(a^2*x^2))/a
```

maple [A] time = 0.02, size = 74, normalized size = 0.96

$$-\frac{\sqrt{\frac{ax+b}{x}}\left(3a^2x^2\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)+5\sqrt{ax+b}\;a\sqrt{b}\;x+2\sqrt{ax+b}\;b^{\frac{3}{2}}\right)}{4\sqrt{ax+b}\;\sqrt{b}\;x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(3/2)/x^(3/2),x)
```

```
[Out] -1/4*((a*x+b)/x)^(1/2)/x^(3/2)*(3*arctanh((a*x+b)^(1/2)/b^(1/2))*a^2*x^2+5*
(a*x+b)^(1/2)*a*b^(1/2)*x+2*(a*x+b)^(1/2)*b^(3/2))/(a*x+b)^(1/2)/b^(1/2)
```

maxima [B] time = 2.32, size = 117, normalized size = 1.52

$$\frac{3\,a^2\log\left(\frac{\sqrt{a+\frac{b}{x}}\,\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\,\sqrt{x}+\sqrt{b}}\right)}{8\,\sqrt{b}}-\frac{5\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2x^{\frac{3}{2}}-3\,\sqrt{a+\frac{b}{x}}\,a^2b\sqrt{x}}{4\left(\left(a+\frac{b}{x}\right)^2x^2-2\left(a+\frac{b}{x}\right)bx+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 3/8*a^2*log((sqrt(a + b/x)*sqrt(x) - sqrt(b))/(sqrt(a + b/x)*sqrt(x) + sqrt
(b)))/sqrt(b) - 1/4*(5*(a + b/x)^(3/2)*a^2*x^(3/2) - 3*sqrt(a + b/x)*a^2*b*
sqrt(x))/((a + b/x)^2*x^2 - 2*(a + b/x)*b*x + b^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(3/2)/x^(3/2),x)
```

```
[Out] int((a + b/x)^(3/2)/x^(3/2), x)
```

sympy [A] time = 9.03, size = 76, normalized size = 0.99

$$-\frac{5a^{\frac{3}{2}}\sqrt{1+\frac{b}{ax}}}{4\sqrt{x}}-\frac{\sqrt{a}\,b\sqrt{1+\frac{b}{ax}}}{2x^{\frac{3}{2}}}-\frac{3a^2\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\,\sqrt{x}}\right)}{4\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)/x**(3/2),x)
```

```
[Out] -5*a**(3/2)*sqrt(1 + b/(a*x))/(4*sqrt(x)) - sqrt(a)*b*sqrt(1 + b/(a*x))/(2*
x**(3/2)) - 3*a**2*asinh(sqrt(b)/(sqrt(a)*sqrt(x)))/(4*sqrt(b))
```

3.1767 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^{5/2}}\,dx$

Optimal. Leaf size=103

$$\frac{a^3 \tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{8b^{3/2}} - \frac{a^2\sqrt{a+\frac{b}{x}}}{8b\sqrt{x}} - \frac{a\sqrt{a+\frac{b}{x}}}{4x^{3/2}} - \frac{\left(a+\frac{b}{x}\right)^{3/2}}{3x^{3/2}}$$

```
[Out] -1/3*(a+b/x)^(3/2)/x^(3/2)+1/8*a^3*arctanh(b^(1/2)/(a+b/x)^(1/2)/x^(1/2))/b
^(3/2)-1/4*a*(a+b/x)^(1/2)/x^(3/2)-1/8*a^2*(a+b/x)^(1/2)/b/x^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {337, 279, 321, 217, 206}

$$\frac{a^3 \tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{8b^{3/2}} - \frac{a^2\sqrt{a+\frac{b}{x}}}{8b\sqrt{x}} - \frac{a\sqrt{a+\frac{b}{x}}}{4x^{3/2}} - \frac{\left(a+\frac{b}{x}\right)^{3/2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)/x^(5/2),x]
```

```
[Out] -(a*Sqrt[a + b/x])/(4*x^(3/2)) - (a + b/x)^(3/2)/(3*x^(3/2)) - (a^2*Sqrt[a
+ b/x])/(8*b*Sqrt[x]) + (a^3*ArcTanh[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x])])/(8*b
^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 337

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, -Dist[k/c, Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1)
```

```
 + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0] &
& FractionQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^{5/2}}\,dx &= -\left(2\,\text{Subst}\left(\int x^2\left(a+bx^2\right)^{3/2}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)\\
&= -\frac{\left(a+\frac{b}{x}\right)^{3/2}}{3x^{3/2}} - a\,\text{Subst}\left(\int x^2\sqrt{a+bx^2}\,dx, x, \frac{1}{\sqrt{x}}\right)\\
&= -\frac{a\sqrt{a+\frac{b}{x}}}{4x^{3/2}} - \frac{\left(a+\frac{b}{x}\right)^{3/2}}{3x^{3/2}} - \frac{1}{4}a^2\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{\sqrt{x}}\right)\\
&= -\frac{a\sqrt{a+\frac{b}{x}}}{4x^{3/2}} - \frac{\left(a+\frac{b}{x}\right)^{3/2}}{3x^{3/2}} - \frac{a^2\sqrt{a+\frac{b}{x}}}{8b\sqrt{x}} + \frac{a^3\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{\sqrt{x}}\right)}{8b}\\
&= -\frac{a\sqrt{a+\frac{b}{x}}}{4x^{3/2}} - \frac{\left(a+\frac{b}{x}\right)^{3/2}}{3x^{3/2}} - \frac{a^2\sqrt{a+\frac{b}{x}}}{8b\sqrt{x}} + \frac{a^3\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{b}{x}}\,\sqrt{x}}\right)}{8b}\\
&= -\frac{a\sqrt{a+\frac{b}{x}}}{4x^{3/2}} - \frac{\left(a+\frac{b}{x}\right)^{3/2}}{3x^{3/2}} - \frac{a^2\sqrt{a+\frac{b}{x}}}{8b\sqrt{x}} + \frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\,\sqrt{x}}\right)}{8b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.19, size = 90, normalized size = 0.87

$$\frac{\sqrt{a+\frac{b}{x}}\left(\frac{3a^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}\,\sqrt{x}}\right)}{\sqrt{\frac{b}{ax}+1}} - \frac{\sqrt{b}\left(3a^2x^2+14abx+8b^2\right)}{x^{5/2}}\right)}{24b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)/x^(5/2),x]
```

```
[Out] (Sqrt[a + b/x]*(-((Sqrt[b]*(8*b^2 + 14*a*b*x + 3*a^2*x^2))/x^(5/2)) + (3*a^
(5/2)*ArcSinh[Sqrt[b]/(Sqrt[a]*Sqrt[x])])/Sqrt[1 + b/(a*x)]))/(24*b^(3/2))
```

fricas [A] time = 0.82, size = 173, normalized size = 1.68

$$\left[\frac{3\,a^3\sqrt{b}\,x^3\log\left(\frac{ax+2\sqrt{b}\,\sqrt{x}\,\sqrt{\frac{ax+b}{x}}+2b}{x}\right) - 2\left(3\,a^2bx^2+14\,ab^2x+8\,b^3\right)\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}{48\,b^2x^3}, -\frac{3\,a^3\sqrt{-b}\,x^3\arctan\left(\frac{\sqrt{-b}\,\sqrt{x}\,\sqrt{\frac{ax}{b}}\right.}{b}\right.}{}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*a^3*sqrt(b)*x^3*log((a*x + 2*sqrt(b)*sqrt(x)*sqrt((a*x + b)/x) + 2
*b)/x) - 2*(3*a^2*b*x^2 + 14*a*b^2*x + 8*b^3)*sqrt(x)*sqrt((a*x + b)/x))/(b
^2*x^3), -1/24*(3*a^3*sqrt(-b)*x^3*arctan(sqrt(-b)*sqrt(x)*sqrt((a*x + b)/x
```

```
)/b) + (3*a^2*b*x^2 + 14*a*b^2*x + 8*b^3)*sqrt(x)*sqrt((a*x + b)/x))/(b^2*x
^3)]
```

giac [A] time = 0.37, size = 84, normalized size = 0.82

$$-\frac{\frac{3\,a^4\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b}+\frac{3\,(ax+b)^{\frac{5}{2}}a^4+8\,(ax+b)^{\frac{3}{2}}a^4b-3\,\sqrt{ax+b}\,a^4b^2}{a^3bx^3}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] -1/24*(3*a^4*arctan(sqrt(a*x + b)/sqrt(-b))/(sqrt(-b)*b) + (3*(a*x + b)^(5/
2)*a^4 + 8*(a*x + b)^(3/2)*a^4*b - 3*sqrt(a*x + b)*a^4*b^2)/(a^3*b*x^3))/a
```

maple [A] time = 0.02, size = 92, normalized size = 0.89

$$-\frac{\sqrt{\frac{ax+b}{x}}\,\left(-3a^3x^3\,\mathrm{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)+3\sqrt{ax+b}\;a^2\sqrt{b}\;x^2+14\sqrt{ax+b}\;a\,b^{\frac{3}{2}}x+8\sqrt{ax+b}\;b^{\frac{5}{2}}\right)}{24\sqrt{ax+b}\;b^{\frac{3}{2}}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(3/2)/x^(5/2),x)
```

```
[Out] -1/24*((a*x+b)/x)^(1/2)*(-3*arctanh((a*x+b)^(1/2)/b^(1/2))*x^3*a^3+8*(a*x+b
)^(1/2)*b^(5/2)+14*(a*x+b)^(1/2)*a*b^(3/2)*x+3*(a*x+b)^(1/2)*a^2*b^(1/2)*x^
2)/x^(5/2)/b^(3/2)/(a*x+b)^(1/2)
```

maxima [B] time = 2.41, size = 159, normalized size = 1.54

$$-\frac{a^3\log\left(\frac{\sqrt{a+\frac{b}{x}}\,\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\,\sqrt{x}+\sqrt{b}}\right)}{16\,b^{\frac{3}{2}}}-\frac{3\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^3x^{\frac{5}{2}}+8\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^3bx^{\frac{3}{2}}-3\sqrt{a+\frac{b}{x}}\,a^3b^2\sqrt{x}}{24\left(\left(a+\frac{b}{x}\right)^3bx^3-3\left(a+\frac{b}{x}\right)^2b^2x^2+3\left(a+\frac{b}{x}\right)b^3x-b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -1/16*a^3*log((sqrt(a + b/x)*sqrt(x) - sqrt(b))/(sqrt(a + b/x)*sqrt(x) + sq
rt(b)))/b^(3/2) - 1/24*(3*(a + b/x)^(5/2)*a^3*x^(5/2) + 8*(a + b/x)^(3/2)*a
^3*b*x^(3/2) - 3*sqrt(a + b/x)*a^3*b^2*sqrt(x))/((a + b/x)^3*b*x^3 - 3*(a +
 b/x)^2*b^2*x^2 + 3*(a + b/x)*b^3*x - b^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(3/2)/x^(5/2),x)
```

```
[Out] int((a + b/x)^(3/2)/x^(5/2), x)
```

sympy [A] time = 14.32, size = 124, normalized size = 1.20

$$-\frac{a^{\frac{5}{2}}}{8b\sqrt{x}\,\sqrt{1+\frac{b}{ax}}}-\frac{17a^{\frac{3}{2}}}{24x^{\frac{3}{2}}\sqrt{1+\frac{b}{ax}}}-\frac{11\sqrt{a}\,b}{12x^{\frac{5}{2}}\sqrt{1+\frac{b}{ax}}}+\frac{a^3\,\mathrm{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\,\sqrt{x}}\right)}{8b^{\frac{3}{2}}}-\frac{b^2}{3\sqrt{a}\,x^{\frac{7}{2}}\sqrt{1+\frac{b}{ax}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)/x**(5/2),x)
```

```
[Out] -a**(5/2)/(8*b*sqrt(x)*sqrt(1 + b/(a*x))) - 17*a**(3/2)/(24*x**(3/2)*sqrt(1
 + b/(a*x))) - 11*sqrt(a)*b/(12*x**(5/2)*sqrt(1 + b/(a*x))) + a**3*asinh(sq
rt(b)/(sqrt(a)*sqrt(x)))/(8*b**(3/2)) - b**2/(3*sqrt(a)*x**(7/2)*sqrt(1 + b
/(a*x)))
```

3.1768 $\int \left(a+\frac{b}{x}\right)^{5/2} x^{11/2}\,dx$

Optimal. Leaf size=100

$$-\frac{32b^3x^{7/2}\left(a+\frac{b}{x}\right)^{7/2}}{3003a^4}+\frac{16b^2x^{9/2}\left(a+\frac{b}{x}\right)^{7/2}}{429a^3}-\frac{12bx^{11/2}\left(a+\frac{b}{x}\right)^{7/2}}{143a^2}+\frac{2x^{13/2}\left(a+\frac{b}{x}\right)^{7/2}}{13a}$$

```
[Out] -32/3003*b^3*(a+b/x)^(7/2)*x^(7/2)/a^4+16/429*b^2*(a+b/x)^(7/2)*x^(9/2)/a^3
-12/143*b*(a+b/x)^(7/2)*x^(11/2)/a^2+2/13*(a+b/x)^(7/2)*x^(13/2)/a
```

Rubi [A] time = 0.03, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {271, 264}

$$\frac{16b^2x^{9/2}\left(a+\frac{b}{x}\right)^{7/2}}{429a^3}-\frac{32b^3x^{7/2}\left(a+\frac{b}{x}\right)^{7/2}}{3003a^4}-\frac{12bx^{11/2}\left(a+\frac{b}{x}\right)^{7/2}}{143a^2}+\frac{2x^{13/2}\left(a+\frac{b}{x}\right)^{7/2}}{13a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)*x^(11/2),x]
```

```
[Out] (-32*b^3*(a + b/x)^(7/2)*x^(7/2))/(3003*a^4) + (16*b^2*(a + b/x)^(7/2)*x^(9
/2))/(429*a^3) - (12*b*(a + b/x)^(7/2)*x^(11/2))/(143*a^2) + (2*(a + b/x)^(
7/2)*x^(13/2))/(13*a)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \left(a+\frac{b}{x}\right)^{5/2} x^{11/2}\,dx &= \frac{2\left(a+\frac{b}{x}\right)^{7/2}x^{13/2}}{13a}-\frac{(6b)\int \left(a+\frac{b}{x}\right)^{5/2}x^{9/2}\,dx}{13a}\\
&= -\frac{12b\left(a+\frac{b}{x}\right)^{7/2}x^{11/2}}{143a^2}+\frac{2\left(a+\frac{b}{x}\right)^{7/2}x^{13/2}}{13a}+\frac{\left(24b^2\right)\int \left(a+\frac{b}{x}\right)^{5/2}x^{7/2}\,dx}{143a^2}\\
&= \frac{16b^2\left(a+\frac{b}{x}\right)^{7/2}x^{9/2}}{429a^3}-\frac{12b\left(a+\frac{b}{x}\right)^{7/2}x^{11/2}}{143a^2}+\frac{2\left(a+\frac{b}{x}\right)^{7/2}x^{13/2}}{13a}-\frac{\left(16b^3\right)\int \left(a+\frac{b}{x}\right)^{5/2}x^{5/2}\,d}{429a^3}\\
&= -\frac{32b^3\left(a+\frac{b}{x}\right)^{7/2}x^{7/2}}{3003a^4}+\frac{16b^2\left(a+\frac{b}{x}\right)^{7/2}x^{9/2}}{429a^3}-\frac{12b\left(a+\frac{b}{x}\right)^{7/2}x^{11/2}}{143a^2}+\frac{2\left(a+\frac{b}{x}\right)^{7/2}x^{13/2}}{13a}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 60, normalized size = 0.60

$$\frac{2\sqrt{x}\sqrt{a+\frac{b}{x}}(ax+b)^3\left(231a^3x^3-126a^2bx^2+56ab^2x-16b^3\right)}{3003a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)*x^(11/2),x]
```

```
[Out] (2*Sqrt[a + b/x]*Sqrt[x]*(b + a*x)^3*(-16*b^3 + 56*a*b^2*x - 126*a^2*b*x^2
+ 231*a^3*x^3))/(3003*a^4)
```

fricas [A] time = 0.79, size = 82, normalized size = 0.82

$$\frac{2\left(231\,a^6x^6+567\,a^5bx^5+371\,a^4b^2x^4+5\,a^3b^3x^3-6\,a^2b^4x^2+8\,ab^5x-16\,b^6\right)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{3003\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*x^(11/2),x, algorithm="fricas")
```

```
[Out] 2/3003*(231*a^6*x^6 + 567*a^5*b*x^5 + 371*a^4*b^2*x^4 + 5*a^3*b^3*x^3 - 6*a
^2*b^4*x^2 + 8*a*b^5*x - 16*b^6)*sqrt(x)*sqrt((a*x + b)/x)/a^4
```

giac [B] time = 0.30, size = 229, normalized size = 2.29

$$\frac{2}{315}b^2\left(\frac{16\,b^{\frac{9}{2}}}{a^4}+\frac{35\,(ax+b)^{\frac{9}{2}}-135\,(ax+b)^{\frac{7}{2}}b+189\,(ax+b)^{\frac{5}{2}}b^2-105\,(ax+b)^{\frac{3}{2}}b^3}{a^4}\right)\mathrm{sgn}(x)-\frac{4}{3465}ab\left(\frac{128\,b^{\frac{11}{2}}}{a^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*x^(11/2),x, algorithm="giac")
```

```
[Out] 2/315*b^2*(16*b^(9/2)/a^4 + (35*(a*x + b)^(9/2) - 135*(a*x + b)^(7/2)*b + 1
89*(a*x + b)^(5/2)*b^2 - 105*(a*x + b)^(3/2)*b^3)/a^4)*sgn(x) - 4/3465*a*b*
(128*b^(11/2)/a^5 - (315*(a*x + b)^(11/2) - 1540*(a*x + b)^(9/2)*b + 2970*(
a*x + b)^(7/2)*b^2 - 2772*(a*x + b)^(5/2)*b^3 + 1155*(a*x + b)^(3/2)*b^4)/a
^5)*sgn(x) + 2/9009*a^2*(256*b^(13/2)/a^6 + (693*(a*x + b)^(13/2) - 4095*(a
*x + b)^(11/2)*b + 10010*(a*x + b)^(9/2)*b^2 - 12870*(a*x + b)^(7/2)*b^3 +
9009*(a*x + b)^(5/2)*b^4 - 3003*(a*x + b)^(3/2)*b^5)/a^6)*sgn(x)
```

maple [A] time = 0.01, size = 55, normalized size = 0.55

$$\frac{2\,(ax+b)\left(231a^3x^3-126a^2b\,x^2+56a\,b^2x-16b^3\right)\left(\frac{ax+b}{x}\right)^{\frac{5}{2}}x^{\frac{5}{2}}}{3003a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2)*x^(11/2),x)
```

```
[Out] 2/3003*(a*x+b)*(231*a^3*x^3-126*a^2*b*x^2+56*a*b^2*x-16*b^3)*x^(5/2)*((a*x+
b)/x)^(5/2)/a^4
```

maxima [A] time = 1.05, size = 69, normalized size = 0.69

$$\frac{2\left(231\left(a+\frac{b}{x}\right)^{\frac{13}{2}}x^{\frac{13}{2}}-819\left(a+\frac{b}{x}\right)^{\frac{11}{2}}bx^{\frac{11}{2}}+1001\left(a+\frac{b}{x}\right)^{\frac{9}{2}}b^2x^{\frac{9}{2}}-429\left(a+\frac{b}{x}\right)^{\frac{7}{2}}b^3x^{\frac{7}{2}}\right)}{3003\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*x^(11/2),x, algorithm="maxima")
```

```
[Out] 2/3003*(231*(a + b/x)^(13/2)*x^(13/2) - 819*(a + b/x)^(11/2)*b*x^(11/2) + 1
001*(a + b/x)^(9/2)*b^2*x^(9/2) - 429*(a + b/x)^(7/2)*b^3*x^(7/2))/a^4
```

mupad **[B]** time = 1.54, size = 78, normalized size = 0.78

$$\sqrt{a+\frac{b}{x}}\left(\frac{2\,a^2\,x^{13/2}}{13}+\frac{106\,b^2\,x^{9/2}}{429}+\frac{10\,b^3\,x^{7/2}}{3003\,a}-\frac{4\,b^4\,x^{5/2}}{1001\,a^2}+\frac{16\,b^5\,x^{3/2}}{3003\,a^3}-\frac{32\,b^6\,\sqrt{x}}{3003\,a^4}+\frac{54\,a\,b\,x^{11/2}}{143}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)*(a + b/x)^(5/2),x)
```

```
[Out] (a + b/x)^(1/2)*((2*a^2*x^(13/2))/13 + (106*b^2*x^(9/2))/429 + (10*b^3*x^(7
/2))/(3003*a) - (4*b^4*x^(5/2))/(1001*a^2) + (16*b^5*x^(3/2))/(3003*a^3) -
(32*b^6*x^(1/2))/(3003*a^4) + (54*a*b*x^(11/2))/143)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)*x**(11/2),x)
```

```
[Out] Timed out
```

3.1769 $\int \left(a+\frac{b}{x}\right)^{5/2} x^{9/2}\,dx$

Optimal. Leaf size=74

$$\frac{16b^2x^{7/2}\left(a+\frac{b}{x}\right)^{7/2}}{693a^3}-\frac{8bx^{9/2}\left(a+\frac{b}{x}\right)^{7/2}}{99a^2}+\frac{2x^{11/2}\left(a+\frac{b}{x}\right)^{7/2}}{11a}$$

```
[Out] 16/693*b^2*(a+b/x)^(7/2)*x^(7/2)/a^3-8/99*b*(a+b/x)^(7/2)*x^(9/2)/a^2+2/11*
(a+b/x)^(7/2)*x^(11/2)/a
```

Rubi [A] time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {271, 264}

$$\frac{16b^2x^{7/2}\left(a+\frac{b}{x}\right)^{7/2}}{693a^3}-\frac{8bx^{9/2}\left(a+\frac{b}{x}\right)^{7/2}}{99a^2}+\frac{2x^{11/2}\left(a+\frac{b}{x}\right)^{7/2}}{11a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)*x^(9/2),x]
```

```
[Out] (16*b^2*(a + b/x)^(7/2)*x^(7/2))/(693*a^3) - (8*b*(a + b/x)^(7/2)*x^(9/2))/
(99*a^2) + (2*(a + b/x)^(7/2)*x^(11/2))/(11*a)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^{5/2} x^{9/2}\,dx &= \frac{2\left(a+\frac{b}{x}\right)^{7/2}x^{11/2}}{11a}-\frac{(4b)\int \left(a+\frac{b}{x}\right)^{5/2}x^{7/2}\,dx}{11a}\\ &= -\frac{8b\left(a+\frac{b}{x}\right)^{7/2}x^{9/2}}{99a^2}+\frac{2\left(a+\frac{b}{x}\right)^{7/2}x^{11/2}}{11a}+\frac{\left(8b^2\right)\int \left(a+\frac{b}{x}\right)^{5/2}x^{5/2}\,dx}{99a^2}\\ &= \frac{16b^2\left(a+\frac{b}{x}\right)^{7/2}x^{7/2}}{693a^3}-\frac{8b\left(a+\frac{b}{x}\right)^{7/2}x^{9/2}}{99a^2}+\frac{2\left(a+\frac{b}{x}\right)^{7/2}x^{11/2}}{11a}\end{aligned}$$

Mathematica [A] time = 0.04, size = 51, normalized size = 0.69

$$\frac{2}{693}x^{7/2}\sqrt{a+\frac{b}{x}}\left(\frac{b}{ax}+1\right)^3\left(63a^2x^2-28abx+8b^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)*x^(9/2),x]
```

```
[Out] (2*Sqrt[a + b/x]*(1 + b/(a*x))^3*x^(7/2)*(8*b^2 - 28*a*b*x + 63*a^2*x^2))/6
93
```

fricas [A] time = 0.62, size = 71, normalized size = 0.96

$$\frac{2\left(63\,a^5x^5+161\,a^4bx^4+113\,a^3b^2x^3+3\,a^2b^3x^2-4\,ab^4x+8\,b^5\right)\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}{693\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*x^(9/2),x, algorithm="fricas")
```

```
[Out] 2/693*(63*a^5*x^5 + 161*a^4*b*x^4 + 113*a^3*b^2*x^3 + 3*a^2*b^3*x^2 - 4*a*b
^4*x + 8*b^5)*sqrt(x)*sqrt((a*x + b)/x)/a^3
```

giac [B] time = 0.21, size = 194, normalized size = 2.62

$$-\frac{2}{105}\,b^2\left(\frac{8\,b^{\frac{7}{2}}}{a^3}-\frac{15\,(ax+b)^{\frac{7}{2}}-42\,(ax+b)^{\frac{5}{2}}b+35\,(ax+b)^{\frac{3}{2}}b^2}{a^3}\right)\mathrm{sgn}(x)+\frac{4}{315}\,ab\left(\frac{16\,b^{\frac{9}{2}}}{a^4}+\frac{35\,(ax+b)^{\frac{9}{2}}-135\,(ax+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*x^(9/2),x, algorithm="giac")
```

```
[Out] -2/105*b^2*(8*b^(7/2)/a^3 - (15*(a*x + b)^(7/2) - 42*(a*x + b)^(5/2)*b + 35
*(a*x + b)^(3/2)*b^2)/a^3)*sgn(x) + 4/315*a*b*(16*b^(9/2)/a^4 + (35*(a*x +
b)^(9/2) - 135*(a*x + b)^(7/2)*b + 189*(a*x + b)^(5/2)*b^2 - 105*(a*x + b)^
(3/2)*b^3)/a^4)*sgn(x) - 2/3465*a^2*(128*b^(11/2)/a^5 - (315*(a*x + b)^(11/
2) - 1540*(a*x + b)^(9/2)*b + 2970*(a*x + b)^(7/2)*b^2 - 2772*(a*x + b)^(5/
2)*b^3 + 1155*(a*x + b)^(3/2)*b^4)/a^5)*sgn(x)
```

maple [A] time = 0.00, size = 44, normalized size = 0.59

$$\frac{2\,(ax+b)\left(63a^2x^2-28abx+8b^2\right)\left(\frac{ax+b}{x}\right)^{\frac{5}{2}}x^{\frac{5}{2}}}{693a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2)*x^(9/2),x)
```

```
[Out] 2/693*(a*x+b)*(63*a^2*x^2-28*a*b*x+8*b^2)*x^(5/2)*((a*x+b)/x)^(5/2)/a^3
```

maxima [A] time = 1.12, size = 52, normalized size = 0.70

$$\frac{2\left(63\left(a+\frac{b}{x}\right)^{\frac{11}{2}}x^{\frac{11}{2}}-154\left(a+\frac{b}{x}\right)^{\frac{9}{2}}bx^{\frac{9}{2}}+99\left(a+\frac{b}{x}\right)^{\frac{7}{2}}b^2x^{\frac{7}{2}}\right)}{693\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*x^(9/2),x, algorithm="maxima")
```

```
[Out] 2/693*(63*(a + b/x)^(11/2)*x^(11/2) - 154*(a + b/x)^(9/2)*b*x^(9/2) + 99*(a
 + b/x)^(7/2)*b^2*x^(7/2))/a^3
```

mupad [B] time = 1.46, size = 67, normalized size = 0.91

$$\sqrt{a+\frac{b}{x}}\left(\frac{2\,a^2\,x^{11/2}}{11}+\frac{226\,b^2\,x^{7/2}}{693}+\frac{2\,b^3\,x^{5/2}}{231\,a}-\frac{8\,b^4\,x^{3/2}}{693\,a^2}+\frac{16\,b^5\,\sqrt{x}}{693\,a^3}+\frac{46\,a\,b\,x^{9/2}}{99}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)*(a + b/x)^(5/2),x)
```

```
[Out] (a + b/x)^(1/2)*((2*a^2*x^(11/2))/11 + (226*b^2*x^(7/2))/693 + (2*b^3*x^(5/
2))/(231*a) - (8*b^4*x^(3/2))/(693*a^2) + (16*b^5*x^(1/2))/(693*a^3) + (46*
a*b*x^(9/2))/99)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)*x**(9/2),x)
```

```
[Out] Timed out
```

3.1770 $\int \left(a+\frac{b}{x}\right)^{5/2} x^{7/2}\,dx$

Optimal. Leaf size=48

$$\frac{2x^{9/2}\left(a+\frac{b}{x}\right)^{7/2}}{9a}-\frac{4bx^{7/2}\left(a+\frac{b}{x}\right)^{7/2}}{63a^2}$$

```
[Out] -4/63*b*(a+b/x)^(7/2)*x^(7/2)/a^2+2/9*(a+b/x)^(7/2)*x^(9/2)/a
```

Rubi [A] time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {271, 264}

$$\frac{2x^{9/2}\left(a+\frac{b}{x}\right)^{7/2}}{9a}-\frac{4bx^{7/2}\left(a+\frac{b}{x}\right)^{7/2}}{63a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)*x^(7/2),x]
```

```
[Out] (-4*b*(a + b/x)^(7/2)*x^(7/2))/(63*a^2) + (2*(a + b/x)^(7/2)*x^(9/2))/(9*a)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^{5/2} x^{7/2}\,dx &= \frac{2\left(a+\frac{b}{x}\right)^{7/2}x^{9/2}}{9a}-\frac{(2b)\int \left(a+\frac{b}{x}\right)^{5/2}x^{5/2}\,dx}{9a}\\ &= -\frac{4b\left(a+\frac{b}{x}\right)^{7/2}x^{7/2}}{63a^2}+\frac{2\left(a+\frac{b}{x}\right)^{7/2}x^{9/2}}{9a}\end{aligned}$$

Mathematica [A] time = 0.03, size = 38, normalized size = 0.79

$$\frac{2\sqrt{x}\sqrt{a+\frac{b}{x}}\,(ax+b)^3(7ax-2b)}{63a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)*x^(7/2),x]
```

```
[Out] (2*Sqrt[a + b/x]*Sqrt[x]*(b + a*x)^3*(-2*b + 7*a*x))/(63*a^2)
```

fricas [A] time = 0.80, size = 59, normalized size = 1.23

$$\frac{2\left(7\,a^4x^4+19\,a^3bx^3+15\,a^2b^2x^2+ab^3x-2\,b^4\right)\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}{63\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/63*(7*a^4*x^4 + 19*a^3*b*x^3 + 15*a^2*b^2*x^2 + a*b^3*x - 2*b^4)*sqrt(x)*
sqrt((a*x + b)/x)/a^2
```

giac [B] time = 0.20, size = 157, normalized size = 3.27

$$\frac{2}{15}\,b^2\left(\frac{2\,b^{\frac{5}{2}}}{a^2}+\frac{3\,(ax+b)^{\frac{5}{2}}-5\,(ax+b)^{\frac{3}{2}}b}{a^2}\right)\mathrm{sgn}(x)-\frac{4}{105}\,ab\left(\frac{8\,b^{\frac{7}{2}}}{a^3}-\frac{15\,(ax+b)^{\frac{7}{2}}-42\,(ax+b)^{\frac{5}{2}}b+35\,(ax+b)^{\frac{3}{2}}b^2}{a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*x^(7/2),x, algorithm="giac")
```

```
[Out] 2/15*b^2*(2*b^(5/2)/a^2 + (3*(a*x + b)^(5/2) - 5*(a*x + b)^(3/2)*b)/a^2)*sg
n(x) - 4/105*a*b*(8*b^(7/2)/a^3 - (15*(a*x + b)^(7/2) - 42*(a*x + b)^(5/2)*
b + 35*(a*x + b)^(3/2)*b^2)/a^3)*sgn(x) + 2/315*a^2*(16*b^(9/2)/a^4 + (35*(
a*x + b)^(9/2) - 135*(a*x + b)^(7/2)*b + 189*(a*x + b)^(5/2)*b^2 - 105*(a*x
 + b)^(3/2)*b^3)/a^4)*sgn(x)
```

maple [A] time = 0.00, size = 33, normalized size = 0.69

$$\frac{2\,(ax+b)\,(7ax-2b)\left(\frac{ax+b}{x}\right)^{\frac{5}{2}}x^{\frac{5}{2}}}{63a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2)*x^(7/2),x)
```

```
[Out] 2/63*(a*x+b)*(7*a*x-2*b)*x^(5/2)*((a*x+b)/x)^(5/2)/a^2
```

maxima [A] time = 1.10, size = 35, normalized size = 0.73

$$\frac{2\left(7\left(a+\frac{b}{x}\right)^{\frac{9}{2}}x^{\frac{9}{2}}-9\left(a+\frac{b}{x}\right)^{\frac{7}{2}}bx^{\frac{7}{2}}\right)}{63\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/63*(7*(a + b/x)^(9/2)*x^(9/2) - 9*(a + b/x)^(7/2)*b*x^(7/2))/a^2
```

mupad [B] time = 1.45, size = 56, normalized size = 1.17

$$\sqrt{a+\frac{b}{x}}\left(\frac{2\,a^2\,x^{9/2}}{9}+\frac{10\,b^2\,x^{5/2}}{21}+\frac{2\,b^3\,x^{3/2}}{63\,a}-\frac{4\,b^4\,\sqrt{x}}{63\,a^2}+\frac{38\,a\,b\,x^{7/2}}{63}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(a + b/x)^(5/2),x)
```

```
[Out] (a + b/x)^(1/2)*((2*a^2*x^(9/2))/9 + (10*b^2*x^(5/2))/21 + (2*b^3*x^(3/2))/
(63*a) - (4*b^4*x^(1/2))/(63*a^2) + (38*a*b*x^(7/2))/63)
```

sympy [**B**] time = 98.84, size = 114, normalized size = 2.38

$$\frac{2a^2\sqrt{b}\,x^4\sqrt{\frac{ax}{b}+1}}{9}+\frac{38ab^{\frac{3}{2}}x^3\sqrt{\frac{ax}{b}+1}}{63}+\frac{10b^{\frac{5}{2}}x^2\sqrt{\frac{ax}{b}+1}}{21}+\frac{2b^{\frac{7}{2}}x\sqrt{\frac{ax}{b}+1}}{63a}-\frac{4b^{\frac{9}{2}}\sqrt{\frac{ax}{b}+1}}{63a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)*x**(7/2),x)
```

```
[Out] 2*a**2*sqrt(b)*x**4*sqrt(a*x/b + 1)/9 + 38*a*b**(3/2)*x**3*sqrt(a*x/b + 1)/
63 + 10*b**(5/2)*x**2*sqrt(a*x/b + 1)/21 + 2*b**(7/2)*x*sqrt(a*x/b + 1)/(63
*a) - 4*b**(9/2)*sqrt(a*x/b + 1)/(63*a**2)
```

3.1771 $\int \left(a+\frac{b}{x}\right)^{5/2} x^{5/2}\,dx$

Optimal. Leaf size=23

$$\frac{2x^{7/2}\left(a+\frac{b}{x}\right)^{7/2}}{7a}$$

```
[Out] 2/7*(a+b/x)^(7/2)*x^(7/2)/a
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {264}

$$\frac{2x^{7/2}\left(a+\frac{b}{x}\right)^{7/2}}{7a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)*x^(5/2),x]
```

```
[Out] (2*(a + b/x)^(7/2)*x^(7/2))/(7*a)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^{5/2}\,dx = \frac{2\left(a+\frac{b}{x}\right)^{7/2} x^{7/2}}{7a}$$

Mathematica [A] time = 0.03, size = 23, normalized size = 1.00

$$\frac{2x^{7/2}\left(a+\frac{b}{x}\right)^{7/2}}{7a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)*x^(5/2),x]
```

```
[Out] (2*(a + b/x)^(7/2)*x^(7/2))/(7*a)
```

fricas [B] time = 0.83, size = 46, normalized size = 2.00

$$\frac{2\left(a^3x^3+3\,a^2bx^2+3\,ab^2x+b^3\right)\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}{7\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/7*(a^3*x^3 + 3*a^2*b*x^2 + 3*a*b^2*x + b^3)*sqrt(x)*sqrt((a*x + b)/x)/a
```

giac [B] time = 0.20, size = 120, normalized size = 5.22

$$\frac{2}{3}b^2\left(\frac{(ax+b)^{\frac{3}{2}}}{a}-\frac{b^{\frac{3}{2}}}{a}\right)\text{sgn}(x)+\frac{4}{15}ab\left(\frac{2\,b^{\frac{5}{2}}}{a^2}+\frac{3\,(ax+b)^{\frac{5}{2}}-5\,(ax+b)^{\frac{3}{2}}b}{a^2}\right)\text{sgn}(x)-\frac{2}{105}a^2\left(\frac{8\,b^{\frac{7}{2}}}{a^3}-\frac{15\,(ax+b)^{\frac{7}{2}}-42}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*x^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*b^2*((a*x + b)^(3/2)/a - b^(3/2)/a)*sgn(x) + 4/15*a*b*(2*b^(5/2)/a^2 +
(3*(a*x + b)^(5/2) - 5*(a*x + b)^(3/2)*b)/a^2)*sgn(x) - 2/105*a^2*(8*b^(7/2
)/a^3 - (15*(a*x + b)^(7/2) - 42*(a*x + b)^(5/2)*b + 35*(a*x + b)^(3/2)*b^2
)/a^3)*sgn(x)
```

maple [A] time = 0.00, size = 25, normalized size = 1.09

$$\frac{2\,(ax+b)\left(\frac{ax+b}{x}\right)^{\frac{5}{2}}x^{\frac{5}{2}}}{7a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2)*x^(5/2),x)
```

```
[Out] 2/7*(a*x+b)*((a*x+b)/x)^(5/2)*x^(5/2)/a
```

maxima [A] time = 1.05, size = 17, normalized size = 0.74

$$\frac{2\left(a+\frac{b}{x}\right)^{\frac{7}{2}}x^{\frac{7}{2}}}{7\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/7*(a + b/x)^(7/2)*x^(7/2)/a
```

mupad [B] time = 1.38, size = 45, normalized size = 1.96

$$\sqrt{a+\frac{b}{x}}\left(\frac{2\,a^2\,x^{7/2}}{7}+\frac{6\,b^2\,x^{3/2}}{7}+\frac{2\,b^3\,\sqrt{x}}{7\,a}+\frac{6\,a\,b\,x^{5/2}}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b/x)^(5/2),x)
```

```
[Out] (a + b/x)^(1/2)*((2*a^2*x^(7/2))/7 + (6*b^2*x^(3/2))/7 + (2*b^3*x^(1/2))/(7
*a) + (6*a*b*x^(5/2))/7)
```

sympy [B] time = 40.24, size = 88, normalized size = 3.83

$$\frac{2a^2\sqrt{b}\,x^3\sqrt{\frac{ax}{b}+1}}{7}+\frac{6ab^{\frac{3}{2}}x^2\sqrt{\frac{ax}{b}+1}}{7}+\frac{6b^{\frac{5}{2}}x\sqrt{\frac{ax}{b}+1}}{7}+\frac{2b^{\frac{7}{2}}\sqrt{\frac{ax}{b}+1}}{7a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)*x**(5/2),x)
```

```
[Out] 2*a**2*sqrt(b)*x**3*sqrt(a*x/b + 1)/7 + 6*a*b**(3/2)*x**2*sqrt(a*x/b + 1)/7
 + 6*b**(5/2)*x*sqrt(a*x/b + 1)/7 + 2*b**(7/2)*sqrt(a*x/b + 1)/(7*a)
```

3.1772 $\int \left(a+\frac{b}{x}\right)^{5/2} x^{3/2}\,dx$

Optimal. Leaf size=93

$$-2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)+2b^2\sqrt{x}\sqrt{a+\frac{b}{x}}+\frac{2}{3}bx^{3/2}\left(a+\frac{b}{x}\right)^{3/2}+\frac{2}{5}x^{5/2}\left(a+\frac{b}{x}\right)^{5/2}$$

```
[Out] 2/3*b*(a+b/x)^(3/2)*x^(3/2)+2/5*(a+b/x)^(5/2)*x^(5/2)-2*b^(5/2)*arctanh(b^(
1/2)/(a+b/x)^(1/2)/x^(1/2))+2*b^2*(a+b/x)^(1/2)*x^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {337, 277, 217, 206}

$$2b^2\sqrt{x}\sqrt{a+\frac{b}{x}}-2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)+\frac{2}{3}bx^{3/2}\left(a+\frac{b}{x}\right)^{3/2}+\frac{2}{5}x^{5/2}\left(a+\frac{b}{x}\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)*x^(3/2),x]
```

```
[Out] 2*b^2*Sqrt[a + b/x]*Sqrt[x] + (2*b*(a + b/x)^(3/2)*x^(3/2))/3 + (2*(a + b/x
)^(5/2)*x^(5/2))/5 - 2*b^(5/2)*ArcTanh[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x])]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 337

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, -Dist[k/c, Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1)
 + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0] &
& FractionQ[m]
```

Rubi steps

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^{3/2}\,dx = -\left(2\,\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{5/2}}{x^6}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= \frac{2}{5}\left(a+\frac{b}{x}\right)^{5/2} x^{5/2} - (2b)\,\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x^4}\,dx, x, \frac{1}{\sqrt{x}}\right)$$

$$= \frac{2}{3}b\left(a+\frac{b}{x}\right)^{3/2} x^{3/2} + \frac{2}{5}\left(a+\frac{b}{x}\right)^{5/2} x^{5/2} - \left(2b^2\right)\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2}\,dx, x, \frac{1}{\sqrt{x}}\right)$$

$$= 2b^2\sqrt{a+\frac{b}{x}}\,\sqrt{x} + \frac{2}{3}b\left(a+\frac{b}{x}\right)^{3/2} x^{3/2} + \frac{2}{5}\left(a+\frac{b}{x}\right)^{5/2} x^{5/2} - \left(2b^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x\right.$$

$$= 2b^2\sqrt{a+\frac{b}{x}}\,\sqrt{x} + \frac{2}{3}b\left(a+\frac{b}{x}\right)^{3/2} x^{3/2} + \frac{2}{5}\left(a+\frac{b}{x}\right)^{5/2} x^{5/2} - \left(2b^3\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, -\right.$$

$$= 2b^2\sqrt{a+\frac{b}{x}}\,\sqrt{x} + \frac{2}{3}b\left(a+\frac{b}{x}\right)^{3/2} x^{3/2} + \frac{2}{5}\left(a+\frac{b}{x}\right)^{5/2} x^{5/2} - 2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\,\sqrt{x}}\right)$$

Mathematica [C] time = 0.02, size = 56, normalized size = 0.60

$$\frac{2a^2x^{5/2}\sqrt{a+\frac{b}{x}}\;{}_2F_1\left(-\frac{5}{2},-\frac{5}{2};-\frac{3}{2};-\frac{b}{ax}\right)}{5\sqrt{\frac{b}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)*x^(3/2),x]
```

```
[Out] (2*a^2*Sqrt[a + b/x]*x^(5/2)*Hypergeometric2F1[-5/2, -5/2, -3/2, -(b/(a*x))
])/(5*Sqrt[1 + b/(a*x)])
```

fricas [A] time = 0.90, size = 142, normalized size = 1.53

$$\left[b^{\frac{5}{2}}\log\left(\frac{ax-2\sqrt{b}\,\sqrt{x}\,\sqrt{\frac{ax+b}{x}}+2b}{x}\right)+\frac{2}{15}\left(3a^2x^2+11abx+23b^2\right)\sqrt{x}\,\sqrt{\frac{ax+b}{x}}, 2\sqrt{-b}\,b^2\arctan\left(\frac{\sqrt{-b}\,\sqrt{x}\,\sqrt{\frac{ax}{x}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*x^(3/2),x, algorithm="fricas")
```

```
[Out] [b^(5/2)*log((a*x - 2*sqrt(b)*sqrt(x)*sqrt((a*x + b)/x) + 2*b)/x) + 2/15*(3
*a^2*x^2 + 11*a*b*x + 23*b^2)*sqrt(x)*sqrt((a*x + b)/x), 2*sqrt(-b)*b^2*arc
tan(sqrt(-b)*sqrt(x)*sqrt((a*x + b)/x)/b) + 2/15*(3*a^2*x^2 + 11*a*b*x + 23
*b^2)*sqrt(x)*sqrt((a*x + b)/x)]
```

giac [A] time = 0.26, size = 56, normalized size = 0.60

$$\frac{2b^3\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}+\frac{2}{5}(ax+b)^{\frac{5}{2}}+\frac{2}{3}(ax+b)^{\frac{3}{2}}b+2\sqrt{ax+b}\,b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*x^(3/2),x, algorithm="giac")
```

```
[Out] 2*b^3*arctan(sqrt(a*x + b)/sqrt(-b))/sqrt(-b) + 2/5*(a*x + b)^(5/2) + 2/3*(
a*x + b)^(3/2)*b + 2*sqrt(a*x + b)*b^2
```

maple [A] time = 0.01, size = 81, normalized size = 0.87

$$-\frac{2\sqrt{\frac{ax+b}{x}}\left(-3\sqrt{ax+b}\,a^2x^2+15b^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)-11\sqrt{ax+b}\,abx-23\sqrt{ax+b}\,b^2\right)\sqrt{x}}{15\sqrt{ax+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2)*x^(3/2),x)
```

```
[Out] -2/15*((a*x+b)/x)^(1/2)*x^(1/2)*(-3*x^2*a^2*(a*x+b)^(1/2)+15*b^(5/2)*arctan
h((a*x+b)^(1/2)/b^(1/2))-11*x*a*b*(a*x+b)^(1/2)-23*(a*x+b)^(1/2)*b^2)/(a*x+
b)^(1/2)
```

maxima [A] time = 2.34, size = 91, normalized size = 0.98

$$\frac{2}{5}\left(a+\frac{b}{x}\right)^{\frac{5}{2}}x^{\frac{5}{2}}+\frac{2}{3}\left(a+\frac{b}{x}\right)^{\frac{3}{2}}bx^{\frac{3}{2}}+b^{\frac{5}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}+\sqrt{b}}\right)+2\sqrt{a+\frac{b}{x}}\,b^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*(a + b/x)^(5/2)*x^(5/2) + 2/3*(a + b/x)^(3/2)*b*x^(3/2) + b^(5/2)*log((
sqrt(a + b/x)*sqrt(x) - sqrt(b))/(sqrt(a + b/x)*sqrt(x) + sqrt(b))) + 2*sqr
t(a + b/x)*b^2*sqrt(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\left(a+\frac{b}{x}\right)^{5/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b/x)^(5/2),x)
```

```
[Out] int(x^(3/2)*(a + b/x)^(5/2), x)
```

sympy [A] time = 26.95, size = 97, normalized size = 1.04

$$\frac{2a^2\sqrt{b}\,x^2\sqrt{\frac{ax}{b}+1}}{5}+\frac{22ab^{\frac{3}{2}}x\sqrt{\frac{ax}{b}+1}}{15}+\frac{46b^{\frac{5}{2}}\sqrt{\frac{ax}{b}+1}}{15}+b^{\frac{5}{2}}\log\left(\frac{ax}{b}\right)-2b^{\frac{5}{2}}\log\left(\sqrt{\frac{ax}{b}+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)*x**(3/2),x)
```

```
[Out] 2*a**2*sqrt(b)*x**2*sqrt(a*x/b + 1)/5 + 22*a*b**(3/2)*x*sqrt(a*x/b + 1)/15
+ 46*b**(5/2)*sqrt(a*x/b + 1)/15 + b**(5/2)*log(a*x/b) - 2*b**(5/2)*log(sqr
t(a*x/b + 1) + 1)
```

3.1773 $\int \left(a+\frac{b}{x}\right)^{5/2}\sqrt{x}\,dx$

Optimal. Leaf size=94

$$-5ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)-\frac{5b^2\sqrt{a+\frac{b}{x}}}{\sqrt{x}}+\frac{2}{3}x^{3/2}\left(a+\frac{b}{x}\right)^{5/2}+\frac{10}{3}b\sqrt{x}\left(a+\frac{b}{x}\right)^{3/2}$$

```
[Out] 2/3*(a+b/x)^(5/2)*x^(3/2)-5*a*b^(3/2)*arctanh(b^(1/2)/(a+b/x)^(1/2)/x^(1/2)
)-5*b^2*(a+b/x)^(1/2)/x^(1/2)+10/3*b*(a+b/x)^(3/2)*x^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {337, 277, 195, 217, 206}

$$-\frac{5b^2\sqrt{a+\frac{b}{x}}}{\sqrt{x}}-5ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)+\frac{2}{3}x^{3/2}\left(a+\frac{b}{x}\right)^{5/2}+\frac{10}{3}b\sqrt{x}\left(a+\frac{b}{x}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)*Sqrt[x],x]
```

```
[Out] (-5*b^2*Sqrt[a + b/x])/Sqrt[x] + (10*b*(a + b/x)^(3/2)*Sqrt[x])/3 + (2*(a +
 b/x)^(5/2)*x^(3/2))/3 - 5*a*b^(3/2)*ArcTanh[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x]
)]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 337

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, -Dist[k/c, Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1)
```

```
 + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0] &
& FractionQ[m]
```

Rubi steps

$$\int \left(a+\frac{b}{x}\right)^{5/2}\sqrt{x}\,dx = -\left(2\,\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{5/2}}{x^4}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= \frac{2}{3}\left(a+\frac{b}{x}\right)^{5/2}x^{3/2} - \frac{1}{3}(10b)\,\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x^2}\,dx, x, \frac{1}{\sqrt{x}}\right)$$

$$= \frac{10}{3}b\left(a+\frac{b}{x}\right)^{3/2}\sqrt{x} + \frac{2}{3}\left(a+\frac{b}{x}\right)^{5/2}x^{3/2} - \left(10b^2\right)\text{Subst}\left(\int \sqrt{a+bx^2}\,dx, x, \frac{1}{\sqrt{x}}\right)$$

$$= -\frac{5b^2\sqrt{a+\frac{b}{x}}}{\sqrt{x}} + \frac{10}{3}b\left(a+\frac{b}{x}\right)^{3/2}\sqrt{x} + \frac{2}{3}\left(a+\frac{b}{x}\right)^{5/2}x^{3/2} - \left(5ab^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,d$$

$$= -\frac{5b^2\sqrt{a+\frac{b}{x}}}{\sqrt{x}} + \frac{10}{3}b\left(a+\frac{b}{x}\right)^{3/2}\sqrt{x} + \frac{2}{3}\left(a+\frac{b}{x}\right)^{5/2}x^{3/2} - \left(5ab^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x\right.$$

$$= -\frac{5b^2\sqrt{a+\frac{b}{x}}}{\sqrt{x}} + \frac{10}{3}b\left(a+\frac{b}{x}\right)^{3/2}\sqrt{x} + \frac{2}{3}\left(a+\frac{b}{x}\right)^{5/2}x^{3/2} - 5ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)$$

Mathematica [C] time = 0.03, size = 56, normalized size = 0.60

$$\frac{2a^2x^{3/2}\sqrt{a+\frac{b}{x}}\;{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};-\frac{b}{ax}\right)}{3\sqrt{\frac{b}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)*Sqrt[x],x]
```

```
[Out] (2*a^2*Sqrt[a + b/x]*x^(3/2)*Hypergeometric2F1[-5/2, -3/2, -1/2, -(b/(a*x))
])/(3*Sqrt[1 + b/(a*x)])
```

fricas [A] time = 0.88, size = 154, normalized size = 1.64

$$\left[\frac{15\,ab^{\frac{3}{2}}x\log\left(\frac{ax-2\sqrt{b}\sqrt{x}\sqrt{\frac{ax+b}{x}}+2b}{x}\right)+2\left(2a^2x^2+14abx-3b^2\right)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{6x}, \frac{15\,a\sqrt{-b}\,bx\arctan\left(\frac{\sqrt{-b}\sqrt{x}\sqrt{\frac{ax+b}{x}}}{b}\right)+}{3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*a*b^(3/2)*x*log((a*x - 2*sqrt(b)*sqrt(x)*sqrt((a*x + b)/x) + 2*b)/
x) + 2*(2*a^2*x^2 + 14*a*b*x - 3*b^2)*sqrt(x)*sqrt((a*x + b)/x))/x, 1/3*(15
*a*sqrt(-b)*b*x*arctan(sqrt(-b)*sqrt(x)*sqrt((a*x + b)/x)/b) + (2*a^2*x^2 +
 14*a*b*x - 3*b^2)*sqrt(x)*sqrt((a*x + b)/x))/x]
```

giac [A] time = 0.32, size = 74, normalized size = 0.79

$$\frac{\frac{15\,a^2b^2\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{\sqrt{-b}} + 2\,(ax+b)^{\frac{3}{2}}a^2 + 12\,\sqrt{ax+b}\,a^2b - \frac{3\,\sqrt{ax+b}\,ab^2}{x}}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*x^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(15*a^2*b^2*arctan(sqrt(a*x + b)/sqrt(-b))/sqrt(-b) + 2*(a*x + b)^(3/2)
*a^2 + 12*sqrt(a*x + b)*a^2*b - 3*sqrt(a*x + b)*a*b^2/x)/a
```

maple [A] time = 0.02, size = 91, normalized size = 0.97

$$-\frac{\sqrt{\frac{ax+b}{x}}\,\left(15a\,b^2x\,\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right) - 2\sqrt{ax+b}\;a^2\sqrt{b}\;x^2 - 14\sqrt{ax+b}\;a\,b^{\frac{3}{2}}x + 3\sqrt{ax+b}\;b^{\frac{5}{2}}\right)}{3\sqrt{ax+b}\;\sqrt{b}\;\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2)*x^(1/2),x)
```

```
[Out] -1/3*((a*x+b)/x)^(1/2)/x^(1/2)*(-2*(a*x+b)^(1/2)*a^2*b^(1/2)*x^2+15*arctanh
((a*x+b)^(1/2)/b^(1/2))*x*a*b^2-14*(a*x+b)^(1/2)*a*b^(3/2)*x+3*(a*x+b)^(1/2
)*b^(5/2))/(a*x+b)^(1/2)/b^(1/2)
```

maxima [A] time = 2.26, size = 111, normalized size = 1.18

$$\frac{2}{3}\left(a+\frac{b}{x}\right)^{\frac{3}{2}}ax^{\frac{3}{2}} + \frac{5}{2}\,ab^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}\,\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\,\sqrt{x}+\sqrt{b}}\right) + 4\sqrt{a+\frac{b}{x}}\,ab\sqrt{x} - \frac{\sqrt{a+\frac{b}{x}}\,ab^2\sqrt{x}}{\left(a+\frac{b}{x}\right)x-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(a + b/x)^(3/2)*a*x^(3/2) + 5/2*a*b^(3/2)*log((sqrt(a + b/x)*sqrt(x) -
sqrt(b))/(sqrt(a + b/x)*sqrt(x) + sqrt(b))) + 4*sqrt(a + b/x)*a*b*sqrt(x) -
 sqrt(a + b/x)*a*b^2*sqrt(x)/((a + b/x)*x - b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\left(a+\frac{b}{x}\right)^{5/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b/x)^(5/2),x)
```

```
[Out] int(x^(1/2)*(a + b/x)^(5/2), x)
```

sympy [A] time = 17.42, size = 99, normalized size = 1.05

$$\frac{2a^2\sqrt{b}\,x\sqrt{\frac{ax}{b}+1}}{3} + \frac{14ab^{\frac{3}{2}}\sqrt{\frac{ax}{b}+1}}{3} + \frac{5ab^{\frac{3}{2}}\log\left(\frac{ax}{b}\right)}{2} - 5ab^{\frac{3}{2}}\log\left(\sqrt{\frac{ax}{b}+1}+1\right) - \frac{b^{\frac{5}{2}}\sqrt{\frac{ax}{b}+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)*x**(1/2),x)
```

```
[Out] 2*a**2*sqrt(b)*x*sqrt(a*x/b + 1)/3 + 14*a*b**(3/2)*sqrt(a*x/b + 1)/3 + 5*a*
b**(3/2)*log(a*x/b)/2 - 5*a*b**(3/2)*log(sqrt(a*x/b + 1) + 1) - b**(5/2)*sq
rt(a*x/b + 1)/x
```

3.1774 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\sqrt{x}}\,dx$

Optimal. Leaf size=97

$$-\frac{15}{4}a^2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)+2\sqrt{x}\left(a+\frac{b}{x}\right)^{5/2}-\frac{5b\left(a+\frac{b}{x}\right)^{3/2}}{2\sqrt{x}}-\frac{15ab\sqrt{a+\frac{b}{x}}}{4\sqrt{x}}$$

```
[Out] -15/4*a^2*arctanh(b^(1/2)/(a+b/x)^(1/2)/x^(1/2))*b^(1/2)-5/2*b*(a+b/x)^(3/2
)/x^(1/2)-15/4*a*b*(a+b/x)^(1/2)/x^(1/2)+2*(a+b/x)^(5/2)*x^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {337, 277, 195, 217, 206}

$$-\frac{15}{4}a^2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)+2\sqrt{x}\left(a+\frac{b}{x}\right)^{5/2}-\frac{5b\left(a+\frac{b}{x}\right)^{3/2}}{2\sqrt{x}}-\frac{15ab\sqrt{a+\frac{b}{x}}}{4\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)/Sqrt[x],x]
```

```
[Out] (-15*a*b*Sqrt[a + b/x])/(4*Sqrt[x]) - (5*b*(a + b/x)^(3/2))/(2*Sqrt[x]) + 2
*(a + b/x)^(5/2)*Sqrt[x] - (15*a^2*Sqrt[b]*ArcTanh[Sqrt[b]/(Sqrt[a + b/x]*S
qrt[x])])/4
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 337

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, -Dist[k/c, Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1)
```

```
 + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0] &
& FractionQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\sqrt{x}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{5/2}}{x^2}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)\\
&= 2\left(a+\frac{b}{x}\right)^{5/2}\sqrt{x} - (10b)\,\text{Subst}\left(\int \left(a+bx^2\right)^{3/2}\,dx, x, \frac{1}{\sqrt{x}}\right)\\
&= -\frac{5b\left(a+\frac{b}{x}\right)^{3/2}}{2\sqrt{x}} + 2\left(a+\frac{b}{x}\right)^{5/2}\sqrt{x} - \frac{1}{2}(15ab)\,\text{Subst}\left(\int \sqrt{a+bx^2}\,dx, x, \frac{1}{\sqrt{x}}\right)\\
&= -\frac{15ab\sqrt{a+\frac{b}{x}}}{4\sqrt{x}} - \frac{5b\left(a+\frac{b}{x}\right)^{3/2}}{2\sqrt{x}} + 2\left(a+\frac{b}{x}\right)^{5/2}\sqrt{x} - \frac{1}{4}\left(15a^2b\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{\sqrt{x}}\right)\\
&= -\frac{15ab\sqrt{a+\frac{b}{x}}}{4\sqrt{x}} - \frac{5b\left(a+\frac{b}{x}\right)^{3/2}}{2\sqrt{x}} + 2\left(a+\frac{b}{x}\right)^{5/2}\sqrt{x} - \frac{1}{4}\left(15a^2b\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)\\
&= -\frac{15ab\sqrt{a+\frac{b}{x}}}{4\sqrt{x}} - \frac{5b\left(a+\frac{b}{x}\right)^{3/2}}{2\sqrt{x}} + 2\left(a+\frac{b}{x}\right)^{5/2}\sqrt{x} - \frac{15}{4}a^2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)
\end{aligned}$$

Mathematica [C] time = 0.03, size = 54, normalized size = 0.56

$$\frac{2a^2\sqrt{x}\sqrt{a+\frac{b}{x}}\;{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};-\frac{b}{ax}\right)}{\sqrt{\frac{b}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)/Sqrt[x],x]
```

```
[Out] (2*a^2*Sqrt[a + b/x]*Sqrt[x]*Hypergeometric2F1[-5/2, -1/2, 1/2, -(b/(a*x))]
)/Sqrt[1 + b/(a*x)]
```

fricas [A] time = 0.85, size = 161, normalized size = 1.66

$$\left[\frac{15\,a^2\sqrt{b}\,x^2\log\left(\frac{ax-2\sqrt{b}\sqrt{x}\sqrt{\frac{ax+b}{x}}+2b}{x}\right)+2\left(8\,a^2x^2-9\,abx-2\,b^2\right)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{8\,x^2},\ \frac{15\,a^2\sqrt{-b}\,x^2\arctan\left(\frac{\sqrt{-b}\sqrt{x}\sqrt{\frac{ax+b}{x}}}{b}\right)+}{4\,x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(15*a^2*sqrt(b)*x^2*log((a*x - 2*sqrt(b)*sqrt(x)*sqrt((a*x + b)/x) + 2
*b)/x) + 2*(8*a^2*x^2 - 9*a*b*x - 2*b^2)*sqrt(x)*sqrt((a*x + b)/x))/x^2, 1/
4*(15*a^2*sqrt(-b)*x^2*arctan(sqrt(-b)*sqrt(x)*sqrt((a*x + b)/x)/b) + (8*a^
2*x^2 - 9*a*b*x - 2*b^2)*sqrt(x)*sqrt((a*x + b)/x))/x^2]
```

giac [A] time = 0.30, size = 80, normalized size = 0.82

$$\frac{\frac{15\,a^3 b \arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{\sqrt{-b}} + 8\sqrt{ax+b}\,a^3 - \frac{9\,(ax+b)^{\frac{3}{2}}a^3 b - 7\sqrt{ax+b}\,a^3 b^2}{a^2 x^2}}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(15*a^3*b*arctan(sqrt(a*x + b)/sqrt(-b))/sqrt(-b) + 8*sqrt(a*x + b)*a^3
 - (9*(a*x + b)^(3/2)*a^3*b - 7*sqrt(a*x + b)*a^3*b^2)/(a^2*x^2))/a
```

maple [A] time = 0.02, size = 93, normalized size = 0.96

$$-\frac{\sqrt{\frac{ax+b}{x}}\left(15a^2 b\,x^2 \operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right) - 8\sqrt{ax+b}\,a^2\sqrt{b}\,x^2 + 9\sqrt{ax+b}\,a\,b^{\frac{3}{2}}x + 2\sqrt{ax+b}\,b^{\frac{5}{2}}\right)}{4\sqrt{ax+b}\,\sqrt{b}\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2)/x^(1/2),x)
```

```
[Out] -1/4*((a*x+b)/x)^(1/2)/x^(3/2)*(15*arctanh((a*x+b)^(1/2)/b^(1/2))*a^2*b*x^2
-8*(a*x+b)^(1/2)*a^2*b^(1/2)*x^2+9*(a*x+b)^(1/2)*a*b^(3/2)*x+2*(a*x+b)^(1/2
)*b^(5/2))/(a*x+b)^(1/2)/b^(1/2)
```

maxima [A] time = 2.36, size = 137, normalized size = 1.41

$$\frac{15}{8}a^2\sqrt{b}\,\log\left(\frac{\sqrt{a+\frac{b}{x}}\,\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\,\sqrt{x}+\sqrt{b}}\right) + 2\sqrt{a+\frac{b}{x}}\,a^2\sqrt{x} - \frac{9\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2 b x^{\frac{3}{2}} - 7\sqrt{a+\frac{b}{x}}\,a^2 b^2\sqrt{x}}{4\left(\left(a+\frac{b}{x}\right)^2 x^2 - 2\left(a+\frac{b}{x}\right)bx + b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 15/8*a^2*sqrt(b)*log((sqrt(a + b/x)*sqrt(x) - sqrt(b))/(sqrt(a + b/x)*sqrt(
x) + sqrt(b))) + 2*sqrt(a + b/x)*a^2*sqrt(x) - 1/4*(9*(a + b/x)^(3/2)*a^2*b
*x^(3/2) - 7*sqrt(a + b/x)*a^2*b^2*sqrt(x))/((a + b/x)^2*x^2 - 2*(a + b/x)*
b*x + b^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(5/2)/x^(1/2),x)
```

```
[Out] int((a + b/x)^(5/2)/x^(1/2), x)
```

sympy [A] time = 13.99, size = 126, normalized size = 1.30

$$\frac{2a^{\frac{5}{2}}\sqrt{x}}{\sqrt{1+\frac{b}{ax}}} - \frac{a^{\frac{3}{2}}b}{4\sqrt{x}\sqrt{1+\frac{b}{ax}}} - \frac{11\sqrt{a}\,b^2}{4x^{\frac{3}{2}}\sqrt{1+\frac{b}{ax}}} - \frac{15a^2\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right)}{4} - \frac{b^3}{2\sqrt{a}\,x^{\frac{5}{2}}\sqrt{1+\frac{b}{ax}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)/x**(1/2),x)
```

```
[Out] 2*a**(5/2)*sqrt(x)/sqrt(1 + b/(a*x)) - a**(3/2)*b/(4*sqrt(x)*sqrt(1 + b/(a*
x))) - 11*sqrt(a)*b**2/(4*x**(3/2)*sqrt(1 + b/(a*x))) - 15*a**2*sqrt(b)*asi
nh(sqrt(b)/(sqrt(a)*sqrt(x)))/4 - b**3/(2*sqrt(a)*x**(5/2)*sqrt(1 + b/(a*x)
))
```

3.1775 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^{3/2}}\,dx$

Optimal. Leaf size=100

$$-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{8\sqrt{b}}-\frac{5a^2\sqrt{a+\frac{b}{x}}}{8\sqrt{x}}-\frac{5a\left(a+\frac{b}{x}\right)^{3/2}}{12\sqrt{x}}-\frac{\left(a+\frac{b}{x}\right)^{5/2}}{3\sqrt{x}}$$

```
[Out] -5/8*a^3*arctanh(b^(1/2)/(a+b/x)^(1/2)/x^(1/2))/b^(1/2)-5/12*a*(a+b/x)^(3/2
)/x^(1/2)-1/3*(a+b/x)^(5/2)/x^(1/2)-5/8*a^2*(a+b/x)^(1/2)/x^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {337, 195, 217, 206}

$$-\frac{5a^2\sqrt{a+\frac{b}{x}}}{8\sqrt{x}}-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{8\sqrt{b}}-\frac{5a\left(a+\frac{b}{x}\right)^{3/2}}{12\sqrt{x}}-\frac{\left(a+\frac{b}{x}\right)^{5/2}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)/x^(3/2),x]
```

```
[Out] (-5*a^2*Sqrt[a + b/x])/(8*Sqrt[x]) - (5*a*(a + b/x)^(3/2))/(12*Sqrt[x]) - (
a + b/x)^(5/2)/(3*Sqrt[x]) - (5*a^3*ArcTanh[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x])
])/(8*Sqrt[b])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 337

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, -Dist[k/c, Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1)
 + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0] &
& FractionQ[m]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^{3/2}}\,dx = -\left(2\,\text{Subst}\left(\int \left(a+bx^2\right)^{5/2}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$
$$= -\frac{\left(a+\frac{b}{x}\right)^{5/2}}{3\sqrt{x}} - \frac{1}{3}(5a)\,\text{Subst}\left(\int \left(a+bx^2\right)^{3/2}\,dx, x, \frac{1}{\sqrt{x}}\right)$$
$$= -\frac{5a\left(a+\frac{b}{x}\right)^{3/2}}{12\sqrt{x}} - \frac{\left(a+\frac{b}{x}\right)^{5/2}}{3\sqrt{x}} - \frac{1}{4}\left(5a^2\right)\text{Subst}\left(\int \sqrt{a+bx^2}\,dx, x, \frac{1}{\sqrt{x}}\right)$$
$$= -\frac{5a^2\sqrt{a+\frac{b}{x}}}{8\sqrt{x}} - \frac{5a\left(a+\frac{b}{x}\right)^{3/2}}{12\sqrt{x}} - \frac{\left(a+\frac{b}{x}\right)^{5/2}}{3\sqrt{x}} - \frac{1}{8}\left(5a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{\sqrt{x}}\right)$$
$$= -\frac{5a^2\sqrt{a+\frac{b}{x}}}{8\sqrt{x}} - \frac{5a\left(a+\frac{b}{x}\right)^{3/2}}{12\sqrt{x}} - \frac{\left(a+\frac{b}{x}\right)^{5/2}}{3\sqrt{x}} - \frac{1}{8}\left(5a^3\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)$$
$$= -\frac{5a^2\sqrt{a+\frac{b}{x}}}{8\sqrt{x}} - \frac{5a\left(a+\frac{b}{x}\right)^{3/2}}{12\sqrt{x}} - \frac{\left(a+\frac{b}{x}\right)^{5/2}}{3\sqrt{x}} - \frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)}{8\sqrt{b}}$$

Mathematica [A] time = 0.22, size = 85, normalized size = 0.85

$$\frac{1}{24}\sqrt{a+\frac{b}{x}}\left(-\frac{15a^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right)}{\sqrt{b}\sqrt{\frac{b}{ax}+1}} - \frac{33a^2x^2+26abx+8b^2}{x^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)/x^(3/2),x]
```

```
[Out] (Sqrt[a + b/x]*(-((8*b^2 + 26*a*b*x + 33*a^2*x^2)/x^(5/2)) - (15*a^(5/2)*Ar
cSinh[Sqrt[b]/(Sqrt[a]*Sqrt[x])])/(Sqrt[b]*Sqrt[1 + b/(a*x)])))/24
```

fricas [A] time = 0.69, size = 174, normalized size = 1.74

$$\left[\frac{15\,a^3\sqrt{b}\,x^3\log\left(\frac{ax-2\sqrt{b}\sqrt{x}\sqrt{\frac{ax+b}{x}}+2b}{x}\right) - 2\left(33\,a^2bx^2+26\,ab^2x+8\,b^3\right)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{48\,bx^3}, \frac{15\,a^3\sqrt{-b}\,x^3\arctan\left(\frac{\sqrt{-b}\sqrt{x}\sqrt{\frac{ax+}{x}}}{b}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*a^3*sqrt(b)*x^3*log((a*x - 2*sqrt(b)*sqrt(x)*sqrt((a*x + b)/x) +
2*b)/x) - 2*(33*a^2*b*x^2 + 26*a*b^2*x + 8*b^3)*sqrt(x)*sqrt((a*x + b)/x))/
(b*x^3), 1/24*(15*a^3*sqrt(-b)*x^3*arctan(sqrt(-b)*sqrt(x)*sqrt((a*x + b)/x
)/b) - (33*a^2*b*x^2 + 26*a*b^2*x + 8*b^3)*sqrt(x)*sqrt((a*x + b)/x))/(b*x^
3)]
```

giac [A] time = 0.34, size = 79, normalized size = 0.79

$$\frac{\frac{15\,a^4\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{\sqrt{-b}} - \frac{33\,(ax+b)^{\frac{5}{2}}a^4-40\,(ax+b)^{\frac{3}{2}}a^4b+15\,\sqrt{ax+b}\,a^4b^2}{a^3x^3}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] 1/24*(15*a^4*arctan(sqrt(a*x + b)/sqrt(-b))/sqrt(-b) - (33*(a*x + b)^(5/2)*
a^4 - 40*(a*x + b)^(3/2)*a^4*b + 15*sqrt(a*x + b)*a^4*b^2)/(a^3*x^3))/a
```

maple [A] time = 0.02, size = 92, normalized size = 0.92

$$-\frac{\sqrt{\frac{ax+b}{x}}\left(15a^3x^3\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)+33\sqrt{ax+b}\,a^2\sqrt{b}\,x^2+26\sqrt{ax+b}\,a\,b^{\frac{3}{2}}x+8\sqrt{ax+b}\,b^{\frac{5}{2}}\right)}{24\sqrt{ax+b}\,\sqrt{b}\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2)/x^(3/2),x)
```

```
[Out] -1/24*((a*x+b)/x)^(1/2)/x^(5/2)*(15*arctanh((a*x+b)^(1/2)/b^(1/2))*x^3*a^3+
33*(a*x+b)^(1/2)*a^2*b^(1/2)*x^2+26*(a*x+b)^(1/2)*a*b^(3/2)*x+8*(a*x+b)^(1/
2)*b^(5/2))/(a*x+b)^(1/2)/b^(1/2)
```

maxima [B] time = 2.48, size = 156, normalized size = 1.56

$$\frac{5\,a^3\log\left(\frac{\sqrt{a+\frac{b}{x}}\,\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\,\sqrt{x}+\sqrt{b}}\right)}{16\,\sqrt{b}}-\frac{33\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^3x^{\frac{5}{2}}-40\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^3bx^{\frac{3}{2}}+15\sqrt{a+\frac{b}{x}}\,a^3b^2\sqrt{x}}{24\left(\left(a+\frac{b}{x}\right)^3x^3-3\left(a+\frac{b}{x}\right)^2bx^2+3\left(a+\frac{b}{x}\right)b^2x-b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 5/16*a^3*log((sqrt(a + b/x)*sqrt(x) - sqrt(b))/(sqrt(a + b/x)*sqrt(x) + sqr
t(b)))/sqrt(b) - 1/24*(33*(a + b/x)^(5/2)*a^3*x^(5/2) - 40*(a + b/x)^(3/2)*
a^3*b*x^(3/2) + 15*sqrt(a + b/x)*a^3*b^2*sqrt(x))/((a + b/x)^3*x^3 - 3*(a +
 b/x)^2*b*x^2 + 3*(a + b/x)*b^2*x - b^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(5/2)/x^(3/2),x)
```

```
[Out] int((a + b/x)^(5/2)/x^(3/2), x)
```

sympy [A] time = 14.40, size = 104, normalized size = 1.04

$$-\frac{11a^{\frac{5}{2}}\sqrt{1+\frac{b}{ax}}}{8\sqrt{x}}-\frac{13a^{\frac{3}{2}}b\sqrt{1+\frac{b}{ax}}}{12x^{\frac{3}{2}}}-\frac{\sqrt{a}\,b^2\sqrt{1+\frac{b}{ax}}}{3x^{\frac{5}{2}}}-\frac{5a^3\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\,\sqrt{x}}\right)}{8\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)/x**(3/2),x)
```

```
[Out] -11*a**(5/2)*sqrt(1 + b/(a*x))/(8*sqrt(x)) - 13*a**(3/2)*b*sqrt(1 + b/(a*x)
)/(12*x**(3/2)) - sqrt(a)*b**2*sqrt(1 + b/(a*x))/(3*x**(5/2)) - 5*a**3*asin
h(sqrt(b)/(sqrt(a)*sqrt(x)))/(8*sqrt(b))
```

3.1776 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^{5/2}}\,dx$

Optimal. Leaf size=126

$$\frac{5a^4\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{64b^{3/2}}-\frac{5a^3\sqrt{a+\frac{b}{x}}}{64b\sqrt{x}}-\frac{5a^2\sqrt{a+\frac{b}{x}}}{32x^{3/2}}-\frac{5a\left(a+\frac{b}{x}\right)^{3/2}}{24x^{3/2}}-\frac{\left(a+\frac{b}{x}\right)^{5/2}}{4x^{3/2}}$$

```
[Out] -5/24*a*(a+b/x)^(3/2)/x^(3/2)-1/4*(a+b/x)^(5/2)/x^(3/2)+5/64*a^4*arctanh(b^
(1/2)/(a+b/x)^(1/2)/x^(1/2))/b^(3/2)-5/32*a^2*(a+b/x)^(1/2)/x^(3/2)-5/64*a^
3*(a+b/x)^(1/2)/b/x^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {337, 279, 321, 217, 206}

$$\frac{5a^4\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{64b^{3/2}}-\frac{5a^2\sqrt{a+\frac{b}{x}}}{32x^{3/2}}-\frac{5a^3\sqrt{a+\frac{b}{x}}}{64b\sqrt{x}}-\frac{5a\left(a+\frac{b}{x}\right)^{3/2}}{24x^{3/2}}-\frac{\left(a+\frac{b}{x}\right)^{5/2}}{4x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)/x^(5/2),x]
```

```
[Out] (-5*a^2*Sqrt[a + b/x])/(32*x^(3/2)) - (5*a*(a + b/x)^(3/2))/(24*x^(3/2)) -
(a + b/x)^(5/2)/(4*x^(3/2)) - (5*a^3*Sqrt[a + b/x])/(64*b*Sqrt[x]) + (5*a^4
*ArcTanh[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x])])/(64*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 337

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, -Dist[k/c, Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1)
 + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0] &
& FractionQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^{5/2}}\,dx &= -\left(2\,\text{Subst}\left(\int x^2\left(a+bx^2\right)^{5/2}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)\\
&= -\frac{\left(a+\frac{b}{x}\right)^{5/2}}{4x^{3/2}} - \frac{1}{4}(5a)\,\text{Subst}\left(\int x^2\left(a+bx^2\right)^{3/2}\,dx, x, \frac{1}{\sqrt{x}}\right)\\
&= -\frac{5a\left(a+\frac{b}{x}\right)^{3/2}}{24x^{3/2}} - \frac{\left(a+\frac{b}{x}\right)^{5/2}}{4x^{3/2}} - \frac{1}{8}\left(5a^2\right)\text{Subst}\left(\int x^2\sqrt{a+bx^2}\,dx, x, \frac{1}{\sqrt{x}}\right)\\
&= -\frac{5a^2\sqrt{a+\frac{b}{x}}}{32x^{3/2}} - \frac{5a\left(a+\frac{b}{x}\right)^{3/2}}{24x^{3/2}} - \frac{\left(a+\frac{b}{x}\right)^{5/2}}{4x^{3/2}} - \frac{1}{32}\left(5a^3\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{\sqrt{x}}\right)\\
&= -\frac{5a^2\sqrt{a+\frac{b}{x}}}{32x^{3/2}} - \frac{5a\left(a+\frac{b}{x}\right)^{3/2}}{24x^{3/2}} - \frac{\left(a+\frac{b}{x}\right)^{5/2}}{4x^{3/2}} - \frac{5a^3\sqrt{a+\frac{b}{x}}}{64b\sqrt{x}} + \frac{\left(5a^4\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{\sqrt{}}\right.}{64b}\\
&= -\frac{5a^2\sqrt{a+\frac{b}{x}}}{32x^{3/2}} - \frac{5a\left(a+\frac{b}{x}\right)^{3/2}}{24x^{3/2}} - \frac{\left(a+\frac{b}{x}\right)^{5/2}}{4x^{3/2}} - \frac{5a^3\sqrt{a+\frac{b}{x}}}{64b\sqrt{x}} + \frac{\left(5a^4\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{}{\sqrt{a+}}\right.}{64b}\\
&= -\frac{5a^2\sqrt{a+\frac{b}{x}}}{32x^{3/2}} - \frac{5a\left(a+\frac{b}{x}\right)^{3/2}}{24x^{3/2}} - \frac{\left(a+\frac{b}{x}\right)^{5/2}}{4x^{3/2}} - \frac{5a^3\sqrt{a+\frac{b}{x}}}{64b\sqrt{x}} + \frac{5a^4\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)}{64b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.22, size = 101, normalized size = 0.80

$$\frac{\sqrt{a+\frac{b}{x}}\left(\frac{15a^{7/2}\sinh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right)}{\sqrt{\frac{b}{ax}+1}} - \frac{\sqrt{b}\left(15a^3x^3+118a^2bx^2+136ab^2x+48b^3\right)}{x^{7/2}}\right)}{192b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)/x^(5/2),x]
```

```
[Out] (Sqrt[a + b/x]*(-((Sqrt[b]*(48*b^3 + 136*a*b^2*x + 118*a^2*b*x^2 + 15*a^3*x
^3))/x^(7/2)) + (15*a^(7/2)*ArcSinh[Sqrt[b]/(Sqrt[a]*Sqrt[x])])/Sqrt[1 + b/
(a*x)]))/(192*b^(3/2))
```

fricas [A] time = 0.88, size = 195, normalized size = 1.55

$$\left[\frac{15\,a^4\sqrt{b}\,x^4\log\left(\frac{ax+2\sqrt{b}\sqrt{x}\sqrt{\frac{ax+b}{x}}+2b}{x}\right) - 2\left(15\,a^3bx^3+118\,a^2b^2x^2+136\,ab^3x+48\,b^4\right)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{384\,b^2x^4}, -\frac{15\,a^4\sqrt{-b}\,x^4}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/384*(15*a^4*sqrt(b)*x^4*log((a*x + 2*sqrt(b)*sqrt(x)*sqrt((a*x + b)/x) +
 2*b)/x) - 2*(15*a^3*b*x^3 + 118*a^2*b^2*x^2 + 136*a*b^3*x + 48*b^4)*sqrt(x
)*sqrt((a*x + b)/x))/(b^2*x^4), -1/192*(15*a^4*sqrt(-b)*x^4*arctan(sqrt(-b)
*sqrt(x)*sqrt((a*x + b)/x)/b) + (15*a^3*b*x^3 + 118*a^2*b^2*x^2 + 136*a*b^3
*x + 48*b^4)*sqrt(x)*sqrt((a*x + b)/x))/(b^2*x^4)]
```

giac [A] time = 0.37, size = 99, normalized size = 0.79

$$-\frac{\frac{15\,a^5\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b}+\frac{15\,(ax+b)^{\frac{7}{2}}a^5+73\,(ax+b)^{\frac{5}{2}}a^5b-55\,(ax+b)^{\frac{3}{2}}a^5b^2+15\,\sqrt{ax+b}\,a^5b^3}{a^4bx^4}}{192\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] -1/192*(15*a^5*arctan(sqrt(a*x + b)/sqrt(-b))/(sqrt(-b)*b) + (15*(a*x + b)^
(7/2)*a^5 + 73*(a*x + b)^(5/2)*a^5*b - 55*(a*x + b)^(3/2)*a^5*b^2 + 15*sqrt
(a*x + b)*a^5*b^3)/(a^4*b*x^4))/a
```

maple [A] time = 0.02, size = 110, normalized size = 0.87

$$-\frac{\sqrt{\frac{ax+b}{x}}\left(-15a^4x^4\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)+15\sqrt{ax+b}\;a^3\sqrt{b}\;x^3+118\sqrt{ax+b}\;a^2b^{\frac{3}{2}}x^2+136\sqrt{ax+b}\;a\,b^{\frac{5}{2}}x+48\sqrt{ax+\right.}}{192\sqrt{ax+b}\;b^{\frac{3}{2}}x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2)/x^(5/2),x)
```

```
[Out] -1/192*((a*x+b)/x)^(1/2)*(-15*arctanh((a*x+b)^(1/2)/b^(1/2))*x^4*a^4+48*b^(
7/2)*(a*x+b)^(1/2)+136*x*a*b^(5/2)*(a*x+b)^(1/2)+118*x^2*a^2*b^(3/2)*(a*x+b
)^(1/2)+15*x^3*a^3*b^(1/2)*(a*x+b)^(1/2))/x^(7/2)/b^(3/2)/(a*x+b)^(1/2)
```

maxima [B] time = 2.36, size = 194, normalized size = 1.54

$$-\frac{5\,a^4\log\left(\frac{\sqrt{a+\frac{b}{x}}\,\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\,\sqrt{x}+\sqrt{b}}\right)}{128\,b^{\frac{3}{2}}}-\frac{15\left(a+\frac{b}{x}\right)^{\frac{7}{2}}a^4x^{\frac{7}{2}}+73\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^4bx^{\frac{5}{2}}-55\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^4b^2x^{\frac{3}{2}}+15\sqrt{a+\frac{b}{x}}\,a^4b^3\sqrt{x}}{192\left(\left(a+\frac{b}{x}\right)^4bx^4-4\left(a+\frac{b}{x}\right)^3b^2x^3+6\left(a+\frac{b}{x}\right)^2b^3x^2-4\left(a+\frac{b}{x}\right)b^4x+b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -5/128*a^4*log((sqrt(a + b/x)*sqrt(x) - sqrt(b))/(sqrt(a + b/x)*sqrt(x) + s
qrt(b)))/b^(3/2) - 1/192*(15*(a + b/x)^(7/2)*a^4*x^(7/2) + 73*(a + b/x)^(5/
2)*a^4*b*x^(5/2) - 55*(a + b/x)^(3/2)*a^4*b^2*x^(3/2) + 15*sqrt(a + b/x)*a^
4*b^3*sqrt(x))/((a + b/x)^4*b*x^4 - 4*(a + b/x)^3*b^2*x^3 + 6*(a + b/x)^2*b
^3*x^2 - 4*(a + b/x)*b^4*x + b^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(5/2)/x^(5/2),x)
```

```
[Out] int((a + b/x)^(5/2)/x^(5/2), x)
```

sympy [**A**] time = 17.40, size = 155, normalized size = 1.23

$$-\frac{5a^{\frac{7}{2}}}{64b\sqrt{x}\sqrt{1+\frac{b}{ax}}}-\frac{133a^{\frac{5}{2}}}{192x^{\frac{3}{2}}\sqrt{1+\frac{b}{ax}}}-\frac{127a^{\frac{3}{2}}b}{96x^{\frac{5}{2}}\sqrt{1+\frac{b}{ax}}}-\frac{23\sqrt{a}\,b^2}{24x^{\frac{7}{2}}\sqrt{1+\frac{b}{ax}}}+\frac{5a^4\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right)}{64b^{\frac{3}{2}}}-\frac{b^3}{4\sqrt{a}\,x^{\frac{9}{2}}\sqrt{1+\frac{b}{ax}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)/x**(5/2),x)
```

```
[Out] -5*a**(7/2)/(64*b*sqrt(x)*sqrt(1 + b/(a*x))) - 133*a**(5/2)/(192*x**(3/2)*s
qrt(1 + b/(a*x))) - 127*a**(3/2)*b/(96*x**(5/2)*sqrt(1 + b/(a*x))) - 23*sqr
t(a)*b**2/(24*x**(7/2)*sqrt(1 + b/(a*x))) + 5*a**4*asinh(sqrt(b)/(sqrt(a)*s
qrt(x)))/(64*b**(3/2)) - b**3/(4*sqrt(a)*x**(9/2)*sqrt(1 + b/(a*x)))
```

3.1777 $\int \frac{x^{7/2}}{\sqrt{a+\frac{b}{x}}}\,dx$

Optimal. Leaf size=126

$$\frac{256b^4\sqrt{x}\sqrt{a+\frac{b}{x}}}{315a^5}-\frac{128b^3x^{3/2}\sqrt{a+\frac{b}{x}}}{315a^4}+\frac{32b^2x^{5/2}\sqrt{a+\frac{b}{x}}}{105a^3}-\frac{16bx^{7/2}\sqrt{a+\frac{b}{x}}}{63a^2}+\frac{2x^{9/2}\sqrt{a+\frac{b}{x}}}{9a}$$

```
[Out] -128/315*b^3*x^(3/2)*(a+b/x)^(1/2)/a^4+32/105*b^2*x^(5/2)*(a+b/x)^(1/2)/a^3
-16/63*b*x^(7/2)*(a+b/x)^(1/2)/a^2+2/9*x^(9/2)*(a+b/x)^(1/2)/a+256/315*b^4*
(a+b/x)^(1/2)*x^(1/2)/a^5
```

Rubi [A] time = 0.04, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {271, 264}

$$\frac{32b^2x^{5/2}\sqrt{a+\frac{b}{x}}}{105a^3}-\frac{128b^3x^{3/2}\sqrt{a+\frac{b}{x}}}{315a^4}+\frac{256b^4\sqrt{x}\sqrt{a+\frac{b}{x}}}{315a^5}-\frac{16bx^{7/2}\sqrt{a+\frac{b}{x}}}{63a^2}+\frac{2x^{9/2}\sqrt{a+\frac{b}{x}}}{9a}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/Sqrt[a + b/x],x]
```

```
[Out] (256*b^4*Sqrt[a + b/x]*Sqrt[x])/(315*a^5) - (128*b^3*Sqrt[a + b/x]*x^(3/2))
/(315*a^4) + (32*b^2*Sqrt[a + b/x]*x^(5/2))/(105*a^3) - (16*b*Sqrt[a + b/x]
*x^(7/2))/(63*a^2) + (2*Sqrt[a + b/x]*x^(9/2))/(9*a)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^{7/2}}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{2\sqrt{a+\frac{b}{x}}\,x^{9/2}}{9a} - \frac{(8b)\int \frac{x^{5/2}}{\sqrt{a+\frac{b}{x}}}\,dx}{9a}$$

$$= -\frac{16b\sqrt{a+\frac{b}{x}}\,x^{7/2}}{63a^2} + \frac{2\sqrt{a+\frac{b}{x}}\,x^{9/2}}{9a} + \frac{\left(16b^2\right)\int \frac{x^{3/2}}{\sqrt{a+\frac{b}{x}}}\,dx}{21a^2}$$

$$= \frac{32b^2\sqrt{a+\frac{b}{x}}\,x^{5/2}}{105a^3} - \frac{16b\sqrt{a+\frac{b}{x}}\,x^{7/2}}{63a^2} + \frac{2\sqrt{a+\frac{b}{x}}\,x^{9/2}}{9a} - \frac{\left(64b^3\right)\int \frac{\sqrt{x}}{\sqrt{a+\frac{b}{x}}}\,dx}{105a^3}$$

$$= -\frac{128b^3\sqrt{a+\frac{b}{x}}\,x^{3/2}}{315a^4} + \frac{32b^2\sqrt{a+\frac{b}{x}}\,x^{5/2}}{105a^3} - \frac{16b\sqrt{a+\frac{b}{x}}\,x^{7/2}}{63a^2} + \frac{2\sqrt{a+\frac{b}{x}}\,x^{9/2}}{9a} + \frac{\left(128b^4\right)\int \frac{1}{\sqrt{a+\frac{b}{x}}}}{315a^4}$$

$$= \frac{256b^4\sqrt{a+\frac{b}{x}}\,\sqrt{x}}{315a^5} - \frac{128b^3\sqrt{a+\frac{b}{x}}\,x^{3/2}}{315a^4} + \frac{32b^2\sqrt{a+\frac{b}{x}}\,x^{5/2}}{105a^3} - \frac{16b\sqrt{a+\frac{b}{x}}\,x^{7/2}}{63a^2} + \frac{2\sqrt{a+\frac{b}{x}}\,x^{9/2}}{9a}$$

Mathematica [A] time = 0.04, size = 64, normalized size = 0.51

$$\frac{2\sqrt{x}\,\sqrt{a+\frac{b}{x}}\left(35a^4x^4 - 40a^3bx^3 + 48a^2b^2x^2 - 64ab^3x + 128b^4\right)}{315a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/Sqrt[a + b/x],x]
```

```
[Out] (2*Sqrt[a + b/x]*Sqrt[x]*(128*b^4 - 64*a*b^3*x + 48*a^2*b^2*x^2 - 40*a^3*b*
x^3 + 35*a^4*x^4))/(315*a^5)
```

fricas [A] time = 0.80, size = 60, normalized size = 0.48

$$\frac{2\left(35\,a^4x^4 - 40\,a^3bx^3 + 48\,a^2b^2x^2 - 64\,ab^3x + 128\,b^4\right)\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}{315\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*a^4*x^4 - 40*a^3*b*x^3 + 48*a^2*b^2*x^2 - 64*a*b^3*x + 128*b^4)*s
qrt(x)*sqrt((a*x + b)/x)/a^5
```

giac [A] time = 0.16, size = 73, normalized size = 0.58

$$\frac{2\sqrt{ax+b}\,b^4}{a^5} - \frac{256\,b^{\frac{9}{2}}}{315\,a^5} + \frac{2\left(35\,(ax+b)^{\frac{9}{2}} - 180\,(ax+b)^{\frac{7}{2}}b + 378\,(ax+b)^{\frac{5}{2}}b^2 - 420\,(ax+b)^{\frac{3}{2}}b^3\right)}{315\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(a*x + b)*b^4/a^5 - 256/315*b^(9/2)/a^5 + 2/315*(35*(a*x + b)^(9/2) -
 180*(a*x + b)^(7/2)*b + 378*(a*x + b)^(5/2)*b^2 - 420*(a*x + b)^(3/2)*b^3)
/a^5
```

maple [A] time = 0.01, size = 66, normalized size = 0.52

$$\frac{2\,(ax+b)\left(35a^4x^4 - 40a^3x^3b + 48a^2x^2b^2 - 64ax\,b^3 + 128b^4\right)}{315\sqrt{\frac{ax+b}{x}}\,a^5\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(a+b/x)^(1/2),x)
```

```
[Out] 2/315*(a*x+b)*(35*a^4*x^4-40*a^3*b*x^3+48*a^2*b^2*x^2-64*a*b^3*x+128*b^4)/a
^5/x^(1/2)/((a*x+b)/x)^(1/2)
```

maxima [A] time = 1.01, size = 86, normalized size = 0.68

$$\frac{2\left(35\left(a+\frac{b}{x}\right)^{\frac{9}{2}}x^{\frac{9}{2}}-180\left(a+\frac{b}{x}\right)^{\frac{7}{2}}bx^{\frac{7}{2}}+378\left(a+\frac{b}{x}\right)^{\frac{5}{2}}b^2x^{\frac{5}{2}}-420\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^3x^{\frac{3}{2}}+315\sqrt{a+\frac{b}{x}}\,b^4\sqrt{x}\right)}{315\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*(a + b/x)^(9/2)*x^(9/2) - 180*(a + b/x)^(7/2)*b*x^(7/2) + 378*(a
+ b/x)^(5/2)*b^2*x^(5/2) - 420*(a + b/x)^(3/2)*b^3*x^(3/2) + 315*sqrt(a + b
/x)*b^4*sqrt(x))/a^5
```

mupad [B] time = 1.49, size = 61, normalized size = 0.48

$$\sqrt{a+\frac{b}{x}}\left(\frac{2\,x^{9/2}}{9\,a}-\frac{16\,b\,x^{7/2}}{63\,a^2}+\frac{32\,b^2\,x^{5/2}}{105\,a^3}-\frac{128\,b^3\,x^{3/2}}{315\,a^4}+\frac{256\,b^4\,\sqrt{x}}{315\,a^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(a + b/x)^(1/2),x)
```

```
[Out] (a + b/x)^(1/2)*((2*x^(9/2))/(9*a) - (16*b*x^(7/2))/(63*a^2) + (32*b^2*x^(5
/2))/(105*a^3) - (128*b^3*x^(3/2))/(315*a^4) + (256*b^4*x^(1/2))/(315*a^5))
```

sympy [B] time = 31.83, size = 692, normalized size = 5.49

$$\frac{70a^8b^{\frac{33}{2}}x^8\sqrt{\frac{ax}{b}+1}}{315a^9b^{16}x^4+1260a^8b^{17}x^3+1890a^7b^{18}x^2+1260a^6b^{19}x+315a^5b^{20}}+\frac{200a^7b^{\frac{35}{2}}x^7\sqrt{\frac{ax}{b}+1}}{315a^9b^{16}x^4+1260a^8b^{17}x^3+1890a^7b^{18}x^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(a+b/x)**(1/2),x)
```

```
[Out] 70*a**8*b**(33/2)*x**8*sqrt(a*x/b + 1)/(315*a**9*b**16*x**4 + 1260*a**8*b**
17*x**3 + 1890*a**7*b**18*x**2 + 1260*a**6*b**19*x + 315*a**5*b**20) + 200*
a**7*b**(35/2)*x**7*sqrt(a*x/b + 1)/(315*a**9*b**16*x**4 + 1260*a**8*b**17*
x**3 + 1890*a**7*b**18*x**2 + 1260*a**6*b**19*x + 315*a**5*b**20) + 196*a**
6*b**(37/2)*x**6*sqrt(a*x/b + 1)/(315*a**9*b**16*x**4 + 1260*a**8*b**17*x**
3 + 1890*a**7*b**18*x**2 + 1260*a**6*b**19*x + 315*a**5*b**20) + 56*a**5*b*
*(39/2)*x**5*sqrt(a*x/b + 1)/(315*a**9*b**16*x**4 + 1260*a**8*b**17*x**3 +
1890*a**7*b**18*x**2 + 1260*a**6*b**19*x + 315*a**5*b**20) + 70*a**4*b**(41
/2)*x**4*sqrt(a*x/b + 1)/(315*a**9*b**16*x**4 + 1260*a**8*b**17*x**3 + 1890
*a**7*b**18*x**2 + 1260*a**6*b**19*x + 315*a**5*b**20) + 560*a**3*b**(43/2)
*x**3*sqrt(a*x/b + 1)/(315*a**9*b**16*x**4 + 1260*a**8*b**17*x**3 + 1890*a*
*7*b**18*x**2 + 1260*a**6*b**19*x + 315*a**5*b**20) + 1120*a**2*b**(45/2)*x
**2*sqrt(a*x/b + 1)/(315*a**9*b**16*x**4 + 1260*a**8*b**17*x**3 + 1890*a**7
*b**18*x**2 + 1260*a**6*b**19*x + 315*a**5*b**20) + 896*a*b**(47/2)*x*sqrt(
a*x/b + 1)/(315*a**9*b**16*x**4 + 1260*a**8*b**17*x**3 + 1890*a**7*b**18*x*
*2 + 1260*a**6*b**19*x + 315*a**5*b**20) + 256*b**(49/2)*sqrt(a*x/b + 1)/(3
15*a**9*b**16*x**4 + 1260*a**8*b**17*x**3 + 1890*a**7*b**18*x**2 + 1260*a**
6*b**19*x + 315*a**5*b**20)
```

3.1778 $\int \frac{x^{5/2}}{\sqrt{a+\frac{b}{x}}}\,dx$

Optimal. Leaf size=100

$$-\frac{32b^3\sqrt{x}\sqrt{a+\frac{b}{x}}}{35a^4}+\frac{16b^2x^{3/2}\sqrt{a+\frac{b}{x}}}{35a^3}-\frac{12bx^{5/2}\sqrt{a+\frac{b}{x}}}{35a^2}+\frac{2x^{7/2}\sqrt{a+\frac{b}{x}}}{7a}$$

```
[Out] 16/35*b^2*x^(3/2)*(a+b/x)^(1/2)/a^3-12/35*b*x^(5/2)*(a+b/x)^(1/2)/a^2+2/7*x
^(7/2)*(a+b/x)^(1/2)/a-32/35*b^3*(a+b/x)^(1/2)*x^(1/2)/a^4
```

Rubi [A] time = 0.03, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {271, 264}

$$\frac{16b^2x^{3/2}\sqrt{a+\frac{b}{x}}}{35a^3}-\frac{32b^3\sqrt{x}\sqrt{a+\frac{b}{x}}}{35a^4}-\frac{12bx^{5/2}\sqrt{a+\frac{b}{x}}}{35a^2}+\frac{2x^{7/2}\sqrt{a+\frac{b}{x}}}{7a}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/Sqrt[a + b/x],x]
```

```
[Out] (-32*b^3*Sqrt[a + b/x]*Sqrt[x])/(35*a^4) + (16*b^2*Sqrt[a + b/x]*x^(3/2))/(
35*a^3) - (12*b*Sqrt[a + b/x]*x^(5/2))/(35*a^2) + (2*Sqrt[a + b/x]*x^(7/2))
/(7*a)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{\sqrt{a+\frac{b}{x}}}\,dx &= \frac{2\sqrt{a+\frac{b}{x}}x^{7/2}}{7a}-\frac{(6b)\int \frac{x^{3/2}}{\sqrt{a+\frac{b}{x}}}\,dx}{7a}\\
&= -\frac{12b\sqrt{a+\frac{b}{x}}x^{5/2}}{35a^2}+\frac{2\sqrt{a+\frac{b}{x}}x^{7/2}}{7a}+\frac{\left(24b^2\right)\int \frac{\sqrt{x}}{\sqrt{a+\frac{b}{x}}}\,dx}{35a^2}\\
&= \frac{16b^2\sqrt{a+\frac{b}{x}}x^{3/2}}{35a^3}-\frac{12b\sqrt{a+\frac{b}{x}}x^{5/2}}{35a^2}+\frac{2\sqrt{a+\frac{b}{x}}x^{7/2}}{7a}-\frac{\left(16b^3\right)\int \frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\,dx}{35a^3}\\
&= -\frac{32b^3\sqrt{a+\frac{b}{x}}\sqrt{x}}{35a^4}+\frac{16b^2\sqrt{a+\frac{b}{x}}x^{3/2}}{35a^3}-\frac{12b\sqrt{a+\frac{b}{x}}x^{5/2}}{35a^2}+\frac{2\sqrt{a+\frac{b}{x}}x^{7/2}}{7a}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.53

$$\frac{2\sqrt{x}\sqrt{a+\frac{b}{x}}\left(5a^3x^3-6a^2bx^2+8ab^2x-16b^3\right)}{35a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/Sqrt[a + b/x],x]
```

```
[Out] (2*Sqrt[a + b/x]*Sqrt[x]*(-16*b^3 + 8*a*b^2*x - 6*a^2*b*x^2 + 5*a^3*x^3))/(
35*a^4)
```

fricas [A] time = 0.58, size = 49, normalized size = 0.49

$$\frac{2\left(5\,a^3x^3-6\,a^2bx^2+8\,ab^2x-16\,b^3\right)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{35\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*a^3*x^3 - 6*a^2*b*x^2 + 8*a*b^2*x - 16*b^3)*sqrt(x)*sqrt((a*x + b)/
x)/a^4
```

giac [A] time = 0.17, size = 61, normalized size = 0.61

$$-\frac{2\sqrt{ax+b}\,b^3}{a^4}+\frac{32\,b^{\frac{7}{2}}}{35\,a^4}+\frac{2\left(5\,(ax+b)^{\frac{7}{2}}-21\,(ax+b)^{\frac{5}{2}}b+35\,(ax+b)^{\frac{3}{2}}b^2\right)}{35\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(a*x + b)*b^3/a^4 + 32/35*b^(7/2)/a^4 + 2/35*(5*(a*x + b)^(7/2) - 21
*(a*x + b)^(5/2)*b + 35*(a*x + b)^(3/2)*b^2)/a^4
```

maple [A] time = 0.00, size = 55, normalized size = 0.55

$$\frac{2\,(ax+b)\left(5a^3x^3-6a^2b\,x^2+8a\,b^2x-16b^3\right)}{35\sqrt{\frac{ax+b}{x}}\;a^4\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a+b/x)^(1/2),x)
```

```
[Out] 2/35*(a*x+b)*(5*a^3*x^3-6*a^2*b*x^2+8*a*b^2*x-16*b^3)/a^4/x^(1/2)/((a*x+b)/
x)^(1/2)
```

maxima [A] time = 1.07, size = 69, normalized size = 0.69

$$\frac{2\left(5\left(a+\frac{b}{x}\right)^{\frac{7}{2}}x^{\frac{7}{2}}-21\left(a+\frac{b}{x}\right)^{\frac{5}{2}}bx^{\frac{5}{2}}+35\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^2x^{\frac{3}{2}}-35\sqrt{a+\frac{b}{x}}\,b^3\sqrt{x}\right)}{35\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/35*(5*(a + b/x)^(7/2)*x^(7/2) - 21*(a + b/x)^(5/2)*b*x^(5/2) + 35*(a + b/
x)^(3/2)*b^2*x^(3/2) - 35*sqrt(a + b/x)*b^3*sqrt(x))/a^4
```

mupad [B] time = 1.43, size = 50, normalized size = 0.50

$$\sqrt{a+\frac{b}{x}}\left(\frac{2\,x^{7/2}}{7\,a}-\frac{12\,b\,x^{5/2}}{35\,a^2}+\frac{16\,b^2\,x^{3/2}}{35\,a^3}-\frac{32\,b^3\,\sqrt{x}}{35\,a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b/x)^(1/2),x)
```

```
[Out] (a + b/x)^(1/2)*((2*x^(7/2))/(7*a) - (12*b*x^(5/2))/(35*a^2) + (16*b^2*x^(3
/2))/(35*a^3) - (32*b^3*x^(1/2))/(35*a^4))
```

sympy [B] time = 11.93, size = 452, normalized size = 4.52

$$\frac{10a^6b^{\frac{19}{2}}x^6\sqrt{\frac{ax}{b}+1}}{35a^7b^9x^3+105a^6b^{10}x^2+105a^5b^{11}x+35a^4b^{12}}+\frac{18a^5b^{\frac{21}{2}}x^5\sqrt{\frac{ax}{b}+1}}{35a^7b^9x^3+105a^6b^{10}x^2+105a^5b^{11}x+35a^4b^{12}}+\frac{}{35a^7b^9x^3+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(a+b/x)**(1/2),x)
```

```
[Out] 10*a**6*b**(19/2)*x**6*sqrt(a*x/b + 1)/(35*a**7*b**9*x**3 + 105*a**6*b**10*
x**2 + 105*a**5*b**11*x + 35*a**4*b**12) + 18*a**5*b**(21/2)*x**5*sqrt(a*x/
b + 1)/(35*a**7*b**9*x**3 + 105*a**6*b**10*x**2 + 105*a**5*b**11*x + 35*a**
4*b**12) + 10*a**4*b**(23/2)*x**4*sqrt(a*x/b + 1)/(35*a**7*b**9*x**3 + 105*
a**6*b**10*x**2 + 105*a**5*b**11*x + 35*a**4*b**12) - 10*a**3*b**(25/2)*x**
3*sqrt(a*x/b + 1)/(35*a**7*b**9*x**3 + 105*a**6*b**10*x**2 + 105*a**5*b**11
*x + 35*a**4*b**12) - 60*a**2*b**(27/2)*x**2*sqrt(a*x/b + 1)/(35*a**7*b**9*
x**3 + 105*a**6*b**10*x**2 + 105*a**5*b**11*x + 35*a**4*b**12) - 80*a*b**(2
9/2)*x*sqrt(a*x/b + 1)/(35*a**7*b**9*x**3 + 105*a**6*b**10*x**2 + 105*a**5*
b**11*x + 35*a**4*b**12) - 32*b**(31/2)*sqrt(a*x/b + 1)/(35*a**7*b**9*x**3
+ 105*a**6*b**10*x**2 + 105*a**5*b**11*x + 35*a**4*b**12)
```

3.1779 $\int \frac{x^{3/2}}{\sqrt{a+\frac{b}{x}}}\,dx$

Optimal. Leaf size=74

$$\frac{16b^2\sqrt{x}\sqrt{a+\frac{b}{x}}}{15a^3}-\frac{8bx^{3/2}\sqrt{a+\frac{b}{x}}}{15a^2}+\frac{2x^{5/2}\sqrt{a+\frac{b}{x}}}{5a}$$

```
[Out] -8/15*b*x^(3/2)*(a+b/x)^(1/2)/a^2+2/5*x^(5/2)*(a+b/x)^(1/2)/a+16/15*b^2*(a+
b/x)^(1/2)*x^(1/2)/a^3
```

Rubi [A] time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {271, 264}

$$\frac{16b^2\sqrt{x}\sqrt{a+\frac{b}{x}}}{15a^3}-\frac{8bx^{3/2}\sqrt{a+\frac{b}{x}}}{15a^2}+\frac{2x^{5/2}\sqrt{a+\frac{b}{x}}}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[a + b/x],x]
```

```
[Out] (16*b^2*Sqrt[a + b/x]*Sqrt[x])/(15*a^3) - (8*b*Sqrt[a + b/x]*x^(3/2))/(15*a
^2) + (2*Sqrt[a + b/x]*x^(5/2))/(5*a)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{3/2}}{\sqrt{a+\frac{b}{x}}}\,dx &= \frac{2\sqrt{a+\frac{b}{x}}\,x^{5/2}}{5a}-\frac{(4b)\int \frac{\sqrt{x}}{\sqrt{a+\frac{b}{x}}}\,dx}{5a}\\ &= -\frac{8b\sqrt{a+\frac{b}{x}}\,x^{3/2}}{15a^2}+\frac{2\sqrt{a+\frac{b}{x}}\,x^{5/2}}{5a}+\frac{\left(8b^2\right)\int \frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\,dx}{15a^2}\\ &= \frac{16b^2\sqrt{a+\frac{b}{x}}\sqrt{x}}{15a^3}-\frac{8b\sqrt{a+\frac{b}{x}}\,x^{3/2}}{15a^2}+\frac{2\sqrt{a+\frac{b}{x}}\,x^{5/2}}{5a}\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.57

$$\frac{2\sqrt{x}\sqrt{a+\frac{b}{x}}\left(3a^2x^2-4abx+8b^2\right)}{15a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[a + b/x],x]
```

```
[Out] (2*Sqrt[a + b/x]*Sqrt[x]*(8*b^2 - 4*a*b*x + 3*a^2*x^2))/(15*a^3)
```

fricas [A] time = 0.78, size = 38, normalized size = 0.51

$$\frac{2\left(3\,a^2x^2-4\,abx+8\,b^2\right)\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}{15\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*a^2*x^2 - 4*a*b*x + 8*b^2)*sqrt(x)*sqrt((a*x + b)/x)/a^3
```

giac [A] time = 0.16, size = 49, normalized size = 0.66

$$\frac{2\sqrt{ax+b}\,b^2}{a^3}-\frac{16\,b^{\frac{5}{2}}}{15\,a^3}+\frac{2\left(3\,(ax+b)^{\frac{5}{2}}-10\,(ax+b)^{\frac{3}{2}}b\right)}{15\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(a*x + b)*b^2/a^3 - 16/15*b^(5/2)/a^3 + 2/15*(3*(a*x + b)^(5/2) - 10*
(a*x + b)^(3/2)*b)/a^3
```

maple [A] time = 0.00, size = 44, normalized size = 0.59

$$\frac{2\,(ax+b)\left(3a^2x^2-4abx+8b^2\right)}{15\sqrt{\frac{ax+b}{x}}\;a^3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a+b/x)^(1/2),x)
```

```
[Out] 2/15*(a*x+b)*(3*a^2*x^2-4*a*b*x+8*b^2)/a^3/x^(1/2)/((a*x+b)/x)^(1/2)
```

maxima [A] time = 1.05, size = 52, normalized size = 0.70

$$\frac{2\left(3\left(a+\frac{b}{x}\right)^{\frac{5}{2}}x^{\frac{5}{2}}-10\left(a+\frac{b}{x}\right)^{\frac{3}{2}}bx^{\frac{3}{2}}+15\sqrt{a+\frac{b}{x}}\,b^2\sqrt{x}\right)}{15\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*(a + b/x)^(5/2)*x^(5/2) - 10*(a + b/x)^(3/2)*b*x^(3/2) + 15*sqrt(a
+ b/x)*b^2*sqrt(x))/a^3
```

mupad [B] time = 1.44, size = 39, normalized size = 0.53

$$\sqrt{a+\frac{b}{x}}\,\left(\frac{2\,x^{5/2}}{5\,a}-\frac{8\,b\,x^{3/2}}{15\,a^2}+\frac{16\,b^2\,\sqrt{x}}{15\,a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b/x)^(1/2),x)
```

```
[Out] (a + b/x)^(1/2)*((2*x^(5/2))/(5*a) - (8*b*x^(3/2))/(15*a^2) + (16*b^2*x^(1/
2))/(15*a^3))
```

sympy [**B**] time = 3.83, size = 260, normalized size = 3.51

$$\frac{6a^4b^{\frac{9}{2}}x^4\sqrt{\frac{ax}{b}+1}}{15a^5b^4x^2+30a^4b^5x+15a^3b^6}+\frac{4a^3b^{\frac{11}{2}}x^3\sqrt{\frac{ax}{b}+1}}{15a^5b^4x^2+30a^4b^5x+15a^3b^6}+\frac{6a^2b^{\frac{13}{2}}x^2\sqrt{\frac{ax}{b}+1}}{15a^5b^4x^2+30a^4b^5x+15a^3b^6}+\frac{24ab^{\frac{15}{2}}x\sqrt{\frac{a}{b}}}{15a^5b^4x^2+30a^4b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(a+b/x)**(1/2),x)
```

```
[Out] 6*a**4*b**(9/2)*x**4*sqrt(a*x/b + 1)/(15*a**5*b**4*x**2 + 30*a**4*b**5*x +
15*a**3*b**6) + 4*a**3*b**(11/2)*x**3*sqrt(a*x/b + 1)/(15*a**5*b**4*x**2 +
30*a**4*b**5*x + 15*a**3*b**6) + 6*a**2*b**(13/2)*x**2*sqrt(a*x/b + 1)/(15*
a**5*b**4*x**2 + 30*a**4*b**5*x + 15*a**3*b**6) + 24*a*b**(15/2)*x*sqrt(a*x
/b + 1)/(15*a**5*b**4*x**2 + 30*a**4*b**5*x + 15*a**3*b**6) + 16*b**(17/2)*
sqrt(a*x/b + 1)/(15*a**5*b**4*x**2 + 30*a**4*b**5*x + 15*a**3*b**6)
```

3.1780 $\int \frac{\sqrt{x}}{\sqrt{a+\frac{b}{x}}}\,dx$

Optimal. Leaf size=48

$$\frac{2x^{3/2}\sqrt{a+\frac{b}{x}}}{3a}-\frac{4b\sqrt{x}\sqrt{a+\frac{b}{x}}}{3a^2}$$

```
[Out] 2/3*x^(3/2)*(a+b/x)^(1/2)/a-4/3*b*(a+b/x)^(1/2)*x^(1/2)/a^2
```

Rubi [A] time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {271, 264}

$$\frac{2x^{3/2}\sqrt{a+\frac{b}{x}}}{3a}-\frac{4b\sqrt{x}\sqrt{a+\frac{b}{x}}}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[a + b/x],x]
```

```
[Out] (-4*b*Sqrt[a + b/x]*Sqrt[x])/(3*a^2) + (2*Sqrt[a + b/x]*x^(3/2))/(3*a)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{\sqrt{a+\frac{b}{x}}}\,dx &= \frac{2\sqrt{a+\frac{b}{x}}\,x^{3/2}}{3a}-\frac{(2b)\int \frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\,dx}{3a}\\ &= -\frac{4b\sqrt{a+\frac{b}{x}}\sqrt{x}}{3a^2}+\frac{2\sqrt{a+\frac{b}{x}}\,x^{3/2}}{3a}\end{aligned}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 0.62

$$\frac{2\sqrt{x}\sqrt{a+\frac{b}{x}}\,(ax-2b)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[a + b/x],x]
```

```
[Out] (2*Sqrt[a + b/x]*Sqrt[x]*(-2*b + a*x))/(3*a^2)
```

fricas [A] time = 0.84, size = 26, normalized size = 0.54

$$\frac{2\,(ax-2\,b)\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(a+b/x)^(1/2),x, algorithm="fricas")
[Out] 2/3*(a*x - 2*b)*sqrt(x)*sqrt((a*x + b)/x)/a^2
```

giac [A] time = 0.19, size = 34, normalized size = 0.71

$$\frac{2\,(ax+b)^{\frac{3}{2}}}{3\,a^2}-\frac{2\,\sqrt{ax+b}\,b}{a^2}+\frac{4\,b^{\frac{3}{2}}}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(a+b/x)^(1/2),x, algorithm="giac")
[Out] 2/3*(a*x + b)^(3/2)/a^2 - 2*sqrt(a*x + b)*b/a^2 + 4/3*b^(3/2)/a^2
```

maple [A] time = 0.00, size = 32, normalized size = 0.67

$$\frac{2\,(ax+b)\,(ax-2b)}{3\sqrt{\frac{ax+b}{x}}\;a^2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a+b/x)^(1/2),x)
[Out] 2/3*(a*x+b)*(a*x-2*b)/a^2/x^(1/2)/((a*x+b)/x)^(1/2)
```

maxima [A] time = 1.05, size = 34, normalized size = 0.71

$$\frac{2\left(\left(a+\frac{b}{x}\right)^{\frac{3}{2}}x^{\frac{3}{2}}-3\,\sqrt{a+\frac{b}{x}}\,b\sqrt{x}\right)}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(a+b/x)^(1/2),x, algorithm="maxima")
[Out] 2/3*((a + b/x)^(3/2)*x^(3/2) - 3*sqrt(a + b/x)*b*sqrt(x))/a^2
```

mupad [B] time = 1.42, size = 28, normalized size = 0.58

$$\sqrt{a+\frac{b}{x}}\,\left(\frac{2\,x^{3/2}}{3\,a}-\frac{4\,b\,\sqrt{x}}{3\,a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b/x)^(1/2),x)
[Out] (a + b/x)^(1/2)*((2*x^(3/2))/(3*a) - (4*b*x^(1/2))/(3*a^2))
```

sympy [A] time = 1.41, size = 42, normalized size = 0.88

$$\frac{2\sqrt{b}\,x\sqrt{\frac{ax}{b}+1}}{3a}-\frac{4b^{\frac{3}{2}}\sqrt{\frac{ax}{b}+1}}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(a+b/x)**(1/2),x)
[Out] 2*sqrt(b)*x*sqrt(a*x/b + 1)/(3*a) - 4*b**(3/2)*sqrt(a*x/b + 1)/(3*a**2)
```

3.1781 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\,dx$

Optimal. Leaf size=21

$$\frac{2\sqrt{x}\sqrt{a+\frac{b}{x}}}{a}$$

```
[Out] 2*(a+b/x)^(1/2)*x^(1/2)/a
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {264}

$$\frac{2\sqrt{x}\sqrt{a+\frac{b}{x}}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x]*Sqrt[x]),x]
```

```
[Out] (2*Sqrt[a + b/x]*Sqrt[x])/a
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\,dx = \frac{2\sqrt{a+\frac{b}{x}}\sqrt{x}}{a}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{2\sqrt{x}\sqrt{a+\frac{b}{x}}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x]*Sqrt[x]),x]
```

```
[Out] (2*Sqrt[a + b/x]*Sqrt[x])/a
```

fricas [A] time = 0.58, size = 19, normalized size = 0.90

$$\frac{2\sqrt{x}\sqrt{\frac{ax+b}{x}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x)*sqrt((a*x + b)/x)/a
```

giac [A] time = 0.18, size = 21, normalized size = 1.00

$$\frac{2\sqrt{ax+b}}{a} - \frac{2\sqrt{b}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(1/2)/x^(1/2),x, algorithm="giac")
[Out] 2*sqrt(a*x + b)/a - 2*sqrt(b)/a
```

maple [A] time = 0.00, size = 25, normalized size = 1.19

$$\frac{2ax+2b}{a\sqrt{\frac{ax+b}{x}}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(1/2)/x^(1/2),x)
[Out] 2*(a*x+b)/a/((a*x+b)/x)^(1/2)/x^(1/2)
```

maxima [A] time = 1.03, size = 17, normalized size = 0.81

$$\frac{2\sqrt{a+\frac{b}{x}}\,\sqrt{x}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(1/2)/x^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(a + b/x)*sqrt(x)/a
```

mupad [B] time = 1.41, size = 17, normalized size = 0.81

$$\frac{2\sqrt{x}\,\sqrt{a+\frac{b}{x}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b/x)^(1/2)),x)
[Out] (2*x^(1/2)*(a + b/x)^(1/2))/a
```

sympy [A] time = 1.72, size = 17, normalized size = 0.81

$$\frac{2\sqrt{b}\,\sqrt{\frac{ax}{b}+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(1/2)/x**(1/2),x)
[Out] 2*sqrt(b)*sqrt(a*x/b + 1)/a
```

3.1782 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\, x^{3/2}}\, dx$

Optimal. Leaf size=30

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{\sqrt{b}}$$

```
[Out] -2*arctanh(b^(1/2)/(a+b/x)^(1/2)/x^(1/2))/b^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {337, 217, 206}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x]*x^(3/2)),x]
```

```
[Out] (-2*ArcTanh[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x])])/Sqrt[b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 337

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, -Dist[k/c, Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1)
 + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0] &
& FractionQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a+\frac{b}{x}}\, x^{3/2}}\, dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\, dx, x, \frac{1}{\sqrt{x}}\right)\right) \\
&= -\left(2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\, dx, x, \frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)\right) \\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)}{\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 1.80

$$-\frac{2\sqrt{a}\,\sqrt{\frac{b}{ax}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}\,\sqrt{x}}\right)}{\sqrt{b}\,\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x]*x^(3/2)),x]
```

```
[Out] (-2*Sqrt[a]*Sqrt[1 + b/(a*x)]*ArcSinh[Sqrt[b]/(Sqrt[a]*Sqrt[x])])/(Sqrt[b]*
Sqrt[a + b/x])
```

fricas [A] time = 0.77, size = 70, normalized size = 2.33

$$\left[\frac{\log\left(\frac{ax-2\,\sqrt{b}\,\sqrt{x}\,\sqrt{\frac{ax+b}{x}}+2\,b}{x}\right)}{\sqrt{b}},\ \frac{2\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}{b}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(1/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [log((a*x - 2*sqrt(b)*sqrt(x)*sqrt((a*x + b)/x) + 2*b)/x)/sqrt(b), 2*sqrt(-
b)*arctan(sqrt(-b)*sqrt(x)*sqrt((a*x + b)/x)/b)/b]
```

giac [A] time = 0.17, size = 39, normalized size = 1.30

$$\frac{2\,\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}-\frac{2\,\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(1/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(a*x + b)/sqrt(-b))/sqrt(-b) - 2*arctan(sqrt(b)/sqrt(-b))/sqrt
(-b)
```

maple [A] time = 0.01, size = 39, normalized size = 1.30

$$-\frac{2\sqrt{\frac{ax+b}{x}}\,\sqrt{x}\,\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)}{\sqrt{ax+b}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(1/2)/x^(3/2),x)
```

```
[Out] -2*((a*x+b)/x)^(1/2)*x^(1/2)/(a*x+b)^(1/2)/b^(1/2)*arctanh((a*x+b)^(1/2)/b^
(1/2))
```

maxima [A] time = 2.37, size = 44, normalized size = 1.47

$$\frac{\log\left(\frac{\sqrt{a+\frac{b}{x}}\,\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\,\sqrt{x}+\sqrt{b}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(1/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] log((sqrt(a + b/x)*sqrt(x) - sqrt(b))/(sqrt(a + b/x)*sqrt(x) + sqrt(b)))/sq
rt(b)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x^{3/2}\sqrt{a+\frac{b}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b/x)^(1/2)),x)
```

```
[Out] int(1/(x^(3/2)*(a + b/x)^(1/2)), x)
```

sympy **[A]** time = 3.24, size = 24, normalized size = 0.80

$$-\frac{2\,\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\,\sqrt{x}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(1/2)/x**(3/2),x)
```

```
[Out] -2*asinh(sqrt(b)/(sqrt(a)*sqrt(x)))/sqrt(b)
```

3.1783 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\, x^{5/2}}\, dx$

Optimal. Leaf size=52

$$\frac{a \tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{b^{3/2}} - \frac{\sqrt{a+\frac{b}{x}}}{b\sqrt{x}}$$

```
[Out] a*arctanh(b^(1/2)/(a+b/x)^(1/2)/x^(1/2))/b^(3/2)-(a+b/x)^(1/2)/b/x^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {337, 321, 217, 206}

$$\frac{a \tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{b^{3/2}} - \frac{\sqrt{a+\frac{b}{x}}}{b\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x]*x^(5/2)),x]
```

```
[Out] -(Sqrt[a + b/x]/(b*Sqrt[x])) + (a*ArcTanh[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x])])
/b^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 337

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, -Dist[k/c, Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1)
 + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0] &
& FractionQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+\frac{b}{x}}\,x^{5/2}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)\\ &= -\frac{\sqrt{a+\frac{b}{x}}}{b\sqrt{x}} + \frac{a\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{\sqrt{x}}\right)}{b}\\ &= -\frac{\sqrt{a+\frac{b}{x}}}{b\sqrt{x}} + \frac{a\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{b}{x}}\,\sqrt{x}}\right)}{b}\\ &= -\frac{\sqrt{a+\frac{b}{x}}}{b\sqrt{x}} + \frac{a\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\,\sqrt{x}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 77, normalized size = 1.48

$$\frac{a^{3/2}x^{3/2}\sqrt{\frac{b}{ax}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}\,\sqrt{x}}\right) - \sqrt{b}\,(ax+b)}{b^{3/2}x^{3/2}\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x]*x^(5/2)),x]
```

```
[Out] (-(Sqrt[b]*(b + a*x)) + a^(3/2)*Sqrt[1 + b/(a*x)]*x^(3/2)*ArcSinh[Sqrt[b]/(
Sqrt[a]*Sqrt[x])])/(b^(3/2)*Sqrt[a + b/x]*x^(3/2))
```

fricas [A] time = 0.95, size = 121, normalized size = 2.33

$$\left[\frac{a\sqrt{b}\,x\log\left(\frac{ax+2\,\sqrt{b}\,\sqrt{x}\,\sqrt{\frac{ax+b}{x}}+2\,b}{x}\right) - 2\,b\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}{2\,b^2x}, -\frac{a\sqrt{-b}\,x\arctan\left(\frac{\sqrt{-b}\,\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}{b}\right) + b\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}{b^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(1/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2*(a*sqrt(b)*x*log((a*x + 2*sqrt(b)*sqrt(x)*sqrt((a*x + b)/x) + 2*b)/x)
- 2*b*sqrt(x)*sqrt((a*x + b)/x))/(b^2*x), -(a*sqrt(-b)*x*arctan(sqrt(-b)*sq
rt(x)*sqrt((a*x + b)/x)/b) + b*sqrt(x)*sqrt((a*x + b)/x))/(b^2*x)]
```

giac [A] time = 0.21, size = 47, normalized size = 0.90

$$-\frac{\frac{a^2\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b} + \frac{\sqrt{ax+b}\,a}{bx}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(1/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] -(a^2*arctan(sqrt(a*x + b)/sqrt(-b))/(sqrt(-b)*b) + sqrt(a*x + b)*a/(b*x))/
a
```

maple [A] time = 0.01, size = 55, normalized size = 1.06

$$-\frac{\sqrt{\frac{ax+b}{x}}\left(-ax\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)+\sqrt{ax+b}\sqrt{b}\right)}{\sqrt{ax+b}\,b^{\frac{3}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(1/2)/x^(5/2),x)
```

```
[Out] -((a*x+b)/x)^(1/2)*(-arctanh((a*x+b)^(1/2)/b^(1/2))*x*a+(a*x+b)^(1/2)*b^(1/
2))/x^(1/2)/b^(3/2)/(a*x+b)^(1/2)
```

maxima [A] time = 2.42, size = 80, normalized size = 1.54

$$-\frac{\sqrt{a+\frac{b}{x}}\,a\sqrt{x}}{\left(a+\frac{b}{x}\right)bx-b^2}-\frac{a\log\left(\frac{\sqrt{a+\frac{b}{x}}\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}+\sqrt{b}}\right)}{2\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(1/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -sqrt(a + b/x)*a*sqrt(x)/((a + b/x)*b*x - b^2) - 1/2*a*log((sqrt(a + b/x)*s
qrt(x) - sqrt(b))/(sqrt(a + b/x)*sqrt(x) + sqrt(b)))/b^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^{5/2}\sqrt{a+\frac{b}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b/x)^(1/2)),x)
```

```
[Out] int(1/(x^(5/2)*(a + b/x)^(1/2)), x)
```

sympy [A] time = 8.69, size = 44, normalized size = 0.85

$$-\frac{\sqrt{a}\sqrt{1+\frac{b}{ax}}}{b\sqrt{x}}+\frac{a\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(1/2)/x**(5/2),x)
```

```
[Out] -sqrt(a)*sqrt(1 + b/(a*x))/(b*sqrt(x)) + a*asinh(sqrt(b)/(sqrt(a)*sqrt(x)))
/b**(3/2)
```

3.1784 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\, x^{7/2}}\, dx$

Optimal. Leaf size=83

$$-\frac{3a^2 \tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{4b^{5/2}} + \frac{3a\sqrt{a+\frac{b}{x}}}{4b^2\sqrt{x}} - \frac{\sqrt{a+\frac{b}{x}}}{2bx^{3/2}}$$

```
[Out] -3/4*a^2*arctanh(b^(1/2)/(a+b/x)^(1/2)/x^(1/2))/b^(5/2)-1/2*(a+b/x)^(1/2)/b
/x^(3/2)+3/4*a*(a+b/x)^(1/2)/b^2/x^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {337, 321, 217, 206}

$$-\frac{3a^2 \tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{4b^{5/2}} + \frac{3a\sqrt{a+\frac{b}{x}}}{4b^2\sqrt{x}} - \frac{\sqrt{a+\frac{b}{x}}}{2bx^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x]*x^(7/2)),x]
```

```
[Out] -Sqrt[a + b/x]/(2*b*x^(3/2)) + (3*a*Sqrt[a + b/x])/(4*b^2*Sqrt[x]) - (3*a^2
*ArcTanh[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x])])/(4*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 337

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, -Dist[k/c, Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1)
 + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0] &
& FractionQ[m]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\,x^{7/2}}\,dx = -\left(2\,\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$
$$= -\frac{\sqrt{a+\frac{b}{x}}}{2bx^{3/2}} + \frac{(3a)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{\sqrt{x}}\right)}{2b}$$
$$= -\frac{\sqrt{a+\frac{b}{x}}}{2bx^{3/2}} + \frac{3a\sqrt{a+\frac{b}{x}}}{4b^2\sqrt{x}} - \frac{\left(3a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{\sqrt{x}}\right)}{4b^2}$$
$$= -\frac{\sqrt{a+\frac{b}{x}}}{2bx^{3/2}} + \frac{3a\sqrt{a+\frac{b}{x}}}{4b^2\sqrt{x}} - \frac{\left(3a^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{b}{x}}\,\sqrt{x}}\right)}{4b^2}$$
$$= -\frac{\sqrt{a+\frac{b}{x}}}{2bx^{3/2}} + \frac{3a\sqrt{a+\frac{b}{x}}}{4b^2\sqrt{x}} - \frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\,\sqrt{x}}\right)}{4b^{5/2}}$$

Mathematica [A] time = 0.07, size = 93, normalized size = 1.12

$$\frac{\sqrt{b}\left(3a^2x^2+abx-2b^2\right)-3a^{5/2}x^{5/2}\sqrt{\frac{b}{ax}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}\,\sqrt{x}}\right)}{4b^{5/2}x^{5/2}\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x]*x^(7/2)),x]
```

```
[Out] (Sqrt[b]*(-2*b^2 + a*b*x + 3*a^2*x^2) - 3*a^(5/2)*Sqrt[1 + b/(a*x)]*x^(5/2)
*ArcSinh[Sqrt[b]/(Sqrt[a]*Sqrt[x])])/(4*b^(5/2)*Sqrt[a + b/x]*x^(5/2))
```

fricas [A] time = 0.81, size = 151, normalized size = 1.82

$$\left[\frac{3\,a^2\sqrt{b}\,x^2\log\left(\frac{ax-2\,\sqrt{b}\,\sqrt{x}\,\sqrt{\frac{ax+b}{x}}+2\,b}{x}\right)+2\left(3\,abx-2\,b^2\right)\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}{8\,b^3x^2},\ \frac{3\,a^2\sqrt{-b}\,x^2\arctan\left(\frac{\sqrt{-b}\,\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}{b}\right)+\left(3\,abx-2\right.}{4\,b^3x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(1/2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*a^2*sqrt(b)*x^2*log((a*x - 2*sqrt(b)*sqrt(x)*sqrt((a*x + b)/x) + 2*
b)/x) + 2*(3*a*b*x - 2*b^2)*sqrt(x)*sqrt((a*x + b)/x))/(b^3*x^2), 1/4*(3*a^
2*sqrt(-b)*x^2*arctan(sqrt(-b)*sqrt(x)*sqrt((a*x + b)/x)/b) + (3*a*b*x - 2*
b^2)*sqrt(x)*sqrt((a*x + b)/x))/(b^3*x^2)]
```

giac [A] time = 0.32, size = 69, normalized size = 0.83

$$\frac{\frac{3\,a^3\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b^2}+\frac{3\,(ax+b)^{\frac{3}{2}}a^3-5\,\sqrt{ax+b}\,a^3b}{a^2b^2x^2}}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(1/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*a^3*arctan(sqrt(a*x + b)/sqrt(-b))/(sqrt(-b)*b^2) + (3*(a*x + b)^(3/
2)*a^3 - 5*sqrt(a*x + b)*a^3*b)/(a^2*b^2*x^2))/a
```

maple [A] time = 0.01, size = 74, normalized size = 0.89

$$-\frac{\sqrt{\frac{ax+b}{x}}\left(3a^2x^2\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)-3\sqrt{ax+b}\;a\sqrt{b}\;x+2\sqrt{ax+b}\;b^{\frac{3}{2}}\right)}{4\sqrt{ax+b}\;b^{\frac{5}{2}}x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(1/2)/x^(7/2),x)
```

```
[Out] -1/4*((a*x+b)/x)^(1/2)/x^(3/2)/b^(5/2)*(3*arctanh((a*x+b)^(1/2)/b^(1/2))*a^
2*x^2-3*(a*x+b)^(1/2)*a*b^(1/2)*x+2*(a*x+b)^(1/2)*b^(3/2))/(a*x+b)^(1/2)
```

maxima [A] time = 2.31, size = 122, normalized size = 1.47

$$\frac{3\,a^2\log\left(\frac{\sqrt{a+\frac{b}{x}}\,\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\,\sqrt{x}+\sqrt{b}}\right)}{8\,b^{\frac{5}{2}}}+\frac{3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2x^{\frac{3}{2}}-5\sqrt{a+\frac{b}{x}}\,a^2b\sqrt{x}}{4\left(\left(a+\frac{b}{x}\right)^2b^2x^2-2\left(a+\frac{b}{x}\right)b^3x+b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(1/2)/x^(7/2),x, algorithm="maxima")
```

```
[Out] 3/8*a^2*log((sqrt(a + b/x)*sqrt(x) - sqrt(b))/(sqrt(a + b/x)*sqrt(x) + sqrt
(b)))/b^(5/2) + 1/4*(3*(a + b/x)^(3/2)*a^2*x^(3/2) - 5*sqrt(a + b/x)*a^2*b*
sqrt(x))/((a + b/x)^2*b^2*x^2 - 2*(a + b/x)*b^3*x + b^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{7/2}\sqrt{a+\frac{b}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a + b/x)^(1/2)),x)
```

```
[Out] int(1/(x^(7/2)*(a + b/x)^(1/2)), x)
```

sympy [A] time = 22.61, size = 102, normalized size = 1.23

$$\frac{3a^{\frac{3}{2}}}{4b^2\sqrt{x}\,\sqrt{1+\frac{b}{ax}}}+\frac{\sqrt{a}}{4bx^{\frac{3}{2}}\sqrt{1+\frac{b}{ax}}}-\frac{3a^2\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\,\sqrt{x}}\right)}{4b^{\frac{5}{2}}}-\frac{1}{2\sqrt{a}\,x^{\frac{5}{2}}\sqrt{1+\frac{b}{ax}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(1/2)/x**(7/2),x)
```

```
[Out] 3*a**(3/2)/(4*b**2*sqrt(x)*sqrt(1 + b/(a*x))) + sqrt(a)/(4*b*x**(3/2)*sqrt(
1 + b/(a*x))) - 3*a**2*asinh(sqrt(b)/(sqrt(a)*sqrt(x)))/(4*b**(5/2)) - 1/(2
*sqrt(a)*x**(5/2)*sqrt(1 + b/(a*x)))
```

3.1785 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\, x^{9/2}}\, dx$

Optimal. Leaf size=109

$$\frac{5a^3 \tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{8b^{7/2}} - \frac{5a^2\sqrt{a+\frac{b}{x}}}{8b^3\sqrt{x}} + \frac{5a\sqrt{a+\frac{b}{x}}}{12b^2x^{3/2}} - \frac{\sqrt{a+\frac{b}{x}}}{3bx^{5/2}}$$

```
[Out] 5/8*a^3*arctanh(b^(1/2)/(a+b/x)^(1/2)/x^(1/2))/b^(7/2)-1/3*(a+b/x)^(1/2)/b/
x^(5/2)+5/12*a*(a+b/x)^(1/2)/b^2/x^(3/2)-5/8*a^2*(a+b/x)^(1/2)/b^3/x^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {337, 321, 217, 206}

$$-\frac{5a^2\sqrt{a+\frac{b}{x}}}{8b^3\sqrt{x}} + \frac{5a^3 \tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{8b^{7/2}} + \frac{5a\sqrt{a+\frac{b}{x}}}{12b^2x^{3/2}} - \frac{\sqrt{a+\frac{b}{x}}}{3bx^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x]*x^(9/2)),x]
```

```
[Out] -Sqrt[a + b/x]/(3*b*x^(5/2)) + (5*a*Sqrt[a + b/x])/(12*b^2*x^(3/2)) - (5*a^
2*Sqrt[a + b/x])/(8*b^3*Sqrt[x]) + (5*a^3*ArcTanh[Sqrt[b]/(Sqrt[a + b/x]*Sq
rt[x])])/(8*b^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 337

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, -Dist[k/c, Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1)
 + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0] &
& FractionQ[m]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\,x^{9/2}}\,dx = -\left(2\,\text{Subst}\left(\int \frac{x^6}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$
$$= -\frac{\sqrt{a+\frac{b}{x}}}{3bx^{5/2}} + \frac{(5a)\,\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{\sqrt{x}}\right)}{3b}$$
$$= -\frac{\sqrt{a+\frac{b}{x}}}{3bx^{5/2}} + \frac{5a\sqrt{a+\frac{b}{x}}}{12b^2x^{3/2}} - \frac{\left(5a^2\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{\sqrt{x}}\right)}{4b^2}$$
$$= -\frac{\sqrt{a+\frac{b}{x}}}{3bx^{5/2}} + \frac{5a\sqrt{a+\frac{b}{x}}}{12b^2x^{3/2}} - \frac{5a^2\sqrt{a+\frac{b}{x}}}{8b^3\sqrt{x}} + \frac{\left(5a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{\sqrt{x}}\right)}{8b^3}$$
$$= -\frac{\sqrt{a+\frac{b}{x}}}{3bx^{5/2}} + \frac{5a\sqrt{a+\frac{b}{x}}}{12b^2x^{3/2}} - \frac{5a^2\sqrt{a+\frac{b}{x}}}{8b^3\sqrt{x}} + \frac{\left(5a^3\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{b}{x}}\,\sqrt{x}}\right)}{8b^3}$$
$$= -\frac{\sqrt{a+\frac{b}{x}}}{3bx^{5/2}} + \frac{5a\sqrt{a+\frac{b}{x}}}{12b^2x^{3/2}} - \frac{5a^2\sqrt{a+\frac{b}{x}}}{8b^3\sqrt{x}} + \frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\,\sqrt{x}}\right)}{8b^{7/2}}$$

Mathematica [A] time = 0.08, size = 106, normalized size = 0.97

$$\frac{15a^{7/2}x^{7/2}\sqrt{\frac{b}{ax}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}\,\sqrt{x}}\right) - \sqrt{b}\left(15a^3x^3 + 5a^2bx^2 - 2ab^2x + 8b^3\right)}{24b^{7/2}x^{7/2}\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x]*x^(9/2)),x]
```

```
[Out] (-(Sqrt[b]*(8*b^3 - 2*a*b^2*x + 5*a^2*b*x^2 + 15*a^3*x^3)) + 15*a^(7/2)*Sqr
t[1 + b/(a*x)]*x^(7/2)*ArcSinh[Sqrt[b]/(Sqrt[a]*Sqrt[x])])/(24*b^(7/2)*Sqrt
[a + b/x]*x^(7/2))
```

fricas [A] time = 0.90, size = 173, normalized size = 1.59

$$\left[\frac{15\,a^3\sqrt{b}\,x^3\log\left(\frac{ax+2\,\sqrt{b}\,\sqrt{x}\,\sqrt{\frac{ax+b}{x}}+2\,b}{x}\right) - 2\left(15\,a^2bx^2 - 10\,ab^2x + 8\,b^3\right)\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}{48\,b^4x^3}, -\frac{15\,a^3\sqrt{-b}\,x^3\arctan\left(\frac{\sqrt{-b}\,\sqrt{x}}{b}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(1/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*a^3*sqrt(b)*x^3*log((a*x + 2*sqrt(b)*sqrt(x)*sqrt((a*x + b)/x) +
2*b)/x) - 2*(15*a^2*b*x^2 - 10*a*b^2*x + 8*b^3)*sqrt(x)*sqrt((a*x + b)/x))/
(b^4*x^3), -1/24*(15*a^3*sqrt(-b)*x^3*arctan(sqrt(-b)*sqrt(x)*sqrt((a*x + b
)/x)/b) + (15*a^2*b*x^2 - 10*a*b^2*x + 8*b^3)*sqrt(x)*sqrt((a*x + b)/x))/(b
^4*x^3)]
```

giac [A] time = 0.23, size = 84, normalized size = 0.77

$$-\frac{\frac{15\,a^4\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b^3} + \frac{15\,(ax+b)^{\frac{5}{2}}a^4 - 40\,(ax+b)^{\frac{3}{2}}a^4b + 33\,\sqrt{ax+b}\,a^4b^2}{a^3b^3x^3}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(1/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] -1/24*(15*a^4*arctan(sqrt(a*x + b)/sqrt(-b))/(sqrt(-b)*b^3) + (15*(a*x + b)
^(5/2)*a^4 - 40*(a*x + b)^(3/2)*a^4*b + 33*sqrt(a*x + b)*a^4*b^2)/(a^3*b^3*
x^3))/a
```

maple [A] time = 0.01, size = 92, normalized size = 0.84

$$-\frac{\sqrt{\frac{ax+b}{x}}\left(-15a^3x^3\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)+15\sqrt{ax+b}\,a^2\sqrt{b}\,x^2-10\sqrt{ax+b}\,a\,b^{\frac{3}{2}}x+8\sqrt{ax+b}\,b^{\frac{5}{2}}\right)}{24\sqrt{ax+b}\,b^{\frac{7}{2}}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(1/2)/x^(9/2),x)
```

```
[Out] -1/24*((a*x+b)/x)^(1/2)*(-15*arctanh((a*x+b)^(1/2)/b^(1/2))*x^3*a^3+8*(a*x+
b)^(1/2)*b^(5/2)-10*(a*x+b)^(1/2)*a*b^(3/2)*x+15*(a*x+b)^(1/2)*a^2*b^(1/2)*
x^2)/x^(5/2)/b^(7/2)/(a*x+b)^(1/2)
```

maxima [A] time = 2.37, size = 161, normalized size = 1.48

$$-\frac{5\,a^3\log\left(\frac{\sqrt{a+\frac{b}{x}}\,\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\,\sqrt{x}+\sqrt{b}}\right)}{16\,b^{\frac{7}{2}}}-\frac{15\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^3x^{\frac{5}{2}}-40\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^3bx^{\frac{3}{2}}+33\sqrt{a+\frac{b}{x}}\,a^3b^2\sqrt{x}}{24\left(\left(a+\frac{b}{x}\right)^3b^3x^3-3\left(a+\frac{b}{x}\right)^2b^4x^2+3\left(a+\frac{b}{x}\right)b^5x-b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(1/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] -5/16*a^3*log((sqrt(a + b/x)*sqrt(x) - sqrt(b))/(sqrt(a + b/x)*sqrt(x) + sq
rt(b)))/b^(7/2) - 1/24*(15*(a + b/x)^(5/2)*a^3*x^(5/2) - 40*(a + b/x)^(3/2)
*a^3*b*x^(3/2) + 33*sqrt(a + b/x)*a^3*b^2*sqrt(x))/((a + b/x)^3*b^3*x^3 - 3
*(a + b/x)^2*b^4*x^2 + 3*(a + b/x)*b^5*x - b^6)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{9/2}\sqrt{a+\frac{b}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(9/2)*(a + b/x)^(1/2)),x)
```

```
[Out] int(1/(x^(9/2)*(a + b/x)^(1/2)), x)
```

sympy [A] time = 56.43, size = 129, normalized size = 1.18

$$-\frac{5a^{\frac{5}{2}}}{8b^3\sqrt{x}\sqrt{1+\frac{b}{ax}}}-\frac{5a^{\frac{3}{2}}}{24b^2x^{\frac{3}{2}}\sqrt{1+\frac{b}{ax}}}+\frac{\sqrt{a}}{12bx^{\frac{5}{2}}\sqrt{1+\frac{b}{ax}}}+\frac{5a^3\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right)}{8b^{\frac{7}{2}}}-\frac{1}{3\sqrt{a}\,x^{\frac{7}{2}}\sqrt{1+\frac{b}{ax}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(1/2)/x**(9/2),x)
```

```
[Out] -5*a**(5/2)/(8*b**3*sqrt(x)*sqrt(1 + b/(a*x))) - 5*a**(3/2)/(24*b**2*x**(3/
2)*sqrt(1 + b/(a*x))) + sqrt(a)/(12*b*x**(5/2)*sqrt(1 + b/(a*x))) + 5*a**3*
asinh(sqrt(b)/(sqrt(a)*sqrt(x)))/(8*b**(7/2)) - 1/(3*sqrt(a)*x**(7/2)*sqrt(
1 + b/(a*x)))
```

3.1786 $\int \frac{x^{5/2}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$

Optimal. Leaf size=126

$$-\frac{256b^4}{35a^5\sqrt{x}\sqrt{a+\frac{b}{x}}}-\frac{128b^3\sqrt{x}}{35a^4\sqrt{a+\frac{b}{x}}}+\frac{32b^2x^{3/2}}{35a^3\sqrt{a+\frac{b}{x}}}-\frac{16bx^{5/2}}{35a^2\sqrt{a+\frac{b}{x}}}+\frac{2x^{7/2}}{7a\sqrt{a+\frac{b}{x}}}$$

```
[Out] 32/35*b^2*x^(3/2)/a^3/(a+b/x)^(1/2)-16/35*b*x^(5/2)/a^2/(a+b/x)^(1/2)+2/7*x
^(7/2)/a/(a+b/x)^(1/2)-256/35*b^4/a^5/(a+b/x)^(1/2)/x^(1/2)-128/35*b^3*x^(1
/2)/a^4/(a+b/x)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {271, 264}

$$\frac{32b^2x^{3/2}}{35a^3\sqrt{a+\frac{b}{x}}}-\frac{256b^4}{35a^5\sqrt{x}\sqrt{a+\frac{b}{x}}}-\frac{128b^3\sqrt{x}}{35a^4\sqrt{a+\frac{b}{x}}}-\frac{16bx^{5/2}}{35a^2\sqrt{a+\frac{b}{x}}}+\frac{2x^{7/2}}{7a\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b/x)^(3/2),x]
```

```
[Out] (-256*b^4)/(35*a^5*Sqrt[a + b/x]*Sqrt[x]) - (128*b^3*Sqrt[x])/(35*a^4*Sqrt[
a + b/x]) + (32*b^2*x^(3/2))/(35*a^3*Sqrt[a + b/x]) - (16*b*x^(5/2))/(35*a^
2*Sqrt[a + b/x]) + (2*x^(7/2))/(7*a*Sqrt[a + b/x])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^{5/2}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{2x^{7/2}}{7a\sqrt{a+\frac{b}{x}}} - \frac{(8b)\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx}{7a}$$

$$= -\frac{16bx^{5/2}}{35a^2\sqrt{a+\frac{b}{x}}} + \frac{2x^{7/2}}{7a\sqrt{a+\frac{b}{x}}} + \frac{\left(48b^2\right)\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx}{35a^2}$$

$$= \frac{32b^2x^{3/2}}{35a^3\sqrt{a+\frac{b}{x}}} - \frac{16bx^{5/2}}{35a^2\sqrt{a+\frac{b}{x}}} + \frac{2x^{7/2}}{7a\sqrt{a+\frac{b}{x}}} - \frac{\left(64b^3\right)\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\sqrt{x}}\,dx}{35a^3}$$

$$= -\frac{128b^3\sqrt{x}}{35a^4\sqrt{a+\frac{b}{x}}} + \frac{32b^2x^{3/2}}{35a^3\sqrt{a+\frac{b}{x}}} - \frac{16bx^{5/2}}{35a^2\sqrt{a+\frac{b}{x}}} + \frac{2x^{7/2}}{7a\sqrt{a+\frac{b}{x}}} + \frac{\left(128b^4\right)\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}x^{3/2}}\,dx}{35a^4}$$

$$= -\frac{256b^4}{35a^5\sqrt{a+\frac{b}{x}}\sqrt{x}} - \frac{128b^3\sqrt{x}}{35a^4\sqrt{a+\frac{b}{x}}} + \frac{32b^2x^{3/2}}{35a^3\sqrt{a+\frac{b}{x}}} - \frac{16bx^{5/2}}{35a^2\sqrt{a+\frac{b}{x}}} + \frac{2x^{7/2}}{7a\sqrt{a+\frac{b}{x}}}$$

Mathematica [A] time = 0.02, size = 64, normalized size = 0.51

$$\frac{2\left(5a^4x^4 - 8a^3bx^3 + 16a^2b^2x^2 - 64ab^3x - 128b^4\right)}{35a^5\sqrt{x}\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b/x)^(3/2),x]
```

```
[Out] (2*(-128*b^4 - 64*a*b^3*x + 16*a^2*b^2*x^2 - 8*a^3*b*x^3 + 5*a^4*x^4))/(35*
a^5*Sqrt[a + b/x]*Sqrt[x])
```

fricas [A] time = 0.59, size = 70, normalized size = 0.56

$$\frac{2\left(5\,a^4x^4 - 8\,a^3bx^3 + 16\,a^2b^2x^2 - 64\,ab^3x - 128\,b^4\right)\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}{35\left(a^6x + a^5b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(a+b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*a^4*x^4 - 8*a^3*b*x^3 + 16*a^2*b^2*x^2 - 64*a*b^3*x - 128*b^4)*sqrt
(x)*sqrt((a*x + b)/x)/(a^6*x + a^5*b)
```

giac [A] time = 0.18, size = 85, normalized size = 0.67

$$\frac{256\,b^{\frac{7}{2}}}{35\,a^5} - \frac{2\,b^4}{\sqrt{ax+b}\,a^5} + \frac{2\left(5\,(ax+b)^{\frac{7}{2}}a^{30} - 28\,(ax+b)^{\frac{5}{2}}a^{30}b + 70\,(ax+b)^{\frac{3}{2}}a^{30}b^2 - 140\,\sqrt{ax+b}\,a^{30}b^3\right)}{35\,a^{35}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(a+b/x)^(3/2),x, algorithm="giac")
```

```
[Out] 256/35*b^(7/2)/a^5 - 2*b^4/(sqrt(a*x + b)*a^5) + 2/35*(5*(a*x + b)^(7/2)*a^
30 - 28*(a*x + b)^(5/2)*a^30*b + 70*(a*x + b)^(3/2)*a^30*b^2 - 140*sqrt(a*x
 + b)*a^30*b^3)/a^35
```

maple [A] time = 0.01, size = 66, normalized size = 0.52

$$\frac{2\,(ax+b)\left(5a^4x^4-8a^3x^3b+16a^2x^2b^2-64axb^3-128b^4\right)}{35\left(\frac{ax+b}{x}\right)^{\frac{3}{2}}a^5x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a+b/x)^(3/2),x)
```

```
[Out] 2/35*(a*x+b)*(5*a^4*x^4-8*a^3*b*x^3+16*a^2*b^2*x^2-64*a*b^3*x-128*b^4)/a^5/
x^(3/2)/((a*x+b)/x)^(3/2)
```

maxima [A] time = 0.99, size = 90, normalized size = 0.71

$$-\frac{2b^4}{\sqrt{a+\frac{b}{x}}\,a^5\sqrt{x}}+\frac{2\left(5\left(a+\frac{b}{x}\right)^{\frac{7}{2}}x^{\frac{7}{2}}-28\left(a+\frac{b}{x}\right)^{\frac{5}{2}}bx^{\frac{5}{2}}+70\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^2x^{\frac{3}{2}}-140\sqrt{a+\frac{b}{x}}\,b^3\sqrt{x}\right)}{35\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(a+b/x)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*b^4/(sqrt(a + b/x)*a^5*sqrt(x)) + 2/35*(5*(a + b/x)^(7/2)*x^(7/2) - 28*(
a + b/x)^(5/2)*b*x^(5/2) + 70*(a + b/x)^(3/2)*b^2*x^(3/2) - 140*sqrt(a + b/
x)*b^3*sqrt(x))/a^5
```

mupad [B] time = 1.63, size = 67, normalized size = 0.53

$$-\frac{2\sqrt{x}\,\sqrt{\frac{b+a\,x}{x}}\,\left(-5\,a^4\,x^4+8\,a^3\,b\,x^3-16\,a^2\,b^2\,x^2+64\,a\,b^3\,x+128\,b^4\right)}{35\,a^5\,\left(b+a\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b/x)^(3/2),x)
```

```
[Out] -(2*x^(1/2)*((b + a*x)/x)^(1/2)*(128*b^4 - 5*a^4*x^4 + 8*a^3*b*x^3 - 16*a^2
*b^2*x^2 + 64*a*b^3*x))/(35*a^5*(b + a*x))
```

sympy [B] time = 14.00, size = 614, normalized size = 4.87

$$\frac{10a^7b^{\frac{33}{2}}x^7\sqrt{\frac{ax}{b}+1}}{35a^9b^{16}x^4+140a^8b^{17}x^3+210a^7b^{18}x^2+140a^6b^{19}x+35a^5b^{20}}+\frac{14a^6b^{\frac{35}{2}}x^6\sqrt{\frac{ax}{b}+1}}{35a^9b^{16}x^4+140a^8b^{17}x^3+210a^7b^{18}x^2+140a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(a+b/x)**(3/2),x)
```

```
[Out] 10*a**7*b**(33/2)*x**7*sqrt(a*x/b + 1)/(35*a**9*b**16*x**4 + 140*a**8*b**17
*x**3 + 210*a**7*b**18*x**2 + 140*a**6*b**19*x + 35*a**5*b**20) + 14*a**6*b
**(35/2)*x**6*sqrt(a*x/b + 1)/(35*a**9*b**16*x**4 + 140*a**8*b**17*x**3 + 2
10*a**7*b**18*x**2 + 140*a**6*b**19*x + 35*a**5*b**20) + 14*a**5*b**(37/2)*
x**5*sqrt(a*x/b + 1)/(35*a**9*b**16*x**4 + 140*a**8*b**17*x**3 + 210*a**7*b
**18*x**2 + 140*a**6*b**19*x + 35*a**5*b**20) - 70*a**4*b**(39/2)*x**4*sqrt
(a*x/b + 1)/(35*a**9*b**16*x**4 + 140*a**8*b**17*x**3 + 210*a**7*b**18*x**2
```

```
 + 140*a**6*b**19*x + 35*a**5*b**20) - 560*a**3*b**(41/2)*x**3*sqrt(a*x/b +
 1)/(35*a**9*b**16*x**4 + 140*a**8*b**17*x**3 + 210*a**7*b**18*x**2 + 140*a
**6*b**19*x + 35*a**5*b**20) - 1120*a**2*b**(43/2)*x**2*sqrt(a*x/b + 1)/(35
*a**9*b**16*x**4 + 140*a**8*b**17*x**3 + 210*a**7*b**18*x**2 + 140*a**6*b**
19*x + 35*a**5*b**20) - 896*a*b**(45/2)*x*sqrt(a*x/b + 1)/(35*a**9*b**16*x*
*4 + 140*a**8*b**17*x**3 + 210*a**7*b**18*x**2 + 140*a**6*b**19*x + 35*a**5
*b**20) - 256*b**(47/2)*sqrt(a*x/b + 1)/(35*a**9*b**16*x**4 + 140*a**8*b**1
7*x**3 + 210*a**7*b**18*x**2 + 140*a**6*b**19*x + 35*a**5*b**20)
```

3.1787 $\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$

Optimal. Leaf size=100

$$\frac{32b^3}{5a^4\sqrt{x}\sqrt{a+\frac{b}{x}}}+\frac{16b^2\sqrt{x}}{5a^3\sqrt{a+\frac{b}{x}}}-\frac{4bx^{3/2}}{5a^2\sqrt{a+\frac{b}{x}}}+\frac{2x^{5/2}}{5a\sqrt{a+\frac{b}{x}}}$$

```
[Out] -4/5*b*x^(3/2)/a^2/(a+b/x)^(1/2)+2/5*x^(5/2)/a/(a+b/x)^(1/2)+32/5*b^3/a^4/(
a+b/x)^(1/2)/x^(1/2)+16/5*b^2*x^(1/2)/a^3/(a+b/x)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {271, 264}

$$\frac{32b^3}{5a^4\sqrt{x}\sqrt{a+\frac{b}{x}}}+\frac{16b^2\sqrt{x}}{5a^3\sqrt{a+\frac{b}{x}}}-\frac{4bx^{3/2}}{5a^2\sqrt{a+\frac{b}{x}}}+\frac{2x^{5/2}}{5a\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b/x)^(3/2),x]
```

```
[Out] (32*b^3)/(5*a^4*Sqrt[a + b/x]*Sqrt[x]) + (16*b^2*Sqrt[x])/(5*a^3*Sqrt[a + b
/x]) - (4*b*x^(3/2))/(5*a^2*Sqrt[a + b/x]) + (2*x^(5/2))/(5*a*Sqrt[a + b/x]
)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{2x^{5/2}}{5a\sqrt{a+\frac{b}{x}}} - \frac{(6b)\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx}{5a}$$

$$= -\frac{4bx^{3/2}}{5a^2\sqrt{a+\frac{b}{x}}} + \frac{2x^{5/2}}{5a\sqrt{a+\frac{b}{x}}} + \frac{\left(8b^2\right)\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\sqrt{x}}\,dx}{5a^2}$$

$$= \frac{16b^2\sqrt{x}}{5a^3\sqrt{a+\frac{b}{x}}} - \frac{4bx^{3/2}}{5a^2\sqrt{a+\frac{b}{x}}} + \frac{2x^{5/2}}{5a\sqrt{a+\frac{b}{x}}} - \frac{\left(16b^3\right)\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}x^{3/2}}\,dx}{5a^3}$$

$$= \frac{32b^3}{5a^4\sqrt{a+\frac{b}{x}}\sqrt{x}} + \frac{16b^2\sqrt{x}}{5a^3\sqrt{a+\frac{b}{x}}} - \frac{4bx^{3/2}}{5a^2\sqrt{a+\frac{b}{x}}} + \frac{2x^{5/2}}{5a\sqrt{a+\frac{b}{x}}}$$

Mathematica [A] time = 0.01, size = 52, normalized size = 0.52

$$\frac{2\left(a^3x^3 - 2a^2bx^2 + 8ab^2x + 16b^3\right)}{5a^4\sqrt{x}\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b/x)^(3/2),x]
```

```
[Out] (2*(16*b^3 + 8*a*b^2*x - 2*a^2*b*x^2 + a^3*x^3))/(5*a^4*Sqrt[a + b/x]*Sqrt[
x])
```

fricas [A] time = 0.83, size = 58, normalized size = 0.58

$$\frac{2\left(a^3x^3 - 2\,a^2bx^2 + 8\,ab^2x + 16\,b^3\right)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{5\left(a^5x + a^4b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(a+b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(a^3*x^3 - 2*a^2*b*x^2 + 8*a*b^2*x + 16*b^3)*sqrt(x)*sqrt((a*x + b)/x)/
(a^5*x + a^4*b)
```

giac [A] time = 0.21, size = 69, normalized size = 0.69

$$-\frac{32\,b^{\frac{5}{2}}}{5\,a^4} + \frac{2\,b^3}{\sqrt{ax+b}\,a^4} + \frac{2\left((ax+b)^{\frac{5}{2}}a^{16} - 5\,(ax+b)^{\frac{3}{2}}a^{16}b + 15\sqrt{ax+b}\,a^{16}b^2\right)}{5\,a^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(a+b/x)^(3/2),x, algorithm="giac")
```

```
[Out] -32/5*b^(5/2)/a^4 + 2*b^3/(sqrt(a*x + b)*a^4) + 2/5*((a*x + b)^(5/2)*a^16 -
 5*(a*x + b)^(3/2)*a^16*b + 15*sqrt(a*x + b)*a^16*b^2)/a^20
```

maple [A] time = 0.01, size = 54, normalized size = 0.54

$$\frac{2\,(ax+b)\left(a^3x^3 - 2a^2b\,x^2 + 8a\,b^2x + 16b^3\right)}{5\left(\frac{ax+b}{x}\right)^{\frac{3}{2}}a^4x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a+b/x)^(3/2),x)
```

```
[Out] 2/5*(a*x+b)*(a^3*x^3-2*a^2*b*x^2+8*a*b^2*x+16*b^3)/a^4/x^(3/2)/((a*x+b)/x)^
(3/2)
```

maxima [A] time = 1.04, size = 72, normalized size = 0.72

$$\frac{2\,b^3}{\sqrt{a+\frac{b}{x}}\,a^4\sqrt{x}}+\frac{2\left(\left(a+\frac{b}{x}\right)^{\frac{5}{2}}x^{\frac{5}{2}}-5\left(a+\frac{b}{x}\right)^{\frac{3}{2}}bx^{\frac{3}{2}}+15\sqrt{a+\frac{b}{x}}\,b^2\sqrt{x}\right)}{5\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(a+b/x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*b^3/(sqrt(a + b/x)*a^4*sqrt(x)) + 2/5*((a + b/x)^(5/2)*x^(5/2) - 5*(a + b
/x)^(3/2)*b*x^(3/2) + 15*sqrt(a + b/x)*b^2*sqrt(x))/a^4
```

mupad [B] time = 1.52, size = 59, normalized size = 0.59

$$\frac{\sqrt{a+\frac{b}{x}}\,\left(\frac{2\,x^{7/2}}{5\,a^2}-\frac{4\,b\,x^{5/2}}{5\,a^3}+\frac{16\,b^2\,x^{3/2}}{5\,a^4}+\frac{32\,b^3\,\sqrt{x}}{5\,a^5}\right)}{x+\frac{b}{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b/x)^(3/2),x)
```

```
[Out] ((a + b/x)^(1/2)*((2*x^(7/2))/(5*a^2) - (4*b*x^(5/2))/(5*a^3) + (16*b^2*x^(
3/2))/(5*a^4) + (32*b^3*x^(1/2))/(5*a^5)))/(x + b/a)
```

sympy [B] time = 5.10, size = 320, normalized size = 3.20

$$\frac{2a^5b^{\frac{19}{2}}x^5\sqrt{\frac{ax}{b}+1}}{5a^7b^9x^3+15a^6b^{10}x^2+15a^5b^{11}x+5a^4b^{12}}+\frac{10a^3b^{\frac{23}{2}}x^3\sqrt{\frac{ax}{b}+1}}{5a^7b^9x^3+15a^6b^{10}x^2+15a^5b^{11}x+5a^4b^{12}}+\frac{60a^2b^{\frac{25}{2}}x^2}{5a^7b^9x^3+15a^6b^{10}x^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(a+b/x)**(3/2),x)
```

```
[Out] 2*a**5*b**(19/2)*x**5*sqrt(a*x/b + 1)/(5*a**7*b**9*x**3 + 15*a**6*b**10*x**
2 + 15*a**5*b**11*x + 5*a**4*b**12) + 10*a**3*b**(23/2)*x**3*sqrt(a*x/b + 1
)/(5*a**7*b**9*x**3 + 15*a**6*b**10*x**2 + 15*a**5*b**11*x + 5*a**4*b**12)
+ 60*a**2*b**(25/2)*x**2*sqrt(a*x/b + 1)/(5*a**7*b**9*x**3 + 15*a**6*b**10*
x**2 + 15*a**5*b**11*x + 5*a**4*b**12) + 80*a*b**(27/2)*x*sqrt(a*x/b + 1)/(
5*a**7*b**9*x**3 + 15*a**6*b**10*x**2 + 15*a**5*b**11*x + 5*a**4*b**12) + 3
2*b**(29/2)*sqrt(a*x/b + 1)/(5*a**7*b**9*x**3 + 15*a**6*b**10*x**2 + 15*a**
5*b**11*x + 5*a**4*b**12)
```

3.1788 $\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$

Optimal. Leaf size=74

$$-\frac{16b^2}{3a^3\sqrt{x}\sqrt{a+\frac{b}{x}}}-\frac{8b\sqrt{x}}{3a^2\sqrt{a+\frac{b}{x}}}+\frac{2x^{3/2}}{3a\sqrt{a+\frac{b}{x}}}$$

```
[Out] 2/3*x^(3/2)/a/(a+b/x)^(1/2)-16/3*b^2/a^3/(a+b/x)^(1/2)/x^(1/2)-8/3*b*x^(1/2
)/a^2/(a+b/x)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {271, 264}

$$-\frac{16b^2}{3a^3\sqrt{x}\sqrt{a+\frac{b}{x}}}-\frac{8b\sqrt{x}}{3a^2\sqrt{a+\frac{b}{x}}}+\frac{2x^{3/2}}{3a\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b/x)^(3/2),x]
```

```
[Out] (-16*b^2)/(3*a^3*Sqrt[a + b/x]*Sqrt[x]) - (8*b*Sqrt[x])/(3*a^2*Sqrt[a + b/x
]) + (2*x^(3/2))/(3*a*Sqrt[a + b/x])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx &= \frac{2x^{3/2}}{3a\sqrt{a+\frac{b}{x}}}-\frac{(4b)\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\sqrt{x}}\,dx}{3a}\\ &= -\frac{8b\sqrt{x}}{3a^2\sqrt{a+\frac{b}{x}}}+\frac{2x^{3/2}}{3a\sqrt{a+\frac{b}{x}}}+\frac{\left(8b^2\right)\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}x^{3/2}}\,dx}{3a^2}\\ &= -\frac{16b^2}{3a^3\sqrt{a+\frac{b}{x}}\sqrt{x}}-\frac{8b\sqrt{x}}{3a^2\sqrt{a+\frac{b}{x}}}+\frac{2x^{3/2}}{3a\sqrt{a+\frac{b}{x}}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.55

$$\frac{2\left(a^2x^2-4abx-8b^2\right)}{3a^3\sqrt{x}\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b/x)^(3/2),x]
```

```
[Out] (2*(-8*b^2 - 4*a*b*x + a^2*x^2))/(3*a^3*Sqrt[a + b/x]*Sqrt[x])
```

fricas [A] time = 0.73, size = 47, normalized size = 0.64

$$\frac{2\left(a^2x^2 - 4\,abx - 8\,b^2\right)\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}{3\left(a^4x + a^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(a+b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(a^2*x^2 - 4*a*b*x - 8*b^2)*sqrt(x)*sqrt((a*x + b)/x)/(a^4*x + a^3*b)
```

giac [A] time = 0.17, size = 54, normalized size = 0.73

$$\frac{16\,b^{\frac{3}{2}}}{3\,a^3} - \frac{2\,b^2}{\sqrt{ax+b}\,a^3} + \frac{2\left((ax+b)^{\frac{3}{2}}a^6 - 6\,\sqrt{ax+b}\,a^6b\right)}{3\,a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(a+b/x)^(3/2),x, algorithm="giac")
```

```
[Out] 16/3*b^(3/2)/a^3 - 2*b^2/(sqrt(a*x + b)*a^3) + 2/3*((a*x + b)^(3/2)*a^6 - 6
*sqrt(a*x + b)*a^6*b)/a^9
```

maple [A] time = 0.01, size = 43, normalized size = 0.58

$$\frac{2\,(ax+b)\left(a^2x^2 - 4abx - 8b^2\right)}{3\left(\frac{ax+b}{x}\right)^{\frac{3}{2}}a^3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a+b/x)^(3/2),x)
```

```
[Out] 2/3*(a*x+b)*(a^2*x^2-4*a*b*x-8*b^2)/a^3/x^(3/2)/((a*x+b)/x)^(3/2)
```

maxima [A] time = 1.17, size = 55, normalized size = 0.74

$$\frac{2\left(\left(a+\frac{b}{x}\right)^{\frac{3}{2}}x^{\frac{3}{2}} - 6\,\sqrt{a+\frac{b}{x}}\,b\sqrt{x}\right)}{3\,a^3} - \frac{2\,b^2}{\sqrt{a+\frac{b}{x}}\,a^3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(a+b/x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*((a + b/x)^(3/2)*x^(3/2) - 6*sqrt(a + b/x)*b*sqrt(x))/a^3 - 2*b^2/(sqrt
(a + b/x)*a^3*sqrt(x))
```

mupad [B] time = 1.49, size = 49, normalized size = 0.66

$$-\frac{\sqrt{a+\frac{b}{x}}\,\left(\frac{8\,b\,x^{3/2}}{3\,a^3} - \frac{2\,x^{5/2}}{3\,a^2} + \frac{16\,b^2\,\sqrt{x}}{3\,a^4}\right)}{x+\frac{b}{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b/x)^(3/2),x)
```

```
[Out] -((a + b/x)^(1/2)*((8*b*x^(3/2))/(3*a^3) - (2*x^(5/2))/(3*a^2) + (16*b^2*x^
(1/2))/(3*a^4)))/(x + b/a)
```

sympy [**B**] time = 2.68, size = 206, normalized size = 2.78

$$\frac{2a^3b^{\frac{9}{2}}x^3\sqrt{\frac{ax}{b}+1}}{3a^5b^4x^2+6a^4b^5x+3a^3b^6}-\frac{6a^2b^{\frac{11}{2}}x^2\sqrt{\frac{ax}{b}+1}}{3a^5b^4x^2+6a^4b^5x+3a^3b^6}-\frac{24ab^{\frac{13}{2}}x\sqrt{\frac{ax}{b}+1}}{3a^5b^4x^2+6a^4b^5x+3a^3b^6}-\frac{16b^{\frac{15}{2}}\sqrt{\frac{ax}{b}+1}}{3a^5b^4x^2+6a^4b^5x+3a^3b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(a+b/x)**(3/2),x)
```

```
[Out] 2*a**3*b**(9/2)*x**3*sqrt(a*x/b + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x + 3*
a**3*b**6) - 6*a**2*b**(11/2)*x**2*sqrt(a*x/b + 1)/(3*a**5*b**4*x**2 + 6*a*
*4*b**5*x + 3*a**3*b**6) - 24*a*b**(13/2)*x*sqrt(a*x/b + 1)/(3*a**5*b**4*x*
*2 + 6*a**4*b**5*x + 3*a**3*b**6) - 16*b**(15/2)*sqrt(a*x/b + 1)/(3*a**5*b*
*4*x**2 + 6*a**4*b**5*x + 3*a**3*b**6)
```

3.1789 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\sqrt{x}}\,dx$

Optimal. Leaf size=44

$$\frac{4b}{a^2\sqrt{x}\sqrt{a+\frac{b}{x}}}+\frac{2\sqrt{x}}{a\sqrt{a+\frac{b}{x}}}$$

```
[Out] 4*b/a^2/(a+b/x)^(1/2)/x^(1/2)+2*x^(1/2)/a/(a+b/x)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {271, 264}

$$\frac{4b}{a^2\sqrt{x}\sqrt{a+\frac{b}{x}}}+\frac{2\sqrt{x}}{a\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(3/2)*Sqrt[x]),x]
```

```
[Out] (4*b)/(a^2*Sqrt[a + b/x]*Sqrt[x]) + (2*Sqrt[x])/(a*Sqrt[a + b/x])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\sqrt{x}}\,dx &= \frac{2\sqrt{x}}{a\sqrt{a+\frac{b}{x}}}-\frac{(2b)\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}x^{3/2}}\,dx}{a}\\ &= \frac{4b}{a^2\sqrt{a+\frac{b}{x}}\sqrt{x}}+\frac{2\sqrt{x}}{a\sqrt{a+\frac{b}{x}}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.64

$$\frac{2(ax+2b)}{a^2\sqrt{x}\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(3/2)*Sqrt[x]),x]
```

```
[Out] (2*(2*b + a*x))/(a^2*Sqrt[a + b/x]*Sqrt[x])
```

fricas [A] time = 0.65, size = 36, normalized size = 0.82

$$\frac{2\,(ax+2\,b)\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}{a^3x+a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/x^(1/2),x, algorithm="fricas")
[Out] 2*(a*x + 2*b)*sqrt(x)*sqrt((a*x + b)/x)/(a^3*x + a^2*b)
```

giac [A] time = 0.17, size = 38, normalized size = 0.86

$$\frac{2\left(\frac{\sqrt{ax+b}}{a}+\frac{b}{\sqrt{ax+b}\,a}\right)}{a}-\frac{4\sqrt{b}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/x^(1/2),x, algorithm="giac")
[Out] 2*(sqrt(a*x + b)/a + b/(sqrt(a*x + b)*a))/a - 4*sqrt(b)/a^2
```

maple [A] time = 0.00, size = 32, normalized size = 0.73

$$\frac{2\,(ax+b)\,(ax+2b)}{\left(\frac{ax+b}{x}\right)^{\frac{3}{2}}a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(3/2)/x^(1/2),x)
[Out] 2*(a*x+b)*(a*x+2*b)/a^2/x^(3/2)/((a*x+b)/x)^(3/2)
```

maxima [A] time = 1.11, size = 36, normalized size = 0.82

$$\frac{2\sqrt{a+\frac{b}{x}}\,\sqrt{x}}{a^2}+\frac{2\,b}{\sqrt{a+\frac{b}{x}}\,a^2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/x^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(a + b/x)*sqrt(x)/a^2 + 2*b/(sqrt(a + b/x)*a^2*sqrt(x))
```

mupad [B] time = 1.44, size = 37, normalized size = 0.84

$$\frac{\sqrt{a+\frac{b}{x}}\,\left(\frac{2\,x^{3/2}}{a^2}+\frac{4\,b\,\sqrt{x}}{a^3}\right)}{x+\frac{b}{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b/x)^(3/2)),x)
[Out] ((a + b/x)^(1/2)*((2*x^(3/2))/a^2 + (4*b*x^(1/2))/a^3))/(x + b/a)
```

sympy [**A**] time = 3.82, size = 39, normalized size = 0.89

$$\frac{2x}{a\sqrt{b}\sqrt{\frac{ax}{b}+1}}+\frac{4\sqrt{b}}{a^2\sqrt{\frac{ax}{b}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(3/2)/x**(1/2),x)
[Out] 2*x/(a*sqrt(b)*sqrt(a*x/b + 1)) + 4*sqrt(b)/(a**2*sqrt(a*x/b + 1))
```

3.1790 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{3/2}}\,dx$

Optimal. Leaf size=21

$$-\frac{2}{a\sqrt{x}\sqrt{a+\frac{b}{x}}}$$

```
[Out] -2/a/(a+b/x)^(1/2)/x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {264}

$$-\frac{2}{a\sqrt{x}\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(3/2)*x^(3/2)),x]
```

```
[Out] -2/(a*Sqrt[a + b/x]*Sqrt[x])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{3/2}}\,dx = -\frac{2}{a\sqrt{a+\frac{b}{x}}\sqrt{x}}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$-\frac{2}{a\sqrt{x}\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(3/2)*x^(3/2)),x]
```

```
[Out] -2/(a*Sqrt[a + b/x]*Sqrt[x])
```

fricas [A] time = 0.78, size = 27, normalized size = 1.29

$$-\frac{2\sqrt{x}\sqrt{\frac{ax+b}{x}}}{a^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(x)*sqrt((a*x + b)/x)/(a^2*x + a*b)
```

giac [A] time = 0.17, size = 21, normalized size = 1.00

$$-\frac{2}{\sqrt{ax+b}\,a}+\frac{2}{a\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/x^(3/2),x, algorithm="giac")
[Out] -2/(sqrt(a*x + b)*a) + 2/(a*sqrt(b))
```

maple [A] time = 0.00, size = 25, normalized size = 1.19

$$-\frac{2\,(ax+b)}{\left(\frac{ax+b}{x}\right)^{\frac{3}{2}}\,a\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(3/2)/x^(3/2),x)
[Out] -2*(a*x+b)/a/((a*x+b)/x)^(3/2)/x^(3/2)
```

maxima [A] time = 1.10, size = 17, normalized size = 0.81

$$-\frac{2}{\sqrt{a+\frac{b}{x}}\,a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/x^(3/2),x, algorithm="maxima")
[Out] -2/(sqrt(a + b/x)*a*sqrt(x))
```

mupad [B] time = 1.43, size = 25, normalized size = 1.19

$$-\frac{2\sqrt{x}\,\sqrt{a+\frac{b}{x}}}{x\,a^2+b\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b/x)^(3/2)),x)
[Out] -(2*x^(1/2)*(a + b/x)^(1/2))/(a*b + a^2*x)
```

sympy [A] time = 5.40, size = 19, normalized size = 0.90

$$-\frac{2}{a\sqrt{b}\,\sqrt{\frac{ax}{b}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(3/2)/x**(3/2),x)
[Out] -2/(a*sqrt(b)*sqrt(a*x/b + 1))
```

3.1791 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{5/2}}\,dx$

Optimal. Leaf size=52

$$\frac{2}{b\sqrt{x}\sqrt{a+\frac{b}{x}}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{b^{3/2}}$$

```
[Out] -2*arctanh(b^(1/2)/(a+b/x)^(1/2)/x^(1/2))/b^(3/2)+2/b/(a+b/x)^(1/2)/x^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {337, 288, 217, 206}

$$\frac{2}{b\sqrt{x}\sqrt{a+\frac{b}{x}}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(3/2)*x^(5/2)),x]
```

```
[Out] 2/(b*Sqrt[a + b/x]*Sqrt[x]) - (2*ArcTanh[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x])])/
b^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 337

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, -Dist[k/c, Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1)
 + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0] &
& FractionQ[m]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{5/2}}\,dx = -\left(2\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^2\right)^{3/2}}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= \frac{2}{b\sqrt{a+\frac{b}{x}}\sqrt{x}} - \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{\sqrt{x}}\right)}{b}$$

$$= \frac{2}{b\sqrt{a+\frac{b}{x}}\sqrt{x}} - \frac{2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)}{b}$$

$$= \frac{2}{b\sqrt{a+\frac{b}{x}}\sqrt{x}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)}{b^{3/2}}$$

Mathematica [A] time = 0.09, size = 73, normalized size = 1.40

$$\frac{2\sqrt{b} - 2\sqrt{a}\sqrt{x}\sqrt{\frac{b}{ax}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right)}{b^{3/2}\sqrt{x}\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(3/2)*x^(5/2)),x]
```

```
[Out] (2*Sqrt[b] - 2*Sqrt[a]*Sqrt[1 + b/(a*x)]*Sqrt[x]*ArcSinh[Sqrt[b]/(Sqrt[a]*S
qrt[x])])/(b^(3/2)*Sqrt[a + b/x]*Sqrt[x])
```

fricas [A] time = 1.18, size = 138, normalized size = 2.65

$$\left[\frac{(ax+b)\sqrt{b}\,\log\left(\frac{ax-2\sqrt{b}\sqrt{x}\sqrt{\frac{ax+b}{x}}+2b}{x}\right)+2b\sqrt{x}\sqrt{\frac{ax+b}{x}}}{ab^2x+b^3}, \frac{2\left((ax+b)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\sqrt{x}\sqrt{\frac{ax+b}{x}}}{b}\right)+b\sqrt{x}\sqrt{\frac{ax+b}{x}}\right)}{ab^2x+b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [((a*x + b)*sqrt(b)*log((a*x - 2*sqrt(b)*sqrt(x)*sqrt((a*x + b)/x) + 2*b)/x
) + 2*b*sqrt(x)*sqrt((a*x + b)/x))/(a*b^2*x + b^3), 2*((a*x + b)*sqrt(-b)*a
rctan(sqrt(-b)*sqrt(x)*sqrt((a*x + b)/x)/b) + b*sqrt(x)*sqrt((a*x + b)/x))/
(a*b^2*x + b^3)]
```

giac [A] time = 0.17, size = 67, normalized size = 1.29

$$\frac{2\,\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b} - \frac{2\left(\sqrt{b}\,\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+\sqrt{-b}\right)}{\sqrt{-b}\,b^{\frac{3}{2}}} + \frac{2}{\sqrt{ax+b}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(a*x + b)/sqrt(-b))/(sqrt(-b)*b) - 2*(sqrt(b)*arctan(sqrt(b)/s
qrt(-b)) + sqrt(-b))/(sqrt(-b)*b^(3/2)) + 2/(sqrt(a*x + b)*b)
```

maple [A] time = 0.01, size = 52, normalized size = 1.00

$$\frac{2\sqrt{\frac{ax+b}{x}}\left(-\sqrt{ax+b}\ \operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)+\sqrt{b}\right)\sqrt{x}}{(ax+b)\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(3/2)/x^(5/2),x)
```

```
[Out] 2*((a*x+b)/x)^(1/2)*x^(1/2)*(-arctanh((a*x+b)^(1/2)/b^(1/2))*(a*x+b)^(1/2)+
b^(1/2))/b^(3/2)/(a*x+b)
```

maxima [A] time = 2.39, size = 62, normalized size = 1.19

$$\frac{\log\left(\frac{\sqrt{a+\frac{b}{x}}\,\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\,\sqrt{x}+\sqrt{b}}\right)}{b^{\frac{3}{2}}}+\frac{2}{\sqrt{a+\frac{b}{x}}\,b\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] log((sqrt(a + b/x)*sqrt(x) - sqrt(b))/(sqrt(a + b/x)*sqrt(x) + sqrt(b)))/b^
(3/2) + 2/(sqrt(a + b/x)*b*sqrt(x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^{5/2}\left(a+\frac{b}{x}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b/x)^(3/2)),x)
```

```
[Out] int(1/(x^(5/2)*(a + b/x)^(3/2)), x)
```

sympy [B] time = 13.99, size = 146, normalized size = 2.81

$$\frac{ab^2x\log\left(\frac{ax}{b}\right)}{ab^{\frac{7}{2}}x+b^{\frac{9}{2}}}-\frac{2ab^2x\log\left(\sqrt{\frac{ax}{b}+1}+1\right)}{ab^{\frac{7}{2}}x+b^{\frac{9}{2}}}+\frac{2b^3\sqrt{\frac{ax}{b}+1}}{ab^{\frac{7}{2}}x+b^{\frac{9}{2}}}+\frac{b^3\log\left(\frac{ax}{b}\right)}{ab^{\frac{7}{2}}x+b^{\frac{9}{2}}}-\frac{2b^3\log\left(\sqrt{\frac{ax}{b}+1}+1\right)}{ab^{\frac{7}{2}}x+b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(3/2)/x**(5/2),x)
```

```
[Out] a*b**2*x*log(a*x/b)/(a*b**(7/2)*x + b**(9/2)) - 2*a*b**2*x*log(sqrt(a*x/b +
 1) + 1)/(a*b**(7/2)*x + b**(9/2)) + 2*b**3*sqrt(a*x/b + 1)/(a*b**(7/2)*x +
 b**(9/2)) + b**3*log(a*x/b)/(a*b**(7/2)*x + b**(9/2)) - 2*b**3*log(sqrt(a*
x/b + 1) + 1)/(a*b**(7/2)*x + b**(9/2))
```

3.1792 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{7/2}}\,dx$

Optimal. Leaf size=74

$$\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{b^{5/2}}-\frac{3\sqrt{a+\frac{b}{x}}}{b^2\sqrt{x}}+\frac{2}{bx^{3/2}\sqrt{a+\frac{b}{x}}}$$

```
[Out] 3*a*arctanh(b^(1/2)/(a+b/x)^(1/2)/x^(1/2))/b^(5/2)+2/b/x^(3/2)/(a+b/x)^(1/2
)-3*(a+b/x)^(1/2)/b^2/x^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {337, 288, 321, 217, 206}

$$-\frac{3\sqrt{a+\frac{b}{x}}}{b^2\sqrt{x}}+\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{b^{5/2}}+\frac{2}{bx^{3/2}\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(3/2)*x^(7/2)),x]
```

```
[Out] 2/(b*Sqrt[a + b/x]*x^(3/2)) - (3*Sqrt[a + b/x])/(b^2*Sqrt[x]) + (3*a*ArcTan
h[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x])])/b^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 337

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, -Dist[k/c, Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1)
```

```
 + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0] &
& FractionQ[m]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{7/2}}\,dx = -\left(2\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^2\right)^{3/2}}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= \frac{2}{b\sqrt{a+\frac{b}{x}}\,x^{3/2}} - \frac{6\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{\sqrt{x}}\right)}{b}$$

$$= \frac{2}{b\sqrt{a+\frac{b}{x}}\,x^{3/2}} - \frac{3\sqrt{a+\frac{b}{x}}}{b^2\sqrt{x}} + \frac{(3a)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{\sqrt{x}}\right)}{b^2}$$

$$= \frac{2}{b\sqrt{a+\frac{b}{x}}\,x^{3/2}} - \frac{3\sqrt{a+\frac{b}{x}}}{b^2\sqrt{x}} + \frac{(3a)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{b}{x}}\,\sqrt{x}}\right)}{b^2}$$

$$= \frac{2}{b\sqrt{a+\frac{b}{x}}\,x^{3/2}} - \frac{3\sqrt{a+\frac{b}{x}}}{b^2\sqrt{x}} + \frac{3a\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\,\sqrt{x}}\right)}{b^{5/2}}$$

Mathematica [C] time = 0.02, size = 56, normalized size = 0.76

$$-\frac{2\sqrt{\frac{b}{ax}+1}\;{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};-\frac{b}{ax}\right)}{5ax^{5/2}\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(3/2)*x^(7/2)),x]
```

```
[Out] (-2*Sqrt[1 + b/(a*x)]*Hypergeometric2F1[3/2, 5/2, 7/2, -(b/(a*x))])/(5*a*Sq
rt[a + b/x]*x^(5/2))
```

fricas [A] time = 1.07, size = 179, normalized size = 2.42

$$\left[\frac{3\left(a^2x^2+abx\right)\sqrt{b}\,\log\left(\frac{ax+2\sqrt{b}\sqrt{x}\sqrt{\frac{ax+b}{x}}+2b}{x}\right) - 2\left(3abx+b^2\right)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{2\left(ab^3x^2+b^4x\right)}, -\frac{3\left(a^2x^2+abx\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\sqrt{x}\sqrt{\frac{ax}{b}}}{b}\right.}{ab^3x^2+b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*(a^2*x^2 + a*b*x)*sqrt(b)*log((a*x + 2*sqrt(b)*sqrt(x)*sqrt((a*x +
b)/x) + 2*b)/x) - 2*(3*a*b*x + b^2)*sqrt(x)*sqrt((a*x + b)/x))/(a*b^3*x^2 +
 b^4*x), -(3*(a^2*x^2 + a*b*x)*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)*sqrt((a*x +
 b)/x)/b) + (3*a*b*x + b^2)*sqrt(x)*sqrt((a*x + b)/x))/(a*b^3*x^2 + b^4*x)]
```

giac [A] time = 0.27, size = 64, normalized size = 0.86

$$-\frac{3\,a\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b^2}-\frac{3\,(ax+b)a-2\,ab}{\left((ax+b)^{\frac{3}{2}}-\sqrt{ax+b}\,b\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] -3*a*arctan(sqrt(a*x + b)/sqrt(-b))/(sqrt(-b)*b^2) - (3*(a*x + b)*a - 2*a*b
)/(((a*x + b)^(3/2) - sqrt(a*x + b)*b)*b^2)
```

maple [A] time = 0.02, size = 61, normalized size = 0.82

$$-\frac{\sqrt{\frac{ax+b}{x}}\,\left(-3\sqrt{ax+b}\,ax\,\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)+3a\sqrt{b}\,x+b^{\frac{3}{2}}\right)}{(ax+b)\,b^{\frac{5}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(3/2)/x^(7/2),x)
```

```
[Out] -((a*x+b)/x)^(1/2)*(-3*arctanh((a*x+b)^(1/2)/b^(1/2))*(a*x+b)^(1/2)*x*a+3*x
*a*b^(1/2)+b^(3/2))/x^(1/2)/(a*x+b)/b^(5/2)
```

maxima [A] time = 2.40, size = 101, normalized size = 1.36

$$-\frac{3\left(a+\frac{b}{x}\right)ax-2\,ab}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^2x^{\frac{3}{2}}-\sqrt{a+\frac{b}{x}}\,b^3\sqrt{x}}-\frac{3\,a\log\left(\frac{\sqrt{a+\frac{b}{x}}\,\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\,\sqrt{x}+\sqrt{b}}\right)}{2\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/x^(7/2),x, algorithm="maxima")
```

```
[Out] -(3*(a + b/x)*a*x - 2*a*b)/((a + b/x)^(3/2)*b^2*x^(3/2) - sqrt(a + b/x)*b^3
*sqrt(x)) - 3/2*a*log((sqrt(a + b/x)*sqrt(x) - sqrt(b))/(sqrt(a + b/x)*sqrt
(x) + sqrt(b)))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{7/2}\left(a+\frac{b}{x}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a + b/x)^(3/2)),x)
```

```
[Out] int(1/(x^(7/2)*(a + b/x)^(3/2)), x)
```

sympy [A] time = 38.02, size = 73, normalized size = 0.99

$$-\frac{3\sqrt{a}}{b^2\sqrt{x}\,\sqrt{1+\frac{b}{ax}}}+\frac{3a\,\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\,\sqrt{x}}\right)}{b^{\frac{5}{2}}}-\frac{1}{\sqrt{a}\,bx^{\frac{3}{2}}\sqrt{1+\frac{b}{ax}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(3/2)/x**(7/2),x)

[Out] -3*sqrt(a)/(b**2*sqrt(x)*sqrt(1 + b/(a*x))) + 3*a*asinh(sqrt(b)/(sqrt(a)*sq
rt(x)))/b**(5/2) - 1/(sqrt(a)*b*x**(3/2)*sqrt(1 + b/(a*x)))
```

3.1793 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{9/2}}\,dx$

Optimal. Leaf size=104

$$-\frac{15a^2 \tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{4b^{7/2}} + \frac{15a\sqrt{a+\frac{b}{x}}}{4b^3\sqrt{x}} - \frac{5\sqrt{a+\frac{b}{x}}}{2b^2x^{3/2}} + \frac{2}{bx^{5/2}\sqrt{a+\frac{b}{x}}}$$

```
[Out] -15/4*a^2*arctanh(b^(1/2)/(a+b/x)^(1/2)/x^(1/2))/b^(7/2)+2/b/x^(5/2)/(a+b/x
)^(1/2)-5/2*(a+b/x)^(1/2)/b^2/x^(3/2)+15/4*a*(a+b/x)^(1/2)/b^3/x^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {337, 288, 321, 217, 206}

$$-\frac{15a^2 \tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{4b^{7/2}} - \frac{5\sqrt{a+\frac{b}{x}}}{2b^2x^{3/2}} + \frac{15a\sqrt{a+\frac{b}{x}}}{4b^3\sqrt{x}} + \frac{2}{bx^{5/2}\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(3/2)*x^(9/2)),x]
```

```
[Out] 2/(b*Sqrt[a + b/x]*x^(5/2)) - (5*Sqrt[a + b/x])/(2*b^2*x^(3/2)) + (15*a*Sqr
t[a + b/x])/(4*b^3*Sqrt[x]) - (15*a^2*ArcTanh[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x
])])/(4*b^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 337

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, -Dist[k/c, Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1)
 + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0] &
& FractionQ[m]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{9/2}}\,dx = -\left(2\,\text{Subst}\left(\int \frac{x^6}{\left(a+bx^2\right)^{3/2}}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= \frac{2}{b\sqrt{a+\frac{b}{x}}\,x^{5/2}} - \frac{10\,\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{\sqrt{x}}\right)}{b}$$

$$= \frac{2}{b\sqrt{a+\frac{b}{x}}\,x^{5/2}} - \frac{5\sqrt{a+\frac{b}{x}}}{2b^2x^{3/2}} + \frac{(15a)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{\sqrt{x}}\right)}{2b^2}$$

$$= \frac{2}{b\sqrt{a+\frac{b}{x}}\,x^{5/2}} - \frac{5\sqrt{a+\frac{b}{x}}}{2b^2x^{3/2}} + \frac{15a\sqrt{a+\frac{b}{x}}}{4b^3\sqrt{x}} - \frac{\left(15a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{\sqrt{x}}\right)}{4b^3}$$

$$= \frac{2}{b\sqrt{a+\frac{b}{x}}\,x^{5/2}} - \frac{5\sqrt{a+\frac{b}{x}}}{2b^2x^{3/2}} + \frac{15a\sqrt{a+\frac{b}{x}}}{4b^3\sqrt{x}} - \frac{\left(15a^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{b}{x}}\,\sqrt{x}}\right)}{4b^3}$$

$$= \frac{2}{b\sqrt{a+\frac{b}{x}}\,x^{5/2}} - \frac{5\sqrt{a+\frac{b}{x}}}{2b^2x^{3/2}} + \frac{15a\sqrt{a+\frac{b}{x}}}{4b^3\sqrt{x}} - \frac{15a^2\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\,\sqrt{x}}\right)}{4b^{7/2}}$$

Mathematica [C] time = 0.02, size = 56, normalized size = 0.54

$$-\frac{2\sqrt{\frac{b}{ax}+1}\;{}_2F_1\left(\frac{3}{2},\frac{7}{2};\frac{9}{2};-\frac{b}{ax}\right)}{7ax^{7/2}\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(3/2)*x^(9/2)),x]
```

```
[Out] (-2*Sqrt[1 + b/(a*x)]*Hypergeometric2F1[3/2, 7/2, 9/2, -(b/(a*x))])/(7*a*Sq
rt[a + b/x]*x^(7/2))
```

fricas [A] time = 0.95, size = 217, normalized size = 2.09

$$\left[\frac{15\left(a^3x^3+a^2bx^2\right)\sqrt{b}\,\log\left(\frac{ax-2\,\sqrt{b}\,\sqrt{x}\,\sqrt{\frac{ax+b}{x}}+2\,b}{x}\right)+2\left(15\,a^2bx^2+5\,ab^2x-2\,b^3\right)\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}{8\left(ab^4x^3+b^5x^2\right)},\ \frac{15\left(a^3x^3+a^2bx^2\right)\sqrt{-b}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] [1/8*(15*(a^3*x^3 + a^2*b*x^2)*sqrt(b)*log((a*x - 2*sqrt(b)*sqrt(x)*sqrt((a
*x + b)/x) + 2*b)/x) + 2*(15*a^2*b*x^2 + 5*a*b^2*x - 2*b^3)*sqrt(x)*sqrt((a
*x + b)/x))/(a*b^4*x^3 + b^5*x^2), 1/4*(15*(a^3*x^3 + a^2*b*x^2)*sqrt(-b)*a
rctan(sqrt(-b)*sqrt(x)*sqrt((a*x + b)/x)/b) + (15*a^2*b*x^2 + 5*a*b^2*x - 2
*b^3)*sqrt(x)*sqrt((a*x + b)/x))/(a*b^4*x^3 + b^5*x^2)]
```

giac [A] time = 0.24, size = 80, normalized size = 0.77

$$\frac{15\,a^2\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{4\sqrt{-b}\,b^3}+\frac{2\,a^2}{\sqrt{ax+b}\,b^3}+\frac{7\,(ax+b)^{\frac{3}{2}}a^2-9\sqrt{ax+b}\,a^2b}{4\,a^2b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] 15/4*a^2*arctan(sqrt(a*x + b)/sqrt(-b))/(sqrt(-b)*b^3) + 2*a^2/(sqrt(a*x +
b)*b^3) + 1/4*(7*(a*x + b)^(3/2)*a^2 - 9*sqrt(a*x + b)*a^2*b)/(a^2*b^3*x^2)
```

maple [A] time = 0.02, size = 78, normalized size = 0.75

$$-\frac{\sqrt{\frac{ax+b}{x}}\,\left(15\sqrt{ax+b}\;a^2x^2\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)-15a^2\sqrt{b}\;x^2-5a\,b^{\frac{3}{2}}x+2b^{\frac{5}{2}}\right)}{4\,(ax+b)\,b^{\frac{7}{2}}x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(3/2)/x^(9/2),x)
```

```
[Out] -1/4*((a*x+b)/x)^(1/2)/x^(3/2)*(15*arctanh((a*x+b)^(1/2)/b^(1/2))*(a*x+b)^(
1/2)*x^2*a^2-5*b^(3/2)*x*a-15*a^2*x^2*b^(1/2)+2*b^(5/2))/(a*x+b)/b^(7/2)
```

maxima [A] time = 2.40, size = 143, normalized size = 1.38

$$\frac{15\left(a+\frac{b}{x}\right)^2a^2x^2-25\left(a+\frac{b}{x}\right)a^2bx+8\,a^2b^2}{4\left(\left(a+\frac{b}{x}\right)^{\frac{5}{2}}b^3x^{\frac{5}{2}}-2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^4x^{\frac{3}{2}}+\sqrt{a+\frac{b}{x}}\,b^5\sqrt{x}\right)}+\frac{15\,a^2\log\left(\frac{\sqrt{a+\frac{b}{x}}\,\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\,\sqrt{x}+\sqrt{b}}\right)}{8\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] 1/4*(15*(a + b/x)^2*a^2*x^2 - 25*(a + b/x)*a^2*b*x + 8*a^2*b^2)/((a + b/x)^
(5/2)*b^3*x^(5/2) - 2*(a + b/x)^(3/2)*b^4*x^(3/2) + sqrt(a + b/x)*b^5*sqrt(
x)) + 15/8*a^2*log((sqrt(a + b/x)*sqrt(x) - sqrt(b))/(sqrt(a + b/x)*sqrt(x)
 + sqrt(b)))/b^(7/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{9/2}\left(a+\frac{b}{x}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(9/2)*(a + b/x)^(3/2)),x)
```

```
[Out] int(1/(x^(9/2)*(a + b/x)^(3/2)), x)
```

sympy [**A**] time = 85.83, size = 107, normalized size = 1.03

$$\frac{15a^{\frac{3}{2}}}{4b^3\sqrt{x}\sqrt{1+\frac{b}{ax}}}+\frac{5\sqrt{a}}{4b^2x^{\frac{3}{2}}\sqrt{1+\frac{b}{ax}}}-\frac{15a^2\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right)}{4b^{\frac{7}{2}}}-\frac{1}{2\sqrt{a}\,bx^{\frac{5}{2}}\sqrt{1+\frac{b}{ax}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(3/2)/x**(9/2),x)
```

```
[Out] 15*a**(3/2)/(4*b**3*sqrt(x)*sqrt(1 + b/(a*x))) + 5*sqrt(a)/(4*b**2*x**(3/2)
*sqrt(1 + b/(a*x))) - 15*a**2*asinh(sqrt(b)/(sqrt(a)*sqrt(x)))/(4*b**(7/2))
 - 1/(2*sqrt(a)*b*x**(5/2)*sqrt(1 + b/(a*x)))
```

3.1794 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{11/2}}\,dx$

Optimal. Leaf size=130

$$\frac{35a^3 \tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{8b^{9/2}} - \frac{35a^2\sqrt{a+\frac{b}{x}}}{8b^4\sqrt{x}} + \frac{35a\sqrt{a+\frac{b}{x}}}{12b^3x^{3/2}} - \frac{7\sqrt{a+\frac{b}{x}}}{3b^2x^{5/2}} + \frac{2}{bx^{7/2}\sqrt{a+\frac{b}{x}}}$$

```
[Out] 35/8*a^3*arctanh(b^(1/2)/(a+b/x)^(1/2)/x^(1/2))/b^(9/2)+2/b/x^(7/2)/(a+b/x)
^(1/2)-7/3*(a+b/x)^(1/2)/b^2/x^(5/2)+35/12*a*(a+b/x)^(1/2)/b^3/x^(3/2)-35/8
*a^2*(a+b/x)^(1/2)/b^4/x^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {337, 288, 321, 217, 206}

$$-\frac{35a^2\sqrt{a+\frac{b}{x}}}{8b^4\sqrt{x}} + \frac{35a^3 \tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{8b^{9/2}} + \frac{35a\sqrt{a+\frac{b}{x}}}{12b^3x^{3/2}} - \frac{7\sqrt{a+\frac{b}{x}}}{3b^2x^{5/2}} + \frac{2}{bx^{7/2}\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(3/2)*x^(11/2)),x]
```

```
[Out] 2/(b*Sqrt[a + b/x]*x^(7/2)) - (7*Sqrt[a + b/x])/(3*b^2*x^(5/2)) + (35*a*Sqr
t[a + b/x])/(12*b^3*x^(3/2)) - (35*a^2*Sqrt[a + b/x])/(8*b^4*Sqrt[x]) + (35
*a^3*ArcTanh[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x])])/(8*b^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 337

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, -Dist[k/c, Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1)
 + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0] &
& FractionQ[m]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{11/2}}\,dx = -\left(2\,\text{Subst}\left(\int \frac{x^8}{\left(a+bx^2\right)^{3/2}}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= \frac{2}{b\sqrt{a+\frac{b}{x}}\,x^{7/2}} - \frac{14\,\text{Subst}\left(\int \frac{x^6}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{\sqrt{x}}\right)}{b}$$

$$= \frac{2}{b\sqrt{a+\frac{b}{x}}\,x^{7/2}} - \frac{7\sqrt{a+\frac{b}{x}}}{3b^2x^{5/2}} + \frac{(35a)\,\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{\sqrt{x}}\right)}{3b^2}$$

$$= \frac{2}{b\sqrt{a+\frac{b}{x}}\,x^{7/2}} - \frac{7\sqrt{a+\frac{b}{x}}}{3b^2x^{5/2}} + \frac{35a\sqrt{a+\frac{b}{x}}}{12b^3x^{3/2}} - \frac{\left(35a^2\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{\sqrt{x}}\right)}{4b^3}$$

$$= \frac{2}{b\sqrt{a+\frac{b}{x}}\,x^{7/2}} - \frac{7\sqrt{a+\frac{b}{x}}}{3b^2x^{5/2}} + \frac{35a\sqrt{a+\frac{b}{x}}}{12b^3x^{3/2}} - \frac{35a^2\sqrt{a+\frac{b}{x}}}{8b^4\sqrt{x}} + \frac{\left(35a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x\right.}{8b^4}$$

$$= \frac{2}{b\sqrt{a+\frac{b}{x}}\,x^{7/2}} - \frac{7\sqrt{a+\frac{b}{x}}}{3b^2x^{5/2}} + \frac{35a\sqrt{a+\frac{b}{x}}}{12b^3x^{3/2}} - \frac{35a^2\sqrt{a+\frac{b}{x}}}{8b^4\sqrt{x}} + \frac{\left(35a^3\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, -\right.}{8b^4}$$

$$= \frac{2}{b\sqrt{a+\frac{b}{x}}\,x^{7/2}} - \frac{7\sqrt{a+\frac{b}{x}}}{3b^2x^{5/2}} + \frac{35a\sqrt{a+\frac{b}{x}}}{12b^3x^{3/2}} - \frac{35a^2\sqrt{a+\frac{b}{x}}}{8b^4\sqrt{x}} + \frac{35a^3\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\,\sqrt{x}}\right)}{8b^{9/2}}$$

Mathematica [C] time = 0.02, size = 56, normalized size = 0.43

$$-\frac{2\sqrt{\frac{b}{ax}+1}\;{}_2F_1\left(\frac{3}{2},\frac{9}{2};\frac{11}{2};-\frac{b}{ax}\right)}{9ax^{9/2}\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(3/2)*x^(11/2)),x]
```

```
[Out] (-2*Sqrt[1 + b/(a*x)]*Hypergeometric2F1[3/2, 9/2, 11/2, -(b/(a*x))])/(9*a*S
qrt[a + b/x]*x^(9/2))
```

fricas [A] time = 0.62, size = 239, normalized size = 1.84

$$\left[\frac{105\left(a^4x^4+a^3bx^3\right)\sqrt{b}\,\log\left(\frac{ax+2\sqrt{b}\sqrt{x}\sqrt{\frac{ax+b}{x}}+2b}{x}\right) - 2\left(105\,a^3bx^3+35\,a^2b^2x^2-14\,ab^3x+8\,b^4\right)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{48\left(ab^5x^4+b^6x^3\right)}, -\frac{105\left(a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/x^(11/2),x, algorithm="fricas")
```

```
[Out] [1/48*(105*(a^4*x^4 + a^3*b*x^3)*sqrt(b)*log((a*x + 2*sqrt(b)*sqrt(x)*sqrt(
(a*x + b)/x) + 2*b)/x) - 2*(105*a^3*b*x^3 + 35*a^2*b^2*x^2 - 14*a*b^3*x + 8
*b^4)*sqrt(x)*sqrt((a*x + b)/x))/(a*b^5*x^4 + b^6*x^3), -1/24*(105*(a^4*x^4
 + a^3*b*x^3)*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)*sqrt((a*x + b)/x)/b) + (105*
a^3*b*x^3 + 35*a^2*b^2*x^2 - 14*a*b^3*x + 8*b^4)*sqrt(x)*sqrt((a*x + b)/x))
/(a*b^5*x^4 + b^6*x^3)]
```

giac [A] time = 0.26, size = 95, normalized size = 0.73

$$-\frac{35\,a^3\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{8\sqrt{-b}\,b^4}-\frac{2\,a^3}{\sqrt{ax+b}\,b^4}-\frac{57\,(ax+b)^{\frac{5}{2}}a^3-136\,(ax+b)^{\frac{3}{2}}a^3b+87\sqrt{ax+b}\,a^3b^2}{24\,a^3b^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/x^(11/2),x, algorithm="giac")
```

```
[Out] -35/8*a^3*arctan(sqrt(a*x + b)/sqrt(-b))/(sqrt(-b)*b^4) - 2*a^3/(sqrt(a*x +
 b)*b^4) - 1/24*(57*(a*x + b)^(5/2)*a^3 - 136*(a*x + b)^(3/2)*a^3*b + 87*sq
rt(a*x + b)*a^3*b^2)/(a^3*b^4*x^3)
```

maple [A] time = 0.02, size = 89, normalized size = 0.68

$$\frac{\sqrt{\frac{ax+b}{x}}\,\left(105\sqrt{ax+b}\;a^3x^3\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)-105a^3\sqrt{b}\;x^3-35a^2b^{\frac{3}{2}}x^2+14a\,b^{\frac{5}{2}}x-8b^{\frac{7}{2}}\right)}{24\,(ax+b)\,b^{\frac{9}{2}}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(3/2)/x^(11/2),x)
```

```
[Out] 1/24*((a*x+b)/x)^(1/2)/x^(5/2)*(105*(a*x+b)^(1/2)*arctanh((a*x+b)^(1/2)/b^(
1/2))*x^3*a^3+14*b^(5/2)*x*a-35*b^(3/2)*x^2*a^2-105*a^3*x^3*b^(1/2)-8*b^(7/
2))/(a*x+b)/b^(9/2)
```

maxima [A] time = 2.26, size = 181, normalized size = 1.39

$$-\frac{105\left(a+\frac{b}{x}\right)^{\frac{7}{2}}a^3x^3-280\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^3bx^2+231\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^3b^2x-48\,a^3b^3}{24\left(\left(a+\frac{b}{x}\right)^{\frac{7}{2}}b^4x^{\frac{7}{2}}-3\left(a+\frac{b}{x}\right)^{\frac{5}{2}}b^5x^{\frac{5}{2}}+3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^6x^{\frac{3}{2}}-\sqrt{a+\frac{b}{x}}\,b^7\sqrt{x}\right)}-\frac{35\,a^3\log\left(\frac{\sqrt{a+\frac{b}{x}}\,\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\,\sqrt{x}+\sqrt{b}}\right)}{16\,b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/x^(11/2),x, algorithm="maxima")
```

```
[Out] -1/24*(105*(a + b/x)^3*a^3*x^3 - 280*(a + b/x)^2*a^3*b*x^2 + 231*(a + b/x)*
a^3*b^2*x - 48*a^3*b^3)/((a + b/x)^(7/2)*b^4*x^(7/2) - 3*(a + b/x)^(5/2)*b^
5*x^(5/2) + 3*(a + b/x)^(3/2)*b^6*x^(3/2) - sqrt(a + b/x)*b^7*sqrt(x)) - 35
/16*a^3*log((sqrt(a + b/x)*sqrt(x) - sqrt(b))/(sqrt(a + b/x)*sqrt(x) + sqrt
(b)))/b^(9/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{11/2}\left(a+\frac{b}{x}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(11/2)*(a + b/x)^(3/2)),x)
[Out] int(1/(x^(11/2)*(a + b/x)^(3/2)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(3/2)/x**(11/2),x)
[Out] Timed out
```

3.1795 $\int \frac{x^{5/2}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$

Optimal. Leaf size=152

$$-\frac{512b^5}{21a^6x^{3/2}\left(a+\frac{b}{x}\right)^{3/2}}-\frac{256b^4}{7a^5\sqrt{x}\left(a+\frac{b}{x}\right)^{3/2}}-\frac{64b^3\sqrt{x}}{7a^4\left(a+\frac{b}{x}\right)^{3/2}}+\frac{32b^2x^{3/2}}{21a^3\left(a+\frac{b}{x}\right)^{3/2}}-\frac{4bx^{5/2}}{7a^2\left(a+\frac{b}{x}\right)^{3/2}}+\frac{2x^{7/2}}{7a\left(a+\frac{b}{x}\right)^{3/2}}$$

```
[Out] -512/21*b^5/a^6/(a+b/x)^(3/2)/x^(3/2)+32/21*b^2*x^(3/2)/a^3/(a+b/x)^(3/2)-4
/7*b*x^(5/2)/a^2/(a+b/x)^(3/2)+2/7*x^(7/2)/a/(a+b/x)^(3/2)-256/7*b^4/a^5/(a
+b/x)^(3/2)/x^(1/2)-64/7*b^3*x^(1/2)/a^4/(a+b/x)^(3/2)
```

Rubi [A] time = 0.06, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {271, 264}

$$-\frac{512b^5}{21a^6x^{3/2}\left(a+\frac{b}{x}\right)^{3/2}}+\frac{32b^2x^{3/2}}{21a^3\left(a+\frac{b}{x}\right)^{3/2}}-\frac{256b^4}{7a^5\sqrt{x}\left(a+\frac{b}{x}\right)^{3/2}}-\frac{64b^3\sqrt{x}}{7a^4\left(a+\frac{b}{x}\right)^{3/2}}-\frac{4bx^{5/2}}{7a^2\left(a+\frac{b}{x}\right)^{3/2}}+\frac{2x^{7/2}}{7a\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b/x)^(5/2),x]
```

```
[Out] (-512*b^5)/(21*a^6*(a + b/x)^(3/2)*x^(3/2)) - (256*b^4)/(7*a^5*(a + b/x)^(3
/2)*Sqrt[x]) - (64*b^3*Sqrt[x])/(7*a^4*(a + b/x)^(3/2)) + (32*b^2*x^(3/2))/
(21*a^3*(a + b/x)^(3/2)) - (4*b*x^(5/2))/(7*a^2*(a + b/x)^(3/2)) + (2*x^(7/
2))/(7*a*(a + b/x)^(3/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^{5/2}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{2x^{7/2}}{7a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{(10b)\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx}{7a}$$

$$= -\frac{4bx^{5/2}}{7a^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{2x^{7/2}}{7a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{\left(16b^2\right)\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx}{7a^2}$$

$$= \frac{32b^2x^{3/2}}{21a^3\left(a+\frac{b}{x}\right)^{3/2}} - \frac{4bx^{5/2}}{7a^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{2x^{7/2}}{7a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{\left(32b^3\right)\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\sqrt{x}}\,dx}{7a^3}$$

$$= -\frac{64b^3\sqrt{x}}{7a^4\left(a+\frac{b}{x}\right)^{3/2}} + \frac{32b^2x^{3/2}}{21a^3\left(a+\frac{b}{x}\right)^{3/2}} - \frac{4bx^{5/2}}{7a^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{2x^{7/2}}{7a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{\left(128b^4\right)\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^{3/2}}\,dx}{7a^4}$$

$$= -\frac{256b^4}{7a^5\left(a+\frac{b}{x}\right)^{3/2}\sqrt{x}} - \frac{64b^3\sqrt{x}}{7a^4\left(a+\frac{b}{x}\right)^{3/2}} + \frac{32b^2x^{3/2}}{21a^3\left(a+\frac{b}{x}\right)^{3/2}} - \frac{4bx^{5/2}}{7a^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{2x^{7/2}}{7a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{\left(2\right.}{}$$

$$= -\frac{512b^5}{21a^6\left(a+\frac{b}{x}\right)^{3/2}x^{3/2}} - \frac{256b^4}{7a^5\left(a+\frac{b}{x}\right)^{3/2}\sqrt{x}} - \frac{64b^3\sqrt{x}}{7a^4\left(a+\frac{b}{x}\right)^{3/2}} + \frac{32b^2x^{3/2}}{21a^3\left(a+\frac{b}{x}\right)^{3/2}} - \frac{4bx^{5/2}}{7a^2\left(a+\frac{b}{x}\right)^{3/2}}$$

Mathematica [A] time = 0.02, size = 82, normalized size = 0.54

$$\frac{2\left(3a^5x^5 - 6a^4bx^4 + 16a^3b^2x^3 - 96a^2b^3x^2 - 384ab^4x - 256b^5\right)}{21a^6\sqrt{x}\sqrt{a+\frac{b}{x}}\,(ax+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b/x)^(5/2),x]
```

```
[Out] (2*(-256*b^5 - 384*a*b^4*x - 96*a^2*b^3*x^2 + 16*a^3*b^2*x^3 - 6*a^4*b*x^4
+ 3*a^5*x^5))/(21*a^6*Sqrt[a + b/x]*Sqrt[x]*(b + a*x))
```

fricas [A] time = 0.87, size = 92, normalized size = 0.61

$$\frac{2\left(3\,a^5x^5 - 6\,a^4bx^4 + 16\,a^3b^2x^3 - 96\,a^2b^3x^2 - 384\,ab^4x - 256\,b^5\right)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{21\left(a^8x^2 + 2\,a^7bx + a^6b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(a+b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*a^5*x^5 - 6*a^4*b*x^4 + 16*a^3*b^2*x^3 - 96*a^2*b^3*x^2 - 384*a*b^4
*x - 256*b^5)*sqrt(x)*sqrt((a*x + b)/x)/(a^8*x^2 + 2*a^7*b*x + a^6*b^2)
```

giac [A] time = 0.19, size = 98, normalized size = 0.64

$$\frac{512\,b^{\frac{7}{2}}}{21\,a^6} - \frac{2\left(15\,(ax+b)b^4 - b^5\right)}{3\,(ax+b)^{\frac{3}{2}}a^6} + \frac{2\left(3\,(ax+b)^{\frac{7}{2}}a^{36} - 21\,(ax+b)^{\frac{5}{2}}a^{36}b + 70\,(ax+b)^{\frac{3}{2}}a^{36}b^2 - 210\sqrt{ax+b}\,a^{36}b^3\right)}{21\,a^{42}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(a+b/x)^(5/2),x, algorithm="giac")
```

```
[Out] 512/21*b^(7/2)/a^6 - 2/3*(15*(a*x + b)*b^4 - b^5)/((a*x + b)^(3/2)*a^6) + 2
/21*(3*(a*x + b)^(7/2)*a^36 - 21*(a*x + b)^(5/2)*a^36*b + 70*(a*x + b)^(3/2
)*a^36*b^2 - 210*sqrt(a*x + b)*a^36*b^3)/a^42
```

maple [A] time = 0.01, size = 77, normalized size = 0.51

$$\frac{2\,(ax+b)\left(3a^5x^5-6a^4b\,x^4+16a^3b^2x^3-96a^2b^3x^2-384a\,b^4x-256b^5\right)}{21\left(\frac{ax+b}{x}\right)^{\frac{5}{2}}a^6x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a+b/x)^(5/2),x)
```

```
[Out] 2/21*(a*x+b)*(3*a^5*x^5-6*a^4*b*x^4+16*a^3*b^2*x^3-96*a^2*b^3*x^2-384*a*b^4
*x-256*b^5)/a^6/x^(5/2)/((a*x+b)/x)^(5/2)
```

maxima [A] time = 1.13, size = 106, normalized size = 0.70

$$\frac{2\left(3\left(a+\frac{b}{x}\right)^{\frac{7}{2}}x^{\frac{7}{2}}-21\left(a+\frac{b}{x}\right)^{\frac{5}{2}}bx^{\frac{5}{2}}+70\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^2x^{\frac{3}{2}}-210\sqrt{a+\frac{b}{x}}\,b^3\sqrt{x}\right)}{21\,a^6}-\frac{2\left(15\left(a+\frac{b}{x}\right)b^4x-b^5\right)}{3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^6x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(a+b/x)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/21*(3*(a + b/x)^(7/2)*x^(7/2) - 21*(a + b/x)^(5/2)*b*x^(5/2) + 70*(a + b/
x)^(3/2)*b^2*x^(3/2) - 210*sqrt(a + b/x)*b^3*sqrt(x))/a^6 - 2/3*(15*(a + b/
x)*b^4*x - b^5)/((a + b/x)^(3/2)*a^6*x^(3/2))
```

mupad [B] time = 1.67, size = 78, normalized size = 0.51

$$-\frac{2\sqrt{x}\,\sqrt{\frac{b+a\,x}{x}}\,\left(-3\,a^5\,x^5+6\,a^4\,b\,x^4-16\,a^3\,b^2\,x^3+96\,a^2\,b^3\,x^2+384\,a\,b^4\,x+256\,b^5\right)}{21\,a^6\,(b+a\,x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b/x)^(5/2),x)
```

```
[Out] -(2*x^(1/2)*((b + a*x)/x)^(1/2)*(256*b^5 - 3*a^5*x^5 + 6*a^4*b*x^4 + 96*a^2
*b^3*x^2 - 16*a^3*b^2*x^3 + 384*a*b^4*x))/(21*a^6*(b + a*x)^2)
```

sympy [B] time = 14.65, size = 799, normalized size = 5.26

$$\frac{6a^8b^{\frac{51}{2}}x^8\sqrt{\frac{ax}{b}+1}}{21a^{11}b^{25}x^5+105a^{10}b^{26}x^4+210a^9b^{27}x^3+210a^8b^{28}x^2+105a^7b^{29}x+21a^6b^{30}}+\frac{6}{21a^{11}b^{25}x^5+105a^{10}b^{26}x^4+21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(a+b/x)**(5/2),x)
```

```
[Out] 6*a**8*b**(51/2)*x**8*sqrt(a*x/b + 1)/(21*a**11*b**25*x**5 + 105*a**10*b**2
6*x**4 + 210*a**9*b**27*x**3 + 210*a**8*b**28*x**2 + 105*a**7*b**29*x + 21*
a**6*b**30) + 6*a**7*b**(53/2)*x**7*sqrt(a*x/b + 1)/(21*a**11*b**25*x**5 +
```

```
105*a**10*b**26*x**4 + 210*a**9*b**27*x**3 + 210*a**8*b**28*x**2 + 105*a**7
*b**29*x + 21*a**6*b**30) + 14*a**6*b**(55/2)*x**6*sqrt(a*x/b + 1)/(21*a**1
1*b**25*x**5 + 105*a**10*b**26*x**4 + 210*a**9*b**27*x**3 + 210*a**8*b**28*
x**2 + 105*a**7*b**29*x + 21*a**6*b**30) - 126*a**5*b**(57/2)*x**5*sqrt(a*x
/b + 1)/(21*a**11*b**25*x**5 + 105*a**10*b**26*x**4 + 210*a**9*b**27*x**3 +
 210*a**8*b**28*x**2 + 105*a**7*b**29*x + 21*a**6*b**30) - 1260*a**4*b**(59
/2)*x**4*sqrt(a*x/b + 1)/(21*a**11*b**25*x**5 + 105*a**10*b**26*x**4 + 210*
a**9*b**27*x**3 + 210*a**8*b**28*x**2 + 105*a**7*b**29*x + 21*a**6*b**30) -
 3360*a**3*b**(61/2)*x**3*sqrt(a*x/b + 1)/(21*a**11*b**25*x**5 + 105*a**10*
b**26*x**4 + 210*a**9*b**27*x**3 + 210*a**8*b**28*x**2 + 105*a**7*b**29*x +
 21*a**6*b**30) - 4032*a**2*b**(63/2)*x**2*sqrt(a*x/b + 1)/(21*a**11*b**25*
x**5 + 105*a**10*b**26*x**4 + 210*a**9*b**27*x**3 + 210*a**8*b**28*x**2 + 1
05*a**7*b**29*x + 21*a**6*b**30) - 2304*a*b**(65/2)*x*sqrt(a*x/b + 1)/(21*a
**11*b**25*x**5 + 105*a**10*b**26*x**4 + 210*a**9*b**27*x**3 + 210*a**8*b**
28*x**2 + 105*a**7*b**29*x + 21*a**6*b**30) - 512*b**(67/2)*sqrt(a*x/b + 1)
/(21*a**11*b**25*x**5 + 105*a**10*b**26*x**4 + 210*a**9*b**27*x**3 + 210*a*
*8*b**28*x**2 + 105*a**7*b**29*x + 21*a**6*b**30)
```

3.1796 $\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$

Optimal. Leaf size=126

$$\frac{256b^4}{15a^5x^{3/2}\left(a+\frac{b}{x}\right)^{3/2}}+\frac{128b^3}{5a^4\sqrt{x}\left(a+\frac{b}{x}\right)^{3/2}}+\frac{32b^2\sqrt{x}}{5a^3\left(a+\frac{b}{x}\right)^{3/2}}-\frac{16bx^{3/2}}{15a^2\left(a+\frac{b}{x}\right)^{3/2}}+\frac{2x^{5/2}}{5a\left(a+\frac{b}{x}\right)^{3/2}}$$

```
[Out] 256/15*b^4/a^5/(a+b/x)^(3/2)/x^(3/2)-16/15*b*x^(3/2)/a^2/(a+b/x)^(3/2)+2/5*
x^(5/2)/a/(a+b/x)^(3/2)+128/5*b^3/a^4/(a+b/x)^(3/2)/x^(1/2)+32/5*b^2*x^(1/2
)/a^3/(a+b/x)^(3/2)
```

Rubi [A] time = 0.05, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {271, 264}

$$\frac{256b^4}{15a^5x^{3/2}\left(a+\frac{b}{x}\right)^{3/2}}+\frac{128b^3}{5a^4\sqrt{x}\left(a+\frac{b}{x}\right)^{3/2}}+\frac{32b^2\sqrt{x}}{5a^3\left(a+\frac{b}{x}\right)^{3/2}}-\frac{16bx^{3/2}}{15a^2\left(a+\frac{b}{x}\right)^{3/2}}+\frac{2x^{5/2}}{5a\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b/x)^(5/2),x]
```

```
[Out] (256*b^4)/(15*a^5*(a + b/x)^(3/2)*x^(3/2)) + (128*b^3)/(5*a^4*(a + b/x)^(3/
2)*Sqrt[x]) + (32*b^2*Sqrt[x])/(5*a^3*(a + b/x)^(3/2)) - (16*b*x^(3/2))/(15
*a^2*(a + b/x)^(3/2)) + (2*x^(5/2))/(5*a*(a + b/x)^(3/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{2x^{5/2}}{5a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{(8b)\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx}{5a}$$

$$= -\frac{16bx^{3/2}}{15a^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{2x^{5/2}}{5a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{\left(16b^2\right)\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\sqrt{x}}\,dx}{5a^2}$$

$$= \frac{32b^2\sqrt{x}}{5a^3\left(a+\frac{b}{x}\right)^{3/2}} - \frac{16bx^{3/2}}{15a^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{2x^{5/2}}{5a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{\left(64b^3\right)\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^{3/2}}\,dx}{5a^3}$$

$$= \frac{128b^3}{5a^4\left(a+\frac{b}{x}\right)^{3/2}\sqrt{x}} + \frac{32b^2\sqrt{x}}{5a^3\left(a+\frac{b}{x}\right)^{3/2}} - \frac{16bx^{3/2}}{15a^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{2x^{5/2}}{5a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{\left(128b^4\right)\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^{5/2}}}{5a^4}$$

$$= \frac{256b^4}{15a^5\left(a+\frac{b}{x}\right)^{3/2}x^{3/2}} + \frac{128b^3}{5a^4\left(a+\frac{b}{x}\right)^{3/2}\sqrt{x}} + \frac{32b^2\sqrt{x}}{5a^3\left(a+\frac{b}{x}\right)^{3/2}} - \frac{16bx^{3/2}}{15a^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{2x^{5/2}}{5a\left(a+\frac{b}{x}\right)^{3/2}}$$

Mathematica [A] time = 0.03, size = 71, normalized size = 0.56

$$\frac{2\sqrt{x}\sqrt{a+\frac{b}{x}}\left(3a^4x^4 - 8a^3bx^3 + 48a^2b^2x^2 + 192ab^3x + 128b^4\right)}{15a^5(ax+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b/x)^(5/2),x]
```

```
[Out] (2*Sqrt[a + b/x]*Sqrt[x]*(128*b^4 + 192*a*b^3*x + 48*a^2*b^2*x^2 - 8*a^3*b*
x^3 + 3*a^4*x^4))/(15*a^5*(b + a*x)^2)
```

fricas [A] time = 0.92, size = 81, normalized size = 0.64

$$\frac{2\left(3\,a^4x^4 - 8\,a^3bx^3 + 48\,a^2b^2x^2 + 192\,ab^3x + 128\,b^4\right)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{15\left(a^7x^2 + 2\,a^6bx + a^5b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(a+b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*a^4*x^4 - 8*a^3*b*x^3 + 48*a^2*b^2*x^2 + 192*a*b^3*x + 128*b^4)*sqr
t(x)*sqrt((a*x + b)/x)/(a^7*x^2 + 2*a^6*b*x + a^5*b^2)
```

giac [A] time = 0.18, size = 83, normalized size = 0.66

$$-\frac{256\,b^{\frac{5}{2}}}{15\,a^5} + \frac{2\left(12\,(ax+b)b^3 - b^4\right)}{3\,(ax+b)^{\frac{3}{2}}a^5} + \frac{2\left(3\,(ax+b)^{\frac{5}{2}}a^{20} - 20\,(ax+b)^{\frac{3}{2}}a^{20}b + 90\sqrt{ax+b}\,a^{20}b^2\right)}{15\,a^{25}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(a+b/x)^(5/2),x, algorithm="giac")
```

```
[Out] -256/15*b^(5/2)/a^5 + 2/3*(12*(a*x + b)*b^3 - b^4)/((a*x + b)^(3/2)*a^5) +
2/15*(3*(a*x + b)^(5/2)*a^20 - 20*(a*x + b)^(3/2)*a^20*b + 90*sqrt(a*x + b)
*a^20*b^2)/a^25
```

maple [A] time = 0.00, size = 66, normalized size = 0.52

$$\frac{2\,(ax+b)\left(3a^4x^4-8a^3x^3b+48a^2x^2b^2+192ax\,b^3+128b^4\right)}{15\left(\frac{ax+b}{x}\right)^{\frac{5}{2}}a^5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a+b/x)^(5/2),x)
```

```
[Out] 2/15*(a*x+b)*(3*a^4*x^4-8*a^3*b*x^3+48*a^2*b^2*x^2+192*a*b^3*x+128*b^4)/a^5
/x^(5/2)/((a*x+b)/x)^(5/2)
```

maxima [A] time = 1.04, size = 89, normalized size = 0.71

$$\frac{2\left(3\left(a+\frac{b}{x}\right)^{\frac{5}{2}}x^{\frac{5}{2}}-20\left(a+\frac{b}{x}\right)^{\frac{3}{2}}bx^{\frac{3}{2}}+90\sqrt{a+\frac{b}{x}}\,b^2\sqrt{x}\right)}{15\,a^5}+\frac{2\left(12\left(a+\frac{b}{x}\right)b^3x-b^4\right)}{3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^5x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(a+b/x)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*(a + b/x)^(5/2)*x^(5/2) - 20*(a + b/x)^(3/2)*b*x^(3/2) + 90*sqrt(a
+ b/x)*b^2*sqrt(x))/a^5 + 2/3*(12*(a + b/x)*b^3*x - b^4)/((a + b/x)^(3/2)*a
^5*x^(3/2))
```

mupad [B] time = 1.57, size = 81, normalized size = 0.64

$$\frac{\sqrt{a+\frac{b}{x}}\left(\frac{2\,x^{9/2}}{5\,a^3}-\frac{16\,b\,x^{7/2}}{15\,a^4}+\frac{32\,b^2\,x^{5/2}}{5\,a^5}+\frac{128\,b^3\,x^{3/2}}{5\,a^6}+\frac{256\,b^4\,\sqrt{x}}{15\,a^7}\right)}{x^2+\frac{b^2}{a^2}+\frac{2\,b\,x}{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b/x)^(5/2),x)
```

```
[Out] ((a + b/x)^(1/2)*((2*x^(9/2))/(5*a^3) - (16*b*x^(7/2))/(15*a^4) + (32*b^2*x
^(5/2))/(5*a^5) + (128*b^3*x^(3/2))/(5*a^6) + (256*b^4*x^(1/2))/(15*a^7)))/
(x^2 + b^2/a^2 + (2*b*x)/a)
```

sympy [B] time = 8.80, size = 536, normalized size = 4.25

$$\frac{6a^6b^{\frac{33}{2}}x^6\sqrt{\frac{ax}{b}+1}}{15a^9b^{16}x^4+60a^8b^{17}x^3+90a^7b^{18}x^2+60a^6b^{19}x+15a^5b^{20}}-\frac{4a^5b^{\frac{35}{2}}x^5\sqrt{\frac{ax}{b}+1}}{15a^9b^{16}x^4+60a^8b^{17}x^3+90a^7b^{18}x^2+60a^6b^{19}x+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(a+b/x)**(5/2),x)
```

```
[Out] 6*a**6*b**(33/2)*x**6*sqrt(a*x/b + 1)/(15*a**9*b**16*x**4 + 60*a**8*b**17*x
**3 + 90*a**7*b**18*x**2 + 60*a**6*b**19*x + 15*a**5*b**20) - 4*a**5*b**(35
/2)*x**5*sqrt(a*x/b + 1)/(15*a**9*b**16*x**4 + 60*a**8*b**17*x**3 + 90*a**7
*b**18*x**2 + 60*a**6*b**19*x + 15*a**5*b**20) + 70*a**4*b**(37/2)*x**4*sqr
t(a*x/b + 1)/(15*a**9*b**16*x**4 + 60*a**8*b**17*x**3 + 90*a**7*b**18*x**2
```

```
+ 60*a**6*b**19*x + 15*a**5*b**20) + 560*a**3*b**(39/2)*x**3*sqrt(a*x/b + 1
)/(15*a**9*b**16*x**4 + 60*a**8*b**17*x**3 + 90*a**7*b**18*x**2 + 60*a**6*b
**19*x + 15*a**5*b**20) + 1120*a**2*b**(41/2)*x**2*sqrt(a*x/b + 1)/(15*a**9
*b**16*x**4 + 60*a**8*b**17*x**3 + 90*a**7*b**18*x**2 + 60*a**6*b**19*x + 1
5*a**5*b**20) + 896*a*b**(43/2)*x*sqrt(a*x/b + 1)/(15*a**9*b**16*x**4 + 60*
a**8*b**17*x**3 + 90*a**7*b**18*x**2 + 60*a**6*b**19*x + 15*a**5*b**20) + 2
56*b**(45/2)*sqrt(a*x/b + 1)/(15*a**9*b**16*x**4 + 60*a**8*b**17*x**3 + 90*
a**7*b**18*x**2 + 60*a**6*b**19*x + 15*a**5*b**20)
```

3.1797 $\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$

Optimal. Leaf size=96

$$-\frac{32b^3}{3a^4x^{3/2}\left(a+\frac{b}{x}\right)^{3/2}}-\frac{16b^2}{a^3\sqrt{x}\left(a+\frac{b}{x}\right)^{3/2}}-\frac{4b\sqrt{x}}{a^2\left(a+\frac{b}{x}\right)^{3/2}}+\frac{2x^{3/2}}{3a\left(a+\frac{b}{x}\right)^{3/2}}$$

```
[Out] -32/3*b^3/a^4/(a+b/x)^(3/2)/x^(3/2)+2/3*x^(3/2)/a/(a+b/x)^(3/2)-16*b^2/a^3/
(a+b/x)^(3/2)/x^(1/2)-4*b*x^(1/2)/a^2/(a+b/x)^(3/2)
```

Rubi [A] time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {271, 264}

$$-\frac{32b^3}{3a^4x^{3/2}\left(a+\frac{b}{x}\right)^{3/2}}-\frac{16b^2}{a^3\sqrt{x}\left(a+\frac{b}{x}\right)^{3/2}}-\frac{4b\sqrt{x}}{a^2\left(a+\frac{b}{x}\right)^{3/2}}+\frac{2x^{3/2}}{3a\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b/x)^(5/2),x]
```

```
[Out] (-32*b^3)/(3*a^4*(a + b/x)^(3/2)*x^(3/2)) - (16*b^2)/(a^3*(a + b/x)^(3/2)*S
qrt[x]) - (4*b*Sqrt[x])/(a^2*(a + b/x)^(3/2)) + (2*x^(3/2))/(3*a*(a + b/x)^
(3/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{2x^{3/2}}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{(2b)\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\sqrt{x}}\,dx}{a}$$

$$= -\frac{4b\sqrt{x}}{a^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{2x^{3/2}}{3a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{\left(8b^2\right)\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^{3/2}}\,dx}{a^2}$$

$$= -\frac{16b^2}{a^3\left(a+\frac{b}{x}\right)^{3/2}\sqrt{x}} - \frac{4b\sqrt{x}}{a^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{2x^{3/2}}{3a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{\left(16b^3\right)\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^{5/2}}\,dx}{a^3}$$

$$= -\frac{32b^3}{3a^4\left(a+\frac{b}{x}\right)^{3/2}x^{3/2}} - \frac{16b^2}{a^3\left(a+\frac{b}{x}\right)^{3/2}\sqrt{x}} - \frac{4b\sqrt{x}}{a^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{2x^{3/2}}{3a\left(a+\frac{b}{x}\right)^{3/2}}$$

Mathematica [A] time = 0.02, size = 59, normalized size = 0.61

$$\frac{2\left(a^3x^3 - 6a^2bx^2 - 24ab^2x - 16b^3\right)}{3a^4\sqrt{x}\sqrt{a+\frac{b}{x}}\,(ax+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b/x)^(5/2),x]
```

```
[Out] (2*(-16*b^3 - 24*a*b^2*x - 6*a^2*b*x^2 + a^3*x^3))/(3*a^4*Sqrt[a + b/x]*Sqr
t[x]*(b + a*x))
```

fricas [A] time = 1.20, size = 69, normalized size = 0.72

$$\frac{2\left(a^3x^3 - 6\,a^2bx^2 - 24\,ab^2x - 16\,b^3\right)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{3\left(a^6x^2 + 2\,a^5bx + a^4b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(a+b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(a^3*x^3 - 6*a^2*b*x^2 - 24*a*b^2*x - 16*b^3)*sqrt(x)*sqrt((a*x + b)/x)
/(a^6*x^2 + 2*a^5*b*x + a^4*b^2)
```

giac [A] time = 0.19, size = 67, normalized size = 0.70

$$\frac{32\,b^{\frac{3}{2}}}{3\,a^4} - \frac{2\left(9\,(ax+b)b^2 - b^3\right)}{3\,(ax+b)^{\frac{3}{2}}a^4} + \frac{2\left((ax+b)^{\frac{3}{2}}a^8 - 9\sqrt{ax+b}\,a^8b\right)}{3\,a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(a+b/x)^(5/2),x, algorithm="giac")
```

```
[Out] 32/3*b^(3/2)/a^4 - 2/3*(9*(a*x + b)*b^2 - b^3)/((a*x + b)^(3/2)*a^4) + 2/3*
((a*x + b)^(3/2)*a^8 - 9*sqrt(a*x + b)*a^8*b)/a^12
```

maple [A] time = 0.00, size = 54, normalized size = 0.56

$$\frac{2\,(ax+b)\left(a^3x^3-6a^2b\,x^2-24a\,b^2x-16b^3\right)}{3\left(\frac{ax+b}{x}\right)^{\frac{5}{2}}a^4x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a+b/x)^(5/2),x)
```

```
[Out] 2/3*(a*x+b)*(a^3*x^3-6*a^2*b*x^2-24*a*b^2*x-16*b^3)/a^4/x^(5/2)/((a*x+b)/x)
^(5/2)
```

maxima [A] time = 0.97, size = 71, normalized size = 0.74

$$\frac{2\left(\left(a+\frac{b}{x}\right)^{\frac{3}{2}}x^{\frac{3}{2}}-9\sqrt{a+\frac{b}{x}}\,b\sqrt{x}\right)}{3\,a^4}-\frac{2\left(9\left(a+\frac{b}{x}\right)b^2x-b^3\right)}{3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^4x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(a+b/x)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*((a + b/x)^(3/2)*x^(3/2) - 9*sqrt(a + b/x)*b*sqrt(x))/a^4 - 2/3*(9*(a +
 b/x)*b^2*x - b^3)/((a + b/x)^(3/2)*a^4*x^(3/2))
```

mupad [B] time = 1.52, size = 71, normalized size = 0.74

$$-\frac{\sqrt{a+\frac{b}{x}}\,\left(\frac{4\,b\,x^{5/2}}{a^4}-\frac{2\,x^{7/2}}{3\,a^3}+\frac{16\,b^2\,x^{3/2}}{a^5}+\frac{32\,b^3\,\sqrt{x}}{3\,a^6}\right)}{x^2+\frac{b^2}{a^2}+\frac{2\,b\,x}{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b/x)^(5/2),x)
```

```
[Out] -((a + b/x)^(1/2)*((4*b*x^(5/2))/a^4 - (2*x^(7/2))/(3*a^3) + (16*b^2*x^(3/2
))/a^5 + (32*b^3*x^(1/2))/(3*a^6)))/(x^2 + b^2/a^2 + (2*b*x)/a)
```

sympy [B] time = 6.45, size = 320, normalized size = 3.33

$$\frac{2a^4b^{\frac{19}{2}}x^4\sqrt{\frac{ax}{b}+1}}{3a^7b^9x^3+9a^6b^{10}x^2+9a^5b^{11}x+3a^4b^{12}}-\frac{10a^3b^{\frac{21}{2}}x^3\sqrt{\frac{ax}{b}+1}}{3a^7b^9x^3+9a^6b^{10}x^2+9a^5b^{11}x+3a^4b^{12}}-\frac{60a^2b^{\frac{23}{2}}x^2\sqrt{\frac{ax}{b}+}}{3a^7b^9x^3+9a^6b^{10}x^2+9a^5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(a+b/x)**(5/2),x)
```

```
[Out] 2*a**4*b**(19/2)*x**4*sqrt(a*x/b + 1)/(3*a**7*b**9*x**3 + 9*a**6*b**10*x**2
 + 9*a**5*b**11*x + 3*a**4*b**12) - 10*a**3*b**(21/2)*x**3*sqrt(a*x/b + 1)/
(3*a**7*b**9*x**3 + 9*a**6*b**10*x**2 + 9*a**5*b**11*x + 3*a**4*b**12) - 60
*a**2*b**(23/2)*x**2*sqrt(a*x/b + 1)/(3*a**7*b**9*x**3 + 9*a**6*b**10*x**2
 + 9*a**5*b**11*x + 3*a**4*b**12) - 80*a*b**(25/2)*x*sqrt(a*x/b + 1)/(3*a**7
*b**9*x**3 + 9*a**6*b**10*x**2 + 9*a**5*b**11*x + 3*a**4*b**12) - 32*b**(27
/2)*sqrt(a*x/b + 1)/(3*a**7*b**9*x**3 + 9*a**6*b**10*x**2 + 9*a**5*b**11*x
 + 3*a**4*b**12)
```

3.1798 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\sqrt{x}}\,dx$

Optimal. Leaf size=70

$$\frac{16b^2}{3a^3x^{3/2}\left(a+\frac{b}{x}\right)^{3/2}}+\frac{8b}{a^2\sqrt{x}\left(a+\frac{b}{x}\right)^{3/2}}+\frac{2\sqrt{x}}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

```
[Out] 16/3*b^2/a^3/(a+b/x)^(3/2)/x^(3/2)+8*b/a^2/(a+b/x)^(3/2)/x^(1/2)+2*x^(1/2)/
a/(a+b/x)^(3/2)
```

Rubi [A] time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {271, 264}

$$\frac{16b^2}{3a^3x^{3/2}\left(a+\frac{b}{x}\right)^{3/2}}+\frac{8b}{a^2\sqrt{x}\left(a+\frac{b}{x}\right)^{3/2}}+\frac{2\sqrt{x}}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(5/2)*Sqrt[x]),x]
```

```
[Out] (16*b^2)/(3*a^3*(a + b/x)^(3/2)*x^(3/2)) + (8*b)/(a^2*(a + b/x)^(3/2)*Sqrt[
x]) + (2*Sqrt[x])/(a*(a + b/x)^(3/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\sqrt{x}}\,dx &= \frac{2\sqrt{x}}{a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{(4b)\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^{3/2}}\,dx}{a}\\ &= \frac{8b}{a^2\left(a+\frac{b}{x}\right)^{3/2}\sqrt{x}}+\frac{2\sqrt{x}}{a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{\left(8b^2\right)\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^{5/2}}\,dx}{a^2}\\ &= \frac{16b^2}{3a^3\left(a+\frac{b}{x}\right)^{3/2}x^{3/2}}+\frac{8b}{a^2\left(a+\frac{b}{x}\right)^{3/2}\sqrt{x}}+\frac{2\sqrt{x}}{a\left(a+\frac{b}{x}\right)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 49, normalized size = 0.70

$$\frac{2\sqrt{x}\sqrt{a+\frac{b}{x}}\left(3a^2x^2+12abx+8b^2\right)}{3a^3(ax+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(5/2)*Sqrt[x]),x]
```

```
[Out] (2*Sqrt[a + b/x]*Sqrt[x]*(8*b^2 + 12*a*b*x + 3*a^2*x^2))/(3*a^3*(b + a*x)^2
)
```

fricas [A] time = 0.94, size = 59, normalized size = 0.84

$$\frac{2\left(3\,a^2x^2+12\,abx+8\,b^2\right)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{3\left(a^5x^2+2\,a^4bx+a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*a^2*x^2 + 12*a*b*x + 8*b^2)*sqrt(x)*sqrt((a*x + b)/x)/(a^5*x^2 + 2*a
^4*b*x + a^3*b^2)
```

giac [A] time = 0.20, size = 47, normalized size = 0.67

$$\frac{2\sqrt{ax+b}}{a^3}-\frac{16\sqrt{b}}{3\,a^3}+\frac{2\left(6\,(ax+b)b-b^2\right)}{3\,(ax+b)^{\frac{3}{2}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(a*x + b)/a^3 - 16/3*sqrt(b)/a^3 + 2/3*(6*(a*x + b)*b - b^2)/((a*x +
b)^(3/2)*a^3)
```

maple [A] time = 0.01, size = 44, normalized size = 0.63

$$\frac{2\,(ax+b)\left(3a^2x^2+12abx+8b^2\right)}{3\left(\frac{ax+b}{x}\right)^{\frac{5}{2}}a^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(5/2)/x^(1/2),x)
```

```
[Out] 2/3*(a*x+b)*(3*a^2*x^2+12*a*b*x+8*b^2)/a^3/x^(5/2)/((a*x+b)/x)^(5/2)
```

maxima [A] time = 1.06, size = 52, normalized size = 0.74

$$\frac{2\sqrt{a+\frac{b}{x}}\sqrt{x}}{a^3}+\frac{2\left(6\left(a+\frac{b}{x}\right)bx-b^2\right)}{3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(a + b/x)*sqrt(x)/a^3 + 2/3*(6*(a + b/x)*b*x - b^2)/((a + b/x)^(3/2)*
a^3*x^(3/2))
```

mupad [**B**] time = 1.54, size = 43, normalized size = 0.61

$$\frac{2\sqrt{x}\,\sqrt{a+\frac{b}{x}}\,\left(3\,a^2\,x^2+12\,a\,b\,x+8\,b^2\right)}{3\,a^3\,\left(b+a\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b/x)^(5/2)),x)
```

```
[Out] (2*x^(1/2)*(a + b/x)^(1/2)*(8*b^2 + 3*a^2*x^2 + 12*a*b*x))/(3*a^3*(b + a*x)
^2)
```

sympy [**B**] time = 7.54, size = 151, normalized size = 2.16

$$\frac{6a^2b^{\frac{9}{2}}x^2\sqrt{\frac{ax}{b}+1}}{3a^5b^4x^2+6a^4b^5x+3a^3b^6}+\frac{24ab^{\frac{11}{2}}x\sqrt{\frac{ax}{b}+1}}{3a^5b^4x^2+6a^4b^5x+3a^3b^6}+\frac{16b^{\frac{13}{2}}\sqrt{\frac{ax}{b}+1}}{3a^5b^4x^2+6a^4b^5x+3a^3b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(5/2)/x**(1/2),x)
```

```
[Out] 6*a**2*b**(9/2)*x**2*sqrt(a*x/b + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x + 3*
a**3*b**6) + 24*a*b**(11/2)*x*sqrt(a*x/b + 1)/(3*a**5*b**4*x**2 + 6*a**4*b*
*5*x + 3*a**3*b**6) + 16*b**(13/2)*sqrt(a*x/b + 1)/(3*a**5*b**4*x**2 + 6*a*
*4*b**5*x + 3*a**3*b**6)
```

3.1799 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{3/2}}\,dx$

Optimal. Leaf size=46

$$-\frac{4b}{3a^2x^{3/2}\left(a+\frac{b}{x}\right)^{3/2}}-\frac{2}{a\sqrt{x}\left(a+\frac{b}{x}\right)^{3/2}}$$

```
[Out] -4/3*b/a^2/(a+b/x)^(3/2)/x^(3/2)-2/a/(a+b/x)^(3/2)/x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {271, 264}

$$-\frac{4b}{3a^2x^{3/2}\left(a+\frac{b}{x}\right)^{3/2}}-\frac{2}{a\sqrt{x}\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(5/2)*x^(3/2)),x]
```

```
[Out] (-4*b)/(3*a^2*(a + b/x)^(3/2)*x^(3/2)) - 2/(a*(a + b/x)^(3/2)*Sqrt[x])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^{3/2}}\,dx &= -\frac{2}{a\left(a+\frac{b}{x}\right)^{3/2}\sqrt{x}}+\frac{(2b)\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^{5/2}}\,dx}{a}\\ &= -\frac{4b}{3a^2\left(a+\frac{b}{x}\right)^{3/2}x^{3/2}}-\frac{2}{a\left(a+\frac{b}{x}\right)^{3/2}\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 38, normalized size = 0.83

$$-\frac{2\sqrt{x}\sqrt{a+\frac{b}{x}}\,(3ax+2b)}{3a^2(ax+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(5/2)*x^(3/2)),x]
```

```
[Out] (-2*Sqrt[a + b/x]*Sqrt[x]*(2*b + 3*a*x))/(3*a^2*(b + a*x)^2)
```

fricas [A] time = 0.68, size = 48, normalized size = 1.04

$$-\frac{2\,(3\,ax+2\,b)\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}{3\left(a^4x^2+2\,a^3bx+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*(3*a*x + 2*b)*sqrt(x)*sqrt((a*x + b)/x)/(a^4*x^2 + 2*a^3*b*x + a^2*b^2
)
```

giac [A] time = 0.19, size = 29, normalized size = 0.63

$$-\frac{2\,(3\,ax+2\,b)}{3\,(ax+b)^{\frac{3}{2}}a^2}+\frac{4}{3\,a^2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*(3*a*x + 2*b)/((a*x + b)^(3/2)*a^2) + 4/3/(a^2*sqrt(b))
```

maple [A] time = 0.00, size = 33, normalized size = 0.72

$$-\frac{2\,(ax+b)\,(3ax+2b)}{3\left(\frac{ax+b}{x}\right)^{\frac{5}{2}}a^2x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(5/2)/x^(3/2),x)
```

```
[Out] -2/3*(a*x+b)*(3*a*x+2*b)/a^2/x^(5/2)/((a*x+b)/x)^(5/2)
```

maxima [A] time = 1.07, size = 31, normalized size = 0.67

$$-\frac{2\left(3\left(a+\frac{b}{x}\right)x-b\right)}{3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] -2/3*(3*(a + b/x)*x - b)/((a + b/x)^(3/2)*a^2*x^(3/2))
```

mupad [B] time = 1.49, size = 49, normalized size = 1.07

$$-\frac{\sqrt{a+\frac{b}{x}}\,\left(\frac{2\,x^{3/2}}{a^3}+\frac{4\,b\,\sqrt{x}}{3\,a^4}\right)}{x^2+\frac{b^2}{a^2}+\frac{2\,b\,x}{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b/x)^(5/2)),x)
```

```
[Out] -((a + b/x)^(1/2)*((2*x^(3/2))/a^3 + (4*b*x^(1/2))/(3*a^4)))/(x^2 + b^2/a^2
 + (2*b*x)/a)
```

sympy [**B**] time = 14.29, size = 94, normalized size = 2.04

$$-\frac{6ax}{3a^3\sqrt{b}\,x\sqrt{\frac{ax}{b}+1}+3a^2b^{\frac{3}{2}}\sqrt{\frac{ax}{b}+1}}-\frac{4b}{3a^3\sqrt{b}\,x\sqrt{\frac{ax}{b}+1}+3a^2b^{\frac{3}{2}}\sqrt{\frac{ax}{b}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(5/2)/x**(3/2),x)
```

```
[Out] -6*a*x/(3*a**3*sqrt(b)*x*sqrt(a*x/b + 1) + 3*a**2*b**(3/2)*sqrt(a*x/b + 1))
 - 4*b/(3*a**3*sqrt(b)*x*sqrt(a*x/b + 1) + 3*a**2*b**(3/2)*sqrt(a*x/b + 1))
```

3.1800 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{5/2}}\,dx$

Optimal. Leaf size=23

$$-\frac{2}{3ax^{3/2}\left(a+\frac{b}{x}\right)^{3/2}}$$

```
[Out] -2/3/a/(a+b/x)^(3/2)/x^(3/2)
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {264}

$$-\frac{2}{3ax^{3/2}\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(5/2)*x^(5/2)),x]
```

```
[Out] -2/(3*a*(a + b/x)^(3/2)*x^(3/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{5/2}}\,dx = -\frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2} x^{3/2}}$$

Mathematica [A] time = 0.02, size = 23, normalized size = 1.00

$$-\frac{2}{3ax^{3/2}\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(5/2)*x^(5/2)),x]
```

```
[Out] -2/(3*a*(a + b/x)^(3/2)*x^(3/2))
```

fricas [B] time = 0.98, size = 38, normalized size = 1.65

$$-\frac{2\sqrt{x}\sqrt{\frac{ax+b}{x}}}{3\left(a^3x^2+2\,a^2bx+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(x)*sqrt((a*x + b)/x)/(a^3*x^2 + 2*a^2*b*x + a*b^2)
```

giac [A] time = 0.18, size = 21, normalized size = 0.91

$$-\frac{2}{3\,(ax+b)^{\frac{3}{2}}a}+\frac{2}{3\,ab^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/x^(5/2),x, algorithm="giac")
[Out] -2/3/((a*x + b)^(3/2)*a) + 2/3/(a*b^(3/2))
```

maple [A] time = 0.00, size = 25, normalized size = 1.09

$$-\frac{2\,(ax+b)}{3\left(\frac{ax+b}{x}\right)^{\frac{5}{2}}a\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(5/2)/x^(5/2),x)
[Out] -2/3*(a*x+b)/a/((a*x+b)/x)^(5/2)/x^(5/2)
```

maxima [A] time = 0.90, size = 17, normalized size = 0.74

$$-\frac{2}{3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/x^(5/2),x, algorithm="maxima")
[Out] -2/3/((a + b/x)^(3/2)*a*x^(3/2))
```

mupad [B] time = 1.49, size = 38, normalized size = 1.65

$$-\frac{2\sqrt{x}\,\sqrt{a+\frac{b}{x}}}{3\left(a^3x^2+2a^2bx+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b/x)^(5/2)),x)
[Out] -(2*x^(1/2)*(a + b/x)^(1/2))/(3*(a*b^2 + a^3*x^2 + 2*a^2*b*x))
```

sympy [B] time = 20.02, size = 42, normalized size = 1.83

$$-\frac{2}{3a^2\sqrt{b}\,x\sqrt{\frac{ax}{b}+1}+3ab^{\frac{3}{2}}\sqrt{\frac{ax}{b}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(5/2)/x**(5/2),x)
[Out] -2/(3*a**2*sqrt(b)*x*sqrt(a*x/b + 1) + 3*a*b**(3/2)*sqrt(a*x/b + 1))
```

3.1801 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{7/2}}\,dx$

Optimal. Leaf size=75

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{b^{5/2}}+\frac{2}{b^2\sqrt{x}\sqrt{a+\frac{b}{x}}}+\frac{2}{3bx^{3/2}\left(a+\frac{b}{x}\right)^{3/2}}$$

```
[Out] 2/3/b/(a+b/x)^(3/2)/x^(3/2)-2*arctanh(b^(1/2)/(a+b/x)^(1/2)/x^(1/2))/b^(5/2
)+2/b^2/(a+b/x)^(1/2)/x^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {337, 288, 217, 206}

$$\frac{2}{b^2\sqrt{x}\sqrt{a+\frac{b}{x}}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{b^{5/2}}+\frac{2}{3bx^{3/2}\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(5/2)*x^(7/2)),x]
```

```
[Out] 2/(3*b*(a + b/x)^(3/2)*x^(3/2)) + 2/(b^2*Sqrt[a + b/x]*Sqrt[x]) - (2*ArcTan
h[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x])])/b^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 337

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, -Dist[k/c, Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1)
 + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0] &
& FractionQ[m]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{7/2}}\,dx = -\left(2\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^2\right)^{5/2}}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= \frac{2}{3b\left(a+\frac{b}{x}\right)^{3/2} x^{3/2}} - \frac{2\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^2\right)^{3/2}}\,dx, x, \frac{1}{\sqrt{x}}\right)}{b}$$

$$= \frac{2}{3b\left(a+\frac{b}{x}\right)^{3/2} x^{3/2}} + \frac{2}{b^2\sqrt{a+\frac{b}{x}}\sqrt{x}} - \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{\sqrt{x}}\right)}{b^2}$$

$$= \frac{2}{3b\left(a+\frac{b}{x}\right)^{3/2} x^{3/2}} + \frac{2}{b^2\sqrt{a+\frac{b}{x}}\sqrt{x}} - \frac{2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)}{b^2}$$

$$= \frac{2}{3b\left(a+\frac{b}{x}\right)^{3/2} x^{3/2}} + \frac{2}{b^2\sqrt{a+\frac{b}{x}}\sqrt{x}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)}{b^{5/2}}$$

Mathematica [A] time = 0.18, size = 94, normalized size = 1.25

$$\frac{2\left(\sqrt{b}\sqrt{x}\,(3ax+4b) - 3\sqrt{a}\,x\sqrt{\frac{b}{ax}+1}\,(ax+b)\sinh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right)\right)}{3b^{5/2}x\sqrt{a+\frac{b}{x}}\,(ax+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(5/2)*x^(7/2)),x]
```

```
[Out] (2*(Sqrt[b]*Sqrt[x]*(4*b + 3*a*x) - 3*Sqrt[a]*Sqrt[1 + b/(a*x)]*x*(b + a*x)
*ArcSinh[Sqrt[b]/(Sqrt[a]*Sqrt[x])]))/(3*b^(5/2)*Sqrt[a + b/x]*x*(b + a*x))
```

fricas [A] time = 0.69, size = 205, normalized size = 2.73

$$\left[\frac{3\left(a^2x^2+2abx+b^2\right)\sqrt{b}\,\log\left(\frac{ax-2\sqrt{b}\sqrt{x}\sqrt{\frac{ax+b}{x}}+2b}{x}\right)+2\left(3abx+4b^2\right)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{3\left(a^2b^3x^2+2ab^4x+b^5\right)}, \frac{2\left(3\left(a^2x^2+2abx+b^2\right)\sqrt{-b}\right.}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(a^2*x^2 + 2*a*b*x + b^2)*sqrt(b)*log((a*x - 2*sqrt(b)*sqrt(x)*sqrt
((a*x + b)/x) + 2*b)/x) + 2*(3*a*b*x + 4*b^2)*sqrt(x)*sqrt((a*x + b)/x))/(a
^2*b^3*x^2 + 2*a*b^4*x + b^5), 2/3*(3*(a^2*x^2 + 2*a*b*x + b^2)*sqrt(-b)*ar
ctan(sqrt(-b)*sqrt(x)*sqrt((a*x + b)/x)/b) + (3*a*b*x + 4*b^2)*sqrt(x)*sqrt
((a*x + b)/x))/(a^2*b^3*x^2 + 2*a*b^4*x + b^5)]
```

giac [A] time = 0.19, size = 78, normalized size = 1.04

$$\frac{2\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b^2} - \frac{2\left(3\sqrt{b}\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+4\sqrt{-b}\right)}{3\sqrt{-b}\,b^{\frac{5}{2}}} + \frac{2\,(3ax+4b)}{3\,(ax+b)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(a*x + b)/sqrt(-b))/(sqrt(-b)*b^2) - 2/3*(3*sqrt(b)*arctan(sqr
t(b)/sqrt(-b)) + 4*sqrt(-b))/(sqrt(-b)*b^(5/2)) + 2/3*(3*a*x + 4*b)/((a*x +
 b)^(3/2)*b^2)
```

maple [A] time = 0.02, size = 85, normalized size = 1.13

$$-\frac{2\sqrt{\frac{ax+b}{x}}\left(3\sqrt{ax+b}\,ax\,\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)-3a\sqrt{b}\,x+3\sqrt{ax+b}\,b\,\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)-4b^{\frac{3}{2}}\right)\sqrt{x}}{3\left(ax+b\right)^{2}b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(5/2)/x^(7/2),x)
```

```
[Out] -2/3*((a*x+b)/x)^(1/2)*x^(1/2)/b^(5/2)*(3*arctanh((a*x+b)^(1/2)/b^(1/2))*(a
*x+b)^(1/2)*x*a+3*arctanh((a*x+b)^(1/2)/b^(1/2))*b*(a*x+b)^(1/2)-3*a*b^(1/2
)*x-4*b^(3/2))/(a*x+b)^2
```

maxima [A] time = 2.34, size = 74, normalized size = 0.99

$$\frac{\log\left(\frac{\sqrt{a+\frac{b}{x}}\,\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\,\sqrt{x}+\sqrt{b}}\right)}{b^{\frac{5}{2}}}+\frac{2\left(3\left(a+\frac{b}{x}\right)x+b\right)}{3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^{2}x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/x^(7/2),x, algorithm="maxima")
```

```
[Out] log((sqrt(a + b/x)*sqrt(x) - sqrt(b))/(sqrt(a + b/x)*sqrt(x) + sqrt(b)))/b^
(5/2) + 2/3*(3*(a + b/x)*x + b)/((a + b/x)^(3/2)*b^2*x^(3/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{7/2}\left(a+\frac{b}{x}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a + b/x)^(5/2)),x)
```

```
[Out] int(1/(x^(7/2)*(a + b/x)^(5/2)), x)
```

sympy [B] time = 51.16, size = 697, normalized size = 9.29

$$\frac{3a^{3}b^{4}x^{3}\log\left(\frac{ax}{b}\right)}{3a^{3}b^{\frac{13}{2}}x^{3}+9a^{2}b^{\frac{15}{2}}x^{2}+9ab^{\frac{17}{2}}x+3b^{\frac{19}{2}}}-\frac{6a^{3}b^{4}x^{3}\log\left(\sqrt{\frac{ax}{b}+1}+1\right)}{3a^{3}b^{\frac{13}{2}}x^{3}+9a^{2}b^{\frac{15}{2}}x^{2}+9ab^{\frac{17}{2}}x+3b^{\frac{19}{2}}}+\frac{6a^{2}b^{5}x^{2}\sqrt{\frac{ax}{b}+1}}{3a^{3}b^{\frac{13}{2}}x^{3}+9a^{2}b^{\frac{15}{2}}x^{2}+9ab^{\frac{17}{2}}x+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(5/2)/x**(7/2),x)
```

```
[Out] 3*a**3*b**4*x**3*log(a*x/b)/(3*a**3*b**(13/2)*x**3 + 9*a**2*b**(15/2)*x**2
 + 9*a*b**(17/2)*x + 3*b**(19/2)) - 6*a**3*b**4*x**3*log(sqrt(a*x/b + 1) + 1
)/(3*a**3*b**(13/2)*x**3 + 9*a**2*b**(15/2)*x**2 + 9*a*b**(17/2)*x + 3*b**(
19/2)) + 6*a**2*b**5*x**2*sqrt(a*x/b + 1)/(3*a**3*b**(13/2)*x**3 + 9*a**2*b
```

```
**(15/2)*x**2 + 9*a*b**(17/2)*x + 3*b**(19/2)) + 9*a**2*b**5*x**2*log(a*x/b
)/(3*a**3*b**(13/2)*x**3 + 9*a**2*b**(15/2)*x**2 + 9*a*b**(17/2)*x + 3*b**(
19/2)) - 18*a**2*b**5*x**2*log(sqrt(a*x/b + 1) + 1)/(3*a**3*b**(13/2)*x**3
+ 9*a**2*b**(15/2)*x**2 + 9*a*b**(17/2)*x + 3*b**(19/2)) + 14*a*b**6*x*sqrt
(a*x/b + 1)/(3*a**3*b**(13/2)*x**3 + 9*a**2*b**(15/2)*x**2 + 9*a*b**(17/2)*
x + 3*b**(19/2)) + 9*a*b**6*x*log(a*x/b)/(3*a**3*b**(13/2)*x**3 + 9*a**2*b*
*(15/2)*x**2 + 9*a*b**(17/2)*x + 3*b**(19/2)) - 18*a*b**6*x*log(sqrt(a*x/b
+ 1) + 1)/(3*a**3*b**(13/2)*x**3 + 9*a**2*b**(15/2)*x**2 + 9*a*b**(17/2)*x
+ 3*b**(19/2)) + 8*b**7*sqrt(a*x/b + 1)/(3*a**3*b**(13/2)*x**3 + 9*a**2*b**
(15/2)*x**2 + 9*a*b**(17/2)*x + 3*b**(19/2)) + 3*b**7*log(a*x/b)/(3*a**3*b*
*(13/2)*x**3 + 9*a**2*b**(15/2)*x**2 + 9*a*b**(17/2)*x + 3*b**(19/2)) - 6*b
**7*log(sqrt(a*x/b + 1) + 1)/(3*a**3*b**(13/2)*x**3 + 9*a**2*b**(15/2)*x**2
 + 9*a*b**(17/2)*x + 3*b**(19/2))
```

3.1802 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{9/2}}\,dx$

Optimal. Leaf size=99

$$\frac{5a\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{b^{7/2}}-\frac{5\sqrt{a+\frac{b}{x}}}{b^3\sqrt{x}}+\frac{10}{3b^2x^{3/2}\sqrt{a+\frac{b}{x}}}+\frac{2}{3bx^{5/2}\left(a+\frac{b}{x}\right)^{3/2}}$$

```
[Out] 2/3/b/(a+b/x)^(3/2)/x^(5/2)+5*a*arctanh(b^(1/2)/(a+b/x)^(1/2)/x^(1/2))/b^(7
/2)+10/3/b^2/x^(3/2)/(a+b/x)^(1/2)-5*(a+b/x)^(1/2)/b^3/x^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {337, 288, 321, 217, 206}

$$\frac{10}{3b^2x^{3/2}\sqrt{a+\frac{b}{x}}}-\frac{5\sqrt{a+\frac{b}{x}}}{b^3\sqrt{x}}+\frac{5a\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{b^{7/2}}+\frac{2}{3bx^{5/2}\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(5/2)*x^(9/2)),x]
```

```
[Out] 2/(3*b*(a + b/x)^(3/2)*x^(5/2)) + 10/(3*b^2*Sqrt[a + b/x]*x^(3/2)) - (5*Sqr
t[a + b/x])/(b^3*Sqrt[x]) + (5*a*ArcTanh[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x])])/
b^(7/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 337

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, -Dist[k/c, Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1)
 + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0] &
& FractionQ[m]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{9/2}}\,dx = -\left(2\,\text{Subst}\left(\int \frac{x^6}{\left(a+bx^2\right)^{5/2}}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= \frac{2}{3b\left(a+\frac{b}{x}\right)^{3/2} x^{5/2}} - \frac{10\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^2\right)^{3/2}}\,dx, x, \frac{1}{\sqrt{x}}\right)}{3b}$$

$$= \frac{2}{3b\left(a+\frac{b}{x}\right)^{3/2} x^{5/2}} + \frac{10}{3b^2\sqrt{a+\frac{b}{x}}\,x^{3/2}} - \frac{10\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{\sqrt{x}}\right)}{b^2}$$

$$= \frac{2}{3b\left(a+\frac{b}{x}\right)^{3/2} x^{5/2}} + \frac{10}{3b^2\sqrt{a+\frac{b}{x}}\,x^{3/2}} - \frac{5\sqrt{a+\frac{b}{x}}}{b^3\sqrt{x}} + \frac{(5a)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{\sqrt{x}}\right)}{b^3}$$

$$= \frac{2}{3b\left(a+\frac{b}{x}\right)^{3/2} x^{5/2}} + \frac{10}{3b^2\sqrt{a+\frac{b}{x}}\,x^{3/2}} - \frac{5\sqrt{a+\frac{b}{x}}}{b^3\sqrt{x}} + \frac{(5a)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{b}{x}}\,\sqrt{x}}\right)}{b^3}$$

$$= \frac{2}{3b\left(a+\frac{b}{x}\right)^{3/2} x^{5/2}} + \frac{10}{3b^2\sqrt{a+\frac{b}{x}}\,x^{3/2}} - \frac{5\sqrt{a+\frac{b}{x}}}{b^3\sqrt{x}} + \frac{5a\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\,\sqrt{x}}\right)}{b^{7/2}}$$

Mathematica [C] time = 0.02, size = 56, normalized size = 0.57

$$-\frac{2\sqrt{\frac{b}{ax}+1}\;{}_2F_1\left(\frac{5}{2},\frac{7}{2};\frac{9}{2};-\frac{b}{ax}\right)}{7a^2x^{7/2}\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(5/2)*x^(9/2)),x]
```

```
[Out] (-2*Sqrt[1 + b/(a*x)]*Hypergeometric2F1[5/2, 7/2, 9/2, -(b/(a*x))])/(7*a^2*
Sqrt[a + b/x]*x^(7/2))
```

fricas [A] time = 0.89, size = 249, normalized size = 2.52

$$\left[\frac{15\left(a^3x^3+2\,a^2bx^2+ab^2x\right)\sqrt{b}\,\log\left(\frac{ax+2\,\sqrt{b}\,\sqrt{x}\,\sqrt{\frac{ax+b}{x}}+2\,b}{x}\right) - 2\left(15\,a^2bx^2+20\,ab^2x+3\,b^3\right)\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}{6\left(a^2b^4x^3+2\,ab^5x^2+b^6x\right)}, -\frac{15\left(a^3x^3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*(a^3*x^3 + 2*a^2*b*x^2 + a*b^2*x)*sqrt(b)*log((a*x + 2*sqrt(b)*sqr
t(x)*sqrt((a*x + b)/x) + 2*b)/x) - 2*(15*a^2*b*x^2 + 20*a*b^2*x + 3*b^3)*sq
rt(x)*sqrt((a*x + b)/x))/(a^2*b^4*x^3 + 2*a*b^5*x^2 + b^6*x), -1/3*(15*(a^3
*x^3 + 2*a^2*b*x^2 + a*b^2*x)*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)*sqrt((a*x +
b)/x)/b) + (15*a^2*b*x^2 + 20*a*b^2*x + 3*b^3)*sqrt(x)*sqrt((a*x + b)/x))/(
a^2*b^4*x^3 + 2*a*b^5*x^2 + b^6*x)]
```

giac [A] time = 0.25, size = 65, normalized size = 0.66

$$-\frac{5\,a\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b^3}-\frac{2\,(6\,(ax+b)a+ab)}{3\,(ax+b)^{\frac{3}{2}}b^3}-\frac{\sqrt{ax+b}}{b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] -5*a*arctan(sqrt(a*x + b)/sqrt(-b))/(sqrt(-b)*b^3) - 2/3*(6*(a*x + b)*a + a
*b)/((a*x + b)^(3/2)*b^3) - sqrt(a*x + b)/(b^3*x)
```

maple [A] time = 0.02, size = 102, normalized size = 1.03

$$-\frac{\sqrt{\frac{ax+b}{x}}\,\left(-15\sqrt{ax+b}\;a^2x^2\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)+15a^2\sqrt{b}\;x^2-15\sqrt{ax+b}\;abx\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)+20a\,b^{\frac{3}{2}}x+3b^{\frac{5}{2}}\right)}{3\,(ax+b)^2\,b^{\frac{7}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(5/2)/x^(9/2),x)
```

```
[Out] -1/3*((a*x+b)/x)^(1/2)*(-15*arctanh((a*x+b)^(1/2)/b^(1/2))*(a*x+b)^(1/2)*x^
2*a^2+3*b^(5/2)+20*a*b^(3/2)*x+15*a^2*b^(1/2)*x^2-15*arctanh((a*x+b)^(1/2)/
b^(1/2))*x*a*b*(a*x+b)^(1/2))/x^(1/2)/(a*x+b)^2/b^(7/2)
```

maxima [A] time = 2.37, size = 119, normalized size = 1.20

$$-\frac{15\left(a+\frac{b}{x}\right)^2ax^2-10\left(a+\frac{b}{x}\right)abx-2\,ab^2}{3\left(\left(a+\frac{b}{x}\right)^{\frac{5}{2}}b^3x^{\frac{5}{2}}-\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^4x^{\frac{3}{2}}\right)}-\frac{5\,a\log\left(\frac{\sqrt{a+\frac{b}{x}}\,\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\,\sqrt{x}+\sqrt{b}}\right)}{2\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] -1/3*(15*(a + b/x)^2*a*x^2 - 10*(a + b/x)*a*b*x - 2*a*b^2)/((a + b/x)^(5/2)
*b^3*x^(5/2) - (a + b/x)^(3/2)*b^4*x^(3/2)) - 5/2*a*log((sqrt(a + b/x)*sqrt
(x) - sqrt(b))/(sqrt(a + b/x)*sqrt(x) + sqrt(b)))/b^(7/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{9/2}\left(a+\frac{b}{x}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(9/2)*(a + b/x)^(5/2)),x)
```

```
[Out] int(1/(x^(9/2)*(a + b/x)^(5/2)), x)
```

sympy [**B**] time = 133.44, size = 818, normalized size = 8.26

$$-\frac{15a^4b^{13}x^4\log\left(\frac{ax}{b}\right)}{6a^3b^{\frac{33}{2}}x^4+18a^2b^{\frac{35}{2}}x^3+18ab^{\frac{37}{2}}x^2+6b^{\frac{39}{2}}x}+\frac{30a^4b^{13}x^4\log\left(\sqrt{\frac{ax}{b}+1}+1\right)}{6a^3b^{\frac{33}{2}}x^4+18a^2b^{\frac{35}{2}}x^3+18ab^{\frac{37}{2}}x^2+6b^{\frac{39}{2}}x}-\frac{30a^3b^{14}}{6a^3b^{\frac{33}{2}}x^4+18a^2b^{\frac{35}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(5/2)/x**(9/2),x)
```

```
[Out] -15*a**4*b**13*x**4*log(a*x/b)/(6*a**3*b**(33/2)*x**4 + 18*a**2*b**(35/2)*x
**3 + 18*a*b**(37/2)*x**2 + 6*b**(39/2)*x) + 30*a**4*b**13*x**4*log(sqrt(a*
x/b + 1) + 1)/(6*a**3*b**(33/2)*x**4 + 18*a**2*b**(35/2)*x**3 + 18*a*b**(37
/2)*x**2 + 6*b**(39/2)*x) - 30*a**3*b**14*x**3*sqrt(a*x/b + 1)/(6*a**3*b**(
33/2)*x**4 + 18*a**2*b**(35/2)*x**3 + 18*a*b**(37/2)*x**2 + 6*b**(39/2)*x)
- 45*a**3*b**14*x**3*log(a*x/b)/(6*a**3*b**(33/2)*x**4 + 18*a**2*b**(35/2)*
x**3 + 18*a*b**(37/2)*x**2 + 6*b**(39/2)*x) + 90*a**3*b**14*x**3*log(sqrt(a
*x/b + 1) + 1)/(6*a**3*b**(33/2)*x**4 + 18*a**2*b**(35/2)*x**3 + 18*a*b**(3
7/2)*x**2 + 6*b**(39/2)*x) - 70*a**2*b**15*x**2*sqrt(a*x/b + 1)/(6*a**3*b**
(33/2)*x**4 + 18*a**2*b**(35/2)*x**3 + 18*a*b**(37/2)*x**2 + 6*b**(39/2)*x)
 - 45*a**2*b**15*x**2*log(a*x/b)/(6*a**3*b**(33/2)*x**4 + 18*a**2*b**(35/2)
*x**3 + 18*a*b**(37/2)*x**2 + 6*b**(39/2)*x) + 90*a**2*b**15*x**2*log(sqrt(
a*x/b + 1) + 1)/(6*a**3*b**(33/2)*x**4 + 18*a**2*b**(35/2)*x**3 + 18*a*b**(
37/2)*x**2 + 6*b**(39/2)*x) - 46*a*b**16*x*sqrt(a*x/b + 1)/(6*a**3*b**(33/2
)*x**4 + 18*a**2*b**(35/2)*x**3 + 18*a*b**(37/2)*x**2 + 6*b**(39/2)*x) - 15
*a*b**16*x*log(a*x/b)/(6*a**3*b**(33/2)*x**4 + 18*a**2*b**(35/2)*x**3 + 18*
a*b**(37/2)*x**2 + 6*b**(39/2)*x) + 30*a*b**16*x*log(sqrt(a*x/b + 1) + 1)/(
6*a**3*b**(33/2)*x**4 + 18*a**2*b**(35/2)*x**3 + 18*a*b**(37/2)*x**2 + 6*b*
*(39/2)*x) - 6*b**17*sqrt(a*x/b + 1)/(6*a**3*b**(33/2)*x**4 + 18*a**2*b**(3
5/2)*x**3 + 18*a*b**(37/2)*x**2 + 6*b**(39/2)*x)
```

3.1803 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{11/2}}\,dx$

Optimal. Leaf size=129

$$-\frac{35a^2 \tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{4b^{9/2}}+\frac{35a\sqrt{a+\frac{b}{x}}}{4b^4\sqrt{x}}-\frac{35\sqrt{a+\frac{b}{x}}}{6b^3x^{3/2}}+\frac{14}{3b^2x^{5/2}\sqrt{a+\frac{b}{x}}}+\frac{2}{3bx^{7/2}\left(a+\frac{b}{x}\right)^{3/2}}$$

```
[Out] 2/3/b/(a+b/x)^(3/2)/x^(7/2)-35/4*a^2*arctanh(b^(1/2)/(a+b/x)^(1/2)/x^(1/2))
/b^(9/2)+14/3/b^2/x^(5/2)/(a+b/x)^(1/2)-35/6*(a+b/x)^(1/2)/b^3/x^(3/2)+35/4
*a*(a+b/x)^(1/2)/b^4/x^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {337, 288, 321, 217, 206}

$$-\frac{35a^2 \tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{4b^{9/2}}-\frac{35\sqrt{a+\frac{b}{x}}}{6b^3x^{3/2}}+\frac{14}{3b^2x^{5/2}\sqrt{a+\frac{b}{x}}}+\frac{35a\sqrt{a+\frac{b}{x}}}{4b^4\sqrt{x}}+\frac{2}{3bx^{7/2}\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(5/2)*x^(11/2)),x]
```

```
[Out] 2/(3*b*(a + b/x)^(3/2)*x^(7/2)) + 14/(3*b^2*Sqrt[a + b/x]*x^(5/2)) - (35*Sq
rt[a + b/x])/(6*b^3*x^(3/2)) + (35*a*Sqrt[a + b/x])/(4*b^4*Sqrt[x]) - (35*a
^2*ArcTanh[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x])])/(4*b^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 337

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, -Dist[k/c, Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1)
 + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0] &
& FractionQ[m]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{11/2}}\,dx = -\left(2\,\text{Subst}\left(\int \frac{x^8}{\left(a+bx^2\right)^{5/2}}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= \frac{2}{3b\left(a+\frac{b}{x}\right)^{3/2} x^{7/2}} - \frac{14\,\text{Subst}\left(\int \frac{x^6}{\left(a+bx^2\right)^{3/2}}\,dx, x, \frac{1}{\sqrt{x}}\right)}{3b}$$

$$= \frac{2}{3b\left(a+\frac{b}{x}\right)^{3/2} x^{7/2}} + \frac{14}{3b^2\sqrt{a+\frac{b}{x}}\,x^{5/2}} - \frac{70\,\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{\sqrt{x}}\right)}{3b^2}$$

$$= \frac{2}{3b\left(a+\frac{b}{x}\right)^{3/2} x^{7/2}} + \frac{14}{3b^2\sqrt{a+\frac{b}{x}}\,x^{5/2}} - \frac{35\sqrt{a+\frac{b}{x}}}{6b^3x^{3/2}} + \frac{(35a)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{\sqrt{x}}\right)}{2b^3}$$

$$= \frac{2}{3b\left(a+\frac{b}{x}\right)^{3/2} x^{7/2}} + \frac{14}{3b^2\sqrt{a+\frac{b}{x}}\,x^{5/2}} - \frac{35\sqrt{a+\frac{b}{x}}}{6b^3x^{3/2}} + \frac{35a\sqrt{a+\frac{b}{x}}}{4b^4\sqrt{x}} - \frac{\left(35a^2\right)\text{Subst}\left(\int \frac{}{\sqrt{}}\right.}{4b^4}$$

$$= \frac{2}{3b\left(a+\frac{b}{x}\right)^{3/2} x^{7/2}} + \frac{14}{3b^2\sqrt{a+\frac{b}{x}}\,x^{5/2}} - \frac{35\sqrt{a+\frac{b}{x}}}{6b^3x^{3/2}} + \frac{35a\sqrt{a+\frac{b}{x}}}{4b^4\sqrt{x}} - \frac{\left(35a^2\right)\text{Subst}\left(\int \frac{}{1-}\right.}{4}$$

$$= \frac{2}{3b\left(a+\frac{b}{x}\right)^{3/2} x^{7/2}} + \frac{14}{3b^2\sqrt{a+\frac{b}{x}}\,x^{5/2}} - \frac{35\sqrt{a+\frac{b}{x}}}{6b^3x^{3/2}} + \frac{35a\sqrt{a+\frac{b}{x}}}{4b^4\sqrt{x}} - \frac{35a^2\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{a+\frac{}{}}}\right.}{4b^{9/2}}$$

Mathematica [C] time = 0.02, size = 56, normalized size = 0.43

$$-\frac{2\sqrt{\frac{b}{ax}+1}\;{}_2F_1\left(\frac{5}{2},\frac{9}{2};\frac{11}{2};-\frac{b}{ax}\right)}{9a^2x^{9/2}\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(5/2)*x^(11/2)),x]
```

```
[Out] (-2*Sqrt[1 + b/(a*x)]*Hypergeometric2F1[5/2, 9/2, 11/2, -(b/(a*x))])/(9*a^2
*Sqrt[a + b/x]*x^(9/2))
```

fricas [A] time = 0.80, size = 283, normalized size = 2.19

$$\left[\frac{105\left(a^4x^4+2\,a^3bx^3+a^2b^2x^2\right)\sqrt{b}\,\log\left(\frac{ax-2\,\sqrt{b}\,\sqrt{x}\,\sqrt{\frac{ax+b}{x}}+2\,b}{x}\right)+2\left(105\,a^3bx^3+140\,a^2b^2x^2+21\,ab^3x-6\,b^4\right)\sqrt{x}\,\sqrt{\frac{a}{}}}{24\left(a^2b^5x^4+2\,ab^6x^3+b^7x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/x^(11/2),x, algorithm="fricas")
```

```
[Out] [1/24*(105*(a^4*x^4 + 2*a^3*b*x^3 + a^2*b^2*x^2)*sqrt(b)*log((a*x - 2*sqrt(
b)*sqrt(x)*sqrt((a*x + b)/x) + 2*b)/x) + 2*(105*a^3*b*x^3 + 140*a^2*b^2*x^2
 + 21*a*b^3*x - 6*b^4)*sqrt(x)*sqrt((a*x + b)/x))/(a^2*b^5*x^4 + 2*a*b^6*x^
3 + b^7*x^2), 1/12*(105*(a^4*x^4 + 2*a^3*b*x^3 + a^2*b^2*x^2)*sqrt(-b)*arct
an(sqrt(-b)*sqrt(x)*sqrt((a*x + b)/x)/b) + (105*a^3*b*x^3 + 140*a^2*b^2*x^2
 + 21*a*b^3*x - 6*b^4)*sqrt(x)*sqrt((a*x + b)/x))/(a^2*b^5*x^4 + 2*a*b^6*x^
3 + b^7*x^2)]
```

giac [A] time = 0.28, size = 93, normalized size = 0.72

$$\frac{35\,a^2\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{4\,\sqrt{-b}\,b^4}+\frac{2\left(9\,(ax+b)a^2+a^2b\right)}{3\,(ax+b)^{\frac{3}{2}}b^4}+\frac{11\,(ax+b)^{\frac{3}{2}}a^2-13\,\sqrt{ax+b}\,a^2b}{4\,a^2b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/x^(11/2),x, algorithm="giac")
```

```
[Out] 35/4*a^2*arctan(sqrt(a*x + b)/sqrt(-b))/(sqrt(-b)*b^4) + 2/3*(9*(a*x + b)*a
^2 + a^2*b)/((a*x + b)^(3/2)*b^4) + 1/4*(11*(a*x + b)^(3/2)*a^2 - 13*sqrt(a
*x + b)*a^2*b)/(a^2*b^4*x^2)
```

maple [A] time = 0.02, size = 117, normalized size = 0.91

$$-\frac{\sqrt{\frac{ax+b}{x}}\,\left(105\sqrt{ax+b}\;a^3x^3\,\mathrm{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)-105a^3\sqrt{b}\;x^3+105\sqrt{ax+b}\;a^2b\,x^2\,\mathrm{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)-140a^2b^{\frac{3}{2}}x^2-\right.}{12\,(ax+b)^2\,b^{\frac{9}{2}}x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(5/2)/x^(11/2),x)
```

```
[Out] -1/12*((a*x+b)/x)^(1/2)/x^(3/2)*(105*(a*x+b)^(1/2)*arctanh((a*x+b)^(1/2)/b^
(1/2))*x^3*a^3+105*arctanh((a*x+b)^(1/2)/b^(1/2))*x^2*a^2*b*(a*x+b)^(1/2)-1
05*a^3*b^(1/2)*x^3-140*a^2*b^(3/2)*x^2-21*a*b^(5/2)*x+6*b^(7/2))/(a*x+b)^2/
b^(9/2)
```

maxima [A] time = 2.50, size = 163, normalized size = 1.26

$$\frac{105\left(a+\frac{b}{x}\right)^3a^2x^3-175\left(a+\frac{b}{x}\right)^2a^2bx^2+56\left(a+\frac{b}{x}\right)a^2b^2x+8\,a^2b^3}{12\left(\left(a+\frac{b}{x}\right)^{\frac{7}{2}}b^4x^{\frac{7}{2}}-2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}b^5x^{\frac{5}{2}}+\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^6x^{\frac{3}{2}}\right)}+\frac{35\,a^2\log\left(\frac{\sqrt{a+\frac{b}{x}}\,\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\,\sqrt{x}+\sqrt{b}}\right)}{8\,b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/x^(11/2),x, algorithm="maxima")
```

```
[Out] 1/12*(105*(a + b/x)^3*a^2*x^3 - 175*(a + b/x)^2*a^2*b*x^2 + 56*(a + b/x)*a^
2*b^2*x + 8*a^2*b^3)/((a + b/x)^(7/2)*b^4*x^(7/2) - 2*(a + b/x)^(5/2)*b^5*x
^(5/2) + (a + b/x)^(3/2)*b^6*x^(3/2)) + 35/8*a^2*log((sqrt(a + b/x)*sqrt(x)
 - sqrt(b))/(sqrt(a + b/x)*sqrt(x) + sqrt(b)))/b^(9/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{11/2}\left(a+\frac{b}{x}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(11/2)*(a + b/x)^(5/2)),x)
```

```
[Out] int(1/(x^(11/2)*(a + b/x)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(5/2)/x**(11/2),x)
```

```
[Out] Timed out
```

3.1804 $\int \left(a + \frac{b}{x^2}\right) x^6 \, dx$

Optimal. Leaf size=17

$$\frac{ax^7}{7} + \frac{bx^5}{5}$$

```
[Out] 1/5*b*x^5+1/7*a*x^7
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$\frac{ax^7}{7} + \frac{bx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*x^6,x]
```

```
[Out] (b*x^5)/5 + (a*x^7)/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \left(a + \frac{b}{x^2}\right) x^6 \, dx &= \int \left(bx^4 + ax^6\right) dx \\ &= \frac{bx^5}{5} + \frac{ax^7}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^7}{7} + \frac{bx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*x^6,x]
```

```
[Out] (b*x^5)/5 + (a*x^7)/7
```

fricas [A] time = 0.90, size = 13, normalized size = 0.76

$$\frac{1}{7} ax^7 + \frac{1}{5} bx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^6,x, algorithm="fricas")
```

```
[Out] 1/7*a*x^7 + 1/5*b*x^5
```

giac [A] time = 0.18, size = 13, normalized size = 0.76

$$\frac{1}{7} ax^7 + \frac{1}{5} bx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^6,x, algorithm="giac")
[Out] 1/7*a*x^7 + 1/5*b*x^5
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{7}a\,x^7+\frac{1}{5}b\,x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^6,x)
[Out] 1/5*b*x^5+1/7*a*x^7
```

maxima [A] time = 1.13, size = 13, normalized size = 0.76

$$\frac{1}{7}\,ax^7+\frac{1}{5}\,bx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^6,x, algorithm="maxima")
[Out] 1/7*a*x^7 + 1/5*b*x^5
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{a\,x^7}{7}+\frac{b\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b/x^2),x)
[Out] (a*x^7)/7 + (b*x^5)/5
```

sympy [A] time = 0.06, size = 12, normalized size = 0.71

$$\frac{ax^7}{7}+\frac{bx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**6,x)
[Out] a*x**7/7 + b*x**5/5
```

3.1805 $\int \left(a + \frac{b}{x^2}\right) x^5 \, dx$

Optimal. Leaf size=17

$$\frac{ax^6}{6} + \frac{bx^4}{4}$$

```
[Out] 1/4*b*x^4+1/6*a*x^6
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$\frac{ax^6}{6} + \frac{bx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*x^5,x]
```

```
[Out] (b*x^4)/4 + (a*x^6)/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \left(a + \frac{b}{x^2}\right) x^5 \, dx &= \int \left(bx^3 + ax^5\right) dx \\ &= \frac{bx^4}{4} + \frac{ax^6}{6}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^6}{6} + \frac{bx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*x^5,x]
```

```
[Out] (b*x^4)/4 + (a*x^6)/6
```

fricas [A] time = 0.60, size = 13, normalized size = 0.76

$$\frac{1}{6} ax^6 + \frac{1}{4} bx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^5,x, algorithm="fricas")
```

```
[Out] 1/6*a*x^6 + 1/4*b*x^4
```

giac [A] time = 0.16, size = 13, normalized size = 0.76

$$\frac{1}{6} ax^6 + \frac{1}{4} bx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^5,x, algorithm="giac")
[Out] 1/6*a*x^6 + 1/4*b*x^4
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{6}a\,x^6+\frac{1}{4}b\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^5,x)
[Out] 1/4*b*x^4+1/6*a*x^6
```

maxima [A] time = 1.02, size = 13, normalized size = 0.76

$$\frac{1}{6}\,ax^6+\frac{1}{4}\,bx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^5,x, algorithm="maxima")
[Out] 1/6*a*x^6 + 1/4*b*x^4
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{a\,x^6}{6}+\frac{b\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b/x^2),x)
[Out] (a*x^6)/6 + (b*x^4)/4
```

sympy [A] time = 0.06, size = 12, normalized size = 0.71

$$\frac{ax^6}{6}+\frac{bx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**5,x)
[Out] a*x**6/6 + b*x**4/4
```

3.1806 $\int \left(a + \frac{b}{x^2}\right) x^4 \, dx$

Optimal. Leaf size=17

$$\frac{ax^5}{5} + \frac{bx^3}{3}$$

```
[Out] 1/3*b*x^3+1/5*a*x^5
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$\frac{ax^5}{5} + \frac{bx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*x^4,x]
```

```
[Out] (b*x^3)/3 + (a*x^5)/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \left(a + \frac{b}{x^2}\right) x^4 \, dx &= \int \left(bx^2 + ax^4\right) dx \\ &= \frac{bx^3}{3} + \frac{ax^5}{5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^5}{5} + \frac{bx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*x^4,x]
```

```
[Out] (b*x^3)/3 + (a*x^5)/5
```

fricas [A] time = 0.69, size = 13, normalized size = 0.76

$$\frac{1}{5} ax^5 + \frac{1}{3} bx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^4,x, algorithm="fricas")
```

```
[Out] 1/5*a*x^5 + 1/3*b*x^3
```

giac [A] time = 0.15, size = 13, normalized size = 0.76

$$\frac{1}{5} ax^5 + \frac{1}{3} bx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^4,x, algorithm="giac")
[Out] 1/5*a*x^5 + 1/3*b*x^3
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{5}a\,x^5+\frac{1}{3}b\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^4,x)
[Out] 1/3*b*x^3+1/5*a*x^5
```

maxima [A] time = 1.06, size = 13, normalized size = 0.76

$$\frac{1}{5}\,ax^5+\frac{1}{3}\,bx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^4,x, algorithm="maxima")
[Out] 1/5*a*x^5 + 1/3*b*x^3
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{a\,x^5}{5}+\frac{b\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b/x^2),x)
[Out] (a*x^5)/5 + (b*x^3)/3
```

sympy [A] time = 0.06, size = 12, normalized size = 0.71

$$\frac{ax^5}{5}+\frac{bx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**4,x)
[Out] a*x**5/5 + b*x**3/3
```

3.1807 $\int \left(a + \frac{b}{x^2}\right) x^3 \, dx$

Optimal. Leaf size=17

$$\frac{ax^4}{4} + \frac{bx^2}{2}$$

```
[Out] 1/2*b*x^2+1/4*a*x^4
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$\frac{ax^4}{4} + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*x^3,x]
```

```
[Out] (b*x^2)/2 + (a*x^4)/4
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \left(a + \frac{b}{x^2}\right) x^3 \, dx &= \int \left(bx + ax^3\right) dx \\ &= \frac{bx^2}{2} + \frac{ax^4}{4}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^4}{4} + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*x^3,x]
```

```
[Out] (b*x^2)/2 + (a*x^4)/4
```

fricas [A] time = 0.75, size = 13, normalized size = 0.76

$$\frac{1}{4} ax^4 + \frac{1}{2} bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^3,x, algorithm="fricas")
```

```
[Out] 1/4*a*x^4 + 1/2*b*x^2
```

giac [A] time = 0.18, size = 13, normalized size = 0.76

$$\frac{1}{4} ax^4 + \frac{1}{2} bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^3,x, algorithm="giac")
[Out] 1/4*a*x^4 + 1/2*b*x^2
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{4}a\,x^4+\frac{1}{2}b\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^3,x)
[Out] 1/2*b*x^2+1/4*a*x^4
```

maxima [A] time = 1.03, size = 13, normalized size = 0.76

$$\frac{1}{4}\,ax^4+\frac{1}{2}\,bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^3,x, algorithm="maxima")
[Out] 1/4*a*x^4 + 1/2*b*x^2
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{a\,x^4}{4}+\frac{b\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b/x^2),x)
[Out] (a*x^4)/4 + (b*x^2)/2
```

sympy [A] time = 0.06, size = 12, normalized size = 0.71

$$\frac{ax^4}{4}+\frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**3,x)
[Out] a*x**4/4 + b*x**2/2
```

3.1808 $\int \left(a + \frac{b}{x^2}\right) x^2 \, dx$

Optimal. Leaf size=12

$$\frac{ax^3}{3} + bx$$

```
[Out] b*x+1/3*a*x^3
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$\frac{ax^3}{3} + bx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*x^2,x]
```

```
[Out] b*x + (a*x^3)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \left(a + \frac{b}{x^2}\right) x^2 \, dx &= \int \left(b + ax^2\right) dx \\ &= bx + \frac{ax^3}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\frac{ax^3}{3} + bx$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*x^2,x]
```

```
[Out] b*x + (a*x^3)/3
```

fricas [A] time = 0.84, size = 10, normalized size = 0.83

$$\frac{1}{3} ax^3 + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^2,x, algorithm="fricas")
```

```
[Out] 1/3*a*x^3 + b*x
```

giac [A] time = 0.15, size = 10, normalized size = 0.83

$$\frac{1}{3} ax^3 + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^2,x, algorithm="giac")
[Out] 1/3*a*x^3 + b*x
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$\frac{1}{3}a\,x^3 + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^2,x)
[Out] b*x+1/3*a*x^3
```

maxima [A] time = 1.04, size = 10, normalized size = 0.83

$$\frac{1}{3}ax^3 + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^2,x, algorithm="maxima")
[Out] 1/3*a*x^3 + b*x
```

mupad [B] time = 0.02, size = 10, normalized size = 0.83

$$\frac{a\,x^3}{3} + b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b/x^2),x)
[Out] b*x + (a*x^3)/3
```

sympy [A] time = 0.06, size = 8, normalized size = 0.67

$$\frac{ax^3}{3} + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**2,x)
[Out] a*x**3/3 + b*x
```

3.1809 $\int \left(a + \frac{b}{x^2}\right) x\, dx$

Optimal. Leaf size=13

$$\frac{ax^2}{2} + b\log(x)$$

```
[Out] 1/2*a*x^2+b*ln(x)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {14}

$$\frac{ax^2}{2} + b\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*x,x]
```

```
[Out] (a*x^2)/2 + b*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \left(a + \frac{b}{x^2}\right) x\, dx &= \int \left(\frac{b}{x} + ax\right) dx \\ &= \frac{ax^2}{2} + b\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{ax^2}{2} + b\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*x,x]
```

```
[Out] (a*x^2)/2 + b*Log[x]
```

fricas [A] time = 0.82, size = 11, normalized size = 0.85

$$\frac{1}{2}\, ax^2 + b\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x,x, algorithm="fricas")
```

```
[Out] 1/2*a*x^2 + b*log(x)
```

giac [A] time = 0.17, size = 12, normalized size = 0.92

$$\frac{1}{2}\, ax^2 + b\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x,x, algorithm="giac")
[Out] 1/2*a*x^2 + b*log(abs(x))
```

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$\frac{a\,x^2}{2} + b\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x,x)
[Out] 1/2*a*x^2+b*ln(x)
```

maxima [A] time = 1.00, size = 14, normalized size = 1.08

$$\frac{1}{2}ax^2 + \frac{1}{2}b\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x,x, algorithm="maxima")
[Out] 1/2*a*x^2 + 1/2*b*log(x^2)
```

mupad [B] time = 0.02, size = 11, normalized size = 0.85

$$\frac{a\,x^2}{2} + b\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b/x^2),x)
[Out] (a*x^2)/2 + b*log(x)
```

sympy [A] time = 0.09, size = 10, normalized size = 0.77

$$\frac{ax^2}{2} + b\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x,x)
[Out] a*x**2/2 + b*log(x)
```

3.1810 $\int \left(a + \frac{b}{x^2}\right) dx$

Optimal. Leaf size=10

$$ax - \frac{b}{x}$$

```
[Out] -b/x+a*x
```

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$ax - \frac{b}{x}$$

Antiderivative was successfully verified.

```
[In] Int[a + b/x^2,x]
```

```
[Out] -(b/x) + a*x
```

Rubi steps

$$\int \left(a + \frac{b}{x^2}\right) dx = -\frac{b}{x} + ax$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$ax - \frac{b}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b/x^2,x]
```

```
[Out] -(b/x) + a*x
```

fricas [A] time = 0.89, size = 13, normalized size = 1.30

$$\frac{ax^2 - b}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b/x^2,x, algorithm="fricas")
```

```
[Out] (a*x^2 - b)/x
```

giac [A] time = 0.18, size = 10, normalized size = 1.00

$$ax - \frac{b}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b/x^2,x, algorithm="giac")
```

```
[Out] a*x - b/x
```

maple [A] time = 0.00, size = 11, normalized size = 1.10

$$ax - \frac{b}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b/x^2,x)
[Out] -b/x+a*x
```

maxima [**A**] time = 1.06, size = 10, normalized size = 1.00

$$ax - \frac{b}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b/x^2,x, algorithm="maxima")
[Out] a*x - b/x
```

mupad [**B**] time = 0.02, size = 10, normalized size = 1.00

$$a\,x - \frac{b}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b/x^2,x)
[Out] a*x - b/x
```

sympy [**A**] time = 0.08, size = 5, normalized size = 0.50

$$ax - \frac{b}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b/x**2,x)
[Out] a*x - b/x
```

3.1811 $\int \frac{a+\frac{b}{x^2}}{x}\,dx$

Optimal. Leaf size=13

$$a\log(x)-\frac{b}{2x^2}$$

```
[Out] -1/2*b/x^2+a*ln(x)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$a\log(x)-\frac{b}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/x,x]
```

```
[Out] -b/(2*x^2) + a*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+\frac{b}{x^2}}{x}\,dx &= \int \left(\frac{b}{x^3}+\frac{a}{x}\right)dx\\ &= -\frac{b}{2x^2}+a\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$a\log(x)-\frac{b}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/x,x]
```

```
[Out] -1/2*b/x^2 + a*Log[x]
```

fricas [A] time = 0.86, size = 17, normalized size = 1.31

$$\frac{2\,ax^2\log(x)-b}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*x^2*log(x) - b)/x^2
```

giac [A] time = 0.15, size = 20, normalized size = 1.54

$$\frac{1}{2}\,a\log\left(x^2\right)-\frac{ax^2+b}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x,x, algorithm="giac")
[Out] 1/2*a*log(x^2) - 1/2*(a*x^2 + b)/x^2
```

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$a \ln(x) - \frac{b}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/x,x)
[Out] -1/2*b/x^2+a*ln(x)
```

maxima [A] time = 1.10, size = 14, normalized size = 1.08

$$\frac{1}{2}\, a \log\left(x^2\right) - \frac{b}{2\, x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x,x, algorithm="maxima")
[Out] 1/2*a*log(x^2) - 1/2*b/x^2
```

mupad [B] time = 0.04, size = 11, normalized size = 0.85

$$a\ \ln(x) - \frac{b}{2\, x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/x,x)
[Out] a*log(x) - b/(2*x^2)
```

sympy [A] time = 0.11, size = 10, normalized size = 0.77

$$a \log(x) - \frac{b}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/x,x)
[Out] a*log(x) - b/(2*x**2)
```

3.1812 $\int \frac{a+\frac{b}{x^2}}{x^2}\,dx$

Optimal. Leaf size=15

$$-\frac{a}{x}-\frac{b}{3x^3}$$

```
[Out] -1/3*b/x^3-a/x
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$-\frac{a}{x}-\frac{b}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/x^2,x]
```

```
[Out] -b/(3*x^3) - a/x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+\frac{b}{x^2}}{x^2}\,dx &= \int \left(\frac{b}{x^4}+\frac{a}{x^2}\right)dx \\ &= -\frac{b}{3x^3}-\frac{a}{x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$-\frac{a}{x}-\frac{b}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/x^2,x]
```

```
[Out] -1/3*b/x^3 - a/x
```

fricas [A] time = 0.74, size = 13, normalized size = 0.87

$$-\frac{3\,ax^2+b}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*a*x^2 + b)/x^3
```

giac [A] time = 0.15, size = 13, normalized size = 0.87

$$-\frac{3\,ax^2+b}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^2,x, algorithm="giac")
[Out] -1/3*(3*a*x^2 + b)/x^3
```

maple [A] time = 0.01, size = 14, normalized size = 0.93

$$-\frac{a}{x}-\frac{b}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/x^2,x)
[Out] -1/3*b/x^3-a/x
```

maxima [A] time = 1.03, size = 13, normalized size = 0.87

$$-\frac{3\,ax^2+b}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^2,x, algorithm="maxima")
[Out] -1/3*(3*a*x^2 + b)/x^3
```

mupad [B] time = 0.02, size = 13, normalized size = 0.87

$$-\frac{3\,a\,x^2+b}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/x^2,x)
[Out] -(b + 3*a*x^2)/(3*x^3)
```

sympy [A] time = 0.12, size = 14, normalized size = 0.93

$$\frac{-3ax^2-b}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/x**2,x)
[Out] (-3*a*x**2 - b)/(3*x**3)
```

3.1813 $\int \frac{a+\frac{b}{x^2}}{x^3}\,dx$

Optimal. Leaf size=17

$$-\frac{a}{2x^2}-\frac{b}{4x^4}$$

```
[Out] -1/4*b/x^4-1/2*a/x^2
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$-\frac{a}{2x^2}-\frac{b}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/x^3,x]
```

```
[Out] -b/(4*x^4) - a/(2*x^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+\frac{b}{x^2}}{x^3}\,dx &= \int \left(\frac{b}{x^5}+\frac{a}{x^3}\right)dx\\ &= -\frac{b}{4x^4}-\frac{a}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$-\frac{a}{2x^2}-\frac{b}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/x^3,x]
```

```
[Out] -1/4*b/x^4 - a/(2*x^2)
```

fricas [A] time = 0.85, size = 13, normalized size = 0.76

$$-\frac{2\,ax^2+b}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*x^2 + b)/x^4
```

giac [A] time = 0.15, size = 13, normalized size = 0.76

$$-\frac{2\,ax^2+b}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^3,x, algorithm="giac")
[Out] -1/4*(2*a*x^2 + b)/x^4
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$-\frac{a}{2x^2}-\frac{b}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/x^3,x)
[Out] -1/4*b/x^4-1/2*a/x^2
```

maxima [A] time = 1.03, size = 14, normalized size = 0.82

$$-\frac{\left(a+\frac{b}{x^2}\right)^2}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^3,x, algorithm="maxima")
[Out] -1/4*(a + b/x^2)^2/b
```

mupad [B] time = 0.03, size = 13, normalized size = 0.76

$$-\frac{2\,a\,x^2+b}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/x^3,x)
[Out] -(b + 2*a*x^2)/(4*x^4)
```

sympy [A] time = 0.12, size = 14, normalized size = 0.82

$$\frac{-2ax^2-b}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/x**3,x)
[Out] (-2*a*x**2 - b)/(4*x**4)
```

3.1814 $\int \frac{a+\frac{b}{x^2}}{x^4}\,dx$

Optimal. Leaf size=17

$$-\frac{a}{3x^3}-\frac{b}{5x^5}$$

```
[Out] -1/5*b/x^5-1/3*a/x^3
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$-\frac{a}{3x^3}-\frac{b}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/x^4,x]
```

```
[Out] -b/(5*x^5) - a/(3*x^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+\frac{b}{x^2}}{x^4}\,dx &= \int \left(\frac{b}{x^6}+\frac{a}{x^4}\right)dx \\ &= -\frac{b}{5x^5}-\frac{a}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$-\frac{a}{3x^3}-\frac{b}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/x^4,x]
```

```
[Out] -1/5*b/x^5 - a/(3*x^3)
```

fricas [A] time = 0.79, size = 15, normalized size = 0.88

$$-\frac{5\,ax^2+3\,b}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/15*(5*a*x^2 + 3*b)/x^5
```

giac [A] time = 0.15, size = 15, normalized size = 0.88

$$-\frac{5\,ax^2+3\,b}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^4,x, algorithm="giac")
[Out] -1/15*(5*a*x^2 + 3*b)/x^5
```

maple [A] time = 0.01, size = 14, normalized size = 0.82

$$-\frac{a}{3x^3}-\frac{b}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/x^4,x)
[Out] -1/5*b/x^5-1/3*a/x^3
```

maxima [A] time = 1.03, size = 15, normalized size = 0.88

$$-\frac{5\,ax^2+3\,b}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^4,x, algorithm="maxima")
[Out] -1/15*(5*a*x^2 + 3*b)/x^5
```

mupad [B] time = 0.03, size = 15, normalized size = 0.88

$$-\frac{5\,a\,x^2+3\,b}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/x^4,x)
[Out] -(3*b + 5*a*x^2)/(15*x^5)
```

sympy [A] time = 0.14, size = 15, normalized size = 0.88

$$\frac{-5ax^2-3b}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/x**4,x)
[Out] (-5*a*x**2 - 3*b)/(15*x**5)
```

3.1815 $\int \frac{a+\frac{b}{x^2}}{x^5}\,dx$

Optimal. Leaf size=17

$$-\frac{a}{4x^4}-\frac{b}{6x^6}$$

```
[Out] -1/6*b/x^6-1/4*a/x^4
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$-\frac{a}{4x^4}-\frac{b}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/x^5,x]
```

```
[Out] -b/(6*x^6) - a/(4*x^4)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+\frac{b}{x^2}}{x^5}\,dx &= \int \left(\frac{b}{x^7}+\frac{a}{x^5}\right)dx\\ &= -\frac{b}{6x^6}-\frac{a}{4x^4}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$-\frac{a}{4x^4}-\frac{b}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/x^5,x]
```

```
[Out] -1/6*b/x^6 - a/(4*x^4)
```

fricas [A] time = 0.61, size = 15, normalized size = 0.88

$$-\frac{3\,ax^2+2\,b}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/12*(3*a*x^2 + 2*b)/x^6
```

giac [A] time = 0.20, size = 15, normalized size = 0.88

$$-\frac{3\,ax^2+2\,b}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^5,x, algorithm="giac")
[Out] -1/12*(3*a*x^2 + 2*b)/x^6
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$-\frac{a}{4x^4}-\frac{b}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/x^5,x)
[Out] -1/6*b/x^6-1/4*a/x^4
```

maxima [A] time = 1.08, size = 15, normalized size = 0.88

$$-\frac{3\,ax^2+2\,b}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^5,x, algorithm="maxima")
[Out] -1/12*(3*a*x^2 + 2*b)/x^6
```

mupad [B] time = 0.03, size = 15, normalized size = 0.88

$$-\frac{3\,a\,x^2+2\,b}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/x^5,x)
[Out] -(2*b + 3*a*x^2)/(12*x^6)
```

sympy [A] time = 0.14, size = 15, normalized size = 0.88

$$\frac{-3ax^2-2b}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/x**5,x)
[Out] (-3*a*x**2 - 2*b)/(12*x**6)
```

3.1816 $\int \frac{a+\frac{b}{x^2}}{x^6}\,dx$

Optimal. Leaf size=17

$$-\frac{a}{5x^5}-\frac{b}{7x^7}$$

```
[Out] -1/7*b/x^7-1/5*a/x^5
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$-\frac{a}{5x^5}-\frac{b}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/x^6,x]
```

```
[Out] -b/(7*x^7) - a/(5*x^5)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+\frac{b}{x^2}}{x^6}\,dx &= \int \left(\frac{b}{x^8}+\frac{a}{x^6}\right)dx\\ &= -\frac{b}{7x^7}-\frac{a}{5x^5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$-\frac{a}{5x^5}-\frac{b}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/x^6,x]
```

```
[Out] -1/7*b/x^7 - a/(5*x^5)
```

fricas [A] time = 0.81, size = 15, normalized size = 0.88

$$-\frac{7\,ax^2+5\,b}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^6,x, algorithm="fricas")
```

```
[Out] -1/35*(7*a*x^2 + 5*b)/x^7
```

giac [A] time = 0.16, size = 15, normalized size = 0.88

$$-\frac{7\,ax^2+5\,b}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^6,x, algorithm="giac")
[Out] -1/35*(7*a*x^2 + 5*b)/x^7
```

maple [A] time = 0.01, size = 14, normalized size = 0.82

$$-\frac{a}{5x^5}-\frac{b}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/x^6,x)
[Out] -1/7*b/x^7-1/5*a/x^5
```

maxima [A] time = 1.00, size = 15, normalized size = 0.88

$$-\frac{7\,ax^2+5\,b}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^6,x, algorithm="maxima")
[Out] -1/35*(7*a*x^2 + 5*b)/x^7
```

mupad [B] time = 0.03, size = 15, normalized size = 0.88

$$-\frac{7\,a\,x^2+5\,b}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/x^6,x)
[Out] -(5*b + 7*a*x^2)/(35*x^7)
```

sympy [A] time = 0.16, size = 15, normalized size = 0.88

$$\frac{-7ax^2-5b}{35x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/x**6,x)
[Out] (-7*a*x**2 - 5*b)/(35*x**7)
```

3.1817 $\int \left(a + \frac{b}{x^2}\right)^2 x^6 \, dx$

Optimal. Leaf size=30

$$\frac{a^2 x^7}{7} + \frac{2}{5} abx^5 + \frac{b^2 x^3}{3}$$

```
[Out] 1/3*b^2*x^3+2/5*a*b*x^5+1/7*a^2*x^7
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 270}

$$\frac{a^2 x^7}{7} + \frac{2}{5} abx^5 + \frac{b^2 x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^2*x^6,x]
```

```
[Out] (b^2*x^3)/3 + (2*a*b*x^5)/5 + (a^2*x^7)/7
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(a + \frac{b}{x^2}\right)^2 x^6 \, dx &= \int x^2 \left(b + ax^2\right)^2 \, dx \\
&= \int \left(b^2 x^2 + 2abx^4 + a^2 x^6\right) \, dx \\
&= \frac{b^2 x^3}{3} + \frac{2}{5} abx^5 + \frac{a^2 x^7}{7}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2 x^7}{7} + \frac{2}{5} abx^5 + \frac{b^2 x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^2*x^6,x]
```

```
[Out] (b^2*x^3)/3 + (2*a*b*x^5)/5 + (a^2*x^7)/7
```

fricas [A] time = 0.77, size = 24, normalized size = 0.80

$$\frac{1}{7} a^2 x^7 + \frac{2}{5} abx^5 + \frac{1}{3} b^2 x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2*x^6,x, algorithm="fricas")
[Out] 1/7*a^2*x^7 + 2/5*a*b*x^5 + 1/3*b^2*x^3
```

giac [A] time = 0.22, size = 24, normalized size = 0.80

$$\frac{1}{7}a^2x^7 + \frac{2}{5}abx^5 + \frac{1}{3}b^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2*x^6,x, algorithm="giac")
[Out] 1/7*a^2*x^7 + 2/5*a*b*x^5 + 1/3*b^2*x^3
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$\frac{1}{7}a^2x^7 + \frac{2}{5}ab\,x^5 + \frac{1}{3}b^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^2*x^6,x)
[Out] 1/3*b^2*x^3+2/5*a*b*x^5+1/7*a^2*x^7
```

maxima [A] time = 0.80, size = 24, normalized size = 0.80

$$\frac{1}{7}a^2x^7 + \frac{2}{5}abx^5 + \frac{1}{3}b^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2*x^6,x, algorithm="maxima")
[Out] 1/7*a^2*x^7 + 2/5*a*b*x^5 + 1/3*b^2*x^3
```

mupad [B] time = 0.04, size = 24, normalized size = 0.80

$$\frac{a^2\,x^7}{7} + \frac{2\,a\,b\,x^5}{5} + \frac{b^2\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b/x^2)^2,x)
[Out] (a^2*x^7)/7 + (b^2*x^3)/3 + (2*a*b*x^5)/5
```

sympy [A] time = 0.07, size = 26, normalized size = 0.87

$$\frac{a^2x^7}{7} + \frac{2abx^5}{5} + \frac{b^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**2*x**6,x)
[Out] a**2*x**7/7 + 2*a*b*x**5/5 + b**2*x**3/3
```

3.1818 $\int \left(a + \frac{b}{x^2}\right)^2 x^5 \, dx$

Optimal. Leaf size=16

$$\frac{\left(ax^2 + b\right)^3}{6a}$$

```
[Out] 1/6*(a*x^2+b)^3/a
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 261}

$$\frac{\left(ax^2 + b\right)^3}{6a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^2*x^5,x]
```

```
[Out] (b + a*x^2)^3/(6*a)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(a + \frac{b}{x^2}\right)^2 x^5 \, dx &= \int x \left(b + ax^2\right)^2 \, dx \\ &= \frac{\left(b + ax^2\right)^3}{6a}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{\left(ax^2 + b\right)^3}{6a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^2*x^5,x]
```

```
[Out] (b + a*x^2)^3/(6*a)
```

fricas [A] time = 0.76, size = 24, normalized size = 1.50

$$\frac{1}{6} a^2 x^6 + \frac{1}{2} abx^4 + \frac{1}{2} b^2 x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2*x^5,x, algorithm="fricas")
[Out] 1/6*a^2*x^6 + 1/2*a*b*x^4 + 1/2*b^2*x^2
```

giac [A] time = 0.16, size = 24, normalized size = 1.50

$$\frac{1}{6}a^2x^6 + \frac{1}{2}abx^4 + \frac{1}{2}b^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2*x^5,x, algorithm="giac")
[Out] 1/6*a^2*x^6 + 1/2*a*b*x^4 + 1/2*b^2*x^2
```

maple [A] time = 0.00, size = 25, normalized size = 1.56

$$\frac{1}{6}a^2x^6 + \frac{1}{2}ab\,x^4 + \frac{1}{2}b^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^2*x^5,x)
[Out] 1/6*a^2*x^6+1/2*a*b*x^4+1/2*b^2*x^2
```

maxima [A] time = 0.88, size = 24, normalized size = 1.50

$$\frac{1}{6}a^2x^6 + \frac{1}{2}abx^4 + \frac{1}{2}b^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2*x^5,x, algorithm="maxima")
[Out] 1/6*a^2*x^6 + 1/2*a*b*x^4 + 1/2*b^2*x^2
```

mupad [B] time = 0.03, size = 24, normalized size = 1.50

$$\frac{a^2\,x^6}{6} + \frac{a\,b\,x^4}{2} + \frac{b^2\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b/x^2)^2,x)
[Out] (a^2*x^6)/6 + (b^2*x^2)/2 + (a*b*x^4)/2
```

sympy [B] time = 0.07, size = 24, normalized size = 1.50

$$\frac{a^2x^6}{6} + \frac{abx^4}{2} + \frac{b^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**2*x**5,x)
[Out] a**2*x**6/6 + a*b*x**4/2 + b**2*x**2/2
```

3.1819 $\int \left(a + \frac{b}{x^2}\right)^2 x^4\, dx$

Optimal. Leaf size=25

$$\frac{a^2x^5}{5} + \frac{2}{3}abx^3 + b^2x$$

```
[Out] b^2*x+2/3*a*b*x^3+1/5*a^2*x^5
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 194}

$$\frac{a^2x^5}{5} + \frac{2}{3}abx^3 + b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^2*x^4,x]
```

```
[Out] b^2*x + (2*a*b*x^3)/3 + (a^2*x^5)/5
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(a + \frac{b}{x^2}\right)^2 x^4\, dx &= \int \left(b + ax^2\right)^2\, dx \\ &= \int \left(b^2 + 2abx^2 + a^2x^4\right)\, dx \\ &= b^2x + \frac{2}{3}abx^3 + \frac{a^2x^5}{5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$\frac{a^2x^5}{5} + \frac{2}{3}abx^3 + b^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^2*x^4,x]
```

```
[Out] b^2*x + (2*a*b*x^3)/3 + (a^2*x^5)/5
```

fricas [A] time = 0.61, size = 21, normalized size = 0.84

$$\frac{1}{5}\, a^2x^5 + \frac{2}{3}\, abx^3 + b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2*x^4,x, algorithm="fricas")
[Out] 1/5*a^2*x^5 + 2/3*a*b*x^3 + b^2*x
```

giac [A] time = 0.18, size = 21, normalized size = 0.84

$$\frac{1}{5}a^2x^5 + \frac{2}{3}abx^3 + b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2*x^4,x, algorithm="giac")
[Out] 1/5*a^2*x^5 + 2/3*a*b*x^3 + b^2*x
```

maple [A] time = 0.00, size = 22, normalized size = 0.88

$$\frac{1}{5}a^2x^5 + \frac{2}{3}ab\,x^3 + b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^2*x^4,x)
[Out] b^2*x+2/3*a*b*x^3+1/5*a^2*x^5
```

maxima [A] time = 0.85, size = 21, normalized size = 0.84

$$\frac{1}{5}a^2x^5 + \frac{2}{3}abx^3 + b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2*x^4,x, algorithm="maxima")
[Out] 1/5*a^2*x^5 + 2/3*a*b*x^3 + b^2*x
```

mupad [B] time = 0.03, size = 21, normalized size = 0.84

$$\frac{a^2\,x^5}{5} + \frac{2\,a\,b\,x^3}{3} + b^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b/x^2)^2,x)
[Out] b^2*x + (a^2*x^5)/5 + (2*a*b*x^3)/3
```

sympy [A] time = 0.07, size = 22, normalized size = 0.88

$$\frac{a^2x^5}{5} + \frac{2abx^3}{3} + b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**2*x**4,x)
[Out] a**2*x**5/5 + 2*a*b*x**3/3 + b**2*x
```

3.1820 $\int \left(a + \frac{b}{x^2}\right)^2 x^3\, dx$

Optimal. Leaf size=23

$$\frac{a^2x^4}{4} + abx^2 + b^2\log(x)$$

```
[Out] a*b*x^2+1/4*a^2*x^4+b^2*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 266, 43}

$$\frac{a^2x^4}{4} + abx^2 + b^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^2*x^3,x]
```

```
[Out] a*b*x^2 + (a^2*x^4)/4 + b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a + \frac{b}{x^2}\right)^2 x^3\, dx &= \int \frac{\left(b + ax^2\right)^2}{x}\, dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{(b + ax)^2}{x}\, dx, x, x^2\right) \\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(2ab + \frac{b^2}{x} + a^2x\right) dx, x, x^2\right) \\
&= abx^2 + \frac{a^2x^4}{4} + b^2\log(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$\frac{a^2x^4}{4} + abx^2 + b^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^2*x^3,x]
[Out] a*b*x^2 + (a^2*x^4)/4 + b^2*Log[x]
```

fricas [A] time = 0.65, size = 21, normalized size = 0.91

$$\frac{1}{4}a^2x^4 + abx^2 + b^2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2*x^3,x, algorithm="fricas")
[Out] 1/4*a^2*x^4 + a*b*x^2 + b^2*log(x)
```

giac [A] time = 0.17, size = 22, normalized size = 0.96

$$\frac{1}{4}a^2x^4 + abx^2 + b^2\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2*x^3,x, algorithm="giac")
[Out] 1/4*a^2*x^4 + a*b*x^2 + b^2*log(abs(x))
```

maple [A] time = 0.00, size = 22, normalized size = 0.96

$$\frac{a^2x^4}{4} + ab\,x^2 + b^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^2*x^3,x)
[Out] a*b*x^2+1/4*a^2*x^4+b^2*ln(x)
```

maxima [A] time = 0.92, size = 24, normalized size = 1.04

$$\frac{1}{4}a^2x^4 + abx^2 + \frac{1}{2}b^2\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2*x^3,x, algorithm="maxima")
[Out] 1/4*a^2*x^4 + a*b*x^2 + 1/2*b^2*log(x^2)
```

mupad [B] time = 1.04, size = 21, normalized size = 0.91

$$b^2\,\ln(x) + \frac{a^2\,x^4}{4} + a\,b\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b/x^2)^2,x)
[Out] b^2*log(x) + (a^2*x^4)/4 + a*b*x^2
```

sympy [A] time = 0.11, size = 20, normalized size = 0.87

$$\frac{a^2x^4}{4} + abx^2 + b^2\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**2*x**3,x)
[Out] a**2*x**4/4 + a*b*x**2 + b**2*log(x)
```

3.1821 $\int \left(a + \frac{b}{x^2}\right)^2 x^2\, dx$

Optimal. Leaf size=24

$$\frac{a^2x^3}{3} + 2abx - \frac{b^2}{x}$$

```
[Out] -b^2/x+2*a*b*x+1/3*a^2*x^3
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 270}

$$\frac{a^2x^3}{3} + 2abx - \frac{b^2}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^2*x^2,x]
```

```
[Out] -(b^2/x) + 2*a*b*x + (a^2*x^3)/3
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a + \frac{b}{x^2}\right)^2 x^2\, dx &= \int \frac{\left(b + ax^2\right)^2}{x^2}\, dx \\ &= \int \left(2ab + \frac{b^2}{x^2} + a^2x^2\right) dx \\ &= -\frac{b^2}{x} + 2abx + \frac{a^2x^3}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$\frac{a^2x^3}{3} + 2abx - \frac{b^2}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^2*x^2,x]
```

```
[Out] -(b^2/x) + 2*a*b*x + (a^2*x^3)/3
```

fricas [A] time = 0.69, size = 25, normalized size = 1.04

$$\frac{a^2x^4 + 6\,abx^2 - 3\,b^2}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2*x^2,x, algorithm="fricas")
[Out] 1/3*(a^2*x^4 + 6*a*b*x^2 - 3*b^2)/x
```

giac [**A**] time = 0.15, size = 22, normalized size = 0.92

$$\frac{1}{3}\,a^2x^3 + 2\,abx - \frac{b^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2*x^2,x, algorithm="giac")
[Out] 1/3*a^2*x^3 + 2*a*b*x - b^2/x
```

maple [**A**] time = 0.00, size = 23, normalized size = 0.96

$$\frac{a^2x^3}{3} + 2abx - \frac{b^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^2*x^2,x)
[Out] -b^2/x+2*a*b*x+1/3*a^2*x^3
```

maxima [**A**] time = 0.90, size = 22, normalized size = 0.92

$$\frac{1}{3}\,a^2x^3 + 2\,abx - \frac{b^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2*x^2,x, algorithm="maxima")
[Out] 1/3*a^2*x^3 + 2*a*b*x - b^2/x
```

mupad [**B**] time = 0.03, size = 22, normalized size = 0.92

$$\frac{a^2\,x^3}{3} - \frac{b^2}{x} + 2\,a\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b/x^2)^2,x)
[Out] (a^2*x^3)/3 - b^2/x + 2*a*b*x
```

sympy [**A**] time = 0.10, size = 19, normalized size = 0.79

$$\frac{a^2x^3}{3} + 2abx - \frac{b^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**2*x**2,x)
[Out] a**2*x**3/3 + 2*a*b*x - b**2/x
```

3.1822 $\int \left(a + \frac{b}{x^2}\right)^2 x\,dx$

Optimal. Leaf size=27

$$\frac{a^2x^2}{2} + 2ab\log(x) - \frac{b^2}{2x^2}$$

```
[Out] -1/2*b^2/x^2+1/2*a^2*x^2+2*a*b*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {263, 266, 43}

$$\frac{a^2x^2}{2} + 2ab\log(x) - \frac{b^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^2*x,x]
```

```
[Out] -b^2/(2*x^2) + (a^2*x^2)/2 + 2*a*b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a + \frac{b}{x^2}\right)^2 x\,dx &= \int \frac{\left(b + ax^2\right)^2}{x^3}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{(b + ax)^2}{x^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(a^2 + \frac{b^2}{x^2} + \frac{2ab}{x}\right)\,dx, x, x^2\right) \\
&= -\frac{b^2}{2x^2} + \frac{a^2x^2}{2} + 2ab\log(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.00

$$\frac{a^2x^2}{2} + 2ab\log(x) - \frac{b^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^2*x,x]
[Out] -1/2*b^2/x^2 + (a^2*x^2)/2 + 2*a*b*Log[x]
```

fricas [A] time = 0.83, size = 27, normalized size = 1.00

$$\frac{a^2x^4 + 4\,abx^2\log(x) - b^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2*x,x, algorithm="fricas")
[Out] 1/2*(a^2*x^4 + 4*a*b*x^2*log(x) - b^2)/x^2
```

giac [A] time = 0.16, size = 24, normalized size = 0.89

$$\frac{1}{2}\,a^2x^2 + 2\,ab\log\left(|x|\right) - \frac{b^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2*x,x, algorithm="giac")
[Out] 1/2*a^2*x^2 + 2*a*b*log(abs(x)) - 1/2*b^2/x^2
```

maple [A] time = 0.01, size = 24, normalized size = 0.89

$$\frac{a^2x^2}{2} + 2ab\ln(x) - \frac{b^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^2*x,x)
[Out] -1/2*b^2/x^2+1/2*a^2*x^2+2*a*b*ln(x)
```

maxima [A] time = 0.88, size = 24, normalized size = 0.89

$$\frac{1}{2}\,a^2x^2 + ab\log\left(x^2\right) - \frac{b^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2*x,x, algorithm="maxima")
[Out] 1/2*a^2*x^2 + a*b*log(x^2) - 1/2*b^2/x^2
```

mupad [B] time = 1.06, size = 23, normalized size = 0.85

$$\frac{a^2\,x^2}{2} - \frac{b^2}{2\,x^2} + 2\,a\,b\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b/x^2)^2,x)
[Out] (a^2*x^2)/2 - b^2/(2*x^2) + 2*a*b*log(x)
```

sympy [A] time = 0.14, size = 24, normalized size = 0.89

$$\frac{a^2x^2}{2} + 2ab\log\left(x\right) - \frac{b^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**2*x,x)
[Out] a**2*x**2/2 + 2*a*b*log(x) - b**2/(2*x**2)
```

3.1823 $\int \left(a + \frac{b}{x^2}\right)^2 dx$

Optimal. Leaf size=23

$$a^2 x - \frac{2ab}{x} - \frac{b^2}{3x^3}$$

```
[Out] -1/3*b^2/x^3-2*a*b/x+a^2*x
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {193, 270}

$$a^2 x - \frac{2ab}{x} - \frac{b^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^2,x]
```

```
[Out] -b^2/(3*x^3) - (2*a*b)/x + a^2*x
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(a + \frac{b}{x^2}\right)^2 dx &= \int \frac{\left(b + ax^2\right)^2}{x^4} dx \\
&= \int \left(a^2 + \frac{b^2}{x^4} + \frac{2ab}{x^2}\right) dx \\
&= -\frac{b^2}{3x^3} - \frac{2ab}{x} + a^2 x
\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$a^2 x - \frac{2ab}{x} - \frac{b^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^2,x]
```

```
[Out] -1/3*b^2/x^3 - (2*a*b)/x + a^2*x
```

fricas [A] time = 0.74, size = 26, normalized size = 1.13

$$\frac{3\,a^2 x^4 - 6\,abx^2 - b^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2,x, algorithm="fricas")
[Out] 1/3*(3*a^2*x^4 - 6*a*b*x^2 - b^2)/x^3
```

giac [**A**] time = 0.15, size = 22, normalized size = 0.96

$$a^2x - \frac{6\,abx^2 + b^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2,x, algorithm="giac")
[Out] a^2*x - 1/3*(6*a*b*x^2 + b^2)/x^3
```

maple [**A**] time = 0.01, size = 22, normalized size = 0.96

$$a^2x - \frac{2ab}{x} - \frac{b^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^2,x)
[Out] -1/3*b^2/x^3-2*a*b/x+a^2*x
```

maxima [**A**] time = 0.86, size = 21, normalized size = 0.91

$$a^2x - \frac{2\,ab}{x} - \frac{b^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2,x, algorithm="maxima")
[Out] a^2*x - 2*a*b/x - 1/3*b^2/x^3
```

mupad [**B**] time = 0.03, size = 24, normalized size = 1.04

$$a^2\,x - \frac{\frac{b^2}{3} + 2\,a\,b\,x^2}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)^2,x)
[Out] a^2*x - (b^2/3 + 2*a*b*x^2)/x^3
```

sympy [**A**] time = 0.14, size = 22, normalized size = 0.96

$$a^2x + \frac{-6abx^2 - b^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**2,x)
[Out] a**2*x + (-6*a*b*x**2 - b**2)/(3*x**3)
```

3.1824 $\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x}\,dx$

Optimal. Leaf size=24

$$a^2\log(x)-\frac{ab}{x^2}-\frac{b^2}{4x^4}$$

```
[Out] -1/4*b^2/x^4-a*b/x^2+a^2*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 266, 43}

$$a^2\log(x)-\frac{ab}{x^2}-\frac{b^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^2/x,x]
```

```
[Out] -b^2/(4*x^4) - (a*b)/x^2 + a^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x}\,dx &= \int \frac{\left(b+ax^2\right)^2}{x^5}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{(b+ax)^2}{x^3}\,dx, x, x^2\right) \\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{b^2}{x^3}+\frac{2ab}{x^2}+\frac{a^2}{x}\right)\,dx, x, x^2\right) \\
&= -\frac{b^2}{4x^4}-\frac{ab}{x^2}+a^2\log(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$a^2\log(x)-\frac{ab}{x^2}-\frac{b^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^2/x,x]
[Out] -1/4*b^2/x^4 - (a*b)/x^2 + a^2*Log[x]
```

fricas [A] time = 0.75, size = 28, normalized size = 1.17

$$\frac{4\,a^2x^4\log(x) - 4\,abx^2 - b^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2/x,x, algorithm="fricas")
[Out] 1/4*(4*a^2*x^4*log(x) - 4*a*b*x^2 - b^2)/x^4
```

giac [A] time = 0.16, size = 34, normalized size = 1.42

$$\frac{1}{2}\,a^2\log\left(x^2\right) - \frac{3\,a^2x^4 + 4\,abx^2 + b^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2/x,x, algorithm="giac")
[Out] 1/2*a^2*log(x^2) - 1/4*(3*a^2*x^4 + 4*a*b*x^2 + b^2)/x^4
```

maple [A] time = 0.01, size = 23, normalized size = 0.96

$$a^2\ln(x) - \frac{ab}{x^2} - \frac{b^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^2/x,x)
[Out] -1/4*b^2/x^4-a*b/x^2+a^2*ln(x)
```

maxima [A] time = 0.88, size = 26, normalized size = 1.08

$$\frac{1}{2}\,a^2\log\left(x^2\right) - \frac{4\,abx^2 + b^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2/x,x, algorithm="maxima")
[Out] 1/2*a^2*log(x^2) - 1/4*(4*a*b*x^2 + b^2)/x^4
```

mupad [B] time = 0.05, size = 24, normalized size = 1.00

$$a^2\,\ln(x) - \frac{\frac{b^2}{4} + a\,b\,x^2}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)^2/x,x)
[Out] a^2*log(x) - (b^2/4 + a*b*x^2)/x^4
```

sympy [A] time = 0.18, size = 24, normalized size = 1.00

$$a^2\log(x) + \frac{-4abx^2 - b^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**2/x,x)
[Out] a**2*log(x) + (-4*a*b*x**2 - b**2)/(4*x**4)
```

3.1825 $\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^2}\,dx$

Optimal. Leaf size=28

$$-\frac{a^2}{x}-\frac{2ab}{3x^3}-\frac{b^2}{5x^5}$$

```
[Out] -1/5*b^2/x^5-2/3*a*b/x^3-a^2/x
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 270}

$$-\frac{a^2}{x}-\frac{2ab}{3x^3}-\frac{b^2}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^2/x^2,x]
```

```
[Out] -b^2/(5*x^5) - (2*a*b)/(3*x^3) - a^2/x
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^2}\,dx &= \int \frac{\left(b+ax^2\right)^2}{x^6}\,dx \\ &= \int \left(\frac{b^2}{x^6}+\frac{2ab}{x^4}+\frac{a^2}{x^2}\right)dx \\ &= -\frac{b^2}{5x^5}-\frac{2ab}{3x^3}-\frac{a^2}{x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 28, normalized size = 1.00

$$-\frac{a^2}{x}-\frac{2ab}{3x^3}-\frac{b^2}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^2/x^2,x]
```

```
[Out] -1/5*b^2/x^5 - (2*a*b)/(3*x^3) - a^2/x
```

fricas [A] time = 0.77, size = 26, normalized size = 0.93

$$-\frac{15\,a^2x^4+10\,abx^2+3\,b^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2/x^2,x, algorithm="fricas")
[Out] -1/15*(15*a^2*x^4 + 10*a*b*x^2 + 3*b^2)/x^5
```

giac [A] time = 0.15, size = 26, normalized size = 0.93

$$-\frac{15\,a^2x^4+10\,abx^2+3\,b^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2/x^2,x, algorithm="giac")
[Out] -1/15*(15*a^2*x^4 + 10*a*b*x^2 + 3*b^2)/x^5
```

maple [A] time = 0.01, size = 25, normalized size = 0.89

$$-\frac{a^2}{x}-\frac{2ab}{3x^3}-\frac{b^2}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^2/x^2,x)
[Out] -1/5*b^2/x^5-2/3*a*b/x^3-a^2/x
```

maxima [A] time = 0.83, size = 26, normalized size = 0.93

$$-\frac{15\,a^2x^4+10\,abx^2+3\,b^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2/x^2,x, algorithm="maxima")
[Out] -1/15*(15*a^2*x^4 + 10*a*b*x^2 + 3*b^2)/x^5
```

mupad [B] time = 0.04, size = 25, normalized size = 0.89

$$-\frac{a^2\,x^4+\frac{2\,a\,b\,x^2}{3}+\frac{b^2}{5}}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)^2/x^2,x)
[Out] -(b^2/5 + a^2*x^4 + (2*a*b*x^2)/3)/x^5
```

sympy [A] time = 0.19, size = 27, normalized size = 0.96

$$\frac{-15a^2x^4-10abx^2-3b^2}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**2/x**2,x)
[Out] (-15*a**2*x**4 - 10*a*b*x**2 - 3*b**2)/(15*x**5)
```

3.1826 $\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^3}\,dx$

Optimal. Leaf size=16

$$-\frac{\left(a+\frac{b}{x^2}\right)^3}{6b}$$

```
[Out] -1/6*(a+b/x^2)^3/b
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$-\frac{\left(a+\frac{b}{x^2}\right)^3}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^2/x^3,x]
```

```
[Out] -(a + b/x^2)^3/(6*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^3}\,dx = -\frac{\left(a+\frac{b}{x^2}\right)^3}{6b}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.88

$$-\frac{a^2}{2x^2} - \frac{ab}{2x^4} - \frac{b^2}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^2/x^3,x]
```

```
[Out] -1/6*b^2/x^6 - (a*b)/(2*x^4) - a^2/(2*x^2)
```

fricas [A] time = 0.74, size = 24, normalized size = 1.50

$$-\frac{3\,a^2x^4 + 3\,abx^2 + b^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/6*(3*a^2*x^4 + 3*a*b*x^2 + b^2)/x^6
```

giac [A] time = 0.15, size = 24, normalized size = 1.50

$$-\frac{3\,a^2x^4+3\,abx^2+b^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2/x^3,x, algorithm="giac")
[Out] -1/6*(3*a^2*x^4 + 3*a*b*x^2 + b^2)/x^6
```

maple [A] time = 0.00, size = 25, normalized size = 1.56

$$-\frac{a^2}{2x^2}-\frac{ab}{2x^4}-\frac{b^2}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^2/x^3,x)
[Out] -1/2*a*b/x^4-1/2*a^2/x^2-1/6*b^2/x^6
```

maxima [A] time = 0.83, size = 14, normalized size = 0.88

$$-\frac{\left(a+\frac{b}{x^2}\right)^3}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2/x^3,x, algorithm="maxima")
[Out] -1/6*(a + b/x^2)^3/b
```

mupad [B] time = 0.04, size = 26, normalized size = 1.62

$$-\frac{\frac{a^2x^4}{2}+\frac{a\,b\,x^2}{2}+\frac{b^2}{6}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)^2/x^3,x)
[Out] -(b^2/6 + (a^2*x^4)/2 + (a*b*x^2)/2)/x^6
```

sympy [B] time = 0.19, size = 26, normalized size = 1.62

$$\frac{-3a^2x^4-3abx^2-b^2}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**2/x**3,x)
[Out] (-3*a**2*x**4 - 3*a*b*x**2 - b**2)/(6*x**6)
```

3.1827 $\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^4}\,dx$

Optimal. Leaf size=30

$$-\frac{a^2}{3x^3}-\frac{2ab}{5x^5}-\frac{b^2}{7x^7}$$

```
[Out] -1/7*b^2/x^7-2/5*a*b/x^5-1/3*a^2/x^3
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 270}

$$-\frac{a^2}{3x^3}-\frac{2ab}{5x^5}-\frac{b^2}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^2/x^4,x]
```

```
[Out] -b^2/(7*x^7) - (2*a*b)/(5*x^5) - a^2/(3*x^3)
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^4}\,dx &= \int \frac{\left(b+ax^2\right)^2}{x^8}\,dx \\ &= \int \left(\frac{b^2}{x^8}+\frac{2ab}{x^6}+\frac{a^2}{x^4}\right)dx \\ &= -\frac{b^2}{7x^7}-\frac{2ab}{5x^5}-\frac{a^2}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$-\frac{a^2}{3x^3}-\frac{2ab}{5x^5}-\frac{b^2}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^2/x^4,x]
```

```
[Out] -1/7*b^2/x^7 - (2*a*b)/(5*x^5) - a^2/(3*x^3)
```

fricas [A] time = 0.66, size = 26, normalized size = 0.87

$$-\frac{35\,a^2x^4+42\,abx^2+15\,b^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2/x^4,x, algorithm="fricas")
[Out] -1/105*(35*a^2*x^4 + 42*a*b*x^2 + 15*b^2)/x^7
```

giac [A] time = 0.17, size = 26, normalized size = 0.87

$$-\frac{35\,a^2x^4+42\,abx^2+15\,b^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2/x^4,x, algorithm="giac")
[Out] -1/105*(35*a^2*x^4 + 42*a*b*x^2 + 15*b^2)/x^7
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$-\frac{a^2}{3x^3}-\frac{2ab}{5x^5}-\frac{b^2}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^2/x^4,x)
[Out] -1/7*b^2/x^7-2/5*a*b/x^5-1/3*a^2/x^3
```

maxima [A] time = 0.88, size = 26, normalized size = 0.87

$$-\frac{35\,a^2x^4+42\,abx^2+15\,b^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2/x^4,x, algorithm="maxima")
[Out] -1/105*(35*a^2*x^4 + 42*a*b*x^2 + 15*b^2)/x^7
```

mupad [B] time = 0.04, size = 26, normalized size = 0.87

$$-\frac{\frac{a^2x^4}{3}+\frac{2\,a\,b\,x^2}{5}+\frac{b^2}{7}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)^2/x^4,x)
[Out] -(b^2/7 + (a^2*x^4)/3 + (2*a*b*x^2)/5)/x^7
```

sympy [A] time = 0.21, size = 27, normalized size = 0.90

$$\frac{-35a^2x^4-42abx^2-15b^2}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**2/x**4,x)
[Out] (-35*a**2*x**4 - 42*a*b*x**2 - 15*b**2)/(105*x**7)
```

3.1828 $\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^5}\,dx$

Optimal. Leaf size=30

$$-\frac{a^2}{4x^4}-\frac{ab}{3x^6}-\frac{b^2}{8x^8}$$

```
[Out] -1/8*b^2/x^8-1/3*a*b/x^6-1/4*a^2/x^4
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 266, 43}

$$-\frac{a^2}{4x^4}-\frac{ab}{3x^6}-\frac{b^2}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^2/x^5,x]
```

```
[Out] -b^2/(8*x^8) - (a*b)/(3*x^6) - a^2/(4*x^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^5}\,dx &= \int \frac{\left(b+ax^2\right)^2}{x^9}\,dx\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(b+ax)^2}{x^5}\,dx, x, x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{b^2}{x^5}+\frac{2ab}{x^4}+\frac{a^2}{x^3}\right)\,dx, x, x^2\right)\\ &= -\frac{b^2}{8x^8}-\frac{ab}{3x^6}-\frac{a^2}{4x^4}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$-\frac{a^2}{4x^4}-\frac{ab}{3x^6}-\frac{b^2}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^2/x^5,x]
[Out] -1/8*b^2/x^8 - (a*b)/(3*x^6) - a^2/(4*x^4)
```

fricas [A] time = 0.84, size = 26, normalized size = 0.87

$$-\frac{6\,a^2x^4+8\,abx^2+3\,b^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2/x^5,x, algorithm="fricas")
[Out] -1/24*(6*a^2*x^4 + 8*a*b*x^2 + 3*b^2)/x^8
```

giac [A] time = 0.15, size = 26, normalized size = 0.87

$$-\frac{6\,a^2x^4+8\,abx^2+3\,b^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2/x^5,x, algorithm="giac")
[Out] -1/24*(6*a^2*x^4 + 8*a*b*x^2 + 3*b^2)/x^8
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$-\frac{a^2}{4x^4}-\frac{ab}{3x^6}-\frac{b^2}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^2/x^5,x)
[Out] -1/8*b^2/x^8-1/3*a*b/x^6-1/4*a^2/x^4
```

maxima [A] time = 0.88, size = 26, normalized size = 0.87

$$-\frac{6\,a^2x^4+8\,abx^2+3\,b^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2/x^5,x, algorithm="maxima")
[Out] -1/24*(6*a^2*x^4 + 8*a*b*x^2 + 3*b^2)/x^8
```

mupad [B] time = 0.04, size = 26, normalized size = 0.87

$$-\frac{\frac{a^2x^4}{4}+\frac{a\,b\,x^2}{3}+\frac{b^2}{8}}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)^2/x^5,x)
[Out] -(b^2/8 + (a^2*x^4)/4 + (a*b*x^2)/3)/x^8
```

sympy [A] time = 0.22, size = 27, normalized size = 0.90

$$\frac{-6a^2x^4-8abx^2-3b^2}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**2/x**5,x)
[Out] (-6*a**2*x**4 - 8*a*b*x**2 - 3*b**2)/(24*x**8)
```

3.1829 $\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^6}\,dx$

Optimal. Leaf size=30

$$-\frac{a^2}{5x^5}-\frac{2ab}{7x^7}-\frac{b^2}{9x^9}$$

```
[Out] -1/9*b^2/x^9-2/7*a*b/x^7-1/5*a^2/x^5
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 270}

$$-\frac{a^2}{5x^5}-\frac{2ab}{7x^7}-\frac{b^2}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^2/x^6,x]
```

```
[Out] -b^2/(9*x^9) - (2*a*b)/(7*x^7) - a^2/(5*x^5)
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^6}\,dx &= \int \frac{\left(b+ax^2\right)^2}{x^{10}}\,dx \\ &= \int \left(\frac{b^2}{x^{10}}+\frac{2ab}{x^8}+\frac{a^2}{x^6}\right)dx \\ &= -\frac{b^2}{9x^9}-\frac{2ab}{7x^7}-\frac{a^2}{5x^5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$-\frac{a^2}{5x^5}-\frac{2ab}{7x^7}-\frac{b^2}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^2/x^6,x]
```

```
[Out] -1/9*b^2/x^9 - (2*a*b)/(7*x^7) - a^2/(5*x^5)
```

fricas [A] time = 0.63, size = 26, normalized size = 0.87

$$-\frac{63\,a^2x^4+90\,abx^2+35\,b^2}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2/x^6,x, algorithm="fricas")
[Out] -1/315*(63*a^2*x^4 + 90*a*b*x^2 + 35*b^2)/x^9
```

giac [A] time = 0.15, size = 26, normalized size = 0.87

$$-\frac{63\,a^2x^4+90\,abx^2+35\,b^2}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2/x^6,x, algorithm="giac")
[Out] -1/315*(63*a^2*x^4 + 90*a*b*x^2 + 35*b^2)/x^9
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$-\frac{a^2}{5x^5}-\frac{2ab}{7x^7}-\frac{b^2}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^2/x^6,x)
[Out] -1/9*b^2/x^9-2/7*a*b/x^7-1/5*a^2/x^5
```

maxima [A] time = 0.88, size = 26, normalized size = 0.87

$$-\frac{63\,a^2x^4+90\,abx^2+35\,b^2}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2/x^6,x, algorithm="maxima")
[Out] -1/315*(63*a^2*x^4 + 90*a*b*x^2 + 35*b^2)/x^9
```

mupad [B] time = 0.04, size = 26, normalized size = 0.87

$$-\frac{\frac{a^2x^4}{5}+\frac{2\,a\,b\,x^2}{7}+\frac{b^2}{9}}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)^2/x^6,x)
[Out] -(b^2/9 + (a^2*x^4)/5 + (2*a*b*x^2)/7)/x^9
```

sympy [A] time = 0.23, size = 27, normalized size = 0.90

$$\frac{-63a^2x^4-90abx^2-35b^2}{315x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**2/x**6,x)
[Out] (-63*a**2*x**4 - 90*a*b*x**2 - 35*b**2)/(315*x**9)
```

3.1830 $\int \left(a + \frac{b}{x^2}\right)^3 x^6\, dx$

Optimal. Leaf size=35

$$\frac{a^3x^7}{7} + \frac{3}{5}a^2bx^5 + ab^2x^3 + b^3x$$

```
[Out] b^3*x+a*b^2*x^3+3/5*a^2*b*x^5+1/7*a^3*x^7
```

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 194}

$$\frac{3}{5}a^2bx^5 + \frac{a^3x^7}{7} + ab^2x^3 + b^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^3*x^6,x]
```

```
[Out] b^3*x + a*b^2*x^3 + (3*a^2*b*x^5)/5 + (a^3*x^7)/7
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(a + \frac{b}{x^2}\right)^3 x^6\, dx &= \int \left(b + ax^2\right)^3\, dx \\ &= \int \left(b^3 + 3ab^2x^2 + 3a^2bx^4 + a^3x^6\right)\, dx \\ &= b^3x + ab^2x^3 + \frac{3}{5}a^2bx^5 + \frac{a^3x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 35, normalized size = 1.00

$$\frac{a^3x^7}{7} + \frac{3}{5}a^2bx^5 + ab^2x^3 + b^3x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^3*x^6,x]
```

```
[Out] b^3*x + a*b^2*x^3 + (3*a^2*b*x^5)/5 + (a^3*x^7)/7
```

fricas [A] time = 1.31, size = 31, normalized size = 0.89

$$\frac{1}{7}\,a^3x^7 + \frac{3}{5}\,a^2bx^5 + ab^2x^3 + b^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^3*x^6,x, algorithm="fricas")
[Out] 1/7*a^3*x^7 + 3/5*a^2*b*x^5 + a*b^2*x^3 + b^3*x
```

giac [A] time = 0.15, size = 31, normalized size = 0.89

$$\frac{1}{7}a^3x^7 + \frac{3}{5}a^2bx^5 + ab^2x^3 + b^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^3*x^6,x, algorithm="giac")
[Out] 1/7*a^3*x^7 + 3/5*a^2*b*x^5 + a*b^2*x^3 + b^3*x
```

maple [A] time = 0.00, size = 32, normalized size = 0.91

$$\frac{1}{7}a^3x^7 + \frac{3}{5}a^2b\,x^5 + a\,b^2x^3 + b^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^3*x^6,x)
[Out] b^3*x+a*b^2*x^3+3/5*a^2*b*x^5+1/7*a^3*x^7
```

maxima [A] time = 0.87, size = 31, normalized size = 0.89

$$\frac{1}{7}a^3x^7 + \frac{3}{5}a^2bx^5 + ab^2x^3 + b^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^3*x^6,x, algorithm="maxima")
[Out] 1/7*a^3*x^7 + 3/5*a^2*b*x^5 + a*b^2*x^3 + b^3*x
```

mupad [B] time = 0.04, size = 31, normalized size = 0.89

$$\frac{a^3\,x^7}{7} + \frac{3\,a^2\,b\,x^5}{5} + a\,b^2\,x^3 + b^3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b/x^2)^3,x)
[Out] b^3*x + (a^3*x^7)/7 + a*b^2*x^3 + (3*a^2*b*x^5)/5
```

sympy [A] time = 0.07, size = 32, normalized size = 0.91

$$\frac{a^3x^7}{7} + \frac{3a^2bx^5}{5} + ab^2x^3 + b^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**3*x**6,x)
[Out] a**3*x**7/7 + 3*a**2*b*x**5/5 + a*b**2*x**3 + b**3*x
```

3.1831 $\int \left(a + \frac{b}{x^2}\right)^3 x^5\, dx$

Optimal. Leaf size=39

$$\frac{a^3x^6}{6} + \frac{3}{4}a^2bx^4 + \frac{3}{2}ab^2x^2 + b^3\log(x)$$

```
[Out] 3/2*a*b^2*x^2+3/4*a^2*b*x^4+1/6*a^3*x^6+b^3*ln(x)
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 266, 43}

$$\frac{3}{4}a^2bx^4 + \frac{a^3x^6}{6} + \frac{3}{2}ab^2x^2 + b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^3*x^5,x]
```

```
[Out] (3*a*b^2*x^2)/2 + (3*a^2*b*x^4)/4 + (a^3*x^6)/6 + b^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a + \frac{b}{x^2}\right)^3 x^5\, dx &= \int \frac{\left(b + ax^2\right)^3}{x}\, dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{(b + ax)^3}{x}\, dx, x, x^2\right) \\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(3ab^2 + \frac{b^3}{x} + 3a^2bx + a^3x^2\right) dx, x, x^2\right) \\
&= \frac{3}{2}ab^2x^2 + \frac{3}{4}a^2bx^4 + \frac{a^3x^6}{6} + b^3\log(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 39, normalized size = 1.00

$$\frac{a^3x^6}{6} + \frac{3}{4}a^2bx^4 + \frac{3}{2}ab^2x^2 + b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^3*x^5,x]
[Out] (3*a*b^2*x^2)/2 + (3*a^2*b*x^4)/4 + (a^3*x^6)/6 + b^3*Log[x]
```

fricas [A] time = 0.89, size = 33, normalized size = 0.85

$$\frac{1}{6}a^3x^6 + \frac{3}{4}a^2bx^4 + \frac{3}{2}ab^2x^2 + b^3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^3*x^5,x, algorithm="fricas")
[Out] 1/6*a^3*x^6 + 3/4*a^2*b*x^4 + 3/2*a*b^2*x^2 + b^3*log(x)
```

giac [A] time = 0.16, size = 36, normalized size = 0.92

$$\frac{1}{6}a^3x^6 + \frac{3}{4}a^2bx^4 + \frac{3}{2}ab^2x^2 + \frac{1}{2}b^3\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^3*x^5,x, algorithm="giac")
[Out] 1/6*a^3*x^6 + 3/4*a^2*b*x^4 + 3/2*a*b^2*x^2 + 1/2*b^3*log(x^2)
```

maple [A] time = 0.00, size = 34, normalized size = 0.87

$$\frac{a^3x^6}{6} + \frac{3a^2b\,x^4}{4} + \frac{3a\,b^2x^2}{2} + b^3\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^3*x^5,x)
[Out] 3/2*a*b^2*x^2+3/4*a^2*b*x^4+1/6*a^3*x^6+b^3*ln(x)
```

maxima [A] time = 0.87, size = 36, normalized size = 0.92

$$\frac{1}{6}a^3x^6 + \frac{3}{4}a^2bx^4 + \frac{3}{2}ab^2x^2 + \frac{1}{2}b^3\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^3*x^5,x, algorithm="maxima")
[Out] 1/6*a^3*x^6 + 3/4*a^2*b*x^4 + 3/2*a*b^2*x^2 + 1/2*b^3*log(x^2)
```

mupad [B] time = 0.04, size = 33, normalized size = 0.85

$$b^3\,\ln(x) + \frac{a^3\,x^6}{6} + \frac{3\,a\,b^2\,x^2}{2} + \frac{3\,a^2\,b\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b/x^2)^3,x)
[Out] b^3*log(x) + (a^3*x^6)/6 + (3*a*b^2*x^2)/2 + (3*a^2*b*x^4)/4
```

sympy [A] time = 0.12, size = 37, normalized size = 0.95

$$\frac{a^3x^6}{6} + \frac{3a^2bx^4}{4} + \frac{3ab^2x^2}{2} + b^3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**3*x**5,x)
[Out] a**3*x**6/6 + 3*a**2*b*x**4/4 + 3*a*b**2*x**2/2 + b**3*log(x)
```

3.1832 $\int \left(a + \frac{b}{x^2}\right)^3 x^4\, dx$

Optimal. Leaf size=34

$$\frac{a^3x^5}{5} + a^2bx^3 + 3ab^2x - \frac{b^3}{x}$$

```
[Out] -b^3/x+3*a*b^2*x+a^2*b*x^3+1/5*a^3*x^5
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 270}

$$a^2bx^3 + \frac{a^3x^5}{5} + 3ab^2x - \frac{b^3}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^3*x^4,x]
```

```
[Out] -(b^3/x) + 3*a*b^2*x + a^2*b*x^3 + (a^3*x^5)/5
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a + \frac{b}{x^2}\right)^3 x^4\, dx &= \int \frac{\left(b + ax^2\right)^3}{x^2}\, dx \\ &= \int \left(3ab^2 + \frac{b^3}{x^2} + 3a^2bx^2 + a^3x^4\right) dx \\ &= -\frac{b^3}{x} + 3ab^2x + a^2bx^3 + \frac{a^3x^5}{5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 34, normalized size = 1.00

$$\frac{a^3x^5}{5} + a^2bx^3 + 3ab^2x - \frac{b^3}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^3*x^4,x]
```

```
[Out] -(b^3/x) + 3*a*b^2*x + a^2*b*x^3 + (a^3*x^5)/5
```

fricas [A] time = 0.53, size = 36, normalized size = 1.06

$$\frac{a^3x^6 + 5\,a^2bx^4 + 15\,ab^2x^2 - 5\,b^3}{5\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^3*x^4,x, algorithm="fricas")
[Out] 1/5*(a^3*x^6 + 5*a^2*b*x^4 + 15*a*b^2*x^2 - 5*b^3)/x
```

giac [A] time = 0.17, size = 32, normalized size = 0.94

$$\frac{1}{5}a^3x^5 + a^2bx^3 + 3ab^2x - \frac{b^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^3*x^4,x, algorithm="giac")
[Out] 1/5*a^3*x^5 + a^2*b*x^3 + 3*a*b^2*x - b^3/x
```

maple [A] time = 0.00, size = 33, normalized size = 0.97

$$\frac{a^3x^5}{5} + a^2b\,x^3 + 3a\,b^2x - \frac{b^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^3*x^4,x)
[Out] -b^3/x+3*a*b^2*x+a^2*b*x^3+1/5*a^3*x^5
```

maxima [A] time = 0.91, size = 32, normalized size = 0.94

$$\frac{1}{5}a^3x^5 + a^2bx^3 + 3ab^2x - \frac{b^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^3*x^4,x, algorithm="maxima")
[Out] 1/5*a^3*x^5 + a^2*b*x^3 + 3*a*b^2*x - b^3/x
```

mupad [B] time = 0.04, size = 32, normalized size = 0.94

$$\frac{a^3\,x^5}{5} - \frac{b^3}{x} + a^2\,b\,x^3 + 3\,a\,b^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b/x^2)^3,x)
[Out] (a^3*x^5)/5 - b^3/x + a^2*b*x^3 + 3*a*b^2*x
```

sympy [A] time = 0.12, size = 29, normalized size = 0.85

$$\frac{a^3x^5}{5} + a^2bx^3 + 3ab^2x - \frac{b^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**3*x**4,x)
[Out] a**3*x**5/5 + a**2*b*x**3 + 3*a*b**2*x - b**3/x
```

3.1833 $\int \left(a + \frac{b}{x^2}\right)^3 x^3\, dx$

Optimal. Leaf size=40

$$\frac{a^3x^4}{4} + \frac{3}{2}a^2bx^2 + 3ab^2\log(x) - \frac{b^3}{2x^2}$$

```
[Out] -1/2*b^3/x^2+3/2*a^2*b*x^2+1/4*a^3*x^4+3*a*b^2*ln(x)
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 266, 43}

$$\frac{3}{2}a^2bx^2 + \frac{a^3x^4}{4} + 3ab^2\log(x) - \frac{b^3}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^3*x^3,x]
```

```
[Out] -b^3/(2*x^2) + (3*a^2*b*x^2)/2 + (a^3*x^4)/4 + 3*a*b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a + \frac{b}{x^2}\right)^3 x^3\, dx &= \int \frac{\left(b + ax^2\right)^3}{x^3}\, dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{(b + ax)^3}{x^2}\, dx, x, x^2\right) \\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(3a^2b + \frac{b^3}{x^2} + \frac{3ab^2}{x} + a^3x\right) dx, x, x^2\right) \\
&= -\frac{b^3}{2x^2} + \frac{3}{2}a^2bx^2 + \frac{a^3x^4}{4} + 3ab^2\log(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.00

$$\frac{a^3x^4}{4} + \frac{3}{2}a^2bx^2 + 3ab^2\log(x) - \frac{b^3}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^3*x^3,x]
[Out] -1/2*b^3/x^2 + (3*a^2*b*x^2)/2 + (a^3*x^4)/4 + 3*a*b^2*Log[x]
```

fricas [A] time = 0.95, size = 38, normalized size = 0.95

$$\frac{a^3x^6 + 6\,a^2bx^4 + 12\,ab^2x^2\log(x) - 2\,b^3}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^3*x^3,x, algorithm="fricas")
[Out] 1/4*(a^3*x^6 + 6*a^2*b*x^4 + 12*a*b^2*x^2*log(x) - 2*b^3)/x^2
```

giac [A] time = 0.15, size = 35, normalized size = 0.88

$$\frac{1}{4}\,a^3x^4 + \frac{3}{2}\,a^2bx^2 + 3\,ab^2\log\left(|x|\right) - \frac{b^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^3*x^3,x, algorithm="giac")
[Out] 1/4*a^3*x^4 + 3/2*a^2*b*x^2 + 3*a*b^2*log(abs(x)) - 1/2*b^3/x^2
```

maple [A] time = 0.01, size = 35, normalized size = 0.88

$$\frac{a^3x^4}{4} + \frac{3a^2b\,x^2}{2} + 3a\,b^2\ln(x) - \frac{b^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^3*x^3,x)
[Out] -1/2*b^3/x^2+3/2*a^2*b*x^2+1/4*a^3*x^4+3*a*b^2*ln(x)
```

maxima [A] time = 0.88, size = 36, normalized size = 0.90

$$\frac{1}{4}\,a^3x^4 + \frac{3}{2}\,a^2bx^2 + \frac{3}{2}\,ab^2\log\left(x^2\right) - \frac{b^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^3*x^3,x, algorithm="maxima")
[Out] 1/4*a^3*x^4 + 3/2*a^2*b*x^2 + 3/2*a*b^2*log(x^2) - 1/2*b^3/x^2
```

mupad [B] time = 0.04, size = 34, normalized size = 0.85

$$\frac{a^3\,x^4}{4} - \frac{b^3}{2\,x^2} + \frac{3\,a^2\,b\,x^2}{2} + 3\,a\,b^2\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b/x^2)^3,x)
[Out] (a^3*x^4)/4 - b^3/(2*x^2) + (3*a^2*b*x^2)/2 + 3*a*b^2*log(x)
```

sympy [A] time = 0.15, size = 37, normalized size = 0.92

$$\frac{a^3x^4}{4} + \frac{3a^2bx^2}{2} + 3ab^2\log(x) - \frac{b^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**3*x**3,x)
[Out] a**3*x**4/4 + 3*a**2*b*x**2/2 + 3*a*b**2*log(x) - b**3/(2*x**2)
```

3.1834 $\int \left(a+\frac{b}{x^2}\right)^3 x^2\,dx$

Optimal. Leaf size=37

$$\frac{a^3x^3}{3}+3a^2bx-\frac{3ab^2}{x}-\frac{b^3}{3x^3}$$

```
[Out] -1/3*b^3/x^3-3*a*b^2/x+3*a^2*b*x+1/3*a^3*x^3
```

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 270}

$$3a^2bx+\frac{a^3x^3}{3}-\frac{3ab^2}{x}-\frac{b^3}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^3*x^2,x]
```

```
[Out] -b^3/(3*x^3) - (3*a*b^2)/x + 3*a^2*b*x + (a^3*x^3)/3
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x^2}\right)^3 x^2\,dx &= \int \frac{\left(b+ax^2\right)^3}{x^4}\,dx\\ &= \int \left(3a^2b+\frac{b^3}{x^4}+\frac{3ab^2}{x^2}+a^3x^2\right)dx\\ &= -\frac{b^3}{3x^3}-\frac{3ab^2}{x}+3a^2bx+\frac{a^3x^3}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 37, normalized size = 1.00

$$\frac{a^3x^3}{3}+3a^2bx-\frac{3ab^2}{x}-\frac{b^3}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^3*x^2,x]
```

```
[Out] -1/3*b^3/x^3 - (3*a*b^2)/x + 3*a^2*b*x + (a^3*x^3)/3
```

fricas [A] time = 0.76, size = 36, normalized size = 0.97

$$\frac{a^3x^6+9\,a^2bx^4-9\,ab^2x^2-b^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^3*x^2,x, algorithm="fricas")
[Out] 1/3*(a^3*x^6 + 9*a^2*b*x^4 - 9*a*b^2*x^2 - b^3)/x^3
```

giac [A] time = 0.19, size = 34, normalized size = 0.92

$$\frac{1}{3}\,a^3x^3 + 3\,a^2bx - \frac{9\,ab^2x^2 + b^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^3*x^2,x, algorithm="giac")
[Out] 1/3*a^3*x^3 + 3*a^2*b*x - 1/3*(9*a*b^2*x^2 + b^3)/x^3
```

maple [A] time = 0.01, size = 34, normalized size = 0.92

$$\frac{a^3x^3}{3} + 3a^2bx - \frac{3a\,b^2}{x} - \frac{b^3}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^3*x^2,x)
[Out] -1/3*b^3/x^3-3*a*b^2/x+3*a^2*b*x+1/3*a^3*x^3
```

maxima [A] time = 0.82, size = 34, normalized size = 0.92

$$\frac{1}{3}\,a^3x^3 + 3\,a^2bx - \frac{9\,ab^2x^2 + b^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^3*x^2,x, algorithm="maxima")
[Out] 1/3*a^3*x^3 + 3*a^2*b*x - 1/3*(9*a*b^2*x^2 + b^3)/x^3
```

mupad [B] time = 1.07, size = 36, normalized size = 0.97

$$\frac{a^3\,x^3}{3} - \frac{\frac{b^3}{3} + 3\,a\,b^2\,x^2}{x^3} + 3\,a^2\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b/x^2)^3,x)
[Out] (a^3*x^3)/3 - (b^3/3 + 3*a*b^2*x^2)/x^3 + 3*a^2*b*x
```

sympy [A] time = 0.16, size = 36, normalized size = 0.97

$$\frac{a^3x^3}{3} + 3a^2bx + \frac{-9ab^2x^2 - b^3}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**3*x**2,x)
[Out] a**3*x**3/3 + 3*a**2*b*x + (-9*a*b**2*x**2 - b**3)/(3*x**3)
```

3.1835 $\int \left(a + \frac{b}{x^2}\right)^3 x\,dx$

Optimal. Leaf size=40

$$\frac{a^3x^2}{2} + 3a^2b\log(x) - \frac{3ab^2}{2x^2} - \frac{b^3}{4x^4}$$

```
[Out] -1/4*b^3/x^4-3/2*a*b^2/x^2+1/2*a^3*x^2+3*a^2*b*ln(x)
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {263, 266, 43}

$$3a^2b\log(x) + \frac{a^3x^2}{2} - \frac{3ab^2}{2x^2} - \frac{b^3}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^3*x,x]
```

```
[Out] -b^3/(4*x^4) - (3*a*b^2)/(2*x^2) + (a^3*x^2)/2 + 3*a^2*b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a + \frac{b}{x^2}\right)^3 x\,dx &= \int \frac{\left(b + ax^2\right)^3}{x^5}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{(b + ax)^3}{x^3}\,dx, x, x^2\right) \\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(a^3 + \frac{b^3}{x^3} + \frac{3ab^2}{x^2} + \frac{3a^2b}{x}\right)\,dx, x, x^2\right) \\
&= -\frac{b^3}{4x^4} - \frac{3ab^2}{2x^2} + \frac{a^3x^2}{2} + 3a^2b\log(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.00

$$\frac{a^3x^2}{2} + 3a^2b\log(x) - \frac{3ab^2}{2x^2} - \frac{b^3}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^3*x,x]
```

```
[Out] -1/4*b^3/x^4 - (3*a*b^2)/(2*x^2) + (a^3*x^2)/2 + 3*a^2*b*Log[x]
```

fricas [A] time = 0.91, size = 39, normalized size = 0.98

$$\frac{2\,a^3x^6 + 12\,a^2bx^4\log(x) - 6\,ab^2x^2 - b^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^3*x,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^3*x^6 + 12*a^2*b*x^4*log(x) - 6*a*b^2*x^2 - b^3)/x^4
```

giac [A] time = 0.15, size = 36, normalized size = 0.90

$$\frac{1}{2}\,a^3x^2 + 3\,a^2b\log\left(|x|\right) - \frac{6\,ab^2x^2 + b^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^3*x,x, algorithm="giac")
```

```
[Out] 1/2*a^3*x^2 + 3*a^2*b*log(abs(x)) - 1/4*(6*a*b^2*x^2 + b^3)/x^4
```

maple [A] time = 0.01, size = 35, normalized size = 0.88

$$\frac{a^3x^2}{2} + 3a^2b\ln(x) - \frac{3a\,b^2}{2x^2} - \frac{b^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^3*x,x)
```

```
[Out] -1/4*b^3/x^4-3/2*a*b^2/x^2+1/2*a^3*x^2+3*a^2*b*ln(x)
```

maxima [A] time = 0.88, size = 37, normalized size = 0.92

$$\frac{1}{2}\,a^3x^2 + \frac{3}{2}\,a^2b\log\left(x^2\right) - \frac{6\,ab^2x^2 + b^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^3*x,x, algorithm="maxima")
```

```
[Out] 1/2*a^3*x^2 + 3/2*a^2*b*log(x^2) - 1/4*(6*a*b^2*x^2 + b^3)/x^4
```

mupad [B] time = 0.04, size = 37, normalized size = 0.92

$$\frac{a^3\,x^2}{2} - \frac{\frac{b^3}{4} + \frac{3\,a\,b^2\,x^2}{2}}{x^4} + 3\,a^2\,b\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b/x^2)^3,x)
```

```
[Out] (a^3*x^2)/2 - (b^3/4 + (3*a*b^2*x^2)/2)/x^4 + 3*a^2*b*log(x)
```

sympy [A] time = 0.20, size = 37, normalized size = 0.92

$$\frac{a^3x^2}{2} + 3a^2b\log(x) + \frac{-6ab^2x^2 - b^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**3*x,x)
```

```
[Out] a**3*x**2/2 + 3*a**2*b*log(x) + (-6*a*b**2*x**2 - b**3)/(4*x**4)
```

3.1836 $\int \left(a+\frac{b}{x^2}\right)^3 dx$

Optimal. Leaf size=34

$$a^3x-\frac{3a^2b}{x}-\frac{ab^2}{x^3}-\frac{b^3}{5x^5}$$

```
[Out] -1/5*b^3/x^5-a*b^2/x^3-3*a^2*b/x+a^3*x
```

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {193, 270}

$$-\frac{3a^2b}{x}+a^3x-\frac{ab^2}{x^3}-\frac{b^3}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^3,x]
```

```
[Out] -b^3/(5*x^5) - (a*b^2)/x^3 - (3*a^2*b)/x + a^3*x
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x^2}\right)^3 dx &= \int \frac{\left(b+ax^2\right)^3}{x^6} dx \\ &= \int \left(a^3+\frac{b^3}{x^6}+\frac{3ab^2}{x^4}+\frac{3a^2b}{x^2}\right) dx \\ &= -\frac{b^3}{5x^5}-\frac{ab^2}{x^3}-\frac{3a^2b}{x}+a^3x\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.00

$$a^3x-\frac{3a^2b}{x}-\frac{ab^2}{x^3}-\frac{b^3}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^3,x]
```

```
[Out] -1/5*b^3/x^5 - (a*b^2)/x^3 - (3*a^2*b)/x + a^3*x
```

fricas [A] time = 1.18, size = 37, normalized size = 1.09

$$\frac{5\,a^3x^6-15\,a^2bx^4-5\,ab^2x^2-b^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^3,x, algorithm="fricas")
[Out] 1/5*(5*a^3*x^6 - 15*a^2*b*x^4 - 5*a*b^2*x^2 - b^3)/x^5
```

giac [**A**] time = 0.16, size = 33, normalized size = 0.97

$$a^3x - \frac{15\,a^2bx^4 + 5\,ab^2x^2 + b^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^3,x, algorithm="giac")
[Out] a^3*x - 1/5*(15*a^2*b*x^4 + 5*a*b^2*x^2 + b^3)/x^5
```

maple [**A**] time = 0.01, size = 33, normalized size = 0.97

$$a^3x - \frac{3a^2b}{x} - \frac{a\,b^2}{x^3} - \frac{b^3}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^3,x)
[Out] -1/5*b^3/x^5-a*b^2/x^3-3*a^2*b/x+a^3*x
```

maxima [**A**] time = 0.88, size = 32, normalized size = 0.94

$$a^3x - \frac{3\,a^2b}{x} - \frac{ab^2}{x^3} - \frac{b^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^3,x, algorithm="maxima")
[Out] a^3*x - 3*a^2*b/x - a*b^2/x^3 - 1/5*b^3/x^5
```

mupad [**B**] time = 0.03, size = 34, normalized size = 1.00

$$a^3\,x - \frac{3\,a^2\,b\,x^4 + a\,b^2\,x^2 + \frac{b^3}{5}}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)^3,x)
[Out] a^3*x - (b^3/5 + a*b^2*x^2 + 3*a^2*b*x^4)/x^5
```

sympy [**A**] time = 0.21, size = 34, normalized size = 1.00

$$a^3x + \frac{-15a^2bx^4 - 5ab^2x^2 - b^3}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**3,x)
[Out] a**3*x + (-15*a**2*b*x**4 - 5*a*b**2*x**2 - b**3)/(5*x**5)
```

3.1837 $\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x}\,dx$

Optimal. Leaf size=39

$$a^3\log(x)-\frac{3a^2b}{2x^2}-\frac{3ab^2}{4x^4}-\frac{b^3}{6x^6}$$

```
[Out] -1/6*b^3/x^6-3/4*a*b^2/x^4-3/2*a^2*b/x^2+a^3*ln(x)
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 266, 43}

$$-\frac{3a^2b}{2x^2}+a^3\log(x)-\frac{3ab^2}{4x^4}-\frac{b^3}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^3/x,x]
```

```
[Out] -b^3/(6*x^6) - (3*a*b^2)/(4*x^4) - (3*a^2*b)/(2*x^2) + a^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x}\,dx &= \int \frac{\left(b+ax^2\right)^3}{x^7}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{(b+ax)^3}{x^4}\,dx, x, x^2\right) \\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{b^3}{x^4}+\frac{3ab^2}{x^3}+\frac{3a^2b}{x^2}+\frac{a^3}{x}\right)\,dx, x, x^2\right) \\
&= -\frac{b^3}{6x^6}-\frac{3ab^2}{4x^4}-\frac{3a^2b}{2x^2}+a^3\log(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 39, normalized size = 1.00

$$a^3\log(x)-\frac{3a^2b}{2x^2}-\frac{3ab^2}{4x^4}-\frac{b^3}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^3/x,x]
```

```
[Out] -1/6*b^3/x^6 - (3*a*b^2)/(4*x^4) - (3*a^2*b)/(2*x^2) + a^3*Log[x]
```

fricas [A] time = 0.57, size = 39, normalized size = 1.00

$$\frac{12\,a^3x^6\log(x) - 18\,a^2bx^4 - 9\,ab^2x^2 - 2\,b^3}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^3/x,x, algorithm="fricas")
```

```
[Out] 1/12*(12*a^3*x^6*log(x) - 18*a^2*b*x^4 - 9*a*b^2*x^2 - 2*b^3)/x^6
```

giac [A] time = 0.18, size = 47, normalized size = 1.21

$$\frac{1}{2}\,a^3\log\left(x^2\right) - \frac{11\,a^3x^6 + 18\,a^2bx^4 + 9\,ab^2x^2 + 2\,b^3}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^3/x,x, algorithm="giac")
```

```
[Out] 1/2*a^3*log(x^2) - 1/12*(11*a^3*x^6 + 18*a^2*b*x^4 + 9*a*b^2*x^2 + 2*b^3)/x
^6
```

maple [A] time = 0.01, size = 34, normalized size = 0.87

$$a^3\ln(x) - \frac{3a^2b}{2x^2} - \frac{3a\,b^2}{4x^4} - \frac{b^3}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^3/x,x)
```

```
[Out] -1/6*b^3/x^6-3/4*a*b^2/x^4-3/2*a^2*b/x^2+a^3*ln(x)
```

maxima [A] time = 0.89, size = 39, normalized size = 1.00

$$\frac{1}{2}\,a^3\log\left(x^2\right) - \frac{18\,a^2bx^4 + 9\,ab^2x^2 + 2\,b^3}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^3/x,x, algorithm="maxima")
```

```
[Out] 1/2*a^3*log(x^2) - 1/12*(18*a^2*b*x^4 + 9*a*b^2*x^2 + 2*b^3)/x^6
```

mupad [B] time = 0.05, size = 36, normalized size = 0.92

$$a^3\,\ln(x) - \frac{\frac{3\,a^2\,b\,x^4}{2} + \frac{3\,a\,b^2\,x^2}{4} + \frac{b^3}{6}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)^3/x,x)
```

```
[Out] a^3*log(x) - (b^3/6 + (3*a*b^2*x^2)/4 + (3*a^2*b*x^4)/2)/x^6
```

sympy [A] time = 0.26, size = 37, normalized size = 0.95

$$a^3\log(x) + \frac{-18a^2bx^4 - 9ab^2x^2 - 2b^3}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**3/x,x)
[Out] a**3*log(x) + (-18*a**2*b*x**4 - 9*a*b**2*x**2 - 2*b**3)/(12*x**6)
```

3.1838 $\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^2}\,dx$

Optimal. Leaf size=39

$$-\frac{a^3}{x}-\frac{a^2 b}{x^3}-\frac{3ab^2}{5x^5}-\frac{b^3}{7x^7}$$

```
[Out] -1/7*b^3/x^7-3/5*a*b^2/x^5-a^2*b/x^3-a^3/x
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 270}

$$-\frac{a^2 b}{x^3}-\frac{a^3}{x}-\frac{3ab^2}{5x^5}-\frac{b^3}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^3/x^2,x]
```

```
[Out] -b^3/(7*x^7) - (3*a*b^2)/(5*x^5) - (a^2*b)/x^3 - a^3/x
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^2}\,dx &= \int \frac{\left(b+ax^2\right)^3}{x^8}\,dx \\ &= \int \left(\frac{b^3}{x^8}+\frac{3ab^2}{x^6}+\frac{3a^2 b}{x^4}+\frac{a^3}{x^2}\right)dx \\ &= -\frac{b^3}{7x^7}-\frac{3ab^2}{5x^5}-\frac{a^2 b}{x^3}-\frac{a^3}{x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 39, normalized size = 1.00

$$-\frac{a^3}{x}-\frac{a^2 b}{x^3}-\frac{3ab^2}{5x^5}-\frac{b^3}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^3/x^2,x]
```

```
[Out] -1/7*b^3/x^7 - (3*a*b^2)/(5*x^5) - (a^2*b)/x^3 - a^3/x
```

fricas [A] time = 0.77, size = 37, normalized size = 0.95

$$-\frac{35\,a^3x^6+35\,a^2bx^4+21\,ab^2x^2+5\,b^3}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^3/x^2,x, algorithm="fricas")
[Out] -1/35*(35*a^3*x^6 + 35*a^2*b*x^4 + 21*a*b^2*x^2 + 5*b^3)/x^7
```

giac [A] time = 0.15, size = 37, normalized size = 0.95

$$-\frac{35\,a^3x^6+35\,a^2bx^4+21\,ab^2x^2+5\,b^3}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^3/x^2,x, algorithm="giac")
[Out] -1/35*(35*a^3*x^6 + 35*a^2*b*x^4 + 21*a*b^2*x^2 + 5*b^3)/x^7
```

maple [A] time = 0.00, size = 36, normalized size = 0.92

$$-\frac{a^3}{x}-\frac{a^2b}{x^3}-\frac{3a\,b^2}{5x^5}-\frac{b^3}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^3/x^2,x)
[Out] -1/7*b^3/x^7-3/5*a*b^2/x^5-a^2*b/x^3-a^3/x
```

maxima [A] time = 0.90, size = 37, normalized size = 0.95

$$-\frac{35\,a^3x^6+35\,a^2bx^4+21\,ab^2x^2+5\,b^3}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^3/x^2,x, algorithm="maxima")
[Out] -1/35*(35*a^3*x^6 + 35*a^2*b*x^4 + 21*a*b^2*x^2 + 5*b^3)/x^7
```

mupad [B] time = 0.03, size = 35, normalized size = 0.90

$$-\frac{a^3\,x^6+a^2\,b\,x^4+\frac{3\,a\,b^2\,x^2}{5}+\frac{b^3}{7}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)^3/x^2,x)
[Out] -(b^3/7 + a^3*x^6 + (3*a*b^2*x^2)/5 + a^2*b*x^4)/x^7
```

sympy [A] time = 0.26, size = 39, normalized size = 1.00

$$\frac{-35a^3x^6-35a^2bx^4-21ab^2x^2-5b^3}{35x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**3/x**2,x)
[Out] (-35*a**3*x**6 - 35*a**2*b*x**4 - 21*a*b**2*x**2 - 5*b**3)/(35*x**7)
```

3.1839 $\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^3}\,dx$

Optimal. Leaf size=16

$$-\frac{\left(a+\frac{b}{x^2}\right)^4}{8b}$$

```
[Out] -1/8*(a+b/x^2)^4/b
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$-\frac{\left(a+\frac{b}{x^2}\right)^4}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^3/x^3,x]
```

```
[Out] -(a + b/x^2)^4/(8*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^3}\,dx = -\frac{\left(a+\frac{b}{x^2}\right)^4}{8b}$$

Mathematica [B] time = 0.01, size = 43, normalized size = 2.69

$$-\frac{a^3}{2x^2}-\frac{3a^2b}{4x^4}-\frac{ab^2}{2x^6}-\frac{b^3}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^3/x^3,x]
```

```
[Out] -1/8*b^3/x^8 - (a*b^2)/(2*x^6) - (3*a^2*b)/(4*x^4) - a^3/(2*x^2)
```

fricas [B] time = 0.78, size = 35, normalized size = 2.19

$$-\frac{4\,a^3x^6+6\,a^2bx^4+4\,ab^2x^2+b^3}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^3/x^3,x, algorithm="fricas")
```

```
[Out] -1/8*(4*a^3*x^6 + 6*a^2*b*x^4 + 4*a*b^2*x^2 + b^3)/x^8
```

giac [**B**] time = 0.15, size = 35, normalized size = 2.19

$$-\frac{4\,a^3x^6+6\,a^2bx^4+4\,ab^2x^2+b^3}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^3/x^3,x, algorithm="giac")
[Out] -1/8*(4*a^3*x^6 + 6*a^2*b*x^4 + 4*a*b^2*x^2 + b^3)/x^8
```

maple [**B**] time = 0.00, size = 36, normalized size = 2.25

$$-\frac{a^3}{2x^2}-\frac{3a^2b}{4x^4}-\frac{a\,b^2}{2x^6}-\frac{b^3}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^3/x^3,x)
[Out] -3/4*a^2*b/x^4-1/8*b^3/x^8-1/2*a^3/x^2-1/2*a*b^2/x^6
```

maxima [**A**] time = 0.85, size = 14, normalized size = 0.88

$$-\frac{\left(a+\frac{b}{x^2}\right)^4}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^3/x^3,x, algorithm="maxima")
[Out] -1/8*(a + b/x^2)^4/b
```

mupad [**B**] time = 0.03, size = 37, normalized size = 2.31

$$-\frac{\frac{a^3x^6}{2}+\frac{3\,a^2\,b\,x^4}{4}+\frac{a\,b^2\,x^2}{2}+\frac{b^3}{8}}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)^3/x^3,x)
[Out] -(b^3/8 + (a^3*x^6)/2 + (a*b^2*x^2)/2 + (3*a^2*b*x^4)/4)/x^8
```

sympy [**B**] time = 0.30, size = 37, normalized size = 2.31

$$\frac{-4a^3x^6-6a^2bx^4-4ab^2x^2-b^3}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**3/x**3,x)
[Out] (-4*a**3*x**6 - 6*a**2*b*x**4 - 4*a*b**2*x**2 - b**3)/(8*x**8)
```

3.1840 $\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^4}\,dx$

Optimal. Leaf size=43

$$-\frac{a^3}{3x^3}-\frac{3a^2b}{5x^5}-\frac{3ab^2}{7x^7}-\frac{b^3}{9x^9}$$

```
[Out] -1/9*b^3/x^9-3/7*a*b^2/x^7-3/5*a^2*b/x^5-1/3*a^3/x^3
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 270}

$$-\frac{3a^2b}{5x^5}-\frac{a^3}{3x^3}-\frac{3ab^2}{7x^7}-\frac{b^3}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^3/x^4,x]
```

```
[Out] -b^3/(9*x^9) - (3*a*b^2)/(7*x^7) - (3*a^2*b)/(5*x^5) - a^3/(3*x^3)
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^4}\,dx &= \int \frac{\left(b+ax^2\right)^3}{x^{10}}\,dx\\ &= \int \left(\frac{b^3}{x^{10}}+\frac{3ab^2}{x^8}+\frac{3a^2b}{x^6}+\frac{a^3}{x^4}\right)dx\\ &= -\frac{b^3}{9x^9}-\frac{3ab^2}{7x^7}-\frac{3a^2b}{5x^5}-\frac{a^3}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$-\frac{a^3}{3x^3}-\frac{3a^2b}{5x^5}-\frac{3ab^2}{7x^7}-\frac{b^3}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^3/x^4,x]
```

```
[Out] -1/9*b^3/x^9 - (3*a*b^2)/(7*x^7) - (3*a^2*b)/(5*x^5) - a^3/(3*x^3)
```

fricas [A] time = 0.83, size = 37, normalized size = 0.86

$$-\frac{105\,a^3x^6+189\,a^2bx^4+135\,ab^2x^2+35\,b^3}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^3/x^4,x, algorithm="fricas")
[Out] -1/315*(105*a^3*x^6 + 189*a^2*b*x^4 + 135*a*b^2*x^2 + 35*b^3)/x^9
```

giac [A] time = 0.15, size = 37, normalized size = 0.86

$$-\frac{105\,a^3x^6+189\,a^2bx^4+135\,ab^2x^2+35\,b^3}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^3/x^4,x, algorithm="giac")
[Out] -1/315*(105*a^3*x^6 + 189*a^2*b*x^4 + 135*a*b^2*x^2 + 35*b^3)/x^9
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$-\frac{a^3}{3x^3}-\frac{3a^2b}{5x^5}-\frac{3a\,b^2}{7x^7}-\frac{b^3}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^3/x^4,x)
[Out] -1/9*b^3/x^9-3/7*a*b^2/x^7-3/5*a^2*b/x^5-1/3*a^3/x^3
```

maxima [A] time = 0.87, size = 37, normalized size = 0.86

$$-\frac{105\,a^3x^6+189\,a^2bx^4+135\,ab^2x^2+35\,b^3}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^3/x^4,x, algorithm="maxima")
[Out] -1/315*(105*a^3*x^6 + 189*a^2*b*x^4 + 135*a*b^2*x^2 + 35*b^3)/x^9
```

mupad [B] time = 0.03, size = 37, normalized size = 0.86

$$-\frac{\frac{a^3x^6}{3}+\frac{3a^2bx^4}{5}+\frac{3ab^2x^2}{7}+\frac{b^3}{9}}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)^3/x^4,x)
[Out] -(b^3/9 + (a^3*x^6)/3 + (3*a*b^2*x^2)/7 + (3*a^2*b*x^4)/5)/x^9
```

sympy [A] time = 0.29, size = 39, normalized size = 0.91

$$\frac{-105a^3x^6-189a^2bx^4-135ab^2x^2-35b^3}{315x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**3/x**4,x)
[Out] (-105*a**3*x**6 - 189*a**2*b*x**4 - 135*a*b**2*x**2 - 35*b**3)/(315*x**9)
```

3.1841 $\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^5}\,dx$

Optimal. Leaf size=40

$$\frac{a\left(ax^2+b\right)^4}{40b^2x^8}-\frac{\left(ax^2+b\right)^4}{10bx^{10}}$$

```
[Out] -1/10*(a*x^2+b)^4/b/x^10+1/40*a*(a*x^2+b)^4/b^2/x^8
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {263, 266, 45, 37}

$$\frac{a\left(ax^2+b\right)^4}{40b^2x^8}-\frac{\left(ax^2+b\right)^4}{10bx^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^3/x^5,x]
```

```
[Out] -(b + a*x^2)^4/(10*b*x^10) + (a*(b + a*x^2)^4)/(40*b^2*x^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^5}\,dx &= \int \frac{\left(b+ax^2\right)^3}{x^{11}}\,dx \\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(b+ax)^3}{x^6}\,dx, x, x^2\right) \\ &= -\frac{\left(b+ax^2\right)^4}{10bx^{10}} - \frac{a\,\text{Subst}\left(\int \frac{(b+ax)^3}{x^5}\,dx, x, x^2\right)}{10b} \\ &= -\frac{\left(b+ax^2\right)^4}{10bx^{10}} + \frac{a\left(b+ax^2\right)^4}{40b^2x^8}\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.08

$$-\frac{a^3}{4x^4} - \frac{a^2b}{2x^6} - \frac{3ab^2}{8x^8} - \frac{b^3}{10x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^3/x^5,x]
[Out] -1/10*b^3/x^10 - (3*a*b^2)/(8*x^8) - (a^2*b)/(2*x^6) - a^3/(4*x^4)
```

fricas [A] time = 1.07, size = 37, normalized size = 0.92

$$-\frac{10\,a^3x^6 + 20\,a^2bx^4 + 15\,ab^2x^2 + 4\,b^3}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^3/x^5,x, algorithm="fricas")
[Out] -1/40*(10*a^3*x^6 + 20*a^2*b*x^4 + 15*a*b^2*x^2 + 4*b^3)/x^10
```

giac [A] time = 0.19, size = 37, normalized size = 0.92

$$-\frac{10\,a^3x^6 + 20\,a^2bx^4 + 15\,ab^2x^2 + 4\,b^3}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^3/x^5,x, algorithm="giac")
[Out] -1/40*(10*a^3*x^6 + 20*a^2*b*x^4 + 15*a*b^2*x^2 + 4*b^3)/x^10
```

maple [A] time = 0.00, size = 36, normalized size = 0.90

$$-\frac{a^3}{4x^4} - \frac{a^2b}{2x^6} - \frac{3a\,b^2}{8x^8} - \frac{b^3}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^3/x^5,x)
[Out] -1/4*a^3/x^4-3/8*a*b^2/x^8-1/2*a^2*b/x^6-1/10*b^3/x^10
```

maxima [A] time = 0.87, size = 37, normalized size = 0.92

$$-\frac{10\,a^3x^6 + 20\,a^2bx^4 + 15\,ab^2x^2 + 4\,b^3}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^3/x^5,x, algorithm="maxima")
[Out] -1/40*(10*a^3*x^6 + 20*a^2*b*x^4 + 15*a*b^2*x^2 + 4*b^3)/x^10
```

mupad [B] time = 0.03, size = 37, normalized size = 0.92

$$-\frac{\frac{a^3 x^6}{4}+\frac{a^2 b x^4}{2}+\frac{3 a b^2 x^2}{8}+\frac{b^3}{10}}{x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)^3/x^5,x)
[Out] -(b^3/10 + (a^3*x^6)/4 + (3*a*b^2*x^2)/8 + (a^2*b*x^4)/2)/x^10
```

sympy [A] time = 0.31, size = 39, normalized size = 0.98

$$\frac{-10a^3x^6 - 20a^2bx^4 - 15ab^2x^2 - 4b^3}{40x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**3/x**5,x)
[Out] (-10*a**3*x**6 - 20*a**2*b*x**4 - 15*a*b**2*x**2 - 4*b**3)/(40*x**10)
```

3.1842 $\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^6}\,dx$

Optimal. Leaf size=43

$$-\frac{a^3}{5x^5}-\frac{3a^2b}{7x^7}-\frac{ab^2}{3x^9}-\frac{b^3}{11x^{11}}$$

```
[Out] -1/11*b^3/x^11-1/3*a*b^2/x^9-3/7*a^2*b/x^7-1/5*a^3/x^5
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 270}

$$-\frac{3a^2b}{7x^7}-\frac{a^3}{5x^5}-\frac{ab^2}{3x^9}-\frac{b^3}{11x^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^3/x^6,x]
```

```
[Out] -b^3/(11*x^11) - (a*b^2)/(3*x^9) - (3*a^2*b)/(7*x^7) - a^3/(5*x^5)
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^6}\,dx &= \int \frac{\left(b+ax^2\right)^3}{x^{12}}\,dx\\ &= \int \left(\frac{b^3}{x^{12}}+\frac{3ab^2}{x^{10}}+\frac{3a^2b}{x^8}+\frac{a^3}{x^6}\right)dx\\ &= -\frac{b^3}{11x^{11}}-\frac{ab^2}{3x^9}-\frac{3a^2b}{7x^7}-\frac{a^3}{5x^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 1.00

$$-\frac{a^3}{5x^5}-\frac{3a^2b}{7x^7}-\frac{ab^2}{3x^9}-\frac{b^3}{11x^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^3/x^6,x]
```

```
[Out] -1/11*b^3/x^11 - (a*b^2)/(3*x^9) - (3*a^2*b)/(7*x^7) - a^3/(5*x^5)
```

fricas [A] time = 0.76, size = 37, normalized size = 0.86

$$-\frac{231\,a^3x^6+495\,a^2bx^4+385\,ab^2x^2+105\,b^3}{1155\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^3/x^6,x, algorithm="fricas")
[Out] -1/1155*(231*a^3*x^6 + 495*a^2*b*x^4 + 385*a*b^2*x^2 + 105*b^3)/x^11
```

giac [A] time = 0.16, size = 37, normalized size = 0.86

$$-\frac{231\,a^3x^6+495\,a^2bx^4+385\,ab^2x^2+105\,b^3}{1155\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^3/x^6,x, algorithm="giac")
[Out] -1/1155*(231*a^3*x^6 + 495*a^2*b*x^4 + 385*a*b^2*x^2 + 105*b^3)/x^11
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$-\frac{a^3}{5x^5}-\frac{3a^2b}{7x^7}-\frac{a\,b^2}{3x^9}-\frac{b^3}{11x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^3/x^6,x)
[Out] -1/11*b^3/x^11-1/3*a*b^2/x^9-3/7*a^2*b/x^7-1/5*a^3/x^5
```

maxima [A] time = 0.83, size = 37, normalized size = 0.86

$$-\frac{231\,a^3x^6+495\,a^2bx^4+385\,ab^2x^2+105\,b^3}{1155\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^3/x^6,x, algorithm="maxima")
[Out] -1/1155*(231*a^3*x^6 + 495*a^2*b*x^4 + 385*a*b^2*x^2 + 105*b^3)/x^11
```

mupad [B] time = 0.03, size = 37, normalized size = 0.86

$$-\frac{\frac{a^3x^6}{5}+\frac{3a^2bx^4}{7}+\frac{ab^2x^2}{3}+\frac{b^3}{11}}{x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)^3/x^6,x)
[Out] -(b^3/11 + (a^3*x^6)/5 + (a*b^2*x^2)/3 + (3*a^2*b*x^4)/7)/x^11
```

sympy [A] time = 0.32, size = 39, normalized size = 0.91

$$\frac{-231a^3x^6-495a^2bx^4-385ab^2x^2-105b^3}{1155x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**3/x**6,x)
[Out] (-231*a**3*x**6 - 495*a**2*b*x**4 - 385*a*b**2*x**2 - 105*b**3)/(1155*x**11
)
```

3.1843 $\int \frac{x^6}{a+\frac{b}{x^2}}\,dx$

Optimal. Leaf size=68

$$\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{a^{9/2}}-\frac{b^3x}{a^4}+\frac{b^2x^3}{3a^3}-\frac{bx^5}{5a^2}+\frac{x^7}{7a}$$

```
[Out] -b^3*x/a^4+1/3*b^2*x^3/a^3-1/5*b*x^5/a^2+1/7*x^7/a+b^(7/2)*arctan(x*a^(1/2)
/b^(1/2))/a^(9/2)
```

Rubi [A] time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 302, 205}

$$\frac{b^2x^3}{3a^3}-\frac{b^3x}{a^4}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{a^{9/2}}-\frac{bx^5}{5a^2}+\frac{x^7}{7a}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b/x^2),x]
```

```
[Out] -((b^3*x)/a^4) + (b^2*x^3)/(3*a^3) - (b*x^5)/(5*a^2) + x^7/(7*a) + (b^(7/2)
*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/a^(9/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{a+\frac{b}{x^2}}\,dx &= \int \frac{x^8}{b+ax^2}\,dx\\ &= \int\left(-\frac{b^3}{a^4}+\frac{b^2x^2}{a^3}-\frac{bx^4}{a^2}+\frac{x^6}{a}+\frac{b^4}{a^4\left(b+ax^2\right)}\right)dx\\ &= -\frac{b^3x}{a^4}+\frac{b^2x^3}{3a^3}-\frac{bx^5}{5a^2}+\frac{x^7}{7a}+\frac{b^4\int\frac{1}{b+ax^2}\,dx}{a^4}\\ &= -\frac{b^3x}{a^4}+\frac{b^2x^3}{3a^3}-\frac{bx^5}{5a^2}+\frac{x^7}{7a}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{a^{9/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 68, normalized size = 1.00

$$\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{a^{9/2}} - \frac{b^3x}{a^4} + \frac{b^2x^3}{3a^3} - \frac{bx^5}{5a^2} + \frac{x^7}{7a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b/x^2),x]
```

```
[Out] -((b^3*x)/a^4) + (b^2*x^3)/(3*a^3) - (b*x^5)/(5*a^2) + x^7/(7*a) + (b^(7/2)
*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/a^(9/2)
```

fricas [A] time = 0.83, size = 148, normalized size = 2.18

$$\left[\frac{30\,a^3x^7 - 42\,a^2bx^5 + 70\,ab^2x^3 + 105\,b^3\sqrt{-\frac{b}{a}}\,\log\left(\frac{ax^2+2\,ax\sqrt{-\frac{b}{a}}-b}{ax^2+b}\right) - 210\,b^3x}{210\,a^4},\ \frac{15\,a^3x^7 - 21\,a^2bx^5 + 35\,ab^2x^3 + 1}{10}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(a+b/x^2),x, algorithm="fricas")
```

```
[Out] [1/210*(30*a^3*x^7 - 42*a^2*b*x^5 + 70*a*b^2*x^3 + 105*b^3*sqrt(-b/a)*log((
a*x^2 + 2*a*x*sqrt(-b/a) - b)/(a*x^2 + b)) - 210*b^3*x)/a^4, 1/105*(15*a^3*
x^7 - 21*a^2*b*x^5 + 35*a*b^2*x^3 + 105*b^3*sqrt(b/a)*arctan(a*x*sqrt(b/a)/
b) - 105*b^3*x)/a^4]
```

giac [A] time = 0.16, size = 65, normalized size = 0.96

$$\frac{b^4\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^4} + \frac{15\,a^6x^7 - 21\,a^5bx^5 + 35\,a^4b^2x^3 - 105\,a^3b^3x}{105\,a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(a+b/x^2),x, algorithm="giac")
```

```
[Out] b^4*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^4) + 1/105*(15*a^6*x^7 - 21*a^5*b*x^
5 + 35*a^4*b^2*x^3 - 105*a^3*b^3*x)/a^7
```

maple [A] time = 0.00, size = 60, normalized size = 0.88

$$\frac{x^7}{7a} - \frac{b\,x^5}{5a^2} + \frac{b^2x^3}{3a^3} + \frac{b^4\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^4} - \frac{b^3x}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a+b/x^2),x)
```

```
[Out] 1/7/a*x^7-1/5*b*x^5/a^2+1/3/a^3*b^2*x^3-1/a^4*b^3*x+b^4/a^4/(a*b)^(1/2)*arc
tan(x*a/(a*b)^(1/2))
```

maxima [A] time = 1.84, size = 60, normalized size = 0.88

$$\frac{b^4\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^4} + \frac{15\,a^3x^7 - 21\,a^2bx^5 + 35\,ab^2x^3 - 105\,b^3x}{105\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(a+b/x^2),x, algorithm="maxima")
```

```
[Out] b^4*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^4) + 1/105*(15*a^3*x^7 - 21*a^2*b*x^
5 + 35*a*b^2*x^3 - 105*b^3*x)/a^4
```

mupad [B] time = 1.10, size = 54, normalized size = 0.79

$$\frac{x^7}{7\,a}-\frac{b\,x^5}{5\,a^2}-\frac{b^3\,x}{a^4}+\frac{b^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{a^{9/2}}+\frac{b^2\,x^3}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b/x^2),x)
```

```
[Out] x^7/(7*a) - (b*x^5)/(5*a^2) - (b^3*x)/a^4 + (b^(7/2)*atan((a^(1/2)*x)/b^(1/
2)))/a^(9/2) + (b^2*x^3)/(3*a^3)
```

sympy [A] time = 0.21, size = 107, normalized size = 1.57

$$-\frac{\sqrt{-\frac{b^7}{a^9}}\,\log\left(-\frac{a^4\sqrt{-\frac{b^7}{a^9}}}{b^3}+x\right)}{2}+\frac{\sqrt{-\frac{b^7}{a^9}}\,\log\left(\frac{a^4\sqrt{-\frac{b^7}{a^9}}}{b^3}+x\right)}{2}+\frac{x^7}{7a}-\frac{bx^5}{5a^2}+\frac{b^2x^3}{3a^3}-\frac{b^3x}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(a+b/x**2),x)
```

```
[Out] -sqrt(-b**7/a**9)*log(-a**4*sqrt(-b**7/a**9)/b**3 + x)/2 + sqrt(-b**7/a**9)
*log(a**4*sqrt(-b**7/a**9)/b**3 + x)/2 + x**7/(7*a) - b*x**5/(5*a**2) + b**
2*x**3/(3*a**3) - b**3*x/a**4
```

3.1844 $\int \frac{x^5}{a+\frac{b}{x^2}}\,dx$

Optimal. Leaf size=53

$$-\frac{b^3\log\left(ax^2+b\right)}{2a^4}+\frac{b^2x^2}{2a^3}-\frac{bx^4}{4a^2}+\frac{x^6}{6a}$$

```
[Out] 1/2*b^2*x^2/a^3-1/4*b*x^4/a^2+1/6*x^6/a-1/2*b^3*ln(a*x^2+b)/a^4
```

Rubi [A] time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 266, 43}

$$\frac{b^2x^2}{2a^3}-\frac{b^3\log\left(ax^2+b\right)}{2a^4}-\frac{bx^4}{4a^2}+\frac{x^6}{6a}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b/x^2),x]
```

```
[Out] (b^2*x^2)/(2*a^3) - (b*x^4)/(4*a^2) + x^6/(6*a) - (b^3*Log[b + a*x^2])/(2*a
^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{a+\frac{b}{x^2}}\,dx &= \int \frac{x^7}{b+ax^2}\,dx\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{b+ax}\,dx, x, x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{b^2}{a^3}-\frac{bx}{a^2}+\frac{x^2}{a}-\frac{b^3}{a^3(b+ax)}\right)\,dx, x, x^2\right)\\ &= \frac{b^2x^2}{2a^3}-\frac{bx^4}{4a^2}+\frac{x^6}{6a}-\frac{b^3\log\left(b+ax^2\right)}{2a^4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 53, normalized size = 1.00

$$-\frac{b^3\log\left(ax^2+b\right)}{2a^4}+\frac{b^2x^2}{2a^3}-\frac{bx^4}{4a^2}+\frac{x^6}{6a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b/x^2),x]
```

```
[Out] (b^2*x^2)/(2*a^3) - (b*x^4)/(4*a^2) + x^6/(6*a) - (b^3*Log[b + a*x^2])/(2*a
^4)
```

fricas [A] time = 0.86, size = 45, normalized size = 0.85

$$\frac{2\,a^3x^6 - 3\,a^2bx^4 + 6\,ab^2x^2 - 6\,b^3\log\left(ax^2+b\right)}{12\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/x^2),x, algorithm="fricas")
```

```
[Out] 1/12*(2*a^3*x^6 - 3*a^2*b*x^4 + 6*a*b^2*x^2 - 6*b^3*log(a*x^2 + b))/a^4
```

giac [A] time = 0.16, size = 47, normalized size = 0.89

$$-\frac{b^3\log\left(\left|ax^2+b\right|\right)}{2\,a^4} + \frac{2\,a^2x^6 - 3\,abx^4 + 6\,b^2x^2}{12\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/x^2),x, algorithm="giac")
```

```
[Out] -1/2*b^3*log(abs(a*x^2 + b))/a^4 + 1/12*(2*a^2*x^6 - 3*a*b*x^4 + 6*b^2*x^2)
/a^3
```

maple [A] time = 0.00, size = 46, normalized size = 0.87

$$\frac{x^6}{6a} - \frac{b\,x^4}{4a^2} + \frac{b^2x^2}{2a^3} - \frac{b^3\ln\left(a\,x^2+b\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a+b/x^2),x)
```

```
[Out] 1/2/a^3*b^2*x^2-1/4/a^2*b*x^4+1/6/a*x^6-1/2*b^3*ln(a*x^2+b)/a^4
```

maxima [A] time = 0.88, size = 46, normalized size = 0.87

$$-\frac{b^3\log\left(ax^2+b\right)}{2\,a^4} + \frac{2\,a^2x^6 - 3\,abx^4 + 6\,b^2x^2}{12\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/x^2),x, algorithm="maxima")
```

```
[Out] -1/2*b^3*log(a*x^2 + b)/a^4 + 1/12*(2*a^2*x^6 - 3*a*b*x^4 + 6*b^2*x^2)/a^3
```

mupad [B] time = 0.05, size = 45, normalized size = 0.85

$$\frac{x^6}{6\,a} - \frac{b\,x^4}{4\,a^2} - \frac{b^3\,\ln\left(a\,x^2+b\right)}{2\,a^4} + \frac{b^2\,x^2}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b/x^2),x)
```

```
[Out] x^6/(6*a) - (b*x^4)/(4*a^2) - (b^3*log(b + a*x^2))/(2*a^4) + (b^2*x^2)/(2*a
^3)
```

sympy [**A**] time = 0.17, size = 44, normalized size = 0.83

$$\frac{x^6}{6a} - \frac{bx^4}{4a^2} + \frac{b^2x^2}{2a^3} - \frac{b^3\log\left(ax^2+b\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a+b/x**2),x)
```

```
[Out] x**6/(6*a) - b*x**4/(4*a**2) + b**2*x**2/(2*a**3) - b**3*log(a*x**2 + b)/(2
*a**4)
```

3.1845 $\int \frac{x^4}{a+\frac{b}{x^2}}\,dx$

Optimal. Leaf size=55

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{a^{7/2}}+\frac{b^2x}{a^3}-\frac{bx^3}{3a^2}+\frac{x^5}{5a}$$

```
[Out] b^2*x/a^3-1/3*b*x^3/a^2+1/5*x^5/a-b^(5/2)*arctan(x*a^(1/2)/b^(1/2))/a^(7/2)
```

Rubi [A] time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 302, 205}

$$\frac{b^2x}{a^3}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{a^{7/2}}-\frac{bx^3}{3a^2}+\frac{x^5}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b/x^2),x]
```

```
[Out] (b^2*x)/a^3 - (b*x^3)/(3*a^2) + x^5/(5*a) - (b^(5/2)*ArcTan[(Sqrt[a]*x)/Sqr
t[b]])/a^(7/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{a+\frac{b}{x^2}}\,dx &= \int \frac{x^6}{b+ax^2}\,dx\\ &= \int\left(\frac{b^2}{a^3}-\frac{bx^2}{a^2}+\frac{x^4}{a}-\frac{b^3}{a^3\left(b+ax^2\right)}\right)dx\\ &= \frac{b^2x}{a^3}-\frac{bx^3}{3a^2}+\frac{x^5}{5a}-\frac{b^3\int\frac{1}{b+ax^2}\,dx}{a^3}\\ &= \frac{b^2x}{a^3}-\frac{bx^3}{3a^2}+\frac{x^5}{5a}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{a^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 55, normalized size = 1.00

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{a^{7/2}}+\frac{b^2x}{a^3}-\frac{bx^3}{3a^2}+\frac{x^5}{5a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b/x^2),x]
```

```
[Out] (b^2*x)/a^3 - (b*x^3)/(3*a^2) + x^5/(5*a) - (b^(5/2)*ArcTan[(Sqrt[a]*x)/Sqr
t[b]])/a^(7/2)
```

fricas [A] time = 0.98, size = 126, normalized size = 2.29

$$\left[\frac{6\,a^2x^5 - 10\,abx^3 + 15\,b^2\sqrt{-\frac{b}{a}}\,\log\left(\frac{ax^2 - 2\,ax\sqrt{-\frac{b}{a}} - b}{ax^2 + b}\right) + 30\,b^2x}{30\,a^3},\ \frac{3\,a^2x^5 - 5\,abx^3 - 15\,b^2\sqrt{\frac{b}{a}}\,\arctan\left(\frac{ax\sqrt{\frac{b}{a}}}{b}\right) + 15\,b^2}{15\,a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/x^2),x, algorithm="fricas")
```

```
[Out] [1/30*(6*a^2*x^5 - 10*a*b*x^3 + 15*b^2*sqrt(-b/a)*log((a*x^2 - 2*a*x*sqrt(-
b/a) - b)/(a*x^2 + b)) + 30*b^2*x)/a^3, 1/15*(3*a^2*x^5 - 5*a*b*x^3 - 15*b^
2*sqrt(b/a)*arctan(a*x*sqrt(b/a)/b) + 15*b^2*x)/a^3]
```

giac [A] time = 0.18, size = 55, normalized size = 1.00

$$-\frac{b^3\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^3} + \frac{3\,a^4x^5 - 5\,a^3bx^3 + 15\,a^2b^2x}{15\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/x^2),x, algorithm="giac")
```

```
[Out] -b^3*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/15*(3*a^4*x^5 - 5*a^3*b*x^3
+ 15*a^2*b^2*x)/a^5
```

maple [A] time = 0.00, size = 49, normalized size = 0.89

$$\frac{x^5}{5a} - \frac{b\,x^3}{3a^2} - \frac{b^3\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^3} + \frac{b^2x}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a+b/x^2),x)
```

```
[Out] 1/5/a*x^5-1/3/a^2*b*x^3+1/a^3*b^2*x-b^3/a^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2
)*a*x)
```

maxima [A] time = 2.00, size = 50, normalized size = 0.91

$$-\frac{b^3\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^3} + \frac{3\,a^2x^5 - 5\,abx^3 + 15\,b^2x}{15\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/x^2),x, algorithm="maxima")
```

```
[Out] -b^3*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/15*(3*a^2*x^5 - 5*a*b*x^3 +
15*b^2*x)/a^3
```

mupad [B] time = 0.06, size = 43, normalized size = 0.78

$$\frac{x^5}{5\,a}-\frac{b\,x^3}{3\,a^2}+\frac{b^2\,x}{a^3}-\frac{b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b/x^2),x)
```

```
[Out] x^5/(5*a) - (b*x^3)/(3*a^2) + (b^2*x)/a^3 - (b^(5/2)*atan((a^(1/2)*x)/b^(1/
2)))/a^(7/2)
```

sympy [A] time = 0.19, size = 95, normalized size = 1.73

$$\frac{\sqrt{-\frac{b^5}{a^7}}\,\log\left(-\frac{a^3\sqrt{-\frac{b^5}{a^7}}}{b^2}+x\right)}{2}-\frac{\sqrt{-\frac{b^5}{a^7}}\,\log\left(\frac{a^3\sqrt{-\frac{b^5}{a^7}}}{b^2}+x\right)}{2}+\frac{x^5}{5a}-\frac{bx^3}{3a^2}+\frac{b^2x}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a+b/x**2),x)
```

```
[Out] sqrt(-b**5/a**7)*log(-a**3*sqrt(-b**5/a**7)/b**2 + x)/2 - sqrt(-b**5/a**7)*
log(a**3*sqrt(-b**5/a**7)/b**2 + x)/2 + x**5/(5*a) - b*x**3/(3*a**2) + b**2
*x/a**3
```

3.1846 $\int \frac{x^3}{a+\frac{b}{x^2}}\,dx$

Optimal. Leaf size=40

$$\frac{b^2\log\left(ax^2+b\right)}{2a^3}-\frac{bx^2}{2a^2}+\frac{x^4}{4a}$$

```
[Out] -1/2*b*x^2/a^2+1/4*x^4/a+1/2*b^2*ln(a*x^2+b)/a^3
```

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 266, 43}

$$\frac{b^2\log\left(ax^2+b\right)}{2a^3}-\frac{bx^2}{2a^2}+\frac{x^4}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b/x^2),x]
```

```
[Out] -(b*x^2)/(2*a^2) + x^4/(4*a) + (b^2*Log[b + a*x^2])/(2*a^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{a+\frac{b}{x^2}}\,dx &= \int \frac{x^5}{b+ax^2}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{b+ax}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{b}{a^2}+\frac{x}{a}+\frac{b^2}{a^2(b+ax)}\right)\,dx, x, x^2\right)\\ &= -\frac{bx^2}{2a^2}+\frac{x^4}{4a}+\frac{b^2\log\left(b+ax^2\right)}{2a^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.00

$$\frac{b^2\log\left(ax^2+b\right)}{2a^3}-\frac{bx^2}{2a^2}+\frac{x^4}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b/x^2),x]
[Out] -1/2*(b*x^2)/a^2 + x^4/(4*a) + (b^2*Log[b + a*x^2])/(2*a^3)
```

fricas [A] time = 0.78, size = 33, normalized size = 0.82

$$\frac{a^2x^4 - 2\,abx^2 + 2\,b^2\log\left(ax^2+b\right)}{4\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x^2),x, algorithm="fricas")
[Out] 1/4*(a^2*x^4 - 2*a*b*x^2 + 2*b^2*log(a*x^2 + b))/a^3
```

giac [A] time = 0.15, size = 35, normalized size = 0.88

$$\frac{b^2\log\left(\left|ax^2+b\right|\right)}{2\,a^3} + \frac{ax^4 - 2\,bx^2}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x^2),x, algorithm="giac")
[Out] 1/2*b^2*log(abs(a*x^2 + b))/a^3 + 1/4*(a*x^4 - 2*b*x^2)/a^2
```

maple [A] time = 0.00, size = 35, normalized size = 0.88

$$\frac{x^4}{4a} - \frac{b\,x^2}{2a^2} + \frac{b^2\ln\left(a\,x^2+b\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b/x^2),x)
[Out] -1/2/a^2*b*x^2+1/4/a*x^4+1/2*b^2*ln(a*x^2+b)/a^3
```

maxima [A] time = 0.83, size = 34, normalized size = 0.85

$$\frac{b^2\log\left(ax^2+b\right)}{2\,a^3} + \frac{ax^4 - 2\,bx^2}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x^2),x, algorithm="maxima")
[Out] 1/2*b^2*log(a*x^2 + b)/a^3 + 1/4*(a*x^4 - 2*b*x^2)/a^2
```

mupad [B] time = 1.09, size = 33, normalized size = 0.82

$$\frac{2\,b^2\,\ln\left(a\,x^2+b\right) + a^2\,x^4 - 2\,a\,b\,x^2}{4\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b/x^2),x)
[Out] (2*b^2*log(b + a*x^2) + a^2*x^4 - 2*a*b*x^2)/(4*a^3)
```

sympy [A] time = 0.16, size = 32, normalized size = 0.80

$$\frac{x^4}{4a} - \frac{bx^2}{2a^2} + \frac{b^2\log\left(ax^2+b\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b/x**2),x)
[Out] x**4/(4*a) - b*x**2/(2*a**2) + b**2*log(a*x**2 + b)/(2*a**3)
```

3.1847 $\int \frac{x^2}{a+\frac{b}{x^2}}\,dx$

Optimal. Leaf size=42

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{a^{5/2}}-\frac{bx}{a^2}+\frac{x^3}{3a}$$

```
[Out] -b*x/a^2+1/3*x^3/a+b^(3/2)*arctan(x*a^(1/2)/b^(1/2))/a^(5/2)
```

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 302, 205}

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{a^{5/2}}-\frac{bx}{a^2}+\frac{x^3}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b/x^2),x]
```

```
[Out] -((b*x)/a^2) + x^3/(3*a) + (b^(3/2)*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/a^(5/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{a+\frac{b}{x^2}}\,dx &= \int \frac{x^4}{b+ax^2}\,dx\\ &= \int\left(-\frac{b}{a^2}+\frac{x^2}{a}+\frac{b^2}{a^2\left(b+ax^2\right)}\right)dx\\ &= -\frac{bx}{a^2}+\frac{x^3}{3a}+\frac{b^2\int\frac{1}{b+ax^2}\,dx}{a^2}\\ &= -\frac{bx}{a^2}+\frac{x^3}{3a}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{a^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 1.00

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{a^{5/2}}-\frac{bx}{a^2}+\frac{x^3}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b/x^2),x]
```

```
[Out] -((b*x)/a^2) + x^3/(3*a) + (b^(3/2)*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/a^(5/2)
```

fricas [A] time = 0.68, size = 99, normalized size = 2.36

$$\left[\frac{2\,ax^3+3\,b\sqrt{-\frac{b}{a}}\,\log\left(\frac{ax^2+2\,ax\sqrt{-\frac{b}{a}}-b}{ax^2+b}\right)-6\,bx}{6\,a^2},\ \frac{ax^3+3\,b\sqrt{\frac{b}{a}}\,\arctan\left(\frac{ax\sqrt{\frac{b}{a}}}{b}\right)-3\,bx}{3\,a^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*a*x^3 + 3*b*sqrt(-b/a)*log((a*x^2 + 2*a*x*sqrt(-b/a) - b)/(a*x^2 +
b)) - 6*b*x)/a^2, 1/3*(a*x^3 + 3*b*sqrt(b/a)*arctan(a*x*sqrt(b/a)/b) - 3*b*
x)/a^2]
```

giac [A] time = 0.16, size = 40, normalized size = 0.95

$$\frac{b^2\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}+\frac{a^2x^3-3\,abx}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^2),x, algorithm="giac")
```

```
[Out] b^2*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(a^2*x^3 - 3*a*b*x)/a^3
```

maple [A] time = 0.00, size = 38, normalized size = 0.90

$$\frac{x^3}{3a}+\frac{b^2\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^2}-\frac{bx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b/x^2),x)
```

```
[Out] 1/3/a*x^3-1/a^2*b*x+1/a^2*b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*a*x)
```

maxima [A] time = 1.85, size = 37, normalized size = 0.88

$$\frac{b^2\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}+\frac{ax^3-3\,bx}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^2),x, algorithm="maxima")
```

```
[Out] b^2*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(a*x^3 - 3*b*x)/a^2
```

mupad [B] time = 0.05, size = 32, normalized size = 0.76

$$\frac{x^3}{3\,a}+\frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{a^{5/2}}-\frac{b\,x}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b/x^2),x)
```

```
[Out] x^3/(3*a) + (b^(3/2)*atan((a^(1/2)*x)/b^(1/2)))/a^(5/2) - (b*x)/a^2
```

sympy [**B**] time = 0.18, size = 80, normalized size = 1.90

$$-\frac{\sqrt{-\frac{b^3}{a^5}}\,\log\left(-\frac{a^2\sqrt{-\frac{b^3}{a^5}}}{b}+x\right)}{2}+\frac{\sqrt{-\frac{b^3}{a^5}}\,\log\left(\frac{a^2\sqrt{-\frac{b^3}{a^5}}}{b}+x\right)}{2}+\frac{x^3}{3a}-\frac{bx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b/x**2),x)
```

```
[Out] -sqrt(-b**3/a**5)*log(-a**2*sqrt(-b**3/a**5)/b + x)/2 + sqrt(-b**3/a**5)*lo
g(a**2*sqrt(-b**3/a**5)/b + x)/2 + x**3/(3*a) - b*x/a**2
```

3.1848 $\int \frac{x}{a+\frac{b}{x^2}}\,dx$

Optimal. Leaf size=27

$$\frac{x^2}{2a}-\frac{b\log\left(ax^2+b\right)}{2a^2}$$

```
[Out] 1/2*x^2/a-1/2*b*ln(a*x^2+b)/a^2
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {263, 266, 43}

$$\frac{x^2}{2a}-\frac{b\log\left(ax^2+b\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b/x^2),x]
```

```
[Out] x^2/(2*a) - (b*Log[b + a*x^2])/(2*a^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{a+\frac{b}{x^2}}\,dx &= \int \frac{x^3}{b+ax^2}\,dx\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{b+ax}\,dx, x, x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{a}-\frac{b}{a(b+ax)}\right)\,dx, x, x^2\right)\\ &= \frac{x^2}{2a}-\frac{b\log\left(b+ax^2\right)}{2a^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{x^2}{2a}-\frac{b\log\left(ax^2+b\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b/x^2),x]
[Out] x^2/(2*a) - (b*Log[b + a*x^2])/(2*a^2)
```

fricas [A] time = 0.98, size = 22, normalized size = 0.81

$$\frac{ax^2 - b\log\left(ax^2+b\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^2),x, algorithm="fricas")
[Out] 1/2*(a*x^2 - b*log(a*x^2 + b))/a^2
```

giac [A] time = 0.18, size = 24, normalized size = 0.89

$$\frac{x^2}{2\,a} - \frac{b\log\left(\left|ax^2+b\right|\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^2),x, algorithm="giac")
[Out] 1/2*x^2/a - 1/2*b*log(abs(a*x^2 + b))/a^2
```

maple [A] time = 0.00, size = 24, normalized size = 0.89

$$\frac{x^2}{2a} - \frac{b\ln\left(a\,x^2+b\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b/x^2),x)
[Out] 1/2/a*x^2-1/2*b*ln(a*x^2+b)/a^2
```

maxima [A] time = 0.79, size = 23, normalized size = 0.85

$$\frac{x^2}{2\,a} - \frac{b\log\left(ax^2+b\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^2),x, algorithm="maxima")
[Out] 1/2*x^2/a - 1/2*b*log(a*x^2 + b)/a^2
```

mupad [B] time = 0.04, size = 22, normalized size = 0.81

$$-\frac{b\,\ln\left(a\,x^2+b\right) - a\,x^2}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b/x^2),x)
[Out] -(b*log(b + a*x^2) - a*x^2)/(2*a^2)
```

sympy [A] time = 0.15, size = 20, normalized size = 0.74

$$\frac{x^2}{2a} - \frac{b\log\left(ax^2+b\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x**2),x)
[Out] x**2/(2*a) - b*log(a*x**2 + b)/(2*a**2)
```

3.1849 $\int \frac{1}{a+\frac{b}{x^2}}\,dx$

Optimal. Leaf size=31

$$\frac{x}{a}-\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{a^{3/2}}$$

```
[Out] x/a-arctan(x*a^(1/2)/b^(1/2))*b^(1/2)/a^(3/2)
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {193, 321, 205}

$$\frac{x}{a}-\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^(-1),x]
```

```
[Out] x/a - (Sqrt[b]*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/a^(3/2)
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+\frac{b}{x^2}}\,dx &= \int \frac{x^2}{b+ax^2}\,dx\\ &= \frac{x}{a}-\frac{b\int \frac{1}{b+ax^2}\,dx}{a}\\ &= \frac{x}{a}-\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{a^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{x}{a}-\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^(-1),x]
```

```
[Out] x/a - (Sqrt[b]*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/a^(3/2)
```

fricas [A] time = 0.76, size = 82, normalized size = 2.65

$$\left[\frac{\sqrt{-\frac{b}{a}}\,\log\left(\frac{ax^2-2\,ax\sqrt{-\frac{b}{a}}-b}{ax^2+b}\right)+2\,x}{2\,a},-\frac{\sqrt{\frac{b}{a}}\,\arctan\left(\frac{ax\sqrt{\frac{b}{a}}}{b}\right)-x}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-b/a)*log((a*x^2 - 2*a*x*sqrt(-b/a) - b)/(a*x^2 + b)) + 2*x)/a,
-(sqrt(b/a)*arctan(a*x*sqrt(b/a)/b) - x)/a]
```

giac [A] time = 0.15, size = 26, normalized size = 0.84

$$-\frac{b\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}+\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2),x, algorithm="giac")
```

```
[Out] -b*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a) + x/a
```

maple [A] time = 0.00, size = 27, normalized size = 0.87

$$-\frac{b\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}\;a}+\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2),x)
```

```
[Out] 1/a*x-1/a*b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*a*x)
```

maxima [A] time = 1.93, size = 26, normalized size = 0.84

$$-\frac{b\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}+\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2),x, algorithm="maxima")
```

```
[Out] -b*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a) + x/a
```

mupad [B] time = 1.09, size = 23, normalized size = 0.74

$$\frac{x}{a}-\frac{\sqrt{b}\,\operatorname{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x^2),x)
```

```
[Out] x/a - (b^(1/2)*atan((a^(1/2)*x)/b^(1/2)))/a^(3/2)
```

sympy [B] time = 0.16, size = 56, normalized size = 1.81

$$\frac{\sqrt{-\frac{b}{a^3}}\,\log\left(-a\sqrt{-\frac{b}{a^3}}+x\right)}{2}-\frac{\sqrt{-\frac{b}{a^3}}\,\log\left(a\sqrt{-\frac{b}{a^3}}+x\right)}{2}+\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2),x)
```

```
[Out] sqrt(-b/a**3)*log(-a*sqrt(-b/a**3) + x)/2 - sqrt(-b/a**3)*log(a*sqrt(-b/a**
3) + x)/2 + x/a
```

3.1850 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)x}\,dx$

Optimal. Leaf size=15

$$\frac{\log\left(ax^2+b\right)}{2a}$$

```
[Out] 1/2*ln(a*x^2+b)/a
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 260}

$$\frac{\log\left(ax^2+b\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)*x),x]
```

```
[Out] Log[b + a*x^2]/(2*a)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x^2}\right)x}\,dx &= \int \frac{x}{b+ax^2}\,dx \\ &= \frac{\log\left(b+ax^2\right)}{2a}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log\left(ax^2+b\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)*x),x]
```

```
[Out] Log[b + a*x^2]/(2*a)
```

fricas [A] time = 0.80, size = 13, normalized size = 0.87

$$\frac{\log\left(ax^2+b\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)/x,x, algorithm="fricas")
[Out] 1/2*log(a*x^2 + b)/a
```

giac [A] time = 0.15, size = 14, normalized size = 0.93

$$\frac{\log\left(\left|ax^2+b\right|\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)/x,x, algorithm="giac")
[Out] 1/2*log(abs(a*x^2 + b))/a
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{\ln\left(a\,x^2+b\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)/x,x)
[Out] 1/2*ln(a*x^2+b)/a
```

maxima [A] time = 0.86, size = 13, normalized size = 0.87

$$\frac{\log\left(ax^2+b\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)/x,x, algorithm="maxima")
[Out] 1/2*log(a*x^2 + b)/a
```

mupad [B] time = 1.06, size = 13, normalized size = 0.87

$$\frac{\ln\left(a\,x^2+b\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b/x^2)),x)
[Out] log(b + a*x^2)/(2*a)
```

sympy [A] time = 0.12, size = 10, normalized size = 0.67

$$\frac{\log\left(ax^2+b\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)/x,x)
[Out] log(a*x**2 + b)/(2*a)
```

3.1851 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^2}\,dx$

Optimal. Leaf size=24

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\,\sqrt{b}}$$

```
[Out] arctan(x*a^(1/2)/b^(1/2))/a^(1/2)/b^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)*x^2),x]
```

```
[Out] ArcTan[(Sqrt[a]*x)/Sqrt[b]]/(Sqrt[a]*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^2}\,dx &= \int \frac{1}{b+ax^2}\,dx\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\,\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)*x^2),x]
```

```
[Out] ArcTan[(Sqrt[a]*x)/Sqrt[b]]/(Sqrt[a]*Sqrt[b])
```

fricas [A] time = 0.78, size = 67, normalized size = 2.79

$$\left[-\frac{\sqrt{-ab}\,\log\left(\frac{ax^2-2\sqrt{-ab}\,x-b}{ax^2+b}\right)}{2\,ab},\ \frac{\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{b}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)/x^2,x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b)*log((a*x^2 - 2*sqrt(-a*b)*x - b)/(a*x^2 + b))/(a*b), sqrt(
a*b)*arctan(sqrt(a*b)*x/b)/(a*b)]
```

giac [**A**] time = 0.19, size = 15, normalized size = 0.62

$$\frac{\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)/x^2,x, algorithm="giac")
```

```
[Out] arctan(a*x/sqrt(a*b))/sqrt(a*b)
```

maple [**A**] time = 0.00, size = 16, normalized size = 0.67

$$\frac{\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)/x^2,x)
```

```
[Out] 1/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*a*x)
```

maxima [**A**] time = 1.93, size = 15, normalized size = 0.62

$$\frac{\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)/x^2,x, algorithm="maxima")
```

```
[Out] arctan(a*x/sqrt(a*b))/sqrt(a*b)
```

mupad [**B**] time = 0.05, size = 16, normalized size = 0.67

$$\frac{\operatorname{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b/x^2)),x)
```

```
[Out] atan((a^(1/2)*x)/b^(1/2))/(a^(1/2)*b^(1/2))
```

sympy [**B**] time = 0.14, size = 53, normalized size = 2.21

$$-\frac{\sqrt{-\frac{1}{ab}}\,\log\left(-b\sqrt{-\frac{1}{ab}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ab}}\,\log\left(b\sqrt{-\frac{1}{ab}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)/x**2,x)
```

```
[Out] -sqrt(-1/(a*b))*log(-b*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*log(b*sqrt(-1
/(a*b)) + x)/2
```

3.1852 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^3}\,dx$

Optimal. Leaf size=15

$$-\frac{\log\left(a+\frac{b}{x^2}\right)}{2b}$$

```
[Out] -1/2*ln(a+b/x^2)/b
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {260}

$$-\frac{\log\left(a+\frac{b}{x^2}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)*x^3),x]
```

```
[Out] -Log[a + b/x^2]/(2*b)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^3}\,dx = -\frac{\log\left(a+\frac{b}{x^2}\right)}{2b}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.47

$$\frac{\log(x)}{b} - \frac{\log\left(ax^2+b\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)*x^3),x]
```

```
[Out] Log[x]/b - Log[b + a*x^2]/(2*b)
```

fricas [A] time = 0.66, size = 18, normalized size = 1.20

$$-\frac{\log\left(ax^2+b\right)-2\,\log(x)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(log(a*x^2 + b) - 2*log(x))/b
```

giac [A] time = 0.17, size = 24, normalized size = 1.60

$$\frac{\log\left(x^2\right)}{2\,b}-\frac{\log\left(\left|a x^2+b\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)/x^3,x, algorithm="giac")
[Out] 1/2*log(x^2)/b - 1/2*log(abs(a*x^2 + b))/b
```

maple [A] time = 0.00, size = 21, normalized size = 1.40

$$\frac{\ln(x)}{b}-\frac{\ln\left(a\,x^2+b\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)/x^3,x)
[Out] -1/2/b*ln(a*x^2+b)+1/b*ln(x)
```

maxima [A] time = 0.77, size = 13, normalized size = 0.87

$$-\frac{\log\left(a+\frac{b}{x^2}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)/x^3,x, algorithm="maxima")
[Out] -1/2*log(a + b/x^2)/b
```

mupad [B] time = 0.07, size = 18, normalized size = 1.20

$$-\frac{\ln\left(a\,x^2+b\right)-2\,\ln(x)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b/x^2)),x)
[Out] -(log(b + a*x^2) - 2*log(x))/(2*b)
```

sympy [A] time = 0.22, size = 15, normalized size = 1.00

$$\frac{\log(x)}{b}-\frac{\log\left(x^2+\frac{b}{a}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)/x**3,x)
[Out] log(x)/b - log(x**2 + b/a)/(2*b)
```

3.1853 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^4}\,dx$

Optimal. Leaf size=34

$$-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{b^{3/2}}-\frac{1}{bx}$$

```
[Out] -1/b/x-arctan(x*a^(1/2)/b^(1/2))*a^(1/2)/b^(3/2)
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 325, 205}

$$-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{b^{3/2}}-\frac{1}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)*x^4),x]
```

```
[Out] -(1/(b*x)) - (Sqrt[a]*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/b^(3/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^4}\,dx &= \int \frac{1}{x^2\left(b+ax^2\right)}\,dx\\ &= -\frac{1}{bx}-\frac{a\int \frac{1}{b+ax^2}\,dx}{b}\\ &= -\frac{1}{bx}-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.00

$$-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{b^{3/2}}-\frac{1}{bx}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)*x^4),x]
```

```
[Out] -(1/(b*x)) - (Sqrt[a]*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/b^(3/2)
```

fricas [A] time = 0.83, size = 82, normalized size = 2.41

$$\left[\frac{x\sqrt{-\frac{a}{b}}\log\left(\frac{ax^2-2\,bx\sqrt{-\frac{a}{b}}-b}{ax^2+b}\right)-2}{2\,bx}, -\frac{x\sqrt{\frac{a}{b}}\arctan\left(x\sqrt{\frac{a}{b}}\right)+1}{bx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/2*(x*sqrt(-a/b)*log((a*x^2 - 2*b*x*sqrt(-a/b) - b)/(a*x^2 + b)) - 2)/(b*
x), -(x*sqrt(a/b)*arctan(x*sqrt(a/b)) + 1)/(b*x)]
```

giac [A] time = 0.15, size = 29, normalized size = 0.85

$$-\frac{a\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}-\frac{1}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)/x^4,x, algorithm="giac")
```

```
[Out] -a*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b) - 1/(b*x)
```

maple [A] time = 0.00, size = 30, normalized size = 0.88

$$-\frac{a\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}\;b}-\frac{1}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)/x^4,x)
```

```
[Out] -1/b/x-a/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*a*x)
```

maxima [A] time = 1.84, size = 29, normalized size = 0.85

$$-\frac{a\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}-\frac{1}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)/x^4,x, algorithm="maxima")
```

```
[Out] -a*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b) - 1/(b*x)
```

mupad [B] time = 0.05, size = 26, normalized size = 0.76

$$-\frac{1}{b\,x}-\frac{\sqrt{a}\,\operatorname{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b/x^2)),x)
```

```
[Out] - 1/(b*x) - (a^(1/2)*atan((a^(1/2)*x)/b^(1/2)))/b^(3/2)
```

sympy [**B**] time = 0.19, size = 65, normalized size = 1.91

$$\frac{\sqrt{-\frac{a}{b^3}}\,\log\left(x-\frac{b^2\sqrt{-\frac{a}{b^3}}}{a}\right)}{2}-\frac{\sqrt{-\frac{a}{b^3}}\,\log\left(x+\frac{b^2\sqrt{-\frac{a}{b^3}}}{a}\right)}{2}-\frac{1}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)/x**4,x)
```

```
[Out] sqrt(-a/b**3)*log(x - b**2*sqrt(-a/b**3)/a)/2 - sqrt(-a/b**3)*log(x + b**2*
sqrt(-a/b**3)/a)/2 - 1/(b*x)
```

3.1854 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^5}\,dx$

Optimal. Leaf size=35

$$\frac{a\log\left(ax^2+b\right)}{2b^2}-\frac{a\log(x)}{b^2}-\frac{1}{2bx^2}$$

```
[Out] -1/2/b/x^2-a*ln(x)/b^2+1/2*a*ln(a*x^2+b)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 266, 44}

$$\frac{a\log\left(ax^2+b\right)}{2b^2}-\frac{a\log(x)}{b^2}-\frac{1}{2bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)*x^5),x]
```

```
[Out] -1/(2*b*x^2) - (a*Log[x])/b^2 + (a*Log[b + a*x^2])/(2*b^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^5}\,dx &= \int \frac{1}{x^3\left(b+ax^2\right)}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2(b+ax)}\,dx, x, x^2\right) \\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{bx^2}-\frac{a}{b^2x}+\frac{a^2}{b^2(b+ax)}\right)dx, x, x^2\right) \\
&= -\frac{1}{2bx^2}-\frac{a\log(x)}{b^2}+\frac{a\log\left(b+ax^2\right)}{2b^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.00

$$\frac{a\log\left(ax^2+b\right)}{2b^2}-\frac{a\log(x)}{b^2}-\frac{1}{2bx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)*x^5),x]
```

```
[Out] -1/2*1/(b*x^2) - (a*Log[x])/b^2 + (a*Log[b + a*x^2])/(2*b^2)
```

fricas [A] time = 0.88, size = 33, normalized size = 0.94

$$\frac{ax^2 \log\left(ax^2+b\right) - 2\,ax^2 \log(x) - b}{2\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/2*(a*x^2*log(a*x^2 + b) - 2*a*x^2*log(x) - b)/(b^2*x^2)
```

giac [A] time = 0.17, size = 43, normalized size = 1.23

$$-\frac{a\log\left(x^2\right)}{2\,b^2} + \frac{a\log\left(\left|ax^2+b\right|\right)}{2\,b^2} + \frac{ax^2-b}{2\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)/x^5,x, algorithm="giac")
```

```
[Out] -1/2*a*log(x^2)/b^2 + 1/2*a*log(abs(a*x^2 + b))/b^2 + 1/2*(a*x^2 - b)/(b^2*
x^2)
```

maple [A] time = 0.01, size = 32, normalized size = 0.91

$$-\frac{a\ln(x)}{b^2} + \frac{a\ln\left(a\,x^2+b\right)}{2b^2} - \frac{1}{2b\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)/x^5,x)
```

```
[Out] -1/2/b/x^2-a/b^2*ln(x)+1/2*a*ln(a*x^2+b)/b^2
```

maxima [A] time = 0.85, size = 33, normalized size = 0.94

$$\frac{a\log\left(ax^2+b\right)}{2\,b^2} - \frac{a\log\left(x^2\right)}{2\,b^2} - \frac{1}{2\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*a*log(a*x^2 + b)/b^2 - 1/2*a*log(x^2)/b^2 - 1/2/(b*x^2)
```

mupad [B] time = 0.07, size = 31, normalized size = 0.89

$$\frac{a\,\ln\left(a\,x^2+b\right)}{2\,b^2} - \frac{1}{2\,b\,x^2} - \frac{a\,\ln(x)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b/x^2)),x)
```

```
[Out] (a*log(b + a*x^2))/(2*b^2) - 1/(2*b*x^2) - (a*log(x))/b^2
```

sympy [**A**] time = 0.28, size = 31, normalized size = 0.89

$$-\frac{a\log(x)}{b^2}+\frac{a\log\left(x^2+\frac{b}{a}\right)}{2b^2}-\frac{1}{2bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)/x**5,x)
[Out] -a*log(x)/b**2 + a*log(x**2 + b/a)/(2*b**2) - 1/(2*b*x**2)
```

3.1855 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^6}\,dx$

Optimal. Leaf size=43

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{b^{5/2}}+\frac{a}{b^2x}-\frac{1}{3bx^3}$$

```
[Out] -1/3/b/x^3+a/b^2/x+a^(3/2)*arctan(x*a^(1/2)/b^(1/2))/b^(5/2)
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 325, 205}

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{b^{5/2}}+\frac{a}{b^2x}-\frac{1}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)*x^6),x]
```

```
[Out] -1/(3*b*x^3) + a/(b^2*x) + (a^(3/2)*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/b^(5/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^6}\,dx &= \int \frac{1}{x^4\left(b+ax^2\right)}\,dx\\ &= -\frac{1}{3bx^3}-\frac{a\int \frac{1}{x^2\left(b+ax^2\right)}\,dx}{b}\\ &= -\frac{1}{3bx^3}+\frac{a}{b^2x}+\frac{a^2\int \frac{1}{b+ax^2}\,dx}{b^2}\\ &= -\frac{1}{3bx^3}+\frac{a}{b^2x}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{b^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 1.00

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{b^{5/2}}+\frac{a}{b^2x}-\frac{1}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)*x^6),x]
```

```
[Out] -1/3*1/(b*x^3) + a/(b^2*x) + (a^(3/2)*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/b^(5/2)
```

fricas [A] time = 1.04, size = 106, normalized size = 2.47

$$\left[\frac{3\,ax^3\sqrt{-\frac{a}{b}}\,\log\left(\frac{ax^2+2\,bx\sqrt{-\frac{a}{b}}-b}{ax^2+b}\right)+6\,ax^2-2\,b}{6\,b^2x^3},\frac{3\,ax^3\sqrt{\frac{a}{b}}\,\arctan\left(x\sqrt{\frac{a}{b}}\right)+3\,ax^2-b}{3\,b^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/6*(3*a*x^3*sqrt(-a/b)*log((a*x^2 + 2*b*x*sqrt(-a/b) - b)/(a*x^2 + b)) +
6*a*x^2 - 2*b)/(b^2*x^3), 1/3*(3*a*x^3*sqrt(a/b)*arctan(x*sqrt(a/b)) + 3*a*
x^2 - b)/(b^2*x^3)]
```

giac [A] time = 0.16, size = 40, normalized size = 0.93

$$\frac{a^2\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}+\frac{3\,ax^2-b}{3\,b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)/x^6,x, algorithm="giac")
```

```
[Out] a^2*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(3*a*x^2 - b)/(b^2*x^3)
```

maple [A] time = 0.01, size = 39, normalized size = 0.91

$$\frac{a^2\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^2}+\frac{a}{b^2x}-\frac{1}{3b\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)/x^6,x)
```

```
[Out] a^2/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*a*x)-1/3/b/x^3+a/b^2/x
```

maxima [A] time = 1.91, size = 40, normalized size = 0.93

$$\frac{a^2\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}+\frac{3\,ax^2-b}{3\,b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)/x^6,x, algorithm="maxima")
```

```
[Out] a^2*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(3*a*x^2 - b)/(b^2*x^3)
```

mupad [B] time = 1.08, size = 37, normalized size = 0.86

$$\frac{a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{b^{5/2}}-\frac{\frac{1}{3\,b}-\frac{a\,x^2}{b^2}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b/x^2)),x)
```

```
[Out] (a^(3/2)*atan((a^(1/2)*x)/b^(1/2)))/b^(5/2) - (1/(3*b) - (a*x^2)/b^2)/x^3
```

sympy [B] time = 0.23, size = 87, normalized size = 2.02

$$-\frac{\sqrt{-\frac{a^3}{b^5}}\,\log\left(x-\frac{b^3\sqrt{-\frac{a^3}{b^5}}}{a^2}\right)}{2}+\frac{\sqrt{-\frac{a^3}{b^5}}\,\log\left(x+\frac{b^3\sqrt{-\frac{a^3}{b^5}}}{a^2}\right)}{2}+\frac{3ax^2-b}{3b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)/x**6,x)
```

```
[Out] -sqrt(-a**3/b**5)*log(x - b**3*sqrt(-a**3/b**5)/a**2)/2 + sqrt(-a**3/b**5)*
log(x + b**3*sqrt(-a**3/b**5)/a**2)/2 + (3*a*x**2 - b)/(3*b**2*x**3)
```

3.1856 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^7}\,dx$

Optimal. Leaf size=49

$$-\frac{a^2\log\left(ax^2+b\right)}{2b^3}+\frac{a^2\log(x)}{b^3}+\frac{a}{2b^2x^2}-\frac{1}{4bx^4}$$

```
[Out] -1/4/b/x^4+1/2*a/b^2/x^2+a^2*ln(x)/b^3-1/2*a^2*ln(a*x^2+b)/b^3
```

Rubi [A] time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 266, 44}

$$-\frac{a^2\log\left(ax^2+b\right)}{2b^3}+\frac{a^2\log(x)}{b^3}+\frac{a}{2b^2x^2}-\frac{1}{4bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)*x^7),x]
```

```
[Out] -1/(4*b*x^4) + a/(2*b^2*x^2) + (a^2*Log[x])/b^3 - (a^2*Log[b + a*x^2])/(2*b
^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^7}\,dx &= \int \frac{1}{x^5\left(b+ax^2\right)}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3(b+ax)}\,dx, x, x^2\right) \\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{bx^3}-\frac{a}{b^2x^2}+\frac{a^2}{b^3x}-\frac{a^3}{b^3(b+ax)}\right)dx, x, x^2\right) \\
&= -\frac{1}{4bx^4}+\frac{a}{2b^2x^2}+\frac{a^2\log(x)}{b^3}-\frac{a^2\log\left(b+ax^2\right)}{2b^3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.00

$$-\frac{a^2\log\left(ax^2+b\right)}{2b^3}+\frac{a^2\log(x)}{b^3}+\frac{a}{2b^2x^2}-\frac{1}{4bx^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)*x^7),x]
```

```
[Out] -1/4*1/(b*x^4) + a/(2*b^2*x^2) + (a^2*Log[x])/b^3 - (a^2*Log[b + a*x^2])/(2
*b^3)
```

fricas [A] time = 1.01, size = 45, normalized size = 0.92

$$-\frac{2\,a^2x^4\log\left(ax^2+b\right)-4\,a^2x^4\log(x)-2\,abx^2+b^2}{4\,b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)/x^7,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^2*x^4*log(a*x^2 + b) - 4*a^2*x^4*log(x) - 2*a*b*x^2 + b^2)/(b^3*x
^4)
```

giac [A] time = 0.15, size = 57, normalized size = 1.16

$$\frac{a^2\log\left(x^2\right)}{2\,b^3}-\frac{a^2\log\left(\left|ax^2+b\right|\right)}{2\,b^3}-\frac{3\,a^2x^4-2\,abx^2+b^2}{4\,b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)/x^7,x, algorithm="giac")
```

```
[Out] 1/2*a^2*log(x^2)/b^3 - 1/2*a^2*log(abs(a*x^2 + b))/b^3 - 1/4*(3*a^2*x^4 - 2
*a*b*x^2 + b^2)/(b^3*x^4)
```

maple [A] time = 0.01, size = 44, normalized size = 0.90

$$\frac{a^2\ln(x)}{b^3}-\frac{a^2\ln\left(a\,x^2+b\right)}{2b^3}+\frac{a}{2b^2x^2}-\frac{1}{4b\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)/x^7,x)
```

```
[Out] -1/4/b/x^4+1/2*a/b^2/x^2+a^2/b^3*ln(x)-1/2*a^2*ln(a*x^2+b)/b^3
```

maxima [A] time = 0.86, size = 47, normalized size = 0.96

$$-\frac{a^2\log\left(ax^2+b\right)}{2\,b^3}+\frac{a^2\log\left(x^2\right)}{2\,b^3}+\frac{2\,ax^2-b}{4\,b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/2*a^2*log(a*x^2 + b)/b^3 + 1/2*a^2*log(x^2)/b^3 + 1/4*(2*a*x^2 - b)/(b^2
*x^4)
```

mupad [B] time = 0.07, size = 46, normalized size = 0.94

$$\frac{a^2\,\ln(x)}{b^3}-\frac{a^2\,\ln\left(a\,x^2+b\right)}{2\,b^3}-\frac{\frac{1}{4\,b}-\frac{a\,x^2}{2\,b^2}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b/x^2)),x)
```

```
[Out] (a^2*log(x))/b^3 - (a^2*log(b + a*x^2))/(2*b^3) - (1/(4*b) - (a*x^2)/(2*b^2
))/x^4
```

sympy [**A**] time = 0.32, size = 42, normalized size = 0.86

$$\frac{a^2 \log(x)}{b^3} - \frac{a^2 \log\left(x^2 + \frac{b}{a}\right)}{2b^3} + \frac{2ax^2 - b}{4b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)/x**7,x)
```

```
[Out] a**2*log(x)/b**3 - a**2*log(x**2 + b/a)/(2*b**3) + (2*a*x**2 - b)/(4*b**2*x
**4)
```

3.1857 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^8}\,dx$

Optimal. Leaf size=58

$$-\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{b^{7/2}}-\frac{a^2}{b^3x}+\frac{a}{3b^2x^3}-\frac{1}{5bx^5}$$

```
[Out] -1/5/b/x^5+1/3*a/b^2/x^3-a^2/b^3/x-a^(5/2)*arctan(x*a^(1/2)/b^(1/2))/b^(7/2
)
```

Rubi [A] time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 325, 205}

$$-\frac{a^2}{b^3x}-\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{b^{7/2}}+\frac{a}{3b^2x^3}-\frac{1}{5bx^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)*x^8),x]
```

```
[Out] -1/(5*b*x^5) + a/(3*b^2*x^3) - a^2/(b^3*x) - (a^(5/2)*ArcTan[(Sqrt[a]*x)/Sq
rt[b]])/b^(7/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^8}\,dx = \int \frac{1}{x^6\left(b+ax^2\right)}\,dx$$

$$= -\frac{1}{5bx^5} - \frac{a\int \frac{1}{x^4\left(b+ax^2\right)}\,dx}{b}$$

$$= -\frac{1}{5bx^5} + \frac{a}{3b^2x^3} + \frac{a^2\int \frac{1}{x^2\left(b+ax^2\right)}\,dx}{b^2}$$

$$= -\frac{1}{5bx^5} + \frac{a}{3b^2x^3} - \frac{a^2}{b^3x} - \frac{a^3\int \frac{1}{b+ax^2}\,dx}{b^3}$$

$$= -\frac{1}{5bx^5} + \frac{a}{3b^2x^3} - \frac{a^2}{b^3x} - \frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{b^{7/2}}$$

Mathematica [A] time = 0.03, size = 58, normalized size = 1.00

$$-\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{b^{7/2}} - \frac{a^2}{b^3x} + \frac{a}{3b^2x^3} - \frac{1}{5bx^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)*x^8),x]
```

```
[Out] -1/5*1/(b*x^5) + a/(3*b^2*x^3) - a^2/(b^3*x) - (a^(5/2)*ArcTan[(Sqrt[a]*x)/
Sqrt[b]])/b^(7/2)
```

fricas [A] time = 0.89, size = 132, normalized size = 2.28

$$\left[\frac{15\,a^2x^5\sqrt{-\frac{a}{b}}\,\log\left(\frac{ax^2-2\,bx\sqrt{-\frac{a}{b}}-b}{ax^2+b}\right) - 30\,a^2x^4 + 10\,abx^2 - 6\,b^2}{30\,b^3x^5}, -\frac{15\,a^2x^5\sqrt{\frac{a}{b}}\,\arctan\left(x\sqrt{\frac{a}{b}}\right) + 15\,a^2x^4 - 5\,abx^2 + 3}{15\,b^3x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)/x^8,x, algorithm="fricas")
```

```
[Out] [1/30*(15*a^2*x^5*sqrt(-a/b)*log((a*x^2 - 2*b*x*sqrt(-a/b) - b)/(a*x^2 + b)
) - 30*a^2*x^4 + 10*a*b*x^2 - 6*b^2)/(b^3*x^5), -1/15*(15*a^2*x^5*sqrt(a/b)
*arctan(x*sqrt(a/b)) + 15*a^2*x^4 - 5*a*b*x^2 + 3*b^2)/(b^3*x^5)]
```

giac [A] time = 0.15, size = 52, normalized size = 0.90

$$-\frac{a^3\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3} - \frac{15\,a^2x^4 - 5\,abx^2 + 3\,b^2}{15\,b^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)/x^8,x, algorithm="giac")
```

```
[Out] -a^3*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/15*(15*a^2*x^4 - 5*a*b*x^2 +
 3*b^2)/(b^3*x^5)
```

maple [A] time = 0.01, size = 52, normalized size = 0.90

$$-\frac{a^3\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^3} - \frac{a^2}{b^3x} + \frac{a}{3b^2x^3} - \frac{1}{5b\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)/x^8,x)
```

```
[Out] -a^3/b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*a*x)-1/5/b/x^5-a^2/b^3/x+1/3*a/b^
2/x^3
```

maxima [A] time = 1.89, size = 52, normalized size = 0.90

$$-\frac{a^3 \arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}\, b^3} - \frac{15\, a^2x^4 - 5\, abx^2 + 3\, b^2}{15\, b^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)/x^8,x, algorithm="maxima")
```

```
[Out] -a^3*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/15*(15*a^2*x^4 - 5*a*b*x^2 +
 3*b^2)/(b^3*x^5)
```

mupad [B] time = 0.05, size = 48, normalized size = 0.83

$$-\frac{\frac{1}{5\,b} - \frac{a\,x^2}{3\,b^2} + \frac{a^2\,x^4}{b^3}}{x^5} - \frac{a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(a + b/x^2)),x)
```

```
[Out] - (1/(5*b) - (a*x^2)/(3*b^2) + (a^2*x^4)/b^3)/x^5 - (a^(5/2)*atan((a^(1/2)*
x)/b^(1/2)))/b^(7/2)
```

sympy [B] time = 0.28, size = 100, normalized size = 1.72

$$\frac{\sqrt{-\frac{a^5}{b^7}}\,\log\left(x - \frac{b^4\sqrt{-\frac{a^5}{b^7}}}{a^3}\right)}{2} - \frac{\sqrt{-\frac{a^5}{b^7}}\,\log\left(x + \frac{b^4\sqrt{-\frac{a^5}{b^7}}}{a^3}\right)}{2} + \frac{-15a^2x^4 + 5abx^2 - 3b^2}{15b^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)/x**8,x)
```

```
[Out] sqrt(-a**5/b**7)*log(x - b**4*sqrt(-a**5/b**7)/a**3)/2 - sqrt(-a**5/b**7)*l
og(x + b**4*sqrt(-a**5/b**7)/a**3)/2 + (-15*a**2*x**4 + 5*a*b*x**2 - 3*b**2
)/(15*b**3*x**5)
```

3.1858 $\int \frac{x^6}{\left(a+\frac{b}{x^2}\right)^2}\,dx$

Optimal. Leaf size=92

$$\frac{9b^{7/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2a^{11/2}}-\frac{9b^3x}{2a^5}+\frac{3b^2x^3}{2a^4}-\frac{9bx^5}{10a^3}+\frac{9x^7}{14a^2}-\frac{x^9}{2a\left(ax^2+b\right)}$$

```
[Out] -9/2*b^3*x/a^5+3/2*b^2*x^3/a^4-9/10*b*x^5/a^3+9/14*x^7/a^2-1/2*x^9/a/(a*x^2
+b)+9/2*b^(7/2)*arctan(x*a^(1/2)/b^(1/2))/a^(11/2)
```

Rubi [A] time = 0.04, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {263, 288, 302, 205}

$$\frac{3b^2x^3}{2a^4}-\frac{9b^3x}{2a^5}+\frac{9b^{7/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2a^{11/2}}-\frac{9bx^5}{10a^3}+\frac{9x^7}{14a^2}-\frac{x^9}{2a\left(ax^2+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b/x^2)^2,x]
```

```
[Out] (-9*b^3*x)/(2*a^5) + (3*b^2*x^3)/(2*a^4) - (9*b*x^5)/(10*a^3) + (9*x^7)/(14
*a^2) - x^9/(2*a*(b + a*x^2)) + (9*b^(7/2)*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(2*
a^(11/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{x^6}{\left(a+\frac{b}{x^2}\right)^2}\,dx = \int \frac{x^{10}}{\left(b+ax^2\right)^2}\,dx$$

$$= -\frac{x^9}{2a\left(b+ax^2\right)} + \frac{9\int \frac{x^8}{b+ax^2}\,dx}{2a}$$

$$= -\frac{x^9}{2a\left(b+ax^2\right)} + \frac{9\int \left(-\frac{b^3}{a^4} + \frac{b^2x^2}{a^3} - \frac{bx^4}{a^2} + \frac{x^6}{a} + \frac{b^4}{a^4\left(b+ax^2\right)}\right)dx}{2a}$$

$$= -\frac{9b^3x}{2a^5} + \frac{3b^2x^3}{2a^4} - \frac{9bx^5}{10a^3} + \frac{9x^7}{14a^2} - \frac{x^9}{2a\left(b+ax^2\right)} + \frac{\left(9b^4\right)\int \frac{1}{b+ax^2}\,dx}{2a^5}$$

$$= -\frac{9b^3x}{2a^5} + \frac{3b^2x^3}{2a^4} - \frac{9bx^5}{10a^3} + \frac{9x^7}{14a^2} - \frac{x^9}{2a\left(b+ax^2\right)} + \frac{9b^{7/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2a^{11/2}}$$

Mathematica [A] time = 0.06, size = 82, normalized size = 0.89

$$\frac{9b^{7/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2a^{11/2}} + \frac{x\left(10a^3x^6 - 28a^2bx^4 - \frac{35b^4}{ax^2+b} + 70ab^2x^2 - 280b^3\right)}{70a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b/x^2)^2,x]
```

```
[Out] (x*(-280*b^3 + 70*a*b^2*x^2 - 28*a^2*b*x^4 + 10*a^3*x^6 - (35*b^4)/(b + a*x
^2)))/(70*a^5) + (9*b^(7/2)*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(2*a^(11/2))
```

fricas [A] time = 0.86, size = 212, normalized size = 2.30

$$\left[\frac{20\,a^4x^9 - 36\,a^3bx^7 + 84\,a^2b^2x^5 - 420\,ab^3x^3 - 630\,b^4x + 315\left(ab^3x^2+b^4\right)\sqrt{-\frac{b}{a}}\,\log\left(\frac{ax^2+2\,ax\sqrt{-\frac{b}{a}}-b}{ax^2+b}\right)}{140\left(a^6x^2+a^5b\right)}, \frac{10\,a^4x^9 -}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(a+b/x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/140*(20*a^4*x^9 - 36*a^3*b*x^7 + 84*a^2*b^2*x^5 - 420*a*b^3*x^3 - 630*b^
4*x + 315*(a*b^3*x^2 + b^4)*sqrt(-b/a)*log((a*x^2 + 2*a*x*sqrt(-b/a) - b)/(
a*x^2 + b)))/(a^6*x^2 + a^5*b), 1/70*(10*a^4*x^9 - 18*a^3*b*x^7 + 42*a^2*b^
2*x^5 - 210*a*b^3*x^3 - 315*b^4*x + 315*(a*b^3*x^2 + b^4)*sqrt(b/a)*arctan(
a*x*sqrt(b/a)/b))/(a^6*x^2 + a^5*b)]
```

giac [A] time = 0.17, size = 84, normalized size = 0.91

$$\frac{9\,b^4\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a^5} - \frac{b^4x}{2\left(ax^2+b\right)a^5} + \frac{5\,a^{12}x^7 - 14\,a^{11}bx^5 + 35\,a^{10}b^2x^3 - 140\,a^9b^3x}{35\,a^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(a+b/x^2)^2,x, algorithm="giac")
```

```
[Out] 9/2*b^4*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^5) - 1/2*b^4*x/((a*x^2 + b)*a^5)
 + 1/35*(5*a^12*x^7 - 14*a^11*b*x^5 + 35*a^10*b^2*x^3 - 140*a^9*b^3*x)/a^14
```

maple [A] time = 0.01, size = 78, normalized size = 0.85

$$\frac{x^7}{7a^2}-\frac{2b\,x^5}{5a^3}+\frac{b^2x^3}{a^4}-\frac{b^4x}{2\left(a\,x^2+b\right)a^5}+\frac{9b^4\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^5}-\frac{4b^3x}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a+b/x^2)^2,x)
```

```
[Out] 1/7*x^7/a^2-2/5/a^3*b*x^5+1/a^4*b^2*x^3-4/a^5*b^3*x-1/2/a^5*b^4*x/(a*x^2+b)
+9/2/a^5*b^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*a*x)
```

maxima [A] time = 2.00, size = 82, normalized size = 0.89

$$-\frac{b^4x}{2\left(a^6x^2+a^5b\right)}+\frac{9\,b^4\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^5}+\frac{5\,a^3x^7-14\,a^2bx^5+35\,ab^2x^3-140\,b^3x}{35\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(a+b/x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*b^4*x/(a^6*x^2 + a^5*b) + 9/2*b^4*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^5
) + 1/35*(5*a^3*x^7 - 14*a^2*b*x^5 + 35*a*b^2*x^3 - 140*b^3*x)/a^5
```

mupad [B] time = 1.10, size = 77, normalized size = 0.84

$$\frac{x^7}{7\,a^2}-\frac{2\,b\,x^5}{5\,a^3}-\frac{4\,b^3\,x}{a^5}+\frac{9\,b^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2\,a^{11/2}}+\frac{b^2\,x^3}{a^4}-\frac{b^4\,x}{2\,\left(a^6\,x^2+b\,a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b/x^2)^2,x)
```

```
[Out] x^7/(7*a^2) - (2*b*x^5)/(5*a^3) - (4*b^3*x)/a^5 + (9*b^(7/2)*atan((a^(1/2)*
x)/b^(1/2)))/(2*a^(11/2)) + (b^2*x^3)/a^4 - (b^4*x)/(2*(a^5*b + a^6*x^2))
```

sympy [A] time = 0.35, size = 134, normalized size = 1.46

$$-\frac{b^4x}{2a^6x^2+2a^5b}-\frac{9\sqrt{-\frac{b^7}{a^{11}}}\,\log\left(-\frac{a^5\sqrt{-\frac{b^7}{a^{11}}}}{b^3}+x\right)}{4}+\frac{9\sqrt{-\frac{b^7}{a^{11}}}\,\log\left(\frac{a^5\sqrt{-\frac{b^7}{a^{11}}}}{b^3}+x\right)}{4}+\frac{x^7}{7a^2}-\frac{2bx^5}{5a^3}+\frac{b^2x^3}{a^4}-\frac{4b^3x}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(a+b/x**2)**2,x)
```

```
[Out] -b**4*x/(2*a**6*x**2 + 2*a**5*b) - 9*sqrt(-b**7/a**11)*log(-a**5*sqrt(-b**7
/a**11)/b**3 + x)/4 + 9*sqrt(-b**7/a**11)*log(a**5*sqrt(-b**7/a**11)/b**3 +
 x)/4 + x**7/(7*a**2) - 2*b*x**5/(5*a**3) + b**2*x**3/a**4 - 4*b**3*x/a**5
```

3.1859 $\int \frac{x^5}{\left(a+\frac{b}{x^2}\right)^2}\,dx$

Optimal. Leaf size=70

$$-\frac{b^4}{2a^5\left(ax^2+b\right)}-\frac{2b^3\log\left(ax^2+b\right)}{a^5}+\frac{3b^2x^2}{2a^4}-\frac{bx^4}{2a^3}+\frac{x^6}{6a^2}$$

```
[Out] 3/2*b^2*x^2/a^4-1/2*b*x^4/a^3+1/6*x^6/a^2-1/2*b^4/a^5/(a*x^2+b)-2*b^3*ln(a*
x^2+b)/a^5
```

Rubi [A] time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 266, 43}

$$\frac{3b^2x^2}{2a^4}-\frac{b^4}{2a^5\left(ax^2+b\right)}-\frac{2b^3\log\left(ax^2+b\right)}{a^5}-\frac{bx^4}{2a^3}+\frac{x^6}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b/x^2)^2,x]
```

```
[Out] (3*b^2*x^2)/(2*a^4) - (b*x^4)/(2*a^3) + x^6/(6*a^2) - b^4/(2*a^5*(b + a*x^2
)) - (2*b^3*Log[b + a*x^2])/a^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(a+\frac{b}{x^2}\right)^2}\,dx &= \int \frac{x^9}{\left(b+ax^2\right)^2}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{(b+ax)^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{3b^2}{a^4}-\frac{2bx}{a^3}+\frac{x^2}{a^2}+\frac{b^4}{a^4(b+ax)^2}-\frac{4b^3}{a^4(b+ax)}\right)dx, x, x^2\right) \\
&= \frac{3b^2x^2}{2a^4}-\frac{bx^4}{2a^3}+\frac{x^6}{6a^2}-\frac{b^4}{2a^5\left(b+ax^2\right)}-\frac{2b^3\log\left(b+ax^2\right)}{a^5}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 60, normalized size = 0.86

$$\frac{a^3x^6 - 3a^2bx^4 - \frac{3b^4}{ax^2+b} - 12b^3\log\left(ax^2+b\right) + 9ab^2x^2}{6a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b/x^2)^2,x]
```

```
[Out] (9*a*b^2*x^2 - 3*a^2*b*x^4 + a^3*x^6 - (3*b^4)/(b + a*x^2) - 12*b^3*Log[b +
 a*x^2])/(6*a^5)
```

fricas [A] time = 0.94, size = 81, normalized size = 1.16

$$\frac{a^4x^8 - 2\,a^3bx^6 + 6\,a^2b^2x^4 + 9\,ab^3x^2 - 3\,b^4 - 12\left(ab^3x^2 + b^4\right)\log\left(ax^2+b\right)}{6\left(a^6x^2 + a^5b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(a^4*x^8 - 2*a^3*b*x^6 + 6*a^2*b^2*x^4 + 9*a*b^3*x^2 - 3*b^4 - 12*(a*b^
3*x^2 + b^4)*log(a*x^2 + b))/(a^6*x^2 + a^5*b)
```

giac [A] time = 0.16, size = 80, normalized size = 1.14

$$-\frac{2\,b^3\log\left(\left|ax^2+b\right|\right)}{a^5} + \frac{a^4x^6 - 3\,a^3bx^4 + 9\,a^2b^2x^2}{6\,a^6} + \frac{4\,ab^3x^2 + 3\,b^4}{2\left(ax^2+b\right)a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/x^2)^2,x, algorithm="giac")
```

```
[Out] -2*b^3*log(abs(a*x^2 + b))/a^5 + 1/6*(a^4*x^6 - 3*a^3*b*x^4 + 9*a^2*b^2*x^2
)/a^6 + 1/2*(4*a*b^3*x^2 + 3*b^4)/((a*x^2 + b)*a^5)
```

maple [A] time = 0.01, size = 63, normalized size = 0.90

$$\frac{x^6}{6a^2} - \frac{b\,x^4}{2a^3} + \frac{3b^2x^2}{2a^4} - \frac{b^4}{2\left(a\,x^2+b\right)a^5} - \frac{2b^3\ln\left(a\,x^2+b\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a+b/x^2)^2,x)
```

```
[Out] 3/2/a^4*b^2*x^2-1/2/a^3*b*x^4+1/6/a^2*x^6-1/2*b^4/a^5/(a*x^2+b)-2*b^3*ln(a*
x^2+b)/a^5
```

maxima [A] time = 0.88, size = 65, normalized size = 0.93

$$-\frac{b^4}{2\left(a^6x^2 + a^5b\right)} - \frac{2\,b^3\log\left(ax^2+b\right)}{a^5} + \frac{a^2x^6 - 3\,abx^4 + 9\,b^2x^2}{6\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*b^4/(a^6*x^2 + a^5*b) - 2*b^3*log(a*x^2 + b)/a^5 + 1/6*(a^2*x^6 - 3*a*
b*x^4 + 9*b^2*x^2)/a^4
```

mupad [B] time = 0.04, size = 68, normalized size = 0.97

$$\frac{x^6}{6\,a^2}-\frac{b^4}{2\,a\,\left(a^5\,x^2+b\,a^4\right)}-\frac{b\,x^4}{2\,a^3}-\frac{2\,b^3\,\ln\left(a\,x^2+b\right)}{a^5}+\frac{3\,b^2\,x^2}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b/x^2)^2,x)
```

```
[Out] x^6/(6*a^2) - b^4/(2*a*(a^4*b + a^5*x^2)) - (b*x^4)/(2*a^3) - (2*b^3*log(b
+ a*x^2))/a^5 + (3*b^2*x^2)/(2*a^4)
```

sympy [A] time = 0.29, size = 66, normalized size = 0.94

$$-\frac{b^4}{2a^6x^2+2a^5b}+\frac{x^6}{6a^2}-\frac{bx^4}{2a^3}+\frac{3b^2x^2}{2a^4}-\frac{2b^3\log\left(ax^2+b\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a+b/x**2)**2,x)
```

```
[Out] -b**4/(2*a**6*x**2 + 2*a**5*b) + x**6/(6*a**2) - b*x**4/(2*a**3) + 3*b**2*x
**2/(2*a**4) - 2*b**3*log(a*x**2 + b)/a**5
```

3.1860 $\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^2}\,dx$

Optimal. Leaf size=79

$$-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2a^{9/2}}+\frac{7b^2x}{2a^4}-\frac{7bx^3}{6a^3}+\frac{7x^5}{10a^2}-\frac{x^7}{2a\left(ax^2+b\right)}$$

```
[Out] 7/2*b^2*x/a^4-7/6*b*x^3/a^3+7/10*x^5/a^2-1/2*x^7/a/(a*x^2+b)-7/2*b^(5/2)*ar
ctan(x*a^(1/2)/b^(1/2))/a^(9/2)
```

Rubi [A] time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {263, 288, 302, 205}

$$\frac{7b^2x}{2a^4}-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2a^{9/2}}-\frac{7bx^3}{6a^3}+\frac{7x^5}{10a^2}-\frac{x^7}{2a\left(ax^2+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b/x^2)^2,x]
```

```
[Out] (7*b^2*x)/(2*a^4) - (7*b*x^3)/(6*a^3) + (7*x^5)/(10*a^2) - x^7/(2*a*(b + a*
x^2)) - (7*b^(5/2)*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(2*a^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^2}\,dx = \int \frac{x^8}{\left(b+ax^2\right)^2}\,dx$$

$$= -\frac{x^7}{2a\left(b+ax^2\right)} + \frac{7\int \frac{x^6}{b+ax^2}\,dx}{2a}$$

$$= -\frac{x^7}{2a\left(b+ax^2\right)} + \frac{7\int \left(\frac{b^2}{a^3} - \frac{bx^2}{a^2} + \frac{x^4}{a} - \frac{b^3}{a^3\left(b+ax^2\right)}\right)dx}{2a}$$

$$= \frac{7b^2x}{2a^4} - \frac{7bx^3}{6a^3} + \frac{7x^5}{10a^2} - \frac{x^7}{2a\left(b+ax^2\right)} - \frac{\left(7b^3\right)\int \frac{1}{b+ax^2}\,dx}{2a^4}$$

$$= \frac{7b^2x}{2a^4} - \frac{7bx^3}{6a^3} + \frac{7x^5}{10a^2} - \frac{x^7}{2a\left(b+ax^2\right)} - \frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2a^{9/2}}$$

Mathematica [A] time = 0.05, size = 71, normalized size = 0.90

$$\frac{x\left(6a^2x^4 + \frac{15b^3}{ax^2+b} - 20abx^2 + 90b^2\right)}{30a^4} - \frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b/x^2)^2,x]
```

```
[Out] (x*(90*b^2 - 20*a*b*x^2 + 6*a^2*x^4 + (15*b^3)/(b + a*x^2)))/(30*a^4) - (7*
b^(5/2)*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(2*a^(9/2))
```

fricas [A] time = 0.89, size = 190, normalized size = 2.41

$$\left[\frac{12\,a^3x^7 - 28\,a^2bx^5 + 140\,ab^2x^3 + 210\,b^3x + 105\left(ab^2x^2 + b^3\right)\sqrt{-\frac{b}{a}}\,\log\left(\frac{ax^2 - 2\,ax\sqrt{-\frac{b}{a}} - b}{ax^2+b}\right)}{60\left(a^5x^2 + a^4b\right)}, \frac{6\,a^3x^7 - 14\,a^2bx^5 + 70}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*a^3*x^7 - 28*a^2*b*x^5 + 140*a*b^2*x^3 + 210*b^3*x + 105*(a*b^2*x
^2 + b^3)*sqrt(-b/a)*log((a*x^2 - 2*a*x*sqrt(-b/a) - b)/(a*x^2 + b)))/(a^5*
x^2 + a^4*b), 1/30*(6*a^3*x^7 - 14*a^2*b*x^5 + 70*a*b^2*x^3 + 105*b^3*x - 1
05*(a*b^2*x^2 + b^3)*sqrt(b/a)*arctan(a*x*sqrt(b/a)/b))/(a^5*x^2 + a^4*b)]
```

giac [A] time = 0.16, size = 73, normalized size = 0.92

$$-\frac{7\,b^3\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a^4} + \frac{b^3x}{2\left(ax^2+b\right)a^4} + \frac{3\,a^8x^5 - 10\,a^7bx^3 + 45\,a^6b^2x}{15\,a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/x^2)^2,x, algorithm="giac")
```

```
[Out] -7/2*b^3*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^4) + 1/2*b^3*x/((a*x^2 + b)*a^4
) + 1/15*(3*a^8*x^5 - 10*a^7*b*x^3 + 45*a^6*b^2*x)/a^10
```

maple [A] time = 0.01, size = 68, normalized size = 0.86

$$\frac{x^5}{5a^2} - \frac{2b\,x^3}{3a^3} + \frac{b^3x}{2\left(a\,x^2+b\right)a^4} - \frac{7b^3\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^4} + \frac{3b^2x}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a+b/x^2)^2,x)
```

```
[Out] 1/5/a^2*x^5-2/3/a^3*b*x^3+3/a^4*b^2*x+1/2/a^4*b^3*x/(a*x^2+b)-7/2/a^4*b^3/(
a*b)^(1/2)*arctan(1/(a*b)^(1/2)*a*x)
```

maxima [A] time = 1.88, size = 71, normalized size = 0.90

$$\frac{b^3x}{2\left(a^5x^2+a^4b\right)} - \frac{7\,b^3\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^4} + \frac{3\,a^2x^5 - 10\,abx^3 + 45\,b^2x}{15\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*x/(a^5*x^2 + a^4*b) - 7/2*b^3*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^4)
 + 1/15*(3*a^2*x^5 - 10*a*b*x^3 + 45*b^2*x)/a^4
```

mupad [B] time = 0.05, size = 66, normalized size = 0.84

$$\frac{x^5}{5\,a^2} - \frac{2\,b\,x^3}{3\,a^3} + \frac{3\,b^2\,x}{a^4} - \frac{7\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2\,a^{9/2}} + \frac{b^3\,x}{2\,\left(a^5\,x^2+b\,a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b/x^2)^2,x)
```

```
[Out] x^5/(5*a^2) - (2*b*x^3)/(3*a^3) + (3*b^2*x)/a^4 - (7*b^(5/2)*atan((a^(1/2)*
x)/b^(1/2)))/(2*a^(9/2)) + (b^3*x)/(2*(a^4*b + a^5*x^2))
```

sympy [A] time = 0.32, size = 124, normalized size = 1.57

$$\frac{b^3x}{2a^5x^2+2a^4b} + \frac{7\sqrt{-\frac{b^5}{a^9}}\,\log\left(-\frac{a^4\sqrt{-\frac{b^5}{a^9}}}{b^2}+x\right)}{4} - \frac{7\sqrt{-\frac{b^5}{a^9}}\,\log\left(\frac{a^4\sqrt{-\frac{b^5}{a^9}}}{b^2}+x\right)}{4} + \frac{x^5}{5a^2} - \frac{2bx^3}{3a^3} + \frac{3b^2x}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a+b/x**2)**2,x)
```

```
[Out] b**3*x/(2*a**5*x**2 + 2*a**4*b) + 7*sqrt(-b**5/a**9)*log(-a**4*sqrt(-b**5/a
**9)/b**2 + x)/4 - 7*sqrt(-b**5/a**9)*log(a**4*sqrt(-b**5/a**9)/b**2 + x)/4
 + x**5/(5*a**2) - 2*b*x**3/(3*a**3) + 3*b**2*x/a**4
```

3.1861 $\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^2}\,dx$

Optimal. Leaf size=57

$$\frac{b^3}{2a^4\left(ax^2+b\right)}+\frac{3b^2\log\left(ax^2+b\right)}{2a^4}-\frac{bx^2}{a^3}+\frac{x^4}{4a^2}$$

```
[Out] -b*x^2/a^3+1/4*x^4/a^2+1/2*b^3/a^4/(a*x^2+b)+3/2*b^2*ln(a*x^2+b)/a^4
```

Rubi [A] time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 266, 43}

$$\frac{b^3}{2a^4\left(ax^2+b\right)}+\frac{3b^2\log\left(ax^2+b\right)}{2a^4}-\frac{bx^2}{a^3}+\frac{x^4}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b/x^2)^2,x]
```

```
[Out] -((b*x^2)/a^3) + x^4/(4*a^2) + b^3/(2*a^4*(b + a*x^2)) + (3*b^2*Log[b + a*x
^2])/(2*a^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^2}\,dx &= \int \frac{x^7}{\left(b+ax^2\right)^2}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{(b+ax)^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{2b}{a^3}+\frac{x}{a^2}-\frac{b^3}{a^3(b+ax)^2}+\frac{3b^2}{a^3(b+ax)}\right)dx, x, x^2\right) \\
&= -\frac{bx^2}{a^3}+\frac{x^4}{4a^2}+\frac{b^3}{2a^4\left(b+ax^2\right)}+\frac{3b^2\log\left(b+ax^2\right)}{2a^4}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.86

$$\frac{a^2x^4 + \frac{2b^3}{ax^2+b} + 6b^2\log\left(ax^2+b\right) - 4abx^2}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b/x^2)^2,x]
```

```
[Out] (-4*a*b*x^2 + a^2*x^4 + (2*b^3)/(b + a*x^2) + 6*b^2*Log[b + a*x^2])/(4*a^4)
```

fricas [A] time = 0.96, size = 70, normalized size = 1.23

$$\frac{a^3x^6 - 3\,a^2bx^4 - 4\,ab^2x^2 + 2\,b^3 + 6\left(ab^2x^2 + b^3\right)\log\left(ax^2+b\right)}{4\left(a^5x^2 + a^4b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(a^3*x^6 - 3*a^2*b*x^4 - 4*a*b^2*x^2 + 2*b^3 + 6*(a*b^2*x^2 + b^3)*log(
a*x^2 + b))/(a^5*x^2 + a^4*b)
```

giac [A] time = 0.17, size = 55, normalized size = 0.96

$$\frac{3\,b^2\log\left(\left|ax^2+b\right|\right)}{2\,a^4} + \frac{b^3}{2\left(ax^2+b\right)a^4} + \frac{a^2x^4 - 4\,abx^2}{4\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x^2)^2,x, algorithm="giac")
```

```
[Out] 3/2*b^2*log(abs(a*x^2 + b))/a^4 + 1/2*b^3/((a*x^2 + b)*a^4) + 1/4*(a^2*x^4
- 4*a*b*x^2)/a^4
```

maple [A] time = 0.01, size = 52, normalized size = 0.91

$$\frac{x^4}{4a^2} - \frac{b\,x^2}{a^3} + \frac{b^3}{2\left(a\,x^2+b\right)a^4} + \frac{3b^2\ln\left(a\,x^2+b\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b/x^2)^2,x)
```

```
[Out] -1/a^3*b*x^2+1/4/a^2*x^4+1/2*b^3/a^4/(a*x^2+b)+3/2*b^2*ln(a*x^2+b)/a^4
```

maxima [A] time = 0.78, size = 54, normalized size = 0.95

$$\frac{b^3}{2\left(a^5x^2 + a^4b\right)} + \frac{3\,b^2\log\left(ax^2+b\right)}{2\,a^4} + \frac{ax^4 - 4\,bx^2}{4\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^3/(a^5*x^2 + a^4*b) + 3/2*b^2*log(a*x^2 + b)/a^4 + 1/4*(a*x^4 - 4*b*x
^2)/a^3
```

mupad [B] time = 0.05, size = 57, normalized size = 1.00

$$\frac{x^4}{4\,a^2} + \frac{b^3}{2\,a\,\left(a^4\,x^2 + b\,a^3\right)} - \frac{b\,x^2}{a^3} + \frac{3\,b^2\,\ln\left(a\,x^2+b\right)}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b/x^2)^2,x)
```

```
[Out] x^4/(4*a^2) + b^3/(2*a*(a^3*b + a^4*x^2)) - (b*x^2)/a^3 + (3*b^2*log(b + a*
x^2))/(2*a^4)
```

sympy [**A**] time = 0.27, size = 53, normalized size = 0.93

$$\frac{b^3}{2a^5x^2 + 2a^4b} + \frac{x^4}{4a^2} - \frac{bx^2}{a^3} + \frac{3b^2 \log\left(ax^2 + b\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b/x**2)**2,x)
```

```
[Out] b**3/(2*a**5*x**2 + 2*a**4*b) + x**4/(4*a**2) - b*x**2/a**3 + 3*b**2*log(a*
x**2 + b)/(2*a**4)
```

3.1862 $\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^2}\,dx$

Optimal. Leaf size=66

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2a^{7/2}}-\frac{5bx}{2a^3}+\frac{5x^3}{6a^2}-\frac{x^5}{2a\left(ax^2+b\right)}$$

```
[Out] -5/2*b*x/a^3+5/6*x^3/a^2-1/2*x^5/a/(a*x^2+b)+5/2*b^(3/2)*arctan(x*a^(1/2)/b
^(1/2))/a^(7/2)
```

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {263, 288, 302, 205}

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2a^{7/2}}-\frac{5bx}{2a^3}+\frac{5x^3}{6a^2}-\frac{x^5}{2a\left(ax^2+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b/x^2)^2,x]
```

```
[Out] (-5*b*x)/(2*a^3) + (5*x^3)/(6*a^2) - x^5/(2*a*(b + a*x^2)) + (5*b^(3/2)*Arc
Tan[(Sqrt[a]*x)/Sqrt[b]])/(2*a^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^2}\,dx = \int \frac{x^6}{\left(b+ax^2\right)^2}\,dx$$

$$= -\frac{x^5}{2a\left(b+ax^2\right)} + \frac{5\int \frac{x^4}{b+ax^2}\,dx}{2a}$$

$$= -\frac{x^5}{2a\left(b+ax^2\right)} + \frac{5\int \left(-\frac{b}{a^2}+\frac{x^2}{a}+\frac{b^2}{a^2\left(b+ax^2\right)}\right)dx}{2a}$$

$$= -\frac{5bx}{2a^3} + \frac{5x^3}{6a^2} - \frac{x^5}{2a\left(b+ax^2\right)} + \frac{\left(5b^2\right)\int \frac{1}{b+ax^2}\,dx}{2a^3}$$

$$= -\frac{5bx}{2a^3} + \frac{5x^3}{6a^2} - \frac{x^5}{2a\left(b+ax^2\right)} + \frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2a^{7/2}}$$

Mathematica [A] time = 0.05, size = 60, normalized size = 0.91

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2a^{7/2}} + \frac{x\left(-\frac{3b^2}{ax^2+b}+2ax^2-12b\right)}{6a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b/x^2)^2,x]
```

```
[Out] (x*(-12*b + 2*a*x^2 - (3*b^2)/(b + a*x^2)))/(6*a^3) + (5*b^(3/2)*ArcTan[(Sq
rt[a]*x)/Sqrt[b]])/(2*a^(7/2))
```

fricas [A] time = 0.79, size = 164, normalized size = 2.48

$$\left[\frac{4\,a^2x^5 - 20\,abx^3 - 30\,b^2x + 15\left(abx^2+b^2\right)\sqrt{-\frac{b}{a}}\,\log\left(\frac{ax^2+2\,ax\sqrt{-\frac{b}{a}}-b}{ax^2+b}\right)}{12\left(a^4x^2+a^3b\right)}, \frac{2\,a^2x^5 - 10\,abx^3 - 15\,b^2x + 15\left(abx^2+b\right.}{6\left(a^4x^2+a^3b\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*a^2*x^5 - 20*a*b*x^3 - 30*b^2*x + 15*(a*b*x^2 + b^2)*sqrt(-b/a)*lo
g((a*x^2 + 2*a*x*sqrt(-b/a) - b)/(a*x^2 + b)))/(a^4*x^2 + a^3*b), 1/6*(2*a^
2*x^5 - 10*a*b*x^3 - 15*b^2*x + 15*(a*b*x^2 + b^2)*sqrt(b/a)*arctan(a*x*sqr
t(b/a)/b))/(a^4*x^2 + a^3*b)]
```

giac [A] time = 0.15, size = 61, normalized size = 0.92

$$\frac{5\,b^2\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a^3} - \frac{b^2x}{2\left(ax^2+b\right)a^3} + \frac{a^4x^3 - 6\,a^3bx}{3\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^2)^2,x, algorithm="giac")
```

```
[Out] 5/2*b^2*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^3) - 1/2*b^2*x/((a*x^2 + b)*a^3)
 + 1/3*(a^4*x^3 - 6*a^3*b*x)/a^6
```

maple [A] time = 0.01, size = 57, normalized size = 0.86

$$\frac{x^3}{3a^2} - \frac{b^2 x}{2\left(a\,x^2+b\right)a^3} + \frac{5b^2 \arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^3} - \frac{2bx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b/x^2)^2,x)
```

```
[Out] 1/3/a^2*x^3-2/a^3*b*x-1/2/a^3*b^2*x/(a*x^2+b)+5/2/a^3*b^2/(a*b)^(1/2)*arcta
n(1/(a*b)^(1/2)*a*x)
```

maxima [A] time = 1.93, size = 59, normalized size = 0.89

$$-\frac{b^2 x}{2\left(a^4x^2+a^3b\right)} + \frac{5\,b^2 \arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^3} + \frac{ax^3-6\,bx}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*b^2*x/(a^4*x^2 + a^3*b) + 5/2*b^2*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^3
) + 1/3*(a*x^3 - 6*b*x)/a^3
```

mupad [B] time = 0.07, size = 56, normalized size = 0.85

$$\frac{x^3}{3\,a^2} + \frac{5\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2\,a^{7/2}} - \frac{b^2\,x}{2\,\left(a^4\,x^2+b\,a^3\right)} - \frac{2\,b\,x}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b/x^2)^2,x)
```

```
[Out] x^3/(3*a^2) + (5*b^(3/2)*atan((a^(1/2)*x)/b^(1/2)))/(2*a^(7/2)) - (b^2*x)/(
2*(a^3*b + a^4*x^2)) - (2*b*x)/a^3
```

sympy [A] time = 0.30, size = 107, normalized size = 1.62

$$-\frac{b^2x}{2a^4x^2+2a^3b} - \frac{5\sqrt{-\frac{b^3}{a^7}}\,\log\left(-\frac{a^3\sqrt{-\frac{b^3}{a^7}}}{b}+x\right)}{4} + \frac{5\sqrt{-\frac{b^3}{a^7}}\,\log\left(\frac{a^3\sqrt{-\frac{b^3}{a^7}}}{b}+x\right)}{4} + \frac{x^3}{3a^2} - \frac{2bx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b/x**2)**2,x)
```

```
[Out] -b**2*x/(2*a**4*x**2 + 2*a**3*b) - 5*sqrt(-b**3/a**7)*log(-a**3*sqrt(-b**3/
a**7)/b + x)/4 + 5*sqrt(-b**3/a**7)*log(a**3*sqrt(-b**3/a**7)/b + x)/4 + x*
*3/(3*a**2) - 2*b*x/a**3
```

3.1863 $\int \frac{x}{\left(a+\frac{b}{x^2}\right)^2}\,dx$

Optimal. Leaf size=44

$$-\frac{b^2}{2a^3\left(ax^2+b\right)}-\frac{b\log\left(ax^2+b\right)}{a^3}+\frac{x^2}{2a^2}$$

```
[Out] 1/2*x^2/a^2-1/2*b^2/a^3/(a*x^2+b)-b*ln(a*x^2+b)/a^3
```

Rubi [A] time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {263, 266, 43}

$$-\frac{b^2}{2a^3\left(ax^2+b\right)}-\frac{b\log\left(ax^2+b\right)}{a^3}+\frac{x^2}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b/x^2)^2,x]
```

```
[Out] x^2/(2*a^2) - b^2/(2*a^3*(b + a*x^2)) - (b*Log[b + a*x^2])/a^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(a+\frac{b}{x^2}\right)^2}\,dx &= \int \frac{x^5}{\left(b+ax^2\right)^2}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{(b+ax)^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{a^2}+\frac{b^2}{a^2(b+ax)^2}-\frac{2b}{a^2(b+ax)}\right)dx, x, x^2\right) \\
&= \frac{x^2}{2a^2}-\frac{b^2}{2a^3\left(b+ax^2\right)}-\frac{b\log\left(b+ax^2\right)}{a^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 0.86

$$\frac{-\frac{b^2}{ax^2+b} - 2b\log\left(ax^2+b\right) + ax^2}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b/x^2)^2,x]
```

```
[Out] (a*x^2 - b^2/(b + a*x^2) - 2*b*Log[b + a*x^2])/(2*a^3)
```

fricas [A] time = 0.68, size = 56, normalized size = 1.27

$$\frac{a^2x^4 + abx^2 - b^2 - 2\left(abx^2+b^2\right)\log\left(ax^2+b\right)}{2\left(a^4x^2+a^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*x^4 + a*b*x^2 - b^2 - 2*(a*b*x^2 + b^2)*log(a*x^2 + b))/(a^4*x^2 +
 a^3*b)
```

giac [A] time = 0.15, size = 41, normalized size = 0.93

$$\frac{x^2}{2\,a^2} - \frac{b\log\left(\left|ax^2+b\right|\right)}{a^3} - \frac{b^2}{2\left(ax^2+b\right)a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2/a^2 - b*log(abs(a*x^2 + b))/a^3 - 1/2*b^2/((a*x^2 + b)*a^3)
```

maple [A] time = 0.01, size = 41, normalized size = 0.93

$$\frac{x^2}{2a^2} - \frac{b^2}{2\left(a\,x^2+b\right)a^3} - \frac{b\ln\left(a\,x^2+b\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b/x^2)^2,x)
```

```
[Out] 1/2/a^2*x^2-1/2*b^2/a^3/(a*x^2+b)-b*ln(a*x^2+b)/a^3
```

maxima [A] time = 0.90, size = 43, normalized size = 0.98

$$-\frac{b^2}{2\left(a^4x^2+a^3b\right)} + \frac{x^2}{2\,a^2} - \frac{b\log\left(ax^2+b\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*b^2/(a^4*x^2 + a^3*b) + 1/2*x^2/a^2 - b*log(a*x^2 + b)/a^3
```

mupad [B] time = 1.08, size = 45, normalized size = 1.02

$$\frac{x^2}{2\,a^2} - \frac{b^2}{2\,\left(a^4\,x^2+b\,a^3\right)} - \frac{b\,\ln\left(a\,x^2+b\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b/x^2)^2,x)
```

```
[Out] x^2/(2*a^2) - b^2/(2*(a^3*b + a^4*x^2)) - (b*log(b + a*x^2))/a^3
```

sympy [**A**] time = 0.25, size = 39, normalized size = 0.89

$$-\frac{b^2}{2a^4x^2+2a^3b}+\frac{x^2}{2a^2}-\frac{b\log\left(ax^2+b\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x**2)**2,x)
```

```
[Out] -b**2/(2*a**4*x**2 + 2*a**3*b) + x**2/(2*a**2) - b*log(a*x**2 + b)/a**3
```

3.1864 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2}\,dx$

Optimal. Leaf size=55

$$-\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2a^{5/2}}+\frac{3x}{2a^2}-\frac{x^3}{2a\left(ax^2+b\right)}$$

```
[Out] 3/2*x/a^2-1/2*x^3/a/(a*x^2+b)-3/2*arctan(x*a^(1/2)/b^(1/2))*b^(1/2)/a^(5/2)
```

Rubi [A] time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {193, 288, 321, 205}

$$-\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2a^{5/2}}+\frac{3x}{2a^2}-\frac{x^3}{2a\left(ax^2+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^(-2),x]
```

```
[Out] (3*x)/(2*a^2) - x^3/(2*a*(b + a*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[a]*x)/Sqrt[
b]])/(2*a^(5/2))
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2}\,dx = \int \frac{x^4}{\left(b+ax^2\right)^2}\,dx$$

$$= -\frac{x^3}{2a\left(b+ax^2\right)} + \frac{3\int \frac{x^2}{b+ax^2}\,dx}{2a}$$

$$= \frac{3x}{2a^2} - \frac{x^3}{2a\left(b+ax^2\right)} - \frac{(3b)\int \frac{1}{b+ax^2}\,dx}{2a^2}$$

$$= \frac{3x}{2a^2} - \frac{x^3}{2a\left(b+ax^2\right)} - \frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2a^{5/2}}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.93

$$-\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2a^{5/2}} + \frac{bx}{2a^2\left(ax^2+b\right)} + \frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^(-2),x]
```

```
[Out] x/a^2 + (b*x)/(2*a^2*(b + a*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[a]*x)/Sqrt[b]])
/(2*a^(5/2))
```

fricas [A] time = 0.90, size = 136, normalized size = 2.47

$$\left[\frac{4\,ax^3+3\left(ax^2+b\right)\sqrt{-\frac{b}{a}}\,\log\left(\frac{ax^2-2\,ax\sqrt{-\frac{b}{a}}-b}{ax^2+b}\right)+6\,bx}{4\left(a^3x^2+a^2b\right)},\ \frac{2\,ax^3-3\left(ax^2+b\right)\sqrt{\frac{b}{a}}\,\arctan\left(\frac{ax\sqrt{\frac{b}{a}}}{b}\right)+3\,bx}{2\left(a^3x^2+a^2b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a*x^3 + 3*(a*x^2 + b)*sqrt(-b/a)*log((a*x^2 - 2*a*x*sqrt(-b/a) - b)
/(a*x^2 + b)) + 6*b*x)/(a^3*x^2 + a^2*b), 1/2*(2*a*x^3 - 3*(a*x^2 + b)*sqrt
(b/a)*arctan(a*x*sqrt(b/a)/b) + 3*b*x)/(a^3*x^2 + a^2*b)]
```

giac [A] time = 0.18, size = 42, normalized size = 0.76

$$-\frac{3\,b\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a^2} + \frac{x}{a^2} + \frac{bx}{2\left(ax^2+b\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^2,x, algorithm="giac")
```

```
[Out] -3/2*b*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^2) + x/a^2 + 1/2*b*x/((a*x^2 + b)
*a^2)
```

maple [A] time = 0.01, size = 43, normalized size = 0.78

$$\frac{bx}{2\left(a\,x^2+b\right)a^2} - \frac{3b\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^2} + \frac{x}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^2,x)
```

```
[Out] 1/a^2*x+1/2/a^2*b*x/(a*x^2+b)-3/2/a^2*b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*a*
x)
```

maxima [A] time = 1.98, size = 45, normalized size = 0.82

$$\frac{bx}{2\left(a^3x^2+a^2b\right)}-\frac{3\,b\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a^2}+\frac{x}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*x/(a^3*x^2 + a^2*b) - 3/2*b*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^2) + x
/a^2
```

mupad [B] time = 1.10, size = 43, normalized size = 0.78

$$\frac{x}{a^2}+\frac{b\,x}{2\left(a^3\,x^2+b\,a^2\right)}-\frac{3\sqrt{b}\,\operatorname{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x^2)^2,x)
```

```
[Out] x/a^2 + (b*x)/(2*(a^2*b + a^3*x^2)) - (3*b^(1/2)*atan((a^(1/2)*x)/b^(1/2)))
/(2*a^(5/2))
```

sympy [A] time = 0.28, size = 83, normalized size = 1.51

$$\frac{bx}{2a^3x^2+2a^2b}+\frac{3\sqrt{-\frac{b}{a^5}}\,\log\left(-a^2\sqrt{-\frac{b}{a^5}}+x\right)}{4}-\frac{3\sqrt{-\frac{b}{a^5}}\,\log\left(a^2\sqrt{-\frac{b}{a^5}}+x\right)}{4}+\frac{x}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**2,x)
```

```
[Out] b*x/(2*a**3*x**2 + 2*a**2*b) + 3*sqrt(-b/a**5)*log(-a**2*sqrt(-b/a**5) + x)
/4 - 3*sqrt(-b/a**5)*log(a**2*sqrt(-b/a**5) + x)/4 + x/a**2
```

3.1865 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x}\,dx$

Optimal. Leaf size=33

$$\frac{b}{2a^2\left(ax^2+b\right)}+\frac{\log\left(ax^2+b\right)}{2a^2}$$

```
[Out] 1/2*b/a^2/(a*x^2+b)+1/2*ln(a*x^2+b)/a^2
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 266, 43}

$$\frac{b}{2a^2\left(ax^2+b\right)}+\frac{\log\left(ax^2+b\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)^2*x),x]
```

```
[Out] b/(2*a^2*(b + a*x^2)) + Log[b + a*x^2]/(2*a^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x}\,dx &= \int \frac{x^3}{\left(b+ax^2\right)^2}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{(b+ax)^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{b}{a(b+ax)^2}+\frac{1}{a(b+ax)}\right)dx, x, x^2\right) \\
&= \frac{b}{2a^2\left(b+ax^2\right)}+\frac{\log\left(b+ax^2\right)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.82

$$\frac{\frac{b}{ax^2+b}+\log\left(ax^2+b\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)^2*x),x]
[Out] (b/(b + a*x^2) + Log[b + a*x^2])/(2*a^2)
```

fricas [A] time = 0.80, size = 35, normalized size = 1.06

$$\frac{\left(ax^2+b\right)\log\left(ax^2+b\right)+b}{2\left(a^3x^2+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^2/x,x, algorithm="fricas")
[Out] 1/2*((a*x^2 + b)*log(a*x^2 + b) + b)/(a^3*x^2 + a^2*b)
```

giac [A] time = 0.15, size = 32, normalized size = 0.97

$$-\frac{x^2}{2\left(ax^2+b\right)a}+\frac{\log\left(\left|ax^2+b\right|\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^2/x,x, algorithm="giac")
[Out] -1/2*x^2/((a*x^2 + b)*a) + 1/2*log(abs(a*x^2 + b))/a^2
```

maple [A] time = 0.01, size = 30, normalized size = 0.91

$$\frac{b}{2\left(a\,x^2+b\right)a^2}+\frac{\ln\left(a\,x^2+b\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^2/x,x)
[Out] 1/2*b/a^2/(a*x^2+b)+1/2*ln(a*x^2+b)/a^2
```

maxima [A] time = 0.88, size = 32, normalized size = 0.97

$$\frac{b}{2\left(a^3x^2+a^2b\right)}+\frac{\log\left(ax^2+b\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^2/x,x, algorithm="maxima")
[Out] 1/2*b/(a^3*x^2 + a^2*b) + 1/2*log(a*x^2 + b)/a^2
```

mupad [B] time = 0.04, size = 29, normalized size = 0.88

$$\frac{\ln\left(a\,x^2+b\right)}{2\,a^2}+\frac{b}{2\,a^2\,\left(a\,x^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b/x^2)^2),x)
[Out] log(b + a*x^2)/(2*a^2) + b/(2*a^2*(b + a*x^2))
```

sympy [**A**] time = 0.21, size = 29, normalized size = 0.88

$$\frac{b}{2a^3x^2 + 2a^2b} + \frac{\log\left(ax^2 + b\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**2/x,x)
```

```
[Out] b/(2*a**3*x**2 + 2*a**2*b) + log(a*x**2 + b)/(2*a**2)
```

3.1866 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^2}\,dx$

Optimal. Leaf size=45

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}} - \frac{x}{2a\left(ax^2+b\right)}$$

```
[Out] -1/2*x/a/(a*x^2+b)+1/2*arctan(x*a^(1/2)/b^(1/2))/a^(3/2)/b^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 288, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}} - \frac{x}{2a\left(ax^2+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)^2*x^2),x]
```

```
[Out] -x/(2*a*(b + a*x^2)) + ArcTan[(Sqrt[a]*x)/Sqrt[b]]/(2*a^(3/2)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^2}\,dx &= \int \frac{x^2}{\left(b+ax^2\right)^2}\,dx \\
&= -\frac{x}{2a\left(b+ax^2\right)} + \frac{\int \frac{1}{b+ax^2}\,dx}{2a} \\
&= -\frac{x}{2a\left(b+ax^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}} - \frac{x}{2a\left(ax^2+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)^2*x^2),x]
[Out] -1/2*x/(a*(b + a*x^2)) + ArcTan[(Sqrt[a]*x)/Sqrt[b]]/(2*a^(3/2)*Sqrt[b])
```

fricas [A] time = 0.91, size = 120, normalized size = 2.67

$$\left[-\frac{2\,abx + \left(ax^2+b\right)\sqrt{-ab}\,\log\left(\frac{ax^2-2\sqrt{-ab}\,x-b}{ax^2+b}\right)}{4\left(a^3bx^2+a^2b^2\right)}, -\frac{abx - \left(ax^2+b\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{b}\right)}{2\left(a^3bx^2+a^2b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^2/x^2,x, algorithm="fricas")
[Out] [-1/4*(2*a*b*x + (a*x^2 + b)*sqrt(-a*b)*log((a*x^2 - 2*sqrt(-a*b)*x - b)/(a
*x^2 + b)))/(a^3*b*x^2 + a^2*b^2), -1/2*(a*b*x - (a*x^2 + b)*sqrt(a*b)*arct
an(sqrt(a*b)*x/b))/(a^3*b*x^2 + a^2*b^2)]
```

giac [A] time = 0.15, size = 35, normalized size = 0.78

$$\frac{\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a} - \frac{x}{2\left(ax^2+b\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^2/x^2,x, algorithm="giac")
[Out] 1/2*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/2*x/((a*x^2 + b)*a)
```

maple [A] time = 0.01, size = 36, normalized size = 0.80

$$-\frac{x}{2\left(a\,x^2+b\right)a} + \frac{\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^2/x^2,x)
[Out] -1/2*x/a/(a*x^2+b)+1/2/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*a*x)
```

maxima [A] time = 1.99, size = 36, normalized size = 0.80

$$-\frac{x}{2\left(a^2x^2+ab\right)} + \frac{\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^2/x^2,x, algorithm="maxima")
[Out] -1/2*x/(a^2*x^2 + a*b) + 1/2*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a)
```

mupad [B] time = 0.04, size = 33, normalized size = 0.73

$$\frac{\operatorname{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2\,a^{3/2}\,\sqrt{b}}-\frac{x}{2\,a\,\left(a\,x^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b/x^2)^2),x)
```

```
[Out] atan((a^(1/2)*x)/b^(1/2))/(2*a^(3/2)*b^(1/2)) - x/(2*a*(b + a*x^2))
```

sympy [B] time = 0.22, size = 78, normalized size = 1.73

$$-\frac{x}{2a^2x^2+2ab}-\frac{\sqrt{-\frac{1}{a^3b}}\,\log\left(-ab\sqrt{-\frac{1}{a^3b}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^3b}}\,\log\left(ab\sqrt{-\frac{1}{a^3b}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**2/x**2,x)
```

```
[Out] -x/(2*a**2*x**2 + 2*a*b) - sqrt(-1/(a**3*b))*log(-a*b*sqrt(-1/(a**3*b)) + x
)/4 + sqrt(-1/(a**3*b))*log(a*b*sqrt(-1/(a**3*b)) + x)/4
```

3.1867 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^3}\,dx$

Optimal. Leaf size=16

$$\frac{1}{2b\left(a+\frac{b}{x^2}\right)}$$

```
[Out] 1/2/b/(a+b/x^2)
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$\frac{1}{2b\left(a+\frac{b}{x^2}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)^2*x^3),x]
```

```
[Out] 1/(2*b*(a + b/x^2))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^3}\,dx = \frac{1}{2b\left(a+\frac{b}{x^2}\right)}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{2a\left(ax^2+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)^2*x^3),x]
```

```
[Out] -1/2*1/(a*(b + a*x^2))
```

fricas [A] time = 0.58, size = 15, normalized size = 0.94

$$-\frac{1}{2\left(a^2x^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/2/(a^2*x^2 + a*b)
```

giac [A] time = 0.15, size = 14, normalized size = 0.88

$$-\frac{1}{2\left(ax^2+b\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^2/x^3,x, algorithm="giac")
[Out] -1/2/((a*x^2 + b)*a)
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$-\frac{1}{2\left(a\,x^2+b\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^2/x^3,x)
[Out] -1/2/a/(a*x^2+b)
```

maxima [A] time = 0.84, size = 14, normalized size = 0.88

$$\frac{1}{2\left(a+\frac{b}{x^2}\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^2/x^3,x, algorithm="maxima")
[Out] 1/2/((a + b/x^2)*b)
```

mupad [B] time = 0.02, size = 14, normalized size = 0.88

$$-\frac{1}{2\,a\,\left(a\,x^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b/x^2)^2),x)
[Out] -1/(2*a*(b + a*x^2))
```

sympy [A] time = 0.17, size = 15, normalized size = 0.94

$$-\frac{1}{2a^2x^2+2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**2/x**3,x)
[Out] -1/(2*a**2*x**2 + 2*a*b)
```

3.1868 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^4}\,dx$

Optimal. Leaf size=45

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2\sqrt{a}\,b^{3/2}}+\frac{x}{2b\left(ax^2+b\right)}$$

```
[Out] 1/2*x/b/(a*x^2+b)+1/2*arctan(x*a^(1/2)/b^(1/2))/b^(3/2)/a^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2\sqrt{a}\,b^{3/2}}+\frac{x}{2b\left(ax^2+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)^2*x^4),x]
```

```
[Out] x/(2*b*(b + a*x^2)) + ArcTan[(Sqrt[a]*x)/Sqrt[b]]/(2*Sqrt[a]*b^(3/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^4}\,dx &= \int \frac{1}{\left(b+ax^2\right)^2}\,dx\\ &= \frac{x}{2b\left(b+ax^2\right)}+\frac{\int \frac{1}{b+ax^2}\,dx}{2b}\\ &= \frac{x}{2b\left(b+ax^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2\sqrt{a}\,b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 45, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2\sqrt{a}\,b^{3/2}}+\frac{x}{2b\left(ax^2+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)^2*x^4),x]
```

```
[Out] x/(2*b*(b + a*x^2)) + ArcTan[(Sqrt[a]*x)/Sqrt[b]]/(2*Sqrt[a]*b^(3/2))
```

fricas [A] time = 1.51, size = 120, normalized size = 2.67

$$\left[\frac{2\,abx-\left(ax^2+b\right)\sqrt{-ab}\,\log\left(\frac{ax^2-2\sqrt{-ab}\,x-b}{ax^2+b}\right)}{4\left(a^2b^2x^2+ab^3\right)},\frac{abx+\left(ax^2+b\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{b}\right)}{2\left(a^2b^2x^2+ab^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^2/x^4,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b*x - (a*x^2 + b)*sqrt(-a*b)*log((a*x^2 - 2*sqrt(-a*b)*x - b)/(a*
x^2 + b)))/(a^2*b^2*x^2 + a*b^3), 1/2*(a*b*x + (a*x^2 + b)*sqrt(a*b)*arctan
(sqrt(a*b)*x/b))/(a^2*b^2*x^2 + a*b^3)]
```

giac [A] time = 0.16, size = 35, normalized size = 0.78

$$\frac{\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b}+\frac{x}{2\left(ax^2+b\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/2*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b) + 1/2*x/((a*x^2 + b)*b)
```

maple [A] time = 0.00, size = 36, normalized size = 0.80

$$\frac{x}{2\left(a\,x^2+b\right)b}+\frac{\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^2/x^4,x)
```

```
[Out] 1/2*x/b/(a*x^2+b)+1/2/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*a*x)
```

maxima [A] time = 1.99, size = 35, normalized size = 0.78

$$\frac{x}{2\left(abx^2+b^2\right)}+\frac{\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/2*x/(a*b*x^2 + b^2) + 1/2*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b)
```

mupad [B] time = 0.04, size = 33, normalized size = 0.73

$$\frac{x}{2\,b\,\left(a\,x^2+b\right)}+\frac{\operatorname{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2\,\sqrt{a}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b/x^2)^2),x)
```

```
[Out] x/(2*b*(b + a*x^2)) + atan((a^(1/2)*x)/b^(1/2))/(2*a^(1/2)*b^(3/2))
```

sympy [B] time = 0.23, size = 78, normalized size = 1.73

$$\frac{x}{2abx^2+2b^2}-\frac{\sqrt{-\frac{1}{ab^3}}\,\log\left(-b^2\sqrt{-\frac{1}{ab^3}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{ab^3}}\,\log\left(b^2\sqrt{-\frac{1}{ab^3}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**2/x**4,x)
```

```
[Out] x/(2*a*b*x**2 + 2*b**2) - sqrt(-1/(a*b**3))*log(-b**2*sqrt(-1/(a*b**3)) + x
)/4 + sqrt(-1/(a*b**3))*log(b**2*sqrt(-1/(a*b**3)) + x)/4
```

3.1869 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^5}\,dx$

Optimal. Leaf size=38

$$-\frac{\log\left(ax^2+b\right)}{2b^2}+\frac{1}{2b\left(ax^2+b\right)}+\frac{\log(x)}{b^2}$$

```
[Out] 1/2/b/(a*x^2+b)+ln(x)/b^2-1/2*ln(a*x^2+b)/b^2
```

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 266, 44}

$$-\frac{\log\left(ax^2+b\right)}{2b^2}+\frac{1}{2b\left(ax^2+b\right)}+\frac{\log(x)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)^2*x^5),x]
```

```
[Out] 1/(2*b*(b + a*x^2)) + Log[x]/b^2 - Log[b + a*x^2]/(2*b^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^5}\,dx &= \int \frac{1}{x\left(b+ax^2\right)^2}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(b+ax)^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{b^2 x}-\frac{a}{b(b+ax)^2}-\frac{a}{b^2(b+ax)}\right)dx, x, x^2\right) \\
&= \frac{1}{2b\left(b+ax^2\right)}+\frac{\log(x)}{b^2}-\frac{\log\left(b+ax^2\right)}{2b^2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.87

$$\frac{\frac{b}{ax^2+b} - \log\left(ax^2+b\right) + 2\log(x)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)^2*x^5),x]
```

```
[Out] (b/(b + a*x^2) + 2*Log[x] - Log[b + a*x^2])/(2*b^2)
```

fricas [A] time = 0.82, size = 47, normalized size = 1.24

$$-\frac{\left(ax^2+b\right)\log\left(ax^2+b\right) - 2\left(ax^2+b\right)\log(x) - b}{2\left(ab^2x^2+b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^2/x^5,x, algorithm="fricas")
```

```
[Out] -1/2*((a*x^2 + b)*log(a*x^2 + b) - 2*(a*x^2 + b)*log(x) - b)/(a*b^2*x^2 + b
^3)
```

giac [A] time = 0.15, size = 47, normalized size = 1.24

$$\frac{\log\left(x^2\right)}{2\,b^2} - \frac{\log\left(\left|ax^2+b\right|\right)}{2\,b^2} + \frac{ax^2+2\,b}{2\left(ax^2+b\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^2/x^5,x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/b^2 - 1/2*log(abs(a*x^2 + b))/b^2 + 1/2*(a*x^2 + 2*b)/((a*x^2
+ b)*b^2)
```

maple [A] time = 0.01, size = 35, normalized size = 0.92

$$\frac{1}{2\left(a\,x^2+b\right)b} + \frac{\ln(x)}{b^2} - \frac{\ln\left(a\,x^2+b\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^2/x^5,x)
```

```
[Out] 1/2/b/(a*x^2+b)+1/b^2*ln(x)-1/2*ln(a*x^2+b)/b^2
```

maxima [A] time = 0.81, size = 37, normalized size = 0.97

$$\frac{1}{2\left(abx^2+b^2\right)} - \frac{\log\left(ax^2+b\right)}{2\,b^2} + \frac{\log\left(x^2\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/2/(a*b*x^2 + b^2) - 1/2*log(a*x^2 + b)/b^2 + 1/2*log(x^2)/b^2
```

mupad [B] time = 0.06, size = 34, normalized size = 0.89

$$\frac{\ln(x)}{b^2} + \frac{1}{2\,b\,\left(a\,x^2+b\right)} - \frac{\ln\left(a\,x^2+b\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b/x^2)^2),x)
```

```
[Out] log(x)/b^2 + 1/(2*b*(b + a*x^2)) - log(b + a*x^2)/(2*b^2)
```

sympy [**A**] time = 0.31, size = 34, normalized size = 0.89

$$\frac{1}{2abx^2 + 2b^2} + \frac{\log(x)}{b^2} - \frac{\log\left(x^2 + \frac{b}{a}\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**2/x**5,x)
```

```
[Out] 1/(2*a*b*x**2 + 2*b**2) + log(x)/b**2 - log(x**2 + b/a)/(2*b**2)
```

3.1870 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^6}\,dx$

Optimal. Leaf size=57

$$-\frac{3\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2b^{5/2}}+\frac{1}{2bx\left(ax^2+b\right)}-\frac{3}{2b^2x}$$

```
[Out] -3/2/b^2/x+1/2/b/x/(a*x^2+b)-3/2*arctan(x*a^(1/2)/b^(1/2))*a^(1/2)/b^(5/2)
```

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {263, 290, 325, 205}

$$-\frac{3\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2b^{5/2}}+\frac{1}{2bx\left(ax^2+b\right)}-\frac{3}{2b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)^2*x^6),x]
```

```
[Out] -3/(2*b^2*x) + 1/(2*b*x*(b + a*x^2)) - (3*Sqrt[a]*ArcTan[(Sqrt[a]*x)/Sqrt[b
]])/(2*b^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^6}\,dx = \int \frac{1}{x^2\left(b+ax^2\right)^2}\,dx$$

$$= \frac{1}{2bx\left(b+ax^2\right)} + \frac{3\int \frac{1}{x^2\left(b+ax^2\right)}\,dx}{2b}$$

$$= -\frac{3}{2b^2x} + \frac{1}{2bx\left(b+ax^2\right)} - \frac{(3a)\int \frac{1}{b+ax^2}\,dx}{2b^2}$$

$$= -\frac{3}{2b^2x} + \frac{1}{2bx\left(b+ax^2\right)} - \frac{3\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2b^{5/2}}$$

Mathematica [A] time = 0.04, size = 54, normalized size = 0.95

$$-\frac{3\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2b^{5/2}} - \frac{ax}{2b^2\left(ax^2+b\right)} - \frac{1}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)^2*x^6),x]
```

```
[Out] -(1/(b^2*x)) - (a*x)/(2*b^2*(b + a*x^2)) - (3*Sqrt[a]*ArcTan[(Sqrt[a]*x)/Sq
rt[b]])/(2*b^(5/2))
```

fricas [A] time = 1.14, size = 136, normalized size = 2.39

$$\left[-\frac{6\,ax^2 - 3\left(ax^3+bx\right)\sqrt{-\frac{a}{b}}\,\log\left(\frac{ax^2-2\,bx\sqrt{-\frac{a}{b}}-b}{ax^2+b}\right)+4\,b}{4\left(ab^2x^3+b^3x\right)}, -\frac{3\,ax^2+3\left(ax^3+bx\right)\sqrt{\frac{a}{b}}\,\arctan\left(x\sqrt{\frac{a}{b}}\right)+2\,b}{2\left(ab^2x^3+b^3x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^2/x^6,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*a*x^2 - 3*(a*x^3 + b*x)*sqrt(-a/b)*log((a*x^2 - 2*b*x*sqrt(-a/b) -
 b)/(a*x^2 + b)) + 4*b)/(a*b^2*x^3 + b^3*x), -1/2*(3*a*x^2 + 3*(a*x^3 + b*x
)*sqrt(a/b)*arctan(x*sqrt(a/b)) + 2*b)/(a*b^2*x^3 + b^3*x)]
```

giac [A] time = 0.15, size = 47, normalized size = 0.82

$$-\frac{3\,a\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b^2} - \frac{3\,ax^2+2\,b}{2\left(ax^3+bx\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^2/x^6,x, algorithm="giac")
```

```
[Out] -3/2*a*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b^2) - 1/2*(3*a*x^2 + 2*b)/((a*x^3
 + b*x)*b^2)
```

maple [A] time = 0.01, size = 46, normalized size = 0.81

$$-\frac{ax}{2\left(a\,x^2+b\right)b^2} - \frac{3a\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^2} - \frac{1}{b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^2/x^6,x)
```

```
[Out] -1/2*a/b^2*x/(a*x^2+b)-3/2*a/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*a*x)-1/b^
2/x
```

maxima [A] time = 1.95, size = 49, normalized size = 0.86

$$-\frac{3\,ax^2+2\,b}{2\left(ab^2x^3+b^3x\right)}-\frac{3\,a\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^2/x^6,x, algorithm="maxima")
```

```
[Out] -1/2*(3*a*x^2 + 2*b)/(a*b^2*x^3 + b^3*x) - 3/2*a*arctan(a*x/sqrt(a*b))/(sqr
t(a*b)*b^2)
```

mupad [B] time = 1.11, size = 44, normalized size = 0.77

$$-\frac{\frac{1}{b}+\frac{3\,a\,x^2}{2\,b^2}}{a\,x^3+b\,x}-\frac{3\,\sqrt{a}\,\operatorname{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b/x^2)^2),x)
```

```
[Out] - (1/b + (3*a*x^2)/(2*b^2))/(b*x + a*x^3) - (3*a^(1/2)*atan((a^(1/2)*x)/b^(
1/2)))/(2*b^(5/2))
```

sympy [A] time = 0.31, size = 92, normalized size = 1.61

$$\frac{3\sqrt{-\frac{a}{b^5}}\,\log\left(x-\frac{b^3\sqrt{-\frac{a}{b^5}}}{a}\right)}{4}-\frac{3\sqrt{-\frac{a}{b^5}}\,\log\left(x+\frac{b^3\sqrt{-\frac{a}{b^5}}}{a}\right)}{4}+\frac{-3ax^2-2b}{2ab^2x^3+2b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**2/x**6,x)
```

```
[Out] 3*sqrt(-a/b**5)*log(x - b**3*sqrt(-a/b**5)/a)/4 - 3*sqrt(-a/b**5)*log(x + b
**3*sqrt(-a/b**5)/a)/4 + (-3*a*x**2 - 2*b)/(2*a*b**2*x**3 + 2*b**3*x)
```

3.1871 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^7}\,dx$

Optimal. Leaf size=49

$$\frac{a\log\left(ax^2+b\right)}{b^3}-\frac{2a\log(x)}{b^3}-\frac{a}{2b^2\left(ax^2+b\right)}-\frac{1}{2b^2x^2}$$

```
[Out] -1/2/b^2/x^2-1/2*a/b^2/(a*x^2+b)-2*a*ln(x)/b^3+a*ln(a*x^2+b)/b^3
```

Rubi [A] time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 266, 44}

$$-\frac{a}{2b^2\left(ax^2+b\right)}+\frac{a\log\left(ax^2+b\right)}{b^3}-\frac{2a\log(x)}{b^3}-\frac{1}{2b^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)^2*x^7),x]
```

```
[Out] -1/(2*b^2*x^2) - a/(2*b^2*(b + a*x^2)) - (2*a*Log[x])/b^3 + (a*Log[b + a*x^
2])/b^3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^7}\,dx &= \int \frac{1}{x^3\left(b+ax^2\right)^2}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2(b+ax)^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{b^2x^2}-\frac{2a}{b^3x}+\frac{a^2}{b^2(b+ax)^2}+\frac{2a^2}{b^3(b+ax)}\right)dx, x, x^2\right) \\
&= -\frac{1}{2b^2x^2}-\frac{a}{2b^2\left(b+ax^2\right)}-\frac{2a\log(x)}{b^3}+\frac{a\log\left(b+ax^2\right)}{b^3}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 41, normalized size = 0.84

$$-\frac{b\left(\frac{a}{ax^2+b}+\frac{1}{x^2}\right)-2a\log\left(ax^2+b\right)+4a\log(x)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)^2*x^7),x]
```

```
[Out] -1/2*(b*(x^(-2) + a/(b + a*x^2)) + 4*a*Log[x] - 2*a*Log[b + a*x^2])/b^3
```

fricas [A] time = 0.94, size = 73, normalized size = 1.49

$$-\frac{2\,abx^2+b^2-2\left(a^2x^4+abx^2\right)\log\left(ax^2+b\right)+4\left(a^2x^4+abx^2\right)\log(x)}{2\left(ab^3x^4+b^4x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^2/x^7,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*x^2 + b^2 - 2*(a^2*x^4 + a*b*x^2)*log(a*x^2 + b) + 4*(a^2*x^4 +
 a*b*x^2)*log(x))/(a*b^3*x^4 + b^4*x^2)
```

giac [A] time = 0.16, size = 51, normalized size = 1.04

$$-\frac{a\log\left(x^2\right)}{b^3}+\frac{a\log\left(\left|ax^2+b\right|\right)}{b^3}-\frac{2\,ax^2+b}{2\left(ax^4+bx^2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^2/x^7,x, algorithm="giac")
```

```
[Out] -a*log(x^2)/b^3 + a*log(abs(a*x^2 + b))/b^3 - 1/2*(2*a*x^2 + b)/((a*x^4 + b
*x^2)*b^2)
```

maple [A] time = 0.01, size = 46, normalized size = 0.94

$$-\frac{a}{2\left(a\,x^2+b\right)b^2}-\frac{2a\ln(x)}{b^3}+\frac{a\ln\left(a\,x^2+b\right)}{b^3}-\frac{1}{2b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^2/x^7,x)
```

```
[Out] -1/2/b^2/x^2-1/2*a/b^2/(a*x^2+b)-2*a/b^3*ln(x)+a*ln(a*x^2+b)/b^3
```

maxima [A] time = 0.91, size = 52, normalized size = 1.06

$$-\frac{2\,ax^2+b}{2\left(ab^2x^4+b^3x^2\right)}+\frac{a\log\left(ax^2+b\right)}{b^3}-\frac{a\log\left(x^2\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^2/x^7,x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*x^2 + b)/(a*b^2*x^4 + b^3*x^2) + a*log(a*x^2 + b)/b^3 - a*log(x^2
)/b^3
```

mupad [B] time = 1.12, size = 51, normalized size = 1.04

$$\frac{a\,\ln\left(a\,x^2+b\right)}{b^3}-\frac{\frac{1}{2\,b}+\frac{a\,x^2}{b^2}}{a\,x^4+b\,x^2}-\frac{2\,a\,\ln(x)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b/x^2)^2),x)
```

```
[Out] (a*log(b + a*x^2))/b^3 - (1/(2*b) + (a*x^2)/b^2)/(a*x^4 + b*x^2) - (2*a*log
(x))/b^3
```

sympy [**A**] time = 0.39, size = 51, normalized size = 1.04

$$-\frac{2a\log(x)}{b^3}+\frac{a\log\left(x^2+\frac{b}{a}\right)}{b^3}+\frac{-2ax^2-b}{2ab^2x^4+2b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**2/x**7,x)
```

```
[Out] -2*a*log(x)/b**3 + a*log(x**2 + b/a)/b**3 + (-2*a*x**2 - b)/(2*a*b**2*x**4
+ 2*b**3*x**2)
```

3.1872 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^8}\,dx$

Optimal. Leaf size=68

$$\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2b^{7/2}}+\frac{5a}{2b^3x}+\frac{1}{2bx^3\left(ax^2+b\right)}-\frac{5}{6b^2x^3}$$

```
[Out] -5/6/b^2/x^3+5/2*a/b^3/x+1/2/b/x^3/(a*x^2+b)+5/2*a^(3/2)*arctan(x*a^(1/2)/b
^(1/2))/b^(7/2)
```

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {263, 290, 325, 205}

$$\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2b^{7/2}}+\frac{5a}{2b^3x}+\frac{1}{2bx^3\left(ax^2+b\right)}-\frac{5}{6b^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)^2*x^8),x]
```

```
[Out] -5/(6*b^2*x^3) + (5*a)/(2*b^3*x) + 1/(2*b*x^3*(b + a*x^2)) + (5*a^(3/2)*Arc
Tan[(Sqrt[a]*x)/Sqrt[b]])/(2*b^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^8}\,dx = \int \frac{1}{x^4\left(b+ax^2\right)^2}\,dx$$

$$= \frac{1}{2bx^3\left(b+ax^2\right)} + \frac{5\int \frac{1}{x^4\left(b+ax^2\right)}\,dx}{2b}$$

$$= -\frac{5}{6b^2x^3} + \frac{1}{2bx^3\left(b+ax^2\right)} - \frac{(5a)\int \frac{1}{x^2\left(b+ax^2\right)}\,dx}{2b^2}$$

$$= -\frac{5}{6b^2x^3} + \frac{5a}{2b^3x} + \frac{1}{2bx^3\left(b+ax^2\right)} + \frac{\left(5a^2\right)\int \frac{1}{b+ax^2}\,dx}{2b^3}$$

$$= -\frac{5}{6b^2x^3} + \frac{5a}{2b^3x} + \frac{1}{2bx^3\left(b+ax^2\right)} + \frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2b^{7/2}}$$

Mathematica [A] time = 0.05, size = 67, normalized size = 0.99

$$\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2b^{7/2}} + \frac{a^2x}{2b^3\left(ax^2+b\right)} + \frac{2a}{b^3x} - \frac{1}{3b^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)^2*x^8),x]
```

```
[Out] -1/3*1/(b^2*x^3) + (2*a)/(b^3*x) + (a^2*x)/(2*b^3*(b + a*x^2)) + (5*a^(3/2)
*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(2*b^(7/2))
```

fricas [A] time = 0.85, size = 172, normalized size = 2.53

$$\left[\frac{30\,a^2x^4 + 20\,abx^2 + 15\left(a^2x^5 + abx^3\right)\sqrt{-\frac{a}{b}}\,\log\left(\frac{ax^2+2\,bx\sqrt{-\frac{a}{b}}-b}{ax^2+b}\right) - 4\,b^2}{12\left(ab^3x^5 + b^4x^3\right)}, \frac{15\,a^2x^4 + 10\,abx^2 + 15\left(a^2x^5 + abx^3\right)\sqrt{\frac{a}{b}}}{6\left(ab^3x^5 + b^4x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^2/x^8,x, algorithm="fricas")
```

```
[Out] [1/12*(30*a^2*x^4 + 20*a*b*x^2 + 15*(a^2*x^5 + a*b*x^3)*sqrt(-a/b)*log((a*x
^2 + 2*b*x*sqrt(-a/b) - b)/(a*x^2 + b)) - 4*b^2)/(a*b^3*x^5 + b^4*x^3), 1/6
*(15*a^2*x^4 + 10*a*b*x^2 + 15*(a^2*x^5 + a*b*x^3)*sqrt(a/b)*arctan(x*sqrt(
a/b)) - 2*b^2)/(a*b^3*x^5 + b^4*x^3)]
```

giac [A] time = 0.17, size = 59, normalized size = 0.87

$$\frac{5\,a^2\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b^3} + \frac{a^2x}{2\left(ax^2+b\right)b^3} + \frac{6\,ax^2-b}{3\,b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^2/x^8,x, algorithm="giac")
```

```
[Out] 5/2*a^2*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/2*a^2*x/((a*x^2 + b)*b^3)
 + 1/3*(6*a*x^2 - b)/(b^3*x^3)
```

maple [A] time = 0.01, size = 59, normalized size = 0.87

$$\frac{a^2 x}{2\left(a\,x^2+b\right)b^3}+\frac{5a^2\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^3}+\frac{2a}{b^3x}-\frac{1}{3b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^2/x^8,x)
```

```
[Out] 1/2*a^2/b^3*x/(a*x^2+b)+5/2*a^2/b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*a*x)-1
/3/b^2/x^3+2*a/b^3/x
```

maxima [A] time = 1.89, size = 64, normalized size = 0.94

$$\frac{15\,a^2x^4+10\,abx^2-2\,b^2}{6\left(ab^3x^5+b^4x^3\right)}+\frac{5\,a^2\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^2/x^8,x, algorithm="maxima")
```

```
[Out] 1/6*(15*a^2*x^4 + 10*a*b*x^2 - 2*b^2)/(a*b^3*x^5 + b^4*x^3) + 5/2*a^2*arcta
n(a*x/sqrt(a*b))/(sqrt(a*b)*b^3)
```

mupad [B] time = 1.17, size = 58, normalized size = 0.85

$$\frac{\frac{5\,a\,x^2}{3\,b^2}-\frac{1}{3\,b}+\frac{5\,a^2\,x^4}{2\,b^3}}{a\,x^5+b\,x^3}+\frac{5\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\;x}{\sqrt{b}}\right)}{2\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(a + b/x^2)^2),x)
```

```
[Out] ((5*a*x^2)/(3*b^2) - 1/(3*b) + (5*a^2*x^4)/(2*b^3))/(a*x^5 + b*x^3) + (5*a^
(3/2)*atan((a^(1/2)*x)/b^(1/2)))/(2*b^(7/2))
```

sympy [A] time = 0.37, size = 114, normalized size = 1.68

$$-\frac{5\sqrt{-\frac{a^3}{b^7}}\,\log\left(x-\frac{b^4\sqrt{-\frac{a^3}{b^7}}}{a^2}\right)}{4}+\frac{5\sqrt{-\frac{a^3}{b^7}}\,\log\left(x+\frac{b^4\sqrt{-\frac{a^3}{b^7}}}{a^2}\right)}{4}+\frac{15a^2x^4+10abx^2-2b^2}{6ab^3x^5+6b^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**2/x**8,x)
```

```
[Out] -5*sqrt(-a**3/b**7)*log(x - b**4*sqrt(-a**3/b**7)/a**2)/4 + 5*sqrt(-a**3/b*
*7)*log(x + b**4*sqrt(-a**3/b**7)/a**2)/4 + (15*a**2*x**4 + 10*a*b*x**2 - 2
*b**2)/(6*a*b**3*x**5 + 6*b**4*x**3)
```

3.1873 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^9}\,dx$

Optimal. Leaf size=66

$$-\frac{3a^2\log\left(ax^2+b\right)}{2b^4}+\frac{3a^2\log(x)}{b^4}+\frac{a^2}{2b^3\left(ax^2+b\right)}+\frac{a}{b^3x^2}-\frac{1}{4b^2x^4}$$

```
[Out] -1/4/b^2/x^4+a/b^3/x^2+1/2*a^2/b^3/(a*x^2+b)+3*a^2*ln(x)/b^4-3/2*a^2*ln(a*x
^2+b)/b^4
```

Rubi [A] time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 266, 44}

$$\frac{a^2}{2b^3\left(ax^2+b\right)}-\frac{3a^2\log\left(ax^2+b\right)}{2b^4}+\frac{3a^2\log(x)}{b^4}+\frac{a}{b^3x^2}-\frac{1}{4b^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)^2*x^9),x]
```

```
[Out] -1/(4*b^2*x^4) + a/(b^3*x^2) + a^2/(2*b^3*(b + a*x^2)) + (3*a^2*Log[x])/b^4
 - (3*a^2*Log[b + a*x^2])/(2*b^4)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^9}\,dx &= \int \frac{1}{x^5\left(b+ax^2\right)^2}\,dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^3(b+ax)^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{b^2x^3}-\frac{2a}{b^3x^2}+\frac{3a^2}{b^4x}-\frac{a^3}{b^3(b+ax)^2}-\frac{3a^3}{b^4(b+ax)}\right)dx, x, x^2\right) \\
&= -\frac{1}{4b^2x^4}+\frac{a}{b^3x^2}+\frac{a^2}{2b^3\left(b+ax^2\right)}+\frac{3a^2\log(x)}{b^4}-\frac{3a^2\log\left(b+ax^2\right)}{2b^4}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 57, normalized size = 0.86

$$\frac{-6a^2\log\left(ax^2+b\right)+b\left(\frac{2a^2}{ax^2+b}+\frac{4a}{x^2}-\frac{b}{x^4}\right)+12a^2\log(x)}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)^2*x^9),x]
```

```
[Out] (b*(-(b/x^4) + (4*a)/x^2 + (2*a^2)/(b + a*x^2)) + 12*a^2*Log[x] - 6*a^2*Log
[b + a*x^2])/(4*b^4)
```

fricas [A] time = 0.83, size = 90, normalized size = 1.36

$$\frac{6\,a^2bx^4+3\,ab^2x^2-b^3-6\left(a^3x^6+a^2bx^4\right)\log\left(ax^2+b\right)+12\left(a^3x^6+a^2bx^4\right)\log(x)}{4\left(ab^4x^6+b^5x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^2/x^9,x, algorithm="fricas")
```

```
[Out] 1/4*(6*a^2*b*x^4 + 3*a*b^2*x^2 - b^3 - 6*(a^3*x^6 + a^2*b*x^4)*log(a*x^2 +
b) + 12*(a^3*x^6 + a^2*b*x^4)*log(x))/(a*b^4*x^6 + b^5*x^4)
```

giac [A] time = 0.15, size = 86, normalized size = 1.30

$$\frac{3\,a^2\log\left(x^2\right)}{2\,b^4}-\frac{3\,a^2\log\left(\left|ax^2+b\right|\right)}{2\,b^4}+\frac{3\,a^3x^2+4\,a^2b}{2\left(ax^2+b\right)b^4}-\frac{9\,a^2x^4-4\,abx^2+b^2}{4\,b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^2/x^9,x, algorithm="giac")
```

```
[Out] 3/2*a^2*log(x^2)/b^4 - 3/2*a^2*log(abs(a*x^2 + b))/b^4 + 1/2*(3*a^3*x^2 + 4
*a^2*b)/((a*x^2 + b)*b^4) - 1/4*(9*a^2*x^4 - 4*a*b*x^2 + b^2)/(b^4*x^4)
```

maple [A] time = 0.01, size = 61, normalized size = 0.92

$$\frac{a^2}{2\left(a\,x^2+b\right)b^3}+\frac{3a^2\ln(x)}{b^4}-\frac{3a^2\ln\left(a\,x^2+b\right)}{2b^4}+\frac{a}{b^3x^2}-\frac{1}{4b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^2/x^9,x)
```

```
[Out] -1/4/b^2/x^4+a/b^3/x^2+1/2*a^2/b^3/(a*x^2+b)+3*a^2/b^4*ln(x)-3/2*a^2*ln(a*x
^2+b)/b^4
```

maxima [A] time = 0.88, size = 70, normalized size = 1.06

$$\frac{6\,a^2x^4+3\,abx^2-b^2}{4\left(ab^3x^6+b^4x^4\right)}-\frac{3\,a^2\log\left(ax^2+b\right)}{2\,b^4}+\frac{3\,a^2\log\left(x^2\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^2/x^9,x, algorithm="maxima")
```

```
[Out] 1/4*(6*a^2*x^4 + 3*a*b*x^2 - b^2)/(a*b^3*x^6 + b^4*x^4) - 3/2*a^2*log(a*x^2
 + b)/b^4 + 3/2*a^2*log(x^2)/b^4
```

mupad [B] time = 0.08, size = 67, normalized size = 1.02

$$\frac{\frac{3\,a\,x^2}{4\,b^2}-\frac{1}{4\,b}+\frac{3\,a^2\,x^4}{2\,b^3}}{a\,x^6+b\,x^4}-\frac{3\,a^2\,\ln\left(a\,x^2+b\right)}{2\,b^4}+\frac{3\,a^2\,\ln(x)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(a + b/x^2)^2),x)
```

```
[Out] ((3*a*x^2)/(4*b^2) - 1/(4*b) + (3*a^2*x^4)/(2*b^3))/(a*x^6 + b*x^4) - (3*a^
2*log(b + a*x^2))/(2*b^4) + (3*a^2*log(x))/b^4
```

sympy [A] time = 0.46, size = 68, normalized size = 1.03

$$\frac{3a^2\log(x)}{b^4}-\frac{3a^2\log\left(x^2+\frac{b}{a}\right)}{2b^4}+\frac{6a^2x^4+3abx^2-b^2}{4ab^3x^6+4b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**2/x**9,x)
```

```
[Out] 3*a**2*log(x)/b**4 - 3*a**2*log(x**2 + b/a)/(2*b**4) + (6*a**2*x**4 + 3*a*b
*x**2 - b**2)/(4*a*b**3*x**6 + 4*b**4*x**4)
```

3.1874 $\int \frac{x^5}{\left(a+\frac{b}{x^2}\right)^3}\,dx$

Optimal. Leaf size=87

$$\frac{b^5}{4a^6\left(ax^2+b\right)^2}-\frac{5b^4}{2a^6\left(ax^2+b\right)}-\frac{5b^3\log\left(ax^2+b\right)}{a^6}+\frac{3b^2x^2}{a^5}-\frac{3bx^4}{4a^4}+\frac{x^6}{6a^3}$$

```
[Out] 3*b^2*x^2/a^5-3/4*b*x^4/a^4+1/6*x^6/a^3+1/4*b^5/a^6/(a*x^2+b)^2-5/2*b^4/a^6
/(a*x^2+b)-5*b^3*ln(a*x^2+b)/a^6
```

Rubi [A] time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 266, 43}

$$\frac{3b^2x^2}{a^5}-\frac{5b^4}{2a^6\left(ax^2+b\right)}+\frac{b^5}{4a^6\left(ax^2+b\right)^2}-\frac{5b^3\log\left(ax^2+b\right)}{a^6}-\frac{3bx^4}{4a^4}+\frac{x^6}{6a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b/x^2)^3,x]
```

```
[Out] (3*b^2*x^2)/a^5 - (3*b*x^4)/(4*a^4) + x^6/(6*a^3) + b^5/(4*a^6*(b + a*x^2)^
2) - (5*b^4)/(2*a^6*(b + a*x^2)) - (5*b^3*Log[b + a*x^2])/a^6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(a+\frac{b}{x^2}\right)^3}\,dx &= \int \frac{x^{11}}{\left(b+ax^2\right)^3}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^5}{(b+ax)^3}\,dx, x, x^2\right) \\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{6b^2}{a^5}-\frac{3bx}{a^4}+\frac{x^2}{a^3}-\frac{b^5}{a^5(b+ax)^3}+\frac{5b^4}{a^5(b+ax)^2}-\frac{10b^3}{a^5(b+ax)}\right)dx, x, x^2\right) \\
&= \frac{3b^2x^2}{a^5}-\frac{3bx^4}{4a^4}+\frac{x^6}{6a^3}+\frac{b^5}{4a^6\left(b+ax^2\right)^2}-\frac{5b^4}{2a^6\left(b+ax^2\right)}-\frac{5b^3\log\left(b+ax^2\right)}{a^6}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 71, normalized size = 0.82

$$\frac{2a^3x^6 - 9a^2bx^4 - \frac{3b^4\left(10ax^2+9b\right)}{\left(ax^2+b\right)^2} - 60b^3\log\left(ax^2+b\right) + 36ab^2x^2}{12a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b/x^2)^3,x]
```

```
[Out] (36*a*b^2*x^2 - 9*a^2*b*x^4 + 2*a^3*x^6 - (3*b^4*(9*b + 10*a*x^2))/(b + a*x
^2)^2 - 60*b^3*Log[b + a*x^2])/(12*a^6)
```

fricas [A] time = 0.73, size = 115, normalized size = 1.32

$$\frac{2\,a^5x^{10} - 5\,a^4bx^8 + 20\,a^3b^2x^6 + 63\,a^2b^3x^4 + 6\,ab^4x^2 - 27\,b^5 - 60\left(a^2b^3x^4 + 2\,ab^4x^2 + b^5\right)\log\left(ax^2+b\right)}{12\left(a^8x^4 + 2\,a^7bx^2 + a^6b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(2*a^5*x^10 - 5*a^4*b*x^8 + 20*a^3*b^2*x^6 + 63*a^2*b^3*x^4 + 6*a*b^4*
x^2 - 27*b^5 - 60*(a^2*b^3*x^4 + 2*a*b^4*x^2 + b^5)*log(a*x^2 + b))/(a^8*x^
4 + 2*a^7*b*x^2 + a^6*b^2)
```

giac [A] time = 0.17, size = 92, normalized size = 1.06

$$-\frac{5\,b^3\log\left(\left|ax^2+b\right|\right)}{a^6} + \frac{30\,a^2b^3x^4 + 50\,ab^4x^2 + 21\,b^5}{4\left(ax^2+b\right)^2a^6} + \frac{2\,a^6x^6 - 9\,a^5bx^4 + 36\,a^4b^2x^2}{12\,a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/x^2)^3,x, algorithm="giac")
```

```
[Out] -5*b^3*log(abs(a*x^2 + b))/a^6 + 1/4*(30*a^2*b^3*x^4 + 50*a*b^4*x^2 + 21*b^
5)/((a*x^2 + b)^2*a^6) + 1/12*(2*a^6*x^6 - 9*a^5*b*x^4 + 36*a^4*b^2*x^2)/a^
9
```

maple [A] time = 0.01, size = 80, normalized size = 0.92

$$\frac{x^6}{6a^3} - \frac{3b\,x^4}{4a^4} + \frac{3b^2x^2}{a^5} + \frac{b^5}{4\left(a\,x^2+b\right)^2a^6} - \frac{5b^4}{2\left(a\,x^2+b\right)a^6} - \frac{5b^3\ln\left(a\,x^2+b\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a+b/x^2)^3,x)
```

```
[Out] 3/a^5*b^2*x^2-3/4/a^4*b*x^4+1/6*x^6/a^3+1/4*b^5/a^6/(a*x^2+b)^2-5/2*b^4/a^6
/(a*x^2+b)-5*b^3*ln(a*x^2+b)/a^6
```

maxima [A] time = 0.88, size = 89, normalized size = 1.02

$$-\frac{10\,ab^4x^2 + 9\,b^5}{4\left(a^8x^4 + 2\,a^7bx^2 + a^6b^2\right)} - \frac{5\,b^3\log\left(ax^2+b\right)}{a^6} + \frac{2\,a^2x^6 - 9\,abx^4 + 36\,b^2x^2}{12\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(10*a*b^4*x^2 + 9*b^5)/(a^8*x^4 + 2*a^7*b*x^2 + a^6*b^2) - 5*b^3*log(a
*x^2 + b)/a^6 + 1/12*(2*a^2*x^6 - 9*a*b*x^4 + 36*b^2*x^2)/a^5
```

mupad **[B]** time = 0.06, size = 90, normalized size = 1.03

$$\frac{x^6}{6\,a^3}-\frac{\frac{9\,b^5}{4\,a}+\frac{5\,b^4\,x^2}{2}}{a^7\,x^4+2\,a^6\,b\,x^2+a^5\,b^2}-\frac{3\,b\,x^4}{4\,a^4}-\frac{5\,b^3\,\ln\left(a\,x^2+b\right)}{a^6}+\frac{3\,b^2\,x^2}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b/x^2)^3,x)
```

```
[Out] x^6/(6*a^3) - ((9*b^5)/(4*a) + (5*b^4*x^2)/2)/(a^5*b^2 + a^7*x^4 + 2*a^6*b*
x^2) - (3*b*x^4)/(4*a^4) - (5*b^3*log(b + a*x^2))/a^6 + (3*b^2*x^2)/a^5
```

sympy **[A]** time = 0.47, size = 92, normalized size = 1.06

$$\frac{-10ab^4x^2-9b^5}{4a^8x^4+8a^7bx^2+4a^6b^2}+\frac{x^6}{6a^3}-\frac{3bx^4}{4a^4}+\frac{3b^2x^2}{a^5}-\frac{5b^3\log\left(ax^2+b\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a+b/x**2)**3,x)
```

```
[Out] (-10*a*b**4*x**2 - 9*b**5)/(4*a**8*x**4 + 8*a**7*b*x**2 + 4*a**6*b**2) + x*
*6/(6*a**3) - 3*b*x**4/(4*a**4) + 3*b**2*x**2/a**5 - 5*b**3*log(a*x**2 + b)
/a**6
```

3.1875 $\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^3}\,dx$

Optimal. Leaf size=98

$$-\frac{63b^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8a^{11/2}}+\frac{63b^2x}{8a^5}-\frac{21bx^3}{8a^4}+\frac{63x^5}{40a^3}-\frac{9x^7}{8a^2\left(ax^2+b\right)}-\frac{x^9}{4a\left(ax^2+b\right)^2}$$

```
[Out] 63/8*b^2*x/a^5-21/8*b*x^3/a^4+63/40*x^5/a^3-1/4*x^9/a/(a*x^2+b)^2-9/8*x^7/a
^2/(a*x^2+b)-63/8*b^(5/2)*arctan(x*a^(1/2)/b^(1/2))/a^(11/2)
```

Rubi [A] time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {263, 288, 302, 205}

$$\frac{63b^2x}{8a^5}-\frac{63b^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8a^{11/2}}-\frac{9x^7}{8a^2\left(ax^2+b\right)}-\frac{21bx^3}{8a^4}+\frac{63x^5}{40a^3}-\frac{x^9}{4a\left(ax^2+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b/x^2)^3,x]
```

```
[Out] (63*b^2*x)/(8*a^5) - (21*b*x^3)/(8*a^4) + (63*x^5)/(40*a^3) - x^9/(4*a*(b +
 a*x^2)^2) - (9*x^7)/(8*a^2*(b + a*x^2)) - (63*b^(5/2)*ArcTan[(Sqrt[a]*x)/S
qrt[b]])/(8*a^(11/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^3}\,dx = \int \frac{x^{10}}{\left(b+ax^2\right)^3}\,dx$$

$$= -\frac{x^9}{4a\left(b+ax^2\right)^2} + \frac{9\int \frac{x^8}{\left(b+ax^2\right)^2}\,dx}{4a}$$

$$= -\frac{x^9}{4a\left(b+ax^2\right)^2} - \frac{9x^7}{8a^2\left(b+ax^2\right)} + \frac{63\int \frac{x^6}{b+ax^2}\,dx}{8a^2}$$

$$= -\frac{x^9}{4a\left(b+ax^2\right)^2} - \frac{9x^7}{8a^2\left(b+ax^2\right)} + \frac{63\int \left(\frac{b^2}{a^3} - \frac{bx^2}{a^2} + \frac{x^4}{a} - \frac{b^3}{a^3\left(b+ax^2\right)}\right)dx}{8a^2}$$

$$= \frac{63b^2x}{8a^5} - \frac{21bx^3}{8a^4} + \frac{63x^5}{40a^3} - \frac{x^9}{4a\left(b+ax^2\right)^2} - \frac{9x^7}{8a^2\left(b+ax^2\right)} - \frac{\left(63b^3\right)\int \frac{1}{b+ax^2}\,dx}{8a^5}$$

$$= \frac{63b^2x}{8a^5} - \frac{21bx^3}{8a^4} + \frac{63x^5}{40a^3} - \frac{x^9}{4a\left(b+ax^2\right)^2} - \frac{9x^7}{8a^2\left(b+ax^2\right)} - \frac{63b^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8a^{11/2}}$$

Mathematica [A] time = 0.05, size = 88, normalized size = 0.90

$$\frac{8a^4x^9 - 24a^3bx^7 + 168a^2b^2x^5 + 525ab^3x^3 + 315b^4x}{40a^5\left(ax^2+b\right)^2} - \frac{63b^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8a^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b/x^2)^3,x]
```

```
[Out] (315*b^4*x + 525*a*b^3*x^3 + 168*a^2*b^2*x^5 - 24*a^3*b*x^7 + 8*a^4*x^9)/(4
0*a^5*(b + a*x^2)^2) - (63*b^(5/2)*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(8*a^(11/2)
)
```

fricas [A] time = 0.84, size = 256, normalized size = 2.61

$$\left[\frac{16\,a^4x^9 - 48\,a^3bx^7 + 336\,a^2b^2x^5 + 1050\,ab^3x^3 + 630\,b^4x + 315\left(a^2b^2x^4 + 2\,ab^3x^2 + b^4\right)\sqrt{-\frac{b}{a}}\,\log\left(\frac{ax^2 - 2\,ax\sqrt{-\frac{b}{a}}}{ax^2+b}\right.}{80\left(a^7x^4 + 2\,a^6bx^2 + a^5b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/80*(16*a^4*x^9 - 48*a^3*b*x^7 + 336*a^2*b^2*x^5 + 1050*a*b^3*x^3 + 630*b
^4*x + 315*(a^2*b^2*x^4 + 2*a*b^3*x^2 + b^4)*sqrt(-b/a)*log((a*x^2 - 2*a*x*
sqrt(-b/a) - b)/(a*x^2 + b)))/(a^7*x^4 + 2*a^6*b*x^2 + a^5*b^2), 1/40*(8*a^
4*x^9 - 24*a^3*b*x^7 + 168*a^2*b^2*x^5 + 525*a*b^3*x^3 + 315*b^4*x - 315*(a
^2*b^2*x^4 + 2*a*b^3*x^2 + b^4)*sqrt(b/a)*arctan(a*x*sqrt(b/a)/b))/(a^7*x^4
 + 2*a^6*b*x^2 + a^5*b^2)]
```

giac [A] time = 0.16, size = 84, normalized size = 0.86

$$-\frac{63\,b^3\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\sqrt{ab}\,a^5} + \frac{17\,ab^3x^3 + 15\,b^4x}{8\left(ax^2+b\right)^2a^5} + \frac{a^{12}x^5 - 5\,a^{11}bx^3 + 30\,a^{10}b^2x}{5\,a^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/x^2)^3,x, algorithm="giac")
```

```
[Out] -63/8*b^3*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^5) + 1/8*(17*a*b^3*x^3 + 15*b^
4*x)/((a*x^2 + b)^2*a^5) + 1/5*(a^12*x^5 - 5*a^11*b*x^3 + 30*a^10*b^2*x)/a^
15
```

maple [A] time = 0.01, size = 88, normalized size = 0.90

$$\frac{x^5}{5a^3}+\frac{17b^3x^3}{8\left(a\,x^2+b\right)^2a^4}-\frac{b\,x^3}{a^4}+\frac{15b^4x}{8\left(a\,x^2+b\right)^2a^5}-\frac{63b^3\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\sqrt{ab}\;a^5}+\frac{6b^2x}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a+b/x^2)^3,x)
```

```
[Out] 1/5/a^3*x^5-1/a^4*b*x^3+6/a^5*b^2*x+17/8/a^4*b^3/(a*x^2+b)^2*x^3+15/8/a^5*b
^4/(a*x^2+b)^2*x-63/8/a^5*b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*a*x)
```

maxima [A] time = 1.87, size = 93, normalized size = 0.95

$$\frac{17\,ab^3x^3+15\,b^4x}{8\left(a^7x^4+2\,a^6bx^2+a^5b^2\right)}-\frac{63\,b^3\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^5}+\frac{a^2x^5-5\,abx^3+30\,b^2x}{5\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(17*a*b^3*x^3 + 15*b^4*x)/(a^7*x^4 + 2*a^6*b*x^2 + a^5*b^2) - 63/8*b^3*
arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^5) + 1/5*(a^2*x^5 - 5*a*b*x^3 + 30*b^2*x
)/a^5
```

mupad [B] time = 1.11, size = 87, normalized size = 0.89

$$\frac{\frac{15\,b^4\,x}{8}+\frac{17\,a\,b^3\,x^3}{8}}{a^7\,x^4+2\,a^6\,b\,x^2+a^5\,b^2}+\frac{x^5}{5\,a^3}-\frac{b\,x^3}{a^4}+\frac{6\,b^2\,x}{a^5}-\frac{63\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8\,a^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b/x^2)^3,x)
```

```
[Out] ((15*b^4*x)/8 + (17*a*b^3*x^3)/8)/(a^5*b^2 + a^7*x^4 + 2*a^6*b*x^2) + x^5/(
5*a^3) - (b*x^3)/a^4 + (6*b^2*x)/a^5 - (63*b^(5/2)*atan((a^(1/2)*x)/b^(1/2)
))/(8*a^(11/2))
```

sympy [A] time = 0.49, size = 144, normalized size = 1.47

$$\frac{63\sqrt{-\frac{b^5}{a^{11}}}\,\log\left(-\frac{a^5\sqrt{-\frac{b^5}{a^{11}}}}{b^2}+x\right)}{16}-\frac{63\sqrt{-\frac{b^5}{a^{11}}}\,\log\left(\frac{a^5\sqrt{-\frac{b^5}{a^{11}}}}{b^2}+x\right)}{16}+\frac{17ab^3x^3+15b^4x}{8a^7x^4+16a^6bx^2+8a^5b^2}+\frac{x^5}{5a^3}-\frac{bx^3}{a^4}+\frac{6b^2x}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a+b/x**2)**3,x)
```

```
[Out] 63*sqrt(-b**5/a**11)*log(-a**5*sqrt(-b**5/a**11)/b**2 + x)/16 - 63*sqrt(-b*
*5/a**11)*log(a**5*sqrt(-b**5/a**11)/b**2 + x)/16 + (17*a*b**3*x**3 + 15*b*
*4*x)/(8*a**7*x**4 + 16*a**6*b*x**2 + 8*a**5*b**2) + x**5/(5*a**3) - b*x**3
/a**4 + 6*b**2*x/a**5
```

3.1876 $\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^3}\,dx$

Optimal. Leaf size=74

$$-\frac{b^4}{4a^5\left(ax^2+b\right)^2}+\frac{2b^3}{a^5\left(ax^2+b\right)}+\frac{3b^2\log\left(ax^2+b\right)}{a^5}-\frac{3bx^2}{2a^4}+\frac{x^4}{4a^3}$$

```
[Out] -3/2*b*x^2/a^4+1/4*x^4/a^3-1/4*b^4/a^5/(a*x^2+b)^2+2*b^3/a^5/(a*x^2+b)+3*b^
2*ln(a*x^2+b)/a^5
```

Rubi [A] time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 266, 43}

$$-\frac{b^4}{4a^5\left(ax^2+b\right)^2}+\frac{2b^3}{a^5\left(ax^2+b\right)}+\frac{3b^2\log\left(ax^2+b\right)}{a^5}-\frac{3bx^2}{2a^4}+\frac{x^4}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b/x^2)^3,x]
```

```
[Out] (-3*b*x^2)/(2*a^4) + x^4/(4*a^3) - b^4/(4*a^5*(b + a*x^2)^2) + (2*b^3)/(a^5
*(b + a*x^2)) + (3*b^2*Log[b + a*x^2])/a^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^3}\,dx &= \int \frac{x^9}{\left(b+ax^2\right)^3}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{(b+ax)^3}\,dx, x, x^2\right) \\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{3b}{a^4}+\frac{x}{a^3}+\frac{b^4}{a^4(b+ax)^3}-\frac{4b^3}{a^4(b+ax)^2}+\frac{6b^2}{a^4(b+ax)}\right)dx, x, x^2\right) \\
&= -\frac{3bx^2}{2a^4}+\frac{x^4}{4a^3}-\frac{b^4}{4a^5\left(b+ax^2\right)^2}+\frac{2b^3}{a^5\left(b+ax^2\right)}+\frac{3b^2\log\left(b+ax^2\right)}{a^5}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 58, normalized size = 0.78

$$\frac{a^2x^4 + \frac{b^3\left(8ax^2+7b\right)}{\left(ax^2+b\right)^2} + 12b^2\log\left(ax^2+b\right) - 6abx^2}{4a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b/x^2)^3,x]
```

```
[Out] (-6*a*b*x^2 + a^2*x^4 + (b^3*(7*b + 8*a*x^2))/(b + a*x^2)^2 + 12*b^2*Log[b
+ a*x^2])/(4*a^5)
```

fricas [A] time = 0.75, size = 103, normalized size = 1.39

$$\frac{a^4x^8 - 4\,a^3bx^6 - 11\,a^2b^2x^4 + 2\,ab^3x^2 + 7\,b^4 + 12\left(a^2b^2x^4 + 2\,ab^3x^2 + b^4\right)\log\left(ax^2+b\right)}{4\left(a^7x^4 + 2\,a^6bx^2 + a^5b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(a^4*x^8 - 4*a^3*b*x^6 - 11*a^2*b^2*x^4 + 2*a*b^3*x^2 + 7*b^4 + 12*(a^2
*b^2*x^4 + 2*a*b^3*x^2 + b^4)*log(a*x^2 + b))/(a^7*x^4 + 2*a^6*b*x^2 + a^5*
b^2)
```

giac [A] time = 0.16, size = 69, normalized size = 0.93

$$\frac{3\,b^2\log\left(\left|ax^2+b\right|\right)}{a^5} + \frac{a^3x^4 - 6\,a^2bx^2}{4\,a^6} + \frac{8\,ab^3x^2 + 7\,b^4}{4\left(ax^2+b\right)^2a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x^2)^3,x, algorithm="giac")
```

```
[Out] 3*b^2*log(abs(a*x^2 + b))/a^5 + 1/4*(a^3*x^4 - 6*a^2*b*x^2)/a^6 + 1/4*(8*a*
b^3*x^2 + 7*b^4)/((a*x^2 + b)^2*a^5)
```

maple [A] time = 0.01, size = 69, normalized size = 0.93

$$\frac{x^4}{4a^3} - \frac{3b\,x^2}{2a^4} - \frac{b^4}{4\left(a\,x^2+b\right)^2a^5} + \frac{2b^3}{\left(a\,x^2+b\right)a^5} + \frac{3b^2\ln\left(a\,x^2+b\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b/x^2)^3,x)
```

```
[Out] -3/2/a^4*b*x^2+1/4/a^3*x^4-1/4*b^4/a^5/(a*x^2+b)^2+2*b^3/a^5/(a*x^2+b)+3*b^
2*ln(a*x^2+b)/a^5
```

maxima [A] time = 0.83, size = 77, normalized size = 1.04

$$\frac{8\,ab^3x^2 + 7\,b^4}{4\left(a^7x^4 + 2\,a^6bx^2 + a^5b^2\right)} + \frac{3\,b^2\log\left(ax^2+b\right)}{a^5} + \frac{ax^4 - 6\,bx^2}{4\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(8*a*b^3*x^2 + 7*b^4)/(a^7*x^4 + 2*a^6*b*x^2 + a^5*b^2) + 3*b^2*log(a*x
^2 + b)/a^5 + 1/4*(a*x^4 - 6*b*x^2)/a^4
```

mupad [B] time = 1.13, size = 78, normalized size = 1.05

$$\frac{\frac{7b^4}{4a}+2b^3x^2}{a^6x^4+2a^5bx^2+a^4b^2}+\frac{x^4}{4a^3}-\frac{3bx^2}{2a^4}+\frac{3b^2\ln\left(ax^2+b\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b/x^2)^3,x)
```

```
[Out] ((7*b^4)/(4*a) + 2*b^3*x^2)/(a^4*b^2 + a^6*x^4 + 2*a^5*b*x^2) + x^4/(4*a^3)
 - (3*b*x^2)/(2*a^4) + (3*b^2*log(b + a*x^2))/a^5
```

sympy [A] time = 0.44, size = 78, normalized size = 1.05

$$\frac{8ab^3x^2+7b^4}{4a^7x^4+8a^6bx^2+4a^5b^2}+\frac{x^4}{4a^3}-\frac{3bx^2}{2a^4}+\frac{3b^2\log\left(ax^2+b\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b/x**2)**3,x)
```

```
[Out] (8*a*b**3*x**2 + 7*b**4)/(4*a**7*x**4 + 8*a**6*b*x**2 + 4*a**5*b**2) + x**4
/(4*a**3) - 3*b*x**2/(2*a**4) + 3*b**2*log(a*x**2 + b)/a**5
```

3.1877 $\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^3}\,dx$

Optimal. Leaf size=85

$$\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8a^{9/2}}-\frac{35bx}{8a^4}+\frac{35x^3}{24a^3}-\frac{7x^5}{8a^2\left(ax^2+b\right)}-\frac{x^7}{4a\left(ax^2+b\right)^2}$$

```
[Out] -35/8*b*x/a^4+35/24*x^3/a^3-1/4*x^7/a/(a*x^2+b)^2-7/8*x^5/a^2/(a*x^2+b)+35/
8*b^(3/2)*arctan(x*a^(1/2)/b^(1/2))/a^(9/2)
```

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {263, 288, 302, 205}

$$\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8a^{9/2}}-\frac{7x^5}{8a^2\left(ax^2+b\right)}-\frac{35bx}{8a^4}+\frac{35x^3}{24a^3}-\frac{x^7}{4a\left(ax^2+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b/x^2)^3,x]
```

```
[Out] (-35*b*x)/(8*a^4) + (35*x^3)/(24*a^3) - x^7/(4*a*(b + a*x^2)^2) - (7*x^5)/(
8*a^2*(b + a*x^2)) + (35*b^(3/2)*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(8*a^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^3}\,dx = \int \frac{x^8}{\left(b+ax^2\right)^3}\,dx$$

$$= -\frac{x^7}{4a\left(b+ax^2\right)^2} + \frac{7\int \frac{x^6}{\left(b+ax^2\right)^2}\,dx}{4a}$$

$$= -\frac{x^7}{4a\left(b+ax^2\right)^2} - \frac{7x^5}{8a^2\left(b+ax^2\right)} + \frac{35\int \frac{x^4}{b+ax^2}\,dx}{8a^2}$$

$$= -\frac{x^7}{4a\left(b+ax^2\right)^2} - \frac{7x^5}{8a^2\left(b+ax^2\right)} + \frac{35\int \left(-\frac{b}{a^2}+\frac{x^2}{a}+\frac{b^2}{a^2\left(b+ax^2\right)}\right)dx}{8a^2}$$

$$= -\frac{35bx}{8a^4} + \frac{35x^3}{24a^3} - \frac{x^7}{4a\left(b+ax^2\right)^2} - \frac{7x^5}{8a^2\left(b+ax^2\right)} + \frac{\left(35b^2\right)\int \frac{1}{b+ax^2}\,dx}{8a^4}$$

$$= -\frac{35bx}{8a^4} + \frac{35x^3}{24a^3} - \frac{x^7}{4a\left(b+ax^2\right)^2} - \frac{7x^5}{8a^2\left(b+ax^2\right)} + \frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8a^{9/2}}$$

Mathematica [A] time = 0.05, size = 77, normalized size = 0.91

$$\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8a^{9/2}} - \frac{-8a^3x^7+56a^2bx^5+175ab^2x^3+105b^3x}{24a^4\left(ax^2+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b/x^2)^3,x]
```

```
[Out] -1/24*(105*b^3*x + 175*a*b^2*x^3 + 56*a^2*b*x^5 - 8*a^3*x^7)/(a^4*(b + a*x^
2)^2) + (35*b^(3/2)*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(8*a^(9/2))
```

fricas [A] time = 0.83, size = 230, normalized size = 2.71

$$\left[\frac{16\,a^3x^7-112\,a^2bx^5-350\,ab^2x^3-210\,b^3x+105\left(a^2bx^4+2\,ab^2x^2+b^3\right)\sqrt{-\frac{b}{a}}\,\log\left(\frac{ax^2+2\,ax\sqrt{-\frac{b}{a}}-b}{ax^2+b}\right)}{48\left(a^6x^4+2\,a^5bx^2+a^4b^2\right)}, \frac{8\,a^3x^7-56}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*a^3*x^7 - 112*a^2*b*x^5 - 350*a*b^2*x^3 - 210*b^3*x + 105*(a^2*b*
x^4 + 2*a*b^2*x^2 + b^3)*sqrt(-b/a)*log((a*x^2 + 2*a*x*sqrt(-b/a) - b)/(a*x
^2 + b)))/(a^6*x^4 + 2*a^5*b*x^2 + a^4*b^2), 1/24*(8*a^3*x^7 - 56*a^2*b*x^5
 - 175*a*b^2*x^3 - 105*b^3*x + 105*(a^2*b*x^4 + 2*a*b^2*x^2 + b^3)*sqrt(b/a
)*arctan(a*x*sqrt(b/a)/b))/(a^6*x^4 + 2*a^5*b*x^2 + a^4*b^2)]
```

giac [A] time = 0.16, size = 73, normalized size = 0.86

$$\frac{35\,b^2\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\sqrt{ab}\,a^4} - \frac{13\,ab^2x^3+11\,b^3x}{8\left(ax^2+b\right)^2a^4} + \frac{a^6x^3-9\,a^5bx}{3\,a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^2)^3,x, algorithm="giac")
```

```
[Out] 35/8*b^2*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^4) - 1/8*(13*a*b^2*x^3 + 11*b^3
*x)/((a*x^2 + b)^2*a^4) + 1/3*(a^6*x^3 - 9*a^5*b*x)/a^9
```

maple [A] time = 0.01, size = 77, normalized size = 0.91

$$-\frac{13b^2x^3}{8\left(a\,x^2+b\right)^2a^3}+\frac{x^3}{3a^3}-\frac{11b^3x}{8\left(a\,x^2+b\right)^2a^4}+\frac{35b^2\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\sqrt{ab}\;a^4}-\frac{3bx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b/x^2)^3,x)
```

```
[Out] 1/3/a^3*x^3-3/a^4*b*x-13/8/a^3*b^2/(a*x^2+b)^2*x^3-11/8/a^4*b^3/(a*x^2+b)^2
*x+35/8/a^4*b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*a*x)
```

maxima [A] time = 1.76, size = 82, normalized size = 0.96

$$-\frac{13\,ab^2x^3+11\,b^3x}{8\left(a^6x^4+2\,a^5bx^2+a^4b^2\right)}+\frac{35\,b^2\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^4}+\frac{ax^3-9\,bx}{3\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(13*a*b^2*x^3 + 11*b^3*x)/(a^6*x^4 + 2*a^5*b*x^2 + a^4*b^2) + 35/8*b^2
*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^4) + 1/3*(a*x^3 - 9*b*x)/a^4
```

mupad [B] time = 1.14, size = 77, normalized size = 0.91

$$\frac{x^3}{3\,a^3}-\frac{\frac{11\,b^3\,x}{8}+\frac{13\,a\,b^2\,x^3}{8}}{a^6\,x^4+2\,a^5\,b\,x^2+a^4\,b^2}+\frac{35\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8\,a^{9/2}}-\frac{3\,b\,x}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b/x^2)^3,x)
```

```
[Out] x^3/(3*a^3) - ((11*b^3*x)/8 + (13*a*b^2*x^3)/8)/(a^4*b^2 + a^6*x^4 + 2*a^5*
b*x^2) + (35*b^(3/2)*atan((a^(1/2)*x)/b^(1/2)))/(8*a^(9/2)) - (3*b*x)/a^4
```

sympy [A] time = 0.48, size = 133, normalized size = 1.56

$$-\frac{35\sqrt{-\frac{b^3}{a^9}}\,\log\left(-\frac{a^4\sqrt{-\frac{b^3}{a^9}}}{b}+x\right)}{16}+\frac{35\sqrt{-\frac{b^3}{a^9}}\,\log\left(\frac{a^4\sqrt{-\frac{b^3}{a^9}}}{b}+x\right)}{16}+\frac{-13ab^2x^3-11b^3x}{8a^6x^4+16a^5bx^2+8a^4b^2}+\frac{x^3}{3a^3}-\frac{3bx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b/x**2)**3,x)
```

```
[Out] -35*sqrt(-b**3/a**9)*log(-a**4*sqrt(-b**3/a**9)/b + x)/16 + 35*sqrt(-b**3/a
**9)*log(a**4*sqrt(-b**3/a**9)/b + x)/16 + (-13*a*b**2*x**3 - 11*b**3*x)/(8
*a**6*x**4 + 16*a**5*b*x**2 + 8*a**4*b**2) + x**3/(3*a**3) - 3*b*x/a**4
```

3.1878 $\int \frac{x}{\left(a+\frac{b}{x^2}\right)^3}\,dx$

Optimal. Leaf size=65

$$\frac{b^3}{4a^4\left(ax^2+b\right)^2}-\frac{3b^2}{2a^4\left(ax^2+b\right)}-\frac{3b\log\left(ax^2+b\right)}{2a^4}+\frac{x^2}{2a^3}$$

```
[Out] 1/2*x^2/a^3+1/4*b^3/a^4/(a*x^2+b)^2-3/2*b^2/a^4/(a*x^2+b)-3/2*b*ln(a*x^2+b)
/a^4
```

Rubi [A] time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {263, 266, 43}

$$\frac{b^3}{4a^4\left(ax^2+b\right)^2}-\frac{3b^2}{2a^4\left(ax^2+b\right)}-\frac{3b\log\left(ax^2+b\right)}{2a^4}+\frac{x^2}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b/x^2)^3,x]
```

```
[Out] x^2/(2*a^3) + b^3/(4*a^4*(b + a*x^2)^2) - (3*b^2)/(2*a^4*(b + a*x^2)) - (3*
b*Log[b + a*x^2])/(2*a^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(a+\frac{b}{x^2}\right)^3}\,dx &= \int \frac{x^7}{\left(b+ax^2\right)^3}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{(b+ax)^3}\,dx, x, x^2\right) \\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{a^3}-\frac{b^3}{a^3(b+ax)^3}+\frac{3b^2}{a^3(b+ax)^2}-\frac{3b}{a^3(b+ax)}\right)dx, x, x^2\right) \\
&= \frac{x^2}{2a^3}+\frac{b^3}{4a^4\left(b+ax^2\right)^2}-\frac{3b^2}{2a^4\left(b+ax^2\right)}-\frac{3b\log\left(b+ax^2\right)}{2a^4}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 48, normalized size = 0.74

$$-\frac{\frac{b^2\left(6ax^2+5b\right)}{\left(ax^2+b\right)^2}+6b\log\left(ax^2+b\right)-2ax^2}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b/x^2)^3,x]
```

```
[Out] -1/4*(-2*a*x^2 + (b^2*(5*b + 6*a*x^2))/(b + a*x^2)^2 + 6*b*Log[b + a*x^2])/
a^4
```

fricas [A] time = 0.99, size = 91, normalized size = 1.40

$$\frac{2\,a^3x^6+4\,a^2bx^4-4\,ab^2x^2-5\,b^3-6\left(a^2bx^4+2\,ab^2x^2+b^3\right)\log\left(ax^2+b\right)}{4\left(a^6x^4+2\,a^5bx^2+a^4b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^3*x^6 + 4*a^2*b*x^4 - 4*a*b^2*x^2 - 5*b^3 - 6*(a^2*b*x^4 + 2*a*b^2
*x^2 + b^3)*log(a*x^2 + b))/(a^6*x^4 + 2*a^5*b*x^2 + a^4*b^2)
```

giac [A] time = 0.16, size = 53, normalized size = 0.82

$$\frac{x^2}{2\,a^3}-\frac{3\,b\log\left(\left|ax^2+b\right|\right)}{2\,a^4}-\frac{6\,ab^2x^2+5\,b^3}{4\left(ax^2+b\right)^2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*x^2/a^3 - 3/2*b*log(abs(a*x^2 + b))/a^4 - 1/4*(6*a*b^2*x^2 + 5*b^3)/((a
*x^2 + b)^2*a^4)
```

maple [A] time = 0.01, size = 58, normalized size = 0.89

$$\frac{x^2}{2a^3}+\frac{b^3}{4\left(a\,x^2+b\right)^2a^4}-\frac{3b^2}{2\left(a\,x^2+b\right)a^4}-\frac{3b\ln\left(a\,x^2+b\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b/x^2)^3,x)
```

```
[Out] 1/2/a^3*x^2+1/4*b^3/a^4/(a*x^2+b)^2-3/2*b^2/a^4/(a*x^2+b)-3/2*b*ln(a*x^2+b)
/a^4
```

maxima [A] time = 0.86, size = 66, normalized size = 1.02

$$-\frac{6\,ab^2x^2+5\,b^3}{4\left(a^6x^4+2\,a^5bx^2+a^4b^2\right)}+\frac{x^2}{2\,a^3}-\frac{3\,b\log\left(ax^2+b\right)}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(6*a*b^2*x^2 + 5*b^3)/(a^6*x^4 + 2*a^5*b*x^2 + a^4*b^2) + 1/2*x^2/a^3
- 3/2*b*log(a*x^2 + b)/a^4
```

mupad [**B**] time = 1.11, size = 68, normalized size = 1.05

$$\frac{x^2}{2\,a^3}-\frac{\frac{5\,b^3}{4\,a}+\frac{3\,b^2\,x^2}{2}}{a^5\,x^4+2\,a^4\,b\,x^2+a^3\,b^2}-\frac{3\,b\,\ln\left(a\,x^2+b\right)}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b/x^2)^3,x)
```

```
[Out] x^2/(2*a^3) - ((5*b^3)/(4*a) + (3*b^2*x^2)/2)/(a^3*b^2 + a^5*x^4 + 2*a^4*b*
x^2) - (3*b*log(b + a*x^2))/(2*a^4)
```

sympy [**A**] time = 0.40, size = 68, normalized size = 1.05

$$\frac{-6ab^2x^2-5b^3}{4a^6x^4+8a^5bx^2+4a^4b^2}+\frac{x^2}{2a^3}-\frac{3b\log\left(ax^2+b\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x**2)**3,x)
```

```
[Out] (-6*a*b**2*x**2 - 5*b**3)/(4*a**6*x**4 + 8*a**5*b*x**2 + 4*a**4*b**2) + x**
2/(2*a**3) - 3*b*log(a*x**2 + b)/(2*a**4)
```

3.1879 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3}\,dx$

Optimal. Leaf size=74

$$-\frac{15\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8a^{7/2}}+\frac{15x}{8a^3}-\frac{5x^3}{8a^2\left(ax^2+b\right)}-\frac{x^5}{4a\left(ax^2+b\right)^2}$$

```
[Out] 15/8*x/a^3-1/4*x^5/a/(a*x^2+b)^2-5/8*x^3/a^2/(a*x^2+b)-15/8*arctan(x*a^(1/2
)/b^(1/2))*b^(1/2)/a^(7/2)
```

Rubi [A] time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {193, 288, 321, 205}

$$-\frac{5x^3}{8a^2\left(ax^2+b\right)}-\frac{15\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8a^{7/2}}+\frac{15x}{8a^3}-\frac{x^5}{4a\left(ax^2+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^(-3),x]
```

```
[Out] (15*x)/(8*a^3) - x^5/(4*a*(b + a*x^2)^2) - (5*x^3)/(8*a^2*(b + a*x^2)) - (1
5*Sqrt[b]*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(8*a^(7/2))
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3}\,dx = \int \frac{x^6}{\left(b+ax^2\right)^3}\,dx$$

$$= -\frac{x^5}{4a\left(b+ax^2\right)^2} + \frac{5\int \frac{x^4}{\left(b+ax^2\right)^2}\,dx}{4a}$$

$$= -\frac{x^5}{4a\left(b+ax^2\right)^2} - \frac{5x^3}{8a^2\left(b+ax^2\right)} + \frac{15\int \frac{x^2}{b+ax^2}\,dx}{8a^2}$$

$$= \frac{15x}{8a^3} - \frac{x^5}{4a\left(b+ax^2\right)^2} - \frac{5x^3}{8a^2\left(b+ax^2\right)} - \frac{(15b)\int \frac{1}{b+ax^2}\,dx}{8a^3}$$

$$= \frac{15x}{8a^3} - \frac{x^5}{4a\left(b+ax^2\right)^2} - \frac{5x^3}{8a^2\left(b+ax^2\right)} - \frac{15\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8a^{7/2}}$$

Mathematica [A] time = 0.05, size = 66, normalized size = 0.89

$$\frac{8a^2x^5+25abx^3+15b^2x}{8a^3\left(ax^2+b\right)^2} - \frac{15\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^(-3),x]
```

```
[Out] (15*b^2*x + 25*a*b*x^3 + 8*a^2*x^5)/(8*a^3*(b + a*x^2)^2) - (15*Sqrt[b]*Arc
Tan[(Sqrt[a]*x)/Sqrt[b]])/(8*a^(7/2))
```

fricas [A] time = 0.83, size = 202, normalized size = 2.73

$$\left[\frac{16\,a^2x^5+50\,abx^3+30\,b^2x+15\left(a^2x^4+2\,abx^2+b^2\right)\sqrt{-\frac{b}{a}}\,\log\left(\frac{ax^2-2\,ax\sqrt{-\frac{b}{a}}-b}{ax^2+b}\right)}{16\left(a^5x^4+2\,a^4bx^2+a^3b^2\right)}, \frac{8\,a^2x^5+25\,abx^3+15\,b^2x-1}{8\left(a^5x^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*a^2*x^5 + 50*a*b*x^3 + 30*b^2*x + 15*(a^2*x^4 + 2*a*b*x^2 + b^2)*
sqrt(-b/a)*log((a*x^2 - 2*a*x*sqrt(-b/a) - b)/(a*x^2 + b)))/(a^5*x^4 + 2*a^
4*b*x^2 + a^3*b^2), 1/8*(8*a^2*x^5 + 25*a*b*x^3 + 15*b^2*x - 15*(a^2*x^4 +
2*a*b*x^2 + b^2)*sqrt(b/a)*arctan(a*x*sqrt(b/a)/b))/(a^5*x^4 + 2*a^4*b*x^2
+ a^3*b^2)]
```

giac [A] time = 0.15, size = 54, normalized size = 0.73

$$-\frac{15\,b\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\sqrt{ab}\,a^3} + \frac{x}{a^3} + \frac{9\,abx^3+7\,b^2x}{8\left(ax^2+b\right)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^3,x, algorithm="giac")
```

```
[Out] -15/8*b*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^3) + x/a^3 + 1/8*(9*a*b*x^3 + 7*
b^2*x)/((a*x^2 + b)^2*a^3)
```

maple [A] time = 0.01, size = 63, normalized size = 0.85

$$\frac{9b\,x^3}{8\left(a\,x^2+b\right)^2a^2}+\frac{7b^2x}{8\left(a\,x^2+b\right)^2a^3}-\frac{15b\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\sqrt{ab}\;a^3}+\frac{x}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^3,x)
```

```
[Out] 1/a^3*x+9/8/a^2*b/(a*x^2+b)^2*x^3+7/8/a^3*b^2/(a*x^2+b)^2*x-15/8/a^3*b/(a*b
)^(1/2)*arctan(1/(a*b)^(1/2)*a*x)
```

maxima [A] time = 1.97, size = 68, normalized size = 0.92

$$\frac{9\,abx^3+7\,b^2x}{8\left(a^5x^4+2\,a^4bx^2+a^3b^2\right)}-\frac{15\,b\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^3}+\frac{x}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(9*a*b*x^3 + 7*b^2*x)/(a^5*x^4 + 2*a^4*b*x^2 + a^3*b^2) - 15/8*b*arctan
(a*x/sqrt(a*b))/(sqrt(a*b)*a^3) + x/a^3
```

mupad [B] time = 1.36, size = 64, normalized size = 0.86

$$\frac{\frac{7\,b^2\,x}{8}+\frac{9\,a\,b\,x^3}{8}}{a^5\,x^4+2\,a^4\,b\,x^2+a^3\,b^2}+\frac{x}{a^3}-\frac{15\,\sqrt{b}\,\operatorname{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x^2)^3,x)
```

```
[Out] ((7*b^2*x)/8 + (9*a*b*x^3)/8)/(a^3*b^2 + a^5*x^4 + 2*a^4*b*x^2) + x/a^3 - (
15*b^(1/2)*atan((a^(1/2)*x)/b^(1/2)))/(8*a^(7/2))
```

sympy [A] time = 0.43, size = 107, normalized size = 1.45

$$\frac{15\sqrt{-\frac{b}{a^7}}\,\log\left(-a^3\sqrt{-\frac{b}{a^7}}+x\right)}{16}-\frac{15\sqrt{-\frac{b}{a^7}}\,\log\left(a^3\sqrt{-\frac{b}{a^7}}+x\right)}{16}+\frac{9abx^3+7b^2x}{8a^5x^4+16a^4bx^2+8a^3b^2}+\frac{x}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**3,x)
```

```
[Out] 15*sqrt(-b/a**7)*log(-a**3*sqrt(-b/a**7) + x)/16 - 15*sqrt(-b/a**7)*log(a**
3*sqrt(-b/a**7) + x)/16 + (9*a*b*x**3 + 7*b**2*x)/(8*a**5*x**4 + 16*a**4*b*
x**2 + 8*a**3*b**2) + x/a**3
```

3.1880 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x}\,dx$

Optimal. Leaf size=49

$$-\frac{b^2}{4a^3\left(ax^2+b\right)^2}+\frac{b}{a^3\left(ax^2+b\right)}+\frac{\log\left(ax^2+b\right)}{2a^3}$$

```
[Out] -1/4*b^2/a^3/(a*x^2+b)^2+b/a^3/(a*x^2+b)+1/2*ln(a*x^2+b)/a^3
```

Rubi [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 266, 43}

$$-\frac{b^2}{4a^3\left(ax^2+b\right)^2}+\frac{b}{a^3\left(ax^2+b\right)}+\frac{\log\left(ax^2+b\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)^3*x),x]
```

```
[Out] -b^2/(4*a^3*(b + a*x^2)^2) + b/(a^3*(b + a*x^2)) + Log[b + a*x^2]/(2*a^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x}\,dx &= \int \frac{x^5}{\left(b+ax^2\right)^3}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{(b+ax)^3}\,dx, x, x^2\right) \\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{b^2}{a^2(b+ax)^3}-\frac{2b}{a^2(b+ax)^2}+\frac{1}{a^2(b+ax)}\right)dx, x, x^2\right) \\
&= -\frac{b^2}{4a^3\left(b+ax^2\right)^2}+\frac{b}{a^3\left(b+ax^2\right)}+\frac{\log\left(b+ax^2\right)}{2a^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.80

$$\frac{\frac{b\left(4ax^2+3b\right)}{\left(ax^2+b\right)^2}+2\log\left(ax^2+b\right)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)^3*x),x]
```

```
[Out] ((b*(3*b + 4*a*x^2))/(b + a*x^2)^2 + 2*Log[b + a*x^2])/(4*a^3)
```

fricas [A] time = 1.27, size = 69, normalized size = 1.41

$$\frac{4\,abx^2+3\,b^2+2\left(a^2x^4+2\,abx^2+b^2\right)\log\left(ax^2+b\right)}{4\left(a^5x^4+2\,a^4bx^2+a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^3/x,x, algorithm="fricas")
```

```
[Out] 1/4*(4*a*b*x^2 + 3*b^2 + 2*(a^2*x^4 + 2*a*b*x^2 + b^2)*log(a*x^2 + b))/(a^5
*x^4 + 2*a^4*b*x^2 + a^3*b^2)
```

giac [A] time = 0.15, size = 42, normalized size = 0.86

$$\frac{\log\left(\left|ax^2+b\right|\right)}{2\,a^3}-\frac{3\,ax^4+2\,bx^2}{4\left(ax^2+b\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^3/x,x, algorithm="giac")
```

```
[Out] 1/2*log(abs(a*x^2 + b))/a^3 - 1/4*(3*a*x^4 + 2*b*x^2)/((a*x^2 + b)^2*a^2)
```

maple [A] time = 0.01, size = 46, normalized size = 0.94

$$-\frac{b^2}{4\left(a\,x^2+b\right)^2a^3}+\frac{b}{\left(a\,x^2+b\right)a^3}+\frac{\ln\left(a\,x^2+b\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^3/x,x)
```

```
[Out] -1/4*b^2/a^3/(a*x^2+b)^2+b/a^3/(a*x^2+b)+1/2*ln(a*x^2+b)/a^3
```

maxima [A] time = 0.86, size = 55, normalized size = 1.12

$$\frac{4\,abx^2+3\,b^2}{4\left(a^5x^4+2\,a^4bx^2+a^3b^2\right)}+\frac{\log\left(ax^2+b\right)}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^3/x,x, algorithm="maxima")
```

```
[Out] 1/4*(4*a*b*x^2 + 3*b^2)/(a^5*x^4 + 2*a^4*b*x^2 + a^3*b^2) + 1/2*log(a*x^2 +
 b)/a^3
```

mupad [B] time = 1.11, size = 52, normalized size = 1.06

$$\frac{\frac{3\,b^2}{4\,a^3}+\frac{b\,x^2}{a^2}}{a^2\,x^4+2\,a\,b\,x^2+b^2}+\frac{\ln\left(a\,x^2+b\right)}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b/x^2)^3),x)
```

```
[Out] ((3*b^2)/(4*a^3) + (b*x^2)/a^2)/(b^2 + a^2*x^4 + 2*a*b*x^2) + log(b + a*x^2
)/(2*a^3)
```

sympy [**A**] time = 0.35, size = 53, normalized size = 1.08

$$\frac{4abx^2 + 3b^2}{4a^5x^4 + 8a^4bx^2 + 4a^3b^2} + \frac{\log\left(ax^2 + b\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**3/x,x)
```

```
[Out] (4*a*b*x**2 + 3*b**2)/(4*a**5*x**4 + 8*a**4*b*x**2 + 4*a**3*b**2) + log(a*x
**2 + b)/(2*a**3)
```

3.1881 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^2}\,dx$

Optimal. Leaf size=64

$$\frac{3\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8a^{5/2}\sqrt{b}} - \frac{3x}{8a^2\left(ax^2+b\right)} - \frac{x^3}{4a\left(ax^2+b\right)^2}$$

```
[Out] -1/4*x^3/a/(a*x^2+b)^2-3/8*x/a^2/(a*x^2+b)+3/8*arctan(x*a^(1/2)/b^(1/2))/a^
(5/2)/b^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 288, 205}

$$-\frac{3x}{8a^2\left(ax^2+b\right)} + \frac{3\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8a^{5/2}\sqrt{b}} - \frac{x^3}{4a\left(ax^2+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)^3*x^2),x]
```

```
[Out] -x^3/(4*a*(b + a*x^2)^2) - (3*x)/(8*a^2*(b + a*x^2)) + (3*ArcTan[(Sqrt[a]*x
)/Sqrt[b]])/(8*a^(5/2)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^2}\,dx = \int \frac{x^4}{\left(b+ax^2\right)^3}\,dx$$

$$= -\frac{x^3}{4a\left(b+ax^2\right)^2} + \frac{3\int \frac{x^2}{\left(b+ax^2\right)^2}\,dx}{4a}$$

$$= -\frac{x^3}{4a\left(b+ax^2\right)^2} - \frac{3x}{8a^2\left(b+ax^2\right)} + \frac{3\int \frac{1}{b+ax^2}\,dx}{8a^2}$$

$$= -\frac{x^3}{4a\left(b+ax^2\right)^2} - \frac{3x}{8a^2\left(b+ax^2\right)} + \frac{3\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8a^{5/2}\sqrt{b}}$$

Mathematica [A] time = 0.04, size = 55, normalized size = 0.86

$$\frac{3\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8a^{5/2}\sqrt{b}} - \frac{5ax^3+3bx}{8a^2\left(ax^2+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)^3*x^2),x]
```

```
[Out] -1/8*(3*b*x + 5*a*x^3)/(a^2*(b + a*x^2)^2) + (3*ArcTan[(Sqrt[a]*x)/Sqrt[b]]
)/(8*a^(5/2)*Sqrt[b])
```

fricas [A] time = 0.75, size = 188, normalized size = 2.94

$$\left[-\frac{10\,a^2bx^3+6\,ab^2x+3\left(a^2x^4+2\,abx^2+b^2\right)\sqrt{-ab}\,\log\left(\frac{ax^2-2\sqrt{-ab}\,x-b}{ax^2+b}\right)}{16\left(a^5bx^4+2\,a^4b^2x^2+a^3b^3\right)}, -\frac{5\,a^2bx^3+3\,ab^2x-3\left(a^2x^4+2\,abx^2+\right.}{8\left(a^5bx^4+2\,a^4b^2x^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^3/x^2,x, algorithm="fricas")
```

```
[Out] [-1/16*(10*a^2*b*x^3 + 6*a*b^2*x + 3*(a^2*x^4 + 2*a*b*x^2 + b^2)*sqrt(-a*b)
*log((a*x^2 - 2*sqrt(-a*b)*x - b)/(a*x^2 + b)))/(a^5*b*x^4 + 2*a^4*b^2*x^2
+ a^3*b^3), -1/8*(5*a^2*b*x^3 + 3*a*b^2*x - 3*(a^2*x^4 + 2*a*b*x^2 + b^2)*s
qrt(a*b)*arctan(sqrt(a*b)*x/b))/(a^5*b*x^4 + 2*a^4*b^2*x^2 + a^3*b^3)]
```

giac [A] time = 0.17, size = 45, normalized size = 0.70

$$\frac{3\,\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\sqrt{ab}\,a^2} - \frac{5\,ax^3+3\,bx}{8\left(ax^2+b\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^3/x^2,x, algorithm="giac")
```

```
[Out] 3/8*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/8*(5*a*x^3 + 3*b*x)/((a*x^2 +
 b)^2*a^2)
```

maple [A] time = 0.01, size = 47, normalized size = 0.73

$$\frac{3\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\sqrt{ab}\,a^2}+\frac{-\frac{5x^3}{8a}-\frac{3bx}{8a^2}}{\left(a\,x^2+b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^3/x^2,x)
```

```
[Out] (-5/8/a*x^3-3/8/a^2*b*x)/(a*x^2+b)^2+3/8/a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/
2)*a*x)
```

maxima [A] time = 1.92, size = 59, normalized size = 0.92

$$-\frac{5\,ax^3+3\,bx}{8\left(a^4x^4+2\,a^3bx^2+a^2b^2\right)}+\frac{3\,\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^3/x^2,x, algorithm="maxima")
```

```
[Out] -1/8*(5*a*x^3 + 3*b*x)/(a^4*x^4 + 2*a^3*b*x^2 + a^2*b^2) + 3/8*arctan(a*x/s
qrt(a*b))/(sqrt(a*b)*a^2)
```

mupad [B] time = 1.13, size = 56, normalized size = 0.88

$$\frac{3\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8\,a^{5/2}\,\sqrt{b}}-\frac{\frac{5\,x^3}{8\,a}+\frac{3\,b\,x}{8\,a^2}}{a^2\,x^4+2\,a\,b\,x^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b/x^2)^3),x)
```

```
[Out] (3*atan((a^(1/2)*x)/b^(1/2)))/(8*a^(5/2)*b^(1/2)) - ((5*x^3)/(8*a) + (3*b*x
)/(8*a^2))/(b^2 + a^2*x^4 + 2*a*b*x^2)
```

sympy [A] time = 0.37, size = 110, normalized size = 1.72

$$-\frac{3\sqrt{-\frac{1}{a^5b}}\,\log\left(-a^2b\sqrt{-\frac{1}{a^5b}}+x\right)}{16}+\frac{3\sqrt{-\frac{1}{a^5b}}\,\log\left(a^2b\sqrt{-\frac{1}{a^5b}}+x\right)}{16}+\frac{-5ax^3-3bx}{8a^4x^4+16a^3bx^2+8a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**3/x**2,x)
```

```
[Out] -3*sqrt(-1/(a**5*b))*log(-a**2*b*sqrt(-1/(a**5*b)) + x)/16 + 3*sqrt(-1/(a**
5*b))*log(a**2*b*sqrt(-1/(a**5*b)) + x)/16 + (-5*a*x**3 - 3*b*x)/(8*a**4*x**
*4 + 16*a**3*b*x**2 + 8*a**2*b**2)
```

3.1882 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^3}\,dx$

Optimal. Leaf size=16

$$\frac{1}{4b\left(a+\frac{b}{x^2}\right)^2}$$

```
[Out] 1/4/b/(a+b/x^2)^2
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$\frac{1}{4b\left(a+\frac{b}{x^2}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)^3*x^3),x]
```

```
[Out] 1/(4*b*(a + b/x^2)^2)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^3}\,dx = \frac{1}{4b\left(a+\frac{b}{x^2}\right)^2}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.50

$$-\frac{2ax^2+b}{4a^2\left(ax^2+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)^3*x^3),x]
```

```
[Out] -1/4*(b + 2*a*x^2)/(a^2*(b + a*x^2)^2)
```

fricas [B] time = 0.92, size = 36, normalized size = 2.25

$$-\frac{2\,ax^2+b}{4\left(a^4x^4+2\,a^3bx^2+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^3/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*x^2 + b)/(a^4*x^4 + 2*a^3*b*x^2 + a^2*b^2)
```

giac [A] time = 0.16, size = 22, normalized size = 1.38

$$-\frac{2\,ax^2+b}{4\left(ax^2+b\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^3/x^3,x, algorithm="giac")
[Out] -1/4*(2*a*x^2 + b)/((a*x^2 + b)^2*a^2)
```

maple [B] time = 0.01, size = 31, normalized size = 1.94

$$\frac{b}{4\left(a\,x^2+b\right)^2a^2}-\frac{1}{2\left(a\,x^2+b\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^3/x^3,x)
[Out] 1/4/a^2*b/(a*x^2+b)^2-1/2/a^2/(a*x^2+b)
```

maxima [A] time = 0.87, size = 14, normalized size = 0.88

$$\frac{1}{4\left(a+\frac{b}{x^2}\right)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^3/x^3,x, algorithm="maxima")
[Out] 1/4/((a + b/x^2)^2*b)
```

mupad [B] time = 1.08, size = 37, normalized size = 2.31

$$-\frac{\frac{b}{4a^2}+\frac{x^2}{2a}}{a^2x^4+2\,a\,b\,x^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b/x^2)^3),x)
[Out] -(b/(4*a^2) + x^2/(2*a))/(b^2 + a^2*x^4 + 2*a*b*x^2)
```

sympy [B] time = 0.29, size = 36, normalized size = 2.25

$$\frac{-2ax^2-b}{4a^4x^4+8a^3bx^2+4a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**3/x**3,x)
[Out] (-2*a*x**2 - b)/(4*a**4*x**4 + 8*a**3*b*x**2 + 4*a**2*b**2)
```

3.1883 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^4}\,dx$

Optimal. Leaf size=65

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2}}+\frac{x}{8ab\left(ax^2+b\right)}-\frac{x}{4a\left(ax^2+b\right)^2}$$

```
[Out] -1/4*x/a/(a*x^2+b)^2+1/8*x/a/b/(a*x^2+b)+1/8*arctan(x*a^(1/2)/b^(1/2))/a^(3
/2)/b^(3/2)
```

Rubi [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {263, 288, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2}}+\frac{x}{8ab\left(ax^2+b\right)}-\frac{x}{4a\left(ax^2+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)^3*x^4),x]
```

```
[Out] -x/(4*a*(b + a*x^2)^2) + x/(8*a*b*(b + a*x^2)) + ArcTan[(Sqrt[a]*x)/Sqrt[b]
]/(8*a^(3/2)*b^(3/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^4}\,dx = \int \frac{x^2}{\left(b+ax^2\right)^3}\,dx$$

$$= -\frac{x}{4a\left(b+ax^2\right)^2} + \frac{\int \frac{1}{\left(b+ax^2\right)^2}\,dx}{4a}$$

$$= -\frac{x}{4a\left(b+ax^2\right)^2} + \frac{x}{8ab\left(b+ax^2\right)} + \frac{\int \frac{1}{b+ax^2}\,dx}{8ab}$$

$$= -\frac{x}{4a\left(b+ax^2\right)^2} + \frac{x}{8ab\left(b+ax^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2}}$$

Mathematica [A] time = 0.03, size = 58, normalized size = 0.89

$$\frac{\frac{\sqrt{a}\,\sqrt{b}\,x\left(ax^2-b\right)}{\left(ax^2+b\right)^2} + \tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)^3*x^4),x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*x*(-b + a*x^2))/(b + a*x^2)^2 + ArcTan[(Sqrt[a]*x)/Sqrt[b
]])/(8*a^(3/2)*b^(3/2))
```

fricas [A] time = 0.77, size = 190, normalized size = 2.92

$$\left[\frac{2\,a^2bx^3 - 2\,ab^2x - \left(a^2x^4 + 2\,abx^2 + b^2\right)\sqrt{-ab}\,\log\left(\frac{ax^2 - 2\,\sqrt{-ab}\,x - b}{ax^2+b}\right)}{16\left(a^4b^2x^4 + 2\,a^3b^3x^2 + a^2b^4\right)}, \frac{a^2bx^3 - ab^2x + \left(a^2x^4 + 2\,abx^2 + b^2\right)\sqrt{ab}\ \text{arct}}{8\left(a^4b^2x^4 + 2\,a^3b^3x^2 + a^2b^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^3/x^4,x, algorithm="fricas")
```

```
[Out] [1/16*(2*a^2*b*x^3 - 2*a*b^2*x - (a^2*x^4 + 2*a*b*x^2 + b^2)*sqrt(-a*b)*log
((a*x^2 - 2*sqrt(-a*b)*x - b)/(a*x^2 + b)))/(a^4*b^2*x^4 + 2*a^3*b^3*x^2 +
a^2*b^4), 1/8*(a^2*b*x^3 - a*b^2*x + (a^2*x^4 + 2*a*b*x^2 + b^2)*sqrt(a*b)*
arctan(sqrt(a*b)*x/b))/(a^4*b^2*x^4 + 2*a^3*b^3*x^2 + a^2*b^4)]
```

giac [A] time = 0.15, size = 50, normalized size = 0.77

$$\frac{\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\sqrt{ab}\,ab} + \frac{ax^3 - bx}{8\left(ax^2+b\right)^2 ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^3/x^4,x, algorithm="giac")
```

```
[Out] 1/8*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a*b) + 1/8*(a*x^3 - b*x)/((a*x^2 + b)^
2*a*b)
```

maple [A] time = 0.01, size = 49, normalized size = 0.75

$$\frac{\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\sqrt{ab}\,ab}+\frac{\frac{x^3}{8b}-\frac{x}{8a}}{\left(a\,x^2+b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^3/x^4,x)
```

```
[Out] (1/8/b*x^3-1/8/a*x)/(a*x^2+b)^2+1/8/b/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*a*
x)
```

maxima [A] time = 1.86, size = 62, normalized size = 0.95

$$\frac{ax^3-bx}{8\left(a^3bx^4+2\,a^2b^2x^2+ab^3\right)}+\frac{\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\sqrt{ab}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/8*(a*x^3 - b*x)/(a^3*b*x^4 + 2*a^2*b^2*x^2 + a*b^3) + 1/8*arctan(a*x/sqrt
(a*b))/(sqrt(a*b)*a*b)
```

mupad [B] time = 1.12, size = 55, normalized size = 0.85

$$\frac{\operatorname{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8\,a^{3/2}\,b^{3/2}}-\frac{\frac{x}{8\,a}-\frac{x^3}{8\,b}}{a^2\,x^4+2\,a\,b\,x^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b/x^2)^3),x)
```

```
[Out] atan((a^(1/2)*x)/b^(1/2))/(8*a^(3/2)*b^(3/2)) - (x/(8*a) - x^3/(8*b))/(b^2
+ a^2*x^4 + 2*a*b*x^2)
```

sympy [B] time = 0.33, size = 110, normalized size = 1.69

$$-\frac{\sqrt{-\frac{1}{a^3b^3}}\,\log\left(-ab^2\sqrt{-\frac{1}{a^3b^3}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^3b^3}}\,\log\left(ab^2\sqrt{-\frac{1}{a^3b^3}}+x\right)}{16}+\frac{ax^3-bx}{8a^3bx^4+16a^2b^2x^2+8ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**3/x**4,x)
```

```
[Out] -sqrt(-1/(a**3*b**3))*log(-a*b**2*sqrt(-1/(a**3*b**3)) + x)/16 + sqrt(-1/(a
**3*b**3))*log(a*b**2*sqrt(-1/(a**3*b**3)) + x)/16 + (a*x**3 - b*x)/(8*a**3
*b*x**4 + 16*a**2*b**2*x**2 + 8*a*b**3)
```

3.1884 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^5}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{4a\left(ax^2+b\right)^2}$$

```
[Out] -1/4/a/(a*x^2+b)^2
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 261}

$$-\frac{1}{4a\left(ax^2+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)^3*x^5),x]
```

```
[Out] -1/(4*a*(b + a*x^2)^2)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^5}\,dx &= \int \frac{x}{\left(b+ax^2\right)^3}\,dx \\ &= -\frac{1}{4a\left(b+ax^2\right)^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{4a\left(ax^2+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)^3*x^5),x]
```

```
[Out] -1/4*1/(a*(b + a*x^2)^2)
```

fricas [A] time = 1.07, size = 26, normalized size = 1.62

$$-\frac{1}{4\left(a^3x^4+2\,a^2bx^2+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^3/x^5,x, algorithm="fricas")
[Out] -1/4/(a^3*x^4 + 2*a^2*b*x^2 + a*b^2)
```

giac [A] time = 0.15, size = 14, normalized size = 0.88

$$-\frac{1}{4\left(ax^2+b\right)^2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^3/x^5,x, algorithm="giac")
[Out] -1/4/((a*x^2 + b)^2*a)
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$-\frac{1}{4\left(a\,x^2+b\right)^2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^3/x^5,x)
[Out] -1/4/a/(a*x^2+b)^2
```

maxima [A] time = 0.67, size = 26, normalized size = 1.62

$$-\frac{1}{4\left(a^3x^4+2\,a^2bx^2+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^3/x^5,x, algorithm="maxima")
[Out] -1/4/(a^3*x^4 + 2*a^2*b*x^2 + a*b^2)
```

mupad [B] time = 1.07, size = 28, normalized size = 1.75

$$-\frac{1}{4\,a^3\,x^4+8\,a^2\,b\,x^2+4\,a\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b/x^2)^3),x)
[Out] -1/(4*a*b^2 + 4*a^3*x^4 + 8*a^2*b*x^2)
```

sympy [A] time = 0.26, size = 27, normalized size = 1.69

$$-\frac{1}{4a^3x^4+8a^2bx^2+4ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**3/x**5,x)
[Out] -1/(4*a**3*x**4 + 8*a**2*b*x**2 + 4*a*b**2)
```

3.1885 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^6}\,dx$

Optimal. Leaf size=62

$$\frac{3\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8\sqrt{a}\,b^{5/2}}+\frac{3x}{8b^2\left(ax^2+b\right)}+\frac{x}{4b\left(ax^2+b\right)^2}$$

```
[Out] 1/4*x/b/(a*x^2+b)^2+3/8*x/b^2/(a*x^2+b)+3/8*arctan(x*a^(1/2)/b^(1/2))/b^(5/
2)/a^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 199, 205}

$$\frac{3x}{8b^2\left(ax^2+b\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8\sqrt{a}\,b^{5/2}}+\frac{x}{4b\left(ax^2+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)^3*x^6),x]
```

```
[Out] x/(4*b*(b + a*x^2)^2) + (3*x)/(8*b^2*(b + a*x^2)) + (3*ArcTan[(Sqrt[a]*x)/S
qrt[b]])/(8*Sqrt[a]*b^(5/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^6}\,dx = \int \frac{1}{\left(b+ax^2\right)^3}\,dx$$

$$= \frac{x}{4b\left(b+ax^2\right)^2} + \frac{3\int \frac{1}{\left(b+ax^2\right)^2}\,dx}{4b}$$

$$= \frac{x}{4b\left(b+ax^2\right)^2} + \frac{3x}{8b^2\left(b+ax^2\right)} + \frac{3\int \frac{1}{b+ax^2}\,dx}{8b^2}$$

$$= \frac{x}{4b\left(b+ax^2\right)^2} + \frac{3x}{8b^2\left(b+ax^2\right)} + \frac{3\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8\sqrt{a}\,b^{5/2}}$$

Mathematica [A] time = 0.05, size = 55, normalized size = 0.89

$$\frac{3\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8\sqrt{a}\,b^{5/2}} + \frac{3ax^3+5bx}{8b^2\left(ax^2+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)^3*x^6),x]
```

```
[Out] (5*b*x + 3*a*x^3)/(8*b^2*(b + a*x^2)^2) + (3*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(
8*Sqrt[a]*b^(5/2))
```

fricas [A] time = 0.65, size = 188, normalized size = 3.03

$$\left[\frac{6\,a^2bx^3+10\,ab^2x-3\left(a^2x^4+2\,abx^2+b^2\right)\sqrt{-ab}\,\log\left(\frac{ax^2-2\sqrt{-ab}\,x-b}{ax^2+b}\right)}{16\left(a^3b^3x^4+2\,a^2b^4x^2+ab^5\right)}, \frac{3\,a^2bx^3+5\,ab^2x+3\left(a^2x^4+2\,abx^2+b^{2}\right.}{8\left(a^3b^3x^4+2\,a^2b^4x^2+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^3/x^6,x, algorithm="fricas")
```

```
[Out] [1/16*(6*a^2*b*x^3 + 10*a*b^2*x - 3*(a^2*x^4 + 2*a*b*x^2 + b^2)*sqrt(-a*b)*
log((a*x^2 - 2*sqrt(-a*b)*x - b)/(a*x^2 + b)))/(a^3*b^3*x^4 + 2*a^2*b^4*x^2
 + a*b^5), 1/8*(3*a^2*b*x^3 + 5*a*b^2*x + 3*(a^2*x^4 + 2*a*b*x^2 + b^2)*sqr
t(a*b)*arctan(sqrt(a*b)*x/b))/(a^3*b^3*x^4 + 2*a^2*b^4*x^2 + a*b^5)]
```

giac [A] time = 0.22, size = 45, normalized size = 0.73

$$\frac{3\,\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\sqrt{ab}\,b^2} + \frac{3\,ax^3+5\,bx}{8\left(ax^2+b\right)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^3/x^6,x, algorithm="giac")
```

```
[Out] 3/8*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/8*(3*a*x^3 + 5*b*x)/((a*x^2 +
 b)^2*b^2)
```

maple [A] time = 0.00, size = 51, normalized size = 0.82

$$\frac{x}{4\left(a\,x^2+b\right)^2 b}+\frac{3x}{8\left(a\,x^2+b\right)b^2}+\frac{3\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\sqrt{ab}\;b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^3/x^6,x)
```

```
[Out] 1/4*x/b/(a*x^2+b)^2+3/8*x/b^2/(a*x^2+b)+3/8/b^2/(a*b)^(1/2)*arctan(1/(a*b)^
(1/2)*a*x)
```

maxima [A] time = 1.95, size = 58, normalized size = 0.94

$$\frac{3\,ax^3+5\,bx}{8\left(a^2b^2x^4+2\,ab^3x^2+b^4\right)}+\frac{3\,\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^3/x^6,x, algorithm="maxima")
```

```
[Out] 1/8*(3*a*x^3 + 5*b*x)/(a^2*b^2*x^4 + 2*a*b^3*x^2 + b^4) + 3/8*arctan(a*x/sq
rt(a*b))/(sqrt(a*b)*b^2)
```

mupad [B] time = 0.07, size = 55, normalized size = 0.89

$$\frac{\frac{5\,x}{8\,b}+\frac{3\,a\,x^3}{8\,b^2}}{a^2\,x^4+2\,a\,b\,x^2+b^2}+\frac{3\,\mathrm{atan}\left(\frac{\sqrt{a}\;x}{\sqrt{b}}\right)}{8\,\sqrt{a}\;b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b/x^2)^3),x)
```

```
[Out] ((5*x)/(8*b) + (3*a*x^3)/(8*b^2))/(b^2 + a^2*x^4 + 2*a*b*x^2) + (3*atan((a^
(1/2)*x)/b^(1/2)))/(8*a^(1/2)*b^(5/2))
```

sympy [A] time = 0.34, size = 105, normalized size = 1.69

$$-\frac{3\sqrt{-\frac{1}{ab^5}}\,\log\left(-b^3\sqrt{-\frac{1}{ab^5}}+x\right)}{16}+\frac{3\sqrt{-\frac{1}{ab^5}}\,\log\left(b^3\sqrt{-\frac{1}{ab^5}}+x\right)}{16}+\frac{3ax^3+5bx}{8a^2b^2x^4+16ab^3x^2+8b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**3/x**6,x)
```

```
[Out] -3*sqrt(-1/(a*b**5))*log(-b**3*sqrt(-1/(a*b**5)) + x)/16 + 3*sqrt(-1/(a*b**
5))*log(b**3*sqrt(-1/(a*b**5)) + x)/16 + (3*a*x**3 + 5*b*x)/(8*a**2*b**2*x**
*4 + 16*a*b**3*x**2 + 8*b**4)
```

3.1886 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^7}\,dx$

Optimal. Leaf size=54

$$-\frac{\log\left(ax^2+b\right)}{2b^3}+\frac{1}{2b^2\left(ax^2+b\right)}+\frac{1}{4b\left(ax^2+b\right)^2}+\frac{\log(x)}{b^3}$$

```
[Out] 1/4/b/(a*x^2+b)^2+1/2/b^2/(a*x^2+b)+ln(x)/b^3-1/2*ln(a*x^2+b)/b^3
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 266, 44}

$$\frac{1}{2b^2\left(ax^2+b\right)}-\frac{\log\left(ax^2+b\right)}{2b^3}+\frac{1}{4b\left(ax^2+b\right)^2}+\frac{\log(x)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)^3*x^7),x]
```

```
[Out] 1/(4*b*(b + a*x^2)^2) + 1/(2*b^2*(b + a*x^2)) + Log[x]/b^3 - Log[b + a*x^2]
/(2*b^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^7}\,dx &= \int \frac{1}{x\left(b+ax^2\right)^3}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(b+ax)^3}\,dx, x, x^2\right) \\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{b^3 x}-\frac{a}{b(b+ax)^3}-\frac{a}{b^2(b+ax)^2}-\frac{a}{b^3(b+ax)}\right)\,dx, x, x^2\right) \\
&= \frac{1}{4b\left(b+ax^2\right)^2}+\frac{1}{2b^2\left(b+ax^2\right)}+\frac{\log(x)}{b^3}-\frac{\log\left(b+ax^2\right)}{2b^3}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 43, normalized size = 0.80

$$\frac{\frac{b\left(2ax^2+3b\right)}{\left(ax^2+b\right)^2}-2\log\left(ax^2+b\right)+4\log(x)}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)^3*x^7),x]
```

```
[Out] ((b*(3*b + 2*a*x^2))/(b + a*x^2)^2 + 4*Log[x] - 2*Log[b + a*x^2])/(4*b^3)
```

fricas [A] time = 0.79, size = 90, normalized size = 1.67

$$\frac{2\,abx^2+3\,b^2-2\left(a^2x^4+2\,abx^2+b^2\right)\log\left(ax^2+b\right)+4\left(a^2x^4+2\,abx^2+b^2\right)\log(x)}{4\left(a^2b^3x^4+2\,ab^4x^2+b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^3/x^7,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*b*x^2 + 3*b^2 - 2*(a^2*x^4 + 2*a*b*x^2 + b^2)*log(a*x^2 + b) + 4*(
a^2*x^4 + 2*a*b*x^2 + b^2)*log(x))/(a^2*b^3*x^4 + 2*a*b^4*x^2 + b^5)
```

giac [A] time = 0.15, size = 59, normalized size = 1.09

$$\frac{\log\left(x^2\right)}{2\,b^3}-\frac{\log\left(\left|ax^2+b\right|\right)}{2\,b^3}+\frac{3\,a^2x^4+8\,abx^2+6\,b^2}{4\left(ax^2+b\right)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^3/x^7,x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/b^3 - 1/2*log(abs(a*x^2 + b))/b^3 + 1/4*(3*a^2*x^4 + 8*a*b*x^2
 + 6*b^2)/((a*x^2 + b)^2*b^3)
```

maple [A] time = 0.01, size = 49, normalized size = 0.91

$$\frac{1}{4\left(a\,x^2+b\right)^2b}+\frac{1}{2\left(a\,x^2+b\right)b^2}+\frac{\ln(x)}{b^3}-\frac{\ln\left(a\,x^2+b\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^3/x^7,x)
```

```
[Out] 1/4/b/(a*x^2+b)^2+1/2/b^2/(a*x^2+b)+1/b^3*ln(x)-1/2*ln(a*x^2+b)/b^3
```

maxima [A] time = 0.89, size = 60, normalized size = 1.11

$$\frac{2\,ax^2+3\,b}{4\left(a^2b^2x^4+2\,ab^3x^2+b^4\right)}-\frac{\log\left(ax^2+b\right)}{2\,b^3}+\frac{\log\left(x^2\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^3/x^7,x, algorithm="maxima")
```

```
[Out] 1/4*(2*a*x^2 + 3*b)/(a^2*b^2*x^4 + 2*a*b^3*x^2 + b^4) - 1/2*log(a*x^2 + b)/
b^3 + 1/2*log(x^2)/b^3
```

mupad [B] time = 1.14, size = 56, normalized size = 1.04

$$\frac{\ln(x)}{b^3}+\frac{\frac{3}{4\,b}+\frac{a\,x^2}{2\,b^2}}{a^2\,x^4+2\,a\,b\,x^2+b^2}-\frac{\ln\left(a\,x^2+b\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b/x^2)^3),x)
```

```
[Out] log(x)/b^3 + (3/(4*b) + (a*x^2)/(2*b^2))/(b^2 + a^2*x^4 + 2*a*b*x^2) - log(
b + a*x^2)/(2*b^3)
```

sympy [A] time = 0.43, size = 56, normalized size = 1.04

$$\frac{2ax^2+3b}{4a^2b^2x^4+8ab^3x^2+4b^4}+\frac{\log(x)}{b^3}-\frac{\log\left(x^2+\frac{b}{a}\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**3/x**7,x)
```

```
[Out] (2*a*x**2 + 3*b)/(4*a**2*b**2*x**4 + 8*a*b**3*x**2 + 4*b**4) + log(x)/b**3
- log(x**2 + b/a)/(2*b**3)
```

3.1887 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^8}\,dx$

Optimal. Leaf size=76

$$-\frac{15\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8b^{7/2}}+\frac{5}{8b^2x\left(ax^2+b\right)}+\frac{1}{4bx\left(ax^2+b\right)^2}-\frac{15}{8b^3x}$$

```
[Out] -15/8/b^3/x+1/4/b/x/(a*x^2+b)^2+5/8/b^2/x/(a*x^2+b)-15/8*arctan(x*a^(1/2)/b
^(1/2))*a^(1/2)/b^(7/2)
```

Rubi [A] time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {263, 290, 325, 205}

$$\frac{5}{8b^2x\left(ax^2+b\right)}-\frac{15\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8b^{7/2}}+\frac{1}{4bx\left(ax^2+b\right)^2}-\frac{15}{8b^3x}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)^3*x^8),x]
```

```
[Out] -15/(8*b^3*x) + 1/(4*b*x*(b + a*x^2)^2) + 5/(8*b^2*x*(b + a*x^2)) - (15*Sqr
t[a]*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(8*b^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^8}\,dx = \int \frac{1}{x^2\left(b+ax^2\right)^3}\,dx$$

$$= \frac{1}{4bx\left(b+ax^2\right)^2} + \frac{5\int \frac{1}{x^2\left(b+ax^2\right)^2}\,dx}{4b}$$

$$= \frac{1}{4bx\left(b+ax^2\right)^2} + \frac{5}{8b^2x\left(b+ax^2\right)} + \frac{15\int \frac{1}{x^2\left(b+ax^2\right)}\,dx}{8b^2}$$

$$= -\frac{15}{8b^3x} + \frac{1}{4bx\left(b+ax^2\right)^2} + \frac{5}{8b^2x\left(b+ax^2\right)} - \frac{(15a)\int \frac{1}{b+ax^2}\,dx}{8b^3}$$

$$= -\frac{15}{8b^3x} + \frac{1}{4bx\left(b+ax^2\right)^2} + \frac{5}{8b^2x\left(b+ax^2\right)} - \frac{15\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8b^{7/2}}$$

Mathematica [A] time = 0.04, size = 68, normalized size = 0.89

$$-\frac{15a^2x^4+25abx^2+8b^2}{8b^3x\left(ax^2+b\right)^2} - \frac{15\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)^3*x^8),x]
```

```
[Out] -1/8*(8*b^2 + 25*a*b*x^2 + 15*a^2*x^4)/(b^3*x*(b + a*x^2)^2) - (15*Sqrt[a]*
ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(8*b^(7/2))
```

fricas [A] time = 0.81, size = 202, normalized size = 2.66

$$\left[-\frac{30\,a^2x^4+50\,abx^2-15\left(a^2x^5+2\,abx^3+b^2x\right)\sqrt{-\frac{a}{b}}\,\log\left(\frac{ax^2-2\,bx\sqrt{-\frac{a}{b}}-b}{ax^2+b}\right)+16\,b^2}{16\left(a^2b^3x^5+2\,ab^4x^3+b^5x\right)}, -\frac{15\,a^2x^4+25\,abx^2+15\left(a^2\right.}{8\left(a^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^3/x^8,x, algorithm="fricas")
```

```
[Out] [-1/16*(30*a^2*x^4 + 50*a*b*x^2 - 15*(a^2*x^5 + 2*a*b*x^3 + b^2*x)*sqrt(-a/
b)*log((a*x^2 - 2*b*x*sqrt(-a/b) - b)/(a*x^2 + b)) + 16*b^2)/(a^2*b^3*x^5 +
 2*a*b^4*x^3 + b^5*x), -1/8*(15*a^2*x^4 + 25*a*b*x^2 + 15*(a^2*x^5 + 2*a*b*
x^3 + b^2*x)*sqrt(a/b)*arctan(x*sqrt(a/b)) + 8*b^2)/(a^2*b^3*x^5 + 2*a*b^4*
x^3 + b^5*x)]
```

giac [A] time = 0.16, size = 57, normalized size = 0.75

$$-\frac{15\,a\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\sqrt{ab}\,b^3} - \frac{7\,a^2x^3+9\,abx}{8\left(ax^2+b\right)^2b^3} - \frac{1}{b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^3/x^8,x, algorithm="giac")
```

```
[Out] -15/8*a*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/8*(7*a^2*x^3 + 9*a*b*x)/(
(a*x^2 + b)^2*b^3) - 1/(b^3*x)
```

maple [A] time = 0.01, size = 66, normalized size = 0.87

$$-\frac{7a^2x^3}{8\left(a\,x^2+b\right)^2b^3}-\frac{9ax}{8\left(a\,x^2+b\right)^2b^2}-\frac{15a\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\sqrt{ab}\,b^3}-\frac{1}{b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^3/x^8,x)
```

```
[Out] -7/8*a^2/b^3/(a*x^2+b)^2*x^3-9/8*a/b^2/(a*x^2+b)^2*x-15/8*a/b^3/(a*b)^(1/2)
*arctan(1/(a*b)^(1/2)*a*x)-1/b^3/x
```

maxima [A] time = 1.96, size = 71, normalized size = 0.93

$$-\frac{15\,a^2x^4+25\,abx^2+8\,b^2}{8\left(a^2b^3x^5+2\,ab^4x^3+b^5x\right)}-\frac{15\,a\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^3/x^8,x, algorithm="maxima")
```

```
[Out] -1/8*(15*a^2*x^4 + 25*a*b*x^2 + 8*b^2)/(a^2*b^3*x^5 + 2*a*b^4*x^3 + b^5*x)
- 15/8*a*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b^3)
```

mupad [B] time = 1.19, size = 66, normalized size = 0.87

$$-\frac{\frac{1}{b}+\frac{25\,a\,x^2}{8\,b^2}+\frac{15\,a^2\,x^4}{8\,b^3}}{a^2\,x^5+2\,a\,b\,x^3+b^2\,x}-\frac{15\,\sqrt{a}\,\operatorname{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(a + b/x^2)^3),x)
```

```
[Out] - (1/b + (25*a*x^2)/(8*b^2) + (15*a^2*x^4)/(8*b^3))/(b^2*x + a^2*x^5 + 2*a*
b*x^3) - (15*a^(1/2)*atan((a^(1/2)*x)/b^(1/2)))/(8*b^(7/2))
```

sympy [A] time = 0.43, size = 116, normalized size = 1.53

$$\frac{15\sqrt{-\frac{a}{b^7}}\,\log\left(x-\frac{b^4\sqrt{-\frac{a}{b^7}}}{a}\right)}{16}-\frac{15\sqrt{-\frac{a}{b^7}}\,\log\left(x+\frac{b^4\sqrt{-\frac{a}{b^7}}}{a}\right)}{16}+\frac{-15a^2x^4-25abx^2-8b^2}{8a^2b^3x^5+16ab^4x^3+8b^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**3/x**8,x)
```

```
[Out] 15*sqrt(-a/b**7)*log(x - b**4*sqrt(-a/b**7)/a)/16 - 15*sqrt(-a/b**7)*log(x
+ b**4*sqrt(-a/b**7)/a)/16 + (-15*a**2*x**4 - 25*a*b*x**2 - 8*b**2)/(8*a**2
*b**3*x**5 + 16*a*b**4*x**3 + 8*b**5*x)
```

3.1888 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^9}\,dx$

Optimal. Leaf size=67

$$\frac{3a\log\left(ax^2+b\right)}{2b^4}-\frac{3a\log(x)}{b^4}-\frac{a}{b^3\left(ax^2+b\right)}-\frac{a}{4b^2\left(ax^2+b\right)^2}-\frac{1}{2b^3x^2}$$

```
[Out] -1/2/b^3/x^2-1/4*a/b^2/(a*x^2+b)^2-a/b^3/(a*x^2+b)-3*a*ln(x)/b^4+3/2*a*ln(a
*x^2+b)/b^4
```

Rubi [A] time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 266, 44}

$$-\frac{a}{b^3\left(ax^2+b\right)}-\frac{a}{4b^2\left(ax^2+b\right)^2}+\frac{3a\log\left(ax^2+b\right)}{2b^4}-\frac{3a\log(x)}{b^4}-\frac{1}{2b^3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)^3*x^9),x]
```

```
[Out] -1/(2*b^3*x^2) - a/(4*b^2*(b + a*x^2)^2) - a/(b^3*(b + a*x^2)) - (3*a*Log[x
])/b^4 + (3*a*Log[b + a*x^2])/(2*b^4)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^9}\,dx &= \int \frac{1}{x^3\left(b+ax^2\right)^3}\,dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2(b+ax)^3}\,dx, x, x^2\right) \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{b^3x^2}-\frac{3a}{b^4x}+\frac{a^2}{b^2(b+ax)^3}+\frac{2a^2}{b^3(b+ax)^2}+\frac{3a^2}{b^4(b+ax)}\right)dx, x, x^2\right) \\
&= -\frac{1}{2b^3x^2}-\frac{a}{4b^2\left(b+ax^2\right)^2}-\frac{a}{b^3\left(b+ax^2\right)}-\frac{3a\log(x)}{b^4}+\frac{3a\log\left(b+ax^2\right)}{2b^4}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 59, normalized size = 0.88

$$-\frac{\frac{b\left(6a^2x^4+9abx^2+2b^2\right)}{x^2\left(ax^2+b\right)^2}-6a\log\left(ax^2+b\right)+12a\log(x)}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)^3*x^9),x]
```

```
[Out] -1/4*((b*(2*b^2 + 9*a*b*x^2 + 6*a^2*x^4))/(x^2*(b + a*x^2)^2) + 12*a*Log[x]
 - 6*a*Log[b + a*x^2])/b^4
```

fricas [A] time = 0.87, size = 119, normalized size = 1.78

$$-\frac{6\,a^2bx^4+9\,ab^2x^2+2\,b^3-6\left(a^3x^6+2\,a^2bx^4+ab^2x^2\right)\log\left(ax^2+b\right)+12\left(a^3x^6+2\,a^2bx^4+ab^2x^2\right)\log(x)}{4\left(a^2b^4x^6+2\,ab^5x^4+b^6x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^3/x^9,x, algorithm="fricas")
```

```
[Out] -1/4*(6*a^2*b*x^4 + 9*a*b^2*x^2 + 2*b^3 - 6*(a^3*x^6 + 2*a^2*b*x^4 + a*b^2*
x^2)*log(a*x^2 + b) + 12*(a^3*x^6 + 2*a^2*b*x^4 + a*b^2*x^2)*log(x))/(a^2*b
^4*x^6 + 2*a*b^5*x^4 + b^6*x^2)
```

giac [A] time = 0.16, size = 82, normalized size = 1.22

$$-\frac{3\,a\log\left(x^2\right)}{2\,b^4}+\frac{3\,a\log\left(\left|ax^2+b\right|\right)}{2\,b^4}-\frac{9\,a^3x^4+22\,a^2bx^2+14\,ab^2}{4\left(ax^2+b\right)^2b^4}+\frac{3\,ax^2-b}{2\,b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^3/x^9,x, algorithm="giac")
```

```
[Out] -3/2*a*log(x^2)/b^4 + 3/2*a*log(abs(a*x^2 + b))/b^4 - 1/4*(9*a^3*x^4 + 22*a
^2*b*x^2 + 14*a*b^2)/((a*x^2 + b)^2*b^4) + 1/2*(3*a*x^2 - b)/(b^4*x^2)
```

maple [A] time = 0.01, size = 62, normalized size = 0.93

$$-\frac{a}{4\left(a\,x^2+b\right)^2b^2}-\frac{a}{\left(a\,x^2+b\right)b^3}-\frac{3a\ln(x)}{b^4}+\frac{3a\ln\left(a\,x^2+b\right)}{2b^4}-\frac{1}{2b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^3/x^9,x)
```

```
[Out] -1/2/b^3/x^2-1/4*a/b^2/(a*x^2+b)^2-a/b^3/(a*x^2+b)-3*a/b^4*ln(x)+3/2*a*ln(a
*x^2+b)/b^4
```

maxima [A] time = 0.90, size = 77, normalized size = 1.15

$$-\frac{6\,a^2x^4+9\,abx^2+2\,b^2}{4\left(a^2b^3x^6+2\,ab^4x^4+b^5x^2\right)}+\frac{3\,a\log\left(ax^2+b\right)}{2\,b^4}-\frac{3\,a\log\left(x^2\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^3/x^9,x, algorithm="maxima")
```

```
[Out] -1/4*(6*a^2*x^4 + 9*a*b*x^2 + 2*b^2)/(a^2*b^3*x^6 + 2*a*b^4*x^4 + b^5*x^2)
 + 3/2*a*log(a*x^2 + b)/b^4 - 3/2*a*log(x^2)/b^4
```

mupad [B] time = 0.08, size = 75, normalized size = 1.12

$$\frac{3\,a\,\ln\left(a\,x^2+b\right)}{2\,b^4}-\frac{\frac{1}{2\,b}+\frac{9\,a\,x^2}{4\,b^2}+\frac{3\,a^2\,x^4}{2\,b^3}}{a^2\,x^6+2\,a\,b\,x^4+b^2\,x^2}-\frac{3\,a\,\ln(x)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(a + b/x^2)^3),x)
```

```
[Out] (3*a*log(b + a*x^2))/(2*b^4) - (1/(2*b) + (9*a*x^2)/(4*b^2) + (3*a^2*x^4)/(
2*b^3))/(a^2*x^6 + b^2*x^2 + 2*a*b*x^4) - (3*a*log(x))/b^4
```

sympy [A] time = 0.53, size = 80, normalized size = 1.19

$$-\frac{3a\log(x)}{b^4}+\frac{3a\log\left(x^2+\frac{b}{a}\right)}{2b^4}+\frac{-6a^2x^4-9abx^2-2b^2}{4a^2b^3x^6+8ab^4x^4+4b^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**3/x**9,x)
```

```
[Out] -3*a*log(x)/b**4 + 3*a*log(x**2 + b/a)/(2*b**4) + (-6*a**2*x**4 - 9*a*b*x**
2 - 2*b**2)/(4*a**2*b**3*x**6 + 8*a*b**4*x**4 + 4*b**5*x**2)
```

3.1889 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^{10}}\,dx$

Optimal. Leaf size=87

$$\frac{35a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8b^{9/2}}+\frac{35a}{8b^4x}+\frac{7}{8b^2x^3\left(ax^2+b\right)}+\frac{1}{4bx^3\left(ax^2+b\right)^2}-\frac{35}{24b^3x^3}$$

```
[Out] -35/24/b^3/x^3+35/8*a/b^4/x+1/4/b/x^3/(a*x^2+b)^2+7/8/b^2/x^3/(a*x^2+b)+35/
8*a^(3/2)*arctan(x*a^(1/2)/b^(1/2))/b^(9/2)
```

Rubi [A] time = 0.03, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {263, 290, 325, 205}

$$\frac{35a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8b^{9/2}}+\frac{7}{8b^2x^3\left(ax^2+b\right)}+\frac{35a}{8b^4x}+\frac{1}{4bx^3\left(ax^2+b\right)^2}-\frac{35}{24b^3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)^3*x^10),x]
```

```
[Out] -35/(24*b^3*x^3) + (35*a)/(8*b^4*x) + 1/(4*b*x^3*(b + a*x^2)^2) + 7/(8*b^2*
x^3*(b + a*x^2)) + (35*a^(3/2)*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(8*b^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^{10}}\,dx = \int \frac{1}{x^4\left(b+ax^2\right)^3}\,dx$$

$$= \frac{1}{4bx^3\left(b+ax^2\right)^2} + \frac{7\int \frac{1}{x^4\left(b+ax^2\right)^2}\,dx}{4b}$$

$$= \frac{1}{4bx^3\left(b+ax^2\right)^2} + \frac{7}{8b^2x^3\left(b+ax^2\right)} + \frac{35\int \frac{1}{x^4\left(b+ax^2\right)}\,dx}{8b^2}$$

$$= -\frac{35}{24b^3x^3} + \frac{1}{4bx^3\left(b+ax^2\right)^2} + \frac{7}{8b^2x^3\left(b+ax^2\right)} - \frac{(35a)\int \frac{1}{x^2\left(b+ax^2\right)}\,dx}{8b^3}$$

$$= -\frac{35}{24b^3x^3} + \frac{35a}{8b^4x} + \frac{1}{4bx^3\left(b+ax^2\right)^2} + \frac{7}{8b^2x^3\left(b+ax^2\right)} + \frac{\left(35a^2\right)\int \frac{1}{b+ax^2}\,dx}{8b^4}$$

$$= -\frac{35}{24b^3x^3} + \frac{35a}{8b^4x} + \frac{1}{4bx^3\left(b+ax^2\right)^2} + \frac{7}{8b^2x^3\left(b+ax^2\right)} + \frac{35a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8b^{9/2}}$$

Mathematica [A] time = 0.05, size = 79, normalized size = 0.91

$$\frac{35a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8b^{9/2}} + \frac{105a^3x^6+175a^2bx^4+56ab^2x^2-8b^3}{24b^4x^3\left(ax^2+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)^3*x^10),x]
```

```
[Out] (-8*b^3 + 56*a*b^2*x^2 + 175*a^2*b*x^4 + 105*a^3*x^6)/(24*b^4*x^3*(b + a*x^
2)^2) + (35*a^(3/2)*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(8*b^(9/2))
```

fricas [A] time = 0.85, size = 238, normalized size = 2.74

$$\left[\frac{210\,a^3x^6+350\,a^2bx^4+112\,ab^2x^2-16\,b^3+105\left(a^3x^7+2\,a^2bx^5+ab^2x^3\right)\sqrt{-\frac{a}{b}}\,\log\left(\frac{ax^2+2\,bx\sqrt{-\frac{a}{b}}-b}{ax^2+b}\right)}{48\left(a^2b^4x^7+2\,ab^5x^5+b^6x^3\right)}, \frac{105\,a^3x^6}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^3/x^10,x, algorithm="fricas")
```

```
[Out] [1/48*(210*a^3*x^6 + 350*a^2*b*x^4 + 112*a*b^2*x^2 - 16*b^3 + 105*(a^3*x^7
+ 2*a^2*b*x^5 + a*b^2*x^3)*sqrt(-a/b)*log((a*x^2 + 2*b*x*sqrt(-a/b) - b)/(a
*x^2 + b)))/(a^2*b^4*x^7 + 2*a*b^5*x^5 + b^6*x^3), 1/24*(105*a^3*x^6 + 175*
a^2*b*x^4 + 56*a*b^2*x^2 - 8*b^3 + 105*(a^3*x^7 + 2*a^2*b*x^5 + a*b^2*x^3)*
sqrt(a/b)*arctan(x*sqrt(a/b)))/(a^2*b^4*x^7 + 2*a*b^5*x^5 + b^6*x^3)]
```

giac [A] time = 0.15, size = 71, normalized size = 0.82

$$\frac{35\,a^2\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\sqrt{ab}\,b^4} + \frac{11\,a^3x^3+13\,a^2bx}{8\left(ax^2+b\right)^2b^4} + \frac{9\,ax^2-b}{3\,b^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^3/x^10,x, algorithm="giac")
```

```
[Out] 35/8*a^2*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/8*(11*a^3*x^3 + 13*a^2*b
*x)/((a*x^2 + b)^2*b^4) + 1/3*(9*a*x^2 - b)/(b^4*x^3)
```

maple [A] time = 0.02, size = 79, normalized size = 0.91

$$\frac{11a^3x^3}{8\left(a\,x^2+b\right)^2b^4}+\frac{13a^2x}{8\left(a\,x^2+b\right)^2b^3}+\frac{35a^2\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\sqrt{ab}\;b^4}+\frac{3a}{b^4x}-\frac{1}{3b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^3/x^10,x)
```

```
[Out] 11/8/b^4*a^3/(a*x^2+b)^2*x^3+13/8/b^3*a^2/(a*x^2+b)^2*x+35/8/b^4*a^2/(a*b)^
(1/2)*arctan(1/(a*b)^(1/2)*a*x)-1/3/b^3/x^3+3*a/b^4/x
```

maxima [A] time = 2.03, size = 86, normalized size = 0.99

$$\frac{105\,a^3x^6+175\,a^2bx^4+56\,ab^2x^2-8\,b^3}{24\left(a^2b^4x^7+2\,ab^5x^5+b^6x^3\right)}+\frac{35\,a^2\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^3/x^10,x, algorithm="maxima")
```

```
[Out] 1/24*(105*a^3*x^6 + 175*a^2*b*x^4 + 56*a*b^2*x^2 - 8*b^3)/(a^2*b^4*x^7 + 2*
a*b^5*x^5 + b^6*x^3) + 35/8*a^2*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b^4)
```

mupad [B] time = 1.17, size = 80, normalized size = 0.92

$$\frac{\frac{7\,a\,x^2}{3\,b^2}-\frac{1}{3\,b}+\frac{175\,a^2\,x^4}{24\,b^3}+\frac{35\,a^3\,x^6}{8\,b^4}}{a^2\,x^7+2\,a\,b\,x^5+b^2\,x^3}+\frac{35\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^10*(a + b/x^2)^3),x)
```

```
[Out] ((7*a*x^2)/(3*b^2) - 1/(3*b) + (175*a^2*x^4)/(24*b^3) + (35*a^3*x^6)/(8*b^4
))/(a^2*x^7 + b^2*x^3 + 2*a*b*x^5) + (35*a^(3/2)*atan((a^(1/2)*x)/b^(1/2)))
/(8*b^(9/2))
```

sympy [A] time = 0.49, size = 138, normalized size = 1.59

$$-\frac{35\sqrt{-\frac{a^3}{b^9}}\,\log\left(x-\frac{b^5\sqrt{-\frac{a^3}{b^9}}}{a^2}\right)}{16}+\frac{35\sqrt{-\frac{a^3}{b^9}}\,\log\left(x+\frac{b^5\sqrt{-\frac{a^3}{b^9}}}{a^2}\right)}{16}+\frac{105a^3x^6+175a^2bx^4+56ab^2x^2-8b^3}{24a^2b^4x^7+48ab^5x^5+24b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**3/x**10,x)
```

```
[Out] -35*sqrt(-a**3/b**9)*log(x - b**5*sqrt(-a**3/b**9)/a**2)/16 + 35*sqrt(-a**3
/b**9)*log(x + b**5*sqrt(-a**3/b**9)/a**2)/16 + (105*a**3*x**6 + 175*a**2*b
*x**4 + 56*a*b**2*x**2 - 8*b**3)/(24*a**2*b**4*x**7 + 48*a*b**5*x**5 + 24*b
**6*x**3)
```

3.1890 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^{11}}\,dx$

Optimal. Leaf size=86

$$-\frac{3a^2\log\left(ax^2+b\right)}{b^5}+\frac{6a^2\log(x)}{b^5}+\frac{3a^2}{2b^4\left(ax^2+b\right)}+\frac{a^2}{4b^3\left(ax^2+b\right)^2}+\frac{3a}{2b^4x^2}-\frac{1}{4b^3x^4}$$

```
[Out] -1/4/b^3/x^4+3/2*a/b^4/x^2+1/4*a^2/b^3/(a*x^2+b)^2+3/2*a^2/b^4/(a*x^2+b)+6*
a^2*ln(x)/b^5-3*a^2*ln(a*x^2+b)/b^5
```

Rubi [A] time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 266, 44}

$$\frac{3a^2}{2b^4\left(ax^2+b\right)}+\frac{a^2}{4b^3\left(ax^2+b\right)^2}-\frac{3a^2\log\left(ax^2+b\right)}{b^5}+\frac{6a^2\log(x)}{b^5}+\frac{3a}{2b^4x^2}-\frac{1}{4b^3x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)^3*x^11),x]
```

```
[Out] -1/(4*b^3*x^4) + (3*a)/(2*b^4*x^2) + a^2/(4*b^3*(b + a*x^2)^2) + (3*a^2)/(2
*b^4*(b + a*x^2)) + (6*a^2*Log[x])/b^5 - (3*a^2*Log[b + a*x^2])/b^5
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^{11}}\,dx &= \int \frac{1}{x^5\left(b+ax^2\right)^3}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3(b+ax)^3}\,dx, x, x^2\right) \\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{b^3x^3}-\frac{3a}{b^4x^2}+\frac{6a^2}{b^5x}-\frac{a^3}{b^3(b+ax)^3}-\frac{3a^3}{b^4(b+ax)^2}-\frac{6a^3}{b^5(b+ax)}\right)dx, x, x^2\right) \\
&= -\frac{1}{4b^3x^4}+\frac{3a}{2b^4x^2}+\frac{a^2}{4b^3\left(b+ax^2\right)^2}+\frac{3a^2}{2b^4\left(b+ax^2\right)}+\frac{6a^2\log(x)}{b^5}-\frac{3a^2\log\left(b+ax^2\right)}{b^5}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 74, normalized size = 0.86

$$\frac{-12a^2\log\left(ax^2+b\right)+24a^2\log(x)+\frac{b\left(12a^3x^6+18a^2bx^4+4ab^2x^2-b^3\right)}{x^4\left(ax^2+b\right)^2}}{4b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)^3*x^11),x]
```

```
[Out] ((b*(-b^3 + 4*a*b^2*x^2 + 18*a^2*b*x^4 + 12*a^3*x^6))/(x^4*(b + a*x^2)^2) +
 24*a^2*Log[x] - 12*a^2*Log[b + a*x^2])/(4*b^5)
```

fricas [A] time = 0.81, size = 134, normalized size = 1.56

$$\frac{12\,a^3bx^6+18\,a^2b^2x^4+4\,ab^3x^2-b^4-12\left(a^4x^8+2\,a^3bx^6+a^2b^2x^4\right)\log\left(ax^2+b\right)+24\left(a^4x^8+2\,a^3bx^6+a^2b^2x^4\right)}{4\left(a^2b^5x^8+2\,ab^6x^6+b^7x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^3/x^11,x, algorithm="fricas")
```

```
[Out] 1/4*(12*a^3*b*x^6 + 18*a^2*b^2*x^4 + 4*a*b^3*x^2 - b^4 - 12*(a^4*x^8 + 2*a^
3*b*x^6 + a^2*b^2*x^4)*log(a*x^2 + b) + 24*(a^4*x^8 + 2*a^3*b*x^6 + a^2*b^2
*x^4)*log(x))/(a^2*b^5*x^8 + 2*a*b^6*x^6 + b^7*x^4)
```

giac [A] time = 0.15, size = 80, normalized size = 0.93

$$\frac{3\,a^2\log\left(x^2\right)}{b^5}-\frac{3\,a^2\log\left(\left|ax^2+b\right|\right)}{b^5}+\frac{12\,a^3x^6+18\,a^2bx^4+4\,ab^2x^2-b^3}{4\left(ax^4+bx^2\right)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^3/x^11,x, algorithm="giac")
```

```
[Out] 3*a^2*log(x^2)/b^5 - 3*a^2*log(abs(a*x^2 + b))/b^5 + 1/4*(12*a^3*x^6 + 18*a
^2*b*x^4 + 4*a*b^2*x^2 - b^3)/((a*x^4 + b*x^2)^2*b^4)
```

maple [A] time = 0.01, size = 79, normalized size = 0.92

$$\frac{a^2}{4\left(a\,x^2+b\right)^2b^3}+\frac{3a^2}{2\left(a\,x^2+b\right)b^4}+\frac{6a^2\ln(x)}{b^5}-\frac{3a^2\ln\left(a\,x^2+b\right)}{b^5}+\frac{3a}{2b^4x^2}-\frac{1}{4b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^3/x^11,x)
```

```
[Out] -1/4/b^3/x^4+3/2*a/b^4/x^2+1/4*a^2/b^3/(a*x^2+b)^2+3/2*a^2/b^4/(a*x^2+b)+6*
a^2/b^5*ln(x)-3*a^2*ln(a*x^2+b)/b^5
```

maxima [A] time = 0.93, size = 92, normalized size = 1.07

$$\frac{12\,a^3x^6+18\,a^2bx^4+4\,ab^2x^2-b^3}{4\left(a^2b^4x^8+2\,ab^5x^6+b^6x^4\right)}-\frac{3\,a^2\log\left(ax^2+b\right)}{b^5}+\frac{3\,a^2\log\left(x^2\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^3/x^11,x, algorithm="maxima")
```

```
[Out] 1/4*(12*a^3*x^6 + 18*a^2*b*x^4 + 4*a*b^2*x^2 - b^3)/(a^2*b^4*x^8 + 2*a*b^5*
x^6 + b^6*x^4) - 3*a^2*log(a*x^2 + b)/b^5 + 3*a^2*log(x^2)/b^5
```

mupad [B] time = 1.15, size = 88, normalized size = 1.02

$$\frac{\frac{a\,x^2}{b^2}-\frac{1}{4\,b}+\frac{9\,a^2\,x^4}{2\,b^3}+\frac{3\,a^3\,x^6}{b^4}}{a^2\,x^8+2\,a\,b\,x^6+b^2\,x^4}-\frac{3\,a^2\,\ln\left(a\,x^2+b\right)}{b^5}+\frac{6\,a^2\,\ln(x)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^11*(a + b/x^2)^3),x)
```

```
[Out] ((a*x^2)/b^2 - 1/(4*b) + (9*a^2*x^4)/(2*b^3) + (3*a^3*x^6)/b^4)/(a^2*x^8 +
b^2*x^4 + 2*a*b*x^6) - (3*a^2*log(b + a*x^2))/b^5 + (6*a^2*log(x))/b^5
```

sympy [A] time = 0.57, size = 90, normalized size = 1.05

$$\frac{6a^2\log(x)}{b^5}-\frac{3a^2\log\left(x^2+\frac{b}{a}\right)}{b^5}+\frac{12a^3x^6+18a^2bx^4+4ab^2x^2-b^3}{4a^2b^4x^8+8ab^5x^6+4b^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**3/x**11,x)
```

```
[Out] 6*a**2*log(x)/b**5 - 3*a**2*log(x**2 + b/a)/b**5 + (12*a**3*x**6 + 18*a**2*
b*x**4 + 4*a*b**2*x**2 - b**3)/(4*a**2*b**4*x**8 + 8*a*b**5*x**6 + 4*b**6*x
**4)
```

3.1891 $\int \sqrt{a+\frac{b}{x^2}}\, x^3\, dx$

Optimal. Leaf size=71

$$-\frac{b^2 \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{8a^{3/2}}+\frac{bx^2\sqrt{a+\frac{b}{x^2}}}{8a}+\frac{1}{4}x^4\sqrt{a+\frac{b}{x^2}}$$

```
[Out] -1/8*b^2*arctanh((a+b/x^2)^(1/2)/a^(1/2))/a^(3/2)+1/8*b*x^2*(a+b/x^2)^(1/2)
/a+1/4*x^4*(a+b/x^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {266, 47, 51, 63, 208}

$$-\frac{b^2 \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{8a^{3/2}}+\frac{1}{4}x^4\sqrt{a+\frac{b}{x^2}}+\frac{bx^2\sqrt{a+\frac{b}{x^2}}}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^2]*x^3,x]
```

```
[Out] (b*Sqrt[a + b/x^2]*x^2)/(8*a) + (Sqrt[a + b/x^2]*x^4)/4 - (b^2*ArcTanh[Sqrt
[a + b/x^2]/Sqrt[a]])/(8*a^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a+\frac{b}{x^2}}\, x^3\, dx &= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^3}\, dx, x, \frac{1}{x^2}\right)\right) \\
&= \frac{1}{4}\sqrt{a+\frac{b}{x^2}}\, x^4 - \frac{1}{8}b\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\, dx, x, \frac{1}{x^2}\right) \\
&= \frac{b\sqrt{a+\frac{b}{x^2}}\, x^2}{8a} + \frac{1}{4}\sqrt{a+\frac{b}{x^2}}\, x^4 + \frac{b^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\, dx, x, \frac{1}{x^2}\right)}{16a} \\
&= \frac{b\sqrt{a+\frac{b}{x^2}}\, x^2}{8a} + \frac{1}{4}\sqrt{a+\frac{b}{x^2}}\, x^4 + \frac{b\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\, dx, x, \sqrt{a+\frac{b}{x^2}}\right)}{8a} \\
&= \frac{b\sqrt{a+\frac{b}{x^2}}\, x^2}{8a} + \frac{1}{4}\sqrt{a+\frac{b}{x^2}}\, x^4 - \frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{8a^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 88, normalized size = 1.24

$$x\sqrt{a+\frac{b}{x^2}}\left(\frac{bx}{8a}+\frac{x^3}{4}\right) - \frac{b^2x\sqrt{a+\frac{b}{x^2}}\,\log\left(\sqrt{a}\,\sqrt{ax^2+b}+ax\right)}{8a^{3/2}\sqrt{ax^2+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^2]*x^3,x]
```

```
[Out] Sqrt[a + b/x^2]*x*((b*x)/(8*a) + x^3/4) - (b^2*Sqrt[a + b/x^2]*x*Log[a*x +
Sqrt[a]*Sqrt[b + a*x^2]])/(8*a^(3/2)*Sqrt[b + a*x^2])
```

fricas [A] time = 0.89, size = 152, normalized size = 2.14

$$\left[\frac{\sqrt{a}\,b^2\log\left(-2\,ax^2+2\,\sqrt{a}\,x^2\sqrt{\frac{ax^2+b}{x^2}}-b\right)+2\left(2\,a^2x^4+abx^2\right)\sqrt{\frac{ax^2+b}{x^2}}}{16\,a^2},\ \frac{\sqrt{-a}\,b^2\arctan\left(\frac{\sqrt{-a}\,x^2\sqrt{\frac{ax^2+b}{x^2}}}{ax^2+b}\right)+\left(2\,a^2x^{4}\right.}{8\,a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2)*x^3,x, algorithm="fricas")
```

```
[Out] [1/16*(sqrt(a)*b^2*log(-2*a*x^2 + 2*sqrt(a)*x^2*sqrt((a*x^2 + b)/x^2) - b)
+ 2*(2*a^2*x^4 + a*b*x^2)*sqrt((a*x^2 + b)/x^2))/a^2, 1/8*(sqrt(-a)*b^2*arc
tan(sqrt(-a)*x^2*sqrt((a*x^2 + b)/x^2)/(a*x^2 + b)) + (2*a^2*x^4 + a*b*x^2)
*sqrt((a*x^2 + b)/x^2))/a^2]
```

giac [A] time = 0.18, size = 69, normalized size = 0.97

$$\frac{1}{8}\sqrt{ax^2+b}\left(2x^2\mathrm{sgn}(x)+\frac{b\mathrm{sgn}(x)}{a}\right)x+\frac{b^2\log\left(\left|-\sqrt{a}\,x+\sqrt{ax^2+b}\right|\right)\mathrm{sgn}(x)}{8a^{\frac{3}{2}}}-\frac{b^2\log\left(|b|\right)\mathrm{sgn}(x)}{16a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2)*x^3,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(a*x^2 + b)*(2*x^2*sgn(x) + b*sgn(x)/a)*x + 1/8*b^2*log(abs(-sqrt(a
)*x + sqrt(a*x^2 + b)))*sgn(x)/a^(3/2) - 1/16*b^2*log(abs(b))*sgn(x)/a^(3/2
)
```

maple [A] time = 0.02, size = 82, normalized size = 1.15

$$\frac{\sqrt{\frac{a\,x^2+b}{x^2}}\left(-b^2\ln\left(\sqrt{a}\,x+\sqrt{a\,x^2+b}\right)-\sqrt{a\,x^2+b}\,\sqrt{a}\,bx+2\left(a\,x^2+b\right)^{\frac{3}{2}}\sqrt{a}\,x\right)x}{8\sqrt{a\,x^2+b}\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^(1/2)*x^3,x)
```

```
[Out] 1/8*((a*x^2+b)/x^2)^(1/2)*x*(2*x*(a*x^2+b)^(3/2)*a^(1/2)-(a*x^2+b)^(1/2)*a^
(1/2)*x*b-ln(x*a^(1/2)+(a*x^2+b)^(1/2))*b^2)/(a*x^2+b)^(1/2)/a^(3/2)
```

maxima [A] time = 1.97, size = 100, normalized size = 1.41

$$\frac{b^2\log\left(\frac{\sqrt{a+\frac{b}{x^2}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^2}}+\sqrt{a}}\right)}{16a^{\frac{3}{2}}}+\frac{\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}b^2+\sqrt{a+\frac{b}{x^2}}\,ab^2}{8\left(\left(a+\frac{b}{x^2}\right)^2a-2\left(a+\frac{b}{x^2}\right)a^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2)*x^3,x, algorithm="maxima")
```

```
[Out] 1/16*b^2*log((sqrt(a + b/x^2) - sqrt(a))/(sqrt(a + b/x^2) + sqrt(a)))/a^(3/
2) + 1/8*((a + b/x^2)^(3/2)*b^2 + sqrt(a + b/x^2)*a*b^2)/((a + b/x^2)^2*a -
 2*(a + b/x^2)*a^2 + a^3)
```

mupad [B] time = 1.61, size = 54, normalized size = 0.76

$$\frac{x^4\sqrt{a+\frac{b}{x^2}}}{8}-\frac{b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{8\,a^{3/2}}+\frac{x^4\left(a+\frac{b}{x^2}\right)^{3/2}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b/x^2)^(1/2),x)
```

```
[Out] (x^4*(a + b/x^2)^(1/2))/8 - (b^2*atanh((a + b/x^2)^(1/2)/a^(1/2)))/(8*a^(3/
2)) + (x^4*(a + b/x^2)^(3/2))/(8*a)
```

sympy [A] time = 3.93, size = 92, normalized size = 1.30

$$\frac{ax^5}{4\sqrt{b}\sqrt{\frac{ax^2}{b}+1}}+\frac{3\sqrt{b}\,x^3}{8\sqrt{\frac{ax^2}{b}+1}}+\frac{b^{\frac{3}{2}}x}{8a\sqrt{\frac{ax^2}{b}+1}}-\frac{b^2\,\mathrm{asinh}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**(1/2)*x**3,x)
```

```
[Out] a*x**5/(4*sqrt(b)*sqrt(a*x**2/b + 1)) + 3*sqrt(b)*x**3/(8*sqrt(a*x**2/b + 1
)) + b**(3/2)*x/(8*a*sqrt(a*x**2/b + 1)) - b**2*asinh(sqrt(a)*x/sqrt(b))/(8
*a**(3/2))
```

3.1892 $\int \sqrt{a+\frac{b}{x^2}}\, x^2\, dx$

Optimal. Leaf size=21

$$\frac{x^3\left(a+\frac{b}{x^2}\right)^{3/2}}{3a}$$

```
[Out] 1/3*(a+b/x^2)^(3/2)*x^3/a
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {264}

$$\frac{x^3\left(a+\frac{b}{x^2}\right)^{3/2}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^2]*x^2,x]
```

```
[Out] ((a + b/x^2)^(3/2)*x^3)/(3*a)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{a+\frac{b}{x^2}}\, x^2\, dx = \frac{\left(a+\frac{b}{x^2}\right)^{3/2} x^3}{3a}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.24

$$\frac{x\sqrt{a+\frac{b}{x^2}}\left(ax^2+b\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^2]*x^2,x]
```

```
[Out] (Sqrt[a + b/x^2]*x*(b + a*x^2))/(3*a)
```

fricas [A] time = 0.89, size = 27, normalized size = 1.29

$$\frac{\left(ax^3+bx\right)\sqrt{\frac{ax^2+b}{x^2}}}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2)*x^2,x, algorithm="fricas")
```

```
[Out] 1/3*(a*x^3 + b*x)*sqrt((a*x^2 + b)/x^2)/a
```

giac [A] time = 0.15, size = 27, normalized size = 1.29

$$\frac{\left(ax^2+b\right)^{\frac{3}{2}}\operatorname{sgn}(x)}{3\,a}-\frac{b^{\frac{3}{2}}\operatorname{sgn}(x)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2)*x^2,x, algorithm="giac")
[Out] 1/3*(a*x^2 + b)^(3/2)*sgn(x)/a - 1/3*b^(3/2)*sgn(x)/a
```

maple [A] time = 0.00, size = 27, normalized size = 1.29

$$\frac{\left(a\,x^2+b\right)\sqrt{\frac{a\,x^2+b}{x^2}}\,x}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^(1/2)*x^2,x)
[Out] 1/3*(a*x^2+b)/a*x*((a*x^2+b)/x^2)^(1/2)
```

maxima [A] time = 0.88, size = 17, normalized size = 0.81

$$\frac{\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}x^3}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2)*x^2,x, algorithm="maxima")
[Out] 1/3*(a + b/x^2)^(3/2)*x^3/a
```

mupad [B] time = 1.24, size = 23, normalized size = 1.10

$$\sqrt{a+\frac{b}{x^2}}\,\left(\frac{x^3}{3}+\frac{b\,x}{3\,a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b/x^2)^(1/2),x)
[Out] (a + b/x^2)^(1/2)*(x^3/3 + (b*x)/(3*a))
```

sympy [B] time = 0.85, size = 41, normalized size = 1.95

$$\frac{\sqrt{b}\,x^2\sqrt{\frac{ax^2}{b}+1}}{3}+\frac{b^{\frac{3}{2}}\sqrt{\frac{ax^2}{b}+1}}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**(1/2)*x**2,x)
[Out] sqrt(b)*x**2*sqrt(a*x**2/b + 1)/3 + b**(3/2)*sqrt(a*x**2/b + 1)/(3*a)
```

3.1893 $\int \sqrt{a+\frac{b}{x^2}}\,x\,dx$

Optimal. Leaf size=47

$$\frac{1}{2}x^2\sqrt{a+\frac{b}{x^2}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

```
[Out] 1/2*b*arctanh((a+b/x^2)^(1/2)/a^(1/2))/a^(1/2)+1/2*(a+b/x^2)^(1/2)*x^2
```

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {266, 47, 63, 208}

$$\frac{1}{2}x^2\sqrt{a+\frac{b}{x^2}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^2]*x,x]
```

```
[Out] (Sqrt[a + b/x^2]*x^2)/2 + (b*ArcTanh[Sqrt[a + b/x^2]/Sqrt[a]])/(2*Sqrt[a])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+\frac{b}{x^2}}\,x\,dx &= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx, x, \frac{1}{x^2}\right)\right)\\ &= \frac{1}{2}\sqrt{a+\frac{b}{x^2}}\,x^2 - \frac{1}{4}b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x^2}\right)\\ &= \frac{1}{2}\sqrt{a+\frac{b}{x^2}}\,x^2 - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x^2}}\right)\\ &= \frac{1}{2}\sqrt{a+\frac{b}{x^2}}\,x^2 + \frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{2\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 58, normalized size = 1.23

$$\frac{1}{2}x\sqrt{a+\frac{b}{x^2}}\left(\frac{b\log\left(\sqrt{a}\,\sqrt{ax^2+b}+ax\right)}{\sqrt{a}\,\sqrt{ax^2+b}}+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^2]*x,x]
```

```
[Out] (Sqrt[a + b/x^2]*x*(x + (b*Log[a*x + Sqrt[a]*Sqrt[b + a*x^2]])/(Sqrt[a]*Sqr
t[b + a*x^2])))/2
```

fricas [A] time = 0.83, size = 127, normalized size = 2.70

$$\left[\frac{2\,ax^2\sqrt{\frac{ax^2+b}{x^2}}+\sqrt{a}\,b\log\left(-2\,ax^2-2\,\sqrt{a}\,x^2\sqrt{\frac{ax^2+b}{x^2}}-b\right)}{4\,a}, \frac{ax^2\sqrt{\frac{ax^2+b}{x^2}}-\sqrt{-a}\,b\arctan\left(\frac{\sqrt{-a}\,x^2\sqrt{\frac{ax^2+b}{x^2}}}{ax^2+b}\right)}{2\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2)*x,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*x^2*sqrt((a*x^2 + b)/x^2) + sqrt(a)*b*log(-2*a*x^2 - 2*sqrt(a)*x^
2*sqrt((a*x^2 + b)/x^2) - b))/a, 1/2*(a*x^2*sqrt((a*x^2 + b)/x^2) - sqrt(-a
)*b*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + b)/x^2)/(a*x^2 + b)))/a]
```

giac [A] time = 0.19, size = 52, normalized size = 1.11

$$\frac{b\log\left(|b|\right)\operatorname{sgn}(x)}{4\sqrt{a}}+\frac{1}{2}\left(\sqrt{ax^2+b}\,x-\frac{b\log\left(\left|-\sqrt{a}\,x+\sqrt{ax^2+b}\right|\right)}{\sqrt{a}}\right)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2)*x,x, algorithm="giac")
```

```
[Out] 1/4*b*log(abs(b))*sgn(x)/sqrt(a) + 1/2*(sqrt(a*x^2 + b)*x - b*log(abs(-sqrt
(a)*x + sqrt(a*x^2 + b)))/sqrt(a))*sgn(x)
```

maple [A] time = 0.00, size = 62, normalized size = 1.32

$$\frac{\sqrt{\frac{a\,x^2+b}{x^2}}\,\left(b\ln\left(\sqrt{a}\,x+\sqrt{a\,x^2+b}\right)+\sqrt{a\,x^2+b}\,\sqrt{a}\,x\right)x}{2\sqrt{a\,x^2+b}\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^(1/2)*x,x)
```

```
[Out] 1/2*((a*x^2+b)/x^2)^(1/2)*x*(x*(a*x^2+b)^(1/2)*a^(1/2)+b*ln(a^(1/2)*x+(a*x^
2+b)^(1/2)))/(a*x^2+b)^(1/2)/a^(1/2)
```

maxima [A] time = 2.05, size = 53, normalized size = 1.13

$$\frac{1}{2}\sqrt{a+\frac{b}{x^2}}\,x^2-\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x^2}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^2}}+\sqrt{a}}\right)}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2)*x,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a + b/x^2)*x^2 - 1/4*b*log((sqrt(a + b/x^2) - sqrt(a))/(sqrt(a + b
/x^2) + sqrt(a)))/sqrt(a)
```

mupad [B] time = 1.38, size = 35, normalized size = 0.74

$$\frac{x^2\sqrt{a+\frac{b}{x^2}}}{2}+\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b/x^2)^(1/2),x)
```

```
[Out] (x^2*(a + b/x^2)^(1/2))/2 + (b*atanh((a + b/x^2)^(1/2)/a^(1/2)))/(2*a^(1/2)
)
```

sympy [A] time = 2.06, size = 41, normalized size = 0.87

$$\frac{\sqrt{b}\,x\sqrt{\frac{ax^2}{b}+1}}{2}+\frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**(1/2)*x,x)
```

```
[Out] sqrt(b)*x*sqrt(a*x**2/b + 1)/2 + b*asinh(sqrt(a)*x/sqrt(b))/(2*sqrt(a))
```

3.1894 $\int \sqrt{a+\frac{b}{x^2}}\,dx$

Optimal. Leaf size=42

$$x\sqrt{a+\frac{b}{x^2}}-\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)$$

```
[Out] -arctanh(b^(1/2)/x/(a+b/x^2)^(1/2))*b^(1/2)+(a+b/x^2)^(1/2)*x
```

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {242, 277, 217, 206}

$$x\sqrt{a+\frac{b}{x^2}}-\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^2],x]
```

```
[Out] Sqrt[a + b/x^2]*x - Sqrt[b]*ArcTanh[Sqrt[b]/(Sqrt[a + b/x^2]*x)]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+\frac{b}{x^2}}\,dx &= -\operatorname{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2}\,dx, x, \frac{1}{x}\right)\\ &= \sqrt{a+\frac{b}{x^2}}\,x - b\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{x}\right)\\ &= \sqrt{a+\frac{b}{x^2}}\,x - b\operatorname{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x}\right)\\ &= \sqrt{a+\frac{b}{x^2}}\,x - \sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 62, normalized size = 1.48

$$x\sqrt{a+\frac{b}{x^2}} - \frac{\sqrt{b}\,x\sqrt{a+\frac{b}{x^2}}\,\tanh^{-1}\left(\frac{\sqrt{ax^2+b}}{\sqrt{b}}\right)}{\sqrt{ax^2+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^2],x]
```

```
[Out] Sqrt[a + b/x^2]*x - (Sqrt[b]*Sqrt[a + b/x^2]*x*ArcTanh[Sqrt[b + a*x^2]/Sqrt
[b]])/Sqrt[b + a*x^2]
```

fricas [A] time = 0.92, size = 108, normalized size = 2.57

$$\left[x\sqrt{\frac{ax^2+b}{x^2}} + \frac{1}{2}\sqrt{b}\log\left(-\frac{ax^2 - 2\sqrt{b}\,x\sqrt{\frac{ax^2+b}{x^2}} + 2b}{x^2}\right), x\sqrt{\frac{ax^2+b}{x^2}} + \sqrt{-b}\arctan\left(\frac{\sqrt{-b}\,x\sqrt{\frac{ax^2+b}{x^2}}}{ax^2+b}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [x*sqrt((a*x^2 + b)/x^2) + 1/2*sqrt(b)*log(-(a*x^2 - 2*sqrt(b)*x*sqrt((a*x^
2 + b)/x^2) + 2*b)/x^2), x*sqrt((a*x^2 + b)/x^2) + sqrt(-b)*arctan(sqrt(-b)
*x*sqrt((a*x^2 + b)/x^2)/(a*x^2 + b))]
```

giac [B] time = 0.16, size = 69, normalized size = 1.64

$$\frac{b\arctan\left(\frac{\sqrt{ax^2+b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}} + \sqrt{ax^2+b}\operatorname{sgn}(x) - \frac{\left(b\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right) + \sqrt{-b}\sqrt{b}\right)\operatorname{sgn}(x)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] b*arctan(sqrt(a*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) + sqrt(a*x^2 + b)*sgn(x)
 - (b*arctan(sqrt(b)/sqrt(-b)) + sqrt(-b)*sqrt(b))*sgn(x)/sqrt(-b)
```

maple [A] time = 0.01, size = 63, normalized size = 1.50

$$-\frac{\sqrt{\frac{a\,x^2+b}{x^2}}\left(\sqrt{b}\,\ln\left(\frac{2b+2\sqrt{a\,x^2+b}\,\sqrt{b}}{x}\right) - \sqrt{a\,x^2+b}\right)x}{\sqrt{a\,x^2+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^(1/2),x)
```

```
[Out] -((a*x^2+b)/x^2)^(1/2)*x*(b^(1/2)*ln(2*(b^(1/2)*(a*x^2+b)^(1/2)+b)/x)-(a*x^
2+b)^(1/2))/(a*x^2+b)^(1/2)
```

maxima [A] time = 1.97, size = 53, normalized size = 1.26

$$\sqrt{a+\frac{b}{x^2}}\,x+\frac{1}{2}\,\sqrt{b}\,\log\left(\frac{\sqrt{a+\frac{b}{x^2}}\,x-\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x+\sqrt{b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a + b/x^2)*x + 1/2*sqrt(b)*log((sqrt(a + b/x^2)*x - sqrt(b))/(sqrt(a +
 b/x^2)*x + sqrt(b)))
```

mupad [B] time = 1.14, size = 55, normalized size = 1.31

$$x\sqrt{a+\frac{b}{x^2}}+\frac{\sqrt{b}\,\operatorname{asin}\left(\frac{\sqrt{b}\,1\mathrm{i}}{\sqrt{a}\,x}\right)\sqrt{a+\frac{b}{x^2}}\,1\mathrm{i}}{\sqrt{a}\,\sqrt{\frac{b}{a\,x^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)^(1/2),x)
```

```
[Out] x*(a + b/x^2)^(1/2) + (b^(1/2)*asin((b^(1/2)*1i)/(a^(1/2)*x))*(a + b/x^2)^(
1/2)*1i)/(a^(1/2)*(b/(a*x^2) + 1)^(1/2))
```

sympy [A] time = 1.58, size = 56, normalized size = 1.33

$$\frac{\sqrt{a}\,x}{\sqrt{1+\frac{b}{a x^2}}}-\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\,x}\right)+\frac{b}{\sqrt{a}\,x\sqrt{1+\frac{b}{a x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**(1/2),x)
```

```
[Out] sqrt(a)*x/sqrt(1 + b/(a*x**2)) - sqrt(b)*asinh(sqrt(b)/(sqrt(a)*x)) + b/(sq
rt(a)*x*sqrt(1 + b/(a*x**2)))
```

3.1895 $\int \frac{\sqrt{a+\frac{b}{x^2}}}{x}\,dx$

Optimal. Leaf size=38

$$\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)-\sqrt{a+\frac{b}{x^2}}$$

```
[Out] arctanh((a+b/x^2)^(1/2)/a^(1/2))*a^(1/2)-(a+b/x^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 50, 63, 208}

$$\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)-\sqrt{a+\frac{b}{x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^2]/x,x]
```

```
[Out] -Sqrt[a + b/x^2] + Sqrt[a]*ArcTanh[Sqrt[a + b/x^2]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+\frac{b}{x^2}}}{x}\,dx &= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x^2}\right)\right)\\ &= -\sqrt{a+\frac{b}{x^2}} - \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x^2}\right)\\ &= -\sqrt{a+\frac{b}{x^2}} - \frac{a\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x^2}}\right)}{b}\\ &= -\sqrt{a+\frac{b}{x^2}} + \sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)\end{aligned}$$

Mathematica [A] time = 0.07, size = 69, normalized size = 1.82

$$-\frac{\sqrt{a+\frac{b}{x^2}}\left(-\sqrt{a}\,\sqrt{b}\,x\sqrt{\frac{ax^2}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)+ax^2+b\right)}{ax^2+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^2]/x,x]
```

```
[Out] -((Sqrt[a + b/x^2]*(b + a*x^2 - Sqrt[a]*Sqrt[b]*x*Sqrt[1 + (a*x^2)/b]*ArcSi
nh[(Sqrt[a]*x)/Sqrt[b]]))/(b + a*x^2))
```

fricas [A] time = 0.82, size = 109, normalized size = 2.87

$$\left[\frac{1}{2}\sqrt{a}\,\log\left(-2\,ax^2-2\sqrt{a}\,x^2\sqrt{\frac{ax^2+b}{x^2}}-b\right)-\sqrt{\frac{ax^2+b}{x^2}}, -\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}\,x^2\sqrt{\frac{ax^2+b}{x^2}}}{ax^2+b}\right)-\sqrt{\frac{ax^2+b}{x^2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log(-2*a*x^2 - 2*sqrt(a)*x^2*sqrt((a*x^2 + b)/x^2) - b) - sqrt
((a*x^2 + b)/x^2), -sqrt(-a)*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + b)/x^2)/(a*x
^2 + b)) - sqrt((a*x^2 + b)/x^2)]
```

giac [B] time = 0.22, size = 61, normalized size = 1.61

$$-\frac{1}{2}\sqrt{a}\,\log\left(\left(\sqrt{a}\,x-\sqrt{ax^2+b}\right)^2\right)\text{sgn}(x)+\frac{2\sqrt{a}\,b\,\text{sgn}(x)}{\left(\sqrt{a}\,x-\sqrt{ax^2+b}\right)^2-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a)*log((sqrt(a)*x - sqrt(a*x^2 + b))^2)*sgn(x) + 2*sqrt(a)*b*sgn(
x)/((sqrt(a)*x - sqrt(a*x^2 + b))^2 - b)
```

maple [B] time = 0.01, size = 83, normalized size = 2.18

$$-\frac{\sqrt{\frac{a\,x^2+b}{x^2}}\left(-abx\ln\left(\sqrt{a}\,x+\sqrt{a\,x^2+b}\right)-\sqrt{a\,x^2+b}\,a^{\frac{3}{2}}x^2+\left(a\,x^2+b\right)^{\frac{3}{2}}\sqrt{a}\right)}{\sqrt{a\,x^2+b}\,\sqrt{a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^(1/2)/x,x)
```

```
[Out] -((a*x^2+b)/x^2)^(1/2)*(-(a*x^2+b)^(1/2)*a^(3/2)*x^2+(a*x^2+b)^(3/2)*a^(1/2
)-ln(a^(1/2)*x+(a*x^2+b)^(1/2))*x*a*b)/(a*x^2+b)^(1/2)/b/a^(1/2)
```

maxima [A] time = 1.93, size = 49, normalized size = 1.29

$$-\frac{1}{2}\sqrt{a}\,\log\left(\frac{\sqrt{a+\frac{b}{x^2}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^2}}+\sqrt{a}}\right)-\sqrt{a+\frac{b}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(a)*log((sqrt(a + b/x^2) - sqrt(a))/(sqrt(a + b/x^2) + sqrt(a))) -
 sqrt(a + b/x^2)
```

mupad [B] time = 1.30, size = 30, normalized size = 0.79

$$\sqrt{a}\,\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)-\sqrt{a+\frac{b}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)^(1/2)/x,x)
```

```
[Out] a^(1/2)*atanh((a + b/x^2)^(1/2)/a^(1/2)) - (a + b/x^2)^(1/2)
```

sympy [A] time = 1.59, size = 56, normalized size = 1.47

$$\sqrt{a}\,\operatorname{asinh}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)-\frac{ax}{\sqrt{b}\sqrt{\frac{ax^2}{b}+1}}-\frac{\sqrt{b}}{x\sqrt{\frac{ax^2}{b}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**(1/2)/x,x)
```

```
[Out] sqrt(a)*asinh(sqrt(a)*x/sqrt(b)) - a*x/(sqrt(b)*sqrt(a*x**2/b + 1)) - sqrt(
b)/(x*sqrt(a*x**2/b + 1))
```

3.1896 $\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^2}\,dx$

Optimal. Leaf size=50

$$-\frac{\sqrt{a+\frac{b}{x^2}}}{2x}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{2\sqrt{b}}$$

```
[Out] -1/2*a*arctanh(b^(1/2)/x/(a+b/x^2)^(1/2))/b^(1/2)-1/2*(a+b/x^2)^(1/2)/x
```

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {335, 195, 217, 206}

$$-\frac{\sqrt{a+\frac{b}{x^2}}}{2x}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^2]/x^2,x]
```

```
[Out] -Sqrt[a + b/x^2]/(2*x) - (a*ArcTanh[Sqrt[b]/(Sqrt[a + b/x^2]*x)])/(2*Sqrt[b
])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^2}\,dx &= -\operatorname{Subst}\left(\int \sqrt{a+bx^2}\,dx, x, \frac{1}{x}\right)\\ &= -\frac{\sqrt{a+\frac{b}{x^2}}}{2x} - \frac{1}{2}a\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{x}\right)\\ &= -\frac{\sqrt{a+\frac{b}{x^2}}}{2x} - \frac{1}{2}a\operatorname{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x}\right)\\ &= -\frac{\sqrt{a+\frac{b}{x^2}}}{2x} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x}\right)}{2\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 68, normalized size = 1.36

$$-\frac{\sqrt{a+\frac{b}{x^2}}\left(ax^2\sqrt{\frac{ax^2}{b}+1}\,\tanh^{-1}\left(\sqrt{\frac{ax^2}{b}+1}\right)+ax^2+b\right)}{2x\left(ax^2+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^2]/x^2,x]
```

```
[Out] -1/2*(Sqrt[a + b/x^2]*(b + a*x^2 + a*x^2*Sqrt[1 + (a*x^2)/b]*ArcTanh[Sqrt[1
 + (a*x^2)/b]]))/(x*(b + a*x^2))
```

fricas [A] time = 0.86, size = 129, normalized size = 2.58

$$\left[\frac{a\sqrt{b}\,x\log\left(-\frac{ax^2-2\sqrt{b}\,x\sqrt{\frac{ax^2+b}{x^2}}+2b}{x^2}\right)-2b\sqrt{\frac{ax^2+b}{x^2}}}{4bx}, \frac{a\sqrt{-b}\,x\arctan\left(\frac{\sqrt{-b}\,x\sqrt{\frac{ax^2+b}{x^2}}}{ax^2+b}\right)-b\sqrt{\frac{ax^2+b}{x^2}}}{2bx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/4*(a*sqrt(b)*x*log(-(a*x^2 - 2*sqrt(b)*x*sqrt((a*x^2 + b)/x^2) + 2*b)/x^
2) - 2*b*sqrt((a*x^2 + b)/x^2))/(b*x), 1/2*(a*sqrt(-b)*x*arctan(sqrt(-b)*x*
sqrt((a*x^2 + b)/x^2)/(a*x^2 + b)) - b*sqrt((a*x^2 + b)/x^2))/(b*x)]
```

giac [A] time = 0.21, size = 50, normalized size = 1.00

$$\frac{\frac{a^2\arctan\left(\frac{\sqrt{ax^2+b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}} - \frac{\sqrt{ax^2+b}\,a\operatorname{sgn}(x)}{x^2}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*(a^2*arctan(sqrt(a*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) - sqrt(a*x^2 + b)
*a*sgn(x)/x^2)/a
```

maple [B] time = 0.01, size = 85, normalized size = 1.70

$$-\frac{\sqrt{\frac{a\,x^2+b}{x^2}}\left(a\sqrt{b}\,x^2\ln\left(\frac{2b+2\sqrt{a\,x^2+b}\,\sqrt{b}}{x}\right)-\sqrt{a\,x^2+b}\,a\,x^2+\left(a\,x^2+b\right)^{\frac{3}{2}}\right)}{2\sqrt{a\,x^2+b}\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^(1/2)/x^2,x)
```

```
[Out] -1/2*((a*x^2+b)/x^2)^(1/2)/x*(b^(1/2)*ln(2*(b+(a*x^2+b)^(1/2)*b^(1/2))/x)*x
^2*a-(a*x^2+b)^(1/2)*x^2*a+(a*x^2+b)^(3/2))/(a*x^2+b)^(1/2)/b
```

maxima [A] time = 1.89, size = 73, normalized size = 1.46

$$-\frac{\sqrt{a+\frac{b}{x^2}}\,ax}{2\left(\left(a+\frac{b}{x^2}\right)x^2-b\right)}+\frac{a\log\left(\frac{\sqrt{a+\frac{b}{x^2}}\,x-\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x+\sqrt{b}}\right)}{4\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(a + b/x^2)*a*x/((a + b/x^2)*x^2 - b) + 1/4*a*log((sqrt(a + b/x^2)
*x - sqrt(b))/(sqrt(a + b/x^2)*x + sqrt(b)))/sqrt(b)
```

mupad [B] time = 1.41, size = 39, normalized size = 0.78

$$-\frac{\sqrt{a+\frac{b}{x^2}}}{2\,x}-\frac{a\,\ln\left(\sqrt{a+\frac{b}{x^2}}+\frac{\sqrt{b}}{x}\right)}{2\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)^(1/2)/x^2,x)
```

```
[Out] - (a + b/x^2)^(1/2)/(2*x) - (a*log((a + b/x^2)^(1/2) + b^(1/2)/x))/(2*b^(1/
2))
```

sympy [A] time = 2.25, size = 42, normalized size = 0.84

$$-\frac{\sqrt{a}\,\sqrt{1+\frac{b}{ax^2}}}{2x}-\frac{a\,\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\,x}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**(1/2)/x**2,x)
```

```
[Out] -sqrt(a)*sqrt(1 + b/(a*x**2))/(2*x) - a*asinh(sqrt(b)/(sqrt(a)*x))/(2*sqrt(
b))
```

3.1897 $\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^3}\,dx$

Optimal. Leaf size=18

$$-\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{3b}$$

```
[Out] -1/3*(a+b/x^2)^(3/2)/b
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$-\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^2]/x^3,x]
```

```
[Out] -(a + b/x^2)^(3/2)/(3*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^3}\,dx = -\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{3b}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$-\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^2]/x^3,x]
```

```
[Out] -1/3*(a + b/x^2)^(3/2)/b
```

fricas [A] time = 0.99, size = 28, normalized size = 1.56

$$-\frac{\left(ax^2+b\right)\sqrt{\frac{ax^2+b}{x^2}}}{3\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/3*(a*x^2 + b)*sqrt((a*x^2 + b)/x^2)/(b*x^2)
```

giac [B] time = 0.20, size = 63, normalized size = 3.50

$$\frac{2\left(3\left(\sqrt{a}\,x-\sqrt{ax^2+b}\right)^4 a^{\frac{3}{2}}\mathrm{sgn}(x)+a^{\frac{3}{2}}b^2\mathrm{sgn}(x)\right)}{3\left(\left(\sqrt{a}\,x-\sqrt{ax^2+b}\right)^2-b\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 2/3*(3*(sqrt(a)*x - sqrt(a*x^2 + b))^4*a^(3/2)*sgn(x) + a^(3/2)*b^2*sgn(x))
/((sqrt(a)*x - sqrt(a*x^2 + b))^2 - b)^3
```

maple [A] time = 0.00, size = 29, normalized size = 1.61

$$-\frac{\left(a\,x^2+b\right)\sqrt{\frac{a\,x^2+b}{x^2}}}{3b\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^(1/2)/x^3,x)
```

```
[Out] -1/3*(a*x^2+b)/x^2/b*((a*x^2+b)/x^2)^(1/2)
```

maxima [A] time = 0.86, size = 14, normalized size = 0.78

$$-\frac{\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/3*(a + b/x^2)^(3/2)/b
```

mupad [B] time = 1.25, size = 24, normalized size = 1.33

$$-\frac{\sqrt{a+\frac{b}{x^2}}\,\left(a\,x^2+b\right)}{3\,b\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)^(1/2)/x^3,x)
```

```
[Out] -((a + b/x^2)^(1/2)*(b + a*x^2))/(3*b*x^2)
```

sympy [B] time = 1.10, size = 42, normalized size = 2.33

$$-\frac{a^{\frac{3}{2}}\sqrt{1+\frac{b}{ax^2}}}{3b}-\frac{\sqrt{a}\,\sqrt{1+\frac{b}{ax^2}}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**(1/2)/x**3,x)
```

```
[Out] -a**(3/2)*sqrt(1 + b/(a*x**2))/(3*b) - sqrt(a)*sqrt(1 + b/(a*x**2))/(3*x**2
)
```

3.1898 $\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^4}\,dx$

Optimal. Leaf size=74

$$\frac{a^2 \tanh^{-1}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{8b^{3/2}} - \frac{a\sqrt{a+\frac{b}{x^2}}}{8bx} - \frac{\sqrt{a+\frac{b}{x^2}}}{4x^3}$$

```
[Out] 1/8*a^2*arctanh(b^(1/2)/x/(a+b/x^2)^(1/2))/b^(3/2)-1/4*(a+b/x^2)^(1/2)/x^3-
1/8*a*(a+b/x^2)^(1/2)/b/x
```

Rubi [A] time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {335, 279, 321, 217, 206}

$$\frac{a^2 \tanh^{-1}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{8b^{3/2}} - \frac{a\sqrt{a+\frac{b}{x^2}}}{8bx} - \frac{\sqrt{a+\frac{b}{x^2}}}{4x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^2]/x^4,x]
```

```
[Out] -Sqrt[a + b/x^2]/(4*x^3) - (a*Sqrt[a + b/x^2])/(8*b*x) + (a^2*ArcTanh[Sqrt[
b]/(Sqrt[a + b/x^2]*x)])/(8*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
```

egerQ[m]

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^4}\,dx &= -\text{Subst}\left(\int x^2\sqrt{a+bx^2}\,dx, x, \frac{1}{x}\right)\\ &= -\frac{\sqrt{a+\frac{b}{x^2}}}{4x^3} - \frac{1}{4}a\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{x}\right)\\ &= -\frac{\sqrt{a+\frac{b}{x^2}}}{4x^3} - \frac{a\sqrt{a+\frac{b}{x^2}}}{8bx} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{x}\right)}{8b}\\ &= -\frac{\sqrt{a+\frac{b}{x^2}}}{4x^3} - \frac{a\sqrt{a+\frac{b}{x^2}}}{8bx} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x}\right)}{8b}\\ &= -\frac{\sqrt{a+\frac{b}{x^2}}}{4x^3} - \frac{a\sqrt{a+\frac{b}{x^2}}}{8bx} + \frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x}\right)}{8b^{3/2}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 47, normalized size = 0.64

$$-\frac{a^2x\sqrt{a+\frac{b}{x^2}}\left(ax^2+b\right)\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{ax^2}{b}+1\right)}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^2]/x^4,x]
```

```
[Out] -1/3*(a^2*Sqrt[a + b/x^2]*x*(b + a*x^2)*Hypergeometric2F1[3/2, 3, 5/2, 1 +
(a*x^2)/b])/b^3
```

fricas [A] time = 0.97, size = 158, normalized size = 2.14

$$\left[\frac{a^2\sqrt{b}\,x^3\log\left(-\frac{ax^2+2\sqrt{b}\,x\sqrt{\frac{ax^2+b}{x^2}}+2b}{x^2}\right) - 2\left(abx^2+2b^2\right)\sqrt{\frac{ax^2+b}{x^2}}}{16\,b^2x^3}, -\frac{a^2\sqrt{-b}\,x^3\arctan\left(\frac{\sqrt{-b}\,x\sqrt{\frac{ax^2+b}{x^2}}}{ax^2+b}\right) + \left(abx^2+2b^2\right.}{8\,b^2x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/16*(a^2*sqrt(b)*x^3*log(-(a*x^2 + 2*sqrt(b)*x*sqrt((a*x^2 + b)/x^2) + 2*
b)/x^2) - 2*(a*b*x^2 + 2*b^2)*sqrt((a*x^2 + b)/x^2))/(b^2*x^3), -1/8*(a^2*s
qrt(-b)*x^3*arctan(sqrt(-b)*x*sqrt((a*x^2 + b)/x^2)/(a*x^2 + b)) + (a*b*x^2
 + 2*b^2)*sqrt((a*x^2 + b)/x^2))/(b^2*x^3)]
```

giac [A] time = 0.23, size = 78, normalized size = 1.05

$$-\frac{\frac{a^3\arctan\left(\frac{\sqrt{ax^2+b}}{\sqrt{-b}}\right)\text{sgn}(x)}{\sqrt{-b}\,b} + \frac{\left(ax^2+b\right)^{\frac{3}{2}}a^3\text{sgn}(x)+\sqrt{ax^2+b}\,a^3b\,\text{sgn}(x)}{a^2bx^4}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/8*(a^3*arctan(sqrt(a*x^2 + b)/sqrt(-b))*sgn(x)/(sqrt(-b)*b) + ((a*x^2 +
b)^(3/2)*a^3*sgn(x) + sqrt(a*x^2 + b)*a^3*b*sgn(x))/(a^2*b*x^4))/a
```

maple [A] time = 0.01, size = 106, normalized size = 1.43

$$\frac{\sqrt{\frac{a\,x^2+b}{x^2}}\left(a^2\sqrt{b}\,x^4\ln\left(\frac{2b+2\sqrt{a\,x^2+b}\,\sqrt{b}}{x}\right)-\sqrt{a\,x^2+b}\,a^2x^4+\left(a\,x^2+b\right)^{\frac{3}{2}}a\,x^2-2\left(a\,x^2+b\right)^{\frac{3}{2}}b\right)}{8\sqrt{a\,x^2+b}\,b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^(1/2)/x^4,x)
```

```
[Out] 1/8*((a*x^2+b)/x^2)^(1/2)/x^3*(b^(1/2)*ln(2*(b+(a*x^2+b)^(1/2)*b^(1/2))/x)*
x^4*a^2-(a*x^2+b)^(1/2)*x^4*a^2+(a*x^2+b)^(3/2)*x^2*a-2*(a*x^2+b)^(3/2)*b)/
(a*x^2+b)^(1/2)/b^2
```

maxima [A] time = 1.98, size = 114, normalized size = 1.54

$$-\frac{a^2\log\left(\frac{\sqrt{a+\frac{b}{x^2}}\,x-\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x+\sqrt{b}}\right)}{16\,b^{\frac{3}{2}}}-\frac{\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}a^2x^3+\sqrt{a+\frac{b}{x^2}}\,a^2bx}{8\left(\left(a+\frac{b}{x^2}\right)^2bx^4-2\left(a+\frac{b}{x^2}\right)b^2x^2+b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/16*a^2*log((sqrt(a + b/x^2)*x - sqrt(b))/(sqrt(a + b/x^2)*x + sqrt(b)))/
b^(3/2) - 1/8*((a + b/x^2)^(3/2)*a^2*x^3 + sqrt(a + b/x^2)*a^2*b*x)/((a + b
/x^2)^2*b*x^4 - 2*(a + b/x^2)*b^2*x^2 + b^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)^(1/2)/x^4,x)
```

```
[Out] int((a + b/x^2)^(1/2)/x^4, x)
```

sympy [A] time = 4.08, size = 92, normalized size = 1.24

$$-\frac{a^{\frac{3}{2}}}{8bx\sqrt{1+\frac{b}{ax^2}}}-\frac{3\sqrt{a}}{8x^3\sqrt{1+\frac{b}{ax^2}}}+\frac{a^2\,\mathrm{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\,x}\right)}{8b^{\frac{3}{2}}}-\frac{b}{4\sqrt{a}\,x^5\sqrt{1+\frac{b}{ax^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**(1/2)/x**4,x)
```

```
[Out] -a**(3/2)/(8*b*x*sqrt(1 + b/(a*x**2))) - 3*sqrt(a)/(8*x**3*sqrt(1 + b/(a*x*
*2))) + a**2*asinh(sqrt(b)/(sqrt(a)*x))/(8*b**(3/2)) - b/(4*sqrt(a)*x**5*sq
rt(1 + b/(a*x**2)))
```

3.1899 $\int \left(a + \frac{b}{x^2}\right)^{3/2} x^3\, dx$

Optimal. Leaf size=68

$$\frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{8\sqrt{a}} + \frac{3}{8}bx^2\sqrt{a+\frac{b}{x^2}} + \frac{1}{4}x^4\left(a+\frac{b}{x^2}\right)^{3/2}$$

```
[Out] 1/4*(a+b/x^2)^(3/2)*x^4+3/8*b^2*arctanh((a+b/x^2)^(1/2)/a^(1/2))/a^(1/2)+3/
8*b*x^2*(a+b/x^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 47, 63, 208}

$$\frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{8\sqrt{a}} + \frac{1}{4}x^4\left(a+\frac{b}{x^2}\right)^{3/2} + \frac{3}{8}bx^2\sqrt{a+\frac{b}{x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^(3/2)*x^3,x]
```

```
[Out] (3*b*Sqrt[a + b/x^2]*x^2)/8 + ((a + b/x^2)^(3/2)*x^4)/4 + (3*b^2*ArcTanh[Sq
rt[a + b/x^2]/Sqrt[a]])/(8*Sqrt[a])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x^2}\right)^{3/2} x^3\,dx &= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^3}\,dx, x, \frac{1}{x^2}\right)\right)\\ &= \frac{1}{4}\left(a+\frac{b}{x^2}\right)^{3/2} x^4 - \frac{1}{8}(3b)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx, x, \frac{1}{x^2}\right)\\ &= \frac{3}{8}b\sqrt{a+\frac{b}{x^2}}\,x^2 + \frac{1}{4}\left(a+\frac{b}{x^2}\right)^{3/2} x^4 - \frac{1}{16}\left(3b^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x^2}\right)\\ &= \frac{3}{8}b\sqrt{a+\frac{b}{x^2}}\,x^2 + \frac{1}{4}\left(a+\frac{b}{x^2}\right)^{3/2} x^4 - \frac{1}{8}(3b)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x^2}}\right)\\ &= \frac{3}{8}b\sqrt{a+\frac{b}{x^2}}\,x^2 + \frac{1}{4}\left(a+\frac{b}{x^2}\right)^{3/2} x^4 + \frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{8\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.15, size = 66, normalized size = 0.97

$$\frac{1}{8}x\sqrt{a+\frac{b}{x^2}}\left(\frac{3b^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\,\sqrt{\frac{ax^2}{b}+1}} + 2ax^3 + 5bx\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^(3/2)*x^3,x]
```

```
[Out] (Sqrt[a + b/x^2]*x*(5*b*x + 2*a*x^3 + (3*b^(3/2)*ArcSinh[(Sqrt[a]*x)/Sqrt[b
]])/(Sqrt[a]*Sqrt[1 + (a*x^2)/b])))/8
```

fricas [A] time = 0.76, size = 157, normalized size = 2.31

$$\left[\frac{3\sqrt{a}\,b^2\log\left(-2ax^2-2\sqrt{a}\,x^2\sqrt{\frac{ax^2+b}{x^2}}-b\right)+2\left(2a^2x^4+5abx^2\right)\sqrt{\frac{ax^2+b}{x^2}}}{16a}, -\frac{3\sqrt{-a}\,b^2\arctan\left(\frac{\sqrt{-a}\,x^2\sqrt{\frac{ax^2+b}{x^2}}}{ax^2+b}\right)-\left(2a\right.}{8a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(3/2)*x^3,x, algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(a)*b^2*log(-2*a*x^2 - 2*sqrt(a)*x^2*sqrt((a*x^2 + b)/x^2) - b
) + 2*(2*a^2*x^4 + 5*a*b*x^2)*sqrt((a*x^2 + b)/x^2))/a, -1/8*(3*sqrt(-a)*b^
2*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + b)/x^2)/(a*x^2 + b)) - (2*a^2*x^4 + 5*a
*b*x^2)*sqrt((a*x^2 + b)/x^2))/a]
```

giac [A] time = 0.19, size = 68, normalized size = 1.00

$$-\frac{3\,b^2\log\left(\left|-\sqrt{a}\,x+\sqrt{ax^2+b}\right|\right)\operatorname{sgn}(x)}{8\sqrt{a}}+\frac{3\,b^2\log\left(|b|\right)\operatorname{sgn}(x)}{16\sqrt{a}}+\frac{1}{8}\left(2\,ax^2\operatorname{sgn}(x)+5\,b\operatorname{sgn}(x)\right)\sqrt{ax^2+b}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(3/2)*x^3,x, algorithm="giac")
```

```
[Out] -3/8*b^2*log(abs(-sqrt(a)*x + sqrt(a*x^2 + b)))*sgn(x)/sqrt(a) + 3/16*b^2*l
og(abs(b))*sgn(x)/sqrt(a) + 1/8*(2*a*x^2*sgn(x) + 5*b*sgn(x))*sqrt(a*x^2 +
b)*x
```

maple [A] time = 0.00, size = 84, normalized size = 1.24

$$\frac{\left(\frac{a x^2+b}{x^2}\right)^{\frac{3}{2}}\left(3b^2\ln\left(\sqrt{a}\,x+\sqrt{a\,x^2+b}\right)+3\sqrt{a\,x^2+b}\,\sqrt{a}\,bx+2\left(a\,x^2+b\right)^{\frac{3}{2}}\sqrt{a}\,x\right)x^3}{8\left(a\,x^2+b\right)^{\frac{3}{2}}\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^(3/2)*x^3,x)
```

```
[Out] 1/8*((a*x^2+b)/x^2)^(3/2)*x^3*(2*(a*x^2+b)^(3/2)*a^(1/2)*x+3*(a*x^2+b)^(1/2
)*a^(1/2)*b*x+3*b^2*ln(a^(1/2)*x+(a*x^2+b)^(1/2)))/(a*x^2+b)^(3/2)/a^(1/2)
```

maxima [A] time = 1.93, size = 98, normalized size = 1.44

$$-\frac{3\,b^2\log\left(\frac{\sqrt{a+\frac{b}{x^2}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^2}}+\sqrt{a}}\right)}{16\sqrt{a}}+\frac{5\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}b^2-3\sqrt{a+\frac{b}{x^2}}\,ab^2}{8\left(\left(a+\frac{b}{x^2}\right)^2-2\left(a+\frac{b}{x^2}\right)a+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(3/2)*x^3,x, algorithm="maxima")
```

```
[Out] -3/16*b^2*log((sqrt(a + b/x^2) - sqrt(a))/(sqrt(a + b/x^2) + sqrt(a)))/sqrt
(a) + 1/8*(5*(a + b/x^2)^(3/2)*b^2 - 3*sqrt(a + b/x^2)*a*b^2)/((a + b/x^2)^
2 - 2*(a + b/x^2)*a + a^2)
```

mupad [B] time = 1.65, size = 52, normalized size = 0.76

$$\frac{5\,x^4\left(a+\frac{b}{x^2}\right)^{3/2}}{8}+\frac{3\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{8\sqrt{a}}-\frac{3\,a\,x^4\sqrt{a+\frac{b}{x^2}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b/x^2)^(3/2),x)
```

```
[Out] (5*x^4*(a + b/x^2)^(3/2))/8 + (3*b^2*atanh((a + b/x^2)^(1/2)/a^(1/2)))/(8*a
^(1/2)) - (3*a*x^4*(a + b/x^2)^(1/2))/8
```

sympy [A] time = 3.53, size = 70, normalized size = 1.03

$$\frac{a\sqrt{b}\,x^3\sqrt{\frac{ax^2}{b}+1}}{4}+\frac{5b^{\frac{3}{2}}x\sqrt{\frac{ax^2}{b}+1}}{8}+\frac{3b^2\,\mathrm{asinh}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**(3/2)*x**3,x)
```

```
[Out] a*sqrt(b)*x**3*sqrt(a*x**2/b + 1)/4 + 5*b**(3/2)*x*sqrt(a*x**2/b + 1)/8 + 3
*b**2*asinh(sqrt(a)*x/sqrt(b))/(8*sqrt(a))
```

3.1900 $\int \left(a+\frac{b}{x^2}\right)^{3/2} x^2\,dx$

Optimal. Leaf size=61

$$-b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)+bx\sqrt{a+\frac{b}{x^2}}+\frac{1}{3}x^3\left(a+\frac{b}{x^2}\right)^{3/2}$$

```
[Out] 1/3*(a+b/x^2)^(3/2)*x^3-b^(3/2)*arctanh(b^(1/2)/x/(a+b/x^2)^(1/2))+b*x*(a+b
/x^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {335, 277, 217, 206}

$$-b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)+\frac{1}{3}x^3\left(a+\frac{b}{x^2}\right)^{3/2}+bx\sqrt{a+\frac{b}{x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^(3/2)*x^2,x]
```

```
[Out] b*Sqrt[a + b/x^2]*x + ((a + b/x^2)^(3/2)*x^3)/3 - b^(3/2)*ArcTanh[Sqrt[b]/(
Sqrt[a + b/x^2]*x)]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x^2}\right)^{3/2} x^2\,dx &= -\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x^4}\,dx, x, \frac{1}{x}\right)\\ &= \frac{1}{3}\left(a+\frac{b}{x^2}\right)^{3/2} x^3 - b\,\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2}\,dx, x, \frac{1}{x}\right)\\ &= b\sqrt{a+\frac{b}{x^2}}\,x + \frac{1}{3}\left(a+\frac{b}{x^2}\right)^{3/2} x^3 - b^2\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{x}\right)\\ &= b\sqrt{a+\frac{b}{x^2}}\,x + \frac{1}{3}\left(a+\frac{b}{x^2}\right)^{3/2} x^3 - b^2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x}\right)\\ &= b\sqrt{a+\frac{b}{x^2}}\,x + \frac{1}{3}\left(a+\frac{b}{x^2}\right)^{3/2} x^3 - b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x}\right)\end{aligned}$$

Mathematica [A] time = 0.06, size = 74, normalized size = 1.21

$$\frac{x\sqrt{a+\frac{b}{x^2}}\left(\sqrt{ax^2+b}\left(ax^2+4b\right)-3b^{3/2}\tanh^{-1}\left(\frac{\sqrt{ax^2+b}}{\sqrt{b}}\right)\right)}{3\sqrt{ax^2+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^(3/2)*x^2,x]
```

```
[Out] (Sqrt[a + b/x^2]*x*(Sqrt[b + a*x^2]*(4*b + a*x^2) - 3*b^(3/2)*ArcTanh[Sqrt[
b + a*x^2]/Sqrt[b]]))/(3*Sqrt[b + a*x^2])
```

fricas [A] time = 0.94, size = 129, normalized size = 2.11

$$\left[\frac{1}{2}\,b^{\frac{3}{2}}\log\left(-\frac{ax^2-2\sqrt{b}\,x\sqrt{\frac{ax^2+b}{x^2}}+2\,b}{x^2}\right)+\frac{1}{3}\left(ax^3+4\,bx\right)\sqrt{\frac{ax^2+b}{x^2}}\,,\ \sqrt{-b}\,b\arctan\left(\frac{\sqrt{-b}\,x\sqrt{\frac{ax^2+b}{x^2}}}{ax^2+b}\right)+\frac{1}{3}\left(ax^3+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(3/2)*x^2,x, algorithm="fricas")
```

```
[Out] [1/2*b^(3/2)*log(-(a*x^2 - 2*sqrt(b)*x*sqrt((a*x^2 + b)/x^2) + 2*b)/x^2) +
1/3*(a*x^3 + 4*b*x)*sqrt((a*x^2 + b)/x^2), sqrt(-b)*b*arctan(sqrt(-b)*x*sqr
t((a*x^2 + b)/x^2)/(a*x^2 + b)) + 1/3*(a*x^3 + 4*b*x)*sqrt((a*x^2 + b)/x^2)
]
```

giac [A] time = 0.18, size = 89, normalized size = 1.46

$$\frac{b^2\arctan\left(\frac{\sqrt{ax^2+b}}{\sqrt{-b}}\right)\text{sgn}(x)}{\sqrt{-b}}+\frac{1}{3}\left(ax^2+b\right)^{\frac{3}{2}}\text{sgn}(x)+\sqrt{ax^2+b}\,b\,\text{sgn}(x)-\frac{\left(3\,b^2\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+4\sqrt{-b}\,b^{\frac{3}{2}}\right)\text{sgn}(x)}{3\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(3/2)*x^2,x, algorithm="giac")
```

```
[Out] b^2*arctan(sqrt(a*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) + 1/3*(a*x^2 + b)^(3/2
)*sgn(x) + sqrt(a*x^2 + b)*b*sgn(x) - 1/3*(3*b^2*arctan(sqrt(b)/sqrt(-b)) +
 4*sqrt(-b)*b^(3/2))*sgn(x)/sqrt(-b)
```

maple [A] time = 0.01, size = 76, normalized size = 1.25

$$\frac{\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{3}{2}}\left(-3b^{\frac{3}{2}}\ln\left(\frac{2b+2\sqrt{a\,x^2+b}\,\sqrt{b}}{x}\right)+3\sqrt{a\,x^2+b}\,b+\left(a\,x^2+b\right)^{\frac{3}{2}}\right)x^3}{3\left(a\,x^2+b\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^(3/2)*x^2,x)
```

```
[Out] 1/3*((a*x^2+b)/x^2)^(3/2)*x^3*((a*x^2+b)^(3/2)-3*b^(3/2)*ln(2*(b+(a*x^2+b)^
(1/2)*b^(1/2))/x)+3*(a*x^2+b)^(1/2)*b)/(a*x^2+b)^(3/2)
```

maxima [A] time = 1.86, size = 68, normalized size = 1.11

$$\frac{1}{3}\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}x^3+\sqrt{a+\frac{b}{x^2}}\,bx+\frac{1}{2}\,b^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x^2}}\,x-\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x+\sqrt{b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(3/2)*x^2,x, algorithm="maxima")
```

```
[Out] 1/3*(a + b/x^2)^(3/2)*x^3 + sqrt(a + b/x^2)*b*x + 1/2*b^(3/2)*log((sqrt(a +
 b/x^2)*x - sqrt(b))/(sqrt(a + b/x^2)*x + sqrt(b)))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^2\left(a+\frac{b}{x^2}\right)^{3/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b/x^2)^(3/2),x)
```

```
[Out] int(x^2*(a + b/x^2)^(3/2), x)
```

sympy [A] time = 2.42, size = 78, normalized size = 1.28

$$\frac{a\sqrt{b}\,x^2\sqrt{\frac{ax^2}{b}+1}}{3}+\frac{4b^{\frac{3}{2}}\sqrt{\frac{ax^2}{b}+1}}{3}+\frac{b^{\frac{3}{2}}\log\left(\frac{ax^2}{b}\right)}{2}-b^{\frac{3}{2}}\log\left(\sqrt{\frac{ax^2}{b}+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**(3/2)*x**2,x)
```

```
[Out] a*sqrt(b)*x**2*sqrt(a*x**2/b + 1)/3 + 4*b**(3/2)*sqrt(a*x**2/b + 1)/3 + b**
(3/2)*log(a*x**2/b)/2 - b**(3/2)*log(sqrt(a*x**2/b + 1) + 1)
```

3.1901 $\int \left(a + \frac{b}{x^2}\right)^{3/2} x\,dx$

Optimal. Leaf size=63

$$\frac{1}{2}x^2\left(a+\frac{b}{x^2}\right)^{3/2} - \frac{3}{2}b\sqrt{a+\frac{b}{x^2}} + \frac{3}{2}\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)$$

```
[Out] 1/2*(a+b/x^2)^(3/2)*x^2+3/2*b*arctanh((a+b/x^2)^(1/2)/a^(1/2))*a^(1/2)-3/2*
b*(a+b/x^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {266, 47, 50, 63, 208}

$$\frac{1}{2}x^2\left(a+\frac{b}{x^2}\right)^{3/2} - \frac{3}{2}b\sqrt{a+\frac{b}{x^2}} + \frac{3}{2}\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^(3/2)*x,x]
```

```
[Out] (-3*b*Sqrt[a + b/x^2])/2 + ((a + b/x^2)^(3/2)*x^2)/2 + (3*Sqrt[a]*b*ArcTanh
[Sqrt[a + b/x^2]/Sqrt[a]])/2
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a+\frac{b}{x^2}\right)^{3/2} x\,dx &= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2}\,dx, x, \frac{1}{x^2}\right)\right) \\
&= \frac{1}{2}\left(a+\frac{b}{x^2}\right)^{3/2} x^2 - \frac{1}{4}(3b)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x^2}\right) \\
&= -\frac{3}{2}b\sqrt{a+\frac{b}{x^2}} + \frac{1}{2}\left(a+\frac{b}{x^2}\right)^{3/2} x^2 - \frac{1}{4}(3ab)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x^2}\right) \\
&= -\frac{3}{2}b\sqrt{a+\frac{b}{x^2}} + \frac{1}{2}\left(a+\frac{b}{x^2}\right)^{3/2} x^2 - \frac{1}{2}(3a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x^2}}\right) \\
&= -\frac{3}{2}b\sqrt{a+\frac{b}{x^2}} + \frac{1}{2}\left(a+\frac{b}{x^2}\right)^{3/2} x^2 + \frac{3}{2}\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 47, normalized size = 0.75

$$-\frac{b\sqrt{a+\frac{b}{x^2}}\;{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{ax^2}{b}\right)}{\sqrt{\frac{ax^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^(3/2)*x,x]
```

```
[Out] -((b*Sqrt[a + b/x^2]*Hypergeometric2F1[-3/2, -1/2, 1/2, -((a*x^2)/b)])/Sqrt
[1 + (a*x^2)/b])
```

fricas [A] time = 0.84, size = 129, normalized size = 2.05

$$\left[\frac{3}{4}\sqrt{a}\,b\log\left(-2\,ax^2-2\sqrt{a}\,x^2\sqrt{\frac{ax^2+b}{x^2}}-b\right)+\frac{1}{2}\left(ax^2-2\,b\right)\sqrt{\frac{ax^2+b}{x^2}},-\frac{3}{2}\sqrt{-a}\,b\arctan\left(\frac{\sqrt{-a}\,x^2\sqrt{\frac{ax^2+b}{x^2}}}{ax^2+b}\right)+\frac{1}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(3/2)*x,x, algorithm="fricas")
```

```
[Out] [3/4*sqrt(a)*b*log(-2*a*x^2 - 2*sqrt(a)*x^2*sqrt((a*x^2 + b)/x^2) - b) + 1/
2*(a*x^2 - 2*b)*sqrt((a*x^2 + b)/x^2), -3/2*sqrt(-a)*b*arctan(sqrt(-a)*x^2*
sqrt((a*x^2 + b)/x^2)/(a*x^2 + b)) + 1/2*(a*x^2 - 2*b)*sqrt((a*x^2 + b)/x^2
)]
```

giac [A] time = 0.28, size = 79, normalized size = 1.25

$$\frac{1}{2}\sqrt{ax^2+b}\,ax\,\text{sgn}(x) - \frac{3}{4}\sqrt{a}\,b\log\left(\left(\sqrt{a}\,x-\sqrt{ax^2+b}\right)^2\right)\text{sgn}(x) + \frac{2\sqrt{a}\,b^2\text{sgn}(x)}{\left(\sqrt{a}\,x-\sqrt{ax^2+b}\right)^2-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(3/2)*x,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(a*x^2 + b)*a*x*sgn(x) - 3/4*sqrt(a)*b*log((sqrt(a)*x - sqrt(a*x^2
+ b))^2)*sgn(x) + 2*sqrt(a)*b^2*sgn(x)/((sqrt(a)*x - sqrt(a*x^2 + b))^2 - b
)
```

maple [B] time = 0.01, size = 107, normalized size = 1.70

$$-\frac{\left(\frac{a x^2+b}{x^2}\right)^{\frac{3}{2}}\left(-3a\,b^2 x\ln\left(\sqrt{a}\,x+\sqrt{a\,x^2+b}\right)-3\sqrt{a\,x^2+b}\;a^{\frac{3}{2}}b\,x^2-2\left(a\,x^2+b\right)^{\frac{3}{2}}a^{\frac{3}{2}}x^2+2\left(a\,x^2+b\right)^{\frac{5}{2}}\sqrt{a}\right)x^2}{2\left(a\,x^2+b\right)^{\frac{3}{2}}\sqrt{a}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^(3/2)*x,x)
```

```
[Out] -1/2*((a*x^2+b)/x^2)^(3/2)*x^2*(-2*(a*x^2+b)^(3/2)*a^(3/2)*x^2+2*(a*x^2+b)^
(5/2)*a^(1/2)-3*(a*x^2+b)^(1/2)*a^(3/2)*x^2*b-3*ln(a^(1/2)*x+(a*x^2+b)^(1/2
))*x*a*b^2)/(a*x^2+b)^(3/2)/b/a^(1/2)
```

maxima [A] time = 1.97, size = 66, normalized size = 1.05

$$\frac{1}{2}\sqrt{a+\frac{b}{x^2}}\,ax^2-\frac{3}{4}\sqrt{a}\,b\log\left(\frac{\sqrt{a+\frac{b}{x^2}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^2}}+\sqrt{a}}\right)-\sqrt{a+\frac{b}{x^2}}\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(3/2)*x,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a + b/x^2)*a*x^2 - 3/4*sqrt(a)*b*log((sqrt(a + b/x^2) - sqrt(a))/(
sqrt(a + b/x^2) + sqrt(a))) - sqrt(a + b/x^2)*b
```

mupad [B] time = 1.56, size = 48, normalized size = 0.76

$$\frac{a\,x^2\sqrt{a+\frac{b}{x^2}}}{2}-b\sqrt{a+\frac{b}{x^2}}+\frac{3\sqrt{a}\;b\,\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b/x^2)^(3/2),x)
```

```
[Out] (a*x^2*(a + b/x^2)^(1/2))/2 - b*(a + b/x^2)^(1/2) + (3*a^(1/2)*b*atanh((a +
 b/x^2)^(1/2)/a^(1/2)))/2
```

sympy [A] time = 2.72, size = 88, normalized size = 1.40

$$\frac{3\sqrt{a}\,b\,\operatorname{asinh}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2}+\frac{a^2x^3}{2\sqrt{b}\,\sqrt{\frac{ax^2}{b}+1}}-\frac{a\sqrt{b}\,x}{2\sqrt{\frac{ax^2}{b}+1}}-\frac{b^{\frac{3}{2}}}{x\sqrt{\frac{ax^2}{b}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**(3/2)*x,x)
```

```
[Out] 3*sqrt(a)*b*asinh(sqrt(a)*x/sqrt(b))/2 + a**2*x**3/(2*sqrt(b)*sqrt(a*x**2/b
 + 1)) - a*sqrt(b)*x/(2*sqrt(a*x**2/b + 1)) - b**(3/2)/(x*sqrt(a*x**2/b + 1
))
```

3.1902 $\int \left(a + \frac{b}{x^2}\right)^{3/2} dx$

Optimal. Leaf size=64

$$x\left(a + \frac{b}{x^2}\right)^{3/2} - \frac{3b\sqrt{a + \frac{b}{x^2}}}{2x} - \frac{3}{2}a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}}{x\sqrt{a + \frac{b}{x^2}}}\right)$$

```
[Out] (a+b/x^2)^(3/2)*x-3/2*a*arctanh(b^(1/2)/x/(a+b/x^2)^(1/2))*b^(1/2)-3/2*b*(a
+b/x^2)^(1/2)/x
```

Rubi [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {242, 277, 195, 217, 206}

$$x\left(a + \frac{b}{x^2}\right)^{3/2} - \frac{3b\sqrt{a + \frac{b}{x^2}}}{2x} - \frac{3}{2}a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}}{x\sqrt{a + \frac{b}{x^2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^(3/2),x]
```

```
[Out] (-3*b*Sqrt[a + b/x^2])/(2*x) + (a + b/x^2)^(3/2)*x - (3*a*Sqrt[b]*ArcTanh[S
qrt[b]/(Sqrt[a + b/x^2]*x)])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x^2}\right)^{3/2} dx &= -\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x^2}\, dx, x, \frac{1}{x}\right)\\ &= \left(a+\frac{b}{x^2}\right)^{3/2} x - (3b)\,\text{Subst}\left(\int \sqrt{a+bx^2}\, dx, x, \frac{1}{x}\right)\\ &= -\frac{3b\sqrt{a+\frac{b}{x^2}}}{2x} + \left(a+\frac{b}{x^2}\right)^{3/2} x - \frac{1}{2}(3ab)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\, dx, x, \frac{1}{x}\right)\\ &= -\frac{3b\sqrt{a+\frac{b}{x^2}}}{2x} + \left(a+\frac{b}{x^2}\right)^{3/2} x - \frac{1}{2}(3ab)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\, dx, x, \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x}\right)\\ &= -\frac{3b\sqrt{a+\frac{b}{x^2}}}{2x} + \left(a+\frac{b}{x^2}\right)^{3/2} x - \frac{3}{2}a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x}\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 47, normalized size = 0.73

$$\frac{ax^3\left(a+\frac{b}{x^2}\right)^{3/2}\left(ax^2+b\right)\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{ax^2}{b}+1\right)}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^(3/2),x]
```

```
[Out] (a*(a + b/x^2)^(3/2)*x^3*(b + a*x^2)*Hypergeometric2F1[2, 5/2, 7/2, 1 + (a*
x^2)/b])/(5*b^2)
```

fricas [A] time = 1.00, size = 142, normalized size = 2.22

$$\left[\frac{3\,a\sqrt{b}\,x\log\left(-\frac{ax^2-2\sqrt{b}\,x\sqrt{\frac{ax^2+b}{x^2}}+2\,b}{x^2}\right)+2\left(2\,ax^2-b\right)\sqrt{\frac{ax^2+b}{x^2}}}{4\,x},\ \frac{3\,a\sqrt{-b}\,x\arctan\left(\frac{\sqrt{-b}\,x\sqrt{\frac{ax^2+b}{x^2}}}{ax^2+b}\right)+\left(2\,ax^2-b\right)\sqrt{\frac{ax^2}{x^2}}}{2\,x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*a*sqrt(b)*x*log(-(a*x^2 - 2*sqrt(b)*x*sqrt((a*x^2 + b)/x^2) + 2*b)/
x^2) + 2*(2*a*x^2 - b)*sqrt((a*x^2 + b)/x^2))/x, 1/2*(3*a*sqrt(-b)*x*arctan
(sqrt(-b)*x*sqrt((a*x^2 + b)/x^2)/(a*x^2 + b)) + (2*a*x^2 - b)*sqrt((a*x^2
+ b)/x^2))/x]
```

giac [A] time = 0.21, size = 69, normalized size = 1.08

$$\frac{\frac{3\,a^2b\arctan\left(\frac{\sqrt{ax^2+b}}{\sqrt{-b}}\right)\text{sgn}(x)}{\sqrt{-b}}+2\sqrt{ax^2+b}\,a^2\text{sgn}(x)-\frac{\sqrt{ax^2+b}\,ab\text{sgn}(x)}{x^2}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(3*a^2*b*arctan(sqrt(a*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) + 2*sqrt(a*x^
2 + b)*a^2*sgn(x) - sqrt(a*x^2 + b)*a*b*sgn(x)/x^2)/a
```

maple [A] time = 0.01, size = 100, normalized size = 1.56

$$-\frac{\left(\frac{a x^2+b}{x^2}\right)^{\frac{3}{2}}\left(3a\,b^{\frac{3}{2}}x^2\ln\left(\frac{2b+2\sqrt{a x^2+b}\,\sqrt{b}}{x}\right)-3\sqrt{a x^2+b}\,ab\,x^2-\left(a x^2+b\right)^{\frac{3}{2}}a x^2+\left(a x^2+b\right)^{\frac{5}{2}}\right)x}{2\left(a x^2+b\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^(3/2),x)
```

```
[Out] -1/2*((a*x^2+b)/x^2)^(3/2)*x*(-(a*x^2+b)^(3/2)*a*x^2+3*b^(3/2)*ln(2*(b+(a*x
^2+b)^(1/2)*b^(1/2))/x)*x^2*a+(a*x^2+b)^(5/2)-3*(a*x^2+b)^(1/2)*x^2*a*b)/(a
*x^2+b)^(3/2)/b
```

maxima [A] time = 1.94, size = 86, normalized size = 1.34

$$\sqrt{a+\frac{b}{x^2}}\,ax-\frac{\sqrt{a+\frac{b}{x^2}}\,abx}{2\left(\left(a+\frac{b}{x^2}\right)x^2-b\right)}+\frac{3}{4}\,a\sqrt{b}\,\log\left(\frac{\sqrt{a+\frac{b}{x^2}}\,x-\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x+\sqrt{b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] sqrt(a + b/x^2)*a*x - 1/2*sqrt(a + b/x^2)*a*b*x/((a + b/x^2)*x^2 - b) + 3/4
*a*sqrt(b)*log((sqrt(a + b/x^2)*x - sqrt(b))/(sqrt(a + b/x^2)*x + sqrt(b)))
```

mupad [B] time = 1.31, size = 36, normalized size = 0.56

$$\frac{x\left(a x^2+b\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b}{a x^2}\right)}{\left(\frac{b}{a}+x^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)^(3/2),x)
```

```
[Out] (x*(b + a*x^2)^(3/2)*hypergeom([-3/2, -1/2], 1/2, -b/(a*x^2)))/(b/a + x^2)^
(3/2)
```

sympy [A] time = 2.81, size = 88, normalized size = 1.38

$$\frac{a^{\frac{3}{2}}x}{\sqrt{1+\frac{b}{ax^2}}}+\frac{\sqrt{a}\,b}{2x\sqrt{1+\frac{b}{ax^2}}}-\frac{3a\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\,x}\right)}{2}-\frac{b^2}{2\sqrt{a}\,x^3\sqrt{1+\frac{b}{ax^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**(3/2),x)
```

```
[Out] a**(3/2)*x/sqrt(1 + b/(a*x**2)) + sqrt(a)*b/(2*x*sqrt(1 + b/(a*x**2))) - 3*
a*sqrt(b)*asinh(sqrt(b)/(sqrt(a)*x))/2 - b**2/(2*sqrt(a)*x**3*sqrt(1 + b/(a
*x**2)))
```

3.1903 $\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=54

$$a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)-a\sqrt{a+\frac{b}{x^2}}-\frac{1}{3}\left(a+\frac{b}{x^2}\right)^{3/2}$$

```
[Out] -1/3*(a+b/x^2)^(3/2)+a^(3/2)*arctanh((a+b/x^2)^(1/2)/a^(1/2))-a*(a+b/x^2)^(
1/2)
```

Rubi [A] time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 50, 63, 208}

$$a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)-a\sqrt{a+\frac{b}{x^2}}-\frac{1}{3}\left(a+\frac{b}{x^2}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^(3/2)/x,x]
```

```
[Out] -(a*Sqrt[a + b/x^2]) - (a + b/x^2)^(3/2)/3 + a^(3/2)*ArcTanh[Sqrt[a + b/x^2
]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x}\,dx &= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, \frac{1}{x^2}\right)\right)\\ &= -\frac{1}{3}\left(a+\frac{b}{x^2}\right)^{3/2} - \frac{1}{2}a\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x^2}\right)\\ &= -a\sqrt{a+\frac{b}{x^2}} - \frac{1}{3}\left(a+\frac{b}{x^2}\right)^{3/2} - \frac{1}{2}a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x^2}\right)\\ &= -a\sqrt{a+\frac{b}{x^2}} - \frac{1}{3}\left(a+\frac{b}{x^2}\right)^{3/2} - \frac{a^2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x^2}}\right)}{b}\\ &= -a\sqrt{a+\frac{b}{x^2}} - \frac{1}{3}\left(a+\frac{b}{x^2}\right)^{3/2} + a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)\end{aligned}$$

Mathematica [C] time = 0.03, size = 52, normalized size = 0.96

$$-\frac{b\sqrt{a+\frac{b}{x^2}}\;{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};-\frac{ax^2}{b}\right)}{3x^2\sqrt{\frac{ax^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^(3/2)/x,x]
```

```
[Out] -1/3*(b*Sqrt[a + b/x^2]*Hypergeometric2F1[-3/2, -3/2, -1/2, -((a*x^2)/b)])/
(x^2*Sqrt[1 + (a*x^2)/b])
```

fricas [A] time = 0.87, size = 141, normalized size = 2.61

$$\left[\frac{3\,a^{\frac{3}{2}}x^2\log\left(-2\,ax^2-2\,\sqrt{a}\,x^2\sqrt{\frac{ax^2+b}{x^2}}-b\right)-2\left(4\,ax^2+b\right)\sqrt{\frac{ax^2+b}{x^2}}}{6\,x^2}, -\frac{3\,\sqrt{-a}\,ax^2\arctan\left(\frac{\sqrt{-a}\,x^2\sqrt{\frac{ax^2+b}{x^2}}}{ax^2+b}\right)+\left(4\,ax^2+\right.}{3\,x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/6*(3*a^(3/2)*x^2*log(-2*a*x^2 - 2*sqrt(a)*x^2*sqrt((a*x^2 + b)/x^2) - b)
 - 2*(4*a*x^2 + b)*sqrt((a*x^2 + b)/x^2))/x^2, -1/3*(3*sqrt(-a)*a*x^2*arcta
n(sqrt(-a)*x^2*sqrt((a*x^2 + b)/x^2)/(a*x^2 + b)) + (4*a*x^2 + b)*sqrt((a*x
^2 + b)/x^2))/x^2]
```

giac [B] time = 0.49, size = 122, normalized size = 2.26

$$-\frac{1}{2}\,a^{\frac{3}{2}}\log\left(\left(\sqrt{a}\,x-\sqrt{ax^2+b}\right)^2\right)\mathrm{sgn}(x)+\frac{4\left(3\left(\sqrt{a}\,x-\sqrt{ax^2+b}\right)^4 a^{\frac{3}{2}}b\,\mathrm{sgn}(x)-3\left(\sqrt{a}\,x-\sqrt{ax^2+b}\right)^2 a^{\frac{3}{2}}b^2\mathrm{sgn}(x)\right.}{3\left(\left(\sqrt{a}\,x-\sqrt{ax^2+b}\right)^2-b\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] -1/2*a^(3/2)*log((sqrt(a)*x - sqrt(a*x^2 + b))^2)*sgn(x) + 4/3*(3*(sqrt(a)*
x - sqrt(a*x^2 + b))^4*a^(3/2)*b*sgn(x) - 3*(sqrt(a)*x - sqrt(a*x^2 + b))^2
*a^(3/2)*b^2*sgn(x) + 2*a^(3/2)*b^3*sgn(x))/((sqrt(a)*x - sqrt(a*x^2 + b))^
2 - b)^3
```

maple [B] time = 0.01, size = 125, normalized size = 2.31

$$-\frac{\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{3}{2}}\left(-3a^2b^2x^3\ln\left(\sqrt{a}\,x+\sqrt{a\,x^2+b}\right)-3\sqrt{a\,x^2+b}\;a^{\frac{5}{2}}b\,x^4-2\left(a\,x^2+b\right)^{\frac{3}{2}}a^{\frac{5}{2}}x^4+2\left(a\,x^2+b\right)^{\frac{5}{2}}a^{\frac{3}{2}}x^2+\right.}{3\left(a\,x^2+b\right)^{\frac{3}{2}}\sqrt{a}\;b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^(3/2)/x,x)
```

```
[Out] -1/3*((a*x^2+b)/x^2)^(3/2)*(-2*(a*x^2+b)^(3/2)*a^(5/2)*x^4+2*(a*x^2+b)^(5/2
)*a^(3/2)*x^2-3*(a*x^2+b)^(1/2)*a^(5/2)*x^4*b-3*ln(a^(1/2)*x+(a*x^2+b)^(1/2
))*x^3*a^2*b^2+(a*x^2+b)^(5/2)*b*a^(1/2))/(a*x^2+b)^(3/2)/b^2/a^(1/2)
```

maxima [A] time = 1.94, size = 61, normalized size = 1.13

$$-\frac{1}{2}\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x^2}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^2}}+\sqrt{a}}\right)-\frac{1}{3}\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}-\sqrt{a+\frac{b}{x^2}}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] -1/2*a^(3/2)*log((sqrt(a + b/x^2) - sqrt(a))/(sqrt(a + b/x^2) + sqrt(a))) -
 1/3*(a + b/x^2)^(3/2) - sqrt(a + b/x^2)*a
```

mupad [B] time = 1.41, size = 42, normalized size = 0.78

$$a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)-a\sqrt{a+\frac{b}{x^2}}-\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)^(3/2)/x,x)
```

```
[Out] a^(3/2)*atanh((a + b/x^2)^(1/2)/a^(1/2)) - a*(a + b/x^2)^(1/2) - (a + b/x^2
)^(3/2)/3
```

sympy [A] time = 2.34, size = 78, normalized size = 1.44

$$-\frac{4a^{\frac{3}{2}}\sqrt{1+\frac{b}{ax^2}}}{3}-\frac{a^{\frac{3}{2}}\log\left(\frac{b}{ax^2}\right)}{2}+a^{\frac{3}{2}}\log\left(\sqrt{1+\frac{b}{ax^2}}+1\right)-\frac{\sqrt{a}\,b\sqrt{1+\frac{b}{ax^2}}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**(3/2)/x,x)
```

```
[Out] -4*a**(3/2)*sqrt(1 + b/(a*x**2))/3 - a**(3/2)*log(b/(a*x**2))/2 + a**(3/2)*
log(sqrt(1 + b/(a*x**2)) + 1) - sqrt(a)*b*sqrt(1 + b/(a*x**2))/(3*x**2)
```

3.1904 $\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=71

$$-\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{8\sqrt{b}}-\frac{3a\sqrt{a+\frac{b}{x^2}}}{8x}-\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{4x}$$

```
[Out] -1/4*(a+b/x^2)^(3/2)/x-3/8*a^2*arctanh(b^(1/2)/x/(a+b/x^2)^(1/2))/b^(1/2)-3
/8*a*(a+b/x^2)^(1/2)/x
```

Rubi [A] time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {335, 195, 217, 206}

$$-\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{8\sqrt{b}}-\frac{3a\sqrt{a+\frac{b}{x^2}}}{8x}-\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^(3/2)/x^2,x]
```

```
[Out] (-3*a*Sqrt[a + b/x^2])/(8*x) - (a + b/x^2)^(3/2)/(4*x) - (3*a^2*ArcTanh[Sqr
t[b]/(Sqrt[a + b/x^2]*x)])/(8*Sqrt[b])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^2}\,dx &= -\operatorname{Subst}\left(\int \left(a+bx^2\right)^{3/2}\,dx, x, \frac{1}{x}\right)\\ &= -\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{4x} - \frac{1}{4}(3a)\operatorname{Subst}\left(\int \sqrt{a+bx^2}\,dx, x, \frac{1}{x}\right)\\ &= -\frac{3a\sqrt{a+\frac{b}{x^2}}}{8x} - \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{4x} - \frac{1}{8}\left(3a^2\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{x}\right)\\ &= -\frac{3a\sqrt{a+\frac{b}{x^2}}}{8x} - \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{4x} - \frac{1}{8}\left(3a^2\right)\operatorname{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x}\right)\\ &= -\frac{3a\sqrt{a+\frac{b}{x^2}}}{8x} - \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{4x} - \frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x}\right)}{8\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 85, normalized size = 1.20

$$-\frac{\sqrt{a+\frac{b}{x^2}}\left(3a^2x^4\sqrt{\frac{ax^2}{b}+1}\,\tanh^{-1}\left(\sqrt{\frac{ax^2}{b}+1}\right)+5a^2x^4+7abx^2+2b^2\right)}{8x^3\left(ax^2+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^(3/2)/x^2,x]
```

```
[Out] -1/8*(Sqrt[a + b/x^2]*(2*b^2 + 7*a*b*x^2 + 5*a^2*x^4 + 3*a^2*x^4*Sqrt[1 + (
a*x^2)/b]*ArcTanh[Sqrt[1 + (a*x^2)/b]]))/(x^3*(b + a*x^2))
```

fricas [A] time = 0.90, size = 163, normalized size = 2.30

$$\left[\frac{3\,a^2\sqrt{b}\,x^3\log\left(-\frac{ax^2-2\,\sqrt{b}\,x\sqrt{\frac{ax^2+b}{x^2}}+2\,b}{x^2}\right)-2\left(5\,abx^2+2\,b^2\right)\sqrt{\frac{ax^2+b}{x^2}}}{16\,bx^3},\ \frac{3\,a^2\sqrt{-b}\,x^3\arctan\left(\frac{\sqrt{-b}\,x\sqrt{\frac{ax^2+b}{x^2}}}{ax^2+b}\right)-\left(5\,abx^2+\right.}{8\,bx^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/16*(3*a^2*sqrt(b)*x^3*log(-(a*x^2 - 2*sqrt(b)*x*sqrt((a*x^2 + b)/x^2) +
2*b)/x^2) - 2*(5*a*b*x^2 + 2*b^2)*sqrt((a*x^2 + b)/x^2))/(b*x^3), 1/8*(3*a^
2*sqrt(-b)*x^3*arctan(sqrt(-b)*x*sqrt((a*x^2 + b)/x^2)/(a*x^2 + b)) - (5*a*
b*x^2 + 2*b^2)*sqrt((a*x^2 + b)/x^2))/(b*x^3)]
```

giac [A] time = 0.27, size = 76, normalized size = 1.07

$$\frac{\frac{3\,a^3\arctan\left(\frac{\sqrt{ax^2+b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}}-\frac{5\left(ax^2+b\right)^{\frac{3}{2}}a^3\operatorname{sgn}(x)-3\,\sqrt{ax^2+b}\,a^3b\operatorname{sgn}(x)}{a^2x^4}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/8*(3*a^3*arctan(sqrt(a*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) - (5*(a*x^2 + b
)^(3/2)*a^3*sgn(x) - 3*sqrt(a*x^2 + b)*a^3*b*sgn(x))/(a^2*x^4))/a
```

maple [B] time = 0.01, size = 125, normalized size = 1.76

$$-\frac{\left(\frac{a x^2+b}{x^2}\right)^{\frac{3}{2}}\left(3a^2b^{\frac{3}{2}}x^4\ln\left(\frac{2b+2\sqrt{a x^2+b}\,\sqrt{b}}{x}\right)-3\sqrt{a x^2+b}\,a^2b x^4-\left(a x^2+b\right)^{\frac{3}{2}}a^2x^4+\left(a x^2+b\right)^{\frac{5}{2}}a x^2+2\left(a x^2+b\right)^{\frac{5}{2}}\right.}{8\left(a x^2+b\right)^{\frac{3}{2}}b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^(3/2)/x^2,x)
```

```
[Out] -1/8*((a*x^2+b)/x^2)^(3/2)/x*(3*b^(3/2)*ln(2*(b+(a*x^2+b)^(1/2)*b^(1/2))/x)
*x^4*a^2-(a*x^2+b)^(3/2)*x^4*a^2+(a*x^2+b)^(5/2)*x^2*a-3*(a*x^2+b)^(1/2)*x^
4*a^2*b+2*(a*x^2+b)^(5/2)*b)/(a*x^2+b)^(3/2)/b^2
```

maxima [B] time = 1.94, size = 113, normalized size = 1.59

$$\frac{3\,a^2\log\left(\frac{\sqrt{a+\frac{b}{x^2}}\,x-\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x+\sqrt{b}}\right)}{16\sqrt{b}}-\frac{5\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}a^2x^3-3\sqrt{a+\frac{b}{x^2}}\,a^2bx}{8\left(\left(a+\frac{b}{x^2}\right)^2x^4-2\left(a+\frac{b}{x^2}\right)bx^2+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 3/16*a^2*log((sqrt(a + b/x^2)*x - sqrt(b))/(sqrt(a + b/x^2)*x + sqrt(b)))/s
qrt(b) - 1/8*(5*(a + b/x^2)^(3/2)*a^2*x^3 - 3*sqrt(a + b/x^2)*a^2*b*x)/((a
+ b/x^2)^2*x^4 - 2*(a + b/x^2)*b*x^2 + b^2)
```

mupad [B] time = 1.53, size = 39, normalized size = 0.55

$$-\frac{\left(a x^2+b\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b}{a x^2}\right)}{x\left(\frac{b}{a}+x^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)^(3/2)/x^2,x)
```

```
[Out] -((b + a*x^2)^(3/2)*hypergeom([-3/2, 1/2], 3/2, -b/(a*x^2)))/(x*(b/a + x^2)
^(3/2))
```

sympy [A] time = 3.65, size = 71, normalized size = 1.00

$$-\frac{5a^{\frac{3}{2}}\sqrt{1+\frac{b}{ax^2}}}{8x}-\frac{\sqrt{a}\,b\sqrt{1+\frac{b}{ax^2}}}{4x^3}-\frac{3a^2\,\mathrm{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\,x}\right)}{8\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**(3/2)/x**2,x)
```

```
[Out] -5*a**(3/2)*sqrt(1 + b/(a*x**2))/(8*x) - sqrt(a)*b*sqrt(1 + b/(a*x**2))/(4*
x**3) - 3*a**2*asinh(sqrt(b)/(sqrt(a)*x))/(8*sqrt(b))
```

3.1905 $\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=18

$$-\frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{5b}$$

```
[Out] -1/5*(a+b/x^2)^(5/2)/b
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$-\frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^(3/2)/x^3,x]
```

```
[Out] -(a + b/x^2)^(5/2)/(5*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^3}\,dx = -\frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{5b}$$

Mathematica [A] time = 0.03, size = 28, normalized size = 1.56

$$-\frac{\left(a+\frac{b}{x^2}\right)^{3/2}\left(ax^2+b\right)}{5bx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^(3/2)/x^3,x]
```

```
[Out] -1/5*((a + b/x^2)^(3/2)*(b + a*x^2))/(b*x^2)
```

fricas [B] time = 1.02, size = 39, normalized size = 2.17

$$-\frac{\left(a^2x^4+2\,abx^2+b^2\right)\sqrt{\frac{ax^2+b}{x^2}}}{5\,bx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/5*(a^2*x^4 + 2*a*b*x^2 + b^2)*sqrt((a*x^2 + b)/x^2)/(b*x^4)
```

giac [B] time = 0.22, size = 92, normalized size = 5.11

$$\frac{2\left(5\left(\sqrt{a}\,x-\sqrt{ax^2+b}\right)^8 a^{\frac{5}{2}}\mathrm{sgn}(x)+10\left(\sqrt{a}\,x-\sqrt{ax^2+b}\right)^4 a^{\frac{5}{2}}b^2\mathrm{sgn}(x)+a^{\frac{5}{2}}b^4\mathrm{sgn}(x)\right)}{5\left(\left(\sqrt{a}\,x-\sqrt{ax^2+b}\right)^2-b\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 2/5*(5*(sqrt(a)*x - sqrt(a*x^2 + b))^8*a^(5/2)*sgn(x) + 10*(sqrt(a)*x - sqr
t(a*x^2 + b))^4*a^(5/2)*b^2*sgn(x) + a^(5/2)*b^4*sgn(x))/((sqrt(a)*x - sqrt
(a*x^2 + b))^2 - b)^5
```

maple [A] time = 0.00, size = 29, normalized size = 1.61

$$-\frac{\left(a\,x^2+b\right)\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{3}{2}}}{5b\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^(3/2)/x^3,x)
```

```
[Out] -1/5*(a*x^2+b)/x^2/b*((a*x^2+b)/x^2)^(3/2)
```

maxima [A] time = 0.86, size = 14, normalized size = 0.78

$$-\frac{\left(a+\frac{b}{x^2}\right)^{\frac{5}{2}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/5*(a + b/x^2)^(5/2)/b
```

mupad [B] time = 1.46, size = 26, normalized size = 1.44

$$-\frac{\sqrt{a+\frac{b}{x^2}}\,\left(a\,x^2+b\right)^2}{5\,b\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)^(3/2)/x^3,x)
```

```
[Out] -((a + b/x^2)^(1/2)*(b + a*x^2)^2)/(5*b*x^4)
```

sympy [B] time = 1.18, size = 68, normalized size = 3.78

$$-\frac{a^{\frac{5}{2}}\sqrt{1+\frac{b}{ax^2}}}{5b}-\frac{2a^{\frac{3}{2}}\sqrt{1+\frac{b}{ax^2}}}{5x^2}-\frac{\sqrt{a}\,b\sqrt{1+\frac{b}{ax^2}}}{5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**(3/2)/x**3,x)
```

```
[Out] -a**(5/2)*sqrt(1 + b/(a*x**2))/(5*b) - 2*a**(3/2)*sqrt(1 + b/(a*x**2))/(5*x
**2) - sqrt(a)*b*sqrt(1 + b/(a*x**2))/(5*x**4)
```

3.1906 $\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=95

$$\frac{a^3 \tanh^{-1}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{16b^{3/2}} - \frac{a^2\sqrt{a+\frac{b}{x^2}}}{16bx} - \frac{a\sqrt{a+\frac{b}{x^2}}}{8x^3} - \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{6x^3}$$

```
[Out] -1/6*(a+b/x^2)^(3/2)/x^3+1/16*a^3*arctanh(b^(1/2)/x/(a+b/x^2)^(1/2))/b^(3/2
)-1/8*a*(a+b/x^2)^(1/2)/x^3-1/16*a^2*(a+b/x^2)^(1/2)/b/x
```

Rubi [A] time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {335, 279, 321, 217, 206}

$$\frac{a^3 \tanh^{-1}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{16b^{3/2}} - \frac{a^2\sqrt{a+\frac{b}{x^2}}}{16bx} - \frac{a\sqrt{a+\frac{b}{x^2}}}{8x^3} - \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^(3/2)/x^4,x]
```

```
[Out] -(a*Sqrt[a + b/x^2])/(8*x^3) - (a + b/x^2)^(3/2)/(6*x^3) - (a^2*Sqrt[a + b/
x^2])/(16*b*x) + (a^3*ArcTanh[Sqrt[b]/(Sqrt[a + b/x^2]*x)])/(16*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
```

egerQ[m]

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^4}\,dx &= -\text{Subst}\left(\int x^2\left(a+bx^2\right)^{3/2}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{6x^3} - \frac{1}{2}a\,\text{Subst}\left(\int x^2\sqrt{a+bx^2}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{a\sqrt{a+\frac{b}{x^2}}}{8x^3} - \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{6x^3} - \frac{1}{8}a^2\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{a\sqrt{a+\frac{b}{x^2}}}{8x^3} - \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{6x^3} - \frac{a^2\sqrt{a+\frac{b}{x^2}}}{16bx} + \frac{a^3\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{x}\right)}{16b}\\
&= -\frac{a\sqrt{a+\frac{b}{x^2}}}{8x^3} - \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{6x^3} - \frac{a^2\sqrt{a+\frac{b}{x^2}}}{16bx} + \frac{a^3\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x}\right)}{16b}\\
&= -\frac{a\sqrt{a+\frac{b}{x^2}}}{8x^3} - \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{6x^3} - \frac{a^2\sqrt{a+\frac{b}{x^2}}}{16bx} + \frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x}\right)}{16b^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 49, normalized size = 0.52

$$\frac{a^3x^3\left(a+\frac{b}{x^2}\right)^{3/2}\left(ax^2+b\right)\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};\frac{ax^2}{b}+1\right)}{5b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^(3/2)/x^4,x]
```

```
[Out] (a^3*(a + b/x^2)^(3/2)*x^3*(b + a*x^2)*Hypergeometric2F1[5/2, 4, 7/2, 1 + (
a*x^2)/b])/(5*b^4)
```

fricas [A] time = 0.86, size = 184, normalized size = 1.94

$$\left[\frac{3\,a^3\sqrt{b}\,x^5\log\left(-\frac{ax^2+2\,\sqrt{b}\,x\sqrt{\frac{ax^2+b}{x^2}}+2\,b}{x^2}\right) - 2\left(3\,a^2bx^4+14\,ab^2x^2+8\,b^3\right)\sqrt{\frac{ax^2+b}{x^2}}}{96\,b^2x^5}, -\frac{3\,a^3\sqrt{-b}\,x^5\arctan\left(\frac{\sqrt{-b}\,x\sqrt{\frac{ax^2+b}{x^2}}}{ax^2+b}\right)}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/96*(3*a^3*sqrt(b)*x^5*log(-(a*x^2 + 2*sqrt(b)*x*sqrt((a*x^2 + b)/x^2) +
2*b)/x^2) - 2*(3*a^2*b*x^4 + 14*a*b^2*x^2 + 8*b^3)*sqrt((a*x^2 + b)/x^2))/(
b^2*x^5), -1/48*(3*a^3*sqrt(-b)*x^5*arctan(sqrt(-b)*x*sqrt((a*x^2 + b)/x^2)
/(a*x^2 + b)) + (3*a^2*b*x^4 + 14*a*b^2*x^2 + 8*b^3)*sqrt((a*x^2 + b)/x^2))
/(b^2*x^5)]
```

giac [A] time = 0.24, size = 100, normalized size = 1.05

$$-\frac{\frac{3\,a^4\arctan\left(\frac{\sqrt{ax^2+b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}\,b}+\frac{3\left(ax^2+b\right)^{\frac{5}{2}}a^4\mathrm{sgn}(x)+8\left(ax^2+b\right)^{\frac{3}{2}}a^4b\,\mathrm{sgn}(x)-3\sqrt{ax^2+b}\,a^4b^2\mathrm{sgn}(x)}{a^3bx^6}}{48\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/48*(3*a^4*arctan(sqrt(a*x^2 + b)/sqrt(-b))*sgn(x)/(sqrt(-b)*b) + (3*(a*x
^2 + b)^(5/2)*a^4*sgn(x) + 8*(a*x^2 + b)^(3/2)*a^4*b*sgn(x) - 3*sqrt(a*x^2
+ b)*a^4*b^2*sgn(x))/(a^3*b*x^6))/a
```

maple [A] time = 0.01, size = 145, normalized size = 1.53

$$\frac{\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{3}{2}}\left(3a^3b^{\frac{3}{2}}x^6\ln\left(\frac{2b+2\sqrt{a\,x^2+b}\,\sqrt{b}}{x}\right)-3\sqrt{a\,x^2+b}\,a^3b\,x^6-\left(a\,x^2+b\right)^{\frac{3}{2}}a^3x^6+\left(a\,x^2+b\right)^{\frac{5}{2}}a^2x^4+2\left(a\,x^2+b\right)\right.}{48\left(a\,x^2+b\right)^{\frac{3}{2}}b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^(3/2)/x^4,x)
```

```
[Out] 1/48*((a*x^2+b)/x^2)^(3/2)/x^3*(-(a*x^2+b)^(3/2)*x^6*a^3+3*b^(3/2)*ln(2*(b+
(a*x^2+b)^(1/2)*b^(1/2))/x)*x^6*a^3+(a*x^2+b)^(5/2)*x^4*a^2-3*(a*x^2+b)^(1/
2)*x^6*a^3*b+2*(a*x^2+b)^(5/2)*x^2*a*b-8*(a*x^2+b)^(5/2)*b^2)/(a*x^2+b)^(3/
2)/b^3
```

maxima [B] time = 2.00, size = 155, normalized size = 1.63

$$-\frac{a^3\log\left(\frac{\sqrt{a+\frac{b}{x^2}}\,x-\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x+\sqrt{b}}\right)}{32\,b^{\frac{3}{2}}}-\frac{3\left(a+\frac{b}{x^2}\right)^{\frac{5}{2}}a^3x^5+8\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}a^3bx^3-3\sqrt{a+\frac{b}{x^2}}\,a^3b^2x}{48\left(\left(a+\frac{b}{x^2}\right)^3bx^6-3\left(a+\frac{b}{x^2}\right)^2b^2x^4+3\left(a+\frac{b}{x^2}\right)b^3x^2-b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/32*a^3*log((sqrt(a + b/x^2)*x - sqrt(b))/(sqrt(a + b/x^2)*x + sqrt(b)))/
b^(3/2) - 1/48*(3*(a + b/x^2)^(5/2)*a^3*x^5 + 8*(a + b/x^2)^(3/2)*a^3*b*x^3
 - 3*sqrt(a + b/x^2)*a^3*b^2*x)/((a + b/x^2)^3*b*x^6 - 3*(a + b/x^2)^2*b^2*
x^4 + 3*(a + b/x^2)*b^3*x^2 - b^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)^(3/2)/x^4,x)
```

```
[Out] int((a + b/x^2)^(3/2)/x^4, x)
```

sympy [A] time = 5.97, size = 119, normalized size = 1.25

$$-\frac{a^{\frac{5}{2}}}{16bx\sqrt{1+\frac{b}{ax^2}}}-\frac{17a^{\frac{3}{2}}}{48x^3\sqrt{1+\frac{b}{ax^2}}}-\frac{11\sqrt{a}\,b}{24x^5\sqrt{1+\frac{b}{ax^2}}}+\frac{a^3\,\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\,x}\right)}{16b^{\frac{3}{2}}}-\frac{b^2}{6\sqrt{a}\,x^7\sqrt{1+\frac{b}{ax^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**(3/2)/x**4,x)
```

```
[Out] -a**(5/2)/(16*b*x*sqrt(1 + b/(a*x**2))) - 17*a**(3/2)/(48*x**3*sqrt(1 + b/(
a*x**2))) - 11*sqrt(a)*b/(24*x**5*sqrt(1 + b/(a*x**2))) + a**3*asinh(sqrt(b
)/(sqrt(a)*x))/(16*b**(3/2)) - b**2/(6*sqrt(a)*x**7*sqrt(1 + b/(a*x**2)))
```

3.1907 $\int \left(a+\frac{b}{x^2}\right)^{5/2} x^3\,dx$

Optimal. Leaf size=86

$$-\frac{15}{8}b^2\sqrt{a+\frac{b}{x^2}}+\frac{15}{8}\sqrt{a}\,b^2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)+\frac{5}{8}bx^2\left(a+\frac{b}{x^2}\right)^{3/2}+\frac{1}{4}x^4\left(a+\frac{b}{x^2}\right)^{5/2}$$

```
[Out] 5/8*b*(a+b/x^2)^(3/2)*x^2+1/4*(a+b/x^2)^(5/2)*x^4+15/8*b^2*arctanh((a+b/x^2
)^(1/2)/a^(1/2))*a^(1/2)-15/8*b^2*(a+b/x^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {266, 47, 50, 63, 208}

$$-\frac{15}{8}b^2\sqrt{a+\frac{b}{x^2}}+\frac{15}{8}\sqrt{a}\,b^2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)+\frac{1}{4}x^4\left(a+\frac{b}{x^2}\right)^{5/2}+\frac{5}{8}bx^2\left(a+\frac{b}{x^2}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^(5/2)*x^3,x]
```

```
[Out] (-15*b^2*Sqrt[a + b/x^2])/8 + (5*b*(a + b/x^2)^(3/2)*x^2)/8 + ((a + b/x^2)^
(5/2)*x^4)/4 + (15*Sqrt[a]*b^2*ArcTanh[Sqrt[a + b/x^2]/Sqrt[a]])/8
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a+\frac{b}{x^2}\right)^{5/2} x^3\,dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^3}\,dx, x, \frac{1}{x^2}\right)\right) \\
&= \frac{1}{4}\left(a+\frac{b}{x^2}\right)^{5/2} x^4 - \frac{1}{8}(5b)\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2}\,dx, x, \frac{1}{x^2}\right) \\
&= \frac{5}{8}b\left(a+\frac{b}{x^2}\right)^{3/2} x^2 + \frac{1}{4}\left(a+\frac{b}{x^2}\right)^{5/2} x^4 - \frac{1}{16}\left(15b^2\right)\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x^2}\right) \\
&= -\frac{15}{8}b^2\sqrt{a+\frac{b}{x^2}} + \frac{5}{8}b\left(a+\frac{b}{x^2}\right)^{3/2} x^2 + \frac{1}{4}\left(a+\frac{b}{x^2}\right)^{5/2} x^4 - \frac{1}{16}\left(15ab^2\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\right. \\
&= -\frac{15}{8}b^2\sqrt{a+\frac{b}{x^2}} + \frac{5}{8}b\left(a+\frac{b}{x^2}\right)^{3/2} x^2 + \frac{1}{4}\left(a+\frac{b}{x^2}\right)^{5/2} x^4 - \frac{1}{8}(15ab)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x\right. \\
&= -\frac{15}{8}b^2\sqrt{a+\frac{b}{x^2}} + \frac{5}{8}b\left(a+\frac{b}{x^2}\right)^{3/2} x^2 + \frac{1}{4}\left(a+\frac{b}{x^2}\right)^{5/2} x^4 + \frac{15}{8}\sqrt{a}\,b^2 \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 49, normalized size = 0.57

$$-\frac{b^2\sqrt{a+\frac{b}{x^2}}\;{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};-\frac{ax^2}{b}\right)}{\sqrt{\frac{ax^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^(5/2)*x^3,x]
```

```
[Out] -((b^2*Sqrt[a + b/x^2]*Hypergeometric2F1[-5/2, -1/2, 1/2, -((a*x^2)/b)])/Sq
rt[1 + (a*x^2)/b])
```

fricas [A] time = 0.89, size = 157, normalized size = 1.83

$$\left[\frac{15}{16}\sqrt{a}\,b^2\log\left(-2\,ax^2-2\sqrt{a}\,x^2\sqrt{\frac{ax^2+b}{x^2}}-b\right)+\frac{1}{8}\left(2\,a^2x^4+9\,abx^2-8\,b^2\right)\sqrt{\frac{ax^2+b}{x^2}}, -\frac{15}{8}\sqrt{-a}\,b^2\arctan\left(\frac{\sqrt{-a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(5/2)*x^3,x, algorithm="fricas")
```

```
[Out] [15/16*sqrt(a)*b^2*log(-2*a*x^2 - 2*sqrt(a)*x^2*sqrt((a*x^2 + b)/x^2) - b)
+ 1/8*(2*a^2*x^4 + 9*a*b*x^2 - 8*b^2)*sqrt((a*x^2 + b)/x^2), -15/8*sqrt(-a)
*b^2*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + b)/x^2)/(a*x^2 + b)) + 1/8*(2*a^2*x^
4 + 9*a*b*x^2 - 8*b^2)*sqrt((a*x^2 + b)/x^2)]
```

giac [A] time = 0.25, size = 95, normalized size = 1.10

$$-\frac{15}{16}\sqrt{a}\,b^2\log\left(\left(\sqrt{a}\,x-\sqrt{ax^2+b}\right)^2\right)\operatorname{sgn}(x)+\frac{2\sqrt{a}\,b^3\operatorname{sgn}(x)}{\left(\sqrt{a}\,x-\sqrt{ax^2+b}\right)^2-b}+\frac{1}{8}\left(2\,a^2x^2\operatorname{sgn}(x)+9\,ab\operatorname{sgn}(x)\right)\sqrt{ax^2+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(5/2)*x^3,x, algorithm="giac")
```

```
[Out] -15/16*sqrt(a)*b^2*log((sqrt(a)*x - sqrt(a*x^2 + b))^2)*sgn(x) + 2*sqrt(a)*
b^3*sgn(x)/((sqrt(a)*x - sqrt(a*x^2 + b))^2 - b) + 1/8*(2*a^2*x^2*sgn(x) +
9*a*b*sgn(x))*sqrt(a*x^2 + b)*x
```

maple [A] time = 0.01, size = 127, normalized size = 1.48

$$\frac{\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{5}{2}}\left(15a\,b^3x\ln\left(\sqrt{a}\;x+\sqrt{a\,x^2+b}\right)+15\sqrt{a\,x^2+b}\;a^{\frac{3}{2}}b^2x^2+10\left(a\,x^2+b\right)^{\frac{3}{2}}a^{\frac{3}{2}}b\,x^2+8\left(a\,x^2+b\right)^{\frac{5}{2}}a^{\frac{3}{2}}x^2-\right.}{8\left(a\,x^2+b\right)^{\frac{5}{2}}\sqrt{a}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^(5/2)*x^3,x)
```

```
[Out] 1/8*((a*x^2+b)/x^2)^(5/2)*x^4*(8*(a*x^2+b)^(5/2)*a^(3/2)*x^2+10*a^(3/2)*(a*
x^2+b)^(3/2)*x^2*b+15*a^(3/2)*(a*x^2+b)^(1/2)*x^2*b^2-8*(a*x^2+b)^(7/2)*a^(
1/2)+15*ln(a^(1/2)*x+(a*x^2+b)^(1/2))*x*a*b^3)/(a*x^2+b)^(5/2)/b/a^(1/2)
```

maxima [A] time = 1.80, size = 115, normalized size = 1.34

$$-\frac{15}{16}\sqrt{a}\,b^2\log\left(\frac{\sqrt{a+\frac{b}{x^2}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^2}}+\sqrt{a}}\right)-\sqrt{a+\frac{b}{x^2}}\,b^2+\frac{9\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}ab^2-7\sqrt{a+\frac{b}{x^2}}\,a^2b^2}{8\left(\left(a+\frac{b}{x^2}\right)^2-2\left(a+\frac{b}{x^2}\right)a+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(5/2)*x^3,x, algorithm="maxima")
```

```
[Out] -15/16*sqrt(a)*b^2*log((sqrt(a + b/x^2) - sqrt(a))/(sqrt(a + b/x^2) + sqrt(
a))) - sqrt(a + b/x^2)*b^2 + 1/8*(9*(a + b/x^2)^(3/2)*a*b^2 - 7*sqrt(a + b/
x^2)*a^2*b^2)/((a + b/x^2)^2 - 2*(a + b/x^2)*a + a^2)
```

mupad [B] time = 1.84, size = 72, normalized size = 0.84

$$\frac{9\,a\,x^4\left(a+\frac{b}{x^2}\right)^{3/2}}{8}-b^2\sqrt{a+\frac{b}{x^2}}-\frac{7\,a^2x^4\sqrt{a+\frac{b}{x^2}}}{8}-\frac{\sqrt{a}\;b^2\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x^2}}\;1\mathrm{i}}{\sqrt{a}}\right)15\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b/x^2)^(5/2),x)
```

```
[Out] (9*a*x^4*(a + b/x^2)^(3/2))/8 - (a^(1/2)*b^2*atan(((a + b/x^2)^(1/2)*1i)/a^
(1/2))*15i)/8 - b^2*(a + b/x^2)^(1/2) - (7*a^2*x^4*(a + b/x^2)^(1/2))/8
```

sympy [A] time = 4.67, size = 117, normalized size = 1.36

$$\frac{15\sqrt{a}\,b^2\,\mathrm{asinh}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8}+\frac{a^3x^5}{4\sqrt{b}\sqrt{\frac{ax^2}{b}+1}}+\frac{11a^2\sqrt{b}\,x^3}{8\sqrt{\frac{ax^2}{b}+1}}+\frac{ab^{\frac{3}{2}}x}{8\sqrt{\frac{ax^2}{b}+1}}-\frac{b^{\frac{5}{2}}}{x\sqrt{\frac{ax^2}{b}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**(5/2)*x**3,x)
```

```
[Out] 15*sqrt(a)*b**2*asinh(sqrt(a)*x/sqrt(b))/8 + a**3*x**5/(4*sqrt(b)*sqrt(a*x*
*2/b + 1)) + 11*a**2*sqrt(b)*x**3/(8*sqrt(a*x**2/b + 1)) + a*b**(3/2)*x/(8*
sqrt(a*x**2/b + 1)) - b**(5/2)/(x*sqrt(a*x**2/b + 1))
```

3.1908 $\int \left(a+\frac{b}{x^2}\right)^{5/2} x^2\,dx$

Optimal. Leaf size=88

$$-\frac{5}{2}ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)-\frac{5b^2\sqrt{a+\frac{b}{x^2}}}{2x}+\frac{5}{3}bx\left(a+\frac{b}{x^2}\right)^{3/2}+\frac{1}{3}x^3\left(a+\frac{b}{x^2}\right)^{5/2}$$

```
[Out] 5/3*b*(a+b/x^2)^(3/2)*x+1/3*(a+b/x^2)^(5/2)*x^3-5/2*a*b^(3/2)*arctanh(b^(1/
2)/x/(a+b/x^2)^(1/2))-5/2*b^2*(a+b/x^2)^(1/2)/x
```

Rubi [A] time = 0.04, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {335, 277, 195, 217, 206}

$$-\frac{5b^2\sqrt{a+\frac{b}{x^2}}}{2x}-\frac{5}{2}ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)+\frac{1}{3}x^3\left(a+\frac{b}{x^2}\right)^{5/2}+\frac{5}{3}bx\left(a+\frac{b}{x^2}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^(5/2)*x^2,x]
```

```
[Out] (-5*b^2*Sqrt[a + b/x^2])/(2*x) + (5*b*(a + b/x^2)^(3/2)*x)/3 + ((a + b/x^2)
^(5/2)*x^3)/3 - (5*a*b^(3/2)*ArcTanh[Sqrt[b]/(Sqrt[a + b/x^2]*x)])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)^{5/2} x^2\,dx = -\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{5/2}}{x^4}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{3}\left(a+\frac{b}{x^2}\right)^{5/2} x^3 - \frac{1}{3}(5b)\,\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{5}{3}b\left(a+\frac{b}{x^2}\right)^{3/2} x + \frac{1}{3}\left(a+\frac{b}{x^2}\right)^{5/2} x^3 - \left(5b^2\right)\text{Subst}\left(\int \sqrt{a+bx^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{5b^2\sqrt{a+\frac{b}{x^2}}}{2x} + \frac{5}{3}b\left(a+\frac{b}{x^2}\right)^{3/2} x + \frac{1}{3}\left(a+\frac{b}{x^2}\right)^{5/2} x^3 - \frac{1}{2}\left(5ab^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx$$

$$= -\frac{5b^2\sqrt{a+\frac{b}{x^2}}}{2x} + \frac{5}{3}b\left(a+\frac{b}{x^2}\right)^{3/2} x + \frac{1}{3}\left(a+\frac{b}{x^2}\right)^{5/2} x^3 - \frac{1}{2}\left(5ab^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x\right.$$

$$= -\frac{5b^2\sqrt{a+\frac{b}{x^2}}}{2x} + \frac{5}{3}b\left(a+\frac{b}{x^2}\right)^{3/2} x + \frac{1}{3}\left(a+\frac{b}{x^2}\right)^{5/2} x^3 - \frac{5}{2}ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x}\right)$$

Mathematica [C] time = 0.01, size = 47, normalized size = 0.53

$$\frac{ax^5\left(a+\frac{b}{x^2}\right)^{5/2}\left(ax^2+b\right)\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};\frac{ax^2}{b}+1\right)}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^(5/2)*x^2,x]
```

```
[Out] (a*(a + b/x^2)^(5/2)*x^5*(b + a*x^2)*Hypergeometric2F1[2, 7/2, 9/2, 1 + (a*
x^2)/b])/(7*b^2)
```

fricas [A] time = 0.92, size = 165, normalized size = 1.88

$$\left[\frac{15\,ab^{\frac{3}{2}}x\log\left(-\frac{ax^2-2\sqrt{b}\,x\sqrt{\frac{ax^2+b}{x^2}}+2b}{x^2}\right)+2\left(2a^2x^4+14abx^2-3b^2\right)\sqrt{\frac{ax^2+b}{x^2}}}{12x}, \frac{15\,a\sqrt{-b}\,bx\arctan\left(\frac{\sqrt{-b}\,x\sqrt{\frac{ax^2+b}{x^2}}}{ax^2+b}\right)+}{6x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(5/2)*x^2,x, algorithm="fricas")
```

```
[Out] [1/12*(15*a*b^(3/2)*x*log(-(a*x^2 - 2*sqrt(b)*x*sqrt((a*x^2 + b)/x^2) + 2*b
)/x^2) + 2*(2*a^2*x^4 + 14*a*b*x^2 - 3*b^2)*sqrt((a*x^2 + b)/x^2))/x, 1/6*(
15*a*sqrt(-b)*b*x*arctan(sqrt(-b)*x*sqrt((a*x^2 + b)/x^2)/(a*x^2 + b)) + (2
*a^2*x^4 + 14*a*b*x^2 - 3*b^2)*sqrt((a*x^2 + b)/x^2))/x]
```

giac [A] time = 0.22, size = 90, normalized size = 1.02

$$\frac{\frac{15\,a^2b^2\arctan\left(\frac{\sqrt{ax^2+b}}{\sqrt{-b}}\right)\text{sgn}(x)}{\sqrt{-b}} + 2\left(ax^2+b\right)^{\frac{3}{2}}a^2\text{sgn}(x) + 12\sqrt{ax^2+b}\,a^2b\text{sgn}(x) - \frac{3\sqrt{ax^2+b}\,ab^2\text{sgn}(x)}{x^2}}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(5/2)*x^2,x, algorithm="giac")
```

```
[Out] 1/6*(15*a^2*b^2*arctan(sqrt(a*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) + 2*(a*x^2
 + b)^(3/2)*a^2*sgn(x) + 12*sqrt(a*x^2 + b)*a^2*b*sgn(x) - 3*sqrt(a*x^2 + b
)*a*b^2*sgn(x)/x^2)/a
```

maple [A] time = 0.01, size = 122, normalized size = 1.39

$$-\frac{\left(\frac{a x^2+b}{x^2}\right)^{\frac{5}{2}}\left(15a\,b^{\frac{5}{2}}x^2\ln\left(\frac{2b+2\sqrt{a x^2+b}\,\sqrt{b}}{x}\right)-15\sqrt{a x^2+b}\,a\,b^2x^2-5\left(a x^2+b\right)^{\frac{3}{2}}ab\,x^2-3\left(a x^2+b\right)^{\frac{5}{2}}a\,x^2+3\left(a x^2+\right.\right.}{6\left(a x^2+b\right)^{\frac{5}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^(5/2)*x^2,x)
```

```
[Out] -1/6*((a*x^2+b)/x^2)^(5/2)*x^3*(-3*(a*x^2+b)^(5/2)*a*x^2+15*b^(5/2)*ln(2*(b
+(a*x^2+b)^(1/2)*b^(1/2))/x)*x^2*a+3*(a*x^2+b)^(7/2)-5*(a*x^2+b)^(3/2)*x^2*
a*b-15*(a*x^2+b)^(1/2)*x^2*a*b^2)/(a*x^2+b)^(5/2)/b
```

maxima [A] time = 1.80, size = 105, normalized size = 1.19

$$\frac{1}{3}\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}ax^3+2\sqrt{a+\frac{b}{x^2}}\,abx-\frac{\sqrt{a+\frac{b}{x^2}}\,ab^2x}{2\left(\left(a+\frac{b}{x^2}\right)x^2-b\right)}+\frac{5}{4}ab^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x^2}}\,x-\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x+\sqrt{b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(5/2)*x^2,x, algorithm="maxima")
```

```
[Out] 1/3*(a + b/x^2)^(3/2)*a*x^3 + 2*sqrt(a + b/x^2)*a*b*x - 1/2*sqrt(a + b/x^2)
*a*b^2*x/((a + b/x^2)*x^2 - b) + 5/4*a*b^(3/2)*log((sqrt(a + b/x^2)*x - sqr
t(b))/(sqrt(a + b/x^2)*x + sqrt(b)))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(a+\frac{b}{x^2}\right)^{5/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b/x^2)^(5/2),x)
```

```
[Out] int(x^2*(a + b/x^2)^(5/2), x)
```

sympy [A] time = 4.00, size = 112, normalized size = 1.27

$$\frac{a^2\sqrt{b}\,x^2\sqrt{\frac{ax^2}{b}+1}}{3}+\frac{7ab^{\frac{3}{2}}\sqrt{\frac{ax^2}{b}+1}}{3}+\frac{5ab^{\frac{3}{2}}\log\left(\frac{ax^2}{b}\right)}{4}-\frac{5ab^{\frac{3}{2}}\log\left(\sqrt{\frac{ax^2}{b}+1}+1\right)}{2}-\frac{b^{\frac{5}{2}}\sqrt{\frac{ax^2}{b}+1}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**(5/2)*x**2,x)
```

```
[Out] a**2*sqrt(b)*x**2*sqrt(a*x**2/b + 1)/3 + 7*a*b**(3/2)*sqrt(a*x**2/b + 1)/3
+ 5*a*b**(3/2)*log(a*x**2/b)/4 - 5*a*b**(3/2)*log(sqrt(a*x**2/b + 1) + 1)/2
 - b**(5/2)*sqrt(a*x**2/b + 1)/(2*x**2)
```

3.1909 $\int \left(a+\frac{b}{x^2}\right)^{5/2} x\,dx$

Optimal. Leaf size=80

$$\frac{5}{2}a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)+\frac{1}{2}x^2\left(a+\frac{b}{x^2}\right)^{5/2}-\frac{5}{6}b\left(a+\frac{b}{x^2}\right)^{3/2}-\frac{5}{2}ab\sqrt{a+\frac{b}{x^2}}$$

```
[Out] -5/6*b*(a+b/x^2)^(3/2)+1/2*(a+b/x^2)^(5/2)*x^2+5/2*a^(3/2)*b*arctanh((a+b/x
^2)^(1/2)/a^(1/2))-5/2*a*b*(a+b/x^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {266, 47, 50, 63, 208}

$$\frac{5}{2}a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)+\frac{1}{2}x^2\left(a+\frac{b}{x^2}\right)^{5/2}-\frac{5}{6}b\left(a+\frac{b}{x^2}\right)^{3/2}-\frac{5}{2}ab\sqrt{a+\frac{b}{x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^(5/2)*x,x]
```

```
[Out] (-5*a*b*Sqrt[a + b/x^2])/2 - (5*b*(a + b/x^2)^(3/2))/6 + ((a + b/x^2)^(5/2)
*x^2)/2 + (5*a^(3/2)*b*ArcTanh[Sqrt[a + b/x^2]/Sqrt[a]])/2
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a+\frac{b}{x^2}\right)^{5/2} x\,dx &= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^2}\,dx, x, \frac{1}{x^2}\right)\right) \\
&= \frac{1}{2}\left(a+\frac{b}{x^2}\right)^{5/2} x^2 - \frac{1}{4}(5b)\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, \frac{1}{x^2}\right) \\
&= -\frac{5}{6}b\left(a+\frac{b}{x^2}\right)^{3/2} + \frac{1}{2}\left(a+\frac{b}{x^2}\right)^{5/2} x^2 - \frac{1}{4}(5ab)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x^2}\right) \\
&= -\frac{5}{2}ab\sqrt{a+\frac{b}{x^2}} - \frac{5}{6}b\left(a+\frac{b}{x^2}\right)^{3/2} + \frac{1}{2}\left(a+\frac{b}{x^2}\right)^{5/2} x^2 - \frac{1}{4}\left(5a^2b\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \right. \\
&= -\frac{5}{2}ab\sqrt{a+\frac{b}{x^2}} - \frac{5}{6}b\left(a+\frac{b}{x^2}\right)^{3/2} + \frac{1}{2}\left(a+\frac{b}{x^2}\right)^{5/2} x^2 - \frac{1}{2}\left(5a^2\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a\right. \\
&= -\frac{5}{2}ab\sqrt{a+\frac{b}{x^2}} - \frac{5}{6}b\left(a+\frac{b}{x^2}\right)^{3/2} + \frac{1}{2}\left(a+\frac{b}{x^2}\right)^{5/2} x^2 + \frac{5}{2}a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.68

$$-\frac{b^2\sqrt{a+\frac{b}{x^2}}\;{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};-\frac{ax^2}{b}\right)}{3x^2\sqrt{\frac{ax^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^(5/2)*x,x]
```

```
[Out] -1/3*(b^2*Sqrt[a + b/x^2]*Hypergeometric2F1[-5/2, -3/2, -1/2, -((a*x^2)/b)]
)/(x^2*Sqrt[1 + (a*x^2)/b])
```

fricas [A] time = 0.60, size = 170, normalized size = 2.12

$$\left[\frac{15\,a^{\frac{3}{2}}bx^2\log\left(-2\,ax^2-2\,\sqrt{a}\,x^2\sqrt{\frac{ax^2+b}{x^2}}-b\right)+2\left(3\,a^2x^4-14\,abx^2-2\,b^2\right)\sqrt{\frac{ax^2+b}{x^2}}}{12\,x^2},\ -\frac{15\,\sqrt{-a}\,abx^2\arctan\left(\frac{\sqrt{-a}\,x^2\sqrt{}}{ax^2+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(5/2)*x,x, algorithm="fricas")
```

```
[Out] [1/12*(15*a^(3/2)*b*x^2*log(-2*a*x^2 - 2*sqrt(a)*x^2*sqrt((a*x^2 + b)/x^2)
- b) + 2*(3*a^2*x^4 - 14*a*b*x^2 - 2*b^2)*sqrt((a*x^2 + b)/x^2))/x^2, -1/6*
(15*sqrt(-a)*a*b*x^2*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + b)/x^2)/(a*x^2 + b))
 - (3*a^2*x^4 - 14*a*b*x^2 - 2*b^2)*sqrt((a*x^2 + b)/x^2))/x^2]
```

giac [B] time = 0.50, size = 142, normalized size = 1.78

$$\frac{1}{2}\sqrt{ax^2+b}\,a^2x\operatorname{sgn}(x)-\frac{5}{4}a^{\frac{3}{2}}b\log\left(\left(\sqrt{a}\,x-\sqrt{ax^2+b}\right)^2\right)\operatorname{sgn}(x)+\frac{2\left(9\left(\sqrt{a}\,x-\sqrt{ax^2+b}\right)^4a^{\frac{3}{2}}b^2\operatorname{sgn}(x)-12\left(\sqrt{}\right.\right.}{3\left(\left(\sqrt{a}\,x-\sqrt{}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(5/2)*x,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(a*x^2 + b)*a^2*x*sgn(x) - 5/4*a^(3/2)*b*log((sqrt(a)*x - sqrt(a*x^
2 + b))^2)*sgn(x) + 2/3*(9*(sqrt(a)*x - sqrt(a*x^2 + b))^4*a^(3/2)*b^2*sgn(
x) - 12*(sqrt(a)*x - sqrt(a*x^2 + b))^2*a^(3/2)*b^3*sgn(x) + 7*a^(3/2)*b^4*
sgn(x))/((sqrt(a)*x - sqrt(a*x^2 + b))^2 - b)^3
```

maple [B] time = 0.01, size = 149, normalized size = 1.86

$$-\frac{\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{5}{2}}\left(-15a^2b^3x^3\ln\left(\sqrt{a}\,x+\sqrt{a\,x^2+b}\right)-15\sqrt{a\,x^2+b}\,a^{\frac{5}{2}}b^2x^4-10\left(a\,x^2+b\right)^{\frac{3}{2}}a^{\frac{5}{2}}b\,x^4-8\left(a\,x^2+b\right)^{\frac{5}{2}}a^{\frac{5}{2}}\right.}{6\left(a\,x^2+b\right)^{\frac{5}{2}}\sqrt{a}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^(5/2)*x,x)
```

```
[Out] -1/6*((a*x^2+b)/x^2)^(5/2)*x^2*(-8*(a*x^2+b)^(5/2)*a^(5/2)*x^4+8*(a*x^2+b)^
(7/2)*a^(3/2)*x^2-10*(a*x^2+b)^(3/2)*a^(5/2)*x^4*b-15*(a*x^2+b)^(1/2)*a^(5/
2)*x^4*b^2+2*(a*x^2+b)^(7/2)*b*a^(1/2)-15*ln(a^(1/2)*x+(a*x^2+b)^(1/2))*x^3
*a^2*b^3)/(a*x^2+b)^(5/2)/b^2/a^(1/2)
```

maxima [A] time = 1.87, size = 81, normalized size = 1.01

$$\frac{1}{2}\sqrt{a+\frac{b}{x^2}}\,a^2x^2-\frac{5}{4}a^{\frac{3}{2}}b\log\left(\frac{\sqrt{a+\frac{b}{x^2}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^2}}+\sqrt{a}}\right)-\frac{1}{3}\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}b-2\sqrt{a+\frac{b}{x^2}}\,ab$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(5/2)*x,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a + b/x^2)*a^2*x^2 - 5/4*a^(3/2)*b*log((sqrt(a + b/x^2) - sqrt(a))
/(sqrt(a + b/x^2) + sqrt(a))) - 1/3*(a + b/x^2)^(3/2)*b - 2*sqrt(a + b/x^2)
*a*b
```

mupad [B] time = 1.75, size = 66, normalized size = 0.82

$$\frac{a^2x^2\sqrt{a+\frac{b}{x^2}}}{2}-\frac{b\left(a+\frac{b}{x^2}\right)^{3/2}}{3}-2\,a\,b\sqrt{a+\frac{b}{x^2}}-\frac{a^{3/2}\,b\operatorname{atan}\left(\frac{\sqrt{a+\frac{b}{x^2}}\,1\mathrm{i}}{\sqrt{a}}\right)5\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b/x^2)^(5/2),x)
```

```
[Out] (a^2*x^2*(a + b/x^2)^(1/2))/2 - (b*(a + b/x^2)^(3/2))/3 - (a^(3/2)*b*atan((
(a + b/x^2)^(1/2)*1i)/a^(1/2))*5i)/2 - 2*a*b*(a + b/x^2)^(1/2)
```

sympy [**A**] time = 4.00, size = 112, normalized size = 1.40

$$\frac{a^{\frac{5}{2}}x^2\sqrt{1+\frac{b}{ax^2}}}{2}-\frac{7a^{\frac{3}{2}}b\sqrt{1+\frac{b}{ax^2}}}{3}-\frac{5a^{\frac{3}{2}}b\log\left(\frac{b}{ax^2}\right)}{4}+\frac{5a^{\frac{3}{2}}b\log\left(\sqrt{1+\frac{b}{ax^2}}+1\right)}{2}-\frac{\sqrt{a}\,b^2\sqrt{1+\frac{b}{ax^2}}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**(5/2)*x,x)
```

```
[Out] a**(5/2)*x**2*sqrt(1 + b/(a*x**2))/2 - 7*a**(3/2)*b*sqrt(1 + b/(a*x**2))/3
- 5*a**(3/2)*b*log(b/(a*x**2))/4 + 5*a**(3/2)*b*log(sqrt(1 + b/(a*x**2)) +
1)/2 - sqrt(a)*b**2*sqrt(1 + b/(a*x**2))/(3*x**2)
```

3.1910 $\int \left(a+\frac{b}{x^2}\right)^{5/2} dx$

Optimal. Leaf size=86

$$-\frac{15}{8}a^2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)+x\left(a+\frac{b}{x^2}\right)^{5/2}-\frac{5b\left(a+\frac{b}{x^2}\right)^{3/2}}{4x}-\frac{15ab\sqrt{a+\frac{b}{x^2}}}{8x}$$

```
[Out] -5/4*b*(a+b/x^2)^(3/2)/x+(a+b/x^2)^(5/2)*x-15/8*a^2*arctanh(b^(1/2)/x/(a+b/
x^2)^(1/2))*b^(1/2)-15/8*a*b*(a+b/x^2)^(1/2)/x
```

Rubi [A] time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {242, 277, 195, 217, 206}

$$-\frac{15}{8}a^2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)+x\left(a+\frac{b}{x^2}\right)^{5/2}-\frac{5b\left(a+\frac{b}{x^2}\right)^{3/2}}{4x}-\frac{15ab\sqrt{a+\frac{b}{x^2}}}{8x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^(5/2),x]
```

```
[Out] (-15*a*b*Sqrt[a + b/x^2])/(8*x) - (5*b*(a + b/x^2)^(3/2))/(4*x) + (a + b/x^
2)^(5/2)*x - (15*a^2*Sqrt[b]*ArcTanh[Sqrt[b]/(Sqrt[a + b/x^2]*x)])/8
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \left(a+\frac{b}{x^2}\right)^{5/2} dx &= -\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{5/2}}{x^2}\, dx, x, \frac{1}{x}\right) \\
&= \left(a+\frac{b}{x^2}\right)^{5/2} x - (5b)\,\text{Subst}\left(\int \left(a+bx^2\right)^{3/2} dx, x, \frac{1}{x}\right) \\
&= -\frac{5b\left(a+\frac{b}{x^2}\right)^{3/2}}{4x} + \left(a+\frac{b}{x^2}\right)^{5/2} x - \frac{1}{4}(15ab)\,\text{Subst}\left(\int \sqrt{a+bx^2}\, dx, x, \frac{1}{x}\right) \\
&= -\frac{15ab\sqrt{a+\frac{b}{x^2}}}{8x} - \frac{5b\left(a+\frac{b}{x^2}\right)^{3/2}}{4x} + \left(a+\frac{b}{x^2}\right)^{5/2} x - \frac{1}{8}\left(15a^2b\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\, dx, x, \frac{1}{x}\right) \\
&= -\frac{15ab\sqrt{a+\frac{b}{x^2}}}{8x} - \frac{5b\left(a+\frac{b}{x^2}\right)^{3/2}}{4x} + \left(a+\frac{b}{x^2}\right)^{5/2} x - \frac{1}{8}\left(15a^2b\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\, dx, x, \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x}\right) \\
&= -\frac{15ab\sqrt{a+\frac{b}{x^2}}}{8x} - \frac{5b\left(a+\frac{b}{x^2}\right)^{3/2}}{4x} + \left(a+\frac{b}{x^2}\right)^{5/2} x - \frac{15}{8}a^2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 49, normalized size = 0.57

$$-\frac{a^2x^5\left(a+\frac{b}{x^2}\right)^{5/2}\left(ax^2+b\right)\,{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};\frac{ax^2}{b}+1\right)}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^(5/2),x]
```

```
[Out] -1/7*(a^2*(a + b/x^2)^(5/2)*x^5*(b + a*x^2)*Hypergeometric2F1[3, 7/2, 9/2,
1 + (a*x^2)/b])/b^3
```

fricas [A] time = 0.99, size = 172, normalized size = 2.00

$$\left[\frac{15\,a^2\sqrt{b}\,x^3\log\left(-\frac{ax^2-2\,\sqrt{b}\,x\sqrt{\frac{ax^2+b}{x^2}}+2\,b}{x^2}\right)+2\left(8\,a^2x^4-9\,abx^2-2\,b^2\right)\sqrt{\frac{ax^2+b}{x^2}}}{16\,x^3},\ \frac{15\,a^2\sqrt{-b}\,x^3\arctan\left(\frac{\sqrt{-b}\,x\sqrt{\frac{ax^2+b}{x^2}}}{ax^2+b}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/16*(15*a^2*sqrt(b)*x^3*log(-(a*x^2 - 2*sqrt(b)*x*sqrt((a*x^2 + b)/x^2) +
 2*b)/x^2) + 2*(8*a^2*x^4 - 9*a*b*x^2 - 2*b^2)*sqrt((a*x^2 + b)/x^2))/x^3,
1/8*(15*a^2*sqrt(-b)*x^3*arctan(sqrt(-b)*x*sqrt((a*x^2 + b)/x^2)/(a*x^2 + b
)) + (8*a^2*x^4 - 9*a*b*x^2 - 2*b^2)*sqrt((a*x^2 + b)/x^2))/x^3]
```

giac [A] time = 0.22, size = 96, normalized size = 1.12

$$\frac{\frac{15\,a^3b\arctan\left(\frac{\sqrt{ax^2+b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}}+8\sqrt{ax^2+b}\,a^3\mathrm{sgn}(x)-\frac{9\left(ax^2+b\right)^{\frac{3}{2}}a^3b\mathrm{sgn}(x)-7\sqrt{ax^2+b}\,a^3b^2\mathrm{sgn}(x)}{a^2x^4}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/8*(15*a^3*b*arctan(sqrt(a*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) + 8*sqrt(a*x
^2 + b)*a^3*sgn(x) - (9*(a*x^2 + b)^(3/2)*a^3*b*sgn(x) - 7*sqrt(a*x^2 + b)*
a^3*b^2*sgn(x))/(a^2*x^4))/a
```

maple [B] time = 0.01, size = 144, normalized size = 1.67

$$-\frac{\left(\frac{a x^2+b}{x^2}\right)^{\frac{5}{2}}\left(15a^2b^{\frac{5}{2}}x^4\ln\left(\frac{2b+2\sqrt{a x^2+b}\ \sqrt{b}}{x}\right)-15\sqrt{a x^2+b}\ a^2b^2x^4-5\left(a x^2+b\right)^{\frac{3}{2}}a^2b x^4-3\left(a x^2+b\right)^{\frac{5}{2}}a^2x^4+3\right.}{8\left(a x^2+b\right)^{\frac{5}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^(5/2),x)
```

```
[Out] -1/8*((a*x^2+b)/x^2)^(5/2)*x*(-3*(a*x^2+b)^(5/2)*a^2*x^4+3*(a*x^2+b)^(7/2)*
x^2*a-5*(a*x^2+b)^(3/2)*x^4*a^2*b+15*b^(5/2)*ln(2*(b+(a*x^2+b)^(1/2)*b^(1/2
))/x)*x^4*a^2-15*(a*x^2+b)^(1/2)*x^4*a^2*b^2+2*(a*x^2+b)^(7/2)*b)/(a*x^2+b)
^(5/2)/b^2
```

maxima [A] time = 1.90, size = 130, normalized size = 1.51

$$\sqrt{a+\frac{b}{x^2}}\,a^2x+\frac{15}{16}a^2\sqrt{b}\,\log\left(\frac{\sqrt{a+\frac{b}{x^2}}\,x-\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x+\sqrt{b}}\right)-\frac{9\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}a^2bx^3-7\sqrt{a+\frac{b}{x^2}}\,a^2b^2x}{8\left(\left(a+\frac{b}{x^2}\right)^2x^4-2\left(a+\frac{b}{x^2}\right)bx^2+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] sqrt(a + b/x^2)*a^2*x + 15/16*a^2*sqrt(b)*log((sqrt(a + b/x^2)*x - sqrt(b))
/(sqrt(a + b/x^2)*x + sqrt(b))) - 1/8*(9*(a + b/x^2)^(3/2)*a^2*b*x^3 - 7*sq
rt(a + b/x^2)*a^2*b^2*x)/((a + b/x^2)^2*x^4 - 2*(a + b/x^2)*b*x^2 + b^2)
```

mupad [B] time = 1.49, size = 36, normalized size = 0.42

$$\frac{x\left(a x^2+b\right)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b}{a x^2}\right)}{\left(\frac{b}{a}+x^2\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)^(5/2),x)
```

```
[Out] (x*(b + a*x^2)^(5/2)*hypergeom([-5/2, -1/2], 1/2, -b/(a*x^2)))/(b/a + x^2)^
(5/2)
```

sympy [A] time = 4.44, size = 117, normalized size = 1.36

$$\frac{a^{\frac{5}{2}}x}{\sqrt{1+\frac{b}{ax^2}}}-\frac{a^{\frac{3}{2}}b}{8x\sqrt{1+\frac{b}{ax^2}}}-\frac{11\sqrt{a}\,b^2}{8x^3\sqrt{1+\frac{b}{ax^2}}}-\frac{15a^2\sqrt{b}\ \operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\,x}\right)}{8}-\frac{b^3}{4\sqrt{a}\,x^5\sqrt{1+\frac{b}{ax^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**(5/2),x)
```

```
[Out] a**(5/2)*x/sqrt(1 + b/(a*x**2)) - a**(3/2)*b/(8*x*sqrt(1 + b/(a*x**2))) - 1
1*sqrt(a)*b**2/(8*x**3*sqrt(1 + b/(a*x**2))) - 15*a**2*sqrt(b)*asinh(sqrt(b
)/(sqrt(a)*x))/8 - b**3/(4*sqrt(a)*x**5*sqrt(1 + b/(a*x**2)))
```

3.1911 $\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x}\,dx$

Optimal. Leaf size=72

$$a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)-a^2\sqrt{a+\frac{b}{x^2}}-\frac{1}{3}a\left(a+\frac{b}{x^2}\right)^{3/2}-\frac{1}{5}\left(a+\frac{b}{x^2}\right)^{5/2}$$

```
[Out] -1/3*a*(a+b/x^2)^(3/2)-1/5*(a+b/x^2)^(5/2)+a^(5/2)*arctanh((a+b/x^2)^(1/2)/
a^(1/2))-a^2*(a+b/x^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 50, 63, 208}

$$-a^2\sqrt{a+\frac{b}{x^2}}+a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)-\frac{1}{3}a\left(a+\frac{b}{x^2}\right)^{3/2}-\frac{1}{5}\left(a+\frac{b}{x^2}\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^(5/2)/x,x]
```

```
[Out] -(a^2*Sqrt[a + b/x^2]) - (a*(a + b/x^2)^(3/2))/3 - (a + b/x^2)^(5/2)/5 + a^
(5/2)*ArcTanh[Sqrt[a + b/x^2]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x}\,dx &= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= -\frac{1}{5}\left(a+\frac{b}{x^2}\right)^{5/2} - \frac{1}{2}a\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, \frac{1}{x^2}\right)\\
&= -\frac{1}{3}a\left(a+\frac{b}{x^2}\right)^{3/2} - \frac{1}{5}\left(a+\frac{b}{x^2}\right)^{5/2} - \frac{1}{2}a^2\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x^2}\right)\\
&= -a^2\sqrt{a+\frac{b}{x^2}} - \frac{1}{3}a\left(a+\frac{b}{x^2}\right)^{3/2} - \frac{1}{5}\left(a+\frac{b}{x^2}\right)^{5/2} - \frac{1}{2}a^3\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x^2}\right)\\
&= -a^2\sqrt{a+\frac{b}{x^2}} - \frac{1}{3}a\left(a+\frac{b}{x^2}\right)^{3/2} - \frac{1}{5}\left(a+\frac{b}{x^2}\right)^{5/2} - \frac{a^3\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x^2}}\right)}{b}\\
&= -a^2\sqrt{a+\frac{b}{x^2}} - \frac{1}{3}a\left(a+\frac{b}{x^2}\right)^{3/2} - \frac{1}{5}\left(a+\frac{b}{x^2}\right)^{5/2} + a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.75

$$-\frac{b^2\sqrt{a+\frac{b}{x^2}}\;{}_2F_1\left(-\frac{5}{2},-\frac{5}{2};-\frac{3}{2};-\frac{ax^2}{b}\right)}{5x^4\sqrt{\frac{ax^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^(5/2)/x,x]
```

```
[Out] -1/5*(b^2*Sqrt[a + b/x^2]*Hypergeometric2F1[-5/2, -5/2, -3/2, -((a*x^2)/b)]
)/(x^4*Sqrt[1 + (a*x^2)/b])
```

fricas [A] time = 0.90, size = 169, normalized size = 2.35

$$\left[\frac{15\,a^{\frac{5}{2}}x^4\log\left(-2\,ax^2-2\,\sqrt{a}\,x^2\sqrt{\frac{ax^2+b}{x^2}}-b\right)-2\left(23\,a^2x^4+11\,abx^2+3\,b^2\right)\sqrt{\frac{ax^2+b}{x^2}}}{30\,x^4},-\frac{15\,\sqrt{-a}\,a^2x^4\arctan\left(\frac{\sqrt{-a}\,x^2\sqrt{}}{ax^2+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [1/30*(15*a^(5/2)*x^4*log(-2*a*x^2 - 2*sqrt(a)*x^2*sqrt((a*x^2 + b)/x^2) -
b) - 2*(23*a^2*x^4 + 11*a*b*x^2 + 3*b^2)*sqrt((a*x^2 + b)/x^2))/x^4, -1/15*
(15*sqrt(-a)*a^2*x^4*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + b)/x^2)/(a*x^2 + b))
 + (23*a^2*x^4 + 11*a*b*x^2 + 3*b^2)*sqrt((a*x^2 + b)/x^2))/x^4]
```

giac [B] time = 1.06, size = 180, normalized size = 2.50

$$-\frac{1}{2}\,a^{\frac{5}{2}}\log\left(\left(\sqrt{a}\,x-\sqrt{ax^2+b}\right)^2\right)\mathrm{sgn}(x)+\frac{2\left(45\left(\sqrt{a}\,x-\sqrt{ax^2+b}\right)^8a^{\frac{5}{2}}b\mathrm{sgn}(x)-90\left(\sqrt{a}\,x-\sqrt{ax^2+b}\right)^6a^{\frac{5}{2}}b^2\mathrm{sgn}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(5/2)/x,x, algorithm="giac")
```

```
[Out] -1/2*a^(5/2)*log((sqrt(a)*x - sqrt(a*x^2 + b))^2)*sgn(x) + 2/15*(45*(sqrt(a
)*x - sqrt(a*x^2 + b))^8*a^(5/2)*b*sgn(x) - 90*(sqrt(a)*x - sqrt(a*x^2 + b)
)^6*a^(5/2)*b^2*sgn(x) + 140*(sqrt(a)*x - sqrt(a*x^2 + b))^4*a^(5/2)*b^3*sg
n(x) - 70*(sqrt(a)*x - sqrt(a*x^2 + b))^2*a^(5/2)*b^4*sgn(x) + 23*a^(5/2)*b
^5*sgn(x))/((sqrt(a)*x - sqrt(a*x^2 + b))^2 - b)^5
```

maple [B] time = 0.01, size = 166, normalized size = 2.31

$$\frac{\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{5}{2}}\left(15a^3b^3x^5\ln\left(\sqrt{a}\,x+\sqrt{a\,x^2+b}\right)+15\sqrt{a\,x^2+b}\;a^{\frac{7}{2}}b^2x^6+10\left(a\,x^2+b\right)^{\frac{3}{2}}a^{\frac{7}{2}}b\,x^6+8\left(a\,x^2+b\right)^{\frac{5}{2}}a^{\frac{7}{2}}x^6\right.}{15\left(a\,x^2+b\right)^{\frac{5}{2}}\sqrt{a}\;b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^(5/2)/x,x)
```

```
[Out] 1/15*((a*x^2+b)/x^2)^(5/2)*(8*a^(7/2)*(a*x^2+b)^(5/2)*x^6+10*a^(7/2)*(a*x^2
+b)^(3/2)*x^6*b+15*a^(7/2)*(a*x^2+b)^(1/2)*x^6*b^2-8*a^(5/2)*(a*x^2+b)^(7/2
)*x^4-2*a^(3/2)*(a*x^2+b)^(7/2)*x^2*b+15*ln(a^(1/2)*x+(a*x^2+b)^(1/2))*x^5*
a^3*b^3-3*(a*x^2+b)^(7/2)*b^2*a^(1/2))/(a*x^2+b)^(5/2)/b^3/a^(1/2)
```

maxima [A] time = 1.83, size = 75, normalized size = 1.04

$$-\frac{1}{2}\,a^{\frac{5}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x^2}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^2}}+\sqrt{a}}\right)-\frac{1}{5}\left(a+\frac{b}{x^2}\right)^{\frac{5}{2}}-\frac{1}{3}\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}a-\sqrt{a+\frac{b}{x^2}}\,a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] -1/2*a^(5/2)*log((sqrt(a + b/x^2) - sqrt(a))/(sqrt(a + b/x^2) + sqrt(a))) -
 1/5*(a + b/x^2)^(5/2) - 1/3*(a + b/x^2)^(3/2)*a - sqrt(a + b/x^2)*a^2
```

mupad [B] time = 1.65, size = 60, normalized size = 0.83

$$-\frac{a\left(a+\frac{b}{x^2}\right)^{3/2}}{3}-\frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{5}-a^2\sqrt{a+\frac{b}{x^2}}-a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x^2}}\,1\mathrm{i}}{\sqrt{a}}\right)1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)^(5/2)/x,x)
```

```
[Out] - a^(5/2)*atan(((a + b/x^2)^(1/2)*1i)/a^(1/2))*1i - (a*(a + b/x^2)^(3/2))/3
 - (a + b/x^2)^(5/2)/5 - a^2*(a + b/x^2)^(1/2)
```

sympy [A] time = 4.10, size = 105, normalized size = 1.46

$$-\frac{23a^{\frac{5}{2}}\sqrt{1+\frac{b}{ax^2}}}{15}-\frac{a^{\frac{5}{2}}\log\left(\frac{b}{ax^2}\right)}{2}+a^{\frac{5}{2}}\log\left(\sqrt{1+\frac{b}{ax^2}}+1\right)-\frac{11a^{\frac{3}{2}}b\sqrt{1+\frac{b}{ax^2}}}{15x^2}-\frac{\sqrt{a}\,b^2\sqrt{1+\frac{b}{ax^2}}}{5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**(5/2)/x,x)
```

```
[Out] -23*a**(5/2)*sqrt(1 + b/(a*x**2))/15 - a**(5/2)*log(b/(a*x**2))/2 + a**(5/2
)*log(sqrt(1 + b/(a*x**2)) + 1) - 11*a**(3/2)*b*sqrt(1 + b/(a*x**2))/(15*x*
*2) - sqrt(a)*b**2*sqrt(1 + b/(a*x**2))/(5*x**4)
```

3.1912 $\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^2}\,dx$

Optimal. Leaf size=92

$$-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{16\sqrt{b}}-\frac{5a^2\sqrt{a+\frac{b}{x^2}}}{16x}-\frac{5a\left(a+\frac{b}{x^2}\right)^{3/2}}{24x}-\frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{6x}$$

```
[Out] -5/24*a*(a+b/x^2)^(3/2)/x-1/6*(a+b/x^2)^(5/2)/x-5/16*a^3*arctanh(b^(1/2)/x/
(a+b/x^2)^(1/2))/b^(1/2)-5/16*a^2*(a+b/x^2)^(1/2)/x
```

Rubi [A] time = 0.04, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {335, 195, 217, 206}

$$-\frac{5a^2\sqrt{a+\frac{b}{x^2}}}{16x}-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{16\sqrt{b}}-\frac{5a\left(a+\frac{b}{x^2}\right)^{3/2}}{24x}-\frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{6x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^(5/2)/x^2,x]
```

```
[Out] (-5*a^2*Sqrt[a + b/x^2])/(16*x) - (5*a*(a + b/x^2)^(3/2))/(24*x) - (a + b/x
^2)^(5/2)/(6*x) - (5*a^3*ArcTanh[Sqrt[b]/(Sqrt[a + b/x^2]*x)])/(16*Sqrt[b])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^2}\,dx &= -\operatorname{Subst}\left(\int \left(a+bx^2\right)^{5/2}\,dx, x, \frac{1}{x}\right) \\
&= -\frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{6x} - \frac{1}{6}(5a)\operatorname{Subst}\left(\int \left(a+bx^2\right)^{3/2}\,dx, x, \frac{1}{x}\right) \\
&= -\frac{5a\left(a+\frac{b}{x^2}\right)^{3/2}}{24x} - \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{6x} - \frac{1}{8}\left(5a^2\right)\operatorname{Subst}\left(\int \sqrt{a+bx^2}\,dx, x, \frac{1}{x}\right) \\
&= -\frac{5a^2\sqrt{a+\frac{b}{x^2}}}{16x} - \frac{5a\left(a+\frac{b}{x^2}\right)^{3/2}}{24x} - \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{6x} - \frac{1}{16}\left(5a^3\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{x}\right) \\
&= -\frac{5a^2\sqrt{a+\frac{b}{x^2}}}{16x} - \frac{5a\left(a+\frac{b}{x^2}\right)^{3/2}}{24x} - \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{6x} - \frac{1}{16}\left(5a^3\right)\operatorname{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{b}{x^2}}}\right) \\
&= -\frac{5a^2\sqrt{a+\frac{b}{x^2}}}{16x} - \frac{5a\left(a+\frac{b}{x^2}\right)^{3/2}}{24x} - \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{6x} - \frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x}\right)}{16\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 96, normalized size = 1.04

$$-\frac{\sqrt{a+\frac{b}{x^2}}\left(15a^3x^6\sqrt{\frac{ax^2}{b}+1}\,\tanh^{-1}\left(\sqrt{\frac{ax^2}{b}+1}\right)+33a^3x^6+59a^2bx^4+34ab^2x^2+8b^3\right)}{48x^5\left(ax^2+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^(5/2)/x^2,x]
```

```
[Out] -1/48*(Sqrt[a + b/x^2]*(8*b^3 + 34*a*b^2*x^2 + 59*a^2*b*x^4 + 33*a^3*x^6 +
15*a^3*x^6*Sqrt[1 + (a*x^2)/b]*ArcTanh[Sqrt[1 + (a*x^2)/b]]))/(x^5*(b + a*x
^2))
```

fricas [A] time = 0.81, size = 185, normalized size = 2.01

$$\left[\frac{15\,a^3\sqrt{b}\,x^5\log\left(-\frac{ax^2-2\sqrt{b}\,x\sqrt{\frac{ax^2+b}{x^2}}+2b}{x^2}\right)-2\left(33\,a^2bx^4+26\,ab^2x^2+8\,b^3\right)\sqrt{\frac{ax^2+b}{x^2}}}{96\,bx^5},\ \frac{15\,a^3\sqrt{-b}\,x^5\arctan\left(\frac{\sqrt{-b}\,x\sqrt{a}}{ax^2+b}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/96*(15*a^3*sqrt(b)*x^5*log(-(a*x^2 - 2*sqrt(b)*x*sqrt((a*x^2 + b)/x^2) +
 2*b)/x^2) - 2*(33*a^2*b*x^4 + 26*a*b^2*x^2 + 8*b^3)*sqrt((a*x^2 + b)/x^2))
/(b*x^5), 1/48*(15*a^3*sqrt(-b)*x^5*arctan(sqrt(-b)*x*sqrt((a*x^2 + b)/x^2)
/(a*x^2 + b)) - (33*a^2*b*x^4 + 26*a*b^2*x^2 + 8*b^3)*sqrt((a*x^2 + b)/x^2)
)/(b*x^5)]
```

giac [A] time = 0.23, size = 95, normalized size = 1.03

$$\frac{\frac{15\,a^4\arctan\left(\frac{\sqrt{ax^2+b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}}-\frac{33\left(ax^2+b\right)^{\frac{5}{2}}a^4\mathrm{sgn}(x)-40\left(ax^2+b\right)^{\frac{3}{2}}a^4b\mathrm{sgn}(x)+15\sqrt{ax^2+b}\,a^4b^2\mathrm{sgn}(x)}{a^3x^6}}{48\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/48*(15*a^4*arctan(sqrt(a*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) - (33*(a*x^2
+ b)^(5/2)*a^4*sgn(x) - 40*(a*x^2 + b)^(3/2)*a^4*b*sgn(x) + 15*sqrt(a*x^2 +
 b)*a^4*b^2*sgn(x))/(a^3*x^6))/a
```

maple [B] time = 0.01, size = 166, normalized size = 1.80

$$-\frac{\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{5}{2}}\left(15a^3b^{\frac{5}{2}}x^6\ln\left(\frac{2b+2\sqrt{a\,x^2+b}\,\sqrt{b}}{x}\right)-15\sqrt{a\,x^2+b}\,a^3b^2x^6-5\left(a\,x^2+b\right)^{\frac{3}{2}}a^3b\,x^6-3\left(a\,x^2+b\right)^{\frac{5}{2}}a^3x^6+3\left(a\,x\right.\right.}{48\left(a\,x^2+b\right)^{\frac{5}{2}}b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^(5/2)/x^2,x)
```

```
[Out] -1/48*((a*x^2+b)/x^2)^(5/2)/x*(-3*(a*x^2+b)^(5/2)*x^6*a^3+15*b^(5/2)*ln(2*(
b+(a*x^2+b)^(1/2)*b^(1/2))/x)*x^6*a^3+3*(a*x^2+b)^(7/2)*x^4*a^2-5*(a*x^2+b)
^(3/2)*x^6*a^3*b-15*(a*x^2+b)^(1/2)*x^6*a^3*b^2+2*(a*x^2+b)^(7/2)*x^2*a*b+8
*(a*x^2+b)^(7/2)*b^2)/(a*x^2+b)^(5/2)/b^3
```

maxima [B] time = 1.99, size = 152, normalized size = 1.65

$$\frac{5\,a^3\log\left(\frac{\sqrt{a+\frac{b}{x^2}}\,x-\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x+\sqrt{b}}\right)}{32\sqrt{b}}-\frac{33\left(a+\frac{b}{x^2}\right)^{\frac{5}{2}}a^3x^5-40\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}a^3bx^3+15\sqrt{a+\frac{b}{x^2}}\,a^3b^2x}{48\left(\left(a+\frac{b}{x^2}\right)^3x^6-3\left(a+\frac{b}{x^2}\right)^2bx^4+3\left(a+\frac{b}{x^2}\right)b^2x^2-b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] 5/32*a^3*log((sqrt(a + b/x^2)*x - sqrt(b))/(sqrt(a + b/x^2)*x + sqrt(b)))/s
qrt(b) - 1/48*(33*(a + b/x^2)^(5/2)*a^3*x^5 - 40*(a + b/x^2)^(3/2)*a^3*b*x^
3 + 15*sqrt(a + b/x^2)*a^3*b^2*x)/((a + b/x^2)^3*x^6 - 3*(a + b/x^2)^2*b*x^
4 + 3*(a + b/x^2)*b^2*x^2 - b^3)
```

mupad [B] time = 1.79, size = 39, normalized size = 0.42

$$-\frac{\left(a\,x^2+b\right)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b}{a\,x^2}\right)}{x\left(\frac{b}{a}+x^2\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)^(5/2)/x^2,x)
```

```
[Out] -((b + a*x^2)^(5/2)*hypergeom([-5/2, 1/2], 3/2, -b/(a*x^2)))/(x*(b/a + x^2)
^(5/2))
```

sympy [**A**] time = 4.82, size = 99, normalized size = 1.08

$$-\frac{11a^{\frac{5}{2}}\sqrt{1+\frac{b}{ax^2}}}{16x}-\frac{13a^{\frac{3}{2}}b\sqrt{1+\frac{b}{ax^2}}}{24x^3}-\frac{\sqrt{a}\,b^2\sqrt{1+\frac{b}{ax^2}}}{6x^5}-\frac{5a^3\,\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\,x}\right)}{16\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**(5/2)/x**2,x)
```

```
[Out] -11*a**(5/2)*sqrt(1 + b/(a*x**2))/(16*x) - 13*a**(3/2)*b*sqrt(1 + b/(a*x**2
))/(24*x**3) - sqrt(a)*b**2*sqrt(1 + b/(a*x**2))/(6*x**5) - 5*a**3*asinh(sq
rt(b)/(sqrt(a)*x))/(16*sqrt(b))
```

3.1913 $\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^3}\,dx$

Optimal. Leaf size=18

$$-\frac{\left(a+\frac{b}{x^2}\right)^{7/2}}{7b}$$

```
[Out] -1/7*(a+b/x^2)^(7/2)/b
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$-\frac{\left(a+\frac{b}{x^2}\right)^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^(5/2)/x^3,x]
```

```
[Out] -(a + b/x^2)^(7/2)/(7*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^3}\,dx = -\frac{\left(a+\frac{b}{x^2}\right)^{7/2}}{7b}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.56

$$-\frac{\left(a+\frac{b}{x^2}\right)^{5/2}\left(ax^2+b\right)}{7bx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^(5/2)/x^3,x]
```

```
[Out] -1/7*((a + b/x^2)^(5/2)*(b + a*x^2))/(b*x^2)
```

fricas [B] time = 1.05, size = 50, normalized size = 2.78

$$-\frac{\left(a^3x^6+3\,a^2bx^4+3\,ab^2x^2+b^3\right)\sqrt{\frac{ax^2+b}{x^2}}}{7\,bx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/7*(a^3*x^6 + 3*a^2*b*x^4 + 3*a*b^2*x^2 + b^3)*sqrt((a*x^2 + b)/x^2)/(b*x
^6)
```

giac [B] time = 0.26, size = 121, normalized size = 6.72

$$\frac{2\left(7\left(\sqrt{a}\,x-\sqrt{ax^2+b}\right)^{12}a^{\frac{7}{2}}\mathrm{sgn}(x)+35\left(\sqrt{a}\,x-\sqrt{ax^2+b}\right)^{8}a^{\frac{7}{2}}b^2\mathrm{sgn}(x)+21\left(\sqrt{a}\,x-\sqrt{ax^2+b}\right)^{4}a^{\frac{7}{2}}b^4\mathrm{sgn}(x)\right.}{7\left(\left(\sqrt{a}\,x-\sqrt{ax^2+b}\right)^2-b\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] 2/7*(7*(sqrt(a)*x - sqrt(a*x^2 + b))^12*a^(7/2)*sgn(x) + 35*(sqrt(a)*x - sq
rt(a*x^2 + b))^8*a^(7/2)*b^2*sgn(x) + 21*(sqrt(a)*x - sqrt(a*x^2 + b))^4*a^
(7/2)*b^4*sgn(x) + a^(7/2)*b^6*sgn(x))/((sqrt(a)*x - sqrt(a*x^2 + b))^2 - b
)^7
```

maple [A] time = 0.00, size = 29, normalized size = 1.61

$$-\frac{\left(a\,x^2+b\right)\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{5}{2}}}{7b\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^(5/2)/x^3,x)
```

```
[Out] -1/7*(a*x^2+b)/x^2/b*((a*x^2+b)/x^2)^(5/2)
```

maxima [A] time = 0.88, size = 14, normalized size = 0.78

$$-\frac{\left(a+\frac{b}{x^2}\right)^{\frac{7}{2}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/7*(a + b/x^2)^(7/2)/b
```

mupad [B] time = 1.75, size = 68, normalized size = 3.78

$$-\frac{a^3\sqrt{a+\frac{b}{x^2}}}{7\,b}-\frac{3\,a^2\sqrt{a+\frac{b}{x^2}}}{7\,x^2}-\frac{b^2\sqrt{a+\frac{b}{x^2}}}{7\,x^6}-\frac{3\,a\,b\sqrt{a+\frac{b}{x^2}}}{7\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)^(5/2)/x^3,x)
```

```
[Out] - (a^3*(a + b/x^2)^(1/2))/(7*b) - (3*a^2*(a + b/x^2)^(1/2))/(7*x^2) - (b^2*
(a + b/x^2)^(1/2))/(7*x^6) - (3*a*b*(a + b/x^2)^(1/2))/(7*x^4)
```

sympy [A] time = 3.63, size = 88, normalized size = 4.89

$$\begin{cases}-\frac{a^3\sqrt{a+\frac{b}{x^2}}}{7b}-\frac{3a^2\sqrt{a+\frac{b}{x^2}}}{7x^2}-\frac{3ab\sqrt{a+\frac{b}{x^2}}}{7x^4}-\frac{b^2\sqrt{a+\frac{b}{x^2}}}{7x^6} & \text{for } b\neq 0\\ -\frac{a^{\frac{5}{2}}}{2x^2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**(5/2)/x**3,x)
```

```
[Out] Piecewise((-a**3*sqrt(a + b/x**2)/(7*b) - 3*a**2*sqrt(a + b/x**2)/(7*x**2)
- 3*a*b*sqrt(a + b/x**2)/(7*x**4) - b**2*sqrt(a + b/x**2)/(7*x**6), Ne(b, 0
)), (-a**(5/2)/(2*x**2), True))
```

3.1914 $\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^4}\,dx$

Optimal. Leaf size=116

$$\frac{5a^4\tanh^{-1}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{128b^{3/2}}-\frac{5a^3\sqrt{a+\frac{b}{x^2}}}{128bx}-\frac{5a^2\sqrt{a+\frac{b}{x^2}}}{64x^3}-\frac{5a\left(a+\frac{b}{x^2}\right)^{3/2}}{48x^3}-\frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{8x^3}$$

```
[Out] -5/48*a*(a+b/x^2)^(3/2)/x^3-1/8*(a+b/x^2)^(5/2)/x^3+5/128*a^4*arctanh(b^(1/
2)/x/(a+b/x^2)^(1/2))/b^(3/2)-5/64*a^2*(a+b/x^2)^(1/2)/x^3-5/128*a^3*(a+b/x
^2)^(1/2)/b/x
```

Rubi [A] time = 0.06, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {335, 279, 321, 217, 206}

$$\frac{5a^4\tanh^{-1}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{128b^{3/2}}-\frac{5a^3\sqrt{a+\frac{b}{x^2}}}{128bx}-\frac{5a^2\sqrt{a+\frac{b}{x^2}}}{64x^3}-\frac{5a\left(a+\frac{b}{x^2}\right)^{3/2}}{48x^3}-\frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{8x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^(5/2)/x^4,x]
```

```
[Out] (-5*a^2*Sqrt[a + b/x^2])/(64*x^3) - (5*a*(a + b/x^2)^(3/2))/(48*x^3) - (a +
 b/x^2)^(5/2)/(8*x^3) - (5*a^3*Sqrt[a + b/x^2])/(128*b*x) + (5*a^4*ArcTanh[
Sqrt[b]/(Sqrt[a + b/x^2]*x)])/(128*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^4}\,dx &= -\text{Subst}\left(\int x^2\left(a+bx^2\right)^{5/2}\,dx, x, \frac{1}{x}\right) \\
&= -\frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{8x^3} - \frac{1}{8}(5a)\,\text{Subst}\left(\int x^2\left(a+bx^2\right)^{3/2}\,dx, x, \frac{1}{x}\right) \\
&= -\frac{5a\left(a+\frac{b}{x^2}\right)^{3/2}}{48x^3} - \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{8x^3} - \frac{1}{16}\left(5a^2\right)\text{Subst}\left(\int x^2\sqrt{a+bx^2}\,dx, x, \frac{1}{x}\right) \\
&= -\frac{5a^2\sqrt{a+\frac{b}{x^2}}}{64x^3} - \frac{5a\left(a+\frac{b}{x^2}\right)^{3/2}}{48x^3} - \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{8x^3} - \frac{1}{64}\left(5a^3\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{x}\right) \\
&= -\frac{5a^2\sqrt{a+\frac{b}{x^2}}}{64x^3} - \frac{5a\left(a+\frac{b}{x^2}\right)^{3/2}}{48x^3} - \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{8x^3} - \frac{5a^3\sqrt{a+\frac{b}{x^2}}}{128bx} + \frac{\left(5a^4\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{x}\right)}{128b} \\
&= -\frac{5a^2\sqrt{a+\frac{b}{x^2}}}{64x^3} - \frac{5a\left(a+\frac{b}{x^2}\right)^{3/2}}{48x^3} - \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{8x^3} - \frac{5a^3\sqrt{a+\frac{b}{x^2}}}{128bx} + \frac{\left(5a^4\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+\frac{b}{x^2}}}\right)}{128b} \\
&= -\frac{5a^2\sqrt{a+\frac{b}{x^2}}}{64x^3} - \frac{5a\left(a+\frac{b}{x^2}\right)^{3/2}}{48x^3} - \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{8x^3} - \frac{5a^3\sqrt{a+\frac{b}{x^2}}}{128bx} + \frac{5a^4\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x}\right)}{128b^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 49, normalized size = 0.42

$$-\frac{a^4x^5\left(a+\frac{b}{x^2}\right)^{5/2}\left(ax^2+b\right)\,{}_2F_1\left(\frac{7}{2},5;\frac{9}{2};\frac{ax^2}{b}+1\right)}{7b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^(5/2)/x^4,x]
```

```
[Out] -1/7*(a^4*(a + b/x^2)^(5/2)*x^5*(b + a*x^2)*Hypergeometric2F1[7/2, 5, 9/2,
1 + (a*x^2)/b])/b^5
```

fricas [A] time = 0.98, size = 206, normalized size = 1.78

$$\left[\frac{15\,a^4\sqrt{b}\,x^7\log\left(-\frac{ax^2+2\,\sqrt{b}\,x\sqrt{\frac{ax^2+b}{x^2}}+2\,b}{x^2}\right) - 2\left(15\,a^3bx^6+118\,a^2b^2x^4+136\,ab^3x^2+48\,b^4\right)\sqrt{\frac{ax^2+b}{x^2}}}{768\,b^2x^7}, -\frac{15\,a^4\sqrt{-b}\,x^7\,\text{ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/768*(15*a^4*sqrt(b)*x^7*log(-(a*x^2 + 2*sqrt(b)*x*sqrt((a*x^2 + b)/x^2)
+ 2*b)/x^2) - 2*(15*a^3*b*x^6 + 118*a^2*b^2*x^4 + 136*a*b^3*x^2 + 48*b^4)*s
qrt((a*x^2 + b)/x^2))/(b^2*x^7), -1/384*(15*a^4*sqrt(-b)*x^7*arctan(sqrt(-b
)*x*sqrt((a*x^2 + b)/x^2)/(a*x^2 + b)) + (15*a^3*b*x^6 + 118*a^2*b^2*x^4 +
136*a*b^3*x^2 + 48*b^4)*sqrt((a*x^2 + b)/x^2))/(b^2*x^7)]
```

giac [A] time = 0.25, size = 119, normalized size = 1.03

$$-\frac{\frac{15\,a^5\arctan\left(\frac{\sqrt{ax^2+b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}\,b}+\frac{15\left(ax^2+b\right)^{\frac{7}{2}}a^5\mathrm{sgn}(x)+73\left(ax^2+b\right)^{\frac{5}{2}}a^5b\mathrm{sgn}(x)-55\left(ax^2+b\right)^{\frac{3}{2}}a^5b^2\mathrm{sgn}(x)+15\sqrt{ax^2+b}\,a^5b^3\mathrm{sgn}(x)}{a^4bx^8}}{384\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/384*(15*a^5*arctan(sqrt(a*x^2 + b)/sqrt(-b))*sgn(x)/(sqrt(-b)*b) + (15*(
a*x^2 + b)^(7/2)*a^5*sgn(x) + 73*(a*x^2 + b)^(5/2)*a^5*b*sgn(x) - 55*(a*x^2
 + b)^(3/2)*a^5*b^2*sgn(x) + 15*sqrt(a*x^2 + b)*a^5*b^3*sgn(x))/(a^4*b*x^8)
)/a
```

maple [B] time = 0.02, size = 186, normalized size = 1.60

$$\frac{\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{5}{2}}\left(15a^4b^{\frac{5}{2}}x^8\ln\left(\frac{2b+2\sqrt{a\,x^2+b}\,\sqrt{b}}{x}\right)-15\sqrt{a\,x^2+b}\,a^4b^2x^8-5\left(a\,x^2+b\right)^{\frac{3}{2}}a^4b\,x^8-3\left(a\,x^2+b\right)^{\frac{5}{2}}a^4x^8+3\left(a\right.\right.}{384\left(a\,x^2+b\right)^{\frac{5}{2}}b^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^(5/2)/x^4,x)
```

```
[Out] 1/384*((a*x^2+b)/x^2)^(5/2)/x^3*(-3*(a*x^2+b)^(5/2)*x^8*a^4+3*(a*x^2+b)^(7/
2)*x^6*a^3-5*(a*x^2+b)^(3/2)*x^8*a^4*b+15*b^(5/2)*ln(2*(b+(a*x^2+b)^(1/2)*b
^(1/2))/x)*x^8*a^4-15*(a*x^2+b)^(1/2)*x^8*a^4*b^2+2*(a*x^2+b)^(7/2)*x^4*a^2
*b+8*(a*x^2+b)^(7/2)*x^2*a*b^2-48*(a*x^2+b)^(7/2)*b^3)/(a*x^2+b)^(5/2)/b^4
```

maxima [B] time = 1.97, size = 190, normalized size = 1.64

$$-\frac{5\,a^4\log\left(\frac{\sqrt{a+\frac{b}{x^2}}\,x-\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x+\sqrt{b}}\right)}{256\,b^{\frac{3}{2}}}-\frac{15\left(a+\frac{b}{x^2}\right)^{\frac{7}{2}}a^4x^7+73\left(a+\frac{b}{x^2}\right)^{\frac{5}{2}}a^4bx^5-55\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}a^4b^2x^3+15\sqrt{a+\frac{b}{x^2}}\,a^4b^3x}{384\left(\left(a+\frac{b}{x^2}\right)^4bx^8-4\left(a+\frac{b}{x^2}\right)^3b^2x^6+6\left(a+\frac{b}{x^2}\right)^2b^3x^4-4\left(a+\frac{b}{x^2}\right)b^4x^2+b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] -5/256*a^4*log((sqrt(a + b/x^2)*x - sqrt(b))/(sqrt(a + b/x^2)*x + sqrt(b)))
/b^(3/2) - 1/384*(15*(a + b/x^2)^(7/2)*a^4*x^7 + 73*(a + b/x^2)^(5/2)*a^4*b
*x^5 - 55*(a + b/x^2)^(3/2)*a^4*b^2*x^3 + 15*sqrt(a + b/x^2)*a^4*b^3*x)/((a
 + b/x^2)^4*b*x^8 - 4*(a + b/x^2)^3*b^2*x^6 + 6*(a + b/x^2)^2*b^3*x^4 - 4*(
a + b/x^2)*b^4*x^2 + b^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)^(5/2)/x^4,x)
```

```
[Out] int((a + b/x^2)^(5/2)/x^4, x)
```

sympy **[A]** time = 8.19, size = 150, normalized size = 1.29

$$-\frac{5a^{\frac{7}{2}}}{128bx\sqrt{1+\frac{b}{ax^2}}}-\frac{133a^{\frac{5}{2}}}{384x^3\sqrt{1+\frac{b}{ax^2}}}-\frac{127a^{\frac{3}{2}}b}{192x^5\sqrt{1+\frac{b}{ax^2}}}-\frac{23\sqrt{a}\,b^2}{48x^7\sqrt{1+\frac{b}{ax^2}}}+\frac{5a^4\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\,x}\right)}{128b^{\frac{3}{2}}}-\frac{b^3}{8\sqrt{a}\,x^9\sqrt{1+\frac{b}{ax^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**(5/2)/x**4,x)
```

```
[Out] -5*a**(7/2)/(128*b*x*sqrt(1 + b/(a*x**2))) - 133*a**(5/2)/(384*x**3*sqrt(1
+ b/(a*x**2))) - 127*a**(3/2)*b/(192*x**5*sqrt(1 + b/(a*x**2))) - 23*sqrt(a
)*b**2/(48*x**7*sqrt(1 + b/(a*x**2))) + 5*a**4*asinh(sqrt(b)/(sqrt(a)*x))/(
128*b**(3/2)) - b**3/(8*sqrt(a)*x**9*sqrt(1 + b/(a*x**2)))
```

3.1915 $\int \frac{x^3}{\sqrt{a+\frac{b}{x^2}}}\,dx$

Optimal. Leaf size=74

$$\frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{8a^{5/2}} - \frac{3bx^2\sqrt{a+\frac{b}{x^2}}}{8a^2} + \frac{x^4\sqrt{a+\frac{b}{x^2}}}{4a}$$

```
[Out] 3/8*b^2*arctanh((a+b/x^2)^(1/2)/a^(1/2))/a^(5/2)-3/8*b*x^2*(a+b/x^2)^(1/2)/
a^2+1/4*x^4*(a+b/x^2)^(1/2)/a
```

Rubi [A] time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 51, 63, 208}

$$\frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{8a^{5/2}} - \frac{3bx^2\sqrt{a+\frac{b}{x^2}}}{8a^2} + \frac{x^4\sqrt{a+\frac{b}{x^2}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b/x^2],x]
```

```
[Out] (-3*b*Sqrt[a + b/x^2]*x^2)/(8*a^2) + (Sqrt[a + b/x^2]*x^4)/(4*a) + (3*b^2*A
rcTanh[Sqrt[a + b/x^2]/Sqrt[a]])/(8*a^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x^2}}}\,dx = -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= \frac{\sqrt{a+\frac{b}{x^2}}\,x^4}{4a} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x^2}\right)}{8a}$$

$$= -\frac{3b\sqrt{a+\frac{b}{x^2}}\,x^2}{8a^2} + \frac{\sqrt{a+\frac{b}{x^2}}\,x^4}{4a} - \frac{\left(3b^2\right)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x^2}\right)}{16a^2}$$

$$= -\frac{3b\sqrt{a+\frac{b}{x^2}}\,x^2}{8a^2} + \frac{\sqrt{a+\frac{b}{x^2}}\,x^4}{4a} - \frac{(3b)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x^2}}\right)}{8a^2}$$

$$= -\frac{3b\sqrt{a+\frac{b}{x^2}}\,x^2}{8a^2} + \frac{\sqrt{a+\frac{b}{x^2}}\,x^4}{4a} + \frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{8a^{5/2}}$$

Mathematica [A] time = 0.03, size = 87, normalized size = 1.18

$$\frac{\sqrt{a}\,x\left(2a^2x^4 - abx^2 - 3b^2\right) + 3b^2\sqrt{ax^2+b}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+b}}\right)}{8a^{5/2}x\sqrt{a+\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + b/x^2],x]
```

```
[Out] (Sqrt[a]*x*(-3*b^2 - a*b*x^2 + 2*a^2*x^4) + 3*b^2*Sqrt[b + a*x^2]*ArcTanh[(
Sqrt[a]*x)/Sqrt[b + a*x^2]])/(8*a^(5/2)*Sqrt[a + b/x^2]*x)
```

fricas [A] time = 1.21, size = 157, normalized size = 2.12

$$\left[\frac{3\sqrt{a}\,b^2\log\left(-2\,ax^2 - 2\sqrt{a}\,x^2\sqrt{\frac{ax^2+b}{x^2}} - b\right) + 2\left(2\,a^2x^4 - 3\,abx^2\right)\sqrt{\frac{ax^2+b}{x^2}}}{16\,a^3}, -\frac{3\sqrt{-a}\,b^2\arctan\left(\frac{\sqrt{-a}\,x^2\sqrt{\frac{ax^2+b}{x^2}}}{ax^2+b}\right) - \left(2\,a\right.}{8\,a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(a)*b^2*log(-2*a*x^2 - 2*sqrt(a)*x^2*sqrt((a*x^2 + b)/x^2) - b
) + 2*(2*a^2*x^4 - 3*a*b*x^2)*sqrt((a*x^2 + b)/x^2))/a^3, -1/8*(3*sqrt(-a)*
b^2*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + b)/x^2)/(a*x^2 + b)) - (2*a^2*x^4 - 3
*a*b*x^2)*sqrt((a*x^2 + b)/x^2))/a^3]
```

giac [A] time = 0.24, size = 84, normalized size = 1.14

$$\frac{1}{8}\sqrt{ax^4+bx^2}\left(\frac{2\,x^2}{a} - \frac{3\,b}{a^2}\right) - \frac{3\,b^2\log\left(\left|-2\left(\sqrt{a}\,x^2 - \sqrt{ax^4+bx^2}\right)\sqrt{a} - b\right|\right)}{16\,a^{\frac{5}{2}}} + \frac{3\,b^2\log\left(|b|\right)}{16\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(a*x^4 + b*x^2)*(2*x^2/a - 3*b/a^2) - 3/16*b^2*log(abs(-2*(sqrt(a)*
x^2 - sqrt(a*x^4 + b*x^2))*sqrt(a) - b))/a^(5/2) + 3/16*b^2*log(abs(b))/a^(
5/2)
```

maple [A] time = 0.01, size = 87, normalized size = 1.18

$$\frac{\sqrt{a\,x^2+b}\,\left(2\sqrt{a\,x^2+b}\;a^{\frac{5}{2}}x^3+3a\,b^2\ln\left(\sqrt{a}\;x+\sqrt{a\,x^2+b}\right)-3\sqrt{a\,x^2+b}\;a^{\frac{3}{2}}bx\right)}{8\sqrt{\frac{a\,x^2+b}{x^2}}\;a^{\frac{7}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b/x^2)^(1/2),x)
```

```
[Out] 1/8*(a*x^2+b)^(1/2)*(2*x^3*(a*x^2+b)^(1/2)*a^(5/2)-3*(a*x^2+b)^(1/2)*a^(3/2
)*x*b+3*ln(a^(1/2)*x+(a*x^2+b)^(1/2))*a*b^2)/((a*x^2+b)/x^2)^(1/2)/x/a^(7/2
)
```

maxima [A] time = 1.89, size = 104, normalized size = 1.41

$$-\frac{3\,b^2\log\left(\frac{\sqrt{a+\frac{b}{x^2}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^2}}+\sqrt{a}}\right)}{16\,a^{\frac{5}{2}}}-\frac{3\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}b^2-5\sqrt{a+\frac{b}{x^2}}\,ab^2}{8\left(\left(a+\frac{b}{x^2}\right)^2a^2-2\left(a+\frac{b}{x^2}\right)a^3+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/16*b^2*log((sqrt(a + b/x^2) - sqrt(a))/(sqrt(a + b/x^2) + sqrt(a)))/a^(5
/2) - 1/8*(3*(a + b/x^2)^(3/2)*b^2 - 5*sqrt(a + b/x^2)*a*b^2)/((a + b/x^2)^
2*a^2 - 2*(a + b/x^2)*a^3 + a^4)
```

mupad [B] time = 1.58, size = 57, normalized size = 0.77

$$\frac{3\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{8\,a^{5/2}}+\frac{5\,x^4\sqrt{a+\frac{b}{x^2}}}{8\,a}-\frac{3\,x^4\left(a+\frac{b}{x^2}\right)^{3/2}}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b/x^2)^(1/2),x)
```

```
[Out] (3*b^2*atanh((a + b/x^2)^(1/2)/a^(1/2)))/(8*a^(5/2)) + (5*x^4*(a + b/x^2)^(
1/2))/(8*a) - (3*x^4*(a + b/x^2)^(3/2))/(8*a^2)
```

sympy [A] time = 4.87, size = 95, normalized size = 1.28

$$\frac{x^5}{4\sqrt{b}\sqrt{\frac{ax^2}{b}+1}}-\frac{\sqrt{b}\,x^3}{8a\sqrt{\frac{ax^2}{b}+1}}-\frac{3b^{\frac{3}{2}}x}{8a^2\sqrt{\frac{ax^2}{b}+1}}+\frac{3b^2\,\mathrm{asinh}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b/x**2)**(1/2),x)
```

```
[Out] x**5/(4*sqrt(b)*sqrt(a*x**2/b + 1)) - sqrt(b)*x**3/(8*a*sqrt(a*x**2/b + 1))
 - 3*b**(3/2)*x/(8*a**2*sqrt(a*x**2/b + 1)) + 3*b**2*asinh(sqrt(a)*x/sqrt(b
))/(8*a**(5/2))
```

3.1916 $\int \frac{x}{\sqrt{a+\frac{b}{x^2}}}\,dx$

Optimal. Leaf size=50

$$\frac{x^2\sqrt{a+\frac{b}{x^2}}}{2a} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{2a^{3/2}}$$

```
[Out] -1/2*b*arctanh((a+b/x^2)^(1/2)/a^(1/2))/a^(3/2)+1/2*x^2*(a+b/x^2)^(1/2)/a
```

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {266, 51, 63, 208}

$$\frac{x^2\sqrt{a+\frac{b}{x^2}}}{2a} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{2a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b/x^2],x]
```

```
[Out] (Sqrt[a + b/x^2]*x^2)/(2*a) - (b*ArcTanh[Sqrt[a + b/x^2]/Sqrt[a]])/(2*a^(3/
2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+\frac{b}{x^2}}}\,dx = -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x^2}\right)\right)$$
$$= \frac{\sqrt{a+\frac{b}{x^2}}\,x^2}{2a} + \frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x^2}\right)}{4a}$$
$$= \frac{\sqrt{a+\frac{b}{x^2}}\,x^2}{2a} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x^2}}\right)}{2a}$$
$$= \frac{\sqrt{a+\frac{b}{x^2}}\,x^2}{2a} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{2a^{3/2}}$$

Mathematica [A] time = 0.02, size = 71, normalized size = 1.42

$$\frac{\sqrt{a}\,x\left(ax^2+b\right) - b\sqrt{ax^2+b}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+b}}\right)}{2a^{3/2}x\sqrt{a+\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b/x^2],x]
```

```
[Out] (Sqrt[a]*x*(b + a*x^2) - b*Sqrt[b + a*x^2]*ArcTanh[(Sqrt[a]*x)/Sqrt[b + a*x
^2]])/(2*a^(3/2)*Sqrt[a + b/x^2]*x)
```

fricas [A] time = 0.90, size = 126, normalized size = 2.52

$$\left[\frac{2\,ax^2\sqrt{\frac{ax^2+b}{x^2}} + \sqrt{a}\,b\log\left(-2\,ax^2 + 2\,\sqrt{a}\,x^2\sqrt{\frac{ax^2+b}{x^2}} - b\right)}{4\,a^2}, \frac{ax^2\sqrt{\frac{ax^2+b}{x^2}} + \sqrt{-a}\,b\arctan\left(\frac{\sqrt{-a}\,x^2\sqrt{\frac{ax^2+b}{x^2}}}{ax^2+b}\right)}{2\,a^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*x^2*sqrt((a*x^2 + b)/x^2) + sqrt(a)*b*log(-2*a*x^2 + 2*sqrt(a)*x^
2*sqrt((a*x^2 + b)/x^2) - b))/a^2, 1/2*(a*x^2*sqrt((a*x^2 + b)/x^2) + sqrt(
-a)*b*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + b)/x^2)/(a*x^2 + b)))/a^2]
```

giac [A] time = 0.25, size = 68, normalized size = 1.36

$$\frac{b\log\left(\left|-2\left(\sqrt{a}\,x^2 - \sqrt{ax^4+bx^2}\right)\sqrt{a} - b\right|\right)}{4\,a^{\frac{3}{2}}} - \frac{b\log\left(|b|\right)}{4\,a^{\frac{3}{2}}} + \frac{\sqrt{ax^4+bx^2}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*b*log(abs(-2*(sqrt(a)*x^2 - sqrt(a*x^4 + b*x^2))*sqrt(a) - b))/a^(3/2)
- 1/4*b*log(abs(b))/a^(3/2) + 1/2*sqrt(a*x^4 + b*x^2)/a
```

maple [A] time = 0.01, size = 66, normalized size = 1.32

$$\frac{\sqrt{a\,x^2+b}\,\left(-ab\ln\left(\sqrt{a}\,x+\sqrt{a\,x^2+b}\right)+\sqrt{a\,x^2+b}\,a^{\frac{3}{2}}x\right)}{2\sqrt{\frac{a\,x^2+b}{x^2}}\,a^{\frac{5}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b/x^2)^(1/2),x)
```

```
[Out] 1/2*(a*x^2+b)^(1/2)*(x*(a*x^2+b)^(1/2)*a^(3/2)-b*ln(a^(1/2)*x+(a*x^2+b)^(1/
2))*a)/((a*x^2+b)/x^2)^(1/2)/x/a^(5/2)
```

maxima [A] time = 1.98, size = 68, normalized size = 1.36

$$\frac{\sqrt{a+\frac{b}{x^2}}\,b}{2\left(\left(a+\frac{b}{x^2}\right)a-a^2\right)}+\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x^2}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^2}}+\sqrt{a}}\right)}{4\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a + b/x^2)*b/((a + b/x^2)*a - a^2) + 1/4*b*log((sqrt(a + b/x^2) -
sqrt(a))/(sqrt(a + b/x^2) + sqrt(a)))/a^(3/2)
```

mupad [B] time = 1.43, size = 38, normalized size = 0.76

$$\frac{x^2\sqrt{a+\frac{b}{x^2}}}{2\,a}-\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{2\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b/x^2)^(1/2),x)
```

```
[Out] (x^2*(a + b/x^2)^(1/2))/(2*a) - (b*atanh((a + b/x^2)^(1/2)/a^(1/2)))/(2*a^(
3/2))
```

sympy [A] time = 2.74, size = 42, normalized size = 0.84

$$\frac{\sqrt{b}\,x\sqrt{\frac{ax^2}{b}+1}}{2a}-\frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x**2)**(1/2),x)
```

```
[Out] sqrt(b)*x*sqrt(a*x**2/b + 1)/(2*a) - b*asinh(sqrt(a)*x/sqrt(b))/(2*a**(3/2)
)
```

3.1917 $\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x}\,dx$

Optimal. Leaf size=24

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] arctanh((a+b/x^2)^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {266, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x^2]*x),x]
```

```
[Out] ArcTanh[Sqrt[a + b/x^2]/Sqrt[a]]/Sqrt[a]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x}\,dx = -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x^2}}\right)}{b}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Mathematica [B] time = 0.02, size = 50, normalized size = 2.08

$$\frac{\sqrt{ax^2+b}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+b}}\right)}{\sqrt{a}\,x\sqrt{a+\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x^2]*x),x]
```

```
[Out] (Sqrt[b + a*x^2]*ArcTanh[(Sqrt[a]*x)/Sqrt[b + a*x^2]])/(Sqrt[a]*Sqrt[a + b/
x^2]*x)
```

fricas [B] time = 0.85, size = 80, normalized size = 3.33

$$\left[\frac{\log\left(-2\,ax^2 - 2\,\sqrt{a}\,x^2\sqrt{\frac{ax^2+b}{x^2}} - b\right)}{2\,\sqrt{a}}, -\frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}\,x^2\sqrt{\frac{ax^2+b}{x^2}}}{ax^2+b}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*log(-2*a*x^2 - 2*sqrt(a)*x^2*sqrt((a*x^2 + b)/x^2) - b)/sqrt(a), -sqrt
(-a)*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + b)/x^2)/(a*x^2 + b))/a]
```

giac [B] time = 0.25, size = 48, normalized size = 2.00

$$-\frac{\log\left(\left|-2\left(\sqrt{a}\,x^2 - \sqrt{ax^4+bx^2}\right)\sqrt{a} - b\right|\right)}{2\,\sqrt{a}} + \frac{\log\left(|b|\right)}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -1/2*log(abs(-2*(sqrt(a)*x^2 - sqrt(a*x^4 + b*x^2))*sqrt(a) - b))/sqrt(a) +
 1/2*log(abs(b))/sqrt(a)
```

maple [B] time = 0.00, size = 46, normalized size = 1.92

$$\frac{\sqrt{a\,x^2+b}\,\ln\left(\sqrt{a}\,x + \sqrt{a\,x^2+b}\right)}{\sqrt{\frac{a\,x^2+b}{x^2}}\,\sqrt{a}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^(1/2)/x,x)
```

```
[Out] 1/((a*x^2+b)/x^2)^(1/2)/x*(a*x^2+b)^(1/2)*ln(a^(1/2)*x+(a*x^2+b)^(1/2))/a^(
1/2)
```

maxima [B] time = 1.90, size = 37, normalized size = 1.54

$$-\frac{\log\left(\frac{\sqrt{a+\frac{b}{x^2}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^2}}+\sqrt{a}}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -1/2*log((sqrt(a + b/x^2) - sqrt(a))/(sqrt(a + b/x^2) + sqrt(a)))/sqrt(a)
```

mupad [B] time = 1.31, size = 18, normalized size = 0.75

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b/x^2)^(1/2)),x)
```

```
[Out] atanh((a + b/x^2)^(1/2)/a^(1/2))/a^(1/2)
```

sympy [A] time = 1.29, size = 17, normalized size = 0.71

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**(1/2)/x,x)
```

```
[Out] asinh(sqrt(a)*x/sqrt(b))/sqrt(a)
```

3.1918 $\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x^3}\,dx$

Optimal. Leaf size=16

$$-\frac{\sqrt{a+\frac{b}{x^2}}}{b}$$

```
[Out] -(a+b/x^2)^(1/2)/b
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$-\frac{\sqrt{a+\frac{b}{x^2}}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x^2]*x^3),x]
```

```
[Out] -(Sqrt[a + b/x^2]/b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x^3}\,dx = -\frac{\sqrt{a+\frac{b}{x^2}}}{b}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$-\frac{\sqrt{a+\frac{b}{x^2}}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x^2]*x^3),x]
```

```
[Out] -(Sqrt[a + b/x^2]/b)
```

fricas [A] time = 0.76, size = 18, normalized size = 1.12

$$-\frac{\sqrt{\frac{ax^2+b}{x^2}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] -sqrt((a*x^2 + b)/x^2)/b
```

giac [A] time = 0.19, size = 18, normalized size = 1.12

$$-\frac{\sqrt{\frac{ax^2+b}{x^2}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(1/2)/x^3,x, algorithm="giac")
[Out] -sqrt((a*x^2 + b)/x^2)/b
```

maple [A] time = 0.00, size = 29, normalized size = 1.81

$$-\frac{a\,x^2+b}{\sqrt{\frac{a\,x^2+b}{x^2}}\;b\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^(1/2)/x^3,x)
[Out] -(a*x^2+b)/x^2/b/((a*x^2+b)/x^2)^(1/2)
```

maxima [A] time = 0.86, size = 14, normalized size = 0.88

$$-\frac{\sqrt{a+\frac{b}{x^2}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(1/2)/x^3,x, algorithm="maxima")
[Out] -sqrt(a + b/x^2)/b
```

mupad [B] time = 1.21, size = 14, normalized size = 0.88

$$-\frac{\sqrt{a+\frac{b}{x^2}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b/x^2)^(1/2)),x)
[Out] -(a + b/x^2)^(1/2)/b
```

sympy [A] time = 1.39, size = 26, normalized size = 1.62

$$\begin{cases} -\frac{\sqrt{a+\frac{b}{x^2}}}{b} & \text{for } b \neq 0 \\ -\frac{1}{2\sqrt{a}\,x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**(1/2)/x**3,x)
[Out] Piecewise((-sqrt(a + b/x**2)/b, Ne(b, 0)), (-1/(2*sqrt(a)*x**2), True))
```

3.1919 $\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x^5}\,dx$

Optimal. Leaf size=35

$$\frac{a\sqrt{a+\frac{b}{x^2}}}{b^2}-\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{3b^2}$$

```
[Out] -1/3*(a+b/x^2)^(3/2)/b^2+a*(a+b/x^2)^(1/2)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{a\sqrt{a+\frac{b}{x^2}}}{b^2}-\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x^2]*x^5),x]
```

```
[Out] (a*Sqrt[a + b/x^2])/b^2 - (a + b/x^2)^(3/2)/(3*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x^5}\,dx &= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}}\,dx, x, \frac{1}{x^2}\right)\right)\\ &= -\left(\frac{1}{2}\,\text{Subst}\left(\int\left(-\frac{a}{b\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b}\right)dx, x, \frac{1}{x^2}\right)\right)\\ &= \frac{a\sqrt{a+\frac{b}{x^2}}}{b^2}-\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{3b^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 0.89

$$\frac{\sqrt{a+\frac{b}{x^2}}\left(2ax^2-b\right)}{3b^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x^2]*x^5),x]
```

```
[Out] (Sqrt[a + b/x^2]*(-b + 2*a*x^2))/(3*b^2*x^2)
```

fricas [A] time = 0.82, size = 31, normalized size = 0.89

$$\frac{\left(2\,ax^2-b\right)\sqrt{\frac{ax^2+b}{x^2}}}{3\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/3*(2*a*x^2 - b)*sqrt((a*x^2 + b)/x^2)/(b^2*x^2)
```

giac [A] time = 0.21, size = 57, normalized size = 1.63

$$\frac{3\left(\sqrt{a}\,x^2-\sqrt{ax^4+bx^2}\right)\sqrt{a}\,+b}{3\left(\sqrt{a}\,x^2-\sqrt{ax^4+bx^2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/3*(3*(sqrt(a)*x^2 - sqrt(a*x^4 + b*x^2))*sqrt(a) + b)/(sqrt(a)*x^2 - sqrt
(a*x^4 + b*x^2))^3
```

maple [A] time = 0.01, size = 39, normalized size = 1.11

$$\frac{\left(a\,x^2+b\right)\left(2a\,x^2-b\right)}{3\sqrt{\frac{a\,x^2+b}{x^2}}\,b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^(1/2)/x^5,x)
```

```
[Out] 1/3*(a*x^2+b)*(2*a*x^2-b)/x^4/b^2/((a*x^2+b)/x^2)^(1/2)
```

maxima [A] time = 0.87, size = 29, normalized size = 0.83

$$-\frac{\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}}{3\,b^2}+\frac{\sqrt{a+\frac{b}{x^2}}\,a}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/3*(a + b/x^2)^(3/2)/b^2 + sqrt(a + b/x^2)*a/b^2
```

mupad [B] time = 1.24, size = 25, normalized size = 0.71

$$-\frac{\sqrt{a+\frac{b}{x^2}}\,\left(b-2\,a\,x^2\right)}{3\,b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b/x^2)^(1/2)),x)
```

```
[Out] -((a + b/x^2)^(1/2)*(b - 2*a*x^2))/(3*b^2*x^2)
```

sympy [**B**] time = 1.62, size = 231, normalized size = 6.60

$$\frac{2a^{\frac{7}{2}}b^{\frac{3}{2}}x^4\sqrt{\frac{ax^2}{b}+1}}{3a^{\frac{5}{2}}b^3x^5+3a^{\frac{3}{2}}b^4x^3}+\frac{a^{\frac{5}{2}}b^{\frac{5}{2}}x^2\sqrt{\frac{ax^2}{b}+1}}{3a^{\frac{5}{2}}b^3x^5+3a^{\frac{3}{2}}b^4x^3}-\frac{a^{\frac{3}{2}}b^{\frac{7}{2}}\sqrt{\frac{ax^2}{b}+1}}{3a^{\frac{5}{2}}b^3x^5+3a^{\frac{3}{2}}b^4x^3}-\frac{2a^4bx^5}{3a^{\frac{5}{2}}b^3x^5+3a^{\frac{3}{2}}b^4x^3}-\frac{2a^3b^2x^3}{3a^{\frac{5}{2}}b^3x^5+3a^{\frac{3}{2}}b^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**(1/2)/x**5,x)
```

```
[Out] 2*a**(7/2)*b**(3/2)*x**4*sqrt(a*x**2/b + 1)/(3*a**(5/2)*b**3*x**5 + 3*a**(3
/2)*b**4*x**3) + a**(5/2)*b**(5/2)*x**2*sqrt(a*x**2/b + 1)/(3*a**(5/2)*b**3
*x**5 + 3*a**(3/2)*b**4*x**3) - a**(3/2)*b**(7/2)*sqrt(a*x**2/b + 1)/(3*a**
(5/2)*b**3*x**5 + 3*a**(3/2)*b**4*x**3) - 2*a**4*b*x**5/(3*a**(5/2)*b**3*x*
*5 + 3*a**(3/2)*b**4*x**3) - 2*a**3*b**2*x**3/(3*a**(5/2)*b**3*x**5 + 3*a**
(3/2)*b**4*x**3)
```

3.1920 $\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x^7}\,dx$

Optimal. Leaf size=57

$$-\frac{a^2\sqrt{a+\frac{b}{x^2}}}{b^3}-\frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{5b^3}+\frac{2a\left(a+\frac{b}{x^2}\right)^{3/2}}{3b^3}$$

```
[Out] 2/3*a*(a+b/x^2)^(3/2)/b^3-1/5*(a+b/x^2)^(5/2)/b^3-a^2*(a+b/x^2)^(1/2)/b^3
```

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{a^2\sqrt{a+\frac{b}{x^2}}}{b^3}-\frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{5b^3}+\frac{2a\left(a+\frac{b}{x^2}\right)^{3/2}}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x^2]*x^7),x]
```

```
[Out] -((a^2*Sqrt[a + b/x^2])/b^3) + (2*a*(a + b/x^2)^(3/2))/(3*b^3) - (a + b/x^2
)^(5/2)/(5*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x^7}\,dx &= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx}}\,dx, x, \frac{1}{x^2}\right)\right)\\ &= -\left(\frac{1}{2}\,\text{Subst}\left(\int \left(\frac{a^2}{b^2\sqrt{a+bx}}-\frac{2a\sqrt{a+bx}}{b^2}+\frac{(a+bx)^{3/2}}{b^2}\right)dx, x, \frac{1}{x^2}\right)\right)\\ &= -\frac{a^2\sqrt{a+\frac{b}{x^2}}}{b^3}+\frac{2a\left(a+\frac{b}{x^2}\right)^{3/2}}{3b^3}-\frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{5b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.74

$$-\frac{\sqrt{a+\frac{b}{x^2}}\left(8a^2x^4-4abx^2+3b^2\right)}{15b^3x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x^2]*x^7),x]
```

```
[Out] -1/15*(Sqrt[a + b/x^2]*(3*b^2 - 4*a*b*x^2 + 8*a^2*x^4))/(b^3*x^4)
```

fricas [A] time = 1.13, size = 42, normalized size = 0.74

$$-\frac{\left(8\,a^2x^4-4\,abx^2+3\,b^2\right)\sqrt{\frac{ax^2+b}{x^2}}}{15\,b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] -1/15*(8*a^2*x^4 - 4*a*b*x^2 + 3*b^2)*sqrt((a*x^2 + b)/x^2)/(b^3*x^4)
```

giac [A] time = 0.22, size = 90, normalized size = 1.58

$$\frac{20\left(\sqrt{a}\,x^2-\sqrt{ax^4+bx^2}\right)^2a+15\left(\sqrt{a}\,x^2-\sqrt{ax^4+bx^2}\right)\sqrt{a}\,b+3\,b^2}{15\left(\sqrt{a}\,x^2-\sqrt{ax^4+bx^2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/15*(20*(sqrt(a)*x^2 - sqrt(a*x^4 + b*x^2))^2*a + 15*(sqrt(a)*x^2 - sqrt(a
*x^4 + b*x^2))*sqrt(a)*b + 3*b^2)/(sqrt(a)*x^2 - sqrt(a*x^4 + b*x^2))^5
```

maple [A] time = 0.01, size = 50, normalized size = 0.88

$$-\frac{\left(a\,x^2+b\right)\left(8a^2x^4-4ab\,x^2+3b^2\right)}{15\sqrt{\frac{a\,x^2+b}{x^2}}\;b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^(1/2)/x^7,x)
```

```
[Out] -1/15*(a*x^2+b)*(8*a^2*x^4-4*a*b*x^2+3*b^2)/x^6/b^3/((a*x^2+b)/x^2)^(1/2)
```

maxima [A] time = 0.87, size = 47, normalized size = 0.82

$$-\frac{\left(a+\frac{b}{x^2}\right)^{\frac{5}{2}}}{5\,b^3}+\frac{2\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}a}{3\,b^3}-\frac{\sqrt{a+\frac{b}{x^2}}\,a^2}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/5*(a + b/x^2)^(5/2)/b^3 + 2/3*(a + b/x^2)^(3/2)*a/b^3 - sqrt(a + b/x^2)*
a^2/b^3
```

mupad [B] time = 1.28, size = 56, normalized size = 0.98

$$-\frac{3\,b^2\sqrt{a+\frac{b}{x^2}}+8\,a^2\,x^4\sqrt{a+\frac{b}{x^2}}-4\,a\,b\,x^2\sqrt{a+\frac{b}{x^2}}}{15\,b^3\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b/x^2)^(1/2)),x)
```

```
[Out] -(3*b^2*(a + b/x^2)^(1/2) + 8*a^2*x^4*(a + b/x^2)^(1/2) - 4*a*b*x^2*(a + b/
x^2)^(1/2))/(15*b^3*x^4)
```

sympy [B] time = 2.31, size = 750, normalized size = 13.16

$$-\frac{8a^{\frac{15}{2}}b^{\frac{9}{2}}x^{10}\sqrt{\frac{ax^2}{b}+1}}{15a^{\frac{11}{2}}b^7x^{11}+45a^{\frac{9}{2}}b^8x^9+45a^{\frac{7}{2}}b^9x^7+15a^{\frac{5}{2}}b^{10}x^5}-\frac{20a^{\frac{13}{2}}b^{\frac{11}{2}}x^8\sqrt{\frac{ax^2}{b}+1}}{15a^{\frac{11}{2}}b^7x^{11}+45a^{\frac{9}{2}}b^8x^9+45a^{\frac{7}{2}}b^9x^7+15a^{\frac{5}{2}}b^{10}x^5}-\frac{}{15a^{\frac{11}{2}}b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**(1/2)/x**7,x)
```

```
[Out] -8*a**(15/2)*b**(9/2)*x**10*sqrt(a*x**2/b + 1)/(15*a**(11/2)*b**7*x**11 + 4
5*a**(9/2)*b**8*x**9 + 45*a**(7/2)*b**9*x**7 + 15*a**(5/2)*b**10*x**5) - 20
*a**(13/2)*b**(11/2)*x**8*sqrt(a*x**2/b + 1)/(15*a**(11/2)*b**7*x**11 + 45*
a**(9/2)*b**8*x**9 + 45*a**(7/2)*b**9*x**7 + 15*a**(5/2)*b**10*x**5) - 15*a
**(11/2)*b**(13/2)*x**6*sqrt(a*x**2/b + 1)/(15*a**(11/2)*b**7*x**11 + 45*a*
*(9/2)*b**8*x**9 + 45*a**(7/2)*b**9*x**7 + 15*a**(5/2)*b**10*x**5) - 5*a**(
9/2)*b**(15/2)*x**4*sqrt(a*x**2/b + 1)/(15*a**(11/2)*b**7*x**11 + 45*a**(9/
2)*b**8*x**9 + 45*a**(7/2)*b**9*x**7 + 15*a**(5/2)*b**10*x**5) - 5*a**(7/2)
*b**(17/2)*x**2*sqrt(a*x**2/b + 1)/(15*a**(11/2)*b**7*x**11 + 45*a**(9/2)*b
**8*x**9 + 45*a**(7/2)*b**9*x**7 + 15*a**(5/2)*b**10*x**5) - 3*a**(5/2)*b**
(19/2)*sqrt(a*x**2/b + 1)/(15*a**(11/2)*b**7*x**11 + 45*a**(9/2)*b**8*x**9
+ 45*a**(7/2)*b**9*x**7 + 15*a**(5/2)*b**10*x**5) + 8*a**8*b**4*x**11/(15*a
**(11/2)*b**7*x**11 + 45*a**(9/2)*b**8*x**9 + 45*a**(7/2)*b**9*x**7 + 15*a*
*(5/2)*b**10*x**5) + 24*a**7*b**5*x**9/(15*a**(11/2)*b**7*x**11 + 45*a**(9/
2)*b**8*x**9 + 45*a**(7/2)*b**9*x**7 + 15*a**(5/2)*b**10*x**5) + 24*a**6*b*
*6*x**7/(15*a**(11/2)*b**7*x**11 + 45*a**(9/2)*b**8*x**9 + 45*a**(7/2)*b**9
*x**7 + 15*a**(5/2)*b**10*x**5) + 8*a**5*b**7*x**5/(15*a**(11/2)*b**7*x**11
 + 45*a**(9/2)*b**8*x**9 + 45*a**(7/2)*b**9*x**7 + 15*a**(5/2)*b**10*x**5)
```

3.1921 $\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x^9}\,dx$

Optimal. Leaf size=75

$$\frac{a^3\sqrt{a+\frac{b}{x^2}}}{b^4}-\frac{a^2\left(a+\frac{b}{x^2}\right)^{3/2}}{b^4}-\frac{\left(a+\frac{b}{x^2}\right)^{7/2}}{7b^4}+\frac{3a\left(a+\frac{b}{x^2}\right)^{5/2}}{5b^4}$$

```
[Out] -a^2*(a+b/x^2)^(3/2)/b^4+3/5*a*(a+b/x^2)^(5/2)/b^4-1/7*(a+b/x^2)^(7/2)/b^4+
a^3*(a+b/x^2)^(1/2)/b^4
```

Rubi [A] time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{a^2\left(a+\frac{b}{x^2}\right)^{3/2}}{b^4}+\frac{a^3\sqrt{a+\frac{b}{x^2}}}{b^4}-\frac{\left(a+\frac{b}{x^2}\right)^{7/2}}{7b^4}+\frac{3a\left(a+\frac{b}{x^2}\right)^{5/2}}{5b^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x^2]*x^9),x]
```

```
[Out] (a^3*Sqrt[a + b/x^2])/b^4 - (a^2*(a + b/x^2)^(3/2))/b^4 + (3*a*(a + b/x^2)^
(5/2))/(5*b^4) - (a + b/x^2)^(7/2)/(7*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x^9}\,dx &= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{\sqrt{a+bx}}\,dx, x, \frac{1}{x^2}\right)\right)\\ &= -\left(\frac{1}{2}\,\text{Subst}\left(\int\left(-\frac{a^3}{b^3\sqrt{a+bx}}+\frac{3a^2\sqrt{a+bx}}{b^3}-\frac{3a(a+bx)^{3/2}}{b^3}+\frac{(a+bx)^{5/2}}{b^3}\right)dx, x, \frac{1}{x^2}\right)\right)\\ &= \frac{a^3\sqrt{a+\frac{b}{x^2}}}{b^4}-\frac{a^2\left(a+\frac{b}{x^2}\right)^{3/2}}{b^4}+\frac{3a\left(a+\frac{b}{x^2}\right)^{5/2}}{5b^4}-\frac{\left(a+\frac{b}{x^2}\right)^{7/2}}{7b^4}\end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.71

$$\frac{\sqrt{a+\frac{b}{x^2}}\left(16a^3x^6-8a^2bx^4+6ab^2x^2-5b^3\right)}{35b^4x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x^2]*x^9),x]
```

```
[Out] (Sqrt[a + b/x^2]*(-5*b^3 + 6*a*b^2*x^2 - 8*a^2*b*x^4 + 16*a^3*x^6))/(35*b^4
*x^6)
```

fricas [A] time = 1.42, size = 53, normalized size = 0.71

$$\frac{\left(16\,a^3x^6 - 8\,a^2bx^4 + 6\,ab^2x^2 - 5\,b^3\right)\sqrt{\frac{ax^2+b}{x^2}}}{35\,b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(1/2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/35*(16*a^3*x^6 - 8*a^2*b*x^4 + 6*a*b^2*x^2 - 5*b^3)*sqrt((a*x^2 + b)/x^2)
/(b^4*x^6)
```

giac [A] time = 0.23, size = 123, normalized size = 1.64

$$\frac{70\left(\sqrt{a}\,x^2 - \sqrt{ax^4+bx^2}\right)^3 a^{\frac{3}{2}} + 84\left(\sqrt{a}\,x^2 - \sqrt{ax^4+bx^2}\right)^2 ab + 35\left(\sqrt{a}\,x^2 - \sqrt{ax^4+bx^2}\right)\sqrt{a}\,b^2 + 5\,b^3}{35\left(\sqrt{a}\,x^2 - \sqrt{ax^4+bx^2}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] 1/35*(70*(sqrt(a)*x^2 - sqrt(a*x^4 + b*x^2))^3*a^(3/2) + 84*(sqrt(a)*x^2 -
sqrt(a*x^4 + b*x^2))^2*a*b + 35*(sqrt(a)*x^2 - sqrt(a*x^4 + b*x^2))*sqrt(a)
*b^2 + 5*b^3)/(sqrt(a)*x^2 - sqrt(a*x^4 + b*x^2))^7
```

maple [A] time = 0.01, size = 61, normalized size = 0.81

$$\frac{\left(a\,x^2+b\right)\left(16a^3x^6 - 8a^2b\,x^4 + 6a\,b^2x^2 - 5b^3\right)}{35\sqrt{\frac{a\,x^2+b}{x^2}}\;b^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^(1/2)/x^9,x)
```

```
[Out] 1/35*(a*x^2+b)*(16*a^3*x^6-8*a^2*b*x^4+6*a*b^2*x^2-5*b^3)/x^8/b^4/((a*x^2+b
)/x^2)^(1/2)
```

maxima [A] time = 0.90, size = 63, normalized size = 0.84

$$-\frac{\left(a+\frac{b}{x^2}\right)^{\frac{7}{2}}}{7\,b^4} + \frac{3\left(a+\frac{b}{x^2}\right)^{\frac{5}{2}}a}{5\,b^4} - \frac{\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}a^2}{b^4} + \frac{\sqrt{a+\frac{b}{x^2}}\,a^3}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(1/2)/x^9,x, algorithm="maxima")
```

```
[Out] -1/7*(a + b/x^2)^(7/2)/b^4 + 3/5*(a + b/x^2)^(5/2)*a/b^4 - (a + b/x^2)^(3/2
)*a^2/b^4 + sqrt(a + b/x^2)*a^3/b^4
```

mupad [B] time = 1.32, size = 73, normalized size = 0.97

$$\frac{16\,a^3\sqrt{a+\frac{b}{x^2}}}{35\,b^4} - \frac{\sqrt{a+\frac{b}{x^2}}}{7\,b\,x^6} + \frac{6\,a\sqrt{a+\frac{b}{x^2}}}{35\,b^2\,x^4} - \frac{8\,a^2\sqrt{a+\frac{b}{x^2}}}{35\,b^3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(a + b/x^2)^(1/2)),x)
```

```
[Out] (16*a^3*(a + b/x^2)^(1/2))/(35*b^4) - (a + b/x^2)^(1/2)/(7*b*x^6) + (6*a*(a
 + b/x^2)^(1/2))/(35*b^2*x^4) - (8*a^2*(a + b/x^2)^(1/2))/(35*b^3*x^2)
```

sympy [**B**] time = 3.56, size = 1969, normalized size = 26.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**(1/2)/x**9,x)
```

```
[Out] 16*a**(25/2)*b**(23/2)*x**18*sqrt(a*x**2/b + 1)/(35*a**(19/2)*b**15*x**19 +
 210*a**(17/2)*b**16*x**17 + 525*a**(15/2)*b**17*x**15 + 700*a**(13/2)*b**1
8*x**13 + 525*a**(11/2)*b**19*x**11 + 210*a**(9/2)*b**20*x**9 + 35*a**(7/2)
*b**21*x**7) + 88*a**(23/2)*b**(25/2)*x**16*sqrt(a*x**2/b + 1)/(35*a**(19/2
)*b**15*x**19 + 210*a**(17/2)*b**16*x**17 + 525*a**(15/2)*b**17*x**15 + 700
*a**(13/2)*b**18*x**13 + 525*a**(11/2)*b**19*x**11 + 210*a**(9/2)*b**20*x**
9 + 35*a**(7/2)*b**21*x**7) + 198*a**(21/2)*b**(27/2)*x**14*sqrt(a*x**2/b +
 1)/(35*a**(19/2)*b**15*x**19 + 210*a**(17/2)*b**16*x**17 + 525*a**(15/2)*b
**17*x**15 + 700*a**(13/2)*b**18*x**13 + 525*a**(11/2)*b**19*x**11 + 210*a*
*(9/2)*b**20*x**9 + 35*a**(7/2)*b**21*x**7) + 231*a**(19/2)*b**(29/2)*x**12
*sqrt(a*x**2/b + 1)/(35*a**(19/2)*b**15*x**19 + 210*a**(17/2)*b**16*x**17 +
 525*a**(15/2)*b**17*x**15 + 700*a**(13/2)*b**18*x**13 + 525*a**(11/2)*b**1
9*x**11 + 210*a**(9/2)*b**20*x**9 + 35*a**(7/2)*b**21*x**7) + 140*a**(17/2)
*b**(31/2)*x**10*sqrt(a*x**2/b + 1)/(35*a**(19/2)*b**15*x**19 + 210*a**(17/
2)*b**16*x**17 + 525*a**(15/2)*b**17*x**15 + 700*a**(13/2)*b**18*x**13 + 52
5*a**(11/2)*b**19*x**11 + 210*a**(9/2)*b**20*x**9 + 35*a**(7/2)*b**21*x**7)
 + 21*a**(15/2)*b**(33/2)*x**8*sqrt(a*x**2/b + 1)/(35*a**(19/2)*b**15*x**19
 + 210*a**(17/2)*b**16*x**17 + 525*a**(15/2)*b**17*x**15 + 700*a**(13/2)*b*
*18*x**13 + 525*a**(11/2)*b**19*x**11 + 210*a**(9/2)*b**20*x**9 + 35*a**(7/
2)*b**21*x**7) - 42*a**(13/2)*b**(35/2)*x**6*sqrt(a*x**2/b + 1)/(35*a**(19/
2)*b**15*x**19 + 210*a**(17/2)*b**16*x**17 + 525*a**(15/2)*b**17*x**15 + 70
0*a**(13/2)*b**18*x**13 + 525*a**(11/2)*b**19*x**11 + 210*a**(9/2)*b**20*x*
*9 + 35*a**(7/2)*b**21*x**7) - 47*a**(11/2)*b**(37/2)*x**4*sqrt(a*x**2/b +
1)/(35*a**(19/2)*b**15*x**19 + 210*a**(17/2)*b**16*x**17 + 525*a**(15/2)*b*
*17*x**15 + 700*a**(13/2)*b**18*x**13 + 525*a**(11/2)*b**19*x**11 + 210*a**
(9/2)*b**20*x**9 + 35*a**(7/2)*b**21*x**7) - 24*a**(9/2)*b**(39/2)*x**2*sqr
t(a*x**2/b + 1)/(35*a**(19/2)*b**15*x**19 + 210*a**(17/2)*b**16*x**17 + 525
*a**(15/2)*b**17*x**15 + 700*a**(13/2)*b**18*x**13 + 525*a**(11/2)*b**19*x*
*11 + 210*a**(9/2)*b**20*x**9 + 35*a**(7/2)*b**21*x**7) - 5*a**(7/2)*b**(41
/2)*sqrt(a*x**2/b + 1)/(35*a**(19/2)*b**15*x**19 + 210*a**(17/2)*b**16*x**1
7 + 525*a**(15/2)*b**17*x**15 + 700*a**(13/2)*b**18*x**13 + 525*a**(11/2)*b
**19*x**11 + 210*a**(9/2)*b**20*x**9 + 35*a**(7/2)*b**21*x**7) - 16*a**13*b
**11*x**19/(35*a**(19/2)*b**15*x**19 + 210*a**(17/2)*b**16*x**17 + 525*a**(
15/2)*b**17*x**15 + 700*a**(13/2)*b**18*x**13 + 525*a**(11/2)*b**19*x**11 +
 210*a**(9/2)*b**20*x**9 + 35*a**(7/2)*b**21*x**7) - 96*a**12*b**12*x**17/(
35*a**(19/2)*b**15*x**19 + 210*a**(17/2)*b**16*x**17 + 525*a**(15/2)*b**17*
x**15 + 700*a**(13/2)*b**18*x**13 + 525*a**(11/2)*b**19*x**11 + 210*a**(9/2
)*b**20*x**9 + 35*a**(7/2)*b**21*x**7) - 240*a**11*b**13*x**15/(35*a**(19/2
)*b**15*x**19 + 210*a**(17/2)*b**16*x**17 + 525*a**(15/2)*b**17*x**15 + 700
*a**(13/2)*b**18*x**13 + 525*a**(11/2)*b**19*x**11 + 210*a**(9/2)*b**20*x**
9 + 35*a**(7/2)*b**21*x**7) - 320*a**10*b**14*x**13/(35*a**(19/2)*b**15*x**
19 + 210*a**(17/2)*b**16*x**17 + 525*a**(15/2)*b**17*x**15 + 700*a**(13/2)*
b**18*x**13 + 525*a**(11/2)*b**19*x**11 + 210*a**(9/2)*b**20*x**9 + 35*a**(
7/2)*b**21*x**7) - 240*a**9*b**15*x**11/(35*a**(19/2)*b**15*x**19 + 210*a**
(17/2)*b**16*x**17 + 525*a**(15/2)*b**17*x**15 + 700*a**(13/2)*b**18*x**13
+ 525*a**(11/2)*b**19*x**11 + 210*a**(9/2)*b**20*x**9 + 35*a**(7/2)*b**21*x
```

```
**7) - 96*a**8*b**16*x**9/(35*a**(19/2)*b**15*x**19 + 210*a**(17/2)*b**16*x
**17 + 525*a**(15/2)*b**17*x**15 + 700*a**(13/2)*b**18*x**13 + 525*a**(11/2
)*b**19*x**11 + 210*a**(9/2)*b**20*x**9 + 35*a**(7/2)*b**21*x**7) - 16*a**7
*b**17*x**7/(35*a**(19/2)*b**15*x**19 + 210*a**(17/2)*b**16*x**17 + 525*a**
(15/2)*b**17*x**15 + 700*a**(13/2)*b**18*x**13 + 525*a**(11/2)*b**19*x**11
+ 210*a**(9/2)*b**20*x**9 + 35*a**(7/2)*b**21*x**7)
```

3.1922 $\int \frac{x^4}{\sqrt{a+\frac{b}{x^2}}}\,dx$

Optimal. Leaf size=66

$$\frac{8b^2x\sqrt{a+\frac{b}{x^2}}}{15a^3}-\frac{4bx^3\sqrt{a+\frac{b}{x^2}}}{15a^2}+\frac{x^5\sqrt{a+\frac{b}{x^2}}}{5a}$$

```
[Out] 8/15*b^2*x*(a+b/x^2)^(1/2)/a^3-4/15*b*x^3*(a+b/x^2)^(1/2)/a^2+1/5*x^5*(a+b/
x^2)^(1/2)/a
```

Rubi [A] time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 191}

$$\frac{8b^2x\sqrt{a+\frac{b}{x^2}}}{15a^3}-\frac{4bx^3\sqrt{a+\frac{b}{x^2}}}{15a^2}+\frac{x^5\sqrt{a+\frac{b}{x^2}}}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a + b/x^2],x]
```

```
[Out] (8*b^2*Sqrt[a + b/x^2]*x)/(15*a^3) - (4*b*Sqrt[a + b/x^2]*x^3)/(15*a^2) + (
Sqrt[a + b/x^2]*x^5)/(5*a)
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt{a+\frac{b}{x^2}}}\,dx &= \frac{\sqrt{a+\frac{b}{x^2}}\,x^5}{5a}-\frac{(4b)\int \frac{x^2}{\sqrt{a+\frac{b}{x^2}}}\,dx}{5a}\\ &= -\frac{4b\sqrt{a+\frac{b}{x^2}}\,x^3}{15a^2}+\frac{\sqrt{a+\frac{b}{x^2}}\,x^5}{5a}+\frac{\left(8b^2\right)\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}\,dx}{15a^2}\\ &= \frac{8b^2\sqrt{a+\frac{b}{x^2}}\,x}{15a^3}-\frac{4b\sqrt{a+\frac{b}{x^2}}\,x^3}{15a^2}+\frac{\sqrt{a+\frac{b}{x^2}}\,x^5}{5a}\end{aligned}$$

Mathematica [A] time = 0.03, size = 40, normalized size = 0.61

$$\frac{x\sqrt{a+\frac{b}{x^2}}\left(3a^2x^4-4abx^2+8b^2\right)}{15a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a + b/x^2],x]
```

```
[Out] (Sqrt[a + b/x^2]*x*(8*b^2 - 4*a*b*x^2 + 3*a^2*x^4))/(15*a^3)
```

fricas [A] time = 0.54, size = 40, normalized size = 0.61

$$\frac{\left(3\,a^2x^5 - 4\,abx^3 + 8\,b^2x\right)\sqrt{\frac{ax^2+b}{x^2}}}{15\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*a^2*x^5 - 4*a*b*x^3 + 8*b^2*x)*sqrt((a*x^2 + b)/x^2)/a^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,
0,4]%%%}] at parameters values [-13,6.59772427287,-13]Warning, choosing roo
t of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at par
ameters values [88,60.9680557316,-81]Warning, choosing root of [1,0,%%%{-2,
[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [-
43,80.6998703005,-84]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{
-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [26,27.1915548402,
44]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%
%%{1,[2,0,4]%%%}] at parameters values [56,99.5462650466,-1]Warning, choosi
ng root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}]
at parameters values [67,64.8330923682,-48](1/5*a^4*(-sqrt(b)/a*sign(x)+sqr
t(a*x^2+b)/a/sign(x))^5+2*a^2*b*(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign(x)
)^3+2*a*sqrt(b)*b*(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign(x))^2*sign(-sqrt
(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign(x))+b^2*(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b
)/a/sign(x))-2/3*a^2*b*(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign(x))^3-2*b^2
*(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign(x))-2*a*sqrt(b)*b*(-sqrt(b)/a*sig
n(x)+sqrt(a*x^2+b)/a/sign(x))^2*sign(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/a/sig
n(x))+a^3*sqrt(b)*(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign(x))^4*sign(-sqrt
(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign(x)))/a^2+b^2/a^2*(-sqrt(b)/a*sign(x)+sqr
t(a*x^2+b)/a/sign(x))
```

maple [A] time = 0.01, size = 50, normalized size = 0.76

$$\frac{\left(a\,x^2+b\right)\left(3a^2x^4 - 4ab\,x^2 + 8b^2\right)}{15\sqrt{\frac{a\,x^2+b}{x^2}}\;a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a+b/x^2)^(1/2),x)
```

```
[Out] 1/15*(a*x^2+b)*(3*a^2*x^4-4*a*b*x^2+8*b^2)/a^3/x/((a*x^2+b)/x^2)^(1/2)
```

maxima [A] time = 0.79, size = 50, normalized size = 0.76

$$\frac{3\left(a+\frac{b}{x^2}\right)^{\frac{5}{2}}x^5 - 10\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}bx^3 + 15\sqrt{a+\frac{b}{x^2}}\,b^2x}{15\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*(a + b/x^2)^(5/2)*x^5 - 10*(a + b/x^2)^(3/2)*b*x^3 + 15*sqrt(a + b/
x^2)*b^2*x)/a^3
```

mupad **[B]** time = 1.56, size = 38, normalized size = 0.58

$$\frac{x^5\sqrt{a+\frac{b}{x^2}}\left(3a^2+\frac{8b^2}{x^4}-\frac{4ab}{x^2}\right)}{15a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b/x^2)^(1/2),x)
```

```
[Out] (x^5*(a + b/x^2)^(1/2)*(3*a^2 + (8*b^2)/x^4 - (4*a*b)/x^2))/(15*a^3)
```

sympy **[B]** time = 1.25, size = 279, normalized size = 4.23

$$\frac{3a^4b^{\frac{9}{2}}x^8\sqrt{\frac{ax^2}{b}+1}}{15a^5b^4x^4+30a^4b^5x^2+15a^3b^6}+\frac{2a^3b^{\frac{11}{2}}x^6\sqrt{\frac{ax^2}{b}+1}}{15a^5b^4x^4+30a^4b^5x^2+15a^3b^6}+\frac{3a^2b^{\frac{13}{2}}x^4\sqrt{\frac{ax^2}{b}+1}}{15a^5b^4x^4+30a^4b^5x^2+15a^3b^6}+\frac{12ab^{\frac{15}{2}}x^2}{15a^5b^4x^4+30a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a+b/x**2)**(1/2),x)
```

```
[Out] 3*a**4*b**(9/2)*x**8*sqrt(a*x**2/b + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x
**2 + 15*a**3*b**6) + 2*a**3*b**(11/2)*x**6*sqrt(a*x**2/b + 1)/(15*a**5*b**
4*x**4 + 30*a**4*b**5*x**2 + 15*a**3*b**6) + 3*a**2*b**(13/2)*x**4*sqrt(a*x
**2/b + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**2 + 15*a**3*b**6) + 12*a*b*
*(15/2)*x**2*sqrt(a*x**2/b + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**2 + 15
*a**3*b**6) + 8*b**(17/2)*sqrt(a*x**2/b + 1)/(15*a**5*b**4*x**4 + 30*a**4*b
**5*x**2 + 15*a**3*b**6)
```

3.1923 $\int \frac{x^2}{\sqrt{a+\frac{b}{x^2}}}\,dx$

Optimal. Leaf size=42

$$\frac{x^3\sqrt{a+\frac{b}{x^2}}}{3a}-\frac{2bx\sqrt{a+\frac{b}{x^2}}}{3a^2}$$

```
[Out] -2/3*b*x*(a+b/x^2)^(1/2)/a^2+1/3*x^3*(a+b/x^2)^(1/2)/a
```

Rubi [A] time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 191}

$$\frac{x^3\sqrt{a+\frac{b}{x^2}}}{3a}-\frac{2bx\sqrt{a+\frac{b}{x^2}}}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b/x^2],x]
```

```
[Out] (-2*b*Sqrt[a + b/x^2]*x)/(3*a^2) + (Sqrt[a + b/x^2]*x^3)/(3*a)
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{a+\frac{b}{x^2}}}\,dx &= \frac{\sqrt{a+\frac{b}{x^2}}\,x^3}{3a}-\frac{(2b)\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}\,dx}{3a}\\ &= -\frac{2b\sqrt{a+\frac{b}{x^2}}\,x}{3a^2}+\frac{\sqrt{a+\frac{b}{x^2}}\,x^3}{3a}\end{aligned}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 0.67

$$\frac{x\sqrt{a+\frac{b}{x^2}}\left(ax^2-2b\right)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b/x^2],x]
```

```
[Out] (Sqrt[a + b/x^2]*x*(-2*b + a*x^2))/(3*a^2)
```

fricas [**A**] time = 0.87, size = 28, normalized size = 0.67

$$\frac{\left(ax^3 - 2\,bx\right)\sqrt{\frac{ax^2+b}{x^2}}}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(a*x^3 - 2*b*x)*sqrt((a*x^2 + b)/x^2)/a^2
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,
0,4]%%%}] at parameters values [-58,43.858863886,56]Warning, choosing root
of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at param
eters values [97,57.5616762415,-86]Warning, choosing root of [1,0,%%%{-2,[1
,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [25,
56.1797324405,2]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0
,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [11,64.4576321822,46]Wa
rning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,
[2,0,4]%%%}] at parameters values [14,83.5125112212,-22]Warning, choosing r
oot of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at p
arameters values [-94,31.5331678749,18]Warning, choosing root of [1,0,%%%{-
2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values
[-90,36.3617133629,39]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%
{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [35,89.0139720016
,56]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,
%%%{1,[2,0,4]%%%}] at parameters values [83,3.24575260049,-90]Warning, choo
sing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}
] at parameters values [-10,94.741137628,73]Warning, choosing root of [1,0,
%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters va
lues [-36,54.1710346672,-32]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%
%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [49,64.8237
054832,-70]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]
%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [66,42.6786879087,-74]Warnin
g, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0
,4]%%%}] at parameters values [22,42.3987513026,-16]Warning, choosing root
of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at param
eters values [-50,8.71320336033,72]Warning, choosing root of [1,0,%%%{-2,[1
,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [-50
,9.53292093519,-48]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4
,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [26,23.6350279255,-2
9]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%
%{1,[2,0,4]%%%}] at parameters values [-35,43.0853207698,64]Warning, choosi
ng root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}]
at parameters values [-75,39.8355747634,33]Warning, choosing root of [1,0,%
%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters val
ues [43,27.7044391828,74]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+
%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [2,63.07268949
97,-2]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},
0,%%%{1,[2,0,4]%%%}] at parameters values [28,8.94420680963,99]Warning, cho
osing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%
```

```
}] at parameters values [-41,5.78430139413,-1]Warning, choosing root of [1,
0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters
values [2,16.0506848684,66]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%
}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [-23,15.6387
684788,93]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%
%%},0,%%%{1,[2,0,4]%%%}] at parameters values [-86,54.2482937952,57]1/a*(-b
*sqrt(b)/3/a*sign(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign(x))+1/3*(a*(-sqrt
(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign(x)+sqrt(b)*sign(-sqrt(b)/a*sign(x)+sqrt(
a*x^2+b)/a/sign(x))/a)*sign(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign(x)+sqrt
(b)*sign(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign(x))/a))^3/a/sign(-sqrt(b)/
a*sign(x)+sqrt(a*x^2+b)/a/sign(x)+sqrt(b)*sign(-sqrt(b)/a*sign(x)+sqrt(a*x^
2+b)/a/sign(x))/a))-b/a*(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign(x))
```

maple [A] time = 0.01, size = 38, normalized size = 0.90

$$\frac{\left(a\,x^2+b\right)\left(a\,x^2-2b\right)}{3\sqrt{\frac{a\,x^2+b}{x^2}}\;a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b/x^2)^(1/2),x)
```

```
[Out] 1/3*(a*x^2+b)*(a*x^2-2*b)/a^2/x/((a*x^2+b)/x^2)^(1/2)
```

maxima [A] time = 0.87, size = 32, normalized size = 0.76

$$\frac{\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}x^3-3\sqrt{a+\frac{b}{x^2}}\,bx}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*((a + b/x^2)^(3/2)*x^3 - 3*sqrt(a + b/x^2)*b*x)/a^2
```

mupad [B] time = 1.40, size = 25, normalized size = 0.60

$$\frac{x^3\sqrt{a+\frac{b}{x^2}}\,\left(a-\frac{2\,b}{x^2}\right)}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b/x^2)^(1/2),x)
```

```
[Out] (x^3*(a + b/x^2)^(1/2)*(a - (2*b)/x^2))/(3*a^2)
```

sympy [A] time = 0.86, size = 46, normalized size = 1.10

$$\frac{\sqrt{b}\,x^2\sqrt{\frac{ax^2}{b}+1}}{3a}-\frac{2b^{\frac{3}{2}}\sqrt{\frac{ax^2}{b}+1}}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b/x**2)**(1/2),x)
```

```
[Out] sqrt(b)*x**2*sqrt(a*x**2/b + 1)/(3*a) - 2*b**(3/2)*sqrt(a*x**2/b + 1)/(3*a*
*2)
```

3.1924 $\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}\,dx$

Optimal. Leaf size=16

$$\frac{x\sqrt{a+\frac{b}{x^2}}}{a}$$

```
[Out] x*(a+b/x^2)^(1/2)/a
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {191}

$$\frac{x\sqrt{a+\frac{b}{x^2}}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b/x^2],x]
```

```
[Out] (Sqrt[a + b/x^2]*x)/a
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}\,dx = \frac{\sqrt{a+\frac{b}{x^2}}\,x}{a}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{x\sqrt{a+\frac{b}{x^2}}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b/x^2],x]
```

```
[Out] (Sqrt[a + b/x^2]*x)/a
```

fricas [A] time = 0.87, size = 18, normalized size = 1.12

$$\frac{x\sqrt{\frac{ax^2+b}{x^2}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] x*sqrt((a*x^2 + b)/x^2)/a
```

giac [A] time = 0.16, size = 28, normalized size = 1.75

$$-\frac{\sqrt{b}\,\mathrm{sgn}(x)}{a}+\frac{\sqrt{ax^2+b}}{a\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(1/2),x, algorithm="giac")
[Out] -sqrt(b)*sgn(x)/a + sqrt(a*x^2 + b)/(a*sgn(x))
```

maple [A] time = 0.00, size = 28, normalized size = 1.75

$$\frac{a\,x^2+b}{\sqrt{\frac{a\,x^2+b}{x^2}}\,ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^(1/2),x)
[Out] (a*x^2+b)/a/x/((a*x^2+b)/x^2)^(1/2)
```

maxima [A] time = 0.86, size = 14, normalized size = 0.88

$$\frac{\sqrt{a+\frac{b}{x^2}}\,x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(1/2),x, algorithm="maxima")
[Out] sqrt(a + b/x^2)*x/a
```

mupad [B] time = 1.17, size = 39, normalized size = 2.44

$$\frac{x\sqrt{\frac{a\,x^2}{b}+1}}{\sqrt{a+\frac{b}{x^2}}\left(\sqrt{\frac{a\,x^2}{b}+1}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x^2)^(1/2),x)
[Out] (x*((a*x^2)/b + 1)^(1/2))/((a + b/x^2)^(1/2)*(((a*x^2)/b + 1)^(1/2) + 1))
```

sympy [A] time = 0.78, size = 17, normalized size = 1.06

$$\frac{\sqrt{b}\sqrt{\frac{ax^2}{b}+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**(1/2),x)
[Out] sqrt(b)*sqrt(a*x**2/b + 1)/a
```

3.1925 $\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x^2}\,dx$

Optimal. Leaf size=28

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{\sqrt{b}}$$

```
[Out] -arctanh(b^(1/2)/x/(a+b/x^2)^(1/2))/b^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {335, 217, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x^2]*x^2),x]
```

```
[Out] -(ArcTanh[Sqrt[b]/(Sqrt[a + b/x^2]*x)]/Sqrt[b])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x^2}\,dx &= -\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{x}\right) \\
&= -\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x}\right) \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x}\right)}{\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 1.79

$$-\frac{\sqrt{ax^2+b}\,\tanh^{-1}\left(\frac{\sqrt{ax^2+b}}{\sqrt{b}}\right)}{\sqrt{b}\,x\sqrt{a+\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x^2]*x^2),x]
```

```
[Out] -((Sqrt[b + a*x^2]*ArcTanh[Sqrt[b + a*x^2]/Sqrt[b]])/(Sqrt[b]*Sqrt[a + b/x^
2]*x))
```

fricas [A] time = 0.81, size = 79, normalized size = 2.82

$$\left[\frac{\log\left(-\frac{ax^2-2\sqrt{b}\,x\sqrt{\frac{ax^2+b}{x^2}}+2b}{x^2}\right)}{2\sqrt{b}},\ \frac{\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x\sqrt{\frac{ax^2+b}{x^2}}}{ax^2+b}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*log(-(a*x^2 - 2*sqrt(b)*x*sqrt((a*x^2 + b)/x^2) + 2*b)/x^2)/sqrt(b), s
qrt(-b)*arctan(sqrt(-b)*x*sqrt((a*x^2 + b)/x^2)/(a*x^2 + b))/b]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [sign(t_nostep),sign(t_nostep+sq
rt(b)/a*sign(t_nostep))]sym2poly/r2sym(const gen & e,const index_m & i,cons
t vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.01, size = 52, normalized size = 1.86

$$-\frac{\sqrt{a\,x^2+b}\,\ln\left(\frac{2b+2\sqrt{a\,x^2+b}\,\sqrt{b}}{x}\right)}{\sqrt{\frac{a\,x^2+b}{x^2}}\,\sqrt{b}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^(1/2)/x^2,x)
```

```
[Out] -1/((a*x^2+b)/x^2)^(1/2)/x*(a*x^2+b)^(1/2)/b^(1/2)*ln(2*(b+(a*x^2+b)^(1/2)*
b^(1/2))/x)
```

maxima [A] time = 1.84, size = 41, normalized size = 1.46

$$\frac{\log\left(\frac{\sqrt{a+\frac{b}{x^2}}\,x-\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x+\sqrt{b}}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*log((sqrt(a + b/x^2)*x - sqrt(b))/(sqrt(a + b/x^2)*x + sqrt(b)))/sqrt(b
)
```

mupad [B] time = 1.40, size = 23, normalized size = 0.82

$$-\frac{\ln\left(\sqrt{a+\frac{b}{x^2}}+\frac{\sqrt{b}}{x}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b/x^2)^(1/2)),x)
```

```
[Out] -log((a + b/x^2)^(1/2) + b^(1/2)/x)/b^(1/2)
```

sympy [A] time = 1.47, size = 19, normalized size = 0.68

$$-\frac{\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\,x}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**(1/2)/x**2,x)
```

```
[Out] -asinh(sqrt(b)/(sqrt(a)*x))/sqrt(b)
```

3.1926 $\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x^4}\,dx$

Optimal. Leaf size=53

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{2b^{3/2}}-\frac{\sqrt{a+\frac{b}{x^2}}}{2bx}$$

```
[Out] 1/2*a*arctanh(b^(1/2)/x/(a+b/x^2)^(1/2))/b^(3/2)-1/2*(a+b/x^2)^(1/2)/b/x
```

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {335, 321, 217, 206}

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{2b^{3/2}}-\frac{\sqrt{a+\frac{b}{x^2}}}{2bx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x^2]*x^4),x]
```

```
[Out] -Sqrt[a + b/x^2]/(2*b*x) + (a*ArcTanh[Sqrt[b]/(Sqrt[a + b/x^2]*x)])/(2*b^(3
/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x^4}\,dx = -\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{\sqrt{a+\frac{b}{x^2}}}{2bx} + \frac{a\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{x}\right)}{2b}$$
$$= -\frac{\sqrt{a+\frac{b}{x^2}}}{2bx} + \frac{a\operatorname{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x}\right)}{2b}$$
$$= -\frac{\sqrt{a+\frac{b}{x^2}}}{2bx} + \frac{a\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x}\right)}{2b^{3/2}}$$

Mathematica [A] time = 0.05, size = 71, normalized size = 1.34

$$\frac{a\left(ax^2+b\right)\left(\frac{\tanh^{-1}\left(\sqrt{\frac{ax^2}{b}+1}\right)}{2\sqrt{\frac{ax^2}{b}+1}} - \frac{b}{2ax^2}\right)}{b^2x\sqrt{a+\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x^2]*x^4),x]
```

```
[Out] (a*(b + a*x^2)*(-1/2*b/(a*x^2) + ArcTanh[Sqrt[1 + (a*x^2)/b]]/(2*Sqrt[1 + (
a*x^2)/b])))/(b^2*Sqrt[a + b/x^2]*x)
```

fricas [A] time = 0.87, size = 128, normalized size = 2.42

$$\left[\frac{a\sqrt{b}\,x\log\left(-\frac{ax^2+2\sqrt{b}\,x\sqrt{\frac{ax^2+b}{x^2}}+2b}{x^2}\right) - 2b\sqrt{\frac{ax^2+b}{x^2}}}{4b^2x}, -\frac{a\sqrt{-b}\,x\arctan\left(\frac{\sqrt{-b}\,x\sqrt{\frac{ax^2+b}{x^2}}}{ax^2+b}\right) + b\sqrt{\frac{ax^2+b}{x^2}}}{2b^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/4*(a*sqrt(b)*x*log(-(a*x^2 + 2*sqrt(b)*x*sqrt((a*x^2 + b)/x^2) + 2*b)/x^
2) - 2*b*sqrt((a*x^2 + b)/x^2))/(b^2*x), -1/2*(a*sqrt(-b)*x*arctan(sqrt(-b)
*x*sqrt((a*x^2 + b)/x^2)/(a*x^2 + b)) + b*sqrt((a*x^2 + b)/x^2))/(b^2*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
```

```
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [sign(t_nostep),sign(t_nostep+sq
rt(b)/a*sign(t_nostep))]sym2poly/r2sym(const gen & e,const index_m & i,cons
t vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 73, normalized size = 1.38

$$-\frac{\sqrt{a\,x^2+b}\,\left(-ab\,x^2\ln\left(\frac{2b+2\sqrt{a\,x^2+b}\,\sqrt{b}}{x}\right)+\sqrt{a\,x^2+b}\,b^{\frac{3}{2}}\right)}{2\sqrt{\frac{a\,x^2+b}{x^2}}\,b^{\frac{5}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^(1/2)/x^4,x)
```

```
[Out] -1/2*(a*x^2+b)^(1/2)*(-a*ln(2*(b+(a*x^2+b)^(1/2)*b^(1/2))/x)*x^2*b+(a*x^2+b
)^(1/2)*b^(3/2))/((a*x^2+b)/x^2)^(1/2)/x^3/b^(5/2)
```

maxima [A] time = 1.99, size = 76, normalized size = 1.43

$$-\frac{\sqrt{a+\frac{b}{x^2}}\,ax}{2\left(\left(a+\frac{b}{x^2}\right)bx^2-b^2\right)}-\frac{a\log\left(\frac{\sqrt{a+\frac{b}{x^2}}\,x-\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x+\sqrt{b}}\right)}{4\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(a + b/x^2)*a*x/((a + b/x^2)*b*x^2 - b^2) - 1/4*a*log((sqrt(a + b/
x^2)*x - sqrt(b))/(sqrt(a + b/x^2)*x + sqrt(b)))/b^(3/2)
```

mupad [B] time = 1.57, size = 60, normalized size = 1.13

$$\begin{cases} -\frac{1}{3\sqrt{a}\,x^3} & \text{if } b=0 \\ \frac{a\,\ln\left(2\sqrt{a+\frac{b}{x^2}}+\frac{2\sqrt{b}}{x}\right)}{2b^{3/2}}-\frac{\sqrt{a+\frac{b}{x^2}}}{2\,b\,x} & \text{if } b\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b/x^2)^(1/2)),x)
```

```
[Out] piecewise(b == 0, -1/(3*a^(1/2)*x^3), b ~= 0, (a*log(2*(a + b/x^2)^(1/2) +
(2*b^(1/2))/x))/(2*b^(3/2)) - (a + b/x^2)^(1/2)/(2*b*x))
```

sympy [A] time = 2.68, size = 42, normalized size = 0.79

$$-\frac{\sqrt{a}\,\sqrt{1+\frac{b}{ax^2}}}{2bx}+\frac{a\,\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\,x}\right)}{2b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**(1/2)/x**4,x)
```

```
[Out] -sqrt(a)*sqrt(1 + b/(a*x**2))/(2*b*x) + a*asinh(sqrt(b)/(sqrt(a)*x))/(2*b**
(3/2))
```

3.1927 $\int \frac{1}{\sqrt{-a+\frac{b}{x^2}}\,x}\,dx$

Optimal. Leaf size=27

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{b}{x^2}-a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -arctan((-a+b/x^2)^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {266, 63, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{b}{x^2}-a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-a + b/x^2]*x),x]
```

```
[Out] -(ArcTan[Sqrt[-a + b/x^2]/Sqrt[a]]/Sqrt[a])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{\sqrt{-a+\frac{b}{x^2}}\,x}\,dx = -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx}}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+\frac{b}{x^2}}\right)}{b}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{-a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Mathematica [B] time = 0.04, size = 56, normalized size = 2.07

$$\frac{\sqrt{ax^2-b}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2-b}}\right)}{\sqrt{a}\,x\sqrt{\frac{b}{x^2}-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-a + b/x^2]*x),x]
```

```
[Out] (Sqrt[-b + a*x^2]*ArcTanh[(Sqrt[a]*x)/Sqrt[-b + a*x^2]])/(Sqrt[a]*Sqrt[-a +
 b/x^2]*x)
```

fricas [A] time = 1.03, size = 88, normalized size = 3.26

$$\left[-\frac{\sqrt{-a}\,\log\left(2\,ax^2-2\,\sqrt{-a}\,x^2\sqrt{-\frac{ax^2-b}{x^2}}-b\right)}{2\,a}, -\frac{\arctan\left(\frac{\sqrt{a}\,x^2\sqrt{-\frac{ax^2-b}{x^2}}}{ax^2-b}\right)}{\sqrt{a}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a)*log(2*a*x^2 - 2*sqrt(-a)*x^2*sqrt(-(a*x^2 - b)/x^2) - b)/a,
-arctan(sqrt(a)*x^2*sqrt(-(a*x^2 - b)/x^2)/(a*x^2 - b))/sqrt(a)]
```

giac [B] time = 0.25, size = 55, normalized size = 2.04

$$-\frac{\log\left(\left|2\left(\sqrt{-a}\,x^2-\sqrt{-ax^4+bx^2}\right)\sqrt{-a}+b\right|\right)}{2\sqrt{-a}}+\frac{\log\left(|b|\right)}{2\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(2*(sqrt(-a)*x^2 - sqrt(-a*x^4 + b*x^2))*sqrt(-a) + b))/sqrt(-a
) + 1/2*log(abs(b))/sqrt(-a)
```

maple [B] time = 0.01, size = 50, normalized size = 1.85

$$\frac{\sqrt{-a\,x^2+b}\,\arctan\left(\frac{\sqrt{a}\,x}{\sqrt{-a\,x^2+b}}\right)}{\sqrt{-\frac{a\,x^2-b}{x^2}}\,\sqrt{a}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a+b/x^2)^(1/2),x)
```

```
[Out] 1/(-(a*x^2-b)/x^2)^(1/2)/x*(-a*x^2+b)^(1/2)/a^(1/2)*arctan(a^(1/2)*x/(-a*x^
2+b)^(1/2))
```

maxima [A] time = 1.98, size = 21, normalized size = 0.78

$$-\frac{\arctan\left(\frac{\sqrt{-a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -arctan(sqrt(-a + b/x^2)/sqrt(a))/sqrt(a)
```

mupad [B] time = 1.35, size = 21, normalized size = 0.78

$$-\frac{\operatorname{atan}\left(\frac{\sqrt{\frac{b}{x^2}-a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b/x^2 - a)^(1/2)),x)
```

```
[Out] -atan((b/x^2 - a)^(1/2)/a^(1/2))/a^(1/2)
```

sympy [A] time = 1.31, size = 46, normalized size = 1.70

$$\begin{cases} -\frac{i\operatorname{acosh}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}} & \text{for } \left|\frac{ax^2}{b}\right| > 1 \\ \frac{\operatorname{asin}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b/x**2)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(sqrt(a)*x/sqrt(b))/sqrt(a), Abs(a*x**2/b) > 1), (asin(s
qrt(a)*x/sqrt(b))/sqrt(a), True))
```

3.1928 $\int \frac{1}{\sqrt{2+\frac{b}{x^2}}\,x^2}\,dx$

Optimal. Leaf size=20

$$-\frac{\text{csch}^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{b}}\right)}{\sqrt{b}}$$

```
[Out] -arccsch(x*2^(1/2)/b^(1/2))/b^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {335, 215}

$$-\frac{\text{csch}^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{b}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 + b/x^2]*x^2),x]
```

```
[Out] -(ArcCsch[(Sqrt[2]*x)/Sqrt[b]]/Sqrt[b])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+\frac{b}{x^2}}\,x^2}\,dx = -\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{\text{csch}^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{b}}\right)}{\sqrt{b}}$$

Mathematica [B] time = 0.02, size = 50, normalized size = 2.50

$$-\frac{\sqrt{b+2x^2}\,\tanh^{-1}\left(\frac{\sqrt{b+2x^2}}{\sqrt{b}}\right)}{\sqrt{b}\,x\sqrt{\frac{b}{x^2}+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 + b/x^2]*x^2),x]
```

```
[Out] -((Sqrt[b + 2*x^2]*ArcTanh[Sqrt[b + 2*x^2]/Sqrt[b]])/(Sqrt[b]*Sqrt[2 + b/x^
2]*x))
```

fricas [B] time = 1.02, size = 75, normalized size = 3.75

$$\left[\frac{\log\left(-\frac{x^2-\sqrt{b}\,x\sqrt{\frac{2x^2+b}{x^2}}+b}{x^2}\right)}{2\sqrt{b}},\ \frac{\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x\sqrt{\frac{2x^2+b}{x^2}}}{2x^2+b}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(2+b/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(x^2 - sqrt(b)*x*sqrt((2*x^2 + b)/x^2) + b)/x^2)/sqrt(b), sqrt(-b
)*arctan(sqrt(-b)*x*sqrt((2*x^2 + b)/x^2)/(2*x^2 + b))/b]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(2+b/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [sign(t_nostep),sign(t_nostep+sq
rt(b)/2*sign(t_nostep))]sym2poly/r2sym(const gen & e,const index_m & i,cons
t vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.01, size = 52, normalized size = 2.60

$$-\frac{\sqrt{2x^2+b}\,\ln\left(\frac{2b+2\sqrt{2x^2+b}\,\sqrt{b}}{x}\right)}{\sqrt{\frac{2x^2+b}{x^2}}\,\sqrt{b}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(2+b/x^2)^(1/2),x)
```

```
[Out] -1/((2*x^2+b)/x^2)^(1/2)/x*(2*x^2+b)^(1/2)/b^(1/2)*ln(2*(b^(1/2)*(2*x^2+b)^
(1/2)+b)/x)
```

maxima [B] time = 1.96, size = 41, normalized size = 2.05

$$\frac{\log\left(\frac{x\sqrt{\frac{b}{x^2}+2}-\sqrt{b}}{x\sqrt{\frac{b}{x^2}+2}+\sqrt{b}}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(2+b/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*log((x*sqrt(b/x^2 + 2) - sqrt(b))/(x*sqrt(b/x^2 + 2) + sqrt(b)))/sqrt(b
)
```

mupad [B] time = 1.27, size = 17, normalized size = 0.85

$$-\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}}{2x}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b/x^2 + 2)^(1/2)),x)
```

```
[Out] -asinh((2^(1/2)*b^(1/2))/(2*x))/b^(1/2)
```

sympy [A] time = 1.32, size = 20, normalized size = 1.00

$$-\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}}{2x}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(2+b/x**2)**(1/2),x)
```

```
[Out] -asinh(sqrt(2)*sqrt(b)/(2*x))/sqrt(b)
```

3.1929 $\int \frac{1}{\sqrt{2-\frac{b}{x^2}}\,x^2}\,dx$

Optimal. Leaf size=20

$$-\frac{\csc^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{b}}\right)}{\sqrt{b}}$$

```
[Out] -arccsc(x*2^(1/2)/b^(1/2))/b^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {335, 216}

$$-\frac{\csc^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{b}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - b/x^2]*x^2),x]
```

```
[Out] -(ArcCsc[(Sqrt[2]*x)/Sqrt[b]]/Sqrt[b])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-\frac{b}{x^2}}\,x^2}\,dx = -\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{\csc^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{b}}\right)}{\sqrt{b}}$$

Mathematica [B] time = 0.02, size = 54, normalized size = 2.70

$$\frac{\sqrt{2x^2-b}\,\tan^{-1}\left(\frac{\sqrt{2x^2-b}}{\sqrt{b}}\right)}{\sqrt{b}\,x\sqrt{2-\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - b/x^2]*x^2),x]
```

```
[Out] (Sqrt[-b + 2*x^2]*ArcTan[Sqrt[-b + 2*x^2]/Sqrt[b]])/(Sqrt[b]*Sqrt[2 - b/x^2
]*x)
```

fricas [B] time = 0.78, size = 84, normalized size = 4.20

$$\left[-\frac{\sqrt{-b}\,\log\left(-\frac{x^2-\sqrt{-b}\,x\sqrt{\frac{2x^2-b}{x^2}}-b}{x^2}\right)}{2b},\ -\frac{\arctan\left(\frac{\sqrt{b}\,x\sqrt{\frac{2x^2-b}{x^2}}}{2x^2-b}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(2-b/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b)*log(-(x^2 - sqrt(-b)*x*sqrt((2*x^2 - b)/x^2) - b)/x^2)/b, -a
rctan(sqrt(b)*x*sqrt((2*x^2 - b)/x^2)/(2*x^2 - b))/sqrt(b)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(2-b/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [sign(t_nostep),sign(t_nostep+sq
rt(-b)/2*sign(t_nostep))]sym2poly/r2sym(const gen & e,const index_m & i,con
st vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.01, size = 64, normalized size = 3.20

$$-\frac{\sqrt{2x^2-b}\,\ln\left(\frac{-2b+2\sqrt{-b}\,\sqrt{2x^2-b}}{x}\right)}{\sqrt{\frac{2x^2-b}{x^2}}\,\sqrt{-b}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(2-b/x^2)^(1/2),x)
```

```
[Out] -1/((2*x^2-b)/x^2)^(1/2)/x*(2*x^2-b)^(1/2)/(-b)^(1/2)*ln(2*((-b)^(1/2)*(2*x
^2-b)^(1/2)-b)/x)
```

maxima [A] time = 1.97, size = 20, normalized size = 1.00

$$\frac{\arctan\left(\frac{x\sqrt{-\frac{b}{x^2}+2}}{\sqrt{b}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(2-b/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arctan(x*sqrt(-b/x^2 + 2)/sqrt(b))/sqrt(b)
```

mupad [B] time = 1.25, size = 21, normalized size = 1.05

$$-\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{-b}}{2x}\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(2 - b/x^2)^(1/2)),x)
```

```
[Out] -asinh((2^(1/2)*(-b)^(1/2))/(2*x))/(-b)^(1/2)
```

sympy [**A**] time = 1.33, size = 49, normalized size = 2.45

$$\begin{cases} \dfrac{i\,\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{b}}{2x}\right)}{\sqrt{b}} & \text{for } \dfrac{\left|\frac{b}{x^2}\right|}{2} > 1 \\ -\dfrac{\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{b}}{2x}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(2-b/x**2)**(1/2),x)
```

```
[Out] Piecewise((I*acosh(sqrt(2)*sqrt(b)/(2*x))/sqrt(b), Abs(b/x**2)/2 > 1), (-as
in(sqrt(2)*sqrt(b)/(2*x))/sqrt(b), True))
```

3.1930 $\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=95

$$\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{8a^{7/2}}-\frac{15b^2}{8a^3\sqrt{a+\frac{b}{x^2}}}-\frac{5bx^2}{8a^2\sqrt{a+\frac{b}{x^2}}}+\frac{x^4}{4a\sqrt{a+\frac{b}{x^2}}}$$

```
[Out] 15/8*b^2*arctanh((a+b/x^2)^(1/2)/a^(1/2))/a^(7/2)-15/8*b^2/a^3/(a+b/x^2)^(1
/2)-5/8*b*x^2/a^2/(a+b/x^2)^(1/2)+1/4*x^4/a/(a+b/x^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 93, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 51, 63, 208}

$$\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{8a^{7/2}}+\frac{5x^4\sqrt{a+\frac{b}{x^2}}}{4a^2}-\frac{15bx^2\sqrt{a+\frac{b}{x^2}}}{8a^3}-\frac{x^4}{a\sqrt{a+\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b/x^2)^(3/2),x]
```

```
[Out] (-15*b*Sqrt[a + b/x^2]*x^2)/(8*a^3) - x^4/(a*Sqrt[a + b/x^2]) + (5*Sqrt[a +
 b/x^2]*x^4)/(4*a^2) + (15*b^2*ArcTanh[Sqrt[a + b/x^2]/Sqrt[a]])/(8*a^(7/2)
)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx = -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3(a+bx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= -\frac{x^4}{a\sqrt{a+\frac{b}{x^2}}} - \frac{5\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx, x, \frac{1}{x^2}\right)}{2a}$$

$$= -\frac{x^4}{a\sqrt{a+\frac{b}{x^2}}} + \frac{5\sqrt{a+\frac{b}{x^2}}\,x^4}{4a^2} + \frac{(15b)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x^2}\right)}{8a^2}$$

$$= -\frac{15b\sqrt{a+\frac{b}{x^2}}\,x^2}{8a^3} - \frac{x^4}{a\sqrt{a+\frac{b}{x^2}}} + \frac{5\sqrt{a+\frac{b}{x^2}}\,x^4}{4a^2} - \frac{\left(15b^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x^2}\right)}{16a^3}$$

$$= -\frac{15b\sqrt{a+\frac{b}{x^2}}\,x^2}{8a^3} - \frac{x^4}{a\sqrt{a+\frac{b}{x^2}}} + \frac{5\sqrt{a+\frac{b}{x^2}}\,x^4}{4a^2} - \frac{(15b)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x^2}}\right)}{8a^3}$$

$$= -\frac{15b\sqrt{a+\frac{b}{x^2}}\,x^2}{8a^3} - \frac{x^4}{a\sqrt{a+\frac{b}{x^2}}} + \frac{5\sqrt{a+\frac{b}{x^2}}\,x^4}{4a^2} + \frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{8a^{7/2}}$$

Mathematica [A] time = 0.06, size = 86, normalized size = 0.91

$$\frac{\sqrt{a}\,x\left(2a^2x^4 - 5abx^2 - 15b^2\right) + 15b^{5/2}\sqrt{\frac{ax^2}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8a^{7/2}x\sqrt{a+\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b/x^2)^(3/2),x]
```

```
[Out] (Sqrt[a]*x*(-15*b^2 - 5*a*b*x^2 + 2*a^2*x^4) + 15*b^(5/2)*Sqrt[1 + (a*x^2)/
b]*ArcSinh[(Sqrt[a]*x)/Sqrt[b]])/(8*a^(7/2)*Sqrt[a + b/x^2]*x)
```

fricas [A] time = 0.90, size = 221, normalized size = 2.33

$$\left[\frac{15\left(ab^2x^2+b^3\right)\sqrt{a}\,\log\left(-2\,ax^2 - 2\,\sqrt{a}\,x^2\sqrt{\frac{ax^2+b}{x^2}} - b\right) + 2\left(2\,a^3x^6 - 5\,a^2bx^4 - 15\,ab^2x^2\right)\sqrt{\frac{ax^2+b}{x^2}}}{16\left(a^5x^2+a^4b\right)}, -\frac{15\left(ab^2x^2+b^3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(15*(a*b^2*x^2 + b^3)*sqrt(a)*log(-2*a*x^2 - 2*sqrt(a)*x^2*sqrt((a*x^
2 + b)/x^2) - b) + 2*(2*a^3*x^6 - 5*a^2*b*x^4 - 15*a*b^2*x^2)*sqrt((a*x^2 +
 b)/x^2))/(a^5*x^2 + a^4*b), -1/8*(15*(a*b^2*x^2 + b^3)*sqrt(-a)*arctan(sqr
```

```
t(-a)*x^2*sqrt((a*x^2 + b)/x^2)/(a*x^2 + b)) - (2*a^3*x^6 - 5*a^2*b*x^4 - 1
5*a*b^2*x^2)*sqrt((a*x^2 + b)/x^2))/(a^5*x^2 + a^4*b)]
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Unable to divide, perhaps due to rounding error%%%{%%%{-2,[1]%%%},[2
,1,2]%%%}+%%%{%%{[4,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,1,3]%%%}+%%%{-2,[0,1,4]%%
%} / %%%{%%%{1,[2]%%%},[2,0,0]%%%}+%%%{%%{[%%%{-2,[1]%%%},0]:[1,0,%%%{-1,[1
]%%%}]%%},[1,0,1]%%%}+%%%{%%%{1,[1]%%%},[0,0,2]%%%} Error: Bad Argument Val
ue
```

maple [**A**] time = 0.01, size = 87, normalized size = 0.92

$$\frac{\left(a\,x^2+b\right)\left(2a^{\frac{7}{2}}x^5-5a^{\frac{5}{2}}b\,x^3-15a^{\frac{3}{2}}b^2x+15\sqrt{a\,x^2+b}\;a\,b^2\ln\left(\sqrt{a}\;x+\sqrt{a\,x^2+b}\right)\right)}{8\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{3}{2}}a^{\frac{9}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b/x^2)^(3/2),x)
```

```
[Out] 1/8*(a*x^2+b)*(2*x^5*a^(7/2)-5*a^(5/2)*x^3*b-15*a^(3/2)*x*b^2+15*(a*x^2+b)^
(1/2)*ln(a^(1/2)*x+(a*x^2+b)^(1/2))*a*b^2)/((a*x^2+b)/x^2)^(3/2)/x^3/a^(9/2
)
```

maxima [**A**] time = 1.96, size = 122, normalized size = 1.28

$$-\frac{15\left(a+\frac{b}{x^2}\right)^2b^2-25\left(a+\frac{b}{x^2}\right)ab^2+8\,a^2b^2}{8\left(\left(a+\frac{b}{x^2}\right)^{\frac{5}{2}}a^3-2\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}a^4+\sqrt{a+\frac{b}{x^2}}\,a^5\right)}-\frac{15\,b^2\log\left(\frac{\sqrt{a+\frac{b}{x^2}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^2}}+\sqrt{a}}\right)}{16\,a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*(15*(a + b/x^2)^2*b^2 - 25*(a + b/x^2)*a*b^2 + 8*a^2*b^2)/((a + b/x^2)
^(5/2)*a^3 - 2*(a + b/x^2)^(3/2)*a^4 + sqrt(a + b/x^2)*a^5) - 15/16*b^2*log
((sqrt(a + b/x^2) - sqrt(a))/(sqrt(a + b/x^2) + sqrt(a)))/a^(7/2)
```

mupad [**B**] time = 1.83, size = 75, normalized size = 0.79

$$\frac{15\,b^2\,\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{8\,a^{7/2}}-\frac{15\,b^2}{8\,a^3\sqrt{a+\frac{b}{x^2}}}+\frac{x^4}{4\,a\sqrt{a+\frac{b}{x^2}}}-\frac{5\,b\,x^2}{8\,a^2\sqrt{a+\frac{b}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b/x^2)^(3/2),x)
```

```
[Out] (15*b^2*atanh((a + b/x^2)^(1/2)/a^(1/2)))/(8*a^(7/2)) - (15*b^2)/(8*a^3*(a
+ b/x^2)^(1/2)) + x^4/(4*a*(a + b/x^2)^(1/2)) - (5*b*x^2)/(8*a^2*(a + b/x^2
)^(1/2))
```

sympy [A] time = 6.33, size = 100, normalized size = 1.05

$$\frac{x^5}{4a\sqrt{b}\sqrt{\frac{ax^2}{b}+1}} - \frac{5\sqrt{b}\,x^3}{8a^2\sqrt{\frac{ax^2}{b}+1}} - \frac{15b^{\frac{3}{2}}x}{8a^3\sqrt{\frac{ax^2}{b}+1}} + \frac{15b^2\,\operatorname{asinh}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b/x**2)**(3/2),x)
```

```
[Out] x**5/(4*a*sqrt(b)*sqrt(a*x**2/b + 1)) - 5*sqrt(b)*x**3/(8*a**2*sqrt(a*x**2/
b + 1)) - 15*b**(3/2)*x/(8*a**3*sqrt(a*x**2/b + 1)) + 15*b**2*asinh(sqrt(a)
*x/sqrt(b))/(8*a**(7/2))
```

3.1931 $\int \frac{x}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=69

$$-\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{2a^{5/2}}+\frac{3b}{2a^2\sqrt{a+\frac{b}{x^2}}}+\frac{x^2}{2a\sqrt{a+\frac{b}{x^2}}}$$

```
[Out] -3/2*b*arctanh((a+b/x^2)^(1/2)/a^(1/2))/a^(5/2)+3/2*b/a^2/(a+b/x^2)^(1/2)+1
/2*x^2/a/(a+b/x^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {266, 51, 63, 208}

$$\frac{3x^2\sqrt{a+\frac{b}{x^2}}}{2a^2}-\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{x^2}{a\sqrt{a+\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b/x^2)^(3/2),x]
```

```
[Out] -(x^2/(a*Sqrt[a + b/x^2])) + (3*Sqrt[a + b/x^2]*x^2)/(2*a^2) - (3*b*ArcTanh
[Sqrt[a + b/x^2]/Sqrt[a]])/(2*a^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx = -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= -\frac{x^2}{a\sqrt{a+\frac{b}{x^2}}} - \frac{3\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x^2}\right)}{2a}$$

$$= -\frac{x^2}{a\sqrt{a+\frac{b}{x^2}}} + \frac{3\sqrt{a+\frac{b}{x^2}}\,x^2}{2a^2} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x^2}\right)}{4a^2}$$

$$= -\frac{x^2}{a\sqrt{a+\frac{b}{x^2}}} + \frac{3\sqrt{a+\frac{b}{x^2}}\,x^2}{2a^2} + \frac{3\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x^2}}\right)}{2a^2}$$

$$= -\frac{x^2}{a\sqrt{a+\frac{b}{x^2}}} + \frac{3\sqrt{a+\frac{b}{x^2}}\,x^2}{2a^2} - \frac{3b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{2a^{5/2}}$$

Mathematica [A] time = 0.04, size = 74, normalized size = 1.07

$$\frac{\sqrt{a}\,x\left(ax^2+3b\right) - 3b^{3/2}\sqrt{\frac{ax^2}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2a^{5/2}x\sqrt{a+\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b/x^2)^(3/2),x]
```

```
[Out] (Sqrt[a]*x*(3*b + a*x^2) - 3*b^(3/2)*Sqrt[1 + (a*x^2)/b]*ArcSinh[(Sqrt[a]*x
)/Sqrt[b]])/(2*a^(5/2)*Sqrt[a + b/x^2]*x)
```

fricas [A] time = 0.63, size = 192, normalized size = 2.78

$$\left[\frac{3\left(abx^2+b^2\right)\sqrt{a}\,\log\left(-2\,ax^2+2\,\sqrt{a}\,x^2\sqrt{\frac{ax^2+b}{x^2}}-b\right)+2\left(a^2x^4+3\,abx^2\right)\sqrt{\frac{ax^2+b}{x^2}}}{4\left(a^4x^2+a^3b\right)}, \frac{3\left(abx^2+b^2\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}}{2\left(a^4\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(a*b*x^2 + b^2)*sqrt(a)*log(-2*a*x^2 + 2*sqrt(a)*x^2*sqrt((a*x^2 +
b)/x^2) - b) + 2*(a^2*x^4 + 3*a*b*x^2)*sqrt((a*x^2 + b)/x^2))/(a^4*x^2 + a^
3*b), 1/2*(3*(a*b*x^2 + b^2)*sqrt(-a)*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + b)/
x^2)/(a*x^2 + b)) + (a^2*x^4 + 3*a*b*x^2)*sqrt((a*x^2 + b)/x^2))/(a^4*x^2 +
 a^3*b)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Unable to divide, perhaps due to rounding error%%%{%%%{-2,[1]%%%},[2
,1,2]%%%}+%%%{%%{[4,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,1,3]%%%}+%%%{-2,[0,1,4]%%
%} / %%%{%%%{1,[2]%%%},[2,0,0]%%%}+%%%{%%{[%%%{-2,[1]%%%},0]:[1,0,%%%{-1,[1
]%%%}]%%},[1,0,1]%%%}+%%%{%%%{1,[1]%%%},[0,0,2]%%%} Error: Bad Argument Val
ue
```

maple [A] time = 0.01, size = 73, normalized size = 1.06

$$\frac{\left(a\,x^2+b\right)\left(a^{\frac{5}{2}}x^3+3a^{\frac{3}{2}}bx-3\sqrt{a\,x^2+b}\;ab\ln\left(\sqrt{a}\;x+\sqrt{a\,x^2+b}\right)\right)}{2\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{3}{2}}a^{\frac{7}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b/x^2)^(3/2),x)
```

```
[Out] 1/2*(a*x^2+b)*(x^3*a^(5/2)+3*a^(3/2)*x*b-3*ln(a^(1/2)*x+(a*x^2+b)^(1/2))*(a
*x^2+b)^(1/2)*a*b)/((a*x^2+b)/x^2)^(3/2)/x^3/a^(7/2)
```

maxima [A] time = 1.82, size = 86, normalized size = 1.25

$$\frac{3\left(a+\frac{b}{x^2}\right)b-2\,ab}{2\left(\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}a^2-\sqrt{a+\frac{b}{x^2}}\,a^3\right)}+\frac{3\,b\log\left(\frac{\sqrt{a+\frac{b}{x^2}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^2}}+\sqrt{a}}\right)}{4\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(3*(a + b/x^2)*b - 2*a*b)/((a + b/x^2)^(3/2)*a^2 - sqrt(a + b/x^2)*a^3)
 + 3/4*b*log((sqrt(a + b/x^2) - sqrt(a))/(sqrt(a + b/x^2) + sqrt(a)))/a^(5/
2)
```

mupad [B] time = 1.61, size = 53, normalized size = 0.77

$$\frac{3\,b}{2\,a^2\sqrt{a+\frac{b}{x^2}}}+\frac{x^2}{2\,a\sqrt{a+\frac{b}{x^2}}}-\frac{3\,b\,\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{2\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b/x^2)^(3/2),x)
```

```
[Out] (3*b)/(2*a^2*(a + b/x^2)^(1/2)) + x^2/(2*a*(a + b/x^2)^(1/2)) - (3*b*atanh(
(a + b/x^2)^(1/2)/a^(1/2)))/(2*a^(5/2))
```

sympy [A] time = 3.57, size = 71, normalized size = 1.03

$$\frac{x^3}{2a\sqrt{b}\sqrt{\frac{ax^2}{b}+1}}+\frac{3\sqrt{b}\,x}{2a^2\sqrt{\frac{ax^2}{b}+1}}-\frac{3b\,\operatorname{asinh}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x**2)**(3/2),x)
```

```
[Out] x**3/(2*a*sqrt(b)*sqrt(a*x**2/b + 1)) + 3*sqrt(b)*x/(2*a**2*sqrt(a*x**2/b +
 1)) - 3*b*asinh(sqrt(a)*x/sqrt(b))/(2*a**(5/2))
```

3.1932 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x}\,dx$

Optimal. Leaf size=41

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{a\sqrt{a+\frac{b}{x^2}}}$$

```
[Out] arctanh((a+b/x^2)^(1/2)/a^(1/2))/a^(3/2)-1/a/(a+b/x^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 51, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{a\sqrt{a+\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)^(3/2)*x),x]
```

```
[Out] -(1/(a*Sqrt[a + b/x^2])) + ArcTanh[Sqrt[a + b/x^2]/Sqrt[a]]/a^(3/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x}\,dx = -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= -\frac{1}{a\sqrt{a+\frac{b}{x^2}}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x^2}\right)}{2a}$$

$$= -\frac{1}{a\sqrt{a+\frac{b}{x^2}}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x^2}}\right)}{ab}$$

$$= -\frac{1}{a\sqrt{a+\frac{b}{x^2}}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{a^{3/2}}$$

Mathematica [A] time = 0.06, size = 62, normalized size = 1.51

$$\frac{\sqrt{b}\sqrt{\frac{ax^2}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right) - \sqrt{a}\,x}{a^{3/2}x\sqrt{a+\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)^(3/2)*x),x]
```

```
[Out] (-(Sqrt[a]*x) + Sqrt[b]*Sqrt[1 + (a*x^2)/b]*ArcSinh[(Sqrt[a]*x)/Sqrt[b]])/(
a^(3/2)*Sqrt[a + b/x^2]*x)
```

fricas [B] time = 0.83, size = 163, normalized size = 3.98

$$\left[-\frac{2\,ax^2\sqrt{\frac{ax^2+b}{x^2}} - \left(ax^2+b\right)\sqrt{a}\,\log\left(-2\,ax^2 - 2\sqrt{a}\,x^2\sqrt{\frac{ax^2+b}{x^2}} - b\right)}{2\left(a^3x^2+a^2b\right)}, -\frac{ax^2\sqrt{\frac{ax^2+b}{x^2}} + \left(ax^2+b\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}\,x^2}{ax^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*x^2*sqrt((a*x^2 + b)/x^2) - (a*x^2 + b)*sqrt(a)*log(-2*a*x^2 - 2
*sqrt(a)*x^2*sqrt((a*x^2 + b)/x^2) - b))/(a^3*x^2 + a^2*b), -(a*x^2*sqrt((a
*x^2 + b)/x^2) + (a*x^2 + b)*sqrt(-a)*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + b)/
x^2)/(a*x^2 + b)))/(a^3*x^2 + a^2*b)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Unable to divide, perhaps due to rounding error%%%{%%%{-2,[1]%%%},[2
,1,2]%%%}+%%%{%%{[4,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,1,3]%%%}+%%%{-2,[0,1,4]%%
%} / %%%{%%%{1,[2]%%%},[2,0,0]%%%}+%%%{%%{[%%%{-2,[1]%%%},0]:[1,0,%%%{-1,[1
]%%%}]%%},[1,0,1]%%%}+%%%{%%%{1,[1]%%%},[0,0,2]%%%} Error: Bad Argument Val
ue
```

maple [A] time = 0.01, size = 63, normalized size = 1.54

$$-\frac{\left(a\,x^2+b\right)\left(a^{\frac{3}{2}}x-\sqrt{a\,x^2+b}\;a\ln\left(\sqrt{a}\;x+\sqrt{a\,x^2+b}\right)\right)}{\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{3}{2}}a^{\frac{5}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^(3/2)/x,x)
```

```
[Out] -(a*x^2+b)*(x*a^(3/2)-ln(a^(1/2)*x+(a*x^2+b)^(1/2))*a*(a*x^2+b)^(1/2))/((a*
x^2+b)/x^2)^(3/2)/x^3/a^(5/2)
```

maxima [A] time = 1.98, size = 52, normalized size = 1.27

$$-\frac{\log\left(\frac{\sqrt{a+\frac{b}{x^2}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^2}}+\sqrt{a}}\right)}{2\,a^{\frac{3}{2}}}-\frac{1}{\sqrt{a+\frac{b}{x^2}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] -1/2*log((sqrt(a + b/x^2) - sqrt(a))/(sqrt(a + b/x^2) + sqrt(a)))/a^(3/2) -
 1/(sqrt(a + b/x^2)*a)
```

mupad [B] time = 1.32, size = 33, normalized size = 0.80

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{a\sqrt{a+\frac{b}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b/x^2)^(3/2)),x)
```

```
[Out] atanh((a + b/x^2)^(1/2)/a^(1/2))/a^(3/2) - 1/(a*(a + b/x^2)^(1/2))
```

sympy [B] time = 2.18, size = 187, normalized size = 4.56

$$-\frac{2a^3x^2\sqrt{1+\frac{b}{ax^2}}}{2a^{\frac{9}{2}}x^2+2a^{\frac{7}{2}}b}-\frac{a^3x^2\log\left(\frac{b}{ax^2}\right)}{2a^{\frac{9}{2}}x^2+2a^{\frac{7}{2}}b}+\frac{2a^3x^2\log\left(\sqrt{1+\frac{b}{ax^2}}+1\right)}{2a^{\frac{9}{2}}x^2+2a^{\frac{7}{2}}b}-\frac{a^2b\log\left(\frac{b}{ax^2}\right)}{2a^{\frac{9}{2}}x^2+2a^{\frac{7}{2}}b}+\frac{2a^2b\log\left(\sqrt{1+\frac{b}{ax^2}}+1\right)}{2a^{\frac{9}{2}}x^2+2a^{\frac{7}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**(3/2)/x,x)
```

```
[Out] -2*a**3*x**2*sqrt(1 + b/(a*x**2))/(2*a**(9/2)*x**2 + 2*a**(7/2)*b) - a**3*x
**2*log(b/(a*x**2))/(2*a**(9/2)*x**2 + 2*a**(7/2)*b) + 2*a**3*x**2*log(sqrt
(1 + b/(a*x**2)) + 1)/(2*a**(9/2)*x**2 + 2*a**(7/2)*b) - a**2*b*log(b/(a*x*
*2))/(2*a**(9/2)*x**2 + 2*a**(7/2)*b) + 2*a**2*b*log(sqrt(1 + b/(a*x**2)) +
 1)/(2*a**(9/2)*x**2 + 2*a**(7/2)*b)
```

3.1933 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^3}\,dx$

Optimal. Leaf size=15

$$\frac{1}{b\sqrt{a+\frac{b}{x^2}}}$$

```
[Out] 1/b/(a+b/x^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$\frac{1}{b\sqrt{a+\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)^(3/2)*x^3),x]
```

```
[Out] 1/(b*Sqrt[a + b/x^2])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^3}\,dx = \frac{1}{b\sqrt{a+\frac{b}{x^2}}}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{1}{b\sqrt{a+\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)^(3/2)*x^3),x]
```

```
[Out] 1/(b*Sqrt[a + b/x^2])
```

fricas [B] time = 0.99, size = 29, normalized size = 1.93

$$\frac{x^2\sqrt{\frac{ax^2+b}{x^2}}}{abx^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] x^2*sqrt((a*x^2 + b)/x^2)/(a*b*x^2 + b^2)
```

giac [B] time = 0.19, size = 44, normalized size = 2.93

$$\frac{1}{\left(\left(\sqrt{a}\,x^2-\sqrt{ax^4+bx^2}\right)\sqrt{a}+b\right)\sqrt{a}}-\frac{1}{\sqrt{a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/(((sqrt(a)*x^2 - sqrt(a*x^4 + b*x^2))*sqrt(a) + b)*sqrt(a)) - 1/(sqrt(a)*
b)
```

maple [B] time = 0.00, size = 28, normalized size = 1.87

$$\frac{a\,x^2+b}{\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{3}{2}}\,b\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^(3/2)/x^3,x)
```

```
[Out] (a*x^2+b)/b/x^2/((a*x^2+b)/x^2)^(3/2)
```

maxima [A] time = 0.88, size = 13, normalized size = 0.87

$$\frac{1}{\sqrt{a+\frac{b}{x^2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/(sqrt(a + b/x^2)*b)
```

mupad [B] time = 1.25, size = 18, normalized size = 1.20

$$\frac{\sqrt{x^2}}{b\,\sqrt{a\,x^2+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b/x^2)^(3/2)),x)
```

```
[Out] (x^2)^(1/2)/(b*(b + a*x^2)^(1/2))
```

sympy [A] time = 2.37, size = 26, normalized size = 1.73

$$\begin{cases}\frac{1}{b\sqrt{a+\frac{b}{x^2}}} & \text{for } b\neq 0\\ -\frac{1}{2a^{\frac{3}{2}}x^2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**(3/2)/x**3,x)
```

```
[Out] Piecewise((1/(b*sqrt(a + b/x**2)), Ne(b, 0)), (-1/(2*a**(3/2)*x**2), True))
```

3.1934 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^5}\,dx$

Optimal. Leaf size=34

$$-\frac{a}{b^2\sqrt{a+\frac{b}{x^2}}}-\frac{\sqrt{a+\frac{b}{x^2}}}{b^2}$$

```
[Out] -a/b^2/(a+b/x^2)^(1/2)-(a+b/x^2)^(1/2)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{a}{b^2\sqrt{a+\frac{b}{x^2}}}-\frac{\sqrt{a+\frac{b}{x^2}}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)^(3/2)*x^5),x]
```

```
[Out] -(a/(b^2*Sqrt[a + b/x^2])) - Sqrt[a + b/x^2]/b^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^5}\,dx &= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{x}{(a+bx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)\right)\\ &= -\left(\frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^{3/2}}+\frac{1}{b\sqrt{a+bx}}\right)dx, x, \frac{1}{x^2}\right)\right)\\ &= -\frac{a}{b^2\sqrt{a+\frac{b}{x^2}}}-\frac{\sqrt{a+\frac{b}{x^2}}}{b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.82

$$\frac{-2ax^2-b}{b^2x^2\sqrt{a+\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)^(3/2)*x^5),x]
[Out] (-b - 2*a*x^2)/(b^2*Sqrt[a + b/x^2]*x^2)
```

fricas [A] time = 0.75, size = 37, normalized size = 1.09

$$-\frac{\left(2\,ax^2+b\right)\sqrt{\frac{ax^2+b}{x^2}}}{ab^2x^2+b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(3/2)/x^5,x, algorithm="fricas")
[Out] -(2*a*x^2 + b)*sqrt((a*x^2 + b)/x^2)/(a*b^2*x^2 + b^3)
```

giac [A] time = 0.21, size = 28, normalized size = 0.82

$$-\frac{\frac{2\,ax^2}{b^2}+\frac{1}{b}}{\sqrt{ax^4+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(3/2)/x^5,x, algorithm="giac")
[Out] -(2*a*x^2/b^2 + 1/b)/sqrt(a*x^4 + b*x^2)
```

maple [A] time = 0.00, size = 37, normalized size = 1.09

$$-\frac{\left(a\,x^2+b\right)\left(2a\,x^2+b\right)}{\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{3}{2}}b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^(3/2)/x^5,x)
[Out] -(a*x^2+b)*(2*a*x^2+b)/x^4/b^2/((a*x^2+b)/x^2)^(3/2)
```

maxima [A] time = 0.89, size = 30, normalized size = 0.88

$$-\frac{\sqrt{a+\frac{b}{x^2}}}{b^2}-\frac{a}{\sqrt{a+\frac{b}{x^2}}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(3/2)/x^5,x, algorithm="maxima")
[Out] -sqrt(a + b/x^2)/b^2 - a/(sqrt(a + b/x^2)*b^2)
```

mupad [B] time = 1.22, size = 36, normalized size = 1.06

$$-\frac{x\sqrt{a+\frac{b}{x^2}}\,\left(\frac{1}{b}+\frac{2\,a\,x^2}{b^2}\right)}{a\,x^3+b\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b/x^2)^(3/2)),x)
[Out] -(x*(a + b/x^2)^(1/2)*(1/b + (2*a*x^2)/b^2))/(b*x + a*x^3)
```

sympy [**A**] time = 3.38, size = 48, normalized size = 1.41

$$\begin{cases} -\frac{2a}{b^2\sqrt{a+\frac{b}{x^2}}} - \frac{1}{bx^2\sqrt{a+\frac{b}{x^2}}} & \text{for } b \neq 0 \\ -\frac{1}{4a^{\frac{3}{2}}x^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**(3/2)/x**5,x)
```

```
[Out] Piecewise((-2*a/(b**2*sqrt(a + b/x**2)) - 1/(b*x**2*sqrt(a + b/x**2)), Ne(b
, 0)), (-1/(4*a**(3/2)*x**4), True))
```

3.1935 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^7}\,dx$

Optimal. Leaf size=54

$$\frac{a^2}{b^3\sqrt{a+\frac{b}{x^2}}}+\frac{2a\sqrt{a+\frac{b}{x^2}}}{b^3}-\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{3b^3}$$

```
[Out] -1/3*(a+b/x^2)^(3/2)/b^3+a^2/b^3/(a+b/x^2)^(1/2)+2*a*(a+b/x^2)^(1/2)/b^3
```

Rubi [A] time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{a^2}{b^3\sqrt{a+\frac{b}{x^2}}}+\frac{2a\sqrt{a+\frac{b}{x^2}}}{b^3}-\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)^(3/2)*x^7),x]
```

```
[Out] a^2/(b^3*Sqrt[a + b/x^2]) + (2*a*Sqrt[a + b/x^2])/b^3 - (a + b/x^2)^(3/2)/(
3*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^7}\,dx &= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{(a+bx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)\right)\\ &= -\left(\frac{1}{2}\,\text{Subst}\left(\int\left(\frac{a^2}{b^2(a+bx)^{3/2}}-\frac{2a}{b^2\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b^2}\right)dx, x, \frac{1}{x^2}\right)\right)\\ &= \frac{a^2}{b^3\sqrt{a+\frac{b}{x^2}}}+\frac{2a\sqrt{a+\frac{b}{x^2}}}{b^3}-\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{3b^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.78

$$\frac{8a^2x^4+4abx^2-b^2}{3b^3x^4\sqrt{a+\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)^(3/2)*x^7),x]
```

```
[Out] (-b^2 + 4*a*b*x^2 + 8*a^2*x^4)/(3*b^3*Sqrt[a + b/x^2]*x^4)
```

fricas [A] time = 0.84, size = 54, normalized size = 1.00

$$\frac{\left(8\,a^2x^4+4\,abx^2-b^2\right)\sqrt{\frac{ax^2+b}{x^2}}}{3\left(ab^3x^4+b^4x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/3*(8*a^2*x^4 + 4*a*b*x^2 - b^2)*sqrt((a*x^2 + b)/x^2)/(a*b^3*x^4 + b^4*x^
2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] integrate(1/((a + b/x^2)^(3/2)*x^7), x)
```

maple [A] time = 0.01, size = 50, normalized size = 0.93

$$\frac{\left(a\,x^2+b\right)\left(8a^2x^4+4ab\,x^2-b^2\right)}{3\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{3}{2}}b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^(3/2)/x^7,x)
```

```
[Out] 1/3*(a*x^2+b)*(8*a^2*x^4+4*a*b*x^2-b^2)/x^6/b^3/((a*x^2+b)/x^2)^(3/2)
```

maxima [A] time = 0.88, size = 46, normalized size = 0.85

$$-\frac{\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}}{3\,b^3}+\frac{2\sqrt{a+\frac{b}{x^2}}\,a}{b^3}+\frac{a^2}{\sqrt{a+\frac{b}{x^2}}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/3*(a + b/x^2)^(3/2)/b^3 + 2*sqrt(a + b/x^2)*a/b^3 + a^2/(sqrt(a + b/x^2)
*b^3)
```

mupad [B] time = 1.34, size = 47, normalized size = 0.87

$$\frac{\sqrt{a+\frac{b}{x^2}}\,\left(8\,a^2\,x^4+4\,a\,b\,x^2-b^2\right)}{3\,b^3\,x^2\,\left(a\,x^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b/x^2)^(3/2)),x)
```

```
[Out] ((a + b/x^2)^(1/2)*(8*a^2*x^4 - b^2 + 4*a*b*x^2))/(3*b^3*x^2*(b + a*x^2))
```

sympy [**B**] time = 2.60, size = 423, normalized size = 7.83

$$\frac{8a^{\frac{9}{2}}b^{\frac{7}{2}}x^6\sqrt{\frac{ax^2}{b}+1}}{3a^{\frac{7}{2}}b^6x^7+6a^{\frac{5}{2}}b^7x^5+3a^{\frac{3}{2}}b^8x^3}+\frac{12a^{\frac{7}{2}}b^{\frac{9}{2}}x^4\sqrt{\frac{ax^2}{b}+1}}{3a^{\frac{7}{2}}b^6x^7+6a^{\frac{5}{2}}b^7x^5+3a^{\frac{3}{2}}b^8x^3}+\frac{3a^{\frac{5}{2}}b^{\frac{11}{2}}x^2\sqrt{\frac{ax^2}{b}+1}}{3a^{\frac{7}{2}}b^6x^7+6a^{\frac{5}{2}}b^7x^5+3a^{\frac{3}{2}}b^8x^3}-\frac{a^{\frac{3}{2}}b^{\frac{13}{2}}\sqrt{}}{3a^{\frac{7}{2}}b^6x^7+6a^{\frac{5}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**(3/2)/x**7,x)
```

```
[Out] 8*a**(9/2)*b**(7/2)*x**6*sqrt(a*x**2/b + 1)/(3*a**(7/2)*b**6*x**7 + 6*a**(5
/2)*b**7*x**5 + 3*a**(3/2)*b**8*x**3) + 12*a**(7/2)*b**(9/2)*x**4*sqrt(a*x*
*2/b + 1)/(3*a**(7/2)*b**6*x**7 + 6*a**(5/2)*b**7*x**5 + 3*a**(3/2)*b**8*x*
*3) + 3*a**(5/2)*b**(11/2)*x**2*sqrt(a*x**2/b + 1)/(3*a**(7/2)*b**6*x**7 +
6*a**(5/2)*b**7*x**5 + 3*a**(3/2)*b**8*x**3) - a**(3/2)*b**(13/2)*sqrt(a*x*
*2/b + 1)/(3*a**(7/2)*b**6*x**7 + 6*a**(5/2)*b**7*x**5 + 3*a**(3/2)*b**8*x*
*3) - 8*a**5*b**3*x**7/(3*a**(7/2)*b**6*x**7 + 6*a**(5/2)*b**7*x**5 + 3*a**
(3/2)*b**8*x**3) - 16*a**4*b**4*x**5/(3*a**(7/2)*b**6*x**7 + 6*a**(5/2)*b**
7*x**5 + 3*a**(3/2)*b**8*x**3) - 8*a**3*b**5*x**3/(3*a**(7/2)*b**6*x**7 + 6
*a**(5/2)*b**7*x**5 + 3*a**(3/2)*b**8*x**3)
```

3.1936 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^9}\,dx$

Optimal. Leaf size=73

$$-\frac{a^3}{b^4\sqrt{a+\frac{b}{x^2}}}-\frac{3a^2\sqrt{a+\frac{b}{x^2}}}{b^4}+\frac{a\left(a+\frac{b}{x^2}\right)^{3/2}}{b^4}-\frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{5b^4}$$

```
[Out] a*(a+b/x^2)^(3/2)/b^4-1/5*(a+b/x^2)^(5/2)/b^4-a^3/b^4/(a+b/x^2)^(1/2)-3*a^2
*(a+b/x^2)^(1/2)/b^4
```

Rubi [A] time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{a^3}{b^4\sqrt{a+\frac{b}{x^2}}}-\frac{3a^2\sqrt{a+\frac{b}{x^2}}}{b^4}+\frac{a\left(a+\frac{b}{x^2}\right)^{3/2}}{b^4}-\frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{5b^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)^(3/2)*x^9),x]
```

```
[Out] -(a^3/(b^4*Sqrt[a + b/x^2])) - (3*a^2*Sqrt[a + b/x^2])/b^4 + (a*(a + b/x^2)
^(3/2))/b^4 - (a + b/x^2)^(5/2)/(5*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^9}\,dx &= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{(a+bx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)\right)\\ &= -\left(\frac{1}{2}\,\text{Subst}\left(\int\left(-\frac{a^3}{b^3(a+bx)^{3/2}}+\frac{3a^2}{b^3\sqrt{a+bx}}-\frac{3a\sqrt{a+bx}}{b^3}+\frac{(a+bx)^{3/2}}{b^3}\right)dx, x, \frac{1}{x^2}\right)\right)\\ &= -\frac{a^3}{b^4\sqrt{a+\frac{b}{x^2}}}-\frac{3a^2\sqrt{a+\frac{b}{x^2}}}{b^4}+\frac{a\left(a+\frac{b}{x^2}\right)^{3/2}}{b^4}-\frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{5b^4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 53, normalized size = 0.73

$$\frac{-16a^3x^6-8a^2bx^4+2ab^2x^2-b^3}{5b^4x^6\sqrt{a+\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)^(3/2)*x^9),x]
```

```
[Out] (-b^3 + 2*a*b^2*x^2 - 8*a^2*b*x^4 - 16*a^3*x^6)/(5*b^4*Sqrt[a + b/x^2]*x^6)
```

fricas [A] time = 0.84, size = 63, normalized size = 0.86

$$-\frac{\left(16\,a^3x^6+8\,a^2bx^4-2\,ab^2x^2+b^3\right)\sqrt{\frac{ax^2+b}{x^2}}}{5\left(ab^4x^6+b^5x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] -1/5*(16*a^3*x^6 + 8*a^2*b*x^4 - 2*a*b^2*x^2 + b^3)*sqrt((a*x^2 + b)/x^2)/(
a*b^4*x^6 + b^5*x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] integrate(1/((a + b/x^2)^(3/2)*x^9), x)
```

maple [A] time = 0.01, size = 59, normalized size = 0.81

$$-\frac{\left(a\,x^2+b\right)\left(16a^3x^6+8a^2b\,x^4-2a\,b^2x^2+b^3\right)}{5\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{3}{2}}b^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^(3/2)/x^9,x)
```

```
[Out] -1/5*(a*x^2+b)*(16*a^3*x^6+8*a^2*b*x^4-2*a*b^2*x^2+b^3)/x^8/b^4/((a*x^2+b)/
x^2)^(3/2)
```

maxima [A] time = 0.87, size = 63, normalized size = 0.86

$$-\frac{\left(a+\frac{b}{x^2}\right)^{\frac{5}{2}}}{5\,b^4}+\frac{\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}a}{b^4}-\frac{3\sqrt{a+\frac{b}{x^2}}\,a^2}{b^4}-\frac{a^3}{\sqrt{a+\frac{b}{x^2}}\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] -1/5*(a + b/x^2)^(5/2)/b^4 + (a + b/x^2)^(3/2)*a/b^4 - 3*sqrt(a + b/x^2)*a^
2/b^4 - a^3/(sqrt(a + b/x^2)*b^4)
```

mupad [B] time = 1.41, size = 56, normalized size = 0.77

$$-\frac{\sqrt{a+\frac{b}{x^2}}\,\left(16\,a^3\,x^6+8\,a^2\,b\,x^4-2\,a\,b^2\,x^2+b^3\right)}{5\,b^4\,x^4\,\left(a\,x^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(a + b/x^2)^(3/2)),x)
```

```
[Out] -((a + b/x^2)^(1/2)*(b^3 + 16*a^3*x^6 - 2*a*b^2*x^2 + 8*a^2*b*x^4))/(5*b^4*
x^4*(b + a*x^2))
```

sympy [**B**] time = 4.25, size = 1844, normalized size = 25.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**(3/2)/x**9,x)
```

```
[Out] -16*a**(21/2)*b**(23/2)*x**16*sqrt(a*x**2/b + 1)/(5*a**(17/2)*b**15*x**17 +
 30*a**(15/2)*b**16*x**15 + 75*a**(13/2)*b**17*x**13 + 100*a**(11/2)*b**18*
x**11 + 75*a**(9/2)*b**19*x**9 + 30*a**(7/2)*b**20*x**7 + 5*a**(5/2)*b**21*
x**5) - 88*a**(19/2)*b**(25/2)*x**14*sqrt(a*x**2/b + 1)/(5*a**(17/2)*b**15*
x**17 + 30*a**(15/2)*b**16*x**15 + 75*a**(13/2)*b**17*x**13 + 100*a**(11/2)
*b**18*x**11 + 75*a**(9/2)*b**19*x**9 + 30*a**(7/2)*b**20*x**7 + 5*a**(5/2)
*b**21*x**5) - 198*a**(17/2)*b**(27/2)*x**12*sqrt(a*x**2/b + 1)/(5*a**(17/2
)*b**15*x**17 + 30*a**(15/2)*b**16*x**15 + 75*a**(13/2)*b**17*x**13 + 100*a
**(11/2)*b**18*x**11 + 75*a**(9/2)*b**19*x**9 + 30*a**(7/2)*b**20*x**7 + 5*
a**(5/2)*b**21*x**5) - 231*a**(15/2)*b**(29/2)*x**10*sqrt(a*x**2/b + 1)/(5*
a**(17/2)*b**15*x**17 + 30*a**(15/2)*b**16*x**15 + 75*a**(13/2)*b**17*x**13
 + 100*a**(11/2)*b**18*x**11 + 75*a**(9/2)*b**19*x**9 + 30*a**(7/2)*b**20*x
**7 + 5*a**(5/2)*b**21*x**5) - 145*a**(13/2)*b**(31/2)*x**8*sqrt(a*x**2/b +
 1)/(5*a**(17/2)*b**15*x**17 + 30*a**(15/2)*b**16*x**15 + 75*a**(13/2)*b**1
7*x**13 + 100*a**(11/2)*b**18*x**11 + 75*a**(9/2)*b**19*x**9 + 30*a**(7/2)*
b**20*x**7 + 5*a**(5/2)*b**21*x**5) - 46*a**(11/2)*b**(33/2)*x**6*sqrt(a*x*
*2/b + 1)/(5*a**(17/2)*b**15*x**17 + 30*a**(15/2)*b**16*x**15 + 75*a**(13/2
)*b**17*x**13 + 100*a**(11/2)*b**18*x**11 + 75*a**(9/2)*b**19*x**9 + 30*a**
(7/2)*b**20*x**7 + 5*a**(5/2)*b**21*x**5) - 8*a**(9/2)*b**(35/2)*x**4*sqrt(
a*x**2/b + 1)/(5*a**(17/2)*b**15*x**17 + 30*a**(15/2)*b**16*x**15 + 75*a**(
13/2)*b**17*x**13 + 100*a**(11/2)*b**18*x**11 + 75*a**(9/2)*b**19*x**9 + 30
*a**(7/2)*b**20*x**7 + 5*a**(5/2)*b**21*x**5) - 3*a**(7/2)*b**(37/2)*x**2*s
qrt(a*x**2/b + 1)/(5*a**(17/2)*b**15*x**17 + 30*a**(15/2)*b**16*x**15 + 75*
a**(13/2)*b**17*x**13 + 100*a**(11/2)*b**18*x**11 + 75*a**(9/2)*b**19*x**9
 + 30*a**(7/2)*b**20*x**7 + 5*a**(5/2)*b**21*x**5) - a**(5/2)*b**(39/2)*sqrt
(a*x**2/b + 1)/(5*a**(17/2)*b**15*x**17 + 30*a**(15/2)*b**16*x**15 + 75*a**
(13/2)*b**17*x**13 + 100*a**(11/2)*b**18*x**11 + 75*a**(9/2)*b**19*x**9 + 3
0*a**(7/2)*b**20*x**7 + 5*a**(5/2)*b**21*x**5) + 16*a**11*b**11*x**17/(5*a*
*(17/2)*b**15*x**17 + 30*a**(15/2)*b**16*x**15 + 75*a**(13/2)*b**17*x**13 +
 100*a**(11/2)*b**18*x**11 + 75*a**(9/2)*b**19*x**9 + 30*a**(7/2)*b**20*x**
7 + 5*a**(5/2)*b**21*x**5) + 96*a**10*b**12*x**15/(5*a**(17/2)*b**15*x**17
 + 30*a**(15/2)*b**16*x**15 + 75*a**(13/2)*b**17*x**13 + 100*a**(11/2)*b**18
*x**11 + 75*a**(9/2)*b**19*x**9 + 30*a**(7/2)*b**20*x**7 + 5*a**(5/2)*b**21
*x**5) + 240*a**9*b**13*x**13/(5*a**(17/2)*b**15*x**17 + 30*a**(15/2)*b**16
*x**15 + 75*a**(13/2)*b**17*x**13 + 100*a**(11/2)*b**18*x**11 + 75*a**(9/2)
*b**19*x**9 + 30*a**(7/2)*b**20*x**7 + 5*a**(5/2)*b**21*x**5) + 320*a**8*b*
*14*x**11/(5*a**(17/2)*b**15*x**17 + 30*a**(15/2)*b**16*x**15 + 75*a**(13/2
)*b**17*x**13 + 100*a**(11/2)*b**18*x**11 + 75*a**(9/2)*b**19*x**9 + 30*a**
(7/2)*b**20*x**7 + 5*a**(5/2)*b**21*x**5) + 240*a**7*b**15*x**9/(5*a**(17/2
)*b**15*x**17 + 30*a**(15/2)*b**16*x**15 + 75*a**(13/2)*b**17*x**13 + 100*a
**(11/2)*b**18*x**11 + 75*a**(9/2)*b**19*x**9 + 30*a**(7/2)*b**20*x**7 + 5*
a**(5/2)*b**21*x**5) + 96*a**6*b**16*x**7/(5*a**(17/2)*b**15*x**17 + 30*a**
(15/2)*b**16*x**15 + 75*a**(13/2)*b**17*x**13 + 100*a**(11/2)*b**18*x**11 +
 75*a**(9/2)*b**19*x**9 + 30*a**(7/2)*b**20*x**7 + 5*a**(5/2)*b**21*x**5) +
 16*a**5*b**17*x**5/(5*a**(17/2)*b**15*x**17 + 30*a**(15/2)*b**16*x**15 + 7
5*a**(13/2)*b**17*x**13 + 100*a**(11/2)*b**18*x**11 + 75*a**(9/2)*b**19*x**
9 + 30*a**(7/2)*b**20*x**7 + 5*a**(5/2)*b**21*x**5)
```

3.1937 $\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=88

$$\frac{16b^2x\sqrt{a+\frac{b}{x^2}}}{5a^4}-\frac{8b^2x}{5a^3\sqrt{a+\frac{b}{x^2}}}-\frac{2bx^3}{5a^2\sqrt{a+\frac{b}{x^2}}}+\frac{x^5}{5a\sqrt{a+\frac{b}{x^2}}}$$

```
[Out] -8/5*b^2*x/a^3/(a+b/x^2)^(1/2)-2/5*b*x^3/a^2/(a+b/x^2)^(1/2)+1/5*x^5/a/(a+b
/x^2)^(1/2)+16/5*b^2*x*(a+b/x^2)^(1/2)/a^4
```

Rubi [A] time = 0.03, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {271, 192, 191}

$$\frac{16b^2x\sqrt{a+\frac{b}{x^2}}}{5a^4}-\frac{8b^2x}{5a^3\sqrt{a+\frac{b}{x^2}}}-\frac{2bx^3}{5a^2\sqrt{a+\frac{b}{x^2}}}+\frac{x^5}{5a\sqrt{a+\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b/x^2)^(3/2),x]
```

```
[Out] (-8*b^2*x)/(5*a^3*Sqrt[a + b/x^2]) + (16*b^2*Sqrt[a + b/x^2]*x)/(5*a^4) - (
2*b*x^3)/(5*a^2*Sqrt[a + b/x^2]) + x^5/(5*a*Sqrt[a + b/x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx = \frac{x^5}{5a\sqrt{a+\frac{b}{x^2}}} - \frac{(6b)\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx}{5a}$$

$$= -\frac{2bx^3}{5a^2\sqrt{a+\frac{b}{x^2}}} + \frac{x^5}{5a\sqrt{a+\frac{b}{x^2}}} + \frac{\left(8b^2\right)\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx}{5a^2}$$

$$= -\frac{8b^2x}{5a^3\sqrt{a+\frac{b}{x^2}}} - \frac{2bx^3}{5a^2\sqrt{a+\frac{b}{x^2}}} + \frac{x^5}{5a\sqrt{a+\frac{b}{x^2}}} + \frac{\left(16b^2\right)\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}\,dx}{5a^3}$$

$$= -\frac{8b^2x}{5a^3\sqrt{a+\frac{b}{x^2}}} + \frac{16b^2\sqrt{a+\frac{b}{x^2}}\,x}{5a^4} - \frac{2bx^3}{5a^2\sqrt{a+\frac{b}{x^2}}} + \frac{x^5}{5a\sqrt{a+\frac{b}{x^2}}}$$

Mathematica [A] time = 0.03, size = 52, normalized size = 0.59

$$\frac{a^3x^6 - 2a^2bx^4 + 8ab^2x^2 + 16b^3}{5a^4x\sqrt{a+\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b/x^2)^(3/2),x]
```

```
[Out] (16*b^3 + 8*a*b^2*x^2 - 2*a^2*b*x^4 + a^3*x^6)/(5*a^4*Sqrt[a + b/x^2]*x)
```

fricas [A] time = 0.85, size = 62, normalized size = 0.70

$$\frac{\left(a^3x^7 - 2\,a^2bx^5 + 8\,ab^2x^3 + 16\,b^3x\right)\sqrt{\frac{ax^2+b}{x^2}}}{5\left(a^5x^2 + a^4b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5*(a^3*x^7 - 2*a^2*b*x^5 + 8*a*b^2*x^3 + 16*b^3*x)*sqrt((a*x^2 + b)/x^2)/
(a^5*x^2 + a^4*b)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,
0,4]%%%}] at parameters values [17,97.031638477,-16]Warning, choosing root
of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at param
eters values [-30,21.9788511144,56]Warning, choosing root of [1,0,%%%{-2,[1
,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [66,
51.1688777674,-43]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,
[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [28,28.9556208979,28]
```

```
Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{
1,[2,0,4]%%%}] at parameters values [-71,67.2689954746,93]Warning, choosing
 root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at
 parameters values [-13,34.9002507734,-2]Warning, choosing root of [1,0,%%%
{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters value
s [-32,42.2093715044,-53]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+
%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [-68,89.604722
4309,-49]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%
%},0,%%%{1,[2,0,4]%%%}] at parameters values [-67,31.1777770203,-77]Warning
, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,
4]%%%}] at parameters values [98,37.8441393101,-21]b^3/a^4/(a*(-sqrt(b)/a*s
ign(x)+sqrt(a*x^2+b)/a/sign(x))+sqrt(b)*sign(-sqrt(b)/a*sign(x)+sqrt(a*x^2+
b)/a/sign(x)))+(1/5*a^26*(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign(x))^5+2*a
^24*b*(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign(x))^3+2*a^23*sqrt(b)*b*(-sqr
t(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign(x))^2*sign(-sqrt(b)/a*sign(x)+sqrt(a*x^
2+b)/a/sign(x))+a^22*b^2*(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign(x))-a^24*
b*(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign(x))^3-3*a^22*b^2*(-sqrt(b)/a*sig
n(x)+sqrt(a*x^2+b)/a/sign(x))+3*a^22*b^2*(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/
a/sign(x))-3*a^23*sqrt(b)*b*(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign(x))^2*
sign(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign(x))+a^25*sqrt(b)*(-sqrt(b)/a*s
ign(x)+sqrt(a*x^2+b)/a/sign(x))^4*sign(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/a/s
ign(x)))/a^25-b^2*sqrt(b)/a^4*sign(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign(
x))
```

maple [A] time = 0.01, size = 60, normalized size = 0.68

$$\frac{\left(a\,x^2+b\right)\left(a^3x^6-2a^2b\,x^4+8a\,b^2x^2+16b^3\right)}{5\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{3}{2}}a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a+b/x^2)^(3/2),x)
```

```
[Out] 1/5*(a*x^2+b)*(a^3*x^6-2*a^2*b*x^4+8*a*b^2*x^2+16*b^3)/a^4/x^3/((a*x^2+b)/x
^2)^(3/2)
```

maxima [A] time = 0.87, size = 69, normalized size = 0.78

$$\frac{b^3}{\sqrt{a+\frac{b}{x^2}}\,a^4x}+\frac{\left(a+\frac{b}{x^2}\right)^{\frac{5}{2}}x^5-5\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}bx^3+15\sqrt{a+\frac{b}{x^2}}\,b^2x}{5\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] b^3/(sqrt(a + b/x^2)*a^4*x) + 1/5*((a + b/x^2)^(5/2)*x^5 - 5*(a + b/x^2)^(3
/2)*b*x^3 + 15*sqrt(a + b/x^2)*b^2*x)/a^4
```

mupad [B] time = 1.72, size = 48, normalized size = 0.55

$$\frac{a^3\,x^6-2\,a^2\,b\,x^4+8\,a\,b^2\,x^2+16\,b^3}{5\,a^4\,x\,\sqrt{a+\frac{b}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b/x^2)^(3/2),x)
```

```
[Out] (16*b^3 + a^3*x^6 + 8*a*b^2*x^2 - 2*a^2*b*x^4)/(5*a^4*x*(a + b/x^2)^(1/2))
```

sympy [B] time = 1.64, size = 337, normalized size = 3.83

$$\frac{a^5 b^{\frac{19}{2}} x^{10}\sqrt{\frac{ax^2}{b}+1}}{5a^7b^9x^6+15a^6b^{10}x^4+15a^5b^{11}x^2+5a^4b^{12}}+\frac{5a^3 b^{\frac{23}{2}} x^6\sqrt{\frac{ax^2}{b}+1}}{5a^7b^9x^6+15a^6b^{10}x^4+15a^5b^{11}x^2+5a^4b^{12}}+\frac{30a^2 b^{\frac{25}{2}} x}{5a^7b^9x^6+15a^6b^{10}x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a+b/x**2)**(3/2),x)
```

```
[Out] a**5*b**(19/2)*x**10*sqrt(a*x**2/b + 1)/(5*a**7*b**9*x**6 + 15*a**6*b**10*x
**4 + 15*a**5*b**11*x**2 + 5*a**4*b**12) + 5*a**3*b**(23/2)*x**6*sqrt(a*x**
2/b + 1)/(5*a**7*b**9*x**6 + 15*a**6*b**10*x**4 + 15*a**5*b**11*x**2 + 5*a*
*4*b**12) + 30*a**2*b**(25/2)*x**4*sqrt(a*x**2/b + 1)/(5*a**7*b**9*x**6 + 1
5*a**6*b**10*x**4 + 15*a**5*b**11*x**2 + 5*a**4*b**12) + 40*a*b**(27/2)*x**
2*sqrt(a*x**2/b + 1)/(5*a**7*b**9*x**6 + 15*a**6*b**10*x**4 + 15*a**5*b**11
*x**2 + 5*a**4*b**12) + 16*b**(29/2)*sqrt(a*x**2/b + 1)/(5*a**7*b**9*x**6 +
 15*a**6*b**10*x**4 + 15*a**5*b**11*x**2 + 5*a**4*b**12)
```

3.1938 $\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=62

$$-\frac{8bx\sqrt{a+\frac{b}{x^2}}}{3a^3}+\frac{4bx}{3a^2\sqrt{a+\frac{b}{x^2}}}+\frac{x^3}{3a\sqrt{a+\frac{b}{x^2}}}$$

```
[Out] 4/3*b*x/a^2/(a+b/x^2)^(1/2)+1/3*x^3/a/(a+b/x^2)^(1/2)-8/3*b*x*(a+b/x^2)^(1/
2)/a^3
```

Rubi [A] time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {271, 192, 191}

$$-\frac{8bx\sqrt{a+\frac{b}{x^2}}}{3a^3}+\frac{4bx}{3a^2\sqrt{a+\frac{b}{x^2}}}+\frac{x^3}{3a\sqrt{a+\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b/x^2)^(3/2),x]
```

```
[Out] (4*b*x)/(3*a^2*Sqrt[a + b/x^2]) - (8*b*Sqrt[a + b/x^2]*x)/(3*a^3) + x^3/(3*
a*Sqrt[a + b/x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx = \frac{x^3}{3a\sqrt{a+\frac{b}{x^2}}} - \frac{(4b)\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx}{3a}$$

$$= \frac{4bx}{3a^2\sqrt{a+\frac{b}{x^2}}} + \frac{x^3}{3a\sqrt{a+\frac{b}{x^2}}} - \frac{(8b)\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}\,dx}{3a^2}$$

$$= \frac{4bx}{3a^2\sqrt{a+\frac{b}{x^2}}} - \frac{8b\sqrt{a+\frac{b}{x^2}}\,x}{3a^3} + \frac{x^3}{3a\sqrt{a+\frac{b}{x^2}}}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 0.66

$$\frac{a^2x^4 - 4abx^2 - 8b^2}{3a^3x\sqrt{a+\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b/x^2)^(3/2),x]
```

```
[Out] (-8*b^2 - 4*a*b*x^2 + a^2*x^4)/(3*a^3*Sqrt[a + b/x^2]*x)
```

fricas [A] time = 0.78, size = 51, normalized size = 0.82

$$\frac{\left(a^2x^5 - 4\,abx^3 - 8\,b^2x\right)\sqrt{\frac{ax^2+b}{x^2}}}{3\left(a^4x^2 + a^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(a^2*x^5 - 4*a*b*x^3 - 8*b^2*x)*sqrt((a*x^2 + b)/x^2)/(a^4*x^2 + a^3*b)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,
0,4]%%%}] at parameters values [87,11.8623251389,-77]Warning, choosing root
 of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at para
meters values [42,66.6397423497,-22]Warning, choosing root of [1,0,%%%{-2,[
1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [-7
9,4.84063049592,-53]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-
4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [-8,67.6494641076,-
79]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%
%%{1,[2,0,4]%%%}] at parameters values [-5,8.73413453996,60]Warning, choosi
ng root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}]
at parameters values [81,31.1594383004,58]1/a*(-b^2/a^2/(a*(-sqrt(b)/a*sign
(x)+sqrt(a*x^2+b)/a/sign(x))+sqrt(b)*sign(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/
```

```
a/sign(x)))+(1/3*a^10*(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign(x))^3+a^8*b*
(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign(x))-2*a^8*b*(-sqrt(b)/a*sign(x)+sq
rt(a*x^2+b)/a/sign(x))+a^9*sqrt(b)*(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign
(x))^2*sign(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign(x)))/a^9+b*sqrt(b)/a^2*
sign(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign(x)))
```

maple [A] time = 0.01, size = 49, normalized size = 0.79

$$\frac{\left(a\,x^2+b\right)\left(a^2x^4-4ab\,x^2-8b^2\right)}{3\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{3}{2}}a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b/x^2)^(3/2),x)
```

```
[Out] 1/3*(a*x^2+b)*(a^2*x^4-4*a*b*x^2-8*b^2)/a^3/x^3/((a*x^2+b)/x^2)^(3/2)
```

maxima [A] time = 0.82, size = 53, normalized size = 0.85

$$\frac{\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}x^3-6\sqrt{a+\frac{b}{x^2}}\,bx}{3\,a^3}-\frac{b^2}{\sqrt{a+\frac{b}{x^2}}\,a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*((a + b/x^2)^(3/2)*x^3 - 6*sqrt(a + b/x^2)*b*x)/a^3 - b^2/(sqrt(a + b/x
^2)*a^3*x)
```

mupad [B] time = 1.59, size = 38, normalized size = 0.61

$$-\frac{-a^2x^4+4\,a\,b\,x^2+8\,b^2}{3\,a^3\,x\sqrt{a+\frac{b}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b/x^2)^(3/2),x)
```

```
[Out] -(8*b^2 - a^2*x^4 + 4*a*b*x^2)/(3*a^3*x*(a + b/x^2)^(1/2))
```

sympy [B] time = 1.28, size = 219, normalized size = 3.53

$$\frac{a^3b^{\frac{9}{2}}x^6\sqrt{\frac{ax^2}{b}+1}}{3a^5b^4x^4+6a^4b^5x^2+3a^3b^6}-\frac{3a^2b^{\frac{11}{2}}x^4\sqrt{\frac{ax^2}{b}+1}}{3a^5b^4x^4+6a^4b^5x^2+3a^3b^6}-\frac{12ab^{\frac{13}{2}}x^2\sqrt{\frac{ax^2}{b}+1}}{3a^5b^4x^4+6a^4b^5x^2+3a^3b^6}-\frac{8b^{\frac{15}{2}}\sqrt{\frac{ax^2}{b}+1}}{3a^5b^4x^4+6a^4b^5x^2+3a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b/x**2)**(3/2),x)
```

```
[Out] a**3*b**(9/2)*x**6*sqrt(a*x**2/b + 1)/(3*a**5*b**4*x**4 + 6*a**4*b**5*x**2
+ 3*a**3*b**6) - 3*a**2*b**(11/2)*x**4*sqrt(a*x**2/b + 1)/(3*a**5*b**4*x**4
 + 6*a**4*b**5*x**2 + 3*a**3*b**6) - 12*a*b**(13/2)*x**2*sqrt(a*x**2/b + 1)
/(3*a**5*b**4*x**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6) - 8*b**(15/2)*sqrt(a*x
**2/b + 1)/(3*a**5*b**4*x**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6)
```

3.1939 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=35

$$\frac{2x\sqrt{a+\frac{b}{x^2}}}{a^2}-\frac{x}{a\sqrt{a+\frac{b}{x^2}}}$$

```
[Out] -x/a/(a+b/x^2)^(1/2)+2*x*(a+b/x^2)^(1/2)/a^2
```

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {192, 191}

$$\frac{2x\sqrt{a+\frac{b}{x^2}}}{a^2}-\frac{x}{a\sqrt{a+\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^(-3/2),x]
```

```
[Out] -(x/(a*Sqrt[a + b/x^2])) + (2*Sqrt[a + b/x^2]*x)/a^2
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx &= -\frac{x}{a\sqrt{a+\frac{b}{x^2}}}+\frac{2\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}\,dx}{a}\\ &= -\frac{x}{a\sqrt{a+\frac{b}{x^2}}}+\frac{2\sqrt{a+\frac{b}{x^2}}\,x}{a^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.77

$$\frac{ax^2+2b}{a^2x\sqrt{a+\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^(-3/2),x]
```

```
[Out] (2*b + a*x^2)/(a^2*Sqrt[a + b/x^2]*x)
```

fricas [A] time = 0.79, size = 39, normalized size = 1.11

$$\frac{\left(ax^3 + 2\,bx\right)\sqrt{\frac{ax^2+b}{x^2}}}{a^3x^2 + a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (a*x^3 + 2*b*x)*sqrt((a*x^2 + b)/x^2)/(a^3*x^2 + a^2*b)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,
0,4]%%%}] at parameters values [11,86.0946290595,-72]Warning, choosing root
 of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at para
meters values [79,97.1729265442,77]Warning, choosing root of [1,0,%%%{-2,[1
,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [35,
94.710570252,-15]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[
0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [-46,57.5337947248,-63
]b/a^2/(a*(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign(x))+sqrt(b)*sign(-sqrt(b
)/a*sign(x)+sqrt(a*x^2+b)/a/sign(x)))+(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/a/s
ign(x))/a-sqrt(b)/a^2*sign(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign(x))
```

maple [A] time = 0.00, size = 37, normalized size = 1.06

$$\frac{\left(a\,x^2 + b\right)\left(a\,x^2 + 2b\right)}{\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{3}{2}} a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^(3/2),x)
```

```
[Out] (a*x^2+b)*(a*x^2+2*b)/a^2/x^3/((a*x^2+b)/x^2)^(3/2)
```

maxima [A] time = 0.89, size = 32, normalized size = 0.91

$$\frac{\sqrt{a + \frac{b}{x^2}}\,x}{a^2} + \frac{b}{\sqrt{a + \frac{b}{x^2}}\,a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] sqrt(a + b/x^2)*x/a^2 + b/(sqrt(a + b/x^2)*a^2*x)
```

mupad [B] time = 1.19, size = 36, normalized size = 1.03

$$\frac{a^2\,x^4 + 3\,a\,b\,x^2 + 2\,b^2}{a^2\,x^3\left(a + \frac{b}{x^2}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x^2)^(3/2),x)
```

```
[Out] (2*b^2 + a^2*x^4 + 3*a*b*x^2)/(a^2*x^3*(a + b/x^2)^(3/2))
```

sympy [**A**] time = 0.99, size = 42, normalized size = 1.20

$$\frac{x^2}{a\sqrt{b}\sqrt{\frac{ax^2}{b}+1}}+\frac{2\sqrt{b}}{a^2\sqrt{\frac{ax^2}{b}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**(3/2),x)
```

```
[Out] x**2/(a*sqrt(b)*sqrt(a*x**2/b + 1)) + 2*sqrt(b)/(a**2*sqrt(a*x**2/b + 1))
```

3.1940 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^2}\,dx$

Optimal. Leaf size=19

$$-\frac{1}{ax\sqrt{a+\frac{b}{x^2}}}$$

```
[Out] -1/a/x/(a+b/x^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {264}

$$-\frac{1}{ax\sqrt{a+\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)^(3/2)*x^2),x]
```

```
[Out] -(1/(a*Sqrt[a + b/x^2]*x))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^2}\,dx = -\frac{1}{a\sqrt{a+\frac{b}{x^2}}\,x}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$-\frac{1}{ax\sqrt{a+\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)^(3/2)*x^2),x]
```

```
[Out] -(1/(a*Sqrt[a + b/x^2]*x))
```

fricas [A] time = 0.83, size = 29, normalized size = 1.53

$$-\frac{x\sqrt{\frac{ax^2+b}{x^2}}}{a^2x^2+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] -x*sqrt((a*x^2 + b)/x^2)/(a^2*x^2 + a*b)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,
0,4]%%%}] at parameters values [20,94.4491733639,99]Warning, choosing root
of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at param
eters values [-9,28.5128403376,-66]Warning, choosing root of [1,0,%%%{-2,[1
,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [11,
86.0946290595,-72]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,
[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [79,97.1729265442,77]
a*((b+b)/2/a^2/b/sqrt(b)*sign(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign(x))-1
/a^2/(a*(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign(x))+sqrt(b)*sign(-sqrt(b)/
a*sign(x)+sqrt(a*x^2+b)/a/sign(x))))
```

maple [**A**] time = 0.00, size = 29, normalized size = 1.53

$$-\frac{a\,x^2+b}{\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{3}{2}}\,a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^(3/2)/x^2,x)
```

```
[Out] -(a*x^2+b)/a/x^3/((a*x^2+b)/x^2)^(3/2)
```

maxima [**A**] time = 0.90, size = 17, normalized size = 0.89

$$-\frac{1}{\sqrt{a+\frac{b}{x^2}}\,a x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] -1/(sqrt(a + b/x^2)*a*x)
```

mupad [**B**] time = 1.20, size = 24, normalized size = 1.26

$$-\frac{x\sqrt{a+\frac{b}{x^2}}}{a\left(a\,x^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b/x^2)^(3/2)),x)
```

```
[Out] -(x*(a + b/x^2)^(1/2))/(a*(b + a*x^2))
```

sympy [**A**] time = 1.02, size = 20, normalized size = 1.05

$$-\frac{1}{a\sqrt{b}\sqrt{\frac{ax^2}{b}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**(3/2)/x**2,x)
```

```
[Out] -1/(a*sqrt(b)*sqrt(a*x**2/b + 1))
```

3.1941 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^4}\,dx$

Optimal. Leaf size=47

$$\frac{1}{bx\sqrt{a+\frac{b}{x^2}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{b^{3/2}}$$

```
[Out] -arctanh(b^(1/2)/x/(a+b/x^2)^(1/2))/b^(3/2)+1/b/x/(a+b/x^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {335, 288, 217, 206}

$$\frac{1}{bx\sqrt{a+\frac{b}{x^2}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)^(3/2)*x^4),x]
```

```
[Out] 1/(b*Sqrt[a + b/x^2]*x) - ArcTanh[Sqrt[b]/(Sqrt[a + b/x^2]*x)]/b^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^4}\,dx = -\text{Subst}\left(\int \frac{x^2}{\left(a+bx^2\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{b\sqrt{a+\frac{b}{x^2}}\,x} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{x}\right)}{b}$$

$$= \frac{1}{b\sqrt{a+\frac{b}{x^2}}\,x} - \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x}\right)}{b}$$

$$= \frac{1}{b\sqrt{a+\frac{b}{x^2}}\,x} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x}\right)}{b^{3/2}}$$

Mathematica [C] time = 0.01, size = 36, normalized size = 0.77

$$\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{ax^2}{b}+1\right)}{bx\sqrt{a+\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)^(3/2)*x^4),x]
```

```
[Out] Hypergeometric2F1[-1/2, 1, 1/2, 1 + (a*x^2)/b]/(b*Sqrt[a + b/x^2]*x)
```

fricas [A] time = 1.08, size = 155, normalized size = 3.30

$$\left[\frac{2\,bx\sqrt{\frac{ax^2+b}{x^2}} + \left(ax^2+b\right)\sqrt{b}\,\log\left(-\frac{ax^2-2\,\sqrt{b}\,x\sqrt{\frac{ax^2+b}{x^2}}+2\,b}{x^2}\right)}{2\left(ab^2x^2+b^3\right)}, \frac{bx\sqrt{\frac{ax^2+b}{x^2}} + \left(ax^2+b\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x\sqrt{\frac{ax^2+b}{x^2}}}{ax^2+b}\right)}{ab^2x^2+b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/2*(2*b*x*sqrt((a*x^2 + b)/x^2) + (a*x^2 + b)*sqrt(b)*log(-(a*x^2 - 2*sqr
t(b)*x*sqrt((a*x^2 + b)/x^2) + 2*b)/x^2))/(a*b^2*x^2 + b^3), (b*x*sqrt((a*x
^2 + b)/x^2) + (a*x^2 + b)*sqrt(-b)*arctan(sqrt(-b)*x*sqrt((a*x^2 + b)/x^2)
/(a*x^2 + b)))/(a*b^2*x^2 + b^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [sign(t_nostep),sign(t_nostep+sq
rt(b)/a*sign(t_nostep))]sym2poly/r2sym(const gen & e,const index_m & i,cons
t vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 67, normalized size = 1.43

$$-\frac{\left(a\,x^2+b\right)\left(\sqrt{a\,x^2+b}\;b\ln\left(\frac{2b+2\sqrt{a\,x^2+b}\;\sqrt{b}}{x}\right)-b^{\frac{3}{2}}\right)}{\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{3}{2}}b^{\frac{5}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^(3/2)/x^4,x)
```

```
[Out] -(a*x^2+b)*(ln(2*(b+(a*x^2+b)^(1/2)*b^(1/2))/x)*b*(a*x^2+b)^(1/2)-b^(3/2))/
((a*x^2+b)/x^2)^(3/2)/x^3/b^(5/2)
```

maxima [A] time = 1.79, size = 58, normalized size = 1.23

$$\frac{\log\left(\frac{\sqrt{a+\frac{b}{x^2}}\,x-\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x+\sqrt{b}}\right)}{2\,b^{\frac{3}{2}}}+\frac{1}{\sqrt{a+\frac{b}{x^2}}\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/2*log((sqrt(a + b/x^2)*x - sqrt(b))/(sqrt(a + b/x^2)*x + sqrt(b)))/b^(3/2
) + 1/(sqrt(a + b/x^2)*b*x)
```

mupad [B] time = 1.37, size = 40, normalized size = 0.85

$$\frac{1}{b\,x\sqrt{a+\frac{b}{x^2}}}-\frac{\ln\left(\sqrt{a+\frac{b}{x^2}}+\frac{\sqrt{b}}{x}\right)}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b/x^2)^(3/2)),x)
```

```
[Out] 1/(b*x*(a + b/x^2)^(1/2)) - log((a + b/x^2)^(1/2) + b^(1/2)/x)/b^(3/2)
```

sympy [B] time = 2.37, size = 184, normalized size = 3.91

$$\frac{ab^2x^2\log\left(\frac{ax^2}{b}\right)}{2ab^{\frac{7}{2}}x^2+2b^{\frac{9}{2}}}-\frac{2ab^2x^2\log\left(\sqrt{\frac{ax^2}{b}+1}+1\right)}{2ab^{\frac{7}{2}}x^2+2b^{\frac{9}{2}}}+\frac{2b^3\sqrt{\frac{ax^2}{b}+1}}{2ab^{\frac{7}{2}}x^2+2b^{\frac{9}{2}}}+\frac{b^3\log\left(\frac{ax^2}{b}\right)}{2ab^{\frac{7}{2}}x^2+2b^{\frac{9}{2}}}-\frac{2b^3\log\left(\sqrt{\frac{ax^2}{b}+1}+1\right)}{2ab^{\frac{7}{2}}x^2+2b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**(3/2)/x**4,x)
```

```
[Out] a*b**2*x**2*log(a*x**2/b)/(2*a*b**(7/2)*x**2 + 2*b**(9/2)) - 2*a*b**2*x**2*
log(sqrt(a*x**2/b + 1) + 1)/(2*a*b**(7/2)*x**2 + 2*b**(9/2)) + 2*b**3*sqrt(
a*x**2/b + 1)/(2*a*b**(7/2)*x**2 + 2*b**(9/2)) + b**3*log(a*x**2/b)/(2*a*b*
*(7/2)*x**2 + 2*b**(9/2)) - 2*b**3*log(sqrt(a*x**2/b + 1) + 1)/(2*a*b**(7/2
)*x**2 + 2*b**(9/2))
```

3.1942 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^6}\,dx$

Optimal. Leaf size=71

$$\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{2b^{5/2}} - \frac{3\sqrt{a+\frac{b}{x^2}}}{2b^2x} + \frac{1}{bx^3\sqrt{a+\frac{b}{x^2}}}$$

```
[Out] 3/2*a*arctanh(b^(1/2)/x/(a+b/x^2)^(1/2))/b^(5/2)+1/b/x^3/(a+b/x^2)^(1/2)-3/
2*(a+b/x^2)^(1/2)/b^2/x
```

Rubi [A] time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {335, 288, 321, 217, 206}

$$-\frac{3\sqrt{a+\frac{b}{x^2}}}{2b^2x} + \frac{3a\tanh^{-1}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{2b^{5/2}} + \frac{1}{bx^3\sqrt{a+\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)^(3/2)*x^6),x]
```

```
[Out] 1/(b*Sqrt[a + b/x^2]*x^3) - (3*Sqrt[a + b/x^2])/(2*b^2*x) + (3*a*ArcTanh[Sq
rt[b]/(Sqrt[a + b/x^2]*x)])/(2*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^6}\,dx &= -\operatorname{Subst}\left(\int \frac{x^4}{\left(a+bx^2\right)^{3/2}}\,dx, x, \frac{1}{x}\right) \\
&= \frac{1}{b\sqrt{a+\frac{b}{x^2}}\,x^3} - \frac{3\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{x}\right)}{b} \\
&= \frac{1}{b\sqrt{a+\frac{b}{x^2}}\,x^3} - \frac{3\sqrt{a+\frac{b}{x^2}}}{2b^2x} + \frac{(3a)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{x}\right)}{2b^2} \\
&= \frac{1}{b\sqrt{a+\frac{b}{x^2}}\,x^3} - \frac{3\sqrt{a+\frac{b}{x^2}}}{2b^2x} + \frac{(3a)\operatorname{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x}\right)}{2b^2} \\
&= \frac{1}{b\sqrt{a+\frac{b}{x^2}}\,x^3} - \frac{3\sqrt{a+\frac{b}{x^2}}}{2b^2x} + \frac{3a\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x}\right)}{2b^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 38, normalized size = 0.54

$$-\frac{a\,{}_2F_1\left(-\frac{1}{2}, 2; \frac{1}{2}; \frac{ax^2}{b}+1\right)}{b^2x\sqrt{a+\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)^(3/2)*x^6),x]
```

```
[Out] -((a*Hypergeometric2F1[-1/2, 2, 1/2, 1 + (a*x^2)/b])/(b^2*Sqrt[a + b/x^2]*x
))
```

fricas [A] time = 0.96, size = 190, normalized size = 2.68

$$\left[\frac{3\left(a^2x^3+abx\right)\sqrt{b}\,\log\left(-\frac{ax^2+2\sqrt{b}\,x\sqrt{\frac{ax^2+b}{x^2}}+2b}{x^2}\right) - 2\left(3abx^2+b^2\right)\sqrt{\frac{ax^2+b}{x^2}}}{4\left(ab^3x^3+b^4x\right)}, -\frac{3\left(a^2x^3+abx\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x\sqrt{\frac{ax^2+}{x^2}}}{ax^2+b}\right.}{2\left(ab^3x^3+b^4x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/4*(3*(a^2*x^3 + a*b*x)*sqrt(b)*log(-(a*x^2 + 2*sqrt(b)*x*sqrt((a*x^2 + b
)/x^2) + 2*b)/x^2) - 2*(3*a*b*x^2 + b^2)*sqrt((a*x^2 + b)/x^2))/(a*b^3*x^3
```

```
+ b^4*x), -1/2*(3*(a^2*x^3 + a*b*x)*sqrt(-b)*arctan(sqrt(-b)*x*sqrt((a*x^2
+ b)/x^2)/(a*x^2 + b)) + (3*a*b*x^2 + b^2)*sqrt((a*x^2 + b)/x^2))/(a*b^3*x^
3 + b^4*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [sign(t_nostep),sign(t_nostep+sq
rt(b)/a*sign(t_nostep))]sym2poly/r2sym(const gen & e,const index_m & i,cons
t vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 81, normalized size = 1.14

$$\frac{\left(a\,x^2+b\right)\left(3\sqrt{a\,x^2+b}\;ab\,x^2\ln\left(\frac{2b+2\sqrt{a\,x^2+b}\;\sqrt{b}}{x}\right)-3a\,b^{\frac{3}{2}}x^2-b^{\frac{5}{2}}\right)}{2\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{3}{2}}b^{\frac{7}{2}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^(3/2)/x^6,x)
```

```
[Out] 1/2*(a*x^2+b)*(3*(a*x^2+b)^(1/2)*ln(2*(b+(a*x^2+b)^(1/2)*b^(1/2))/x)*x^2*a*
b-3*b^(3/2)*x^2*a-b^(5/2))/((a*x^2+b)/x^2)^(3/2)/x^5/b^(7/2)
```

maxima [A] time = 1.96, size = 97, normalized size = 1.37

$$-\frac{3\left(a+\frac{b}{x^2}\right)ax^2-2\,ab}{2\left(\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}b^2x^3-\sqrt{a+\frac{b}{x^2}}\,b^3x\right)}-\frac{3\,a\log\left(\frac{\sqrt{a+\frac{b}{x^2}}\,x-\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x+\sqrt{b}}\right)}{4\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] -1/2*(3*(a + b/x^2)*a*x^2 - 2*a*b)/((a + b/x^2)^(3/2)*b^2*x^3 - sqrt(a + b/
x^2)*b^3*x) - 3/4*a*log((sqrt(a + b/x^2)*x - sqrt(b))/(sqrt(a + b/x^2)*x +
sqrt(b)))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^6\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b/x^2)^(3/2)),x)
```

```
[Out] int(1/(x^6*(a + b/x^2)^(3/2)), x)
```

sympy [**A**] time = 4.14, size = 73, normalized size = 1.03

$$-\frac{3\sqrt{a}}{2b^2x\sqrt{1+\frac{b}{ax^2}}}+\frac{3a\,\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\,x}\right)}{2b^{\frac{5}{2}}}-\frac{1}{2\sqrt{a}\,bx^3\sqrt{1+\frac{b}{ax^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**(3/2)/x**6,x)
```

```
[Out] -3*sqrt(a)/(2*b**2*x*sqrt(1 + b/(a*x**2))) + 3*a*asinh(sqrt(b)/(sqrt(a)*x))
/(2*b**(5/2)) - 1/(2*sqrt(a)*b*x**3*sqrt(1 + b/(a*x**2)))
```

3.1943 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^8}\,dx$

Optimal. Leaf size=95

$$-\frac{15a^2 \tanh^{-1}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{8b^{7/2}}+\frac{15a\sqrt{a+\frac{b}{x^2}}}{8b^3x}-\frac{5\sqrt{a+\frac{b}{x^2}}}{4b^2x^3}+\frac{1}{bx^5\sqrt{a+\frac{b}{x^2}}}$$

```
[Out] -15/8*a^2*arctanh(b^(1/2)/x/(a+b/x^2)^(1/2))/b^(7/2)+1/b/x^5/(a+b/x^2)^(1/2
)-5/4*(a+b/x^2)^(1/2)/b^2/x^3+15/8*a*(a+b/x^2)^(1/2)/b^3/x
```

Rubi [A] time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {335, 288, 321, 217, 206}

$$-\frac{15a^2 \tanh^{-1}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{8b^{7/2}}+\frac{15a\sqrt{a+\frac{b}{x^2}}}{8b^3x}-\frac{5\sqrt{a+\frac{b}{x^2}}}{4b^2x^3}+\frac{1}{bx^5\sqrt{a+\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)^(3/2)*x^8),x]
```

```
[Out] 1/(b*Sqrt[a + b/x^2]*x^5) - (5*Sqrt[a + b/x^2])/(4*b^2*x^3) + (15*a*Sqrt[a
+ b/x^2])/(8*b^3*x) - (15*a^2*ArcTanh[Sqrt[b]/(Sqrt[a + b/x^2]*x)])/(8*b^(7
/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^8}\,dx = -\text{Subst}\left(\int \frac{x^6}{\left(a+bx^2\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{b\sqrt{a+\frac{b}{x^2}}\,x^5} - \frac{5\,\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{x}\right)}{b}$$

$$= \frac{1}{b\sqrt{a+\frac{b}{x^2}}\,x^5} - \frac{5\sqrt{a+\frac{b}{x^2}}}{4b^2x^3} + \frac{(15a)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{x}\right)}{4b^2}$$

$$= \frac{1}{b\sqrt{a+\frac{b}{x^2}}\,x^5} - \frac{5\sqrt{a+\frac{b}{x^2}}}{4b^2x^3} + \frac{15a\sqrt{a+\frac{b}{x^2}}}{8b^3x} - \frac{\left(15a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{x}\right)}{8b^3}$$

$$= \frac{1}{b\sqrt{a+\frac{b}{x^2}}\,x^5} - \frac{5\sqrt{a+\frac{b}{x^2}}}{4b^2x^3} + \frac{15a\sqrt{a+\frac{b}{x^2}}}{8b^3x} - \frac{\left(15a^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x}\right)}{8b^3}$$

$$= \frac{1}{b\sqrt{a+\frac{b}{x^2}}\,x^5} - \frac{5\sqrt{a+\frac{b}{x^2}}}{4b^2x^3} + \frac{15a\sqrt{a+\frac{b}{x^2}}}{8b^3x} - \frac{15a^2\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x}\right)}{8b^{7/2}}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.41

$$\frac{a^2\,{}_2F_1\left(-\frac{1}{2}, 3; \frac{1}{2}; \frac{ax^2}{b}+1\right)}{b^3x\sqrt{a+\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)^(3/2)*x^8),x]
```

```
[Out] (a^2*Hypergeometric2F1[-1/2, 3, 1/2, 1 + (a*x^2)/b])/(b^3*Sqrt[a + b/x^2]*x
)
```

fricas [A] time = 0.91, size = 228, normalized size = 2.40

$$\left[\frac{15\left(a^3x^5+a^2bx^3\right)\sqrt{b}\,\log\left(-\frac{ax^2-2\sqrt{b}\,x\sqrt{\frac{ax^2+b}{x^2}}+2b}{x^2}\right)+2\left(15\,a^2bx^4+5\,ab^2x^2-2\,b^3\right)\sqrt{\frac{ax^2+b}{x^2}}}{16\left(ab^4x^5+b^5x^3\right)}, \frac{15\left(a^3x^5+a^2bx^3\right)\sqrt{-b}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] [1/16*(15*(a^3*x^5 + a^2*b*x^3)*sqrt(b)*log(-(a*x^2 - 2*sqrt(b)*x*sqrt((a*x
^2 + b)/x^2) + 2*b)/x^2) + 2*(15*a^2*b*x^4 + 5*a*b^2*x^2 - 2*b^3)*sqrt((a*x
^2 + b)/x^2))/(a*b^4*x^5 + b^5*x^3), 1/8*(15*(a^3*x^5 + a^2*b*x^3)*sqrt(-b)
*arctan(sqrt(-b)*x*sqrt((a*x^2 + b)/x^2)/(a*x^2 + b)) + (15*a^2*b*x^4 + 5*a
*b^2*x^2 - 2*b^3)*sqrt((a*x^2 + b)/x^2))/(a*b^4*x^5 + b^5*x^3)]
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [sign(t_nostep),sign(t_nostep+sq
rt(b)/a*sign(t_nostep))]sym2poly/r2sym(const gen & e,const index_m & i,cons
t vecteur & l) Error: Bad Argument Value
```

maple [**A**] time = 0.01, size = 94, normalized size = 0.99

$$-\frac{\left(a\,x^2+b\right)\left(15\sqrt{a\,x^2+b}\;a^2b\,x^4\ln\left(\frac{2b+2\sqrt{a\,x^2+b}\;\sqrt{b}}{x}\right)-15a^2b^{\frac{3}{2}}x^4-5a\,b^{\frac{5}{2}}x^2+2b^{\frac{7}{2}}\right)}{8\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{3}{2}}b^{\frac{9}{2}}x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^(3/2)/x^8,x)
```

```
[Out] -1/8*(a*x^2+b)*(15*(a*x^2+b)^(1/2)*ln(2*(b+(a*x^2+b)^(1/2)*b^(1/2))/x)*x^4*
a^2*b-15*b^(3/2)*x^4*a^2-5*b^(5/2)*x^2*a+2*b^(7/2))/((a*x^2+b)/x^2)^(3/2)/x
^7/b^(9/2)
```

maxima [**A**] time = 1.99, size = 139, normalized size = 1.46

$$\frac{15\left(a+\frac{b}{x^2}\right)^2a^2x^4-25\left(a+\frac{b}{x^2}\right)a^2bx^2+8\,a^2b^2}{8\left(\left(a+\frac{b}{x^2}\right)^{\frac{5}{2}}b^3x^5-2\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}b^4x^3+\sqrt{a+\frac{b}{x^2}}\,b^5x\right)}+\frac{15\,a^2\log\left(\frac{\sqrt{a+\frac{b}{x^2}}\,x-\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x+\sqrt{b}}\right)}{16\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] 1/8*(15*(a + b/x^2)^2*a^2*x^4 - 25*(a + b/x^2)*a^2*b*x^2 + 8*a^2*b^2)/((a +
 b/x^2)^(5/2)*b^3*x^5 - 2*(a + b/x^2)^(3/2)*b^4*x^3 + sqrt(a + b/x^2)*b^5*x
) + 15/16*a^2*log((sqrt(a + b/x^2)*x - sqrt(b))/(sqrt(a + b/x^2)*x + sqrt(b
)))/b^(7/2)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^8\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(a + b/x^2)^(3/2)),x)
```

```
[Out] int(1/(x^8*(a + b/x^2)^(3/2)), x)
```

sympy [**A**] time = 7.00, size = 102, normalized size = 1.07

$$\frac{15a^{\frac{3}{2}}}{8b^3x\sqrt{1+\frac{b}{ax^2}}}+\frac{5\sqrt{a}}{8b^2x^3\sqrt{1+\frac{b}{ax^2}}}-\frac{15a^2\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\,x}\right)}{8b^{\frac{7}{2}}}-\frac{1}{4\sqrt{a}\,bx^5\sqrt{1+\frac{b}{ax^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**(3/2)/x**8,x)
```

```
[Out] 15*a**(3/2)/(8*b**3*x*sqrt(1 + b/(a*x**2))) + 5*sqrt(a)/(8*b**2*x**3*sqrt(1
 + b/(a*x**2))) - 15*a**2*asinh(sqrt(b)/(sqrt(a)*x))/(8*b**(7/2)) - 1/(4*sq
rt(a)*b*x**5*sqrt(1 + b/(a*x**2)))
```

3.1944 $\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx$

Optimal. Leaf size=116

$$\frac{35b^2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{8a^{9/2}}-\frac{35b^2}{8a^4\sqrt{a+\frac{b}{x^2}}}-\frac{35b^2}{24a^3\left(a+\frac{b}{x^2}\right)^{3/2}}-\frac{7bx^2}{8a^2\left(a+\frac{b}{x^2}\right)^{3/2}}+\frac{x^4}{4a\left(a+\frac{b}{x^2}\right)^{3/2}}$$

```
[Out] -35/24*b^2/a^3/(a+b/x^2)^(3/2)-7/8*b*x^2/a^2/(a+b/x^2)^(3/2)+1/4*x^4/a/(a+b
/x^2)^(3/2)+35/8*b^2*arctanh((a+b/x^2)^(1/2)/a^(1/2))/a^(9/2)-35/8*b^2/a^4/
(a+b/x^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 51, 63, 208}

$$\frac{35b^2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{8a^{9/2}}+\frac{35x^4\sqrt{a+\frac{b}{x^2}}}{12a^3}-\frac{7x^4}{3a^2\sqrt{a+\frac{b}{x^2}}}-\frac{35bx^2\sqrt{a+\frac{b}{x^2}}}{8a^4}-\frac{x^4}{3a\left(a+\frac{b}{x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b/x^2)^(5/2),x]
```

```
[Out] (-35*b*Sqrt[a + b/x^2]*x^2)/(8*a^4) - x^4/(3*a*(a + b/x^2)^(3/2)) - (7*x^4)
/(3*a^2*Sqrt[a + b/x^2]) + (35*Sqrt[a + b/x^2]*x^4)/(12*a^3) + (35*b^2*ArcT
anh[Sqrt[a + b/x^2]/Sqrt[a]])/(8*a^(9/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx = -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3(a+bx)^{5/2}}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= -\frac{x^4}{3a\left(a+\frac{b}{x^2}\right)^{3/2}} - \frac{7\,\text{Subst}\left(\int \frac{1}{x^3(a+bx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)}{6a}$$

$$= -\frac{x^4}{3a\left(a+\frac{b}{x^2}\right)^{3/2}} - \frac{7x^4}{3a^2\sqrt{a+\frac{b}{x^2}}} - \frac{35\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx, x, \frac{1}{x^2}\right)}{6a^2}$$

$$= -\frac{x^4}{3a\left(a+\frac{b}{x^2}\right)^{3/2}} - \frac{7x^4}{3a^2\sqrt{a+\frac{b}{x^2}}} + \frac{35\sqrt{a+\frac{b}{x^2}}\,x^4}{12a^3} + \frac{(35b)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x^2}\right)}{8a^3}$$

$$= -\frac{35b\sqrt{a+\frac{b}{x^2}}\,x^2}{8a^4} - \frac{x^4}{3a\left(a+\frac{b}{x^2}\right)^{3/2}} - \frac{7x^4}{3a^2\sqrt{a+\frac{b}{x^2}}} + \frac{35\sqrt{a+\frac{b}{x^2}}\,x^4}{12a^3} - \frac{\left(35b^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\right.}{16a^4}$$

$$= -\frac{35b\sqrt{a+\frac{b}{x^2}}\,x^2}{8a^4} - \frac{x^4}{3a\left(a+\frac{b}{x^2}\right)^{3/2}} - \frac{7x^4}{3a^2\sqrt{a+\frac{b}{x^2}}} + \frac{35\sqrt{a+\frac{b}{x^2}}\,x^4}{12a^3} - \frac{(35b)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx\right.}{8a^4}$$

$$= -\frac{35b\sqrt{a+\frac{b}{x^2}}\,x^2}{8a^4} - \frac{x^4}{3a\left(a+\frac{b}{x^2}\right)^{3/2}} - \frac{7x^4}{3a^2\sqrt{a+\frac{b}{x^2}}} + \frac{35\sqrt{a+\frac{b}{x^2}}\,x^4}{12a^3} + \frac{35b^2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{8a^{9/2}}$$

Mathematica [A] time = 0.23, size = 112, normalized size = 0.97

$$\frac{\sqrt{a}\left(6a^3x^6 - 21a^2bx^4 - 140ab^2x^2 - 105b^3\right) + \frac{105b^{5/2}\left(ax^2+b\right)\sqrt{\frac{ax^2}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{x}}{24a^{9/2}\sqrt{a+\frac{b}{x^2}}\left(ax^2+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b/x^2)^(5/2),x]
```

```
[Out] (Sqrt[a]*(-105*b^3 - 140*a*b^2*x^2 - 21*a^2*b*x^4 + 6*a^3*x^6) + (105*b^(5/
2)*(b + a*x^2)*Sqrt[1 + (a*x^2)/b]*ArcSinh[(Sqrt[a]*x)/Sqrt[b]])/x)/(24*a^(
9/2)*Sqrt[a + b/x^2]*(b + a*x^2))
```

fricas [A] time = 1.04, size = 287, normalized size = 2.47

$$\left[\frac{105\left(a^2b^2x^4 + 2ab^3x^2 + b^4\right)\sqrt{a}\,\log\left(-2ax^2 - 2\sqrt{a}\,x^2\sqrt{\frac{ax^2+b}{x^2}} - b\right) + 2\left(6a^4x^8 - 21a^3bx^6 - 140a^2b^2x^4 - 105ab^3\right.}{48\left(a^7x^4 + 2a^6bx^2 + a^5b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(105*(a^2*b^2*x^4 + 2*a*b^3*x^2 + b^4)*sqrt(a)*log(-2*a*x^2 - 2*sqrt(
a)*x^2*sqrt((a*x^2 + b)/x^2) - b) + 2*(6*a^4*x^8 - 21*a^3*b*x^6 - 140*a^2*b
^2*x^4 - 105*a*b^3*x^2)*sqrt((a*x^2 + b)/x^2))/(a^7*x^4 + 2*a^6*b*x^2 + a^5
*b^2), -1/24*(105*(a^2*b^2*x^4 + 2*a*b^3*x^2 + b^4)*sqrt(-a)*arctan(sqrt(-a
)*x^2*sqrt((a*x^2 + b)/x^2)/(a*x^2 + b)) - (6*a^4*x^8 - 21*a^3*b*x^6 - 140*
a^2*b^2*x^4 - 105*a*b^3*x^2)*sqrt((a*x^2 + b)/x^2))/(a^7*x^4 + 2*a^6*b*x^2
+ a^5*b^2)]
```

giac [B] time = 0.37, size = 196, normalized size = 1.69

$$\frac{1}{8}\sqrt{ax^4+bx^2}\left(\frac{2x^2}{a^3}-\frac{11b}{a^4}\right)-\frac{35b^2\log\left(\left|-2\left(\sqrt{a}\,x^2-\sqrt{ax^4+bx^2}\right)\sqrt{a}-b\right|\right)}{16a^{\frac{9}{2}}}+\frac{5\left(21b^2\log(|b|)+32b^2\right)}{48a^{\frac{9}{2}}}-\frac{12\left(\sqrt{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(a*x^4 + b*x^2)*(2*x^2/a^3 - 11*b/a^4) - 35/16*b^2*log(abs(-2*(sqrt
(a)*x^2 - sqrt(a*x^4 + b*x^2))*sqrt(a) - b))/a^(9/2) + 5/48*(21*b^2*log(abs
(b)) + 32*b^2)/a^(9/2) - 1/3*(12*(sqrt(a)*x^2 - sqrt(a*x^4 + b*x^2))^2*a*b^
3 + 21*(sqrt(a)*x^2 - sqrt(a*x^4 + b*x^2))*sqrt(a)*b^4 + 10*b^5)/(((sqrt(a)
*x^2 - sqrt(a*x^4 + b*x^2))*sqrt(a) + b)^3*a^(9/2))
```

maple [A] time = 0.02, size = 98, normalized size = 0.84

$$\frac{\left(ax^2+b\right)\left(6a^{\frac{9}{2}}x^7-21a^{\frac{7}{2}}bx^5-140a^{\frac{5}{2}}b^2x^3-105a^{\frac{3}{2}}b^3x+105\left(ax^2+b\right)^{\frac{3}{2}}ab^2\ln\left(\sqrt{a}\,x+\sqrt{ax^2+b}\right)\right)}{24\left(\frac{ax^2+b}{x^2}\right)^{\frac{5}{2}}a^{\frac{11}{2}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b/x^2)^(5/2),x)
```

```
[Out] 1/24*(a*x^2+b)*(6*x^7*a^(9/2)-21*a^(7/2)*x^5*b-140*a^(5/2)*x^3*b^2-105*a^(3
/2)*x*b^3+105*ln(a^(1/2)*x+(a*x^2+b)^(1/2))*(a*x^2+b)^(3/2)*a*b^2)/((a*x^2+
b)/x^2)^(5/2)/x^5/a^(11/2)
```

maxima [A] time = 1.90, size = 139, normalized size = 1.20

$$-\frac{105\left(a+\frac{b}{x^2}\right)^3b^2-175\left(a+\frac{b}{x^2}\right)^2ab^2+56\left(a+\frac{b}{x^2}\right)a^2b^2+8a^3b^2}{24\left(\left(a+\frac{b}{x^2}\right)^{\frac{7}{2}}a^4-2\left(a+\frac{b}{x^2}\right)^{\frac{5}{2}}a^5+\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}a^6\right)}-\frac{35b^2\log\left(\frac{\sqrt{a+\frac{b}{x^2}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^2}}+\sqrt{a}}\right)}{16a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/24*(105*(a + b/x^2)^3*b^2 - 175*(a + b/x^2)^2*a*b^2 + 56*(a + b/x^2)*a^2
*b^2 + 8*a^3*b^2)/((a + b/x^2)^(7/2)*a^4 - 2*(a + b/x^2)^(5/2)*a^5 + (a + b
/x^2)^(3/2)*a^6) - 35/16*b^2*log((sqrt(a + b/x^2) - sqrt(a))/(sqrt(a + b/x^
2) + sqrt(a)))/a^(9/2)
```

mupad [B] time = 1.92, size = 95, normalized size = 0.82

$$\frac{35\,b^2\,\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{8\,a^{9/2}}-\frac{35\,b^2}{6\,a^3\left(a+\frac{b}{x^2}\right)^{3/2}}+\frac{x^4}{4\,a\left(a+\frac{b}{x^2}\right)^{3/2}}-\frac{7\,b\,x^2}{8\,a^2\left(a+\frac{b}{x^2}\right)^{3/2}}-\frac{35\,b^3}{8\,a^4\,x^2\left(a+\frac{b}{x^2}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b/x^2)^(5/2),x)
```

```
[Out] (35*b^2*atanh((a + b/x^2)^(1/2)/a^(1/2)))/(8*a^(9/2)) - (35*b^2)/(6*a^3*(a
+ b/x^2)^(3/2)) + x^4/(4*a*(a + b/x^2)^(3/2)) - (7*b*x^2)/(8*a^2*(a + b/x^2
)^(3/2)) - (35*b^3)/(8*a^4*x^2*(a + b/x^2)^(3/2))
```

sympy [B] time = 8.92, size = 432, normalized size = 3.72

$$\frac{6a^{\frac{89}{2}}b^{75}x^7}{24a^{\frac{93}{2}}b^{\frac{151}{2}}x^2\sqrt{\frac{ax^2}{b}+1}+24a^{\frac{91}{2}}b^{\frac{153}{2}}\sqrt{\frac{ax^2}{b}+1}}-\frac{21a^{\frac{87}{2}}b^{76}x^5}{24a^{\frac{93}{2}}b^{\frac{151}{2}}x^2\sqrt{\frac{ax^2}{b}+1}+24a^{\frac{91}{2}}b^{\frac{153}{2}}\sqrt{\frac{ax^2}{b}+1}}-\frac{140a^{\frac{8}{}}}{24a^{\frac{93}{2}}b^{\frac{151}{2}}x^2\sqrt{\frac{ax^2}{b}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b/x**2)**(5/2),x)
```

```
[Out] 6*a**(89/2)*b**75*x**7/(24*a**(93/2)*b**(151/2)*x**2*sqrt(a*x**2/b + 1) + 2
4*a**(91/2)*b**(153/2)*sqrt(a*x**2/b + 1)) - 21*a**(87/2)*b**76*x**5/(24*a*
*(93/2)*b**(151/2)*x**2*sqrt(a*x**2/b + 1) + 24*a**(91/2)*b**(153/2)*sqrt(a
*x**2/b + 1)) - 140*a**(85/2)*b**77*x**3/(24*a**(93/2)*b**(151/2)*x**2*sqrt
(a*x**2/b + 1) + 24*a**(91/2)*b**(153/2)*sqrt(a*x**2/b + 1)) - 105*a**(83/2
)*b**78*x/(24*a**(93/2)*b**(151/2)*x**2*sqrt(a*x**2/b + 1) + 24*a**(91/2)*b
**(153/2)*sqrt(a*x**2/b + 1)) + 105*a**42*b**(155/2)*x**2*sqrt(a*x**2/b + 1
)*asinh(sqrt(a)*x/sqrt(b))/(24*a**(93/2)*b**(151/2)*x**2*sqrt(a*x**2/b + 1)
 + 24*a**(91/2)*b**(153/2)*sqrt(a*x**2/b + 1)) + 105*a**41*b**(157/2)*sqrt(
a*x**2/b + 1)*asinh(sqrt(a)*x/sqrt(b))/(24*a**(93/2)*b**(151/2)*x**2*sqrt(a
*x**2/b + 1) + 24*a**(91/2)*b**(153/2)*sqrt(a*x**2/b + 1))
```

3.1945 $\int \frac{x}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx$

Optimal. Leaf size=88

$$-\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{2a^{7/2}}+\frac{5b}{2a^3\sqrt{a+\frac{b}{x^2}}}+\frac{5b}{6a^2\left(a+\frac{b}{x^2}\right)^{3/2}}+\frac{x^2}{2a\left(a+\frac{b}{x^2}\right)^{3/2}}$$

```
[Out] 5/6*b/a^2/(a+b/x^2)^(3/2)+1/2*x^2/a/(a+b/x^2)^(3/2)-5/2*b*arctanh((a+b/x^2)
^(1/2)/a^(1/2))/a^(7/2)+5/2*b/a^3/(a+b/x^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 92, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {266, 51, 63, 208}

$$\frac{5x^2\sqrt{a+\frac{b}{x^2}}}{2a^3}-\frac{5x^2}{3a^2\sqrt{a+\frac{b}{x^2}}}-\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{2a^{7/2}}-\frac{x^2}{3a\left(a+\frac{b}{x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b/x^2)^(5/2),x]
```

```
[Out] -x^2/(3*a*(a + b/x^2)^(3/2)) - (5*x^2)/(3*a^2*Sqrt[a + b/x^2]) + (5*Sqrt[a
+ b/x^2]*x^2)/(2*a^3) - (5*b*ArcTanh[Sqrt[a + b/x^2]/Sqrt[a]])/(2*a^(7/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx = -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{5/2}}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= -\frac{x^2}{3a\left(a+\frac{b}{x^2}\right)^{3/2}} - \frac{5\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)}{6a}$$

$$= -\frac{x^2}{3a\left(a+\frac{b}{x^2}\right)^{3/2}} - \frac{5x^2}{3a^2\sqrt{a+\frac{b}{x^2}}} - \frac{5\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x^2}\right)}{2a^2}$$

$$= -\frac{x^2}{3a\left(a+\frac{b}{x^2}\right)^{3/2}} - \frac{5x^2}{3a^2\sqrt{a+\frac{b}{x^2}}} + \frac{5\sqrt{a+\frac{b}{x^2}}\,x^2}{2a^3} + \frac{(5b)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x^2}\right)}{4a^3}$$

$$= -\frac{x^2}{3a\left(a+\frac{b}{x^2}\right)^{3/2}} - \frac{5x^2}{3a^2\sqrt{a+\frac{b}{x^2}}} + \frac{5\sqrt{a+\frac{b}{x^2}}\,x^2}{2a^3} + \frac{5\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x^2}}\right)}{2a^3}$$

$$= -\frac{x^2}{3a\left(a+\frac{b}{x^2}\right)^{3/2}} - \frac{5x^2}{3a^2\sqrt{a+\frac{b}{x^2}}} + \frac{5\sqrt{a+\frac{b}{x^2}}\,x^2}{2a^3} - \frac{5b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{2a^{7/2}}$$

Mathematica [A] time = 0.17, size = 101, normalized size = 1.15

$$\frac{\sqrt{a}\left(3a^2x^4+20abx^2+15b^2\right) - \frac{15b^{3/2}\left(ax^2+b\right)\sqrt{\frac{ax^2}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{x}}{6a^{7/2}\sqrt{a+\frac{b}{x^2}}\left(ax^2+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b/x^2)^(5/2),x]
```

```
[Out] (Sqrt[a]*(15*b^2 + 20*a*b*x^2 + 3*a^2*x^4) - (15*b^(3/2)*(b + a*x^2)*Sqrt[1
 + (a*x^2)/b]*ArcSinh[(Sqrt[a]*x)/Sqrt[b]])/x)/(6*a^(7/2)*Sqrt[a + b/x^2]*(
b + a*x^2))
```

fricas [A] time = 0.81, size = 260, normalized size = 2.95

$$\left[\frac{15\left(a^2bx^4+2\,ab^2x^2+b^3\right)\sqrt{a}\,\log\left(-2\,ax^2+2\,\sqrt{a}\,x^2\sqrt{\frac{ax^2+b}{x^2}}-b\right)+2\left(3\,a^3x^6+20\,a^2bx^4+15\,ab^2x^2\right)\sqrt{\frac{ax^2+b}{x^2}}}{12\left(a^6x^4+2\,a^5bx^2+a^4b^2\right)}, \frac{15\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(15*(a^2*b*x^4 + 2*a*b^2*x^2 + b^3)*sqrt(a)*log(-2*a*x^2 + 2*sqrt(a)*
x^2*sqrt((a*x^2 + b)/x^2) - b) + 2*(3*a^3*x^6 + 20*a^2*b*x^4 + 15*a*b^2*x^2
)*sqrt((a*x^2 + b)/x^2))/(a^6*x^4 + 2*a^5*b*x^2 + a^4*b^2), 1/6*(15*(a^2*b*
x^4 + 2*a*b^2*x^2 + b^3)*sqrt(-a)*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + b)/x^2)
/(a*x^2 + b)) + (3*a^3*x^6 + 20*a^2*b*x^4 + 15*a*b^2*x^2)*sqrt((a*x^2 + b)/
x^2))/(a^6*x^4 + 2*a^5*b*x^2 + a^4*b^2)]
```

giac [B] time = 0.32, size = 178, normalized size = 2.02

$$\frac{5\,b\log\left(\left|-2\left(\sqrt{a}\,x^2-\sqrt{ax^4+bx^2}\right)\sqrt{a}-b\right|\right)}{4\,a^{\frac{7}{2}}}-\frac{15\,b\log\left(|b|\right)+28\,b}{12\,a^{\frac{7}{2}}}+\frac{\sqrt{ax^4+bx^2}}{2\,a^3}+\frac{9\left(\sqrt{a}\,x^2-\sqrt{ax^4+bx^2}\right)^2ab^2}{3\left(\left(\sqrt{a}\,x^2-\sqrt{}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^2)^(5/2),x, algorithm="giac")
```

```
[Out] 5/4*b*log(abs(-2*(sqrt(a)*x^2 - sqrt(a*x^4 + b*x^2))*sqrt(a) - b))/a^(7/2)
- 1/12*(15*b*log(abs(b)) + 28*b)/a^(7/2) + 1/2*sqrt(a*x^4 + b*x^2)/a^3 + 1/
3*(9*(sqrt(a)*x^2 - sqrt(a*x^4 + b*x^2))^2*a*b^2 + 15*(sqrt(a)*x^2 - sqrt(a
*x^4 + b*x^2))*sqrt(a)*b^3 + 7*b^4)/(((sqrt(a)*x^2 - sqrt(a*x^4 + b*x^2))*s
qrt(a) + b)^3*a^(7/2))
```

maple [A] time = 0.01, size = 85, normalized size = 0.97

$$\frac{\left(a\,x^2+b\right)\left(3a^{\frac{7}{2}}x^5+20a^{\frac{5}{2}}b\,x^3+15a^{\frac{3}{2}}b^2x-15\left(a\,x^2+b\right)^{\frac{3}{2}}ab\ln\left(\sqrt{a}\,x+\sqrt{a\,x^2+b}\right)\right)}{6\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{5}{2}}a^{\frac{9}{2}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b/x^2)^(5/2),x)
```

```
[Out] 1/6*(a*x^2+b)*(3*a^(7/2)*x^5+20*a^(5/2)*b*x^3+15*a^(3/2)*b^2*x-15*ln(a^(1/2
)*x+(a*x^2+b)^(1/2))*(a*x^2+b)^(3/2)*a*b)/((a*x^2+b)/x^2)^(5/2)/x^5/a^(9/2)
```

maxima [A] time = 1.89, size = 101, normalized size = 1.15

$$\frac{15\left(a+\frac{b}{x^2}\right)^2b-10\left(a+\frac{b}{x^2}\right)ab-2\,a^2b}{6\left(\left(a+\frac{b}{x^2}\right)^{\frac{5}{2}}a^3-\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}a^4\right)}+\frac{5\,b\log\left(\frac{\sqrt{a+\frac{b}{x^2}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^2}}+\sqrt{a}}\right)}{4\,a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(15*(a + b/x^2)^2*b - 10*(a + b/x^2)*a*b - 2*a^2*b)/((a + b/x^2)^(5/2)*
a^3 - (a + b/x^2)^(3/2)*a^4) + 5/4*b*log((sqrt(a + b/x^2) - sqrt(a))/(sqrt(
a + b/x^2) + sqrt(a)))/a^(7/2)
```

mupad [B] time = 1.64, size = 73, normalized size = 0.83

$$\frac{10\,b}{3\,a^2\left(a+\frac{b}{x^2}\right)^{3/2}}+\frac{x^2}{2\,a\left(a+\frac{b}{x^2}\right)^{3/2}}-\frac{5\,b\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{2\,a^{7/2}}+\frac{5\,b^2}{2\,a^3\,x^2\left(a+\frac{b}{x^2}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b/x^2)^(5/2),x)
```

```
[Out] (10*b)/(3*a^2*(a + b/x^2)^(3/2)) + x^2/(2*a*(a + b/x^2)^(3/2)) - (5*b*atanh
((a + b/x^2)^(1/2)/a^(1/2)))/(2*a^(7/2)) + (5*b^2)/(2*a^3*x^2*(a + b/x^2)^(
3/2))
```

sympy [**B**] time = 5.72, size = 819, normalized size = 9.31

$$\frac{6a^{17}x^8\sqrt{1+\frac{b}{ax^2}}}{12a^{\frac{39}{2}}x^6+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^2+12a^{\frac{33}{2}}b^3}+\frac{46a^{16}bx^6\sqrt{1+\frac{b}{ax^2}}}{12a^{\frac{39}{2}}x^6+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^2+12a^{\frac{33}{2}}b^3}+\frac{15a^{16}bx^6}{12a^{\frac{39}{2}}x^6+36a^{\frac{37}{2}}bx^4+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x**2)**(5/2),x)
```

```
[Out] 6*a**17*x**8*sqrt(1 + b/(a*x**2))/(12*a**(39/2)*x**6 + 36*a**(37/2)*b*x**4
+ 36*a**(35/2)*b**2*x**2 + 12*a**(33/2)*b**3) + 46*a**16*b*x**6*sqrt(1 + b/
(a*x**2))/(12*a**(39/2)*x**6 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**2
 + 12*a**(33/2)*b**3) + 15*a**16*b*x**6*log(b/(a*x**2))/(12*a**(39/2)*x**6
+ 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**2 + 12*a**(33/2)*b**3) - 30*a*
*16*b*x**6*log(sqrt(1 + b/(a*x**2)) + 1)/(12*a**(39/2)*x**6 + 36*a**(37/2)*
b*x**4 + 36*a**(35/2)*b**2*x**2 + 12*a**(33/2)*b**3) + 70*a**15*b**2*x**4*s
qrt(1 + b/(a*x**2))/(12*a**(39/2)*x**6 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)
*b**2*x**2 + 12*a**(33/2)*b**3) + 45*a**15*b**2*x**4*log(b/(a*x**2))/(12*a*
*(39/2)*x**6 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**2 + 12*a**(33/2)*
b**3) - 90*a**15*b**2*x**4*log(sqrt(1 + b/(a*x**2)) + 1)/(12*a**(39/2)*x**6
 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**2 + 12*a**(33/2)*b**3) + 30*a
**14*b**3*x**2*sqrt(1 + b/(a*x**2))/(12*a**(39/2)*x**6 + 36*a**(37/2)*b*x**
4 + 36*a**(35/2)*b**2*x**2 + 12*a**(33/2)*b**3) + 45*a**14*b**3*x**2*log(b/
(a*x**2))/(12*a**(39/2)*x**6 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**2
 + 12*a**(33/2)*b**3) - 90*a**14*b**3*x**2*log(sqrt(1 + b/(a*x**2)) + 1)/(1
2*a**(39/2)*x**6 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**2 + 12*a**(33
/2)*b**3) + 15*a**13*b**4*log(b/(a*x**2))/(12*a**(39/2)*x**6 + 36*a**(37/2)
*b*x**4 + 36*a**(35/2)*b**2*x**2 + 12*a**(33/2)*b**3) - 30*a**13*b**4*log(s
qrt(1 + b/(a*x**2)) + 1)/(12*a**(39/2)*x**6 + 36*a**(37/2)*b*x**4 + 36*a**(
35/2)*b**2*x**2 + 12*a**(33/2)*b**3)
```

3.1946 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x}\,dx$

Optimal. Leaf size=59

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{1}{a^2\sqrt{a+\frac{b}{x^2}}} - \frac{1}{3a\left(a+\frac{b}{x^2}\right)^{3/2}}$$

```
[Out] -1/3/a/(a+b/x^2)^(3/2)+arctanh((a+b/x^2)^(1/2)/a^(1/2))/a^(5/2)-1/a^2/(a+b/
x^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 51, 63, 208}

$$-\frac{1}{a^2\sqrt{a+\frac{b}{x^2}}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{1}{3a\left(a+\frac{b}{x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)^(5/2)*x),x]
```

```
[Out] -1/(3*a*(a + b/x^2)^(3/2)) - 1/(a^2*Sqrt[a + b/x^2]) + ArcTanh[Sqrt[a + b/x
^2]/Sqrt[a]]/a^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x}\,dx = -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= -\frac{1}{3a\left(a+\frac{b}{x^2}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)}{2a}$$

$$= -\frac{1}{3a\left(a+\frac{b}{x^2}\right)^{3/2}} - \frac{1}{a^2\sqrt{a+\frac{b}{x^2}}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x^2}\right)}{2a^2}$$

$$= -\frac{1}{3a\left(a+\frac{b}{x^2}\right)^{3/2}} - \frac{1}{a^2\sqrt{a+\frac{b}{x^2}}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x^2}}\right)}{a^2 b}$$

$$= -\frac{1}{3a\left(a+\frac{b}{x^2}\right)^{3/2}} - \frac{1}{a^2\sqrt{a+\frac{b}{x^2}}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{a^{5/2}}$$

Mathematica [A] time = 0.18, size = 91, normalized size = 1.54

$$\frac{\frac{3\sqrt{b}\left(ax^2+b\right)\sqrt{\frac{ax^2}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{x} - \sqrt{a}\,\left(4ax^2+3b\right)}{3a^{5/2}\sqrt{a+\frac{b}{x^2}}\,\left(ax^2+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)^(5/2)*x),x]
```

```
[Out] (-(Sqrt[a]*(3*b + 4*a*x^2)) + (3*Sqrt[b]*(b + a*x^2)*Sqrt[1 + (a*x^2)/b]*Ar
cSinh[(Sqrt[a]*x)/Sqrt[b]])/x)/(3*a^(5/2)*Sqrt[a + b/x^2]*(b + a*x^2))
```

fricas [B] time = 1.13, size = 232, normalized size = 3.93

$$\left[\frac{3\left(a^2x^4+2abx^2+b^2\right)\sqrt{a}\,\log\left(-2ax^2-2\sqrt{a}\,x^2\sqrt{\frac{ax^2+b}{x^2}}-b\right)-2\left(4a^2x^4+3abx^2\right)\sqrt{\frac{ax^2+b}{x^2}}}{6\left(a^5x^4+2a^4bx^2+a^3b^2\right)}, -\frac{3\left(a^2x^4+2abx^2+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [1/6*(3*(a^2*x^4 + 2*a*b*x^2 + b^2)*sqrt(a)*log(-2*a*x^2 - 2*sqrt(a)*x^2*sq
rt((a*x^2 + b)/x^2) - b) - 2*(4*a^2*x^4 + 3*a*b*x^2)*sqrt((a*x^2 + b)/x^2))
/(a^5*x^4 + 2*a^4*b*x^2 + a^3*b^2), -1/3*(3*(a^2*x^4 + 2*a*b*x^2 + b^2)*sqr
t(-a)*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + b)/x^2)/(a*x^2 + b)) + (4*a^2*x^4 +
 3*a*b*x^2)*sqrt((a*x^2 + b)/x^2))/(a^5*x^4 + 2*a^4*b*x^2 + a^3*b^2)]
```

giac [B] time = 0.33, size = 154, normalized size = 2.61

$$\frac{3\log(|b|)+8}{6a^{\frac{5}{2}}}-\frac{\log\left(\left|-2\left(\sqrt{a}\,x^2-\sqrt{ax^4+bx^2}\right)\sqrt{a}-b\right|\right)}{2a^{\frac{5}{2}}}-\frac{6\left(\sqrt{a}\,x^2-\sqrt{ax^4+bx^2}\right)^2ab+9\left(\sqrt{a}\,x^2-\sqrt{ax^4+bx}\right.}{3\left(\left(\sqrt{a}\,x^2-\sqrt{ax^4+bx^2}\right)\sqrt{a}+b\right)^3a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(5/2)/x,x, algorithm="giac")
```

```
[Out] 1/6*(3*log(abs(b)) + 8)/a^(5/2) - 1/2*log(abs(-2*(sqrt(a)*x^2 - sqrt(a*x^4
+ b*x^2))*sqrt(a) - b))/a^(5/2) - 1/3*(6*(sqrt(a)*x^2 - sqrt(a*x^4 + b*x^2)
)^2*a*b + 9*(sqrt(a)*x^2 - sqrt(a*x^4 + b*x^2))*sqrt(a)*b^2 + 4*b^3)/(((sqr
t(a)*x^2 - sqrt(a*x^4 + b*x^2))*sqrt(a) + b)^3*a^(5/2))
```

maple [A] time = 0.01, size = 73, normalized size = 1.24

$$-\frac{\left(ax^2+b\right)\left(4a^{\frac{5}{2}}x^3+3a^{\frac{3}{2}}bx-3\left(ax^2+b\right)^{\frac{3}{2}}a\ln\left(\sqrt{a}\,x+\sqrt{ax^2+b}\right)\right)}{3\left(\frac{ax^2+b}{x^2}\right)^{\frac{5}{2}}a^{\frac{7}{2}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^(5/2)/x,x)
```

```
[Out] -1/3*(a*x^2+b)*(4*a^(5/2)*x^3+3*a^(3/2)*b*x-3*ln(a^(1/2)*x+(a*x^2+b)^(1/2))
*(a*x^2+b)^(3/2)*a)/((a*x^2+b)/x^2)^(5/2)/x^5/a^(7/2)
```

maxima [A] time = 1.99, size = 62, normalized size = 1.05

$$-\frac{\log\left(\frac{\sqrt{a+\frac{b}{x^2}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^2}}+\sqrt{a}}\right)}{2a^{\frac{5}{2}}}-\frac{4a+\frac{3b}{x^2}}{3\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] -1/2*log((sqrt(a + b/x^2) - sqrt(a))/(sqrt(a + b/x^2) + sqrt(a)))/a^(5/2) -
 1/3*(4*a + 3*b/x^2)/((a + b/x^2)^(3/2)*a^2)
```

mupad [B] time = 1.39, size = 47, normalized size = 0.80

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{\frac{a+\frac{b}{x^2}}{a^2}+\frac{1}{3a}}{\left(a+\frac{b}{x^2}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b/x^2)^(5/2)),x)
```

```
[Out] atanh((a + b/x^2)^(1/2)/a^(1/2))/a^(5/2) - ((a + b/x^2)/a^2 + 1/(3*a))/(a +
 b/x^2)^(3/2)
```

sympy [**B**] time = 3.50, size = 743, normalized size = 12.59

$$-\frac{8a^7x^6\sqrt{1+\frac{b}{ax^2}}}{6a^{\frac{19}{2}}x^6+18a^{\frac{17}{2}}bx^4+18a^{\frac{15}{2}}b^2x^2+6a^{\frac{13}{2}}b^3}-\frac{3a^7x^6\log\left(\frac{b}{ax^2}\right)}{6a^{\frac{19}{2}}x^6+18a^{\frac{17}{2}}bx^4+18a^{\frac{15}{2}}b^2x^2+6a^{\frac{13}{2}}b^3}+\frac{6a^7x^6\log\left(\sqrt{1+}\right.}{6a^{\frac{19}{2}}x^6+18a^{\frac{17}{2}}bx^4+18a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**(5/2)/x,x)
```

```
[Out] -8*a**7*x**6*sqrt(1 + b/(a*x**2))/(6*a**(19/2)*x**6 + 18*a**(17/2)*b*x**4 +
 18*a**(15/2)*b**2*x**2 + 6*a**(13/2)*b**3) - 3*a**7*x**6*log(b/(a*x**2))/(
6*a**(19/2)*x**6 + 18*a**(17/2)*b*x**4 + 18*a**(15/2)*b**2*x**2 + 6*a**(13/
2)*b**3) + 6*a**7*x**6*log(sqrt(1 + b/(a*x**2)) + 1)/(6*a**(19/2)*x**6 + 18
*a**(17/2)*b*x**4 + 18*a**(15/2)*b**2*x**2 + 6*a**(13/2)*b**3) - 14*a**6*b*
x**4*sqrt(1 + b/(a*x**2))/(6*a**(19/2)*x**6 + 18*a**(17/2)*b*x**4 + 18*a**(
15/2)*b**2*x**2 + 6*a**(13/2)*b**3) - 9*a**6*b*x**4*log(b/(a*x**2))/(6*a**(
19/2)*x**6 + 18*a**(17/2)*b*x**4 + 18*a**(15/2)*b**2*x**2 + 6*a**(13/2)*b**
3) + 18*a**6*b*x**4*log(sqrt(1 + b/(a*x**2)) + 1)/(6*a**(19/2)*x**6 + 18*a*
*(17/2)*b*x**4 + 18*a**(15/2)*b**2*x**2 + 6*a**(13/2)*b**3) - 6*a**5*b**2*x
**2*sqrt(1 + b/(a*x**2))/(6*a**(19/2)*x**6 + 18*a**(17/2)*b*x**4 + 18*a**(1
5/2)*b**2*x**2 + 6*a**(13/2)*b**3) - 9*a**5*b**2*x**2*log(b/(a*x**2))/(6*a*
*(19/2)*x**6 + 18*a**(17/2)*b*x**4 + 18*a**(15/2)*b**2*x**2 + 6*a**(13/2)*b
**3) + 18*a**5*b**2*x**2*log(sqrt(1 + b/(a*x**2)) + 1)/(6*a**(19/2)*x**6 +
18*a**(17/2)*b*x**4 + 18*a**(15/2)*b**2*x**2 + 6*a**(13/2)*b**3) - 3*a**4*b
**3*log(b/(a*x**2))/(6*a**(19/2)*x**6 + 18*a**(17/2)*b*x**4 + 18*a**(15/2)*
b**2*x**2 + 6*a**(13/2)*b**3) + 6*a**4*b**3*log(sqrt(1 + b/(a*x**2)) + 1)/(
6*a**(19/2)*x**6 + 18*a**(17/2)*b*x**4 + 18*a**(15/2)*b**2*x**2 + 6*a**(13/
2)*b**3)
```

3.1947 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^3}\,dx$

Optimal. Leaf size=18

$$\frac{1}{3b\left(a+\frac{b}{x^2}\right)^{3/2}}$$

```
[Out] 1/3/b/(a+b/x^2)^(3/2)
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$\frac{1}{3b\left(a+\frac{b}{x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)^(5/2)*x^3),x]
```

```
[Out] 1/(3*b*(a + b/x^2)^(3/2))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^3}\,dx = \frac{1}{3b\left(a+\frac{b}{x^2}\right)^{3/2}}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.56

$$\frac{ax^2+b}{3bx^2\left(a+\frac{b}{x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)^(5/2)*x^3),x]
```

```
[Out] (b + a*x^2)/(3*b*(a + b/x^2)^(5/2)*x^2)
```

fricas [B] time = 0.78, size = 41, normalized size = 2.28

$$\frac{x^4\sqrt{\frac{ax^2+b}{x^2}}}{3\left(a^2bx^4+2\,ab^2x^2+b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/3*x^4*sqrt((a*x^2 + b)/x^2)/(a^2*b*x^4 + 2*a*b^2*x^2 + b^3)
```

giac [B] time = 0.20, size = 110, normalized size = 6.11

$$-\frac{1}{3\,a^{\frac{3}{2}}b}+\frac{3\left(\sqrt{a}\,x^2-\sqrt{ax^4+bx^2}\right)^2a^{\frac{3}{2}}+3\left(\sqrt{a}\,x^2-\sqrt{ax^4+bx^2}\right)ab+\sqrt{a}\,b^2}{3\left(\left(\sqrt{a}\,x^2-\sqrt{ax^4+bx^2}\right)\sqrt{a}+b\right)^3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/3/(a^(3/2)*b) + 1/3*(3*(sqrt(a)*x^2 - sqrt(a*x^4 + b*x^2))^2*a^(3/2) + 3
*(sqrt(a)*x^2 - sqrt(a*x^4 + b*x^2))*a*b + sqrt(a)*b^2)/(((sqrt(a)*x^2 - sq
rt(a*x^4 + b*x^2))*sqrt(a) + b)^3*a^2)
```

maple [A] time = 0.01, size = 29, normalized size = 1.61

$$\frac{a\,x^2+b}{3\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{5}{2}}b\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^(5/2)/x^3,x)
```

```
[Out] 1/3*(a*x^2+b)/b/x^2/((a*x^2+b)/x^2)^(5/2)
```

maxima [A] time = 0.76, size = 14, normalized size = 0.78

$$\frac{1}{3\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/3/((a + b/x^2)^(3/2)*b)
```

mupad [B] time = 1.25, size = 14, normalized size = 0.78

$$\frac{1}{3\,b\left(a+\frac{b}{x^2}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b/x^2)^(5/2)),x)
```

```
[Out] 1/(3*b*(a + b/x^2)^(3/2))
```

sympy [A] time = 3.78, size = 48, normalized size = 2.67

$$\begin{cases}\dfrac{1}{3ab\sqrt{a+\frac{b}{x^2}}+\frac{3b^2\sqrt{a+\frac{b}{x^2}}}{x^2}} & \text{for } b\neq 0\\ -\dfrac{1}{2a^{\frac{5}{2}}x^2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**(5/2)/x**3,x)

[Out] Piecewise((1/(3*a*b*sqrt(a + b/x**2) + 3*b**2*sqrt(a + b/x**2)/x**2), Ne(b,
 0)), (-1/(2*a**(5/2)*x**2), True))
```

3.1948 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^5}\,dx$

Optimal. Leaf size=35

$$\frac{1}{b^2\sqrt{a+\frac{b}{x^2}}}-\frac{a}{3b^2\left(a+\frac{b}{x^2}\right)^{3/2}}$$

```
[Out] -1/3*a/b^2/(a+b/x^2)^(3/2)+1/b^2/(a+b/x^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{1}{b^2\sqrt{a+\frac{b}{x^2}}}-\frac{a}{3b^2\left(a+\frac{b}{x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)^(5/2)*x^5),x]
```

```
[Out] -a/(3*b^2*(a + b/x^2)^(3/2)) + 1/(b^2*Sqrt[a + b/x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^5}\,dx &= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{x}{(a+bx)^{5/2}}\,dx, x, \frac{1}{x^2}\right)\right)\\ &= -\left(\frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^{5/2}}+\frac{1}{b(a+bx)^{3/2}}\right)\,dx, x, \frac{1}{x^2}\right)\right)\\ &= -\frac{a}{3b^2\left(a+\frac{b}{x^2}\right)^{3/2}}+\frac{1}{b^2\sqrt{a+\frac{b}{x^2}}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 1.06

$$\frac{2ax^2+3b}{3b^2\sqrt{a+\frac{b}{x^2}}\left(ax^2+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)^(5/2)*x^5),x]
```

```
[Out] (3*b + 2*a*x^2)/(3*b^2*Sqrt[a + b/x^2]*(b + a*x^2))
```

fricas [A] time = 0.79, size = 53, normalized size = 1.51

$$\frac{\left(2\,ax^4+3\,bx^2\right)\sqrt{\frac{ax^2+b}{x^2}}}{3\left(a^2b^2x^4+2\,ab^3x^2+b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/3*(2*a*x^4 + 3*b*x^2)*sqrt((a*x^2 + b)/x^2)/(a^2*b^2*x^4 + 2*a*b^3*x^2 +
b^4)
```

giac [B] time = 0.24, size = 78, normalized size = 2.23

$$-\frac{2}{3\sqrt{a}\,b^2}+\frac{3\left(\sqrt{a}\,x^2-\sqrt{ax^4+bx^2}\right)a+2\sqrt{a}\,b}{3\left(\left(\sqrt{a}\,x^2-\sqrt{ax^4+bx^2}\right)\sqrt{a}+b\right)^3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] -2/3/(sqrt(a)*b^2) + 1/3*(3*(sqrt(a)*x^2 - sqrt(a*x^4 + b*x^2))*a + 2*sqrt(
a)*b)/(((sqrt(a)*x^2 - sqrt(a*x^4 + b*x^2))*sqrt(a) + b)^3*a)
```

maple [A] time = 0.00, size = 39, normalized size = 1.11

$$\frac{\left(a\,x^2+b\right)\left(2a\,x^2+3b\right)}{3\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{5}{2}}b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^(5/2)/x^5,x)
```

```
[Out] 1/3*(a*x^2+b)/x^4*(2*a*x^2+3*b)/b^2/((a*x^2+b)/x^2)^(5/2)
```

maxima [A] time = 0.92, size = 29, normalized size = 0.83

$$\frac{1}{\sqrt{a+\frac{b}{x^2}}\,b^2}-\frac{a}{3\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/(sqrt(a + b/x^2)*b^2) - 1/3*a/((a + b/x^2)^(3/2)*b^2)
```

mupad [B] time = 1.26, size = 27, normalized size = 0.77

$$\frac{2\,a\,x^2+3\,b}{3\,b^2\,x^2\left(a+\frac{b}{x^2}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b/x^2)^(5/2)),x)
```

```
[Out] (3*b + 2*a*x^2)/(3*b^2*x^2*(a + b/x^2)^(3/2))
```

sympy [**A**] time = 5.70, size = 94, normalized size = 2.69

$$\begin{cases} \frac{2ax^2}{3ab^2x^2\sqrt{a+\frac{b}{x^2}}+3b^3\sqrt{a+\frac{b}{x^2}}}+\frac{3b}{3ab^2x^2\sqrt{a+\frac{b}{x^2}}+3b^3\sqrt{a+\frac{b}{x^2}}} & \text{for } b\neq 0 \\ -\frac{1}{4a^{\frac{5}{2}}x^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**(5/2)/x**5,x)
```

```
[Out] Piecewise((2*a*x**2/(3*a*b**2*x**2*sqrt(a + b/x**2) + 3*b**3*sqrt(a + b/x**
2)) + 3*b/(3*a*b**2*x**2*sqrt(a + b/x**2) + 3*b**3*sqrt(a + b/x**2)), Ne(b,
 0)), (-1/(4*a**(5/2)*x**4), True))
```

3.1949 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^7}\,dx$

Optimal. Leaf size=55

$$\frac{a^2}{3b^3\left(a+\frac{b}{x^2}\right)^{3/2}}-\frac{2a}{b^3\sqrt{a+\frac{b}{x^2}}}-\frac{\sqrt{a+\frac{b}{x^2}}}{b^3}$$

```
[Out] 1/3*a^2/b^3/(a+b/x^2)^(3/2)-2*a/b^3/(a+b/x^2)^(1/2)-(a+b/x^2)^(1/2)/b^3
```

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{a^2}{3b^3\left(a+\frac{b}{x^2}\right)^{3/2}}-\frac{2a}{b^3\sqrt{a+\frac{b}{x^2}}}-\frac{\sqrt{a+\frac{b}{x^2}}}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)^(5/2)*x^7),x]
```

```
[Out] a^2/(3*b^3*(a + b/x^2)^(3/2)) - (2*a)/(b^3*Sqrt[a + b/x^2]) - Sqrt[a + b/x^
2]/b^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^7}\,dx &= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{(a+bx)^{5/2}}\,dx, x, \frac{1}{x^2}\right)\right)\\ &= -\left(\frac{1}{2}\,\text{Subst}\left(\int \left(\frac{a^2}{b^2(a+bx)^{5/2}}-\frac{2a}{b^2(a+bx)^{3/2}}+\frac{1}{b^2\sqrt{a+bx}}\right)dx, x, \frac{1}{x^2}\right)\right)\\ &= \frac{a^2}{3b^3\left(a+\frac{b}{x^2}\right)^{3/2}}-\frac{2a}{b^3\sqrt{a+\frac{b}{x^2}}}-\frac{\sqrt{a+\frac{b}{x^2}}}{b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 48, normalized size = 0.87

$$-\frac{\sqrt{a+\frac{b}{x^2}}\left(8a^2x^4+12abx^2+3b^2\right)}{3b^3\left(ax^2+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)^(5/2)*x^7),x]
```

```
[Out] -1/3*(Sqrt[a + b/x^2]*(3*b^2 + 12*a*b*x^2 + 8*a^2*x^4))/(b^3*(b + a*x^2)^2)
```

fricas [A] time = 1.32, size = 61, normalized size = 1.11

$$-\frac{\left(8\,a^2x^4+12\,abx^2+3\,b^2\right)\sqrt{\frac{ax^2+b}{x^2}}}{3\left(a^2b^3x^4+2\,ab^4x^2+b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(5/2)/x^7,x, algorithm="fricas")
```

```
[Out] -1/3*(8*a^2*x^4 + 12*a*b*x^2 + 3*b^2)*sqrt((a*x^2 + b)/x^2)/(a^2*b^3*x^4 +
2*a*b^4*x^2 + b^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{\frac{5}{2}}x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(5/2)/x^7,x, algorithm="giac")
```

```
[Out] integrate(1/((a + b/x^2)^(5/2)*x^7), x)
```

maple [A] time = 0.01, size = 50, normalized size = 0.91

$$-\frac{\left(a\,x^2+b\right)\left(8a^2x^4+12ab\,x^2+3b^2\right)}{3\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{5}{2}}b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^(5/2)/x^7,x)
```

```
[Out] -1/3*(a*x^2+b)*(8*a^2*x^4+12*a*b*x^2+3*b^2)/x^6/b^3/((a*x^2+b)/x^2)^(5/2)
```

maxima [A] time = 0.92, size = 47, normalized size = 0.85

$$-\frac{\sqrt{a+\frac{b}{x^2}}}{b^3}-\frac{2\,a}{\sqrt{a+\frac{b}{x^2}}\,b^3}+\frac{a^2}{3\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(5/2)/x^7,x, algorithm="maxima")
```

```
[Out] -sqrt(a + b/x^2)/b^3 - 2*a/(sqrt(a + b/x^2)*b^3) + 1/3*a^2/((a + b/x^2)^(3/
2)*b^3)
```

mupad [B] time = 1.29, size = 42, normalized size = 0.76

$$-\frac{\sqrt{a+\frac{b}{x^2}}\,\left(\frac{8\,a^2\,x^4}{3}+4\,a\,b\,x^2+b^2\right)}{b^3\left(a\,x^2+b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b/x^2)^(5/2)),x)
```

```
[Out] -((a + b/x^2)^(1/2)*(b^2 + (8*a^2*x^4)/3 + 4*a*b*x^2))/(b^3*(b + a*x^2)^2)
```

sympy [A] time = 8.36, size = 153, normalized size = 2.78

$$\begin{cases} -\dfrac{8a^2x^4}{3ab^3x^4\sqrt{a+\frac{b}{x^2}}+3b^4x^2\sqrt{a+\frac{b}{x^2}}} - \dfrac{12abx^2}{3ab^3x^4\sqrt{a+\frac{b}{x^2}}+3b^4x^2\sqrt{a+\frac{b}{x^2}}} - \dfrac{3b^2}{3ab^3x^4\sqrt{a+\frac{b}{x^2}}+3b^4x^2\sqrt{a+\frac{b}{x^2}}} & \text{for } b \neq 0 \\ -\dfrac{1}{6a^{\frac{5}{2}}x^6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**(5/2)/x**7,x)
```

```
[Out] Piecewise((-8*a**2*x**4/(3*a*b**3*x**4*sqrt(a + b/x**2) + 3*b**4*x**2*sqrt(
a + b/x**2)) - 12*a*b*x**2/(3*a*b**3*x**4*sqrt(a + b/x**2) + 3*b**4*x**2*sq
rt(a + b/x**2)) - 3*b**2/(3*a*b**3*x**4*sqrt(a + b/x**2) + 3*b**4*x**2*sqrt
(a + b/x**2)), Ne(b, 0)), (-1/(6*a**(5/2)*x**6), True))
```

3.1950 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^9}\,dx$

Optimal. Leaf size=76

$$-\frac{a^3}{3b^4\left(a+\frac{b}{x^2}\right)^{3/2}}+\frac{3a^2}{b^4\sqrt{a+\frac{b}{x^2}}}+\frac{3a\sqrt{a+\frac{b}{x^2}}}{b^4}-\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{3b^4}$$

```
[Out] -1/3*a^3/b^4/(a+b/x^2)^(3/2)-1/3*(a+b/x^2)^(3/2)/b^4+3*a^2/b^4/(a+b/x^2)^(1
/2)+3*a*(a+b/x^2)^(1/2)/b^4
```

Rubi [A] time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{a^3}{3b^4\left(a+\frac{b}{x^2}\right)^{3/2}}+\frac{3a^2}{b^4\sqrt{a+\frac{b}{x^2}}}+\frac{3a\sqrt{a+\frac{b}{x^2}}}{b^4}-\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{3b^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)^(5/2)*x^9),x]
```

```
[Out] -a^3/(3*b^4*(a + b/x^2)^(3/2)) + (3*a^2)/(b^4*Sqrt[a + b/x^2]) + (3*a*Sqrt[
a + b/x^2])/b^4 - (a + b/x^2)^(3/2)/(3*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^9}\,dx &= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{(a+bx)^{5/2}}\,dx, x, \frac{1}{x^2}\right)\right)\\ &= -\left(\frac{1}{2}\,\text{Subst}\left(\int\left(-\frac{a^3}{b^3(a+bx)^{5/2}}+\frac{3a^2}{b^3(a+bx)^{3/2}}-\frac{3a}{b^3\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b^3}\right)dx, x, \frac{1}{x^2}\right)\right)\\ &= -\frac{a^3}{3b^4\left(a+\frac{b}{x^2}\right)^{3/2}}+\frac{3a^2}{b^4\sqrt{a+\frac{b}{x^2}}}+\frac{3a\sqrt{a+\frac{b}{x^2}}}{b^4}-\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{3b^4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 62, normalized size = 0.82

$$\frac{16a^3x^6 + 24a^2bx^4 + 6ab^2x^2 - b^3}{3b^4x^4\sqrt{a + \frac{b}{x^2}}\left(ax^2 + b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)^(5/2)*x^9),x]
```

```
[Out] (-b^3 + 6*a*b^2*x^2 + 24*a^2*b*x^4 + 16*a^3*x^6)/(3*b^4*Sqrt[a + b/x^2]*x^4
*(b + a*x^2))
```

fricas [A] time = 0.63, size = 76, normalized size = 1.00

$$\frac{\left(16\,a^3x^6 + 24\,a^2bx^4 + 6\,ab^2x^2 - b^3\right)\sqrt{\frac{ax^2+b}{x^2}}}{3\left(a^2b^4x^6 + 2\,ab^5x^4 + b^6x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(5/2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/3*(16*a^3*x^6 + 24*a^2*b*x^4 + 6*a*b^2*x^2 - b^3)*sqrt((a*x^2 + b)/x^2)/(
a^2*b^4*x^6 + 2*a*b^5*x^4 + b^6*x^2)
```

giac [A] time = 0.22, size = 58, normalized size = 0.76

$$\frac{2\left(4\,x^2\left(\frac{2\,a^3x^2}{b^4} + \frac{3\,a^2}{b^3}\right) + \frac{3\,a}{b^2}\right)x^2 - \frac{1}{b}}{3\left(ax^4 + bx^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(5/2)/x^9,x, algorithm="giac")
```

```
[Out] 1/3*(2*(4*x^2*(2*a^3*x^2/b^4 + 3*a^2/b^3) + 3*a/b^2)*x^2 - 1/b)/(a*x^4 + b*
x^2)^(3/2)
```

maple [A] time = 0.01, size = 61, normalized size = 0.80

$$\frac{\left(a\,x^2 + b\right)\left(16a^3x^6 + 24a^2b\,x^4 + 6a\,b^2x^2 - b^3\right)}{3\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{5}{2}}b^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^(5/2)/x^9,x)
```

```
[Out] 1/3*(a*x^2+b)*(16*a^3*x^6+24*a^2*b*x^4+6*a*b^2*x^2-b^3)/x^8/b^4/((a*x^2+b)/
x^2)^(5/2)
```

maxima [A] time = 0.89, size = 64, normalized size = 0.84

$$-\frac{\left(a + \frac{b}{x^2}\right)^{\frac{3}{2}}}{3\,b^4} + \frac{3\sqrt{a + \frac{b}{x^2}}\,a}{b^4} + \frac{3\,a^2}{\sqrt{a + \frac{b}{x^2}}\,b^4} - \frac{a^3}{3\left(a + \frac{b}{x^2}\right)^{\frac{3}{2}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(5/2)/x^9,x, algorithm="maxima")
```

```
[Out] -1/3*(a + b/x^2)^(3/2)/b^4 + 3*sqrt(a + b/x^2)*a/b^4 + 3*a^2/(sqrt(a + b/x^
2)*b^4) - 1/3*a^3/((a + b/x^2)^(3/2)*b^4)
```

mupad **[B]** time = 1.36, size = 58, normalized size = 0.76

$$\frac{\sqrt{a+\frac{b}{x^2}}\,\left(16\,a^3\,x^6+24\,a^2\,b\,x^4+6\,a\,b^2\,x^2-b^3\right)}{3\,b^4\,x^2\,\left(a\,x^2+b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(a + b/x^2)^(5/2)),x)
```

```
[Out] ((a + b/x^2)^(1/2)*(16*a^3*x^6 - b^3 + 6*a*b^2*x^2 + 24*a^2*b*x^4))/(3*b^4*
x^2*(b + a*x^2)^2)
```

sympy **[A]** time = 12.15, size = 201, normalized size = 2.64

$$\begin{cases} \frac{16a^3x^6}{3ab^4x^6\sqrt{a+\frac{b}{x^2}}+3b^5x^4\sqrt{a+\frac{b}{x^2}}}+\frac{24a^2bx^4}{3ab^4x^6\sqrt{a+\frac{b}{x^2}}+3b^5x^4\sqrt{a+\frac{b}{x^2}}}+\frac{6ab^2x^2}{3ab^4x^6\sqrt{a+\frac{b}{x^2}}+3b^5x^4\sqrt{a+\frac{b}{x^2}}}-\frac{b^3}{3ab^4x^6\sqrt{a+\frac{b}{x^2}}+3b^5x^4\sqrt{a+\frac{b}{x^2}}} & \text{for } b \\ -\frac{1}{8a^{\frac{5}{2}}x^8} & \text{othe} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**(5/2)/x**9,x)
```

```
[Out] Piecewise((16*a**3*x**6/(3*a*b**4*x**6*sqrt(a + b/x**2) + 3*b**5*x**4*sqrt(
a + b/x**2)) + 24*a**2*b*x**4/(3*a*b**4*x**6*sqrt(a + b/x**2) + 3*b**5*x**4
*sqrt(a + b/x**2)) + 6*a*b**2*x**2/(3*a*b**4*x**6*sqrt(a + b/x**2) + 3*b**5
*x**4*sqrt(a + b/x**2)) - b**3/(3*a*b**4*x**6*sqrt(a + b/x**2) + 3*b**5*x**
4*sqrt(a + b/x**2)), Ne(b, 0)), (-1/(8*a**(5/2)*x**8), True))
```

3.1951 $\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx$

Optimal. Leaf size=82

$$-\frac{16bx\sqrt{a+\frac{b}{x^2}}}{3a^4}+\frac{8bx}{3a^3\sqrt{a+\frac{b}{x^2}}}+\frac{2bx}{3a^2\left(a+\frac{b}{x^2}\right)^{3/2}}+\frac{x^3}{3a\left(a+\frac{b}{x^2}\right)^{3/2}}$$

```
[Out] 2/3*b*x/a^2/(a+b/x^2)^(3/2)+1/3*x^3/a/(a+b/x^2)^(3/2)+8/3*b*x/a^3/(a+b/x^2)
^(1/2)-16/3*b*x*(a+b/x^2)^(1/2)/a^4
```

Rubi [A] time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {271, 192, 191}

$$-\frac{16bx\sqrt{a+\frac{b}{x^2}}}{3a^4}+\frac{8bx}{3a^3\sqrt{a+\frac{b}{x^2}}}+\frac{2bx}{3a^2\left(a+\frac{b}{x^2}\right)^{3/2}}+\frac{x^3}{3a\left(a+\frac{b}{x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b/x^2)^(5/2),x]
```

```
[Out] (2*b*x)/(3*a^2*(a + b/x^2)^(3/2)) + (8*b*x)/(3*a^3*Sqrt[a + b/x^2]) - (16*b
*Sqrt[a + b/x^2]*x)/(3*a^4) + x^3/(3*a*(a + b/x^2)^(3/2))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx = \frac{x^3}{3a\left(a+\frac{b}{x^2}\right)^{3/2}} - \frac{(2b)\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx}{a}$$

$$= \frac{2bx}{3a^2\left(a+\frac{b}{x^2}\right)^{3/2}} + \frac{x^3}{3a\left(a+\frac{b}{x^2}\right)^{3/2}} - \frac{(8b)\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx}{3a^2}$$

$$= \frac{2bx}{3a^2\left(a+\frac{b}{x^2}\right)^{3/2}} + \frac{8bx}{3a^3\sqrt{a+\frac{b}{x^2}}} + \frac{x^3}{3a\left(a+\frac{b}{x^2}\right)^{3/2}} - \frac{(16b)\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}\,dx}{3a^3}$$

$$= \frac{2bx}{3a^2\left(a+\frac{b}{x^2}\right)^{3/2}} + \frac{8bx}{3a^3\sqrt{a+\frac{b}{x^2}}} - \frac{16b\sqrt{a+\frac{b}{x^2}}\,x}{3a^4} + \frac{x^3}{3a\left(a+\frac{b}{x^2}\right)^{3/2}}$$

Mathematica [A] time = 0.03, size = 61, normalized size = 0.74

$$\frac{a^3x^6 - 6a^2bx^4 - 24ab^2x^2 - 16b^3}{3a^4x\sqrt{a+\frac{b}{x^2}}\left(ax^2+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b/x^2)^(5/2),x]
```

```
[Out] (-16*b^3 - 24*a*b^2*x^2 - 6*a^2*b*x^4 + a^3*x^6)/(3*a^4*Sqrt[a + b/x^2]*x*(
b + a*x^2))
```

fricas [A] time = 1.04, size = 73, normalized size = 0.89

$$\frac{\left(a^3x^7 - 6\,a^2bx^5 - 24\,ab^2x^3 - 16\,b^3x\right)\sqrt{\frac{ax^2+b}{x^2}}}{3\left(a^6x^4 + 2\,a^5bx^2 + a^4b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(a^3*x^7 - 6*a^2*b*x^5 - 24*a*b^2*x^3 - 16*b^3*x)*sqrt((a*x^2 + b)/x^2)
/(a^6*x^4 + 2*a^5*b*x^2 + a^4*b^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,
0,4]%%%}] at parameters values [-37,79.8475059881,-6]Warning, choosing root
 of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at para
meters values [95,91.1380034634,2]Warning, choosing root of [1,0,%%%{-2,[1,
0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [70,7
```

```
7.1190297226,8]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,
1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [-26,73.1070263633,63]Wa
rning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,
[2,0,4]%%%}] at parameters values [-51,40.7394986684,-52]Warning, choosing
root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at
parameters values [42,71.9334104825,-88]Warning, choosing root of [1,0,%%%{
-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values
 [-98,19.1478250967,-53]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%
%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [-65,12.0642261
468,-46]1/a*((1/3*a^18*(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign(x))^3+a^16*
b*(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign(x))-3*a^16*b*(-sqrt(b)/a*sign(x)
+sqrt(a*x^2+b)/a/sign(x))+a^17*sqrt(b)*(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/a/
sign(x))^2*sign(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign(x)))/a^18+1/3*(-9*a
^2*b^2*(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign(x))^2-9*b^3+b^3-18*a*sqrt(b
)*b^2*(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign(x))*sign(-sqrt(b)/a*sign(x)+
sqrt(a*x^2+b)/a/sign(x)))/a^3/(a*(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign(x
))+sqrt(b)*sign(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign(x)))^3+8*b*sqrt(b)/
3/a^3*sign(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign(x)))
```

maple [A] time = 0.01, size = 60, normalized size = 0.73

$$\frac{\left(a\,x^2+b\right)\left(a^3x^6-6a^2b\,x^4-24a\,b^2x^2-16b^3\right)}{3\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{5}{2}}a^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b/x^2)^(5/2),x)
```

```
[Out] 1/3*(a*x^2+b)*(a^3*x^6-6*a^2*b*x^4-24*a*b^2*x^2-16*b^3)/a^4/x^5/((a*x^2+b)/
x^2)^(5/2)
```

maxima [A] time = 0.91, size = 71, normalized size = 0.87

$$\frac{\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}x^3-9\sqrt{a+\frac{b}{x^2}}\,bx}{3\,a^4}-\frac{9\left(a+\frac{b}{x^2}\right)b^2x^2-b^3}{3\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*((a + b/x^2)^(3/2)*x^3 - 9*sqrt(a + b/x^2)*b*x)/a^4 - 1/3*(9*(a + b/x^2
)*b^2*x^2 - b^3)/((a + b/x^2)^(3/2)*a^4*x^3)
```

mupad [B] time = 1.73, size = 81, normalized size = 0.99

$$\frac{6\,a^2\left(a+\frac{b}{x^2}\right)-24\,a\left(a+\frac{b}{x^2}\right)^2+16\left(a+\frac{b}{x^2}\right)^3+a^3}{\left(\frac{3\,a^5}{b\,x}-\frac{3\,a^4\left(a+\frac{b}{x^2}\right)}{b\,x}\right)\left(a+\frac{b}{x^2}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b/x^2)^(5/2),x)
```

```
[Out] (6*a^2*(a + b/x^2) - 24*a*(a + b/x^2)^2 + 16*(a + b/x^2)^3 + a^3)/(((3*a^5)
/(b*x) - (3*a^4*(a + b/x^2))/(b*x))*(a + b/x^2)^(3/2))
```

sympy [**B**] time = 1.88, size = 337, normalized size = 4.11

$$\frac{a^4 b^{\frac{19}{2}} x^8 \sqrt{\frac{ax^2}{b}+1}}{3a^7b^9x^6+9a^6b^{10}x^4+9a^5b^{11}x^2+3a^4b^{12}} - \frac{5a^3 b^{\frac{21}{2}} x^6 \sqrt{\frac{ax^2}{b}+1}}{3a^7b^9x^6+9a^6b^{10}x^4+9a^5b^{11}x^2+3a^4b^{12}} - \frac{30a^2 b^{\frac{23}{2}} x^4 \sqrt{\frac{ax^2}{b}+}}{3a^7b^9x^6+9a^6b^{10}x^4+9a^5b^{1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b/x**2)**(5/2),x)
```

```
[Out] a**4*b**(19/2)*x**8*sqrt(a*x**2/b + 1)/(3*a**7*b**9*x**6 + 9*a**6*b**10*x**
4 + 9*a**5*b**11*x**2 + 3*a**4*b**12) - 5*a**3*b**(21/2)*x**6*sqrt(a*x**2/b
 + 1)/(3*a**7*b**9*x**6 + 9*a**6*b**10*x**4 + 9*a**5*b**11*x**2 + 3*a**4*b*
*12) - 30*a**2*b**(23/2)*x**4*sqrt(a*x**2/b + 1)/(3*a**7*b**9*x**6 + 9*a**6
*b**10*x**4 + 9*a**5*b**11*x**2 + 3*a**4*b**12) - 40*a*b**(25/2)*x**2*sqrt(
a*x**2/b + 1)/(3*a**7*b**9*x**6 + 9*a**6*b**10*x**4 + 9*a**5*b**11*x**2 + 3
*a**4*b**12) - 16*b**(27/2)*sqrt(a*x**2/b + 1)/(3*a**7*b**9*x**6 + 9*a**6*b
**10*x**4 + 9*a**5*b**11*x**2 + 3*a**4*b**12)
```

3.1952 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx$

Optimal. Leaf size=58

$$\frac{8x\sqrt{a+\frac{b}{x^2}}}{3a^3}-\frac{4x}{3a^2\sqrt{a+\frac{b}{x^2}}}-\frac{x}{3a\left(a+\frac{b}{x^2}\right)^{3/2}}$$

```
[Out] -1/3*x/a/(a+b/x^2)^(3/2)-4/3*x/a^2/(a+b/x^2)^(1/2)+8/3*x*(a+b/x^2)^(1/2)/a^
3
```

Rubi [A] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {192, 191}

$$\frac{8x\sqrt{a+\frac{b}{x^2}}}{3a^3}-\frac{4x}{3a^2\sqrt{a+\frac{b}{x^2}}}-\frac{x}{3a\left(a+\frac{b}{x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^(-5/2),x]
```

```
[Out] -x/(3*a*(a + b/x^2)^(3/2)) - (4*x)/(3*a^2*Sqrt[a + b/x^2]) + (8*Sqrt[a + b/
x^2]*x)/(3*a^3)
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx &= -\frac{x}{3a\left(a+\frac{b}{x^2}\right)^{3/2}}+\frac{4\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx}{3a}\\ &= -\frac{x}{3a\left(a+\frac{b}{x^2}\right)^{3/2}}-\frac{4x}{3a^2\sqrt{a+\frac{b}{x^2}}}+\frac{8\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}\,dx}{3a^2}\\ &= -\frac{x}{3a\left(a+\frac{b}{x^2}\right)^{3/2}}-\frac{4x}{3a^2\sqrt{a+\frac{b}{x^2}}}+\frac{8\sqrt{a+\frac{b}{x^2}}\,x}{3a^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.88

$$\frac{3a^2x^4+12abx^2+8b^2}{3a^3x\sqrt{a+\frac{b}{x^2}}\left(ax^2+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^(-5/2),x]
```

```
[Out] (8*b^2 + 12*a*b*x^2 + 3*a^2*x^4)/(3*a^3*Sqrt[a + b/x^2]*x*(b + a*x^2))
```

fricas [A] time = 0.64, size = 63, normalized size = 1.09

$$\frac{\left(3\,a^2x^5+12\,abx^3+8\,b^2x\right)\sqrt{\frac{ax^2+b}{x^2}}}{3\left(a^5x^4+2\,a^4bx^2+a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^2*x^5 + 12*a*b*x^3 + 8*b^2*x)*sqrt((a*x^2 + b)/x^2)/(a^5*x^4 + 2*a
^4*b*x^2 + a^3*b^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [sign(t_nostep),sign(t_nostep+sq
rt(b)/a*sign(t_nostep))]sym2poly/r2sym(const gen & e,const index_m & i,cons
t vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 50, normalized size = 0.86

$$\frac{\left(a\,x^2+b\right)\left(3a^2x^4+12ab\,x^2+8b^2\right)}{3\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{5}{2}}a^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^(5/2),x)
```

```
[Out] 1/3*(a*x^2+b)*(3*a^2*x^4+12*a*b*x^2+8*b^2)/a^3/x^5/((a*x^2+b)/x^2)^(5/2)
```

maxima [A] time = 0.81, size = 51, normalized size = 0.88

$$\frac{\sqrt{a+\frac{b}{x^2}}\,x}{a^3}+\frac{6\left(a+\frac{b}{x^2}\right)bx^2-b^2}{3\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] sqrt(a + b/x^2)*x/a^3 + 1/3*(6*(a + b/x^2)*b*x^2 - b^2)/((a + b/x^2)^(3/2)*
a^3*x^3)
```

mupad **[B]** time = 1.33, size = 43, normalized size = 0.74

$$\frac{x\left(\frac{a\,x^2}{b}+1\right)^{5/2}\sqrt{x^{10}}\;{}_2\mathrm{F}_1\left(\frac{5}{2},3;\;4;\;-\frac{a\,x^2}{b}\right)}{6\left(a\,x^2+b\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x^2)^(5/2),x)
```

```
[Out] (x*((a*x^2)/b + 1)^(5/2)*(x^10)^(1/2)*hypergeom([5/2, 3], 4, -(a*x^2)/b))/(
6*(b + a*x^2)^(5/2))
```

sympy **[B]** time = 1.74, size = 163, normalized size = 2.81

$$\frac{3a^2b^{\frac{9}{2}}x^4\sqrt{\frac{ax^2}{b}+1}}{3a^5b^4x^4+6a^4b^5x^2+3a^3b^6}+\frac{12ab^{\frac{11}{2}}x^2\sqrt{\frac{ax^2}{b}+1}}{3a^5b^4x^4+6a^4b^5x^2+3a^3b^6}+\frac{8b^{\frac{13}{2}}\sqrt{\frac{ax^2}{b}+1}}{3a^5b^4x^4+6a^4b^5x^2+3a^3b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**(5/2),x)
```

```
[Out] 3*a**2*b**(9/2)*x**4*sqrt(a*x**2/b + 1)/(3*a**5*b**4*x**4 + 6*a**4*b**5*x**
2 + 3*a**3*b**6) + 12*a*b**(11/2)*x**2*sqrt(a*x**2/b + 1)/(3*a**5*b**4*x**4
 + 6*a**4*b**5*x**2 + 3*a**3*b**6) + 8*b**(13/2)*sqrt(a*x**2/b + 1)/(3*a**5
*b**4*x**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6)
```

3.1953 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^2}\,dx$

Optimal. Leaf size=42

$$-\frac{2b}{3a^2x^3\left(a+\frac{b}{x^2}\right)^{3/2}}-\frac{1}{ax\left(a+\frac{b}{x^2}\right)^{3/2}}$$

```
[Out] -2/3*b/a^2/(a+b/x^2)^(3/2)/x^3-1/a/(a+b/x^2)^(3/2)/x
```

Rubi [A] time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$-\frac{2b}{3a^2x^3\left(a+\frac{b}{x^2}\right)^{3/2}}-\frac{1}{ax\left(a+\frac{b}{x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)^(5/2)*x^2),x]
```

```
[Out] (-2*b)/(3*a^2*(a + b/x^2)^(3/2)*x^3) - 1/(a*(a + b/x^2)^(3/2)*x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}x^2}\,dx &= -\frac{1}{a\left(a+\frac{b}{x^2}\right)^{3/2}x}+\frac{(2b)\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}x^4}\,dx}{a}\\ &= -\frac{2b}{3a^2\left(a+\frac{b}{x^2}\right)^{3/2}x^3}-\frac{1}{a\left(a+\frac{b}{x^2}\right)^{3/2}x}\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 0.90

$$-\frac{x\sqrt{a+\frac{b}{x^2}}\left(3ax^2+2b\right)}{3a^2\left(ax^2+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)^(5/2)*x^2),x]
```

```
[Out] -1/3*(Sqrt[a + b/x^2]*x*(2*b + 3*a*x^2))/(a^2*(b + a*x^2)^2)
```

fricas [**A**] time = 0.95, size = 52, normalized size = 1.24

$$-\frac{\left(3\,ax^3+2\,bx\right)\sqrt{\frac{ax^2+b}{x^2}}}{3\left(a^4x^4+2\,a^3bx^2+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*a*x^3 + 2*b*x)*sqrt((a*x^2 + b)/x^2)/(a^4*x^4 + 2*a^3*b*x^2 + a^2*b
^2)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,
0,4]%%%}] at parameters values [-90,36.3617133629,39]Warning, choosing root
 of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at para
meters values [35,89.0139720016,56]Warning, choosing root of [1,0,%%%{-2,[1
,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [83,
3.24575260049,-90]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,
[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [-10,94.741137628,73]
Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{
1,[2,0,4]%%%}] at parameters values [-36,54.1710346672,-32]Warning, choosin
g root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] a
t parameters values [49,64.8237054832,-70]a*((b+b)/3/a^3/b/sqrt(b)*sign(-sq
rt(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign(x))+1/3*(-3*a^2*(-sqrt(b)/a*sign(x)+sq
rt(a*x^2+b)/a/sign(x))^2-3*b-6*a*sqrt(b)*(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/
a/sign(x))*sign(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign(x))+b)/a^3/(a*(-sqr
t(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign(x))+sqrt(b)*sign(-sqrt(b)/a*sign(x)+sqr
t(a*x^2+b)/a/sign(x)))^3)
```

maple [**A**] time = 0.01, size = 39, normalized size = 0.93

$$-\frac{\left(a\,x^2+b\right)\left(3a\,x^2+2b\right)}{3\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{5}{2}}a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^(5/2)/x^2,x)
```

```
[Out] -1/3*(a*x^2+b)*(3*a*x^2+2*b)/a^2/x^5/((a*x^2+b)/x^2)^(5/2)
```

maxima [**A**] time = 0.87, size = 33, normalized size = 0.79

$$-\frac{3\left(a+\frac{b}{x^2}\right)x^2-b}{3\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] -1/3*(3*(a + b/x^2)*x^2 - b)/((a + b/x^2)^(3/2)*a^2*x^3)
```

mupad [**B**] time = 1.27, size = 34, normalized size = 0.81

$$-\frac{x\sqrt{a+\frac{b}{x^2}}\,\left(3\,a\,x^2+2\,b\right)}{3\,a^2\left(a\,x^2+b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b/x^2)^(5/2)),x)
```

```
[Out] -(x*(a + b/x^2)^(1/2)*(2*b + 3*a*x^2))/(3*a^2*(b + a*x^2)^2)
```

sympy [**B**] time = 1.86, size = 105, normalized size = 2.50

$$-\frac{3ax^2}{3a^3\sqrt{b}\,x^2\sqrt{\frac{ax^2}{b}+1}+3a^2b^{\frac{3}{2}}\sqrt{\frac{ax^2}{b}+1}}-\frac{2b}{3a^3\sqrt{b}\,x^2\sqrt{\frac{ax^2}{b}+1}+3a^2b^{\frac{3}{2}}\sqrt{\frac{ax^2}{b}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**(5/2)/x**2,x)
```

```
[Out] -3*a*x**2/(3*a**3*sqrt(b)*x**2*sqrt(a*x**2/b + 1) + 3*a**2*b**(3/2)*sqrt(a*
x**2/b + 1)) - 2*b/(3*a**3*sqrt(b)*x**2*sqrt(a*x**2/b + 1) + 3*a**2*b**(3/2
)*sqrt(a*x**2/b + 1))
```

3.1954 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^4}\,dx$

Optimal. Leaf size=21

$$-\frac{1}{3ax^3\left(a+\frac{b}{x^2}\right)^{3/2}}$$

```
[Out] -1/3/a/(a+b/x^2)^(3/2)/x^3
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {264}

$$-\frac{1}{3ax^3\left(a+\frac{b}{x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)^(5/2)*x^4),x]
```

```
[Out] -1/(3*a*(a + b/x^2)^(3/2)*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^4}\,dx = -\frac{1}{3a\left(a+\frac{b}{x^2}\right)^{3/2} x^3}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.33

$$-\frac{ax^2+b}{3ax^5\left(a+\frac{b}{x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)^(5/2)*x^4),x]
```

```
[Out] -1/3*(b + a*x^2)/(a*(a + b/x^2)^(5/2)*x^5)
```

fricas [B] time = 0.69, size = 40, normalized size = 1.90

$$-\frac{x\sqrt{\frac{ax^2+b}{x^2}}}{3\left(a^3x^4+2\,a^2bx^2+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*x*sqrt((a*x^2 + b)/x^2)/(a^3*x^4 + 2*a^2*b*x^2 + a*b^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,
0,4]%%%}] at parameters values [32,76.6146142203,-62]Warning, choosing root
 of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at para
meters values [66,99.6590219955,-30]Warning, choosing root of [1,0,%%%{-2,[
1,0,2]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [-5
0,17.713730142,-44]Warning, choosing root of [1,0,%%%{-2,[1,0,2]%%%}+%%%{-4
,[0,1,0]%%%},0,%%%{1,[2,0,4]%%%}] at parameters values [45,14.5515509131,62
](b+b)/6/a/b^2/sqrt(b)*sign(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign(x))-1/3
/(a*(-sqrt(b)/a*sign(x)+sqrt(a*x^2+b)/a/sign(x))+sqrt(b)*sign(-sqrt(b)/a*si
gn(x)+sqrt(a*x^2+b)/a/sign(x)))^3/a
```

maple [A] time = 0.00, size = 29, normalized size = 1.38

$$-\frac{a\,x^2+b}{3\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{5}{2}}\,a\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^(5/2)/x^4,x)
```

```
[Out] -1/3*(a*x^2+b)/a/x^5/((a*x^2+b)/x^2)^(5/2)
```

maxima [A] time = 0.92, size = 17, normalized size = 0.81

$$-\frac{1}{3\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/3/((a + b/x^2)^(3/2)*a*x^3)
```

mupad [B] time = 1.20, size = 17, normalized size = 0.81

$$-\frac{1}{3\,a\,x^3\left(a+\frac{b}{x^2}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b/x^2)^(5/2)),x)
```

```
[Out] -1/(3*a*x^3*(a + b/x^2)^(3/2))
```

sympy [B] time = 1.96, size = 48, normalized size = 2.29

$$-\frac{1}{3a^2\sqrt{b}\,x^2\sqrt{\frac{ax^2}{b}+1}+3ab^{\frac{3}{2}}\sqrt{\frac{ax^2}{b}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**(5/2)/x**4,x)
```

```
[Out] -1/(3*a**2*sqrt(b)*x**2*sqrt(a*x**2/b + 1) + 3*a*b**(3/2)*sqrt(a*x**2/b + 1
))
```

3.1955 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^6}\,dx$

Optimal. Leaf size=68

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{b^{5/2}}+\frac{1}{b^2 x\sqrt{a+\frac{b}{x^2}}}+\frac{1}{3bx^3\left(a+\frac{b}{x^2}\right)^{3/2}}$$

```
[Out] 1/3/b/(a+b/x^2)^(3/2)/x^3-arctanh(b^(1/2)/x/(a+b/x^2)^(1/2))/b^(5/2)+1/b^2/
x/(a+b/x^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {335, 288, 217, 206}

$$\frac{1}{b^2 x\sqrt{a+\frac{b}{x^2}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{b^{5/2}}+\frac{1}{3bx^3\left(a+\frac{b}{x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)^(5/2)*x^6),x]
```

```
[Out] 1/(3*b*(a + b/x^2)^(3/2)*x^3) + 1/(b^2*Sqrt[a + b/x^2]*x) - ArcTanh[Sqrt[b]
/(Sqrt[a + b/x^2]*x)]/b^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}x^6}\,dx = -\operatorname{Subst}\left(\int \frac{x^4}{\left(a+bx^2\right)^{5/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{3b\left(a+\frac{b}{x^2}\right)^{3/2}x^3} - \frac{\operatorname{Subst}\left(\int \frac{x^2}{\left(a+bx^2\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{b}$$

$$= \frac{1}{3b\left(a+\frac{b}{x^2}\right)^{3/2}x^3} + \frac{1}{b^2\sqrt{a+\frac{b}{x^2}}\,x} - \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{x}\right)}{b^2}$$

$$= \frac{1}{3b\left(a+\frac{b}{x^2}\right)^{3/2}x^3} + \frac{1}{b^2\sqrt{a+\frac{b}{x^2}}\,x} - \frac{\operatorname{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x}\right)}{b^2}$$

$$= \frac{1}{3b\left(a+\frac{b}{x^2}\right)^{3/2}x^3} + \frac{1}{b^2\sqrt{a+\frac{b}{x^2}}\,x} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x}\right)}{b^{5/2}}$$

Mathematica [C] time = 0.01, size = 46, normalized size = 0.68

$$\frac{\left(ax^2+b\right)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{ax^2}{b}+1\right)}{3bx^5\left(a+\frac{b}{x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)^(5/2)*x^6),x]
```

```
[Out] ((b + a*x^2)*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (a*x^2)/b])/(3*b*(a + b/x
^2)^(5/2)*x^5)
```

fricas [A] time = 0.96, size = 226, normalized size = 3.32

$$\left[\frac{3\left(a^2x^4+2abx^2+b^2\right)\sqrt{b}\,\log\left(-\frac{ax^2-2\sqrt{b}\,x\sqrt{\frac{ax^2+b}{x^2}}+2b}{x^2}\right)+2\left(3abx^3+4b^2x\right)\sqrt{\frac{ax^2+b}{x^2}}}{6\left(a^2b^3x^4+2ab^4x^2+b^5\right)}, \frac{3\left(a^2x^4+2abx^2+b^2\right)\sqrt{-b}}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(5/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/6*(3*(a^2*x^4 + 2*a*b*x^2 + b^2)*sqrt(b)*log(-(a*x^2 - 2*sqrt(b)*x*sqrt(
(a*x^2 + b)/x^2) + 2*b)/x^2) + 2*(3*a*b*x^3 + 4*b^2*x)*sqrt((a*x^2 + b)/x^2
))/(a^2*b^3*x^4 + 2*a*b^4*x^2 + b^5), 1/3*(3*(a^2*x^4 + 2*a*b*x^2 + b^2)*sq
rt(-b)*arctan(sqrt(-b)*x*sqrt((a*x^2 + b)/x^2)/(a*x^2 + b)) + (3*a*b*x^3 +
4*b^2*x)*sqrt((a*x^2 + b)/x^2))/(a^2*b^3*x^4 + 2*a*b^4*x^2 + b^5)]
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(5/2)/x^6,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [sign(t_nostep),sign(t_nostep+sq
rt(b)/a*sign(t_nostep))]sym2poly/r2sym(const gen & e,const index_m & i,cons
t vecteur & l) Error: Bad Argument Value
```

maple [**A**] time = 0.01, size = 77, normalized size = 1.13

$$\frac{\left(a\,x^2+b\right)\left(3a\,b^{\frac{3}{2}}x^2+4b^{\frac{5}{2}}-3\left(a\,x^2+b\right)^{\frac{3}{2}}b\ln\left(\frac{2b+2\sqrt{a\,x^2+b}\,\sqrt{b}}{x}\right)\right)}{3\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{5}{2}}b^{\frac{7}{2}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^(5/2)/x^6,x)
```

```
[Out] 1/3*(a*x^2+b)*(3*a*b^(3/2)*x^2+4*b^(5/2)-3*ln(2*(b+(a*x^2+b)^(1/2)*b^(1/2))
/x)*(a*x^2+b)^(3/2)*b)/((a*x^2+b)/x^2)^(5/2)/x^5/b^(7/2)
```

maxima [**A**] time = 1.98, size = 73, normalized size = 1.07

$$\frac{\log\left(\frac{\sqrt{a+\frac{b}{x^2}}\,x-\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x+\sqrt{b}}\right)}{2\,b^{\frac{5}{2}}}+\frac{3\left(a+\frac{b}{x^2}\right)x^2+b}{3\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(5/2)/x^6,x, algorithm="maxima")
```

```
[Out] 1/2*log((sqrt(a + b/x^2)*x - sqrt(b))/(sqrt(a + b/x^2)*x + sqrt(b)))/b^(5/2
) + 1/3*(3*(a + b/x^2)*x^2 + b)/((a + b/x^2)^(3/2)*b^2*x^3)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^6\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b/x^2)^(5/2)),x)
```

```
[Out] int(1/(x^6*(a + b/x^2)^(5/2)), x)
```

sympy [**B**] time = 4.59, size = 740, normalized size = 10.88

$$\frac{3a^3b^4x^6\log\left(\frac{ax^2}{b}\right)}{6a^3b^{\frac{13}{2}}x^6+18a^2b^{\frac{15}{2}}x^4+18ab^{\frac{17}{2}}x^2+6b^{\frac{19}{2}}}-\frac{6a^3b^4x^6\log\left(\sqrt{\frac{ax^2}{b}+1}+1\right)}{6a^3b^{\frac{13}{2}}x^6+18a^2b^{\frac{15}{2}}x^4+18ab^{\frac{17}{2}}x^2+6b^{\frac{19}{2}}}+\frac{6a^2b^5x^4\sqrt{\frac{ax^2}{b}+}}{6a^3b^{\frac{13}{2}}x^6+18a^2b^{\frac{15}{2}}x^4+18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**(5/2)/x**6,x)
```

```
[Out] 3*a**3*b**4*x**6*log(a*x**2/b)/(6*a**3*b**(13/2)*x**6 + 18*a**2*b**(15/2)*x
**4 + 18*a*b**(17/2)*x**2 + 6*b**(19/2)) - 6*a**3*b**4*x**6*log(sqrt(a*x**2
/b + 1) + 1)/(6*a**3*b**(13/2)*x**6 + 18*a**2*b**(15/2)*x**4 + 18*a*b**(17/
2)*x**2 + 6*b**(19/2)) + 6*a**2*b**5*x**4*sqrt(a*x**2/b + 1)/(6*a**3*b**(13
/2)*x**6 + 18*a**2*b**(15/2)*x**4 + 18*a*b**(17/2)*x**2 + 6*b**(19/2)) + 9*
a**2*b**5*x**4*log(a*x**2/b)/(6*a**3*b**(13/2)*x**6 + 18*a**2*b**(15/2)*x**
4 + 18*a*b**(17/2)*x**2 + 6*b**(19/2)) - 18*a**2*b**5*x**4*log(sqrt(a*x**2/
b + 1) + 1)/(6*a**3*b**(13/2)*x**6 + 18*a**2*b**(15/2)*x**4 + 18*a*b**(17/2
)*x**2 + 6*b**(19/2)) + 14*a*b**6*x**2*sqrt(a*x**2/b + 1)/(6*a**3*b**(13/2)
*x**6 + 18*a**2*b**(15/2)*x**4 + 18*a*b**(17/2)*x**2 + 6*b**(19/2)) + 9*a*b
**6*x**2*log(a*x**2/b)/(6*a**3*b**(13/2)*x**6 + 18*a**2*b**(15/2)*x**4 + 18
*a*b**(17/2)*x**2 + 6*b**(19/2)) - 18*a*b**6*x**2*log(sqrt(a*x**2/b + 1) +
1)/(6*a**3*b**(13/2)*x**6 + 18*a**2*b**(15/2)*x**4 + 18*a*b**(17/2)*x**2 +
6*b**(19/2)) + 8*b**7*sqrt(a*x**2/b + 1)/(6*a**3*b**(13/2)*x**6 + 18*a**2*b
**(15/2)*x**4 + 18*a*b**(17/2)*x**2 + 6*b**(19/2)) + 3*b**7*log(a*x**2/b)/(
6*a**3*b**(13/2)*x**6 + 18*a**2*b**(15/2)*x**4 + 18*a*b**(17/2)*x**2 + 6*b*
*(19/2)) - 6*b**7*log(sqrt(a*x**2/b + 1) + 1)/(6*a**3*b**(13/2)*x**6 + 18*a
**2*b**(15/2)*x**4 + 18*a*b**(17/2)*x**2 + 6*b**(19/2))
```

3.1956 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^8}\,dx$

Optimal. Leaf size=95

$$\frac{5a\tanh^{-1}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{2b^{7/2}}-\frac{5\sqrt{a+\frac{b}{x^2}}}{2b^3x}+\frac{5}{3b^2x^3\sqrt{a+\frac{b}{x^2}}}+\frac{1}{3bx^5\left(a+\frac{b}{x^2}\right)^{3/2}}$$

```
[Out] 1/3/b/(a+b/x^2)^(3/2)/x^5+5/2*a*arctanh(b^(1/2)/x/(a+b/x^2)^(1/2))/b^(7/2)+
5/3/b^2/x^3/(a+b/x^2)^(1/2)-5/2*(a+b/x^2)^(1/2)/b^3/x
```

Rubi [A] time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {335, 288, 321, 217, 206}

$$-\frac{5\sqrt{a+\frac{b}{x^2}}}{2b^3x}+\frac{5}{3b^2x^3\sqrt{a+\frac{b}{x^2}}}+\frac{5a\tanh^{-1}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{2b^{7/2}}+\frac{1}{3bx^5\left(a+\frac{b}{x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^2)^(5/2)*x^8),x]
```

```
[Out] 1/(3*b*(a + b/x^2)^(3/2)*x^5) + 5/(3*b^2*Sqrt[a + b/x^2]*x^3) - (5*Sqrt[a +
 b/x^2])/(2*b^3*x) + (5*a*ArcTanh[Sqrt[b]/(Sqrt[a + b/x^2]*x)])/(2*b^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^8}\,dx = -\text{Subst}\left(\int \frac{x^6}{\left(a+bx^2\right)^{5/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{3b\left(a+\frac{b}{x^2}\right)^{3/2} x^5} - \frac{5\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^2\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3b}$$

$$= \frac{1}{3b\left(a+\frac{b}{x^2}\right)^{3/2} x^5} + \frac{5}{3b^2\sqrt{a+\frac{b}{x^2}}\,x^3} - \frac{5\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{x}\right)}{b^2}$$

$$= \frac{1}{3b\left(a+\frac{b}{x^2}\right)^{3/2} x^5} + \frac{5}{3b^2\sqrt{a+\frac{b}{x^2}}\,x^3} - \frac{5\sqrt{a+\frac{b}{x^2}}}{2b^3 x} + \frac{(5a)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{x}\right)}{2b^3}$$

$$= \frac{1}{3b\left(a+\frac{b}{x^2}\right)^{3/2} x^5} + \frac{5}{3b^2\sqrt{a+\frac{b}{x^2}}\,x^3} - \frac{5\sqrt{a+\frac{b}{x^2}}}{2b^3 x} + \frac{(5a)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x}\right)}{2b^3}$$

$$= \frac{1}{3b\left(a+\frac{b}{x^2}\right)^{3/2} x^5} + \frac{5}{3b^2\sqrt{a+\frac{b}{x^2}}\,x^3} - \frac{5\sqrt{a+\frac{b}{x^2}}}{2b^3 x} + \frac{5a\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x}\right)}{2b^{7/2}}$$

Mathematica [C] time = 0.01, size = 47, normalized size = 0.49

$$-\frac{a\left(ax^2+b\right)\,{}_2F_1\left(-\frac{3}{2}, 2; -\frac{1}{2}; \frac{ax^2}{b}+1\right)}{3b^2x^5\left(a+\frac{b}{x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^2)^(5/2)*x^8),x]
```

```
[Out] -1/3*(a*(b + a*x^2)*Hypergeometric2F1[-3/2, 2, -1/2, 1 + (a*x^2)/b])/(b^2*(
a + b/x^2)^(5/2)*x^5)
```

fricas [A] time = 1.16, size = 260, normalized size = 2.74

$$\left[\frac{15\left(a^3x^5+2\,a^2bx^3+ab^2x\right)\sqrt{b}\,\log\left(-\frac{ax^2+2\,\sqrt{b}\,x\sqrt{\frac{ax^2+b}{x^2}}+2\,b}{x^2}\right) - 2\left(15\,a^2bx^4+20\,ab^2x^2+3\,b^3\right)\sqrt{\frac{ax^2+b}{x^2}}}{12\left(a^2b^4x^5+2\,ab^5x^3+b^6x\right)}, \quad 15\left(a^3x^5\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(5/2)/x^8,x, algorithm="fricas")
```

```
[Out] [1/12*(15*(a^3*x^5 + 2*a^2*b*x^3 + a*b^2*x)*sqrt(b)*log(-(a*x^2 + 2*sqrt(b)
*x*sqrt((a*x^2 + b)/x^2) + 2*b)/x^2) - 2*(15*a^2*b*x^4 + 20*a*b^2*x^2 + 3*b
^3)*sqrt((a*x^2 + b)/x^2))/(a^2*b^4*x^5 + 2*a*b^5*x^3 + b^6*x), -1/6*(15*(a
^3*x^5 + 2*a^2*b*x^3 + a*b^2*x)*sqrt(-b)*arctan(sqrt(-b)*x*sqrt((a*x^2 + b)
/x^2)/(a*x^2 + b)) + (15*a^2*b*x^4 + 20*a*b^2*x^2 + 3*b^3)*sqrt((a*x^2 + b)
/x^2))/(a^2*b^4*x^5 + 2*a*b^5*x^3 + b^6*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(5/2)/x^8,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [sign(t_nostep),sign(t_nostep+sq
rt(b)/a*sign(t_nostep))]sym2poly/r2sym(const gen & e,const index_m & i,cons
t vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 92, normalized size = 0.97

$$-\frac{\left(a\,x^2+b\right)\left(15a^2b^{\frac{3}{2}}x^4+20a\,b^{\frac{5}{2}}x^2-15\left(a\,x^2+b\right)^{\frac{3}{2}}ab\,x^2\ln\left(\frac{2b+2\sqrt{a\,x^2+b}\,\sqrt{b}}{x}\right)+3b^{\frac{7}{2}}\right)}{6\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{5}{2}}b^{\frac{9}{2}}x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^(5/2)/x^8,x)
```

```
[Out] -1/6*(a*x^2+b)*(15*a^2*b^(3/2)*x^4+20*a*b^(5/2)*x^2-15*ln(2*(b+(a*x^2+b)^(1
/2)*b^(1/2))/x)*(a*x^2+b)^(3/2)*x^2*a*b+3*b^(7/2))/((a*x^2+b)/x^2)^(5/2)/x^
7/b^(9/2)
```

maxima [A] time = 1.90, size = 117, normalized size = 1.23

$$-\frac{15\left(a+\frac{b}{x^2}\right)^2ax^4-10\left(a+\frac{b}{x^2}\right)abx^2-2\,ab^2}{6\left(\left(a+\frac{b}{x^2}\right)^{\frac{5}{2}}b^3x^5-\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}b^4x^3\right)}-\frac{5\,a\log\left(\frac{\sqrt{a+\frac{b}{x^2}}\,x-\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x+\sqrt{b}}\right)}{4\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(5/2)/x^8,x, algorithm="maxima")
```

```
[Out] -1/6*(15*(a + b/x^2)^2*a*x^4 - 10*(a + b/x^2)*a*b*x^2 - 2*a*b^2)/((a + b/x^
2)^(5/2)*b^3*x^5 - (a + b/x^2)^(3/2)*b^4*x^3) - 5/4*a*log((sqrt(a + b/x^2)*
x - sqrt(b))/(sqrt(a + b/x^2)*x + sqrt(b)))/b^(7/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^8\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(a + b/x^2)^(5/2)),x)
```

```
[Out] int(1/(x^8*(a + b/x^2)^(5/2)), x)
```

sympy [**B**] time = 7.01, size = 864, normalized size = 9.09

$$-\frac{15a^4b^{13}x^8\log\left(\frac{ax^2}{b}\right)}{12a^3b^{\frac{33}{2}}x^8+36a^2b^{\frac{35}{2}}x^6+36ab^{\frac{37}{2}}x^4+12b^{\frac{39}{2}}x^2}+\frac{30a^4b^{13}x^8\log\left(\sqrt{\frac{ax^2}{b}+1}+1\right)}{12a^3b^{\frac{33}{2}}x^8+36a^2b^{\frac{35}{2}}x^6+36ab^{\frac{37}{2}}x^4+12b^{\frac{39}{2}}x^2}-\frac{}{12a^3b^{\frac{33}{2}}x^8+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**(5/2)/x**8,x)
```

```
[Out] -15*a**4*b**13*x**8*log(a*x**2/b)/(12*a**3*b**(33/2)*x**8 + 36*a**2*b**(35/
2)*x**6 + 36*a*b**(37/2)*x**4 + 12*b**(39/2)*x**2) + 30*a**4*b**13*x**8*log
(sqrt(a*x**2/b + 1) + 1)/(12*a**3*b**(33/2)*x**8 + 36*a**2*b**(35/2)*x**6 +
 36*a*b**(37/2)*x**4 + 12*b**(39/2)*x**2) - 30*a**3*b**14*x**6*sqrt(a*x**2/
b + 1)/(12*a**3*b**(33/2)*x**8 + 36*a**2*b**(35/2)*x**6 + 36*a*b**(37/2)*x*
*4 + 12*b**(39/2)*x**2) - 45*a**3*b**14*x**6*log(a*x**2/b)/(12*a**3*b**(33/
2)*x**8 + 36*a**2*b**(35/2)*x**6 + 36*a*b**(37/2)*x**4 + 12*b**(39/2)*x**2)
 + 90*a**3*b**14*x**6*log(sqrt(a*x**2/b + 1) + 1)/(12*a**3*b**(33/2)*x**8 +
 36*a**2*b**(35/2)*x**6 + 36*a*b**(37/2)*x**4 + 12*b**(39/2)*x**2) - 70*a**
2*b**15*x**4*sqrt(a*x**2/b + 1)/(12*a**3*b**(33/2)*x**8 + 36*a**2*b**(35/2)
*x**6 + 36*a*b**(37/2)*x**4 + 12*b**(39/2)*x**2) - 45*a**2*b**15*x**4*log(a
*x**2/b)/(12*a**3*b**(33/2)*x**8 + 36*a**2*b**(35/2)*x**6 + 36*a*b**(37/2)*
x**4 + 12*b**(39/2)*x**2) + 90*a**2*b**15*x**4*log(sqrt(a*x**2/b + 1) + 1)/
(12*a**3*b**(33/2)*x**8 + 36*a**2*b**(35/2)*x**6 + 36*a*b**(37/2)*x**4 + 12
*b**(39/2)*x**2) - 46*a*b**16*x**2*sqrt(a*x**2/b + 1)/(12*a**3*b**(33/2)*x*
*8 + 36*a**2*b**(35/2)*x**6 + 36*a*b**(37/2)*x**4 + 12*b**(39/2)*x**2) - 15
*a*b**16*x**2*log(a*x**2/b)/(12*a**3*b**(33/2)*x**8 + 36*a**2*b**(35/2)*x**
6 + 36*a*b**(37/2)*x**4 + 12*b**(39/2)*x**2) + 30*a*b**16*x**2*log(sqrt(a*x
**2/b + 1) + 1)/(12*a**3*b**(33/2)*x**8 + 36*a**2*b**(35/2)*x**6 + 36*a*b**
(37/2)*x**4 + 12*b**(39/2)*x**2) - 6*b**17*sqrt(a*x**2/b + 1)/(12*a**3*b**(
33/2)*x**8 + 36*a**2*b**(35/2)*x**6 + 36*a*b**(37/2)*x**4 + 12*b**(39/2)*x*
*2)
```

3.1957 $\int \frac{\sqrt[3]{1+\frac{1}{x^2}}}{x^3}\,dx$

Optimal. Leaf size=13

$$-\frac{3}{8}\left(\frac{1}{x^2}+1\right)^{4/3}$$

```
[Out] -3/8*(1+1/x^2)^(4/3)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$-\frac{3}{8}\left(\frac{1}{x^2}+1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^(-2))^(1/3)/x^3,x]
```

```
[Out] (-3*(1 + x^(-2))^(4/3))/8
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt[3]{1+\frac{1}{x^2}}}{x^3}\,dx = -\frac{3}{8}\left(1+\frac{1}{x^2}\right)^{4/3}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$-\frac{3}{8}\left(\frac{1}{x^2}+1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^(-2))^(1/3)/x^3,x]
```

```
[Out] (-3*(1 + x^(-2))^(4/3))/8
```

fricas [B] time = 0.99, size = 21, normalized size = 1.62

$$-\frac{3\left(x^2+1\right)\left(\frac{x^2+1}{x^2}\right)^{\frac{1}{3}}}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x^2)^(1/3)/x^3,x, algorithm="fricas")
```

```
[Out] -3/8*(x^2 + 1)*((x^2 + 1)/x^2)^(1/3)/x^2
```

giac [B] time = 0.16, size = 21, normalized size = 1.62

$$-\frac{3\left(x^2+1\right)\left(\frac{x^2+1}{x^2}\right)^{\frac{1}{3}}}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x^2)^(1/3)/x^3,x, algorithm="giac")
[Out] -3/8*(x^2 + 1)*((x^2 + 1)/x^2)^(1/3)/x^2
```

maple [B] time = 0.00, size = 22, normalized size = 1.69

$$-\frac{3\left(x^2+1\right)\left(\frac{x^2+1}{x^2}\right)^{\frac{1}{3}}}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+1/x^2)^(1/3)/x^3,x)
[Out] -3/8/x^2*(x^2+1)*((x^2+1)/x^2)^(1/3)
```

maxima [A] time = 0.83, size = 9, normalized size = 0.69

$$-\frac{3}{8}\left(\frac{1}{x^2}+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x^2)^(1/3)/x^3,x, algorithm="maxima")
[Out] -3/8*(1/x^2 + 1)^(4/3)
```

mupad [B] time = 1.35, size = 17, normalized size = 1.31

$$-\frac{3\left(\frac{1}{x^2}+1\right)^{1/3}\left(x^2+1\right)}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x^2 + 1)^(1/3)/x^3,x)
[Out] -(3*(1/x^2 + 1)^(1/3)*(x^2 + 1))/(8*x^2)
```

sympy [B] time = 1.10, size = 31, normalized size = 2.38

$$-\frac{3\sqrt[3]{1+\frac{1}{x^2}}}{8}-\frac{3\sqrt[3]{1+\frac{1}{x^2}}}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x**2)**(1/3)/x**3,x)
[Out] -3*(1 + x**(-2))**(1/3)/8 - 3*(1 + x**(-2))**(1/3)/(8*x**2)
```

3.1958 $\int \frac{\left(1+\frac{1}{x^2}\right)^{5/3}}{x^3}\,dx$

Optimal. Leaf size=13

$$-\frac{3}{16}\left(\frac{1}{x^2}+1\right)^{8/3}$$

```
[Out] -3/16*(1+1/x^2)^(8/3)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$-\frac{3}{16}\left(\frac{1}{x^2}+1\right)^{8/3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^(-2))^(5/3)/x^3,x]
```

```
[Out] (-3*(1 + x^(-2))^(8/3))/16
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(1+\frac{1}{x^2}\right)^{5/3}}{x^3}\,dx = -\frac{3}{16}\left(1+\frac{1}{x^2}\right)^{8/3}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.62

$$-\frac{3\left(\frac{1}{x^2}+1\right)^{5/3}\left(x^2+1\right)}{16x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^(-2))^(5/3)/x^3,x]
```

```
[Out] (-3*(1 + x^(-2))^(5/3)*(1 + x^2))/(16*x^2)
```

fricas [B] time = 0.64, size = 26, normalized size = 2.00

$$-\frac{3\left(x^4+2x^2+1\right)\left(\frac{x^2+1}{x^2}\right)^{\frac{2}{3}}}{16\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x^2)^(5/3)/x^3,x, algorithm="fricas")
```

```
[Out] -3/16*(x^4 + 2*x^2 + 1)*((x^2 + 1)/x^2)^(2/3)/x^4
```

giac [B] time = 0.17, size = 23, normalized size = 1.77

$$-\frac{3\left(x^2+1\right)^2\left(\frac{x^2+1}{x^2}\right)^{\frac{2}{3}}}{16\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x^2)^(5/3)/x^3,x, algorithm="giac")
[Out] -3/16*(x^2 + 1)^2*((x^2 + 1)/x^2)^(2/3)/x^4
```

maple [B] time = 0.00, size = 22, normalized size = 1.69

$$-\frac{3\left(x^2+1\right)\left(\frac{x^2+1}{x^2}\right)^{\frac{5}{3}}}{16x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+1/x^2)^(5/3)/x^3,x)
[Out] -3/16/x^2*(x^2+1)*((x^2+1)/x^2)^(5/3)
```

maxima [A] time = 0.85, size = 9, normalized size = 0.69

$$-\frac{3}{16}\left(\frac{1}{x^2}+1\right)^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x^2)^(5/3)/x^3,x, algorithm="maxima")
[Out] -3/16*(1/x^2 + 1)^(8/3)
```

mupad [B] time = 1.42, size = 19, normalized size = 1.46

$$-\frac{3\left(\frac{1}{x^2}+1\right)^{2/3}\left(x^2+1\right)^2}{16\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x^2 + 1)^(5/3)/x^3,x)
[Out] -(3*(1/x^2 + 1)^(2/3)*(x^2 + 1)^2)/(16*x^4)
```

sympy [B] time = 2.29, size = 48, normalized size = 3.69

$$-\frac{3\left(1+\frac{1}{x^2}\right)^{\frac{2}{3}}}{16}-\frac{3\left(1+\frac{1}{x^2}\right)^{\frac{2}{3}}}{8x^2}-\frac{3\left(1+\frac{1}{x^2}\right)^{\frac{2}{3}}}{16x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x**2)**(5/3)/x**3,x)
[Out] -3*(1 + x**(-2))**(2/3)/16 - 3*(1 + x**(-2))**(2/3)/(8*x**2) - 3*(1 + x**(-
2))**(2/3)/(16*x**4)
```

3.1959 $\int \left(1+\frac{b}{x^2}\right)^{3/2} (cx)^m \, dx$

Optimal. Leaf size=44

$$\frac{(cx)^{m+1}\, {}_2F_1\left(-\frac{3}{2}, \frac{1}{2}(-m-1); \frac{1-m}{2}; -\frac{b}{x^2}\right)}{c(m+1)}$$

```
[Out] (c*x)^(1+m)*hypergeom([-3/2, -1/2-1/2*m],[1/2-1/2*m],-b/x^2)/c/(1+m)
```

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {339, 364}

$$\frac{(cx)^{m+1}\, {}_2F_1\left(-\frac{3}{2}, \frac{1}{2}(-m-1); \frac{1-m}{2}; -\frac{b}{x^2}\right)}{c(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + b/x^2)^(3/2)*(c*x)^m,x]
```

```
[Out] ((c*x)^(1 + m)*Hypergeometric2F1[-3/2, (-1 - m)/2, (1 - m)/2, -(b/x^2)])/(c
*(1 + m))
```

Rule 339

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Dist[((c
*x)^(m + 1)*(1/x)^(m + 1))/c, Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x
], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \left(1+\frac{b}{x^2}\right)^{3/2} (cx)^m \, dx = -\frac{\left(\left(\frac{1}{x}\right)^{1+m} (cx)^{1+m}\right) \text{Subst}\left(\int x^{-2-m} \left(1+bx^2\right)^{3/2} dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{(cx)^{1+m}\, {}_2F_1\left(-\frac{3}{2}, \frac{1}{2}(-1-m); \frac{1-m}{2}; -\frac{b}{x^2}\right)}{c(1+m)}$$

Mathematica [A] time = 0.03, size = 63, normalized size = 1.43

$$\frac{b\sqrt{\frac{b}{x^2}+1}\,(cx)^m\, {}_2F_1\left(-\frac{3}{2}, \frac{m}{2}-1; \frac{m}{2}; -\frac{x^2}{b}\right)}{(m-2)x\sqrt{\frac{b+x^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + b/x^2)^(3/2)*(c*x)^m,x]
```

```
[Out] (b*Sqrt[1 + b/x^2]*(c*x)^m*Hypergeometric2F1[-3/2, -1 + m/2, m/2, -(x^2/b)]
)/((-2 + m)*x*Sqrt[(b + x^2)/b])
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(x^2+b\right)(cx)^m\sqrt{\frac{x^2+b}{x^2}}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+b/x^2)^(3/2)*(c*x)^m,x, algorithm="fricas")
[Out] integral((x^2 + b)*(c*x)^m*sqrt((x^2 + b)/x^2)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx)^m \left(\frac{b}{x^2}+1\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+b/x^2)^(3/2)*(c*x)^m,x, algorithm="giac")
[Out] integrate((c*x)^m*(b/x^2 + 1)^(3/2), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \left(\frac{b}{x^2}+1\right)^{\frac{3}{2}} (cx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+b/x^2)^(3/2)*(c*x)^m,x)
[Out] int((1+b/x^2)^(3/2)*(c*x)^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx)^m \left(\frac{b}{x^2}+1\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+b/x^2)^(3/2)*(c*x)^m,x, algorithm="maxima")
[Out] integrate((c*x)^m*(b/x^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (c\,x)^m \left(\frac{b}{x^2}+1\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(b/x^2 + 1)^(3/2),x)
[Out] int((c*x)^m*(b/x^2 + 1)^(3/2), x)
```

sympy [C] time = 5.29, size = 56, normalized size = 1.27

$$-\frac{c^m x x^m \Gamma\left(-\frac{m}{2}-\frac{1}{2}\right) {}_2F_1\left(\left.\begin{matrix}-\frac{3}{2}, -\frac{m}{2}-\frac{1}{2} \\ \frac{1}{2}-\frac{m}{2}\end{matrix}\right| \frac{b e^{i\pi}}{x^2}\right)}{2\Gamma\left(\frac{1}{2}-\frac{m}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+b/x**2)**(3/2)*(c*x)**m,x)
```

```
[Out] -c**m*x*x**m*gamma(-m/2 - 1/2)*hyper((-3/2, -m/2 - 1/2), (1/2 - m/2,), b*ex
p_polar(I*pi)/x**2)/(2*gamma(1/2 - m/2))
```

3.1960 $\int \sqrt{1+\frac{b}{x^2}}\,(cx)^m\,dx$

Optimal. Leaf size=44

$$\frac{(cx)^{m+1}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}(-m-1);\frac{1-m}{2};-\frac{b}{x^2}\right)}{c(m+1)}$$

```
[Out] (c*x)^(1+m)*hypergeom([-1/2, -1/2-1/2*m],[1/2-1/2*m],-b/x^2)/c/(1+m)
```

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {339, 364}

$$\frac{(cx)^{m+1}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}(-m-1);\frac{1-m}{2};-\frac{b}{x^2}\right)}{c(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + b/x^2]*(c*x)^m,x]
```

```
[Out] ((c*x)^(1 + m)*Hypergeometric2F1[-1/2, (-1 - m)/2, (1 - m)/2, -(b/x^2)])/(c
*(1 + m))
```

Rule 339

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Dist[((c
*x)^(m + 1)*(1/x)^(m + 1))/c, Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x
], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \sqrt{1+\frac{b}{x^2}}\,(cx)^m\,dx = -\frac{\left(\left(\frac{1}{x}\right)^{1+m}(cx)^{1+m}\right)\text{Subst}\left(\int x^{-2-m}\sqrt{1+bx^2}\,dx,x,\frac{1}{x}\right)}{c}$$
$$= \frac{(cx)^{1+m}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}(-1-m);\frac{1-m}{2};-\frac{b}{x^2}\right)}{c(1+m)}$$

Mathematica [A] time = 0.02, size = 58, normalized size = 1.32

$$\frac{x\sqrt{\frac{b}{x^2}+1}\,(cx)^m\,{}_2F_1\left(-\frac{1}{2},\frac{m}{2};\frac{m}{2}+1;-\frac{x^2}{b}\right)}{m\sqrt{\frac{x^2}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + b/x^2]*(c*x)^m,x]
```

```
[Out] (Sqrt[1 + b/x^2]*x*(c*x)^m*Hypergeometric2F1[-1/2, m/2, 1 + m/2, -(x^2/b)])
/(m*Sqrt[1 + x^2/b])
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left((cx)^m \sqrt{\frac{x^2+b}{x^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+b/x^2)^(1/2)*(c*x)^m,x, algorithm="fricas")
```

```
[Out] integral((c*x)^m*sqrt((x^2 + b)/x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx)^m \sqrt{\frac{b}{x^2}+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+b/x^2)^(1/2)*(c*x)^m,x, algorithm="giac")
```

```
[Out] integrate((c*x)^m*sqrt(b/x^2 + 1), x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{b}{x^2}+1}\ (cx)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+1)^(1/2)*(c*x)^m,x)
```

```
[Out] int((b/x^2+1)^(1/2)*(c*x)^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx)^m \sqrt{\frac{b}{x^2}+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+b/x^2)^(1/2)*(c*x)^m,x, algorithm="maxima")
```

```
[Out] integrate((c*x)^m*sqrt(b/x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (c\,x)^m \sqrt{\frac{b}{x^2}+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(b/x^2 + 1)^(1/2),x)
```

```
[Out] int((c*x)^m*(b/x^2 + 1)^(1/2), x)
```

sympy [C] time = 2.04, size = 48, normalized size = 1.09

$$-\frac{\sqrt{b}\, c^m x^m \Gamma\left(-\frac{m}{2}\right) {}_2F_1\left(\begin{matrix}-\frac{1}{2}, \frac{m}{2} \\ \frac{m}{2}+1\end{matrix}\middle| \frac{x^2 e^{i\pi}}{b}\right)}{2\Gamma\left(1-\frac{m}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+b/x**2)**(1/2)*(c*x)**m,x)
```

```
[Out] -sqrt(b)*c**m*x**m*gamma(-m/2)*hyper((-1/2, m/2), (m/2 + 1,), x**2*exp_pola
r(I*pi)/b)/(2*gamma(1 - m/2))
```

3.1961 $\int \frac{(cx)^m}{\sqrt{1+\frac{b}{x^2}}}\,dx$

Optimal. Leaf size=44

$$\frac{(cx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-m-1);\frac{1-m}{2};-\frac{b}{x^2}\right)}{c(m+1)}$$

```
[Out] (c*x)^(1+m)*hypergeom([1/2, -1/2-1/2*m],[1/2-1/2*m],-b/x^2)/c/(1+m)
```

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {339, 364}

$$\frac{(cx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-m-1);\frac{1-m}{2};-\frac{b}{x^2}\right)}{c(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m/Sqrt[1 + b/x^2],x]
```

```
[Out] ((c*x)^(1 + m)*Hypergeometric2F1[1/2, (-1 - m)/2, (1 - m)/2, -(b/x^2)])/(c*
(1 + m))
```

Rule 339

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Dist[((c
*x)^(m + 1)*(1/x)^(m + 1))/c, Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x
], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(cx)^m}{\sqrt{1+\frac{b}{x^2}}}\,dx = -\frac{\left(\left(\frac{1}{x}\right)^{1+m}(cx)^{1+m}\right)\text{Subst}\left(\int \frac{x^{-2-m}}{\sqrt{1+bx^2}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{(cx)^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1-m);\frac{1-m}{2};-\frac{b}{x^2}\right)}{c(1+m)}$$

Mathematica [A] time = 0.02, size = 64, normalized size = 1.45

$$\frac{x\sqrt{\frac{x^2}{b}+1}\,(cx)^m\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+2}{2}+1;-\frac{x^2}{b}\right)}{(m+2)\sqrt{\frac{b}{x^2}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m/Sqrt[1 + b/x^2],x]
```

```
[Out] (x*(c*x)^m*Sqrt[1 + x^2/b]*Hypergeometric2F1[1/2, (2 + m)/2, 1 + (2 + m)/2,
 -(x^2/b)])/((2 + m)*Sqrt[1 + b/x^2])
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(cx)^m x^2 \sqrt{\frac{x^2+b}{x^2}}}{x^2+b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(1+b/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((c*x)^m*x^2*sqrt((x^2 + b)/x^2)/(x^2 + b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{\sqrt{\frac{b}{x^2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(1+b/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^m/sqrt(b/x^2 + 1), x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{\sqrt{\frac{b}{x^2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(b/x^2+1)^(1/2),x)
```

```
[Out] int((c*x)^m/(b/x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{\sqrt{\frac{b}{x^2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(1+b/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^m/sqrt(b/x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c\,x)^m}{\sqrt{\frac{b}{x^2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(b/x^2 + 1)^(1/2),x)
```

```
[Out] int((c*x)^m/(b/x^2 + 1)^(1/2), x)
```

sympy [C] time = 1.41, size = 54, normalized size = 1.23

$$-\frac{c^m x x^m \Gamma\left(-\frac{m}{2}-\frac{1}{2}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, -\frac{m}{2}-\frac{1}{2} \\ \frac{1}{2}-\frac{m}{2}\end{matrix}\right| \frac{b e^{i\pi}}{x^2}\right)}{2\Gamma\left(\frac{1}{2}-\frac{m}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m/(1+b/x**2)**(1/2),x)
```

```
[Out] -c**m*x*x**m*gamma(-m/2 - 1/2)*hyper((1/2, -m/2 - 1/2), (1/2 - m/2,), b*exp
_polar(I*pi)/x**2)/(2*gamma(1/2 - m/2))
```

3.1962 $\int \frac{(cx)^m}{\left(1+\frac{b}{x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=44

$$\frac{(cx)^{m+1}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}(-m-1);\frac{1-m}{2};-\frac{b}{x^2}\right)}{c(m+1)}$$

```
[Out] (c*x)^(1+m)*hypergeom([3/2, -1/2-1/2*m],[1/2-1/2*m],-b/x^2)/c/(1+m)
```

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {339, 364}

$$\frac{(cx)^{m+1}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}(-m-1);\frac{1-m}{2};-\frac{b}{x^2}\right)}{c(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m/(1 + b/x^2)^(3/2),x]
```

```
[Out] ((c*x)^(1 + m)*Hypergeometric2F1[3/2, (-1 - m)/2, (1 - m)/2, -(b/x^2)])/(c*
(1 + m))
```

Rule 339

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Dist[((c
*x)^(m + 1)*(1/x)^(m + 1))/c, Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x
], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(cx)^m}{\left(1+\frac{b}{x^2}\right)^{3/2}}\,dx = -\frac{\left(\left(\frac{1}{x}\right)^{1+m}(cx)^{1+m}\right)\text{Subst}\left(\int \frac{x^{-2-m}}{\left(1+bx^2\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{(cx)^{1+m}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}(-1-m);\frac{1-m}{2};-\frac{b}{x^2}\right)}{c(1+m)}$$

Mathematica [A] time = 0.04, size = 67, normalized size = 1.52

$$\frac{x^3\sqrt{\frac{b+x^2}{b}}\,(cx)^m\,{}_2F_1\left(\frac{3}{2},\frac{m}{2}+2;\frac{m}{2}+3;-\frac{x^2}{b}\right)}{b(m+4)\sqrt{\frac{b}{x^2}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m/(1 + b/x^2)^(3/2),x]
```

```
[Out] (x^3*(c*x)^m*Sqrt[(b + x^2)/b]*Hypergeometric2F1[3/2, 2 + m/2, 3 + m/2, -(x
^2/b)])/(b*(4 + m)*Sqrt[1 + b/x^2])
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(cx)^m x^4 \sqrt{\frac{x^2+b}{x^2}}}{x^4+2bx^2+b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(1+b/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x)^m*x^4*sqrt((x^2 + b)/x^2)/(x^4 + 2*b*x^2 + b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{\left(\frac{b}{x^2}+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(1+b/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^m/(b/x^2 + 1)^(3/2), x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{\left(\frac{b}{x^2}+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(b/x^2+1)^(3/2),x)
```

```
[Out] int((c*x)^m/(b/x^2+1)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{\left(\frac{b}{x^2}+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(1+b/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^m/(b/x^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c\,x)^m}{\left(\frac{b}{x^2}+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(b/x^2 + 1)^(3/2),x)
```

```
[Out] int((c*x)^m/(b/x^2 + 1)^(3/2), x)
```

sympy [C] time = 2.48, size = 54, normalized size = 1.23

$$-\frac{c^m x x^m \Gamma\left(-\frac{m}{2}-\frac{1}{2}\right) {}_2F_1\left(\begin{matrix}\frac{3}{2}, -\frac{m}{2}-\frac{1}{2} \\ \frac{1}{2}-\frac{m}{2}\end{matrix}\middle| \frac{be^{i\pi}}{x^2}\right)}{2\Gamma\left(\frac{1}{2}-\frac{m}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m/(1+b/x**2)**(3/2),x)
```

```
[Out] -c**m*x*x**m*gamma(-m/2 - 1/2)*hyper((3/2, -m/2 - 1/2), (1/2 - m/2,), b*exp
_polar(I*pi)/x**2)/(2*gamma(1/2 - m/2))
```

3.1963 $\int \left(1+\frac{b}{x^2}\right)^p (cx)^m\,dx$

Optimal. Leaf size=44

$$\frac{(cx)^{m+1}\,{}_2F_1\left(\frac{1}{2}(-m-1),-p;\frac{1-m}{2};-\frac{b}{x^2}\right)}{c(m+1)}$$

```
[Out] (c*x)^(1+m)*hypergeom([-p, -1/2-1/2*m],[1/2-1/2*m],-b/x^2)/c/(1+m)
```

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {339, 364}

$$\frac{(cx)^{m+1}\,{}_2F_1\left(\frac{1}{2}(-m-1),-p;\frac{1-m}{2};-\frac{b}{x^2}\right)}{c(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + b/x^2)^p*(c*x)^m,x]
```

```
[Out] ((c*x)^(1 + m)*Hypergeometric2F1[(-1 - m)/2, -p, (1 - m)/2, -(b/x^2)])/(c*(
1 + m))
```

Rule 339

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Dist[((c
*x)^(m + 1)*(1/x)^(m + 1))/c, Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x
], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \left(1+\frac{b}{x^2}\right)^p (cx)^m\,dx = -\frac{\left(\left(\frac{1}{x}\right)^{1+m}(cx)^{1+m}\right)\text{Subst}\left(\int x^{-2-m}\left(1+bx^2\right)^p\,dx,x,\frac{1}{x}\right)}{c}$$
$$= \frac{(cx)^{1+m}\,{}_2F_1\left(\frac{1}{2}(-1-m),-p;\frac{1-m}{2};-\frac{b}{x^2}\right)}{c(1+m)}$$

Mathematica [A] time = 0.02, size = 71, normalized size = 1.61

$$\frac{x\left(\frac{b}{x^2}+1\right)^p\left(\frac{x^2}{b}+1\right)^{-p}(cx)^m\,{}_2F_1\left(\frac{1}{2}(m-2p+1),-p;\frac{1}{2}(m-2p+1)+1;-\frac{x^2}{b}\right)}{m-2p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + b/x^2)^p*(c*x)^m,x]
```

```
[Out] ((1 + b/x^2)^p*x*(c*x)^m*Hypergeometric2F1[(1 + m - 2*p)/2, -p, 1 + (1 + m
- 2*p)/2, -(x^2/b)])/((1 + m - 2*p)*(1 + x^2/b)^p)
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left((cx)^m\left(\frac{x^2+b}{x^2}\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+b/x^2)^p*(c*x)^m,x, algorithm="fricas")
```

```
[Out] integral((c*x)^m*((x^2 + b)/x^2)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx)^m\left(\frac{b}{x^2}+1\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+b/x^2)^p*(c*x)^m,x, algorithm="giac")
```

```
[Out] integrate((c*x)^m*(b/x^2 + 1)^p, x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int (cx)^m\left(\frac{b}{x^2}+1\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2+1)^p*(c*x)^m,x)
```

```
[Out] int((b/x^2+1)^p*(c*x)^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx)^m\left(\frac{b}{x^2}+1\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+b/x^2)^p*(c*x)^m,x, algorithm="maxima")
```

```
[Out] integrate((c*x)^m*(b/x^2 + 1)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (c\,x)^m\left(\frac{b}{x^2}+1\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(b/x^2 + 1)^p,x)
```

```
[Out] int((c*x)^m*(b/x^2 + 1)^p, x)
```

sympy [C] time = 17.75, size = 54, normalized size = 1.23

$$-\frac{c^m x x^m \Gamma\left(-\frac{m}{2}-\frac{1}{2}\right){}_2F_1\left(\begin{matrix}-p, -\frac{m}{2}-\frac{1}{2}\\ \frac{1}{2}-\frac{m}{2}\end{matrix}\middle| \frac{be^{i\pi}}{x^2}\right)}{2\Gamma\left(\frac{1}{2}-\frac{m}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+b/x**2)**p*(c*x)**m,x)
```

```
[Out] -c**m*x*x**m*gamma(-m/2 - 1/2)*hyper((-p, -m/2 - 1/2), (1/2 - m/2,), b*exp_
polar(I*pi)/x**2)/(2*gamma(1/2 - m/2))
```

3.1964 $\int \left(a+\frac{b}{x^2}\right)^p (cx)^m\,dx$

Optimal. Leaf size=70

$$\frac{(cx)^{m+1}\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{2}(-m-1),-p;\frac{1-m}{2};-\frac{b}{ax^2}\right)}{c(m+1)}$$

```
[Out] (a+b/x^2)^p*(c*x)^(1+m)*hypergeom([-p, -1/2-1/2*m],[1/2-1/2*m],-b/a/x^2)/c/
(1+m)/((1+b/a/x^2)^p)
```

Rubi [A] time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {339, 365, 364}

$$\frac{(cx)^{m+1}\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{2}(-m-1),-p;\frac{1-m}{2};-\frac{b}{ax^2}\right)}{c(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^p*(c*x)^m,x]
```

```
[Out] ((a + b/x^2)^p*(c*x)^(1 + m)*Hypergeometric2F1[(-1 - m)/2, -p, (1 - m)/2, -
(b/(a*x^2))])/(c*(1 + m)*(1 + b/(a*x^2))^p)
```

Rule 339

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Dist[((c
*x)^(m + 1)*(1/x)^(m + 1))/c, Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x
], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x^2}\right)^p (cx)^m\,dx &= -\frac{\left(\left(\frac{1}{x}\right)^{1+m}(cx)^{1+m}\right)\text{Subst}\left(\int x^{-2-m}\left(a+bx^2\right)^p\,dx,x,\frac{1}{x}\right)}{c}\\ &= -\frac{\left(\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(\frac{1}{x}\right)^{1+m}(cx)^{1+m}\right)\text{Subst}\left(\int x^{-2-m}\left(1+\frac{bx^2}{a}\right)^p\,dx,x,\frac{1}{x}\right)}{c}\\ &= \frac{\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}(cx)^{1+m}\,{}_2F_1\left(\frac{1}{2}(-1-m),-p;\frac{1-m}{2};-\frac{b}{ax^2}\right)}{c(1+m)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 73, normalized size = 1.04

$$\frac{x(cx)^m \left(a+\frac{b}{x^2}\right)^p \left(\frac{ax^2}{b}+1\right)^{-p} {}_2F_1\left(\frac{1}{2}(m-2p+1), -p; \frac{1}{2}(m-2p+1)+1; -\frac{ax^2}{b}\right)}{m-2p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^p*(c*x)^m,x]
```

```
[Out] ((a + b/x^2)^p*x*(c*x)^m*Hypergeometric2F1[(1 + m - 2*p)/2, -p, 1 + (1 + m
- 2*p)/2, -((a*x^2)/b)])/((1 + m - 2*p)*(1 + (a*x^2)/b)^p)
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left((cx)^m \left(\frac{ax^2+b}{x^2}\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c*x)^m,x, algorithm="fricas")
```

```
[Out] integral((c*x)^m*((a*x^2 + b)/x^2)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx)^m \left(a+\frac{b}{x^2}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c*x)^m,x, algorithm="giac")
```

```
[Out] integrate((c*x)^m*(a + b/x^2)^p, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int (cx)^m \left(a+\frac{b}{x^2}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^p*(c*x)^m,x)
```

```
[Out] int((a+b/x^2)^p*(c*x)^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx)^m \left(a+\frac{b}{x^2}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c*x)^m,x, algorithm="maxima")
```

```
[Out] integrate((c*x)^m*(a + b/x^2)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x)^m \left(a+\frac{b}{x^2}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(a + b/x^2)^p,x)
```

```
[Out] int((c*x)^m*(a + b/x^2)^p, x)
```

sympy [C] time = 22.27, size = 60, normalized size = 0.86

$$-\frac{a^p c^m x x^m \Gamma\left(-\frac{m}{2}-\frac{1}{2}\right) {}_2F_1\left(\begin{matrix}-p, -\frac{m}{2}-\frac{1}{2} \\ \frac{1}{2}-\frac{m}{2}\end{matrix}\middle| \frac{b e^{i\pi}}{a x^2}\right)}{2\Gamma\left(\frac{1}{2}-\frac{m}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c*x)**m,x)
```

```
[Out] -a**p*c**m*x*x**m*gamma(-m/2 - 1/2)*hyper((-p, -m/2 - 1/2), (1/2 - m/2,), b
*exp_polar(I*pi)/(a*x**2))/(2*gamma(1/2 - m/2))
```

3.1965 $\int \frac{x^5}{a+\frac{b}{x^3}}\,dx$

Optimal. Leaf size=40

$$\frac{b^2 \log\left(ax^3+b\right)}{3a^3} - \frac{bx^3}{3a^2} + \frac{x^6}{6a}$$

```
[Out] -1/3*b*x^3/a^2+1/6*x^6/a+1/3*b^2*ln(a*x^3+b)/a^3
```

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 266, 43}

$$\frac{b^2 \log\left(ax^3+b\right)}{3a^3} - \frac{bx^3}{3a^2} + \frac{x^6}{6a}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b/x^3),x]
```

```
[Out] -(b*x^3)/(3*a^2) + x^6/(6*a) + (b^2*Log[b + a*x^3])/(3*a^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{a+\frac{b}{x^3}}\,dx &= \int \frac{x^8}{b+ax^3}\,dx \\
&= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{b+ax}\,dx, x, x^3\right) \\
&= \frac{1}{3}\,\text{Subst}\left(\int \left(-\frac{b}{a^2} + \frac{x}{a} + \frac{b^2}{a^2(b+ax)}\right)\,dx, x, x^3\right) \\
&= -\frac{bx^3}{3a^2} + \frac{x^6}{6a} + \frac{b^2 \log\left(b+ax^3\right)}{3a^3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.00

$$\frac{b^2 \log\left(ax^3+b\right)}{3a^3} - \frac{bx^3}{3a^2} + \frac{x^6}{6a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b/x^3),x]
[Out] -1/3*(b*x^3)/a^2 + x^6/(6*a) + (b^2*Log[b + a*x^3])/(3*a^3)
```

fricas [A] time = 0.88, size = 33, normalized size = 0.82

$$\frac{a^2x^6 - 2\,abx^3 + 2\,b^2\log\left(ax^3 + b\right)}{6\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/x^3),x, algorithm="fricas")
[Out] 1/6*(a^2*x^6 - 2*a*b*x^3 + 2*b^2*log(a*x^3 + b))/a^3
```

giac [A] time = 0.16, size = 35, normalized size = 0.88

$$\frac{b^2\log\left(\left|ax^3 + b\right|\right)}{3\,a^3} + \frac{ax^6 - 2\,bx^3}{6\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/x^3),x, algorithm="giac")
[Out] 1/3*b^2*log(abs(a*x^3 + b))/a^3 + 1/6*(a*x^6 - 2*b*x^3)/a^2
```

maple [A] time = 0.00, size = 35, normalized size = 0.88

$$\frac{x^6}{6a} - \frac{b\,x^3}{3a^2} + \frac{b^2\ln\left(a\,x^3 + b\right)}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a+b/x^3),x)
[Out] -1/3/a^2*b*x^3+1/6/a*x^6+1/3*b^2*ln(a*x^3+b)/a^3
```

maxima [A] time = 0.88, size = 34, normalized size = 0.85

$$\frac{b^2\log\left(ax^3 + b\right)}{3\,a^3} + \frac{ax^6 - 2\,bx^3}{6\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/x^3),x, algorithm="maxima")
[Out] 1/3*b^2*log(a*x^3 + b)/a^3 + 1/6*(a*x^6 - 2*b*x^3)/a^2
```

mupad [B] time = 0.06, size = 33, normalized size = 0.82

$$\frac{2\,b^2\,\ln\left(a\,x^3 + b\right) + a^2\,x^6 - 2\,a\,b\,x^3}{6\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b/x^3),x)
[Out] (2*b^2*log(b + a*x^3) + a^2*x^6 - 2*a*b*x^3)/(6*a^3)
```

sympy [A] time = 0.23, size = 32, normalized size = 0.80

$$\frac{x^6}{6a} - \frac{bx^3}{3a^2} + \frac{b^2\log\left(ax^3 + b\right)}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a+b/x**3),x)
[Out] x**6/(6*a) - b*x**3/(3*a**2) + b**2*log(a*x**3 + b)/(3*a**3)
```

3.1966 $\int \frac{x^4}{a+\frac{b}{x^3}}\,dx$

Optimal. Leaf size=136

$$\frac{b^{5/3}\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)}{6a^{8/3}}-\frac{b^{5/3}\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{3a^{8/3}}-\frac{b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt{3}\,a^{8/3}}-\frac{bx^2}{2a^2}+\frac{x^5}{5a}$$

```
[Out] -1/2*b*x^2/a^2+1/5*x^5/a-1/3*b^(5/3)*ln(b^(1/3)+a^(1/3)*x)/a^(8/3)+1/6*b^(5
/3)*ln(b^(2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/a^(8/3)-1/3*b^(5/3)*arctan(1/
3*(b^(1/3)-2*a^(1/3)*x)/b^(1/3)*3^(1/2))/a^(8/3)*3^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {263, 302, 292, 31, 634, 617, 204, 628}

$$\frac{b^{5/3}\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)}{6a^{8/3}}-\frac{b^{5/3}\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{3a^{8/3}}-\frac{b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt{3}\,a^{8/3}}-\frac{bx^2}{2a^2}+\frac{x^5}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b/x^3),x]
```

```
[Out] -(b*x^2)/(2*a^2) + x^5/(5*a) - (b^(5/3)*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqr
t[3]*b^(1/3))])/(Sqrt[3]*a^(8/3)) - (b^(5/3)*Log[b^(1/3) + a^(1/3)*x])/(3*a
^(8/3)) + (b^(5/3)*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(6*a^(8/
3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{a+\frac{b}{x^3}}\,dx &= \int \frac{x^7}{b+ax^3}\,dx \\
&= \int \left(-\frac{bx}{a^2}+\frac{x^4}{a}+\frac{b^2 x}{a^2\left(b+ax^3\right)}\right)dx \\
&= -\frac{bx^2}{2a^2}+\frac{x^5}{5a}+\frac{b^2\int \frac{x}{b+ax^3}\,dx}{a^2} \\
&= -\frac{bx^2}{2a^2}+\frac{x^5}{5a}-\frac{b^{5/3}\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{a}\,x}\,dx}{3a^{7/3}}+\frac{b^{5/3}\int \frac{\sqrt[3]{b}+\sqrt[3]{a}\,x}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{3a^{7/3}} \\
&= -\frac{bx^2}{2a^2}+\frac{x^5}{5a}-\frac{b^{5/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{3a^{8/3}}+\frac{b^{5/3}\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2a^{2/3}x}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{6a^{8/3}}+\frac{b^2\int \frac{1}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{2a^{7/3}} \\
&= -\frac{bx^2}{2a^2}+\frac{x^5}{5a}-\frac{b^{5/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{3a^{8/3}}+\frac{b^{5/3}\log\left(b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2\right)}{6a^{8/3}}+\frac{b^{5/3}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,\right.}{a^{8/3}} \\
&= -\frac{bx^2}{2a^2}+\frac{x^5}{5a}-\frac{b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt{3}\,a^{8/3}}-\frac{b^{5/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{3a^{8/3}}+\frac{b^{5/3}\log\left(b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2\right)}{6a^{8/3}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 122, normalized size = 0.90

$$\frac{5b^{5/3}\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)-15a^{2/3}bx^2+6a^{5/3}x^5-10b^{5/3}\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)-10\sqrt{3}\,b^{5/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}\,x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{30a^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b/x^3),x]
```

```
[Out] (-15*a^(2/3)*b*x^2 + 6*a^(5/3)*x^5 - 10*Sqrt[3]*b^(5/3)*ArcTan[(1 - (2*a^(1
/3)*x)/b^(1/3))/Sqrt[3]] - 10*b^(5/3)*Log[b^(1/3) + a^(1/3)*x] + 5*b^(5/3)*
Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(30*a^(8/3))
```

fricas [A] time = 0.83, size = 140, normalized size = 1.03

$$\frac{6\,ax^5 - 15\,bx^2 + 10\sqrt{3}\,b\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,ax\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}} + \sqrt{3}\,b}{3\,b}\right) - 5\,b\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx^2 - ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}} - b\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right) + 1}{30\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/x^3),x, algorithm="fricas")
```

```
[Out] 1/30*(6*a*x^5 - 15*b*x^2 + 10*sqrt(3)*b*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt
(3)*a*x*(-b^2/a^2)^(1/3) + sqrt(3)*b)/b) - 5*b*(-b^2/a^2)^(1/3)*log(b*x^2 -
 a*x*(-b^2/a^2)^(2/3) - b*(-b^2/a^2)^(1/3)) + 10*b*(-b^2/a^2)^(1/3)*log(b*x
 + a*(-b^2/a^2)^(2/3)))/a^2
```

giac [A] time = 0.17, size = 132, normalized size = 0.97

$$-\frac{b\left(-\frac{b}{a}\right)^{\frac{2}{3}}\log\left(\left|x - \left(-\frac{b}{a}\right)^{\frac{1}{3}}\right|\right)}{3\,a^2} - \frac{\sqrt{3}\,\left(-a^2b\right)^{\frac{2}{3}}\,b\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,a^4} + \frac{\left(-a^2b\right)^{\frac{2}{3}}\,b\log\left(x^2 + x\left(-\frac{b}{a}\right)^{\frac{1}{3}} + \left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\,a^4} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/x^3),x, algorithm="giac")
```

```
[Out] -1/3*b*(-b/a)^(2/3)*log(abs(x - (-b/a)^(1/3)))/a^2 - 1/3*sqrt(3)*(-a^2*b)^(
2/3)*b*arctan(1/3*sqrt(3)*(2*x + (-b/a)^(1/3))/(-b/a)^(1/3))/a^4 + 1/6*(-a^
2*b)^(2/3)*b*log(x^2 + x*(-b/a)^(1/3) + (-b/a)^(2/3))/a^4 + 1/10*(2*a^4*x^5
 - 5*a^3*b*x^2)/a^5
```

maple [A] time = 0.01, size = 117, normalized size = 0.86

$$\frac{x^5}{5a} - \frac{b\,x^2}{2a^2} + \frac{\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{b}{a}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}a^3} - \frac{b^2\ln\left(x + \left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}a^3} + \frac{b^2\ln\left(x^2 - \left(\frac{b}{a}\right)^{\frac{1}{3}}x + \left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\left(\frac{b}{a}\right)^{\frac{1}{3}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a+b/x^3),x)
```

```
[Out] 1/5/a*x^5-1/2/a^2*b*x^2-1/3/a^3*b^2/(1/a*b)^(1/3)*ln(x+(1/a*b)^(1/3))+1/6/a
^3*b^2/(1/a*b)^(1/3)*ln(x^2-(1/a*b)^(1/3)*x+(1/a*b)^(2/3))+1/3/a^3*b^2*3^(1
/2)/(1/a*b)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/a*b)^(1/3)*x-1))
```

maxima [A] time = 1.99, size = 125, normalized size = 0.92

$$\frac{\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,a^3\left(\frac{b}{a}\right)^{\frac{1}{3}}} + \frac{b^2\log\left(x^2 - x\left(\frac{b}{a}\right)^{\frac{1}{3}} + \left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\,a^3\left(\frac{b}{a}\right)^{\frac{1}{3}}} - \frac{b^2\log\left(x + \left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\,a^3\left(\frac{b}{a}\right)^{\frac{1}{3}}} + \frac{2\,ax^5 - 5\,bx^2}{10\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/x^3),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*x - (b/a)^(1/3))/(b/a)^(1/3))/(a^3*(b
/a)^(1/3)) + 1/6*b^2*log(x^2 - x*(b/a)^(1/3) + (b/a)^(2/3))/(a^3*(b/a)^(1/3
)) - 1/3*b^2*log(x + (b/a)^(1/3))/(a^3*(b/a)^(1/3)) + 1/10*(2*a*x^5 - 5*b*x
^2)/a^2
```

mupad **[B]** time = 1.32, size = 143, normalized size = 1.05

$$\frac{x^5}{5a}-\frac{b\,x^2}{2\,a^2}+\frac{(-b)^{5/3}\,\ln\left(\frac{b^4\,x}{a^3}-\frac{(-b)^{13/3}}{a^{10/3}}\right)}{3\,a^{8/3}}-\frac{(-b)^{5/3}\,\ln\left(\frac{b^4\,x}{a^3}-\frac{(-b)^{13/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{a^{10/3}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,a^{8/3}}+\frac{(-b)^{5/3}\,\ln\left(\frac{b^4\,x}{a^3}-\frac{9\,(-b)^{13/3}\left(-\right.\right.}{a^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b/x^3),x)
```

```
[Out] x^5/(5*a) - (b*x^2)/(2*a^2) + ((-b)^(5/3)*log((b^4*x)/a^3 - (-b)^(13/3)/a^(
10/3)))/(3*a^(8/3)) - ((-b)^(5/3)*log((b^4*x)/a^3 - ((-b)^(13/3)*((3^(1/2)*
1i)/2 + 1/2)^2)/a^(10/3))*((3^(1/2)*1i)/2 + 1/2))/(3*a^(8/3)) + ((-b)^(5/3)
*log((b^4*x)/a^3 - (9*(-b)^(13/3)*((3^(1/2)*1i)/6 - 1/6)^2)/a^(10/3))*((3^(
1/2)*1i)/6 - 1/6))/a^(8/3)
```

sympy **[A]** time = 0.26, size = 44, normalized size = 0.32

$$\operatorname{RootSum}\left(27t^3a^8+b^5,\left(t\mapsto t\log\left(\frac{9t^2a^5}{b^3}+x\right)\right)\right)+\frac{x^5}{5a}-\frac{bx^2}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a+b/x**3),x)
```

```
[Out] RootSum(27*_t**3*a**8 + b**5, Lambda(_t, _t*log(9*_t**2*a**5/b**3 + x))) +
x**5/(5*a) - b*x**2/(2*a**2)
```

3.1967 $\int \frac{x^3}{a+\frac{b}{x^3}}\,dx$

Optimal. Leaf size=132

$$-\frac{b^{4/3}\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)}{6a^{7/3}}+\frac{b^{4/3}\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{3a^{7/3}}-\frac{b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt{3}\,a^{7/3}}-\frac{bx}{a^2}+\frac{x^4}{4a}$$

```
[Out] -b*x/a^2+1/4*x^4/a+1/3*b^(4/3)*ln(b^(1/3)+a^(1/3)*x)/a^(7/3)-1/6*b^(4/3)*ln
(b^(2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/a^(7/3)-1/3*b^(4/3)*arctan(1/3*(b^(
1/3)-2*a^(1/3)*x)/b^(1/3)*3^(1/2))/a^(7/3)*3^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {263, 302, 200, 31, 634, 617, 204, 628}

$$-\frac{b^{4/3}\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)}{6a^{7/3}}+\frac{b^{4/3}\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{3a^{7/3}}-\frac{b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt{3}\,a^{7/3}}-\frac{bx}{a^2}+\frac{x^4}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b/x^3),x]
```

```
[Out] -((b*x)/a^2) + x^4/(4*a) - (b^(4/3)*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]
*b^(1/3))])/(Sqrt[3]*a^(7/3)) + (b^(4/3)*Log[b^(1/3) + a^(1/3)*x])/(3*a^(7/
3)) - (b^(4/3)*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(6*a^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{a+\frac{b}{x^3}}\,dx &= \int \frac{x^6}{b+ax^3}\,dx \\
&= \int \left(-\frac{b}{a^2}+\frac{x^3}{a}+\frac{b^2}{a^2\left(b+ax^3\right)}\right)dx \\
&= -\frac{bx}{a^2}+\frac{x^4}{4a}+\frac{b^2\int \frac{1}{b+ax^3}\,dx}{a^2} \\
&= -\frac{bx}{a^2}+\frac{x^4}{4a}+\frac{b^{4/3}\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{a}\,x}\,dx}{3a^2}+\frac{b^{4/3}\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}\,x}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{3a^2} \\
&= -\frac{bx}{a^2}+\frac{x^4}{4a}+\frac{b^{4/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{3a^{7/3}}-\frac{b^{4/3}\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2a^{2/3}x}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{6a^{7/3}}+\frac{b^{5/3}\int \frac{1}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{2a^2} \\
&= -\frac{bx}{a^2}+\frac{x^4}{4a}+\frac{b^{4/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{3a^{7/3}}-\frac{b^{4/3}\log\left(b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2\right)}{6a^{7/3}}+\frac{b^{4/3}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x\right.}{a^{7/3}} \\
&= -\frac{bx}{a^2}+\frac{x^4}{4a}-\frac{b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt{3}\,a^{7/3}}+\frac{b^{4/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{3a^{7/3}}-\frac{b^{4/3}\log\left(b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2\right)}{6a^{7/3}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 120, normalized size = 0.91

$$\frac{-2b^{4/3}\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)+3a^{4/3}x^4+4b^{4/3}\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)-4\sqrt{3}\,b^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}\,x}{\sqrt[3]{b}}}{\sqrt{3}}\right)-12\sqrt[3]{a}\,bx}{12a^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b/x^3),x]
```

```
[Out] (-12*a^(1/3)*b*x + 3*a^(4/3)*x^4 - 4*Sqrt[3]*b^(4/3)*ArcTan[(1 - (2*a^(1/3)
*x)/b^(1/3))/Sqrt[3]] + 4*b^(4/3)*Log[b^(1/3) + a^(1/3)*x] - 2*b^(4/3)*Log[
b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(12*a^(7/3))
```

fricas [A] time = 0.95, size = 108, normalized size = 0.82

$$\frac{3\,ax^4+4\sqrt{3}\,b\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,ax\left(\frac{b}{a}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3\,b}\right)-2\,b\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(x^2-x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)+4\,b\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-1}{12\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x^3),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*x^4 + 4*sqrt(3)*b*(b/a)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(b/a)^(2/
3) - sqrt(3)*b)/b) - 2*b*(b/a)^(1/3)*log(x^2 - x*(b/a)^(1/3) + (b/a)^(2/3))
 + 4*b*(b/a)^(1/3)*log(x + (b/a)^(1/3)) - 12*b*x)/a^2
```

giac [A] time = 0.17, size = 129, normalized size = 0.98

$$-\frac{b\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right|\right)}{3\,a^2}+\frac{\sqrt{3}\,\left(-a^2b\right)^{\frac{1}{3}}\,b\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,a^3}+\frac{\left(-a^2b\right)^{\frac{1}{3}}\,b\log\left(x^2+x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\,a^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x^3),x, algorithm="giac")
```

```
[Out] -1/3*b*(-b/a)^(1/3)*log(abs(x - (-b/a)^(1/3)))/a^2 + 1/3*sqrt(3)*(-a^2*b)^(
1/3)*b*arctan(1/3*sqrt(3)*(2*x + (-b/a)^(1/3))/(-b/a)^(1/3))/a^3 + 1/6*(-a^
2*b)^(1/3)*b*log(x^2 + x*(-b/a)^(1/3) + (-b/a)^(2/3))/a^3 + 1/4*(a^3*x^4 -
4*a^2*b*x)/a^4
```

maple [A] time = 0.00, size = 115, normalized size = 0.87

$$\frac{x^4}{4a}-\frac{bx}{a^2}+\frac{\sqrt{3}\;b^2\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{b}{a}\right)^{\frac{2}{3}}a^3}+\frac{b^2\ln\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{2}{3}}a^3}-\frac{b^2\ln\left(x^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}x+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\left(\frac{b}{a}\right)^{\frac{2}{3}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b/x^3),x)
```

```
[Out] 1/4/a*x^4-1/a^2*b*x+1/3/a^3*b^2/(1/a*b)^(2/3)*ln(x+(1/a*b)^(1/3))-1/6/a^3*b
^2/(1/a*b)^(2/3)*ln(x^2-(1/a*b)^(1/3)*x+(1/a*b)^(2/3))+1/3/a^3*b^2/(1/a*b)^
(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/a*b)^(1/3)*x-1))
```

maxima [A] time = 1.93, size = 122, normalized size = 0.92

$$\frac{\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,a^3\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{b^2\log\left(x^2-x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\,a^3\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{b^2\log\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\,a^3\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{ax^4-4\,bx}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x^3),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*x - (b/a)^(1/3))/(b/a)^(1/3))/(a^3*(b
/a)^(2/3)) - 1/6*b^2*log(x^2 - x*(b/a)^(1/3) + (b/a)^(2/3))/(a^3*(b/a)^(2/3
)) + 1/3*b^2*log(x + (b/a)^(1/3))/(a^3*(b/a)^(2/3)) + 1/4*(a*x^4 - 4*b*x)/a
^2
```

mupad [B] time = 0.20, size = 119, normalized size = 0.90

$$\frac{x^4}{4\,a}+\frac{b^{4/3}\,\ln\left(3\,b^2\,x+\frac{3\,b^{7/3}}{a^{1/3}}\right)}{3\,a^{7/3}}-\frac{b\,x}{a^2}-\frac{b^{4/3}\,\ln\left(3\,b^2\,x-\frac{3\,b^{7/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{a^{1/3}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,a^{7/3}}+\frac{b^{4/3}\,\ln\left(3\,b^2\,x+\frac{9\,b^{7/3}\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{a^{1/3}}\right)\left(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b/x^3),x)
```

```
[Out] x^4/(4*a) + (b^(4/3)*log(3*b^2*x + (3*b^(7/3))/a^(1/3)))/(3*a^(7/3)) - (b*x
)/a^2 - (b^(4/3)*log(3*b^2*x - (3*b^(7/3)*((3^(1/2)*1i)/2 + 1/2))/a^(1/3))*
((3^(1/2)*1i)/2 + 1/2))/(3*a^(7/3)) + (b^(4/3)*log(3*b^2*x + (9*b^(7/3)*((3
^(1/2)*1i)/6 - 1/6))/a^(1/3))*((3^(1/2)*1i)/6 - 1/6))/a^(7/3)
```

sympy [A] time = 0.21, size = 37, normalized size = 0.28

$$\operatorname{RootSum}\left(27t^3a^7-b^4,\left(t\mapsto t\log\left(\frac{3ta^2}{b}+x\right)\right)\right)+\frac{x^4}{4a}-\frac{bx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b/x**3),x)
```

```
[Out] RootSum(27*_t**3*a**7 - b**4, Lambda(_t, _t*log(3*_t*a**2/b + x))) + x**4/(
4*a) - b*x/a**2
```

3.1968 $\int \frac{x^2}{a+\frac{b}{x^3}}\,dx$

Optimal. Leaf size=27

$$\frac{x^3}{3a}-\frac{b\log\left(ax^3+b\right)}{3a^2}$$

```
[Out] 1/3*x^3/a-1/3*b*ln(a*x^3+b)/a^2
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 266, 43}

$$\frac{x^3}{3a}-\frac{b\log\left(ax^3+b\right)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b/x^3),x]
```

```
[Out] x^3/(3*a) - (b*Log[b + a*x^3])/(3*a^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{a+\frac{b}{x^3}}\,dx &= \int \frac{x^5}{b+ax^3}\,dx\\ &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{b+ax}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{1}{a}-\frac{b}{a(b+ax)}\right)\,dx, x, x^3\right)\\ &= \frac{x^3}{3a}-\frac{b\log\left(b+ax^3\right)}{3a^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.00

$$\frac{x^3}{3a}-\frac{b\log\left(ax^3+b\right)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b/x^3),x]
[Out] x^3/(3*a) - (b*Log[b + a*x^3])/(3*a^2)
```

fricas [A] time = 0.82, size = 22, normalized size = 0.81

$$\frac{ax^3 - b\log\left(ax^3 + b\right)}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^3),x, algorithm="fricas")
[Out] 1/3*(a*x^3 - b*log(a*x^3 + b))/a^2
```

giac [A] time = 0.20, size = 24, normalized size = 0.89

$$\frac{x^3}{3\,a} - \frac{b\log\left(\left|ax^3 + b\right|\right)}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^3),x, algorithm="giac")
[Out] 1/3*x^3/a - 1/3*b*log(abs(a*x^3 + b))/a^2
```

maple [A] time = 0.00, size = 24, normalized size = 0.89

$$\frac{x^3}{3a} - \frac{b\ln\left(a\,x^3 + b\right)}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b/x^3),x)
[Out] 1/3/a*x^3-1/3*b*ln(a*x^3+b)/a^2
```

maxima [A] time = 0.86, size = 23, normalized size = 0.85

$$\frac{x^3}{3\,a} - \frac{b\log\left(ax^3 + b\right)}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^3),x, algorithm="maxima")
[Out] 1/3*x^3/a - 1/3*b*log(a*x^3 + b)/a^2
```

mupad [B] time = 1.12, size = 22, normalized size = 0.81

$$-\frac{b\,\ln\left(a\,x^3 + b\right) - a\,x^3}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b/x^3),x)
[Out] -(b*log(b + a*x^3) - a*x^3)/(3*a^2)
```

sympy [A] time = 0.24, size = 20, normalized size = 0.74

$$\frac{x^3}{3a} - \frac{b\log\left(ax^3 + b\right)}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b/x**3),x)
[Out] x**3/(3*a) - b*log(a*x**3 + b)/(3*a**2)
```

3.1969 $\int \frac{x}{a+\frac{b}{x^3}}\,dx$

Optimal. Leaf size=124

$$-\frac{b^{2/3}\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)}{6a^{5/3}}+\frac{b^{2/3}\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{3a^{5/3}}+\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt{3}\,a^{5/3}}+\frac{x^2}{2a}$$

```
[Out] 1/2*x^2/a+1/3*b^(2/3)*ln(b^(1/3)+a^(1/3)*x)/a^(5/3)-1/6*b^(2/3)*ln(b^(2/3)-
a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/a^(5/3)+1/3*b^(2/3)*arctan(1/3*(b^(1/3)-2*a^
(1/3)*x)/b^(1/3)*3^(1/2))/a^(5/3)*3^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {263, 321, 292, 31, 634, 617, 204, 628}

$$-\frac{b^{2/3}\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)}{6a^{5/3}}+\frac{b^{2/3}\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{3a^{5/3}}+\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt{3}\,a^{5/3}}+\frac{x^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b/x^3),x]
```

```
[Out] x^2/(2*a) + (b^(2/3)*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))])/(Sq
rt[3]*a^(5/3)) + (b^(2/3)*Log[b^(1/3) + a^(1/3)*x])/(3*a^(5/3)) - (b^(2/3)*
Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(6*a^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{a+\frac{b}{x^3}}\,dx &= \int \frac{x^4}{b+ax^3}\,dx \\
&= \frac{x^2}{2a} - \frac{b\int \frac{x}{b+ax^3}\,dx}{a} \\
&= \frac{x^2}{2a} + \frac{b^{2/3}\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{a}\,x}\,dx}{3a^{4/3}} - \frac{b^{2/3}\int \frac{\sqrt[3]{b}+\sqrt[3]{a}\,x}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{3a^{4/3}} \\
&= \frac{x^2}{2a} + \frac{b^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{3a^{5/3}} - \frac{b^{2/3}\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2a^{2/3}x}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{6a^{5/3}} - \frac{b\int \frac{1}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{2a^{4/3}} \\
&= \frac{x^2}{2a} + \frac{b^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{3a^{5/3}} - \frac{b^{2/3}\log\left(b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2\right)}{6a^{5/3}} - \frac{b^{2/3}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\right.}{a^{5/3}} \\
&= \frac{x^2}{2a} + \frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt{3}\,a^{5/3}} + \frac{b^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{3a^{5/3}} - \frac{b^{2/3}\log\left(b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2\right)}{6a^{5/3}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 111, normalized size = 0.90

$$\frac{-b^{2/3}\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)+3a^{2/3}x^2+2b^{2/3}\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)+2\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}\,x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{6a^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b/x^3),x]
```

```
[Out] (3*a^(2/3)*x^2 + 2*Sqrt[3]*b^(2/3)*ArcTan[(1 - (2*a^(1/3)*x)/b^(1/3))/Sqrt[
3]] + 2*b^(2/3)*Log[b^(1/3) + a^(1/3)*x] - b^(2/3)*Log[b^(2/3) - a^(1/3)*b^
(1/3)*x + a^(2/3)*x^2])/(6*a^(5/3))
```

fricas [A] time = 0.87, size = 123, normalized size = 0.99

$$\frac{3\,x^2-2\sqrt{3}\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,ax\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}-\sqrt{3}\,b}{3\,b}\right)-\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}+b\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right)+2\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx+a\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^3),x, algorithm="fricas")
```

```
[Out] 1/6*(3*x^2 - 2*sqrt(3)*(b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(b^2/a^2)^
(1/3) - sqrt(3)*b)/b) - (b^2/a^2)^(1/3)*log(b*x^2 - a*x*(b^2/a^2)^(2/3) + b
*(b^2/a^2)^(1/3)) + 2*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)^(2/3)))/a
```

giac [A] time = 0.17, size = 114, normalized size = 0.92

$$\frac{x^2}{2\,a}+\frac{\left(-\frac{b}{a}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right|\right)}{3\,a}+\frac{\sqrt{3}\left(-a^2b\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,a^3}-\frac{\left(-a^2b\right)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^3),x, algorithm="giac")
```

```
[Out] 1/2*x^2/a + 1/3*(-b/a)^(2/3)*log(abs(x - (-b/a)^(1/3)))/a + 1/3*sqrt(3)*(-a
^2*b)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-b/a)^(1/3))/(-b/a)^(1/3))/a^3 - 1/6
*(-a^2*b)^(2/3)*log(x^2 + x*(-b/a)^(1/3) + (-b/a)^(2/3))/a^3
```

maple [A] time = 0.00, size = 102, normalized size = 0.82

$$\frac{x^2}{2a}-\frac{\sqrt{3}\;b\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}a^2}+\frac{b\ln\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}a^2}-\frac{b\ln\left(x^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}x+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\left(\frac{b}{a}\right)^{\frac{1}{3}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b/x^3),x)
```

```
[Out] 1/2/a*x^2+1/3/a^2*b/(1/a*b)^(1/3)*ln(x+(1/a*b)^(1/3))-1/6/a^2*b/(1/a*b)^(1/
3)*ln(x^2-(1/a*b)^(1/3)*x+(1/a*b)^(2/3))-1/3/a^2*b*3^(1/2)/(1/a*b)^(1/3)*ar
ctan(1/3*3^(1/2)*(2/(1/a*b)^(1/3)*x-1))
```

maxima [A] time = 1.92, size = 109, normalized size = 0.88

$$\frac{x^2}{2\,a}-\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,a^2\left(\frac{b}{a}\right)^{\frac{1}{3}}}-\frac{b\log\left(x^2-x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\,a^2\left(\frac{b}{a}\right)^{\frac{1}{3}}}+\frac{b\log\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\,a^2\left(\frac{b}{a}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^3),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/a - 1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*x - (b/a)^(1/3))/(b/a)^(1/3
))/(a^2*(b/a)^(1/3)) - 1/6*b*log(x^2 - x*(b/a)^(1/3) + (b/a)^(2/3))/(a^2*(b
/a)^(1/3)) + 1/3*b*log(x + (b/a)^(1/3))/(a^2*(b/a)^(1/3))
```

mupad **[B]** time = 0.19, size = 120, normalized size = 0.97

$$\frac{x^2}{2\,a}+\frac{b^{2/3}\,\ln\left(\frac{b^{7/3}}{a^{4/3}}+\frac{b^2\,x}{a}\right)}{3\,a^{5/3}}-\frac{b^{2/3}\,\ln\left(\frac{b^2\,x}{a}+\frac{b^{7/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{a^{4/3}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,a^{5/3}}+\frac{b^{2/3}\,\ln\left(\frac{b^2\,x}{a}+\frac{9\,b^{7/3}\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2}{a^{4/3}}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}}{6}\right.}{a^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b/x^3),x)
```

```
[Out] x^2/(2*a) + (b^(2/3)*log(b^(7/3)/a^(4/3) + (b^2*x)/a))/(3*a^(5/3)) - (b^(2/
3)*log((b^2*x)/a + (b^(7/3)*((3^(1/2)*1i)/2 + 1/2)^2)/a^(4/3))*((3^(1/2)*1i
)/2 + 1/2))/(3*a^(5/3)) + (b^(2/3)*log((b^2*x)/a + (9*b^(7/3)*((3^(1/2)*1i)
/6 - 1/6)^2)/a^(4/3))*((3^(1/2)*1i)/6 - 1/6))/a^(5/3)
```

sympy **[A]** time = 0.19, size = 32, normalized size = 0.26

$$\mathrm{RootSum}\left(27t^3a^5-b^2,\left(t\mapsto t\log\left(\frac{9t^2a^3}{b}+x\right)\right)\right)+\frac{x^2}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x**3),x)
```

```
[Out] RootSum(27*_t**3*a**5 - b**2, Lambda(_t, _t*log(9*_t**2*a**3/b + x))) + x**
2/(2*a)
```

3.1970 $\int \frac{1}{a+\frac{b}{x^3}}\,dx$

Optimal. Leaf size=119

$$\frac{\sqrt[3]{b}\,\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)}{6a^{4/3}}-\frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{3a^{4/3}}+\frac{\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt{3}\,a^{4/3}}+\frac{x}{a}$$

```
[Out] x/a-1/3*b^(1/3)*ln(b^(1/3)+a^(1/3)*x)/a^(4/3)+1/6*b^(1/3)*ln(b^(2/3)-a^(1/3
)*b^(1/3)*x+a^(2/3)*x^2)/a^(4/3)+1/3*b^(1/3)*arctan(1/3*(b^(1/3)-2*a^(1/3)*
x)/b^(1/3)*3^(1/2))/a^(4/3)*3^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.889, Rules used = {193, 321, 200, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{b}\,\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)}{6a^{4/3}}-\frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{3a^{4/3}}+\frac{\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt{3}\,a^{4/3}}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^3)^(-1),x]
```

```
[Out] x/a + (b^(1/3)*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))])/(Sqrt[3]*
a^(4/3)) - (b^(1/3)*Log[b^(1/3) + a^(1/3)*x])/(3*a^(4/3)) + (b^(1/3)*Log[b^
(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(6*a^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{a+\frac{b}{x^3}}\,dx &= \int \frac{x^3}{b+ax^3}\,dx \\
&= \frac{x}{a} - \frac{b\int \frac{1}{b+ax^3}\,dx}{a} \\
&= \frac{x}{a} - \frac{\sqrt[3]{b}\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{a}\,x}\,dx}{3a} - \frac{\sqrt[3]{b}\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}\,x}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{3a} \\
&= \frac{x}{a} - \frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{3a^{4/3}} + \frac{\sqrt[3]{b}\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2a^{2/3}x}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{6a^{4/3}} - \frac{b^{2/3}\int \frac{1}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{2a} \\
&= \frac{x}{a} - \frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{3a^{4/3}} + \frac{\sqrt[3]{b}\,\log\left(b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2\right)}{6a^{4/3}} - \frac{\sqrt[3]{b}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\right.}{a^{4/3}} \\
&= \frac{x}{a} + \frac{\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt{3}\,a^{4/3}} - \frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{3a^{4/3}} + \frac{\sqrt[3]{b}\,\log\left(b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2\right)}{6a^{4/3}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 108, normalized size = 0.91

$$\frac{\sqrt[3]{b}\,\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)-2\sqrt[3]{b}\,\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)+2\sqrt{3}\,\sqrt[3]{b}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}\,x}{\sqrt[3]{b}}}{\sqrt{3}}\right)+6\sqrt[3]{a}\,x}{6a^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^3)^(-1),x]
```

```
[Out] (6*a^(1/3)*x + 2*Sqrt[3]*b^(1/3)*ArcTan[(1 - (2*a^(1/3)*x)/b^(1/3))/Sqrt[3]
] - 2*b^(1/3)*Log[b^(1/3) + a^(1/3)*x] + b^(1/3)*Log[b^(2/3) - a^(1/3)*b^(1
/3)*x + a^(2/3)*x^2])/(6*a^(4/3))
```

fricas [A] time = 0.85, size = 106, normalized size = 0.89

$$\frac{2\sqrt{3}\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b}{a}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)-\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)+2\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(x-\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)+6x}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*(-b/a)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b/a)^(2/3) - sqrt(3
)*b)/b) - (-b/a)^(1/3)*log(x^2 + x*(-b/a)^(1/3) + (-b/a)^(2/3)) + 2*(-b/a)^
(1/3)*log(x - (-b/a)^(1/3)) + 6*x)/a
```

giac [A] time = 0.16, size = 111, normalized size = 0.93

$$\frac{\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right|\right)}{3a}+\frac{x}{a}-\frac{\sqrt{3}\left(-a^2b\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3a^2}-\frac{\left(-a^2b\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3),x, algorithm="giac")
```

```
[Out] 1/3*(-b/a)^(1/3)*log(abs(x - (-b/a)^(1/3)))/a + x/a - 1/3*sqrt(3)*(-a^2*b)^
(1/3)*arctan(1/3*sqrt(3)*(2*x + (-b/a)^(1/3))/(-b/a)^(1/3))/a^2 - 1/6*(-a^2
*b)^(1/3)*log(x^2 + x*(-b/a)^(1/3) + (-b/a)^(2/3))/a^2
```

maple [A] time = 0.00, size = 99, normalized size = 0.83

$$\frac{x}{a}-\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{b}{a}\right)^{\frac{2}{3}}a^2}-\frac{b\ln\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{2}{3}}a^2}+\frac{b\ln\left(x^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}x+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\left(\frac{b}{a}\right)^{\frac{2}{3}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^3),x)
```

```
[Out] 1/a*x-1/3/a^2*b/(1/a*b)^(2/3)*ln(x+(1/a*b)^(1/3))+1/6/a^2*b/(1/a*b)^(2/3)*l
n(x^2-(1/a*b)^(1/3)*x+(1/a*b)^(2/3))-1/3/a^2*b/(1/a*b)^(2/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2/(1/a*b)^(1/3)*x-1))
```

maxima [A] time = 1.81, size = 106, normalized size = 0.89

$$\frac{x}{a}-\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3a^2\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{b\log\left(x^2-x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a^2\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{b\log\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a^2\left(\frac{b}{a}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3),x, algorithm="maxima")
```

```
[Out] x/a - 1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*x - (b/a)^(1/3))/(b/a)^(1/3))/(a^
2*(b/a)^(2/3)) + 1/6*b*log(x^2 - x*(b/a)^(1/3) + (b/a)^(2/3))/(a^2*(b/a)^(2
/3)) - 1/3*b*log(x + (b/a)^(1/3))/(a^2*(b/a)^(2/3))
```

mupad **[B]** time = 1.32, size = 114, normalized size = 0.96

$$\frac{x}{a}+\frac{(-b)^{1/3}\,\ln\left((-b)^{4/3}+a^{1/3}\,b\,x\right)}{3\,a^{4/3}}-\frac{(-b)^{1/3}\,\ln\left(3\,a^{2/3}\,(-b)^{4/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-3\,a\,b\,x\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,a^{4/3}}+\frac{(-b)^{1/3}\,\ln\left(9\,a^{2/}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x^3),x)
```

```
[Out] x/a + ((-b)^(1/3)*log((-b)^(4/3) + a^(1/3)*b*x))/(3*a^(4/3)) - ((-b)^(1/3)*
log(3*a^(2/3)*(-b)^(4/3)*((3^(1/2)*1i)/2 + 1/2) - 3*a*b*x)*((3^(1/2)*1i)/2
+ 1/2))/(3*a^(4/3)) + ((-b)^(1/3)*log(9*a^(2/3)*(-b)^(4/3)*((3^(1/2)*1i)/6
- 1/6) + 3*a*b*x)*((3^(1/2)*1i)/6 - 1/6))/a^(4/3)
```

sympy **[A]** time = 0.18, size = 22, normalized size = 0.18

$$\mathrm{RootSum}\left(27t^3a^4+b,\left(t\mapsto t\log\left(-3ta+x\right)\right)\right)+\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**3),x)
```

```
[Out] RootSum(27*_t**3*a**4 + b, Lambda(_t, _t*log(-3*_t*a + x))) + x/a
```

3.1971 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)x}\,dx$

Optimal. Leaf size=15

$$\frac{\log\left(ax^3+b\right)}{3a}$$

```
[Out] 1/3*ln(a*x^3+b)/a
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {263, 260}

$$\frac{\log\left(ax^3+b\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^3)*x),x]
```

```
[Out] Log[b + a*x^3]/(3*a)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x^3}\right)x}\,dx &= \int \frac{x^2}{b+ax^3}\,dx \\ &= \frac{\log\left(b+ax^3\right)}{3a}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log\left(ax^3+b\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^3)*x),x]
```

```
[Out] Log[b + a*x^3]/(3*a)
```

fricas [A] time = 0.87, size = 13, normalized size = 0.87

$$\frac{\log\left(ax^3+b\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)/x,x, algorithm="fricas")
[Out] 1/3*log(a*x^3 + b)/a
```

giac [A] time = 0.15, size = 14, normalized size = 0.93

$$\frac{\log\left(\left|ax^3+b\right|\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)/x,x, algorithm="giac")
[Out] 1/3*log(abs(a*x^3 + b))/a
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{\ln\left(a\,x^3+b\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^3)/x,x)
[Out] 1/3*ln(a*x^3+b)/a
```

maxima [A] time = 0.89, size = 13, normalized size = 0.87

$$\frac{\log\left(ax^3+b\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)/x,x, algorithm="maxima")
[Out] 1/3*log(a*x^3 + b)/a
```

mupad [B] time = 0.03, size = 13, normalized size = 0.87

$$\frac{\ln\left(a\,x^3+b\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b/x^3)),x)
[Out] log(b + a*x^3)/(3*a)
```

sympy [A] time = 0.16, size = 10, normalized size = 0.67

$$\frac{\log\left(ax^3+b\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**3)/x,x)
[Out] log(a*x**3 + b)/(3*a)
```

3.1972 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^2}\,dx$

Optimal. Leaf size=115

$$\frac{\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)}{6a^{2/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}}$$

```
[Out] -1/3*ln(b^(1/3)+a^(1/3)*x)/a^(2/3)/b^(1/3)+1/6*ln(b^(2/3)-a^(1/3)*b^(1/3)*x
+a^(2/3)*x^2)/a^(2/3)/b^(1/3)-1/3*arctan(1/3*(b^(1/3)-2*a^(1/3)*x)/b^(1/3)*
3^(1/2))/a^(2/3)/b^(1/3)*3^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {263, 292, 31, 634, 617, 204, 628}

$$\frac{\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)}{6a^{2/3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^3)*x^2),x]
```

```
[Out] -(ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))]/(Sqrt[3]*a^(2/3)*b^(1/3
))) - Log[b^(1/3) + a^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + Log[b^(2/3) - a^(1/3)*
b^(1/3)*x + a^(2/3)*x^2]/(6*a^(2/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^2}\,dx = \int \frac{x}{b+ax^3}\,dx$$

$$= -\frac{\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{a}\,x}\,dx}{3\sqrt[3]{a}\,\sqrt[3]{b}} + \frac{\int \frac{\sqrt[3]{b}+\sqrt[3]{a}\,x}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{3\sqrt[3]{a}\,\sqrt[3]{b}}$$

$$= -\frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{3a^{2/3}\sqrt[3]{b}} + \frac{\int \frac{1}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{2\sqrt[3]{a}} + \frac{\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2a^{2/3}x}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{6a^{2/3}\sqrt[3]{b}}$$

$$= -\frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{3a^{2/3}\sqrt[3]{b}} + \frac{\log\left(b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{a}\,x}{\sqrt[3]{b}}\right)}{a^{2/3}\sqrt[3]{b}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}} - \frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{3a^{2/3}\sqrt[3]{b}} + \frac{\log\left(b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}$$

Mathematica [A] time = 0.01, size = 89, normalized size = 0.77

$$\frac{\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)-2\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)-2\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}\,x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{6a^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^3)*x^2),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(1 - (2*a^(1/3)*x)/b^(1/3))/Sqrt[3]] - 2*Log[b^(1/3) + a
^(1/3)*x] + Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(6*a^(2/3)*b^(1
/3))
```

fricas [A] time = 0.87, size = 304, normalized size = 2.64

$$\left[\frac{3\sqrt{\frac{1}{3}}\,ab\sqrt{\frac{\left(-a^2b\right)^{\frac{1}{3}}}{b}}\,\log\left(\frac{2a^2x^3-ab+3\sqrt{\frac{1}{3}}\left(abx+2\left(-a^2b\right)^{\frac{2}{3}}x^2+\left(-a^2b\right)^{\frac{1}{3}}b\right)\sqrt{\frac{\left(-a^2b\right)^{\frac{1}{3}}}{b}}-3\left(-a^2b\right)^{\frac{2}{3}}x}{ax^3+b}\right)+\left(-a^2b\right)^{\frac{2}{3}}\log\left(a^2x^2+\left(-a^2b\right.\right.\right.}{6\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)/x^2,x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt((-a^2*b)^(1/3)/b)*log((2*a^2*x^3 - a*b + 3*sqrt(
1/3)*(a*b*x + 2*(-a^2*b)^(2/3)*x^2 + (-a^2*b)^(1/3)*b)*sqrt((-a^2*b)^(1/3)/
b) - 3*(-a^2*b)^(2/3)*x)/(a*x^3 + b)) + (-a^2*b)^(2/3)*log(a^2*x^2 + (-a^2*
b)^(1/3)*a*x + (-a^2*b)^(2/3)) - 2*(-a^2*b)^(2/3)*log(a*x - (-a^2*b)^(1/3))
)/(a^2*b), 1/6*(6*sqrt(1/3)*a*b*sqrt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2
*a*x + (-a^2*b)^(1/3))*sqrt(-(-a^2*b)^(1/3)/b)/a) + (-a^2*b)^(2/3)*log(a^2*
x^2 + (-a^2*b)^(1/3)*a*x + (-a^2*b)^(2/3)) - 2*(-a^2*b)^(2/3)*log(a*x - (-a
^2*b)^(1/3)))/(a^2*b)]
```

giac [A] time = 0.21, size = 112, normalized size = 0.97

$$-\frac{\left(-\frac{b}{a}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right|\right)}{3\,b}-\frac{\sqrt{3}\,\left(-a^2b\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,a^2b}+\frac{\left(-a^2b\right)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)/x^2,x, algorithm="giac")
```

```
[Out] -1/3*(-b/a)^(2/3)*log(abs(x - (-b/a)^(1/3)))/b - 1/3*sqrt(3)*(-a^2*b)^(2/3)
*arctan(1/3*sqrt(3)*(2*x + (-b/a)^(1/3))/(-b/a)^(1/3))/(a^2*b) + 1/6*(-a^2*
b)^(2/3)*log(x^2 + x*(-b/a)^(1/3) + (-b/a)^(2/3))/(a^2*b)
```

maple [A] time = 0.00, size = 91, normalized size = 0.79

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}a}-\frac{\ln\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}a}+\frac{\ln\left(x^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}x+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\left(\frac{b}{a}\right)^{\frac{1}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^3)/x^2,x)
```

```
[Out] -1/3/a/(1/a*b)^(1/3)*ln(x+(1/a*b)^(1/3))+1/6/a/(1/a*b)^(1/3)*ln(x^2-(1/a*b)
^(1/3)*x+(1/a*b)^(2/3))+1/3*3^(1/2)/a/(1/a*b)^(1/3)*arctan(1/3*3^(1/2)*(2/(
1/a*b)^(1/3)*x-1))
```

maxima [A] time = 1.90, size = 98, normalized size = 0.85

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,a\left(\frac{b}{a}\right)^{\frac{1}{3}}}+\frac{\log\left(x^2-x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\,a\left(\frac{b}{a}\right)^{\frac{1}{3}}}-\frac{\log\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\,a\left(\frac{b}{a}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (b/a)^(1/3))/(b/a)^(1/3))/(a*(b/a)^(1
/3)) + 1/6*log(x^2 - x*(b/a)^(1/3) + (b/a)^(2/3))/(a*(b/a)^(1/3)) - 1/3*log
(x + (b/a)^(1/3))/(a*(b/a)^(1/3))
```

mupad **[B]** time = 1.35, size = 111, normalized size = 0.97

$$\frac{\ln\left(a^{1/3}\,x-(-b)^{1/3}\right)}{3\,a^{2/3}\,(-b)^{1/3}}+\frac{\ln\left(a\,x-\frac{a^{2/3}\,(-b)^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,(-b)^{1/3}}-\frac{\ln\left(a\,x-\frac{a^{2/3}\,(-b)^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,(-b)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b/x^3)),x)
```

```
[Out] log(a^(1/3)*x - (-b)^(1/3))/(3*a^(2/3)*(-b)^(1/3)) + (log(a*x - (a^(2/3)*(-
b)^(1/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(6*a^(2/3)*(-b)^(1/3)) -
(log(a*x - (a^(2/3)*(-b)^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/(6*
a^(2/3)*(-b)^(1/3))
```

sympy **[A]** time = 0.16, size = 24, normalized size = 0.21

$$\mathrm{RootSum}\left(27t^3a^2b+1,\left(t\mapsto t\log\left(9t^2ab+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**3)/x**2,x)
```

```
[Out] RootSum(27*_t**3*a**2*b + 1, Lambda(_t, _t*log(9*_t**2*a*b + x)))
```

3.1973 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^3}\,dx$

Optimal. Leaf size=115

$$-\frac{\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)}{6\sqrt[3]{a}\,b^{2/3}}+\frac{\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}\,b^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{2/3}}$$

```
[Out] 1/3*ln(b^(1/3)+a^(1/3)*x)/a^(1/3)/b^(2/3)-1/6*ln(b^(2/3)-a^(1/3)*b^(1/3)*x+
a^(2/3)*x^2)/a^(1/3)/b^(2/3)-1/3*arctan(1/3*(b^(1/3)-2*a^(1/3)*x)/b^(1/3)*3
^(1/2))/a^(1/3)/b^(2/3)*3^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {263, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)}{6\sqrt[3]{a}\,b^{2/3}}+\frac{\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}\,b^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^3)*x^3),x]
```

```
[Out] -(ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))]/(Sqrt[3]*a^(1/3)*b^(2/3
))) + Log[b^(1/3) + a^(1/3)*x]/(3*a^(1/3)*b^(2/3)) - Log[b^(2/3) - a^(1/3)*
b^(1/3)*x + a^(2/3)*x^2]/(6*a^(1/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^3}\,dx = \int \frac{1}{b+ax^3}\,dx$$

$$= \frac{\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{a}\,x}\,dx}{3b^{2/3}} + \frac{\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}\,x}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{3b^{2/3}}$$

$$= \frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}} - \frac{\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2a^{2/3}x}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{6\sqrt[3]{a}\,b^{2/3}} + \frac{\int \frac{1}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{2\sqrt[3]{b}}$$

$$= \frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}} - \frac{\log\left(b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2\right)}{6\sqrt[3]{a}\,b^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{a}\,x}{\sqrt[3]{b}}\right)}{\sqrt[3]{a}\,b^{2/3}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{2/3}} + \frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}} - \frac{\log\left(b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2\right)}{6\sqrt[3]{a}\,b^{2/3}}$$

Mathematica [A] time = 0.01, size = 89, normalized size = 0.77

$$-\frac{\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)-2\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)+2\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}\,x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{6\sqrt[3]{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^3)*x^3),x]
```

```
[Out] -1/6*(2*Sqrt[3]*ArcTan[(1 - (2*a^(1/3)*x)/b^(1/3))/Sqrt[3]] - 2*Log[b^(1/3)
 + a^(1/3)*x] + Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(a^(1/3)*b^
(2/3))
```

fricas [A] time = 0.85, size = 299, normalized size = 2.60

$$\left[\frac{3\sqrt{\frac{1}{3}}\,ab\sqrt{-\frac{\left(ab^2\right)^{\frac{1}{3}}}{a}}\,\log\left(\frac{2\,abx^3-3\left(ab^2\right)^{\frac{1}{3}}bx-b^2+3\sqrt{\frac{1}{3}}\left(2\,abx^2+\left(ab^2\right)^{\frac{2}{3}}x-\left(ab^2\right)^{\frac{1}{3}}b\right)\sqrt{-\frac{\left(ab^2\right)^{\frac{1}{3}}}{a}}}{ax^3+b}\right)-\left(ab^2\right)^{\frac{2}{3}}\log\left(abx^2-\left(ab^2\right)^{\frac{2}{3}}x+\right.\right.}{6\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)/x^3,x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt(-(a*b^2)^(1/3)/a)*log((2*a*b*x^3 - 3*(a*b^2)^(1/
3)*b*x - b^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a*b^2)^(2/3)*x - (a*b^2)^(1/3)*b)*
sqrt(-(a*b^2)^(1/3)/a))/(a*x^3 + b)) - (a*b^2)^(2/3)*log(a*b*x^2 - (a*b^2)^
(2/3)*x + (a*b^2)^(1/3)*b) + 2*(a*b^2)^(2/3)*log(a*b*x + (a*b^2)^(2/3)))/(a
*b^2), 1/6*(6*sqrt(1/3)*a*b*sqrt((a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*(a*b^
2)^(2/3)*x - (a*b^2)^(1/3)*b)*sqrt((a*b^2)^(1/3)/a)/b^2) - (a*b^2)^(2/3)*lo
g(a*b*x^2 - (a*b^2)^(2/3)*x + (a*b^2)^(1/3)*b) + 2*(a*b^2)^(2/3)*log(a*b*x
+ (a*b^2)^(2/3)))/(a*b^2)]
```

giac [A] time = 0.17, size = 112, normalized size = 0.97

$$-\frac{\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right|\right)}{3\,b}+\frac{\sqrt{3}\left(-a^2b\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,ab}+\frac{\left(-a^2b\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)/x^3,x, algorithm="giac")
```

```
[Out] -1/3*(-b/a)^(1/3)*log(abs(x - (-b/a)^(1/3)))/b + 1/3*sqrt(3)*(-a^2*b)^(1/3)
*arctan(1/3*sqrt(3)*(2*x + (-b/a)^(1/3))/(-b/a)^(1/3))/(a*b) + 1/6*(-a^2*b)
^(1/3)*log(x^2 + x*(-b/a)^(1/3) + (-b/a)^(2/3))/(a*b)
```

maple [A] time = 0.00, size = 91, normalized size = 0.79

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{b}{a}\right)^{\frac{2}{3}}a}+\frac{\ln\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{2}{3}}a}-\frac{\ln\left(x^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}x+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\left(\frac{b}{a}\right)^{\frac{2}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^3)/x^3,x)
```

```
[Out] 1/3/a/(1/a*b)^(2/3)*ln(x+(1/a*b)^(1/3))-1/6/a/(1/a*b)^(2/3)*ln(x^2-(1/a*b)^
(1/3)*x+(1/a*b)^(2/3))+1/3/a/(1/a*b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1
/a*b)^(1/3)*x-1))
```

maxima [A] time = 1.98, size = 98, normalized size = 0.85

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\log\left(x^2-x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\,a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\log\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\,a\left(\frac{b}{a}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)/x^3,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (b/a)^(1/3))/(b/a)^(1/3))/(a*(b/a)^(2
/3)) - 1/6*log(x^2 - x*(b/a)^(1/3) + (b/a)^(2/3))/(a*(b/a)^(2/3)) + 1/3*log
(x + (b/a)^(1/3))/(a*(b/a)^(2/3))
```

mupad [B] time = 1.30, size = 99, normalized size = 0.86

$$\frac{\ln\left(a^{1/3}\,x+b^{1/3}\right)}{3\,a^{1/3}\,b^{2/3}}+\frac{\ln\left(3\,a^2\,x+\frac{3\,a^{5/3}\,b^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{1/3}\,b^{2/3}}-\frac{\ln\left(3\,a^2\,x-\frac{3\,a^{5/3}\,b^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{1/3}\,b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b/x^3)),x)
```

```
[Out] log(a^(1/3)*x + b^(1/3))/(3*a^(1/3)*b^(2/3)) + (log(3*a^2*x + (3*a^(5/3)*b^
(1/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(6*a^(1/3)*b^(2/3)) - (log(3*a
^2*x - (3*a^(5/3)*b^(1/3)*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(6*a^(1/3)
*b^(2/3))
```

sympy [A] time = 0.19, size = 20, normalized size = 0.17

$$\operatorname{RootSum}\left(27t^3ab^2-1,\left(t\mapsto t\log\left(3tb+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**3)/x**3,x)
```

```
[Out] RootSum(27*_t**3*a*b**2 - 1, Lambda(_t, _t*log(3*_t*b + x)))
```

3.1974 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^4}\,dx$

Optimal. Leaf size=15

$$-\frac{\log\left(a+\frac{b}{x^3}\right)}{3b}$$

```
[Out] -1/3*ln(a+b/x^3)/b
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {260}

$$-\frac{\log\left(a+\frac{b}{x^3}\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^3)*x^4),x]
```

```
[Out] -Log[a + b/x^3]/(3*b)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^4}\,dx = -\frac{\log\left(a+\frac{b}{x^3}\right)}{3b}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.47

$$\frac{\log(x)}{b} - \frac{\log\left(ax^3+b\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^3)*x^4),x]
```

```
[Out] Log[x]/b - Log[b + a*x^3]/(3*b)
```

fricas [A] time = 0.85, size = 18, normalized size = 1.20

$$-\frac{\log\left(ax^3+b\right)-3\,\log(x)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(log(a*x^3 + b) - 3*log(x))/b
```

giac [A] time = 0.15, size = 22, normalized size = 1.47

$$-\frac{\log\left(\left|ax^3+b\right|\right)}{3\,b}+\frac{\log\left(|x|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)/x^4,x, algorithm="giac")
[Out] -1/3*log(abs(a*x^3 + b))/b + log(abs(x))/b
```

maple [A] time = 0.00, size = 21, normalized size = 1.40

$$\frac{\ln(x)}{b}-\frac{\ln\left(a\,x^3+b\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^3)/x^4,x)
[Out] -1/3/b*ln(a*x^3+b)+1/b*ln(x)
```

maxima [A] time = 0.89, size = 13, normalized size = 0.87

$$-\frac{\log\left(a+\frac{b}{x^3}\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)/x^4,x, algorithm="maxima")
[Out] -1/3*log(a + b/x^3)/b
```

mupad [B] time = 1.18, size = 18, normalized size = 1.20

$$-\frac{\ln\left(a\,x^3+b\right)-3\,\ln(x)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b/x^3)),x)
[Out] -(log(b + a*x^3) - 3*log(x))/(3*b)
```

sympy [A] time = 0.32, size = 15, normalized size = 1.00

$$\frac{\log\left(x\right)}{b}-\frac{\log\left(x^3+\frac{b}{a}\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**3)/x**4,x)
[Out] log(x)/b - log(x**3 + b/a)/(3*b)
```

3.1975 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^5}\,dx$

Optimal. Leaf size=122

$$-\frac{\sqrt[3]{a}\,\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)}{6b^{4/3}}+\frac{\sqrt[3]{a}\,\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{3b^{4/3}}+\frac{\sqrt[3]{a}\,\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt{3}\,b^{4/3}}-\frac{1}{bx}$$

```
[Out] -1/b/x+1/3*a^(1/3)*ln(b^(1/3)+a^(1/3)*x)/b^(4/3)-1/6*a^(1/3)*ln(b^(2/3)-a^(
1/3)*b^(1/3)*x+a^(2/3)*x^2)/b^(4/3)+1/3*a^(1/3)*arctan(1/3*(b^(1/3)-2*a^(1/
3)*x)/b^(1/3)*3^(1/2))/b^(4/3)*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {263, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{\sqrt[3]{a}\,\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)}{6b^{4/3}}+\frac{\sqrt[3]{a}\,\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{3b^{4/3}}+\frac{\sqrt[3]{a}\,\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt{3}\,b^{4/3}}-\frac{1}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^3)*x^5),x]
```

```
[Out] -(1/(b*x)) + (a^(1/3)*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))])/(S
qrt[3]*b^(4/3)) + (a^(1/3)*Log[b^(1/3) + a^(1/3)*x])/(3*b^(4/3)) - (a^(1/3)
*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(6*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^5}\,dx &= \int \frac{1}{x^2\left(b+ax^3\right)}\,dx \\
&= -\frac{1}{bx} - \frac{a\int \frac{x}{b+ax^3}\,dx}{b} \\
&= -\frac{1}{bx} + \frac{a^{2/3}\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{a}\,x}\,dx}{3b^{4/3}} - \frac{a^{2/3}\int \frac{\sqrt[3]{b}+\sqrt[3]{a}\,x}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{3b^{4/3}} \\
&= -\frac{1}{bx} + \frac{\sqrt[3]{a}\,\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{3b^{4/3}} - \frac{\sqrt[3]{a}\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2a^{2/3}x}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{6b^{4/3}} - \frac{a^{2/3}\int \frac{1}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{2b} \\
&= -\frac{1}{bx} + \frac{\sqrt[3]{a}\,\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{3b^{4/3}} - \frac{\sqrt[3]{a}\,\log\left(b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2\right)}{6b^{4/3}} - \frac{\sqrt[3]{a}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx\right)}{b^{4/3}} \\
&= -\frac{1}{bx} + \frac{\sqrt[3]{a}\,\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt{3}\,b^{4/3}} + \frac{\sqrt[3]{a}\,\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{3b^{4/3}} - \frac{\sqrt[3]{a}\,\log\left(b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2\right)}{6b^{4/3}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 114, normalized size = 0.93

$$\frac{-\sqrt[3]{a}\,x\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)+2\sqrt[3]{a}\,x\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)+2\sqrt{3}\,\sqrt[3]{a}\,x\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}\,x}{\sqrt[3]{b}}}{\sqrt{3}}\right)-6\sqrt[3]{b}}{6b^{4/3}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^3)*x^5),x]
```

```
[Out] (-6*b^(1/3) + 2*Sqrt[3]*a^(1/3)*x*ArcTan[(1 - (2*a^(1/3)*x)/b^(1/3))/Sqrt[3
]] + 2*a^(1/3)*x*Log[b^(1/3) + a^(1/3)*x] - a^(1/3)*x*Log[b^(2/3) - a^(1/3)
*b^(1/3)*x + a^(2/3)*x^2])/(6*b^(4/3)*x)
```

fricas [A] time = 0.78, size = 103, normalized size = 0.84

$$-\frac{2\sqrt{3}\,x\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\,x\left(\frac{a}{b}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+x\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(ax^2-bx\left(\frac{a}{b}\right)^{\frac{2}{3}}+b\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-2\,x\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(ax+b\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+}{6\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)/x^5,x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*x*(a/b)^(1/3)*arctan(2/3*sqrt(3)*x*(a/b)^(1/3) - 1/3*sqrt(3
)) + x*(a/b)^(1/3)*log(a*x^2 - b*x*(a/b)^(2/3) + b*(a/b)^(1/3)) - 2*x*(a/b)
^(1/3)*log(a*x + b*(a/b)^(2/3)) + 6)/(b*x)
```

giac [A] time = 0.21, size = 121, normalized size = 0.99

$$\frac{a\left(-\frac{b}{a}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right|\right)}{3\,b^2}+\frac{\sqrt{3}\,\left(-a^2b\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,ab^2}-\frac{\left(-a^2b\right)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\,ab^2}-\frac{1}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)/x^5,x, algorithm="giac")
```

```
[Out] 1/3*a*(-b/a)^(2/3)*log(abs(x - (-b/a)^(1/3)))/b^2 + 1/3*sqrt(3)*(-a^2*b)^(2
/3)*arctan(1/3*sqrt(3)*(2*x + (-b/a)^(1/3))/(-b/a)^(1/3))/(a*b^2) - 1/6*(-a
^2*b)^(2/3)*log(x^2 + x*(-b/a)^(1/3) + (-b/a)^(2/3))/(a*b^2) - 1/(b*x)
```

maple [A] time = 0.00, size = 99, normalized size = 0.81

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}b}+\frac{\ln\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}b}-\frac{\ln\left(x^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}x+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\left(\frac{b}{a}\right)^{\frac{1}{3}}b}-\frac{1}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^3)/x^5,x)
```

```
[Out] 1/3/b/(1/a*b)^(1/3)*ln(x+(1/a*b)^(1/3))-1/6/b/(1/a*b)^(1/3)*ln(x^2-(1/a*b)^
(1/3)*x+(1/a*b)^(2/3))-1/3/b*3^(1/2)/(1/a*b)^(1/3)*arctan(1/3*3^(1/2)*(2/(1
/a*b)^(1/3)*x-1))-1/b/x
```

maxima [A] time = 1.95, size = 106, normalized size = 0.87

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,b\left(\frac{b}{a}\right)^{\frac{1}{3}}}-\frac{\log\left(x^2-x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{b}{a}\right)^{\frac{1}{3}}}+\frac{\log\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{b}{a}\right)^{\frac{1}{3}}}-\frac{1}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)/x^5,x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (b/a)^(1/3))/(b/a)^(1/3))/(b*(b/a)^(
1/3)) - 1/6*log(x^2 - x*(b/a)^(1/3) + (b/a)^(2/3))/(b*(b/a)^(1/3)) + 1/3*lo
g(x + (b/a)^(1/3))/(b*(b/a)^(1/3)) - 1/(b*x)
```

mupad **[B]** time = 0.25, size = 102, normalized size = 0.84

$$\frac{a^{1/3}\,\ln\left(a^{1/3}\,x+b^{1/3}\right)}{3\,b^{4/3}}-\frac{1}{b\,x}-\frac{a^{1/3}\,\ln\left(4\,a^{1/3}\,x-2\,b^{1/3}+\sqrt{3}\;b^{1/3}\,2\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\;1\mathrm{i}}{2}\right)}{3\,b^{4/3}}+\frac{a^{1/3}\,\ln\left(4\,a^{1/3}\,x-2\,b^{1/3}-\sqrt{3}\;b^{1}\right.}{b^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b/x^3)),x)
```

```
[Out] (a^(1/3)*log(a^(1/3)*x + b^(1/3)))/(3*b^(4/3)) - 1/(b*x) - (a^(1/3)*log(3^(
1/2)*b^(1/3)*2i + 4*a^(1/3)*x - 2*b^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(3*b^(4/
3)) + (a^(1/3)*log(4*a^(1/3)*x - 3^(1/2)*b^(1/3)*2i - 2*b^(1/3))*((3^(1/2)*
1i)/6 - 1/6))/b^(4/3)
```

sympy **[A]** time = 0.23, size = 29, normalized size = 0.24

$$\mathrm{RootSum}\left(27t^3b^4-a,\left(t\mapsto t\log\left(\frac{9t^2b^3}{a}+x\right)\right)\right)-\frac{1}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**3)/x**5,x)
```

```
[Out] RootSum(27*_t**3*b**4 - a, Lambda(_t, _t*log(9*_t**2*b**3/a + x))) - 1/(b*x
)
```

3.1976 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^6}\,dx$

Optimal. Leaf size=124

$$\frac{a^{2/3}\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)}{6b^{5/3}}-\frac{a^{2/3}\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{3b^{5/3}}+\frac{a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt{3}\,b^{5/3}}-\frac{1}{2bx^2}$$

```
[Out] -1/2/b/x^2-1/3*a^(2/3)*ln(b^(1/3)+a^(1/3)*x)/b^(5/3)+1/6*a^(2/3)*ln(b^(2/3)
-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/b^(5/3)+1/3*a^(2/3)*arctan(1/3*(b^(1/3)-2*a
^(1/3)*x)/b^(1/3)*3^(1/2))/b^(5/3)*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {263, 325, 200, 31, 634, 617, 204, 628}

$$\frac{a^{2/3}\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)}{6b^{5/3}}-\frac{a^{2/3}\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{3b^{5/3}}+\frac{a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt{3}\,b^{5/3}}-\frac{1}{2bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^3)*x^6),x]
```

```
[Out] -1/(2*b*x^2) + (a^(2/3)*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))])/
(Sqrt[3]*b^(5/3)) - (a^(2/3)*Log[b^(1/3) + a^(1/3)*x])/(3*b^(5/3)) + (a^(2/
3)*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(6*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^6}\,dx = \int \frac{1}{x^3\left(b+ax^3\right)}\,dx$$

$$= -\frac{1}{2bx^2} - \frac{a\int \frac{1}{b+ax^3}\,dx}{b}$$

$$= -\frac{1}{2bx^2} - \frac{a\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{a}\,x}\,dx}{3b^{5/3}} - \frac{a\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}\,x}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{3b^{5/3}}$$

$$= -\frac{1}{2bx^2} - \frac{a^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{3b^{5/3}} + \frac{a^{2/3}\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2a^{2/3}x}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{6b^{5/3}} - \frac{a\int \frac{1}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{2b^{4/3}}$$

$$= -\frac{1}{2bx^2} - \frac{a^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{3b^{5/3}} + \frac{a^{2/3}\log\left(b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2\right)}{6b^{5/3}} - \frac{a^{2/3}\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\right.}{b^{5/3}}$$

$$= -\frac{1}{2bx^2} + \frac{a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt{3}\,b^{5/3}} - \frac{a^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{3b^{5/3}} + \frac{a^{2/3}\log\left(b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x\right.}{6b^{5/3}}$$

Mathematica [A] time = 0.03, size = 119, normalized size = 0.96

$$\frac{a^{2/3}x^2\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)-2a^{2/3}x^2\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)+2\sqrt{3}\,a^{2/3}x^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}\,x}{\sqrt[3]{b}}}{\sqrt{3}}\right)-3b^{2/3}}{6b^{5/3}x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^3)*x^6),x]
```

```
[Out] (-3*b^(2/3) + 2*Sqrt[3]*a^(2/3)*x^2*ArcTan[(1 - (2*a^(1/3)*x)/b^(1/3))/Sqrt
[3]] - 2*a^(2/3)*x^2*Log[b^(1/3) + a^(1/3)*x] + a^(2/3)*x^2*Log[b^(2/3) - a
^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(6*b^(5/3)*x^2)
```

fricas [A] time = 0.76, size = 143, normalized size = 1.15

$$\frac{2\sqrt{3}\,x^2\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,bx\left(-\frac{a^2}{b^2}\right)^{\frac{2}{3}}-\sqrt{3}\,a}{3a}\right)-x^2\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(a^2x^2+abx\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}+b^2\left(-\frac{a^2}{b^2}\right)^{\frac{2}{3}}\right)+2\,x^2\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(a}{6\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)/x^6,x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*x^2*(-a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a^2/b^2)^(2
/3) - sqrt(3)*a)/a) - x^2*(-a^2/b^2)^(1/3)*log(a^2*x^2 + a*b*x*(-a^2/b^2)^(
1/3) + b^2*(-a^2/b^2)^(2/3)) + 2*x^2*(-a^2/b^2)^(1/3)*log(a*x - b*(-a^2/b^2
)^(1/3)) - 3)/(b*x^2)
```

giac [A] time = 0.16, size = 115, normalized size = 0.93

$$\frac{a\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right|\right)}{3\,b^2}-\frac{\sqrt{3}\,\left(-a^2b\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,b^2}-\frac{\left(-a^2b\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\,b^2}-\frac{1}{2\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)/x^6,x, algorithm="giac")
```

```
[Out] 1/3*a*(-b/a)^(1/3)*log(abs(x - (-b/a)^(1/3)))/b^2 - 1/3*sqrt(3)*(-a^2*b)^(1
/3)*arctan(1/3*sqrt(3)*(2*x + (-b/a)^(1/3))/(-b/a)^(1/3))/b^2 - 1/6*(-a^2*b
)^(1/3)*log(x^2 + x*(-b/a)^(1/3) + (-b/a)^(2/3))/b^2 - 1/2/(b*x^2)
```

maple [A] time = 0.00, size = 99, normalized size = 0.80

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{b}{a}\right)^{\frac{2}{3}}b}-\frac{\ln\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{2}{3}}b}+\frac{\ln\left(x^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}x+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\left(\frac{b}{a}\right)^{\frac{2}{3}}b}-\frac{1}{2b\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^3)/x^6,x)
```

```
[Out] -1/3/b/(1/a*b)^(2/3)*ln(x+(1/a*b)^(1/3))+1/6/b/(1/a*b)^(2/3)*ln(x^2-(1/a*b)
^(1/3)*x+(1/a*b)^(2/3))-1/3/b/(1/a*b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(
1/a*b)^(1/3)*x-1))-1/2/b/x^2
```

maxima [A] time = 1.99, size = 106, normalized size = 0.85

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,b\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\log\left(x^2-x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\log\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{1}{2\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)/x^6,x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (b/a)^(1/3))/(b/a)^(1/3))/(b*(b/a)^(
2/3)) + 1/6*log(x^2 - x*(b/a)^(1/3) + (b/a)^(2/3))/(b*(b/a)^(2/3)) - 1/3*lo
g(x + (b/a)^(1/3))/(b*(b/a)^(2/3)) - 1/2/(b*x^2)
```

mupad **[B]** time = 1.34, size = 128, normalized size = 1.03

$$\frac{a^{2/3}\,\ln\left((-b)^{7/3}-a^{1/3}\,b^2\,x\right)}{3\,(-b)^{5/3}}-\frac{1}{2\,b\,x^2}-\frac{a^{2/3}\,\ln\left(3\,a^3\,b^2\,x+3\,a^{8/3}\,(-b)^{7/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,(-b)^{5/3}}+\frac{a^{2/3}\,\ln\left(3\,a^3\,b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b/x^3)),x)
```

```
[Out] (a^(2/3)*log((-b)^(7/3) - a^(1/3)*b^2*x))/(3*(-b)^(5/3)) - 1/(2*b*x^2) - (a
^(2/3)*log(3*a^3*b^2*x + 3*a^(8/3)*(-b)^(7/3)*((3^(1/2)*1i)/2 + 1/2))*((3^(
1/2)*1i)/2 + 1/2))/(3*(-b)^(5/3)) + (a^(2/3)*log(3*a^3*b^2*x - 9*a^(8/3)*(-
b)^(7/3)*((3^(1/2)*1i)/6 - 1/6))*((3^(1/2)*1i)/6 - 1/6))/(-b)^(5/3)
```

sympy **[A]** time = 0.32, size = 32, normalized size = 0.26

$$\mathrm{RootSum}\left(27t^3b^5+a^2,\left(t\mapsto t\log\left(-\frac{3tb^2}{a}+x\right)\right)\right)-\frac{1}{2bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**3)/x**6,x)
```

```
[Out] RootSum(27*_t**3*b**5 + a**2, Lambda(_t, _t*log(-3*_t*b**2/a + x))) - 1/(2*
b*x**2)
```

3.1977 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^7}\,dx$

Optimal. Leaf size=35

$$\frac{a\log\left(ax^3+b\right)}{3b^2}-\frac{a\log(x)}{b^2}-\frac{1}{3bx^3}$$

```
[Out] -1/3/b/x^3-a*ln(x)/b^2+1/3*a*ln(a*x^3+b)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 266, 44}

$$\frac{a\log\left(ax^3+b\right)}{3b^2}-\frac{a\log(x)}{b^2}-\frac{1}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^3)*x^7),x]
```

```
[Out] -1/(3*b*x^3) - (a*Log[x])/b^2 + (a*Log[b + a*x^3])/(3*b^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^7}\,dx &= \int \frac{1}{x^4\left(b+ax^3\right)}\,dx \\
&= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x^2(b+ax)}\,dx, x, x^3\right) \\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{1}{bx^2}-\frac{a}{b^2x}+\frac{a^2}{b^2(b+ax)}\right)dx, x, x^3\right) \\
&= -\frac{1}{3bx^3}-\frac{a\log(x)}{b^2}+\frac{a\log\left(b+ax^3\right)}{3b^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.00

$$\frac{a\log\left(ax^3+b\right)}{3b^2}-\frac{a\log(x)}{b^2}-\frac{1}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^3)*x^7),x]
```

```
[Out] -1/3*1/(b*x^3) - (a*Log[x])/b^2 + (a*Log[b + a*x^3])/(3*b^2)
```

fricas [A] time = 0.83, size = 33, normalized size = 0.94

$$\frac{ax^3 \log\left(ax^3+b\right) - 3\,ax^3 \log(x) - b}{3\,b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)/x^7,x, algorithm="fricas")
```

```
[Out] 1/3*(a*x^3*log(a*x^3 + b) - 3*a*x^3*log(x) - b)/(b^2*x^3)
```

giac [A] time = 0.19, size = 42, normalized size = 1.20

$$\frac{a\log\left(\left|ax^3+b\right|\right)}{3\,b^2} - \frac{a\log\left(|x|\right)}{b^2} + \frac{ax^3-b}{3\,b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)/x^7,x, algorithm="giac")
```

```
[Out] 1/3*a*log(abs(a*x^3 + b))/b^2 - a*log(abs(x))/b^2 + 1/3*(a*x^3 - b)/(b^2*x^
3)
```

maple [A] time = 0.01, size = 32, normalized size = 0.91

$$-\frac{a\ln(x)}{b^2} + \frac{a\ln\left(a\,x^3+b\right)}{3b^2} - \frac{1}{3b\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^3)/x^7,x)
```

```
[Out] -1/3/b/x^3-a/b^2*ln(x)+1/3*a*ln(a*x^3+b)/b^2
```

maxima [A] time = 0.82, size = 33, normalized size = 0.94

$$\frac{a\log\left(ax^3+b\right)}{3\,b^2} - \frac{a\log\left(x^3\right)}{3\,b^2} - \frac{1}{3\,bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)/x^7,x, algorithm="maxima")
```

```
[Out] 1/3*a*log(a*x^3 + b)/b^2 - 1/3*a*log(x^3)/b^2 - 1/3/(b*x^3)
```

mupad [B] time = 1.16, size = 31, normalized size = 0.89

$$\frac{a\,\ln\left(a\,x^3+b\right)}{3\,b^2} - \frac{1}{3\,b\,x^3} - \frac{a\,\ln(x)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b/x^3)),x)
```

```
[Out] (a*log(b + a*x^3))/(3*b^2) - 1/(3*b*x^3) - (a*log(x))/b^2
```

sympy [**A**] time = 0.44, size = 31, normalized size = 0.89

$$-\frac{a\log(x)}{b^2}+\frac{a\log\left(x^3+\frac{b}{a}\right)}{3b^2}-\frac{1}{3bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**3)/x**7,x)
[Out] -a*log(x)/b**2 + a*log(x**3 + b/a)/(3*b**2) - 1/(3*b*x**3)
```

3.1978 $\int \frac{x^5}{\left(a+\frac{b}{x^3}\right)^2}\,dx$

Optimal. Leaf size=56

$$\frac{b^3}{3a^4\left(ax^3+b\right)}+\frac{b^2\log\left(ax^3+b\right)}{a^4}-\frac{2bx^3}{3a^3}+\frac{x^6}{6a^2}$$

```
[Out] -2/3*b*x^3/a^3+1/6*x^6/a^2+1/3*b^3/a^4/(a*x^3+b)+b^2*ln(a*x^3+b)/a^4
```

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 266, 43}

$$\frac{b^3}{3a^4\left(ax^3+b\right)}+\frac{b^2\log\left(ax^3+b\right)}{a^4}-\frac{2bx^3}{3a^3}+\frac{x^6}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b/x^3)^2,x]
```

```
[Out] (-2*b*x^3)/(3*a^3) + x^6/(6*a^2) + b^3/(3*a^4*(b + a*x^3)) + (b^2*Log[b + a
*x^3])/a^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(a+\frac{b}{x^3}\right)^2}\,dx &= \int \frac{x^{11}}{\left(b+ax^3\right)^2}\,dx \\
&= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^3}{(b+ax)^2}\,dx, x, x^3\right) \\
&= \frac{1}{3}\,\text{Subst}\left(\int \left(-\frac{2b}{a^3}+\frac{x}{a^2}-\frac{b^3}{a^3(b+ax)^2}+\frac{3b^2}{a^3(b+ax)}\right)\,dx, x, x^3\right) \\
&= -\frac{2bx^3}{3a^3}+\frac{x^6}{6a^2}+\frac{b^3}{3a^4\left(b+ax^3\right)}+\frac{b^2\log\left(b+ax^3\right)}{a^4}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.88

$$\frac{a^2x^6 + \frac{2b^3}{ax^3+b} + 6b^2\log\left(ax^3+b\right) - 4abx^3}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b/x^3)^2,x]
```

```
[Out] (-4*a*b*x^3 + a^2*x^6 + (2*b^3)/(b + a*x^3) + 6*b^2*Log[b + a*x^3])/(6*a^4)
```

fricas [A] time = 0.82, size = 70, normalized size = 1.25

$$\frac{a^3x^9 - 3\,a^2bx^6 - 4\,ab^2x^3 + 2\,b^3 + 6\left(ab^2x^3 + b^3\right)\log\left(ax^3+b\right)}{6\left(a^5x^3 + a^4b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/x^3)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(a^3*x^9 - 3*a^2*b*x^6 - 4*a*b^2*x^3 + 2*b^3 + 6*(a*b^2*x^3 + b^3)*log(
a*x^3 + b))/(a^5*x^3 + a^4*b)
```

giac [A] time = 0.16, size = 54, normalized size = 0.96

$$\frac{b^2\log\left(\left|ax^3+b\right|\right)}{a^4} + \frac{b^3}{3\left(ax^3+b\right)a^4} + \frac{a^2x^6 - 4\,abx^3}{6\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/x^3)^2,x, algorithm="giac")
```

```
[Out] b^2*log(abs(a*x^3 + b))/a^4 + 1/3*b^3/((a*x^3 + b)*a^4) + 1/6*(a^2*x^6 - 4*
a*b*x^3)/a^4
```

maple [A] time = 0.01, size = 51, normalized size = 0.91

$$\frac{x^6}{6a^2} - \frac{2b\,x^3}{3a^3} + \frac{b^3}{3\left(a\,x^3+b\right)a^4} + \frac{b^2\ln\left(a\,x^3+b\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a+b/x^3)^2,x)
```

```
[Out] -2/3/a^3*b*x^3+1/6/a^2*x^6+1/3*b^3/a^4/(a*x^3+b)+b^2*ln(a*x^3+b)/a^4
```

maxima [A] time = 0.85, size = 53, normalized size = 0.95

$$\frac{b^3}{3\left(a^5x^3 + a^4b\right)} + \frac{b^2\log\left(ax^3+b\right)}{a^4} + \frac{ax^6 - 4\,bx^3}{6\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/x^3)^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^3/(a^5*x^3 + a^4*b) + b^2*log(a*x^3 + b)/a^4 + 1/6*(a*x^6 - 4*b*x^3)/
a^3
```

mupad [B] time = 1.13, size = 56, normalized size = 1.00

$$\frac{x^6}{6\,a^2} + \frac{b^3}{3\,a\,\left(a^4\,x^3 + b\,a^3\right)} - \frac{2\,b\,x^3}{3\,a^3} + \frac{b^2\,\ln\left(a\,x^3+b\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b/x^3)^2,x)
```

```
[Out] x^6/(6*a^2) + b^3/(3*a*(a^3*b + a^4*x^3)) - (2*b*x^3)/(3*a^3) + (b^2*log(b
+ a*x^3))/a^4
```

sympy [**A**] time = 0.40, size = 53, normalized size = 0.95

$$\frac{b^3}{3a^5x^3 + 3a^4b} + \frac{x^6}{6a^2} - \frac{2bx^3}{3a^3} + \frac{b^2 \log\left(ax^3 + b\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a+b/x**3)**2,x)
```

```
[Out] b**3/(3*a**5*x**3 + 3*a**4*b) + x**6/(6*a**2) - 2*b*x**3/(3*a**3) + b**2*lo
g(a*x**3 + b)/a**4
```

3.1979 $\int \frac{x^4}{\left(a+\frac{b}{x^3}\right)^2}\,dx$

Optimal. Leaf size=157

$$\frac{4b^{5/3}\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)}{9a^{11/3}}-\frac{8b^{5/3}\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{9a^{11/3}}-\frac{8b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{3\sqrt{3}\,a^{11/3}}-\frac{4bx^2}{3a^3}+\frac{8x^5}{15a^2}-\frac{x^8}{3a\left(ax^3+b\right)}$$

```
[Out] -4/3*b*x^2/a^3+8/15*x^5/a^2-1/3*x^8/a/(a*x^3+b)-8/9*b^(5/3)*ln(b^(1/3)+a^(1
/3)*x)/a^(11/3)+4/9*b^(5/3)*ln(b^(2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/a^(11
/3)-8/9*b^(5/3)*arctan(1/3*(b^(1/3)-2*a^(1/3)*x)/b^(1/3)*3^(1/2))/a^(11/3)*
3^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {263, 288, 302, 292, 31, 634, 617, 204, 628}

$$\frac{4b^{5/3}\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)}{9a^{11/3}}-\frac{8b^{5/3}\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{9a^{11/3}}-\frac{8b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{3\sqrt{3}\,a^{11/3}}-\frac{4bx^2}{3a^3}+\frac{8x^5}{15a^2}-\frac{x^8}{3a\left(ax^3+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b/x^3)^2,x]
```

```
[Out] (-4*b*x^2)/(3*a^3) + (8*x^5)/(15*a^2) - x^8/(3*a*(b + a*x^3)) - (8*b^(5/3)*
ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))])/(3*Sqrt[3]*a^(11/3)) - (
8*b^(5/3)*Log[b^(1/3) + a^(1/3)*x])/(9*a^(11/3)) + (4*b^(5/3)*Log[b^(2/3) -
 a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(9*a^(11/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
```

```
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^4}{\left(a+\frac{b}{x^3}\right)^2}\,dx = \int \frac{x^{10}}{\left(b+ax^3\right)^2}\,dx$$

$$= -\frac{x^8}{3a\left(b+ax^3\right)} + \frac{8\int \frac{x^7}{b+ax^3}\,dx}{3a}$$

$$= -\frac{x^8}{3a\left(b+ax^3\right)} + \frac{8\int \left(-\frac{bx}{a^2}+\frac{x^4}{a}+\frac{b^2x}{a^2\left(b+ax^3\right)}\right)dx}{3a}$$

$$= -\frac{4bx^2}{3a^3} + \frac{8x^5}{15a^2} - \frac{x^8}{3a\left(b+ax^3\right)} + \frac{\left(8b^2\right)\int \frac{x}{b+ax^3}\,dx}{3a^3}$$

$$= -\frac{4bx^2}{3a^3} + \frac{8x^5}{15a^2} - \frac{x^8}{3a\left(b+ax^3\right)} - \frac{\left(8b^{5/3}\right)\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{a}\,x}\,dx}{9a^{10/3}} + \frac{\left(8b^{5/3}\right)\int \frac{\sqrt[3]{b}+\sqrt[3]{a}\,x}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{9a^{10/3}}$$

$$= -\frac{4bx^2}{3a^3} + \frac{8x^5}{15a^2} - \frac{x^8}{3a\left(b+ax^3\right)} - \frac{8b^{5/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{9a^{11/3}} + \frac{\left(4b^{5/3}\right)\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2a^{2/3}x}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{9a^{11/3}} + \frac{\left(4b$$

$$= -\frac{4bx^2}{3a^3} + \frac{8x^5}{15a^2} - \frac{x^8}{3a\left(b+ax^3\right)} - \frac{8b^{5/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{9a^{11/3}} + \frac{4b^{5/3}\log\left(b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2\right)}{9a^{11/3}} -$$

$$= -\frac{4bx^2}{3a^3} + \frac{8x^5}{15a^2} - \frac{x^8}{3a\left(b+ax^3\right)} - \frac{8b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{3\sqrt{3}\,a^{11/3}} - \frac{8b^{5/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{9a^{11/3}} + \frac{4b^{5/3}\log\left($$

Mathematica [A] time = 0.11, size = 144, normalized size = 0.92

$$\frac{20b^{5/3}\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)-\frac{15a^{2/3}b^2x^2}{ax^3+b}-45a^{2/3}bx^2+9a^{5/3}x^5-40b^{5/3}\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)-40\sqrt{3}\,b^{5/3}\tan^{-}}{45a^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b/x^3)^2,x]
```

```
[Out] (-45*a^(2/3)*b*x^2 + 9*a^(5/3)*x^5 - (15*a^(2/3)*b^2*x^2)/(b + a*x^3) - 40*
Sqrt[3]*b^(5/3)*ArcTan[(1 - (2*a^(1/3)*x)/b^(1/3))/Sqrt[3]] - 40*b^(5/3)*Lo
g[b^(1/3) + a^(1/3)*x] + 20*b^(5/3)*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/
3)*x^2])/(45*a^(11/3))
```

fricas [A] time = 0.63, size = 190, normalized size = 1.21

$$\frac{9\,a^2x^8-36\,abx^5-60\,b^2x^2+40\,\sqrt{3}\left(abx^3+b^2\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,ax\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+\sqrt{3}\,b}{3\,b}\right)-20\left(abx^3+b^2\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b\right.}{45\left(a^4x^3+a^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/x^3)^2,x, algorithm="fricas")
```

```
[Out] 1/45*(9*a^2*x^8 - 36*a*b*x^5 - 60*b^2*x^2 + 40*sqrt(3)*(a*b*x^3 + b^2)*(-b^
2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(1/3) + sqrt(3)*b)/b) - 2
0*(a*b*x^3 + b^2)*(-b^2/a^2)^(1/3)*log(b*x^2 - a*x*(-b^2/a^2)^(2/3) - b*(-b
^2/a^2)^(1/3)) + 40*(a*b*x^3 + b^2)*(-b^2/a^2)^(1/3)*log(b*x + a*(-b^2/a^2)
^(2/3)))/(a^4*x^3 + a^3*b)
```

giac [A] time = 0.20, size = 151, normalized size = 0.96

$$-\frac{b^2x^2}{3\left(ax^3+b\right)a^3}-\frac{8\,b\left(-\frac{b}{a}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3}-\frac{8\sqrt{3}\,\left(-a^2b\right)^{\frac{2}{3}}\,b\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{9\,a^5}+\frac{4\,\left(-a^2b\right)^{\frac{2}{3}}\,b\log\left(x^2+\right.}{9\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/x^3)^2,x, algorithm="giac")
```

```
[Out] -1/3*b^2*x^2/((a*x^3 + b)*a^3) - 8/9*b*(-b/a)^(2/3)*log(abs(x - (-b/a)^(1/3
)))/a^3 - 8/9*sqrt(3)*(-a^2*b)^(2/3)*b*arctan(1/3*sqrt(3)*(2*x + (-b/a)^(1/
3))/(-b/a)^(1/3))/a^5 + 4/9*(-a^2*b)^(2/3)*b*log(x^2 + x*(-b/a)^(1/3) + (-b
/a)^(2/3))/a^5 + 1/5*(a^8*x^5 - 5*a^7*b*x^2)/a^10
```

maple [A] time = 0.01, size = 137, normalized size = 0.87

$$\frac{x^5}{5a^2}-\frac{b^2x^2}{3\left(a\,x^3+b\right)a^3}-\frac{b\,x^2}{a^3}+\frac{8\sqrt{3}\;b^2\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{b}{a}\right)^{\frac{1}{3}}a^4}-\frac{8b^2\ln\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{9\left(\frac{b}{a}\right)^{\frac{1}{3}}a^4}+\frac{4b^2\ln\left(x^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}x+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{9\left(\frac{b}{a}\right)^{\frac{1}{3}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a+b/x^3)^2,x)
```

```
[Out] 1/5/a^2*x^5-1/a^3*b*x^2-1/3/a^3*b^2*x^2/(a*x^3+b)-8/9/a^4*b^2/(1/a*b)^(1/3)
*ln(x+(1/a*b)^(1/3))+4/9/a^4*b^2/(1/a*b)^(1/3)*ln(x^2-(1/a*b)^(1/3)*x+(1/a*
b)^(2/3))+8/9/a^4*b^2*3^(1/2)/(1/a*b)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/a*b)^(
1/3)*x-1))
```

maxima [A] time = 1.80, size = 147, normalized size = 0.94

$$-\frac{b^2x^2}{3\left(a^4x^3+a^3b\right)}+\frac{8\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{9\,a^4\left(\frac{b}{a}\right)^{\frac{1}{3}}}+\frac{4\,b^2\log\left(x^2-x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{9\,a^4\left(\frac{b}{a}\right)^{\frac{1}{3}}}-\frac{8\,b^2\log\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{9\,a^4\left(\frac{b}{a}\right)^{\frac{1}{3}}}+\frac{ax^5-5}{5\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/x^3)^2,x, algorithm="maxima")
```

```
[Out] -1/3*b^2*x^2/(a^4*x^3 + a^3*b) + 8/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*x -
(b/a)^(1/3))/(b/a)^(1/3))/(a^4*(b/a)^(1/3)) + 4/9*b^2*log(x^2 - x*(b/a)^(1/
3) + (b/a)^(2/3))/(a^4*(b/a)^(1/3)) - 8/9*b^2*log(x + (b/a)^(1/3))/(a^4*(b/
a)^(1/3)) + 1/5*(a*x^5 - 5*b*x^2)/a^3
```

mupad [B] time = 1.29, size = 172, normalized size = 1.10

$$\frac{x^5}{5a^2}-\frac{b^2x^2}{3\left(a^4x^3+ba^3\right)}-\frac{bx^2}{a^3}+\frac{8(-b)^{5/3}\ln\left(\frac{64b^4x}{9a^5}-\frac{64(-b)^{13/3}}{9a^{16/3}}\right)}{9a^{11/3}}+\frac{8(-b)^{5/3}\ln\left(\frac{64b^4x}{9a^5}-\frac{64(-b)^{13/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{9a^{16/3}}\right)\left(-\frac{1}{2}+\frac{\sqrt{}}{}\right.}{9a^{11/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b/x^3)^2,x)
```

```
[Out] x^5/(5*a^2) - (b^2*x^2)/(3*(a^3*b + a^4*x^3)) - (b*x^2)/a^3 + (8*(-b)^(5/3)
*log((64*b^4*x)/(9*a^5) - (64*(-b)^(13/3))/(9*a^(16/3))))/(9*a^(11/3)) + (8
*(-b)^(5/3)*log((64*b^4*x)/(9*a^5) - (64*(-b)^(13/3)*((3^(1/2)*1i)/2 - 1/2)
^2)/(9*a^(16/3)))*((3^(1/2)*1i)/2 - 1/2))/(9*a^(11/3)) - (8*(-b)^(5/3)*log(
(64*b^4*x)/(9*a^5) - (64*(-b)^(13/3)*((3^(1/2)*1i)/2 + 1/2)^2)/(9*a^(16/3))
)*((3^(1/2)*1i)/2 + 1/2))/(9*a^(11/3))
```

sympy [A] time = 0.41, size = 70, normalized size = 0.45

$$-\frac{b^2x^2}{3a^4x^3+3a^3b}+\operatorname{RootSum}\left(729t^3a^{11}+512b^5,\left(t\mapsto t\log\left(\frac{81t^2a^7}{64b^3}+x\right)\right)\right)+\frac{x^5}{5a^2}-\frac{bx^2}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a+b/x**3)**2,x)
```

```
[Out] -b**2*x**2/(3*a**4*x**3 + 3*a**3*b) + RootSum(729*_t**3*a**11 + 512*b**5, L
ambda(_t, _t*log(81*_t**2*a**7/(64*b**3) + x))) + x**5/(5*a**2) - b*x**2/a*
*3
```

3.1980 $\int \frac{x^3}{\left(a+\frac{b}{x^3}\right)^2}\,dx$

Optimal. Leaf size=155

$$-\frac{7b^{4/3}\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)}{18a^{10/3}}+\frac{7b^{4/3}\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{9a^{10/3}}-\frac{7b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{3\sqrt{3}\,a^{10/3}}-\frac{7bx}{3a^3}+\frac{7x^4}{12a^2}-\frac{x^7}{3a\left(ax^3+b\right.}$$

```
[Out] -7/3*b*x/a^3+7/12*x^4/a^2-1/3*x^7/a/(a*x^3+b)+7/9*b^(4/3)*ln(b^(1/3)+a^(1/3
)*x)/a^(10/3)-7/18*b^(4/3)*ln(b^(2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/a^(10/
3)-7/9*b^(4/3)*arctan(1/3*(b^(1/3)-2*a^(1/3)*x)/b^(1/3)*3^(1/2))/a^(10/3)*3
^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {263, 288, 302, 200, 31, 634, 617, 204, 628}

$$-\frac{7b^{4/3}\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)}{18a^{10/3}}+\frac{7b^{4/3}\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{9a^{10/3}}-\frac{7b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{3\sqrt{3}\,a^{10/3}}-\frac{7bx}{3a^3}+\frac{7x^4}{12a^2}-\frac{x^7}{3a\left(ax^3+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b/x^3)^2,x]
```

```
[Out] (-7*b*x)/(3*a^3) + (7*x^4)/(12*a^2) - x^7/(3*a*(b + a*x^3)) - (7*b^(4/3)*Ar
cTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))])/(3*Sqrt[3]*a^(10/3)) + (7*
b^(4/3)*Log[b^(1/3) + a^(1/3)*x])/(9*a^(10/3)) - (7*b^(4/3)*Log[b^(2/3) - a
^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(18*a^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^3}{\left(a+\frac{b}{x^3}\right)^2}\,dx = \int \frac{x^9}{\left(b+ax^3\right)^2}\,dx$$

$$= -\frac{x^7}{3a\left(b+ax^3\right)} + \frac{7\int \frac{x^6}{b+ax^3}\,dx}{3a}$$

$$= -\frac{x^7}{3a\left(b+ax^3\right)} + \frac{7\int \left(-\frac{b}{a^2}+\frac{x^3}{a}+\frac{b^2}{a^2\left(b+ax^3\right)}\right)dx}{3a}$$

$$= -\frac{7bx}{3a^3} + \frac{7x^4}{12a^2} - \frac{x^7}{3a\left(b+ax^3\right)} + \frac{\left(7b^2\right)\int \frac{1}{b+ax^3}\,dx}{3a^3}$$

$$= -\frac{7bx}{3a^3} + \frac{7x^4}{12a^2} - \frac{x^7}{3a\left(b+ax^3\right)} + \frac{\left(7b^{4/3}\right)\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{a}\,x}\,dx}{9a^3} + \frac{\left(7b^{4/3}\right)\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}\,x}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{9a^3}$$

$$= -\frac{7bx}{3a^3} + \frac{7x^4}{12a^2} - \frac{x^7}{3a\left(b+ax^3\right)} + \frac{7b^{4/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{9a^{10/3}} - \frac{\left(7b^{4/3}\right)\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2a^{2/3}x}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{18a^{10/3}} + \frac{\left(7$$

$$= -\frac{7bx}{3a^3} + \frac{7x^4}{12a^2} - \frac{x^7}{3a\left(b+ax^3\right)} + \frac{7b^{4/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{9a^{10/3}} - \frac{7b^{4/3}\log\left(b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2\right)}{18a^{10/3}}$$

$$= -\frac{7bx}{3a^3} + \frac{7x^4}{12a^2} - \frac{x^7}{3a\left(b+ax^3\right)} - \frac{7b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{3\sqrt{3}\,a^{10/3}} + \frac{7b^{4/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{9a^{10/3}} - \frac{7b^{4/3}\log}{}$$

Mathematica [A] time = 0.11, size = 140, normalized size = 0.90

$$\frac{-14b^{4/3}\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)+9a^{4/3}x^4+28b^{4/3}\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)-28\sqrt{3}\,b^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}\,x}{\sqrt[3]{b}}}{\sqrt{3}}\right)-\frac{12\sqrt[3]{a}\,b^{}}{ax^3+b}}{36a^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b/x^3)^2,x]
```

```
[Out] (-72*a^(1/3)*b*x + 9*a^(4/3)*x^4 - (12*a^(1/3)*b^2*x)/(b + a*x^3) - 28*Sqrt
[3]*b^(4/3)*ArcTan[(1 - (2*a^(1/3)*x)/b^(1/3))/Sqrt[3]] + 28*b^(4/3)*Log[b^
(1/3) + a^(1/3)*x] - 14*b^(4/3)*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x
^2])/(36*a^(10/3))
```

fricas [A] time = 0.74, size = 158, normalized size = 1.02

$$\frac{9\,a^2x^7-63\,abx^4-84\,b^2x+28\sqrt{3}\left(abx^3+b^2\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,ax\left(\frac{b}{a}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3\,b}\right)-14\left(abx^3+b^2\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(x^2-x\right.}{36\left(a^4x^3+a^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x^3)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(9*a^2*x^7 - 63*a*b*x^4 - 84*b^2*x + 28*sqrt(3)*(a*b*x^3 + b^2)*(b/a)^
(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(b/a)^(2/3) - sqrt(3)*b)/b) - 14*(a*b*x^3 +
 b^2)*(b/a)^(1/3)*log(x^2 - x*(b/a)^(1/3) + (b/a)^(2/3)) + 28*(a*b*x^3 + b^
2)*(b/a)^(1/3)*log(x + (b/a)^(1/3)))/(a^4*x^3 + a^3*b)
```

giac [A] time = 0.24, size = 147, normalized size = 0.95

$$-\frac{7\,b\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3}-\frac{b^2x}{3\left(ax^3+b\right)a^3}+\frac{7\sqrt{3}\,\left(-a^2b\right)^{\frac{1}{3}}b\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{9\,a^4}+\frac{7\,\left(-a^2b\right)^{\frac{1}{3}}b\log\left(x^2+x\left($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x^3)^2,x, algorithm="giac")
```

```
[Out] -7/9*b*(-b/a)^(1/3)*log(abs(x - (-b/a)^(1/3)))/a^3 - 1/3*b^2*x/((a*x^3 + b)
*a^3) + 7/9*sqrt(3)*(-a^2*b)^(1/3)*b*arctan(1/3*sqrt(3)*(2*x + (-b/a)^(1/3)
)/(-b/a)^(1/3))/a^4 + 7/18*(-a^2*b)^(1/3)*b*log(x^2 + x*(-b/a)^(1/3) + (-b/
a)^(2/3))/a^4 + 1/4*(a^6*x^4 - 8*a^5*b*x)/a^8
```

maple [A] time = 0.01, size = 133, normalized size = 0.86

$$\frac{x^4}{4a^2}-\frac{b^2x}{3\left(a\,x^3+b\right)a^3}-\frac{2bx}{a^3}+\frac{7\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{b}{a}\right)^{\frac{2}{3}}a^4}+\frac{7b^2\ln\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{9\left(\frac{b}{a}\right)^{\frac{2}{3}}a^4}-\frac{7b^2\ln\left(x^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}x+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{18\left(\frac{b}{a}\right)^{\frac{2}{3}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b/x^3)^2,x)
```

```
[Out] 1/4/a^2*x^4-2/a^3*b*x-1/3/a^3*b^2*x/(a*x^3+b)+7/9/a^4*b^2/(1/a*b)^(2/3)*ln(
x+(1/a*b)^(1/3))-7/18/a^4*b^2/(1/a*b)^(2/3)*ln(x^2-(1/a*b)^(1/3)*x+(1/a*b)^
(2/3))+7/9/a^4*b^2/(1/a*b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/a*b)^(1/3
)*x-1))
```

maxima [A] time = 2.01, size = 143, normalized size = 0.92

$$-\frac{b^2x}{3\left(a^4x^3+a^3b\right)}+\frac{7\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{9\,a^4\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{7\,b^2\log\left(x^2-x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{18\,a^4\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{7\,b^2\log\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{9\,a^4\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{ax^4-8\,bx}{4\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x^3)^2,x, algorithm="maxima")
```

```
[Out] -1/3*b^2*x/(a^4*x^3 + a^3*b) + 7/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*x - (b
/a)^(1/3))/(b/a)^(1/3))/(a^4*(b/a)^(2/3)) - 7/18*b^2*log(x^2 - x*(b/a)^(1/3
) + (b/a)^(2/3))/(a^4*(b/a)^(2/3)) + 7/9*b^2*log(x + (b/a)^(1/3))/(a^4*(b/a
)^(2/3)) + 1/4*(a*x^4 - 8*b*x)/a^3
```

mupad [B] time = 0.22, size = 152, normalized size = 0.98

$$\frac{x^4}{4\,a^2}-\frac{b^2\,x}{3\left(a^4\,x^3+b\,a^3\right)}+\frac{7\,b^{4/3}\,\ln\left(\frac{7\,b^{7/3}}{a^{4/3}}+\frac{7\,b^2\,x}{a}\right)}{9\,a^{10/3}}-\frac{2\,b\,x}{a^3}+\frac{7\,b^{4/3}\,\ln\left(\frac{7\,b^2\,x}{a}+\frac{7\,b^{7/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{a^{4/3}}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,a^{10/3}}-\frac{7\,b^{4/3}\,\ln\left(\frac{7\,b^2\,x}{a}+\frac{7\,b^{7/3}\left(-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{a^{4/3}}\right)\left(-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,a^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b/x^3)^2,x)
```

```
[Out] x^4/(4*a^2) - (b^2*x)/(3*(a^3*b + a^4*x^3)) + (7*b^(4/3)*log((7*b^(7/3))/a^
(4/3) + (7*b^2*x)/a))/(9*a^(10/3)) - (2*b*x)/a^3 + (7*b^(4/3)*log((7*b^2*x)
/a + (7*b^(7/3)*((3^(1/2)*1i)/2 - 1/2))/a^(4/3))*((3^(1/2)*1i)/2 - 1/2))/(9
*a^(10/3)) - (7*b^(4/3)*log((7*b^2*x)/a - (7*b^(7/3)*((3^(1/2)*1i)/2 + 1/2)
)/a^(4/3))*((3^(1/2)*1i)/2 + 1/2))/(9*a^(10/3))
```

sympy [A] time = 0.39, size = 65, normalized size = 0.42

$$-\frac{b^2x}{3a^4x^3+3a^3b}+\operatorname{RootSum}\left(729t^3a^{10}-343b^4,\left(t\mapsto t\log\left(\frac{9ta^3}{7b}+x\right)\right)\right)+\frac{x^4}{4a^2}-\frac{2bx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b/x**3)**2,x)
```

```
[Out] -b**2*x/(3*a**4*x**3 + 3*a**3*b) + RootSum(729*_t**3*a**10 - 343*b**4, Lamb
da(_t, _t*log(9*_t*a**3/(7*b) + x))) + x**4/(4*a**2) - 2*b*x/a**3
```

3.1981 $\int \frac{x^2}{\left(a+\frac{b}{x^3}\right)^2}\,dx$

Optimal. Leaf size=46

$$-\frac{b^2}{3a^3\left(ax^3+b\right)}-\frac{2b\log\left(ax^3+b\right)}{3a^3}+\frac{x^3}{3a^2}$$

```
[Out] 1/3*x^3/a^2-1/3*b^2/a^3/(a*x^3+b)-2/3*b*ln(a*x^3+b)/a^3
```

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 266, 43}

$$-\frac{b^2}{3a^3\left(ax^3+b\right)}-\frac{2b\log\left(ax^3+b\right)}{3a^3}+\frac{x^3}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b/x^3)^2,x]
```

```
[Out] x^3/(3*a^2) - b^2/(3*a^3*(b + a*x^3)) - (2*b*Log[b + a*x^3])/(3*a^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(a+\frac{b}{x^3}\right)^2}\,dx &= \int \frac{x^8}{\left(b+ax^3\right)^2}\,dx \\
&= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{(b+ax)^2}\,dx, x, x^3\right) \\
&= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{1}{a^2}+\frac{b^2}{a^2(b+ax)^2}-\frac{2b}{a^2(b+ax)}\right)dx, x, x^3\right) \\
&= \frac{x^3}{3a^2}-\frac{b^2}{3a^3\left(b+ax^3\right)}-\frac{2b\log\left(b+ax^3\right)}{3a^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 0.83

$$\frac{-\frac{b^2}{ax^3+b} - 2b\log\left(ax^3+b\right) + ax^3}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b/x^3)^2,x]
```

```
[Out] (a*x^3 - b^2/(b + a*x^3) - 2*b*Log[b + a*x^3])/(3*a^3)
```

fricas [A] time = 0.97, size = 56, normalized size = 1.22

$$\frac{a^2x^6 + abx^3 - b^2 - 2\left(abx^3+b^2\right)\log\left(ax^3+b\right)}{3\left(a^4x^3+a^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^3)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(a^2*x^6 + a*b*x^3 - b^2 - 2*(a*b*x^3 + b^2)*log(a*x^3 + b))/(a^4*x^3 +
 a^3*b)
```

giac [A] time = 0.21, size = 41, normalized size = 0.89

$$\frac{x^3}{3\,a^2} - \frac{2\,b\log\left(\left|ax^3+b\right|\right)}{3\,a^3} - \frac{b^2}{3\left(ax^3+b\right)a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^3)^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3/a^2 - 2/3*b*log(abs(a*x^3 + b))/a^3 - 1/3*b^2/((a*x^3 + b)*a^3)
```

maple [A] time = 0.01, size = 41, normalized size = 0.89

$$\frac{x^3}{3a^2} - \frac{b^2}{3\left(a\,x^3+b\right)a^3} - \frac{2b\ln\left(a\,x^3+b\right)}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b/x^3)^2,x)
```

```
[Out] 1/3/a^2*x^3-1/3*b^2/a^3/(a*x^3+b)-2/3*b*ln(a*x^3+b)/a^3
```

maxima [A] time = 0.90, size = 43, normalized size = 0.93

$$-\frac{b^2}{3\left(a^4x^3+a^3b\right)} + \frac{x^3}{3\,a^2} - \frac{2\,b\log\left(ax^3+b\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^3)^2,x, algorithm="maxima")
```

```
[Out] -1/3*b^2/(a^4*x^3 + a^3*b) + 1/3*x^3/a^2 - 2/3*b*log(a*x^3 + b)/a^3
```

mupad [B] time = 0.05, size = 45, normalized size = 0.98

$$\frac{x^3}{3\,a^2} - \frac{b^2}{3\,\left(a^4\,x^3+b\,a^3\right)} - \frac{2\,b\,\ln\left(a\,x^3+b\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b/x^3)^2,x)
```

```
[Out] x^3/(3*a^2) - b^2/(3*(a^3*b + a^4*x^3)) - (2*b*log(b + a*x^3))/(3*a^3)
```

sympy [**A**] time = 0.37, size = 42, normalized size = 0.91

$$-\frac{b^2}{3a^4x^3+3a^3b}+\frac{x^3}{3a^2}-\frac{2b\log\left(ax^3+b\right)}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b/x**3)**2,x)
```

```
[Out] -b**2/(3*a**4*x**3 + 3*a**3*b) + x**3/(3*a**2) - 2*b*log(a*x**3 + b)/(3*a**
3)
```

3.1982 $\int \frac{x}{\left(a+\frac{b}{x^3}\right)^2}\,dx$

Optimal. Leaf size=146

$$-\frac{5b^{2/3}\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)}{18a^{8/3}}+\frac{5b^{2/3}\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{9a^{8/3}}+\frac{5b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{3\sqrt{3}\,a^{8/3}}+\frac{5x^2}{6a^2}-\frac{x^5}{3a\left(ax^3+b\right)}$$

```
[Out] 5/6*x^2/a^2-1/3*x^5/a/(a*x^3+b)+5/9*b^(2/3)*ln(b^(1/3)+a^(1/3)*x)/a^(8/3)-5
/18*b^(2/3)*ln(b^(2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/a^(8/3)+5/9*b^(2/3)*a
rctan(1/3*(b^(1/3)-2*a^(1/3)*x)/b^(1/3)*3^(1/2))/a^(8/3)*3^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.818$, Rules used = {263, 288, 321, 292, 31, 634, 617, 204, 628}

$$-\frac{5b^{2/3}\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)}{18a^{8/3}}+\frac{5b^{2/3}\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{9a^{8/3}}+\frac{5b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{3\sqrt{3}\,a^{8/3}}+\frac{5x^2}{6a^2}-\frac{x^5}{3a\left(ax^3+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b/x^3)^2,x]
```

```
[Out] (5*x^2)/(6*a^2) - x^5/(3*a*(b + a*x^3)) + (5*b^(2/3)*ArcTan[(b^(1/3) - 2*a^
(1/3)*x)/(Sqrt[3]*b^(1/3))])/(3*Sqrt[3]*a^(8/3)) + (5*b^(2/3)*Log[b^(1/3) +
 a^(1/3)*x])/(9*a^(8/3)) - (5*b^(2/3)*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(
2/3)*x^2])/(18*a^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
```

```
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x}{\left(a+\frac{b}{x^3}\right)^2}\,dx = \int \frac{x^7}{\left(b+ax^3\right)^2}\,dx$$

$$= -\frac{x^5}{3a\left(b+ax^3\right)} + \frac{5\int \frac{x^4}{b+ax^3}\,dx}{3a}$$

$$= \frac{5x^2}{6a^2} - \frac{x^5}{3a\left(b+ax^3\right)} - \frac{(5b)\int \frac{x}{b+ax^3}\,dx}{3a^2}$$

$$= \frac{5x^2}{6a^2} - \frac{x^5}{3a\left(b+ax^3\right)} + \frac{\left(5b^{2/3}\right)\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{a}\,x}\,dx}{9a^{7/3}} - \frac{\left(5b^{2/3}\right)\int \frac{\sqrt[3]{b}+\sqrt[3]{a}\,x}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{9a^{7/3}}$$

$$= \frac{5x^2}{6a^2} - \frac{x^5}{3a\left(b+ax^3\right)} + \frac{5b^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{9a^{8/3}} - \frac{\left(5b^{2/3}\right)\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2a^{2/3}x}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{18a^{8/3}} - \frac{(5b)\int \frac{}{b^{2/3}-\sqrt[3]{a}}}{6a^{7/}}$$

$$= \frac{5x^2}{6a^2} - \frac{x^5}{3a\left(b+ax^3\right)} + \frac{5b^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{9a^{8/3}} - \frac{5b^{2/3}\log\left(b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2\right)}{18a^{8/3}} - \frac{\left(5b^{2/3}\right)\mathrm{S}}{}$$

$$= \frac{5x^2}{6a^2} - \frac{x^5}{3a\left(b+ax^3\right)} + \frac{5b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{3\sqrt{3}\,a^{8/3}} + \frac{5b^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{9a^{8/3}} - \frac{5b^{2/3}\log\left(b^{2/3}-\sqrt[3]{a}\right.}{18a^{8/3}}$$

Mathematica [A] time = 0.09, size = 131, normalized size = 0.90

$$\frac{-5b^{2/3}\log\left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}\right)+\frac{6a^{2/3}bx^2}{ax^3+b}+9a^{2/3}x^2+10b^{2/3}\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)+10\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{18a^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b/x^3)^2,x]
```

```
[Out] (9*a^(2/3)*x^2 + (6*a^(2/3)*b*x^2)/(b + a*x^3) + 10*Sqrt[3]*b^(2/3)*ArcTan[
(1 - (2*a^(1/3)*x)/b^(1/3))/Sqrt[3]] + 10*b^(2/3)*Log[b^(1/3) + a^(1/3)*x]
- 5*b^(2/3)*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(18*a^(8/3))
```

fricas [A] time = 0.92, size = 163, normalized size = 1.12

$$\frac{9\,ax^5+15\,bx^2-10\sqrt{3}\left(ax^3+b\right)\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,ax\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}-\sqrt{3}\,b}{3\,b}\right)-5\left(ax^3+b\right)\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}+b\left(\right.\right.}{18\left(a^3x^3+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^3)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(9*a*x^5 + 15*b*x^2 - 10*sqrt(3)*(a*x^3 + b)*(b^2/a^2)^(1/3)*arctan(1/
3*(2*sqrt(3)*a*x*(b^2/a^2)^(1/3) - sqrt(3)*b)/b) - 5*(a*x^3 + b)*(b^2/a^2)^
(1/3)*log(b*x^2 - a*x*(b^2/a^2)^(2/3) + b*(b^2/a^2)^(1/3)) + 10*(a*x^3 + b)
*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)^(2/3)))/(a^3*x^3 + a^2*b)
```

giac [A] time = 0.17, size = 132, normalized size = 0.90

$$\frac{x^2}{2\,a^2}+\frac{bx^2}{3\left(ax^3+b\right)a^2}+\frac{5\left(-\frac{b}{a}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2}+\frac{5\sqrt{3}\left(-a^2b\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{9\,a^4}-\frac{5\left(-a^2b\right)^{\frac{2}{3}}\log\left(x^2+\right.}{18\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^3)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2/a^2 + 1/3*b*x^2/((a*x^3 + b)*a^2) + 5/9*(-b/a)^(2/3)*log(abs(x - (-
b/a)^(1/3)))/a^2 + 5/9*sqrt(3)*(-a^2*b)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-b
/a)^(1/3))/(-b/a)^(1/3))/a^4 - 5/18*(-a^2*b)^(2/3)*log(x^2 + x*(-b/a)^(1/3)
 + (-b/a)^(2/3))/a^4
```

maple [A] time = 0.01, size = 120, normalized size = 0.82

$$\frac{b\,x^2}{3\left(a\,x^3+b\right)a^2}+\frac{x^2}{2a^2}-\frac{5\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{b}{a}\right)^{\frac{1}{3}}a^3}+\frac{5b\ln\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{9\left(\frac{b}{a}\right)^{\frac{1}{3}}a^3}-\frac{5b\ln\left(x^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}x+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{18\left(\frac{b}{a}\right)^{\frac{1}{3}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b/x^3)^2,x)
```

```
[Out] 1/2/a^2*x^2+1/3/a^2*b*x^2/(a*x^3+b)+5/9/a^3*b/(1/a*b)^(1/3)*ln(x+(1/a*b)^(1
/3))-5/18/a^3*b/(1/a*b)^(1/3)*ln(x^2-(1/a*b)^(1/3)*x+(1/a*b)^(2/3))-5/9/a^3
*b*3^(1/2)/(1/a*b)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/a*b)^(1/3)*x-1))
```

maxima [A] time = 1.97, size = 130, normalized size = 0.89

$$\frac{bx^2}{3\left(a^3x^3+a^2b\right)}+\frac{x^2}{2a^2}-\frac{5\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{9a^3\left(\frac{b}{a}\right)^{\frac{1}{3}}}-\frac{5b\log\left(x^2-x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{18a^3\left(\frac{b}{a}\right)^{\frac{1}{3}}}+\frac{5b\log\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{9a^3\left(\frac{b}{a}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^3)^2,x, algorithm="maxima")
```

```
[Out] 1/3*b*x^2/(a^3*x^3 + a^2*b) + 1/2*x^2/a^2 - 5/9*sqrt(3)*b*arctan(1/3*sqrt(3
)*(2*x - (b/a)^(1/3))/(b/a)^(1/3))/(a^3*(b/a)^(1/3)) - 5/18*b*log(x^2 - x*(
b/a)^(1/3) + (b/a)^(2/3))/(a^3*(b/a)^(1/3)) + 5/9*b*log(x + (b/a)^(1/3))/(a
^3*(b/a)^(1/3))
```

mupad [B] time = 1.25, size = 139, normalized size = 0.95

$$\frac{x^2}{2a^2}+\frac{5b^{2/3}\,\ln\left(a^{1/3}x+b^{1/3}\right)}{9a^{8/3}}+\frac{bx^2}{3\left(a^3x^3+ba^2\right)}+\frac{5b^{2/3}\,\ln\left(\frac{25b^2x}{9a^3}+\frac{25b^{7/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{9a^{10/3}}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9a^{8/3}}-\frac{5b^{2/3}\,\ln\left(\frac{25b^2}{9a^3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b/x^3)^2,x)
```

```
[Out] x^2/(2*a^2) + (5*b^(2/3)*log(a^(1/3)*x + b^(1/3)))/(9*a^(8/3)) + (b*x^2)/(3
*(a^2*b + a^3*x^3)) + (5*b^(2/3)*log((25*b^2*x)/(9*a^3) + (25*b^(7/3)*((3^(
1/2)*1i)/2 - 1/2)^2)/(9*a^(10/3)))*((3^(1/2)*1i)/2 - 1/2))/(9*a^(8/3)) - (5
*b^(2/3)*log((25*b^2*x)/(9*a^3) + (25*b^(7/3)*((3^(1/2)*1i)/2 + 1/2)^2)/(9*
a^(10/3)))*((3^(1/2)*1i)/2 + 1/2))/(9*a^(8/3))
```

sympy [A] time = 0.42, size = 58, normalized size = 0.40

$$\frac{bx^2}{3a^3x^3+3a^2b}+\operatorname{RootSum}\left(729t^3a^8-125b^2,\left(t\mapsto t\log\left(\frac{81t^2a^5}{25b}+x\right)\right)\right)+\frac{x^2}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x**3)**2,x)
```

```
[Out] b*x**2/(3*a**3*x**3 + 3*a**2*b) + RootSum(729*_t**3*a**8 - 125*b**2, Lambda
(_t, _t*log(81*_t**2*a**5/(25*b) + x))) + x**2/(2*a**2)
```

3.1983 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2}\,dx$

Optimal. Leaf size=144

$$\frac{2\sqrt[3]{b}\,\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)}{9a^{7/3}}-\frac{4\sqrt[3]{b}\,\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{9a^{7/3}}+\frac{4\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{3\sqrt{3}\,a^{7/3}}+\frac{4x}{3a^2}-\frac{x^4}{3a\left(ax^3+b\right)}$$

```
[Out] 4/3*x/a^2-1/3*x^4/a/(a*x^3+b)-4/9*b^(1/3)*ln(b^(1/3)+a^(1/3)*x)/a^(7/3)+2/9
*b^(1/3)*ln(b^(2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/a^(7/3)+4/9*b^(1/3)*arct
an(1/3*(b^(1/3)-2*a^(1/3)*x)/b^(1/3)*3^(1/2))/a^(7/3)*3^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {193, 288, 321, 200, 31, 634, 617, 204, 628}

$$\frac{2\sqrt[3]{b}\,\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)}{9a^{7/3}}-\frac{4\sqrt[3]{b}\,\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{9a^{7/3}}+\frac{4\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{3\sqrt{3}\,a^{7/3}}+\frac{4x}{3a^2}-\frac{x^4}{3a\left(ax^3+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^3)^(-2),x]
```

```
[Out] (4*x)/(3*a^2) - x^4/(3*a*(b + a*x^3)) + (4*b^(1/3)*ArcTan[(b^(1/3) - 2*a^(1
/3)*x)/(Sqrt[3]*b^(1/3))])/(3*Sqrt[3]*a^(7/3)) - (4*b^(1/3)*Log[b^(1/3) + a
^(1/3)*x])/(9*a^(7/3)) + (2*b^(1/3)*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/
3)*x^2])/(9*a^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2}\,dx = \int \frac{x^6}{\left(b+ax^3\right)^2}\,dx$$

$$= -\frac{x^4}{3a\left(b+ax^3\right)} + \frac{4\int \frac{x^3}{b+ax^3}\,dx}{3a}$$

$$= \frac{4x}{3a^2} - \frac{x^4}{3a\left(b+ax^3\right)} - \frac{(4b)\int \frac{1}{b+ax^3}\,dx}{3a^2}$$

$$= \frac{4x}{3a^2} - \frac{x^4}{3a\left(b+ax^3\right)} - \frac{\left(4\sqrt[3]{b}\right)\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{a}\,x}\,dx}{9a^2} - \frac{\left(4\sqrt[3]{b}\right)\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}\,x}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{9a^2}$$

$$= \frac{4x}{3a^2} - \frac{x^4}{3a\left(b+ax^3\right)} - \frac{4\sqrt[3]{b}\,\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{9a^{7/3}} + \frac{\left(2\sqrt[3]{b}\right)\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2a^{2/3}x}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{9a^{7/3}} - \frac{\left(2b^{2/3}\right)\int \frac{}{b^{2/3}-\sqrt[3]{}}}{3a}$$

$$= \frac{4x}{3a^2} - \frac{x^4}{3a\left(b+ax^3\right)} - \frac{4\sqrt[3]{b}\,\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{9a^{7/3}} + \frac{2\sqrt[3]{b}\,\log\left(b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2\right)}{9a^{7/3}} - \frac{\left(4\sqrt[3]{b}\right)\mathrm{Su}}{}$$

$$= \frac{4x}{3a^2} - \frac{x^4}{3a\left(b+ax^3\right)} + \frac{4\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{3\sqrt{3}\,a^{7/3}} - \frac{4\sqrt[3]{b}\,\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{9a^{7/3}} + \frac{2\sqrt[3]{b}\,\log\left(b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{}\right.}{9a^{7/3}}$$

Mathematica [A] time = 0.12, size = 127, normalized size = 0.88

$$\frac{2\sqrt[3]{b}\log\left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)+\frac{3\sqrt[3]{a}bx}{ax^3+b}-4\sqrt[3]{b}\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)+4\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)+9\sqrt[3]{a}x}{9a^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^3)^(-2),x]
```

```
[Out] (9*a^(1/3)*x + (3*a^(1/3)*b*x)/(b + a*x^3) + 4*Sqrt[3]*b^(1/3)*ArcTan[(1 -
(2*a^(1/3)*x)/b^(1/3))/Sqrt[3]] - 4*b^(1/3)*Log[b^(1/3) + a^(1/3)*x] + 2*b^
(1/3)*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(9*a^(7/3))
```

fricas [A] time = 1.06, size = 146, normalized size = 1.01

$$\frac{9\,ax^4+4\sqrt{3}\left(ax^3+b\right)\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b}{a}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)-2\left(ax^3+b\right)\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)+4\left(}{9\left(a^3x^3+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^2,x, algorithm="fricas")
```

```
[Out] 1/9*(9*a*x^4 + 4*sqrt(3)*(a*x^3 + b)*(-b/a)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x
*(-b/a)^(2/3) - sqrt(3)*b)/b) - 2*(a*x^3 + b)*(-b/a)^(1/3)*log(x^2 + x*(-b/
a)^(1/3) + (-b/a)^(2/3)) + 4*(a*x^3 + b)*(-b/a)^(1/3)*log(x - (-b/a)^(1/3))
 + 12*b*x)/(a^3*x^3 + a^2*b)
```

giac [A] time = 0.21, size = 127, normalized size = 0.88

$$\frac{4\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2}+\frac{x}{a^2}+\frac{bx}{3\left(ax^3+b\right)a^2}-\frac{4\sqrt{3}\left(-a^2b\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{9\,a^3}-\frac{2\left(-a^2b\right)^{\frac{1}{3}}\log\left(x^2+x\right.}{9\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^2,x, algorithm="giac")
```

```
[Out] 4/9*(-b/a)^(1/3)*log(abs(x - (-b/a)^(1/3)))/a^2 + x/a^2 + 1/3*b*x/((a*x^3 +
 b)*a^2) - 4/9*sqrt(3)*(-a^2*b)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-b/a)^(1/3
))/(-b/a)^(1/3))/a^3 - 2/9*(-a^2*b)^(1/3)*log(x^2 + x*(-b/a)^(1/3) + (-b/a)
^(2/3))/a^3
```

maple [A] time = 0.01, size = 115, normalized size = 0.80

$$\frac{bx}{3\left(a\,x^3+b\right)a^2}+\frac{x}{a^2}-\frac{4\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{b}{a}\right)^{\frac{2}{3}}a^3}-\frac{4b\ln\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{9\left(\frac{b}{a}\right)^{\frac{2}{3}}a^3}+\frac{2b\ln\left(x^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}x+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{9\left(\frac{b}{a}\right)^{\frac{2}{3}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^3)^2,x)
```

```
[Out] 1/a^2*x+1/3/a^2*b*x/(a*x^3+b)-4/9/a^3*b/(1/a*b)^(2/3)*ln(x+(1/a*b)^(1/3))+2
/9/a^3*b/(1/a*b)^(2/3)*ln(x^2-(1/a*b)^(1/3)*x+(1/a*b)^(2/3))-4/9/a^3*b/(1/a
*b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/a*b)^(1/3)*x-1))
```

maxima [A] time = 1.98, size = 125, normalized size = 0.87

$$\frac{bx}{3\left(a^3x^3+a^2b\right)}+\frac{x}{a^2}-\frac{4\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{9a^3\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{2b\log\left(x^2-x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{9a^3\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{4b\log\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{9a^3\left(\frac{b}{a}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^2,x, algorithm="maxima")
```

```
[Out] 1/3*b*x/(a^3*x^3 + a^2*b) + x/a^2 - 4/9*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*x -
 (b/a)^(1/3))/(b/a)^(1/3))/(a^3*(b/a)^(2/3)) + 2/9*b*log(x^2 - x*(b/a)^(1/3
) + (b/a)^(2/3))/(a^3*(b/a)^(2/3)) - 4/9*b*log(x + (b/a)^(1/3))/(a^3*(b/a)^
(2/3))
```

mupad [B] time = 1.32, size = 132, normalized size = 0.92

$$\frac{x}{a^2}+\frac{4\,(-b)^{1/3}\,\ln\left((-b)^{4/3}+a^{1/3}\,b\,x\right)}{9\,a^{7/3}}+\frac{b\,x}{3\,\left(a^3\,x^3+b\,a^2\right)}-\frac{4\,(-b)^{1/3}\,\ln\left(4\,b\,x-\frac{4\,(-b)^{4/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{a^{1/3}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,a^{7/3}}+\frac{(-b)^{1/3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x^3)^2,x)
```

```
[Out] x/a^2 + (4*(-b)^(1/3)*log((-b)^(4/3) + a^(1/3)*b*x))/(9*a^(7/3)) + (b*x)/(3
*(a^2*b + a^3*x^3)) - (4*(-b)^(1/3)*log(4*b*x - (4*(-b)^(4/3)*((3^(1/2)*1i)
/2 + 1/2))/a^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(9*a^(7/3)) + ((-b)^(1/3)*log(4
*b*x + (9*(-b)^(4/3)*((3^(1/2)*2i)/9 - 2/9))/a^(1/3))*((3^(1/2)*2i)/9 - 2/9
))/a^(7/3)
```

sympy [A] time = 0.34, size = 48, normalized size = 0.33

$$\frac{bx}{3a^3x^3+3a^2b}+\operatorname{RootSum}\left(729t^3a^7+64b,\left(t\mapsto t\log\left(-\frac{9ta^2}{4}+x\right)\right)\right)+\frac{x}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**3)**2,x)
```

```
[Out] b*x/(3*a**3*x**3 + 3*a**2*b) + RootSum(729*_t**3*a**7 + 64*b, Lambda(_t, _t
*log(-9*_t*a**2/4 + x))) + x/a**2
```

3.1984 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x}\,dx$

Optimal. Leaf size=33

$$\frac{b}{3a^2\left(ax^3+b\right)}+\frac{\log\left(ax^3+b\right)}{3a^2}$$

```
[Out] 1/3*b/a^2/(a*x^3+b)+1/3*ln(a*x^3+b)/a^2
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 266, 43}

$$\frac{b}{3a^2\left(ax^3+b\right)}+\frac{\log\left(ax^3+b\right)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^3)^2*x),x]
```

```
[Out] b/(3*a^2*(b + a*x^3)) + Log[b + a*x^3]/(3*a^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x}\,dx &= \int \frac{x^5}{\left(b+ax^3\right)^2}\,dx \\
&= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x}{(b+ax)^2}\,dx, x, x^3\right) \\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(-\frac{b}{a(b+ax)^2}+\frac{1}{a(b+ax)}\right)dx, x, x^3\right) \\
&= \frac{b}{3a^2\left(b+ax^3\right)}+\frac{\log\left(b+ax^3\right)}{3a^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.82

$$\frac{\frac{b}{ax^3+b}+\log\left(ax^3+b\right)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^3)^2*x),x]
[Out] (b/(b + a*x^3) + Log[b + a*x^3])/(3*a^2)
```

fricas [A] time = 0.66, size = 35, normalized size = 1.06

$$\frac{\left(ax^3+b\right)\log\left(ax^3+b\right)+b}{3\left(a^3x^3+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^2/x,x, algorithm="fricas")
[Out] 1/3*((a*x^3 + b)*log(a*x^3 + b) + b)/(a^3*x^3 + a^2*b)
```

giac [A] time = 0.17, size = 32, normalized size = 0.97

$$-\frac{x^3}{3\left(ax^3+b\right)a}+\frac{\log\left(\left|ax^3+b\right|\right)}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^2/x,x, algorithm="giac")
[Out] -1/3*x^3/((a*x^3 + b)*a) + 1/3*log(abs(a*x^3 + b))/a^2
```

maple [A] time = 0.00, size = 30, normalized size = 0.91

$$\frac{b}{3\left(a\,x^3+b\right)a^2}+\frac{\ln\left(a\,x^3+b\right)}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^3)^2/x,x)
[Out] 1/3*b/a^2/(a*x^3+b)+1/3*ln(a*x^3+b)/a^2
```

maxima [A] time = 0.89, size = 32, normalized size = 0.97

$$\frac{b}{3\left(a^3x^3+a^2b\right)}+\frac{\log\left(ax^3+b\right)}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^2/x,x, algorithm="maxima")
[Out] 1/3*b/(a^3*x^3 + a^2*b) + 1/3*log(a*x^3 + b)/a^2
```

mupad [B] time = 0.05, size = 29, normalized size = 0.88

$$\frac{\ln\left(a\,x^3+b\right)}{3\,a^2}+\frac{b}{3\,a^2\,\left(a\,x^3+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b/x^3)^2),x)
[Out] log(b + a*x^3)/(3*a^2) + b/(3*a^2*(b + a*x^3))
```

sympy [**A**] time = 0.38, size = 29, normalized size = 0.88

$$\frac{b}{3a^3x^3 + 3a^2b} + \frac{\log\left(ax^3 + b\right)}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**3)**2/x,x)
```

```
[Out] b/(3*a**3*x**3 + 3*a**2*b) + log(a*x**3 + b)/(3*a**2)
```

3.1985 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^2}\,dx$

Optimal. Leaf size=136

$$\frac{\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)}{9a^{5/3}\sqrt[3]{b}}-\frac{2\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{9a^{5/3}\sqrt[3]{b}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{3\sqrt{3}\,a^{5/3}\sqrt[3]{b}}-\frac{x^2}{3a\left(ax^3+b\right)}$$

```
[Out] -1/3*x^2/a/(a*x^3+b)-2/9*ln(b^(1/3)+a^(1/3)*x)/a^(5/3)/b^(1/3)+1/9*ln(b^(2/
3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/a^(5/3)/b^(1/3)-2/9*arctan(1/3*(b^(1/3)-2
*a^(1/3)*x)/b^(1/3)*3^(1/2))/a^(5/3)/b^(1/3)*3^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {263, 288, 292, 31, 634, 617, 204, 628}

$$\frac{\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)}{9a^{5/3}\sqrt[3]{b}}-\frac{2\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{9a^{5/3}\sqrt[3]{b}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{3\sqrt{3}\,a^{5/3}\sqrt[3]{b}}-\frac{x^2}{3a\left(ax^3+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^3)^2*x^2),x]
```

```
[Out] -x^2/(3*a*(b + a*x^3)) - (2*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3)
)])/(3*Sqrt[3]*a^(5/3)*b^(1/3)) - (2*Log[b^(1/3) + a^(1/3)*x])/(9*a^(5/3)*b
^(1/3)) + Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2]/(9*a^(5/3)*b^(1/3)
)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
```

```
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^2}\,dx = \int \frac{x^4}{\left(b+ax^3\right)^2}\,dx$$

$$= -\frac{x^2}{3a\left(b+ax^3\right)} + \frac{2\int \frac{x}{b+ax^3}\,dx}{3a}$$

$$= -\frac{x^2}{3a\left(b+ax^3\right)} - \frac{2\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{a}\,x}\,dx}{9a^{4/3}\sqrt[3]{b}} + \frac{2\int \frac{\sqrt[3]{b}+\sqrt[3]{a}\,x}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{9a^{4/3}\sqrt[3]{b}}$$

$$= -\frac{x^2}{3a\left(b+ax^3\right)} - \frac{2\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{9a^{5/3}\sqrt[3]{b}} + \frac{\int \frac{1}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{3a^{4/3}} + \frac{\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2a^{2/3}x}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{9a^{5/3}\sqrt[3]{b}}$$

$$= -\frac{x^2}{3a\left(b+ax^3\right)} - \frac{2\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{9a^{5/3}\sqrt[3]{b}} + \frac{\log\left(b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2\right)}{9a^{5/3}\sqrt[3]{b}} + \frac{2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\right.}{3a^{5/3}}$$

$$= -\frac{x^2}{3a\left(b+ax^3\right)} - \frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{3\sqrt{3}\,a^{5/3}\sqrt[3]{b}} - \frac{2\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{9a^{5/3}\sqrt[3]{b}} + \frac{\log\left(b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2\right.}{9a^{5/3}\sqrt[3]{b}}$$

Mathematica [A] time = 0.08, size = 119, normalized size = 0.88

$$\frac{\frac{\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)}{\sqrt[3]{b}} - \frac{3a^{2/3}x^2}{ax^3+b} - \frac{2\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{\sqrt[3]{b}} - \frac{2\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}\,x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{9a^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^3)^2*x^2),x]
```

```
[Out] ((-3*a^(2/3)*x^2)/(b + a*x^3) - (2*Sqrt[3]*ArcTan[(1 - (2*a^(1/3)*x)/b^(1/3
))/Sqrt[3]])/b^(1/3) - (2*Log[b^(1/3) + a^(1/3)*x])/b^(1/3) + Log[b^(2/3) -
 a^(1/3)*b^(1/3)*x + a^(2/3)*x^2]/b^(1/3))/(9*a^(5/3))
```

fricas [A] time = 1.19, size = 400, normalized size = 2.94

$$\left[-\frac{3\,a^2bx^2-3\sqrt{\frac{1}{3}}\left(a^2bx^3+ab^2\right)\sqrt{\frac{\left(-a^2b\right)^{\frac{1}{3}}}{b}}\,\log\left(\frac{2\,a^2x^3-ab+3\sqrt{\frac{1}{3}}\left(abx+2\left(-a^2b\right)^{\frac{2}{3}}x^2+\left(-a^2b\right)^{\frac{1}{3}}b\right)\sqrt{\frac{\left(-a^2b\right)^{\frac{1}{3}}}{b}}-3\left(-a^2b\right)^{\frac{2}{3}}x}{ax^3+b}\right)-\left(ax^3+\right.}{9\left(a^4bx^3+a^3b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^2/x^2,x, algorithm="fricas")
```

```
[Out] [-1/9*(3*a^2*b*x^2 - 3*sqrt(1/3)*(a^2*b*x^3 + a*b^2)*sqrt((-a^2*b)^(1/3)/b)
*log((2*a^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a^2*b)^(2/3)*x^2 + (-a^2*b
)^(1/3)*b)*sqrt((-a^2*b)^(1/3)/b) - 3*(-a^2*b)^(2/3)*x)/(a*x^3 + b)) - (a*x
^3 + b)*(-a^2*b)^(2/3)*log(a^2*x^2 + (-a^2*b)^(1/3)*a*x + (-a^2*b)^(2/3)) +
 2*(a*x^3 + b)*(-a^2*b)^(2/3)*log(a*x - (-a^2*b)^(1/3)))/(a^4*b*x^3 + a^3*b
^2), -1/9*(3*a^2*b*x^2 - 6*sqrt(1/3)*(a^2*b*x^3 + a*b^2)*sqrt(-(-a^2*b)^(1/
3)/b)*arctan(sqrt(1/3)*(2*a*x + (-a^2*b)^(1/3))*sqrt(-(-a^2*b)^(1/3)/b)/a)
- (a*x^3 + b)*(-a^2*b)^(2/3)*log(a^2*x^2 + (-a^2*b)^(1/3)*a*x + (-a^2*b)^(2
/3)) + 2*(a*x^3 + b)*(-a^2*b)^(2/3)*log(a*x - (-a^2*b)^(1/3)))/(a^4*b*x^3 +
 a^3*b^2)]
```

giac [A] time = 0.20, size = 132, normalized size = 0.97

$$-\frac{x^2}{3\left(ax^3+b\right)a}-\frac{2\left(-\frac{b}{a}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right|\right)}{9\,ab}-\frac{2\sqrt{3}\left(-a^2b\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{9\,a^3b}+\frac{\left(-a^2b\right)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\right.}{9\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^2/x^2,x, algorithm="giac")
```

```
[Out] -1/3*x^2/((a*x^3 + b)*a) - 2/9*(-b/a)^(2/3)*log(abs(x - (-b/a)^(1/3)))/(a*b
) - 2/9*sqrt(3)*(-a^2*b)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-b/a)^(1/3))/(-b/
a)^(1/3))/(a^3*b) + 1/9*(-a^2*b)^(2/3)*log(x^2 + x*(-b/a)^(1/3) + (-b/a)^(2
/3))/(a^3*b)
```

maple [A] time = 0.01, size = 108, normalized size = 0.79

$$-\frac{x^2}{3\left(a\,x^3+b\right)a}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{b}{a}\right)^{\frac{1}{3}}a^2}-\frac{2\ln\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{9\left(\frac{b}{a}\right)^{\frac{1}{3}}a^2}+\frac{\ln\left(x^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}x+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{9\left(\frac{b}{a}\right)^{\frac{1}{3}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^3)^2/x^2,x)
```

```
[Out] -1/3*x^2/a/(a*x^3+b)-2/9/a^2/(1/a*b)^(1/3)*ln(x+(1/a*b)^(1/3))+1/9/a^2/(1/a
*b)^(1/3)*ln(x^2-(1/a*b)^(1/3)*x+(1/a*b)^(2/3))+2/9/a^2*3^(1/2)/(1/a*b)^(1/
3)*arctan(1/3*3^(1/2)*(2/(1/a*b)^(1/3)*x-1))
```

maxima [A] time = 1.98, size = 116, normalized size = 0.85

$$-\frac{x^2}{3\left(a^2x^3+ab\right)}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{9a^2\left(\frac{b}{a}\right)^{\frac{1}{3}}}+\frac{\log\left(x^2-x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{9a^2\left(\frac{b}{a}\right)^{\frac{1}{3}}}-\frac{2\log\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{9a^2\left(\frac{b}{a}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^2/x^2,x, algorithm="maxima")
```

```
[Out] -1/3*x^2/(a^2*x^3 + a*b) + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (b/a)^(1/3
))/(b/a)^(1/3))/(a^2*(b/a)^(1/3)) + 1/9*log(x^2 - x*(b/a)^(1/3) + (b/a)^(2/
3))/(a^2*(b/a)^(1/3)) - 2/9*log(x + (b/a)^(1/3))/(a^2*(b/a)^(1/3))
```

mupad [B] time = 0.25, size = 138, normalized size = 1.01

$$\frac{2\ln\left(\frac{4b^{1/3}}{9(-a)^{4/3}}+\frac{4x}{9a}\right)}{9(-a)^{5/3}b^{1/3}}-\frac{x^2}{3a\left(ax^3+b\right)}+\frac{\ln\left(\frac{4x}{9a}+\frac{b^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{9(-a)^{4/3}}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{9(-a)^{5/3}b^{1/3}}-\frac{\ln\left(\frac{4x}{9a}+\frac{b^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{9(-a)^{4/3}}\right)\left(1+\sqrt{3}\,1\right.}{9(-a)^{5/3}b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b/x^3)^2),x)
```

```
[Out] (2*log((4*b^(1/3))/(9*(-a)^(4/3)) + (4*x)/(9*a)))/(9*(-a)^(5/3)*b^(1/3)) -
x^2/(3*a*(b + a*x^3)) + (log((4*x)/(9*a) + (b^(1/3)*(3^(1/2)*1i - 1)^2)/(9*
(-a)^(4/3)))*(3^(1/2)*1i - 1))/(9*(-a)^(5/3)*b^(1/3)) - (log((4*x)/(9*a) +
(b^(1/3)*(3^(1/2)*1i + 1)^2)/(9*(-a)^(4/3)))*(3^(1/2)*1i + 1))/(9*(-a)^(5/3
)*b^(1/3))
```

sympy [A] time = 0.32, size = 44, normalized size = 0.32

$$-\frac{x^2}{3a^2x^3+3ab}+\operatorname{RootSum}\left(729t^3a^5b+8,\left(t\mapsto t\log\left(\frac{81t^2a^3b}{4}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**3)**2/x**2,x)
```

```
[Out] -x**2/(3*a**2*x**3 + 3*a*b) + RootSum(729*_t**3*a**5*b + 8, Lambda(_t, _t*l
og(81*_t**2*a**3*b/4 + x)))
```

3.1986 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^3}\,dx$

Optimal. Leaf size=134

$$-\frac{\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)}{18a^{4/3}b^{2/3}}+\frac{\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{9a^{4/3}b^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{3\sqrt{3}\,a^{4/3}b^{2/3}}-\frac{x}{3a\left(ax^3+b\right)}$$

```
[Out] -1/3*x/a/(a*x^3+b)+1/9*ln(b^(1/3)+a^(1/3)*x)/a^(4/3)/b^(2/3)-1/18*ln(b^(2/3
)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/a^(4/3)/b^(2/3)-1/9*arctan(1/3*(b^(1/3)-2*
a^(1/3)*x)/b^(1/3)*3^(1/2))/a^(4/3)/b^(2/3)*3^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {263, 288, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)}{18a^{4/3}b^{2/3}}+\frac{\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{9a^{4/3}b^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{3\sqrt{3}\,a^{4/3}b^{2/3}}-\frac{x}{3a\left(ax^3+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^3)^2*x^3),x]
```

```
[Out] -x/(3*a*(b + a*x^3)) - ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))]/(3
*Sqrt[3]*a^(4/3)*b^(2/3)) + Log[b^(1/3) + a^(1/3)*x]/(9*a^(4/3)*b^(2/3)) -
Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2]/(18*a^(4/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^3}\,dx = \int \frac{x^3}{\left(b+ax^3\right)^2}\,dx$$

$$= -\frac{x}{3a\left(b+ax^3\right)} + \frac{\int \frac{1}{b+ax^3}\,dx}{3a}$$

$$= -\frac{x}{3a\left(b+ax^3\right)} + \frac{\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{a}\,x}\,dx}{9ab^{2/3}} + \frac{\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}\,x}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{9ab^{2/3}}$$

$$= -\frac{x}{3a\left(b+ax^3\right)} + \frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{9a^{4/3}b^{2/3}} - \frac{\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2a^{2/3}x}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{18a^{4/3}b^{2/3}} + \frac{\int \frac{1}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{6a\sqrt[3]{b}}$$

$$= -\frac{x}{3a\left(b+ax^3\right)} + \frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{9a^{4/3}b^{2/3}} - \frac{\log\left(b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2\right)}{18a^{4/3}b^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,}{3a^{4/3}b^{2/}}$$

$$= -\frac{x}{3a\left(b+ax^3\right)} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{3\sqrt{3}\,a^{4/3}b^{2/3}} + \frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{9a^{4/3}b^{2/3}} - \frac{\log\left(b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2\right)}{18a^{4/3}b^{2/3}}$$

Mathematica [A] time = 0.07, size = 118, normalized size = 0.88

$$\frac{-\frac{\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)}{b^{2/3}} + \frac{2\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{b^{2/3}} - \frac{2\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}\,x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{b^{2/3}} - \frac{6\sqrt[3]{a}\,x}{ax^3+b}}{18a^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^3)^2*x^3),x]
```

```
[Out] ((-6*a^(1/3)*x)/(b + a*x^3) - (2*Sqrt[3]*ArcTan[(1 - (2*a^(1/3)*x)/b^(1/3))
/Sqrt[3]])/b^(2/3) + (2*Log[b^(1/3) + a^(1/3)*x])/b^(2/3) - Log[b^(2/3) - a
^(1/3)*b^(1/3)*x + a^(2/3)*x^2]/b^(2/3))/(18*a^(4/3))
```

fricas [A] time = 1.27, size = 391, normalized size = 2.92

$$\left[-\frac{6\,ab^2x-3\,\sqrt{\frac{1}{3}}\left(a^2bx^3+ab^2\right)\sqrt{-\frac{\left(ab^2\right)^{\frac{1}{3}}}{a}}\,\log\left(\frac{2\,abx^3-3\left(ab^2\right)^{\frac{1}{3}}bx-b^2+3\,\sqrt{\frac{1}{3}}\left(2\,abx^2+\left(ab^2\right)^{\frac{2}{3}}x-\left(ab^2\right)^{\frac{1}{3}}b\right)\sqrt{-\frac{\left(ab^2\right)^{\frac{1}{3}}}{a}}}{ax^3+b}\right)+\left(ax^3+b\right)\right.}{18\left(a^3b^2x^3+a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^2/x^3,x, algorithm="fricas")
```

```
[Out] [-1/18*(6*a*b^2*x - 3*sqrt(1/3)*(a^2*b*x^3 + a*b^2)*sqrt(-(a*b^2)^(1/3)/a)*
log((2*a*b*x^3 - 3*(a*b^2)^(1/3)*b*x - b^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a*b^
2)^(2/3)*x - (a*b^2)^(1/3)*b)*sqrt(-(a*b^2)^(1/3)/a))/(a*x^3 + b)) + (a*x^3
 + b)*(a*b^2)^(2/3)*log(a*b*x^2 - (a*b^2)^(2/3)*x + (a*b^2)^(1/3)*b) - 2*(a
*x^3 + b)*(a*b^2)^(2/3)*log(a*b*x + (a*b^2)^(2/3)))/(a^3*b^2*x^3 + a^2*b^3)
, -1/18*(6*a*b^2*x - 6*sqrt(1/3)*(a^2*b*x^3 + a*b^2)*sqrt((a*b^2)^(1/3)/a)*
arctan(sqrt(1/3)*(2*(a*b^2)^(2/3)*x - (a*b^2)^(1/3)*b)*sqrt((a*b^2)^(1/3)/a
)/b^2) + (a*x^3 + b)*(a*b^2)^(2/3)*log(a*b*x^2 - (a*b^2)^(2/3)*x + (a*b^2)^
(1/3)*b) - 2*(a*x^3 + b)*(a*b^2)^(2/3)*log(a*b*x + (a*b^2)^(2/3)))/(a^3*b^2
*x^3 + a^2*b^3)]
```

giac [A] time = 0.17, size = 130, normalized size = 0.97

$$-\frac{\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right|\right)}{9\,ab}-\frac{x}{3\left(ax^3+b\right)a}+\frac{\sqrt{3}\,\left(-a^2b\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{9\,a^2b}+\frac{\left(-a^2b\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\left(-\right.\right.}{18\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^2/x^3,x, algorithm="giac")
```

```
[Out] -1/9*(-b/a)^(1/3)*log(abs(x - (-b/a)^(1/3)))/(a*b) - 1/3*x/((a*x^3 + b)*a)
+ 1/9*sqrt(3)*(-a^2*b)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-b/a)^(1/3))/(-b/a)
^(1/3))/(a^2*b) + 1/18*(-a^2*b)^(1/3)*log(x^2 + x*(-b/a)^(1/3) + (-b/a)^(2/
3))/(a^2*b)
```

maple [A] time = 0.01, size = 106, normalized size = 0.79

$$-\frac{x}{3\left(a\,x^3+b\right)a}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{b}{a}\right)^{\frac{2}{3}}a^2}+\frac{\ln\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{9\left(\frac{b}{a}\right)^{\frac{2}{3}}a^2}-\frac{\ln\left(x^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}x+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{18\left(\frac{b}{a}\right)^{\frac{2}{3}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^3)^2/x^3,x)
```

```
[Out] -1/3*x/a/(a*x^3+b)+1/9/a^2/(1/a*b)^(2/3)*ln(x+(1/a*b)^(1/3))-1/18/a^2/(1/a*
b)^(2/3)*ln(x^2-(1/a*b)^(1/3)*x+(1/a*b)^(2/3))+1/9/a^2/(1/a*b)^(2/3)*3^(1/2
)*arctan(1/3*3^(1/2)*(2/(1/a*b)^(1/3)*x-1))
```

maxima [A] time = 1.98, size = 114, normalized size = 0.85

$$-\frac{x}{3\left(a^2x^3+ab\right)}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{9a^2\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\log\left(x^2-x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{18a^2\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\log\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{9a^2\left(\frac{b}{a}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/3*x/(a^2*x^3 + a*b) + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (b/a)^(1/3))
/(b/a)^(1/3))/(a^2*(b/a)^(2/3)) - 1/18*log(x^2 - x*(b/a)^(1/3) + (b/a)^(2/3
))/(a^2*(b/a)^(2/3)) + 1/9*log(x + (b/a)^(1/3))/(a^2*(b/a)^(2/3))
```

mupad [B] time = 1.33, size = 108, normalized size = 0.81

$$\frac{\ln\left(a^{1/3}x+b^{1/3}\right)}{9a^{4/3}b^{2/3}}-\frac{x}{3a\left(ax^3+b\right)}+\frac{\ln\left(ax+\frac{a^{2/3}b^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{18a^{4/3}b^{2/3}}-\frac{\ln\left(ax-\frac{a^{2/3}b^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(1+\sqrt{3}\right.}{18a^{4/3}b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b/x^3)^2),x)
```

```
[Out] log(a^(1/3)*x + b^(1/3))/(9*a^(4/3)*b^(2/3)) - x/(3*a*(b + a*x^3)) + (log(a
*x + (a^(2/3)*b^(1/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(18*a^(4/3)*b^
(2/3)) - (log(a*x - (a^(2/3)*b^(1/3)*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))
/(18*a^(4/3)*b^(2/3))
```

sympy [A] time = 0.31, size = 39, normalized size = 0.29

$$-\frac{x}{3a^2x^3+3ab}+\operatorname{RootSum}\left(729t^3a^4b^2-1,\left(t\mapsto t\log\left(9tab+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**3)**2/x**3,x)
```

```
[Out] -x/(3*a**2*x**3 + 3*a*b) + RootSum(729*_t**3*a**4*b**2 - 1, Lambda(_t, _t*l
og(9*_t*a*b + x)))
```

3.1987 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^4}\,dx$

Optimal. Leaf size=16

$$\frac{1}{3b\left(a+\frac{b}{x^3}\right)}$$

```
[Out] 1/3/b/(a+b/x^3)
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$\frac{1}{3b\left(a+\frac{b}{x^3}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^3)^2*x^4),x]
```

```
[Out] 1/(3*b*(a + b/x^3))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^4}\,dx = \frac{1}{3b\left(a+\frac{b}{x^3}\right)}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$-\frac{1}{3a\left(ax^3+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^3)^2*x^4),x]
```

```
[Out] -1/3*1/(a*(b + a*x^3))
```

fricas [A] time = 0.84, size = 15, normalized size = 0.94

$$-\frac{1}{3\left(a^2x^3+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^2/x^4,x, algorithm="fricas")
```

```
[Out] -1/3/(a^2*x^3 + a*b)
```

giac [A] time = 0.16, size = 14, normalized size = 0.88

$$-\frac{1}{3\left(ax^3+b\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^2/x^4,x, algorithm="giac")
[Out] -1/3/((a*x^3 + b)*a)
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$-\frac{1}{3\left(a\,x^3+b\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^3)^2/x^4,x)
[Out] -1/3/a/(a*x^3+b)
```

maxima [A] time = 0.85, size = 14, normalized size = 0.88

$$\frac{1}{3\left(a+\frac{b}{x^3}\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^2/x^4,x, algorithm="maxima")
[Out] 1/3/((a + b/x^3)*b)
```

mupad [B] time = 1.10, size = 14, normalized size = 0.88

$$-\frac{1}{3\,a\,\left(a\,x^3+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b/x^3)^2),x)
[Out] -1/(3*a*(b + a*x^3))
```

sympy [A] time = 0.27, size = 15, normalized size = 0.94

$$-\frac{1}{3a^2x^3+3ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**3)**2/x**4,x)
[Out] -1/(3*a**2*x**3 + 3*a*b)
```

3.1988 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^5}\,dx$

Optimal. Leaf size=136

$$\frac{\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)}{18a^{2/3}b^{4/3}}-\frac{\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{9a^{2/3}b^{4/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{3\sqrt{3}\,a^{2/3}b^{4/3}}+\frac{x^2}{3b\left(ax^3+b\right)}$$

```
[Out] 1/3*x^2/b/(a*x^3+b)-1/9*ln(b^(1/3)+a^(1/3)*x)/a^(2/3)/b^(4/3)+1/18*ln(b^(2/
3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/a^(2/3)/b^(4/3)-1/9*arctan(1/3*(b^(1/3)-2
*a^(1/3)*x)/b^(1/3)*3^(1/2))/a^(2/3)/b^(4/3)*3^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {263, 290, 292, 31, 634, 617, 204, 628}

$$\frac{\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)}{18a^{2/3}b^{4/3}}-\frac{\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{9a^{2/3}b^{4/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{3\sqrt{3}\,a^{2/3}b^{4/3}}+\frac{x^2}{3b\left(ax^3+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^3)^2*x^5),x]
```

```
[Out] x^2/(3*b*(b + a*x^3)) - ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))]/(
3*Sqrt[3]*a^(2/3)*b^(4/3)) - Log[b^(1/3) + a^(1/3)*x]/(9*a^(2/3)*b^(4/3)) +
 Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2]/(18*a^(2/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
```

```
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^5}\,dx &= \int \frac{x}{\left(b+ax^3\right)^2}\,dx \\
&= \frac{x^2}{3b\left(b+ax^3\right)} + \frac{\int \frac{x}{b+ax^3}\,dx}{3b} \\
&= \frac{x^2}{3b\left(b+ax^3\right)} - \frac{\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{a}\,x}\,dx}{9\sqrt[3]{a}\,b^{4/3}} + \frac{\int \frac{\sqrt[3]{b}+\sqrt[3]{a}\,x}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{9\sqrt[3]{a}\,b^{4/3}} \\
&= \frac{x^2}{3b\left(b+ax^3\right)} - \frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{9a^{2/3}b^{4/3}} + \frac{\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2a^{2/3}x}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{18a^{2/3}b^{4/3}} + \frac{\int \frac{1}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{6\sqrt[3]{a}\,b} \\
&= \frac{x^2}{3b\left(b+ax^3\right)} - \frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{9a^{2/3}b^{4/3}} + \frac{\log\left(b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2\right)}{18a^{2/3}b^{4/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x\right.}{3a^{2/3}b^{4/3}} \\
&= \frac{x^2}{3b\left(b+ax^3\right)} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{3\sqrt{3}\,a^{2/3}b^{4/3}} - \frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{9a^{2/3}b^{4/3}} + \frac{\log\left(b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2\right)}{18a^{2/3}b^{4/3}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 119, normalized size = 0.88

$$\frac{\frac{\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)}{a^{2/3}} - \frac{2\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{a^{2/3}} - \frac{2\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}\,x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{a^{2/3}} + \frac{6\sqrt[3]{b}\,x^2}{ax^3+b}}{18b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^3)^2*x^5),x]
```

```
[Out] ((6*b^(1/3)*x^2)/(b + a*x^3) - (2*Sqrt[3]*ArcTan[(1 - (2*a^(1/3)*x)/b^(1/3)
)/Sqrt[3]])/a^(2/3) - (2*Log[b^(1/3) + a^(1/3)*x])/a^(2/3) + Log[b^(2/3) -
a^(1/3)*b^(1/3)*x + a^(2/3)*x^2]/a^(2/3))/(18*b^(4/3))
```

fricas [A] time = 0.90, size = 402, normalized size = 2.96

$$\left[\frac{6\,a^2bx^2+3\,\sqrt{\frac{1}{3}}\left(a^2bx^3+ab^2\right)\sqrt{\frac{\left(-a^2b\right)^{\frac{1}{3}}}{b}}\,\log\left(\frac{2\,a^2x^3-ab+3\,\sqrt{\frac{1}{3}}\left(abx+2\left(-a^2b\right)^{\frac{2}{3}}x^2+\left(-a^2b\right)^{\frac{1}{3}}b\right)\sqrt{\frac{\left(-a^2b\right)^{\frac{1}{3}}}{b}}-3\left(-a^2b\right)^{\frac{2}{3}}x}{ax^3+b}\right)+\left(ax^3+b\right.}{18\left(a^3b^2x^3+a^2b^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^2/x^5,x, algorithm="fricas")
```

```
[Out] [1/18*(6*a^2*b*x^2 + 3*sqrt(1/3)*(a^2*b*x^3 + a*b^2)*sqrt((-a^2*b)^(1/3)/b)
*log((2*a^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a^2*b)^(2/3)*x^2 + (-a^2*b
)^(1/3)*b)*sqrt((-a^2*b)^(1/3)/b) - 3*(-a^2*b)^(2/3)*x)/(a*x^3 + b)) + (a*x
^3 + b)*(-a^2*b)^(2/3)*log(a^2*x^2 + (-a^2*b)^(1/3)*a*x + (-a^2*b)^(2/3)) -
 2*(a*x^3 + b)*(-a^2*b)^(2/3)*log(a*x - (-a^2*b)^(1/3)))/(a^3*b^2*x^3 + a^2
*b^3), 1/18*(6*a^2*b*x^2 + 6*sqrt(1/3)*(a^2*b*x^3 + a*b^2)*sqrt(-(-a^2*b)^(
1/3)/b)*arctan(sqrt(1/3)*(2*a*x + (-a^2*b)^(1/3))*sqrt(-(-a^2*b)^(1/3)/b)/a
) + (a*x^3 + b)*(-a^2*b)^(2/3)*log(a^2*x^2 + (-a^2*b)^(1/3)*a*x + (-a^2*b)^
(2/3)) - 2*(a*x^3 + b)*(-a^2*b)^(2/3)*log(a*x - (-a^2*b)^(1/3)))/(a^3*b^2*x
^3 + a^2*b^3)]
```

giac [A] time = 0.18, size = 129, normalized size = 0.95

$$\frac{x^2}{3\left(ax^3+b\right)b}-\frac{\left(-\frac{b}{a}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right|\right)}{9\,b^2}-\frac{\sqrt{3}\,\left(-a^2b\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{9\,a^2b^2}+\frac{\left(-a^2b\right)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\left(-\frac{b}{a}\right)\right.}{18\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^2/x^5,x, algorithm="giac")
```

```
[Out] 1/3*x^2/((a*x^3 + b)*b) - 1/9*(-b/a)^(2/3)*log(abs(x - (-b/a)^(1/3)))/b^2 -
 1/9*sqrt(3)*(-a^2*b)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-b/a)^(1/3))/(-b/a)^
(1/3))/(a^2*b^2) + 1/18*(-a^2*b)^(2/3)*log(x^2 + x*(-b/a)^(1/3) + (-b/a)^(2
/3))/(a^2*b^2)
```

maple [A] time = 0.00, size = 117, normalized size = 0.86

$$\frac{x^2}{3\left(a\,x^3+b\right)b}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{b}{a}\right)^{\frac{1}{3}}ab}-\frac{\ln\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{9\left(\frac{b}{a}\right)^{\frac{1}{3}}ab}+\frac{\ln\left(x^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}x+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{18\left(\frac{b}{a}\right)^{\frac{1}{3}}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^3)^2/x^5,x)
```

```
[Out] 1/3*x^2/b/(a*x^3+b)-1/9/b/a/(1/a*b)^(1/3)*ln(x+(1/a*b)^(1/3))+1/18/b/a/(1/a
*b)^(1/3)*ln(x^2-(1/a*b)^(1/3)*x+(1/a*b)^(2/3))+1/9/b*3^(1/2)/a/(1/a*b)^(1/
3)*arctan(1/3*3^(1/2)*(2/(1/a*b)^(1/3)*x-1))
```

maxima [A] time = 1.86, size = 124, normalized size = 0.91

$$\frac{x^2}{3\left(abx^3+b^2\right)}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{9ab\left(\frac{b}{a}\right)^{\frac{1}{3}}}+\frac{\log\left(x^2-x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{18ab\left(\frac{b}{a}\right)^{\frac{1}{3}}}-\frac{\log\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{9ab\left(\frac{b}{a}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/3*x^2/(a*b*x^3 + b^2) + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (b/a)^(1/3)
)/(b/a)^(1/3))/(a*b*(b/a)^(1/3)) + 1/18*log(x^2 - x*(b/a)^(1/3) + (b/a)^(2/
3))/(a*b*(b/a)^(1/3)) - 1/9*log(x + (b/a)^(1/3))/(a*b*(b/a)^(1/3))
```

mupad [B] time = 1.32, size = 138, normalized size = 1.01

$$\frac{x^2}{3b\left(ax^3+b\right)}+\frac{(-1)^{1/3}\ln\left(\frac{(-1)^{2/3}a^{2/3}}{9b^{5/3}}+\frac{ax}{9b^2}\right)}{9a^{2/3}b^{4/3}}-\frac{(-1)^{1/3}\ln\left((-1)^{2/3}b^{1/3}-2a^{1/3}x+(-1)^{1/6}\sqrt{3}\,b^{1/3}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9a^{2/3}b^{4/3}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b/x^3)^2),x)
```

```
[Out] x^2/(3*b*(b + a*x^3)) + ((-1)^(1/3)*log(((-1)^(2/3)*a^(2/3))/(9*b^(5/3)) +
(a*x)/(9*b^2)))/(9*a^(2/3)*b^(4/3)) - ((-1)^(1/3)*log((-1)^(2/3)*b^(1/3) -
2*a^(1/3)*x + (-1)^(1/6)*3^(1/2)*b^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(9*a^(2/3
)*b^(4/3)) + ((-1)^(1/3)*log(2*a^(1/3)*x - (-1)^(2/3)*b^(1/3) + (-1)^(1/6)*
3^(1/2)*b^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(9*a^(2/3)*b^(4/3))
```

sympy [A] time = 0.50, size = 44, normalized size = 0.32

$$\frac{x^2}{3abx^3+3b^2}+\operatorname{RootSum}\left(729t^3a^2b^4+1,\left(t\mapsto t\log\left(81t^2ab^3+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**3)**2/x**5,x)
```

```
[Out] x**2/(3*a*b*x**3 + 3*b**2) + RootSum(729*_t**3*a**2*b**4 + 1, Lambda(_t, _t
*log(81*_t**2*a*b**3 + x)))
```

3.1989 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^6}\,dx$

Optimal. Leaf size=134

$$-\frac{\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)}{9\sqrt[3]{a}\,b^{5/3}}+\frac{2\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{9\sqrt[3]{a}\,b^{5/3}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{3\sqrt{3}\,\sqrt[3]{a}\,b^{5/3}}+\frac{x}{3b\left(ax^3+b\right)}$$

```
[Out] 1/3*x/b/(a*x^3+b)+2/9*ln(b^(1/3)+a^(1/3)*x)/a^(1/3)/b^(5/3)-1/9*ln(b^(2/3)-
a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/a^(1/3)/b^(5/3)-2/9*arctan(1/3*(b^(1/3)-2*a^
(1/3)*x)/b^(1/3)*3^(1/2))/a^(1/3)/b^(5/3)*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {263, 199, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)}{9\sqrt[3]{a}\,b^{5/3}}+\frac{2\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{9\sqrt[3]{a}\,b^{5/3}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{3\sqrt{3}\,\sqrt[3]{a}\,b^{5/3}}+\frac{x}{3b\left(ax^3+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^3)^2*x^6),x]
```

```
[Out] x/(3*b*(b + a*x^3)) - (2*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))])
/(3*Sqrt[3]*a^(1/3)*b^(5/3)) + (2*Log[b^(1/3) + a^(1/3)*x])/(9*a^(1/3)*b^(5
/3)) - Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2]/(9*a^(1/3)*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^6}\,dx = \int \frac{1}{\left(b+ax^3\right)^2}\,dx$$

$$= \frac{x}{3b\left(b+ax^3\right)} + \frac{2\int \frac{1}{b+ax^3}\,dx}{3b}$$

$$= \frac{x}{3b\left(b+ax^3\right)} + \frac{2\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{a}\,x}\,dx}{9b^{5/3}} + \frac{2\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}\,x}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{9b^{5/3}}$$

$$= \frac{x}{3b\left(b+ax^3\right)} + \frac{2\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{9\sqrt[3]{a}\,b^{5/3}} - \frac{\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2a^{2/3}x}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{9\sqrt[3]{a}\,b^{5/3}} + \frac{\int \frac{1}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{3b^{4/3}}$$

$$= \frac{x}{3b\left(b+ax^3\right)} + \frac{2\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{9\sqrt[3]{a}\,b^{5/3}} - \frac{\log\left(b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2\right)}{9\sqrt[3]{a}\,b^{5/3}} + \frac{2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,d\right.}{3\sqrt[3]{a}\,b^{5}}$$

$$= \frac{x}{3b\left(b+ax^3\right)} - \frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{3\sqrt{3}\,\sqrt[3]{a}\,b^{5/3}} + \frac{2\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x\right)}{9\sqrt[3]{a}\,b^{5/3}} - \frac{\log\left(b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2\right)}{9\sqrt[3]{a}\,b^{5/3}}$$

Mathematica [A] time = 0.07, size = 118, normalized size = 0.88

$$\frac{-\frac{\log\left(a^{2/3}x^2-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)}{\sqrt[3]{a}} + \frac{3b^{2/3}x}{ax^3+b} + \frac{2\log\left(\sqrt[3]{a}\,x+\sqrt[3]{b}\right)}{\sqrt[3]{a}} - \frac{2\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}\,x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{9b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^3)^2*x^6),x]
```

```
[Out] ((3*b^(2/3)*x)/(b + a*x^3) - (2*Sqrt[3]*ArcTan[(1 - (2*a^(1/3)*x)/b^(1/3))/
Sqrt[3]])/a^(1/3) + (2*Log[b^(1/3) + a^(1/3)*x])/a^(1/3) - Log[b^(2/3) - a^
(1/3)*b^(1/3)*x + a^(2/3)*x^2]/a^(1/3))/(9*b^(5/3))
```

fricas [A] time = 0.85, size = 389, normalized size = 2.90

$$\left[\frac{3\,ab^2x+3\sqrt{\frac{1}{3}}\left(a^2bx^3+ab^2\right)\sqrt{-\frac{\left(ab^2\right)^{\frac{1}{3}}}{a}}\,\log\left(\frac{2\,abx^3-3\left(ab^2\right)^{\frac{1}{3}}bx-b^2+3\sqrt{\frac{1}{3}}\left(2\,abx^2+\left(ab^2\right)^{\frac{2}{3}}x-\left(ab^2\right)^{\frac{1}{3}}b\right)\sqrt{-\frac{\left(ab^2\right)^{\frac{1}{3}}}{a}}}{ax^3+b}\right)-\left(ax^3+b\right)\left(a\right.}{9\left(a^2b^3x^3+ab^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^2/x^6,x, algorithm="fricas")
```

```
[Out] [1/9*(3*a*b^2*x + 3*sqrt(1/3)*(a^2*b*x^3 + a*b^2)*sqrt(-(a*b^2)^(1/3)/a)*lo
g((2*a*b*x^3 - 3*(a*b^2)^(1/3)*b*x - b^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a*b^2)
^(2/3)*x - (a*b^2)^(1/3)*b)*sqrt(-(a*b^2)^(1/3)/a))/(a*x^3 + b)) - (a*x^3 +
 b)*(a*b^2)^(2/3)*log(a*b*x^2 - (a*b^2)^(2/3)*x + (a*b^2)^(1/3)*b) + 2*(a*x
^3 + b)*(a*b^2)^(2/3)*log(a*b*x + (a*b^2)^(2/3)))/(a^2*b^3*x^3 + a*b^4), 1/
9*(3*a*b^2*x + 6*sqrt(1/3)*(a^2*b*x^3 + a*b^2)*sqrt((a*b^2)^(1/3)/a)*arctan
(sqrt(1/3)*(2*(a*b^2)^(2/3)*x - (a*b^2)^(1/3)*b)*sqrt((a*b^2)^(1/3)/a)/b^2)
 - (a*x^3 + b)*(a*b^2)^(2/3)*log(a*b*x^2 - (a*b^2)^(2/3)*x + (a*b^2)^(1/3)*
b) + 2*(a*x^3 + b)*(a*b^2)^(2/3)*log(a*b*x + (a*b^2)^(2/3)))/(a^2*b^3*x^3 +
 a*b^4)]
```

giac [A] time = 0.17, size = 127, normalized size = 0.95

$$-\frac{2\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right|\right)}{9\,b^2}+\frac{x}{3\left(ax^3+b\right)b}+\frac{2\sqrt{3}\left(-a^2b\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{9\,ab^2}+\frac{\left(-a^2b\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\right.}{9\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^2/x^6,x, algorithm="giac")
```

```
[Out] -2/9*(-b/a)^(1/3)*log(abs(x - (-b/a)^(1/3)))/b^2 + 1/3*x/((a*x^3 + b)*b) +
2/9*sqrt(3)*(-a^2*b)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-b/a)^(1/3))/(-b/a)^(
1/3))/(a*b^2) + 1/9*(-a^2*b)^(1/3)*log(x^2 + x*(-b/a)^(1/3) + (-b/a)^(2/3))
/(a*b^2)
```

maple [A] time = 0.00, size = 115, normalized size = 0.86

$$\frac{x}{3\left(a\,x^3+b\right)b}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{b}{a}\right)^{\frac{2}{3}}ab}+\frac{2\ln\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{9\left(\frac{b}{a}\right)^{\frac{2}{3}}ab}-\frac{\ln\left(x^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}x+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{9\left(\frac{b}{a}\right)^{\frac{2}{3}}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^3)^2/x^6,x)
```

```
[Out] 1/3*x/b/(a*x^3+b)+2/9/b/a/(1/a*b)^(2/3)*ln(x+(1/a*b)^(1/3))-1/9/b/a/(1/a*b)
^(2/3)*ln(x^2-(1/a*b)^(1/3)*x+(1/a*b)^(2/3))+2/9/b/a/(1/a*b)^(2/3)*3^(1/2)*
arctan(1/3*3^(1/2)*(2/(1/a*b)^(1/3)*x-1))
```

maxima [A] time = 1.89, size = 122, normalized size = 0.91

$$\frac{x}{3\left(abx^3+b^2\right)}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{9ab\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\log\left(x^2-x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{9ab\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{2\log\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{9ab\left(\frac{b}{a}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^2/x^6,x, algorithm="maxima")
```

```
[Out] 1/3*x/(a*b*x^3 + b^2) + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (b/a)^(1/3))/
(b/a)^(1/3))/(a*b*(b/a)^(2/3)) - 1/9*log(x^2 - x*(b/a)^(1/3) + (b/a)^(2/3))
/(a*b*(b/a)^(2/3)) + 2/9*log(x + (b/a)^(1/3))/(a*b*(b/a)^(2/3))
```

mupad [B] time = 0.20, size = 128, normalized size = 0.96

$$\frac{x}{3\,b\,\left(a\,x^3+b\right)}+\frac{2\,\ln\left(\frac{2\,a^{5/3}}{b^{2/3}}+\frac{2\,a^2\,x}{b}\right)}{9\,a^{1/3}\,b^{5/3}}+\frac{\ln\left(\frac{2\,a^2\,x}{b}+\frac{a^{5/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{b^{2/3}}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{9\,a^{1/3}\,b^{5/3}}-\frac{\ln\left(\frac{2\,a^2\,x}{b}-\frac{a^{5/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{b^{2/3}}\right)\left(1+\sqrt{3}\right.}{9\,a^{1/3}\,b^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b/x^3)^2),x)
```

```
[Out] x/(3*b*(b + a*x^3)) + (2*log((2*a^(5/3))/b^(2/3) + (2*a^2*x)/b))/(9*a^(1/3)
*b^(5/3)) + (log((2*a^2*x)/b + (a^(5/3)*(3^(1/2)*1i - 1))/b^(2/3))*(3^(1/2)
*1i - 1))/(9*a^(1/3)*b^(5/3)) - (log((2*a^2*x)/b - (a^(5/3)*(3^(1/2)*1i + 1
))/b^(2/3))*(3^(1/2)*1i + 1))/(9*a^(1/3)*b^(5/3))
```

sympy [A] time = 0.33, size = 39, normalized size = 0.29

$$\frac{x}{3abx^3+3b^2}+\operatorname{RootSum}\left(729t^3ab^5-8,\left(t\mapsto t\log\left(\frac{9tb^2}{2}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**3)**2/x**6,x)
```

```
[Out] x/(3*a*b*x**3 + 3*b**2) + RootSum(729*_t**3*a*b**5 - 8, Lambda(_t, _t*log(9
*_t*b**2/2 + x)))
```

3.1990 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^7}\,dx$

Optimal. Leaf size=38

$$-\frac{\log\left(ax^3+b\right)}{3b^2}+\frac{1}{3b\left(ax^3+b\right)}+\frac{\log(x)}{b^2}$$

```
[Out] 1/3/b/(a*x^3+b)+ln(x)/b^2-1/3*ln(a*x^3+b)/b^2
```

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 266, 44}

$$-\frac{\log\left(ax^3+b\right)}{3b^2}+\frac{1}{3b\left(ax^3+b\right)}+\frac{\log(x)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^3)^2*x^7),x]
```

```
[Out] 1/(3*b*(b + a*x^3)) + Log[x]/b^2 - Log[b + a*x^3]/(3*b^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^7}\,dx &= \int \frac{1}{x\left(b+ax^3\right)^2}\,dx \\
&= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x(b+ax)^2}\,dx, x, x^3\right) \\
&= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{1}{b^2 x}-\frac{a}{b(b+ax)^2}-\frac{a}{b^2(b+ax)}\right)dx, x, x^3\right) \\
&= \frac{1}{3b\left(b+ax^3\right)}+\frac{\log(x)}{b^2}-\frac{\log\left(b+ax^3\right)}{3b^2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.87

$$\frac{\frac{b}{ax^3+b} - \log\left(ax^3+b\right) + 3\log(x)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^3)^2*x^7),x]
```

```
[Out] (b/(b + a*x^3) + 3*Log[x] - Log[b + a*x^3])/(3*b^2)
```

fricas [A] time = 0.84, size = 47, normalized size = 1.24

$$-\frac{\left(ax^3+b\right)\log\left(ax^3+b\right) - 3\left(ax^3+b\right)\log(x) - b}{3\left(ab^2x^3+b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^2/x^7,x, algorithm="fricas")
```

```
[Out] -1/3*((a*x^3 + b)*log(a*x^3 + b) - 3*(a*x^3 + b)*log(x) - b)/(a*b^2*x^3 + b
^3)
```

giac [A] time = 0.16, size = 45, normalized size = 1.18

$$-\frac{\log\left(\left|ax^3+b\right|\right)}{3\,b^2} + \frac{\log\left(|x|\right)}{b^2} + \frac{ax^3+2\,b}{3\left(ax^3+b\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^2/x^7,x, algorithm="giac")
```

```
[Out] -1/3*log(abs(a*x^3 + b))/b^2 + log(abs(x))/b^2 + 1/3*(a*x^3 + 2*b)/((a*x^3
+ b)*b^2)
```

maple [A] time = 0.01, size = 35, normalized size = 0.92

$$\frac{1}{3\left(a\,x^3+b\right)b} + \frac{\ln(x)}{b^2} - \frac{\ln\left(a\,x^3+b\right)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^3)^2/x^7,x)
```

```
[Out] 1/3/b/(a*x^3+b)+1/b^2*ln(x)-1/3*ln(a*x^3+b)/b^2
```

maxima [A] time = 0.89, size = 37, normalized size = 0.97

$$\frac{1}{3\left(abx^3+b^2\right)} - \frac{\log\left(ax^3+b\right)}{3\,b^2} + \frac{\log\left(x^3\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^2/x^7,x, algorithm="maxima")
```

```
[Out] 1/3/(a*b*x^3 + b^2) - 1/3*log(a*x^3 + b)/b^2 + 1/3*log(x^3)/b^2
```

mupad [B] time = 0.07, size = 34, normalized size = 0.89

$$\frac{\ln(x)}{b^2} + \frac{1}{3\,b\,\left(a\,x^3+b\right)} - \frac{\ln\left(a\,x^3+b\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b/x^3)^2),x)
```

```
[Out] log(x)/b^2 + 1/(3*b*(b + a*x^3)) - log(b + a*x^3)/(3*b^2)
```

sympy [**A**] time = 0.43, size = 34, normalized size = 0.89

$$\frac{1}{3abx^3 + 3b^2} + \frac{\log(x)}{b^2} - \frac{\log\left(x^3 + \frac{b}{a}\right)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**3)**2/x**7,x)
```

```
[Out] 1/(3*a*b*x**3 + 3*b**2) + log(x)/b**2 - log(x**3 + b/a)/(3*b**2)
```

3.1991 $\int \sqrt{a+\frac{b}{x^3}}\, x^5\, dx$

Optimal. Leaf size=71

$$-\frac{b^2 \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{12a^{3/2}}+\frac{bx^3\sqrt{a+\frac{b}{x^3}}}{12a}+\frac{1}{6}x^6\sqrt{a+\frac{b}{x^3}}$$

```
[Out] -1/12*b^2*arctanh((a+b/x^3)^(1/2)/a^(1/2))/a^(3/2)+1/12*b*x^3*(a+b/x^3)^(1/
2)/a+1/6*x^6*(a+b/x^3)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {266, 47, 51, 63, 208}

$$-\frac{b^2 \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{12a^{3/2}}+\frac{1}{6}x^6\sqrt{a+\frac{b}{x^3}}+\frac{bx^3\sqrt{a+\frac{b}{x^3}}}{12a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^3]*x^5,x]
```

```
[Out] (b*Sqrt[a + b/x^3]*x^3)/(12*a) + (Sqrt[a + b/x^3]*x^6)/6 - (b^2*ArcTanh[Sqr
t[a + b/x^3]/Sqrt[a]])/(12*a^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a+\frac{b}{x^3}}\, x^5\, dx &= -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^3}\, dx, x, \frac{1}{x^3}\right)\right) \\
&= \frac{1}{6}\sqrt{a+\frac{b}{x^3}}\, x^6 - \frac{1}{12} b\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\, dx, x, \frac{1}{x^3}\right) \\
&= \frac{b\sqrt{a+\frac{b}{x^3}}\, x^3}{12a} + \frac{1}{6}\sqrt{a+\frac{b}{x^3}}\, x^6 + \frac{b^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\, dx, x, \frac{1}{x^3}\right)}{24a} \\
&= \frac{b\sqrt{a+\frac{b}{x^3}}\, x^3}{12a} + \frac{1}{6}\sqrt{a+\frac{b}{x^3}}\, x^6 + \frac{b\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\, dx, x, \sqrt{a+\frac{b}{x^3}}\right)}{12a} \\
&= \frac{b\sqrt{a+\frac{b}{x^3}}\, x^3}{12a} + \frac{1}{6}\sqrt{a+\frac{b}{x^3}}\, x^6 - \frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{12a^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 83, normalized size = 1.17

$$\frac{x^{3/2}\sqrt{a+\frac{b}{x^3}}\left(\sqrt{a}\, x^{3/2}\left(2ax^3+b\right) - \frac{b^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\, x^{3/2}}{\sqrt{b}}\right)}{\sqrt{\frac{ax^3}{b}+1}}\right)}{12a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^3]*x^5,x]
```

```
[Out] (Sqrt[a + b/x^3]*x^(3/2)*(Sqrt[a]*x^(3/2)*(b + 2*a*x^3) - (b^(3/2)*ArcSinh[
(Sqrt[a]*x^(3/2))/Sqrt[b]])/Sqrt[1 + (a*x^3)/b]))/(12*a^(3/2))
```

fricas [A] time = 1.17, size = 175, normalized size = 2.46

$$\left[\frac{\sqrt{a}\, b^2 \log\left(-8\,a^2x^6 - 8\,abx^3 - b^2 + 4\left(2\,ax^6 + bx^3\right)\sqrt{a}\,\sqrt{\frac{ax^3+b}{x^3}}\right) + 4\left(2\,a^2x^6 + abx^3\right)\sqrt{\frac{ax^3+b}{x^3}}}{48\,a^2}, \frac{\sqrt{-a}\, b^2 \arctan\left(\frac{2\sqrt{-}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b/x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(sqrt(a)*b^2*log(-8*a^2*x^6 - 8*a*b*x^3 - b^2 + 4*(2*a*x^6 + b*x^3)*s
qrt(a)*sqrt((a*x^3 + b)/x^3)) + 4*(2*a^2*x^6 + a*b*x^3)*sqrt((a*x^3 + b)/x^
3))/a^2, 1/24*(sqrt(-a)*b^2*arctan(2*sqrt(-a)*x^3*sqrt((a*x^3 + b)/x^3)/(2*
a*x^3 + b)) + 2*(2*a^2*x^6 + a*b*x^3)*sqrt((a*x^3 + b)/x^3))/a^2]
```

giac [A] time = 0.26, size = 55, normalized size = 0.77

$$\frac{1}{12}\sqrt{ax^4+bx}\left(2x^3+\frac{b}{a}\right)x+\frac{b^2\arctan\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{-a}}\right)}{12\sqrt{-a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b/x^3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(a*x^4 + b*x)*(2*x^3 + b/a)*x + 1/12*b^2*arctan(sqrt(a + b/x^3)/sq
rt(-a))/(sqrt(-a)*a)
```

maple [C] time = 0.68, size = 3560, normalized size = 50.14

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b/x^3)^(1/2),x)
```

```
[Out] 1/12*((a*x^3+b)/x^3)^(1/2)*x^2/a^3*(I*(a*x^4+b*x)^(1/2)*(1/a^2*x*(-a*x+(-a^
2*b)^(1/3))*(I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))*(I*3^(1/2)*(-a^
2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3)))^(1/2)*3^(1/2)*x*a^2*b+12*I*(-(I*3^(1/2)-3
)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)
+2*a*x+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/
2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))
)^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))
^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2
))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*3^(1/2)*x*a*b^2-6*I*(-(I*3^(1/2)-3)
*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)+
2*a*x+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2
)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))
^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^
(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2)
)/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*3^(1/2)*b^2+6*I*(-(I*3^(1/2)-3)*x*a/
(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x
+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a
^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2
)*EllipticF((-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2),
((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*3^(1/2)*x^
2*a^2*b^2+2*I*(a*x^4+b*x)^(1/2)*(1/a^2*x*(-a*x+(-a^2*b)^(1/3))*(I*3^(1/2)*(
-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))*(I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b
)^(1/3)))^(1/2)*3^(1/2)*x^4*a^3+6*I*(-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x
+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))/(1
+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(
-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^
(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3
^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*3^(1/2)*b^2-6*
(-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*
(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(
1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(
-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a
^2*b)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3
))^(1/2))*x^2*a^2*b^2-12*I*(-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)
^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))/(1+I*3^(1/2
))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(
1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*
x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)
```

```
/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*3^(1/2)*x*a*b^2+6*(-(I*
3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2
*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*
((I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*
b)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b
)^(1/3)))^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1
+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a^2*b^2-6*x^4*(a*x^4+b*x)^(1/2)*a^3*(
1/a^2*x*(-a*x+(-a^2*b)^(1/3))*(I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3
))*(I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3)))^(1/2)+12*(-(I*3^(1/2)-3
)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)
+2*a*x+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/
2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))
)^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^
(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^
2*b)^(1/3)*x*a*b^2-12*(-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3
)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-
a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3))
/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*a/
(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*
3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)
*x*a*b^2-6*I*(-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)
*((I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2
*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3))/(I*3^(1/
2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*a/(I*3^(1/2
)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3
)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*3^(1/2)*x^2*a^2*b^2-6*(
-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(
-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1
/2)*((I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-
a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^
2*b)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3)
)^(1/2))*(-a^2*b)^(2/3)*b^2+6*(-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2
*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))/(1+I*3^(
1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b
)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)
-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)
-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*
b)^(2/3)*b^2-3*b*x*(a*x^4+b*x)^(1/2)*a^2*(1/a^2*x*(-a*x+(-a^2*b)^(1/3))*(I*
3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))*(I*3^(1/2)*(-a^2*b)^(1/3)-2*a*
x-(-a^2*b)^(1/3)))^(1/2))/(x*(a*x^3+b))^(1/2)/(I*3^(1/2)-3)/(1/a^2*x*(-a*x+
(-a^2*b)^(1/3))*(I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))*(I*3^(1/2)*
(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3)))^(1/2)
```

maxima [A] time = 1.91, size = 100, normalized size = 1.41

$$\frac{b^2 \log\left(\frac{\sqrt{a+\frac{b}{x^3}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^3}}+\sqrt{a}}\right)}{24\,a^{\frac{3}{2}}}+\frac{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}b^2+\sqrt{a+\frac{b}{x^3}}\,ab^2}{12\left(\left(a+\frac{b}{x^3}\right)^2 a-2\left(a+\frac{b}{x^3}\right)a^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b/x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/24*b^2*log((sqrt(a + b/x^3) - sqrt(a))/(sqrt(a + b/x^3) + sqrt(a)))/a^(3/
2) + 1/12*((a + b/x^3)^(3/2)*b^2 + sqrt(a + b/x^3)*a*b^2)/((a + b/x^3)^2*a
- 2*(a + b/x^3)*a^2 + a^3)
```

mupad [B] time = 1.77, size = 76, normalized size = 1.07

$$\frac{x^6\sqrt{a+\frac{b}{x^3}}}{6}+\frac{b^2\ln\left(x^6\left(\sqrt{a+\frac{b}{x^3}}-\sqrt{a}\right)^3\left(\sqrt{a+\frac{b}{x^3}}+\sqrt{a}\right)\right)}{24\,a^{3/2}}+\frac{b\,x^3\sqrt{a+\frac{b}{x^3}}}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b/x^3)^(1/2),x)
```

```
[Out] (x^6*(a + b/x^3)^(1/2))/6 + (b^2*log(x^6*((a + b/x^3)^(1/2) - a^(1/2))^3*((
a + b/x^3)^(1/2) + a^(1/2))))/(24*a^(3/2)) + (b*x^3*(a + b/x^3)^(1/2))/(12*
a)
```

sympy [A] time = 4.53, size = 100, normalized size = 1.41

$$\frac{a x^{\frac{15}{2}}}{6\sqrt{b}\sqrt{\frac{ax^3}{b}+1}}+\frac{\sqrt{b}\,x^{\frac{9}{2}}}{4\sqrt{\frac{ax^3}{b}+1}}+\frac{b^{\frac{3}{2}}x^{\frac{3}{2}}}{12a\sqrt{\frac{ax^3}{b}+1}}-\frac{b^2\operatorname{asinh}\left(\frac{\sqrt{a}\,x^{\frac{3}{2}}}{\sqrt{b}}\right)}{12a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b/x**3)**(1/2),x)
```

```
[Out] a*x**(15/2)/(6*sqrt(b)*sqrt(a*x**3/b + 1)) + sqrt(b)*x**(9/2)/(4*sqrt(a*x**
3/b + 1)) + b**(3/2)*x**(3/2)/(12*a*sqrt(a*x**3/b + 1)) - b**2*asinh(sqrt(a
)*x**(3/2)/sqrt(b))/(12*a**(3/2))
```

3.1992 $\int \sqrt{a+\frac{b}{x^3}}\,x^2\,dx$

Optimal. Leaf size=47

$$\frac{1}{3}x^3\sqrt{a+\frac{b}{x^3}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

```
[Out] 1/3*b*arctanh((a+b/x^3)^(1/2)/a^(1/2))/a^(1/2)+1/3*x^3*(a+b/x^3)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 47, 63, 208}

$$\frac{1}{3}x^3\sqrt{a+\frac{b}{x^3}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^3]*x^2,x]
```

```
[Out] (Sqrt[a + b/x^3]*x^3)/3 + (b*ArcTanh[Sqrt[a + b/x^3]/Sqrt[a]])/(3*Sqrt[a])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+\frac{b}{x^3}}\,x^2\,dx &= -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx, x, \frac{1}{x^3}\right)\right)\\ &= \frac{1}{3}\sqrt{a+\frac{b}{x^3}}\,x^3 - \frac{1}{6}b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x^3}\right)\\ &= \frac{1}{3}\sqrt{a+\frac{b}{x^3}}\,x^3 - \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x^3}}\right)\\ &= \frac{1}{3}\sqrt{a+\frac{b}{x^3}}\,x^3 + \frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{3\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.11, size = 68, normalized size = 1.45

$$\frac{1}{3}x^{3/2}\sqrt{a+\frac{b}{x^3}}\left(\frac{\sqrt{b}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{b}}\right)}{\sqrt{a}\,\sqrt{\frac{ax^3}{b}+1}}+x^{3/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^3]*x^2,x]
```

```
[Out] (Sqrt[a + b/x^3]*x^(3/2)*(x^(3/2) + (Sqrt[b]*ArcSinh[(Sqrt[a]*x^(3/2))/Sqrt
[b]])/(Sqrt[a]*Sqrt[1 + (a*x^3)/b])))/3
```

fricas [A] time = 0.86, size = 150, normalized size = 3.19

$$\left[\frac{4\,ax^3\sqrt{\frac{ax^3+b}{x^3}}+\sqrt{a}\,b\log\left(-8\,a^2x^6-8\,abx^3-b^2-4\left(2\,ax^6+bx^3\right)\sqrt{a}\,\sqrt{\frac{ax^3+b}{x^3}}\right)}{12\,a},\ \frac{2\,ax^3\sqrt{\frac{ax^3+b}{x^3}}-\sqrt{-a}\,b\arctan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b/x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*a*x^3*sqrt((a*x^3 + b)/x^3) + sqrt(a)*b*log(-8*a^2*x^6 - 8*a*b*x^3
 - b^2 - 4*(2*a*x^6 + b*x^3)*sqrt(a)*sqrt((a*x^3 + b)/x^3)))/a, 1/6*(2*a*x^
3*sqrt((a*x^3 + b)/x^3) - sqrt(-a)*b*arctan(2*sqrt(-a)*x^3*sqrt((a*x^3 + b)
/x^3)/(2*a*x^3 + b)))/a]
```

giac [A] time = 0.26, size = 39, normalized size = 0.83

$$\frac{1}{3}\sqrt{ax^4+bx}\,x - \frac{b\arctan\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{-a}}\right)}{3\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b/x^3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(a*x^4 + b*x)*x - 1/3*b*arctan(sqrt(a + b/x^3)/sqrt(-a))/sqrt(-a)
```

maple [C] time = 0.03, size = 3340, normalized size = 71.06

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b/x^3)^(1/2),x)
```

```
[Out] -1/3*((a*x^3+b)/x^3)^(1/2)*x^2/a^2*(6*I*3^(1/2)*(-(I*3^(1/2)-3)*x*a/(I*3^(1
/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*
b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(
1/3)-2*a*x-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*Ellip
ticF((-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2),((I*3^(
1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a^2*b-6*I*3^(
1/2)*(-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(
1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3
)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-
a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a
*x+(-a^2*b)^(1/3)))^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(
1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a^2*b-12*I*(-a^2*b)^(1/3)*3
^(1/2)*(-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3
^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1
/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/
(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-
a*x+(-a^2*b)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^
(1/2)-3))^(1/2))*x*a*b+12*I*(-a^2*b)^(1/3)*3^(1/2)*(-(I*3^(1/2)-3)*x*a/(I*3
^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a
^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b
)^(1/3)-2*a*x-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*El
lipticPi((-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2),(I*
3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1
/2)-3))^(1/2))*x*a*b+6*I*(-a^2*b)^(2/3)*3^(1/2)*(-(I*3^(1/2)-3)*x*a/(I*3^(1
/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*
b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(
1/3)-2*a*x-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*Ellip
ticF((-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2),((I*3^(
1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*b-6*I*(-a^2*b)^(2
/3)*3^(1/2)*(-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*
((I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*
b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3))/(I*3^(1/2
)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*a/(I*3^(1/2)
-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)
*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*b-6*(-(I*3^(1/2)-3)*x*a/
(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x
+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a
^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2
)*EllipticF((-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2),
((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a^2*b+
6*(-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2
)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))
^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x
+(-a^2*b)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+
(-a^2*b)^(1/3)))^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2
)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a^2*b+12*(-a^2*b)^(1/3)*(-(I*3
^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*
b)^(1/3)+2*a*x+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*(
(I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b
)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^
(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/
2))*x*a*b-12*(-a^2*b)^(1/3)*(-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b
```

```
)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))/(1+I*3^(1/
2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^
(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3
)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3
),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x*a*b-6*
(-a^2*b)^(2/3)*(-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/
2)*((I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a
^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3))/(I*3^(
1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*a/(I*3^(1/
2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2
))/(I*3^(1/2)-3))^(1/2))*b+6*(-a^2*b)^(2/3)*(-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-
1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(
1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)
-2*a*x-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticP
i((-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2),(I*3^(1/2)
-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))
^(1/2))*b-I*3^(1/2)*(a*x^4+b*x)^(1/2)*(1/a^2*x*(-a*x+(-a^2*b)^(1/3))*(I*3^(
1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))*(I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(
-a^2*b)^(1/3)))^(1/2)*x*a^2+3*x*(a*x^4+b*x)^(1/2)*a^2*(1/a^2*x*(-a*x+(-a^2*
b)^(1/3))*(I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))*(I*3^(1/2)*(-a^2*
b)^(1/3)-2*a*x-(-a^2*b)^(1/3)))^(1/2))/(x*(a*x^3+b))^(1/2)/(I*3^(1/2)-3)/(1
/a^2*x*(-a*x+(-a^2*b)^(1/3))*(I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3)
)*(I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3)))^(1/2)
```

maxima [A] time = 1.92, size = 53, normalized size = 1.13

$$\frac{1}{3}\sqrt{a+\frac{b}{x^3}}\,x^3-\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x^3}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^3}}+\sqrt{a}}\right)}{6\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b/x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(a + b/x^3)*x^3 - 1/6*b*log((sqrt(a + b/x^3) - sqrt(a))/(sqrt(a + b
/x^3) + sqrt(a)))/sqrt(a)
```

mupad [B] time = 1.60, size = 56, normalized size = 1.19

$$\frac{x^3\sqrt{a+\frac{b}{x^3}}}{3}+\frac{b\,\ln\left(x^6\left(\sqrt{a+\frac{b}{x^3}}-\sqrt{a}\right)\left(\sqrt{a+\frac{b}{x^3}}+\sqrt{a}\right)^3\right)}{6\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b/x^3)^(1/2),x)
```

```
[Out] (x^3*(a + b/x^3)^(1/2))/3 + (b*log(x^6*((a + b/x^3)^(1/2) - a^(1/2))*((a +
b/x^3)^(1/2) + a^(1/2))^3))/(6*a^(1/2))
```

sympy [A] time = 2.44, size = 48, normalized size = 1.02

$$\frac{\sqrt{b}\,x^{\frac{3}{2}}\sqrt{\frac{ax^3}{b}+1}}{3}+\frac{b\,\operatorname{asinh}\left(\frac{\sqrt{a}\,x^{\frac{3}{2}}}{\sqrt{b}}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b/x**3)**(1/2),x)
```

```
[Out] sqrt(b)*x**(3/2)*sqrt(a*x**3/b + 1)/3 + b*asinh(sqrt(a)*x**(3/2)/sqrt(b))/(
3*sqrt(a))
```

3.1993 $\int \frac{\sqrt{a+\frac{b}{x^3}}}{x}\,dx$

Optimal. Leaf size=43

$$\frac{2}{3}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)-\frac{2}{3}\sqrt{a+\frac{b}{x^3}}$$

```
[Out] 2/3*arctanh((a+b/x^3)^(1/2)/a^(1/2))*a^(1/2)-2/3*(a+b/x^3)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 50, 63, 208}

$$\frac{2}{3}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)-\frac{2}{3}\sqrt{a+\frac{b}{x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^3]/x,x]
```

```
[Out] (-2*Sqrt[a + b/x^3])/3 + (2*Sqrt[a]*ArcTanh[Sqrt[a + b/x^3]/Sqrt[a]])/3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+\frac{b}{x^3}}}{x}\,dx &= -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x^3}\right)\right)\\ &= -\frac{2}{3}\sqrt{a+\frac{b}{x^3}} - \frac{1}{3}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x^3}\right)\\ &= -\frac{2}{3}\sqrt{a+\frac{b}{x^3}} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x^3}}\right)}{3b}\\ &= -\frac{2}{3}\sqrt{a+\frac{b}{x^3}} + \frac{2}{3}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)\end{aligned}$$

Mathematica [A] time = 0.10, size = 64, normalized size = 1.49

$$\frac{2}{3}\sqrt{a+\frac{b}{x^3}}\left(\frac{\sqrt{a}\,x^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{\frac{ax^3}{b}+1}} - 1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^3]/x,x]
```

```
[Out] (2*Sqrt[a + b/x^3]*(-1 + (Sqrt[a]*x^(3/2)*ArcSinh[(Sqrt[a]*x^(3/2))/Sqrt[b]
])/(Sqrt[b]*Sqrt[1 + (a*x^3)/b])))/3
```

fricas [A] time = 1.17, size = 131, normalized size = 3.05

$$\left[\frac{1}{6}\sqrt{a}\,\log\left(-8\,a^2x^6 - 8\,abx^3 - b^2 - 4\left(2\,ax^6 + bx^3\right)\sqrt{a}\,\sqrt{\frac{ax^3+b}{x^3}}\right) - \frac{2}{3}\sqrt{\frac{ax^3+b}{x^3}}, -\frac{1}{3}\sqrt{-a}\,\arctan\left(\frac{2\sqrt{-a}\,x^3}{2\,ax^3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/6*sqrt(a)*log(-8*a^2*x^6 - 8*a*b*x^3 - b^2 - 4*(2*a*x^6 + b*x^3)*sqrt(a)
*sqrt((a*x^3 + b)/x^3)) - 2/3*sqrt((a*x^3 + b)/x^3), -1/3*sqrt(-a)*arctan(2
*sqrt(-a)*x^3*sqrt((a*x^3 + b)/x^3)/(2*a*x^3 + b)) - 2/3*sqrt((a*x^3 + b)/x
^3)]
```

giac [A] time = 0.24, size = 36, normalized size = 0.84

$$-\frac{2\,a\arctan\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{-a}}\right)}{3\sqrt{-a}} - \frac{2}{3}\sqrt{a+\frac{b}{x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -2/3*a*arctan(sqrt(a + b/x^3)/sqrt(-a))/sqrt(-a) - 2/3*sqrt(a + b/x^3)
```

maple [C] time = 0.03, size = 3339, normalized size = 77.65

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^3)^(1/2)/x,x)
```

```
[Out] -2/3*((a*x^3+b)/x^3)^(1/2)/a*(6*I*3^(1/2)*(-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)
/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/
3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)-2
*a*x-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((
-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2),((I*3^(1/2)+3
)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^4*a^2-6*I*3^(1/2)*(-(
I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a
^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2
)*((I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^
2*b)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2
*b)^(1/3)))^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/
(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^3*a+12*I*(-a^2*b)^(1/3)*3^(1/2)*(-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/
(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3
))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)-2*
a*x-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticPi((
-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2),(I*3^(1/2)-1)
/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1
/2))*x^3*a+6*I*(-a^2*b)^(2/3)*3^(1/2)*(-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a
*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))/
(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x
-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*
3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I
*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2-6*I*(-a^2*b)^(2/3)*3^(1
/2)*(-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1
/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)
))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a
*x+(-a^2*b)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*
x+(-a^2*b)^(1/3)))^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1
/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2-6*(-(I*3^(1/2)-3)*x*a/(I*3^(
1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2
*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^
(1/3)-2*a*x-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*Elli
pticF((-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2),((I*3^
(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^4*a^2+6*(-(I*3
^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*
b)^(1/3)+2*a*x+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*(
(I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b
)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)
^(1/3)))^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+
I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^4*a^2+12*(-a^2*b)^(1/3)*(-(I*3^(1/2)-3)*
x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)+2
*a*x+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)
*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^
(1/2)*EllipticF((-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1
/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^3*a-
12*(-a^2*b)^(1/3)*(-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^
(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+
(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3))/(I*
3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*a/(I*3
^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1
```

```
/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^3*a-6*(-a^2*b)^(
2/3)*(-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(
1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3
)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-
a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*
x+(-a^2*b)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1
/2)-3))^(1/2))*x^2+6*(-a^2*b)^(2/3)*(-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x
+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))/(1
+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(
-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticPi((-(I*3
^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2),(I*3^(1/2)-1)/(I*3
^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*
x^2+I*3^(1/2)*(a*x^4+b*x)^(1/2)*(1/a^2*x*(-a*x+(-a^2*b)^(1/3))*(I*3^(1/2)*(
-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))*(I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b
)^(1/3)))^(1/2)*a-3*(a*x^4+b*x)^(1/2)*a*(1/a^2*x*(-a*x+(-a^2*b)^(1/3))*(I*3
^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))*(I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x
-(-a^2*b)^(1/3)))^(1/2))/(x*(a*x^3+b))^(1/2)/(I*3^(1/2)-3)/(1/a^2*x*(-a*x+(
-a^2*b)^(1/3))*(I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))*(I*3^(1/2)*(
-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3)))^(1/2)
```

maxima [A] time = 1.86, size = 49, normalized size = 1.14

$$-\frac{1}{3}\sqrt{a}\log\left(\frac{\sqrt{a+\frac{b}{x^3}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^3}}+\sqrt{a}}\right)-\frac{2}{3}\sqrt{a+\frac{b}{x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(a)*log((sqrt(a + b/x^3) - sqrt(a))/(sqrt(a + b/x^3) + sqrt(a))) -
 2/3*sqrt(a + b/x^3)
```

mupad [B] time = 1.38, size = 31, normalized size = 0.72

$$\frac{2\sqrt{a}\,\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{3}-\frac{2\sqrt{a+\frac{b}{x^3}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^3)^(1/2)/x,x)
```

```
[Out] (2*a^(1/2)*atanh((a + b/x^3)^(1/2)/a^(1/2)))/3 - (2*(a + b/x^3)^(1/2))/3
```

sympy [B] time = 1.75, size = 76, normalized size = 1.77

$$\frac{2\sqrt{a}\,\operatorname{asinh}\left(\frac{\sqrt{a}\,x^{\frac{3}{2}}}{\sqrt{b}}\right)}{3}-\frac{2ax^{\frac{3}{2}}}{3\sqrt{b}\sqrt{\frac{ax^3}{b}+1}}-\frac{2\sqrt{b}}{3x^{\frac{3}{2}}\sqrt{\frac{ax^3}{b}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**3)**(1/2)/x,x)
```

```
[Out] 2*sqrt(a)*asinh(sqrt(a)*x**(3/2)/sqrt(b))/3 - 2*a*x**(3/2)/(3*sqrt(b)*sqrt(
a*x**3/b + 1)) - 2*sqrt(b)/(3*x**(3/2)*sqrt(a*x**3/b + 1))
```

3.1994 $\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^4}\,dx$

Optimal. Leaf size=18

$$-\frac{2\left(a+\frac{b}{x^3}\right)^{3/2}}{9b}$$

```
[Out] -2/9*(a+b/x^3)^(3/2)/b
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$-\frac{2\left(a+\frac{b}{x^3}\right)^{3/2}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^3]/x^4,x]
```

```
[Out] (-2*(a + b/x^3)^(3/2))/(9*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^4}\,dx = -\frac{2\left(a+\frac{b}{x^3}\right)^{3/2}}{9b}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$-\frac{2\left(a+\frac{b}{x^3}\right)^{3/2}}{9b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^3]/x^4,x]
```

```
[Out] (-2*(a + b/x^3)^(3/2))/(9*b)
```

fricas [A] time = 0.87, size = 28, normalized size = 1.56

$$-\frac{2\left(ax^3+b\right)\sqrt{\frac{ax^3+b}{x^3}}}{9\,bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -2/9*(a*x^3 + b)*sqrt((a*x^3 + b)/x^3)/(b*x^3)
```

giac [A] time = 0.18, size = 14, normalized size = 0.78

$$-\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(1/2)/x^4,x, algorithm="giac")
[Out] -2/9*(a + b/x^3)^(3/2)/b
```

maple [A] time = 0.00, size = 29, normalized size = 1.61

$$-\frac{2\left(a\,x^3+b\right)\sqrt{\frac{a\,x^3+b}{x^3}}}{9b\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^3)^(1/2)/x^4,x)
[Out] -2/9/x^3*(a*x^3+b)/b*((a*x^3+b)/x^3)^(1/2)
```

maxima [A] time = 0.79, size = 14, normalized size = 0.78

$$-\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(1/2)/x^4,x, algorithm="maxima")
[Out] -2/9*(a + b/x^3)^(3/2)/b
```

mupad [B] time = 1.43, size = 24, normalized size = 1.33

$$-\frac{2\sqrt{a+\frac{b}{x^3}}\,\left(a\,x^3+b\right)}{9\,b\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^3)^(1/2)/x^4,x)
[Out] -(2*(a + b/x^3)^(1/2)*(b + a*x^3))/(9*b*x^3)
```

sympy [B] time = 1.23, size = 46, normalized size = 2.56

$$-\frac{2a^{\frac{3}{2}}\sqrt{1+\frac{b}{ax^3}}}{9b}-\frac{2\sqrt{a}\,\sqrt{1+\frac{b}{ax^3}}}{9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**3)**(1/2)/x**4,x)
[Out] -2*a**(3/2)*sqrt(1 + b/(a*x**3))/(9*b) - 2*sqrt(a)*sqrt(1 + b/(a*x**3))/(9*
x**3)
```

3.1995 $\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^7}\,dx$

Optimal. Leaf size=38

$$\frac{2a\left(a+\frac{b}{x^3}\right)^{3/2}}{9b^2}-\frac{2\left(a+\frac{b}{x^3}\right)^{5/2}}{15b^2}$$

```
[Out] 2/9*a*(a+b/x^3)^(3/2)/b^2-2/15*(a+b/x^3)^(5/2)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{2a\left(a+\frac{b}{x^3}\right)^{3/2}}{9b^2}-\frac{2\left(a+\frac{b}{x^3}\right)^{5/2}}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^3]/x^7,x]
```

```
[Out] (2*a*(a + b/x^3)^(3/2))/(9*b^2) - (2*(a + b/x^3)^(5/2))/(15*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^7}\,dx &= -\left(\frac{1}{3}\,\text{Subst}\left(\int x\sqrt{a+bx}\,dx, x, \frac{1}{x^3}\right)\right)\\ &= -\left(\frac{1}{3}\,\text{Subst}\left(\int\left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx, x, \frac{1}{x^3}\right)\right)\\ &= \frac{2a\left(a+\frac{b}{x^3}\right)^{3/2}}{9b^2}-\frac{2\left(a+\frac{b}{x^3}\right)^{5/2}}{15b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.00

$$\frac{2\sqrt{a+\frac{b}{x^3}}\left(ax^3+b\right)\left(2ax^3-3b\right)}{45b^2x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^3]/x^7,x]
```

```
[Out] (2*Sqrt[a + b/x^3]*(b + a*x^3)*(-3*b + 2*a*x^3))/(45*b^2*x^6)
```

fricas [A] time = 1.45, size = 42, normalized size = 1.11

$$\frac{2\left(2\,a^2x^6 - abx^3 - 3\,b^2\right)\sqrt{\frac{ax^3+b}{x^3}}}{45\,b^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(1/2)/x^7,x, algorithm="fricas")
[Out] 2/45*(2*a^2*x^6 - a*b*x^3 - 3*b^2)*sqrt((a*x^3 + b)/x^3)/(b^2*x^6)
```

giac [A] time = 0.19, size = 29, normalized size = 0.76

$$-\frac{2\left(3\left(a+\frac{b}{x^3}\right)^{\frac{5}{2}} - 5\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}a\right)}{45\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(1/2)/x^7,x, algorithm="giac")
[Out] -2/45*(3*(a + b/x^3)^(5/2) - 5*(a + b/x^3)^(3/2)*a)/b^2
```

maple [A] time = 0.00, size = 39, normalized size = 1.03

$$\frac{2\left(a\,x^3+b\right)\left(2a\,x^3-3b\right)\sqrt{\frac{a\,x^3+b}{x^3}}}{45b^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^3)^(1/2)/x^7,x)
[Out] 2/45*(a*x^3+b)*(2*a*x^3-3*b)*((a*x^3+b)/x^3)^(1/2)/b^2/x^6
```

maxima [A] time = 0.92, size = 30, normalized size = 0.79

$$-\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{5}{2}}}{15\,b^2} + \frac{2\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}a}{9\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(1/2)/x^7,x, algorithm="maxima")
[Out] -2/15*(a + b/x^3)^(5/2)/b^2 + 2/9*(a + b/x^3)^(3/2)*a/b^2
```

mupad [B] time = 1.67, size = 37, normalized size = 0.97

$$-\frac{2\sqrt{a+\frac{b}{x^3}}\,\left(-2\,a^2\,x^6 + a\,b\,x^3 + 3\,b^2\right)}{45\,b^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^3)^(1/2)/x^7,x)
[Out] -(2*(a + b/x^3)^(1/2)*(3*b^2 - 2*a^2*x^6 + a*b*x^3))/(45*b^2*x^6)
```

sympy [B] time = 1.80, size = 313, normalized size = 8.24

$$\frac{4a^{\frac{11}{2}}b^{\frac{3}{2}}x^9\sqrt{\frac{ax^3}{b}+1}}{45a^{\frac{7}{2}}b^3x^{\frac{21}{2}} + 45a^{\frac{5}{2}}b^4x^{\frac{15}{2}}} + \frac{2a^{\frac{9}{2}}b^{\frac{5}{2}}x^6\sqrt{\frac{ax^3}{b}+1}}{45a^{\frac{7}{2}}b^3x^{\frac{21}{2}} + 45a^{\frac{5}{2}}b^4x^{\frac{15}{2}}} - \frac{8a^{\frac{7}{2}}b^{\frac{7}{2}}x^3\sqrt{\frac{ax^3}{b}+1}}{45a^{\frac{7}{2}}b^3x^{\frac{21}{2}} + 45a^{\frac{5}{2}}b^4x^{\frac{15}{2}}} - \frac{6a^{\frac{5}{2}}b^{\frac{9}{2}}\sqrt{\frac{ax^3}{b}+1}}{45a^{\frac{7}{2}}b^3x^{\frac{21}{2}} + 45a^{\frac{5}{2}}b^4x^{\frac{15}{2}}} - \frac{}{45a^{\frac{7}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**3)**(1/2)/x**7,x)
```

```
[Out] 4*a**(11/2)*b**(3/2)*x**9*sqrt(a*x**3/b + 1)/(45*a**(7/2)*b**3*x**(21/2) +
45*a**(5/2)*b**4*x**(15/2)) + 2*a**(9/2)*b**(5/2)*x**6*sqrt(a*x**3/b + 1)/(
45*a**(7/2)*b**3*x**(21/2) + 45*a**(5/2)*b**4*x**(15/2)) - 8*a**(7/2)*b**(7
/2)*x**3*sqrt(a*x**3/b + 1)/(45*a**(7/2)*b**3*x**(21/2) + 45*a**(5/2)*b**4*
x**(15/2)) - 6*a**(5/2)*b**(9/2)*sqrt(a*x**3/b + 1)/(45*a**(7/2)*b**3*x**(2
1/2) + 45*a**(5/2)*b**4*x**(15/2)) - 4*a**6*b*x**(21/2)/(45*a**(7/2)*b**3*x
**(21/2) + 45*a**(5/2)*b**4*x**(15/2)) - 4*a**5*b**2*x**(15/2)/(45*a**(7/2)
*b**3*x**(21/2) + 45*a**(5/2)*b**4*x**(15/2))
```

3.1996 $\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^{10}}\,dx$

Optimal. Leaf size=59

$$-\frac{2a^2\left(a+\frac{b}{x^3}\right)^{3/2}}{9b^3}-\frac{2\left(a+\frac{b}{x^3}\right)^{7/2}}{21b^3}+\frac{4a\left(a+\frac{b}{x^3}\right)^{5/2}}{15b^3}$$

```
[Out] -2/9*a^2*(a+b/x^3)^(3/2)/b^3+4/15*a*(a+b/x^3)^(5/2)/b^3-2/21*(a+b/x^3)^(7/2
)/b^3
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{2a^2\left(a+\frac{b}{x^3}\right)^{3/2}}{9b^3}-\frac{2\left(a+\frac{b}{x^3}\right)^{7/2}}{21b^3}+\frac{4a\left(a+\frac{b}{x^3}\right)^{5/2}}{15b^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^3]/x^10,x]
```

```
[Out] (-2*a^2*(a + b/x^3)^(3/2))/(9*b^3) + (4*a*(a + b/x^3)^(5/2))/(15*b^3) - (2*
(a + b/x^3)^(7/2))/(21*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^{10}}\,dx &= -\left(\frac{1}{3}\,\text{Subst}\left(\int x^2\sqrt{a+bx}\,dx, x, \frac{1}{x^3}\right)\right)\\ &= -\left(\frac{1}{3}\,\text{Subst}\left(\int\left(\frac{a^2\sqrt{a+bx}}{b^2}-\frac{2a(a+bx)^{3/2}}{b^2}+\frac{(a+bx)^{5/2}}{b^2}\right)dx, x, \frac{1}{x^3}\right)\right)\\ &= -\frac{2a^2\left(a+\frac{b}{x^3}\right)^{3/2}}{9b^3}+\frac{4a\left(a+\frac{b}{x^3}\right)^{5/2}}{15b^3}-\frac{2\left(a+\frac{b}{x^3}\right)^{7/2}}{21b^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 0.83

$$-\frac{2\sqrt{a+\frac{b}{x^3}}\left(ax^3+b\right)\left(8a^2x^6-12abx^3+15b^2\right)}{315b^3x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^3]/x^10,x]
```

```
[Out] (-2*Sqrt[a + b/x^3]*(b + a*x^3)*(15*b^2 - 12*a*b*x^3 + 8*a^2*x^6))/(315*b^3
*x^9)
```

fricas [A] time = 0.96, size = 53, normalized size = 0.90

$$-\frac{2\left(8\,a^3x^9-4\,a^2bx^6+3\,ab^2x^3+15\,b^3\right)\sqrt{\frac{ax^3+b}{x^3}}}{315\,b^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(1/2)/x^10,x, algorithm="fricas")
```

```
[Out] -2/315*(8*a^3*x^9 - 4*a^2*b*x^6 + 3*a*b^2*x^3 + 15*b^3)*sqrt((a*x^3 + b)/x^
3)/(b^3*x^9)
```

giac [A] time = 0.22, size = 43, normalized size = 0.73

$$-\frac{2\left(15\left(a+\frac{b}{x^3}\right)^{\frac{7}{2}}-42\left(a+\frac{b}{x^3}\right)^{\frac{5}{2}}a+35\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}a^2\right)}{315\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(1/2)/x^10,x, algorithm="giac")
```

```
[Out] -2/315*(15*(a + b/x^3)^(7/2) - 42*(a + b/x^3)^(5/2)*a + 35*(a + b/x^3)^(3/2
)*a^2)/b^3
```

maple [A] time = 0.01, size = 50, normalized size = 0.85

$$-\frac{2\left(a\,x^3+b\right)\left(8a^2x^6-12ab\,x^3+15b^2\right)\sqrt{\frac{a\,x^3+b}{x^3}}}{315b^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^3)^(1/2)/x^10,x)
```

```
[Out] -2/315*(a*x^3+b)*(8*a^2*x^6-12*a*b*x^3+15*b^2)*((a*x^3+b)/x^3)^(1/2)/b^3/x^
9
```

maxima [A] time = 0.89, size = 47, normalized size = 0.80

$$-\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{7}{2}}}{21\,b^3}+\frac{4\left(a+\frac{b}{x^3}\right)^{\frac{5}{2}}a}{15\,b^3}-\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}a^2}{9\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(1/2)/x^10,x, algorithm="maxima")
```

```
[Out] -2/21*(a + b/x^3)^(7/2)/b^3 + 4/15*(a + b/x^3)^(5/2)*a/b^3 - 2/9*(a + b/x^3
)^(3/2)*a^2/b^3
```

mupad [B] time = 1.94, size = 70, normalized size = 1.19

$$\frac{8\,a^2\sqrt{a+\frac{b}{x^3}}}{315\,b^2\,x^3}-\frac{16\,a^3\sqrt{a+\frac{b}{x^3}}}{315\,b^3}-\frac{2\,a\sqrt{a+\frac{b}{x^3}}}{105\,b\,x^6}-\frac{2\sqrt{a+\frac{b}{x^3}}}{21\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^3)^(1/2)/x^10,x)
```

```
[Out] (8*a^2*(a + b/x^3)^(1/2))/(315*b^2*x^3) - (16*a^3*(a + b/x^3)^(1/2))/(315*b
^3) - (2*a*(a + b/x^3)^(1/2))/(105*b*x^6) - (2*(a + b/x^3)^(1/2))/(21*x^9)
```

sympy [**B**] time = 2.91, size = 913, normalized size = 15.47

$$-\frac{16a^{\frac{19}{2}}b^{\frac{9}{2}}x^{18}\sqrt{\frac{ax^3}{b}+1}}{315a^{\frac{13}{2}}b^7x^{\frac{39}{2}}+945a^{\frac{11}{2}}b^8x^{\frac{33}{2}}+945a^{\frac{9}{2}}b^9x^{\frac{27}{2}}+315a^{\frac{7}{2}}b^{10}x^{\frac{21}{2}}}-\frac{40a^{\frac{17}{2}}b^{\frac{11}{2}}x^{15}\sqrt{\frac{ax^3}{b}+1}}{315a^{\frac{13}{2}}b^7x^{\frac{39}{2}}+945a^{\frac{11}{2}}b^8x^{\frac{33}{2}}+945a^{\frac{9}{2}}b^9x^{\frac{27}{2}}+315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**3)**(1/2)/x**10,x)
```

```
[Out] -16*a**(19/2)*b**(9/2)*x**18*sqrt(a*x**3/b + 1)/(315*a**(13/2)*b**7*x**(39/
2) + 945*a**(11/2)*b**8*x**(33/2) + 945*a**(9/2)*b**9*x**(27/2) + 315*a**(7
/2)*b**10*x**(21/2)) - 40*a**(17/2)*b**(11/2)*x**15*sqrt(a*x**3/b + 1)/(315
*a**(13/2)*b**7*x**(39/2) + 945*a**(11/2)*b**8*x**(33/2) + 945*a**(9/2)*b**
9*x**(27/2) + 315*a**(7/2)*b**10*x**(21/2)) - 30*a**(15/2)*b**(13/2)*x**12*
sqrt(a*x**3/b + 1)/(315*a**(13/2)*b**7*x**(39/2) + 945*a**(11/2)*b**8*x**(3
3/2) + 945*a**(9/2)*b**9*x**(27/2) + 315*a**(7/2)*b**10*x**(21/2)) - 40*a**
(13/2)*b**(15/2)*x**9*sqrt(a*x**3/b + 1)/(315*a**(13/2)*b**7*x**(39/2) + 94
5*a**(11/2)*b**8*x**(33/2) + 945*a**(9/2)*b**9*x**(27/2) + 315*a**(7/2)*b**
10*x**(21/2)) - 100*a**(11/2)*b**(17/2)*x**6*sqrt(a*x**3/b + 1)/(315*a**(13
/2)*b**7*x**(39/2) + 945*a**(11/2)*b**8*x**(33/2) + 945*a**(9/2)*b**9*x**(2
7/2) + 315*a**(7/2)*b**10*x**(21/2)) - 96*a**(9/2)*b**(19/2)*x**3*sqrt(a*x*
*3/b + 1)/(315*a**(13/2)*b**7*x**(39/2) + 945*a**(11/2)*b**8*x**(33/2) + 94
5*a**(9/2)*b**9*x**(27/2) + 315*a**(7/2)*b**10*x**(21/2)) - 30*a**(7/2)*b**
(21/2)*sqrt(a*x**3/b + 1)/(315*a**(13/2)*b**7*x**(39/2) + 945*a**(11/2)*b**
8*x**(33/2) + 945*a**(9/2)*b**9*x**(27/2) + 315*a**(7/2)*b**10*x**(21/2)) +
 16*a**10*b**4*x**(39/2)/(315*a**(13/2)*b**7*x**(39/2) + 945*a**(11/2)*b**8
*x**(33/2) + 945*a**(9/2)*b**9*x**(27/2) + 315*a**(7/2)*b**10*x**(21/2)) +
48*a**9*b**5*x**(33/2)/(315*a**(13/2)*b**7*x**(39/2) + 945*a**(11/2)*b**8*x
**(33/2) + 945*a**(9/2)*b**9*x**(27/2) + 315*a**(7/2)*b**10*x**(21/2)) + 48
*a**8*b**6*x**(27/2)/(315*a**(13/2)*b**7*x**(39/2) + 945*a**(11/2)*b**8*x**
(33/2) + 945*a**(9/2)*b**9*x**(27/2) + 315*a**(7/2)*b**10*x**(21/2)) + 16*a
**7*b**7*x**(21/2)/(315*a**(13/2)*b**7*x**(39/2) + 945*a**(11/2)*b**8*x**(3
3/2) + 945*a**(9/2)*b**9*x**(27/2) + 315*a**(7/2)*b**10*x**(21/2))
```

3.1997 $\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^{13}}\,dx$

Optimal. Leaf size=80

$$\frac{2a^3\left(a+\frac{b}{x^3}\right)^{3/2}}{9b^4}-\frac{2a^2\left(a+\frac{b}{x^3}\right)^{5/2}}{5b^4}-\frac{2\left(a+\frac{b}{x^3}\right)^{9/2}}{27b^4}+\frac{2a\left(a+\frac{b}{x^3}\right)^{7/2}}{7b^4}$$

```
[Out] 2/9*a^3*(a+b/x^3)^(3/2)/b^4-2/5*a^2*(a+b/x^3)^(5/2)/b^4+2/7*a*(a+b/x^3)^(7/
2)/b^4-2/27*(a+b/x^3)^(9/2)/b^4
```

Rubi [A] time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{2a^2\left(a+\frac{b}{x^3}\right)^{5/2}}{5b^4}+\frac{2a^3\left(a+\frac{b}{x^3}\right)^{3/2}}{9b^4}-\frac{2\left(a+\frac{b}{x^3}\right)^{9/2}}{27b^4}+\frac{2a\left(a+\frac{b}{x^3}\right)^{7/2}}{7b^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^3]/x^13,x]
```

```
[Out] (2*a^3*(a + b/x^3)^(3/2))/(9*b^4) - (2*a^2*(a + b/x^3)^(5/2))/(5*b^4) + (2*
a*(a + b/x^3)^(7/2))/(7*b^4) - (2*(a + b/x^3)^(9/2))/(27*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^{13}}\,dx &= -\left(\frac{1}{3}\,\text{Subst}\left(\int x^3\sqrt{a+bx}\,dx, x, \frac{1}{x^3}\right)\right)\\ &= -\left(\frac{1}{3}\,\text{Subst}\left(\int\left(-\frac{a^3\sqrt{a+bx}}{b^3}+\frac{3a^2(a+bx)^{3/2}}{b^3}-\frac{3a(a+bx)^{5/2}}{b^3}+\frac{(a+bx)^{7/2}}{b^3}\right)dx, x, \frac{1}{x^3}\right)\right)\\ &= \frac{2a^3\left(a+\frac{b}{x^3}\right)^{3/2}}{9b^4}-\frac{2a^2\left(a+\frac{b}{x^3}\right)^{5/2}}{5b^4}+\frac{2a\left(a+\frac{b}{x^3}\right)^{7/2}}{7b^4}-\frac{2\left(a+\frac{b}{x^3}\right)^{9/2}}{27b^4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 60, normalized size = 0.75

$$\frac{2\sqrt{a+\frac{b}{x^3}}\left(ax^3+b\right)\left(16a^3x^9-24a^2bx^6+30ab^2x^3-35b^3\right)}{945b^4x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^3]/x^13,x]
```

```
[Out] (2*Sqrt[a + b/x^3]*(b + a*x^3)*(-35*b^3 + 30*a*b^2*x^3 - 24*a^2*b*x^6 + 16*
a^3*x^9))/(945*b^4*x^12)
```

fricas [A] time = 1.24, size = 64, normalized size = 0.80

$$\frac{2\left(16\,a^4x^{12}-8\,a^3bx^9+6\,a^2b^2x^6-5\,ab^3x^3-35\,b^4\right)\sqrt{\frac{ax^3+b}{x^3}}}{945\,b^4x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(1/2)/x^13,x, algorithm="fricas")
```

```
[Out] 2/945*(16*a^4*x^12 - 8*a^3*b*x^9 + 6*a^2*b^2*x^6 - 5*a*b^3*x^3 - 35*b^4)*sq
rt((a*x^3 + b)/x^3)/(b^4*x^12)
```

giac [A] time = 0.20, size = 57, normalized size = 0.71

$$-\frac{2\left(35\left(a+\frac{b}{x^3}\right)^{\frac{9}{2}}-135\left(a+\frac{b}{x^3}\right)^{\frac{7}{2}}a+189\left(a+\frac{b}{x^3}\right)^{\frac{5}{2}}a^2-105\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}a^3\right)}{945\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(1/2)/x^13,x, algorithm="giac")
```

```
[Out] -2/945*(35*(a + b/x^3)^(9/2) - 135*(a + b/x^3)^(7/2)*a + 189*(a + b/x^3)^(5
/2)*a^2 - 105*(a + b/x^3)^(3/2)*a^3)/b^4
```

maple [A] time = 0.01, size = 61, normalized size = 0.76

$$\frac{2\left(a\,x^3+b\right)\left(16a^3x^9-24a^2b\,x^6+30a\,b^2x^3-35b^3\right)\sqrt{\frac{a\,x^3+b}{x^3}}}{945b^4x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^3)^(1/2)/x^13,x)
```

```
[Out] 2/945*(a*x^3+b)*(16*a^3*x^9-24*a^2*b*x^6+30*a*b^2*x^3-35*b^3)*((a*x^3+b)/x^
3)^(1/2)/x^12/b^4
```

maxima [A] time = 0.81, size = 64, normalized size = 0.80

$$-\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{9}{2}}}{27\,b^4}+\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{7}{2}}a}{7\,b^4}-\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{5}{2}}a^2}{5\,b^4}+\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}a^3}{9\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(1/2)/x^13,x, algorithm="maxima")
```

```
[Out] -2/27*(a + b/x^3)^(9/2)/b^4 + 2/7*(a + b/x^3)^(7/2)*a/b^4 - 2/5*(a + b/x^3)
^(5/2)*a^2/b^4 + 2/9*(a + b/x^3)^(3/2)*a^3/b^4
```

mupad [B] time = 2.34, size = 90, normalized size = 1.12

$$\frac{32\,a^4\sqrt{a+\frac{b}{x^3}}}{945\,b^4}-\frac{2\sqrt{a+\frac{b}{x^3}}}{27\,x^{12}}-\frac{2\,a\sqrt{a+\frac{b}{x^3}}}{189\,b\,x^9}-\frac{16\,a^3\sqrt{a+\frac{b}{x^3}}}{945\,b^3\,x^3}+\frac{4\,a^2\sqrt{a+\frac{b}{x^3}}}{315\,b^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^3)^(1/2)/x^13,x)
```

```
[Out] (32*a^4*(a + b/x^3)^(1/2))/(945*b^4) - (2*(a + b/x^3)^(1/2))/(27*x^12) - (2
*a*(a + b/x^3)^(1/2))/(189*b*x^9) - (16*a^3*(a + b/x^3)^(1/2))/(945*b^3*x^3
) + (4*a^2*(a + b/x^3)^(1/2))/(315*b^2*x^6)
```

sympy [**B**] time = 4.28, size = 2317, normalized size = 28.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**3)**(1/2)/x**13,x)
```

```
[Out] 32*a**(29/2)*b**(23/2)*x**30*sqrt(a*x**3/b + 1)/(945*a**(21/2)*b**15*x**(63
/2) + 5670*a**(19/2)*b**16*x**(57/2) + 14175*a**(17/2)*b**17*x**(51/2) + 18
900*a**(15/2)*b**18*x**(45/2) + 14175*a**(13/2)*b**19*x**(39/2) + 5670*a**(
11/2)*b**20*x**(33/2) + 945*a**(9/2)*b**21*x**(27/2)) + 176*a**(27/2)*b**(2
5/2)*x**27*sqrt(a*x**3/b + 1)/(945*a**(21/2)*b**15*x**(63/2) + 5670*a**(19/
2)*b**16*x**(57/2) + 14175*a**(17/2)*b**17*x**(51/2) + 18900*a**(15/2)*b**1
8*x**(45/2) + 14175*a**(13/2)*b**19*x**(39/2) + 5670*a**(11/2)*b**20*x**(33
/2) + 945*a**(9/2)*b**21*x**(27/2)) + 396*a**(25/2)*b**(27/2)*x**24*sqrt(a*
x**3/b + 1)/(945*a**(21/2)*b**15*x**(63/2) + 5670*a**(19/2)*b**16*x**(57/2)
 + 14175*a**(17/2)*b**17*x**(51/2) + 18900*a**(15/2)*b**18*x**(45/2) + 1417
5*a**(13/2)*b**19*x**(39/2) + 5670*a**(11/2)*b**20*x**(33/2) + 945*a**(9/2)
*b**21*x**(27/2)) + 462*a**(23/2)*b**(29/2)*x**21*sqrt(a*x**3/b + 1)/(945*a
**(21/2)*b**15*x**(63/2) + 5670*a**(19/2)*b**16*x**(57/2) + 14175*a**(17/2)
*b**17*x**(51/2) + 18900*a**(15/2)*b**18*x**(45/2) + 14175*a**(13/2)*b**19*
x**(39/2) + 5670*a**(11/2)*b**20*x**(33/2) + 945*a**(9/2)*b**21*x**(27/2))
+ 210*a**(21/2)*b**(31/2)*x**18*sqrt(a*x**3/b + 1)/(945*a**(21/2)*b**15*x**
(63/2) + 5670*a**(19/2)*b**16*x**(57/2) + 14175*a**(17/2)*b**17*x**(51/2) +
 18900*a**(15/2)*b**18*x**(45/2) + 14175*a**(13/2)*b**19*x**(39/2) + 5670*a
**(11/2)*b**20*x**(33/2) + 945*a**(9/2)*b**21*x**(27/2)) - 378*a**(19/2)*b*
*(33/2)*x**15*sqrt(a*x**3/b + 1)/(945*a**(21/2)*b**15*x**(63/2) + 5670*a**(
19/2)*b**16*x**(57/2) + 14175*a**(17/2)*b**17*x**(51/2) + 18900*a**(15/2)*b
**18*x**(45/2) + 14175*a**(13/2)*b**19*x**(39/2) + 5670*a**(11/2)*b**20*x**
(33/2) + 945*a**(9/2)*b**21*x**(27/2)) - 1134*a**(17/2)*b**(35/2)*x**12*sqr
t(a*x**3/b + 1)/(945*a**(21/2)*b**15*x**(63/2) + 5670*a**(19/2)*b**16*x**(5
7/2) + 14175*a**(17/2)*b**17*x**(51/2) + 18900*a**(15/2)*b**18*x**(45/2) +
14175*a**(13/2)*b**19*x**(39/2) + 5670*a**(11/2)*b**20*x**(33/2) + 945*a**(
9/2)*b**21*x**(27/2)) - 1494*a**(15/2)*b**(37/2)*x**9*sqrt(a*x**3/b + 1)/(9
45*a**(21/2)*b**15*x**(63/2) + 5670*a**(19/2)*b**16*x**(57/2) + 14175*a**(1
7/2)*b**17*x**(51/2) + 18900*a**(15/2)*b**18*x**(45/2) + 14175*a**(13/2)*b*
*19*x**(39/2) + 5670*a**(11/2)*b**20*x**(33/2) + 945*a**(9/2)*b**21*x**(27/
2)) - 1098*a**(13/2)*b**(39/2)*x**6*sqrt(a*x**3/b + 1)/(945*a**(21/2)*b**15
*x**(63/2) + 5670*a**(19/2)*b**16*x**(57/2) + 14175*a**(17/2)*b**17*x**(51/
2) + 18900*a**(15/2)*b**18*x**(45/2) + 14175*a**(13/2)*b**19*x**(39/2) + 56
70*a**(11/2)*b**20*x**(33/2) + 945*a**(9/2)*b**21*x**(27/2)) - 430*a**(11/2
)*b**(41/2)*x**3*sqrt(a*x**3/b + 1)/(945*a**(21/2)*b**15*x**(63/2) + 5670*a
**(19/2)*b**16*x**(57/2) + 14175*a**(17/2)*b**17*x**(51/2) + 18900*a**(15/2
)*b**18*x**(45/2) + 14175*a**(13/2)*b**19*x**(39/2) + 5670*a**(11/2)*b**20*
x**(33/2) + 945*a**(9/2)*b**21*x**(27/2)) - 70*a**(9/2)*b**(43/2)*sqrt(a*x*
*3/b + 1)/(945*a**(21/2)*b**15*x**(63/2) + 5670*a**(19/2)*b**16*x**(57/2) +
 14175*a**(17/2)*b**17*x**(51/2) + 18900*a**(15/2)*b**18*x**(45/2) + 14175*
a**(13/2)*b**19*x**(39/2) + 5670*a**(11/2)*b**20*x**(33/2) + 945*a**(9/2)*b
**21*x**(27/2)) - 32*a**15*b**11*x**(63/2)/(945*a**(21/2)*b**15*x**(63/2) +
 5670*a**(19/2)*b**16*x**(57/2) + 14175*a**(17/2)*b**17*x**(51/2) + 18900*a
**(15/2)*b**18*x**(45/2) + 14175*a**(13/2)*b**19*x**(39/2) + 5670*a**(11/2)
*b**20*x**(33/2) + 945*a**(9/2)*b**21*x**(27/2)) - 192*a**14*b**12*x**(57/2
```

```
)/(945*a**(21/2)*b**15*x**(63/2) + 5670*a**(19/2)*b**16*x**(57/2) + 14175*a
**(17/2)*b**17*x**(51/2) + 18900*a**(15/2)*b**18*x**(45/2) + 14175*a**(13/2
)*b**19*x**(39/2) + 5670*a**(11/2)*b**20*x**(33/2) + 945*a**(9/2)*b**21*x**
(27/2)) - 480*a**13*b**13*x**(51/2)/(945*a**(21/2)*b**15*x**(63/2) + 5670*a
**(19/2)*b**16*x**(57/2) + 14175*a**(17/2)*b**17*x**(51/2) + 18900*a**(15/2
)*b**18*x**(45/2) + 14175*a**(13/2)*b**19*x**(39/2) + 5670*a**(11/2)*b**20*
x**(33/2) + 945*a**(9/2)*b**21*x**(27/2)) - 640*a**12*b**14*x**(45/2)/(945*
a**(21/2)*b**15*x**(63/2) + 5670*a**(19/2)*b**16*x**(57/2) + 14175*a**(17/2
)*b**17*x**(51/2) + 18900*a**(15/2)*b**18*x**(45/2) + 14175*a**(13/2)*b**19
*x**(39/2) + 5670*a**(11/2)*b**20*x**(33/2) + 945*a**(9/2)*b**21*x**(27/2))
 - 480*a**11*b**15*x**(39/2)/(945*a**(21/2)*b**15*x**(63/2) + 5670*a**(19/2
)*b**16*x**(57/2) + 14175*a**(17/2)*b**17*x**(51/2) + 18900*a**(15/2)*b**18
*x**(45/2) + 14175*a**(13/2)*b**19*x**(39/2) + 5670*a**(11/2)*b**20*x**(33/
2) + 945*a**(9/2)*b**21*x**(27/2)) - 192*a**10*b**16*x**(33/2)/(945*a**(21/
2)*b**15*x**(63/2) + 5670*a**(19/2)*b**16*x**(57/2) + 14175*a**(17/2)*b**17
*x**(51/2) + 18900*a**(15/2)*b**18*x**(45/2) + 14175*a**(13/2)*b**19*x**(39
/2) + 5670*a**(11/2)*b**20*x**(33/2) + 945*a**(9/2)*b**21*x**(27/2)) - 32*a
**9*b**17*x**(27/2)/(945*a**(21/2)*b**15*x**(63/2) + 5670*a**(19/2)*b**16*x
**(57/2) + 14175*a**(17/2)*b**17*x**(51/2) + 18900*a**(15/2)*b**18*x**(45/2
) + 14175*a**(13/2)*b**19*x**(39/2) + 5670*a**(11/2)*b**20*x**(33/2) + 945*
a**(9/2)*b**21*x**(27/2))
```

3.1998 $\int \sqrt{a+\frac{b}{x^3}}\, x^7\, dx$

Optimal. Leaf size=291

$$-\frac{21b^2x^2\sqrt{a+\frac{b}{x^3}}}{320a^2}-\frac{7\,3^{3/4}\sqrt{2+\sqrt{3}}\;b^{8/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{320a^2\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}+\frac{1}{8}x$$

```
[Out] -21/320*b^2*x^2*(a+b/x^3)^(1/2)/a^2+3/80*b*x^5*(a+b/x^3)^(1/2)/a+1/8*x^8*(a
+b/x^3)^(1/2)-7/320*3^(3/4)*b^(8/3)*(a^(1/3)+b^(1/3)/x)*EllipticF((b^(1/3)/
x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*
6^(1/2)+1/2*2^(1/2))*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^
(1/3)*(1+3^(1/2)))^2)^(1/2)/a^2/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x
)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {335, 277, 325, 218}

$$-\frac{21b^2x^2\sqrt{a+\frac{b}{x^3}}}{320a^2}-\frac{7\,3^{3/4}\sqrt{2+\sqrt{3}}\;b^{8/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{320a^2\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}+\frac{1}{8}x$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^3]*x^7,x]
```

```
[Out] (-21*b^2*Sqrt[a + b/x^3]*x^2)/(320*a^2) + (3*b*Sqrt[a + b/x^3]*x^5)/(80*a)
+ (Sqrt[a + b/x^3]*x^8)/8 - (7*3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(8/3)*(a^(1/3) +
 b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3
)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(320*a^2*Sqrt[a
 + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)/x)^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

```
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \sqrt{a+\frac{b}{x^3}}\, x^7\, dx = -\text{Subst}\left(\int \frac{\sqrt{a+bx^3}}{x^9}\, dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{8}\sqrt{a+\frac{b}{x^3}}\, x^8 - \frac{1}{16}(3b)\,\text{Subst}\left(\int \frac{1}{x^6\sqrt{a+bx^3}}\, dx, x, \frac{1}{x}\right)$$

$$= \frac{3b\sqrt{a+\frac{b}{x^3}}\, x^5}{80a} + \frac{1}{8}\sqrt{a+\frac{b}{x^3}}\, x^8 + \frac{\left(21b^2\right)\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx^3}}\, dx, x, \frac{1}{x}\right)}{160a}$$

$$= -\frac{21b^2\sqrt{a+\frac{b}{x^3}}\, x^2}{320a^2} + \frac{3b\sqrt{a+\frac{b}{x^3}}\, x^5}{80a} + \frac{1}{8}\sqrt{a+\frac{b}{x^3}}\, x^8 - \frac{\left(21b^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^3}}\, dx, x, \frac{1}{x}\right)}{640a^2}$$

$$= -\frac{21b^2\sqrt{a+\frac{b}{x^3}}\, x^2}{320a^2} + \frac{3b\sqrt{a+\frac{b}{x^3}}\, x^5}{80a} + \frac{1}{8}\sqrt{a+\frac{b}{x^3}}\, x^8 - \frac{7\,3^{3/4}\sqrt{2+\sqrt{3}}\, b^{8/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{-}}{320a^2\sqrt{}}$$

Mathematica [C] time = 0.08, size = 96, normalized size = 0.33

$$\frac{x^2\sqrt{a+\frac{b}{x^3}}\left(\sqrt{\frac{ax^3}{b}+1}\left(10a^2x^6+3abx^3-7b^2\right)+7b^2\,{}_2F_1\left(-\frac{1}{2},\frac{1}{6};\frac{7}{6};-\frac{ax^3}{b}\right)\right)}{80a^2\sqrt{\frac{ax^3}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^3]*x^7,x]
```

```
[Out] (Sqrt[a + b/x^3]*x^2*(Sqrt[1 + (a*x^3)/b]*(-7*b^2 + 3*a*b*x^3 + 10*a^2*x^6)
 + 7*b^2*Hypergeometric2F1[-1/2, 1/6, 7/6, -((a*x^3)/b)]))/(80*a^2*Sqrt[1 +
 (a*x^3)/b])
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(x^7\sqrt{\frac{ax^3+b}{x^3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b/x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^7*sqrt((a*x^3 + b)/x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + \frac{b}{x^3}}\, x^7\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b/x^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/x^3)*x^7, x)
```

maple [B] time = 0.03, size = 2226, normalized size = 7.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a+b/x^3)^(1/2),x)
```

```
[Out] -1/320*((a*x^3+b)/x^3)^(1/2)*x^2/a^3/(-a^2*b)^(1/3)*(-40*I*(-a^2*b)^(1/3)*3
^(1/2)*(a*x^4+b*x)^(1/2)*(1/a^2*x*(-a*x+(-a^2*b)^(1/3))*(I*3^(1/2)*(-a^2*b)
^(1/3)+2*a*x+(-a^2*b)^(1/3))*(I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3)
))^(1/2)*x^6*a^3+42*I*3^(1/2)*(-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2
*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))/(1+I*3^(
1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b
)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-
3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)
-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a^2*b^3-84*I*(-a^2*b)^(1/3)*3^(
1/2)*(-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(
1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3
)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-
a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*
x+(-a^2*b)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1
/2)-3))^(1/2))*x*a*b^3+120*x^6*(a*x^4+b*x)^(1/2)*a^3*(-a^2*b)^(1/3)*(1/a^2*
x*(-a*x+(-a^2*b)^(1/3))*(I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))*(I*
3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3)))^(1/2)+42*I*(-a^2*b)^(2/3)*3^(
1/2)*(-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(
1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3
)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-
a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*
x+(-a^2*b)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1
/2)-3))^(1/2))*b^3-12*I*(-a^2*b)^(1/3)*3^(1/2)*(a*x^4+b*x)^(1/2)*(1/a^2*x*(
-a*x+(-a^2*b)^(1/3))*(I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))*(I*3^(
1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3)))^(1/2)*x^3*a^2*b-42*(-(I*3^(1/2)-
3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3
)+2*a*x+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1
/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)
))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))
^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2
*a^2*b^3+84*(-a^2*b)^(1/3)*(-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)
^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))/(1+I*3^(1/2
))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(
1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*
x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)
/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x*a*b^3-42*(-a^2*b)^(2/3)*(-(I*3^(1/2)
-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/
```

```
3)+2*a*x+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(
1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3
)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))
)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*b^
3+36*b*x^3*(a*x^4+b*x)^(1/2)*a^2*(-a^2*b)^(1/3)*(1/a^2*x*(-a*x+(-a^2*b)^(1/
3))*(I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))*(I*3^(1/2)*(-a^2*b)^(1/
3)-2*a*x-(-a^2*b)^(1/3)))^(1/2)+21*I*(-a^2*b)^(1/3)*3^(1/2)*(a*x^4+b*x)^(1/
2)*(1/a^2*x*(-a*x+(-a^2*b)^(1/3))*(I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^
(1/3))*(I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3)))^(1/2)*a*b^2-63*b^2*
(a*x^4+b*x)^(1/2)*a*(-a^2*b)^(1/3)*(1/a^2*x*(-a*x+(-a^2*b)^(1/3))*(I*3^(1/2
)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))*(I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^
2*b)^(1/3)))^(1/2))/(x*(a*x^3+b))^(1/2)/(I*3^(1/2)-3)/(1/a^2*x*(-a*x+(-a^2*
b)^(1/3))*(I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))*(I*3^(1/2)*(-a^2*
b)^(1/3)-2*a*x-(-a^2*b)^(1/3)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + \frac{b}{x^3}}\, x^7\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b/x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x^3)*x^7, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^7 \sqrt{a + \frac{b}{x^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b/x^3)^(1/2),x)
```

```
[Out] int(x^7*(a + b/x^3)^(1/2), x)
```

sympy [A] time = 1.59, size = 48, normalized size = 0.16

$$-\frac{\sqrt{a}\, x^8 \Gamma\left(-\frac{8}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{8}{3}, -\frac{1}{2} \\ -\frac{5}{3} \end{matrix} \middle| \frac{b e^{i\pi}}{a x^3}\right)}{3\Gamma\left(-\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(a+b/x**3)**(1/2),x)
```

```
[Out] -sqrt(a)*x**8*gamma(-8/3)*hyper((-8/3, -1/2), (-5/3,), b*exp_polar(I*pi)/(a
*x**3))/(3*gamma(-5/3))
```

3.1999 $\int \sqrt{a+\frac{b}{x^3}}\,x^4\,dx$

Optimal. Leaf size=267

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{5/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{20a\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}+\frac{1}{5}x^5\sqrt{a+\frac{b}{x^3}}+\frac{3bx^2\sqrt{a}}{20a}$$

```
[Out] 3/20*b*x^2*(a+b/x^3)^(1/2)/a+1/5*x^5*(a+b/x^3)^(1/2)+1/20*3^(3/4)*b^(5/3)*(
a^(1/3)+b^(1/3)/x)*EllipticF((b^(1/3)/x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)/x+a^(
1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)+b^(2/3
)/x^2-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a/(a+b/x^
3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1
/2)
```

Rubi [A] time = 0.12, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {335, 277, 325, 218}

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{5/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{20a\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}+\frac{1}{5}x^5\sqrt{a+\frac{b}{x^3}}+\frac{3bx^2\sqrt{a}}{20a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^3]*x^4,x]
```

```
[Out] (3*b*Sqrt[a + b/x^3]*x^2)/(20*a) + (Sqrt[a + b/x^3]*x^5)/5 + (3^(3/4)*Sqrt[
2 + Sqrt[3]]*b^(5/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a
^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[
((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -
7 - 4*Sqrt[3]])/(20*a*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \sqrt{a+\frac{b}{x^3}}\,x^4\,dx = -\text{Subst}\left(\int \frac{\sqrt{a+bx^3}}{x^6}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{5}\sqrt{a+\frac{b}{x^3}}\,x^5 - \frac{1}{10}(3b)\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{3b\sqrt{a+\frac{b}{x^3}}\,x^2}{20a} + \frac{1}{5}\sqrt{a+\frac{b}{x^3}}\,x^5 + \frac{\left(3b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{40a}$$

$$= \frac{3b\sqrt{a+\frac{b}{x^3}}\,x^2}{20a} + \frac{1}{5}\sqrt{a+\frac{b}{x^3}}\,x^5 + \frac{3^{3/4}\sqrt{2+\sqrt{3}}\;b^{5/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\,\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\right.}{20a\sqrt{a+\frac{b}{x^3}}\,\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{}}{x}\right.}}}$$

Mathematica [C] time = 0.06, size = 80, normalized size = 0.30

$$\frac{x^2\sqrt{a+\frac{b}{x^3}}\left(\left(ax^3+b\right)\sqrt{\frac{ax^3}{b}+1} - b\,{}_2F_1\left(-\frac{1}{2},\frac{1}{6};\frac{7}{6};-\frac{ax^3}{b}\right)\right)}{5a\sqrt{\frac{ax^3}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^3]*x^4,x]
```

```
[Out] (Sqrt[a + b/x^3]*x^2*((b + a*x^3)*Sqrt[1 + (a*x^3)/b] - b*Hypergeometric2F1
[-1/2, 1/6, 7/6, -((a*x^3)/b)]))/(5*a*Sqrt[1 + (a*x^3)/b])
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(x^4\sqrt{\frac{ax^3+b}{x^3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b/x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^4*sqrt((a*x^3 + b)/x^3), x)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + \frac{b}{x^3}}\, x^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b/x^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/x^3)*x^4, x)
```

maple [**B**] time = 0.03, size = 2010, normalized size = 7.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b/x^3)^(1/2),x)
```

```
[Out] 1/20*((a*x^3+b)/x^3)^(1/2)*x^2/a^2/(-a^2*b)^(1/3)*(6*I*(-(I*3^(1/2)-3)/(I*3
^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)
+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1
/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/
2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)
,((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*3^(1/2)*x
^2*a^2*b^2-12*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1
/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-
a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3
^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)
-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1
/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*3^(1/2)*x*a*b^2+4*I*(a*x^4+b*x)^(
1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))
*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*(-a^2*b)^(1/
3)*3^(1/2)*x^3*a^2+6*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*
a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(
-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3
))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*
3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1
+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*3^(1/2)*b^2-6*(-(I*3^(1/2)
-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*
b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*
x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/
3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*
x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x
^2*a^2*b^2+12*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2
)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^
2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(
1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1
)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2
))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*x*a*b^2-12*x^3*(a*x^4+b*x)^(1/2)*a^
2*(-a^2*b)^(1/3)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a
^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)-
6*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*
3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))
^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*
x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^
2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2
)-3))^(1/2))*(-a^2*b)^(2/3)*b^2+3*I*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3)
)*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b
)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*(-a^2*b)^(1/3)*3^(1/2)*a*b-9*b*(a*x^4+
b*x)^(1/2)*a*(-a^2*b)^(1/3)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b
```

```
)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^
2*x)^(1/2))/((a*x^3+b)*x)^(1/2)/(I*3^(1/2)-3)/((-a*x+(-a^2*b)^(1/3))*(2*a*x
+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-
(-a^2*b)^(1/3))/a^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + \frac{b}{x^3}}\, x^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b/x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x^3)*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 \sqrt{a + \frac{b}{x^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b/x^3)^(1/2),x)
```

```
[Out] int(x^4*(a + b/x^3)^(1/2), x)
```

sympy [A] time = 1.29, size = 48, normalized size = 0.18

$$-\frac{\sqrt{a}\, x^5 \Gamma\left(-\frac{5}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{3}, -\frac{1}{2} \\ -\frac{2}{3} \end{matrix} \middle| \frac{b e^{i\pi}}{a x^3}\right)}{3\Gamma\left(-\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b/x**3)**(1/2),x)
```

```
[Out] -sqrt(a)*x**5*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*exp_polar(I*pi)/(a
*x**3))/(3*gamma(-2/3))
```

3.2000 $\int \sqrt{a+\frac{b}{x^3}}\,x\,dx$

Optimal. Leaf size=242

$$\frac{1}{2}x^2\sqrt{a+\frac{b}{x^3}}-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{2\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

```
[Out] 1/2*x^2*(a+b/x^3)^(1/2)-1/2*3^(3/4)*b^(2/3)*(a^(1/3)+b^(1/3)/x)*EllipticF((
b^(1/3)/x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*
I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/(b^(1
/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/
3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {335, 277, 218}

$$\frac{1}{2}x^2\sqrt{a+\frac{b}{x^3}}-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{2\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^3]*x,x]
```

```
[Out] (Sqrt[a + b/x^3]*x^2)/2 - (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(2/3)*(a^(1/3) + b^(
1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(2*Sqrt[a + b/x^3]*
Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]
)
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \sqrt{a+\frac{b}{x^3}}\,x\,dx = -\text{Subst}\left(\int \frac{\sqrt{a+bx^3}}{x^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}\sqrt{a+\frac{b}{x^3}}\,x^2 - \frac{1}{4}(3b)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}\sqrt{a+\frac{b}{x^3}}\,x^2 - \frac{3^{3/4}\sqrt{2+\sqrt{3}}\;b^{2/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\right.}{2\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

Mathematica [C] time = 0.01, size = 49, normalized size = 0.20

$$\frac{2x^2\sqrt{a+\frac{b}{x^3}}\;{}_2F_1\left(-\frac{1}{2},\frac{1}{6};\frac{7}{6};-\frac{ax^3}{b}\right)}{\sqrt{\frac{ax^3}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^3]*x,x]
```

```
[Out] (2*Sqrt[a + b/x^3]*x^2*Hypergeometric2F1[-1/2, 1/6, 7/6, -((a*x^3)/b)])/Sqr
t[1 + (a*x^3)/b]
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(x\sqrt{\frac{ax^3+b}{x^3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x*sqrt((a*x^3 + b)/x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+\frac{b}{x^3}}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/x^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/x^3)*x, x)
```

maple [B] time = 0.03, size = 1786, normalized size = 7.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b/x^3)^(1/2),x)
```

```
[Out] -1/2*((a*x^3+b)/x^3)^(1/2)*x^2/(-a^2*b)^(1/3)/a*(6*I*3^(1/2)*(-(I*3^(1/2)-3
)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)
^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+
I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)
))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)
^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2
*a^2*b-12*I*(-a^2*b)^(1/3)*3^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^
2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+
I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(
-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^
(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3
^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x*a*b+6*I*(-a^2*b)^(2/3)*3^(1
/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+
I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)
))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-
a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-
a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1
/2)-3))^(1/2))*b-6*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)
^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x
+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(
I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1
/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3
^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a^2*b+12*(-a^2*b)^(1/3)*(-(I*3^(1/2)-3)/(
I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1
/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3
^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^
(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1
/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x*a*b-
6*(-a^2*b)^(2/3)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(
1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(
-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*
3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2
)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(
1/2))/(I*3^(1/2)-3))^(1/2))*b-I*(-a^2*b)^(1/3)*3^(1/2)*(a*x^4+b*x)^(1/2)*((
-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*
x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*a+3*(a*x^4+b*x)^(1/
2)*a*(-a^2*b)^(1/3)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+
(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/
2))/((a*x^3+b)*x)^(1/2)/(I*3^(1/2)-3)/((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/
2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)
^(1/3))/a^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + \frac{b}{x^3}}\, x\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x^3)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\sqrt{a+\frac{b}{x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b/x^3)^(1/2),x)
```

```
[Out] int(x*(a + b/x^3)^(1/2), x)
```

sympy [A] time = 1.09, size = 44, normalized size = 0.18

$$-\frac{\sqrt{a}\,x^2\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\ \frac{1}{3}\end{matrix}\middle|\frac{be^{i\pi}}{ax^3}\right)}{3\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/x**3)**(1/2),x)
```

```
[Out] -sqrt(a)*x**2*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*exp_polar(I*pi)/(a*
x**3))/(3*gamma(1/3))
```

3.2001 $\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^2}\,dx$

Optimal. Leaf size=243

$$-\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\;a\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{5\sqrt[3]{b}\,\sqrt{a+\frac{b}{x^3}}\,\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}-\frac{2\sqrt{a+\frac{b}{x^3}}}{5x}$$

```
[Out] -2/5*(a+b/x^3)^(1/2)/x-2/5*3^(3/4)*a*(a^(1/3)+b^(1/3)/x)*EllipticF((b^(1/3)
/x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2
*6^(1/2)+1/2*2^(1/2))*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+a
^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(1/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1
/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {335, 195, 218}

$$-\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\;a\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{5\sqrt[3]{b}\,\sqrt{a+\frac{b}{x^3}}\,\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}-\frac{2\sqrt{a+\frac{b}{x^3}}}{5x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^3]/x^2,x]
```

```
[Out] (-2*Sqrt[a + b/x^3])/(5*x) - (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(a^(1/3) + b^(1
/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(5*b^(1/3)*Sqrt[a +
b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)/x)^2])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^2}\,dx = -\text{Subst}\left(\int \sqrt{a+bx^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2\sqrt{a+\frac{b}{x^3}}}{5x} - \frac{1}{5}(3a)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2\sqrt{a+\frac{b}{x^3}}}{5x} - \frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\;a\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-\right.}{5\sqrt[3]{b}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.21

$$-\frac{2\sqrt{a+\frac{b}{x^3}}\;{}_2F_1\left(-\frac{5}{6},-\frac{1}{2};\frac{1}{6};-\frac{ax^3}{b}\right)}{5x\sqrt{\frac{ax^3}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^3]/x^2,x]
```

```
[Out] (-2*Sqrt[a + b/x^3]*Hypergeometric2F1[-5/6, -1/2, 1/6, -((a*x^3)/b)])/(5*x*
Sqrt[1 + (a*x^3)/b])
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\frac{ax^3+b}{x^3}}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt((a*x^3 + b)/x^3)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/x^3)/x^2, x)
```

maple [B] time = 0.03, size = 1785, normalized size = 7.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^3)^(1/2)/x^2,x)
```

```
[Out] -2/5*((a*x^3+b)/x^3)^(1/2)/x/(-a^2*b)^(1/3)*(6*I*3^(1/2)*(-(I*3^(1/2)-3)/(I
*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/
3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^
(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(
1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/
2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^5*a^2
-12*I*(-a^2*b)^(1/3)*3^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(
1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1
/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b
)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-
3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)
-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^4*a+6*I*(-a^2*b)^(2/3)*3^(1/2)*(-
(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1
/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/
2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-
a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)
^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3)
)^(1/2))*x^3-6*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/
2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a
^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^
(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-
1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/
2))/(I*3^(1/2)-3))^(1/2))*x^5*a^2+12*(-a^2*b)^(1/3)*(-(I*3^(1/2)-3)/(I*3^(1
/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-
a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)
*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*
EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((
I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^4*a-6*(-a^
2*b)^(2/3)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*(
(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b
)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2
)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(
-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/
(I*3^(1/2)-3))^(1/2))*x^3+I*(-a^2*b)^(1/3)*3^(1/2)*(a*x^4+b*x)^(1/2)*((-a*x
+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*
3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)-3*(a*x^4+b*x)^(1/2)*(-a
^2*b)^(1/3)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)
^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2))/((a*
x^3+b)*x)^(1/2)/(I*3^(1/2)-3)/((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2
*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/
a^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x^3)/x^2, x)
```

mupad [B] time = 1.41, size = 40, normalized size = 0.16

$$-\frac{\sqrt{a+\frac{b}{x^3}}\ {}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{1}{3};\ \frac{4}{3};\ -\frac{b}{a\,x^3}\right)}{x\sqrt{\frac{b}{a\,x^3}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^3)^(1/2)/x^2,x)
```

```
[Out] -((a + b/x^3)^(1/2)*hypergeom([-1/2, 1/3], 4/3, -b/(a*x^3)))/(x*(b/(a*x^3)
+ 1)^(1/2))
```

sympy [A] time = 1.09, size = 39, normalized size = 0.16

$$-\frac{\sqrt{a}\,\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\,\frac{be^{i\pi}}{ax^3}\right)}{3x\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**3)**(1/2)/x**2,x)
```

```
[Out] -sqrt(a)*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*exp_polar(I*pi)/(a*x**3))/
(3*x*gamma(4/3))
```

3.2002 $\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^5}\,dx$

Optimal. Leaf size=267

$$\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^2\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{55b^{4/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}-\frac{6a\sqrt{a+\frac{b}{x^3}}}{55bx}-\frac{2\sqrt{a+\frac{b}{x^3}}}{11x^4}$$

```
[Out] -2/11*(a+b/x^3)^(1/2)/x^4-6/55*a*(a+b/x^3)^(1/2)/b/x+4/55*3^(3/4)*a^2*(a^(1
/3)+b^(1/3)/x)*EllipticF((b^(1/3)/x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3)
*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)+b^(2/3)/x^
2-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(4/3)/(a+b/
x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^
(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {335, 279, 321, 218}

$$\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^2\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{55b^{4/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}-\frac{6a\sqrt{a+\frac{b}{x^3}}}{55bx}-\frac{2\sqrt{a+\frac{b}{x^3}}}{11x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^3]/x^5,x]
```

```
[Out] (-2*Sqrt[a + b/x^3])/(11*x^4) - (6*a*Sqrt[a + b/x^3])/(55*b*x) + (4*3^(3/4)
*Sqrt[2 + Sqrt[3]]*a^2*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 -
(a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSi
n[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)],
 -7 - 4*Sqrt[3]])/(55*b^(4/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1
/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^5}\,dx = -\text{Subst}\left(\int x^3\sqrt{a+bx^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2\sqrt{a+\frac{b}{x^3}}}{11x^4} - \frac{1}{11}(3a)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2\sqrt{a+\frac{b}{x^3}}}{11x^4} - \frac{6a\sqrt{a+\frac{b}{x^3}}}{55bx} + \frac{\left(6a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{55b}$$

$$= -\frac{2\sqrt{a+\frac{b}{x^3}}}{11x^4} - \frac{6a\sqrt{a+\frac{b}{x^3}}}{55bx} + \frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^2\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{(1-}{(1+}\right.\right.}{55b^{4/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

Mathematica [C] time = 0.02, size = 51, normalized size = 0.19

$$-\frac{2\sqrt{a+\frac{b}{x^3}}\;{}_2F_1\left(-\frac{11}{6},-\frac{1}{2};-\frac{5}{6};-\frac{ax^3}{b}\right)}{11x^4\sqrt{\frac{ax^3}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^3]/x^5,x]
```

```
[Out] (-2*Sqrt[a + b/x^3]*Hypergeometric2F1[-11/6, -1/2, -5/6, -((a*x^3)/b)])/(11
*x^4*Sqrt[1 + (a*x^3)/b])
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\frac{ax^3+b}{x^3}}}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] integral(sqrt((a*x^3 + b)/x^3)/x^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/x^3)/x^5, x)
```

maple [B] time = 0.04, size = 2002, normalized size = 7.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^3)^(1/2)/x^5,x)
```

```
[Out] -2/55*((a*x^3+b)/x^3)^(1/2)/x^4/(-a^2*b)^(1/3)*(-12*I*(-(I*3^(1/2)-3)/(I*3^
(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+
(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/
2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2
)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),
((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*3^(1/2)*x^
8*a^3+24*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*(
(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b
)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2
)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(
-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/
(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*3^(1/2)*x^7*a^2-12*I*(-(I*3^(1/2)-3)/(
I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1
/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3
^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^
(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1
/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*
b)^(2/3)*3^(1/2)*x^6*a+12*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3
))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2)
)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(
1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/
(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)
/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^8*a^3-24*(-(I*3^(1/2)-3)/(I*3^(1/2)-
1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*
b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a
^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*Elli
pticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^
(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*x
^7*a^2+12*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((
2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)
^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)
-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-
a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(
I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*x^6*a+3*I*(a*x^4+b*x)^(1/2)*((-a*x+(-a^
2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/
2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*(-a^2*b)^(1/3)*3^(1/2)*x^3*a
-9*a*(a*x^4+b*x)^(1/2)*x^3*(-a^2*b)^(1/3)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3
^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^
```

```
2*b)^(1/3))/a^2*x)^(1/2)+5*I*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*
x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)
-(-a^2*b)^(1/3))/a^2*x)^(1/2)*(-a^2*b)^(1/3)*3^(1/2)*b-15*(a*x^4+b*x)^(1/2)
*b*(-a^2*b)^(1/3)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-
a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)
)/((a*x^3+b)*x)^(1/2)/b/(I*3^(1/2)-3)/((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/
2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)
^(1/3))/a^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x^3)/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^3)^(1/2)/x^5,x)
```

```
[Out] int((a + b/x^3)^(1/2)/x^5, x)
```

sympy [A] time = 1.34, size = 41, normalized size = 0.15

$$-\frac{\sqrt{a}\,\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{be^{i\pi}}{ax^3}\right)}{3x^4\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**3)**(1/2)/x**5,x)
```

```
[Out] -sqrt(a)*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*exp_polar(I*pi)/(a*x**3))/
(3*x**4*gamma(7/3))
```

3.2003 $\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^8}\,dx$

Optimal. Leaf size=291

$$\frac{48a^2\sqrt{a+\frac{b}{x^3}}}{935b^2x}-\frac{32\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^3\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{935b^{7/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}-\frac{2\sqrt{a+}}{17x^{}}$$

```
[Out] -2/17*(a+b/x^3)^(1/2)/x^7-6/187*a*(a+b/x^3)^(1/2)/b/x^4+48/935*a^2*(a+b/x^3
)^(1/2)/b^2/x-32/935*3^(3/4)*a^3*(a^(1/3)+b^(1/3)/x)*EllipticF((b^(1/3)/x+a
^(1/3)*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(
1/2)+1/2*2^(1/2))*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^(1/
3)*(1+3^(1/2)))^2)^(1/2)/b^(7/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/
x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {335, 279, 321, 218}

$$\frac{48a^2\sqrt{a+\frac{b}{x^3}}}{935b^2x}-\frac{32\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^3\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{935b^{7/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}-\frac{6a\sqrt{a}}{187b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^3]/x^8,x]
```

```
[Out] (-2*Sqrt[a + b/x^3])/(17*x^7) - (6*a*Sqrt[a + b/x^3])/(187*b*x^4) + (48*a^2
*Sqrt[a + b/x^3])/(935*b^2*x) - (32*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^3*(a^(1/3)
+ b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/
3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(935*b^(7/3)*S
qrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)/x)^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
```

```
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^8}\,dx = -\text{Subst}\left(\int x^6\sqrt{a+bx^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2\sqrt{a+\frac{b}{x^3}}}{17x^7} - \frac{1}{17}(3a)\,\text{Subst}\left(\int \frac{x^6}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2\sqrt{a+\frac{b}{x^3}}}{17x^7} - \frac{6a\sqrt{a+\frac{b}{x^3}}}{187bx^4} + \frac{\left(24a^2\right)\text{Subst}\left(\int \frac{x^3}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{187b}$$

$$= -\frac{2\sqrt{a+\frac{b}{x^3}}}{17x^7} - \frac{6a\sqrt{a+\frac{b}{x^3}}}{187bx^4} + \frac{48a^2\sqrt{a+\frac{b}{x^3}}}{935b^2x} - \frac{\left(48a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{935b^2}$$

$$= -\frac{2\sqrt{a+\frac{b}{x^3}}}{17x^7} - \frac{6a\sqrt{a+\frac{b}{x^3}}}{187bx^4} + \frac{48a^2\sqrt{a+\frac{b}{x^3}}}{935b^2x} - \frac{32\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^3\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{}}{}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\right.}}}{935b^{7/3}\sqrt{a+\frac{b}{x^3}}\sqrt{}}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.18

$$-\frac{2\sqrt{a+\frac{b}{x^3}}\;{}_2F_1\left(-\frac{17}{6},-\frac{1}{2};-\frac{11}{6};-\frac{ax^3}{b}\right)}{17x^7\sqrt{\frac{ax^3}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^3]/x^8,x]
```

```
[Out] (-2*Sqrt[a + b/x^3]*Hypergeometric2F1[-17/6, -1/2, -11/6, -((a*x^3)/b)])/(1
7*x^7*Sqrt[1 + (a*x^3)/b])
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\frac{ax^3+b}{x^3}}}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(1/2)/x^8,x, algorithm="fricas")
```

```
[Out] integral(sqrt((a*x^3 + b)/x^3)/x^8, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(1/2)/x^8,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/x^3)/x^8, x)
```

maple [B] time = 0.04, size = 2222, normalized size = 7.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^3)^(1/2)/x^8,x)
```

```
[Out] 2/935*((a*x^3+b)/x^3)^(1/2)/x^7/(-a^2*b)^(1/3)*(-96*I*(-(I*3^(1/2)-3)/(I*3^
(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+
(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/
2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2
)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),
((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*3^(1/2)*x^
11*a^4+192*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)
*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2
*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1
/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)
/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2)
)/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*3^(1/2)*x^10*a^3-96*I*(-(I*3^(1/2)-3
)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)
^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+
I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)
))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)
^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a
^2*b)^(2/3)*3^(1/2)*x^9*a^2+96*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)
^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^
(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2
*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2
)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/
2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^11*a^4-192*(-(I*3^(1/2)-3)/(I*3
^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)
+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1
/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/
2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)
,((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^
(1/3)*x^10*a^3+96*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^
(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+
(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I
*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/
2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^
(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*x^9*a^2+24*I*(a*x^4+b*x)^(1/2)*
((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*
a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*(-a^2*b)^(1/3)*3^
```

```
(1/2)*x^6*a^2-72*a^2*(a*x^4+b*x)^(1/2)*x^6*(-a^2*b)^(1/3)*((-a*x+(-a^2*b)^(
1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a
^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)-15*I*(a*x^4+b*x)^(1/2)*((-a*x+(-a^
2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/
2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*(-a^2*b)^(1/3)*3^(1/2)*x^3*a
*b+45*a*(a*x^4+b*x)^(1/2)*x^3*b*(-a^2*b)^(1/3)*((-a*x+(-a^2*b)^(1/3))*(2*a*
x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)
-(-a^2*b)^(1/3))/a^2*x)^(1/2)-55*I*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))
*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)
^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*(-a^2*b)^(1/3)*3^(1/2)*b^2+165*(a*x^4+b
*x)^(1/2)*b^2*(-a^2*b)^(1/3)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*
b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a
^2*x)^(1/2))/((a*x^3+b)*x)^(1/2)/b^2/(I*3^(1/2)-3)/((-a*x+(-a^2*b)^(1/3))*(
2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(
1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \frac{b}{x^3}}}{x^8}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(1/2)/x^8,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x^3)/x^8, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a + \frac{b}{x^3}}}{x^8}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^3)^(1/2)/x^8,x)
```

```
[Out] int((a + b/x^3)^(1/2)/x^8, x)
```

sympy [A] time = 1.72, size = 41, normalized size = 0.14

$$-\frac{\sqrt{a}\,\Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{7}{3} \\ \frac{10}{3} \end{matrix}\middle| \frac{be^{i\pi}}{ax^3}\right)}{3x^7\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**3)**(1/2)/x**8,x)
```

```
[Out] -sqrt(a)*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), b*exp_polar(I*pi)/(a*x**3))
/(3*x**7*gamma(10/3))
```

3.2004 $\int \sqrt{a+\frac{b}{x^3}}\, x^6\, dx$

Optimal. Leaf size=563

$$\frac{5\,3^{3/4} b^{7/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{56\sqrt{2}\,a^{5/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}-\frac{15\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{7/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{224a}$$

```
[Out] -15/112*b^2*x*(a+b/x^3)^(1/2)/a^2+3/56*b*x^4*(a+b/x^3)^(1/2)/a+1/7*x^7*(a+b
/x^3)^(1/2)+15/112*b^(7/3)*(a+b/x^3)^(1/2)/a^2/(b^(1/3)/x+a^(1/3)*(1+3^(1/2
)))+5/112*3^(3/4)*b^(7/3)*(a^(1/3)+b^(1/3)/x)*EllipticF((b^(1/3)/x+a^(1/3)*
(1-3^(1/2)))/(b^(1/3)/x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((a^(2/3)+b^(2/
3)/x^2-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(5/3)*
2^(1/2)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+
3^(1/2)))^2)^(1/2)-15/224*3^(1/4)*b^(7/3)*(a^(1/3)+b^(1/3)/x)*EllipticE((b^
(1/3)/x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)
*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/(b^(1/3
)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(5/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)
+b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 563, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {335, 277, 325, 303, 218, 1877}

$$-\frac{15b^2x\sqrt{a+\frac{b}{x^3}}}{112a^2}+\frac{15b^{7/3}\sqrt{a+\frac{b}{x^3}}}{112a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}+\frac{5\,3^{3/4} b^{7/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\right.}{56\sqrt{2}\,a^{5/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^3]*x^6,x]
```

```
[Out] (15*b^(7/3)*Sqrt[a + b/x^3])/(112*a^2*((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x))
- (15*b^2*Sqrt[a + b/x^3]*x)/(112*a^2) + (3*b*Sqrt[a + b/x^3]*x^4)/(56*a) +
 (Sqrt[a + b/x^3]*x^7)/7 - (15*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(7/3)*(a^(1/3) +
 b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)/x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3
)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(224*a^(5/3)*Sq
rt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)/x)^2]) + (5*3^(3/4)*b^(7/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) +
b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*E
llipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)/x)], -7 - 4*Sqrt[3]])/(56*Sqrt[2]*a^(5/3)*Sqrt[a + b/x^3]*Sqrt[(a
^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
```

```
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \sqrt{a+\frac{b}{x^3}}\,x^6\,dx = -\operatorname{Subst}\left(\int \frac{\sqrt{a+bx^3}}{x^8}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{7}\sqrt{a+\frac{b}{x^3}}\,x^7 - \frac{1}{14}(3b)\operatorname{Subst}\left(\int \frac{1}{x^5\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{3b\sqrt{a+\frac{b}{x^3}}\,x^4}{56a} + \frac{1}{7}\sqrt{a+\frac{b}{x^3}}\,x^7 + \frac{\left(15b^2\right)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{112a}$$
$$= -\frac{15b^2\sqrt{a+\frac{b}{x^3}}\,x}{112a^2} + \frac{3b\sqrt{a+\frac{b}{x^3}}\,x^4}{56a} + \frac{1}{7}\sqrt{a+\frac{b}{x^3}}\,x^7 + \frac{\left(15b^3\right)\operatorname{Subst}\left(\int \frac{x}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{224a^2}$$
$$= -\frac{15b^2\sqrt{a+\frac{b}{x^3}}\,x}{112a^2} + \frac{3b\sqrt{a+\frac{b}{x^3}}\,x^4}{56a} + \frac{1}{7}\sqrt{a+\frac{b}{x^3}}\,x^7 + \frac{\left(15b^{8/3}\right)\operatorname{Subst}\left(\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right.}{224a^2}$$
$$= \frac{15b^{7/3}\sqrt{a+\frac{b}{x^3}}}{112a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)} - \frac{15b^2\sqrt{a+\frac{b}{x^3}}\,x}{112a^2} + \frac{3b\sqrt{a+\frac{b}{x^3}}\,x^4}{56a} + \frac{1}{7}\sqrt{a+\frac{b}{x^3}}\,x^7 - \frac{15\sqrt[4]{3}\sqrt{2}}{}$$

Mathematica [C] time = 0.05, size = 80, normalized size = 0.14

$$\frac{x^4\sqrt{a+\frac{b}{x^3}}\left(\left(ax^3+b\right)\sqrt{\frac{ax^3}{b}+1} - b\,{}_2F_1\left(-\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; -\frac{ax^3}{b}\right)\right)}{7a\sqrt{\frac{ax^3}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^3]*x^6,x]
```

```
[Out] (Sqrt[a + b/x^3]*x^4*((b + a*x^3)*Sqrt[1 + (a*x^3)/b] - b*Hypergeometric2F1
[-1/2, 5/6, 11/6, -((a*x^3)/b)]))/(7*a*Sqrt[1 + (a*x^3)/b])
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(x^6\sqrt{\frac{ax^3+b}{x^3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b/x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^6*sqrt((a*x^3 + b)/x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+\frac{b}{x^3}}\,x^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b/x^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/x^3)*x^6, x)
```

maple [B] time = 0.03, size = 2799, normalized size = 4.97

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a+b/x^3)^(1/2),x)
```

```
[Out] 1/56*((a*x^3+b)/x^3)^(1/2)*x^2/a^3*(8*I*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(
1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a
^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*3^(1/2)*x^5*a^3+30*I*(-(I*3^(1/2)-
3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b
)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x
+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3
)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x
)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-
a^2*b)^(2/3)*3^(1/2)*x*b^2-15*I*(-a^2*b)^(1/3)*3^(1/2)*x^2*a*b^2-24*(a*x^4+
b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^
(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*x^5*a^
3-30*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x
+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3
)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(
-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(
-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(
1/2)-3))^(1/2))*(-a^2*b)^(1/3)*x^2*a*b^2+45*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(
-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(
1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b
)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*Elliptic
E((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2
)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*x^2*a
*b^2+3*I*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)
^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2
*x)^(1/2)*3^(1/2)*x^2*a^2*b-15*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*
b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*
3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a
^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1
/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(
1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*3^(1/2)*x^2*a*b^
2+60*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x
+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3
)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(
-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(
-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(
1/2)-3))^(1/2))*(-a^2*b)^(2/3)*x*b^2-90*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x
+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3)
)/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1
/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-
(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)
*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*x*b^2-15*
I*(-a^2*b)^(2/3)*3^(1/2)*x*b^2+30*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2
*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I
*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-
a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(
1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^
(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b^3-45*(-(I*3^(1/2)-3)/(I*3^
(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+
(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/
2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2
)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),
((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b^3+15*I
```

```
*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3
^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^
(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x
+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2
*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)
-3))^(1/2))*3^(1/2)*a*b^3-9*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x
+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-
(-a^2*b)^(1/3))/a^2*x)^(1/2)*x^2*a^2*b-15*I*3^(1/2)*x^3*a^2*b^2+45*b^2*x^3*
a^2+45*(-a^2*b)^(1/3)*x^2*a*b^2+45*(-a^2*b)^(2/3)*x*b^2)/((a*x^3+b)*x)^(1/2
)/(I*3^(1/2)-3)/((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^
2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + \frac{b}{x^3}}\, x^6\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b/x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x^3)*x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^6 \sqrt{a + \frac{b}{x^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b/x^3)^(1/2),x)
```

```
[Out] int(x^6*(a + b/x^3)^(1/2), x)
```

sympy [A] time = 1.61, size = 48, normalized size = 0.09

$$-\frac{\sqrt{a}\, x^7 \Gamma\left(-\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{7}{3}, -\frac{1}{2} \\ -\frac{4}{3} \end{matrix} \middle| \frac{be^{i\pi}}{ax^3}\right)}{3\Gamma\left(-\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(a+b/x**3)**(1/2),x)
```

```
[Out] -sqrt(a)*x**7*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*exp_polar(I*pi)/(a
*x**3))/(3*gamma(-4/3))
```

3.2005 $\int \sqrt{a+\frac{b}{x^3}}\, x^3\, dx$

Optimal. Leaf size=539

$$-\frac{3^{3/4} b^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\, F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{4\sqrt{2}\, a^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}+\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\, b^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right.}{16}$$

```
[Out] 3/8*b*x*(a+b/x^3)^(1/2)/a+1/4*x^4*(a+b/x^3)^(1/2)-3/8*b^(4/3)*(a+b/x^3)^(1/
2)/a/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))-1/8*3^(3/4)*b^(4/3)*(a^(1/3)+b^(1/3)/x
)*EllipticF((b^(1/3)/x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))
,I*3^(1/2)+2*I)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^(1/3)
*(1+3^(1/2)))^2)^(1/2)/a^(2/3)*2^(1/2)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^
(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+3/16*3^(1/4)*b^(4/3)*(a^(
1/3)+b^(1/3)/x)*EllipticE((b^(1/3)/x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3
)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)+b^(2/3)/x
^2-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(a+b
/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)
^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 539, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {335, 277, 325, 303, 218, 1877}

$$-\frac{3^{3/4} b^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\, F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{4\sqrt{2}\, a^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}+\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\, b^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right.}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^3]*x^3,x]
```

```
[Out] (-3*b^(4/3)*Sqrt[a + b/x^3])/(8*a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)) + (3
*b*Sqrt[a + b/x^3]*x)/(8*a) + (Sqrt[a + b/x^3]*x^4)/4 + (3*3^(1/4)*Sqrt[2 -
 Sqrt[3]]*b^(4/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1
/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticE[ArcSin[((1
 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 -
 4*Sqrt[3]])/(16*a^(2/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x
))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]) - (3^(3/4)*b^(4/3)*(a^(1/3) + b^
(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(4*Sqrt[2]*a^(2/3)
*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)/x)^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
```

```
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
 + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \sqrt{a+\frac{b}{x^3}}\,x^3\,dx = -\operatorname{Subst}\left(\int \frac{\sqrt{a+bx^3}}{x^5}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{4}\sqrt{a+\frac{b}{x^3}}\,x^4 - \frac{1}{8}(3b)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{3b\sqrt{a+\frac{b}{x^3}}\,x}{8a} + \frac{1}{4}\sqrt{a+\frac{b}{x^3}}\,x^4 - \frac{\left(3b^2\right)\operatorname{Subst}\left(\int \frac{x}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{16a}$$
$$= \frac{3b\sqrt{a+\frac{b}{x^3}}\,x}{8a} + \frac{1}{4}\sqrt{a+\frac{b}{x^3}}\,x^4 - \frac{\left(3b^{5/3}\right)\operatorname{Subst}\left(\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{16a} - \frac{\left(3\sqrt{\frac{1}{2}\left(2-\sqrt{3}\right.\right.}$$
$$= -\frac{3b^{4/3}\sqrt{a+\frac{b}{x^3}}}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)} + \frac{3b\sqrt{a+\frac{b}{x^3}}\,x}{8a} + \frac{1}{4}\sqrt{a+\frac{b}{x^3}}\,x^4 + \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{4/3}\left(\sqrt[3]{a}+\right.}{}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.09

$$\frac{2x^4\sqrt{a+\frac{b}{x^3}}\;{}_2F_1\left(-\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; -\frac{ax^3}{b}\right)}{5\sqrt{\frac{ax^3}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^3]*x^3,x]
```

```
[Out] (2*Sqrt[a + b/x^3]*x^4*Hypergeometric2F1[-1/2, 5/6, 11/6, -((a*x^3)/b)])/(5
*Sqrt[1 + (a*x^3)/b])
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(x^3\sqrt{\frac{ax^3+b}{x^3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b/x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^3*sqrt((a*x^3 + b)/x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+\frac{b}{x^3}}\,x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b/x^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/x^3)*x^3, x)
```

maple [B] time = 0.03, size = 2579, normalized size = 4.78

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b/x^3)^(1/2),x)
```

```
[Out] 1/4*((a*x^3+b)/x^3)^(1/2)*x^2/a^2*(3*I*(-a^2*b)^(1/3)*3^(1/2)*x^2*a*b-3*I*(
-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(
1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1
/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(
-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b
)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3
))^(1/2))*3^(1/2)*a*b^2+6*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3
))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2)
)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(
1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/
(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)
/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*x^2*a*b-9*(-(I*3^(1/2)-
3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b
)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x
+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3
)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x
)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-
a^2*b)^(1/3)*x^2*a*b+I*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^
(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2
*b)^(1/3))/a^2*x)^(1/2)*3^(1/2)*x^2*a^2+3*I*3^(1/2)*x^3*a^2*b-12*(-(I*3^(1/
2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^
2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*
a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(
1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*
a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))
*(-a^2*b)^(2/3)*x*b+18*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*
a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(
-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3
))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*
3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1
+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*x*b+3*I*(-(I*3^(1/2)-3)/(I
*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/
3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^
(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(
1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/
2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b
)^(1/3)*3^(1/2)*x^2*a*b-6*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3
))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2)
)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(
1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/
(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)
/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b^2+9*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/
(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^
(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*
b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*Ellipti
cE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/
2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b^2+3*I*(-a^2*b)^
(2/3)*3^(1/2)*x*b-3*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/
2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)
^(1/3))/a^2*x)^(1/2)*x^2*a^2-6*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*
b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*
3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a
^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1
/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(
1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*3^(1/2)*x*b-9*a^
2*b*x^3-9*(-a^2*b)^(1/3)*x^2*a*b-9*(-a^2*b)^(2/3)*x*b)/((a*x^3+b)*x)^(1/2)/
```

```
(I*3^(1/2)-3)/((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*
b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + \frac{b}{x^3}}\, x^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b/x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x^3)*x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \sqrt{a + \frac{b}{x^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b/x^3)^(1/2),x)
```

```
[Out] int(x^3*(a + b/x^3)^(1/2), x)
```

sympy [A] time = 1.36, size = 48, normalized size = 0.09

$$-\frac{\sqrt{a}\, x^4 \Gamma\left(-\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{4}{3}, -\frac{1}{2} \\ -\frac{1}{3} \end{matrix} \middle| \frac{b e^{i\pi}}{a x^3}\right)}{3\Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b/x**3)**(1/2),x)
```

```
[Out] -sqrt(a)*x**4*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*exp_polar(I*pi)/(a
*x**3))/(3*gamma(-1/3))
```

3.2006 $\int \sqrt{a+\frac{b}{x^3}}\,dx$

Optimal. Leaf size=507

$$-\frac{\sqrt{2}\,3^{3/4}\sqrt[3]{a}\,\sqrt[3]{b}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)\quad 3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,\sqrt[3]{b}\left(\sqrt[3]{}\right.}{\sqrt{a+\frac{b}{x^3}}\,\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}+$$

```
[Out] x*(a+b/x^3)^(1/2)-3*b^(1/3)*(a+b/x^3)^(1/2)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))
-3^(3/4)*a^(1/3)*b^(1/3)*(a^(1/3)+b^(1/3)/x)*EllipticF((b^(1/3)/x+a^(1/3)*(
1-3^(1/2)))/(b^(1/3)/x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3
)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/(
a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))
^2)^(1/2)+3/2*3^(1/4)*a^(1/3)*b^(1/3)*(a^(1/3)+b^(1/3)/x)*EllipticE((b^(1/3
)/x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/
2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+
a^(1/3)*(1+3^(1/2)))^2)^(1/2)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/
(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 507, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {242, 277, 303, 218, 1877}

$$-\frac{\sqrt{2}\,3^{3/4}\sqrt[3]{a}\,\sqrt[3]{b}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)\quad 3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,\sqrt[3]{b}\left(\sqrt[3]{}\right.}{\sqrt{a+\frac{b}{x^3}}\,\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^3],x]
```

```
[Out] (-3*b^(1/3)*Sqrt[a + b/x^3])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x) + Sqrt[a +
 b/x^3]*x + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*b^(1/3)*(a^(1/3) + b^(1/3)
/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)/x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(2*Sqrt[a + b/x^3]*Sqrt
[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]) -
(Sqrt[2]*3^(3/4)*a^(1/3)*b^(1/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2
/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*Ellip
ticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)/x)], -7 - 4*Sqrt[3]])/(Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/
3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \sqrt{a+\frac{b}{x^3}}\,dx = -\text{Subst}\left(\int \frac{\sqrt{a+bx^3}}{x^2}\,dx, x, \frac{1}{x}\right)$$

$$= \sqrt{a+\frac{b}{x^3}}\,x - \frac{1}{2}(3b)\,\text{Subst}\left(\int \frac{x}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)$$

$$= \sqrt{a+\frac{b}{x^3}}\,x - \frac{1}{2}\left(3b^{2/3}\right)\text{Subst}\left(\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right) - \frac{\left(3\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,b^{2/3}\right)\text{Sub}}{\sqrt{2}}$$

$$= -\frac{3\sqrt[3]{b}\sqrt{a+\frac{b}{x^3}}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}} + \sqrt{a+\frac{b}{x^3}}\,x + \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}{2\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\right.}}}$$

Mathematica [C] time = 0.01, size = 47, normalized size = 0.09

$$-\frac{2x\sqrt{a+\frac{b}{x^3}}\ {}_2F_1\left(-\frac{1}{2},-\frac{1}{6};\frac{5}{6};-\frac{ax^3}{b}\right)}{\sqrt{\frac{ax^3}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^3],x]
```

```
[Out] (-2*Sqrt[a + b/x^3]*x*Hypergeometric2F1[-1/2, -1/6, 5/6, -((a*x^3)/b)])/Sqr
t[1 + (a*x^3)/b]
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{\frac{ax^3+b}{x^3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt((a*x^3 + b)/x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+\frac{b}{x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/x^3), x)
```

maple [B] time = 0.03, size = 2864, normalized size = 5.65

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^3)^(1/2),x)
```

```
[Out] -2*((a*x^3+b)/x^3)^(1/2)*x/a*(3*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2
*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I
*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-
a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(
1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^
(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*3^(1/2)*((a*x^3+b)*x)^(1/2)*a*
b+I*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*
(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*3^(1/2)*a*b-6
*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3
^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^
(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x
+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2
*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)
-3))^(1/2))*(-a^2*b)^(1/3)*((a*x^3+b)*x)^(1/2)*x^2*a+9*(-(I*3^(1/2)-3)/(I*3
^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)
+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1
/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/
2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)
```

```
,((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^
(1/3)*((a*x^3+b)*x)^(1/2)*x^2*a+6*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a
^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1
+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-
(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3
^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*
3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*3^(1/2)*((a*x
^3+b)*x)^(1/2)*x+12*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x
)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*
x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/
(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(
1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*
3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*((a*x^3+b)*x)^(1/2)*x-18*(-(I
*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2
)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)
*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^
2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(
1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^
(1/2))*(-a^2*b)^(2/3)*((a*x^3+b)*x)^(1/2)*x+I*((-a*x+(-a^2*b)^(1/3))*(2*a*x
+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-
(-a^2*b)^(1/3))/a^2*x)^(1/2)*3^(1/2)*x^3*a^2-3*I*3^(1/2)*((a*x^3+b)*x)^(1/2
)*x^3*a^2+6*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*
((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*
b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/
2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/
(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))
/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^(1/2)*a*b-9*(-(I*3^(1/2)-3)/(I*3^(1/2)
-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2
*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-
a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*Ell
ipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3
^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^(
1/2)*a*b-3*I*(-a^2*b)^(2/3)*3^(1/2)*((a*x^3+b)*x)^(1/2)*x-3*((-a*x+(-a^2*b)
^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(
-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*x^3*a^2-3*I*(-a^2*b)^(1/3)*3^(1/
2)*((a*x^3+b)*x)^(1/2)*x^2*a+9*((a*x^3+b)*x)^(1/2)*x^3*a^2-3*I*(-(I*3^(1/2)
-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*
b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*
x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/
3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*
x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(
-a^2*b)^(1/3)*3^(1/2)*((a*x^3+b)*x)^(1/2)*x^2*a+9*(-a^2*b)^(1/3)*((a*x^3+b)
*x)^(1/2)*x^2*a+9*(-a^2*b)^(2/3)*((a*x^3+b)*x)^(1/2)*x-3*((-a*x+(-a^2*b)^(1
/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^
2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*a*b)/(a*x^3+b)/(I*3^(1/2)-3)/((-a*x
+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*
3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + \frac{b}{x^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x^3), x)
```

mupad [**B**] time = 1.29, size = 38, normalized size = 0.07

$$-\frac{2\,x\,\sqrt{a+\frac{b}{x^3}}\;{}_2\mathrm{F}_1\left(-\frac{1}{2},-\frac{1}{6};\;\frac{5}{6};\;-\frac{a\,x^3}{b}\right)}{\sqrt{\frac{a\,x^3}{b}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^3)^(1/2),x)
```

```
[Out] -(2*x*(a + b/x^3)^(1/2)*hypergeom([-1/2, -1/6], 5/6, -(a*x^3)/b))/((a*x^3)/
b + 1)^(1/2)
```

sympy [**A**] time = 1.00, size = 42, normalized size = 0.08

$$-\frac{\sqrt{a}\,x\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\,\frac{be^{i\pi}}{ax^3}\right)}{3\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**3)**(1/2),x)
```

```
[Out] -sqrt(a)*x*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*exp_polar(I*pi)/(a*x**
3))/(3*gamma(2/3))
```

3.2007 $\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^3}\,dx$

Optimal. Leaf size=517

$$-\frac{2\sqrt{2}\,3^{3/4}a^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)\quad 3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}\right.}{7b^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}+$$

```
[Out] -2/7*(a+b/x^3)^(1/2)/x^2-6/7*a*(a+b/x^3)^(1/2)/b^(2/3)/(b^(1/3)/x+a^(1/3)*(
1+3^(1/2)))-2/7*3^(3/4)*a^(4/3)*(a^(1/3)+b^(1/3)/x)*EllipticF((b^(1/3)/x+a^
(1/3)*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*(
(a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^
(1/2)/b^(2/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/(b^(1/3)/x+a^(1/
3)*(1+3^(1/2)))^2)^(1/2)+3/7*3^(1/4)*a^(4/3)*(a^(1/3)+b^(1/3)/x)*EllipticE(
(b^(1/3)/x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2
*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/(b^(
1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1
/3)+b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 517, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {335, 279, 303, 218, 1877}

$$-\frac{2\sqrt{2}\,3^{3/4}a^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)\quad 3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}\right.}{7b^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^3]/x^3,x]
```

```
[Out] (-6*a*Sqrt[a + b/x^3])/(7*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)) - (2
*Sqrt[a + b/x^3])/(7*x^2) + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(a^(1/3) +
 b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)/x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3
)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(7*b^(2/3)*Sqrt
[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)/x)^2]) - (2*Sqrt[2]*3^(3/4)*a^(4/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2
/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x
)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(7*b^(2/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1
/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
```

```
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^3}\,dx = -\text{Subst}\left(\int x\sqrt{a+bx^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2\sqrt{a+\frac{b}{x^3}}}{7x^2} - \frac{1}{7}(3a)\,\text{Subst}\left(\int \frac{x}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2\sqrt{a+\frac{b}{x^3}}}{7x^2} - \frac{(3a)\,\text{Subst}\left(\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{7\sqrt[3]{b}} - \frac{\left(3\sqrt{2\left(2-\sqrt{3}\right)}\,a^{4/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^3}}\,d\right.}{7\sqrt[3]{b}}$$

$$= -\frac{6a\sqrt{a+\frac{b}{x^3}}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)} - \frac{2\sqrt{a+\frac{b}{x^3}}}{7x^2} + \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}{7b^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(}{\left(\left(1+\sqrt{3}\right.\right.}}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.10

$$-\frac{2\sqrt{a+\frac{b}{x^3}}\,{}_2F_1\left(-\frac{7}{6},-\frac{1}{2};-\frac{1}{6};-\frac{ax^3}{b}\right)}{7x^2\sqrt{\frac{ax^3}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^3]/x^3,x]
```

```
[Out] (-2*Sqrt[a + b/x^3]*Hypergeometric2F1[-7/6, -1/2, -1/6, -((a*x^3)/b)])/(7*x
^2*Sqrt[1 + (a*x^3)/b])
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\frac{ax^3+b}{x^3}}}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt((a*x^3 + b)/x^3)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/x^3)/x^3, x)
```

maple [B] time = 0.04, size = 3309, normalized size = 6.40

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^3)^(1/2)/x^3,x)
```

```
[Out] 2/21*((a*x^3+b)/x^3)^(1/2)/x^3*(4*I*((a*x^3+b)*x)^(1/2)*3^(1/2)*(a*x^4+b*x)
^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3
))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*x^3*a-18*I
*((a*x^3+b)*x)^(1/2)*3^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(
1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1
/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b
)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-
3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)
-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^4*a*b+18*I*((a*x^3+b)*x)^(1/2)*3^
(1/2)*(-a^2*b)^(2/3)*x^5+36*((a*x^3+b)*x)^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-
1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*
b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a
^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*Elli
pticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^
(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*x
^6*a-54*((a*x^3+b)*x)^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1
/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/
```

```
2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)
^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3
)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-
1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*x^6*a+18*I*((a*x^3+b)
*x)^(1/2)*3^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^
(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+
(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I
*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/
2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^
(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*x^6*a-72*((a*x^3+b)*x)^(1/2)*(-
(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1
/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/
2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-
a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)
^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3)
)^(1/2))*(-a^2*b)^(2/3)*x^5+108*((a*x^3+b)*x)^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1
/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-
a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)
*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*
EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((
I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/
3)*x^5-13*I*3^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+
(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/
2)*x^4*a*b+18*I*((a*x^3+b)*x)^(1/2)*3^(1/2)*(-a^2*b)^(1/3)*x^6*a-36*((a*x^3
+b)*x)^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)
*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2
*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1
/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)
/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2)
)/(I*3^(1/2)-3))^(1/2))*x^4*a*b+54*((a*x^3+b)*x)^(1/2)*(-(I*3^(1/2)-3)/(I*3
^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)
+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1
/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/
2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)
,((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^4*a*b-1
3*I*3^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)
^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*x^7*a
^2-54*((a*x^3+b)*x)^(1/2)*x^7*a^2+39*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2
)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^
(1/3))/a^2*x)^(1/2)*x^7*a^2+18*I*((a*x^3+b)*x)^(1/2)*3^(1/2)*x^7*a^2-54*((a
*x^3+b)*x)^(1/2)*(-a^2*b)^(1/3)*x^6*a-3*I*((a*x^3+b)*x)^(1/2)*3^(1/2)*(a*x^
4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b
)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*b-54
*((a*x^3+b)*x)^(1/2)*(-a^2*b)^(2/3)*x^5-12*((a*x^3+b)*x)^(1/2)*(a*x^4+b*x)^
(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3)
)*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*x^3*a+39*((
-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*
x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*x^4*a*b-36*I*((a*x^
3+b)*x)^(1/2)*3^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a
*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-
a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3)
)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3
^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+
I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*x^5+9*((a*x^3+b)*x)^(1/2)*(
a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a
^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*
b)/(a*x^3+b)/b/(I*3^(1/2)-3)/((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*
b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a
^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x^3)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^3)^(1/2)/x^3,x)
```

```
[Out] int((a + b/x^3)^(1/2)/x^3, x)
```

sympy [A] time = 1.27, size = 41, normalized size = 0.08

$$-\frac{\sqrt{a}\,\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{be^{i\pi}}{ax^3}\right)}{3x^2\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**3)**(1/2)/x**3,x)
```

```
[Out] -sqrt(a)*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*exp_polar(I*pi)/(a*x**3))/
(3*x**2*gamma(5/3))
```

3.2008 $\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^6}\,dx$

Optimal. Leaf size=541

$$\frac{8\sqrt{2}\,3^{3/4}a^{7/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{91b^{5/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}-\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{7/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{9}$$

```
[Out] -2/13*(a+b/x^3)^(1/2)/x^5-6/91*a*(a+b/x^3)^(1/2)/b/x^2+24/91*a^2*(a+b/x^3)^
(1/2)/b^(5/3)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))+8/91*3^(3/4)*a^(7/3)*(a^(1/3)
+b^(1/3)/x)*EllipticF((b^(1/3)/x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3)*(1
+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/
(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(
a^(1/3)+b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-12/91*3^(1/4)*a
^(7/3)*(a^(1/3)+b^(1/3)/x)*EllipticE((b^(1/3)/x+a^(1/3)*(1-3^(1/2)))/(b^(1/
3)/x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3
)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b
^(5/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3
^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 541, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {335, 279, 321, 303, 218, 1877}

$$\frac{24a^2\sqrt{a+\frac{b}{x^3}}}{91b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}+\frac{8\sqrt{2}\,3^{3/4}a^{7/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{91b^{5/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}-$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^3]/x^6,x]
```

```
[Out] (24*a^2*Sqrt[a + b/x^3])/(91*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)) -
 (2*Sqrt[a + b/x^3])/(13*x^5) - (6*a*Sqrt[a + b/x^3])/(91*b*x^2) - (12*3^(1
/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)
/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*Elliptic
E[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)/x)], -7 - 4*Sqrt[3]])/(91*b^(5/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3)
 + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]) + (8*Sqrt[2]*3^(3/4)
*a^(7/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/
3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3
])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3
]])/(91*b^(5/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)/x)^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^6}\,dx = -\operatorname{Subst}\left(\int x^4\sqrt{a+bx^3}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{2\sqrt{a+\frac{b}{x^3}}}{13x^5} - \frac{1}{13}(3a)\operatorname{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{2\sqrt{a+\frac{b}{x^3}}}{13x^5} - \frac{6a\sqrt{a+\frac{b}{x^3}}}{91bx^2} + \frac{\left(12a^2\right)\operatorname{Subst}\left(\int \frac{x}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{91b}$$
$$= -\frac{2\sqrt{a+\frac{b}{x^3}}}{13x^5} - \frac{6a\sqrt{a+\frac{b}{x^3}}}{91bx^2} + \frac{\left(12a^2\right)\operatorname{Subst}\left(\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{91b^{4/3}} + \frac{\left(12\sqrt{2\left(2-\sqrt{3}\right)}\,a^{7/3}\right)}{9}$$
$$= \frac{24a^2\sqrt{a+\frac{b}{x^3}}}{91b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)} - \frac{2\sqrt{a+\frac{b}{x^3}}}{13x^5} - \frac{6a\sqrt{a+\frac{b}{x^3}}}{91bx^2} - \frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{7/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{-}}{91b^{5/3}\sqrt{}}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.09

$$-\frac{2\sqrt{a+\frac{b}{x^3}}\;{}_2F_1\left(-\frac{13}{6},-\frac{1}{2};-\frac{7}{6};-\frac{ax^3}{b}\right)}{13x^5\sqrt{\frac{ax^3}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^3]/x^6,x]
```

```
[Out] (-2*Sqrt[a + b/x^3]*Hypergeometric2F1[-13/6, -1/2, -7/6, -((a*x^3)/b)])/(13
*x^5*Sqrt[1 + (a*x^3)/b])
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\frac{ax^3+b}{x^3}}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] integral(sqrt((a*x^3 + b)/x^3)/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/x^3)/x^6, x)
```

maple [B] time = 0.04, size = 3556, normalized size = 6.57

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^3)^(1/2)/x^6,x)
```

```
[Out] 2/91*((a*x^3+b)/x^3)^(1/2)/x^6*(4*I*3^(1/2)*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*
b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)
*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*((a*x^3+b)*x)^(1/2)*x^6*a^2+72
*(-a^2*b)^(2/3)*((a*x^3+b)*x)^(1/2)*x^8*a+72*(-a^2*b)^(1/3)*((a*x^3+b)*x)^(
1/2)*x^9*a^2-24*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^
2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*x
^7*a^2*b+21*(a*x^4+b*x)^(1/2)*b^2*((a*x^3+b)*x)^(1/2)*((-a*x+(-a^2*b)^(1/3)
)*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b
)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)-7*I*3^(1/2)*(a*x^4+b*x)^(1/2)*((-a*x+(
-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^
(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*((a*x^3+b)*x)^(1/2)*b^2-2
4*I*(-a^2*b)^(2/3)*3^(1/2)*((a*x^3+b)*x)^(1/2)*x^8*a-24*I*(-a^2*b)^(1/3)*3^
(1/2)*((a*x^3+b)*x)^(1/2)*x^9*a^2+8*I*3^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x
+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-
(-a^2*b)^(1/3))/a^2*x)^(1/2)*x^7*a^2*b+9*a*(a*x^4+b*x)^(1/2)*x^3*b*((a*x^3+
b)*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)
^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)-12*a^
2*(a*x^4+b*x)^(1/2)*x^6*((a*x^3+b)*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I
*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-
a^2*b)^(1/3))/a^2*x)^(1/2)+8*I*3^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1
/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b
)^(1/3))/a^2*x)^(1/2)*x^10*a^3-24*I*3^(1/2)*((a*x^3+b)*x)^(1/2)*x^10*a^3-24
*I*(-a^2*b)^(1/3)*3^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3
))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2)
)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(
1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/
(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)
/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^(1/2)*x^9*a^2+48*I*(-a^2
*b)^(2/3)*3^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^
(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+
(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I
*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/
2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^
(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^(1/2)*x^8*a+24*I*a^2*b*3^(1/2)*(
-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(
1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1
/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(
-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b
)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3
))^(1/2))*((a*x^3+b)*x)^(1/2)*x^7-24*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2
)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^
(1/3))/a^2*x)^(1/2)*x^10*a^3+72*((a*x^3+b)*x)^(1/2)*x^10*a^3-72*a^2*b*(-(I*
3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)
*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*
((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2
*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1
/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(
1/2))*((a*x^3+b)*x)^(1/2)*x^7-144*(-a^2*b)^(2/3)*(-(I*3^(1/2)-3)/(I*3^(1/2)
-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2
*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-
a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*Ell
ipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3
```

```
^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^(
1/2)*x^8*a+48*a^2*b*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x
)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*
x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/
(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(
1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*
3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^(1/2)*x^7+72*(-a^2*b)^(1/3)*(-
(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1
/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/
2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-
a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)
^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3)
)^(1/2))*((a*x^3+b)*x)^(1/2)*x^9*a^2+96*(-a^2*b)^(2/3)*(-(I*3^(1/2)-3)/(I*3
^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)
+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1
/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/
2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)
,((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b
)*x)^(1/2)*x^8*a-3*I*3^(1/2)*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*
x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)
-(-a^2*b)^(1/3))/a^2*x)^(1/2)*((a*x^3+b)*x)^(1/2)*x^3*a*b-48*(-a^2*b)^(1/3)
*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3
^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^
(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x
+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2
*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)
-3))^(1/2))*((a*x^3+b)*x)^(1/2)*x^9*a^2)/(a*x^3+b)/b^2/(I*3^(1/2)-3)/((-a*x
+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*
3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x^3)/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^3)^(1/2)/x^6,x)
```

```
[Out] int((a + b/x^3)^(1/2)/x^6, x)
```

sympy [A] time = 1.46, size = 41, normalized size = 0.08

$$-\frac{\sqrt{a}\,\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2}, \frac{5}{3} \\ \frac{8}{3}\end{matrix}\right| \frac{be^{i\pi}}{ax^3}\right)}{3x^5\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**3)**(1/2)/x**6,x)
```

```
[Out] -sqrt(a)*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*exp_polar(I*pi)/(a*x**3))/
(3*x**5*gamma(8/3))
```

3.2009 $\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^9}\,dx$

Optimal. Leaf size=565

$$-\frac{80\sqrt{2}\,3^{3/4}a^{10/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{1729b^{8/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}+\frac{120\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{10/3}\left(\sqrt[3]{}\right.}{}$$

```
[Out] -2/19*(a+b/x^3)^(1/2)/x^8-6/247*a*(a+b/x^3)^(1/2)/b/x^5+60/1729*a^2*(a+b/x^
3)^(1/2)/b^2/x^2-240/1729*a^3*(a+b/x^3)^(1/2)/b^(8/3)/(b^(1/3)/x+a^(1/3)*(1
+3^(1/2)))-80/1729*3^(3/4)*a^(10/3)*(a^(1/3)+b^(1/3)/x)*EllipticF((b^(1/3)/
x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/
2)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))
^2)^(1/2)/b^(8/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/(b^(1/3)/x+a
^(1/3)*(1+3^(1/2)))^2)^(1/2)+120/1729*3^(1/4)*a^(10/3)*(a^(1/3)+b^(1/3)/x)*
EllipticE((b^(1/3)/x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3)*(1+3^(1/2))),I
*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/
3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(8/3)/(a+b/x^3)^(1/2)/(a^(
1/3)*(a^(1/3)+b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 565, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {335, 279, 321, 303, 218, 1877}

$$-\frac{240a^3\sqrt{a+\frac{b}{x^3}}}{1729b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}+\frac{60a^2\sqrt{a+\frac{b}{x^3}}}{1729b^2x^2}-\frac{80\sqrt{2}\,3^{3/4}a^{10/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{}\right.\right.}{1729b^{8/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^3]/x^9,x]
```

```
[Out] (-240*a^3*Sqrt[a + b/x^3])/(1729*b^(8/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x
)) - (2*Sqrt[a + b/x^3])/(19*x^8) - (6*a*Sqrt[a + b/x^3])/(247*b*x^5) + (60
*a^2*Sqrt[a + b/x^3])/(1729*b^2*x^2) + (120*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(10
/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^
(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(
1729*b^(8/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)/x)^2]) - (80*Sqrt[2]*3^(3/4)*a^(10/3)*(a^(1/3) + b^
(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(1729*b^(8/3)*Sqrt
[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)/x)^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^9}\,dx = -\text{Subst}\left(\int x^7\sqrt{a+bx^3}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{2\sqrt{a+\frac{b}{x^3}}}{19x^8} - \frac{1}{19}(3a)\,\text{Subst}\left(\int \frac{x^7}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{2\sqrt{a+\frac{b}{x^3}}}{19x^8} - \frac{6a\sqrt{a+\frac{b}{x^3}}}{247bx^5} + \frac{\left(30a^2\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{247b}$$
$$= -\frac{2\sqrt{a+\frac{b}{x^3}}}{19x^8} - \frac{6a\sqrt{a+\frac{b}{x^3}}}{247bx^5} + \frac{60a^2\sqrt{a+\frac{b}{x^3}}}{1729b^2x^2} - \frac{\left(120a^3\right)\text{Subst}\left(\int \frac{x}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{1729b^2}$$
$$= -\frac{2\sqrt{a+\frac{b}{x^3}}}{19x^8} - \frac{6a\sqrt{a+\frac{b}{x^3}}}{247bx^5} + \frac{60a^2\sqrt{a+\frac{b}{x^3}}}{1729b^2x^2} - \frac{\left(120a^3\right)\text{Subst}\left(\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{1729b^{7/3}} - \frac{\left(120$$

$$= -\frac{240a^3\sqrt{a+\frac{b}{x^3}}}{1729b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)} - \frac{2\sqrt{a+\frac{b}{x^3}}}{19x^8} - \frac{6a\sqrt{a+\frac{b}{x^3}}}{247bx^5} + \frac{60a^2\sqrt{a+\frac{b}{x^3}}}{1729b^2x^2} + \frac{120\sqrt[4]{3}\sqrt{2-\sqrt{}}}{}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.09

$$-\frac{2\sqrt{a+\frac{b}{x^3}}\;{}_2F_1\left(-\frac{19}{6},-\frac{1}{2};-\frac{13}{6};-\frac{ax^3}{b}\right)}{19x^8\sqrt{\frac{ax^3}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^3]/x^9,x]
```

```
[Out] (-2*Sqrt[a + b/x^3]*Hypergeometric2F1[-19/6, -1/2, -13/6, -((a*x^3)/b)])/(1
9*x^8*Sqrt[1 + (a*x^3)/b])
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\frac{ax^3+b}{x^3}}}{x^9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(1/2)/x^9,x, algorithm="fricas")
```

```
[Out] integral(sqrt((a*x^3 + b)/x^3)/x^9, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/x^3)/x^9, x)
```

maple [B] time = 0.04, size = 3788, normalized size = 6.70

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^3)^(1/2)/x^9,x)
```

```
[Out] -2/1729*((a*x^3+b)/x^3)^(1/2)/x^9*(40*I*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(
1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a
^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*3^(1/2)*((a*x^3+b)*x)^(1/2)*x^9*a^
3+720*(-a^2*b)^(2/3)*((a*x^3+b)*x)^(1/2)*x^11*a^2+720*(-a^2*b)^(1/3)*((a*x^
3+b)*x)^(1/2)*x^12*a^3-240*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)
^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2
*x)^(1/2)*x^10*a^3*b-273*(a*x^4+b*x)^(1/2)*b^3*((a*x^3+b)*x)^(1/2)*((-a*x+(
-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^
(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)+80*I*((-a*x+(-a^2*b)^(1/3
))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*
b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*3^(1/2)*x^10*a^3*b+91*I*(a*x^4+b*x)^(
1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))
*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*3^(1/2)*((a*
x^3+b)*x)^(1/2)*b^3-240*I*(-a^2*b)^(2/3)*3^(1/2)*((a*x^3+b)*x)^(1/2)*x^11*a
^2+90*a^2*(a*x^4+b*x)^(1/2)*x^6*b*((a*x^3+b)*x)^(1/2)*((-a*x+(-a^2*b)^(1/3)
)*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b
)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)-63*a*(a*x^4+b*x)^(1/2)*x^3*b^2*((a*x^3
+b)*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b
)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)-240*
I*(-a^2*b)^(1/3)*3^(1/2)*((a*x^3+b)*x)^(1/2)*x^12*a^3+21*I*(a*x^4+b*x)^(1/2
)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-
2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*3^(1/2)*((a*x^3
+b)*x)^(1/2)*x^3*a*b^2-720*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/
3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2
))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^
(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)
/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1
)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a^3*b*((a*x^3+b)*x)^(1/2)*x^10-480*(-
(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1
/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/
2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-
a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)
^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3)
)^(1/2))*(-a^2*b)^(1/3)*((a*x^3+b)*x)^(1/2)*x^12*a^3+720*(-(I*3^(1/2)-3)/(I
*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/
3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^
(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(
1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/
2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b
)^(1/3)*((a*x^3+b)*x)^(1/2)*x^12*a^3+960*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*
x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3
))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(
1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((
-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3
)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*((a*x^3+
b)*x)^(1/2)*x^11*a^2-1440*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3
))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2)
)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(
1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/
(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)
/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*((a*x^3+b)*x)^(1/2)*x^1
```

```
1*a^2+480*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((
2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)
^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)
-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-
a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(
I*3^(1/2)-3))^(1/2))*a^3*b*((a*x^3+b)*x)^(1/2)*x^10-30*I*(a*x^4+b*x)^(1/2)*
((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*
a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*3^(1/2)*((a*x^3+b
)*x)^(1/2)*x^6*a^2*b+240*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/
3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2
))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^
(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)
/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1
)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a^3*b*3^(1/2)*((a*x^3+b)*x)^(1/2)*x^1
0-240*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*
a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(
1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1
)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*
x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*
3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*3^(1/2)*((a*x^3+b)*x)^(1/2)*x^12*a^3+480*
I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*
3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))
^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*
x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^
2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2
)-3))^(1/2))*(-a^2*b)^(2/3)*3^(1/2)*((a*x^3+b)*x)^(1/2)*x^11*a^2-120*a^3*(a
*x^4+b*x)^(1/2)*x^9*((a*x^3+b)*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(
1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*
b)^(1/3))/a^2*x)^(1/2)+80*I*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b
)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^
2*x)^(1/2)*3^(1/2)*x^13*a^4-240*I*3^(1/2)*((a*x^3+b)*x)^(1/2)*x^13*a^4-240*
((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*
a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*x^13*a^4+720*((a*
x^3+b)*x)^(1/2)*x^13*a^4)/(a*x^3+b)/b^3/(I*3^(1/2)-3)/((-a*x+(-a^2*b)^(1/3)
)*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b
)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(1/2)/x^9,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x^3)/x^9, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^3)^(1/2)/x^9,x)
```

```
[Out] int((a + b/x^3)^(1/2)/x^9, x)
```

sympy [**A**] time = 1.80, size = 41, normalized size = 0.07

$$-\frac{\sqrt{a}\,\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2}, \frac{8}{3} \\ \frac{11}{3}\end{matrix}\right| \frac{be^{i\pi}}{ax^3}\right)}{3x^8\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**3)**(1/2)/x**9,x)
```

```
[Out] -sqrt(a)*gamma(8/3)*hyper((-1/2, 8/3), (11/3,), b*exp_polar(I*pi)/(a*x**3))
/(3*x**8*gamma(11/3))
```

3.2010 $\int \left(a+\frac{b}{x^3}\right)^{3/2} x^5\,dx$

Optimal. Leaf size=68

$$\frac{b^2 \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{4\sqrt{a}}+\frac{1}{4}bx^3\sqrt{a+\frac{b}{x^3}}+\frac{1}{6}x^6\left(a+\frac{b}{x^3}\right)^{3/2}$$

```
[Out] 1/6*(a+b/x^3)^(3/2)*x^6+1/4*b^2*arctanh((a+b/x^3)^(1/2)/a^(1/2))/a^(1/2)+1/
4*b*x^3*(a+b/x^3)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 47, 63, 208}

$$\frac{b^2 \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{4\sqrt{a}}+\frac{1}{6}x^6\left(a+\frac{b}{x^3}\right)^{3/2}+\frac{1}{4}bx^3\sqrt{a+\frac{b}{x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^3)^(3/2)*x^5,x]
```

```
[Out] (b*Sqrt[a + b/x^3]*x^3)/4 + ((a + b/x^3)^(3/2)*x^6)/6 + (b^2*ArcTanh[Sqrt[a
 + b/x^3]/Sqrt[a]])/(4*Sqrt[a])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \left(a+\frac{b}{x^3}\right)^{3/2} x^5\,dx = -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^3}\,dx, x, \frac{1}{x^3}\right)\right)$$
$$= \frac{1}{6}\left(a+\frac{b}{x^3}\right)^{3/2} x^6 - \frac{1}{4}b\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx, x, \frac{1}{x^3}\right)$$
$$= \frac{1}{4}b\sqrt{a+\frac{b}{x^3}}\,x^3 + \frac{1}{6}\left(a+\frac{b}{x^3}\right)^{3/2} x^6 - \frac{1}{8}b^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x^3}\right)$$
$$= \frac{1}{4}b\sqrt{a+\frac{b}{x^3}}\,x^3 + \frac{1}{6}\left(a+\frac{b}{x^3}\right)^{3/2} x^6 - \frac{1}{4}b\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x^3}}\right)$$
$$= \frac{1}{4}b\sqrt{a+\frac{b}{x^3}}\,x^3 + \frac{1}{6}\left(a+\frac{b}{x^3}\right)^{3/2} x^6 + \frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

Mathematica [A] time = 0.19, size = 80, normalized size = 1.18

$$\frac{1}{12}x^{3/2}\sqrt{a+\frac{b}{x^3}}\left(\frac{3b^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{b}}\right)}{\sqrt{a}\,\sqrt{\frac{ax^3}{b}+1}} + x^{3/2}\left(2ax^3+5b\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^3)^(3/2)*x^5,x]
```

```
[Out] (Sqrt[a + b/x^3]*x^(3/2)*(x^(3/2)*(5*b + 2*a*x^3) + (3*b^(3/2)*ArcSinh[(Sqr
t[a]*x^(3/2))/Sqrt[b]])/(Sqrt[a]*Sqrt[1 + (a*x^3)/b])))/12
```

fricas [A] time = 1.18, size = 179, normalized size = 2.63

$$\left[\frac{3\sqrt{a}\,b^2\log\left(-8\,a^2x^6-8\,abx^3-b^2-4\left(2\,ax^6+bx^3\right)\sqrt{a}\,\sqrt{\frac{ax^3+b}{x^3}}\right)+4\left(2\,a^2x^6+5\,abx^3\right)\sqrt{\frac{ax^3+b}{x^3}}}{48\,a}, -\frac{3\sqrt{-a}\,b^2\,\text{arc}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(3/2)*x^5,x, algorithm="fricas")
```

```
[Out] [1/48*(3*sqrt(a)*b^2*log(-8*a^2*x^6 - 8*a*b*x^3 - b^2 - 4*(2*a*x^6 + b*x^3)
*sqrt(a)*sqrt((a*x^3 + b)/x^3)) + 4*(2*a^2*x^6 + 5*a*b*x^3)*sqrt((a*x^3 + b
)/x^3))/a, -1/24*(3*sqrt(-a)*b^2*arctan(2*sqrt(-a)*x^3*sqrt((a*x^3 + b)/x^3
)/(2*a*x^3 + b)) - 2*(2*a^2*x^6 + 5*a*b*x^3)*sqrt((a*x^3 + b)/x^3))/a]
```

giac [A] time = 0.32, size = 51, normalized size = 0.75

$$\frac{1}{12}\sqrt{ax^4+bx}\left(2\,ax^3+5\,b\right)x - \frac{b^2\arctan\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{-a}}\right)}{4\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(3/2)*x^5,x, algorithm="giac")
```

```
[Out] 1/12*sqrt(a*x^4 + b*x)*(2*a*x^3 + 5*b)*x - 1/4*b^2*arctan(sqrt(a + b/x^3)/s
qrt(-a))/sqrt(-a)
```

maple [C] time = 0.01, size = 3569, normalized size = 52.49

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^3)^(3/2)*x^5,x)
```

```
[Out] 1/12*((a*x^3+b)/x^3)^(3/2)*x^5/a^2*(36*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*
x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3
))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(
1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((
-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3
)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*3^(1/2)*
x*a*b^2+5*I*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2
*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/
a^2*x)^(1/2)*3^(1/2)*x*a^2*b+18*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2
*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I
*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-
a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticPi((-(I*3^
(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),(I*3^(1/2)-1)/(I*3^
(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(
-a^2*b)^(2/3)*3^(1/2)*b^2+2*I*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a
*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3
)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*3^(1/2)*x^4*a^3-18*I*(-(I*3^(1/2)-3)/(I*3^(1
/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-
a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)
*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*
EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((
I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/
3)*3^(1/2)*b^2-36*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x
)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*
x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/
(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)/(I*3^
(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3
^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*
3^(1/2)*x*a*b^2+18*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)
^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x
+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(
I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1
/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3
^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a^2*b^2+18*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1
)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b
)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^
2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*Ellip
ticPi((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),(I*3^(
1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)
-3))^(1/2))*3^(1/2)*x^2*a^2*b^2-18*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^
2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+
I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(
-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticPi((-(I*3
^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),(I*3^(1/2)-1)/(I*3
^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*
x^2*a^2*b^2-6*x^4*(a*x^4+b*x)^(1/2)*a^3*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(
1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*
b)^(1/3))/a^2*x)^(1/2)-36*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3
```

```
))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2)
)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(
1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/
(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)
/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*x*a*b^2+36*(-(I*3^(1/2)
-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*
b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*
x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/
3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a
*x)^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(
1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*x*a*b^2-18*I*(-(I*3^(1/2)-3)/(I*
3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3
)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(
1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1
/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2
),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*3^(1/2)*
x^2*a^2*b^2+18*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/
2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a
^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^
(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-
1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/
2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*b^2-18*(-(I*3^(1/2)-3)/(I*3^(1/2)-
1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*
b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a
^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*Elli
pticPi((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),(I*3^
(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2
)-3))^(1/2))*(-a^2*b)^(2/3)*b^2-15*b*x*(a*x^4+b*x)^(1/2)*a^2*((-a*x+(-a^2*b
)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*
(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2))/(a*x^3+b)/((a*x^3+b)*x)^(1/2)/
(I*3^(1/2)-3)/((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*
b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)
```

maxima [A] time = 1.86, size = 98, normalized size = 1.44

$$-\frac{b^2 \log\left(\frac{\sqrt{a+\frac{b}{x^3}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^3}}+\sqrt{a}}\right)}{8\sqrt{a}}+\frac{5\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}b^2-3\sqrt{a+\frac{b}{x^3}}\,ab^2}{12\left(\left(a+\frac{b}{x^3}\right)^2-2\left(a+\frac{b}{x^3}\right)a+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(3/2)*x^5,x, algorithm="maxima")
```

```
[Out] -1/8*b^2*log((sqrt(a + b/x^3) - sqrt(a))/(sqrt(a + b/x^3) + sqrt(a)))/sqrt(
a) + 1/12*(5*(a + b/x^3)^(3/2)*b^2 - 3*sqrt(a + b/x^3)*a*b^2)/((a + b/x^3)^
2 - 2*(a + b/x^3)*a + a^2)
```

mupad [B] time = 1.86, size = 74, normalized size = 1.09

$$\frac{a\,x^6\sqrt{a+\frac{b}{x^3}}}{6}+\frac{5\,b\,x^3\sqrt{a+\frac{b}{x^3}}}{12}+\frac{b^2\,\ln\left(x^6\left(\sqrt{a+\frac{b}{x^3}}-\sqrt{a}\right)\left(\sqrt{a+\frac{b}{x^3}}+\sqrt{a}\right)^3\right)}{8\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b/x^3)^(3/2),x)
```

```
[Out] (a*x^6*(a + b/x^3)^(1/2))/6 + (5*b*x^3*(a + b/x^3)^(1/2))/12 + (b^2*log(x^6
*((a + b/x^3)^(1/2) - a^(1/2))*((a + b/x^3)^(1/2) + a^(1/2))^3))/(8*a^(1/2)
)
```

sympy [**A**] time = 3.87, size = 76, normalized size = 1.12

$$\frac{a\sqrt{b}\,x^{\frac{9}{2}}\sqrt{\frac{ax^3}{b}+1}}{6}+\frac{5b^{\frac{3}{2}}x^{\frac{3}{2}}\sqrt{\frac{ax^3}{b}+1}}{12}+\frac{b^2\,\mathrm{asinh}\left(\frac{\sqrt{a}\,x^{\frac{3}{2}}}{\sqrt{b}}\right)}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**3)**(3/2)*x**5,x)
```

```
[Out] a*sqrt(b)*x**(9/2)*sqrt(a*x**3/b + 1)/6 + 5*b**(3/2)*x**(3/2)*sqrt(a*x**3/b
 + 1)/12 + b**2*asinh(sqrt(a)*x**(3/2)/sqrt(b))/(4*sqrt(a))
```

3.2011 $\int \left(a+\frac{b}{x^3}\right)^{3/2} x^2\,dx$

Optimal. Leaf size=58

$$\frac{1}{3}x^3\left(a+\frac{b}{x^3}\right)^{3/2}-b\sqrt{a+\frac{b}{x^3}}+\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)$$

```
[Out] 1/3*(a+b/x^3)^(3/2)*x^3+b*arctanh((a+b/x^3)^(1/2)/a^(1/2))*a^(1/2)-b*(a+b/x
^3)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {266, 47, 50, 63, 208}

$$\frac{1}{3}x^3\left(a+\frac{b}{x^3}\right)^{3/2}-b\sqrt{a+\frac{b}{x^3}}+\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^3)^(3/2)*x^2,x]
```

```
[Out] -(b*Sqrt[a + b/x^3]) + ((a + b/x^3)^(3/2)*x^3)/3 + Sqrt[a]*b*ArcTanh[Sqrt[a
 + b/x^3]/Sqrt[a]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a+\frac{b}{x^3}\right)^{3/2} x^2\,dx &= -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2}\,dx, x, \frac{1}{x^3}\right)\right) \\
&= \frac{1}{3}\left(a+\frac{b}{x^3}\right)^{3/2} x^3 - \frac{1}{2}b\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x^3}\right) \\
&= -b\sqrt{a+\frac{b}{x^3}} + \frac{1}{3}\left(a+\frac{b}{x^3}\right)^{3/2} x^3 - \frac{1}{2}(ab)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x^3}\right) \\
&= -b\sqrt{a+\frac{b}{x^3}} + \frac{1}{3}\left(a+\frac{b}{x^3}\right)^{3/2} x^3 - a\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x^3}}\right) \\
&= -b\sqrt{a+\frac{b}{x^3}} + \frac{1}{3}\left(a+\frac{b}{x^3}\right)^{3/2} x^3 + \sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [C] time = 0.02, size = 49, normalized size = 0.84

$$-\frac{2b\sqrt{a+\frac{b}{x^3}}\;{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{ax^3}{b}\right)}{3\sqrt{\frac{ax^3}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^3)^(3/2)*x^2,x]
```

```
[Out] (-2*b*Sqrt[a + b/x^3]*Hypergeometric2F1[-3/2, -1/2, 1/2, -((a*x^3)/b)])/(3*
Sqrt[1 + (a*x^3)/b])
```

fricas [A] time = 1.17, size = 151, normalized size = 2.60

$$\left[\frac{1}{4}\sqrt{a}\,b\log\left(-8\,a^2x^6-8\,abx^3-b^2-4\left(2\,ax^6+bx^3\right)\sqrt{a}\,\sqrt{\frac{ax^3+b}{x^3}}\right)+\frac{1}{3}\left(ax^3-2\,b\right)\sqrt{\frac{ax^3+b}{x^3}}, -\frac{1}{2}\sqrt{-a}\,b\arctan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(3/2)*x^2,x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(a)*b*log(-8*a^2*x^6 - 8*a*b*x^3 - b^2 - 4*(2*a*x^6 + b*x^3)*sqrt(
a)*sqrt((a*x^3 + b)/x^3)) + 1/3*(a*x^3 - 2*b)*sqrt((a*x^3 + b)/x^3), -1/2*s
qrt(-a)*b*arctan(2*sqrt(-a)*x^3*sqrt((a*x^3 + b)/x^3)/(2*a*x^3 + b)) + 1/3*
(a*x^3 - 2*b)*sqrt((a*x^3 + b)/x^3)]
```

giac [A] time = 0.35, size = 55, normalized size = 0.95

$$\frac{1}{3}\sqrt{ax^4+bx}\,ax-\frac{1}{3}\left(\frac{3\,a\arctan\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{a+\frac{b}{x^3}}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(3/2)*x^2,x, algorithm="giac")
```

```
[Out] 1/3*sqrt(a*x^4 + b*x)*a*x - 1/3*(3*a*arctan(sqrt(a + b/x^3)/sqrt(-a))/sqrt(
-a) + 2*sqrt(a + b/x^3))*b
```

maple [C] time = 0.02, size = 3559, normalized size = 61.36

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^3)^(3/2)*x^2,x)
```

```
[Out] -1/3*((a*x^3+b)/x^3)^(3/2)*x^3/a*(-18*I*(-a^2*b)^(2/3)*3^(1/2)*(-(I*3^(1/2)
-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*
b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*
x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/
3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*
x)^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(
1/2))/(I*3^(1/2)-3))^(1/2))*x^2*b-I*3^(1/2)*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*
b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)
*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*x^3*a^2-36*I*(-a^2*b)^(1/3)*3^
(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*
x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/
3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/
(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+
(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^
(1/2)-3))^(1/2))*x^3*a*b-18*I*3^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(
-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/
(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3
)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticPi((-(
I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),(I*3^(1/2)-1)/(
I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2
))*x^4*a^2*b+36*I*(-a^2*b)^(1/3)*3^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*
x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3
))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(
1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticPi(
(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),(I*3^(1/2)-1
)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(
1/2))*x^3*a*b+18*I*3^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/
3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2
))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^
(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)
/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1
)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^4*a^2*b-18*(-(I*3^(1/2)-3)/(I*3^(1/
2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a
^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*
(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*E
llipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I
*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^4*a^2*b+18*
(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^
(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(
1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+
(-a^2*b)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2
*b)^(1/3))*a*x)^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)
-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^4*a^2*b+36*(-a^2*b)^(1/3)*(-(I*3^
(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(
-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((
-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b
```

```
)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3
))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/
2))*x^3*a*b-36*(-a^2*b)^(1/3)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^
(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(
1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*
b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2
)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),(I*3^(1/2)-1)/(I*3^(1/2
)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^3*a
*b-18*(-a^2*b)^(2/3)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*
x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a
*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))
/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^
(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I
*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*b+18*(-a^2*b)^(2/3)*(-(I*3^(1/2)-3)/(I*
3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3
)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(
1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1
/2)*EllipticPi((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/
2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(
I*3^(1/2)-3))^(1/2))*x^2*b+18*I*(-a^2*b)^(2/3)*3^(1/2)*(-(I*3^(1/2)-3)/(I*3
^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)
+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1
/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/
2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)
,((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*b+3*(
a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a
^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*
x^3*a^2+2*I*3^(1/2)*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/
2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)
^(1/3))/a^2*x)^(1/2)*a*b-6*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+
I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(
-a^2*b)^(1/3))/a^2*x)^(1/2)*a*b)/(a*x^3+b)/((a*x^3+b)*x)^(1/2)/(I*3^(1/2)-3
)/((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-
2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)
```

maxima [A] time = 1.96, size = 66, normalized size = 1.14

$$\frac{1}{3}\sqrt{a+\frac{b}{x^3}}\,ax^3-\frac{1}{2}\sqrt{a}\,b\log\left(\frac{\sqrt{a+\frac{b}{x^3}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^3}}+\sqrt{a}}\right)-\frac{2}{3}\sqrt{a+\frac{b}{x^3}}\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(3/2)*x^2,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(a + b/x^3)*a*x^3 - 1/2*sqrt(a)*b*log((sqrt(a + b/x^3) - sqrt(a))/(
sqrt(a + b/x^3) + sqrt(a))) - 2/3*sqrt(a + b/x^3)*b
```

mupad [B] time = 1.79, size = 69, normalized size = 1.19

$$\frac{a\,x^3\sqrt{a+\frac{b}{x^3}}}{3}-\frac{2\,b\sqrt{a+\frac{b}{x^3}}}{3}+\frac{\sqrt{a}\;b\;\ln\left(x^6\left(\sqrt{a+\frac{b}{x^3}}-\sqrt{a}\right)\left(\sqrt{a+\frac{b}{x^3}}+\sqrt{a}\right)^3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b/x^3)^(3/2),x)
```

```
[Out] (a*x^3*(a + b/x^3)^(1/2))/3 - (2*b*(a + b/x^3)^(1/2))/3 + (a^(1/2)*b*log(x^
6*((a + b/x^3)^(1/2) - a^(1/2))*((a + b/x^3)^(1/2) + a^(1/2))^3))/2
```

sympy [**B**] time = 3.00, size = 100, normalized size = 1.72

$$\sqrt{a}\,b\,\operatorname{asinh}\left(\frac{\sqrt{a}\,x^{\frac{3}{2}}}{\sqrt{b}}\right)+\frac{a^2x^{\frac{9}{2}}}{3\sqrt{b}\,\sqrt{\frac{ax^3}{b}+1}}-\frac{a\sqrt{b}\,x^{\frac{3}{2}}}{3\sqrt{\frac{ax^3}{b}+1}}-\frac{2b^{\frac{3}{2}}}{3x^{\frac{3}{2}}\sqrt{\frac{ax^3}{b}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**3)**(3/2)*x**2,x)
```

```
[Out] sqrt(a)*b*asinh(sqrt(a)*x**(3/2)/sqrt(b)) + a**2*x**(9/2)/(3*sqrt(b)*sqrt(a
*x**3/b + 1)) - a*sqrt(b)*x**(3/2)/(3*sqrt(a*x**3/b + 1)) - 2*b**(3/2)/(3*x
**(3/2)*sqrt(a*x**3/b + 1))
```

3.2012 $\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=59

$$\frac{2}{3}a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)-\frac{2}{3}a\sqrt{a+\frac{b}{x^3}}-\frac{2}{9}\left(a+\frac{b}{x^3}\right)^{3/2}$$

```
[Out] -2/9*(a+b/x^3)^(3/2)+2/3*a^(3/2)*arctanh((a+b/x^3)^(1/2)/a^(1/2))-2/3*a*(a+
b/x^3)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 50, 63, 208}

$$\frac{2}{3}a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)-\frac{2}{3}a\sqrt{a+\frac{b}{x^3}}-\frac{2}{9}\left(a+\frac{b}{x^3}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^3)^(3/2)/x,x]
```

```
[Out] (-2*a*Sqrt[a + b/x^3])/3 - (2*(a + b/x^3)^(3/2))/9 + (2*a^(3/2)*ArcTanh[Sqr
t[a + b/x^3]/Sqrt[a]])/3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x}\,dx &= -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, \frac{1}{x^3}\right)\right)\\ &= -\frac{2}{9}\left(a+\frac{b}{x^3}\right)^{3/2} - \frac{1}{3}a\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x^3}\right)\\ &= -\frac{2}{3}a\sqrt{a+\frac{b}{x^3}} - \frac{2}{9}\left(a+\frac{b}{x^3}\right)^{3/2} - \frac{1}{3}a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x^3}\right)\\ &= -\frac{2}{3}a\sqrt{a+\frac{b}{x^3}} - \frac{2}{9}\left(a+\frac{b}{x^3}\right)^{3/2} - \frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x^3}}\right)}{3b}\\ &= -\frac{2}{3}a\sqrt{a+\frac{b}{x^3}} - \frac{2}{9}\left(a+\frac{b}{x^3}\right)^{3/2} + \frac{2}{3}a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)\end{aligned}$$

Mathematica [C] time = 0.02, size = 52, normalized size = 0.88

$$-\frac{2b\sqrt{a+\frac{b}{x^3}}\;{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};-\frac{ax^3}{b}\right)}{9x^3\sqrt{\frac{ax^3}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^3)^(3/2)/x,x]
```

```
[Out] (-2*b*Sqrt[a + b/x^3]*Hypergeometric2F1[-3/2, -3/2, -1/2, -((a*x^3)/b)])/(9
*x^3*Sqrt[1 + (a*x^3)/b])
```

fricas [A] time = 1.08, size = 164, normalized size = 2.78

$$\left[\frac{3\,a^{\frac{3}{2}}x^3\log\left(-8\,a^2x^6-8\,abx^3-b^2-4\left(2\,ax^6+bx^3\right)\sqrt{a}\,\sqrt{\frac{ax^3+b}{x^3}}\right)-4\left(4\,ax^3+b\right)\sqrt{\frac{ax^3+b}{x^3}}}{18\,x^3}, -\frac{3\,\sqrt{-a}\,ax^3\arctan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/18*(3*a^(3/2)*x^3*log(-8*a^2*x^6 - 8*a*b*x^3 - b^2 - 4*(2*a*x^6 + b*x^3)
*sqrt(a)*sqrt((a*x^3 + b)/x^3)) - 4*(4*a*x^3 + b)*sqrt((a*x^3 + b)/x^3))/x^
3, -1/9*(3*sqrt(-a)*a*x^3*arctan(2*sqrt(-a)*x^3*sqrt((a*x^3 + b)/x^3)/(2*a*
x^3 + b)) + 2*(4*a*x^3 + b)*sqrt((a*x^3 + b)/x^3))/x^3]
```

giac [A] time = 0.28, size = 50, normalized size = 0.85

$$-\frac{2\,a^2\arctan\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{-a}}\right)}{3\sqrt{-a}} - \frac{2}{9}\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}} - \frac{2}{3}\sqrt{a+\frac{b}{x^3}}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(3/2)/x,x, algorithm="giac")
```

```
[Out] -2/3*a^2*arctan(sqrt(a + b/x^3)/sqrt(-a))/sqrt(-a) - 2/9*(a + b/x^3)^(3/2)
- 2/3*sqrt(a + b/x^3)*a
```

maple **[C]** time = 0.05, size = 3535, normalized size = 59.92

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^3)^(3/2)/x,x)
```

```
[Out] -2/9*((a*x^3+b)/x^3)^(3/2)*(-36*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2
*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I
*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-
a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(
1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^
(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*3^(1/2)*x^6*a-1
8*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+
I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)
))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-
a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(
-a^2*b)^(1/3))*a*x)^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(
1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*3^(1/2)*x^5+18*I
*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3
^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^
(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x
+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2
*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)
-3))^(1/2))*(-a^2*b)^(2/3)*3^(1/2)*x^5+36*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(
-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(
1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b
)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*Elliptic
Pi((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),(I*3^(1/2
)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3)
)^(1/2))*(-a^2*b)^(1/3)*3^(1/2)*x^6*a+18*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-
a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1
/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)
^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF
((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)
+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*3^(1/2)*x^7*a^2-18*(-
(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1
/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/
2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-
a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)
^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3)
)^(1/2))*x^7*a^2+4*I*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1
/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b
)^(1/3))/a^2*x)^(1/2)*3^(1/2)*x^3*a+18*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+
(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))
/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/
3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticPi((-
(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),(I*3^(1/2)-1)/
(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/
2))*x^7*a^2+36*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/
2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a
^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^
(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-
1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/
2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*x^6*a-36*(-(I*3^(1/2)-3)/(I*3^(1/2
```

```
)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^
2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(
-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*El
lipticPi((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),(I*
3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1
/2)-3))^(1/2))*(-a^2*b)^(1/3)*x^6*a-18*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+
(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))
/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/
3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(
I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*
(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*x^5+18*(-(
I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/
2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2
)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a
^2*b)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)
^(1/3))*a*x)^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)
/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*x^5-18*I*(-(I*3^(1/2)-3
)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)
^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+
I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)
))^(1/2)*EllipticPi((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x
)^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/
2))/(I*3^(1/2)-3))^(1/2))*3^(1/2)*x^7*a^2-12*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2
*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2
)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*x^3*a+I*(a*x^4+b*x)^(1/2)*((-
a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x
+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*3^(1/2)*b-3*(a*x^4+b
*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(
1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*b)/(a*x
^3+b)/((a*x^3+b)*x)^(1/2)/(I*3^(1/2)-3)/((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(
1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*
b)^(1/3))/a^2*x)^(1/2)
```

maxima [A] time = 1.97, size = 61, normalized size = 1.03

$$-\frac{1}{3}\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x^3}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^3}}+\sqrt{a}}\right)-\frac{2}{9}\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}-\frac{2}{3}\sqrt{a+\frac{b}{x^3}}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] -1/3*a^(3/2)*log((sqrt(a + b/x^3) - sqrt(a))/(sqrt(a + b/x^3) + sqrt(a))) -
 2/9*(a + b/x^3)^(3/2) - 2/3*sqrt(a + b/x^3)*a
```

mupad [B] time = 1.62, size = 43, normalized size = 0.73

$$\frac{2\,a^{3/2}\,\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{3}-\frac{2\,a\,\sqrt{a+\frac{b}{x^3}}}{3}-\frac{2\left(a+\frac{b}{x^3}\right)^{3/2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^3)^(3/2)/x,x)
```

```
[Out] (2*a^(3/2)*atanh((a + b/x^3)^(1/2)/a^(1/2)))/3 - (2*a*(a + b/x^3)^(1/2))/3
 - (2*(a + b/x^3)^(3/2))/9
```

sympy [**A**] time = 2.38, size = 83, normalized size = 1.41

$$-\frac{8a^{\frac{3}{2}}\sqrt{1+\frac{b}{ax^3}}}{9}-\frac{a^{\frac{3}{2}}\log\left(\frac{b}{ax^3}\right)}{3}+\frac{2a^{\frac{3}{2}}\log\left(\sqrt{1+\frac{b}{ax^3}}+1\right)}{3}-\frac{2\sqrt{a}\,b\sqrt{1+\frac{b}{ax^3}}}{9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**3)**(3/2)/x,x)
```

```
[Out] -8*a**(3/2)*sqrt(1 + b/(a*x**3))/9 - a**(3/2)*log(b/(a*x**3))/3 + 2*a**(3/2
)*log(sqrt(1 + b/(a*x**3)) + 1)/3 - 2*sqrt(a)*b*sqrt(1 + b/(a*x**3))/(9*x**
3)
```

3.2013 $\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=18

$$-\frac{2\left(a+\frac{b}{x^3}\right)^{5/2}}{15b}$$

```
[Out] -2/15*(a+b/x^3)^(5/2)/b
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$-\frac{2\left(a+\frac{b}{x^3}\right)^{5/2}}{15b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^3)^(3/2)/x^4,x]
```

```
[Out] (-2*(a + b/x^3)^(5/2))/(15*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^4}\,dx = -\frac{2\left(a+\frac{b}{x^3}\right)^{5/2}}{15b}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.56

$$-\frac{2\left(a+\frac{b}{x^3}\right)^{3/2}\left(ax^3+b\right)}{15bx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^3)^(3/2)/x^4,x]
```

```
[Out] (-2*(a + b/x^3)^(3/2)*(b + a*x^3))/(15*b*x^3)
```

fricas [B] time = 1.05, size = 39, normalized size = 2.17

$$-\frac{2\left(a^2x^6+2\,abx^3+b^2\right)\sqrt{\frac{ax^3+b}{x^3}}}{15\,bx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] -2/15*(a^2*x^6 + 2*a*b*x^3 + b^2)*sqrt((a*x^3 + b)/x^3)/(b*x^6)
```

giac [A] time = 0.18, size = 14, normalized size = 0.78

$$-\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{5}{2}}}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(3/2)/x^4,x, algorithm="giac")
[Out] -2/15*(a + b/x^3)^(5/2)/b
```

maple [A] time = 0.00, size = 29, normalized size = 1.61

$$-\frac{2\left(a\,x^3+b\right)\left(\frac{a\,x^3+b}{x^3}\right)^{\frac{3}{2}}}{15b\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^3)^(3/2)/x^4,x)
[Out] -2/15*(a*x^3+b)/x^3/b*((a*x^3+b)/x^3)^(3/2)
```

maxima [A] time = 0.85, size = 14, normalized size = 0.78

$$-\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{5}{2}}}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(3/2)/x^4,x, algorithm="maxima")
[Out] -2/15*(a + b/x^3)^(5/2)/b
```

mupad [B] time = 1.89, size = 26, normalized size = 1.44

$$-\frac{2\sqrt{a+\frac{b}{x^3}}\,\left(a\,x^3+b\right)^2}{15\,b\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^3)^(3/2)/x^4,x)
[Out] -(2*(a + b/x^3)^(1/2)*(b + a*x^3)^2)/(15*b*x^6)
```

sympy [B] time = 1.36, size = 71, normalized size = 3.94

$$-\frac{2a^{\frac{5}{2}}\sqrt{1+\frac{b}{ax^3}}}{15b}-\frac{4a^{\frac{3}{2}}\sqrt{1+\frac{b}{ax^3}}}{15x^3}-\frac{2\sqrt{a}\,b\sqrt{1+\frac{b}{ax^3}}}{15x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**3)**(3/2)/x**4,x)
[Out] -2*a**(5/2)*sqrt(1 + b/(a*x**3))/(15*b) - 4*a**(3/2)*sqrt(1 + b/(a*x**3))/(
15*x**3) - 2*sqrt(a)*b*sqrt(1 + b/(a*x**3))/(15*x**6)
```

3.2014 $\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=38

$$\frac{2a\left(a+\frac{b}{x^3}\right)^{5/2}}{15b^2}-\frac{2\left(a+\frac{b}{x^3}\right)^{7/2}}{21b^2}$$

```
[Out] 2/15*a*(a+b/x^3)^(5/2)/b^2-2/21*(a+b/x^3)^(7/2)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{2a\left(a+\frac{b}{x^3}\right)^{5/2}}{15b^2}-\frac{2\left(a+\frac{b}{x^3}\right)^{7/2}}{21b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^3)^(3/2)/x^7,x]
```

```
[Out] (2*a*(a + b/x^3)^(5/2))/(15*b^2) - (2*(a + b/x^3)^(7/2))/(21*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^7}\,dx &= -\left(\frac{1}{3}\,\text{Subst}\left(\int x(a+bx)^{3/2}\,dx, x, \frac{1}{x^3}\right)\right)\\ &= -\left(\frac{1}{3}\,\text{Subst}\left(\int \left(-\frac{a(a+bx)^{3/2}}{b}+\frac{(a+bx)^{5/2}}{b}\right)dx, x, \frac{1}{x^3}\right)\right)\\ &= \frac{2a\left(a+\frac{b}{x^3}\right)^{5/2}}{15b^2}-\frac{2\left(a+\frac{b}{x^3}\right)^{7/2}}{21b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.05

$$\frac{2\sqrt{a+\frac{b}{x^3}}\left(ax^3+b\right)^2\left(2ax^3-5b\right)}{105b^2x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^3)^(3/2)/x^7,x]
```

```
[Out] (2*Sqrt[a + b/x^3]*(b + a*x^3)^2*(-5*b + 2*a*x^3))/(105*b^2*x^9)
```

fricas [A] time = 0.84, size = 53, normalized size = 1.39

$$\frac{2\left(2\,a^3x^9 - a^2bx^6 - 8\,ab^2x^3 - 5\,b^3\right)\sqrt{\frac{ax^3+b}{x^3}}}{105\,b^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] 2/105*(2*a^3*x^9 - a^2*b*x^6 - 8*a*b^2*x^3 - 5*b^3)*sqrt((a*x^3 + b)/x^3)/(
b^2*x^9)
```

giac [A] time = 0.20, size = 29, normalized size = 0.76

$$-\frac{2\left(5\left(a+\frac{b}{x^3}\right)^{\frac{7}{2}} - 7\left(a+\frac{b}{x^3}\right)^{\frac{5}{2}}a\right)}{105\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] -2/105*(5*(a + b/x^3)^(7/2) - 7*(a + b/x^3)^(5/2)*a)/b^2
```

maple [A] time = 0.01, size = 39, normalized size = 1.03

$$\frac{2\left(a\,x^3+b\right)\left(2a\,x^3-5b\right)\left(\frac{a\,x^3+b}{x^3}\right)^{\frac{3}{2}}}{105b^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^3)^(3/2)/x^7,x)
```

```
[Out] 2/105*(a*x^3+b)*(2*a*x^3-5*b)*((a*x^3+b)/x^3)^(3/2)/b^2/x^6
```

maxima [A] time = 0.87, size = 30, normalized size = 0.79

$$-\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{7}{2}}}{21\,b^2} + \frac{2\left(a+\frac{b}{x^3}\right)^{\frac{5}{2}}a}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] -2/21*(a + b/x^3)^(7/2)/b^2 + 2/15*(a + b/x^3)^(5/2)*a/b^2
```

mupad [B] time = 2.36, size = 68, normalized size = 1.79

$$\frac{4\,a^3\sqrt{a+\frac{b}{x^3}}}{105\,b^2} - \frac{2\,b\sqrt{a+\frac{b}{x^3}}}{21\,x^9} - \frac{16\,a\sqrt{a+\frac{b}{x^3}}}{105\,x^6} - \frac{2\,a^2\sqrt{a+\frac{b}{x^3}}}{105\,b\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^3)^(3/2)/x^7,x)
```

```
[Out] (4*a^3*(a + b/x^3)^(1/2))/(105*b^2) - (2*b*(a + b/x^3)^(1/2))/(21*x^9) - (1
6*a*(a + b/x^3)^(1/2))/(105*x^6) - (2*a^2*(a + b/x^3)^(1/2))/(105*b*x^3)
```

sympy [B] time = 2.05, size = 371, normalized size = 9.76

$$\frac{4a^{\frac{15}{2}}b^{\frac{3}{2}}x^{12}\sqrt{\frac{ax^3}{b}+1}}{105a^{\frac{9}{2}}b^3x^{\frac{27}{2}}+105a^{\frac{7}{2}}b^4x^{\frac{21}{2}}}+\frac{2a^{\frac{13}{2}}b^{\frac{5}{2}}x^9\sqrt{\frac{ax^3}{b}+1}}{105a^{\frac{9}{2}}b^3x^{\frac{27}{2}}+105a^{\frac{7}{2}}b^4x^{\frac{21}{2}}}-\frac{18a^{\frac{11}{2}}b^{\frac{7}{2}}x^6\sqrt{\frac{ax^3}{b}+1}}{105a^{\frac{9}{2}}b^3x^{\frac{27}{2}}+105a^{\frac{7}{2}}b^4x^{\frac{21}{2}}}-\frac{26a^{\frac{9}{2}}b^{\frac{9}{2}}x^3\sqrt{\frac{ax^3}{b}+1}}{105a^{\frac{9}{2}}b^3x^{\frac{27}{2}}+105a^{\frac{7}{2}}b^4x^{\frac{2}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**3)**(3/2)/x**7,x)
```

```
[Out] 4*a**(15/2)*b**(3/2)*x**12*sqrt(a*x**3/b + 1)/(105*a**(9/2)*b**3*x**(27/2)
+ 105*a**(7/2)*b**4*x**(21/2)) + 2*a**(13/2)*b**(5/2)*x**9*sqrt(a*x**3/b +
1)/(105*a**(9/2)*b**3*x**(27/2) + 105*a**(7/2)*b**4*x**(21/2)) - 18*a**(11/
2)*b**(7/2)*x**6*sqrt(a*x**3/b + 1)/(105*a**(9/2)*b**3*x**(27/2) + 105*a**(
7/2)*b**4*x**(21/2)) - 26*a**(9/2)*b**(9/2)*x**3*sqrt(a*x**3/b + 1)/(105*a*
*(9/2)*b**3*x**(27/2) + 105*a**(7/2)*b**4*x**(21/2)) - 10*a**(7/2)*b**(11/2
)*sqrt(a*x**3/b + 1)/(105*a**(9/2)*b**3*x**(27/2) + 105*a**(7/2)*b**4*x**(2
1/2)) - 4*a**8*b*x**(27/2)/(105*a**(9/2)*b**3*x**(27/2) + 105*a**(7/2)*b**4
*x**(21/2)) - 4*a**7*b**2*x**(21/2)/(105*a**(9/2)*b**3*x**(27/2) + 105*a**(
7/2)*b**4*x**(21/2))
```

3.2015 $\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^{10}}\,dx$

Optimal. Leaf size=59

$$-\frac{2a^2\left(a+\frac{b}{x^3}\right)^{5/2}}{15b^3}-\frac{2\left(a+\frac{b}{x^3}\right)^{9/2}}{27b^3}+\frac{4a\left(a+\frac{b}{x^3}\right)^{7/2}}{21b^3}$$

```
[Out] -2/15*a^2*(a+b/x^3)^(5/2)/b^3+4/21*a*(a+b/x^3)^(7/2)/b^3-2/27*(a+b/x^3)^(9/
2)/b^3
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{2a^2\left(a+\frac{b}{x^3}\right)^{5/2}}{15b^3}-\frac{2\left(a+\frac{b}{x^3}\right)^{9/2}}{27b^3}+\frac{4a\left(a+\frac{b}{x^3}\right)^{7/2}}{21b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^3)^(3/2)/x^10,x]
```

```
[Out] (-2*a^2*(a + b/x^3)^(5/2))/(15*b^3) + (4*a*(a + b/x^3)^(7/2))/(21*b^3) - (2
*(a + b/x^3)^(9/2))/(27*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^{10}}\,dx &= -\left(\frac{1}{3}\,\text{Subst}\left(\int x^2(a+bx)^{3/2}\,dx, x, \frac{1}{x^3}\right)\right)\\ &= -\left(\frac{1}{3}\,\text{Subst}\left(\int\left(\frac{a^2(a+bx)^{3/2}}{b^2}-\frac{2a(a+bx)^{5/2}}{b^2}+\frac{(a+bx)^{7/2}}{b^2}\right)dx, x, \frac{1}{x^3}\right)\right)\\ &= -\frac{2a^2\left(a+\frac{b}{x^3}\right)^{5/2}}{15b^3}+\frac{4a\left(a+\frac{b}{x^3}\right)^{7/2}}{21b^3}-\frac{2\left(a+\frac{b}{x^3}\right)^{9/2}}{27b^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 0.86

$$-\frac{2\sqrt{a+\frac{b}{x^3}}\left(ax^3+b\right)^2\left(8a^2x^6-20abx^3+35b^2\right)}{945b^3x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^3)^(3/2)/x^10,x]
```

```
[Out] (-2*Sqrt[a + b/x^3]*(b + a*x^3)^2*(35*b^2 - 20*a*b*x^3 + 8*a^2*x^6))/(945*b
^3*x^12)
```

fricas [A] time = 0.82, size = 64, normalized size = 1.08

$$-\frac{2\left(8\,a^4x^{12}-4\,a^3bx^9+3\,a^2b^2x^6+50\,ab^3x^3+35\,b^4\right)\sqrt{\frac{ax^3+b}{x^3}}}{945\,b^3x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(3/2)/x^10,x, algorithm="fricas")
```

```
[Out] -2/945*(8*a^4*x^12 - 4*a^3*b*x^9 + 3*a^2*b^2*x^6 + 50*a*b^3*x^3 + 35*b^4)*s
qrt((a*x^3 + b)/x^3)/(b^3*x^12)
```

giac [A] time = 0.20, size = 43, normalized size = 0.73

$$-\frac{2\left(35\left(a+\frac{b}{x^3}\right)^{\frac{9}{2}}-90\left(a+\frac{b}{x^3}\right)^{\frac{7}{2}}a+63\left(a+\frac{b}{x^3}\right)^{\frac{5}{2}}a^2\right)}{945\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(3/2)/x^10,x, algorithm="giac")
```

```
[Out] -2/945*(35*(a + b/x^3)^(9/2) - 90*(a + b/x^3)^(7/2)*a + 63*(a + b/x^3)^(5/2
)*a^2)/b^3
```

maple [A] time = 0.01, size = 50, normalized size = 0.85

$$-\frac{2\left(a\,x^3+b\right)\left(8a^2x^6-20ab\,x^3+35b^2\right)\left(\frac{a\,x^3+b}{x^3}\right)^{\frac{3}{2}}}{945b^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^3)^(3/2)/x^10,x)
```

```
[Out] -2/945*(a*x^3+b)*(8*a^2*x^6-20*a*b*x^3+35*b^2)*((a*x^3+b)/x^3)^(3/2)/b^3/x^
9
```

maxima [A] time = 0.82, size = 47, normalized size = 0.80

$$-\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{9}{2}}}{27\,b^3}+\frac{4\left(a+\frac{b}{x^3}\right)^{\frac{7}{2}}a}{21\,b^3}-\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{5}{2}}a^2}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(3/2)/x^10,x, algorithm="maxima")
```

```
[Out] -2/27*(a + b/x^3)^(9/2)/b^3 + 4/21*(a + b/x^3)^(7/2)*a/b^3 - 2/15*(a + b/x^
3)^(5/2)*a^2/b^3
```

mupad [B] time = 2.85, size = 88, normalized size = 1.49

$$\frac{8\,a^3\sqrt{a+\frac{b}{x^3}}}{945\,b^2\,x^3}-\frac{2\,b\sqrt{a+\frac{b}{x^3}}}{27\,x^{12}}-\frac{16\,a^4\sqrt{a+\frac{b}{x^3}}}{945\,b^3}-\frac{20\,a\sqrt{a+\frac{b}{x^3}}}{189\,x^9}-\frac{2\,a^2\sqrt{a+\frac{b}{x^3}}}{315\,b\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^3)^(3/2)/x^10,x)
```

```
[Out] (8*a^3*(a + b/x^3)^(1/2))/(945*b^2*x^3) - (2*b*(a + b/x^3)^(1/2))/(27*x^12)
 - (16*a^4*(a + b/x^3)^(1/2))/(945*b^3) - (20*a*(a + b/x^3)^(1/2))/(189*x^9
) - (2*a^2*(a + b/x^3)^(1/2))/(315*b*x^6)
```

sympy [**B**] time = 3.24, size = 1001, normalized size = 16.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**3)**(3/2)/x**10,x)
```

```
[Out] -16*a**(23/2)*b**(9/2)*x**21*sqrt(a*x**3/b + 1)/(945*a**(15/2)*b**7*x**(45/
2) + 2835*a**(13/2)*b**8*x**(39/2) + 2835*a**(11/2)*b**9*x**(33/2) + 945*a*
*(9/2)*b**10*x**(27/2)) - 40*a**(21/2)*b**(11/2)*x**18*sqrt(a*x**3/b + 1)/(
945*a**(15/2)*b**7*x**(45/2) + 2835*a**(13/2)*b**8*x**(39/2) + 2835*a**(11/
2)*b**9*x**(33/2) + 945*a**(9/2)*b**10*x**(27/2)) - 30*a**(19/2)*b**(13/2)*
x**15*sqrt(a*x**3/b + 1)/(945*a**(15/2)*b**7*x**(45/2) + 2835*a**(13/2)*b**
8*x**(39/2) + 2835*a**(11/2)*b**9*x**(33/2) + 945*a**(9/2)*b**10*x**(27/2))
 - 110*a**(17/2)*b**(15/2)*x**12*sqrt(a*x**3/b + 1)/(945*a**(15/2)*b**7*x**
(45/2) + 2835*a**(13/2)*b**8*x**(39/2) + 2835*a**(11/2)*b**9*x**(33/2) + 94
5*a**(9/2)*b**10*x**(27/2)) - 380*a**(15/2)*b**(17/2)*x**9*sqrt(a*x**3/b +
1)/(945*a**(15/2)*b**7*x**(45/2) + 2835*a**(13/2)*b**8*x**(39/2) + 2835*a**
(11/2)*b**9*x**(33/2) + 945*a**(9/2)*b**10*x**(27/2)) - 516*a**(13/2)*b**(1
9/2)*x**6*sqrt(a*x**3/b + 1)/(945*a**(15/2)*b**7*x**(45/2) + 2835*a**(13/2)
*b**8*x**(39/2) + 2835*a**(11/2)*b**9*x**(33/2) + 945*a**(9/2)*b**10*x**(27
/2)) - 310*a**(11/2)*b**(21/2)*x**3*sqrt(a*x**3/b + 1)/(945*a**(15/2)*b**7*
x**(45/2) + 2835*a**(13/2)*b**8*x**(39/2) + 2835*a**(11/2)*b**9*x**(33/2) +
 945*a**(9/2)*b**10*x**(27/2)) - 70*a**(9/2)*b**(23/2)*sqrt(a*x**3/b + 1)/(
945*a**(15/2)*b**7*x**(45/2) + 2835*a**(13/2)*b**8*x**(39/2) + 2835*a**(11/
2)*b**9*x**(33/2) + 945*a**(9/2)*b**10*x**(27/2)) + 16*a**12*b**4*x**(45/2)
/(945*a**(15/2)*b**7*x**(45/2) + 2835*a**(13/2)*b**8*x**(39/2) + 2835*a**(1
1/2)*b**9*x**(33/2) + 945*a**(9/2)*b**10*x**(27/2)) + 48*a**11*b**5*x**(39/
2)/(945*a**(15/2)*b**7*x**(45/2) + 2835*a**(13/2)*b**8*x**(39/2) + 2835*a**
(11/2)*b**9*x**(33/2) + 945*a**(9/2)*b**10*x**(27/2)) + 48*a**10*b**6*x**(3
3/2)/(945*a**(15/2)*b**7*x**(45/2) + 2835*a**(13/2)*b**8*x**(39/2) + 2835*a
**(11/2)*b**9*x**(33/2) + 945*a**(9/2)*b**10*x**(27/2)) + 16*a**9*b**7*x**(
27/2)/(945*a**(15/2)*b**7*x**(45/2) + 2835*a**(13/2)*b**8*x**(39/2) + 2835*
a**(11/2)*b**9*x**(33/2) + 945*a**(9/2)*b**10*x**(27/2))
```

3.2016 $\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^{13}}\,dx$

Optimal. Leaf size=80

$$\frac{2a^3\left(a+\frac{b}{x^3}\right)^{5/2}}{15b^4}-\frac{2a^2\left(a+\frac{b}{x^3}\right)^{7/2}}{7b^4}-\frac{2\left(a+\frac{b}{x^3}\right)^{11/2}}{33b^4}+\frac{2a\left(a+\frac{b}{x^3}\right)^{9/2}}{9b^4}$$

```
[Out] 2/15*a^3*(a+b/x^3)^(5/2)/b^4-2/7*a^2*(a+b/x^3)^(7/2)/b^4+2/9*a*(a+b/x^3)^(9
/2)/b^4-2/33*(a+b/x^3)^(11/2)/b^4
```

Rubi [A] time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{2a^2\left(a+\frac{b}{x^3}\right)^{7/2}}{7b^4}+\frac{2a^3\left(a+\frac{b}{x^3}\right)^{5/2}}{15b^4}-\frac{2\left(a+\frac{b}{x^3}\right)^{11/2}}{33b^4}+\frac{2a\left(a+\frac{b}{x^3}\right)^{9/2}}{9b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^3)^(3/2)/x^13,x]
```

```
[Out] (2*a^3*(a + b/x^3)^(5/2))/(15*b^4) - (2*a^2*(a + b/x^3)^(7/2))/(7*b^4) + (2
*a*(a + b/x^3)^(9/2))/(9*b^4) - (2*(a + b/x^3)^(11/2))/(33*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^{13}}\,dx &= -\left(\frac{1}{3}\,\text{Subst}\left(\int x^3(a+bx)^{3/2}\,dx, x, \frac{1}{x^3}\right)\right)\\ &= -\left(\frac{1}{3}\,\text{Subst}\left(\int\left(-\frac{a^3(a+bx)^{3/2}}{b^3}+\frac{3a^2(a+bx)^{5/2}}{b^3}-\frac{3a(a+bx)^{7/2}}{b^3}+\frac{(a+bx)^{9/2}}{b^3}\right)dx, x, \frac{1}{x^3}\right)\right)\\ &= \frac{2a^3\left(a+\frac{b}{x^3}\right)^{5/2}}{15b^4}-\frac{2a^2\left(a+\frac{b}{x^3}\right)^{7/2}}{7b^4}+\frac{2a\left(a+\frac{b}{x^3}\right)^{9/2}}{9b^4}-\frac{2\left(a+\frac{b}{x^3}\right)^{11/2}}{33b^4}\end{aligned}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 0.78

$$\frac{2\sqrt{a+\frac{b}{x^3}}\left(ax^3+b\right)^2\left(16a^3x^9-40a^2bx^6+70ab^2x^3-105b^3\right)}{3465b^4x^{15}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^3)^(3/2)/x^13,x]
```

```
[Out] (2*Sqrt[a + b/x^3]*(b + a*x^3)^2*(-105*b^3 + 70*a*b^2*x^3 - 40*a^2*b*x^6 +
16*a^3*x^9))/(3465*b^4*x^15)
```

fricas [A] time = 0.79, size = 75, normalized size = 0.94

$$\frac{2\left(16\,a^5x^{15}-8\,a^4bx^{12}+6\,a^3b^2x^9-5\,a^2b^3x^6-140\,ab^4x^3-105\,b^5\right)\sqrt{\frac{ax^3+b}{x^3}}}{3465\,b^4x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(3/2)/x^13,x, algorithm="fricas")
```

```
[Out] 2/3465*(16*a^5*x^15 - 8*a^4*b*x^12 + 6*a^3*b^2*x^9 - 5*a^2*b^3*x^6 - 140*a*
b^4*x^3 - 105*b^5)*sqrt((a*x^3 + b)/x^3)/(b^4*x^15)
```

giac [A] time = 0.20, size = 57, normalized size = 0.71

$$-\frac{2\left(105\left(a+\frac{b}{x^3}\right)^{\frac{11}{2}}-385\left(a+\frac{b}{x^3}\right)^{\frac{9}{2}}a+495\left(a+\frac{b}{x^3}\right)^{\frac{7}{2}}a^2-231\left(a+\frac{b}{x^3}\right)^{\frac{5}{2}}a^3\right)}{3465\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(3/2)/x^13,x, algorithm="giac")
```

```
[Out] -2/3465*(105*(a + b/x^3)^(11/2) - 385*(a + b/x^3)^(9/2)*a + 495*(a + b/x^3)
^(7/2)*a^2 - 231*(a + b/x^3)^(5/2)*a^3)/b^4
```

maple [A] time = 0.01, size = 61, normalized size = 0.76

$$\frac{2\left(a\,x^3+b\right)\left(16a^3x^9-40a^2b\,x^6+70a\,b^2x^3-105b^3\right)\left(\frac{a\,x^3+b}{x^3}\right)^{\frac{3}{2}}}{3465b^4x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^3)^(3/2)/x^13,x)
```

```
[Out] 2/3465*(a*x^3+b)*(16*a^3*x^9-40*a^2*b*x^6+70*a*b^2*x^3-105*b^3)*((a*x^3+b)/
x^3)^(3/2)/x^12/b^4
```

maxima [A] time = 0.82, size = 64, normalized size = 0.80

$$-\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{11}{2}}}{33\,b^4}+\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{9}{2}}a}{9\,b^4}-\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{7}{2}}a^2}{7\,b^4}+\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{5}{2}}a^3}{15\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^(3/2)/x^13,x, algorithm="maxima")
```

```
[Out] -2/33*(a + b/x^3)^(11/2)/b^4 + 2/9*(a + b/x^3)^(9/2)*a/b^4 - 2/7*(a + b/x^3
)^(7/2)*a^2/b^4 + 2/15*(a + b/x^3)^(5/2)*a^3/b^4
```

mupad [B] time = 3.43, size = 108, normalized size = 1.35

$$\frac{32\,a^5\sqrt{a+\frac{b}{x^3}}}{3465\,b^4}-\frac{2\,b\sqrt{a+\frac{b}{x^3}}}{33\,x^{15}}-\frac{8\,a\sqrt{a+\frac{b}{x^3}}}{99\,x^{12}}-\frac{16\,a^4\sqrt{a+\frac{b}{x^3}}}{3465\,b^3\,x^3}+\frac{4\,a^3\sqrt{a+\frac{b}{x^3}}}{1155\,b^2\,x^6}-\frac{2\,a^2\sqrt{a+\frac{b}{x^3}}}{693\,b\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^3)^(3/2)/x^13,x)
```

```
[Out] (32*a^5*(a + b/x^3)^(1/2))/(3465*b^4) - (2*b*(a + b/x^3)^(1/2))/(33*x^15) -
 (8*a*(a + b/x^3)^(1/2))/(99*x^12) - (16*a^4*(a + b/x^3)^(1/2))/(3465*b^3*x
^3) + (4*a^3*(a + b/x^3)^(1/2))/(1155*b^2*x^6) - (2*a^2*(a + b/x^3)^(1/2))/
(693*b*x^9)
```

sympy [**B**] time = 4.75, size = 2317, normalized size = 28.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**3)**(3/2)/x**13,x)
```

```
[Out] 32*a**(33/2)*b**(23/2)*x**33*sqrt(a*x**3/b + 1)/(3465*a**(23/2)*b**15*x**(6
9/2) + 20790*a**(21/2)*b**16*x**(63/2) + 51975*a**(19/2)*b**17*x**(57/2) +
69300*a**(17/2)*b**18*x**(51/2) + 51975*a**(15/2)*b**19*x**(45/2) + 20790*a
**(13/2)*b**20*x**(39/2) + 3465*a**(11/2)*b**21*x**(33/2)) + 176*a**(31/2)*
b**(25/2)*x**30*sqrt(a*x**3/b + 1)/(3465*a**(23/2)*b**15*x**(69/2) + 20790*
a**(21/2)*b**16*x**(63/2) + 51975*a**(19/2)*b**17*x**(57/2) + 69300*a**(17/
2)*b**18*x**(51/2) + 51975*a**(15/2)*b**19*x**(45/2) + 20790*a**(13/2)*b**2
0*x**(39/2) + 3465*a**(11/2)*b**21*x**(33/2)) + 396*a**(29/2)*b**(27/2)*x**
27*sqrt(a*x**3/b + 1)/(3465*a**(23/2)*b**15*x**(69/2) + 20790*a**(21/2)*b**
16*x**(63/2) + 51975*a**(19/2)*b**17*x**(57/2) + 69300*a**(17/2)*b**18*x**(
51/2) + 51975*a**(15/2)*b**19*x**(45/2) + 20790*a**(13/2)*b**20*x**(39/2) +
 3465*a**(11/2)*b**21*x**(33/2)) + 462*a**(27/2)*b**(29/2)*x**24*sqrt(a*x**
3/b + 1)/(3465*a**(23/2)*b**15*x**(69/2) + 20790*a**(21/2)*b**16*x**(63/2)
+ 51975*a**(19/2)*b**17*x**(57/2) + 69300*a**(17/2)*b**18*x**(51/2) + 51975
*a**(15/2)*b**19*x**(45/2) + 20790*a**(13/2)*b**20*x**(39/2) + 3465*a**(11/
2)*b**21*x**(33/2)) - 1848*a**(23/2)*b**(33/2)*x**18*sqrt(a*x**3/b + 1)/(34
65*a**(23/2)*b**15*x**(69/2) + 20790*a**(21/2)*b**16*x**(63/2) + 51975*a**(
19/2)*b**17*x**(57/2) + 69300*a**(17/2)*b**18*x**(51/2) + 51975*a**(15/2)*b
**19*x**(45/2) + 20790*a**(13/2)*b**20*x**(39/2) + 3465*a**(11/2)*b**21*x**
(33/2)) - 5544*a**(21/2)*b**(35/2)*x**15*sqrt(a*x**3/b + 1)/(3465*a**(23/2)
*b**15*x**(69/2) + 20790*a**(21/2)*b**16*x**(63/2) + 51975*a**(19/2)*b**17*
x**(57/2) + 69300*a**(17/2)*b**18*x**(51/2) + 51975*a**(15/2)*b**19*x**(45/
2) + 20790*a**(13/2)*b**20*x**(39/2) + 3465*a**(11/2)*b**21*x**(33/2)) - 88
44*a**(19/2)*b**(37/2)*x**12*sqrt(a*x**3/b + 1)/(3465*a**(23/2)*b**15*x**(6
9/2) + 20790*a**(21/2)*b**16*x**(63/2) + 51975*a**(19/2)*b**17*x**(57/2) +
69300*a**(17/2)*b**18*x**(51/2) + 51975*a**(15/2)*b**19*x**(45/2) + 20790*a
**(13/2)*b**20*x**(39/2) + 3465*a**(11/2)*b**21*x**(33/2)) - 8448*a**(17/2)
*b**(39/2)*x**9*sqrt(a*x**3/b + 1)/(3465*a**(23/2)*b**15*x**(69/2) + 20790*
a**(21/2)*b**16*x**(63/2) + 51975*a**(19/2)*b**17*x**(57/2) + 69300*a**(17/
2)*b**18*x**(51/2) + 51975*a**(15/2)*b**19*x**(45/2) + 20790*a**(13/2)*b**2
0*x**(39/2) + 3465*a**(11/2)*b**21*x**(33/2)) - 4840*a**(15/2)*b**(41/2)*x*
*6*sqrt(a*x**3/b + 1)/(3465*a**(23/2)*b**15*x**(69/2) + 20790*a**(21/2)*b**
16*x**(63/2) + 51975*a**(19/2)*b**17*x**(57/2) + 69300*a**(17/2)*b**18*x**(
51/2) + 51975*a**(15/2)*b**19*x**(45/2) + 20790*a**(13/2)*b**20*x**(39/2) +
 3465*a**(11/2)*b**21*x**(33/2)) - 1540*a**(13/2)*b**(43/2)*x**3*sqrt(a*x**
3/b + 1)/(3465*a**(23/2)*b**15*x**(69/2) + 20790*a**(21/2)*b**16*x**(63/2)
+ 51975*a**(19/2)*b**17*x**(57/2) + 69300*a**(17/2)*b**18*x**(51/2) + 51975
*a**(15/2)*b**19*x**(45/2) + 20790*a**(13/2)*b**20*x**(39/2) + 3465*a**(11/
2)*b**21*x**(33/2)) - 210*a**(11/2)*b**(45/2)*sqrt(a*x**3/b + 1)/(3465*a**(
23/2)*b**15*x**(69/2) + 20790*a**(21/2)*b**16*x**(63/2) + 51975*a**(19/2)*b
**17*x**(57/2) + 69300*a**(17/2)*b**18*x**(51/2) + 51975*a**(15/2)*b**19*x*
*(45/2) + 20790*a**(13/2)*b**20*x**(39/2) + 3465*a**(11/2)*b**21*x**(33/2))
 - 32*a**17*b**11*x**(69/2)/(3465*a**(23/2)*b**15*x**(69/2) + 20790*a**(21/
2)*b**16*x**(63/2) + 51975*a**(19/2)*b**17*x**(57/2) + 69300*a**(17/2)*b**1
```

```
8*x**(51/2) + 51975*a**(15/2)*b**19*x**(45/2) + 20790*a**(13/2)*b**20*x**(3
9/2) + 3465*a**(11/2)*b**21*x**(33/2)) - 192*a**16*b**12*x**(63/2)/(3465*a*
*(23/2)*b**15*x**(69/2) + 20790*a**(21/2)*b**16*x**(63/2) + 51975*a**(19/2)
*b**17*x**(57/2) + 69300*a**(17/2)*b**18*x**(51/2) + 51975*a**(15/2)*b**19*
x**(45/2) + 20790*a**(13/2)*b**20*x**(39/2) + 3465*a**(11/2)*b**21*x**(33/2
)) - 480*a**15*b**13*x**(57/2)/(3465*a**(23/2)*b**15*x**(69/2) + 20790*a**(
21/2)*b**16*x**(63/2) + 51975*a**(19/2)*b**17*x**(57/2) + 69300*a**(17/2)*b
**18*x**(51/2) + 51975*a**(15/2)*b**19*x**(45/2) + 20790*a**(13/2)*b**20*x*
*(39/2) + 3465*a**(11/2)*b**21*x**(33/2)) - 640*a**14*b**14*x**(51/2)/(3465
*a**(23/2)*b**15*x**(69/2) + 20790*a**(21/2)*b**16*x**(63/2) + 51975*a**(19
/2)*b**17*x**(57/2) + 69300*a**(17/2)*b**18*x**(51/2) + 51975*a**(15/2)*b**
19*x**(45/2) + 20790*a**(13/2)*b**20*x**(39/2) + 3465*a**(11/2)*b**21*x**(3
3/2)) - 480*a**13*b**15*x**(45/2)/(3465*a**(23/2)*b**15*x**(69/2) + 20790*a
**(21/2)*b**16*x**(63/2) + 51975*a**(19/2)*b**17*x**(57/2) + 69300*a**(17/2
)*b**18*x**(51/2) + 51975*a**(15/2)*b**19*x**(45/2) + 20790*a**(13/2)*b**20
*x**(39/2) + 3465*a**(11/2)*b**21*x**(33/2)) - 192*a**12*b**16*x**(39/2)/(3
465*a**(23/2)*b**15*x**(69/2) + 20790*a**(21/2)*b**16*x**(63/2) + 51975*a**
(19/2)*b**17*x**(57/2) + 69300*a**(17/2)*b**18*x**(51/2) + 51975*a**(15/2)*
b**19*x**(45/2) + 20790*a**(13/2)*b**20*x**(39/2) + 3465*a**(11/2)*b**21*x*
*(33/2)) - 32*a**11*b**17*x**(33/2)/(3465*a**(23/2)*b**15*x**(69/2) + 20790
*a**(21/2)*b**16*x**(63/2) + 51975*a**(19/2)*b**17*x**(57/2) + 69300*a**(17
/2)*b**18*x**(51/2) + 51975*a**(15/2)*b**19*x**(45/2) + 20790*a**(13/2)*b**
20*x**(39/2) + 3465*a**(11/2)*b**21*x**(33/2))
```

3.2017 $\int \frac{x^5}{\sqrt{a+\frac{b}{x^3}}}\,dx$

Optimal. Leaf size=74

$$\frac{b^2 \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{4a^{5/2}} - \frac{bx^3\sqrt{a+\frac{b}{x^3}}}{4a^2} + \frac{x^6\sqrt{a+\frac{b}{x^3}}}{6a}$$

```
[Out] 1/4*b^2*arctanh((a+b/x^3)^(1/2)/a^(1/2))/a^(5/2)-1/4*b*x^3*(a+b/x^3)^(1/2)/
a^2+1/6*x^6*(a+b/x^3)^(1/2)/a
```

Rubi [A] time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 51, 63, 208}

$$\frac{b^2 \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{4a^{5/2}} - \frac{bx^3\sqrt{a+\frac{b}{x^3}}}{4a^2} + \frac{x^6\sqrt{a+\frac{b}{x^3}}}{6a}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[a + b/x^3],x]
```

```
[Out] -(b*Sqrt[a + b/x^3]*x^3)/(4*a^2) + (Sqrt[a + b/x^3]*x^6)/(6*a) + (b^2*ArcTa
nh[Sqrt[a + b/x^3]/Sqrt[a]])/(4*a^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{\sqrt{a+\frac{b}{x^3}}}\,dx = -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx, x, \frac{1}{x^3}\right)\right)$$
$$= \frac{\sqrt{a+\frac{b}{x^3}}\,x^6}{6a} + \frac{b\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x^3}\right)}{4a}$$
$$= -\frac{b\sqrt{a+\frac{b}{x^3}}\,x^3}{4a^2} + \frac{\sqrt{a+\frac{b}{x^3}}\,x^6}{6a} - \frac{b^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x^3}\right)}{8a^2}$$
$$= -\frac{b\sqrt{a+\frac{b}{x^3}}\,x^3}{4a^2} + \frac{\sqrt{a+\frac{b}{x^3}}\,x^6}{6a} - \frac{b\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x^3}}\right)}{4a^2}$$
$$= -\frac{b\sqrt{a+\frac{b}{x^3}}\,x^3}{4a^2} + \frac{\sqrt{a+\frac{b}{x^3}}\,x^6}{6a} + \frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{4a^{5/2}}$$

Mathematica [A] time = 0.04, size = 97, normalized size = 1.31

$$\frac{\sqrt{a}\,x^{3/2}\left(2a^2x^6 - abx^3 - 3b^2\right) + 3b^2\sqrt{ax^3+b}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{ax^3+b}}\right)}{12a^{5/2}x^{3/2}\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[a + b/x^3],x]
```

```
[Out] (Sqrt[a]*x^(3/2)*(-3*b^2 - a*b*x^3 + 2*a^2*x^6) + 3*b^2*Sqrt[b + a*x^3]*Arc
Tanh[(Sqrt[a]*x^(3/2))/Sqrt[b + a*x^3]])/(12*a^(5/2)*Sqrt[a + b/x^3]*x^(3/2
))
```

fricas [A] time = 1.14, size = 179, normalized size = 2.42

$$\left[\frac{3\sqrt{a}\,b^2\log\left(-8\,a^2x^6 - 8\,abx^3 - b^2 - 4\left(2\,ax^6 + bx^3\right)\sqrt{a}\,\sqrt{\frac{ax^3+b}{x^3}}\right) + 4\left(2\,a^2x^6 - 3\,abx^3\right)\sqrt{\frac{ax^3+b}{x^3}}}{48\,a^3}, -\frac{3\sqrt{-a}\,b^2\arctan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*sqrt(a)*b^2*log(-8*a^2*x^6 - 8*a*b*x^3 - b^2 - 4*(2*a*x^6 + b*x^3)
*sqrt(a)*sqrt((a*x^3 + b)/x^3)) + 4*(2*a^2*x^6 - 3*a*b*x^3)*sqrt((a*x^3 + b
)/x^3))/a^3, -1/24*(3*sqrt(-a)*b^2*arctan(2*sqrt(-a)*x^3*sqrt((a*x^3 + b)/x
^3)/(2*a*x^3 + b)) - 2*(2*a^2*x^6 - 3*a*b*x^3)*sqrt((a*x^3 + b)/x^3))/a^3]
```

giac [A] time = 0.23, size = 99, normalized size = 1.34

$$-\frac{1}{12}\,b^2\left(\frac{3\,\arctan\left(\frac{\sqrt{\frac{ax^3+b}{x^3}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2} - \frac{5\,a\sqrt{\frac{ax^3+b}{x^3}} - \frac{3\left(ax^3+b\right)\sqrt{\frac{ax^3+b}{x^3}}}{x^3}}{\left(a-\frac{ax^3+b}{x^3}\right)^2a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/x^3)^(1/2),x, algorithm="giac")
```

```
[Out] -1/12*b^2*(3*arctan(sqrt((a*x^3 + b)/x^3)/sqrt(-a))/(sqrt(-a)*a^2) - (5*a*s
qrt((a*x^3 + b)/x^3) - 3*(a*x^3 + b)*sqrt((a*x^3 + b)/x^3)/x^3)/((a - (a*x^
3 + b)/x^3)^2*a^2))
```

maple [C] time = 0.03, size = 3567, normalized size = 48.20

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a+b/x^3)^(1/2),x)
```

```
[Out] 1/12/((a*x^3+b)/x^3)^(1/2)/x*(a*x^3+b)/a^4*(-18*I*(-(I*3^(1/2)-3)/(I*3^(1/2
)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^
2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(
-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*El
lipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*
3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*3^(1/2)*x^2*a^
2*b^2+2*I*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b
)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^
2*x)^(1/2)*3^(1/2)*x^4*a^3-18*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b
)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3
^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^
2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/
2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1
/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*3^(1/2)*b^2-3*I*(
a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a
^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*
3^(1/2)*x*a^2*b+36*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*
x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a
*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))
/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^
(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I
*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*3^(1/2)*x*a*b^2-36*I*(-(I*3^
(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(
-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((
-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b
)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/
3))*a*x)^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+
I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*3^(1/2)*x*a*b^2+18*(-(I*3^(
1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-
a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-
2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)
^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)
)*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2
))*x^2*a^2*b^2+18*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x
)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*
x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/
(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)/(I*3^
(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3
^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*3^(1/2)*x^2*a^2
*b^2-18*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*
a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(
1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1
)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a
*x+(-a^2*b)^(1/3))*a*x)^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I
*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a^2*b^2-6*x^4*(a*x^4+b*
```

```
x)^(1/2)*a^3*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b
)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)-36*(
-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(
1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1
/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(
-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b
)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3
))^(1/2))*(-a^2*b)^(1/3)*x*a*b^2+36*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a
^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1
+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-
(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticPi((-(I*
3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),(I*3^(1/2)-1)/(I*
3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))
*(-a^2*b)^(1/3)*x*a*b^2+18*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(
1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1
/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b
)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)
-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)
-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*
b)^(2/3)*3^(1/2)*b^2+18*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))
*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/
(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/
3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I
*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(
1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*b^2-18*(-(I*3^(1/2)-3)/(I
*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/
3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^
(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(
1/2)*EllipticPi((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1
/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/
(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*b^2+9*b*x*(a*x^4+b*x)^(1/2)*a^2*((-a*x
+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*
3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2))/((a*x^3+b)*x)^(1/2)/(I
*3^(1/2)-3)/((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)
^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)
```

maxima [A] time = 2.01, size = 104, normalized size = 1.41

$$-\frac{b^2 \log\left(\frac{\sqrt{a+\frac{b}{x^3}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^3}}+\sqrt{a}}\right)}{8\,a^{\frac{5}{2}}}-\frac{3\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}b^2-5\sqrt{a+\frac{b}{x^3}}\,ab^2}{12\left(\left(a+\frac{b}{x^3}\right)^2a^2-2\left(a+\frac{b}{x^3}\right)a^3+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*b^2*log((sqrt(a + b/x^3) - sqrt(a))/(sqrt(a + b/x^3) + sqrt(a)))/a^(5/
2) - 1/12*(3*(a + b/x^3)^(3/2)*b^2 - 5*sqrt(a + b/x^3)*a*b^2)/((a + b/x^3)^
2*a^2 - 2*(a + b/x^3)*a^3 + a^4)
```

mupad [B] time = 1.74, size = 79, normalized size = 1.07

$$\frac{x^6\sqrt{a+\frac{b}{x^3}}}{6\,a}+\frac{b^2\,\ln\left(x^6\left(\sqrt{a+\frac{b}{x^3}}-\sqrt{a}\right)\left(\sqrt{a+\frac{b}{x^3}}+\sqrt{a}\right)^3\right)}{8\,a^{5/2}}-\frac{b\,x^3\sqrt{a+\frac{b}{x^3}}}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b/x^3)^(1/2),x)
```

```
[Out] (x^6*(a + b/x^3)^(1/2))/(6*a) + (b^2*log(x^6*((a + b/x^3)^(1/2) - a^(1/2))*
((a + b/x^3)^(1/2) + a^(1/2))^3))/(8*a^(5/2)) - (b*x^3*(a + b/x^3)^(1/2))/(
4*a^2)
```

sympy [**A**] time = 5.79, size = 102, normalized size = 1.38

$$\frac{x^{\frac{15}{2}}}{6\sqrt{b}\sqrt{\frac{ax^3}{b}+1}} - \frac{\sqrt{b}\,x^{\frac{9}{2}}}{12a\sqrt{\frac{ax^3}{b}+1}} - \frac{b^{\frac{3}{2}}x^{\frac{3}{2}}}{4a^2\sqrt{\frac{ax^3}{b}+1}} + \frac{b^2\,\operatorname{asinh}\left(\frac{\sqrt{a}\,x^{\frac{3}{2}}}{\sqrt{b}}\right)}{4a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a+b/x**3)**(1/2),x)
```

```
[Out] x**(15/2)/(6*sqrt(b)*sqrt(a*x**3/b + 1)) - sqrt(b)*x**(9/2)/(12*a*sqrt(a*x*
*3/b + 1)) - b**(3/2)*x**(3/2)/(4*a**2*sqrt(a*x**3/b + 1)) + b**2*asinh(sqr
t(a)*x**(3/2)/sqrt(b))/(4*a**(5/2))
```

3.2018 $\int \frac{x^2}{\sqrt{a+\frac{b}{x^3}}}\,dx$

Optimal. Leaf size=50

$$\frac{x^3\sqrt{a+\frac{b}{x^3}}}{3a} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{3a^{3/2}}$$

```
[Out] -1/3*b*arctanh((a+b/x^3)^(1/2)/a^(1/2))/a^(3/2)+1/3*x^3*(a+b/x^3)^(1/2)/a
```

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 51, 63, 208}

$$\frac{x^3\sqrt{a+\frac{b}{x^3}}}{3a} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{3a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b/x^3],x]
```

```
[Out] (Sqrt[a + b/x^3]*x^3)/(3*a) - (b*ArcTanh[Sqrt[a + b/x^3]/Sqrt[a]])/(3*a^(3/
2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x^3}}}\,dx = -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x^3}\right)\right)$$
$$= \frac{\sqrt{a+\frac{b}{x^3}}\,x^3}{3a} + \frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x^3}\right)}{6a}$$
$$= \frac{\sqrt{a+\frac{b}{x^3}}\,x^3}{3a} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x^3}}\right)}{3a}$$
$$= \frac{\sqrt{a+\frac{b}{x^3}}\,x^3}{3a} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{3a^{3/2}}$$

Mathematica [A] time = 0.03, size = 81, normalized size = 1.62

$$\frac{\sqrt{a}\,x^{3/2}\left(ax^3+b\right) - b\sqrt{ax^3+b}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{ax^3+b}}\right)}{3a^{3/2}x^{3/2}\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b/x^3],x]
```

```
[Out] (Sqrt[a]*x^(3/2)*(b + a*x^3) - b*Sqrt[b + a*x^3]*ArcTanh[(Sqrt[a]*x^(3/2))/
Sqrt[b + a*x^3]])/(3*a^(3/2)*Sqrt[a + b/x^3]*x^(3/2))
```

fricas [A] time = 1.25, size = 149, normalized size = 2.98

$$\left[\frac{4\,ax^3\sqrt{\frac{ax^3+b}{x^3}} + \sqrt{a}\,b\log\left(-8\,a^2x^6 - 8\,abx^3 - b^2 + 4\left(2\,ax^6+bx^3\right)\sqrt{a}\,\sqrt{\frac{ax^3+b}{x^3}}\right)}{12\,a^2}, \frac{2\,ax^3\sqrt{\frac{ax^3+b}{x^3}} + \sqrt{-a}\,b\arctan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*a*x^3*sqrt((a*x^3 + b)/x^3) + sqrt(a)*b*log(-8*a^2*x^6 - 8*a*b*x^3
 - b^2 + 4*(2*a*x^6 + b*x^3)*sqrt(a)*sqrt((a*x^3 + b)/x^3)))/a^2, 1/6*(2*a*
x^3*sqrt((a*x^3 + b)/x^3) + sqrt(-a)*b*arctan(2*sqrt(-a)*x^3*sqrt((a*x^3 +
b)/x^3)/(2*a*x^3 + b)))/a^2]
```

giac [A] time = 0.25, size = 67, normalized size = 1.34

$$\frac{1}{3}\,b\left(\frac{\arctan\left(\frac{\sqrt{\frac{ax^3+b}{x^3}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a} - \frac{\sqrt{\frac{ax^3+b}{x^3}}}{\left(a-\frac{ax^3+b}{x^3}\right)a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*b*(arctan(sqrt((a*x^3 + b)/x^3)/sqrt(-a))/(sqrt(-a)*a) - sqrt((a*x^3 +
b)/x^3)/((a - (a*x^3 + b)/x^3)*a))
```

maple [C] time = 0.01, size = 3347, normalized size = 66.94

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b/x^3)^(1/2),x)
```

```
[Out] 1/3/((a*x^3+b)/x^3)^(1/2)/x*(a*x^3+b)/a^3*(6*I*3^(1/2)*(-(I*3^(1/2)-3)/(I*3
^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)
+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1
/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/
2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)
,((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a^2*b
-6*I*3^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)
*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2
*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1
/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)/(I*3^(1/2)-1
)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+
3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a^2*b-12*I*(-a^2*b
)^(1/3)*3^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1
/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-
a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3
^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)
-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1
/2))/(I*3^(1/2)-3))^(1/2))*x*a*b+12*I*(-a^2*b)^(1/3)*3^(1/2)*(-(I*3^(1/2)-3
)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)
^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+
I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)
))^(1/2)*EllipticPi((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x
)^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/
2))/(I*3^(1/2)-3))^(1/2))*x*a*b+6*I*(-a^2*b)^(2/3)*3^(1/2)*(-(I*3^(1/2)-3)/
(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(
1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*
3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))
^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(
1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*b-6*(
-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(
1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1
/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(
-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b
)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3
))^(1/2))*x^2*a^2*b-6*I*(-a^2*b)^(2/3)*3^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1
)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b
)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^
2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*Ellip
ticPi((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),(I*3^(
1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)
-3))^(1/2))*b+6*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1
/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-
a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3
^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)/(I*3^(1/2
)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/
2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a^2*b+12*(-a^2*
b)^(1/3)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2
*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^
```

```
(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-
1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a
*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I
*3^(1/2)-3))^(1/2))*x*a*b-12*(-a^2*b)^(1/3)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(
-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(
1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b
)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*Elliptic
Pi((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),(I*3^(1/2
)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3)
)^(1/2))*x*a*b-6*(-a^2*b)^(2/3)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b
)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3
^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^
2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/
2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1
/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*b+6*(-a^2*b)^(2/3)*(-(I*3^(1/2)-
3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b
)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x
+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3
)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*
x)^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1
/2))/(I*3^(1/2)-3))^(1/2))*b+I*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^
2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))
/a^2*x)^(1/2)*(a*x^4+b*x)^(1/2)*3^(1/2)*x*a^2-3*x*(a*x^4+b*x)^(1/2)*a^2*((-
a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x
+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2))/((a*x^3+b)*x)^(1/2)
/(I*3^(1/2)-3)/((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2
*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)
```

maxima [A] time = 1.93, size = 68, normalized size = 1.36

$$\frac{\sqrt{a+\frac{b}{x^3}}\,b}{3\left(\left(a+\frac{b}{x^3}\right)a-a^2\right)}+\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x^3}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^3}}+\sqrt{a}}\right)}{6\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(a + b/x^3)*b/((a + b/x^3)*a - a^2) + 1/6*b*log((sqrt(a + b/x^3) -
sqrt(a))/(sqrt(a + b/x^3) + sqrt(a)))/a^(3/2)
```

mupad [B] time = 1.65, size = 59, normalized size = 1.18

$$\frac{b\,\ln\left(x^6\left(\sqrt{a+\frac{b}{x^3}}-\sqrt{a}\right)^3\left(\sqrt{a+\frac{b}{x^3}}+\sqrt{a}\right)\right)}{6\,a^{3/2}}+\frac{x^3\sqrt{a+\frac{b}{x^3}}}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b/x^3)^(1/2),x)
```

```
[Out] (b*log(x^6*((a + b/x^3)^(1/2) - a^(1/2))^3*((a + b/x^3)^(1/2) + a^(1/2))))/
(6*a^(3/2)) + (x^3*(a + b/x^3)^(1/2))/(3*a)
```

sympy [A] time = 2.63, size = 49, normalized size = 0.98

$$\frac{\sqrt{b}\,x^{\frac{3}{2}}\sqrt{\frac{ax^3}{b}+1}}{3a}-\frac{b\,\operatorname{asinh}\left(\frac{\sqrt{a}\,x^{\frac{3}{2}}}{\sqrt{b}}\right)}{3a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b/x**3)**(1/2),x)
```

```
[Out] sqrt(b)*x**(3/2)*sqrt(a*x**3/b + 1)/(3*a) - b*asinh(sqrt(a)*x**(3/2)/sqrt(b
))/(3*a**(3/2))
```

3.2019 $\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x}\,dx$

Optimal. Leaf size=27

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

```
[Out] 2/3*arctanh((a+b/x^3)^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {266, 63, 208}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x^3]*x),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b/x^3]/Sqrt[a]])/(3*Sqrt[a])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x}\,dx = -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x^3}\right)\right)$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x^3}}\right)}{3b}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Mathematica [B] time = 0.02, size = 59, normalized size = 2.19

$$\frac{2\sqrt{ax^3+b}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{ax^3+b}}\right)}{3\sqrt{a}\,x^{3/2}\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x^3]*x),x]
```

```
[Out] (2*Sqrt[b + a*x^3]*ArcTanh[(Sqrt[a]*x^(3/2))/Sqrt[b + a*x^3]])/(3*Sqrt[a]*S
qrt[a + b/x^3]*x^(3/2))
```

fricas [B] time = 1.18, size = 102, normalized size = 3.78

$$\left[\frac{\log\left(-8\,a^2x^6 - 8\,abx^3 - b^2 - 4\left(2\,ax^6 + bx^3\right)\sqrt{a}\,\sqrt{\frac{ax^3+b}{x^3}}\right)}{6\,\sqrt{a}}, -\frac{\sqrt{-a}\,\arctan\left(\frac{2\,\sqrt{-a}\,x^3\sqrt{\frac{ax^3+b}{x^3}}}{2\,ax^3+b}\right)}{3\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*log(-8*a^2*x^6 - 8*a*b*x^3 - b^2 - 4*(2*a*x^6 + b*x^3)*sqrt(a)*sqrt((a
*x^3 + b)/x^3))/sqrt(a), -1/3*sqrt(-a)*arctan(2*sqrt(-a)*x^3*sqrt((a*x^3 +
b)/x^3)/(2*a*x^3 + b))/a]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/x^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a + b/x^3)*x), x)
```

maple [C] time = 0.01, size = 480, normalized size = 17.78

$$-\frac{4\left(a\,x^3+b\right)\left(i\sqrt{3}-1\right)\sqrt{-\frac{\left(i\sqrt{3}-3\right)ax}{\left(i\sqrt{3}-1\right)\left(-ax+\left(-a^2b\right)^{\frac{1}{3}}\right)}}\left(-ax+\left(-a^2b\right)^{\frac{1}{3}}\right)^2\sqrt{\frac{2ax+i\sqrt{3}\left(-a^2b\right)^{\frac{1}{3}}+\left(-a^2b\right)^{\frac{1}{3}}}{\left(1+i\sqrt{3}\right)\left(-ax+\left(-a^2b\right)^{\frac{1}{3}}\right)}}\sqrt{\frac{-2ax+i\sqrt{3}\left(-a^2b\right)^{\frac{1}{3}}}{\left(i\sqrt{3}-1\right)\left(-ax+\left(-\right.\right.}}}{\sqrt{\frac{a\,x^3+b}{x^3}}\sqrt{\left(a\,x^3+b\right)x}\left(i\sqrt{3}-3\right)\sqrt{\frac{\left(-ax-\right.}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b/x^3)^(1/2),x)
```

```
[Out] -4/((a*x^3+b)/x^3)^(1/2)/x*(a*x^3+b)*(I*3^(1/2)-1)*(-(I*3^(1/2)-3)/(I*3^(1/
2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*(-a*x+(-a^2*b)^(1/3))^2*((2*a*x+I*3^
(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(
1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+
(-a^2*b)^(1/3)))^(1/2)/a^2*(EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(
-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(
1/2)-3))^(1/2))-EllipticPi((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/
3))*a*x)^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+
I*3^(1/2))/(I*3^(1/2)-3))^(1/2)))/((a*x^3+b)*x)^(1/2)/(I*3^(1/2)-3)/((-a*x+
(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3
^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)
```

maxima [A] time = 1.91, size = 37, normalized size = 1.37

$$-\frac{\log\left(\frac{\sqrt{a+\frac{b}{x^3}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^3}}+\sqrt{a}}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*log((sqrt(a + b/x^3) - sqrt(a))/(sqrt(a + b/x^3) + sqrt(a)))/sqrt(a)
```

mupad [B] time = 1.31, size = 19, normalized size = 0.70

$$\frac{2\,\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b/x^3)^(1/2)),x)
```

```
[Out] (2*atanh((a + b/x^3)^(1/2)/a^(1/2)))/(3*a^(1/2))
```

sympy [A] time = 1.37, size = 24, normalized size = 0.89

$$\frac{2\,\operatorname{asinh}\left(\frac{\sqrt{a}\,x^{\frac{3}{2}}}{\sqrt{b}}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/x**3)**(1/2),x)
```

```
[Out] 2*asinh(sqrt(a)*x**(3/2)/sqrt(b))/(3*sqrt(a))
```

3.2020 $\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^4}\,dx$

Optimal. Leaf size=18

$$-\frac{2\sqrt{a+\frac{b}{x^3}}}{3b}$$

```
[Out] -2/3*(a+b/x^3)^(1/2)/b
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$-\frac{2\sqrt{a+\frac{b}{x^3}}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x^3]*x^4),x]
```

```
[Out] (-2*Sqrt[a + b/x^3])/(3*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^4}\,dx = -\frac{2\sqrt{a+\frac{b}{x^3}}}{3b}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$-\frac{2\sqrt{a+\frac{b}{x^3}}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x^3]*x^4),x]
```

```
[Out] (-2*Sqrt[a + b/x^3])/(3*b)
```

fricas [A] time = 0.83, size = 18, normalized size = 1.00

$$-\frac{2\sqrt{\frac{ax^3+b}{x^3}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b/x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt((a*x^3 + b)/x^3)/b
```

giac [A] time = 0.15, size = 18, normalized size = 1.00

$$-\frac{2\sqrt{\frac{ax^3+b}{x^3}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b/x^3)^(1/2),x, algorithm="giac")
[Out] -2/3*sqrt((a*x^3 + b)/x^3)/b
```

maple [A] time = 0.01, size = 29, normalized size = 1.61

$$-\frac{2\left(a\,x^3+b\right)}{3\sqrt{\frac{a\,x^3+b}{x^3}}\;b\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a+b/x^3)^(1/2),x)
[Out] -2/3*(a*x^3+b)/x^3/b/((a*x^3+b)/x^3)^(1/2)
```

maxima [A] time = 0.79, size = 14, normalized size = 0.78

$$-\frac{2\sqrt{a+\frac{b}{x^3}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b/x^3)^(1/2),x, algorithm="maxima")
[Out] -2/3*sqrt(a + b/x^3)/b
```

mupad [B] time = 1.18, size = 14, normalized size = 0.78

$$-\frac{2\sqrt{a+\frac{b}{x^3}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b/x^3)^(1/2)),x)
[Out] -(2*(a + b/x^3)^(1/2))/(3*b)
```

sympy [A] time = 2.04, size = 29, normalized size = 1.61

$$\begin{cases} -\frac{2\sqrt{a+\frac{b}{x^3}}}{3b} & \text{for } b \neq 0 \\ -\frac{1}{3\sqrt{a}\,x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a+b/x**3)**(1/2),x)
[Out] Piecewise((-2*sqrt(a + b/x**3)/(3*b), Ne(b, 0)), (-1/(3*sqrt(a)*x**3), True
))
```

3.2021 $\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^7}\,dx$

Optimal. Leaf size=38

$$\frac{2a\sqrt{a+\frac{b}{x^3}}}{3b^2}-\frac{2\left(a+\frac{b}{x^3}\right)^{3/2}}{9b^2}$$

```
[Out] -2/9*(a+b/x^3)^(3/2)/b^2+2/3*a*(a+b/x^3)^(1/2)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{2a\sqrt{a+\frac{b}{x^3}}}{3b^2}-\frac{2\left(a+\frac{b}{x^3}\right)^{3/2}}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x^3]*x^7),x]
```

```
[Out] (2*a*Sqrt[a + b/x^3])/(3*b^2) - (2*(a + b/x^3)^(3/2))/(9*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^7}\,dx &= -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}}\,dx, x, \frac{1}{x^3}\right)\right)\\ &= -\left(\frac{1}{3}\,\text{Subst}\left(\int\left(-\frac{a}{b\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b}\right)dx, x, \frac{1}{x^3}\right)\right)\\ &= \frac{2a\sqrt{a+\frac{b}{x^3}}}{3b^2}-\frac{2\left(a+\frac{b}{x^3}\right)^{3/2}}{9b^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 0.82

$$\frac{2\sqrt{a+\frac{b}{x^3}}\left(2ax^3-b\right)}{9b^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x^3]*x^7),x]
```

```
[Out] (2*Sqrt[a + b/x^3]*(-b + 2*a*x^3))/(9*b^2*x^3)
```

fricas [A] time = 0.81, size = 31, normalized size = 0.82

$$\frac{2\left(2\,ax^3-b\right)\sqrt{\frac{ax^3+b}{x^3}}}{9\,b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(a+b/x^3)^(1/2),x, algorithm="fricas")
[Out] 2/9*(2*a*x^3 - b)*sqrt((a*x^3 + b)/x^3)/(b^2*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(a+b/x^3)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(a + b/x^3)*x^7), x)
```

maple [A] time = 0.00, size = 39, normalized size = 1.03

$$\frac{2\left(a\,x^3+b\right)\left(2a\,x^3-b\right)}{9\sqrt{\frac{a\,x^3+b}{x^3}}\,b^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(a+b/x^3)^(1/2),x)
[Out] 2/9*(a*x^3+b)*(2*a*x^3-b)/x^6/b^2/((a*x^3+b)/x^3)^(1/2)
```

maxima [A] time = 0.84, size = 30, normalized size = 0.79

$$-\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}{9\,b^2}+\frac{2\sqrt{a+\frac{b}{x^3}}\,a}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(a+b/x^3)^(1/2),x, algorithm="maxima")
[Out] -2/9*(a + b/x^3)^(3/2)/b^2 + 2/3*sqrt(a + b/x^3)*a/b^2
```

mupad [B] time = 1.25, size = 25, normalized size = 0.66

$$-\frac{2\sqrt{a+\frac{b}{x^3}}\,\left(b-2\,a\,x^3\right)}{9\,b^2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b/x^3)^(1/2)),x)
[Out] -(2*(a + b/x^3)^(1/2)*(b - 2*a*x^3))/(9*b^2*x^3)
```

sympy [B] time = 1.96, size = 255, normalized size = 6.71

$$\frac{4a^{\frac{7}{2}}b^{\frac{3}{2}}x^6\sqrt{\frac{ax^3}{b}+1}}{9a^{\frac{5}{2}}b^3x^{\frac{15}{2}}+9a^{\frac{3}{2}}b^4x^{\frac{9}{2}}}+\frac{2a^{\frac{5}{2}}b^{\frac{5}{2}}x^3\sqrt{\frac{ax^3}{b}+1}}{9a^{\frac{5}{2}}b^3x^{\frac{15}{2}}+9a^{\frac{3}{2}}b^4x^{\frac{9}{2}}}-\frac{2a^{\frac{3}{2}}b^{\frac{7}{2}}\sqrt{\frac{ax^3}{b}+1}}{9a^{\frac{5}{2}}b^3x^{\frac{15}{2}}+9a^{\frac{3}{2}}b^4x^{\frac{9}{2}}}-\frac{4a^4bx^{\frac{15}{2}}}{9a^{\frac{5}{2}}b^3x^{\frac{15}{2}}+9a^{\frac{3}{2}}b^4x^{\frac{9}{2}}}-\frac{4a^3b^2x^{\frac{9}{2}}}{9a^{\frac{5}{2}}b^3x^{\frac{15}{2}}+9a^{\frac{3}{2}}b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(a+b/x**3)**(1/2),x)
```

```
[Out] 4*a**(7/2)*b**(3/2)*x**6*sqrt(a*x**3/b + 1)/(9*a**(5/2)*b**3*x**(15/2) + 9*
a**(3/2)*b**4*x**(9/2)) + 2*a**(5/2)*b**(5/2)*x**3*sqrt(a*x**3/b + 1)/(9*a*
*(5/2)*b**3*x**(15/2) + 9*a**(3/2)*b**4*x**(9/2)) - 2*a**(3/2)*b**(7/2)*sqr
t(a*x**3/b + 1)/(9*a**(5/2)*b**3*x**(15/2) + 9*a**(3/2)*b**4*x**(9/2)) - 4*
a**4*b*x**(15/2)/(9*a**(5/2)*b**3*x**(15/2) + 9*a**(3/2)*b**4*x**(9/2)) - 4
*a**3*b**2*x**(9/2)/(9*a**(5/2)*b**3*x**(15/2) + 9*a**(3/2)*b**4*x**(9/2))
```

3.2022 $\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\, x^{10}}\, dx$

Optimal. Leaf size=59

$$-\frac{2a^2\sqrt{a+\frac{b}{x^3}}}{3b^3}-\frac{2\left(a+\frac{b}{x^3}\right)^{5/2}}{15b^3}+\frac{4a\left(a+\frac{b}{x^3}\right)^{3/2}}{9b^3}$$

```
[Out] 4/9*a*(a+b/x^3)^(3/2)/b^3-2/15*(a+b/x^3)^(5/2)/b^3-2/3*a^2*(a+b/x^3)^(1/2)/
b^3
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{2a^2\sqrt{a+\frac{b}{x^3}}}{3b^3}-\frac{2\left(a+\frac{b}{x^3}\right)^{5/2}}{15b^3}+\frac{4a\left(a+\frac{b}{x^3}\right)^{3/2}}{9b^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x^3]*x^10),x]
```

```
[Out] (-2*a^2*Sqrt[a + b/x^3])/(3*b^3) + (4*a*(a + b/x^3)^(3/2))/(9*b^3) - (2*(a
+ b/x^3)^(5/2))/(15*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\, x^{10}}\, dx &= -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx}}\, dx, x, \frac{1}{x^3}\right)\right)\\ &= -\left(\frac{1}{3}\,\text{Subst}\left(\int \left(\frac{a^2}{b^2\sqrt{a+bx}}-\frac{2a\sqrt{a+bx}}{b^2}+\frac{(a+bx)^{3/2}}{b^2}\right) dx, x, \frac{1}{x^3}\right)\right)\\ &= -\frac{2a^2\sqrt{a+\frac{b}{x^3}}}{3b^3}+\frac{4a\left(a+\frac{b}{x^3}\right)^{3/2}}{9b^3}-\frac{2\left(a+\frac{b}{x^3}\right)^{5/2}}{15b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.71

$$-\frac{2\sqrt{a+\frac{b}{x^3}}\left(8a^2x^6-4abx^3+3b^2\right)}{45b^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x^3]*x^10),x]
[Out] (-2*Sqrt[a + b/x^3]*(3*b^2 - 4*a*b*x^3 + 8*a^2*x^6))/(45*b^3*x^6)
```

fricas [A] time = 0.80, size = 42, normalized size = 0.71

$$-\frac{2\left(8\,a^2x^6-4\,abx^3+3\,b^2\right)\sqrt{\frac{ax^3+b}{x^3}}}{45\,b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(a+b/x^3)^(1/2),x, algorithm="fricas")
[Out] -2/45*(8*a^2*x^6 - 4*a*b*x^3 + 3*b^2)*sqrt((a*x^3 + b)/x^3)/(b^3*x^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(a+b/x^3)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(a + b/x^3)*x^10), x)
```

maple [A] time = 0.00, size = 50, normalized size = 0.85

$$-\frac{2\left(a\,x^3+b\right)\left(8a^2x^6-4ab\,x^3+3b^2\right)}{45\sqrt{\frac{a\,x^3+b}{x^3}}\,b^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(a+b/x^3)^(1/2),x)
[Out] -2/45*(a*x^3+b)*(8*a^2*x^6-4*a*b*x^3+3*b^2)/x^9/b^3/((a*x^3+b)/x^3)^(1/2)
```

maxima [A] time = 0.88, size = 47, normalized size = 0.80

$$-\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{5}{2}}}{15\,b^3}+\frac{4\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}a}{9\,b^3}-\frac{2\sqrt{a+\frac{b}{x^3}}\,a^2}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(a+b/x^3)^(1/2),x, algorithm="maxima")
[Out] -2/15*(a + b/x^3)^(5/2)/b^3 + 4/9*(a + b/x^3)^(3/2)*a/b^3 - 2/3*sqrt(a + b/
x^3)*a^2/b^3
```

mupad [B] time = 1.35, size = 56, normalized size = 0.95

$$-\frac{6\,b^2\sqrt{a+\frac{b}{x^3}}+16\,a^2\,x^6\sqrt{a+\frac{b}{x^3}}-8\,a\,b\,x^3\sqrt{a+\frac{b}{x^3}}}{45\,b^3\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^10*(a + b/x^3)^(1/2)),x)
```

```
[Out] -(6*b^2*(a + b/x^3)^(1/2) + 16*a^2*x^6*(a + b/x^3)^(1/2) - 8*a*b*x^3*(a + b
/x^3)^(1/2))/(45*b^3*x^6)
```

sympy [B] time = 3.19, size = 824, normalized size = 13.97

$$-\frac{16a^{\frac{15}{2}}b^{\frac{9}{2}}x^{15}\sqrt{\frac{ax^3}{b}+1}}{45a^{\frac{11}{2}}b^7x^{\frac{33}{2}}+135a^{\frac{9}{2}}b^8x^{\frac{27}{2}}+135a^{\frac{7}{2}}b^9x^{\frac{21}{2}}+45a^{\frac{5}{2}}b^{10}x^{\frac{15}{2}}}-\frac{40a^{\frac{13}{2}}b^{\frac{11}{2}}x^{12}\sqrt{\frac{ax^3}{b}+1}}{45a^{\frac{11}{2}}b^7x^{\frac{33}{2}}+135a^{\frac{9}{2}}b^8x^{\frac{27}{2}}+135a^{\frac{7}{2}}b^9x^{\frac{21}{2}}+45a^{\frac{5}{2}}b^{10}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(a+b/x**3)**(1/2),x)
```

```
[Out] -16*a**(15/2)*b**(9/2)*x**15*sqrt(a*x**3/b + 1)/(45*a**(11/2)*b**7*x**(33/2
) + 135*a**(9/2)*b**8*x**(27/2) + 135*a**(7/2)*b**9*x**(21/2) + 45*a**(5/2)
*b**10*x**(15/2)) - 40*a**(13/2)*b**(11/2)*x**12*sqrt(a*x**3/b + 1)/(45*a**
(11/2)*b**7*x**(33/2) + 135*a**(9/2)*b**8*x**(27/2) + 135*a**(7/2)*b**9*x**
(21/2) + 45*a**(5/2)*b**10*x**(15/2)) - 30*a**(11/2)*b**(13/2)*x**9*sqrt(a*
x**3/b + 1)/(45*a**(11/2)*b**7*x**(33/2) + 135*a**(9/2)*b**8*x**(27/2) + 13
5*a**(7/2)*b**9*x**(21/2) + 45*a**(5/2)*b**10*x**(15/2)) - 10*a**(9/2)*b**(
15/2)*x**6*sqrt(a*x**3/b + 1)/(45*a**(11/2)*b**7*x**(33/2) + 135*a**(9/2)*b
**8*x**(27/2) + 135*a**(7/2)*b**9*x**(21/2) + 45*a**(5/2)*b**10*x**(15/2))
 - 10*a**(7/2)*b**(17/2)*x**3*sqrt(a*x**3/b + 1)/(45*a**(11/2)*b**7*x**(33/2
) + 135*a**(9/2)*b**8*x**(27/2) + 135*a**(7/2)*b**9*x**(21/2) + 45*a**(5/2)
*b**10*x**(15/2)) - 6*a**(5/2)*b**(19/2)*sqrt(a*x**3/b + 1)/(45*a**(11/2)*b
**7*x**(33/2) + 135*a**(9/2)*b**8*x**(27/2) + 135*a**(7/2)*b**9*x**(21/2) +
 45*a**(5/2)*b**10*x**(15/2)) + 16*a**8*b**4*x**(33/2)/(45*a**(11/2)*b**7*x
**(33/2) + 135*a**(9/2)*b**8*x**(27/2) + 135*a**(7/2)*b**9*x**(21/2) + 45*a
**(5/2)*b**10*x**(15/2)) + 48*a**7*b**5*x**(27/2)/(45*a**(11/2)*b**7*x**(33
/2) + 135*a**(9/2)*b**8*x**(27/2) + 135*a**(7/2)*b**9*x**(21/2) + 45*a**(5/
2)*b**10*x**(15/2)) + 48*a**6*b**6*x**(21/2)/(45*a**(11/2)*b**7*x**(33/2) +
 135*a**(9/2)*b**8*x**(27/2) + 135*a**(7/2)*b**9*x**(21/2) + 45*a**(5/2)*b*
*10*x**(15/2)) + 16*a**5*b**7*x**(15/2)/(45*a**(11/2)*b**7*x**(33/2) + 135*
a**(9/2)*b**8*x**(27/2) + 135*a**(7/2)*b**9*x**(21/2) + 45*a**(5/2)*b**10*x
**(15/2))
```

3.2023 $\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\, x^{13}}\, dx$

Optimal. Leaf size=80

$$\frac{2a^3\sqrt{a+\frac{b}{x^3}}}{3b^4} - \frac{2a^2\left(a+\frac{b}{x^3}\right)^{3/2}}{3b^4} - \frac{2\left(a+\frac{b}{x^3}\right)^{7/2}}{21b^4} + \frac{2a\left(a+\frac{b}{x^3}\right)^{5/2}}{5b^4}$$

```
[Out] -2/3*a^2*(a+b/x^3)^(3/2)/b^4+2/5*a*(a+b/x^3)^(5/2)/b^4-2/21*(a+b/x^3)^(7/2)
/b^4+2/3*a^3*(a+b/x^3)^(1/2)/b^4
```

Rubi [A] time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{2a^2\left(a+\frac{b}{x^3}\right)^{3/2}}{3b^4} + \frac{2a^3\sqrt{a+\frac{b}{x^3}}}{3b^4} - \frac{2\left(a+\frac{b}{x^3}\right)^{7/2}}{21b^4} + \frac{2a\left(a+\frac{b}{x^3}\right)^{5/2}}{5b^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x^3]*x^13),x]
```

```
[Out] (2*a^3*Sqrt[a + b/x^3])/(3*b^4) - (2*a^2*(a + b/x^3)^(3/2))/(3*b^4) + (2*a*
(a + b/x^3)^(5/2))/(5*b^4) - (2*(a + b/x^3)^(7/2))/(21*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\, x^{13}}\, dx &= -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{x^3}{\sqrt{a+bx}}\, dx, x, \frac{1}{x^3}\right)\right) \\
&= -\left(\frac{1}{3}\,\text{Subst}\left(\int \left(-\frac{a^3}{b^3\sqrt{a+bx}} + \frac{3a^2\sqrt{a+bx}}{b^3} - \frac{3a(a+bx)^{3/2}}{b^3} + \frac{(a+bx)^{5/2}}{b^3}\right) dx, x, \frac{1}{x^3}\right)\right) \\
&= \frac{2a^3\sqrt{a+\frac{b}{x^3}}}{3b^4} - \frac{2a^2\left(a+\frac{b}{x^3}\right)^{3/2}}{3b^4} + \frac{2a\left(a+\frac{b}{x^3}\right)^{5/2}}{5b^4} - \frac{2\left(a+\frac{b}{x^3}\right)^{7/2}}{21b^4}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.66

$$\frac{2\sqrt{a+\frac{b}{x^3}}\left(16a^3x^9 - 8a^2bx^6 + 6ab^2x^3 - 5b^3\right)}{105b^4x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x^3]*x^13),x]
```

```
[Out] (2*Sqrt[a + b/x^3]*(-5*b^3 + 6*a*b^2*x^3 - 8*a^2*b*x^6 + 16*a^3*x^9))/(105*
b^4*x^9)
```

fricas [A] time = 0.92, size = 53, normalized size = 0.66

$$\frac{2\left(16\,a^3x^9 - 8\,a^2bx^6 + 6\,ab^2x^3 - 5\,b^3\right)\sqrt{\frac{ax^3+b}{x^3}}}{105\,b^4x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(a+b/x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(16*a^3*x^9 - 8*a^2*b*x^6 + 6*a*b^2*x^3 - 5*b^3)*sqrt((a*x^3 + b)/x^3
)/(b^4*x^9)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + \frac{b}{x^3}}\,x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(a+b/x^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a + b/x^3)*x^13), x)
```

maple [A] time = 0.01, size = 61, normalized size = 0.76

$$\frac{2\left(a\,x^3+b\right)\left(16a^3x^9 - 8a^2b\,x^6 + 6a\,b^2x^3 - 5b^3\right)}{105\sqrt{\frac{a\,x^3+b}{x^3}}\;b^4x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^13/(a+b/x^3)^(1/2),x)
```

```
[Out] 2/105*(a*x^3+b)*(16*a^3*x^9-8*a^2*b*x^6+6*a*b^2*x^3-5*b^3)/x^12/b^4/((a*x^3
+b)/x^3)^(1/2)
```

maxima [A] time = 0.82, size = 64, normalized size = 0.80

$$-\frac{2\left(a + \frac{b}{x^3}\right)^{\frac{7}{2}}}{21\,b^4} + \frac{2\left(a + \frac{b}{x^3}\right)^{\frac{5}{2}}a}{5\,b^4} - \frac{2\left(a + \frac{b}{x^3}\right)^{\frac{3}{2}}a^2}{3\,b^4} + \frac{2\sqrt{a + \frac{b}{x^3}}\,a^3}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(a+b/x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/21*(a + b/x^3)^(7/2)/b^4 + 2/5*(a + b/x^3)^(5/2)*a/b^4 - 2/3*(a + b/x^3)
^(3/2)*a^2/b^4 + 2/3*sqrt(a + b/x^3)*a^3/b^4
```

mupad [B] time = 1.44, size = 73, normalized size = 0.91

$$\frac{32\,a^3\sqrt{a + \frac{b}{x^3}}}{105\,b^4} - \frac{2\sqrt{a + \frac{b}{x^3}}}{21\,b\,x^9} + \frac{4\,a\sqrt{a + \frac{b}{x^3}}}{35\,b^2\,x^6} - \frac{16\,a^2\sqrt{a + \frac{b}{x^3}}}{105\,b^3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^13*(a + b/x^3)^(1/2)),x)
```

```
[Out] (32*a^3*(a + b/x^3)^(1/2))/(105*b^4) - (2*(a + b/x^3)^(1/2))/(21*b*x^9) + (
4*a*(a + b/x^3)^(1/2))/(35*b^2*x^6) - (16*a^2*(a + b/x^3)^(1/2))/(105*b^3*x
^3)
```

sympy [**B**] time = 4.74, size = 2183, normalized size = 27.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**13/(a+b/x**3)**(1/2),x)
```

```
[Out] 32*a**(25/2)*b**(23/2)*x**27*sqrt(a*x**3/b + 1)/(105*a**(19/2)*b**15*x**(57
/2) + 630*a**(17/2)*b**16*x**(51/2) + 1575*a**(15/2)*b**17*x**(45/2) + 2100
*a**(13/2)*b**18*x**(39/2) + 1575*a**(11/2)*b**19*x**(33/2) + 630*a**(9/2)*
b**20*x**(27/2) + 105*a**(7/2)*b**21*x**(21/2)) + 176*a**(23/2)*b**(25/2)*x
**24*sqrt(a*x**3/b + 1)/(105*a**(19/2)*b**15*x**(57/2) + 630*a**(17/2)*b**1
6*x**(51/2) + 1575*a**(15/2)*b**17*x**(45/2) + 2100*a**(13/2)*b**18*x**(39/
2) + 1575*a**(11/2)*b**19*x**(33/2) + 630*a**(9/2)*b**20*x**(27/2) + 105*a*
*(7/2)*b**21*x**(21/2)) + 396*a**(21/2)*b**(27/2)*x**21*sqrt(a*x**3/b + 1)/
(105*a**(19/2)*b**15*x**(57/2) + 630*a**(17/2)*b**16*x**(51/2) + 1575*a**(1
5/2)*b**17*x**(45/2) + 2100*a**(13/2)*b**18*x**(39/2) + 1575*a**(11/2)*b**1
9*x**(33/2) + 630*a**(9/2)*b**20*x**(27/2) + 105*a**(7/2)*b**21*x**(21/2))
+ 462*a**(19/2)*b**(29/2)*x**18*sqrt(a*x**3/b + 1)/(105*a**(19/2)*b**15*x**
(57/2) + 630*a**(17/2)*b**16*x**(51/2) + 1575*a**(15/2)*b**17*x**(45/2) + 2
100*a**(13/2)*b**18*x**(39/2) + 1575*a**(11/2)*b**19*x**(33/2) + 630*a**(9/
2)*b**20*x**(27/2) + 105*a**(7/2)*b**21*x**(21/2)) + 280*a**(17/2)*b**(31/2
)*x**15*sqrt(a*x**3/b + 1)/(105*a**(19/2)*b**15*x**(57/2) + 630*a**(17/2)*b
**16*x**(51/2) + 1575*a**(15/2)*b**17*x**(45/2) + 2100*a**(13/2)*b**18*x**(
39/2) + 1575*a**(11/2)*b**19*x**(33/2) + 630*a**(9/2)*b**20*x**(27/2) + 105
*a**(7/2)*b**21*x**(21/2)) + 42*a**(15/2)*b**(33/2)*x**12*sqrt(a*x**3/b + 1
)/(105*a**(19/2)*b**15*x**(57/2) + 630*a**(17/2)*b**16*x**(51/2) + 1575*a**
(15/2)*b**17*x**(45/2) + 2100*a**(13/2)*b**18*x**(39/2) + 1575*a**(11/2)*b*
*19*x**(33/2) + 630*a**(9/2)*b**20*x**(27/2) + 105*a**(7/2)*b**21*x**(21/2)
) - 84*a**(13/2)*b**(35/2)*x**9*sqrt(a*x**3/b + 1)/(105*a**(19/2)*b**15*x**
(57/2) + 630*a**(17/2)*b**16*x**(51/2) + 1575*a**(15/2)*b**17*x**(45/2) + 2
100*a**(13/2)*b**18*x**(39/2) + 1575*a**(11/2)*b**19*x**(33/2) + 630*a**(9/
2)*b**20*x**(27/2) + 105*a**(7/2)*b**21*x**(21/2)) - 94*a**(11/2)*b**(37/2)
*x**6*sqrt(a*x**3/b + 1)/(105*a**(19/2)*b**15*x**(57/2) + 630*a**(17/2)*b**
16*x**(51/2) + 1575*a**(15/2)*b**17*x**(45/2) + 2100*a**(13/2)*b**18*x**(39
/2) + 1575*a**(11/2)*b**19*x**(33/2) + 630*a**(9/2)*b**20*x**(27/2) + 105*a
**(7/2)*b**21*x**(21/2)) - 48*a**(9/2)*b**(39/2)*x**3*sqrt(a*x**3/b + 1)/(1
05*a**(19/2)*b**15*x**(57/2) + 630*a**(17/2)*b**16*x**(51/2) + 1575*a**(15/
2)*b**17*x**(45/2) + 2100*a**(13/2)*b**18*x**(39/2) + 1575*a**(11/2)*b**19*
x**(33/2) + 630*a**(9/2)*b**20*x**(27/2) + 105*a**(7/2)*b**21*x**(21/2)) -
10*a**(7/2)*b**(41/2)*sqrt(a*x**3/b + 1)/(105*a**(19/2)*b**15*x**(57/2) + 6
30*a**(17/2)*b**16*x**(51/2) + 1575*a**(15/2)*b**17*x**(45/2) + 2100*a**(13
/2)*b**18*x**(39/2) + 1575*a**(11/2)*b**19*x**(33/2) + 630*a**(9/2)*b**20*x
**(27/2) + 105*a**(7/2)*b**21*x**(21/2)) - 32*a**13*b**11*x**(57/2)/(105*a*
*(19/2)*b**15*x**(57/2) + 630*a**(17/2)*b**16*x**(51/2) + 1575*a**(15/2)*b*
*17*x**(45/2) + 2100*a**(13/2)*b**18*x**(39/2) + 1575*a**(11/2)*b**19*x**(3
3/2) + 630*a**(9/2)*b**20*x**(27/2) + 105*a**(7/2)*b**21*x**(21/2)) - 192*a
**12*b**12*x**(51/2)/(105*a**(19/2)*b**15*x**(57/2) + 630*a**(17/2)*b**16*x
**(51/2) + 1575*a**(15/2)*b**17*x**(45/2) + 2100*a**(13/2)*b**18*x**(39/2)
+ 1575*a**(11/2)*b**19*x**(33/2) + 630*a**(9/2)*b**20*x**(27/2) + 105*a**(7
/2)*b**21*x**(21/2)) - 480*a**11*b**13*x**(45/2)/(105*a**(19/2)*b**15*x**(5
7/2) + 630*a**(17/2)*b**16*x**(51/2) + 1575*a**(15/2)*b**17*x**(45/2) + 210
0*a**(13/2)*b**18*x**(39/2) + 1575*a**(11/2)*b**19*x**(33/2) + 630*a**(9/2)
*b**20*x**(27/2) + 105*a**(7/2)*b**21*x**(21/2)) - 640*a**10*b**14*x**(39/2
```

```
)/(105*a**(19/2)*b**15*x**(57/2) + 630*a**(17/2)*b**16*x**(51/2) + 1575*a**
(15/2)*b**17*x**(45/2) + 2100*a**(13/2)*b**18*x**(39/2) + 1575*a**(11/2)*b*
*19*x**(33/2) + 630*a**(9/2)*b**20*x**(27/2) + 105*a**(7/2)*b**21*x**(21/2)
) - 480*a**9*b**15*x**(33/2)/(105*a**(19/2)*b**15*x**(57/2) + 630*a**(17/2)
*b**16*x**(51/2) + 1575*a**(15/2)*b**17*x**(45/2) + 2100*a**(13/2)*b**18*x*
*(39/2) + 1575*a**(11/2)*b**19*x**(33/2) + 630*a**(9/2)*b**20*x**(27/2) + 1
05*a**(7/2)*b**21*x**(21/2)) - 192*a**8*b**16*x**(27/2)/(105*a**(19/2)*b**1
5*x**(57/2) + 630*a**(17/2)*b**16*x**(51/2) + 1575*a**(15/2)*b**17*x**(45/2
) + 2100*a**(13/2)*b**18*x**(39/2) + 1575*a**(11/2)*b**19*x**(33/2) + 630*a
**(9/2)*b**20*x**(27/2) + 105*a**(7/2)*b**21*x**(21/2)) - 32*a**7*b**17*x**
(21/2)/(105*a**(19/2)*b**15*x**(57/2) + 630*a**(17/2)*b**16*x**(51/2) + 157
5*a**(15/2)*b**17*x**(45/2) + 2100*a**(13/2)*b**18*x**(39/2) + 1575*a**(11/
2)*b**19*x**(33/2) + 630*a**(9/2)*b**20*x**(27/2) + 105*a**(7/2)*b**21*x**(
21/2))
```

3.2024 $\int \frac{x^7}{\sqrt{a+\frac{b}{x^3}}}\,dx$

Optimal. Leaf size=294

$$\frac{91b^2x^2\sqrt{a+\frac{b}{x^3}}}{320a^3}-\frac{13bx^5\sqrt{a+\frac{b}{x^3}}}{80a^2}+\frac{91\sqrt{2+\sqrt{3}}\,b^{8/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-\right.}{320\sqrt[4]{3}\,a^3\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

```
[Out] 91/320*b^2*x^2*(a+b/x^3)^(1/2)/a^3-13/80*b*x^5*(a+b/x^3)^(1/2)/a^2+1/8*x^8*
(a+b/x^3)^(1/2)/a+91/960*b^(8/3)*(a^(1/3)+b^(1/3)/x)*EllipticF((b^(1/3)/x+a
^(1/3)*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(
1/2)+1/2*2^(1/2))*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^(1/
3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^3/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1
/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {335, 325, 218}

$$\frac{91b^2x^2\sqrt{a+\frac{b}{x^3}}}{320a^3}+\frac{91\sqrt{2+\sqrt{3}}\,b^{8/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{320\sqrt[4]{3}\,a^3\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}-\frac{13bx^5\sqrt{}}{80}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/Sqrt[a + b/x^3],x]
```

```
[Out] (91*b^2*Sqrt[a + b/x^3]*x^2)/(320*a^3) - (13*b*Sqrt[a + b/x^3]*x^5)/(80*a^2
) + (Sqrt[a + b/x^3]*x^8)/(8*a) + (91*Sqrt[2 + Sqrt[3]]*b^(8/3)*(a^(1/3) +
b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)
/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(320*3^(1/4)*a^3
*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)/x)^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{\sqrt{a+\frac{b}{x^3}}}\,dx &= -\text{Subst}\left(\int \frac{1}{x^9\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right) \\
&= \frac{\sqrt{a+\frac{b}{x^3}}\,x^8}{8a} + \frac{(13b)\,\text{Subst}\left(\int \frac{1}{x^6\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{16a} \\
&= -\frac{13b\sqrt{a+\frac{b}{x^3}}\,x^5}{80a^2} + \frac{\sqrt{a+\frac{b}{x^3}}\,x^8}{8a} - \frac{\left(91b^2\right)\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{160a^2} \\
&= \frac{91b^2\sqrt{a+\frac{b}{x^3}}\,x^2}{320a^3} - \frac{13b\sqrt{a+\frac{b}{x^3}}\,x^5}{80a^2} + \frac{\sqrt{a+\frac{b}{x^3}}\,x^8}{8a} + \frac{\left(91b^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{640a^3} \\
&= \frac{91b^2\sqrt{a+\frac{b}{x^3}}\,x^2}{320a^3} - \frac{13b\sqrt{a+\frac{b}{x^3}}\,x^5}{80a^2} + \frac{\sqrt{a+\frac{b}{x^3}}\,x^8}{8a} + \frac{91\sqrt{2+\sqrt{3}}\;b^{8/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-}{\left(1+\sqrt{3}\right)\sqrt[3]{}}}}{320\sqrt[4]{3}\,a^3\sqrt{a+\frac{}{x}}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 91, normalized size = 0.31

$$\frac{40a^3x^9 - 12a^2bx^6 - 91b^3\sqrt{\frac{ax^3}{b}+1}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{ax^3}{b}\right) + 39ab^2x^3 + 91b^3}{320a^3x\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/Sqrt[a + b/x^3],x]
```

```
[Out] (91*b^3 + 39*a*b^2*x^3 - 12*a^2*b*x^6 + 40*a^3*x^9 - 91*b^3*Sqrt[1 + (a*x^3
)/b]*Hypergeometric2F1[1/6, 1/2, 7/6, -((a*x^3)/b)])/(320*a^3*Sqrt[a + b/x^
3]*x)
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^{10}\sqrt{\frac{ax^3+b}{x^3}}}{ax^3+b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(a+b/x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^10*sqrt((a*x^3 + b)/x^3)/(a*x^3 + b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(a+b/x^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^7/sqrt(a + b/x^3), x)
```

maple [B] time = 0.03, size = 2233, normalized size = 7.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a+b/x^3)^(1/2),x)
```

```
[Out] 1/320/((a*x^3+b)/x^3)^(1/2)/x*(a*x^3+b)/(-a^2*b)^(1/3)/a^4*(40*I*(a*x^4+b*x
)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/
3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*(-a^2*b)^
(1/3)*3^(1/2)*x^6*a^3+182*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1
/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/
2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)
^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3
)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-
1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*3^(1/2)*x^2*a^2*b^3-120*(a*x^4+b*x)^
(1/2)*(-a^2*b)^(1/3)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)
+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1
/2)*a^3*x^6-364*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^
(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+
(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I
*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/
2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^
(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*3^(1/2)*x*a*b^3-52*I*(a*x^4+b*x
)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/
3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*(-a^2*b)^
(1/3)*3^(1/2)*x^3*a^2*b+182*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^
(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(
1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*
b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)
-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2
)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*3^(1/2)*b^3-182*(-(
I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/
2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2
)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a
^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^
(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))
^(1/2))*x^2*a^2*b^3+364*(-a^2*b)^(1/3)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+
(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))
/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/
3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(
I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*
(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x*a*b^3+156*(a*x^4+b*x)^(
1/2)*(-a^2*b)^(1/3)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+
(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/
2)*a^2*b*x^3-182*(-a^2*b)^(2/3)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b
)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3
^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^
```

```
2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/
2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1
/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*b^3+91*I*(a*x^4+b*x)^(1/2)*((-a*
x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I
*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*(-a^2*b)^(1/3)*3^(1/2)
*a*b^2-273*(a*x^4+b*x)^(1/2)*(-a^2*b)^(1/3)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I
*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-
a^2*b)^(1/3))/a^2*x)^(1/2)*a*b^2)/((a*x^3+b)*x)^(1/2)/(I*3^(1/2)-3)/((-a*x+
(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3
^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(a+b/x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^7/sqrt(a + b/x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b/x^3)^(1/2),x)
```

```
[Out] int(x^7/(a + b/x^3)^(1/2), x)
```

sympy [A] time = 1.46, size = 46, normalized size = 0.16

$$-\frac{x^8\Gamma\left(-\frac{8}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{8}{3}, \frac{1}{2}\\ -\frac{5}{3}\end{matrix}\right| \frac{be^{i\pi}}{ax^3}\right)}{3\sqrt{a}\,\Gamma\left(-\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(a+b/x**3)**(1/2),x)
```

```
[Out] -x**8*gamma(-8/3)*hyper((-8/3, 1/2), (-5/3,), b*exp_polar(I*pi)/(a*x**3))/(
3*sqrt(a)*gamma(-5/3))
```

3.2025 $\int \frac{x^4}{\sqrt{a+\frac{b}{x^3}}}\,dx$

Optimal. Leaf size=270

$$-\frac{7bx^2\sqrt{a+\frac{b}{x^3}}}{20a^2}-\frac{7\sqrt{2+\sqrt{3}}\,b^{5/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{20\sqrt[4]{3}\,a^2\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}+\frac{x^5\sqrt{a+}}{5a}$$

```
[Out] -7/20*b*x^2*(a+b/x^3)^(1/2)/a^2+1/5*x^5*(a+b/x^3)^(1/2)/a-7/60*b^(5/3)*(a^(
1/3)+b^(1/3)/x)*EllipticF((b^(1/3)/x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3
)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)+b^(2/3)/x
^2-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^2/
(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2))
)^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {335, 325, 218}

$$-\frac{7\sqrt{2+\sqrt{3}}\,b^{5/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{20\sqrt[4]{3}\,a^2\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}-\frac{7bx^2\sqrt{a+\frac{b}{x^3}}}{20a^2}+\frac{x^5\sqrt{a+}}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a + b/x^3],x]
```

```
[Out] (-7*b*Sqrt[a + b/x^3]*x^2)/(20*a^2) + (Sqrt[a + b/x^3]*x^5)/(5*a) - (7*Sqrt
[2 + Sqrt[3]]*b^(5/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (
a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin
[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)],
-7 - 4*Sqrt[3]])/(20*3^(1/4)*a^2*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b
^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{a+\frac{b}{x^3}}}\,dx = -\text{Subst}\left(\int \frac{1}{x^6\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt{a+\frac{b}{x^3}}\,x^5}{5a} + \frac{(7b)\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{10a}$$

$$= -\frac{7b\sqrt{a+\frac{b}{x^3}}\,x^2}{20a^2} + \frac{\sqrt{a+\frac{b}{x^3}}\,x^5}{5a} - \frac{\left(7b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{40a^2}$$

$$= -\frac{7b\sqrt{a+\frac{b}{x^3}}\,x^2}{20a^2} + \frac{\sqrt{a+\frac{b}{x^3}}\,x^5}{5a} - \frac{7\sqrt{2+\sqrt{3}}\;b^{5/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{(1-}{(1+}\right.\right.}{20\sqrt[4]{3}\,a^2\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

Mathematica [C] time = 0.03, size = 80, normalized size = 0.30

$$\frac{4a^2x^6 + 7b^2\sqrt{\frac{ax^3}{b}+1}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{ax^3}{b}\right) - 3abx^3 - 7b^2}{20a^2x\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a + b/x^3],x]
```

```
[Out] (-7*b^2 - 3*a*b*x^3 + 4*a^2*x^6 + 7*b^2*Sqrt[1 + (a*x^3)/b]*Hypergeometric2
F1[1/6, 1/2, 7/6, -((a*x^3)/b)])/(20*a^2*Sqrt[a + b/x^3]*x)
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^7\sqrt{\frac{ax^3+b}{x^3}}}{ax^3+b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^7*sqrt((a*x^3 + b)/x^3)/(a*x^3 + b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/x^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(a + b/x^3), x)
```

maple [B] time = 0.01, size = 2017, normalized size = 7.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a+b/x^3)^(1/2),x)
```

```
[Out] -1/20/((a*x^3+b)/x^3)^(1/2)/x*(a*x^3+b)/a^3/(-a^2*b)^(1/3)*(14*I*3^(1/2)*(-
(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1
/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/
2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-
a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)
^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3)
)^(1/2))*x^2*a^2*b^2-28*I*(-a^2*b)^(1/3)*3^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)
-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2
*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-
a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*Ell
ipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3
^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x*a*b^2+14*I*(-
a^2*b)^(2/3)*3^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*
x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a
*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))
/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^
(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I
*3^(1/2))/(I*3^(1/2)-3))^(1/2))*b^2-4*I*(-a^2*b)^(1/3)*3^(1/2)*(a*x^4+b*x)^
(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3)
)*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*x^3*a^2-14*
(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^
(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(
1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+
(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*
b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-
3))^(1/2))*x^2*a^2*b^2+28*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3
))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2)
)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(
1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/
(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)
/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*x*a*b^2-14*(-(I*3^(1/2)
-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*
b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*
x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/
3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*
x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(
-a^2*b)^(2/3)*b^2+12*(a*x^4+b*x)^(1/2)*(-a^2*b)^(1/3)*((-a*x+(-a^2*b)^(1/3)
)*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b
)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*a^2*x^3+7*I*(-a^2*b)^(1/3)*3^(1/2)*(a*
x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2
*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*a*
b-21*(a*x^4+b*x)^(1/2)*(-a^2*b)^(1/3)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/
2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)
^(1/3))/a^2*x)^(1/2)*a*b)/((a*x^3+b)*x)^(1/2)/(I*3^(1/2)-3)/((-a*x+(-a^2*b)
^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(
-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(a + b/x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b/x^3)^(1/2),x)
```

```
[Out] int(x^4/(a + b/x^3)^(1/2), x)
```

sympy [A] time = 1.20, size = 46, normalized size = 0.17

$$-\frac{x^5\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3}, \frac{1}{2} \\ -\frac{2}{3}\end{matrix}\middle| \frac{be^{i\pi}}{ax^3}\right)}{3\sqrt{a}\,\Gamma\left(-\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a+b/x**3)**(1/2),x)
```

```
[Out] -x**5*gamma(-5/3)*hyper((-5/3, 1/2), (-2/3,), b*exp_polar(I*pi)/(a*x**3))/(
3*sqrt(a)*gamma(-2/3))
```

3.2026 $\int \frac{x}{\sqrt{a+\frac{b}{x^3}}}\,dx$

Optimal. Leaf size=248

$$\frac{\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\,a\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}+\frac{x^2\sqrt{a+\frac{b}{x^3}}}{2a}$$

```
[Out] 1/2*x^2*(a+b/x^3)^(1/2)/a+1/6*b^(2/3)*(a^(1/3)+b^(1/3)/x)*EllipticF((b^(1/3
)/x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/
2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+
a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b
^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {335, 325, 218}

$$\frac{\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\,a\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}+\frac{x^2\sqrt{a+\frac{b}{x^3}}}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b/x^3],x]
```

```
[Out] (Sqrt[a + b/x^3]*x^2)/(2*a) + (Sqrt[2 + Sqrt[3]]*b^(2/3)*(a^(1/3) + b^(1/3)
/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*a*Sqrt[a + b
/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
/x)^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+\frac{b}{x^3}}}\,dx = -\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt{a+\frac{b}{x^3}}\,x^2}{2a} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{4a}$$

$$= \frac{\sqrt{a+\frac{b}{x^3}}\,x^2}{2a} + \frac{\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-\right.}{2\sqrt[4]{3}\,a\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

Mathematica [C] time = 0.02, size = 64, normalized size = 0.26

$$\frac{-b\sqrt{\frac{ax^3}{b}+1}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{ax^3}{b}\right)+ax^3+b}{2ax\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b/x^3],x]
```

```
[Out] (b + a*x^3 - b*Sqrt[1 + (a*x^3)/b]*Hypergeometric2F1[1/6, 1/2, 7/6, -((a*x^3)/b)])/(2*a*Sqrt[a + b/x^3]*x)
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^4\sqrt{\frac{ax^3+b}{x^3}}}{ax^3+b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^4*sqrt((a*x^3 + b)/x^3)/(a*x^3 + b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(a + b/x^3), x)
```

maple [B] time = 0.01, size = 1793, normalized size = 7.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b/x^3)^(1/2),x)
```

```
[Out] 1/2/((a*x^3+b)/x^3)^(1/2)/x*(a*x^3+b)/a^2/(-a^2*b)^(1/3)*(2*I*(-(I*3^(1/2)-
3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b
)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x
+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3
)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x
)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*3^
(1/2)*x^2*a^2*b-4*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x
)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*
x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/
(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(
1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*
3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*3^(1/2)*x*a*b+2*I*(-(I*3^(1/2
)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2
*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a
*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1
/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a
*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*
(-a^2*b)^(2/3)*3^(1/2)*b-2*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/
3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2
))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^
(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)
/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1
)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a^2*b+4*(-a^2*b)^(1/3)*(-(I*3^(1/
2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^
2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*
a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(
1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*
a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))
*x*a*b-2*(-a^2*b)^(2/3)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))
*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/
(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/
3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I
*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(
1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*b+I*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1
/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^
2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*(-a^2*b)^(1/3)*3^(1/2)*a-3*(a*x^4+b
*x)^(1/2)*(-a^2*b)^(1/3)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(
1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x
)^(1/2)*a)/((a*x^3+b)*x)^(1/2)/(I*3^(1/2)-3)/((-a*x+(-a^2*b)^(1/3))*(2*a*x+
I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(
-a^2*b)^(1/3))/a^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(a + b/x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b/x^3)^(1/2),x)
```

```
[Out] int(x/(a + b/x^3)^(1/2), x)
```

sympy [A] time = 1.03, size = 42, normalized size = 0.17

$$-\frac{x^2\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3}, \frac{1}{2}\\ \frac{1}{3}\end{matrix}\middle| \frac{be^{i\pi}}{ax^3}\right)}{3\sqrt{a}\,\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x**3)**(1/2),x)
```

```
[Out] -x**2*gamma(-2/3)*hyper((-2/3, 1/2), (1/3,), b*exp_polar(I*pi)/(a*x**3))/(3
*sqrt(a)*gamma(1/3))
```

3.2027 $\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^2}\,dx$

Optimal. Leaf size=221

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,\sqrt[3]{b}\,\sqrt{a+\frac{b}{x^3}}\,\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

```
[Out] -2/3*(a^(1/3)+b^(1/3)/x)*EllipticF((b^(1/3)/x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)
/x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)+
b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(
3/4)/b^(1/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/(b^(1/3)/x+a^(1/3
)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {335, 218}

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,\sqrt[3]{b}\,\sqrt{a+\frac{b}{x^3}}\,\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x^3]*x^2),x]
```

```
[Out] (-2*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (
a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin
[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)],
-7 - 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) +
b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^2}\,dx = -\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,\sqrt[3]{b}\,\sqrt{a+\frac{b}{x^3}}\,\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

Mathematica [C] time = 0.01, size = 49, normalized size = 0.22

$$\frac{2\sqrt{\frac{ax^3}{b}+1}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{ax^3}{b}\right)}{x\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x^3]*x^2),x]
```

```
[Out] (2*Sqrt[1 + (a*x^3)/b]*Hypergeometric2F1[1/6, 1/2, 7/6, -((a*x^3)/b)])/(Sqr
t[a + b/x^3]*x)
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x\sqrt{\frac{ax^3+b}{x^3}}}{ax^3+b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b/x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x*sqrt((a*x^3 + b)/x^3)/(a*x^3 + b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b/x^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a + b/x^3)*x^2), x)
```

maple [B] time = 0.01, size = 437, normalized size = 1.98

$$-\frac{4\left(a\,x^3+b\right)\sqrt{-\frac{\left(i\sqrt{3}-3\right)ax}{\left(i\sqrt{3}-1\right)\left(-ax+\left(-a^2b\right)^{\frac{1}{3}}\right)}}\sqrt{\frac{2ax+i\sqrt{3}\left(-a^2b\right)^{\frac{1}{3}}+\left(-a^2b\right)^{\frac{1}{3}}}{\left(1+i\sqrt{3}\right)\left(-ax+\left(-a^2b\right)^{\frac{1}{3}}\right)}}\sqrt{\frac{-2ax+i\sqrt{3}\left(-a^2b\right)^{\frac{1}{3}}-\left(-a^2b\right)^{\frac{1}{3}}}{\left(i\sqrt{3}-1\right)\left(-ax+\left(-a^2b\right)^{\frac{1}{3}}\right)}}\left(i\sqrt{3}\,a^2x^2-a^2x^2-\right.}{\sqrt{\frac{a\,x^3+b}{x^3}}\left(-a^2b\right)^{\frac{1}{3}}\sqrt{\left(a\,x^3+b\right)x}\left(i\sqrt{3}-3\right)\sqrt{\frac{\left(-ax+\left(\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b/x^3)^(1/2),x)
```

```
[Out] -4/((a*x^3+b)/x^3)^(1/2)/x*(a*x^3+b)/(-a^2*b)^(1/3)/a*(-(I*3^(1/2)-3)/(I*3^
(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+
(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/
2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2
)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),
((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(I*3^(1/2)
*x^2*a^2-2*I*(-a^2*b)^(1/3)*3^(1/2)*x*a+I*(-a^2*b)^(2/3)*3^(1/2)-a^2*x^2+2*
(-a^2*b)^(1/3)*x*a-(-a^2*b)^(2/3))/((a*x^3+b)*x)^(1/2)/(I*3^(1/2)-3)/((-a*x
+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*
3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + \frac{b}{x^3}}\, x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b/x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a + b/x^3)*x^2), x)
```

mupad [B] time = 1.28, size = 40, normalized size = 0.18

$$-\frac{\sqrt{\frac{b}{a\,x^3}+1}\;{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ -\frac{b}{a\,x^3}\right)}{x\sqrt{a+\frac{b}{x^3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b/x^3)^(1/2)),x)
```

```
[Out] -((b/(a*x^3) + 1)^(1/2)*hypergeom([1/3, 1/2], 4/3, -b/(a*x^3)))/(x*(a + b/x
^3)^(1/2))
```

sympy [A] time = 1.15, size = 37, normalized size = 0.17

$$-\frac{\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|\frac{be^{i\pi}}{ax^3}\right)}{3\sqrt{a}\,x\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b/x**3)**(1/2),x)
```

```
[Out] -gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*exp_polar(I*pi)/(a*x**3))/(3*sqrt(a
)*x*gamma(4/3))
```

3.2028 $\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^5}\,dx$

Optimal. Leaf size=246

$$\frac{4\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\,b^{4/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}-\frac{2\sqrt{a+\frac{b}{x^3}}}{5bx}$$

```
[Out] -2/5*(a+b/x^3)^(1/2)/b/x+4/15*a*(a^(1/3)+b^(1/3)/x)*EllipticF((b^(1/3)/x+a^
(1/3)*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1
/2)+1/2*2^(1/2))*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^(1/3
)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(4/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b
^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {335, 321, 218}

$$\frac{4\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\,b^{4/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}-\frac{2\sqrt{a+\frac{b}{x^3}}}{5bx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x^3]*x^5),x]
```

```
[Out] (-2*Sqrt[a + b/x^3])/(5*b*x) + (4*Sqrt[2 + Sqrt[3]]*a*(a^(1/3) + b^(1/3)/x)
*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*b^(4/3)*Sqrt[a
+ b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)/x)^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^5}\,dx = -\text{Subst}\left(\int \frac{x^3}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2\sqrt{a+\frac{b}{x^3}}}{5bx} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{5b}$$

$$= -\frac{2\sqrt{a+\frac{b}{x^3}}}{5bx} + \frac{4\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle| -7-4\right.}{5\sqrt[4]{3}\,b^{4/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.21

$$-\frac{2\sqrt{\frac{ax^3}{b}+1}\;{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};-\frac{ax^3}{b}\right)}{5x^4\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x^3]*x^5),x]
```

```
[Out] (-2*Sqrt[1 + (a*x^3)/b]*Hypergeometric2F1[-5/6, 1/2, 1/6, -((a*x^3)/b)])/(5
*Sqrt[a + b/x^3]*x^4)
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\frac{ax^3+b}{x^3}}}{ax^5+bx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a+b/x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt((a*x^3 + b)/x^3)/(a*x^5 + b*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a+b/x^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a + b/x^3)*x^5), x)
```

maple [B] time = 0.02, size = 1795, normalized size = 7.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(a+b/x^3)^(1/2),x)
```

```
[Out] -2/5/((a*x^3+b)/x^3)^(1/2)/x^4*(a*x^3+b)/(-a^2*b)^(1/3)/b*(-4*I*(-(I*3^(1/2
)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2
*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a
*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1
/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a
*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*
3^(1/2)*x^5*a^2+8*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x
)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*
x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/
(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(
1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*
3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*3^(1/2)*x^4*a-4*I*(-(I*3^(1/2
)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2
*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a
*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1
/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a
*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*
(-a^2*b)^(2/3)*3^(1/2)*x^3+4*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(
1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1
/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b
)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-
3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)
-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^5*a^2-8*(-a^2*b)^(1/3)*(-(I*3^(1/
2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^
2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*
a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(
1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*
a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))
*x^4*a+4*(-a^2*b)^(2/3)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))
*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/
(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/
3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I
*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(
1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^3+I*(-a^2*b)^(1/3)*3^(1/2)*(a*x^4+b*x)
^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3
))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)-3*(a*x^4+b
*x)^(1/2)*(-a^2*b)^(1/3)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(
1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x
)^(1/2))/((a*x^3+b)*x)^(1/2)/(I*3^(1/2)-3)/((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*
3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a
^2*b)^(1/3))/a^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a+b/x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a + b/x^3)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^5 \sqrt{a + \frac{b}{x^3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b/x^3)^(1/2)),x)
```

```
[Out] int(1/(x^5*(a + b/x^3)^(1/2)), x)
```

sympy [A] time = 1.35, size = 39, normalized size = 0.16

$$-\frac{\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{4}{3} \\ \frac{7}{3}\end{matrix}\middle| \frac{be^{i\pi}}{ax^3}\right)}{3\sqrt{a}\, x^4 \Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(a+b/x**3)**(1/2),x)
```

```
[Out] -gamma(4/3)*hyper((1/2, 4/3), (7/3,), b*exp_polar(I*pi)/(a*x**3))/(3*sqrt(a
)*x**4*gamma(7/3))
```

3.2029 $\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^8}\,dx$

Optimal. Leaf size=270

$$-\frac{32\sqrt{2+\sqrt{3}}\,a^2\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{55\sqrt[4]{3}\,b^{7/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}+\frac{16a\sqrt{a+\frac{b}{x^3}}}{55b^2x}-\frac{2\sqrt{a+}}{11bx^4}$$

```
[Out] -2/11*(a+b/x^3)^(1/2)/b/x^4+16/55*a*(a+b/x^3)^(1/2)/b^2/x-32/165*a^2*(a^(1/
3)+b^(1/3)/x)*EllipticF((b^(1/3)/x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3)*
(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)+b^(2/3)/x^2
-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(7/3
)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2
)))^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {335, 321, 218}

$$-\frac{32\sqrt{2+\sqrt{3}}\,a^2\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{55\sqrt[4]{3}\,b^{7/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}+\frac{16a\sqrt{a+\frac{b}{x^3}}}{55b^2x}-\frac{2\sqrt{a+}}{11bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x^3]*x^8),x]
```

```
[Out] (-2*Sqrt[a + b/x^3])/(11*b*x^4) + (16*a*Sqrt[a + b/x^3])/(55*b^2*x) - (32*S
qrt[2 + Sqrt[3]]*a^2*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a
^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[
((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -
7 - 4*Sqrt[3]])/(55*3^(1/4)*b^(7/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3)
+ b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^8}\,dx &= -\text{Subst}\left(\int \frac{x^6}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right) \\
&= -\frac{2\sqrt{a+\frac{b}{x^3}}}{11bx^4} + \frac{(8a)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{11b} \\
&= -\frac{2\sqrt{a+\frac{b}{x^3}}}{11bx^4} + \frac{16a\sqrt{a+\frac{b}{x^3}}}{55b^2x} - \frac{\left(16a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{55b^2} \\
&= -\frac{2\sqrt{a+\frac{b}{x^3}}}{11bx^4} + \frac{16a\sqrt{a+\frac{b}{x^3}}}{55b^2x} - \frac{32\sqrt{2+\sqrt{3}}\,a^2\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\,\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right.}{\left(1+\sqrt{3}\right.}\right.\right.}{55\sqrt[4]{3}\,b^{7/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.19

$$-\frac{2\sqrt{\frac{ax^3}{b}+1}\;{}_2F_1\left(-\frac{11}{6},\frac{1}{2};-\frac{5}{6};-\frac{ax^3}{b}\right)}{11x^7\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x^3]*x^8),x]
```

```
[Out] (-2*Sqrt[1 + (a*x^3)/b]*Hypergeometric2F1[-11/6, 1/2, -5/6, -((a*x^3)/b)])/
(11*Sqrt[a + b/x^3]*x^7)
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\frac{ax^3+b}{x^3}}}{ax^8+bx^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(a+b/x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt((a*x^3 + b)/x^3)/(a*x^8 + b*x^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(a+b/x^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a + b/x^3)*x^8), x)
```

maple [B] time = 0.02, size = 2009, normalized size = 7.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(a+b/x^3)^(1/2),x)
```

```
[Out] 2/55/((a*x^3+b)/x^3)^(1/2)/x^7*(a*x^3+b)/(-a^2*b)^(1/3)/b^2*(-32*I*(-(I*3^(
1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-
a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-
2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)
^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)
)*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2
))*3^(1/2)*x^8*a^3+64*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))
*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/
(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/
3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I
*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(
1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*3^(1/2)*x^7*a^2-32*I*(-(I
*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2
)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)
*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^
2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(
1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^
(1/2))*(-a^2*b)^(2/3)*3^(1/2)*x^6*a+32*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+
(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))
/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/
3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(
I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*
(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^8*a^3-64*(-(I*3^(1/2)-3
)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)
^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+
I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)
))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)
^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a
^2*b)^(1/3)*x^7*a^2+32*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*
a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(
-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3
))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*
3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1
+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*x^6*a+8*I*(a*x^4+b*x)^(1/2
)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-
2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*(-a^2*b)^(1/3)*
3^(1/2)*x^3*a-24*(a*x^4+b*x)^(1/2)*(-a^2*b)^(1/3)*((-a*x+(-a^2*b)^(1/3))*(2
*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1
/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*a*x^3-5*I*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b
)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*
(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*(-a^2*b)^(1/3)*3^(1/2)*b+15*(a*
x^4+b*x)^(1/2)*(-a^2*b)^(1/3)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2
*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/
a^2*x)^(1/2)*b)/((a*x^3+b)*x)^(1/2)/(I*3^(1/2)-3)/((-a*x+(-a^2*b)^(1/3))*(2
*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1
/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(a+b/x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a + b/x^3)*x^8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^8\sqrt{a+\frac{b}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(a + b/x^3)^(1/2)),x)
```

```
[Out] int(1/(x^8*(a + b/x^3)^(1/2)), x)
```

sympy [A] time = 1.68, size = 39, normalized size = 0.14

$$-\frac{\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\right|\frac{be^{i\pi}}{ax^3}\right)}{3\sqrt{a}\,x^7\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(a+b/x**3)**(1/2),x)
```

```
[Out] -gamma(7/3)*hyper((1/2, 7/3), (10/3,), b*exp_polar(I*pi)/(a*x**3))/(3*sqrt(
a)*x**7*gamma(10/3))
```

3.2030 $\int \frac{x^6}{\sqrt{a+\frac{b}{x^3}}}\,dx$

Optimal. Leaf size=566

$$-\frac{55b^{7/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{56\sqrt{2}\,\sqrt[4]{3}\,a^{8/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}+\frac{55\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{7/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{224}$$

```
[Out] 55/112*b^2*x*(a+b/x^3)^(1/2)/a^3-11/56*b*x^4*(a+b/x^3)^(1/2)/a^2+1/7*x^7*(a
+b/x^3)^(1/2)/a-55/112*b^(7/3)*(a+b/x^3)^(1/2)/a^3/(b^(1/3)/x+a^(1/3)*(1+3^
(1/2)))-55/336*b^(7/3)*(a^(1/3)+b^(1/3)/x)*EllipticF((b^(1/3)/x+a^(1/3)*(1-
3^(1/2)))/(b^(1/3)/x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((a^(2/3)+b^(2/3)/
x^2-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(
8/3)*2^(1/2)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/(b^(1/3)/x+a^(1/3
)*(1+3^(1/2)))^2)^(1/2)+55/224*3^(1/4)*b^(7/3)*(a^(1/3)+b^(1/3)/x)*Elliptic
E((b^(1/3)/x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)
+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/(b
^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(8/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^
(1/3)+b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 566, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {335, 325, 303, 218, 1877}

$$\frac{55b^2x\sqrt{a+\frac{b}{x^3}}}{112a^3}-\frac{55b^{7/3}\sqrt{a+\frac{b}{x^3}}}{112a^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}-\frac{55b^{7/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\right|$$

Antiderivative was successfully verified.

```
[In] Int[x^6/Sqrt[a + b/x^3],x]
```

```
[Out] (-55*b^(7/3)*Sqrt[a + b/x^3])/(112*a^3*((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x))
 + (55*b^2*Sqrt[a + b/x^3]*x)/(112*a^3) - (11*b*Sqrt[a + b/x^3]*x^4)/(56*a^
2) + (Sqrt[a + b/x^3]*x^7)/(7*a) + (55*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(7/3)*(a
^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3)
 + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(224*a^
(8/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)/x)^2]) - (55*b^(7/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) +
 b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*
EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(56*Sqrt[2]*3^(1/4)*a^(8/3)*Sqrt[a + b/x^3
]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^
2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^6}{\sqrt{a+\frac{b}{x^3}}}\,dx = -\operatorname{Subst}\left(\int \frac{1}{x^8\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt{a+\frac{b}{x^3}}\,x^7}{7a} + \frac{(11b)\operatorname{Subst}\left(\int \frac{1}{x^5\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{14a}$$

$$= -\frac{11b\sqrt{a+\frac{b}{x^3}}\,x^4}{56a^2} + \frac{\sqrt{a+\frac{b}{x^3}}\,x^7}{7a} - \frac{\left(55b^2\right)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{112a^2}$$

$$= \frac{55b^2\sqrt{a+\frac{b}{x^3}}\,x}{112a^3} - \frac{11b\sqrt{a+\frac{b}{x^3}}\,x^4}{56a^2} + \frac{\sqrt{a+\frac{b}{x^3}}\,x^7}{7a} - \frac{\left(55b^3\right)\operatorname{Subst}\left(\int \frac{x}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{224a^3}$$

$$= \frac{55b^2\sqrt{a+\frac{b}{x^3}}\,x}{112a^3} - \frac{11b\sqrt{a+\frac{b}{x^3}}\,x^4}{56a^2} + \frac{\sqrt{a+\frac{b}{x^3}}\,x^7}{7a} - \frac{\left(55b^{8/3}\right)\operatorname{Subst}\left(\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{224a^3}$$

$$= -\frac{55b^{7/3}\sqrt{a+\frac{b}{x^3}}}{112a^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)} + \frac{55b^2\sqrt{a+\frac{b}{x^3}}\,x}{112a^3} - \frac{11b\sqrt{a+\frac{b}{x^3}}\,x^4}{56a^2} + \frac{\sqrt{a+\frac{b}{x^3}}\,x^7}{7a} + \frac{55\sqrt[4]{3}\sqrt{2}}{}$$

Mathematica [C] time = 0.03, size = 79, normalized size = 0.14

$$\frac{8a^2x^7 + 11b^2x\sqrt{\frac{ax^3}{b}+1}\;{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};-\frac{ax^3}{b}\right) - 3abx^4 - 11b^2x}{56a^2\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/Sqrt[a + b/x^3],x]
```

```
[Out] (-11*b^2*x - 3*a*b*x^4 + 8*a^2*x^7 + 11*b^2*x*Sqrt[1 + (a*x^3)/b]*Hypergeom
etric2F1[1/2, 5/6, 11/6, -((a*x^3)/b)])/(56*a^2*Sqrt[a + b/x^3])
```

fricas [F] time = 1.11, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^9\sqrt{\frac{ax^3+b}{x^3}}}{ax^3+b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(a+b/x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^9*sqrt((a*x^3 + b)/x^3)/(a*x^3 + b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(a+b/x^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/sqrt(a + b/x^3), x)
```

maple [B] time = 0.04, size = 2806, normalized size = 4.96

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a+b/x^3)^(1/2),x)
```

```
[Out] -1/56/((a*x^3+b)/x^3)^(1/2)/x*(a*x^3+b)/a^4*(110*I*(-a^2*b)^(2/3)*3^(1/2)*(
-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(
1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1
/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(
-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b
)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3
))^(1/2))*x*b^2-8*I*3^(1/2)*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x
+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-
(-a^2*b)^(1/3))/a^2*x)^(1/2)*x^5*a^3-55*I*(-a^2*b)^(1/3)*3^(1/2)*x^2*a*b^2-
110*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+
I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)
))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-
a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-
a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1
/2)-3))^(1/2))*(-a^2*b)^(1/3)*x^2*a*b^2+165*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(
-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(
1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b
)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*Elliptic
E((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2
)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*x^2*a
*b^2+11*I*3^(1/2)*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)
*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(
1/3))/a^2*x)^(1/2)*x^2*a^2*b+24*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2
*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1
/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*a^3*x^5+220*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/
(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^
(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*
b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*Ellipti
cF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/
2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*x*b^
2-330*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*
x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/
3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/
(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+
(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^
(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*x*b^2+55*I*3^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/
2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a
^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*
(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*E
llipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I
*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b^3-55*I*(-
a^2*b)^(2/3)*3^(1/2)*x*b^2-55*I*3^(1/2)*x^3*a^2*b^2+110*(-(I*3^(1/2)-3)/(I*
3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3
)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(
1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1
/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2
),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b^3-16
5*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*
3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))
^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*
x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^
```

```
2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2
)-3))^(1/2))*a*b^3-55*I*(-a^2*b)^(1/3)*3^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1
)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b
)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^
2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*Ellip
ticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(
1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a*b^2-33*(a*x
^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*
b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*a^2
*b*x^2+165*a^2*b^2*x^3+165*(-a^2*b)^(1/3)*a*b^2*x^2+165*(-a^2*b)^(2/3)*b^2*
x)/((a*x^3+b)*x)^(1/2)/(I*3^(1/2)-3)/((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2
)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^
(1/3))/a^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(a+b/x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/sqrt(a + b/x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b/x^3)^(1/2),x)
```

```
[Out] int(x^6/(a + b/x^3)^(1/2), x)
```

sympy [A] time = 1.31, size = 46, normalized size = 0.08

$$-\frac{x^7\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{7}{3}, \frac{1}{2} \\ -\frac{4}{3}\end{matrix}\right| \frac{be^{i\pi}}{ax^3}\right)}{3\sqrt{a}\,\Gamma\left(-\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(a+b/x**3)**(1/2),x)
```

```
[Out] -x**7*gamma(-7/3)*hyper((-7/3, 1/2), (-4/3,), b*exp_polar(I*pi)/(a*x**3))/(
3*sqrt(a)*gamma(-4/3))
```

3.2031 $\int \frac{x^3}{\sqrt{a+\frac{b}{x^3}}}\,dx$

Optimal. Leaf size=542

$$\frac{5b^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{4\sqrt{2}\,\sqrt[4]{3}\,a^{5/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}-\frac{5\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^2}{\left(\left(1\right.\right.}}}{16a^{5/3}\sqrt{a}}$$

```
[Out] -5/8*b*x*(a+b/x^3)^(1/2)/a^2+1/4*x^4*(a+b/x^3)^(1/2)/a+5/8*b^(4/3)*(a+b/x^3
)^(1/2)/a^2/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))+5/24*b^(4/3)*(a^(1/3)+b^(1/3)/x
)*EllipticF((b^(1/3)/x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))
,I*3^(1/2)+2*I)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^(1/3)
*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(5/3)*2^(1/2)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^
(1/3)+b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-5/16*3^(1/4)*b^(4
/3)*(a^(1/3)+b^(1/3)/x)*EllipticE((b^(1/3)/x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)/
x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)+b
^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(5
/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1
/2)))^2)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 542, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {335, 325, 303, 218, 1877}

$$\frac{5b^{4/3}\sqrt{a+\frac{b}{x^3}}}{8a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}+\frac{5b^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{4\sqrt{2}\,\sqrt[4]{3}\,a^{5/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}-\frac{5\sqrt[4]{3}\sqrt{2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b/x^3],x]
```

```
[Out] (5*b^(4/3)*Sqrt[a + b/x^3])/(8*a^2*((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)) - (
5*b*Sqrt[a + b/x^3]*x)/(8*a^2) + (Sqrt[a + b/x^3]*x^4)/(4*a) - (5*3^(1/4)*S
qrt[2 - Sqrt[3]]*b^(4/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2
 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticE[Arc
Sin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)
], -7 - 4*Sqrt[3]])/(16*a^(5/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]) + (5*b^(4/3)*(a^(1/3) + b
^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/
x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(4*Sqrt[2]*3^(1/4
)*a^(5/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)/x)^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x^3}}}\,dx = -\operatorname{Subst}\left(\int \frac{1}{x^5\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt{a+\frac{b}{x^3}}\,x^4}{4a} + \frac{(5b)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{8a}$$

$$= -\frac{5b\sqrt{a+\frac{b}{x^3}}\,x}{8a^2} + \frac{\sqrt{a+\frac{b}{x^3}}\,x^4}{4a} + \frac{\left(5b^2\right)\operatorname{Subst}\left(\int \frac{x}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{16a^2}$$

$$= -\frac{5b\sqrt{a+\frac{b}{x^3}}\,x}{8a^2} + \frac{\sqrt{a+\frac{b}{x^3}}\,x^4}{4a} + \frac{\left(5b^{5/3}\right)\operatorname{Subst}\left(\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{16a^2} + \frac{\left(5\sqrt{\frac{1}{2}\left(2-\sqrt{3}\right)}\,b^{5/3}\right.}{}$$

$$= \frac{5b^{4/3}\sqrt{a+\frac{b}{x^3}}}{8a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)} - \frac{5b\sqrt{a+\frac{b}{x^3}}\,x}{8a^2} + \frac{\sqrt{a+\frac{b}{x^3}}\,x^4}{4a} - \frac{5\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^2}{\left((1\right.}}}{16a^{5/3}\sqrt{a}}$$

Mathematica [C] time = 0.02, size = 62, normalized size = 0.11

$$\frac{x\left(-b\sqrt{\frac{ax^3}{b}+1}\;{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};-\frac{ax^3}{b}\right)+ax^3+b\right)}{4a\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + b/x^3],x]
```

```
[Out] (x*(b + a*x^3 - b*Sqrt[1 + (a*x^3)/b]*Hypergeometric2F1[1/2, 5/6, 11/6, -((
a*x^3)/b)]))/(4*a*Sqrt[a + b/x^3])
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^6\sqrt{\frac{ax^3+b}{x^3}}}{ax^3+b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^6*sqrt((a*x^3 + b)/x^3)/(a*x^3 + b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(a + b/x^3), x)
```

maple [B] time = 0.01, size = 2586, normalized size = 4.77

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b/x^3)^(1/2),x)
```

```
[Out] 1/4/((a*x^3+b)/x^3)^(1/2)/x*(a*x^3+b)/a^3*(5*I*3^(1/2)*(-(I*3^(1/2)-3)/(I*3
^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)
+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1
/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/
2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)
,((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b^2+10*
I*(-a^2*b)^(2/3)*3^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)
)*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))
/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1
/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(
I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/
(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x*b-10*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-
a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1
/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)
^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF
((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)
+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*x^2*a*
b+15*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x
+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3
)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(
-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(
-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(
1/2)-3))^(1/2))*(-a^2*b)^(1/3)*x^2*a*b-5*I*3^(1/2)*x^3*a^2*b+20*(-(I*3^(1/2
)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2
*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a
*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1
/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a
*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*
(-a^2*b)^(2/3)*x*b-30*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a
*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-
a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3)
)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3
^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+
I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*x*b-5*I*(-a^2*b)^(2/3)*3^(1
/2)*x*b-5*I*(-a^2*b)^(1/3)*3^(1/2)*x^2*a*b+I*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2
*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2
)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*3^(1/2)*a^2*x^2+10*(-(I*3^(1/
2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^
2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*
a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(
1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*
a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))
*a*b^2-15*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((
2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)
^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)
-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-
a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(
I*3^(1/2)-3))^(1/2))*a*b^2-5*I*(-a^2*b)^(1/3)*3^(1/2)*(-(I*3^(1/2)-3)/(I*3^
(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+
(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/
2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2
)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),
```

```
((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a*b-3*
(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-
a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)
*a^2*x^2+15*a^2*b*x^3+15*(-a^2*b)^(1/3)*a*b*x^2+15*(-a^2*b)^(2/3)*b*x)/((a*
x^3+b)*x)^(1/2)/(I*3^(1/2)-3)/((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2
*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/
a^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(a + b/x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b/x^3)^(1/2),x)
```

```
[Out] int(x^3/(a + b/x^3)^(1/2), x)
```

sympy [A] time = 1.14, size = 46, normalized size = 0.08

$$-\frac{x^4\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{4}{3}, \frac{1}{2} \\ -\frac{1}{3}\end{matrix}\right| \frac{be^{i\pi}}{ax^3}\right)}{3\sqrt{a}\,\Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b/x**3)**(1/2),x)
```

```
[Out] -x**4*gamma(-4/3)*hyper((-4/3, 1/2), (-1/3,), b*exp_polar(I*pi)/(a*x**3))/(
3*sqrt(a)*gamma(-1/3))
```

3.2032 $\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}\,dx$

Optimal. Leaf size=513

$$-\frac{\sqrt{2}\,\sqrt[3]{b}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,a^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{b}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{2a}$$

```
[Out] x*(a+b/x^3)^(1/2)/a-b^(1/3)*(a+b/x^3)^(1/2)/a/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)
))-1/3*b^(1/3)*(a^(1/3)+b^(1/3)/x)*EllipticF((b^(1/3)/x+a^(1/3)*(1-3^(1/2)))
)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)+b^(2/3)/
x^2-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(
2/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(
1/2)))^2)^(1/2)+1/2*3^(1/4)*b^(1/3)*(a^(1/3)+b^(1/3)/x)*EllipticE((b^(1/3)/
x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*
6^(1/2)-1/2*2^(1/2))*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^
(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/
3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 513, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {242, 325, 303, 218, 1877}

$$-\frac{\sqrt{2}\,\sqrt[3]{b}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,a^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{b}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b/x^3],x]
```

```
[Out] -((b^(1/3)*Sqrt[a + b/x^3])/(a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x))) + (Sqr
t[a + b/x^3]*x)/a + (3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*(a^(1/3) + b^(1/3)/x
)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)/x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(2*a^(2/3)*Sqrt[a + b/x^3
]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^
2]) - (Sqrt[2]*b^(1/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 -
(a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSi
n[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)],
 -7 - 4*Sqrt[3]])/(3^(1/4)*a^(2/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) +
 b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
```

```
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
 + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}\,dx = -\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt{a+\frac{b}{x^3}}\,x}{a} - \frac{b\,\text{Subst}\left(\int \frac{x}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= \frac{\sqrt{a+\frac{b}{x^3}}\,x}{a} - \frac{b^{2/3}\,\text{Subst}\left(\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{2a} - \frac{\left(\sqrt{\frac{1}{2}\left(2-\sqrt{3}\right)}\,b^{2/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^3}}\,dx, x\right.}{a^{2/3}}$$

$$= -\frac{\sqrt[3]{b}\sqrt{a+\frac{b}{x^3}}}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)} + \frac{\sqrt{a+\frac{b}{x^3}}\,x}{a} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{b}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,E\left(\text{s}\right.}{2a^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\right.}}$$

Mathematica [C] time = 0.01, size = 49, normalized size = 0.10

$$\frac{2x\sqrt{\frac{ax^3}{b}+1}\ {}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};-\frac{ax^3}{b}\right)}{5\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b/x^3],x]
```

```
[Out] (2*x*Sqrt[1 + (a*x^3)/b]*Hypergeometric2F1[1/2, 5/6, 11/6, -((a*x^3)/b)])/(
5*Sqrt[a + b/x^3])
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3\sqrt{\frac{ax^3+b}{x^3}}}{ax^3+b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^3*sqrt((a*x^3 + b)/x^3)/(a*x^3 + b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a + b/x^3), x)
```

maple [B] time = 0.01, size = 2374, normalized size = 4.63

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^3)^(1/2),x)
```

```
[Out] -2/((a*x^3+b)/x^3)^(1/2)/x*(a*x^3+b)/a^2*(-I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/
(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^
(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*
b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*Ellipti
cE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/
2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*3^(1
/2)*x^2*a+2*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2
)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^
2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(
1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1
)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2
))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*3^(1/2)*x-2*(-(I*3^(1/2)-3)/(I*3^(1
/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-
a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)
*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*
EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((
I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/
```

```
3)*x^2*a+3*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*(
(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b
)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2
)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(
-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/
(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*x^2*a+I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/
(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^
(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*
b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*Ellipti
cE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/
2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*3^(1/2)*a*b+4*(-(I*
3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)
*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*
((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2
*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1
/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(
1/2))*(-a^2*b)^(2/3)*x-6*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)
)*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))
/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1
/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(
I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/
(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*x-I*3^(1/2)*x^3*a^2+2*(-
(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1
/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/
2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-
a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)
^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3)
)^(1/2))*a*b-3*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/
2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a
^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^
(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-
1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/
2))/(I*3^(1/2)-3))^(1/2))*a*b-I*(-a^2*b)^(1/3)*3^(1/2)*x^2*a-I*(-a^2*b)^(2/
3)*3^(1/2)*x+3*a^2*x^3+3*(-a^2*b)^(1/3)*x^2*a+3*(-a^2*b)^(2/3)*x)/((a*x^3+b
)*x)^(1/2)/(I*3^(1/2)-3)/((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(
1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x
)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a + b/x^3), x)
```

mupad [B] time = 1.16, size = 38, normalized size = 0.07

$$\frac{2\,x\,\sqrt{\frac{a\,x^3}{b}+1}\;{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{5}{6};\ \frac{11}{6};\ -\frac{a\,x^3}{b}\right)}{5\,\sqrt{a+\frac{b}{x^3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x^3)^(1/2),x)
```

```
[Out] (2*x*((a*x^3)/b + 1)^(1/2)*hypergeom([1/2, 5/6], 11/6, -(a*x^3)/b))/(5*(a +
 b/x^3)^(1/2))
```

sympy [**A**] time = 1.04, size = 41, normalized size = 0.08

$$-\frac{x\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3}, \frac{1}{2} \\ \frac{2}{3}\end{matrix}\right| \frac{be^{i\pi}}{ax^3}\right)}{3\sqrt{a}\,\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**3)**(1/2),x)
```

```
[Out] -x*gamma(-1/3)*hyper((-1/3, 1/2), (2/3,), b*exp_polar(I*pi)/(a*x**3))/(3*sq
rt(a)*gamma(2/3))
```

3.2033 $\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^3}\,dx$

Optimal. Leaf size=491

$$-\frac{2\sqrt{2}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}\sqrt{a+\frac{b}{x^3}}\,\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}+\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{}}{b^{2/3}}$$

```
[Out] -2*(a+b/x^3)^(1/2)/b^(2/3)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))-2/3*a^(1/3)*(a^(
1/3)+b^(1/3)/x)*EllipticF((b^(1/3)/x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3
)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)
/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(a+b/x^3)^(1/2
)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+3^(
1/4)*a^(1/3)*(a^(1/3)+b^(1/3)/x)*EllipticE((b^(1/3)/x+a^(1/3)*(1-3^(1/2)))/
(b^(1/3)/x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((
a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(
1/2)/b^(2/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/(b^(1/3)/x+a^(1/3
)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 491, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {335, 303, 218, 1877}

$$-\frac{2\sqrt{2}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}\sqrt{a+\frac{b}{x^3}}\,\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}+\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{}}{b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x^3]*x^3),x]
```

```
[Out] (-2*Sqrt[a + b/x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)) + (3^(1/
4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/
x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticE
[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)/x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b
^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]) - (2*Sqrt[2]*a^(1/3)*(a^
(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) +
 b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(3^(1/4)
*b^(2/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)/x)^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
```

```
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^3}\,dx = -\text{Subst}\left(\int \frac{x}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{\sqrt[3]{b}} - \frac{\left(\sqrt{2\left(2-\sqrt{3}\right)}\,\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{\sqrt[3]{b}}$$

$$= -\frac{2\sqrt{a+\frac{b}{x^3}}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\;\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,E\left(\sin^{-1}\left(\right.\right.}{b^{2/3}\sqrt{a+\frac{b}{x^3}}\,\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

Mathematica [C] time = 0.01, size = 49, normalized size = 0.10

$$-\frac{2\sqrt{\frac{ax^3}{b}+1}\;{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};-\frac{ax^3}{b}\right)}{x^2\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x^3]*x^3),x]
```

```
[Out] (-2*Sqrt[1 + (a*x^3)/b]*Hypergeometric2F1[-1/6, 1/2, 5/6, -((a*x^3)/b)])/(S
qrt[a + b/x^3]*x^2)
```

fricas [F] time = 1.06, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\frac{ax^3+b}{x^3}}}{ax^3+b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b/x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt((a*x^3 + b)/x^3)/(a*x^3 + b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b/x^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a + b/x^3)*x^3), x)
```

maple [B] time = 0.02, size = 2860, normalized size = 5.82

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b/x^3)^(1/2),x)
```

```
[Out] -2/((a*x^3+b)/x^3)^(1/2)/x^2/a*(I*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(
-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/
3))/a^2*x)^(1/2)*3^(1/2)*x^3*a^2-2*I*3^(1/2)*((a*x^3+b)*x)^(1/2)*x^3*a^2-4*
(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^
(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(
1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+
(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*
b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-
3))^(1/2))*(-a^2*b)^(1/3)*((a*x^3+b)*x)^(1/2)*x^2*a+6*(-(I*3^(1/2)-3)/(I*3^
(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+
(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/
2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2
)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),
((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(
1/3)*((a*x^3+b)*x)^(1/2)*x^2*a-2*I*(-a^2*b)^(1/3)*3^(1/2)*((a*x^3+b)*x)^(1/
2)*x^2*a+8*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*(
(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b
)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2
)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(
-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/
(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*((a*x^3+b)*x)^(1/2)*x-12*(-(I*3^(1/2)-
3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b
)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x
+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3
)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x
)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-
a^2*b)^(2/3)*((a*x^3+b)*x)^(1/2)*x+2*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+
(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))
/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/
3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(
```

```
I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*
(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*3^(1/2)*((a*x^3+b)*x)^(1/
2)*a*b+I*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1
/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*3^(1/2)*
a*b+4*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*
x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/
3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/
(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+
(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^
(1/2)-3))^(1/2))*((a*x^3+b)*x)^(1/2)*a*b-6*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-
a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1
/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)
^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE
((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)
+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^(1/2)*a
*b-2*I*(-a^2*b)^(2/3)*3^(1/2)*((a*x^3+b)*x)^(1/2)*x-3*((-a*x+(-a^2*b)^(1/3)
)*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b
)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*x^3*a^2-2*I*(-(I*3^(1/2)-3)/(I*3^(1/2)
-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2
*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-
a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*Ell
ipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3
^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*
3^(1/2)*((a*x^3+b)*x)^(1/2)*x^2*a+6*((a*x^3+b)*x)^(1/2)*x^3*a^2+4*I*(-(I*3^
(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(
-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((
-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b
)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3
))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/
2))*(-a^2*b)^(2/3)*3^(1/2)*((a*x^3+b)*x)^(1/2)*x+6*(-a^2*b)^(1/3)*((a*x^3+b
)*x)^(1/2)*x^2*a+6*(-a^2*b)^(2/3)*((a*x^3+b)*x)^(1/2)*x-3*((-a*x+(-a^2*b)^(
1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a
^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*a*b)/b/(I*3^(1/2)-3)/((-a*x+(-a^2*
b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)
*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b/x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a + b/x^3)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b/x^3)^(1/2)),x)
```

```
[Out] int(1/(x^3*(a + b/x^3)^(1/2)), x)
```

sympy [**A**] time = 1.20, size = 39, normalized size = 0.08

$$-\frac{\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{2}{3} \\ \frac{5}{3}\end{matrix}\right| \frac{be^{i\pi}}{ax^3}\right)}{3\sqrt{a}\,x^2\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b/x**3)**(1/2),x)
```

```
[Out] -gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*exp_polar(I*pi)/(a*x**3))/(3*sqrt(a
)*x**2*gamma(5/3))
```

3.2034 $\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^6}\,dx$

Optimal. Leaf size=520

$$\frac{8\sqrt{2}\,a^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{7\sqrt[4]{3}\,b^{5/3}\sqrt{a+\frac{b}{x^3}}\,\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{7b}$$

```
[Out] -2/7*(a+b/x^3)^(1/2)/b/x^2+8/7*a*(a+b/x^3)^(1/2)/b^(5/3)/(b^(1/3)/x+a^(1/3)
*(1+3^(1/2)))+8/21*a^(4/3)*(a^(1/3)+b^(1/3)/x)*EllipticF((b^(1/3)/x+a^(1/3)
*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2
/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
*3^(3/4)/b^(5/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/(b^(1/3)/x+a^
(1/3)*(1+3^(1/2)))^2)^(1/2)-4/7*3^(1/4)*a^(4/3)*(a^(1/3)+b^(1/3)/x)*Ellipti
cE((b^(1/3)/x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3)*(1+3^(1/2))),I*3^(1/2
)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/(
b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a
^(1/3)+b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 520, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {335, 321, 303, 218, 1877}

$$\frac{8\sqrt{2}\,a^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{7\sqrt[4]{3}\,b^{5/3}\sqrt{a+\frac{b}{x^3}}\,\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x^3]*x^6),x]
```

```
[Out] (8*a*Sqrt[a + b/x^3])/(7*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)) - (2*
Sqrt[a + b/x^3])/(7*b*x^2) - (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(a^(1/3)
+ b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)/x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/
3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(7*b^(5/3)*Sqr
t[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)/x)^2]) + (8*Sqrt[2]*a^(4/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b
^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*El
lipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)/x)], -7 - 4*Sqrt[3]])/(7*3^(1/4)*b^(5/3)*Sqrt[a + b/x^3]*Sqrt[(a^(
1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
```

```
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^6}\,dx = -\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2\sqrt{a+\frac{b}{x^3}}}{7bx^2} + \frac{(4a)\,\text{Subst}\left(\int \frac{x}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{7b}$$

$$= -\frac{2\sqrt{a+\frac{b}{x^3}}}{7bx^2} + \frac{(4a)\,\text{Subst}\left(\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{7b^{4/3}} + \frac{\left(4\sqrt{2\left(2-\sqrt{3}\right)}\,a^{4/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^3}}\right.}{7b^{4/3}}$$

$$= \frac{8a\sqrt{a+\frac{b}{x^3}}}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)} - \frac{2\sqrt{a+\frac{b}{x^3}}}{7bx^2} - \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}{7b^{5/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}}{\left(\left(1+\sqrt{}\right.\right.}}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.10

$$-\frac{2\sqrt{\frac{ax^3}{b}+1}\ {}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};-\frac{ax^3}{b}\right)}{7x^5\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x^3]*x^6),x]
```

```
[Out] (-2*Sqrt[1 + (a*x^3)/b]*Hypergeometric2F1[-7/6, 1/2, -1/6, -((a*x^3)/b)])/(
7*Sqrt[a + b/x^3]*x^5)
```

fricas [F] time = 1.11, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\frac{ax^3+b}{x^3}}}{ax^6+bx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(a+b/x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt((a*x^3 + b)/x^3)/(a*x^6 + b*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(a+b/x^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a + b/x^3)*x^6), x)
```

maple [B] time = 0.02, size = 3300, normalized size = 6.35

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(a+b/x^3)^(1/2),x)
```

```
[Out] 2/21/((a*x^3+b)/x^3)^(1/2)/x^6*(8*I*3^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I
*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-
a^2*b)^(1/3))/a^2*x)^(1/2)*x^7*a^2-24*I*(-a^2*b)^(1/3)*3^(1/2)*((a*x^3+b)*x
)^(1/2)*x^6*a-48*((a*x^3+b)*x)^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-
a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(
1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)
-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*
3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I
*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*x^6*a+72*((a
*x^3+b)*x)^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(
1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(
-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*
3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2
)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(
1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*x^6*a+48*I*(-a^2*b)^(2/3)*3^(1/2
)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*
3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))
```

```
^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*
x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^
2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2
)-3))^(1/2))*((a*x^3+b)*x)^(1/2)*x^5+96*((a*x^3+b)*x)^(1/2)*(-(I*3^(1/2)-3)
/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^
(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I
*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))
)^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^
(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^
2*b)^(2/3)*x^5-144*((a*x^3+b)*x)^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+
(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))
/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/
3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(
I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*
(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*x^5-24*I*(
-a^2*b)^(2/3)*3^(1/2)*((a*x^3+b)*x)^(1/2)*x^5+4*I*((a*x^3+b)*x)^(1/2)*3^(1/
2)*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)
+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1
/2)*x^3*a-24*I*3^(1/2)*((a*x^3+b)*x)^(1/2)*x^7*a^2+48*((a*x^3+b)*x)^(1/2)*(
-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(
1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1
/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(
-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b
)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3
))^(1/2))*x^4*a*b-72*((a*x^3+b)*x)^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*
x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3
))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(
1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((
-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3
)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^4*a*b+24*I*3^(1/2)*(-
(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1
/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/
2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-
a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)
^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3)
)^(1/2))*((a*x^3+b)*x)^(1/2)*x^4*a*b-24*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(
1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*
b)^(1/3))/a^2*x)^(1/2)*x^7*a^2+72*((a*x^3+b)*x)^(1/2)*x^7*a^2+72*((a*x^3+b)
*x)^(1/2)*(-a^2*b)^(1/3)*x^6*a-24*I*(-a^2*b)^(1/3)*3^(1/2)*(-(I*3^(1/2)-3)/
(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(
1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*
3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))
^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(
1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x
^3+b)*x)^(1/2)*x^6*a+8*I*3^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-
a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3
))/a^2*x)^(1/2)*x^4*a*b+72*((a*x^3+b)*x)^(1/2)*(-a^2*b)^(2/3)*x^5-12*((a*x^
3+b)*x)^(1/2)*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a
^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3)
)/a^2*x)^(1/2)*x^3*a-24*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1
/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)
^(1/2)*x^4*a*b-3*I*((a*x^3+b)*x)^(1/2)*3^(1/2)*(a*x^4+b*x)^(1/2)*((-a*x+(-a
^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1
/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*b+9*((a*x^3+b)*x)^(1/2)*(a*
x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2
*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*b)
/b^2/(I*3^(1/2)-3)/((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(
-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(a+b/x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a + b/x^3)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^6\sqrt{a+\frac{b}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b/x^3)^(1/2)),x)
```

```
[Out] int(1/(x^6*(a + b/x^3)^(1/2)), x)
```

sympy [A] time = 1.47, size = 39, normalized size = 0.08

$$-\frac{\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\right|\frac{be^{i\pi}}{ax^3}\right)}{3\sqrt{a}\,x^5\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(a+b/x**3)**(1/2),x)
```

```
[Out] -gamma(5/3)*hyper((1/2, 5/3), (8/3,), b*exp_polar(I*pi)/(a*x**3))/(3*sqrt(a
)*x**5*gamma(8/3))
```

3.2035 $\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^9}\,dx$

Optimal. Leaf size=544

$$-\frac{80\sqrt{2}\,a^{7/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{91\sqrt[4]{3}\,b^{8/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}+\frac{40\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{7/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right.}{91}$$

```
[Out] -2/13*(a+b/x^3)^(1/2)/b/x^5+20/91*a*(a+b/x^3)^(1/2)/b^2/x^2-80/91*a^2*(a+b/
x^3)^(1/2)/b^(8/3)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))-80/273*a^(7/3)*(a^(1/3)+
b^(1/3)/x)*EllipticF((b^(1/3)/x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3)*(1+
3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/(
b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(8/3)/(a+b/x^3)^(1/2)/(a^
(1/3)*(a^(1/3)+b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+40/91*3^
(1/4)*a^(7/3)*(a^(1/3)+b^(1/3)/x)*EllipticE((b^(1/3)/x+a^(1/3)*(1-3^(1/2)))
/(b^(1/3)/x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*(
(a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^
(1/2)/b^(8/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/(b^(1/3)/x+a^(1/
3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 544, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {335, 321, 303, 218, 1877}

$$-\frac{80a^2\sqrt{a+\frac{b}{x^3}}}{91b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}-\frac{80\sqrt{2}\,a^{7/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{91\sqrt[4]{3}\,b^{8/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}+$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x^3]*x^9),x]
```

```
[Out] (-80*a^2*Sqrt[a + b/x^3])/(91*b^(8/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x))
- (2*Sqrt[a + b/x^3])/(13*b*x^5) + (20*a*Sqrt[a + b/x^3])/(91*b^2*x^2) + (4
0*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b
^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*El
lipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)/x)], -7 - 4*Sqrt[3]])/(91*b^(8/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a
^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]) - (80*Sqrt[2]*
a^(7/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3
))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3]
)*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]
])/(91*3^(1/4)*b^(8/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^9}\,dx = -\operatorname{Subst}\left(\int \frac{x^7}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2\sqrt{a+\frac{b}{x^3}}}{13bx^5} + \frac{(10a)\operatorname{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{13b}$$

$$= -\frac{2\sqrt{a+\frac{b}{x^3}}}{13bx^5} + \frac{20a\sqrt{a+\frac{b}{x^3}}}{91b^2x^2} - \frac{\left(40a^2\right)\operatorname{Subst}\left(\int \frac{x}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{91b^2}$$

$$= -\frac{2\sqrt{a+\frac{b}{x^3}}}{13bx^5} + \frac{20a\sqrt{a+\frac{b}{x^3}}}{91b^2x^2} - \frac{\left(40a^2\right)\operatorname{Subst}\left(\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{91b^{7/3}} - \frac{\left(40\sqrt{2\left(2-\sqrt{3}\right)}a^{7}\right.}{}$$

$$= -\frac{80a^2\sqrt{a+\frac{b}{x^3}}}{91b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)} - \frac{2\sqrt{a+\frac{b}{x^3}}}{13bx^5} + \frac{20a\sqrt{a+\frac{b}{x^3}}}{91b^2x^2} + \frac{40\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{7/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right.}{}$$

$$91b$$

Mathematica [C] time = 0.02, size = 51, normalized size = 0.09

$$-\frac{2\sqrt{\frac{ax^3}{b}+1}\;{}_2F_1\left(-\frac{13}{6},\frac{1}{2};-\frac{7}{6};-\frac{ax^3}{b}\right)}{13x^8\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x^3]*x^9),x]
```

```
[Out] (-2*Sqrt[1 + (a*x^3)/b]*Hypergeometric2F1[-13/6, 1/2, -7/6, -((a*x^3)/b)])/
(13*Sqrt[a + b/x^3]*x^8)
```

fricas [F] time = 1.08, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\frac{ax^3+b}{x^3}}}{ax^9+bx^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(a+b/x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt((a*x^3 + b)/x^3)/(a*x^9 + b*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(a+b/x^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a + b/x^3)*x^9), x)
```

maple [B] time = 0.02, size = 3547, normalized size = 6.52

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(a+b/x^3)^(1/2),x)
```

```
[Out] -2/273/((a*x^3+b)/x^3)^(1/2)/x^9*(40*I*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1
/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^
2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*3^(1/2)*((a*x^3+b)*x)^(1/2)*x^6*a^2
+720*(-a^2*b)^(2/3)*((a*x^3+b)*x)^(1/2)*x^8*a+720*(-a^2*b)^(1/3)*((a*x^3+b)
*x)^(1/2)*x^9*a^2-240*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3
)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(
1/2)*x^7*a^2*b-63*(a*x^4+b*x)^(1/2)*b^2*((a*x^3+b)*x)^(1/2)*((-a*x+(-a^2*b)
^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(
-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)-240*I*(-a^2*b)^(1/3)*3^(1/2)*((a
*x^3+b)*x)^(1/2)*x^9*a^2+80*I*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2
*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/
a^2*x)^(1/2)*3^(1/2)*x^7*a^2*b+21*I*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3)
)*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b
)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*3^(1/2)*((a*x^3+b)*x)^(1/2)*b^2-240*I*
(-a^2*b)^(2/3)*3^(1/2)*((a*x^3+b)*x)^(1/2)*x^8*a+90*a*(a*x^4+b*x)^(1/2)*x^3
*b*((a*x^3+b)*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/
3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^
(1/2)-240*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*
((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*
b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/
2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/
(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))
/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*3^(1/2)*((a*x^3+b)*x)^(1/2)*x^9*a^2+4
80*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x
+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3
)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(
-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(
-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(
1/2)-3))^(1/2))*(-a^2*b)^(2/3)*3^(1/2)*((a*x^3+b)*x)^(1/2)*x^8*a+240*I*(-(I
*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2
)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)
*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^
2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(
1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^
(1/2))*a^2*b*3^(1/2)*((a*x^3+b)*x)^(1/2)*x^7-120*a^2*(a*x^4+b*x)^(1/2)*x^6*
((a*x^3+b)*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+
(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/
2)+80*I*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/
3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*3^(1/2)*x
^10*a^3-240*I*3^(1/2)*((a*x^3+b)*x)^(1/2)*x^10*a^3-30*I*(a*x^4+b*x)^(1/2)*(
(-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a
*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*3^(1/2)*((a*x^3+b)
*x)^(1/2)*x^3*a*b-240*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3
)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(
1/2)*x^10*a^3+720*((a*x^3+b)*x)^(1/2)*x^10*a^3-720*a^2*b*(-(I*3^(1/2)-3)/(I
*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/
3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^
(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(
1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/
2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3
+b)*x)^(1/2)*x^7-1440*(-a^2*b)^(2/3)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-
a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(
```

```
1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)
-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*
3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I
*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^(1/2)*x^8*a+4
80*a^2*b*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2
*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^
(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-
1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a
*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I
*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^(1/2)*x^7+720*(-a^2*b)^(1/3)*(-(I*3^(1/2)
-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*
b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*
x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/
3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*
x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(
(a*x^3+b)*x)^(1/2)*x^9*a^2+960*(-a^2*b)^(2/3)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)
/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)
^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2
*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*Ellipt
icF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1
/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^(1/2
)*x^8*a-480*(-a^2*b)^(1/3)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/
3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2
))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^
(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)
/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1
)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^(1/2)*x^9*a^2)/b^3/(I*3
^(1/2)-3)/((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(
1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(a+b/x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a + b/x^3)*x^9), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^9\,\sqrt{a+\frac{b}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(a + b/x^3)^(1/2)),x)
```

```
[Out] int(1/(x^9*(a + b/x^3)^(1/2)), x)
```

sympy [A] time = 1.79, size = 39, normalized size = 0.07

$$-\frac{\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{8}{3} \\ \frac{11}{3}\end{matrix}\right| \frac{be^{i\pi}}{ax^3}\right)}{3\sqrt{a}\,x^8\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(a+b/x**3)**(1/2),x)
```

```
[Out] -gamma(8/3)*hyper((1/2, 8/3), (11/3,), b*exp_polar(I*pi)/(a*x**3))/(3*sqrt(
a)*x**8*gamma(11/3))
```

3.2036 $\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^{12}}\,dx$

Optimal. Leaf size=568

$$\frac{1280\sqrt{2}\,a^{10/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{1729\sqrt[4]{3}\,b^{11/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}-\frac{640\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{10/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{1729\sqrt[4]{3}\,b^{11/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

```
[Out] -2/19*(a+b/x^3)^(1/2)/b/x^8+32/247*a*(a+b/x^3)^(1/2)/b^2/x^5-320/1729*a^2*(
a+b/x^3)^(1/2)/b^3/x^2+1280/1729*a^3*(a+b/x^3)^(1/2)/b^(11/3)/(b^(1/3)/x+a^
(1/3)*(1+3^(1/2)))+1280/5187*a^(10/3)*(a^(1/3)+b^(1/3)/x)*EllipticF((b^(1/3
)/x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(
1/2)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)
))^2)^(1/2)*3^(3/4)/b^(11/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/(
b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-640/1729*3^(1/4)*a^(10/3)*(a^(1/3)+
b^(1/3)/x)*EllipticE((b^(1/3)/x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3)*(1+
3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)+b^(2/3)/x^2-a^
(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(11/3)/(a+b/x^3
)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/
2)
```

Rubi [A] time = 0.35, antiderivative size = 568, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {335, 321, 303, 218, 1877}

$$\frac{1280a^3\sqrt{a+\frac{b}{x^3}}}{1729b^{11/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}-\frac{320a^2\sqrt{a+\frac{b}{x^3}}}{1729b^3x^2}+\frac{1280\sqrt{2}\,a^{10/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{}}{\left(1+\sqrt{3}\right)\sqrt[3]{}}\right.\right.}{1729\sqrt[4]{3}\,b^{11/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x^3]*x^12),x]
```

```
[Out] (1280*a^3*Sqrt[a + b/x^3])/(1729*b^(11/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)/
x)) - (2*Sqrt[a + b/x^3])/(19*b*x^8) + (32*a*Sqrt[a + b/x^3])/(247*b^2*x^5)
 - (320*a^2*Sqrt[a + b/x^3])/(1729*b^3*x^2) - (640*3^(1/4)*Sqrt[2 - Sqrt[3]
]*a^(10/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(
1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticE[ArcSin[((1 - Sqrt
[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt
[3]])/(1729*b^(11/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]) + (1280*Sqrt[2]*a^(10/3)*(a^(1/3) +
b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)
/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(1729*3^(1/4)*b^
(11/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)/x)^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^{12}}\,dx = -\operatorname{Subst}\left(\int \frac{x^{10}}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{2\sqrt{a+\frac{b}{x^3}}}{19bx^8} + \frac{(16a)\operatorname{Subst}\left(\int \frac{x^7}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{19b}$$
$$= -\frac{2\sqrt{a+\frac{b}{x^3}}}{19bx^8} + \frac{32a\sqrt{a+\frac{b}{x^3}}}{247b^2x^5} - \frac{\left(160a^2\right)\operatorname{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{247b^2}$$
$$= -\frac{2\sqrt{a+\frac{b}{x^3}}}{19bx^8} + \frac{32a\sqrt{a+\frac{b}{x^3}}}{247b^2x^5} - \frac{320a^2\sqrt{a+\frac{b}{x^3}}}{1729b^3x^2} + \frac{\left(640a^3\right)\operatorname{Subst}\left(\int \frac{x}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{1729b^3}$$
$$= -\frac{2\sqrt{a+\frac{b}{x^3}}}{19bx^8} + \frac{32a\sqrt{a+\frac{b}{x^3}}}{247b^2x^5} - \frac{320a^2\sqrt{a+\frac{b}{x^3}}}{1729b^3x^2} + \frac{\left(640a^3\right)\operatorname{Subst}\left(\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{1729b^{10/3}} +$$
$$= \frac{1280a^3\sqrt{a+\frac{b}{x^3}}}{1729b^{11/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)} - \frac{2\sqrt{a+\frac{b}{x^3}}}{19bx^8} + \frac{32a\sqrt{a+\frac{b}{x^3}}}{247b^2x^5} - \frac{320a^2\sqrt{a+\frac{b}{x^3}}}{1729b^3x^2} - \frac{640\sqrt[4]{3}\sqrt{2}}{}$$

Mathematica [C] time = 0.02, size = 51, normalized size = 0.09

$$-\frac{2\sqrt{\frac{ax^3}{b}+1}\;{}_2F_1\left(-\frac{19}{6}, \frac{1}{2}; -\frac{13}{6}; -\frac{ax^3}{b}\right)}{19x^{11}\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x^3]*x^12),x]
```

```
[Out] (-2*Sqrt[1 + (a*x^3)/b]*Hypergeometric2F1[-19/6, 1/2, -13/6, -((a*x^3)/b)])
/(19*Sqrt[a + b/x^3]*x^11)
```

fricas [F] time = 1.15, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\frac{ax^3+b}{x^3}}}{ax^{12}+bx^9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^12/(a+b/x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt((a*x^3 + b)/x^3)/(a*x^12 + b*x^9), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^12/(a+b/x^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a + b/x^3)*x^12), x)
```

maple [B] time = 0.02, size = 3779, normalized size = 6.65

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^12/(a+b/x^3)^(1/2),x)
```

```
[Out] 2/5187/((a*x^3+b)/x^3)^(1/2)/x^12*(-3840*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-
a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1
/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)
^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE
((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)
+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^(1/2)*(
-a^2*b)^(1/3)*3^(1/2)*x^12*a^3+11520*(-a^2*b)^(2/3)*((a*x^3+b)*x)^(1/2)*x^1
1*a^2+11520*(-a^2*b)^(1/3)*((a*x^3+b)*x)^(1/2)*x^12*a^3-3840*((-a*x+(-a^2*b
)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*
(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*x^10*a^3*b+819*(a*x^4+b*x)^(1/2
)*b^3*((a*x^3+b)*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^
(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*
x)^(1/2)+1280*I*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^
2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*3
^(1/2)*x^10*a^3*b-273*I*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3
^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^
2*b)^(1/3))/a^2*x)^(1/2)*((a*x^3+b)*x)^(1/2)*3^(1/2)*b^3-3840*I*((a*x^3+b)*
x)^(1/2)*(-a^2*b)^(2/3)*3^(1/2)*x^11*a^2-3840*I*((a*x^3+b)*x)^(1/2)*(-a^2*b
)^(1/3)*3^(1/2)*x^12*a^3+1440*a^2*(a*x^4+b*x)^(1/2)*x^6*b*((a*x^3+b)*x)^(1/
2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(
-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)-1008*a*(a*x^4+
b*x)^(1/2)*x^3*b^2*((a*x^3+b)*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1
/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b
)^(1/3))/a^2*x)^(1/2)-11520*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1
/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/
2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)
^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3
)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-
1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a^3*b*((a*x^3+b)*x)^(1/2)*x^10-7680*
(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^
(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(
1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+
(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*
b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-
3))^(1/2))*(-a^2*b)^(1/3)*((a*x^3+b)*x)^(1/2)*x^12*a^3+11520*(-(I*3^(1/2)-3
)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)
^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+
I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)
))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)
^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a
^2*b)^(1/3)*((a*x^3+b)*x)^(1/2)*x^12*a^3+15360*(-(I*3^(1/2)-3)/(I*3^(1/2)-1
)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b
)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^
2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*Ellip
ticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(
1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*((
a*x^3+b)*x)^(1/2)*x^11*a^2-23040*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*
b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*
3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a
^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1
/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(
```

```
1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*((a*x^3+b)*x)^(1
/2)*x^11*a^2+7680*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^
(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+
(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I
*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/
2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^
(1/2))/(I*3^(1/2)-3))^(1/2))*a^3*b*((a*x^3+b)*x)^(1/2)*x^10+7680*I*(-(I*3^(
1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-
a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-
2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)
^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)
)*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2
))*((a*x^3+b)*x)^(1/2)*(-a^2*b)^(2/3)*3^(1/2)*x^11*a^2+3840*I*(-(I*3^(1/2)-
3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b
)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x
+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3
)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x
)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((
a*x^3+b)*x)^(1/2)*a^3*b*3^(1/2)*x^10-480*I*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b
)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*
(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*((a*x^3+b)*x)^(1/2)*3^(1/2)*x^6
*a^2*b+336*I*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^
2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))
/a^2*x)^(1/2)*((a*x^3+b)*x)^(1/2)*3^(1/2)*x^3*a*b^2-1920*a^3*(a*x^4+b*x)^(1
/2)*x^9*((a*x^3+b)*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b
)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^
2*x)^(1/2)+1280*I*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-
a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)
*3^(1/2)*x^13*a^4-3840*I*((a*x^3+b)*x)^(1/2)*3^(1/2)*x^13*a^4-3840*((-a*x+(
-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^
(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*x^13*a^4+11520*((a*x^3+b)
*x)^(1/2)*x^13*a^4+640*I*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*
3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a
^2*b)^(1/3))/a^2*x)^(1/2)*((a*x^3+b)*x)^(1/2)*3^(1/2)*x^9*a^3)/b^4/(I*3^(1/
2)-3)/((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3)
)*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^12/(a+b/x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a + b/x^3)*x^12), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{12}\sqrt{a+\frac{b}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^12*(a + b/x^3)^(1/2)),x)
```

```
[Out] int(1/(x^12*(a + b/x^3)^(1/2)), x)
```

sympy [**A**] time = 2.25, size = 39, normalized size = 0.07

$$-\frac{\Gamma\left(\frac{11}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{11}{3} \\ \frac{14}{3}\end{matrix}\right| \frac{be^{i\pi}}{ax^3}\right)}{3\sqrt{a}\, x^{11}\Gamma\left(\frac{14}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**12/(a+b/x**3)**(1/2),x)
```

```
[Out] -gamma(11/3)*hyper((1/2, 11/3), (14/3,), b*exp_polar(I*pi)/(a*x**3))/(3*sqr
t(a)*x**11*gamma(14/3))
```

3.2037 $\int \frac{x^5}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$

Optimal. Leaf size=95

$$\frac{5b^2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{4a^{7/2}}-\frac{5b^2}{4a^3\sqrt{a+\frac{b}{x^3}}}-\frac{5bx^3}{12a^2\sqrt{a+\frac{b}{x^3}}}+\frac{x^6}{6a\sqrt{a+\frac{b}{x^3}}}$$

```
[Out] 5/4*b^2*arctanh((a+b/x^3)^(1/2)/a^(1/2))/a^(7/2)-5/4*b^2/a^3/(a+b/x^3)^(1/2
)-5/12*b*x^3/a^2/(a+b/x^3)^(1/2)+1/6*x^6/a/(a+b/x^3)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 51, 63, 208}

$$\frac{5b^2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{4a^{7/2}}+\frac{5x^6\sqrt{a+\frac{b}{x^3}}}{6a^2}-\frac{5bx^3\sqrt{a+\frac{b}{x^3}}}{4a^3}-\frac{2x^6}{3a\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b/x^3)^(3/2),x]
```

```
[Out] (-5*b*Sqrt[a + b/x^3]*x^3)/(4*a^3) - (2*x^6)/(3*a*Sqrt[a + b/x^3]) + (5*Sqr
t[a + b/x^3]*x^6)/(6*a^2) + (5*b^2*ArcTanh[Sqrt[a + b/x^3]/Sqrt[a]])/(4*a^(
7/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^3(a+bx)^{3/2}}\,dx, x, \frac{1}{x^3}\right)\right)$$

$$= -\frac{2x^6}{3a\sqrt{a+\frac{b}{x^3}}} - \frac{5\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx, x, \frac{1}{x^3}\right)}{3a}$$

$$= -\frac{2x^6}{3a\sqrt{a+\frac{b}{x^3}}} + \frac{5\sqrt{a+\frac{b}{x^3}}\,x^6}{6a^2} + \frac{(5b)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x^3}\right)}{4a^2}$$

$$= -\frac{5b\sqrt{a+\frac{b}{x^3}}\,x^3}{4a^3} - \frac{2x^6}{3a\sqrt{a+\frac{b}{x^3}}} + \frac{5\sqrt{a+\frac{b}{x^3}}\,x^6}{6a^2} - \frac{\left(5b^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x^3}\right)}{8a^3}$$

$$= -\frac{5b\sqrt{a+\frac{b}{x^3}}\,x^3}{4a^3} - \frac{2x^6}{3a\sqrt{a+\frac{b}{x^3}}} + \frac{5\sqrt{a+\frac{b}{x^3}}\,x^6}{6a^2} - \frac{(5b)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x^3}}\right)}{4a^3}$$

$$= -\frac{5b\sqrt{a+\frac{b}{x^3}}\,x^3}{4a^3} - \frac{2x^6}{3a\sqrt{a+\frac{b}{x^3}}} + \frac{5\sqrt{a+\frac{b}{x^3}}\,x^6}{6a^2} + \frac{5b^2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{4a^{7/2}}$$

Mathematica [A] time = 0.06, size = 96, normalized size = 1.01

$$\frac{\sqrt{a}\,x^{3/2}\left(2a^2x^6 - 5abx^3 - 15b^2\right) + 15b^{5/2}\sqrt{\frac{ax^3}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{b}}\right)}{12a^{7/2}x^{3/2}\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b/x^3)^(3/2),x]
```

```
[Out] (Sqrt[a]*x^(3/2)*(-15*b^2 - 5*a*b*x^3 + 2*a^2*x^6) + 15*b^(5/2)*Sqrt[1 + (a
*x^3)/b]*ArcSinh[(Sqrt[a]*x^(3/2))/Sqrt[b]])/(12*a^(7/2)*Sqrt[a + b/x^3]*x^
(3/2))
```

fricas [A] time = 1.18, size = 243, normalized size = 2.56

$$\left[\frac{15\left(ab^2x^3+b^3\right)\sqrt{a}\,\log\left(-8\,a^2x^6 - 8\,abx^3 - b^2 - 4\left(2\,ax^6+bx^3\right)\sqrt{a}\,\sqrt{\frac{ax^3+b}{x^3}}\right) + 4\left(2\,a^3x^9 - 5\,a^2bx^6 - 15\,ab^2x^3\right.}{48\left(a^5x^3+a^4b\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/x^3)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*(a*b^2*x^3 + b^3)*sqrt(a)*log(-8*a^2*x^6 - 8*a*b*x^3 - b^2 - 4*(2
*a*x^6 + b*x^3)*sqrt(a)*sqrt((a*x^3 + b)/x^3)) + 4*(2*a^3*x^9 - 5*a^2*b*x^6
```

```
 - 15*a*b^2*x^3)*sqrt((a*x^3 + b)/x^3))/(a^5*x^3 + a^4*b), -1/24*(15*(a*b^2
*x^3 + b^3)*sqrt(-a)*arctan(2*sqrt(-a)*x^3*sqrt((a*x^3 + b)/x^3)/(2*a*x^3 +
 b)) - 2*(2*a^3*x^9 - 5*a^2*b*x^6 - 15*a*b^2*x^3)*sqrt((a*x^3 + b)/x^3))/(a
^5*x^3 + a^4*b)]
```

giac [A] time = 0.25, size = 117, normalized size = 1.23

$$-\frac{1}{12}\,b^2\left(\frac{15\arctan\left(\frac{\sqrt{\frac{ax^3+b}{x^3}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^3}+\frac{8}{a^3\sqrt{\frac{ax^3+b}{x^3}}}-\frac{9\,a\sqrt{\frac{ax^3+b}{x^3}}-\frac{7\left(ax^3+b\right)\sqrt{\frac{ax^3+b}{x^3}}}{x^3}}{\left(a-\frac{ax^3+b}{x^3}\right)^2a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/x^3)^(3/2),x, algorithm="giac")
```

```
[Out] -1/12*b^2*(15*arctan(sqrt((a*x^3 + b)/x^3)/sqrt(-a))/(sqrt(-a)*a^3) + 8/(a^
3*sqrt((a*x^3 + b)/x^3)) - (9*a*sqrt((a*x^3 + b)/x^3) - 7*(a*x^3 + b)*sqrt(
(a*x^3 + b)/x^3)/x^3)/((a - (a*x^3 + b)/x^3)^2*a^3))
```

maple [C] time = 0.05, size = 3910, normalized size = 41.16

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a+b/x^3)^(3/2),x)
```

```
[Out] 1/12/((a*x^3+b)/x^3)^(3/2)/x^5*(a*x^3+b)/a^5*(-8*I*((-a*x+(-a^2*b)^(1/3))*(
2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(
1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*3^(1/2)*x^2*a^2*b^2-7*I*(a*x^4+b*x)^(1/2)
*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2
*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*3^(1/2)*((a*x^3+
b)*x)^(1/2)*x*a^2*b+90*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)
)*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))
/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1
/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)/
(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),
((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(
2/3)*3^(1/2)*((a*x^3+b)*x)^(1/2)*b^2+180*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-
a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1
/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)
^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF
((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)
+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*3^(1/2
)*((a*x^3+b)*x)^(1/2)*x*a*b^2+90*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^
2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+
I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(
-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticPi((-(I*3
^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),(I*3^(1/2)-1)/(I*3
^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*
3^(1/2)*((a*x^3+b)*x)^(1/2)*x^2*a^2*b^2-90*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/
(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^
(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*
b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*Ellipti
cF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/
2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*3^(1
/2)*((a*x^3+b)*x)^(1/2)*b^2+90*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)
^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^
```

```
(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2
*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2
)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/
2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^(1/2)*x^2*a^2*b^2+2
*I*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)
+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1
/2)*3^(1/2)*((a*x^3+b)*x)^(1/2)*x^4*a^3-90*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-
a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1
/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)
^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticP
i((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),(I*3^(1/2)
-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))
^(1/2))*((a*x^3+b)*x)^(1/2)*x^2*a^2*b^2-6*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)
^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(
-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*((a*x^3+b)*x)^(1/2)*x^4*a^3-180*
(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^
(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(
1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+
(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*
b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-
3))^(1/2))*(-a^2*b)^(1/3)*((a*x^3+b)*x)^(1/2)*x*a*b^2+180*(-(I*3^(1/2)-3)/(
I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1
/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3
^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^
(1/2)*EllipticPi((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(
1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))
/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*((a*x^3+b)*x)^(1/2)*x*a*b^2-180*I*(-(
I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/
2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2
)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a
^2*b)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)
^(1/3))*a*x)^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)
/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*3^(1/2)*((a*x^3+b)*x)^(
1/2)*x*a*b^2+90*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1
/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-
a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3
^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)
-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1
/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*((a*x^3+b)*x)^(1/2)*b^2-90*(-(I*3
^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*
(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*(
(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*
b)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1
/3))*a*x)^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1
+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*((a*x^3+b)*x)^(1/2)*b^2-90
*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I
*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3))
)^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a
*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a
^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/
2)-3))^(1/2))*3^(1/2)*((a*x^3+b)*x)^(1/2)*x^2*a^2*b^2+21*(a*x^4+b*x)^(1/2)*
((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*
a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*((a*x^3+b)*x)^(1/
2)*x*a^2*b+24*x^2*b^2*a^2*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^
(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*
x)^(1/2))/(I*3^(1/2)-3)/((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1
/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)
^(1/2)
```

maxima [A] time = 1.98, size = 122, normalized size = 1.28

$$-\frac{15\left(a+\frac{b}{x^3}\right)^2 b^2-25\left(a+\frac{b}{x^3}\right)ab^2+8a^2b^2}{12\left(\left(a+\frac{b}{x^3}\right)^{\frac{5}{2}}a^3-2\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}a^4+\sqrt{a+\frac{b}{x^3}}\,a^5\right)}-\frac{5b^2\log\left(\frac{\sqrt{a+\frac{b}{x^3}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^3}}+\sqrt{a}}\right)}{8a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/x^3)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/12*(15*(a + b/x^3)^2*b^2 - 25*(a + b/x^3)*a*b^2 + 8*a^2*b^2)/((a + b/x^3
)^(5/2)*a^3 - 2*(a + b/x^3)^(3/2)*a^4 + sqrt(a + b/x^3)*a^5) - 5/8*b^2*log(
(sqrt(a + b/x^3) - sqrt(a))/(sqrt(a + b/x^3) + sqrt(a)))/a^(7/2)
```

mupad [B] time = 1.93, size = 96, normalized size = 1.01

$$\frac{x^6\sqrt{a+\frac{b}{x^3}}}{6a^2}-\frac{2b^2}{3a^3\sqrt{a+\frac{b}{x^3}}}+\frac{5b^2\,\ln\left(x^6\left(\sqrt{a+\frac{b}{x^3}}-\sqrt{a}\right)\left(\sqrt{a+\frac{b}{x^3}}+\sqrt{a}\right)^3\right)}{8a^{7/2}}-\frac{7bx^3\sqrt{a+\frac{b}{x^3}}}{12a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b/x^3)^(3/2),x)
```

```
[Out] (x^6*(a + b/x^3)^(1/2))/(6*a^2) - (2*b^2)/(3*a^3*(a + b/x^3)^(1/2)) + (5*b^
2*log(x^6*((a + b/x^3)^(1/2) - a^(1/2))*((a + b/x^3)^(1/2) + a^(1/2))^3))/(
8*a^(7/2)) - (7*b*x^3*(a + b/x^3)^(1/2))/(12*a^3)
```

sympy [A] time = 6.16, size = 110, normalized size = 1.16

$$\frac{x^{\frac{15}{2}}}{6a\sqrt{b}\sqrt{\frac{ax^3}{b}+1}}-\frac{5\sqrt{b}\,x^{\frac{9}{2}}}{12a^2\sqrt{\frac{ax^3}{b}+1}}-\frac{5b^{\frac{3}{2}}x^{\frac{3}{2}}}{4a^3\sqrt{\frac{ax^3}{b}+1}}+\frac{5b^2\,\operatorname{asinh}\left(\frac{\sqrt{a}\,x^{\frac{3}{2}}}{\sqrt{b}}\right)}{4a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a+b/x**3)**(3/2),x)
```

```
[Out] x**(15/2)/(6*a*sqrt(b)*sqrt(a*x**3/b + 1)) - 5*sqrt(b)*x**(9/2)/(12*a**2*sq
rt(a*x**3/b + 1)) - 5*b**(3/2)*x**(3/2)/(4*a**3*sqrt(a*x**3/b + 1)) + 5*b**
2*asinh(sqrt(a)*x**(3/2)/sqrt(b))/(4*a**(7/2))
```

3.2038 $\int \frac{x^2}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$

Optimal. Leaf size=64

$$-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{b}{a^2\sqrt{a+\frac{b}{x^3}}}+\frac{x^3}{3a\sqrt{a+\frac{b}{x^3}}}$$

```
[Out] -b*arctanh((a+b/x^3)^(1/2)/a^(1/2))/a^(5/2)+b/a^2/(a+b/x^3)^(1/2)+1/3*x^3/a
/(a+b/x^3)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 51, 63, 208}

$$\frac{x^3\sqrt{a+\frac{b}{x^3}}}{a^2}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{2x^3}{3a\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b/x^3)^(3/2),x]
```

```
[Out] (-2*x^3)/(3*a*Sqrt[a + b/x^3]) + (Sqrt[a + b/x^3]*x^3)/a^2 - (b*ArcTanh[Sqr
t[a + b/x^3]/Sqrt[a]])/a^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^2}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx, x, \frac{1}{x^3}\right)\right)$$

$$= -\frac{2x^3}{3a\sqrt{a+\frac{b}{x^3}}} - \frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x^3}\right)}{a}$$

$$= -\frac{2x^3}{3a\sqrt{a+\frac{b}{x^3}}} + \frac{\sqrt{a+\frac{b}{x^3}}\,x^3}{a^2} + \frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x^3}\right)}{2a^2}$$

$$= -\frac{2x^3}{3a\sqrt{a+\frac{b}{x^3}}} + \frac{\sqrt{a+\frac{b}{x^3}}\,x^3}{a^2} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x^3}}\right)}{a^2}$$

$$= -\frac{2x^3}{3a\sqrt{a+\frac{b}{x^3}}} + \frac{\sqrt{a+\frac{b}{x^3}}\,x^3}{a^2} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{a^{5/2}}$$

Mathematica [A] time = 0.04, size = 84, normalized size = 1.31

$$\frac{\sqrt{a}\,x^{3/2}\left(ax^3+3b\right) - 3b^{3/2}\sqrt{\frac{ax^3}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{b}}\right)}{3a^{5/2}x^{3/2}\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b/x^3)^(3/2),x]
```

```
[Out] (Sqrt[a]*x^(3/2)*(3*b + a*x^3) - 3*b^(3/2)*Sqrt[1 + (a*x^3)/b]*ArcSinh[(Sqr
t[a]*x^(3/2))/Sqrt[b]])/(3*a^(5/2)*Sqrt[a + b/x^3]*x^(3/2))
```

fricas [A] time = 1.18, size = 215, normalized size = 3.36

$$\left[\frac{3\left(abx^3+b^2\right)\sqrt{a}\,\log\left(-8\,a^2x^6-8\,abx^3-b^2+4\left(2\,ax^6+bx^3\right)\sqrt{a}\,\sqrt{\frac{ax^3+b}{x^3}}\right)+4\left(a^2x^6+3\,abx^3\right)\sqrt{\frac{ax^3+b}{x^3}}}{12\left(a^4x^3+a^3b\right)}, \frac{3\left(abx^3+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^3)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a*b*x^3 + b^2)*sqrt(a)*log(-8*a^2*x^6 - 8*a*b*x^3 - b^2 + 4*(2*a*
x^6 + b*x^3)*sqrt(a)*sqrt((a*x^3 + b)/x^3)) + 4*(a^2*x^6 + 3*a*b*x^3)*sqrt(
(a*x^3 + b)/x^3))/(a^4*x^3 + a^3*b), 1/6*(3*(a*b*x^3 + b^2)*sqrt(-a)*arctan
(2*sqrt(-a)*x^3*sqrt((a*x^3 + b)/x^3)/(2*a*x^3 + b)) + 2*(a^2*x^6 + 3*a*b*x
^3)*sqrt((a*x^3 + b)/x^3))/(a^4*x^3 + a^3*b)]
```

giac [A] time = 0.24, size = 97, normalized size = 1.52

$$\frac{1}{3}b\left(\frac{3\arctan\left(\frac{\sqrt{\frac{ax^3+b}{x^3}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{2a-\frac{3\left(ax^3+b\right)}{x^3}}{\left(a\sqrt{\frac{ax^3+b}{x^3}}-\frac{\left(ax^3+b\right)\sqrt{\frac{ax^3+b}{x^3}}}{x^3}\right)a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*b*(3*arctan(sqrt((a*x^3 + b)/x^3)/sqrt(-a))/(sqrt(-a)*a^2) + (2*a - 3*(
a*x^3 + b)/x^3)/((a*sqrt((a*x^3 + b)/x^3) - (a*x^3 + b)*sqrt((a*x^3 + b)/x^
3)/x^3)*a^2))
```

maple [C] time = 0.02, size = 3664, normalized size = 57.25

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b/x^3)^(3/2),x)
```

```
[Out] 1/3/((a*x^3+b)/x^3)^(3/2)/x^5*(a*x^3+b)/a^4*(18*I*(-(I*3^(1/2)-3)/(I*3^(1/2
)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^
2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(
-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*El
lipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*
3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*3^(1/2)*((a*x^
3+b)*x)^(1/2)*x^2*a^2*b+I*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I
*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-
a^2*b)^(1/3))/a^2*x)^(1/2)*3^(1/2)*((a*x^3+b)*x)^(1/2)*x*a^2+18*I*(-(I*3^(1
/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a
^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2
*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^
(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))
*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)
)*(-a^2*b)^(2/3)*3^(1/2)*((a*x^3+b)*x)^(1/2)*b-36*I*(-(I*3^(1/2)-3)/(I*3^(1
/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-
a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)
*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*
EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((
I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/
3)*3^(1/2)*((a*x^3+b)*x)^(1/2)*x*a*b-18*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a
*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/
3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^
(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticPi
((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),(I*3^(1/2)-
1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^
(1/2))*3^(1/2)*((a*x^3+b)*x)^(1/2)*x^2*a^2*b-18*(-(I*3^(1/2)-3)/(I*3^(1/2)-
1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*
b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a
^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*Elli
pticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^
(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^(1
/2)*x^2*a^2*b-18*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)
^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x
+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(
```

```
I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)/(I*3^(
1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^
(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*3
^(1/2)*((a*x^3+b)*x)^(1/2)*b+18*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b
)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3
^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^
2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1
/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),(I*3^(1/2)-1)/(I*3^(1
/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a
*x^3+b)*x)^(1/2)*x^2*a^2*b+36*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^
(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(
1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*
b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)
-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2
)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*((a*x^3+b)*x)^(1/2)
*x*a*b-36*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((
2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)
^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)
-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(
-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*
(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*((a*x^3+b)
*x)^(1/2)*x*a*b+2*I*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+
(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/
2)*3^(1/2)*x^2*a^2*b-18*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))
*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/
(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/
3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I
*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(
1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*((a*x^3+b)*x)^(1/2)*b+18*
(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^
(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(
1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+
(-a^2*b)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2
*b)^(1/3))*a*x)^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)
-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*((a*x^3+b)*x)^(1/2)*
b+36*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a
*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1
/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)
/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*
x+(-a^2*b)^(1/3))*a*x)^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*
3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*3^(1/2)*((a*x
^3+b)*x)^(1/2)*x*a*b-3*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^
(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2
*b)^(1/3))/a^2*x)^(1/2)*((a*x^3+b)*x)^(1/2)*x*a^2-6*x^2*b*a^2*((-a*x+(-a^2*
b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)
*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2))/(I*3^(1/2)-3)/((-a*x+(-a^2*b)
^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(
-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)
```

maxima [A] time = 1.96, size = 86, normalized size = 1.34

$$\frac{3\left(a+\frac{b}{x^3}\right)b-2ab}{3\left(\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}a^2-\sqrt{a+\frac{b}{x^3}}\,a^3\right)}+\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x^3}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^3}}+\sqrt{a}}\right)}{2\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^3)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*(a + b/x^3)*b - 2*a*b)/((a + b/x^3)^(3/2)*a^2 - sqrt(a + b/x^3)*a^3)
 + 1/2*b*log((sqrt(a + b/x^3) - sqrt(a))/(sqrt(a + b/x^3) + sqrt(a)))/a^(5/
2)
```

mupad [B] time = 1.76, size = 73, normalized size = 1.14

$$\frac{b}{a^2\sqrt{a+\frac{b}{x^3}}}+\frac{b\,\ln\left(x^6\left(\sqrt{a+\frac{b}{x^3}}-\sqrt{a}\right)^3\left(\sqrt{a+\frac{b}{x^3}}+\sqrt{a}\right)\right)}{2\,a^{5/2}}+\frac{x^3}{3\,a\sqrt{a+\frac{b}{x^3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b/x^3)^(3/2),x)
```

```
[Out] b/(a^2*(a + b/x^3)^(1/2)) + (b*log(x^6*((a + b/x^3)^(1/2) - a^(1/2))^3*((a
+ b/x^3)^(1/2) + a^(1/2))))/(2*a^(5/2)) + x^3/(3*a*(a + b/x^3)^(1/2))
```

sympy [A] time = 3.58, size = 73, normalized size = 1.14

$$\frac{x^{\frac{9}{2}}}{3a\sqrt{b}\sqrt{\frac{ax^3}{b}+1}}+\frac{\sqrt{b}\,x^{\frac{3}{2}}}{a^2\sqrt{\frac{ax^3}{b}+1}}-\frac{b\,\operatorname{asinh}\left(\frac{\sqrt{a}\,x^{\frac{3}{2}}}{\sqrt{b}}\right)}{a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b/x**3)**(3/2),x)
```

```
[Out] x**(9/2)/(3*a*sqrt(b)*sqrt(a*x**3/b + 1)) + sqrt(b)*x**(3/2)/(a**2*sqrt(a*x
**3/b + 1)) - b*asinh(sqrt(a)*x**(3/2)/sqrt(b))/a**(5/2)
```

3.2039 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x}\,dx$

Optimal. Leaf size=46

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{3a^{3/2}}-\frac{2}{3a\sqrt{a+\frac{b}{x^3}}}$$

```
[Out] 2/3*arctanh((a+b/x^3)^(1/2)/a^(1/2))/a^(3/2)-2/3/a/(a+b/x^3)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 51, 63, 208}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{3a^{3/2}}-\frac{2}{3a\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^3)^(3/2)*x),x]
```

```
[Out] -2/(3*a*Sqrt[a + b/x^3]) + (2*ArcTanh[Sqrt[a + b/x^3]/Sqrt[a]])/(3*a^(3/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x}\,dx = -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \frac{1}{x^3}\right)\right)$$

$$= -\frac{2}{3a\sqrt{a+\frac{b}{x^3}}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x^3}\right)}{3a}$$

$$= -\frac{2}{3a\sqrt{a+\frac{b}{x^3}}} - \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x^3}}\right)}{3ab}$$

$$= -\frac{2}{3a\sqrt{a+\frac{b}{x^3}}} + \frac{2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{3a^{3/2}}$$

Mathematica [A] time = 0.08, size = 75, normalized size = 1.63

$$\frac{2\left(\sqrt{b}\sqrt{\frac{ax^3}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{b}}\right) - \sqrt{a}\,x^{3/2}\right)}{3a^{3/2}x^{3/2}\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^3)^(3/2)*x),x]
```

```
[Out] (2*(-(Sqrt[a]*x^(3/2)) + Sqrt[b]*Sqrt[1 + (a*x^3)/b]*ArcSinh[(Sqrt[a]*x^(3/
2))/Sqrt[b]]))/(3*a^(3/2)*Sqrt[a + b/x^3]*x^(3/2))
```

fricas [B] time = 1.39, size = 186, normalized size = 4.04

$$\left[-\frac{4\,ax^3\sqrt{\frac{ax^3+b}{x^3}} - \left(ax^3+b\right)\sqrt{a}\,\log\left(-8\,a^2x^6 - 8\,abx^3 - b^2 - 4\left(2\,ax^6+bx^3\right)\sqrt{a}\,\sqrt{\frac{ax^3+b}{x^3}}\right)}{6\left(a^3x^3+a^2b\right)}, -\frac{2\,ax^3\sqrt{\frac{ax^3+b}{x^3}} + \left(ax\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [-1/6*(4*a*x^3*sqrt((a*x^3 + b)/x^3) - (a*x^3 + b)*sqrt(a)*log(-8*a^2*x^6 -
 8*a*b*x^3 - b^2 - 4*(2*a*x^6 + b*x^3)*sqrt(a)*sqrt((a*x^3 + b)/x^3)))/(a^3
*x^3 + a^2*b), -1/3*(2*a*x^3*sqrt((a*x^3 + b)/x^3) + (a*x^3 + b)*sqrt(-a)*a
rctan(2*sqrt(-a)*x^3*sqrt((a*x^3 + b)/x^3)/(2*a*x^3 + b)))/(a^3*x^3 + a^2*b
)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}} x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate(1/((a + b/x^3)^(3/2)*x), x)
```

maple [C] time = 0.01, size = 3438, normalized size = 74.74

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^3)^(3/2)/x,x)
```

```
[Out] -2/3/((a*x^3+b)/x^3)^(3/2)/x^5*(a*x^3+b)/a^3*(6*I*(-(I*3^(1/2)-3)/(I*3^(1/2
)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^
2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(
-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*El
lipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*
3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^
(1/2)*3^(1/2)*x^2*a^2-6*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3
))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2)
)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(
1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)
/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3)
,((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b
)*x)^(1/2)*3^(1/2)*x^2*a^2-12*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b
)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3
^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^
2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/
2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1
/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^(1/2)*(-a^2*b)^(1/
3)*3^(1/2)*x*a+12*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x
)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*
x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/
(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)/(I*3^
(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3
^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^(
1/2)*(-a^2*b)^(1/3)*3^(1/2)*x*a+6*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a
^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1
+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-
(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3
^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*
3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^(1/2)*(-a^2*b)
^(2/3)*3^(1/2)-6*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(
1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(
-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*
3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2
)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(
1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^(1/2)*x^2*a^2-6*I*(-(I*3^(1/2)-3)
/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^
(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I
*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))
)^(1/2)*EllipticPi((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)
^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2
))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^(1/2)*(-a^2*b)^(2/3)*3^(1/2)+6*(-(I*
3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)
*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*
((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2
*b)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(
1/3))*a*x)^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(
1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^(1/2)*x^2*a^2+12*(-(I*3^(1
/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a
```

```
^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2
*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^
(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))
*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)
)*((a*x^3+b)*x)^(1/2)*(-a^2*b)^(1/3)*x*a-12*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(
-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(
1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b
)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*Elliptic
Pi((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),(I*3^(1/2
)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3)
)^(1/2))*((a*x^3+b)*x)^(1/2)*(-a^2*b)^(1/3)*x*a-6*(-(I*3^(1/2)-3)/(I*3^(1/2
)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^
2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(
-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*El
lipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*
3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^
(1/2)*(-a^2*b)^(2/3)+6*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*
a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(
-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3
))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)/(I
*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((
I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x
)^(1/2)*(-a^2*b)^(2/3)+I*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(
1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x
)^(1/2)*3^(1/2)*x^2*a^2-3*x^2*a^2*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(
-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/
3))/a^2*x)^(1/2))/(I*3^(1/2)-3)/((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a
^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3)
)/a^2*x)^(1/2)
```

maxima [A] time = 1.92, size = 52, normalized size = 1.13

$$-\frac{\log\left(\frac{\sqrt{a+\frac{b}{x^3}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^3}}+\sqrt{a}}\right)}{3\,a^{\frac{3}{2}}}-\frac{2}{3\sqrt{a+\frac{b}{x^3}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] -1/3*log((sqrt(a + b/x^3) - sqrt(a))/(sqrt(a + b/x^3) + sqrt(a)))/a^(3/2) -
 2/3/(sqrt(a + b/x^3)*a)
```

mupad [B] time = 1.39, size = 34, normalized size = 0.74

$$\frac{2\,\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{3\,a^{3/2}}-\frac{2}{3\,a\sqrt{a+\frac{b}{x^3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b/x^3)^(3/2)),x)
```

```
[Out] (2*atanh((a + b/x^3)^(1/2)/a^(1/2)))/(3*a^(3/2)) - 2/(3*a*(a + b/x^3)^(1/2)
)
```

sympy [**B**] time = 2.22, size = 187, normalized size = 4.07

$$-\frac{2a^3x^3\sqrt{1+\frac{b}{ax^3}}}{3a^{\frac{9}{2}}x^3+3a^{\frac{7}{2}}b}-\frac{a^3x^3\log\left(\frac{b}{ax^3}\right)}{3a^{\frac{9}{2}}x^3+3a^{\frac{7}{2}}b}+\frac{2a^3x^3\log\left(\sqrt{1+\frac{b}{ax^3}}+1\right)}{3a^{\frac{9}{2}}x^3+3a^{\frac{7}{2}}b}-\frac{a^2b\log\left(\frac{b}{ax^3}\right)}{3a^{\frac{9}{2}}x^3+3a^{\frac{7}{2}}b}+\frac{2a^2b\log\left(\sqrt{1+\frac{b}{ax^3}}+1\right)}{3a^{\frac{9}{2}}x^3+3a^{\frac{7}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**3)**(3/2)/x,x)
```

```
[Out] -2*a**3*x**3*sqrt(1 + b/(a*x**3))/(3*a**(9/2)*x**3 + 3*a**(7/2)*b) - a**3*x
**3*log(b/(a*x**3))/(3*a**(9/2)*x**3 + 3*a**(7/2)*b) + 2*a**3*x**3*log(sqrt
(1 + b/(a*x**3)) + 1)/(3*a**(9/2)*x**3 + 3*a**(7/2)*b) - a**2*b*log(b/(a*x*
*3))/(3*a**(9/2)*x**3 + 3*a**(7/2)*b) + 2*a**2*b*log(sqrt(1 + b/(a*x**3)) +
 1)/(3*a**(9/2)*x**3 + 3*a**(7/2)*b)
```

3.2040 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^4}\,dx$

Optimal. Leaf size=18

$$\frac{2}{3b\sqrt{a+\frac{b}{x^3}}}$$

```
[Out] 2/3/b/(a+b/x^3)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$\frac{2}{3b\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^3)^(3/2)*x^4),x]
```

```
[Out] 2/(3*b*Sqrt[a + b/x^3])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^4}\,dx = \frac{2}{3b\sqrt{a+\frac{b}{x^3}}}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{2}{3b\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^3)^(3/2)*x^4),x]
```

```
[Out] 2/(3*b*Sqrt[a + b/x^3])
```

fricas [B] time = 0.88, size = 30, normalized size = 1.67

$$\frac{2x^3\sqrt{\frac{ax^3+b}{x^3}}}{3\left(abx^3+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] 2/3*x^3*sqrt((a*x^3 + b)/x^3)/(a*b*x^3 + b^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}} x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(3/2)/x^4,x, algorithm="giac")
[Out] integrate(1/((a + b/x^3)^(3/2)*x^4), x)
```

maple [A] time = 0.01, size = 29, normalized size = 1.61

$$\frac{\frac{2a\,x^3}{3}+\frac{2b}{3}}{\left(\frac{a\,x^3+b}{x^3}\right)^{\frac{3}{2}} b\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^3)^(3/2)/x^4,x)
[Out] 2/3*(a*x^3+b)/x^3/b/((a*x^3+b)/x^3)^(3/2)
```

maxima [A] time = 0.83, size = 14, normalized size = 0.78

$$\frac{2}{3\sqrt{a+\frac{b}{x^3}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(3/2)/x^4,x, algorithm="maxima")
[Out] 2/3/(sqrt(a + b/x^3)*b)
```

mupad [B] time = 1.22, size = 14, normalized size = 0.78

$$\frac{2}{3\,b\sqrt{a+\frac{b}{x^3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b/x^3)^(3/2)),x)
[Out] 2/(3*b*(a + b/x^3)^(1/2))
```

sympy [A] time = 3.20, size = 27, normalized size = 1.50

$$\begin{cases} \frac{2}{3b\sqrt{a+\frac{b}{x^3}}} & \text{for } b \neq 0 \\ -\frac{1}{3a^{\frac{3}{2}}x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**3)**(3/2)/x**4,x)
[Out] Piecewise((2/(3*b*sqrt(a + b/x**3)), Ne(b, 0)), (-1/(3*a**(3/2)*x**3), True
))
```

3.2041 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^7}\,dx$

Optimal. Leaf size=38

$$-\frac{2a}{3b^2\sqrt{a+\frac{b}{x^3}}}-\frac{2\sqrt{a+\frac{b}{x^3}}}{3b^2}$$

```
[Out] -2/3*a/b^2/(a+b/x^3)^(1/2)-2/3*(a+b/x^3)^(1/2)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{2a}{3b^2\sqrt{a+\frac{b}{x^3}}}-\frac{2\sqrt{a+\frac{b}{x^3}}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^3)^(3/2)*x^7),x]
```

```
[Out] (-2*a)/(3*b^2*Sqrt[a + b/x^3]) - (2*Sqrt[a + b/x^3])/(3*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^7}\,dx &= -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{x}{(a+bx)^{3/2}}\,dx, x, \frac{1}{x^3}\right)\right)\\ &= -\left(\frac{1}{3}\,\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^{3/2}}+\frac{1}{b\sqrt{a+bx}}\right)dx, x, \frac{1}{x^3}\right)\right)\\ &= -\frac{2a}{3b^2\sqrt{a+\frac{b}{x^3}}}-\frac{2\sqrt{a+\frac{b}{x^3}}}{3b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.76

$$-\frac{2\left(2ax^3+b\right)}{3b^2x^3\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^3)^(3/2)*x^7),x]
[Out] (-2*(b + 2*a*x^3))/(3*b^2*Sqrt[a + b/x^3]*x^3)
```

fricas [A] time = 0.96, size = 37, normalized size = 0.97

$$-\frac{2\left(2\,ax^3+b\right)\sqrt{\frac{ax^3+b}{x^3}}}{3\left(ab^2x^3+b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(3/2)/x^7,x, algorithm="fricas")
[Out] -2/3*(2*a*x^3 + b)*sqrt((a*x^3 + b)/x^3)/(a*b^2*x^3 + b^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(3/2)/x^7,x, algorithm="giac")
[Out] integrate(1/((a + b/x^3)^(3/2)*x^7), x)
```

maple [A] time = 0.01, size = 37, normalized size = 0.97

$$-\frac{2\left(a\,x^3+b\right)\left(2a\,x^3+b\right)}{3\left(\frac{a\,x^3+b}{x^3}\right)^{\frac{3}{2}}b^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^3)^(3/2)/x^7,x)
[Out] -2/3*(a*x^3+b)*(2*a*x^3+b)/x^6/b^2/((a*x^3+b)/x^3)^(3/2)
```

maxima [A] time = 0.87, size = 30, normalized size = 0.79

$$-\frac{2\sqrt{a+\frac{b}{x^3}}}{3\,b^2}-\frac{2\,a}{3\sqrt{a+\frac{b}{x^3}}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(3/2)/x^7,x, algorithm="maxima")
[Out] -2/3*sqrt(a + b/x^3)/b^2 - 2/3*a/(sqrt(a + b/x^3)*b^2)
```

mupad [B] time = 1.25, size = 31, normalized size = 0.82

$$-\frac{2\sqrt{a+\frac{b}{x^3}}\,\left(2\,a\,x^3+b\right)}{3\,b^2\,\left(a\,x^3+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b/x^3)^(3/2)),x)
[Out] -(2*(a + b/x^3)^(1/2)*(b + 2*a*x^3))/(3*b^2*(b + a*x^3))
```

sympy [**A**] time = 6.15, size = 51, normalized size = 1.34

$$\begin{cases} -\frac{4a}{3b^2\sqrt{a+\frac{b}{x^3}}} - \frac{2}{3bx^3\sqrt{a+\frac{b}{x^3}}} & \text{for } b \neq 0 \\ -\frac{1}{6a^{\frac{3}{2}}x^6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**3)**(3/2)/x**7,x)
```

```
[Out] Piecewise((-4*a/(3*b**2*sqrt(a + b/x**3)) - 2/(3*b*x**3*sqrt(a + b/x**3)),
Ne(b, 0)), (-1/(6*a**(3/2)*x**6), True))
```

3.2042 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^{10}}\,dx$

Optimal. Leaf size=59

$$\frac{2a^2}{3b^3\sqrt{a+\frac{b}{x^3}}}+\frac{4a\sqrt{a+\frac{b}{x^3}}}{3b^3}-\frac{2\left(a+\frac{b}{x^3}\right)^{3/2}}{9b^3}$$

```
[Out] -2/9*(a+b/x^3)^(3/2)/b^3+2/3*a^2/b^3/(a+b/x^3)^(1/2)+4/3*a*(a+b/x^3)^(1/2)/
b^3
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{2a^2}{3b^3\sqrt{a+\frac{b}{x^3}}}+\frac{4a\sqrt{a+\frac{b}{x^3}}}{3b^3}-\frac{2\left(a+\frac{b}{x^3}\right)^{3/2}}{9b^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^3)^(3/2)*x^10),x]
```

```
[Out] (2*a^2)/(3*b^3*Sqrt[a + b/x^3]) + (4*a*Sqrt[a + b/x^3])/(3*b^3) - (2*(a + b
/x^3)^(3/2))/(9*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^{10}}\,dx &= -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{(a+bx)^{3/2}}\,dx, x, \frac{1}{x^3}\right)\right)\\ &= -\left(\frac{1}{3}\,\text{Subst}\left(\int \left(\frac{a^2}{b^2(a+bx)^{3/2}}-\frac{2a}{b^2\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b^2}\right)dx, x, \frac{1}{x^3}\right)\right)\\ &= \frac{2a^2}{3b^3\sqrt{a+\frac{b}{x^3}}}+\frac{4a\sqrt{a+\frac{b}{x^3}}}{3b^3}-\frac{2\left(a+\frac{b}{x^3}\right)^{3/2}}{9b^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.71

$$\frac{2\left(8a^2x^6+4abx^3-b^2\right)}{9b^3x^6\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^3)^(3/2)*x^10),x]
```

```
[Out] (2*(-b^2 + 4*a*b*x^3 + 8*a^2*x^6))/(9*b^3*Sqrt[a + b/x^3]*x^6)
```

fricas [A] time = 0.87, size = 54, normalized size = 0.92

$$\frac{2\left(8\,a^2x^6+4\,abx^3-b^2\right)\sqrt{\frac{ax^3+b}{x^3}}}{9\left(ab^3x^6+b^4x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(3/2)/x^10,x, algorithm="fricas")
```

```
[Out] 2/9*(8*a^2*x^6 + 4*a*b*x^3 - b^2)*sqrt((a*x^3 + b)/x^3)/(a*b^3*x^6 + b^4*x^
3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(3/2)/x^10,x, algorithm="giac")
```

```
[Out] integrate(1/((a + b/x^3)^(3/2)*x^10), x)
```

maple [A] time = 0.01, size = 50, normalized size = 0.85

$$\frac{2\left(a\,x^3+b\right)\left(8a^2x^6+4ab\,x^3-b^2\right)}{9\left(\frac{a\,x^3+b}{x^3}\right)^{\frac{3}{2}}b^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^3)^(3/2)/x^10,x)
```

```
[Out] 2/9*(a*x^3+b)*(8*a^2*x^6+4*a*b*x^3-b^2)/x^9/b^3/((a*x^3+b)/x^3)^(3/2)
```

maxima [A] time = 0.84, size = 47, normalized size = 0.80

$$-\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}{9\,b^3}+\frac{4\sqrt{a+\frac{b}{x^3}}\,a}{3\,b^3}+\frac{2\,a^2}{3\sqrt{a+\frac{b}{x^3}}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(3/2)/x^10,x, algorithm="maxima")
```

```
[Out] -2/9*(a + b/x^3)^(3/2)/b^3 + 4/3*sqrt(a + b/x^3)*a/b^3 + 2/3*a^2/(sqrt(a +
b/x^3)*b^3)
```

mupad [B] time = 1.36, size = 62, normalized size = 1.05

$$\frac{10\,a\sqrt{a+\frac{b}{x^3}}}{9\,b^3}-\frac{2\sqrt{a+\frac{b}{x^3}}}{9\,b^2\,x^3}+\frac{2\,a^2\,x^3\sqrt{a+\frac{b}{x^3}}}{3\,b^3\left(a\,x^3+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^10*(a + b/x^3)^(3/2)),x)
```

```
[Out] (10*a*(a + b/x^3)^(1/2))/(9*b^3) - (2*(a + b/x^3)^(1/2))/(9*b^2*x^3) + (2*a
^2*x^3*(a + b/x^3)^(1/2))/(3*b^3*(b + a*x^3))
```

sympy [**B**] time = 3.48, size = 466, normalized size = 7.90

$$\frac{16a^{\frac{9}{2}}b^{\frac{7}{2}}x^9\sqrt{\frac{ax^3}{b}+1}}{9a^{\frac{7}{2}}b^6x^{\frac{21}{2}}+18a^{\frac{5}{2}}b^7x^{\frac{15}{2}}+9a^{\frac{3}{2}}b^8x^{\frac{9}{2}}}+\frac{24a^{\frac{7}{2}}b^{\frac{9}{2}}x^6\sqrt{\frac{ax^3}{b}+1}}{9a^{\frac{7}{2}}b^6x^{\frac{21}{2}}+18a^{\frac{5}{2}}b^7x^{\frac{15}{2}}+9a^{\frac{3}{2}}b^8x^{\frac{9}{2}}}+\frac{6a^{\frac{5}{2}}b^{\frac{11}{2}}x^3\sqrt{\frac{ax^3}{b}+1}}{9a^{\frac{7}{2}}b^6x^{\frac{21}{2}}+18a^{\frac{5}{2}}b^7x^{\frac{15}{2}}+9a^{\frac{3}{2}}b^8x^{\frac{9}{2}}}-\frac{}{9a^{\frac{7}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**3)**(3/2)/x**10,x)
```

```
[Out] 16*a**(9/2)*b**(7/2)*x**9*sqrt(a*x**3/b + 1)/(9*a**(7/2)*b**6*x**(21/2) + 1
8*a**(5/2)*b**7*x**(15/2) + 9*a**(3/2)*b**8*x**(9/2)) + 24*a**(7/2)*b**(9/2
)*x**6*sqrt(a*x**3/b + 1)/(9*a**(7/2)*b**6*x**(21/2) + 18*a**(5/2)*b**7*x**
(15/2) + 9*a**(3/2)*b**8*x**(9/2)) + 6*a**(5/2)*b**(11/2)*x**3*sqrt(a*x**3/
b + 1)/(9*a**(7/2)*b**6*x**(21/2) + 18*a**(5/2)*b**7*x**(15/2) + 9*a**(3/2)
*b**8*x**(9/2)) - 2*a**(3/2)*b**(13/2)*sqrt(a*x**3/b + 1)/(9*a**(7/2)*b**6*
x**(21/2) + 18*a**(5/2)*b**7*x**(15/2) + 9*a**(3/2)*b**8*x**(9/2)) - 16*a**
5*b**3*x**(21/2)/(9*a**(7/2)*b**6*x**(21/2) + 18*a**(5/2)*b**7*x**(15/2) +
9*a**(3/2)*b**8*x**(9/2)) - 32*a**4*b**4*x**(15/2)/(9*a**(7/2)*b**6*x**(21/
2) + 18*a**(5/2)*b**7*x**(15/2) + 9*a**(3/2)*b**8*x**(9/2)) - 16*a**3*b**5*
x**(9/2)/(9*a**(7/2)*b**6*x**(21/2) + 18*a**(5/2)*b**7*x**(15/2) + 9*a**(3/
2)*b**8*x**(9/2))
```

3.2043 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^{13}}\,dx$

Optimal. Leaf size=78

$$-\frac{2a^3}{3b^4\sqrt{a+\frac{b}{x^3}}}-\frac{2a^2\sqrt{a+\frac{b}{x^3}}}{b^4}+\frac{2a\left(a+\frac{b}{x^3}\right)^{3/2}}{3b^4}-\frac{2\left(a+\frac{b}{x^3}\right)^{5/2}}{15b^4}$$

```
[Out] 2/3*a*(a+b/x^3)^(3/2)/b^4-2/15*(a+b/x^3)^(5/2)/b^4-2/3*a^3/b^4/(a+b/x^3)^(1
/2)-2*a^2*(a+b/x^3)^(1/2)/b^4
```

Rubi [A] time = 0.04, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{2a^3}{3b^4\sqrt{a+\frac{b}{x^3}}}-\frac{2a^2\sqrt{a+\frac{b}{x^3}}}{b^4}+\frac{2a\left(a+\frac{b}{x^3}\right)^{3/2}}{3b^4}-\frac{2\left(a+\frac{b}{x^3}\right)^{5/2}}{15b^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^3)^(3/2)*x^13),x]
```

```
[Out] (-2*a^3)/(3*b^4*Sqrt[a + b/x^3]) - (2*a^2*Sqrt[a + b/x^3])/b^4 + (2*a*(a +
b/x^3)^(3/2))/(3*b^4) - (2*(a + b/x^3)^(5/2))/(15*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^{13}}\,dx &= -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{x^3}{(a+bx)^{3/2}}\,dx, x, \frac{1}{x^3}\right)\right)\\ &= -\left(\frac{1}{3}\,\text{Subst}\left(\int \left(-\frac{a^3}{b^3(a+bx)^{3/2}}+\frac{3a^2}{b^3\sqrt{a+bx}}-\frac{3a\sqrt{a+bx}}{b^3}+\frac{(a+bx)^{3/2}}{b^3}\right)dx, x, \frac{1}{x^3}\right)\right)\\ &= -\frac{2a^3}{3b^4\sqrt{a+\frac{b}{x^3}}}-\frac{2a^2\sqrt{a+\frac{b}{x^3}}}{b^4}+\frac{2a\left(a+\frac{b}{x^3}\right)^{3/2}}{3b^4}-\frac{2\left(a+\frac{b}{x^3}\right)^{5/2}}{15b^4}\end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 0.65

$$-\frac{2\left(16a^3x^9+8a^2bx^6-2ab^2x^3+b^3\right)}{15b^4x^9\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^3)^(3/2)*x^13),x]
```

```
[Out] (-2*(b^3 - 2*a*b^2*x^3 + 8*a^2*b*x^6 + 16*a^3*x^9))/(15*b^4*Sqrt[a + b/x^3]
*x^9)
```

fricas [A] time = 0.95, size = 63, normalized size = 0.81

$$-\frac{2\left(16\,a^3x^9+8\,a^2bx^6-2\,ab^2x^3+b^3\right)\sqrt{\frac{ax^3+b}{x^3}}}{15\left(ab^4x^9+b^5x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(3/2)/x^13,x, algorithm="fricas")
```

```
[Out] -2/15*(16*a^3*x^9 + 8*a^2*b*x^6 - 2*a*b^2*x^3 + b^3)*sqrt((a*x^3 + b)/x^3)/
(a*b^4*x^9 + b^5*x^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(3/2)/x^13,x, algorithm="giac")
```

```
[Out] integrate(1/((a + b/x^3)^(3/2)*x^13), x)
```

maple [A] time = 0.01, size = 59, normalized size = 0.76

$$-\frac{2\left(a\,x^3+b\right)\left(16a^3x^9+8a^2b\,x^6-2a\,b^2x^3+b^3\right)}{15\left(\frac{a\,x^3+b}{x^3}\right)^{\frac{3}{2}}b^4x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^3)^(3/2)/x^13,x)
```

```
[Out] -2/15*(a*x^3+b)*(16*a^3*x^9+8*a^2*b*x^6-2*a*b^2*x^3+b^3)/x^12/b^4/((a*x^3+b
)/x^3)^(3/2)
```

maxima [A] time = 0.90, size = 64, normalized size = 0.82

$$-\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{5}{2}}}{15\,b^4}+\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}a}{3\,b^4}-\frac{2\sqrt{a+\frac{b}{x^3}}\,a^2}{b^4}-\frac{2\,a^3}{3\sqrt{a+\frac{b}{x^3}}\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(3/2)/x^13,x, algorithm="maxima")
```

```
[Out] -2/15*(a + b/x^3)^(5/2)/b^4 + 2/3*(a + b/x^3)^(3/2)*a/b^4 - 2*sqrt(a + b/x^
3)*a^2/b^4 - 2/3*a^3/(sqrt(a + b/x^3)*b^4)
```

mupad [B] time = 1.52, size = 82, normalized size = 1.05

$$\frac{2\,a\sqrt{a+\frac{b}{x^3}}}{5\,b^3\,x^3}-\frac{2\sqrt{a+\frac{b}{x^3}}}{15\,b^2\,x^6}-\frac{22\,a^2\sqrt{a+\frac{b}{x^3}}}{15\,b^4}-\frac{2\,a^3\,x^3\sqrt{a+\frac{b}{x^3}}}{3\,b^4\left(a\,x^3+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^13*(a + b/x^3)^(3/2)),x)
```

```
[Out] (2*a*(a + b/x^3)^(1/2))/(5*b^3*x^3) - (2*(a + b/x^3)^(1/2))/(15*b^2*x^6) -
(22*a^2*(a + b/x^3)^(1/2))/(15*b^4) - (2*a^3*x^3*(a + b/x^3)^(1/2))/(3*b^4*
(b + a*x^3))
```

sympy [B] time = 5.17, size = 2048, normalized size = 26.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**3)**(3/2)/x**13,x)
```

```
[Out] -32*a**(21/2)*b**(23/2)*x**24*sqrt(a*x**3/b + 1)/(15*a**(17/2)*b**15*x**(51
/2) + 90*a**(15/2)*b**16*x**(45/2) + 225*a**(13/2)*b**17*x**(39/2) + 300*a*
*(11/2)*b**18*x**(33/2) + 225*a**(9/2)*b**19*x**(27/2) + 90*a**(7/2)*b**20*
x**(21/2) + 15*a**(5/2)*b**21*x**(15/2)) - 176*a**(19/2)*b**(25/2)*x**21*sq
rt(a*x**3/b + 1)/(15*a**(17/2)*b**15*x**(51/2) + 90*a**(15/2)*b**16*x**(45/
2) + 225*a**(13/2)*b**17*x**(39/2) + 300*a**(11/2)*b**18*x**(33/2) + 225*a*
*(9/2)*b**19*x**(27/2) + 90*a**(7/2)*b**20*x**(21/2) + 15*a**(5/2)*b**21*x*
*(15/2)) - 396*a**(17/2)*b**(27/2)*x**18*sqrt(a*x**3/b + 1)/(15*a**(17/2)*b
**15*x**(51/2) + 90*a**(15/2)*b**16*x**(45/2) + 225*a**(13/2)*b**17*x**(39/
2) + 300*a**(11/2)*b**18*x**(33/2) + 225*a**(9/2)*b**19*x**(27/2) + 90*a**(
7/2)*b**20*x**(21/2) + 15*a**(5/2)*b**21*x**(15/2)) - 462*a**(15/2)*b**(29/
2)*x**15*sqrt(a*x**3/b + 1)/(15*a**(17/2)*b**15*x**(51/2) + 90*a**(15/2)*b*
*16*x**(45/2) + 225*a**(13/2)*b**17*x**(39/2) + 300*a**(11/2)*b**18*x**(33/
2) + 225*a**(9/2)*b**19*x**(27/2) + 90*a**(7/2)*b**20*x**(21/2) + 15*a**(5/
2)*b**21*x**(15/2)) - 290*a**(13/2)*b**(31/2)*x**12*sqrt(a*x**3/b + 1)/(15*
a**(17/2)*b**15*x**(51/2) + 90*a**(15/2)*b**16*x**(45/2) + 225*a**(13/2)*b*
*17*x**(39/2) + 300*a**(11/2)*b**18*x**(33/2) + 225*a**(9/2)*b**19*x**(27/2
) + 90*a**(7/2)*b**20*x**(21/2) + 15*a**(5/2)*b**21*x**(15/2)) - 92*a**(11/
2)*b**(33/2)*x**9*sqrt(a*x**3/b + 1)/(15*a**(17/2)*b**15*x**(51/2) + 90*a**
(15/2)*b**16*x**(45/2) + 225*a**(13/2)*b**17*x**(39/2) + 300*a**(11/2)*b**1
8*x**(33/2) + 225*a**(9/2)*b**19*x**(27/2) + 90*a**(7/2)*b**20*x**(21/2) +
15*a**(5/2)*b**21*x**(15/2)) - 16*a**(9/2)*b**(35/2)*x**6*sqrt(a*x**3/b + 1
)/(15*a**(17/2)*b**15*x**(51/2) + 90*a**(15/2)*b**16*x**(45/2) + 225*a**(13
/2)*b**17*x**(39/2) + 300*a**(11/2)*b**18*x**(33/2) + 225*a**(9/2)*b**19*x*
*(27/2) + 90*a**(7/2)*b**20*x**(21/2) + 15*a**(5/2)*b**21*x**(15/2)) - 6*a*
*(7/2)*b**(37/2)*x**3*sqrt(a*x**3/b + 1)/(15*a**(17/2)*b**15*x**(51/2) + 90
*a**(15/2)*b**16*x**(45/2) + 225*a**(13/2)*b**17*x**(39/2) + 300*a**(11/2)*
b**18*x**(33/2) + 225*a**(9/2)*b**19*x**(27/2) + 90*a**(7/2)*b**20*x**(21/2
) + 15*a**(5/2)*b**21*x**(15/2)) - 2*a**(5/2)*b**(39/2)*sqrt(a*x**3/b + 1)/
(15*a**(17/2)*b**15*x**(51/2) + 90*a**(15/2)*b**16*x**(45/2) + 225*a**(13/2
)*b**17*x**(39/2) + 300*a**(11/2)*b**18*x**(33/2) + 225*a**(9/2)*b**19*x**(
27/2) + 90*a**(7/2)*b**20*x**(21/2) + 15*a**(5/2)*b**21*x**(15/2)) + 32*a**
11*b**11*x**(51/2)/(15*a**(17/2)*b**15*x**(51/2) + 90*a**(15/2)*b**16*x**(4
5/2) + 225*a**(13/2)*b**17*x**(39/2) + 300*a**(11/2)*b**18*x**(33/2) + 225*
a**(9/2)*b**19*x**(27/2) + 90*a**(7/2)*b**20*x**(21/2) + 15*a**(5/2)*b**21*
x**(15/2)) + 192*a**10*b**12*x**(45/2)/(15*a**(17/2)*b**15*x**(51/2) + 90*a
**(15/2)*b**16*x**(45/2) + 225*a**(13/2)*b**17*x**(39/2) + 300*a**(11/2)*b*
*18*x**(33/2) + 225*a**(9/2)*b**19*x**(27/2) + 90*a**(7/2)*b**20*x**(21/2)
+ 15*a**(5/2)*b**21*x**(15/2)) + 480*a**9*b**13*x**(39/2)/(15*a**(17/2)*b**
15*x**(51/2) + 90*a**(15/2)*b**16*x**(45/2) + 225*a**(13/2)*b**17*x**(39/2)
 + 300*a**(11/2)*b**18*x**(33/2) + 225*a**(9/2)*b**19*x**(27/2) + 90*a**(7/
2)*b**20*x**(21/2) + 15*a**(5/2)*b**21*x**(15/2)) + 640*a**8*b**14*x**(33/2
)/(15*a**(17/2)*b**15*x**(51/2) + 90*a**(15/2)*b**16*x**(45/2) + 225*a**(13
/2)*b**17*x**(39/2) + 300*a**(11/2)*b**18*x**(33/2) + 225*a**(9/2)*b**19*x*
*(27/2) + 90*a**(7/2)*b**20*x**(21/2) + 15*a**(5/2)*b**21*x**(15/2)) + 480*
```

```
a**7*b**15*x**(27/2)/(15*a**(17/2)*b**15*x**(51/2) + 90*a**(15/2)*b**16*x**
(45/2) + 225*a**(13/2)*b**17*x**(39/2) + 300*a**(11/2)*b**18*x**(33/2) + 22
5*a**(9/2)*b**19*x**(27/2) + 90*a**(7/2)*b**20*x**(21/2) + 15*a**(5/2)*b**2
1*x**(15/2)) + 192*a**6*b**16*x**(21/2)/(15*a**(17/2)*b**15*x**(51/2) + 90*
a**(15/2)*b**16*x**(45/2) + 225*a**(13/2)*b**17*x**(39/2) + 300*a**(11/2)*b
**18*x**(33/2) + 225*a**(9/2)*b**19*x**(27/2) + 90*a**(7/2)*b**20*x**(21/2)
 + 15*a**(5/2)*b**21*x**(15/2)) + 32*a**5*b**17*x**(15/2)/(15*a**(17/2)*b**
15*x**(51/2) + 90*a**(15/2)*b**16*x**(45/2) + 225*a**(13/2)*b**17*x**(39/2)
 + 300*a**(11/2)*b**18*x**(33/2) + 225*a**(9/2)*b**19*x**(27/2) + 90*a**(7/
2)*b**20*x**(21/2) + 15*a**(5/2)*b**21*x**(15/2))
```

3.2044 $\int \frac{x^7}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$

Optimal. Leaf size=315

$$\frac{1729b^2x^2\sqrt{a+\frac{b}{x^3}}}{960a^4}-\frac{247bx^5\sqrt{a+\frac{b}{x^3}}}{240a^3}+\frac{19x^8\sqrt{a+\frac{b}{x^3}}}{24a^2}+\frac{1729\sqrt{2+\sqrt{3}}\,b^{8/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}F\left(\sin$$
$$960\sqrt[4]{3}\,a^4\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+$$
$$\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}$$

```
[Out] -2/3*x^8/a/(a+b/x^3)^(1/2)+1729/960*b^2*x^2*(a+b/x^3)^(1/2)/a^4-247/240*b*x
^5*(a+b/x^3)^(1/2)/a^3+19/24*x^8*(a+b/x^3)^(1/2)/a^2+1729/2880*b^(8/3)*(a^(
1/3)+b^(1/3)/x)*EllipticF((b^(1/3)/x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3
)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)+b^(2/3)/x
^2-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^4/
(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2))
)^2)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {335, 290, 325, 218}

$$\frac{1729b^2x^2\sqrt{a+\frac{b}{x^3}}}{960a^4}+\frac{1729\sqrt{2+\sqrt{3}}\,b^{8/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{960\sqrt[4]{3}\,a^4\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}+$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b/x^3)^(3/2),x]
```

```
[Out] (1729*b^2*Sqrt[a + b/x^3]*x^2)/(960*a^4) - (247*b*Sqrt[a + b/x^3]*x^5)/(240
*a^3) - (2*x^8)/(3*a*Sqrt[a + b/x^3]) + (19*Sqrt[a + b/x^3]*x^8)/(24*a^2) +
 (1729*Sqrt[2 + Sqrt[3]]*b^(8/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2
/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*Ellip
ticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)/x)], -7 - 4*Sqrt[3]])/(960*3^(1/4)*a^4*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*
(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
```

```
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
 + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{x^7}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = -\operatorname{Subst}\left(\int \frac{1}{x^9\left(a+bx^3\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2x^8}{3a\sqrt{a+\frac{b}{x^3}}} - \frac{19\operatorname{Subst}\left(\int \frac{1}{x^9\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{3a}$$

$$= -\frac{2x^8}{3a\sqrt{a+\frac{b}{x^3}}} + \frac{19\sqrt{a+\frac{b}{x^3}}\,x^8}{24a^2} + \frac{(247b)\operatorname{Subst}\left(\int \frac{1}{x^6\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{48a^2}$$

$$= -\frac{247b\sqrt{a+\frac{b}{x^3}}\,x^5}{240a^3} - \frac{2x^8}{3a\sqrt{a+\frac{b}{x^3}}} + \frac{19\sqrt{a+\frac{b}{x^3}}\,x^8}{24a^2} - \frac{\left(1729b^2\right)\operatorname{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{480a^3}$$

$$= \frac{1729b^2\sqrt{a+\frac{b}{x^3}}\,x^2}{960a^4} - \frac{247b\sqrt{a+\frac{b}{x^3}}\,x^5}{240a^3} - \frac{2x^8}{3a\sqrt{a+\frac{b}{x^3}}} + \frac{19\sqrt{a+\frac{b}{x^3}}\,x^8}{24a^2} + \frac{\left(1729b^3\right)\operatorname{Subst}\left(\int \frac{}{\sqrt{}}\right.}{1920a^4}$$

$$= \frac{1729b^2\sqrt{a+\frac{b}{x^3}}\,x^2}{960a^4} - \frac{247b\sqrt{a+\frac{b}{x^3}}\,x^5}{240a^3} - \frac{2x^8}{3a\sqrt{a+\frac{b}{x^3}}} + \frac{19\sqrt{a+\frac{b}{x^3}}\,x^8}{24a^2} + \frac{1729\sqrt{2+\sqrt{3}}\,b^{8/3}\left(\right.}{}$$

Mathematica [C] time = 0.03, size = 91, normalized size = 0.29

$$\frac{120a^3x^9 - 228a^2bx^6 - 1729b^3\sqrt{\frac{ax^3}{b}+1}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{ax^3}{b}\right) + 741ab^2x^3 + 1729b^3}{960a^4x\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b/x^3)^(3/2),x]
```

```
[Out] (1729*b^3 + 741*a*b^2*x^3 - 228*a^2*b*x^6 + 120*a^3*x^9 - 1729*b^3*Sqrt[1 +
 (a*x^3)/b]*Hypergeometric2F1[1/6, 1/2, 7/6, -((a*x^3)/b)])/(960*a^4*Sqrt[a
 + b/x^3]*x)
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^{13}\sqrt{\frac{ax^3+b}{x^3}}}{a^2x^6+2\,abx^3+b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(a+b/x^3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x^13*sqrt((a*x^3 + b)/x^3)/(a^2*x^6 + 2*a*b*x^3 + b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(a+b/x^3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^7/(a + b/x^3)^(3/2), x)
```

maple [B] time = 0.04, size = 2540, normalized size = 8.06

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a+b/x^3)^(3/2),x)
```

```
[Out] 1/960/((a*x^3+b)/x^3)^(3/2)/x^5*(a*x^3+b)/(-a^2*b)^(1/3)/a^5*(120*I*(a*x^4+
b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^
(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*(-a^2*
b)^(1/3)*3^(1/2)*((a*x^3+b)*x)^(1/2)*x^6*a^3+3458*I*(-(I*3^(1/2)-3)/(I*3^(1
/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-
a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)
*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*
EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((
I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*3^(1/2)*((a*
x^3+b)*x)^(1/2)*x^2*a^2*b^3-360*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2
*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1
/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*(-a^2*b)^(1/3)*((a*x^3+b)*x)^(1/2)*x^6*a^3
-6916*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*
a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(
1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1
)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*
x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*
3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*3^(1/2)*((a*x^3+b)*x)^(1/2)*x*a*b^3-348*I
*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(
-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2
)*(-a^2*b)^(1/3)*3^(1/2)*((a*x^3+b)*x)^(1/2)*x^3*a^2*b+3458*I*(-(I*3^(1/2)-
3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b
)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x
+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3
```

```
)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x
)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-
a^2*b)^(2/3)*3^(1/2)*((a*x^3+b)*x)^(1/2)*b^3-3458*(-(I*3^(1/2)-3)/(I*3^(1/2
)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^
2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(
-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*El
lipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*
3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^
(1/2)*x^2*a^2*b^3+6916*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*
a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(
-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3
))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*
3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1
+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*((a*x^3+b)*x)^(1/2)*x*a*b^
3+1044*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(
1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x
)^(1/2)*(-a^2*b)^(1/3)*((a*x^3+b)*x)^(1/2)*x^3*a^2*b-3458*(-(I*3^(1/2)-3)/(
I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1
/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3
^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^
(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1
/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*
b)^(2/3)*((a*x^3+b)*x)^(1/2)*b^3+1089*I*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(
1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a
^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*(-a^2*b)^(1/3)*3^(1/2)*((a*x^3+b)*
x)^(1/2)*a*b^2+640*I*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)
+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1
/2)*(-a^2*b)^(1/3)*3^(1/2)*x*a*b^3-3267*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(
1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a
^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*(-a^2*b)^(1/3)*((a*x^3+b)*x)^(1/2)
*a*b^2-1920*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)
^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*(-a^2
*b)^(1/3)*x*a*b^3)/(I*3^(1/2)-3)/((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-
a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3
))/a^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(a+b/x^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^7/(a + b/x^3)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b/x^3)^(3/2),x)
```

```
[Out] int(x^7/(a + b/x^3)^(3/2), x)
```

sympy [**A**] time = 1.59, size = 46, normalized size = 0.15

$$-\frac{x^8\Gamma\left(-\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{8}{3}, \frac{3}{2} \\ -\frac{5}{3}\end{matrix}\middle| \frac{be^{i\pi}}{ax^3}\right)}{3a^{\frac{3}{2}}\Gamma\left(-\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(a+b/x**3)**(3/2),x)
```

```
[Out] -x**8*gamma(-8/3)*hyper((-8/3, 3/2), (-5/3,), b*exp_polar(I*pi)/(a*x**3))/(
3*a**(3/2)*gamma(-5/3))
```

3.2045 $\int \frac{x^4}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$

Optimal. Leaf size=291

$$-\frac{91bx^2\sqrt{a+\frac{b}{x^3}}}{60a^3}+\frac{13x^5\sqrt{a+\frac{b}{x^3}}}{15a^2}-\frac{91\sqrt{2+\sqrt{3}}\,b^{5/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-$$
$$60\sqrt[4]{3}\,a^3\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}$$

```
[Out] -2/3*x^5/a/(a+b/x^3)^(1/2)-91/60*b*x^2*(a+b/x^3)^(1/2)/a^3+13/15*x^5*(a+b/x
^3)^(1/2)/a^2-91/180*b^(5/3)*(a^(1/3)+b^(1/3)/x)*EllipticF((b^(1/3)/x+a^(1/
3)*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)
+1/2*2^(1/2))*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(
1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^3/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/
x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {335, 290, 325, 218}

$$-\frac{91\sqrt{2+\sqrt{3}}\,b^{5/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{60\sqrt[4]{3}\,a^3\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}+\frac{13x^5\sqrt{a+\frac{b}{x^3}}}{15a^2}-\frac{91bx^2\sqrt{}}{60a}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b/x^3)^(3/2),x]
```

```
[Out] (-91*b*Sqrt[a + b/x^3]*x^2)/(60*a^3) - (2*x^5)/(3*a*Sqrt[a + b/x^3]) + (13*
Sqrt[a + b/x^3]*x^5)/(15*a^2) - (91*Sqrt[2 + Sqrt[3]]*b^(5/3)*(a^(1/3) + b^
(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(60*3^(1/4)*a^3*Sq
rt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)/x)^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
```

```
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{x^4}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = -\text{Subst}\left(\int \frac{1}{x^6\left(a+bx^3\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2x^5}{3a\sqrt{a+\frac{b}{x^3}}} - \frac{13\,\text{Subst}\left(\int \frac{1}{x^6\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{3a}$$

$$= -\frac{2x^5}{3a\sqrt{a+\frac{b}{x^3}}} + \frac{13\sqrt{a+\frac{b}{x^3}}\,x^5}{15a^2} + \frac{(91b)\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{30a^2}$$

$$= -\frac{91b\sqrt{a+\frac{b}{x^3}}\,x^2}{60a^3} - \frac{2x^5}{3a\sqrt{a+\frac{b}{x^3}}} + \frac{13\sqrt{a+\frac{b}{x^3}}\,x^5}{15a^2} - \frac{\left(91b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{120a^3}$$

$$= -\frac{91b\sqrt{a+\frac{b}{x^3}}\,x^2}{60a^3} - \frac{2x^5}{3a\sqrt{a+\frac{b}{x^3}}} + \frac{13\sqrt{a+\frac{b}{x^3}}\,x^5}{15a^2} - \frac{91\sqrt{2+\sqrt{3}}\,b^{5/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}{60\sqrt[4]{3}\,a^3\sqrt{a+}}$$

Mathematica [C] time = 0.04, size = 80, normalized size = 0.27

$$\frac{12a^2x^6 + 91b^2\sqrt{\frac{ax^3}{b}+1}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{ax^3}{b}\right) - 39abx^3 - 91b^2}{60a^3x\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b/x^3)^(3/2),x]
```

```
[Out] (-91*b^2 - 39*a*b*x^3 + 12*a^2*x^6 + 91*b^2*Sqrt[1 + (a*x^3)/b]*Hypergeomet
ric2F1[1/6, 1/2, 7/6, -((a*x^3)/b)])/(60*a^3*Sqrt[a + b/x^3]*x)
```

fricas [F] time = 1.18, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^{10}\sqrt{\frac{ax^3+b}{x^3}}}{a^2x^6+2\,abx^3+b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/x^3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x^10*sqrt((a*x^3 + b)/x^3)/(a^2*x^6 + 2*a*b*x^3 + b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/x^3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/(a + b/x^3)^(3/2), x)
```

maple [B] time = 0.02, size = 2302, normalized size = 7.91

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a+b/x^3)^(3/2),x)
```

```
[Out] -1/60/((a*x^3+b)/x^3)^(3/2)/x^5*(a*x^3+b)/a^4/(-a^2*b)^(1/3)*(182*I*3^(1/2)
*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3
^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^
(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x
+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2
*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)
-3))^(1/2))*((a*x^3+b)*x)^(1/2)*x^2*a^2*b^2-364*I*(-a^2*b)^(1/3)*3^(1/2)*(-
(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1
/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/
2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-
a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)
^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3)
)^(1/2))*((a*x^3+b)*x)^(1/2)*x*a*b^2+182*I*(-a^2*b)^(2/3)*3^(1/2)*(-(I*3^(1
/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a
^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2
*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^
(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))
*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)
)*((a*x^3+b)*x)^(1/2)*b^2-12*I*(-a^2*b)^(1/3)*3^(1/2)*(a*x^4+b*x)^(1/2)*((-
a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x
+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*((a*x^3+b)*x)^(1/2)*
x^3*a^2-182*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*
((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*
b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/
2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/
(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))
/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^(1/2)*x^2*a^2*b^2+364*(-(I*3^(1/2)-3)/
(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(
1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*
```

```
3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))
^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(
1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2
*b)^(1/3)*((a*x^3+b)*x)^(1/2)*x*a*b^2-182*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a
*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/
3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^
(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF(
(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+
3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*((a*x^3
+b)*x)^(1/2)*b^2+36*(-a^2*b)^(1/3)*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))
*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)
^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*((a*x^3+b)*x)^(1/2)*x^3*a^2+51*I*(-a^2*
b)^(1/3)*3^(1/2)*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*
(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1
/3))/a^2*x)^(1/2)*((a*x^3+b)*x)^(1/2)*a*b+40*I*(-a^2*b)^(1/3)*3^(1/2)*((-a*
x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I
*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*x*a*b^2-153*(-a^2*b)^(
1/3)*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/
3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^
(1/2)*((a*x^3+b)*x)^(1/2)*a*b-120*(-a^2*b)^(1/3)*((-a*x+(-a^2*b)^(1/3))*(2*
a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/
3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*x*a*b^2)/(I*3^(1/2)-3)/((-a*x+(-a^2*b)^(1/3
))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*
b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/x^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(a + b/x^3)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b/x^3)^(3/2),x)
```

```
[Out] int(x^4/(a + b/x^3)^(3/2), x)
```

sympy [A] time = 1.38, size = 46, normalized size = 0.16

$$-\frac{x^5\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{5}{3}, \frac{3}{2} \\ -\frac{2}{3}\end{matrix}\right| \frac{be^{i\pi}}{ax^3}\right)}{3a^{\frac{3}{2}}\Gamma\left(-\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a+b/x**3)**(3/2),x)
```

```
[Out] -x**5*gamma(-5/3)*hyper((-5/3, 3/2), (-2/3,), b*exp_polar(I*pi)/(a*x**3))/(
3*a**(3/2)*gamma(-2/3))
```

3.2046 $\int \frac{x}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$

Optimal. Leaf size=269

$$\frac{7x^2\sqrt{a+\frac{b}{x^3}}}{6a^2}+\frac{7\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{6\sqrt[4]{3}\,a^2\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}-\frac{2x^2}{3a\sqrt{a+\frac{b}{x^3}}}$$

```
[Out] -2/3*x^2/a/(a+b/x^3)^(1/2)+7/6*x^2*(a+b/x^3)^(1/2)/a^2+7/18*b^(2/3)*(a^(1/3
)+b^(1/3)/x)*EllipticF((b^(1/3)/x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3)*(
1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)+b^(2/3)/x^2-
a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^2/(a+
b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2
)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {335, 290, 325, 218}

$$\frac{7\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{6\sqrt[4]{3}\,a^2\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}+\frac{7x^2\sqrt{a+\frac{b}{x^3}}}{6a^2}-\frac{2x^2}{3a\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b/x^3)^(3/2),x]
```

```
[Out] (-2*x^2)/(3*a*Sqrt[a + b/x^3]) + (7*Sqrt[a + b/x^3]*x^2)/(6*a^2) + (7*Sqrt[
2 + Sqrt[3]]*b^(2/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a
^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[
((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -
7 - 4*Sqrt[3]])/(6*3^(1/4)*a^2*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(
1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
```

```
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{x}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = -\text{Subst}\left(\int \frac{1}{x^3\left(a+bx^3\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2x^2}{3a\sqrt{a+\frac{b}{x^3}}} - \frac{7\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{3a}$$

$$= -\frac{2x^2}{3a\sqrt{a+\frac{b}{x^3}}} + \frac{7\sqrt{a+\frac{b}{x^3}}\,x^2}{6a^2} + \frac{(7b)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{12a^2}$$

$$= -\frac{2x^2}{3a\sqrt{a+\frac{b}{x^3}}} + \frac{7\sqrt{a+\frac{b}{x^3}}\,x^2}{6a^2} + \frac{7\sqrt{2+\sqrt{3}}\;b^{2/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{(1}{(1}\right.\right.}{6\sqrt[4]{3}\,a^2\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}}}$$

Mathematica [C] time = 0.04, size = 67, normalized size = 0.25

$$\frac{-7b\sqrt{\frac{ax^3}{b}+1}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{ax^3}{b}\right)+3ax^3+7b}{6a^2x\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b/x^3)^(3/2),x]
```

```
[Out] (7*b + 3*a*x^3 - 7*b*Sqrt[1 + (a*x^3)/b]*Hypergeometric2F1[1/6, 1/2, 7/6, -
((a*x^3)/b)])/(6*a^2*Sqrt[a + b/x^3]*x)
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^7\sqrt{\frac{ax^3+b}{x^3}}}{a^2x^6+2\,abx^3+b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x^7*sqrt((a*x^3 + b)/x^3)/(a^2*x^6 + 2*a*b*x^3 + b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/(a + b/x^3)^(3/2), x)
```

maple [B] time = 0.01, size = 2052, normalized size = 7.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b/x^3)^(3/2),x)
```

```
[Out] 1/6/((a*x^3+b)/x^3)^(3/2)/x^5*(a*x^3+b)/a^3/(-a^2*b)^(1/3)*(14*I*(-(I*3^(1/
2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^
2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*
a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(
1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*
a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))
*3^(1/2)*((a*x^3+b)*x)^(1/2)*x^2*a^2*b-28*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(
-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(
1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b
)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*Elliptic
F((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2
)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*3^(1/
2)*((a*x^3+b)*x)^(1/2)*x*a*b+14*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2
*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I
*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-
a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(
1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^
(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*3^(1/2)*((a*x^3
+b)*x)^(1/2)*b-14*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^
(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+
(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I
*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/
2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^
(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^(1/2)*x^2*a^2*b+28*(-(I*3^(1/2)-
3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b
)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x
+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3
)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x
)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-
a^2*b)^(1/3)*((a*x^3+b)*x)^(1/2)*x*a*b-14*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a
*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/
3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^
(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF(
(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+
3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*((a*x^3
+b)*x)^(1/2)*b+3*I*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2
```

```
)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^
(1/3))/a^2*x)^(1/2)*(-a^2*b)^(1/3)*3^(1/2)*((a*x^3+b)*x)^(1/2)*a+4*I*((-a*x
+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*
3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*(-a^2*b)^(1/3)*3^(1/2)*
x*a*b-9*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^
(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*
x)^(1/2)*(-a^2*b)^(1/3)*((a*x^3+b)*x)^(1/2)*a-12*((-a*x+(-a^2*b)^(1/3))*(2*
a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/
3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*(-a^2*b)^(1/3)*x*a*b)/(I*3^(1/2)-3)/((-a*x+
(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3
^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(a + b/x^3)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b/x^3)^(3/2),x)
```

```
[Out] int(x/(a + b/x^3)^(3/2), x)
```

sympy [A] time = 1.20, size = 42, normalized size = 0.16

$$-\frac{x^2\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{3}{2}\\ \frac{1}{3}\end{matrix}\right|\frac{be^{i\pi}}{ax^3}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x**3)**(3/2),x)
```

```
[Out] -x**2*gamma(-2/3)*hyper((-2/3, 3/2), (1/3,), b*exp_polar(I*pi)/(a*x**3))/(3
*a**(3/2)*gamma(1/3))
```

3.2047 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^2}\,dx$

Optimal. Leaf size=248

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,a\sqrt[3]{b}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}-\frac{2}{3ax\sqrt{a+\frac{b}{x^3}}}$$

```
[Out] -2/3/a/x/(a+b/x^3)^(1/2)-2/9*(a^(1/3)+b^(1/3)/x)*EllipticF((b^(1/3)/x+a^(1/
3)*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)
+1/2*2^(1/2))*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(
1+3^(1/2)))^2)^(1/2)*3^(3/4)/a/b^(1/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^
(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {335, 199, 218}

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,a\sqrt[3]{b}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}-\frac{2}{3ax\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^3)^(3/2)*x^2),x]
```

```
[Out] -2/(3*a*Sqrt[a + b/x^3]*x) - (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1/3)/x)*Sqr
t[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a*b^(1/3)*Sqrt[a +
b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)/x)^2])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^2}\,dx = -\text{Subst}\left(\int \frac{1}{\left(a+bx^3\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2}{3a\sqrt{a+\frac{b}{x^3}}\,x} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{3a}$$

$$= -\frac{2}{3a\sqrt{a+\frac{b}{x^3}}\,x} - \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\right|-}{3\sqrt[4]{3}\,a\sqrt[3]{b}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

Mathematica [C] time = 0.02, size = 57, normalized size = 0.23

$$\frac{2\left(\sqrt{\frac{ax^3}{b}+1}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{ax^3}{b}\right)-1\right)}{3ax\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^3)^(3/2)*x^2),x]
```

```
[Out] (2*(-1 + Sqrt[1 + (a*x^3)/b]*Hypergeometric2F1[1/6, 1/2, 7/6, -((a*x^3)/b)]
))/(3*a*Sqrt[a + b/x^3]*x)
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^4\sqrt{\frac{ax^3+b}{x^3}}}{a^2x^6+2\,abx^3+b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(x^4*sqrt((a*x^3 + b)/x^3)/(a^2*x^6 + 2*a*b*x^3 + b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}} x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a + b/x^3)^(3/2)*x^2), x)
```

maple [B] time = 0.01, size = 1822, normalized size = 7.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^3)^(3/2)/x^2,x)
```

```
[Out] -2/3/((a*x^3+b)/x^3)^(3/2)/x^5*(a*x^3+b)/a^2/(-a^2*b)^(1/3)*(2*I*((a*x^3+b)
*x)^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((
2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)
^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)
-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-
a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(
I*3^(1/2)-3))^(1/2))*3^(1/2)*x^2*a^2-4*I*((a*x^3+b)*x)^(1/2)*(-(I*3^(1/2)-3
)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)
^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+
I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)
))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)
^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a
^2*b)^(1/3)*3^(1/2)*x*a+2*I*((a*x^3+b)*x)^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-
1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*
b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a
^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*Elli
pticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^
(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*3
^(1/2)-2*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2
*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^
(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-
1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a
*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I
*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^(1/2)*x^2*a^2+4*(-(I*3^(1/2)-3)/(I*3^(1/2
)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^
2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(
-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*El
lipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*
3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^
(1/2)*(-a^2*b)^(1/3)*x*a-2*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/
3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2
))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^
(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)
/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1
)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^(1/2)*(-a^2*b)^(2/3)+I*
((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*
a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*(-a^2*b)^(1/3)*3^
(1/2)*x*a-3*x*a*(-a^2*b)^(1/3)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^
2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))
/a^2*x)^(1/2))/(I*3^(1/2)-3)/((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*
b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a
^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}} x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a + b/x^3)^(3/2)*x^2), x)
```

mupad [B] time = 1.44, size = 40, normalized size = 0.16

$$-\frac{\left(\frac{b}{a\,x^3}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{3}{2};\ \frac{4}{3};\ -\frac{b}{a\,x^3}\right)}{x\left(a+\frac{b}{x^3}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b/x^3)^(3/2)),x)
```

```
[Out] -((b/(a*x^3) + 1)^(3/2)*hypergeom([1/3, 3/2], 4/3, -b/(a*x^3)))/(x*(a + b/x
^3)^(3/2))
```

sympy [A] time = 1.42, size = 37, normalized size = 0.15

$$-\frac{\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{4}{3}\end{matrix}\right|\frac{be^{i\pi}}{ax^3}\right)}{3a^{\frac{3}{2}}x\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**3)**(3/2)/x**2,x)
```

```
[Out] -gamma(1/3)*hyper((1/3, 3/2), (4/3,), b*exp_polar(I*pi)/(a*x**3))/(3*a**(3/
2)*x*gamma(4/3))
```

3.2048 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^5}\,dx$

Optimal. Leaf size=245

$$\frac{2}{3bx\sqrt{a+\frac{b}{x^3}}}-\frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,b^{4/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

```
[Out] 2/3/b/x/(a+b/x^3)^(1/2)-4/9*(a^(1/3)+b^(1/3)/x)*EllipticF((b^(1/3)/x+a^(1/3
)*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+
1/2*2^(1/2))*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1
+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(4/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/
3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {335, 288, 218}

$$\frac{2}{3bx\sqrt{a+\frac{b}{x^3}}}-\frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,b^{4/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^3)^(3/2)*x^5),x]
```

```
[Out] 2/(3*b*Sqrt[a + b/x^3]*x) - (4*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1/3)/x)*Sqrt
[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(4/3)*Sqrt[a + b/x
^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x
)^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^5}\,dx = -\text{Subst}\left(\int \frac{x^3}{\left(a+bx^3\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2}{3b\sqrt{a+\frac{b}{x^3}}\,x} - \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{3b}$$

$$= \frac{2}{3b\sqrt{a+\frac{b}{x^3}}\,x} - \frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle| -\right.}{3\sqrt[4]{3}\,b^{4/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

Mathematica [C] time = 0.04, size = 58, normalized size = 0.24

$$\frac{4\sqrt{\frac{ax^3}{b}+1}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{ax^3}{b}\right)+2}{3bx\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^3)^(3/2)*x^5),x]
```

```
[Out] (2 + 4*Sqrt[1 + (a*x^3)/b]*Hypergeometric2F1[1/6, 1/2, 7/6, -((a*x^3)/b)])/
(3*b*Sqrt[a + b/x^3]*x)
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x\sqrt{\frac{ax^3+b}{x^3}}}{a^2x^6+2\,abx^3+b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] integral(x*sqrt((a*x^3 + b)/x^3)/(a^2*x^6 + 2*a*b*x^3 + b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}} x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(1/((a + b/x^3)^(3/2)*x^5), x)
```

maple [B] time = 0.02, size = 1825, normalized size = 7.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^3)^(3/2)/x^5,x)
```

```
[Out] -2/3/((a*x^3+b)/x^3)^(3/2)/x^5*(a*x^3+b)/(-a^2*b)^(1/3)/a/b*(4*I*((a*x^3+b)
*x)^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((
2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)
^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)
-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-
a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(
I*3^(1/2)-3))^(1/2))*3^(1/2)*x^2*a^2-8*I*((a*x^3+b)*x)^(1/2)*(-(I*3^(1/2)-3
)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)
^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+
I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)
))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)
^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a
^2*b)^(1/3)*3^(1/2)*x*a+4*I*((a*x^3+b)*x)^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-
1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*
b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a
^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*Elli
pticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^
(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*3
^(1/2)-4*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2
*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^
(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-
1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a
*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I
*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^(1/2)*x^2*a^2+8*(-(I*3^(1/2)-3)/(I*3^(1/2
)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^
2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(
-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*El
lipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*
3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^
(1/2)*(-a^2*b)^(1/3)*x*a-4*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/
3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2
))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^
(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)
/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1
)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^(1/2)*(-a^2*b)^(2/3)-I*
((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*
a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*(-a^2*b)^(1/3)*3^
(1/2)*x*a+3*(-a^2*b)^(1/3)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)
^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2
*x)^(1/2)*a*x)/(I*3^(1/2)-3)/((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*
b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a
^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}} x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(1/((a + b/x^3)^(3/2)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^5\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b/x^3)^(3/2)),x)
```

```
[Out] int(1/(x^5*(a + b/x^3)^(3/2)), x)
```

sympy [A] time = 2.15, size = 39, normalized size = 0.16

$$-\frac{\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{4}{3},\frac{3}{2}\\ \frac{7}{3}\end{matrix}\right|\frac{be^{i\pi}}{ax^3}\right)}{3a^{\frac{3}{2}}x^4\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**3)**(3/2)/x**5,x)
```

```
[Out] -gamma(4/3)*hyper((4/3, 3/2), (7/3,), b*exp_polar(I*pi)/(a*x**3))/(3*a**(3/
2)*x**4*gamma(7/3))
```

3.2049 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^8}\,dx$

Optimal. Leaf size=267

$$\frac{32\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{15\sqrt[4]{3}\,b^{7/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}-\frac{16\sqrt{a+\frac{b}{x^3}}}{15b^2x}+\frac{2}{3bx^4\sqrt{a+\frac{b}{x^3}}}$$

```
[Out] 2/3/b/x^4/(a+b/x^3)^(1/2)-16/15*(a+b/x^3)^(1/2)/b^2/x+32/45*a*(a^(1/3)+b^(1
/3)/x)*EllipticF((b^(1/3)/x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3)*(1+3^(1
/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)+b^(2/3)/x^2-a^(1/3
)*b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(7/3)/(a+b/
x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^
(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {335, 288, 321, 218}

$$\frac{32\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{15\sqrt[4]{3}\,b^{7/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}-\frac{16\sqrt{a+\frac{b}{x^3}}}{15b^2x}+\frac{2}{3bx^4\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^3)^(3/2)*x^8),x]
```

```
[Out] 2/(3*b*Sqrt[a + b/x^3]*x^4) - (16*Sqrt[a + b/x^3])/(15*b^2*x) + (32*Sqrt[2
+ Sqrt[3]]*a*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b
^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sq
rt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sq
rt[3]])/(15*3^(1/4)*b^(7/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3
)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^8}\,dx = -\text{Subst}\left(\int \frac{x^6}{\left(a+bx^3\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2}{3b\sqrt{a+\frac{b}{x^3}}\,x^4} - \frac{8\,\text{Subst}\left(\int \frac{x^3}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{3b}$$

$$= \frac{2}{3b\sqrt{a+\frac{b}{x^3}}\,x^4} - \frac{16\sqrt{a+\frac{b}{x^3}}}{15b^2x} + \frac{(16a)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{15b^2}$$

$$= \frac{2}{3b\sqrt{a+\frac{b}{x^3}}\,x^4} - \frac{16\sqrt{a+\frac{b}{x^3}}}{15b^2x} + \frac{32\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\right.\right.}{15\sqrt[4]{3}\,b^{7/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\right.}}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.20

$$-\frac{2\sqrt{\frac{ax^3}{b}+1}\;{}_2F_1\left(-\frac{5}{6},\frac{3}{2};\frac{1}{6};-\frac{ax^3}{b}\right)}{5bx^4\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^3)^(3/2)*x^8),x]
```

```
[Out] (-2*Sqrt[1 + (a*x^3)/b]*Hypergeometric2F1[-5/6, 3/2, 1/6, -((a*x^3)/b)])/(5
*b*Sqrt[a + b/x^3]*x^4)
```

fricas [F] time = 1.09, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\frac{ax^3+b}{x^3}}}{a^2x^8+2\,abx^5+b^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] integral(sqrt((a*x^3 + b)/x^3)/(a^2*x^8 + 2*a*b*x^5 + b^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}} x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] integrate(1/((a + b/x^3)^(3/2)*x^8), x)
```

maple [B] time = 0.02, size = 2056, normalized size = 7.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^3)^(3/2)/x^8,x)
```

```
[Out] 2/15/((a*x^3+b)/x^3)^(3/2)/x^8*(a*x^3+b)/(-a^2*b)^(1/3)/b^2*(32*I*3^(1/2)*(
-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(
1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1
/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(
-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b
)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3
))^(1/2))*((a*x^3+b)*x)^(1/2)*x^5*a^2-64*I*(-a^2*b)^(1/3)*3^(1/2)*(-(I*3^(1
/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a
^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2
*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^
(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))
*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)
)*((a*x^3+b)*x)^(1/2)*x^4*a+32*I*(-a^2*b)^(2/3)*3^(1/2)*(-(I*3^(1/2)-3)/(I*
3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3
)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(
1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1
/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2
),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+
b)*x)^(1/2)*x^3-32*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)
^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x
+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(
I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1
/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3
^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^(1/2)*x^5*a^2+64*(-a^2*b)^(1/3)
*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3
^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^
(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x
+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2
*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)
-3))^(1/2))*((a*x^3+b)*x)^(1/2)*x^4*a-32*(-a^2*b)^(2/3)*(-(I*3^(1/2)-3)/(I*
3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3
)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(
1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1
/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2
),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+
b)*x)^(1/2)*x^3-5*I*(-a^2*b)^(1/3)*3^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*
```

```
3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a
^2*b)^(1/3))/a^2*x)^(1/2)*x^4*a+15*(-a^2*b)^(1/3)*((-a*x+(-a^2*b)^(1/3))*(2
*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1
/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*x^4*a-3*I*(-a^2*b)^(1/3)*3^(1/2)*(a*x^4+b*
x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1
/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*((a*x^3+
b)*x)^(1/2)+9*(-a^2*b)^(1/3)*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*
x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)
-(-a^2*b)^(1/3))/a^2*x)^(1/2)*((a*x^3+b)*x)^(1/2))/(I*3^(1/2)-3)/((-a*x+(-a
^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1
/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}} x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] integrate(1/((a + b/x^3)^(3/2)*x^8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^8\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(a + b/x^3)^(3/2)),x)
```

```
[Out] int(1/(x^8*(a + b/x^3)^(3/2)), x)
```

sympy [A] time = 2.25, size = 39, normalized size = 0.15

$$-\frac{\Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\left.\begin{matrix}\frac{3}{2}, \frac{7}{3} \\ \frac{10}{3}\end{matrix}\right| \frac{be^{i\pi}}{ax^3}\right)}{3a^{\frac{3}{2}}x^7\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**3)**(3/2)/x**8,x)
```

```
[Out] -gamma(7/3)*hyper((3/2, 7/3), (10/3,), b*exp_polar(I*pi)/(a*x**3))/(3*a**(3
/2)*x**7*gamma(10/3))
```

3.2050 $\int \frac{x^6}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$

Optimal. Leaf size=587

$$-\frac{935b^{7/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{168\sqrt{2}\,\sqrt[4]{3}\,a^{11/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}+\frac{935\sqrt{2-\sqrt{3}}\,b^{7/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{}}{224\,3^{3/4}a}$$

```
[Out] -2/3*x^7/a/(a+b/x^3)^(1/2)+935/336*b^2*x*(a+b/x^3)^(1/2)/a^4-187/168*b*x^4*
(a+b/x^3)^(1/2)/a^3+17/21*x^7*(a+b/x^3)^(1/2)/a^2-935/336*b^(7/3)*(a+b/x^3)
^(1/2)/a^4/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))-935/1008*b^(7/3)*(a^(1/3)+b^(1/3
)/x)*EllipticF((b^(1/3)/x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3)*(1+3^(1/2
))),I*3^(1/2)+2*I)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^(1
/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(11/3)*2^(1/2)/(a+b/x^3)^(1/2)/(a^(1/3)
*(a^(1/3)+b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+935/672*b^(7/
3)*(a^(1/3)+b^(1/3)/x)*EllipticE((b^(1/3)/x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)/x
+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)+b^
(2/3)/x^2-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/
4)/a^(11/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/(b^(1/3)/x+a^(1/3)
*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 587, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {335, 290, 325, 303, 218, 1877}

$$\frac{935b^2x\sqrt{a+\frac{b}{x^3}}}{336a^4}-\frac{935b^{7/3}\sqrt{a+\frac{b}{x^3}}}{336a^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}-\frac{935b^{7/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\right|}{168\sqrt{2}\,\sqrt[4]{3}\,a^{11/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b/x^3)^(3/2),x]
```

```
[Out] (-935*b^(7/3)*Sqrt[a + b/x^3])/(336*a^4*((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)
) + (935*b^2*Sqrt[a + b/x^3]*x)/(336*a^4) - (187*b*Sqrt[a + b/x^3]*x^4)/(16
8*a^3) - (2*x^7)/(3*a*Sqrt[a + b/x^3]) + (17*Sqrt[a + b/x^3]*x^7)/(21*a^2)
+ (935*Sqrt[2 - Sqrt[3]]*b^(7/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2
/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*Ellip
ticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)/x)], -7 - 4*Sqrt[3]])/(224*3^(3/4)*a^(11/3)*Sqrt[a + b/x^3]*Sqrt[(a^(
1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]) - (935*
b^(7/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3
))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3]
)*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]
])/(168*Sqrt[2]*3^(1/4)*a^(11/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b
^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^6}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = -\operatorname{Subst}\left(\int \frac{1}{x^8\left(a+bx^3\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2x^7}{3a\sqrt{a+\frac{b}{x^3}}} - \frac{17\operatorname{Subst}\left(\int \frac{1}{x^8\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{3a}$$

$$= -\frac{2x^7}{3a\sqrt{a+\frac{b}{x^3}}} + \frac{17\sqrt{a+\frac{b}{x^3}}\,x^7}{21a^2} + \frac{(187b)\operatorname{Subst}\left(\int \frac{1}{x^5\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{42a^2}$$

$$= -\frac{187b\sqrt{a+\frac{b}{x^3}}\,x^4}{168a^3} - \frac{2x^7}{3a\sqrt{a+\frac{b}{x^3}}} + \frac{17\sqrt{a+\frac{b}{x^3}}\,x^7}{21a^2} - \frac{\left(935b^2\right)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{336a^3}$$

$$= \frac{935b^2\sqrt{a+\frac{b}{x^3}}\,x}{336a^4} - \frac{187b\sqrt{a+\frac{b}{x^3}}\,x^4}{168a^3} - \frac{2x^7}{3a\sqrt{a+\frac{b}{x^3}}} + \frac{17\sqrt{a+\frac{b}{x^3}}\,x^7}{21a^2} - \frac{\left(935b^3\right)\operatorname{Subst}\left(\int \frac{x}{\sqrt{a+bx}}\right.}{672a^4}$$

$$= \frac{935b^2\sqrt{a+\frac{b}{x^3}}\,x}{336a^4} - \frac{187b\sqrt{a+\frac{b}{x^3}}\,x^4}{168a^3} - \frac{2x^7}{3a\sqrt{a+\frac{b}{x^3}}} + \frac{17\sqrt{a+\frac{b}{x^3}}\,x^7}{21a^2} - \frac{\left(935b^{8/3}\right)\operatorname{Subst}\left(\int \frac{\left(1-\sqrt{}\right.}{672a^4}\right.$$

$$= -\frac{935b^{7/3}\sqrt{a+\frac{b}{x^3}}}{336a^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)} + \frac{935b^2\sqrt{a+\frac{b}{x^3}}\,x}{336a^4} - \frac{187b\sqrt{a+\frac{b}{x^3}}\,x^4}{168a^3} - \frac{2x^7}{3a\sqrt{a+\frac{b}{x^3}}} + \frac{17\sqrt{a+}}{21}$$

Mathematica [C] time = 0.05, size = 78, normalized size = 0.13

$$\frac{x\left(16a^2x^6 - 187b^2\sqrt{\frac{ax^3}{b}+1}\;{}_2F_1\left(\frac{5}{6},\frac{3}{2};\frac{11}{6};-\frac{ax^3}{b}\right) - 34abx^3 + 187b^2\right)}{112a^3\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b/x^3)^(3/2),x]
```

```
[Out] (x*(187*b^2 - 34*a*b*x^3 + 16*a^2*x^6 - 187*b^2*Sqrt[1 + (a*x^3)/b]*Hyperge
ometric2F1[5/6, 3/2, 11/6, -((a*x^3)/b)]))/(112*a^3*Sqrt[a + b/x^3])
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^{12}\sqrt{\frac{ax^3+b}{x^3}}}{a^2x^6+2\,abx^3+b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(a+b/x^3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x^12*sqrt((a*x^3 + b)/x^3)/(a^2*x^6 + 2*a*b*x^3 + b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(a+b/x^3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/(a + b/x^3)^(3/2), x)
```

maple [B] time = 0.04, size = 3182, normalized size = 5.42

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a+b/x^3)^(3/2),x)
```

```
[Out] -1/168/((a*x^3+b)/x^3)^(3/2)/x^5*(a*x^3+b)/a^5*(-935*I*(-a^2*b)^(2/3)*3^(1/
2)*((a*x^3+b)*x)^(1/2)*x*b^2+75*I*3^(1/2)*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)
^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(
-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*((a*x^3+b)*x)^(1/2)*x^2*a^2*b-93
5*I*(-a^2*b)^(1/3)*3^(1/2)*((a*x^3+b)*x)^(1/2)*x^2*a*b^2-1870*(-a^2*b)^(1/3
)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*
3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))
^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*
x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^
2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2
)-3))^(1/2))*((a*x^3+b)*x)^(1/2)*x^2*a*b^2+2805*(-a^2*b)^(1/3)*(-(I*3^(1/2)
-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*
b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*
x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/
3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*
x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(
(a*x^3+b)*x)^(1/2)*x^2*a*b^2+1870*I*(-a^2*b)^(2/3)*3^(1/2)*(-(I*3^(1/2)-3)/
(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(
1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*
3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))
^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(
1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x
^3+b)*x)^(1/2)*x*b^2+72*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3
^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^
2*b)^(1/3))/a^2*x)^(1/2)*((a*x^3+b)*x)^(1/2)*x^5*a^3+3740*(-a^2*b)^(2/3)*(-
(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1
/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/
2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-
a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)
^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3)
)^(1/2))*((a*x^3+b)*x)^(1/2)*x*b^2-5610*(-a^2*b)^(2/3)*(-(I*3^(1/2)-3)/(I*3
^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)
+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1
/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/
2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)
,((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b
)*x)^(1/2)*x*b^2+112*I*3^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^
2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))
/a^2*x)^(1/2)*x^3*a^2*b^2+935*I*3^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x
+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3)
```

```
)/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1
/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-
(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)
*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^(1/2)*a*b^
3-935*I*3^(1/2)*((a*x^3+b)*x)^(1/2)*x^3*a^2*b^2-24*I*3^(1/2)*(a*x^4+b*x)^(1
/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*
(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*((a*x^3+b)*x)
^(1/2)*x^5*a^3+1870*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x
)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*
x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/
(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(
1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*
3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^(1/2)*a*b^3-2805*(-(I*3^(1/2)-
3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b
)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x
+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3
)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x
)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((
a*x^3+b)*x)^(1/2)*a*b^3-935*I*(-a^2*b)^(1/3)*3^(1/2)*(-(I*3^(1/2)-3)/(I*3^(
1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(
-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2
)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)
*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),(
(I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*
x)^(1/2)*x^2*a*b^2-225*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^
(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2
*b)^(1/3))/a^2*x)^(1/2)*((a*x^3+b)*x)^(1/2)*x^2*a^2*b-336*((-a*x+(-a^2*b)^(
1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a
^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*x^3*a^2*b^2+2805*((a*x^3+b)*x)^(1/
2)*x^3*a^2*b^2+2805*(-a^2*b)^(1/3)*((a*x^3+b)*x)^(1/2)*x^2*a*b^2+2805*(-a^2
*b)^(2/3)*((a*x^3+b)*x)^(1/2)*x*b^2)/(I*3^(1/2)-3)/((-a*x+(-a^2*b)^(1/3))*(
2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(
1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(a+b/x^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(a + b/x^3)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b/x^3)^(3/2),x)
```

```
[Out] int(x^6/(a + b/x^3)^(3/2), x)
```

sympy [**A**] time = 1.56, size = 46, normalized size = 0.08

$$-\frac{x^7\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{7}{3}, \frac{3}{2} \\ -\frac{4}{3}\end{matrix}\right| \frac{be^{i\pi}}{ax^3}\right)}{3a^{\frac{3}{2}}\Gamma\left(-\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(a+b/x**3)**(3/2),x)
```

```
[Out] -x**7*gamma(-7/3)*hyper((-7/3, 3/2), (-4/3,), b*exp_polar(I*pi)/(a*x**3))/(
3*a**(3/2)*gamma(-4/3))
```

3.2051 $\int \frac{x^3}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$

Optimal. Leaf size=563

$$\frac{55b^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{12\sqrt{2}\,\sqrt[4]{3}\,a^{8/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}-\frac{55\sqrt{2-\sqrt{3}}\,b^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}}{\left(\left(1+\right.\right.}}{16\,3^{3/4}a^{8/3}\sqrt{}}$$

```
[Out] -2/3*x^4/a/(a+b/x^3)^(1/2)-55/24*b*x*(a+b/x^3)^(1/2)/a^3+11/12*x^4*(a+b/x^3
)^(1/2)/a^2+55/24*b^(4/3)*(a+b/x^3)^(1/2)/a^3/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)
))+55/72*b^(4/3)*(a^(1/3)+b^(1/3)/x)*EllipticF((b^(1/3)/x+a^(1/3)*(1-3^(1/2
)))/(b^(1/3)/x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((a^(2/3)+b^(2/3)/x^2-a^
(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(8/3)*2
^(1/2)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3
^(1/2)))^2)^(1/2)-55/48*b^(4/3)*(a^(1/3)+b^(1/3)/x)*EllipticE((b^(1/3)/x+a^
(1/3)*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1
/2)-1/2*2^(1/2))*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^(1/3
)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(8/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b
^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 563, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {335, 290, 325, 303, 218, 1877}

$$\frac{55b^{4/3}\sqrt{a+\frac{b}{x^3}}}{24a^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}+\frac{55b^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{12\sqrt{2}\,\sqrt[4]{3}\,a^{8/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}-\frac{55\sqrt{2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b/x^3)^(3/2),x]
```

```
[Out] (55*b^(4/3)*Sqrt[a + b/x^3])/(24*a^3*((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)) -
 (55*b*Sqrt[a + b/x^3]*x)/(24*a^3) - (2*x^4)/(3*a*Sqrt[a + b/x^3]) + (11*Sq
rt[a + b/x^3]*x^4)/(12*a^2) - (55*Sqrt[2 - Sqrt[3]]*b^(4/3)*(a^(1/3) + b^(1
/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)/x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(16*3^(3/4)*a^(8/3)*
Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)/x)^2]) + (55*b^(4/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/
3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)/x)], -7 - 4*Sqrt[3]])/(12*Sqrt[2]*3^(1/4)*a^(8/3)*Sqrt[a + b/x^3]*Sqrt
[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^3}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = -\operatorname{Subst}\left(\int \frac{1}{x^5\left(a+bx^3\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2x^4}{3a\sqrt{a+\frac{b}{x^3}}} - \frac{11\operatorname{Subst}\left(\int \frac{1}{x^5\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{3a}$$

$$= -\frac{2x^4}{3a\sqrt{a+\frac{b}{x^3}}} + \frac{11\sqrt{a+\frac{b}{x^3}}\,x^4}{12a^2} + \frac{(55b)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{24a^2}$$

$$= -\frac{55b\sqrt{a+\frac{b}{x^3}}\,x}{24a^3} - \frac{2x^4}{3a\sqrt{a+\frac{b}{x^3}}} + \frac{11\sqrt{a+\frac{b}{x^3}}\,x^4}{12a^2} + \frac{\left(55b^2\right)\operatorname{Subst}\left(\int \frac{x}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{48a^3}$$

$$= -\frac{55b\sqrt{a+\frac{b}{x^3}}\,x}{24a^3} - \frac{2x^4}{3a\sqrt{a+\frac{b}{x^3}}} + \frac{11\sqrt{a+\frac{b}{x^3}}\,x^4}{12a^2} + \frac{\left(55b^{5/3}\right)\operatorname{Subst}\left(\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{48a^3} +$$

$$= \frac{55b^{4/3}\sqrt{a+\frac{b}{x^3}}}{24a^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)} - \frac{55b\sqrt{a+\frac{b}{x^3}}\,x}{24a^3} - \frac{2x^4}{3a\sqrt{a+\frac{b}{x^3}}} + \frac{11\sqrt{a+\frac{b}{x^3}}\,x^4}{12a^2} - \frac{55\sqrt{2-\sqrt{3}}\,b^{4/}}{}$$

Mathematica [C] time = 0.03, size = 65, normalized size = 0.12

$$\frac{x\left(11b\sqrt{\frac{ax^3}{b}+1}\;{}_2F_1\left(\frac{5}{6},\frac{3}{2};\frac{11}{6};-\frac{ax^3}{b}\right)+2ax^3-11b\right)}{8a^2\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b/x^3)^(3/2),x]
```

```
[Out] (x*(-11*b + 2*a*x^3 + 11*b*Sqrt[1 + (a*x^3)/b]*Hypergeometric2F1[5/6, 3/2,
11/6, -((a*x^3)/b)]))/(8*a^2*Sqrt[a + b/x^3])
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^9\sqrt{\frac{ax^3+b}{x^3}}}{a^2x^6+2\,abx^3+b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x^3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x^9*sqrt((a*x^3 + b)/x^3)/(a^2*x^6 + 2*a*b*x^3 + b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x^3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(a + b/x^3)^(3/2), x)
```

maple [B] time = 0.02, size = 2936, normalized size = 5.21

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b/x^3)^(3/2),x)
```

```
[Out] 1/12/((a*x^3+b)/x^3)^(3/2)/x^5*(a*x^3+b)/a^4*(55*I*3^(1/2)*(-(I*3^(1/2)-3)/
(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(
1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*
3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))
^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(
1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x
^3+b)*x)^(1/2)*a*b^2-55*I*(-a^2*b)^(2/3)*3^(1/2)*((a*x^3+b)*x)^(1/2)*x*b-11
0*(-a^2*b)^(1/3)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(
1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(
-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*
3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2
)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(
1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^(1/2)*x^2*a*b+165*(-a^2*b)^(1/3)*
(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^
(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(
1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+
(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*
b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-
3))^(1/2))*((a*x^3+b)*x)^(1/2)*x^2*a*b-55*I*(-a^2*b)^(1/3)*3^(1/2)*(-(I*3^(
1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-
a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-
2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)
^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)
)*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2
))*((a*x^3+b)*x)^(1/2)*x^2*a*b+220*(-a^2*b)^(2/3)*(-(I*3^(1/2)-3)/(I*3^(1/2
)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^
2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(
-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*El
lipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*
3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^
(1/2)*x*b-330*(-a^2*b)^(2/3)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(
1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1
/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b
)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-
3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)
-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^(1/2)*x*b+8*I*3^(1/2)
*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2
*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*x^3*a^2*b+3*I*3^
(1/2)*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1
/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)
^(1/2)*((a*x^3+b)*x)^(1/2)*x^2*a^2+110*I*(-a^2*b)^(2/3)*3^(1/2)*(-(I*3^(1/2
)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2
*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a
*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1
/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a
*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*
((a*x^3+b)*x)^(1/2)*x*b-55*I*3^(1/2)*((a*x^3+b)*x)^(1/2)*x^3*a^2*b+110*(-(I
```

```
*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2
)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)
*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^
2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(
1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^
(1/2))*((a*x^3+b)*x)^(1/2)*a*b^2-165*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-
a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(
1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)
-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*
3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I
*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^(1/2)*a*b^2-5
5*I*(-a^2*b)^(1/3)*3^(1/2)*((a*x^3+b)*x)^(1/2)*x^2*a*b-9*(a*x^4+b*x)^(1/2)*
((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*
a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*((a*x^3+b)*x)^(1/
2)*x^2*a^2-24*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*
b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*x^3
*a^2*b+165*((a*x^3+b)*x)^(1/2)*x^3*a^2*b+165*(-a^2*b)^(1/3)*((a*x^3+b)*x)^(
1/2)*x^2*a*b+165*(-a^2*b)^(2/3)*((a*x^3+b)*x)^(1/2)*x*b)/(I*3^(1/2)-3)/((-a
*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+
I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(a + b/x^3)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b/x^3)^(3/2),x)
```

```
[Out] int(x^3/(a + b/x^3)^(3/2), x)
```

sympy [A] time = 1.45, size = 46, normalized size = 0.08

$$-\frac{x^4\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3}, \frac{3}{2} \\ -\frac{1}{3}\end{matrix}\middle| \frac{be^{i\pi}}{ax^3}\right)}{3a^{\frac{3}{2}}\Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b/x**3)**(3/2),x)
```

```
[Out] -x**4*gamma(-4/3)*hyper((-4/3, 3/2), (-1/3,), b*exp_polar(I*pi)/(a*x**3))/(
3*a**(3/2)*gamma(-1/3))
```

3.2052 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$

Optimal. Leaf size=539

$$-\frac{5\sqrt{2}\,\sqrt[3]{b}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,a^{5/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}+\frac{5\sqrt{2-\sqrt{3}}\,\sqrt[3]{b}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{2\,3^{3/}}$$

```
[Out] -2/3*x/a/(a+b/x^3)^(1/2)+5/3*x*(a+b/x^3)^(1/2)/a^2-5/3*b^(1/3)*(a+b/x^3)^(1
/2)/a^2/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))-5/9*b^(1/3)*(a^(1/3)+b^(1/3)/x)*Ell
ipticF((b^(1/3)/x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3)*(1+3^(1/2))),I*3^
(1/2)+2*I)*2^(1/2)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^(1
/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(5/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)
+b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+5/6*b^(1/3)*(a^(1/3)+b
^(1/3)/x)*EllipticE((b^(1/3)/x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3)*(1+3
^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)+b^(2/3)/x^2-a^(
1/3)*b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(5/3)/(a
+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^
2)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 539, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {242, 290, 325, 303, 218, 1877}

$$-\frac{5\sqrt{2}\,\sqrt[3]{b}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,a^{5/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}+\frac{5\sqrt{2-\sqrt{3}}\,\sqrt[3]{b}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{2\,3^{3/}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^3)^(-3/2),x]
```

```
[Out] (-5*b^(1/3)*Sqrt[a + b/x^3])/(3*a^2*((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)) -
(2*x)/(3*a*Sqrt[a + b/x^3]) + (5*Sqrt[a + b/x^3]*x)/(3*a^2) + (5*Sqrt[2 - S
qrt[3]]*b^(1/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3
)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticE[ArcSin[((1 -
 Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4
*Sqrt[3]])/(2*3^(3/4)*a^(5/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1
/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]) - (5*Sqrt[2]*b^(1/3)*(a^(1/
3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^
(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*
a^(5/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)/x)^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
```

```
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = -\operatorname{Subst}\left(\int \frac{1}{x^2\left(a+bx^3\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2x}{3a\sqrt{a+\frac{b}{x^3}}} - \frac{5\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{3a}$$

$$= -\frac{2x}{3a\sqrt{a+\frac{b}{x^3}}} + \frac{5\sqrt{a+\frac{b}{x^3}}\,x}{3a^2} - \frac{(5b)\operatorname{Subst}\left(\int \frac{x}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{6a^2}$$

$$= -\frac{2x}{3a\sqrt{a+\frac{b}{x^3}}} + \frac{5\sqrt{a+\frac{b}{x^3}}\,x}{3a^2} - \frac{\left(5b^{2/3}\right)\operatorname{Subst}\left(\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{6a^2} - \frac{\left(5\sqrt{\frac{1}{2}\left(2-\sqrt{3}\right)}\right.}{}$$

$$= -\frac{5\sqrt[3]{b}\,\sqrt{a+\frac{b}{x^3}}}{3a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)} - \frac{2x}{3a\sqrt{a+\frac{b}{x^3}}} + \frac{5\sqrt{a+\frac{b}{x^3}}\,x}{3a^2} + \frac{5\sqrt{2-\sqrt{3}}\,\sqrt[3]{b}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{}}{}$$

$$2\,3^{3/4}a$$

Mathematica [C] time = 0.03, size = 53, normalized size = 0.10

$$\frac{x - x\sqrt{\frac{ax^3}{b}+1}\;{}_2F_1\left(\frac{5}{6}, \frac{3}{2}; \frac{11}{6}; -\frac{ax^3}{b}\right)}{a\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^3)^(-3/2),x]
```

```
[Out] (x - x*Sqrt[1 + (a*x^3)/b]*Hypergeometric2F1[5/6, 3/2, 11/6, -((a*x^3)/b)])
/(a*Sqrt[a + b/x^3])
```

fricas [F] time = 1.09, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^6\sqrt{\frac{ax^3+b}{x^3}}}{a^2x^6+2\,abx^3+b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x^6*sqrt((a*x^3 + b)/x^3)/(a^2*x^6 + 2*a*b*x^3 + b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a + b/x^3)^(-3/2), x)
```

maple [B] time = 0.02, size = 2700, normalized size = 5.01

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^3)^(3/2),x)
```

```
[Out] -2/3/((a*x^3+b)/x^3)^(3/2)/x^5*(a*x^3+b)/a^3*(10*I*(-(I*3^(1/2)-3)/(I*3^(1/
2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a
^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*
(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*E
llipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I
*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*3^(1/2)*(-a^2
*b)^(2/3)*((a*x^3+b)*x)^(1/2)*x-5*I*3^(1/2)*(-a^2*b)^(2/3)*((a*x^3+b)*x)^(1
/2)*x-10*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2
*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^
(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-
1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a
*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I
*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*((a*x^3+b)*x)^(1/2)*x^2*a+I*((-a*x+(-a^2
*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2
)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*3^(1/2)*x^3*a^2+15*(-(I*3^(1/
2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^
2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*
a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(
1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*
a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))
*(-a^2*b)^(1/3)*((a*x^3+b)*x)^(1/2)*x^2*a+20*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/
(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^
(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*
b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*Ellipti
cF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/
2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*((a*
x^3+b)*x)^(1/2)*x-30*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*
x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a
*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))
/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^
(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I
*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*((a*x^3+b)*x)^(1/2)*x-5*I*(-
(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1
/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/
2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-
a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)
^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3)
)^(1/2))*3^(1/2)*(-a^2*b)^(1/3)*((a*x^3+b)*x)^(1/2)*x^2*a-5*I*3^(1/2)*((a*x
^3+b)*x)^(1/2)*x^3*a^2+10*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3
))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2)
)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(
1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/
(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)
/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^(1/2)*a*b-15*(-(I*3^(1/2
)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2
*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a
*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1
/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a
*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*
```

```
((a*x^3+b)*x)^(1/2)*a*b+5*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1
/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/
2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)
^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3
)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-
1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*3^(1/2)*((a*x^3+b)*x)^(1/2)*a*b-3*((
-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*
x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*x^3*a^2-5*I*3^(1/2)
*(-a^2*b)^(1/3)*((a*x^3+b)*x)^(1/2)*x^2*a+15*((a*x^3+b)*x)^(1/2)*x^3*a^2+15
*(-a^2*b)^(1/3)*((a*x^3+b)*x)^(1/2)*x^2*a+15*(-a^2*b)^(2/3)*((a*x^3+b)*x)^(
1/2)*x)/(I*3^(1/2)-3)/((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3
)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(
1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x^3)^(-3/2), x)
```

mupad [B] time = 1.27, size = 38, normalized size = 0.07

$$\frac{2\,x\left(\frac{a\,x^3}{b}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{11}{6};\ \frac{17}{6};\ -\frac{a\,x^3}{b}\right)}{11\left(a+\frac{b}{x^3}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x^3)^(3/2),x)
```

```
[Out] (2*x*((a*x^3)/b + 1)^(3/2)*hypergeom([3/2, 11/6], 17/6, -(a*x^3)/b))/(11*(a
 + b/x^3)^(3/2))
```

sympy [A] time = 1.24, size = 41, normalized size = 0.08

$$-\frac{x\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},\frac{3}{2}\\ \frac{2}{3}\end{matrix}\right|\frac{be^{i\pi}}{ax^3}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**3)**(3/2),x)
```

```
[Out] -x*gamma(-1/3)*hyper((-1/3, 3/2), (2/3,), b*exp_polar(I*pi)/(a*x**3))/(3*a*
*(3/2)*gamma(2/3))
```

3.2053 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^3}\,dx$

Optimal. Leaf size=520

$$\frac{2\sqrt{2}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,a^{2/3}b^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}} - \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\right.}}}{3^{3/4}a^{2/3}b^{2/3}\sqrt{a+}}$$

```
[Out] -2/3/a/x^2/(a+b/x^3)^(1/2)+2/3*(a+b/x^3)^(1/2)/a/b^(2/3)/(b^(1/3)/x+a^(1/3)
*(1+3^(1/2)))+2/9*(a^(1/3)+b^(1/3)/x)*EllipticF((b^(1/3)/x+a^(1/3)*(1-3^(1/
2)))/(b^(1/3)/x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)+b^(2/
3)/x^2-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/
a^(2/3)/b^(2/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/(b^(1/3)/x+a^(
1/3)*(1+3^(1/2)))^2)^(1/2)-1/3*(a^(1/3)+b^(1/3)/x)*EllipticE((b^(1/3)/x+a^(
1/3)*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/
2)-1/2*2^(1/2))*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^(1/3)
*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(2/3)/b^(2/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^
(1/3)+b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 520, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {335, 290, 303, 218, 1877}

$$\frac{2\sqrt{2}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,a^{2/3}b^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}} - \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\right.}}}{3^{3/4}a^{2/3}b^{2/3}\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^3)^(3/2)*x^3),x]
```

```
[Out] (2*Sqrt[a + b/x^3])/(3*a*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)) - 2/(
3*a*Sqrt[a + b/x^3]*x^2) - (Sqrt[2 - Sqrt[3]]*(a^(1/3) + b^(1/3)/x)*Sqrt[(a
^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)/x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(3^(3/4)*a^(2/3)*b^(2/3)*Sqrt[a +
b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)/x)^2]) + (2*Sqrt[2]*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (
a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin
[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)],
-7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(2/3)*b^(2/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(
a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^3}\,dx = -\text{Subst}\left(\int \frac{x}{\left(a+bx^3\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2}{3a\sqrt{a+\frac{b}{x^3}}\,x^2} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{3a}$$

$$= -\frac{2}{3a\sqrt{a+\frac{b}{x^3}}\,x^2} + \frac{\text{Subst}\left(\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{3a\sqrt[3]{b}} + \frac{\sqrt{2\left(2-\sqrt{3}\right)}\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^3}}\,d\right.}{3a^{2/3}\sqrt[3]{b}}$$

$$= \frac{2\sqrt{a+\frac{b}{x^3}}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)} - \frac{2}{3a\sqrt{a+\frac{b}{x^3}}\,x^2} - \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right.}}}{3^{3/4}a^{2/3}b^{2/3}\sqrt{a+\frac{b}{x^3}}}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.10

$$\frac{2x\sqrt{\frac{ax^3}{b}+1}\ {}_2F_1\left(\frac{5}{6},\frac{3}{2};\frac{11}{6};-\frac{ax^3}{b}\right)}{5b\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^3)^(3/2)*x^3),x]
```

```
[Out] (2*x*Sqrt[1 + (a*x^3)/b]*Hypergeometric2F1[5/6, 3/2, 11/6, -((a*x^3)/b)])/(
5*b*Sqrt[a + b/x^3])
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3\sqrt{\frac{ax^3+b}{x^3}}}{a^2x^6+2\,abx^3+b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(x^3*sqrt((a*x^3 + b)/x^3)/(a^2*x^6 + 2*a*b*x^3 + b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a + b/x^3)^(3/2)*x^3), x)
```

maple [B] time = 0.01, size = 2703, normalized size = 5.20

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^3)^(3/2)/x^3,x)
```

```
[Out] 2/3/((a*x^3+b)/x^3)^(3/2)/x^5*(a*x^3+b)/a^2*(-2*I*(-a^2*b)^(1/3)*3^(1/2)*((
a*x^3+b)*x)^(1/2)*x^2*a+2*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1
/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/
2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)
^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3
)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-
1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*3^(1/2)*((a*x^3+b)*x)^(1/2)*a*b-4*(-
(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1
/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/
2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-
a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)
^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3)
)^(1/2))*(-a^2*b)^(1/3)*((a*x^3+b)*x)^(1/2)*x^2*a+6*(-(I*3^(1/2)-3)/(I*3^(1
/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-
a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)
*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*
```

```
EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((
I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/
3)*((a*x^3+b)*x)^(1/2)*x^2*a+4*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*
b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*
3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a
^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1
/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(
1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*3^(1/2)*((a*x^3+
b)*x)^(1/2)*x+8*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1
/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-
a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3
^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)
-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1
/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*((a*x^3+b)*x)^(1/2)*x-12*(-(I*3^(
1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-
a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-
2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)
^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)
)*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2
))*(-a^2*b)^(2/3)*((a*x^3+b)*x)^(1/2)*x-2*I*(-a^2*b)^(2/3)*3^(1/2)*((a*x^3+
b)*x)^(1/2)*x-2*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^
(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+
(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I
*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/
2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^
(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*3^(1/2)*((a*x^3+b)*x)^(1/2)*x^2
*a-2*I*3^(1/2)*((a*x^3+b)*x)^(1/2)*x^3*a^2+4*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/
(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^
(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*
b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*Ellipti
cF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/
2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^(1/2)
*a*b-6*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a
*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1
/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)
/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x
+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3
^(1/2)-3))^(1/2))*((a*x^3+b)*x)^(1/2)*a*b+I*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I
*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-
a^2*b)^(1/3))/a^2*x)^(1/2)*3^(1/2)*a^2*x^3+6*((a*x^3+b)*x)^(1/2)*a^2*x^3-3*
((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*
a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*a^2*x^3+6*(-a^2*b
)^(1/3)*((a*x^3+b)*x)^(1/2)*a*x^2+6*(-a^2*b)^(2/3)*((a*x^3+b)*x)^(1/2)*x)/b
/(I*3^(1/2)-3)/((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2
*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}} x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((a + b/x^3)^(3/2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b/x^3)^(3/2)),x)
```

```
[Out] int(1/(x^3*(a + b/x^3)^(3/2)), x)
```

sympy [A] time = 1.44, size = 39, normalized size = 0.08

$$-\frac{\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{2}{3}, \frac{3}{2} \\ \frac{5}{3}\end{matrix}\right| \frac{be^{i\pi}}{ax^3}\right)}{3a^{\frac{3}{2}}x^2\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**3)**(3/2)/x**3,x)
```

```
[Out] -gamma(2/3)*hyper((2/3, 3/2), (5/3,), b*exp_polar(I*pi)/(a*x**3))/(3*a**(3/
2)*x**2*gamma(5/3))
```

3.2054 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^6}\,dx$

Optimal. Leaf size=517

$$-\frac{8\sqrt{2}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,b^{5/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}+\frac{4\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{3^{3/4}}$$

```
[Out] 2/3/b/x^2/(a+b/x^3)^(1/2)-8/3*(a+b/x^3)^(1/2)/b^(5/3)/(b^(1/3)/x+a^(1/3)*(1
+3^(1/2)))-8/9*a^(1/3)*(a^(1/3)+b^(1/3)/x)*EllipticF((b^(1/3)/x+a^(1/3)*(1-
3^(1/2)))/(b^(1/3)/x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)+
b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(
3/4)/b^(5/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/(b^(1/3)/x+a^(1/3
)*(1+3^(1/2)))^2)^(1/2)+4/3*a^(1/3)*(a^(1/3)+b^(1/3)/x)*EllipticE((b^(1/3)/
x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*
6^(1/2)-1/2*2^(1/2))*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^
(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/b^(5/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/
3)+b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 517, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {335, 288, 303, 218, 1877}

$$-\frac{8\sqrt{2}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,b^{5/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}+\frac{4\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{3^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^3)^(3/2)*x^6),x]
```

```
[Out] (-8*Sqrt[a + b/x^3])/(3*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)) + 2/(3
*b*Sqrt[a + b/x^3]*x^2) + (4*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)/x
)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)/x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(3^(3/4)*b^(5/3)*Sqrt[a +
 b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)/x)^2]) - (8*Sqrt[2]*a^(1/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3
)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)/x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(5/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*
(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^6}\,dx = -\text{Subst}\left(\int \frac{x^4}{\left(a+bx^3\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2}{3b\sqrt{a+\frac{b}{x^3}}\,x^2} - \frac{4\,\text{Subst}\left(\int \frac{x}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{3b}$$

$$= \frac{2}{3b\sqrt{a+\frac{b}{x^3}}\,x^2} - \frac{4\,\text{Subst}\left(\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{3b^{4/3}} - \frac{\left(4\sqrt{2\left(2-\sqrt{3}\right)}\,\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+b}}\right.}{3b^{4/3}}$$

$$= -\frac{8\sqrt{a+\frac{b}{x^3}}}{3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)} + \frac{2}{3b\sqrt{a+\frac{b}{x^3}}\,x^2} + \frac{4\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{}}{}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\right.}}}{3^{3/4}b^{5/3}\sqrt{a+\frac{b}{x^3}}}$$

Mathematica [C] time = 0.03, size = 52, normalized size = 0.10

$$-\frac{2\sqrt{\frac{ax^3}{b}+1}\ {}_2F_1\left(-\frac{1}{6},\frac{3}{2};\frac{5}{6};-\frac{ax^3}{b}\right)}{bx^2\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^3)^(3/2)*x^6),x]
```

```
[Out] (-2*Sqrt[1 + (a*x^3)/b]*Hypergeometric2F1[-1/6, 3/2, 5/6, -((a*x^3)/b)])/(b
*Sqrt[a + b/x^3]*x^2)
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\frac{ax^3+b}{x^3}}}{a^2x^6+2\,abx^3+b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] integral(sqrt((a*x^3 + b)/x^3)/(a^2*x^6 + 2*a*b*x^3 + b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate(1/((a + b/x^3)^(3/2)*x^6), x)
```

maple [B] time = 0.02, size = 2867, normalized size = 5.55

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^3)^(3/2)/x^6,x)
```

```
[Out] -2/3/((a*x^3+b)/x^3)^(3/2)/x^5*(a*x^3+b)/a*(16*I*(-(I*3^(1/2)-3)/(I*3^(1/2)
-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2
*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-
a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*Ell
ipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3
^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*
3^(1/2)*((a*x^3+b)*x)^(1/2)*x-8*I*(-a^2*b)^(2/3)*3^(1/2)*((a*x^3+b)*x)^(1/2
)*x-16*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a
*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1
/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)
/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x
+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3
^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*((a*x^3+b)*x)^(1/2)*x^2*a+24*(-(I*3^(1/2)-
3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b
)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x
+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3
```

```
)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x
)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-
a^2*b)^(1/3)*((a*x^3+b)*x)^(1/2)*x^2*a-8*I*3^(1/2)*((a*x^3+b)*x)^(1/2)*x^3*
a^2+32*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a
*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1
/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)
/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x
+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3
^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*((a*x^3+b)*x)^(1/2)*x-48*(-(I*3^(1/2)-3)/(
I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1
/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3
^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^
(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1
/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*
b)^(2/3)*((a*x^3+b)*x)^(1/2)*x-8*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^
2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+
I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(
-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^
(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3
^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*3^(1/2)*((a*x^
3+b)*x)^(1/2)*x^2*a+3*I*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1
/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)
^(1/2)*3^(1/2)*a*b+16*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a
*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-
a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3)
)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3
^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+
I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^(1/2)*a*b-24*(-(I*3^(1/2)-3)
/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^
(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I
*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))
)^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^
(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*
x^3+b)*x)^(1/2)*a*b+8*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))
*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/
(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/
3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I
*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(
1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*3^(1/2)*((a*x^3+b)*x)^(1/2)*a*b-12*((-a*
x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I
*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*a^2*x^3-8*I*(-a^2*b)^(
1/3)*3^(1/2)*((a*x^3+b)*x)^(1/2)*x^2*a+24*((a*x^3+b)*x)^(1/2)*a^2*x^3+4*I*(
(-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a
*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*3^(1/2)*x^3*a^2+24
*(-a^2*b)^(1/3)*((a*x^3+b)*x)^(1/2)*a*x^2+24*(-a^2*b)^(2/3)*((a*x^3+b)*x)^(
1/2)*x-9*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1
/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*a*b)/b^2
/(I*3^(1/2)-3)/((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2
*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}} x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(1/((a + b/x^3)^(3/2)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^6\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b/x^3)^(3/2)),x)
```

```
[Out] int(1/(x^6*(a + b/x^3)^(3/2)), x)
```

sympy [A] time = 1.84, size = 39, normalized size = 0.08

$$-\frac{\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2}, \frac{5}{3} \\ \frac{8}{3}\end{matrix}\right| \frac{be^{i\pi}}{ax^3}\right)}{3a^{\frac{3}{2}}x^5\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**3)**(3/2)/x**6,x)
```

```
[Out] -gamma(5/3)*hyper((3/2, 5/3), (8/3,), b*exp_polar(I*pi)/(a*x**3))/(3*a**(3/
2)*x**5*gamma(8/3))
```

3.2055 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^9}\,dx$

Optimal. Leaf size=541

$$\frac{80\sqrt{2}\,a^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{21\sqrt[4]{3}\,b^{8/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}-\frac{40\sqrt{2-\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{}}{7\,3^{3/4}b^{8}}$$

```
[Out] 2/3/b/x^5/(a+b/x^3)^(1/2)-20/21*(a+b/x^3)^(1/2)/b^2/x^2+80/21*a*(a+b/x^3)^(
1/2)/b^(8/3)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))+80/63*a^(4/3)*(a^(1/3)+b^(1/3)
/x)*EllipticF((b^(1/3)/x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)
)),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/(b^(1/3)
/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(8/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(
a^(1/3)+b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-40/21*a^(4/3)*(
a^(1/3)+b^(1/3)/x)*EllipticE((b^(1/3)/x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)/x+a^(
1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)+b^(2/3
)/x^2-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/b
^(8/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3
^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 541, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {335, 288, 321, 303, 218, 1877}

$$\frac{80\sqrt{2}\,a^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{21\sqrt[4]{3}\,b^{8/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}-\frac{40\sqrt{2-\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{}}{7\,3^{3/4}b^{8}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^3)^(3/2)*x^9),x]
```

```
[Out] (80*a*Sqrt[a + b/x^3])/(21*b^(8/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)) + 2
/(3*b*Sqrt[a + b/x^3]*x^5) - (20*Sqrt[a + b/x^3])/(21*b^2*x^2) - (40*Sqrt[2
 - Sqrt[3]]*a^(4/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^
(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticE[ArcSin[(
(1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7
 - 4*Sqrt[3]])/(7*3^(3/4)*b^(8/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) +
b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]) + (80*Sqrt[2]*a^(4/3)*(
a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3)
 + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(21*3^
(1/4)*b^(8/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)/x)^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
```

```
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^9}\,dx = -\operatorname{Subst}\left(\int \frac{x^7}{\left(a+bx^3\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2}{3b\sqrt{a+\frac{b}{x^3}}\,x^5} - \frac{10\operatorname{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{3b}$$

$$= \frac{2}{3b\sqrt{a+\frac{b}{x^3}}\,x^5} - \frac{20\sqrt{a+\frac{b}{x^3}}}{21b^2x^2} + \frac{(40a)\operatorname{Subst}\left(\int \frac{x}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{21b^2}$$

$$= \frac{2}{3b\sqrt{a+\frac{b}{x^3}}\,x^5} - \frac{20\sqrt{a+\frac{b}{x^3}}}{21b^2x^2} + \frac{(40a)\operatorname{Subst}\left(\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{21b^{7/3}} + \frac{\left(40\sqrt{2\left(2-\sqrt{3}\right)}\right.}{}$$

$$= \frac{80a\sqrt{a+\frac{b}{x^3}}}{21b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)} + \frac{2}{3b\sqrt{a+\frac{b}{x^3}}\,x^5} - \frac{20\sqrt{a+\frac{b}{x^3}}}{21b^2x^2} - \frac{40\sqrt{2-\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right.}{}$$

$$7\,3$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.10

$$-\frac{2\sqrt{\frac{ax^3}{b}+1}\;{}_2F_1\left(-\frac{7}{6},\frac{3}{2};-\frac{1}{6};-\frac{ax^3}{b}\right)}{7bx^5\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^3)^(3/2)*x^9),x]
```

```
[Out] (-2*Sqrt[1 + (a*x^3)/b]*Hypergeometric2F1[-7/6, 3/2, -1/6, -((a*x^3)/b)])/(
7*b*Sqrt[a + b/x^3]*x^5)
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\frac{ax^3+b}{x^3}}}{a^2x^9+2\,abx^6+b^2x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] integral(sqrt((a*x^3 + b)/x^3)/(a^2*x^9 + 2*a*b*x^6 + b^2*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}} x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] integrate(1/((a + b/x^3)^(3/2)*x^9), x)
```

maple [B] time = 0.02, size = 3307, normalized size = 6.11

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^3)^(3/2)/x^9,x)
```

```
[Out] 2/21/((a*x^3+b)/x^3)^(3/2)/x^9*(a*x^3+b)*(29*I*3^(1/2)*((-a*x+(-a^2*b)^(1/3
))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*
b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*x^4*a*b-80*I*(-a^2*b)^(1/3)*3^(1/2)*(
-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(
1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1
/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(
-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b
)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3
))^(1/2))*((a*x^3+b)*x)^(1/2)*x^6*a-160*((a*x^3+b)*x)^(1/2)*(-(I*3^(1/2)-3)
/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^
(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I
*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))
)^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^
(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^
2*b)^(1/3)*x^6*a+240*((a*x^3+b)*x)^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*
x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3
))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(
1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((
-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3
)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*x^6*a-3*
I*((a*x^3+b)*x)^(1/2)*3^(1/2)*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a
*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3
)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*b+320*((a*x^3+b)*x)^(1/2)*(-(I*3^(1/2)-3)/(I
*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/
3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^
(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(
1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/
2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b
)^(2/3)*x^5-480*((a*x^3+b)*x)^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a
^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1
+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-
(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3
^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*
3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(2/3)*x^5+80*I*3^(1
/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+
I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)
))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-
a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-
a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1
/2)-3))^(1/2))*((a*x^3+b)*x)^(1/2)*x^4*a*b+36*I*3^(1/2)*((-a*x+(-a^2*b)^(1/
3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2
*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*x^7*a^2-80*I*(-a^2*b)^(1/3)*3^(1/2)*
((a*x^3+b)*x)^(1/2)*x^6*a+160*((a*x^3+b)*x)^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2
)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^
2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(
-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*El
lipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*
3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^4*a*b-240*((
a*x^3+b)*x)^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^
```

```
(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+
(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I
*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/
2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^
(1/2))/(I*3^(1/2)-3))^(1/2))*x^4*a*b-80*I*3^(1/2)*((a*x^3+b)*x)^(1/2)*x^7*a
^2-108*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3
))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*x^7*a^2+24
0*((a*x^3+b)*x)^(1/2)*x^7*a^2+240*((a*x^3+b)*x)^(1/2)*(-a^2*b)^(1/3)*x^6*a-
80*I*(-a^2*b)^(2/3)*3^(1/2)*((a*x^3+b)*x)^(1/2)*x^5+160*I*(-a^2*b)^(2/3)*3^
(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*
x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/
3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/
(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+
(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^
(1/2)-3))^(1/2))*((a*x^3+b)*x)^(1/2)*x^5+240*((a*x^3+b)*x)^(1/2)*(-a^2*b)^(
2/3)*x^5-12*((a*x^3+b)*x)^(1/2)*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2
*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1
/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*x^3*a-87*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3
^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^
2*b)^(1/3))/a^2*x)^(1/2)*x^4*a*b+4*I*((a*x^3+b)*x)^(1/2)*3^(1/2)*(a*x^4+b*x
)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/
3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*x^3*a+9*(
(a*x^3+b)*x)^(1/2)*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2
)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^
(1/3))/a^2*x)^(1/2)*b)/b^3/(I*3^(1/2)-3)/((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^
(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2
*b)^(1/3))/a^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}} x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] integrate(1/((a + b/x^3)^(3/2)*x^9), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^9\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(a + b/x^3)^(3/2)),x)
```

```
[Out] int(1/(x^9*(a + b/x^3)^(3/2)), x)
```

sympy [A] time = 2.55, size = 39, normalized size = 0.07

$$-\frac{\Gamma\left(\frac{8}{3}\right) {}_2F_1\left(\left.\begin{matrix}\frac{3}{2}, \frac{8}{3} \\ \frac{11}{3}\end{matrix}\right| \frac{be^{i\pi}}{ax^3}\right)}{3a^{\frac{3}{2}}x^8\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**3)**(3/2)/x**9,x)
```

```
[Out] -gamma(8/3)*hyper((3/2, 8/3), (11/3,), b*exp_polar(I*pi)/(a*x**3))/(3*a**(3
/2)*x**8*gamma(11/3))
```

3.2056 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^{12}}\,dx$

Optimal. Leaf size=565

$$-\frac{1280\sqrt{2}\,a^{7/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{273\sqrt[4]{3}\,b^{11/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}+\frac{640\sqrt{2-\sqrt{3}}\,a^{7/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right.}{91\,3}$$

```
[Out] 2/3/b/x^8/(a+b/x^3)^(1/2)-32/39*(a+b/x^3)^(1/2)/b^2/x^5+320/273*a*(a+b/x^3)
^(1/2)/b^3/x^2-1280/273*a^2*(a+b/x^3)^(1/2)/b^(11/3)/(b^(1/3)/x+a^(1/3)*(1+
3^(1/2)))-1280/819*a^(7/3)*(a^(1/3)+b^(1/3)/x)*EllipticF((b^(1/3)/x+a^(1/3)
*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2
/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
*3^(3/4)/b^(11/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/(b^(1/3)/x+a
^(1/3)*(1+3^(1/2)))^2)^(1/2)+640/273*a^(7/3)*(a^(1/3)+b^(1/3)/x)*EllipticE(
(b^(1/3)/x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)/x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2
*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/(b^(
1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/b^(11/3)/(a+b/x^3)^(1/2)/(a^(1
/3)*(a^(1/3)+b^(1/3)/x)/(b^(1/3)/x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 565, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {335, 288, 321, 303, 218, 1877}

$$-\frac{1280a^2\sqrt{a+\frac{b}{x^3}}}{273b^{11/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}-\frac{1280\sqrt{2}\,a^{7/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{\right.}}{273\sqrt[4]{3}\,b^{11/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^3)^(3/2)*x^12),x]
```

```
[Out] (-1280*a^2*Sqrt[a + b/x^3])/(273*b^(11/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)/
x)) + 2/(3*b*Sqrt[a + b/x^3]*x^8) - (32*Sqrt[a + b/x^3])/(39*b^2*x^5) + (32
0*a*Sqrt[a + b/x^3])/(273*b^3*x^2) + (640*Sqrt[2 - Sqrt[3]]*a^(7/3)*(a^(1/3
) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(
1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(91*3^(3/4)*
b^(11/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)/x)^2]) - (1280*Sqrt[2]*a^(7/3)*(a^(1/3) + b^(1/3)/x)*Sq
rt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(273*3^(1/4)*b^(11/3)*Sqrt[a
+ b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)/x)^2])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^{12}}\,dx = -\operatorname{Subst}\left(\int \frac{x^{10}}{\left(a+bx^3\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2}{3b\sqrt{a+\frac{b}{x^3}}\,x^8} - \frac{16\operatorname{Subst}\left(\int \frac{x^7}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{3b}$$

$$= \frac{2}{3b\sqrt{a+\frac{b}{x^3}}\,x^8} - \frac{32\sqrt{a+\frac{b}{x^3}}}{39b^2x^5} + \frac{(160a)\operatorname{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{39b^2}$$

$$= \frac{2}{3b\sqrt{a+\frac{b}{x^3}}\,x^8} - \frac{32\sqrt{a+\frac{b}{x^3}}}{39b^2x^5} + \frac{320a\sqrt{a+\frac{b}{x^3}}}{273b^3x^2} - \frac{\left(640a^2\right)\operatorname{Subst}\left(\int \frac{x}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{273b^3}$$

$$= \frac{2}{3b\sqrt{a+\frac{b}{x^3}}\,x^8} - \frac{32\sqrt{a+\frac{b}{x^3}}}{39b^2x^5} + \frac{320a\sqrt{a+\frac{b}{x^3}}}{273b^3x^2} - \frac{\left(640a^2\right)\operatorname{Subst}\left(\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx, x, \frac{1}{x}\right)}{273b^{10/3}}$$

$$= -\frac{1280a^2\sqrt{a+\frac{b}{x^3}}}{273b^{11/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)} + \frac{2}{3b\sqrt{a+\frac{b}{x^3}}\,x^8} - \frac{32\sqrt{a+\frac{b}{x^3}}}{39b^2x^5} + \frac{320a\sqrt{a+\frac{b}{x^3}}}{273b^3x^2} + \frac{640\sqrt{}}{}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.10

$$-\frac{2\sqrt{\frac{ax^3}{b}+1}\;{}_2F_1\left(-\frac{13}{6}, \frac{3}{2}; -\frac{7}{6}; -\frac{ax^3}{b}\right)}{13bx^8\sqrt{a+\frac{b}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^3)^(3/2)*x^12),x]
```

```
[Out] (-2*Sqrt[1 + (a*x^3)/b]*Hypergeometric2F1[-13/6, 3/2, -7/6, -((a*x^3)/b)])/
(13*b*Sqrt[a + b/x^3]*x^8)
```

fricas [F] time = 1.08, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\frac{ax^3+b}{x^3}}}{a^2x^{12}+2\,abx^9+b^2x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(3/2)/x^12,x, algorithm="fricas")
```

```
[Out] integral(sqrt((a*x^3 + b)/x^3)/(a^2*x^12 + 2*a*b*x^9 + b^2*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}} x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(3/2)/x^12,x, algorithm="giac")
```

```
[Out] integrate(1/((a + b/x^3)^(3/2)*x^12), x)
```

maple [B] time = 0.02, size = 3554, normalized size = 6.29

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^3)^(3/2)/x^12,x)
```

```
[Out] -2/273/((a*x^3+b)/x^3)^(3/2)/x^12*(a*x^3+b)*(92*I*3^(1/2)*(a*x^4+b*x)^(1/2)
*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2
*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*((a*x^3+b)*x)^(1
/2)*x^6*a^2-69*I*3^(1/2)*(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*
3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a
^2*b)^(1/3))/a^2*x)^(1/2)*((a*x^3+b)*x)^(1/2)*x^3*a*b+3840*(-a^2*b)^(2/3)*(
(a*x^3+b)*x)^(1/2)*x^8*a+3840*(-a^2*b)^(1/3)*((a*x^3+b)*x)^(1/2)*x^9*a^2-13
71*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(
-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*x^7*a^2*b-63*(
a*x^4+b*x)^(1/2)*b^2*((a*x^3+b)*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^
(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2
*b)^(1/3))/a^2*x)^(1/2)-1280*I*(-a^2*b)^(1/3)*3^(1/2)*((a*x^3+b)*x)^(1/2)*x
^9*a^2+457*I*3^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)
+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1
/2)*x^7*a^2*b-1280*I*(-a^2*b)^(2/3)*3^(1/2)*((a*x^3+b)*x)^(1/2)*x^8*a+21*I*
(a*x^4+b*x)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-
a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)
*3^(1/2)*((a*x^3+b)*x)^(1/2)*b^2+207*a*(a*x^4+b*x)^(1/2)*x^3*b*((a*x^3+b)*x
)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/
3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)-1280*I*(-
a^2*b)^(1/3)*3^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*
x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a
*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))
/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^
(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I
*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^(1/2)*x^9*a^2+2560*I*(-a^2*b)
^(2/3)*3^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/
2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a
^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^
(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-
1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/
2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^(1/2)*x^8*a+1280*I*a^2*b*3^(1/2)*(-
(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1
/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/
2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-
a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)
^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3)
)^(1/2))*((a*x^3+b)*x)^(1/2)*x^7-276*a^2*(a*x^4+b*x)^(1/2)*x^6*((a*x^3+b)*x
)^(1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/
3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)+548*I*3^(
1/2)*((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))
*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/a^2*x)^(1/2)*x^10*a^3-128
0*I*3^(1/2)*((a*x^3+b)*x)^(1/2)*x^10*a^3-1644*((-a*x+(-a^2*b)^(1/3))*(2*a*x
+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-
(-a^2*b)^(1/3))/a^2*x)^(1/2)*x^10*a^3+3840*((a*x^3+b)*x)^(1/2)*x^10*a^3-384
0*a^2*b*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*
a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(
```

```
1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1
)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*
x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*
3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^(1/2)*x^7-7680*(-a^2*b)^(2/3)*(-(I*3^(1/2)
-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*
b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*
x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/
3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*
x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(
(a*x^3+b)*x)^(1/2)*x^8*a+2560*a^2*b*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a
^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1
+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-
(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3
^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*
3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^(1/2)*x^7+3840
*(-a^2*b)^(1/3)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1
/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-
a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3
^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)
-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1
/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3+b)*x)^(1/2)*x^9*a^2+5120*(-a^2*b)^(2/3)*
(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^
(1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(
1/2)*((-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+
(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*
b)^(1/3))*a*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-
3))^(1/2))*((a*x^3+b)*x)^(1/2)*x^8*a-2560*(-a^2*b)^(1/3)*(-(I*3^(1/2)-3)/(I
*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/2)*((2*a*x+I*3^(1/2)*(-a^2*b)^(1/
3)+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((-2*a*x+I*3^
(1/2)*(-a^2*b)^(1/3)-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(
1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))*a*x)^(1/
2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((a*x^3
+b)*x)^(1/2)*x^9*a^2)/b^4/(I*3^(1/2)-3)/((-a*x+(-a^2*b)^(1/3))*(2*a*x+I*3^(
1/2)*(-a^2*b)^(1/3)+(-a^2*b)^(1/3))*(-2*a*x+I*3^(1/2)*(-a^2*b)^(1/3)-(-a^2*
b)^(1/3))/a^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}} x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^3)^(3/2)/x^12,x, algorithm="maxima")
```

```
[Out] integrate(1/((a + b/x^3)^(3/2)*x^12), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{12}\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^12*(a + b/x^3)^(3/2)),x)
```

```
[Out] int(1/(x^12*(a + b/x^3)^(3/2)), x)
```

sympy [**A**] time = 3.06, size = 39, normalized size = 0.07

$$-\frac{\Gamma\left(\frac{11}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2}, \frac{11}{3} \\ \frac{14}{3}\end{matrix}\right| \frac{be^{i\pi}}{ax^3}\right)}{3a^{\frac{3}{2}}x^{11}\Gamma\left(\frac{14}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**3)**(3/2)/x**12,x)
```

```
[Out] -gamma(11/3)*hyper((3/2, 11/3), (14/3,), b*exp_polar(I*pi)/(a*x**3))/(3*a**
(3/2)*x**11*gamma(14/3))
```

3.2057 $\int \frac{1}{a+\frac{b}{x^4}}\,dx$

Optimal. Leaf size=190

$$\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}x^2+\sqrt{b}\right)}{4\sqrt{2}a^{5/4}}-\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}x^2+\sqrt{b}\right)}{4\sqrt{2}a^{5/4}}+\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}\right)}{2\sqrt{2}a^{5/4}}-\frac{\sqrt[4]{b}\tan^{-1}}{2}$$

```
[Out] x/a-1/4*b^(1/4)*arctan(-1+a^(1/4)*x*2^(1/2)/b^(1/4))/a^(5/4)*2^(1/2)-1/4*b^
(1/4)*arctan(1+a^(1/4)*x*2^(1/2)/b^(1/4))/a^(5/4)*2^(1/2)+1/8*b^(1/4)*ln(-a
^(1/4)*b^(1/4)*x*2^(1/2)+x^2*a^(1/2)+b^(1/2))/a^(5/4)*2^(1/2)-1/8*b^(1/4)*l
n(a^(1/4)*b^(1/4)*x*2^(1/2)+x^2*a^(1/2)+b^(1/2))/a^(5/4)*2^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.889$, Rules used = {193, 321, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}x^2+\sqrt{b}\right)}{4\sqrt{2}a^{5/4}}-\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}x^2+\sqrt{b}\right)}{4\sqrt{2}a^{5/4}}+\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}\right)}{2\sqrt{2}a^{5/4}}-\frac{\sqrt[4]{b}\tan^{-1}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^4)^(-1),x]
```

```
[Out] x/a + (b^(1/4)*ArcTan[1 - (Sqrt[2]*a^(1/4)*x)/b^(1/4)])/(2*Sqrt[2]*a^(5/4))
 - (b^(1/4)*ArcTan[1 + (Sqrt[2]*a^(1/4)*x)/b^(1/4)])/(2*Sqrt[2]*a^(5/4)) +
(b^(1/4)*Log[Sqrt[b] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[a]*x^2])/(4*Sqrt[2]
*a^(5/4)) - (b^(1/4)*Log[Sqrt[b] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[a]*x^2]
)/(4*Sqrt[2]*a^(5/4))
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{a+\frac{b}{x^4}}\,dx = \int \frac{x^4}{b+ax^4}\,dx$$

$$= \frac{x}{a} - \frac{b\int \frac{1}{b+ax^4}\,dx}{a}$$

$$= \frac{x}{a} - \frac{\sqrt{b}\int \frac{\sqrt{b}-\sqrt{a}\,x^2}{b+ax^4}\,dx}{2a} - \frac{\sqrt{b}\int \frac{\sqrt{b}+\sqrt{a}\,x^2}{b+ax^4}\,dx}{2a}$$

$$= \frac{x}{a} + \frac{\sqrt[4]{b}\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}+2x}{-\frac{\sqrt{b}}{\sqrt{a}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}-x^2}\,dx}{4\sqrt{2}\,a^{5/4}} + \frac{\sqrt[4]{b}\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}-2x}{-\frac{\sqrt{b}}{\sqrt{a}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}-x^2}\,dx}{4\sqrt{2}\,a^{5/4}} - \frac{\sqrt{b}\int \frac{1}{\frac{\sqrt{b}}{\sqrt{a}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+x^2}\,dx}{4a^{3/2}} - \frac{\sqrt{b}\int \frac{}{\frac{\sqrt{b}}{\sqrt{a}}+}}{4a}$$

$$= \frac{x}{a} + \frac{\sqrt[4]{b}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}\,x^2\right)}{4\sqrt{2}\,a^{5/4}} - \frac{\sqrt[4]{b}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}\,x^2\right)}{4\sqrt{2}\,a^{5/4}} - \frac{\sqrt[4]{b}\ \text{Subst}}{}$$

$$= \frac{x}{a} + \frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)}{2\sqrt{2}\,a^{5/4}} - \frac{\sqrt[4]{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)}{2\sqrt{2}\,a^{5/4}} + \frac{\sqrt[4]{b}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}\,x^2\right)}{4\sqrt{2}\,a^{5/4}}$$

Mathematica [A] time = 0.06, size = 173, normalized size = 0.91

$$\frac{\sqrt{2}\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}\,x^2+\sqrt{b}\right)-\sqrt{2}\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}\,x^2+\sqrt{b}\right)+2\sqrt{2}\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}}{\sqrt[4]{}}\right.}{8a^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^4)^(-1),x]
```

```
[Out] (8*a^(1/4)*x + 2*Sqrt[2]*b^(1/4)*ArcTan[1 - (Sqrt[2]*a^(1/4)*x)/b^(1/4)] -
2*Sqrt[2]*b^(1/4)*ArcTan[1 + (Sqrt[2]*a^(1/4)*x)/b^(1/4)] + Sqrt[2]*b^(1/4)
*Log[Sqrt[b] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[a]*x^2] - Sqrt[2]*b^(1/4)*L
og[Sqrt[b] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[a]*x^2])/(8*a^(5/4))
```

fricas [A] time = 0.85, size = 119, normalized size = 0.63

$$-\frac{4\,a\left(-\frac{b}{a^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{a^4x\left(-\frac{b}{a^5}\right)^{\frac{3}{4}}-\sqrt{a^2\sqrt{-\frac{b}{a^5}}+x^2}\,a^4\left(-\frac{b}{a^5}\right)^{\frac{3}{4}}}{b}\right)+a\left(-\frac{b}{a^5}\right)^{\frac{1}{4}}\log\left(a\left(-\frac{b}{a^5}\right)^{\frac{1}{4}}+x\right)-a\left(-\frac{b}{a^5}\right)^{\frac{1}{4}}\log\left(-a\left(-\frac{b}{a^5}\right)^{\frac{1}{4}}+\right.}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^4),x, algorithm="fricas")
```

```
[Out] -1/4*(4*a*(-b/a^5)^(1/4)*arctan(-(a^4*x*(-b/a^5)^(3/4) - sqrt(a^2*sqrt(-b/a
^5) + x^2)*a^4*(-b/a^5)^(3/4))/b) + a*(-b/a^5)^(1/4)*log(a*(-b/a^5)^(1/4) +
 x) - a*(-b/a^5)^(1/4)*log(-a*(-b/a^5)^(1/4) + x) - 4*x)/a
```

giac [A] time = 0.16, size = 172, normalized size = 0.91

$$\frac{x}{a}-\frac{\sqrt{2}\,\left(a^3b\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)}{2\left(\frac{b}{a}\right)^{\frac{1}{4}}}\right)}{4\,a^2}-\frac{\sqrt{2}\,\left(a^3b\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)}{2\left(\frac{b}{a}\right)^{\frac{1}{4}}}\right)}{4\,a^2}-\frac{\sqrt{2}\,\left(a^3b\right)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}\,x\left(\frac{b}{a}\right)^{\frac{1}{4}}+\right.}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^4),x, algorithm="giac")
```

```
[Out] x/a - 1/4*sqrt(2)*(a^3*b)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(b/a)^(1/
4))/(b/a)^(1/4))/a^2 - 1/4*sqrt(2)*(a^3*b)^(1/4)*arctan(1/2*sqrt(2)*(2*x -
sqrt(2)*(b/a)^(1/4))/(b/a)^(1/4))/a^2 - 1/8*sqrt(2)*(a^3*b)^(1/4)*log(x^2 +
 sqrt(2)*x*(b/a)^(1/4) + sqrt(b/a))/a^2 + 1/8*sqrt(2)*(a^3*b)^(1/4)*log(x^2
 - sqrt(2)*x*(b/a)^(1/4) + sqrt(b/a))/a^2
```

maple [A] time = 0.01, size = 133, normalized size = 0.70

$$\frac{x}{a}-\frac{\left(\frac{b}{a}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{b}{a}\right)^{\frac{1}{4}}}-1\right)}{4a}-\frac{\left(\frac{b}{a}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{b}{a}\right)^{\frac{1}{4}}}+1\right)}{4a}-\frac{\left(\frac{b}{a}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x^2+\left(\frac{b}{a}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{b}{a}}}{x^2-\left(\frac{b}{a}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{b}{a}}}\right)}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^4),x)
```

```
[Out] 1/a*x-1/8/a*(1/a*b)^(1/4)*2^(1/2)*ln((x^2+(1/a*b)^(1/4)*x*2^(1/2)+(1/a*b)^(
1/2))/(x^2-(1/a*b)^(1/4)*x*2^(1/2)+(1/a*b)^(1/2)))-1/4/a*(1/a*b)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(1/a*b)^(1/4)*x+1)-1/4/a*(1/a*b)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(1/a*b)^(1/4)*x-1)
```

maxima [A] time = 2.00, size = 179, normalized size = 0.94

$$\frac{2\sqrt{2}\,\sqrt{b}\,\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{a}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\sqrt{2}\,\sqrt{b}\,\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{a}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,b^{\frac{1}{4}}\log\left(\sqrt{a}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{b}\right)}{a^{\frac{1}{4}}}-\frac{\sqrt{2}\,b^{\frac{1}{4}}\log\left(\sqrt{a}\,x^2-\sqrt{2}\right.}{a^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^4),x, algorithm="maxima")
```

```
[Out] -1/8*(2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2)*(2*sqrt(a)*x + sqrt(2)*a^(1/4)*b
^(1/4))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*sqrt(b)*ar
ctan(1/2*sqrt(2)*(2*sqrt(a)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(
b)))/sqrt(sqrt(a)*sqrt(b)) + sqrt(2)*b^(1/4)*log(sqrt(a)*x^2 + sqrt(2)*a^(1
/4)*b^(1/4)*x + sqrt(b))/a^(1/4) - sqrt(2)*b^(1/4)*log(sqrt(a)*x^2 - sqrt(2
)*a^(1/4)*b^(1/4)*x + sqrt(b))/a^(1/4))/a + x/a
```

mupad **[B]** time = 0.10, size = 48, normalized size = 0.25

$$\frac{x}{a} - \frac{(-b)^{1/4}\,\operatorname{atan}\left(\frac{a^{1/4}\,x}{(-b)^{1/4}}\right)}{2\,a^{5/4}} - \frac{(-b)^{1/4}\,\operatorname{atanh}\left(\frac{a^{1/4}\,x}{(-b)^{1/4}}\right)}{2\,a^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x^4),x)
```

```
[Out] x/a - ((-b)^(1/4)*atan((a^(1/4)*x)/(-b)^(1/4)))/(2*a^(5/4)) - ((-b)^(1/4)*a
tanh((a^(1/4)*x)/(-b)^(1/4)))/(2*a^(5/4))
```

sympy **[A]** time = 0.21, size = 22, normalized size = 0.12

$$\operatorname{RootSum}\left(256t^4a^5 + b, \left(t \mapsto t\log\left(-4ta + x\right)\right)\right) + \frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**4),x)
```

```
[Out] RootSum(256*_t**4*a**5 + b, Lambda(_t, _t*log(-4*_t*a + x))) + x/a
```

3.2058 $\int \sqrt{a+\frac{b}{x^4}}\, x^3\, dx$

Optimal. Leaf size=47

$$\frac{1}{4}x^4\sqrt{a+\frac{b}{x^4}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

```
[Out] 1/4*b*arctanh((a+b/x^4)^(1/2)/a^(1/2))/a^(1/2)+1/4*x^4*(a+b/x^4)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 47, 63, 208}

$$\frac{1}{4}x^4\sqrt{a+\frac{b}{x^4}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^4]*x^3,x]
```

```
[Out] (Sqrt[a + b/x^4]*x^4)/4 + (b*ArcTanh[Sqrt[a + b/x^4]/Sqrt[a]])/(4*Sqrt[a])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+\frac{b}{x^4}}\, x^3\, dx &= -\left(\frac{1}{4}\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\, dx, x, \frac{1}{x^4}\right)\right)\\ &= \frac{1}{4}\sqrt{a+\frac{b}{x^4}}\, x^4 - \frac{1}{8}b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\, dx, x, \frac{1}{x^4}\right)\\ &= \frac{1}{4}\sqrt{a+\frac{b}{x^4}}\, x^4 - \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\, dx, x, \sqrt{a+\frac{b}{x^4}}\right)\\ &= \frac{1}{4}\sqrt{a+\frac{b}{x^4}}\, x^4 + \frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{4\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.14, size = 62, normalized size = 1.32

$$\frac{1}{4}x^2\sqrt{a+\frac{b}{x^4}}\left(\frac{\sqrt{b}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^2}{\sqrt{b}}\right)}{\sqrt{a}\,\sqrt{\frac{ax^4}{b}+1}}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^4]*x^3,x]
```

```
[Out] (Sqrt[a + b/x^4]*x^2*(x^2 + (Sqrt[b]*ArcSinh[(Sqrt[a]*x^2)/Sqrt[b]])/(Sqrt[
a]*Sqrt[1 + (a*x^4)/b])))/4
```

fricas [A] time = 0.75, size = 127, normalized size = 2.70

$$\left[\frac{2\,ax^4\sqrt{\frac{ax^4+b}{x^4}}+\sqrt{a}\,b\log\left(-2\,ax^4-2\,\sqrt{a}\,x^4\sqrt{\frac{ax^4+b}{x^4}}-b\right)}{8\,a},\ \frac{ax^4\sqrt{\frac{ax^4+b}{x^4}}-\sqrt{-a}\,b\arctan\left(\frac{\sqrt{-a}\,x^4\sqrt{\frac{ax^4+b}{x^4}}}{ax^4+b}\right)}{4\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b/x^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*a*x^4*sqrt((a*x^4 + b)/x^4) + sqrt(a)*b*log(-2*a*x^4 - 2*sqrt(a)*x^
4*sqrt((a*x^4 + b)/x^4) - b))/a, 1/4*(a*x^4*sqrt((a*x^4 + b)/x^4) - sqrt(-a
)*b*arctan(sqrt(-a)*x^4*sqrt((a*x^4 + b)/x^4)/(a*x^4 + b)))/a]
```

giac [A] time = 0.17, size = 41, normalized size = 0.87

$$\frac{1}{4}\sqrt{ax^4+b}\,x^2-\frac{b\log\left(\left|-\sqrt{a}\,x^2+\sqrt{ax^4+b}\right|\right)}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b/x^4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(a*x^4 + b)*x^2 - 1/4*b*log(abs(-sqrt(a)*x^2 + sqrt(a*x^4 + b)))/sq
rt(a)
```

maple [A] time = 0.02, size = 68, normalized size = 1.45

$$\frac{\sqrt{\frac{a\,x^4+b}{x^4}}\,\left(\sqrt{a\,x^4+b}\,\sqrt{a}\,x^2+b\ln\left(\sqrt{a}\,x^2+\sqrt{a\,x^4+b}\right)\right)x^2}{4\sqrt{a\,x^4+b}\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b/x^4)^(1/2),x)
```

```
[Out] 1/4*((a*x^4+b)/x^4)^(1/2)*x^2*(x^2*(a*x^4+b)^(1/2)*a^(1/2)+b*ln(x^2*a^(1/2)
+(a*x^4+b)^(1/2)))/(a*x^4+b)^(1/2)/a^(1/2)
```

maxima [A] time = 1.95, size = 53, normalized size = 1.13

$$\frac{1}{4}\sqrt{a+\frac{b}{x^4}}\,x^4-\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x^4}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^4}}+\sqrt{a}}\right)}{8\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b/x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(a + b/x^4)*x^4 - 1/8*b*log((sqrt(a + b/x^4) - sqrt(a))/(sqrt(a + b
/x^4) + sqrt(a)))/sqrt(a)
```

mupad [B] time = 1.57, size = 35, normalized size = 0.74

$$\frac{x^4\sqrt{a+\frac{b}{x^4}}}{4}+\frac{b\,\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b/x^4)^(1/2),x)
```

```
[Out] (x^4*(a + b/x^4)^(1/2))/4 + (b*atanh((a + b/x^4)^(1/2)/a^(1/2)))/(4*a^(1/2)
)
```

sympy [A] time = 2.50, size = 44, normalized size = 0.94

$$\frac{\sqrt{b}\,x^2\sqrt{\frac{ax^4}{b}+1}}{4}+\frac{b\,\operatorname{asinh}\left(\frac{\sqrt{a}\,x^2}{\sqrt{b}}\right)}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b/x**4)**(1/2),x)
```

```
[Out] sqrt(b)*x**2*sqrt(a*x**4/b + 1)/4 + b*asinh(sqrt(a)*x**2/sqrt(b))/(4*sqrt(a
))
```

3.2059 $\int \sqrt{a+\frac{b}{x^4}}\,x\,dx$

Optimal. Leaf size=49

$$\frac{1}{2}x^2\sqrt{a+\frac{b}{x^4}}-\frac{1}{2}\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}}{x^2\sqrt{a+\frac{b}{x^4}}}\right)$$

```
[Out] -1/2*arctanh(b^(1/2)/x^2/(a+b/x^4)^(1/2))*b^(1/2)+1/2*x^2*(a+b/x^4)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {335, 275, 277, 217, 206}

$$\frac{1}{2}x^2\sqrt{a+\frac{b}{x^4}}-\frac{1}{2}\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}}{x^2\sqrt{a+\frac{b}{x^4}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^4]*x,x]
```

```
[Out] (Sqrt[a + b/x^4]*x^2)/2 - (Sqrt[b]*ArcTanh[Sqrt[b]/(Sqrt[a + b/x^4]*x^2)])/
2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+\frac{b}{x^4}}\,x\,dx &= -\operatorname{Subst}\left(\int \frac{\sqrt{a+bx^4}}{x^3}\,dx, x, \frac{1}{x}\right)\\ &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2}\,dx, x, \frac{1}{x^2}\right)\right)\\ &= \frac{1}{2}\sqrt{a+\frac{b}{x^4}}\,x^2 - \frac{1}{2}b\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{x^2}\right)\\ &= \frac{1}{2}\sqrt{a+\frac{b}{x^4}}\,x^2 - \frac{1}{2}b\operatorname{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{b}{x^4}}\,x^2}\right)\\ &= \frac{1}{2}\sqrt{a+\frac{b}{x^4}}\,x^2 - \frac{1}{2}\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^4}}\,x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.06, size = 66, normalized size = 1.35

$$\frac{x^2\sqrt{a+\frac{b}{x^4}}\left(\sqrt{ax^4+b}-\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{ax^4+b}}{\sqrt{b}}\right)\right)}{2\sqrt{ax^4+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^4]*x,x]
```

```
[Out] (Sqrt[a + b/x^4]*x^2*(Sqrt[b + a*x^4] - Sqrt[b]*ArcTanh[Sqrt[b + a*x^4]/Sqr
t[b]]))/(2*Sqrt[b + a*x^4])
```

fricas [A] time = 1.19, size = 112, normalized size = 2.29

$$\left[\frac{1}{2}\,x^2\sqrt{\frac{ax^4+b}{x^4}} + \frac{1}{4}\,\sqrt{b}\,\log\left(\frac{ax^4 - 2\,\sqrt{b}\,x^2\sqrt{\frac{ax^4+b}{x^4}} + 2\,b}{x^4}\right), \frac{1}{2}\,x^2\sqrt{\frac{ax^4+b}{x^4}} + \frac{1}{2}\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x^2\sqrt{\frac{ax^4+b}{x^4}}}{b}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/x^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*x^2*sqrt((a*x^4 + b)/x^4) + 1/4*sqrt(b)*log((a*x^4 - 2*sqrt(b)*x^2*sqr
t((a*x^4 + b)/x^4) + 2*b)/x^4), 1/2*x^2*sqrt((a*x^4 + b)/x^4) + 1/2*sqrt(-b
)*arctan(sqrt(-b)*x^2*sqrt((a*x^4 + b)/x^4)/b)]
```

giac [A] time = 0.15, size = 36, normalized size = 0.73

$$\frac{b\arctan\left(\frac{\sqrt{ax^4+b}}{\sqrt{-b}}\right)}{2\sqrt{-b}} + \frac{1}{2}\sqrt{ax^4+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/x^4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*b*arctan(sqrt(a*x^4 + b)/sqrt(-b))/sqrt(-b) + 1/2*sqrt(a*x^4 + b)
```

maple [A] time = 0.01, size = 64, normalized size = 1.31

$$\frac{\sqrt{\frac{a\,x^4+b}{x^4}}\left(-\sqrt{b}\,\ln\left(\frac{2b+2\sqrt{a\,x^4+b}\,\sqrt{b}}{x^2}\right)+\sqrt{a\,x^4+b}\right)x^2}{2\sqrt{a\,x^4+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b/x^4)^(1/2),x)
```

```
[Out] 1/2*((a*x^4+b)/x^4)^(1/2)*x^2*(-b^(1/2)*ln(2*(b^(1/2)*(a*x^4+b)^(1/2)+b)/x^
2)+(a*x^4+b)^(1/2))/(a*x^4+b)^(1/2)
```

maxima [A] time = 2.04, size = 60, normalized size = 1.22

$$\frac{1}{2}\sqrt{a+\frac{b}{x^4}}\,x^2+\frac{1}{4}\sqrt{b}\,\log\left(\frac{\sqrt{a+\frac{b}{x^4}}\,x^2-\sqrt{b}}{\sqrt{a+\frac{b}{x^4}}\,x^2+\sqrt{b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a + b/x^4)*x^2 + 1/4*sqrt(b)*log((sqrt(a + b/x^4)*x^2 - sqrt(b))/(
sqrt(a + b/x^4)*x^2 + sqrt(b)))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\sqrt{a+\frac{b}{x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b/x^4)^(1/2),x)
```

```
[Out] int(x*(a + b/x^4)^(1/2), x)
```

sympy [A] time = 1.79, size = 66, normalized size = 1.35

$$\frac{\sqrt{a}\,x^2}{2\sqrt{1+\frac{b}{ax^4}}}-\frac{\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\,x^2}\right)}{2}+\frac{b}{2\sqrt{a}\,x^2\sqrt{1+\frac{b}{ax^4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/x**4)**(1/2),x)
```

```
[Out] sqrt(a)*x**2/(2*sqrt(1 + b/(a*x**4))) - sqrt(b)*asinh(sqrt(b)/(sqrt(a)*x**2
))/2 + b/(2*sqrt(a)*x**2*sqrt(1 + b/(a*x**4)))
```

3.2060 $\int \frac{\sqrt{a+\frac{b}{x^4}}}{x}\,dx$

Optimal. Leaf size=43

$$\frac{1}{2}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)-\frac{1}{2}\sqrt{a+\frac{b}{x^4}}$$

```
[Out] 1/2*arctanh((a+b/x^4)^(1/2)/a^(1/2))*a^(1/2)-1/2*(a+b/x^4)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 50, 63, 208}

$$\frac{1}{2}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)-\frac{1}{2}\sqrt{a+\frac{b}{x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^4]/x,x]
```

```
[Out] -Sqrt[a + b/x^4]/2 + (Sqrt[a]*ArcTanh[Sqrt[a + b/x^4]/Sqrt[a]])/2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+\frac{b}{x^4}}}{x}\,dx &= -\left(\frac{1}{4}\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x^4}\right)\right)\\ &= -\frac{1}{2}\sqrt{a+\frac{b}{x^4}} - \frac{1}{4}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x^4}\right)\\ &= -\frac{1}{2}\sqrt{a+\frac{b}{x^4}} - \frac{a\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x^4}}\right)}{2b}\\ &= -\frac{1}{2}\sqrt{a+\frac{b}{x^4}} + \frac{1}{2}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)\end{aligned}$$

Mathematica [A] time = 0.12, size = 60, normalized size = 1.40

$$\frac{1}{2}\sqrt{a+\frac{b}{x^4}}\left(\frac{\sqrt{a}\,x^2\sinh^{-1}\left(\frac{\sqrt{a}\,x^2}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{\frac{ax^4}{b}+1}}-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^4]/x,x]
```

```
[Out] (Sqrt[a + b/x^4]*(-1 + (Sqrt[a]*x^2*ArcSinh[(Sqrt[a]*x^2)/Sqrt[b]])/(Sqrt[b
]*Sqrt[1 + (a*x^4)/b])))/2
```

fricas [A] time = 0.89, size = 109, normalized size = 2.53

$$\left[\frac{1}{4}\sqrt{a}\,\log\left(-2\,ax^4-2\sqrt{a}\,x^4\sqrt{\frac{ax^4+b}{x^4}}-b\right)-\frac{1}{2}\sqrt{\frac{ax^4+b}{x^4}}, -\frac{1}{2}\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}\,x^4\sqrt{\frac{ax^4+b}{x^4}}}{ax^4+b}\right)-\frac{1}{2}\sqrt{\frac{ax^4+b}{x^4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(a)*log(-2*a*x^4 - 2*sqrt(a)*x^4*sqrt((a*x^4 + b)/x^4) - b) - 1/2*
sqrt((a*x^4 + b)/x^4), -1/2*sqrt(-a)*arctan(sqrt(-a)*x^4*sqrt((a*x^4 + b)/x
^4)/(a*x^4 + b)) - 1/2*sqrt((a*x^4 + b)/x^4)]
```

giac [A] time = 0.20, size = 60, normalized size = 1.40

$$-\frac{1}{4}\sqrt{a}\,\log\left(\left(\sqrt{a}\,x^2-\sqrt{ax^4+b}\right)^2\right)+\frac{\sqrt{a}\,b}{\left(\sqrt{a}\,x^2-\sqrt{ax^4+b}\right)^2-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(a)*log((sqrt(a)*x^2 - sqrt(a*x^4 + b))^2) + sqrt(a)*b/((sqrt(a)*x
^2 - sqrt(a*x^4 + b))^2 - b)
```

maple [B] time = 0.01, size = 80, normalized size = 1.86

$$-\frac{\sqrt{\frac{a\,x^4+b}{x^4}}\,\left(-\sqrt{a\,x^4+b}\,a\,x^4-\sqrt{a}\,b\,x^2\ln\left(\sqrt{a}\,x^2+\sqrt{a\,x^4+b}\right)+\left(a\,x^4+b\right)^{\frac{3}{2}}\right)}{2\sqrt{a\,x^4+b}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^4)^(1/2)/x,x)
```

```
[Out] -1/2*((a*x^4+b)/x^4)^(1/2)*(-a*x^4*(a*x^4+b)^(1/2)-a^(1/2)*ln(a^(1/2)*x^2+(
a*x^4+b)^(1/2))*x^2*b+(a*x^4+b)^(3/2))/(a*x^4+b)^(1/2)/b
```

maxima [A] time = 1.99, size = 49, normalized size = 1.14

$$-\frac{1}{4}\sqrt{a}\,\log\left(\frac{\sqrt{a+\frac{b}{x^4}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^4}}+\sqrt{a}}\right)-\frac{1}{2}\sqrt{a+\frac{b}{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(a)*log((sqrt(a + b/x^4) - sqrt(a))/(sqrt(a + b/x^4) + sqrt(a))) -
 1/2*sqrt(a + b/x^4)
```

mupad [B] time = 1.34, size = 31, normalized size = 0.72

$$\frac{\sqrt{a}\,\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{2}-\frac{\sqrt{a+\frac{b}{x^4}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^4)^(1/2)/x,x)
```

```
[Out] (a^(1/2)*atanh((a + b/x^4)^(1/2)/a^(1/2)))/2 - (a + b/x^4)^(1/2)/2
```

sympy [A] time = 1.78, size = 66, normalized size = 1.53

$$\frac{\sqrt{a}\,\operatorname{asinh}\left(\frac{\sqrt{a}\,x^2}{\sqrt{b}}\right)}{2}-\frac{ax^2}{2\sqrt{b}\,\sqrt{\frac{ax^4}{b}+1}}-\frac{\sqrt{b}}{2x^2\sqrt{\frac{ax^4}{b}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**4)**(1/2)/x,x)
```

```
[Out] sqrt(a)*asinh(sqrt(a)*x**2/sqrt(b))/2 - a*x**2/(2*sqrt(b)*sqrt(a*x**4/b + 1
)) - sqrt(b)/(2*x**2*sqrt(a*x**4/b + 1))
```

3.2061 $\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^3}\,dx$

Optimal. Leaf size=50

$$-\frac{\sqrt{a+\frac{b}{x^4}}}{4x^2}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}}{x^2\sqrt{a+\frac{b}{x^4}}}\right)}{4\sqrt{b}}$$

```
[Out] -1/4*a*arctanh(b^(1/2)/x^2/(a+b/x^4)^(1/2))/b^(1/2)-1/4*(a+b/x^4)^(1/2)/x^2
```

Rubi [A] time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {335, 275, 195, 217, 206}

$$-\frac{\sqrt{a+\frac{b}{x^4}}}{4x^2}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}}{x^2\sqrt{a+\frac{b}{x^4}}}\right)}{4\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^4]/x^3,x]
```

```
[Out] -Sqrt[a + b/x^4]/(4*x^2) - (a*ArcTanh[Sqrt[b]/(Sqrt[a + b/x^4]*x^2)])/(4*Sq
rt[b])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
 + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^3}\,dx &= -\operatorname{Subst}\left(\int x\sqrt{a+bx^4}\,dx, x, \frac{1}{x}\right) \\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \sqrt{a+bx^2}\,dx, x, \frac{1}{x^2}\right)\right) \\
&= -\frac{\sqrt{a+\frac{b}{x^4}}}{4x^2} - \frac{1}{4}a\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{x^2}\right) \\
&= -\frac{\sqrt{a+\frac{b}{x^4}}}{4x^2} - \frac{1}{4}a\operatorname{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{b}{x^4}}\,x^2}\right) \\
&= -\frac{\sqrt{a+\frac{b}{x^4}}}{4x^2} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^4}}\,x^2}\right)}{4\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 68, normalized size = 1.36

$$-\frac{\sqrt{a+\frac{b}{x^4}}\left(ax^4\sqrt{\frac{ax^4}{b}+1}\,\tanh^{-1}\left(\sqrt{\frac{ax^4}{b}+1}\right)+ax^4+b\right)}{4x^2\left(ax^4+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^4]/x^3,x]
```

```
[Out] -1/4*(Sqrt[a + b/x^4]*(b + a*x^4 + a*x^4*Sqrt[1 + (a*x^4)/b]*ArcTanh[Sqrt[1
 + (a*x^4)/b]]))/(x^2*(b + a*x^4))
```

fricas [A] time = 0.86, size = 130, normalized size = 2.60

$$\left[\frac{a\sqrt{b}\,x^2\log\left(\frac{ax^4-2\sqrt{b}\,x^2\sqrt{\frac{ax^4+b}{x^4}}+2b}{x^4}\right)-2b\sqrt{\frac{ax^4+b}{x^4}}}{8bx^2}, \frac{a\sqrt{-b}\,x^2\arctan\left(\frac{\sqrt{-b}\,x^2\sqrt{\frac{ax^4+b}{x^4}}}{b}\right)-b\sqrt{\frac{ax^4+b}{x^4}}}{4bx^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(a*sqrt(b)*x^2*log((a*x^4 - 2*sqrt(b)*x^2*sqrt((a*x^4 + b)/x^4) + 2*b)
/x^4) - 2*b*sqrt((a*x^4 + b)/x^4))/(b*x^2), 1/4*(a*sqrt(-b)*x^2*arctan(sqrt
(-b)*x^2*sqrt((a*x^4 + b)/x^4)/b) - b*sqrt((a*x^4 + b)/x^4))/(b*x^2)]
```

giac [A] time = 0.15, size = 46, normalized size = 0.92

$$\frac{\frac{a^2\arctan\left(\frac{\sqrt{ax^4+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}-\frac{\sqrt{ax^4+b}\,a}{x^4}}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(a^2*arctan(sqrt(a*x^4 + b)/sqrt(-b))/sqrt(-b) - sqrt(a*x^4 + b)*a/x^4)
/a
```

maple [B] time = 0.01, size = 90, normalized size = 1.80

$$-\frac{\sqrt{\frac{a\,x^4+b}{x^4}}\left(ab\,x^4\ln\left(\frac{2b+2\sqrt{a\,x^4+b}\,\sqrt{b}}{x^2}\right)-\sqrt{a\,x^4+b}\,a\sqrt{b}\,x^4+\left(a\,x^4+b\right)^{\frac{3}{2}}\sqrt{b}\right)}{4\sqrt{a\,x^4+b}\,b^{\frac{3}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^4)^(1/2)/x^3,x)
```

```
[Out] -1/4*((a*x^4+b)/x^4)^(1/2)/x^2*(-a*(a*x^4+b)^(1/2)*x^4*b^(1/2)+a*ln(2*(b+(a
*x^4+b)^(1/2)*b^(1/2))/x^2)*x^4*b+(a*x^4+b)^(3/2)*b^(1/2))/(a*x^4+b)^(1/2)/
b^(3/2)
```

maxima [B] time = 1.87, size = 79, normalized size = 1.58

$$-\frac{\sqrt{a+\frac{b}{x^4}}\,ax^2}{4\left(\left(a+\frac{b}{x^4}\right)x^4-b\right)}+\frac{a\log\left(\frac{\sqrt{a+\frac{b}{x^4}}\,x^2-\sqrt{b}}{\sqrt{a+\frac{b}{x^4}}\,x^2+\sqrt{b}}\right)}{8\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(a + b/x^4)*a*x^2/((a + b/x^4)*x^4 - b) + 1/8*a*log((sqrt(a + b/x^
4)*x^2 - sqrt(b))/(sqrt(a + b/x^4)*x^2 + sqrt(b)))/sqrt(b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^4)^(1/2)/x^3,x)
```

```
[Out] int((a + b/x^4)^(1/2)/x^3, x)
```

sympy [A] time = 2.45, size = 46, normalized size = 0.92

$$-\frac{\sqrt{a}\,\sqrt{1+\frac{b}{ax^4}}}{4x^2}-\frac{a\,\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\,x^2}\right)}{4\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**4)**(1/2)/x**3,x)
```

```
[Out] -sqrt(a)*sqrt(1 + b/(a*x**4))/(4*x**2) - a*asinh(sqrt(b)/(sqrt(a)*x**2))/(4
*sqrt(b))
```

3.2062 $\int \sqrt{a+\frac{b}{x^4}}\, x^2\, dx$

Optimal. Leaf size=107

$$\frac{1}{3}x^3\sqrt{a+\frac{b}{x^4}} - \frac{b^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{a+\frac{b}{x^4}}}$$

```
[Out] 1/3*x^3*(a+b/x^4)^(1/2)-1/3*b^(3/4)*(cos(2*arccot(a^(1/4)*x/b^(1/4)))^2)^(1
/2)/cos(2*arccot(a^(1/4)*x/b^(1/4)))*EllipticF(sin(2*arccot(a^(1/4)*x/b^(1/
4))),1/2*2^(1/2))*(a^(1/2)+b^(1/2)/x^2)*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)
^(1/2)/a^(1/4)/(a+b/x^4)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {335, 277, 220}

$$\frac{1}{3}x^3\sqrt{a+\frac{b}{x^4}} - \frac{b^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{a+\frac{b}{x^4}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^4]*x^2,x]
```

```
[Out] (Sqrt[a + b/x^4]*x^3)/3 - (b^(3/4)*Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)
^2]*(Sqrt[a] + Sqrt[b]/x^2)*EllipticF[2*ArcCot[(a^(1/4)*x)/b^(1/4)], 1/2])/
(3*a^(1/4)*Sqrt[a + b/x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \sqrt{a+\frac{b}{x^4}}\,x^2\,dx = -\operatorname{Subst}\left(\int \frac{\sqrt{a+bx^4}}{x^4}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{3}\sqrt{a+\frac{b}{x^4}}\,x^3 - \frac{1}{3}(2b)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{3}\sqrt{a+\frac{b}{x^4}}\,x^3 - \frac{b^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{a+\frac{b}{x^4}}}$$

Mathematica [C] time = 0.25, size = 93, normalized size = 0.87

$$\frac{1}{3}x^2\sqrt{a+\frac{b}{x^4}}\left(x - \frac{2ib\sqrt{\frac{ax^4}{b}+1}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,x\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\left(ax^4+b\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^4]*x^2,x]
```

```
[Out] (Sqrt[a + b/x^4]*x^2*(x - ((2*I)*b*Sqrt[1 + (a*x^4)/b]*EllipticF[I*ArcSinh[
Sqrt[(I*Sqrt[a])/Sqrt[b]]*x], -1])/(Sqrt[(I*Sqrt[a])/Sqrt[b]]*(b + a*x^4)))
)/3
```

fricas [F] time = 1.16, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2\sqrt{\frac{ax^4+b}{x^4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b/x^4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^2*sqrt((a*x^4 + b)/x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+\frac{b}{x^4}}\,x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b/x^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/x^4)*x^2, x)
```

maple [C] time = 0.05, size = 130, normalized size = 1.21

$$\frac{\sqrt{\frac{a\,x^4+b}{x^4}}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,a\,x^5 + \sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,bx + 2\sqrt{-\frac{i\sqrt{a}\,x^2-\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}\,x^2+\sqrt{b}}{\sqrt{b}}}\,b\,\text{EllipticF}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,x, i\right)\right)x^2}{3\left(a\,x^4+b\right)\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b/x^4)^(1/2),x)
```

```
[Out] 1/3*((a*x^4+b)/x^4)^(1/2)*x^2*((I*a^(1/2)/b^(1/2))^(1/2)*x^5*a+2*b*(-(I*a^(
1/2)*x^2-b^(1/2))/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*El
lipticF(x*(I*a^(1/2)/b^(1/2))^(1/2),I)+(I*a^(1/2)/b^(1/2))^(1/2)*x*b)/(a*x^
4+b)/(I*a^(1/2)/b^(1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + \frac{b}{x^4}}\, x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b/x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x^4)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \sqrt{a + \frac{b}{x^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b/x^4)^(1/2),x)
```

```
[Out] int(x^2*(a + b/x^4)^(1/2), x)
```

sympy [C] time = 1.30, size = 44, normalized size = 0.41

$$-\frac{\sqrt{a}\, x^3 \Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, -\frac{1}{2} \\ \frac{1}{4} \end{matrix} \middle| \frac{be^{i\pi}}{ax^4}\right)}{4\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b/x**4)**(1/2),x)
```

```
[Out] -sqrt(a)*x**3*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*exp_polar(I*pi)/(a*
x**4))/(4*gamma(1/4))
```

3.2063 $\int \sqrt{a+\frac{b}{x^4}}\,dx$

Optimal. Leaf size=224

$$x\sqrt{a+\frac{b}{x^4}}-\frac{2\sqrt{b}\sqrt{a+\frac{b}{x^4}}}{x\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)}-\frac{\sqrt[4]{a}\sqrt[4]{b}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt{a+\frac{b}{x^4}}}+\frac{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\right.}{\sqrt{a+}}$$

```
[Out] x*(a+b/x^4)^(1/2)-2*b^(1/2)*(a+b/x^4)^(1/2)/x/(a^(1/2)+b^(1/2)/x^2)+2*a^(1/
4)*b^(1/4)*(cos(2*arccot(a^(1/4)*x/b^(1/4)))^2)^(1/2)/cos(2*arccot(a^(1/4)*
x/b^(1/4)))*EllipticE(sin(2*arccot(a^(1/4)*x/b^(1/4))),1/2*2^(1/2))*(a^(1/2
)+b^(1/2)/x^2)*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)/(a+b/x^4)^(1/2)-a^
(1/4)*b^(1/4)*(cos(2*arccot(a^(1/4)*x/b^(1/4)))^2)^(1/2)/cos(2*arccot(a^(1/
4)*x/b^(1/4)))*EllipticF(sin(2*arccot(a^(1/4)*x/b^(1/4))),1/2*2^(1/2))*(a^(
1/2)+b^(1/2)/x^2)*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)/(a+b/x^4)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {242, 277, 305, 220, 1196}

$$x\sqrt{a+\frac{b}{x^4}}-\frac{2\sqrt{b}\sqrt{a+\frac{b}{x^4}}}{x\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)}-\frac{\sqrt[4]{a}\sqrt[4]{b}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt{a+\frac{b}{x^4}}}+\frac{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\right.}{\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^4],x]
```

```
[Out] (-2*Sqrt[b]*Sqrt[a + b/x^4])/((Sqrt[a] + Sqrt[b]/x^2)*x) + Sqrt[a + b/x^4]*
x + (2*a^(1/4)*b^(1/4)*Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a]
 + Sqrt[b]/x^2)*EllipticE[2*ArcCot[(a^(1/4)*x)/b^(1/4)], 1/2])/Sqrt[a + b/x
^4] - (a^(1/4)*b^(1/4)*Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a]
 + Sqrt[b]/x^2)*EllipticF[2*ArcCot[(a^(1/4)*x)/b^(1/4)], 1/2])/Sqrt[a + b/x
^4]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \sqrt{a+\frac{b}{x^4}}\,dx = -\text{Subst}\left(\int \frac{\sqrt{a+bx^4}}{x^2}\,dx, x, \frac{1}{x}\right)$$

$$= \sqrt{a+\frac{b}{x^4}}\,x - (2b)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right)$$

$$= \sqrt{a+\frac{b}{x^4}}\,x - \left(2\sqrt{a}\,\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right) + \left(2\sqrt{a}\,\sqrt{b}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx, x,\right.$$

$$= -\frac{2\sqrt{b}\,\sqrt{a+\frac{b}{x^4}}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)x} + \sqrt{a+\frac{b}{x^4}}\,x + \frac{2\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{\sqrt{a+\frac{b}{x^4}}} - \frac{\sqrt[4]{a}\,\sqrt[4]{}}{}$$

Mathematica [C] time = 0.01, size = 47, normalized size = 0.21

$$-\frac{x\sqrt{a+\frac{b}{x^4}}\;{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};-\frac{ax^4}{b}\right)}{\sqrt{\frac{ax^4}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^4],x]
```

```
[Out] -((Sqrt[a + b/x^4]*x*Hypergeometric2F1[-1/2, -1/4, 3/4, -((a*x^4)/b)])/Sqrt
[1 + (a*x^4)/b])
```

fricas [F] time = 1.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{\frac{ax^4+b}{x^4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt((a*x^4 + b)/x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+\frac{b}{x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/x^4), x)
```

maple [C] time = 0.01, size = 202, normalized size = 0.90

$$\frac{\sqrt{\frac{a\,x^4+b}{x^4}}\left(-\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,a\,x^4-2i\sqrt{-\frac{i\sqrt{a}\,x^2-\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}\,x^2+\sqrt{b}}{\sqrt{b}}}\,\sqrt{a}\,\sqrt{b}\,x\,\text{EllipticE}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,x,i\right)+2i\sqrt{-\frac{i\sqrt{a}\,x^2-\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\right.}{\left(a\,x^4+b\right)\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^4)^(1/2),x)
```

```
[Out] ((a*x^4+b)/x^4)^(1/2)*x*(2*I*a^(1/2)*b^(1/2)*(-(I*a^(1/2)*x^2-b^(1/2))/b^(1
/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*x*EllipticF((I*a^(1/2)/b
^(1/2))^(1/2)*x,I)-2*I*a^(1/2)*b^(1/2)*(-(I*a^(1/2)*x^2-b^(1/2))/b^(1/2))^(
1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*x*EllipticE((I*a^(1/2)/b^(1/2)
)^(1/2)*x,I)-(I*a^(1/2)/b^(1/2))^(1/2)*x^4*a-(I*a^(1/2)/b^(1/2))^(1/2)*b)/(
a*x^4+b)/(I*a^(1/2)/b^(1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+\frac{b}{x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x^4), x)
```

mupad [B] time = 1.23, size = 38, normalized size = 0.17

$$-\frac{x\sqrt{a+\frac{b}{x^4}}\;{}_2\mathrm{F}_1\left(-\frac{1}{2},-\frac{1}{4};\;\frac{3}{4};\;-\frac{a\,x^4}{b}\right)}{\sqrt{\frac{a\,x^4}{b}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^4)^(1/2),x)
```

```
[Out] -(x*(a + b/x^4)^(1/2)*hypergeom([-1/2, -1/4], 3/4, -(a*x^4)/b))/((a*x^4)/b
+ 1)^(1/2)
```

sympy [C] time = 1.17, size = 42, normalized size = 0.19

$$-\frac{\sqrt{a}\,x\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\right|\frac{be^{i\pi}}{ax^4}\right)}{4\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**4)**(1/2),x)
```

```
[Out] -sqrt(a)*x*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*exp_polar(I*pi)/(a*x**
4))/(4*gamma(3/4))
```

3.2064 $\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^2}\,dx$

Optimal. Leaf size=107

$$-\frac{a^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+\frac{b}{x^4}}}-\frac{\sqrt{a+\frac{b}{x^4}}}{3x}$$

```
[Out] -1/3*(a+b/x^4)^(1/2)/x-1/3*a^(3/4)*(cos(2*arccot(a^(1/4)*x/b^(1/4)))^2)^(1/
2)/cos(2*arccot(a^(1/4)*x/b^(1/4)))*EllipticF(sin(2*arccot(a^(1/4)*x/b^(1/4
))),1/2*2^(1/2))*(a^(1/2)+b^(1/2)/x^2)*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^
(1/2)/b^(1/4)/(a+b/x^4)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {335, 195, 220}

$$-\frac{a^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+\frac{b}{x^4}}}-\frac{\sqrt{a+\frac{b}{x^4}}}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^4]/x^2,x]
```

```
[Out] -Sqrt[a + b/x^4]/(3*x) - (a^(3/4)*Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^
2]*(Sqrt[a] + Sqrt[b]/x^2)*EllipticF[2*ArcCot[(a^(1/4)*x)/b^(1/4)], 1/2])/(
3*b^(1/4)*Sqrt[a + b/x^4])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^2}\,dx = -\operatorname{Subst}\left(\int \sqrt{a+bx^4}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{\sqrt{a+\frac{b}{x^4}}}{3x} - \frac{1}{3}(2a)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{\sqrt{a+\frac{b}{x^4}}}{3x} - \frac{a^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+\frac{b}{x^4}}}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.48

$$-\frac{\sqrt{a+\frac{b}{x^4}}\;{}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{4};-\frac{ax^4}{b}\right)}{3x\sqrt{\frac{ax^4}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^4]/x^2,x]
```

```
[Out] -1/3*(Sqrt[a + b/x^4]*Hypergeometric2F1[-3/4, -1/2, 1/4, -((a*x^4)/b)])/(x*
Sqrt[1 + (a*x^4)/b])
```

fricas [F] time = 1.25, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\frac{ax^4+b}{x^4}}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt((a*x^4 + b)/x^4)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/x^4)/x^2, x)
```

maple [C] time = 0.02, size = 132, normalized size = 1.23

$$-\frac{\sqrt{\frac{a\,x^4+b}{x^4}}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,a\,x^4 - 2\sqrt{-\frac{i\sqrt{a}\,x^2-\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}\,x^2+\sqrt{b}}{\sqrt{b}}}\,a\,x^3\,\text{EllipticF}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,x, i\right) + \sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,b\right)}{3\left(a\,x^4+b\right)\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^4)^(1/2)/x^2,x)
```

```
[Out] -1/3*((a*x^4+b)/x^4)^(1/2)*(-2*a*(-(I*a^(1/2)*x^2-b^(1/2))/b^(1/2))^(1/2)*(
(I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*EllipticF((I*a^(1/2)/b^(1/2))^(1/2)*
x,I)*x^3+(I*a^(1/2)/b^(1/2))^(1/2)*a*x^4+(I*a^(1/2)/b^(1/2))^(1/2)*b)/x/(a*
x^4+b)/(I*a^(1/2)/b^(1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x^4)/x^2, x)
```

mupad [B] time = 1.34, size = 39, normalized size = 0.36

$$-\frac{\sqrt{a\,x^4+b}\;{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{1}{4};\;\frac{5}{4};\;-\frac{b}{a\,x^4}\right)}{x\sqrt{\frac{b}{a}+x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^4)^(1/2)/x^2,x)
```

```
[Out] -((b + a*x^4)^(1/2)*hypergeom([-1/2, 1/4], 5/4, -b/(a*x^4)))/(x*(b/a + x^4)
^(1/2))
```

sympy [C] time = 1.21, size = 39, normalized size = 0.36

$$-\frac{\sqrt{a}\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\right|\frac{be^{i\pi}}{ax^4}\right)}{4x\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**4)**(1/2)/x**2,x)
```

```
[Out] -sqrt(a)*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*exp_polar(I*pi)/(a*x**4))/
(4*x*gamma(5/4))
```

3.2065 $\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^4}\,dx$

Optimal. Leaf size=236

$$-\frac{a^{5/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+\frac{b}{x^4}}}+\frac{2a^{5/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+\frac{b}{x^4}}}-\frac{\sqrt{a+}}{5x}$$

```
[Out] -1/5*(a+b/x^4)^(1/2)/x^3-2/5*a*(a+b/x^4)^(1/2)/x/b^(1/2)/(a^(1/2)+b^(1/2)/x
^2)+2/5*a^(5/4)*(cos(2*arccot(a^(1/4)*x/b^(1/4)))^2)^(1/2)/cos(2*arccot(a^(
1/4)*x/b^(1/4)))*EllipticE(sin(2*arccot(a^(1/4)*x/b^(1/4))),1/2*2^(1/2))*(a
^(1/2)+b^(1/2)/x^2)*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)/b^(3/4)/(a+b/
x^4)^(1/2)-1/5*a^(5/4)*(cos(2*arccot(a^(1/4)*x/b^(1/4)))^2)^(1/2)/cos(2*arc
cot(a^(1/4)*x/b^(1/4)))*EllipticF(sin(2*arccot(a^(1/4)*x/b^(1/4))),1/2*2^(1
/2))*(a^(1/2)+b^(1/2)/x^2)*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)/b^(3/4
)/(a+b/x^4)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {335, 279, 305, 220, 1196}

$$-\frac{a^{5/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+\frac{b}{x^4}}}+\frac{2a^{5/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+\frac{b}{x^4}}}-\frac{2a}{5\sqrt{b}\,x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^4]/x^4,x]
```

```
[Out] -Sqrt[a + b/x^4]/(5*x^3) - (2*a*Sqrt[a + b/x^4])/(5*Sqrt[b]*(Sqrt[a] + Sqrt
[b]/x^2)*x) + (2*a^(5/4)*Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[
a] + Sqrt[b]/x^2)*EllipticE[2*ArcCot[(a^(1/4)*x)/b^(1/4)], 1/2])/(5*b^(3/4)
*Sqrt[a + b/x^4]) - (a^(5/4)*Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(S
qrt[a] + Sqrt[b]/x^2)*EllipticF[2*ArcCot[(a^(1/4)*x)/b^(1/4)], 1/2])/(5*b^(
3/4)*Sqrt[a + b/x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^4}\,dx = -\text{Subst}\left(\int x^2\sqrt{a+bx^4}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\sqrt{a+\frac{b}{x^4}}}{5x^3} - \frac{1}{5}(2a)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\sqrt{a+\frac{b}{x^4}}}{5x^3} - \frac{\left(2a^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right)}{5\sqrt{b}} + \frac{\left(2a^{3/2}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right)}{5\sqrt{b}}$$

$$= -\frac{\sqrt{a+\frac{b}{x^4}}}{5x^3} - \frac{2a\sqrt{a+\frac{b}{x^4}}}{5\sqrt{b}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)x} + \frac{2a^{5/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+\frac{b}{x^4}}} - \frac{a^{5/4}\sqrt{}}{}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.22

$$-\frac{\sqrt{a+\frac{b}{x^4}}\;{}_2F_1\left(-\frac{5}{4},-\frac{1}{2};-\frac{1}{4};-\frac{ax^4}{b}\right)}{5x^3\sqrt{\frac{ax^4}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^4]/x^4,x]
```

```
[Out] -1/5*(Sqrt[a + b/x^4]*Hypergeometric2F1[-5/4, -1/2, -1/4, -((a*x^4)/b)])/(x
^3*Sqrt[1 + (a*x^4)/b])
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\frac{ax^4+b}{x^4}}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt((a*x^4 + b)/x^4)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/x^4)/x^4, x)
```

maple [C] time = 0.02, size = 235, normalized size = 1.00

$$\frac{\sqrt{\frac{a\,x^4+b}{x^4}}\left(-2\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;a^2\sqrt{b}\;x^8-2i\sqrt{-\frac{i\sqrt{a}\,x^2-\sqrt{b}}{\sqrt{b}}}\;\sqrt{\frac{i\sqrt{a}\,x^2+\sqrt{b}}{\sqrt{b}}}\;a^{\frac{3}{2}}b\,x^5\,\text{EllipticE}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;x,i\right)+2i\sqrt{-\frac{i\sqrt{a}\,x^2-\sqrt{b}}{\sqrt{b}}}\;\sqrt{}}{5\left(a\,x^4+b\right)\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;b^{\frac{3}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^4)^(1/2)/x^4,x)
```

```
[Out] 1/5*((a*x^4+b)/x^4)^(1/2)*(2*I*a^(3/2)*(-(I*a^(1/2)*x^2-b^(1/2))/b^(1/2))^(
1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*x^5*b*EllipticF((I*a^(1/2)/b^(
1/2))^(1/2)*x,I)-2*I*a^(3/2)*(-(I*a^(1/2)*x^2-b^(1/2))/b^(1/2))^(1/2)*((I*a
^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*x^5*b*EllipticE((I*a^(1/2)/b^(1/2))^(1/2
)*x,I)-2*b^(1/2)*(I*a^(1/2)/b^(1/2))^(1/2)*x^8*a^2-3*b^(3/2)*(I*a^(1/2)/b^(
1/2))^(1/2)*x^4*a-b^(5/2)*(I*a^(1/2)/b^(1/2))^(1/2))/x^3/(a*x^4+b)/b^(3/2)/
(I*a^(1/2)/b^(1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x^4)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^4)^(1/2)/x^4,x)
```

```
[Out] int((a + b/x^4)^(1/2)/x^4, x)
```

sympy [C] time = 1.44, size = 41, normalized size = 0.17

$$-\frac{\sqrt{a}\,\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{be^{i\pi}}{ax^4}\right)}{4x^3\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**4)**(1/2)/x**4,x)
```

```
[Out] -sqrt(a)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*exp_polar(I*pi)/(a*x**4))/
(4*x**3*gamma(7/4))
```

3.2066 $\int \left(a+\frac{b}{x^4}\right)^{3/2} x^3\,dx$

Optimal. Leaf size=63

$$\frac{1}{4}x^4\left(a+\frac{b}{x^4}\right)^{3/2}-\frac{3}{4}b\sqrt{a+\frac{b}{x^4}}+\frac{3}{4}\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)$$

```
[Out] 1/4*(a+b/x^4)^(3/2)*x^4+3/4*b*arctanh((a+b/x^4)^(1/2)/a^(1/2))*a^(1/2)-3/4*
b*(a+b/x^4)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {266, 47, 50, 63, 208}

$$\frac{1}{4}x^4\left(a+\frac{b}{x^4}\right)^{3/2}-\frac{3}{4}b\sqrt{a+\frac{b}{x^4}}+\frac{3}{4}\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^4)^(3/2)*x^3,x]
```

```
[Out] (-3*b*Sqrt[a + b/x^4])/4 + ((a + b/x^4)^(3/2)*x^4)/4 + (3*Sqrt[a]*b*ArcTanh
[Sqrt[a + b/x^4]/Sqrt[a]])/4
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a+\frac{b}{x^4}\right)^{3/2} x^3\,dx &= -\left(\frac{1}{4}\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2}\,dx, x, \frac{1}{x^4}\right)\right) \\
&= \frac{1}{4}\left(a+\frac{b}{x^4}\right)^{3/2} x^4 - \frac{1}{8}(3b)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x^4}\right) \\
&= -\frac{3}{4}b\sqrt{a+\frac{b}{x^4}} + \frac{1}{4}\left(a+\frac{b}{x^4}\right)^{3/2} x^4 - \frac{1}{8}(3ab)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x^4}\right) \\
&= -\frac{3}{4}b\sqrt{a+\frac{b}{x^4}} + \frac{1}{4}\left(a+\frac{b}{x^4}\right)^{3/2} x^4 - \frac{1}{4}(3a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x^4}}\right) \\
&= -\frac{3}{4}b\sqrt{a+\frac{b}{x^4}} + \frac{1}{4}\left(a+\frac{b}{x^4}\right)^{3/2} x^4 + \frac{3}{4}\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [C] time = 0.02, size = 49, normalized size = 0.78

$$-\frac{b\sqrt{a+\frac{b}{x^4}}\;{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{ax^4}{b}\right)}{2\sqrt{\frac{ax^4}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^4)^(3/2)*x^3,x]
```

```
[Out] -1/2*(b*Sqrt[a + b/x^4]*Hypergeometric2F1[-3/2, -1/2, 1/2, -((a*x^4)/b)])/S
qrt[1 + (a*x^4)/b]
```

fricas [A] time = 0.93, size = 129, normalized size = 2.05

$$\left[\frac{3}{8}\sqrt{a}\,b\log\left(-2\,ax^4-2\sqrt{a}\,x^4\sqrt{\frac{ax^4+b}{x^4}}-b\right)+\frac{1}{4}\left(ax^4-2\,b\right)\sqrt{\frac{ax^4+b}{x^4}},-\frac{3}{4}\sqrt{-a}\,b\arctan\left(\frac{\sqrt{-a}\,x^4\sqrt{\frac{ax^4+b}{x^4}}}{ax^4+b}\right)+\frac{1}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(3/2)*x^3,x, algorithm="fricas")
```

```
[Out] [3/8*sqrt(a)*b*log(-2*a*x^4 - 2*sqrt(a)*x^4*sqrt((a*x^4 + b)/x^4) - b) + 1/
4*(a*x^4 - 2*b)*sqrt((a*x^4 + b)/x^4), -3/4*sqrt(-a)*b*arctan(sqrt(-a)*x^4*
sqrt((a*x^4 + b)/x^4)/(a*x^4 + b)) + 1/4*(a*x^4 - 2*b)*sqrt((a*x^4 + b)/x^4
)]
```

giac [A] time = 0.22, size = 78, normalized size = 1.24

$$\frac{1}{4}\sqrt{ax^4+b}\,ax^2-\frac{3}{8}\sqrt{a}\,b\log\left(\left(\sqrt{a}\,x^2-\sqrt{ax^4+b}\right)^2\right)+\frac{\sqrt{a}\,b^2}{\left(\sqrt{a}\,x^2-\sqrt{ax^4+b}\right)^2-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(3/2)*x^3,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(a*x^4 + b)*a*x^2 - 3/8*sqrt(a)*b*log((sqrt(a)*x^2 - sqrt(a*x^4 + b
))^2) + sqrt(a)*b^2/((sqrt(a)*x^2 - sqrt(a*x^4 + b))^2 - b)
```

maple [A] time = 0.02, size = 82, normalized size = 1.30

$$\frac{\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{3}{2}}\left(\sqrt{a\,x^4+b}\;a\,x^4+3\sqrt{a}\;b\,x^2\ln\left(\sqrt{a}\;x^2+\sqrt{a\,x^4+b}\right)-2\sqrt{a\,x^4+b}\;b\right)x^4}{4\left(a\,x^4+b\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^4)^(3/2)*x^3,x)
```

```
[Out] 1/4*((a*x^4+b)/x^4)^(3/2)*x^4*((a*x^4+b)^(1/2)*a*x^4+3*a^(1/2)*b*x^2*ln(a^(
1/2)*x^2+(a*x^4+b)^(1/2))-2*b*(a*x^4+b)^(1/2))/(a*x^4+b)^(3/2)
```

maxima [A] time = 2.02, size = 66, normalized size = 1.05

$$\frac{1}{4}\sqrt{a+\frac{b}{x^4}}\,ax^4-\frac{3}{8}\sqrt{a}\,b\log\left(\frac{\sqrt{a+\frac{b}{x^4}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^4}}+\sqrt{a}}\right)-\frac{1}{2}\sqrt{a+\frac{b}{x^4}}\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(3/2)*x^3,x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(a + b/x^4)*a*x^4 - 3/8*sqrt(a)*b*log((sqrt(a + b/x^4) - sqrt(a))/(
sqrt(a + b/x^4) + sqrt(a))) - 1/2*sqrt(a + b/x^4)*b
```

mupad [B] time = 1.74, size = 48, normalized size = 0.76

$$\frac{a\,x^4\sqrt{a+\frac{b}{x^4}}}{4}-\frac{b\sqrt{a+\frac{b}{x^4}}}{2}+\frac{3\sqrt{a}\;b\,\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b/x^4)^(3/2),x)
```

```
[Out] (a*x^4*(a + b/x^4)^(1/2))/4 - (b*(a + b/x^4)^(1/2))/2 + (3*a^(1/2)*b*atanh(
(a + b/x^4)^(1/2)/a^(1/2)))/4
```

sympy [A] time = 3.28, size = 95, normalized size = 1.51

$$\frac{3\sqrt{a}\,b\,\operatorname{asinh}\left(\frac{\sqrt{a}\,x^2}{\sqrt{b}}\right)}{4}+\frac{a^2x^6}{4\sqrt{b}\,\sqrt{\frac{ax^4}{b}+1}}-\frac{a\sqrt{b}\,x^2}{4\sqrt{\frac{ax^4}{b}+1}}-\frac{b^{\frac{3}{2}}}{2x^2\sqrt{\frac{ax^4}{b}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**4)**(3/2)*x**3,x)
```

```
[Out] 3*sqrt(a)*b*asinh(sqrt(a)*x**2/sqrt(b))/4 + a**2*x**6/(4*sqrt(b)*sqrt(a*x**
4/b + 1)) - a*sqrt(b)*x**2/(4*sqrt(a*x**4/b + 1)) - b**(3/2)/(2*x**2*sqrt(a
*x**4/b + 1))
```

3.2067 $\int \left(a + \frac{b}{x^4}\right)^{3/2} x\,dx$

Optimal. Leaf size=69

$$\frac{1}{2}x^2\left(a+\frac{b}{x^4}\right)^{3/2} - \frac{3b\sqrt{a+\frac{b}{x^4}}}{4x^2} - \frac{3}{4}a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}}{x^2\sqrt{a+\frac{b}{x^4}}}\right)$$

```
[Out] 1/2*(a+b/x^4)^(3/2)*x^2-3/4*a*arctanh(b^(1/2)/x^2/(a+b/x^4)^(1/2))*b^(1/2)-
3/4*b*(a+b/x^4)^(1/2)/x^2
```

Rubi [A] time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {335, 275, 277, 195, 217, 206}

$$\frac{1}{2}x^2\left(a+\frac{b}{x^4}\right)^{3/2} - \frac{3b\sqrt{a+\frac{b}{x^4}}}{4x^2} - \frac{3}{4}a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}}{x^2\sqrt{a+\frac{b}{x^4}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^4)^(3/2)*x,x]
```

```
[Out] (-3*b*Sqrt[a + b/x^4])/(4*x^2) + ((a + b/x^4)^(3/2)*x^2)/2 - (3*a*Sqrt[b]*A
rcTanh[Sqrt[b]/(Sqrt[a + b/x^4]*x^2)])/4
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \left(a+\frac{b}{x^4}\right)^{3/2} x\,dx &= -\text{Subst}\left(\int \frac{\left(a+bx^4\right)^{3/2}}{x^3}\,dx, x, \frac{1}{x}\right) \\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x^2}\,dx, x, \frac{1}{x^2}\right)\right) \\
&= \frac{1}{2}\left(a+\frac{b}{x^4}\right)^{3/2} x^2 - \frac{1}{2}(3b)\,\text{Subst}\left(\int \sqrt{a+bx^2}\,dx, x, \frac{1}{x^2}\right) \\
&= -\frac{3b\sqrt{a+\frac{b}{x^4}}}{4x^2} + \frac{1}{2}\left(a+\frac{b}{x^4}\right)^{3/2} x^2 - \frac{1}{4}(3ab)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{x^2}\right) \\
&= -\frac{3b\sqrt{a+\frac{b}{x^4}}}{4x^2} + \frac{1}{2}\left(a+\frac{b}{x^4}\right)^{3/2} x^2 - \frac{1}{4}(3ab)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{b}{x^4}}\,x^2}\right) \\
&= -\frac{3b\sqrt{a+\frac{b}{x^4}}}{4x^2} + \frac{1}{2}\left(a+\frac{b}{x^4}\right)^{3/2} x^2 - \frac{3}{4}a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^4}}\,x^2}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 47, normalized size = 0.68

$$\frac{ax^6\left(a+\frac{b}{x^4}\right)^{3/2}\left(ax^4+b\right)\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{ax^4}{b}+1\right)}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^4)^(3/2)*x,x]
```

```
[Out] (a*(a + b/x^4)^(3/2)*x^6*(b + a*x^4)*Hypergeometric2F1[2, 5/2, 7/2, 1 + (a*
x^4)/b])/(10*b^2)
```

fricas [A] time = 0.73, size = 143, normalized size = 2.07

$$\left[\frac{3\,a\sqrt{b}\,x^2\log\left(\frac{ax^4-2\sqrt{b}\,x^2\sqrt{\frac{ax^4+b}{x^4}}+2b}{x^4}\right)+2\left(2\,ax^4-b\right)\sqrt{\frac{ax^4+b}{x^4}}}{8\,x^2},\ \frac{3\,a\sqrt{-b}\,x^2\arctan\left(\frac{\sqrt{-b}\,x^2\sqrt{\frac{ax^4+b}{x^4}}}{b}\right)+\left(2\,ax^4-b\right)\sqrt{\frac{a}{}}}{4\,x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(3/2)*x,x, algorithm="fricas")
```

```
[Out] [1/8*(3*a*sqrt(b)*x^2*log((a*x^4 - 2*sqrt(b)*x^2*sqrt((a*x^4 + b)/x^4) + 2*
b)/x^4) + 2*(2*a*x^4 - b)*sqrt((a*x^4 + b)/x^4))/x^2, 1/4*(3*a*sqrt(-b)*x^2
*arctan(sqrt(-b)*x^2*sqrt((a*x^4 + b)/x^4)/b) + (2*a*x^4 - b)*sqrt((a*x^4 +
 b)/x^4))/x^2]
```

giac [A] time = 0.16, size = 63, normalized size = 0.91

$$\frac{\frac{3\,a^2 b \arctan\left(\frac{\sqrt{a x^4+b}}{\sqrt{-b}}\right)}{\sqrt{-b}} + 2\sqrt{a x^4+b}\,a^2 - \frac{\sqrt{a x^4+b}\,a b}{x^4}}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(3/2)*x,x, algorithm="giac")
```

```
[Out] 1/4*(3*a^2*b*arctan(sqrt(a*x^4 + b)/sqrt(-b))/sqrt(-b) + 2*sqrt(a*x^4 + b)*
a^2 - sqrt(a*x^4 + b)*a*b/x^4)/a
```

maple [A] time = 0.02, size = 86, normalized size = 1.25

$$\frac{\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{3}{2}}\left(-3a\sqrt{b}\,x^4 \ln\left(\frac{2b+2\sqrt{a\,x^4+b}\,\sqrt{b}}{x^2}\right) + 2\sqrt{a\,x^4+b}\,a\,x^4 - \sqrt{a\,x^4+b}\,b\right)x^2}{4\left(a\,x^4+b\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^4)^(3/2)*x,x)
```

```
[Out] 1/4*((a*x^4+b)/x^4)^(3/2)*x^2*(-3*a*b^(1/2)*ln(2*(b+(a*x^4+b)^(1/2)*b^(1/2)
)/x^2)*x^4+2*(a*x^4+b)^(1/2)*a*x^4-(a*x^4+b)^(1/2)*b)/(a*x^4+b)^(3/2)
```

maxima [A] time = 1.97, size = 95, normalized size = 1.38

$$\frac{1}{2}\sqrt{a+\frac{b}{x^4}}\,a x^2 - \frac{\sqrt{a+\frac{b}{x^4}}\,a b x^2}{4\left(\left(a+\frac{b}{x^4}\right)x^4 - b\right)} + \frac{3}{8}\,a\sqrt{b}\,\log\left(\frac{\sqrt{a+\frac{b}{x^4}}\,x^2 - \sqrt{b}}{\sqrt{a+\frac{b}{x^4}}\,x^2 + \sqrt{b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(3/2)*x,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a + b/x^4)*a*x^2 - 1/4*sqrt(a + b/x^4)*a*b*x^2/((a + b/x^4)*x^4 -
b) + 3/8*a*sqrt(b)*log((sqrt(a + b/x^4)*x^2 - sqrt(b))/(sqrt(a + b/x^4)*x^2
 + sqrt(b)))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(a+\frac{b}{x^4}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b/x^4)^(3/2),x)
```

```
[Out] int(x*(a + b/x^4)^(3/2), x)
```

sympy [A] time = 3.08, size = 95, normalized size = 1.38

$$\frac{a^{\frac{3}{2}}x^2}{2\sqrt{1+\frac{b}{a x^4}}} + \frac{\sqrt{a}\,b}{4x^2\sqrt{1+\frac{b}{a x^4}}} - \frac{3a\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\,x^2}\right)}{4} - \frac{b^2}{4\sqrt{a}\,x^6\sqrt{1+\frac{b}{a x^4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**4)**(3/2)*x,x)
```

```
[Out] a**(3/2)*x**2/(2*sqrt(1 + b/(a*x**4))) + sqrt(a)*b/(4*x**2*sqrt(1 + b/(a*x*
*4))) - 3*a*sqrt(b)*asinh(sqrt(b)/(sqrt(a)*x**2))/4 - b**2/(4*sqrt(a)*x**6*
sqrt(1 + b/(a*x**4)))
```

3.2068 $\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=59

$$\frac{1}{2}a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)-\frac{1}{2}a\sqrt{a+\frac{b}{x^4}}-\frac{1}{6}\left(a+\frac{b}{x^4}\right)^{3/2}$$

```
[Out] -1/6*(a+b/x^4)^(3/2)+1/2*a^(3/2)*arctanh((a+b/x^4)^(1/2)/a^(1/2))-1/2*a*(a+
b/x^4)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 50, 63, 208}

$$\frac{1}{2}a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)-\frac{1}{2}a\sqrt{a+\frac{b}{x^4}}-\frac{1}{6}\left(a+\frac{b}{x^4}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^4)^(3/2)/x,x]
```

```
[Out] -(a*Sqrt[a + b/x^4])/2 - (a + b/x^4)^(3/2)/6 + (a^(3/2)*ArcTanh[Sqrt[a + b/
x^4]/Sqrt[a]])/2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x}\,dx &= -\left(\frac{1}{4}\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, \frac{1}{x^4}\right)\right)\\ &= -\frac{1}{6}\left(a+\frac{b}{x^4}\right)^{3/2} - \frac{1}{4}a\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x^4}\right)\\ &= -\frac{1}{2}a\sqrt{a+\frac{b}{x^4}} - \frac{1}{6}\left(a+\frac{b}{x^4}\right)^{3/2} - \frac{1}{4}a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x^4}\right)\\ &= -\frac{1}{2}a\sqrt{a+\frac{b}{x^4}} - \frac{1}{6}\left(a+\frac{b}{x^4}\right)^{3/2} - \frac{a^2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x^4}}\right)}{2b}\\ &= -\frac{1}{2}a\sqrt{a+\frac{b}{x^4}} - \frac{1}{6}\left(a+\frac{b}{x^4}\right)^{3/2} + \frac{1}{2}a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)\end{aligned}$$

Mathematica [C] time = 0.02, size = 52, normalized size = 0.88

$$-\frac{b\sqrt{a+\frac{b}{x^4}}\;{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};-\frac{ax^4}{b}\right)}{6x^4\sqrt{\frac{ax^4}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^4)^(3/2)/x,x]
```

```
[Out] -1/6*(b*Sqrt[a + b/x^4]*Hypergeometric2F1[-3/2, -3/2, -1/2, -((a*x^4)/b)])/
(x^4*Sqrt[1 + (a*x^4)/b])
```

fricas [A] time = 0.84, size = 141, normalized size = 2.39

$$\left[\frac{3\,a^{\frac{3}{2}}x^4\log\left(-2\,ax^4-2\,\sqrt{a}\,x^4\sqrt{\frac{ax^4+b}{x^4}}-b\right)-2\left(4\,ax^4+b\right)\sqrt{\frac{ax^4+b}{x^4}}}{12\,x^4}, -\frac{3\,\sqrt{-a}\,ax^4\arctan\left(\frac{\sqrt{-a}\,x^4\sqrt{\frac{ax^4+b}{x^4}}}{ax^4+b}\right)+\left(4\,ax^4+\right.}{6\,x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/12*(3*a^(3/2)*x^4*log(-2*a*x^4 - 2*sqrt(a)*x^4*sqrt((a*x^4 + b)/x^4) - b
) - 2*(4*a*x^4 + b)*sqrt((a*x^4 + b)/x^4))/x^4, -1/6*(3*sqrt(-a)*a*x^4*arct
an(sqrt(-a)*x^4*sqrt((a*x^4 + b)/x^4)/(a*x^4 + b)) + (4*a*x^4 + b)*sqrt((a*
x^4 + b)/x^4))/x^4]
```

giac [B] time = 0.25, size = 122, normalized size = 2.07

$$-\frac{1}{4}\,a^{\frac{3}{2}}\log\left(\left(\sqrt{a}\,x^2-\sqrt{ax^4+b}\right)^2\right)+\frac{2\left(3\left(\sqrt{a}\,x^2-\sqrt{ax^4+b}\right)^4a^{\frac{3}{2}}b-3\left(\sqrt{a}\,x^2-\sqrt{ax^4+b}\right)^2a^{\frac{3}{2}}b^2+2\,a^{\frac{3}{2}}b^3\right)}{3\left(\left(\sqrt{a}\,x^2-\sqrt{ax^4+b}\right)^2-b\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(3/2)/x,x, algorithm="giac")
```

```
[Out] -1/4*a^(3/2)*log((sqrt(a)*x^2 - sqrt(a*x^4 + b))^2) + 2/3*(3*(sqrt(a)*x^2 -
 sqrt(a*x^4 + b))^4*a^(3/2)*b - 3*(sqrt(a)*x^2 - sqrt(a*x^4 + b))^2*a^(3/2)
*b^2 + 2*a^(3/2)*b^3)/((sqrt(a)*x^2 - sqrt(a*x^4 + b))^2 - b)^3
```

maple [A] time = 0.02, size = 78, normalized size = 1.32

$$-\frac{\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{3}{2}}\left(-3a^{\frac{3}{2}}x^6\ln\left(\sqrt{a}\;x^2+\sqrt{a\,x^4+b}\right)+4\sqrt{a\,x^4+b}\;a\,x^4+\sqrt{a\,x^4+b}\;b\right)}{6\left(a\,x^4+b\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^4)^(3/2)/x,x)
```

```
[Out] -1/6*((a*x^4+b)/x^4)^(3/2)*(-3*a^(3/2)*ln(a^(1/2)*x^2+(a*x^4+b)^(1/2))*x^6+
4*(a*x^4+b)^(1/2)*a*x^4+(a*x^4+b)^(1/2)*b)/(a*x^4+b)^(3/2)
```

maxima [A] time = 1.95, size = 61, normalized size = 1.03

$$-\frac{1}{4}\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x^4}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^4}}+\sqrt{a}}\right)-\frac{1}{6}\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}}-\frac{1}{2}\sqrt{a+\frac{b}{x^4}}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] -1/4*a^(3/2)*log((sqrt(a + b/x^4) - sqrt(a))/(sqrt(a + b/x^4) + sqrt(a))) -
 1/6*(a + b/x^4)^(3/2) - 1/2*sqrt(a + b/x^4)*a
```

mupad [B] time = 1.50, size = 43, normalized size = 0.73

$$\frac{a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{2}-\frac{a\sqrt{a+\frac{b}{x^4}}}{2}-\frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^4)^(3/2)/x,x)
```

```
[Out] (a^(3/2)*atanh((a + b/x^4)^(1/2)/a^(1/2)))/2 - (a*(a + b/x^4)^(1/2))/2 - (a
 + b/x^4)^(3/2)/6
```

sympy [A] time = 2.48, size = 80, normalized size = 1.36

$$-\frac{2a^{\frac{3}{2}}\sqrt{1+\frac{b}{ax^4}}}{3}-\frac{a^{\frac{3}{2}}\log\left(\frac{b}{ax^4}\right)}{4}+\frac{a^{\frac{3}{2}}\log\left(\sqrt{1+\frac{b}{ax^4}}+1\right)}{2}-\frac{\sqrt{a}\,b\sqrt{1+\frac{b}{ax^4}}}{6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**4)**(3/2)/x,x)
```

```
[Out] -2*a**(3/2)*sqrt(1 + b/(a*x**4))/3 - a**(3/2)*log(b/(a*x**4))/4 + a**(3/2)*
log(sqrt(1 + b/(a*x**4)) + 1)/2 - sqrt(a)*b*sqrt(1 + b/(a*x**4))/(6*x**4)
```

3.2069 $\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=71

$$-\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}}{x^2\sqrt{a+\frac{b}{x^4}}}\right)}{16\sqrt{b}}-\frac{3a\sqrt{a+\frac{b}{x^4}}}{16x^2}-\frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{8x^2}$$

```
[Out] -1/8*(a+b/x^4)^(3/2)/x^2-3/16*a^2*arctanh(b^(1/2)/x^2/(a+b/x^4)^(1/2))/b^(1
/2)-3/16*a*(a+b/x^4)^(1/2)/x^2
```

Rubi [A] time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {335, 275, 195, 217, 206}

$$-\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}}{x^2\sqrt{a+\frac{b}{x^4}}}\right)}{16\sqrt{b}}-\frac{3a\sqrt{a+\frac{b}{x^4}}}{16x^2}-\frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^4)^(3/2)/x^3,x]
```

```
[Out] (-3*a*Sqrt[a + b/x^4])/(16*x^2) - (a + b/x^4)^(3/2)/(8*x^2) - (3*a^2*ArcTan
h[Sqrt[b]/(Sqrt[a + b/x^4]*x^2)])/(16*Sqrt[b])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
```

egerQ[m]

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^3}\,dx &= -\text{Subst}\left(\int x\left(a+bx^4\right)^{3/2}\,dx, x, \frac{1}{x}\right) \\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \left(a+bx^2\right)^{3/2}\,dx, x, \frac{1}{x^2}\right)\right) \\
&= -\frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{8x^2} - \frac{1}{8}(3a)\,\text{Subst}\left(\int \sqrt{a+bx^2}\,dx, x, \frac{1}{x^2}\right) \\
&= -\frac{3a\sqrt{a+\frac{b}{x^4}}}{16x^2} - \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{8x^2} - \frac{1}{16}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{x^2}\right) \\
&= -\frac{3a\sqrt{a+\frac{b}{x^4}}}{16x^2} - \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{8x^2} - \frac{1}{16}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{b}{x^4}}\,x^2}\right) \\
&= -\frac{3a\sqrt{a+\frac{b}{x^4}}}{16x^2} - \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{8x^2} - \frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^4}}\,x^2}\right)}{16\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 85, normalized size = 1.20

$$-\frac{\sqrt{a+\frac{b}{x^4}}\left(3a^2x^8\sqrt{\frac{ax^4}{b}+1}\,\tanh^{-1}\left(\sqrt{\frac{ax^4}{b}+1}\right)+5a^2x^8+7abx^4+2b^2\right)}{16x^6\left(ax^4+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^4)^(3/2)/x^3,x]
```

```
[Out] -1/16*(Sqrt[a + b/x^4]*(2*b^2 + 7*a*b*x^4 + 5*a^2*x^8 + 3*a^2*x^8*Sqrt[1 +
(a*x^4)/b]*ArcTanh[Sqrt[1 + (a*x^4)/b]]))/(x^6*(b + a*x^4))
```

fricas [A] time = 1.04, size = 160, normalized size = 2.25

$$\left[\frac{3\,a^2\sqrt{b}\,x^6\log\left(\frac{ax^4-2\,\sqrt{b}\,x^2\sqrt{\frac{ax^4+b}{x^4}}+2\,b}{x^4}\right)-2\left(5\,abx^4+2\,b^2\right)\sqrt{\frac{ax^4+b}{x^4}}}{32\,bx^6},\ \frac{3\,a^2\sqrt{-b}\,x^6\arctan\left(\frac{\sqrt{-b}\,x^2\sqrt{\frac{ax^4+b}{x^4}}}{b}\right)-\left(5\,abx^4+\right.}{16\,bx^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/32*(3*a^2*sqrt(b)*x^6*log((a*x^4 - 2*sqrt(b)*x^2*sqrt((a*x^4 + b)/x^4) +
 2*b)/x^4) - 2*(5*a*b*x^4 + 2*b^2)*sqrt((a*x^4 + b)/x^4))/(b*x^6), 1/16*(3*
a^2*sqrt(-b)*x^6*arctan(sqrt(-b)*x^2*sqrt((a*x^4 + b)/x^4)/b) - (5*a*b*x^4
+ 2*b^2)*sqrt((a*x^4 + b)/x^4))/(b*x^6)]
```

giac [A] time = 0.16, size = 70, normalized size = 0.99

$$\frac{\frac{3\,a^3\arctan\left(\frac{\sqrt{ax^4+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}-\frac{5\left(ax^4+b\right)^{\frac{3}{2}}a^3-3\sqrt{ax^4+b}\,a^3b}{a^2x^8}}{16\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/16*(3*a^3*arctan(sqrt(a*x^4 + b)/sqrt(-b))/sqrt(-b) - (5*(a*x^4 + b)^(3/2
)*a^3 - 3*sqrt(a*x^4 + b)*a^3*b)/(a^2*x^8))/a
```

maple [A] time = 0.02, size = 93, normalized size = 1.31

$$-\frac{\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{3}{2}}\left(3a^2x^8\ln\left(\frac{2b+2\sqrt{a\,x^4+b}\,\sqrt{b}}{x^2}\right)+5\sqrt{a\,x^4+b}\;a\sqrt{b}\;x^4+2\sqrt{a\,x^4+b}\;b^{\frac{3}{2}}\right)}{16\left(a\,x^4+b\right)^{\frac{3}{2}}\sqrt{b}\;x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^4)^(3/2)/x^3,x)
```

```
[Out] -1/16*((a*x^4+b)/x^4)^(3/2)/x^2*(3*a^2*ln(2*(b+(a*x^4+b)^(1/2)*b^(1/2))/x^2
)*x^8+5*(a*x^4+b)^(1/2)*a*b^(1/2)*x^4+2*b^(3/2)*(a*x^4+b)^(1/2))/(a*x^4+b)^
(3/2)/b^(1/2)
```

maxima [B] time = 1.88, size = 119, normalized size = 1.68

$$\frac{3\,a^2\log\left(\frac{\sqrt{a+\frac{b}{x^4}}\,x^2-\sqrt{b}}{\sqrt{a+\frac{b}{x^4}}\,x^2+\sqrt{b}}\right)}{32\sqrt{b}}-\frac{5\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}}a^2x^6-3\sqrt{a+\frac{b}{x^4}}\,a^2bx^2}{16\left(\left(a+\frac{b}{x^4}\right)^2x^8-2\left(a+\frac{b}{x^4}\right)bx^4+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 3/32*a^2*log((sqrt(a + b/x^4)*x^2 - sqrt(b))/(sqrt(a + b/x^4)*x^2 + sqrt(b)
))/sqrt(b) - 1/16*(5*(a + b/x^4)^(3/2)*a^2*x^6 - 3*sqrt(a + b/x^4)*a^2*b*x^
2)/((a + b/x^4)^2*x^8 - 2*(a + b/x^4)*b*x^4 + b^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^4)^(3/2)/x^3,x)
```

```
[Out] int((a + b/x^4)^(3/2)/x^3, x)
```

sympy [A] time = 3.63, size = 75, normalized size = 1.06

$$-\frac{5a^{\frac{3}{2}}\sqrt{1+\frac{b}{ax^4}}}{16x^2}-\frac{\sqrt{a}\,b\sqrt{1+\frac{b}{ax^4}}}{8x^6}-\frac{3a^2\,\mathrm{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\,x^2}\right)}{16\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**4)**(3/2)/x**3,x)
```

```
[Out] -5*a**(3/2)*sqrt(1 + b/(a*x**4))/(16*x**2) - sqrt(a)*b*sqrt(1 + b/(a*x**4))
/(8*x**6) - 3*a**2*asinh(sqrt(b)/(sqrt(a)*x**2))/(16*sqrt(b))
```

3.2070 $\int \left(a+\frac{b}{x^4}\right)^{3/2} x^2\,dx$

Optimal. Leaf size=126

$$-\frac{2a^{3/4}b^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt{a+\frac{b}{x^4}}}-\frac{2b\sqrt{a+\frac{b}{x^4}}}{3x}+\frac{1}{3}x^3\left(a+\frac{b}{x^4}\right)^{3/2}$$

```
[Out] 1/3*(a+b/x^4)^(3/2)*x^3-2/3*b*(a+b/x^4)^(1/2)/x-2/3*a^(3/4)*b^(3/4)*(cos(2*
arccot(a^(1/4)*x/b^(1/4)))^2)^(1/2)/cos(2*arccot(a^(1/4)*x/b^(1/4)))*Ellipt
icF(sin(2*arccot(a^(1/4)*x/b^(1/4))),1/2*2^(1/2))*(a^(1/2)+b^(1/2)/x^2)*((a
+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)/(a+b/x^4)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {335, 277, 195, 220}

$$-\frac{2a^{3/4}b^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt{a+\frac{b}{x^4}}}+\frac{1}{3}x^3\left(a+\frac{b}{x^4}\right)^{3/2}-\frac{2b\sqrt{a+\frac{b}{x^4}}}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^4)^(3/2)*x^2,x]
```

```
[Out] (-2*b*Sqrt[a + b/x^4])/(3*x) + ((a + b/x^4)^(3/2)*x^3)/3 - (2*a^(3/4)*b^(3/
4)*Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a] + Sqrt[b]/x^2)*Elli
pticF[2*ArcCot[(a^(1/4)*x)/b^(1/4)], 1/2])/(3*Sqrt[a + b/x^4])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
```

egerQ[m]

Rubi steps

$$\int \left(a+\frac{b}{x^4}\right)^{3/2} x^2\,dx = -\text{Subst}\left(\int \frac{\left(a+bx^4\right)^{3/2}}{x^4}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{3}\left(a+\frac{b}{x^4}\right)^{3/2} x^3 - (2b)\,\text{Subst}\left(\int \sqrt{a+bx^4}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{2b\sqrt{a+\frac{b}{x^4}}}{3x} + \frac{1}{3}\left(a+\frac{b}{x^4}\right)^{3/2} x^3 - \frac{1}{3}(4ab)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{2b\sqrt{a+\frac{b}{x^4}}}{3x} + \frac{1}{3}\left(a+\frac{b}{x^4}\right)^{3/2} x^3 - \frac{2a^{3/4}b^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right.}{3\sqrt{a+\frac{b}{x^4}}}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.41

$$-\frac{b\sqrt{a+\frac{b}{x^4}}\;{}_2F_1\left(-\frac{3}{2},-\frac{3}{4};\frac{1}{4};-\frac{ax^4}{b}\right)}{3x\sqrt{\frac{ax^4}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^4)^(3/2)*x^2,x]
```

```
[Out] -1/3*(b*Sqrt[a + b/x^4]*Hypergeometric2F1[-3/2, -3/4, 1/4, -((a*x^4)/b)])/(
x*Sqrt[1 + (a*x^4)/b])
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ax^4+b\right)\sqrt{\frac{ax^4+b}{x^4}}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(3/2)*x^2,x, algorithm="fricas")
```

```
[Out] integral((a*x^4 + b)*sqrt((a*x^4 + b)/x^4)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+\frac{b}{x^4}\right)^{\frac{3}{2}} x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(3/2)*x^2,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^4)^(3/2)*x^2, x)
```

maple [C] time = 0.01, size = 138, normalized size = 1.10

$$\frac{\left(\frac{a x^4+b}{x^4}\right)^{\frac{3}{2}}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\, a^2x^8+4\sqrt{-\frac{i\sqrt{a}\, x^2-\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}\, x^2+\sqrt{b}}{\sqrt{b}}}\, ab\, x^3\,\text{EllipticF}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\, x, i\right)-\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\, b^2\right)x^3}{3\left(a x^4+b\right)^2\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^4)^(3/2)*x^2,x)
```

```
[Out] 1/3*((a*x^4+b)/x^4)^(3/2)*x^3*((I*a^(1/2)/b^(1/2))^(1/2)*x^8*a^2+4*a*b*(-(I
*a^(1/2)*x^2-b^(1/2))/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2
)*EllipticF((I*a^(1/2)/b^(1/2))^(1/2)*x,I)*x^3-(I*a^(1/2)/b^(1/2))^(1/2)*b^
2)/(a*x^4+b)^2/(I*a^(1/2)/b^(1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+\frac{b}{x^4}\right)^{\frac{3}{2}} x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(3/2)*x^2,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x^4)^(3/2)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(a+\frac{b}{x^4}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b/x^4)^(3/2),x)
```

```
[Out] int(x^2*(a + b/x^4)^(3/2), x)
```

sympy [C] time = 1.61, size = 44, normalized size = 0.35

$$-\frac{a^{\frac{3}{2}}x^3\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{2}, -\frac{3}{4}\\ \frac{1}{4}\end{matrix}\right| \frac{be^{i\pi}}{ax^4}\right)}{4\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**4)**(3/2)*x**2,x)
```

```
[Out] -a**(3/2)*x**3*gamma(-3/4)*hyper((-3/2, -3/4), (1/4,), b*exp_polar(I*pi)/(a
*x**4))/(4*gamma(1/4))
```

3.2071 $\int \left(a+\frac{b}{x^4}\right)^{3/2} dx$

Optimal. Leaf size=250

$$-\frac{6a^{5/4}\sqrt[4]{b}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5\sqrt{a+\frac{b}{x^4}}}+\frac{12a^{5/4}\sqrt[4]{b}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right)|$$

```
[Out] (a+b/x^4)^(3/2)*x-6/5*b*(a+b/x^4)^(1/2)/x^3-12/5*a*b^(1/2)*(a+b/x^4)^(1/2)/
x/(a^(1/2)+b^(1/2)/x^2)+12/5*a^(5/4)*b^(1/4)*(cos(2*arccot(a^(1/4)*x/b^(1/4
)))^2)^(1/2)/cos(2*arccot(a^(1/4)*x/b^(1/4)))*EllipticE(sin(2*arccot(a^(1/4
)*x/b^(1/4))),1/2*2^(1/2))*(a^(1/2)+b^(1/2)/x^2)*((a+b/x^4)/(a^(1/2)+b^(1/2
)/x^2)^2)^(1/2)/(a+b/x^4)^(1/2)-6/5*a^(5/4)*b^(1/4)*(cos(2*arccot(a^(1/4)*x
/b^(1/4)))^2)^(1/2)/cos(2*arccot(a^(1/4)*x/b^(1/4)))*EllipticF(sin(2*arccot
(a^(1/4)*x/b^(1/4))),1/2*2^(1/2))*(a^(1/2)+b^(1/2)/x^2)*((a+b/x^4)/(a^(1/2)
+b^(1/2)/x^2)^2)^(1/2)/(a+b/x^4)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {242, 277, 279, 305, 220, 1196}

$$-\frac{6a^{5/4}\sqrt[4]{b}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5\sqrt{a+\frac{b}{x^4}}}+\frac{12a^{5/4}\sqrt[4]{b}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right)|$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^4)^(3/2),x]
```

```
[Out] (-6*b*Sqrt[a + b/x^4])/(5*x^3) - (12*a*Sqrt[b]*Sqrt[a + b/x^4])/(5*(Sqrt[a]
 + Sqrt[b]/x^2)*x) + (a + b/x^4)^(3/2)*x + (12*a^(5/4)*b^(1/4)*Sqrt[(a + b/
x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a] + Sqrt[b]/x^2)*EllipticE[2*ArcCot[
(a^(1/4)*x)/b^(1/4)], 1/2])/(5*Sqrt[a + b/x^4]) - (6*a^(5/4)*b^(1/4)*Sqrt[(
a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a] + Sqrt[b]/x^2)*EllipticF[2*A
rcCot[(a^(1/4)*x)/b^(1/4)], 1/2])/(5*Sqrt[a + b/x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \left(a+\frac{b}{x^4}\right)^{3/2} dx = -\text{Subst}\left(\int \frac{\left(a+bx^4\right)^{3/2}}{x^2}\, dx, x, \frac{1}{x}\right)$$

$$= \left(a+\frac{b}{x^4}\right)^{3/2} x - (6b)\,\text{Subst}\left(\int x^2\sqrt{a+bx^4}\, dx, x, \frac{1}{x}\right)$$

$$= -\frac{6b\sqrt{a+\frac{b}{x^4}}}{5x^3} + \left(a+\frac{b}{x^4}\right)^{3/2} x - \frac{1}{5}(12ab)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\, dx, x, \frac{1}{x}\right)$$

$$= -\frac{6b\sqrt{a+\frac{b}{x^4}}}{5x^3} + \left(a+\frac{b}{x^4}\right)^{3/2} x - \frac{1}{5}\left(12a^{3/2}\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\, dx, x, \frac{1}{x}\right) + \frac{1}{5}\left(12a^{3/2}\sqrt{b}\right)$$

$$= -\frac{6b\sqrt{a+\frac{b}{x^4}}}{5x^3} - \frac{12a\sqrt{b}\sqrt{a+\frac{b}{x^4}}}{5\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)x} + \left(a+\frac{b}{x^4}\right)^{3/2} x + \frac{12a^{5/4}\sqrt[4]{b}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\,\text{co}\right.}{5\sqrt{a+\frac{b}{x^4}}}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.21

$$-\frac{b\sqrt{a+\frac{b}{x^4}}\;{}_2F_1\left(-\frac{3}{2},-\frac{5}{4};-\frac{1}{4};-\frac{ax^4}{b}\right)}{5x^3\sqrt{\frac{ax^4}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^4)^(3/2),x]
```

```
[Out] -1/5*(b*Sqrt[a + b/x^4]*Hypergeometric2F1[-3/2, -5/4, -1/4, -((a*x^4)/b)])/
(x^3*Sqrt[1 + (a*x^4)/b])
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ax^4+b\right)\sqrt{\frac{ax^4+b}{x^4}}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(3/2),x, algorithm="fricas")
[Out] integral((a*x^4 + b)*sqrt((a*x^4 + b)/x^4)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+\frac{b}{x^4}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(3/2),x, algorithm="giac")
[Out] integrate((a + b/x^4)^(3/2), x)
```

maple [C] time = 0.02, size = 228, normalized size = 0.91

$$-\frac{\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{3}{2}}\left(7\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;a^2x^8+12i\sqrt{-\frac{i\sqrt{a}\;x^2-\sqrt{b}}{\sqrt{b}}}\;\sqrt{\frac{i\sqrt{a}\;x^2+\sqrt{b}}{\sqrt{b}}}\;a^{\frac{3}{2}}\sqrt{b}\;x^5\,\text{EllipticE}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;x,i\right)-12i\sqrt{-\frac{i\sqrt{a}\;x^2-\sqrt{b}}{\sqrt{b}}}\;\sqrt{}}{5\left(a\,x^4+b\right)^2\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^4)^(3/2),x)
[Out] -1/5*((a*x^4+b)/x^4)^(3/2)*x*(-12*I*a^(3/2)*b^(1/2)*(-(I*a^(1/2)*x^2-b^(1/2
))/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*x^5*EllipticF((I*
a^(1/2)/b^(1/2))^(1/2)*x,I)+12*I*a^(3/2)*b^(1/2)*(-(I*a^(1/2)*x^2-b^(1/2))/
b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*x^5*EllipticE((I*a^(
1/2)/b^(1/2))^(1/2)*x,I)+7*(I*a^(1/2)/b^(1/2))^(1/2)*a^2*x^8+8*(I*a^(1/2)/b
^(1/2))^(1/2)*x^4*a*b+(I*a^(1/2)/b^(1/2))^(1/2)*b^2)/(a*x^4+b)^2/(I*a^(1/2)
/b^(1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+\frac{b}{x^4}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(3/2),x, algorithm="maxima")
[Out] integrate((a + b/x^4)^(3/2), x)
```

mupad [B] time = 1.42, size = 38, normalized size = 0.15

$$-\frac{x\left(a+\frac{b}{x^4}\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{5}{4};\,-\frac{1}{4};\,-\frac{a\,x^4}{b}\right)}{5\left(\frac{a\,x^4}{b}+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^4)^(3/2),x)
```

```
[Out] -(x*(a + b/x^4)^(3/2)*hypergeom([-3/2, -5/4], -1/4, -(a*x^4)/b))/(5*((a*x^4
)/b + 1)^(3/2))
```

sympy [C] time = 1.38, size = 42, normalized size = 0.17

$$-\frac{a^{\frac{3}{2}} x \Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{2}, -\frac{1}{4} \\ \frac{3}{4} \end{matrix} \middle| \frac{be^{i\pi}}{ax^4}\right)}{4\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**4)**(3/2),x)
```

```
[Out] -a**(3/2)*x*gamma(-1/4)*hyper((-3/2, -1/4), (3/4,), b*exp_polar(I*pi)/(a*x*
**4))/(4*gamma(3/4))
```

3.2072 $\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=126

$$-\frac{2a^{7/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{b}\sqrt{a+\frac{b}{x^4}}}-\frac{2a\sqrt{a+\frac{b}{x^4}}}{7x}-\frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{7x}$$

```
[Out] -1/7*(a+b/x^4)^(3/2)/x-2/7*a*(a+b/x^4)^(1/2)/x-2/7*a^(7/4)*(cos(2*arccot(a^
(1/4)*x/b^(1/4)))^2)^(1/2)/cos(2*arccot(a^(1/4)*x/b^(1/4)))*EllipticF(sin(2
*arccot(a^(1/4)*x/b^(1/4))),1/2*2^(1/2))*(a^(1/2)+b^(1/2)/x^2)*((a+b/x^4)/(
a^(1/2)+b^(1/2)/x^2)^2)^(1/2)/b^(1/4)/(a+b/x^4)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {335, 195, 220}

$$-\frac{2a^{7/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{b}\sqrt{a+\frac{b}{x^4}}}-\frac{2a\sqrt{a+\frac{b}{x^4}}}{7x}-\frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{7x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^4)^(3/2)/x^2,x]
```

```
[Out] (-2*a*Sqrt[a + b/x^4])/(7*x) - (a + b/x^4)^(3/2)/(7*x) - (2*a^(7/4)*Sqrt[(a
 + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a] + Sqrt[b]/x^2)*EllipticF[2*Ar
cCot[(a^(1/4)*x)/b^(1/4)], 1/2])/(7*b^(1/4)*Sqrt[a + b/x^4])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^2}\,dx = -\text{Subst}\left(\int \left(a+bx^4\right)^{3/2}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{7x} - \frac{1}{7}(6a)\,\text{Subst}\left(\int \sqrt{a+bx^4}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{2a\sqrt{a+\frac{b}{x^4}}}{7x} - \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{7x} - \frac{1}{7}\left(4a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{2a\sqrt{a+\frac{b}{x^4}}}{7x} - \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{7x} - \frac{2a^{7/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{b}\sqrt{a+\frac{b}{x^4}}}$$

Mathematica [C] time = 0.03, size = 52, normalized size = 0.41

$$-\frac{b\sqrt{a+\frac{b}{x^4}}\;{}_2F_1\left(-\frac{7}{4},-\frac{3}{2};-\frac{3}{4};-\frac{ax^4}{b}\right)}{7x^5\sqrt{\frac{ax^4}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^4)^(3/2)/x^2,x]
```

```
[Out] -1/7*(b*Sqrt[a + b/x^4]*Hypergeometric2F1[-7/4, -3/2, -3/4, -((a*x^4)/b)])/
(x^5*Sqrt[1 + (a*x^4)/b])
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ax^4+b\right)\sqrt{\frac{ax^4+b}{x^4}}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral((a*x^4 + b)*sqrt((a*x^4 + b)/x^4)/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^4)^(3/2)/x^2, x)
```

maple [C] time = 0.02, size = 158, normalized size = 1.25

$$\frac{\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{3}{2}}\left(-3\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;a^2x^8+4\sqrt{-\frac{i\sqrt{a}\;x^2-\sqrt{b}}{\sqrt{b}}}\;\sqrt{\frac{i\sqrt{a}\;x^2+\sqrt{b}}{\sqrt{b}}}\;a^2x^7\,\text{EllipticF}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;x,i\right)-4\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;ab\,x^4-\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;b^2\right)}{7\left(a\,x^4+b\right)^2\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^4)^(3/2)/x^2,x)
```

```
[Out] 1/7*((a*x^4+b)/x^4)^(3/2)*(4*a^2*(-(I*a^(1/2)*x^2-b^(1/2))/b^(1/2))^(1/2)*(
(I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*EllipticF((I*a^(1/2)/b^(1/2))^(1/2)*
x,I)*x^7-3*(I*a^(1/2)/b^(1/2))^(1/2)*a^2*x^8-4*(I*a^(1/2)/b^(1/2))^(1/2)*a*
b*x^4-(I*a^(1/2)/b^(1/2))^(1/2)*b^2)/x/(a*x^4+b)^2/(I*a^(1/2)/b^(1/2))^(1/2
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x^4)^(3/2)/x^2, x)
```

mupad [B] time = 1.57, size = 39, normalized size = 0.31

$$-\frac{\left(a\,x^4+b\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{4};\ \frac{5}{4};\ -\frac{b}{a\,x^4}\right)}{x\left(\frac{b}{a}+x^4\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^4)^(3/2)/x^2,x)
```

```
[Out] -((b + a*x^4)^(3/2)*hypergeom([-3/2, 1/4], 5/4, -b/(a*x^4)))/(x*(b/a + x^4)
^(3/2))
```

sympy [C] time = 1.42, size = 39, normalized size = 0.31

$$-\frac{a^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\right|\frac{be^{i\pi}}{ax^4}\right)}{4x\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**4)**(3/2)/x**2,x)
```

```
[Out] -a**(3/2)*gamma(1/4)*hyper((-3/2, 1/4), (5/4,), b*exp_polar(I*pi)/(a*x**4))
/(4*x*gamma(5/4))
```

3.2073 $\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=257

$$-\frac{2a^{9/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+\frac{b}{x^4}}}+\frac{4a^{9/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+\frac{b}{x^4}}}-\frac{4a^2}{15\sqrt{b}\,x}$$

```
[Out] -1/9*(a+b/x^4)^(3/2)/x^3-2/15*a*(a+b/x^4)^(1/2)/x^3-4/15*a^2*(a+b/x^4)^(1/2
)/x/b^(1/2)/(a^(1/2)+b^(1/2)/x^2)+4/15*a^(9/4)*(cos(2*arccot(a^(1/4)*x/b^(1
/4)))^2)^(1/2)/cos(2*arccot(a^(1/4)*x/b^(1/4)))*EllipticE(sin(2*arccot(a^(1
/4)*x/b^(1/4))),1/2*2^(1/2))*(a^(1/2)+b^(1/2)/x^2)*((a+b/x^4)/(a^(1/2)+b^(1
/2)/x^2)^2)^(1/2)/b^(3/4)/(a+b/x^4)^(1/2)-2/15*a^(9/4)*(cos(2*arccot(a^(1/4
)*x/b^(1/4)))^2)^(1/2)/cos(2*arccot(a^(1/4)*x/b^(1/4)))*EllipticF(sin(2*arc
cot(a^(1/4)*x/b^(1/4))),1/2*2^(1/2))*(a^(1/2)+b^(1/2)/x^2)*((a+b/x^4)/(a^(1
/2)+b^(1/2)/x^2)^2)^(1/2)/b^(3/4)/(a+b/x^4)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {335, 279, 305, 220, 1196}

$$-\frac{2a^{9/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+\frac{b}{x^4}}}+\frac{4a^{9/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+\frac{b}{x^4}}}-\frac{4a^2}{15\sqrt{b}\,x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^4)^(3/2)/x^4,x]
```

```
[Out] (-2*a*Sqrt[a + b/x^4])/(15*x^3) - (a + b/x^4)^(3/2)/(9*x^3) - (4*a^2*Sqrt[a
 + b/x^4])/(15*Sqrt[b]*(Sqrt[a] + Sqrt[b]/x^2)*x) + (4*a^(9/4)*Sqrt[(a + b/
x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a] + Sqrt[b]/x^2)*EllipticE[2*ArcCot[
(a^(1/4)*x)/b^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a + b/x^4]) - (2*a^(9/4)*Sqrt[
(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a] + Sqrt[b]/x^2)*EllipticF[2*
ArcCot[(a^(1/4)*x)/b^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a + b/x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^4}\,dx = -\text{Subst}\left(\int x^2\left(a+bx^4\right)^{3/2}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{9x^3} - \frac{1}{3}(2a)\,\text{Subst}\left(\int x^2\sqrt{a+bx^4}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2a\sqrt{a+\frac{b}{x^4}}}{15x^3} - \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{9x^3} - \frac{1}{15}\left(4a^2\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2a\sqrt{a+\frac{b}{x^4}}}{15x^3} - \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{9x^3} - \frac{\left(4a^{5/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right)}{15\sqrt{b}} + \frac{\left(4a^{5/2}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\right.}{15\sqrt{b}}$$

$$= -\frac{2a\sqrt{a+\frac{b}{x^4}}}{15x^3} - \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{9x^3} - \frac{4a^2\sqrt{a+\frac{b}{x^4}}}{15\sqrt{b}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)x} + \frac{4a^{9/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\,\text{co}\right.}{15b^{3/4}\sqrt{a+\frac{b}{x^4}}}$$

Mathematica [C] time = 0.03, size = 52, normalized size = 0.20

$$-\frac{b\sqrt{a+\frac{b}{x^4}}\;{}_2F_1\left(-\frac{9}{4},-\frac{3}{2};-\frac{5}{4};-\frac{ax^4}{b}\right)}{9x^7\sqrt{\frac{ax^4}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^4)^(3/2)/x^4,x]
```

```
[Out] -1/9*(b*Sqrt[a + b/x^4]*Hypergeometric2F1[-9/4, -3/2, -5/4, -((a*x^4)/b)])/
(x^7*Sqrt[1 + (a*x^4)/b])
```

fricas [F] time = 1.18, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ax^4+b\right)\sqrt{\frac{ax^4+b}{x^4}}}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral((a*x^4 + b)*sqrt((a*x^4 + b)/x^4)/x^8, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^4)^(3/2)/x^4, x)
```

maple [C] time = 0.02, size = 257, normalized size = 1.00

$$-\frac{\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{3}{2}}\left(12\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;a^3\sqrt{b}\;x^{12}+12i\sqrt{-\frac{i\sqrt{a}\;x^2-\sqrt{b}}{\sqrt{b}}}\;\sqrt{\frac{i\sqrt{a}\;x^2+\sqrt{b}}{\sqrt{b}}}\;a^{\frac{5}{2}}b\,x^9\,\text{EllipticE}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;x,i\right)-12i\sqrt{-\frac{i\sqrt{a}\;x^2-\sqrt{b}}{\sqrt{b}}}\right.}{45\left(a\,x^4+b\right)^2\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^4)^(3/2)/x^4,x)
```

```
[Out] -1/45*((a*x^4+b)/x^4)^(3/2)*(-12*I*a^(5/2)*(-(I*a^(1/2)*x^2-b^(1/2))/b^(1/2
))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*x^9*b*EllipticF((I*a^(1/2)
/b^(1/2))^(1/2)*x,I)+12*I*a^(5/2)*(-(I*a^(1/2)*x^2-b^(1/2))/b^(1/2))^(1/2)*
((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*x^9*b*EllipticE((I*a^(1/2)/b^(1/2))
^(1/2)*x,I)+12*(I*a^(1/2)/b^(1/2))^(1/2)*b^(1/2)*x^12*a^3+23*(I*a^(1/2)/b^(
1/2))^(1/2)*b^(3/2)*x^8*a^2+16*(I*a^(1/2)/b^(1/2))^(1/2)*b^(5/2)*x^4*a+5*(I
*a^(1/2)/b^(1/2))^(1/2)*b^(7/2))/x^3/(a*x^4+b)^2/b^(3/2)/(I*a^(1/2)/b^(1/2)
)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x^4)^(3/2)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^4)^(3/2)/x^4,x)
```

```
[Out] int((a + b/x^4)^(3/2)/x^4, x)
```

sympy [C] time = 1.51, size = 41, normalized size = 0.16

$$-\frac{a^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{2}, \frac{3}{4} \\ \frac{7}{4}\end{matrix}\right| \frac{be^{i\pi}}{ax^4}\right)}{4x^3\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**4)**(3/2)/x**4,x)
```

```
[Out] -a**(3/2)*gamma(3/4)*hyper((-3/2, 3/4), (7/4,), b*exp_polar(I*pi)/(a*x**4))
/(4*x**3*gamma(7/4))
```

3.2074 $\int \left(a+\frac{b}{x^4}\right)^{5/2} x^3\,dx$

Optimal. Leaf size=80

$$\frac{5}{4}a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)+\frac{1}{4}x^4\left(a+\frac{b}{x^4}\right)^{5/2}-\frac{5}{12}b\left(a+\frac{b}{x^4}\right)^{3/2}-\frac{5}{4}ab\sqrt{a+\frac{b}{x^4}}$$

```
[Out] -5/12*b*(a+b/x^4)^(3/2)+1/4*(a+b/x^4)^(5/2)*x^4+5/4*a^(3/2)*b*arctanh((a+b/
x^4)^(1/2)/a^(1/2))-5/4*a*b*(a+b/x^4)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {266, 47, 50, 63, 208}

$$\frac{5}{4}a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)+\frac{1}{4}x^4\left(a+\frac{b}{x^4}\right)^{5/2}-\frac{5}{12}b\left(a+\frac{b}{x^4}\right)^{3/2}-\frac{5}{4}ab\sqrt{a+\frac{b}{x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^4)^(5/2)*x^3,x]
```

```
[Out] (-5*a*b*Sqrt[a + b/x^4])/4 - (5*b*(a + b/x^4)^(3/2))/12 + ((a + b/x^4)^(5/2
)*x^4)/4 + (5*a^(3/2)*b*ArcTanh[Sqrt[a + b/x^4]/Sqrt[a]])/4
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a+\frac{b}{x^4}\right)^{5/2} x^3\,dx &= -\left(\frac{1}{4}\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^2}\,dx, x, \frac{1}{x^4}\right)\right) \\
&= \frac{1}{4}\left(a+\frac{b}{x^4}\right)^{5/2} x^4 - \frac{1}{8}(5b)\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, \frac{1}{x^4}\right) \\
&= -\frac{5}{12}b\left(a+\frac{b}{x^4}\right)^{3/2} + \frac{1}{4}\left(a+\frac{b}{x^4}\right)^{5/2} x^4 - \frac{1}{8}(5ab)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x^4}\right) \\
&= -\frac{5}{4}ab\sqrt{a+\frac{b}{x^4}} - \frac{5}{12}b\left(a+\frac{b}{x^4}\right)^{3/2} + \frac{1}{4}\left(a+\frac{b}{x^4}\right)^{5/2} x^4 - \frac{1}{8}\left(5a^2b\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,d\right. \\
&= -\frac{5}{4}ab\sqrt{a+\frac{b}{x^4}} - \frac{5}{12}b\left(a+\frac{b}{x^4}\right)^{3/2} + \frac{1}{4}\left(a+\frac{b}{x^4}\right)^{5/2} x^4 - \frac{1}{4}\left(5a^2\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x\right. \\
&= -\frac{5}{4}ab\sqrt{a+\frac{b}{x^4}} - \frac{5}{12}b\left(a+\frac{b}{x^4}\right)^{3/2} + \frac{1}{4}\left(a+\frac{b}{x^4}\right)^{5/2} x^4 + \frac{5}{4}a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.68

$$-\frac{b^2\sqrt{a+\frac{b}{x^4}}\;{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};-\frac{ax^4}{b}\right)}{6x^4\sqrt{\frac{ax^4}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^4)^(5/2)*x^3,x]
```

```
[Out] -1/6*(b^2*Sqrt[a + b/x^4]*Hypergeometric2F1[-5/2, -3/2, -1/2, -((a*x^4)/b)]
)/(x^4*Sqrt[1 + (a*x^4)/b])
```

fricas [A] time = 0.57, size = 170, normalized size = 2.12

$$\left[\frac{15\,a^{\frac{3}{2}}bx^4\log\left(-2\,ax^4-2\,\sqrt{a}\,x^4\sqrt{\frac{ax^4+b}{x^4}}-b\right)+2\left(3\,a^2x^8-14\,abx^4-2\,b^2\right)\sqrt{\frac{ax^4+b}{x^4}}}{24\,x^4},\ -\frac{15\,\sqrt{-a}\,abx^4\arctan\left(\frac{\sqrt{-a}}{a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(5/2)*x^3,x, algorithm="fricas")
```

```
[Out] [1/24*(15*a^(3/2)*b*x^4*log(-2*a*x^4 - 2*sqrt(a)*x^4*sqrt((a*x^4 + b)/x^4)
- b) + 2*(3*a^2*x^8 - 14*a*b*x^4 - 2*b^2)*sqrt((a*x^4 + b)/x^4))/x^4, -1/12
*(15*sqrt(-a)*a*b*x^4*arctan(sqrt(-a)*x^4*sqrt((a*x^4 + b)/x^4)/(a*x^4 + b)
) - (3*a^2*x^8 - 14*a*b*x^4 - 2*b^2)*sqrt((a*x^4 + b)/x^4))/x^4]
```

giac [B] time = 0.27, size = 142, normalized size = 1.78

$$\frac{1}{4}\sqrt{ax^4+b}\,a^2x^2-\frac{5}{8}a^{\frac{3}{2}}b\log\left(\left(\sqrt{a}\,x^2-\sqrt{ax^4+b}\right)^2\right)+\frac{9\left(\sqrt{a}\,x^2-\sqrt{ax^4+b}\right)^4a^{\frac{3}{2}}b^2-12\left(\sqrt{a}\,x^2-\sqrt{ax^4+b}\right)^2a^{\frac{3}{2}}b^3}{3\left(\left(\sqrt{a}\,x^2-\sqrt{ax^4+b}\right)^2-b\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(5/2)*x^3,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(a*x^4 + b)*a^2*x^2 - 5/8*a^(3/2)*b*log((sqrt(a)*x^2 - sqrt(a*x^4 +
 b))^2) + 1/3*(9*(sqrt(a)*x^2 - sqrt(a*x^4 + b))^4*a^(3/2)*b^2 - 12*(sqrt(a
)*x^2 - sqrt(a*x^4 + b))^2*a^(3/2)*b^3 + 7*a^(3/2)*b^4)/((sqrt(a)*x^2 - sqr
t(a*x^4 + b))^2 - b)^3
```

maple [A] time = 0.02, size = 103, normalized size = 1.29

$$\frac{\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{5}{2}}\left(3\sqrt{a\,x^4+b}\;a^2x^8+15a^{\frac{3}{2}}b\,x^6\ln\left(\sqrt{a}\;x^2+\sqrt{a\,x^4+b}\right)-14\sqrt{a\,x^4+b}\;ab\,x^4-2\sqrt{a\,x^4+b}\;b^2\right)x^4}{12\left(a\,x^4+b\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^4)^(5/2)*x^3,x)
```

```
[Out] 1/12*((a*x^4+b)/x^4)^(5/2)*x^4*(3*a^2*x^8*(a*x^4+b)^(1/2)+15*a^(3/2)*b*ln(a
^(1/2)*x^2+(a*x^4+b)^(1/2))*x^6-14*a*b*(a*x^4+b)^(1/2)*x^4-2*b^2*(a*x^4+b)^
(1/2))/(a*x^4+b)^(5/2)
```

maxima [A] time = 1.92, size = 81, normalized size = 1.01

$$\frac{1}{4}\sqrt{a+\frac{b}{x^4}}\,a^2x^4-\frac{5}{8}a^{\frac{3}{2}}b\log\left(\frac{\sqrt{a+\frac{b}{x^4}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^4}}+\sqrt{a}}\right)-\frac{1}{6}\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}}b-\sqrt{a+\frac{b}{x^4}}\,ab$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(5/2)*x^3,x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(a + b/x^4)*a^2*x^4 - 5/8*a^(3/2)*b*log((sqrt(a + b/x^4) - sqrt(a))
/(sqrt(a + b/x^4) + sqrt(a))) - 1/6*(a + b/x^4)^(3/2)*b - sqrt(a + b/x^4)*a
*b
```

mupad [B] time = 2.04, size = 66, normalized size = 0.82

$$\frac{a^2x^4\sqrt{a+\frac{b}{x^4}}}{4}-\frac{b\left(a+\frac{b}{x^4}\right)^{3/2}}{6}-a\,b\sqrt{a+\frac{b}{x^4}}-\frac{a^{3/2}\,b\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x^4}}\;1\mathrm{i}}{\sqrt{a}}\right)5\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b/x^4)^(5/2),x)
```

```
[Out] (a^2*x^4*(a + b/x^4)^(1/2))/4 - (b*(a + b/x^4)^(3/2))/6 - (a^(3/2)*b*atan((
(a + b/x^4)^(1/2)*1i)/a^(1/2))*5i)/4 - a*b*(a + b/x^4)^(1/2)
```

sympy [A] time = 4.60, size = 112, normalized size = 1.40

$$\frac{a^{\frac{5}{2}}x^4\sqrt{1+\frac{b}{ax^4}}}{4}-\frac{7a^{\frac{3}{2}}b\sqrt{1+\frac{b}{ax^4}}}{6}-\frac{5a^{\frac{3}{2}}b\log\left(\frac{b}{ax^4}\right)}{8}+\frac{5a^{\frac{3}{2}}b\log\left(\sqrt{1+\frac{b}{ax^4}}+1\right)}{4}-\frac{\sqrt{a}\,b^2\sqrt{1+\frac{b}{ax^4}}}{6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**4)**(5/2)*x**3,x)
```

```
[Out] a**(5/2)*x**4*sqrt(1 + b/(a*x**4))/4 - 7*a**(3/2)*b*sqrt(1 + b/(a*x**4))/6
- 5*a**(3/2)*b*log(b/(a*x**4))/8 + 5*a**(3/2)*b*log(sqrt(1 + b/(a*x**4)) +
1)/4 - sqrt(a)*b**2*sqrt(1 + b/(a*x**4))/(6*x**4)
```

3.2075 $\int \left(a+\frac{b}{x^4}\right)^{5/2} x\,dx$

Optimal. Leaf size=91

$$-\frac{15}{16}a^2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}}{x^2\sqrt{a+\frac{b}{x^4}}}\right)+\frac{1}{2}x^2\left(a+\frac{b}{x^4}\right)^{5/2}-\frac{5b\left(a+\frac{b}{x^4}\right)^{3/2}}{8x^2}-\frac{15ab\sqrt{a+\frac{b}{x^4}}}{16x^2}$$

```
[Out] -5/8*b*(a+b/x^4)^(3/2)/x^2+1/2*(a+b/x^4)^(5/2)*x^2-15/16*a^2*arctanh(b^(1/2
)/x^2/(a+b/x^4)^(1/2))*b^(1/2)-15/16*a*b*(a+b/x^4)^(1/2)/x^2
```

Rubi [A] time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {335, 275, 277, 195, 217, 206}

$$-\frac{15}{16}a^2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}}{x^2\sqrt{a+\frac{b}{x^4}}}\right)+\frac{1}{2}x^2\left(a+\frac{b}{x^4}\right)^{5/2}-\frac{5b\left(a+\frac{b}{x^4}\right)^{3/2}}{8x^2}-\frac{15ab\sqrt{a+\frac{b}{x^4}}}{16x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^4)^(5/2)*x,x]
```

```
[Out] (-15*a*b*Sqrt[a + b/x^4])/(16*x^2) - (5*b*(a + b/x^4)^(3/2))/(8*x^2) + ((a
+ b/x^4)^(5/2)*x^2)/2 - (15*a^2*Sqrt[b]*ArcTanh[Sqrt[b]/(Sqrt[a + b/x^4]*x^
2)])/16
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

```
nomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \left(a+\frac{b}{x^4}\right)^{5/2} x\,dx = -\text{Subst}\left(\int \frac{\left(a+bx^4\right)^{5/2}}{x^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{5/2}}{x^2}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= \frac{1}{2}\left(a+\frac{b}{x^4}\right)^{5/2}x^2 - \frac{1}{2}(5b)\,\text{Subst}\left(\int \left(a+bx^2\right)^{3/2}\,dx, x, \frac{1}{x^2}\right)$$

$$= -\frac{5b\left(a+\frac{b}{x^4}\right)^{3/2}}{8x^2} + \frac{1}{2}\left(a+\frac{b}{x^4}\right)^{5/2}x^2 - \frac{1}{8}(15ab)\,\text{Subst}\left(\int \sqrt{a+bx^2}\,dx, x, \frac{1}{x^2}\right)$$

$$= -\frac{15ab\sqrt{a+\frac{b}{x^4}}}{16x^2} - \frac{5b\left(a+\frac{b}{x^4}\right)^{3/2}}{8x^2} + \frac{1}{2}\left(a+\frac{b}{x^4}\right)^{5/2}x^2 - \frac{1}{16}\left(15a^2b\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx\right.$$

$$= -\frac{15ab\sqrt{a+\frac{b}{x^4}}}{16x^2} - \frac{5b\left(a+\frac{b}{x^4}\right)^{3/2}}{8x^2} + \frac{1}{2}\left(a+\frac{b}{x^4}\right)^{5/2}x^2 - \frac{1}{16}\left(15a^2b\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,\right.$$

$$= -\frac{15ab\sqrt{a+\frac{b}{x^4}}}{16x^2} - \frac{5b\left(a+\frac{b}{x^4}\right)^{3/2}}{8x^2} + \frac{1}{2}\left(a+\frac{b}{x^4}\right)^{5/2}x^2 - \frac{15}{16}a^2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^4}}\,x^2}\right)$$

Mathematica [C] time = 0.02, size = 49, normalized size = 0.54

$$-\frac{a^2x^{10}\left(a+\frac{b}{x^4}\right)^{5/2}\left(ax^4+b\right)\,{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};\frac{ax^4}{b}+1\right)}{14b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^4)^(5/2)*x,x]
```

```
[Out] -1/14*(a^2*(a + b/x^4)^(5/2)*x^10*(b + a*x^4)*Hypergeometric2F1[3, 7/2, 9/2
, 1 + (a*x^4)/b])/b^3
```

fricas [A] time = 0.88, size = 169, normalized size = 1.86

$$\left[\frac{15\,a^2\sqrt{b}\,x^6\log\left(\frac{ax^4-2\sqrt{b}\,x^2\sqrt{\frac{ax^4+b}{x^4}}+2b}{x^4}\right)+2\left(8\,a^2x^8-9\,abx^4-2\,b^2\right)\sqrt{\frac{ax^4+b}{x^4}}}{32\,x^6},\ \frac{15\,a^2\sqrt{-b}\,x^6\arctan\left(\frac{\sqrt{-b}\,x^2\sqrt{\frac{ax^4+b}{x^4}}}{b}\right)}{16}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(5/2)*x,x, algorithm="fricas")
```

```
[Out] [1/32*(15*a^2*sqrt(b)*x^6*log((a*x^4 - 2*sqrt(b)*x^2*sqrt((a*x^4 + b)/x^4)
+ 2*b)/x^4) + 2*(8*a^2*x^8 - 9*a*b*x^4 - 2*b^2)*sqrt((a*x^4 + b)/x^4))/x^6,
 1/16*(15*a^2*sqrt(-b)*x^6*arctan(sqrt(-b)*x^2*sqrt((a*x^4 + b)/x^4)/b) + (
8*a^2*x^8 - 9*a*b*x^4 - 2*b^2)*sqrt((a*x^4 + b)/x^4))/x^6]
```

giac [A] time = 0.19, size = 88, normalized size = 0.97

$$\frac{\frac{15\,a^3 b \arctan\left(\frac{\sqrt{a x^4+b}}{\sqrt{-b}}\right)}{\sqrt{-b}} + 8\sqrt{a x^4+b}\,a^3 - \frac{9\left(a x^4+b\right)^{\frac{3}{2}} a^3 b - 7\sqrt{a x^4+b}\,a^3 b^2}{a^2 x^8}}{16\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(5/2)*x,x, algorithm="giac")
```

```
[Out] 1/16*(15*a^3*b*arctan(sqrt(a*x^4 + b)/sqrt(-b))/sqrt(-b) + 8*sqrt(a*x^4 + b
)*a^3 - (9*(a*x^4 + b)^(3/2)*a^3*b - 7*sqrt(a*x^4 + b)*a^3*b^2)/(a^2*x^8))/
a
```

maple [A] time = 0.02, size = 108, normalized size = 1.19

$$\frac{\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{5}{2}}\left(-15a^2\sqrt{b}\,x^8\ln\left(\frac{2b+2\sqrt{a\,x^4+b}\,\sqrt{b}}{x^2}\right) + 8\sqrt{a\,x^4+b}\,a^2x^8 - 9\sqrt{a\,x^4+b}\,ab\,x^4 - 2\sqrt{a\,x^4+b}\,b^2\right)x^2}{16\left(a\,x^4+b\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^4)^(5/2)*x,x)
```

```
[Out] 1/16*((a*x^4+b)/x^4)^(5/2)*x^2*(-15*a^2*b^(1/2)*ln(2*(b+(a*x^4+b)^(1/2)*b^(
1/2))/x^2)*x^8+8*(a*x^4+b)^(1/2)*a^2*x^8-9*(a*x^4+b)^(1/2)*a*b*x^4-2*(a*x^4
+b)^(1/2)*b^2)/(a*x^4+b)^(5/2)
```

maxima [A] time = 1.96, size = 139, normalized size = 1.53

$$\frac{1}{2}\sqrt{a+\frac{b}{x^4}}\,a^2x^2 + \frac{15}{32}\,a^2\sqrt{b}\,\log\left(\frac{\sqrt{a+\frac{b}{x^4}}\,x^2-\sqrt{b}}{\sqrt{a+\frac{b}{x^4}}\,x^2+\sqrt{b}}\right) - \frac{9\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}}a^2bx^6 - 7\sqrt{a+\frac{b}{x^4}}\,a^2b^2x^2}{16\left(\left(a+\frac{b}{x^4}\right)^2x^8 - 2\left(a+\frac{b}{x^4}\right)bx^4 + b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(5/2)*x,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a + b/x^4)*a^2*x^2 + 15/32*a^2*sqrt(b)*log((sqrt(a + b/x^4)*x^2 -
sqrt(b))/(sqrt(a + b/x^4)*x^2 + sqrt(b))) - 1/16*(9*(a + b/x^4)^(3/2)*a^2*b
*x^6 - 7*sqrt(a + b/x^4)*a^2*b^2*x^2)/((a + b/x^4)^2*x^8 - 2*(a + b/x^4)*b*
x^4 + b^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(a+\frac{b}{x^4}\right)^{5/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b/x^4)^(5/2),x)
```

```
[Out] int(x*(a + b/x^4)^(5/2), x)
```

sympy [**A**] time = 4.97, size = 124, normalized size = 1.36

$$\frac{a^{\frac{5}{2}}x^2}{2\sqrt{1+\frac{b}{ax^4}}}-\frac{a^{\frac{3}{2}}b}{16x^2\sqrt{1+\frac{b}{ax^4}}}-\frac{11\sqrt{a}\,b^2}{16x^6\sqrt{1+\frac{b}{ax^4}}}-\frac{15a^2\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\,x^2}\right)}{16}-\frac{b^3}{8\sqrt{a}\,x^{10}\sqrt{1+\frac{b}{ax^4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**4)**(5/2)*x,x)
```

```
[Out] a**(5/2)*x**2/(2*sqrt(1 + b/(a*x**4))) - a**(3/2)*b/(16*x**2*sqrt(1 + b/(a*
x**4))) - 11*sqrt(a)*b**2/(16*x**6*sqrt(1 + b/(a*x**4))) - 15*a**2*sqrt(b)*
asinh(sqrt(b)/(sqrt(a)*x**2))/16 - b**3/(8*sqrt(a)*x**10*sqrt(1 + b/(a*x**4
)))
```

3.2076 $\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x}\,dx$

Optimal. Leaf size=77

$$\frac{1}{2}a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)-\frac{1}{2}a^2\sqrt{a+\frac{b}{x^4}}-\frac{1}{6}a\left(a+\frac{b}{x^4}\right)^{3/2}-\frac{1}{10}\left(a+\frac{b}{x^4}\right)^{5/2}$$

```
[Out] -1/6*a*(a+b/x^4)^(3/2)-1/10*(a+b/x^4)^(5/2)+1/2*a^(5/2)*arctanh((a+b/x^4)^(
1/2)/a^(1/2))-1/2*a^2*(a+b/x^4)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 50, 63, 208}

$$-\frac{1}{2}a^2\sqrt{a+\frac{b}{x^4}}+\frac{1}{2}a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)-\frac{1}{6}a\left(a+\frac{b}{x^4}\right)^{3/2}-\frac{1}{10}\left(a+\frac{b}{x^4}\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^4)^(5/2)/x,x]
```

```
[Out] -(a^2*Sqrt[a + b/x^4])/2 - (a*(a + b/x^4)^(3/2))/6 - (a + b/x^4)^(5/2)/10 +
 (a^(5/2)*ArcTanh[Sqrt[a + b/x^4]/Sqrt[a]])/2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x}\,dx &= -\left(\frac{1}{4}\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x}\,dx, x, \frac{1}{x^4}\right)\right)\\
&= -\frac{1}{10}\left(a+\frac{b}{x^4}\right)^{5/2} - \frac{1}{4}a\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, \frac{1}{x^4}\right)\\
&= -\frac{1}{6}a\left(a+\frac{b}{x^4}\right)^{3/2} - \frac{1}{10}\left(a+\frac{b}{x^4}\right)^{5/2} - \frac{1}{4}a^2\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x^4}\right)\\
&= -\frac{1}{2}a^2\sqrt{a+\frac{b}{x^4}} - \frac{1}{6}a\left(a+\frac{b}{x^4}\right)^{3/2} - \frac{1}{10}\left(a+\frac{b}{x^4}\right)^{5/2} - \frac{1}{4}a^3\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x^4}\right)\\
&= -\frac{1}{2}a^2\sqrt{a+\frac{b}{x^4}} - \frac{1}{6}a\left(a+\frac{b}{x^4}\right)^{3/2} - \frac{1}{10}\left(a+\frac{b}{x^4}\right)^{5/2} - \frac{a^3\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x^4}}\right)}{2b}\\
&= -\frac{1}{2}a^2\sqrt{a+\frac{b}{x^4}} - \frac{1}{6}a\left(a+\frac{b}{x^4}\right)^{3/2} - \frac{1}{10}\left(a+\frac{b}{x^4}\right)^{5/2} + \frac{1}{2}a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [C] time = 0.04, size = 54, normalized size = 0.70

$$-\frac{b^2\sqrt{a+\frac{b}{x^4}}\;{}_2F_1\left(-\frac{5}{2},-\frac{5}{2};-\frac{3}{2};-\frac{ax^4}{b}\right)}{10x^8\sqrt{\frac{ax^4}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^4)^(5/2)/x,x]
```

```
[Out] -1/10*(b^2*Sqrt[a + b/x^4]*Hypergeometric2F1[-5/2, -5/2, -3/2, -((a*x^4)/b)
])/(x^8*Sqrt[1 + (a*x^4)/b])
```

fricas [A] time = 1.27, size = 169, normalized size = 2.19

$$\left[\frac{15\,a^{\frac{5}{2}}x^8\log\left(-2\,ax^4-2\,\sqrt{a}\,x^4\sqrt{\frac{ax^4+b}{x^4}}-b\right)-2\left(23\,a^2x^8+11\,abx^4+3\,b^2\right)\sqrt{\frac{ax^4+b}{x^4}}}{60\,x^8},-\frac{15\,\sqrt{-a}\,a^2x^8\arctan\left(\frac{\sqrt{-a}\,x}{a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [1/60*(15*a^(5/2)*x^8*log(-2*a*x^4 - 2*sqrt(a)*x^4*sqrt((a*x^4 + b)/x^4) -
b) - 2*(23*a^2*x^8 + 11*a*b*x^4 + 3*b^2)*sqrt((a*x^4 + b)/x^4))/x^8, -1/30*
(15*sqrt(-a)*a^2*x^8*arctan(sqrt(-a)*x^4*sqrt((a*x^4 + b)/x^4)/(a*x^4 + b))
 + (23*a^2*x^8 + 11*a*b*x^4 + 3*b^2)*sqrt((a*x^4 + b)/x^4))/x^8]
```

giac [B] time = 0.28, size = 180, normalized size = 2.34

$$-\frac{1}{4}\,a^{\frac{5}{2}}\log\left(\left(\sqrt{a}\,x^2-\sqrt{ax^4+b}\right)^2\right)+\frac{45\left(\sqrt{a}\,x^2-\sqrt{ax^4+b}\right)^8a^{\frac{5}{2}}b-90\left(\sqrt{a}\,x^2-\sqrt{ax^4+b}\right)^6a^{\frac{5}{2}}b^2+140\left(\sqrt{a}\,x^2-\right.}{15\left(\left(\sqrt{a}\,x^2-\sqrt{ax^4+b}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(5/2)/x,x, algorithm="giac")
```

```
[Out] -1/4*a^(5/2)*log((sqrt(a)*x^2 - sqrt(a*x^4 + b))^2) + 1/15*(45*(sqrt(a)*x^2
 - sqrt(a*x^4 + b))^8*a^(5/2)*b - 90*(sqrt(a)*x^2 - sqrt(a*x^4 + b))^6*a^(5
/2)*b^2 + 140*(sqrt(a)*x^2 - sqrt(a*x^4 + b))^4*a^(5/2)*b^3 - 70*(sqrt(a)*x
^2 - sqrt(a*x^4 + b))^2*a^(5/2)*b^4 + 23*a^(5/2)*b^5)/((sqrt(a)*x^2 - sqrt(
a*x^4 + b))^2 - b)^5
```

maple [A] time = 0.02, size = 99, normalized size = 1.29

$$-\frac{\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{5}{2}}\left(-15a^{\frac{5}{2}}x^{10}\ln\left(\sqrt{a}\,x^2+\sqrt{a\,x^4+b}\right)+23\sqrt{a\,x^4+b}\,a^2x^8+11\sqrt{a\,x^4+b}\,ab\,x^4+3\sqrt{a\,x^4+b}\,b^2\right)}{30\left(a\,x^4+b\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^4)^(5/2)/x,x)
```

```
[Out] -1/30*((a*x^4+b)/x^4)^(5/2)*(-15*a^(5/2)*ln(a^(1/2)*x^2+(a*x^4+b)^(1/2))*x^
10+23*(a*x^4+b)^(1/2)*a^2*x^8+11*(a*x^4+b)^(1/2)*a*b*x^4+3*(a*x^4+b)^(1/2)*
b^2)/(a*x^4+b)^(5/2)
```

maxima [A] time = 2.01, size = 75, normalized size = 0.97

$$-\frac{1}{4}a^{\frac{5}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x^4}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^4}}+\sqrt{a}}\right)-\frac{1}{10}\left(a+\frac{b}{x^4}\right)^{\frac{5}{2}}-\frac{1}{6}\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}}a-\frac{1}{2}\sqrt{a+\frac{b}{x^4}}\,a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] -1/4*a^(5/2)*log((sqrt(a + b/x^4) - sqrt(a))/(sqrt(a + b/x^4) + sqrt(a))) -
 1/10*(a + b/x^4)^(5/2) - 1/6*(a + b/x^4)^(3/2)*a - 1/2*sqrt(a + b/x^4)*a^2
```

mupad [B] time = 1.87, size = 60, normalized size = 0.78

$$-\frac{a\left(a+\frac{b}{x^4}\right)^{3/2}}{6}-\frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{10}-\frac{a^2\sqrt{a+\frac{b}{x^4}}}{2}-\frac{a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x^4}}\,1\mathrm{i}}{\sqrt{a}}\right)1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^4)^(5/2)/x,x)
```

```
[Out] - (a^(5/2)*atan(((a + b/x^4)^(1/2)*1i)/a^(1/2))*1i)/2 - (a*(a + b/x^4)^(3/2
))/6 - (a + b/x^4)^(5/2)/10 - (a^2*(a + b/x^4)^(1/2))/2
```

sympy [A] time = 6.30, size = 107, normalized size = 1.39

$$-\frac{23a^{\frac{5}{2}}\sqrt{1+\frac{b}{ax^4}}}{30}-\frac{a^{\frac{5}{2}}\log\left(\frac{b}{ax^4}\right)}{4}+\frac{a^{\frac{5}{2}}\log\left(\sqrt{1+\frac{b}{ax^4}}+1\right)}{2}-\frac{11a^{\frac{3}{2}}b\sqrt{1+\frac{b}{ax^4}}}{30x^4}-\frac{\sqrt{a}\,b^2\sqrt{1+\frac{b}{ax^4}}}{10x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**4)**(5/2)/x,x)
```

```
[Out] -23*a**(5/2)*sqrt(1 + b/(a*x**4))/30 - a**(5/2)*log(b/(a*x**4))/4 + a**(5/2
)*log(sqrt(1 + b/(a*x**4)) + 1)/2 - 11*a**(3/2)*b*sqrt(1 + b/(a*x**4))/(30*
x**4) - sqrt(a)*b**2*sqrt(1 + b/(a*x**4))/(10*x**8)
```

3.2077 $\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^3}\,dx$

Optimal. Leaf size=92

$$-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}}{x^2\sqrt{a+\frac{b}{x^4}}}\right)}{32\sqrt{b}}-\frac{5a^2\sqrt{a+\frac{b}{x^4}}}{32x^2}-\frac{5a\left(a+\frac{b}{x^4}\right)^{3/2}}{48x^2}-\frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{12x^2}$$

```
[Out] -5/48*a*(a+b/x^4)^(3/2)/x^2-1/12*(a+b/x^4)^(5/2)/x^2-5/32*a^3*arctanh(b^(1/
2)/x^2/(a+b/x^4)^(1/2))/b^(1/2)-5/32*a^2*(a+b/x^4)^(1/2)/x^2
```

Rubi [A] time = 0.07, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {335, 275, 195, 217, 206}

$$-\frac{5a^2\sqrt{a+\frac{b}{x^4}}}{32x^2}-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}}{x^2\sqrt{a+\frac{b}{x^4}}}\right)}{32\sqrt{b}}-\frac{5a\left(a+\frac{b}{x^4}\right)^{3/2}}{48x^2}-\frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{12x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^4)^(5/2)/x^3,x]
```

```
[Out] (-5*a^2*Sqrt[a + b/x^4])/(32*x^2) - (5*a*(a + b/x^4)^(3/2))/(48*x^2) - (a +
 b/x^4)^(5/2)/(12*x^2) - (5*a^3*ArcTanh[Sqrt[b]/(Sqrt[a + b/x^4]*x^2)])/(32
*Sqrt[b])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
```

egerQ[m]

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^3}\,dx &= -\text{Subst}\left(\int x\left(a+bx^4\right)^{5/2}\,dx, x, \frac{1}{x}\right) \\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \left(a+bx^2\right)^{5/2}\,dx, x, \frac{1}{x^2}\right)\right) \\
&= -\frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{12x^2} - \frac{1}{12}(5a)\,\text{Subst}\left(\int \left(a+bx^2\right)^{3/2}\,dx, x, \frac{1}{x^2}\right) \\
&= -\frac{5a\left(a+\frac{b}{x^4}\right)^{3/2}}{48x^2} - \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{12x^2} - \frac{1}{16}\left(5a^2\right)\text{Subst}\left(\int \sqrt{a+bx^2}\,dx, x, \frac{1}{x^2}\right) \\
&= -\frac{5a^2\sqrt{a+\frac{b}{x^4}}}{32x^2} - \frac{5a\left(a+\frac{b}{x^4}\right)^{3/2}}{48x^2} - \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{12x^2} - \frac{1}{32}\left(5a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{x^2}\right) \\
&= -\frac{5a^2\sqrt{a+\frac{b}{x^4}}}{32x^2} - \frac{5a\left(a+\frac{b}{x^4}\right)^{3/2}}{48x^2} - \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{12x^2} - \frac{1}{32}\left(5a^3\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{b}{x^4}}\,x^2}\right) \\
&= -\frac{5a^2\sqrt{a+\frac{b}{x^4}}}{32x^2} - \frac{5a\left(a+\frac{b}{x^4}\right)^{3/2}}{48x^2} - \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{12x^2} - \frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^4}}\,x^2}\right)}{32\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 96, normalized size = 1.04

$$-\frac{\sqrt{a+\frac{b}{x^4}}\left(15a^3x^{12}\sqrt{\frac{ax^4}{b}+1}\,\tanh^{-1}\left(\sqrt{\frac{ax^4}{b}+1}\right)+33a^3x^{12}+59a^2bx^8+34ab^2x^4+8b^3\right)}{96x^{10}\left(ax^4+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^4)^(5/2)/x^3,x]
```

```
[Out] -1/96*(Sqrt[a + b/x^4]*(8*b^3 + 34*a*b^2*x^4 + 59*a^2*b*x^8 + 33*a^3*x^12 +
 15*a^3*x^12*Sqrt[1 + (a*x^4)/b]*ArcTanh[Sqrt[1 + (a*x^4)/b]]))/(x^10*(b +
a*x^4))
```

fricas [A] time = 1.38, size = 182, normalized size = 1.98

$$\left[\frac{15\,a^3\sqrt{b}\,x^{10}\log\left(\frac{ax^4-2\,\sqrt{b}\,x^2\sqrt{\frac{ax^4+b}{x^4}}+2\,b}{x^4}\right)-2\left(33\,a^2bx^8+26\,ab^2x^4+8\,b^3\right)\sqrt{\frac{ax^4+b}{x^4}}}{192\,bx^{10}},\ \frac{15\,a^3\sqrt{-b}\,x^{10}\arctan\left(\frac{\sqrt{-b}\,x^2\sqrt{\frac{ax^4}{x}}\right.}{b}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/192*(15*a^3*sqrt(b)*x^10*log((a*x^4 - 2*sqrt(b)*x^2*sqrt((a*x^4 + b)/x^4
) + 2*b)/x^4) - 2*(33*a^2*b*x^8 + 26*a*b^2*x^4 + 8*b^3)*sqrt((a*x^4 + b)/x^
4))/(b*x^10), 1/96*(15*a^3*sqrt(-b)*x^10*arctan(sqrt(-b)*x^2*sqrt((a*x^4 +
b)/x^4)/b) - (33*a^2*b*x^8 + 26*a*b^2*x^4 + 8*b^3)*sqrt((a*x^4 + b)/x^4))/(
b*x^10)]
```

giac [A] time = 0.17, size = 87, normalized size = 0.95

$$\frac{\frac{15\,a^4\arctan\left(\frac{\sqrt{ax^4+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}-\frac{33\left(ax^4+b\right)^{\frac{5}{2}}a^4-40\left(ax^4+b\right)^{\frac{3}{2}}a^4b+15\sqrt{ax^4+b}\,a^4b^2}{a^3x^{12}}}{96\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/96*(15*a^4*arctan(sqrt(a*x^4 + b)/sqrt(-b))/sqrt(-b) - (33*(a*x^4 + b)^(5
/2)*a^4 - 40*(a*x^4 + b)^(3/2)*a^4*b + 15*sqrt(a*x^4 + b)*a^4*b^2)/(a^3*x^1
2))/a
```

maple [A] time = 0.02, size = 113, normalized size = 1.23

$$-\frac{\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{5}{2}}\left(15a^3x^{12}\ln\left(\frac{2b+2\sqrt{a\,x^4+b}\,\sqrt{b}}{x^2}\right)+33\sqrt{a\,x^4+b}\;a^2\sqrt{b}\;x^8+26\sqrt{a\,x^4+b}\;a\,b^{\frac{3}{2}}x^4+8\sqrt{a\,x^4+b}\;b^{\frac{5}{2}}\right)}{96\left(a\,x^4+b\right)^{\frac{5}{2}}\sqrt{b}\;x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^4)^(5/2)/x^3,x)
```

```
[Out] -1/96*((a*x^4+b)/x^4)^(5/2)/x^2*(15*a^3*ln(2*(b+(a*x^4+b)^(1/2)*b^(1/2))/x^
2)*x^12+33*a^2*(a*x^4+b)^(1/2)*x^8*b^(1/2)+26*a*b^(3/2)*(a*x^4+b)^(1/2)*x^4
+8*b^(5/2)*(a*x^4+b)^(1/2))/(a*x^4+b)^(5/2)/b^(1/2)
```

maxima [B] time = 1.99, size = 158, normalized size = 1.72

$$\frac{5\,a^3\log\left(\frac{\sqrt{a+\frac{b}{x^4}}\,x^2-\sqrt{b}}{\sqrt{a+\frac{b}{x^4}}\,x^2+\sqrt{b}}\right)}{64\sqrt{b}}-\frac{33\left(a+\frac{b}{x^4}\right)^{\frac{5}{2}}a^3x^{10}-40\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}}a^3bx^6+15\sqrt{a+\frac{b}{x^4}}\,a^3b^2x^2}{96\left(\left(a+\frac{b}{x^4}\right)^3x^{12}-3\left(a+\frac{b}{x^4}\right)^2bx^8+3\left(a+\frac{b}{x^4}\right)b^2x^4-b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] 5/64*a^3*log((sqrt(a + b/x^4)*x^2 - sqrt(b))/(sqrt(a + b/x^4)*x^2 + sqrt(b)
))/sqrt(b) - 1/96*(33*(a + b/x^4)^(5/2)*a^3*x^10 - 40*(a + b/x^4)^(3/2)*a^3
*b*x^6 + 15*sqrt(a + b/x^4)*a^3*b^2*x^2)/((a + b/x^4)^3*x^12 - 3*(a + b/x^4
)^2*b*x^8 + 3*(a + b/x^4)*b^2*x^4 - b^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^4)^(5/2)/x^3,x)
```

```
[Out] int((a + b/x^4)^(5/2)/x^3, x)
```

sympy [A] time = 6.23, size = 102, normalized size = 1.11

$$-\frac{11a^{\frac{5}{2}}\sqrt{1+\frac{b}{ax^4}}}{32x^2}-\frac{13a^{\frac{3}{2}}b\sqrt{1+\frac{b}{ax^4}}}{48x^6}-\frac{\sqrt{a}\,b^2\sqrt{1+\frac{b}{ax^4}}}{12x^{10}}-\frac{5a^3\,\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\,x^2}\right)}{32\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**4)**(5/2)/x**3,x)
```

```
[Out] -11*a**(5/2)*sqrt(1 + b/(a*x**4))/(32*x**2) - 13*a**(3/2)*b*sqrt(1 + b/(a*x
**4))/(48*x**6) - sqrt(a)*b**2*sqrt(1 + b/(a*x**4))/(12*x**10) - 5*a**3*asi
nh(sqrt(b)/(sqrt(a)*x**2))/(32*sqrt(b))
```

3.2078 $\int \left(a+\frac{b}{x^4}\right)^{5/2} x^2\,dx$

Optimal. Leaf size=146

$$-\frac{20a^{7/4}b^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21\sqrt{a+\frac{b}{x^4}}}-\frac{10b\left(a+\frac{b}{x^4}\right)^{3/2}}{21x}-\frac{20ab\sqrt{a+\frac{b}{x^4}}}{21x}+\frac{1}{3}x^3\left(a+\frac{b}{x^4}\right)^{5/2}$$

```
[Out] -10/21*b*(a+b/x^4)^(3/2)/x+1/3*(a+b/x^4)^(5/2)*x^3-20/21*a*b*(a+b/x^4)^(1/2
)/x-20/21*a^(7/4)*b^(3/4)*(cos(2*arccot(a^(1/4)*x/b^(1/4)))^2)^(1/2)/cos(2*
arccot(a^(1/4)*x/b^(1/4)))*EllipticF(sin(2*arccot(a^(1/4)*x/b^(1/4))),1/2*2
^(1/2))*(a^(1/2)+b^(1/2)/x^2)*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)/(a+
b/x^4)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {335, 277, 195, 220}

$$-\frac{20a^{7/4}b^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21\sqrt{a+\frac{b}{x^4}}}+\frac{1}{3}x^3\left(a+\frac{b}{x^4}\right)^{5/2}-\frac{10b\left(a+\frac{b}{x^4}\right)^{3/2}}{21x}-\frac{20ab\sqrt{a+\frac{b}{x^4}}}{21x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^4)^(5/2)*x^2,x]
```

```
[Out] (-20*a*b*Sqrt[a + b/x^4])/(21*x) - (10*b*(a + b/x^4)^(3/2))/(21*x) + ((a +
b/x^4)^(5/2)*x^3)/3 - (20*a^(7/4)*b^(3/4)*Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[
b]/x^2)^2]*(Sqrt[a] + Sqrt[b]/x^2)*EllipticF[2*ArcCot[(a^(1/4)*x)/b^(1/4)],
 1/2])/(21*Sqrt[a + b/x^4])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \left(a+\frac{b}{x^4}\right)^{5/2} x^2\,dx = -\text{Subst}\left(\int \frac{\left(a+bx^4\right)^{5/2}}{x^4}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{3}\left(a+\frac{b}{x^4}\right)^{5/2} x^3 - \frac{1}{3}(10b)\,\text{Subst}\left(\int \left(a+bx^4\right)^{3/2}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{10b\left(a+\frac{b}{x^4}\right)^{3/2}}{21x} + \frac{1}{3}\left(a+\frac{b}{x^4}\right)^{5/2} x^3 - \frac{1}{7}(20ab)\,\text{Subst}\left(\int \sqrt{a+bx^4}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{20ab\sqrt{a+\frac{b}{x^4}}}{21x} - \frac{10b\left(a+\frac{b}{x^4}\right)^{3/2}}{21x} + \frac{1}{3}\left(a+\frac{b}{x^4}\right)^{5/2} x^3 - \frac{1}{21}\left(40a^2b\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx\right.$$

$$= -\frac{20ab\sqrt{a+\frac{b}{x^4}}}{21x} - \frac{10b\left(a+\frac{b}{x^4}\right)^{3/2}}{21x} + \frac{1}{3}\left(a+\frac{b}{x^4}\right)^{5/2} x^3 - \frac{20a^{7/4}b^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)}{21\sqrt{a+\frac{b}{x^4}}}$$

Mathematica [C] time = 0.03, size = 54, normalized size = 0.37

$$-\frac{b^2\sqrt{a+\frac{b}{x^4}}\;{}_2F_1\left(-\frac{5}{2},-\frac{7}{4};-\frac{3}{4};-\frac{ax^4}{b}\right)}{7x^5\sqrt{\frac{ax^4}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^4)^(5/2)*x^2,x]
```

```
[Out] -1/7*(b^2*Sqrt[a + b/x^4]*Hypergeometric2F1[-5/2, -7/4, -3/4, -((a*x^4)/b)]
)/(x^5*Sqrt[1 + (a*x^4)/b])
```

fricas [F] time = 1.14, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2x^8+2\,abx^4+b^2\right)\sqrt{\frac{ax^4+b}{x^4}}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(5/2)*x^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*x^8 + 2*a*b*x^4 + b^2)*sqrt((a*x^4 + b)/x^4)/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+\frac{b}{x^4}\right)^{\frac{5}{2}} x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(5/2)*x^2,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^4)^(5/2)*x^2, x)
```

maple [C] time = 0.02, size = 181, normalized size = 1.24

$$\frac{\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{5}{2}}\left(7\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;a^3x^{12}-9\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;a^2b\,x^8+40\sqrt{-\frac{i\sqrt{a}\;x^2-\sqrt{b}}{\sqrt{b}}}\;\sqrt{\frac{i\sqrt{a}\;x^2+\sqrt{b}}{\sqrt{b}}}\;a^2b\,x^7\,\text{EllipticF}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;x,i\right)-19\sqrt{i}$$
$$21\left(a\,x^4+b\right)^3\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^4)^(5/2)*x^2,x)
```

```
[Out] 1/21*((a*x^4+b)/x^4)^(5/2)*x^3*(7*(I*a^(1/2)/b^(1/2))^(1/2)*x^12*a^3+40*a^2
*b*(-(I*a^(1/2)*x^2-b^(1/2))/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2
))^(1/2)*EllipticF((I*a^(1/2)/b^(1/2))^(1/2)*x,I)*x^7-9*(I*a^(1/2)/b^(1/2))
^(1/2)*x^8*a^2*b-19*(I*a^(1/2)/b^(1/2))^(1/2)*x^4*a*b^2-3*(I*a^(1/2)/b^(1/2
))^(1/2)*b^3)/(a*x^4+b)^3/(I*a^(1/2)/b^(1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+\frac{b}{x^4}\right)^{\frac{5}{2}} x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(5/2)*x^2,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x^4)^(5/2)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(a+\frac{b}{x^4}\right)^{5/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b/x^4)^(5/2),x)
```

```
[Out] int(x^2*(a + b/x^4)^(5/2), x)
```

sympy [C] time = 2.24, size = 44, normalized size = 0.30

$$-\frac{a^{\frac{5}{2}}x^3\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{5}{2},-\frac{3}{4}\\ \frac{1}{4}\end{matrix}\right|\frac{be^{i\pi}}{ax^4}\right)}{4\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**4)**(5/2)*x**2,x)
```

```
[Out] -a**(5/2)*x**3*gamma(-3/4)*hyper((-5/2, -3/4), (1/4,), b*exp_polar(I*pi)/(a
*x**4))/(4*gamma(1/4))
```

3.2079 $\int \left(a+\frac{b}{x^4}\right)^{5/2} dx$

Optimal. Leaf size=272

$$-\frac{4a^{9/4}\sqrt[4]{b}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt{a+\frac{b}{x^4}}}+\frac{8a^{9/4}\sqrt[4]{b}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt{a+\frac{b}{x^4}}}-$$

```
[Out] -10/9*b*(a+b/x^4)^(3/2)/x^3+(a+b/x^4)^(5/2)*x-4/3*a*b*(a+b/x^4)^(1/2)/x^3-8
/3*a^2*b^(1/2)*(a+b/x^4)^(1/2)/x/(a^(1/2)+b^(1/2)/x^2)+8/3*a^(9/4)*b^(1/4)*
(cos(2*arccot(a^(1/4)*x/b^(1/4)))^2)^(1/2)/cos(2*arccot(a^(1/4)*x/b^(1/4)))
*EllipticE(sin(2*arccot(a^(1/4)*x/b^(1/4))),1/2*2^(1/2))*(a^(1/2)+b^(1/2)/x
^2)*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)/(a+b/x^4)^(1/2)-4/3*a^(9/4)*b
^(1/4)*(cos(2*arccot(a^(1/4)*x/b^(1/4)))^2)^(1/2)/cos(2*arccot(a^(1/4)*x/b^
(1/4)))*EllipticF(sin(2*arccot(a^(1/4)*x/b^(1/4))),1/2*2^(1/2))*(a^(1/2)+b^
(1/2)/x^2)*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)/(a+b/x^4)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {242, 277, 279, 305, 220, 1196}

$$-\frac{8a^2\sqrt{b}\sqrt{a+\frac{b}{x^4}}}{3x\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)}-\frac{4a^{9/4}\sqrt[4]{b}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt{a+\frac{b}{x^4}}}+\frac{8a^{9/4}\sqrt[4]{b}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\right.}{3\sqrt{a+\frac{b}{x^4}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^4)^(5/2),x]
```

```
[Out] (-4*a*b*Sqrt[a + b/x^4])/(3*x^3) - (10*b*(a + b/x^4)^(3/2))/(9*x^3) - (8*a^
2*Sqrt[b]*Sqrt[a + b/x^4])/(3*(Sqrt[a] + Sqrt[b]/x^2)*x) + (a + b/x^4)^(5/2
)*x + (8*a^(9/4)*b^(1/4)*Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[
a] + Sqrt[b]/x^2)*EllipticE[2*ArcCot[(a^(1/4)*x)/b^(1/4)], 1/2])/(3*Sqrt[a
+ b/x^4]) - (4*a^(9/4)*b^(1/4)*Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*
(Sqrt[a] + Sqrt[b]/x^2)*EllipticF[2*ArcCot[(a^(1/4)*x)/b^(1/4)], 1/2])/(3*S
qrt[a + b/x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

```
nomialQ[a, b, c, n, m, p, x]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \left(a+\frac{b}{x^4}\right)^{5/2} dx = -\text{Subst}\left(\int \frac{\left(a+bx^4\right)^{5/2}}{x^2}\, dx, x, \frac{1}{x}\right)$$

$$= \left(a+\frac{b}{x^4}\right)^{5/2} x - (10b)\,\text{Subst}\left(\int x^2\left(a+bx^4\right)^{3/2}\, dx, x, \frac{1}{x}\right)$$

$$= -\frac{10b\left(a+\frac{b}{x^4}\right)^{3/2}}{9x^3} + \left(a+\frac{b}{x^4}\right)^{5/2} x - \frac{1}{3}(20ab)\,\text{Subst}\left(\int x^2\sqrt{a+bx^4}\, dx, x, \frac{1}{x}\right)$$

$$= -\frac{4ab\sqrt{a+\frac{b}{x^4}}}{3x^3} - \frac{10b\left(a+\frac{b}{x^4}\right)^{3/2}}{9x^3} + \left(a+\frac{b}{x^4}\right)^{5/2} x - \frac{1}{3}\left(8a^2b\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\, dx, x, \frac{1}{x}\right)$$

$$= -\frac{4ab\sqrt{a+\frac{b}{x^4}}}{3x^3} - \frac{10b\left(a+\frac{b}{x^4}\right)^{3/2}}{9x^3} + \left(a+\frac{b}{x^4}\right)^{5/2} x - \frac{1}{3}\left(8a^{5/2}\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\, dx, x\right.$$

$$= -\frac{4ab\sqrt{a+\frac{b}{x^4}}}{3x^3} - \frac{10b\left(a+\frac{b}{x^4}\right)^{3/2}}{9x^3} - \frac{8a^2\sqrt{b}\sqrt{a+\frac{b}{x^4}}}{3\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)x} + \left(a+\frac{b}{x^4}\right)^{5/2} x + \frac{8a^{9/4}\sqrt[4]{b}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)}}}{}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.20

$$-\frac{b^2\sqrt{a+\frac{b}{x^4}}\;{}_2F_1\left(-\frac{5}{2},-\frac{9}{4};-\frac{5}{4};-\frac{ax^4}{b}\right)}{9x^7\sqrt{\frac{ax^4}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^4)^(5/2),x]
```

```
[Out] -1/9*(b^2*Sqrt[a + b/x^4]*Hypergeometric2F1[-5/2, -9/4, -5/4, -((a*x^4)/b)]
)/(x^7*Sqrt[1 + (a*x^4)/b])
```

fricas **[F]** time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2x^8 + 2\,abx^4 + b^2\right)\sqrt{\frac{ax^4+b}{x^4}}}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*x^8 + 2*a*b*x^4 + b^2)*sqrt((a*x^4 + b)/x^4)/x^8, x)
```

giac **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + \frac{b}{x^4}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a + b/x^4)^(5/2), x)
```

maple **[C]** time = 0.02, size = 251, normalized size = 0.92

$$\frac{\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{5}{2}}\left(-15\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;a^3x^{12} - 24i\sqrt{-\frac{i\sqrt{a}\;x^2-\sqrt{b}}{\sqrt{b}}}\;\sqrt{\frac{i\sqrt{a}\;x^2+\sqrt{b}}{\sqrt{b}}}\;a^{\frac{5}{2}}\sqrt{b}\;x^9\,\text{EllipticE}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;x, i\right) + 24i\sqrt{-\frac{i\sqrt{a}\;x^2-\sqrt{b}}{\sqrt{b}}}\;\sqrt{}}{9\left(a\,x^4+b\right)^3\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^4)^(5/2),x)
```

```
[Out] 1/9*((a*x^4+b)/x^4)^(5/2)*x*(24*I*a^(5/2)*b^(1/2)*(-(I*a^(1/2)*x^2-b^(1/2))
/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*x^9*EllipticF((I*a^
(1/2)/b^(1/2))^(1/2)*x,I)-24*I*a^(5/2)*b^(1/2)*(-(I*a^(1/2)*x^2-b^(1/2))/b^
(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*x^9*EllipticE((I*a^(1/
2)/b^(1/2))^(1/2)*x,I)-15*(I*a^(1/2)/b^(1/2))^(1/2)*a^3*x^12-19*(I*a^(1/2)/
b^(1/2))^(1/2)*a^2*b*x^8-5*(I*a^(1/2)/b^(1/2))^(1/2)*a*b^2*x^4-(I*a^(1/2)/b
^(1/2))^(1/2)*b^3)/(a*x^4+b)^3/(I*a^(1/2)/b^(1/2))^(1/2)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + \frac{b}{x^4}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x^4)^(5/2), x)
```

mupad **[B]** time = 1.71, size = 38, normalized size = 0.14

$$-\frac{x\left(a + \frac{b}{x^4}\right)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2}, -\frac{9}{4}; -\frac{5}{4}; -\frac{a\,x^4}{b}\right)}{9\left(\frac{a\,x^4}{b}+1\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^4)^(5/2),x)
```

```
[Out] -(x*(a + b/x^4)^(5/2)*hypergeom([-5/2, -9/4], -5/4, -(a*x^4)/b))/(9*((a*x^4
)/b + 1)^(5/2))
```

sympy [C] time = 1.98, size = 42, normalized size = 0.15

$$-\frac{a^{\frac{5}{2}} x \Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{2}, -\frac{1}{4} \\ \frac{3}{4} \end{matrix} \middle| \frac{b e^{i\pi}}{a x^4}\right)}{4\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**4)**(5/2),x)
```

```
[Out] -a**(5/2)*x*gamma(-1/4)*hyper((-5/2, -1/4), (3/4,), b*exp_polar(I*pi)/(a*x*
**4))/(4*gamma(3/4))
```

3.2080 $\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^2}\,dx$

Optimal. Leaf size=147

$$-\frac{20a^{11/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77\sqrt[4]{b}\sqrt{a+\frac{b}{x^4}}}-\frac{20a^2\sqrt{a+\frac{b}{x^4}}}{77x}-\frac{10a\left(a+\frac{b}{x^4}\right)^{3/2}}{77x}-\frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{11x}$$

```
[Out] -10/77*a*(a+b/x^4)^(3/2)/x-1/11*(a+b/x^4)^(5/2)/x-20/77*a^2*(a+b/x^4)^(1/2)
/x-20/77*a^(11/4)*(cos(2*arccot(a^(1/4)*x/b^(1/4)))^2)^(1/2)/cos(2*arccot(a
^(1/4)*x/b^(1/4)))*EllipticF(sin(2*arccot(a^(1/4)*x/b^(1/4))),1/2*2^(1/2))*
(a^(1/2)+b^(1/2)/x^2)*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)/b^(1/4)/(a+
b/x^4)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {335, 195, 220}

$$-\frac{20a^2\sqrt{a+\frac{b}{x^4}}}{77x}-\frac{20a^{11/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77\sqrt[4]{b}\sqrt{a+\frac{b}{x^4}}}-\frac{10a\left(a+\frac{b}{x^4}\right)^{3/2}}{77x}-\frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{11x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^4)^(5/2)/x^2,x]
```

```
[Out] (-20*a^2*Sqrt[a + b/x^4])/(77*x) - (10*a*(a + b/x^4)^(3/2))/(77*x) - (a + b
/x^4)^(5/2)/(11*x) - (20*a^(11/4)*Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^
2]*(Sqrt[a] + Sqrt[b]/x^2)*EllipticF[2*ArcCot[(a^(1/4)*x)/b^(1/4)], 1/2])/(
77*b^(1/4)*Sqrt[a + b/x^4])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^2}\,dx = -\text{Subst}\left(\int \left(a+bx^4\right)^{5/2}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{11x} - \frac{1}{11}(10a)\,\text{Subst}\left(\int \left(a+bx^4\right)^{3/2}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{10a\left(a+\frac{b}{x^4}\right)^{3/2}}{77x} - \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{11x} - \frac{1}{77}\left(60a^2\right)\text{Subst}\left(\int \sqrt{a+bx^4}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{20a^2\sqrt{a+\frac{b}{x^4}}}{77x} - \frac{10a\left(a+\frac{b}{x^4}\right)^{3/2}}{77x} - \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{11x} - \frac{1}{77}\left(40a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{20a^2\sqrt{a+\frac{b}{x^4}}}{77x} - \frac{10a\left(a+\frac{b}{x^4}\right)^{3/2}}{77x} - \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{11x} - \frac{20a^{11/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.37

$$-\frac{b^2\sqrt{a+\frac{b}{x^4}}\;{}_2F_1\left(-\frac{11}{4}, -\frac{5}{2}; -\frac{7}{4}; -\frac{ax^4}{b}\right)}{11x^9\sqrt{\frac{ax^4}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^4)^(5/2)/x^2,x]
```

```
[Out] -1/11*(b^2*Sqrt[a + b/x^4]*Hypergeometric2F1[-11/4, -5/2, -7/4, -((a*x^4)/b
)])/(x^9*Sqrt[1 + (a*x^4)/b])
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2x^8+2\,abx^4+b^2\right)\sqrt{\frac{ax^4+b}{x^4}}}{x^{10}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*x^8 + 2*a*b*x^4 + b^2)*sqrt((a*x^4 + b)/x^4)/x^10, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{\frac{5}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^4)^(5/2)/x^2, x)
```

maple [C] time = 0.02, size = 180, normalized size = 1.22

$$-\frac{\left(\frac{a x^4+b}{x^4}\right)^{\frac{5}{2}}\left(37\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\ a^3x^{12}-40\sqrt{-\frac{i\sqrt{a}\ x^2-\sqrt{b}}{\sqrt{b}}}\ \sqrt{\frac{i\sqrt{a}\ x^2+\sqrt{b}}{\sqrt{b}}}\ a^3x^{11}\,\text{EllipticF}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\ x, i\right)+61\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\ a^2b\,x^8+31\sqrt{}\right.}{77\left(a\,x^4+b\right)^3\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\ x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^4)^(5/2)/x^2,x)
```

```
[Out] -1/77*((a*x^4+b)/x^4)^(5/2)*(-40*a^3*(-(I*a^(1/2)*x^2-b^(1/2))/b^(1/2))^(1/
2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*EllipticF((I*a^(1/2)/b^(1/2))^(1
/2)*x,I)*x^11+37*(I*a^(1/2)/b^(1/2))^(1/2)*a^3*x^12+61*(I*a^(1/2)/b^(1/2))^
(1/2)*a^2*b*x^8+31*(I*a^(1/2)/b^(1/2))^(1/2)*a*b^2*x^4+7*(I*a^(1/2)/b^(1/2)
)^(1/2)*b^3)/x/(a*x^4+b)^3/(I*a^(1/2)/b^(1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{\frac{5}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x^4)^(5/2)/x^2, x)
```

mupad [B] time = 2.00, size = 39, normalized size = 0.27

$$-\frac{\left(a\,x^4+b\right)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},\frac{1}{4};\ \frac{5}{4};\ -\frac{b}{a\,x^4}\right)}{x\left(\frac{b}{a}+x^4\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^4)^(5/2)/x^2,x)
```

```
[Out] -((b + a*x^4)^(5/2)*hypergeom([-5/2, 1/4], 5/4, -b/(a*x^4)))/(x*(b/a + x^4)
^(5/2))
```

sympy [C] time = 2.21, size = 39, normalized size = 0.27

$$-\frac{a^{\frac{5}{2}}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{5}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\right|\frac{be^{i\pi}}{ax^4}\right)}{4x\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**4)**(5/2)/x**2,x)
```

```
[Out] -a**(5/2)*gamma(1/4)*hyper((-5/2, 1/4), (5/4,), b*exp_polar(I*pi)/(a*x**4))
/(4*x*gamma(5/4))
```

3.2081 $\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^4}\,dx$

Optimal. Leaf size=278

$$-\frac{4a^{13/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{39b^{3/4}\sqrt{a+\frac{b}{x^4}}}+\frac{8a^{13/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{39b^{3/4}\sqrt{a+\frac{b}{x^4}}}-\frac{}{39}$$

```
[Out] -10/117*a*(a+b/x^4)^(3/2)/x^3-1/13*(a+b/x^4)^(5/2)/x^3-4/39*a^2*(a+b/x^4)^(
1/2)/x^3-8/39*a^3*(a+b/x^4)^(1/2)/x/b^(1/2)/(a^(1/2)+b^(1/2)/x^2)+8/39*a^(1
3/4)*(cos(2*arccot(a^(1/4)*x/b^(1/4)))^2)^(1/2)/cos(2*arccot(a^(1/4)*x/b^(1
/4)))*EllipticE(sin(2*arccot(a^(1/4)*x/b^(1/4))),1/2*2^(1/2))*(a^(1/2)+b^(1
/2)/x^2)*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)/b^(3/4)/(a+b/x^4)^(1/2)-
4/39*a^(13/4)*(cos(2*arccot(a^(1/4)*x/b^(1/4)))^2)^(1/2)/cos(2*arccot(a^(1/
4)*x/b^(1/4)))*EllipticF(sin(2*arccot(a^(1/4)*x/b^(1/4))),1/2*2^(1/2))*(a^(
1/2)+b^(1/2)/x^2)*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)/b^(3/4)/(a+b/x^
4)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {335, 279, 305, 220, 1196}

$$-\frac{4a^{13/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{39b^{3/4}\sqrt{a+\frac{b}{x^4}}}+\frac{8a^{13/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{39b^{3/4}\sqrt{a+\frac{b}{x^4}}}-\frac{}{39}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^4)^(5/2)/x^4,x]
```

```
[Out] (-4*a^2*Sqrt[a + b/x^4])/(39*x^3) - (10*a*(a + b/x^4)^(3/2))/(117*x^3) - (a
 + b/x^4)^(5/2)/(13*x^3) - (8*a^3*Sqrt[a + b/x^4])/(39*Sqrt[b]*(Sqrt[a] + S
qrt[b]/x^2)*x) + (8*a^(13/4)*Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(S
qrt[a] + Sqrt[b]/x^2)*EllipticE[2*ArcCot[(a^(1/4)*x)/b^(1/4)], 1/2])/(39*b^
(3/4)*Sqrt[a + b/x^4]) - (4*a^(13/4)*Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^
2)^2]*(Sqrt[a] + Sqrt[b]/x^2)*EllipticF[2*ArcCot[(a^(1/4)*x)/b^(1/4)], 1/2]
)/(39*b^(3/4)*Sqrt[a + b/x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^4}\,dx = -\text{Subst}\left(\int x^2\left(a+bx^4\right)^{5/2}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{13x^3} - \frac{1}{13}(10a)\,\text{Subst}\left(\int x^2\left(a+bx^4\right)^{3/2}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{10a\left(a+\frac{b}{x^4}\right)^{3/2}}{117x^3} - \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{13x^3} - \frac{1}{39}\left(20a^2\right)\text{Subst}\left(\int x^2\sqrt{a+bx^4}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{4a^2\sqrt{a+\frac{b}{x^4}}}{39x^3} - \frac{10a\left(a+\frac{b}{x^4}\right)^{3/2}}{117x^3} - \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{13x^3} - \frac{1}{39}\left(8a^3\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{4a^2\sqrt{a+\frac{b}{x^4}}}{39x^3} - \frac{10a\left(a+\frac{b}{x^4}\right)^{3/2}}{117x^3} - \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{13x^3} - \frac{\left(8a^{7/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right)}{39\sqrt{b}} + \frac{\left(8a^{7/2}\right)\text{S}}{}$$

$$= -\frac{4a^2\sqrt{a+\frac{b}{x^4}}}{39x^3} - \frac{10a\left(a+\frac{b}{x^4}\right)^{3/2}}{117x^3} - \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{13x^3} - \frac{8a^3\sqrt{a+\frac{b}{x^4}}}{39\sqrt{b}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)x} + \frac{8a^{13/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{}\right.}{39b}$$

Mathematica [C] time = 0.01, size = 54, normalized size = 0.19

$$-\frac{b^2\sqrt{a+\frac{b}{x^4}}\;{}_2F_1\left(-\frac{13}{4}, -\frac{5}{2}; -\frac{9}{4}; -\frac{ax^4}{b}\right)}{13x^{11}\sqrt{\frac{ax^4}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^4)^(5/2)/x^4,x]
```

```
[Out] -1/13*(b^2*Sqrt[a + b/x^4]*Hypergeometric2F1[-13/4, -5/2, -9/4, -((a*x^4)/b
)])/(x^11*Sqrt[1 + (a*x^4)/b])
```

fricas [F] time = 1.10, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2x^8+2\,abx^4+b^2\right)\sqrt{\frac{ax^4+b}{x^4}}}{x^{12}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^2*x^8 + 2*a*b*x^4 + b^2)*sqrt((a*x^4 + b)/x^4)/x^12, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{\frac{5}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^4)^(5/2)/x^4, x)
```

maple [C] time = 0.03, size = 279, normalized size = 1.00

$$\frac{\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{5}{2}}\left(-24\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;a^4\sqrt{b}\;x^{16}-24i\sqrt{-\frac{i\sqrt{a}\;x^2-\sqrt{b}}{\sqrt{b}}}\;\sqrt{\frac{i\sqrt{a}\;x^2+\sqrt{b}}{\sqrt{b}}}\;a^{\frac{7}{2}}b\,x^{13}\,\text{EllipticE}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;x,i\right)+24i\sqrt{-\frac{i\sqrt{a}\;x^2-}{\sqrt{b}}}\right.}{117\left(a\,x^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^4)^(5/2)/x^4,x)
```

```
[Out] 1/117*((a*x^4+b)/x^4)^(5/2)*(24*I*a^(7/2)*(-(I*a^(1/2)*x^2-b^(1/2))/b^(1/2)
)^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*x^13*b*EllipticF((I*a^(1/2)
/b^(1/2))^(1/2)*x,I)-24*I*a^(7/2)*(-(I*a^(1/2)*x^2-b^(1/2))/b^(1/2))^(1/2)*
((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*x^13*b*EllipticE((I*a^(1/2)/b^(1/2)
)^(1/2)*x,I)-24*b^(1/2)*(I*a^(1/2)/b^(1/2))^(1/2)*x^16*a^4-55*b^(3/2)*(I*a^
(1/2)/b^(1/2))^(1/2)*x^12*a^3-59*b^(5/2)*(I*a^(1/2)/b^(1/2))^(1/2)*x^8*a^2-
37*b^(7/2)*(I*a^(1/2)/b^(1/2))^(1/2)*x^4*a-9*b^(9/2)*(I*a^(1/2)/b^(1/2))^(1
/2))/x^3/(a*x^4+b)^3/b^(3/2)/(I*a^(1/2)/b^(1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{\frac{5}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^4)^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x^4)^(5/2)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^4)^(5/2)/x^4,x)
```

```
[Out] int((a + b/x^4)^(5/2)/x^4, x)
```

sympy [C] time = 1.89, size = 41, normalized size = 0.15

$$-\frac{a^{\frac{5}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{5}{2}, \frac{3}{4} \\ \frac{7}{4}\end{matrix}\right| \frac{be^{i\pi}}{ax^4}\right)}{4x^3\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**4)**(5/2)/x**4,x)
```

```
[Out] -a**(5/2)*gamma(3/4)*hyper((-5/2, 3/4), (7/4,), b*exp_polar(I*pi)/(a*x**4))
/(4*x**3*gamma(7/4))
```

3.2082 $\int \frac{x^3}{\sqrt{a+\frac{b}{x^4}}}\,dx$

Optimal. Leaf size=50

$$\frac{x^4\sqrt{a+\frac{b}{x^4}}}{4a}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{4a^{3/2}}$$

```
[Out] -1/4*b*arctanh((a+b/x^4)^(1/2)/a^(1/2))/a^(3/2)+1/4*x^4*(a+b/x^4)^(1/2)/a
```

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 51, 63, 208}

$$\frac{x^4\sqrt{a+\frac{b}{x^4}}}{4a}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{4a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b/x^4],x]
```

```
[Out] (Sqrt[a + b/x^4]*x^4)/(4*a) - (b*ArcTanh[Sqrt[a + b/x^4]/Sqrt[a]])/(4*a^(3/
2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x^4}}}\,dx = -\left(\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x^4}\right)\right)$$
$$= \frac{\sqrt{a+\frac{b}{x^4}}\,x^4}{4a} + \frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x^4}\right)}{8a}$$
$$= \frac{\sqrt{a+\frac{b}{x^4}}\,x^4}{4a} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x^4}}\right)}{4a}$$
$$= \frac{\sqrt{a+\frac{b}{x^4}}\,x^4}{4a} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{4a^{3/2}}$$

Mathematica [A] time = 0.04, size = 75, normalized size = 1.50

$$\frac{\sqrt{a}\,x^2\left(ax^4+b\right) - b\sqrt{ax^4+b}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x^2}{\sqrt{ax^4+b}}\right)}{4a^{3/2}x^2\sqrt{a+\frac{b}{x^4}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + b/x^4],x]
```

```
[Out] (Sqrt[a]*x^2*(b + a*x^4) - b*Sqrt[b + a*x^4]*ArcTanh[(Sqrt[a]*x^2)/Sqrt[b +
 a*x^4]])/(4*a^(3/2)*Sqrt[a + b/x^4]*x^2)
```

fricas [A] time = 1.14, size = 126, normalized size = 2.52

$$\left[\frac{2\,ax^4\sqrt{\frac{ax^4+b}{x^4}} + \sqrt{a}\,b\log\left(-2\,ax^4 + 2\,\sqrt{a}\,x^4\sqrt{\frac{ax^4+b}{x^4}} - b\right)}{8\,a^2}, \frac{ax^4\sqrt{\frac{ax^4+b}{x^4}} + \sqrt{-a}\,b\arctan\left(\frac{\sqrt{-a}\,x^4\sqrt{\frac{ax^4+b}{x^4}}}{ax^4+b}\right)}{4\,a^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*a*x^4*sqrt((a*x^4 + b)/x^4) + sqrt(a)*b*log(-2*a*x^4 + 2*sqrt(a)*x^
4*sqrt((a*x^4 + b)/x^4) - b))/a^2, 1/4*(a*x^4*sqrt((a*x^4 + b)/x^4) + sqrt(
-a)*b*arctan(sqrt(-a)*x^4*sqrt((a*x^4 + b)/x^4)/(a*x^4 + b)))/a^2]
```

giac [A] time = 0.20, size = 53, normalized size = 1.06

$$\frac{\sqrt{ax^4+b}\,x^2}{4\,a} + \frac{b\log\left(\left|-\sqrt{a}\,x^2 + \sqrt{ax^4+b}\right|\right)}{4\,a^{\frac{3}{2}}} - \frac{b\log\left(|b|\right)}{8\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x^4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(a*x^4 + b)*x^2/a + 1/4*b*log(abs(-sqrt(a)*x^2 + sqrt(a*x^4 + b)))/
a^(3/2) - 1/8*b*log(abs(b))/a^(3/2)
```

maple [A] time = 0.02, size = 70, normalized size = 1.40

$$\frac{\sqrt{a\,x^4+b}\,\left(\sqrt{a\,x^4+b}\,a^{\frac{3}{2}}x^2-ab\ln\left(\sqrt{a}\,x^2+\sqrt{a\,x^4+b}\right)\right)}{4\sqrt{\frac{a\,x^4+b}{x^4}}\,a^{\frac{5}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b/x^4)^(1/2),x)
```

```
[Out] 1/4*(a*x^4+b)^(1/2)*(x^2*(a*x^4+b)^(1/2)*a^(3/2)-b*ln(a^(1/2)*x^2+(a*x^4+b)
^(1/2))*a)/((a*x^4+b)/x^4)^(1/2)/x^2/a^(5/2)
```

maxima [A] time = 1.96, size = 68, normalized size = 1.36

$$\frac{\sqrt{a+\frac{b}{x^4}}\,b}{4\left(\left(a+\frac{b}{x^4}\right)a-a^2\right)}+\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x^4}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^4}}+\sqrt{a}}\right)}{8\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(a + b/x^4)*b/((a + b/x^4)*a - a^2) + 1/8*b*log((sqrt(a + b/x^4) -
sqrt(a))/(sqrt(a + b/x^4) + sqrt(a)))/a^(3/2)
```

mupad [B] time = 1.57, size = 38, normalized size = 0.76

$$\frac{x^4\sqrt{a+\frac{b}{x^4}}}{4\,a}-\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{4\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b/x^4)^(1/2),x)
```

```
[Out] (x^4*(a + b/x^4)^(1/2))/(4*a) - (b*atanh((a + b/x^4)^(1/2)/a^(1/2)))/(4*a^(
3/2))
```

sympy [A] time = 2.76, size = 46, normalized size = 0.92

$$\frac{\sqrt{b}\,x^2\sqrt{\frac{ax^4}{b}+1}}{4a}-\frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}\,x^2}{\sqrt{b}}\right)}{4a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b/x**4)**(1/2),x)
```

```
[Out] sqrt(b)*x**2*sqrt(a*x**4/b + 1)/(4*a) - b*asinh(sqrt(a)*x**2/sqrt(b))/(4*a*
*(3/2))
```

3.2083 $\int \frac{x}{\sqrt{a+\frac{b}{x^4}}}\,dx$

Optimal. Leaf size=21

$$\frac{x^2\sqrt{a+\frac{b}{x^4}}}{2a}$$

```
[Out] 1/2*x^2*(a+b/x^4)^(1/2)/a
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {264}

$$\frac{x^2\sqrt{a+\frac{b}{x^4}}}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b/x^4],x]
```

```
[Out] (Sqrt[a + b/x^4]*x^2)/(2*a)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+\frac{b}{x^4}}}\,dx = \frac{\sqrt{a+\frac{b}{x^4}}\,x^2}{2a}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{x^2\sqrt{a+\frac{b}{x^4}}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b/x^4],x]
```

```
[Out] (Sqrt[a + b/x^4]*x^2)/(2*a)
```

fricas [A] time = 1.33, size = 21, normalized size = 1.00

$$\frac{x^2\sqrt{\frac{ax^4+b}{x^4}}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*sqrt((a*x^4 + b)/x^4)/a
```

giac [A] time = 0.19, size = 23, normalized size = 1.10

$$\frac{\sqrt{ax^4+b}}{2\,a}-\frac{\sqrt{b}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^4)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(a*x^4 + b)/a - 1/2*sqrt(b)/a
```

maple [A] time = 0.01, size = 29, normalized size = 1.38

$$\frac{a\,x^4+b}{2\sqrt{\frac{a\,x^4+b}{x^4}}\,a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b/x^4)^(1/2),x)
[Out] 1/2*(a*x^4+b)/a/x^2/((a*x^4+b)/x^4)^(1/2)
```

maxima [A] time = 0.87, size = 17, normalized size = 0.81

$$\frac{\sqrt{a+\frac{b}{x^4}}\,x^2}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^4)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(a + b/x^4)*x^2/a
```

mupad [B] time = 1.32, size = 17, normalized size = 0.81

$$\frac{x^2\,\sqrt{a+\frac{b}{x^4}}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b/x^4)^(1/2),x)
[Out] (x^2*(a + b/x^4)^(1/2))/(2*a)
```

sympy [A] time = 0.87, size = 19, normalized size = 0.90

$$\frac{\sqrt{b}\,\sqrt{\frac{ax^4}{b}+1}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x**4)**(1/2),x)
[Out] sqrt(b)*sqrt(a*x**4/b + 1)/(2*a)
```

3.2084 $\int \frac{1}{\sqrt{a+\frac{b}{x^4}}\,x}\,dx$

Optimal. Leaf size=27

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

```
[Out] 1/2*arctanh((a+b/x^4)^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {266, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x^4]*x),x]
```

```
[Out] ArcTanh[Sqrt[a + b/x^4]/Sqrt[a]]/(2*Sqrt[a])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}\,x}\,dx = -\left(\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x^4}\right)\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x^4}}\right)}{2b}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Mathematica [B] time = 0.03, size = 55, normalized size = 2.04

$$\frac{\sqrt{ax^4+b}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x^2}{\sqrt{ax^4+b}}\right)}{2\sqrt{a}\,x^2\sqrt{a+\frac{b}{x^4}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x^4]*x),x]
```

```
[Out] (Sqrt[b + a*x^4]*ArcTanh[(Sqrt[a]*x^2)/Sqrt[b + a*x^4]])/(2*Sqrt[a]*Sqrt[a
+ b/x^4]*x^2)
```

fricas [A] time = 1.12, size = 80, normalized size = 2.96

$$\left[\frac{\log\left(-2\,ax^4 - 2\,\sqrt{a}\,x^4\sqrt{\frac{ax^4+b}{x^4}} - b\right)}{4\,\sqrt{a}}, -\frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}\,x^4\sqrt{\frac{ax^4+b}{x^4}}}{ax^4+b}\right)}{2\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/x^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-2*a*x^4 - 2*sqrt(a)*x^4*sqrt((a*x^4 + b)/x^4) - b)/sqrt(a), -1/2*
sqrt(-a)*arctan(sqrt(-a)*x^4*sqrt((a*x^4 + b)/x^4)/(a*x^4 + b))/a]
```

giac [A] time = 0.21, size = 34, normalized size = 1.26

$$-\frac{\log\left(\left|-\sqrt{a}\,x^2 + \sqrt{ax^4+b}\right|\right)}{2\,\sqrt{a}} + \frac{\log\left(|b|\right)}{4\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/x^4)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(-sqrt(a)*x^2 + sqrt(a*x^4 + b)))/sqrt(a) + 1/4*log(abs(b))/sqr
t(a)
```

maple [B] time = 0.01, size = 49, normalized size = 1.81

$$\frac{\sqrt{a\,x^4+b}\,\ln\left(\sqrt{a}\,x^2 + \sqrt{a\,x^4+b}\right)}{2\sqrt{\frac{a\,x^4+b}{x^4}}\,\sqrt{a}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b/x^4)^(1/2),x)
```

```
[Out] 1/2/((a*x^4+b)/x^4)^(1/2)/x^2*(a*x^4+b)^(1/2)*ln(a^(1/2)*x^2+(a*x^4+b)^(1/2
))/a^(1/2)
```

maxima [A] time = 1.97, size = 37, normalized size = 1.37

$$-\frac{\log\left(\frac{\sqrt{a+\frac{b}{x^4}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^4}}+\sqrt{a}}\right)}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*log((sqrt(a + b/x^4) - sqrt(a))/(sqrt(a + b/x^4) + sqrt(a)))/sqrt(a)
```

mupad [B] time = 1.30, size = 19, normalized size = 0.70

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b/x^4)^(1/2)),x)
```

```
[Out] atanh((a + b/x^4)^(1/2)/a^(1/2))/(2*a^(1/2))
```

sympy [A] time = 1.45, size = 20, normalized size = 0.74

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{a}\,x^2}{\sqrt{b}}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/x**4)**(1/2),x)
```

```
[Out] asinh(sqrt(a)*x**2/sqrt(b))/(2*sqrt(a))
```

3.2085 $\int \frac{1}{\sqrt{a+\frac{b}{x^4}}\,x^3}\,dx$

Optimal. Leaf size=30

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}}{x^2\sqrt{a+\frac{b}{x^4}}}\right)}{2\sqrt{b}}$$

```
[Out] -1/2*arctanh(b^(1/2)/x^2/(a+b/x^4)^(1/2))/b^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {335, 275, 217, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}}{x^2\sqrt{a+\frac{b}{x^4}}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x^4]*x^3),x]
```

```
[Out] -ArcTanh[Sqrt[b]/(Sqrt[a + b/x^4]*x^2)]/(2*Sqrt[b])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+\frac{b}{x^4}}\,x^3}\,dx &= -\operatorname{Subst}\left(\int \frac{x}{\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right)\\ &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{x^2}\right)\right)\\ &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{b}{x^4}}\,x^2}\right)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^4}}\,x^2}\right)}{2\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 52, normalized size = 1.73

$$-\frac{\sqrt{ax^4+b}\,\tanh^{-1}\left(\frac{\sqrt{ax^4+b}}{\sqrt{b}}\right)}{2\sqrt{b}\,x^2\sqrt{a+\frac{b}{x^4}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x^4]*x^3),x]
```

```
[Out] -1/2*(Sqrt[b + a*x^4]*ArcTanh[Sqrt[b + a*x^4]/Sqrt[b]])/(Sqrt[b]*Sqrt[a + b
/x^4]*x^2)
```

fricas [A] time = 1.05, size = 77, normalized size = 2.57

$$\left[\frac{\log\left(\frac{ax^4-2\sqrt{b}\,x^2\sqrt{\frac{ax^4+b}{x^4}}+2b}{x^4}\right)}{4\sqrt{b}}, \frac{\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x^2\sqrt{\frac{ax^4+b}{x^4}}}{b}\right)}{2b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b/x^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log((a*x^4 - 2*sqrt(b)*x^2*sqrt((a*x^4 + b)/x^4) + 2*b)/x^4)/sqrt(b),
1/2*sqrt(-b)*arctan(sqrt(-b)*x^2*sqrt((a*x^4 + b)/x^4)/b)/b]
```

giac [A] time = 0.17, size = 41, normalized size = 1.37

$$\frac{\arctan\left(\frac{\sqrt{ax^4+b}}{\sqrt{-b}}\right)}{2\sqrt{-b}} - \frac{\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)}{2\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b/x^4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*arctan(sqrt(a*x^4 + b)/sqrt(-b))/sqrt(-b) - 1/2*arctan(sqrt(b)/sqrt(-b)
)/sqrt(-b)
```

maple [B] time = 0.01, size = 52, normalized size = 1.73

$$-\frac{\sqrt{a\,x^4+b}\,\ln\left(\frac{2b+2\sqrt{a\,x^4+b}\,\sqrt{b}}{x^2}\right)}{2\sqrt{\frac{a\,x^4+b}{x^4}}\,\sqrt{b}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b/x^4)^(1/2),x)
```

```
[Out] -1/2/((a*x^4+b)/x^4)^(1/2)/x^2*(a*x^4+b)^(1/2)/b^(1/2)*ln(2*(b+(a*x^4+b)^(1
/2)*b^(1/2))/x^2)
```

maxima [B] time = 1.97, size = 45, normalized size = 1.50

$$\frac{\log\left(\frac{\sqrt{a+\frac{b}{x^4}}\,x^2-\sqrt{b}}{\sqrt{a+\frac{b}{x^4}}\,x^2+\sqrt{b}}\right)}{4\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b/x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*log((sqrt(a + b/x^4)*x^2 - sqrt(b))/(sqrt(a + b/x^4)*x^2 + sqrt(b)))/sq
rt(b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{x^4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b/x^4)^(1/2)),x)
```

```
[Out] int(1/(x^3*(a + b/x^4)^(1/2)), x)
```

sympy [A] time = 1.63, size = 22, normalized size = 0.73

$$-\frac{\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\,x^2}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b/x**4)**(1/2),x)
```

```
[Out] -asinh(sqrt(b)/(sqrt(a)*x**2))/(2*sqrt(b))
```

3.2086 $\int \frac{x^2}{\sqrt{a+\frac{b}{x^4}}}\,dx$

Optimal. Leaf size=110

$$\frac{b^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+\frac{b}{x^4}}}+\frac{x^3\sqrt{a+\frac{b}{x^4}}}{3a}$$

```
[Out] 1/3*x^3*(a+b/x^4)^(1/2)/a+1/6*b^(3/4)*(cos(2*arccot(a^(1/4)*x/b^(1/4)))^2)^
(1/2)/cos(2*arccot(a^(1/4)*x/b^(1/4)))*EllipticF(sin(2*arccot(a^(1/4)*x/b^(
1/4))),1/2*2^(1/2))*(a^(1/2)+b^(1/2)/x^2)*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^
2)^(1/2)/a^(5/4)/(a+b/x^4)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {335, 325, 220}

$$\frac{b^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+\frac{b}{x^4}}}+\frac{x^3\sqrt{a+\frac{b}{x^4}}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b/x^4],x]
```

```
[Out] (Sqrt[a + b/x^4]*x^3)/(3*a) + (b^(3/4)*Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/
x^2)^2]*(Sqrt[a] + Sqrt[b]/x^2)*EllipticF[2*ArcCot[(a^(1/4)*x)/b^(1/4)], 1/
2])/(6*a^(5/4)*Sqrt[a + b/x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x^4}}}\,dx = -\text{Subst}\left(\int \frac{1}{x^4\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{\sqrt{a+\frac{b}{x^4}}\,x^3}{3a} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right)}{3a}$$
$$= \frac{\sqrt{a+\frac{b}{x^4}}\,x^3}{3a} + \frac{b^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+\frac{b}{x^4}}}$$

Mathematica [C] time = 0.02, size = 64, normalized size = 0.58

$$\frac{-b\sqrt{\frac{ax^4}{b}+1}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{ax^4}{b}\right)+ax^4+b}{3ax\sqrt{a+\frac{b}{x^4}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b/x^4],x]
```

```
[Out] (b + a*x^4 - b*Sqrt[1 + (a*x^4)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((a*x^
4)/b)])/(3*a*Sqrt[a + b/x^4]*x)
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^6\sqrt{\frac{ax^4+b}{x^4}}}{ax^4+b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^6*sqrt((a*x^4 + b)/x^4)/(a*x^4 + b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x^4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(a + b/x^4), x)
```

maple [C] time = 0.01, size = 124, normalized size = 1.13

$$\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,a\,x^5+\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,bx-\sqrt{-\frac{i\sqrt{a}\,x^2-\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}\,x^2+\sqrt{b}}{\sqrt{b}}}\,b\,\text{EllipticF}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,x, i\right)}{3\sqrt{\frac{a\,x^4+b}{x^4}}\,\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b/x^4)^(1/2),x)
```

```
[Out] 1/3*((I*a^(1/2)/b^(1/2))^(1/2)*a*x^5-b*(-(I*a^(1/2)*x^2-b^(1/2))/b^(1/2))^(
1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*EllipticF((I*a^(1/2)/b^(1/2))^
(1/2)*x,I)+(I*a^(1/2)/b^(1/2))^(1/2)*b*x)/((a*x^4+b)/x^4)^(1/2)/x^2/a/(I*a^
(1/2)/b^(1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x^4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(a + b/x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x^4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b/x^4)^(1/2),x)
```

```
[Out] int(x^2/(a + b/x^4)^(1/2), x)
```

sympy [C] time = 1.30, size = 42, normalized size = 0.38

$$-\frac{x^3\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{4}, \frac{1}{2}\\ \frac{1}{4}\end{matrix}\right| \frac{be^{i\pi}}{ax^4}\right)}{4\sqrt{a}\,\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b/x**4)**(1/2),x)
```

```
[Out] -x**3*gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), b*exp_polar(I*pi)/(a*x**4))/(4
*sqrt(a)*gamma(1/4))
```

3.2087 $\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}\,dx$

Optimal. Leaf size=231

$$-\frac{\sqrt[4]{b}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2a^{3/4}\sqrt{a+\frac{b}{x^4}}}+\frac{\sqrt[4]{b}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+\frac{b}{x^4}}}+\frac{x\sqrt{a+}}{a}$$

```
[Out] x*(a+b/x^4)^(1/2)/a-b^(1/2)*(a+b/x^4)^(1/2)/a/x/(a^(1/2)+b^(1/2)/x^2)+b^(1/
4)*(cos(2*arccot(a^(1/4)*x/b^(1/4)))^2)^(1/2)/cos(2*arccot(a^(1/4)*x/b^(1/4
)))*EllipticE(sin(2*arccot(a^(1/4)*x/b^(1/4))),1/2*2^(1/2))*(a^(1/2)+b^(1/2
)/x^2)*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)/a^(3/4)/(a+b/x^4)^(1/2)-1/
2*b^(1/4)*(cos(2*arccot(a^(1/4)*x/b^(1/4)))^2)^(1/2)/cos(2*arccot(a^(1/4)*x
/b^(1/4)))*EllipticF(sin(2*arccot(a^(1/4)*x/b^(1/4))),1/2*2^(1/2))*(a^(1/2)
+b^(1/2)/x^2)*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)/a^(3/4)/(a+b/x^4)^(
1/2)
```

Rubi [A] time = 0.10, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {242, 325, 305, 220, 1196}

$$-\frac{\sqrt[4]{b}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2a^{3/4}\sqrt{a+\frac{b}{x^4}}}+\frac{\sqrt[4]{b}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+\frac{b}{x^4}}}+\frac{x\sqrt{a+}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b/x^4],x]
```

```
[Out] -((Sqrt[b]*Sqrt[a + b/x^4])/(a*(Sqrt[a] + Sqrt[b]/x^2)*x)) + (Sqrt[a + b/x^
4]*x)/a + (b^(1/4)*Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a] + S
qrt[b]/x^2)*EllipticE[2*ArcCot[(a^(1/4)*x)/b^(1/4)], 1/2])/(a^(3/4)*Sqrt[a
+ b/x^4]) - (b^(1/4)*Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a] +
 Sqrt[b]/x^2)*EllipticF[2*ArcCot[(a^(1/4)*x)/b^(1/4)], 1/2])/(2*a^(3/4)*Sqr
t[a + b/x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}\,dx = -\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt{a+\frac{b}{x^4}}\,x}{a} - \frac{b\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{\sqrt{a+\frac{b}{x^4}}\,x}{a} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right)}{\sqrt{a}} + \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right)}{\sqrt{a}}$$

$$= -\frac{\sqrt{b}\sqrt{a+\frac{b}{x^4}}}{a\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)x} + \frac{\sqrt{a+\frac{b}{x^4}}\,x}{a} + \frac{\sqrt[4]{b}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+\frac{b}{x^4}}} - \frac{\sqrt[4]{b}\sqrt{\frac{a}{\left(\sqrt{a}\right.}}}{}$$

Mathematica [C] time = 0.01, size = 49, normalized size = 0.21

$$\frac{x\sqrt{\frac{ax^4}{b}+1}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{ax^4}{b}\right)}{3\sqrt{a+\frac{b}{x^4}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b/x^4],x]
```

```
[Out] (x*Sqrt[1 + (a*x^4)/b]*Hypergeometric2F1[1/2, 3/4, 7/4, -((a*x^4)/b)])/(3*S
qrt[a + b/x^4])
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^4\sqrt{\frac{ax^4+b}{x^4}}}{ax^4+b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^4*sqrt((a*x^4 + b)/x^4)/(a*x^4 + b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a + b/x^4), x)
```

maple [C] time = 0.01, size = 113, normalized size = 0.49

$$\frac{i\sqrt{-\frac{i\sqrt{a}\,x^2-\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}\,x^2+\sqrt{b}}{\sqrt{b}}}\,\left(-\text{EllipticE}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,x,i\right)+\text{EllipticF}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,x,i\right)\right)\sqrt{b}}{\sqrt{\frac{a\,x^4+b}{x^4}}\,\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,\sqrt{a}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^4)^(1/2),x)
```

```
[Out] I/((a*x^4+b)/x^4)^(1/2)/x^2*b^(1/2)/(I*a^(1/2)/b^(1/2))^(1/2)*(-(I*a^(1/2)*
x^2-b^(1/2))/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)/a^(1/2)
*(EllipticF((I*a^(1/2)/b^(1/2))^(1/2)*x,I)-EllipticE((I*a^(1/2)/b^(1/2))^(1
/2)*x,I))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a + b/x^4), x)
```

mupad [B] time = 1.19, size = 43, normalized size = 0.19

$$\frac{x\,\sqrt{\frac{a\,x^4}{b}+1}\,\sqrt{x^4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\ \frac{7}{4};\ -\frac{a\,x^4}{b}\right)}{3\,\sqrt{a\,x^4+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x^4)^(1/2),x)
```

```
[Out] (x*((a*x^4)/b + 1)^(1/2)*(x^4)^(1/2)*hypergeom([1/2, 3/4], 7/4, -(a*x^4)/b)
)/(3*(b + a*x^4)^(1/2))
```

sympy [C] time = 1.11, size = 41, normalized size = 0.18

$$-\frac{x\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\right|\frac{be^{i\pi}}{ax^4}\right)}{4\sqrt{a}\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**4)**(1/2),x)

[Out] -x*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), b*exp_polar(I*pi)/(a*x**4))/(4*sq
rt(a)*gamma(3/4))
```

3.2088 $\int \frac{1}{\sqrt{a+\frac{b}{x^4}}\,x^2}\,dx$

Optimal. Leaf size=88

$$-\frac{\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{a+\frac{b}{x^4}}}$$

```
[Out] -1/2*(cos(2*arccot(a^(1/4)*x/b^(1/4)))^2)^(1/2)/cos(2*arccot(a^(1/4)*x/b^(1
/4)))*EllipticF(sin(2*arccot(a^(1/4)*x/b^(1/4))),1/2*2^(1/2))*(a^(1/2)+b^(1
/2)/x^2)*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)/a^(1/4)/b^(1/4)/(a+b/x^4
)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {335, 220}

$$-\frac{\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{a+\frac{b}{x^4}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x^4]*x^2),x]
```

```
[Out] -(Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a] + Sqrt[b]/x^2)*Ellip
ticF[2*ArcCot[(a^(1/4)*x)/b^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*Sqrt[a + b/x^4
])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}\,x^2}\,dx = -\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{a+\frac{b}{x^4}}}$$

Mathematica [C] time = 0.06, size = 77, normalized size = 0.88

$$-\frac{i\sqrt{\frac{ax^4}{b}+1}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,x\right)\middle|-1\right)}{x^2\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,\sqrt{a+\frac{b}{x^4}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x^4]*x^2),x]
```

```
[Out] ((-I)*Sqrt[1 + (a*x^4)/b]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[a])/Sqrt[b]]*x],
 -1])/(Sqrt[(I*Sqrt[a])/Sqrt[b]]*Sqrt[a + b/x^4]*x^2)
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2\sqrt{\frac{ax^4+b}{x^4}}}{ax^4+b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b/x^4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^2*sqrt((a*x^4 + b)/x^4)/(a*x^4 + b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b/x^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a + b/x^4)*x^2), x)
```

maple [C] time = 0.01, size = 86, normalized size = 0.98

$$\frac{\sqrt{-\frac{i\sqrt{a}\,x^2-\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}\,x^2+\sqrt{b}}{\sqrt{b}}}\,\text{EllipticF}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,x,i\right)}{\sqrt{\frac{a\,x^4+b}{x^4}}\,\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b/x^4)^(1/2),x)
```

```
[Out] 1/((a*x^4+b)/x^4)^(1/2)/x^2/(I*a^(1/2)/b^(1/2))^(1/2)*(-(I*a^(1/2)*x^2-b^(1
/2))/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*EllipticF((I*a^
(1/2)/b^(1/2))^(1/2)*x,I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b/x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a + b/x^4)*x^2), x)
```

mupad [B] time = 1.26, size = 39, normalized size = 0.44

$$-\frac{\sqrt{\frac{b}{a}+x^4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ -\frac{b}{a\,x^4}\right)}{x\,\sqrt{a\,x^4+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b/x^4)^(1/2)),x)
```

```
[Out] -((b/a + x^4)^(1/2)*hypergeom([1/4, 1/2], 5/4, -b/(a*x^4)))/(x*(b + a*x^4)^
(1/2))
```

sympy [C] time = 1.24, size = 37, normalized size = 0.42

$$-\frac{\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{be^{i\pi}}{ax^4}\right)}{4\sqrt{a}\,x\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b/x**4)**(1/2),x)
```

```
[Out] -gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*exp_polar(I*pi)/(a*x**4))/(4*sqrt(a
)*x*gamma(5/4))
```

3.2089 $\int \frac{1}{\sqrt{a+\frac{b}{x^4}}\,x^4}\,dx$

Optimal. Leaf size=212

$$-\frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+\frac{b}{x^4}}}+\frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+\frac{b}{x^4}}}-\frac{\sqrt{a+}}{\sqrt{b}\,x\left(\sqrt{a}\right.}$$

```
[Out] -(a+b/x^4)^(1/2)/x/b^(1/2)/(a^(1/2)+b^(1/2)/x^2)+a^(1/4)*(cos(2*arccot(a^(1
/4)*x/b^(1/4)))^2)^(1/2)/cos(2*arccot(a^(1/4)*x/b^(1/4)))*EllipticE(sin(2*a
rccot(a^(1/4)*x/b^(1/4))),1/2*2^(1/2))*(a^(1/2)+b^(1/2)/x^2)*((a+b/x^4)/(a^
(1/2)+b^(1/2)/x^2)^2)^(1/2)/b^(3/4)/(a+b/x^4)^(1/2)-1/2*a^(1/4)*(cos(2*arcc
ot(a^(1/4)*x/b^(1/4)))^2)^(1/2)/cos(2*arccot(a^(1/4)*x/b^(1/4)))*EllipticF(
sin(2*arccot(a^(1/4)*x/b^(1/4))),1/2*2^(1/2))*(a^(1/2)+b^(1/2)/x^2)*((a+b/x
^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)/b^(3/4)/(a+b/x^4)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {335, 305, 220, 1196}

$$-\frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+\frac{b}{x^4}}}+\frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+\frac{b}{x^4}}}-\frac{\sqrt{a+}}{\sqrt{b}\,x\left(\sqrt{a}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x^4]*x^4),x]
```

```
[Out] -(Sqrt[a + b/x^4]/(Sqrt[b]*(Sqrt[a] + Sqrt[b]/x^2)*x)) + (a^(1/4)*Sqrt[(a +
 b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a] + Sqrt[b]/x^2)*EllipticE[2*ArcC
ot[(a^(1/4)*x)/b^(1/4)], 1/2])/(b^(3/4)*Sqrt[a + b/x^4]) - (a^(1/4)*Sqrt[(a
 + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a] + Sqrt[b]/x^2)*EllipticF[2*Ar
cCot[(a^(1/4)*x)/b^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b/x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}\,x^4}\,dx = -\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\sqrt{a}\ \text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right)}{\sqrt{b}} + \frac{\sqrt{a}\ \text{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right)}{\sqrt{b}}$$

$$= -\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{b}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)x} + \frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+\frac{b}{x^4}}} - \frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}}$$

Mathematica [C] time = 0.01, size = 49, normalized size = 0.23

$$-\frac{\sqrt{\frac{ax^4}{b}+1}\ {}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{ax^4}{b}\right)}{x^3\sqrt{a+\frac{b}{x^4}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x^4]*x^4),x]
```

```
[Out] -((Sqrt[1 + (a*x^4)/b]*Hypergeometric2F1[-1/4, 1/2, 3/4, -((a*x^4)/b)])/(Sq
rt[a + b/x^4]*x^3))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\frac{ax^4+b}{x^4}}}{ax^4+b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b/x^4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt((a*x^4 + b)/x^4)/(a*x^4 + b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b/x^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a + b/x^4)*x^4), x)
```

maple [C] time = 0.01, size = 199, normalized size = 0.94

$$\frac{-\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\ a\sqrt{b}\ x^4 - i\sqrt{-\frac{i\sqrt{a}\ x^2-\sqrt{b}}{\sqrt{b}}}\ \sqrt{\frac{i\sqrt{a}\ x^2+\sqrt{b}}{\sqrt{b}}}\ \sqrt{a}\ bx\,\text{EllipticE}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\ x, i\right) + i\sqrt{-\frac{i\sqrt{a}\ x^2-\sqrt{b}}{\sqrt{b}}}\ \sqrt{\frac{i\sqrt{a}\ x^2+\sqrt{b}}{\sqrt{b}}}\ \sqrt{a}\ bx}{\sqrt{\frac{a\,x^4+b}{x^4}}\ \sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\ b^{\frac{3}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a+b/x^4)^(1/2),x)
```

```
[Out] (I*a^(1/2)*(-(I*a^(1/2)*x^2-b^(1/2))/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2)
)/b^(1/2))^(1/2)*x*b*EllipticF((I*a^(1/2)/b^(1/2))^(1/2)*x,I)-I*a^(1/2)*(-(
I*a^(1/2)*x^2-b^(1/2))/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/
2)*x*b*EllipticE((I*a^(1/2)/b^(1/2))^(1/2)*x,I)-b^(1/2)*(I*a^(1/2)/b^(1/2))
^(1/2)*x^4*a-b^(3/2)*(I*a^(1/2)/b^(1/2))^(1/2))/((a*x^4+b)/x^4)^(1/2)/x^3/b
^(3/2)/(I*a^(1/2)/b^(1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b/x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a + b/x^4)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{x^4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b/x^4)^(1/2)),x)
```

```
[Out] int(1/(x^4*(a + b/x^4)^(1/2)), x)
```

sympy [C] time = 1.49, size = 39, normalized size = 0.18

$$-\frac{\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{3}{4}\\ \frac{7}{4}\end{matrix}\right| \frac{be^{i\pi}}{ax^4}\right)}{4\sqrt{a}\,x^3\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a+b/x**4)**(1/2),x)
```

```
[Out] -gamma(3/4)*hyper((1/2, 3/4), (7/4,), b*exp_polar(I*pi)/(a*x**4))/(4*sqrt(a
)*x**3*gamma(7/4))
```

3.2090 $\int \frac{x^3}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx$

Optimal. Leaf size=69

$$-\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{4a^{5/2}}+\frac{3b}{4a^2\sqrt{a+\frac{b}{x^4}}}+\frac{x^4}{4a\sqrt{a+\frac{b}{x^4}}}$$

```
[Out] -3/4*b*arctanh((a+b/x^4)^(1/2)/a^(1/2))/a^(5/2)+3/4*b/a^2/(a+b/x^4)^(1/2)+1
/4*x^4/a/(a+b/x^4)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 51, 63, 208}

$$\frac{3x^4\sqrt{a+\frac{b}{x^4}}}{4a^2}-\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{4a^{5/2}}-\frac{x^4}{2a\sqrt{a+\frac{b}{x^4}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b/x^4)^(3/2),x]
```

```
[Out] -x^4/(2*a*Sqrt[a + b/x^4]) + (3*Sqrt[a + b/x^4]*x^4)/(4*a^2) - (3*b*ArcTanh
[Sqrt[a + b/x^4]/Sqrt[a]])/(4*a^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx = -\left(\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx, x, \frac{1}{x^4}\right)\right)$$

$$= -\frac{x^4}{2a\sqrt{a+\frac{b}{x^4}}} - \frac{3\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x^4}\right)}{4a}$$

$$= -\frac{x^4}{2a\sqrt{a+\frac{b}{x^4}}} + \frac{3\sqrt{a+\frac{b}{x^4}}\,x^4}{4a^2} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x^4}\right)}{8a^2}$$

$$= -\frac{x^4}{2a\sqrt{a+\frac{b}{x^4}}} + \frac{3\sqrt{a+\frac{b}{x^4}}\,x^4}{4a^2} + \frac{3\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x^4}}\right)}{4a^2}$$

$$= -\frac{x^4}{2a\sqrt{a+\frac{b}{x^4}}} + \frac{3\sqrt{a+\frac{b}{x^4}}\,x^4}{4a^2} - \frac{3b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{4a^{5/2}}$$

Mathematica [A] time = 0.06, size = 78, normalized size = 1.13

$$\frac{\sqrt{a}\,x^2\left(ax^4+3b\right) - 3b^{3/2}\sqrt{\frac{ax^4}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^2}{\sqrt{b}}\right)}{4a^{5/2}x^2\sqrt{a+\frac{b}{x^4}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b/x^4)^(3/2),x]
```

```
[Out] (Sqrt[a]*x^2*(3*b + a*x^4) - 3*b^(3/2)*Sqrt[1 + (a*x^4)/b]*ArcSinh[(Sqrt[a]
*x^2)/Sqrt[b]])/(4*a^(5/2)*Sqrt[a + b/x^4]*x^2)
```

fricas [A] time = 1.10, size = 192, normalized size = 2.78

$$\left[\frac{3\left(abx^4+b^2\right)\sqrt{a}\,\log\left(-2\,ax^4+2\,\sqrt{a}\,x^4\sqrt{\frac{ax^4+b}{x^4}}-b\right)+2\left(a^2x^8+3\,abx^4\right)\sqrt{\frac{ax^4+b}{x^4}}}{8\left(a^4x^4+a^3b\right)}, \frac{3\left(abx^4+b^2\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}}{}\right.}{4\left(a^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x^4)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*(a*b*x^4 + b^2)*sqrt(a)*log(-2*a*x^4 + 2*sqrt(a)*x^4*sqrt((a*x^4 +
b)/x^4) - b) + 2*(a^2*x^8 + 3*a*b*x^4)*sqrt((a*x^4 + b)/x^4))/(a^4*x^4 + a^
3*b), 1/4*(3*(a*b*x^4 + b^2)*sqrt(-a)*arctan(sqrt(-a)*x^4*sqrt((a*x^4 + b)/
x^4)/(a*x^4 + b)) + (a^2*x^8 + 3*a*b*x^4)*sqrt((a*x^4 + b)/x^4))/(a^4*x^4 +
 a^3*b)]
```

giac [A] time = 0.25, size = 64, normalized size = 0.93

$$\frac{\left(\frac{x^4}{a}+\frac{3\,b}{a^2}\right)x^2}{4\sqrt{a x^4+b}}+\frac{3\,b\log\left(\left|-\sqrt{a}\,x^2+\sqrt{a x^4+b}\right|\right)}{4\,a^{\frac{5}{2}}}-\frac{3\,b\log\left(|b|\right)}{8\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x^4)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(x^4/a + 3*b/a^2)*x^2/sqrt(a*x^4 + b) + 3/4*b*log(abs(-sqrt(a)*x^2 + sq
rt(a*x^4 + b)))/a^(5/2) - 3/8*b*log(abs(b))/a^(5/2)
```

maple [A] time = 0.02, size = 79, normalized size = 1.14

$$\frac{\left(a\,x^4+b\right)\left(a^{\frac{7}{2}}x^6+3a^{\frac{5}{2}}b\,x^2-3\sqrt{a\,x^4+b}\;a^2b\ln\left(\sqrt{a}\;x^2+\sqrt{a\,x^4+b}\right)\right)}{4\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{3}{2}}a^{\frac{9}{2}}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b/x^4)^(3/2),x)
```

```
[Out] 1/4*(a*x^4+b)*(x^6*a^(7/2)+3*x^2*b*a^(5/2)-3*b*ln(a^(1/2)*x^2+(a*x^4+b)^(1/
2))*a^2*(a*x^4+b)^(1/2))/((a*x^4+b)/x^4)^(3/2)/x^6/a^(9/2)
```

maxima [A] time = 1.96, size = 86, normalized size = 1.25

$$\frac{3\left(a+\frac{b}{x^4}\right)b-2\,ab}{4\left(\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}}a^2-\sqrt{a+\frac{b}{x^4}}\,a^3\right)}+\frac{3\,b\log\left(\frac{\sqrt{a+\frac{b}{x^4}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^4}}+\sqrt{a}}\right)}{8\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x^4)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*(a + b/x^4)*b - 2*a*b)/((a + b/x^4)^(3/2)*a^2 - sqrt(a + b/x^4)*a^3)
 + 3/8*b*log((sqrt(a + b/x^4) - sqrt(a))/(sqrt(a + b/x^4) + sqrt(a)))/a^(5/
2)
```

mupad [B] time = 1.76, size = 53, normalized size = 0.77

$$\frac{3\,b}{4\,a^2\sqrt{a+\frac{b}{x^4}}}+\frac{x^4}{4\,a\sqrt{a+\frac{b}{x^4}}}-\frac{3\,b\,\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{4\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b/x^4)^(3/2),x)
```

```
[Out] (3*b)/(4*a^2*(a + b/x^4)^(1/2)) + x^4/(4*a*(a + b/x^4)^(1/2)) - (3*b*atanh(
(a + b/x^4)^(1/2)/a^(1/2)))/(4*a^(5/2))
```

sympy [A] time = 4.01, size = 75, normalized size = 1.09

$$\frac{x^6}{4a\sqrt{b}\sqrt{\frac{ax^4}{b}+1}}+\frac{3\sqrt{b}\,x^2}{4a^2\sqrt{\frac{ax^4}{b}+1}}-\frac{3b\,\operatorname{asinh}\left(\frac{\sqrt{a}\,x^2}{\sqrt{b}}\right)}{4a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b/x**4)**(3/2),x)
```

```
[Out] x**6/(4*a*sqrt(b)*sqrt(a*x**4/b + 1)) + 3*sqrt(b)*x**2/(4*a**2*sqrt(a*x**4/
b + 1)) - 3*b*asinh(sqrt(a)*x**2/sqrt(b))/(4*a**(5/2))
```

3.2091 $\int \frac{x}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx$

Optimal. Leaf size=40

$$\frac{x^2\sqrt{a+\frac{b}{x^4}}}{a^2}-\frac{x^2}{2a\sqrt{a+\frac{b}{x^4}}}$$

```
[Out] -1/2*x^2/a/(a+b/x^4)^(1/2)+x^2*(a+b/x^4)^(1/2)/a^2
```

Rubi [A] time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {273, 264}

$$\frac{x^2\sqrt{a+\frac{b}{x^4}}}{a^2}-\frac{x^2}{2a\sqrt{a+\frac{b}{x^4}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b/x^4)^(3/2),x]
```

```
[Out] -x^2/(2*a*Sqrt[a + b/x^4]) + (Sqrt[a + b/x^4]*x^2)/a^2
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx &= -\frac{x^2}{2a\sqrt{a+\frac{b}{x^4}}}+\frac{2\int \frac{x}{\sqrt{a+\frac{b}{x^4}}}\,dx}{a}\\ &= -\frac{x^2}{2a\sqrt{a+\frac{b}{x^4}}}+\frac{\sqrt{a+\frac{b}{x^4}}\,x^2}{a^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 0.75

$$\frac{ax^4+2b}{2a^2x^2\sqrt{a+\frac{b}{x^4}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b/x^4)^(3/2),x]
[Out] (2*b + a*x^4)/(2*a^2*Sqrt[a + b/x^4]*x^2)
```

fricas [A] time = 0.84, size = 42, normalized size = 1.05

$$\frac{\left(ax^6 + 2\,bx^2\right)\sqrt{\frac{ax^4+b}{x^4}}}{2\left(a^3x^4 + a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^4)^(3/2),x, algorithm="fricas")
[Out] 1/2*(a*x^6 + 2*b*x^2)*sqrt((a*x^4 + b)/x^4)/(a^3*x^4 + a^2*b)
```

giac [A] time = 0.22, size = 42, normalized size = 1.05

$$\frac{\frac{\sqrt{ax^4+b}}{a} + \frac{b}{\sqrt{ax^4+b}\,a}}{2\,a} - \frac{\sqrt{b}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^4)^(3/2),x, algorithm="giac")
[Out] 1/2*(sqrt(a*x^4 + b)/a + b/(sqrt(a*x^4 + b)*a))/a - sqrt(b)/a^2
```

maple [A] time = 0.00, size = 38, normalized size = 0.95

$$\frac{\left(a\,x^4 + b\right)\left(a\,x^4 + 2b\right)}{2\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{3}{2}}\,a^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b/x^4)^(3/2),x)
[Out] 1/2*(a*x^4+b)*(a*x^4+2*b)/a^2/x^6/((a*x^4+b)/x^4)^(3/2)
```

maxima [A] time = 0.82, size = 36, normalized size = 0.90

$$\frac{\sqrt{a + \frac{b}{x^4}}\,x^2}{2\,a^2} + \frac{b}{2\sqrt{a + \frac{b}{x^4}}\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^4)^(3/2),x, algorithm="maxima")
[Out] 1/2*sqrt(a + b/x^4)*x^2/a^2 + 1/2*b/(sqrt(a + b/x^4)*a^2*x^2)
```

mupad [B] time = 1.39, size = 24, normalized size = 0.60

$$\frac{\frac{a\,x^4}{2} + b}{a^2\,x^2\sqrt{a + \frac{b}{x^4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b/x^4)^(3/2),x)
[Out] (b + (a*x^4)/2)/(a^2*x^2*(a + b/x^4)^(1/2))
```

sympy [**A**] time = 1.21, size = 42, normalized size = 1.05

$$\frac{x^4}{2a\sqrt{b}\sqrt{\frac{ax^4}{b}+1}}+\frac{\sqrt{b}}{a^2\sqrt{\frac{ax^4}{b}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x**4)**(3/2),x)
```

```
[Out] x**4/(2*a*sqrt(b)*sqrt(a*x**4/b + 1)) + sqrt(b)/(a**2*sqrt(a*x**4/b + 1))
```

3.2092 $\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2} x}\,dx$

Optimal. Leaf size=46

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{2a^{3/2}} - \frac{1}{2a\sqrt{a+\frac{b}{x^4}}}$$

```
[Out] 1/2*arctanh((a+b/x^4)^(1/2)/a^(1/2))/a^(3/2)-1/2/a/(a+b/x^4)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 51, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{2a^{3/2}} - \frac{1}{2a\sqrt{a+\frac{b}{x^4}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^4)^(3/2)*x),x]
```

```
[Out] -1/(2*a*Sqrt[a + b/x^4]) + ArcTanh[Sqrt[a + b/x^4]/Sqrt[a]]/(2*a^(3/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2} x}\,dx = -\left(\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \frac{1}{x^4}\right)\right)$$

$$= -\frac{1}{2a\sqrt{a+\frac{b}{x^4}}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x^4}\right)}{4a}$$

$$= -\frac{1}{2a\sqrt{a+\frac{b}{x^4}}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x^4}}\right)}{2ab}$$

$$= -\frac{1}{2a\sqrt{a+\frac{b}{x^4}}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{2a^{3/2}}$$

Mathematica [A] time = 0.09, size = 69, normalized size = 1.50

$$\frac{\sqrt{b}\sqrt{\frac{ax^4}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}x^2}{\sqrt{b}}\right) - \sqrt{a}x^2}{2a^{3/2}x^2\sqrt{a+\frac{b}{x^4}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^4)^(3/2)*x),x]
```

```
[Out] (-(Sqrt[a]*x^2) + Sqrt[b]*Sqrt[1 + (a*x^4)/b]*ArcSinh[(Sqrt[a]*x^2)/Sqrt[b]
])/(2*a^(3/2)*Sqrt[a + b/x^4]*x^2)
```

fricas [B] time = 0.82, size = 163, normalized size = 3.54

$$\left[-\frac{2\,ax^4\sqrt{\frac{ax^4+b}{x^4}} - \left(ax^4+b\right)\sqrt{a}\,\log\left(-2\,ax^4 - 2\,\sqrt{a}\,x^4\sqrt{\frac{ax^4+b}{x^4}} - b\right)}{4\left(a^3x^4+a^2b\right)}, -\frac{ax^4\sqrt{\frac{ax^4+b}{x^4}} + \left(ax^4+b\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^4)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*x^4*sqrt((a*x^4 + b)/x^4) - (a*x^4 + b)*sqrt(a)*log(-2*a*x^4 - 2
*sqrt(a)*x^4*sqrt((a*x^4 + b)/x^4) - b))/(a^3*x^4 + a^2*b), -1/2*(a*x^4*sqr
t((a*x^4 + b)/x^4) + (a*x^4 + b)*sqrt(-a)*arctan(sqrt(-a)*x^4*sqrt((a*x^4 +
 b)/x^4)/(a*x^4 + b)))/(a^3*x^4 + a^2*b)]
```

giac [A] time = 0.22, size = 51, normalized size = 1.11

$$-\frac{x^2}{2\sqrt{ax^4+b}\,a} - \frac{\log\left(\left|-\sqrt{a}\,x^2+\sqrt{ax^4+b}\right|\right)}{2\,a^{\frac{3}{2}}} + \frac{\log\left(|b|\right)}{4\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^4)^(3/2)/x,x, algorithm="giac")
```

```
[Out] -1/2*x^2/(sqrt(a*x^4 + b)*a) - 1/2*log(abs(-sqrt(a)*x^2 + sqrt(a*x^4 + b)))
/a^(3/2) + 1/4*log(abs(b))/a^(3/2)
```

maple [A] time = 0.01, size = 67, normalized size = 1.46

$$-\frac{\left(a\,x^4+b\right)\left(a^{\frac{3}{2}}x^2-\sqrt{a\,x^4+b}\;a\ln\left(\sqrt{a}\;x^2+\sqrt{a\,x^4+b}\right)\right)}{2\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{3}{2}}a^{\frac{5}{2}}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^4)^(3/2)/x,x)
```

```
[Out] -1/2*(a*x^4+b)*(x^2*a^(3/2)-ln(a^(1/2)*x^2+(a*x^4+b)^(1/2))*a*(a*x^4+b)^(1/
2))/((a*x^4+b)/x^4)^(3/2)/x^6/a^(5/2)
```

maxima [A] time = 1.93, size = 52, normalized size = 1.13

$$-\frac{\log\left(\frac{\sqrt{a+\frac{b}{x^4}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^4}}+\sqrt{a}}\right)}{4\,a^{\frac{3}{2}}}-\frac{1}{2\sqrt{a+\frac{b}{x^4}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^4)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] -1/4*log((sqrt(a + b/x^4) - sqrt(a))/(sqrt(a + b/x^4) + sqrt(a)))/a^(3/2) -
 1/2/(sqrt(a + b/x^4)*a)
```

mupad [B] time = 1.36, size = 34, normalized size = 0.74

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{2\,a^{3/2}}-\frac{1}{2\,a\sqrt{a+\frac{b}{x^4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b/x^4)^(3/2)),x)
```

```
[Out] atanh((a + b/x^4)^(1/2)/a^(1/2))/(2*a^(3/2)) - 1/(2*a*(a + b/x^4)^(1/2))
```

sympy [B] time = 2.33, size = 187, normalized size = 4.07

$$-\frac{2a^3x^4\sqrt{1+\frac{b}{ax^4}}}{4a^{\frac{9}{2}}x^4+4a^{\frac{7}{2}}b}-\frac{a^3x^4\log\left(\frac{b}{ax^4}\right)}{4a^{\frac{9}{2}}x^4+4a^{\frac{7}{2}}b}+\frac{2a^3x^4\log\left(\sqrt{1+\frac{b}{ax^4}}+1\right)}{4a^{\frac{9}{2}}x^4+4a^{\frac{7}{2}}b}-\frac{a^2b\log\left(\frac{b}{ax^4}\right)}{4a^{\frac{9}{2}}x^4+4a^{\frac{7}{2}}b}+\frac{2a^2b\log\left(\sqrt{1+\frac{b}{ax^4}}+1\right)}{4a^{\frac{9}{2}}x^4+4a^{\frac{7}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**4)**(3/2)/x,x)
```

```
[Out] -2*a**3*x**4*sqrt(1 + b/(a*x**4))/(4*a**(9/2)*x**4 + 4*a**(7/2)*b) - a**3*x
**4*log(b/(a*x**4))/(4*a**(9/2)*x**4 + 4*a**(7/2)*b) + 2*a**3*x**4*log(sqrt
(1 + b/(a*x**4)) + 1)/(4*a**(9/2)*x**4 + 4*a**(7/2)*b) - a**2*b*log(b/(a*x*
*4))/(4*a**(9/2)*x**4 + 4*a**(7/2)*b) + 2*a**2*b*log(sqrt(1 + b/(a*x**4)) +
 1)/(4*a**(9/2)*x**4 + 4*a**(7/2)*b)
```

3.2093 $\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2} x^3}\,dx$

Optimal. Leaf size=21

$$-\frac{1}{2ax^2\sqrt{a+\frac{b}{x^4}}}$$

```
[Out] -1/2/a/x^2/(a+b/x^4)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {264}

$$-\frac{1}{2ax^2\sqrt{a+\frac{b}{x^4}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^4)^(3/2)*x^3),x]
```

```
[Out] -1/(2*a*Sqrt[a + b/x^4]*x^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2} x^3}\,dx = -\frac{1}{2a\sqrt{a+\frac{b}{x^4}}\,x^2}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$-\frac{1}{2ax^2\sqrt{a+\frac{b}{x^4}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^4)^(3/2)*x^3),x]
```

```
[Out] -1/2*1/(a*Sqrt[a + b/x^4]*x^2)
```

fricas [A] time = 0.84, size = 31, normalized size = 1.48

$$-\frac{x^2\sqrt{\frac{ax^4+b}{x^4}}}{2\left(a^2x^4+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^4)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*x^2*sqrt((a*x^4 + b)/x^4)/(a^2*x^4 + a*b)
```

giac [A] time = 0.17, size = 23, normalized size = 1.10

$$-\frac{1}{2\sqrt{ax^4+b}\,a}+\frac{1}{2a\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^4)^(3/2)/x^3,x, algorithm="giac")
[Out] -1/2/(sqrt(a*x^4 + b)*a) + 1/2/(a*sqrt(b))
```

maple [A] time = 0.00, size = 29, normalized size = 1.38

$$-\frac{a\,x^4+b}{2\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{3}{2}}a\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^4)^(3/2)/x^3,x)
[Out] -1/2*(a*x^4+b)/a/x^6/((a*x^4+b)/x^4)^(3/2)
```

maxima [A] time = 0.88, size = 17, normalized size = 0.81

$$-\frac{1}{2\sqrt{a+\frac{b}{x^4}}\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^4)^(3/2)/x^3,x, algorithm="maxima")
[Out] -1/2/(sqrt(a + b/x^4)*a*x^2)
```

mupad [B] time = 1.22, size = 26, normalized size = 1.24

$$-\frac{x^2\sqrt{a+\frac{b}{x^4}}}{2a\left(a\,x^4+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b/x^4)^(3/2)),x)
[Out] -(x^2*(a + b/x^4)^(1/2))/(2*a*(b + a*x^4))
```

sympy [A] time = 1.40, size = 22, normalized size = 1.05

$$-\frac{1}{2a\sqrt{b}\sqrt{\frac{ax^4}{b}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**4)**(3/2)/x**3,x)
[Out] -1/(2*a*sqrt(b)*sqrt(a*x**4/b + 1))
```

3.2094 $\int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx$

Optimal. Leaf size=131

$$\frac{5b^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{12a^{9/4}\sqrt{a+\frac{b}{x^4}}}+\frac{5x^3\sqrt{a+\frac{b}{x^4}}}{6a^2}-\frac{x^3}{2a\sqrt{a+\frac{b}{x^4}}}$$

```
[Out] -1/2*x^3/a/(a+b/x^4)^(1/2)+5/6*x^3*(a+b/x^4)^(1/2)/a^2+5/12*b^(3/4)*(cos(2*
arccot(a^(1/4)*x/b^(1/4)))^2)^(1/2)/cos(2*arccot(a^(1/4)*x/b^(1/4)))*Ellipt
icF(sin(2*arccot(a^(1/4)*x/b^(1/4))),1/2*2^(1/2))*(a^(1/2)+b^(1/2)/x^2)*((a
+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)/a^(9/4)/(a+b/x^4)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {335, 290, 325, 220}

$$\frac{5b^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{12a^{9/4}\sqrt{a+\frac{b}{x^4}}}+\frac{5x^3\sqrt{a+\frac{b}{x^4}}}{6a^2}-\frac{x^3}{2a\sqrt{a+\frac{b}{x^4}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b/x^4)^(3/2),x]
```

```
[Out] -x^3/(2*a*Sqrt[a + b/x^4]) + (5*Sqrt[a + b/x^4]*x^3)/(6*a^2) + (5*b^(3/4)*S
qrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a] + Sqrt[b]/x^2)*Elliptic
F[2*ArcCot[(a^(1/4)*x)/b^(1/4)], 1/2])/(12*a^(9/4)*Sqrt[a + b/x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
```

egerQ[m]

Rubi steps

$$\int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx = -\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^4\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{x^3}{2a\sqrt{a+\frac{b}{x^4}}} - \frac{5\,\text{Subst}\left(\int \frac{1}{x^4\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{x^3}{2a\sqrt{a+\frac{b}{x^4}}} + \frac{5\sqrt{a+\frac{b}{x^4}}\,x^3}{6a^2} + \frac{(5b)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right)}{6a^2}$$

$$= -\frac{x^3}{2a\sqrt{a+\frac{b}{x^4}}} + \frac{5\sqrt{a+\frac{b}{x^4}}\,x^3}{6a^2} + \frac{5b^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{12a^{9/4}\sqrt{a+\frac{b}{x^4}}}$$

Mathematica [C] time = 0.02, size = 67, normalized size = 0.51

$$\frac{-5b\sqrt{\frac{ax^4}{b}+1}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{ax^4}{b}\right)+2ax^4+5b}{6a^2x\sqrt{a+\frac{b}{x^4}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b/x^4)^(3/2),x]
```

```
[Out] (5*b + 2*a*x^4 - 5*b*Sqrt[1 + (a*x^4)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -
((a*x^4)/b)])/(6*a^2*Sqrt[a + b/x^4]*x)
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^{10}\sqrt{\frac{ax^4+b}{x^4}}}{a^2x^8+2\,abx^4+b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^4)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x^10*sqrt((a*x^4 + b)/x^4)/(a^2*x^8 + 2*a*b*x^4 + b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(a + b/x^4)^(3/2), x)
```

maple [C] time = 0.02, size = 133, normalized size = 1.02

$$\frac{\left(a\,x^4+b\right)\left(2\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;a\,x^5+5\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;bx-5\sqrt{-\frac{i\sqrt{a}\;x^2-\sqrt{b}}{\sqrt{b}}}\;\sqrt{\frac{i\sqrt{a}\;x^2+\sqrt{b}}{\sqrt{b}}}\;b\,\text{EllipticF}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;x,i\right)\right)}{6\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{3}{2}}\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;a^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b/x^4)^(3/2),x)
```

```
[Out] 1/6*(a*x^4+b)*(2*(I*a^(1/2)/b^(1/2))^(1/2)*a*x^5-5*b*(-(I*a^(1/2)*x^2-b^(1/
2))/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*EllipticF((I*a^(
1/2)/b^(1/2))^(1/2)*x,I)+5*(I*a^(1/2)/b^(1/2))^(1/2)*b*x)/((a*x^4+b)/x^4)^(
3/2)/x^6/a^2/(I*a^(1/2)/b^(1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(a + b/x^4)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b/x^4)^(3/2),x)
```

```
[Out] int(x^2/(a + b/x^4)^(3/2), x)
```

sympy [C] time = 1.29, size = 42, normalized size = 0.32

$$-\frac{x^3\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{4},\frac{3}{2}\\ \frac{1}{4}\end{matrix}\right|\frac{be^{i\pi}}{ax^4}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b/x**4)**(3/2),x)
```

```
[Out] -x**3*gamma(-3/4)*hyper((-3/4, 3/2), (1/4,), b*exp_polar(I*pi)/(a*x**4))/(4
*a**(3/2)*gamma(1/4))
```

3.2095 $\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx$

Optimal. Leaf size=258

$$-\frac{3\sqrt[4]{b}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{4a^{7/4}\sqrt{a+\frac{b}{x^4}}}+\frac{3\sqrt[4]{b}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2a^{7/4}\sqrt{a+\frac{b}{x^4}}}+\frac{3x\sqrt{a-}}{2a^2}$$

```
[Out] -1/2*x/a/(a+b/x^4)^(1/2)+3/2*x*(a+b/x^4)^(1/2)/a^2-3/2*b^(1/2)*(a+b/x^4)^(1
/2)/a^2/x/(a^(1/2)+b^(1/2)/x^2)+3/2*b^(1/4)*(cos(2*arccot(a^(1/4)*x/b^(1/4)
))^2)^(1/2)/cos(2*arccot(a^(1/4)*x/b^(1/4)))*EllipticE(sin(2*arccot(a^(1/4)
*x/b^(1/4))),1/2*2^(1/2))*(a^(1/2)+b^(1/2)/x^2)*((a+b/x^4)/(a^(1/2)+b^(1/2)
/x^2)^2)^(1/2)/a^(7/4)/(a+b/x^4)^(1/2)-3/4*b^(1/4)*(cos(2*arccot(a^(1/4)*x/
b^(1/4)))^2)^(1/2)/cos(2*arccot(a^(1/4)*x/b^(1/4)))*EllipticF(sin(2*arccot(
a^(1/4)*x/b^(1/4))),1/2*2^(1/2))*(a^(1/2)+b^(1/2)/x^2)*((a+b/x^4)/(a^(1/2)+
b^(1/2)/x^2)^2)^(1/2)/a^(7/4)/(a+b/x^4)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {242, 290, 325, 305, 220, 1196}

$$\frac{3x\sqrt{a+\frac{b}{x^4}}}{2a^2}-\frac{3\sqrt{b}\sqrt{a+\frac{b}{x^4}}}{2a^2x\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)}-\frac{3\sqrt[4]{b}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{4a^{7/4}\sqrt{a+\frac{b}{x^4}}}+\frac{3\sqrt[4]{b}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)}{2a^{7/4}\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^4)^(-3/2),x]
```

```
[Out] (-3*Sqrt[b]*Sqrt[a + b/x^4])/(2*a^2*(Sqrt[a] + Sqrt[b]/x^2)*x) - x/(2*a*Sqr
t[a + b/x^4]) + (3*Sqrt[a + b/x^4]*x)/(2*a^2) + (3*b^(1/4)*Sqrt[(a + b/x^4)
/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a] + Sqrt[b]/x^2)*EllipticE[2*ArcCot[(a^(
1/4)*x)/b^(1/4)], 1/2])/(2*a^(7/4)*Sqrt[a + b/x^4]) - (3*b^(1/4)*Sqrt[(a +
b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a] + Sqrt[b]/x^2)*EllipticF[2*ArcCo
t[(a^(1/4)*x)/b^(1/4)], 1/2])/(4*a^(7/4)*Sqrt[a + b/x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx = -\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^4\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{x}{2a\sqrt{a+\frac{b}{x^4}}} - \frac{3\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{x}{2a\sqrt{a+\frac{b}{x^4}}} + \frac{3\sqrt{a+\frac{b}{x^4}}\,x}{2a^2} - \frac{(3b)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right)}{2a^2}$$

$$= -\frac{x}{2a\sqrt{a+\frac{b}{x^4}}} + \frac{3\sqrt{a+\frac{b}{x^4}}\,x}{2a^2} - \frac{\left(3\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right)}{2a^{3/2}} + \frac{\left(3\sqrt{b}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\right.}{2a^{3/2}}$$

$$= -\frac{3\sqrt{b}\sqrt{a+\frac{b}{x^4}}}{2a^2\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)x} - \frac{x}{2a\sqrt{a+\frac{b}{x^4}}} + \frac{3\sqrt{a+\frac{b}{x^4}}\,x}{2a^2} + \frac{3\sqrt[4]{b}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\cot^{-}\right.}{2a^{7/4}\sqrt{a+\frac{b}{x^4}}}$$

Mathematica [C] time = 0.03, size = 53, normalized size = 0.21

$$\frac{x - x\sqrt{\frac{ax^4}{b}+1}\;{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{ax^4}{b}\right)}{a\sqrt{a+\frac{b}{x^4}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^4)^(-3/2),x]
```

```
[Out] (x - x*Sqrt[1 + (a*x^4)/b]*Hypergeometric2F1[3/4, 3/2, 7/4, -((a*x^4)/b)])/
(a*Sqrt[a + b/x^4])
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^8\sqrt{\frac{ax^4+b}{x^4}}}{a^2x^8+2\,abx^4+b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^4)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x^8*sqrt((a*x^4 + b)/x^4)/(a^2*x^8 + 2*a*b*x^4 + b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a + b/x^4)^(-3/2), x)
```

maple [C] time = 0.01, size = 187, normalized size = 0.72

$$-\frac{\left(a\,x^4+b\right)\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;a^{\frac{3}{2}}x^3+3i\sqrt{-\frac{i\sqrt{a}\;x^2-\sqrt{b}}{\sqrt{b}}}\;\sqrt{\frac{i\sqrt{a}\;x^2+\sqrt{b}}{\sqrt{b}}}\;a\sqrt{b}\;\text{EllipticE}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;x, i\right)-3i\sqrt{-\frac{i\sqrt{a}\;x^2-\sqrt{b}}{\sqrt{b}}}\;\sqrt{\frac{i\sqrt{a}\;x^2+\sqrt{}}{\sqrt{b}}}\right.}{2\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{3}{2}}\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;a^{\frac{5}{2}}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^4)^(3/2),x)
```

```
[Out] -1/2*(a*x^4+b)*(x^3*a^(3/2)*(I*a^(1/2)/b^(1/2))^(1/2)-3*I*b^(1/2)*(-(I*a^(1
/2)*x^2-b^(1/2))/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*a*E
llipticF((I*a^(1/2)/b^(1/2))^(1/2)*x,I)+3*I*b^(1/2)*(-(I*a^(1/2)*x^2-b^(1/2
))/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*a*EllipticE((I*a^
(1/2)/b^(1/2))^(1/2)*x,I))/((a*x^4+b)/x^4)^(3/2)/x^6/a^(5/2)/(I*a^(1/2)/b^(
1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x^4)^(-3/2), x)
```

mupad [B] time = 1.23, size = 43, normalized size = 0.17

$$\frac{x\left(\frac{a\,x^4}{b}+1\right)^{3/2}\sqrt{x^{12}}\;{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{7}{4};\;\frac{11}{4};\;-\frac{a\,x^4}{b}\right)}{7\left(a\,x^4+b\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x^4)^(3/2),x)
```

```
[Out] (x*((a*x^4)/b + 1)^(3/2)*(x^12)^(1/2)*hypergeom([3/2, 7/4], 11/4, -(a*x^4)/
b))/(7*(b + a*x^4)^(3/2))
```

sympy [C] time = 1.30, size = 41, normalized size = 0.16

$$-\frac{x\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{4}, \frac{3}{2} \\ \frac{3}{4}\end{matrix}\right| \frac{be^{i\pi}}{ax^4}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**4)**(3/2),x)
```

```
[Out] -x*gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), b*exp_polar(I*pi)/(a*x**4))/(4*a*
*(3/2)*gamma(3/4))
```

3.2096 $\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2} x^2}\,dx$

Optimal. Leaf size=110

$$-\frac{\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{b}\sqrt{a+\frac{b}{x^4}}}-\frac{1}{2ax\sqrt{a+\frac{b}{x^4}}}$$

```
[Out] -1/2/a/x/(a+b/x^4)^(1/2)-1/4*(cos(2*arccot(a^(1/4)*x/b^(1/4)))^2)^(1/2)/cos
(2*arccot(a^(1/4)*x/b^(1/4)))*EllipticF(sin(2*arccot(a^(1/4)*x/b^(1/4))),1/
2*2^(1/2))*(a^(1/2)+b^(1/2)/x^2)*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)/
a^(5/4)/b^(1/4)/(a+b/x^4)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {335, 199, 220}

$$-\frac{\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{b}\sqrt{a+\frac{b}{x^4}}}-\frac{1}{2ax\sqrt{a+\frac{b}{x^4}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^4)^(3/2)*x^2),x]
```

```
[Out] -1/(2*a*Sqrt[a + b/x^4]*x) - (Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(
Sqrt[a] + Sqrt[b]/x^2)*EllipticF[2*ArcCot[(a^(1/4)*x)/b^(1/4)], 1/2])/(4*a^
(5/4)*b^(1/4)*Sqrt[a + b/x^4])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2} x^2}\,dx = -\text{Subst}\left(\int \frac{1}{\left(a+bx^4\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{2a\sqrt{a+\frac{b}{x^4}}\,x} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{1}{2a\sqrt{a+\frac{b}{x^4}}\,x} - \frac{\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{b}\sqrt{a+\frac{b}{x^4}}}$$

Mathematica [C] time = 0.02, size = 57, normalized size = 0.52

$$\frac{\sqrt{\frac{ax^4}{b}+1}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{ax^4}{b}\right)-1}{2ax\sqrt{a+\frac{b}{x^4}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^4)^(3/2)*x^2),x]
```

```
[Out] (-1 + Sqrt[1 + (a*x^4)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((a*x^4)/b)])/(
2*a*Sqrt[a + b/x^4]*x)
```

fricas [F] time = 1.14, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^6\sqrt{\frac{ax^4+b}{x^4}}}{a^2x^8+2\,abx^4+b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^4)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(x^6*sqrt((a*x^4 + b)/x^4)/(a^2*x^8 + 2*a*b*x^4 + b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}} x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^4)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a + b/x^4)^(3/2)*x^2), x)
```

maple [C] time = 0.01, size = 113, normalized size = 1.03

$$-\frac{\left(a\,x^4+b\right)\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;x-\sqrt{-\frac{i\sqrt{a}\;x^2-\sqrt{b}}{\sqrt{b}}}\;\sqrt{\frac{i\sqrt{a}\;x^2+\sqrt{b}}{\sqrt{b}}}\;\text{EllipticF}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;x, i\right)\right)}{2\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{3}{2}}\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;a\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^4)^(3/2)/x^2,x)
```

```
[Out] -1/2*(a*x^4+b)*(-(-(I*a^(1/2)*x^2-b^(1/2))/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b
^(1/2))/b^(1/2))^(1/2)*EllipticF((I*a^(1/2)/b^(1/2))^(1/2)*x,I)+(I*a^(1/2)/
b^(1/2))^(1/2)*x)/((a*x^4+b)/x^4)^(3/2)/x^6/a/(I*a^(1/2)/b^(1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}} x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^4)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a + b/x^4)^(3/2)*x^2), x)
```

mupad [B] time = 1.37, size = 39, normalized size = 0.35

$$-\frac{\left(\frac{b}{a}+x^4\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{2};\ \frac{5}{4};\ -\frac{b}{a\,x^4}\right)}{x\left(a\,x^4+b\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b/x^4)^(3/2)),x)
```

```
[Out] -((b/a + x^4)^(3/2)*hypergeom([1/4, 3/2], 5/4, -b/(a*x^4)))/(x*(b + a*x^4)^
(3/2))
```

sympy [C] time = 1.50, size = 37, normalized size = 0.34

$$-\frac{\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{be^{i\pi}}{ax^4}\right)}{4a^{\frac{3}{2}}x\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**4)**(3/2)/x**2,x)
```

```
[Out] -gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*exp_polar(I*pi)/(a*x**4))/(4*a**(3/
2)*x*gamma(5/4))
```

3.2097 $\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2} x^4}\,dx$

Optimal. Leaf size=241

$$\frac{\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{4a^{3/4}b^{3/4}\sqrt{a+\frac{b}{x^4}}}-\frac{\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+\frac{b}{x^4}}}-\frac{1}{2ax^3\sqrt{a+\frac{b}{x^4}}}+$$

```
[Out] -1/2/a/x^3/(a+b/x^4)^(1/2)+1/2*(a+b/x^4)^(1/2)/a/x/b^(1/2)/(a^(1/2)+b^(1/2)
/x^2)-1/2*(cos(2*arccot(a^(1/4)*x/b^(1/4)))^2)^(1/2)/cos(2*arccot(a^(1/4)*x
/b^(1/4)))*EllipticE(sin(2*arccot(a^(1/4)*x/b^(1/4))),1/2*2^(1/2))*(a^(1/2)
+b^(1/2)/x^2)*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)/a^(3/4)/b^(3/4)/(a+
b/x^4)^(1/2)+1/4*(cos(2*arccot(a^(1/4)*x/b^(1/4)))^2)^(1/2)/cos(2*arccot(a^
(1/4)*x/b^(1/4)))*EllipticF(sin(2*arccot(a^(1/4)*x/b^(1/4))),1/2*2^(1/2))*(
a^(1/2)+b^(1/2)/x^2)*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)/a^(3/4)/b^(3
/4)/(a+b/x^4)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {335, 290, 305, 220, 1196}

$$\frac{\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{4a^{3/4}b^{3/4}\sqrt{a+\frac{b}{x^4}}}-\frac{\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+\frac{b}{x^4}}}+\frac{\sqrt{a+\frac{b}{x^4}}}{2a\sqrt{b}\,x\left(\sqrt{a}+$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^4)^(3/2)*x^4),x]
```

```
[Out] -1/(2*a*Sqrt[a + b/x^4]*x^3) + Sqrt[a + b/x^4]/(2*a*Sqrt[b]*(Sqrt[a] + Sqrt
[b]/x^2)*x) - (Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a] + Sqrt[
b]/x^2)*EllipticE[2*ArcCot[(a^(1/4)*x)/b^(1/4)], 1/2])/(2*a^(3/4)*b^(3/4)*S
qrt[a + b/x^4]) + (Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a] + S
qrt[b]/x^2)*EllipticF[2*ArcCot[(a^(1/4)*x)/b^(1/4)], 1/2])/(4*a^(3/4)*b^(3/
4)*Sqrt[a + b/x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2} x^4}\,dx = -\text{Subst}\left(\int \frac{x^2}{\left(a+bx^4\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{2a\sqrt{a+\frac{b}{x^4}}\,x^3} + \frac{\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{1}{2a\sqrt{a+\frac{b}{x^4}}\,x^3} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right)}{2\sqrt{a}\,\sqrt{b}} - \frac{\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right)}{2\sqrt{a}\,\sqrt{b}}$$

$$= -\frac{1}{2a\sqrt{a+\frac{b}{x^4}}\,x^3} + \frac{\sqrt{a+\frac{b}{x^4}}}{2a\sqrt{b}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)x} - \frac{\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+\frac{b}{x^4}}} + \cdots$$

Mathematica [C] time = 0.03, size = 52, normalized size = 0.22

$$\frac{x\sqrt{\frac{ax^4}{b}+1}\;{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{ax^4}{b}\right)}{3b\sqrt{a+\frac{b}{x^4}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^4)^(3/2)*x^4),x]
```

```
[Out] (x*Sqrt[1 + (a*x^4)/b]*Hypergeometric2F1[3/4, 3/2, 7/4, -((a*x^4)/b)])/(3*b
*Sqrt[a + b/x^4])
```

fricas [F] time = 1.09, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^4\sqrt{\frac{ax^4+b}{x^4}}}{a^2x^8+2\,abx^4+b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^4)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(x^4*sqrt((a*x^4 + b)/x^4)/(a^2*x^8 + 2*a*b*x^4 + b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}} x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^4)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(1/((a + b/x^4)^(3/2)*x^4), x)
```

maple [C] time = 0.01, size = 187, normalized size = 0.78

$$\frac{\left(a\,x^4+b\right)\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,\sqrt{a}\,\sqrt{b}\,x^3+i\sqrt{-\frac{i\sqrt{a}\,x^2-\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}\,x^2+\sqrt{b}}{\sqrt{b}}}\,b\,\text{EllipticE}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,x,i\right)-i\sqrt{-\frac{i\sqrt{a}\,x^2-\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}\,x^2+}{\sqrt{b}}}\right.}{2\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{3}{2}}\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,\sqrt{a}\,b^{\frac{3}{2}}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^4)^(3/2)/x^4,x)
```

```
[Out] 1/2*(a*x^4+b)*(x^3*b^(1/2)*(I*a^(1/2)/b^(1/2))^(1/2)*a^(1/2)-I*(-(I*a^(1/2)
*x^2-b^(1/2))/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*b*Elli
pticF((I*a^(1/2)/b^(1/2))^(1/2)*x,I)+I*(-(I*a^(1/2)*x^2-b^(1/2))/b^(1/2))^(
1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*b*EllipticE((I*a^(1/2)/b^(1/2)
)^(1/2)*x,I))/((a*x^4+b)/x^4)^(3/2)/x^6/b^(3/2)/(I*a^(1/2)/b^(1/2))^(1/2)/a
^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}} x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^4)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(1/((a + b/x^4)^(3/2)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b/x^4)^(3/2)),x)
```

```
[Out] int(1/(x^4*(a + b/x^4)^(3/2)), x)
```

sympy [C] time = 1.71, size = 39, normalized size = 0.16

$$-\frac{\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\right|\frac{be^{i\pi}}{ax^4}\right)}{4a^{\frac{3}{2}}x^3\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**4)**(3/2)/x**4,x)
```

```
[Out] -gamma(3/4)*hyper((3/4, 3/2), (7/4,), b*exp_polar(I*pi)/(a*x**4))/(4*a**(3/
2)*x**3*gamma(7/4))
```

3.2098 $\int \frac{x^3}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx$

Optimal. Leaf size=88

$$-\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{4a^{7/2}}+\frac{5b}{4a^3\sqrt{a+\frac{b}{x^4}}}+\frac{5b}{12a^2\left(a+\frac{b}{x^4}\right)^{3/2}}+\frac{x^4}{4a\left(a+\frac{b}{x^4}\right)^{3/2}}$$

```
[Out] 5/12*b/a^2/(a+b/x^4)^(3/2)+1/4*x^4/a/(a+b/x^4)^(3/2)-5/4*b*arctanh((a+b/x^4
)^(1/2)/a^(1/2))/a^(7/2)+5/4*b/a^3/(a+b/x^4)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 51, 63, 208}

$$\frac{5x^4\sqrt{a+\frac{b}{x^4}}}{4a^3}-\frac{5x^4}{6a^2\sqrt{a+\frac{b}{x^4}}}-\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{4a^{7/2}}-\frac{x^4}{6a\left(a+\frac{b}{x^4}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b/x^4)^(5/2),x]
```

```
[Out] -x^4/(6*a*(a + b/x^4)^(3/2)) - (5*x^4)/(6*a^2*Sqrt[a + b/x^4]) + (5*Sqrt[a
+ b/x^4]*x^4)/(4*a^3) - (5*b*ArcTanh[Sqrt[a + b/x^4]/Sqrt[a]])/(4*a^(7/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx = -\left(\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{5/2}}\,dx, x, \frac{1}{x^4}\right)\right)$$

$$= -\frac{x^4}{6a\left(a+\frac{b}{x^4}\right)^{3/2}} - \frac{5\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx, x, \frac{1}{x^4}\right)}{12a}$$

$$= -\frac{x^4}{6a\left(a+\frac{b}{x^4}\right)^{3/2}} - \frac{5x^4}{6a^2\sqrt{a+\frac{b}{x^4}}} - \frac{5\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x^4}\right)}{4a^2}$$

$$= -\frac{x^4}{6a\left(a+\frac{b}{x^4}\right)^{3/2}} - \frac{5x^4}{6a^2\sqrt{a+\frac{b}{x^4}}} + \frac{5\sqrt{a+\frac{b}{x^4}}\,x^4}{4a^3} + \frac{(5b)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x^4}\right)}{8a^3}$$

$$= -\frac{x^4}{6a\left(a+\frac{b}{x^4}\right)^{3/2}} - \frac{5x^4}{6a^2\sqrt{a+\frac{b}{x^4}}} + \frac{5\sqrt{a+\frac{b}{x^4}}\,x^4}{4a^3} + \frac{5\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x^4}}\right)}{4a^3}$$

$$= -\frac{x^4}{6a\left(a+\frac{b}{x^4}\right)^{3/2}} - \frac{5x^4}{6a^2\sqrt{a+\frac{b}{x^4}}} + \frac{5\sqrt{a+\frac{b}{x^4}}\,x^4}{4a^3} - \frac{5b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{4a^{7/2}}$$

Mathematica [A] time = 0.23, size = 103, normalized size = 1.17

$$\frac{\sqrt{a}\left(3a^2x^8+20abx^4+15b^2\right) - \frac{15b^{3/2}\left(ax^4+b\right)\sqrt{\frac{ax^4}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}x^2}{\sqrt{b}}\right)}{x^2}}{12a^{7/2}\sqrt{a+\frac{b}{x^4}}\left(ax^4+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b/x^4)^(5/2),x]
```

```
[Out] (Sqrt[a]*(15*b^2 + 20*a*b*x^4 + 3*a^2*x^8) - (15*b^(3/2)*(b + a*x^4)*Sqrt[1
 + (a*x^4)/b]*ArcSinh[(Sqrt[a]*x^2)/Sqrt[b]])/x^2)/(12*a^(7/2)*Sqrt[a + b/x
^4]*(b + a*x^4))
```

fricas [A] time = 0.85, size = 260, normalized size = 2.95

$$\left[\frac{15\left(a^2bx^8+2\,ab^2x^4+b^3\right)\sqrt{a}\,\log\left(-2\,ax^4+2\,\sqrt{a}\,x^4\sqrt{\frac{ax^4+b}{x^4}}-b\right)+2\left(3\,a^3x^{12}+20\,a^2bx^8+15\,ab^2x^4\right)\sqrt{\frac{ax^4+b}{x^4}}}{24\left(a^6x^8+2\,a^5bx^4+a^4b^2\right)}, \frac{15}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x^4)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(15*(a^2*b*x^8 + 2*a*b^2*x^4 + b^3)*sqrt(a)*log(-2*a*x^4 + 2*sqrt(a)*
x^4*sqrt((a*x^4 + b)/x^4) - b) + 2*(3*a^3*x^12 + 20*a^2*b*x^8 + 15*a*b^2*x^
4)*sqrt((a*x^4 + b)/x^4))/(a^6*x^8 + 2*a^5*b*x^4 + a^4*b^2), 1/12*(15*(a^2*
b*x^8 + 2*a*b^2*x^4 + b^3)*sqrt(-a)*arctan(sqrt(-a)*x^4*sqrt((a*x^4 + b)/x^
4)/(a*x^4 + b)) + (3*a^3*x^12 + 20*a^2*b*x^8 + 15*a*b^2*x^4)*sqrt((a*x^4 +
b)/x^4))/(a^6*x^8 + 2*a^5*b*x^4 + a^4*b^2)]
```

giac [A] time = 0.23, size = 78, normalized size = 0.89

$$\frac{\left(\left(\frac{3x^4}{a}+\frac{20b}{a^2}\right)x^4+\frac{15b^2}{a^3}\right)x^2}{12\left(ax^4+b\right)^{\frac{3}{2}}}+\frac{5b\log\left(\left|-\sqrt{a}\,x^2+\sqrt{ax^4+b}\right|\right)}{4a^{\frac{7}{2}}}-\frac{5b\log\left(|b|\right)}{8a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x^4)^(5/2),x, algorithm="giac")
```

```
[Out] 1/12*((3*x^4/a + 20*b/a^2)*x^4 + 15*b^2/a^3)*x^2/(a*x^4 + b)^(3/2) + 5/4*b*
log(abs(-sqrt(a)*x^2 + sqrt(a*x^4 + b)))/a^(7/2) - 5/8*b*log(abs(b))/a^(7/2
)
```

maple [B] time = 0.07, size = 282, normalized size = 3.20

$$\frac{\left(a\,x^4+b\right)^{\frac{5}{2}}\left(3\sqrt{a\,x^4+b}\;a^{\frac{15}{2}}x^{10}-15a^7b\,x^8\ln\left(\sqrt{a}\;x^2+\sqrt{a\,x^4+b}\right)+14\sqrt{-\frac{\left(-a\,x^2+\sqrt{-ab}\right)\left(a\,x^2+\sqrt{-ab}\right)}{a}}\;a^{\frac{13}{2}}b\,x^6+6\right.}{12\left(\frac{a\,x}{}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b/x^4)^(5/2),x)
```

```
[Out] 1/12*(a*x^4+b)^(5/2)*(3*a^(15/2)*(a*x^4+b)^(1/2)*x^10+14*a^(13/2)*(-(-a*x^2
+(-a*b)^(1/2))/a*(a*x^2+(-a*b)^(1/2)))^(1/2)*x^6*b+6*a^(13/2)*b*(a*x^4+b)^(
1/2)*x^6-15*ln(a^(1/2)*x^2+(a*x^4+b)^(1/2))*x^8*a^7*b+12*a^(11/2)*(-(-a*x^2
+(-a*b)^(1/2))/a*(a*x^2+(-a*b)^(1/2)))^(1/2)*b^2*x^2+3*a^(11/2)*b^2*(a*x^4+
b)^(1/2)*x^2-30*ln(a^(1/2)*x^2+(a*x^4+b)^(1/2))*a^6*b^2*x^4-15*ln(a^(1/2)*x
^2+(a*x^4+b)^(1/2))*a^5*b^3)/a^(13/2)/((a*x^4+b)/x^4)^(5/2)/x^10/(-a*x^2+(-
a*b)^(1/2))^2/(a*x^2+(-a*b)^(1/2))^2
```

maxima [A] time = 1.98, size = 101, normalized size = 1.15

$$\frac{15\left(a+\frac{b}{x^4}\right)^2b-10\left(a+\frac{b}{x^4}\right)ab-2\,a^2b}{12\left(\left(a+\frac{b}{x^4}\right)^{\frac{5}{2}}a^3-\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}}a^4\right)}+\frac{5b\log\left(\frac{\sqrt{a+\frac{b}{x^4}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^4}}+\sqrt{a}}\right)}{8a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/x^4)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(15*(a + b/x^4)^2*b - 10*(a + b/x^4)*a*b - 2*a^2*b)/((a + b/x^4)^(5/2)
*a^3 - (a + b/x^4)^(3/2)*a^4) + 5/8*b*log((sqrt(a + b/x^4) - sqrt(a))/(sqrt
(a + b/x^4) + sqrt(a)))/a^(7/2)
```

mupad [**B**] time = 1.89, size = 73, normalized size = 0.83

$$\frac{5\,b}{3\,a^2\left(a+\frac{b}{x^4}\right)^{3/2}}+\frac{x^4}{4\,a\left(a+\frac{b}{x^4}\right)^{3/2}}-\frac{5\,b\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{4\,a^{7/2}}+\frac{5\,b^2}{4\,a^3\,x^4\left(a+\frac{b}{x^4}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b/x^4)^(5/2),x)
```

```
[Out] (5*b)/(3*a^2*(a + b/x^4)^(3/2)) + x^4/(4*a*(a + b/x^4)^(3/2)) - (5*b*atanh(
(a + b/x^4)^(1/2)/a^(1/2)))/(4*a^(7/2)) + (5*b^2)/(4*a^3*x^4*(a + b/x^4)^(3
/2))
```

sympy [**B**] time = 6.20, size = 819, normalized size = 9.31

$$\frac{6a^{17}x^{16}\sqrt{1+\frac{b}{ax^4}}}{24a^{\frac{39}{2}}x^{12}+72a^{\frac{37}{2}}bx^8+72a^{\frac{35}{2}}b^2x^4+24a^{\frac{33}{2}}b^3}+\frac{46a^{16}bx^{12}\sqrt{1+\frac{b}{ax^4}}}{24a^{\frac{39}{2}}x^{12}+72a^{\frac{37}{2}}bx^8+72a^{\frac{35}{2}}b^2x^4+24a^{\frac{33}{2}}b^3}+\frac{15a^{16}b}{24a^{\frac{39}{2}}x^{12}+72a^{\frac{37}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b/x**4)**(5/2),x)
```

```
[Out] 6*a**17*x**16*sqrt(1 + b/(a*x**4))/(24*a**(39/2)*x**12 + 72*a**(37/2)*b*x**
8 + 72*a**(35/2)*b**2*x**4 + 24*a**(33/2)*b**3) + 46*a**16*b*x**12*sqrt(1 +
 b/(a*x**4))/(24*a**(39/2)*x**12 + 72*a**(37/2)*b*x**8 + 72*a**(35/2)*b**2*
x**4 + 24*a**(33/2)*b**3) + 15*a**16*b*x**12*log(b/(a*x**4))/(24*a**(39/2)*
x**12 + 72*a**(37/2)*b*x**8 + 72*a**(35/2)*b**2*x**4 + 24*a**(33/2)*b**3) -
 30*a**16*b*x**12*log(sqrt(1 + b/(a*x**4)) + 1)/(24*a**(39/2)*x**12 + 72*a*
*(37/2)*b*x**8 + 72*a**(35/2)*b**2*x**4 + 24*a**(33/2)*b**3) + 70*a**15*b**
2*x**8*sqrt(1 + b/(a*x**4))/(24*a**(39/2)*x**12 + 72*a**(37/2)*b*x**8 + 72*
a**(35/2)*b**2*x**4 + 24*a**(33/2)*b**3) + 45*a**15*b**2*x**8*log(b/(a*x**4
))/(24*a**(39/2)*x**12 + 72*a**(37/2)*b*x**8 + 72*a**(35/2)*b**2*x**4 + 24*
a**(33/2)*b**3) - 90*a**15*b**2*x**8*log(sqrt(1 + b/(a*x**4)) + 1)/(24*a**(
39/2)*x**12 + 72*a**(37/2)*b*x**8 + 72*a**(35/2)*b**2*x**4 + 24*a**(33/2)*b
**3) + 30*a**14*b**3*x**4*sqrt(1 + b/(a*x**4))/(24*a**(39/2)*x**12 + 72*a**
(37/2)*b*x**8 + 72*a**(35/2)*b**2*x**4 + 24*a**(33/2)*b**3) + 45*a**14*b**3
*x**4*log(b/(a*x**4))/(24*a**(39/2)*x**12 + 72*a**(37/2)*b*x**8 + 72*a**(35
/2)*b**2*x**4 + 24*a**(33/2)*b**3) - 90*a**14*b**3*x**4*log(sqrt(1 + b/(a*x
**4)) + 1)/(24*a**(39/2)*x**12 + 72*a**(37/2)*b*x**8 + 72*a**(35/2)*b**2*x*
*4 + 24*a**(33/2)*b**3) + 15*a**13*b**4*log(b/(a*x**4))/(24*a**(39/2)*x**12
 + 72*a**(37/2)*b*x**8 + 72*a**(35/2)*b**2*x**4 + 24*a**(33/2)*b**3) - 30*a
**13*b**4*log(sqrt(1 + b/(a*x**4)) + 1)/(24*a**(39/2)*x**12 + 72*a**(37/2)*
b*x**8 + 72*a**(35/2)*b**2*x**4 + 24*a**(33/2)*b**3)
```

3.2099 $\int \frac{x}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx$

Optimal. Leaf size=64

$$\frac{4x^2\sqrt{a+\frac{b}{x^4}}}{3a^3}-\frac{2x^2}{3a^2\sqrt{a+\frac{b}{x^4}}}-\frac{x^2}{6a\left(a+\frac{b}{x^4}\right)^{3/2}}$$

```
[Out] -1/6*x^2/a/(a+b/x^4)^(3/2)-2/3*x^2/a^2/(a+b/x^4)^(1/2)+4/3*x^2*(a+b/x^4)^(1
/2)/a^3
```

Rubi [A] time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {273, 264}

$$\frac{4x^2\sqrt{a+\frac{b}{x^4}}}{3a^3}-\frac{2x^2}{3a^2\sqrt{a+\frac{b}{x^4}}}-\frac{x^2}{6a\left(a+\frac{b}{x^4}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b/x^4)^(5/2),x]
```

```
[Out] -x^2/(6*a*(a + b/x^4)^(3/2)) - (2*x^2)/(3*a^2*Sqrt[a + b/x^4]) + (4*Sqrt[a
+ b/x^4]*x^2)/(3*a^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx &= -\frac{x^2}{6a\left(a+\frac{b}{x^4}\right)^{3/2}}+\frac{4\int \frac{x}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx}{3a}\\ &= -\frac{x^2}{6a\left(a+\frac{b}{x^4}\right)^{3/2}}-\frac{2x^2}{3a^2\sqrt{a+\frac{b}{x^4}}}+\frac{8\int \frac{x}{\sqrt{a+\frac{b}{x^4}}}\,dx}{3a^2}\\ &= -\frac{x^2}{6a\left(a+\frac{b}{x^4}\right)^{3/2}}-\frac{2x^2}{3a^2\sqrt{a+\frac{b}{x^4}}}+\frac{4\sqrt{a+\frac{b}{x^4}}\,x^2}{3a^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 0.80

$$\frac{3a^2x^8+12abx^4+8b^2}{6a^3x^2\sqrt{a+\frac{b}{x^4}}\left(ax^4+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b/x^4)^(5/2),x]
```

```
[Out] (8*b^2 + 12*a*b*x^4 + 3*a^2*x^8)/(6*a^3*Sqrt[a + b/x^4]*x^2*(b + a*x^4))
```

fricas [A] time = 0.67, size = 65, normalized size = 1.02

$$\frac{\left(3\,a^2x^{10}+12\,abx^6+8\,b^2x^2\right)\sqrt{\frac{ax^4+b}{x^4}}}{6\left(a^5x^8+2\,a^4bx^4+a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^4)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*a^2*x^10 + 12*a*b*x^6 + 8*b^2*x^2)*sqrt((a*x^4 + b)/x^4)/(a^5*x^8 +
2*a^4*b*x^4 + a^3*b^2)
```

giac [A] time = 0.19, size = 53, normalized size = 0.83

$$\frac{\sqrt{ax^4+b}}{2\,a^3}-\frac{4\,\sqrt{b}}{3\,a^3}+\frac{6\left(ax^4+b\right)b-b^2}{6\left(ax^4+b\right)^{\frac{3}{2}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^4)^(5/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(a*x^4 + b)/a^3 - 4/3*sqrt(b)/a^3 + 1/6*(6*(a*x^4 + b)*b - b^2)/((a
*x^4 + b)^(3/2)*a^3)
```

maple [A] time = 0.01, size = 50, normalized size = 0.78

$$\frac{\left(a\,x^4+b\right)\left(3x^8a^2+12ab\,x^4+8b^2\right)}{6\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{5}{2}}a^3x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b/x^4)^(5/2),x)
```

```
[Out] 1/6*(a*x^4+b)*(3*a^2*x^8+12*a*b*x^4+8*b^2)/a^3/x^10/((a*x^4+b)/x^4)^(5/2)
```

maxima [A] time = 0.88, size = 54, normalized size = 0.84

$$\frac{\sqrt{a+\frac{b}{x^4}}\,x^2}{2\,a^3}+\frac{6\left(a+\frac{b}{x^4}\right)bx^4-b^2}{6\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}}a^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^4)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a + b/x^4)*x^2/a^3 + 1/6*(6*(a + b/x^4)*b*x^4 - b^2)/((a + b/x^4)^
(3/2)*a^3*x^6)
```

mupad [B] time = 1.48, size = 38, normalized size = 0.59

$$\frac{3\,a^2\,x^8+12\,a\,b\,x^4+8\,b^2}{6\,a^3\,x^6\left(a+\frac{b}{x^4}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b/x^4)^(5/2),x)
```

```
[Out] (8*b^2 + 3*a^2*x^8 + 12*a*b*x^4)/(6*a^3*x^6*(a + b/x^4)^(3/2))
```

sympy [B] time = 1.66, size = 163, normalized size = 2.55

$$\frac{3a^2b^{\frac{9}{2}}x^8\sqrt{\frac{ax^4}{b}+1}}{6a^5b^4x^8+12a^4b^5x^4+6a^3b^6}+\frac{12ab^{\frac{11}{2}}x^4\sqrt{\frac{ax^4}{b}+1}}{6a^5b^4x^8+12a^4b^5x^4+6a^3b^6}+\frac{8b^{\frac{13}{2}}\sqrt{\frac{ax^4}{b}+1}}{6a^5b^4x^8+12a^4b^5x^4+6a^3b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x**4)**(5/2),x)
```

```
[Out] 3*a**2*b**(9/2)*x**8*sqrt(a*x**4/b + 1)/(6*a**5*b**4*x**8 + 12*a**4*b**5*x*
**4 + 6*a**3*b**6) + 12*a*b**(11/2)*x**4*sqrt(a*x**4/b + 1)/(6*a**5*b**4*x**
8 + 12*a**4*b**5*x**4 + 6*a**3*b**6) + 8*b**(13/2)*sqrt(a*x**4/b + 1)/(6*a*
*5*b**4*x**8 + 12*a**4*b**5*x**4 + 6*a**3*b**6)
```

3.2100 $\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2} x}\,dx$

Optimal. Leaf size=64

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{2a^{5/2}} - \frac{1}{2a^2\sqrt{a+\frac{b}{x^4}}} - \frac{1}{6a\left(a+\frac{b}{x^4}\right)^{3/2}}$$

```
[Out] -1/6/a/(a+b/x^4)^(3/2)+1/2*arctanh((a+b/x^4)^(1/2)/a^(1/2))/a^(5/2)-1/2/a^2
/(a+b/x^4)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 51, 63, 208}

$$-\frac{1}{2a^2\sqrt{a+\frac{b}{x^4}}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{2a^{5/2}} - \frac{1}{6a\left(a+\frac{b}{x^4}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^4)^(5/2)*x),x]
```

```
[Out] -1/(6*a*(a + b/x^4)^(3/2)) - 1/(2*a^2*Sqrt[a + b/x^4]) + ArcTanh[Sqrt[a + b
/x^4]/Sqrt[a]]/(2*a^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2} x}\,dx = -\left(\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, \frac{1}{x^4}\right)\right)$$

$$= -\frac{1}{6a\left(a+\frac{b}{x^4}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \frac{1}{x^4}\right)}{4a}$$

$$= -\frac{1}{6a\left(a+\frac{b}{x^4}\right)^{3/2}} - \frac{1}{2a^2\sqrt{a+\frac{b}{x^4}}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x^4}\right)}{4a^2}$$

$$= -\frac{1}{6a\left(a+\frac{b}{x^4}\right)^{3/2}} - \frac{1}{2a^2\sqrt{a+\frac{b}{x^4}}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x^4}}\right)}{2a^2b}$$

$$= -\frac{1}{6a\left(a+\frac{b}{x^4}\right)^{3/2}} - \frac{1}{2a^2\sqrt{a+\frac{b}{x^4}}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{2a^{5/2}}$$

Mathematica [A] time = 0.24, size = 93, normalized size = 1.45

$$\frac{\frac{3\sqrt{b}\left(ax^4+b\right)\sqrt{\frac{ax^4}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}x^2}{\sqrt{b}}\right)}{x^2} - \sqrt{a}\left(4ax^4+3b\right)}{6a^{5/2}\sqrt{a+\frac{b}{x^4}}\left(ax^4+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^4)^(5/2)*x),x]
```

```
[Out] (-(Sqrt[a]*(3*b + 4*a*x^4)) + (3*Sqrt[b]*(b + a*x^4)*Sqrt[1 + (a*x^4)/b]*Ar
cSinh[(Sqrt[a]*x^2)/Sqrt[b]])/x^2)/(6*a^(5/2)*Sqrt[a + b/x^4]*(b + a*x^4))
```

fricas [B] time = 0.80, size = 232, normalized size = 3.62

$$\left[\frac{3\left(a^2x^8+2abx^4+b^2\right)\sqrt{a}\,\log\left(-2ax^4-2\sqrt{a}\,x^4\sqrt{\frac{ax^4+b}{x^4}}-b\right)-2\left(4a^2x^8+3abx^4\right)\sqrt{\frac{ax^4+b}{x^4}}}{12\left(a^5x^8+2a^4bx^4+a^3b^2\right)}, -\frac{3\left(a^2x^8+2abx^4\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^4)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a^2*x^8 + 2*a*b*x^4 + b^2)*sqrt(a)*log(-2*a*x^4 - 2*sqrt(a)*x^4*s
qrt((a*x^4 + b)/x^4) - b) - 2*(4*a^2*x^8 + 3*a*b*x^4)*sqrt((a*x^4 + b)/x^4)
)/(a^5*x^8 + 2*a^4*b*x^4 + a^3*b^2), -1/6*(3*(a^2*x^8 + 2*a*b*x^4 + b^2)*sq
rt(-a)*arctan(sqrt(-a)*x^4*sqrt((a*x^4 + b)/x^4)/(a*x^4 + b)) + (4*a^2*x^8
+ 3*a*b*x^4)*sqrt((a*x^4 + b)/x^4))/(a^5*x^8 + 2*a^4*b*x^4 + a^3*b^2)]
```

giac [A] time = 0.26, size = 63, normalized size = 0.98

$$-\frac{\left(\frac{4x^4}{a}+\frac{3b}{a^2}\right)x^2}{6\left(ax^4+b\right)^{\frac{3}{2}}}-\frac{\log\left(\left|-\sqrt{a}\,x^2+\sqrt{ax^4+b}\right|\right)}{2\,a^{\frac{5}{2}}}+\frac{\log\left(|b|\right)}{4\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^4)^(5/2)/x,x, algorithm="giac")
```

```
[Out] -1/6*(4*x^4/a + 3*b/a^2)*x^2/(a*x^4 + b)^(3/2) - 1/2*log(abs(-sqrt(a)*x^2 +
 sqrt(a*x^4 + b)))/a^(5/2) + 1/4*log(abs(b))/a^(5/2)
```

maple [B] time = 0.03, size = 221, normalized size = 3.45

$$-\frac{\left(a\,x^4+b\right)^{\frac{5}{2}}\left(-3a^5x^8\ln\left(\sqrt{a}\;x^2+\sqrt{a\,x^4+b}\right)+4\sqrt{-\frac{\left(-a\,x^2+\sqrt{-ab}\right)\left(a\,x^2+\sqrt{-ab}\right)}{a}}\;a^{\frac{9}{2}}x^6-6a^4b\,x^4\ln\left(\sqrt{a}\;x^2+\sqrt{a\,x^4+\right.\right.}}{6\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{5}{2}}\left(-a\,x^2+\sqrt{-ab}\right)^2\left(a\,x^2+\sqrt{-ab}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^4)^(5/2)/x,x)
```

```
[Out] -1/6*(a*x^4+b)^(5/2)*(4*a^(9/2)*(-(-a*x^2+(-a*b)^(1/2))*(a*x^2+(-a*b)^(1/2)
)/a)^(1/2)*x^6-3*ln(a^(1/2)*x^2+(a*x^4+b)^(1/2))*x^8*a^5+3*a^(7/2)*b*(-(-a*
x^2+(-a*b)^(1/2))*(a*x^2+(-a*b)^(1/2))/a)^(1/2)*x^2-6*a^4*b*ln(a^(1/2)*x^2+
(a*x^4+b)^(1/2))*x^4-3*a^3*b^2*ln(a^(1/2)*x^2+(a*x^4+b)^(1/2)))/a^(7/2)/((a
*x^4+b)/x^4)^(5/2)/x^10/(-a*x^2+(-a*b)^(1/2))^2/(a*x^2+(-a*b)^(1/2))^2
```

maxima [A] time = 1.92, size = 62, normalized size = 0.97

$$-\frac{\log\left(\frac{\sqrt{a+\frac{b}{x^4}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^4}}+\sqrt{a}}\right)}{4\,a^{\frac{5}{2}}}-\frac{4\,a+\frac{3\,b}{x^4}}{6\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^4)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] -1/4*log((sqrt(a + b/x^4) - sqrt(a))/(sqrt(a + b/x^4) + sqrt(a)))/a^(5/2) -
 1/6*(4*a + 3*b/x^4)/((a + b/x^4)^(3/2)*a^2)
```

mupad [B] time = 1.38, size = 48, normalized size = 0.75

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{2\,a^{5/2}}-\frac{\frac{a+\frac{b}{x^4}}{a^2}+\frac{1}{3\,a}}{2\left(a+\frac{b}{x^4}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b/x^4)^(5/2)),x)
```

```
[Out] atanh((a + b/x^4)^(1/2)/a^(1/2))/(2*a^(5/2)) - ((a + b/x^4)/a^2 + 1/(3*a))/
(2*(a + b/x^4)^(3/2))
```

sympy [B] time = 3.69, size = 743, normalized size = 11.61

$$-\frac{8a^7x^{12}\sqrt{1+\frac{b}{ax^4}}}{12a^{\frac{19}{2}}x^{12}+36a^{\frac{17}{2}}bx^8+36a^{\frac{15}{2}}b^2x^4+12a^{\frac{13}{2}}b^3}-\frac{3a^7x^{12}\log\left(\frac{b}{ax^4}\right)}{12a^{\frac{19}{2}}x^{12}+36a^{\frac{17}{2}}bx^8+36a^{\frac{15}{2}}b^2x^4+12a^{\frac{13}{2}}b^3}+\frac{6a^7x^{12}}{12a^{\frac{19}{2}}x^{12}+36a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**4)**(5/2)/x,x)
```

```
[Out] -8*a**7*x**12*sqrt(1 + b/(a*x**4))/(12*a**(19/2)*x**12 + 36*a**(17/2)*b*x**
8 + 36*a**(15/2)*b**2*x**4 + 12*a**(13/2)*b**3) - 3*a**7*x**12*log(b/(a*x**
4))/(12*a**(19/2)*x**12 + 36*a**(17/2)*b*x**8 + 36*a**(15/2)*b**2*x**4 + 12
*a**(13/2)*b**3) + 6*a**7*x**12*log(sqrt(1 + b/(a*x**4)) + 1)/(12*a**(19/2)
*x**12 + 36*a**(17/2)*b*x**8 + 36*a**(15/2)*b**2*x**4 + 12*a**(13/2)*b**3)
 - 14*a**6*b*x**8*sqrt(1 + b/(a*x**4))/(12*a**(19/2)*x**12 + 36*a**(17/2)*b*
x**8 + 36*a**(15/2)*b**2*x**4 + 12*a**(13/2)*b**3) - 9*a**6*b*x**8*log(b/(a
*x**4))/(12*a**(19/2)*x**12 + 36*a**(17/2)*b*x**8 + 36*a**(15/2)*b**2*x**4
 + 12*a**(13/2)*b**3) + 18*a**6*b*x**8*log(sqrt(1 + b/(a*x**4)) + 1)/(12*a**
(19/2)*x**12 + 36*a**(17/2)*b*x**8 + 36*a**(15/2)*b**2*x**4 + 12*a**(13/2)*
b**3) - 6*a**5*b**2*x**4*sqrt(1 + b/(a*x**4))/(12*a**(19/2)*x**12 + 36*a**(
17/2)*b*x**8 + 36*a**(15/2)*b**2*x**4 + 12*a**(13/2)*b**3) - 9*a**5*b**2*x*
*4*log(b/(a*x**4))/(12*a**(19/2)*x**12 + 36*a**(17/2)*b*x**8 + 36*a**(15/2)
*b**2*x**4 + 12*a**(13/2)*b**3) + 18*a**5*b**2*x**4*log(sqrt(1 + b/(a*x**4)
) + 1)/(12*a**(19/2)*x**12 + 36*a**(17/2)*b*x**8 + 36*a**(15/2)*b**2*x**4 +
 12*a**(13/2)*b**3) - 3*a**4*b**3*log(b/(a*x**4))/(12*a**(19/2)*x**12 + 36*
a**(17/2)*b*x**8 + 36*a**(15/2)*b**2*x**4 + 12*a**(13/2)*b**3) + 6*a**4*b**
3*log(sqrt(1 + b/(a*x**4)) + 1)/(12*a**(19/2)*x**12 + 36*a**(17/2)*b*x**8 +
 36*a**(15/2)*b**2*x**4 + 12*a**(13/2)*b**3)
```

3.2101 $\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2} x^3}\,dx$

Optimal. Leaf size=44

$$-\frac{b}{3a^2x^6\left(a+\frac{b}{x^4}\right)^{3/2}}-\frac{1}{2ax^2\left(a+\frac{b}{x^4}\right)^{3/2}}$$

```
[Out] -1/3*b/a^2/(a+b/x^4)^(3/2)/x^6-1/2/a/(a+b/x^4)^(3/2)/x^2
```

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {271, 264}

$$-\frac{b}{3a^2x^6\left(a+\frac{b}{x^4}\right)^{3/2}}-\frac{1}{2ax^2\left(a+\frac{b}{x^4}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^4)^(5/2)*x^3),x]
```

```
[Out] -b/(3*a^2*(a + b/x^4)^(3/2)*x^6) - 1/(2*a*(a + b/x^4)^(3/2)*x^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2}x^3}\,dx &= -\frac{1}{2a\left(a+\frac{b}{x^4}\right)^{3/2}x^2}+\frac{(2b)\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2}x^7}\,dx}{a}\\ &= -\frac{b}{3a^2\left(a+\frac{b}{x^4}\right)^{3/2}x^6}-\frac{1}{2a\left(a+\frac{b}{x^4}\right)^{3/2}x^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 0.91

$$\frac{-3ax^4-2b}{6a^2x^2\sqrt{a+\frac{b}{x^4}}\left(ax^4+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^4)^(5/2)*x^3),x]
```

```
[Out] (-2*b - 3*a*x^4)/(6*a^2*Sqrt[a + b/x^4]*x^2*(b + a*x^4))
```

fricas [A] time = 0.78, size = 54, normalized size = 1.23

$$-\frac{\left(3\,ax^6+2\,bx^2\right)\sqrt{\frac{ax^4+b}{x^4}}}{6\left(a^4x^8+2\,a^3bx^4+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^4)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/6*(3*a*x^6 + 2*b*x^2)*sqrt((a*x^4 + b)/x^4)/(a^4*x^8 + 2*a^3*b*x^4 + a^2
*b^2)
```

giac [A] time = 0.19, size = 33, normalized size = 0.75

$$-\frac{3\,ax^4+2\,b}{6\left(ax^4+b\right)^{\frac{3}{2}}a^2}+\frac{1}{3\,a^2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^4)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/6*(3*a*x^4 + 2*b)/((a*x^4 + b)^(3/2)*a^2) + 1/3/(a^2*sqrt(b))
```

maple [A] time = 0.01, size = 39, normalized size = 0.89

$$-\frac{\left(a\,x^4+b\right)\left(3a\,x^4+2b\right)}{6\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{5}{2}}a^2x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^4)^(5/2)/x^3,x)
```

```
[Out] -1/6*(a*x^4+b)*(3*a*x^4+2*b)/a^2/x^10/((a*x^4+b)/x^4)^(5/2)
```

maxima [A] time = 0.87, size = 33, normalized size = 0.75

$$-\frac{3\left(a+\frac{b}{x^4}\right)x^4-b}{6\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}}a^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^4)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/6*(3*(a + b/x^4)*x^4 - b)/((a + b/x^4)^(3/2)*a^2*x^6)
```

mupad [B] time = 1.27, size = 36, normalized size = 0.82

$$-\frac{x^2\sqrt{a+\frac{b}{x^4}}\,\left(3\,a\,x^4+2\,b\right)}{6\,a^2\left(a\,x^4+b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b/x^4)^(5/2)),x)
```

```
[Out] -(x^2*(a + b/x^4)^(1/2)*(2*b + 3*a*x^4))/(6*a^2*(b + a*x^4)^2)
```

sympy [**B**] time = 1.92, size = 105, normalized size = 2.39

$$-\frac{3ax^4}{6a^3\sqrt{b}\,x^4\sqrt{\frac{ax^4}{b}+1}+6a^2b^{\frac{3}{2}}\sqrt{\frac{ax^4}{b}+1}}-\frac{2b}{6a^3\sqrt{b}\,x^4\sqrt{\frac{ax^4}{b}+1}+6a^2b^{\frac{3}{2}}\sqrt{\frac{ax^4}{b}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**4)**(5/2)/x**3,x)
```

```
[Out] -3*a*x**4/(6*a**3*sqrt(b)*x**4*sqrt(a*x**4/b + 1) + 6*a**2*b**(3/2)*sqrt(a*
x**4/b + 1)) - 2*b/(6*a**3*sqrt(b)*x**4*sqrt(a*x**4/b + 1) + 6*a**2*b**(3/2
)*sqrt(a*x**4/b + 1))
```

3.2102 $\int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx$

Optimal. Leaf size=152

$$\frac{5b^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{8a^{13/4}\sqrt{a+\frac{b}{x^4}}}+\frac{5x^3\sqrt{a+\frac{b}{x^4}}}{4a^3}-\frac{3x^3}{4a^2\sqrt{a+\frac{b}{x^4}}}-\frac{x^3}{6a\left(a+\frac{b}{x^4}\right)^{3/2}}$$

```
[Out] -1/6*x^3/a/(a+b/x^4)^(3/2)-3/4*x^3/a^2/(a+b/x^4)^(1/2)+5/4*x^3*(a+b/x^4)^(1
/2)/a^3+5/8*b^(3/4)*(cos(2*arccot(a^(1/4)*x/b^(1/4)))^2)^(1/2)/cos(2*arccot
(a^(1/4)*x/b^(1/4)))*EllipticF(sin(2*arccot(a^(1/4)*x/b^(1/4))),1/2*2^(1/2)
)*(a^(1/2)+b^(1/2)/x^2)*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)/a^(13/4)/
(a+b/x^4)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {335, 290, 325, 220}

$$\frac{5b^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{8a^{13/4}\sqrt{a+\frac{b}{x^4}}}+\frac{5x^3\sqrt{a+\frac{b}{x^4}}}{4a^3}-\frac{3x^3}{4a^2\sqrt{a+\frac{b}{x^4}}}-\frac{x^3}{6a\left(a+\frac{b}{x^4}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b/x^4)^(5/2),x]
```

```
[Out] -x^3/(6*a*(a + b/x^4)^(3/2)) - (3*x^3)/(4*a^2*Sqrt[a + b/x^4]) + (5*Sqrt[a
+ b/x^4]*x^3)/(4*a^3) + (5*b^(3/4)*Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)
^2]*(Sqrt[a] + Sqrt[b]/x^2)*EllipticF[2*ArcCot[(a^(1/4)*x)/b^(1/4)], 1/2])/
(8*a^(13/4)*Sqrt[a + b/x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx = -\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^4\right)^{5/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{x^3}{6a\left(a+\frac{b}{x^4}\right)^{3/2}} - \frac{3\,\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^4\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{x^3}{6a\left(a+\frac{b}{x^4}\right)^{3/2}} - \frac{3x^3}{4a^2\sqrt{a+\frac{b}{x^4}}} - \frac{15\,\text{Subst}\left(\int \frac{1}{x^4\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right)}{4a^2}$$

$$= -\frac{x^3}{6a\left(a+\frac{b}{x^4}\right)^{3/2}} - \frac{3x^3}{4a^2\sqrt{a+\frac{b}{x^4}}} + \frac{5\sqrt{a+\frac{b}{x^4}}\,x^3}{4a^3} + \frac{(5b)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right)}{4a^3}$$

$$= -\frac{x^3}{6a\left(a+\frac{b}{x^4}\right)^{3/2}} - \frac{3x^3}{4a^2\sqrt{a+\frac{b}{x^4}}} + \frac{5\sqrt{a+\frac{b}{x^4}}\,x^3}{4a^3} + \frac{5b^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{}}{\sqrt[4]{}}\right.\right.}{8a^{13/4}\sqrt{a+\frac{b}{x^4}}}$$

Mathematica [C] time = 0.07, size = 94, normalized size = 0.62

$$\frac{4a^2x^8 - 15b\left(ax^4+b\right)\sqrt{\frac{ax^4}{b}+1}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{ax^4}{b}\right) + 21abx^4 + 15b^2}{12a^3x\sqrt{a+\frac{b}{x^4}}\left(ax^4+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b/x^4)^(5/2),x]
```

```
[Out] (15*b^2 + 21*a*b*x^4 + 4*a^2*x^8 - 15*b*(b + a*x^4)*Sqrt[1 + (a*x^4)/b]*Hyp
ergeometric2F1[1/4, 1/2, 5/4, -((a*x^4)/b)])/(12*a^3*Sqrt[a + b/x^4]*x*(b +
 a*x^4))
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^{14}\sqrt{\frac{ax^4+b}{x^4}}}{a^3x^{12}+3\,a^2bx^8+3\,ab^2x^4+b^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^4)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(x^14*sqrt((a*x^4 + b)/x^4)/(a^3*x^12 + 3*a^2*b*x^8 + 3*a*b^2*x^4 +
 b^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^4)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(a + b/x^4)^(5/2), x)
```

maple [C] time = 0.02, size = 304, normalized size = 2.00

$$\frac{4\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,a^3x^{13}+25\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,a^2b\,x^9-15\sqrt{-\frac{i\sqrt{a}\,x^2-\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}\,x^2+\sqrt{b}}{\sqrt{b}}}\,a^2b\,x^8\,\text{EllipticF}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,x,i\right)+36\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,a\,b^2x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b/x^4)^(5/2),x)
```

```
[Out] 1/12*(4*(I*a^(1/2)/b^(1/2))^(1/2)*x^13*a^3-15*(-(I*a^(1/2)*x^2-b^(1/2))/b^(
1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*EllipticF((I*a^(1/2)/b^
(1/2))^(1/2)*x,I)*x^8*a^2*b+25*(I*a^(1/2)/b^(1/2))^(1/2)*x^9*a^2*b-30*(-(I*
a^(1/2)*x^2-b^(1/2))/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)
*EllipticF((I*a^(1/2)/b^(1/2))^(1/2)*x,I)*x^4*a*b^2+36*(I*a^(1/2)/b^(1/2))^
(1/2)*x^5*a*b^2-15*(-(I*a^(1/2)*x^2-b^(1/2))/b^(1/2))^(1/2)*((I*a^(1/2)*x^2
+b^(1/2))/b^(1/2))^(1/2)*EllipticF((I*a^(1/2)/b^(1/2))^(1/2)*x,I)*b^3+15*(I
*a^(1/2)/b^(1/2))^(1/2)*x*b^3)/a^3/((a*x^4+b)/x^4)^(5/2)/x^10/(I*a^(1/2)/b^
(1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^4)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(a + b/x^4)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b/x^4)^(5/2),x)
```

```
[Out] int(x^2/(a + b/x^4)^(5/2), x)
```

sympy [C] time = 1.51, size = 42, normalized size = 0.28

$$-\frac{x^3\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{4},\frac{5}{2}\\ \frac{1}{4}\end{matrix}\right|\frac{be^{i\pi}}{ax^4}\right)}{4a^{\frac{5}{2}}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b/x**4)**(5/2),x)
```

```
[Out] -x**3*gamma(-3/4)*hyper((-3/4, 5/2), (1/4,), b*exp_polar(I*pi)/(a*x**4))/(4
*a**(5/2)*gamma(1/4))
```

3.2103 $\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx$

Optimal. Leaf size=277

$$-\frac{7\sqrt[4]{b}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{8a^{11/4}\sqrt{a+\frac{b}{x^4}}}+\frac{7\sqrt[4]{b}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{4a^{11/4}\sqrt{a+\frac{b}{x^4}}}+\frac{7x\sqrt{}}{}$$

```
[Out] -1/6*x/a/(a+b/x^4)^(3/2)-7/12*x/a^2/(a+b/x^4)^(1/2)+7/4*x*(a+b/x^4)^(1/2)/a
^3-7/4*b^(1/2)*(a+b/x^4)^(1/2)/a^3/x/(a^(1/2)+b^(1/2)/x^2)+7/4*b^(1/4)*(cos
(2*arccot(a^(1/4)*x/b^(1/4)))^2)^(1/2)/cos(2*arccot(a^(1/4)*x/b^(1/4)))*Ell
ipticE(sin(2*arccot(a^(1/4)*x/b^(1/4))),1/2*2^(1/2))*(a^(1/2)+b^(1/2)/x^2)*
((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)/a^(11/4)/(a+b/x^4)^(1/2)-7/8*b^(1
/4)*(cos(2*arccot(a^(1/4)*x/b^(1/4)))^2)^(1/2)/cos(2*arccot(a^(1/4)*x/b^(1/
4)))*EllipticF(sin(2*arccot(a^(1/4)*x/b^(1/4))),1/2*2^(1/2))*(a^(1/2)+b^(1/
2)/x^2)*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)/a^(11/4)/(a+b/x^4)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {242, 290, 325, 305, 220, 1196}

$$\frac{7x\sqrt{a+\frac{b}{x^4}}}{4a^3}-\frac{7x}{12a^2\sqrt{a+\frac{b}{x^4}}}-\frac{7\sqrt{b}\sqrt{a+\frac{b}{x^4}}}{4a^3x\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)}-\frac{7\sqrt[4]{b}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{8a^{11/4}\sqrt{a+\frac{b}{x^4}}}+\frac{7\sqrt[4]{b}\sqrt{\frac{}{\left(\sqrt{a}\right.}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^4)^(-5/2),x]
```

```
[Out] (-7*Sqrt[b]*Sqrt[a + b/x^4])/(4*a^3*(Sqrt[a] + Sqrt[b]/x^2)*x) - x/(6*a*(a
+ b/x^4)^(3/2)) - (7*x)/(12*a^2*Sqrt[a + b/x^4]) + (7*Sqrt[a + b/x^4]*x)/(4
*a^3) + (7*b^(1/4)*Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a] + S
qrt[b]/x^2)*EllipticE[2*ArcCot[(a^(1/4)*x)/b^(1/4)], 1/2])/(4*a^(11/4)*Sqrt
[a + b/x^4]) - (7*b^(1/4)*Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt
[a] + Sqrt[b]/x^2)*EllipticF[2*ArcCot[(a^(1/4)*x)/b^(1/4)], 1/2])/(8*a^(11/
4)*Sqrt[a + b/x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
```

```
]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx = -\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^4\right)^{5/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{x}{6a\left(a+\frac{b}{x^4}\right)^{3/2}} - \frac{7\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^4\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{6a}$$

$$= -\frac{x}{6a\left(a+\frac{b}{x^4}\right)^{3/2}} - \frac{7x}{12a^2\sqrt{a+\frac{b}{x^4}}} - \frac{7\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right)}{4a^2}$$

$$= -\frac{x}{6a\left(a+\frac{b}{x^4}\right)^{3/2}} - \frac{7x}{12a^2\sqrt{a+\frac{b}{x^4}}} + \frac{7\sqrt{a+\frac{b}{x^4}}\,x}{4a^3} - \frac{(7b)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right)}{4a^3}$$

$$= -\frac{x}{6a\left(a+\frac{b}{x^4}\right)^{3/2}} - \frac{7x}{12a^2\sqrt{a+\frac{b}{x^4}}} + \frac{7\sqrt{a+\frac{b}{x^4}}\,x}{4a^3} - \frac{\left(7\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right)}{4a^{5/2}} + \frac{\left(7\sqrt{b}\right)}{}$$

$$= -\frac{7\sqrt{b}\,\sqrt{a+\frac{b}{x^4}}}{4a^3\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)x} - \frac{x}{6a\left(a+\frac{b}{x^4}\right)^{3/2}} - \frac{7x}{12a^2\sqrt{a+\frac{b}{x^4}}} + \frac{7\sqrt{a+\frac{b}{x^4}}\,x}{4a^3} + \frac{7\sqrt[4]{b}\,\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}\right.}{4a^{11/4}}$$

Mathematica [C] time = 0.05, size = 81, normalized size = 0.29

$$\frac{-7x\left(ax^4+b\right)\sqrt{\frac{ax^4}{b}+1}\ {}_2F_1\left(\frac{3}{4},\frac{5}{2};\frac{7}{4};-\frac{ax^4}{b}\right)+3ax^5+7bx}{3a^2\sqrt{a+\frac{b}{x^4}}\left(ax^4+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^4)^(-5/2),x]
```

```
[Out] (7*b*x + 3*a*x^5 - 7*x*(b + a*x^4)*Sqrt[1 + (a*x^4)/b]*Hypergeometric2F1[3/
4, 5/2, 7/4, -((a*x^4)/b)])/(3*a^2*Sqrt[a + b/x^4]*(b + a*x^4))
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^{12}\sqrt{\frac{ax^4+b}{x^4}}}{a^3x^{12}+3\,a^2bx^8+3\,ab^2x^4+b^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^4)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(x^12*sqrt((a*x^4 + b)/x^4)/(a^3*x^12 + 3*a^2*b*x^8 + 3*a*b^2*x^4 +
 b^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^4)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a + b/x^4)^(-5/2), x)
```

maple [C] time = 0.02, size = 503, normalized size = 1.82

$$-\frac{9\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\ a^{\frac{9}{2}}x^{11}+21i\sqrt{-\frac{i\sqrt{a}\ x^2-\sqrt{b}}{\sqrt{b}}}\ \sqrt{\frac{i\sqrt{a}\ x^2+\sqrt{b}}{\sqrt{b}}}\ a^4\sqrt{b}\ x^8\,\text{EllipticE}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\ x,i\right)-21i\sqrt{-\frac{i\sqrt{a}\ x^2-\sqrt{b}}{\sqrt{b}}}\ \sqrt{\frac{i\sqrt{a}\ x^2+\sqrt{}}{\sqrt{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^4)^(5/2),x)
```

```
[Out] -1/12*(9*(I*a^(1/2)/b^(1/2))^(1/2)*a^(9/2)*x^11-21*I*(-(I*a^(1/2)*x^2-b^(1/
2))/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*EllipticF((I*a^(
1/2)/b^(1/2))^(1/2)*x,I)*b^(1/2)*x^8*a^4+21*I*(-(I*a^(1/2)*x^2-b^(1/2))/b^(
1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*EllipticE((I*a^(1/2)/b^
(1/2))^(1/2)*x,I)*b^(1/2)*x^8*a^4+16*(I*a^(1/2)/b^(1/2))^(1/2)*a^(7/2)*x^7*
b-42*I*(-(I*a^(1/2)*x^2-b^(1/2))/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^
(1/2))^(1/2)*EllipticF((I*a^(1/2)/b^(1/2))^(1/2)*x,I)*b^(3/2)*x^4*a^3+42*I*
(-(I*a^(1/2)*x^2-b^(1/2))/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^
(1/2)*EllipticE((I*a^(1/2)/b^(1/2))^(1/2)*x,I)*b^(3/2)*x^4*a^3+7*(I*a^(1/2)
/b^(1/2))^(1/2)*a^(5/2)*x^3*b^2-21*I*(-(I*a^(1/2)*x^2-b^(1/2))/b^(1/2))^(1/
2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*EllipticF((I*a^(1/2)/b^(1/2))^(1
```

```
/2)*x,I)*b^(5/2)*a^2+21*I*(-(I*a^(1/2)*x^2-b^(1/2))/b^(1/2))^(1/2)*((I*a^(1
/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*EllipticE((I*a^(1/2)/b^(1/2))^(1/2)*x,I)*b^
(5/2)*a^2)/a^(9/2)/((a*x^4+b)/x^4)^(5/2)/x^10/(I*a^(1/2)/b^(1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^4)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x^4)^(-5/2), x)
```

mupad [B] time = 1.30, size = 43, normalized size = 0.16

$$\frac{x\left(\frac{a\,x^4}{b}+1\right)^{5/2}\sqrt{x^{20}}\;{}_2\mathrm{F}_1\left(\frac{5}{2},\frac{11}{4};\;\frac{15}{4};\;-\frac{a\,x^4}{b}\right)}{11\left(a\,x^4+b\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x^4)^(5/2),x)
```

```
[Out] (x*((a*x^4)/b + 1)^(5/2)*(x^20)^(1/2)*hypergeom([5/2, 11/4], 15/4, -(a*x^4)
/b))/(11*(b + a*x^4)^(5/2))
```

sympy [C] time = 1.55, size = 41, normalized size = 0.15

$$-\frac{x\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{4},\frac{5}{2}\\ \frac{3}{4}\end{matrix}\right|\frac{be^{i\pi}}{ax^4}\right)}{4a^{\frac{5}{2}}\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**4)**(5/2),x)
```

```
[Out] -x*gamma(-1/4)*hyper((-1/4, 5/2), (3/4,), b*exp_polar(I*pi)/(a*x**4))/(4*a*
*(5/2)*gamma(3/4))
```

3.2104 $\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2} x^2}\,dx$

Optimal. Leaf size=131

$$-\frac{5\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{24a^{9/4}\sqrt[4]{b}\sqrt{a+\frac{b}{x^4}}}-\frac{5}{12a^2x\sqrt{a+\frac{b}{x^4}}}-\frac{1}{6ax\left(a+\frac{b}{x^4}\right)^{3/2}}$$

```
[Out] -1/6/a/(a+b/x^4)^(3/2)/x-5/12/a^2/x/(a+b/x^4)^(1/2)-5/24*(cos(2*arccot(a^(1
/4)*x/b^(1/4)))^2)^(1/2)/cos(2*arccot(a^(1/4)*x/b^(1/4)))*EllipticF(sin(2*a
rccot(a^(1/4)*x/b^(1/4))),1/2*2^(1/2))*(a^(1/2)+b^(1/2)/x^2)*((a+b/x^4)/(a^
(1/2)+b^(1/2)/x^2)^2)^(1/2)/a^(9/4)/b^(1/4)/(a+b/x^4)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {335, 199, 220}

$$-\frac{5}{12a^2x\sqrt{a+\frac{b}{x^4}}}-\frac{5\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{24a^{9/4}\sqrt[4]{b}\sqrt{a+\frac{b}{x^4}}}-\frac{1}{6ax\left(a+\frac{b}{x^4}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^4)^(5/2)*x^2),x]
```

```
[Out] -1/(6*a*(a + b/x^4)^(3/2)*x) - 5/(12*a^2*Sqrt[a + b/x^4]*x) - (5*Sqrt[(a +
b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a] + Sqrt[b]/x^2)*EllipticF[2*ArcCo
t[(a^(1/4)*x)/b^(1/4)], 1/2])/(24*a^(9/4)*b^(1/4)*Sqrt[a + b/x^4])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2}x^2}\,dx = -\text{Subst}\left(\int \frac{1}{\left(a+bx^4\right)^{5/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{6a\left(a+\frac{b}{x^4}\right)^{3/2}x} - \frac{5\,\text{Subst}\left(\int \frac{1}{\left(a+bx^4\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{6a}$$

$$= -\frac{1}{6a\left(a+\frac{b}{x^4}\right)^{3/2}x} - \frac{5}{12a^2\sqrt{a+\frac{b}{x^4}}\,x} - \frac{5\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right)}{12a^2}$$

$$= -\frac{1}{6a\left(a+\frac{b}{x^4}\right)^{3/2}x} - \frac{5}{12a^2\sqrt{a+\frac{b}{x^4}}\,x} - \frac{5\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{24a^{9/4}\sqrt[4]{b}\sqrt{a+\frac{b}{x^4}}}$$

Mathematica [C] time = 0.06, size = 82, normalized size = 0.63

$$\frac{5\left(ax^4+b\right)\sqrt{\frac{ax^4}{b}+1}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{ax^4}{b}\right)-7ax^4-5b}{12a^2x\sqrt{a+\frac{b}{x^4}}\left(ax^4+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^4)^(5/2)*x^2),x]
```

```
[Out] (-5*b - 7*a*x^4 + 5*(b + a*x^4)*Sqrt[1 + (a*x^4)/b]*Hypergeometric2F1[1/4,
1/2, 5/4, -((a*x^4)/b)])/(12*a^2*Sqrt[a + b/x^4]*x*(b + a*x^4))
```

fricas [F] time = 1.14, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^{10}\sqrt{\frac{ax^4+b}{x^4}}}{a^3x^{12}+3\,a^2bx^8+3\,ab^2x^4+b^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^4)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(x^10*sqrt((a*x^4 + b)/x^4)/(a^3*x^12 + 3*a^2*b*x^8 + 3*a*b^2*x^4 +
 b^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{\frac{5}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^4)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a + b/x^4)^(5/2)*x^2), x)
```

maple [C] time = 0.02, size = 279, normalized size = 2.13

$$-\frac{7\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,a^2x^9-5\sqrt{-\frac{i\sqrt{a}\,x^2-\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}\,x^2+\sqrt{b}}{\sqrt{b}}}\,a^2x^8\,\text{EllipticF}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,x,i\right)+12\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,abx^5-10\sqrt{-\frac{i\sqrt{a}\,x^2-\sqrt{b}}{\sqrt{b}}}\,\sqrt{}}{12\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^4)^(5/2)/x^2,x)
```

```
[Out] -1/12*(-5*(-(I*a^(1/2)*x^2-b^(1/2))/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))
/b^(1/2))^(1/2)*EllipticF((I*a^(1/2)/b^(1/2))^(1/2)*x,I)*x^8*a^2+7*(I*a^(1/
2)/b^(1/2))^(1/2)*x^9*a^2-10*(-(I*a^(1/2)*x^2-b^(1/2))/b^(1/2))^(1/2)*((I*a
^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*EllipticF((I*a^(1/2)/b^(1/2))^(1/2)*x,I)
*x^4*a*b+12*(I*a^(1/2)/b^(1/2))^(1/2)*x^5*a*b-5*(-(I*a^(1/2)*x^2-b^(1/2))/b
^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*EllipticF((I*a^(1/2)/
b^(1/2))^(1/2)*x,I)*b^2+5*(I*a^(1/2)/b^(1/2))^(1/2)*x*b^2)/a^2/((a*x^4+b)/x
^4)^(5/2)/x^10/(I*a^(1/2)/b^(1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{\frac{5}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^4)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a + b/x^4)^(5/2)*x^2), x)
```

mupad [B] time = 1.44, size = 39, normalized size = 0.30

$$-\frac{\left(\frac{b}{a}+x^4\right)^{5/2}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{5}{2};\ \frac{5}{4};\ -\frac{b}{a\,x^4}\right)}{x\left(a\,x^4+b\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b/x^4)^(5/2)),x)
```

```
[Out] -((b/a + x^4)^(5/2)*hypergeom([1/4, 5/2], 5/4, -b/(a*x^4)))/(x*(b + a*x^4)^
(5/2))
```

sympy [C] time = 1.85, size = 37, normalized size = 0.28

$$-\frac{\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{5}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{be^{i\pi}}{ax^4}\right)}{4a^{\frac{5}{2}}x\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**4)**(5/2)/x**2,x)
```

```
[Out] -gamma(1/4)*hyper((1/4, 5/2), (5/4,), b*exp_polar(I*pi)/(a*x**4))/(4*a**(5/
2)*x*gamma(5/4))
```

3.2105 $\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2} x^4}\,dx$

Optimal. Leaf size=262

$$\frac{\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{8a^{7/4}b^{3/4}\sqrt{a+\frac{b}{x^4}}}-\frac{\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{4a^{7/4}b^{3/4}\sqrt{a+\frac{b}{x^4}}}-\frac{1}{4a^2x^3\sqrt{a+\frac{b}{x^4}}}+\frac{}{4a}$$

```
[Out] -1/6/a/(a+b/x^4)^(3/2)/x^3-1/4/a^2/x^3/(a+b/x^4)^(1/2)+1/4*(a+b/x^4)^(1/2)/
a^2/x/b^(1/2)/(a^(1/2)+b^(1/2)/x^2)-1/4*(cos(2*arccot(a^(1/4)*x/b^(1/4)))^2
)^(1/2)/cos(2*arccot(a^(1/4)*x/b^(1/4)))*EllipticE(sin(2*arccot(a^(1/4)*x/b
^(1/4))),1/2*2^(1/2))*(a^(1/2)+b^(1/2)/x^2)*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2
)^2)^(1/2)/a^(7/4)/b^(3/4)/(a+b/x^4)^(1/2)+1/8*(cos(2*arccot(a^(1/4)*x/b^(1
/4)))^2)^(1/2)/cos(2*arccot(a^(1/4)*x/b^(1/4)))*EllipticF(sin(2*arccot(a^(1
/4)*x/b^(1/4))),1/2*2^(1/2))*(a^(1/2)+b^(1/2)/x^2)*((a+b/x^4)/(a^(1/2)+b^(1
/2)/x^2)^2)^(1/2)/a^(7/4)/b^(3/4)/(a+b/x^4)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {335, 290, 305, 220, 1196}

$$\frac{\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)F\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{8a^{7/4}b^{3/4}\sqrt{a+\frac{b}{x^4}}}-\frac{\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{4a^{7/4}b^{3/4}\sqrt{a+\frac{b}{x^4}}}+\frac{\sqrt{a+\frac{b}{x^4}}}{4a^2\sqrt{b}\,x\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^4)^(5/2)*x^4),x]
```

```
[Out] -1/(6*a*(a + b/x^4)^(3/2)*x^3) - 1/(4*a^2*Sqrt[a + b/x^4]*x^3) + Sqrt[a + b
/x^4]/(4*a^2*Sqrt[b]*(Sqrt[a] + Sqrt[b]/x^2)*x) - (Sqrt[(a + b/x^4)/(Sqrt[a
] + Sqrt[b]/x^2)^2]*(Sqrt[a] + Sqrt[b]/x^2)*EllipticE[2*ArcCot[(a^(1/4)*x)/
b^(1/4)], 1/2])/(4*a^(7/4)*b^(3/4)*Sqrt[a + b/x^4]) + (Sqrt[(a + b/x^4)/(Sq
rt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a] + Sqrt[b]/x^2)*EllipticF[2*ArcCot[(a^(1/4)
*x)/b^(1/4)], 1/2])/(8*a^(7/4)*b^(3/4)*Sqrt[a + b/x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2} x^4}\,dx = -\text{Subst}\left(\int \frac{x^2}{\left(a+bx^4\right)^{5/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{6a\left(a+\frac{b}{x^4}\right)^{3/2} x^3} - \frac{\text{Subst}\left(\int \frac{x^2}{\left(a+bx^4\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{1}{6a\left(a+\frac{b}{x^4}\right)^{3/2} x^3} - \frac{1}{4a^2\sqrt{a+\frac{b}{x^4}}\,x^3} + \frac{\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right)}{4a^2}$$

$$= -\frac{1}{6a\left(a+\frac{b}{x^4}\right)^{3/2} x^3} - \frac{1}{4a^2\sqrt{a+\frac{b}{x^4}}\,x^3} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \frac{1}{x}\right)}{4a^{3/2}\sqrt{b}} - \frac{\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx\right.}{4a^{3/2}\sqrt{b}}$$

$$= -\frac{1}{6a\left(a+\frac{b}{x^4}\right)^{3/2} x^3} - \frac{1}{4a^2\sqrt{a+\frac{b}{x^4}}\,x^3} + \frac{\sqrt{a+\frac{b}{x^4}}}{4a^2\sqrt{b}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)x} - \frac{\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)}{4a^{7/4}b^{3/4}\sqrt{}}$$

Mathematica [C] time = 0.06, size = 77, normalized size = 0.29

$$\frac{x\left(ax^4+b\right)\sqrt{\frac{ax^4}{b}+1}\;{}_2F_1\left(\frac{3}{4},\frac{5}{2};\frac{7}{4};-\frac{ax^4}{b}\right)-bx}{3ab\sqrt{a+\frac{b}{x^4}}\left(ax^4+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^4)^(5/2)*x^4),x]
```

```
[Out] (-(b*x) + x*(b + a*x^4)*Sqrt[1 + (a*x^4)/b]*Hypergeometric2F1[3/4, 5/2, 7/4
, -((a*x^4)/b)])/(3*a*b*Sqrt[a + b/x^4]*(b + a*x^4))
```

fricas [F] time = 1.09, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^8\sqrt{\frac{ax^4+b}{x^4}}}{a^3x^{12}+3\,a^2bx^8+3\,ab^2x^4+b^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^4)^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(x^8*sqrt((a*x^4 + b)/x^4)/(a^3*x^12 + 3*a^2*b*x^8 + 3*a*b^2*x^4 +
b^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{\frac{5}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^4)^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(1/((a + b/x^4)^(5/2)*x^4), x)
```

maple [C] time = 0.02, size = 503, normalized size = 1.92

$$-\frac{-3\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;a^{\frac{7}{2}}\sqrt{b}\;x^{11}-3i\sqrt{-\frac{i\sqrt{a}\;x^2-\sqrt{b}}{\sqrt{b}}}\;\sqrt{\frac{i\sqrt{a}\;x^2+\sqrt{b}}{\sqrt{b}}}\;a^3b\,x^8\,\text{EllipticE}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\;x,i\right)+3i\sqrt{-\frac{i\sqrt{a}\;x^2-\sqrt{b}}{\sqrt{b}}}\;\sqrt{\frac{i\sqrt{a}\;x^2+\sqrt{b}}{\sqrt{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^4)^(5/2)/x^4,x)
```

```
[Out] -1/12*(-3*a^(7/2)*b^(1/2)*(I*a^(1/2)/b^(1/2))^(1/2)*x^11+3*I*(-(I*a^(1/2)*x
^2-b^(1/2))/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*Elliptic
F((I*a^(1/2)/b^(1/2))^(1/2)*x,I)*x^8*a^3*b-3*I*(-(I*a^(1/2)*x^2-b^(1/2))/b^
(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*EllipticE((I*a^(1/2)/b
^(1/2))^(1/2)*x,I)*x^8*a^3*b-4*a^(5/2)*b^(3/2)*(I*a^(1/2)/b^(1/2))^(1/2)*x^
7+6*I*(-(I*a^(1/2)*x^2-b^(1/2))/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(
1/2))^(1/2)*EllipticF((I*a^(1/2)/b^(1/2))^(1/2)*x,I)*x^4*a^2*b^2-6*I*(-(I*a
^(1/2)*x^2-b^(1/2))/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*
EllipticE((I*a^(1/2)/b^(1/2))^(1/2)*x,I)*x^4*a^2*b^2-a^(3/2)*b^(5/2)*(I*a^(
1/2)/b^(1/2))^(1/2)*x^3+3*I*(-(I*a^(1/2)*x^2-b^(1/2))/b^(1/2))^(1/2)*((I*a^
(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*EllipticF((I*a^(1/2)/b^(1/2))^(1/2)*x,I)*
a*b^3-3*I*(-(I*a^(1/2)*x^2-b^(1/2))/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))
/b^(1/2))^(1/2)*EllipticE((I*a^(1/2)/b^(1/2))^(1/2)*x,I)*a*b^3)/a^(5/2)/((a
*x^4+b)/x^4)^(5/2)/x^10/b^(3/2)/(I*a^(1/2)/b^(1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{\frac{5}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^4)^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(1/((a + b/x^4)^(5/2)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b/x^4)^(5/2)),x)
```

```
[Out] int(1/(x^4*(a + b/x^4)^(5/2)), x)
```

sympy [C] time = 2.11, size = 39, normalized size = 0.15

$$-\frac{\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{5}{2}\\ \frac{7}{4}\end{matrix}\right|\frac{be^{i\pi}}{ax^4}\right)}{4a^{\frac{5}{2}}x^3\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**4)**(5/2)/x**4,x)
```

```
[Out] -gamma(3/4)*hyper((3/4, 5/2), (7/4,), b*exp_polar(I*pi)/(a*x**4))/(4*a**(5/
2)*x**3*gamma(7/4))
```

3.2106 $\int \frac{1}{a+\frac{b}{x^5}}\,dx$

Optimal. Leaf size=310

$$\frac{\left(1-\sqrt{5}\right)\sqrt[5]{b}\,\log\left(a^{2/5}x^2-\frac{1}{2}\left(1-\sqrt{5}\right)\sqrt[5]{a}\,\sqrt[5]{b}\,x+b^{2/5}\right)}{20a^{6/5}}+\frac{\left(1+\sqrt{5}\right)\sqrt[5]{b}\,\log\left(a^{2/5}x^2-\frac{1}{2}\left(1+\sqrt{5}\right)\sqrt[5]{a}\,\sqrt[5]{b}\,x+b^{2/5}\right)}{20a^{6/5}}-$$

```
[Out] x/a-1/5*b^(1/5)*ln(b^(1/5)+a^(1/5)*x)/a^(6/5)+1/20*b^(1/5)*ln(b^(2/5)+a^(2/
5)*x^2-1/2*a^(1/5)*b^(1/5)*x*(-5^(1/2)+1))*(-5^(1/2)+1)/a^(6/5)+1/20*b^(1/5
)*ln(b^(2/5)+a^(2/5)*x^2-1/2*a^(1/5)*b^(1/5)*x*(5^(1/2)+1))*(5^(1/2)+1)/a^(
6/5)-1/10*b^(1/5)*arctan(1/5*a^(1/5)*x*(50+10*5^(1/2))^(1/2)/b^(1/5)-1/5*(2
5+10*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)/a^(6/5)-1/10*b^(1/5)*arctan(1/5*(
25-10*5^(1/2))^(1/2)+2*a^(1/5)*x*2^(1/2)/(5+5^(1/2))^(1/2)/b^(1/5))*(10+2*5
^(1/2))^(1/2)/a^(6/5)
```

Rubi [A] time = 0.66, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.889, Rules used = {193, 321, 201, 634, 618, 204, 628, 31}

$$\frac{\left(1-\sqrt{5}\right)\sqrt[5]{b}\,\log\left(a^{2/5}x^2-\frac{1}{2}\left(1-\sqrt{5}\right)\sqrt[5]{a}\,\sqrt[5]{b}\,x+b^{2/5}\right)}{20a^{6/5}}+\frac{\left(1+\sqrt{5}\right)\sqrt[5]{b}\,\log\left(a^{2/5}x^2-\frac{1}{2}\left(1+\sqrt{5}\right)\sqrt[5]{a}\,\sqrt[5]{b}\,x+b^{2/5}\right)}{20a^{6/5}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^5)^(-1),x]
```

```
[Out] x/a - (Sqrt[(5 + Sqrt[5])/2]*b^(1/5)*ArcTan[Sqrt[(5 - 2*Sqrt[5])/5] + (2*Sq
rt[2/(5 + Sqrt[5])]*a^(1/5)*x)/b^(1/5)])/(5*a^(6/5)) + (Sqrt[(5 - Sqrt[5])/
2]*b^(1/5)*ArcTan[Sqrt[(5 + 2*Sqrt[5])/5] - (Sqrt[(2*(5 + Sqrt[5]))/5]*a^(1
/5)*x)/b^(1/5)])/(5*a^(6/5)) - (b^(1/5)*Log[b^(1/5) + a^(1/5)*x])/(5*a^(6/5
)) + ((1 - Sqrt[5])*b^(1/5)*Log[b^(2/5) - ((1 - Sqrt[5])*a^(1/5)*b^(1/5)*x)
/2 + a^(2/5)*x^2])/(20*a^(6/5)) + ((1 + Sqrt[5])*b^(1/5)*Log[b^(2/5) - ((1
+ Sqrt[5])*a^(1/5)*b^(1/5)*x)/2 + a^(2/5)*x^2])/(20*a^(6/5))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[((2*k -
 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]; (r*Int[1
/(r + s*x), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 1)/2}], x], x]
] /; FreeQ[{a, b}, x] && IGtQ[(n - 3)/2, 0] && PosQ[a/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{a+\frac{b}{x^5}}\,dx = \int \frac{x^5}{b+ax^5}\,dx$$

$$= \frac{x}{a} - \frac{b\int \frac{1}{b+ax^5}\,dx}{a}$$

$$= \frac{x}{a} - \frac{\sqrt[5]{b}\int \frac{1}{\sqrt[5]{b}+\sqrt[5]{a}\,x}\,dx}{5a} - \frac{\left(2\sqrt[5]{b}\right)\int \frac{\sqrt[5]{b}-\frac{1}{4}\left(1-\sqrt{5}\right)\sqrt[5]{a}\,x}{b^{2/5}-\frac{1}{2}\left(1-\sqrt{5}\right)\sqrt[5]{a}\,\sqrt[5]{b}\,x+a^{2/5}x^2}\,dx}{5a} - \frac{\left(2\sqrt[5]{b}\right)\int \frac{\sqrt[5]{b}-\frac{1}{4}\left(1+\sqrt{5}\right)\sqrt[5]{a}\,x}{b^{2/5}-\frac{1}{2}\left(1+\sqrt{5}\right)\sqrt[5]{a}\,\sqrt[5]{b}\,x+a^{2/5}x^2}\,dx}{5a}$$

$$= \frac{x}{a} - \frac{\sqrt[5]{b}\,\log\left(\sqrt[5]{b}+\sqrt[5]{a}\,x\right)}{5a^{6/5}} + \frac{\left(\left(1-\sqrt{5}\right)\sqrt[5]{b}\right)\int \frac{-\frac{1}{2}\left(1-\sqrt{5}\right)\sqrt[5]{a}\,\sqrt[5]{b}+2a^{2/5}x}{b^{2/5}-\frac{1}{2}\left(1-\sqrt{5}\right)\sqrt[5]{a}\,\sqrt[5]{b}\,x+a^{2/5}x^2}\,dx}{20a^{6/5}} + \frac{\left(\left(1+\sqrt{5}\right)\sqrt[5]{b}\right)\int}{}$$

$$= \frac{x}{a} - \frac{\sqrt[5]{b}\,\log\left(\sqrt[5]{b}+\sqrt[5]{a}\,x\right)}{5a^{6/5}} + \frac{\left(1+\sqrt{5}\right)\sqrt[5]{b}\,\log\left(2b^{2/5}-\sqrt[5]{a}\,\sqrt[5]{b}\,x-\sqrt{5}\,\sqrt[5]{a}\,\sqrt[5]{b}\,x+2a^{2/5}x^2\right)}{20a^{6/5}} + \frac{\left(1}{}$$

$$= \frac{x}{a} + \frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\,\sqrt[5]{b}\,\tan^{-1}\left(\frac{\sqrt{5+\sqrt{5}}\left(\left(1+\sqrt{5}\right)\sqrt[5]{b}-4\sqrt[5]{a}\,x\right)}{2\sqrt{10}\,\sqrt[5]{b}}\right)}{5a^{6/5}} - \frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\,\sqrt[5]{b}\,\tan^{-1}\left(\sqrt{\frac{1}{5}\left(5-2\sqrt{}\right.\right.}\right.}{5a^{6/5}}$$

Mathematica [A] time = 0.29, size = 267, normalized size = 0.86

$$-\left(\sqrt{5}-1\right)\sqrt[5]{b}\,\log\left(a^{2/5}x^2+\frac{1}{2}\left(\sqrt{5}-1\right)\sqrt[5]{a}\,\sqrt[5]{b}\,x+b^{2/5}\right)+\left(1+\sqrt{5}\right)\sqrt[5]{b}\,\log\left(a^{2/5}x^2-\frac{1}{2}\left(1+\sqrt{5}\right)\sqrt[5]{a}\,\sqrt[5]{b}\,x+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^5)^(-1),x]
```

```
[Out] (20*a^(1/5)*x - 2*Sqrt[2*(5 + Sqrt[5])]*b^(1/5)*ArcTan[((-1 + Sqrt[5])*b^(1
/5) + 4*a^(1/5)*x)/(Sqrt[2*(5 + Sqrt[5])]*b^(1/5))] - 2*Sqrt[10 - 2*Sqrt[5]
]*b^(1/5)*ArcTan[(-((1 + Sqrt[5])*b^(1/5)) + 4*a^(1/5)*x)/(Sqrt[10 - 2*Sqrt
[5]]*b^(1/5))] - 4*b^(1/5)*Log[b^(1/5) + a^(1/5)*x] - (-1 + Sqrt[5])*b^(1/5
)*Log[b^(2/5) + ((-1 + Sqrt[5])*a^(1/5)*b^(1/5)*x)/2 + a^(2/5)*x^2] + (1 +
Sqrt[5])*b^(1/5)*Log[b^(2/5) - ((1 + Sqrt[5])*a^(1/5)*b^(1/5)*x)/2 + a^(2/5
)*x^2])/(20*a^(6/5))
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [**A**] time = 0.20, size = 268, normalized size = 0.86

$$\frac{\left(-\frac{b}{a}\right)^{\frac{1}{5}}\log\left(\left|x-\left(-\frac{b}{a}\right)^{\frac{1}{5}}\right|\right)}{5\,a}+\frac{x}{a}-\frac{\left(-a^4b\right)^{\frac{1}{5}}\sqrt{2\sqrt{5}+10}\,\arctan\left(-\frac{\left(\sqrt{5}-1\right)\left(-\frac{b}{a}\right)^{\frac{1}{5}}-4x}{\sqrt{2\sqrt{5}+10}\left(-\frac{b}{a}\right)^{\frac{1}{5}}}\right)}{10\,a^2}-\frac{\left(-a^4b\right)^{\frac{1}{5}}\sqrt{-2\sqrt{5}+10}\,\arctan\left($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^5),x, algorithm="giac")
```

```
[Out] 1/5*(-b/a)^(1/5)*log(abs(x - (-b/a)^(1/5)))/a + x/a - 1/10*(-a^4*b)^(1/5)*s
qrt(2*sqrt(5) + 10)*arctan(-((sqrt(5) - 1)*(-b/a)^(1/5) - 4*x)/(sqrt(2*sqrt
(5) + 10)*(-b/a)^(1/5)))/a^2 - 1/10*(-a^4*b)^(1/5)*sqrt(-2*sqrt(5) + 10)*ar
ctan(((sqrt(5) + 1)*(-b/a)^(1/5) + 4*x)/(sqrt(-2*sqrt(5) + 10)*(-b/a)^(1/5)
))/a^2 - 1/5*(-a^4*b)^(1/5)*log(x^2 + 1/2*x*(sqrt(5)*(-b/a)^(1/5) + (-b/a)^
(1/5)) + (-b/a)^(2/5))/(a^2*(sqrt(5) - 1)) + 1/5*(-a^4*b)^(1/5)*log(x^2 - 1
/2*x*(sqrt(5)*(-b/a)^(1/5) - (-b/a)^(1/5)) + (-b/a)^(2/5))/(a^2*(sqrt(5) +
1))
```

maple [**B**] time = 0.09, size = 911, normalized size = 2.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^5),x)
```

```
[Out] 1/a*x+1/a^2*b/(1/a*b)^(4/5)/(5^(1/2)-5)/(5+5^(1/2))*ln((1/a*b)^(1/5)*x*5^(1
/2)+2*(1/a*b)^(2/5)-(1/a*b)^(1/5)*x+2*x^2)*5^(1/2)-1/a^2*b/(1/a*b)^(4/5)/(5
^(1/2)-5)/(5+5^(1/2))*ln((1/a*b)^(1/5)*x*5^(1/2)+2*(1/a*b)^(2/5)-(1/a*b)^(1
/5)*x+2*x^2)+20/a^2*b/(1/a*b)^(3/5)/(5^(1/2)-5)/(5+5^(1/2))/(10*(1/a*b)^(2/
5)+2*(1/a*b)^(2/5)*5^(1/2))^(1/2)*arctan(1/(10*(1/a*b)^(2/5)+2*(1/a*b)^(2/5
)*5^(1/2))^(1/2)*(1/a*b)^(1/5)*5^(1/2)-1/(10*(1/a*b)^(2/5)+2*(1/a*b)^(2/5)*
5^(1/2))^(1/2)*(1/a*b)^(1/5)+4*x/(10*(1/a*b)^(2/5)+2*(1/a*b)^(2/5)*5^(1/2))
^(1/2))+4/a^2*b/(1/a*b)^(3/5)/(5^(1/2)-5)/(5+5^(1/2))/(10*(1/a*b)^(2/5)+2*(
1/a*b)^(2/5)*5^(1/2))^(1/2)*arctan(1/(10*(1/a*b)^(2/5)+2*(1/a*b)^(2/5)*5^(1
/2))^(1/2)*(1/a*b)^(1/5)*5^(1/2)-1/(10*(1/a*b)^(2/5)+2*(1/a*b)^(2/5)*5^(1/2
))^(1/2)*(1/a*b)^(1/5)+4*x/(10*(1/a*b)^(2/5)+2*(1/a*b)^(2/5)*5^(1/2))^(1/2)
```

```
)*5^(1/2)+4/a^2*b/(1/a*b)^(4/5)/(5^(1/2)-5)/(5+5^(1/2))*ln(x+(1/a*b)^(1/5))
-1/a^2*b/(1/a*b)^(4/5)/(5^(1/2)-5)/(5+5^(1/2))*ln(-(1/a*b)^(1/5)*x*5^(1/2)+
2*(1/a*b)^(2/5)-(1/a*b)^(1/5)*x+2*x^2)*5^(1/2)-1/a^2*b/(1/a*b)^(4/5)/(5^(1/
2)-5)/(5+5^(1/2))*ln(-(1/a*b)^(1/5)*x*5^(1/2)+2*(1/a*b)^(2/5)-(1/a*b)^(1/5)
*x+2*x^2)+20/a^2*b/(1/a*b)^(3/5)/(5^(1/2)-5)/(5+5^(1/2))/(10*(1/a*b)^(2/5)-
2*(1/a*b)^(2/5)*5^(1/2))^(1/2)*arctan(-1/(10*(1/a*b)^(2/5)-2*(1/a*b)^(2/5)*
5^(1/2))^(1/2)*(1/a*b)^(1/5)*5^(1/2)-1/(10*(1/a*b)^(2/5)-2*(1/a*b)^(2/5)*5^
(1/2))^(1/2)*(1/a*b)^(1/5)+4*x/(10*(1/a*b)^(2/5)-2*(1/a*b)^(2/5)*5^(1/2))^(
1/2))-4/a^2*b/(1/a*b)^(3/5)/(5^(1/2)-5)/(5+5^(1/2))/(10*(1/a*b)^(2/5)-2*(1/
a*b)^(2/5)*5^(1/2))^(1/2)*arctan(-1/(10*(1/a*b)^(2/5)-2*(1/a*b)^(2/5)*5^(1/
2))^(1/2)*(1/a*b)^(1/5)*5^(1/2)-1/(10*(1/a*b)^(2/5)-2*(1/a*b)^(2/5)*5^(1/2)
)^(1/2)*(1/a*b)^(1/5)+4*x/(10*(1/a*b)^(2/5)-2*(1/a*b)^(2/5)*5^(1/2))^(1/2))
*5^(1/2)
```

maxima [A] time = 2.03, size = 250, normalized size = 0.81

$$\frac{2\sqrt{5}\,b^{\frac{1}{5}}\left(\sqrt{5}+1\right)\arctan\left(\frac{4a^{\frac{2}{5}}x+a^{\frac{1}{5}}b^{\frac{1}{5}}\left(\sqrt{5}-1\right)}{a^{\frac{1}{5}}b^{\frac{1}{5}}\sqrt{2\sqrt{5}+10}}\right)}{a^{\frac{1}{5}}\sqrt{2\sqrt{5}+10}}+\frac{2\sqrt{5}\,b^{\frac{1}{5}}\left(\sqrt{5}-1\right)\arctan\left(\frac{4a^{\frac{2}{5}}x-a^{\frac{1}{5}}b^{\frac{1}{5}}\left(\sqrt{5}+1\right)}{a^{\frac{1}{5}}b^{\frac{1}{5}}\sqrt{-2\sqrt{5}+10}}\right)}{a^{\frac{1}{5}}\sqrt{-2\sqrt{5}+10}}-\frac{b^{\frac{1}{5}}\left(\sqrt{5}+3\right)\log\left(2a^{\frac{2}{5}}x^2-a^{\frac{1}{5}}b^{\frac{1}{5}}x\left(\sqrt{5}+1\right)+2b^{\frac{2}{5}}\right.}{a^{\frac{1}{5}}\left(\sqrt{5}+1\right)}$$

$$-\frac{}{10\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^5),x, algorithm="maxima")
```

```
[Out] -1/10*(2*sqrt(5)*b^(1/5)*(sqrt(5) + 1)*arctan((4*a^(2/5)*x + a^(1/5)*b^(1/5
)*(sqrt(5) - 1))/(a^(1/5)*b^(1/5)*sqrt(2*sqrt(5) + 10)))/(a^(1/5)*sqrt(2*sq
rt(5) + 10)) + 2*sqrt(5)*b^(1/5)*(sqrt(5) - 1)*arctan((4*a^(2/5)*x - a^(1/5
)*b^(1/5)*(sqrt(5) + 1))/(a^(1/5)*b^(1/5)*sqrt(-2*sqrt(5) + 10)))/(a^(1/5)*
sqrt(-2*sqrt(5) + 10)) - b^(1/5)*(sqrt(5) + 3)*log(2*a^(2/5)*x^2 - a^(1/5)*
b^(1/5)*x*(sqrt(5) + 1) + 2*b^(2/5))/(a^(1/5)*(sqrt(5) + 1)) - b^(1/5)*(sqr
t(5) - 3)*log(2*a^(2/5)*x^2 + a^(1/5)*b^(1/5)*x*(sqrt(5) - 1) + 2*b^(2/5))/
(a^(1/5)*(sqrt(5) - 1)) + 2*b^(1/5)*log(a^(1/5)*x + b^(1/5))/a^(1/5))/a + x
/a
```

mupad [B] time = 1.63, size = 290, normalized size = 0.94

$$\frac{x}{a}+\frac{(-b)^{1/5}\,\ln\left((-b)^{16/5}+a^{1/5}\,b^3\,x\right)}{5\,a^{6/5}}-\frac{(-b)^{1/5}\,\ln\left(25\,a^{4/5}\,(-b)^{16/5}\left(\frac{\sqrt{-2\,\sqrt{5}-10}}{20}-\frac{\sqrt{5}}{20}+\frac{1}{20}\right)-5\,a\,b^3\,x\right)\left(\frac{\sqrt{-2\,\sqrt{5}-10}}{20}\right.}{a^{6/5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x^5),x)
```

```
[Out] x/a + ((-b)^(1/5)*log((-b)^(16/5) + a^(1/5)*b^3*x))/(5*a^(6/5)) - ((-b)^(1/
5)*log(25*a^(4/5)*(-b)^(16/5)*((- 2*5^(1/2) - 10)^(1/2)/20 - 5^(1/2)/20 + 1
/20) - 5*a*b^3*x)*((- 2*5^(1/2) - 10)^(1/2)/20 - 5^(1/2)/20 + 1/20))/a^(6/5
) + ((-b)^(1/5)*log(25*a^(4/5)*(-b)^(16/5)*(5^(1/2)/20 + (- 2*5^(1/2) - 10)
^(1/2)/20 - 1/20) + 5*a*b^3*x)*(5^(1/2)/20 + (- 2*5^(1/2) - 10)^(1/2)/20 -
1/20))/a^(6/5) - ((-b)^(1/5)*log(25*a^(4/5)*(-b)^(16/5)*(5^(1/2)/20 - (2*5^
(1/2) - 10)^(1/2)/20 + 1/20) - 5*a*b^3*x)*(5^(1/2)/20 - (2*5^(1/2) - 10)^(1
/2)/20 + 1/20))/a^(6/5) - ((-b)^(1/5)*log(25*a^(4/5)*(-b)^(16/5)*(5^(1/2)/2
0 + (2*5^(1/2) - 10)^(1/2)/20 + 1/20) - 5*a*b^3*x)*(5^(1/2)/20 + (2*5^(1/2)
 - 10)^(1/2)/20 + 1/20))/a^(6/5)
```

sympy [A] time = 0.20, size = 22, normalized size = 0.07

$$\operatorname{RootSum}\left(3125t^5a^6+b,\left(t\mapsto t\log\left(-5ta+x\right)\right)\right)+\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**5),x)
[Out] RootSum(3125*_t**5*a**6 + b, Lambda(_t, _t*log(-5*_t*a + x))) + x/a
```

3.2107 $\int \frac{1}{\sqrt{a+\frac{b}{x^5}}\,x}\,dx$

Optimal. Leaf size=27

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^5}}}{\sqrt{a}}\right)}{5\sqrt{a}}$$

```
[Out] 2/5*arctanh((a+b/x^5)^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {266, 63, 208}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^5}}}{\sqrt{a}}\right)}{5\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x^5]*x),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b/x^5]/Sqrt[a]])/(5*Sqrt[a])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+\frac{b}{x^5}}\,x}\,dx = -\left(\frac{1}{5}\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x^5}\right)\right)$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x^5}}\right)}{5b}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x^5}}}{\sqrt{a}}\right)}{5\sqrt{a}}$$

Mathematica [B] time = 0.02, size = 59, normalized size = 2.19

$$\frac{2\sqrt{ax^5+b}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x^{5/2}}{\sqrt{ax^5+b}}\right)}{5\sqrt{a}\,x^{5/2}\sqrt{a+\frac{b}{x^5}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x^5]*x),x]
```

```
[Out] (2*Sqrt[b + a*x^5]*ArcTanh[(Sqrt[a]*x^(5/2))/Sqrt[b + a*x^5]])/(5*Sqrt[a]*S
qrt[a + b/x^5]*x^(5/2))
```

fricas [B] time = 2.14, size = 102, normalized size = 3.78

$$\left[\frac{\log\left(-8\,a^2x^{10}-8\,abx^5-b^2-4\left(2\,ax^{10}+bx^5\right)\sqrt{a}\,\sqrt{\frac{ax^5+b}{x^5}}\right)}{10\,\sqrt{a}}, -\frac{\sqrt{-a}\,\arctan\left(\frac{2\,\sqrt{-a}\,x^5\sqrt{\frac{ax^5+b}{x^5}}}{2\,ax^5+b}\right)}{5\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/x^5)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/10*log(-8*a^2*x^10 - 8*a*b*x^5 - b^2 - 4*(2*a*x^10 + b*x^5)*sqrt(a)*sqrt
((a*x^5 + b)/x^5))/sqrt(a), -1/5*sqrt(-a)*arctan(2*sqrt(-a)*x^5*sqrt((a*x^5
 + b)/x^5)/(2*a*x^5 + b))/a]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x^5}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/x^5)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a + b/x^5)*x), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x^5}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b/x^5)^(1/2),x)
[Out] int(1/x/(a+b/x^5)^(1/2),x)
```

maxima [A] time = 1.93, size = 37, normalized size = 1.37

$$-\frac{\log\left(\frac{\sqrt{a+\frac{b}{x^5}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^5}}+\sqrt{a}}\right)}{5\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/x^5)^(1/2),x, algorithm="maxima")
[Out] -1/5*log((sqrt(a + b/x^5) - sqrt(a))/(sqrt(a + b/x^5) + sqrt(a)))/sqrt(a)
```

mupad [B] time = 1.35, size = 19, normalized size = 0.70

$$\frac{2\,\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x^5}}}{\sqrt{a}}\right)}{5\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b/x^5)^(1/2)),x)
[Out] (2*atanh((a + b/x^5)^(1/2)/a^(1/2)))/(5*a^(1/2))
```

sympy [A] time = 1.44, size = 24, normalized size = 0.89

$$\frac{2\,\operatorname{asinh}\left(\frac{\sqrt{a}\,x^{\frac{5}{2}}}{\sqrt{b}}\right)}{5\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/x**5)**(1/2),x)
[Out] 2*asinh(sqrt(a)*x**(5/2)/sqrt(b))/(5*sqrt(a))
```

3.2108 $\int \frac{1}{\sqrt{-a+\frac{b}{x^5}}\, x}\, dx$

Optimal. Leaf size=29

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{\frac{b}{x^5}-a}}{\sqrt{a}}\right)}{5\sqrt{a}}$$

```
[Out] -2/5*arctan((-a+b/x^5)^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {266, 63, 205}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{\frac{b}{x^5}-a}}{\sqrt{a}}\right)}{5\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-a + b/x^5]*x),x]
```

```
[Out] (-2*ArcTan[Sqrt[-a + b/x^5]/Sqrt[a]])/(5*Sqrt[a])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{\sqrt{-a+\frac{b}{x^5}}\,x}\,dx = -\left(\frac{1}{5}\,\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx}}\,dx, x, \frac{1}{x^5}\right)\right)$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+\frac{b}{x^5}}\right)}{5b}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{-a+\frac{b}{x^5}}}{\sqrt{a}}\right)}{5\sqrt{a}}$$

Mathematica [B] time = 0.04, size = 65, normalized size = 2.24

$$\frac{2\sqrt{ax^5-b}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x^{5/2}}{\sqrt{ax^5-b}}\right)}{5\sqrt{a}\,x^{5/2}\sqrt{\frac{b}{x^5}-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-a + b/x^5]*x),x]
```

```
[Out] (2*Sqrt[-b + a*x^5]*ArcTanh[(Sqrt[a]*x^(5/2))/Sqrt[-b + a*x^5]])/(5*Sqrt[a]
*Sqrt[-a + b/x^5]*x^(5/2))
```

fricas [A] time = 2.78, size = 111, normalized size = 3.83

$$\left[-\frac{\sqrt{-a}\,\log\left(-8\,a^2x^{10}+8\,abx^5-b^2+4\left(2\,ax^{10}-bx^5\right)\sqrt{-a}\,\sqrt{-\frac{ax^5-b}{x^5}}\right)}{10\,a}, -\frac{\arctan\left(\frac{2\,\sqrt{a}\,x^5\sqrt{-\frac{ax^5-b}{x^5}}}{2\,ax^5-b}\right)}{5\,\sqrt{a}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b/x^5)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/10*sqrt(-a)*log(-8*a^2*x^10 + 8*a*b*x^5 - b^2 + 4*(2*a*x^10 - b*x^5)*sq
rt(-a)*sqrt(-(a*x^5 - b)/x^5))/a, -1/5*arctan(2*sqrt(a)*x^5*sqrt(-(a*x^5 -
b)/x^5)/(2*a*x^5 - b))/sqrt(a)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a+\frac{b}{x^5}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b/x^5)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-a + b/x^5)*x), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a+\frac{b}{x^5}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a+b/x^5)^(1/2),x)
```

```
[Out] int(1/x/(-a+b/x^5)^(1/2),x)
```

maxima [A] time = 1.96, size = 21, normalized size = 0.72

$$-\frac{2\arctan\left(\frac{\sqrt{-a+\frac{b}{x^5}}}{\sqrt{a}}\right)}{5\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b/x^5)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/5*arctan(sqrt(-a + b/x^5)/sqrt(a))/sqrt(a)
```

mupad [B] time = 1.31, size = 21, normalized size = 0.72

$$-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{\frac{b}{x^5}-a}}{\sqrt{a}}\right)}{5\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b/x^5 - a)^(1/2)),x)
```

```
[Out] -(2*atan((b/x^5 - a)^(1/2)/a^(1/2)))/(5*a^(1/2))
```

sympy [A] time = 1.58, size = 60, normalized size = 2.07

$$\begin{cases} -\frac{2i\,\mathrm{acosh}\left(\frac{\sqrt{a}x^{\frac{5}{2}}}{\sqrt{b}}\right)}{5\sqrt{a}} & \text{for } \left|\frac{ax^5}{b}\right| > 1 \\ \frac{2\,\mathrm{asin}\left(\frac{\sqrt{a}x^{\frac{5}{2}}}{\sqrt{b}}\right)}{5\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b/x**5)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(a)*x**(5/2)/sqrt(b))/(5*sqrt(a)), Abs(a*x**5/b)
> 1), (2*asin(sqrt(a)*x**(5/2)/sqrt(b))/(5*sqrt(a)), True))
```

3.2109 $\int \frac{1}{a+\frac{b}{x^6}}\,dx$

Optimal. Leaf size=220

$$\frac{\sqrt[6]{b}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{a}x^2+\sqrt[3]{b}\right)}{4\sqrt{3}a^{7/6}}-\frac{\sqrt[6]{b}\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{a}x^2+\sqrt[3]{b}\right)}{4\sqrt{3}a^{7/6}}-\frac{\sqrt[6]{b}\tan^{-1}\left(\frac{\sqrt[6]{a}x}{\sqrt[6]{b}}\right)}{3a^{7/6}}+\frac{\sqrt[6]{b}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}x}{\sqrt[6]{b}}\right)}{6a^{7/6}}$$

```
[Out] x/a-1/3*b^(1/6)*arctan(a^(1/6)*x/b^(1/6))/a^(7/6)+1/6*b^(1/6)*arctan((-2*a^
(1/6)*x+b^(1/6)*3^(1/2))/b^(1/6))/a^(7/6)-1/6*b^(1/6)*arctan((2*a^(1/6)*x+b
^(1/6)*3^(1/2))/b^(1/6))/a^(7/6)+1/12*b^(1/6)*ln(b^(1/3)+a^(1/3)*x^2-a^(1/6
)*b^(1/6)*x*3^(1/2))/a^(7/6)*3^(1/2)-1/12*b^(1/6)*ln(b^(1/3)+a^(1/3)*x^2+a^
(1/6)*b^(1/6)*x*3^(1/2))/a^(7/6)*3^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.889$, Rules used = {193, 321, 209, 634, 618, 204, 628, 205}

$$\frac{\sqrt[6]{b}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{a}x^2+\sqrt[3]{b}\right)}{4\sqrt{3}a^{7/6}}-\frac{\sqrt[6]{b}\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{a}x^2+\sqrt[3]{b}\right)}{4\sqrt{3}a^{7/6}}-\frac{\sqrt[6]{b}\tan^{-1}\left(\frac{\sqrt[6]{a}x}{\sqrt[6]{b}}\right)}{3a^{7/6}}+\frac{\sqrt[6]{b}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}x}{\sqrt[6]{b}}\right)}{6a^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^6)^(-1),x]
```

```
[Out] x/a - (b^(1/6)*ArcTan[(a^(1/6)*x)/b^(1/6)])/(3*a^(7/6)) + (b^(1/6)*ArcTan[(
Sqrt[3]*b^(1/6) - 2*a^(1/6)*x)/b^(1/6)])/(6*a^(7/6)) - (b^(1/6)*ArcTan[(Sqr
t[3]*b^(1/6) + 2*a^(1/6)*x)/b^(1/6)])/(6*a^(7/6)) + (b^(1/6)*Log[b^(1/3) -
Sqrt[3]*a^(1/6)*b^(1/6)*x + a^(1/3)*x^2])/(4*Sqrt[3]*a^(7/6)) - (b^(1/6)*Lo
g[b^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*x + a^(1/3)*x^2])/(4*Sqrt[3]*a^(7/6))
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{a+\frac{b}{x^6}}\,dx = \int \frac{x^6}{b+ax^6}\,dx$$

$$= \frac{x}{a} - \frac{b\int \frac{1}{b+ax^6}\,dx}{a}$$

$$= \frac{x}{a} - \frac{\sqrt[6]{b}\int \frac{\sqrt[6]{b}-\frac{1}{2}\sqrt{3}\,\sqrt[6]{a}\,x}{\sqrt[3]{b}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}\,x^2}\,dx}{3a} - \frac{\sqrt[6]{b}\int \frac{\sqrt[6]{b}+\frac{1}{2}\sqrt{3}\,\sqrt[6]{a}\,x}{\sqrt[3]{b}+\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}\,x^2}\,dx}{3a} - \frac{\sqrt[3]{b}\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{a}\,x^2}\,dx}{3a}$$

$$= \frac{x}{a} - \frac{\sqrt[6]{b}\,\tan^{-1}\left(\frac{\sqrt[6]{a}\,x}{\sqrt[6]{b}}\right)}{3a^{7/6}} + \frac{\sqrt[6]{b}\int \frac{-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}+2\sqrt[3]{a}\,x}{\sqrt[3]{b}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}\,x^2}\,dx}{4\sqrt{3}\,a^{7/6}} - \frac{\sqrt[6]{b}\int \frac{\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}+2\sqrt[3]{a}\,x}{\sqrt[3]{b}+\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}\,x^2}\,dx}{4\sqrt{3}\,a^{7/6}} - \frac{\sqrt[3]{b}\int \frac{}{\sqrt[3]{b}-}}$$

$$= \frac{x}{a} - \frac{\sqrt[6]{b}\,\tan^{-1}\left(\frac{\sqrt[6]{a}\,x}{\sqrt[6]{b}}\right)}{3a^{7/6}} + \frac{\sqrt[6]{b}\,\log\left(\sqrt[3]{b}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}\,x^2\right)}{4\sqrt{3}\,a^{7/6}} - \frac{\sqrt[6]{b}\,\log\left(\sqrt[3]{b}+\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}\,\right.}{4\sqrt{3}\,a^{7/6}}$$

$$= \frac{x}{a} - \frac{\sqrt[6]{b}\,\tan^{-1}\left(\frac{\sqrt[6]{a}\,x}{\sqrt[6]{b}}\right)}{3a^{7/6}} + \frac{\sqrt[6]{b}\,\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{a}\,x}{\sqrt[6]{b}}\right)}{6a^{7/6}} - \frac{\sqrt[6]{b}\,\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{a}\,x}{\sqrt[6]{b}}\right)}{6a^{7/6}} + \frac{\sqrt[6]{b}\,\log\left(\sqrt[3]{b}-\sqrt{3}\,\sqrt[6]{}\right.}{4\sqrt{3}\,a}$$

Mathematica [A] time = 0.08, size = 182, normalized size = 0.83

$$\frac{\sqrt{3}\,\sqrt[6]{b}\,\log\left(-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}\,x^2+\sqrt[3]{b}\right)-\sqrt{3}\,\sqrt[6]{b}\,\log\left(\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{a}\,x^2+\sqrt[3]{b}\right)-4\sqrt[6]{b}\,\tan^{-1}\left(\frac{\sqrt[6]{a}\,x}{\sqrt[6]{b}}\right)+2\sqrt[6]{b}}{12a^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^6)^(-1),x]
```

```
[Out] (12*a^(1/6)*x - 4*b^(1/6)*ArcTan[(a^(1/6)*x)/b^(1/6)] + 2*b^(1/6)*ArcTan[Sq
rt[3] - (2*a^(1/6)*x)/b^(1/6)] - 2*b^(1/6)*ArcTan[Sqrt[3] + (2*a^(1/6)*x)/b
^(1/6)] + Sqrt[3]*b^(1/6)*Log[b^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*x + a^(1/3)
*x^2] - Sqrt[3]*b^(1/6)*Log[b^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*x + a^(1/3)*x
^2])/(12*a^(7/6))
```

fricas [B] time = 1.30, size = 314, normalized size = 1.43

$$-\frac{4\sqrt{3}\,a\left(-\frac{b}{a^7}\right)^{\frac{1}{6}}\arctan\left(-\frac{2\sqrt{3}\,a^6x\left(-\frac{b}{a^7}\right)^{\frac{5}{6}}-2\sqrt{3}\sqrt{a^2\left(-\frac{b}{a^7}\right)^{\frac{1}{3}}+ax\left(-\frac{b}{a^7}\right)^{\frac{1}{6}}+x^2}\,a^6\left(-\frac{b}{a^7}\right)^{\frac{5}{6}}-\sqrt{3}\,b}{3\,b}\right)+4\sqrt{3}\,a\left(-\frac{b}{a^7}\right)^{\frac{1}{6}}\arctan\left(-\frac{2\sqrt{3}}{}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^6),x, algorithm="fricas")
```

```
[Out] -1/12*(4*sqrt(3)*a*(-b/a^7)^(1/6)*arctan(-1/3*(2*sqrt(3)*a^6*x*(-b/a^7)^(5/
6) - 2*sqrt(3)*sqrt(a^2*(-b/a^7)^(1/3) + a*x*(-b/a^7)^(1/6) + x^2)*a^6*(-b/
a^7)^(5/6) - sqrt(3)*b)/b) + 4*sqrt(3)*a*(-b/a^7)^(1/6)*arctan(-1/3*(2*sqrt
(3)*a^6*x*(-b/a^7)^(5/6) - 2*sqrt(3)*sqrt(a^2*(-b/a^7)^(1/3) - a*x*(-b/a^7)
^(1/6) + x^2)*a^6*(-b/a^7)^(5/6) + sqrt(3)*b)/b) + a*(-b/a^7)^(1/6)*log(a^2
*(-b/a^7)^(1/3) + a*x*(-b/a^7)^(1/6) + x^2) - a*(-b/a^7)^(1/6)*log(a^2*(-b/
a^7)^(1/3) - a*x*(-b/a^7)^(1/6) + x^2) + 2*a*(-b/a^7)^(1/6)*log(a*(-b/a^7)^
(1/6) + x) - 2*a*(-b/a^7)^(1/6)*log(-a*(-b/a^7)^(1/6) + x) - 12*x)/a
```

giac [A] time = 0.17, size = 180, normalized size = 0.82

$$\frac{x}{a}-\frac{\sqrt{3}\,\left(a^5b\right)^{\frac{1}{6}}\log\left(x^2+\sqrt{3}\,x\left(\frac{b}{a}\right)^{\frac{1}{6}}+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{12\,a^2}+\frac{\sqrt{3}\,\left(a^5b\right)^{\frac{1}{6}}\log\left(x^2-\sqrt{3}\,x\left(\frac{b}{a}\right)^{\frac{1}{6}}+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{12\,a^2}-\frac{\left(a^5b\right)^{\frac{1}{6}}\arctan\left(\frac{2\,x+\sqrt{3}\left(\right.}{\left(\frac{b}{a}\right)^{\frac{1}{6}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^6),x, algorithm="giac")
```

```
[Out] x/a - 1/12*sqrt(3)*(a^5*b)^(1/6)*log(x^2 + sqrt(3)*x*(b/a)^(1/6) + (b/a)^(1
/3))/a^2 + 1/12*sqrt(3)*(a^5*b)^(1/6)*log(x^2 - sqrt(3)*x*(b/a)^(1/6) + (b/
a)^(1/3))/a^2 - 1/6*(a^5*b)^(1/6)*arctan((2*x + sqrt(3)*(b/a)^(1/6))/(b/a)^
(1/6))/a^2 - 1/6*(a^5*b)^(1/6)*arctan((2*x - sqrt(3)*(b/a)^(1/6))/(b/a)^(1/
6))/a^2 - 1/3*(a^5*b)^(1/6)*arctan(x/(b/a)^(1/6))/a^2
```

maple [A] time = 0.10, size = 164, normalized size = 0.75

$$\frac{x}{a}-\frac{\left(\frac{b}{a}\right)^{\frac{1}{6}}\arctan\left(\frac{x}{\left(\frac{b}{a}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\left(\frac{b}{a}\right)^{\frac{1}{6}}\arctan\left(\frac{2x}{\left(\frac{b}{a}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6a}-\frac{\left(\frac{b}{a}\right)^{\frac{1}{6}}\arctan\left(\frac{2x}{\left(\frac{b}{a}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6a}+\frac{\sqrt{3}\,\left(\frac{b}{a}\right)^{\frac{1}{6}}\ln\left(x^2-\sqrt{3}\,\left(\frac{b}{a}\right)^{\frac{1}{6}}x\right.}{12a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^6),x)
```

```
[Out] 1/a*x+1/12/a*3^(1/2)*(1/a*b)^(1/6)*ln(x^2-3^(1/2)*(1/a*b)^(1/6)*x+(1/a*b)^(
1/3))-1/6/a*(1/a*b)^(1/6)*arctan(2*x/(1/a*b)^(1/6)-3^(1/2))-1/12/a*3^(1/2)*
(1/a*b)^(1/6)*ln(x^2+3^(1/2)*(1/a*b)^(1/6)*x+(1/a*b)^(1/3))-1/6/a*(1/a*b)^(
1/6)*arctan(2*x/(1/a*b)^(1/6)+3^(1/2))-1/3/a*(1/a*b)^(1/6)*arctan(x/(1/a*b)
^(1/6))
```

maxima [A] time = 1.77, size = 194, normalized size = 0.88

$$-\frac{\frac{\sqrt{3}\,b^{\frac{1}{6}}\log\left(a^{\frac{1}{3}}x^2+\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}x+b^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}}-\frac{\sqrt{3}\,b^{\frac{1}{6}}\log\left(a^{\frac{1}{3}}x^2-\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}x+b^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}}+\frac{4\,b^{\frac{1}{3}}\arctan\left(\frac{a^{\frac{1}{3}}x}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{2\,b^{\frac{1}{3}}\arctan\left(\frac{2a^{\frac{1}{3}}x+\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{2\,b^{\frac{1}{3}}\arctan}{}}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^6),x, algorithm="maxima")
```

```
[Out] -1/12*(sqrt(3)*b^(1/6)*log(a^(1/3)*x^2 + sqrt(3)*a^(1/6)*b^(1/6)*x + b^(1/3
))/a^(1/6) - sqrt(3)*b^(1/6)*log(a^(1/3)*x^2 - sqrt(3)*a^(1/6)*b^(1/6)*x +
b^(1/3))/a^(1/6) + 4*b^(1/3)*arctan(a^(1/3)*x/sqrt(a^(1/3)*b^(1/3)))/sqrt(a
^(1/3)*b^(1/3)) + 2*b^(1/3)*arctan((2*a^(1/3)*x + sqrt(3)*a^(1/6)*b^(1/6))/
sqrt(a^(1/3)*b^(1/3)))/sqrt(a^(1/3)*b^(1/3)) + 2*b^(1/3)*arctan((2*a^(1/3)*
x - sqrt(3)*a^(1/6)*b^(1/6))/sqrt(a^(1/3)*b^(1/3)))/sqrt(a^(1/3)*b^(1/3)))/
a + x/a
```

mupad [B] time = 1.23, size = 227, normalized size = 1.03

$$\frac{x}{a}+\frac{(-b)^{1/6}\,\mathrm{atan}\left(\frac{a^{1/6}\,x\,1\mathrm{i}}{(-b)^{1/6}}\right)\,1\mathrm{i}}{3\,a^{7/6}}+\frac{(-b)^{1/6}\,\mathrm{atan}\left(\frac{(-b)^{25/6}\,x\,1\mathrm{i}}{a^{1/6}\left(\frac{(-b)^{13/3}}{a^{1/3}}+\frac{\sqrt{3}\,(-b)^{13/3}\,1\mathrm{i}}{a^{1/3}}\right)}+\frac{\sqrt{3}\,(-b)^{25/6}\,x}{a^{1/6}\left(\frac{(-b)^{13/3}}{a^{1/3}}+\frac{\sqrt{3}\,(-b)^{13/3}\,1\mathrm{i}}{a^{1/3}}\right)}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)1\mathrm{i}\quad(-b}{3\,a^{7/6}}-\frac{}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x^6),x)
```

```
[Out] x/a + ((-b)^(1/6)*atan((a^(1/6)*x*1i)/(-b)^(1/6))*1i)/(3*a^(7/6)) + ((-b)^(
1/6)*atan(((-b)^(25/6)*x*1i)/(a^(1/6)*((-b)^(13/3)/a^(1/3) + (3^(1/2)*(-b)^
(13/3)*1i)/a^(1/3))) + (3^(1/2)*(-b)^(25/6)*x)/(a^(1/6)*((-b)^(13/3)/a^(1/3
) + (3^(1/2)*(-b)^(13/3)*1i)/a^(1/3))))*((3^(1/2)*1i)/2 - 1/2)*1i)/(3*a^(7/
6)) - ((-b)^(1/6)*atan(((-b)^(25/6)*x*1i)/(a^(1/6)*((-b)^(13/3)/a^(1/3) - (
3^(1/2)*(-b)^(13/3)*1i)/a^(1/3))) - (3^(1/2)*(-b)^(25/6)*x)/(a^(1/6)*((-b)^
(13/3)/a^(1/3) - (3^(1/2)*(-b)^(13/3)*1i)/a^(1/3))))*((3^(1/2)*1i)/2 + 1/2)
*1i)/(3*a^(7/6))
```

sympy [A] time = 0.21, size = 22, normalized size = 0.10

$$\mathrm{RootSum}\left(46656t^6a^7+b,\left(t\mapsto t\log\left(-6ta+x\right)\right)\right)+\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**6),x)
```

```
[Out] RootSum(46656*_t**6*a**7 + b, Lambda(_t, _t*log(-6*_t*a + x))) + x/a
```

3.2110 $\int \frac{1}{a+\frac{b}{x^8}}\,dx$

Optimal. Leaf size=272

$$-\frac{\sqrt[8]{b}\,\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{-a}\,x^2+\sqrt[4]{b}\right)}{8\sqrt{2}\,(-a)^{9/8}}+\frac{\sqrt[8]{b}\,\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{-a}\,x^2+\sqrt[4]{b}\right)}{8\sqrt{2}\,(-a)^{9/8}}+\frac{\sqrt[8]{b}\,\tan^{-1}\left(\frac{\sqrt[8]{-a}\,x}{\sqrt[8]{b}}\right)}{4(-a)^{9/8}}-\frac{\sqrt[8]{b}}{}$$

```
[Out] x/a+1/4*b^(1/8)*arctan((-a)^(1/8)*x/b^(1/8))/(-a)^(9/8)+1/4*b^(1/8)*arctanh
((-a)^(1/8)*x/b^(1/8))/(-a)^(9/8)+1/8*b^(1/8)*arctan(-1+(-a)^(1/8)*x*2^(1/2
)/b^(1/8))/(-a)^(9/8)*2^(1/2)+1/8*b^(1/8)*arctan(1+(-a)^(1/8)*x*2^(1/2)/b^(
1/8))/(-a)^(9/8)*2^(1/2)-1/16*b^(1/8)*ln(b^(1/4)+(-a)^(1/4)*x^2-(-a)^(1/8)*
b^(1/8)*x*2^(1/2))/(-a)^(9/8)*2^(1/2)+1/16*b^(1/8)*ln(b^(1/4)+(-a)^(1/4)*x^
2+(-a)^(1/8)*b^(1/8)*x*2^(1/2))/(-a)^(9/8)*2^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 1.333$, Rules used = {193, 321, 214, 212, 208, 205, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\sqrt[8]{b}\,\log\left(-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{-a}\,x^2+\sqrt[4]{b}\right)}{8\sqrt{2}\,(-a)^{9/8}}+\frac{\sqrt[8]{b}\,\log\left(\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{-a}\,x^2+\sqrt[4]{b}\right)}{8\sqrt{2}\,(-a)^{9/8}}+\frac{\sqrt[8]{b}\,\tan^{-1}\left(\frac{\sqrt[8]{-a}\,x}{\sqrt[8]{b}}\right)}{4(-a)^{9/8}}-\frac{\sqrt[8]{b}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^8)^(-1),x]
```

```
[Out] x/a + (b^(1/8)*ArcTan[((-a)^(1/8)*x)/b^(1/8)])/(4*(-a)^(9/8)) - (b^(1/8)*Ar
cTan[1 - (Sqrt[2]*(-a)^(1/8)*x)/b^(1/8)])/(4*Sqrt[2]*(-a)^(9/8)) + (b^(1/8)
*ArcTan[1 + (Sqrt[2]*(-a)^(1/8)*x)/b^(1/8)])/(4*Sqrt[2]*(-a)^(9/8)) + (b^(1
/8)*ArcTanh[((-a)^(1/8)*x)/b^(1/8)])/(4*(-a)^(9/8)) - (b^(1/8)*Log[b^(1/4)
- Sqrt[2]*(-a)^(1/8)*b^(1/8)*x + (-a)^(1/4)*x^2])/(8*Sqrt[2]*(-a)^(9/8)) +
(b^(1/8)*Log[b^(1/4) + Sqrt[2]*(-a)^(1/8)*b^(1/8)*x + (-a)^(1/4)*x^2])/(8*S
qrt[2]*(-a)^(9/8))
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{a+\frac{b}{x^8}}\,dx = \int \frac{x^8}{b+ax^8}\,dx$$

$$= \frac{x}{a} - \frac{b\int \frac{1}{b+ax^8}\,dx}{a}$$

$$= \frac{x}{a} - \frac{\sqrt{b}\int \frac{1}{\sqrt{b}-\sqrt{-a}\,x^4}\,dx}{2a} - \frac{\sqrt{b}\int \frac{1}{\sqrt{b}+\sqrt{-a}\,x^4}\,dx}{2a}$$

$$= \frac{x}{a} - \frac{\sqrt[4]{b}\int \frac{1}{\sqrt[4]{b}-\sqrt[4]{-a}\,x^2}\,dx}{4a} - \frac{\sqrt[4]{b}\int \frac{1}{\sqrt[4]{b}+\sqrt[4]{-a}\,x^2}\,dx}{4a} - \frac{\sqrt[4]{b}\int \frac{\sqrt[4]{b}-\sqrt[4]{-a}\,x^2}{\sqrt{b}+\sqrt{-a}\,x^4}\,dx}{4a} - \frac{\sqrt[4]{b}\int \frac{\sqrt[4]{b}+\sqrt[4]{-a}\,x^2}{\sqrt{b}+\sqrt{-a}\,x^4}\,dx}{4a}$$

$$= \frac{x}{a} + \frac{\sqrt[8]{b}\,\tan^{-1}\left(\frac{\sqrt[8]{-a}\,x}{\sqrt[8]{b}}\right)}{4(-a)^{9/8}} + \frac{\sqrt[8]{b}\,\tanh^{-1}\left(\frac{\sqrt[8]{-a}\,x}{\sqrt[8]{b}}\right)}{4(-a)^{9/8}} - \frac{\sqrt[8]{b}\int \frac{\frac{\sqrt{2}\,\sqrt[8]{b}}{\sqrt[8]{-a}}+2x}{-\frac{\sqrt[4]{b}}{\sqrt[4]{-a}}-\frac{\sqrt{2}\,\sqrt[8]{b}\,x}{\sqrt[8]{-a}}-x^2}\,dx}{8\sqrt{2}\,(-a)^{9/8}} - \frac{\sqrt[8]{b}\int \frac{\frac{\sqrt{2}\,\sqrt[8]{b}}{\sqrt[8]{-a}}-2x}{-\frac{\sqrt[4]{b}}{\sqrt[4]{-a}}+\frac{\sqrt{2}\,\sqrt[8]{b}\,x}{\sqrt[8]{-a}}-x^2}\,dx}{8\sqrt{2}\,(-a)^{9/8}}$$

$$= \frac{x}{a} + \frac{\sqrt[8]{b}\,\tan^{-1}\left(\frac{\sqrt[8]{-a}\,x}{\sqrt[8]{b}}\right)}{4(-a)^{9/8}} + \frac{\sqrt[8]{b}\,\tanh^{-1}\left(\frac{\sqrt[8]{-a}\,x}{\sqrt[8]{b}}\right)}{4(-a)^{9/8}} - \frac{\sqrt[8]{b}\,\log\left(\sqrt[4]{b}-\sqrt{2}\,\sqrt[8]{-a}\,\sqrt[8]{b}\,x+\sqrt[4]{-a}\,x^2\right)}{8\sqrt{2}\,(-a)^{9/8}} + \frac{\sqrt[8]{b}}{}$$

$$= \frac{x}{a} + \frac{\sqrt[8]{b}\,\tan^{-1}\left(\frac{\sqrt[8]{-a}\,x}{\sqrt[8]{b}}\right)}{4(-a)^{9/8}} - \frac{\sqrt[8]{b}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[8]{-a}\,x}{\sqrt[8]{b}}\right)}{4\sqrt{2}\,(-a)^{9/8}} + \frac{\sqrt[8]{b}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[8]{-a}\,x}{\sqrt[8]{b}}\right)}{4\sqrt{2}\,(-a)^{9/8}} + \frac{\sqrt[8]{b}\,\tanh^{-1}\left(\frac{\sqrt[8]{}}{}\right.}{4(-a)^{9/8}}$$

Mathematica [A] time = 0.34, size = 367, normalized size = 1.35

$$\frac{\sqrt[8]{b}\,\sin\left(\frac{\pi}{8}\right)\log\left(-2\sqrt[8]{a}\,\sqrt[8]{b}\,x\sin\left(\frac{\pi}{8}\right)+\sqrt[4]{a}\,x^2+\sqrt[4]{b}\right)-\sqrt[8]{b}\,\sin\left(\frac{\pi}{8}\right)\log\left(2\sqrt[8]{a}\,\sqrt[8]{b}\,x\sin\left(\frac{\pi}{8}\right)+\sqrt[4]{a}\,x^2+\sqrt[4]{b}\right)+\sqrt[8]{b}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^8)^(-1),x]
```

```
[Out] (8*a^(1/8)*x - 2*b^(1/8)*ArcTan[(a^(1/8)*x*Sec[Pi/8])/b^(1/8) - Tan[Pi/8]]*
Cos[Pi/8] - 2*b^(1/8)*ArcTan[(a^(1/8)*x*Sec[Pi/8])/b^(1/8) + Tan[Pi/8]]*Cos
[Pi/8] + b^(1/8)*Cos[Pi/8]*Log[b^(1/4) + a^(1/4)*x^2 - 2*a^(1/8)*b^(1/8)*x*
Cos[Pi/8]] - b^(1/8)*Cos[Pi/8]*Log[b^(1/4) + a^(1/4)*x^2 + 2*a^(1/8)*b^(1/8
)*x*Cos[Pi/8]] + 2*b^(1/8)*ArcTan[Cot[Pi/8] - (a^(1/8)*x*Csc[Pi/8])/b^(1/8)
]*Sin[Pi/8] - 2*b^(1/8)*ArcTan[Cot[Pi/8] + (a^(1/8)*x*Csc[Pi/8])/b^(1/8)]*S
in[Pi/8] + b^(1/8)*Log[b^(1/4) + a^(1/4)*x^2 - 2*a^(1/8)*b^(1/8)*x*Sin[Pi/8
]]*Sin[Pi/8] - b^(1/8)*Log[b^(1/4) + a^(1/4)*x^2 + 2*a^(1/8)*b^(1/8)*x*Sin[
Pi/8]]*Sin[Pi/8])/(8*a^(9/8))
```

fricas [B] time = 1.01, size = 385, normalized size = 1.42

$$\frac{4\sqrt{2}\,a\left(-\frac{b}{a^9}\right)^{\frac{1}{8}}\arctan\left(-\frac{\sqrt{2}\,a^8x\left(-\frac{b}{a^9}\right)^{\frac{7}{8}}-\sqrt{2}\,\sqrt{\sqrt{2}\,ax\left(-\frac{b}{a^9}\right)^{\frac{1}{8}}+a^2\left(-\frac{b}{a^9}\right)^{\frac{1}{4}}+x^2}\,a^8\left(-\frac{b}{a^9}\right)^{\frac{7}{8}}-b}{b}\right)+4\sqrt{2}\,a\left(-\frac{b}{a^9}\right)^{\frac{1}{8}}\arctan\left(-\frac{\sqrt{2}\,a^8x\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^8),x, algorithm="fricas")
```

```
[Out] -1/16*(4*sqrt(2)*a*(-b/a^9)^(1/8)*arctan(-(sqrt(2)*a^8*x*(-b/a^9)^(7/8) - s
qrt(2)*sqrt(sqrt(2)*a*x*(-b/a^9)^(1/8) + a^2*(-b/a^9)^(1/4) + x^2)*a^8*(-b/
```

```
a^9)^(7/8) - b)/b) + 4*sqrt(2)*a*(-b/a^9)^(1/8)*arctan(-(sqrt(2)*a^8*x*(-b/
a^9)^(7/8) - sqrt(2)*sqrt(-sqrt(2)*a*x*(-b/a^9)^(1/8) + a^2*(-b/a^9)^(1/4)
+ x^2)*a^8*(-b/a^9)^(7/8) + b)/b) + sqrt(2)*a*(-b/a^9)^(1/8)*log(sqrt(2)*a*
x*(-b/a^9)^(1/8) + a^2*(-b/a^9)^(1/4) + x^2) - sqrt(2)*a*(-b/a^9)^(1/8)*log
(-sqrt(2)*a*x*(-b/a^9)^(1/8) + a^2*(-b/a^9)^(1/4) + x^2) + 8*a*(-b/a^9)^(1/
8)*arctan(-(a^8*x*(-b/a^9)^(7/8) - sqrt(a^2*(-b/a^9)^(1/4) + x^2)*a^8*(-b/a
^9)^(7/8))/b) + 2*a*(-b/a^9)^(1/8)*log(a*(-b/a^9)^(1/8) + x) - 2*a*(-b/a^9)
^(1/8)*log(-a*(-b/a^9)^(1/8) + x) - 16*x)/a
```

giac [B] time = 0.21, size = 442, normalized size = 1.62

$$\frac{x}{a}-\frac{\left(\frac{b}{a}\right)^{\frac{1}{8}}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}}\right)}{4a\sqrt{-2\sqrt{2}+4}}-\frac{\left(\frac{b}{a}\right)^{\frac{1}{8}}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}}\right)}{4a\sqrt{-2\sqrt{2}+4}}-\frac{\left(\frac{b}{a}\right)^{\frac{1}{8}}\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}}\right)}{4a\sqrt{2\sqrt{2}+4}}-\frac{\left(\frac{b}{a}\right)^{\frac{1}{8}}\arctan\left($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^8),x, algorithm="giac")
```

```
[Out] x/a - 1/4*(b/a)^(1/8)*arctan((2*x + sqrt(-sqrt(2) + 2)*(b/a)^(1/8))/(sqrt(s
qrt(2) + 2)*(b/a)^(1/8)))/(a*sqrt(-2*sqrt(2) + 4)) - 1/4*(b/a)^(1/8)*arctan
((2*x - sqrt(-sqrt(2) + 2)*(b/a)^(1/8))/(sqrt(sqrt(2) + 2)*(b/a)^(1/8)))/(a
*sqrt(-2*sqrt(2) + 4)) - 1/4*(b/a)^(1/8)*arctan((2*x + sqrt(sqrt(2) + 2)*(b
/a)^(1/8))/(sqrt(-sqrt(2) + 2)*(b/a)^(1/8)))/(a*sqrt(2*sqrt(2) + 4)) - 1/4*
(b/a)^(1/8)*arctan((2*x - sqrt(sqrt(2) + 2)*(b/a)^(1/8))/(sqrt(-sqrt(2) + 2
)*(b/a)^(1/8)))/(a*sqrt(2*sqrt(2) + 4)) - 1/8*(b/a)^(1/8)*log(x^2 + x*sqrt(
sqrt(2) + 2)*(b/a)^(1/8) + (b/a)^(1/4))/(a*sqrt(-2*sqrt(2) + 4)) + 1/8*(b/a
)^(1/8)*log(x^2 - x*sqrt(sqrt(2) + 2)*(b/a)^(1/8) + (b/a)^(1/4))/(a*sqrt(-2
*sqrt(2) + 4)) - 1/8*(b/a)^(1/8)*log(x^2 + x*sqrt(-sqrt(2) + 2)*(b/a)^(1/8)
 + (b/a)^(1/4))/(a*sqrt(2*sqrt(2) + 4)) + 1/8*(b/a)^(1/8)*log(x^2 - x*sqrt(
-sqrt(2) + 2)*(b/a)^(1/8) + (b/a)^(1/4))/(a*sqrt(2*sqrt(2) + 4))
```

maple [C] time = 0.02, size = 34, normalized size = 0.12

$$\frac{x}{a}-\frac{b\ln\left(-\operatorname{RootOf}\left(a_Z^8+b\right)+x\right)}{8a^2\operatorname{RootOf}\left(a_Z^8+b\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^8),x)
```

```
[Out] 1/a*x-1/8/a^2*b*sum(1/_R^7*ln(-_R+x),_R=RootOf(_Z^8*a+b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{8}b\left(\frac{2\left(\frac{b}{a}\right)^{\frac{1}{8}}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}}\right)}{b\sqrt{-2\sqrt{2}+4}}+\frac{2\left(\frac{b}{a}\right)^{\frac{1}{8}}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}}\right)}{b\sqrt{-2\sqrt{2}+4}}+\frac{2\left(\frac{b}{a}\right)^{\frac{1}{8}}\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}}\right)}{b\sqrt{2\sqrt{2}+4}}+\frac{2\left(\frac{b}{a}\right)^{\frac{1}{8}}\arctan\left($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^8),x, algorithm="maxima")
```

```
[Out] -b*integrate(1/(a*x^8 + b), x)/a + x/a
```

mupad [B] time = 1.21, size = 115, normalized size = 0.42

$$\frac{x}{a}-\frac{(-b)^{1/8}\operatorname{atan}\left(\frac{a^{1/8}x}{(-b)^{1/8}}\right)}{4a^{9/8}}+\frac{(-b)^{1/8}\operatorname{atan}\left(\frac{a^{1/8}x\,1\mathrm{i}}{(-b)^{1/8}}\right)1\mathrm{i}}{4a^{9/8}}+\frac{\sqrt{2}\,(-b)^{1/8}\operatorname{atan}\left(\frac{\sqrt{2}\,a^{1/8}x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-b)^{1/8}}\right)\left(-\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)}{a^{9/8}}+\frac{\sqrt{2}\,(-b)^{1/8}\,\mathrm{ata}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x^8),x)
```

```
[Out] x/a - ((-b)^(1/8)*atan((a^(1/8)*x)/(-b)^(1/8)))/(4*a^(9/8)) + ((-b)^(1/8)*a
tan((a^(1/8)*x*1i)/(-b)^(1/8))*1i)/(4*a^(9/8)) - (2^(1/2)*(-b)^(1/8)*atan((
2^(1/2)*a^(1/8)*x*(1/2 - 1i/2))/(-b)^(1/8))*(1/8 + 1i/8))/a^(9/8) - (2^(1/2
)*(-b)^(1/8)*atan((2^(1/2)*a^(1/8)*x*(1/2 + 1i/2))/(-b)^(1/8))*(1/8 - 1i/8)
)/a^(9/8)
```

sympy [A] time = 0.22, size = 22, normalized size = 0.08

$$\operatorname{RootSum}\left(16777216t^8a^9+b,\left(t\mapsto t\log\left(-8ta+x\right)\right)\right)+\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**8),x)
```

```
[Out] RootSum(16777216*_t**8*a**9 + b, Lambda(_t, _t*log(-8*_t*a + x))) + x/a
```

3.2111 $\int \left(a + b\sqrt{x}\right) x^4 \, dx$

Optimal. Leaf size=19

$$\frac{ax^5}{5} + \frac{2}{11} bx^{11/2}$$

```
[Out] 1/5*a*x^5+2/11*b*x^(11/2)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$\frac{ax^5}{5} + \frac{2}{11} bx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])*x^4,x]
```

```
[Out] (a*x^5)/5 + (2*b*x^(11/2))/11
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \left(a + b\sqrt{x}\right) x^4 \, dx &= \int \left(ax^4 + bx^{9/2}\right) dx \\ &= \frac{ax^5}{5} + \frac{2}{11} bx^{11/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{ax^5}{5} + \frac{2}{11} bx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])*x^4,x]
```

```
[Out] (a*x^5)/5 + (2*b*x^(11/2))/11
```

fricas [A] time = 1.17, size = 13, normalized size = 0.68

$$\frac{2}{11} bx^{\frac{11}{2}} + \frac{1}{5} ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/11*b*x^(11/2) + 1/5*a*x^5
```

giac [A] time = 0.15, size = 13, normalized size = 0.68

$$\frac{2}{11} bx^{\frac{11}{2}} + \frac{1}{5} ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*x^(1/2)),x, algorithm="giac")
```

```
[Out] 2/11*b*x^(11/2) + 1/5*a*x^5
```

maple [A] time = 0.00, size = 14, normalized size = 0.74

$$\frac{2b\,x^{\frac{11}{2}}}{11}+\frac{a\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*x^(1/2)),x)
```

```
[Out] 1/5*a*x^5+2/11*b*x^(11/2)
```

maxima [B] time = 0.78, size = 166, normalized size = 8.74

$$\frac{2\left(b\sqrt{x}+a\right)^{11}}{11\,b^{10}}-\frac{9\left(b\sqrt{x}+a\right)^{10}a}{5\,b^{10}}+\frac{8\left(b\sqrt{x}+a\right)^{9}a^2}{b^{10}}-\frac{21\left(b\sqrt{x}+a\right)^{8}a^3}{b^{10}}+\frac{36\left(b\sqrt{x}+a\right)^{7}a^4}{b^{10}}-\frac{42\left(b\sqrt{x}+a\right)^{6}a^5}{b^{10}}+\frac{16}{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2/11*(b*sqrt(x) + a)^11/b^10 - 9/5*(b*sqrt(x) + a)^10*a/b^10 + 8*(b*sqrt(x)
 + a)^9*a^2/b^10 - 21*(b*sqrt(x) + a)^8*a^3/b^10 + 36*(b*sqrt(x) + a)^7*a^4
/b^10 - 42*(b*sqrt(x) + a)^6*a^5/b^10 + 168/5*(b*sqrt(x) + a)^5*a^6/b^10 -
18*(b*sqrt(x) + a)^4*a^7/b^10 + 6*(b*sqrt(x) + a)^3*a^8/b^10 - (b*sqrt(x) +
 a)^2*a^9/b^10
```

mupad [B] time = 0.03, size = 13, normalized size = 0.68

$$\frac{a\,x^5}{5}+\frac{2\,b\,x^{11/2}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^(1/2)),x)
```

```
[Out] (a*x^5)/5 + (2*b*x^(11/2))/11
```

sympy [A] time = 1.48, size = 15, normalized size = 0.79

$$\frac{ax^5}{5}+\frac{2bx^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*x**(1/2)),x)
```

```
[Out] a*x**5/5 + 2*b*x**(11/2)/11
```

3.2112 $\int \left(a + b\sqrt{x}\right) x^3 \, dx$

Optimal. Leaf size=19

$$\frac{ax^4}{4} + \frac{2}{9}bx^{9/2}$$

```
[Out] 1/4*a*x^4+2/9*b*x^(9/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$\frac{ax^4}{4} + \frac{2}{9}bx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])*x^3,x]
```

```
[Out] (a*x^4)/4 + (2*b*x^(9/2))/9
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \left(a + b\sqrt{x}\right) x^3 \, dx &= \int \left(ax^3 + bx^{7/2}\right) dx \\ &= \frac{ax^4}{4} + \frac{2}{9}bx^{9/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{ax^4}{4} + \frac{2}{9}bx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])*x^3,x]
```

```
[Out] (a*x^4)/4 + (2*b*x^(9/2))/9
```

fricas [A] time = 1.24, size = 13, normalized size = 0.68

$$\frac{2}{9}\, bx^{\frac{9}{2}} + \frac{1}{4}\, ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/9*b*x^(9/2) + 1/4*a*x^4
```

giac [A] time = 0.15, size = 13, normalized size = 0.68

$$\frac{2}{9}\, bx^{\frac{9}{2}} + \frac{1}{4}\, ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^(1/2)),x, algorithm="giac")
[Out] 2/9*b*x^(9/2) + 1/4*a*x^4
```

maple [A] time = 0.00, size = 14, normalized size = 0.74

$$\frac{2b\,x^{\frac{9}{2}}}{9}+\frac{a\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*x^(1/2)),x)
[Out] 1/4*a*x^4+2/9*b*x^(9/2)
```

maxima [B] time = 0.92, size = 132, normalized size = 6.95

$$\frac{2\left(b\sqrt{x}+a\right)^9}{9\,b^8}-\frac{7\left(b\sqrt{x}+a\right)^8 a}{4\,b^8}+\frac{6\left(b\sqrt{x}+a\right)^7 a^2}{b^8}-\frac{35\left(b\sqrt{x}+a\right)^6 a^3}{3\,b^8}+\frac{14\left(b\sqrt{x}+a\right)^5 a^4}{b^8}-\frac{21\left(b\sqrt{x}+a\right)^4 a^5}{2\,b^8}+\frac{14\left($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^(1/2)),x, algorithm="maxima")
[Out] 2/9*(b*sqrt(x) + a)^9/b^8 - 7/4*(b*sqrt(x) + a)^8*a/b^8 + 6*(b*sqrt(x) + a)
^7*a^2/b^8 - 35/3*(b*sqrt(x) + a)^6*a^3/b^8 + 14*(b*sqrt(x) + a)^5*a^4/b^8
- 21/2*(b*sqrt(x) + a)^4*a^5/b^8 + 14/3*(b*sqrt(x) + a)^3*a^6/b^8 - (b*sqrt
(x) + a)^2*a^7/b^8
```

mupad [B] time = 0.03, size = 13, normalized size = 0.68

$$\frac{a\,x^4}{4}+\frac{2\,b\,x^{9/2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^(1/2)),x)
[Out] (a*x^4)/4 + (2*b*x^(9/2))/9
```

sympy [A] time = 1.41, size = 15, normalized size = 0.79

$$\frac{ax^4}{4}+\frac{2bx^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*x**(1/2)),x)
[Out] a*x**4/4 + 2*b*x**(9/2)/9
```

3.2113 $\int \left(a + b\sqrt{x}\right) x^2\, dx$

Optimal. Leaf size=19

$$\frac{ax^3}{3} + \frac{2}{7}bx^{7/2}$$

```
[Out] 1/3*a*x^3+2/7*b*x^(7/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$\frac{ax^3}{3} + \frac{2}{7}bx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])*x^2,x]
```

```
[Out] (a*x^3)/3 + (2*b*x^(7/2))/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \left(a + b\sqrt{x}\right) x^2\, dx &= \int \left(ax^2 + bx^{5/2}\right) dx \\ &= \frac{ax^3}{3} + \frac{2}{7}bx^{7/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{ax^3}{3} + \frac{2}{7}bx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])*x^2,x]
```

```
[Out] (a*x^3)/3 + (2*b*x^(7/2))/7
```

fricas [A] time = 0.68, size = 13, normalized size = 0.68

$$\frac{2}{7}\, bx^{\frac{7}{2}} + \frac{1}{3}\, ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/7*b*x^(7/2) + 1/3*a*x^3
```

giac [A] time = 0.15, size = 13, normalized size = 0.68

$$\frac{2}{7}\, bx^{\frac{7}{2}} + \frac{1}{3}\, ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^(1/2)),x, algorithm="giac")
```

```
[Out] 2/7*b*x^(7/2) + 1/3*a*x^3
```

maple [A] time = 0.00, size = 14, normalized size = 0.74

$$\frac{2b\,x^{\frac{7}{2}}}{7}+\frac{a\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*x^(1/2)),x)
```

```
[Out] 1/3*a*x^3+2/7*b*x^(7/2)
```

maxima [B] time = 0.89, size = 98, normalized size = 5.16

$$\frac{2\left(b\sqrt{x}+a\right)^7}{7\,b^6}-\frac{5\left(b\sqrt{x}+a\right)^6 a}{3\,b^6}+\frac{4\left(b\sqrt{x}+a\right)^5 a^2}{b^6}-\frac{5\left(b\sqrt{x}+a\right)^4 a^3}{b^6}+\frac{10\left(b\sqrt{x}+a\right)^3 a^4}{3\,b^6}-\frac{\left(b\sqrt{x}+a\right)^2 a^5}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2/7*(b*sqrt(x) + a)^7/b^6 - 5/3*(b*sqrt(x) + a)^6*a/b^6 + 4*(b*sqrt(x) + a)
^5*a^2/b^6 - 5*(b*sqrt(x) + a)^4*a^3/b^6 + 10/3*(b*sqrt(x) + a)^3*a^4/b^6 -
 (b*sqrt(x) + a)^2*a^5/b^6
```

mupad [B] time = 0.02, size = 13, normalized size = 0.68

$$\frac{a\,x^3}{3}+\frac{2\,b\,x^{7/2}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^(1/2)),x)
```

```
[Out] (a*x^3)/3 + (2*b*x^(7/2))/7
```

sympy [A] time = 1.35, size = 15, normalized size = 0.79

$$\frac{ax^3}{3}+\frac{2bx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*x**(1/2)),x)
```

```
[Out] a*x**3/3 + 2*b*x**(7/2)/7
```

3.2114 $\int \left(a + b\sqrt{x}\right) x\, dx$

Optimal. Leaf size=19

$$\frac{ax^2}{2} + \frac{2}{5}bx^{5/2}$$

```
[Out] 1/2*a*x^2+2/5*b*x^(5/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$\frac{ax^2}{2} + \frac{2}{5}bx^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])*x,x]
```

```
[Out] (a*x^2)/2 + (2*b*x^(5/2))/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \left(a + b\sqrt{x}\right) x\, dx &= \int \left(ax + bx^{3/2}\right) dx \\ &= \frac{ax^2}{2} + \frac{2}{5}bx^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{ax^2}{2} + \frac{2}{5}bx^{5/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])*x,x]
```

```
[Out] (a*x^2)/2 + (2*b*x^(5/2))/5
```

fricas [A] time = 1.08, size = 13, normalized size = 0.68

$$\frac{2}{5}\, bx^{\frac{5}{2}} + \frac{1}{2}\, ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/5*b*x^(5/2) + 1/2*a*x^2
```

giac [A] time = 0.15, size = 13, normalized size = 0.68

$$\frac{2}{5}\, bx^{\frac{5}{2}} + \frac{1}{2}\, ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^(1/2)),x, algorithm="giac")
```

```
[Out] 2/5*b*x^(5/2) + 1/2*a*x^2
```

maple [A] time = 0.00, size = 14, normalized size = 0.74

$$\frac{2b\,x^{\frac{5}{2}}}{5}+\frac{a\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*x^(1/2)),x)
```

```
[Out] 1/2*a*x^2+2/5*b*x^(5/2)
```

maxima [B] time = 0.92, size = 64, normalized size = 3.37

$$\frac{2\left(b\sqrt{x}+a\right)^5}{5\,b^4}-\frac{3\left(b\sqrt{x}+a\right)^4 a}{2\,b^4}+\frac{2\left(b\sqrt{x}+a\right)^3 a^2}{b^4}-\frac{\left(b\sqrt{x}+a\right)^2 a^3}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2/5*(b*sqrt(x) + a)^5/b^4 - 3/2*(b*sqrt(x) + a)^4*a/b^4 + 2*(b*sqrt(x) + a)
^3*a^2/b^4 - (b*sqrt(x) + a)^2*a^3/b^4
```

mupad [B] time = 0.02, size = 13, normalized size = 0.68

$$\frac{a\,x^2}{2}+\frac{2\,b\,x^{5/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^(1/2)),x)
```

```
[Out] (a*x^2)/2 + (2*b*x^(5/2))/5
```

sympy [A] time = 1.19, size = 15, normalized size = 0.79

$$\frac{ax^2}{2}+\frac{2bx^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x**(1/2)),x)
```

```
[Out] a*x**2/2 + 2*b*x**(5/2)/5
```

3.2115 $\int \left(a + b\sqrt{x}\right)\, dx$

Optimal. Leaf size=14

$$ax + \frac{2}{3}bx^{3/2}$$

```
[Out] a*x+2/3*b*x^(3/2)
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$ax + \frac{2}{3}bx^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sqrt[x],x]
```

```
[Out] a*x + (2*b*x^(3/2))/3
```

Rubi steps

$$\int \left(a + b\sqrt{x}\right)\, dx = ax + \frac{2}{3}bx^{3/2}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$ax + \frac{2}{3}bx^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sqrt[x],x]
```

```
[Out] a*x + (2*b*x^(3/2))/3
```

fricas [A] time = 1.20, size = 10, normalized size = 0.71

$$\frac{2}{3}\, bx^{\frac{3}{2}} + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*b*x^(3/2) + a*x
```

giac [A] time = 0.16, size = 10, normalized size = 0.71

$$\frac{2}{3}\, bx^{\frac{3}{2}} + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*b*x^(3/2) + a*x
```

maple [A] time = 0.00, size = 11, normalized size = 0.79

$$\frac{2b\, x^{\frac{3}{2}}}{3} + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*x^(1/2),x)
[Out] a*x+2/3*b*x^(3/2)
```

maxima [A] time = 0.86, size = 10, normalized size = 0.71

$$\frac{2}{3} bx^{\frac{3}{2}} + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*x^(1/2),x, algorithm="maxima")
[Out] 2/3*b*x^(3/2) + a*x
```

mupad [B] time = 0.02, size = 10, normalized size = 0.71

$$a\,x + \frac{2\,b\,x^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*x^(1/2),x)
[Out] a*x + (2*b*x^(3/2))/3
```

sympy [A] time = 0.06, size = 12, normalized size = 0.86

$$ax + \frac{2bx^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*x**(1/2),x)
[Out] a*x + 2*b*x**(3/2)/3
```

3.2116 $\int \frac{a+b\sqrt{x}}{x}\,dx$

Optimal. Leaf size=13

$$a\log(x)+2b\sqrt{x}$$

```
[Out] a*ln(x)+2*b*x^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$a\log(x)+2b\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])/x,x]
```

```
[Out] 2*b*Sqrt[x] + a*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sqrt{x}}{x}\,dx &= \int \left(\frac{a}{x}+\frac{b}{\sqrt{x}}\right)dx\\ &= 2b\sqrt{x}+a\log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$a\log(x)+2b\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])/x,x]
```

```
[Out] 2*b*Sqrt[x] + a*Log[x]
```

fricas [A] time = 1.07, size = 14, normalized size = 1.08

$$2\,a\log\left(\sqrt{x}\right)+2\,b\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))/x,x, algorithm="fricas")
```

```
[Out] 2*a*log(sqrt(x)) + 2*b*sqrt(x)
```

giac [A] time = 0.18, size = 12, normalized size = 0.92

$$a\log\left(|x|\right)+2\,b\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))/x,x, algorithm="giac")
```

```
[Out] a*log(abs(x)) + 2*b*sqrt(x)
```

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$a\ln(x) + 2b\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))/x,x)
```

```
[Out] a*ln(x)+2*b*x^(1/2)
```

maxima [A] time = 0.92, size = 11, normalized size = 0.85

$$a\log(x) + 2\,b\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))/x,x, algorithm="maxima")
```

```
[Out] a*log(x) + 2*b*sqrt(x)
```

mupad [B] time = 0.05, size = 11, normalized size = 0.85

$$2\,b\,\sqrt{x} + a\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))/x,x)
```

```
[Out] 2*b*x^(1/2) + a*log(x)
```

sympy [A] time = 0.18, size = 12, normalized size = 0.92

$$a\log(x) + 2b\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))/x,x)
```

```
[Out] a*log(x) + 2*b*sqrt(x)
```

3.2117 $\int \frac{a+b\sqrt{x}}{x^2}\,dx$

Optimal. Leaf size=15

$$-\frac{a}{x}-\frac{2b}{\sqrt{x}}$$

```
[Out] -a/x-2*b/x^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$-\frac{a}{x}-\frac{2b}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])/x^2,x]
```

```
[Out] -(a/x) - (2*b)/Sqrt[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sqrt{x}}{x^2}\,dx &= \int \left(\frac{a}{x^2}+\frac{b}{x^{3/2}}\right)dx\\ &= -\frac{a}{x}-\frac{2b}{\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$-\frac{a}{x}-\frac{2b}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])/x^2,x]
```

```
[Out] -(a/x) - (2*b)/Sqrt[x]
```

fricas [A] time = 1.01, size = 13, normalized size = 0.87

$$-\frac{2\,b\sqrt{x}+a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))/x^2,x, algorithm="fricas")
```

```
[Out] -(2*b*sqrt(x) + a)/x
```

giac [A] time = 0.15, size = 13, normalized size = 0.87

$$-\frac{2\,b\sqrt{x}+a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))/x^2,x, algorithm="giac")
[Out] -(2*b*sqrt(x) + a)/x
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$-\frac{2b}{\sqrt{x}} - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))/x^2,x)
[Out] -a/x-2*b/x^(1/2)
```

maxima [A] time = 0.79, size = 13, normalized size = 0.87

$$-\frac{2\,b\sqrt{x} + a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))/x^2,x, algorithm="maxima")
[Out] -(2*b*sqrt(x) + a)/x
```

mupad [B] time = 0.04, size = 13, normalized size = 0.87

$$-\frac{a}{x} - \frac{2\,b}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))/x^2,x)
[Out] - a/x - (2*b)/x^(1/2)
```

sympy [A] time = 0.44, size = 12, normalized size = 0.80

$$-\frac{a}{x} - \frac{2b}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))/x**2,x)
[Out] -a/x - 2*b/sqrt(x)
```

3.2118 $\int \frac{a+b\sqrt{x}}{x^3}\,dx$

Optimal. Leaf size=19

$$-\frac{a}{2x^2}-\frac{2b}{3x^{3/2}}$$

```
[Out] -1/2*a/x^2-2/3*b/x^(3/2)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$-\frac{a}{2x^2}-\frac{2b}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])/x^3,x]
```

```
[Out] -a/(2*x^2) - (2*b)/(3*x^(3/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sqrt{x}}{x^3}\,dx &= \int \left(\frac{a}{x^3}+\frac{b}{x^{5/2}}\right)dx\\ &= -\frac{a}{2x^2}-\frac{2b}{3x^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$-\frac{a}{2x^2}-\frac{2b}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])/x^3,x]
```

```
[Out] -1/2*a/x^2 - (2*b)/(3*x^(3/2))
```

fricas [A] time = 0.98, size = 15, normalized size = 0.79

$$-\frac{4\,b\sqrt{x}+3\,a}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))/x^3,x, algorithm="fricas")
```

```
[Out] -1/6*(4*b*sqrt(x) + 3*a)/x^2
```

giac [A] time = 0.17, size = 15, normalized size = 0.79

$$-\frac{4\,b\sqrt{x}+3\,a}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))/x^3,x, algorithm="giac")
[Out] -1/6*(4*b*sqrt(x) + 3*a)/x^2
```

maple [A] time = 0.00, size = 14, normalized size = 0.74

$$-\frac{2b}{3x^{\frac{3}{2}}} - \frac{a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))/x^3,x)
[Out] -1/2*a/x^2-2/3*b/x^(3/2)
```

maxima [A] time = 0.84, size = 15, normalized size = 0.79

$$-\frac{4\,b\sqrt{x} + 3\,a}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))/x^3,x, algorithm="maxima")
[Out] -1/6*(4*b*sqrt(x) + 3*a)/x^2
```

mupad [B] time = 0.03, size = 13, normalized size = 0.68

$$-\frac{a}{2\,x^2} - \frac{2\,b}{3\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))/x^3,x)
[Out] - a/(2*x^2) - (2*b)/(3*x^(3/2))
```

sympy [A] time = 0.66, size = 17, normalized size = 0.89

$$-\frac{a}{2x^2} - \frac{2b}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))/x**3,x)
[Out] -a/(2*x**2) - 2*b/(3*x**(3/2))
```

3.2119 $\int \frac{a+b\sqrt{x}}{x^4}\,dx$

Optimal. Leaf size=19

$$-\frac{a}{3x^3}-\frac{2b}{5x^{5/2}}$$

```
[Out] -1/3*a/x^3-2/5*b/x^(5/2)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$-\frac{a}{3x^3}-\frac{2b}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])/x^4,x]
```

```
[Out] -a/(3*x^3) - (2*b)/(5*x^(5/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sqrt{x}}{x^4}\,dx &= \int \left(\frac{a}{x^4}+\frac{b}{x^{7/2}}\right)dx\\ &= -\frac{a}{3x^3}-\frac{2b}{5x^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$-\frac{a}{3x^3}-\frac{2b}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])/x^4,x]
```

```
[Out] -1/3*a/x^3 - (2*b)/(5*x^(5/2))
```

fricas [A] time = 1.10, size = 15, normalized size = 0.79

$$-\frac{6\,b\sqrt{x}+5\,a}{15\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))/x^4,x, algorithm="fricas")
```

```
[Out] -1/15*(6*b*sqrt(x) + 5*a)/x^3
```

giac [A] time = 0.16, size = 15, normalized size = 0.79

$$-\frac{6\,b\sqrt{x}+5\,a}{15\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))/x^4,x, algorithm="giac")
[Out] -1/15*(6*b*sqrt(x) + 5*a)/x^3
```

maple [A] time = 0.00, size = 14, normalized size = 0.74

$$-\frac{2b}{5x^{\frac{5}{2}}} - \frac{a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))/x^4,x)
[Out] -1/3*a/x^3-2/5*b/x^(5/2)
```

maxima [A] time = 0.88, size = 15, normalized size = 0.79

$$-\frac{6\,b\sqrt{x} + 5\,a}{15\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))/x^4,x, algorithm="maxima")
[Out] -1/15*(6*b*sqrt(x) + 5*a)/x^3
```

mupad [B] time = 0.03, size = 15, normalized size = 0.79

$$-\frac{5\,a + 6\,b\,\sqrt{x}}{15\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))/x^4,x)
[Out] -(5*a + 6*b*x^(1/2))/(15*x^3)
```

sympy [A] time = 0.97, size = 17, normalized size = 0.89

$$-\frac{a}{3x^3} - \frac{2b}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))/x**4,x)
[Out] -a/(3*x**3) - 2*b/(5*x**(5/2))
```

3.2120 $\int \left(a + b\sqrt{x}\right)^2 x^4\, dx$

Optimal. Leaf size=32

$$\frac{a^2x^5}{5} + \frac{4}{11}abx^{11/2} + \frac{b^2x^6}{6}$$

```
[Out] 1/5*a^2*x^5+4/11*a*b*x^(11/2)+1/6*b^2*x^6
```

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{a^2x^5}{5} + \frac{4}{11}abx^{11/2} + \frac{b^2x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^2*x^4,x]
```

```
[Out] (a^2*x^5)/5 + (4*a*b*x^(11/2))/11 + (b^2*x^6)/6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a + b\sqrt{x}\right)^2 x^4\, dx &= 2\,\text{Subst}\left(\int x^9 (a + bx)^2\, dx, x, \sqrt{x}\right) \\
&= 2\,\text{Subst}\left(\int \left(a^2x^9 + 2abx^{10} + b^2x^{11}\right)\, dx, x, \sqrt{x}\right) \\
&= \frac{a^2x^5}{5} + \frac{4}{11}abx^{11/2} + \frac{b^2x^6}{6}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 0.88

$$\frac{1}{330}x^5\left(66a^2 + 120ab\sqrt{x} + 55b^2x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^2*x^4,x]
```

```
[Out] (x^5*(66*a^2 + 120*a*b*Sqrt[x] + 55*b^2*x))/330
```

fricas [A] time = 0.87, size = 24, normalized size = 0.75

$$\frac{1}{6}\,b^2x^6 + \frac{4}{11}\,abx^{\frac{11}{2}} + \frac{1}{5}\,a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*x^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/6*b^2*x^6 + 4/11*a*b*x^(11/2) + 1/5*a^2*x^5
```

giac [A] time = 0.15, size = 24, normalized size = 0.75

$$\frac{1}{6}\,b^2x^6+\frac{4}{11}\,abx^{\frac{11}{2}}+\frac{1}{5}\,a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*x^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/6*b^2*x^6 + 4/11*a*b*x^(11/2) + 1/5*a^2*x^5
```

maple [A] time = 0.00, size = 25, normalized size = 0.78

$$\frac{b^2x^6}{6}+\frac{4ab\,x^{\frac{11}{2}}}{11}+\frac{a^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*x^(1/2))^2,x)
```

```
[Out] 1/5*a^2*x^5+4/11*a*b*x^(11/2)+1/6*b^2*x^6
```

maxima [B] time = 0.85, size = 166, normalized size = 5.19

$$\frac{\left(b\sqrt{x}+a\right)^{12}}{6\,b^{10}}-\frac{18\left(b\sqrt{x}+a\right)^{11}a}{11\,b^{10}}+\frac{36\left(b\sqrt{x}+a\right)^{10}a^2}{5\,b^{10}}-\frac{56\left(b\sqrt{x}+a\right)^{9}a^3}{3\,b^{10}}+\frac{63\left(b\sqrt{x}+a\right)^{8}a^4}{2\,b^{10}}-\frac{36\left(b\sqrt{x}+a\right)^{7}a^5}{b^{10}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*x^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(b*sqrt(x) + a)^12/b^10 - 18/11*(b*sqrt(x) + a)^11*a/b^10 + 36/5*(b*sqr
t(x) + a)^10*a^2/b^10 - 56/3*(b*sqrt(x) + a)^9*a^3/b^10 + 63/2*(b*sqrt(x) +
 a)^8*a^4/b^10 - 36*(b*sqrt(x) + a)^7*a^5/b^10 + 28*(b*sqrt(x) + a)^6*a^6/b
^10 - 72/5*(b*sqrt(x) + a)^5*a^7/b^10 + 9/2*(b*sqrt(x) + a)^4*a^8/b^10 - 2/
3*(b*sqrt(x) + a)^3*a^9/b^10
```

mupad [B] time = 1.10, size = 24, normalized size = 0.75

$$\frac{a^2\,x^5}{5}+\frac{b^2\,x^6}{6}+\frac{4\,a\,b\,x^{11/2}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^(1/2))^2,x)
```

```
[Out] (a^2*x^5)/5 + (b^2*x^6)/6 + (4*a*b*x^(11/2))/11
```

sympy [A] time = 1.46, size = 27, normalized size = 0.84

$$\frac{a^2x^5}{5}+\frac{4abx^{\frac{11}{2}}}{11}+\frac{b^2x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*x**(1/2))**2,x)
```

```
[Out] a**2*x**5/5 + 4*a*b*x**(11/2)/11 + b**2*x**6/6
```

3.2121 $\int \left(a + b\sqrt{x}\right)^2 x^3\, dx$

Optimal. Leaf size=32

$$\frac{a^2x^4}{4} + \frac{4}{9}abx^{9/2} + \frac{b^2x^5}{5}$$

```
[Out] 1/4*a^2*x^4+4/9*a*b*x^(9/2)+1/5*b^2*x^5
```

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{a^2x^4}{4} + \frac{4}{9}abx^{9/2} + \frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^2*x^3,x]
```

```
[Out] (a^2*x^4)/4 + (4*a*b*x^(9/2))/9 + (b^2*x^5)/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a + b\sqrt{x}\right)^2 x^3\, dx &= 2\,\text{Subst}\left(\int x^7 (a + bx)^2\, dx, x, \sqrt{x}\right) \\
&= 2\,\text{Subst}\left(\int \left(a^2x^7 + 2abx^8 + b^2x^9\right)\, dx, x, \sqrt{x}\right) \\
&= \frac{a^2x^4}{4} + \frac{4}{9}abx^{9/2} + \frac{b^2x^5}{5}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 1.00

$$\frac{a^2x^4}{4} + \frac{4}{9}abx^{9/2} + \frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^2*x^3,x]
```

```
[Out] (a^2*x^4)/4 + (4*a*b*x^(9/2))/9 + (b^2*x^5)/5
```

fricas [A] time = 1.01, size = 24, normalized size = 0.75

$$\frac{1}{5}\, b^2x^5 + \frac{4}{9}\, abx^{\frac{9}{2}} + \frac{1}{4}\, a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/5*b^2*x^5 + 4/9*a*b*x^(9/2) + 1/4*a^2*x^4
```

giac [A] time = 0.15, size = 24, normalized size = 0.75

$$\frac{1}{5}\,b^2x^5+\frac{4}{9}\,abx^{\frac{9}{2}}+\frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/5*b^2*x^5 + 4/9*a*b*x^(9/2) + 1/4*a^2*x^4
```

maple [A] time = 0.00, size = 25, normalized size = 0.78

$$\frac{b^2x^5}{5}+\frac{4ab\,x^{\frac{9}{2}}}{9}+\frac{a^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*x^(1/2))^2,x)
```

```
[Out] 1/4*a^2*x^4+4/9*a*b*x^(9/2)+1/5*b^2*x^5
```

maxima [B] time = 0.87, size = 132, normalized size = 4.12

$$\frac{\left(b\sqrt{x}+a\right)^{10}}{5\,b^8}-\frac{14\left(b\sqrt{x}+a\right)^{9}a}{9\,b^8}+\frac{21\left(b\sqrt{x}+a\right)^{8}a^2}{4\,b^8}-\frac{10\left(b\sqrt{x}+a\right)^{7}a^3}{b^8}+\frac{35\left(b\sqrt{x}+a\right)^{6}a^4}{3\,b^8}-\frac{42\left(b\sqrt{x}+a\right)^{5}a^5}{5\,b^8}+\frac{7}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/5*(b*sqrt(x) + a)^10/b^8 - 14/9*(b*sqrt(x) + a)^9*a/b^8 + 21/4*(b*sqrt(x)
 + a)^8*a^2/b^8 - 10*(b*sqrt(x) + a)^7*a^3/b^8 + 35/3*(b*sqrt(x) + a)^6*a^4
/b^8 - 42/5*(b*sqrt(x) + a)^5*a^5/b^8 + 7/2*(b*sqrt(x) + a)^4*a^6/b^8 - 2/3
*(b*sqrt(x) + a)^3*a^7/b^8
```

mupad [B] time = 0.03, size = 24, normalized size = 0.75

$$\frac{a^2\,x^4}{4}+\frac{b^2\,x^5}{5}+\frac{4\,a\,b\,x^{9/2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^(1/2))^2,x)
```

```
[Out] (a^2*x^4)/4 + (b^2*x^5)/5 + (4*a*b*x^(9/2))/9
```

sympy [A] time = 0.90, size = 27, normalized size = 0.84

$$\frac{a^2x^4}{4}+\frac{4abx^{\frac{9}{2}}}{9}+\frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*x**(1/2))**2,x)
```

```
[Out] a**2*x**4/4 + 4*a*b*x**(9/2)/9 + b**2*x**5/5
```

3.2122 $\int \left(a + b\sqrt{x}\right)^2 x^2\, dx$

Optimal. Leaf size=32

$$\frac{a^2x^3}{3} + \frac{4}{7}abx^{7/2} + \frac{b^2x^4}{4}$$

```
[Out] 1/3*a^2*x^3+4/7*a*b*x^(7/2)+1/4*b^2*x^4
```

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{a^2x^3}{3} + \frac{4}{7}abx^{7/2} + \frac{b^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^2*x^2,x]
```

```
[Out] (a^2*x^3)/3 + (4*a*b*x^(7/2))/7 + (b^2*x^4)/4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a + b\sqrt{x}\right)^2 x^2\, dx &= 2\,\text{Subst}\left(\int x^5(a + bx)^2\, dx, x, \sqrt{x}\right) \\
&= 2\,\text{Subst}\left(\int \left(a^2x^5 + 2abx^6 + b^2x^7\right)\, dx, x, \sqrt{x}\right) \\
&= \frac{a^2x^3}{3} + \frac{4}{7}abx^{7/2} + \frac{b^2x^4}{4}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 0.88

$$\frac{1}{84}x^3\left(28a^2 + 48ab\sqrt{x} + 21b^2x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^2*x^2,x]
```

```
[Out] (x^3*(28*a^2 + 48*a*b*Sqrt[x] + 21*b^2*x))/84
```

fricas [A] time = 0.78, size = 24, normalized size = 0.75

$$\frac{1}{4}\,b^2x^4 + \frac{4}{7}\,abx^{\frac{7}{2}} + \frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/4*b^2*x^4 + 4/7*a*b*x^(7/2) + 1/3*a^2*x^3
```

giac [A] time = 0.15, size = 24, normalized size = 0.75

$$\frac{1}{4}\,b^2x^4+\frac{4}{7}\,abx^{\frac{7}{2}}+\frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/4*b^2*x^4 + 4/7*a*b*x^(7/2) + 1/3*a^2*x^3
```

maple [A] time = 0.00, size = 25, normalized size = 0.78

$$\frac{b^2x^4}{4}+\frac{4ab\,x^{\frac{7}{2}}}{7}+\frac{a^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*x^(1/2))^2,x)
```

```
[Out] 1/3*a^2*x^3+4/7*a*b*x^(7/2)+1/4*b^2*x^4
```

maxima [B] time = 0.81, size = 98, normalized size = 3.06

$$\frac{\left(b\sqrt{x}+a\right)^8}{4\,b^6}-\frac{10\left(b\sqrt{x}+a\right)^7a}{7\,b^6}+\frac{10\left(b\sqrt{x}+a\right)^6a^2}{3\,b^6}-\frac{4\left(b\sqrt{x}+a\right)^5a^3}{b^6}+\frac{5\left(b\sqrt{x}+a\right)^4a^4}{2\,b^6}-\frac{2\left(b\sqrt{x}+a\right)^3a^5}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b*sqrt(x) + a)^8/b^6 - 10/7*(b*sqrt(x) + a)^7*a/b^6 + 10/3*(b*sqrt(x)
+ a)^6*a^2/b^6 - 4*(b*sqrt(x) + a)^5*a^3/b^6 + 5/2*(b*sqrt(x) + a)^4*a^4/b^
6 - 2/3*(b*sqrt(x) + a)^3*a^5/b^6
```

mupad [B] time = 1.09, size = 24, normalized size = 0.75

$$\frac{a^2\,x^3}{3}+\frac{b^2\,x^4}{4}+\frac{4\,a\,b\,x^{7/2}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^(1/2))^2,x)
```

```
[Out] (a^2*x^3)/3 + (b^2*x^4)/4 + (4*a*b*x^(7/2))/7
```

sympy [A] time = 0.49, size = 27, normalized size = 0.84

$$\frac{a^2x^3}{3}+\frac{4abx^{\frac{7}{2}}}{7}+\frac{b^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*x**(1/2))**2,x)
```

```
[Out] a**2*x**3/3 + 4*a*b*x**(7/2)/7 + b**2*x**4/4
```

3.2123 $\int \left(a + b\sqrt{x}\right)^2 x\,dx$

Optimal. Leaf size=32

$$\frac{a^2x^2}{2} + \frac{4}{5}abx^{5/2} + \frac{b^2x^3}{3}$$

```
[Out] 1/2*a^2*x^2+4/5*a*b*x^(5/2)+1/3*b^2*x^3
```

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{a^2x^2}{2} + \frac{4}{5}abx^{5/2} + \frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^2*x,x]
```

```
[Out] (a^2*x^2)/2 + (4*a*b*x^(5/2))/5 + (b^2*x^3)/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a + b\sqrt{x}\right)^2 x\,dx &= 2\,\text{Subst}\left(\int x^3(a + bx)^2\,dx, x, \sqrt{x}\right) \\
&= 2\,\text{Subst}\left(\int \left(a^2x^3 + 2abx^4 + b^2x^5\right)\,dx, x, \sqrt{x}\right) \\
&= \frac{a^2x^2}{2} + \frac{4}{5}abx^{5/2} + \frac{b^2x^3}{3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.88

$$\frac{1}{30}x^2\left(15a^2 + 24ab\sqrt{x} + 10b^2x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^2*x,x]
```

```
[Out] (x^2*(15*a^2 + 24*a*b*Sqrt[x] + 10*b^2*x))/30
```

fricas [A] time = 1.08, size = 24, normalized size = 0.75

$$\frac{1}{3}\,b^2x^3 + \frac{4}{5}\,abx^{\frac{5}{2}} + \frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^(1/2))^2,x, algorithm="fricas")
[Out] 1/3*b^2*x^3 + 4/5*a*b*x^(5/2) + 1/2*a^2*x^2
```

giac [A] time = 0.16, size = 24, normalized size = 0.75

$$\frac{1}{3}\,b^2x^3+\frac{4}{5}\,abx^{\frac{5}{2}}+\frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^(1/2))^2,x, algorithm="giac")
[Out] 1/3*b^2*x^3 + 4/5*a*b*x^(5/2) + 1/2*a^2*x^2
```

maple [A] time = 0.00, size = 25, normalized size = 0.78

$$\frac{b^2x^3}{3}+\frac{4ab\,x^{\frac{5}{2}}}{5}+\frac{a^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*x^(1/2))^2,x)
[Out] 1/2*a^2*x^2+4/5*a*b*x^(5/2)+1/3*b^2*x^3
```

maxima [B] time = 0.90, size = 64, normalized size = 2.00

$$\frac{\left(b\sqrt{x}+a\right)^6}{3\,b^4}-\frac{6\left(b\sqrt{x}+a\right)^5a}{5\,b^4}+\frac{3\left(b\sqrt{x}+a\right)^4a^2}{2\,b^4}-\frac{2\left(b\sqrt{x}+a\right)^3a^3}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^(1/2))^2,x, algorithm="maxima")
[Out] 1/3*(b*sqrt(x) + a)^6/b^4 - 6/5*(b*sqrt(x) + a)^5*a/b^4 + 3/2*(b*sqrt(x) +
a)^4*a^2/b^4 - 2/3*(b*sqrt(x) + a)^3*a^3/b^4
```

mupad [B] time = 0.03, size = 24, normalized size = 0.75

$$\frac{a^2\,x^2}{2}+\frac{b^2\,x^3}{3}+\frac{4\,a\,b\,x^{5/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^(1/2))^2,x)
[Out] (a^2*x^2)/2 + (b^2*x^3)/3 + (4*a*b*x^(5/2))/5
```

sympy [A] time = 0.30, size = 27, normalized size = 0.84

$$\frac{a^2x^2}{2}+\frac{4abx^{\frac{5}{2}}}{5}+\frac{b^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x**(1/2))**2,x)
[Out] a**2*x**2/2 + 4*a*b*x**(5/2)/5 + b**2*x**3/3
```

3.2124 $\int \left(a + b\sqrt{x}\right)^2 \, dx$

Optimal. Leaf size=27

$$a^2 x + \frac{4}{3} a b x^{3/2} + \frac{b^2 x^2}{2}$$

```
[Out] a^2*x+4/3*a*b*x^(3/2)+1/2*b^2*x^2
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {190, 43}

$$a^2 x + \frac{4}{3} a b x^{3/2} + \frac{b^2 x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^2,x]
```

```
[Out] a^2*x + (4*a*b*x^(3/2))/3 + (b^2*x^2)/2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}
\int \left(a + b\sqrt{x}\right)^2 \, dx &= 2\,\text{Subst}\left(\int x(a + bx)^2 \, dx, x, \sqrt{x}\right) \\
&= 2\,\text{Subst}\left(\int \left(a^2 x + 2abx^2 + b^2 x^3\right) \, dx, x, \sqrt{x}\right) \\
&= a^2 x + \frac{4}{3} a b x^{3/2} + \frac{b^2 x^2}{2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$a^2 x + \frac{4}{3} a b x^{3/2} + \frac{b^2 x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^2,x]
```

```
[Out] a^2*x + (4*a*b*x^(3/2))/3 + (b^2*x^2)/2
```

fricas [A] time = 1.47, size = 21, normalized size = 0.78

$$\frac{1}{2} b^2 x^2 + \frac{4}{3} a b x^{\frac{3}{2}} + a^2 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^2,x, algorithm="fricas")
[Out] 1/2*b^2*x^2 + 4/3*a*b*x^(3/2) + a^2*x
```

giac [A] time = 0.15, size = 21, normalized size = 0.78

$$\frac{1}{2}b^2x^2 + \frac{4}{3}abx^{\frac{3}{2}} + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^2,x, algorithm="giac")
[Out] 1/2*b^2*x^2 + 4/3*a*b*x^(3/2) + a^2*x
```

maple [A] time = 0.00, size = 22, normalized size = 0.81

$$\frac{b^2x^2}{2} + \frac{4ab\,x^{\frac{3}{2}}}{3} + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^2,x)
[Out] a^2*x+4/3*a*b*x^(3/2)+1/2*b^2*x^2
```

maxima [A] time = 0.89, size = 21, normalized size = 0.78

$$\frac{1}{2}b^2x^2 + \frac{4}{3}abx^{\frac{3}{2}} + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^2,x, algorithm="maxima")
[Out] 1/2*b^2*x^2 + 4/3*a*b*x^(3/2) + a^2*x
```

mupad [B] time = 0.03, size = 21, normalized size = 0.78

$$a^2\,x + \frac{b^2\,x^2}{2} + \frac{4\,a\,b\,x^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^2,x)
[Out] a^2*x + (b^2*x^2)/2 + (4*a*b*x^(3/2))/3
```

sympy [A] time = 0.18, size = 24, normalized size = 0.89

$$a^2x + \frac{4abx^{\frac{3}{2}}}{3} + \frac{b^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**2,x)
[Out] a**2*x + 4*a*b*x**(3/2)/3 + b**2*x**2/2
```

3.2125 $\int \frac{\left(a+b\sqrt{x}\right)^2}{x}\,dx$

Optimal. Leaf size=21

$$a^2\log(x)+4ab\sqrt{x}+b^2x$$

```
[Out] b^2*x+a^2*ln(x)+4*a*b*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$a^2\log(x)+4ab\sqrt{x}+b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^2/x,x]
```

```
[Out] 4*a*b*Sqrt[x] + b^2*x + a^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt{x}\right)^2}{x}\,dx &= 2\,\text{Subst}\left(\int \frac{(a+bx)^2}{x}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(2ab+\frac{a^2}{x}+b^2x\right)\,dx, x, \sqrt{x}\right)\\ &= 4ab\sqrt{x}+b^2x+a^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.04, size = 21, normalized size = 1.00

$$a^2\log(x)+4ab\sqrt{x}+b^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^2/x,x]
```

```
[Out] 4*a*b*Sqrt[x] + b^2*x + a^2*Log[x]
```

fricas [A] time = 1.00, size = 22, normalized size = 1.05

$$b^2x+2\,a^2\log\left(\sqrt{x}\right)+4\,ab\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^2/x,x, algorithm="fricas")
[Out] b^2*x + 2*a^2*log(sqrt(x)) + 4*a*b*sqrt(x)
```

giac [A] time = 0.15, size = 20, normalized size = 0.95

$$b^2x + a^2\log(|x|) + 4\,ab\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^2/x,x, algorithm="giac")
[Out] b^2*x + a^2*log(abs(x)) + 4*a*b*sqrt(x)
```

maple [A] time = 0.00, size = 20, normalized size = 0.95

$$a^2\ln(x) + b^2x + 4ab\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^2/x,x)
[Out] b^2*x+a^2*ln(x)+4*a*b*x^(1/2)
```

maxima [A] time = 0.88, size = 19, normalized size = 0.90

$$b^2x + a^2\log(x) + 4\,ab\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^2/x,x, algorithm="maxima")
[Out] b^2*x + a^2*log(x) + 4*a*b*sqrt(x)
```

mupad [B] time = 1.09, size = 22, normalized size = 1.05

$$2\,a^2\,\ln\left(\sqrt{x}\right) + b^2\,x + 4\,a\,b\,\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^2/x,x)
[Out] 2*a^2*log(x^(1/2)) + b^2*x + 4*a*b*x^(1/2)
```

sympy [A] time = 0.31, size = 20, normalized size = 0.95

$$a^2\log(x) + 4ab\sqrt{x} + b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**2/x,x)
[Out] a**2*log(x) + 4*a*b*sqrt(x) + b**2*x
```

3.2126 $\int \frac{\left(a+b\sqrt{x}\right)^2}{x^2}\,dx$

Optimal. Leaf size=24

$$-\frac{a^2}{x}-\frac{4ab}{\sqrt{x}}+b^2\log(x)$$

```
[Out] -a^2/x+b^2*ln(x)-4*a*b/x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{a^2}{x}-\frac{4ab}{\sqrt{x}}+b^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^2/x^2,x]
```

```
[Out] -(a^2/x) - (4*a*b)/Sqrt[x] + b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt{x}\right)^2}{x^2}\,dx &= 2\,\text{Subst}\left(\int \frac{(a+bx)^2}{x^3}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(\frac{a^2}{x^3}+\frac{2ab}{x^2}+\frac{b^2}{x}\right)dx, x, \sqrt{x}\right)\\ &= -\frac{a^2}{x}-\frac{4ab}{\sqrt{x}}+b^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 24, normalized size = 1.00

$$-\frac{a^2}{x}-\frac{4ab}{\sqrt{x}}+b^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^2/x^2,x]
```

```
[Out] -(a^2/x) - (4*a*b)/Sqrt[x] + b^2*Log[x]
```

fricas [A] time = 1.24, size = 27, normalized size = 1.12

$$\frac{2\,b^2 x \log\left(\sqrt{x}\right) - 4\,ab\sqrt{x} - a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^2/x^2,x, algorithm="fricas")
[Out] (2*b^2*x*log(sqrt(x)) - 4*a*b*sqrt(x) - a^2)/x
```

giac [A] time = 0.15, size = 24, normalized size = 1.00

$$b^2 \log\left(|x|\right) - \frac{4\,ab\sqrt{x} + a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^2/x^2,x, algorithm="giac")
[Out] b^2*log(abs(x)) - (4*a*b*sqrt(x) + a^2)/x
```

maple [A] time = 0.00, size = 23, normalized size = 0.96

$$b^2 \ln(x) - \frac{4ab}{\sqrt{x}} - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^2/x^2,x)
[Out] -a^2/x+b^2*ln(x)-4*a*b/x^(1/2)
```

maxima [A] time = 0.98, size = 23, normalized size = 0.96

$$b^2 \log(x) - \frac{4\,ab\sqrt{x} + a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^2/x^2,x, algorithm="maxima")
[Out] b^2*log(x) - (4*a*b*sqrt(x) + a^2)/x
```

mupad [B] time = 1.08, size = 26, normalized size = 1.08

$$2\,b^2 \ln\left(\sqrt{x}\right) - \frac{a^2 + 4\,a\,b\,\sqrt{x}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^2/x^2,x)
[Out] 2*b^2*log(x^(1/2)) - (a^2 + 4*a*b*x^(1/2))/x
```

sympy [A] time = 0.45, size = 20, normalized size = 0.83

$$-\frac{a^2}{x} - \frac{4ab}{\sqrt{x}} + b^2 \log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**2/x**2,x)
[Out] -a**2/x - 4*a*b/sqrt(x) + b**2*log(x)
```

3.2127 $\int \frac{\left(a+b\sqrt{x}\right)^2}{x^3}\,dx$

Optimal. Leaf size=30

$$-\frac{a^2}{2x^2}-\frac{4ab}{3x^{3/2}}-\frac{b^2}{x}$$

```
[Out] -1/2*a^2/x^2-4/3*a*b/x^(3/2)-b^2/x
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{a^2}{2x^2}-\frac{4ab}{3x^{3/2}}-\frac{b^2}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^2/x^3,x]
```

```
[Out] -a^2/(2*x^2) - (4*a*b)/(3*x^(3/2)) - b^2/x
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt{x}\right)^2}{x^3}\,dx &= 2\,\text{Subst}\left(\int \frac{(a+bx)^2}{x^5}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(\frac{a^2}{x^5}+\frac{2ab}{x^4}+\frac{b^2}{x^3}\right)dx, x, \sqrt{x}\right)\\ &= -\frac{a^2}{2x^2}-\frac{4ab}{3x^{3/2}}-\frac{b^2}{x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$-\frac{a^2}{2x^2}-\frac{4ab}{3x^{3/2}}-\frac{b^2}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^2/x^3,x]
```

```
[Out] -1/2*a^2/x^2 - (4*a*b)/(3*x^(3/2)) - b^2/x
```

fricas [A] time = 1.14, size = 24, normalized size = 0.80

$$-\frac{6\,b^2x+8\,ab\sqrt{x}+3\,a^2}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^2/x^3,x, algorithm="fricas")
[Out] -1/6*(6*b^2*x + 8*a*b*sqrt(x) + 3*a^2)/x^2
```

giac [A] time = 0.16, size = 24, normalized size = 0.80

$$-\frac{6\,b^2x+8\,ab\sqrt{x}\,+3\,a^2}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^2/x^3,x, algorithm="giac")
[Out] -1/6*(6*b^2*x + 8*a*b*sqrt(x) + 3*a^2)/x^2
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$-\frac{b^2}{x}-\frac{4ab}{3x^{\frac{3}{2}}}-\frac{a^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^2/x^3,x)
[Out] -1/2*a^2/x^2-4/3*a*b/x^(3/2)-b^2/x
```

maxima [A] time = 0.81, size = 24, normalized size = 0.80

$$-\frac{6\,b^2x+8\,ab\sqrt{x}\,+3\,a^2}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^2/x^3,x, algorithm="maxima")
[Out] -1/6*(6*b^2*x + 8*a*b*sqrt(x) + 3*a^2)/x^2
```

mupad [B] time = 0.03, size = 24, normalized size = 0.80

$$-\frac{6\,b^2\,x+3\,a^2+8\,a\,b\,\sqrt{x}}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^2/x^3,x)
[Out] -(6*b^2*x + 3*a^2 + 8*a*b*x^(1/2))/(6*x^2)
```

sympy [A] time = 0.70, size = 26, normalized size = 0.87

$$-\frac{a^2}{2x^2}-\frac{4ab}{3x^{\frac{3}{2}}}-\frac{b^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**2/x**3,x)
[Out] -a**2/(2*x**2) - 4*a*b/(3*x**(3/2)) - b**2/x
```

3.2128 $\int \frac{\left(a+b\sqrt{x}\right)^2}{x^4}\,dx$

Optimal. Leaf size=32

$$-\frac{a^2}{3x^3}-\frac{4ab}{5x^{5/2}}-\frac{b^2}{2x^2}$$

```
[Out] -1/3*a^2/x^3-4/5*a*b/x^(5/2)-1/2*b^2/x^2
```

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{a^2}{3x^3}-\frac{4ab}{5x^{5/2}}-\frac{b^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^2/x^4,x]
```

```
[Out] -a^2/(3*x^3) - (4*a*b)/(5*x^(5/2)) - b^2/(2*x^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt{x}\right)^2}{x^4}\,dx &= 2\,\text{Subst}\left(\int \frac{(a+bx)^2}{x^7}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(\frac{a^2}{x^7}+\frac{2ab}{x^6}+\frac{b^2}{x^5}\right)dx, x, \sqrt{x}\right)\\ &= -\frac{a^2}{3x^3}-\frac{4ab}{5x^{5/2}}-\frac{b^2}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.88

$$-\frac{10a^2+24ab\sqrt{x}+15b^2x}{30x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^2/x^4,x]
```

```
[Out] -1/30*(10*a^2 + 24*a*b*Sqrt[x] + 15*b^2*x)/x^3
```

fricas [A] time = 0.84, size = 24, normalized size = 0.75

$$-\frac{15\,b^2x+24\,ab\sqrt{x}+10\,a^2}{30\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^2/x^4,x, algorithm="fricas")
[Out] -1/30*(15*b^2*x + 24*a*b*sqrt(x) + 10*a^2)/x^3
```

giac [**A**] time = 0.16, size = 24, normalized size = 0.75

$$-\frac{15\,b^2x+24\,ab\sqrt{x}+10\,a^2}{30\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^2/x^4,x, algorithm="giac")
[Out] -1/30*(15*b^2*x + 24*a*b*sqrt(x) + 10*a^2)/x^3
```

maple [**A**] time = 0.00, size = 25, normalized size = 0.78

$$-\frac{b^2}{2x^2}-\frac{4ab}{5x^{\frac{5}{2}}}-\frac{a^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^2/x^4,x)
[Out] -1/3*a^2/x^3-4/5*a*b/x^(5/2)-1/2*b^2/x^2
```

maxima [**A**] time = 0.84, size = 24, normalized size = 0.75

$$-\frac{15\,b^2x+24\,ab\sqrt{x}+10\,a^2}{30\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^2/x^4,x, algorithm="maxima")
[Out] -1/30*(15*b^2*x + 24*a*b*sqrt(x) + 10*a^2)/x^3
```

mupad [**B**] time = 0.03, size = 24, normalized size = 0.75

$$-\frac{15\,b^2\,x+10\,a^2+24\,a\,b\,\sqrt{x}}{30\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^2/x^4,x)
[Out] -(15*b^2*x + 10*a^2 + 24*a*b*x^(1/2))/(30*x^3)
```

sympy [**A**] time = 1.06, size = 29, normalized size = 0.91

$$-\frac{a^2}{3x^3}-\frac{4ab}{5x^{\frac{5}{2}}}-\frac{b^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**2/x**4,x)
[Out] -a**2/(3*x**3) - 4*a*b/(5*x**(5/2)) - b**2/(2*x**2)
```

3.2129 $\int \frac{\left(a+b\sqrt{x}\right)^2}{x^5}\,dx$

Optimal. Leaf size=32

$$-\frac{a^2}{4x^4}-\frac{4ab}{7x^{7/2}}-\frac{b^2}{3x^3}$$

```
[Out] -1/4*a^2/x^4-4/7*a*b/x^(7/2)-1/3*b^2/x^3
```

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{a^2}{4x^4}-\frac{4ab}{7x^{7/2}}-\frac{b^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^2/x^5,x]
```

```
[Out] -a^2/(4*x^4) - (4*a*b)/(7*x^(7/2)) - b^2/(3*x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt{x}\right)^2}{x^5}\,dx &= 2\,\text{Subst}\left(\int \frac{(a+bx)^2}{x^9}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(\frac{a^2}{x^9}+\frac{2ab}{x^8}+\frac{b^2}{x^7}\right)dx, x, \sqrt{x}\right)\\ &= -\frac{a^2}{4x^4}-\frac{4ab}{7x^{7/2}}-\frac{b^2}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 28, normalized size = 0.88

$$-\frac{21a^2+48ab\sqrt{x}+28b^2x}{84x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^2/x^5,x]
```

```
[Out] -1/84*(21*a^2 + 48*a*b*Sqrt[x] + 28*b^2*x)/x^4
```

fricas [A] time = 0.90, size = 24, normalized size = 0.75

$$-\frac{28\,b^2x+48\,ab\sqrt{x}+21\,a^2}{84\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^2/x^5,x, algorithm="fricas")
[Out] -1/84*(28*b^2*x + 48*a*b*sqrt(x) + 21*a^2)/x^4
```

giac [A] time = 0.16, size = 24, normalized size = 0.75

$$-\frac{28\,b^2x+48\,ab\sqrt{x}+21\,a^2}{84\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^2/x^5,x, algorithm="giac")
[Out] -1/84*(28*b^2*x + 48*a*b*sqrt(x) + 21*a^2)/x^4
```

maple [A] time = 0.00, size = 25, normalized size = 0.78

$$-\frac{b^2}{3x^3}-\frac{4ab}{7x^{\frac{7}{2}}}-\frac{a^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^2/x^5,x)
[Out] -1/4*a^2/x^4-4/7*a*b/x^(7/2)-1/3*b^2/x^3
```

maxima [A] time = 0.85, size = 24, normalized size = 0.75

$$-\frac{28\,b^2x+48\,ab\sqrt{x}+21\,a^2}{84\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^2/x^5,x, algorithm="maxima")
[Out] -1/84*(28*b^2*x + 48*a*b*sqrt(x) + 21*a^2)/x^4
```

mupad [B] time = 0.03, size = 24, normalized size = 0.75

$$-\frac{28\,b^2\,x+21\,a^2+48\,a\,b\,\sqrt{x}}{84\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^2/x^5,x)
[Out] -(28*b^2*x + 21*a^2 + 48*a*b*x^(1/2))/(84*x^4)
```

sympy [A] time = 1.36, size = 29, normalized size = 0.91

$$-\frac{a^2}{4x^4}-\frac{4ab}{7x^{\frac{7}{2}}}-\frac{b^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**2/x**5,x)
[Out] -a**2/(4*x**4) - 4*a*b/(7*x**(7/2)) - b**2/(3*x**3)
```

3.2130 $\int \left(a + b\sqrt{x}\right)^3 x^4\, dx$

Optimal. Leaf size=47

$$\frac{a^3x^5}{5} + \frac{6}{11}a^2bx^{11/2} + \frac{1}{2}ab^2x^6 + \frac{2}{13}b^3x^{13/2}$$

```
[Out] 1/5*a^3*x^5+6/11*a^2*b*x^(11/2)+1/2*a*b^2*x^6+2/13*b^3*x^(13/2)
```

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{6}{11}a^2bx^{11/2} + \frac{a^3x^5}{5} + \frac{1}{2}ab^2x^6 + \frac{2}{13}b^3x^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^3*x^4,x]
```

```
[Out] (a^3*x^5)/5 + (6*a^2*b*x^(11/2))/11 + (a*b^2*x^6)/2 + (2*b^3*x^(13/2))/13
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a + b\sqrt{x}\right)^3 x^4\, dx &= 2\,\text{Subst}\left(\int x^9(a + bx)^3\, dx, x, \sqrt{x}\right) \\
&= 2\,\text{Subst}\left(\int \left(a^3x^9 + 3a^2bx^{10} + 3ab^2x^{11} + b^3x^{12}\right) dx, x, \sqrt{x}\right) \\
&= \frac{a^3x^5}{5} + \frac{6}{11}a^2bx^{11/2} + \frac{1}{2}ab^2x^6 + \frac{2}{13}b^3x^{13/2}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 47, normalized size = 1.00

$$\frac{a^3x^5}{5} + \frac{6}{11}a^2bx^{11/2} + \frac{1}{2}ab^2x^6 + \frac{2}{13}b^3x^{13/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^3*x^4,x]
```

```
[Out] (a^3*x^5)/5 + (6*a^2*b*x^(11/2))/11 + (a*b^2*x^6)/2 + (2*b^3*x^(13/2))/13
```

fricas [A] time = 0.93, size = 41, normalized size = 0.87

$$\frac{1}{2}\,ab^2x^6 + \frac{1}{5}\,a^3x^5 + \frac{2}{143}\left(11\,b^3x^6 + 39\,a^2bx^5\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*x^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 1/2*a*b^2*x^6 + 1/5*a^3*x^5 + 2/143*(11*b^3*x^6 + 39*a^2*b*x^5)*sqrt(x)
```

giac [A] time = 0.15, size = 35, normalized size = 0.74

$$\frac{2}{13}b^3x^{\frac{13}{2}} + \frac{1}{2}ab^2x^6 + \frac{6}{11}a^2bx^{\frac{11}{2}} + \frac{1}{5}a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*x^(1/2))^3,x, algorithm="giac")
```

```
[Out] 2/13*b^3*x^(13/2) + 1/2*a*b^2*x^6 + 6/11*a^2*b*x^(11/2) + 1/5*a^3*x^5
```

maple [A] time = 0.00, size = 36, normalized size = 0.77

$$\frac{2b^3x^{\frac{13}{2}}}{13} + \frac{a\,b^2x^6}{2} + \frac{6a^2b\,x^{\frac{11}{2}}}{11} + \frac{a^3x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*x^(1/2))^3,x)
```

```
[Out] 1/5*a^3*x^5+6/11*a^2*b*x^(11/2)+1/2*a*b^2*x^6+2/13*b^3*x^(13/2)
```

maxima [B] time = 0.92, size = 166, normalized size = 3.53

$$\frac{2\left(b\sqrt{x}+a\right)^{13}}{13\,b^{10}} - \frac{3\left(b\sqrt{x}+a\right)^{12}a}{2\,b^{10}} + \frac{72\left(b\sqrt{x}+a\right)^{11}a^2}{11\,b^{10}} - \frac{84\left(b\sqrt{x}+a\right)^{10}a^3}{5\,b^{10}} + \frac{28\left(b\sqrt{x}+a\right)^{9}a^4}{b^{10}} - \frac{63\left(b\sqrt{x}+a\right)^{8}a^5}{2\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*x^(1/2))^3,x, algorithm="maxima")
```

```
[Out] 2/13*(b*sqrt(x) + a)^13/b^10 - 3/2*(b*sqrt(x) + a)^12*a/b^10 + 72/11*(b*sqr
t(x) + a)^11*a^2/b^10 - 84/5*(b*sqrt(x) + a)^10*a^3/b^10 + 28*(b*sqrt(x) +
a)^9*a^4/b^10 - 63/2*(b*sqrt(x) + a)^8*a^5/b^10 + 24*(b*sqrt(x) + a)^7*a^6/
b^10 - 12*(b*sqrt(x) + a)^6*a^7/b^10 + 18/5*(b*sqrt(x) + a)^5*a^8/b^10 - 1/
2*(b*sqrt(x) + a)^4*a^9/b^10
```

mupad [B] time = 0.04, size = 35, normalized size = 0.74

$$\frac{a^3\,x^5}{5} + \frac{2\,b^3\,x^{13/2}}{13} + \frac{a\,b^2\,x^6}{2} + \frac{6\,a^2\,b\,x^{11/2}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^(1/2))^3,x)
```

```
[Out] (a^3*x^5)/5 + (2*b^3*x^(13/2))/13 + (a*b^2*x^6)/2 + (6*a^2*b*x^(11/2))/11
```

sympy [A] time = 2.09, size = 42, normalized size = 0.89

$$\frac{a^3x^5}{5} + \frac{6a^2bx^{\frac{11}{2}}}{11} + \frac{ab^2x^6}{2} + \frac{2b^3x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*x**(1/2))**3,x)
```

```
[Out] a**3*x**5/5 + 6*a**2*b*x**(11/2)/11 + a*b**2*x**6/2 + 2*b**3*x**(13/2)/13
```

3.2131 $\int \left(a+b\sqrt{x}\right)^3 x^3\,dx$

Optimal. Leaf size=47

$$\frac{a^3x^4}{4}+\frac{2}{3}a^2bx^{9/2}+\frac{3}{5}ab^2x^5+\frac{2}{11}b^3x^{11/2}$$

```
[Out] 1/4*a^3*x^4+2/3*a^2*b*x^(9/2)+3/5*a*b^2*x^5+2/11*b^3*x^(11/2)
```

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{2}{3}a^2bx^{9/2}+\frac{a^3x^4}{4}+\frac{3}{5}ab^2x^5+\frac{2}{11}b^3x^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^3*x^3,x]
```

```
[Out] (a^3*x^4)/4 + (2*a^2*b*x^(9/2))/3 + (3*a*b^2*x^5)/5 + (2*b^3*x^(11/2))/11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sqrt{x}\right)^3 x^3\,dx &= 2\,\text{Subst}\left(\int x^7(a+bx)^3\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(a^3x^7+3a^2bx^8+3ab^2x^9+b^3x^{10}\right)\,dx, x, \sqrt{x}\right)\\ &= \frac{a^3x^4}{4}+\frac{2}{3}a^2bx^{9/2}+\frac{3}{5}ab^2x^5+\frac{2}{11}b^3x^{11/2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 47, normalized size = 1.00

$$\frac{a^3x^4}{4}+\frac{2}{3}a^2bx^{9/2}+\frac{3}{5}ab^2x^5+\frac{2}{11}b^3x^{11/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^3*x^3,x]
```

```
[Out] (a^3*x^4)/4 + (2*a^2*b*x^(9/2))/3 + (3*a*b^2*x^5)/5 + (2*b^3*x^(11/2))/11
```

fricas [A] time = 1.00, size = 41, normalized size = 0.87

$$\frac{3}{5}\,ab^2x^5+\frac{1}{4}\,a^3x^4+\frac{2}{33}\left(3\,b^3x^5+11\,a^2bx^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 3/5*a*b^2*x^5 + 1/4*a^3*x^4 + 2/33*(3*b^3*x^5 + 11*a^2*b*x^4)*sqrt(x)
```

giac [A] time = 0.16, size = 35, normalized size = 0.74

$$\frac{2}{11}b^3x^{\frac{11}{2}}+\frac{3}{5}ab^2x^5+\frac{2}{3}a^2bx^{\frac{9}{2}}+\frac{1}{4}a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^(1/2))^3,x, algorithm="giac")
```

```
[Out] 2/11*b^3*x^(11/2) + 3/5*a*b^2*x^5 + 2/3*a^2*b*x^(9/2) + 1/4*a^3*x^4
```

maple [A] time = 0.00, size = 36, normalized size = 0.77

$$\frac{2b^3x^{\frac{11}{2}}}{11}+\frac{3a\,b^2x^5}{5}+\frac{2a^2b\,x^{\frac{9}{2}}}{3}+\frac{a^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*x^(1/2))^3,x)
```

```
[Out] 1/4*a^3*x^4+2/3*a^2*b*x^(9/2)+3/5*a*b^2*x^5+2/11*b^3*x^(11/2)
```

maxima [B] time = 0.94, size = 132, normalized size = 2.81

$$\frac{2\left(b\sqrt{x}+a\right)^{11}}{11\,b^8}-\frac{7\left(b\sqrt{x}+a\right)^{10}a}{5\,b^8}+\frac{14\left(b\sqrt{x}+a\right)^{9}a^2}{3\,b^8}-\frac{35\left(b\sqrt{x}+a\right)^{8}a^3}{4\,b^8}+\frac{10\left(b\sqrt{x}+a\right)^{7}a^4}{b^8}-\frac{7\left(b\sqrt{x}+a\right)^{6}a^5}{b^8}+\frac{14}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^(1/2))^3,x, algorithm="maxima")
```

```
[Out] 2/11*(b*sqrt(x) + a)^11/b^8 - 7/5*(b*sqrt(x) + a)^10*a/b^8 + 14/3*(b*sqrt(x
) + a)^9*a^2/b^8 - 35/4*(b*sqrt(x) + a)^8*a^3/b^8 + 10*(b*sqrt(x) + a)^7*a^
4/b^8 - 7*(b*sqrt(x) + a)^6*a^5/b^8 + 14/5*(b*sqrt(x) + a)^5*a^6/b^8 - 1/2*
(b*sqrt(x) + a)^4*a^7/b^8
```

mupad [B] time = 0.04, size = 35, normalized size = 0.74

$$\frac{a^3\,x^4}{4}+\frac{2\,b^3\,x^{11/2}}{11}+\frac{3\,a\,b^2\,x^5}{5}+\frac{2\,a^2\,b\,x^{9/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^(1/2))^3,x)
```

```
[Out] (a^3*x^4)/4 + (2*b^3*x^(11/2))/11 + (3*a*b^2*x^5)/5 + (2*a^2*b*x^(9/2))/3
```

sympy [A] time = 1.92, size = 44, normalized size = 0.94

$$\frac{a^3x^4}{4}+\frac{2a^2bx^{\frac{9}{2}}}{3}+\frac{3ab^2x^5}{5}+\frac{2b^3x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*x**(1/2))**3,x)
```

```
[Out] a**3*x**4/4 + 2*a**2*b*x**(9/2)/3 + 3*a*b**2*x**5/5 + 2*b**3*x**(11/2)/11
```

3.2132 $\int \left(a + b\sqrt{x}\right)^3 x^2\, dx$

Optimal. Leaf size=47

$$\frac{a^3x^3}{3} + \frac{6}{7}a^2bx^{7/2} + \frac{3}{4}ab^2x^4 + \frac{2}{9}b^3x^{9/2}$$

```
[Out] 1/3*a^3*x^3+6/7*a^2*b*x^(7/2)+3/4*a*b^2*x^4+2/9*b^3*x^(9/2)
```

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{6}{7}a^2bx^{7/2} + \frac{a^3x^3}{3} + \frac{3}{4}ab^2x^4 + \frac{2}{9}b^3x^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^3*x^2,x]
```

```
[Out] (a^3*x^3)/3 + (6*a^2*b*x^(7/2))/7 + (3*a*b^2*x^4)/4 + (2*b^3*x^(9/2))/9
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a + b\sqrt{x}\right)^3 x^2\, dx &= 2\,\text{Subst}\left(\int x^5(a + bx)^3\, dx, x, \sqrt{x}\right) \\
&= 2\,\text{Subst}\left(\int \left(a^3x^5 + 3a^2bx^6 + 3ab^2x^7 + b^3x^8\right)\, dx, x, \sqrt{x}\right) \\
&= \frac{a^3x^3}{3} + \frac{6}{7}a^2bx^{7/2} + \frac{3}{4}ab^2x^4 + \frac{2}{9}b^3x^{9/2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 1.00

$$\frac{a^3x^3}{3} + \frac{6}{7}a^2bx^{7/2} + \frac{3}{4}ab^2x^4 + \frac{2}{9}b^3x^{9/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^3*x^2,x]
```

```
[Out] (a^3*x^3)/3 + (6*a^2*b*x^(7/2))/7 + (3*a*b^2*x^4)/4 + (2*b^3*x^(9/2))/9
```

fricas [A] time = 0.69, size = 41, normalized size = 0.87

$$\frac{3}{4}\,ab^2x^4 + \frac{1}{3}\,a^3x^3 + \frac{2}{63}\left(7\,b^3x^4 + 27\,a^2bx^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 3/4*a*b^2*x^4 + 1/3*a^3*x^3 + 2/63*(7*b^3*x^4 + 27*a^2*b*x^3)*sqrt(x)
```

giac [A] time = 0.19, size = 35, normalized size = 0.74

$$\frac{2}{9}b^3x^{\frac{9}{2}} + \frac{3}{4}ab^2x^4 + \frac{6}{7}a^2bx^{\frac{7}{2}} + \frac{1}{3}a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^(1/2))^3,x, algorithm="giac")
```

```
[Out] 2/9*b^3*x^(9/2) + 3/4*a*b^2*x^4 + 6/7*a^2*b*x^(7/2) + 1/3*a^3*x^3
```

maple [A] time = 0.00, size = 36, normalized size = 0.77

$$\frac{2b^3x^{\frac{9}{2}}}{9} + \frac{3a\,b^2x^4}{4} + \frac{6a^2b\,x^{\frac{7}{2}}}{7} + \frac{a^3x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*x^(1/2))^3,x)
```

```
[Out] 1/3*a^3*x^3+6/7*a^2*b*x^(7/2)+3/4*a*b^2*x^4+2/9*b^3*x^(9/2)
```

maxima [B] time = 0.93, size = 98, normalized size = 2.09

$$\frac{2\left(b\sqrt{x}+a\right)^9}{9\,b^6} - \frac{5\left(b\sqrt{x}+a\right)^8 a}{4\,b^6} + \frac{20\left(b\sqrt{x}+a\right)^7 a^2}{7\,b^6} - \frac{10\left(b\sqrt{x}+a\right)^6 a^3}{3\,b^6} + \frac{2\left(b\sqrt{x}+a\right)^5 a^4}{b^6} - \frac{\left(b\sqrt{x}+a\right)^4 a^5}{2\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^(1/2))^3,x, algorithm="maxima")
```

```
[Out] 2/9*(b*sqrt(x) + a)^9/b^6 - 5/4*(b*sqrt(x) + a)^8*a/b^6 + 20/7*(b*sqrt(x) +
 a)^7*a^2/b^6 - 10/3*(b*sqrt(x) + a)^6*a^3/b^6 + 2*(b*sqrt(x) + a)^5*a^4/b^
6 - 1/2*(b*sqrt(x) + a)^4*a^5/b^6
```

mupad [B] time = 0.04, size = 35, normalized size = 0.74

$$\frac{a^3\,x^3}{3} + \frac{2\,b^3\,x^{9/2}}{9} + \frac{3\,a\,b^2\,x^4}{4} + \frac{6\,a^2\,b\,x^{7/2}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^(1/2))^3,x)
```

```
[Out] (a^3*x^3)/3 + (2*b^3*x^(9/2))/9 + (3*a*b^2*x^4)/4 + (6*a^2*b*x^(7/2))/7
```

sympy [A] time = 1.94, size = 44, normalized size = 0.94

$$\frac{a^3x^3}{3} + \frac{6a^2bx^{\frac{7}{2}}}{7} + \frac{3ab^2x^4}{4} + \frac{2b^3x^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*x**(1/2))**3,x)
```

```
[Out] a**3*x**3/3 + 6*a**2*b*x**(7/2)/7 + 3*a*b**2*x**4/4 + 2*b**3*x**(9/2)/9
```

3.2133 $\int \left(a+b\sqrt{x}\right)^3 x\,dx$

Optimal. Leaf size=44

$$\frac{a^3x^2}{2}+\frac{6}{5}a^2bx^{5/2}+ab^2x^3+\frac{2}{7}b^3x^{7/2}$$

```
[Out] 1/2*a^3*x^2+6/5*a^2*b*x^(5/2)+a*b^2*x^3+2/7*b^3*x^(7/2)
```

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{6}{5}a^2bx^{5/2}+\frac{a^3x^2}{2}+ab^2x^3+\frac{2}{7}b^3x^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^3*x,x]
```

```
[Out] (a^3*x^2)/2 + (6*a^2*b*x^(5/2))/5 + a*b^2*x^3 + (2*b^3*x^(7/2))/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sqrt{x}\right)^3 x\,dx &= 2\,\text{Subst}\left(\int x^3(a+bx)^3\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(a^3x^3+3a^2bx^4+3ab^2x^5+b^3x^6\right)\,dx, x, \sqrt{x}\right)\\ &= \frac{a^3x^2}{2}+\frac{6}{5}a^2bx^{5/2}+ab^2x^3+\frac{2}{7}b^3x^{7/2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 1.00

$$\frac{a^3x^2}{2}+\frac{6}{5}a^2bx^{5/2}+ab^2x^3+\frac{2}{7}b^3x^{7/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^3*x,x]
```

```
[Out] (a^3*x^2)/2 + (6*a^2*b*x^(5/2))/5 + a*b^2*x^3 + (2*b^3*x^(7/2))/7
```

fricas [A] time = 1.15, size = 40, normalized size = 0.91

$$ab^2x^3+\frac{1}{2}\,a^3x^2+\frac{2}{35}\left(5\,b^3x^3+21\,a^2bx^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^(1/2))^3,x, algorithm="fricas")
```

```
[Out] a*b^2*x^3 + 1/2*a^3*x^2 + 2/35*(5*b^3*x^3 + 21*a^2*b*x^2)*sqrt(x)
```

giac [A] time = 0.16, size = 34, normalized size = 0.77

$$\frac{2}{7}b^3x^{\frac{7}{2}} + ab^2x^3 + \frac{6}{5}a^2bx^{\frac{5}{2}} + \frac{1}{2}a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^(1/2))^3,x, algorithm="giac")
```

```
[Out] 2/7*b^3*x^(7/2) + a*b^2*x^3 + 6/5*a^2*b*x^(5/2) + 1/2*a^3*x^2
```

maple [A] time = 0.00, size = 35, normalized size = 0.80

$$\frac{2b^3x^{\frac{7}{2}}}{7} + a\,b^2x^3 + \frac{6a^2b\,x^{\frac{5}{2}}}{5} + \frac{a^3x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*x^(1/2))^3,x)
```

```
[Out] 1/2*a^3*x^2+6/5*a^2*b*x^(5/2)+a*b^2*x^3+2/7*b^3*x^(7/2)
```

maxima [A] time = 0.84, size = 64, normalized size = 1.45

$$\frac{2\left(b\sqrt{x}+a\right)^7}{7\,b^4} - \frac{\left(b\sqrt{x}+a\right)^6 a}{b^4} + \frac{6\left(b\sqrt{x}+a\right)^5 a^2}{5\,b^4} - \frac{\left(b\sqrt{x}+a\right)^4 a^3}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^(1/2))^3,x, algorithm="maxima")
```

```
[Out] 2/7*(b*sqrt(x) + a)^7/b^4 - (b*sqrt(x) + a)^6*a/b^4 + 6/5*(b*sqrt(x) + a)^5
*a^2/b^4 - 1/2*(b*sqrt(x) + a)^4*a^3/b^4
```

mupad [B] time = 0.04, size = 34, normalized size = 0.77

$$\frac{a^3\,x^2}{2} + \frac{2\,b^3\,x^{7/2}}{7} + a\,b^2\,x^3 + \frac{6\,a^2\,b\,x^{5/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^(1/2))^3,x)
```

```
[Out] (a^3*x^2)/2 + (2*b^3*x^(7/2))/7 + a*b^2*x^3 + (6*a^2*b*x^(5/2))/5
```

sympy [A] time = 1.86, size = 41, normalized size = 0.93

$$\frac{a^3x^2}{2} + \frac{6a^2bx^{\frac{5}{2}}}{5} + ab^2x^3 + \frac{2b^3x^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x**(1/2))**3,x)
```

```
[Out] a**3*x**2/2 + 6*a**2*b*x**(5/2)/5 + a*b**2*x**3 + 2*b**3*x**(7/2)/7
```

3.2134 $\int \left(a+b\sqrt{x}\right)^3\,dx$

Optimal. Leaf size=38

$$\frac{2\left(a+b\sqrt{x}\right)^5}{5b^2}-\frac{a\left(a+b\sqrt{x}\right)^4}{2b^2}$$

```
[Out] -1/2*a*(a+b*x^(1/2))^4/b^2+2/5*(a+b*x^(1/2))^5/b^2
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {190, 43}

$$\frac{2\left(a+b\sqrt{x}\right)^5}{5b^2}-\frac{a\left(a+b\sqrt{x}\right)^4}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^3,x]
```

```
[Out] -(a*(a + b*Sqrt[x])^4)/(2*b^2) + (2*(a + b*Sqrt[x])^5)/(5*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
 - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
  IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sqrt{x}\right)^3\,dx &= 2\,\text{Subst}\left(\int x(a+bx)^3\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(-\frac{a(a+bx)^3}{b}+\frac{(a+bx)^4}{b}\right)\,dx, x, \sqrt{x}\right)\\ &= -\frac{a\left(a+b\sqrt{x}\right)^4}{2b^2}+\frac{2\left(a+b\sqrt{x}\right)^5}{5b^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 0.74

$$-\frac{\left(a-4b\sqrt{x}\right)\left(a+b\sqrt{x}\right)^4}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^3,x]
```

```
[Out] -1/10*((a - 4*b*Sqrt[x])*(a + b*Sqrt[x])^4)/b^2
```

fricas [A] time = 0.68, size = 35, normalized size = 0.92

$$\frac{3}{2}ab^2x^2+a^3x+\frac{2}{5}\left(b^3x^2+5\,a^2bx\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^3,x, algorithm="fricas")
[Out] 3/2*a*b^2*x^2 + a^3*x + 2/5*(b^3*x^2 + 5*a^2*b*x)*sqrt(x)
```

giac [A] time = 0.15, size = 32, normalized size = 0.84

$$\frac{2}{5}b^3x^{\frac{5}{2}} + \frac{3}{2}ab^2x^2 + 2\,a^2bx^{\frac{3}{2}} + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^3,x, algorithm="giac")
[Out] 2/5*b^3*x^(5/2) + 3/2*a*b^2*x^2 + 2*a^2*b*x^(3/2) + a^3*x
```

maple [A] time = 0.00, size = 33, normalized size = 0.87

$$\frac{2b^3x^{\frac{5}{2}}}{5} + \frac{3a\,b^2x^2}{2} + 2a^2b\,x^{\frac{3}{2}} + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^3,x)
[Out] 2/5*x^(5/2)*b^3+3/2*a*b^2*x^2+2*a^2*b*x^(3/2)+a^3*x
```

maxima [A] time = 0.91, size = 32, normalized size = 0.84

$$\frac{2}{5}b^3x^{\frac{5}{2}} + \frac{3}{2}ab^2x^2 + 2\,a^2bx^{\frac{3}{2}} + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^3,x, algorithm="maxima")
[Out] 2/5*b^3*x^(5/2) + 3/2*a*b^2*x^2 + 2*a^2*b*x^(3/2) + a^3*x
```

mupad [B] time = 0.04, size = 32, normalized size = 0.84

$$a^3\,x + \frac{2\,b^3\,x^{5/2}}{5} + \frac{3\,a\,b^2\,x^2}{2} + 2\,a^2\,b\,x^{3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^3,x)
[Out] a^3*x + (2*b^3*x^(5/2))/5 + (3*a*b^2*x^2)/2 + 2*a^2*b*x^(3/2)
```

sympy [A] time = 1.75, size = 39, normalized size = 1.03

$$a^3x + 2a^2bx^{\frac{3}{2}} + \frac{3ab^2x^2}{2} + \frac{2b^3x^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**3,x)
[Out] a**3*x + 2*a**2*b*x**(3/2) + 3*a*b**2*x**2/2 + 2*b**3*x**(5/2)/5
```

3.2135 $\int \frac{\left(a+b\sqrt{x}\right)^3}{x}\,dx$

Optimal. Leaf size=37

$$a^3\log(x)+6a^2b\sqrt{x}+3ab^2x+\frac{2}{3}b^3x^{3/2}$$

```
[Out] 3*a*b^2*x+2/3*b^3*x^(3/2)+a^3*ln(x)+6*a^2*b*x^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$6a^2b\sqrt{x}+a^3\log(x)+3ab^2x+\frac{2}{3}b^3x^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^3/x,x]
```

```
[Out] 6*a^2*b*Sqrt[x] + 3*a*b^2*x + (2*b^3*x^(3/2))/3 + a^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt{x}\right)^3}{x}\,dx &= 2\,\text{Subst}\left(\int \frac{(a+bx)^3}{x}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(3a^2b+\frac{a^3}{x}+3ab^2x+b^3x^2\right)dx, x, \sqrt{x}\right)\\ &= 6a^2b\sqrt{x}+3ab^2x+\frac{2}{3}b^3x^{3/2}+a^3\log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.00

$$a^3\log(x)+6a^2b\sqrt{x}+3ab^2x+\frac{2}{3}b^3x^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^3/x,x]
```

```
[Out] 6*a^2*b*Sqrt[x] + 3*a*b^2*x + (2*b^3*x^(3/2))/3 + a^3*Log[x]
```

fricas [A] time = 1.01, size = 34, normalized size = 0.92

$$3\,ab^2x+2\,a^3\log\left(\sqrt{x}\right)+\frac{2}{3}\left(b^3x+9\,a^2b\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^3/x,x, algorithm="fricas")
[Out] 3*a*b^2*x + 2*a^3*log(sqrt(x)) + 2/3*(b^3*x + 9*a^2*b)*sqrt(x)
```

giac [A] time = 0.16, size = 32, normalized size = 0.86

$$\frac{2}{3}b^3x^{\frac{3}{2}} + 3ab^2x + a^3\log(|x|) + 6a^2b\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^3/x,x, algorithm="giac")
[Out] 2/3*b^3*x^(3/2) + 3*a*b^2*x + a^3*log(abs(x)) + 6*a^2*b*sqrt(x)
```

maple [A] time = 0.00, size = 32, normalized size = 0.86

$$\frac{2b^3x^{\frac{3}{2}}}{3} + a^3\ln(x) + 3a\,b^2x + 6a^2b\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^3/x,x)
[Out] 3*a*b^2*x+2/3*b^3*x^(3/2)+a^3*ln(x)+6*a^2*b*x^(1/2)
```

maxima [A] time = 0.88, size = 31, normalized size = 0.84

$$\frac{2}{3}b^3x^{\frac{3}{2}} + 3ab^2x + a^3\log(x) + 6a^2b\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^3/x,x, algorithm="maxima")
[Out] 2/3*b^3*x^(3/2) + 3*a*b^2*x + a^3*log(x) + 6*a^2*b*sqrt(x)
```

mupad [B] time = 0.04, size = 34, normalized size = 0.92

$$2a^3\,\ln\left(\sqrt{x}\right) + \frac{2b^3x^{3/2}}{3} + 6a^2b\sqrt{x} + 3ab^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^3/x,x)
[Out] 2*a^3*log(x^(1/2)) + (2*b^3*x^(3/2))/3 + 6*a^2*b*x^(1/2) + 3*a*b^2*x
```

sympy [A] time = 0.26, size = 37, normalized size = 1.00

$$a^3\log(x) + 6a^2b\sqrt{x} + 3ab^2x + \frac{2b^3x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**3/x,x)
[Out] a**3*log(x) + 6*a**2*b*sqrt(x) + 3*a*b**2*x + 2*b**3*x**(3/2)/3
```

3.2136 $\int \frac{\left(a+b\sqrt{x}\right)^3}{x^2}\,dx$

Optimal. Leaf size=38

$$-\frac{a^3}{x}-\frac{6a^2b}{\sqrt{x}}+3ab^2\log(x)+2b^3\sqrt{x}$$

```
[Out] -a^3/x+3*a*b^2*ln(x)-6*a^2*b/x^(1/2)+2*b^3*x^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{6a^2b}{\sqrt{x}}-\frac{a^3}{x}+3ab^2\log(x)+2b^3\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^3/x^2,x]
```

```
[Out] -(a^3/x) - (6*a^2*b)/Sqrt[x] + 2*b^3*Sqrt[x] + 3*a*b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt{x}\right)^3}{x^2}\,dx &= 2\,\text{Subst}\left(\int \frac{(a+bx)^3}{x^3}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(b^3+\frac{a^3}{x^3}+\frac{3a^2b}{x^2}+\frac{3ab^2}{x}\right)dx, x, \sqrt{x}\right)\\ &= -\frac{a^3}{x}-\frac{6a^2b}{\sqrt{x}}+2b^3\sqrt{x}+3ab^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 1.00

$$-\frac{a^3}{x}-\frac{6a^2b}{\sqrt{x}}+3ab^2\log(x)+2b^3\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^3/x^2,x]
```

```
[Out] -(a^3/x) - (6*a^2*b)/Sqrt[x] + 2*b^3*Sqrt[x] + 3*a*b^2*Log[x]
```

fricas [A] time = 0.94, size = 38, normalized size = 1.00

$$\frac{6\,ab^2x\log\left(\sqrt{x}\right) - a^3 + 2\left(b^3x - 3\,a^2b\right)\sqrt{x}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^3/x^2,x, algorithm="fricas")
[Out] (6*a*b^2*x*log(sqrt(x)) - a^3 + 2*(b^3*x - 3*a^2*b)*sqrt(x))/x
```

giac [A] time = 0.15, size = 36, normalized size = 0.95

$$3\,ab^2\log\left(|x|\right) + 2\,b^3\sqrt{x} - \frac{6\,a^2b\sqrt{x} + a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^3/x^2,x, algorithm="giac")
[Out] 3*a*b^2*log(abs(x)) + 2*b^3*sqrt(x) - (6*a^2*b*sqrt(x) + a^3)/x
```

maple [A] time = 0.00, size = 35, normalized size = 0.92

$$3a\,b^2\ln(x) + 2b^3\sqrt{x} - \frac{6a^2b}{\sqrt{x}} - \frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^3/x^2,x)
[Out] -a^3/x+3*a*b^2*ln(x)-6*a^2*b/x^(1/2)+2*b^3*x^(1/2)
```

maxima [A] time = 0.89, size = 35, normalized size = 0.92

$$3\,ab^2\log(x) + 2\,b^3\sqrt{x} - \frac{6\,a^2b\sqrt{x} + a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^3/x^2,x, algorithm="maxima")
[Out] 3*a*b^2*log(x) + 2*b^3*sqrt(x) - (6*a^2*b*sqrt(x) + a^3)/x
```

mupad [B] time = 1.10, size = 37, normalized size = 0.97

$$2\,b^3\,\sqrt{x} - \frac{a^3 + 6\,a^2\,b\,\sqrt{x}}{x} + 6\,a\,b^2\,\ln\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^3/x^2,x)
[Out] 2*b^3*x^(1/2) - (a^3 + 6*a^2*b*x^(1/2))/x + 6*a*b^2*log(x^(1/2))
```

sympy [A] time = 0.49, size = 36, normalized size = 0.95

$$-\frac{a^3}{x} - \frac{6a^2b}{\sqrt{x}} + 3ab^2\log\left(x\right) + 2b^3\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**3/x**2,x)
[Out] -a**3/x - 6*a**2*b/sqrt(x) + 3*a*b**2*log(x) + 2*b**3*sqrt(x)
```

3.2137 $\int \frac{\left(a+b\sqrt{x}\right)^3}{x^3}\,dx$

Optimal. Leaf size=21

$$-\frac{\left(a+b\sqrt{x}\right)^4}{2ax^2}$$

```
[Out] -1/2*(a+b*x^(1/2))^4/a/x^2
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {264}

$$-\frac{\left(a+b\sqrt{x}\right)^4}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^3/x^3,x]
```

```
[Out] -(a + b*Sqrt[x])^4/(2*a*x^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\sqrt{x}\right)^3}{x^3}\,dx = -\frac{\left(a+b\sqrt{x}\right)^4}{2ax^2}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$-\frac{\left(a+b\sqrt{x}\right)^4}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^3/x^3,x]
```

```
[Out] -1/2*(a + b*Sqrt[x])^4/(a*x^2)
```

fricas [A] time = 1.08, size = 32, normalized size = 1.52

$$-\frac{6\,ab^2x + a^3 + 4\left(b^3x + a^2b\right)\sqrt{x}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^3/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(6*a*b^2*x + a^3 + 4*(b^3*x + a^2*b)*sqrt(x))/x^2
```

giac [A] time = 0.15, size = 33, normalized size = 1.57

$$-\frac{4\,b^3x^{\frac{3}{2}} + 6\,ab^2x + 4\,a^2b\sqrt{x} + a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^3/x^3,x, algorithm="giac")
[Out] -1/2*(4*b^3*x^(3/2) + 6*a*b^2*x + 4*a^2*b*sqrt(x) + a^3)/x^2
```

maple [B] time = 0.00, size = 36, normalized size = 1.71

$$-\frac{2b^3}{\sqrt{x}}-\frac{3a\,b^2}{x}-\frac{2a^2b}{x^{\frac{3}{2}}}-\frac{a^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^3/x^3,x)
[Out] -2*b^3/x^(1/2)-3*a*b^2/x-2*a^2*b/x^(3/2)-1/2*a^3/x^2
```

maxima [A] time = 0.81, size = 33, normalized size = 1.57

$$-\frac{4\,b^3x^{\frac{3}{2}}+6\,ab^2x+4\,a^2b\sqrt{x}+a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^3/x^3,x, algorithm="maxima")
[Out] -1/2*(4*b^3*x^(3/2) + 6*a*b^2*x + 4*a^2*b*sqrt(x) + a^3)/x^2
```

mupad [B] time = 0.03, size = 33, normalized size = 1.57

$$-\frac{a^3+4\,b^3\,x^{3/2}+4\,a^2\,b\,\sqrt{x}+6\,a\,b^2\,x}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^3/x^3,x)
[Out] -(a^3 + 4*b^3*x^(3/2) + 4*a^2*b*x^(1/2) + 6*a*b^2*x)/(2*x^2)
```

sympy [B] time = 0.69, size = 39, normalized size = 1.86

$$-\frac{a^3}{2x^2}-\frac{2a^2b}{x^{\frac{3}{2}}}-\frac{3ab^2}{x}-\frac{2b^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**3/x**3,x)
[Out] -a**3/(2*x**2) - 2*a**2*b/x**(3/2) - 3*a*b**2/x - 2*b**3/sqrt(x)
```

3.2138 $\int \frac{\left(a+b\sqrt{x}\right)^3}{x^4}\,dx$

Optimal. Leaf size=47

$$-\frac{a^3}{3x^3}-\frac{6a^2b}{5x^{5/2}}-\frac{3ab^2}{2x^2}-\frac{2b^3}{3x^{3/2}}$$

```
[Out] -1/3*a^3/x^3-6/5*a^2*b/x^(5/2)-3/2*a*b^2/x^2-2/3*b^3/x^(3/2)
```

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{6a^2b}{5x^{5/2}}-\frac{a^3}{3x^3}-\frac{3ab^2}{2x^2}-\frac{2b^3}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^3/x^4,x]
```

```
[Out] -a^3/(3*x^3) - (6*a^2*b)/(5*x^(5/2)) - (3*a*b^2)/(2*x^2) - (2*b^3)/(3*x^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt{x}\right)^3}{x^4}\,dx &= 2\,\text{Subst}\left(\int \frac{(a+bx)^3}{x^7}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(\frac{a^3}{x^7}+\frac{3a^2b}{x^6}+\frac{3ab^2}{x^5}+\frac{b^3}{x^4}\right)dx, x, \sqrt{x}\right)\\ &= -\frac{a^3}{3x^3}-\frac{6a^2b}{5x^{5/2}}-\frac{3ab^2}{2x^2}-\frac{2b^3}{3x^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 0.87

$$-\frac{10a^3+36a^2b\sqrt{x}+45ab^2x+20b^3x^{3/2}}{30x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^3/x^4,x]
```

```
[Out] -1/30*(10*a^3 + 36*a^2*b*Sqrt[x] + 45*a*b^2*x + 20*b^3*x^(3/2))/x^3
```

fricas [A] time = 1.05, size = 36, normalized size = 0.77

$$-\frac{45\,ab^2x+10\,a^3+4\left(5\,b^3x+9\,a^2b\right)\sqrt{x}}{30\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^3/x^4,x, algorithm="fricas")
[Out] -1/30*(45*a*b^2*x + 10*a^3 + 4*(5*b^3*x + 9*a^2*b)*sqrt(x))/x^3
```

giac [A] time = 0.15, size = 35, normalized size = 0.74

$$-\frac{20\,b^3x^{\frac{3}{2}}+45\,ab^2x+36\,a^2b\sqrt{x}+10\,a^3}{30\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^3/x^4,x, algorithm="giac")
[Out] -1/30*(20*b^3*x^(3/2) + 45*a*b^2*x + 36*a^2*b*sqrt(x) + 10*a^3)/x^3
```

maple [A] time = 0.00, size = 36, normalized size = 0.77

$$-\frac{2b^3}{3x^{\frac{3}{2}}}-\frac{3a\,b^2}{2x^2}-\frac{6a^2b}{5x^{\frac{5}{2}}}-\frac{a^3}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^3/x^4,x)
[Out] -1/3*a^3/x^3-6/5*a^2*b/x^(5/2)-3/2*a*b^2/x^2-2/3*b^3/x^(3/2)
```

maxima [A] time = 0.84, size = 35, normalized size = 0.74

$$-\frac{20\,b^3x^{\frac{3}{2}}+45\,ab^2x+36\,a^2b\sqrt{x}+10\,a^3}{30\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^3/x^4,x, algorithm="maxima")
[Out] -1/30*(20*b^3*x^(3/2) + 45*a*b^2*x + 36*a^2*b*sqrt(x) + 10*a^3)/x^3
```

mupad [B] time = 1.07, size = 35, normalized size = 0.74

$$-\frac{10\,a^3+20\,b^3\,x^{3/2}+36\,a^2\,b\,\sqrt{x}+45\,a\,b^2\,x}{30\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^3/x^4,x)
[Out] -(10*a^3 + 20*b^3*x^(3/2) + 36*a^2*b*x^(1/2) + 45*a*b^2*x)/(30*x^3)
```

sympy [A] time = 1.10, size = 46, normalized size = 0.98

$$-\frac{a^3}{3x^3}-\frac{6a^2b}{5x^{\frac{5}{2}}}-\frac{3ab^2}{2x^2}-\frac{2b^3}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**3/x**4,x)
[Out] -a**3/(3*x**3) - 6*a**2*b/(5*x**(5/2)) - 3*a*b**2/(2*x**2) - 2*b**3/(3*x**(
3/2))
```

3.2139 $\int \frac{\left(a+b\sqrt{x}\right)^3}{x^5}\,dx$

Optimal. Leaf size=45

$$-\frac{a^3}{4x^4}-\frac{6a^2b}{7x^{7/2}}-\frac{ab^2}{x^3}-\frac{2b^3}{5x^{5/2}}$$

```
[Out] -1/4*a^3/x^4-6/7*a^2*b/x^(7/2)-a*b^2/x^3-2/5*b^3/x^(5/2)
```

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{6a^2b}{7x^{7/2}}-\frac{a^3}{4x^4}-\frac{ab^2}{x^3}-\frac{2b^3}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^3/x^5,x]
```

```
[Out] -a^3/(4*x^4) - (6*a^2*b)/(7*x^(7/2)) - (a*b^2)/x^3 - (2*b^3)/(5*x^(5/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt{x}\right)^3}{x^5}\,dx &= 2\,\text{Subst}\left(\int \frac{(a+bx)^3}{x^9}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(\frac{a^3}{x^9}+\frac{3a^2b}{x^8}+\frac{3ab^2}{x^7}+\frac{b^3}{x^6}\right)dx, x, \sqrt{x}\right)\\ &= -\frac{a^3}{4x^4}-\frac{6a^2b}{7x^{7/2}}-\frac{ab^2}{x^3}-\frac{2b^3}{5x^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 0.91

$$-\frac{35a^3+120a^2b\sqrt{x}+140ab^2x+56b^3x^{3/2}}{140x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^3/x^5,x]
```

```
[Out] -1/140*(35*a^3 + 120*a^2*b*Sqrt[x] + 140*a*b^2*x + 56*b^3*x^(3/2))/x^4
```

fricas [A] time = 1.02, size = 36, normalized size = 0.80

$$-\frac{140\,ab^2x+35\,a^3+8\left(7\,b^3x+15\,a^2b\right)\sqrt{x}}{140\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^3/x^5,x, algorithm="fricas")
[Out] -1/140*(140*a*b^2*x + 35*a^3 + 8*(7*b^3*x + 15*a^2*b)*sqrt(x))/x^4
```

giac [A] time = 0.15, size = 35, normalized size = 0.78

$$-\frac{56\,b^3x^{\frac{3}{2}}+140\,ab^2x+120\,a^2b\sqrt{x}\,+35\,a^3}{140\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^3/x^5,x, algorithm="giac")
[Out] -1/140*(56*b^3*x^(3/2) + 140*a*b^2*x + 120*a^2*b*sqrt(x) + 35*a^3)/x^4
```

maple [A] time = 0.00, size = 36, normalized size = 0.80

$$-\frac{2b^3}{5x^{\frac{5}{2}}}-\frac{a\,b^2}{x^3}-\frac{6a^2b}{7x^{\frac{7}{2}}}-\frac{a^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^3/x^5,x)
[Out] -1/4*a^3/x^4-6/7*a^2*b/x^(7/2)-a*b^2/x^3-2/5*b^3/x^(5/2)
```

maxima [A] time = 0.80, size = 35, normalized size = 0.78

$$-\frac{56\,b^3x^{\frac{3}{2}}+140\,ab^2x+120\,a^2b\sqrt{x}\,+35\,a^3}{140\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^3/x^5,x, algorithm="maxima")
[Out] -1/140*(56*b^3*x^(3/2) + 140*a*b^2*x + 120*a^2*b*sqrt(x) + 35*a^3)/x^4
```

mupad [B] time = 0.03, size = 35, normalized size = 0.78

$$-\frac{35\,a^3+56\,b^3\,x^{3/2}+120\,a^2\,b\,\sqrt{x}\,+140\,a\,b^2\,x}{140\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^3/x^5,x)
[Out] -(35*a^3 + 56*b^3*x^(3/2) + 120*a^2*b*x^(1/2) + 140*a*b^2*x)/(140*x^4)
```

sympy [A] time = 1.55, size = 42, normalized size = 0.93

$$-\frac{a^3}{4x^4}-\frac{6a^2b}{7x^{\frac{7}{2}}}-\frac{ab^2}{x^3}-\frac{2b^3}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**3/x**5,x)
[Out] -a**3/(4*x**4) - 6*a**2*b/(7*x**(7/2)) - a*b**2/x**3 - 2*b**3/(5*x**(5/2))
```

3.2140 $\int \frac{\left(a+b\sqrt{x}\right)^3}{x^6}\,dx$

Optimal. Leaf size=47

$$-\frac{a^3}{5x^5}-\frac{2a^2b}{3x^{9/2}}-\frac{3ab^2}{4x^4}-\frac{2b^3}{7x^{7/2}}$$

```
[Out] -1/5*a^3/x^5-2/3*a^2*b/x^(9/2)-3/4*a*b^2/x^4-2/7*b^3/x^(7/2)
```

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{2a^2b}{3x^{9/2}}-\frac{a^3}{5x^5}-\frac{3ab^2}{4x^4}-\frac{2b^3}{7x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^3/x^6,x]
```

```
[Out] -a^3/(5*x^5) - (2*a^2*b)/(3*x^(9/2)) - (3*a*b^2)/(4*x^4) - (2*b^3)/(7*x^(7/
2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt{x}\right)^3}{x^6}\,dx &= 2\,\text{Subst}\left(\int \frac{(a+bx)^3}{x^{11}}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(\frac{a^3}{x^{11}}+\frac{3a^2b}{x^{10}}+\frac{3ab^2}{x^9}+\frac{b^3}{x^8}\right)dx, x, \sqrt{x}\right)\\ &= -\frac{a^3}{5x^5}-\frac{2a^2b}{3x^{9/2}}-\frac{3ab^2}{4x^4}-\frac{2b^3}{7x^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 0.87

$$-\frac{84a^3+280a^2b\sqrt{x}+315ab^2x+120b^3x^{3/2}}{420x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^3/x^6,x]
```

```
[Out] -1/420*(84*a^3 + 280*a^2*b*Sqrt[x] + 315*a*b^2*x + 120*b^3*x^(3/2))/x^5
```

fricas [A] time = 0.88, size = 36, normalized size = 0.77

$$-\frac{315\,ab^2x+84\,a^3+40\left(3\,b^3x+7\,a^2b\right)\sqrt{x}}{420\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^3/x^6,x, algorithm="fricas")
[Out] -1/420*(315*a*b^2*x + 84*a^3 + 40*(3*b^3*x + 7*a^2*b)*sqrt(x))/x^5
```

giac [A] time = 0.16, size = 35, normalized size = 0.74

$$-\frac{120\,b^3x^{\frac{3}{2}}+315\,ab^2x+280\,a^2b\sqrt{x}+84\,a^3}{420\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^3/x^6,x, algorithm="giac")
[Out] -1/420*(120*b^3*x^(3/2) + 315*a*b^2*x + 280*a^2*b*sqrt(x) + 84*a^3)/x^5
```

maple [A] time = 0.00, size = 36, normalized size = 0.77

$$-\frac{2b^3}{7x^{\frac{7}{2}}}-\frac{3a\,b^2}{4x^4}-\frac{2a^2b}{3x^{\frac{9}{2}}}-\frac{a^3}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^3/x^6,x)
[Out] -1/5*a^3/x^5-2/3*a^2*b/x^(9/2)-3/4*a*b^2/x^4-2/7*b^3/x^(7/2)
```

maxima [A] time = 0.90, size = 35, normalized size = 0.74

$$-\frac{120\,b^3x^{\frac{3}{2}}+315\,ab^2x+280\,a^2b\sqrt{x}+84\,a^3}{420\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^3/x^6,x, algorithm="maxima")
[Out] -1/420*(120*b^3*x^(3/2) + 315*a*b^2*x + 280*a^2*b*sqrt(x) + 84*a^3)/x^5
```

mupad [B] time = 0.03, size = 35, normalized size = 0.74

$$-\frac{84\,a^3+120\,b^3\,x^{3/2}+280\,a^2\,b\,\sqrt{x}+315\,a\,b^2\,x}{420\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^3/x^6,x)
[Out] -(84*a^3 + 120*b^3*x^(3/2) + 280*a^2*b*x^(1/2) + 315*a*b^2*x)/(420*x^5)
```

sympy [A] time = 2.22, size = 46, normalized size = 0.98

$$-\frac{a^3}{5x^5}-\frac{2a^2b}{3x^{\frac{9}{2}}}-\frac{3ab^2}{4x^4}-\frac{2b^3}{7x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**3/x**6,x)
[Out] -a**3/(5*x**5) - 2*a**2*b/(3*x**(9/2)) - 3*a*b**2/(4*x**4) - 2*b**3/(7*x**(
7/2))
```

3.2141 $\int \left(a + b\sqrt{x}\right)^5 x^4\, dx$

Optimal. Leaf size=75

$$\frac{a^5x^5}{5} + \frac{10}{11}a^4bx^{11/2} + \frac{5}{3}a^3b^2x^6 + \frac{20}{13}a^2b^3x^{13/2} + \frac{5}{7}ab^4x^7 + \frac{2}{15}b^5x^{15/2}$$

```
[Out] 1/5*a^5*x^5+10/11*a^4*b*x^(11/2)+5/3*a^3*b^2*x^6+20/13*a^2*b^3*x^(13/2)+5/7
*a*b^4*x^7+2/15*b^5*x^(15/2)
```

Rubi [A] time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{20}{13}a^2b^3x^{13/2} + \frac{5}{3}a^3b^2x^6 + \frac{10}{11}a^4bx^{11/2} + \frac{a^5x^5}{5} + \frac{5}{7}ab^4x^7 + \frac{2}{15}b^5x^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^5*x^4,x]
```

```
[Out] (a^5*x^5)/5 + (10*a^4*b*x^(11/2))/11 + (5*a^3*b^2*x^6)/3 + (20*a^2*b^3*x^(1
3/2))/13 + (5*a*b^4*x^7)/7 + (2*b^5*x^(15/2))/15
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a + b\sqrt{x}\right)^5 x^4\, dx &= 2\,\text{Subst}\left(\int x^9(a + bx)^5\, dx, x, \sqrt{x}\right) \\
&= 2\,\text{Subst}\left(\int \left(a^5x^9 + 5a^4bx^{10} + 10a^3b^2x^{11} + 10a^2b^3x^{12} + 5ab^4x^{13} + b^5x^{14}\right) dx, x, \sqrt{x}\right) \\
&= \frac{a^5x^5}{5} + \frac{10}{11}a^4bx^{11/2} + \frac{5}{3}a^3b^2x^6 + \frac{20}{13}a^2b^3x^{13/2} + \frac{5}{7}ab^4x^7 + \frac{2}{15}b^5x^{15/2}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 75, normalized size = 1.00

$$\frac{a^5x^5}{5} + \frac{10}{11}a^4bx^{11/2} + \frac{5}{3}a^3b^2x^6 + \frac{20}{13}a^2b^3x^{13/2} + \frac{5}{7}ab^4x^7 + \frac{2}{15}b^5x^{15/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^5*x^4,x]
```

```
[Out] (a^5*x^5)/5 + (10*a^4*b*x^(11/2))/11 + (5*a^3*b^2*x^6)/3 + (20*a^2*b^3*x^(1
3/2))/13 + (5*a*b^4*x^7)/7 + (2*b^5*x^(15/2))/15
```

fricas [A] time = 0.95, size = 63, normalized size = 0.84

$$\frac{5}{7}ab^4x^7+\frac{5}{3}a^3b^2x^6+\frac{1}{5}a^5x^5+\frac{2}{2145}\left(143\,b^5x^7+1650\,a^2b^3x^6+975\,a^4bx^5\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*x^(1/2))^5,x, algorithm="fricas")
```

```
[Out] 5/7*a*b^4*x^7 + 5/3*a^3*b^2*x^6 + 1/5*a^5*x^5 + 2/2145*(143*b^5*x^7 + 1650*
a^2*b^3*x^6 + 975*a^4*b*x^5)*sqrt(x)
```

giac [A] time = 0.15, size = 57, normalized size = 0.76

$$\frac{2}{15}b^5x^{\frac{15}{2}}+\frac{5}{7}ab^4x^7+\frac{20}{13}a^2b^3x^{\frac{13}{2}}+\frac{5}{3}a^3b^2x^6+\frac{10}{11}a^4bx^{\frac{11}{2}}+\frac{1}{5}a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*x^(1/2))^5,x, algorithm="giac")
```

```
[Out] 2/15*b^5*x^(15/2) + 5/7*a*b^4*x^7 + 20/13*a^2*b^3*x^(13/2) + 5/3*a^3*b^2*x^
6 + 10/11*a^4*b*x^(11/2) + 1/5*a^5*x^5
```

maple [A] time = 0.00, size = 58, normalized size = 0.77

$$\frac{2b^5x^{\frac{15}{2}}}{15}+\frac{5a\,b^4x^7}{7}+\frac{20a^2b^3x^{\frac{13}{2}}}{13}+\frac{5a^3b^2x^6}{3}+\frac{10a^4b\,x^{\frac{11}{2}}}{11}+\frac{a^5x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*x^(1/2))^5,x)
```

```
[Out] 1/5*a^5*x^5+10/11*a^4*b*x^(11/2)+5/3*a^3*b^2*x^6+20/13*a^2*b^3*x^(13/2)+5/7
*a*b^4*x^7+2/15*b^5*x^(15/2)
```

maxima [B] time = 0.94, size = 166, normalized size = 2.21

$$\frac{2\left(b\sqrt{x}+a\right)^{15}}{15\,b^{10}}-\frac{9\left(b\sqrt{x}+a\right)^{14}a}{7\,b^{10}}+\frac{72\left(b\sqrt{x}+a\right)^{13}a^2}{13\,b^{10}}-\frac{14\left(b\sqrt{x}+a\right)^{12}a^3}{b^{10}}+\frac{252\left(b\sqrt{x}+a\right)^{11}a^4}{11\,b^{10}}-\frac{126\left(b\sqrt{x}+a\right)^{1}}{5\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*x^(1/2))^5,x, algorithm="maxima")
```

```
[Out] 2/15*(b*sqrt(x) + a)^15/b^10 - 9/7*(b*sqrt(x) + a)^14*a/b^10 + 72/13*(b*sqr
t(x) + a)^13*a^2/b^10 - 14*(b*sqrt(x) + a)^12*a^3/b^10 + 252/11*(b*sqrt(x)
+ a)^11*a^4/b^10 - 126/5*(b*sqrt(x) + a)^10*a^5/b^10 + 56/3*(b*sqrt(x) + a)
^9*a^6/b^10 - 9*(b*sqrt(x) + a)^8*a^7/b^10 + 18/7*(b*sqrt(x) + a)^7*a^8/b^1
0 - 1/3*(b*sqrt(x) + a)^6*a^9/b^10
```

mupad [B] time = 0.03, size = 57, normalized size = 0.76

$$\frac{a^5\,x^5}{5}+\frac{2\,b^5\,x^{15/2}}{15}+\frac{5\,a\,b^4\,x^7}{7}+\frac{10\,a^4\,b\,x^{11/2}}{11}+\frac{5\,a^3\,b^2\,x^6}{3}+\frac{20\,a^2\,b^3\,x^{13/2}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^(1/2))^5,x)
```

```
[Out] (a^5*x^5)/5 + (2*b^5*x^(15/2))/15 + (5*a*b^4*x^7)/7 + (10*a^4*b*x^(11/2))/1
1 + (5*a^3*b^2*x^6)/3 + (20*a^2*b^3*x^(13/2))/13
```

sympy [**A**] time = 2.73, size = 73, normalized size = 0.97

$$\frac{a^5x^5}{5} + \frac{10a^4bx^{\frac{11}{2}}}{11} + \frac{5a^3b^2x^6}{3} + \frac{20a^2b^3x^{\frac{13}{2}}}{13} + \frac{5ab^4x^7}{7} + \frac{2b^5x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*x**(1/2))**5,x)
```

```
[Out] a**5*x**5/5 + 10*a**4*b*x**(11/2)/11 + 5*a**3*b**2*x**6/3 + 20*a**2*b**3*x*
*(13/2)/13 + 5*a*b**4*x**7/7 + 2*b**5*x**(15/2)/15
```

3.2142 $\int \left(a + b\sqrt{x}\right)^5 x^3\, dx$

Optimal. Leaf size=73

$$\frac{a^5x^4}{4} + \frac{10}{9}a^4bx^{9/2} + 2a^3b^2x^5 + \frac{20}{11}a^2b^3x^{11/2} + \frac{5}{6}ab^4x^6 + \frac{2}{13}b^5x^{13/2}$$

```
[Out] 1/4*a^5*x^4+10/9*a^4*b*x^(9/2)+2*a^3*b^2*x^5+20/11*a^2*b^3*x^(11/2)+5/6*a*b
^4*x^6+2/13*b^5*x^(13/2)
```

Rubi [A] time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{20}{11}a^2b^3x^{11/2} + 2a^3b^2x^5 + \frac{10}{9}a^4bx^{9/2} + \frac{a^5x^4}{4} + \frac{5}{6}ab^4x^6 + \frac{2}{13}b^5x^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^5*x^3,x]
```

```
[Out] (a^5*x^4)/4 + (10*a^4*b*x^(9/2))/9 + 2*a^3*b^2*x^5 + (20*a^2*b^3*x^(11/2))/
11 + (5*a*b^4*x^6)/6 + (2*b^5*x^(13/2))/13
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a + b\sqrt{x}\right)^5 x^3\, dx &= 2\,\text{Subst}\left(\int x^7(a + bx)^5\, dx, x, \sqrt{x}\right) \\
&= 2\,\text{Subst}\left(\int \left(a^5x^7 + 5a^4bx^8 + 10a^3b^2x^9 + 10a^2b^3x^{10} + 5ab^4x^{11} + b^5x^{12}\right) dx, x, \sqrt{x}\right) \\
&= \frac{a^5x^4}{4} + \frac{10}{9}a^4bx^{9/2} + 2a^3b^2x^5 + \frac{20}{11}a^2b^3x^{11/2} + \frac{5}{6}ab^4x^6 + \frac{2}{13}b^5x^{13/2}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 73, normalized size = 1.00

$$\frac{a^5x^4}{4} + \frac{10}{9}a^4bx^{9/2} + 2a^3b^2x^5 + \frac{20}{11}a^2b^3x^{11/2} + \frac{5}{6}ab^4x^6 + \frac{2}{13}b^5x^{13/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^5*x^3,x]
```

```
[Out] (a^5*x^4)/4 + (10*a^4*b*x^(9/2))/9 + 2*a^3*b^2*x^5 + (20*a^2*b^3*x^(11/2))/
11 + (5*a*b^4*x^6)/6 + (2*b^5*x^(13/2))/13
```

fricas [A] time = 1.00, size = 63, normalized size = 0.86

$$\frac{5}{6}ab^4x^6 + 2a^3b^2x^5 + \frac{1}{4}a^5x^4 + \frac{2}{1287}\left(99b^5x^6 + 1170a^2b^3x^5 + 715a^4bx^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^(1/2))^5,x, algorithm="fricas")
```

```
[Out] 5/6*a*b^4*x^6 + 2*a^3*b^2*x^5 + 1/4*a^5*x^4 + 2/1287*(99*b^5*x^6 + 1170*a^2
*b^3*x^5 + 715*a^4*b*x^4)*sqrt(x)
```

giac [A] time = 0.16, size = 57, normalized size = 0.78

$$\frac{2}{13}b^5x^{\frac{13}{2}} + \frac{5}{6}ab^4x^6 + \frac{20}{11}a^2b^3x^{\frac{11}{2}} + 2a^3b^2x^5 + \frac{10}{9}a^4bx^{\frac{9}{2}} + \frac{1}{4}a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^(1/2))^5,x, algorithm="giac")
```

```
[Out] 2/13*b^5*x^(13/2) + 5/6*a*b^4*x^6 + 20/11*a^2*b^3*x^(11/2) + 2*a^3*b^2*x^5
+ 10/9*a^4*b*x^(9/2) + 1/4*a^5*x^4
```

maple [A] time = 0.00, size = 58, normalized size = 0.79

$$\frac{2b^5x^{\frac{13}{2}}}{13} + \frac{5a\,b^4x^6}{6} + \frac{20a^2b^3x^{\frac{11}{2}}}{11} + 2a^3b^2x^5 + \frac{10a^4b\,x^{\frac{9}{2}}}{9} + \frac{a^5x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*x^(1/2))^5,x)
```

```
[Out] 1/4*a^5*x^4+10/9*a^4*b*x^(9/2)+2*a^3*b^2*x^5+20/11*a^2*b^3*x^(11/2)+5/6*a*b
^4*x^6+2/13*b^5*x^(13/2)
```

maxima [B] time = 0.90, size = 132, normalized size = 1.81

$$\frac{2\left(b\sqrt{x}+a\right)^{13}}{13\,b^8} - \frac{7\left(b\sqrt{x}+a\right)^{12}a}{6\,b^8} + \frac{42\left(b\sqrt{x}+a\right)^{11}a^2}{11\,b^8} - \frac{7\left(b\sqrt{x}+a\right)^{10}a^3}{b^8} + \frac{70\left(b\sqrt{x}+a\right)^{9}a^4}{9\,b^8} - \frac{21\left(b\sqrt{x}+a\right)^{8}a^5}{4\,b^8} + \frac{2\left(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^(1/2))^5,x, algorithm="maxima")
```

```
[Out] 2/13*(b*sqrt(x) + a)^13/b^8 - 7/6*(b*sqrt(x) + a)^12*a/b^8 + 42/11*(b*sqrt(
x) + a)^11*a^2/b^8 - 7*(b*sqrt(x) + a)^10*a^3/b^8 + 70/9*(b*sqrt(x) + a)^9*
a^4/b^8 - 21/4*(b*sqrt(x) + a)^8*a^5/b^8 + 2*(b*sqrt(x) + a)^7*a^6/b^8 - 1/
3*(b*sqrt(x) + a)^6*a^7/b^8
```

mupad [B] time = 0.03, size = 57, normalized size = 0.78

$$\frac{a^5\,x^4}{4} + \frac{2\,b^5\,x^{13/2}}{13} + \frac{5\,a\,b^4\,x^6}{6} + \frac{10\,a^4\,b\,x^{9/2}}{9} + 2\,a^3\,b^2\,x^5 + \frac{20\,a^2\,b^3\,x^{11/2}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^(1/2))^5,x)
```

```
[Out] (a^5*x^4)/4 + (2*b^5*x^(13/2))/13 + (5*a*b^4*x^6)/6 + (10*a^4*b*x^(9/2))/9
+ 2*a^3*b^2*x^5 + (20*a^2*b^3*x^(11/2))/11
```

sympy [**A**] time = 2.56, size = 71, normalized size = 0.97

$$\frac{a^5x^4}{4} + \frac{10a^4bx^{\frac{9}{2}}}{9} + 2a^3b^2x^5 + \frac{20a^2b^3x^{\frac{11}{2}}}{11} + \frac{5ab^4x^6}{6} + \frac{2b^5x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*x**(1/2))**5,x)
```

```
[Out] a**5*x**4/4 + 10*a**4*b*x**(9/2)/9 + 2*a**3*b**2*x**5 + 20*a**2*b**3*x**(11
/2)/11 + 5*a*b**4*x**6/6 + 2*b**5*x**(13/2)/13
```

3.2143 $\int \left(a+b\sqrt{x}\right)^5 x^2\,dx$

Optimal. Leaf size=72

$$\frac{a^5x^3}{3}+\frac{10}{7}a^4bx^{7/2}+\frac{5}{2}a^3b^2x^4+\frac{20}{9}a^2b^3x^{9/2}+ab^4x^5+\frac{2}{11}b^5x^{11/2}$$

```
[Out] 1/3*a^5*x^3+10/7*a^4*b*x^(7/2)+5/2*a^3*b^2*x^4+20/9*a^2*b^3*x^(9/2)+a*b^4*x
^5+2/11*b^5*x^(11/2)
```

Rubi [A] time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{20}{9}a^2b^3x^{9/2}+\frac{5}{2}a^3b^2x^4+\frac{10}{7}a^4bx^{7/2}+\frac{a^5x^3}{3}+ab^4x^5+\frac{2}{11}b^5x^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^5*x^2,x]
```

```
[Out] (a^5*x^3)/3 + (10*a^4*b*x^(7/2))/7 + (5*a^3*b^2*x^4)/2 + (20*a^2*b^3*x^(9/2
))/9 + a*b^4*x^5 + (2*b^5*x^(11/2))/11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sqrt{x}\right)^5 x^2\,dx &= 2\,\text{Subst}\left(\int x^5(a+bx)^5\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(a^5x^5+5a^4bx^6+10a^3b^2x^7+10a^2b^3x^8+5ab^4x^9+b^5x^{10}\right)\,dx, x, \sqrt{x}\right)\\ &= \frac{a^5x^3}{3}+\frac{10}{7}a^4bx^{7/2}+\frac{5}{2}a^3b^2x^4+\frac{20}{9}a^2b^3x^{9/2}+ab^4x^5+\frac{2}{11}b^5x^{11/2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 72, normalized size = 1.00

$$\frac{a^5x^3}{3}+\frac{10}{7}a^4bx^{7/2}+\frac{5}{2}a^3b^2x^4+\frac{20}{9}a^2b^3x^{9/2}+ab^4x^5+\frac{2}{11}b^5x^{11/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^5*x^2,x]
```

```
[Out] (a^5*x^3)/3 + (10*a^4*b*x^(7/2))/7 + (5*a^3*b^2*x^4)/2 + (20*a^2*b^3*x^(9/2
))/9 + a*b^4*x^5 + (2*b^5*x^(11/2))/11
```

fricas [A] time = 0.97, size = 62, normalized size = 0.86

$$ab^4x^5+\frac{5}{2}a^3b^2x^4+\frac{1}{3}a^5x^3+\frac{2}{693}\left(63b^5x^5+770a^2b^3x^4+495a^4bx^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^(1/2))^5,x, algorithm="fricas")
```

```
[Out] a*b^4*x^5 + 5/2*a^3*b^2*x^4 + 1/3*a^5*x^3 + 2/693*(63*b^5*x^5 + 770*a^2*b^3
*x^4 + 495*a^4*b*x^3)*sqrt(x)
```

giac [A] time = 0.16, size = 56, normalized size = 0.78

$$\frac{2}{11}b^5x^{\frac{11}{2}}+ab^4x^5+\frac{20}{9}a^2b^3x^{\frac{9}{2}}+\frac{5}{2}a^3b^2x^4+\frac{10}{7}a^4bx^{\frac{7}{2}}+\frac{1}{3}a^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^(1/2))^5,x, algorithm="giac")
```

```
[Out] 2/11*b^5*x^(11/2) + a*b^4*x^5 + 20/9*a^2*b^3*x^(9/2) + 5/2*a^3*b^2*x^4 + 10
/7*a^4*b*x^(7/2) + 1/3*a^5*x^3
```

maple [A] time = 0.00, size = 57, normalized size = 0.79

$$\frac{2b^5x^{\frac{11}{2}}}{11}+ab^4x^5+\frac{20a^2b^3x^{\frac{9}{2}}}{9}+\frac{5a^3b^2x^4}{2}+\frac{10a^4bx^{\frac{7}{2}}}{7}+\frac{a^5x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*x^(1/2))^5,x)
```

```
[Out] 1/3*a^5*x^3+10/7*a^4*b*x^(7/2)+5/2*a^3*b^2*x^4+20/9*a^2*b^3*x^(9/2)+a*b^4*x
^5+2/11*b^5*x^(11/2)
```

maxima [A] time = 0.90, size = 98, normalized size = 1.36

$$\frac{2\left(b\sqrt{x}+a\right)^{11}}{11b^6}-\frac{\left(b\sqrt{x}+a\right)^{10}a}{b^6}+\frac{20\left(b\sqrt{x}+a\right)^9a^2}{9b^6}-\frac{5\left(b\sqrt{x}+a\right)^8a^3}{2b^6}+\frac{10\left(b\sqrt{x}+a\right)^7a^4}{7b^6}-\frac{\left(b\sqrt{x}+a\right)^6a^5}{3b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^(1/2))^5,x, algorithm="maxima")
```

```
[Out] 2/11*(b*sqrt(x) + a)^11/b^6 - (b*sqrt(x) + a)^10*a/b^6 + 20/9*(b*sqrt(x) +
a)^9*a^2/b^6 - 5/2*(b*sqrt(x) + a)^8*a^3/b^6 + 10/7*(b*sqrt(x) + a)^7*a^4/b
^6 - 1/3*(b*sqrt(x) + a)^6*a^5/b^6
```

mupad [B] time = 0.03, size = 56, normalized size = 0.78

$$\frac{a^5x^3}{3}+\frac{2b^5x^{11/2}}{11}+ab^4x^5+\frac{10a^4bx^{7/2}}{7}+\frac{5a^3b^2x^4}{2}+\frac{20a^2b^3x^{9/2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^(1/2))^5,x)
```

```
[Out] (a^5*x^3)/3 + (2*b^5*x^(11/2))/11 + a*b^4*x^5 + (10*a^4*b*x^(7/2))/7 + (5*a
^3*b^2*x^4)/2 + (20*a^2*b^3*x^(9/2))/9
```

sympy [A] time = 2.34, size = 70, normalized size = 0.97

$$\frac{a^5x^3}{3}+\frac{10a^4bx^{\frac{7}{2}}}{7}+\frac{5a^3b^2x^4}{2}+\frac{20a^2b^3x^{\frac{9}{2}}}{9}+ab^4x^5+\frac{2b^5x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*x**(1/2))**5,x)
```

```
[Out] a**5*x**3/3 + 10*a**4*b*x**(7/2)/7 + 5*a**3*b**2*x**4/2 + 20*a**2*b**3*x**(
9/2)/9 + a*b**4*x**5 + 2*b**5*x**(11/2)/11
```

3.2144 $\int \left(a+b\sqrt{x}\right)^5 x\,dx$

Optimal. Leaf size=80

$$-\frac{a^3\left(a+b\sqrt{x}\right)^6}{3b^4}+\frac{6a^2\left(a+b\sqrt{x}\right)^7}{7b^4}+\frac{2\left(a+b\sqrt{x}\right)^9}{9b^4}-\frac{3a\left(a+b\sqrt{x}\right)^8}{4b^4}$$

```
[Out] -1/3*a^3*(a+b*x^(1/2))^6/b^4+6/7*a^2*(a+b*x^(1/2))^7/b^4-3/4*a*(a+b*x^(1/2)
)^8/b^4+2/9*(a+b*x^(1/2))^9/b^4
```

Rubi [A] time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{6a^2\left(a+b\sqrt{x}\right)^7}{7b^4}-\frac{a^3\left(a+b\sqrt{x}\right)^6}{3b^4}+\frac{2\left(a+b\sqrt{x}\right)^9}{9b^4}-\frac{3a\left(a+b\sqrt{x}\right)^8}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^5*x,x]
```

```
[Out] -(a^3*(a + b*Sqrt[x])^6)/(3*b^4) + (6*a^2*(a + b*Sqrt[x])^7)/(7*b^4) - (3*a
*(a + b*Sqrt[x])^8)/(4*b^4) + (2*(a + b*Sqrt[x])^9)/(9*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sqrt{x}\right)^5 x\,dx &= 2\,\text{Subst}\left(\int x^3(a+bx)^5\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int\left(-\frac{a^3(a+bx)^5}{b^3}+\frac{3a^2(a+bx)^6}{b^3}-\frac{3a(a+bx)^7}{b^3}+\frac{(a+bx)^8}{b^3}\right)dx, x, \sqrt{x}\right)\\ &= -\frac{a^3\left(a+b\sqrt{x}\right)^6}{3b^4}+\frac{6a^2\left(a+b\sqrt{x}\right)^7}{7b^4}-\frac{3a\left(a+b\sqrt{x}\right)^8}{4b^4}+\frac{2\left(a+b\sqrt{x}\right)^9}{9b^4}\end{aligned}$$

Mathematica [A] time = 0.02, size = 73, normalized size = 0.91

$$\frac{a^5x^2}{2}+2a^4bx^{5/2}+\frac{10}{3}a^3b^2x^3+\frac{20}{7}a^2b^3x^{7/2}+\frac{5}{4}ab^4x^4+\frac{2}{9}b^5x^{9/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^5*x,x]
```

```
[Out] (a^5*x^2)/2 + 2*a^4*b*x^(5/2) + (10*a^3*b^2*x^3)/3 + (20*a^2*b^3*x^(7/2))/7
 + (5*a*b^4*x^4)/4 + (2*b^5*x^(9/2))/9
```

fricas [A] time = 1.26, size = 63, normalized size = 0.79

$$\frac{5}{4}ab^4x^4+\frac{10}{3}a^3b^2x^3+\frac{1}{2}a^5x^2+\frac{2}{63}\left(7b^5x^4+90a^2b^3x^3+63a^4bx^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^(1/2))^5,x, algorithm="fricas")
```

```
[Out] 5/4*a*b^4*x^4 + 10/3*a^3*b^2*x^3 + 1/2*a^5*x^2 + 2/63*(7*b^5*x^4 + 90*a^2*b
^3*x^3 + 63*a^4*b*x^2)*sqrt(x)
```

giac [A] time = 0.16, size = 57, normalized size = 0.71

$$\frac{2}{9}b^5x^{\frac{9}{2}}+\frac{5}{4}ab^4x^4+\frac{20}{7}a^2b^3x^{\frac{7}{2}}+\frac{10}{3}a^3b^2x^3+2a^4bx^{\frac{5}{2}}+\frac{1}{2}a^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^(1/2))^5,x, algorithm="giac")
```

```
[Out] 2/9*b^5*x^(9/2) + 5/4*a*b^4*x^4 + 20/7*a^2*b^3*x^(7/2) + 10/3*a^3*b^2*x^3 +
 2*a^4*b*x^(5/2) + 1/2*a^5*x^2
```

maple [A] time = 0.00, size = 58, normalized size = 0.72

$$\frac{2b^5x^{\frac{9}{2}}}{9}+\frac{5a\,b^4x^4}{4}+\frac{20a^2b^3x^{\frac{7}{2}}}{7}+\frac{10a^3b^2x^3}{3}+2a^4b\,x^{\frac{5}{2}}+\frac{a^5x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*x^(1/2))^5,x)
```

```
[Out] 2/9*x^(9/2)*b^5+5/4*a*b^4*x^4+20/7*x^(7/2)*a^2*b^3+10/3*a^3*b^2*x^3+2*x^(5/
2)*a^4*b+1/2*a^5*x^2
```

maxima [A] time = 0.84, size = 64, normalized size = 0.80

$$\frac{2\left(b\sqrt{x}+a\right)^9}{9\,b^4}-\frac{3\left(b\sqrt{x}+a\right)^8a}{4\,b^4}+\frac{6\left(b\sqrt{x}+a\right)^7a^2}{7\,b^4}-\frac{\left(b\sqrt{x}+a\right)^6a^3}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^(1/2))^5,x, algorithm="maxima")
```

```
[Out] 2/9*(b*sqrt(x) + a)^9/b^4 - 3/4*(b*sqrt(x) + a)^8*a/b^4 + 6/7*(b*sqrt(x) +
a)^7*a^2/b^4 - 1/3*(b*sqrt(x) + a)^6*a^3/b^4
```

mupad [B] time = 0.03, size = 57, normalized size = 0.71

$$\frac{a^5\,x^2}{2}+\frac{2\,b^5\,x^{9/2}}{9}+\frac{5\,a\,b^4\,x^4}{4}+2\,a^4\,b\,x^{5/2}+\frac{10\,a^3\,b^2\,x^3}{3}+\frac{20\,a^2\,b^3\,x^{7/2}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^(1/2))^5,x)
```

```
[Out] (a^5*x^2)/2 + (2*b^5*x^(9/2))/9 + (5*a*b^4*x^4)/4 + 2*a^4*b*x^(5/2) + (10*a
^3*b^2*x^3)/3 + (20*a^2*b^3*x^(7/2))/7
```

sympy [A] time = 2.34, size = 71, normalized size = 0.89

$$\frac{a^5x^2}{2}+2a^4bx^{\frac{5}{2}}+\frac{10a^3b^2x^3}{3}+\frac{20a^2b^3x^{\frac{7}{2}}}{7}+\frac{5ab^4x^4}{4}+\frac{2b^5x^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x**(1/2))**5,x)
```

```
[Out] a**5*x**2/2 + 2*a**4*b*x**(5/2) + 10*a**3*b**2*x**3/3 + 20*a**2*b**3*x**(7/
2)/7 + 5*a*b**4*x**4/4 + 2*b**5*x**(9/2)/9
```

3.2145 $\int \left(a+b\sqrt{x}\right)^5 dx$

Optimal. Leaf size=38

$$\frac{2\left(a+b\sqrt{x}\right)^7}{7b^2}-\frac{a\left(a+b\sqrt{x}\right)^6}{3b^2}$$

```
[Out] -1/3*a*(a+b*x^(1/2))^6/b^2+2/7*(a+b*x^(1/2))^7/b^2
```

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {190, 43}

$$\frac{2\left(a+b\sqrt{x}\right)^7}{7b^2}-\frac{a\left(a+b\sqrt{x}\right)^6}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^5,x]
```

```
[Out] -(a*(a + b*Sqrt[x])^6)/(3*b^2) + (2*(a + b*Sqrt[x])^7)/(7*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sqrt{x}\right)^5 dx &= 2\,\text{Subst}\left(\int x(a+bx)^5\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(-\frac{a(a+bx)^5}{b}+\frac{(a+bx)^6}{b}\right) dx, x, \sqrt{x}\right)\\ &= -\frac{a\left(a+b\sqrt{x}\right)^6}{3b^2}+\frac{2\left(a+b\sqrt{x}\right)^7}{7b^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 0.74

$$-\frac{\left(a-6b\sqrt{x}\right)\left(a+b\sqrt{x}\right)^6}{21b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^5,x]
```

```
[Out] -1/21*((a - 6*b*Sqrt[x])*(a + b*Sqrt[x])^6)/b^2
```

fricas [A] time = 0.90, size = 58, normalized size = 1.53

$$\frac{5}{3}\,ab^4x^3+5\,a^3b^2x^2+a^5x+\frac{2}{21}\left(3\,b^5x^3+42\,a^2b^3x^2+35\,a^4bx\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^5,x, algorithm="fricas")
```

```
[Out] 5/3*a*b^4*x^3 + 5*a^3*b^2*x^2 + a^5*x + 2/21*(3*b^5*x^3 + 42*a^2*b^3*x^2 +
35*a^4*b*x)*sqrt(x)
```

giac [A] time = 0.16, size = 54, normalized size = 1.42

$$\frac{2}{7}\,b^5x^{\frac{7}{2}} + \frac{5}{3}\,ab^4x^3 + 4\,a^2b^3x^{\frac{5}{2}} + 5\,a^3b^2x^2 + \frac{10}{3}\,a^4bx^{\frac{3}{2}} + a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^5,x, algorithm="giac")
```

```
[Out] 2/7*b^5*x^(7/2) + 5/3*a*b^4*x^3 + 4*a^2*b^3*x^(5/2) + 5*a^3*b^2*x^2 + 10/3*
a^4*b*x^(3/2) + a^5*x
```

maple [A] time = 0.00, size = 55, normalized size = 1.45

$$\frac{2b^5x^{\frac{7}{2}}}{7} + \frac{5a\,b^4x^3}{3} + 4a^2b^3x^{\frac{5}{2}} + 5a^3b^2x^2 + \frac{10a^4b\,x^{\frac{3}{2}}}{3} + a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^5,x)
```

```
[Out] 2/7*x^(7/2)*b^5+5/3*a*b^4*x^3+4*x^(5/2)*a^2*b^3+5*a^3*b^2*x^2+10/3*x^(3/2)*
a^4*b+a^5*x
```

maxima [A] time = 0.84, size = 54, normalized size = 1.42

$$\frac{2}{7}\,b^5x^{\frac{7}{2}} + \frac{5}{3}\,ab^4x^3 + 4\,a^2b^3x^{\frac{5}{2}} + 5\,a^3b^2x^2 + \frac{10}{3}\,a^4bx^{\frac{3}{2}} + a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^5,x, algorithm="maxima")
```

```
[Out] 2/7*b^5*x^(7/2) + 5/3*a*b^4*x^3 + 4*a^2*b^3*x^(5/2) + 5*a^3*b^2*x^2 + 10/3*
a^4*b*x^(3/2) + a^5*x
```

mupad [B] time = 0.02, size = 54, normalized size = 1.42

$$a^5\,x + \frac{2\,b^5\,x^{7/2}}{7} + \frac{5\,a\,b^4\,x^3}{3} + \frac{10\,a^4\,b\,x^{3/2}}{3} + 5\,a^3\,b^2\,x^2 + 4\,a^2\,b^3\,x^{5/2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^5,x)
```

```
[Out] a^5*x + (2*b^5*x^(7/2))/7 + (5*a*b^4*x^3)/3 + (10*a^4*b*x^(3/2))/3 + 5*a^3*
b^2*x^2 + 4*a^2*b^3*x^(5/2)
```

sympy [B] time = 2.24, size = 66, normalized size = 1.74

$$a^5x + \frac{10a^4bx^{\frac{3}{2}}}{3} + 5a^3b^2x^2 + 4a^2b^3x^{\frac{5}{2}} + \frac{5ab^4x^3}{3} + \frac{2b^5x^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**5,x)
```

```
[Out] a**5*x + 10*a**4*b*x**(3/2)/3 + 5*a**3*b**2*x**2 + 4*a**2*b**3*x**(5/2) + 5
*a*b**4*x**3/3 + 2*b**5*x**(7/2)/7
```

3.2146 $\int \frac{\left(a+b\sqrt{x}\right)^5}{x}\,dx$

Optimal. Leaf size=65

$$a^5\log(x)+10a^4b\sqrt{x}+10a^3b^2x+\frac{20}{3}a^2b^3x^{3/2}+\frac{5}{2}ab^4x^2+\frac{2}{5}b^5x^{5/2}$$

```
[Out] 10*a^3*b^2*x+20/3*a^2*b^3*x^(3/2)+5/2*a*b^4*x^2+2/5*b^5*x^(5/2)+a^5*ln(x)+1
0*a^4*b*x^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{20}{3}a^2b^3x^{3/2}+10a^3b^2x+10a^4b\sqrt{x}+a^5\log(x)+\frac{5}{2}ab^4x^2+\frac{2}{5}b^5x^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^5/x,x]
```

```
[Out] 10*a^4*b*Sqrt[x] + 10*a^3*b^2*x + (20*a^2*b^3*x^(3/2))/3 + (5*a*b^4*x^2)/2
+ (2*b^5*x^(5/2))/5 + a^5*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt{x}\right)^5}{x}\,dx &= 2\,\text{Subst}\left(\int \frac{(a+bx)^5}{x}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(5a^4b+\frac{a^5}{x}+10a^3b^2x+10a^2b^3x^2+5ab^4x^3+b^5x^4\right)dx, x, \sqrt{x}\right)\\ &= 10a^4b\sqrt{x}+10a^3b^2x+\frac{20}{3}a^2b^3x^{3/2}+\frac{5}{2}ab^4x^2+\frac{2}{5}b^5x^{5/2}+a^5\log(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 65, normalized size = 1.00

$$a^5\log(x)+10a^4b\sqrt{x}+10a^3b^2x+\frac{20}{3}a^2b^3x^{3/2}+\frac{5}{2}ab^4x^2+\frac{2}{5}b^5x^{5/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^5/x,x]
```

```
[Out] 10*a^4*b*Sqrt[x] + 10*a^3*b^2*x + (20*a^2*b^3*x^(3/2))/3 + (5*a*b^4*x^2)/2
+ (2*b^5*x^(5/2))/5 + a^5*Log[x]
```

fricas [A] time = 1.19, size = 57, normalized size = 0.88

$$\frac{5}{2}ab^4x^2+10a^3b^2x+2a^5\log\left(\sqrt{x}\right)+\frac{2}{15}\left(3b^5x^2+50a^2b^3x+75a^4b\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^5/x,x, algorithm="fricas")
```

```
[Out] 5/2*a*b^4*x^2 + 10*a^3*b^2*x + 2*a^5*log(sqrt(x)) + 2/15*(3*b^5*x^2 + 50*a^
2*b^3*x + 75*a^4*b)*sqrt(x)
```

giac [A] time = 0.18, size = 54, normalized size = 0.83

$$\frac{2}{5}b^5x^{\frac{5}{2}}+\frac{5}{2}ab^4x^2+\frac{20}{3}a^2b^3x^{\frac{3}{2}}+10a^3b^2x+a^5\log\left(|x|\right)+10a^4b\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^5/x,x, algorithm="giac")
```

```
[Out] 2/5*b^5*x^(5/2) + 5/2*a*b^4*x^2 + 20/3*a^2*b^3*x^(3/2) + 10*a^3*b^2*x + a^5
*log(abs(x)) + 10*a^4*b*sqrt(x)
```

maple [A] time = 0.00, size = 54, normalized size = 0.83

$$\frac{2b^5x^{\frac{5}{2}}}{5}+\frac{5a\,b^4x^2}{2}+\frac{20a^2b^3x^{\frac{3}{2}}}{3}+a^5\ln(x)+10a^3b^2x+10a^4b\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^5/x,x)
```

```
[Out] 10*a^3*b^2*x+20/3*a^2*b^3*x^(3/2)+5/2*a*b^4*x^2+2/5*b^5*x^(5/2)+a^5*ln(x)+1
0*a^4*b*x^(1/2)
```

maxima [A] time = 0.87, size = 53, normalized size = 0.82

$$\frac{2}{5}b^5x^{\frac{5}{2}}+\frac{5}{2}ab^4x^2+\frac{20}{3}a^2b^3x^{\frac{3}{2}}+10a^3b^2x+a^5\log(x)+10a^4b\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^5/x,x, algorithm="maxima")
```

```
[Out] 2/5*b^5*x^(5/2) + 5/2*a*b^4*x^2 + 20/3*a^2*b^3*x^(3/2) + 10*a^3*b^2*x + a^5
*log(x) + 10*a^4*b*sqrt(x)
```

mupad [B] time = 0.03, size = 56, normalized size = 0.86

$$2\,a^5\,\ln\left(\sqrt{x}\right)+\frac{2\,b^5\,x^{5/2}}{5}+10\,a^3\,b^2\,x+\frac{5\,a\,b^4\,x^2}{2}+10\,a^4\,b\,\sqrt{x}+\frac{20\,a^2\,b^3\,x^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^5/x,x)
```

```
[Out] 2*a^5*log(x^(1/2)) + (2*b^5*x^(5/2))/5 + 10*a^3*b^2*x + (5*a*b^4*x^2)/2 + 1
0*a^4*b*x^(1/2) + (20*a^2*b^3*x^(3/2))/3
```

sympy [A] time = 0.46, size = 66, normalized size = 1.02

$$a^5\log(x)+10a^4b\sqrt{x}+10a^3b^2x+\frac{20a^2b^3x^{\frac{3}{2}}}{3}+\frac{5ab^4x^2}{2}+\frac{2b^5x^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**5/x,x)
```

```
[Out] a**5*log(x) + 10*a**4*b*sqrt(x) + 10*a**3*b**2*x + 20*a**2*b**3*x**(3/2)/3
+ 5*a*b**4*x**2/2 + 2*b**5*x**(5/2)/5
```

3.2147 $\int \frac{\left(a+b\sqrt{x}\right)^5}{x^2}\,dx$

Optimal. Leaf size=62

$$-\frac{a^5}{x}-\frac{10a^4b}{\sqrt{x}}+10a^3b^2\log(x)+20a^2b^3\sqrt{x}+5ab^4x+\frac{2}{3}b^5x^{3/2}$$

```
[Out] -a^5/x+5*a*b^4*x+2/3*b^5*x^(3/2)+10*a^3*b^2*ln(x)-10*a^4*b/x^(1/2)+20*a^2*b
^3*x^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$20a^2b^3\sqrt{x}+10a^3b^2\log(x)-\frac{10a^4b}{\sqrt{x}}-\frac{a^5}{x}+5ab^4x+\frac{2}{3}b^5x^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^5/x^2,x]
```

```
[Out] -(a^5/x) - (10*a^4*b)/Sqrt[x] + 20*a^2*b^3*Sqrt[x] + 5*a*b^4*x + (2*b^5*x^(
3/2))/3 + 10*a^3*b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt{x}\right)^5}{x^2}\,dx &= 2\,\text{Subst}\left(\int \frac{(a+bx)^5}{x^3}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(10a^2b^3+\frac{a^5}{x^3}+\frac{5a^4b}{x^2}+\frac{10a^3b^2}{x}+5ab^4x+b^5x^2\right)dx, x, \sqrt{x}\right)\\ &= -\frac{a^5}{x}-\frac{10a^4b}{\sqrt{x}}+20a^2b^3\sqrt{x}+5ab^4x+\frac{2}{3}b^5x^{3/2}+10a^3b^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.03, size = 62, normalized size = 1.00

$$-\frac{a^5}{x}-\frac{10a^4b}{\sqrt{x}}+10a^3b^2\log(x)+20a^2b^3\sqrt{x}+5ab^4x+\frac{2}{3}b^5x^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^5/x^2,x]
```

```
[Out] -(a^5/x) - (10*a^4*b)/Sqrt[x] + 20*a^2*b^3*Sqrt[x] + 5*a*b^4*x + (2*b^5*x^(
3/2))/3 + 10*a^3*b^2*Log[x]
```

fricas [A] time = 0.83, size = 61, normalized size = 0.98

$$\frac{15\,ab^4x^2 + 60\,a^3b^2x\log\left(\sqrt{x}\right) - 3\,a^5 + 2\left(b^5x^2 + 30\,a^2b^3x - 15\,a^4b\right)\sqrt{x}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^5/x^2,x, algorithm="fricas")
```

```
[Out] 1/3*(15*a*b^4*x^2 + 60*a^3*b^2*x*log(sqrt(x)) - 3*a^5 + 2*(b^5*x^2 + 30*a^2
*b^3*x - 15*a^4*b)*sqrt(x))/x
```

giac [A] time = 0.16, size = 56, normalized size = 0.90

$$\frac{2}{3}\,b^5x^{\frac{3}{2}} + 5\,ab^4x + 10\,a^3b^2\log\left(|x|\right) + 20\,a^2b^3\sqrt{x} - \frac{10\,a^4b\sqrt{x} + a^5}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^5/x^2,x, algorithm="giac")
```

```
[Out] 2/3*b^5*x^(3/2) + 5*a*b^4*x + 10*a^3*b^2*log(abs(x)) + 20*a^2*b^3*sqrt(x) -
 (10*a^4*b*sqrt(x) + a^5)/x
```

maple [A] time = 0.00, size = 55, normalized size = 0.89

$$\frac{2b^5x^{\frac{3}{2}}}{3} + 10a^3b^2\ln(x) + 5a\,b^4x + 20a^2b^3\sqrt{x} - \frac{10a^4b}{\sqrt{x}} - \frac{a^5}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^5/x^2,x)
```

```
[Out] -a^5/x+5*a*b^4*x+2/3*b^5*x^(3/2)+10*a^3*b^2*ln(x)-10*a^4*b/x^(1/2)+20*a^2*b
^3*x^(1/2)
```

maxima [A] time = 0.91, size = 55, normalized size = 0.89

$$\frac{2}{3}\,b^5x^{\frac{3}{2}} + 5\,ab^4x + 10\,a^3b^2\log(x) + 20\,a^2b^3\sqrt{x} - \frac{10\,a^4b\sqrt{x} + a^5}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^5/x^2,x, algorithm="maxima")
```

```
[Out] 2/3*b^5*x^(3/2) + 5*a*b^4*x + 10*a^3*b^2*log(x) + 20*a^2*b^3*sqrt(x) - (10*
a^4*b*sqrt(x) + a^5)/x
```

mupad [B] time = 0.03, size = 57, normalized size = 0.92

$$\frac{2\,b^5\,x^{3/2}}{3} - \frac{a^5 + 10\,a^4\,b\,\sqrt{x}}{x} + 20\,a^3\,b^2\,\ln\left(\sqrt{x}\right) + 20\,a^2\,b^3\,\sqrt{x} + 5\,a\,b^4\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^5/x^2,x)
```

```
[Out] (2*b^5*x^(3/2))/3 - (a^5 + 10*a^4*b*x^(1/2))/x + 20*a^3*b^2*log(x^(1/2)) +
20*a^2*b^3*x^(1/2) + 5*a*b^4*x
```

sympy [**A**] time = 0.65, size = 61, normalized size = 0.98

$$-\frac{a^5}{x}-\frac{10a^4b}{\sqrt{x}}+10a^3b^2\log(x)+20a^2b^3\sqrt{x}+5ab^4x+\frac{2b^5x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**5/x**2,x)
```

```
[Out] -a**5/x - 10*a**4*b/sqrt(x) + 10*a**3*b**2*log(x) + 20*a**2*b**3*sqrt(x) +
5*a*b**4*x + 2*b**5*x**(3/2)/3
```

3.2148 $\int \frac{\left(a+b\sqrt{x}\right)^5}{x^3}\,dx$

Optimal. Leaf size=66

$$-\frac{a^5}{2x^2}-\frac{10a^4b}{3x^{3/2}}-\frac{10a^3b^2}{x}-\frac{20a^2b^3}{\sqrt{x}}+5ab^4\log(x)+2b^5\sqrt{x}$$

```
[Out] -1/2*a^5/x^2-10/3*a^4*b/x^(3/2)-10*a^3*b^2/x+5*a*b^4*ln(x)-20*a^2*b^3/x^(1/
2)+2*b^5*x^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{10a^3b^2}{x}-\frac{20a^2b^3}{\sqrt{x}}-\frac{10a^4b}{3x^{3/2}}-\frac{a^5}{2x^2}+5ab^4\log(x)+2b^5\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^5/x^3,x]
```

```
[Out] -a^5/(2*x^2) - (10*a^4*b)/(3*x^(3/2)) - (10*a^3*b^2)/x - (20*a^2*b^3)/Sqrt[
x] + 2*b^5*Sqrt[x] + 5*a*b^4*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt{x}\right)^5}{x^3}\,dx &= 2\,\text{Subst}\left(\int \frac{(a+bx)^5}{x^5}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(b^5+\frac{a^5}{x^5}+\frac{5a^4b}{x^4}+\frac{10a^3b^2}{x^3}+\frac{10a^2b^3}{x^2}+\frac{5ab^4}{x}\right)dx, x, \sqrt{x}\right)\\ &= -\frac{a^5}{2x^2}-\frac{10a^4b}{3x^{3/2}}-\frac{10a^3b^2}{x}-\frac{20a^2b^3}{\sqrt{x}}+2b^5\sqrt{x}+5ab^4\log(x)\end{aligned}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 1.00

$$-\frac{a^5}{2x^2}-\frac{10a^4b}{3x^{3/2}}-\frac{10a^3b^2}{x}-\frac{20a^2b^3}{\sqrt{x}}+5ab^4\log(x)+2b^5\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^5/x^3,x]
```

```
[Out] -1/2*a^5/x^2 - (10*a^4*b)/(3*x^(3/2)) - (10*a^3*b^2)/x - (20*a^2*b^3)/Sqrt[
x] + 2*b^5*Sqrt[x] + 5*a*b^4*Log[x]
```

fricas [A] time = 0.89, size = 62, normalized size = 0.94

$$\frac{60\,ab^4x^2\log\left(\sqrt{x}\right) - 60\,a^3b^2x - 3\,a^5 + 4\left(3\,b^5x^2 - 30\,a^2b^3x - 5\,a^4b\right)\sqrt{x}}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^5/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*(60*a*b^4*x^2*log(sqrt(x)) - 60*a^3*b^2*x - 3*a^5 + 4*(3*b^5*x^2 - 30*a
^2*b^3*x - 5*a^4*b)*sqrt(x))/x^2
```

giac [A] time = 0.21, size = 58, normalized size = 0.88

$$5\,ab^4\log\left(|x|\right) + 2\,b^5\sqrt{x} - \frac{120\,a^2b^3x^{\frac{3}{2}} + 60\,a^3b^2x + 20\,a^4b\sqrt{x} + 3\,a^5}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^5/x^3,x, algorithm="giac")
```

```
[Out] 5*a*b^4*log(abs(x)) + 2*b^5*sqrt(x) - 1/6*(120*a^2*b^3*x^(3/2) + 60*a^3*b^2
*x + 20*a^4*b*sqrt(x) + 3*a^5)/x^2
```

maple [A] time = 0.00, size = 57, normalized size = 0.86

$$5a\,b^4\ln(x) + 2b^5\sqrt{x} - \frac{20a^2b^3}{\sqrt{x}} - \frac{10a^3b^2}{x} - \frac{10a^4b}{3x^{\frac{3}{2}}} - \frac{a^5}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^5/x^3,x)
```

```
[Out] -1/2*a^5/x^2-10/3*a^4*b/x^(3/2)-10*a^3*b^2/x+5*a*b^4*ln(x)-20*a^2*b^3/x^(1/
2)+2*b^5*x^(1/2)
```

maxima [A] time = 0.78, size = 57, normalized size = 0.86

$$5\,ab^4\log(x) + 2\,b^5\sqrt{x} - \frac{120\,a^2b^3x^{\frac{3}{2}} + 60\,a^3b^2x + 20\,a^4b\sqrt{x} + 3\,a^5}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^5/x^3,x, algorithm="maxima")
```

```
[Out] 5*a*b^4*log(x) + 2*b^5*sqrt(x) - 1/6*(120*a^2*b^3*x^(3/2) + 60*a^3*b^2*x +
20*a^4*b*sqrt(x) + 3*a^5)/x^2
```

mupad [B] time = 0.05, size = 59, normalized size = 0.89

$$2\,b^5\,\sqrt{x} - \frac{\frac{a^5}{2} + 10\,a^3\,b^2\,x + \frac{10\,a^4\,b\,\sqrt{x}}{3} + 20\,a^2\,b^3\,x^{3/2}}{x^2} + 10\,a\,b^4\,\ln\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^5/x^3,x)
```

```
[Out] 2*b^5*x^(1/2) - (a^5/2 + 10*a^3*b^2*x + (10*a^4*b*x^(1/2))/3 + 20*a^2*b^3*x
^(3/2))/x^2 + 10*a*b^4*log(x^(1/2))
```

sympy [**A**] time = 0.74, size = 65, normalized size = 0.98

$$-\frac{a^5}{2x^2} - \frac{10a^4b}{3x^{\frac{3}{2}}} - \frac{10a^3b^2}{x} - \frac{20a^2b^3}{\sqrt{x}} + 5ab^4\log(x) + 2b^5\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**5/x**3,x)
```

```
[Out] -a**5/(2*x**2) - 10*a**4*b/(3*x**(3/2)) - 10*a**3*b**2/x - 20*a**2*b**3/sqr
t(x) + 5*a*b**4*log(x) + 2*b**5*sqrt(x)
```

3.2149 $\int \frac{\left(a+b\sqrt{x}\right)^5}{x^4}\,dx$

Optimal. Leaf size=21

$$-\frac{\left(a+b\sqrt{x}\right)^6}{3ax^3}$$

```
[Out] -1/3*(a+b*x^(1/2))^6/a/x^3
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {264}

$$-\frac{\left(a+b\sqrt{x}\right)^6}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^5/x^4,x]
```

```
[Out] -(a + b*Sqrt[x])^6/(3*a*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^4}\,dx = -\frac{\left(a+b\sqrt{x}\right)^6}{3ax^3}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$-\frac{\left(a+b\sqrt{x}\right)^6}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^5/x^4,x]
```

```
[Out] -1/3*(a + b*Sqrt[x])^6/(a*x^3)
```

fricas [B] time = 0.89, size = 56, normalized size = 2.67

$$-\frac{15\,ab^4x^2 + 15\,a^3b^2x + a^5 + 2\left(3\,b^5x^2 + 10\,a^2b^3x + 3\,a^4b\right)\sqrt{x}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^5/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(15*a*b^4*x^2 + 15*a^3*b^2*x + a^5 + 2*(3*b^5*x^2 + 10*a^2*b^3*x + 3*a
^4*b)*sqrt(x))/x^3
```

giac [B] time = 0.15, size = 55, normalized size = 2.62

$$-\frac{6\,b^5x^{\frac{5}{2}}+15\,ab^4x^2+20\,a^2b^3x^{\frac{3}{2}}+15\,a^3b^2x+6\,a^4b\sqrt{x}+a^5}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^5/x^4,x, algorithm="giac")
```

```
[Out] -1/3*(6*b^5*x^(5/2) + 15*a*b^4*x^2 + 20*a^2*b^3*x^(3/2) + 15*a^3*b^2*x + 6*
a^4*b*sqrt(x) + a^5)/x^3
```

maple [B] time = 0.00, size = 58, normalized size = 2.76

$$-\frac{2b^5}{\sqrt{x}}-\frac{5a\,b^4}{x}-\frac{20a^2b^3}{3x^{\frac{3}{2}}}-\frac{5a^3b^2}{x^2}-\frac{2a^4b}{x^{\frac{5}{2}}}-\frac{a^5}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^5/x^4,x)
```

```
[Out] -2*b^5/x^(1/2)-5*a*b^4/x-20/3*a^2*b^3/x^(3/2)-5*a^3*b^2/x^2-2*a^4*b/x^(5/2)
-1/3*a^5/x^3
```

maxima [B] time = 0.87, size = 55, normalized size = 2.62

$$-\frac{6\,b^5x^{\frac{5}{2}}+15\,ab^4x^2+20\,a^2b^3x^{\frac{3}{2}}+15\,a^3b^2x+6\,a^4b\sqrt{x}+a^5}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^5/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(6*b^5*x^(5/2) + 15*a*b^4*x^2 + 20*a^2*b^3*x^(3/2) + 15*a^3*b^2*x + 6*
a^4*b*sqrt(x) + a^5)/x^3
```

mupad [B] time = 1.11, size = 55, normalized size = 2.62

$$-\frac{a^5+6\,b^5\,x^{5/2}+15\,a^3\,b^2\,x+15\,a\,b^4\,x^2+6\,a^4\,b\,\sqrt{x}+20\,a^2\,b^3\,x^{3/2}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^5/x^4,x)
```

```
[Out] -(a^5 + 6*b^5*x^(5/2) + 15*a^3*b^2*x + 15*a*b^4*x^2 + 6*a^4*b*x^(1/2) + 20*
a^2*b^3*x^(3/2))/(3*x^3)
```

sympy [B] time = 1.11, size = 66, normalized size = 3.14

$$-\frac{a^5}{3x^3}-\frac{2a^4b}{x^{\frac{5}{2}}}-\frac{5a^3b^2}{x^2}-\frac{20a^2b^3}{3x^{\frac{3}{2}}}-\frac{5ab^4}{x}-\frac{2b^5}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**5/x**4,x)
```

```
[Out] -a**5/(3*x**3) - 2*a**4*b/x**(5/2) - 5*a**3*b**2/x**2 - 20*a**2*b**3/(3*x**
(3/2)) - 5*a*b**4/x - 2*b**5/sqrt(x)
```

3.2150 $\int \frac{\left(a+b\sqrt{x}\right)^5}{x^5}\,dx$

Optimal. Leaf size=70

$$-\frac{b^2\left(a+b\sqrt{x}\right)^6}{84a^3x^3}+\frac{b\left(a+b\sqrt{x}\right)^6}{14a^2x^{7/2}}-\frac{\left(a+b\sqrt{x}\right)^6}{4ax^4}$$

```
[Out] -1/4*(a+b*x^(1/2))^6/a/x^4+1/14*b*(a+b*x^(1/2))^6/a^2/x^(7/2)-1/84*b^2*(a+b
*x^(1/2))^6/a^3/x^3
```

Rubi [A] time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {266, 45, 37}

$$-\frac{b^2\left(a+b\sqrt{x}\right)^6}{84a^3x^3}+\frac{b\left(a+b\sqrt{x}\right)^6}{14a^2x^{7/2}}-\frac{\left(a+b\sqrt{x}\right)^6}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^5/x^5,x]
```

```
[Out] -(a + b*Sqrt[x])^6/(4*a*x^4) + (b*(a + b*Sqrt[x])^6)/(14*a^2*x^(7/2)) - (b^
2*(a + b*Sqrt[x])^6)/(84*a^3*x^3)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^5}\,dx = 2\,\text{Subst}\left(\int \frac{(a+bx)^5}{x^9}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{\left(a+b\sqrt{x}\right)^6}{4ax^4} - \frac{b\,\text{Subst}\left(\int \frac{(a+bx)^5}{x^8}\,dx, x, \sqrt{x}\right)}{2a}$$
$$= -\frac{\left(a+b\sqrt{x}\right)^6}{4ax^4} + \frac{b\left(a+b\sqrt{x}\right)^6}{14a^2x^{7/2}} + \frac{b^2\,\text{Subst}\left(\int \frac{(a+bx)^5}{x^7}\,dx, x, \sqrt{x}\right)}{14a^2}$$
$$= -\frac{\left(a+b\sqrt{x}\right)^6}{4ax^4} + \frac{b\left(a+b\sqrt{x}\right)^6}{14a^2x^{7/2}} - \frac{b^2\left(a+b\sqrt{x}\right)^6}{84a^3x^3}$$

Mathematica [A] time = 0.03, size = 65, normalized size = 0.93

$$-\frac{21a^5 + 120a^4b\sqrt{x} + 280a^3b^2x + 336a^2b^3x^{3/2} + 210ab^4x^2 + 56b^5x^{5/2}}{84x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^5/x^5,x]
```

```
[Out] -1/84*(21*a^5 + 120*a^4*b*Sqrt[x] + 280*a^3*b^2*x + 336*a^2*b^3*x^(3/2) + 2
10*a*b^4*x^2 + 56*b^5*x^(5/2))/x^4
```

fricas [A] time = 0.95, size = 58, normalized size = 0.83

$$-\frac{210\,ab^4x^2 + 280\,a^3b^2x + 21\,a^5 + 8\left(7\,b^5x^2 + 42\,a^2b^3x + 15\,a^4b\right)\sqrt{x}}{84\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^5/x^5,x, algorithm="fricas")
```

```
[Out] -1/84*(210*a*b^4*x^2 + 280*a^3*b^2*x + 21*a^5 + 8*(7*b^5*x^2 + 42*a^2*b^3*x
 + 15*a^4*b)*sqrt(x))/x^4
```

giac [A] time = 0.21, size = 57, normalized size = 0.81

$$-\frac{56\,b^5x^{\frac{5}{2}} + 210\,ab^4x^2 + 336\,a^2b^3x^{\frac{3}{2}} + 280\,a^3b^2x + 120\,a^4b\sqrt{x} + 21\,a^5}{84\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^5/x^5,x, algorithm="giac")
```

```
[Out] -1/84*(56*b^5*x^(5/2) + 210*a*b^4*x^2 + 336*a^2*b^3*x^(3/2) + 280*a^3*b^2*x
 + 120*a^4*b*sqrt(x) + 21*a^5)/x^4
```

maple [A] time = 0.00, size = 58, normalized size = 0.83

$$-\frac{2b^5}{3x^{\frac{3}{2}}} - \frac{5a\,b^4}{2x^2} - \frac{4a^2b^3}{x^{\frac{5}{2}}} - \frac{10a^3b^2}{3x^3} - \frac{10a^4b}{7x^{\frac{7}{2}}} - \frac{a^5}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^5/x^5,x)
```

```
[Out] -2/3*b^5/x^(3/2)-5/2*a*b^4/x^2-4*a^2*b^3/x^(5/2)-10/3*a^3*b^2/x^3-10/7*a^4*
b/x^(7/2)-1/4*a^5/x^4
```

maxima [A] time = 0.86, size = 57, normalized size = 0.81

$$-\frac{56\,b^5x^{\frac{5}{2}}+210\,ab^4x^2+336\,a^2b^3x^{\frac{3}{2}}+280\,a^3b^2x+120\,a^4b\sqrt{x}+21\,a^5}{84\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^5/x^5,x, algorithm="maxima")
```

```
[Out] -1/84*(56*b^5*x^(5/2) + 210*a*b^4*x^2 + 336*a^2*b^3*x^(3/2) + 280*a^3*b^2*x
 + 120*a^4*b*sqrt(x) + 21*a^5)/x^4
```

mupad [B] time = 1.10, size = 57, normalized size = 0.81

$$-\frac{21\,a^5+56\,b^5\,x^{5/2}+280\,a^3\,b^2\,x+210\,a\,b^4\,x^2+120\,a^4\,b\,\sqrt{x}+336\,a^2\,b^3\,x^{3/2}}{84\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^5/x^5,x)
```

```
[Out] -(21*a^5 + 56*b^5*x^(5/2) + 280*a^3*b^2*x + 210*a*b^4*x^2 + 120*a^4*b*x^(1/
2) + 336*a^2*b^3*x^(3/2))/(84*x^4)
```

sympy [A] time = 1.64, size = 73, normalized size = 1.04

$$-\frac{a^5}{4x^4}-\frac{10a^4b}{7x^{\frac{7}{2}}}-\frac{10a^3b^2}{3x^3}-\frac{4a^2b^3}{x^{\frac{5}{2}}}-\frac{5ab^4}{2x^2}-\frac{2b^5}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**5/x**5,x)
```

```
[Out] -a**5/(4*x**4) - 10*a**4*b/(7*x**(7/2)) - 10*a**3*b**2/(3*x**3) - 4*a**2*b*
*3/x**(5/2) - 5*a*b**4/(2*x**2) - 2*b**5/(3*x**(3/2))
```

3.2151 $\int \frac{\left(a+b\sqrt{x}\right)^5}{x^6}\,dx$

Optimal. Leaf size=75

$$-\frac{a^5}{5x^5}-\frac{10a^4b}{9x^{9/2}}-\frac{5a^3b^2}{2x^4}-\frac{20a^2b^3}{7x^{7/2}}-\frac{5ab^4}{3x^3}-\frac{2b^5}{5x^{5/2}}$$

```
[Out] -1/5*a^5/x^5-10/9*a^4*b/x^(9/2)-5/2*a^3*b^2/x^4-20/7*a^2*b^3/x^(7/2)-5/3*a*
b^4/x^3-2/5*b^5/x^(5/2)
```

Rubi [A] time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{5a^3b^2}{2x^4}-\frac{20a^2b^3}{7x^{7/2}}-\frac{10a^4b}{9x^{9/2}}-\frac{a^5}{5x^5}-\frac{5ab^4}{3x^3}-\frac{2b^5}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^5/x^6,x]
```

```
[Out] -a^5/(5*x^5) - (10*a^4*b)/(9*x^(9/2)) - (5*a^3*b^2)/(2*x^4) - (20*a^2*b^3)/
(7*x^(7/2)) - (5*a*b^4)/(3*x^3) - (2*b^5)/(5*x^(5/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt{x}\right)^5}{x^6}\,dx &= 2\,\text{Subst}\left(\int \frac{(a+bx)^5}{x^{11}}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(\frac{a^5}{x^{11}}+\frac{5a^4b}{x^{10}}+\frac{10a^3b^2}{x^9}+\frac{10a^2b^3}{x^8}+\frac{5ab^4}{x^7}+\frac{b^5}{x^6}\right)dx, x, \sqrt{x}\right)\\ &= -\frac{a^5}{5x^5}-\frac{10a^4b}{9x^{9/2}}-\frac{5a^3b^2}{2x^4}-\frac{20a^2b^3}{7x^{7/2}}-\frac{5ab^4}{3x^3}-\frac{2b^5}{5x^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 65, normalized size = 0.87

$$-\frac{126a^5+700a^4b\sqrt{x}+1575a^3b^2x+1800a^2b^3x^{3/2}+1050ab^4x^2+252b^5x^{5/2}}{630x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^5/x^6,x]
```

```
[Out] -1/630*(126*a^5 + 700*a^4*b*Sqrt[x] + 1575*a^3*b^2*x + 1800*a^2*b^3*x^(3/2)
 + 1050*a*b^4*x^2 + 252*b^5*x^(5/2))/x^5
```

fricas [A] time = 1.32, size = 58, normalized size = 0.77

$$-\frac{1050\,ab^4x^2+1575\,a^3b^2x+126\,a^5+4\left(63\,b^5x^2+450\,a^2b^3x+175\,a^4b\right)\sqrt{x}}{630\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^5/x^6,x, algorithm="fricas")
```

```
[Out] -1/630*(1050*a*b^4*x^2 + 1575*a^3*b^2*x + 126*a^5 + 4*(63*b^5*x^2 + 450*a^2
*b^3*x + 175*a^4*b)*sqrt(x))/x^5
```

giac [A] time = 0.18, size = 57, normalized size = 0.76

$$-\frac{252\,b^5x^{\frac{5}{2}}+1050\,ab^4x^2+1800\,a^2b^3x^{\frac{3}{2}}+1575\,a^3b^2x+700\,a^4b\sqrt{x}+126\,a^5}{630\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^5/x^6,x, algorithm="giac")
```

```
[Out] -1/630*(252*b^5*x^(5/2) + 1050*a*b^4*x^2 + 1800*a^2*b^3*x^(3/2) + 1575*a^3*
b^2*x + 700*a^4*b*sqrt(x) + 126*a^5)/x^5
```

maple [A] time = 0.00, size = 58, normalized size = 0.77

$$-\frac{2b^5}{5x^{\frac{5}{2}}}-\frac{5a\,b^4}{3x^3}-\frac{20a^2b^3}{7x^{\frac{7}{2}}}-\frac{5a^3b^2}{2x^4}-\frac{10a^4b}{9x^{\frac{9}{2}}}-\frac{a^5}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^5/x^6,x)
```

```
[Out] -1/5*a^5/x^5-10/9*a^4*b/x^(9/2)-5/2*a^3*b^2/x^4-20/7*a^2*b^3/x^(7/2)-5/3*a*
b^4/x^3-2/5*b^5/x^(5/2)
```

maxima [A] time = 0.88, size = 57, normalized size = 0.76

$$-\frac{252\,b^5x^{\frac{5}{2}}+1050\,ab^4x^2+1800\,a^2b^3x^{\frac{3}{2}}+1575\,a^3b^2x+700\,a^4b\sqrt{x}+126\,a^5}{630\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^5/x^6,x, algorithm="maxima")
```

```
[Out] -1/630*(252*b^5*x^(5/2) + 1050*a*b^4*x^2 + 1800*a^2*b^3*x^(3/2) + 1575*a^3*
b^2*x + 700*a^4*b*sqrt(x) + 126*a^5)/x^5
```

mupad [B] time = 0.04, size = 57, normalized size = 0.76

$$-\frac{126\,a^5+252\,b^5\,x^{5/2}+1575\,a^3\,b^2\,x+1050\,a\,b^4\,x^2+700\,a^4\,b\,\sqrt{x}+1800\,a^2\,b^3\,x^{3/2}}{630\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^5/x^6,x)
```

```
[Out] -(126*a^5 + 252*b^5*x^(5/2) + 1575*a^3*b^2*x + 1050*a*b^4*x^2 + 700*a^4*b*x
^(1/2) + 1800*a^2*b^3*x^(3/2))/(630*x^5)
```

sympy [**A**] time = 2.21, size = 75, normalized size = 1.00

$$-\frac{a^5}{5x^5}-\frac{10a^4b}{9x^{\frac{9}{2}}}-\frac{5a^3b^2}{2x^4}-\frac{20a^2b^3}{7x^{\frac{7}{2}}}-\frac{5ab^4}{3x^3}-\frac{2b^5}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**5/x**6,x)
```

```
[Out] -a**5/(5*x**5) - 10*a**4*b/(9*x**(9/2)) - 5*a**3*b**2/(2*x**4) - 20*a**2*b*
**3/(7*x**(7/2)) - 5*a*b**4/(3*x**3) - 2*b**5/(5*x**(5/2))
```

3.2152 $\int \frac{\left(a+b\sqrt{x}\right)^5}{x^7}\,dx$

Optimal. Leaf size=73

$$-\frac{a^5}{6x^6}-\frac{10a^4b}{11x^{11/2}}-\frac{2a^3b^2}{x^5}-\frac{20a^2b^3}{9x^{9/2}}-\frac{5ab^4}{4x^4}-\frac{2b^5}{7x^{7/2}}$$

```
[Out] -1/6*a^5/x^6-10/11*a^4*b/x^(11/2)-2*a^3*b^2/x^5-20/9*a^2*b^3/x^(9/2)-5/4*a*
b^4/x^4-2/7*b^5/x^(7/2)
```

Rubi [A] time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{2a^3b^2}{x^5}-\frac{20a^2b^3}{9x^{9/2}}-\frac{10a^4b}{11x^{11/2}}-\frac{a^5}{6x^6}-\frac{5ab^4}{4x^4}-\frac{2b^5}{7x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^5/x^7,x]
```

```
[Out] -a^5/(6*x^6) - (10*a^4*b)/(11*x^(11/2)) - (2*a^3*b^2)/x^5 - (20*a^2*b^3)/(9
*x^(9/2)) - (5*a*b^4)/(4*x^4) - (2*b^5)/(7*x^(7/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt{x}\right)^5}{x^7}\,dx &= 2\,\text{Subst}\left(\int \frac{(a+bx)^5}{x^{13}}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(\frac{a^5}{x^{13}}+\frac{5a^4b}{x^{12}}+\frac{10a^3b^2}{x^{11}}+\frac{10a^2b^3}{x^{10}}+\frac{5ab^4}{x^9}+\frac{b^5}{x^8}\right)dx, x, \sqrt{x}\right)\\ &= -\frac{a^5}{6x^6}-\frac{10a^4b}{11x^{11/2}}-\frac{2a^3b^2}{x^5}-\frac{20a^2b^3}{9x^{9/2}}-\frac{5ab^4}{4x^4}-\frac{2b^5}{7x^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 65, normalized size = 0.89

$$-\frac{462a^5+2520a^4b\sqrt{x}+5544a^3b^2x+6160a^2b^3x^{3/2}+3465ab^4x^2+792b^5x^{5/2}}{2772x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^5/x^7,x]
```

```
[Out] -1/2772*(462*a^5 + 2520*a^4*b*Sqrt[x] + 5544*a^3*b^2*x + 6160*a^2*b^3*x^(3/
2) + 3465*a*b^4*x^2 + 792*b^5*x^(5/2))/x^6
```

fricas [A] time = 1.02, size = 58, normalized size = 0.79

$$-\frac{3465\,ab^4x^2 + 5544\,a^3b^2x + 462\,a^5 + 8\left(99\,b^5x^2 + 770\,a^2b^3x + 315\,a^4b\right)\sqrt{x}}{2772\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^5/x^7,x, algorithm="fricas")
```

```
[Out] -1/2772*(3465*a*b^4*x^2 + 5544*a^3*b^2*x + 462*a^5 + 8*(99*b^5*x^2 + 770*a^
2*b^3*x + 315*a^4*b)*sqrt(x))/x^6
```

giac [A] time = 0.16, size = 57, normalized size = 0.78

$$-\frac{792\,b^5x^{\frac{5}{2}} + 3465\,ab^4x^2 + 6160\,a^2b^3x^{\frac{3}{2}} + 5544\,a^3b^2x + 2520\,a^4b\sqrt{x} + 462\,a^5}{2772\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^5/x^7,x, algorithm="giac")
```

```
[Out] -1/2772*(792*b^5*x^(5/2) + 3465*a*b^4*x^2 + 6160*a^2*b^3*x^(3/2) + 5544*a^3
*b^2*x + 2520*a^4*b*sqrt(x) + 462*a^5)/x^6
```

maple [A] time = 0.00, size = 58, normalized size = 0.79

$$-\frac{2b^5}{7x^{\frac{7}{2}}} - \frac{5a\,b^4}{4x^4} - \frac{20a^2b^3}{9x^{\frac{9}{2}}} - \frac{2a^3b^2}{x^5} - \frac{10a^4b}{11x^{\frac{11}{2}}} - \frac{a^5}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^5/x^7,x)
```

```
[Out] -1/6*a^5/x^6-10/11*a^4*b/x^(11/2)-2*a^3*b^2/x^5-20/9*a^2*b^3/x^(9/2)-5/4*a*
b^4/x^4-2/7*b^5/x^(7/2)
```

maxima [A] time = 0.88, size = 57, normalized size = 0.78

$$-\frac{792\,b^5x^{\frac{5}{2}} + 3465\,ab^4x^2 + 6160\,a^2b^3x^{\frac{3}{2}} + 5544\,a^3b^2x + 2520\,a^4b\sqrt{x} + 462\,a^5}{2772\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^5/x^7,x, algorithm="maxima")
```

```
[Out] -1/2772*(792*b^5*x^(5/2) + 3465*a*b^4*x^2 + 6160*a^2*b^3*x^(3/2) + 5544*a^3
*b^2*x + 2520*a^4*b*sqrt(x) + 462*a^5)/x^6
```

mupad [B] time = 0.04, size = 57, normalized size = 0.78

$$-\frac{\frac{a^5}{6} + \frac{2\,b^5\,x^{5/2}}{7} + 2\,a^3\,b^2\,x + \frac{5\,a\,b^4\,x^2}{4} + \frac{10\,a^4\,b\,\sqrt{x}}{11} + \frac{20\,a^2\,b^3\,x^{3/2}}{9}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^5/x^7,x)
```

```
[Out] -(a^5/6 + (2*b^5*x^(5/2))/7 + 2*a^3*b^2*x + (5*a*b^4*x^2)/4 + (10*a^4*b*x^(
1/2))/11 + (20*a^2*b^3*x^(3/2))/9)/x^6
```

sympy [**A**] time = 3.21, size = 73, normalized size = 1.00

$$-\frac{a^5}{6x^6}-\frac{10a^4b}{11x^{\frac{11}{2}}}-\frac{2a^3b^2}{x^5}-\frac{20a^2b^3}{9x^{\frac{9}{2}}}-\frac{5ab^4}{4x^4}-\frac{2b^5}{7x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**5/x**7,x)
```

```
[Out] -a**5/(6*x**6) - 10*a**4*b/(11*x**(11/2)) - 2*a**3*b**2/x**5 - 20*a**2*b**3
/(9*x**(9/2)) - 5*a*b**4/(4*x**4) - 2*b**5/(7*x**(7/2))
```

3.2153 $\int \left(a + b\sqrt{x}\right)^{10} x^4\, dx$

Optimal. Leaf size=140

$$\frac{a^{10}x^5}{5}+\frac{20}{11}a^9bx^{11/2}+\frac{15}{2}a^8b^2x^6+\frac{240}{13}a^7b^3x^{13/2}+30a^6b^4x^7+\frac{168}{5}a^5b^5x^{15/2}+\frac{105}{4}a^4b^6x^8+\frac{240}{17}a^3b^7x^{17/2}+5a^2b^8x^9+\frac{20}{19}a$$

```
[Out] 1/5*a^10*x^5+20/11*a^9*b*x^(11/2)+15/2*a^8*b^2*x^6+240/13*a^7*b^3*x^(13/2)+
30*a^6*b^4*x^7+168/5*a^5*b^5*x^(15/2)+105/4*a^4*b^6*x^8+240/17*a^3*b^7*x^(1
7/2)+5*a^2*b^8*x^9+20/19*a*b^9*x^(19/2)+1/10*b^10*x^10
```

Rubi [A] time = 0.10, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{15}{2}a^8b^2x^6+\frac{240}{13}a^7b^3x^{13/2}+30a^6b^4x^7+\frac{168}{5}a^5b^5x^{15/2}+\frac{105}{4}a^4b^6x^8+\frac{240}{17}a^3b^7x^{17/2}+5a^2b^8x^9+\frac{20}{11}a^9bx^{11/2}+\frac{a^{10}x^5}{5}+\frac{20}{19}a$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^10*x^4,x]
```

```
[Out] (a^10*x^5)/5 + (20*a^9*b*x^(11/2))/11 + (15*a^8*b^2*x^6)/2 + (240*a^7*b^3*x
^(13/2))/13 + 30*a^6*b^4*x^7 + (168*a^5*b^5*x^(15/2))/5 + (105*a^4*b^6*x^8)
/4 + (240*a^3*b^7*x^(17/2))/17 + 5*a^2*b^8*x^9 + (20*a*b^9*x^(19/2))/19 + (
b^10*x^10)/10
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \left(a + b\sqrt{x}\right)^{10} x^4\, dx &= 2\,\text{Subst}\left(\int x^9(a + bx)^{10}\, dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(a^{10}x^9 + 10a^9bx^{10} + 45a^8b^2x^{11} + 120a^7b^3x^{12} + 210a^6b^4x^{13} + 252a^5b^5x^{14} + 2\right.\right.\\ &= \frac{a^{10}x^5}{5} + \frac{20}{11}a^9bx^{11/2} + \frac{15}{2}a^8b^2x^6 + \frac{240}{13}a^7b^3x^{13/2} + 30a^6b^4x^7 + \frac{168}{5}a^5b^5x^{15/2} + \frac{105}{4}a^4b^6x^8\end{aligned}$$

Mathematica [A] time = 0.07, size = 140, normalized size = 1.00

$$\frac{a^{10}x^5}{5}+\frac{20}{11}a^9bx^{11/2}+\frac{15}{2}a^8b^2x^6+\frac{240}{13}a^7b^3x^{13/2}+30a^6b^4x^7+\frac{168}{5}a^5b^5x^{15/2}+\frac{105}{4}a^4b^6x^8+\frac{240}{17}a^3b^7x^{17/2}+5a^2b^8x^9+\frac{20}{19}a$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^10*x^4,x]
```

```
[Out] (a^10*x^5)/5 + (20*a^9*b*x^(11/2))/11 + (15*a^8*b^2*x^6)/2 + (240*a^7*b^3*x
^(13/2))/13 + 30*a^6*b^4*x^7 + (168*a^5*b^5*x^(15/2))/5 + (105*a^4*b^6*x^8)
/4 + (240*a^3*b^7*x^(17/2))/17 + 5*a^2*b^8*x^9 + (20*a*b^9*x^(19/2))/19 + (
b^10*x^10)/10
```

fricas [A] time = 1.03, size = 118, normalized size = 0.84

$$\frac{1}{10}b^{10}x^{10}+5a^2b^8x^9+\frac{105}{4}a^4b^6x^8+30a^6b^4x^7+\frac{15}{2}a^8b^2x^6+\frac{1}{5}a^{10}x^5+\frac{4}{230945}\left(60775ab^9x^9+815100a^3b^7x^8+19\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*x^(1/2))^10,x, algorithm="fricas")
```

```
[Out] 1/10*b^10*x^10 + 5*a^2*b^8*x^9 + 105/4*a^4*b^6*x^8 + 30*a^6*b^4*x^7 + 15/2*
a^8*b^2*x^6 + 1/5*a^10*x^5 + 4/230945*(60775*a*b^9*x^9 + 815100*a^3*b^7*x^8
 + 1939938*a^5*b^5*x^7 + 1065900*a^7*b^3*x^6 + 104975*a^9*b*x^5)*sqrt(x)
```

giac [A] time = 0.16, size = 112, normalized size = 0.80

$$\frac{1}{10}b^{10}x^{10}+\frac{20}{19}ab^9x^{\frac{19}{2}}+5a^2b^8x^9+\frac{240}{17}a^3b^7x^{\frac{17}{2}}+\frac{105}{4}a^4b^6x^8+\frac{168}{5}a^5b^5x^{\frac{15}{2}}+30a^6b^4x^7+\frac{240}{13}a^7b^3x^{\frac{13}{2}}+\frac{15}{2}a^8b^2x^6+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*x^(1/2))^10,x, algorithm="giac")
```

```
[Out] 1/10*b^10*x^10 + 20/19*a*b^9*x^(19/2) + 5*a^2*b^8*x^9 + 240/17*a^3*b^7*x^(1
7/2) + 105/4*a^4*b^6*x^8 + 168/5*a^5*b^5*x^(15/2) + 30*a^6*b^4*x^7 + 240/13
*a^7*b^3*x^(13/2) + 15/2*a^8*b^2*x^6 + 20/11*a^9*b*x^(11/2) + 1/5*a^10*x^5
```

maple [A] time = 0.00, size = 113, normalized size = 0.81

$$\frac{b^{10}x^{10}}{10}+\frac{20ab^9x^{\frac{19}{2}}}{19}+5a^2b^8x^9+\frac{240a^3b^7x^{\frac{17}{2}}}{17}+\frac{105a^4b^6x^8}{4}+\frac{168a^5b^5x^{\frac{15}{2}}}{5}+30a^6b^4x^7+\frac{240a^7b^3x^{\frac{13}{2}}}{13}+\frac{15a^8b^2x^6}{2}+\frac{20a^9}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*x^(1/2))^10,x)
```

```
[Out] 1/5*a^10*x^5+20/11*a^9*b*x^(11/2)+15/2*a^8*b^2*x^6+240/13*a^7*b^3*x^(13/2)+
30*a^6*b^4*x^7+168/5*a^5*b^5*x^(15/2)+105/4*a^4*b^6*x^8+240/17*a^3*b^7*x^(1
7/2)+5*a^2*b^8*x^9+20/19*a*b^9*x^(19/2)+1/10*b^10*x^10
```

maxima [A] time = 0.93, size = 166, normalized size = 1.19

$$\frac{\left(b\sqrt{x}+a\right)^{20}}{10\,b^{10}}-\frac{18\left(b\sqrt{x}+a\right)^{19}a}{19\,b^{10}}+\frac{4\left(b\sqrt{x}+a\right)^{18}a^2}{b^{10}}-\frac{168\left(b\sqrt{x}+a\right)^{17}a^3}{17\,b^{10}}+\frac{63\left(b\sqrt{x}+a\right)^{16}a^4}{4\,b^{10}}-\frac{84\left(b\sqrt{x}+a\right)^{15}a^5}{5\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*x^(1/2))^10,x, algorithm="maxima")
```

```
[Out] 1/10*(b*sqrt(x) + a)^20/b^10 - 18/19*(b*sqrt(x) + a)^19*a/b^10 + 4*(b*sqrt(
x) + a)^18*a^2/b^10 - 168/17*(b*sqrt(x) + a)^17*a^3/b^10 + 63/4*(b*sqrt(x)
 + a)^16*a^4/b^10 - 84/5*(b*sqrt(x) + a)^15*a^5/b^10 + 12*(b*sqrt(x) + a)^14
*a^6/b^10 - 72/13*(b*sqrt(x) + a)^13*a^7/b^10 + 3/2*(b*sqrt(x) + a)^12*a^8/
b^10 - 2/11*(b*sqrt(x) + a)^11*a^9/b^10
```

mupad [B] time = 1.09, size = 112, normalized size = 0.80

$$\frac{a^{10}x^5}{5}+\frac{b^{10}x^{10}}{10}+\frac{20a^9bx^{11/2}}{11}+\frac{20ab^9x^{19/2}}{19}+\frac{15a^8b^2x^6}{2}+30a^6b^4x^7+\frac{105a^4b^6x^8}{4}+5a^2b^8x^9+\frac{240a^7b^3x^{13/2}}{13}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^(1/2))^10,x)
```

```
[Out] (a^10*x^5)/5 + (b^10*x^10)/10 + (20*a^9*b*x^(11/2))/11 + (20*a*b^9*x^(19/2)
)/19 + (15*a^8*b^2*x^6)/2 + 30*a^6*b^4*x^7 + (105*a^4*b^6*x^8)/4 + 5*a^2*b^
8*x^9 + (240*a^7*b^3*x^(13/2))/13 + (168*a^5*b^5*x^(15/2))/5 + (240*a^3*b^7
*x^(17/2))/17
```

sympy [**A**] time = 7.82, size = 139, normalized size = 0.99

$$\frac{a^{10}x^5}{5}+\frac{20a^9bx^{\frac{11}{2}}}{11}+\frac{15a^8b^2x^6}{2}+\frac{240a^7b^3x^{\frac{13}{2}}}{13}+30a^6b^4x^7+\frac{168a^5b^5x^{\frac{15}{2}}}{5}+\frac{105a^4b^6x^8}{4}+\frac{240a^3b^7x^{\frac{17}{2}}}{17}+5a^2b^8x^9+\frac{20ab^9x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*x**(1/2))**10,x)
```

```
[Out] a**10*x**5/5 + 20*a**9*b*x**(11/2)/11 + 15*a**8*b**2*x**6/2 + 240*a**7*b**3
*x**(13/2)/13 + 30*a**6*b**4*x**7 + 168*a**5*b**5*x**(15/2)/5 + 105*a**4*b*
*6*x**8/4 + 240*a**3*b**7*x**(17/2)/17 + 5*a**2*b**8*x**9 + 20*a*b**9*x**(1
9/2)/19 + b**10*x**10/10
```

3.2154 $\int \left(a+b\sqrt{x}\right)^{10} x^3\,dx$

Optimal. Leaf size=162

$$-\frac{2a^7\left(a+b\sqrt{x}\right)^{11}}{11b^8}+\frac{7a^6\left(a+b\sqrt{x}\right)^{12}}{6b^8}-\frac{42a^5\left(a+b\sqrt{x}\right)^{13}}{13b^8}+\frac{5a^4\left(a+b\sqrt{x}\right)^{14}}{b^8}-\frac{14a^3\left(a+b\sqrt{x}\right)^{15}}{3b^8}+\frac{21a^2\left(a+b\sqrt{}\right.}{8b^8}$$

```
[Out] -2/11*a^7*(a+b*x^(1/2))^11/b^8+7/6*a^6*(a+b*x^(1/2))^12/b^8-42/13*a^5*(a+b*
x^(1/2))^13/b^8+5*a^4*(a+b*x^(1/2))^14/b^8-14/3*a^3*(a+b*x^(1/2))^15/b^8+21
/8*a^2*(a+b*x^(1/2))^16/b^8-14/17*a*(a+b*x^(1/2))^17/b^8+1/9*(a+b*x^(1/2))^
18/b^8
```

Rubi [A] time = 0.08, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{21a^2\left(a+b\sqrt{x}\right)^{16}}{8b^8}-\frac{14a^3\left(a+b\sqrt{x}\right)^{15}}{3b^8}+\frac{5a^4\left(a+b\sqrt{x}\right)^{14}}{b^8}-\frac{42a^5\left(a+b\sqrt{x}\right)^{13}}{13b^8}+\frac{7a^6\left(a+b\sqrt{x}\right)^{12}}{6b^8}-\frac{2a^7\left(a+b\sqrt{x}\right.}{11b^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^10*x^3,x]
```

```
[Out] (-2*a^7*(a + b*Sqrt[x])^11)/(11*b^8) + (7*a^6*(a + b*Sqrt[x])^12)/(6*b^8) -
 (42*a^5*(a + b*Sqrt[x])^13)/(13*b^8) + (5*a^4*(a + b*Sqrt[x])^14)/b^8 - (1
4*a^3*(a + b*Sqrt[x])^15)/(3*b^8) + (21*a^2*(a + b*Sqrt[x])^16)/(8*b^8) - (
14*a*(a + b*Sqrt[x])^17)/(17*b^8) + (a + b*Sqrt[x])^18/(9*b^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sqrt{x}\right)^{10} x^3\,dx &= 2\,\text{Subst}\left(\int x^7(a+bx)^{10}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int\left(-\frac{a^7(a+bx)^{10}}{b^7}+\frac{7a^6(a+bx)^{11}}{b^7}-\frac{21a^5(a+bx)^{12}}{b^7}+\frac{35a^4(a+bx)^{13}}{b^7}-\frac{35a}{}\right.\right.\\ &= -\frac{2a^7\left(a+b\sqrt{x}\right)^{11}}{11b^8}+\frac{7a^6\left(a+b\sqrt{x}\right)^{12}}{6b^8}-\frac{42a^5\left(a+b\sqrt{x}\right)^{13}}{13b^8}+\frac{5a^4\left(a+b\sqrt{x}\right)^{14}}{b^8}-\frac{14a^{}}{}\end{aligned}$$

Mathematica [A] time = 0.06, size = 136, normalized size = 0.84

$$\frac{a^{10}x^4}{4}+\frac{20}{9}a^9bx^{9/2}+9a^8b^2x^5+\frac{240}{11}a^7b^3x^{11/2}+35a^6b^4x^6+\frac{504}{13}a^5b^5x^{13/2}+30a^4b^6x^7+16a^3b^7x^{15/2}+\frac{45}{8}a^2b^8x^8+\frac{20}{17}ab^{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^10*x^3,x]
```

```
[Out] (a^10*x^4)/4 + (20*a^9*b*x^(9/2))/9 + 9*a^8*b^2*x^5 + (240*a^7*b^3*x^(11/2)
)/11 + 35*a^6*b^4*x^6 + (504*a^5*b^5*x^(13/2))/13 + 30*a^4*b^6*x^7 + 16*a^3
*b^7*x^(15/2) + (45*a^2*b^8*x^8)/8 + (20*a*b^9*x^(17/2))/17 + (b^10*x^9)/9
```

fricas [A] time = 0.96, size = 118, normalized size = 0.73

$$\frac{1}{9}b^{10}x^9+\frac{45}{8}a^2b^8x^8+30a^4b^6x^7+35a^6b^4x^6+9a^8b^2x^5+\frac{1}{4}a^{10}x^4+\frac{4}{21879}\left(6435ab^9x^8+87516a^3b^7x^7+212058a^5b^5x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^(1/2))^10,x, algorithm="fricas")
```

```
[Out] 1/9*b^10*x^9 + 45/8*a^2*b^8*x^8 + 30*a^4*b^6*x^7 + 35*a^6*b^4*x^6 + 9*a^8*b
^2*x^5 + 1/4*a^10*x^4 + 4/21879*(6435*a*b^9*x^8 + 87516*a^3*b^7*x^7 + 21205
8*a^5*b^5*x^6 + 119340*a^7*b^3*x^5 + 12155*a^9*b*x^4)*sqrt(x)
```

giac [A] time = 0.15, size = 112, normalized size = 0.69

$$\frac{1}{9}b^{10}x^9+\frac{20}{17}ab^9x^{\frac{17}{2}}+\frac{45}{8}a^2b^8x^8+16a^3b^7x^{\frac{15}{2}}+30a^4b^6x^7+\frac{504}{13}a^5b^5x^{\frac{13}{2}}+35a^6b^4x^6+\frac{240}{11}a^7b^3x^{\frac{11}{2}}+9a^8b^2x^5+\frac{20}{9}a^9bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^(1/2))^10,x, algorithm="giac")
```

```
[Out] 1/9*b^10*x^9 + 20/17*a*b^9*x^(17/2) + 45/8*a^2*b^8*x^8 + 16*a^3*b^7*x^(15/2
) + 30*a^4*b^6*x^7 + 504/13*a^5*b^5*x^(13/2) + 35*a^6*b^4*x^6 + 240/11*a^7*
b^3*x^(11/2) + 9*a^8*b^2*x^5 + 20/9*a^9*b*x^(9/2) + 1/4*a^10*x^4
```

maple [A] time = 0.00, size = 113, normalized size = 0.70

$$\frac{b^{10}x^9}{9}+\frac{20ab^9x^{\frac{17}{2}}}{17}+\frac{45a^2b^8x^8}{8}+16a^3b^7x^{\frac{15}{2}}+30a^4b^6x^7+\frac{504a^5b^5x^{\frac{13}{2}}}{13}+35a^6b^4x^6+\frac{240a^7b^3x^{\frac{11}{2}}}{11}+9a^8b^2x^5+\frac{20a^9bx^{\frac{9}{2}}}{9}+\frac{a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*x^(1/2))^10,x)
```

```
[Out] 1/9*x^9*b^10+20/17*x^(17/2)*a*b^9+45/8*x^8*a^2*b^8+16*x^(15/2)*a^3*b^7+30*a
^4*b^6*x^7+504/13*x^(13/2)*a^5*b^5+35*x^6*a^6*b^4+240/11*x^(11/2)*a^7*b^3+9
*x^5*a^8*b^2+20/9*x^(9/2)*a^9*b+1/4*a^10*x^4
```

maxima [A] time = 0.87, size = 132, normalized size = 0.81

$$\frac{\left(b\sqrt{x}+a\right)^{18}}{9b^8}-\frac{14\left(b\sqrt{x}+a\right)^{17}a}{17b^8}+\frac{21\left(b\sqrt{x}+a\right)^{16}a^2}{8b^8}-\frac{14\left(b\sqrt{x}+a\right)^{15}a^3}{3b^8}+\frac{5\left(b\sqrt{x}+a\right)^{14}a^4}{b^8}-\frac{42\left(b\sqrt{x}+a\right)^{13}a^5}{13b^8}+\frac{7}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^(1/2))^10,x, algorithm="maxima")
```

```
[Out] 1/9*(b*sqrt(x) + a)^18/b^8 - 14/17*(b*sqrt(x) + a)^17*a/b^8 + 21/8*(b*sqrt(
x) + a)^16*a^2/b^8 - 14/3*(b*sqrt(x) + a)^15*a^3/b^8 + 5*(b*sqrt(x) + a)^14
*a^4/b^8 - 42/13*(b*sqrt(x) + a)^13*a^5/b^8 + 7/6*(b*sqrt(x) + a)^12*a^6/b^
8 - 2/11*(b*sqrt(x) + a)^11*a^7/b^8
```

mupad [B] time = 0.06, size = 112, normalized size = 0.69

$$\frac{a^{10}x^4}{4}+\frac{b^{10}x^9}{9}+\frac{20a^9bx^{9/2}}{9}+\frac{20ab^9x^{17/2}}{17}+9a^8b^2x^5+35a^6b^4x^6+30a^4b^6x^7+\frac{45a^2b^8x^8}{8}+\frac{240a^7b^3x^{11/2}}{11}+\frac{504a^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^(1/2))^10,x)
```

```
[Out] (a^10*x^4)/4 + (b^10*x^9)/9 + (20*a^9*b*x^(9/2))/9 + (20*a*b^9*x^(17/2))/17
 + 9*a^8*b^2*x^5 + 35*a^6*b^4*x^6 + 30*a^4*b^6*x^7 + (45*a^2*b^8*x^8)/8 + (
240*a^7*b^3*x^(11/2))/11 + (504*a^5*b^5*x^(13/2))/13 + 16*a^3*b^7*x^(15/2)
```

sympy [**A**] time = 6.05, size = 136, normalized size = 0.84

$$\frac{a^{10}x^4}{4}+\frac{20a^9bx^{\frac{9}{2}}}{9}+9a^8b^2x^5+\frac{240a^7b^3x^{\frac{11}{2}}}{11}+35a^6b^4x^6+\frac{504a^5b^5x^{\frac{13}{2}}}{13}+30a^4b^6x^7+16a^3b^7x^{\frac{15}{2}}+\frac{45a^2b^8x^8}{8}+\frac{20ab^9x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*x**(1/2))**10,x)
```

```
[Out] a**10*x**4/4 + 20*a**9*b*x**(9/2)/9 + 9*a**8*b**2*x**5 + 240*a**7*b**3*x**(
11/2)/11 + 35*a**6*b**4*x**6 + 504*a**5*b**5*x**(13/2)/13 + 30*a**4*b**6*x*
*7 + 16*a**3*b**7*x**(15/2) + 45*a**2*b**8*x**8/8 + 20*a*b**9*x**(17/2)/17
+ b**10*x**9/9
```

3.2155 $\int \left(a+b\sqrt{x}\right)^{10} x^2\,dx$

Optimal. Leaf size=122

$$-\frac{2a^5\left(a+b\sqrt{x}\right)^{11}}{11b^6}+\frac{5a^4\left(a+b\sqrt{x}\right)^{12}}{6b^6}-\frac{20a^3\left(a+b\sqrt{x}\right)^{13}}{13b^6}+\frac{10a^2\left(a+b\sqrt{x}\right)^{14}}{7b^6}+\frac{\left(a+b\sqrt{x}\right)^{16}}{8b^6}-\frac{2a\left(a+b\sqrt{x}\right)^{15}}{3b^6}$$

```
[Out] -2/11*a^5*(a+b*x^(1/2))^11/b^6+5/6*a^4*(a+b*x^(1/2))^12/b^6-20/13*a^3*(a+b*
x^(1/2))^13/b^6+10/7*a^2*(a+b*x^(1/2))^14/b^6-2/3*a*(a+b*x^(1/2))^15/b^6+1/
8*(a+b*x^(1/2))^16/b^6
```

Rubi [A] time = 0.06, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{10a^2\left(a+b\sqrt{x}\right)^{14}}{7b^6}-\frac{20a^3\left(a+b\sqrt{x}\right)^{13}}{13b^6}+\frac{5a^4\left(a+b\sqrt{x}\right)^{12}}{6b^6}-\frac{2a^5\left(a+b\sqrt{x}\right)^{11}}{11b^6}+\frac{\left(a+b\sqrt{x}\right)^{16}}{8b^6}-\frac{2a\left(a+b\sqrt{x}\right)^{15}}{3b^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^10*x^2,x]
```

```
[Out] (-2*a^5*(a + b*Sqrt[x])^11)/(11*b^6) + (5*a^4*(a + b*Sqrt[x])^12)/(6*b^6) -
 (20*a^3*(a + b*Sqrt[x])^13)/(13*b^6) + (10*a^2*(a + b*Sqrt[x])^14)/(7*b^6)
 - (2*a*(a + b*Sqrt[x])^15)/(3*b^6) + (a + b*Sqrt[x])^16/(8*b^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sqrt{x}\right)^{10} x^2\,dx &= 2\,\text{Subst}\left(\int x^5(a+bx)^{10}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int\left(-\frac{a^5(a+bx)^{10}}{b^5}+\frac{5a^4(a+bx)^{11}}{b^5}-\frac{10a^3(a+bx)^{12}}{b^5}+\frac{10a^2(a+bx)^{13}}{b^5}-\frac{5a(a+\right.\right.\\ &= -\frac{2a^5\left(a+b\sqrt{x}\right)^{11}}{11b^6}+\frac{5a^4\left(a+b\sqrt{x}\right)^{12}}{6b^6}-\frac{20a^3\left(a+b\sqrt{x}\right)^{13}}{13b^6}+\frac{10a^2\left(a+b\sqrt{x}\right)^{14}}{7b^6}-\frac{2a\left(a+\right.}\end{aligned}$$

Mathematica [A] time = 0.05, size = 74, normalized size = 0.61

$$-\frac{\left(a+b\sqrt{x}\right)^{11}\left(a^5-11a^4b\sqrt{x}+66a^3b^2x-286a^2b^3x^{3/2}+1001ab^4x^2-3003b^5x^{5/2}\right)}{24024b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^10*x^2,x]
```

```
[Out] -1/24024*((a + b*Sqrt[x])^11*(a^5 - 11*a^4*b*Sqrt[x] + 66*a^3*b^2*x - 286*a
^2*b^3*x^(3/2) + 1001*a*b^4*x^2 - 3003*b^5*x^(5/2)))/b^6
```

fricas [A] time = 0.87, size = 118, normalized size = 0.97

$$\frac{1}{8}b^{10}x^8+\frac{45}{7}a^2b^8x^7+35a^4b^6x^6+42a^6b^4x^5+\frac{45}{4}a^8b^2x^4+\frac{1}{3}a^{10}x^3+\frac{4}{3003}\left(1001ab^9x^7+13860a^3b^7x^6+34398a^5b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^(1/2))^10,x, algorithm="fricas")
```

```
[Out] 1/8*b^10*x^8 + 45/7*a^2*b^8*x^7 + 35*a^4*b^6*x^6 + 42*a^6*b^4*x^5 + 45/4*a^
8*b^2*x^4 + 1/3*a^10*x^3 + 4/3003*(1001*a*b^9*x^7 + 13860*a^3*b^7*x^6 + 343
98*a^5*b^5*x^5 + 20020*a^7*b^3*x^4 + 2145*a^9*b*x^3)*sqrt(x)
```

giac [A] time = 0.16, size = 112, normalized size = 0.92

$$\frac{1}{8}b^{10}x^8+\frac{4}{3}ab^9x^{\frac{15}{2}}+\frac{45}{7}a^2b^8x^7+\frac{240}{13}a^3b^7x^{\frac{13}{2}}+35a^4b^6x^6+\frac{504}{11}a^5b^5x^{\frac{11}{2}}+42a^6b^4x^5+\frac{80}{3}a^7b^3x^{\frac{9}{2}}+\frac{45}{4}a^8b^2x^4+\frac{20}{7}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^(1/2))^10,x, algorithm="giac")
```

```
[Out] 1/8*b^10*x^8 + 4/3*a*b^9*x^(15/2) + 45/7*a^2*b^8*x^7 + 240/13*a^3*b^7*x^(13
/2) + 35*a^4*b^6*x^6 + 504/11*a^5*b^5*x^(11/2) + 42*a^6*b^4*x^5 + 80/3*a^7*
b^3*x^(9/2) + 45/4*a^8*b^2*x^4 + 20/7*a^9*b*x^(7/2) + 1/3*a^10*x^3
```

maple [A] time = 0.00, size = 113, normalized size = 0.93

$$\frac{b^{10}x^8}{8}+\frac{4ab^9x^{\frac{15}{2}}}{3}+\frac{45a^2b^8x^7}{7}+\frac{240a^3b^7x^{\frac{13}{2}}}{13}+35a^4b^6x^6+\frac{504a^5b^5x^{\frac{11}{2}}}{11}+42a^6b^4x^5+\frac{80a^7b^3x^{\frac{9}{2}}}{3}+\frac{45a^8b^2x^4}{4}+\frac{20a^9bx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*x^(1/2))^10,x)
```

```
[Out] 1/8*x^8*b^10+4/3*x^(15/2)*a*b^9+45/7*x^7*a^2*b^8+240/13*x^(13/2)*a^3*b^7+35
*a^4*b^6*x^6+504/11*x^(11/2)*a^5*b^5+42*x^5*a^6*b^4+80/3*x^(9/2)*a^7*b^3+45
/4*x^4*a^8*b^2+20/7*x^(7/2)*a^9*b+1/3*x^3*a^10
```

maxima [A] time = 0.88, size = 98, normalized size = 0.80

$$\frac{\left(b\sqrt{x}+a\right)^{16}}{8b^6}-\frac{2\left(b\sqrt{x}+a\right)^{15}a}{3b^6}+\frac{10\left(b\sqrt{x}+a\right)^{14}a^2}{7b^6}-\frac{20\left(b\sqrt{x}+a\right)^{13}a^3}{13b^6}+\frac{5\left(b\sqrt{x}+a\right)^{12}a^4}{6b^6}-\frac{2\left(b\sqrt{x}+a\right)^{11}a^5}{11b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^(1/2))^10,x, algorithm="maxima")
```

```
[Out] 1/8*(b*sqrt(x) + a)^16/b^6 - 2/3*(b*sqrt(x) + a)^15*a/b^6 + 10/7*(b*sqrt(x)
 + a)^14*a^2/b^6 - 20/13*(b*sqrt(x) + a)^13*a^3/b^6 + 5/6*(b*sqrt(x) + a)^1
2*a^4/b^6 - 2/11*(b*sqrt(x) + a)^11*a^5/b^6
```

mupad [B] time = 0.06, size = 112, normalized size = 0.92

$$\frac{a^{10}x^3}{3}+\frac{b^{10}x^8}{8}+\frac{20a^9bx^{7/2}}{7}+\frac{4ab^9x^{15/2}}{3}+\frac{45a^8b^2x^4}{4}+42a^6b^4x^5+35a^4b^6x^6+\frac{45a^2b^8x^7}{7}+\frac{80a^7b^3x^{9/2}}{3}+\frac{504a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^(1/2))^10,x)
```

```
[Out] (a^10*x^3)/3 + (b^10*x^8)/8 + (20*a^9*b*x^(7/2))/7 + (4*a*b^9*x^(15/2))/3 +
 (45*a^8*b^2*x^4)/4 + 42*a^6*b^4*x^5 + 35*a^4*b^6*x^6 + (45*a^2*b^8*x^7)/7
+ (80*a^7*b^3*x^(9/2))/3 + (504*a^5*b^5*x^(11/2))/11 + (240*a^3*b^7*x^(13/2
))/13
```

sympy [**A**] time = 4.01, size = 139, normalized size = 1.14

$$\frac{a^{10}x^3}{3}+\frac{20a^9bx^{\frac{7}{2}}}{7}+\frac{45a^8b^2x^4}{4}+\frac{80a^7b^3x^{\frac{9}{2}}}{3}+42a^6b^4x^5+\frac{504a^5b^5x^{\frac{11}{2}}}{11}+35a^4b^6x^6+\frac{240a^3b^7x^{\frac{13}{2}}}{13}+\frac{45a^2b^8x^7}{7}+\frac{4ab^9x^{\frac{15}{2}}}{3}+b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*x**(1/2))**10,x)
```

```
[Out] a**10*x**3/3 + 20*a**9*b*x**(7/2)/7 + 45*a**8*b**2*x**4/4 + 80*a**7*b**3*x*
*(9/2)/3 + 42*a**6*b**4*x**5 + 504*a**5*b**5*x**(11/2)/11 + 35*a**4*b**6*x*
*6 + 240*a**3*b**7*x**(13/2)/13 + 45*a**2*b**8*x**7/7 + 4*a*b**9*x**(15/2)/
3 + b**10*x**8/8
```

3.2156 $\int \left(a+b\sqrt{x}\right)^{10} x\,dx$

Optimal. Leaf size=80

$$-\frac{2a^3\left(a+b\sqrt{x}\right)^{11}}{11b^4}+\frac{a^2\left(a+b\sqrt{x}\right)^{12}}{2b^4}+\frac{\left(a+b\sqrt{x}\right)^{14}}{7b^4}-\frac{6a\left(a+b\sqrt{x}\right)^{13}}{13b^4}$$

```
[Out] -2/11*a^3*(a+b*x^(1/2))^11/b^4+1/2*a^2*(a+b*x^(1/2))^12/b^4-6/13*a*(a+b*x^(
1/2))^13/b^4+1/7*(a+b*x^(1/2))^14/b^4
```

Rubi [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{a^2\left(a+b\sqrt{x}\right)^{12}}{2b^4}-\frac{2a^3\left(a+b\sqrt{x}\right)^{11}}{11b^4}+\frac{\left(a+b\sqrt{x}\right)^{14}}{7b^4}-\frac{6a\left(a+b\sqrt{x}\right)^{13}}{13b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^10*x,x]
```

```
[Out] (-2*a^3*(a + b*Sqrt[x])^11)/(11*b^4) + (a^2*(a + b*Sqrt[x])^12)/(2*b^4) - (
6*a*(a + b*Sqrt[x])^13)/(13*b^4) + (a + b*Sqrt[x])^14/(7*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sqrt{x}\right)^{10} x\,dx &= 2\,\text{Subst}\left(\int x^3(a+bx)^{10}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int\left(-\frac{a^3(a+bx)^{10}}{b^3}+\frac{3a^2(a+bx)^{11}}{b^3}-\frac{3a(a+bx)^{12}}{b^3}+\frac{(a+bx)^{13}}{b^3}\right)dx, x, \sqrt{x}\right)\\ &= -\frac{2a^3\left(a+b\sqrt{x}\right)^{11}}{11b^4}+\frac{a^2\left(a+b\sqrt{x}\right)^{12}}{2b^4}-\frac{6a\left(a+b\sqrt{x}\right)^{13}}{13b^4}+\frac{\left(a+b\sqrt{x}\right)^{14}}{7b^4}\end{aligned}$$

Mathematica [A] time = 0.04, size = 50, normalized size = 0.62

$$-\frac{\left(a+b\sqrt{x}\right)^{11}\left(a^3-11a^2b\sqrt{x}+66ab^2x-286b^3x^{3/2}\right)}{2002b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^10*x,x]
```

```
[Out] -1/2002*((a + b*Sqrt[x])^11*(a^3 - 11*a^2*b*Sqrt[x] + 66*a*b^2*x - 286*b^3*
x^(3/2)))/b^4
```

fricas [A] time = 0.78, size = 118, normalized size = 1.48

$$\frac{1}{7}b^{10}x^7+\frac{15}{2}a^2b^8x^6+42a^4b^6x^5+\frac{105}{2}a^6b^4x^4+15a^8b^2x^3+\frac{1}{2}a^{10}x^2+\frac{4}{1001}\left(385ab^9x^6+5460a^3b^7x^5+14014a^5b^5x^4+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^(1/2))^10,x, algorithm="fricas")
```

```
[Out] 1/7*b^10*x^7 + 15/2*a^2*b^8*x^6 + 42*a^4*b^6*x^5 + 105/2*a^6*b^4*x^4 + 15*a
^8*b^2*x^3 + 1/2*a^10*x^2 + 4/1001*(385*a*b^9*x^6 + 5460*a^3*b^7*x^5 + 1401
4*a^5*b^5*x^4 + 8580*a^7*b^3*x^3 + 1001*a^9*b*x^2)*sqrt(x)
```

giac [A] time = 0.17, size = 112, normalized size = 1.40

$$\frac{1}{7}b^{10}x^7+\frac{20}{13}ab^9x^{\frac{13}{2}}+\frac{15}{2}a^2b^8x^6+\frac{240}{11}a^3b^7x^{\frac{11}{2}}+42a^4b^6x^5+56a^5b^5x^{\frac{9}{2}}+\frac{105}{2}a^6b^4x^4+\frac{240}{7}a^7b^3x^{\frac{7}{2}}+15a^8b^2x^3+4a^9bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^(1/2))^10,x, algorithm="giac")
```

```
[Out] 1/7*b^10*x^7 + 20/13*a*b^9*x^(13/2) + 15/2*a^2*b^8*x^6 + 240/11*a^3*b^7*x^(
11/2) + 42*a^4*b^6*x^5 + 56*a^5*b^5*x^(9/2) + 105/2*a^6*b^4*x^4 + 240/7*a^7
*b^3*x^(7/2) + 15*a^8*b^2*x^3 + 4*a^9*b*x^(5/2) + 1/2*a^10*x^2
```

maple [A] time = 0.00, size = 113, normalized size = 1.41

$$\frac{b^{10}x^7}{7}+\frac{20ab^9x^{\frac{13}{2}}}{13}+\frac{15a^2b^8x^6}{2}+\frac{240a^3b^7x^{\frac{11}{2}}}{11}+42a^4b^6x^5+56a^5b^5x^{\frac{9}{2}}+\frac{105a^6b^4x^4}{2}+\frac{240a^7b^3x^{\frac{7}{2}}}{7}+15a^8b^2x^3+4a^9bx^{\frac{5}{2}}+\frac{a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*x^(1/2))^10,x)
```

```
[Out] 1/7*x^7*b^10+20/13*x^(13/2)*a*b^9+15/2*x^6*a^2*b^8+240/11*x^(11/2)*a^3*b^7+
42*x^5*a^4*b^6+56*x^(9/2)*a^5*b^5+105/2*x^4*a^6*b^4+240/7*x^(7/2)*a^7*b^3+1
5*x^3*a^8*b^2+4*x^(5/2)*a^9*b+1/2*x^2*a^10
```

maxima [A] time = 0.85, size = 64, normalized size = 0.80

$$\frac{\left(b\sqrt{x}+a\right)^{14}}{7\,b^4}-\frac{6\left(b\sqrt{x}+a\right)^{13}a}{13\,b^4}+\frac{\left(b\sqrt{x}+a\right)^{12}a^2}{2\,b^4}-\frac{2\left(b\sqrt{x}+a\right)^{11}a^3}{11\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^(1/2))^10,x, algorithm="maxima")
```

```
[Out] 1/7*(b*sqrt(x) + a)^14/b^4 - 6/13*(b*sqrt(x) + a)^13*a/b^4 + 1/2*(b*sqrt(x)
 + a)^12*a^2/b^4 - 2/11*(b*sqrt(x) + a)^11*a^3/b^4
```

mupad [B] time = 0.06, size = 112, normalized size = 1.40

$$\frac{a^{10}x^2}{2}+\frac{b^{10}x^7}{7}+4a^9bx^{5/2}+\frac{20ab^9x^{13/2}}{13}+15a^8b^2x^3+\frac{105a^6b^4x^4}{2}+42a^4b^6x^5+\frac{15a^2b^8x^6}{2}+\frac{240a^7b^3x^{7/2}}{7}+56a^5b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^(1/2))^10,x)
```

```
[Out] (a^10*x^2)/2 + (b^10*x^7)/7 + 4*a^9*b*x^(5/2) + (20*a*b^9*x^(13/2))/13 + 15
*a^8*b^2*x^3 + (105*a^6*b^4*x^4)/2 + 42*a^4*b^6*x^5 + (15*a^2*b^8*x^6)/2 +
(240*a^7*b^3*x^(7/2))/7 + 56*a^5*b^5*x^(9/2) + (240*a^3*b^7*x^(11/2))/11
```

sympy [**A**] time = 2.76, size = 136, normalized size = 1.70

$$\frac{a^{10}x^2}{2}+4a^9bx^{\frac{5}{2}}+15a^8b^2x^3+\frac{240a^7b^3x^{\frac{7}{2}}}{7}+\frac{105a^6b^4x^4}{2}+56a^5b^5x^{\frac{9}{2}}+42a^4b^6x^5+\frac{240a^3b^7x^{\frac{11}{2}}}{11}+\frac{15a^2b^8x^6}{2}+\frac{20ab^9x^{\frac{13}{2}}}{13}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x**(1/2))**10,x)
```

```
[Out] a**10*x**2/2 + 4*a**9*b*x**(5/2) + 15*a**8*b**2*x**3 + 240*a**7*b**3*x**(7/
2)/7 + 105*a**6*b**4*x**4/2 + 56*a**5*b**5*x**(9/2) + 42*a**4*b**6*x**5 + 2
40*a**3*b**7*x**(11/2)/11 + 15*a**2*b**8*x**6/2 + 20*a*b**9*x**(13/2)/13 +
b**10*x**7/7
```

3.2157 $\int \left(a+b\sqrt{x}\right)^{10}\,dx$

Optimal. Leaf size=38

$$\frac{\left(a+b\sqrt{x}\right)^{12}}{6b^2}-\frac{2a\left(a+b\sqrt{x}\right)^{11}}{11b^2}$$

```
[Out] -2/11*a*(a+b*x^(1/2))^11/b^2+1/6*(a+b*x^(1/2))^12/b^2
```

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {190, 43}

$$\frac{\left(a+b\sqrt{x}\right)^{12}}{6b^2}-\frac{2a\left(a+b\sqrt{x}\right)^{11}}{11b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^10,x]
```

```
[Out] (-2*a*(a + b*Sqrt[x])^11)/(11*b^2) + (a + b*Sqrt[x])^12/(6*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sqrt{x}\right)^{10}\,dx &= 2\,\text{Subst}\left(\int x(a+bx)^{10}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(-\frac{a(a+bx)^{10}}{b}+\frac{(a+bx)^{11}}{b}\right)\,dx, x, \sqrt{x}\right)\\ &= -\frac{2a\left(a+b\sqrt{x}\right)^{11}}{11b^2}+\frac{\left(a+b\sqrt{x}\right)^{12}}{6b^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 28, normalized size = 0.74

$$-\frac{\left(a-11b\sqrt{x}\right)\left(a+b\sqrt{x}\right)^{11}}{66b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^10,x]
```

```
[Out] -1/66*((a - 11*b*Sqrt[x])*(a + b*Sqrt[x])^11)/b^2
```

fricas [B] time = 1.25, size = 113, normalized size = 2.97

$$\frac{1}{6}b^{10}x^6+9a^2b^8x^5+\frac{105}{2}a^4b^6x^4+70a^6b^4x^3+\frac{45}{2}a^8b^2x^2+a^{10}x+\frac{4}{33}\left(15ab^9x^5+220a^3b^7x^4+594a^5b^5x^3+396a^7b^3x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^10,x, algorithm="fricas")
```

```
[Out] 1/6*b^10*x^6 + 9*a^2*b^8*x^5 + 105/2*a^4*b^6*x^4 + 70*a^6*b^4*x^3 + 45/2*a^
8*b^2*x^2 + a^10*x + 4/33*(15*a*b^9*x^5 + 220*a^3*b^7*x^4 + 594*a^5*b^5*x^3
 + 396*a^7*b^3*x^2 + 55*a^9*b*x)*sqrt(x)
```

giac [B] time = 0.15, size = 109, normalized size = 2.87

$$\frac{1}{6}b^{10}x^6+\frac{20}{11}ab^9x^{\frac{11}{2}}+9a^2b^8x^5+\frac{80}{3}a^3b^7x^{\frac{9}{2}}+\frac{105}{2}a^4b^6x^4+72a^5b^5x^{\frac{7}{2}}+70a^6b^4x^3+48a^7b^3x^{\frac{5}{2}}+\frac{45}{2}a^8b^2x^2+\frac{20}{3}a^9bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^10,x, algorithm="giac")
```

```
[Out] 1/6*b^10*x^6 + 20/11*a*b^9*x^(11/2) + 9*a^2*b^8*x^5 + 80/3*a^3*b^7*x^(9/2)
+ 105/2*a^4*b^6*x^4 + 72*a^5*b^5*x^(7/2) + 70*a^6*b^4*x^3 + 48*a^7*b^3*x^(5
/2) + 45/2*a^8*b^2*x^2 + 20/3*a^9*b*x^(3/2) + a^10*x
```

maple [B] time = 0.00, size = 110, normalized size = 2.89

$$\frac{b^{10}x^6}{6}+\frac{20a\,b^9x^{\frac{11}{2}}}{11}+9a^2b^8x^5+\frac{80a^3b^7x^{\frac{9}{2}}}{3}+\frac{105a^4b^6x^4}{2}+72a^5b^5x^{\frac{7}{2}}+70a^6b^4x^3+48a^7b^3x^{\frac{5}{2}}+\frac{45a^8b^2x^2}{2}+\frac{20a^9b\,x^{\frac{3}{2}}}{3}+a^1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^10,x)
```

```
[Out] 1/6*x^6*b^10+20/11*x^(11/2)*a*b^9+9*x^5*a^2*b^8+80/3*x^(9/2)*a^3*b^7+105/2*
x^4*a^4*b^6+72*x^(7/2)*a^5*b^5+70*x^3*a^6*b^4+48*x^(5/2)*a^7*b^3+45/2*x^2*a
^8*b^2+20/3*x^(3/2)*a^9*b+x*a^10
```

maxima [A] time = 0.87, size = 30, normalized size = 0.79

$$\frac{\left(b\sqrt{x}+a\right)^{12}}{6\,b^2}-\frac{2\left(b\sqrt{x}+a\right)^{11}a}{11\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^10,x, algorithm="maxima")
```

```
[Out] 1/6*(b*sqrt(x) + a)^12/b^2 - 2/11*(b*sqrt(x) + a)^11*a/b^2
```

mupad [B] time = 0.06, size = 109, normalized size = 2.87

$$a^{10}\,x+\frac{b^{10}\,x^6}{6}+\frac{20\,a^9\,b\,x^{3/2}}{3}+\frac{20\,a\,b^9\,x^{11/2}}{11}+\frac{45\,a^8\,b^2\,x^2}{2}+70\,a^6\,b^4\,x^3+\frac{105\,a^4\,b^6\,x^4}{2}+9\,a^2\,b^8\,x^5+48\,a^7\,b^3\,x^{5/2}+72\,a^5\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^10,x)
```

```
[Out] a^10*x + (b^10*x^6)/6 + (20*a^9*b*x^(3/2))/3 + (20*a*b^9*x^(11/2))/11 + (45
*a^8*b^2*x^2)/2 + 70*a^6*b^4*x^3 + (105*a^4*b^6*x^4)/2 + 9*a^2*b^8*x^5 + 48
*a^7*b^3*x^(5/2) + 72*a^5*b^5*x^(7/2) + (80*a^3*b^7*x^(9/2))/3
```

sympy [B] time = 1.83, size = 133, normalized size = 3.50

$$a^{10}x+\frac{20a^9bx^{\frac{3}{2}}}{3}+\frac{45a^8b^2x^2}{2}+48a^7b^3x^{\frac{5}{2}}+70a^6b^4x^3+72a^5b^5x^{\frac{7}{2}}+\frac{105a^4b^6x^4}{2}+\frac{80a^3b^7x^{\frac{9}{2}}}{3}+9a^2b^8x^5+\frac{20ab^9x^{\frac{11}{2}}}{11}+\frac{b^{10}x}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**10,x)
```

```
[Out] a**10*x + 20*a**9*b*x**(3/2)/3 + 45*a**8*b**2*x**2/2 + 48*a**7*b**3*x**(5/2
) + 70*a**6*b**4*x**3 + 72*a**5*b**5*x**(7/2) + 105*a**4*b**6*x**4/2 + 80*a
**3*b**7*x**(9/2)/3 + 9*a**2*b**8*x**5 + 20*a*b**9*x**(11/2)/11 + b**10*x**
6/6
```

3.2158 $\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x}\,dx$

Optimal. Leaf size=128

$$a^{10}\log(x)+20a^9b\sqrt{x}+45a^8b^2x+80a^7b^3x^{3/2}+105a^6b^4x^2+\frac{504}{5}a^5b^5x^{5/2}+70a^4b^6x^3+\frac{240}{7}a^3b^7x^{7/2}+\frac{45}{4}a^2b^8x^4+\frac{20}{9}$$

```
[Out] 45*a^8*b^2*x+80*a^7*b^3*x^(3/2)+105*a^6*b^4*x^2+504/5*a^5*b^5*x^(5/2)+70*a^
4*b^6*x^3+240/7*a^3*b^7*x^(7/2)+45/4*a^2*b^8*x^4+20/9*a*b^9*x^(9/2)+1/5*b^1
0*x^5+a^10*ln(x)+20*a^9*b*x^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$80a^7b^3x^{3/2}+105a^6b^4x^2+\frac{504}{5}a^5b^5x^{5/2}+70a^4b^6x^3+\frac{240}{7}a^3b^7x^{7/2}+\frac{45}{4}a^2b^8x^4+45a^8b^2x+20a^9b\sqrt{x}+a^{10}\log(x)+\frac{20}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^10/x,x]
```

```
[Out] 20*a^9*b*Sqrt[x] + 45*a^8*b^2*x + 80*a^7*b^3*x^(3/2) + 105*a^6*b^4*x^2 + (5
04*a^5*b^5*x^(5/2))/5 + 70*a^4*b^6*x^3 + (240*a^3*b^7*x^(7/2))/7 + (45*a^2*
b^8*x^4)/4 + (20*a*b^9*x^(9/2))/9 + (b^10*x^5)/5 + a^10*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x}\,dx &= 2\,\text{Subst}\left(\int \frac{(a+bx)^{10}}{x}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(10a^9b+\frac{a^{10}}{x}+45a^8b^2x+120a^7b^3x^2+210a^6b^4x^3+252a^5b^5x^4+210a^4b^6x^5\right.\right.\\ &= 20a^9b\sqrt{x}+45a^8b^2x+80a^7b^3x^{3/2}+105a^6b^4x^2+\frac{504}{5}a^5b^5x^{5/2}+70a^4b^6x^3+\frac{240}{7}a^3b^7x^{7/2}\end{aligned}$$

Mathematica [A] time = 0.04, size = 128, normalized size = 1.00

$$a^{10}\log(x)+20a^9b\sqrt{x}+45a^8b^2x+80a^7b^3x^{3/2}+105a^6b^4x^2+\frac{504}{5}a^5b^5x^{5/2}+70a^4b^6x^3+\frac{240}{7}a^3b^7x^{7/2}+\frac{45}{4}a^2b^8x^4+\frac{20}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^10/x,x]
```

```
[Out] 20*a^9*b*Sqrt[x] + 45*a^8*b^2*x + 80*a^7*b^3*x^(3/2) + 105*a^6*b^4*x^2 + (5
04*a^5*b^5*x^(5/2))/5 + 70*a^4*b^6*x^3 + (240*a^3*b^7*x^(7/2))/7 + (45*a^2*
b^8*x^4)/4 + (20*a*b^9*x^(9/2))/9 + (b^10*x^5)/5 + a^10*Log[x]
```

fricas [A] time = 1.09, size = 112, normalized size = 0.88

$$\frac{1}{5}b^{10}x^5+\frac{45}{4}a^2b^8x^4+70a^4b^6x^3+105a^6b^4x^2+45a^8b^2x+2a^{10}\log\left(\sqrt{x}\right)+\frac{4}{315}\left(175ab^9x^4+2700a^3b^7x^3+7938a^5b^5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^10/x,x, algorithm="fricas")
```

```
[Out] 1/5*b^10*x^5 + 45/4*a^2*b^8*x^4 + 70*a^4*b^6*x^3 + 105*a^6*b^4*x^2 + 45*a^8
*b^2*x + 2*a^10*log(sqrt(x)) + 4/315*(175*a*b^9*x^4 + 2700*a^3*b^7*x^3 + 79
38*a^5*b^5*x^2 + 6300*a^7*b^3*x + 1575*a^9*b)*sqrt(x)
```

giac [A] time = 0.16, size = 109, normalized size = 0.85

$$\frac{1}{5}b^{10}x^5+\frac{20}{9}ab^9x^{\frac{9}{2}}+\frac{45}{4}a^2b^8x^4+\frac{240}{7}a^3b^7x^{\frac{7}{2}}+70a^4b^6x^3+\frac{504}{5}a^5b^5x^{\frac{5}{2}}+105a^6b^4x^2+80a^7b^3x^{\frac{3}{2}}+45a^8b^2x+a^{10}\log(|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^10/x,x, algorithm="giac")
```

```
[Out] 1/5*b^10*x^5 + 20/9*a*b^9*x^(9/2) + 45/4*a^2*b^8*x^4 + 240/7*a^3*b^7*x^(7/2
) + 70*a^4*b^6*x^3 + 504/5*a^5*b^5*x^(5/2) + 105*a^6*b^4*x^2 + 80*a^7*b^3*x
^(3/2) + 45*a^8*b^2*x + a^10*log(abs(x)) + 20*a^9*b*sqrt(x)
```

maple [A] time = 0.00, size = 109, normalized size = 0.85

$$\frac{b^{10}x^5}{5}+\frac{20ab^9x^{\frac{9}{2}}}{9}+\frac{45a^2b^8x^4}{4}+\frac{240a^3b^7x^{\frac{7}{2}}}{7}+70a^4b^6x^3+\frac{504a^5b^5x^{\frac{5}{2}}}{5}+105a^6b^4x^2+80a^7b^3x^{\frac{3}{2}}+a^{10}\ln(x)+45a^8b^2x+20a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^10/x,x)
```

```
[Out] 45*a^8*b^2*x+80*a^7*b^3*x^(3/2)+105*a^6*b^4*x^2+504/5*a^5*b^5*x^(5/2)+70*a^
4*b^6*x^3+240/7*a^3*b^7*x^(7/2)+45/4*a^2*b^8*x^4+20/9*a*b^9*x^(9/2)+1/5*b^1
0*x^5+a^10*ln(x)+20*a^9*b*x^(1/2)
```

maxima [A] time = 0.85, size = 108, normalized size = 0.84

$$\frac{1}{5}b^{10}x^5+\frac{20}{9}ab^9x^{\frac{9}{2}}+\frac{45}{4}a^2b^8x^4+\frac{240}{7}a^3b^7x^{\frac{7}{2}}+70a^4b^6x^3+\frac{504}{5}a^5b^5x^{\frac{5}{2}}+105a^6b^4x^2+80a^7b^3x^{\frac{3}{2}}+45a^8b^2x+a^{10}\log(x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^10/x,x, algorithm="maxima")
```

```
[Out] 1/5*b^10*x^5 + 20/9*a*b^9*x^(9/2) + 45/4*a^2*b^8*x^4 + 240/7*a^3*b^7*x^(7/2
) + 70*a^4*b^6*x^3 + 504/5*a^5*b^5*x^(5/2) + 105*a^6*b^4*x^2 + 80*a^7*b^3*x
^(3/2) + 45*a^8*b^2*x + a^10*log(x) + 20*a^9*b*sqrt(x)
```

mupad [B] time = 0.06, size = 111, normalized size = 0.87

$$2a^{10}\ln\left(\sqrt{x}\right)+\frac{b^{10}x^5}{5}+45a^8b^2x+20a^9b\sqrt{x}+\frac{20ab^9x^{9/2}}{9}+105a^6b^4x^2+70a^4b^6x^3+\frac{45a^2b^8x^4}{4}+80a^7b^3x^{3/2}+\frac{50}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^10/x,x)
```

```
[Out] 2*a^10*log(x^(1/2)) + (b^10*x^5)/5 + 45*a^8*b^2*x + 20*a^9*b*x^(1/2) + (20*
a*b^9*x^(9/2))/9 + 105*a^6*b^4*x^2 + 70*a^4*b^6*x^3 + (45*a^2*b^8*x^4)/4 +
80*a^7*b^3*x^(3/2) + (504*a^5*b^5*x^(5/2))/5 + (240*a^3*b^7*x^(7/2))/7
```

sympy [**A**] time = 2.12, size = 131, normalized size = 1.02

$$a^{10}\log(x)+20a^9b\sqrt{x}+45a^8b^2x+80a^7b^3x^{\frac{3}{2}}+105a^6b^4x^2+\frac{504a^5b^5x^{\frac{5}{2}}}{5}+70a^4b^6x^3+\frac{240a^3b^7x^{\frac{7}{2}}}{7}+\frac{45a^2b^8x^4}{4}+\frac{20ab^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**10/x,x)
```

```
[Out] a**10*log(x) + 20*a**9*b*sqrt(x) + 45*a**8*b**2*x + 80*a**7*b**3*x**(3/2) +
 105*a**6*b**4*x**2 + 504*a**5*b**5*x**(5/2)/5 + 70*a**4*b**6*x**3 + 240*a*
*3*b**7*x**(7/2)/7 + 45*a**2*b**8*x**4/4 + 20*a*b**9*x**(9/2)/9 + b**10*x**
5/5
```

3.2159 $\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^2}\,dx$

Optimal. Leaf size=123

$$-\frac{a^{10}}{x}-\frac{20a^9b}{\sqrt{x}}+45a^8b^2\log(x)+240a^7b^3\sqrt{x}+210a^6b^4x+168a^5b^5x^{3/2}+105a^4b^6x^2+48a^3b^7x^{5/2}+15a^2b^8x^3+\frac{20}{7}ab^9x^{7/2}+$$

```
[Out] -a^10/x+210*a^6*b^4*x+168*a^5*b^5*x^(3/2)+105*a^4*b^6*x^2+48*a^3*b^7*x^(5/2
)+15*a^2*b^8*x^3+20/7*a*b^9*x^(7/2)+1/4*b^10*x^4+45*a^8*b^2*ln(x)-20*a^9*b/
x^(1/2)+240*a^7*b^3*x^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$168a^5b^5x^{3/2}+105a^4b^6x^2+48a^3b^7x^{5/2}+15a^2b^8x^3+240a^7b^3\sqrt{x}+210a^6b^4x+45a^8b^2\log(x)-\frac{20a^9b}{\sqrt{x}}-\frac{a^{10}}{x}+\frac{20}{7}ab^9x^{7/2}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^10/x^2,x]
```

```
[Out] -(a^10/x) - (20*a^9*b)/Sqrt[x] + 240*a^7*b^3*Sqrt[x] + 210*a^6*b^4*x + 168*
a^5*b^5*x^(3/2) + 105*a^4*b^6*x^2 + 48*a^3*b^7*x^(5/2) + 15*a^2*b^8*x^3 + (
20*a*b^9*x^(7/2))/7 + (b^10*x^4)/4 + 45*a^8*b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^2}\,dx &= 2\,\text{Subst}\left(\int \frac{(a+bx)^{10}}{x^3}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(120a^7b^3+\frac{a^{10}}{x^3}+\frac{10a^9b}{x^2}+\frac{45a^8b^2}{x}+210a^6b^4x+252a^5b^5x^2+210a^4b^6x^3+120\right.\right.\\ &= -\frac{a^{10}}{x}-\frac{20a^9b}{\sqrt{x}}+240a^7b^3\sqrt{x}+210a^6b^4x+168a^5b^5x^{3/2}+105a^4b^6x^2+48a^3b^7x^{5/2}+15a^2b^8\end{aligned}$$

Mathematica [A] time = 0.06, size = 123, normalized size = 1.00

$$-\frac{a^{10}}{x}-\frac{20a^9b}{\sqrt{x}}+45a^8b^2\log(x)+240a^7b^3\sqrt{x}+210a^6b^4x+168a^5b^5x^{3/2}+105a^4b^6x^2+48a^3b^7x^{5/2}+15a^2b^8x^3+\frac{20}{7}ab^9x^{7/2}+$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^10/x^2,x]
```

```
[Out] -(a^10/x) - (20*a^9*b)/Sqrt[x] + 240*a^7*b^3*Sqrt[x] + 210*a^6*b^4*x + 168*
a^5*b^5*x^(3/2) + 105*a^4*b^6*x^2 + 48*a^3*b^7*x^(5/2) + 15*a^2*b^8*x^3 + (
20*a*b^9*x^(7/2))/7 + (b^10*x^4)/4 + 45*a^8*b^2*Log[x]
```

fricas [A] time = 0.63, size = 117, normalized size = 0.95

$$\frac{7\,b^{10}x^5 + 420\,a^2b^8x^4 + 2940\,a^4b^6x^3 + 5880\,a^6b^4x^2 + 2520\,a^8b^2x\log\left(\sqrt{x}\right) - 28\,a^{10} + 16\left(5\,ab^9x^4 + 84\,a^3b^7x^3 +}{28\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^10/x^2,x, algorithm="fricas")
```

```
[Out] 1/28*(7*b^10*x^5 + 420*a^2*b^8*x^4 + 2940*a^4*b^6*x^3 + 5880*a^6*b^4*x^2 +
2520*a^8*b^2*x*log(sqrt(x)) - 28*a^10 + 16*(5*a*b^9*x^4 + 84*a^3*b^7*x^3 +
294*a^5*b^5*x^2 + 420*a^7*b^3*x - 35*a^9*b)*sqrt(x))/x
```

giac [A] time = 0.17, size = 111, normalized size = 0.90

$$\frac{1}{4}\,b^{10}x^4+\frac{20}{7}\,ab^9x^{\frac{7}{2}}+15\,a^2b^8x^3+48\,a^3b^7x^{\frac{5}{2}}+105\,a^4b^6x^2+168\,a^5b^5x^{\frac{3}{2}}+210\,a^6b^4x+45\,a^8b^2\log\left(|x|\right)+240\,a^7b^3\sqrt{x}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^10/x^2,x, algorithm="giac")
```

```
[Out] 1/4*b^10*x^4 + 20/7*a*b^9*x^(7/2) + 15*a^2*b^8*x^3 + 48*a^3*b^7*x^(5/2) + 1
05*a^4*b^6*x^2 + 168*a^5*b^5*x^(3/2) + 210*a^6*b^4*x + 45*a^8*b^2*log(abs(x
)) + 240*a^7*b^3*sqrt(x) - (20*a^9*b*sqrt(x) + a^10)/x
```

maple [A] time = 0.00, size = 110, normalized size = 0.89

$$\frac{b^{10}x^4}{4}+\frac{20a\,b^9x^{\frac{7}{2}}}{7}+15a^2b^8x^3+48a^3b^7x^{\frac{5}{2}}+105a^4b^6x^2+168a^5b^5x^{\frac{3}{2}}+45a^8b^2\ln(x)+210a^6b^4x+240a^7b^3\sqrt{x}-\frac{20a^9b}{\sqrt{x}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^10/x^2,x)
```

```
[Out] -a^10/x+210*a^6*b^4*x+168*a^5*b^5*x^(3/2)+105*a^4*b^6*x^2+48*a^3*b^7*x^(5/2
)+15*a^2*b^8*x^3+20/7*a*b^9*x^(7/2)+1/4*b^10*x^4+45*a^8*b^2*ln(x)-20*a^9*b/
x^(1/2)+240*a^7*b^3*x^(1/2)
```

maxima [A] time = 0.81, size = 110, normalized size = 0.89

$$\frac{1}{4}\,b^{10}x^4+\frac{20}{7}\,ab^9x^{\frac{7}{2}}+15\,a^2b^8x^3+48\,a^3b^7x^{\frac{5}{2}}+105\,a^4b^6x^2+168\,a^5b^5x^{\frac{3}{2}}+210\,a^6b^4x+45\,a^8b^2\log(x)+240\,a^7b^3\sqrt{x}-\frac{2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^10/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^10*x^4 + 20/7*a*b^9*x^(7/2) + 15*a^2*b^8*x^3 + 48*a^3*b^7*x^(5/2) + 1
05*a^4*b^6*x^2 + 168*a^5*b^5*x^(3/2) + 210*a^6*b^4*x + 45*a^8*b^2*log(x) +
240*a^7*b^3*sqrt(x) - (20*a^9*b*sqrt(x) + a^10)/x
```

mupad [B] time = 0.06, size = 112, normalized size = 0.91

$$\frac{b^{10}\,x^4}{4}-\frac{a^{10}+20\,a^9\,b\,\sqrt{x}}{x}+90\,a^8\,b^2\,\ln\left(\sqrt{x}\right)+210\,a^6\,b^4\,x+\frac{20\,a\,b^9\,x^{7/2}}{7}+105\,a^4\,b^6\,x^2+15\,a^2\,b^8\,x^3+240\,a^7\,b^3\,\sqrt{x}+1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^10/x^2,x)
```

```
[Out] (b^10*x^4)/4 - (a^10 + 20*a^9*b*x^(1/2))/x + 90*a^8*b^2*log(x^(1/2)) + 210*
a^6*b^4*x + (20*a*b^9*x^(7/2))/7 + 105*a^4*b^6*x^2 + 15*a^2*b^8*x^3 + 240*a
^7*b^3*x^(1/2) + 168*a^5*b^5*x^(3/2) + 48*a^3*b^7*x^(5/2)
```

sympy [**A**] time = 2.18, size = 124, normalized size = 1.01

$$-\frac{a^{10}}{x}-\frac{20a^9b}{\sqrt{x}}+45a^8b^2\log(x)+240a^7b^3\sqrt{x}+210a^6b^4x+168a^5b^5x^{\frac{3}{2}}+105a^4b^6x^2+48a^3b^7x^{\frac{5}{2}}+15a^2b^8x^3+\frac{20ab^9x^{\frac{7}{2}}}{7}+\frac{b^1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**10/x**2,x)
```

```
[Out] -a**10/x - 20*a**9*b/sqrt(x) + 45*a**8*b**2*log(x) + 240*a**7*b**3*sqrt(x)
+ 210*a**6*b**4*x + 168*a**5*b**5*x**(3/2) + 105*a**4*b**6*x**2 + 48*a**3*b
**7*x**(5/2) + 15*a**2*b**8*x**3 + 20*a*b**9*x**(7/2)/7 + b**10*x**4/4
```

3.2160 $\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^3}\,dx$

Optimal. Leaf size=127

$$-\frac{a^{10}}{2x^2}-\frac{20a^9b}{3x^{3/2}}-\frac{45a^8b^2}{x}-\frac{240a^7b^3}{\sqrt{x}}+210a^6b^4\log(x)+504a^5b^5\sqrt{x}+210a^4b^6x+80a^3b^7x^{3/2}+\frac{45}{2}a^2b^8x^2+4ab^9x^{5/2}+\frac{b^{10}}{3}$$

```
[Out] -1/2*a^10/x^2-20/3*a^9*b/x^(3/2)-45*a^8*b^2/x+210*a^4*b^6*x+80*a^3*b^7*x^(3
/2)+45/2*a^2*b^8*x^2+4*a*b^9*x^(5/2)+1/3*b^10*x^3+210*a^6*b^4*ln(x)-240*a^7
*b^3/x^(1/2)+504*a^5*b^5*x^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$80a^3b^7x^{3/2}+\frac{45}{2}a^2b^8x^2-\frac{45a^8b^2}{x}-\frac{240a^7b^3}{\sqrt{x}}+504a^5b^5\sqrt{x}+210a^4b^6x+210a^6b^4\log(x)-\frac{20a^9b}{3x^{3/2}}-\frac{a^{10}}{2x^2}+4ab^9x^{5/2}+\frac{b^{10}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^10/x^3,x]
```

```
[Out] -a^10/(2*x^2) - (20*a^9*b)/(3*x^(3/2)) - (45*a^8*b^2)/x - (240*a^7*b^3)/Sqr
t[x] + 504*a^5*b^5*Sqrt[x] + 210*a^4*b^6*x + 80*a^3*b^7*x^(3/2) + (45*a^2*b
^8*x^2)/2 + 4*a*b^9*x^(5/2) + (b^10*x^3)/3 + 210*a^6*b^4*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^3}\,dx &= 2\,\text{Subst}\left(\int \frac{(a+bx)^{10}}{x^5}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(252a^5b^5+\frac{a^{10}}{x^5}+\frac{10a^9b}{x^4}+\frac{45a^8b^2}{x^3}+\frac{120a^7b^3}{x^2}+\frac{210a^6b^4}{x}+210a^4b^6x+120\right.\right.\\ &= -\frac{a^{10}}{2x^2}-\frac{20a^9b}{3x^{3/2}}-\frac{45a^8b^2}{x}-\frac{240a^7b^3}{\sqrt{x}}+504a^5b^5\sqrt{x}+210a^4b^6x+80a^3b^7x^{3/2}+\frac{45}{2}a^2b^8x^2\end{aligned}$$

Mathematica [A] time = 0.06, size = 127, normalized size = 1.00

$$-\frac{a^{10}}{2x^2}-\frac{20a^9b}{3x^{3/2}}-\frac{45a^8b^2}{x}-\frac{240a^7b^3}{\sqrt{x}}+210a^6b^4\log(x)+504a^5b^5\sqrt{x}+210a^4b^6x+80a^3b^7x^{3/2}+\frac{45}{2}a^2b^8x^2+4ab^9x^{5/2}+\frac{b^{10}}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^10/x^3,x]
```

```
[Out] -1/2*a^10/x^2 - (20*a^9*b)/(3*x^(3/2)) - (45*a^8*b^2)/x - (240*a^7*b^3)/Sqr
t[x] + 504*a^5*b^5*Sqrt[x] + 210*a^4*b^6*x + 80*a^3*b^7*x^(3/2) + (45*a^2*b
^8*x^2)/2 + 4*a*b^9*x^(5/2) + (b^10*x^3)/3 + 210*a^6*b^4*Log[x]
```

fricas [A] time = 0.96, size = 117, normalized size = 0.92

$$\frac{2\,b^{10}x^5+135\,a^2b^8x^4+1260\,a^4b^6x^3+2520\,a^6b^4x^2\log\left(\sqrt{x}\right)-270\,a^8b^2x-3\,a^{10}+8\left(3\,ab^9x^4+60\,a^3b^7x^3+378\,a}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^10/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^10*x^5 + 135*a^2*b^8*x^4 + 1260*a^4*b^6*x^3 + 2520*a^6*b^4*x^2*log
(sqrt(x)) - 270*a^8*b^2*x - 3*a^10 + 8*(3*a*b^9*x^4 + 60*a^3*b^7*x^3 + 378*
a^5*b^5*x^2 - 180*a^7*b^3*x - 5*a^9*b)*sqrt(x))/x^2
```

giac [A] time = 0.17, size = 111, normalized size = 0.87

$$\frac{1}{3}\,b^{10}x^3+4\,ab^9x^{\frac{5}{2}}+\frac{45}{2}\,a^2b^8x^2+80\,a^3b^7x^{\frac{3}{2}}+210\,a^4b^6x+210\,a^6b^4\log\left(|x|\right)+504\,a^5b^5\sqrt{x}-\frac{1440\,a^7b^3x^{\frac{3}{2}}+270\,a^8b^2x+}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^10/x^3,x, algorithm="giac")
```

```
[Out] 1/3*b^10*x^3 + 4*a*b^9*x^(5/2) + 45/2*a^2*b^8*x^2 + 80*a^3*b^7*x^(3/2) + 21
0*a^4*b^6*x + 210*a^6*b^4*log(abs(x)) + 504*a^5*b^5*sqrt(x) - 1/6*(1440*a^7
*b^3*x^(3/2) + 270*a^8*b^2*x + 40*a^9*b*sqrt(x) + 3*a^10)/x^2
```

maple [A] time = 0.00, size = 110, normalized size = 0.87

$$\frac{b^{10}x^3}{3}+4a\,b^9x^{\frac{5}{2}}+\frac{45a^2b^8x^2}{2}+80a^3b^7x^{\frac{3}{2}}+210a^6b^4\ln(x)+210a^4b^6x+504a^5b^5\sqrt{x}-\frac{240a^7b^3}{\sqrt{x}}-\frac{45a^8b^2}{x}-\frac{20a^9b}{3x^{\frac{3}{2}}}-\frac{a^{10}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^10/x^3,x)
```

```
[Out] -1/2*a^10/x^2-20/3*a^9*b/x^(3/2)-45*a^8*b^2/x+210*a^4*b^6*x+80*a^3*b^7*x^(3
/2)+45/2*a^2*b^8*x^2+4*a*b^9*x^(5/2)+1/3*b^10*x^3+210*a^6*b^4*ln(x)-240*a^7
*b^3/x^(1/2)+504*a^5*b^5*x^(1/2)
```

maxima [A] time = 0.88, size = 110, normalized size = 0.87

$$\frac{1}{3}\,b^{10}x^3+4\,ab^9x^{\frac{5}{2}}+\frac{45}{2}\,a^2b^8x^2+80\,a^3b^7x^{\frac{3}{2}}+210\,a^4b^6x+210\,a^6b^4\log(x)+504\,a^5b^5\sqrt{x}-\frac{1440\,a^7b^3x^{\frac{3}{2}}+270\,a^8b^2x+40}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^10/x^3,x, algorithm="maxima")
```

```
[Out] 1/3*b^10*x^3 + 4*a*b^9*x^(5/2) + 45/2*a^2*b^8*x^2 + 80*a^3*b^7*x^(3/2) + 21
0*a^4*b^6*x + 210*a^6*b^4*log(x) + 504*a^5*b^5*sqrt(x) - 1/6*(1440*a^7*b^3*
x^(3/2) + 270*a^8*b^2*x + 40*a^9*b*sqrt(x) + 3*a^10)/x^2
```

mupad [B] time = 0.05, size = 112, normalized size = 0.88

$$\frac{b^{10}\,x^3}{3}-\frac{\frac{a^{10}}{2}+45\,a^8\,b^2\,x+\frac{20\,a^9\,b\,\sqrt{x}}{3}+240\,a^7\,b^3\,x^{3/2}}{x^2}+420\,a^6\,b^4\,\ln\left(\sqrt{x}\right)+210\,a^4\,b^6\,x+4\,a\,b^9\,x^{5/2}+\frac{45\,a^2\,b^8\,x^2}{2}+504\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^10/x^3,x)
```

```
[Out] (b^10*x^3)/3 - (a^10/2 + 45*a^8*b^2*x + (20*a^9*b*x^(1/2))/3 + 240*a^7*b^3*
x^(3/2))/x^2 + 420*a^6*b^4*log(x^(1/2)) + 210*a^4*b^6*x + 4*a*b^9*x^(5/2) +
 (45*a^2*b^8*x^2)/2 + 504*a^5*b^5*x^(1/2) + 80*a^3*b^7*x^(3/2)
```

sympy [**A**] time = 1.95, size = 128, normalized size = 1.01

$$-\frac{a^{10}}{2x^2}-\frac{20a^9b}{3x^{\frac{3}{2}}}-\frac{45a^8b^2}{x}-\frac{240a^7b^3}{\sqrt{x}}+210a^6b^4\log(x)+504a^5b^5\sqrt{x}+210a^4b^6x+80a^3b^7x^{\frac{3}{2}}+\frac{45a^2b^8x^2}{2}+4ab^9x^{\frac{5}{2}}+\frac{b^{10}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**10/x**3,x)
```

```
[Out] -a**10/(2*x**2) - 20*a**9*b/(3*x**(3/2)) - 45*a**8*b**2/x - 240*a**7*b**3/s
qrt(x) + 210*a**6*b**4*log(x) + 504*a**5*b**5*sqrt(x) + 210*a**4*b**6*x + 8
0*a**3*b**7*x**(3/2) + 45*a**2*b**8*x**2/2 + 4*a*b**9*x**(5/2) + b**10*x**3
/3
```

3.2161 $\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^4}\,dx$

Optimal. Leaf size=127

$$-\frac{a^{10}}{3x^3}-\frac{4a^9b}{x^{5/2}}-\frac{45a^8b^2}{2x^2}-\frac{80a^7b^3}{x^{3/2}}-\frac{210a^6b^4}{x}-\frac{504a^5b^5}{\sqrt{x}}+210a^4b^6\log(x)+240a^3b^7\sqrt{x}+45a^2b^8x+\frac{20}{3}ab^9x^{3/2}+\frac{b^{10}x^2}{2}$$

```
[Out] -1/3*a^10/x^3-4*a^9*b/x^(5/2)-45/2*a^8*b^2/x^2-80*a^7*b^3/x^(3/2)-210*a^6*b
^4/x+45*a^2*b^8*x+20/3*a*b^9*x^(3/2)+1/2*b^10*x^2+210*a^4*b^6*ln(x)-504*a^5
*b^5/x^(1/2)+240*a^3*b^7*x^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{45a^8b^2}{2x^2}-\frac{80a^7b^3}{x^{3/2}}-\frac{210a^6b^4}{x}-\frac{504a^5b^5}{\sqrt{x}}+240a^3b^7\sqrt{x}+45a^2b^8x+210a^4b^6\log(x)-\frac{4a^9b}{x^{5/2}}-\frac{a^{10}}{3x^3}+\frac{20}{3}ab^9x^{3/2}+\frac{b^{10}x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^10/x^4,x]
```

```
[Out] -a^10/(3*x^3) - (4*a^9*b)/x^(5/2) - (45*a^8*b^2)/(2*x^2) - (80*a^7*b^3)/x^(
3/2) - (210*a^6*b^4)/x - (504*a^5*b^5)/Sqrt[x] + 240*a^3*b^7*Sqrt[x] + 45*a
^2*b^8*x + (20*a*b^9*x^(3/2))/3 + (b^10*x^2)/2 + 210*a^4*b^6*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^4}\,dx &= 2\,\text{Subst}\left(\int \frac{(a+bx)^{10}}{x^7}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(120a^3b^7+\frac{a^{10}}{x^7}+\frac{10a^9b}{x^6}+\frac{45a^8b^2}{x^5}+\frac{120a^7b^3}{x^4}+\frac{210a^6b^4}{x^3}+\frac{252a^5b^5}{x^2}+\frac{210a^4b^6}{x}\right.\right.\\ &= -\frac{a^{10}}{3x^3}-\frac{4a^9b}{x^{5/2}}-\frac{45a^8b^2}{2x^2}-\frac{80a^7b^3}{x^{3/2}}-\frac{210a^6b^4}{x}-\frac{504a^5b^5}{\sqrt{x}}+240a^3b^7\sqrt{x}+45a^2b^8x+\frac{20}{3}ab^9x\end{aligned}$$

Mathematica [A] time = 0.07, size = 124, normalized size = 0.98

$$210a^4b^6\log(x)-\frac{2a^{10}+24a^9b\sqrt{x}+135a^8b^2x+480a^7b^3x^{3/2}+1260a^6b^4x^2+3024a^5b^5x^{5/2}-1440a^3b^7x^{7/2}-270a}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^10/x^4,x]
```

```
[Out] -1/6*(2*a^10 + 24*a^9*b*Sqrt[x] + 135*a^8*b^2*x + 480*a^7*b^3*x^(3/2) + 126
0*a^6*b^4*x^2 + 3024*a^5*b^5*x^(5/2) - 1440*a^3*b^7*x^(7/2) - 270*a^2*b^8*x
^4 - 40*a*b^9*x^(9/2) - 3*b^10*x^5)/x^3 + 210*a^4*b^6*Log[x]
```

fricas [A] time = 1.07, size = 117, normalized size = 0.92

$$\frac{3\,b^{10}x^5 + 270\,a^2b^8x^4 + 2520\,a^4b^6x^3\log\left(\sqrt{x}\right) - 1260\,a^6b^4x^2 - 135\,a^8b^2x - 2\,a^{10} + 8\left(5\,ab^9x^4 + 180\,a^3b^7x^3 - 3\right.}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^10/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^10*x^5 + 270*a^2*b^8*x^4 + 2520*a^4*b^6*x^3*log(sqrt(x)) - 1260*a^
6*b^4*x^2 - 135*a^8*b^2*x - 2*a^10 + 8*(5*a*b^9*x^4 + 180*a^3*b^7*x^3 - 378
*a^5*b^5*x^2 - 60*a^7*b^3*x - 3*a^9*b)*sqrt(x))/x^3
```

giac [A] time = 0.25, size = 111, normalized size = 0.87

$$\frac{1}{2}\,b^{10}x^2+\frac{20}{3}\,ab^9x^{\frac{3}{2}}+45\,a^2b^8x+210\,a^4b^6\log\left(|x|\right)+240\,a^3b^7\sqrt{x}-\frac{3024\,a^5b^5x^{\frac{5}{2}}+1260\,a^6b^4x^2+480\,a^7b^3x^{\frac{3}{2}}+135}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^10/x^4,x, algorithm="giac")
```

```
[Out] 1/2*b^10*x^2 + 20/3*a*b^9*x^(3/2) + 45*a^2*b^8*x + 210*a^4*b^6*log(abs(x))
+ 240*a^3*b^7*sqrt(x) - 1/6*(3024*a^5*b^5*x^(5/2) + 1260*a^6*b^4*x^2 + 480*
a^7*b^3*x^(3/2) + 135*a^8*b^2*x + 24*a^9*b*sqrt(x) + 2*a^10)/x^3
```

maple [A] time = 0.00, size = 110, normalized size = 0.87

$$\frac{b^{10}x^2}{2}+\frac{20a\,b^9x^{\frac{3}{2}}}{3}+210a^4b^6\ln(x)+45a^2b^8x+240a^3b^7\sqrt{x}-\frac{504a^5b^5}{\sqrt{x}}-\frac{210a^6b^4}{x}-\frac{80a^7b^3}{x^{\frac{3}{2}}}-\frac{45a^8b^2}{2x^2}-\frac{4a^9b}{x^{\frac{5}{2}}}-\frac{a^{10}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^10/x^4,x)
```

```
[Out] -1/3*a^10/x^3-4*a^9*b/x^(5/2)-45/2*a^8*b^2/x^2-80*a^7*b^3/x^(3/2)-210*a^6*b
^4/x+45*a^2*b^8*x+20/3*a*b^9*x^(3/2)+1/2*b^10*x^2+210*a^4*b^6*ln(x)-504*a^5
*b^5/x^(1/2)+240*a^3*b^7*x^(1/2)
```

maxima [A] time = 0.87, size = 110, normalized size = 0.87

$$\frac{1}{2}\,b^{10}x^2+\frac{20}{3}\,ab^9x^{\frac{3}{2}}+45\,a^2b^8x+210\,a^4b^6\log(x)+240\,a^3b^7\sqrt{x}-\frac{3024\,a^5b^5x^{\frac{5}{2}}+1260\,a^6b^4x^2+480\,a^7b^3x^{\frac{3}{2}}+135\,a}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^10/x^4,x, algorithm="maxima")
```

```
[Out] 1/2*b^10*x^2 + 20/3*a*b^9*x^(3/2) + 45*a^2*b^8*x + 210*a^4*b^6*log(x) + 240
*a^3*b^7*sqrt(x) - 1/6*(3024*a^5*b^5*x^(5/2) + 1260*a^6*b^4*x^2 + 480*a^7*b
^3*x^(3/2) + 135*a^8*b^2*x + 24*a^9*b*sqrt(x) + 2*a^10)/x^3
```

mupad [B] time = 1.10, size = 112, normalized size = 0.88

$$\frac{b^{10}\,x^2}{2}-\frac{\frac{a^{10}}{3}+\frac{45\,a^8\,b^2\,x}{2}+4\,a^9\,b\,\sqrt{x}\,+210\,a^6\,b^4\,x^2+80\,a^7\,b^3\,x^{3/2}+504\,a^5\,b^5\,x^{5/2}}{x^3}+420\,a^4\,b^6\,\ln\left(\sqrt{x}\right)+45\,a^2\,b^8\,x+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^10/x^4,x)
```

```
[Out] (b^10*x^2)/2 - (a^10/3 + (45*a^8*b^2*x)/2 + 4*a^9*b*x^(1/2) + 210*a^6*b^4*x
^2 + 80*a^7*b^3*x^(3/2) + 504*a^5*b^5*x^(5/2))/x^3 + 420*a^4*b^6*log(x^(1/2
)) + 45*a^2*b^8*x + (20*a*b^9*x^(3/2))/3 + 240*a^3*b^7*x^(1/2)
```

sympy [A] time = 2.00, size = 128, normalized size = 1.01

$$-\frac{a^{10}}{3x^3}-\frac{4a^9b}{x^{\frac{5}{2}}}-\frac{45a^8b^2}{2x^2}-\frac{80a^7b^3}{x^{\frac{3}{2}}}-\frac{210a^6b^4}{x}-\frac{504a^5b^5}{\sqrt{x}}+210a^4b^6\log(x)+240a^3b^7\sqrt{x}+45a^2b^8x+\frac{20ab^9x^{\frac{3}{2}}}{3}+\frac{b^{10}x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**10/x**4,x)
```

```
[Out] -a**10/(3*x**3) - 4*a**9*b/x**(5/2) - 45*a**8*b**2/(2*x**2) - 80*a**7*b**3/
x**(3/2) - 210*a**6*b**4/x - 504*a**5*b**5/sqrt(x) + 210*a**4*b**6*log(x) +
 240*a**3*b**7*sqrt(x) + 45*a**2*b**8*x + 20*a*b**9*x**(3/2)/3 + b**10*x**2
/2
```

3.2162 $\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^5}\,dx$

Optimal. Leaf size=122

$$-\frac{a^{10}}{4x^4}-\frac{20a^9b}{7x^{7/2}}-\frac{15a^8b^2}{x^3}-\frac{48a^7b^3}{x^{5/2}}-\frac{105a^6b^4}{x^2}-\frac{168a^5b^5}{x^{3/2}}-\frac{210a^4b^6}{x}-\frac{240a^3b^7}{\sqrt{x}}+45a^2b^8\log(x)+20ab^9\sqrt{x}+b^{10}x$$

```
[Out] -1/4*a^10/x^4-20/7*a^9*b/x^(7/2)-15*a^8*b^2/x^3-48*a^7*b^3/x^(5/2)-105*a^6*
b^4/x^2-168*a^5*b^5/x^(3/2)-210*a^4*b^6/x+b^10*x+45*a^2*b^8*ln(x)-240*a^3*b
^7/x^(1/2)+20*a*b^9*x^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{15a^8b^2}{x^3}-\frac{48a^7b^3}{x^{5/2}}-\frac{105a^6b^4}{x^2}-\frac{168a^5b^5}{x^{3/2}}-\frac{210a^4b^6}{x}-\frac{240a^3b^7}{\sqrt{x}}+45a^2b^8\log(x)-\frac{20a^9b}{7x^{7/2}}-\frac{a^{10}}{4x^4}+20ab^9\sqrt{x}+b^{10}x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^10/x^5,x]
```

```
[Out] -a^10/(4*x^4) - (20*a^9*b)/(7*x^(7/2)) - (15*a^8*b^2)/x^3 - (48*a^7*b^3)/x^
(5/2) - (105*a^6*b^4)/x^2 - (168*a^5*b^5)/x^(3/2) - (210*a^4*b^6)/x - (240*
a^3*b^7)/Sqrt[x] + 20*a*b^9*Sqrt[x] + b^10*x + 45*a^2*b^8*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^5}\,dx &= 2\,\text{Subst}\left(\int \frac{(a+bx)^{10}}{x^9}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(10ab^9+\frac{a^{10}}{x^9}+\frac{10a^9b}{x^8}+\frac{45a^8b^2}{x^7}+\frac{120a^7b^3}{x^6}+\frac{210a^6b^4}{x^5}+\frac{252a^5b^5}{x^4}+\frac{210a^4b}{x^3}\right.\right.\\ &= -\frac{a^{10}}{4x^4}-\frac{20a^9b}{7x^{7/2}}-\frac{15a^8b^2}{x^3}-\frac{48a^7b^3}{x^{5/2}}-\frac{105a^6b^4}{x^2}-\frac{168a^5b^5}{x^{3/2}}-\frac{210a^4b^6}{x}-\frac{240a^3b^7}{\sqrt{x}}+20ab^9\end{aligned}$$

Mathematica [A] time = 0.09, size = 122, normalized size = 1.00

$$-\frac{a^{10}}{4x^4}-\frac{20a^9b}{7x^{7/2}}-\frac{15a^8b^2}{x^3}-\frac{48a^7b^3}{x^{5/2}}-\frac{105a^6b^4}{x^2}-\frac{168a^5b^5}{x^{3/2}}-\frac{210a^4b^6}{x}-\frac{240a^3b^7}{\sqrt{x}}+45a^2b^8\log(x)+20ab^9\sqrt{x}+b^{10}x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^10/x^5,x]
```

```
[Out] -1/4*a^10/x^4 - (20*a^9*b)/(7*x^(7/2)) - (15*a^8*b^2)/x^3 - (48*a^7*b^3)/x^
(5/2) - (105*a^6*b^4)/x^2 - (168*a^5*b^5)/x^(3/2) - (210*a^4*b^6)/x - (240*
a^3*b^7)/Sqrt[x] + 20*a*b^9*Sqrt[x] + b^10*x + 45*a^2*b^8*Log[x]
```

fricas [A] time = 1.04, size = 117, normalized size = 0.96

$$\frac{28\,b^{10}x^5 + 2520\,a^2b^8x^4\log\left(\sqrt{x}\right) - 5880\,a^4b^6x^3 - 2940\,a^6b^4x^2 - 420\,a^8b^2x - 7\,a^{10} + 16\left(35\,ab^9x^4 - 420\,a^3b^7x^3 -}{28\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^10/x^5,x, algorithm="fricas")
```

```
[Out] 1/28*(28*b^10*x^5 + 2520*a^2*b^8*x^4*log(sqrt(x)) - 5880*a^4*b^6*x^3 - 2940
*a^6*b^4*x^2 - 420*a^8*b^2*x - 7*a^10 + 16*(35*a*b^9*x^4 - 420*a^3*b^7*x^3
- 294*a^5*b^5*x^2 - 84*a^7*b^3*x - 5*a^9*b)*sqrt(x))/x^4
```

giac [A] time = 0.16, size = 110, normalized size = 0.90

$$b^{10}x+45\,a^2b^8\log\left(|x|\right)+20\,ab^9\sqrt{x}-\frac{6720\,a^3b^7x^{\frac{7}{2}} + 5880\,a^4b^6x^3 + 4704\,a^5b^5x^{\frac{5}{2}} + 2940\,a^6b^4x^2 + 1344\,a^7b^3x^{\frac{3}{2}} + 420}{28\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^10/x^5,x, algorithm="giac")
```

```
[Out] b^10*x + 45*a^2*b^8*log(abs(x)) + 20*a*b^9*sqrt(x) - 1/28*(6720*a^3*b^7*x^(
7/2) + 5880*a^4*b^6*x^3 + 4704*a^5*b^5*x^(5/2) + 2940*a^6*b^4*x^2 + 1344*a^
7*b^3*x^(3/2) + 420*a^8*b^2*x + 80*a^9*b*sqrt(x) + 7*a^10)/x^4
```

maple [A] time = 0.00, size = 109, normalized size = 0.89

$$45a^2b^8\ln(x)+b^{10}x+20a\,b^9\sqrt{x}-\frac{240a^3b^7}{\sqrt{x}}-\frac{210a^4b^6}{x}-\frac{168a^5b^5}{x^{\frac{3}{2}}}-\frac{105a^6b^4}{x^2}-\frac{48a^7b^3}{x^{\frac{5}{2}}}-\frac{15a^8b^2}{x^3}-\frac{20a^9b}{7x^{\frac{7}{2}}}-\frac{a^{10}}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^10/x^5,x)
```

```
[Out] -1/4*a^10/x^4-20/7*a^9*b/x^(7/2)-15*a^8*b^2/x^3-48*a^7*b^3/x^(5/2)-105*a^6*
b^4/x^2-168*a^5*b^5/x^(3/2)-210*a^4*b^6/x+b^10*x+45*a^2*b^8*ln(x)-240*a^3*b
^7/x^(1/2)+20*a*b^9*x^(1/2)
```

maxima [A] time = 0.89, size = 109, normalized size = 0.89

$$b^{10}x+45\,a^2b^8\log(x)+20\,ab^9\sqrt{x}-\frac{6720\,a^3b^7x^{\frac{7}{2}} + 5880\,a^4b^6x^3 + 4704\,a^5b^5x^{\frac{5}{2}} + 2940\,a^6b^4x^2 + 1344\,a^7b^3x^{\frac{3}{2}} + 420\,a}{28\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^10/x^5,x, algorithm="maxima")
```

```
[Out] b^10*x + 45*a^2*b^8*log(x) + 20*a*b^9*sqrt(x) - 1/28*(6720*a^3*b^7*x^(7/2)
+ 5880*a^4*b^6*x^3 + 4704*a^5*b^5*x^(5/2) + 2940*a^6*b^4*x^2 + 1344*a^7*b^3
*x^(3/2) + 420*a^8*b^2*x + 80*a^9*b*sqrt(x) + 7*a^10)/x^4
```

mupad [B] time = 0.05, size = 111, normalized size = 0.91

$$b^{10}\,x-\frac{\frac{a^{10}}{4}+15\,a^8\,b^2\,x+\frac{20\,a^9\,b\,\sqrt{x}}{7}+105\,a^6\,b^4\,x^2+210\,a^4\,b^6\,x^3+48\,a^7\,b^3\,x^{3/2}+168\,a^5\,b^5\,x^{5/2}+240\,a^3\,b^7\,x^{7/2}}{x^4}+90\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^10/x^5,x)
```

```
[Out] b^10*x - (a^10/4 + 15*a^8*b^2*x + (20*a^9*b*x^(1/2))/7 + 105*a^6*b^4*x^2 +
210*a^4*b^6*x^3 + 48*a^7*b^3*x^(3/2) + 168*a^5*b^5*x^(5/2) + 240*a^3*b^7*x^
(7/2))/x^4 + 90*a^2*b^8*log(x^(1/2)) + 20*a*b^9*x^(1/2)
```

sympy [A] time = 2.01, size = 124, normalized size = 1.02

$$-\frac{a^{10}}{4x^4}-\frac{20a^9b}{7x^{\frac{7}{2}}}-\frac{15a^8b^2}{x^3}-\frac{48a^7b^3}{x^{\frac{5}{2}}}-\frac{105a^6b^4}{x^2}-\frac{168a^5b^5}{x^{\frac{3}{2}}}-\frac{210a^4b^6}{x}-\frac{240a^3b^7}{\sqrt{x}}+45a^2b^8\log(x)+20ab^9\sqrt{x}+b^{10}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**10/x**5,x)
```

```
[Out] -a**10/(4*x**4) - 20*a**9*b/(7*x**(7/2)) - 15*a**8*b**2/x**3 - 48*a**7*b**3
/x**(5/2) - 105*a**6*b**4/x**2 - 168*a**5*b**5/x**(3/2) - 210*a**4*b**6/x -
 240*a**3*b**7/sqrt(x) + 45*a**2*b**8*log(x) + 20*a*b**9*sqrt(x) + b**10*x
```

3.2163 $\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^6}\,dx$

Optimal. Leaf size=130

$$-\frac{a^{10}}{5x^5}-\frac{20a^9b}{9x^{9/2}}-\frac{45a^8b^2}{4x^4}-\frac{240a^7b^3}{7x^{7/2}}-\frac{70a^6b^4}{x^3}-\frac{504a^5b^5}{5x^{5/2}}-\frac{105a^4b^6}{x^2}-\frac{80a^3b^7}{x^{3/2}}-\frac{45a^2b^8}{x}-\frac{20ab^9}{\sqrt{x}}+b^{10}\log(x)$$

```
[Out] -1/5*a^10/x^5-20/9*a^9*b/x^(9/2)-45/4*a^8*b^2/x^4-240/7*a^7*b^3/x^(7/2)-70*
a^6*b^4/x^3-504/5*a^5*b^5/x^(5/2)-105*a^4*b^6/x^2-80*a^3*b^7/x^(3/2)-45*a^2
*b^8/x+b^10*ln(x)-20*a*b^9/x^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{45a^8b^2}{4x^4}-\frac{240a^7b^3}{7x^{7/2}}-\frac{70a^6b^4}{x^3}-\frac{504a^5b^5}{5x^{5/2}}-\frac{105a^4b^6}{x^2}-\frac{80a^3b^7}{x^{3/2}}-\frac{45a^2b^8}{x}-\frac{20a^9b}{9x^{9/2}}-\frac{a^{10}}{5x^5}-\frac{20ab^9}{\sqrt{x}}+b^{10}\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^10/x^6,x]
```

```
[Out] -a^10/(5*x^5) - (20*a^9*b)/(9*x^(9/2)) - (45*a^8*b^2)/(4*x^4) - (240*a^7*b^
3)/(7*x^(7/2)) - (70*a^6*b^4)/x^3 - (504*a^5*b^5)/(5*x^(5/2)) - (105*a^4*b^
6)/x^2 - (80*a^3*b^7)/x^(3/2) - (45*a^2*b^8)/x - (20*a*b^9)/Sqrt[x] + b^10*
Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^6}\,dx &= 2\,\text{Subst}\left(\int \frac{(a+bx)^{10}}{x^{11}}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(\frac{a^{10}}{x^{11}}+\frac{10a^9b}{x^{10}}+\frac{45a^8b^2}{x^9}+\frac{120a^7b^3}{x^8}+\frac{210a^6b^4}{x^7}+\frac{252a^5b^5}{x^6}+\frac{210a^4b^6}{x^5}+\frac{120a^3b\right.\right.\\ &= -\frac{a^{10}}{5x^5}-\frac{20a^9b}{9x^{9/2}}-\frac{45a^8b^2}{4x^4}-\frac{240a^7b^3}{7x^{7/2}}-\frac{70a^6b^4}{x^3}-\frac{504a^5b^5}{5x^{5/2}}-\frac{105a^4b^6}{x^2}-\frac{80a^3b^7}{x^{3/2}}-\frac{45a^2b^8}{x}-\end{aligned}$$

Mathematica [A] time = 0.08, size = 130, normalized size = 1.00

$$-\frac{a^{10}}{5x^5}-\frac{20a^9b}{9x^{9/2}}-\frac{45a^8b^2}{4x^4}-\frac{240a^7b^3}{7x^{7/2}}-\frac{70a^6b^4}{x^3}-\frac{504a^5b^5}{5x^{5/2}}-\frac{105a^4b^6}{x^2}-\frac{80a^3b^7}{x^{3/2}}-\frac{45a^2b^8}{x}-\frac{20ab^9}{\sqrt{x}}+b^{10}\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^10/x^6,x]
```

```
[Out] -1/5*a^10/x^5 - (20*a^9*b)/(9*x^(9/2)) - (45*a^8*b^2)/(4*x^4) - (240*a^7*b^
3)/(7*x^(7/2)) - (70*a^6*b^4)/x^3 - (504*a^5*b^5)/(5*x^(5/2)) - (105*a^4*b^
6)/x^2 - (80*a^3*b^7)/x^(3/2) - (45*a^2*b^8)/x - (20*a*b^9)/Sqrt[x] + b^10*
Log[x]
```

fricas [A] time = 0.92, size = 117, normalized size = 0.90

$$\frac{2520\,b^{10}x^5\log\left(\sqrt{x}\right) - 56700\,a^2b^8x^4 - 132300\,a^4b^6x^3 - 88200\,a^6b^4x^2 - 14175\,a^8b^2x - 252\,a^{10} - 16\left(1575\,ab^9x\right.}{1260\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^10/x^6,x, algorithm="fricas")
```

```
[Out] 1/1260*(2520*b^10*x^5*log(sqrt(x)) - 56700*a^2*b^8*x^4 - 132300*a^4*b^6*x^3
 - 88200*a^6*b^4*x^2 - 14175*a^8*b^2*x - 252*a^10 - 16*(1575*a*b^9*x^4 + 63
00*a^3*b^7*x^3 + 7938*a^5*b^5*x^2 + 2700*a^7*b^3*x + 175*a^9*b)*sqrt(x))/x^
5
```

giac [A] time = 0.28, size = 112, normalized size = 0.86

$$b^{10}\log\left(|x|\right) - \frac{25200\,ab^9x^{\frac{9}{2}} + 56700\,a^2b^8x^4 + 100800\,a^3b^7x^{\frac{7}{2}} + 132300\,a^4b^6x^3 + 127008\,a^5b^5x^{\frac{5}{2}} + 88200\,a^6b^4x^2}{1260\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^10/x^6,x, algorithm="giac")
```

```
[Out] b^10*log(abs(x)) - 1/1260*(25200*a*b^9*x^(9/2) + 56700*a^2*b^8*x^4 + 100800
*a^3*b^7*x^(7/2) + 132300*a^4*b^6*x^3 + 127008*a^5*b^5*x^(5/2) + 88200*a^6*
b^4*x^2 + 43200*a^7*b^3*x^(3/2) + 14175*a^8*b^2*x + 2800*a^9*b*sqrt(x) + 25
2*a^10)/x^5
```

maple [A] time = 0.00, size = 111, normalized size = 0.85

$$b^{10}\ln(x) - \frac{20a\,b^9}{\sqrt{x}} - \frac{45a^2b^8}{x} - \frac{80a^3b^7}{x^{\frac{3}{2}}} - \frac{105a^4b^6}{x^2} - \frac{504a^5b^5}{5x^{\frac{5}{2}}} - \frac{70a^6b^4}{x^3} - \frac{240a^7b^3}{7x^{\frac{7}{2}}} - \frac{45a^8b^2}{4x^4} - \frac{20a^9b}{9x^{\frac{9}{2}}} - \frac{a^{10}}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^10/x^6,x)
```

```
[Out] -1/5*a^10/x^5-20/9*a^9*b/x^(9/2)-45/4*a^8*b^2/x^4-240/7*a^7*b^3/x^(7/2)-70*
a^6*b^4/x^3-504/5*a^5*b^5/x^(5/2)-105*a^4*b^6/x^2-80*a^3*b^7/x^(3/2)-45*a^2
*b^8/x+b^10*ln(x)-20*a*b^9/x^(1/2)
```

maxima [A] time = 0.89, size = 111, normalized size = 0.85

$$b^{10}\log(x) - \frac{25200\,ab^9x^{\frac{9}{2}} + 56700\,a^2b^8x^4 + 100800\,a^3b^7x^{\frac{7}{2}} + 132300\,a^4b^6x^3 + 127008\,a^5b^5x^{\frac{5}{2}} + 88200\,a^6b^4x^2 +}{1260\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^10/x^6,x, algorithm="maxima")
```

```
[Out] b^10*log(x) - 1/1260*(25200*a*b^9*x^(9/2) + 56700*a^2*b^8*x^4 + 100800*a^3*
b^7*x^(7/2) + 132300*a^4*b^6*x^3 + 127008*a^5*b^5*x^(5/2) + 88200*a^6*b^4*x
^2 + 43200*a^7*b^3*x^(3/2) + 14175*a^8*b^2*x + 2800*a^9*b*sqrt(x) + 252*a^1
0)/x^5
```

mupad [B] time = 1.14, size = 114, normalized size = 0.88

$$2\,b^{10}\,\ln\left(\sqrt{x}\right)-\frac{\frac{a^{10}}{5}+\frac{45\,a^8\,b^2\,x}{4}+\frac{20\,a^9\,b\,\sqrt{x}}{9}+20\,a\,b^9\,x^{9/2}+70\,a^6\,b^4\,x^2+105\,a^4\,b^6\,x^3+45\,a^2\,b^8\,x^4+\frac{240\,a^7\,b^3\,x^{3/2}}{7}+\frac{504}{x^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^10/x^6,x)
```

```
[Out] 2*b^10*log(x^(1/2)) - (a^10/5 + (45*a^8*b^2*x)/4 + (20*a^9*b*x^(1/2))/9 + 2
0*a*b^9*x^(9/2) + 70*a^6*b^4*x^2 + 105*a^4*b^6*x^3 + 45*a^2*b^8*x^4 + (240*
a^7*b^3*x^(3/2))/7 + (504*a^5*b^5*x^(5/2))/5 + 80*a^3*b^7*x^(7/2))/x^5
```

sympy [A] time = 2.73, size = 131, normalized size = 1.01

$$-\frac{a^{10}}{5x^5}-\frac{20a^9b}{9x^{\frac{9}{2}}}-\frac{45a^8b^2}{4x^4}-\frac{240a^7b^3}{7x^{\frac{7}{2}}}-\frac{70a^6b^4}{x^3}-\frac{504a^5b^5}{5x^{\frac{5}{2}}}-\frac{105a^4b^6}{x^2}-\frac{80a^3b^7}{x^{\frac{3}{2}}}-\frac{45a^2b^8}{x}-\frac{20ab^9}{\sqrt{x}}+b^{10}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**10/x**6,x)
```

```
[Out] -a**10/(5*x**5) - 20*a**9*b/(9*x**(9/2)) - 45*a**8*b**2/(4*x**4) - 240*a**7
*b**3/(7*x**(7/2)) - 70*a**6*b**4/x**3 - 504*a**5*b**5/(5*x**(5/2)) - 105*a
**4*b**6/x**2 - 80*a**3*b**7/x**(3/2) - 45*a**2*b**8/x - 20*a*b**9/sqrt(x)
+ b**10*log(x)
```

3.2164 $\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^7}\,dx$

Optimal. Leaf size=46

$$\frac{b\left(a+b\sqrt{x}\right)^{11}}{66a^2x^{11/2}}-\frac{\left(a+b\sqrt{x}\right)^{11}}{6ax^6}$$

```
[Out] -1/6*(a+b*x^(1/2))^11/a/x^6+1/66*b*(a+b*x^(1/2))^11/a^2/x^(11/2)
```

Rubi [A] time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {266, 45, 37}

$$\frac{b\left(a+b\sqrt{x}\right)^{11}}{66a^2x^{11/2}}-\frac{\left(a+b\sqrt{x}\right)^{11}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^10/x^7,x]
```

```
[Out] -(a + b*Sqrt[x])^11/(6*a*x^6) + (b*(a + b*Sqrt[x])^11)/(66*a^2*x^(11/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^7}\,dx &= 2\,\text{Subst}\left(\int \frac{(a+bx)^{10}}{x^{13}}\,dx, x, \sqrt{x}\right)\\ &= -\frac{\left(a+b\sqrt{x}\right)^{11}}{6ax^6}-\frac{b\,\text{Subst}\left(\int \frac{(a+bx)^{10}}{x^{12}}\,dx, x, \sqrt{x}\right)}{6a}\\ &= -\frac{\left(a+b\sqrt{x}\right)^{11}}{6ax^6}+\frac{b\left(a+b\sqrt{x}\right)^{11}}{66a^2x^{11/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 0.70

$$\frac{\left(b\sqrt{x}-11a\right)\left(a+b\sqrt{x}\right)^{11}}{66a^2x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^10/x^7,x]
```

```
[Out] ((-11*a + b*Sqrt[x])*(a + b*Sqrt[x])^11)/(66*a^2*x^6)
```

fricas [B] time = 1.20, size = 113, normalized size = 2.46

$$-\frac{66\,b^{10}x^5+1485\,a^2b^8x^4+4620\,a^4b^6x^3+3465\,a^6b^4x^2+594\,a^8b^2x+11\,a^{10}+8\left(55\,ab^9x^4+396\,a^3b^7x^3+594\,a^5b\right.}{66\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^10/x^7,x, algorithm="fricas")
```

```
[Out] -1/66*(66*b^10*x^5 + 1485*a^2*b^8*x^4 + 4620*a^4*b^6*x^3 + 3465*a^6*b^4*x^2
 + 594*a^8*b^2*x + 11*a^10 + 8*(55*a*b^9*x^4 + 396*a^3*b^7*x^3 + 594*a^5*b^
5*x^2 + 220*a^7*b^3*x + 15*a^9*b)*sqrt(x))/x^6
```

giac [B] time = 0.16, size = 112, normalized size = 2.43

$$-\frac{66\,b^{10}x^5+440\,ab^9x^{\frac{9}{2}}+1485\,a^2b^8x^4+3168\,a^3b^7x^{\frac{7}{2}}+4620\,a^4b^6x^3+4752\,a^5b^5x^{\frac{5}{2}}+3465\,a^6b^4x^2+1760\,a^7b^3x^{\frac{3}{2}}}{66\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^10/x^7,x, algorithm="giac")
```

```
[Out] -1/66*(66*b^10*x^5 + 440*a*b^9*x^(9/2) + 1485*a^2*b^8*x^4 + 3168*a^3*b^7*x^
(7/2) + 4620*a^4*b^6*x^3 + 4752*a^5*b^5*x^(5/2) + 3465*a^6*b^4*x^2 + 1760*a
^7*b^3*x^(3/2) + 594*a^8*b^2*x + 120*a^9*b*sqrt(x) + 11*a^10)/x^6
```

maple [B] time = 0.00, size = 113, normalized size = 2.46

$$-\frac{b^{10}}{x}-\frac{20a\,b^9}{3x^{\frac{3}{2}}}-\frac{45a^2b^8}{2x^2}-\frac{48a^3b^7}{x^{\frac{5}{2}}}-\frac{70a^4b^6}{x^3}-\frac{72a^5b^5}{x^{\frac{7}{2}}}-\frac{105a^6b^4}{2x^4}-\frac{80a^7b^3}{3x^{\frac{9}{2}}}-\frac{9a^8b^2}{x^5}-\frac{20a^9b}{11x^{\frac{11}{2}}}-\frac{a^{10}}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^10/x^7,x)
```

```
[Out] -b^10/x-20/3*a*b^9/x^(3/2)-45/2*a^2*b^8/x^2-48*a^3*b^7/x^(5/2)-70*a^4*b^6/x
^3-72*a^5*b^5/x^(7/2)-105/2*a^6*b^4/x^4-80/3*a^7*b^3/x^(9/2)-9*a^8*b^2/x^5-
20/11*a^9*b/x^(11/2)-1/6*a^10/x^6
```

maxima [B] time = 0.91, size = 112, normalized size = 2.43

$$-\frac{66\,b^{10}x^5+440\,ab^9x^{\frac{9}{2}}+1485\,a^2b^8x^4+3168\,a^3b^7x^{\frac{7}{2}}+4620\,a^4b^6x^3+4752\,a^5b^5x^{\frac{5}{2}}+3465\,a^6b^4x^2+1760\,a^7b^3x^{\frac{3}{2}}}{66\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^10/x^7,x, algorithm="maxima")
```

```
[Out] -1/66*(66*b^10*x^5 + 440*a*b^9*x^(9/2) + 1485*a^2*b^8*x^4 + 3168*a^3*b^7*x^
(7/2) + 4620*a^4*b^6*x^3 + 4752*a^5*b^5*x^(5/2) + 3465*a^6*b^4*x^2 + 1760*a
^7*b^3*x^(3/2) + 594*a^8*b^2*x + 120*a^9*b*sqrt(x) + 11*a^10)/x^6
```

mupad [B] time = 1.13, size = 111, normalized size = 2.41

$$-\frac{\frac{a^{10}}{6}+b^{10}x^5+9a^8b^2x+\frac{20a^9b\sqrt{x}}{11}+\frac{20ab^9x^{9/2}}{3}+\frac{105a^6b^4x^2}{2}+70a^4b^6x^3+\frac{45a^2b^8x^4}{2}+\frac{80a^7b^3x^{3/2}}{3}+72a^5b^5x^{5/2}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^10/x^7,x)
```

```
[Out] -(a^10/6 + b^10*x^5 + 9*a^8*b^2*x + (20*a^9*b*x^(1/2))/11 + (20*a*b^9*x^(9/
2))/3 + (105*a^6*b^4*x^2)/2 + 70*a^4*b^6*x^3 + (45*a^2*b^8*x^4)/2 + (80*a^7
*b^3*x^(3/2))/3 + 72*a^5*b^5*x^(5/2) + 48*a^3*b^7*x^(7/2))/x^6
```

sympy [B] time = 3.64, size = 134, normalized size = 2.91

$$-\frac{a^{10}}{6x^6}-\frac{20a^9b}{11x^{\frac{11}{2}}}-\frac{9a^8b^2}{x^5}-\frac{80a^7b^3}{3x^{\frac{9}{2}}}-\frac{105a^6b^4}{2x^4}-\frac{72a^5b^5}{x^{\frac{7}{2}}}-\frac{70a^4b^6}{x^3}-\frac{48a^3b^7}{x^{\frac{5}{2}}}-\frac{45a^2b^8}{2x^2}-\frac{20ab^9}{3x^{\frac{3}{2}}}-\frac{b^{10}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**10/x**7,x)
```

```
[Out] -a**10/(6*x**6) - 20*a**9*b/(11*x**(11/2)) - 9*a**8*b**2/x**5 - 80*a**7*b**
3/(3*x**(9/2)) - 105*a**6*b**4/(2*x**4) - 72*a**5*b**5/x**(7/2) - 70*a**4*b
**6/x**3 - 48*a**3*b**7/x**(5/2) - 45*a**2*b**8/(2*x**2) - 20*a*b**9/(3*x**
(3/2)) - b**10/x
```

3.2165 $\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^8}\,dx$

Optimal. Leaf size=96

$$\frac{b^3\left(a+b\sqrt{x}\right)^{11}}{2002a^4x^{11/2}}-\frac{b^2\left(a+b\sqrt{x}\right)^{11}}{182a^3x^6}+\frac{3b\left(a+b\sqrt{x}\right)^{11}}{91a^2x^{13/2}}-\frac{\left(a+b\sqrt{x}\right)^{11}}{7ax^7}$$

```
[Out] -1/7*(a+b*x^(1/2))^11/a/x^7+3/91*b*(a+b*x^(1/2))^11/a^2/x^(13/2)-1/182*b^2*
(a+b*x^(1/2))^11/a^3/x^6+1/2002*b^3*(a+b*x^(1/2))^11/a^4/x^(11/2)
```

Rubi [A] time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {266, 45, 37}

$$\frac{b^3\left(a+b\sqrt{x}\right)^{11}}{2002a^4x^{11/2}}-\frac{b^2\left(a+b\sqrt{x}\right)^{11}}{182a^3x^6}+\frac{3b\left(a+b\sqrt{x}\right)^{11}}{91a^2x^{13/2}}-\frac{\left(a+b\sqrt{x}\right)^{11}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^10/x^8,x]
```

```
[Out] -(a + b*Sqrt[x])^11/(7*a*x^7) + (3*b*(a + b*Sqrt[x])^11)/(91*a^2*x^(13/2))
- (b^2*(a + b*Sqrt[x])^11)/(182*a^3*x^6) + (b^3*(a + b*Sqrt[x])^11)/(2002*a
^4*x^(11/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^8}\,dx = 2\,\text{Subst}\left(\int \frac{(a+bx)^{10}}{x^{15}}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{\left(a+b\sqrt{x}\right)^{11}}{7ax^7} - \frac{(3b)\,\text{Subst}\left(\int \frac{(a+bx)^{10}}{x^{14}}\,dx, x, \sqrt{x}\right)}{7a}$$
$$= -\frac{\left(a+b\sqrt{x}\right)^{11}}{7ax^7} + \frac{3b\left(a+b\sqrt{x}\right)^{11}}{91a^2x^{13/2}} + \frac{\left(6b^2\right)\text{Subst}\left(\int \frac{(a+bx)^{10}}{x^{13}}\,dx, x, \sqrt{x}\right)}{91a^2}$$
$$= -\frac{\left(a+b\sqrt{x}\right)^{11}}{7ax^7} + \frac{3b\left(a+b\sqrt{x}\right)^{11}}{91a^2x^{13/2}} - \frac{b^2\left(a+b\sqrt{x}\right)^{11}}{182a^3x^6} - \frac{b^3\,\text{Subst}\left(\int \frac{(a+bx)^{10}}{x^{12}}\,dx, x, \sqrt{x}\right)}{182a^3}$$
$$= -\frac{\left(a+b\sqrt{x}\right)^{11}}{7ax^7} + \frac{3b\left(a+b\sqrt{x}\right)^{11}}{91a^2x^{13/2}} - \frac{b^2\left(a+b\sqrt{x}\right)^{11}}{182a^3x^6} + \frac{b^3\left(a+b\sqrt{x}\right)^{11}}{2002a^4x^{11/2}}$$

Mathematica [A] time = 0.01, size = 54, normalized size = 0.56

$$\frac{\left(a+b\sqrt{x}\right)^{11}\left(-286a^3+66a^2b\sqrt{x}-11ab^2x+b^3x^{3/2}\right)}{2002a^4x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^10/x^8,x]
```

```
[Out] ((a + b*Sqrt[x])^11*(-286*a^3 + 66*a^2*b*Sqrt[x] - 11*a*b^2*x + b^3*x^(3/2)
))/(2002*a^4*x^7)
```

fricas [A] time = 0.93, size = 113, normalized size = 1.18

$$-\frac{1001\,b^{10}x^5+30030\,a^2b^8x^4+105105\,a^4b^6x^3+84084\,a^6b^4x^2+15015\,a^8b^2x+286\,a^{10}+8\left(1001\,ab^9x^4+8580}{2002\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^10/x^8,x, algorithm="fricas")
```

```
[Out] -1/2002*(1001*b^10*x^5 + 30030*a^2*b^8*x^4 + 105105*a^4*b^6*x^3 + 84084*a^6
*b^4*x^2 + 15015*a^8*b^2*x + 286*a^10 + 8*(1001*a*b^9*x^4 + 8580*a^3*b^7*x^
3 + 14014*a^5*b^5*x^2 + 5460*a^7*b^3*x + 385*a^9*b)*sqrt(x))/x^7
```

giac [A] time = 0.17, size = 112, normalized size = 1.17

$$-\frac{1001\,b^{10}x^5+8008\,ab^9x^{\frac{9}{2}}+30030\,a^2b^8x^4+68640\,a^3b^7x^{\frac{7}{2}}+105105\,a^4b^6x^3+112112\,a^5b^5x^{\frac{5}{2}}+84084\,a^6b^4x^2+}{2002\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^10/x^8,x, algorithm="giac")
```

```
[Out] -1/2002*(1001*b^10*x^5 + 8008*a*b^9*x^(9/2) + 30030*a^2*b^8*x^4 + 68640*a^3
*b^7*x^(7/2) + 105105*a^4*b^6*x^3 + 112112*a^5*b^5*x^(5/2) + 84084*a^6*b^4*
x^2 + 43680*a^7*b^3*x^(3/2) + 15015*a^8*b^2*x + 3080*a^9*b*sqrt(x) + 286*a^
10)/x^7
```

maple [A] time = 0.00, size = 113, normalized size = 1.18

$$-\frac{b^{10}}{2x^2}-\frac{4a\,b^9}{x^{\frac{5}{2}}}-\frac{15a^2b^8}{x^3}-\frac{240a^3b^7}{7x^{\frac{7}{2}}}-\frac{105a^4b^6}{2x^4}-\frac{56a^5b^5}{x^{\frac{9}{2}}}-\frac{42a^6b^4}{x^5}-\frac{240a^7b^3}{11x^{\frac{11}{2}}}-\frac{15a^8b^2}{2x^6}-\frac{20a^9b}{13x^{\frac{13}{2}}}-\frac{a^{10}}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^10/x^8,x)
```

```
[Out] -1/2*b^10/x^2-4*a*b^9/x^(5/2)-15*a^2*b^8/x^3-240/7*a^3*b^7/x^(7/2)-105/2*a^
4*b^6/x^4-56*a^5*b^5/x^(9/2)-42*a^6*b^4/x^5-240/11*a^7*b^3/x^(11/2)-15/2*a^
8*b^2/x^6-20/13*a^9*b/x^(13/2)-1/7*a^10/x^7
```

maxima [A] time = 0.87, size = 112, normalized size = 1.17

$$-\frac{1001\,b^{10}x^5+8008\,ab^9x^{\frac{9}{2}}+30030\,a^2b^8x^4+68640\,a^3b^7x^{\frac{7}{2}}+105105\,a^4b^6x^3+112112\,a^5b^5x^{\frac{5}{2}}+84084\,a^6b^4x^2+4}{2002\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^10/x^8,x, algorithm="maxima")
```

```
[Out] -1/2002*(1001*b^10*x^5 + 8008*a*b^9*x^(9/2) + 30030*a^2*b^8*x^4 + 68640*a^3
*b^7*x^(7/2) + 105105*a^4*b^6*x^3 + 112112*a^5*b^5*x^(5/2) + 84084*a^6*b^4*
x^2 + 43680*a^7*b^3*x^(3/2) + 15015*a^8*b^2*x + 3080*a^9*b*sqrt(x) + 286*a^
10)/x^7
```

mupad [B] time = 1.13, size = 112, normalized size = 1.17

$$-\frac{\frac{a^{10}}{7}+\frac{b^{10}\,x^5}{2}+\frac{15\,a^8\,b^2\,x}{2}+\frac{20\,a^9\,b\,\sqrt{x}}{13}+4\,a\,b^9\,x^{9/2}+42\,a^6\,b^4\,x^2+\frac{105\,a^4\,b^6\,x^3}{2}+15\,a^2\,b^8\,x^4+\frac{240\,a^7\,b^3\,x^{3/2}}{11}+56\,a^5\,b^5\,x^{5/2}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^10/x^8,x)
```

```
[Out] -(a^10/7 + (b^10*x^5)/2 + (15*a^8*b^2*x)/2 + (20*a^9*b*x^(1/2))/13 + 4*a*b^
9*x^(9/2) + 42*a^6*b^4*x^2 + (105*a^4*b^6*x^3)/2 + 15*a^2*b^8*x^4 + (240*a^
7*b^3*x^(3/2))/11 + 56*a^5*b^5*x^(5/2) + (240*a^3*b^7*x^(7/2))/7)/x^7
```

sympy [A] time = 4.89, size = 138, normalized size = 1.44

$$-\frac{a^{10}}{7x^7}-\frac{20a^9b}{13x^{\frac{13}{2}}}-\frac{15a^8b^2}{2x^6}-\frac{240a^7b^3}{11x^{\frac{11}{2}}}-\frac{42a^6b^4}{x^5}-\frac{56a^5b^5}{x^{\frac{9}{2}}}-\frac{105a^4b^6}{2x^4}-\frac{240a^3b^7}{7x^{\frac{7}{2}}}-\frac{15a^2b^8}{x^3}-\frac{4ab^9}{x^{\frac{5}{2}}}-\frac{b^{10}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**10/x**8,x)
```

```
[Out] -a**10/(7*x**7) - 20*a**9*b/(13*x**(13/2)) - 15*a**8*b**2/(2*x**6) - 240*a*
*7*b**3/(11*x**(11/2)) - 42*a**6*b**4/x**5 - 56*a**5*b**5/x**(9/2) - 105*a*
*4*b**6/(2*x**4) - 240*a**3*b**7/(7*x**(7/2)) - 15*a**2*b**8/x**3 - 4*a*b**
9/x**(5/2) - b**10/(2*x**2)
```

3.2166 $\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^9}\,dx$

Optimal. Leaf size=146

$$\frac{b^5\left(a+b\sqrt{x}\right)^{11}}{24024a^6x^{11/2}}-\frac{b^4\left(a+b\sqrt{x}\right)^{11}}{2184a^5x^6}+\frac{b^3\left(a+b\sqrt{x}\right)^{11}}{364a^4x^{13/2}}-\frac{b^2\left(a+b\sqrt{x}\right)^{11}}{84a^3x^7}+\frac{b\left(a+b\sqrt{x}\right)^{11}}{24a^2x^{15/2}}-\frac{\left(a+b\sqrt{x}\right)^{11}}{8ax^8}$$

```
[Out] -1/8*(a+b*x^(1/2))^11/a/x^8+1/24*b*(a+b*x^(1/2))^11/a^2/x^(15/2)-1/84*b^2*(
a+b*x^(1/2))^11/a^3/x^7+1/364*b^3*(a+b*x^(1/2))^11/a^4/x^(13/2)-1/2184*b^4*
(a+b*x^(1/2))^11/a^5/x^6+1/24024*b^5*(a+b*x^(1/2))^11/a^6/x^(11/2)
```

Rubi [A] time = 0.06, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {266, 45, 37}

$$\frac{b^5\left(a+b\sqrt{x}\right)^{11}}{24024a^6x^{11/2}}-\frac{b^4\left(a+b\sqrt{x}\right)^{11}}{2184a^5x^6}+\frac{b^3\left(a+b\sqrt{x}\right)^{11}}{364a^4x^{13/2}}-\frac{b^2\left(a+b\sqrt{x}\right)^{11}}{84a^3x^7}+\frac{b\left(a+b\sqrt{x}\right)^{11}}{24a^2x^{15/2}}-\frac{\left(a+b\sqrt{x}\right)^{11}}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^10/x^9,x]
```

```
[Out] -(a + b*Sqrt[x])^11/(8*a*x^8) + (b*(a + b*Sqrt[x])^11)/(24*a^2*x^(15/2)) -
(b^2*(a + b*Sqrt[x])^11)/(84*a^3*x^7) + (b^3*(a + b*Sqrt[x])^11)/(364*a^4*x
^(13/2)) - (b^4*(a + b*Sqrt[x])^11)/(2184*a^5*x^6) + (b^5*(a + b*Sqrt[x])^1
1)/(24024*a^6*x^(11/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^9}\,dx = 2\,\text{Subst}\left(\int \frac{(a+bx)^{10}}{x^{17}}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{\left(a+b\sqrt{x}\right)^{11}}{8ax^8} - \frac{(5b)\,\text{Subst}\left(\int \frac{(a+bx)^{10}}{x^{16}}\,dx, x, \sqrt{x}\right)}{8a}$$
$$= -\frac{\left(a+b\sqrt{x}\right)^{11}}{8ax^8} + \frac{b\left(a+b\sqrt{x}\right)^{11}}{24a^2x^{15/2}} + \frac{b^2\,\text{Subst}\left(\int \frac{(a+bx)^{10}}{x^{15}}\,dx, x, \sqrt{x}\right)}{6a^2}$$
$$= -\frac{\left(a+b\sqrt{x}\right)^{11}}{8ax^8} + \frac{b\left(a+b\sqrt{x}\right)^{11}}{24a^2x^{15/2}} - \frac{b^2\left(a+b\sqrt{x}\right)^{11}}{84a^3x^7} - \frac{b^3\,\text{Subst}\left(\int \frac{(a+bx)^{10}}{x^{14}}\,dx, x, \sqrt{x}\right)}{28a^3}$$
$$= -\frac{\left(a+b\sqrt{x}\right)^{11}}{8ax^8} + \frac{b\left(a+b\sqrt{x}\right)^{11}}{24a^2x^{15/2}} - \frac{b^2\left(a+b\sqrt{x}\right)^{11}}{84a^3x^7} + \frac{b^3\left(a+b\sqrt{x}\right)^{11}}{364a^4x^{13/2}} + \frac{b^4\,\text{Subst}\left(\int \frac{(a+bx)^{10}}{x^{13}}\right.}{182a^4}$$
$$= -\frac{\left(a+b\sqrt{x}\right)^{11}}{8ax^8} + \frac{b\left(a+b\sqrt{x}\right)^{11}}{24a^2x^{15/2}} - \frac{b^2\left(a+b\sqrt{x}\right)^{11}}{84a^3x^7} + \frac{b^3\left(a+b\sqrt{x}\right)^{11}}{364a^4x^{13/2}} - \frac{b^4\left(a+b\sqrt{x}\right)^{11}}{2184a^5x^6} - \frac{b}{}$$
$$= -\frac{\left(a+b\sqrt{x}\right)^{11}}{8ax^8} + \frac{b\left(a+b\sqrt{x}\right)^{11}}{24a^2x^{15/2}} - \frac{b^2\left(a+b\sqrt{x}\right)^{11}}{84a^3x^7} + \frac{b^3\left(a+b\sqrt{x}\right)^{11}}{364a^4x^{13/2}} - \frac{b^4\left(a+b\sqrt{x}\right)^{11}}{2184a^5x^6} + \frac{b}{}$$

Mathematica [A] time = 0.03, size = 140, normalized size = 0.96

$$-\frac{a^{10}}{8x^8} - \frac{4a^9b}{3x^{15/2}} - \frac{45a^8b^2}{7x^7} - \frac{240a^7b^3}{13x^{13/2}} - \frac{35a^6b^4}{x^6} - \frac{504a^5b^5}{11x^{11/2}} - \frac{42a^4b^6}{x^5} - \frac{80a^3b^7}{3x^{9/2}} - \frac{45a^2b^8}{4x^4} - \frac{20ab^9}{7x^{7/2}} - \frac{b^{10}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^10/x^9,x]
```

```
[Out] -1/8*a^10/x^8 - (4*a^9*b)/(3*x^(15/2)) - (45*a^8*b^2)/(7*x^7) - (240*a^7*b^
3)/(13*x^(13/2)) - (35*a^6*b^4)/x^6 - (504*a^5*b^5)/(11*x^(11/2)) - (42*a^4
*b^6)/x^5 - (80*a^3*b^7)/(3*x^(9/2)) - (45*a^2*b^8)/(4*x^4) - (20*a*b^9)/(7
*x^(7/2)) - b^10/(3*x^3)
```

fricas [A] time = 0.81, size = 113, normalized size = 0.77

$$-\frac{8008\,b^{10}x^5 + 270270\,a^2b^8x^4 + 1009008\,a^4b^6x^3 + 840840\,a^6b^4x^2 + 154440\,a^8b^2x + 3003\,a^{10} + 32\left(2145\,ab^9x^4 + \right.}{24024\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^10/x^9,x, algorithm="fricas")
```

```
[Out] -1/24024*(8008*b^10*x^5 + 270270*a^2*b^8*x^4 + 1009008*a^4*b^6*x^3 + 840840
*a^6*b^4*x^2 + 154440*a^8*b^2*x + 3003*a^10 + 32*(2145*a*b^9*x^4 + 20020*a^
3*b^7*x^3 + 34398*a^5*b^5*x^2 + 13860*a^7*b^3*x + 1001*a^9*b)*sqrt(x))/x^8
```

giac [A] time = 0.18, size = 112, normalized size = 0.77

$$-\frac{8008\,b^{10}x^5 + 68640\,ab^9x^{\frac{9}{2}} + 270270\,a^2b^8x^4 + 640640\,a^3b^7x^{\frac{7}{2}} + 1009008\,a^4b^6x^3 + 1100736\,a^5b^5x^{\frac{5}{2}} + 840840\,a^6}{24024\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^10/x^9,x, algorithm="giac")
```

```
[Out] -1/24024*(8008*b^10*x^5 + 68640*a*b^9*x^(9/2) + 270270*a^2*b^8*x^4 + 640640
*a^3*b^7*x^(7/2) + 1009008*a^4*b^6*x^3 + 1100736*a^5*b^5*x^(5/2) + 840840*a
```

```
^6*b^4*x^2 + 443520*a^7*b^3*x^(3/2) + 154440*a^8*b^2*x + 32032*a^9*b*sqrt(x
) + 3003*a^10)/x^8
```

maple [A] time = 0.00, size = 113, normalized size = 0.77

$$-\frac{b^{10}}{3x^3}-\frac{20a\,b^9}{7x^{\frac{7}{2}}}-\frac{45a^2b^8}{4x^4}-\frac{80a^3b^7}{3x^{\frac{9}{2}}}-\frac{42a^4b^6}{x^5}-\frac{504a^5b^5}{11x^{\frac{11}{2}}}-\frac{35a^6b^4}{x^6}-\frac{240a^7b^3}{13x^{\frac{13}{2}}}-\frac{45a^8b^2}{7x^7}-\frac{4a^9b}{3x^{\frac{15}{2}}}-\frac{a^{10}}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^10/x^9,x)
```

```
[Out] -1/3*b^10/x^3-20/7*a*b^9/x^(7/2)-45/4*a^2*b^8/x^4-80/3*a^3*b^7/x^(9/2)-42*a
^4*b^6/x^5-504/11*a^5*b^5/x^(11/2)-35*a^6*b^4/x^6-240/13*a^7*b^3/x^(13/2)-4
5/7*a^8*b^2/x^7-4/3*a^9*b/x^(15/2)-1/8*a^10/x^8
```

maxima [A] time = 0.94, size = 112, normalized size = 0.77

$$-\frac{8008\,b^{10}x^5+68640\,ab^9x^{\frac{9}{2}}+270270\,a^2b^8x^4+640640\,a^3b^7x^{\frac{7}{2}}+1009008\,a^4b^6x^3+1100736\,a^5b^5x^{\frac{5}{2}}+840840}{24024\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^10/x^9,x, algorithm="maxima")
```

```
[Out] -1/24024*(8008*b^10*x^5 + 68640*a*b^9*x^(9/2) + 270270*a^2*b^8*x^4 + 640640
*a^3*b^7*x^(7/2) + 1009008*a^4*b^6*x^3 + 1100736*a^5*b^5*x^(5/2) + 840840*a
^6*b^4*x^2 + 443520*a^7*b^3*x^(3/2) + 154440*a^8*b^2*x + 32032*a^9*b*sqrt(x
) + 3003*a^10)/x^8
```

mupad [B] time = 0.09, size = 112, normalized size = 0.77

$$-\frac{\frac{a^{10}}{8}+\frac{b^{10}x^5}{3}+\frac{45a^8b^2x}{7}+\frac{4a^9b\sqrt{x}}{3}+\frac{20ab^9x^{9/2}}{7}+35a^6b^4x^2+42a^4b^6x^3+\frac{45a^2b^8x^4}{4}+\frac{240a^7b^3x^{3/2}}{13}+\frac{504a^5b^5x^{5/2}}{11}+}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^10/x^9,x)
```

```
[Out] -(a^10/8 + (b^10*x^5)/3 + (45*a^8*b^2*x)/7 + (4*a^9*b*x^(1/2))/3 + (20*a*b^
9*x^(9/2))/7 + 35*a^6*b^4*x^2 + 42*a^4*b^6*x^3 + (45*a^2*b^8*x^4)/4 + (240*
a^7*b^3*x^(3/2))/13 + (504*a^5*b^5*x^(5/2))/11 + (80*a^3*b^7*x^(7/2))/3)/x^
8
```

sympy [A] time = 7.27, size = 141, normalized size = 0.97

$$-\frac{a^{10}}{8x^8}-\frac{4a^9b}{3x^{\frac{15}{2}}}-\frac{45a^8b^2}{7x^7}-\frac{240a^7b^3}{13x^{\frac{13}{2}}}-\frac{35a^6b^4}{x^6}-\frac{504a^5b^5}{11x^{\frac{11}{2}}}-\frac{42a^4b^6}{x^5}-\frac{80a^3b^7}{3x^{\frac{9}{2}}}-\frac{45a^2b^8}{4x^4}-\frac{20ab^9}{7x^{\frac{7}{2}}}-\frac{b^{10}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**10/x**9,x)
```

```
[Out] -a**10/(8*x**8) - 4*a**9*b/(3*x**(15/2)) - 45*a**8*b**2/(7*x**7) - 240*a**7
*b**3/(13*x**(13/2)) - 35*a**6*b**4/x**6 - 504*a**5*b**5/(11*x**(11/2)) - 4
2*a**4*b**6/x**5 - 80*a**3*b**7/(3*x**(9/2)) - 45*a**2*b**8/(4*x**4) - 20*a
*b**9/(7*x**(7/2)) - b**10/(3*x**3)
```

3.2167 $\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^{10}}\,dx$

Optimal. Leaf size=136

$$-\frac{a^{10}}{9x^9}-\frac{20a^9b}{17x^{17/2}}-\frac{45a^8b^2}{8x^8}-\frac{16a^7b^3}{x^{15/2}}-\frac{30a^6b^4}{x^7}-\frac{504a^5b^5}{13x^{13/2}}-\frac{35a^4b^6}{x^6}-\frac{240a^3b^7}{11x^{11/2}}-\frac{9a^2b^8}{x^5}-\frac{20ab^9}{9x^{9/2}}-\frac{b^{10}}{4x^4}$$

```
[Out] -1/9*a^10/x^9-20/17*a^9*b/x^(17/2)-45/8*a^8*b^2/x^8-16*a^7*b^3/x^(15/2)-30*
a^6*b^4/x^7-504/13*a^5*b^5/x^(13/2)-35*a^4*b^6/x^6-240/11*a^3*b^7/x^(11/2)-
9*a^2*b^8/x^5-20/9*a*b^9/x^(9/2)-1/4*b^10/x^4
```

Rubi [A] time = 0.07, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{45a^8b^2}{8x^8}-\frac{16a^7b^3}{x^{15/2}}-\frac{30a^6b^4}{x^7}-\frac{504a^5b^5}{13x^{13/2}}-\frac{35a^4b^6}{x^6}-\frac{240a^3b^7}{11x^{11/2}}-\frac{9a^2b^8}{x^5}-\frac{20a^9b}{17x^{17/2}}-\frac{a^{10}}{9x^9}-\frac{20ab^9}{9x^{9/2}}-\frac{b^{10}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^10/x^10,x]
```

```
[Out] -a^10/(9*x^9) - (20*a^9*b)/(17*x^(17/2)) - (45*a^8*b^2)/(8*x^8) - (16*a^7*b
^3)/x^(15/2) - (30*a^6*b^4)/x^7 - (504*a^5*b^5)/(13*x^(13/2)) - (35*a^4*b^6
)/x^6 - (240*a^3*b^7)/(11*x^(11/2)) - (9*a^2*b^8)/x^5 - (20*a*b^9)/(9*x^(9/
2)) - b^10/(4*x^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^{10}}\,dx &= 2\,\text{Subst}\left(\int \frac{(a+bx)^{10}}{x^{19}}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(\frac{a^{10}}{x^{19}}+\frac{10a^9b}{x^{18}}+\frac{45a^8b^2}{x^{17}}+\frac{120a^7b^3}{x^{16}}+\frac{210a^6b^4}{x^{15}}+\frac{252a^5b^5}{x^{14}}+\frac{210a^4b^6}{x^{13}}+\frac{120a^3b}{x^{12}}\right.\right.\\ &= -\frac{a^{10}}{9x^9}-\frac{20a^9b}{17x^{17/2}}-\frac{45a^8b^2}{8x^8}-\frac{16a^7b^3}{x^{15/2}}-\frac{30a^6b^4}{x^7}-\frac{504a^5b^5}{13x^{13/2}}-\frac{35a^4b^6}{x^6}-\frac{240a^3b^7}{11x^{11/2}}-\frac{9a^2b^8}{x^5}-\frac{2}{9}\end{aligned}$$

Mathematica [A] time = 0.06, size = 136, normalized size = 1.00

$$-\frac{a^{10}}{9x^9}-\frac{20a^9b}{17x^{17/2}}-\frac{45a^8b^2}{8x^8}-\frac{16a^7b^3}{x^{15/2}}-\frac{30a^6b^4}{x^7}-\frac{504a^5b^5}{13x^{13/2}}-\frac{35a^4b^6}{x^6}-\frac{240a^3b^7}{11x^{11/2}}-\frac{9a^2b^8}{x^5}-\frac{20ab^9}{9x^{9/2}}-\frac{b^{10}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^10/x^10,x]
```

```
[Out] -1/9*a^10/x^9 - (20*a^9*b)/(17*x^(17/2)) - (45*a^8*b^2)/(8*x^8) - (16*a^7*b
^3)/x^(15/2) - (30*a^6*b^4)/x^7 - (504*a^5*b^5)/(13*x^(13/2)) - (35*a^4*b^6
)/x^6 - (240*a^3*b^7)/(11*x^(11/2)) - (9*a^2*b^8)/x^5 - (20*a*b^9)/(9*x^(9/
2)) - b^10/(4*x^4)
```

fricas [A] time = 0.75, size = 113, normalized size = 0.83

$$-\frac{43758\,b^{10}x^5+1575288\,a^2b^8x^4+6126120\,a^4b^6x^3+5250960\,a^6b^4x^2+984555\,a^8b^2x+19448\,a^{10}+32\left(12155\right.}{175032\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^10/x^10,x, algorithm="fricas")
```

```
[Out] -1/175032*(43758*b^10*x^5 + 1575288*a^2*b^8*x^4 + 6126120*a^4*b^6*x^3 + 525
0960*a^6*b^4*x^2 + 984555*a^8*b^2*x + 19448*a^10 + 32*(12155*a*b^9*x^4 + 11
9340*a^3*b^7*x^3 + 212058*a^5*b^5*x^2 + 87516*a^7*b^3*x + 6435*a^9*b)*sqrt(
x))/x^9
```

giac [A] time = 0.17, size = 112, normalized size = 0.82

$$-\frac{43758\,b^{10}x^5+388960\,ab^9x^{\frac{9}{2}}+1575288\,a^2b^8x^4+3818880\,a^3b^7x^{\frac{7}{2}}+6126120\,a^4b^6x^3+6785856\,a^5b^5x^{\frac{5}{2}}+52}{175032\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^10/x^10,x, algorithm="giac")
```

```
[Out] -1/175032*(43758*b^10*x^5 + 388960*a*b^9*x^(9/2) + 1575288*a^2*b^8*x^4 + 38
18880*a^3*b^7*x^(7/2) + 6126120*a^4*b^6*x^3 + 6785856*a^5*b^5*x^(5/2) + 525
0960*a^6*b^4*x^2 + 2800512*a^7*b^3*x^(3/2) + 984555*a^8*b^2*x + 205920*a^9*
b*sqrt(x) + 19448*a^10)/x^9
```

maple [A] time = 0.00, size = 113, normalized size = 0.83

$$-\frac{b^{10}}{4x^4}-\frac{20a\,b^9}{9x^{\frac{9}{2}}}-\frac{9a^2b^8}{x^5}-\frac{240a^3b^7}{11x^{\frac{11}{2}}}-\frac{35a^4b^6}{x^6}-\frac{504a^5b^5}{13x^{\frac{13}{2}}}-\frac{30a^6b^4}{x^7}-\frac{16a^7b^3}{x^{\frac{15}{2}}}-\frac{45a^8b^2}{8x^8}-\frac{20a^9b}{17x^{\frac{17}{2}}}-\frac{a^{10}}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^10/x^10,x)
```

```
[Out] -1/9*a^10/x^9-20/17*a^9*b/x^(17/2)-45/8*a^8*b^2/x^8-16*a^7*b^3/x^(15/2)-30*
a^6*b^4/x^7-504/13*a^5*b^5/x^(13/2)-35*a^4*b^6/x^6-240/11*a^3*b^7/x^(11/2)-
9*a^2*b^8/x^5-20/9*a*b^9/x^(9/2)-1/4*b^10/x^4
```

maxima [A] time = 0.90, size = 112, normalized size = 0.82

$$-\frac{43758\,b^{10}x^5+388960\,ab^9x^{\frac{9}{2}}+1575288\,a^2b^8x^4+3818880\,a^3b^7x^{\frac{7}{2}}+6126120\,a^4b^6x^3+6785856\,a^5b^5x^{\frac{5}{2}}+52}{175032\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^10/x^10,x, algorithm="maxima")
```

```
[Out] -1/175032*(43758*b^10*x^5 + 388960*a*b^9*x^(9/2) + 1575288*a^2*b^8*x^4 + 38
18880*a^3*b^7*x^(7/2) + 6126120*a^4*b^6*x^3 + 6785856*a^5*b^5*x^(5/2) + 525
0960*a^6*b^4*x^2 + 2800512*a^7*b^3*x^(3/2) + 984555*a^8*b^2*x + 205920*a^9*
b*sqrt(x) + 19448*a^10)/x^9
```

mupad [B] time = 0.09, size = 112, normalized size = 0.82

$$-\frac{\frac{a^{10}}{9}+\frac{b^{10}x^5}{4}+\frac{45a^8b^2x}{8}+\frac{20a^9b\sqrt{x}}{17}+\frac{20ab^9x^{9/2}}{9}+30a^6b^4x^2+35a^4b^6x^3+9a^2b^8x^4+16a^7b^3x^{3/2}+\frac{504a^5b^5x^{5/2}}{13}+}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^10/x^10,x)
```

```
[Out] -(a^10/9 + (b^10*x^5)/4 + (45*a^8*b^2*x)/8 + (20*a^9*b*x^(1/2))/17 + (20*a*
b^9*x^(9/2))/9 + 30*a^6*b^4*x^2 + 35*a^4*b^6*x^3 + 9*a^2*b^8*x^4 + 16*a^7*b
^3*x^(3/2) + (504*a^5*b^5*x^(5/2))/13 + (240*a^3*b^7*x^(7/2))/11)/x^9
```

sympy [A] time = 9.42, size = 138, normalized size = 1.01

$$-\frac{a^{10}}{9x^9}-\frac{20a^9b}{17x^{\frac{17}{2}}}-\frac{45a^8b^2}{8x^8}-\frac{16a^7b^3}{x^{\frac{15}{2}}}-\frac{30a^6b^4}{x^7}-\frac{504a^5b^5}{13x^{\frac{13}{2}}}-\frac{35a^4b^6}{x^6}-\frac{240a^3b^7}{11x^{\frac{11}{2}}}-\frac{9a^2b^8}{x^5}-\frac{20ab^9}{9x^{\frac{9}{2}}}-\frac{b^{10}}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**10/x**10,x)
```

```
[Out] -a**10/(9*x**9) - 20*a**9*b/(17*x**(17/2)) - 45*a**8*b**2/(8*x**8) - 16*a**
7*b**3/x**(15/2) - 30*a**6*b**4/x**7 - 504*a**5*b**5/(13*x**(13/2)) - 35*a*
*4*b**6/x**6 - 240*a**3*b**7/(11*x**(11/2)) - 9*a**2*b**8/x**5 - 20*a*b**9/
(9*x**(9/2)) - b**10/(4*x**4)
```

3.2168 $\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^{11}}\,dx$

Optimal. Leaf size=140

$$-\frac{a^{10}}{10x^{10}}-\frac{20a^9b}{19x^{19/2}}-\frac{5a^8b^2}{x^9}-\frac{240a^7b^3}{17x^{17/2}}-\frac{105a^6b^4}{4x^8}-\frac{168a^5b^5}{5x^{15/2}}-\frac{30a^4b^6}{x^7}-\frac{240a^3b^7}{13x^{13/2}}-\frac{15a^2b^8}{2x^6}-\frac{20ab^9}{11x^{11/2}}-\frac{b^{10}}{5x^5}$$

```
[Out] -1/10*a^10/x^10-20/19*a^9*b/x^(19/2)-5*a^8*b^2/x^9-240/17*a^7*b^3/x^(17/2)-
105/4*a^6*b^4/x^8-168/5*a^5*b^5/x^(15/2)-30*a^4*b^6/x^7-240/13*a^3*b^7/x^(1
3/2)-15/2*a^2*b^8/x^6-20/11*a*b^9/x^(11/2)-1/5*b^10/x^5
```

Rubi [A] time = 0.07, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{5a^8b^2}{x^9}-\frac{240a^7b^3}{17x^{17/2}}-\frac{105a^6b^4}{4x^8}-\frac{168a^5b^5}{5x^{15/2}}-\frac{30a^4b^6}{x^7}-\frac{240a^3b^7}{13x^{13/2}}-\frac{15a^2b^8}{2x^6}-\frac{20a^9b}{19x^{19/2}}-\frac{a^{10}}{10x^{10}}-\frac{20ab^9}{11x^{11/2}}-\frac{b^{10}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^10/x^11,x]
```

```
[Out] -a^10/(10*x^10) - (20*a^9*b)/(19*x^(19/2)) - (5*a^8*b^2)/x^9 - (240*a^7*b^3
)/(17*x^(17/2)) - (105*a^6*b^4)/(4*x^8) - (168*a^5*b^5)/(5*x^(15/2)) - (30*
a^4*b^6)/x^7 - (240*a^3*b^7)/(13*x^(13/2)) - (15*a^2*b^8)/(2*x^6) - (20*a*b
^9)/(11*x^(11/2)) - b^10/(5*x^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^{11}}\,dx &= 2\,\text{Subst}\left(\int \frac{(a+bx)^{10}}{x^{21}}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(\frac{a^{10}}{x^{21}}+\frac{10a^9b}{x^{20}}+\frac{45a^8b^2}{x^{19}}+\frac{120a^7b^3}{x^{18}}+\frac{210a^6b^4}{x^{17}}+\frac{252a^5b^5}{x^{16}}+\frac{210a^4b^6}{x^{15}}+\frac{120}{x}\right.\right.\\ &= -\frac{a^{10}}{10x^{10}}-\frac{20a^9b}{19x^{19/2}}-\frac{5a^8b^2}{x^9}-\frac{240a^7b^3}{17x^{17/2}}-\frac{105a^6b^4}{4x^8}-\frac{168a^5b^5}{5x^{15/2}}-\frac{30a^4b^6}{x^7}-\frac{240a^3b^7}{13x^{13/2}}-\frac{15a^2}{2x}\end{aligned}$$

Mathematica [A] time = 0.06, size = 140, normalized size = 1.00

$$-\frac{a^{10}}{10x^{10}}-\frac{20a^9b}{19x^{19/2}}-\frac{5a^8b^2}{x^9}-\frac{240a^7b^3}{17x^{17/2}}-\frac{105a^6b^4}{4x^8}-\frac{168a^5b^5}{5x^{15/2}}-\frac{30a^4b^6}{x^7}-\frac{240a^3b^7}{13x^{13/2}}-\frac{15a^2b^8}{2x^6}-\frac{20ab^9}{11x^{11/2}}-\frac{b^{10}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^10/x^11,x]
```

```
[Out] -1/10*a^10/x^10 - (20*a^9*b)/(19*x^(19/2)) - (5*a^8*b^2)/x^9 - (240*a^7*b^3
)/(17*x^(17/2)) - (105*a^6*b^4)/(4*x^8) - (168*a^5*b^5)/(5*x^(15/2)) - (30*
a^4*b^6)/x^7 - (240*a^3*b^7)/(13*x^(13/2)) - (15*a^2*b^8)/(2*x^6) - (20*a*b
^9)/(11*x^(11/2)) - b^10/(5*x^5)
```

fricas [A] time = 0.78, size = 113, normalized size = 0.81

$$-\frac{184756\,b^{10}x^5+6928350\,a^2b^8x^4+27713400\,a^4b^6x^3+24249225\,a^6b^4x^2+4618900\,a^8b^2x+92378\,a^{10}+16\left(104\right.}{923780\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^10/x^11,x, algorithm="fricas")
```

```
[Out] -1/923780*(184756*b^10*x^5 + 6928350*a^2*b^8*x^4 + 27713400*a^4*b^6*x^3 + 2
4249225*a^6*b^4*x^2 + 4618900*a^8*b^2*x + 92378*a^10 + 16*(104975*a*b^9*x^4
 + 1065900*a^3*b^7*x^3 + 1939938*a^5*b^5*x^2 + 815100*a^7*b^3*x + 60775*a^9
*b)*sqrt(x))/x^10
```

giac [A] time = 0.19, size = 112, normalized size = 0.80

$$-\frac{184756\,b^{10}x^5+1679600\,ab^9x^{\frac{9}{2}}+6928350\,a^2b^8x^4+17054400\,a^3b^7x^{\frac{7}{2}}+27713400\,a^4b^6x^3+31039008\,a^5b^5x^{\frac{5}{2}}+}{923780\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^10/x^11,x, algorithm="giac")
```

```
[Out] -1/923780*(184756*b^10*x^5 + 1679600*a*b^9*x^(9/2) + 6928350*a^2*b^8*x^4 +
17054400*a^3*b^7*x^(7/2) + 27713400*a^4*b^6*x^3 + 31039008*a^5*b^5*x^(5/2)
+ 24249225*a^6*b^4*x^2 + 13041600*a^7*b^3*x^(3/2) + 4618900*a^8*b^2*x + 972
400*a^9*b*sqrt(x) + 92378*a^10)/x^10
```

maple [A] time = 0.00, size = 113, normalized size = 0.81

$$-\frac{b^{10}}{5x^5}-\frac{20a\,b^9}{11x^{\frac{11}{2}}}-\frac{15a^2b^8}{2x^6}-\frac{240a^3b^7}{13x^{\frac{13}{2}}}-\frac{30a^4b^6}{x^7}-\frac{168a^5b^5}{5x^{\frac{15}{2}}}-\frac{105a^6b^4}{4x^8}-\frac{240a^7b^3}{17x^{\frac{17}{2}}}-\frac{5a^8b^2}{x^9}-\frac{20a^9b}{19x^{\frac{19}{2}}}-\frac{a^{10}}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^10/x^11,x)
```

```
[Out] -1/10*a^10/x^10-20/19*a^9*b/x^(19/2)-5*a^8*b^2/x^9-240/17*a^7*b^3/x^(17/2)-
105/4*a^6*b^4/x^8-168/5*a^5*b^5/x^(15/2)-30*a^4*b^6/x^7-240/13*a^3*b^7/x^(1
3/2)-15/2*a^2*b^8/x^6-20/11*a*b^9/x^(11/2)-1/5*b^10/x^5
```

maxima [A] time = 0.88, size = 112, normalized size = 0.80

$$-\frac{184756\,b^{10}x^5+1679600\,ab^9x^{\frac{9}{2}}+6928350\,a^2b^8x^4+17054400\,a^3b^7x^{\frac{7}{2}}+27713400\,a^4b^6x^3+31039008\,a^5b^5x^{\frac{5}{2}}+}{923780\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^10/x^11,x, algorithm="maxima")
```

```
[Out] -1/923780*(184756*b^10*x^5 + 1679600*a*b^9*x^(9/2) + 6928350*a^2*b^8*x^4 +
17054400*a^3*b^7*x^(7/2) + 27713400*a^4*b^6*x^3 + 31039008*a^5*b^5*x^(5/2)
+ 24249225*a^6*b^4*x^2 + 13041600*a^7*b^3*x^(3/2) + 4618900*a^8*b^2*x + 972
400*a^9*b*sqrt(x) + 92378*a^10)/x^10
```

mupad [B] time = 1.12, size = 112, normalized size = 0.80

$$-\frac{\frac{a^{10}}{10}+\frac{b^{10}x^5}{5}+5a^8b^2x+\frac{20a^9b\sqrt{x}}{19}+\frac{20ab^9x^{9/2}}{11}+\frac{105a^6b^4x^2}{4}+30a^4b^6x^3+\frac{15a^2b^8x^4}{2}+\frac{240a^7b^3x^{3/2}}{17}+\frac{168a^5b^5x^{5/2}}{5}+}{x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^10/x^11,x)
```

```
[Out] -(a^10/10 + (b^10*x^5)/5 + 5*a^8*b^2*x + (20*a^9*b*x^(1/2))/19 + (20*a*b^9*
x^(9/2))/11 + (105*a^6*b^4*x^2)/4 + 30*a^4*b^6*x^3 + (15*a^2*b^8*x^4)/2 + (
240*a^7*b^3*x^(3/2))/17 + (168*a^5*b^5*x^(5/2))/5 + (240*a^3*b^7*x^(7/2))/1
3)/x^10
```

sympy [A] time = 12.38, size = 141, normalized size = 1.01

$$-\frac{a^{10}}{10x^{10}}-\frac{20a^9b}{19x^{\frac{19}{2}}}-\frac{5a^8b^2}{x^9}-\frac{240a^7b^3}{17x^{\frac{17}{2}}}-\frac{105a^6b^4}{4x^8}-\frac{168a^5b^5}{5x^{\frac{15}{2}}}-\frac{30a^4b^6}{x^7}-\frac{240a^3b^7}{13x^{\frac{13}{2}}}-\frac{15a^2b^8}{2x^6}-\frac{20ab^9}{11x^{\frac{11}{2}}}-\frac{b^{10}}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**10/x**11,x)
```

```
[Out] -a**10/(10*x**10) - 20*a**9*b/(19*x**(19/2)) - 5*a**8*b**2/x**9 - 240*a**7*
b**3/(17*x**(17/2)) - 105*a**6*b**4/(4*x**8) - 168*a**5*b**5/(5*x**(15/2))
- 30*a**4*b**6/x**7 - 240*a**3*b**7/(13*x**(13/2)) - 15*a**2*b**8/(2*x**6)
- 20*a*b**9/(11*x**(11/2)) - b**10/(5*x**5)
```

3.2169 $\int \left(a+b\sqrt{x}\right)^{15} x^5\,dx$

Optimal. Leaf size=242

$$-\frac{a^{11}\left(a+b\sqrt{x}\right)^{16}}{8b^{12}}+\frac{22a^{10}\left(a+b\sqrt{x}\right)^{17}}{17b^{12}}-\frac{55a^9\left(a+b\sqrt{x}\right)^{18}}{9b^{12}}+\frac{330a^8\left(a+b\sqrt{x}\right)^{19}}{19b^{12}}-\frac{33a^7\left(a+b\sqrt{x}\right)^{20}}{b^{12}}+\frac{44a^6\left(a+b\right.}{b^{12}}$$

```
[Out] -1/8*a^11*(a+b*x^(1/2))^16/b^12+22/17*a^10*(a+b*x^(1/2))^17/b^12-55/9*a^9*(
a+b*x^(1/2))^18/b^12+330/19*a^8*(a+b*x^(1/2))^19/b^12-33*a^7*(a+b*x^(1/2))^
20/b^12+44*a^6*(a+b*x^(1/2))^21/b^12-42*a^5*(a+b*x^(1/2))^22/b^12+660/23*a^
4*(a+b*x^(1/2))^23/b^12-55/4*a^3*(a+b*x^(1/2))^24/b^12+22/5*a^2*(a+b*x^(1/2
))^25/b^12-11/13*a*(a+b*x^(1/2))^26/b^12+2/27*(a+b*x^(1/2))^27/b^12
```

Rubi [A] time = 0.16, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{22a^2\left(a+b\sqrt{x}\right)^{25}}{5b^{12}}-\frac{55a^3\left(a+b\sqrt{x}\right)^{24}}{4b^{12}}+\frac{660a^4\left(a+b\sqrt{x}\right)^{23}}{23b^{12}}-\frac{42a^5\left(a+b\sqrt{x}\right)^{22}}{b^{12}}+\frac{44a^6\left(a+b\sqrt{x}\right)^{21}}{b^{12}}-\frac{33a^7\left(a+b\sqrt{\right.}}{b^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^15*x^5,x]
```

```
[Out] -(a^11*(a + b*Sqrt[x])^16)/(8*b^12) + (22*a^10*(a + b*Sqrt[x])^17)/(17*b^12
) - (55*a^9*(a + b*Sqrt[x])^18)/(9*b^12) + (330*a^8*(a + b*Sqrt[x])^19)/(19
*b^12) - (33*a^7*(a + b*Sqrt[x])^20)/b^12 + (44*a^6*(a + b*Sqrt[x])^21)/b^1
2 - (42*a^5*(a + b*Sqrt[x])^22)/b^12 + (660*a^4*(a + b*Sqrt[x])^23)/(23*b^1
2) - (55*a^3*(a + b*Sqrt[x])^24)/(4*b^12) + (22*a^2*(a + b*Sqrt[x])^25)/(5*
b^12) - (11*a*(a + b*Sqrt[x])^26)/(13*b^12) + (2*(a + b*Sqrt[x])^27)/(27*b^
12)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sqrt{x}\right)^{15} x^5\,dx &= 2\,\text{Subst}\left(\int x^{11}(a+bx)^{15}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int\left(-\frac{a^{11}(a+bx)^{15}}{b^{11}}+\frac{11a^{10}(a+bx)^{16}}{b^{11}}-\frac{55a^9(a+bx)^{17}}{b^{11}}+\frac{165a^8(a+bx)^{18}}{b^{11}}-\frac{33}{}\right.\right.\\ &= -\frac{a^{11}\left(a+b\sqrt{x}\right)^{16}}{8b^{12}}+\frac{22a^{10}\left(a+b\sqrt{x}\right)^{17}}{17b^{12}}-\frac{55a^9\left(a+b\sqrt{x}\right)^{18}}{9b^{12}}+\frac{330a^8\left(a+b\sqrt{x}\right)^{19}}{19b^{12}}-\frac{33}{}\end{aligned}$$

Mathematica [A] time = 0.12, size = 211, normalized size = 0.87

$$\frac{a^{15}x^6}{6}+\frac{30}{13}a^{14}bx^{13/2}+15a^{13}b^2x^7+\frac{182}{3}a^{12}b^3x^{15/2}+\frac{1365}{8}a^{11}b^4x^8+\frac{6006}{17}a^{10}b^5x^{17/2}+\frac{5005}{9}a^9b^6x^9+\frac{12870}{19}a^8b^7x^{19/2}+\frac{12}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^15*x^5,x]
```

```
[Out] (a^15*x^6)/6 + (30*a^14*b*x^(13/2))/13 + 15*a^13*b^2*x^7 + (182*a^12*b^3*x^
(15/2))/3 + (1365*a^11*b^4*x^8)/8 + (6006*a^10*b^5*x^(17/2))/17 + (5005*a^9
*b^6*x^9)/9 + (12870*a^8*b^7*x^(19/2))/19 + (1287*a^7*b^8*x^10)/2 + (1430*a
^6*b^9*x^(21/2))/3 + 273*a^5*b^10*x^11 + (2730*a^4*b^11*x^(23/2))/23 + (455
*a^3*b^12*x^12)/12 + (42*a^2*b^13*x^(25/2))/5 + (15*a*b^14*x^13)/13 + (2*b^
15*x^(27/2))/27
```

fricas [A] time = 0.57, size = 173, normalized size = 0.71

$$\frac{15}{13}ab^{14}x^{13}+\frac{455}{12}a^3b^{12}x^{12}+273a^5b^{10}x^{11}+\frac{1287}{2}a^7b^8x^{10}+\frac{5005}{9}a^9b^6x^9+\frac{1365}{8}a^{11}b^4x^8+15a^{13}b^2x^7+\frac{1}{6}a^{15}x^6+\frac{}{130}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*x^(1/2))^15,x, algorithm="fricas")
```

```
[Out] 15/13*a*b^14*x^13 + 455/12*a^3*b^12*x^12 + 273*a^5*b^10*x^11 + 1287/2*a^7*b
^8*x^10 + 5005/9*a^9*b^6*x^9 + 1365/8*a^11*b^4*x^8 + 15*a^13*b^2*x^7 + 1/6*
a^15*x^6 + 2/13037895*(482885*b^15*x^13 + 54759159*a^2*b^13*x^12 + 77377072
5*a^4*b^11*x^11 + 3107364975*a^6*b^9*x^10 + 4415729175*a^8*b^7*x^9 + 230310
5805*a^10*b^5*x^8 + 395482815*a^12*b^3*x^7 + 15043725*a^14*b*x^6)*sqrt(x)
```

giac [A] time = 0.21, size = 167, normalized size = 0.69

$$\frac{2}{27}b^{15}x^{\frac{27}{2}}+\frac{15}{13}ab^{14}x^{13}+\frac{42}{5}a^2b^{13}x^{\frac{25}{2}}+\frac{455}{12}a^3b^{12}x^{12}+\frac{2730}{23}a^4b^{11}x^{\frac{23}{2}}+273a^5b^{10}x^{11}+\frac{1430}{3}a^6b^9x^{\frac{21}{2}}+\frac{1287}{2}a^7b^8x^{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*x^(1/2))^15,x, algorithm="giac")
```

```
[Out] 2/27*b^15*x^(27/2) + 15/13*a*b^14*x^13 + 42/5*a^2*b^13*x^(25/2) + 455/12*a^
3*b^12*x^12 + 2730/23*a^4*b^11*x^(23/2) + 273*a^5*b^10*x^11 + 1430/3*a^6*b^
9*x^(21/2) + 1287/2*a^7*b^8*x^10 + 12870/19*a^8*b^7*x^(19/2) + 5005/9*a^9*b
^6*x^9 + 6006/17*a^10*b^5*x^(17/2) + 1365/8*a^11*b^4*x^8 + 182/3*a^12*b^3*x
^(15/2) + 15*a^13*b^2*x^7 + 30/13*a^14*b*x^(13/2) + 1/6*a^15*x^6
```

maple [A] time = 0.00, size = 168, normalized size = 0.69

$$\frac{2b^{15}x^{\frac{27}{2}}}{27}+\frac{15ab^{14}x^{13}}{13}+\frac{42a^2b^{13}x^{\frac{25}{2}}}{5}+\frac{455a^3b^{12}x^{12}}{12}+\frac{2730a^4b^{11}x^{\frac{23}{2}}}{23}+273a^5b^{10}x^{11}+\frac{1430a^6b^9x^{\frac{21}{2}}}{3}+\frac{1287a^7b^8x^{10}}{2}+\frac{1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*x^(1/2))^15,x)
```

```
[Out] 2/27*b^15*x^(27/2)+15/13*x^13*a*b^14+42/5*a^2*b^13*x^(25/2)+455/12*x^12*a^3
*b^12+2730/23*a^4*b^11*x^(23/2)+273*x^11*a^5*b^10+1430/3*a^6*b^9*x^(21/2)+1
287/2*x^10*a^7*b^8+12870/19*a^8*b^7*x^(19/2)+5005/9*x^9*a^9*b^6+6006/17*a^1
0*b^5*x^(17/2)+1365/8*x^8*a^11*b^4+182/3*a^12*b^3*x^(15/2)+15*x^7*a^13*b^2+
30/13*a^14*b*x^(13/2)+1/6*a^15*x^6
```

maxima [A] time = 0.91, size = 200, normalized size = 0.83

$$\frac{2\left(b\sqrt{x}+a\right)^{27}}{27\,b^{12}}-\frac{11\left(b\sqrt{x}+a\right)^{26}a}{13\,b^{12}}+\frac{22\left(b\sqrt{x}+a\right)^{25}a^2}{5\,b^{12}}-\frac{55\left(b\sqrt{x}+a\right)^{24}a^3}{4\,b^{12}}+\frac{660\left(b\sqrt{x}+a\right)^{23}a^4}{23\,b^{12}}-\frac{42\left(b\sqrt{x}+a\right)^{2}}{b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*x^(1/2))^15,x, algorithm="maxima")
```

```
[Out] 2/27*(b*sqrt(x) + a)^27/b^12 - 11/13*(b*sqrt(x) + a)^26*a/b^12 + 22/5*(b*sq
rt(x) + a)^25*a^2/b^12 - 55/4*(b*sqrt(x) + a)^24*a^3/b^12 + 660/23*(b*sqrt(
x) + a)^23*a^4/b^12 - 42*(b*sqrt(x) + a)^22*a^5/b^12 + 44*(b*sqrt(x) + a)^2
1*a^6/b^12 - 33*(b*sqrt(x) + a)^20*a^7/b^12 + 330/19*(b*sqrt(x) + a)^19*a^8
/b^12 - 55/9*(b*sqrt(x) + a)^18*a^9/b^12 + 22/17*(b*sqrt(x) + a)^17*a^10/b^
12 - 1/8*(b*sqrt(x) + a)^16*a^11/b^12
```

mupad [B] time = 0.16, size = 167, normalized size = 0.69

$$\frac{a^{15}\,x^6}{6}+\frac{2\,b^{15}\,x^{27/2}}{27}+\frac{15\,a\,b^{14}\,x^{13}}{13}+\frac{30\,a^{14}\,b\,x^{13/2}}{13}+15\,a^{13}\,b^2\,x^7+\frac{1365\,a^{11}\,b^4\,x^8}{8}+\frac{5005\,a^9\,b^6\,x^9}{9}+\frac{1287\,a^7\,b^8\,x^{10}}{2}+273\,a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^(1/2))^15,x)
```

```
[Out] (a^15*x^6)/6 + (2*b^15*x^(27/2))/27 + (15*a*b^14*x^13)/13 + (30*a^14*b*x^(1
3/2))/13 + 15*a^13*b^2*x^7 + (1365*a^11*b^4*x^8)/8 + (5005*a^9*b^6*x^9)/9 +
 (1287*a^7*b^8*x^10)/2 + 273*a^5*b^10*x^11 + (455*a^3*b^12*x^12)/12 + (182*
a^12*b^3*x^(15/2))/3 + (6006*a^10*b^5*x^(17/2))/17 + (12870*a^8*b^7*x^(19/2
))/19 + (1430*a^6*b^9*x^(21/2))/3 + (2730*a^4*b^11*x^(23/2))/23 + (42*a^2*b
^13*x^(25/2))/5
```

sympy [A] time = 7.54, size = 214, normalized size = 0.88

$$\frac{a^{15}x^6}{6}+\frac{30a^{14}bx^{\frac{13}{2}}}{13}+15a^{13}b^2x^7+\frac{182a^{12}b^3x^{\frac{15}{2}}}{3}+\frac{1365a^{11}b^4x^8}{8}+\frac{6006a^{10}b^5x^{\frac{17}{2}}}{17}+\frac{5005a^9b^6x^9}{9}+\frac{12870a^8b^7x^{\frac{19}{2}}}{19}+\frac{1287a^7}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*x**(1/2))**15,x)
```

```
[Out] a**15*x**6/6 + 30*a**14*b*x**(13/2)/13 + 15*a**13*b**2*x**7 + 182*a**12*b**
3*x**(15/2)/3 + 1365*a**11*b**4*x**8/8 + 6006*a**10*b**5*x**(17/2)/17 + 500
5*a**9*b**6*x**9/9 + 12870*a**8*b**7*x**(19/2)/19 + 1287*a**7*b**8*x**10/2
+ 1430*a**6*b**9*x**(21/2)/3 + 273*a**5*b**10*x**11 + 2730*a**4*b**11*x**(2
3/2)/23 + 455*a**3*b**12*x**12/12 + 42*a**2*b**13*x**(25/2)/5 + 15*a*b**14*
x**13/13 + 2*b**15*x**(27/2)/27
```

3.2170 $\int \left(a+b\sqrt{x}\right)^{15} x^4\,dx$

Optimal. Leaf size=202

$$-\frac{a^9\left(a+b\sqrt{x}\right)^{16}}{8b^{10}}+\frac{18a^8\left(a+b\sqrt{x}\right)^{17}}{17b^{10}}-\frac{4a^7\left(a+b\sqrt{x}\right)^{18}}{b^{10}}+\frac{168a^6\left(a+b\sqrt{x}\right)^{19}}{19b^{10}}-\frac{63a^5\left(a+b\sqrt{x}\right)^{20}}{5b^{10}}+\frac{12a^4\left(a+b\sqrt{x}\right)^{21}}{b^{10}}$$

```
[Out] -1/8*a^9*(a+b*x^(1/2))^16/b^10+18/17*a^8*(a+b*x^(1/2))^17/b^10-4*a^7*(a+b*x
^(1/2))^18/b^10+168/19*a^6*(a+b*x^(1/2))^19/b^10-63/5*a^5*(a+b*x^(1/2))^20/
b^10+12*a^4*(a+b*x^(1/2))^21/b^10-84/11*a^3*(a+b*x^(1/2))^22/b^10+72/23*a^2
*(a+b*x^(1/2))^23/b^10-3/4*a*(a+b*x^(1/2))^24/b^10+2/25*(a+b*x^(1/2))^25/b^
10
```

Rubi [A] time = 0.11, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{72a^2\left(a+b\sqrt{x}\right)^{23}}{23b^{10}}-\frac{84a^3\left(a+b\sqrt{x}\right)^{22}}{11b^{10}}+\frac{12a^4\left(a+b\sqrt{x}\right)^{21}}{b^{10}}-\frac{63a^5\left(a+b\sqrt{x}\right)^{20}}{5b^{10}}+\frac{168a^6\left(a+b\sqrt{x}\right)^{19}}{19b^{10}}-\frac{4a^7\left(a+b\right.}{b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^15*x^4,x]
```

```
[Out] -(a^9*(a + b*Sqrt[x])^16)/(8*b^10) + (18*a^8*(a + b*Sqrt[x])^17)/(17*b^10)
- (4*a^7*(a + b*Sqrt[x])^18)/b^10 + (168*a^6*(a + b*Sqrt[x])^19)/(19*b^10)
- (63*a^5*(a + b*Sqrt[x])^20)/(5*b^10) + (12*a^4*(a + b*Sqrt[x])^21)/b^10 -
 (84*a^3*(a + b*Sqrt[x])^22)/(11*b^10) + (72*a^2*(a + b*Sqrt[x])^23)/(23*b^
10) - (3*a*(a + b*Sqrt[x])^24)/(4*b^10) + (2*(a + b*Sqrt[x])^25)/(25*b^10)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sqrt{x}\right)^{15} x^4\,dx &= 2\,\text{Subst}\left(\int x^9(a+bx)^{15}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int\left(-\frac{a^9(a+bx)^{15}}{b^9}+\frac{9a^8(a+bx)^{16}}{b^9}-\frac{36a^7(a+bx)^{17}}{b^9}+\frac{84a^6(a+bx)^{18}}{b^9}-\frac{126}{}\right.\right.\\ &= -\frac{a^9\left(a+b\sqrt{x}\right)^{16}}{8b^{10}}+\frac{18a^8\left(a+b\sqrt{x}\right)^{17}}{17b^{10}}-\frac{4a^7\left(a+b\sqrt{x}\right)^{18}}{b^{10}}+\frac{168a^6\left(a+b\sqrt{x}\right)^{19}}{19b^{10}}-\frac{63a}{}\end{aligned}$$

Mathematica [A] time = 0.10, size = 122, normalized size = 0.60

$$-\frac{\left(a+b\sqrt{x}\right)^{16}\left(a^9-16a^8b\sqrt{x}+136a^7b^2x-816a^6b^3x^{3/2}+3876a^5b^4x^2-15504a^4b^5x^{5/2}+54264a^3b^6x^3-170\right.}{16343800b^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^15*x^4,x]
```

```
[Out] -1/16343800*((a + b*Sqrt[x])^16*(a^9 - 16*a^8*b*Sqrt[x] + 136*a^7*b^2*x - 8
16*a^6*b^3*x^(3/2) + 3876*a^5*b^4*x^2 - 15504*a^4*b^5*x^(5/2) + 54264*a^3*b
^6*x^3 - 170544*a^2*b^7*x^(7/2) + 490314*a*b^8*x^4 - 1307504*b^9*x^(9/2)))/
b^10
```

fricas [A] time = 1.13, size = 173, normalized size = 0.86

$$\frac{5}{4}ab^{14}x^{12}+\frac{455}{11}a^3b^{12}x^{11}+\frac{3003}{10}a^5b^{10}x^{10}+715a^7b^8x^9+\frac{5005}{8}a^9b^6x^8+195a^{11}b^4x^7+\frac{35}{2}a^{13}b^2x^6+\frac{1}{5}a^{15}x^5+\frac{2}{2042975}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*x^(1/2))^15,x, algorithm="fricas")
```

```
[Out] 5/4*a*b^14*x^12 + 455/11*a^3*b^12*x^11 + 3003/10*a^5*b^10*x^10 + 715*a^7*b^
8*x^9 + 5005/8*a^9*b^6*x^8 + 195*a^11*b^4*x^7 + 35/2*a^13*b^2*x^6 + 1/5*a^1
5*x^5 + 2/2042975*(81719*b^15*x^12 + 9326625*a^2*b^13*x^11 + 132793375*a^4*
b^11*x^10 + 538162625*a^6*b^9*x^9 + 773326125*a^8*b^7*x^8 + 409003595*a^10*
b^5*x^7 + 71504125*a^12*b^3*x^6 + 2785875*a^14*b*x^5)*sqrt(x)
```

giac [A] time = 0.17, size = 167, normalized size = 0.83

$$\frac{2}{25}b^{15}x^{\frac{25}{2}}+\frac{5}{4}ab^{14}x^{12}+\frac{210}{23}a^2b^{13}x^{\frac{23}{2}}+\frac{455}{11}a^3b^{12}x^{11}+130a^4b^{11}x^{\frac{21}{2}}+\frac{3003}{10}a^5b^{10}x^{10}+\frac{10010}{19}a^6b^9x^{\frac{19}{2}}+715a^7b^8x^9+\frac{1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*x^(1/2))^15,x, algorithm="giac")
```

```
[Out] 2/25*b^15*x^(25/2) + 5/4*a*b^14*x^12 + 210/23*a^2*b^13*x^(23/2) + 455/11*a^
3*b^12*x^11 + 130*a^4*b^11*x^(21/2) + 3003/10*a^5*b^10*x^10 + 10010/19*a^6*
b^9*x^(19/2) + 715*a^7*b^8*x^9 + 12870/17*a^8*b^7*x^(17/2) + 5005/8*a^9*b^6
*x^8 + 2002/5*a^10*b^5*x^(15/2) + 195*a^11*b^4*x^7 + 70*a^12*b^3*x^(13/2) +
 35/2*a^13*b^2*x^6 + 30/11*a^14*b*x^(11/2) + 1/5*a^15*x^5
```

maple [A] time = 0.00, size = 168, normalized size = 0.83

$$\frac{2b^{15}x^{\frac{25}{2}}}{25}+\frac{5a\,b^{14}x^{12}}{4}+\frac{210a^2b^{13}x^{\frac{23}{2}}}{23}+\frac{455a^3b^{12}x^{11}}{11}+130a^4b^{11}x^{\frac{21}{2}}+\frac{3003a^5b^{10}x^{10}}{10}+\frac{10010a^6b^9x^{\frac{19}{2}}}{19}+715a^7b^8x^9+\frac{12870}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*x^(1/2))^15,x)
```

```
[Out] 2/25*x^(25/2)*b^15+5/4*x^12*a*b^14+210/23*x^(23/2)*a^2*b^13+455/11*x^11*a^3
*b^12+130*x^(21/2)*a^4*b^11+3003/10*x^10*a^5*b^10+10010/19*x^(19/2)*a^6*b^9
+715*x^9*a^7*b^8+12870/17*x^(17/2)*a^8*b^7+5005/8*a^9*b^6*x^8+2002/5*x^(15/
2)*a^10*b^5+195*x^7*a^11*b^4+70*x^(13/2)*a^12*b^3+35/2*x^6*a^13*b^2+30/11*x
^(11/2)*a^14*b+1/5*a^15*x^5
```

maxima [A] time = 0.93, size = 166, normalized size = 0.82

$$\frac{2\left(b\sqrt{x}+a\right)^{25}}{25\,b^{10}}-\frac{3\left(b\sqrt{x}+a\right)^{24}a}{4\,b^{10}}+\frac{72\left(b\sqrt{x}+a\right)^{23}a^2}{23\,b^{10}}-\frac{84\left(b\sqrt{x}+a\right)^{22}a^3}{11\,b^{10}}+\frac{12\left(b\sqrt{x}+a\right)^{21}a^4}{b^{10}}-\frac{63\left(b\sqrt{x}+a\right)^{20}a^5}{5\,b^{10}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*x^(1/2))^15,x, algorithm="maxima")
```

```
[Out] 2/25*(b*sqrt(x) + a)^25/b^10 - 3/4*(b*sqrt(x) + a)^24*a/b^10 + 72/23*(b*sqr
t(x) + a)^23*a^2/b^10 - 84/11*(b*sqrt(x) + a)^22*a^3/b^10 + 12*(b*sqrt(x) +
 a)^21*a^4/b^10 - 63/5*(b*sqrt(x) + a)^20*a^5/b^10 + 168/19*(b*sqrt(x) + a)
^19*a^6/b^10 - 4*(b*sqrt(x) + a)^18*a^7/b^10 + 18/17*(b*sqrt(x) + a)^17*a^8
/b^10 - 1/8*(b*sqrt(x) + a)^16*a^9/b^10
```

mupad [B] time = 0.16, size = 167, normalized size = 0.83

$$\frac{a^{15}\,x^5}{5}+\frac{2\,b^{15}\,x^{25/2}}{25}+\frac{5\,a\,b^{14}\,x^{12}}{4}+\frac{30\,a^{14}\,b\,x^{11/2}}{11}+\frac{35\,a^{13}\,b^2\,x^6}{2}+195\,a^{11}\,b^4\,x^7+\frac{5005\,a^9\,b^6\,x^8}{8}+715\,a^7\,b^8\,x^9+\frac{3003\,a^5}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^(1/2))^15,x)
```

```
[Out] (a^15*x^5)/5 + (2*b^15*x^(25/2))/25 + (5*a*b^14*x^12)/4 + (30*a^14*b*x^(11/
2))/11 + (35*a^13*b^2*x^6)/2 + 195*a^11*b^4*x^7 + (5005*a^9*b^6*x^8)/8 + 71
5*a^7*b^8*x^9 + (3003*a^5*b^10*x^10)/10 + (455*a^3*b^12*x^11)/11 + 70*a^12*
b^3*x^(13/2) + (2002*a^10*b^5*x^(15/2))/5 + (12870*a^8*b^7*x^(17/2))/17 + (
10010*a^6*b^9*x^(19/2))/19 + 130*a^4*b^11*x^(21/2) + (210*a^2*b^13*x^(23/2)
)/23
```

sympy [A] time = 6.73, size = 211, normalized size = 1.04

$$\frac{a^{15}x^5}{5}+\frac{30a^{14}bx^{\frac{11}{2}}}{11}+\frac{35a^{13}b^2x^6}{2}+70a^{12}b^3x^{\frac{13}{2}}+195a^{11}b^4x^7+\frac{2002a^{10}b^5x^{\frac{15}{2}}}{5}+\frac{5005a^9b^6x^8}{8}+\frac{12870a^8b^7x^{\frac{17}{2}}}{17}+715a^7b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*x**(1/2))**15,x)
```

```
[Out] a**15*x**5/5 + 30*a**14*b*x**(11/2)/11 + 35*a**13*b**2*x**6/2 + 70*a**12*b*
*3*x**(13/2) + 195*a**11*b**4*x**7 + 2002*a**10*b**5*x**(15/2)/5 + 5005*a**
9*b**6*x**8/8 + 12870*a**8*b**7*x**(17/2)/17 + 715*a**7*b**8*x**9 + 10010*a
**6*b**9*x**(19/2)/19 + 3003*a**5*b**10*x**10/10 + 130*a**4*b**11*x**(21/2)
 + 455*a**3*b**12*x**11/11 + 210*a**2*b**13*x**(23/2)/23 + 5*a*b**14*x**12/
4 + 2*b**15*x**(25/2)/25
```

3.2171 $\int \left(a+b\sqrt{x}\right)^{15} x^3\,dx$

Optimal. Leaf size=162

$$-\frac{a^7\left(a+b\sqrt{x}\right)^{16}}{8b^8}+\frac{14a^6\left(a+b\sqrt{x}\right)^{17}}{17b^8}-\frac{7a^5\left(a+b\sqrt{x}\right)^{18}}{3b^8}+\frac{70a^4\left(a+b\sqrt{x}\right)^{19}}{19b^8}-\frac{7a^3\left(a+b\sqrt{x}\right)^{20}}{2b^8}+\frac{2a^2\left(a+b\sqrt{x}\right)^{21}}{b^8}$$

```
[Out] -1/8*a^7*(a+b*x^(1/2))^16/b^8+14/17*a^6*(a+b*x^(1/2))^17/b^8-7/3*a^5*(a+b*x
^(1/2))^18/b^8+70/19*a^4*(a+b*x^(1/2))^19/b^8-7/2*a^3*(a+b*x^(1/2))^20/b^8+
2*a^2*(a+b*x^(1/2))^21/b^8-7/11*a*(a+b*x^(1/2))^22/b^8+2/23*(a+b*x^(1/2))^2
3/b^8
```

Rubi [A] time = 0.09, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{2a^2\left(a+b\sqrt{x}\right)^{21}}{b^8}-\frac{7a^3\left(a+b\sqrt{x}\right)^{20}}{2b^8}+\frac{70a^4\left(a+b\sqrt{x}\right)^{19}}{19b^8}-\frac{7a^5\left(a+b\sqrt{x}\right)^{18}}{3b^8}+\frac{14a^6\left(a+b\sqrt{x}\right)^{17}}{17b^8}-\frac{a^7\left(a+b\sqrt{x}\right)^{16}}{8b^8}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^15*x^3,x]
```

```
[Out] -(a^7*(a + b*Sqrt[x])^16)/(8*b^8) + (14*a^6*(a + b*Sqrt[x])^17)/(17*b^8) -
(7*a^5*(a + b*Sqrt[x])^18)/(3*b^8) + (70*a^4*(a + b*Sqrt[x])^19)/(19*b^8) -
 (7*a^3*(a + b*Sqrt[x])^20)/(2*b^8) + (2*a^2*(a + b*Sqrt[x])^21)/b^8 - (7*a
*(a + b*Sqrt[x])^22)/(11*b^8) + (2*(a + b*Sqrt[x])^23)/(23*b^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sqrt{x}\right)^{15} x^3\,dx &= 2\,\text{Subst}\left(\int x^7(a+bx)^{15}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int\left(-\frac{a^7(a+bx)^{15}}{b^7}+\frac{7a^6(a+bx)^{16}}{b^7}-\frac{21a^5(a+bx)^{17}}{b^7}+\frac{35a^4(a+bx)^{18}}{b^7}-\frac{35a^3(a\right.\right.\\ &= -\frac{a^7\left(a+b\sqrt{x}\right)^{16}}{8b^8}+\frac{14a^6\left(a+b\sqrt{x}\right)^{17}}{17b^8}-\frac{7a^5\left(a+b\sqrt{x}\right)^{18}}{3b^8}+\frac{70a^4\left(a+b\sqrt{x}\right)^{19}}{19b^8}-\frac{7a^3\left(a+\right.}{2}\end{aligned}$$

Mathematica [A] time = 0.08, size = 98, normalized size = 0.60

$$-\frac{\left(a+b\sqrt{x}\right)^{16}\left(a^7-16a^6b\sqrt{x}+136a^5b^2x-816a^4b^3x^{3/2}+3876a^3b^4x^2-15504a^2b^5x^{5/2}+54264ab^6x^3-170544b\right.}{1961256b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^15*x^3,x]
```

```
[Out] -1/1961256*((a + b*Sqrt[x])^16*(a^7 - 16*a^6*b*Sqrt[x] + 136*a^5*b^2*x - 81
6*a^4*b^3*x^(3/2) + 3876*a^3*b^4*x^2 - 15504*a^2*b^5*x^(5/2) + 54264*a*b^6*
x^3 - 170544*b^7*x^(7/2)))/b^8
```

fricas [A] time = 1.08, size = 173, normalized size = 1.07

$$\frac{15}{11}ab^{14}x^{11}+\frac{91}{2}a^3b^{12}x^{10}+\frac{1001}{3}a^5b^{10}x^9+\frac{6435}{8}a^7b^8x^8+715a^9b^6x^7+\frac{455}{2}a^{11}b^4x^6+21a^{13}b^2x^5+\frac{1}{4}a^{15}x^4+\frac{2}{245157}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^(1/2))^15,x, algorithm="fricas")
```

```
[Out] 15/11*a*b^14*x^11 + 91/2*a^3*b^12*x^10 + 1001/3*a^5*b^10*x^9 + 6435/8*a^7*b
^8*x^8 + 715*a^9*b^6*x^7 + 455/2*a^11*b^4*x^6 + 21*a^13*b^2*x^5 + 1/4*a^15*
x^4 + 2/245157*(10659*b^15*x^11 + 1225785*a^2*b^13*x^10 + 17612595*a^4*b^11
*x^9 + 72177105*a^6*b^9*x^8 + 105172353*a^8*b^7*x^7 + 56631267*a^10*b^5*x^6
 + 10140585*a^12*b^3*x^5 + 408595*a^14*b*x^4)*sqrt(x)
```

giac [A] time = 0.16, size = 167, normalized size = 1.03

$$\frac{2}{23}b^{15}x^{\frac{23}{2}}+\frac{15}{11}ab^{14}x^{11}+10a^2b^{13}x^{\frac{21}{2}}+\frac{91}{2}a^3b^{12}x^{10}+\frac{2730}{19}a^4b^{11}x^{\frac{19}{2}}+\frac{1001}{3}a^5b^{10}x^9+\frac{10010}{17}a^6b^9x^{\frac{17}{2}}+\frac{6435}{8}a^7b^8x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^(1/2))^15,x, algorithm="giac")
```

```
[Out] 2/23*b^15*x^(23/2) + 15/11*a*b^14*x^11 + 10*a^2*b^13*x^(21/2) + 91/2*a^3*b^
12*x^10 + 2730/19*a^4*b^11*x^(19/2) + 1001/3*a^5*b^10*x^9 + 10010/17*a^6*b^
9*x^(17/2) + 6435/8*a^7*b^8*x^8 + 858*a^8*b^7*x^(15/2) + 715*a^9*b^6*x^7 +
462*a^10*b^5*x^(13/2) + 455/2*a^11*b^4*x^6 + 910/11*a^12*b^3*x^(11/2) + 21*
a^13*b^2*x^5 + 10/3*a^14*b*x^(9/2) + 1/4*a^15*x^4
```

maple [A] time = 0.00, size = 168, normalized size = 1.04

$$\frac{2b^{15}x^{\frac{23}{2}}}{23}+\frac{15a\,b^{14}x^{11}}{11}+10a^2b^{13}x^{\frac{21}{2}}+\frac{91a^3b^{12}x^{10}}{2}+\frac{2730a^4b^{11}x^{\frac{19}{2}}}{19}+\frac{1001a^5b^{10}x^9}{3}+\frac{10010a^6b^9x^{\frac{17}{2}}}{17}+\frac{6435a^7b^8x^8}{8}+85$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*x^(1/2))^15,x)
```

```
[Out] 2/23*x^(23/2)*b^15+15/11*x^11*a*b^14+10*x^(21/2)*a^2*b^13+91/2*a^3*b^12*x^1
0+2730/19*x^(19/2)*a^4*b^11+1001/3*x^9*a^5*b^10+10010/17*x^(17/2)*a^6*b^9+6
435/8*x^8*a^7*b^8+858*x^(15/2)*a^8*b^7+715*a^9*b^6*x^7+462*x^(13/2)*a^10*b^
5+455/2*x^6*a^11*b^4+910/11*x^(11/2)*a^12*b^3+21*x^5*a^13*b^2+10/3*x^(9/2)*
a^14*b+1/4*x^4*a^15
```

maxima [A] time = 0.89, size = 132, normalized size = 0.81

$$\frac{2\left(b\sqrt{x}+a\right)^{23}}{23\,b^8}-\frac{7\left(b\sqrt{x}+a\right)^{22}a}{11\,b^8}+\frac{2\left(b\sqrt{x}+a\right)^{21}a^2}{b^8}-\frac{7\left(b\sqrt{x}+a\right)^{20}a^3}{2\,b^8}+\frac{70\left(b\sqrt{x}+a\right)^{19}a^4}{19\,b^8}-\frac{7\left(b\sqrt{x}+a\right)^{18}a^5}{3\,b^8}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^(1/2))^15,x, algorithm="maxima")
```

```
[Out] 2/23*(b*sqrt(x) + a)^23/b^8 - 7/11*(b*sqrt(x) + a)^22*a/b^8 + 2*(b*sqrt(x)
 + a)^21*a^2/b^8 - 7/2*(b*sqrt(x) + a)^20*a^3/b^8 + 70/19*(b*sqrt(x) + a)^19
*a^4/b^8 - 7/3*(b*sqrt(x) + a)^18*a^5/b^8 + 14/17*(b*sqrt(x) + a)^17*a^6/b^
8 - 1/8*(b*sqrt(x) + a)^16*a^7/b^8
```

mupad [B] time = 0.16, size = 167, normalized size = 1.03

$$\frac{a^{15}x^4}{4}+\frac{2b^{15}x^{23/2}}{23}+\frac{15ab^{14}x^{11}}{11}+\frac{10a^{14}bx^{9/2}}{3}+21a^{13}b^2x^5+\frac{455a^{11}b^4x^6}{2}+715a^9b^6x^7+\frac{6435a^7b^8x^8}{8}+\frac{1001a^5b^{10}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^(1/2))^15,x)
```

```
[Out] (a^15*x^4)/4 + (2*b^15*x^(23/2))/23 + (15*a*b^14*x^11)/11 + (10*a^14*b*x^(9
/2))/3 + 21*a^13*b^2*x^5 + (455*a^11*b^4*x^6)/2 + 715*a^9*b^6*x^7 + (6435*a
^7*b^8*x^8)/8 + (1001*a^5*b^10*x^9)/3 + (91*a^3*b^12*x^10)/2 + (910*a^12*b^
3*x^(11/2))/11 + 462*a^10*b^5*x^(13/2) + 858*a^8*b^7*x^(15/2) + (10010*a^6*
b^9*x^(17/2))/17 + (2730*a^4*b^11*x^(19/2))/19 + 10*a^2*b^13*x^(21/2)
```

sympy [A] time = 6.35, size = 209, normalized size = 1.29

$$\frac{a^{15}x^4}{4}+\frac{10a^{14}bx^{\frac{9}{2}}}{3}+21a^{13}b^2x^5+\frac{910a^{12}b^3x^{\frac{11}{2}}}{11}+\frac{455a^{11}b^4x^6}{2}+462a^{10}b^5x^{\frac{13}{2}}+715a^9b^6x^7+858a^8b^7x^{\frac{15}{2}}+\frac{6435a^7b^8x^8}{8}+\frac{10}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*x**(1/2))**15,x)
```

```
[Out] a**15*x**4/4 + 10*a**14*b*x**(9/2)/3 + 21*a**13*b**2*x**5 + 910*a**12*b**3*
x**(11/2)/11 + 455*a**11*b**4*x**6/2 + 462*a**10*b**5*x**(13/2) + 715*a**9*
b**6*x**7 + 858*a**8*b**7*x**(15/2) + 6435*a**7*b**8*x**8/8 + 10010*a**6*b*
*9*x**(17/2)/17 + 1001*a**5*b**10*x**9/3 + 2730*a**4*b**11*x**(19/2)/19 + 9
1*a**3*b**12*x**10/2 + 10*a**2*b**13*x**(21/2) + 15*a*b**14*x**11/11 + 2*b*
*15*x**(23/2)/23
```

3.2172 $\int \left(a+b\sqrt{x}\right)^{15} x^2\,dx$

Optimal. Leaf size=122

$$-\frac{a^5\left(a+b\sqrt{x}\right)^{16}}{8b^6}+\frac{10a^4\left(a+b\sqrt{x}\right)^{17}}{17b^6}-\frac{10a^3\left(a+b\sqrt{x}\right)^{18}}{9b^6}+\frac{20a^2\left(a+b\sqrt{x}\right)^{19}}{19b^6}+\frac{2\left(a+b\sqrt{x}\right)^{21}}{21b^6}-\frac{a\left(a+b\sqrt{x}\right)^{20}}{2b^6}$$

```
[Out] -1/8*a^5*(a+b*x^(1/2))^16/b^6+10/17*a^4*(a+b*x^(1/2))^17/b^6-10/9*a^3*(a+b*
x^(1/2))^18/b^6+20/19*a^2*(a+b*x^(1/2))^19/b^6-1/2*a*(a+b*x^(1/2))^20/b^6+2
/21*(a+b*x^(1/2))^21/b^6
```

Rubi [A] time = 0.07, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{20a^2\left(a+b\sqrt{x}\right)^{19}}{19b^6}-\frac{10a^3\left(a+b\sqrt{x}\right)^{18}}{9b^6}+\frac{10a^4\left(a+b\sqrt{x}\right)^{17}}{17b^6}-\frac{a^5\left(a+b\sqrt{x}\right)^{16}}{8b^6}+\frac{2\left(a+b\sqrt{x}\right)^{21}}{21b^6}-\frac{a\left(a+b\sqrt{x}\right)^{20}}{2b^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^15*x^2,x]
```

```
[Out] -(a^5*(a + b*Sqrt[x])^16)/(8*b^6) + (10*a^4*(a + b*Sqrt[x])^17)/(17*b^6) -
(10*a^3*(a + b*Sqrt[x])^18)/(9*b^6) + (20*a^2*(a + b*Sqrt[x])^19)/(19*b^6)
- (a*(a + b*Sqrt[x])^20)/(2*b^6) + (2*(a + b*Sqrt[x])^21)/(21*b^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sqrt{x}\right)^{15} x^2\,dx &= 2\,\text{Subst}\left(\int x^5(a+bx)^{15}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int\left(-\frac{a^5(a+bx)^{15}}{b^5}+\frac{5a^4(a+bx)^{16}}{b^5}-\frac{10a^3(a+bx)^{17}}{b^5}+\frac{10a^2(a+bx)^{18}}{b^5}-\frac{5a(\right.\right.\right.\\ &= -\frac{a^5\left(a+b\sqrt{x}\right)^{16}}{8b^6}+\frac{10a^4\left(a+b\sqrt{x}\right)^{17}}{17b^6}-\frac{10a^3\left(a+b\sqrt{x}\right)^{18}}{9b^6}+\frac{20a^2\left(a+b\sqrt{x}\right)^{19}}{19b^6}-\frac{a\left(\right.}{}\end{aligned}$$

Mathematica [A] time = 0.07, size = 74, normalized size = 0.61

$$-\frac{\left(a+b\sqrt{x}\right)^{16}\left(a^5-16a^4b\sqrt{x}+136a^3b^2x-816a^2b^3x^{3/2}+3876ab^4x^2-15504b^5x^{5/2}\right)}{162792b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^15*x^2,x]
```

```
[Out] -1/162792*((a + b*Sqrt[x])^16*(a^5 - 16*a^4*b*Sqrt[x] + 136*a^3*b^2*x - 816
*a^2*b^3*x^(3/2) + 3876*a*b^4*x^2 - 15504*b^5*x^(5/2)))/b^6
```

fricas [A] time = 0.66, size = 173, normalized size = 1.42

$$\frac{3}{2}\,ab^{14}x^{10}+\frac{455}{9}\,a^3b^{12}x^9+\frac{3003}{8}\,a^5b^{10}x^8+\frac{6435}{7}\,a^7b^8x^7+\frac{5005}{6}\,a^9b^6x^6+273\,a^{11}b^4x^5+\frac{105}{4}\,a^{13}b^2x^4+\frac{1}{3}\,a^{15}x^3+\frac{2}{20349}\,($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^(1/2))^15,x, algorithm="fricas")
```

```
[Out] 3/2*a*b^14*x^10 + 455/9*a^3*b^12*x^9 + 3003/8*a^5*b^10*x^8 + 6435/7*a^7*b^8
*x^7 + 5005/6*a^9*b^6*x^6 + 273*a^11*b^4*x^5 + 105/4*a^13*b^2*x^4 + 1/3*a^1
5*x^3 + 2/20349*(969*b^15*x^10 + 112455*a^2*b^13*x^9 + 1633905*a^4*b^11*x^8
 + 6789783*a^6*b^9*x^7 + 10072755*a^8*b^7*x^6 + 5555277*a^10*b^5*x^5 + 1028
755*a^12*b^3*x^4 + 43605*a^14*b*x^3)*sqrt(x)
```

giac [A] time = 0.16, size = 167, normalized size = 1.37

$$\frac{2}{21}\,b^{15}x^{\frac{21}{2}}+\frac{3}{2}\,ab^{14}x^{10}+\frac{210}{19}\,a^2b^{13}x^{\frac{19}{2}}+\frac{455}{9}\,a^3b^{12}x^9+\frac{2730}{17}\,a^4b^{11}x^{\frac{17}{2}}+\frac{3003}{8}\,a^5b^{10}x^8+\frac{2002}{3}\,a^6b^9x^{\frac{15}{2}}+\frac{6435}{7}\,a^7b^8x^7+9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^(1/2))^15,x, algorithm="giac")
```

```
[Out] 2/21*b^15*x^(21/2) + 3/2*a*b^14*x^10 + 210/19*a^2*b^13*x^(19/2) + 455/9*a^3
*b^12*x^9 + 2730/17*a^4*b^11*x^(17/2) + 3003/8*a^5*b^10*x^8 + 2002/3*a^6*b^
9*x^(15/2) + 6435/7*a^7*b^8*x^7 + 990*a^8*b^7*x^(13/2) + 5005/6*a^9*b^6*x^6
 + 546*a^10*b^5*x^(11/2) + 273*a^11*b^4*x^5 + 910/9*a^12*b^3*x^(9/2) + 105/
4*a^13*b^2*x^4 + 30/7*a^14*b*x^(7/2) + 1/3*a^15*x^3
```

maple [A] time = 0.00, size = 168, normalized size = 1.38

$$\frac{2b^{15}x^{\frac{21}{2}}}{21}+\frac{3a\,b^{14}x^{10}}{2}+\frac{210a^2b^{13}x^{\frac{19}{2}}}{19}+\frac{455a^3b^{12}x^9}{9}+\frac{2730a^4b^{11}x^{\frac{17}{2}}}{17}+\frac{3003a^5b^{10}x^8}{8}+\frac{2002a^6b^9x^{\frac{15}{2}}}{3}+\frac{6435a^7b^8x^7}{7}+990a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*x^(1/2))^15,x)
```

```
[Out] 2/21*x^(21/2)*b^15+3/2*x^10*a*b^14+210/19*x^(19/2)*a^2*b^13+455/9*a^3*b^12*
x^9+2730/17*x^(17/2)*a^4*b^11+3003/8*x^8*a^5*b^10+2002/3*x^(15/2)*a^6*b^9+6
435/7*x^7*a^7*b^8+990*x^(13/2)*a^8*b^7+5005/6*x^6*a^9*b^6+546*x^(11/2)*a^10
*b^5+273*x^5*a^11*b^4+910/9*x^(9/2)*a^12*b^3+105/4*x^4*a^13*b^2+30/7*x^(7/2
)*a^14*b+1/3*x^3*a^15
```

maxima [A] time = 0.91, size = 98, normalized size = 0.80

$$\frac{2\left(b\sqrt{x}+a\right)^{21}}{21\,b^6}-\frac{\left(b\sqrt{x}+a\right)^{20}a}{2\,b^6}+\frac{20\left(b\sqrt{x}+a\right)^{19}a^2}{19\,b^6}-\frac{10\left(b\sqrt{x}+a\right)^{18}a^3}{9\,b^6}+\frac{10\left(b\sqrt{x}+a\right)^{17}a^4}{17\,b^6}-\frac{\left(b\sqrt{x}+a\right)^{16}a^5}{8\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^(1/2))^15,x, algorithm="maxima")
```

```
[Out] 2/21*(b*sqrt(x) + a)^21/b^6 - 1/2*(b*sqrt(x) + a)^20*a/b^6 + 20/19*(b*sqrt(
x) + a)^19*a^2/b^6 - 10/9*(b*sqrt(x) + a)^18*a^3/b^6 + 10/17*(b*sqrt(x) + a
)^17*a^4/b^6 - 1/8*(b*sqrt(x) + a)^16*a^5/b^6
```

mupad [B] time = 0.15, size = 167, normalized size = 1.37

$$\frac{a^{15}\,x^3}{3}+\frac{2\,b^{15}\,x^{21/2}}{21}+\frac{30\,a^{14}\,b\,x^{7/2}}{7}+\frac{3\,a\,b^{14}\,x^{10}}{2}+\frac{105\,a^{13}\,b^2\,x^4}{4}+273\,a^{11}\,b^4\,x^5+\frac{5005\,a^9\,b^6\,x^6}{6}+\frac{6435\,a^7\,b^8\,x^7}{7}+\frac{3003\,a^5}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^(1/2))^15,x)
```

```
[Out] (a^15*x^3)/3 + (2*b^15*x^(21/2))/21 + (30*a^14*b*x^(7/2))/7 + (3*a*b^14*x^1
0)/2 + (105*a^13*b^2*x^4)/4 + 273*a^11*b^4*x^5 + (5005*a^9*b^6*x^6)/6 + (64
35*a^7*b^8*x^7)/7 + (3003*a^5*b^10*x^8)/8 + (455*a^3*b^12*x^9)/9 + (910*a^1
2*b^3*x^(9/2))/9 + 546*a^10*b^5*x^(11/2) + 990*a^8*b^7*x^(13/2) + (2002*a^6
*b^9*x^(15/2))/3 + (2730*a^4*b^11*x^(17/2))/17 + (210*a^2*b^13*x^(19/2))/19
```

sympy [**A**] time = 6.04, size = 212, normalized size = 1.74

$$\frac{a^{15}x^3}{3}+\frac{30a^{14}bx^{\frac{7}{2}}}{7}+\frac{105a^{13}b^2x^4}{4}+\frac{910a^{12}b^3x^{\frac{9}{2}}}{9}+273a^{11}b^4x^5+546a^{10}b^5x^{\frac{11}{2}}+\frac{5005a^9b^6x^6}{6}+990a^8b^7x^{\frac{13}{2}}+\frac{6435a^7b^8}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*x**(1/2))**15,x)
```

```
[Out] a**15*x**3/3 + 30*a**14*b*x**(7/2)/7 + 105*a**13*b**2*x**4/4 + 910*a**12*b*
*3*x**(9/2)/9 + 273*a**11*b**4*x**5 + 546*a**10*b**5*x**(11/2) + 5005*a**9*
b**6*x**6/6 + 990*a**8*b**7*x**(13/2) + 6435*a**7*b**8*x**7/7 + 2002*a**6*b
**9*x**(15/2)/3 + 3003*a**5*b**10*x**8/8 + 2730*a**4*b**11*x**(17/2)/17 + 4
55*a**3*b**12*x**9/9 + 210*a**2*b**13*x**(19/2)/19 + 3*a*b**14*x**10/2 + 2*
b**15*x**(21/2)/21
```

3.2173 $\int \left(a+b\sqrt{x}\right)^{15} x\,dx$

Optimal. Leaf size=80

$$-\frac{a^3\left(a+b\sqrt{x}\right)^{16}}{8b^4}+\frac{6a^2\left(a+b\sqrt{x}\right)^{17}}{17b^4}+\frac{2\left(a+b\sqrt{x}\right)^{19}}{19b^4}-\frac{a\left(a+b\sqrt{x}\right)^{18}}{3b^4}$$

```
[Out] -1/8*a^3*(a+b*x^(1/2))^16/b^4+6/17*a^2*(a+b*x^(1/2))^17/b^4-1/3*a*(a+b*x^(1
/2))^18/b^4+2/19*(a+b*x^(1/2))^19/b^4
```

Rubi [A] time = 0.06, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{6a^2\left(a+b\sqrt{x}\right)^{17}}{17b^4}-\frac{a^3\left(a+b\sqrt{x}\right)^{16}}{8b^4}+\frac{2\left(a+b\sqrt{x}\right)^{19}}{19b^4}-\frac{a\left(a+b\sqrt{x}\right)^{18}}{3b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^15*x,x]
```

```
[Out] -(a^3*(a + b*Sqrt[x])^16)/(8*b^4) + (6*a^2*(a + b*Sqrt[x])^17)/(17*b^4) - (
a*(a + b*Sqrt[x])^18)/(3*b^4) + (2*(a + b*Sqrt[x])^19)/(19*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sqrt{x}\right)^{15} x\,dx &= 2\,\text{Subst}\left(\int x^3(a+bx)^{15}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int\left(-\frac{a^3(a+bx)^{15}}{b^3}+\frac{3a^2(a+bx)^{16}}{b^3}-\frac{3a(a+bx)^{17}}{b^3}+\frac{(a+bx)^{18}}{b^3}\right)dx, x, \sqrt{x}\right)\\ &= -\frac{a^3\left(a+b\sqrt{x}\right)^{16}}{8b^4}+\frac{6a^2\left(a+b\sqrt{x}\right)^{17}}{17b^4}-\frac{a\left(a+b\sqrt{x}\right)^{18}}{3b^4}+\frac{2\left(a+b\sqrt{x}\right)^{19}}{19b^4}\end{aligned}$$

Mathematica [A] time = 0.06, size = 50, normalized size = 0.62

$$-\frac{\left(a+b\sqrt{x}\right)^{16}\left(a^3-16a^2b\sqrt{x}+136ab^2x-816b^3x^{3/2}\right)}{7752b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^15*x,x]
```

```
[Out] -1/7752*((a + b*Sqrt[x])^16*(a^3 - 16*a^2*b*Sqrt[x] + 136*a*b^2*x - 816*b^3
*x^(3/2)))/b^4
```

fricas [B] time = 1.14, size = 173, normalized size = 2.16

$$\frac{5}{3}ab^{14}x^9+\frac{455}{8}a^3b^{12}x^8+429a^5b^{10}x^7+\frac{2145}{2}a^7b^8x^6+1001a^9b^6x^5+\frac{1365}{4}a^{11}b^4x^4+35a^{13}b^2x^3+\frac{1}{2}a^{15}x^2+\frac{2}{969}\left(51\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^(1/2))^15,x, algorithm="fricas")
```

```
[Out] 5/3*a*b^14*x^9 + 455/8*a^3*b^12*x^8 + 429*a^5*b^10*x^7 + 2145/2*a^7*b^8*x^6
 + 1001*a^9*b^6*x^5 + 1365/4*a^11*b^4*x^4 + 35*a^13*b^2*x^3 + 1/2*a^15*x^2
+ 2/969*(51*b^15*x^9 + 5985*a^2*b^13*x^8 + 88179*a^4*b^11*x^7 + 373065*a^6*
b^9*x^6 + 566865*a^8*b^7*x^5 + 323323*a^10*b^5*x^4 + 62985*a^12*b^3*x^3 + 2
907*a^14*b*x^2)*sqrt(x)
```

giac [B] time = 0.16, size = 167, normalized size = 2.09

$$\frac{2}{19}b^{15}x^{\frac{19}{2}}+\frac{5}{3}ab^{14}x^9+\frac{210}{17}a^2b^{13}x^{\frac{17}{2}}+\frac{455}{8}a^3b^{12}x^8+182a^4b^{11}x^{\frac{15}{2}}+429a^5b^{10}x^7+770a^6b^9x^{\frac{13}{2}}+\frac{2145}{2}a^7b^8x^6+1170$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^(1/2))^15,x, algorithm="giac")
```

```
[Out] 2/19*b^15*x^(19/2) + 5/3*a*b^14*x^9 + 210/17*a^2*b^13*x^(17/2) + 455/8*a^3*
b^12*x^8 + 182*a^4*b^11*x^(15/2) + 429*a^5*b^10*x^7 + 770*a^6*b^9*x^(13/2)
+ 2145/2*a^7*b^8*x^6 + 1170*a^8*b^7*x^(11/2) + 1001*a^9*b^6*x^5 + 2002/3*a^
10*b^5*x^(9/2) + 1365/4*a^11*b^4*x^4 + 130*a^12*b^3*x^(7/2) + 35*a^13*b^2*x
^3 + 6*a^14*b*x^(5/2) + 1/2*a^15*x^2
```

maple [B] time = 0.00, size = 168, normalized size = 2.10

$$\frac{2b^{15}x^{\frac{19}{2}}}{19}+\frac{5a\,b^{14}x^9}{3}+\frac{210a^2b^{13}x^{\frac{17}{2}}}{17}+\frac{455a^3b^{12}x^8}{8}+182a^4b^{11}x^{\frac{15}{2}}+429a^5b^{10}x^7+770a^6b^9x^{\frac{13}{2}}+\frac{2145a^7b^8x^6}{2}+1170a^8b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*x^(1/2))^15,x)
```

```
[Out] 2/19*x^(19/2)*b^15+5/3*x^9*a*b^14+210/17*x^(17/2)*a^2*b^13+455/8*x^8*a^3*b^
12+182*x^(15/2)*a^4*b^11+429*x^7*a^5*b^10+770*x^(13/2)*a^6*b^9+2145/2*x^6*a
^7*b^8+1170*x^(11/2)*a^8*b^7+1001*x^5*a^9*b^6+2002/3*x^(9/2)*a^10*b^5+1365/
4*x^4*a^11*b^4+130*x^(7/2)*a^12*b^3+35*x^3*a^13*b^2+6*x^(5/2)*a^14*b+1/2*x^
2*a^15
```

maxima [A] time = 0.89, size = 64, normalized size = 0.80

$$\frac{2\left(b\sqrt{x}+a\right)^{19}}{19\,b^4}-\frac{\left(b\sqrt{x}+a\right)^{18}a}{3\,b^4}+\frac{6\left(b\sqrt{x}+a\right)^{17}a^2}{17\,b^4}-\frac{\left(b\sqrt{x}+a\right)^{16}a^3}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^(1/2))^15,x, algorithm="maxima")
```

```
[Out] 2/19*(b*sqrt(x) + a)^19/b^4 - 1/3*(b*sqrt(x) + a)^18*a/b^4 + 6/17*(b*sqrt(x
) + a)^17*a^2/b^4 - 1/8*(b*sqrt(x) + a)^16*a^3/b^4
```

mupad [B] time = 0.16, size = 167, normalized size = 2.09

$$\frac{a^{15}x^2}{2}+\frac{2\,b^{15}x^{19/2}}{19}+6\,a^{14}\,b\,x^{5/2}+\frac{5\,a\,b^{14}\,x^9}{3}+35\,a^{13}\,b^2\,x^3+\frac{1365\,a^{11}\,b^4\,x^4}{4}+1001\,a^9\,b^6\,x^5+\frac{2145\,a^7\,b^8\,x^6}{2}+429\,a^5\,b^1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^(1/2))^15,x)
```

```
[Out] (a^15*x^2)/2 + (2*b^15*x^(19/2))/19 + 6*a^14*b*x^(5/2) + (5*a*b^14*x^9)/3 +
 35*a^13*b^2*x^3 + (1365*a^11*b^4*x^4)/4 + 1001*a^9*b^6*x^5 + (2145*a^7*b^8
*x^6)/2 + 429*a^5*b^10*x^7 + (455*a^3*b^12*x^8)/8 + 130*a^12*b^3*x^(7/2) +
(2002*a^10*b^5*x^(9/2))/3 + 1170*a^8*b^7*x^(11/2) + 770*a^6*b^9*x^(13/2) +
182*a^4*b^11*x^(15/2) + (210*a^2*b^13*x^(17/2))/17
```

sympy [B] time = 5.79, size = 204, normalized size = 2.55

$$\frac{a^{15}x^2}{2}+6a^{14}bx^{\frac{5}{2}}+35a^{13}b^2x^3+130a^{12}b^3x^{\frac{7}{2}}+\frac{1365a^{11}b^4x^4}{4}+\frac{2002a^{10}b^5x^{\frac{9}{2}}}{3}+1001a^9b^6x^5+1170a^8b^7x^{\frac{11}{2}}+\frac{2145a^7b^8x^6}{2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x**(1/2))**15,x)
```

```
[Out] a**15*x**2/2 + 6*a**14*b*x**(5/2) + 35*a**13*b**2*x**3 + 130*a**12*b**3*x**
(7/2) + 1365*a**11*b**4*x**4/4 + 2002*a**10*b**5*x**(9/2)/3 + 1001*a**9*b**
6*x**5 + 1170*a**8*b**7*x**(11/2) + 2145*a**7*b**8*x**6/2 + 770*a**6*b**9*x
**(13/2) + 429*a**5*b**10*x**7 + 182*a**4*b**11*x**(15/2) + 455*a**3*b**12*
x**8/8 + 210*a**2*b**13*x**(17/2)/17 + 5*a*b**14*x**9/3 + 2*b**15*x**(19/2)
/19
```

3.2174 $\int \left(a+b\sqrt{x}\right)^{15} dx$

Optimal. Leaf size=38

$$\frac{2\left(a+b\sqrt{x}\right)^{17}}{17b^2}-\frac{a\left(a+b\sqrt{x}\right)^{16}}{8b^2}$$

```
[Out] -1/8*a*(a+b*x^(1/2))^16/b^2+2/17*(a+b*x^(1/2))^17/b^2
```

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {190, 43}

$$\frac{2\left(a+b\sqrt{x}\right)^{17}}{17b^2}-\frac{a\left(a+b\sqrt{x}\right)^{16}}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^15,x]
```

```
[Out] -(a*(a + b*Sqrt[x])^16)/(8*b^2) + (2*(a + b*Sqrt[x])^17)/(17*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sqrt{x}\right)^{15} dx &= 2\,\text{Subst}\left(\int x(a+bx)^{15}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(-\frac{a(a+bx)^{15}}{b}+\frac{(a+bx)^{16}}{b}\right) dx, x, \sqrt{x}\right)\\ &= -\frac{a\left(a+b\sqrt{x}\right)^{16}}{8b^2}+\frac{2\left(a+b\sqrt{x}\right)^{17}}{17b^2}\end{aligned}$$

Mathematica [A] time = 0.04, size = 28, normalized size = 0.74

$$-\frac{\left(a-16b\sqrt{x}\right)\left(a+b\sqrt{x}\right)^{16}}{136b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^15,x]
```

```
[Out] -1/136*((a - 16*b*Sqrt[x])*(a + b*Sqrt[x])^16)/b^2
```

fricas [B] time = 1.17, size = 167, normalized size = 4.39

$$\frac{15}{8}ab^{14}x^8+65\,a^3b^{12}x^7+\frac{1001}{2}a^5b^{10}x^6+1287\,a^7b^8x^5+\frac{5005}{4}a^9b^6x^4+455\,a^{11}b^4x^3+\frac{105}{2}a^{13}b^2x^2+a^{15}x+\frac{2}{17}\left(b^{15}x^8+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15,x, algorithm="fricas")
```

```
[Out] 15/8*a*b^14*x^8 + 65*a^3*b^12*x^7 + 1001/2*a^5*b^10*x^6 + 1287*a^7*b^8*x^5
+ 5005/4*a^9*b^6*x^4 + 455*a^11*b^4*x^3 + 105/2*a^13*b^2*x^2 + a^15*x + 2/1
7*(b^15*x^8 + 119*a^2*b^13*x^7 + 1785*a^4*b^11*x^6 + 7735*a^6*b^9*x^5 + 121
55*a^8*b^7*x^4 + 7293*a^10*b^5*x^3 + 1547*a^12*b^3*x^2 + 85*a^14*b*x)*sqrt(
x)
```

giac [B] time = 0.16, size = 164, normalized size = 4.32

$$\frac{2}{17}b^{15}x^{\frac{17}{2}}+\frac{15}{8}ab^{14}x^8+14a^2b^{13}x^{\frac{15}{2}}+65a^3b^{12}x^7+210a^4b^{11}x^{\frac{13}{2}}+\frac{1001}{2}a^5b^{10}x^6+910a^6b^9x^{\frac{11}{2}}+1287a^7b^8x^5+1430a^8b^$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15,x, algorithm="giac")
```

```
[Out] 2/17*b^15*x^(17/2) + 15/8*a*b^14*x^8 + 14*a^2*b^13*x^(15/2) + 65*a^3*b^12*x
^7 + 210*a^4*b^11*x^(13/2) + 1001/2*a^5*b^10*x^6 + 910*a^6*b^9*x^(11/2) + 1
287*a^7*b^8*x^5 + 1430*a^8*b^7*x^(9/2) + 5005/4*a^9*b^6*x^4 + 858*a^10*b^5*
x^(7/2) + 455*a^11*b^4*x^3 + 182*a^12*b^3*x^(5/2) + 105/2*a^13*b^2*x^2 + 10
*a^14*b*x^(3/2) + a^15*x
```

maple [B] time = 0.00, size = 165, normalized size = 4.34

$$\frac{2b^{15}x^{\frac{17}{2}}}{17}+\frac{15ab^{14}x^8}{8}+14a^2b^{13}x^{\frac{15}{2}}+65a^3b^{12}x^7+210a^4b^{11}x^{\frac{13}{2}}+\frac{1001a^5b^{10}x^6}{2}+910a^6b^9x^{\frac{11}{2}}+1287a^7b^8x^5+1430a^8b^7x^{\frac{9}{2}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^15,x)
```

```
[Out] 2/17*x^(17/2)*b^15+15/8*x^8*a*b^14+14*x^(15/2)*a^2*b^13+65*x^7*a^3*b^12+210
*x^(13/2)*a^4*b^11+1001/2*x^6*a^5*b^10+910*x^(11/2)*a^6*b^9+1287*x^5*a^7*b^
8+1430*x^(9/2)*a^8*b^7+5005/4*x^4*a^9*b^6+858*x^(7/2)*a^10*b^5+455*x^3*a^11
*b^4+182*x^(5/2)*a^12*b^3+105/2*x^2*a^13*b^2+10*x^(3/2)*a^14*b+x*a^15
```

maxima [A] time = 0.85, size = 30, normalized size = 0.79

$$\frac{2\left(b\sqrt{x}+a\right)^{17}}{17\,b^2}-\frac{\left(b\sqrt{x}+a\right)^{16}a}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15,x, algorithm="maxima")
```

```
[Out] 2/17*(b*sqrt(x) + a)^17/b^2 - 1/8*(b*sqrt(x) + a)^16*a/b^2
```

mupad [B] time = 0.16, size = 164, normalized size = 4.32

$$a^{15}x+\frac{2\,b^{15}\,x^{17/2}}{17}+10\,a^{14}\,b\,x^{3/2}+\frac{15\,a\,b^{14}\,x^8}{8}+\frac{105\,a^{13}\,b^2\,x^2}{2}+455\,a^{11}\,b^4\,x^3+\frac{5005\,a^9\,b^6\,x^4}{4}+1287\,a^7\,b^8\,x^5+\frac{1001\,a^5\,b^{10}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^15,x)
```

```
[Out] a^15*x + (2*b^15*x^(17/2))/17 + 10*a^14*b*x^(3/2) + (15*a*b^14*x^8)/8 + (10
5*a^13*b^2*x^2)/2 + 455*a^11*b^4*x^3 + (5005*a^9*b^6*x^4)/4 + 1287*a^7*b^8*
x^5 + (1001*a^5*b^10*x^6)/2 + 65*a^3*b^12*x^7 + 182*a^12*b^3*x^(5/2) + 858*
```

```
a^10*b^5*x^(7/2) + 1430*a^8*b^7*x^(9/2) + 910*a^6*b^9*x^(11/2) + 210*a^4*b^
11*x^(13/2) + 14*a^2*b^13*x^(15/2)
```

sympy [**B**] time = 5.43, size = 197, normalized size = 5.18

$$a^{15}x+10a^{14}bx^{\frac{3}{2}}+\frac{105a^{13}b^2x^2}{2}+182a^{12}b^3x^{\frac{5}{2}}+455a^{11}b^4x^3+858a^{10}b^5x^{\frac{7}{2}}+\frac{5005a^9b^6x^4}{4}+1430a^8b^7x^{\frac{9}{2}}+1287a^7b^8x^5+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**15,x)
```

```
[Out] a**15*x + 10*a**14*b*x**(3/2) + 105*a**13*b**2*x**2/2 + 182*a**12*b**3*x**(
5/2) + 455*a**11*b**4*x**3 + 858*a**10*b**5*x**(7/2) + 5005*a**9*b**6*x**4/
4 + 1430*a**8*b**7*x**(9/2) + 1287*a**7*b**8*x**5 + 910*a**6*b**9*x**(11/2)
 + 1001*a**5*b**10*x**6/2 + 210*a**4*b**11*x**(13/2) + 65*a**3*b**12*x**7 +
 14*a**2*b**13*x**(15/2) + 15*a*b**14*x**8/8 + 2*b**15*x**(17/2)/17
```

3.2175 $\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x}\,dx$

Optimal. Leaf size=205

$$a^{15}\log(x)+30a^{14}b\sqrt{x}+105a^{13}b^2x+\frac{910}{3}a^{12}b^3x^{3/2}+\frac{1365}{2}a^{11}b^4x^2+\frac{6006}{5}a^{10}b^5x^{5/2}+\frac{5005}{3}a^9b^6x^3+\frac{12870}{7}a^8b^7x^{7/2}+\frac{6}{}$$

```
[Out] 105*a^13*b^2*x+910/3*a^12*b^3*x^(3/2)+1365/2*a^11*b^4*x^2+6006/5*a^10*b^5*x
^(5/2)+5005/3*a^9*b^6*x^3+12870/7*a^8*b^7*x^(7/2)+6435/4*a^7*b^8*x^4+10010/
9*a^6*b^9*x^(9/2)+3003/5*a^5*b^10*x^5+2730/11*a^4*b^11*x^(11/2)+455/6*a^3*b
^12*x^6+210/13*a^2*b^13*x^(13/2)+15/7*a*b^14*x^7+2/15*b^15*x^(15/2)+a^15*ln
(x)+30*a^14*b*x^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{910}{3}a^{12}b^3x^{3/2}+\frac{1365}{2}a^{11}b^4x^2+\frac{6006}{5}a^{10}b^5x^{5/2}+\frac{5005}{3}a^9b^6x^3+\frac{12870}{7}a^8b^7x^{7/2}+\frac{6435}{4}a^7b^8x^4+\frac{10010}{9}a^6b^9x^{9/2}+\frac{3003}{5}a^{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^15/x,x]
```

```
[Out] 30*a^14*b*Sqrt[x] + 105*a^13*b^2*x + (910*a^12*b^3*x^(3/2))/3 + (1365*a^11*
b^4*x^2)/2 + (6006*a^10*b^5*x^(5/2))/5 + (5005*a^9*b^6*x^3)/3 + (12870*a^8*
b^7*x^(7/2))/7 + (6435*a^7*b^8*x^4)/4 + (10010*a^6*b^9*x^(9/2))/9 + (3003*a
^5*b^10*x^5)/5 + (2730*a^4*b^11*x^(11/2))/11 + (455*a^3*b^12*x^6)/6 + (210*
a^2*b^13*x^(13/2))/13 + (15*a*b^14*x^7)/7 + (2*b^15*x^(15/2))/15 + a^15*Log
[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x}\,dx &= 2\,\text{Subst}\left(\int \frac{(a+bx)^{15}}{x}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(15a^{14}b+\frac{a^{15}}{x}+105a^{13}b^2x+455a^{12}b^3x^2+1365a^{11}b^4x^3+3003a^{10}b^5x^4+5005\right.\right.\\ &= 30a^{14}b\sqrt{x}+105a^{13}b^2x+\frac{910}{3}a^{12}b^3x^{3/2}+\frac{1365}{2}a^{11}b^4x^2+\frac{6006}{5}a^{10}b^5x^{5/2}+\frac{5005}{3}a^9b^6x^3+\frac{1}{}\end{aligned}$$

Mathematica [A] time = 0.11, size = 205, normalized size = 1.00

$$a^{15}\log(x)+30a^{14}b\sqrt{x}+105a^{13}b^2x+\frac{910}{3}a^{12}b^3x^{3/2}+\frac{1365}{2}a^{11}b^4x^2+\frac{6006}{5}a^{10}b^5x^{5/2}+\frac{5005}{3}a^9b^6x^3+\frac{12870}{7}a^8b^7x^{7/2}+\frac{6}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^15/x,x]
```

```
[Out] 30*a^14*b*Sqrt[x] + 105*a^13*b^2*x + (910*a^12*b^3*x^(3/2))/3 + (1365*a^11*
b^4*x^2)/2 + (6006*a^10*b^5*x^(5/2))/5 + (5005*a^9*b^6*x^3)/3 + (12870*a^8*
b^7*x^(7/2))/7 + (6435*a^7*b^8*x^4)/4 + (10010*a^6*b^9*x^(9/2))/9 + (3003*a
^5*b^10*x^5)/5 + (2730*a^4*b^11*x^(11/2))/11 + (455*a^3*b^12*x^6)/6 + (210*
a^2*b^13*x^(13/2))/13 + (15*a*b^14*x^7)/7 + (2*b^15*x^(15/2))/15 + a^15*Log
[x]
```

fricas [A] time = 1.21, size = 167, normalized size = 0.81

$$\frac{15}{7}\,ab^{14}x^7+\frac{455}{6}\,a^3b^{12}x^6+\frac{3003}{5}\,a^5b^{10}x^5+\frac{6435}{4}\,a^7b^8x^4+\frac{5005}{3}\,a^9b^6x^3+\frac{1365}{2}\,a^{11}b^4x^2+105\,a^{13}b^2x+2\,a^{15}\log\left(\sqrt{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x,x, algorithm="fricas")
```

```
[Out] 15/7*a*b^14*x^7 + 455/6*a^3*b^12*x^6 + 3003/5*a^5*b^10*x^5 + 6435/4*a^7*b^8
*x^4 + 5005/3*a^9*b^6*x^3 + 1365/2*a^11*b^4*x^2 + 105*a^13*b^2*x + 2*a^15*l
og(sqrt(x)) + 2/45045*(3003*b^15*x^7 + 363825*a^2*b^13*x^6 + 5589675*a^4*b^
11*x^5 + 25050025*a^6*b^9*x^4 + 41409225*a^8*b^7*x^3 + 27054027*a^10*b^5*x^
2 + 6831825*a^12*b^3*x + 675675*a^14*b)*sqrt(x)
```

giac [A] time = 0.17, size = 164, normalized size = 0.80

$$\frac{2}{15}\,b^{15}x^{\frac{15}{2}}+\frac{15}{7}\,ab^{14}x^7+\frac{210}{13}\,a^2b^{13}x^{\frac{13}{2}}+\frac{455}{6}\,a^3b^{12}x^6+\frac{2730}{11}\,a^4b^{11}x^{\frac{11}{2}}+\frac{3003}{5}\,a^5b^{10}x^5+\frac{10010}{9}\,a^6b^9x^{\frac{9}{2}}+\frac{6435}{4}\,a^7b^8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x,x, algorithm="giac")
```

```
[Out] 2/15*b^15*x^(15/2) + 15/7*a*b^14*x^7 + 210/13*a^2*b^13*x^(13/2) + 455/6*a^3
*b^12*x^6 + 2730/11*a^4*b^11*x^(11/2) + 3003/5*a^5*b^10*x^5 + 10010/9*a^6*b
^9*x^(9/2) + 6435/4*a^7*b^8*x^4 + 12870/7*a^8*b^7*x^(7/2) + 5005/3*a^9*b^6*
x^3 + 6006/5*a^10*b^5*x^(5/2) + 1365/2*a^11*b^4*x^2 + 910/3*a^12*b^3*x^(3/2
) + 105*a^13*b^2*x + a^15*log(abs(x)) + 30*a^14*b*sqrt(x)
```

maple [A] time = 0.00, size = 164, normalized size = 0.80

$$\frac{2b^{15}x^{\frac{15}{2}}}{15}+\frac{15a\,b^{14}x^7}{7}+\frac{210a^2b^{13}x^{\frac{13}{2}}}{13}+\frac{455a^3b^{12}x^6}{6}+\frac{2730a^4b^{11}x^{\frac{11}{2}}}{11}+\frac{3003a^5b^{10}x^5}{5}+\frac{10010a^6b^9x^{\frac{9}{2}}}{9}+\frac{6435a^7b^8x^4}{4}+\frac{1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^15/x,x)
```

```
[Out] 105*a^13*b^2*x+910/3*a^12*b^3*x^(3/2)+1365/2*a^11*b^4*x^2+6006/5*a^10*b^5*x
^(5/2)+5005/3*a^9*b^6*x^3+12870/7*a^8*b^7*x^(7/2)+6435/4*a^7*b^8*x^4+10010/
9*a^6*b^9*x^(9/2)+3003/5*a^5*b^10*x^5+2730/11*a^4*b^11*x^(11/2)+455/6*a^3*b
^12*x^6+210/13*a^2*b^13*x^(13/2)+15/7*a*b^14*x^7+2/15*b^15*x^(15/2)+a^15*ln
(x)+30*a^14*b*x^(1/2)
```

maxima [A] time = 0.86, size = 163, normalized size = 0.80

$$\frac{2}{15}\,b^{15}x^{\frac{15}{2}}+\frac{15}{7}\,ab^{14}x^7+\frac{210}{13}\,a^2b^{13}x^{\frac{13}{2}}+\frac{455}{6}\,a^3b^{12}x^6+\frac{2730}{11}\,a^4b^{11}x^{\frac{11}{2}}+\frac{3003}{5}\,a^5b^{10}x^5+\frac{10010}{9}\,a^6b^9x^{\frac{9}{2}}+\frac{6435}{4}\,a^7b^8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x,x, algorithm="maxima")
```

```
[Out] 2/15*b^15*x^(15/2) + 15/7*a*b^14*x^7 + 210/13*a^2*b^13*x^(13/2) + 455/6*a^3
*b^12*x^6 + 2730/11*a^4*b^11*x^(11/2) + 3003/5*a^5*b^10*x^5 + 10010/9*a^6*b
^9*x^(9/2) + 6435/4*a^7*b^8*x^4 + 12870/7*a^8*b^7*x^(7/2) + 5005/3*a^9*b^6*
x^3 + 6006/5*a^10*b^5*x^(5/2) + 1365/2*a^11*b^4*x^2 + 910/3*a^12*b^3*x^(3/2
) + 105*a^13*b^2*x + a^15*log(x) + 30*a^14*b*sqrt(x)
```

mupad [B] time = 0.16, size = 166, normalized size = 0.81

$$2\,a^{15}\,\ln\left(\sqrt{x}\right)+\frac{2\,b^{15}\,x^{15/2}}{15}+105\,a^{13}\,b^2\,x+30\,a^{14}\,b\,\sqrt{x}+\frac{15\,a\,b^{14}\,x^7}{7}+\frac{1365\,a^{11}\,b^4\,x^2}{2}+\frac{5005\,a^9\,b^6\,x^3}{3}+\frac{6435\,a^7\,b^8\,x^4}{4}+3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^15/x,x)
```

```
[Out] 2*a^15*log(x^(1/2)) + (2*b^15*x^(15/2))/15 + 105*a^13*b^2*x + 30*a^14*b*x^(
1/2) + (15*a*b^14*x^7)/7 + (1365*a^11*b^4*x^2)/2 + (5005*a^9*b^6*x^3)/3 + (
6435*a^7*b^8*x^4)/4 + (3003*a^5*b^10*x^5)/5 + (910*a^12*b^3*x^(3/2))/3 + (4
55*a^3*b^12*x^6)/6 + (6006*a^10*b^5*x^(5/2))/5 + (12870*a^8*b^7*x^(7/2))/7
+ (10010*a^6*b^9*x^(9/2))/9 + (2730*a^4*b^11*x^(11/2))/11 + (210*a^2*b^13*x
^(13/2))/13
```

sympy [A] time = 6.03, size = 211, normalized size = 1.03

$$a^{15}\log(x)+30a^{14}b\sqrt{x}+105a^{13}b^2x+\frac{910a^{12}b^3x^{\frac{3}{2}}}{3}+\frac{1365a^{11}b^4x^2}{2}+\frac{6006a^{10}b^5x^{\frac{5}{2}}}{5}+\frac{5005a^9b^6x^3}{3}+\frac{12870a^8b^7x^{\frac{7}{2}}}{7}+\frac{6435}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**15/x,x)
```

```
[Out] a**15*log(x) + 30*a**14*b*sqrt(x) + 105*a**13*b**2*x + 910*a**12*b**3*x**(3
/2)/3 + 1365*a**11*b**4*x**2/2 + 6006*a**10*b**5*x**(5/2)/5 + 5005*a**9*b**
6*x**3/3 + 12870*a**8*b**7*x**(7/2)/7 + 6435*a**7*b**8*x**4/4 + 10010*a**6*
b**9*x**(9/2)/9 + 3003*a**5*b**10*x**5/5 + 2730*a**4*b**11*x**(11/2)/11 + 4
55*a**3*b**12*x**6/6 + 210*a**2*b**13*x**(13/2)/13 + 15*a*b**14*x**7/7 + 2*
b**15*x**(15/2)/15
```

3.2176 $\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^2}\,dx$

Optimal. Leaf size=192

$$-\frac{a^{15}}{x}-\frac{30a^{14}b}{\sqrt{x}}+105a^{13}b^2\log(x)+910a^{12}b^3\sqrt{x}+1365a^{11}b^4x+2002a^{10}b^5x^{3/2}+\frac{5005}{2}a^9b^6x^2+2574a^8b^7x^{5/2}+2145a$$

```
[Out] -a^15/x+1365*a^11*b^4*x+2002*a^10*b^5*x^(3/2)+5005/2*a^9*b^6*x^2+2574*a^8*b
^7*x^(5/2)+2145*a^7*b^8*x^3+1430*a^6*b^9*x^(7/2)+3003/4*a^5*b^10*x^4+910/3*
a^4*b^11*x^(9/2)+91*a^3*b^12*x^5+210/11*a^2*b^13*x^(11/2)+5/2*a*b^14*x^6+2/
13*b^15*x^(13/2)+105*a^13*b^2*ln(x)-30*a^14*b/x^(1/2)+910*a^12*b^3*x^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$2002a^{10}b^5x^{3/2}+\frac{5005}{2}a^9b^6x^2+2574a^8b^7x^{5/2}+2145a^7b^8x^3+1430a^6b^9x^{7/2}+\frac{3003}{4}a^5b^{10}x^4+\frac{910}{3}a^4b^{11}x^{9/2}+91a^3b^{12}x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^15/x^2,x]
```

```
[Out] -(a^15/x) - (30*a^14*b)/Sqrt[x] + 910*a^12*b^3*Sqrt[x] + 1365*a^11*b^4*x +
2002*a^10*b^5*x^(3/2) + (5005*a^9*b^6*x^2)/2 + 2574*a^8*b^7*x^(5/2) + 2145*
a^7*b^8*x^3 + 1430*a^6*b^9*x^(7/2) + (3003*a^5*b^10*x^4)/4 + (910*a^4*b^11*
x^(9/2))/3 + 91*a^3*b^12*x^5 + (210*a^2*b^13*x^(11/2))/11 + (5*a*b^14*x^6)/
2 + (2*b^15*x^(13/2))/13 + 105*a^13*b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^2}\,dx &= 2\,\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^3}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(455a^{12}b^3+\frac{a^{15}}{x^3}+\frac{15a^{14}b}{x^2}+\frac{105a^{13}b^2}{x}+1365a^{11}b^4x+3003a^{10}b^5x^2+5005\right.\right.\\ &= -\frac{a^{15}}{x}-\frac{30a^{14}b}{\sqrt{x}}+910a^{12}b^3\sqrt{x}+1365a^{11}b^4x+2002a^{10}b^5x^{3/2}+\frac{5005}{2}a^9b^6x^2+2574a^8b^7\end{aligned}$$

Mathematica [A] time = 0.13, size = 192, normalized size = 1.00

$$-\frac{a^{15}}{x}-\frac{30a^{14}b}{\sqrt{x}}+105a^{13}b^2\log(x)+910a^{12}b^3\sqrt{x}+1365a^{11}b^4x+2002a^{10}b^5x^{3/2}+\frac{5005}{2}a^9b^6x^2+2574a^8b^7x^{5/2}+2145a$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^15/x^2,x]
```

```
[Out] -(a^15/x) - (30*a^14*b)/Sqrt[x] + 910*a^12*b^3*Sqrt[x] + 1365*a^11*b^4*x +
2002*a^10*b^5*x^(3/2) + (5005*a^9*b^6*x^2)/2 + 2574*a^8*b^7*x^(5/2) + 2145*
a^7*b^8*x^3 + 1430*a^6*b^9*x^(7/2) + (3003*a^5*b^10*x^4)/4 + (910*a^4*b^11*
x^(9/2))/3 + 91*a^3*b^12*x^5 + (210*a^2*b^13*x^(11/2))/11 + (5*a*b^14*x^6)/
2 + (2*b^15*x^(13/2))/13 + 105*a^13*b^2*Log[x]
```

fricas [A] time = 1.18, size = 172, normalized size = 0.90

$$\frac{4290\,ab^{14}x^7 + 156156\,a^3b^{12}x^6 + 1288287\,a^5b^{10}x^5 + 3680820\,a^7b^8x^4 + 4294290\,a^9b^6x^3 + 2342340\,a^{11}b^4x^2 + 360}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^2,x, algorithm="fricas")
```

```
[Out] 1/1716*(4290*a*b^14*x^7 + 156156*a^3*b^12*x^6 + 1288287*a^5*b^10*x^5 + 3680
820*a^7*b^8*x^4 + 4294290*a^9*b^6*x^3 + 2342340*a^11*b^4*x^2 + 360360*a^13*
b^2*x*log(sqrt(x)) - 1716*a^15 + 8*(33*b^15*x^7 + 4095*a^2*b^13*x^6 + 65065
*a^4*b^11*x^5 + 306735*a^6*b^9*x^4 + 552123*a^8*b^7*x^3 + 429429*a^10*b^5*x
^2 + 195195*a^12*b^3*x - 6435*a^14*b)*sqrt(x))/x
```

giac [A] time = 0.19, size = 166, normalized size = 0.86

$$\frac{2}{13}\,b^{15}x^{\frac{13}{2}} + \frac{5}{2}\,ab^{14}x^6 + \frac{210}{11}\,a^2b^{13}x^{\frac{11}{2}} + 91\,a^3b^{12}x^5 + \frac{910}{3}\,a^4b^{11}x^{\frac{9}{2}} + \frac{3003}{4}\,a^5b^{10}x^4 + 1430\,a^6b^9x^{\frac{7}{2}} + 2145\,a^7b^8x^3 + 2574\,a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^2,x, algorithm="giac")
```

```
[Out] 2/13*b^15*x^(13/2) + 5/2*a*b^14*x^6 + 210/11*a^2*b^13*x^(11/2) + 91*a^3*b^1
2*x^5 + 910/3*a^4*b^11*x^(9/2) + 3003/4*a^5*b^10*x^4 + 1430*a^6*b^9*x^(7/2)
 + 2145*a^7*b^8*x^3 + 2574*a^8*b^7*x^(5/2) + 5005/2*a^9*b^6*x^2 + 2002*a^10
*b^5*x^(3/2) + 1365*a^11*b^4*x + 105*a^13*b^2*log(abs(x)) + 910*a^12*b^3*sq
rt(x) - (30*a^14*b*sqrt(x) + a^15)/x
```

maple [A] time = 0.00, size = 165, normalized size = 0.86

$$\frac{2b^{15}x^{\frac{13}{2}}}{13} + \frac{5a\,b^{14}x^6}{2} + \frac{210a^2b^{13}x^{\frac{11}{2}}}{11} + 91a^3b^{12}x^5 + \frac{910a^4b^{11}x^{\frac{9}{2}}}{3} + \frac{3003a^5b^{10}x^4}{4} + 1430a^6b^9x^{\frac{7}{2}} + 2145a^7b^8x^3 + 2574a^8b^7x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^15/x^2,x)
```

```
[Out] -a^15/x+1365*a^11*b^4*x+2002*a^10*b^5*x^(3/2)+5005/2*a^9*b^6*x^2+2574*a^8*b
^7*x^(5/2)+2145*a^7*b^8*x^3+1430*a^6*b^9*x^(7/2)+3003/4*a^5*b^10*x^4+910/3*
a^4*b^11*x^(9/2)+91*a^3*b^12*x^5+210/11*a^2*b^13*x^(11/2)+5/2*a*b^14*x^6+2/
13*b^15*x^(13/2)+105*a^13*b^2*ln(x)-30*a^14*b/x^(1/2)+910*a^12*b^3*x^(1/2)
```

maxima [A] time = 0.87, size = 165, normalized size = 0.86

$$\frac{2}{13}\,b^{15}x^{\frac{13}{2}} + \frac{5}{2}\,ab^{14}x^6 + \frac{210}{11}\,a^2b^{13}x^{\frac{11}{2}} + 91\,a^3b^{12}x^5 + \frac{910}{3}\,a^4b^{11}x^{\frac{9}{2}} + \frac{3003}{4}\,a^5b^{10}x^4 + 1430\,a^6b^9x^{\frac{7}{2}} + 2145\,a^7b^8x^3 + 2574\,a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^2,x, algorithm="maxima")
```

```
[Out] 2/13*b^15*x^(13/2) + 5/2*a*b^14*x^6 + 210/11*a^2*b^13*x^(11/2) + 91*a^3*b^1
2*x^5 + 910/3*a^4*b^11*x^(9/2) + 3003/4*a^5*b^10*x^4 + 1430*a^6*b^9*x^(7/2)
 + 2145*a^7*b^8*x^3 + 2574*a^8*b^7*x^(5/2) + 5005/2*a^9*b^6*x^2 + 2002*a^10
*b^5*x^(3/2) + 1365*a^11*b^4*x + 105*a^13*b^2*log(x) + 910*a^12*b^3*sqrt(x)
 - (30*a^14*b*sqrt(x) + a^15)/x
```

mupad [B] time = 0.14, size = 167, normalized size = 0.87

$$\frac{2\,b^{15}\,x^{13/2}}{13}-\frac{a^{15}+30\,a^{14}\,b\,\sqrt{x}}{x}+210\,a^{13}\,b^2\,\ln\left(\sqrt{x}\right)+1365\,a^{11}\,b^4\,x+\frac{5\,a\,b^{14}\,x^6}{2}+\frac{5005\,a^9\,b^6\,x^2}{2}+2145\,a^7\,b^8\,x^3+910$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^15/x^2,x)
```

```
[Out] (2*b^15*x^(13/2))/13 - (a^15 + 30*a^14*b*x^(1/2))/x + 210*a^13*b^2*log(x^(1
/2)) + 1365*a^11*b^4*x + (5*a*b^14*x^6)/2 + (5005*a^9*b^6*x^2)/2 + 2145*a^7
*b^8*x^3 + 910*a^12*b^3*x^(1/2) + (3003*a^5*b^10*x^4)/4 + 91*a^3*b^12*x^5 +
 2002*a^10*b^5*x^(3/2) + 2574*a^8*b^7*x^(5/2) + 1430*a^6*b^9*x^(7/2) + (910
*a^4*b^11*x^(9/2))/3 + (210*a^2*b^13*x^(11/2))/11
```

sympy [A] time = 5.81, size = 197, normalized size = 1.03

$$-\frac{a^{15}}{x}-\frac{30a^{14}b}{\sqrt{x}}+105a^{13}b^2\log(x)+910a^{12}b^3\sqrt{x}+1365a^{11}b^4x+2002a^{10}b^5x^{\frac{3}{2}}+\frac{5005a^9b^6x^2}{2}+2574a^8b^7x^{\frac{5}{2}}+2145a^7b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**15/x**2,x)
```

```
[Out] -a**15/x - 30*a**14*b/sqrt(x) + 105*a**13*b**2*log(x) + 910*a**12*b**3*sqrt
(x) + 1365*a**11*b**4*x + 2002*a**10*b**5*x**(3/2) + 5005*a**9*b**6*x**2/2
+ 2574*a**8*b**7*x**(5/2) + 2145*a**7*b**8*x**3 + 1430*a**6*b**9*x**(7/2) +
 3003*a**5*b**10*x**4/4 + 910*a**4*b**11*x**(9/2)/3 + 91*a**3*b**12*x**5 +
210*a**2*b**13*x**(11/2)/11 + 5*a*b**14*x**6/2 + 2*b**15*x**(13/2)/13
```

3.2177 $\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^3}\,dx$

Optimal. Leaf size=190

$$-\frac{a^{15}}{2x^2}-\frac{10a^{14}b}{x^{3/2}}-\frac{105a^{13}b^2}{x}-\frac{910a^{12}b^3}{\sqrt{x}}+1365a^{11}b^4\log(x)+6006a^{10}b^5\sqrt{x}+5005a^9b^6x+4290a^8b^7x^{3/2}+\frac{6435}{2}a^7b^8x^2+2$$

```
[Out] -1/2*a^15/x^2-10*a^14*b/x^(3/2)-105*a^13*b^2/x+5005*a^9*b^6*x+4290*a^8*b^7*
x^(3/2)+6435/2*a^7*b^8*x^2+2002*a^6*b^9*x^(5/2)+1001*a^5*b^10*x^3+390*a^4*b
^11*x^(7/2)+455/4*a^3*b^12*x^4+70/3*a^2*b^13*x^(9/2)+3*a*b^14*x^5+2/11*b^15
*x^(11/2)+1365*a^11*b^4*ln(x)-910*a^12*b^3/x^(1/2)+6006*a^10*b^5*x^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$4290a^8b^7x^{3/2}+\frac{6435}{2}a^7b^8x^2+2002a^6b^9x^{5/2}+1001a^5b^{10}x^3+390a^4b^{11}x^{7/2}+\frac{455}{4}a^3b^{12}x^4+\frac{70}{3}a^2b^{13}x^{9/2}-\frac{105a^{13}b^2}{x}-\frac{910$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^15/x^3,x]
```

```
[Out] -a^15/(2*x^2) - (10*a^14*b)/x^(3/2) - (105*a^13*b^2)/x - (910*a^12*b^3)/Sqr
t[x] + 6006*a^10*b^5*Sqrt[x] + 5005*a^9*b^6*x + 4290*a^8*b^7*x^(3/2) + (643
5*a^7*b^8*x^2)/2 + 2002*a^6*b^9*x^(5/2) + 1001*a^5*b^10*x^3 + 390*a^4*b^11*
x^(7/2) + (455*a^3*b^12*x^4)/4 + (70*a^2*b^13*x^(9/2))/3 + 3*a*b^14*x^5 + (
2*b^15*x^(11/2))/11 + 1365*a^11*b^4*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^3}\,dx &= 2\,\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^5}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int\left(3003a^{10}b^5+\frac{a^{15}}{x^5}+\frac{15a^{14}b}{x^4}+\frac{105a^{13}b^2}{x^3}+\frac{455a^{12}b^3}{x^2}+\frac{1365a^{11}b^4}{x}+5005a^9b^6x\right.\right.\\ &= -\frac{a^{15}}{2x^2}-\frac{10a^{14}b}{x^{3/2}}-\frac{105a^{13}b^2}{x}-\frac{910a^{12}b^3}{\sqrt{x}}+6006a^{10}b^5\sqrt{x}+5005a^9b^6x+4290a^8b^7x^{3/2}+\frac{643}{2}\end{aligned}$$

Mathematica [A] time = 0.14, size = 190, normalized size = 1.00

$$-\frac{a^{15}}{2x^2}-\frac{10a^{14}b}{x^{3/2}}-\frac{105a^{13}b^2}{x}-\frac{910a^{12}b^3}{\sqrt{x}}+1365a^{11}b^4\log(x)+6006a^{10}b^5\sqrt{x}+5005a^9b^6x+4290a^8b^7x^{3/2}+\frac{6435}{2}a^7b^8x^2+2$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^15/x^3,x]
```

```
[Out] -1/2*a^15/x^2 - (10*a^14*b)/x^(3/2) - (105*a^13*b^2)/x - (910*a^12*b^3)/Sqr
t[x] + 6006*a^10*b^5*Sqrt[x] + 5005*a^9*b^6*x + 4290*a^8*b^7*x^(3/2) + (643
5*a^7*b^8*x^2)/2 + 2002*a^6*b^9*x^(5/2) + 1001*a^5*b^10*x^3 + 390*a^4*b^11*
x^(7/2) + (455*a^3*b^12*x^4)/4 + (70*a^2*b^13*x^(9/2))/3 + 3*a*b^14*x^5 + (
2*b^15*x^(11/2))/11 + 1365*a^11*b^4*Log[x]
```

fricas [A] time = 0.88, size = 172, normalized size = 0.91

$$\frac{396\,ab^{14}x^7 + 15015\,a^3b^{12}x^6 + 132132\,a^5b^{10}x^5 + 424710\,a^7b^8x^4 + 660660\,a^9b^6x^3 + 360360\,a^{11}b^4x^2\log\left(\sqrt{x}\right) -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^3,x, algorithm="fricas")
```

```
[Out] 1/132*(396*a*b^14*x^7 + 15015*a^3*b^12*x^6 + 132132*a^5*b^10*x^5 + 424710*a
^7*b^8*x^4 + 660660*a^9*b^6*x^3 + 360360*a^11*b^4*x^2*log(sqrt(x)) - 13860*
a^13*b^2*x - 66*a^15 + 8*(3*b^15*x^7 + 385*a^2*b^13*x^6 + 6435*a^4*b^11*x^5
 + 33033*a^6*b^9*x^4 + 70785*a^8*b^7*x^3 + 99099*a^10*b^5*x^2 - 15015*a^12*
b^3*x - 165*a^14*b)*sqrt(x))/x^2
```

giac [A] time = 0.18, size = 164, normalized size = 0.86

$$\frac{2}{11}\,b^{15}x^{\frac{11}{2}} + 3\,ab^{14}x^5 + \frac{70}{3}\,a^2b^{13}x^{\frac{9}{2}} + \frac{455}{4}\,a^3b^{12}x^4 + 390\,a^4b^{11}x^{\frac{7}{2}} + 1001\,a^5b^{10}x^3 + 2002\,a^6b^9x^{\frac{5}{2}} + \frac{6435}{2}\,a^7b^8x^2 + 4290\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^3,x, algorithm="giac")
```

```
[Out] 2/11*b^15*x^(11/2) + 3*a*b^14*x^5 + 70/3*a^2*b^13*x^(9/2) + 455/4*a^3*b^12*
x^4 + 390*a^4*b^11*x^(7/2) + 1001*a^5*b^10*x^3 + 2002*a^6*b^9*x^(5/2) + 643
5/2*a^7*b^8*x^2 + 4290*a^8*b^7*x^(3/2) + 5005*a^9*b^6*x + 1365*a^11*b^4*log
(abs(x)) + 6006*a^10*b^5*sqrt(x) - 1/2*(1820*a^12*b^3*x^(3/2) + 210*a^13*b^
2*x + 20*a^14*b*sqrt(x) + a^15)/x^2
```

maple [A] time = 0.00, size = 165, normalized size = 0.87

$$\frac{2b^{15}x^{\frac{11}{2}}}{11} + 3a\,b^{14}x^5 + \frac{70a^2b^{13}x^{\frac{9}{2}}}{3} + \frac{455a^3b^{12}x^4}{4} + 390a^4b^{11}x^{\frac{7}{2}} + 1001a^5b^{10}x^3 + 2002a^6b^9x^{\frac{5}{2}} + \frac{6435a^7b^8x^2}{2} + 4290a^8b^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^15/x^3,x)
```

```
[Out] -1/2*a^15/x^2-10*a^14*b/x^(3/2)-105*a^13*b^2/x+5005*a^9*b^6*x+4290*a^8*b^7*
x^(3/2)+6435/2*a^7*b^8*x^2+2002*a^6*b^9*x^(5/2)+1001*a^5*b^10*x^3+390*a^4*b
^11*x^(7/2)+455/4*a^3*b^12*x^4+70/3*a^2*b^13*x^(9/2)+3*a*b^14*x^5+2/11*b^15
*x^(11/2)+1365*a^11*b^4*ln(x)-910*a^12*b^3/x^(1/2)+6006*a^10*b^5*x^(1/2)
```

maxima [A] time = 0.90, size = 163, normalized size = 0.86

$$\frac{2}{11}\,b^{15}x^{\frac{11}{2}} + 3\,ab^{14}x^5 + \frac{70}{3}\,a^2b^{13}x^{\frac{9}{2}} + \frac{455}{4}\,a^3b^{12}x^4 + 390\,a^4b^{11}x^{\frac{7}{2}} + 1001\,a^5b^{10}x^3 + 2002\,a^6b^9x^{\frac{5}{2}} + \frac{6435}{2}\,a^7b^8x^2 + 4290\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^3,x, algorithm="maxima")
```

```
[Out] 2/11*b^15*x^(11/2) + 3*a*b^14*x^5 + 70/3*a^2*b^13*x^(9/2) + 455/4*a^3*b^12*
x^4 + 390*a^4*b^11*x^(7/2) + 1001*a^5*b^10*x^3 + 2002*a^6*b^9*x^(5/2) + 643
5/2*a^7*b^8*x^2 + 4290*a^8*b^7*x^(3/2) + 5005*a^9*b^6*x + 1365*a^11*b^4*log
(x) + 6006*a^10*b^5*sqrt(x) - 1/2*(1820*a^12*b^3*x^(3/2) + 210*a^13*b^2*x +
 20*a^14*b*sqrt(x) + a^15)/x^2
```

mupad [B] time = 0.10, size = 167, normalized size = 0.88

$$\frac{2\,b^{15}\,x^{11/2}}{11}-\frac{\frac{a^{15}}{2}+105\,a^{13}\,b^2\,x+10\,a^{14}\,b\,\sqrt{x}+910\,a^{12}\,b^3\,x^{3/2}}{x^2}+2730\,a^{11}\,b^4\,\ln\left(\sqrt{x}\right)+5005\,a^9\,b^6\,x+3\,a\,b^{14}\,x^5+\frac{6435$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^15/x^3,x)
```

```
[Out] (2*b^15*x^(11/2))/11 - (a^15/2 + 105*a^13*b^2*x + 10*a^14*b*x^(1/2) + 910*a
^12*b^3*x^(3/2))/x^2 + 2730*a^11*b^4*log(x^(1/2)) + 5005*a^9*b^6*x + 3*a*b^
14*x^5 + (6435*a^7*b^8*x^2)/2 + 1001*a^5*b^10*x^3 + 6006*a^10*b^5*x^(1/2) +
 (455*a^3*b^12*x^4)/4 + 4290*a^8*b^7*x^(3/2) + 2002*a^6*b^9*x^(5/2) + 390*a
^4*b^11*x^(7/2) + (70*a^2*b^13*x^(9/2))/3
```

sympy [A] time = 5.63, size = 196, normalized size = 1.03

$$-\frac{a^{15}}{2x^2}-\frac{10a^{14}b}{x^{\frac{3}{2}}}-\frac{105a^{13}b^2}{x}-\frac{910a^{12}b^3}{\sqrt{x}}+1365a^{11}b^4\log(x)+6006a^{10}b^5\sqrt{x}+5005a^9b^6x+4290a^8b^7x^{\frac{3}{2}}+\frac{6435a^7b^8x^2}{2}+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**15/x**3,x)
```

```
[Out] -a**15/(2*x**2) - 10*a**14*b/x**(3/2) - 105*a**13*b**2/x - 910*a**12*b**3/s
qrt(x) + 1365*a**11*b**4*log(x) + 6006*a**10*b**5*sqrt(x) + 5005*a**9*b**6*
x + 4290*a**8*b**7*x**(3/2) + 6435*a**7*b**8*x**2/2 + 2002*a**6*b**9*x**(5/
2) + 1001*a**5*b**10*x**3 + 390*a**4*b**11*x**(7/2) + 455*a**3*b**12*x**4/4
 + 70*a**2*b**13*x**(9/2)/3 + 3*a*b**14*x**5 + 2*b**15*x**(11/2)/11
```

3.2178 $\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^4}\,dx$

Optimal. Leaf size=196

$$-\frac{a^{15}}{3x^3}-\frac{6a^{14}b}{x^{5/2}}-\frac{105a^{13}b^2}{2x^2}-\frac{910a^{12}b^3}{3x^{3/2}}-\frac{1365a^{11}b^4}{x}-\frac{6006a^{10}b^5}{\sqrt{x}}+5005a^9b^6\log(x)+12870a^8b^7\sqrt{x}+6435a^7b^8x+\frac{100}{3}$$

```
[Out] -1/3*a^15/x^3-6*a^14*b/x^(5/2)-105/2*a^13*b^2/x^2-910/3*a^12*b^3/x^(3/2)-13
65*a^11*b^4/x+6435*a^7*b^8*x+10010/3*a^6*b^9*x^(3/2)+3003/2*a^5*b^10*x^2+54
6*a^4*b^11*x^(5/2)+455/3*a^3*b^12*x^3+30*a^2*b^13*x^(7/2)+15/4*a*b^14*x^4+2
/9*b^15*x^(9/2)+5005*a^9*b^6*ln(x)-6006*a^10*b^5/x^(1/2)+12870*a^8*b^7*x^(1
/2)
```

Rubi [A] time = 0.12, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{105a^{13}b^2}{2x^2}-\frac{910a^{12}b^3}{3x^{3/2}}+\frac{10010}{3}a^6b^9x^{3/2}+\frac{3003}{2}a^5b^{10}x^2+546a^4b^{11}x^{5/2}+\frac{455}{3}a^3b^{12}x^3+30a^2b^{13}x^{7/2}-\frac{1365a^{11}b^4}{x}-\frac{60}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^15/x^4,x]
```

```
[Out] -a^15/(3*x^3) - (6*a^14*b)/x^(5/2) - (105*a^13*b^2)/(2*x^2) - (910*a^12*b^3
)/(3*x^(3/2)) - (1365*a^11*b^4)/x - (6006*a^10*b^5)/Sqrt[x] + 12870*a^8*b^7
*Sqrt[x] + 6435*a^7*b^8*x + (10010*a^6*b^9*x^(3/2))/3 + (3003*a^5*b^10*x^2)
/2 + 546*a^4*b^11*x^(5/2) + (455*a^3*b^12*x^3)/3 + 30*a^2*b^13*x^(7/2) + (1
5*a*b^14*x^4)/4 + (2*b^15*x^(9/2))/9 + 5005*a^9*b^6*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^4}\,dx &= 2\,\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^7}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(6435a^8b^7+\frac{a^{15}}{x^7}+\frac{15a^{14}b}{x^6}+\frac{105a^{13}b^2}{x^5}+\frac{455a^{12}b^3}{x^4}+\frac{1365a^{11}b^4}{x^3}+\frac{3003a^{10}}{x^2}\right.\right.\\ &= -\frac{a^{15}}{3x^3}-\frac{6a^{14}b}{x^{5/2}}-\frac{105a^{13}b^2}{2x^2}-\frac{910a^{12}b^3}{3x^{3/2}}-\frac{1365a^{11}b^4}{x}-\frac{6006a^{10}b^5}{\sqrt{x}}+12870a^8b^7\sqrt{x}+643\end{aligned}$$

Mathematica [A] time = 0.14, size = 196, normalized size = 1.00

$$-\frac{a^{15}}{3x^3}-\frac{6a^{14}b}{x^{5/2}}-\frac{105a^{13}b^2}{2x^2}-\frac{910a^{12}b^3}{3x^{3/2}}-\frac{1365a^{11}b^4}{x}-\frac{6006a^{10}b^5}{\sqrt{x}}+5005a^9b^6\log(x)+12870a^8b^7\sqrt{x}+6435a^7b^8x+\frac{100}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^15/x^4,x]
```

```
[Out] -1/3*a^15/x^3 - (6*a^14*b)/x^(5/2) - (105*a^13*b^2)/(2*x^2) - (910*a^12*b^3
)/(3*x^(3/2)) - (1365*a^11*b^4)/x - (6006*a^10*b^5)/Sqrt[x] + 12870*a^8*b^7
*Sqrt[x] + 6435*a^7*b^8*x + (10010*a^6*b^9*x^(3/2))/3 + (3003*a^5*b^10*x^2)
/2 + 546*a^4*b^11*x^(5/2) + (455*a^3*b^12*x^3)/3 + 30*a^2*b^13*x^(7/2) + (1
5*a*b^14*x^4)/4 + (2*b^15*x^(9/2))/9 + 5005*a^9*b^6*Log[x]
```

fricas [A] time = 1.05, size = 171, normalized size = 0.87

$$\frac{135\,ab^{14}x^7 + 5460\,a^3b^{12}x^6 + 54054\,a^5b^{10}x^5 + 231660\,a^7b^8x^4 + 360360\,a^9b^6x^3\log\left(\sqrt{x}\right) - 49140\,a^{11}b^4x^2 - 1890\,a^{13}b^2x - 12\,a^{15} + 8\left(b^{15}x^7 + 135\,a^2b^{13}x^6 + 2457\,a^4b^{11}x^5 + 1501\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^4,x, algorithm="fricas")
```

```
[Out] 1/36*(135*a*b^14*x^7 + 5460*a^3*b^12*x^6 + 54054*a^5*b^10*x^5 + 231660*a^7*
b^8*x^4 + 360360*a^9*b^6*x^3*log(sqrt(x)) - 49140*a^11*b^4*x^2 - 1890*a^13*
b^2*x - 12*a^15 + 8*(b^15*x^7 + 135*a^2*b^13*x^6 + 2457*a^4*b^11*x^5 + 1501
5*a^6*b^9*x^4 + 57915*a^8*b^7*x^3 - 27027*a^10*b^5*x^2 - 1365*a^12*b^3*x -
27*a^14*b)*sqrt(x))/x^3
```

giac [A] time = 0.19, size = 166, normalized size = 0.85

$$\frac{2}{9}\,b^{15}x^{\frac{9}{2}} + \frac{15}{4}\,ab^{14}x^4 + 30\,a^2b^{13}x^{\frac{7}{2}} + \frac{455}{3}\,a^3b^{12}x^3 + 546\,a^4b^{11}x^{\frac{5}{2}} + \frac{3003}{2}\,a^5b^{10}x^2 + \frac{10010}{3}\,a^6b^9x^{\frac{3}{2}} + 6435\,a^7b^8x + 5005\,a^9b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^4,x, algorithm="giac")
```

```
[Out] 2/9*b^15*x^(9/2) + 15/4*a*b^14*x^4 + 30*a^2*b^13*x^(7/2) + 455/3*a^3*b^12*x
^3 + 546*a^4*b^11*x^(5/2) + 3003/2*a^5*b^10*x^2 + 10010/3*a^6*b^9*x^(3/2) +
 6435*a^7*b^8*x + 5005*a^9*b^6*log(abs(x)) + 12870*a^8*b^7*sqrt(x) - 1/6*(3
6036*a^10*b^5*x^(5/2) + 8190*a^11*b^4*x^2 + 1820*a^12*b^3*x^(3/2) + 315*a^1
3*b^2*x + 36*a^14*b*sqrt(x) + 2*a^15)/x^3
```

maple [A] time = 0.00, size = 165, normalized size = 0.84

$$\frac{2b^{15}x^{\frac{9}{2}}}{9} + \frac{15a\,b^{14}x^4}{4} + 30a^2b^{13}x^{\frac{7}{2}} + \frac{455a^3b^{12}x^3}{3} + 546a^4b^{11}x^{\frac{5}{2}} + \frac{3003a^5b^{10}x^2}{2} + \frac{10010a^6b^9x^{\frac{3}{2}}}{3} + 5005a^9b^6\ln(x) + 6435a^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^15/x^4,x)
```

```
[Out] -1/3*a^15/x^3-6*a^14*b/x^(5/2)-105/2*a^13*b^2/x^2-910/3*a^12*b^3/x^(3/2)-13
65*a^11*b^4/x+6435*a^7*b^8*x+10010/3*a^6*b^9*x^(3/2)+3003/2*a^5*b^10*x^2+54
6*a^4*b^11*x^(5/2)+455/3*a^3*b^12*x^3+30*a^2*b^13*x^(7/2)+15/4*a*b^14*x^4+2
/9*b^15*x^(9/2)+5005*a^9*b^6*ln(x)-6006*a^10*b^5/x^(1/2)+12870*a^8*b^7*x^(1
/2)
```

maxima [A] time = 0.90, size = 165, normalized size = 0.84

$$\frac{2}{9}\,b^{15}x^{\frac{9}{2}} + \frac{15}{4}\,ab^{14}x^4 + 30\,a^2b^{13}x^{\frac{7}{2}} + \frac{455}{3}\,a^3b^{12}x^3 + 546\,a^4b^{11}x^{\frac{5}{2}} + \frac{3003}{2}\,a^5b^{10}x^2 + \frac{10010}{3}\,a^6b^9x^{\frac{3}{2}} + 6435\,a^7b^8x + 5005\,a^9b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^4,x, algorithm="maxima")
```

```
[Out] 2/9*b^15*x^(9/2) + 15/4*a*b^14*x^4 + 30*a^2*b^13*x^(7/2) + 455/3*a^3*b^12*x
^3 + 546*a^4*b^11*x^(5/2) + 3003/2*a^5*b^10*x^2 + 10010/3*a^6*b^9*x^(3/2) +
 6435*a^7*b^8*x + 5005*a^9*b^6*log(x) + 12870*a^8*b^7*sqrt(x) - 1/6*(36036*
a^10*b^5*x^(5/2) + 8190*a^11*b^4*x^2 + 1820*a^12*b^3*x^(3/2) + 315*a^13*b^2
*x + 36*a^14*b*sqrt(x) + 2*a^15)/x^3
```

mupad **[B]** time = 1.16, size = 167, normalized size = 0.85

$$\frac{2\,b^{15}\,x^{9/2}}{9}-\frac{\frac{a^{15}}{3}+\frac{105\,a^{13}\,b^2\,x}{2}+6\,a^{14}\,b\,\sqrt{x}+1365\,a^{11}\,b^4\,x^2+\frac{910\,a^{12}\,b^3\,x^{3/2}}{3}+6006\,a^{10}\,b^5\,x^{5/2}}{x^3}+10010\,a^9\,b^6\,\ln\left(\sqrt{x}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^15/x^4,x)
```

```
[Out] (2*b^15*x^(9/2))/9 - (a^15/3 + (105*a^13*b^2*x)/2 + 6*a^14*b*x^(1/2) + 1365
*a^11*b^4*x^2 + (910*a^12*b^3*x^(3/2))/3 + 6006*a^10*b^5*x^(5/2))/x^3 + 100
10*a^9*b^6*log(x^(1/2)) + 6435*a^7*b^8*x + (15*a*b^14*x^4)/4 + (3003*a^5*b^
10*x^2)/2 + (455*a^3*b^12*x^3)/3 + 12870*a^8*b^7*x^(1/2) + (10010*a^6*b^9*x
^(3/2))/3 + 546*a^4*b^11*x^(5/2) + 30*a^2*b^13*x^(7/2)
```

sympy **[A]** time = 5.37, size = 201, normalized size = 1.03

$$-\frac{a^{15}}{3x^3}-\frac{6a^{14}b}{x^{\frac{5}{2}}}-\frac{105a^{13}b^2}{2x^2}-\frac{910a^{12}b^3}{3x^{\frac{3}{2}}}-\frac{1365a^{11}b^4}{x}-\frac{6006a^{10}b^5}{\sqrt{x}}+5005a^9b^6\log(x)+12870a^8b^7\sqrt{x}+6435a^7b^8x+\frac{10}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**15/x**4,x)
```

```
[Out] -a**15/(3*x**3) - 6*a**14*b/x**(5/2) - 105*a**13*b**2/(2*x**2) - 910*a**12*
b**3/(3*x**(3/2)) - 1365*a**11*b**4/x - 6006*a**10*b**5/sqrt(x) + 5005*a**9
*b**6*log(x) + 12870*a**8*b**7*sqrt(x) + 6435*a**7*b**8*x + 10010*a**6*b**9
*x**(3/2)/3 + 3003*a**5*b**10*x**2/2 + 546*a**4*b**11*x**(5/2) + 455*a**3*b
**12*x**3/3 + 30*a**2*b**13*x**(7/2) + 15*a*b**14*x**4/4 + 2*b**15*x**(9/2)
/9
```

3.2179 $\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^6}\,dx$

Optimal. Leaf size=194

$$-\frac{a^{15}}{5x^5}-\frac{10a^{14}b}{3x^{9/2}}-\frac{105a^{13}b^2}{4x^4}-\frac{130a^{12}b^3}{x^{7/2}}-\frac{455a^{11}b^4}{x^3}-\frac{6006a^{10}b^5}{5x^{5/2}}-\frac{5005a^9b^6}{2x^2}-\frac{4290a^8b^7}{x^{3/2}}-\frac{6435a^7b^8}{x}-\frac{10010a^6b^9}{\sqrt{x}}+3003a$$

```
[Out] -1/5*a^15/x^5-10/3*a^14*b/x^(9/2)-105/4*a^13*b^2/x^4-130*a^12*b^3/x^(7/2)-4
55*a^11*b^4/x^3-6006/5*a^10*b^5/x^(5/2)-5005/2*a^9*b^6/x^2-4290*a^8*b^7/x^(
3/2)-6435*a^7*b^8/x+455*a^3*b^12*x+70*a^2*b^13*x^(3/2)+15/2*a*b^14*x^2+2/5*
b^15*x^(5/2)+3003*a^5*b^10*ln(x)-10010*a^6*b^9/x^(1/2)+2730*a^4*b^11*x^(1/2
)
```

Rubi [A] time = 0.12, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{105a^{13}b^2}{4x^4}-\frac{130a^{12}b^3}{x^{7/2}}-\frac{455a^{11}b^4}{x^3}-\frac{6006a^{10}b^5}{5x^{5/2}}-\frac{5005a^9b^6}{2x^2}-\frac{4290a^8b^7}{x^{3/2}}+70a^2b^{13}x^{3/2}-\frac{6435a^7b^8}{x}-\frac{10010a^6b^9}{\sqrt{x}}+2730a^4$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^15/x^6,x]
```

```
[Out] -a^15/(5*x^5) - (10*a^14*b)/(3*x^(9/2)) - (105*a^13*b^2)/(4*x^4) - (130*a^1
2*b^3)/x^(7/2) - (455*a^11*b^4)/x^3 - (6006*a^10*b^5)/(5*x^(5/2)) - (5005*a
^9*b^6)/(2*x^2) - (4290*a^8*b^7)/x^(3/2) - (6435*a^7*b^8)/x - (10010*a^6*b^
9)/Sqrt[x] + 2730*a^4*b^11*Sqrt[x] + 455*a^3*b^12*x + 70*a^2*b^13*x^(3/2) +
 (15*a*b^14*x^2)/2 + (2*b^15*x^(5/2))/5 + 3003*a^5*b^10*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^6}\,dx &= 2\,\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{11}}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(1365a^4b^{11}+\frac{a^{15}}{x^{11}}+\frac{15a^{14}b}{x^{10}}+\frac{105a^{13}b^2}{x^9}+\frac{455a^{12}b^3}{x^8}+\frac{1365a^{11}b^4}{x^7}+\frac{3003a^{10}b^5}{x^6}\right.\right.\\ &= -\frac{a^{15}}{5x^5}-\frac{10a^{14}b}{3x^{9/2}}-\frac{105a^{13}b^2}{4x^4}-\frac{130a^{12}b^3}{x^{7/2}}-\frac{455a^{11}b^4}{x^3}-\frac{6006a^{10}b^5}{5x^{5/2}}-\frac{5005a^9b^6}{2x^2}-\frac{4290a^8b^7}{x^{3/2}}-\end{aligned}$$

Mathematica [A] time = 0.15, size = 194, normalized size = 1.00

$$-\frac{a^{15}}{5x^5}-\frac{10a^{14}b}{3x^{9/2}}-\frac{105a^{13}b^2}{4x^4}-\frac{130a^{12}b^3}{x^{7/2}}-\frac{455a^{11}b^4}{x^3}-\frac{6006a^{10}b^5}{5x^{5/2}}-\frac{5005a^9b^6}{2x^2}-\frac{4290a^8b^7}{x^{3/2}}-\frac{6435a^7b^8}{x}-\frac{10010a^6b^9}{\sqrt{x}}+3003a$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^15/x^6,x]
```

```
[Out] -1/5*a^15/x^5 - (10*a^14*b)/(3*x^(9/2)) - (105*a^13*b^2)/(4*x^4) - (130*a^1
2*b^3)/x^(7/2) - (455*a^11*b^4)/x^3 - (6006*a^10*b^5)/(5*x^(5/2)) - (5005*a
^9*b^6)/(2*x^2) - (4290*a^8*b^7)/x^(3/2) - (6435*a^7*b^8)/x - (10010*a^6*b^
9)/Sqrt[x] + 2730*a^4*b^11*Sqrt[x] + 455*a^3*b^12*x + 70*a^2*b^13*x^(3/2) +
 (15*a*b^14*x^2)/2 + (2*b^15*x^(5/2))/5 + 3003*a^5*b^10*Log[x]
```

fricas [A] time = 0.92, size = 172, normalized size = 0.89

$$\frac{450\,ab^{14}x^7 + 27300\,a^3b^{12}x^6 + 360360\,a^5b^{10}x^5\log\left(\sqrt{x}\right) - 386100\,a^7b^8x^4 - 150150\,a^9b^6x^3 - 27300\,a^{11}b^4x^2 - 1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^6,x, algorithm="fricas")
```

```
[Out] 1/60*(450*a*b^14*x^7 + 27300*a^3*b^12*x^6 + 360360*a^5*b^10*x^5*log(sqrt(x)
) - 386100*a^7*b^8*x^4 - 150150*a^9*b^6*x^3 - 27300*a^11*b^4*x^2 - 1575*a^1
3*b^2*x - 12*a^15 + 8*(3*b^15*x^7 + 525*a^2*b^13*x^6 + 20475*a^4*b^11*x^5 -
 75075*a^6*b^9*x^4 - 32175*a^8*b^7*x^3 - 9009*a^10*b^5*x^2 - 975*a^12*b^3*x
 - 25*a^14*b)*sqrt(x))/x^5
```

giac [A] time = 0.17, size = 166, normalized size = 0.86

$$\frac{2}{5}\,b^{15}x^{\frac{5}{2}} + \frac{15}{2}\,ab^{14}x^2 + 70\,a^2b^{13}x^{\frac{3}{2}} + 455\,a^3b^{12}x + 3003\,a^5b^{10}\log\left(|x|\right) + 2730\,a^4b^{11}\sqrt{x} - \frac{600600\,a^6b^9x^{\frac{9}{2}} + 386100\,a^7}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^6,x, algorithm="giac")
```

```
[Out] 2/5*b^15*x^(5/2) + 15/2*a*b^14*x^2 + 70*a^2*b^13*x^(3/2) + 455*a^3*b^12*x +
 3003*a^5*b^10*log(abs(x)) + 2730*a^4*b^11*sqrt(x) - 1/60*(600600*a^6*b^9*x
^(9/2) + 386100*a^7*b^8*x^4 + 257400*a^8*b^7*x^(7/2) + 150150*a^9*b^6*x^3 +
 72072*a^10*b^5*x^(5/2) + 27300*a^11*b^4*x^2 + 7800*a^12*b^3*x^(3/2) + 1575
*a^13*b^2*x + 200*a^14*b*sqrt(x) + 12*a^15)/x^5
```

maple [A] time = 0.00, size = 165, normalized size = 0.85

$$\frac{2b^{15}x^{\frac{5}{2}}}{5} + \frac{15a\,b^{14}x^2}{2} + 70a^2b^{13}x^{\frac{3}{2}} + 3003a^5b^{10}\ln(x) + 455a^3b^{12}x + 2730a^4b^{11}\sqrt{x} - \frac{10010a^6b^9}{\sqrt{x}} - \frac{6435a^7b^8}{x} - \frac{4290a^8b^7}{x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^15/x^6,x)
```

```
[Out] -1/5*a^15/x^5-10/3*a^14*b/x^(9/2)-105/4*a^13*b^2/x^4-130*a^12*b^3/x^(7/2)-4
55*a^11*b^4/x^3-6006/5*a^10*b^5/x^(5/2)-5005/2*a^9*b^6/x^2-4290*a^8*b^7/x^(
3/2)-6435*a^7*b^8/x+455*a^3*b^12*x+70*a^2*b^13*x^(3/2)+15/2*a*b^14*x^2+2/5*
b^15*x^(5/2)+3003*a^5*b^10*ln(x)-10010*a^6*b^9/x^(1/2)+2730*a^4*b^11*x^(1/2
)
```

maxima [A] time = 0.91, size = 165, normalized size = 0.85

$$\frac{2}{5}\,b^{15}x^{\frac{5}{2}} + \frac{15}{2}\,ab^{14}x^2 + 70\,a^2b^{13}x^{\frac{3}{2}} + 455\,a^3b^{12}x + 3003\,a^5b^{10}\log(x) + 2730\,a^4b^{11}\sqrt{x} - \frac{600600\,a^6b^9x^{\frac{9}{2}} + 386100\,a^7b^}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^6,x, algorithm="maxima")
```

```
[Out] 2/5*b^15*x^(5/2) + 15/2*a*b^14*x^2 + 70*a^2*b^13*x^(3/2) + 455*a^3*b^12*x +
 3003*a^5*b^10*log(x) + 2730*a^4*b^11*sqrt(x) - 1/60*(600600*a^6*b^9*x^(9/2
) + 386100*a^7*b^8*x^4 + 257400*a^8*b^7*x^(7/2) + 150150*a^9*b^6*x^3 + 7207
2*a^10*b^5*x^(5/2) + 27300*a^11*b^4*x^2 + 7800*a^12*b^3*x^(3/2) + 1575*a^13
*b^2*x + 200*a^14*b*sqrt(x) + 12*a^15)/x^5
```

mupad [B] time = 1.12, size = 167, normalized size = 0.86

$$\frac{2\,b^{15}\,x^{5/2}}{5}-\frac{\frac{a^{15}}{5}+\frac{105\,a^{13}\,b^2\,x}{4}+\frac{10\,a^{14}\,b\,\sqrt{x}}{3}+455\,a^{11}\,b^4\,x^2+\frac{5005\,a^9\,b^6\,x^3}{2}+6435\,a^7\,b^8\,x^4+130\,a^{12}\,b^3\,x^{3/2}+\frac{6006\,a^{10}\,b^5\,x^{5/2}}{5}}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^15/x^6,x)
```

```
[Out] (2*b^15*x^(5/2))/5 - (a^15/5 + (105*a^13*b^2*x)/4 + (10*a^14*b*x^(1/2))/3 +
 455*a^11*b^4*x^2 + (5005*a^9*b^6*x^3)/2 + 6435*a^7*b^8*x^4 + 130*a^12*b^3*
x^(3/2) + (6006*a^10*b^5*x^(5/2))/5 + 4290*a^8*b^7*x^(7/2) + 10010*a^6*b^9*
x^(9/2))/x^5 + 6006*a^5*b^10*log(x^(1/2)) + 455*a^3*b^12*x + (15*a*b^14*x^2
)/2 + 2730*a^4*b^11*x^(1/2) + 70*a^2*b^13*x^(3/2)
```

sympy [A] time = 5.05, size = 199, normalized size = 1.03

$$-\frac{a^{15}}{5x^5}-\frac{10a^{14}b}{3x^{\frac{9}{2}}}-\frac{105a^{13}b^2}{4x^4}-\frac{130a^{12}b^3}{x^{\frac{7}{2}}}-\frac{455a^{11}b^4}{x^3}-\frac{6006a^{10}b^5}{5x^{\frac{5}{2}}}-\frac{5005a^9b^6}{2x^2}-\frac{4290a^8b^7}{x^{\frac{3}{2}}}-\frac{6435a^7b^8}{x}-\frac{10010a^6b^9}{\sqrt{x}}+3003a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**15/x**6,x)
```

```
[Out] -a**15/(5*x**5) - 10*a**14*b/(3*x**(9/2)) - 105*a**13*b**2/(4*x**4) - 130*a
**12*b**3/x**(7/2) - 455*a**11*b**4/x**3 - 6006*a**10*b**5/(5*x**(5/2)) - 5
005*a**9*b**6/(2*x**2) - 4290*a**8*b**7/x**(3/2) - 6435*a**7*b**8/x - 10010
*a**6*b**9/sqrt(x) + 3003*a**5*b**10*log(x) + 2730*a**4*b**11*sqrt(x) + 455
*a**3*b**12*x + 70*a**2*b**13*x**(3/2) + 15*a*b**14*x**2/2 + 2*b**15*x**(5/
2)/5
```

3.2180 $\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^7}\,dx$

Optimal. Leaf size=196

$$-\frac{a^{15}}{6x^6}-\frac{30a^{14}b}{11x^{11/2}}-\frac{21a^{13}b^2}{x^5}-\frac{910a^{12}b^3}{9x^{9/2}}-\frac{1365a^{11}b^4}{4x^4}-\frac{858a^{10}b^5}{x^{7/2}}-\frac{5005a^9b^6}{3x^3}-\frac{2574a^8b^7}{x^{5/2}}-\frac{6435a^7b^8}{2x^2}-\frac{10010a^6b^9}{3x^{3/2}}-\frac{300}{}$$

```
[Out] -1/6*a^15/x^6-30/11*a^14*b/x^(11/2)-21*a^13*b^2/x^5-910/9*a^12*b^3/x^(9/2)-
1365/4*a^11*b^4/x^4-858*a^10*b^5/x^(7/2)-5005/3*a^9*b^6/x^3-2574*a^8*b^7/x^
(5/2)-6435/2*a^7*b^8/x^2-10010/3*a^6*b^9/x^(3/2)-3003*a^5*b^10/x+15*a*b^14*
x+2/3*b^15*x^(3/2)+455*a^3*b^12*ln(x)-2730*a^4*b^11/x^(1/2)+210*a^2*b^13*x^
(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{21a^{13}b^2}{x^5}-\frac{910a^{12}b^3}{9x^{9/2}}-\frac{1365a^{11}b^4}{4x^4}-\frac{858a^{10}b^5}{x^{7/2}}-\frac{5005a^9b^6}{3x^3}-\frac{2574a^8b^7}{x^{5/2}}-\frac{6435a^7b^8}{2x^2}-\frac{10010a^6b^9}{3x^{3/2}}-\frac{3003a^5b^{10}}{x}-\frac{2730a}{\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^15/x^7,x]
```

```
[Out] -a^15/(6*x^6) - (30*a^14*b)/(11*x^(11/2)) - (21*a^13*b^2)/x^5 - (910*a^12*b
^3)/(9*x^(9/2)) - (1365*a^11*b^4)/(4*x^4) - (858*a^10*b^5)/x^(7/2) - (5005*
a^9*b^6)/(3*x^3) - (2574*a^8*b^7)/x^(5/2) - (6435*a^7*b^8)/(2*x^2) - (10010
*a^6*b^9)/(3*x^(3/2)) - (3003*a^5*b^10)/x - (2730*a^4*b^11)/Sqrt[x] + 210*a
^2*b^13*Sqrt[x] + 15*a*b^14*x + (2*b^15*x^(3/2))/3 + 455*a^3*b^12*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^7}\,dx &= 2\,\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{13}}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(105a^2b^{13}+\frac{a^{15}}{x^{13}}+\frac{15a^{14}b}{x^{12}}+\frac{105a^{13}b^2}{x^{11}}+\frac{455a^{12}b^3}{x^{10}}+\frac{1365a^{11}b^4}{x^9}+\frac{3003a^{10}b}{x^8}\right.\right.\\ &= -\frac{a^{15}}{6x^6}-\frac{30a^{14}b}{11x^{11/2}}-\frac{21a^{13}b^2}{x^5}-\frac{910a^{12}b^3}{9x^{9/2}}-\frac{1365a^{11}b^4}{4x^4}-\frac{858a^{10}b^5}{x^{7/2}}-\frac{5005a^9b^6}{3x^3}-\frac{2574a^8b^7}{x^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.16, size = 196, normalized size = 1.00

$$-\frac{a^{15}}{6x^6}-\frac{30a^{14}b}{11x^{11/2}}-\frac{21a^{13}b^2}{x^5}-\frac{910a^{12}b^3}{9x^{9/2}}-\frac{1365a^{11}b^4}{4x^4}-\frac{858a^{10}b^5}{x^{7/2}}-\frac{5005a^9b^6}{3x^3}-\frac{2574a^8b^7}{x^{5/2}}-\frac{6435a^7b^8}{2x^2}-\frac{10010a^6b^9}{3x^{3/2}}-\frac{300}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^15/x^7,x]
```

```
[Out] -1/6*a^15/x^6 - (30*a^14*b)/(11*x^(11/2)) - (21*a^13*b^2)/x^5 - (910*a^12*b
^3)/(9*x^(9/2)) - (1365*a^11*b^4)/(4*x^4) - (858*a^10*b^5)/x^(7/2) - (5005*
a^9*b^6)/(3*x^3) - (2574*a^8*b^7)/x^(5/2) - (6435*a^7*b^8)/(2*x^2) - (10010
*a^6*b^9)/(3*x^(3/2)) - (3003*a^5*b^10)/x - (2730*a^4*b^11)/Sqrt[x] + 210*a
^2*b^13*Sqrt[x] + 15*a*b^14*x + (2*b^15*x^(3/2))/3 + 455*a^3*b^12*Log[x]
```

fricas [A] time = 0.82, size = 172, normalized size = 0.88

$$\frac{5940\,ab^{14}x^7+360360\,a^3b^{12}x^6\log\left(\sqrt{x}\right)-1189188\,a^5b^{10}x^5-1274130\,a^7b^8x^4-660660\,a^9b^6x^3-135135\,a^{11}b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^7,x, algorithm="fricas")
```

```
[Out] 1/396*(5940*a*b^14*x^7 + 360360*a^3*b^12*x^6*log(sqrt(x)) - 1189188*a^5*b^1
0*x^5 - 1274130*a^7*b^8*x^4 - 660660*a^9*b^6*x^3 - 135135*a^11*b^4*x^2 - 83
16*a^13*b^2*x - 66*a^15 + 8*(33*b^15*x^7 + 10395*a^2*b^13*x^6 - 135135*a^4*
b^11*x^5 - 165165*a^6*b^9*x^4 - 127413*a^8*b^7*x^3 - 42471*a^10*b^5*x^2 - 5
005*a^12*b^3*x - 135*a^14*b)*sqrt(x))/x^6
```

giac [A] time = 0.18, size = 166, normalized size = 0.85

$$\frac{2}{3}\,b^{15}x^{\frac{3}{2}}+15\,ab^{14}x+455\,a^3b^{12}\log\left(|x|\right)+210\,a^2b^{13}\sqrt{x}-\frac{1081080\,a^4b^{11}x^{\frac{11}{2}}+1189188\,a^5b^{10}x^5+1321320\,a^6b^9x^{\frac{9}{2}}+12}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^7,x, algorithm="giac")
```

```
[Out] 2/3*b^15*x^(3/2) + 15*a*b^14*x + 455*a^3*b^12*log(abs(x)) + 210*a^2*b^13*sq
rt(x) - 1/396*(1081080*a^4*b^11*x^(11/2) + 1189188*a^5*b^10*x^5 + 1321320*a
^6*b^9*x^(9/2) + 1274130*a^7*b^8*x^4 + 1019304*a^8*b^7*x^(7/2) + 660660*a^9
*b^6*x^3 + 339768*a^10*b^5*x^(5/2) + 135135*a^11*b^4*x^2 + 40040*a^12*b^3*x
^(3/2) + 8316*a^13*b^2*x + 1080*a^14*b*sqrt(x) + 66*a^15)/x^6
```

maple [A] time = 0.00, size = 165, normalized size = 0.84

$$\frac{2b^{15}x^{\frac{3}{2}}}{3}+455a^3b^{12}\ln(x)+15a\,b^{14}x+210a^2b^{13}\sqrt{x}-\frac{2730a^4b^{11}}{\sqrt{x}}-\frac{3003a^5b^{10}}{x}-\frac{10010a^6b^9}{3x^{\frac{3}{2}}}-\frac{6435a^7b^8}{2x^2}-\frac{2574a^8b^7}{x^{\frac{5}{2}}}-\frac{500}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^15/x^7,x)
```

```
[Out] -1/6*a^15/x^6-30/11*a^14*b/x^(11/2)-21*a^13*b^2/x^5-910/9*a^12*b^3/x^(9/2)-
1365/4*a^11*b^4/x^4-858*a^10*b^5/x^(7/2)-5005/3*a^9*b^6/x^3-2574*a^8*b^7/x^
(5/2)-6435/2*a^7*b^8/x^2-10010/3*a^6*b^9/x^(3/2)-3003*a^5*b^10/x+15*a*b^14*
x+2/3*b^15*x^(3/2)+455*a^3*b^12*ln(x)-2730*a^4*b^11/x^(1/2)+210*a^2*b^13*x^
(1/2)
```

maxima [A] time = 0.90, size = 165, normalized size = 0.84

$$\frac{2}{3}\,b^{15}x^{\frac{3}{2}}+15\,ab^{14}x+455\,a^3b^{12}\log(x)+210\,a^2b^{13}\sqrt{x}-\frac{1081080\,a^4b^{11}x^{\frac{11}{2}}+1189188\,a^5b^{10}x^5+1321320\,a^6b^9x^{\frac{9}{2}}+127}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^7,x, algorithm="maxima")
```

```
[Out] 2/3*b^15*x^(3/2) + 15*a*b^14*x + 455*a^3*b^12*log(x) + 210*a^2*b^13*sqrt(x)
 - 1/396*(1081080*a^4*b^11*x^(11/2) + 1189188*a^5*b^10*x^5 + 1321320*a^6*b^
9*x^(9/2) + 1274130*a^7*b^8*x^4 + 1019304*a^8*b^7*x^(7/2) + 660660*a^9*b^6*
x^3 + 339768*a^10*b^5*x^(5/2) + 135135*a^11*b^4*x^2 + 40040*a^12*b^3*x^(3/2
) + 8316*a^13*b^2*x + 1080*a^14*b*sqrt(x) + 66*a^15)/x^6
```

mupad **[B]** time = 1.12, size = 167, normalized size = 0.85

$$\frac{2\,b^{15}\,x^{3/2}}{3}-\frac{\frac{a^{15}}{6}+21\,a^{13}\,b^2\,x+\frac{30\,a^{14}\,b\,\sqrt{x}}{11}+\frac{1365\,a^{11}\,b^4\,x^2}{4}+\frac{5005\,a^9\,b^6\,x^3}{3}+\frac{6435\,a^7\,b^8\,x^4}{2}+3003\,a^5\,b^{10}\,x^5+\frac{910\,a^{12}\,b^3\,x^{3/2}}{9}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^15/x^7,x)
```

```
[Out] (2*b^15*x^(3/2))/3 - (a^15/6 + 21*a^13*b^2*x + (30*a^14*b*x^(1/2))/11 + (13
65*a^11*b^4*x^2)/4 + (5005*a^9*b^6*x^3)/3 + (6435*a^7*b^8*x^4)/2 + 3003*a^5
*b^10*x^5 + (910*a^12*b^3*x^(3/2))/9 + 858*a^10*b^5*x^(5/2) + 2574*a^8*b^7*
x^(7/2) + (10010*a^6*b^9*x^(9/2))/3 + 2730*a^4*b^11*x^(11/2))/x^6 + 910*a^3
*b^12*log(x^(1/2)) + 210*a^2*b^13*x^(1/2) + 15*a*b^14*x
```

sympy **[A]** time = 5.04, size = 201, normalized size = 1.03

$$-\frac{a^{15}}{6x^6}-\frac{30a^{14}b}{11x^{\frac{11}{2}}}-\frac{21a^{13}b^2}{x^5}-\frac{910a^{12}b^3}{9x^{\frac{9}{2}}}-\frac{1365a^{11}b^4}{4x^4}-\frac{858a^{10}b^5}{x^{\frac{7}{2}}}-\frac{5005a^9b^6}{3x^3}-\frac{2574a^8b^7}{x^{\frac{5}{2}}}-\frac{6435a^7b^8}{2x^2}-\frac{10010a^6b^9}{3x^{\frac{3}{2}}}-\frac{300}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**15/x**7,x)
```

```
[Out] -a**15/(6*x**6) - 30*a**14*b/(11*x**(11/2)) - 21*a**13*b**2/x**5 - 910*a**1
2*b**3/(9*x**(9/2)) - 1365*a**11*b**4/(4*x**4) - 858*a**10*b**5/x**(7/2) -
5005*a**9*b**6/(3*x**3) - 2574*a**8*b**7/x**(5/2) - 6435*a**7*b**8/(2*x**2)
 - 10010*a**6*b**9/(3*x**(3/2)) - 3003*a**5*b**10/x - 2730*a**4*b**11/sqrt(
x) + 455*a**3*b**12*log(x) + 210*a**2*b**13*sqrt(x) + 15*a*b**14*x + 2*b**1
5*x**(3/2)/3
```

3.2181 $\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^8}\,dx$

Optimal. Leaf size=198

$$-\frac{a^{15}}{7x^7}-\frac{30a^{14}b}{13x^{13/2}}-\frac{35a^{13}b^2}{2x^6}-\frac{910a^{12}b^3}{11x^{11/2}}-\frac{273a^{11}b^4}{x^5}-\frac{2002a^{10}b^5}{3x^{9/2}}-\frac{5005a^9b^6}{4x^4}-\frac{12870a^8b^7}{7x^{7/2}}-\frac{2145a^7b^8}{x^3}-\frac{2002a^6b^9}{x^{5/2}}-\frac{3003a}{2x^2}$$

```
[Out] -1/7*a^15/x^7-30/13*a^14*b/x^(13/2)-35/2*a^13*b^2/x^6-910/11*a^12*b^3/x^(11
/2)-273*a^11*b^4/x^5-2002/3*a^10*b^5/x^(9/2)-5005/4*a^9*b^6/x^4-12870/7*a^8
*b^7/x^(7/2)-2145*a^7*b^8/x^3-2002*a^6*b^9/x^(5/2)-3003/2*a^5*b^10/x^2-910*
a^4*b^11/x^(3/2)-455*a^3*b^12/x+15*a*b^14*ln(x)-210*a^2*b^13/x^(1/2)+2*b^15
*x^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{35a^{13}b^2}{2x^6}-\frac{910a^{12}b^3}{11x^{11/2}}-\frac{273a^{11}b^4}{x^5}-\frac{2002a^{10}b^5}{3x^{9/2}}-\frac{5005a^9b^6}{4x^4}-\frac{12870a^8b^7}{7x^{7/2}}-\frac{2145a^7b^8}{x^3}-\frac{2002a^6b^9}{x^{5/2}}-\frac{3003a^5b^{10}}{2x^2}-\frac{910a^4b^{11}}{x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^15/x^8,x]
```

```
[Out] -a^15/(7*x^7) - (30*a^14*b)/(13*x^(13/2)) - (35*a^13*b^2)/(2*x^6) - (910*a^
12*b^3)/(11*x^(11/2)) - (273*a^11*b^4)/x^5 - (2002*a^10*b^5)/(3*x^(9/2)) -
(5005*a^9*b^6)/(4*x^4) - (12870*a^8*b^7)/(7*x^(7/2)) - (2145*a^7*b^8)/x^3 -
 (2002*a^6*b^9)/x^(5/2) - (3003*a^5*b^10)/(2*x^2) - (910*a^4*b^11)/x^(3/2)
- (455*a^3*b^12)/x - (210*a^2*b^13)/Sqrt[x] + 2*b^15*Sqrt[x] + 15*a*b^14*Lo
g[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^8}\,dx &= 2\,\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{15}}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(b^{15}+\frac{a^{15}}{x^{15}}+\frac{15a^{14}b}{x^{14}}+\frac{105a^{13}b^2}{x^{13}}+\frac{455a^{12}b^3}{x^{12}}+\frac{1365a^{11}b^4}{x^{11}}+\frac{3003a^{10}b^5}{x^{10}}+\frac{500}{}\right.\right.\\ &= -\frac{a^{15}}{7x^7}-\frac{30a^{14}b}{13x^{13/2}}-\frac{35a^{13}b^2}{2x^6}-\frac{910a^{12}b^3}{11x^{11/2}}-\frac{273a^{11}b^4}{x^5}-\frac{2002a^{10}b^5}{3x^{9/2}}-\frac{5005a^9b^6}{4x^4}-\frac{12870a^8b^7}{7x^{7/2}}-\end{aligned}$$

Mathematica [A] time = 0.18, size = 198, normalized size = 1.00

$$-\frac{a^{15}}{7x^7}-\frac{30a^{14}b}{13x^{13/2}}-\frac{35a^{13}b^2}{2x^6}-\frac{910a^{12}b^3}{11x^{11/2}}-\frac{273a^{11}b^4}{x^5}-\frac{2002a^{10}b^5}{3x^{9/2}}-\frac{5005a^9b^6}{4x^4}-\frac{12870a^8b^7}{7x^{7/2}}-\frac{2145a^7b^8}{x^3}-\frac{2002a^6b^9}{x^{5/2}}-\frac{3003a}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^15/x^8,x]
```

```
[Out] -1/7*a^15/x^7 - (30*a^14*b)/(13*x^(13/2)) - (35*a^13*b^2)/(2*x^6) - (910*a^
12*b^3)/(11*x^(11/2)) - (273*a^11*b^4)/x^5 - (2002*a^10*b^5)/(3*x^(9/2)) -
(5005*a^9*b^6)/(4*x^4) - (12870*a^8*b^7)/(7*x^(7/2)) - (2145*a^7*b^8)/x^3 -
 (2002*a^6*b^9)/x^(5/2) - (3003*a^5*b^10)/(2*x^2) - (910*a^4*b^11)/x^(3/2)
- (455*a^3*b^12)/x - (210*a^2*b^13)/Sqrt[x] + 2*b^15*Sqrt[x] + 15*a*b^14*Lo
g[x]
```

fricas [A] time = 1.09, size = 172, normalized size = 0.87

$$\frac{360360\,ab^{14}x^7\log\left(\sqrt{x}\right) - 5465460\,a^3b^{12}x^6 - 18036018\,a^5b^{10}x^5 - 25765740\,a^7b^8x^4 - 15030015\,a^9b^6x^3 - 3279}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^8,x, algorithm="fricas")
```

```
[Out] 1/12012*(360360*a*b^14*x^7*log(sqrt(x)) - 5465460*a^3*b^12*x^6 - 18036018*a
^5*b^10*x^5 - 25765740*a^7*b^8*x^4 - 15030015*a^9*b^6*x^3 - 3279276*a^11*b^
4*x^2 - 210210*a^13*b^2*x - 1716*a^15 + 8*(3003*b^15*x^7 - 315315*a^2*b^13*
x^6 - 1366365*a^4*b^11*x^5 - 3006003*a^6*b^9*x^4 - 2760615*a^8*b^7*x^3 - 10
02001*a^10*b^5*x^2 - 124215*a^12*b^3*x - 3465*a^14*b)*sqrt(x))/x^7
```

giac [A] time = 0.18, size = 168, normalized size = 0.85

$$15\,ab^{14}\log\left(|x|\right)+2\,b^{15}\sqrt{x}-\frac{2522520\,a^2b^{13}x^{\frac{13}{2}}+5465460\,a^3b^{12}x^6+10930920\,a^4b^{11}x^{\frac{11}{2}}+18036018\,a^5b^{10}x^5+2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^8,x, algorithm="giac")
```

```
[Out] 15*a*b^14*log(abs(x)) + 2*b^15*sqrt(x) - 1/12012*(2522520*a^2*b^13*x^(13/2)
 + 5465460*a^3*b^12*x^6 + 10930920*a^4*b^11*x^(11/2) + 18036018*a^5*b^10*x^
5 + 24048024*a^6*b^9*x^(9/2) + 25765740*a^7*b^8*x^4 + 22084920*a^8*b^7*x^(7
/2) + 15030015*a^9*b^6*x^3 + 8016008*a^10*b^5*x^(5/2) + 3279276*a^11*b^4*x^
2 + 993720*a^12*b^3*x^(3/2) + 210210*a^13*b^2*x + 27720*a^14*b*sqrt(x) + 17
16*a^15)/x^7
```

maple [A] time = 0.00, size = 167, normalized size = 0.84

$$15a\,b^{14}\ln(x)+2b^{15}\sqrt{x}-\frac{210a^2b^{13}}{\sqrt{x}}-\frac{455a^3b^{12}}{x}-\frac{910a^4b^{11}}{x^{\frac{3}{2}}}-\frac{3003a^5b^{10}}{2x^2}-\frac{2002a^6b^9}{x^{\frac{5}{2}}}-\frac{2145a^7b^8}{x^3}-\frac{12870a^8b^7}{7x^{\frac{7}{2}}}-\frac{5005a}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^15/x^8,x)
```

```
[Out] -1/7*a^15/x^7-30/13*a^14*b/x^(13/2)-35/2*a^13*b^2/x^6-910/11*a^12*b^3/x^(11
/2)-273*a^11*b^4/x^5-2002/3*a^10*b^5/x^(9/2)-5005/4*a^9*b^6/x^4-12870/7*a^8
*b^7/x^(7/2)-2145*a^7*b^8/x^3-2002*a^6*b^9/x^(5/2)-3003/2*a^5*b^10/x^2-910*
a^4*b^11/x^(3/2)-455*a^3*b^12/x+15*a*b^14*ln(x)-210*a^2*b^13/x^(1/2)+2*b^15
*x^(1/2)
```

maxima [A] time = 0.87, size = 167, normalized size = 0.84

$$15\,ab^{14}\log(x)+2\,b^{15}\sqrt{x}-\frac{2522520\,a^2b^{13}x^{\frac{13}{2}}+5465460\,a^3b^{12}x^6+10930920\,a^4b^{11}x^{\frac{11}{2}}+18036018\,a^5b^{10}x^5+24}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^8,x, algorithm="maxima")
```

```
[Out] 15*a*b^14*log(x) + 2*b^15*sqrt(x) - 1/12012*(2522520*a^2*b^13*x^(13/2) + 54
65460*a^3*b^12*x^6 + 10930920*a^4*b^11*x^(11/2) + 18036018*a^5*b^10*x^5 + 2
4048024*a^6*b^9*x^(9/2) + 25765740*a^7*b^8*x^4 + 22084920*a^8*b^7*x^(7/2) +
 15030015*a^9*b^6*x^3 + 8016008*a^10*b^5*x^(5/2) + 3279276*a^11*b^4*x^2 + 9
93720*a^12*b^3*x^(3/2) + 210210*a^13*b^2*x + 27720*a^14*b*sqrt(x) + 1716*a^
15)/x^7
```

mupad **[B]** time = 1.27, size = 168, normalized size = 0.85

$$2\,b^{15}\,\sqrt{x}-\frac{a^{15}}{7\,x^7}-\frac{30\,a^{14}\,b}{13\,x^{13/2}}-\frac{455\,a^3\,b^{12}}{x}-\frac{3003\,a^5\,b^{10}}{2\,x^2}-\frac{210\,a^2\,b^{13}}{\sqrt{x}}-\frac{2145\,a^7\,b^8}{x^3}-\frac{5005\,a^9\,b^6}{4\,x^4}-\frac{910\,a^4\,b^{11}}{x^{3/2}}-\frac{273\,a^{11}\,b^4}{x^5}-\frac{35}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^15/x^8,x)
```

```
[Out] 2*b^15*x^(1/2) - a^15/(7*x^7) - (30*a^14*b)/(13*x^(13/2)) - (455*a^3*b^12)/
x - (3003*a^5*b^10)/(2*x^2) - (210*a^2*b^13)/x^(1/2) - (2145*a^7*b^8)/x^3 -
 (5005*a^9*b^6)/(4*x^4) - (910*a^4*b^11)/x^(3/2) - (273*a^11*b^4)/x^5 - (35
*a^13*b^2)/(2*x^6) - (2002*a^6*b^9)/x^(5/2) - (12870*a^8*b^7)/(7*x^(7/2)) -
 (2002*a^10*b^5)/(3*x^(9/2)) - (910*a^12*b^3)/(11*x^(11/2)) + 30*a*b^14*log
(x^(1/2))
```

sympy **[A]** time = 5.87, size = 202, normalized size = 1.02

$$-\frac{a^{15}}{7x^7}-\frac{30a^{14}b}{13x^{\frac{13}{2}}}-\frac{35a^{13}b^2}{2x^6}-\frac{910a^{12}b^3}{11x^{\frac{11}{2}}}-\frac{273a^{11}b^4}{x^5}-\frac{2002a^{10}b^5}{3x^{\frac{9}{2}}}-\frac{5005a^9b^6}{4x^4}-\frac{12870a^8b^7}{7x^{\frac{7}{2}}}-\frac{2145a^7b^8}{x^3}-\frac{2002a^6b^9}{x^{\frac{5}{2}}}-\frac{3003a^{5}}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**15/x**8,x)
```

```
[Out] -a**15/(7*x**7) - 30*a**14*b/(13*x**(13/2)) - 35*a**13*b**2/(2*x**6) - 910*
a**12*b**3/(11*x**(11/2)) - 273*a**11*b**4/x**5 - 2002*a**10*b**5/(3*x**(9/
2)) - 5005*a**9*b**6/(4*x**4) - 12870*a**8*b**7/(7*x**(7/2)) - 2145*a**7*b*
*8/x**3 - 2002*a**6*b**9/x**(5/2) - 3003*a**5*b**10/(2*x**2) - 910*a**4*b**
11/x**(3/2) - 455*a**3*b**12/x - 210*a**2*b**13/sqrt(x) + 15*a*b**14*log(x)
 + 2*b**15*sqrt(x)
```

3.2182 $\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^9}\,dx$

Optimal. Leaf size=21

$$-\frac{\left(a+b\sqrt{x}\right)^{16}}{8ax^8}$$

```
[Out] -1/8*(a+b*x^(1/2))^16/a/x^8
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {264}

$$-\frac{\left(a+b\sqrt{x}\right)^{16}}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^15/x^9,x]
```

```
[Out] -(a + b*Sqrt[x])^16/(8*a*x^8)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^9}\,dx = -\frac{\left(a+b\sqrt{x}\right)^{16}}{8ax^8}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$-\frac{\left(a+b\sqrt{x}\right)^{16}}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^15/x^9,x]
```

```
[Out] -1/8*(a + b*Sqrt[x])^16/(a*x^8)
```

fricas [B] time = 0.92, size = 164, normalized size = 7.81

$$-\frac{120\,ab^{14}x^7 + 1820\,a^3b^{12}x^6 + 8008\,a^5b^{10}x^5 + 12870\,a^7b^8x^4 + 8008\,a^9b^6x^3 + 1820\,a^{11}b^4x^2 + 120\,a^{13}b^2x + a^{15}}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^9,x, algorithm="fricas")
```

```
[Out] -1/8*(120*a*b^14*x^7 + 1820*a^3*b^12*x^6 + 8008*a^5*b^10*x^5 + 12870*a^7*b^
8*x^4 + 8008*a^9*b^6*x^3 + 1820*a^11*b^4*x^2 + 120*a^13*b^2*x + a^15 + 16*(
b^15*x^7 + 35*a^2*b^13*x^6 + 273*a^4*b^11*x^5 + 715*a^6*b^9*x^4 + 715*a^8*b
^7*x^3 + 273*a^10*b^5*x^2 + 35*a^12*b^3*x + a^14*b)*sqrt(x))/x^8
```

giac [B] time = 0.19, size = 165, normalized size = 7.86

$$-\frac{16\,b^{15}x^{\frac{15}{2}}+120\,ab^{14}x^{7}+560\,a^{2}b^{13}x^{\frac{13}{2}}+1820\,a^{3}b^{12}x^{6}+4368\,a^{4}b^{11}x^{\frac{11}{2}}+8008\,a^{5}b^{10}x^{5}+11440\,a^{6}b^{9}x^{\frac{9}{2}}+12870$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^9,x, algorithm="giac")
```

```
[Out] -1/8*(16*b^15*x^(15/2) + 120*a*b^14*x^7 + 560*a^2*b^13*x^(13/2) + 1820*a^3*
b^12*x^6 + 4368*a^4*b^11*x^(11/2) + 8008*a^5*b^10*x^5 + 11440*a^6*b^9*x^(9/
2) + 12870*a^7*b^8*x^4 + 11440*a^8*b^7*x^(7/2) + 8008*a^9*b^6*x^3 + 4368*a^
10*b^5*x^(5/2) + 1820*a^11*b^4*x^2 + 560*a^12*b^3*x^(3/2) + 120*a^13*b^2*x
+ 16*a^14*b*sqrt(x) + a^15)/x^8
```

maple [B] time = 0.00, size = 168, normalized size = 8.00

$$-\frac{2b^{15}}{\sqrt{x}}-\frac{15a\,b^{14}}{x}-\frac{70a^{2}b^{13}}{x^{\frac{3}{2}}}-\frac{455a^{3}b^{12}}{2x^{2}}-\frac{546a^{4}b^{11}}{x^{\frac{5}{2}}}-\frac{1001a^{5}b^{10}}{x^{3}}-\frac{1430a^{6}b^{9}}{x^{\frac{7}{2}}}-\frac{6435a^{7}b^{8}}{4x^{4}}-\frac{1430a^{8}b^{7}}{x^{\frac{9}{2}}}-\frac{1001a^{9}b^{6}}{x^{5}}-\frac{546a^{10}}{x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^15/x^9,x)
```

```
[Out] -2*b^15/x^(1/2)-15*a*b^14/x-70*a^2*b^13/x^(3/2)-455/2*a^3*b^12/x^2-546*a^4*
b^11/x^(5/2)-1001*a^5*b^10/x^3-1430*a^6*b^9/x^(7/2)-6435/4*a^7*b^8/x^4-1430
*a^8*b^7/x^(9/2)-1001*a^9*b^6/x^5-546*a^10*b^5/x^(11/2)-455/2*a^11*b^4/x^6-
70*a^12*b^3/x^(13/2)-15*a^13*b^2/x^7-2*a^14*b/x^(15/2)-1/8*a^15/x^8
```

maxima [B] time = 0.96, size = 165, normalized size = 7.86

$$-\frac{16\,b^{15}x^{\frac{15}{2}}+120\,ab^{14}x^{7}+560\,a^{2}b^{13}x^{\frac{13}{2}}+1820\,a^{3}b^{12}x^{6}+4368\,a^{4}b^{11}x^{\frac{11}{2}}+8008\,a^{5}b^{10}x^{5}+11440\,a^{6}b^{9}x^{\frac{9}{2}}+12870$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^9,x, algorithm="maxima")
```

```
[Out] -1/8*(16*b^15*x^(15/2) + 120*a*b^14*x^7 + 560*a^2*b^13*x^(13/2) + 1820*a^3*
b^12*x^6 + 4368*a^4*b^11*x^(11/2) + 8008*a^5*b^10*x^5 + 11440*a^6*b^9*x^(9/
2) + 12870*a^7*b^8*x^4 + 11440*a^8*b^7*x^(7/2) + 8008*a^9*b^6*x^3 + 4368*a^
10*b^5*x^(5/2) + 1820*a^11*b^4*x^2 + 560*a^12*b^3*x^(3/2) + 120*a^13*b^2*x
+ 16*a^14*b*sqrt(x) + a^15)/x^8
```

mupad [B] time = 0.20, size = 167, normalized size = 7.95

$$-\frac{\frac{a^{15}}{8}+2\,b^{15}\,x^{15/2}+15\,a^{13}\,b^{2}\,x+2\,a^{14}\,b\,\sqrt{x}+15\,a\,b^{14}\,x^{7}+\frac{455\,a^{11}\,b^{4}\,x^{2}}{2}+1001\,a^{9}\,b^{6}\,x^{3}+\frac{6435\,a^{7}\,b^{8}\,x^{4}}{4}+1001\,a^{5}\,b^{10}\,x^{5}}{x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^15/x^9,x)
```

```
[Out] -(a^15/8 + 2*b^15*x^(15/2) + 15*a^13*b^2*x + 2*a^14*b*x^(1/2) + 15*a*b^14*x
^7 + (455*a^11*b^4*x^2)/2 + 1001*a^9*b^6*x^3 + (6435*a^7*b^8*x^4)/4 + 1001*
a^5*b^10*x^5 + 70*a^12*b^3*x^(3/2) + (455*a^3*b^12*x^6)/2 + 546*a^10*b^5*x^
(5/2) + 1430*a^8*b^7*x^(7/2) + 1430*a^6*b^9*x^(9/2) + 546*a^4*b^11*x^(11/2)
 + 70*a^2*b^13*x^(13/2))/x^8
```

sympy [B] time = 7.56, size = 197, normalized size = 9.38

$$-\frac{a^{15}}{8x^{8}}-\frac{2a^{14}b}{x^{\frac{15}{2}}}-\frac{15a^{13}b^{2}}{x^{7}}-\frac{70a^{12}b^{3}}{x^{\frac{13}{2}}}-\frac{455a^{11}b^{4}}{2x^{6}}-\frac{546a^{10}b^{5}}{x^{\frac{11}{2}}}-\frac{1001a^{9}b^{6}}{x^{5}}-\frac{1430a^{8}b^{7}}{x^{\frac{9}{2}}}-\frac{6435a^{7}b^{8}}{4x^{4}}-\frac{1430a^{6}b^{9}}{x^{\frac{7}{2}}}-\frac{1001a^{5}b^{10}}{x^{3}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**15/x**9,x)
```

```
[Out] -a**15/(8*x**8) - 2*a**14*b/x**(15/2) - 15*a**13*b**2/x**7 - 70*a**12*b**3/
x**(13/2) - 455*a**11*b**4/(2*x**6) - 546*a**10*b**5/x**(11/2) - 1001*a**9*
b**6/x**5 - 1430*a**8*b**7/x**(9/2) - 6435*a**7*b**8/(4*x**4) - 1430*a**6*b
**9/x**(7/2) - 1001*a**5*b**10/x**3 - 546*a**4*b**11/x**(5/2) - 455*a**3*b*
*12/(2*x**2) - 70*a**2*b**13/x**(3/2) - 15*a*b**14/x - 2*b**15/sqrt(x)
```

3.2183 $\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{10}}\,dx$

Optimal. Leaf size=70

$$-\frac{b^2\left(a+b\sqrt{x}\right)^{16}}{1224a^3x^8}+\frac{2b\left(a+b\sqrt{x}\right)^{16}}{153a^2x^{17/2}}-\frac{\left(a+b\sqrt{x}\right)^{16}}{9ax^9}$$

```
[Out] -1/9*(a+b*x^(1/2))^16/a/x^9+2/153*b*(a+b*x^(1/2))^16/a^2/x^(17/2)-1/1224*b^
2*(a+b*x^(1/2))^16/a^3/x^8
```

Rubi [A] time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {266, 45, 37}

$$-\frac{b^2\left(a+b\sqrt{x}\right)^{16}}{1224a^3x^8}+\frac{2b\left(a+b\sqrt{x}\right)^{16}}{153a^2x^{17/2}}-\frac{\left(a+b\sqrt{x}\right)^{16}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^15/x^10,x]
```

```
[Out] -(a + b*Sqrt[x])^16/(9*a*x^9) + (2*b*(a + b*Sqrt[x])^16)/(153*a^2*x^(17/2))
 - (b^2*(a + b*Sqrt[x])^16)/(1224*a^3*x^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{10}}\,dx = 2\,\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{19}}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{\left(a+b\sqrt{x}\right)^{16}}{9ax^9} - \frac{(2b)\,\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{18}}\,dx, x, \sqrt{x}\right)}{9a}$$
$$= -\frac{\left(a+b\sqrt{x}\right)^{16}}{9ax^9} + \frac{2b\left(a+b\sqrt{x}\right)^{16}}{153a^2x^{17/2}} + \frac{\left(2b^2\right)\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{17}}\,dx, x, \sqrt{x}\right)}{153a^2}$$
$$= -\frac{\left(a+b\sqrt{x}\right)^{16}}{9ax^9} + \frac{2b\left(a+b\sqrt{x}\right)^{16}}{153a^2x^{17/2}} - \frac{b^2\left(a+b\sqrt{x}\right)^{16}}{1224a^3x^8}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.59

$$-\frac{\left(a+b\sqrt{x}\right)^{16}\left(136a^2 - 16ab\sqrt{x} + b^2x\right)}{1224a^3x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^15/x^10,x]
```

```
[Out] -1/1224*((a + b*Sqrt[x])^16*(136*a^2 - 16*a*b*Sqrt[x] + b^2*x))/(a^3*x^9)
```

fricas [B] time = 0.98, size = 168, normalized size = 2.40

$$-\frac{9180\,ab^{14}x^7 + 185640\,a^3b^{12}x^6 + 918918\,a^5b^{10}x^5 + 1575288\,a^7b^8x^4 + 1021020\,a^9b^6x^3 + 238680\,a^{11}b^4x^2 + 16}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^10,x, algorithm="fricas")
```

```
[Out] -1/1224*(9180*a*b^14*x^7 + 185640*a^3*b^12*x^6 + 918918*a^5*b^10*x^5 + 1575
288*a^7*b^8*x^4 + 1021020*a^9*b^6*x^3 + 238680*a^11*b^4*x^2 + 16065*a^13*b^
2*x + 136*a^15 + 16*(51*b^15*x^7 + 3213*a^2*b^13*x^6 + 29835*a^4*b^11*x^5 +
 85085*a^6*b^9*x^4 + 89505*a^8*b^7*x^3 + 35343*a^10*b^5*x^2 + 4641*a^12*b^3
*x + 135*a^14*b)*sqrt(x))/x^9
```

giac [B] time = 0.17, size = 167, normalized size = 2.39

$$-\frac{816\,b^{15}x^{\frac{15}{2}} + 9180\,ab^{14}x^7 + 51408\,a^2b^{13}x^{\frac{13}{2}} + 185640\,a^3b^{12}x^6 + 477360\,a^4b^{11}x^{\frac{11}{2}} + 918918\,a^5b^{10}x^5 + 136136}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^10,x, algorithm="giac")
```

```
[Out] -1/1224*(816*b^15*x^(15/2) + 9180*a*b^14*x^7 + 51408*a^2*b^13*x^(13/2) + 18
5640*a^3*b^12*x^6 + 477360*a^4*b^11*x^(11/2) + 918918*a^5*b^10*x^5 + 136136
0*a^6*b^9*x^(9/2) + 1575288*a^7*b^8*x^4 + 1432080*a^8*b^7*x^(7/2) + 1021020
*a^9*b^6*x^3 + 565488*a^10*b^5*x^(5/2) + 238680*a^11*b^4*x^2 + 74256*a^12*b
^3*x^(3/2) + 16065*a^13*b^2*x + 2160*a^14*b*sqrt(x) + 136*a^15)/x^9
```

maple [B] time = 0.00, size = 168, normalized size = 2.40

$$-\frac{2b^{15}}{3x^{\frac{3}{2}}} - \frac{15a\,b^{14}}{2x^2} - \frac{42a^2b^{13}}{x^{\frac{5}{2}}} - \frac{455a^3b^{12}}{3x^3} - \frac{390a^4b^{11}}{x^{\frac{7}{2}}} - \frac{3003a^5b^{10}}{4x^4} - \frac{10010a^6b^9}{9x^{\frac{9}{2}}} - \frac{1287a^7b^8}{x^5} - \frac{1170a^8b^7}{x^{\frac{11}{2}}} - \frac{5005a^9b^6}{6x^6} - \frac{46}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^15/x^10,x)
```

```
[Out] -2/3*b^15/x^(3/2)-15/2*a*b^14/x^2-42*a^2*b^13/x^(5/2)-455/3*a^3*b^12/x^3-39
0*a^4*b^11/x^(7/2)-3003/4*a^5*b^10/x^4-10010/9*a^6*b^9/x^(9/2)-1287*a^7*b^8
/x^5-1170*a^8*b^7/x^(11/2)-5005/6*a^9*b^6/x^6-462*a^10*b^5/x^(13/2)-195*a^1
1*b^4/x^7-182/3*a^12*b^3/x^(15/2)-105/8*a^13*b^2/x^8-30/17*a^14*b/x^(17/2)-
1/9*a^15/x^9
```

maxima [B] time = 0.91, size = 167, normalized size = 2.39

$$-\frac{816\,b^{15}x^{\frac{15}{2}}+9180\,ab^{14}x^{7}+51408\,a^{2}b^{13}x^{\frac{13}{2}}+185640\,a^{3}b^{12}x^{6}+477360\,a^{4}b^{11}x^{\frac{11}{2}}+918918\,a^{5}b^{10}x^{5}+1361360\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^10,x, algorithm="maxima")
```

```
[Out] -1/1224*(816*b^15*x^(15/2) + 9180*a*b^14*x^7 + 51408*a^2*b^13*x^(13/2) + 18
5640*a^3*b^12*x^6 + 477360*a^4*b^11*x^(11/2) + 918918*a^5*b^10*x^5 + 136136
0*a^6*b^9*x^(9/2) + 1575288*a^7*b^8*x^4 + 1432080*a^8*b^7*x^(7/2) + 1021020
*a^9*b^6*x^3 + 565488*a^10*b^5*x^(5/2) + 238680*a^11*b^4*x^2 + 74256*a^12*b
^3*x^(3/2) + 16065*a^13*b^2*x + 2160*a^14*b*sqrt(x) + 136*a^15)/x^9
```

mupad [B] time = 1.24, size = 167, normalized size = 2.39

$$-\frac{\frac{a^{15}}{9}+\frac{2\,b^{15}\,x^{15/2}}{3}+\frac{105\,a^{13}\,b^{2}\,x}{8}+\frac{30\,a^{14}\,b\,\sqrt{x}}{17}+\frac{15\,a\,b^{14}\,x^{7}}{2}+195\,a^{11}\,b^{4}\,x^{2}+\frac{5005\,a^{9}\,b^{6}\,x^{3}}{6}+1287\,a^{7}\,b^{8}\,x^{4}+\frac{3003\,a^{5}\,b^{10}\,x^{5}}{4}+\frac{18}{x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^15/x^10,x)
```

```
[Out] -(a^15/9 + (2*b^15*x^(15/2))/3 + (105*a^13*b^2*x)/8 + (30*a^14*b*x^(1/2))/1
7 + (15*a*b^14*x^7)/2 + 195*a^11*b^4*x^2 + (5005*a^9*b^6*x^3)/6 + 1287*a^7*
b^8*x^4 + (3003*a^5*b^10*x^5)/4 + (182*a^12*b^3*x^(3/2))/3 + (455*a^3*b^12*
x^6)/3 + 462*a^10*b^5*x^(5/2) + 1170*a^8*b^7*x^(7/2) + (10010*a^6*b^9*x^(9/
2))/9 + 390*a^4*b^11*x^(11/2) + 42*a^2*b^13*x^(13/2))/x^9
```

sympy [B] time = 10.07, size = 209, normalized size = 2.99

$$-\frac{a^{15}}{9x^{9}}-\frac{30a^{14}b}{17x^{\frac{17}{2}}}-\frac{105a^{13}b^{2}}{8x^{8}}-\frac{182a^{12}b^{3}}{3x^{\frac{15}{2}}}-\frac{195a^{11}b^{4}}{x^{7}}-\frac{462a^{10}b^{5}}{x^{\frac{13}{2}}}-\frac{5005a^{9}b^{6}}{6x^{6}}-\frac{1170a^{8}b^{7}}{x^{\frac{11}{2}}}-\frac{1287a^{7}b^{8}}{x^{5}}-\frac{10010a^{6}b^{9}}{9x^{\frac{9}{2}}}-\frac{3003a^{5}}{4x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**15/x**10,x)
```

```
[Out] -a**15/(9*x**9) - 30*a**14*b/(17*x**(17/2)) - 105*a**13*b**2/(8*x**8) - 182
*a**12*b**3/(3*x**(15/2)) - 195*a**11*b**4/x**7 - 462*a**10*b**5/x**(13/2)
- 5005*a**9*b**6/(6*x**6) - 1170*a**8*b**7/x**(11/2) - 1287*a**7*b**8/x**5
- 10010*a**6*b**9/(9*x**(9/2)) - 3003*a**5*b**10/(4*x**4) - 390*a**4*b**11/
x**(7/2) - 455*a**3*b**12/(3*x**3) - 42*a**2*b**13/x**(5/2) - 15*a*b**14/(2
*x**2) - 2*b**15/(3*x**(3/2))
```

3.2184 $\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{11}}\,dx$

Optimal. Leaf size=120

$$-\frac{b^4\left(a+b\sqrt{x}\right)^{16}}{38760a^5x^8}+\frac{2b^3\left(a+b\sqrt{x}\right)^{16}}{4845a^4x^{17/2}}-\frac{b^2\left(a+b\sqrt{x}\right)^{16}}{285a^3x^9}+\frac{2b\left(a+b\sqrt{x}\right)^{16}}{95a^2x^{19/2}}-\frac{\left(a+b\sqrt{x}\right)^{16}}{10ax^{10}}$$

```
[Out] -1/10*(a+b*x^(1/2))^16/a/x^10+2/95*b*(a+b*x^(1/2))^16/a^2/x^(19/2)-1/285*b^
2*(a+b*x^(1/2))^16/a^3/x^9+2/4845*b^3*(a+b*x^(1/2))^16/a^4/x^(17/2)-1/38760
*b^4*(a+b*x^(1/2))^16/a^5/x^8
```

Rubi [A] time = 0.05, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {266, 45, 37}

$$-\frac{b^4\left(a+b\sqrt{x}\right)^{16}}{38760a^5x^8}+\frac{2b^3\left(a+b\sqrt{x}\right)^{16}}{4845a^4x^{17/2}}-\frac{b^2\left(a+b\sqrt{x}\right)^{16}}{285a^3x^9}+\frac{2b\left(a+b\sqrt{x}\right)^{16}}{95a^2x^{19/2}}-\frac{\left(a+b\sqrt{x}\right)^{16}}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^15/x^11,x]
```

```
[Out] -(a + b*Sqrt[x])^16/(10*a*x^10) + (2*b*(a + b*Sqrt[x])^16)/(95*a^2*x^(19/2)
) - (b^2*(a + b*Sqrt[x])^16)/(285*a^3*x^9) + (2*b^3*(a + b*Sqrt[x])^16)/(48
45*a^4*x^(17/2)) - (b^4*(a + b*Sqrt[x])^16)/(38760*a^5*x^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{11}}\,dx &= 2\,\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{21}}\,dx, x, \sqrt{x}\right) \\
&= -\frac{\left(a+b\sqrt{x}\right)^{16}}{10ax^{10}} - \frac{(2b)\,\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{20}}\,dx, x, \sqrt{x}\right)}{5a} \\
&= -\frac{\left(a+b\sqrt{x}\right)^{16}}{10ax^{10}} + \frac{2b\left(a+b\sqrt{x}\right)^{16}}{95a^2x^{19/2}} + \frac{\left(6b^2\right)\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{19}}\,dx, x, \sqrt{x}\right)}{95a^2} \\
&= -\frac{\left(a+b\sqrt{x}\right)^{16}}{10ax^{10}} + \frac{2b\left(a+b\sqrt{x}\right)^{16}}{95a^2x^{19/2}} - \frac{b^2\left(a+b\sqrt{x}\right)^{16}}{285a^3x^9} - \frac{\left(2b^3\right)\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{18}}\,dx, x, \sqrt{x}\right)}{285a^3} \\
&= -\frac{\left(a+b\sqrt{x}\right)^{16}}{10ax^{10}} + \frac{2b\left(a+b\sqrt{x}\right)^{16}}{95a^2x^{19/2}} - \frac{b^2\left(a+b\sqrt{x}\right)^{16}}{285a^3x^9} + \frac{2b^3\left(a+b\sqrt{x}\right)^{16}}{4845a^4x^{17/2}} + \frac{\left(2b^4\right)\text{Subst}\left(\int \frac{(}{}\right.}{484} \\
&= -\frac{\left(a+b\sqrt{x}\right)^{16}}{10ax^{10}} + \frac{2b\left(a+b\sqrt{x}\right)^{16}}{95a^2x^{19/2}} - \frac{b^2\left(a+b\sqrt{x}\right)^{16}}{285a^3x^9} + \frac{2b^3\left(a+b\sqrt{x}\right)^{16}}{4845a^4x^{17/2}} - \frac{b^4\left(a+b\sqrt{x}\right)^{16}}{38760a^5x^8}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 65, normalized size = 0.54

$$-\frac{\left(a+b\sqrt{x}\right)^{16}\left(3876a^4 - 816a^3b\sqrt{x} + 136a^2b^2x - 16ab^3x^{3/2} + b^4x^2\right)}{38760a^5x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^15/x^11,x]
```

```
[Out] -1/38760*((a + b*Sqrt[x])^16*(3876*a^4 - 816*a^3*b*Sqrt[x] + 136*a^2*b^2*x
- 16*a*b^3*x^(3/2) + b^4*x^2))/(a^5*x^10)
```

fricas [A] time = 1.01, size = 168, normalized size = 1.40

$$-\frac{193800\,ab^{14}x^7 + 4408950\,a^3b^{12}x^6 + 23279256\,a^5b^{10}x^5 + 41570100\,a^7b^8x^4 + 27713400\,a^9b^6x^3 + 6613425\,a^{11}b^4x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^11,x, algorithm="fricas")
```

```
[Out] -1/38760*(193800*a*b^14*x^7 + 4408950*a^3*b^12*x^6 + 23279256*a^5*b^10*x^5
+ 41570100*a^7*b^8*x^4 + 27713400*a^9*b^6*x^3 + 6613425*a^11*b^4*x^2 + 4522
00*a^13*b^2*x + 3876*a^15 + 16*(969*b^15*x^7 + 72675*a^2*b^13*x^6 + 734825*
a^4*b^11*x^5 + 2204475*a^6*b^9*x^4 + 2398275*a^8*b^7*x^3 + 969969*a^10*b^5*
x^2 + 129675*a^12*b^3*x + 3825*a^14*b)*sqrt(x))/x^10
```

giac [A] time = 0.18, size = 167, normalized size = 1.39

$$-\frac{15504\,b^{15}x^{\frac{15}{2}} + 193800\,ab^{14}x^7 + 1162800\,a^2b^{13}x^{\frac{13}{2}} + 4408950\,a^3b^{12}x^6 + 11757200\,a^4b^{11}x^{\frac{11}{2}} + 23279256\,a^5b^{10}x^{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^11,x, algorithm="giac")
```

```
[Out] -1/38760*(15504*b^15*x^(15/2) + 193800*a*b^14*x^7 + 1162800*a^2*b^13*x^(13/
2) + 4408950*a^3*b^12*x^6 + 11757200*a^4*b^11*x^(11/2) + 23279256*a^5*b^10*
x^5 + 35271600*a^6*b^9*x^(9/2) + 41570100*a^7*b^8*x^4 + 38372400*a^8*b^7*x^
(7/2) + 27713400*a^9*b^6*x^3 + 15519504*a^10*b^5*x^(5/2) + 6613425*a^11*b^4
```

```
*x^2 + 2074800*a^12*b^3*x^(3/2) + 452200*a^13*b^2*x + 61200*a^14*b*sqrt(x)
+ 3876*a^15)/x^10
```

maple [A] time = 0.00, size = 168, normalized size = 1.40

$$-\frac{2b^{15}}{5x^{\frac{5}{2}}}-\frac{5a\,b^{14}}{x^3}-\frac{30a^2b^{13}}{x^{\frac{7}{2}}}-\frac{455a^3b^{12}}{4x^4}-\frac{910a^4b^{11}}{3x^{\frac{9}{2}}}-\frac{3003a^5b^{10}}{5x^5}-\frac{910a^6b^9}{x^{\frac{11}{2}}}-\frac{2145a^7b^8}{2x^6}-\frac{990a^8b^7}{x^{\frac{13}{2}}}-\frac{715a^9b^6}{x^7}-\frac{2002a^{10}b}{5x^{\frac{15}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^15/x^11,x)
```

```
[Out] -2/5*b^15/x^(5/2)-5*a*b^14/x^3-30*a^2*b^13/x^(7/2)-455/4*a^3*b^12/x^4-910/3
*a^4*b^11/x^(9/2)-3003/5*a^5*b^10/x^5-910*a^6*b^9/x^(11/2)-2145/2*a^7*b^8/x
^6-990*a^8*b^7/x^(13/2)-715*a^9*b^6/x^7-2002/5*a^10*b^5/x^(15/2)-1365/8*a^1
1*b^4/x^8-910/17*a^12*b^3/x^(17/2)-35/3*a^13*b^2/x^9-30/19*a^14*b/x^(19/2)-
1/10*a^15/x^10
```

maxima [A] time = 0.87, size = 167, normalized size = 1.39

$$-\frac{15504\,b^{15}x^{\frac{15}{2}}+193800\,ab^{14}x^7+1162800\,a^2b^{13}x^{\frac{13}{2}}+4408950\,a^3b^{12}x^6+11757200\,a^4b^{11}x^{\frac{11}{2}}+23279256\,a^5b^{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^11,x, algorithm="maxima")
```

```
[Out] -1/38760*(15504*b^15*x^(15/2) + 193800*a*b^14*x^7 + 1162800*a^2*b^13*x^(13/
2) + 4408950*a^3*b^12*x^6 + 11757200*a^4*b^11*x^(11/2) + 23279256*a^5*b^10*
x^5 + 35271600*a^6*b^9*x^(9/2) + 41570100*a^7*b^8*x^4 + 38372400*a^8*b^7*x^
(7/2) + 27713400*a^9*b^6*x^3 + 15519504*a^10*b^5*x^(5/2) + 6613425*a^11*b^4
*x^2 + 2074800*a^12*b^3*x^(3/2) + 452200*a^13*b^2*x + 61200*a^14*b*sqrt(x)
+ 3876*a^15)/x^10
```

mupad [B] time = 0.19, size = 167, normalized size = 1.39

$$-\frac{\frac{a^{15}}{10}+\frac{2\,b^{15}\,x^{15/2}}{5}+\frac{35\,a^{13}\,b^2\,x}{3}+\frac{30\,a^{14}\,b\,\sqrt{x}}{19}+5\,a\,b^{14}\,x^7+\frac{1365\,a^{11}\,b^4\,x^2}{8}+715\,a^9\,b^6\,x^3+\frac{2145\,a^7\,b^8\,x^4}{2}+\frac{3003\,a^5\,b^{10}\,x^5}{5}+\frac{91}{}}{x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^15/x^11,x)
```

```
[Out] -(a^15/10 + (2*b^15*x^(15/2))/5 + (35*a^13*b^2*x)/3 + (30*a^14*b*x^(1/2))/1
9 + 5*a*b^14*x^7 + (1365*a^11*b^4*x^2)/8 + 715*a^9*b^6*x^3 + (2145*a^7*b^8*
x^4)/2 + (3003*a^5*b^10*x^5)/5 + (910*a^12*b^3*x^(3/2))/17 + (455*a^3*b^12*
x^6)/4 + (2002*a^10*b^5*x^(5/2))/5 + 990*a^8*b^7*x^(7/2) + 910*a^6*b^9*x^(9
/2) + (910*a^4*b^11*x^(11/2))/3 + 30*a^2*b^13*x^(13/2))/x^10
```

sympy [A] time = 13.03, size = 211, normalized size = 1.76

$$-\frac{a^{15}}{10x^{10}}-\frac{30a^{14}b}{19x^{\frac{19}{2}}}-\frac{35a^{13}b^2}{3x^9}-\frac{910a^{12}b^3}{17x^{\frac{17}{2}}}-\frac{1365a^{11}b^4}{8x^8}-\frac{2002a^{10}b^5}{5x^{\frac{15}{2}}}-\frac{715a^9b^6}{x^7}-\frac{990a^8b^7}{x^{\frac{13}{2}}}-\frac{2145a^7b^8}{2x^6}-\frac{910a^6b^9}{x^{\frac{11}{2}}}-\frac{3003a}{5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**15/x**11,x)
```

```
[Out] -a**15/(10*x**10) - 30*a**14*b/(19*x**(19/2)) - 35*a**13*b**2/(3*x**9) - 91
0*a**12*b**3/(17*x**(17/2)) - 1365*a**11*b**4/(8*x**8) - 2002*a**10*b**5/(5
*x**(15/2)) - 715*a**9*b**6/x**7 - 990*a**8*b**7/x**(13/2) - 2145*a**7*b**8
/(2*x**6) - 910*a**6*b**9/x**(11/2) - 3003*a**5*b**10/(5*x**5) - 910*a**4*b
**11/(3*x**(9/2)) - 455*a**3*b**12/(4*x**4) - 30*a**2*b**13/x**(7/2) - 5*a*
b**14/x**3 - 2*b**15/(5*x**(5/2))
```

3.2185 $\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{12}}\,dx$

Optimal. Leaf size=170

$$-\frac{b^6\left(a+b\sqrt{x}\right)^{16}}{596904a^7x^8}+\frac{2b^5\left(a+b\sqrt{x}\right)^{16}}{74613a^6x^{17/2}}-\frac{b^4\left(a+b\sqrt{x}\right)^{16}}{4389a^5x^9}+\frac{2b^3\left(a+b\sqrt{x}\right)^{16}}{1463a^4x^{19/2}}-\frac{b^2\left(a+b\sqrt{x}\right)^{16}}{154a^3x^{10}}+\frac{2b\left(a+b\sqrt{x}\right)^{16}}{77a^2x^{21/2}}-\frac{\left(a+\right.}{11}$$

```
[Out] -1/11*(a+b*x^(1/2))^16/a/x^11+2/77*b*(a+b*x^(1/2))^16/a^2/x^(21/2)-1/154*b^
2*(a+b*x^(1/2))^16/a^3/x^10+2/1463*b^3*(a+b*x^(1/2))^16/a^4/x^(19/2)-1/4389
*b^4*(a+b*x^(1/2))^16/a^5/x^9+2/74613*b^5*(a+b*x^(1/2))^16/a^6/x^(17/2)-1/5
96904*b^6*(a+b*x^(1/2))^16/a^7/x^8
```

Rubi [A] time = 0.07, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {266, 45, 37}

$$-\frac{b^6\left(a+b\sqrt{x}\right)^{16}}{596904a^7x^8}+\frac{2b^5\left(a+b\sqrt{x}\right)^{16}}{74613a^6x^{17/2}}-\frac{b^4\left(a+b\sqrt{x}\right)^{16}}{4389a^5x^9}+\frac{2b^3\left(a+b\sqrt{x}\right)^{16}}{1463a^4x^{19/2}}-\frac{b^2\left(a+b\sqrt{x}\right)^{16}}{154a^3x^{10}}+\frac{2b\left(a+b\sqrt{x}\right)^{16}}{77a^2x^{21/2}}-\frac{\left(a+\right.}{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^15/x^12,x]
```

```
[Out] -(a + b*Sqrt[x])^16/(11*a*x^11) + (2*b*(a + b*Sqrt[x])^16)/(77*a^2*x^(21/2)
) - (b^2*(a + b*Sqrt[x])^16)/(154*a^3*x^10) + (2*b^3*(a + b*Sqrt[x])^16)/(1
463*a^4*x^(19/2)) - (b^4*(a + b*Sqrt[x])^16)/(4389*a^5*x^9) + (2*b^5*(a + b
*Sqrt[x])^16)/(74613*a^6*x^(17/2)) - (b^6*(a + b*Sqrt[x])^16)/(596904*a^7*x
^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{12}}\,dx = 2\,\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{23}}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{\left(a+b\sqrt{x}\right)^{16}}{11ax^{11}} - \frac{(6b)\,\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{22}}\,dx, x, \sqrt{x}\right)}{11a}$$
$$= -\frac{\left(a+b\sqrt{x}\right)^{16}}{11ax^{11}} + \frac{2b\left(a+b\sqrt{x}\right)^{16}}{77a^2x^{21/2}} + \frac{\left(10b^2\right)\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{21}}\,dx, x, \sqrt{x}\right)}{77a^2}$$
$$= -\frac{\left(a+b\sqrt{x}\right)^{16}}{11ax^{11}} + \frac{2b\left(a+b\sqrt{x}\right)^{16}}{77a^2x^{21/2}} - \frac{b^2\left(a+b\sqrt{x}\right)^{16}}{154a^3x^{10}} - \frac{\left(2b^3\right)\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{20}}\,dx, x, \sqrt{x}\right)}{77a^3}$$
$$= -\frac{\left(a+b\sqrt{x}\right)^{16}}{11ax^{11}} + \frac{2b\left(a+b\sqrt{x}\right)^{16}}{77a^2x^{21/2}} - \frac{b^2\left(a+b\sqrt{x}\right)^{16}}{154a^3x^{10}} + \frac{2b^3\left(a+b\sqrt{x}\right)^{16}}{1463a^4x^{19/2}} + \frac{\left(6b^4\right)\text{Subst}\left($$
$$= -\frac{\left(a+b\sqrt{x}\right)^{16}}{11ax^{11}} + \frac{2b\left(a+b\sqrt{x}\right)^{16}}{77a^2x^{21/2}} - \frac{b^2\left(a+b\sqrt{x}\right)^{16}}{154a^3x^{10}} + \frac{2b^3\left(a+b\sqrt{x}\right)^{16}}{1463a^4x^{19/2}} - \frac{b^4\left(a+b\sqrt{x}\right)^{1}}{4389a^5x^9}$$
$$= -\frac{\left(a+b\sqrt{x}\right)^{16}}{11ax^{11}} + \frac{2b\left(a+b\sqrt{x}\right)^{16}}{77a^2x^{21/2}} - \frac{b^2\left(a+b\sqrt{x}\right)^{16}}{154a^3x^{10}} + \frac{2b^3\left(a+b\sqrt{x}\right)^{16}}{1463a^4x^{19/2}} - \frac{b^4\left(a+b\sqrt{x}\right)^{1}}{4389a^5x^9}$$
$$= -\frac{\left(a+b\sqrt{x}\right)^{16}}{11ax^{11}} + \frac{2b\left(a+b\sqrt{x}\right)^{16}}{77a^2x^{21/2}} - \frac{b^2\left(a+b\sqrt{x}\right)^{16}}{154a^3x^{10}} + \frac{2b^3\left(a+b\sqrt{x}\right)^{16}}{1463a^4x^{19/2}} - \frac{b^4\left(a+b\sqrt{x}\right)^{1}}{4389a^5x^9}$$

Mathematica [A] time = 0.02, size = 89, normalized size = 0.52

$$-\frac{\left(a+b\sqrt{x}\right)^{16}\left(54264a^6 - 15504a^5b\sqrt{x} + 3876a^4b^2x - 816a^3b^3x^{3/2} + 136a^2b^4x^2 - 16ab^5x^{5/2} + b^6x^3\right)}{596904a^7x^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^15/x^12,x]
```

```
[Out] -1/596904*((a + b*Sqrt[x])^16*(54264*a^6 - 15504*a^5*b*Sqrt[x] + 3876*a^4*b
^2*x - 816*a^3*b^3*x^(3/2) + 136*a^2*b^4*x^2 - 16*a*b^5*x^(5/2) + b^6*x^3))
/(a^7*x^11)
```

fricas [A] time = 0.74, size = 168, normalized size = 0.99

$$-\frac{2238390\,ab^{14}x^7 + 54318264\,a^3b^{12}x^6 + 298750452\,a^5b^{10}x^5 + 548725320\,a^7b^8x^4 + 373438065\,a^9b^6x^3 + 9053}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^12,x, algorithm="fricas")
```

```
[Out] -1/596904*(2238390*a*b^14*x^7 + 54318264*a^3*b^12*x^6 + 298750452*a^5*b^10*
x^5 + 548725320*a^7*b^8*x^4 + 373438065*a^9*b^6*x^3 + 90530440*a^11*b^4*x^2
 + 6267492*a^13*b^2*x + 54264*a^15 + 16*(10659*b^15*x^7 + 870485*a^2*b^13*x
^6 + 9258795*a^4*b^11*x^5 + 28726005*a^6*b^9*x^4 + 32008977*a^8*b^7*x^3 + 1
3180167*a^10*b^5*x^2 + 1786785*a^12*b^3*x + 53295*a^14*b)*sqrt(x))/x^11
```

giac [A] time = 0.17, size = 167, normalized size = 0.98

$$-\frac{170544\,b^{15}x^{\frac{15}{2}} + 2238390\,ab^{14}x^7 + 13927760\,a^2b^{13}x^{\frac{13}{2}} + 54318264\,a^3b^{12}x^6 + 148140720\,a^4b^{11}x^{\frac{11}{2}} + 298750}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^12,x, algorithm="giac")
```

```
[Out] -1/596904*(170544*b^15*x^(15/2) + 2238390*a*b^14*x^7 + 13927760*a^2*b^13*x^
(13/2) + 54318264*a^3*b^12*x^6 + 148140720*a^4*b^11*x^(11/2) + 298750452*a^
5*b^10*x^5 + 459616080*a^6*b^9*x^(9/2) + 548725320*a^7*b^8*x^4 + 512143632*
a^8*b^7*x^(7/2) + 373438065*a^9*b^6*x^3 + 210882672*a^10*b^5*x^(5/2) + 9053
0440*a^11*b^4*x^2 + 28588560*a^12*b^3*x^(3/2) + 6267492*a^13*b^2*x + 852720
*a^14*b*sqrt(x) + 54264*a^15)/x^11
```

maple [A] time = 0.00, size = 168, normalized size = 0.99

$$-\frac{2b^{15}}{7x^{\frac{7}{2}}}-\frac{15a\,b^{14}}{4x^4}-\frac{70a^2b^{13}}{3x^{\frac{9}{2}}}-\frac{91a^3b^{12}}{x^5}-\frac{2730a^4b^{11}}{11x^{\frac{11}{2}}}-\frac{1001a^5b^{10}}{2x^6}-\frac{770a^6b^9}{x^{\frac{13}{2}}}-\frac{6435a^7b^8}{7x^7}-\frac{858a^8b^7}{x^{\frac{15}{2}}}-\frac{5005a^9b^6}{8x^8}-\frac{6006a^{10}b}{17x^{\frac{17}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^15/x^12,x)
```

```
[Out] -2/7*b^15/x^(7/2)-15/4*a*b^14/x^4-70/3*a^2*b^13/x^(9/2)-91*a^3*b^12/x^5-273
0/11*a^4*b^11/x^(11/2)-1001/2*a^5*b^10/x^6-770*a^6*b^9/x^(13/2)-6435/7*a^7*
b^8/x^7-858*a^8*b^7/x^(15/2)-5005/8*a^9*b^6/x^8-6006/17*a^10*b^5/x^(17/2)-4
55/3*a^11*b^4/x^9-910/19*a^12*b^3/x^(19/2)-21/2*a^13*b^2/x^10-10/7*a^14*b/x
^(21/2)-1/11*a^15/x^11
```

maxima [A] time = 0.94, size = 167, normalized size = 0.98

$$-\frac{170544\,b^{15}x^{\frac{15}{2}}+2238390\,ab^{14}x^7+13927760\,a^2b^{13}x^{\frac{13}{2}}+54318264\,a^3b^{12}x^6+148140720\,a^4b^{11}x^{\frac{11}{2}}+298750452}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^12,x, algorithm="maxima")
```

```
[Out] -1/596904*(170544*b^15*x^(15/2) + 2238390*a*b^14*x^7 + 13927760*a^2*b^13*x^
(13/2) + 54318264*a^3*b^12*x^6 + 148140720*a^4*b^11*x^(11/2) + 298750452*a^
5*b^10*x^5 + 459616080*a^6*b^9*x^(9/2) + 548725320*a^7*b^8*x^4 + 512143632*
a^8*b^7*x^(7/2) + 373438065*a^9*b^6*x^3 + 210882672*a^10*b^5*x^(5/2) + 9053
0440*a^11*b^4*x^2 + 28588560*a^12*b^3*x^(3/2) + 6267492*a^13*b^2*x + 852720
*a^14*b*sqrt(x) + 54264*a^15)/x^11
```

mupad [B] time = 1.27, size = 167, normalized size = 0.98

$$-\frac{\frac{a^{15}}{11}+\frac{2\,b^{15}\,x^{15/2}}{7}+\frac{21\,a^{13}\,b^2\,x}{2}+\frac{10\,a^{14}\,b\,\sqrt{x}}{7}+\frac{15\,a\,b^{14}\,x^7}{4}+\frac{455\,a^{11}\,b^4\,x^2}{3}+\frac{5005\,a^9\,b^6\,x^3}{8}+\frac{6435\,a^7\,b^8\,x^4}{7}+\frac{1001\,a^5\,b^{10}\,x^5}{2}+\frac{910\,a^{12}\,b^3}{19}}{x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^15/x^12,x)
```

```
[Out] -(a^15/11 + (2*b^15*x^(15/2))/7 + (21*a^13*b^2*x)/2 + (10*a^14*b*x^(1/2))/7
 + (15*a*b^14*x^7)/4 + (455*a^11*b^4*x^2)/3 + (5005*a^9*b^6*x^3)/8 + (6435*
a^7*b^8*x^4)/7 + (1001*a^5*b^10*x^5)/2 + (910*a^12*b^3*x^(3/2))/19 + 91*a^3
*b^12*x^6 + (6006*a^10*b^5*x^(5/2))/17 + 858*a^8*b^7*x^(7/2) + 770*a^6*b^9*
x^(9/2) + (2730*a^4*b^11*x^(11/2))/11 + (70*a^2*b^13*x^(13/2))/3)/x^11
```

sympy [A] time = 16.78, size = 214, normalized size = 1.26

$$-\frac{a^{15}}{11x^{11}}-\frac{10a^{14}b}{7x^{\frac{21}{2}}}-\frac{21a^{13}b^2}{2x^{10}}-\frac{910a^{12}b^3}{19x^{\frac{19}{2}}}-\frac{455a^{11}b^4}{3x^9}-\frac{6006a^{10}b^5}{17x^{\frac{17}{2}}}-\frac{5005a^9b^6}{8x^8}-\frac{858a^8b^7}{x^{\frac{15}{2}}}-\frac{6435a^7b^8}{7x^7}-\frac{770a^6b^9}{x^{\frac{13}{2}}}-\frac{1001a^5b}{2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**15/x**12,x)

[Out] -a**15/(11*x**11) - 10*a**14*b/(7*x**(21/2)) - 21*a**13*b**2/(2*x**10) - 91
0*a**12*b**3/(19*x**(19/2)) - 455*a**11*b**4/(3*x**9) - 6006*a**10*b**5/(17
*x**(17/2)) - 5005*a**9*b**6/(8*x**8) - 858*a**8*b**7/x**(15/2) - 6435*a**7
*b**8/(7*x**7) - 770*a**6*b**9/x**(13/2) - 1001*a**5*b**10/(2*x**6) - 2730*
a**4*b**11/(11*x**(11/2)) - 91*a**3*b**12/x**5 - 70*a**2*b**13/(3*x**(9/2))
 - 15*a*b**14/(4*x**4) - 2*b**15/(7*x**(7/2))
```

3.2186 $\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{13}}\,dx$

Optimal. Leaf size=220

$$-\frac{b^8\left(a+b\sqrt{x}\right)^{16}}{5883768a^9x^8}+\frac{2b^7\left(a+b\sqrt{x}\right)^{16}}{735471a^8x^{17/2}}-\frac{b^6\left(a+b\sqrt{x}\right)^{16}}{43263a^7x^9}+\frac{2b^5\left(a+b\sqrt{x}\right)^{16}}{14421a^6x^{19/2}}-\frac{b^4\left(a+b\sqrt{x}\right)^{16}}{1518a^5x^{10}}+\frac{2b^3\left(a+b\sqrt{x}\right)^{16}}{759a^4x^{21/2}}-\frac{7b^2}{}$$

```
[Out] -1/12*(a+b*x^(1/2))^16/a/x^12+2/69*b*(a+b*x^(1/2))^16/a^2/x^(23/2)-7/759*b^
2*(a+b*x^(1/2))^16/a^3/x^11+2/759*b^3*(a+b*x^(1/2))^16/a^4/x^(21/2)-1/1518*
b^4*(a+b*x^(1/2))^16/a^5/x^10+2/14421*b^5*(a+b*x^(1/2))^16/a^6/x^(19/2)-1/4
3263*b^6*(a+b*x^(1/2))^16/a^7/x^9+2/735471*b^7*(a+b*x^(1/2))^16/a^8/x^(17/2
)-1/5883768*b^8*(a+b*x^(1/2))^16/a^9/x^8
```

Rubi [A] time = 0.11, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {266, 45, 37}

$$-\frac{b^8\left(a+b\sqrt{x}\right)^{16}}{5883768a^9x^8}+\frac{2b^7\left(a+b\sqrt{x}\right)^{16}}{735471a^8x^{17/2}}-\frac{b^6\left(a+b\sqrt{x}\right)^{16}}{43263a^7x^9}+\frac{2b^5\left(a+b\sqrt{x}\right)^{16}}{14421a^6x^{19/2}}-\frac{b^4\left(a+b\sqrt{x}\right)^{16}}{1518a^5x^{10}}+\frac{2b^3\left(a+b\sqrt{x}\right)^{16}}{759a^4x^{21/2}}-\frac{7b^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^15/x^13,x]
```

```
[Out] -(a + b*Sqrt[x])^16/(12*a*x^12) + (2*b*(a + b*Sqrt[x])^16)/(69*a^2*x^(23/2)
) - (7*b^2*(a + b*Sqrt[x])^16)/(759*a^3*x^11) + (2*b^3*(a + b*Sqrt[x])^16)/
(759*a^4*x^(21/2)) - (b^4*(a + b*Sqrt[x])^16)/(1518*a^5*x^10) + (2*b^5*(a +
 b*Sqrt[x])^16)/(14421*a^6*x^(19/2)) - (b^6*(a + b*Sqrt[x])^16)/(43263*a^7*
x^9) + (2*b^7*(a + b*Sqrt[x])^16)/(735471*a^8*x^(17/2)) - (b^8*(a + b*Sqrt[
x])^16)/(5883768*a^9*x^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{13}}\,dx = 2\,\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{25}}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{\left(a+b\sqrt{x}\right)^{16}}{12ax^{12}} - \frac{(2b)\,\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{24}}\,dx, x, \sqrt{x}\right)}{3a}$$
$$= -\frac{\left(a+b\sqrt{x}\right)^{16}}{12ax^{12}} + \frac{2b\left(a+b\sqrt{x}\right)^{16}}{69a^2x^{23/2}} + \frac{\left(14b^2\right)\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{23}}\,dx, x, \sqrt{x}\right)}{69a^2}$$
$$= -\frac{\left(a+b\sqrt{x}\right)^{16}}{12ax^{12}} + \frac{2b\left(a+b\sqrt{x}\right)^{16}}{69a^2x^{23/2}} - \frac{7b^2\left(a+b\sqrt{x}\right)^{16}}{759a^3x^{11}} - \frac{\left(14b^3\right)\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{22}}\,dx, x, \sqrt{}\right.}{253a^3}$$
$$= -\frac{\left(a+b\sqrt{x}\right)^{16}}{12ax^{12}} + \frac{2b\left(a+b\sqrt{x}\right)^{16}}{69a^2x^{23/2}} - \frac{7b^2\left(a+b\sqrt{x}\right)^{16}}{759a^3x^{11}} + \frac{2b^3\left(a+b\sqrt{x}\right)^{16}}{759a^4x^{21/2}} + \frac{\left(10b^4\right)\text{Subs}}{}$$
$$= -\frac{\left(a+b\sqrt{x}\right)^{16}}{12ax^{12}} + \frac{2b\left(a+b\sqrt{x}\right)^{16}}{69a^2x^{23/2}} - \frac{7b^2\left(a+b\sqrt{x}\right)^{16}}{759a^3x^{11}} + \frac{2b^3\left(a+b\sqrt{x}\right)^{16}}{759a^4x^{21/2}} - \frac{b^4\left(a+b\sqrt{x}\right.}{1518a^5x^{10}}$$
$$= -\frac{\left(a+b\sqrt{x}\right)^{16}}{12ax^{12}} + \frac{2b\left(a+b\sqrt{x}\right)^{16}}{69a^2x^{23/2}} - \frac{7b^2\left(a+b\sqrt{x}\right)^{16}}{759a^3x^{11}} + \frac{2b^3\left(a+b\sqrt{x}\right)^{16}}{759a^4x^{21/2}} - \frac{b^4\left(a+b\sqrt{x}\right.}{1518a^5x^{10}}$$
$$= -\frac{\left(a+b\sqrt{x}\right)^{16}}{12ax^{12}} + \frac{2b\left(a+b\sqrt{x}\right)^{16}}{69a^2x^{23/2}} - \frac{7b^2\left(a+b\sqrt{x}\right)^{16}}{759a^3x^{11}} + \frac{2b^3\left(a+b\sqrt{x}\right)^{16}}{759a^4x^{21/2}} - \frac{b^4\left(a+b\sqrt{x}\right.}{1518a^5x^{10}}$$
$$= -\frac{\left(a+b\sqrt{x}\right)^{16}}{12ax^{12}} + \frac{2b\left(a+b\sqrt{x}\right)^{16}}{69a^2x^{23/2}} - \frac{7b^2\left(a+b\sqrt{x}\right)^{16}}{759a^3x^{11}} + \frac{2b^3\left(a+b\sqrt{x}\right)^{16}}{759a^4x^{21/2}} - \frac{b^4\left(a+b\sqrt{x}\right.}{1518a^5x^{10}}$$
$$= -\frac{\left(a+b\sqrt{x}\right)^{16}}{12ax^{12}} + \frac{2b\left(a+b\sqrt{x}\right)^{16}}{69a^2x^{23/2}} - \frac{7b^2\left(a+b\sqrt{x}\right)^{16}}{759a^3x^{11}} + \frac{2b^3\left(a+b\sqrt{x}\right)^{16}}{759a^4x^{21/2}} - \frac{b^4\left(a+b\sqrt{x}\right.}{1518a^5x^{10}}$$

Mathematica [A] time = 0.06, size = 209, normalized size = 0.95

$$-\frac{a^{15}}{12x^{12}} - \frac{30a^{14}b}{23x^{23/2}} - \frac{105a^{13}b^2}{11x^{11}} - \frac{130a^{12}b^3}{3x^{21/2}} - \frac{273a^{11}b^4}{2x^{10}} - \frac{6006a^{10}b^5}{19x^{19/2}} - \frac{5005a^9b^6}{9x^9} - \frac{12870a^8b^7}{17x^{17/2}} - \frac{6435a^7b^8}{8x^8} - \frac{2002a^6b^9}{3x^{15/2}} -$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^15/x^13,x]
```

```
[Out] -1/12*a^15/x^12 - (30*a^14*b)/(23*x^(23/2)) - (105*a^13*b^2)/(11*x^11) - (1
30*a^12*b^3)/(3*x^(21/2)) - (273*a^11*b^4)/(2*x^10) - (6006*a^10*b^5)/(19*x
^(19/2)) - (5005*a^9*b^6)/(9*x^9) - (12870*a^8*b^7)/(17*x^(17/2)) - (6435*a
^7*b^8)/(8*x^8) - (2002*a^6*b^9)/(3*x^(15/2)) - (429*a^5*b^10)/x^7 - (210*a
^4*b^11)/x^(13/2) - (455*a^3*b^12)/(6*x^6) - (210*a^2*b^13)/(11*x^(11/2)) -
 (3*a*b^14)/x^5 - (2*b^15)/(9*x^(9/2))
```

fricas [A] time = 1.04, size = 168, normalized size = 0.76

$$-\frac{17651304\,ab^{14}x^7 + 446185740\,a^3b^{12}x^6 + 2524136472\,a^5b^{10}x^5 + 4732755885\,a^7b^8x^4 + 3272028760\,a^9b^6x^3 +}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^13,x, algorithm="fricas")
```

```
[Out] -1/5883768*(17651304*a*b^14*x^7 + 446185740*a^3*b^12*x^6 + 2524136472*a^5*b
^10*x^5 + 4732755885*a^7*b^8*x^4 + 3272028760*a^9*b^6*x^3 + 803134332*a^11*
b^4*x^2 + 56163240*a^13*b^2*x + 490314*a^15 + 16*(81719*b^15*x^7 + 7020405*
```

```
a^2*b^13*x^6 + 77224455*a^4*b^11*x^5 + 245402157*a^6*b^9*x^4 + 278397405*a^
8*b^7*x^3 + 116243127*a^10*b^5*x^2 + 15935205*a^12*b^3*x + 479655*a^14*b)*s
qrt(x))/x^12
```

giac [A] time = 0.18, size = 167, normalized size = 0.76

$$-\frac{1307504\,b^{15}x^{\frac{15}{2}}+17651304\,ab^{14}x^{7}+112326480\,a^{2}b^{13}x^{\frac{13}{2}}+446185740\,a^{3}b^{12}x^{6}+1235591280\,a^{4}b^{11}x^{\frac{11}{2}}+2524}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^13,x, algorithm="giac")
```

```
[Out] -1/5883768*(1307504*b^15*x^(15/2) + 17651304*a*b^14*x^7 + 112326480*a^2*b^1
3*x^(13/2) + 446185740*a^3*b^12*x^6 + 1235591280*a^4*b^11*x^(11/2) + 252413
6472*a^5*b^10*x^5 + 3926434512*a^6*b^9*x^(9/2) + 4732755885*a^7*b^8*x^4 + 4
454358480*a^8*b^7*x^(7/2) + 3272028760*a^9*b^6*x^3 + 1859890032*a^10*b^5*x^
(5/2) + 803134332*a^11*b^4*x^2 + 254963280*a^12*b^3*x^(3/2) + 56163240*a^13
*b^2*x + 7674480*a^14*b*sqrt(x) + 490314*a^15)/x^12
```

maple [A] time = 0.00, size = 168, normalized size = 0.76

$$-\frac{2b^{15}}{9x^{\frac{9}{2}}}-\frac{3a\,b^{14}}{x^{5}}-\frac{210a^{2}b^{13}}{11x^{\frac{11}{2}}}-\frac{455a^{3}b^{12}}{6x^{6}}-\frac{210a^{4}b^{11}}{x^{\frac{13}{2}}}-\frac{429a^{5}b^{10}}{x^{7}}-\frac{2002a^{6}b^{9}}{3x^{\frac{15}{2}}}-\frac{6435a^{7}b^{8}}{8x^{8}}-\frac{12870a^{8}b^{7}}{17x^{\frac{17}{2}}}-\frac{5005a^{9}b^{6}}{9x^{9}}-\frac{6006a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^15/x^13,x)
```

```
[Out] -2/9*b^15/x^(9/2)-3*a*b^14/x^5-210/11*a^2*b^13/x^(11/2)-455/6*a^3*b^12/x^6-
210*a^4*b^11/x^(13/2)-429*a^5*b^10/x^7-2002/3*a^6*b^9/x^(15/2)-6435/8*a^7*b
^8/x^8-12870/17*a^8*b^7/x^(17/2)-5005/9*a^9*b^6/x^9-6006/19*a^10*b^5/x^(19/
2)-273/2*a^11*b^4/x^10-130/3*a^12*b^3/x^(21/2)-105/11*a^13*b^2/x^11-30/23*a
^14*b/x^(23/2)-1/12*a^15/x^12
```

maxima [A] time = 0.83, size = 167, normalized size = 0.76

$$-\frac{1307504\,b^{15}x^{\frac{15}{2}}+17651304\,ab^{14}x^{7}+112326480\,a^{2}b^{13}x^{\frac{13}{2}}+446185740\,a^{3}b^{12}x^{6}+1235591280\,a^{4}b^{11}x^{\frac{11}{2}}+2524}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^13,x, algorithm="maxima")
```

```
[Out] -1/5883768*(1307504*b^15*x^(15/2) + 17651304*a*b^14*x^7 + 112326480*a^2*b^1
3*x^(13/2) + 446185740*a^3*b^12*x^6 + 1235591280*a^4*b^11*x^(11/2) + 252413
6472*a^5*b^10*x^5 + 3926434512*a^6*b^9*x^(9/2) + 4732755885*a^7*b^8*x^4 + 4
454358480*a^8*b^7*x^(7/2) + 3272028760*a^9*b^6*x^3 + 1859890032*a^10*b^5*x^
(5/2) + 803134332*a^11*b^4*x^2 + 254963280*a^12*b^3*x^(3/2) + 56163240*a^13
*b^2*x + 7674480*a^14*b*sqrt(x) + 490314*a^15)/x^12
```

mupad [B] time = 1.27, size = 167, normalized size = 0.76

$$-\frac{\frac{a^{15}}{12}+\frac{2\,b^{15}\,x^{15/2}}{9}+\frac{105\,a^{13}\,b^{2}\,x}{11}+\frac{30\,a^{14}\,b\,\sqrt{x}}{23}+3\,a\,b^{14}\,x^{7}+\frac{273\,a^{11}\,b^{4}\,x^{2}}{2}+\frac{5005\,a^{9}\,b^{6}\,x^{3}}{9}+\frac{6435\,a^{7}\,b^{8}\,x^{4}}{8}+429\,a^{5}\,b^{10}\,x^{5}+\frac{130\,a^{1}}{}}{x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^15/x^13,x)
```

```
[Out] -(a^15/12 + (2*b^15*x^(15/2))/9 + (105*a^13*b^2*x)/11 + (30*a^14*b*x^(1/2))
/23 + 3*a*b^14*x^7 + (273*a^11*b^4*x^2)/2 + (5005*a^9*b^6*x^3)/9 + (6435*a^
```

```
7*b^8*x^4)/8 + 429*a^5*b^10*x^5 + (130*a^12*b^3*x^(3/2))/3 + (455*a^3*b^12*
x^6)/6 + (6006*a^10*b^5*x^(5/2))/19 + (12870*a^8*b^7*x^(7/2))/17 + (2002*a^
6*b^9*x^(9/2))/3 + 210*a^4*b^11*x^(11/2) + (210*a^2*b^13*x^(13/2))/11)/x^12
```

sympy [**A**] time = 21.12, size = 214, normalized size = 0.97

$$-\frac{a^{15}}{12x^{12}}-\frac{30a^{14}b}{23x^{\frac{23}{2}}}-\frac{105a^{13}b^2}{11x^{11}}-\frac{130a^{12}b^3}{3x^{\frac{21}{2}}}-\frac{273a^{11}b^4}{2x^{10}}-\frac{6006a^{10}b^5}{19x^{\frac{19}{2}}}-\frac{5005a^9b^6}{9x^9}-\frac{12870a^8b^7}{17x^{\frac{17}{2}}}-\frac{6435a^7b^8}{8x^8}-\frac{2002a^6b^9}{3x^{\frac{15}{2}}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**15/x**13,x)
```

```
[Out] -a**15/(12*x**12) - 30*a**14*b/(23*x**(23/2)) - 105*a**13*b**2/(11*x**11) -
 130*a**12*b**3/(3*x**(21/2)) - 273*a**11*b**4/(2*x**10) - 6006*a**10*b**5/
(19*x**(19/2)) - 5005*a**9*b**6/(9*x**9) - 12870*a**8*b**7/(17*x**(17/2)) -
 6435*a**7*b**8/(8*x**8) - 2002*a**6*b**9/(3*x**(15/2)) - 429*a**5*b**10/x*
*7 - 210*a**4*b**11/x**(13/2) - 455*a**3*b**12/(6*x**6) - 210*a**2*b**13/(1
1*x**(11/2)) - 3*a*b**14/x**5 - 2*b**15/(9*x**(9/2))
```

3.2187 $\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{14}}\,dx$

Optimal. Leaf size=270

$$-\frac{b^{10}\left(a+b\sqrt{x}\right)^{16}}{42493880a^{11}x^8}+\frac{2b^9\left(a+b\sqrt{x}\right)^{16}}{5311735a^{10}x^{17/2}}-\frac{b^8\left(a+b\sqrt{x}\right)^{16}}{312455a^9x^9}+\frac{6b^7\left(a+b\sqrt{x}\right)^{16}}{312455a^8x^{19/2}}-\frac{3b^6\left(a+b\sqrt{x}\right)^{16}}{32890a^7x^{10}}+\frac{6b^5\left(a+b\sqrt{x}\right)^{16}}{16445a^6x^{21/2}}-\frac{2}{}$$

```
[Out] -1/13*(a+b*x^(1/2))^16/a/x^13+2/65*b*(a+b*x^(1/2))^16/a^2/x^(25/2)-3/260*b^
2*(a+b*x^(1/2))^16/a^3/x^12+6/1495*b^3*(a+b*x^(1/2))^16/a^4/x^(23/2)-21/164
45*b^4*(a+b*x^(1/2))^16/a^5/x^11+6/16445*b^5*(a+b*x^(1/2))^16/a^6/x^(21/2)-
3/32890*b^6*(a+b*x^(1/2))^16/a^7/x^10+6/312455*b^7*(a+b*x^(1/2))^16/a^8/x^(
19/2)-1/312455*b^8*(a+b*x^(1/2))^16/a^9/x^9+2/5311735*b^9*(a+b*x^(1/2))^16/
a^10/x^(17/2)-1/42493880*b^10*(a+b*x^(1/2))^16/a^11/x^8
```

Rubi [A] time = 0.15, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {266, 45, 37}

$$-\frac{b^{10}\left(a+b\sqrt{x}\right)^{16}}{42493880a^{11}x^8}+\frac{2b^9\left(a+b\sqrt{x}\right)^{16}}{5311735a^{10}x^{17/2}}-\frac{b^8\left(a+b\sqrt{x}\right)^{16}}{312455a^9x^9}+\frac{6b^7\left(a+b\sqrt{x}\right)^{16}}{312455a^8x^{19/2}}-\frac{3b^6\left(a+b\sqrt{x}\right)^{16}}{32890a^7x^{10}}+\frac{6b^5\left(a+b\sqrt{x}\right)^{16}}{16445a^6x^{21/2}}-\frac{2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^15/x^14,x]
```

```
[Out] -(a + b*Sqrt[x])^16/(13*a*x^13) + (2*b*(a + b*Sqrt[x])^16)/(65*a^2*x^(25/2)
) - (3*b^2*(a + b*Sqrt[x])^16)/(260*a^3*x^12) + (6*b^3*(a + b*Sqrt[x])^16)/
(1495*a^4*x^(23/2)) - (21*b^4*(a + b*Sqrt[x])^16)/(16445*a^5*x^11) + (6*b^5
*(a + b*Sqrt[x])^16)/(16445*a^6*x^(21/2)) - (3*b^6*(a + b*Sqrt[x])^16)/(328
90*a^7*x^10) + (6*b^7*(a + b*Sqrt[x])^16)/(312455*a^8*x^(19/2)) - (b^8*(a +
 b*Sqrt[x])^16)/(312455*a^9*x^9) + (2*b^9*(a + b*Sqrt[x])^16)/(5311735*a^10
*x^(17/2)) - (b^10*(a + b*Sqrt[x])^16)/(42493880*a^11*x^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{14}}\,dx = 2\,\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{27}}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\left(a+b\sqrt{x}\right)^{16}}{13ax^{13}} - \frac{(10b)\,\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{26}}\,dx, x, \sqrt{x}\right)}{13a}$$

$$= -\frac{\left(a+b\sqrt{x}\right)^{16}}{13ax^{13}} + \frac{2b\left(a+b\sqrt{x}\right)^{16}}{65a^2x^{25/2}} + \frac{\left(18b^2\right)\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{25}}\,dx, x, \sqrt{x}\right)}{65a^2}$$

$$= -\frac{\left(a+b\sqrt{x}\right)^{16}}{13ax^{13}} + \frac{2b\left(a+b\sqrt{x}\right)^{16}}{65a^2x^{25/2}} - \frac{3b^2\left(a+b\sqrt{x}\right)^{16}}{260a^3x^{12}} - \frac{\left(6b^3\right)\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{24}}\,dx, x, \sqrt{x}\right.}{65a^3}$$

$$= -\frac{\left(a+b\sqrt{x}\right)^{16}}{13ax^{13}} + \frac{2b\left(a+b\sqrt{x}\right)^{16}}{65a^2x^{25/2}} - \frac{3b^2\left(a+b\sqrt{x}\right)^{16}}{260a^3x^{12}} + \frac{6b^3\left(a+b\sqrt{x}\right)^{16}}{1495a^4x^{23/2}} + \frac{\left(42b^4\right)\text{Subs}}{}$$

$$= -\frac{\left(a+b\sqrt{x}\right)^{16}}{13ax^{13}} + \frac{2b\left(a+b\sqrt{x}\right)^{16}}{65a^2x^{25/2}} - \frac{3b^2\left(a+b\sqrt{x}\right)^{16}}{260a^3x^{12}} + \frac{6b^3\left(a+b\sqrt{x}\right)^{16}}{1495a^4x^{23/2}} - \frac{21b^4\left(a+b\sqrt{}\right.}{16445a^5x}$$

$$= -\frac{\left(a+b\sqrt{x}\right)^{16}}{13ax^{13}} + \frac{2b\left(a+b\sqrt{x}\right)^{16}}{65a^2x^{25/2}} - \frac{3b^2\left(a+b\sqrt{x}\right)^{16}}{260a^3x^{12}} + \frac{6b^3\left(a+b\sqrt{x}\right)^{16}}{1495a^4x^{23/2}} - \frac{21b^4\left(a+b\sqrt{}\right.}{16445a^5x}$$

$$= -\frac{\left(a+b\sqrt{x}\right)^{16}}{13ax^{13}} + \frac{2b\left(a+b\sqrt{x}\right)^{16}}{65a^2x^{25/2}} - \frac{3b^2\left(a+b\sqrt{x}\right)^{16}}{260a^3x^{12}} + \frac{6b^3\left(a+b\sqrt{x}\right)^{16}}{1495a^4x^{23/2}} - \frac{21b^4\left(a+b\sqrt{}\right.}{16445a^5x}$$

$$= -\frac{\left(a+b\sqrt{x}\right)^{16}}{13ax^{13}} + \frac{2b\left(a+b\sqrt{x}\right)^{16}}{65a^2x^{25/2}} - \frac{3b^2\left(a+b\sqrt{x}\right)^{16}}{260a^3x^{12}} + \frac{6b^3\left(a+b\sqrt{x}\right)^{16}}{1495a^4x^{23/2}} - \frac{21b^4\left(a+b\sqrt{}\right.}{16445a^5x}$$

$$= -\frac{\left(a+b\sqrt{x}\right)^{16}}{13ax^{13}} + \frac{2b\left(a+b\sqrt{x}\right)^{16}}{65a^2x^{25/2}} - \frac{3b^2\left(a+b\sqrt{x}\right)^{16}}{260a^3x^{12}} + \frac{6b^3\left(a+b\sqrt{x}\right)^{16}}{1495a^4x^{23/2}} - \frac{21b^4\left(a+b\sqrt{}\right.}{16445a^5x}$$

$$= -\frac{\left(a+b\sqrt{x}\right)^{16}}{13ax^{13}} + \frac{2b\left(a+b\sqrt{x}\right)^{16}}{65a^2x^{25/2}} - \frac{3b^2\left(a+b\sqrt{x}\right)^{16}}{260a^3x^{12}} + \frac{6b^3\left(a+b\sqrt{x}\right)^{16}}{1495a^4x^{23/2}} - \frac{21b^4\left(a+b\sqrt{}\right.}{16445a^5x}$$

Mathematica [A] time = 0.15, size = 207, normalized size = 0.77

$$-\frac{a^{15}}{13x^{13}} - \frac{6a^{14}b}{5x^{25/2}} - \frac{35a^{13}b^2}{4x^{12}} - \frac{910a^{12}b^3}{23x^{23/2}} - \frac{1365a^{11}b^4}{11x^{11}} - \frac{286a^{10}b^5}{x^{21/2}} - \frac{1001a^9b^6}{2x^{10}} - \frac{12870a^8b^7}{19x^{19/2}} - \frac{715a^7b^8}{x^9} - \frac{10010a^6b^9}{17x^{17/2}} - \frac{300}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^15/x^14,x]
```

```
[Out] -1/13*a^15/x^13 - (6*a^14*b)/(5*x^(25/2)) - (35*a^13*b^2)/(4*x^12) - (910*a
^12*b^3)/(23*x^(23/2)) - (1365*a^11*b^4)/(11*x^11) - (286*a^10*b^5)/x^(21/2
) - (1001*a^9*b^6)/(2*x^10) - (12870*a^8*b^7)/(19*x^(19/2)) - (715*a^7*b^8)
/x^9 - (10010*a^6*b^9)/(17*x^(17/2)) - (3003*a^5*b^10)/(8*x^8) - (182*a^4*b
^11)/x^(15/2) - (65*a^3*b^12)/x^7 - (210*a^2*b^13)/(13*x^(13/2)) - (5*a*b^1
4)/(2*x^6) - (2*b^15)/(11*x^(11/2))
```

fricas [A] time = 1.35, size = 168, normalized size = 0.62

$$-\frac{106234700\,ab^{14}x^7 + 2762102200\,a^3b^{12}x^6 + 15951140205\,a^5b^{10}x^5 + 30383124200\,a^7b^8x^4 + 21268186940\,a^9b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^14,x, algorithm="fricas")
```

```
[Out] -1/42493880*(106234700*a*b^14*x^7 + 2762102200*a^3*b^12*x^6 + 15951140205*a
^5*b^10*x^5 + 30383124200*a^7*b^8*x^4 + 21268186940*a^9*b^6*x^3 + 527310420
0*a^11*b^4*x^2 + 371821450*a^13*b^2*x + 3268760*a^15 + 16*(482885*b^15*x^7
+ 42902475*a^2*b^13*x^6 + 483367885*a^4*b^11*x^5 + 1563837275*a^6*b^9*x^4 +
 1799000775*a^8*b^7*x^3 + 759578105*a^10*b^5*x^2 + 105079975*a^12*b^3*x + 3
187041*a^14*b)*sqrt(x))/x^13
```

giac [A] time = 0.20, size = 167, normalized size = 0.62

$$-\frac{7726160\,b^{15}x^{\frac{15}{2}}+106234700\,ab^{14}x^{7}+686439600\,a^{2}b^{13}x^{\frac{13}{2}}+2762102200\,a^{3}b^{12}x^{6}+7733886160\,a^{4}b^{11}x^{\frac{11}{2}}+15}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^14,x, algorithm="giac")
```

```
[Out] -1/42493880*(7726160*b^15*x^(15/2) + 106234700*a*b^14*x^7 + 686439600*a^2*b
^13*x^(13/2) + 2762102200*a^3*b^12*x^6 + 7733886160*a^4*b^11*x^(11/2) + 159
51140205*a^5*b^10*x^5 + 25021396400*a^6*b^9*x^(9/2) + 30383124200*a^7*b^8*x
^4 + 28784012400*a^8*b^7*x^(7/2) + 21268186940*a^9*b^6*x^3 + 12153249680*a^
10*b^5*x^(5/2) + 5273104200*a^11*b^4*x^2 + 1681279600*a^12*b^3*x^(3/2) + 37
1821450*a^13*b^2*x + 50992656*a^14*b*sqrt(x) + 3268760*a^15)/x^13
```

maple [A] time = 0.00, size = 168, normalized size = 0.62

$$-\frac{2b^{15}}{11x^{\frac{11}{2}}}-\frac{5a\,b^{14}}{2x^{6}}-\frac{210a^{2}b^{13}}{13x^{\frac{13}{2}}}-\frac{65a^{3}b^{12}}{x^{7}}-\frac{182a^{4}b^{11}}{x^{\frac{15}{2}}}-\frac{3003a^{5}b^{10}}{8x^{8}}-\frac{10010a^{6}b^{9}}{17x^{\frac{17}{2}}}-\frac{715a^{7}b^{8}}{x^{9}}-\frac{12870a^{8}b^{7}}{19x^{\frac{19}{2}}}-\frac{1001a^{9}b^{6}}{2x^{10}}-\frac{286a^{1}}{x^{\frac{21}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^15/x^14,x)
```

```
[Out] -2/11*b^15/x^(11/2)-5/2*a*b^14/x^6-210/13*a^2*b^13/x^(13/2)-65*a^3*b^12/x^7
-182*a^4*b^11/x^(15/2)-3003/8*a^5*b^10/x^8-10010/17*a^6*b^9/x^(17/2)-715*a^
7*b^8/x^9-12870/19*a^8*b^7/x^(19/2)-1001/2*a^9*b^6/x^10-286*a^10*b^5/x^(21/
2)-1365/11*a^11*b^4/x^11-910/23*a^12*b^3/x^(23/2)-35/4*a^13*b^2/x^12-6/5*a^
14*b/x^(25/2)-1/13*a^15/x^13
```

maxima [A] time = 0.96, size = 167, normalized size = 0.62

$$-\frac{7726160\,b^{15}x^{\frac{15}{2}}+106234700\,ab^{14}x^{7}+686439600\,a^{2}b^{13}x^{\frac{13}{2}}+2762102200\,a^{3}b^{12}x^{6}+7733886160\,a^{4}b^{11}x^{\frac{11}{2}}+15}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^14,x, algorithm="maxima")
```

```
[Out] -1/42493880*(7726160*b^15*x^(15/2) + 106234700*a*b^14*x^7 + 686439600*a^2*b
^13*x^(13/2) + 2762102200*a^3*b^12*x^6 + 7733886160*a^4*b^11*x^(11/2) + 159
51140205*a^5*b^10*x^5 + 25021396400*a^6*b^9*x^(9/2) + 30383124200*a^7*b^8*x
^4 + 28784012400*a^8*b^7*x^(7/2) + 21268186940*a^9*b^6*x^3 + 12153249680*a^
10*b^5*x^(5/2) + 5273104200*a^11*b^4*x^2 + 1681279600*a^12*b^3*x^(3/2) + 37
1821450*a^13*b^2*x + 50992656*a^14*b*sqrt(x) + 3268760*a^15)/x^13
```

mupad [B] time = 1.27, size = 167, normalized size = 0.62

$$-\frac{\frac{a^{15}}{13}+\frac{2\,b^{15}\,x^{15/2}}{11}+\frac{35\,a^{13}\,b^{2}\,x}{4}+\frac{6\,a^{14}\,b\,\sqrt{x}}{5}+\frac{5\,a\,b^{14}\,x^{7}}{2}+\frac{1365\,a^{11}\,b^{4}\,x^{2}}{11}+\frac{1001\,a^{9}\,b^{6}\,x^{3}}{2}+715\,a^{7}\,b^{8}\,x^{4}+\frac{3003\,a^{5}\,b^{10}\,x^{5}}{8}+\frac{910\,a^{12}\,b^{3}}{23}}{x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^15/x^14,x)
```

```
[Out] -(a^15/13 + (2*b^15*x^(15/2))/11 + (35*a^13*b^2*x)/4 + (6*a^14*b*x^(1/2))/5
 + (5*a*b^14*x^7)/2 + (1365*a^11*b^4*x^2)/11 + (1001*a^9*b^6*x^3)/2 + 715*a
^7*b^8*x^4 + (3003*a^5*b^10*x^5)/8 + (910*a^12*b^3*x^(3/2))/23 + 65*a^3*b^1
2*x^6 + 286*a^10*b^5*x^(5/2) + (12870*a^8*b^7*x^(7/2))/19 + (10010*a^6*b^9*
x^(9/2))/17 + 182*a^4*b^11*x^(11/2) + (210*a^2*b^13*x^(13/2))/13)/x^13
```

sympy [A] time = 27.38, size = 212, normalized size = 0.79

$$-\frac{a^{15}}{13x^{13}}-\frac{6a^{14}b}{5x^{\frac{25}{2}}}-\frac{35a^{13}b^2}{4x^{12}}-\frac{910a^{12}b^3}{23x^{\frac{23}{2}}}-\frac{1365a^{11}b^4}{11x^{11}}-\frac{286a^{10}b^5}{x^{\frac{21}{2}}}-\frac{1001a^9b^6}{2x^{10}}-\frac{12870a^8b^7}{19x^{\frac{19}{2}}}-\frac{715a^7b^8}{x^9}-\frac{10010a^6b^9}{17x^{\frac{17}{2}}}-\frac{300}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**15/x**14,x)
```

```
[Out] -a**15/(13*x**13) - 6*a**14*b/(5*x**(25/2)) - 35*a**13*b**2/(4*x**12) - 910
*a**12*b**3/(23*x**(23/2)) - 1365*a**11*b**4/(11*x**11) - 286*a**10*b**5/x*
*(21/2) - 1001*a**9*b**6/(2*x**10) - 12870*a**8*b**7/(19*x**(19/2)) - 715*a
**7*b**8/x**9 - 10010*a**6*b**9/(17*x**(17/2)) - 3003*a**5*b**10/(8*x**8) -
 182*a**4*b**11/x**(15/2) - 65*a**3*b**12/x**7 - 210*a**2*b**13/(13*x**(13/
2)) - 5*a*b**14/(2*x**6) - 2*b**15/(11*x**(11/2))
```

3.2188 $\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{15}}\,dx$

Optimal. Leaf size=211

$$-\frac{a^{15}}{14x^{14}}-\frac{10a^{14}b}{9x^{27/2}}-\frac{105a^{13}b^2}{13x^{13}}-\frac{182a^{12}b^3}{5x^{25/2}}-\frac{455a^{11}b^4}{4x^{12}}-\frac{6006a^{10}b^5}{23x^{23/2}}-\frac{455a^9b^6}{x^{11}}-\frac{4290a^8b^7}{7x^{21/2}}-\frac{1287a^7b^8}{2x^{10}}-\frac{10010a^6b^9}{19x^{19/2}}-\frac{1001}{3}$$

```
[Out] -1/14*a^15/x^14-10/9*a^14*b/x^(27/2)-105/13*a^13*b^2/x^13-182/5*a^12*b^3/x^
(25/2)-455/4*a^11*b^4/x^12-6006/23*a^10*b^5/x^(23/2)-455*a^9*b^6/x^11-4290/
7*a^8*b^7/x^(21/2)-1287/2*a^7*b^8/x^10-10010/19*a^6*b^9/x^(19/2)-1001/3*a^5
*b^10/x^9-2730/17*a^4*b^11/x^(17/2)-455/8*a^3*b^12/x^8-14*a^2*b^13/x^(15/2)
-15/7*a*b^14/x^7-2/13*b^15/x^(13/2)
```

Rubi [A] time = 0.12, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{105a^{13}b^2}{13x^{13}}-\frac{182a^{12}b^3}{5x^{25/2}}-\frac{455a^{11}b^4}{4x^{12}}-\frac{6006a^{10}b^5}{23x^{23/2}}-\frac{455a^9b^6}{x^{11}}-\frac{4290a^8b^7}{7x^{21/2}}-\frac{1287a^7b^8}{2x^{10}}-\frac{10010a^6b^9}{19x^{19/2}}-\frac{1001a^5b^{10}}{3x^9}-\frac{2730a^4b^1}{17x^{17/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^15/x^15,x]
```

```
[Out] -a^15/(14*x^14) - (10*a^14*b)/(9*x^(27/2)) - (105*a^13*b^2)/(13*x^13) - (18
2*a^12*b^3)/(5*x^(25/2)) - (455*a^11*b^4)/(4*x^12) - (6006*a^10*b^5)/(23*x^
(23/2)) - (455*a^9*b^6)/x^11 - (4290*a^8*b^7)/(7*x^(21/2)) - (1287*a^7*b^8)
/(2*x^10) - (10010*a^6*b^9)/(19*x^(19/2)) - (1001*a^5*b^10)/(3*x^9) - (2730
*a^4*b^11)/(17*x^(17/2)) - (455*a^3*b^12)/(8*x^8) - (14*a^2*b^13)/x^(15/2)
- (15*a*b^14)/(7*x^7) - (2*b^15)/(13*x^(13/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{15}}\,dx &= 2\,\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{29}}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(\frac{a^{15}}{x^{29}}+\frac{15a^{14}b}{x^{28}}+\frac{105a^{13}b^2}{x^{27}}+\frac{455a^{12}b^3}{x^{26}}+\frac{1365a^{11}b^4}{x^{25}}+\frac{3003a^{10}b^5}{x^{24}}+\frac{5005a^9b^6}{x^{23}}\right.\right.\\ &= -\frac{a^{15}}{14x^{14}}-\frac{10a^{14}b}{9x^{27/2}}-\frac{105a^{13}b^2}{13x^{13}}-\frac{182a^{12}b^3}{5x^{25/2}}-\frac{455a^{11}b^4}{4x^{12}}-\frac{6006a^{10}b^5}{23x^{23/2}}-\frac{455a^9b^6}{x^{11}}-\frac{4290a^8b^7}{7x^{21/2}}\end{aligned}$$

Mathematica [A] time = 0.13, size = 211, normalized size = 1.00

$$-\frac{a^{15}}{14x^{14}}-\frac{10a^{14}b}{9x^{27/2}}-\frac{105a^{13}b^2}{13x^{13}}-\frac{182a^{12}b^3}{5x^{25/2}}-\frac{455a^{11}b^4}{4x^{12}}-\frac{6006a^{10}b^5}{23x^{23/2}}-\frac{455a^9b^6}{x^{11}}-\frac{4290a^8b^7}{7x^{21/2}}-\frac{1287a^7b^8}{2x^{10}}-\frac{10010a^6b^9}{19x^{19/2}}-\frac{1001}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^15/x^15,x]
```

```
[Out] -1/14*a^15/x^14 - (10*a^14*b)/(9*x^(27/2)) - (105*a^13*b^2)/(13*x^13) - (18
2*a^12*b^3)/(5*x^(25/2)) - (455*a^11*b^4)/(4*x^12) - (6006*a^10*b^5)/(23*x^
(23/2)) - (455*a^9*b^6)/x^11 - (4290*a^8*b^7)/(7*x^(21/2)) - (1287*a^7*b^8)
/(2*x^10) - (10010*a^6*b^9)/(19*x^(19/2)) - (1001*a^5*b^10)/(3*x^9) - (2730
*a^4*b^11)/(17*x^(17/2)) - (455*a^3*b^12)/(8*x^8) - (14*a^2*b^13)/x^(15/2)
- (15*a*b^14)/(7*x^7) - (2*b^15)/(13*x^(13/2))
```

fricas [A] time = 0.88, size = 168, normalized size = 0.80

$$-\frac{521515800\,ab^{14}x^7+13841898525\,a^3b^{12}x^6+81205804680\,a^5b^{10}x^5+156611194740\,a^7b^8x^4+110735188200$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^15,x, algorithm="fricas")
```

```
[Out] -1/243374040*(521515800*a*b^14*x^7 + 13841898525*a^3*b^12*x^6 + 81205804680
*a^5*b^10*x^5 + 156611194740*a^7*b^8*x^4 + 110735188200*a^9*b^6*x^3 + 27683
797050*a^11*b^4*x^2 + 1965713400*a^13*b^2*x + 17383860*a^15 + 16*(2340135*b
^15*x^7 + 212952285*a^2*b^13*x^6 + 2442687975*a^4*b^11*x^5 + 8013730725*a^6
*b^9*x^4 + 9322094925*a^8*b^7*x^3 + 3972023055*a^10*b^5*x^2 + 553675941*a^1
2*b^3*x + 16900975*a^14*b)*sqrt(x))/x^14
```

giac [A] time = 0.18, size = 167, normalized size = 0.79

$$-\frac{37442160\,b^{15}x^{\frac{15}{2}}+521515800\,ab^{14}x^7+3407236560\,a^2b^{13}x^{\frac{13}{2}}+13841898525\,a^3b^{12}x^6+39083007600\,a^4b^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^15,x, algorithm="giac")
```

```
[Out] -1/243374040*(37442160*b^15*x^(15/2) + 521515800*a*b^14*x^7 + 3407236560*a^
2*b^13*x^(13/2) + 13841898525*a^3*b^12*x^6 + 39083007600*a^4*b^11*x^(11/2)
+ 81205804680*a^5*b^10*x^5 + 128219691600*a^6*b^9*x^(9/2) + 156611194740*a^
7*b^8*x^4 + 149153518800*a^8*b^7*x^(7/2) + 110735188200*a^9*b^6*x^3 + 63552
368880*a^10*b^5*x^(5/2) + 27683797050*a^11*b^4*x^2 + 8858815056*a^12*b^3*x^
(3/2) + 1965713400*a^13*b^2*x + 270415600*a^14*b*sqrt(x) + 17383860*a^15)/x
^14
```

maple [A] time = 0.00, size = 168, normalized size = 0.80

$$-\frac{2b^{15}}{13x^{\frac{13}{2}}}-\frac{15a\,b^{14}}{7x^7}-\frac{14a^2b^{13}}{x^{\frac{15}{2}}}-\frac{455a^3b^{12}}{8x^8}-\frac{2730a^4b^{11}}{17x^{\frac{17}{2}}}-\frac{1001a^5b^{10}}{3x^9}-\frac{10010a^6b^9}{19x^{\frac{19}{2}}}-\frac{1287a^7b^8}{2x^{10}}-\frac{4290a^8b^7}{7x^{\frac{21}{2}}}-\frac{455a^9b^6}{x^{11}}-\frac{6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^15/x^15,x)
```

```
[Out] -1/14*a^15/x^14-10/9*a^14*b/x^(27/2)-105/13*a^13*b^2/x^13-182/5*a^12*b^3/x^
(25/2)-455/4*a^11*b^4/x^12-6006/23*a^10*b^5/x^(23/2)-455*a^9*b^6/x^11-4290/
7*a^8*b^7/x^(21/2)-1287/2*a^7*b^8/x^10-10010/19*a^6*b^9/x^(19/2)-1001/3*a^5
*b^10/x^9-2730/17*a^4*b^11/x^(17/2)-455/8*a^3*b^12/x^8-14*a^2*b^13/x^(15/2)
-15/7*a*b^14/x^7-2/13*b^15/x^(13/2)
```

maxima [A] time = 0.90, size = 167, normalized size = 0.79

$$-\frac{37442160\,b^{15}x^{\frac{15}{2}}+521515800\,ab^{14}x^7+3407236560\,a^2b^{13}x^{\frac{13}{2}}+13841898525\,a^3b^{12}x^6+39083007600\,a^4b^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^15,x, algorithm="maxima")
```

```
[Out] -1/243374040*(37442160*b^15*x^(15/2) + 521515800*a*b^14*x^7 + 3407236560*a^
2*b^13*x^(13/2) + 13841898525*a^3*b^12*x^6 + 39083007600*a^4*b^11*x^(11/2)
+ 81205804680*a^5*b^10*x^5 + 128219691600*a^6*b^9*x^(9/2) + 156611194740*a^
7*b^8*x^4 + 149153518800*a^8*b^7*x^(7/2) + 110735188200*a^9*b^6*x^3 + 63552
368880*a^10*b^5*x^(5/2) + 27683797050*a^11*b^4*x^2 + 8858815056*a^12*b^3*x^
(3/2) + 1965713400*a^13*b^2*x + 270415600*a^14*b*sqrt(x) + 17383860*a^15)/x
^14
```

mupad [B] time = 0.20, size = 167, normalized size = 0.79

$$-\frac{\frac{a^{15}}{14}+\frac{2\,b^{15}\,x^{15/2}}{13}+\frac{105\,a^{13}\,b^2\,x}{13}+\frac{10\,a^{14}\,b\,\sqrt{x}}{9}+\frac{15\,a\,b^{14}\,x^7}{7}+\frac{455\,a^{11}\,b^4\,x^2}{4}+455\,a^9\,b^6\,x^3+\frac{1287\,a^7\,b^8\,x^4}{2}+\frac{1001\,a^5\,b^{10}\,x^5}{3}+\frac{182\,a^{12}}{5}}{x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^15/x^15,x)
```

```
[Out] -(a^15/14 + (2*b^15*x^(15/2))/13 + (105*a^13*b^2*x)/13 + (10*a^14*b*x^(1/2)
)/9 + (15*a*b^14*x^7)/7 + (455*a^11*b^4*x^2)/4 + 455*a^9*b^6*x^3 + (1287*a^
7*b^8*x^4)/2 + (1001*a^5*b^10*x^5)/3 + (182*a^12*b^3*x^(3/2))/5 + (455*a^3*
b^12*x^6)/8 + (6006*a^10*b^5*x^(5/2))/23 + (4290*a^8*b^7*x^(7/2))/7 + (1001
0*a^6*b^9*x^(9/2))/19 + (2730*a^4*b^11*x^(11/2))/17 + 14*a^2*b^13*x^(13/2))
/x^14
```

sympy [A] time = 34.85, size = 216, normalized size = 1.02

$$-\frac{a^{15}}{14x^{14}}-\frac{10a^{14}b}{9x^{\frac{27}{2}}}-\frac{105a^{13}b^2}{13x^{13}}-\frac{182a^{12}b^3}{5x^{\frac{25}{2}}}-\frac{455a^{11}b^4}{4x^{12}}-\frac{6006a^{10}b^5}{23x^{\frac{23}{2}}}-\frac{455a^9b^6}{x^{11}}-\frac{4290a^8b^7}{7x^{\frac{21}{2}}}-\frac{1287a^7b^8}{2x^{10}}-\frac{10010a^6b^9}{19x^{\frac{19}{2}}}-\frac{1001}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**15/x**15,x)
```

```
[Out] -a**15/(14*x**14) - 10*a**14*b/(9*x**(27/2)) - 105*a**13*b**2/(13*x**13) -
182*a**12*b**3/(5*x**(25/2)) - 455*a**11*b**4/(4*x**12) - 6006*a**10*b**5/(
23*x**(23/2)) - 455*a**9*b**6/x**11 - 4290*a**8*b**7/(7*x**(21/2)) - 1287*a
**7*b**8/(2*x**10) - 10010*a**6*b**9/(19*x**(19/2)) - 1001*a**5*b**10/(3*x*
*9) - 2730*a**4*b**11/(17*x**(17/2)) - 455*a**3*b**12/(8*x**8) - 14*a**2*b*
*13/x**(15/2) - 15*a*b**14/(7*x**7) - 2*b**15/(13*x**(13/2))
```

3.2189 $\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{16}}\,dx$

Optimal. Leaf size=211

$$-\frac{a^{15}}{15x^{15}}-\frac{30a^{14}b}{29x^{29/2}}-\frac{15a^{13}b^2}{2x^{14}}-\frac{910a^{12}b^3}{27x^{27/2}}-\frac{105a^{11}b^4}{x^{13}}-\frac{6006a^{10}b^5}{25x^{25/2}}-\frac{5005a^9b^6}{12x^{12}}-\frac{12870a^8b^7}{23x^{23/2}}-\frac{585a^7b^8}{x^{11}}-\frac{1430a^6b^9}{3x^{21/2}}-\frac{30}{}$$

```
[Out] -1/15*a^15/x^15-30/29*a^14*b/x^(29/2)-15/2*a^13*b^2/x^14-910/27*a^12*b^3/x^
(27/2)-105*a^11*b^4/x^13-6006/25*a^10*b^5/x^(25/2)-5005/12*a^9*b^6/x^12-128
70/23*a^8*b^7/x^(23/2)-585*a^7*b^8/x^11-1430/3*a^6*b^9/x^(21/2)-3003/10*a^5
*b^10/x^10-2730/19*a^4*b^11/x^(19/2)-455/9*a^3*b^12/x^9-210/17*a^2*b^13/x^(
17/2)-15/8*a*b^14/x^8-2/15*b^15/x^(15/2)
```

Rubi [A] time = 0.11, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{15a^{13}b^2}{2x^{14}}-\frac{910a^{12}b^3}{27x^{27/2}}-\frac{105a^{11}b^4}{x^{13}}-\frac{6006a^{10}b^5}{25x^{25/2}}-\frac{5005a^9b^6}{12x^{12}}-\frac{12870a^8b^7}{23x^{23/2}}-\frac{585a^7b^8}{x^{11}}-\frac{1430a^6b^9}{3x^{21/2}}-\frac{3003a^5b^{10}}{10x^{10}}-\frac{2730a^4}{19x^{19}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^15/x^16,x]
```

```
[Out] -a^15/(15*x^15) - (30*a^14*b)/(29*x^(29/2)) - (15*a^13*b^2)/(2*x^14) - (910
*a^12*b^3)/(27*x^(27/2)) - (105*a^11*b^4)/x^13 - (6006*a^10*b^5)/(25*x^(25/
2)) - (5005*a^9*b^6)/(12*x^12) - (12870*a^8*b^7)/(23*x^(23/2)) - (585*a^7*b
^8)/x^11 - (1430*a^6*b^9)/(3*x^(21/2)) - (3003*a^5*b^10)/(10*x^10) - (2730*
a^4*b^11)/(19*x^(19/2)) - (455*a^3*b^12)/(9*x^9) - (210*a^2*b^13)/(17*x^(17
/2)) - (15*a*b^14)/(8*x^8) - (2*b^15)/(15*x^(15/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{16}}\,dx &= 2\,\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{31}}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(\frac{a^{15}}{x^{31}}+\frac{15a^{14}b}{x^{30}}+\frac{105a^{13}b^2}{x^{29}}+\frac{455a^{12}b^3}{x^{28}}+\frac{1365a^{11}b^4}{x^{27}}+\frac{3003a^{10}b^5}{x^{26}}+\frac{5005a^9}{x^{25}}\right.\right.\\ &= -\frac{a^{15}}{15x^{15}}-\frac{30a^{14}b}{29x^{29/2}}-\frac{15a^{13}b^2}{2x^{14}}-\frac{910a^{12}b^3}{27x^{27/2}}-\frac{105a^{11}b^4}{x^{13}}-\frac{6006a^{10}b^5}{25x^{25/2}}-\frac{5005a^9b^6}{12x^{12}}-\frac{12870a^8}{23x^{23}}\end{aligned}$$

Mathematica [A] time = 0.14, size = 211, normalized size = 1.00

$$-\frac{a^{15}}{15x^{15}}-\frac{30a^{14}b}{29x^{29/2}}-\frac{15a^{13}b^2}{2x^{14}}-\frac{910a^{12}b^3}{27x^{27/2}}-\frac{105a^{11}b^4}{x^{13}}-\frac{6006a^{10}b^5}{25x^{25/2}}-\frac{5005a^9b^6}{12x^{12}}-\frac{12870a^8b^7}{23x^{23/2}}-\frac{585a^7b^8}{x^{11}}-\frac{1430a^6b^9}{3x^{21/2}}-\frac{30}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^15/x^16,x]
```

```
[Out] -1/15*a^15/x^15 - (30*a^14*b)/(29*x^(29/2)) - (15*a^13*b^2)/(2*x^14) - (910
*a^12*b^3)/(27*x^(27/2)) - (105*a^11*b^4)/x^13 - (6006*a^10*b^5)/(25*x^(25/
2)) - (5005*a^9*b^6)/(12*x^12) - (12870*a^8*b^7)/(23*x^(23/2)) - (585*a^7*b
^8)/x^11 - (1430*a^6*b^9)/(3*x^(21/2)) - (3003*a^5*b^10)/(10*x^10) - (2730*
a^4*b^11)/(19*x^(19/2)) - (455*a^3*b^12)/(9*x^9) - (210*a^2*b^13)/(17*x^(17
/2)) - (15*a*b^14)/(8*x^8) - (2*b^15)/(15*x^(15/2))
```

fricas [A] time = 0.83, size = 168, normalized size = 0.80

$$-\frac{2181340125\,ab^{14}x^7+58815393000\,a^3b^{12}x^6+349363434420\,a^5b^{10}x^5+680578119000\,a^7b^8x^4+485226992250$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^16,x, algorithm="fricas")
```

```
[Out] -1/1163381400*(2181340125*a*b^14*x^7 + 58815393000*a^3*b^12*x^6 + 349363434
420*a^5*b^10*x^5 + 680578119000*a^7*b^8*x^4 + 485226992250*a^9*b^6*x^3 + 12
2155047000*a^11*b^4*x^2 + 8725360500*a^13*b^2*x + 77558760*a^15 + 16*(96948
45*b^15*x^7 + 898198875*a^2*b^13*x^6 + 10447471125*a^4*b^11*x^5 + 346590708
75*a^6*b^9*x^4 + 40686735375*a^8*b^7*x^3 + 17468171721*a^10*b^5*x^2 + 24506
41375*a^12*b^3*x + 75218625*a^14*b)*sqrt(x))/x^15
```

giac [A] time = 0.17, size = 167, normalized size = 0.79

$$-\frac{155117520\,b^{15}x^{\frac{15}{2}}+2181340125\,ab^{14}x^7+14371182000\,a^2b^{13}x^{\frac{13}{2}}+58815393000\,a^3b^{12}x^6+167159538000\,a^4b^1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^16,x, algorithm="giac")
```

```
[Out] -1/1163381400*(155117520*b^15*x^(15/2) + 2181340125*a*b^14*x^7 + 1437118200
0*a^2*b^13*x^(13/2) + 58815393000*a^3*b^12*x^6 + 167159538000*a^4*b^11*x^(1
1/2) + 349363434420*a^5*b^10*x^5 + 554545134000*a^6*b^9*x^(9/2) + 680578119
000*a^7*b^8*x^4 + 650987766000*a^8*b^7*x^(7/2) + 485226992250*a^9*b^6*x^3 +
 279490747536*a^10*b^5*x^(5/2) + 122155047000*a^11*b^4*x^2 + 39210262000*a^
12*b^3*x^(3/2) + 8725360500*a^13*b^2*x + 1203498000*a^14*b*sqrt(x) + 775587
60*a^15)/x^15
```

maple [A] time = 0.00, size = 168, normalized size = 0.80

$$-\frac{2b^{15}}{15x^{\frac{15}{2}}}-\frac{15a\,b^{14}}{8x^8}-\frac{210a^2b^{13}}{17x^{\frac{17}{2}}}-\frac{455a^3b^{12}}{9x^9}-\frac{2730a^4b^{11}}{19x^{\frac{19}{2}}}-\frac{3003a^5b^{10}}{10x^{10}}-\frac{1430a^6b^9}{3x^{\frac{21}{2}}}-\frac{585a^7b^8}{x^{11}}-\frac{12870a^8b^7}{23x^{\frac{23}{2}}}-\frac{5005a^9b^6}{12x^{12}}-\frac{60}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^15/x^16,x)
```

```
[Out] -1/15*a^15/x^15-30/29*a^14*b/x^(29/2)-15/2*a^13*b^2/x^14-910/27*a^12*b^3/x^
(27/2)-105*a^11*b^4/x^13-6006/25*a^10*b^5/x^(25/2)-5005/12*a^9*b^6/x^12-128
70/23*a^8*b^7/x^(23/2)-585*a^7*b^8/x^11-1430/3*a^6*b^9/x^(21/2)-3003/10*a^5
*b^10/x^10-2730/19*a^4*b^11/x^(19/2)-455/9*a^3*b^12/x^9-210/17*a^2*b^13/x^(
17/2)-15/8*a*b^14/x^8-2/15*b^15/x^(15/2)
```

maxima [A] time = 0.84, size = 167, normalized size = 0.79

$$-\frac{155117520\,b^{15}x^{\frac{15}{2}}+2181340125\,ab^{14}x^7+14371182000\,a^2b^{13}x^{\frac{13}{2}}+58815393000\,a^3b^{12}x^6+167159538000\,a^4b^1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^16,x, algorithm="maxima")
```

```
[Out] -1/1163381400*(155117520*b^15*x^(15/2) + 2181340125*a*b^14*x^7 + 1437118200
0*a^2*b^13*x^(13/2) + 58815393000*a^3*b^12*x^6 + 167159538000*a^4*b^11*x^(1
1/2) + 349363434420*a^5*b^10*x^5 + 554545134000*a^6*b^9*x^(9/2) + 680578119
000*a^7*b^8*x^4 + 650987766000*a^8*b^7*x^(7/2) + 485226992250*a^9*b^6*x^3 +
 279490747536*a^10*b^5*x^(5/2) + 122155047000*a^11*b^4*x^2 + 39210262000*a^
12*b^3*x^(3/2) + 8725360500*a^13*b^2*x + 1203498000*a^14*b*sqrt(x) + 775587
60*a^15)/x^15
```

mupad **[B]** time = 1.25, size = 167, normalized size = 0.79

$$-\frac{\frac{a^{15}}{15}+\frac{2\,b^{15}\,x^{15/2}}{15}+\frac{15\,a^{13}\,b^2\,x}{2}+\frac{30\,a^{14}\,b\,\sqrt{x}}{29}+\frac{15\,a\,b^{14}\,x^7}{8}+105\,a^{11}\,b^4\,x^2+\frac{5005\,a^9\,b^6\,x^3}{12}+585\,a^7\,b^8\,x^4+\frac{3003\,a^5\,b^{10}\,x^5}{10}+\frac{9}{}}{x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^15/x^16,x)
```

```
[Out] -(a^15/15 + (2*b^15*x^(15/2))/15 + (15*a^13*b^2*x)/2 + (30*a^14*b*x^(1/2))/
29 + (15*a*b^14*x^7)/8 + 105*a^11*b^4*x^2 + (5005*a^9*b^6*x^3)/12 + 585*a^7
*b^8*x^4 + (3003*a^5*b^10*x^5)/10 + (910*a^12*b^3*x^(3/2))/27 + (455*a^3*b^
12*x^6)/9 + (6006*a^10*b^5*x^(5/2))/25 + (12870*a^8*b^7*x^(7/2))/23 + (1430
*a^6*b^9*x^(9/2))/3 + (2730*a^4*b^11*x^(11/2))/19 + (210*a^2*b^13*x^(13/2))
/17)/x^15
```

sympy **[A]** time = 48.09, size = 216, normalized size = 1.02

$$-\frac{a^{15}}{15x^{15}}-\frac{30a^{14}b}{29x^{\frac{29}{2}}}-\frac{15a^{13}b^2}{2x^{14}}-\frac{910a^{12}b^3}{27x^{\frac{27}{2}}}-\frac{105a^{11}b^4}{x^{13}}-\frac{6006a^{10}b^5}{25x^{\frac{25}{2}}}-\frac{5005a^9b^6}{12x^{12}}-\frac{12870a^8b^7}{23x^{\frac{23}{2}}}-\frac{585a^7b^8}{x^{11}}-\frac{1430a^6b^9}{3x^{\frac{21}{2}}}-\frac{300}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**15/x**16,x)
```

```
[Out] -a**15/(15*x**15) - 30*a**14*b/(29*x**(29/2)) - 15*a**13*b**2/(2*x**14) - 9
10*a**12*b**3/(27*x**(27/2)) - 105*a**11*b**4/x**13 - 6006*a**10*b**5/(25*x
**(25/2)) - 5005*a**9*b**6/(12*x**12) - 12870*a**8*b**7/(23*x**(23/2)) - 58
5*a**7*b**8/x**11 - 1430*a**6*b**9/(3*x**(21/2)) - 3003*a**5*b**10/(10*x**1
0) - 2730*a**4*b**11/(19*x**(19/2)) - 455*a**3*b**12/(9*x**9) - 210*a**2*b*
*13/(17*x**(17/2)) - 15*a*b**14/(8*x**8) - 2*b**15/(15*x**(15/2))
```

3.2190 $\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{17}}\,dx$

Optimal. Leaf size=207

$$-\frac{a^{15}}{16x^{16}}-\frac{30a^{14}b}{31x^{31/2}}-\frac{7a^{13}b^2}{x^{15}}-\frac{910a^{12}b^3}{29x^{29/2}}-\frac{195a^{11}b^4}{2x^{14}}-\frac{2002a^{10}b^5}{9x^{27/2}}-\frac{385a^9b^6}{x^{13}}-\frac{2574a^8b^7}{5x^{25/2}}-\frac{2145a^7b^8}{4x^{12}}-\frac{10010a^6b^9}{23x^{23/2}}-\frac{273a^5b}{x^{11}}$$

```
[Out] -1/16*a^15/x^16-30/31*a^14*b/x^(31/2)-7*a^13*b^2/x^15-910/29*a^12*b^3/x^(29
/2)-195/2*a^11*b^4/x^14-2002/9*a^10*b^5/x^(27/2)-385*a^9*b^6/x^13-2574/5*a^
8*b^7/x^(25/2)-2145/4*a^7*b^8/x^12-10010/23*a^6*b^9/x^(23/2)-273*a^5*b^10/x
^11-130*a^4*b^11/x^(21/2)-91/2*a^3*b^12/x^10-210/19*a^2*b^13/x^(19/2)-5/3*a
*b^14/x^9-2/17*b^15/x^(17/2)
```

Rubi [A] time = 0.11, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{7a^{13}b^2}{x^{15}}-\frac{910a^{12}b^3}{29x^{29/2}}-\frac{195a^{11}b^4}{2x^{14}}-\frac{2002a^{10}b^5}{9x^{27/2}}-\frac{385a^9b^6}{x^{13}}-\frac{2574a^8b^7}{5x^{25/2}}-\frac{2145a^7b^8}{4x^{12}}-\frac{10010a^6b^9}{23x^{23/2}}-\frac{273a^5b^{10}}{x^{11}}-\frac{130a^4b^{11}}{x^{21/2}}-\frac{9}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^15/x^17,x]
```

```
[Out] -a^15/(16*x^16) - (30*a^14*b)/(31*x^(31/2)) - (7*a^13*b^2)/x^15 - (910*a^12
*b^3)/(29*x^(29/2)) - (195*a^11*b^4)/(2*x^14) - (2002*a^10*b^5)/(9*x^(27/2)
) - (385*a^9*b^6)/x^13 - (2574*a^8*b^7)/(5*x^(25/2)) - (2145*a^7*b^8)/(4*x^
12) - (10010*a^6*b^9)/(23*x^(23/2)) - (273*a^5*b^10)/x^11 - (130*a^4*b^11)/
x^(21/2) - (91*a^3*b^12)/(2*x^10) - (210*a^2*b^13)/(19*x^(19/2)) - (5*a*b^1
4)/(3*x^9) - (2*b^15)/(17*x^(17/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{17}}\,dx &= 2\,\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{33}}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(\frac{a^{15}}{x^{33}}+\frac{15a^{14}b}{x^{32}}+\frac{105a^{13}b^2}{x^{31}}+\frac{455a^{12}b^3}{x^{30}}+\frac{1365a^{11}b^4}{x^{29}}+\frac{3003a^{10}b^5}{x^{28}}+\frac{5005a^9b^6}{x^{27}}\right.\right.\\ &= -\frac{a^{15}}{16x^{16}}-\frac{30a^{14}b}{31x^{31/2}}-\frac{7a^{13}b^2}{x^{15}}-\frac{910a^{12}b^3}{29x^{29/2}}-\frac{195a^{11}b^4}{2x^{14}}-\frac{2002a^{10}b^5}{9x^{27/2}}-\frac{385a^9b^6}{x^{13}}-\frac{2574a^8b^7}{5x^{25/2}}-\end{aligned}$$

Mathematica [A] time = 0.14, size = 207, normalized size = 1.00

$$-\frac{a^{15}}{16x^{16}}-\frac{30a^{14}b}{31x^{31/2}}-\frac{7a^{13}b^2}{x^{15}}-\frac{910a^{12}b^3}{29x^{29/2}}-\frac{195a^{11}b^4}{2x^{14}}-\frac{2002a^{10}b^5}{9x^{27/2}}-\frac{385a^9b^6}{x^{13}}-\frac{2574a^8b^7}{5x^{25/2}}-\frac{2145a^7b^8}{4x^{12}}-\frac{10010a^6b^9}{23x^{23/2}}-\frac{273a^5b}{x^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^15/x^17,x]
```

```
[Out] -1/16*a^15/x^16 - (30*a^14*b)/(31*x^(31/2)) - (7*a^13*b^2)/x^15 - (910*a^12
*b^3)/(29*x^(29/2)) - (195*a^11*b^4)/(2*x^14) - (2002*a^10*b^5)/(9*x^(27/2)
) - (385*a^9*b^6)/x^13 - (2574*a^8*b^7)/(5*x^(25/2)) - (2145*a^7*b^8)/(4*x^
12) - (10010*a^6*b^9)/(23*x^(23/2)) - (273*a^5*b^10)/x^11 - (130*a^4*b^11)/
x^(21/2) - (91*a^3*b^12)/(2*x^10) - (210*a^2*b^13)/(19*x^(19/2)) - (5*a*b^1
4)/(3*x^9) - (2*b^15)/(17*x^(17/2))
```

fricas [A] time = 0.97, size = 168, normalized size = 0.81

$$-\frac{8014405200\,ab^{14}x^7+218793261960\,a^3b^{12}x^6+1312759571760\,a^5b^{10}x^5+2578634873100\,a^7b^8x^4+1851327}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^17,x, algorithm="fricas")
```

```
[Out] -1/4808643120*(8014405200*a*b^14*x^7 + 218793261960*a^3*b^12*x^6 + 13127595
71760*a^5*b^10*x^5 + 2578634873100*a^7*b^8*x^4 + 1851327601200*a^9*b^6*x^3
+ 468842704200*a^11*b^4*x^2 + 33660501840*a^13*b^2*x + 300540195*a^15 + 32*
(17678835*b^15*x^7 + 1660880025*a^2*b^13*x^6 + 19535112675*a^4*b^11*x^5 + 6
5400159825*a^6*b^9*x^4 + 77359046193*a^8*b^7*x^3 + 33426748355*a^10*b^5*x^2
 + 4715372025*a^12*b^3*x + 145422675*a^14*b)*sqrt(x))/x^16
```

giac [A] time = 0.18, size = 167, normalized size = 0.81

$$-\frac{565722720\,b^{15}x^{\frac{15}{2}}+8014405200\,ab^{14}x^7+53148160800\,a^2b^{13}x^{\frac{13}{2}}+218793261960\,a^3b^{12}x^6+625123605600}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^17,x, algorithm="giac")
```

```
[Out] -1/4808643120*(565722720*b^15*x^(15/2) + 8014405200*a*b^14*x^7 + 5314816080
0*a^2*b^13*x^(13/2) + 218793261960*a^3*b^12*x^6 + 625123605600*a^4*b^11*x^(
11/2) + 1312759571760*a^5*b^10*x^5 + 2092805114400*a^6*b^9*x^(9/2) + 257863
4873100*a^7*b^8*x^4 + 2475489478176*a^8*b^7*x^(7/2) + 1851327601200*a^9*b^6
*x^3 + 1069655947360*a^10*b^5*x^(5/2) + 468842704200*a^11*b^4*x^2 + 1508919
04800*a^12*b^3*x^(3/2) + 33660501840*a^13*b^2*x + 4653525600*a^14*b*sqrt(x)
 + 300540195*a^15)/x^16
```

maple [A] time = 0.00, size = 168, normalized size = 0.81

$$-\frac{2b^{15}}{17x^{\frac{17}{2}}}-\frac{5a\,b^{14}}{3x^9}-\frac{210a^2b^{13}}{19x^{\frac{19}{2}}}-\frac{91a^3b^{12}}{2x^{10}}-\frac{130a^4b^{11}}{x^{\frac{21}{2}}}-\frac{273a^5b^{10}}{x^{11}}-\frac{10010a^6b^9}{23x^{\frac{23}{2}}}-\frac{2145a^7b^8}{4x^{12}}-\frac{2574a^8b^7}{5x^{\frac{25}{2}}}-\frac{385a^9b^6}{x^{13}}-\frac{2002}{9x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))^15/x^17,x)
```

```
[Out] -1/16*a^15/x^16-30/31*a^14*b/x^(31/2)-7*a^13*b^2/x^15-910/29*a^12*b^3/x^(29
/2)-195/2*a^11*b^4/x^14-2002/9*a^10*b^5/x^(27/2)-385*a^9*b^6/x^13-2574/5*a^
8*b^7/x^(25/2)-2145/4*a^7*b^8/x^12-10010/23*a^6*b^9/x^(23/2)-273*a^5*b^10/x
^11-130*a^4*b^11/x^(21/2)-91/2*a^3*b^12/x^10-210/19*a^2*b^13/x^(19/2)-5/3*a
*b^14/x^9-2/17*b^15/x^(17/2)
```

maxima [A] time = 0.87, size = 167, normalized size = 0.81

$$-\frac{565722720\,b^{15}x^{\frac{15}{2}}+8014405200\,ab^{14}x^7+53148160800\,a^2b^{13}x^{\frac{13}{2}}+218793261960\,a^3b^{12}x^6+625123605600}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^15/x^17,x, algorithm="maxima")
```

```
[Out] -1/4808643120*(565722720*b^15*x^(15/2) + 8014405200*a*b^14*x^7 + 5314816080
0*a^2*b^13*x^(13/2) + 218793261960*a^3*b^12*x^6 + 625123605600*a^4*b^11*x^(
11/2) + 1312759571760*a^5*b^10*x^5 + 2092805114400*a^6*b^9*x^(9/2) + 257863
4873100*a^7*b^8*x^4 + 2475489478176*a^8*b^7*x^(7/2) + 1851327601200*a^9*b^6
*x^3 + 1069655947360*a^10*b^5*x^(5/2) + 468842704200*a^11*b^4*x^2 + 1508919
04800*a^12*b^3*x^(3/2) + 33660501840*a^13*b^2*x + 4653525600*a^14*b*sqrt(x)
 + 300540195*a^15)/x^16
```

mupad [B] time = 0.20, size = 167, normalized size = 0.81

$$-\frac{\frac{a^{15}}{16}+\frac{2\,b^{15}\,x^{15/2}}{17}+7\,a^{13}\,b^2\,x+\frac{30\,a^{14}\,b\,\sqrt{x}}{31}+\frac{5\,a\,b^{14}\,x^7}{3}+\frac{195\,a^{11}\,b^4\,x^2}{2}+385\,a^9\,b^6\,x^3+\frac{2145\,a^7\,b^8\,x^4}{4}+273\,a^5\,b^{10}\,x^5+\frac{910\,a^{12}}{2}}{x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^15/x^17,x)
```

```
[Out] -(a^15/16 + (2*b^15*x^(15/2))/17 + 7*a^13*b^2*x + (30*a^14*b*x^(1/2))/31 +
(5*a*b^14*x^7)/3 + (195*a^11*b^4*x^2)/2 + 385*a^9*b^6*x^3 + (2145*a^7*b^8*x
^4)/4 + 273*a^5*b^10*x^5 + (910*a^12*b^3*x^(3/2))/29 + (91*a^3*b^12*x^6)/2
+ (2002*a^10*b^5*x^(5/2))/9 + (2574*a^8*b^7*x^(7/2))/5 + (10010*a^6*b^9*x^(
9/2))/23 + 130*a^4*b^11*x^(11/2) + (210*a^2*b^13*x^(13/2))/19)/x^16
```

sympy [A] time = 51.04, size = 212, normalized size = 1.02

$$-\frac{a^{15}}{16x^{16}}-\frac{30a^{14}b}{31x^{\frac{31}{2}}}-\frac{7a^{13}b^2}{x^{15}}-\frac{910a^{12}b^3}{29x^{\frac{29}{2}}}-\frac{195a^{11}b^4}{2x^{14}}-\frac{2002a^{10}b^5}{9x^{\frac{27}{2}}}-\frac{385a^9b^6}{x^{13}}-\frac{2574a^8b^7}{5x^{\frac{25}{2}}}-\frac{2145a^7b^8}{4x^{12}}-\frac{10010a^6b^9}{23x^{\frac{23}{2}}}-\frac{273a^5b}{x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**15/x**17,x)
```

```
[Out] -a**15/(16*x**16) - 30*a**14*b/(31*x**(31/2)) - 7*a**13*b**2/x**15 - 910*a*
*12*b**3/(29*x**(29/2)) - 195*a**11*b**4/(2*x**14) - 2002*a**10*b**5/(9*x**
(27/2)) - 385*a**9*b**6/x**13 - 2574*a**8*b**7/(5*x**(25/2)) - 2145*a**7*b*
*8/(4*x**12) - 10010*a**6*b**9/(23*x**(23/2)) - 273*a**5*b**10/x**11 - 130*
a**4*b**11/x**(21/2) - 91*a**3*b**12/(2*x**10) - 210*a**2*b**13/(19*x**(19/
2)) - 5*a*b**14/(3*x**9) - 2*b**15/(17*x**(17/2))
```

3.2191 $\int \frac{x^3}{a+b\sqrt{x}}\,dx$

Optimal. Leaf size=107

$$-\frac{2a^7\log\left(a+b\sqrt{x}\right)}{b^8}+\frac{2a^6\sqrt{x}}{b^7}-\frac{a^5x}{b^6}+\frac{2a^4x^{3/2}}{3b^5}-\frac{a^3x^2}{2b^4}+\frac{2a^2x^{5/2}}{5b^3}-\frac{ax^3}{3b^2}+\frac{2x^{7/2}}{7b}$$

```
[Out] -a^5*x/b^6+2/3*a^4*x^(3/2)/b^5-1/2*a^3*x^2/b^4+2/5*a^2*x^(5/2)/b^3-1/3*a*x^
3/b^2+2/7*x^(7/2)/b-2*a^7*ln(a+b*x^(1/2))/b^8+2*a^6*x^(1/2)/b^7
```

Rubi [A] time = 0.07, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{2a^4x^{3/2}}{3b^5}-\frac{a^3x^2}{2b^4}+\frac{2a^2x^{5/2}}{5b^3}+\frac{2a^6\sqrt{x}}{b^7}-\frac{a^5x}{b^6}-\frac{2a^7\log\left(a+b\sqrt{x}\right)}{b^8}-\frac{ax^3}{3b^2}+\frac{2x^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*Sqrt[x]),x]
```

```
[Out] (2*a^6*Sqrt[x])/b^7 - (a^5*x)/b^6 + (2*a^4*x^(3/2))/(3*b^5) - (a^3*x^2)/(2*
b^4) + (2*a^2*x^(5/2))/(5*b^3) - (a*x^3)/(3*b^2) + (2*x^(7/2))/(7*b) - (2*a
^7*Log[a + b*Sqrt[x]])/b^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{a+b\sqrt{x}}\,dx &= 2\,\text{Subst}\left(\int \frac{x^7}{a+bx}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(\frac{a^6}{b^7}-\frac{a^5x}{b^6}+\frac{a^4x^2}{b^5}-\frac{a^3x^3}{b^4}+\frac{a^2x^4}{b^3}-\frac{ax^5}{b^2}+\frac{x^6}{b}-\frac{a^7}{b^7(a+bx)}\right)dx, x, \sqrt{x}\right)\\ &= \frac{2a^6\sqrt{x}}{b^7}-\frac{a^5x}{b^6}+\frac{2a^4x^{3/2}}{3b^5}-\frac{a^3x^2}{2b^4}+\frac{2a^2x^{5/2}}{5b^3}-\frac{ax^3}{3b^2}+\frac{2x^{7/2}}{7b}-\frac{2a^7\log\left(a+b\sqrt{x}\right)}{b^8}\end{aligned}$$

Mathematica [A] time = 0.10, size = 99, normalized size = 0.93

$$\frac{-420a^7\log\left(a+b\sqrt{x}\right)+420a^6b\sqrt{x}-210a^5b^2x+140a^4b^3x^{3/2}-105a^3b^4x^2+84a^2b^5x^{5/2}-70ab^6x^3+60b^7x^{7/2}}{210b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*Sqrt[x]),x]
```

```
[Out] (420*a^6*b*Sqrt[x] - 210*a^5*b^2*x + 140*a^4*b^3*x^(3/2) - 105*a^3*b^4*x^2
+ 84*a^2*b^5*x^(5/2) - 70*a*b^6*x^3 + 60*b^7*x^(7/2) - 420*a^7*Log[a + b*Sq
rt[x]])/(210*b^8)
```

fricas [A] time = 0.92, size = 88, normalized size = 0.82

$$-\frac{70\,ab^6x^3+105\,a^3b^4x^2+210\,a^5b^2x+420\,a^7\log\left(b\sqrt{x}+a\right)-4\left(15\,b^7x^3+21\,a^2b^5x^2+35\,a^4b^3x+105\,a^6b\right)\sqrt{x}}{210\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/210*(70*a*b^6*x^3 + 105*a^3*b^4*x^2 + 210*a^5*b^2*x + 420*a^7*log(b*sqrt
(x) + a) - 4*(15*b^7*x^3 + 21*a^2*b^5*x^2 + 35*a^4*b^3*x + 105*a^6*b)*sqrt(
x))/b^8
```

giac [A] time = 0.15, size = 89, normalized size = 0.83

$$-\frac{2\,a^7\log\left(\left|b\sqrt{x}+a\right|\right)}{b^8}+\frac{60\,b^6x^{\frac{7}{2}}-70\,ab^5x^3+84\,a^2b^4x^{\frac{5}{2}}-105\,a^3b^3x^2+140\,a^4b^2x^{\frac{3}{2}}-210\,a^5bx+420\,a^6\sqrt{x}}{210\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^(1/2)),x, algorithm="giac")
```

```
[Out] -2*a^7*log(abs(b*sqrt(x) + a))/b^8 + 1/210*(60*b^6*x^(7/2) - 70*a*b^5*x^3 +
 84*a^2*b^4*x^(5/2) - 105*a^3*b^3*x^2 + 140*a^4*b^2*x^(3/2) - 210*a^5*b*x +
 420*a^6*sqrt(x))/b^7
```

maple [A] time = 0.00, size = 88, normalized size = 0.82

$$\frac{2x^{\frac{7}{2}}}{7b}-\frac{a\,x^3}{3b^2}+\frac{2a^2x^{\frac{5}{2}}}{5b^3}-\frac{a^3x^2}{2b^4}+\frac{2a^4x^{\frac{3}{2}}}{3b^5}-\frac{2a^7\ln\left(b\sqrt{x}+a\right)}{b^8}-\frac{a^5x}{b^6}+\frac{2a^6\sqrt{x}}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*x^(1/2)),x)
```

```
[Out] -a^5*x/b^6+2/3*a^4*x^(3/2)/b^5-1/2*a^3*x^2/b^4+2/5*a^2*x^(5/2)/b^3-1/3*a/b^
2*x^3+2/7*x^(7/2)/b-2*a^7*ln(a+b*x^(1/2))/b^8+2*a^6*x^(1/2)/b^7
```

maxima [A] time = 0.88, size = 129, normalized size = 1.21

$$-\frac{2\,a^7\log\left(b\sqrt{x}+a\right)}{b^8}+\frac{2\left(b\sqrt{x}+a\right)^7}{7\,b^8}-\frac{7\left(b\sqrt{x}+a\right)^6a}{3\,b^8}+\frac{42\left(b\sqrt{x}+a\right)^5a^2}{5\,b^8}-\frac{35\left(b\sqrt{x}+a\right)^4a^3}{2\,b^8}+\frac{70\left(b\sqrt{x}+a\right)^3a^4}{3\,b^8}-\frac{2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^(1/2)),x, algorithm="maxima")
```

```
[Out] -2*a^7*log(b*sqrt(x) + a)/b^8 + 2/7*(b*sqrt(x) + a)^7/b^8 - 7/3*(b*sqrt(x)
+ a)^6*a/b^8 + 42/5*(b*sqrt(x) + a)^5*a^2/b^8 - 35/2*(b*sqrt(x) + a)^4*a^3/
b^8 + 70/3*(b*sqrt(x) + a)^3*a^4/b^8 - 21*(b*sqrt(x) + a)^2*a^5/b^8 + 14*(b
*sqrt(x) + a)*a^6/b^8
```

mupad [B] time = 0.04, size = 87, normalized size = 0.81

$$\frac{2\,x^{7/2}}{7\,b}-\frac{a\,x^3}{3\,b^2}-\frac{a^5\,x}{b^6}-\frac{2\,a^7\,\ln\left(a+b\,\sqrt{x}\right)}{b^8}-\frac{a^3\,x^2}{2\,b^4}+\frac{2\,a^2\,x^{5/2}}{5\,b^3}+\frac{2\,a^4\,x^{3/2}}{3\,b^5}+\frac{2\,a^6\,\sqrt{x}}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^(1/2)),x)
```

```
[Out] (2*x^(7/2))/(7*b) - (a*x^3)/(3*b^2) - (a^5*x)/b^6 - (2*a^7*log(a + b*x^(1/2
)))/b^8 - (a^3*x^2)/(2*b^4) + (2*a^2*x^(5/2))/(5*b^3) + (2*a^4*x^(3/2))/(3*
b^5) + (2*a^6*x^(1/2))/b^7
```

sympy [**A**] time = 1.09, size = 109, normalized size = 1.02

$$\begin{cases} -\frac{2a^7 \log\left(\frac{a}{b}+\sqrt{x}\right)}{b^8} + \frac{2a^6\sqrt{x}}{b^7} - \frac{a^5 x}{b^6} + \frac{2a^4 x^{\frac{3}{2}}}{3b^5} - \frac{a^3x^2}{2b^4} + \frac{2a^2x^{\frac{5}{2}}}{5b^3} - \frac{ax^3}{3b^2} + \frac{2x^{\frac{7}{2}}}{7b} & \text{for } b \neq 0 \\ \frac{x^4}{4a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*x**(1/2)),x)
```

```
[Out] Piecewise((-2*a**7*log(a/b + sqrt(x))/b**8 + 2*a**6*sqrt(x)/b**7 - a**5*x/b
**6 + 2*a**4*x**(3/2)/(3*b**5) - a**3*x**2/(2*b**4) + 2*a**2*x**(5/2)/(5*b*
*3) - a*x**3/(3*b**2) + 2*x**(7/2)/(7*b), Ne(b, 0)), (x**4/(4*a), True))
```

3.2192 $\int \frac{x^2}{a+b\sqrt{x}}\,dx$

Optimal. Leaf size=79

$$-\frac{2a^5\log\left(a+b\sqrt{x}\right)}{b^6}+\frac{2a^4\sqrt{x}}{b^5}-\frac{a^3x}{b^4}+\frac{2a^2x^{3/2}}{3b^3}-\frac{ax^2}{2b^2}+\frac{2x^{5/2}}{5b}$$

```
[Out] -a^3*x/b^4+2/3*a^2*x^(3/2)/b^3-1/2*a*x^2/b^2+2/5*x^(5/2)/b-2*a^5*ln(a+b*x^(
1/2))/b^6+2*a^4*x^(1/2)/b^5
```

Rubi [A] time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{2a^2x^{3/2}}{3b^3}+\frac{2a^4\sqrt{x}}{b^5}-\frac{a^3x}{b^4}-\frac{2a^5\log\left(a+b\sqrt{x}\right)}{b^6}-\frac{ax^2}{2b^2}+\frac{2x^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*Sqrt[x]),x]
```

```
[Out] (2*a^4*Sqrt[x])/b^5 - (a^3*x)/b^4 + (2*a^2*x^(3/2))/(3*b^3) - (a*x^2)/(2*b^
2) + (2*x^(5/2))/(5*b) - (2*a^5*Log[a + b*Sqrt[x]])/b^6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{a+b\sqrt{x}}\,dx &= 2\,\text{Subst}\left(\int \frac{x^5}{a+bx}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int\left(\frac{a^4}{b^5}-\frac{a^3x}{b^4}+\frac{a^2x^2}{b^3}-\frac{ax^3}{b^2}+\frac{x^4}{b}-\frac{a^5}{b^5(a+bx)}\right)dx, x, \sqrt{x}\right)\\ &= \frac{2a^4\sqrt{x}}{b^5}-\frac{a^3x}{b^4}+\frac{2a^2x^{3/2}}{3b^3}-\frac{ax^2}{2b^2}+\frac{2x^{5/2}}{5b}-\frac{2a^5\log\left(a+b\sqrt{x}\right)}{b^6}\end{aligned}$$

Mathematica [A] time = 0.07, size = 75, normalized size = 0.95

$$\frac{-60a^5\log\left(a+b\sqrt{x}\right)+60a^4b\sqrt{x}-30a^3b^2x+20a^2b^3x^{3/2}-15ab^4x^2+12b^5x^{5/2}}{30b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*Sqrt[x]),x]
```

```
[Out] (60*a^4*b*Sqrt[x] - 30*a^3*b^2*x + 20*a^2*b^3*x^(3/2) - 15*a*b^4*x^2 + 12*b
^5*x^(5/2) - 60*a^5*Log[a + b*Sqrt[x]])/(30*b^6)
```

fricas [A] time = 0.92, size = 66, normalized size = 0.84

$$-\frac{15\,ab^4x^2+30\,a^3b^2x+60\,a^5\log\left(b\sqrt{x}+a\right)-4\left(3\,b^5x^2+5\,a^2b^3x+15\,a^4b\right)\sqrt{x}}{30\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/30*(15*a*b^4*x^2 + 30*a^3*b^2*x + 60*a^5*log(b*sqrt(x) + a) - 4*(3*b^5*x
^2 + 5*a^2*b^3*x + 15*a^4*b)*sqrt(x))/b^6
```

giac [A] time = 0.15, size = 67, normalized size = 0.85

$$-\frac{2\,a^5\log\left(\left|b\sqrt{x}+a\right|\right)}{b^6}+\frac{12\,b^4x^{\frac{5}{2}}-15\,ab^3x^2+20\,a^2b^2x^{\frac{3}{2}}-30\,a^3bx+60\,a^4\sqrt{x}}{30\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^(1/2)),x, algorithm="giac")
```

```
[Out] -2*a^5*log(abs(b*sqrt(x) + a))/b^6 + 1/30*(12*b^4*x^(5/2) - 15*a*b^3*x^2 +
20*a^2*b^2*x^(3/2) - 30*a^3*b*x + 60*a^4*sqrt(x))/b^5
```

maple [A] time = 0.00, size = 66, normalized size = 0.84

$$\frac{2x^{\frac{5}{2}}}{5b}-\frac{a\,x^2}{2b^2}+\frac{2a^2x^{\frac{3}{2}}}{3b^3}-\frac{2a^5\ln\left(b\sqrt{x}+a\right)}{b^6}-\frac{a^3x}{b^4}+\frac{2a^4\sqrt{x}}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^(1/2)+a),x)
```

```
[Out] -a^3*x/b^4+2/3*a^2*x^(3/2)/b^3-1/2*a/b^2*x^2+2/5*x^(5/2)/b-2*a^5*ln(b*x^(1/
2)+a)/b^6+2*a^4*x^(1/2)/b^5
```

maxima [A] time = 0.86, size = 95, normalized size = 1.20

$$-\frac{2\,a^5\log\left(b\sqrt{x}+a\right)}{b^6}+\frac{2\left(b\sqrt{x}+a\right)^5}{5\,b^6}-\frac{5\left(b\sqrt{x}+a\right)^4a}{2\,b^6}+\frac{20\left(b\sqrt{x}+a\right)^3a^2}{3\,b^6}-\frac{10\left(b\sqrt{x}+a\right)^2a^3}{b^6}+\frac{10\left(b\sqrt{x}+a\right)a^4}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^(1/2)),x, algorithm="maxima")
```

```
[Out] -2*a^5*log(b*sqrt(x) + a)/b^6 + 2/5*(b*sqrt(x) + a)^5/b^6 - 5/2*(b*sqrt(x)
+ a)^4*a/b^6 + 20/3*(b*sqrt(x) + a)^3*a^2/b^6 - 10*(b*sqrt(x) + a)^2*a^3/b^
6 + 10*(b*sqrt(x) + a)*a^4/b^6
```

mupad [B] time = 0.03, size = 65, normalized size = 0.82

$$\frac{2\,x^{5/2}}{5\,b}-\frac{a\,x^2}{2\,b^2}-\frac{a^3\,x}{b^4}-\frac{2\,a^5\,\ln\left(a+b\,\sqrt{x}\right)}{b^6}+\frac{2\,a^2\,x^{3/2}}{3\,b^3}+\frac{2\,a^4\,\sqrt{x}}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^(1/2)),x)
```

```
[Out] (2*x^(5/2))/(5*b) - (a*x^2)/(2*b^2) - (a^3*x)/b^4 - (2*a^5*log(a + b*x^(1/2
)))/b^6 + (2*a^2*x^(3/2))/(3*b^3) + (2*a^4*x^(1/2))/b^5
```

sympy [**A**] time = 0.65, size = 82, normalized size = 1.04

$$\begin{cases} -\dfrac{2a^5 \log\left(\frac{a}{b}+\sqrt{x}\right)}{b^6} + \dfrac{2a^4\sqrt{x}}{b^5} - \dfrac{a^3 x}{b^4} + \dfrac{2a^2 x^{\frac{3}{2}}}{3b^3} - \dfrac{ax^2}{2b^2} + \dfrac{2x^{\frac{5}{2}}}{5b} & \text{for } b \neq 0 \\ \dfrac{x^3}{3a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*x**(1/2)),x)
```

```
[Out] Piecewise((-2*a**5*log(a/b + sqrt(x))/b**6 + 2*a**4*sqrt(x)/b**5 - a**3*x/b
**4 + 2*a**2*x**(3/2)/(3*b**3) - a*x**2/(2*b**2) + 2*x**(5/2)/(5*b), Ne(b,
0)), (x**3/(3*a), True))
```

3.2193 $\int \frac{x}{a+b\sqrt{x}}\,dx$

Optimal. Leaf size=51

$$-\frac{2a^3\log\left(a+b\sqrt{x}\right)}{b^4}+\frac{2a^2\sqrt{x}}{b^3}-\frac{ax}{b^2}+\frac{2x^{3/2}}{3b}$$

```
[Out] -a*x/b^2+2/3*x^(3/2)/b-2*a^3*ln(a+b*x^(1/2))/b^4+2*a^2*x^(1/2)/b^3
```

Rubi [A] time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{2a^2\sqrt{x}}{b^3}-\frac{2a^3\log\left(a+b\sqrt{x}\right)}{b^4}-\frac{ax}{b^2}+\frac{2x^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*Sqrt[x]),x]
```

```
[Out] (2*a^2*Sqrt[x])/b^3 - (a*x)/b^2 + (2*x^(3/2))/(3*b) - (2*a^3*Log[a + b*Sqrt
[x]])/b^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{a+b\sqrt{x}}\,dx &= 2\,\text{Subst}\left(\int \frac{x^3}{a+bx}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int\left(\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right)dx, x, \sqrt{x}\right)\\ &= \frac{2a^2\sqrt{x}}{b^3}-\frac{ax}{b^2}+\frac{2x^{3/2}}{3b}-\frac{2a^3\log\left(a+b\sqrt{x}\right)}{b^4}\end{aligned}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 1.00

$$-\frac{2a^3\log\left(a+b\sqrt{x}\right)}{b^4}+\frac{2a^2\sqrt{x}}{b^3}-\frac{ax}{b^2}+\frac{2x^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*Sqrt[x]),x]
```

```
[Out] (2*a^2*Sqrt[x])/b^3 - (a*x)/b^2 + (2*x^(3/2))/(3*b) - (2*a^3*Log[a + b*Sqrt
[x]])/b^4
```

fricas [A] time = 0.74, size = 43, normalized size = 0.84

$$-\frac{3\,ab^2x+6\,a^3\log\left(b\sqrt{x}+a\right)-2\left(b^3x+3\,a^2b\right)\sqrt{x}}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/3*(3*a*b^2*x + 6*a^3*log(b*sqrt(x) + a) - 2*(b^3*x + 3*a^2*b)*sqrt(x))/b
^4
```

giac [A] time = 0.17, size = 45, normalized size = 0.88

$$-\frac{2\,a^3\log\left(\left|b\sqrt{x}+a\right|\right)}{b^4}+\frac{2\,b^2x^{\frac{3}{2}}-3\,abx+6\,a^2\sqrt{x}}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^(1/2)),x, algorithm="giac")
```

```
[Out] -2*a^3*log(abs(b*sqrt(x) + a))/b^4 + 1/3*(2*b^2*x^(3/2) - 3*a*b*x + 6*a^2*s
qrt(x))/b^3
```

maple [A] time = 0.00, size = 44, normalized size = 0.86

$$\frac{2x^{\frac{3}{2}}}{3b}-\frac{2a^3\ln\left(b\sqrt{x}+a\right)}{b^4}-\frac{ax}{b^2}+\frac{2a^2\sqrt{x}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^(1/2)+a),x)
```

```
[Out] -a/b^2*x+2/3/b*x^(3/2)-2*a^3*ln(b*x^(1/2)+a)/b^4+2*a^2*x^(1/2)/b^3
```

maxima [A] time = 0.77, size = 61, normalized size = 1.20

$$-\frac{2\,a^3\log\left(b\sqrt{x}+a\right)}{b^4}+\frac{2\left(b\sqrt{x}+a\right)^3}{3\,b^4}-\frac{3\left(b\sqrt{x}+a\right)^2a}{b^4}+\frac{6\left(b\sqrt{x}+a\right)a^2}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^(1/2)),x, algorithm="maxima")
```

```
[Out] -2*a^3*log(b*sqrt(x) + a)/b^4 + 2/3*(b*sqrt(x) + a)^3/b^4 - 3*(b*sqrt(x) +
a)^2*a/b^4 + 6*(b*sqrt(x) + a)*a^2/b^4
```

mupad [B] time = 0.04, size = 43, normalized size = 0.84

$$\frac{2\,x^{3/2}}{3\,b}-\frac{2\,a^3\,\ln\left(a+b\,\sqrt{x}\right)}{b^4}+\frac{2\,a^2\,\sqrt{x}}{b^3}-\frac{a\,x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^(1/2)),x)
```

```
[Out] (2*x^(3/2))/(3*b) - (2*a^3*log(a + b*x^(1/2)))/b^4 + (2*a^2*x^(1/2))/b^3 -
(a*x)/b^2
```

sympy [A] time = 0.31, size = 54, normalized size = 1.06

$$\begin{cases}-\frac{2a^3\log\left(\frac{a}{b}+\sqrt{x}\right)}{b^4}+\frac{2a^2\sqrt{x}}{b^3}-\frac{ax}{b^2}+\frac{2x^{\frac{3}{2}}}{3b} & \text{for } b\neq 0\\ \frac{x^2}{2a} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x**(1/2)),x)
```

```
[Out] Piecewise((-2*a**3*log(a/b + sqrt(x))/b**4 + 2*a**2*sqrt(x)/b**3 - a*x/b**2
 + 2*x**(3/2)/(3*b), Ne(b, 0)), (x**2/(2*a), True))
```

3.2194 $\int \frac{1}{a+b\sqrt{x}}\,dx$

Optimal. Leaf size=27

$$\frac{2\sqrt{x}}{b}-\frac{2a\log\left(a+b\sqrt{x}\right)}{b^2}$$

```
[Out] -2*a*ln(a+b*x^(1/2))/b^2+2*x^(1/2)/b
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {190, 43}

$$\frac{2\sqrt{x}}{b}-\frac{2a\log\left(a+b\sqrt{x}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^(-1),x]
```

```
[Out] (2*Sqrt[x])/b - (2*a*Log[a + b*Sqrt[x]])/b^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
 - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
  IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\sqrt{x}}\,dx &= 2\,\text{Subst}\left(\int \frac{x}{a+bx}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx, x, \sqrt{x}\right)\\ &= \frac{2\sqrt{x}}{b}-\frac{2a\log\left(a+b\sqrt{x}\right)}{b^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 27, normalized size = 1.00

$$\frac{2\sqrt{x}}{b}-\frac{2a\log\left(a+b\sqrt{x}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^(-1),x]
```

```
[Out] (2*Sqrt[x])/b - (2*a*Log[a + b*Sqrt[x]])/b^2
```

fricas [A] time = 1.06, size = 22, normalized size = 0.81

$$-\frac{2\left(a\log\left(b\sqrt{x}+a\right)-b\sqrt{x}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/2)),x, algorithm="fricas")
[Out] -2*(a*log(b*sqrt(x) + a) - b*sqrt(x))/b^2
```

giac [A] time = 0.19, size = 24, normalized size = 0.89

$$-\frac{2\,a\log\left(\left|b\sqrt{x}+a\right|\right)}{b^2}+\frac{2\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/2)),x, algorithm="giac")
[Out] -2*a*log(abs(b*sqrt(x) + a))/b^2 + 2*sqrt(x)/b
```

maple [B] time = 0.01, size = 57, normalized size = 2.11

$$-\frac{a\ln\left(b\sqrt{x}+a\right)}{b^2}+\frac{a\ln\left(b\sqrt{x}-a\right)}{b^2}-\frac{a\ln\left(b^2x-a^2\right)}{b^2}+\frac{2\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^(1/2)+a),x)
[Out] 2*x^(1/2)/b-a*ln(b*x^(1/2)+a)/b^2+a/b^2*ln(b*x^(1/2)-a)-a*ln(b^2*x-a^2)/b^2
```

maxima [A] time = 0.92, size = 27, normalized size = 1.00

$$-\frac{2\,a\log\left(b\sqrt{x}+a\right)}{b^2}+\frac{2\left(b\sqrt{x}+a\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/2)),x, algorithm="maxima")
[Out] -2*a*log(b*sqrt(x) + a)/b^2 + 2*(b*sqrt(x) + a)/b^2
```

mupad [B] time = 0.04, size = 23, normalized size = 0.85

$$\frac{2\sqrt{x}}{b}-\frac{2\,a\,\ln\left(a+b\sqrt{x}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^(1/2)),x)
[Out] (2*x^(1/2))/b - (2*a*log(a + b*x^(1/2)))/b^2
```

sympy [A] time = 0.21, size = 27, normalized size = 1.00

$$\begin{cases} -\frac{2a\log\left(\frac{a}{b}+\sqrt{x}\right)}{b^2}+\frac{2\sqrt{x}}{b} & \text{for } b\neq 0 \\ \frac{x}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**(1/2)),x)
[Out] Piecewise((-2*a*log(a/b + sqrt(x))/b**2 + 2*sqrt(x)/b, Ne(b, 0)), (x/a, Tru
e))
```

3.2195 $\int \frac{1}{\left(a+b\sqrt{x}\right)x}\,dx$

Optimal. Leaf size=22

$$\frac{\log(x)}{a}-\frac{2\log\left(a+b\sqrt{x}\right)}{a}$$

```
[Out] ln(x)/a-2*ln(a+b*x^(1/2))/a
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 36, 29, 31}

$$\frac{\log(x)}{a}-\frac{2\log\left(a+b\sqrt{x}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sqrt[x])*x),x]
```

```
[Out] (-2*Log[a + b*Sqrt[x]])/a + Log[x]/a
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+b\sqrt{x}\right)x}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx, x, \sqrt{x}\right)\\ &= \frac{2\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, \sqrt{x}\right)}{a}-\frac{(2b)\,\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, \sqrt{x}\right)}{a}\\ &= -\frac{2\log\left(a+b\sqrt{x}\right)}{a}+\frac{\log(x)}{a}\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$\frac{\log(x)}{a}-\frac{2\log\left(a+b\sqrt{x}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sqrt[x])*x),x]
[Out] (-2*Log[a + b*Sqrt[x]])/a + Log[x]/a
```

fricas [A] time = 1.02, size = 20, normalized size = 0.91

$$-\frac{2\left(\log\left(b\sqrt{x}+a\right)-\log\left(\sqrt{x}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^(1/2)),x, algorithm="fricas")
[Out] -2*(log(b*sqrt(x) + a) - log(sqrt(x)))/a
```

giac [A] time = 0.17, size = 22, normalized size = 1.00

$$-\frac{2\,\log\left(\left|b\sqrt{x}+a\right|\right)}{a}+\frac{\log\left(|x|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^(1/2)),x, algorithm="giac")
[Out] -2*log(abs(b*sqrt(x) + a))/a + log(abs(x))/a
```

maple [A] time = 0.01, size = 21, normalized size = 0.95

$$\frac{\ln(x)}{a}-\frac{2\ln\left(b\sqrt{x}+a\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^(1/2)+a),x)
[Out] 1/a*ln(x)-2*ln(b*x^(1/2)+a)/a
```

maxima [A] time = 0.89, size = 20, normalized size = 0.91

$$-\frac{2\,\log\left(b\sqrt{x}+a\right)}{a}+\frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^(1/2)),x, algorithm="maxima")
[Out] -2*log(b*sqrt(x) + a)/a + log(x)/a
```

mupad [B] time = 0.04, size = 17, normalized size = 0.77

$$-\frac{4\,\mathrm{atanh}\left(\frac{2\,b\,\sqrt{x}}{a}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^(1/2))),x)
[Out] -(4*atanh((2*b*x^(1/2))/a + 1))/a
```

sympy [**A**] time = 0.38, size = 37, normalized size = 1.68

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2}{b\sqrt{x}} & \text{for } a = 0 \\ \frac{\log(x)}{a} & \text{for } b = 0 \\ \frac{\log(x)}{a} - \frac{2\log\left(\frac{a}{b}+\sqrt{x}\right)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x**(1/2)),x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (-2/(b*sqrt(x)), Eq(a, 0)), (
log(x)/a, Eq(b, 0)), (log(x)/a - 2*log(a/b + sqrt(x))/a, True))
```

3.2196 $\int \frac{1}{\left(a+b\sqrt{x}\right)x^2}\,dx$

Optimal. Leaf size=47

$$-\frac{2b^2\log\left(a+b\sqrt{x}\right)}{a^3}+\frac{b^2\log(x)}{a^3}+\frac{2b}{a^2\sqrt{x}}-\frac{1}{ax}$$

```
[Out] -1/a/x+b^2*ln(x)/a^3-2*b^2*ln(a+b*x^(1/2))/a^3+2*b/a^2/x^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 44}

$$-\frac{2b^2\log\left(a+b\sqrt{x}\right)}{a^3}+\frac{b^2\log(x)}{a^3}+\frac{2b}{a^2\sqrt{x}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sqrt[x])*x^2),x]
```

```
[Out] -(1/(a*x)) + (2*b)/(a^2*Sqrt[x]) - (2*b^2*Log[a + b*Sqrt[x]])/a^3 + (b^2*Lo
g[x])/a^3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+b\sqrt{x}\right)x^2}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{x^3(a+bx)}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)dx, x, \sqrt{x}\right)\\ &= -\frac{1}{ax}+\frac{2b}{a^2\sqrt{x}}-\frac{2b^2\log\left(a+b\sqrt{x}\right)}{a^3}+\frac{b^2\log(x)}{a^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 44, normalized size = 0.94

$$\frac{-2b^2x\log\left(a+b\sqrt{x}\right)-a\left(a-2b\sqrt{x}\right)+b^2x\log(x)}{a^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sqrt[x])*x^2),x]
```

```
[Out] (-(a*(a - 2*b*Sqrt[x])) - 2*b^2*x*Log[a + b*Sqrt[x]] + b^2*x*Log[x])/(a^3*x
)
```

fricas [A] time = 1.13, size = 43, normalized size = 0.91

$$-\frac{2\,b^2 x \log\left(b\sqrt{x}+a\right)-2\,b^2 x\log\left(\sqrt{x}\right)-2\,ab\sqrt{x}+a^2}{a^3 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^(1/2)),x, algorithm="fricas")
```

```
[Out] -(2*b^2*x*log(b*sqrt(x) + a) - 2*b^2*x*log(sqrt(x)) - 2*a*b*sqrt(x) + a^2)/
(a^3*x)
```

giac [A] time = 0.19, size = 48, normalized size = 1.02

$$-\frac{2\,b^2\log\left(\left|b\sqrt{x}+a\right|\right)}{a^3}+\frac{b^2\log\left(|x|\right)}{a^3}+\frac{2\,ab\sqrt{x}-a^2}{a^3 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^(1/2)),x, algorithm="giac")
```

```
[Out] -2*b^2*log(abs(b*sqrt(x) + a))/a^3 + b^2*log(abs(x))/a^3 + (2*a*b*sqrt(x) -
 a^2)/(a^3*x)
```

maple [A] time = 0.01, size = 44, normalized size = 0.94

$$\frac{b^2\ln(x)}{a^3}-\frac{2b^2\ln\left(b\sqrt{x}+a\right)}{a^3}+\frac{2b}{a^2\sqrt{x}}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^(1/2)+a),x)
```

```
[Out] -1/a/x+1/a^3*b^2*ln(x)-2*b^2*ln(b*x^(1/2)+a)/a^3+2*b/a^2/x^(1/2)
```

maxima [A] time = 0.93, size = 43, normalized size = 0.91

$$-\frac{2\,b^2\log\left(b\sqrt{x}+a\right)}{a^3}+\frac{b^2\log(x)}{a^3}+\frac{2\,b\sqrt{x}-a}{a^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^(1/2)),x, algorithm="maxima")
```

```
[Out] -2*b^2*log(b*sqrt(x) + a)/a^3 + b^2*log(x)/a^3 + (2*b*sqrt(x) - a)/(a^2*x)
```

mupad [B] time = 0.06, size = 39, normalized size = 0.83

$$-\frac{\frac{1}{a}-\frac{2\,b\,\sqrt{x}}{a^2}}{x}-\frac{4\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,\sqrt{x}}{a}+1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^(1/2))),x)
```

```
[Out] - (1/a - (2*b*x^(1/2))/a^2)/x - (4*b^2*atanh((2*b*x^(1/2))/a + 1))/a^3
```

sympy [**A**] time = 1.78, size = 68, normalized size = 1.45

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2}{3bx^{\frac{3}{2}}} & \text{for } a = 0 \\ -\frac{1}{ax} & \text{for } b = 0 \\ -\frac{1}{ax} + \frac{2b}{a^2\sqrt{x}} + \frac{b^2\log(x)}{a^3} - \frac{2b^2\log\left(\frac{a}{b}+\sqrt{x}\right)}{a^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*x**(1/2)),x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (-2/(3*b*x**(3/2)), Eq(a, 0)
), (-1/(a*x), Eq(b, 0)), (-1/(a*x) + 2*b/(a**2*sqrt(x)) + b**2*log(x)/a**3
- 2*b**2*log(a/b + sqrt(x))/a**3, True))
```

3.2197 $\int \frac{1}{\left(a+b\sqrt{x}\right)x^3}\,dx$

Optimal. Leaf size=75

$$-\frac{2b^4\log\left(a+b\sqrt{x}\right)}{a^5}+\frac{b^4\log(x)}{a^5}+\frac{2b^3}{a^4\sqrt{x}}-\frac{b^2}{a^3x}+\frac{2b}{3a^2x^{3/2}}-\frac{1}{2ax^2}$$

```
[Out] -1/2/a/x^2+2/3*b/a^2/x^(3/2)-b^2/a^3/x+b^4*ln(x)/a^5-2*b^4*ln(a+b*x^(1/2))/
a^5+2*b^3/a^4/x^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 44}

$$\frac{2b^3}{a^4\sqrt{x}}-\frac{b^2}{a^3x}-\frac{2b^4\log\left(a+b\sqrt{x}\right)}{a^5}+\frac{b^4\log(x)}{a^5}+\frac{2b}{3a^2x^{3/2}}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sqrt[x])*x^3),x]
```

```
[Out] -1/(2*a*x^2) + (2*b)/(3*a^2*x^(3/2)) - b^2/(a^3*x) + (2*b^3)/(a^4*Sqrt[x])
- (2*b^4*Log[a + b*Sqrt[x]])/a^5 + (b^4*Log[x])/a^5
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+b\sqrt{x}\right)x^3}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{x^5(a+bx)}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(\frac{1}{ax^5}-\frac{b}{a^2x^4}+\frac{b^2}{a^3x^3}-\frac{b^3}{a^4x^2}+\frac{b^4}{a^5x}-\frac{b^5}{a^5(a+bx)}\right)dx, x, \sqrt{x}\right)\\ &= -\frac{1}{2ax^2}+\frac{2b}{3a^2x^{3/2}}-\frac{b^2}{a^3x}+\frac{2b^3}{a^4\sqrt{x}}-\frac{2b^4\log\left(a+b\sqrt{x}\right)}{a^5}+\frac{b^4\log(x)}{a^5}\end{aligned}$$

Mathematica [A] time = 0.07, size = 69, normalized size = 0.92

$$\frac{\frac{a\left(-3a^3+4a^2b\sqrt{x}-6ab^2x+12b^3x^{3/2}\right)}{x^2}-12b^4\log\left(a+b\sqrt{x}\right)+6b^4\log(x)}{6a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sqrt[x])*x^3),x]
```

```
[Out] ((a*(-3*a^3 + 4*a^2*b*Sqrt[x] - 6*a*b^2*x + 12*b^3*x^(3/2)))/x^2 - 12*b^4*L
og[a + b*Sqrt[x]] + 6*b^4*Log[x])/(6*a^5)
```

fricas [A] time = 0.75, size = 69, normalized size = 0.92

$$-\frac{12\,b^4x^2\log\left(b\sqrt{x}+a\right)-12\,b^4x^2\log\left(\sqrt{x}\right)+6\,a^2b^2x+3\,a^4-4\left(3\,ab^3x+a^3b\right)\sqrt{x}}{6\,a^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/6*(12*b^4*x^2*log(b*sqrt(x) + a) - 12*b^4*x^2*log(sqrt(x)) + 6*a^2*b^2*x
 + 3*a^4 - 4*(3*a*b^3*x + a^3*b)*sqrt(x))/(a^5*x^2)
```

giac [A] time = 0.16, size = 69, normalized size = 0.92

$$-\frac{2\,b^4\log\left(\left|b\sqrt{x}+a\right|\right)}{a^5}+\frac{b^4\log\left(|x|\right)}{a^5}+\frac{12\,ab^3x^{\frac{3}{2}}-6\,a^2b^2x+4\,a^3b\sqrt{x}-3\,a^4}{6\,a^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^(1/2)),x, algorithm="giac")
```

```
[Out] -2*b^4*log(abs(b*sqrt(x) + a))/a^5 + b^4*log(abs(x))/a^5 + 1/6*(12*a*b^3*x^
(3/2) - 6*a^2*b^2*x + 4*a^3*b*sqrt(x) - 3*a^4)/(a^5*x^2)
```

maple [A] time = 0.01, size = 66, normalized size = 0.88

$$\frac{b^4\ln(x)}{a^5}-\frac{2b^4\ln\left(b\sqrt{x}+a\right)}{a^5}+\frac{2b^3}{a^4\sqrt{x}}-\frac{b^2}{a^3x}+\frac{2b}{3a^2x^{\frac{3}{2}}}-\frac{1}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^(1/2)+a),x)
```

```
[Out] -1/2/a/x^2+2/3*b/a^2/x^(3/2)-b^2/a^3/x+b^4*ln(x)/a^5-2*b^4*ln(b*x^(1/2)+a)/
a^5+2*b^3/a^4/x^(1/2)
```

maxima [A] time = 0.85, size = 64, normalized size = 0.85

$$-\frac{2\,b^4\log\left(b\sqrt{x}+a\right)}{a^5}+\frac{b^4\log(x)}{a^5}+\frac{12\,b^3x^{\frac{3}{2}}-6\,ab^2x+4\,a^2b\sqrt{x}-3\,a^3}{6\,a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^(1/2)),x, algorithm="maxima")
```

```
[Out] -2*b^4*log(b*sqrt(x) + a)/a^5 + b^4*log(x)/a^5 + 1/6*(12*b^3*x^(3/2) - 6*a*
b^2*x + 4*a^2*b*sqrt(x) - 3*a^3)/(a^4*x^2)
```

mupad [B] time = 0.07, size = 60, normalized size = 0.80

$$-\frac{\frac{1}{2a}-\frac{2\,b\,\sqrt{x}}{3\,a^2}+\frac{b^2\,x}{a^3}-\frac{2\,b^3\,x^{3/2}}{a^4}}{x^2}-\frac{4\,b^4\,\mathrm{atanh}\left(\frac{2\,b\,\sqrt{x}}{a}+1\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^(1/2))),x)
```

```
[Out] - (1/(2*a) - (2*b*x^(1/2))/(3*a^2) + (b^2*x)/a^3 - (2*b^3*x^(3/2))/a^4)/x^2
 - (4*b^4*atanh((2*b*x^(1/2))/a + 1))/a^5
```

sympy [**A**] time = 4.53, size = 99, normalized size = 1.32

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2}{5bx^{\frac{5}{2}}} & \text{for } a = 0 \\ -\frac{1}{2ax^2} & \text{for } b = 0 \\ -\frac{1}{2ax^2} + \frac{2b}{3a^2x^{\frac{3}{2}}} - \frac{b^2}{a^3x} + \frac{2b^3}{a^4\sqrt{x}} + \frac{b^4\log(x)}{a^5} - \frac{2b^4\log\left(\frac{a}{b}+\sqrt{x}\right)}{a^5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*x**(1/2)),x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(5*b*x**(5/2)), Eq(a, 0)
), (-1/(2*a*x**2), Eq(b, 0)), (-1/(2*a*x**2) + 2*b/(3*a**2*x**(3/2)) - b**2
/(a**3*x) + 2*b**3/(a**4*sqrt(x)) + b**4*log(x)/a**5 - 2*b**4*log(a/b + sqr
t(x))/a**5, True))
```

3.2198 $\int \frac{1}{\left(a+b\sqrt{x}\right)x^4}\,dx$

Optimal. Leaf size=103

$$-\frac{2b^6\log\left(a+b\sqrt{x}\right)}{a^7}+\frac{b^6\log(x)}{a^7}+\frac{2b^5}{a^6\sqrt{x}}-\frac{b^4}{a^5x}+\frac{2b^3}{3a^4x^{3/2}}-\frac{b^2}{2a^3x^2}+\frac{2b}{5a^2x^{5/2}}-\frac{1}{3ax^3}$$

```
[Out] -1/3/a/x^3+2/5*b/a^2/x^(5/2)-1/2*b^2/a^3/x^2+2/3*b^3/a^4/x^(3/2)-b^4/a^5/x+
b^6*ln(x)/a^7-2*b^6*ln(a+b*x^(1/2))/a^7+2*b^5/a^6/x^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 44}

$$\frac{2b^3}{3a^4x^{3/2}}-\frac{b^2}{2a^3x^2}+\frac{2b^5}{a^6\sqrt{x}}-\frac{b^4}{a^5x}-\frac{2b^6\log\left(a+b\sqrt{x}\right)}{a^7}+\frac{b^6\log(x)}{a^7}+\frac{2b}{5a^2x^{5/2}}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sqrt[x])*x^4),x]
```

```
[Out] -1/(3*a*x^3) + (2*b)/(5*a^2*x^(5/2)) - b^2/(2*a^3*x^2) + (2*b^3)/(3*a^4*x^(
3/2)) - b^4/(a^5*x) + (2*b^5)/(a^6*Sqrt[x]) - (2*b^6*Log[a + b*Sqrt[x]])/a^
7 + (b^6*Log[x])/a^7
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+b\sqrt{x}\right)x^4}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{x^7(a+bx)}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(\frac{1}{ax^7}-\frac{b}{a^2x^6}+\frac{b^2}{a^3x^5}-\frac{b^3}{a^4x^4}+\frac{b^4}{a^5x^3}-\frac{b^5}{a^6x^2}+\frac{b^6}{a^7x}-\frac{b^7}{a^7(a+bx)}\right)dx, x, \sqrt{x}\right)\\ &= -\frac{1}{3ax^3}+\frac{2b}{5a^2x^{5/2}}-\frac{b^2}{2a^3x^2}+\frac{2b^3}{3a^4x^{3/2}}-\frac{b^4}{a^5x}+\frac{2b^5}{a^6\sqrt{x}}-\frac{2b^6\log\left(a+b\sqrt{x}\right)}{a^7}+\frac{b^6\log(x)}{a^7}\end{aligned}$$

Mathematica [A] time = 0.09, size = 93, normalized size = 0.90

$$\frac{\frac{a\left(-10a^5+12a^4b\sqrt{x}-15a^3b^2x+20a^2b^3x^{3/2}-30ab^4x^2+60b^5x^{5/2}\right)}{x^3}-60b^6\log\left(a+b\sqrt{x}\right)+30b^6\log(x)}{30a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sqrt[x])*x^4),x]
```

```
[Out] ((a*(-10*a^5 + 12*a^4*b*Sqrt[x] - 15*a^3*b^2*x + 20*a^2*b^3*x^(3/2) - 30*a*
b^4*x^2 + 60*b^5*x^(5/2)))/x^3 - 60*b^6*Log[a + b*Sqrt[x]] + 30*b^6*Log[x])
/(30*a^7)
```

fricas [A] time = 1.08, size = 92, normalized size = 0.89

$$-\frac{60\,b^6x^3\log\left(b\sqrt{x}+a\right)-60\,b^6x^3\log\left(\sqrt{x}\right)+30\,a^2b^4x^2+15\,a^4b^2x+10\,a^6-4\left(15\,ab^5x^2+5\,a^3b^3x+3\,a^5b\right)\sqrt{x}}{30\,a^7x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*x^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/30*(60*b^6*x^3*log(b*sqrt(x) + a) - 60*b^6*x^3*log(sqrt(x)) + 30*a^2*b^4
*x^2 + 15*a^4*b^2*x + 10*a^6 - 4*(15*a*b^5*x^2 + 5*a^3*b^3*x + 3*a^5*b)*sqr
t(x))/(a^7*x^3)
```

giac [A] time = 0.15, size = 91, normalized size = 0.88

$$-\frac{2\,b^6\log\left(\left|b\sqrt{x}+a\right|\right)}{a^7}+\frac{b^6\log\left(|x|\right)}{a^7}+\frac{60\,ab^5x^{\frac{5}{2}}-30\,a^2b^4x^2+20\,a^3b^3x^{\frac{3}{2}}-15\,a^4b^2x+12\,a^5b\sqrt{x}-10\,a^6}{30\,a^7x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*x^(1/2)),x, algorithm="giac")
```

```
[Out] -2*b^6*log(abs(b*sqrt(x) + a))/a^7 + b^6*log(abs(x))/a^7 + 1/30*(60*a*b^5*x
^(5/2) - 30*a^2*b^4*x^2 + 20*a^3*b^3*x^(3/2) - 15*a^4*b^2*x + 12*a^5*b*sqrt
(x) - 10*a^6)/(a^7*x^3)
```

maple [A] time = 0.01, size = 88, normalized size = 0.85

$$\frac{b^6\ln(x)}{a^7}-\frac{2b^6\ln\left(b\sqrt{x}+a\right)}{a^7}+\frac{2b^5}{a^6\sqrt{x}}-\frac{b^4}{a^5x}+\frac{2b^3}{3a^4x^{\frac{3}{2}}}-\frac{b^2}{2a^3x^2}+\frac{2b}{5a^2x^{\frac{5}{2}}}-\frac{1}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^(1/2)+a),x)
```

```
[Out] -1/3/a/x^3+2/5*b/a^2/x^(5/2)-1/2*b^2/x^2/a^3+2/3*b^3/a^4/x^(3/2)-b^4/a^5/x+
b^6*ln(x)/a^7-2*b^6*ln(b*x^(1/2)+a)/a^7+2*b^5/a^6/x^(1/2)
```

maxima [A] time = 0.92, size = 86, normalized size = 0.83

$$-\frac{2\,b^6\log\left(b\sqrt{x}+a\right)}{a^7}+\frac{b^6\log(x)}{a^7}+\frac{60\,b^5x^{\frac{5}{2}}-30\,ab^4x^2+20\,a^2b^3x^{\frac{3}{2}}-15\,a^3b^2x+12\,a^4b\sqrt{x}-10\,a^5}{30\,a^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*x^(1/2)),x, algorithm="maxima")
```

```
[Out] -2*b^6*log(b*sqrt(x) + a)/a^7 + b^6*log(x)/a^7 + 1/30*(60*b^5*x^(5/2) - 30*
a*b^4*x^2 + 20*a^2*b^3*x^(3/2) - 15*a^3*b^2*x + 12*a^4*b*sqrt(x) - 10*a^5)/
(a^6*x^3)
```

mupad [B] time = 1.11, size = 82, normalized size = 0.80

$$-\frac{\frac{1}{3\,a}-\frac{2\,b\,\sqrt{x}}{5\,a^2}+\frac{b^2\,x}{2\,a^3}+\frac{b^4\,x^2}{a^5}-\frac{2\,b^3\,x^{3/2}}{3\,a^4}-\frac{2\,b^5\,x^{5/2}}{a^6}}{x^3}-\frac{4\,b^6\,\mathrm{atanh}\left(\frac{2\,b\,\sqrt{x}}{a}+1\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^(1/2))),x)
```

```
[Out] - (1/(3*a) - (2*b*x^(1/2))/(5*a^2) + (b^2*x)/(2*a^3) + (b^4*x^2)/a^5 - (2*b
^3*x^(3/2))/(3*a^4) - (2*b^5*x^(5/2))/a^6)/x^3 - (4*b^6*atanh((2*b*x^(1/2))
/a + 1))/a^7
```

sympy [A] time = 10.19, size = 126, normalized size = 1.22

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2}{7bx^{\frac{7}{2}}} & \text{for } a = 0 \\ -\frac{1}{3ax^3} & \text{for } b = 0 \\ -\frac{1}{3ax^3} + \frac{2b}{5a^2x^{\frac{5}{2}}} - \frac{b^2}{2a^3x^2} + \frac{2b^3}{3a^4x^{\frac{3}{2}}} - \frac{b^4}{a^5x} + \frac{2b^5}{a^6\sqrt{x}} + \frac{b^6\log(x)}{a^7} - \frac{2b^6\log\left(\frac{a}{b}+\sqrt{x}\right)}{a^7} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a+b*x**(1/2)),x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(7*b*x**(7/2)), Eq(a, 0)
), (-1/(3*a*x**3), Eq(b, 0)), (-1/(3*a*x**3) + 2*b/(5*a**2*x**(5/2)) - b**2
/(2*a**3*x**2) + 2*b**3/(3*a**4*x**(3/2)) - b**4/(a**5*x) + 2*b**5/(a**6*sq
rt(x)) + b**6*log(x)/a**7 - 2*b**6*log(a/b + sqrt(x))/a**7, True))
```

3.2199 $\int \frac{x^3}{\left(a+b\sqrt{x}\right)^2}\,dx$

Optimal. Leaf size=111

$$\frac{2a^7}{b^8\left(a+b\sqrt{x}\right)}+\frac{14a^6\log\left(a+b\sqrt{x}\right)}{b^8}-\frac{12a^5\sqrt{x}}{b^7}+\frac{5a^4x}{b^6}-\frac{8a^3x^{3/2}}{3b^5}+\frac{3a^2x^2}{2b^4}-\frac{4ax^{5/2}}{5b^3}+\frac{x^3}{3b^2}$$

```
[Out] 5*a^4*x/b^6-8/3*a^3*x^(3/2)/b^5+3/2*a^2*x^2/b^4-4/5*a*x^(5/2)/b^3+1/3*x^3/b
^2+14*a^6*ln(a+b*x^(1/2))/b^8-12*a^5*x^(1/2)/b^7+2*a^7/b^8/(a+b*x^(1/2))
```

Rubi [A] time = 0.08, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{8a^3x^{3/2}}{3b^5}+\frac{3a^2x^2}{2b^4}+\frac{2a^7}{b^8\left(a+b\sqrt{x}\right)}-\frac{12a^5\sqrt{x}}{b^7}+\frac{5a^4x}{b^6}+\frac{14a^6\log\left(a+b\sqrt{x}\right)}{b^8}-\frac{4ax^{5/2}}{5b^3}+\frac{x^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*Sqrt[x])^2,x]
```

```
[Out] (2*a^7)/(b^8*(a + b*Sqrt[x])) - (12*a^5*Sqrt[x])/b^7 + (5*a^4*x)/b^6 - (8*a
^3*x^(3/2))/(3*b^5) + (3*a^2*x^2)/(2*b^4) - (4*a*x^(5/2))/(5*b^3) + x^3/(3*
b^2) + (14*a^6*Log[a + b*Sqrt[x]])/b^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(a+b\sqrt{x}\right)^2}\,dx &= 2\,\text{Subst}\left(\int \frac{x^7}{(a+bx)^2}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int\left(-\frac{6a^5}{b^7}+\frac{5a^4x}{b^6}-\frac{4a^3x^2}{b^5}+\frac{3a^2x^3}{b^4}-\frac{2ax^4}{b^3}+\frac{x^5}{b^2}-\frac{a^7}{b^7(a+bx)^2}+\frac{7a^6}{b^7(a+bx)}\right)dx, x\right)\\ &= \frac{2a^7}{b^8\left(a+b\sqrt{x}\right)}-\frac{12a^5\sqrt{x}}{b^7}+\frac{5a^4x}{b^6}-\frac{8a^3x^{3/2}}{3b^5}+\frac{3a^2x^2}{2b^4}-\frac{4ax^{5/2}}{5b^3}+\frac{x^3}{3b^2}+\frac{14a^6\log\left(a+b\sqrt{x}\right)}{b^8}\end{aligned}$$

Mathematica [A] time = 0.11, size = 102, normalized size = 0.92

$$\frac{\frac{60a^7}{a+b\sqrt{x}}+420a^6\log\left(a+b\sqrt{x}\right)-360a^5b\sqrt{x}+150a^4b^2x-80a^3b^3x^{3/2}+45a^2b^4x^2-24ab^5x^{5/2}+10b^6x^3}{30b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*Sqrt[x])^2,x]
```

```
[Out] ((60*a^7)/(a + b*Sqrt[x]) - 360*a^5*b*Sqrt[x] + 150*a^4*b^2*x - 80*a^3*b^3*
x^(3/2) + 45*a^2*b^4*x^2 - 24*a*b^5*x^(5/2) + 10*b^6*x^3 + 420*a^6*Log[a +
b*Sqrt[x]])/(30*b^8)
```

fricas [A] time = 0.96, size = 128, normalized size = 1.15

$$\frac{10\,b^8x^4+35\,a^2b^6x^3+105\,a^4b^4x^2-150\,a^6b^2x-60\,a^8+420\left(a^6b^2x-a^8\right)\log\left(b\sqrt{x}+a\right)-4\left(6\,ab^7x^3+14\,a^3b^5\right.}{30\left(b^{10}x-a^2b^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/30*(10*b^8*x^4 + 35*a^2*b^6*x^3 + 105*a^4*b^4*x^2 - 150*a^6*b^2*x - 60*a^
8 + 420*(a^6*b^2*x - a^8)*log(b*sqrt(x) + a) - 4*(6*a*b^7*x^3 + 14*a^3*b^5*
x^2 + 70*a^5*b^3*x - 105*a^7*b)*sqrt(x))/(b^10*x - a^2*b^8)
```

giac [A] time = 0.21, size = 100, normalized size = 0.90

$$\frac{14\,a^6\log\left(\left|b\sqrt{x}+a\right|\right)}{b^8}+\frac{2\,a^7}{\left(b\sqrt{x}+a\right)b^8}+\frac{10\,b^{10}x^3-24\,ab^9x^{\frac{5}{2}}+45\,a^2b^8x^2-80\,a^3b^7x^{\frac{3}{2}}+150\,a^4b^6x-360\,a^5b^5\sqrt{x}}{30\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^(1/2))^2,x, algorithm="giac")
```

```
[Out] 14*a^6*log(abs(b*sqrt(x) + a))/b^8 + 2*a^7/((b*sqrt(x) + a)*b^8) + 1/30*(10
*b^10*x^3 - 24*a*b^9*x^(5/2) + 45*a^2*b^8*x^2 - 80*a^3*b^7*x^(3/2) + 150*a^
4*b^6*x - 360*a^5*b^5*sqrt(x))/b^12
```

maple [A] time = 0.01, size = 94, normalized size = 0.85

$$\frac{x^3}{3b^2}-\frac{4a\,x^{\frac{5}{2}}}{5b^3}+\frac{3a^2x^2}{2b^4}-\frac{8a^3x^{\frac{3}{2}}}{3b^5}+\frac{2a^7}{\left(b\sqrt{x}+a\right)b^8}+\frac{14a^6\ln\left(b\sqrt{x}+a\right)}{b^8}+\frac{5a^4x}{b^6}-\frac{12a^5\sqrt{x}}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^(1/2)+a)^2,x)
```

```
[Out] 5*a^4*x/b^6-8/3*a^3*x^(3/2)/b^5+3/2*a^2*x^2/b^4-4/5*a*x^(5/2)/b^3+1/3/b^2*x
^3+14*a^6*ln(b*x^(1/2)+a)/b^8-12*a^5*x^(1/2)/b^7+2*a^7/b^8/(b*x^(1/2)+a)
```

maxima [A] time = 0.91, size = 129, normalized size = 1.16

$$\frac{14\,a^6\log\left(b\sqrt{x}+a\right)}{b^8}+\frac{\left(b\sqrt{x}+a\right)^6}{3\,b^8}-\frac{14\left(b\sqrt{x}+a\right)^5a}{5\,b^8}+\frac{21\left(b\sqrt{x}+a\right)^4a^2}{2\,b^8}-\frac{70\left(b\sqrt{x}+a\right)^3a^3}{3\,b^8}+\frac{35\left(b\sqrt{x}+a\right)^2a^4}{b^8}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 14*a^6*log(b*sqrt(x) + a)/b^8 + 1/3*(b*sqrt(x) + a)^6/b^8 - 14/5*(b*sqrt(x)
 + a)^5*a/b^8 + 21/2*(b*sqrt(x) + a)^4*a^2/b^8 - 70/3*(b*sqrt(x) + a)^3*a^3
/b^8 + 35*(b*sqrt(x) + a)^2*a^4/b^8 - 42*(b*sqrt(x) + a)*a^5/b^8 + 2*a^7/((
b*sqrt(x) + a)*b^8)
```

mupad [B] time = 1.11, size = 99, normalized size = 0.89

$$\frac{x^3}{3\,b^2}+\frac{2\,a^7}{b\left(a\,b^7+b^8\,\sqrt{x}\right)}-\frac{4\,a\,x^{5/2}}{5\,b^3}+\frac{5\,a^4\,x}{b^6}+\frac{14\,a^6\,\ln\left(a+b\,\sqrt{x}\right)}{b^8}+\frac{3\,a^2\,x^2}{2\,b^4}-\frac{8\,a^3\,x^{3/2}}{3\,b^5}-\frac{12\,a^5\,\sqrt{x}}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^(1/2))^2,x)
```

```
[Out] x^3/(3*b^2) + (2*a^7)/(b*(a*b^7 + b^8*x^(1/2))) - (4*a*x^(5/2))/(5*b^3) + (
5*a^4*x)/b^6 + (14*a^6*log(a + b*x^(1/2)))/b^8 + (3*a^2*x^2)/(2*b^4) - (8*a
^3*x^(3/2))/(3*b^5) - (12*a^5*x^(1/2))/b^7
```

sympy [A] time = 2.61, size = 272, normalized size = 2.45

$$\begin{cases} \frac{420a^7 \log\left(\frac{a}{b}+\sqrt{x}\right)}{30ab^8+30b^9\sqrt{x}} + \frac{420a^7}{30ab^8+30b^9\sqrt{x}} + \frac{420a^6b\sqrt{x}\log\left(\frac{a}{b}+\sqrt{x}\right)}{30ab^8+30b^9\sqrt{x}} - \frac{210a^5b^2x}{30ab^8+30b^9\sqrt{x}} + \frac{70a^4b^3x^{\frac{3}{2}}}{30ab^8+30b^9\sqrt{x}} - \frac{35a^3b^4x^2}{30ab^8+30b^9\sqrt{x}} + \frac{21a^2b^5x^{\frac{5}{2}}}{30ab^8+30b^9\sqrt{x}} \\ \frac{x^4}{4a^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*x**(1/2))**2,x)
```

```
[Out] Piecewise((420*a**7*log(a/b + sqrt(x))/(30*a*b**8 + 30*b**9*sqrt(x)) + 420*
a**7/(30*a*b**8 + 30*b**9*sqrt(x)) + 420*a**6*b*sqrt(x)*log(a/b + sqrt(x))/
(30*a*b**8 + 30*b**9*sqrt(x)) - 210*a**5*b**2*x/(30*a*b**8 + 30*b**9*sqrt(x
)) + 70*a**4*b**3*x**(3/2)/(30*a*b**8 + 30*b**9*sqrt(x)) - 35*a**3*b**4*x**
2/(30*a*b**8 + 30*b**9*sqrt(x)) + 21*a**2*b**5*x**(5/2)/(30*a*b**8 + 30*b**
9*sqrt(x)) - 14*a*b**6*x**3/(30*a*b**8 + 30*b**9*sqrt(x)) + 10*b**7*x**(7/2
)/(30*a*b**8 + 30*b**9*sqrt(x)), Ne(b, 0)), (x**4/(4*a**2), True))
```

3.2200 $\int \frac{x^2}{\left(a+b\sqrt{x}\right)^2}\,dx$

Optimal. Leaf size=83

$$\frac{2a^5}{b^6\left(a+b\sqrt{x}\right)}+\frac{10a^4\log\left(a+b\sqrt{x}\right)}{b^6}-\frac{8a^3\sqrt{x}}{b^5}+\frac{3a^2x}{b^4}-\frac{4ax^{3/2}}{3b^3}+\frac{x^2}{2b^2}$$

```
[Out] 3*a^2*x/b^4-4/3*a*x^(3/2)/b^3+1/2*x^2/b^2+10*a^4*ln(a+b*x^(1/2))/b^6-8*a^3*
x^(1/2)/b^5+2*a^5/b^6/(a+b*x^(1/2))
```

Rubi [A] time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{2a^5}{b^6\left(a+b\sqrt{x}\right)}-\frac{8a^3\sqrt{x}}{b^5}+\frac{3a^2x}{b^4}+\frac{10a^4\log\left(a+b\sqrt{x}\right)}{b^6}-\frac{4ax^{3/2}}{3b^3}+\frac{x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*Sqrt[x])^2,x]
```

```
[Out] (2*a^5)/(b^6*(a + b*Sqrt[x])) - (8*a^3*Sqrt[x])/b^5 + (3*a^2*x)/b^4 - (4*a*
x^(3/2))/(3*b^3) + x^2/(2*b^2) + (10*a^4*Log[a + b*Sqrt[x]])/b^6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a+b\sqrt{x}\right)^2}\,dx &= 2\,\text{Subst}\left(\int \frac{x^5}{(a+bx)^2}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int\left(-\frac{4a^3}{b^5}+\frac{3a^2x}{b^4}-\frac{2ax^2}{b^3}+\frac{x^3}{b^2}-\frac{a^5}{b^5(a+bx)^2}+\frac{5a^4}{b^5(a+bx)}\right)dx, x, \sqrt{x}\right)\\ &= \frac{2a^5}{b^6\left(a+b\sqrt{x}\right)}-\frac{8a^3\sqrt{x}}{b^5}+\frac{3a^2x}{b^4}-\frac{4ax^{3/2}}{3b^3}+\frac{x^2}{2b^2}+\frac{10a^4\log\left(a+b\sqrt{x}\right)}{b^6}\end{aligned}$$

Mathematica [A] time = 0.06, size = 78, normalized size = 0.94

$$\frac{\frac{12a^5}{a+b\sqrt{x}}+60a^4\log\left(a+b\sqrt{x}\right)-48a^3b\sqrt{x}+18a^2b^2x-8ab^3x^{3/2}+3b^4x^2}{6b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*Sqrt[x])^2,x]
```

```
[Out] ((12*a^5)/(a + b*Sqrt[x]) - 48*a^3*b*Sqrt[x] + 18*a^2*b^2*x - 8*a*b^3*x^(3/
2) + 3*b^4*x^2 + 60*a^4*Log[a + b*Sqrt[x]])/(6*b^6)
```

fricas [A] time = 1.08, size = 106, normalized size = 1.28

$$\frac{3\,b^6x^3+15\,a^2b^4x^2-18\,a^4b^2x-12\,a^6+60\left(a^4b^2x-a^6\right)\log\left(b\sqrt{x}+a\right)-4\left(2\,ab^5x^2+10\,a^3b^3x-15\,a^5b\right)\sqrt{x}}{6\left(b^8x-a^2b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^6*x^3 + 15*a^2*b^4*x^2 - 18*a^4*b^2*x - 12*a^6 + 60*(a^4*b^2*x - a
^6)*log(b*sqrt(x) + a) - 4*(2*a*b^5*x^2 + 10*a^3*b^3*x - 15*a^5*b)*sqrt(x))
/(b^8*x - a^2*b^6)
```

giac [A] time = 0.21, size = 78, normalized size = 0.94

$$\frac{10\,a^4\log\left(\left|b\sqrt{x}+a\right|\right)}{b^6}+\frac{2\,a^5}{\left(b\sqrt{x}+a\right)b^6}+\frac{3\,b^6x^2-8\,ab^5x^{\frac{3}{2}}+18\,a^2b^4x-48\,a^3b^3\sqrt{x}}{6\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^(1/2))^2,x, algorithm="giac")
```

```
[Out] 10*a^4*log(abs(b*sqrt(x) + a))/b^6 + 2*a^5/((b*sqrt(x) + a)*b^6) + 1/6*(3*b
^6*x^2 - 8*a*b^5*x^(3/2) + 18*a^2*b^4*x - 48*a^3*b^3*sqrt(x))/b^8
```

maple [A] time = 0.01, size = 72, normalized size = 0.87

$$\frac{x^2}{2b^2}-\frac{4a\,x^{\frac{3}{2}}}{3b^3}+\frac{2a^5}{\left(b\sqrt{x}+a\right)b^6}+\frac{10a^4\ln\left(b\sqrt{x}+a\right)}{b^6}+\frac{3a^2x}{b^4}-\frac{8a^3\sqrt{x}}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^(1/2)+a)^2,x)
```

```
[Out] 3*a^2*x/b^4-4/3*a*x^(3/2)/b^3+1/2*x^2/b^2+10*a^4*ln(b*x^(1/2)+a)/b^6-8*a^3*
x^(1/2)/b^5+2*a^5/b^6/(b*x^(1/2)+a)
```

maxima [A] time = 0.89, size = 95, normalized size = 1.14

$$\frac{10\,a^4\log\left(b\sqrt{x}+a\right)}{b^6}+\frac{\left(b\sqrt{x}+a\right)^4}{2\,b^6}-\frac{10\left(b\sqrt{x}+a\right)^3a}{3\,b^6}+\frac{10\left(b\sqrt{x}+a\right)^2a^2}{b^6}-\frac{20\left(b\sqrt{x}+a\right)a^3}{b^6}+\frac{2\,a^5}{\left(b\sqrt{x}+a\right)b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 10*a^4*log(b*sqrt(x) + a)/b^6 + 1/2*(b*sqrt(x) + a)^4/b^6 - 10/3*(b*sqrt(x)
 + a)^3*a/b^6 + 10*(b*sqrt(x) + a)^2*a^2/b^6 - 20*(b*sqrt(x) + a)*a^3/b^6 +
 2*a^5/((b*sqrt(x) + a)*b^6)
```

mupad [B] time = 0.03, size = 77, normalized size = 0.93

$$\frac{x^2}{2\,b^2}+\frac{2\,a^5}{b\left(a\,b^5+b^6\,\sqrt{x}\right)}+\frac{3\,a^2\,x}{b^4}-\frac{4\,a\,x^{3/2}}{3\,b^3}+\frac{10\,a^4\,\ln\left(a+b\,\sqrt{x}\right)}{b^6}-\frac{8\,a^3\,\sqrt{x}}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^(1/2))^2,x)
```

```
[Out] x^2/(2*b^2) + (2*a^5)/(b*(a*b^5 + b^6*x^(1/2))) + (3*a^2*x)/b^4 - (4*a*x^(3
/2))/(3*b^3) + (10*a^4*log(a + b*x^(1/2)))/b^6 - (8*a^3*x^(1/2))/b^5
```

sympy [**A**] time = 1.26, size = 212, normalized size = 2.55

$$\begin{cases} \frac{60a^5\log\left(\frac{a}{b}+\sqrt{x}\right)}{6ab^6+6b^7\sqrt{x}} + \frac{60a^5}{6ab^6+6b^7\sqrt{x}} + \frac{60a^4b\sqrt{x}\log\left(\frac{a}{b}+\sqrt{x}\right)}{6ab^6+6b^7\sqrt{x}} - \frac{30a^3b^2x}{6ab^6+6b^7\sqrt{x}} + \frac{10a^2b^3x^{\frac{3}{2}}}{6ab^6+6b^7\sqrt{x}} - \frac{5ab^4x^2}{6ab^6+6b^7\sqrt{x}} + \frac{3b^5x^{\frac{5}{2}}}{6ab^6+6b^7\sqrt{x}} & \text{for } b \neq \\ \frac{x^3}{3a^2} & \text{other} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*x**(1/2))**2,x)
```

```
[Out] Piecewise((60*a**5*log(a/b + sqrt(x))/(6*a*b**6 + 6*b**7*sqrt(x)) + 60*a**5
/(6*a*b**6 + 6*b**7*sqrt(x)) + 60*a**4*b*sqrt(x)*log(a/b + sqrt(x))/(6*a*b*
*6 + 6*b**7*sqrt(x)) - 30*a**3*b**2*x/(6*a*b**6 + 6*b**7*sqrt(x)) + 10*a**2
*b**3*x**(3/2)/(6*a*b**6 + 6*b**7*sqrt(x)) - 5*a*b**4*x**2/(6*a*b**6 + 6*b*
*7*sqrt(x)) + 3*b**5*x**(5/2)/(6*a*b**6 + 6*b**7*sqrt(x)), Ne(b, 0)), (x**3
/(3*a**2), True))
```

3.2201 $\int \frac{x}{\left(a+b\sqrt{x}\right)^2}\,dx$

Optimal. Leaf size=54

$$\frac{2a^3}{b^4\left(a+b\sqrt{x}\right)}+\frac{6a^2\log\left(a+b\sqrt{x}\right)}{b^4}-\frac{4a\sqrt{x}}{b^3}+\frac{x}{b^2}$$

```
[Out] x/b^2+6*a^2*ln(a+b*x^(1/2))/b^4-4*a*x^(1/2)/b^3+2*a^3/b^4/(a+b*x^(1/2))
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{2a^3}{b^4\left(a+b\sqrt{x}\right)}+\frac{6a^2\log\left(a+b\sqrt{x}\right)}{b^4}-\frac{4a\sqrt{x}}{b^3}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*Sqrt[x])^2,x]
```

```
[Out] (2*a^3)/(b^4*(a + b*Sqrt[x])) - (4*a*Sqrt[x])/b^3 + x/b^2 + (6*a^2*Log[a +
b*Sqrt[x]])/b^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a+b\sqrt{x}\right)^2}\,dx &= 2\,\text{Subst}\left(\int \frac{x^3}{(a+bx)^2}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int\left(-\frac{2a}{b^3}+\frac{x}{b^2}-\frac{a^3}{b^3(a+bx)^2}+\frac{3a^2}{b^3(a+bx)}\right)dx, x, \sqrt{x}\right)\\ &= \frac{2a^3}{b^4\left(a+b\sqrt{x}\right)}-\frac{4a\sqrt{x}}{b^3}+\frac{x}{b^2}+\frac{6a^2\log\left(a+b\sqrt{x}\right)}{b^4}\end{aligned}$$

Mathematica [A] time = 0.05, size = 50, normalized size = 0.93

$$\frac{\frac{2a^3}{a+b\sqrt{x}}+6a^2\log\left(a+b\sqrt{x}\right)-4ab\sqrt{x}+b^2x}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*Sqrt[x])^2,x]
```

```
[Out] ((2*a^3)/(a + b*Sqrt[x]) - 4*a*b*Sqrt[x] + b^2*x + 6*a^2*Log[a + b*Sqrt[x]]
)/b^4
```

fricas [A] time = 0.73, size = 82, normalized size = 1.52

$$\frac{b^4x^2 - a^2b^2x - 2\,a^4 + 6\left(a^2b^2x - a^4\right)\log\left(b\sqrt{x} + a\right) - 2\left(2\,ab^3x - 3\,a^3b\right)\sqrt{x}}{b^6x - a^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^(1/2))^2,x, algorithm="fricas")
```

```
[Out] (b^4*x^2 - a^2*b^2*x - 2*a^4 + 6*(a^2*b^2*x - a^4)*log(b*sqrt(x) + a) - 2*(
2*a*b^3*x - 3*a^3*b)*sqrt(x))/(b^6*x - a^2*b^4)
```

giac [A] time = 0.15, size = 52, normalized size = 0.96

$$\frac{6\,a^2\log\left(\left|b\sqrt{x} + a\right|\right)}{b^4} + \frac{2\,a^3}{\left(b\sqrt{x} + a\right)b^4} + \frac{b^2x - 4\,ab\sqrt{x}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^(1/2))^2,x, algorithm="giac")
```

```
[Out] 6*a^2*log(abs(b*sqrt(x) + a))/b^4 + 2*a^3/((b*sqrt(x) + a)*b^4) + (b^2*x -
4*a*b*sqrt(x))/b^4
```

maple [A] time = 0.01, size = 49, normalized size = 0.91

$$\frac{2a^3}{\left(b\sqrt{x} + a\right)b^4} + \frac{6a^2\ln\left(b\sqrt{x} + a\right)}{b^4} + \frac{x}{b^2} - \frac{4a\sqrt{x}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^(1/2)+a)^2,x)
```

```
[Out] x/b^2+6*a^2*ln(b*x^(1/2)+a)/b^4-4*a*x^(1/2)/b^3+2*a^3/b^4/(b*x^(1/2)+a)
```

maxima [A] time = 0.85, size = 60, normalized size = 1.11

$$\frac{6\,a^2\log\left(b\sqrt{x} + a\right)}{b^4} + \frac{\left(b\sqrt{x} + a\right)^2}{b^4} - \frac{6\left(b\sqrt{x} + a\right)a}{b^4} + \frac{2\,a^3}{\left(b\sqrt{x} + a\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 6*a^2*log(b*sqrt(x) + a)/b^4 + (b*sqrt(x) + a)^2/b^4 - 6*(b*sqrt(x) + a)*a/
b^4 + 2*a^3/((b*sqrt(x) + a)*b^4)
```

mupad [B] time = 0.04, size = 54, normalized size = 1.00

$$\frac{x}{b^2} + \frac{2\,a^3}{b\left(a\,b^3 + b^4\,\sqrt{x}\right)} - \frac{4\,a\,\sqrt{x}}{b^3} + \frac{6\,a^2\,\ln\left(a + b\,\sqrt{x}\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^(1/2))^2,x)
```

```
[Out] x/b^2 + (2*a^3)/(b*(a*b^3 + b^4*x^(1/2))) - (4*a*x^(1/2))/b^3 + (6*a^2*log(
a + b*x^(1/2)))/b^4
```

sympy [**A**] time = 0.48, size = 134, normalized size = 2.48

$$\begin{cases} \frac{6a^3 \log\left(\frac{a}{b}+\sqrt{x}\right)}{ab^4+b^5\sqrt{x}} + \frac{6a^3}{ab^4+b^5\sqrt{x}} + \frac{6a^2b\sqrt{x}\log\left(\frac{a}{b}+\sqrt{x}\right)}{ab^4+b^5\sqrt{x}} - \frac{3ab^2x}{ab^4+b^5\sqrt{x}} + \frac{b^3x^{\frac{3}{2}}}{ab^4+b^5\sqrt{x}} & \text{for } b \neq 0 \\ \frac{x^2}{2a^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x**(1/2))**2,x)
```

```
[Out] Piecewise((6*a**3*log(a/b + sqrt(x))/(a*b**4 + b**5*sqrt(x)) + 6*a**3/(a*b*
*4 + b**5*sqrt(x)) + 6*a**2*b*sqrt(x)*log(a/b + sqrt(x))/(a*b**4 + b**5*sqr
t(x)) - 3*a*b**2*x/(a*b**4 + b**5*sqrt(x)) + b**3*x**(3/2)/(a*b**4 + b**5*s
qrt(x)), Ne(b, 0)), (x**2/(2*a**2), True))
```

3.2202 $\int \frac{1}{\left(a+b\sqrt{x}\right)^2}\,dx$

Optimal. Leaf size=33

$$\frac{2a}{b^2\left(a+b\sqrt{x}\right)}+\frac{2\log\left(a+b\sqrt{x}\right)}{b^2}$$

```
[Out] 2*ln(a+b*x^(1/2))/b^2+2*a/b^2/(a+b*x^(1/2))
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {190, 43}

$$\frac{2a}{b^2\left(a+b\sqrt{x}\right)}+\frac{2\log\left(a+b\sqrt{x}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^(-2),x]
```

```
[Out] (2*a)/(b^2*(a + b*Sqrt[x])) + (2*Log[a + b*Sqrt[x]])/b^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+b\sqrt{x}\right)^2}\,dx &= 2\,\text{Subst}\left(\int \frac{x}{(a+bx)^2}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^2}+\frac{1}{b(a+bx)}\right)dx, x, \sqrt{x}\right)\\ &= \frac{2a}{b^2\left(a+b\sqrt{x}\right)}+\frac{2\log\left(a+b\sqrt{x}\right)}{b^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 0.88

$$\frac{2\left(\frac{a}{a+b\sqrt{x}}+\log\left(a+b\sqrt{x}\right)\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^(-2),x]
```

```
[Out] (2*(a/(a + b*Sqrt[x]) + Log[a + b*Sqrt[x]]))/b^2
```

fricas [A] time = 0.67, size = 50, normalized size = 1.52

$$\frac{2\left(ab\sqrt{x}-a^2+\left(b^2x-a^2\right)\log\left(b\sqrt{x}+a\right)\right)}{b^4x-a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 2*(a*b*sqrt(x) - a^2 + (b^2*x - a^2)*log(b*sqrt(x) + a))/(b^4*x - a^2*b^2)
```

giac [A] time = 0.17, size = 30, normalized size = 0.91

$$\frac{2\log\left(\left|b\sqrt{x}+a\right|\right)}{b^2}+\frac{2a}{\left(b\sqrt{x}+a\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/2))^2,x, algorithm="giac")
```

```
[Out] 2*log(abs(b*sqrt(x) + a))/b^2 + 2*a/((b*sqrt(x) + a)*b^2)
```

maple [B] time = 0.02, size = 96, normalized size = 2.91

$$-\frac{2a^2}{\left(b^2x-a^2\right)b^2}+\frac{a}{\left(b\sqrt{x}+a\right)b^2}+\frac{a}{\left(b\sqrt{x}-a\right)b^2}+\frac{\ln\left(b\sqrt{x}+a\right)}{b^2}-\frac{\ln\left(b\sqrt{x}-a\right)}{b^2}+\frac{\ln\left(b^2x-a^2\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^(1/2)+a)^2,x)
```

```
[Out] -2*a^2/(b^2*x-a^2)/b^2+ln(b^2*x-a^2)/b^2+a/b^2/(b*x^(1/2)+a)+ln(b*x^(1/2)+a
)/b^2+a/b^2/(b*x^(1/2)-a)-1/b^2*ln(b*x^(1/2)-a)
```

maxima [A] time = 0.88, size = 29, normalized size = 0.88

$$\frac{2\log\left(b\sqrt{x}+a\right)}{b^2}+\frac{2a}{\left(b\sqrt{x}+a\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 2*log(b*sqrt(x) + a)/b^2 + 2*a/((b*sqrt(x) + a)*b^2)
```

mupad [B] time = 1.09, size = 29, normalized size = 0.88

$$\frac{2\ln\left(a+b\sqrt{x}\right)}{b^2}+\frac{2a}{b^2\left(a+b\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^(1/2))^2,x)
```

```
[Out] (2*log(a + b*x^(1/2)))/b^2 + (2*a)/(b^2*(a + b*x^(1/2)))
```

sympy [A] time = 0.63, size = 80, normalized size = 2.42

$$\begin{cases}\frac{2a\log\left(\frac{a}{b}+\sqrt{x}\right)}{ab^2+b^3\sqrt{x}}+\frac{2a}{ab^2+b^3\sqrt{x}}+\frac{2b\sqrt{x}\log\left(\frac{a}{b}+\sqrt{x}\right)}{ab^2+b^3\sqrt{x}} & \text{for } b\neq 0\\ \frac{x}{a^2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**(1/2))**2,x)
```

```
[Out] Piecewise((2*a*log(a/b + sqrt(x))/(a*b**2 + b**3*sqrt(x)) + 2*a/(a*b**2 + b
**3*sqrt(x)) + 2*b*sqrt(x)*log(a/b + sqrt(x))/(a*b**2 + b**3*sqrt(x)), Ne(b
, 0)), (x/a**2, True))
```

3.2203 $\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x}\,dx$

Optimal. Leaf size=38

$$-\frac{2\log\left(a+b\sqrt{x}\right)}{a^2}+\frac{\log(x)}{a^2}+\frac{2}{a\left(a+b\sqrt{x}\right)}$$

```
[Out] ln(x)/a^2-2*ln(a+b*x^(1/2))/a^2+2/a/(a+b*x^(1/2))
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 44}

$$-\frac{2\log\left(a+b\sqrt{x}\right)}{a^2}+\frac{\log(x)}{a^2}+\frac{2}{a\left(a+b\sqrt{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sqrt[x])^2*x),x]
```

```
[Out] 2/(a*(a + b*Sqrt[x])) - (2*Log[a + b*Sqrt[x]])/a^2 + Log[x]/a^2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{x(a+bx)^2}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(\frac{1}{a^2 x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right)dx, x, \sqrt{x}\right)\\ &= \frac{2}{a\left(a+b\sqrt{x}\right)}-\frac{2\log\left(a+b\sqrt{x}\right)}{a^2}+\frac{\log(x)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.04, size = 33, normalized size = 0.87

$$\frac{\frac{2a}{a+b\sqrt{x}}-2\log\left(a+b\sqrt{x}\right)+\log(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sqrt[x])^2*x),x]
```

```
[Out] ((2*a)/(a + b*Sqrt[x]) - 2*Log[a + b*Sqrt[x]] + Log[x])/a^2
```

fricas [A] time = 1.39, size = 67, normalized size = 1.76

$$\frac{2\left(ab\sqrt{x}-a^2-\left(b^2x-a^2\right)\log\left(b\sqrt{x}+a\right)+\left(b^2x-a^2\right)\log\left(\sqrt{x}\right)\right)}{a^2b^2x-a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 2*(a*b*sqrt(x) - a^2 - (b^2*x - a^2)*log(b*sqrt(x) + a) + (b^2*x - a^2)*log
(sqrt(x)))/(a^2*b^2*x - a^4)
```

giac [A] time = 0.22, size = 36, normalized size = 0.95

$$-\frac{2\log\left(\left|b\sqrt{x}+a\right|\right)}{a^2}+\frac{\log\left(|x|\right)}{a^2}+\frac{2}{\left(b\sqrt{x}+a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^(1/2))^2,x, algorithm="giac")
```

```
[Out] -2*log(abs(b*sqrt(x) + a))/a^2 + log(abs(x))/a^2 + 2/((b*sqrt(x) + a)*a)
```

maple [A] time = 0.01, size = 35, normalized size = 0.92

$$\frac{2}{\left(b\sqrt{x}+a\right)a}+\frac{\ln(x)}{a^2}-\frac{2\ln\left(b\sqrt{x}+a\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^(1/2)+a)^2,x)
```

```
[Out] 1/a^2*ln(x)-2*ln(b*x^(1/2)+a)/a^2+2/a/(b*x^(1/2)+a)
```

maxima [A] time = 0.88, size = 34, normalized size = 0.89

$$\frac{2}{ab\sqrt{x}+a^2}-\frac{2\log\left(b\sqrt{x}+a\right)}{a^2}+\frac{\log(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 2/(a*b*sqrt(x) + a^2) - 2*log(b*sqrt(x) + a)/a^2 + log(x)/a^2
```

mupad [B] time = 0.05, size = 32, normalized size = 0.84

$$\frac{2}{a\left(a+b\sqrt{x}\right)}-\frac{4\,\mathrm{atanh}\left(\frac{2b\sqrt{x}}{a}+1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^(1/2))^2),x)
```

```
[Out] 2/(a*(a + b*x^(1/2))) - (4*atanh((2*b*x^(1/2))/a + 1))/a^2
```

sympy [**A**] time = 1.47, size = 151, normalized size = 3.97

$$\begin{cases} \frac{\tilde{\infty}}{x} & \text{for } a = 0 \wedge b = 0 \\ \frac{\log(x)}{a^2} & \text{for } b = 0 \\ -\frac{1}{b^2 x} & \text{for } a = 0 \\ \frac{a\sqrt{x}\,\log(x)}{a^3\sqrt{x}+a^2bx} - \frac{2a\sqrt{x}\,\log\left(\frac{a}{b}+\sqrt{x}\right)}{a^3\sqrt{x}+a^2bx} + \frac{2a\sqrt{x}}{a^3\sqrt{x}+a^2bx} + \frac{bx\log(x)}{a^3\sqrt{x}+a^2bx} - \frac{2bx\log\left(\frac{a}{b}+\sqrt{x}\right)}{a^3\sqrt{x}+a^2bx} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x**(1/2))**2,x)
```

```
[Out] Piecewise((zoo/x, Eq(a, 0) & Eq(b, 0)), (log(x)/a**2, Eq(b, 0)), (-1/(b**2*
x), Eq(a, 0)), (a*sqrt(x)*log(x)/(a**3*sqrt(x) + a**2*b*x) - 2*a*sqrt(x)*lo
g(a/b + sqrt(x))/(a**3*sqrt(x) + a**2*b*x) + 2*a*sqrt(x)/(a**3*sqrt(x) + a*
*2*b*x) + b*x*log(x)/(a**3*sqrt(x) + a**2*b*x) - 2*b*x*log(a/b + sqrt(x))/(
a**3*sqrt(x) + a**2*b*x), True))
```

3.2204 $\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x^2}\,dx$

Optimal. Leaf size=67

$$-\frac{6b^2\log\left(a+b\sqrt{x}\right)}{a^4}+\frac{3b^2\log(x)}{a^4}+\frac{2b^2}{a^3\left(a+b\sqrt{x}\right)}+\frac{4b}{a^3\sqrt{x}}-\frac{1}{a^2x}$$

```
[Out] -1/a^2/x+3*b^2*ln(x)/a^4-6*b^2*ln(a+b*x^(1/2))/a^4+4*b/a^3/x^(1/2)+2*b^2/a^
3/(a+b*x^(1/2))
```

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 44}

$$\frac{2b^2}{a^3\left(a+b\sqrt{x}\right)}-\frac{6b^2\log\left(a+b\sqrt{x}\right)}{a^4}+\frac{3b^2\log(x)}{a^4}+\frac{4b}{a^3\sqrt{x}}-\frac{1}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sqrt[x])^2*x^2),x]
```

```
[Out] (2*b^2)/(a^3*(a + b*Sqrt[x])) - 1/(a^2*x) + (4*b)/(a^3*Sqrt[x]) - (6*b^2*Lo
g[a + b*Sqrt[x]])/a^4 + (3*b^2*Log[x])/a^4
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x^2}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{x^3(a+bx)^2}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int\left(\frac{1}{a^2x^3}-\frac{2b}{a^3x^2}+\frac{3b^2}{a^4x}-\frac{b^3}{a^3(a+bx)^2}-\frac{3b^3}{a^4(a+bx)}\right)dx, x, \sqrt{x}\right)\\ &= \frac{2b^2}{a^3\left(a+b\sqrt{x}\right)}-\frac{1}{a^2x}+\frac{4b}{a^3\sqrt{x}}-\frac{6b^2\log\left(a+b\sqrt{x}\right)}{a^4}+\frac{3b^2\log(x)}{a^4}\end{aligned}$$

Mathematica [A] time = 0.13, size = 60, normalized size = 0.90

$$\frac{a\left(\frac{2b^2}{a+b\sqrt{x}}-\frac{a}{x}+\frac{4b}{\sqrt{x}}\right)-6b^2\log\left(a+b\sqrt{x}\right)+3b^2\log(x)}{a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sqrt[x])^2*x^2),x]
```

```
[Out] (a*((2*b^2)/(a + b*Sqrt[x]) - a/x + (4*b)/Sqrt[x]) - 6*b^2*Log[a + b*Sqrt[x
]] + 3*b^2*Log[x])/a^4
```

fricas [A] time = 1.27, size = 105, normalized size = 1.57

$$-\frac{3\,a^2b^2x - a^4 + 6\left(b^4x^2 - a^2b^2x\right)\log\left(b\sqrt{x} + a\right) - 6\left(b^4x^2 - a^2b^2x\right)\log\left(\sqrt{x}\right) - 2\left(3\,ab^3x - 2\,a^3b\right)\sqrt{x}}{a^4b^2x^2 - a^6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^(1/2))^2,x, algorithm="fricas")
```

```
[Out] -(3*a^2*b^2*x - a^4 + 6*(b^4*x^2 - a^2*b^2*x)*log(b*sqrt(x) + a) - 6*(b^4*x
^2 - a^2*b^2*x)*log(sqrt(x)) - 2*(3*a*b^3*x - 2*a^3*b)*sqrt(x))/(a^4*b^2*x^
2 - a^6*x)
```

giac [A] time = 0.21, size = 67, normalized size = 1.00

$$-\frac{6\,b^2\log\left(\left|b\sqrt{x} + a\right|\right)}{a^4} + \frac{3\,b^2\log\left(|x|\right)}{a^4} + \frac{6\,ab^2x + 3\,a^2b\sqrt{x} - a^3}{\left(b\sqrt{x} + a\right)a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^(1/2))^2,x, algorithm="giac")
```

```
[Out] -6*b^2*log(abs(b*sqrt(x) + a))/a^4 + 3*b^2*log(abs(x))/a^4 + (6*a*b^2*x + 3
*a^2*b*sqrt(x) - a^3)/((b*sqrt(x) + a)*a^4*x)
```

maple [A] time = 0.01, size = 62, normalized size = 0.93

$$\frac{2b^2}{\left(b\sqrt{x} + a\right)a^3} + \frac{3b^2\ln(x)}{a^4} - \frac{6b^2\ln\left(b\sqrt{x} + a\right)}{a^4} + \frac{4b}{a^3\sqrt{x}} - \frac{1}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^(1/2)+a)^2,x)
```

```
[Out] -1/a^2/x+3/a^4*b^2*ln(x)-6*b^2*ln(b*x^(1/2)+a)/a^4+4*b/a^3/x^(1/2)+2*b^2/a^
3/(b*x^(1/2)+a)
```

maxima [A] time = 0.91, size = 63, normalized size = 0.94

$$\frac{6\,b^2x + 3\,ab\sqrt{x} - a^2}{a^3bx^{\frac{3}{2}} + a^4x} - \frac{6\,b^2\log\left(b\sqrt{x} + a\right)}{a^4} + \frac{3\,b^2\log(x)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^(1/2))^2,x, algorithm="maxima")
```

```
[Out] (6*b^2*x + 3*a*b*sqrt(x) - a^2)/(a^3*b*x^(3/2) + a^4*x) - 6*b^2*log(b*sqrt(
x) + a)/a^4 + 3*b^2*log(x)/a^4
```

mupad [B] time = 1.14, size = 57, normalized size = 0.85

$$\frac{\frac{3\,b\,\sqrt{x}}{a^2} - \frac{1}{a} + \frac{6\,b^2\,x}{a^3}}{a\,x + b\,x^{3/2}} - \frac{12\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,\sqrt{x}}{a} + 1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^(1/2))^2),x)
```

```
[Out] ((3*b*x^(1/2))/a^2 - 1/a + (6*b^2*x)/a^3)/(a*x + b*x^(3/2)) - (12*b^2*atanh
((2*b*x^(1/2))/a + 1))/a^4
```

sympy [A] time = 1.82, size = 238, normalized size = 3.55

$$\begin{cases} \frac{\tilde{\infty}}{x^2} & \text{for } a = \\ -\frac{1}{a^2 x} & \text{for } b = \\ -\frac{1}{2b^2x^2} & \text{for } a = \\ -\frac{a^3\sqrt{x}}{a^5 x^{\frac{3}{2}} + a^4 b x^2} + \frac{3a^2 bx}{a^5 x^{\frac{3}{2}} + a^4 b x^2} + \frac{3ab^2 x^{\frac{3}{2}} \log(x)}{a^5 x^{\frac{3}{2}} + a^4 b x^2} - \frac{6ab^2 x^{\frac{3}{2}} \log\left(\frac{a}{b} + \sqrt{x}\right)}{a^5 x^{\frac{3}{2}} + a^4 b x^2} + \frac{6ab^2 x^{\frac{3}{2}}}{a^5 x^{\frac{3}{2}} + a^4 b x^2} + \frac{3b^3 x^2 \log(x)}{a^5 x^{\frac{3}{2}} + a^4 b x^2} - \frac{6b^3 x^2 \log\left(\frac{a}{b} + \sqrt{x}\right)}{a^5 x^{\frac{3}{2}} + a^4 b x^2} & \text{otherv} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*x**(1/2))**2,x)
```

```
[Out] Piecewise((zoo/x**2, Eq(a, 0) & Eq(b, 0)), (-1/(a**2*x), Eq(b, 0)), (-1/(2*
b**2*x**2), Eq(a, 0)), (-a**3*sqrt(x)/(a**5*x**(3/2) + a**4*b*x**2) + 3*a**
2*b*x/(a**5*x**(3/2) + a**4*b*x**2) + 3*a*b**2*x**(3/2)*log(x)/(a**5*x**(3/
2) + a**4*b*x**2) - 6*a*b**2*x**(3/2)*log(a/b + sqrt(x))/(a**5*x**(3/2) + a
**4*b*x**2) + 6*a*b**2*x**(3/2)/(a**5*x**(3/2) + a**4*b*x**2) + 3*b**3*x**2
*log(x)/(a**5*x**(3/2) + a**4*b*x**2) - 6*b**3*x**2*log(a/b + sqrt(x))/(a**
5*x**(3/2) + a**4*b*x**2), True))
```

3.2205 $\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x^3}\,dx$

Optimal. Leaf size=95

$$-\frac{10b^4\log\left(a+b\sqrt{x}\right)}{a^6}+\frac{5b^4\log(x)}{a^6}+\frac{2b^4}{a^5\left(a+b\sqrt{x}\right)}+\frac{8b^3}{a^5\sqrt{x}}-\frac{3b^2}{a^4x}+\frac{4b}{3a^3x^{3/2}}-\frac{1}{2a^2x^2}$$

```
[Out] -1/2/a^2/x^2+4/3*b/a^3/x^(3/2)-3*b^2/a^4/x+5*b^4*ln(x)/a^6-10*b^4*ln(a+b*x^
(1/2))/a^6+8*b^3/a^5/x^(1/2)+2*b^4/a^5/(a+b*x^(1/2))
```

Rubi [A] time = 0.06, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 44}

$$\frac{2b^4}{a^5\left(a+b\sqrt{x}\right)}+\frac{8b^3}{a^5\sqrt{x}}-\frac{3b^2}{a^4x}-\frac{10b^4\log\left(a+b\sqrt{x}\right)}{a^6}+\frac{5b^4\log(x)}{a^6}+\frac{4b}{3a^3x^{3/2}}-\frac{1}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sqrt[x])^2*x^3),x]
```

```
[Out] (2*b^4)/(a^5*(a + b*Sqrt[x])) - 1/(2*a^2*x^2) + (4*b)/(3*a^3*x^(3/2)) - (3*
b^2)/(a^4*x) + (8*b^3)/(a^5*Sqrt[x]) - (10*b^4*Log[a + b*Sqrt[x]])/a^6 + (5
*b^4*Log[x])/a^6
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x^3}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{x^5(a+bx)^2}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(\frac{1}{a^2x^5}-\frac{2b}{a^3x^4}+\frac{3b^2}{a^4x^3}-\frac{4b^3}{a^5x^2}+\frac{5b^4}{a^6x}-\frac{b^5}{a^5(a+bx)^2}-\frac{5b^5}{a^6(a+bx)}\right)dx, x, \sqrt{x}\right)\\ &= \frac{2b^4}{a^5\left(a+b\sqrt{x}\right)}-\frac{1}{2a^2x^2}+\frac{4b}{3a^3x^{3/2}}-\frac{3b^2}{a^4x}+\frac{8b^3}{a^5\sqrt{x}}-\frac{10b^4\log\left(a+b\sqrt{x}\right)}{a^6}+\frac{5b^4\log(x)}{a^6}\end{aligned}$$

Mathematica [A] time = 0.14, size = 91, normalized size = 0.96

$$\frac{\frac{a\left(-3a^4+5a^3b\sqrt{x}-10a^2b^2x+30ab^3x^{3/2}+60b^4x^2\right)}{x^2\left(a+b\sqrt{x}\right)}-60b^4\log\left(a+b\sqrt{x}\right)+30b^4\log(x)}{6a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sqrt[x])^2*x^3),x]
```

```
[Out] ((a*(-3*a^4 + 5*a^3*b*Sqrt[x] - 10*a^2*b^2*x + 30*a*b^3*x^(3/2) + 60*b^4*x^
2))/((a + b*Sqrt[x])*x^2) - 60*b^4*Log[a + b*Sqrt[x]] + 30*b^4*Log[x])/(6*a
^6)
```

fricas [A] time = 1.00, size = 133, normalized size = 1.40

$$-\frac{30\,a^2b^4x^2-15\,a^4b^2x-3\,a^6+60\left(b^6x^3-a^2b^4x^2\right)\log\left(b\sqrt{x}+a\right)-60\left(b^6x^3-a^2b^4x^2\right)\log\left(\sqrt{x}\right)-4\left(15\,ab^5x^2}{6\left(a^6b^2x^3-a^8x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^(1/2))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(30*a^2*b^4*x^2 - 15*a^4*b^2*x - 3*a^6 + 60*(b^6*x^3 - a^2*b^4*x^2)*lo
g(b*sqrt(x) + a) - 60*(b^6*x^3 - a^2*b^4*x^2)*log(sqrt(x)) - 4*(15*a*b^5*x^
2 - 10*a^3*b^3*x - 2*a^5*b)*sqrt(x))/(a^6*b^2*x^3 - a^8*x^2)
```

giac [A] time = 0.16, size = 90, normalized size = 0.95

$$-\frac{10\,b^4\log\left(\left|b\sqrt{x}+a\right|\right)}{a^6}+\frac{5\,b^4\log\left(|x|\right)}{a^6}+\frac{60\,ab^4x^2+30\,a^2b^3x^{\frac{3}{2}}-10\,a^3b^2x+5\,a^4b\sqrt{x}-3\,a^5}{6\left(b\sqrt{x}+a\right)a^6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^(1/2))^2,x, algorithm="giac")
```

```
[Out] -10*b^4*log(abs(b*sqrt(x) + a))/a^6 + 5*b^4*log(abs(x))/a^6 + 1/6*(60*a*b^4
*x^2 + 30*a^2*b^3*x^(3/2) - 10*a^3*b^2*x + 5*a^4*b*sqrt(x) - 3*a^5)/((b*sqr
t(x) + a)*a^6*x^2)
```

maple [A] time = 0.01, size = 84, normalized size = 0.88

$$\frac{2b^4}{\left(b\sqrt{x}+a\right)a^5}+\frac{5b^4\ln(x)}{a^6}-\frac{10b^4\ln\left(b\sqrt{x}+a\right)}{a^6}+\frac{8b^3}{a^5\sqrt{x}}-\frac{3b^2}{a^4x}+\frac{4b}{3a^3x^{\frac{3}{2}}}-\frac{1}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^(1/2)+a)^2,x)
```

```
[Out] -1/2/a^2/x^2+4/3*b/a^3/x^(3/2)-3*b^2/a^4/x+5*b^4*ln(x)/a^6-10*b^4*ln(b*x^(1
/2)+a)/a^6+8*b^3/a^5/x^(1/2)+2*b^4/a^5/(b*x^(1/2)+a)
```

maxima [A] time = 0.91, size = 88, normalized size = 0.93

$$\frac{60\,b^4x^2+30\,ab^3x^{\frac{3}{2}}-10\,a^2b^2x+5\,a^3b\sqrt{x}-3\,a^4}{6\left(a^5bx^{\frac{5}{2}}+a^6x^2\right)}-\frac{10\,b^4\log\left(b\sqrt{x}+a\right)}{a^6}+\frac{5\,b^4\log(x)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(60*b^4*x^2 + 30*a*b^3*x^(3/2) - 10*a^2*b^2*x + 5*a^3*b*sqrt(x) - 3*a^4
)/(a^5*b*x^(5/2) + a^6*x^2) - 10*b^4*log(b*sqrt(x) + a)/a^6 + 5*b^4*log(x)/
a^6
```

mupad [B] time = 1.17, size = 81, normalized size = 0.85

$$\frac{\frac{5\,b\,\sqrt{x}}{6\,a^2}-\frac{1}{2\,a}-\frac{5\,b^2\,x}{3\,a^3}+\frac{10\,b^4\,x^2}{a^5}+\frac{5\,b^3\,x^{3/2}}{a^4}}{a\,x^2+b\,x^{5/2}}-\frac{20\,b^4\,\mathrm{atanh}\left(\frac{2\,b\,\sqrt{x}}{a}+1\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^(1/2))^2),x)
```

```
[Out] ((5*b*x^(1/2))/(6*a^2) - 1/(2*a) - (5*b^2*x)/(3*a^3) + (10*b^4*x^2)/a^5 + (
5*b^3*x^(3/2))/a^4)/(a*x^2 + b*x^(5/2)) - (20*b^4*atanh((2*b*x^(1/2))/a + 1
))/a^6
```

sympy [A] time = 4.07, size = 333, normalized size = 3.51

$$\begin{cases} \frac{\infty}{x^3} \\ -\frac{1}{3b^2x^3} \\ -\frac{1}{2a^2x^2} \\ -\frac{3a^5\sqrt{x}}{6a^7x^{\frac{5}{2}}+6a^6bx^3} + \frac{5a^4bx}{6a^7x^{\frac{5}{2}}+6a^6bx^3} - \frac{10a^3b^2x^{\frac{3}{2}}}{6a^7x^{\frac{5}{2}}+6a^6bx^3} + \frac{30a^2b^3x^2}{6a^7x^{\frac{5}{2}}+6a^6bx^3} + \frac{30ab^4x^{\frac{5}{2}}\log(x)}{6a^7x^{\frac{5}{2}}+6a^6bx^3} - \frac{60ab^4x^{\frac{5}{2}}\log\left(\frac{a}{b}+\sqrt{x}\right)}{6a^7x^{\frac{5}{2}}+6a^6bx^3} + \frac{60ab^4x^{\frac{5}{2}}}{6a^7x^{\frac{5}{2}}+6a^6bx^3} + \frac{30b} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*x**(1/2))**2,x)
```

```
[Out] Piecewise((zoo/x**3, Eq(a, 0) & Eq(b, 0)), (-1/(3*b**2*x**3), Eq(a, 0)), (-
1/(2*a**2*x**2), Eq(b, 0)), (-3*a**5*sqrt(x)/(6*a**7*x**(5/2) + 6*a**6*b*x*
*3) + 5*a**4*b*x/(6*a**7*x**(5/2) + 6*a**6*b*x**3) - 10*a**3*b**2*x**(3/2)/
(6*a**7*x**(5/2) + 6*a**6*b*x**3) + 30*a**2*b**3*x**2/(6*a**7*x**(5/2) + 6*
a**6*b*x**3) + 30*a*b**4*x**(5/2)*log(x)/(6*a**7*x**(5/2) + 6*a**6*b*x**3)
- 60*a*b**4*x**(5/2)*log(a/b + sqrt(x))/(6*a**7*x**(5/2) + 6*a**6*b*x**3) +
 60*a*b**4*x**(5/2)/(6*a**7*x**(5/2) + 6*a**6*b*x**3) + 30*b**5*x**3*log(x)
/(6*a**7*x**(5/2) + 6*a**6*b*x**3) - 60*b**5*x**3*log(a/b + sqrt(x))/(6*a**
7*x**(5/2) + 6*a**6*b*x**3), True))
```

3.2206 $\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x^4}\,dx$

Optimal. Leaf size=123

$$-\frac{14b^6\log\left(a+b\sqrt{x}\right)}{a^8}+\frac{7b^6\log(x)}{a^8}+\frac{2b^6}{a^7\left(a+b\sqrt{x}\right)}+\frac{12b^5}{a^7\sqrt{x}}-\frac{5b^4}{a^6x}+\frac{8b^3}{3a^5x^{3/2}}-\frac{3b^2}{2a^4x^2}+\frac{4b}{5a^3x^{5/2}}-\frac{1}{3a^2x^3}$$

```
[Out] -1/3/a^2/x^3+4/5*b/a^3/x^(5/2)-3/2*b^2/a^4/x^2+8/3*b^3/a^5/x^(3/2)-5*b^4/a^
6/x+7*b^6*ln(x)/a^8-14*b^6*ln(a+b*x^(1/2))/a^8+12*b^5/a^7/x^(1/2)+2*b^6/a^7
/(a+b*x^(1/2))
```

Rubi [A] time = 0.08, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 44}

$$\frac{8b^3}{3a^5x^{3/2}}-\frac{3b^2}{2a^4x^2}+\frac{2b^6}{a^7\left(a+b\sqrt{x}\right)}+\frac{12b^5}{a^7\sqrt{x}}-\frac{5b^4}{a^6x}-\frac{14b^6\log\left(a+b\sqrt{x}\right)}{a^8}+\frac{7b^6\log(x)}{a^8}+\frac{4b}{5a^3x^{5/2}}-\frac{1}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sqrt[x])^2*x^4),x]
```

```
[Out] (2*b^6)/(a^7*(a + b*Sqrt[x])) - 1/(3*a^2*x^3) + (4*b)/(5*a^3*x^(5/2)) - (3*
b^2)/(2*a^4*x^2) + (8*b^3)/(3*a^5*x^(3/2)) - (5*b^4)/(a^6*x) + (12*b^5)/(a^
7*Sqrt[x]) - (14*b^6*Log[a + b*Sqrt[x]])/a^8 + (7*b^6*Log[x])/a^8
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x^4}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{x^7(a+bx)^2}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int\left(\frac{1}{a^2x^7}-\frac{2b}{a^3x^6}+\frac{3b^2}{a^4x^5}-\frac{4b^3}{a^5x^4}+\frac{5b^4}{a^6x^3}-\frac{6b^5}{a^7x^2}+\frac{7b^6}{a^8x}-\frac{b^7}{a^7(a+bx)^2}-\frac{7b\right.\right.\\ &= \frac{2b^6}{a^7\left(a+b\sqrt{x}\right)}-\frac{1}{3a^2x^3}+\frac{4b}{5a^3x^{5/2}}-\frac{3b^2}{2a^4x^2}+\frac{8b^3}{3a^5x^{3/2}}-\frac{5b^4}{a^6x}+\frac{12b^5}{a^7\sqrt{x}}-\frac{14b^6\log\left(a+b\right.}{a^8}\end{aligned}$$

Mathematica [A] time = 0.18, size = 115, normalized size = 0.93

$$\frac{\frac{a\left(-10a^6+14a^5b\sqrt{x}-21a^4b^2x+35a^3b^3x^{3/2}-70a^2b^4x^2+210ab^5x^{5/2}+420b^6x^3\right)}{x^3\left(a+b\sqrt{x}\right)}-420b^6\log\left(a+b\sqrt{x}\right)+210b^6\log(x)}{30a^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sqrt[x])^2*x^4),x]
```

```
[Out] ((a*(-10*a^6 + 14*a^5*b*Sqrt[x] - 21*a^4*b^2*x + 35*a^3*b^3*x^(3/2) - 70*a^
2*b^4*x^2 + 210*a*b^5*x^(5/2) + 420*b^6*x^3))/((a + b*Sqrt[x])*x^3) - 420*b
^6*Log[a + b*Sqrt[x]] + 210*b^6*Log[x])/(30*a^8)
```

fricas [A] time = 0.81, size = 155, normalized size = 1.26

$$-\frac{210\,a^2b^6x^3-105\,a^4b^4x^2-35\,a^6b^2x-10\,a^8+420\left(b^8x^4-a^2b^6x^3\right)\log\left(b\sqrt{x}+a\right)-420\left(b^8x^4-a^2b^6x^3\right)\log\left(\sqrt{x}\right.}{30\left(a^8b^2x^4-a^{10}x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*x^(1/2))^2,x, algorithm="fricas")
```

```
[Out] -1/30*(210*a^2*b^6*x^3 - 105*a^4*b^4*x^2 - 35*a^6*b^2*x - 10*a^8 + 420*(b^8
*x^4 - a^2*b^6*x^3)*log(b*sqrt(x) + a) - 420*(b^8*x^4 - a^2*b^6*x^3)*log(sq
rt(x)) - 4*(105*a*b^7*x^3 - 70*a^3*b^5*x^2 - 14*a^5*b^3*x - 6*a^7*b)*sqrt(x
))/(a^8*b^2*x^4 - a^10*x^3)
```

giac [A] time = 0.16, size = 112, normalized size = 0.91

$$-\frac{14\,b^6\log\left(\left|b\sqrt{x}+a\right|\right)}{a^8}+\frac{7\,b^6\log\left(|x|\right)}{a^8}+\frac{420\,ab^6x^3+210\,a^2b^5x^{\frac{5}{2}}-70\,a^3b^4x^2+35\,a^4b^3x^{\frac{3}{2}}-21\,a^5b^2x+14\,a^6b\sqrt{x}-}{30\left(b\sqrt{x}+a\right)a^8x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*x^(1/2))^2,x, algorithm="giac")
```

```
[Out] -14*b^6*log(abs(b*sqrt(x) + a))/a^8 + 7*b^6*log(abs(x))/a^8 + 1/30*(420*a*b
^6*x^3 + 210*a^2*b^5*x^(5/2) - 70*a^3*b^4*x^2 + 35*a^4*b^3*x^(3/2) - 21*a^5
*b^2*x + 14*a^6*b*sqrt(x) - 10*a^7)/((b*sqrt(x) + a)*a^8*x^3)
```

maple [A] time = 0.01, size = 106, normalized size = 0.86

$$\frac{2b^6}{\left(b\sqrt{x}+a\right)a^7}+\frac{7b^6\ln(x)}{a^8}-\frac{14b^6\ln\left(b\sqrt{x}+a\right)}{a^8}+\frac{12b^5}{a^7\sqrt{x}}-\frac{5b^4}{a^6x}+\frac{8b^3}{3a^5x^{\frac{3}{2}}}-\frac{3b^2}{2a^4x^2}+\frac{4b}{5a^3x^{\frac{5}{2}}}-\frac{1}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^(1/2)+a)^2,x)
```

```
[Out] -1/3/a^2/x^3+4/5*b/a^3/x^(5/2)-3/2*b^2/a^4/x^2+8/3*b^3/a^5/x^(3/2)-5*b^4/a^
6/x+7*b^6*ln(x)/a^8-14*b^6*ln(b*x^(1/2)+a)/a^8+12*b^5/a^7/x^(1/2)+2*b^6/a^7
/(b*x^(1/2)+a)
```

maxima [A] time = 0.85, size = 110, normalized size = 0.89

$$\frac{420\,b^6x^3+210\,ab^5x^{\frac{5}{2}}-70\,a^2b^4x^2+35\,a^3b^3x^{\frac{3}{2}}-21\,a^4b^2x+14\,a^5b\sqrt{x}-10\,a^6}{30\left(a^7bx^{\frac{7}{2}}+a^8x^3\right)}-\frac{14\,b^6\log\left(b\sqrt{x}+a\right)}{a^8}+\frac{7\,b^6\log(x)}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*x^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/30*(420*b^6*x^3 + 210*a*b^5*x^(5/2) - 70*a^2*b^4*x^2 + 35*a^3*b^3*x^(3/2)
 - 21*a^4*b^2*x + 14*a^5*b*sqrt(x) - 10*a^6)/(a^7*b*x^(7/2) + a^8*x^3) - 14
*b^6*log(b*sqrt(x) + a)/a^8 + 7*b^6*log(x)/a^8
```

mupad [B] time = 0.08, size = 103, normalized size = 0.84

$$\frac{\frac{7\,b\,\sqrt{x}}{15\,a^2}-\frac{1}{3\,a}-\frac{7\,b^2\,x}{10\,a^3}-\frac{7\,b^4\,x^2}{3\,a^5}+\frac{7\,b^3\,x^{3/2}}{6\,a^4}+\frac{14\,b^6\,x^3}{a^7}+\frac{7\,b^5\,x^{5/2}}{a^6}}{a\,x^3+b\,x^{7/2}}-\frac{28\,b^6\,\mathrm{atanh}\left(\frac{2\,b\,\sqrt{x}}{a}+1\right)}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^(1/2))^2),x)
```

```
[Out] ((7*b*x^(1/2))/(15*a^2) - 1/(3*a) - (7*b^2*x)/(10*a^3) - (7*b^4*x^2)/(3*a^5
) + (7*b^3*x^(3/2))/(6*a^4) + (14*b^6*x^3)/a^7 + (7*b^5*x^(5/2))/a^6)/(a*x^
3 + b*x^(7/2)) - (28*b^6*atanh((2*b*x^(1/2))/a + 1))/a^8
```

sympy [A] time = 5.65, size = 400, normalized size = 3.25

$$\begin{cases}\frac{\tilde{\infty}}{x^4} \\ -\frac{1}{4b^2x^4} \\ -\frac{1}{3a^2x^3} \\ -\frac{10a^7\sqrt{x}}{30a^9x^{\frac{7}{2}}+30a^8bx^4}+\frac{14a^6bx}{30a^9x^{\frac{7}{2}}+30a^8bx^4}-\frac{21a^5b^2x^{\frac{3}{2}}}{30a^9x^{\frac{7}{2}}+30a^8bx^4}+\frac{35a^4b^3x^2}{30a^9x^{\frac{7}{2}}+30a^8bx^4}-\frac{70a^3b^4x^{\frac{5}{2}}}{30a^9x^{\frac{7}{2}}+30a^8bx^4}+\frac{210a^2b^5x^3}{30a^9x^{\frac{7}{2}}+30a^8bx^4}+\frac{210ab^6x^{\frac{7}{2}}\,\mathrm{lo}}{30a^9x^{\frac{7}{2}}+30}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a+b*x**(1/2))**2,x)
```

```
[Out] Piecewise((zoo/x**4, Eq(a, 0) & Eq(b, 0)), (-1/(4*b**2*x**4), Eq(a, 0)), (-
1/(3*a**2*x**3), Eq(b, 0)), (-10*a**7*sqrt(x)/(30*a**9*x**(7/2) + 30*a**8*b
*x**4) + 14*a**6*b*x/(30*a**9*x**(7/2) + 30*a**8*b*x**4) - 21*a**5*b**2*x**
(3/2)/(30*a**9*x**(7/2) + 30*a**8*b*x**4) + 35*a**4*b**3*x**2/(30*a**9*x**(
7/2) + 30*a**8*b*x**4) - 70*a**3*b**4*x**(5/2)/(30*a**9*x**(7/2) + 30*a**8*
b*x**4) + 210*a**2*b**5*x**3/(30*a**9*x**(7/2) + 30*a**8*b*x**4) + 210*a*b*
*6*x**(7/2)*log(x)/(30*a**9*x**(7/2) + 30*a**8*b*x**4) - 420*a*b**6*x**(7/2
)*log(a/b + sqrt(x))/(30*a**9*x**(7/2) + 30*a**8*b*x**4) + 420*a*b**6*x**(7
/2)/(30*a**9*x**(7/2) + 30*a**8*b*x**4) + 210*b**7*x**4*log(x)/(30*a**9*x**
(7/2) + 30*a**8*b*x**4) - 420*b**7*x**4*log(a/b + sqrt(x))/(30*a**9*x**(7/2
) + 30*a**8*b*x**4), True))
```

3.2207 $\int \frac{x^3}{\left(a+b\sqrt{x}\right)^3}\,dx$

Optimal. Leaf size=114

$$\frac{a^7}{b^8\left(a+b\sqrt{x}\right)^2}-\frac{14a^6}{b^8\left(a+b\sqrt{x}\right)}-\frac{42a^5\log\left(a+b\sqrt{x}\right)}{b^8}+\frac{30a^4\sqrt{x}}{b^7}-\frac{10a^3x}{b^6}+\frac{4a^2x^{3/2}}{b^5}-\frac{3ax^2}{2b^4}+\frac{2x^{5/2}}{5b^3}$$

```
[Out] -10*a^3*x/b^6+4*a^2*x^(3/2)/b^5-3/2*a*x^2/b^4+2/5*x^(5/2)/b^3-42*a^5*ln(a+b
*x^(1/2))/b^8+30*a^4*x^(1/2)/b^7+a^7/b^8/(a+b*x^(1/2))^2-14*a^6/b^8/(a+b*x^
(1/2))
```

Rubi [A] time = 0.09, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{4a^2x^{3/2}}{b^5}+\frac{a^7}{b^8\left(a+b\sqrt{x}\right)^2}-\frac{14a^6}{b^8\left(a+b\sqrt{x}\right)}+\frac{30a^4\sqrt{x}}{b^7}-\frac{10a^3x}{b^6}-\frac{42a^5\log\left(a+b\sqrt{x}\right)}{b^8}-\frac{3ax^2}{2b^4}+\frac{2x^{5/2}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*Sqrt[x])^3,x]
```

```
[Out] a^7/(b^8*(a + b*Sqrt[x])^2) - (14*a^6)/(b^8*(a + b*Sqrt[x])) + (30*a^4*Sqrt
[x])/b^7 - (10*a^3*x)/b^6 + (4*a^2*x^(3/2))/b^5 - (3*a*x^2)/(2*b^4) + (2*x^
(5/2))/(5*b^3) - (42*a^5*Log[a + b*Sqrt[x]])/b^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(a+b\sqrt{x}\right)^3}\,dx &= 2\,\text{Subst}\left(\int \frac{x^7}{(a+bx)^3}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int\left(\frac{15a^4}{b^7}-\frac{10a^3x}{b^6}+\frac{6a^2x^2}{b^5}-\frac{3ax^3}{b^4}+\frac{x^4}{b^3}-\frac{a^7}{b^7(a+bx)^3}+\frac{7a^6}{b^7(a+bx)^2}-\frac{21a^5}{b^7(a+bx)}\right.\right.\\ &= \frac{a^7}{b^8\left(a+b\sqrt{x}\right)^2}-\frac{14a^6}{b^8\left(a+b\sqrt{x}\right)}+\frac{30a^4\sqrt{x}}{b^7}-\frac{10a^3x}{b^6}+\frac{4a^2x^{3/2}}{b^5}-\frac{3ax^2}{2b^4}+\frac{2x^{5/2}}{5b^3}-\frac{42a^5\log\left(a\right.}{b^8}\end{aligned}$$

Mathematica [A] time = 0.13, size = 107, normalized size = 0.94

$$\frac{\frac{10a^7}{\left(a+b\sqrt{x}\right)^2}-\frac{140a^6}{a+b\sqrt{x}}-420a^5\log\left(a+b\sqrt{x}\right)+300a^4b\sqrt{x}-100a^3b^2x+40a^2b^3x^{3/2}-15ab^4x^2+4b^5x^{5/2}}{10b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*Sqrt[x])^3,x]
```

```
[Out] ((10*a^7)/(a + b*Sqrt[x])^2 - (140*a^6)/(a + b*Sqrt[x]) + 300*a^4*b*Sqrt[x]
 - 100*a^3*b^2*x + 40*a^2*b^3*x^(3/2) - 15*a*b^4*x^2 + 4*b^5*x^(5/2) - 420*
a^5*Log[a + b*Sqrt[x]])/(10*b^8)
```

fricas [A] time = 1.25, size = 157, normalized size = 1.38

$$-\frac{15\,ab^8x^4+70\,a^3b^6x^3-185\,a^5b^4x^2-50\,a^7b^2x+130\,a^9+420\left(a^5b^4x^2-2\,a^7b^2x+a^9\right)\log\left(b\sqrt{x}+a\right)-4\left(b^9x\right.}{10\left(b^{12}x^2-2\,a^2b^{10}x+a^4b^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^(1/2))^3,x, algorithm="fricas")
```

```
[Out] -1/10*(15*a*b^8*x^4 + 70*a^3*b^6*x^3 - 185*a^5*b^4*x^2 - 50*a^7*b^2*x + 130
*a^9 + 420*(a^5*b^4*x^2 - 2*a^7*b^2*x + a^9)*log(b*sqrt(x) + a) - 4*(b^9*x^
4 + 8*a^2*b^7*x^3 + 56*a^4*b^5*x^2 - 175*a^6*b^3*x + 105*a^8*b)*sqrt(x))/(b
^12*x^2 - 2*a^2*b^10*x + a^4*b^8)
```

giac [A] time = 0.17, size = 101, normalized size = 0.89

$$-\frac{42\,a^5\log\left(\left|b\sqrt{x}+a\right|\right)}{b^8}-\frac{14\,a^6b\sqrt{x}+13\,a^7}{\left(b\sqrt{x}+a\right)^2b^8}+\frac{4\,b^{12}x^{\frac{5}{2}}-15\,ab^{11}x^2+40\,a^2b^{10}x^{\frac{3}{2}}-100\,a^3b^9x+300\,a^4b^8\sqrt{x}}{10\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^(1/2))^3,x, algorithm="giac")
```

```
[Out] -42*a^5*log(abs(b*sqrt(x) + a))/b^8 - (14*a^6*b*sqrt(x) + 13*a^7)/((b*sqrt(
x) + a)^2*b^8) + 1/10*(4*b^12*x^(5/2) - 15*a*b^11*x^2 + 40*a^2*b^10*x^(3/2)
 - 100*a^3*b^9*x + 300*a^4*b^8*sqrt(x))/b^15
```

maple [A] time = 0.01, size = 99, normalized size = 0.87

$$\frac{2x^{\frac{5}{2}}}{5b^3}+\frac{a^7}{\left(b\sqrt{x}+a\right)^2b^8}-\frac{3a\,x^2}{2b^4}+\frac{4a^2x^{\frac{3}{2}}}{b^5}-\frac{14a^6}{\left(b\sqrt{x}+a\right)b^8}-\frac{42a^5\ln\left(b\sqrt{x}+a\right)}{b^8}-\frac{10a^3x}{b^6}+\frac{30a^4\sqrt{x}}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^(1/2)+a)^3,x)
```

```
[Out] -10*a^3*x/b^6+4*a^2*x^(3/2)/b^5-3/2*a*x^2/b^4+2/5*x^(5/2)/b^3-42*a^5*ln(b*x
^(1/2)+a)/b^8+30*a^4*x^(1/2)/b^7+a^7/b^8/(b*x^(1/2)+a)^2-14*a^6/b^8/(b*x^(1
/2)+a)
```

maxima [A] time = 0.89, size = 128, normalized size = 1.12

$$-\frac{42\,a^5\log\left(b\sqrt{x}+a\right)}{b^8}+\frac{2\left(b\sqrt{x}+a\right)^5}{5\,b^8}-\frac{7\left(b\sqrt{x}+a\right)^4a}{2\,b^8}+\frac{14\left(b\sqrt{x}+a\right)^3a^2}{b^8}-\frac{35\left(b\sqrt{x}+a\right)^2a^3}{b^8}+\frac{70\left(b\sqrt{x}+a\right)a^4}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^(1/2))^3,x, algorithm="maxima")
```

```
[Out] -42*a^5*log(b*sqrt(x) + a)/b^8 + 2/5*(b*sqrt(x) + a)^5/b^8 - 7/2*(b*sqrt(x)
 + a)^4*a/b^8 + 14*(b*sqrt(x) + a)^3*a^2/b^8 - 35*(b*sqrt(x) + a)^2*a^3/b^8
```

```
 + 70*(b*sqrt(x) + a)*a^4/b^8 - 14*a^6/((b*sqrt(x) + a)*b^8) + a^7/((b*sqrt
(x) + a)^2*b^8)
```

mupad [B] time = 1.13, size = 108, normalized size = 0.95

$$\frac{2\,x^{5/2}}{5\,b^3}-\frac{\frac{13\,a^7}{b}+14\,a^6\,\sqrt{x}}{b^9\,x+a^2\,b^7+2\,a\,b^8\,\sqrt{x}}-\frac{3\,a\,x^2}{2\,b^4}-\frac{10\,a^3\,x}{b^6}-\frac{42\,a^5\,\ln\left(a+b\,\sqrt{x}\right)}{b^8}+\frac{4\,a^2\,x^{3/2}}{b^5}+\frac{30\,a^4\,\sqrt{x}}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^(1/2))^3,x)
```

```
[Out] (2*x^(5/2))/(5*b^3) - ((13*a^7)/b + 14*a^6*x^(1/2))/(b^9*x + a^2*b^7 + 2*a*
b^8*x^(1/2)) - (3*a*x^2)/(2*b^4) - (10*a^3*x)/b^6 - (42*a^5*log(a + b*x^(1/
2)))/b^8 + (4*a^2*x^(3/2))/b^5 + (30*a^4*x^(1/2))/b^7
```

sympy [A] time = 3.16, size = 413, normalized size = 3.62

$$\begin{cases}-\frac{420a^7\log\left(\frac{a}{b}+\sqrt{x}\right)}{10a^2b^8+20ab^9\sqrt{x}+10b^{10}x}-\frac{630a^7}{10a^2b^8+20ab^9\sqrt{x}+10b^{10}x}-\frac{840a^6b\sqrt{x}\log\left(\frac{a}{b}+\sqrt{x}\right)}{10a^2b^8+20ab^9\sqrt{x}+10b^{10}x}-\frac{840a^6b\sqrt{x}}{10a^2b^8+20ab^9\sqrt{x}+10b^{10}x}-\frac{420a^5b^2x\log\left(\frac{a}{b}+\sqrt{x}\right.}{10a^2b^8+20ab^9\sqrt{x}+10b^1} \\ \frac{x^4}{4a^3}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*x**(1/2))**3,x)
```

```
[Out] Piecewise((-420*a**7*log(a/b + sqrt(x))/(10*a**2*b**8 + 20*a*b**9*sqrt(x) +
 10*b**10*x) - 630*a**7/(10*a**2*b**8 + 20*a*b**9*sqrt(x) + 10*b**10*x) - 8
40*a**6*b*sqrt(x)*log(a/b + sqrt(x))/(10*a**2*b**8 + 20*a*b**9*sqrt(x) + 10
*b**10*x) - 840*a**6*b*sqrt(x)/(10*a**2*b**8 + 20*a*b**9*sqrt(x) + 10*b**10
*x) - 420*a**5*b**2*x*log(a/b + sqrt(x))/(10*a**2*b**8 + 20*a*b**9*sqrt(x)
+ 10*b**10*x) + 140*a**4*b**3*x**(3/2)/(10*a**2*b**8 + 20*a*b**9*sqrt(x) +
10*b**10*x) - 35*a**3*b**4*x**2/(10*a**2*b**8 + 20*a*b**9*sqrt(x) + 10*b**1
0*x) + 14*a**2*b**5*x**(5/2)/(10*a**2*b**8 + 20*a*b**9*sqrt(x) + 10*b**10*x
) - 7*a*b**6*x**3/(10*a**2*b**8 + 20*a*b**9*sqrt(x) + 10*b**10*x) + 4*b**7*
x**(7/2)/(10*a**2*b**8 + 20*a*b**9*sqrt(x) + 10*b**10*x), Ne(b, 0)), (x**4/
(4*a**3), True))
```

3.2208 $\int \frac{x^2}{\left(a+b\sqrt{x}\right)^3}\,dx$

Optimal. Leaf size=88

$$\frac{a^5}{b^6\left(a+b\sqrt{x}\right)^2}-\frac{10a^4}{b^6\left(a+b\sqrt{x}\right)}-\frac{20a^3\log\left(a+b\sqrt{x}\right)}{b^6}+\frac{12a^2\sqrt{x}}{b^5}-\frac{3ax}{b^4}+\frac{2x^{3/2}}{3b^3}$$

```
[Out] -3*a*x/b^4+2/3*x^(3/2)/b^3-20*a^3*ln(a+b*x^(1/2))/b^6+12*a^2*x^(1/2)/b^5+a^
5/b^6/(a+b*x^(1/2))^2-10*a^4/b^6/(a+b*x^(1/2))
```

Rubi [A] time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{a^5}{b^6\left(a+b\sqrt{x}\right)^2}-\frac{10a^4}{b^6\left(a+b\sqrt{x}\right)}+\frac{12a^2\sqrt{x}}{b^5}-\frac{20a^3\log\left(a+b\sqrt{x}\right)}{b^6}-\frac{3ax}{b^4}+\frac{2x^{3/2}}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*Sqrt[x])^3,x]
```

```
[Out] a^5/(b^6*(a + b*Sqrt[x])^2) - (10*a^4)/(b^6*(a + b*Sqrt[x])) + (12*a^2*Sqrt
[x])/b^5 - (3*a*x)/b^4 + (2*x^(3/2))/(3*b^3) - (20*a^3*Log[a + b*Sqrt[x]])/
b^6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a+b\sqrt{x}\right)^3}\,dx &= 2\,\text{Subst}\left(\int \frac{x^5}{(a+bx)^3}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int\left(\frac{6a^2}{b^5}-\frac{3ax}{b^4}+\frac{x^2}{b^3}-\frac{a^5}{b^5(a+bx)^3}+\frac{5a^4}{b^5(a+bx)^2}-\frac{10a^3}{b^5(a+bx)}\right)dx, x, \sqrt{x}\right)\\ &= \frac{a^5}{b^6\left(a+b\sqrt{x}\right)^2}-\frac{10a^4}{b^6\left(a+b\sqrt{x}\right)}+\frac{12a^2\sqrt{x}}{b^5}-\frac{3ax}{b^4}+\frac{2x^{3/2}}{3b^3}-\frac{20a^3\log\left(a+b\sqrt{x}\right)}{b^6}\end{aligned}$$

Mathematica [A] time = 0.08, size = 83, normalized size = 0.94

$$\frac{\frac{3a^5}{\left(a+b\sqrt{x}\right)^2}-\frac{30a^4}{a+b\sqrt{x}}-60a^3\log\left(a+b\sqrt{x}\right)+36a^2b\sqrt{x}-9ab^2x+2b^3x^{3/2}}{3b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*Sqrt[x])^3,x]
```

```
[Out] ((3*a^5)/(a + b*Sqrt[x])^2 - (30*a^4)/(a + b*Sqrt[x]) + 36*a^2*b*Sqrt[x] -
9*a*b^2*x + 2*b^3*x^(3/2) - 60*a^3*Log[a + b*Sqrt[x]])/(3*b^6)
```

fricas [A] time = 0.90, size = 135, normalized size = 1.53

$$-\frac{9\,ab^6x^3-18\,a^3b^4x^2-24\,a^5b^2x+27\,a^7+60\left(a^3b^4x^2-2\,a^5b^2x+a^7\right)\log\left(b\sqrt{x}+a\right)-2\left(b^7x^3+16\,a^2b^5x^2-50\,a$$
$$3\left(b^{10}x^2-2\,a^2b^8x+a^4b^6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^(1/2))^3,x, algorithm="fricas")
```

```
[Out] -1/3*(9*a*b^6*x^3 - 18*a^3*b^4*x^2 - 24*a^5*b^2*x + 27*a^7 + 60*(a^3*b^4*x^
2 - 2*a^5*b^2*x + a^7)*log(b*sqrt(x) + a) - 2*(b^7*x^3 + 16*a^2*b^5*x^2 - 5
0*a^4*b^3*x + 30*a^6*b)*sqrt(x))/(b^10*x^2 - 2*a^2*b^8*x + a^4*b^6)
```

giac [A] time = 0.16, size = 79, normalized size = 0.90

$$-\frac{20\,a^3\log\left(\left|b\sqrt{x}+a\right|\right)}{b^6}-\frac{10\,a^4b\sqrt{x}+9\,a^5}{\left(b\sqrt{x}+a\right)^2b^6}+\frac{2\,b^6x^{\frac{3}{2}}-9\,ab^5x+36\,a^2b^4\sqrt{x}}{3\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^(1/2))^3,x, algorithm="giac")
```

```
[Out] -20*a^3*log(abs(b*sqrt(x) + a))/b^6 - (10*a^4*b*sqrt(x) + 9*a^5)/((b*sqrt(x
) + a)^2*b^6) + 1/3*(2*b^6*x^(3/2) - 9*a*b^5*x + 36*a^2*b^4*sqrt(x))/b^9
```

maple [A] time = 0.01, size = 77, normalized size = 0.88

$$\frac{a^5}{\left(b\sqrt{x}+a\right)^2b^6}+\frac{2x^{\frac{3}{2}}}{3b^3}-\frac{10a^4}{\left(b\sqrt{x}+a\right)b^6}-\frac{20a^3\ln\left(b\sqrt{x}+a\right)}{b^6}-\frac{3ax}{b^4}+\frac{12a^2\sqrt{x}}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^(1/2)+a)^3,x)
```

```
[Out] -3*a*x/b^4+2/3*x^(3/2)/b^3-20*a^3*ln(b*x^(1/2)+a)/b^6+12*a^2*x^(1/2)/b^5+a^
5/b^6/(b*x^(1/2)+a)^2-10*a^4/b^6/(b*x^(1/2)+a)
```

maxima [A] time = 0.85, size = 94, normalized size = 1.07

$$-\frac{20\,a^3\log\left(b\sqrt{x}+a\right)}{b^6}+\frac{2\left(b\sqrt{x}+a\right)^3}{3\,b^6}-\frac{5\left(b\sqrt{x}+a\right)^2a}{b^6}+\frac{20\left(b\sqrt{x}+a\right)a^2}{b^6}-\frac{10\,a^4}{\left(b\sqrt{x}+a\right)b^6}+\frac{a^5}{\left(b\sqrt{x}+a\right)^2b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^(1/2))^3,x, algorithm="maxima")
```

```
[Out] -20*a^3*log(b*sqrt(x) + a)/b^6 + 2/3*(b*sqrt(x) + a)^3/b^6 - 5*(b*sqrt(x) +
 a)^2*a/b^6 + 20*(b*sqrt(x) + a)*a^2/b^6 - 10*a^4/((b*sqrt(x) + a)*b^6) + a
^5/((b*sqrt(x) + a)^2*b^6)
```

mupad [B] time = 0.04, size = 86, normalized size = 0.98

$$\frac{2\,x^{3/2}}{3\,b^3}-\frac{\frac{9\,a^5}{b}+10\,a^4\,\sqrt{x}}{b^7\,x+a^2\,b^5+2\,a\,b^6\,\sqrt{x}}-\frac{20\,a^3\,\ln\left(a+b\,\sqrt{x}\right)}{b^6}+\frac{12\,a^2\,\sqrt{x}}{b^5}-\frac{3\,a\,x}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^(1/2))^3,x)
```

```
[Out] (2*x^(3/2))/(3*b^3) - ((9*a^5)/b + 10*a^4*x^(1/2))/(b^7*x + a^2*b^5 + 2*a*b
^6*x^(1/2)) - (20*a^3*log(a + b*x^(1/2)))/b^6 + (12*a^2*x^(1/2))/b^5 - (3*a
*x)/b^4
```

sympy [A] time = 1.46, size = 333, normalized size = 3.78

$$\begin{cases} -\frac{60a^5 \log\left(\frac{a}{b}+\sqrt{x}\right)}{3a^2b^6+6ab^7\sqrt{x}+3b^8x} - \frac{90a^5}{3a^2b^6+6ab^7\sqrt{x}+3b^8x} - \frac{120a^4b\sqrt{x}\log\left(\frac{a}{b}+\sqrt{x}\right)}{3a^2b^6+6ab^7\sqrt{x}+3b^8x} - \frac{120a^4b\sqrt{x}}{3a^2b^6+6ab^7\sqrt{x}+3b^8x} - \frac{60a^3b^2x\log\left(\frac{a}{b}+\sqrt{x}\right)}{3a^2b^6+6ab^7\sqrt{x}+3b^8x} + \frac{20a^2}{3a^2b^6+6al} \\ \frac{x^3}{3a^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*x**(1/2))**3,x)
```

```
[Out] Piecewise((-60*a**5*log(a/b + sqrt(x))/(3*a**2*b**6 + 6*a*b**7*sqrt(x) + 3*
b**8*x) - 90*a**5/(3*a**2*b**6 + 6*a*b**7*sqrt(x) + 3*b**8*x) - 120*a**4*b*
sqrt(x)*log(a/b + sqrt(x))/(3*a**2*b**6 + 6*a*b**7*sqrt(x) + 3*b**8*x) - 12
0*a**4*b*sqrt(x)/(3*a**2*b**6 + 6*a*b**7*sqrt(x) + 3*b**8*x) - 60*a**3*b**2
*x*log(a/b + sqrt(x))/(3*a**2*b**6 + 6*a*b**7*sqrt(x) + 3*b**8*x) + 20*a**2
*b**3*x**(3/2)/(3*a**2*b**6 + 6*a*b**7*sqrt(x) + 3*b**8*x) - 5*a*b**4*x**2/
(3*a**2*b**6 + 6*a*b**7*sqrt(x) + 3*b**8*x) + 2*b**5*x**(5/2)/(3*a**2*b**6
+ 6*a*b**7*sqrt(x) + 3*b**8*x), Ne(b, 0)), (x**3/(3*a**3), True))
```

3.2209 $\int \frac{x}{\left(a+b\sqrt{x}\right)^3}\,dx$

Optimal. Leaf size=64

$$\frac{a^3}{b^4\left(a+b\sqrt{x}\right)^2}-\frac{6a^2}{b^4\left(a+b\sqrt{x}\right)}-\frac{6a\log\left(a+b\sqrt{x}\right)}{b^4}+\frac{2\sqrt{x}}{b^3}$$

```
[Out] -6*a*ln(a+b*x^(1/2))/b^4+2*x^(1/2)/b^3+a^3/b^4/(a+b*x^(1/2))^2-6*a^2/b^4/(a
+b*x^(1/2))
```

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{a^3}{b^4\left(a+b\sqrt{x}\right)^2}-\frac{6a^2}{b^4\left(a+b\sqrt{x}\right)}-\frac{6a\log\left(a+b\sqrt{x}\right)}{b^4}+\frac{2\sqrt{x}}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*Sqrt[x])^3,x]
```

```
[Out] a^3/(b^4*(a + b*Sqrt[x])^2) - (6*a^2)/(b^4*(a + b*Sqrt[x])) + (2*Sqrt[x])/b
^3 - (6*a*Log[a + b*Sqrt[x]])/b^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a+b\sqrt{x}\right)^3}\,dx &= 2\,\text{Subst}\left(\int \frac{x^3}{(a+bx)^3}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(\frac{1}{b^3}-\frac{a^3}{b^3(a+bx)^3}+\frac{3a^2}{b^3(a+bx)^2}-\frac{3a}{b^3(a+bx)}\right)dx, x, \sqrt{x}\right)\\ &= \frac{a^3}{b^4\left(a+b\sqrt{x}\right)^2}-\frac{6a^2}{b^4\left(a+b\sqrt{x}\right)}+\frac{2\sqrt{x}}{b^3}-\frac{6a\log\left(a+b\sqrt{x}\right)}{b^4}\end{aligned}$$

Mathematica [A] time = 0.08, size = 57, normalized size = 0.89

$$\frac{\frac{a^3}{\left(a+b\sqrt{x}\right)^2}-\frac{6a^2}{a+b\sqrt{x}}-6a\log\left(a+b\sqrt{x}\right)+2b\sqrt{x}}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*Sqrt[x])^3,x]
```

```
[Out] (a^3/(a + b*Sqrt[x])^2 - (6*a^2)/(a + b*Sqrt[x]) + 2*b*Sqrt[x] - 6*a*Log[a
+ b*Sqrt[x]])/b^4
```

fricas [A] time = 0.89, size = 101, normalized size = 1.58

$$\frac{7\,a^3b^2x - 5\,a^5 - 6\left(ab^4x^2 - 2\,a^3b^2x + a^5\right)\log\left(b\sqrt{x} + a\right) + 2\left(b^5x^2 - 5\,a^2b^3x + 3\,a^4b\right)\sqrt{x}}{b^8x^2 - 2\,a^2b^6x + a^4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^(1/2))^3,x, algorithm="fricas")
```

```
[Out] (7*a^3*b^2*x - 5*a^5 - 6*(a*b^4*x^2 - 2*a^3*b^2*x + a^5)*log(b*sqrt(x) + a)
 + 2*(b^5*x^2 - 5*a^2*b^3*x + 3*a^4*b)*sqrt(x))/(b^8*x^2 - 2*a^2*b^6*x + a^
4*b^4)
```

giac [A] time = 0.15, size = 53, normalized size = 0.83

$$-\frac{6\,a\log\left(\left|b\sqrt{x} + a\right|\right)}{b^4} + \frac{2\sqrt{x}}{b^3} - \frac{6\,a^2b\sqrt{x} + 5\,a^3}{\left(b\sqrt{x} + a\right)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^(1/2))^3,x, algorithm="giac")
```

```
[Out] -6*a*log(abs(b*sqrt(x) + a))/b^4 + 2*sqrt(x)/b^3 - (6*a^2*b*sqrt(x) + 5*a^3
)/((b*sqrt(x) + a)^2*b^4)
```

maple [A] time = 0.01, size = 57, normalized size = 0.89

$$\frac{a^3}{\left(b\sqrt{x} + a\right)^2b^4} - \frac{6a^2}{\left(b\sqrt{x} + a\right)b^4} - \frac{6a\ln\left(b\sqrt{x} + a\right)}{b^4} + \frac{2\sqrt{x}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^(1/2)+a)^3,x)
```

```
[Out] -6*a*ln(b*x^(1/2)+a)/b^4+2*x^(1/2)/b^3+a^3/b^4/(b*x^(1/2)+a)^2-6*a^2/b^4/(b
*x^(1/2)+a)
```

maxima [A] time = 0.85, size = 60, normalized size = 0.94

$$-\frac{6\,a\log\left(b\sqrt{x} + a\right)}{b^4} + \frac{2\left(b\sqrt{x} + a\right)}{b^4} - \frac{6\,a^2}{\left(b\sqrt{x} + a\right)b^4} + \frac{a^3}{\left(b\sqrt{x} + a\right)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^(1/2))^3,x, algorithm="maxima")
```

```
[Out] -6*a*log(b*sqrt(x) + a)/b^4 + 2*(b*sqrt(x) + a)/b^4 - 6*a^2/((b*sqrt(x) + a
)*b^4) + a^3/((b*sqrt(x) + a)^2*b^4)
```

mupad [B] time = 1.15, size = 66, normalized size = 1.03

$$\frac{2\sqrt{x}}{b^3} - \frac{\frac{5\,a^3}{b} + 6\,a^2\sqrt{x}}{b^5\,x + a^2\,b^3 + 2\,a\,b^4\sqrt{x}} - \frac{6\,a\,\ln\left(a + b\sqrt{x}\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^(1/2))^3,x)
```

```
[Out] (2*x^(1/2))/b^3 - ((5*a^3)/b + 6*a^2*x^(1/2))/(b^5*x + a^2*b^3 + 2*a*b^4*x^
(1/2)) - (6*a*log(a + b*x^(1/2)))/b^4
```

sympy [A] time = 0.95, size = 233, normalized size = 3.64

$$\begin{cases} -\frac{6a^3 \log\left(\frac{a}{b}+\sqrt{x}\right)}{a^2b^4+2ab^5\sqrt{x}+b^6x} - \frac{9a^3}{a^2b^4+2ab^5\sqrt{x}+b^6x} - \frac{12a^2b\sqrt{x}\,\log\left(\frac{a}{b}+\sqrt{x}\right)}{a^2b^4+2ab^5\sqrt{x}+b^6x} - \frac{12a^2b\sqrt{x}}{a^2b^4+2ab^5\sqrt{x}+b^6x} - \frac{6ab^2x\log\left(\frac{a}{b}+\sqrt{x}\right)}{a^2b^4+2ab^5\sqrt{x}+b^6x} + \frac{2b^3x^{\frac{3}{2}}}{a^2b^4+2ab^5\sqrt{x}+b^6x} & \text{fo} \\ \frac{x^2}{2a^3} & \text{ot} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x**(1/2))**3,x)
```

```
[Out] Piecewise((-6*a**3*log(a/b + sqrt(x))/(a**2*b**4 + 2*a*b**5*sqrt(x) + b**6*
x) - 9*a**3/(a**2*b**4 + 2*a*b**5*sqrt(x) + b**6*x) - 12*a**2*b*sqrt(x)*log
(a/b + sqrt(x))/(a**2*b**4 + 2*a*b**5*sqrt(x) + b**6*x) - 12*a**2*b*sqrt(x)
/(a**2*b**4 + 2*a*b**5*sqrt(x) + b**6*x) - 6*a*b**2*x*log(a/b + sqrt(x))/(a
**2*b**4 + 2*a*b**5*sqrt(x) + b**6*x) + 2*b**3*x**(3/2)/(a**2*b**4 + 2*a*b*
*5*sqrt(x) + b**6*x), Ne(b, 0)), (x**2/(2*a**3), True))
```

3.2210 $\int \frac{1}{\left(a+b\sqrt{x}\right)^3}\,dx$

Optimal. Leaf size=16

$$\frac{x}{a\left(a+b\sqrt{x}\right)^2}$$

```
[Out] x/a/(a+b*x^(1/2))^2
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {190, 37}

$$\frac{x}{a\left(a+b\sqrt{x}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^(-3),x]
```

```
[Out] x/(a*(a + b*Sqrt[x])^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{x}{(a+bx)^3}\,dx, x, \sqrt{x}\right)$$
$$= \frac{x}{a\left(a+b\sqrt{x}\right)^2}$$

Mathematica [A] time = 0.02, size = 16, normalized size = 1.00

$$\frac{x}{a\left(a+b\sqrt{x}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^(-3),x]
```

```
[Out] x/(a*(a + b*Sqrt[x])^2)
```

fricas [B] time = 0.63, size = 47, normalized size = 2.94

$$-\frac{2\,b^3x^{\frac{3}{2}} - 3\,ab^2x + a^3}{b^6x^2 - 2\,a^2b^4x + a^4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/2))^3,x, algorithm="fricas")
```

```
[Out] -(2*b^3*x^(3/2) - 3*a*b^2*x + a^3)/(b^6*x^2 - 2*a^2*b^4*x + a^4*b^2)
```

giac [A] time = 0.19, size = 22, normalized size = 1.38

$$-\frac{2\,b\sqrt{x}+a}{\left(b\sqrt{x}+a\right)^2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/2))^3,x, algorithm="giac")
```

```
[Out] -(2*b*sqrt(x) + a)/((b*sqrt(x) + a)^2*b^2)
```

maple [B] time = 0.02, size = 131, normalized size = 8.19

$$-3\left(-\frac{a^2}{2\left(b^2x-a^2\right)^2 b^4}-\frac{1}{\left(b^2x-a^2\right)b^4}\right)a\,b^2+\frac{a^3}{2\left(b^2x-a^2\right)^2 b^2}+\frac{a}{2\left(b\sqrt{x}+a\right)^2 b^2}-\frac{a}{2\left(b\sqrt{x}-a\right)^2 b^2}-\frac{1}{\left(b\sqrt{x}+a\right)b^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^(1/2)+a)^3,x)
```

```
[Out] -1/b^2/(b*x^(1/2)+a)+1/2/b^2*a/(b*x^(1/2)+a)^2-1/b^2/(b*x^(1/2)-a)-1/2/b^2*
a/(b*x^(1/2)-a)^2+1/2*a^3/(b^2*x-a^2)^2/b^2-3*a*b^2*(-1/2*a^2/b^4/(b^2*x-a^
2)^2-1/b^4/(b^2*x-a^2))
```

maxima [B] time = 0.76, size = 29, normalized size = 1.81

$$-\frac{2}{\left(b\sqrt{x}+a\right)b^2}+\frac{a}{\left(b\sqrt{x}+a\right)^2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/2))^3,x, algorithm="maxima")
```

```
[Out] -2/((b*sqrt(x) + a)*b^2) + a/((b*sqrt(x) + a)^2*b^2)
```

mupad [B] time = 1.12, size = 34, normalized size = 2.12

$$-\frac{\frac{a}{b^2}+\frac{2\sqrt{x}}{b}}{b^2\,x+a^2+2\,a\,b\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^(1/2))^3,x)
```

```
[Out] -(a/b^2 + (2*x^(1/2))/b)/(b^2*x + a^2 + 2*a*b*x^(1/2))
```

sympy [A] time = 1.15, size = 63, normalized size = 3.94

$$\begin{cases} -\frac{a}{a^2b^2+2ab^3\sqrt{x}+b^4x}-\frac{2b\sqrt{x}}{a^2b^2+2ab^3\sqrt{x}+b^4x} & \text{for } b\neq 0 \\ \frac{x}{a^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**(1/2))**3,x)
```

```
[Out] Piecewise((-a/(a**2*b**2 + 2*a*b**3*sqrt(x) + b**4*x) - 2*b*sqrt(x)/(a**2*b
**2 + 2*a*b**3*sqrt(x) + b**4*x), Ne(b, 0)), (x/a**3, True))
```

3.2211 $\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x}\,dx$

Optimal. Leaf size=53

$$-\frac{2\log\left(a+b\sqrt{x}\right)}{a^3}+\frac{\log(x)}{a^3}+\frac{2}{a^2\left(a+b\sqrt{x}\right)}+\frac{1}{a\left(a+b\sqrt{x}\right)^2}$$

```
[Out] ln(x)/a^3-2*ln(a+b*x^(1/2))/a^3+1/a/(a+b*x^(1/2))^2+2/a^2/(a+b*x^(1/2))
```

Rubi [A] time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 44}

$$\frac{2}{a^2\left(a+b\sqrt{x}\right)}-\frac{2\log\left(a+b\sqrt{x}\right)}{a^3}+\frac{\log(x)}{a^3}+\frac{1}{a\left(a+b\sqrt{x}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sqrt[x])^3*x),x]
```

```
[Out] 1/(a*(a + b*Sqrt[x])^2) + 2/(a^2*(a + b*Sqrt[x])) - (2*Log[a + b*Sqrt[x]])/
a^3 + Log[x]/a^3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{x(a+bx)^3}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(\frac{1}{a^3 x}-\frac{b}{a(a+bx)^3}-\frac{b}{a^2(a+bx)^2}-\frac{b}{a^3(a+bx)}\right)dx, x, \sqrt{x}\right)\\ &= \frac{1}{a\left(a+b\sqrt{x}\right)^2}+\frac{2}{a^2\left(a+b\sqrt{x}\right)}-\frac{2\log\left(a+b\sqrt{x}\right)}{a^3}+\frac{\log(x)}{a^3}\end{aligned}$$

Mathematica [A] time = 0.06, size = 44, normalized size = 0.83

$$\frac{\frac{a\left(3a+2b\sqrt{x}\right)}{\left(a+b\sqrt{x}\right)^2}-2\log\left(a+b\sqrt{x}\right)+\log(x)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sqrt[x])^3*x),x]
```

```
[Out] ((a*(3*a + 2*b*Sqrt[x]))/(a + b*Sqrt[x])^2 - 2*Log[a + b*Sqrt[x]] + Log[x])
/a^3
```

fricas [B] time = 0.99, size = 115, normalized size = 2.17

$$-\frac{a^2b^2x - 3\,a^4 + 2\left(b^4x^2 - 2\,a^2b^2x + a^4\right)\log\left(b\sqrt{x} + a\right) - 2\left(b^4x^2 - 2\,a^2b^2x + a^4\right)\log\left(\sqrt{x}\right) - 2\left(ab^3x - 2\,a^3b\right)\sqrt{x}}{a^3b^4x^2 - 2\,a^5b^2x + a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^(1/2))^3,x, algorithm="fricas")
```

```
[Out] -(a^2*b^2*x - 3*a^4 + 2*(b^4*x^2 - 2*a^2*b^2*x + a^4)*log(b*sqrt(x) + a) -
2*(b^4*x^2 - 2*a^2*b^2*x + a^4)*log(sqrt(x)) - 2*(a*b^3*x - 2*a^3*b)*sqrt(x
))/(a^3*b^4*x^2 - 2*a^5*b^2*x + a^7)
```

giac [A] time = 0.16, size = 48, normalized size = 0.91

$$-\frac{2\,\log\left(\left|b\sqrt{x} + a\right|\right)}{a^3} + \frac{\log\left(|x|\right)}{a^3} + \frac{2\,ab\sqrt{x} + 3\,a^2}{\left(b\sqrt{x} + a\right)^2 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^(1/2))^3,x, algorithm="giac")
```

```
[Out] -2*log(abs(b*sqrt(x) + a))/a^3 + log(abs(x))/a^3 + (2*a*b*sqrt(x) + 3*a^2)/
((b*sqrt(x) + a)^2*a^3)
```

maple [A] time = 0.01, size = 48, normalized size = 0.91

$$\frac{1}{\left(b\sqrt{x} + a\right)^2 a} + \frac{2}{\left(b\sqrt{x} + a\right) a^2} + \frac{\ln(x)}{a^3} - \frac{2\ln\left(b\sqrt{x} + a\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^(1/2)+a)^3,x)
```

```
[Out] 1/a^3*ln(x)-2*ln(b*x^(1/2)+a)/a^3+1/a/(b*x^(1/2)+a)^2+2/a^2/(b*x^(1/2)+a)
```

maxima [A] time = 0.89, size = 54, normalized size = 1.02

$$\frac{2\,b\sqrt{x} + 3\,a}{a^2b^2x + 2\,a^3b\sqrt{x} + a^4} - \frac{2\,\log\left(b\sqrt{x} + a\right)}{a^3} + \frac{\log(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^(1/2))^3,x, algorithm="maxima")
```

```
[Out] (2*b*sqrt(x) + 3*a)/(a^2*b^2*x + 2*a^3*b*sqrt(x) + a^4) - 2*log(b*sqrt(x) +
 a)/a^3 + log(x)/a^3
```

mupad [B] time = 0.07, size = 52, normalized size = 0.98

$$\frac{\frac{3}{a} + \frac{2\,b\,\sqrt{x}}{a^2}}{b^2\,x + a^2 + 2\,a\,b\,\sqrt{x}} - \frac{4\,\mathrm{atanh}\left(\frac{2\,b\,\sqrt{x}}{a} + 1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^(1/2))^3),x)
```

```
[Out] (3/a + (2*b*x^(1/2))/a^2)/(b^2*x + a^2 + 2*a*b*x^(1/2)) - (4*atanh((2*b*x^(
1/2))/a + 1))/a^3
```

sympy [A] time = 2.59, size = 364, normalized size = 6.87

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} \\ \frac{\log(x)}{a^3} \\ -\frac{2}{3b^3x^{\frac{3}{2}}} \\ \frac{a^2\sqrt{x}\log(x)}{a^5\sqrt{x}+2a^4bx+a^3b^2x^{\frac{3}{2}}} - \frac{2a^2\sqrt{x}\log\left(\frac{a}{b}+\sqrt{x}\right)}{a^5\sqrt{x}+2a^4bx+a^3b^2x^{\frac{3}{2}}} + \frac{3a^2\sqrt{x}}{a^5\sqrt{x}+2a^4bx+a^3b^2x^{\frac{3}{2}}} + \frac{2abx\log(x)}{a^5\sqrt{x}+2a^4bx+a^3b^2x^{\frac{3}{2}}} - \frac{4abx\log\left(\frac{a}{b}+\sqrt{x}\right)}{a^5\sqrt{x}+2a^4bx+a^3b^2x^{\frac{3}{2}}} + \frac{2a}{a^5\sqrt{x}+2a^4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x**(1/2))**3,x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (log(x)/a**3, Eq(b, 0)), (-2
/(3*b**3*x**(3/2)), Eq(a, 0)), (a**2*sqrt(x)*log(x)/(a**5*sqrt(x) + 2*a**4*
b*x + a**3*b**2*x**(3/2)) - 2*a**2*sqrt(x)*log(a/b + sqrt(x))/(a**5*sqrt(x)
 + 2*a**4*b*x + a**3*b**2*x**(3/2)) + 3*a**2*sqrt(x)/(a**5*sqrt(x) + 2*a**4
*b*x + a**3*b**2*x**(3/2)) + 2*a*b*x*log(x)/(a**5*sqrt(x) + 2*a**4*b*x + a*
*3*b**2*x**(3/2)) - 4*a*b*x*log(a/b + sqrt(x))/(a**5*sqrt(x) + 2*a**4*b*x +
 a**3*b**2*x**(3/2)) + 2*a*b*x/(a**5*sqrt(x) + 2*a**4*b*x + a**3*b**2*x**(3
/2)) + b**2*x**(3/2)*log(x)/(a**5*sqrt(x) + 2*a**4*b*x + a**3*b**2*x**(3/2)
) - 2*b**2*x**(3/2)*log(a/b + sqrt(x))/(a**5*sqrt(x) + 2*a**4*b*x + a**3*b*
*2*x**(3/2)), True))
```

3.2212 $\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x^2}\,dx$

Optimal. Leaf size=85

$$-\frac{12b^2\log\left(a+b\sqrt{x}\right)}{a^5}+\frac{6b^2\log(x)}{a^5}+\frac{6b^2}{a^4\left(a+b\sqrt{x}\right)}+\frac{6b}{a^4\sqrt{x}}+\frac{b^2}{a^3\left(a+b\sqrt{x}\right)^2}-\frac{1}{a^3x}$$

```
[Out] -1/a^3/x+6*b^2*ln(x)/a^5-12*b^2*ln(a+b*x^(1/2))/a^5+6*b/a^4/x^(1/2)+b^2/a^3
/(a+b*x^(1/2))^2+6*b^2/a^4/(a+b*x^(1/2))
```

Rubi [A] time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 44}

$$\frac{6b^2}{a^4\left(a+b\sqrt{x}\right)}+\frac{b^2}{a^3\left(a+b\sqrt{x}\right)^2}-\frac{12b^2\log\left(a+b\sqrt{x}\right)}{a^5}+\frac{6b^2\log(x)}{a^5}+\frac{6b}{a^4\sqrt{x}}-\frac{1}{a^3x}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sqrt[x])^3*x^2),x]
```

```
[Out] b^2/(a^3*(a + b*Sqrt[x])^2) + (6*b^2)/(a^4*(a + b*Sqrt[x])) - 1/(a^3*x) + (
6*b)/(a^4*Sqrt[x]) - (12*b^2*Log[a + b*Sqrt[x]])/a^5 + (6*b^2*Log[x])/a^5
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x^2}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{x^3(a+bx)^3}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(\frac{1}{a^3x^3}-\frac{3b}{a^4x^2}+\frac{6b^2}{a^5x}-\frac{b^3}{a^3(a+bx)^3}-\frac{3b^3}{a^4(a+bx)^2}-\frac{6b^3}{a^5(a+bx)}\right)dx, x, \sqrt{x}\right)\\ &= \frac{b^2}{a^3\left(a+b\sqrt{x}\right)^2}+\frac{6b^2}{a^4\left(a+b\sqrt{x}\right)}-\frac{1}{a^3x}+\frac{6b}{a^4\sqrt{x}}-\frac{12b^2\log\left(a+b\sqrt{x}\right)}{a^5}+\frac{6b^2\log(x)}{a^5}\end{aligned}$$

Mathematica [A] time = 0.14, size = 77, normalized size = 0.91

$$\frac{\frac{a\left(-a^3+4a^2b\sqrt{x}+18ab^2x+12b^3x^{3/2}\right)}{x\left(a+b\sqrt{x}\right)^2}-12b^2\log\left(a+b\sqrt{x}\right)+6b^2\log(x)}{a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sqrt[x])^3*x^2),x]
```

```
[Out] ((a*(-a^3 + 4*a^2*b*Sqrt[x] + 18*a*b^2*x + 12*b^3*x^(3/2)))/((a + b*Sqrt[x]
)^2*x) - 12*b^2*Log[a + b*Sqrt[x]] + 6*b^2*Log[x])/a^5
```

fricas [B] time = 0.80, size = 155, normalized size = 1.82

$$-\frac{6\,a^2b^4x^2-9\,a^4b^2x+a^6+12\left(b^6x^3-2\,a^2b^4x^2+a^4b^2x\right)\log\left(b\sqrt{x}+a\right)-12\left(b^6x^3-2\,a^2b^4x^2+a^4b^2x\right)\log\left(\sqrt{}\right.}{a^5b^4x^3-2\,a^7b^2x^2+a^9x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^(1/2))^3,x, algorithm="fricas")
```

```
[Out] -(6*a^2*b^4*x^2 - 9*a^4*b^2*x + a^6 + 12*(b^6*x^3 - 2*a^2*b^4*x^2 + a^4*b^2
*x)*log(b*sqrt(x) + a) - 12*(b^6*x^3 - 2*a^2*b^4*x^2 + a^4*b^2*x)*log(sqrt(
x)) - 2*(6*a*b^5*x^2 - 10*a^3*b^3*x + 3*a^5*b)*sqrt(x))/(a^5*b^4*x^3 - 2*a^
7*b^2*x^2 + a^9*x)
```

giac [A] time = 0.16, size = 74, normalized size = 0.87

$$-\frac{12\,b^2\log\left(\left|b\sqrt{x}+a\right|\right)}{a^5}+\frac{6\,b^2\log\left(|x|\right)}{a^5}+\frac{12\,b^3x^{\frac{3}{2}}+18\,ab^2x+4\,a^2b\sqrt{x}-a^3}{\left(bx+a\sqrt{x}\right)^2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^(1/2))^3,x, algorithm="giac")
```

```
[Out] -12*b^2*log(abs(b*sqrt(x) + a))/a^5 + 6*b^2*log(abs(x))/a^5 + (12*b^3*x^(3/
2) + 18*a*b^2*x + 4*a^2*b*sqrt(x) - a^3)/((b*x + a*sqrt(x))^2*a^4)
```

maple [A] time = 0.01, size = 78, normalized size = 0.92

$$\frac{b^2}{\left(b\sqrt{x}+a\right)^2a^3}+\frac{6b^2}{\left(b\sqrt{x}+a\right)a^4}+\frac{6b^2\ln(x)}{a^5}-\frac{12b^2\ln\left(b\sqrt{x}+a\right)}{a^5}+\frac{6b}{a^4\sqrt{x}}-\frac{1}{a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^(1/2)+a)^3,x)
```

```
[Out] -1/a^3/x+6*b^2*ln(x)/a^5-12*b^2*ln(b*x^(1/2)+a)/a^5+6*b/a^4/x^(1/2)+b^2/a^3
/(b*x^(1/2)+a)^2+6*b^2/a^4/(b*x^(1/2)+a)
```

maxima [A] time = 0.89, size = 85, normalized size = 1.00

$$\frac{12\,b^3x^{\frac{3}{2}}+18\,ab^2x+4\,a^2b\sqrt{x}-a^3}{a^4b^2x^2+2\,a^5bx^{\frac{3}{2}}+a^6x}-\frac{12\,b^2\log\left(b\sqrt{x}+a\right)}{a^5}+\frac{6\,b^2\log(x)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^(1/2))^3,x, algorithm="maxima")
```

```
[Out] (12*b^3*x^(3/2) + 18*a*b^2*x + 4*a^2*b*sqrt(x) - a^3)/(a^4*b^2*x^2 + 2*a^5*
b*x^(3/2) + a^6*x) - 12*b^2*log(b*sqrt(x) + a)/a^5 + 6*b^2*log(x)/a^5
```

mupad [B] time = 1.17, size = 79, normalized size = 0.93

$$\frac{\frac{4\,b\,\sqrt{x}}{a^2}-\frac{1}{a}+\frac{18\,b^2\,x}{a^3}+\frac{12\,b^3\,x^{3/2}}{a^4}}{a^2\,x+b^2\,x^2+2\,a\,b\,x^{3/2}}-\frac{24\,b^2\,\operatorname{atanh}\left(\frac{2\,b\,\sqrt{x}}{a}+1\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^(1/2))^3),x)
```

```
[Out] ((4*b*x^(1/2))/a^2 - 1/a + (18*b^2*x)/a^3 + (12*b^3*x^(3/2))/a^4)/(a^2*x +
b^2*x^2 + 2*a*b*x^(3/2)) - (24*b^2*atanh((2*b*x^(1/2))/a + 1))/a^5
```

sympy [A] time = 4.98, size = 481, normalized size = 5.66

$$\begin{cases} \frac{\infty}{x^{\frac{5}{2}}} \\ -\frac{1}{a^3 x} \\ -\frac{2}{5b^3 x^{\frac{5}{2}}} \\ -\frac{a^4\sqrt{x}}{a^7 x^{\frac{3}{2}} + 2a^6 b x^2 + a^5 b^2 x^{\frac{5}{2}}} + \frac{4a^3 bx}{a^7 x^{\frac{3}{2}} + 2a^6 b x^2 + a^5 b^2 x^{\frac{5}{2}}} + \frac{6a^2 b^2 x^{\frac{3}{2}} \log(x)}{a^7 x^{\frac{3}{2}} + 2a^6 b x^2 + a^5 b^2 x^{\frac{5}{2}}} - \frac{12a^2 b^2 x^{\frac{3}{2}} \log\left(\frac{a}{b} + \sqrt{x}\right)}{a^7 x^{\frac{3}{2}} + 2a^6 b x^2 + a^5 b^2 x^{\frac{5}{2}}} + \frac{18a^2 b^2 x^{\frac{3}{2}}}{a^7 x^{\frac{3}{2}} + 2a^6 b x^2 + a^5 b^2 x^{\frac{5}{2}}} + \frac{12ab^3 x}{a^7 x^{\frac{3}{2}} + 2a^6 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*x**(1/2))**3,x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-1/(a**3*x), Eq(b, 0)), (-2
/(5*b**3*x**(5/2)), Eq(a, 0)), (-a**4*sqrt(x)/(a**7*x**(3/2) + 2*a**6*b*x**
2 + a**5*b**2*x**(5/2)) + 4*a**3*b*x/(a**7*x**(3/2) + 2*a**6*b*x**2 + a**5*
b**2*x**(5/2)) + 6*a**2*b**2*x**(3/2)*log(x)/(a**7*x**(3/2) + 2*a**6*b*x**2
 + a**5*b**2*x**(5/2)) - 12*a**2*b**2*x**(3/2)*log(a/b + sqrt(x))/(a**7*x**
(3/2) + 2*a**6*b*x**2 + a**5*b**2*x**(5/2)) + 18*a**2*b**2*x**(3/2)/(a**7*x
**(3/2) + 2*a**6*b*x**2 + a**5*b**2*x**(5/2)) + 12*a*b**3*x**2*log(x)/(a**7
*x**(3/2) + 2*a**6*b*x**2 + a**5*b**2*x**(5/2)) - 24*a*b**3*x**2*log(a/b +
sqrt(x))/(a**7*x**(3/2) + 2*a**6*b*x**2 + a**5*b**2*x**(5/2)) + 12*a*b**3*x
**2/(a**7*x**(3/2) + 2*a**6*b*x**2 + a**5*b**2*x**(5/2)) + 6*b**4*x**(5/2)*
log(x)/(a**7*x**(3/2) + 2*a**6*b*x**2 + a**5*b**2*x**(5/2)) - 12*b**4*x**(5
/2)*log(a/b + sqrt(x))/(a**7*x**(3/2) + 2*a**6*b*x**2 + a**5*b**2*x**(5/2))
, True))
```

3.2213 $\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x^3}\,dx$

Optimal. Leaf size=111

$$-\frac{30b^4\log\left(a+b\sqrt{x}\right)}{a^7}+\frac{15b^4\log(x)}{a^7}+\frac{10b^4}{a^6\left(a+b\sqrt{x}\right)}+\frac{20b^3}{a^6\sqrt{x}}+\frac{b^4}{a^5\left(a+b\sqrt{x}\right)^2}-\frac{6b^2}{a^5x}+\frac{2b}{a^4x^{3/2}}-\frac{1}{2a^3x^2}$$

```
[Out] -1/2/a^3/x^2+2*b/a^4/x^(3/2)-6*b^2/a^5/x+15*b^4*ln(x)/a^7-30*b^4*ln(a+b*x^(
1/2))/a^7+20*b^3/a^6/x^(1/2)+b^4/a^5/(a+b*x^(1/2))^2+10*b^4/a^6/(a+b*x^(1/2
))
```

Rubi [A] time = 0.07, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 44}

$$\frac{10b^4}{a^6\left(a+b\sqrt{x}\right)}+\frac{b^4}{a^5\left(a+b\sqrt{x}\right)^2}+\frac{20b^3}{a^6\sqrt{x}}-\frac{6b^2}{a^5x}-\frac{30b^4\log\left(a+b\sqrt{x}\right)}{a^7}+\frac{15b^4\log(x)}{a^7}+\frac{2b}{a^4x^{3/2}}-\frac{1}{2a^3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sqrt[x])^3*x^3),x]
```

```
[Out] b^4/(a^5*(a + b*Sqrt[x])^2) + (10*b^4)/(a^6*(a + b*Sqrt[x])) - 1/(2*a^3*x^2
) + (2*b)/(a^4*x^(3/2)) - (6*b^2)/(a^5*x) + (20*b^3)/(a^6*Sqrt[x]) - (30*b^
4*Log[a + b*Sqrt[x]])/a^7 + (15*b^4*Log[x])/a^7
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x^3}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^5(a+bx)^3}\,dx, x, \sqrt{x}\right)$$

$$= 2\,\text{Subst}\left(\int \left(\frac{1}{a^3x^5}-\frac{3b}{a^4x^4}+\frac{6b^2}{a^5x^3}-\frac{10b^3}{a^6x^2}+\frac{15b^4}{a^7x}-\frac{b^5}{a^5(a+bx)^3}-\frac{5b^5}{a^6(a+bx)^2}-\frac{15}{a^7(a+\right.\right.$$

$$= \frac{b^4}{a^5\left(a+b\sqrt{x}\right)^2}+\frac{10b^4}{a^6\left(a+b\sqrt{x}\right)}-\frac{1}{2a^3x^2}+\frac{2b}{a^4x^{3/2}}-\frac{6b^2}{a^5x}+\frac{20b^3}{a^6\sqrt{x}}-\frac{30b^4\log\left(a+b\sqrt{x}\right.}{a^7}$$

Mathematica [A] time = 0.18, size = 104, normalized size = 0.94

$$\frac{\frac{a\left(-a^5+2a^4b\sqrt{x}-5a^3b^2x+20a^2b^3x^{3/2}+90ab^4x^2+60b^5x^{5/2}\right)}{x^2\left(a+b\sqrt{x}\right)^2}-60b^4\log\left(a+b\sqrt{x}\right)+30b^4\log(x)}{2a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sqrt[x])^3*x^3),x]
```

```
[Out] ((a*(-a^5 + 2*a^4*b*Sqrt[x] - 5*a^3*b^2*x + 20*a^2*b^3*x^(3/2) + 90*a*b^4*x
^2 + 60*b^5*x^(5/2)))/((a + b*Sqrt[x])^2*x^2) - 60*b^4*Log[a + b*Sqrt[x]] +
 30*b^4*Log[x])/(2*a^7)
```

fricas [A] time = 0.85, size = 182, normalized size = 1.64

$$-\frac{30\,a^2b^6x^3-45\,a^4b^4x^2+10\,a^6b^2x+a^8+60\left(b^8x^4-2\,a^2b^6x^3+a^4b^4x^2\right)\log\left(b\sqrt{x}+a\right)-60\left(b^8x^4-2\,a^2b^6x^3+a\right.}{2\left(a^7b^4x^4-2\,a^9b^2x^3+a^{11}x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^(1/2))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(30*a^2*b^6*x^3 - 45*a^4*b^4*x^2 + 10*a^6*b^2*x + a^8 + 60*(b^8*x^4 -
2*a^2*b^6*x^3 + a^4*b^4*x^2)*log(b*sqrt(x) + a) - 60*(b^8*x^4 - 2*a^2*b^6*x
^3 + a^4*b^4*x^2)*log(sqrt(x)) - 4*(15*a*b^7*x^3 - 25*a^3*b^5*x^2 + 8*a^5*b
^3*x + a^7*b)*sqrt(x))/(a^7*b^4*x^4 - 2*a^9*b^2*x^3 + a^11*x^2)
```

giac [A] time = 0.16, size = 101, normalized size = 0.91

$$-\frac{30\,b^4\log\left(\left|b\sqrt{x}+a\right|\right)}{a^7}+\frac{15\,b^4\log\left(|x|\right)}{a^7}+\frac{60\,ab^5x^{\frac{5}{2}}+90\,a^2b^4x^2+20\,a^3b^3x^{\frac{3}{2}}-5\,a^4b^2x+2\,a^5b\sqrt{x}-a^6}{2\left(b\sqrt{x}+a\right)^2a^7x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^(1/2))^3,x, algorithm="giac")
```

```
[Out] -30*b^4*log(abs(b*sqrt(x) + a))/a^7 + 15*b^4*log(abs(x))/a^7 + 1/2*(60*a*b^
5*x^(5/2) + 90*a^2*b^4*x^2 + 20*a^3*b^3*x^(3/2) - 5*a^4*b^2*x + 2*a^5*b*sqr
t(x) - a^6)/((b*sqrt(x) + a)^2*a^7*x^2)
```

maple [A] time = 0.01, size = 100, normalized size = 0.90

$$\frac{b^4}{\left(b\sqrt{x}+a\right)^2a^5}+\frac{10b^4}{\left(b\sqrt{x}+a\right)a^6}+\frac{15b^4\ln(x)}{a^7}-\frac{30b^4\ln\left(b\sqrt{x}+a\right)}{a^7}+\frac{20b^3}{a^6\sqrt{x}}-\frac{6b^2}{a^5x}+\frac{2b}{a^4x^{\frac{3}{2}}}-\frac{1}{2a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^(1/2)+a)^3,x)
```

```
[Out] -1/2/a^3/x^2+2*b/a^4/x^(3/2)-6*b^2/a^5/x+15*b^4*ln(x)/a^7-30*b^4*ln(b*x^(1/
2)+a)/a^7+20*b^3/a^6/x^(1/2)+b^4/a^5/(b*x^(1/2)+a)^2+10*b^4/a^6/(b*x^(1/2)+
a)
```

maxima [A] time = 0.88, size = 110, normalized size = 0.99

$$\frac{60\,b^5x^{\frac{5}{2}}+90\,ab^4x^2+20\,a^2b^3x^{\frac{3}{2}}-5\,a^3b^2x+2\,a^4b\sqrt{x}-a^5}{2\left(a^6b^2x^3+2\,a^7bx^{\frac{5}{2}}+a^8x^2\right)}-\frac{30\,b^4\log\left(b\sqrt{x}+a\right)}{a^7}+\frac{15\,b^4\log(x)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^(1/2))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(60*b^5*x^(5/2) + 90*a*b^4*x^2 + 20*a^2*b^3*x^(3/2) - 5*a^3*b^2*x + 2*a
^4*b*sqrt(x) - a^5)/(a^6*b^2*x^3 + 2*a^7*b*x^(5/2) + a^8*x^2) - 30*b^4*log(
b*sqrt(x) + a)/a^7 + 15*b^4*log(x)/a^7
```

mupad [B] time = 0.11, size = 102, normalized size = 0.92

$$\frac{\frac{b\sqrt{x}}{a^2}-\frac{1}{2a}-\frac{5b^2x}{2a^3}+\frac{45b^4x^2}{a^5}+\frac{10b^3x^{3/2}}{a^4}+\frac{30b^5x^{5/2}}{a^6}}{a^2x^2+b^2x^3+2abx^{5/2}}-\frac{60b^4\operatorname{atanh}\left(\frac{2b\sqrt{x}}{a}+1\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^(1/2))^3),x)
```

```
[Out] ((b*x^(1/2))/a^2 - 1/(2*a) - (5*b^2*x)/(2*a^3) + (45*b^4*x^2)/a^5 + (10*b^3
*x^(3/2))/a^4 + (30*b^5*x^(5/2))/a^6)/(a^2*x^2 + b^2*x^3 + 2*a*b*x^(5/2)) -
 (60*b^4*atanh((2*b*x^(1/2))/a + 1))/a^7
```

sympy [A] time = 11.06, size = 612, normalized size = 5.51

$$\begin{cases}\frac{\tilde{\infty}}{x^{\frac{7}{2}}} \\ -\frac{2}{7b^3x^{\frac{7}{2}}} \\ -\frac{1}{2a^3x^2} \\ -\frac{a^6\sqrt{x}}{2a^9x^{\frac{5}{2}}+4a^8bx^3+2a^7b^2x^{\frac{7}{2}}}+\frac{2a^5bx}{2a^9x^{\frac{5}{2}}+4a^8bx^3+2a^7b^2x^{\frac{7}{2}}}-\frac{5a^4b^2x^{\frac{3}{2}}}{2a^9x^{\frac{5}{2}}+4a^8bx^3+2a^7b^2x^{\frac{7}{2}}}+\frac{20a^3b^3x^2}{2a^9x^{\frac{5}{2}}+4a^8bx^3+2a^7b^2x^{\frac{7}{2}}}+\frac{30a^2b^4x^{\frac{5}{2}}\log(x)}{2a^9x^{\frac{5}{2}}+4a^8bx^3+2a^7b^2x}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*x**(1/2))**3,x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(7*b**3*x**(7/2)), Eq(a,
 0)), (-1/(2*a**3*x**2), Eq(b, 0)), (-a**6*sqrt(x)/(2*a**9*x**(5/2) + 4*a**
8*b*x**3 + 2*a**7*b**2*x**(7/2)) + 2*a**5*b*x/(2*a**9*x**(5/2) + 4*a**8*b*x
**3 + 2*a**7*b**2*x**(7/2)) - 5*a**4*b**2*x**(3/2)/(2*a**9*x**(5/2) + 4*a**
8*b*x**3 + 2*a**7*b**2*x**(7/2)) + 20*a**3*b**3*x**2/(2*a**9*x**(5/2) + 4*a
**8*b*x**3 + 2*a**7*b**2*x**(7/2)) + 30*a**2*b**4*x**(5/2)*log(x)/(2*a**9*x
**(5/2) + 4*a**8*b*x**3 + 2*a**7*b**2*x**(7/2)) - 60*a**2*b**4*x**(5/2)*log
(a/b + sqrt(x))/(2*a**9*x**(5/2) + 4*a**8*b*x**3 + 2*a**7*b**2*x**(7/2)) +
90*a**2*b**4*x**(5/2)/(2*a**9*x**(5/2) + 4*a**8*b*x**3 + 2*a**7*b**2*x**(7/
2)) + 60*a*b**5*x**3*log(x)/(2*a**9*x**(5/2) + 4*a**8*b*x**3 + 2*a**7*b**2*
x**(7/2)) - 120*a*b**5*x**3*log(a/b + sqrt(x))/(2*a**9*x**(5/2) + 4*a**8*b*
x**3 + 2*a**7*b**2*x**(7/2)) + 60*a*b**5*x**3/(2*a**9*x**(5/2) + 4*a**8*b*x
**3 + 2*a**7*b**2*x**(7/2)) + 30*b**6*x**(7/2)*log(x)/(2*a**9*x**(5/2) + 4*
a**8*b*x**3 + 2*a**7*b**2*x**(7/2)) - 60*b**6*x**(7/2)*log(a/b + sqrt(x))/(
2*a**9*x**(5/2) + 4*a**8*b*x**3 + 2*a**7*b**2*x**(7/2)), True))
```

3.2214 $\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x^4}\,dx$

Optimal. Leaf size=139

$$-\frac{56b^6\log\left(a+b\sqrt{x}\right)}{a^9}+\frac{28b^6\log(x)}{a^9}+\frac{14b^6}{a^8\left(a+b\sqrt{x}\right)}+\frac{42b^5}{a^8\sqrt{x}}+\frac{b^6}{a^7\left(a+b\sqrt{x}\right)^2}-\frac{15b^4}{a^7x}+\frac{20b^3}{3a^6x^{3/2}}-\frac{3b^2}{a^5x^2}+\frac{6b}{5a^4x^{5/2}}-\frac{1}{3a^3x}$$

```
[Out] -1/3/a^3/x^3+6/5*b/a^4/x^(5/2)-3*b^2/a^5/x^2+20/3*b^3/a^6/x^(3/2)-15*b^4/a^
7/x+28*b^6*ln(x)/a^9-56*b^6*ln(a+b*x^(1/2))/a^9+42*b^5/a^8/x^(1/2)+b^6/a^7/
(a+b*x^(1/2))^2+14*b^6/a^8/(a+b*x^(1/2))
```

Rubi [A] time = 0.10, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 44}

$$\frac{20b^3}{3a^6x^{3/2}}-\frac{3b^2}{a^5x^2}+\frac{14b^6}{a^8\left(a+b\sqrt{x}\right)}+\frac{b^6}{a^7\left(a+b\sqrt{x}\right)^2}+\frac{42b^5}{a^8\sqrt{x}}-\frac{15b^4}{a^7x}-\frac{56b^6\log\left(a+b\sqrt{x}\right)}{a^9}+\frac{28b^6\log(x)}{a^9}+\frac{6b}{5a^4x^{5/2}}-\frac{1}{3a^3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sqrt[x])^3*x^4),x]
```

```
[Out] b^6/(a^7*(a + b*Sqrt[x])^2) + (14*b^6)/(a^8*(a + b*Sqrt[x])) - 1/(3*a^3*x^3
) + (6*b)/(5*a^4*x^(5/2)) - (3*b^2)/(a^5*x^2) + (20*b^3)/(3*a^6*x^(3/2)) -
(15*b^4)/(a^7*x) + (42*b^5)/(a^8*Sqrt[x]) - (56*b^6*Log[a + b*Sqrt[x]])/a^9
 + (28*b^6*Log[x])/a^9
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x^4}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{x^7(a+bx)^3}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int\left(\frac{1}{a^3x^7}-\frac{3b}{a^4x^6}+\frac{6b^2}{a^5x^5}-\frac{10b^3}{a^6x^4}+\frac{15b^4}{a^7x^3}-\frac{21b^5}{a^8x^2}+\frac{28b^6}{a^9x}-\frac{b^7}{a^7(a+bx)^3}-\frac{7b^7}{a^8(a+b\right.\right.\\ &= \frac{b^6}{a^7\left(a+b\sqrt{x}\right)^2}+\frac{14b^6}{a^8\left(a+b\sqrt{x}\right)}-\frac{1}{3a^3x^3}+\frac{6b}{5a^4x^{5/2}}-\frac{3b^2}{a^5x^2}+\frac{20b^3}{3a^6x^{3/2}}-\frac{15b^4}{a^7x}+\frac{42b^5}{a^8\sqrt{x}}-\frac{5}{}\end{aligned}$$

Mathematica [A] time = 0.22, size = 128, normalized size = 0.92

$$\frac{\frac{a\left(-5a^7+8a^6b\sqrt{x}-14a^5b^2x+28a^4b^3x^{3/2}-70a^3b^4x^2+280a^2b^5x^{5/2}+1260ab^6x^3+840b^7x^{7/2}\right)}{x^3\left(a+b\sqrt{x}\right)^2}-840b^6\log\left(a+b\sqrt{x}\right)+420b^6\log(x)}{15a^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sqrt[x])^3*x^4),x]
```

```
[Out] ((a*(-5*a^7 + 8*a^6*b*Sqrt[x] - 14*a^5*b^2*x + 28*a^4*b^3*x^(3/2) - 70*a^3*
b^4*x^2 + 280*a^2*b^5*x^(5/2) + 1260*a*b^6*x^3 + 840*b^7*x^(7/2)))/((a + b*
Sqrt[x])^2*x^3) - 840*b^6*Log[a + b*Sqrt[x]] + 420*b^6*Log[x])/(15*a^9)
```

fricas [A] time = 1.25, size = 207, normalized size = 1.49

$$-\frac{420\,a^2b^8x^4-630\,a^4b^6x^3+140\,a^6b^4x^2+35\,a^8b^2x+5\,a^{10}+840\left(b^{10}x^5-2\,a^2b^8x^4+a^4b^6x^3\right)\log\left(b\sqrt{x}+a\right)-}{15\left(a^9b^4x^5-2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*x^(1/2))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(420*a^2*b^8*x^4 - 630*a^4*b^6*x^3 + 140*a^6*b^4*x^2 + 35*a^8*b^2*x +
 5*a^10 + 840*(b^10*x^5 - 2*a^2*b^8*x^4 + a^4*b^6*x^3)*log(b*sqrt(x) + a) -
 840*(b^10*x^5 - 2*a^2*b^8*x^4 + a^4*b^6*x^3)*log(sqrt(x)) - 2*(420*a*b^9*x
^4 - 700*a^3*b^7*x^3 + 224*a^5*b^5*x^2 + 32*a^7*b^3*x + 9*a^9*b)*sqrt(x))/(
a^9*b^4*x^5 - 2*a^11*b^2*x^4 + a^13*x^3)
```

giac [A] time = 0.18, size = 123, normalized size = 0.88

$$-\frac{56\,b^6\log\left(\left|b\sqrt{x}+a\right|\right)}{a^9}+\frac{28\,b^6\log\left(|x|\right)}{a^9}+\frac{840\,ab^7x^{\frac{7}{2}}+1260\,a^2b^6x^3+280\,a^3b^5x^{\frac{5}{2}}-70\,a^4b^4x^2+28\,a^5b^3x^{\frac{3}{2}}-14\,a}{15\left(b\sqrt{x}+a\right)^2a^9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*x^(1/2))^3,x, algorithm="giac")
```

```
[Out] -56*b^6*log(abs(b*sqrt(x) + a))/a^9 + 28*b^6*log(abs(x))/a^9 + 1/15*(840*a*
b^7*x^(7/2) + 1260*a^2*b^6*x^3 + 280*a^3*b^5*x^(5/2) - 70*a^4*b^4*x^2 + 28*
a^5*b^3*x^(3/2) - 14*a^6*b^2*x + 8*a^7*b*sqrt(x) - 5*a^8)/((b*sqrt(x) + a)^
2*a^9*x^3)
```

maple [A] time = 0.01, size = 122, normalized size = 0.88

$$\frac{b^6}{\left(b\sqrt{x}+a\right)^2a^7}+\frac{14b^6}{\left(b\sqrt{x}+a\right)a^8}+\frac{28b^6\ln(x)}{a^9}-\frac{56b^6\ln\left(b\sqrt{x}+a\right)}{a^9}+\frac{42b^5}{a^8\sqrt{x}}-\frac{15b^4}{a^7x}+\frac{20b^3}{3a^6x^{\frac{3}{2}}}-\frac{3b^2}{a^5x^2}+\frac{6b}{5a^4x^{\frac{5}{2}}}-\frac{1}{3a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^(1/2)+a)^3,x)
```

```
[Out] -1/3/a^3/x^3+6/5*b/a^4/x^(5/2)-3*b^2/a^5/x^2+20/3*b^3/a^6/x^(3/2)-15*b^4/a^
7/x+28*b^6*ln(x)/a^9-56*b^6*ln(b*x^(1/2)+a)/a^9+42*b^5/a^8/x^(1/2)+b^6/a^7/
(b*x^(1/2)+a)^2+14*b^6/a^8/(b*x^(1/2)+a)
```

maxima [A] time = 0.97, size = 132, normalized size = 0.95

$$\frac{840\,b^7x^{\frac{7}{2}}+1260\,ab^6x^3+280\,a^2b^5x^{\frac{5}{2}}-70\,a^3b^4x^2+28\,a^4b^3x^{\frac{3}{2}}-14\,a^5b^2x+8\,a^6b\sqrt{x}-5\,a^7}{15\left(a^8b^2x^4+2\,a^9bx^{\frac{7}{2}}+a^{10}x^3\right)}-\frac{56\,b^6\log\left(b\sqrt{x}+\right.}{a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*x^(1/2))^3,x, algorithm="maxima")
```

```
[Out] 1/15*(840*b^7*x^(7/2) + 1260*a*b^6*x^3 + 280*a^2*b^5*x^(5/2) - 70*a^3*b^4*x
^2 + 28*a^4*b^3*x^(3/2) - 14*a^5*b^2*x + 8*a^6*b*sqrt(x) - 5*a^7)/(a^8*b^2*
x^4 + 2*a^9*b*x^(7/2) + a^10*x^3) - 56*b^6*log(b*sqrt(x) + a)/a^9 + 28*b^6*
log(x)/a^9
```

mupad [B] time = 1.22, size = 125, normalized size = 0.90

$$\frac{\frac{8\,b\,\sqrt{x}}{15\,a^2}-\frac{1}{3\,a}-\frac{14\,b^2\,x}{15\,a^3}-\frac{14\,b^4\,x^2}{3\,a^5}+\frac{28\,b^3\,x^{3/2}}{15\,a^4}+\frac{84\,b^6\,x^3}{a^7}+\frac{56\,b^5\,x^{5/2}}{3\,a^6}+\frac{56\,b^7\,x^{7/2}}{a^8}}{a^2\,x^3+b^2\,x^4+2\,a\,b\,x^{7/2}}-\frac{112\,b^6\,\operatorname{atanh}\left(\frac{2\,b\,\sqrt{x}}{a}+1\right)}{a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^(1/2))^3),x)
```

```
[Out] ((8*b*x^(1/2))/(15*a^2) - 1/(3*a) - (14*b^2*x)/(15*a^3) - (14*b^4*x^2)/(3*a
^5) + (28*b^3*x^(3/2))/(15*a^4) + (84*b^6*x^3)/a^7 + (56*b^5*x^(5/2))/(3*a^
6) + (56*b^7*x^(7/2))/a^8)/(a^2*x^3 + b^2*x^4 + 2*a*b*x^(7/2)) - (112*b^6*a
tanh((2*b*x^(1/2))/a + 1))/a^9
```

sympy [A] time = 19.33, size = 707, normalized size = 5.09

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{9}{2}}} & \\ -\frac{1}{3a^3x^3} & \\ -\frac{2}{9b^3x^{\frac{9}{2}}} & \\ -\frac{5a^8\sqrt{x}}{15a^{11}x^{\frac{7}{2}}+30a^{10}bx^4+15a^9b^2x^{\frac{9}{2}}}+\frac{8a^7bx}{15a^{11}x^{\frac{7}{2}}+30a^{10}bx^4+15a^9b^2x^{\frac{9}{2}}}-\frac{14a^6b^2x^{\frac{3}{2}}}{15a^{11}x^{\frac{7}{2}}+30a^{10}bx^4+15a^9b^2x^{\frac{9}{2}}}+\frac{28a^5b^3x^2}{15a^{11}x^{\frac{7}{2}}+30a^{10}bx^4+15a^9b^2x^{\frac{9}{2}}}-\frac{}{15a^{11} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a+b*x**(1/2))**3,x)
```

```
[Out] Piecewise((zoo/x**(9/2), Eq(a, 0) & Eq(b, 0)), (-1/(3*a**3*x**3), Eq(b, 0))
, (-2/(9*b**3*x**(9/2)), Eq(a, 0)), (-5*a**8*sqrt(x)/(15*a**11*x**(7/2) + 3
0*a**10*b*x**4 + 15*a**9*b**2*x**(9/2)) + 8*a**7*b*x/(15*a**11*x**(7/2) + 3
0*a**10*b*x**4 + 15*a**9*b**2*x**(9/2)) - 14*a**6*b**2*x**(3/2)/(15*a**11*x
**(7/2) + 30*a**10*b*x**4 + 15*a**9*b**2*x**(9/2)) + 28*a**5*b**3*x**2/(15*
a**11*x**(7/2) + 30*a**10*b*x**4 + 15*a**9*b**2*x**(9/2)) - 70*a**4*b**4*x*
*(5/2)/(15*a**11*x**(7/2) + 30*a**10*b*x**4 + 15*a**9*b**2*x**(9/2)) + 280*
a**3*b**5*x**3/(15*a**11*x**(7/2) + 30*a**10*b*x**4 + 15*a**9*b**2*x**(9/2)
) + 420*a**2*b**6*x**(7/2)*log(x)/(15*a**11*x**(7/2) + 30*a**10*b*x**4 + 15
*a**9*b**2*x**(9/2)) - 840*a**2*b**6*x**(7/2)*log(a/b + sqrt(x))/(15*a**11*
x**(7/2) + 30*a**10*b*x**4 + 15*a**9*b**2*x**(9/2)) + 1260*a**2*b**6*x**(7/
2)/(15*a**11*x**(7/2) + 30*a**10*b*x**4 + 15*a**9*b**2*x**(9/2)) + 840*a*b*
*7*x**4*log(x)/(15*a**11*x**(7/2) + 30*a**10*b*x**4 + 15*a**9*b**2*x**(9/2)
) - 1680*a*b**7*x**4*log(a/b + sqrt(x))/(15*a**11*x**(7/2) + 30*a**10*b*x**
4 + 15*a**9*b**2*x**(9/2)) + 840*a*b**7*x**4/(15*a**11*x**(7/2) + 30*a**10*
b*x**4 + 15*a**9*b**2*x**(9/2)) + 420*b**8*x**(9/2)*log(x)/(15*a**11*x**(7/
2) + 30*a**10*b*x**4 + 15*a**9*b**2*x**(9/2)) - 840*b**8*x**(9/2)*log(a/b +
 sqrt(x))/(15*a**11*x**(7/2) + 30*a**10*b*x**4 + 15*a**9*b**2*x**(9/2)), Tr
ue))
```

3.2215 $\int \frac{x^4}{\left(a+b\sqrt{x}\right)^5}\,dx$

Optimal. Leaf size=155

$$\frac{a^9}{2b^{10}\left(a+b\sqrt{x}\right)^4}-\frac{6a^8}{b^{10}\left(a+b\sqrt{x}\right)^3}+\frac{36a^7}{b^{10}\left(a+b\sqrt{x}\right)^2}-\frac{168a^6}{b^{10}\left(a+b\sqrt{x}\right)}-\frac{252a^5\log\left(a+b\sqrt{x}\right)}{b^{10}}+\frac{140a^4\sqrt{x}}{b^9}-\frac{35a^3x}{b^8}$$

```
[Out] -35*a^3*x/b^8+10*a^2*x^(3/2)/b^7-5/2*a*x^2/b^6+2/5*x^(5/2)/b^5-252*a^5*ln(a
+b*x^(1/2))/b^10+140*a^4*x^(1/2)/b^9+1/2*a^9/b^10/(a+b*x^(1/2))^4-6*a^8/b^1
0/(a+b*x^(1/2))^3+36*a^7/b^10/(a+b*x^(1/2))^2-168*a^6/b^10/(a+b*x^(1/2))
```

Rubi [A] time = 0.14, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{10a^2x^{3/2}}{b^7}+\frac{a^9}{2b^{10}\left(a+b\sqrt{x}\right)^4}-\frac{6a^8}{b^{10}\left(a+b\sqrt{x}\right)^3}+\frac{36a^7}{b^{10}\left(a+b\sqrt{x}\right)^2}-\frac{168a^6}{b^{10}\left(a+b\sqrt{x}\right)}+\frac{140a^4\sqrt{x}}{b^9}-\frac{35a^3x}{b^8}-\frac{252a^5\log($$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*Sqrt[x])^5,x]
```

```
[Out] a^9/(2*b^10*(a + b*Sqrt[x])^4) - (6*a^8)/(b^10*(a + b*Sqrt[x])^3) + (36*a^7
)/(b^10*(a + b*Sqrt[x])^2) - (168*a^6)/(b^10*(a + b*Sqrt[x])) + (140*a^4*Sq
rt[x])/b^9 - (35*a^3*x)/b^8 + (10*a^2*x^(3/2))/b^7 - (5*a*x^2)/(2*b^6) + (2
*x^(5/2))/(5*b^5) - (252*a^5*Log[a + b*Sqrt[x]])/b^10
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\left(a+b\sqrt{x}\right)^5}\,dx &= 2\,\text{Subst}\left(\int \frac{x^9}{(a+bx)^5}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int\left(\frac{70a^4}{b^9}-\frac{35a^3x}{b^8}+\frac{15a^2x^2}{b^7}-\frac{5ax^3}{b^6}+\frac{x^4}{b^5}-\frac{a^9}{b^9(a+bx)^5}+\frac{9a^8}{b^9(a+bx)^4}-\frac{36a}{b^9(a+\right.\right.\\ &= \frac{a^9}{2b^{10}\left(a+b\sqrt{x}\right)^4}-\frac{6a^8}{b^{10}\left(a+b\sqrt{x}\right)^3}+\frac{36a^7}{b^{10}\left(a+b\sqrt{x}\right)^2}-\frac{168a^6}{b^{10}\left(a+b\sqrt{x}\right)}+\frac{140a^4\sqrt{x}}{b^9}-\frac{35}{b}\end{aligned}$$

Mathematica [A] time = 0.19, size = 150, normalized size = 0.97

$$\frac{-1375a^9-2980a^8b\sqrt{x}+570a^7b^2x+5420a^6b^3x^{3/2}+3875a^5b^4x^2-2520a^5\left(a+b\sqrt{x}\right)^4\log\left(a+b\sqrt{x}\right)+504a}{10b^{10}\left(a+b\sqrt{x}\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*Sqrt[x])^5,x]
```

```
[Out] (-1375*a^9 - 2980*a^8*b*Sqrt[x] + 570*a^7*b^2*x + 5420*a^6*b^3*x^(3/2) + 38
75*a^5*b^4*x^2 + 504*a^4*b^5*x^(5/2) - 84*a^3*b^6*x^3 + 24*a^2*b^7*x^(7/2)
- 9*a*b^8*x^4 + 4*b^9*x^(9/2) - 2520*a^5*(a + b*Sqrt[x])^4*Log[a + b*Sqrt[x
]])/(10*b^10*(a + b*Sqrt[x])^4)
```

fricas [A] time = 1.10, size = 245, normalized size = 1.58

$$-\frac{25\,ab^{12}x^6+250\,a^3b^{10}x^5-1250\,a^5b^8x^4-40\,a^7b^6x^3+3840\,a^9b^4x^2-4240\,a^{11}b^2x+1375\,a^{13}+2520\left(a^5b^8x^4-4\right.}{10\left(b^{18}x^{}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b*x^(1/2))^5,x, algorithm="fricas")
```

```
[Out] -1/10*(25*a*b^12*x^6 + 250*a^3*b^10*x^5 - 1250*a^5*b^8*x^4 - 40*a^7*b^6*x^3
 + 3840*a^9*b^4*x^2 - 4240*a^11*b^2*x + 1375*a^13 + 2520*(a^5*b^8*x^4 - 4*a
^7*b^6*x^3 + 6*a^9*b^4*x^2 - 4*a^11*b^2*x + a^13)*log(b*sqrt(x) + a) - 4*(b
^13*x^6 + 21*a^2*b^11*x^5 + 256*a^4*b^9*x^4 - 1674*a^6*b^7*x^3 + 3066*a^8*b
^5*x^2 - 2310*a^10*b^3*x + 630*a^12*b)*sqrt(x))/(b^18*x^4 - 4*a^2*b^16*x^3
+ 6*a^4*b^14*x^2 - 4*a^6*b^12*x + a^8*b^10)
```

giac [A] time = 0.19, size = 121, normalized size = 0.78

$$-\frac{252\,a^5\log\left(\left|b\sqrt{x}+a\right|\right)}{b^{10}}-\frac{336\,a^6b^3x^{\frac{3}{2}}+936\,a^7b^2x+876\,a^8b\sqrt{x}+275\,a^9}{2\left(b\sqrt{x}+a\right)^4b^{10}}+\frac{4\,b^{20}x^{\frac{5}{2}}-25\,ab^{19}x^2+100\,a^2b^{18}x^{\frac{3}{2}}-3}{10\,b^{25}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b*x^(1/2))^5,x, algorithm="giac")
```

```
[Out] -252*a^5*log(abs(b*sqrt(x) + a))/b^10 - 1/2*(336*a^6*b^3*x^(3/2) + 936*a^7*
b^2*x + 876*a^8*b*sqrt(x) + 275*a^9)/((b*sqrt(x) + a)^4*b^10) + 1/10*(4*b^2
0*x^(5/2) - 25*a*b^19*x^2 + 100*a^2*b^18*x^(3/2) - 350*a^3*b^17*x + 1400*a^
4*b^16*sqrt(x))/b^25
```

maple [A] time = 0.01, size = 134, normalized size = 0.86

$$\frac{a^9}{2\left(b\sqrt{x}+a\right)^4b^{10}}-\frac{6a^8}{\left(b\sqrt{x}+a\right)^3b^{10}}+\frac{2x^{\frac{5}{2}}}{5b^5}+\frac{36a^7}{\left(b\sqrt{x}+a\right)^2b^{10}}-\frac{5a\,x^2}{2b^6}+\frac{10a^2x^{\frac{3}{2}}}{b^7}-\frac{168a^6}{\left(b\sqrt{x}+a\right)b^{10}}-\frac{252a^5\ln\left(b\sqrt{x}+a\right)}{b^{10}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^(1/2)+a)^5,x)
```

```
[Out] -35*a^3*x/b^8+10*a^2*x^(3/2)/b^7-5/2*a*x^2/b^6+2/5*x^(5/2)/b^5-252*a^5*ln(b
*x^(1/2)+a)/b^10+140*a^4*x^(1/2)/b^9+1/2*a^9/b^10/(b*x^(1/2)+a)^4-6*a^8/b^1
0/(b*x^(1/2)+a)^3+36*a^7/b^10/(b*x^(1/2)+a)^2-168*a^6/b^10/(b*x^(1/2)+a)
```

maxima [A] time = 0.85, size = 163, normalized size = 1.05

$$-\frac{252\,a^5\log\left(b\sqrt{x}+a\right)}{b^{10}}+\frac{2\left(b\sqrt{x}+a\right)^5}{5\,b^{10}}-\frac{9\left(b\sqrt{x}+a\right)^4a}{2\,b^{10}}+\frac{24\left(b\sqrt{x}+a\right)^3a^2}{b^{10}}-\frac{84\left(b\sqrt{x}+a\right)^2a^3}{b^{10}}+\frac{252\left(b\sqrt{x}+a\right)a^4}{b^{10}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b*x^(1/2))^5,x, algorithm="maxima")
```

```
[Out] -252*a^5*log(b*sqrt(x) + a)/b^10 + 2/5*(b*sqrt(x) + a)^5/b^10 - 9/2*(b*sqrt
(x) + a)^4*a/b^10 + 24*(b*sqrt(x) + a)^3*a^2/b^10 - 84*(b*sqrt(x) + a)^2*a^
3/b^10 + 252*(b*sqrt(x) + a)*a^4/b^10 - 168*a^6/((b*sqrt(x) + a)*b^10) + 36
*a^7/((b*sqrt(x) + a)^2*b^10) - 6*a^8/((b*sqrt(x) + a)^3*b^10) + 1/2*a^9/((
b*sqrt(x) + a)^4*b^10)
```

mupad **[B]** time = 0.06, size = 148, normalized size = 0.95

$$\frac{2\,x^{5/2}}{5\,b^5}-\frac{\frac{275\,a^9}{2\,b}+438\,a^8\,\sqrt{x}+168\,a^6\,b^2\,x^{3/2}+468\,a^7\,b\,x}{a^4\,b^9+b^{13}\,x^2+6\,a^2\,b^{11}\,x+4\,a\,b^{12}\,x^{3/2}+4\,a^3\,b^{10}\,\sqrt{x}}-\frac{5\,a\,x^2}{2\,b^6}-\frac{35\,a^3\,x}{b^8}-\frac{252\,a^5\,\ln\left(a+b\,\sqrt{x}\right)}{b^{10}}+\frac{10\,a^2\,x^{3/2}}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^(1/2))^5,x)
```

```
[Out] (2*x^(5/2))/(5*b^5) - ((275*a^9)/(2*b) + 438*a^8*x^(1/2) + 168*a^6*b^2*x^(3
/2) + 468*a^7*b*x)/(a^4*b^9 + b^13*x^2 + 6*a^2*b^11*x + 4*a*b^12*x^(3/2) +
4*a^3*b^10*x^(1/2)) - (5*a*x^2)/(2*b^6) - (35*a^3*x)/b^8 - (252*a^5*log(a +
 b*x^(1/2)))/b^10 + (10*a^2*x^(3/2))/b^7 + (140*a^4*x^(1/2))/b^9
```

sympy **[A]** time = 5.95, size = 949, normalized size = 6.12

$$\begin{cases} -\frac{2520a^9\log\left(\frac{a}{b}+\sqrt{x}\right)}{10a^4b^{10}+40a^3b^{11}\sqrt{x}+60a^2b^{12}x+40ab^{13}x^{\frac{3}{2}}+10b^{14}x^2}-\frac{5250a^9}{10a^4b^{10}+40a^3b^{11}\sqrt{x}+60a^2b^{12}x+40ab^{13}x^{\frac{3}{2}}+10b^{14}x^2}-\frac{10080a^8b\sqrt{x}\,\mathrm{lo}}{10a^4b^{10}+40a^3b^{11}\sqrt{x}+60a^2b} \\ \frac{x^5}{5a^5} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a+b*x**(1/2))**5,x)
```

```
[Out] Piecewise((-2520*a**9*log(a/b + sqrt(x))/(10*a**4*b**10 + 40*a**3*b**11*sqr
t(x) + 60*a**2*b**12*x + 40*a*b**13*x**(3/2) + 10*b**14*x**2) - 5250*a**9/(
10*a**4*b**10 + 40*a**3*b**11*sqrt(x) + 60*a**2*b**12*x + 40*a*b**13*x**(3/
2) + 10*b**14*x**2) - 10080*a**8*b*sqrt(x)*log(a/b + sqrt(x))/(10*a**4*b**1
0 + 40*a**3*b**11*sqrt(x) + 60*a**2*b**12*x + 40*a*b**13*x**(3/2) + 10*b**1
4*x**2) - 18480*a**8*b*sqrt(x)/(10*a**4*b**10 + 40*a**3*b**11*sqrt(x) + 60*
a**2*b**12*x + 40*a*b**13*x**(3/2) + 10*b**14*x**2) - 15120*a**7*b**2*x*log
(a/b + sqrt(x))/(10*a**4*b**10 + 40*a**3*b**11*sqrt(x) + 60*a**2*b**12*x +
40*a*b**13*x**(3/2) + 10*b**14*x**2) - 22680*a**7*b**2*x/(10*a**4*b**10 + 4
0*a**3*b**11*sqrt(x) + 60*a**2*b**12*x + 40*a*b**13*x**(3/2) + 10*b**14*x**
2) - 10080*a**6*b**3*x**(3/2)*log(a/b + sqrt(x))/(10*a**4*b**10 + 40*a**3*b
**11*sqrt(x) + 60*a**2*b**12*x + 40*a*b**13*x**(3/2) + 10*b**14*x**2) - 100
80*a**6*b**3*x**(3/2)/(10*a**4*b**10 + 40*a**3*b**11*sqrt(x) + 60*a**2*b**1
2*x + 40*a*b**13*x**(3/2) + 10*b**14*x**2) - 2520*a**5*b**4*x**2*log(a/b +
sqrt(x))/(10*a**4*b**10 + 40*a**3*b**11*sqrt(x) + 60*a**2*b**12*x + 40*a*b*
*13*x**(3/2) + 10*b**14*x**2) + 504*a**4*b**5*x**(5/2)/(10*a**4*b**10 + 40*
a**3*b**11*sqrt(x) + 60*a**2*b**12*x + 40*a*b**13*x**(3/2) + 10*b**14*x**2)
 - 84*a**3*b**6*x**3/(10*a**4*b**10 + 40*a**3*b**11*sqrt(x) + 60*a**2*b**12
*x + 40*a*b**13*x**(3/2) + 10*b**14*x**2) + 24*a**2*b**7*x**(7/2)/(10*a**4*
b**10 + 40*a**3*b**11*sqrt(x) + 60*a**2*b**12*x + 40*a*b**13*x**(3/2) + 10*
b**14*x**2) - 9*a*b**8*x**4/(10*a**4*b**10 + 40*a**3*b**11*sqrt(x) + 60*a**
2*b**12*x + 40*a*b**13*x**(3/2) + 10*b**14*x**2) + 4*b**9*x**(9/2)/(10*a**4
*b**10 + 40*a**3*b**11*sqrt(x) + 60*a**2*b**12*x + 40*a*b**13*x**(3/2) + 10
*b**14*x**2), Ne(b, 0)), (x**5/(5*a**5), True))
```

3.2216 $\int \frac{x^3}{\left(a+b\sqrt{x}\right)^5}\,dx$

Optimal. Leaf size=131

$$\frac{a^7}{2b^8\left(a+b\sqrt{x}\right)^4}-\frac{14a^6}{3b^8\left(a+b\sqrt{x}\right)^3}+\frac{21a^5}{b^8\left(a+b\sqrt{x}\right)^2}-\frac{70a^4}{b^8\left(a+b\sqrt{x}\right)}-\frac{70a^3\log\left(a+b\sqrt{x}\right)}{b^8}+\frac{30a^2\sqrt{x}}{b^7}-\frac{5ax}{b^6}+\frac{2x^{3/2}}{3b^5}$$

```
[Out] -5*a*x/b^6+2/3*x^(3/2)/b^5-70*a^3*ln(a+b*x^(1/2))/b^8+30*a^2*x^(1/2)/b^7+1/
2*a^7/b^8/(a+b*x^(1/2))^4-14/3*a^6/b^8/(a+b*x^(1/2))^3+21*a^5/b^8/(a+b*x^(1
/2))^2-70*a^4/b^8/(a+b*x^(1/2))
```

Rubi [A] time = 0.10, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{a^7}{2b^8\left(a+b\sqrt{x}\right)^4}-\frac{14a^6}{3b^8\left(a+b\sqrt{x}\right)^3}+\frac{21a^5}{b^8\left(a+b\sqrt{x}\right)^2}-\frac{70a^4}{b^8\left(a+b\sqrt{x}\right)}+\frac{30a^2\sqrt{x}}{b^7}-\frac{70a^3\log\left(a+b\sqrt{x}\right)}{b^8}-\frac{5ax}{b^6}+\frac{2x^{3/2}}{3b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*Sqrt[x])^5,x]
```

```
[Out] a^7/(2*b^8*(a + b*Sqrt[x])^4) - (14*a^6)/(3*b^8*(a + b*Sqrt[x])^3) + (21*a^
5)/(b^8*(a + b*Sqrt[x])^2) - (70*a^4)/(b^8*(a + b*Sqrt[x])) + (30*a^2*Sqrt[
x])/b^7 - (5*a*x)/b^6 + (2*x^(3/2))/(3*b^5) - (70*a^3*Log[a + b*Sqrt[x]])/b
^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(a+b\sqrt{x}\right)^5}\,dx &= 2\,\text{Subst}\left(\int \frac{x^7}{(a+bx)^5}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(\frac{15a^2}{b^7}-\frac{5ax}{b^6}+\frac{x^2}{b^5}-\frac{a^7}{b^7(a+bx)^5}+\frac{7a^6}{b^7(a+bx)^4}-\frac{21a^5}{b^7(a+bx)^3}+\frac{35a^4}{b^7(a+bx)^2}-\frac{}{b}\right.\right.\\ &= \frac{a^7}{2b^8\left(a+b\sqrt{x}\right)^4}-\frac{14a^6}{3b^8\left(a+b\sqrt{x}\right)^3}+\frac{21a^5}{b^8\left(a+b\sqrt{x}\right)^2}-\frac{70a^4}{b^8\left(a+b\sqrt{x}\right)}+\frac{30a^2\sqrt{x}}{b^7}-\frac{5ax}{b^6}+\frac{2x}{3}\end{aligned}$$

Mathematica [A] time = 0.12, size = 126, normalized size = 0.96

$$\frac{-319a^7-856a^6b\sqrt{x}-444a^5b^2x+544a^4b^3x^{3/2}+556a^3b^4x^2-420a^3\left(a+b\sqrt{x}\right)^4\log\left(a+b\sqrt{x}\right)+84a^2b^5x^{5/2}-1}{6b^8\left(a+b\sqrt{x}\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*Sqrt[x])^5,x]
```

```
[Out] (-319*a^7 - 856*a^6*b*Sqrt[x] - 444*a^5*b^2*x + 544*a^4*b^3*x^(3/2) + 556*a
^3*b^4*x^2 + 84*a^2*b^5*x^(5/2) - 14*a*b^6*x^3 + 4*b^7*x^(7/2) - 420*a^3*(a
 + b*Sqrt[x])^4*Log[a + b*Sqrt[x]])/(6*b^8*(a + b*Sqrt[x])^4)
```

fricas [B] time = 0.67, size = 223, normalized size = 1.70

$$-\frac{30\,ab^{10}x^5-120\,a^3b^8x^4-366\,a^5b^6x^3+1179\,a^7b^4x^2-1066\,a^9b^2x+319\,a^{11}+420\left(a^3b^8x^4-4\,a^5b^6x^3+6\,a^7b^4\right.}{6\left(b^{16}x^4-4\,a^2b^{14}x^3+6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^(1/2))^5,x, algorithm="fricas")
```

```
[Out] -1/6*(30*a*b^10*x^5 - 120*a^3*b^8*x^4 - 366*a^5*b^6*x^3 + 1179*a^7*b^4*x^2
- 1066*a^9*b^2*x + 319*a^11 + 420*(a^3*b^8*x^4 - 4*a^5*b^6*x^3 + 6*a^7*b^4*
x^2 - 4*a^9*b^2*x + a^11)*log(b*sqrt(x) + a) - 4*(b^11*x^5 + 41*a^2*b^9*x^4
 - 279*a^4*b^7*x^3 + 511*a^6*b^5*x^2 - 385*a^8*b^3*x + 105*a^10*b)*sqrt(x))
/(b^16*x^4 - 4*a^2*b^14*x^3 + 6*a^4*b^12*x^2 - 4*a^6*b^10*x + a^8*b^8)
```

giac [A] time = 0.16, size = 99, normalized size = 0.76

$$-\frac{70\,a^3\log\left(\left|b\sqrt{x}+a\right|\right)}{b^8}-\frac{420\,a^4b^3x^{\frac{3}{2}}+1134\,a^5b^2x+1036\,a^6b\sqrt{x}+319\,a^7}{6\left(b\sqrt{x}+a\right)^4b^8}+\frac{2\,b^{10}x^{\frac{3}{2}}-15\,ab^9x+90\,a^2b^8\sqrt{x}}{3\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^(1/2))^5,x, algorithm="giac")
```

```
[Out] -70*a^3*log(abs(b*sqrt(x) + a))/b^8 - 1/6*(420*a^4*b^3*x^(3/2) + 1134*a^5*b
^2*x + 1036*a^6*b*sqrt(x) + 319*a^7)/((b*sqrt(x) + a)^4*b^8) + 1/3*(2*b^10*
x^(3/2) - 15*a*b^9*x + 90*a^2*b^8*sqrt(x))/b^15
```

maple [A] time = 0.01, size = 112, normalized size = 0.85

$$\frac{a^7}{2\left(b\sqrt{x}+a\right)^4b^8}-\frac{14a^6}{3\left(b\sqrt{x}+a\right)^3b^8}+\frac{21a^5}{\left(b\sqrt{x}+a\right)^2b^8}+\frac{2x^{\frac{3}{2}}}{3b^5}-\frac{70a^4}{\left(b\sqrt{x}+a\right)b^8}-\frac{70a^3\ln\left(b\sqrt{x}+a\right)}{b^8}-\frac{5ax}{b^6}+\frac{30a^2\sqrt{x}}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^(1/2)+a)^5,x)
```

```
[Out] -5*a*x/b^6+2/3*x^(3/2)/b^5-70*a^3*ln(b*x^(1/2)+a)/b^8+30*a^2*x^(1/2)/b^7+1/
2*a^7/b^8/(b*x^(1/2)+a)^4-14/3*a^6/b^8/(b*x^(1/2)+a)^3+21*a^5/b^8/(b*x^(1/2
)+a)^2-70*a^4/b^8/(b*x^(1/2)+a)
```

maxima [A] time = 0.90, size = 129, normalized size = 0.98

$$-\frac{70\,a^3\log\left(b\sqrt{x}+a\right)}{b^8}+\frac{2\left(b\sqrt{x}+a\right)^3}{3\,b^8}-\frac{7\left(b\sqrt{x}+a\right)^2a}{b^8}+\frac{42\left(b\sqrt{x}+a\right)a^2}{b^8}-\frac{70\,a^4}{\left(b\sqrt{x}+a\right)b^8}+\frac{21\,a^5}{\left(b\sqrt{x}+a\right)^2b^8}-\frac{}{3\left(b\sqrt{}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^(1/2))^5,x, algorithm="maxima")
```

```
[Out] -70*a^3*log(b*sqrt(x) + a)/b^8 + 2/3*(b*sqrt(x) + a)^3/b^8 - 7*(b*sqrt(x) +
 a)^2*a/b^8 + 42*(b*sqrt(x) + a)*a^2/b^8 - 70*a^4/((b*sqrt(x) + a)*b^8) + 2
```

```
1*a^5/((b*sqrt(x) + a)^2*b^8) - 14/3*a^6/((b*sqrt(x) + a)^3*b^8) + 1/2*a^7/
((b*sqrt(x) + a)^4*b^8)
```

mupad [B] time = 1.20, size = 126, normalized size = 0.96

$$\frac{2\,x^{3/2}}{3\,b^5}-\frac{\frac{319\,a^7}{6\,b}+\frac{518\,a^6\,\sqrt{x}}{3}+70\,a^4\,b^2\,x^{3/2}+189\,a^5\,b\,x}{a^4\,b^7+b^{11}\,x^2+6\,a^2\,b^9\,x+4\,a\,b^{10}\,x^{3/2}+4\,a^3\,b^8\,\sqrt{x}}-\frac{70\,a^3\,\ln\left(a+b\,\sqrt{x}\right)}{b^8}+\frac{30\,a^2\,\sqrt{x}}{b^7}-\frac{5\,a\,x}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^(1/2))^5,x)
```

```
[Out] (2*x^(3/2))/(3*b^5) - ((319*a^7)/(6*b) + (518*a^6*x^(1/2))/3 + 70*a^4*b^2*x
^(3/2) + 189*a^5*b*x)/(a^4*b^7 + b^11*x^2 + 6*a^2*b^9*x + 4*a*b^10*x^(3/2)
+ 4*a^3*b^8*x^(1/2)) - (70*a^3*log(a + b*x^(1/2)))/b^8 + (30*a^2*x^(1/2))/b
^7 - (5*a*x)/b^6
```

sympy [A] time = 3.03, size = 818, normalized size = 6.24

$$\begin{cases} -\frac{420a^7\log\left(\frac{a}{b}+\sqrt{x}\right)}{6a^4b^8+24a^3b^9\sqrt{x}+36a^2b^{10}x+24ab^{11}x^{\frac{3}{2}}+6b^{12}x^2}-\frac{875a^7}{6a^4b^8+24a^3b^9\sqrt{x}+36a^2b^{10}x+24ab^{11}x^{\frac{3}{2}}+6b^{12}x^2}-\frac{1680a^6b\sqrt{x}\log\left(\frac{a}{b}+\sqrt{x}\right)}{6a^4b^8+24a^3b^9\sqrt{x}+36a^2b^{10}x+24ab^{11}x^{\frac{3}{2}}+} \\ \frac{x^4}{4a^5} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*x**(1/2))**5,x)
```

```
[Out] Piecewise((-420*a**7*log(a/b + sqrt(x))/(6*a**4*b**8 + 24*a**3*b**9*sqrt(x)
 + 36*a**2*b**10*x + 24*a*b**11*x**(3/2) + 6*b**12*x**2) - 875*a**7/(6*a**4
*b**8 + 24*a**3*b**9*sqrt(x) + 36*a**2*b**10*x + 24*a*b**11*x**(3/2) + 6*b*
*12*x**2) - 1680*a**6*b*sqrt(x)*log(a/b + sqrt(x))/(6*a**4*b**8 + 24*a**3*b
**9*sqrt(x) + 36*a**2*b**10*x + 24*a*b**11*x**(3/2) + 6*b**12*x**2) - 3080*
a**6*b*sqrt(x)/(6*a**4*b**8 + 24*a**3*b**9*sqrt(x) + 36*a**2*b**10*x + 24*a
*b**11*x**(3/2) + 6*b**12*x**2) - 2520*a**5*b**2*x*log(a/b + sqrt(x))/(6*a*
*4*b**8 + 24*a**3*b**9*sqrt(x) + 36*a**2*b**10*x + 24*a*b**11*x**(3/2) + 6*
b**12*x**2) - 3780*a**5*b**2*x/(6*a**4*b**8 + 24*a**3*b**9*sqrt(x) + 36*a**
2*b**10*x + 24*a*b**11*x**(3/2) + 6*b**12*x**2) - 1680*a**4*b**3*x**(3/2)*l
og(a/b + sqrt(x))/(6*a**4*b**8 + 24*a**3*b**9*sqrt(x) + 36*a**2*b**10*x + 2
4*a*b**11*x**(3/2) + 6*b**12*x**2) - 1680*a**4*b**3*x**(3/2)/(6*a**4*b**8 +
 24*a**3*b**9*sqrt(x) + 36*a**2*b**10*x + 24*a*b**11*x**(3/2) + 6*b**12*x**
2) - 420*a**3*b**4*x**2*log(a/b + sqrt(x))/(6*a**4*b**8 + 24*a**3*b**9*sqrt
(x) + 36*a**2*b**10*x + 24*a*b**11*x**(3/2) + 6*b**12*x**2) + 84*a**2*b**5*
x**(5/2)/(6*a**4*b**8 + 24*a**3*b**9*sqrt(x) + 36*a**2*b**10*x + 24*a*b**11
*x**(3/2) + 6*b**12*x**2) - 14*a*b**6*x**3/(6*a**4*b**8 + 24*a**3*b**9*sqrt
(x) + 36*a**2*b**10*x + 24*a*b**11*x**(3/2) + 6*b**12*x**2) + 4*b**7*x**(7/
2)/(6*a**4*b**8 + 24*a**3*b**9*sqrt(x) + 36*a**2*b**10*x + 24*a*b**11*x**(3
/2) + 6*b**12*x**2), Ne(b, 0)), (x**4/(4*a**5), True))
```

3.2217 $\int \frac{x^2}{\left(a+b\sqrt{x}\right)^5}\,dx$

Optimal. Leaf size=107

$$\frac{a^5}{2b^6\left(a+b\sqrt{x}\right)^4}-\frac{10a^4}{3b^6\left(a+b\sqrt{x}\right)^3}+\frac{10a^3}{b^6\left(a+b\sqrt{x}\right)^2}-\frac{20a^2}{b^6\left(a+b\sqrt{x}\right)}-\frac{10a\log\left(a+b\sqrt{x}\right)}{b^6}+\frac{2\sqrt{x}}{b^5}$$

```
[Out] -10*a*ln(a+b*x^(1/2))/b^6+2*x^(1/2)/b^5+1/2*a^5/b^6/(a+b*x^(1/2))^4-10/3*a^
4/b^6/(a+b*x^(1/2))^3+10*a^3/b^6/(a+b*x^(1/2))^2-20*a^2/b^6/(a+b*x^(1/2))
```

Rubi [A] time = 0.07, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{a^5}{2b^6\left(a+b\sqrt{x}\right)^4}-\frac{10a^4}{3b^6\left(a+b\sqrt{x}\right)^3}+\frac{10a^3}{b^6\left(a+b\sqrt{x}\right)^2}-\frac{20a^2}{b^6\left(a+b\sqrt{x}\right)}-\frac{10a\log\left(a+b\sqrt{x}\right)}{b^6}+\frac{2\sqrt{x}}{b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*Sqrt[x])^5,x]
```

```
[Out] a^5/(2*b^6*(a + b*Sqrt[x])^4) - (10*a^4)/(3*b^6*(a + b*Sqrt[x])^3) + (10*a^
3)/(b^6*(a + b*Sqrt[x])^2) - (20*a^2)/(b^6*(a + b*Sqrt[x])) + (2*Sqrt[x])/b
^5 - (10*a*Log[a + b*Sqrt[x]])/b^6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a+b\sqrt{x}\right)^5}\,dx &= 2\,\text{Subst}\left(\int \frac{x^5}{(a+bx)^5}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int\left(\frac{1}{b^5}-\frac{a^5}{b^5(a+bx)^5}+\frac{5a^4}{b^5(a+bx)^4}-\frac{10a^3}{b^5(a+bx)^3}+\frac{10a^2}{b^5(a+bx)^2}-\frac{5a}{b^5(a+bx)}\right)\right.\\ &= \frac{a^5}{2b^6\left(a+b\sqrt{x}\right)^4}-\frac{10a^4}{3b^6\left(a+b\sqrt{x}\right)^3}+\frac{10a^3}{b^6\left(a+b\sqrt{x}\right)^2}-\frac{20a^2}{b^6\left(a+b\sqrt{x}\right)}+\frac{2\sqrt{x}}{b^5}-\frac{10a\log\left(\right.}{b}\end{aligned}$$

Mathematica [A] time = 0.09, size = 100, normalized size = 0.93

$$-\frac{77a^5+248a^4b\sqrt{x}+252a^3b^2x+48a^2b^3x^{3/2}-48ab^4x^2+60a\left(a+b\sqrt{x}\right)^4\log\left(a+b\sqrt{x}\right)-12b^5x^{5/2}}{6b^6\left(a+b\sqrt{x}\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*Sqrt[x])^5,x]
```

```
[Out] -1/6*(77*a^5 + 248*a^4*b*Sqrt[x] + 252*a^3*b^2*x + 48*a^2*b^3*x^(3/2) - 48*
a*b^4*x^2 - 12*b^5*x^(5/2) + 60*a*(a + b*Sqrt[x])^4*Log[a + b*Sqrt[x]])/(b^
6*(a + b*Sqrt[x])^4)
```

fricas **[B]** time = 0.84, size = 191, normalized size = 1.79

$$\frac{180\,a^3b^6x^3 - 357\,a^5b^4x^2 + 278\,a^7b^2x - 77\,a^9 - 60\left(ab^8x^4 - 4\,a^3b^6x^3 + 6\,a^5b^4x^2 - 4\,a^7b^2x + a^9\right)\log\left(b\sqrt{x} + a\right) + 4}{6\left(b^{14}x^4 - 4\,a^2b^{12}x^3 + 6\,a^4b^{10}x^2 - 4\,a^6b^8x + a^8b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^(1/2))^5,x, algorithm="fricas")
```

```
[Out] 1/6*(180*a^3*b^6*x^3 - 357*a^5*b^4*x^2 + 278*a^7*b^2*x - 77*a^9 - 60*(a*b^8
*x^4 - 4*a^3*b^6*x^3 + 6*a^5*b^4*x^2 - 4*a^7*b^2*x + a^9)*log(b*sqrt(x) + a
) + 4*(3*b^9*x^4 - 42*a^2*b^7*x^3 + 73*a^4*b^5*x^2 - 55*a^6*b^3*x + 15*a^8*
b)*sqrt(x))/(b^14*x^4 - 4*a^2*b^12*x^3 + 6*a^4*b^10*x^2 - 4*a^6*b^8*x + a^8
*b^6)
```

giac **[A]** time = 0.15, size = 73, normalized size = 0.68

$$-\frac{10\,a\log\left(\left|b\sqrt{x} + a\right|\right)}{b^6} + \frac{2\sqrt{x}}{b^5} - \frac{120\,a^2b^3x^{\frac{3}{2}} + 300\,a^3b^2x + 260\,a^4b\sqrt{x} + 77\,a^5}{6\left(b\sqrt{x} + a\right)^4b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^(1/2))^5,x, algorithm="giac")
```

```
[Out] -10*a*log(abs(b*sqrt(x) + a))/b^6 + 2*sqrt(x)/b^5 - 1/6*(120*a^2*b^3*x^(3/2
) + 300*a^3*b^2*x + 260*a^4*b*sqrt(x) + 77*a^5)/((b*sqrt(x) + a)^4*b^6)
```

maple **[A]** time = 0.01, size = 92, normalized size = 0.86

$$\frac{a^5}{2\left(b\sqrt{x} + a\right)^4b^6} - \frac{10a^4}{3\left(b\sqrt{x} + a\right)^3b^6} + \frac{10a^3}{\left(b\sqrt{x} + a\right)^2b^6} - \frac{20a^2}{\left(b\sqrt{x} + a\right)b^6} - \frac{10a\ln\left(b\sqrt{x} + a\right)}{b^6} + \frac{2\sqrt{x}}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^(1/2)+a)^5,x)
```

```
[Out] -10*a*ln(b*x^(1/2)+a)/b^6+2*x^(1/2)/b^5+1/2*a^5/b^6/(b*x^(1/2)+a)^4-10/3*a^
4/b^6/(b*x^(1/2)+a)^3+10*a^3/b^6/(b*x^(1/2)+a)^2-20*a^2/b^6/(b*x^(1/2)+a)
```

maxima **[A]** time = 0.77, size = 95, normalized size = 0.89

$$-\frac{10\,a\log\left(b\sqrt{x} + a\right)}{b^6} + \frac{2\left(b\sqrt{x} + a\right)}{b^6} - \frac{20\,a^2}{\left(b\sqrt{x} + a\right)b^6} + \frac{10\,a^3}{\left(b\sqrt{x} + a\right)^2b^6} - \frac{10\,a^4}{3\left(b\sqrt{x} + a\right)^3b^6} + \frac{a^5}{2\left(b\sqrt{x} + a\right)^4b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^(1/2))^5,x, algorithm="maxima")
```

```
[Out] -10*a*log(b*sqrt(x) + a)/b^6 + 2*(b*sqrt(x) + a)/b^6 - 20*a^2/((b*sqrt(x) +
 a)*b^6) + 10*a^3/((b*sqrt(x) + a)^2*b^6) - 10/3*a^4/((b*sqrt(x) + a)^3*b^6
) + 1/2*a^5/((b*sqrt(x) + a)^4*b^6)
```

mupad [B] time = 1.17, size = 106, normalized size = 0.99

$$\frac{2\sqrt{x}}{b^5}-\frac{\frac{77\,a^5}{6\,b}+\frac{130\,a^4\sqrt{x}}{3}+20\,a^2\,b^2\,x^{3/2}+50\,a^3\,b\,x}{a^4\,b^5+b^9\,x^2+6\,a^2\,b^7\,x+4\,a\,b^8\,x^{3/2}+4\,a^3\,b^6\,\sqrt{x}}-\frac{10\,a\,\ln\left(a+b\,\sqrt{x}\right)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^(1/2))^5,x)
```

```
[Out] (2*x^(1/2))/b^5 - ((77*a^5)/(6*b) + (130*a^4*x^(1/2))/3 + 20*a^2*b^2*x^(3/2
) + 50*a^3*b*x)/(a^4*b^5 + b^9*x^2 + 6*a^2*b^7*x + 4*a*b^8*x^(3/2) + 4*a^3*
b^6*x^(1/2)) - (10*a*log(a + b*x^(1/2)))/b^6
```

sympy [A] time = 2.17, size = 687, normalized size = 6.42

$$\begin{cases}-\frac{60a^5\log\left(\frac{a}{b}+\sqrt{x}\right)}{6a^4b^6+24a^3b^7\sqrt{x}+36a^2b^8x+24ab^9x^{\frac{3}{2}}+6b^{10}x^2}-\frac{125a^5}{6a^4b^6+24a^3b^7\sqrt{x}+36a^2b^8x+24ab^9x^{\frac{3}{2}}+6b^{10}x^2}-\frac{240a^4b\sqrt{x}\,\log\left(\frac{a}{b}+\sqrt{x}\right)}{6a^4b^6+24a^3b^7\sqrt{x}+36a^2b^8x+24ab^9x^{\frac{3}{2}}+6b^{10}x^2} & \text{for } b\neq 0\\ \frac{x^3}{3a^5} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*x**(1/2))**5,x)
```

```
[Out] Piecewise((-60*a**5*log(a/b + sqrt(x))/(6*a**4*b**6 + 24*a**3*b**7*sqrt(x)
+ 36*a**2*b**8*x + 24*a*b**9*x**(3/2) + 6*b**10*x**2) - 125*a**5/(6*a**4*b*
*6 + 24*a**3*b**7*sqrt(x) + 36*a**2*b**8*x + 24*a*b**9*x**(3/2) + 6*b**10*x
**2) - 240*a**4*b*sqrt(x)*log(a/b + sqrt(x))/(6*a**4*b**6 + 24*a**3*b**7*sq
rt(x) + 36*a**2*b**8*x + 24*a*b**9*x**(3/2) + 6*b**10*x**2) - 440*a**4*b*sq
rt(x)/(6*a**4*b**6 + 24*a**3*b**7*sqrt(x) + 36*a**2*b**8*x + 24*a*b**9*x**(
3/2) + 6*b**10*x**2) - 360*a**3*b**2*x*log(a/b + sqrt(x))/(6*a**4*b**6 + 24
*a**3*b**7*sqrt(x) + 36*a**2*b**8*x + 24*a*b**9*x**(3/2) + 6*b**10*x**2) -
540*a**3*b**2*x/(6*a**4*b**6 + 24*a**3*b**7*sqrt(x) + 36*a**2*b**8*x + 24*a
*b**9*x**(3/2) + 6*b**10*x**2) - 240*a**2*b**3*x**(3/2)*log(a/b + sqrt(x))/
(6*a**4*b**6 + 24*a**3*b**7*sqrt(x) + 36*a**2*b**8*x + 24*a*b**9*x**(3/2) +
 6*b**10*x**2) - 240*a**2*b**3*x**(3/2)/(6*a**4*b**6 + 24*a**3*b**7*sqrt(x)
 + 36*a**2*b**8*x + 24*a*b**9*x**(3/2) + 6*b**10*x**2) - 60*a*b**4*x**2*log
(a/b + sqrt(x))/(6*a**4*b**6 + 24*a**3*b**7*sqrt(x) + 36*a**2*b**8*x + 24*a
*b**9*x**(3/2) + 6*b**10*x**2) + 12*b**5*x**(5/2)/(6*a**4*b**6 + 24*a**3*b*
*7*sqrt(x) + 36*a**2*b**8*x + 24*a*b**9*x**(3/2) + 6*b**10*x**2), Ne(b, 0))
, (x**3/(3*a**5), True))
```

3.2218 $\int \frac{x}{\left(a+b\sqrt{x}\right)^5}\,dx$

Optimal. Leaf size=21

$$\frac{x^2}{2a\left(a+b\sqrt{x}\right)^4}$$

```
[Out] 1/2*x^2/a/(a+b*x^(1/2))^4
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {264}

$$\frac{x^2}{2a\left(a+b\sqrt{x}\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*Sqrt[x])^5,x]
```

```
[Out] x^2/(2*a*(a + b*Sqrt[x])^4)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x}{\left(a+b\sqrt{x}\right)^5}\,dx = \frac{x^2}{2a\left(a+b\sqrt{x}\right)^4}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{x^2}{2a\left(a+b\sqrt{x}\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*Sqrt[x])^5,x]
```

```
[Out] x^2/(2*a*(a + b*Sqrt[x])^4)
```

fricas [B] time = 1.13, size = 108, normalized size = 5.14

$$\frac{10\,ab^6x^3 - 5\,a^3b^4x^2 + 4\,a^5b^2x - a^7 - 4\left(b^7x^3 + a^2b^5x^2\right)\sqrt{x}}{2\left(b^{12}x^4 - 4\,a^2b^{10}x^3 + 6\,a^4b^8x^2 - 4\,a^6b^6x + a^8b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^(1/2))^5,x, algorithm="fricas")
```

```
[Out] 1/2*(10*a*b^6*x^3 - 5*a^3*b^4*x^2 + 4*a^5*b^2*x - a^7 - 4*(b^7*x^3 + a^2*b^
5*x^2)*sqrt(x))/(b^12*x^4 - 4*a^2*b^10*x^3 + 6*a^4*b^8*x^2 - 4*a^6*b^6*x +
a^8*b^4)
```

giac [B] time = 0.15, size = 42, normalized size = 2.00

$$-\frac{4\,b^3x^{\frac{3}{2}}+6\,ab^2x+4\,a^2b\sqrt{x}+a^3}{2\left(b\sqrt{x}+a\right)^4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^(1/2))^5,x, algorithm="giac")
```

```
[Out] -1/2*(4*b^3*x^(3/2) + 6*a*b^2*x + 4*a^2*b*sqrt(x) + a^3)/((b*sqrt(x) + a)^4
*b^4)
```

maple [B] time = 0.01, size = 65, normalized size = 3.10

$$\frac{a^3}{2\left(b\sqrt{x}+a\right)^4b^4}-\frac{2a^2}{\left(b\sqrt{x}+a\right)^3b^4}+\frac{3a}{\left(b\sqrt{x}+a\right)^2b^4}-\frac{2}{\left(b\sqrt{x}+a\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^(1/2)+a)^5,x)
```

```
[Out] 1/2*a^3/b^4/(b*x^(1/2)+a)^4-2/b^4/(b*x^(1/2)+a)-2*a^2/b^4/(b*x^(1/2)+a)^3+3
*a/b^4/(b*x^(1/2)+a)^2
```

maxima [B] time = 0.93, size = 64, normalized size = 3.05

$$-\frac{2}{\left(b\sqrt{x}+a\right)b^4}+\frac{3\,a}{\left(b\sqrt{x}+a\right)^2b^4}-\frac{2\,a^2}{\left(b\sqrt{x}+a\right)^3b^4}+\frac{a^3}{2\left(b\sqrt{x}+a\right)^4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^(1/2))^5,x, algorithm="maxima")
```

```
[Out] -2/((b*sqrt(x) + a)*b^4) + 3*a/((b*sqrt(x) + a)^2*b^4) - 2*a^2/((b*sqrt(x)
+ a)^3*b^4) + 1/2*a^3/((b*sqrt(x) + a)^4*b^4)
```

mupad [B] time = 1.19, size = 77, normalized size = 3.67

$$-\frac{\frac{a^3}{2\,b^4}+\frac{2\,x^{3/2}}{b}+\frac{2\,a^2\,\sqrt{x}}{b^3}+\frac{3\,a\,x}{b^2}}{a^4+b^4\,x^2+6\,a^2\,b^2\,x+4\,a^3\,b\,\sqrt{x}+4\,a\,b^3\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^(1/2))^5,x)
```

```
[Out] -(a^3/(2*b^4) + (2*x^(3/2))/b + (2*a^2*x^(1/2))/b^3 + (3*a*x)/b^2)/(a^4 + b
^4*x^2 + 6*a^2*b^2*x + 4*a^3*b*x^(1/2) + 4*a*b^3*x^(3/2))
```

sympy [A] time = 1.85, size = 253, normalized size = 12.05

$$\begin{cases}-\frac{a^3}{2a^4b^4+8a^3b^5\sqrt{x}+12a^2b^6x+8ab^7x^{\frac{3}{2}}+2b^8x^2}-\frac{4a^2b\sqrt{x}}{2a^4b^4+8a^3b^5\sqrt{x}+12a^2b^6x+8ab^7x^{\frac{3}{2}}+2b^8x^2}-\frac{6ab^2x}{2a^4b^4+8a^3b^5\sqrt{x}+12a^2b^6x+8ab^7x^{\frac{3}{2}}+2b^8x^2}- & \\ \frac{x^2}{2a^5} & \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x**(1/2))**5,x)
```

```
[Out] Piecewise((-a**3/(2*a**4*b**4 + 8*a**3*b**5*sqrt(x) + 12*a**2*b**6*x + 8*a*
b**7*x**(3/2) + 2*b**8*x**2) - 4*a**2*b*sqrt(x)/(2*a**4*b**4 + 8*a**3*b**5*
sqrt(x) + 12*a**2*b**6*x + 8*a*b**7*x**(3/2) + 2*b**8*x**2) - 6*a*b**2*x/(2
*a**4*b**4 + 8*a**3*b**5*sqrt(x) + 12*a**2*b**6*x + 8*a*b**7*x**(3/2) + 2*b
**8*x**2) - 4*b**3*x**(3/2)/(2*a**4*b**4 + 8*a**3*b**5*sqrt(x) + 12*a**2*b*
*6*x + 8*a*b**7*x**(3/2) + 2*b**8*x**2), Ne(b, 0)), (x**2/(2*a**5), True))
```

3.2219 $\int \frac{1}{\left(a+b\sqrt{x}\right)^5}\,dx$

Optimal. Leaf size=38

$$\frac{a}{2b^2\left(a+b\sqrt{x}\right)^4}-\frac{2}{3b^2\left(a+b\sqrt{x}\right)^3}$$

```
[Out] 1/2*a/b^2/(a+b*x^(1/2))^4-2/3/b^2/(a+b*x^(1/2))^3
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {190, 43}

$$\frac{a}{2b^2\left(a+b\sqrt{x}\right)^4}-\frac{2}{3b^2\left(a+b\sqrt{x}\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^(-5),x]
```

```
[Out] a/(2*b^2*(a + b*Sqrt[x])^4) - 2/(3*b^2*(a + b*Sqrt[x])^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+b\sqrt{x}\right)^5}\,dx &= 2\,\text{Subst}\left(\int \frac{x}{(a+bx)^5}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^5}+\frac{1}{b(a+bx)^4}\right)dx, x, \sqrt{x}\right)\\ &= \frac{a}{2b^2\left(a+b\sqrt{x}\right)^4}-\frac{2}{3b^2\left(a+b\sqrt{x}\right)^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 0.74

$$-\frac{a+4b\sqrt{x}}{6b^2\left(a+b\sqrt{x}\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^(-5),x]
```

```
[Out] -1/6*(a + 4*b*Sqrt[x])/(b^2*(a + b*Sqrt[x])^4)
```

fricas [B] time = 1.01, size = 96, normalized size = 2.53

$$\frac{15\,ab^4x^2+10\,a^3b^2x-a^5-4\left(b^5x^2+5\,a^2b^3x\right)\sqrt{x}}{6\left(b^{10}x^4-4\,a^2b^8x^3+6\,a^4b^6x^2-4\,a^6b^4x+a^8b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/2))^5,x, algorithm="fricas")
```

```
[Out] 1/6*(15*a*b^4*x^2 + 10*a^3*b^2*x - a^5 - 4*(b^5*x^2 + 5*a^2*b^3*x)*sqrt(x))
/(b^10*x^4 - 4*a^2*b^8*x^3 + 6*a^4*b^6*x^2 - 4*a^6*b^4*x + a^8*b^2)
```

giac [A] time = 0.16, size = 22, normalized size = 0.58

$$-\frac{4\,b\sqrt{x}+a}{6\left(b\sqrt{x}+a\right)^4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/2))^5,x, algorithm="giac")
```

```
[Out] -1/6*(4*b*sqrt(x) + a)/((b*sqrt(x) + a)^4*b^2)
```

maple [B] time = 0.04, size = 200, normalized size = 5.26

$$-10\left(-\frac{a^2}{4\left(b^2x-a^2\right)^4b^4}-\frac{1}{3\left(b^2x-a^2\right)^3b^4}\right)a^3b^2-5\left(-\frac{a^4}{4\left(b^2x-a^2\right)^4b^6}-\frac{2a^2}{3\left(b^2x-a^2\right)^3b^6}-\frac{1}{2\left(b^2x-a^2\right)^2b^6}\right)a\,b^4+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^(1/2)+a)^5,x)
```

```
[Out] -1/3/b^2/(b*x^(1/2)+a)^3+1/4*a/b^2/(b*x^(1/2)+a)^4-1/3/b^2/(b*x^(1/2)-a)^3-
1/4/b^2*a/(b*x^(1/2)-a)^4+1/4*a^5/(b^2*x-a^2)^4/b^2-10*a^3*b^2*(-1/4*a^2/b^
4/(b^2*x-a^2)^4-1/3/b^4/(b^2*x-a^2)^3)-5*a*b^4*(-1/4*a^4/b^6/(b^2*x-a^2)^4-
1/2/b^6/(b^2*x-a^2)^2-2/3*a^2/b^6/(b^2*x-a^2)^3)
```

maxima [A] time = 0.86, size = 30, normalized size = 0.79

$$-\frac{2}{3\left(b\sqrt{x}+a\right)^3b^2}+\frac{a}{2\left(b\sqrt{x}+a\right)^4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/2))^5,x, algorithm="maxima")
```

```
[Out] -2/3/((b*sqrt(x) + a)^3*b^2) + 1/2*a/((b*sqrt(x) + a)^4*b^2)
```

mupad [B] time = 1.20, size = 57, normalized size = 1.50

$$-\frac{\frac{a}{6\,b^2}+\frac{2\sqrt{x}}{3\,b}}{a^4+b^4\,x^2+6\,a^2\,b^2\,x+4\,a^3\,b\,\sqrt{x}+4\,a\,b^3\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^(1/2))^5,x)
```

```
[Out] -(a/(6*b^2) + (2*x^(1/2))/(3*b))/(a^4 + b^4*x^2 + 6*a^2*b^2*x + 4*a^3*b*x^(
1/2) + 4*a*b^3*x^(3/2))
```

sympy [**A**] time = 1.81, size = 121, normalized size = 3.18

$$\begin{cases} -\dfrac{a}{6a^4b^2+24a^3b^3\sqrt{x}+36a^2b^4x+24ab^5x^{\frac{3}{2}}+6b^6x^2} - \dfrac{4b\sqrt{x}}{6a^4b^2+24a^3b^3\sqrt{x}+36a^2b^4x+24ab^5x^{\frac{3}{2}}+6b^6x^2} & \text{for } b \neq 0 \\ \dfrac{x}{a^5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**(1/2))**5,x)
```

```
[Out] Piecewise((-a/(6*a**4*b**2 + 24*a**3*b**3*sqrt(x) + 36*a**2*b**4*x + 24*a*b
**5*x**(3/2) + 6*b**6*x**2) - 4*b*sqrt(x)/(6*a**4*b**2 + 24*a**3*b**3*sqrt(
x) + 36*a**2*b**4*x + 24*a*b**5*x**(3/2) + 6*b**6*x**2), Ne(b, 0)), (x/a**5
, True))
```

3.2220 $\int \frac{1}{\left(a+b\sqrt{x}\right)^5 x}\,dx$

Optimal. Leaf size=89

$$-\frac{2\log\left(a+b\sqrt{x}\right)}{a^5}+\frac{\log(x)}{a^5}+\frac{2}{a^4\left(a+b\sqrt{x}\right)}+\frac{1}{a^3\left(a+b\sqrt{x}\right)^2}+\frac{2}{3a^2\left(a+b\sqrt{x}\right)^3}+\frac{1}{2a\left(a+b\sqrt{x}\right)^4}$$

```
[Out] ln(x)/a^5-2*ln(a+b*x^(1/2))/a^5+1/2/a/(a+b*x^(1/2))^4+2/3/a^2/(a+b*x^(1/2))
^3+1/a^3/(a+b*x^(1/2))^2+2/a^4/(a+b*x^(1/2))
```

Rubi [A] time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 44}

$$\frac{2}{a^4\left(a+b\sqrt{x}\right)}+\frac{1}{a^3\left(a+b\sqrt{x}\right)^2}+\frac{2}{3a^2\left(a+b\sqrt{x}\right)^3}-\frac{2\log\left(a+b\sqrt{x}\right)}{a^5}+\frac{\log(x)}{a^5}+\frac{1}{2a\left(a+b\sqrt{x}\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sqrt[x])^5*x),x]
```

```
[Out] 1/(2*a*(a + b*Sqrt[x])^4) + 2/(3*a^2*(a + b*Sqrt[x])^3) + 1/(a^3*(a + b*Sqr
t[x])^2) + 2/(a^4*(a + b*Sqrt[x])) - (2*Log[a + b*Sqrt[x]])/a^5 + Log[x]/a^
5
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+b\sqrt{x}\right)^5 x}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{x(a+bx)^5}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(\frac{1}{a^5 x}-\frac{b}{a(a+bx)^5}-\frac{b}{a^2(a+bx)^4}-\frac{b}{a^3(a+bx)^3}-\frac{b}{a^4(a+bx)^2}-\frac{b}{a^5(a+bx)}\right) d\\ &= \frac{1}{2a\left(a+b\sqrt{x}\right)^4}+\frac{2}{3a^2\left(a+b\sqrt{x}\right)^3}+\frac{1}{a^3\left(a+b\sqrt{x}\right)^2}+\frac{2}{a^4\left(a+b\sqrt{x}\right)}-\frac{2\log\left(a+b\sqrt{x}\right)}{a^5}+\end{aligned}$$

Mathematica [A] time = 0.12, size = 71, normalized size = 0.80

$$\frac{\frac{a\left(25a^3+52a^2b\sqrt{x}+42ab^2x+12b^3x^{3/2}\right)}{\left(a+b\sqrt{x}\right)^4}-12\log\left(a+b\sqrt{x}\right)+6\log(x)}{6a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sqrt[x])^5*x),x]
```

```
[Out] ((a*(25*a^3 + 52*a^2*b*Sqrt[x] + 42*a*b^2*x + 12*b^3*x^(3/2)))/(a + b*Sqrt[
x])^4 - 12*Log[a + b*Sqrt[x]] + 6*Log[x])/(6*a^5)
```

fricas [B] time = 1.05, size = 227, normalized size = 2.55

$$-\frac{6\,a^2b^6x^3-21\,a^4b^4x^2+16\,a^6b^2x-25\,a^8+12\left(b^8x^4-4\,a^2b^6x^3+6\,a^4b^4x^2-4\,a^6b^2x+a^8\right)\log\left(b\sqrt{x}+a\right)-12}{6\left(a^5b^8x^4-4\,a^7b^6x^3+6\,a^9b^4x^2-4\,a^{11}b^2x+a^{13}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^(1/2))^5,x, algorithm="fricas")
```

```
[Out] -1/6*(6*a^2*b^6*x^3 - 21*a^4*b^4*x^2 + 16*a^6*b^2*x - 25*a^8 + 12*(b^8*x^4
- 4*a^2*b^6*x^3 + 6*a^4*b^4*x^2 - 4*a^6*b^2*x + a^8)*log(b*sqrt(x) + a) - 1
2*(b^8*x^4 - 4*a^2*b^6*x^3 + 6*a^4*b^4*x^2 - 4*a^6*b^2*x + a^8)*log(sqrt(x)
) - 4*(3*a*b^7*x^3 - 11*a^3*b^5*x^2 + 14*a^5*b^3*x - 12*a^7*b)*sqrt(x))/(a^
5*b^8*x^4 - 4*a^7*b^6*x^3 + 6*a^9*b^4*x^2 - 4*a^11*b^2*x + a^13)
```

giac [A] time = 0.16, size = 69, normalized size = 0.78

$$-\frac{2\,\log\left(\left|b\sqrt{x}+a\right|\right)}{a^5}+\frac{\log\left(|x|\right)}{a^5}+\frac{12\,ab^3x^{\frac{3}{2}}+42\,a^2b^2x+52\,a^3b\sqrt{x}+25\,a^4}{6\left(b\sqrt{x}+a\right)^4a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^(1/2))^5,x, algorithm="giac")
```

```
[Out] -2*log(abs(b*sqrt(x) + a))/a^5 + log(abs(x))/a^5 + 1/6*(12*a*b^3*x^(3/2) +
42*a^2*b^2*x + 52*a^3*b*sqrt(x) + 25*a^4)/((b*sqrt(x) + a)^4*a^5)
```

maple [A] time = 0.01, size = 76, normalized size = 0.85

$$\frac{1}{2\left(b\sqrt{x}+a\right)^4a}+\frac{2}{3\left(b\sqrt{x}+a\right)^3a^2}+\frac{1}{\left(b\sqrt{x}+a\right)^2a^3}+\frac{2}{\left(b\sqrt{x}+a\right)a^4}+\frac{\ln(x)}{a^5}-\frac{2\ln\left(b\sqrt{x}+a\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^(1/2)+a)^5,x)
```

```
[Out] ln(x)/a^5-2*ln(b*x^(1/2)+a)/a^5+1/2/a/(b*x^(1/2)+a)^4+2/3/a^2/(b*x^(1/2)+a)
^3+1/a^3/(b*x^(1/2)+a)^2+2/a^4/(b*x^(1/2)+a)
```

maxima [A] time = 0.93, size = 97, normalized size = 1.09

$$\frac{12\,b^3x^{\frac{3}{2}}+42\,ab^2x+52\,a^2b\sqrt{x}+25\,a^3}{6\left(a^4b^4x^2+4\,a^5b^3x^{\frac{3}{2}}+6\,a^6b^2x+4\,a^7b\sqrt{x}+a^8\right)}-\frac{2\,\log\left(b\sqrt{x}+a\right)}{a^5}+\frac{\log(x)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^(1/2))^5,x, algorithm="maxima")
```

```
[Out] 1/6*(12*b^3*x^(3/2) + 42*a*b^2*x + 52*a^2*b*sqrt(x) + 25*a^3)/(a^4*b^4*x^2
+ 4*a^5*b^3*x^(3/2) + 6*a^6*b^2*x + 4*a^7*b*sqrt(x) + a^8) - 2*log(b*sqrt(x
) + a)/a^5 + log(x)/a^5
```

mupad [B] time = 0.08, size = 94, normalized size = 1.06

$$\frac{\frac{25}{6\,a}+\frac{26\,b\,\sqrt{x}}{3\,a^2}+\frac{7\,b^2\,x}{a^3}+\frac{2\,b^3\,x^{3/2}}{a^4}}{a^4+b^4\,x^2+6\,a^2\,b^2\,x+4\,a^3\,b\,\sqrt{x}+4\,a\,b^3\,x^{3/2}}-\frac{4\,\mathrm{atanh}\left(\frac{2\,b\,\sqrt{x}}{a}+1\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^(1/2))^5),x)
```

```
[Out] (25/(6*a) + (26*b*x^(1/2))/(3*a^2) + (7*b^2*x)/a^3 + (2*b^3*x^(3/2))/a^4)/(
a^4 + b^4*x^2 + 6*a^2*b^2*x + 4*a^3*b*x^(1/2) + 4*a*b^3*x^(3/2)) - (4*atanh
((2*b*x^(1/2))/a + 1))/a^5
```

sympy [A] time = 5.83, size = 1049, normalized size = 11.79

$$\begin{cases}\frac{\tilde{\infty}}{x^{\frac{5}{2}}} \\ \frac{\log(x)}{a^5} \\ -\frac{2}{5b^5x^{\frac{5}{2}}} \\ \frac{6a^4\sqrt{x}\log(x)}{6a^9\sqrt{x}+24a^8bx+36a^7b^2x^{\frac{3}{2}}+24a^6b^3x^2+6a^5b^4x^{\frac{5}{2}}}-\frac{12a^4\sqrt{x}\log\left(\frac{a}{b}+\sqrt{x}\right)}{6a^9\sqrt{x}+24a^8bx+36a^7b^2x^{\frac{3}{2}}+24a^6b^3x^2+6a^5b^4x^{\frac{5}{2}}}+\frac{25a^4\sqrt{x}}{6a^9\sqrt{x}+24a^8bx+36a^7b^2x^{\frac{3}{2}}+24a^6b^3x^2+6a^5\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x**(1/2))**5,x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (log(x)/a**5, Eq(b, 0)), (-2
/(5*b**5*x**(5/2)), Eq(a, 0)), (6*a**4*sqrt(x)*log(x)/(6*a**9*sqrt(x) + 24*
a**8*b*x + 36*a**7*b**2*x**(3/2) + 24*a**6*b**3*x**2 + 6*a**5*b**4*x**(5/2)
) - 12*a**4*sqrt(x)*log(a/b + sqrt(x))/(6*a**9*sqrt(x) + 24*a**8*b*x + 36*a
**7*b**2*x**(3/2) + 24*a**6*b**3*x**2 + 6*a**5*b**4*x**(5/2)) + 25*a**4*sqr
t(x)/(6*a**9*sqrt(x) + 24*a**8*b*x + 36*a**7*b**2*x**(3/2) + 24*a**6*b**3*x
**2 + 6*a**5*b**4*x**(5/2)) + 24*a**3*b*x*log(x)/(6*a**9*sqrt(x) + 24*a**8*
b*x + 36*a**7*b**2*x**(3/2) + 24*a**6*b**3*x**2 + 6*a**5*b**4*x**(5/2)) - 4
8*a**3*b*x*log(a/b + sqrt(x))/(6*a**9*sqrt(x) + 24*a**8*b*x + 36*a**7*b**2*
x**(3/2) + 24*a**6*b**3*x**2 + 6*a**5*b**4*x**(5/2)) + 52*a**3*b*x/(6*a**9*
sqrt(x) + 24*a**8*b*x + 36*a**7*b**2*x**(3/2) + 24*a**6*b**3*x**2 + 6*a**5*
b**4*x**(5/2)) + 36*a**2*b**2*x**(3/2)*log(x)/(6*a**9*sqrt(x) + 24*a**8*b*x
 + 36*a**7*b**2*x**(3/2) + 24*a**6*b**3*x**2 + 6*a**5*b**4*x**(5/2)) - 72*a
**2*b**2*x**(3/2)*log(a/b + sqrt(x))/(6*a**9*sqrt(x) + 24*a**8*b*x + 36*a**
7*b**2*x**(3/2) + 24*a**6*b**3*x**2 + 6*a**5*b**4*x**(5/2)) + 42*a**2*b**2*
x**(3/2)/(6*a**9*sqrt(x) + 24*a**8*b*x + 36*a**7*b**2*x**(3/2) + 24*a**6*b*
*3*x**2 + 6*a**5*b**4*x**(5/2)) + 24*a*b**3*x**2*log(x)/(6*a**9*sqrt(x) + 2
4*a**8*b*x + 36*a**7*b**2*x**(3/2) + 24*a**6*b**3*x**2 + 6*a**5*b**4*x**(5/
2)) - 48*a*b**3*x**2*log(a/b + sqrt(x))/(6*a**9*sqrt(x) + 24*a**8*b*x + 36*
a**7*b**2*x**(3/2) + 24*a**6*b**3*x**2 + 6*a**5*b**4*x**(5/2)) + 12*a*b**3*
x**2/(6*a**9*sqrt(x) + 24*a**8*b*x + 36*a**7*b**2*x**(3/2) + 24*a**6*b**3*x
**2 + 6*a**5*b**4*x**(5/2)) + 6*b**4*x**(5/2)*log(x)/(6*a**9*sqrt(x) + 24*a
**8*b*x + 36*a**7*b**2*x**(3/2) + 24*a**6*b**3*x**2 + 6*a**5*b**4*x**(5/2))
 - 12*b**4*x**(5/2)*log(a/b + sqrt(x))/(6*a**9*sqrt(x) + 24*a**8*b*x + 36*a
**7*b**2*x**(3/2) + 24*a**6*b**3*x**2 + 6*a**5*b**4*x**(5/2)), True))
```

3.2221 $\int \frac{1}{\left(a+b\sqrt{x}\right)^5 x^2}\,dx$

Optimal. Leaf size=126

$$-\frac{30b^2\log\left(a+b\sqrt{x}\right)}{a^7}+\frac{15b^2\log(x)}{a^7}+\frac{20b^2}{a^6\left(a+b\sqrt{x}\right)}+\frac{10b}{a^6\sqrt{x}}+\frac{6b^2}{a^5\left(a+b\sqrt{x}\right)^2}-\frac{1}{a^5x}+\frac{2b^2}{a^4\left(a+b\sqrt{x}\right)^3}+\frac{b^2}{2a^3\left(a+b\sqrt{x}\right)^4}$$

```
[Out] -1/a^5/x+15*b^2*ln(x)/a^7-30*b^2*ln(a+b*x^(1/2))/a^7+10*b/a^6/x^(1/2)+1/2*b
^2/a^3/(a+b*x^(1/2))^4+2*b^2/a^4/(a+b*x^(1/2))^3+6*b^2/a^5/(a+b*x^(1/2))^2+
20*b^2/a^6/(a+b*x^(1/2))
```

Rubi [A] time = 0.09, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 44}

$$\frac{20b^2}{a^6\left(a+b\sqrt{x}\right)}+\frac{6b^2}{a^5\left(a+b\sqrt{x}\right)^2}+\frac{2b^2}{a^4\left(a+b\sqrt{x}\right)^3}+\frac{b^2}{2a^3\left(a+b\sqrt{x}\right)^4}-\frac{30b^2\log\left(a+b\sqrt{x}\right)}{a^7}+\frac{15b^2\log(x)}{a^7}+\frac{10b}{a^6\sqrt{x}}-\frac{}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sqrt[x])^5*x^2),x]
```

```
[Out] b^2/(2*a^3*(a + b*Sqrt[x])^4) + (2*b^2)/(a^4*(a + b*Sqrt[x])^3) + (6*b^2)/(
a^5*(a + b*Sqrt[x])^2) + (20*b^2)/(a^6*(a + b*Sqrt[x])) - 1/(a^5*x) + (10*b
)/(a^6*Sqrt[x]) - (30*b^2*Log[a + b*Sqrt[x]])/a^7 + (15*b^2*Log[x])/a^7
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^5 x^2}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^3(a+bx)^5}\,dx, x, \sqrt{x}\right)$$

$$= 2\,\text{Subst}\left(\int \left(\frac{1}{a^5x^3}-\frac{5b}{a^6x^2}+\frac{15b^2}{a^7x}-\frac{b^3}{a^3(a+bx)^5}-\frac{3b^3}{a^4(a+bx)^4}-\frac{6b^3}{a^5(a+bx)^3}-\frac{10b}{a^6(a+\right.\right.$$

$$= \frac{b^2}{2a^3\left(a+b\sqrt{x}\right)^4}+\frac{2b^2}{a^4\left(a+b\sqrt{x}\right)^3}+\frac{6b^2}{a^5\left(a+b\sqrt{x}\right)^2}+\frac{20b^2}{a^6\left(a+b\sqrt{x}\right)}-\frac{1}{a^5x}+\frac{10b}{a^6\sqrt{x}}-$$

Mathematica [A] time = 0.18, size = 104, normalized size = 0.83

$$\frac{\frac{a\left(-2a^5+12a^4b\sqrt{x}+125a^3b^2x+260a^2b^3x^{3/2}+210ab^4x^2+60b^5x^{5/2}\right)}{x\left(a+b\sqrt{x}\right)^4}-60b^2\log\left(a+b\sqrt{x}\right)+30b^2\log(x)}{2a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sqrt[x])^5*x^2),x]
```

```
[Out] ((a*(-2*a^5 + 12*a^4*b*Sqrt[x] + 125*a^3*b^2*x + 260*a^2*b^3*x^(3/2) + 210*
a*b^4*x^2 + 60*b^5*x^(5/2)))/((a + b*Sqrt[x])^4*x) - 60*b^2*Log[a + b*Sqrt[
x]] + 30*b^2*Log[x])/(2*a^7)
```

fricas [B] time = 0.73, size = 267, normalized size = 2.12

$$-\frac{30\,a^2b^8x^4-105\,a^4b^6x^3+130\,a^6b^4x^2-65\,a^8b^2x+2\,a^{10}+60\left(b^{10}x^5-4\,a^2b^8x^4+6\,a^4b^6x^3-4\,a^6b^4x^2+a^8b^2x\right)\ln}{2\left(a^7b^8x^5-4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^(1/2))^5,x, algorithm="fricas")
```

```
[Out] -1/2*(30*a^2*b^8*x^4 - 105*a^4*b^6*x^3 + 130*a^6*b^4*x^2 - 65*a^8*b^2*x + 2
*a^10 + 60*(b^10*x^5 - 4*a^2*b^8*x^4 + 6*a^4*b^6*x^3 - 4*a^6*b^4*x^2 + a^8*
b^2*x)*log(b*sqrt(x) + a) - 60*(b^10*x^5 - 4*a^2*b^8*x^4 + 6*a^4*b^6*x^3 -
4*a^6*b^4*x^2 + a^8*b^2*x)*log(sqrt(x)) - 4*(15*a*b^9*x^4 - 55*a^3*b^7*x^3
+ 73*a^5*b^5*x^2 - 40*a^7*b^3*x + 5*a^9*b)*sqrt(x))/(a^7*b^8*x^5 - 4*a^9*b^
6*x^4 + 6*a^11*b^4*x^3 - 4*a^13*b^2*x^2 + a^15*x)
```

giac [A] time = 0.17, size = 101, normalized size = 0.80

$$-\frac{30\,b^2\log\left(\left|b\sqrt{x}+a\right|\right)}{a^7}+\frac{15\,b^2\log\left(|x|\right)}{a^7}+\frac{60\,ab^5x^{\frac{5}{2}}+210\,a^2b^4x^2+260\,a^3b^3x^{\frac{3}{2}}+125\,a^4b^2x+12\,a^5b\sqrt{x}-2\,a^6}{2\left(b\sqrt{x}+a\right)^4a^7x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^(1/2))^5,x, algorithm="giac")
```

```
[Out] -30*b^2*log(abs(b*sqrt(x) + a))/a^7 + 15*b^2*log(abs(x))/a^7 + 1/2*(60*a*b^
5*x^(5/2) + 210*a^2*b^4*x^2 + 260*a^3*b^3*x^(3/2) + 125*a^4*b^2*x + 12*a^5*
b*sqrt(x) - 2*a^6)/((b*sqrt(x) + a)^4*a^7*x)
```

maple [A] time = 0.01, size = 113, normalized size = 0.90

$$\frac{b^2}{2\left(b\sqrt{x}+a\right)^4a^3}+\frac{2b^2}{\left(b\sqrt{x}+a\right)^3a^4}+\frac{6b^2}{\left(b\sqrt{x}+a\right)^2a^5}+\frac{20b^2}{\left(b\sqrt{x}+a\right)a^6}+\frac{15b^2\ln(x)}{a^7}-\frac{30b^2\ln\left(b\sqrt{x}+a\right)}{a^7}+\frac{10b}{a^6\sqrt{x}}-\frac{1}{a^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^(1/2)+a)^5,x)
```

```
[Out] -1/a^5/x+15*b^2*ln(x)/a^7-30*b^2*ln(b*x^(1/2)+a)/a^7+10*b/a^6/x^(1/2)+1/2*b
^2/a^3/(b*x^(1/2)+a)^4+2*b^2/a^4/(b*x^(1/2)+a)^3+6*b^2/a^5/(b*x^(1/2)+a)^2+
20*b^2/a^6/(b*x^(1/2)+a)
```

maxima [A] time = 0.93, size = 130, normalized size = 1.03

$$\frac{60\,b^5x^{\frac{5}{2}}+210\,ab^4x^2+260\,a^2b^3x^{\frac{3}{2}}+125\,a^3b^2x+12\,a^4b\sqrt{x}-2\,a^5}{2\left(a^6b^4x^3+4\,a^7b^3x^{\frac{5}{2}}+6\,a^8b^2x^2+4\,a^9bx^{\frac{3}{2}}+a^{10}x\right)}-\frac{30\,b^2\log\left(b\sqrt{x}+a\right)}{a^7}+\frac{15\,b^2\log(x)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^(1/2))^5,x, algorithm="maxima")
```

```
[Out] 1/2*(60*b^5*x^(5/2) + 210*a*b^4*x^2 + 260*a^2*b^3*x^(3/2) + 125*a^3*b^2*x +
 12*a^4*b*sqrt(x) - 2*a^5)/(a^6*b^4*x^3 + 4*a^7*b^3*x^(5/2) + 6*a^8*b^2*x^2
 + 4*a^9*b*x^(3/2) + a^10*x) - 30*b^2*log(b*sqrt(x) + a)/a^7 + 15*b^2*log(x
)/a^7
```

mupad **[B]** time = 0.12, size = 123, normalized size = 0.98

$$\frac{\frac{6\,b\,\sqrt{x}}{a^2}-\frac{1}{a}+\frac{125\,b^2\,x}{2\,a^3}+\frac{105\,b^4\,x^2}{a^5}+\frac{130\,b^3\,x^{3/2}}{a^4}+\frac{30\,b^5\,x^{5/2}}{a^6}}{a^4\,x+b^4\,x^3+4\,a^3\,b\,x^{3/2}+4\,a\,b^3\,x^{5/2}+6\,a^2\,b^2\,x^2}-\frac{60\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,\sqrt{x}}{a}+1\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^(1/2))^5),x)
```

```
[Out] ((6*b*x^(1/2))/a^2 - 1/a + (125*b^2*x)/(2*a^3) + (105*b^4*x^2)/a^5 + (130*b
^3*x^(3/2))/a^4 + (30*b^5*x^(5/2))/a^6)/(a^4*x + b^4*x^3 + 4*a^3*b*x^(3/2)
+ 4*a*b^3*x^(5/2) + 6*a^2*b^2*x^2) - (60*b^2*atanh((2*b*x^(1/2))/a + 1))/a^
7
```

sympy **[A]** time = 12.84, size = 1232, normalized size = 9.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*x**(1/2))**5,x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-1/(a**5*x), Eq(b, 0)), (-2
/(7*b**5*x**(7/2)), Eq(a, 0)), (-2*a**6*sqrt(x)/(2*a**11*x**(3/2) + 8*a**10
*b*x**2 + 12*a**9*b**2*x**(5/2) + 8*a**8*b**3*x**3 + 2*a**7*b**4*x**(7/2))
+ 12*a**5*b*x/(2*a**11*x**(3/2) + 8*a**10*b*x**2 + 12*a**9*b**2*x**(5/2) +
8*a**8*b**3*x**3 + 2*a**7*b**4*x**(7/2)) + 30*a**4*b**2*x**(3/2)*log(x)/(2*
a**11*x**(3/2) + 8*a**10*b*x**2 + 12*a**9*b**2*x**(5/2) + 8*a**8*b**3*x**3
+ 2*a**7*b**4*x**(7/2)) - 60*a**4*b**2*x**(3/2)*log(a/b + sqrt(x))/(2*a**11
*x**(3/2) + 8*a**10*b*x**2 + 12*a**9*b**2*x**(5/2) + 8*a**8*b**3*x**3 + 2*a
**7*b**4*x**(7/2)) + 125*a**4*b**2*x**(3/2)/(2*a**11*x**(3/2) + 8*a**10*b*x
**2 + 12*a**9*b**2*x**(5/2) + 8*a**8*b**3*x**3 + 2*a**7*b**4*x**(7/2)) + 12
0*a**3*b**3*x**2*log(x)/(2*a**11*x**(3/2) + 8*a**10*b*x**2 + 12*a**9*b**2*x
**(5/2) + 8*a**8*b**3*x**3 + 2*a**7*b**4*x**(7/2)) - 240*a**3*b**3*x**2*log
(a/b + sqrt(x))/(2*a**11*x**(3/2) + 8*a**10*b*x**2 + 12*a**9*b**2*x**(5/2)
+ 8*a**8*b**3*x**3 + 2*a**7*b**4*x**(7/2)) + 260*a**3*b**3*x**2/(2*a**11*x*
*(3/2) + 8*a**10*b*x**2 + 12*a**9*b**2*x**(5/2) + 8*a**8*b**3*x**3 + 2*a**7
*b**4*x**(7/2)) + 180*a**2*b**4*x**(5/2)*log(x)/(2*a**11*x**(3/2) + 8*a**10
*b*x**2 + 12*a**9*b**2*x**(5/2) + 8*a**8*b**3*x**3 + 2*a**7*b**4*x**(7/2))
- 360*a**2*b**4*x**(5/2)*log(a/b + sqrt(x))/(2*a**11*x**(3/2) + 8*a**10*b*x
**2 + 12*a**9*b**2*x**(5/2) + 8*a**8*b**3*x**3 + 2*a**7*b**4*x**(7/2)) + 21
0*a**2*b**4*x**(5/2)/(2*a**11*x**(3/2) + 8*a**10*b*x**2 + 12*a**9*b**2*x**(
5/2) + 8*a**8*b**3*x**3 + 2*a**7*b**4*x**(7/2)) + 120*a*b**5*x**3*log(x)/(2
*a**11*x**(3/2) + 8*a**10*b*x**2 + 12*a**9*b**2*x**(5/2) + 8*a**8*b**3*x**3
 + 2*a**7*b**4*x**(7/2)) - 240*a*b**5*x**3*log(a/b + sqrt(x))/(2*a**11*x**(
3/2) + 8*a**10*b*x**2 + 12*a**9*b**2*x**(5/2) + 8*a**8*b**3*x**3 + 2*a**7*b
**4*x**(7/2)) + 60*a*b**5*x**3/(2*a**11*x**(3/2) + 8*a**10*b*x**2 + 12*a**9
*b**2*x**(5/2) + 8*a**8*b**3*x**3 + 2*a**7*b**4*x**(7/2)) + 30*b**6*x**(7/2
)*log(x)/(2*a**11*x**(3/2) + 8*a**10*b*x**2 + 12*a**9*b**2*x**(5/2) + 8*a**
8*b**3*x**3 + 2*a**7*b**4*x**(7/2)) - 60*b**6*x**(7/2)*log(a/b + sqrt(x))/(
2*a**11*x**(3/2) + 8*a**10*b*x**2 + 12*a**9*b**2*x**(5/2) + 8*a**8*b**3*x**
3 + 2*a**7*b**4*x**(7/2)), True))
```

3.2222 $\int \frac{1}{\left(a+b\sqrt{x}\right)^5 x^3}\,dx$

Optimal. Leaf size=156

$$-\frac{140b^4\log\left(a+b\sqrt{x}\right)}{a^9}+\frac{70b^4\log(x)}{a^9}+\frac{70b^4}{a^8\left(a+b\sqrt{x}\right)}+\frac{70b^3}{a^8\sqrt{x}}+\frac{15b^4}{a^7\left(a+b\sqrt{x}\right)^2}-\frac{15b^2}{a^7x}+\frac{10b^4}{3a^6\left(a+b\sqrt{x}\right)^3}+\frac{10b}{3a^6x^{3/2}}+\frac{}{2}$$

```
[Out] -1/2/a^5/x^2+10/3*b/a^6/x^(3/2)-15*b^2/a^7/x+70*b^4*ln(x)/a^9-140*b^4*ln(a+
b*x^(1/2))/a^9+70*b^3/a^8/x^(1/2)+1/2*b^4/a^5/(a+b*x^(1/2))^4+10/3*b^4/a^6/
(a+b*x^(1/2))^3+15*b^4/a^7/(a+b*x^(1/2))^2+70*b^4/a^8/(a+b*x^(1/2))
```

Rubi [A] time = 0.11, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 44}

$$\frac{70b^4}{a^8\left(a+b\sqrt{x}\right)}+\frac{15b^4}{a^7\left(a+b\sqrt{x}\right)^2}+\frac{10b^4}{3a^6\left(a+b\sqrt{x}\right)^3}+\frac{b^4}{2a^5\left(a+b\sqrt{x}\right)^4}+\frac{70b^3}{a^8\sqrt{x}}-\frac{15b^2}{a^7x}-\frac{140b^4\log\left(a+b\sqrt{x}\right)}{a^9}+\frac{70b^4\log}{a^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sqrt[x])^5*x^3),x]
```

```
[Out] b^4/(2*a^5*(a + b*Sqrt[x])^4) + (10*b^4)/(3*a^6*(a + b*Sqrt[x])^3) + (15*b^
4)/(a^7*(a + b*Sqrt[x])^2) + (70*b^4)/(a^8*(a + b*Sqrt[x])) - 1/(2*a^5*x^2)
 + (10*b)/(3*a^6*x^(3/2)) - (15*b^2)/(a^7*x) + (70*b^3)/(a^8*Sqrt[x]) - (14
0*b^4*Log[a + b*Sqrt[x]])/a^9 + (70*b^4*Log[x])/a^9
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+b\sqrt{x}\right)^5 x^3}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{x^5(a+bx)^5}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int\left(\frac{1}{a^5x^5}-\frac{5b}{a^6x^4}+\frac{15b^2}{a^7x^3}-\frac{35b^3}{a^8x^2}+\frac{70b^4}{a^9x}-\frac{b^5}{a^5(a+bx)^5}-\frac{5b^5}{a^6(a+bx)^4}-\frac{15b^5}{a^7(a+b}\right.\right.\\ &= \frac{b^4}{2a^5\left(a+b\sqrt{x}\right)^4}+\frac{10b^4}{3a^6\left(a+b\sqrt{x}\right)^3}+\frac{15b^4}{a^7\left(a+b\sqrt{x}\right)^2}+\frac{70b^4}{a^8\left(a+b\sqrt{x}\right)}-\frac{1}{2a^5x^2}+\frac{10b}{3a^6x^{3/2}}-\end{aligned}$$

Mathematica [A] time = 0.21, size = 128, normalized size = 0.82

$$\frac{\frac{a\left(-3a^7+8a^6b\sqrt{x}-28a^5b^2x+168a^4b^3x^{3/2}+1750a^3b^4x^2+3640a^2b^5x^{5/2}+2940ab^6x^3+840b^7x^{7/2}\right)}{x^2\left(a+b\sqrt{x}\right)^4}-840b^4\log\left(a+b\sqrt{x}\right)+420b^4\log(x)}{6a^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sqrt[x])^5*x^3),x]
```

```
[Out] ((a*(-3*a^7 + 8*a^6*b*Sqrt[x] - 28*a^5*b^2*x + 168*a^4*b^3*x^(3/2) + 1750*a
^3*b^4*x^2 + 3640*a^2*b^5*x^(5/2) + 2940*a*b^6*x^3 + 840*b^7*x^(7/2)))/((a
+ b*Sqrt[x])^4*x^2) - 840*b^4*Log[a + b*Sqrt[x]] + 420*b^4*Log[x])/(6*a^9)
```

fricas [B] time = 1.18, size = 295, normalized size = 1.89

$$-\frac{420\,a^2b^{10}x^5-1470\,a^4b^8x^4+1820\,a^6b^6x^3-875\,a^8b^4x^2+78\,a^{10}b^2x+3\,a^{12}+840\left(b^{12}x^6-4\,a^2b^{10}x^5+6\,a^4b^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^(1/2))^5,x, algorithm="fricas")
```

```
[Out] -1/6*(420*a^2*b^10*x^5 - 1470*a^4*b^8*x^4 + 1820*a^6*b^6*x^3 - 875*a^8*b^4*
x^2 + 78*a^10*b^2*x + 3*a^12 + 840*(b^12*x^6 - 4*a^2*b^10*x^5 + 6*a^4*b^8*x
^4 - 4*a^6*b^6*x^3 + a^8*b^4*x^2)*log(b*sqrt(x) + a) - 840*(b^12*x^6 - 4*a^
2*b^10*x^5 + 6*a^4*b^8*x^4 - 4*a^6*b^6*x^3 + a^8*b^4*x^2)*log(sqrt(x)) - 4*
(210*a*b^11*x^5 - 770*a^3*b^9*x^4 + 1022*a^5*b^7*x^3 - 558*a^7*b^5*x^2 + 85
*a^9*b^3*x + 5*a^11*b)*sqrt(x))/(a^9*b^8*x^6 - 4*a^11*b^6*x^5 + 6*a^13*b^4*
x^4 - 4*a^15*b^2*x^3 + a^17*x^2)
```

giac [A] time = 0.17, size = 119, normalized size = 0.76

$$-\frac{140\,b^4\log\left(\left|b\sqrt{x}+a\right|\right)}{a^9}+\frac{70\,b^4\log\left(|x|\right)}{a^9}+\frac{840\,b^7x^{\frac{7}{2}}+2940\,ab^6x^3+3640\,a^2b^5x^{\frac{5}{2}}+1750\,a^3b^4x^2+168\,a^4b^3x^{\frac{3}{2}}-}{6\left(bx+a\sqrt{x}\right)^4a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^(1/2))^5,x, algorithm="giac")
```

```
[Out] -140*b^4*log(abs(b*sqrt(x) + a))/a^9 + 70*b^4*log(abs(x))/a^9 + 1/6*(840*b^
7*x^(7/2) + 2940*a*b^6*x^3 + 3640*a^2*b^5*x^(5/2) + 1750*a^3*b^4*x^2 + 168*
a^4*b^3*x^(3/2) - 28*a^5*b^2*x + 8*a^6*b*sqrt(x) - 3*a^7)/((b*x + a*sqrt(x)
)^4*a^8)
```

maple [A] time = 0.01, size = 135, normalized size = 0.87

$$\frac{b^4}{2\left(b\sqrt{x}+a\right)^4a^5}+\frac{10b^4}{3\left(b\sqrt{x}+a\right)^3a^6}+\frac{15b^4}{\left(b\sqrt{x}+a\right)^2a^7}+\frac{70b^4}{\left(b\sqrt{x}+a\right)a^8}+\frac{70b^4\ln(x)}{a^9}-\frac{140b^4\ln\left(b\sqrt{x}+a\right)}{a^9}+\frac{70b^3}{a^8\sqrt{x}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^(1/2)+a)^5,x)
```

```
[Out] -1/2/a^5/x^2+10/3*b/a^6/x^(3/2)-15*b^2/a^7/x+70*b^4*ln(x)/a^9-140*b^4*ln(b*
x^(1/2)+a)/a^9+70*b^3/a^8/x^(1/2)+1/2*b^4/a^5/(b*x^(1/2)+a)^4+10/3*b^4/a^6/
(b*x^(1/2)+a)^3+15*b^4/a^7/(b*x^(1/2)+a)^2+70*b^4/a^8/(b*x^(1/2)+a)
```

maxima [A] time = 0.93, size = 154, normalized size = 0.99

$$\frac{840\,b^7x^{\frac{7}{2}}+2940\,ab^6x^3+3640\,a^2b^5x^{\frac{5}{2}}+1750\,a^3b^4x^2+168\,a^4b^3x^{\frac{3}{2}}-28\,a^5b^2x+8\,a^6b\sqrt{x}-3\,a^7}{6\left(a^8b^4x^4+4\,a^9b^3x^{\frac{7}{2}}+6\,a^{10}b^2x^3+4\,a^{11}bx^{\frac{5}{2}}+a^{12}x^2\right)}-\frac{140\,b^4\log\left(b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^(1/2))^5,x, algorithm="maxima")
```

```
[Out] 1/6*(840*b^7*x^(7/2) + 2940*a*b^6*x^3 + 3640*a^2*b^5*x^(5/2) + 1750*a^3*b^4
*x^2 + 168*a^4*b^3*x^(3/2) - 28*a^5*b^2*x + 8*a^6*b*sqrt(x) - 3*a^7)/(a^8*b
^4*x^4 + 4*a^9*b^3*x^(7/2) + 6*a^10*b^2*x^3 + 4*a^11*b*x^(5/2) + a^12*x^2)
- 140*b^4*log(b*sqrt(x) + a)/a^9 + 70*b^4*log(x)/a^9
```

mupad **[B]** time = 1.27, size = 147, normalized size = 0.94

$$\frac{\frac{4\,b\,\sqrt{x}}{3\,a^2}-\frac{1}{2\,a}-\frac{14\,b^2\,x}{3\,a^3}+\frac{875\,b^4\,x^2}{3\,a^5}+\frac{28\,b^3\,x^{3/2}}{a^4}+\frac{490\,b^6\,x^3}{a^7}+\frac{1820\,b^5\,x^{5/2}}{3\,a^6}+\frac{140\,b^7\,x^{7/2}}{a^8}}{a^4\,x^2+b^4\,x^4+4\,a^3\,b\,x^{5/2}+4\,a\,b^3\,x^{7/2}+6\,a^2\,b^2\,x^3}-\frac{280\,b^4\,\operatorname{atanh}\left(\frac{2\,b\,\sqrt{x}}{a}+1\right)}{a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^(1/2))^5),x)
```

```
[Out] ((4*b*x^(1/2))/(3*a^2) - 1/(2*a) - (14*b^2*x)/(3*a^3) + (875*b^4*x^2)/(3*a^
5) + (28*b^3*x^(3/2))/a^4 + (490*b^6*x^3)/a^7 + (1820*b^5*x^(5/2))/(3*a^6)
+ (140*b^7*x^(7/2))/a^8)/(a^4*x^2 + b^4*x^4 + 4*a^3*b*x^(5/2) + 4*a*b^3*x^(
7/2) + 6*a^2*b^2*x^3) - (280*b^4*atanh((2*b*x^(1/2))/a + 1))/a^9
```

sympy **[A]** time = 22.06, size = 1380, normalized size = 8.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*x**(1/2))**5,x)
```

```
[Out] Piecewise((zoo/x**(9/2), Eq(a, 0) & Eq(b, 0)), (-1/(2*a**5*x**2), Eq(b, 0))
, (-2/(9*b**5*x**(9/2)), Eq(a, 0)), (-3*a**8*sqrt(x)/(6*a**13*x**(5/2) + 24
*a**12*b*x**3 + 36*a**11*b**2*x**(7/2) + 24*a**10*b**3*x**4 + 6*a**9*b**4*x
**(9/2)) + 8*a**7*b*x/(6*a**13*x**(5/2) + 24*a**12*b*x**3 + 36*a**11*b**2*x
**(7/2) + 24*a**10*b**3*x**4 + 6*a**9*b**4*x**(9/2)) - 28*a**6*b**2*x**(3/2
)/(6*a**13*x**(5/2) + 24*a**12*b*x**3 + 36*a**11*b**2*x**(7/2) + 24*a**10*b
**3*x**4 + 6*a**9*b**4*x**(9/2)) + 168*a**5*b**3*x**2/(6*a**13*x**(5/2) + 2
4*a**12*b*x**3 + 36*a**11*b**2*x**(7/2) + 24*a**10*b**3*x**4 + 6*a**9*b**4*
x**(9/2)) + 420*a**4*b**4*x**(5/2)*log(x)/(6*a**13*x**(5/2) + 24*a**12*b*x*
*3 + 36*a**11*b**2*x**(7/2) + 24*a**10*b**3*x**4 + 6*a**9*b**4*x**(9/2)) -
840*a**4*b**4*x**(5/2)*log(a/b + sqrt(x))/(6*a**13*x**(5/2) + 24*a**12*b*x*
*3 + 36*a**11*b**2*x**(7/2) + 24*a**10*b**3*x**4 + 6*a**9*b**4*x**(9/2)) +
1750*a**4*b**4*x**(5/2)/(6*a**13*x**(5/2) + 24*a**12*b*x**3 + 36*a**11*b**2
*x**(7/2) + 24*a**10*b**3*x**4 + 6*a**9*b**4*x**(9/2)) + 1680*a**3*b**5*x**
3*log(x)/(6*a**13*x**(5/2) + 24*a**12*b*x**3 + 36*a**11*b**2*x**(7/2) + 24*
a**10*b**3*x**4 + 6*a**9*b**4*x**(9/2)) - 3360*a**3*b**5*x**3*log(a/b + sqr
t(x))/(6*a**13*x**(5/2) + 24*a**12*b*x**3 + 36*a**11*b**2*x**(7/2) + 24*a**
10*b**3*x**4 + 6*a**9*b**4*x**(9/2)) + 3640*a**3*b**5*x**3/(6*a**13*x**(5/2
) + 24*a**12*b*x**3 + 36*a**11*b**2*x**(7/2) + 24*a**10*b**3*x**4 + 6*a**9*
b**4*x**(9/2)) + 2520*a**2*b**6*x**(7/2)*log(x)/(6*a**13*x**(5/2) + 24*a**1
2*b*x**3 + 36*a**11*b**2*x**(7/2) + 24*a**10*b**3*x**4 + 6*a**9*b**4*x**(9/
2)) - 5040*a**2*b**6*x**(7/2)*log(a/b + sqrt(x))/(6*a**13*x**(5/2) + 24*a**
12*b*x**3 + 36*a**11*b**2*x**(7/2) + 24*a**10*b**3*x**4 + 6*a**9*b**4*x**(9
/2)) + 2940*a**2*b**6*x**(7/2)/(6*a**13*x**(5/2) + 24*a**12*b*x**3 + 36*a**
11*b**2*x**(7/2) + 24*a**10*b**3*x**4 + 6*a**9*b**4*x**(9/2)) + 1680*a*b**7
*x**4*log(x)/(6*a**13*x**(5/2) + 24*a**12*b*x**3 + 36*a**11*b**2*x**(7/2) +
 24*a**10*b**3*x**4 + 6*a**9*b**4*x**(9/2)) - 3360*a*b**7*x**4*log(a/b + sq
rt(x))/(6*a**13*x**(5/2) + 24*a**12*b*x**3 + 36*a**11*b**2*x**(7/2) + 24*a*
*10*b**3*x**4 + 6*a**9*b**4*x**(9/2)) + 840*a*b**7*x**4/(6*a**13*x**(5/2) +
 24*a**12*b*x**3 + 36*a**11*b**2*x**(7/2) + 24*a**10*b**3*x**4 + 6*a**9*b**
4*x**(9/2)) + 420*b**8*x**(9/2)*log(x)/(6*a**13*x**(5/2) + 24*a**12*b*x**3
+ 36*a**11*b**2*x**(7/2) + 24*a**10*b**3*x**4 + 6*a**9*b**4*x**(9/2)) - 840
*b**8*x**(9/2)*log(a/b + sqrt(x))/(6*a**13*x**(5/2) + 24*a**12*b*x**3 + 36*
a**11*b**2*x**(7/2) + 24*a**10*b**3*x**4 + 6*a**9*b**4*x**(9/2)), True))
```

3.2223 $\int \frac{x^5}{\left(a+b\sqrt{x}\right)^8}\,dx$

Optimal. Leaf size=203

$$\frac{2a^{11}}{7b^{12}\left(a+b\sqrt{x}\right)^7}-\frac{11a^{10}}{3b^{12}\left(a+b\sqrt{x}\right)^6}+\frac{22a^9}{b^{12}\left(a+b\sqrt{x}\right)^5}-\frac{165a^8}{2b^{12}\left(a+b\sqrt{x}\right)^4}+\frac{220a^7}{b^{12}\left(a+b\sqrt{x}\right)^3}-\frac{462a^6}{b^{12}\left(a+b\sqrt{x}\right)^2}+\frac{}{b^{1}}$$

```
[Out] 36*a^2*x/b^10-16/3*a*x^(3/2)/b^9+1/2*x^2/b^8+660*a^4*ln(a+b*x^(1/2))/b^12-2
40*a^3*x^(1/2)/b^11+2/7*a^11/b^12/(a+b*x^(1/2))^7-11/3*a^10/b^12/(a+b*x^(1/
2))^6+22*a^9/b^12/(a+b*x^(1/2))^5-165/2*a^8/b^12/(a+b*x^(1/2))^4+220*a^7/b^
12/(a+b*x^(1/2))^3-462*a^6/b^12/(a+b*x^(1/2))^2+924*a^5/b^12/(a+b*x^(1/2))
```

Rubi [A] time = 0.20, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{2a^{11}}{7b^{12}\left(a+b\sqrt{x}\right)^7}-\frac{11a^{10}}{3b^{12}\left(a+b\sqrt{x}\right)^6}+\frac{22a^9}{b^{12}\left(a+b\sqrt{x}\right)^5}-\frac{165a^8}{2b^{12}\left(a+b\sqrt{x}\right)^4}+\frac{220a^7}{b^{12}\left(a+b\sqrt{x}\right)^3}-\frac{462a^6}{b^{12}\left(a+b\sqrt{x}\right)^2}+\frac{}{b^{1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*Sqrt[x])^8,x]
```

```
[Out] (2*a^11)/(7*b^12*(a + b*Sqrt[x])^7) - (11*a^10)/(3*b^12*(a + b*Sqrt[x])^6)
+ (22*a^9)/(b^12*(a + b*Sqrt[x])^5) - (165*a^8)/(2*b^12*(a + b*Sqrt[x])^4)
+ (220*a^7)/(b^12*(a + b*Sqrt[x])^3) - (462*a^6)/(b^12*(a + b*Sqrt[x])^2) +
 (924*a^5)/(b^12*(a + b*Sqrt[x])) - (240*a^3*Sqrt[x])/b^11 + (36*a^2*x)/b^1
0 - (16*a*x^(3/2))/(3*b^9) + x^2/(2*b^8) + (660*a^4*Log[a + b*Sqrt[x]])/b^1
2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{\left(a+b\sqrt{x}\right)^8}\,dx = 2\,\text{Subst}\left(\int \frac{x^{11}}{(a+bx)^8}\,dx, x, \sqrt{x}\right)$$

$$= 2\,\text{Subst}\left(\int \left(-\frac{120a^3}{b^{11}}+\frac{36a^2x}{b^{10}}-\frac{8ax^2}{b^9}+\frac{x^3}{b^8}-\frac{a^{11}}{b^{11}(a+bx)^8}+\frac{11a^{10}}{b^{11}(a+bx)^7}-\frac{55a^9}{b^{11}(a+bx)^6}\right.\right.$$

$$= \frac{2a^{11}}{7b^{12}\left(a+b\sqrt{x}\right)^7}-\frac{11a^{10}}{3b^{12}\left(a+b\sqrt{x}\right)^6}+\frac{22a^9}{b^{12}\left(a+b\sqrt{x}\right)^5}-\frac{165a^8}{2b^{12}\left(a+b\sqrt{x}\right)^4}+\frac{220a^7}{b^{12}\left(a+b\sqrt{x}\right.}$$

Mathematica [A] time = 0.27, size = 174, normalized size = 0.86

$$\frac{25961a^{11}+154007a^{10}b\sqrt{x}+365001a^9b^2x+414295a^8b^3x^{3/2}+171745a^7b^4x^2-90993a^6b^5x^{5/2}-127351a^5b^6x^3-}{42b^{12}\left(a+b\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*Sqrt[x])^8,x]
```

```
[Out] (25961*a^11 + 154007*a^10*b*Sqrt[x] + 365001*a^9*b^2*x + 414295*a^8*b^3*x^(
3/2) + 171745*a^7*b^4*x^2 - 90993*a^6*b^5*x^(5/2) - 127351*a^5*b^6*x^3 - 45
913*a^4*b^7*x^(7/2) - 3465*a^3*b^8*x^4 + 385*a^2*b^9*x^(9/2) - 77*a*b^10*x^
5 + 21*b^11*x^(11/2) + 27720*a^4*(a + b*Sqrt[x])^7*Log[a + b*Sqrt[x]])/(42*
b^12*(a + b*Sqrt[x])^7)
```

fricas [B] time = 0.82, size = 370, normalized size = 1.82

$$\frac{21\,b^{18}x^9+1365\,a^2b^{16}x^8-10143\,a^4b^{14}x^7-27195\,a^6b^{12}x^6+227094\,a^8b^{10}x^5-540190\,a^{10}b^8x^4+661465\,a^{12}b^6x^3-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*x^(1/2))^8,x, algorithm="fricas")
```

```
[Out] 1/42*(21*b^18*x^9 + 1365*a^2*b^16*x^8 - 10143*a^4*b^14*x^7 - 27195*a^6*b^12
*x^6 + 227094*a^8*b^10*x^5 - 540190*a^10*b^8*x^4 + 661465*a^12*b^6*x^3 - 45
5091*a^14*b^4*x^2 + 167867*a^16*b^2*x - 25961*a^18 + 27720*(a^4*b^14*x^7 -
7*a^6*b^12*x^6 + 21*a^8*b^10*x^5 - 35*a^10*b^8*x^4 + 35*a^12*b^6*x^3 - 21*a
^14*b^4*x^2 + 7*a^16*b^2*x - a^18)*log(b*sqrt(x) + a) - 8*(28*a*b^17*x^8 +
1064*a^3*b^15*x^7 - 13083*a^5*b^13*x^6 + 48580*a^7*b^11*x^5 - 92323*a^9*b^9
*x^4 + 101376*a^11*b^7*x^3 - 65373*a^13*b^5*x^2 + 23100*a^15*b^3*x - 3465*a
^17*b)*sqrt(x))/(b^26*x^7 - 7*a^2*b^24*x^6 + 21*a^4*b^22*x^5 - 35*a^6*b^20*
x^4 + 35*a^8*b^18*x^3 - 21*a^10*b^16*x^2 + 7*a^12*b^14*x - a^14*b^12)
```

giac [A] time = 0.18, size = 143, normalized size = 0.70

$$\frac{660\,a^4\log\left(\left|b\sqrt{x}+a\right|\right)}{b^{12}}+\frac{38808\,a^5b^6x^3+213444\,a^6b^5x^{\frac{5}{2}}+494340\,a^7b^4x^2+615615\,a^8b^3x^{\frac{3}{2}}+434049\,a^9b^2x+164}{42\left(b\sqrt{x}+a\right)^7b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*x^(1/2))^8,x, algorithm="giac")
```

```
[Out] 660*a^4*log(abs(b*sqrt(x) + a))/b^12 + 1/42*(38808*a^5*b^6*x^3 + 213444*a^6
*b^5*x^(5/2) + 494340*a^7*b^4*x^2 + 615615*a^8*b^3*x^(3/2) + 434049*a^9*b^2
*x + 164087*a^10*b*sqrt(x) + 25961*a^11)/((b*sqrt(x) + a)^7*b^12) + 1/6*(3*
b^24*x^2 - 32*a*b^23*x^(3/2) + 216*a^2*b^22*x - 1440*a^3*b^21*sqrt(x))/b^32
```

maple [A] time = 0.01, size = 174, normalized size = 0.86

$$\frac{2a^{11}}{7\left(b\sqrt{x}+a\right)^7b^{12}}-\frac{11a^{10}}{3\left(b\sqrt{x}+a\right)^6b^{12}}+\frac{22a^9}{\left(b\sqrt{x}+a\right)^5b^{12}}-\frac{165a^8}{2\left(b\sqrt{x}+a\right)^4b^{12}}+\frac{220a^7}{\left(b\sqrt{x}+a\right)^3b^{12}}-\frac{462a^6}{\left(b\sqrt{x}+a\right)^2b^{12}}+\frac{x^2}{2b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^(1/2)+a)^8,x)
```

```
[Out] 36*a^2*x/b^10-16/3*a*x^(3/2)/b^9+1/2*x^2/b^8+660*a^4*ln(b*x^(1/2)+a)/b^12-2
40*a^3*x^(1/2)/b^11+2/7*a^11/b^12/(b*x^(1/2)+a)^7-11/3*a^10/b^12/(b*x^(1/2)
```

```
+a)^6+22*a^9/b^12/(b*x^(1/2)+a)^5-165/2*a^8/b^12/(b*x^(1/2)+a)^4+220*a^7/b^
12/(b*x^(1/2)+a)^3-462*a^6/b^12/(b*x^(1/2)+a)^2+924*a^5/b^12/(b*x^(1/2)+a)
```

maxima [A] time = 0.86, size = 197, normalized size = 0.97

$$\frac{660\,a^4\log\left(b\sqrt{x}+a\right)}{b^{12}}+\frac{\left(b\sqrt{x}+a\right)^4}{2\,b^{12}}-\frac{22\left(b\sqrt{x}+a\right)^3 a}{3\,b^{12}}+\frac{55\left(b\sqrt{x}+a\right)^2 a^2}{b^{12}}-\frac{330\left(b\sqrt{x}+a\right)a^3}{b^{12}}+\frac{924\,a^5}{\left(b\sqrt{x}+a\right)b^{12}}-\frac{}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*x^(1/2))^8,x, algorithm="maxima")
```

```
[Out] 660*a^4*log(b*sqrt(x) + a)/b^12 + 1/2*(b*sqrt(x) + a)^4/b^12 - 22/3*(b*sqrt
(x) + a)^3*a/b^12 + 55*(b*sqrt(x) + a)^2*a^2/b^12 - 330*(b*sqrt(x) + a)*a^3
/b^12 + 924*a^5/((b*sqrt(x) + a)*b^12) - 462*a^6/((b*sqrt(x) + a)^2*b^12) +
 220*a^7/((b*sqrt(x) + a)^3*b^12) - 165/2*a^8/((b*sqrt(x) + a)^4*b^12) + 22
*a^9/((b*sqrt(x) + a)^5*b^12) - 11/3*a^10/((b*sqrt(x) + a)^6*b^12) + 2/7*a^
11/((b*sqrt(x) + a)^7*b^12)
```

mupad [B] time = 1.20, size = 202, normalized size = 1.00

$$\frac{\frac{25961\,a^{11}}{42\,b}+\frac{23441\,a^{10}\sqrt{x}}{6}+11770\,a^7\,b^3\,x^2+924\,a^5\,b^5\,x^3+\frac{29315\,a^8\,b^2\,x^{3/2}}{2}+5082\,a^6\,b^4\,x^{5/2}+\frac{20669\,a^9\,b\,x}{2}}{a^7\,b^{11}+b^{18}\,x^{7/2}+21\,a^5\,b^{13}\,x+7\,a\,b^{17}\,x^3+35\,a^3\,b^{15}\,x^2+7\,a^6\,b^{12}\,\sqrt{x}+35\,a^4\,b^{14}\,x^{3/2}+21\,a^2\,b^{16}\,x^{5/2}}+\frac{x^2}{2\,b^8}+\frac{36}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^(1/2))^8,x)
```

```
[Out] ((25961*a^11)/(42*b) + (23441*a^10*x^(1/2))/6 + 11770*a^7*b^3*x^2 + 924*a^5
*b^5*x^3 + (29315*a^8*b^2*x^(3/2))/2 + 5082*a^6*b^4*x^(5/2) + (20669*a^9*b*
x)/2)/(a^7*b^11 + b^18*x^(7/2) + 21*a^5*b^13*x + 7*a*b^17*x^3 + 35*a^3*b^15
*x^2 + 7*a^6*b^12*x^(1/2) + 35*a^4*b^14*x^(3/2) + 21*a^2*b^16*x^(5/2)) + x^
2/(2*b^8) + (36*a^2*x)/b^10 - (16*a*x^(3/2))/(3*b^9) + (660*a^4*log(a + b*x
^(1/2)))/b^12 - (240*a^3*x^(1/2))/b^11
```

sympy [A] time = 14.04, size = 2048, normalized size = 10.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a+b*x**(1/2))**8,x)
```

```
[Out] Piecewise((27720*a**11*log(a/b + sqrt(x))/(42*a**7*b**12 + 294*a**6*b**13*s
qrt(x) + 882*a**5*b**14*x + 1470*a**4*b**15*x**(3/2) + 1470*a**3*b**16*x**2
 + 882*a**2*b**17*x**(5/2) + 294*a*b**18*x**3 + 42*b**19*x**(7/2)) + 71874*
a**11/(42*a**7*b**12 + 294*a**6*b**13*sqrt(x) + 882*a**5*b**14*x + 1470*a**
4*b**15*x**(3/2) + 1470*a**3*b**16*x**2 + 882*a**2*b**17*x**(5/2) + 294*a*b
**18*x**3 + 42*b**19*x**(7/2)) + 194040*a**10*b*sqrt(x)*log(a/b + sqrt(x))/
(42*a**7*b**12 + 294*a**6*b**13*sqrt(x) + 882*a**5*b**14*x + 1470*a**4*b**1
5*x**(3/2) + 1470*a**3*b**16*x**2 + 882*a**2*b**17*x**(5/2) + 294*a*b**18*x
**3 + 42*b**19*x**(7/2)) + 475398*a**10*b*sqrt(x)/(42*a**7*b**12 + 294*a**6
*b**13*sqrt(x) + 882*a**5*b**14*x + 1470*a**4*b**15*x**(3/2) + 1470*a**3*b*
*16*x**2 + 882*a**2*b**17*x**(5/2) + 294*a*b**18*x**3 + 42*b**19*x**(7/2))
+ 582120*a**9*b**2*x*log(a/b + sqrt(x))/(42*a**7*b**12 + 294*a**6*b**13*sqr
t(x) + 882*a**5*b**14*x + 1470*a**4*b**15*x**(3/2) + 1470*a**3*b**16*x**2 +
 882*a**2*b**17*x**(5/2) + 294*a*b**18*x**3 + 42*b**19*x**(7/2)) + 1329174*
a**9*b**2*x/(42*a**7*b**12 + 294*a**6*b**13*sqrt(x) + 882*a**5*b**14*x + 14
70*a**4*b**15*x**(3/2) + 1470*a**3*b**16*x**2 + 882*a**2*b**17*x**(5/2) + 2
94*a*b**18*x**3 + 42*b**19*x**(7/2)) + 970200*a**8*b**3*x**(3/2)*log(a/b +
```

```
sqrt(x))/(42*a**7*b**12 + 294*a**6*b**13*sqrt(x) + 882*a**5*b**14*x + 1470*
a**4*b**15*x**(3/2) + 1470*a**3*b**16*x**2 + 882*a**2*b**17*x**(5/2) + 294*
a*b**18*x**3 + 42*b**19*x**(7/2)) + 2021250*a**8*b**3*x**(3/2)/(42*a**7*b**
12 + 294*a**6*b**13*sqrt(x) + 882*a**5*b**14*x + 1470*a**4*b**15*x**(3/2) +
 1470*a**3*b**16*x**2 + 882*a**2*b**17*x**(5/2) + 294*a*b**18*x**3 + 42*b**
19*x**(7/2)) + 970200*a**7*b**4*x**2*log(a/b + sqrt(x))/(42*a**7*b**12 + 29
4*a**6*b**13*sqrt(x) + 882*a**5*b**14*x + 1470*a**4*b**15*x**(3/2) + 1470*a
**3*b**16*x**2 + 882*a**2*b**17*x**(5/2) + 294*a*b**18*x**3 + 42*b**19*x**(
7/2)) + 1778700*a**7*b**4*x**2/(42*a**7*b**12 + 294*a**6*b**13*sqrt(x) + 88
2*a**5*b**14*x + 1470*a**4*b**15*x**(3/2) + 1470*a**3*b**16*x**2 + 882*a**2
*b**17*x**(5/2) + 294*a*b**18*x**3 + 42*b**19*x**(7/2)) + 582120*a**6*b**5*
x**(5/2)*log(a/b + sqrt(x))/(42*a**7*b**12 + 294*a**6*b**13*sqrt(x) + 882*a
**5*b**14*x + 1470*a**4*b**15*x**(3/2) + 1470*a**3*b**16*x**2 + 882*a**2*b*
*17*x**(5/2) + 294*a*b**18*x**3 + 42*b**19*x**(7/2)) + 873180*a**6*b**5*x**
(5/2)/(42*a**7*b**12 + 294*a**6*b**13*sqrt(x) + 882*a**5*b**14*x + 1470*a**
4*b**15*x**(3/2) + 1470*a**3*b**16*x**2 + 882*a**2*b**17*x**(5/2) + 294*a*b
**18*x**3 + 42*b**19*x**(7/2)) + 194040*a**5*b**6*x**3*log(a/b + sqrt(x))/(
42*a**7*b**12 + 294*a**6*b**13*sqrt(x) + 882*a**5*b**14*x + 1470*a**4*b**15
*x**(3/2) + 1470*a**3*b**16*x**2 + 882*a**2*b**17*x**(5/2) + 294*a*b**18*x*
*3 + 42*b**19*x**(7/2)) + 194040*a**5*b**6*x**3/(42*a**7*b**12 + 294*a**6*b
**13*sqrt(x) + 882*a**5*b**14*x + 1470*a**4*b**15*x**(3/2) + 1470*a**3*b**1
6*x**2 + 882*a**2*b**17*x**(5/2) + 294*a*b**18*x**3 + 42*b**19*x**(7/2)) +
27720*a**4*b**7*x**(7/2)*log(a/b + sqrt(x))/(42*a**7*b**12 + 294*a**6*b**13
*sqrt(x) + 882*a**5*b**14*x + 1470*a**4*b**15*x**(3/2) + 1470*a**3*b**16*x*
*2 + 882*a**2*b**17*x**(5/2) + 294*a*b**18*x**3 + 42*b**19*x**(7/2)) - 3465
*a**3*b**8*x**4/(42*a**7*b**12 + 294*a**6*b**13*sqrt(x) + 882*a**5*b**14*x
+ 1470*a**4*b**15*x**(3/2) + 1470*a**3*b**16*x**2 + 882*a**2*b**17*x**(5/2)
 + 294*a*b**18*x**3 + 42*b**19*x**(7/2)) + 385*a**2*b**9*x**(9/2)/(42*a**7*
b**12 + 294*a**6*b**13*sqrt(x) + 882*a**5*b**14*x + 1470*a**4*b**15*x**(3/2
) + 1470*a**3*b**16*x**2 + 882*a**2*b**17*x**(5/2) + 294*a*b**18*x**3 + 42*
b**19*x**(7/2)) - 77*a*b**10*x**5/(42*a**7*b**12 + 294*a**6*b**13*sqrt(x) +
 882*a**5*b**14*x + 1470*a**4*b**15*x**(3/2) + 1470*a**3*b**16*x**2 + 882*a
**2*b**17*x**(5/2) + 294*a*b**18*x**3 + 42*b**19*x**(7/2)) + 21*b**11*x**(1
1/2)/(42*a**7*b**12 + 294*a**6*b**13*sqrt(x) + 882*a**5*b**14*x + 1470*a**4
*b**15*x**(3/2) + 1470*a**3*b**16*x**2 + 882*a**2*b**17*x**(5/2) + 294*a*b*
*18*x**3 + 42*b**19*x**(7/2)), Ne(b, 0)), (x**6/(6*a**8), True))
```

3.2224 $\int \frac{x^4}{\left(a+b\sqrt{x}\right)^8}\,dx$

Optimal. Leaf size=172

$$\frac{2a^9}{7b^{10}\left(a+b\sqrt{x}\right)^7}-\frac{3a^8}{b^{10}\left(a+b\sqrt{x}\right)^6}+\frac{72a^7}{5b^{10}\left(a+b\sqrt{x}\right)^5}-\frac{42a^6}{b^{10}\left(a+b\sqrt{x}\right)^4}+\frac{84a^5}{b^{10}\left(a+b\sqrt{x}\right)^3}-\frac{126a^4}{b^{10}\left(a+b\sqrt{x}\right)^2}+\frac{}{b^{10}}$$

```
[Out] x/b^8+72*a^2*ln(a+b*x^(1/2))/b^10-16*a*x^(1/2)/b^9+2/7*a^9/b^10/(a+b*x^(1/2
))^7-3*a^8/b^10/(a+b*x^(1/2))^6+72/5*a^7/b^10/(a+b*x^(1/2))^5-42*a^6/b^10/(
a+b*x^(1/2))^4+84*a^5/b^10/(a+b*x^(1/2))^3-126*a^4/b^10/(a+b*x^(1/2))^2+168
*a^3/b^10/(a+b*x^(1/2))
```

Rubi [A] time = 0.16, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{2a^9}{7b^{10}\left(a+b\sqrt{x}\right)^7}-\frac{3a^8}{b^{10}\left(a+b\sqrt{x}\right)^6}+\frac{72a^7}{5b^{10}\left(a+b\sqrt{x}\right)^5}-\frac{42a^6}{b^{10}\left(a+b\sqrt{x}\right)^4}+\frac{84a^5}{b^{10}\left(a+b\sqrt{x}\right)^3}-\frac{126a^4}{b^{10}\left(a+b\sqrt{x}\right)^2}+\frac{}{b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*Sqrt[x])^8,x]
```

```
[Out] (2*a^9)/(7*b^10*(a + b*Sqrt[x])^7) - (3*a^8)/(b^10*(a + b*Sqrt[x])^6) + (72
*a^7)/(5*b^10*(a + b*Sqrt[x])^5) - (42*a^6)/(b^10*(a + b*Sqrt[x])^4) + (84*
a^5)/(b^10*(a + b*Sqrt[x])^3) - (126*a^4)/(b^10*(a + b*Sqrt[x])^2) + (168*a
^3)/(b^10*(a + b*Sqrt[x])) - (16*a*Sqrt[x])/b^9 + x/b^8 + (72*a^2*Log[a + b
*Sqrt[x]])/b^10
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\left(a+b\sqrt{x}\right)^8}\,dx &= 2\,\text{Subst}\left(\int \frac{x^9}{(a+bx)^8}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(-\frac{8a}{b^9}+\frac{x}{b^8}-\frac{a^9}{b^9(a+bx)^8}+\frac{9a^8}{b^9(a+bx)^7}-\frac{36a^7}{b^9(a+bx)^6}+\frac{84a^6}{b^9(a+bx)^5}-\frac{12}{b^9(a}\right.\right.\\ &= \frac{2a^9}{7b^{10}\left(a+b\sqrt{x}\right)^7}-\frac{3a^8}{b^{10}\left(a+b\sqrt{x}\right)^6}+\frac{72a^7}{5b^{10}\left(a+b\sqrt{x}\right)^5}-\frac{42a^6}{b^{10}\left(a+b\sqrt{x}\right)^4}+\frac{84a^5}{b^{10}\left(a+b\sqrt{x}\right.}\end{aligned}$$

Mathematica [A] time = 0.18, size = 150, normalized size = 0.87

$$\frac{3349a^9 + 20923a^8b\sqrt{x} + 53949a^7b^2x + 72275a^6b^3x^{3/2} + 50225a^5b^4x^2 + 12495a^4b^5x^{5/2} - 4655a^3b^6x^3 - 3185a^2b^}{35b^{10}\left(a + b\sqrt{x}\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*Sqrt[x])^8,x]
```

```
[Out] (3349*a^9 + 20923*a^8*b*Sqrt[x] + 53949*a^7*b^2*x + 72275*a^6*b^3*x^(3/2) +
 50225*a^5*b^4*x^2 + 12495*a^4*b^5*x^(5/2) - 4655*a^3*b^6*x^3 - 3185*a^2*b^
7*x^(7/2) - 315*a*b^8*x^4 + 35*b^9*x^(9/2) + 2520*a^2*(a + b*Sqrt[x])^7*Log
[a + b*Sqrt[x]])/(35*b^10*(a + b*Sqrt[x])^7)
```

fricas [B] time = 1.09, size = 348, normalized size = 2.02

$$\frac{35\,b^{16}x^8 - 245\,a^2b^{14}x^7 - 9555\,a^4b^{12}x^6 + 41405\,a^6b^{10}x^5 - 83720\,a^8b^8x^4 + 94745\,a^{10}b^6x^3 - 62139\,a^{12}b^4x^2 + 2218}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b*x^(1/2))^8,x, algorithm="fricas")
```

```
[Out] 1/35*(35*b^16*x^8 - 245*a^2*b^14*x^7 - 9555*a^4*b^12*x^6 + 41405*a^6*b^10*x
^5 - 83720*a^8*b^8*x^4 + 94745*a^10*b^6*x^3 - 62139*a^12*b^4*x^2 + 22183*a^
14*b^2*x - 3349*a^16 + 2520*(a^2*b^14*x^7 - 7*a^4*b^12*x^6 + 21*a^6*b^10*x^
5 - 35*a^8*b^8*x^4 + 35*a^10*b^6*x^3 - 21*a^12*b^4*x^2 + 7*a^14*b^2*x - a^1
6)*log(b*sqrt(x) + a) - 8*(70*a*b^15*x^7 - 1225*a^3*b^13*x^6 + 4410*a^5*b^1
1*x^5 - 8393*a^7*b^9*x^4 + 9216*a^9*b^7*x^3 - 5943*a^11*b^5*x^2 + 2100*a^13
*b^3*x - 315*a^15*b)*sqrt(x))/(b^24*x^7 - 7*a^2*b^22*x^6 + 21*a^4*b^20*x^5
- 35*a^6*b^18*x^4 + 35*a^8*b^16*x^3 - 21*a^10*b^14*x^2 + 7*a^12*b^12*x - a^
14*b^10)
```

giac [A] time = 0.17, size = 119, normalized size = 0.69

$$\frac{72\,a^2\log\left(\left|b\sqrt{x} + a\right|\right)}{b^{10}} + \frac{b^8x - 16\,ab^7\sqrt{x}}{b^{16}} + \frac{5880\,a^3b^6x^3 + 30870\,a^4b^5x^{\frac{5}{2}} + 69090\,a^5b^4x^2 + 83790\,a^6b^3x^{\frac{3}{2}} + 57834\,a}{35\left(b\sqrt{x} + a\right)^7b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b*x^(1/2))^8,x, algorithm="giac")
```

```
[Out] 72*a^2*log(abs(b*sqrt(x) + a))/b^10 + (b^8*x - 16*a*b^7*sqrt(x))/b^16 + 1/3
5*(5880*a^3*b^6*x^3 + 30870*a^4*b^5*x^(5/2) + 69090*a^5*b^4*x^2 + 83790*a^6
*b^3*x^(3/2) + 57834*a^7*b^2*x + 21483*a^8*b*sqrt(x) + 3349*a^9)/((b*sqrt(x
) + a)^7*b^10)
```

maple [A] time = 0.01, size = 151, normalized size = 0.88

$$\frac{2a^9}{7\left(b\sqrt{x} + a\right)^7b^{10}} - \frac{3a^8}{\left(b\sqrt{x} + a\right)^6b^{10}} + \frac{72a^7}{5\left(b\sqrt{x} + a\right)^5b^{10}} - \frac{42a^6}{\left(b\sqrt{x} + a\right)^4b^{10}} + \frac{84a^5}{\left(b\sqrt{x} + a\right)^3b^{10}} - \frac{126a^4}{\left(b\sqrt{x} + a\right)^2b^{10}} + \frac{1}{(b\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^(1/2)+a)^8,x)
```

```
[Out] x/b^8+72*a^2*ln(b*x^(1/2)+a)/b^10-16*a*x^(1/2)/b^9+2/7*a^9/b^10/(b*x^(1/2)+
a)^7-3*a^8/b^10/(b*x^(1/2)+a)^6+72/5*a^7/b^10/(b*x^(1/2)+a)^5-42*a^6/b^10/(
```

```
b*x^(1/2)+a)^4+84*a^5/b^10/(b*x^(1/2)+a)^3-126*a^4/b^10/(b*x^(1/2)+a)^2+168
*a^3/b^10/(b*x^(1/2)+a)
```

maxima [A] time = 0.88, size = 162, normalized size = 0.94

$$\frac{72\,a^2\log\left(b\sqrt{x}+a\right)}{b^{10}}+\frac{\left(b\sqrt{x}+a\right)^2}{b^{10}}-\frac{18\left(b\sqrt{x}+a\right)a}{b^{10}}+\frac{168\,a^3}{\left(b\sqrt{x}+a\right)b^{10}}-\frac{126\,a^4}{\left(b\sqrt{x}+a\right)^2b^{10}}+\frac{84\,a^5}{\left(b\sqrt{x}+a\right)^3b^{10}}-\frac{42}{\left(b\sqrt{x}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b*x^(1/2))^8,x, algorithm="maxima")
```

```
[Out] 72*a^2*log(b*sqrt(x) + a)/b^10 + (b*sqrt(x) + a)^2/b^10 - 18*(b*sqrt(x) + a
)*a/b^10 + 168*a^3/((b*sqrt(x) + a)*b^10) - 126*a^4/((b*sqrt(x) + a)^2*b^10
) + 84*a^5/((b*sqrt(x) + a)^3*b^10) - 42*a^6/((b*sqrt(x) + a)^4*b^10) + 72/
5*a^7/((b*sqrt(x) + a)^5*b^10) - 3*a^8/((b*sqrt(x) + a)^6*b^10) + 2/7*a^9/(
(b*sqrt(x) + a)^7*b^10)
```

mupad [B] time = 0.09, size = 179, normalized size = 1.04

$$\frac{x}{b^8}+\frac{\frac{3349\,a^9}{35\,b}+\frac{3069\,a^8\,\sqrt{x}}{5}+1974\,a^5\,b^3\,x^2+168\,a^3\,b^5\,x^3+2394\,a^6\,b^2\,x^{3/2}+882\,a^4\,b^4\,x^{5/2}+\frac{8262\,a^7\,b\,x}{5}}{a^7\,b^9+b^{16}\,x^{7/2}+21\,a^5\,b^{11}\,x+7\,a\,b^{15}\,x^3+35\,a^3\,b^{13}\,x^2+7\,a^6\,b^{10}\,\sqrt{x}+35\,a^4\,b^{12}\,x^{3/2}+21\,a^2\,b^{14}\,x^{5/2}}-\frac{16\,a}{b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^(1/2))^8,x)
```

```
[Out] x/b^8 + ((3349*a^9)/(35*b) + (3069*a^8*x^(1/2))/5 + 1974*a^5*b^3*x^2 + 168*
a^3*b^5*x^3 + 2394*a^6*b^2*x^(3/2) + 882*a^4*b^4*x^(5/2) + (8262*a^7*b*x)/5
)/(a^7*b^9 + b^16*x^(7/2) + 21*a^5*b^11*x + 7*a*b^15*x^3 + 35*a^3*b^13*x^2
+ 7*a^6*b^10*x^(1/2) + 35*a^4*b^12*x^(3/2) + 21*a^2*b^14*x^(5/2)) - (16*a*x
^(1/2))/b^9 + (72*a^2*log(a + b*x^(1/2)))/b^10
```

sympy [A] time = 6.98, size = 1839, normalized size = 10.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a+b*x**(1/2))**8,x)
```

```
[Out] Piecewise((2520*a**9*log(a/b + sqrt(x))/(35*a**7*b**10 + 245*a**6*b**11*sqr
t(x) + 735*a**5*b**12*x + 1225*a**4*b**13*x**(3/2) + 1225*a**3*b**14*x**2 +
 735*a**2*b**15*x**(5/2) + 245*a*b**16*x**3 + 35*b**17*x**(7/2)) + 6534*a**
9/(35*a**7*b**10 + 245*a**6*b**11*sqrt(x) + 735*a**5*b**12*x + 1225*a**4*b*
*13*x**(3/2) + 1225*a**3*b**14*x**2 + 735*a**2*b**15*x**(5/2) + 245*a*b**16
*x**3 + 35*b**17*x**(7/2)) + 17640*a**8*b*sqrt(x)*log(a/b + sqrt(x))/(35*a*
*7*b**10 + 245*a**6*b**11*sqrt(x) + 735*a**5*b**12*x + 1225*a**4*b**13*x**(
3/2) + 1225*a**3*b**14*x**2 + 735*a**2*b**15*x**(5/2) + 245*a*b**16*x**3 +
35*b**17*x**(7/2)) + 43218*a**8*b*sqrt(x)/(35*a**7*b**10 + 245*a**6*b**11*s
qrt(x) + 735*a**5*b**12*x + 1225*a**4*b**13*x**(3/2) + 1225*a**3*b**14*x**2
 + 735*a**2*b**15*x**(5/2) + 245*a*b**16*x**3 + 35*b**17*x**(7/2)) + 52920*
a**7*b**2*x*log(a/b + sqrt(x))/(35*a**7*b**10 + 245*a**6*b**11*sqrt(x) + 73
5*a**5*b**12*x + 1225*a**4*b**13*x**(3/2) + 1225*a**3*b**14*x**2 + 735*a**2
*b**15*x**(5/2) + 245*a*b**16*x**3 + 35*b**17*x**(7/2)) + 120834*a**7*b**2*
x/(35*a**7*b**10 + 245*a**6*b**11*sqrt(x) + 735*a**5*b**12*x + 1225*a**4*b*
*13*x**(3/2) + 1225*a**3*b**14*x**2 + 735*a**2*b**15*x**(5/2) + 245*a*b**16
*x**3 + 35*b**17*x**(7/2)) + 88200*a**6*b**3*x**(3/2)*log(a/b + sqrt(x))/(3
5*a**7*b**10 + 245*a**6*b**11*sqrt(x) + 735*a**5*b**12*x + 1225*a**4*b**13*
x**(3/2) + 1225*a**3*b**14*x**2 + 735*a**2*b**15*x**(5/2) + 245*a*b**16*x**
```

```
3 + 35*b**17*x**(7/2)) + 183750*a**6*b**3*x**(3/2)/(35*a**7*b**10 + 245*a**
6*b**11*sqrt(x) + 735*a**5*b**12*x + 1225*a**4*b**13*x**(3/2) + 1225*a**3*b
**14*x**2 + 735*a**2*b**15*x**(5/2) + 245*a*b**16*x**3 + 35*b**17*x**(7/2))
 + 88200*a**5*b**4*x**2*log(a/b + sqrt(x))/(35*a**7*b**10 + 245*a**6*b**11*
sqrt(x) + 735*a**5*b**12*x + 1225*a**4*b**13*x**(3/2) + 1225*a**3*b**14*x**
2 + 735*a**2*b**15*x**(5/2) + 245*a*b**16*x**3 + 35*b**17*x**(7/2)) + 16170
0*a**5*b**4*x**2/(35*a**7*b**10 + 245*a**6*b**11*sqrt(x) + 735*a**5*b**12*x
 + 1225*a**4*b**13*x**(3/2) + 1225*a**3*b**14*x**2 + 735*a**2*b**15*x**(5/2
) + 245*a*b**16*x**3 + 35*b**17*x**(7/2)) + 52920*a**4*b**5*x**(5/2)*log(a/
b + sqrt(x))/(35*a**7*b**10 + 245*a**6*b**11*sqrt(x) + 735*a**5*b**12*x + 1
225*a**4*b**13*x**(3/2) + 1225*a**3*b**14*x**2 + 735*a**2*b**15*x**(5/2) +
245*a*b**16*x**3 + 35*b**17*x**(7/2)) + 79380*a**4*b**5*x**(5/2)/(35*a**7*b
**10 + 245*a**6*b**11*sqrt(x) + 735*a**5*b**12*x + 1225*a**4*b**13*x**(3/2)
 + 1225*a**3*b**14*x**2 + 735*a**2*b**15*x**(5/2) + 245*a*b**16*x**3 + 35*b
**17*x**(7/2)) + 17640*a**3*b**6*x**3*log(a/b + sqrt(x))/(35*a**7*b**10 + 2
45*a**6*b**11*sqrt(x) + 735*a**5*b**12*x + 1225*a**4*b**13*x**(3/2) + 1225*
a**3*b**14*x**2 + 735*a**2*b**15*x**(5/2) + 245*a*b**16*x**3 + 35*b**17*x**
(7/2)) + 17640*a**3*b**6*x**3/(35*a**7*b**10 + 245*a**6*b**11*sqrt(x) + 735
*a**5*b**12*x + 1225*a**4*b**13*x**(3/2) + 1225*a**3*b**14*x**2 + 735*a**2*
b**15*x**(5/2) + 245*a*b**16*x**3 + 35*b**17*x**(7/2)) + 2520*a**2*b**7*x**
(7/2)*log(a/b + sqrt(x))/(35*a**7*b**10 + 245*a**6*b**11*sqrt(x) + 735*a**5
*b**12*x + 1225*a**4*b**13*x**(3/2) + 1225*a**3*b**14*x**2 + 735*a**2*b**15
*x**(5/2) + 245*a*b**16*x**3 + 35*b**17*x**(7/2)) - 315*a*b**8*x**4/(35*a**
7*b**10 + 245*a**6*b**11*sqrt(x) + 735*a**5*b**12*x + 1225*a**4*b**13*x**(3
/2) + 1225*a**3*b**14*x**2 + 735*a**2*b**15*x**(5/2) + 245*a*b**16*x**3 + 3
5*b**17*x**(7/2)) + 35*b**9*x**(9/2)/(35*a**7*b**10 + 245*a**6*b**11*sqrt(x
) + 735*a**5*b**12*x + 1225*a**4*b**13*x**(3/2) + 1225*a**3*b**14*x**2 + 73
5*a**2*b**15*x**(5/2) + 245*a*b**16*x**3 + 35*b**17*x**(7/2)), Ne(b, 0)), (
x**5/(5*a**8), True))
```

3.2225 $\int \frac{x^3}{\left(a+b\sqrt{x}\right)^8}\,dx$

Optimal. Leaf size=157

$$\frac{2a^7}{7b^8\left(a+b\sqrt{x}\right)^7}-\frac{7a^6}{3b^8\left(a+b\sqrt{x}\right)^6}+\frac{42a^5}{5b^8\left(a+b\sqrt{x}\right)^5}-\frac{35a^4}{2b^8\left(a+b\sqrt{x}\right)^4}+\frac{70a^3}{3b^8\left(a+b\sqrt{x}\right)^3}-\frac{21a^2}{b^8\left(a+b\sqrt{x}\right)^2}+\frac{1}{b^8\left(a\right.}$$

```
[Out] 2*ln(a+b*x^(1/2))/b^8+2/7*a^7/b^8/(a+b*x^(1/2))^7-7/3*a^6/b^8/(a+b*x^(1/2))
^6+42/5*a^5/b^8/(a+b*x^(1/2))^5-35/2*a^4/b^8/(a+b*x^(1/2))^4+70/3*a^3/b^8/(
a+b*x^(1/2))^3-21*a^2/b^8/(a+b*x^(1/2))^2+14*a/b^8/(a+b*x^(1/2))
```

Rubi [A] time = 0.11, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{2a^7}{7b^8\left(a+b\sqrt{x}\right)^7}-\frac{7a^6}{3b^8\left(a+b\sqrt{x}\right)^6}+\frac{42a^5}{5b^8\left(a+b\sqrt{x}\right)^5}-\frac{35a^4}{2b^8\left(a+b\sqrt{x}\right)^4}+\frac{70a^3}{3b^8\left(a+b\sqrt{x}\right)^3}-\frac{21a^2}{b^8\left(a+b\sqrt{x}\right)^2}+\frac{1}{b^8\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*Sqrt[x])^8,x]
```

```
[Out] (2*a^7)/(7*b^8*(a + b*Sqrt[x])^7) - (7*a^6)/(3*b^8*(a + b*Sqrt[x])^6) + (42
*a^5)/(5*b^8*(a + b*Sqrt[x])^5) - (35*a^4)/(2*b^8*(a + b*Sqrt[x])^4) + (70*
a^3)/(3*b^8*(a + b*Sqrt[x])^3) - (21*a^2)/(b^8*(a + b*Sqrt[x])^2) + (14*a)/
(b^8*(a + b*Sqrt[x])) + (2*Log[a + b*Sqrt[x]])/b^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(a+b\sqrt{x}\right)^8}\,dx &= 2\,\text{Subst}\left(\int \frac{x^7}{(a+bx)^8}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(-\frac{a^7}{b^7(a+bx)^8}+\frac{7a^6}{b^7(a+bx)^7}-\frac{21a^5}{b^7(a+bx)^6}+\frac{35a^4}{b^7(a+bx)^5}-\frac{35a^3}{b^7(a+bx)^4}+\frac{b^7}{}\right.\right.\\ &= \frac{2a^7}{7b^8\left(a+b\sqrt{x}\right)^7}-\frac{7a^6}{3b^8\left(a+b\sqrt{x}\right)^6}+\frac{42a^5}{5b^8\left(a+b\sqrt{x}\right)^5}-\frac{35a^4}{2b^8\left(a+b\sqrt{x}\right)^4}+\frac{70a^3}{3b^8\left(a+b\sqrt{x}\right.}\end{aligned}$$

Mathematica [A] time = 0.14, size = 102, normalized size = 0.65

$$\frac{\frac{a\left(1089a^6+7203a^5b\sqrt{x}+20139a^4b^2x+30625a^3b^3x^{3/2}+26950a^2b^4x^2+13230ab^5x^{5/2}+2940b^6x^3\right)}{\left(a+b\sqrt{x}\right)^7}+420\log\left(a+b\sqrt{x}\right)}{210b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*Sqrt[x])^8,x]
```

```
[Out] ((a*(1089*a^6 + 7203*a^5*b*Sqrt[x] + 20139*a^4*b^2*x + 30625*a^3*b^3*x^(3/2
) + 26950*a^2*b^4*x^2 + 13230*a*b^5*x^(5/2) + 2940*b^6*x^3))/(a + b*Sqrt[x]
)^7 + 420*Log[a + b*Sqrt[x]])/(210*b^8)
```

fricas [B] time = 1.15, size = 315, normalized size = 2.01

$$-\frac{7350\,a^2b^{12}x^6 - 16905\,a^4b^{10}x^5 + 32585\,a^6b^8x^4 - 34370\,a^8b^6x^3 + 21504\,a^{10}b^4x^2 - 7413\,a^{12}b^2x + 1089\,a^{14} - 420\,(}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^(1/2))^8,x, algorithm="fricas")
```

```
[Out] -1/210*(7350*a^2*b^12*x^6 - 16905*a^4*b^10*x^5 + 32585*a^6*b^8*x^4 - 34370*
a^8*b^6*x^3 + 21504*a^10*b^4*x^2 - 7413*a^12*b^2*x + 1089*a^14 - 420*(b^14*
x^7 - 7*a^2*b^12*x^6 + 21*a^4*b^10*x^5 - 35*a^6*b^8*x^4 + 35*a^8*b^6*x^3 -
21*a^10*b^4*x^2 + 7*a^12*b^2*x - a^14)*log(b*sqrt(x) + a) - 4*(735*a*b^13*x
^6 - 980*a^3*b^11*x^5 + 2891*a^5*b^9*x^4 - 3072*a^7*b^7*x^3 + 1981*a^9*b^5*
x^2 - 700*a^11*b^3*x + 105*a^13*b)*sqrt(x))/(b^22*x^7 - 7*a^2*b^20*x^6 + 21
*a^4*b^18*x^5 - 35*a^6*b^16*x^4 + 35*a^8*b^14*x^3 - 21*a^10*b^12*x^2 + 7*a^
12*b^10*x - a^14*b^8)
```

giac [A] time = 0.16, size = 95, normalized size = 0.61

$$\frac{2\,\log\left(\left|b\sqrt{x}+a\right|\right)}{b^8}+\frac{2940\,ab^5x^3+13230\,a^2b^4x^{\frac{5}{2}}+26950\,a^3b^3x^2+30625\,a^4b^2x^{\frac{3}{2}}+20139\,a^5bx+7203\,a^6\sqrt{x}+\frac{10}{}}{210\left(b\sqrt{x}+a\right)^7b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^(1/2))^8,x, algorithm="giac")
```

```
[Out] 2*log(abs(b*sqrt(x) + a))/b^8 + 1/210*(2940*a*b^5*x^3 + 13230*a^2*b^4*x^(5/
2) + 26950*a^3*b^3*x^2 + 30625*a^4*b^2*x^(3/2) + 20139*a^5*b*x + 7203*a^6*s
qrt(x) + 1089*a^7/b)/((b*sqrt(x) + a)^7*b^7)
```

maple [A] time = 0.01, size = 132, normalized size = 0.84

$$\frac{2a^7}{7\left(b\sqrt{x}+a\right)^7b^8}-\frac{7a^6}{3\left(b\sqrt{x}+a\right)^6b^8}+\frac{42a^5}{5\left(b\sqrt{x}+a\right)^5b^8}-\frac{35a^4}{2\left(b\sqrt{x}+a\right)^4b^8}+\frac{70a^3}{3\left(b\sqrt{x}+a\right)^3b^8}-\frac{21a^2}{\left(b\sqrt{x}+a\right)^2b^8}+\frac{1}{\left(b\sqrt{x}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^(1/2)+a)^8,x)
```

```
[Out] 2*ln(b*x^(1/2)+a)/b^8+2/7*a^7/b^8/(b*x^(1/2)+a)^7-7/3*a^6/b^8/(b*x^(1/2)+a)
^6+42/5*a^5/b^8/(b*x^(1/2)+a)^5-35/2*a^4/b^8/(b*x^(1/2)+a)^4+70/3*a^3/b^8/(
b*x^(1/2)+a)^3-21*a^2/b^8/(b*x^(1/2)+a)^2+14*a/b^8/(b*x^(1/2)+a)
```

maxima [A] time = 0.86, size = 131, normalized size = 0.83

$$\frac{2\,\log\left(b\sqrt{x}+a\right)}{b^8}+\frac{14\,a}{\left(b\sqrt{x}+a\right)b^8}-\frac{21\,a^2}{\left(b\sqrt{x}+a\right)^2b^8}+\frac{70\,a^3}{3\left(b\sqrt{x}+a\right)^3b^8}-\frac{35\,a^4}{2\left(b\sqrt{x}+a\right)^4b^8}+\frac{42\,a^5}{5\left(b\sqrt{x}+a\right)^5b^8}-\frac{7\,a^6}{3\left(b\sqrt{x}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^(1/2))^8,x, algorithm="maxima")
```

```
[Out] 2*log(b*sqrt(x) + a)/b^8 + 14*a/((b*sqrt(x) + a)*b^8) - 21*a^2/((b*sqrt(x)
+ a)^2*b^8) + 70/3*a^3/((b*sqrt(x) + a)^3*b^8) - 35/2*a^4/((b*sqrt(x) + a)^
4*b^8) + 42/5*a^5/((b*sqrt(x) + a)^5*b^8) - 7/3*a^6/((b*sqrt(x) + a)^6*b^8)
 + 2/7*a^7/((b*sqrt(x) + a)^7*b^8)
```

mupad [B] time = 1.23, size = 159, normalized size = 1.01

$$\frac{\frac{363\,a^7}{70\,b^8}+\frac{14\,a\,x^3}{b^2}+\frac{959\,a^5\,x}{10\,b^6}+\frac{385\,a^3\,x^2}{3\,b^4}+\frac{63\,a^2\,x^{5/2}}{b^3}+\frac{875\,a^4\,x^{3/2}}{6\,b^5}+\frac{343\,a^6\,\sqrt{x}}{10\,b^7}}{a^7+b^7\,x^{7/2}+21\,a^5\,b^2\,x+7\,a\,b^6\,x^3+7\,a^6\,b\,\sqrt{x}\,+35\,a^3\,b^4\,x^2+35\,a^4\,b^3\,x^{3/2}+21\,a^2\,b^5\,x^{5/2}}+\frac{2\,\ln\left(a+b\,\sqrt{x}\right)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^(1/2))^8,x)
```

```
[Out] ((363*a^7)/(70*b^8) + (14*a*x^3)/b^2 + (959*a^5*x)/(10*b^6) + (385*a^3*x^2)
/(3*b^4) + (63*a^2*x^(5/2))/b^3 + (875*a^4*x^(3/2))/(6*b^5) + (343*a^6*x^(1
/2))/(10*b^7))/(a^7 + b^7*x^(7/2) + 21*a^5*b^2*x + 7*a*b^6*x^3 + 7*a^6*b*x^
(1/2) + 35*a^3*b^4*x^2 + 35*a^4*b^3*x^(3/2) + 21*a^2*b^5*x^(5/2)) + (2*log(
a + b*x^(1/2)))/b^8
```

sympy [A] time = 6.91, size = 1629, normalized size = 10.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*x**(1/2))**8,x)
```

```
[Out] Piecewise((420*a**7*log(a/b + sqrt(x))/(210*a**7*b**8 + 1470*a**6*b**9*sqrt
(x) + 4410*a**5*b**10*x + 7350*a**4*b**11*x**(3/2) + 7350*a**3*b**12*x**2 +
 4410*a**2*b**13*x**(5/2) + 1470*a*b**14*x**3 + 210*b**15*x**(7/2)) + 1089*
a**7/(210*a**7*b**8 + 1470*a**6*b**9*sqrt(x) + 4410*a**5*b**10*x + 7350*a**
4*b**11*x**(3/2) + 7350*a**3*b**12*x**2 + 4410*a**2*b**13*x**(5/2) + 1470*a
*b**14*x**3 + 210*b**15*x**(7/2)) + 2940*a**6*b*sqrt(x)*log(a/b + sqrt(x))/
(210*a**7*b**8 + 1470*a**6*b**9*sqrt(x) + 4410*a**5*b**10*x + 7350*a**4*b**
11*x**(3/2) + 7350*a**3*b**12*x**2 + 4410*a**2*b**13*x**(5/2) + 1470*a*b**1
4*x**3 + 210*b**15*x**(7/2)) + 7203*a**6*b*sqrt(x)/(210*a**7*b**8 + 1470*a*
*6*b**9*sqrt(x) + 4410*a**5*b**10*x + 7350*a**4*b**11*x**(3/2) + 7350*a**3*
b**12*x**2 + 4410*a**2*b**13*x**(5/2) + 1470*a*b**14*x**3 + 210*b**15*x**(7
/2)) + 8820*a**5*b**2*x*log(a/b + sqrt(x))/(210*a**7*b**8 + 1470*a**6*b**9*
sqrt(x) + 4410*a**5*b**10*x + 7350*a**4*b**11*x**(3/2) + 7350*a**3*b**12*x*
*2 + 4410*a**2*b**13*x**(5/2) + 1470*a*b**14*x**3 + 210*b**15*x**(7/2)) + 2
0139*a**5*b**2*x/(210*a**7*b**8 + 1470*a**6*b**9*sqrt(x) + 4410*a**5*b**10*
x + 7350*a**4*b**11*x**(3/2) + 7350*a**3*b**12*x**2 + 4410*a**2*b**13*x**(5
/2) + 1470*a*b**14*x**3 + 210*b**15*x**(7/2)) + 14700*a**4*b**3*x**(3/2)*lo
g(a/b + sqrt(x))/(210*a**7*b**8 + 1470*a**6*b**9*sqrt(x) + 4410*a**5*b**10*
x + 7350*a**4*b**11*x**(3/2) + 7350*a**3*b**12*x**2 + 4410*a**2*b**13*x**(5
/2) + 1470*a*b**14*x**3 + 210*b**15*x**(7/2)) + 30625*a**4*b**3*x**(3/2)/(2
10*a**7*b**8 + 1470*a**6*b**9*sqrt(x) + 4410*a**5*b**10*x + 7350*a**4*b**11
*x**(3/2) + 7350*a**3*b**12*x**2 + 4410*a**2*b**13*x**(5/2) + 1470*a*b**14*
x**3 + 210*b**15*x**(7/2)) + 14700*a**3*b**4*x**2*log(a/b + sqrt(x))/(210*a
**7*b**8 + 1470*a**6*b**9*sqrt(x) + 4410*a**5*b**10*x + 7350*a**4*b**11*x**
(3/2) + 7350*a**3*b**12*x**2 + 4410*a**2*b**13*x**(5/2) + 1470*a*b**14*x**3
 + 210*b**15*x**(7/2)) + 26950*a**3*b**4*x**2/(210*a**7*b**8 + 1470*a**6*b*
*9*sqrt(x) + 4410*a**5*b**10*x + 7350*a**4*b**11*x**(3/2) + 7350*a**3*b**12
*x**2 + 4410*a**2*b**13*x**(5/2) + 1470*a*b**14*x**3 + 210*b**15*x**(7/2))
+ 8820*a**2*b**5*x**(5/2)*log(a/b + sqrt(x))/(210*a**7*b**8 + 1470*a**6*b**
9*sqrt(x) + 4410*a**5*b**10*x + 7350*a**4*b**11*x**(3/2) + 7350*a**3*b**12*
x**2 + 4410*a**2*b**13*x**(5/2) + 1470*a*b**14*x**3 + 210*b**15*x**(7/2)) +
 13230*a**2*b**5*x**(5/2)/(210*a**7*b**8 + 1470*a**6*b**9*sqrt(x) + 4410*a*
*5*b**10*x + 7350*a**4*b**11*x**(3/2) + 7350*a**3*b**12*x**2 + 4410*a**2*b*
```

```
*13*x**(5/2) + 1470*a*b**14*x**3 + 210*b**15*x**(7/2)) + 2940*a*b**6*x**3*l
og(a/b + sqrt(x))/(210*a**7*b**8 + 1470*a**6*b**9*sqrt(x) + 4410*a**5*b**10
*x + 7350*a**4*b**11*x**(3/2) + 7350*a**3*b**12*x**2 + 4410*a**2*b**13*x**(
5/2) + 1470*a*b**14*x**3 + 210*b**15*x**(7/2)) + 2940*a*b**6*x**3/(210*a**7
*b**8 + 1470*a**6*b**9*sqrt(x) + 4410*a**5*b**10*x + 7350*a**4*b**11*x**(3/
2) + 7350*a**3*b**12*x**2 + 4410*a**2*b**13*x**(5/2) + 1470*a*b**14*x**3 +
210*b**15*x**(7/2)) + 420*b**7*x**(7/2)*log(a/b + sqrt(x))/(210*a**7*b**8 +
 1470*a**6*b**9*sqrt(x) + 4410*a**5*b**10*x + 7350*a**4*b**11*x**(3/2) + 73
50*a**3*b**12*x**2 + 4410*a**2*b**13*x**(5/2) + 1470*a*b**14*x**3 + 210*b**
15*x**(7/2)), Ne(b, 0)), (x**4/(4*a**8), True))
```

3.2226 $\int \frac{x^2}{\left(a+b\sqrt{x}\right)^8}\,dx$

Optimal. Leaf size=43

$$\frac{x^3}{21a^2\left(a+b\sqrt{x}\right)^6}+\frac{2x^3}{7a\left(a+b\sqrt{x}\right)^7}$$

```
[Out] 2/7*x^3/a/(a+b*x^(1/2))^7+1/21*x^3/a^2/(a+b*x^(1/2))^6
```

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {266, 45, 37}

$$\frac{x^3}{21a^2\left(a+b\sqrt{x}\right)^6}+\frac{2x^3}{7a\left(a+b\sqrt{x}\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*Sqrt[x])^8,x]
```

```
[Out] (2*x^3)/(7*a*(a + b*Sqrt[x])^7) + x^3/(21*a^2*(a + b*Sqrt[x])^6)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a+b\sqrt{x}\right)^8}\,dx &= 2\,\text{Subst}\left(\int \frac{x^5}{(a+bx)^8}\,dx, x, \sqrt{x}\right)\\ &= \frac{2x^3}{7a\left(a+b\sqrt{x}\right)^7}+\frac{2\,\text{Subst}\left(\int \frac{x^5}{(a+bx)^7}\,dx, x, \sqrt{x}\right)}{7a}\\ &= \frac{2x^3}{7a\left(a+b\sqrt{x}\right)^7}+\frac{x^3}{21a^2\left(a+b\sqrt{x}\right)^6}\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 0.74

$$\frac{x^3\left(7a+b\sqrt{x}\right)}{21a^2\left(a+b\sqrt{x}\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*Sqrt[x])^8,x]
```

```
[Out] ((7*a + b*Sqrt[x])*x^3)/(21*a^2*(a + b*Sqrt[x])^7)
```

fricas [B] time = 1.08, size = 188, normalized size = 4.37

$$-\frac{21\,b^{12}x^6+231\,a^2b^{10}x^5+105\,a^4b^8x^4+42\,a^6b^6x^3-21\,a^8b^4x^2+7\,a^{10}b^2x-a^{12}-16\left(7\,ab^{11}x^5+14\,a^3b^9x^4+3\,a^5b\right.}{21\left(b^{20}x^7-7\,a^2b^{18}x^6+21\,a^4b^{16}x^5-35\,a^6b^{14}x^4+35\,a^8b^{12}x^3-21\,a^{10}b^{10}x^2+7\,a^{12}b^8x-a^{14}b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^(1/2))^8,x, algorithm="fricas")
```

```
[Out] -1/21*(21*b^12*x^6 + 231*a^2*b^10*x^5 + 105*a^4*b^8*x^4 + 42*a^6*b^6*x^3 -
21*a^8*b^4*x^2 + 7*a^10*b^2*x - a^12 - 16*(7*a*b^11*x^5 + 14*a^3*b^9*x^4 +
3*a^5*b^7*x^3)*sqrt(x))/(b^20*x^7 - 7*a^2*b^18*x^6 + 21*a^4*b^16*x^5 - 35*a
^6*b^14*x^4 + 35*a^8*b^12*x^3 - 21*a^10*b^10*x^2 + 7*a^12*b^8*x - a^14*b^6)
```

giac [A] time = 0.19, size = 64, normalized size = 1.49

$$-\frac{21\,b^5x^{\frac{5}{2}}+35\,ab^4x^2+35\,a^2b^3x^{\frac{3}{2}}+21\,a^3b^2x+7\,a^4b\sqrt{x}+a^5}{21\left(b\sqrt{x}+a\right)^7b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^(1/2))^8,x, algorithm="giac")
```

```
[Out] -1/21*(21*b^5*x^(5/2) + 35*a*b^4*x^2 + 35*a^2*b^3*x^(3/2) + 21*a^3*b^2*x +
7*a^4*b*sqrt(x) + a^5)/((b*sqrt(x) + a)^7*b^6)
```

maple [B] time = 0.01, size = 99, normalized size = 2.30

$$\frac{2a^5}{7\left(b\sqrt{x}+a\right)^7b^6}-\frac{5a^4}{3\left(b\sqrt{x}+a\right)^6b^6}+\frac{4a^3}{\left(b\sqrt{x}+a\right)^5b^6}-\frac{5a^2}{\left(b\sqrt{x}+a\right)^4b^6}+\frac{10a}{3\left(b\sqrt{x}+a\right)^3b^6}-\frac{1}{\left(b\sqrt{x}+a\right)^2b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^(1/2)+a)^8,x)
```

```
[Out] 10/3*a/b^6/(b*x^(1/2)+a)^3+4*a^3/b^6/(b*x^(1/2)+a)^5+2/7*a^5/b^6/(b*x^(1/2)
+a)^7-5*a^2/b^6/(b*x^(1/2)+a)^4-5/3*a^4/b^6/(b*x^(1/2)+a)^6-1/b^6/(b*x^(1/2
)+a)^2
```

maxima [B] time = 0.90, size = 98, normalized size = 2.28

$$-\frac{1}{\left(b\sqrt{x}+a\right)^2b^6}+\frac{10\,a}{3\left(b\sqrt{x}+a\right)^3b^6}-\frac{5\,a^2}{\left(b\sqrt{x}+a\right)^4b^6}+\frac{4\,a^3}{\left(b\sqrt{x}+a\right)^5b^6}-\frac{5\,a^4}{3\left(b\sqrt{x}+a\right)^6b^6}+\frac{2\,a^5}{7\left(b\sqrt{x}+a\right)^7b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^(1/2))^8,x, algorithm="maxima")
```

```
[Out] -1/((b*sqrt(x) + a)^2*b^6) + 10/3*a/((b*sqrt(x) + a)^3*b^6) - 5*a^2/((b*sqr
t(x) + a)^4*b^6) + 4*a^3/((b*sqrt(x) + a)^5*b^6) - 5/3*a^4/((b*sqrt(x) + a)
^6*b^6) + 2/7*a^5/((b*sqrt(x) + a)^7*b^6)
```

mupad [B] time = 0.17, size = 130, normalized size = 3.02

$$-\frac{\frac{a^5}{21\,b^6}+\frac{x^{5/2}}{b}+\frac{5\,a\,x^2}{3\,b^2}+\frac{a^3\,x}{b^4}+\frac{5\,a^2\,x^{3/2}}{3\,b^3}+\frac{a^4\,\sqrt{x}}{3\,b^5}}{a^7+b^7\,x^{7/2}+21\,a^5\,b^2\,x+7\,a\,b^6\,x^3+7\,a^6\,b\,\sqrt{x}+35\,a^3\,b^4\,x^2+35\,a^4\,b^3\,x^{3/2}+21\,a^2\,b^5\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^(1/2))^8,x)
```

```
[Out] -(a^5/(21*b^6) + x^(5/2)/b + (5*a*x^2)/(3*b^2) + (a^3*x)/b^4 + (5*a^2*x^(3/
2))/(3*b^3) + (a^4*x^(1/2))/(3*b^5))/(a^7 + b^7*x^(7/2) + 21*a^5*b^2*x + 7*
a*b^6*x^3 + 7*a^6*b*x^(1/2) + 35*a^3*b^4*x^2 + 35*a^4*b^3*x^(3/2) + 21*a^2*
b^5*x^(5/2))
```

sympy [A] time = 6.49, size = 619, normalized size = 14.40

$$\begin{cases} -\frac{a^5}{21a^7b^6+147a^6b^7\sqrt{x}+441a^5b^8x+735a^4b^9x^{\frac{3}{2}}+735a^3b^{10}x^2+441a^2b^{11}x^{\frac{5}{2}}+147ab^{12}x^3+21b^{13}x^{\frac{7}{2}}} - \frac{7a^{4} & \\ \frac{x^3}{3a^8} & \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*x**(1/2))**8,x)
```

```
[Out] Piecewise((-a**5/(21*a**7*b**6 + 147*a**6*b**7*sqrt(x) + 441*a**5*b**8*x +
735*a**4*b**9*x**(3/2) + 735*a**3*b**10*x**2 + 441*a**2*b**11*x**(5/2) + 14
7*a*b**12*x**3 + 21*b**13*x**(7/2)) - 7*a**4*b*sqrt(x)/(21*a**7*b**6 + 147*
a**6*b**7*sqrt(x) + 441*a**5*b**8*x + 735*a**4*b**9*x**(3/2) + 735*a**3*b**
10*x**2 + 441*a**2*b**11*x**(5/2) + 147*a*b**12*x**3 + 21*b**13*x**(7/2)) -
 21*a**3*b**2*x/(21*a**7*b**6 + 147*a**6*b**7*sqrt(x) + 441*a**5*b**8*x + 7
35*a**4*b**9*x**(3/2) + 735*a**3*b**10*x**2 + 441*a**2*b**11*x**(5/2) + 147
*a*b**12*x**3 + 21*b**13*x**(7/2)) - 35*a**2*b**3*x**(3/2)/(21*a**7*b**6 +
147*a**6*b**7*sqrt(x) + 441*a**5*b**8*x + 735*a**4*b**9*x**(3/2) + 735*a**3
*b**10*x**2 + 441*a**2*b**11*x**(5/2) + 147*a*b**12*x**3 + 21*b**13*x**(7/2
)) - 35*a*b**4*x**2/(21*a**7*b**6 + 147*a**6*b**7*sqrt(x) + 441*a**5*b**8*x
 + 735*a**4*b**9*x**(3/2) + 735*a**3*b**10*x**2 + 441*a**2*b**11*x**(5/2) +
 147*a*b**12*x**3 + 21*b**13*x**(7/2)) - 21*b**5*x**(5/2)/(21*a**7*b**6 + 1
47*a**6*b**7*sqrt(x) + 441*a**5*b**8*x + 735*a**4*b**9*x**(3/2) + 735*a**3*
b**10*x**2 + 441*a**2*b**11*x**(5/2) + 147*a*b**12*x**3 + 21*b**13*x**(7/2)
), Ne(b, 0)), (x**3/(3*a**8), True))
```

3.2227 $\int \frac{x}{\left(a+b\sqrt{x}\right)^8}\,dx$

Optimal. Leaf size=78

$$\frac{2a^3}{7b^4\left(a+b\sqrt{x}\right)^7}-\frac{a^2}{b^4\left(a+b\sqrt{x}\right)^6}+\frac{6a}{5b^4\left(a+b\sqrt{x}\right)^5}-\frac{1}{2b^4\left(a+b\sqrt{x}\right)^4}$$

```
[Out] 2/7*a^3/b^4/(a+b*x^(1/2))^7-a^2/b^4/(a+b*x^(1/2))^6+6/5*a/b^4/(a+b*x^(1/2))
^5-1/2/b^4/(a+b*x^(1/2))^4
```

Rubi [A] time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{2a^3}{7b^4\left(a+b\sqrt{x}\right)^7}-\frac{a^2}{b^4\left(a+b\sqrt{x}\right)^6}+\frac{6a}{5b^4\left(a+b\sqrt{x}\right)^5}-\frac{1}{2b^4\left(a+b\sqrt{x}\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*Sqrt[x])^8,x]
```

```
[Out] (2*a^3)/(7*b^4*(a + b*Sqrt[x])^7) - a^2/(b^4*(a + b*Sqrt[x])^6) + (6*a)/(5*
b^4*(a + b*Sqrt[x])^5) - 1/(2*b^4*(a + b*Sqrt[x])^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a+b\sqrt{x}\right)^8}\,dx &= 2\,\text{Subst}\left(\int \frac{x^3}{(a+bx)^8}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(-\frac{a^3}{b^3(a+bx)^8}+\frac{3a^2}{b^3(a+bx)^7}-\frac{3a}{b^3(a+bx)^6}+\frac{1}{b^3(a+bx)^5}\right)dx, x, \sqrt{x}\right)\\ &= \frac{2a^3}{7b^4\left(a+b\sqrt{x}\right)^7}-\frac{a^2}{b^4\left(a+b\sqrt{x}\right)^6}+\frac{6a}{5b^4\left(a+b\sqrt{x}\right)^5}-\frac{1}{2b^4\left(a+b\sqrt{x}\right)^4}\end{aligned}$$

Mathematica [A] time = 0.05, size = 50, normalized size = 0.64

$$-\frac{a^3+7a^2b\sqrt{x}+21ab^2x+35b^3x^{3/2}}{70b^4\left(a+b\sqrt{x}\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*Sqrt[x])^8,x]
```

```
[Out] -1/70*(a^3 + 7*a^2*b*Sqrt[x] + 21*a*b^2*x + 35*b^3*x^(3/2))/(b^4*(a + b*Sqr
t[x])^7)
```

fricas [B] time = 0.90, size = 177, normalized size = 2.27

$$-\frac{35\,b^{10}x^5+595\,a^2b^8x^4+630\,a^4b^6x^3+14\,a^6b^4x^2+7\,a^8b^2x-a^{10}-32\left(7\,ab^9x^4+26\,a^3b^7x^3+7\,a^5b^5x^2\right)\sqrt{x}}{70\left(b^{18}x^7-7\,a^2b^{16}x^6+21\,a^4b^{14}x^5-35\,a^6b^{12}x^4+35\,a^8b^{10}x^3-21\,a^{10}b^8x^2+7\,a^{12}b^6x-a^{14}b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^(1/2))^8,x, algorithm="fricas")
```

```
[Out] -1/70*(35*b^10*x^5 + 595*a^2*b^8*x^4 + 630*a^4*b^6*x^3 + 14*a^6*b^4*x^2 + 7
*a^8*b^2*x - a^10 - 32*(7*a*b^9*x^4 + 26*a^3*b^7*x^3 + 7*a^5*b^5*x^2)*sqrt(
x))/(b^18*x^7 - 7*a^2*b^16*x^6 + 21*a^4*b^14*x^5 - 35*a^6*b^12*x^4 + 35*a^8
*b^10*x^3 - 21*a^10*b^8*x^2 + 7*a^12*b^6*x - a^14*b^4)
```

giac [A] time = 0.16, size = 42, normalized size = 0.54

$$-\frac{35\,b^3x^{\frac{3}{2}}+21\,ab^2x+7\,a^2b\sqrt{x}+a^3}{70\left(b\sqrt{x}+a\right)^7b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^(1/2))^8,x, algorithm="giac")
```

```
[Out] -1/70*(35*b^3*x^(3/2) + 21*a*b^2*x + 7*a^2*b*sqrt(x) + a^3)/((b*sqrt(x) + a
)^7*b^4)
```

maple [A] time = 0.01, size = 65, normalized size = 0.83

$$\frac{2a^3}{7\left(b\sqrt{x}+a\right)^7b^4}-\frac{a^2}{\left(b\sqrt{x}+a\right)^6b^4}+\frac{6a}{5\left(b\sqrt{x}+a\right)^5b^4}-\frac{1}{2\left(b\sqrt{x}+a\right)^4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^(1/2)+a)^8,x)
```

```
[Out] 2/7*a^3/b^4/(b*x^(1/2)+a)^7-a^2/b^4/(b*x^(1/2)+a)^6+6/5*a/b^4/(b*x^(1/2)+a)
^5-1/2/b^4/(b*x^(1/2)+a)^4
```

maxima [A] time = 0.85, size = 64, normalized size = 0.82

$$-\frac{1}{2\left(b\sqrt{x}+a\right)^4b^4}+\frac{6\,a}{5\left(b\sqrt{x}+a\right)^5b^4}-\frac{a^2}{\left(b\sqrt{x}+a\right)^6b^4}+\frac{2\,a^3}{7\left(b\sqrt{x}+a\right)^7b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^(1/2))^8,x, algorithm="maxima")
```

```
[Out] -1/2/((b*sqrt(x) + a)^4*b^4) + 6/5*a/((b*sqrt(x) + a)^5*b^4) - a^2/((b*sqrt
(x) + a)^6*b^4) + 2/7*a^3/((b*sqrt(x) + a)^7*b^4)
```

mupad [B] time = 1.21, size = 110, normalized size = 1.41

$$-\frac{\frac{a^3}{70\,b^4}+\frac{x^{3/2}}{2\,b}+\frac{a^2\sqrt{x}}{10\,b^3}+\frac{3\,a\,x}{10\,b^2}}{a^7+b^7\,x^{7/2}+21\,a^5\,b^2\,x+7\,a\,b^6\,x^3+7\,a^6\,b\,\sqrt{x}+35\,a^3\,b^4\,x^2+35\,a^4\,b^3\,x^{3/2}+21\,a^2\,b^5\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^(1/2))^8,x)
```

```
[Out] -(a^3/(70*b^4) + x^(3/2)/(2*b) + (a^2*x^(1/2))/(10*b^3) + (3*a*x)/(10*b^2))
/(a^7 + b^7*x^(7/2) + 21*a^5*b^2*x + 7*a*b^6*x^3 + 7*a^6*b*x^(1/2) + 35*a^3
*b^4*x^2 + 35*a^4*b^3*x^(3/2) + 21*a^2*b^5*x^(5/2))
```

sympy [A] time = 6.16, size = 410, normalized size = 5.26

$$\begin{cases} -\dfrac{a^3}{70a^7b^4+490a^6b^5\sqrt{x}+1470a^5b^6x+2450a^4b^7x^{\frac{3}{2}}+2450a^3b^8x^2+1470a^2b^9x^{\frac{5}{2}}+490ab^{10}x^3+70b^{11}x^{\frac{7}{2}}} - \dfrac{7a^{\cdots}}{70a^7b^4+490a^6b^5\sqrt{x}+1470a^5b^6x+2450a^4b^7x^{\frac{3}{2}}} \\ \dfrac{x^2}{2a^8} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x**(1/2))**8,x)
```

```
[Out] Piecewise((-a**3/(70*a**7*b**4 + 490*a**6*b**5*sqrt(x) + 1470*a**5*b**6*x +
 2450*a**4*b**7*x**(3/2) + 2450*a**3*b**8*x**2 + 1470*a**2*b**9*x**(5/2) +
490*a*b**10*x**3 + 70*b**11*x**(7/2)) - 7*a**2*b*sqrt(x)/(70*a**7*b**4 + 49
0*a**6*b**5*sqrt(x) + 1470*a**5*b**6*x + 2450*a**4*b**7*x**(3/2) + 2450*a**
3*b**8*x**2 + 1470*a**2*b**9*x**(5/2) + 490*a*b**10*x**3 + 70*b**11*x**(7/2
)) - 21*a*b**2*x/(70*a**7*b**4 + 490*a**6*b**5*sqrt(x) + 1470*a**5*b**6*x +
 2450*a**4*b**7*x**(3/2) + 2450*a**3*b**8*x**2 + 1470*a**2*b**9*x**(5/2) +
490*a*b**10*x**3 + 70*b**11*x**(7/2)) - 35*b**3*x**(3/2)/(70*a**7*b**4 + 49
0*a**6*b**5*sqrt(x) + 1470*a**5*b**6*x + 2450*a**4*b**7*x**(3/2) + 2450*a**
3*b**8*x**2 + 1470*a**2*b**9*x**(5/2) + 490*a*b**10*x**3 + 70*b**11*x**(7/2
)), Ne(b, 0)), (x**2/(2*a**8), True))
```

3.2228 $\int \frac{1}{\left(a+b\sqrt{x}\right)^8}\,dx$

Optimal. Leaf size=38

$$\frac{2a}{7b^2\left(a+b\sqrt{x}\right)^7}-\frac{1}{3b^2\left(a+b\sqrt{x}\right)^6}$$

```
[Out] 2/7*a/b^2/(a+b*x^(1/2))^7-1/3/b^2/(a+b*x^(1/2))^6
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {190, 43}

$$\frac{2a}{7b^2\left(a+b\sqrt{x}\right)^7}-\frac{1}{3b^2\left(a+b\sqrt{x}\right)^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^(-8),x]
```

```
[Out] (2*a)/(7*b^2*(a + b*Sqrt[x])^7) - 1/(3*b^2*(a + b*Sqrt[x])^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
 - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
  IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+b\sqrt{x}\right)^8}\,dx &= 2\,\text{Subst}\left(\int \frac{x}{(a+bx)^8}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^8}+\frac{1}{b(a+bx)^7}\right)dx, x, \sqrt{x}\right)\\ &= \frac{2a}{7b^2\left(a+b\sqrt{x}\right)^7}-\frac{1}{3b^2\left(a+b\sqrt{x}\right)^6}\end{aligned}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 0.74

$$-\frac{a+7b\sqrt{x}}{21b^2\left(a+b\sqrt{x}\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^(-8),x]
```

```
[Out] -1/21*(a + 7*b*Sqrt[x])/(b^2*(a + b*Sqrt[x])^7)
```

fricas [B] time = 0.94, size = 164, normalized size = 4.32

$$-\frac{7\,b^8x^4+140\,a^2b^6x^3+210\,a^4b^4x^2+28\,a^6b^2x-a^8-16\left(3\,ab^7x^3+14\,a^3b^5x^2+7\,a^5b^3x\right)\sqrt{x}}{21\left(b^{16}x^7-7\,a^2b^{14}x^6+21\,a^4b^{12}x^5-35\,a^6b^{10}x^4+35\,a^8b^8x^3-21\,a^{10}b^6x^2+7\,a^{12}b^4x-a^{14}b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/2))^8,x, algorithm="fricas")
```

```
[Out] -1/21*(7*b^8*x^4 + 140*a^2*b^6*x^3 + 210*a^4*b^4*x^2 + 28*a^6*b^2*x - a^8 -
 16*(3*a*b^7*x^3 + 14*a^3*b^5*x^2 + 7*a^5*b^3*x)*sqrt(x))/(b^16*x^7 - 7*a^2
*b^14*x^6 + 21*a^4*b^12*x^5 - 35*a^6*b^10*x^4 + 35*a^8*b^8*x^3 - 21*a^10*b^
6*x^2 + 7*a^12*b^4*x - a^14*b^2)
```

giac [A] time = 0.18, size = 22, normalized size = 0.58

$$-\frac{7\,b\sqrt{x}+a}{21\left(b\sqrt{x}+a\right)^7b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/2))^8,x, algorithm="giac")
```

```
[Out] -1/21*(7*b*sqrt(x) + a)/((b*sqrt(x) + a)^7*b^2)
```

maple [B] time = 0.07, size = 399, normalized size = 10.50

$$28\left(-\frac{a^2}{7\left(b^2x-a^2\right)^7b^4}-\frac{1}{6\left(b^2x-a^2\right)^6b^4}\right)a^6b^2+70\left(-\frac{a^4}{7\left(b^2x-a^2\right)^7b^6}-\frac{a^2}{3\left(b^2x-a^2\right)^6b^6}-\frac{1}{5\left(b^2x-a^2\right)^5b^6}\right)a^4b^4-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^(1/2)+a)^8,x)
```

```
[Out] -1/7*a^8/(b^2*x-a^2)^7/b^2+b^8*(-a^2/b^10/(b^2*x-a^2)^4-1/3/b^10/(b^2*x-a^2
)^3-6/5*a^4/b^10/(b^2*x-a^2)^5-1/7*a^8/b^10/(b^2*x-a^2)^7-2/3*a^6/b^10/(b^2
*x-a^2)^6)+1/7*a/b^2/(b*x^(1/2)+a)^7+1/7*a/b^2/(b*x^(1/2)-a)^7-1/6/b^2/(b*x
^(1/2)+a)^6+1/6/b^2/(b*x^(1/2)-a)^6+28*b^6*a^2*(-1/4/(b^2*x-a^2)^4/b^8-3/5*
a^2/b^8/(b^2*x-a^2)^5-1/7*a^6/b^8/(b^2*x-a^2)^7-1/2*a^4/b^8/(b^2*x-a^2)^6)+
28*b^2*a^6*(-1/7*a^2/b^4/(b^2*x-a^2)^7-1/6/b^4/(b^2*x-a^2)^6)+70*a^4*b^4*(-
1/5/b^6/(b^2*x-a^2)^5-1/7*a^4/b^6/(b^2*x-a^2)^7-1/3*a^2/b^6/(b^2*x-a^2)^6)
```

maxima [A] time = 0.86, size = 30, normalized size = 0.79

$$-\frac{1}{3\left(b\sqrt{x}+a\right)^6b^2}+\frac{2\,a}{7\left(b\sqrt{x}+a\right)^7b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/2))^8,x, algorithm="maxima")
```

```
[Out] -1/3/((b*sqrt(x) + a)^6*b^2) + 2/7*a/((b*sqrt(x) + a)^7*b^2)
```

mupad [B] time = 1.22, size = 90, normalized size = 2.37

$$-\frac{\frac{a}{21\,b^2}+\frac{\sqrt{x}}{3\,b}}{a^7+b^7\,x^{7/2}+21\,a^5\,b^2\,x+7\,a\,b^6\,x^3+7\,a^6\,b\,\sqrt{x}+35\,a^3\,b^4\,x^2+35\,a^4\,b^3\,x^{3/2}+21\,a^2\,b^5\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^(1/2))^8,x)
```

```
[Out] -(a/(21*b^2) + x^(1/2)/(3*b))/(a^7 + b^7*x^(7/2) + 21*a^5*b^2*x + 7*a*b^6*x
^3 + 7*a^6*b*x^(1/2) + 35*a^3*b^4*x^2 + 35*a^4*b^3*x^(3/2) + 21*a^2*b^5*x^(
5/2))
```

sympy [A] time = 5.67, size = 199, normalized size = 5.24

$$\begin{cases} -\dfrac{a}{21a^7b^2+147a^6b^3\sqrt{x}+441a^5b^4x+735a^4b^5x^{\frac{3}{2}}+735a^3b^6x^2+441a^2b^7x^{\frac{5}{2}}+147ab^8x^3+21b^9x^{\frac{7}{2}}} - \dfrac{7b\sqrt{x}}{21a^7b^2+147a^6b^3\sqrt{x}+441a^5b^4x+735a^4b^5x^{\frac{3}{2}}+735} \\ \dfrac{x}{a^8} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**(1/2))**8,x)
```

```
[Out] Piecewise((-a/(21*a**7*b**2 + 147*a**6*b**3*sqrt(x) + 441*a**5*b**4*x + 735
*a**4*b**5*x**(3/2) + 735*a**3*b**6*x**2 + 441*a**2*b**7*x**(5/2) + 147*a*b
**8*x**3 + 21*b**9*x**(7/2)) - 7*b*sqrt(x)/(21*a**7*b**2 + 147*a**6*b**3*sq
rt(x) + 441*a**5*b**4*x + 735*a**4*b**5*x**(3/2) + 735*a**3*b**6*x**2 + 441
*a**2*b**7*x**(5/2) + 147*a*b**8*x**3 + 21*b**9*x**(7/2)), Ne(b, 0)), (x/a*
*8, True))
```

3.2229 $\int \frac{1}{\left(a+b\sqrt{x}\right)^8 x}\,dx$

Optimal. Leaf size=143

$$-\frac{2\log\left(a+b\sqrt{x}\right)}{a^8}+\frac{\log(x)}{a^8}+\frac{2}{a^7\left(a+b\sqrt{x}\right)}+\frac{1}{a^6\left(a+b\sqrt{x}\right)^2}+\frac{2}{3a^5\left(a+b\sqrt{x}\right)^3}+\frac{1}{2a^4\left(a+b\sqrt{x}\right)^4}+\frac{2}{5a^3\left(a+b\sqrt{x}\right)^5}+$$

```
[Out] ln(x)/a^8-2*ln(a+b*x^(1/2))/a^8+2/7/a/(a+b*x^(1/2))^7+1/3/a^2/(a+b*x^(1/2))
^6+2/5/a^3/(a+b*x^(1/2))^5+1/2/a^4/(a+b*x^(1/2))^4+2/3/a^5/(a+b*x^(1/2))^3+
1/a^6/(a+b*x^(1/2))^2+2/a^7/(a+b*x^(1/2))
```

Rubi [A] time = 0.09, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 44}

$$\frac{2}{a^7\left(a+b\sqrt{x}\right)}+\frac{1}{a^6\left(a+b\sqrt{x}\right)^2}+\frac{2}{3a^5\left(a+b\sqrt{x}\right)^3}+\frac{1}{2a^4\left(a+b\sqrt{x}\right)^4}+\frac{2}{5a^3\left(a+b\sqrt{x}\right)^5}+\frac{1}{3a^2\left(a+b\sqrt{x}\right)^6}-\frac{2\log\left(a+\right.}{a^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sqrt[x])^8*x),x]
```

```
[Out] 2/(7*a*(a + b*Sqrt[x])^7) + 1/(3*a^2*(a + b*Sqrt[x])^6) + 2/(5*a^3*(a + b*S
qrt[x])^5) + 1/(2*a^4*(a + b*Sqrt[x])^4) + 2/(3*a^5*(a + b*Sqrt[x])^3) + 1/
(a^6*(a + b*Sqrt[x])^2) + 2/(a^7*(a + b*Sqrt[x])) - (2*Log[a + b*Sqrt[x]])/
a^8 + Log[x]/a^8
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+b\sqrt{x}\right)^8 x}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{x(a+bx)^8}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int\left(\frac{1}{a^8 x}-\frac{b}{a(a+bx)^8}-\frac{b}{a^2(a+bx)^7}-\frac{b}{a^3(a+bx)^6}-\frac{b}{a^4(a+bx)^5}-\frac{b}{a^5(a+bx)^4}-\right.\right.\\ &= \frac{2}{7a\left(a+b\sqrt{x}\right)^7}+\frac{1}{3a^2\left(a+b\sqrt{x}\right)^6}+\frac{2}{5a^3\left(a+b\sqrt{x}\right)^5}+\frac{1}{2a^4\left(a+b\sqrt{x}\right)^4}+\frac{2}{3a^5\left(a+b\sqrt{x}\right)^3}\end{aligned}$$

Mathematica [A] time = 0.18, size = 106, normalized size = 0.74

$$\frac{\frac{a\left(1089a^6+4683a^5b\sqrt{x}+9639a^4b^2x+11165a^3b^3x^{3/2}+7490a^2b^4x^2+2730ab^5x^{5/2}+420b^6x^3\right)}{\left(a+b\sqrt{x}\right)^7}-420\log\left(a+b\sqrt{x}\right)+210\log(x)}{210a^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sqrt[x])^8*x),x]
```

```
[Out] ((a*(1089*a^6 + 4683*a^5*b*Sqrt[x] + 9639*a^4*b^2*x + 11165*a^3*b^3*x^(3/2)
 + 7490*a^2*b^4*x^2 + 2730*a*b^5*x^(5/2) + 420*b^6*x^3))/(a + b*Sqrt[x])^7
- 420*Log[a + b*Sqrt[x]] + 210*Log[x])/(210*a^8)
```

fricas [B] time = 1.02, size = 398, normalized size = 2.78

$$-\frac{210\,a^2b^{12}x^6 - 1365\,a^4b^{10}x^5 + 3745\,a^6b^8x^4 - 5530\,a^8b^6x^3 + 5964\,a^{10}b^4x^2 - 273\,a^{12}b^2x + 1089\,a^{14} + 420\left(b^{14}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^(1/2))^8,x, algorithm="fricas")
```

```
[Out] -1/210*(210*a^2*b^12*x^6 - 1365*a^4*b^10*x^5 + 3745*a^6*b^8*x^4 - 5530*a^8*
b^6*x^3 + 5964*a^10*b^4*x^2 - 273*a^12*b^2*x + 1089*a^14 + 420*(b^14*x^7 -
7*a^2*b^12*x^6 + 21*a^4*b^10*x^5 - 35*a^6*b^8*x^4 + 35*a^8*b^6*x^3 - 21*a^1
0*b^4*x^2 + 7*a^12*b^2*x - a^14)*log(b*sqrt(x) + a) - 420*(b^14*x^7 - 7*a^2
*b^12*x^6 + 21*a^4*b^10*x^5 - 35*a^6*b^8*x^4 + 35*a^8*b^6*x^3 - 21*a^10*b^4
*x^2 + 7*a^12*b^2*x - a^14)*log(sqrt(x)) - 4*(105*a*b^13*x^6 - 700*a^3*b^11
*x^5 + 1981*a^5*b^9*x^4 - 3072*a^7*b^7*x^3 + 2891*a^9*b^5*x^2 - 980*a^11*b^
3*x + 735*a^13*b)*sqrt(x))/(a^8*b^14*x^7 - 7*a^10*b^12*x^6 + 21*a^12*b^10*x
^5 - 35*a^14*b^8*x^4 + 35*a^16*b^6*x^3 - 21*a^18*b^4*x^2 + 7*a^20*b^2*x - a
^22)
```

giac [A] time = 0.17, size = 102, normalized size = 0.71

$$-\frac{2\,\log\left(\left|b\sqrt{x}+a\right|\right)}{a^8}+\frac{\log\left(|x|\right)}{a^8}+\frac{420\,ab^6x^3+2730\,a^2b^5x^{\frac{5}{2}}+7490\,a^3b^4x^2+11165\,a^4b^3x^{\frac{3}{2}}+9639\,a^5b^2x+4683\,a}{210\left(b\sqrt{x}+a\right)^7a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^(1/2))^8,x, algorithm="giac")
```

```
[Out] -2*log(abs(b*sqrt(x) + a))/a^8 + log(abs(x))/a^8 + 1/210*(420*a*b^6*x^3 + 2
730*a^2*b^5*x^(5/2) + 7490*a^3*b^4*x^2 + 11165*a^4*b^3*x^(3/2) + 9639*a^5*b
^2*x + 4683*a^6*b*sqrt(x) + 1089*a^7)/((b*sqrt(x) + a)^7*a^8)
```

maple [A] time = 0.01, size = 118, normalized size = 0.83

$$\frac{2}{7\left(b\sqrt{x}+a\right)^7a}+\frac{1}{3\left(b\sqrt{x}+a\right)^6a^2}+\frac{2}{5\left(b\sqrt{x}+a\right)^5a^3}+\frac{1}{2\left(b\sqrt{x}+a\right)^4a^4}+\frac{2}{3\left(b\sqrt{x}+a\right)^3a^5}+\frac{1}{\left(b\sqrt{x}+a\right)^2a^6}+\frac{1}{\left(b\sqrt{}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^(1/2)+a)^8,x)
```

```
[Out] ln(x)/a^8-2*ln(b*x^(1/2)+a)/a^8+2/7/a/(b*x^(1/2)+a)^7+1/3/a^2/(b*x^(1/2)+a)
^6+2/5/a^3/(b*x^(1/2)+a)^5+1/2/a^4/(b*x^(1/2)+a)^4+2/3/a^5/(b*x^(1/2)+a)^3+
1/a^6/(b*x^(1/2)+a)^2+2/a^7/(b*x^(1/2)+a)
```

maxima [A] time = 1.01, size = 163, normalized size = 1.14

$$\frac{420\,b^6x^3+2730\,ab^5x^{\frac{5}{2}}+7490\,a^2b^4x^2+11165\,a^3b^3x^{\frac{3}{2}}+9639\,a^4b^2x+4683\,a^5b\sqrt{x}+1089\,a^6}{210\left(a^7b^7x^{\frac{7}{2}}+7\,a^8b^6x^3+21\,a^9b^5x^{\frac{5}{2}}+35\,a^{10}b^4x^2+35\,a^{11}b^3x^{\frac{3}{2}}+21\,a^{12}b^2x+7\,a^{13}b\sqrt{x}+a^{14}\right)}-\frac{2\,\log\left(b\sqrt{x}+\right.}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^(1/2))^8,x, algorithm="maxima")
```

```
[Out] 1/210*(420*b^6*x^3 + 2730*a*b^5*x^(5/2) + 7490*a^2*b^4*x^2 + 11165*a^3*b^3*
x^(3/2) + 9639*a^4*b^2*x + 4683*a^5*b*sqrt(x) + 1089*a^6)/(a^7*b^7*x^(7/2)
+ 7*a^8*b^6*x^3 + 21*a^9*b^5*x^(5/2) + 35*a^10*b^4*x^2 + 35*a^11*b^3*x^(3/2
) + 21*a^12*b^2*x + 7*a^13*b*sqrt(x) + a^14) - 2*log(b*sqrt(x) + a)/a^8 + l
og(x)/a^8
```

mupad [B] time = 1.20, size = 160, normalized size = 1.12

$$\frac{\frac{363}{70\,a}+\frac{223\,b\,\sqrt{x}}{10\,a^2}+\frac{459\,b^2\,x}{10\,a^3}+\frac{107\,b^4\,x^2}{3\,a^5}+\frac{319\,b^3\,x^{3/2}}{6\,a^4}+\frac{2\,b^6\,x^3}{a^7}+\frac{13\,b^5\,x^{5/2}}{a^6}}{a^7+b^7\,x^{7/2}+21\,a^5\,b^2\,x+7\,a\,b^6\,x^3+7\,a^6\,b\,\sqrt{x}+35\,a^3\,b^4\,x^2+35\,a^4\,b^3\,x^{3/2}+21\,a^2\,b^5\,x^{5/2}}-\frac{4\,\mathrm{atanh}\left(\frac{2\,b\,\sqrt{x}}{a}+1\right)}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^(1/2))^8),x)
```

```
[Out] (363/(70*a) + (223*b*x^(1/2))/(10*a^2) + (459*b^2*x)/(10*a^3) + (107*b^4*x^
2)/(3*a^5) + (319*b^3*x^(3/2))/(6*a^4) + (2*b^6*x^3)/a^7 + (13*b^5*x^(5/2))
/a^6)/(a^7 + b^7*x^(7/2) + 21*a^5*b^2*x + 7*a*b^6*x^3 + 7*a^6*b*x^(1/2) + 3
5*a^3*b^4*x^2 + 35*a^4*b^3*x^(3/2) + 21*a^2*b^5*x^(5/2)) - (4*atanh((2*b*x^
(1/2))/a + 1))/a^8
```

sympy [A] time = 11.35, size = 2581, normalized size = 18.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x**(1/2))**8,x)
```

```
[Out] Piecewise((zoo/x**4, Eq(a, 0) & Eq(b, 0)), (log(x)/a**8, Eq(b, 0)), (-1/(4*
b**8*x**4), Eq(a, 0)), (210*a**7*sqrt(x)*log(x)/(210*a**15*sqrt(x) + 1470*a
**14*b*x + 4410*a**13*b**2*x**(3/2) + 7350*a**12*b**3*x**2 + 7350*a**11*b**
4*x**(5/2) + 4410*a**10*b**5*x**3 + 1470*a**9*b**6*x**(7/2) + 210*a**8*b**7
*x**4) - 420*a**7*sqrt(x)*log(a/b + sqrt(x))/(210*a**15*sqrt(x) + 1470*a**1
4*b*x + 4410*a**13*b**2*x**(3/2) + 7350*a**12*b**3*x**2 + 7350*a**11*b**4*x
**(5/2) + 4410*a**10*b**5*x**3 + 1470*a**9*b**6*x**(7/2) + 210*a**8*b**7*x*
*4) + 1089*a**7*sqrt(x)/(210*a**15*sqrt(x) + 1470*a**14*b*x + 4410*a**13*b*
*2*x**(3/2) + 7350*a**12*b**3*x**2 + 7350*a**11*b**4*x**(5/2) + 4410*a**10*
b**5*x**3 + 1470*a**9*b**6*x**(7/2) + 210*a**8*b**7*x**4) + 1470*a**6*b*x*l
og(x)/(210*a**15*sqrt(x) + 1470*a**14*b*x + 4410*a**13*b**2*x**(3/2) + 7350
*a**12*b**3*x**2 + 7350*a**11*b**4*x**(5/2) + 4410*a**10*b**5*x**3 + 1470*a
**9*b**6*x**(7/2) + 210*a**8*b**7*x**4) - 2940*a**6*b*x*log(a/b + sqrt(x))/
(210*a**15*sqrt(x) + 1470*a**14*b*x + 4410*a**13*b**2*x**(3/2) + 7350*a**12
*b**3*x**2 + 7350*a**11*b**4*x**(5/2) + 4410*a**10*b**5*x**3 + 1470*a**9*b*
*6*x**(7/2) + 210*a**8*b**7*x**4) + 4683*a**6*b*x/(210*a**15*sqrt(x) + 1470
*a**14*b*x + 4410*a**13*b**2*x**(3/2) + 7350*a**12*b**3*x**2 + 7350*a**11*b
**4*x**(5/2) + 4410*a**10*b**5*x**3 + 1470*a**9*b**6*x**(7/2) + 210*a**8*b*
*7*x**4) + 4410*a**5*b**2*x**(3/2)*log(x)/(210*a**15*sqrt(x) + 1470*a**14*b
*x + 4410*a**13*b**2*x**(3/2) + 7350*a**12*b**3*x**2 + 7350*a**11*b**4*x**(
5/2) + 4410*a**10*b**5*x**3 + 1470*a**9*b**6*x**(7/2) + 210*a**8*b**7*x**4)
 - 8820*a**5*b**2*x**(3/2)*log(a/b + sqrt(x))/(210*a**15*sqrt(x) + 1470*a**
14*b*x + 4410*a**13*b**2*x**(3/2) + 7350*a**12*b**3*x**2 + 7350*a**11*b**4*
x**(5/2) + 4410*a**10*b**5*x**3 + 1470*a**9*b**6*x**(7/2) + 210*a**8*b**7*x
**4) + 9639*a**5*b**2*x**(3/2)/(210*a**15*sqrt(x) + 1470*a**14*b*x + 4410*a
**13*b**2*x**(3/2) + 7350*a**12*b**3*x**2 + 7350*a**11*b**4*x**(5/2) + 4410
*a**10*b**5*x**3 + 1470*a**9*b**6*x**(7/2) + 210*a**8*b**7*x**4) + 7350*a**
4*b**3*x**2*log(x)/(210*a**15*sqrt(x) + 1470*a**14*b*x + 4410*a**13*b**2*x*
*(3/2) + 7350*a**12*b**3*x**2 + 7350*a**11*b**4*x**(5/2) + 4410*a**10*b**5*
x**3 + 1470*a**9*b**6*x**(7/2) + 210*a**8*b**7*x**4) - 14700*a**4*b**3*x**2
```

```
*log(a/b + sqrt(x))/(210*a**15*sqrt(x) + 1470*a**14*b*x + 4410*a**13*b**2*x
**(3/2) + 7350*a**12*b**3*x**2 + 7350*a**11*b**4*x**(5/2) + 4410*a**10*b**5
*x**3 + 1470*a**9*b**6*x**(7/2) + 210*a**8*b**7*x**4) + 11165*a**4*b**3*x**
2/(210*a**15*sqrt(x) + 1470*a**14*b*x + 4410*a**13*b**2*x**(3/2) + 7350*a**
12*b**3*x**2 + 7350*a**11*b**4*x**(5/2) + 4410*a**10*b**5*x**3 + 1470*a**9*
b**6*x**(7/2) + 210*a**8*b**7*x**4) + 7350*a**3*b**4*x**(5/2)*log(x)/(210*a
**15*sqrt(x) + 1470*a**14*b*x + 4410*a**13*b**2*x**(3/2) + 7350*a**12*b**3*
x**2 + 7350*a**11*b**4*x**(5/2) + 4410*a**10*b**5*x**3 + 1470*a**9*b**6*x**
(7/2) + 210*a**8*b**7*x**4) - 14700*a**3*b**4*x**(5/2)*log(a/b + sqrt(x))/(
210*a**15*sqrt(x) + 1470*a**14*b*x + 4410*a**13*b**2*x**(3/2) + 7350*a**12*
b**3*x**2 + 7350*a**11*b**4*x**(5/2) + 4410*a**10*b**5*x**3 + 1470*a**9*b**
6*x**(7/2) + 210*a**8*b**7*x**4) + 7490*a**3*b**4*x**(5/2)/(210*a**15*sqrt(
x) + 1470*a**14*b*x + 4410*a**13*b**2*x**(3/2) + 7350*a**12*b**3*x**2 + 735
0*a**11*b**4*x**(5/2) + 4410*a**10*b**5*x**3 + 1470*a**9*b**6*x**(7/2) + 21
0*a**8*b**7*x**4) + 4410*a**2*b**5*x**3*log(x)/(210*a**15*sqrt(x) + 1470*a*
*14*b*x + 4410*a**13*b**2*x**(3/2) + 7350*a**12*b**3*x**2 + 7350*a**11*b**4
*x**(5/2) + 4410*a**10*b**5*x**3 + 1470*a**9*b**6*x**(7/2) + 210*a**8*b**7*
x**4) - 8820*a**2*b**5*x**3*log(a/b + sqrt(x))/(210*a**15*sqrt(x) + 1470*a*
*14*b*x + 4410*a**13*b**2*x**(3/2) + 7350*a**12*b**3*x**2 + 7350*a**11*b**4
*x**(5/2) + 4410*a**10*b**5*x**3 + 1470*a**9*b**6*x**(7/2) + 210*a**8*b**7*
x**4) + 2730*a**2*b**5*x**3/(210*a**15*sqrt(x) + 1470*a**14*b*x + 4410*a**1
3*b**2*x**(3/2) + 7350*a**12*b**3*x**2 + 7350*a**11*b**4*x**(5/2) + 4410*a*
*10*b**5*x**3 + 1470*a**9*b**6*x**(7/2) + 210*a**8*b**7*x**4) + 1470*a*b**6
*x**(7/2)*log(x)/(210*a**15*sqrt(x) + 1470*a**14*b*x + 4410*a**13*b**2*x**(
3/2) + 7350*a**12*b**3*x**2 + 7350*a**11*b**4*x**(5/2) + 4410*a**10*b**5*x*
*3 + 1470*a**9*b**6*x**(7/2) + 210*a**8*b**7*x**4) - 2940*a*b**6*x**(7/2)*l
og(a/b + sqrt(x))/(210*a**15*sqrt(x) + 1470*a**14*b*x + 4410*a**13*b**2*x**
(3/2) + 7350*a**12*b**3*x**2 + 7350*a**11*b**4*x**(5/2) + 4410*a**10*b**5*x
**3 + 1470*a**9*b**6*x**(7/2) + 210*a**8*b**7*x**4) + 420*a*b**6*x**(7/2)/(
210*a**15*sqrt(x) + 1470*a**14*b*x + 4410*a**13*b**2*x**(3/2) + 7350*a**12*
b**3*x**2 + 7350*a**11*b**4*x**(5/2) + 4410*a**10*b**5*x**3 + 1470*a**9*b**
6*x**(7/2) + 210*a**8*b**7*x**4) + 210*b**7*x**4*log(x)/(210*a**15*sqrt(x)
+ 1470*a**14*b*x + 4410*a**13*b**2*x**(3/2) + 7350*a**12*b**3*x**2 + 7350*a
**11*b**4*x**(5/2) + 4410*a**10*b**5*x**3 + 1470*a**9*b**6*x**(7/2) + 210*a
**8*b**7*x**4) - 420*b**7*x**4*log(a/b + sqrt(x))/(210*a**15*sqrt(x) + 1470
*a**14*b*x + 4410*a**13*b**2*x**(3/2) + 7350*a**12*b**3*x**2 + 7350*a**11*b
**4*x**(5/2) + 4410*a**10*b**5*x**3 + 1470*a**9*b**6*x**(7/2) + 210*a**8*b*
*7*x**4), True))
```

3.2230 $\int \frac{1}{\left(a+b\sqrt{x}\right)^8 x^2}\,dx$

Optimal. Leaf size=184

$$-\frac{72b^2\log\left(a+b\sqrt{x}\right)}{a^{10}}+\frac{36b^2\log(x)}{a^{10}}+\frac{56b^2}{a^9\left(a+b\sqrt{x}\right)}+\frac{16b}{a^9\sqrt{x}}+\frac{21b^2}{a^8\left(a+b\sqrt{x}\right)^2}-\frac{1}{a^8x}+\frac{10b^2}{a^7\left(a+b\sqrt{x}\right)^3}+\frac{5b^2}{a^6\left(a+b\sqrt{x}\right)^4}$$

```
[Out] -1/a^8/x+36*b^2*ln(x)/a^10-72*b^2*ln(a+b*x^(1/2))/a^10+16*b/a^9/x^(1/2)+2/7
*b^2/a^3/(a+b*x^(1/2))^7+b^2/a^4/(a+b*x^(1/2))^6+12/5*b^2/a^5/(a+b*x^(1/2))
^5+5*b^2/a^6/(a+b*x^(1/2))^4+10*b^2/a^7/(a+b*x^(1/2))^3+21*b^2/a^8/(a+b*x^(
1/2))^2+56*b^2/a^9/(a+b*x^(1/2))
```

Rubi [A] time = 0.16, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 44}

$$\frac{56b^2}{a^9\left(a+b\sqrt{x}\right)}+\frac{21b^2}{a^8\left(a+b\sqrt{x}\right)^2}+\frac{10b^2}{a^7\left(a+b\sqrt{x}\right)^3}+\frac{5b^2}{a^6\left(a+b\sqrt{x}\right)^4}+\frac{12b^2}{5a^5\left(a+b\sqrt{x}\right)^5}+\frac{b^2}{a^4\left(a+b\sqrt{x}\right)^6}+\frac{2b^2}{7a^3\left(a+b\sqrt{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sqrt[x])^8*x^2),x]
```

```
[Out] (2*b^2)/(7*a^3*(a + b*Sqrt[x])^7) + b^2/(a^4*(a + b*Sqrt[x])^6) + (12*b^2)/
(5*a^5*(a + b*Sqrt[x])^5) + (5*b^2)/(a^6*(a + b*Sqrt[x])^4) + (10*b^2)/(a^7
*(a + b*Sqrt[x])^3) + (21*b^2)/(a^8*(a + b*Sqrt[x])^2) + (56*b^2)/(a^9*(a +
 b*Sqrt[x])) - 1/(a^8*x) + (16*b)/(a^9*Sqrt[x]) - (72*b^2*Log[a + b*Sqrt[x]
])/a^10 + (36*b^2*Log[x])/a^10
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+b\sqrt{x}\right)^8 x^2}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{x^3(a+bx)^8}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int\left(\frac{1}{a^8x^3}-\frac{8b}{a^9x^2}+\frac{36b^2}{a^{10}x}-\frac{b^3}{a^3(a+bx)^8}-\frac{3b^3}{a^4(a+bx)^7}-\frac{6b^3}{a^5(a+bx)^6}-\frac{10b^3}{a^6(a+bx}\right.\right.\\ &= \frac{2b^2}{7a^3\left(a+b\sqrt{x}\right)^7}+\frac{b^2}{a^4\left(a+b\sqrt{x}\right)^6}+\frac{12b^2}{5a^5\left(a+b\sqrt{x}\right)^5}+\frac{5b^2}{a^6\left(a+b\sqrt{x}\right)^4}+\frac{10b^2}{a^7\left(a+b\sqrt{x}\right)^3}+\end{aligned}$$

Mathematica [A] time = 0.27, size = 139, normalized size = 0.76

$$\frac{\frac{a\left(-35a^8+315a^7b\sqrt{x}+6534a^6b^2x+28098a^5b^3x^{3/2}+57834a^4b^4x^2+66990a^3b^5x^{5/2}+44940a^2b^6x^3+16380ab^7x^{7/2}+2520b^8x^4\right)}{x\left(a+b\sqrt{x}\right)^7}-2520b^2\log\left(a+\right.}{35a^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sqrt[x])^8*x^2),x]
```

```
[Out] ((a*(-35*a^8 + 315*a^7*b*Sqrt[x] + 6534*a^6*b^2*x + 28098*a^5*b^3*x^(3/2) +
 57834*a^4*b^4*x^2 + 66990*a^3*b^5*x^(5/2) + 44940*a^2*b^6*x^3 + 16380*a*b^
7*x^(7/2) + 2520*b^8*x^4))/((a + b*Sqrt[x])^7*x) - 2520*b^2*Log[a + b*Sqrt[
x]] + 1260*b^2*Log[x])/(35*a^10)
```

fricas [B] time = 1.48, size = 435, normalized size = 2.36

$$-\frac{1260\,a^2b^{14}x^7-8190\,a^4b^{12}x^6+22470\,a^6b^{10}x^5-33495\,a^8b^8x^4+28924\,a^{10}b^6x^3-13888\,a^{12}b^4x^2+3594\,a^{14}b^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^(1/2))^8,x, algorithm="fricas")
```

```
[Out] -1/35*(1260*a^2*b^14*x^7 - 8190*a^4*b^12*x^6 + 22470*a^6*b^10*x^5 - 33495*a
^8*b^8*x^4 + 28924*a^10*b^6*x^3 - 13888*a^12*b^4*x^2 + 3594*a^14*b^2*x - 35
*a^16 + 2520*(b^16*x^8 - 7*a^2*b^14*x^7 + 21*a^4*b^12*x^6 - 35*a^6*b^10*x^5
 + 35*a^8*b^8*x^4 - 21*a^10*b^6*x^3 + 7*a^12*b^4*x^2 - a^14*b^2*x)*log(b*sq
rt(x) + a) - 2520*(b^16*x^8 - 7*a^2*b^14*x^7 + 21*a^4*b^12*x^6 - 35*a^6*b^1
0*x^5 + 35*a^8*b^8*x^4 - 21*a^10*b^6*x^3 + 7*a^12*b^4*x^2 - a^14*b^2*x)*log
(sqrt(x)) - 8*(315*a*b^15*x^7 - 2100*a^3*b^13*x^6 + 5943*a^5*b^11*x^5 - 921
6*a^7*b^9*x^4 + 8393*a^9*b^7*x^3 - 4410*a^11*b^5*x^2 + 1225*a^13*b^3*x - 70
*a^15*b)*sqrt(x))/(a^10*b^14*x^8 - 7*a^12*b^12*x^7 + 21*a^14*b^10*x^6 - 35*
a^16*b^8*x^5 + 35*a^18*b^6*x^4 - 21*a^20*b^4*x^3 + 7*a^22*b^2*x^2 - a^24*x)
```

giac [A] time = 0.21, size = 134, normalized size = 0.73

$$-\frac{72\,b^2\log\left(\left|b\sqrt{x}+a\right|\right)}{a^{10}}+\frac{36\,b^2\log\left(|x|\right)}{a^{10}}+\frac{2520\,ab^8x^4+16380\,a^2b^7x^{\frac{7}{2}}+44940\,a^3b^6x^3+66990\,a^4b^5x^{\frac{5}{2}}+57834}{35\left(b\sqrt{x}+a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^(1/2))^8,x, algorithm="giac")
```

```
[Out] -72*b^2*log(abs(b*sqrt(x) + a))/a^10 + 36*b^2*log(abs(x))/a^10 + 1/35*(2520
*a*b^8*x^4 + 16380*a^2*b^7*x^(7/2) + 44940*a^3*b^6*x^3 + 66990*a^4*b^5*x^(5
/2) + 57834*a^5*b^4*x^2 + 28098*a^6*b^3*x^(3/2) + 6534*a^7*b^2*x + 315*a^8*
b*sqrt(x) - 35*a^9)/((b*sqrt(x) + a)^7*a^10*x)
```

maple [A] time = 0.01, size = 163, normalized size = 0.89

$$\frac{2b^2}{7\left(b\sqrt{x}+a\right)^7a^3}+\frac{b^2}{\left(b\sqrt{x}+a\right)^6a^4}+\frac{12b^2}{5\left(b\sqrt{x}+a\right)^5a^5}+\frac{5b^2}{\left(b\sqrt{x}+a\right)^4a^6}+\frac{10b^2}{\left(b\sqrt{x}+a\right)^3a^7}+\frac{21b^2}{\left(b\sqrt{x}+a\right)^2a^8}+\frac{56b}{\left(b\sqrt{x}+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^(1/2)+a)^8,x)
```

```
[Out] -1/a^8/x+36*b^2*ln(x)/a^10-72*b^2*ln(b*x^(1/2)+a)/a^10+16*b/a^9/x^(1/2)+2/7
*b^2/a^3/(b*x^(1/2)+a)^7+b^2/a^4/(b*x^(1/2)+a)^6+12/5*b^2/a^5/(b*x^(1/2)+a)
```

```
^5+5*b^2/a^6/(b*x^(1/2)+a)^4+10*b^2/a^7/(b*x^(1/2)+a)^3+21*b^2/a^8/(b*x^(1/
2)+a)^2+56*b^2/a^9/(b*x^(1/2)+a)
```

maxima [A] time = 0.98, size = 196, normalized size = 1.07

$$\frac{2520\,b^8x^4+16380\,ab^7x^{\frac{7}{2}}+44940\,a^2b^6x^3+66990\,a^3b^5x^{\frac{5}{2}}+57834\,a^4b^4x^2+28098\,a^5b^3x^{\frac{3}{2}}+6534\,a^6b^2x+315\,a^7}{35\left(a^9b^7x^{\frac{9}{2}}+7\,a^{10}b^6x^4+21\,a^{11}b^5x^{\frac{7}{2}}+35\,a^{12}b^4x^3+35\,a^{13}b^3x^{\frac{5}{2}}+21\,a^{14}b^2x^2+7\,a^{15}bx^{\frac{3}{2}}+a^{16}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^(1/2))^8,x, algorithm="maxima")
```

```
[Out] 1/35*(2520*b^8*x^4 + 16380*a*b^7*x^(7/2) + 44940*a^2*b^6*x^3 + 66990*a^3*b^
5*x^(5/2) + 57834*a^4*b^4*x^2 + 28098*a^5*b^3*x^(3/2) + 6534*a^6*b^2*x + 31
5*a^7*b*sqrt(x) - 35*a^8)/(a^9*b^7*x^(9/2) + 7*a^10*b^6*x^4 + 21*a^11*b^5*x
^(7/2) + 35*a^12*b^4*x^3 + 35*a^13*b^3*x^(5/2) + 21*a^14*b^2*x^2 + 7*a^15*b
*x^(3/2) + a^16*x) - 72*b^2*log(b*sqrt(x) + a)/a^10 + 36*b^2*log(x)/a^10
```

mupad [B] time = 1.26, size = 189, normalized size = 1.03

$$\frac{\frac{9\,b\,\sqrt{x}}{a^2}-\frac{1}{a}+\frac{6534\,b^2\,x}{35\,a^3}+\frac{8262\,b^4\,x^2}{5\,a^5}+\frac{4014\,b^3\,x^{3/2}}{5\,a^4}+\frac{1284\,b^6\,x^3}{a^7}+\frac{1914\,b^5\,x^{5/2}}{a^6}+\frac{72\,b^8\,x^4}{a^9}+\frac{468\,b^7\,x^{7/2}}{a^8}}{a^7\,x+b^7\,x^{9/2}+7\,a\,b^6\,x^4+7\,a^6\,b\,x^{3/2}+21\,a^5\,b^2\,x^2+35\,a^3\,b^4\,x^3+35\,a^4\,b^3\,x^{5/2}+21\,a^2\,b^5\,x^{7/2}}-\frac{144\,b^2\,\operatorname{atanh}\left(\frac{2\,b\,\sqrt{x}}{a}\right.}{a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^(1/2))^8),x)
```

```
[Out] ((9*b*x^(1/2))/a^2 - 1/a + (6534*b^2*x)/(35*a^3) + (8262*b^4*x^2)/(5*a^5) +
 (4014*b^3*x^(3/2))/(5*a^4) + (1284*b^6*x^3)/a^7 + (1914*b^5*x^(5/2))/a^6 +
 (72*b^8*x^4)/a^9 + (468*b^7*x^(7/2))/a^8)/(a^7*x + b^7*x^(9/2) + 7*a*b^6*x
^4 + 7*a^6*b*x^(3/2) + 21*a^5*b^2*x^2 + 35*a^3*b^4*x^3 + 35*a^4*b^3*x^(5/2)
 + 21*a^2*b^5*x^(7/2)) - (144*b^2*atanh((2*b*x^(1/2))/a + 1))/a^10
```

sympy [A] time = 20.42, size = 2854, normalized size = 15.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*x**(1/2))**8,x)
```

```
[Out] Piecewise((zoo/x**5, Eq(a, 0) & Eq(b, 0)), (-1/(a**8*x), Eq(b, 0)), (-1/(5*
b**8*x**5), Eq(a, 0)), (-35*a**9*sqrt(x)/(35*a**17*x**(3/2) + 245*a**16*b*x
**2 + 735*a**15*b**2*x**(5/2) + 1225*a**14*b**3*x**3 + 1225*a**13*b**4*x**(
7/2) + 735*a**12*b**5*x**4 + 245*a**11*b**6*x**(9/2) + 35*a**10*b**7*x**5)
+ 315*a**8*b*x/(35*a**17*x**(3/2) + 245*a**16*b*x**2 + 735*a**15*b**2*x**(5
/2) + 1225*a**14*b**3*x**3 + 1225*a**13*b**4*x**(7/2) + 735*a**12*b**5*x**4
 + 245*a**11*b**6*x**(9/2) + 35*a**10*b**7*x**5) + 1260*a**7*b**2*x**(3/2)*
log(x)/(35*a**17*x**(3/2) + 245*a**16*b*x**2 + 735*a**15*b**2*x**(5/2) + 12
25*a**14*b**3*x**3 + 1225*a**13*b**4*x**(7/2) + 735*a**12*b**5*x**4 + 245*a
**11*b**6*x**(9/2) + 35*a**10*b**7*x**5) - 2520*a**7*b**2*x**(3/2)*log(a/b
+ sqrt(x))/(35*a**17*x**(3/2) + 245*a**16*b*x**2 + 735*a**15*b**2*x**(5/2)
+ 1225*a**14*b**3*x**3 + 1225*a**13*b**4*x**(7/2) + 735*a**12*b**5*x**4 + 2
45*a**11*b**6*x**(9/2) + 35*a**10*b**7*x**5) + 6534*a**7*b**2*x**(3/2)/(35*
a**17*x**(3/2) + 245*a**16*b*x**2 + 735*a**15*b**2*x**(5/2) + 1225*a**14*b*
*3*x**3 + 1225*a**13*b**4*x**(7/2) + 735*a**12*b**5*x**4 + 245*a**11*b**6*x
**(9/2) + 35*a**10*b**7*x**5) + 8820*a**6*b**3*x**2*log(x)/(35*a**17*x**(3/
2) + 245*a**16*b*x**2 + 735*a**15*b**2*x**(5/2) + 1225*a**14*b**3*x**3 + 12
25*a**13*b**4*x**(7/2) + 735*a**12*b**5*x**4 + 245*a**11*b**6*x**(9/2) + 35
*a**10*b**7*x**5) - 17640*a**6*b**3*x**2*log(a/b + sqrt(x))/(35*a**17*x**(3
```

```
/2) + 245*a**16*b*x**2 + 735*a**15*b**2*x**(5/2) + 1225*a**14*b**3*x**3 + 1
225*a**13*b**4*x**(7/2) + 735*a**12*b**5*x**4 + 245*a**11*b**6*x**(9/2) + 3
5*a**10*b**7*x**5) + 28098*a**6*b**3*x**2/(35*a**17*x**(3/2) + 245*a**16*b*
x**2 + 735*a**15*b**2*x**(5/2) + 1225*a**14*b**3*x**3 + 1225*a**13*b**4*x**
(7/2) + 735*a**12*b**5*x**4 + 245*a**11*b**6*x**(9/2) + 35*a**10*b**7*x**5)
 + 26460*a**5*b**4*x**(5/2)*log(x)/(35*a**17*x**(3/2) + 245*a**16*b*x**2 +
735*a**15*b**2*x**(5/2) + 1225*a**14*b**3*x**3 + 1225*a**13*b**4*x**(7/2) +
 735*a**12*b**5*x**4 + 245*a**11*b**6*x**(9/2) + 35*a**10*b**7*x**5) - 5292
0*a**5*b**4*x**(5/2)*log(a/b + sqrt(x))/(35*a**17*x**(3/2) + 245*a**16*b*x*
*2 + 735*a**15*b**2*x**(5/2) + 1225*a**14*b**3*x**3 + 1225*a**13*b**4*x**(7
/2) + 735*a**12*b**5*x**4 + 245*a**11*b**6*x**(9/2) + 35*a**10*b**7*x**5) +
 57834*a**5*b**4*x**(5/2)/(35*a**17*x**(3/2) + 245*a**16*b*x**2 + 735*a**15
*b**2*x**(5/2) + 1225*a**14*b**3*x**3 + 1225*a**13*b**4*x**(7/2) + 735*a**1
2*b**5*x**4 + 245*a**11*b**6*x**(9/2) + 35*a**10*b**7*x**5) + 44100*a**4*b*
*5*x**3*log(x)/(35*a**17*x**(3/2) + 245*a**16*b*x**2 + 735*a**15*b**2*x**(5
/2) + 1225*a**14*b**3*x**3 + 1225*a**13*b**4*x**(7/2) + 735*a**12*b**5*x**4
 + 245*a**11*b**6*x**(9/2) + 35*a**10*b**7*x**5) - 88200*a**4*b**5*x**3*log
(a/b + sqrt(x))/(35*a**17*x**(3/2) + 245*a**16*b*x**2 + 735*a**15*b**2*x**(
5/2) + 1225*a**14*b**3*x**3 + 1225*a**13*b**4*x**(7/2) + 735*a**12*b**5*x**
4 + 245*a**11*b**6*x**(9/2) + 35*a**10*b**7*x**5) + 66990*a**4*b**5*x**3/(3
5*a**17*x**(3/2) + 245*a**16*b*x**2 + 735*a**15*b**2*x**(5/2) + 1225*a**14*
b**3*x**3 + 1225*a**13*b**4*x**(7/2) + 735*a**12*b**5*x**4 + 245*a**11*b**6
*x**(9/2) + 35*a**10*b**7*x**5) + 44100*a**3*b**6*x**(7/2)*log(x)/(35*a**17
*x**(3/2) + 245*a**16*b*x**2 + 735*a**15*b**2*x**(5/2) + 1225*a**14*b**3*x*
*3 + 1225*a**13*b**4*x**(7/2) + 735*a**12*b**5*x**4 + 245*a**11*b**6*x**(9/
2) + 35*a**10*b**7*x**5) - 88200*a**3*b**6*x**(7/2)*log(a/b + sqrt(x))/(35*
a**17*x**(3/2) + 245*a**16*b*x**2 + 735*a**15*b**2*x**(5/2) + 1225*a**14*b*
*3*x**3 + 1225*a**13*b**4*x**(7/2) + 735*a**12*b**5*x**4 + 245*a**11*b**6*x
**(9/2) + 35*a**10*b**7*x**5) + 44940*a**3*b**6*x**(7/2)/(35*a**17*x**(3/2)
 + 245*a**16*b*x**2 + 735*a**15*b**2*x**(5/2) + 1225*a**14*b**3*x**3 + 1225
*a**13*b**4*x**(7/2) + 735*a**12*b**5*x**4 + 245*a**11*b**6*x**(9/2) + 35*a
**10*b**7*x**5) + 26460*a**2*b**7*x**4*log(x)/(35*a**17*x**(3/2) + 245*a**1
6*b*x**2 + 735*a**15*b**2*x**(5/2) + 1225*a**14*b**3*x**3 + 1225*a**13*b**4
*x**(7/2) + 735*a**12*b**5*x**4 + 245*a**11*b**6*x**(9/2) + 35*a**10*b**7*x
**5) - 52920*a**2*b**7*x**4*log(a/b + sqrt(x))/(35*a**17*x**(3/2) + 245*a**
16*b*x**2 + 735*a**15*b**2*x**(5/2) + 1225*a**14*b**3*x**3 + 1225*a**13*b**
4*x**(7/2) + 735*a**12*b**5*x**4 + 245*a**11*b**6*x**(9/2) + 35*a**10*b**7*
x**5) + 16380*a**2*b**7*x**4/(35*a**17*x**(3/2) + 245*a**16*b*x**2 + 735*a*
*15*b**2*x**(5/2) + 1225*a**14*b**3*x**3 + 1225*a**13*b**4*x**(7/2) + 735*a
**12*b**5*x**4 + 245*a**11*b**6*x**(9/2) + 35*a**10*b**7*x**5) + 8820*a*b**
8*x**(9/2)*log(x)/(35*a**17*x**(3/2) + 245*a**16*b*x**2 + 735*a**15*b**2*x*
*(5/2) + 1225*a**14*b**3*x**3 + 1225*a**13*b**4*x**(7/2) + 735*a**12*b**5*x
**4 + 245*a**11*b**6*x**(9/2) + 35*a**10*b**7*x**5) - 17640*a*b**8*x**(9/2)
*log(a/b + sqrt(x))/(35*a**17*x**(3/2) + 245*a**16*b*x**2 + 735*a**15*b**2*
x**(5/2) + 1225*a**14*b**3*x**3 + 1225*a**13*b**4*x**(7/2) + 735*a**12*b**5
*x**4 + 245*a**11*b**6*x**(9/2) + 35*a**10*b**7*x**5) + 2520*a*b**8*x**(9/2
)/(35*a**17*x**(3/2) + 245*a**16*b*x**2 + 735*a**15*b**2*x**(5/2) + 1225*a*
*14*b**3*x**3 + 1225*a**13*b**4*x**(7/2) + 735*a**12*b**5*x**4 + 245*a**11*
b**6*x**(9/2) + 35*a**10*b**7*x**5) + 1260*b**9*x**5*log(x)/(35*a**17*x**(3
/2) + 245*a**16*b*x**2 + 735*a**15*b**2*x**(5/2) + 1225*a**14*b**3*x**3 + 1
225*a**13*b**4*x**(7/2) + 735*a**12*b**5*x**4 + 245*a**11*b**6*x**(9/2) + 3
5*a**10*b**7*x**5) - 2520*b**9*x**5*log(a/b + sqrt(x))/(35*a**17*x**(3/2) +
 245*a**16*b*x**2 + 735*a**15*b**2*x**(5/2) + 1225*a**14*b**3*x**3 + 1225*a
**13*b**4*x**(7/2) + 735*a**12*b**5*x**4 + 245*a**11*b**6*x**(9/2) + 35*a**
10*b**7*x**5), True))
```

3.2231 $\int \frac{1}{\left(a+b\sqrt{x}\right)^8 x^3}\,dx$

Optimal. Leaf size=217

$$-\frac{660b^4\log\left(a+b\sqrt{x}\right)}{a^{12}}+\frac{330b^4\log(x)}{a^{12}}+\frac{420b^4}{a^{11}\left(a+b\sqrt{x}\right)}+\frac{240b^3}{a^{11}\sqrt{x}}+\frac{126b^4}{a^{10}\left(a+b\sqrt{x}\right)^2}-\frac{36b^2}{a^{10}x}+\frac{140b^4}{3a^9\left(a+b\sqrt{x}\right)^3}+\frac{16b}{3a^9x^{3/}}$$

```
[Out] -1/2/a^8/x^2+16/3*b/a^9/x^(3/2)-36*b^2/a^10/x+330*b^4*ln(x)/a^12-660*b^4*ln
(a+b*x^(1/2))/a^12+240*b^3/a^11/x^(1/2)+2/7*b^4/a^5/(a+b*x^(1/2))^7+5/3*b^4
/a^6/(a+b*x^(1/2))^6+6*b^4/a^7/(a+b*x^(1/2))^5+35/2*b^4/a^8/(a+b*x^(1/2))^4
+140/3*b^4/a^9/(a+b*x^(1/2))^3+126*b^4/a^10/(a+b*x^(1/2))^2+420*b^4/a^11/(a
+b*x^(1/2))
```

Rubi [A] time = 0.19, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 44}

$$\frac{420b^4}{a^{11}\left(a+b\sqrt{x}\right)}+\frac{126b^4}{a^{10}\left(a+b\sqrt{x}\right)^2}+\frac{140b^4}{3a^9\left(a+b\sqrt{x}\right)^3}+\frac{35b^4}{2a^8\left(a+b\sqrt{x}\right)^4}+\frac{6b^4}{a^7\left(a+b\sqrt{x}\right)^5}+\frac{5b^4}{3a^6\left(a+b\sqrt{x}\right)^6}+\frac{2b^4}{7a^5\left(a+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sqrt[x])^8*x^3),x]
```

```
[Out] (2*b^4)/(7*a^5*(a + b*Sqrt[x])^7) + (5*b^4)/(3*a^6*(a + b*Sqrt[x])^6) + (6*
b^4)/(a^7*(a + b*Sqrt[x])^5) + (35*b^4)/(2*a^8*(a + b*Sqrt[x])^4) + (140*b^
4)/(3*a^9*(a + b*Sqrt[x])^3) + (126*b^4)/(a^10*(a + b*Sqrt[x])^2) + (420*b^
4)/(a^11*(a + b*Sqrt[x])) - 1/(2*a^8*x^2) + (16*b)/(3*a^9*x^(3/2)) - (36*b^
2)/(a^10*x) + (240*b^3)/(a^11*Sqrt[x]) - (660*b^4*Log[a + b*Sqrt[x]])/a^12
+ (330*b^4*Log[x])/a^12
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+b\sqrt{x}\right)^8 x^3}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{x^5(a+bx)^8}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(\frac{1}{a^8x^5}-\frac{8b}{a^9x^4}+\frac{36b^2}{a^{10}x^3}-\frac{120b^3}{a^{11}x^2}+\frac{330b^4}{a^{12}x}-\frac{b^5}{a^5(a+bx)^8}-\frac{5b^5}{a^6(a+bx)^7}-\frac{15}{a^7(a\right.\right.\right.\\ &= \frac{2b^4}{7a^5\left(a+b\sqrt{x}\right)^7}+\frac{5b^4}{3a^6\left(a+b\sqrt{x}\right)^6}+\frac{6b^4}{a^7\left(a+b\sqrt{x}\right)^5}+\frac{35b^4}{2a^8\left(a+b\sqrt{x}\right)^4}+\frac{140b^4}{3a^9\left(a+b\sqrt{x}\right)^{}}\end{aligned}$$

Mathematica [A] time = 0.36, size = 163, normalized size = 0.75

$$\frac{a\left(-21a^{10}+77a^9b\sqrt{x}-385a^8b^2x+3465a^7b^3x^{3/2}+71874a^6b^4x^2+309078a^5b^5x^{5/2}+636174a^4b^6x^3+736890a^3b^7x^{7/2}+494340a^2b^8x^4+180180ab^9x^{9/2}+27\right.}{x^2\left(a+b\sqrt{x}\right)^7}$$

$$42a^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Sqrt[x])^8*x^3),x]
```

```
[Out] ((a*(-21*a^10 + 77*a^9*b*Sqrt[x] - 385*a^8*b^2*x + 3465*a^7*b^3*x^(3/2) + 7
1874*a^6*b^4*x^2 + 309078*a^5*b^5*x^(5/2) + 636174*a^4*b^6*x^3 + 736890*a^3
*b^7*x^(7/2) + 494340*a^2*b^8*x^4 + 180180*a*b^9*x^(9/2) + 27720*b^10*x^5))
/((a + b*Sqrt[x])^7*x^2) - 27720*b^4*Log[a + b*Sqrt[x]] + 13860*b^4*Log[x])
/(42*a^12)
```

fricas [B] time = 1.02, size = 463, normalized size = 2.13

$$-\frac{13860\,a^2b^{16}x^8-90090\,a^4b^{14}x^7+247170\,a^6b^{12}x^6-368445\,a^8b^{10}x^5+318087\,a^{10}b^8x^4-154532\,a^{12}b^6x^3+36}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^(1/2))^8,x, algorithm="fricas")
```

```
[Out] -1/42*(13860*a^2*b^16*x^8 - 90090*a^4*b^14*x^7 + 247170*a^6*b^12*x^6 - 3684
45*a^8*b^10*x^5 + 318087*a^10*b^8*x^4 - 154532*a^12*b^6*x^3 + 36104*a^14*b^
4*x^2 - 1365*a^16*b^2*x - 21*a^18 + 27720*(b^18*x^9 - 7*a^2*b^16*x^8 + 21*a
^4*b^14*x^7 - 35*a^6*b^12*x^6 + 35*a^8*b^10*x^5 - 21*a^10*b^8*x^4 + 7*a^12*
b^6*x^3 - a^14*b^4*x^2)*log(b*sqrt(x) + a) - 27720*(b^18*x^9 - 7*a^2*b^16*x
^8 + 21*a^4*b^14*x^7 - 35*a^6*b^12*x^6 + 35*a^8*b^10*x^5 - 21*a^10*b^8*x^4
 + 7*a^12*b^6*x^3 - a^14*b^4*x^2)*log(sqrt(x)) - 8*(3465*a*b^17*x^8 - 23100*
a^3*b^15*x^7 + 65373*a^5*b^13*x^6 - 101376*a^7*b^11*x^5 + 92323*a^9*b^9*x^4
 - 48580*a^11*b^7*x^3 + 13083*a^13*b^5*x^2 - 1064*a^15*b^3*x - 28*a^17*b)*s
qrt(x))/(a^12*b^14*x^9 - 7*a^14*b^12*x^8 + 21*a^16*b^10*x^7 - 35*a^18*b^8*x
^6 + 35*a^20*b^6*x^5 - 21*a^22*b^4*x^4 + 7*a^24*b^2*x^3 - a^26*x^2)
```

giac [A] time = 0.17, size = 156, normalized size = 0.72

$$-\frac{660\,b^4\log\left(\left|b\sqrt{x}+a\right|\right)}{a^{12}}+\frac{330\,b^4\log\left(|x|\right)}{a^{12}}+\frac{27720\,ab^{10}x^5+180180\,a^2b^9x^{\frac{9}{2}}+494340\,a^3b^8x^4+736890\,a^4b^7x^{\frac{7}{2}}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^(1/2))^8,x, algorithm="giac")
```

```
[Out] -660*b^4*log(abs(b*sqrt(x) + a))/a^12 + 330*b^4*log(abs(x))/a^12 + 1/42*(27
720*a*b^10*x^5 + 180180*a^2*b^9*x^(9/2) + 494340*a^3*b^8*x^4 + 736890*a^4*b
^7*x^(7/2) + 636174*a^5*b^6*x^3 + 309078*a^6*b^5*x^(5/2) + 71874*a^7*b^4*x^
2 + 3465*a^8*b^3*x^(3/2) - 385*a^9*b^2*x + 77*a^10*b*sqrt(x) - 21*a^11)/((b
*sqrt(x) + a)^7*a^12*x^2)
```

maple [A] time = 0.02, size = 186, normalized size = 0.86

$$\frac{2b^4}{7\left(b\sqrt{x}+a\right)^7a^5}+\frac{5b^4}{3\left(b\sqrt{x}+a\right)^6a^6}+\frac{6b^4}{\left(b\sqrt{x}+a\right)^5a^7}+\frac{35b^4}{2\left(b\sqrt{x}+a\right)^4a^8}+\frac{140b^4}{3\left(b\sqrt{x}+a\right)^3a^9}+\frac{126b^4}{\left(b\sqrt{x}+a\right)^2a^{10}}+\frac{}{\left(b\sqrt{}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^(1/2)+a)^8,x)
```

```
[Out] -1/2/a^8/x^2+16/3*b/a^9/x^(3/2)-36*b^2/a^10/x+330*b^4*ln(x)/a^12-660*b^4*ln
(b*x^(1/2)+a)/a^12+240*b^3/a^11/x^(1/2)+2/7*b^4/a^5/(b*x^(1/2)+a)^7+5/3*b^4
/a^6/(b*x^(1/2)+a)^6+6*b^4/a^7/(b*x^(1/2)+a)^5+35/2*b^4/a^8/(b*x^(1/2)+a)^4
+140/3*b^4/a^9/(b*x^(1/2)+a)^3+126*b^4/a^10/(b*x^(1/2)+a)^2+420*b^4/a^11/(b
*x^(1/2)+a)
```

maxima [A] time = 1.11, size = 220, normalized size = 1.01

$$\frac{27720\,b^{10}x^5 + 180180\,ab^9x^{\frac{9}{2}} + 494340\,a^2b^8x^4 + 736890\,a^3b^7x^{\frac{7}{2}} + 636174\,a^4b^6x^3 + 309078\,a^5b^5x^{\frac{5}{2}} + 71874\,a^6b^4x}{42\left(a^{11}b^7x^{\frac{11}{2}} + 7\,a^{12}b^6x^5 + 21\,a^{13}b^5x^{\frac{9}{2}} + 35\,a^{14}b^4x^4 + 35\,a^{15}b^3x^{\frac{7}{2}} + 21\,a^{16}b^2x^3 +\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^(1/2))^8,x, algorithm="maxima")
```

```
[Out] 1/42*(27720*b^10*x^5 + 180180*a*b^9*x^(9/2) + 494340*a^2*b^8*x^4 + 736890*a
^3*b^7*x^(7/2) + 636174*a^4*b^6*x^3 + 309078*a^5*b^5*x^(5/2) + 71874*a^6*b^
4*x^2 + 3465*a^7*b^3*x^(3/2) - 385*a^8*b^2*x + 77*a^9*b*sqrt(x) - 21*a^10)/
(a^11*b^7*x^(11/2) + 7*a^12*b^6*x^5 + 21*a^13*b^5*x^(9/2) + 35*a^14*b^4*x^4
 + 35*a^15*b^3*x^(7/2) + 21*a^16*b^2*x^3 + 7*a^17*b*x^(5/2) + a^18*x^2) - 6
60*b^4*log(b*sqrt(x) + a)/a^12 + 330*b^4*log(x)/a^12
```

mupad [B] time = 1.60, size = 213, normalized size = 0.98

$$\frac{\frac{11\,b\,\sqrt{x}}{6\,a^2} - \frac{1}{2\,a} - \frac{55\,b^2\,x}{6\,a^3} + \frac{11979\,b^4\,x^2}{7\,a^5} + \frac{165\,b^3\,x^{3/2}}{2\,a^4} + \frac{15147\,b^6\,x^3}{a^7} + \frac{7359\,b^5\,x^{5/2}}{a^6} + \frac{11770\,b^8\,x^4}{a^9} + \frac{17545\,b^7\,x^{7/2}}{a^8} + \frac{660\,b^{10}\,x^5}{a^{11}} + \frac{4290\,b^9\,x^9}{a^{10}}}{a^7\,x^2 + b^7\,x^{11/2} + 7\,a\,b^6\,x^5 + 7\,a^6\,b\,x^{5/2} + 21\,a^5\,b^2\,x^3 + 35\,a^3\,b^4\,x^4 + 35\,a^4\,b^3\,x^{7/2} + 21\,a^2\,b^5\,x^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^(1/2))^8),x)
```

```
[Out] ((11*b*x^(1/2))/(6*a^2) - 1/(2*a) - (55*b^2*x)/(6*a^3) + (11979*b^4*x^2)/(7
*a^5) + (165*b^3*x^(3/2))/(2*a^4) + (15147*b^6*x^3)/a^7 + (7359*b^5*x^(5/2)
)/a^6 + (11770*b^8*x^4)/a^9 + (17545*b^7*x^(7/2))/a^8 + (660*b^10*x^5)/a^11
 + (4290*b^9*x^(9/2))/a^10)/(a^7*x^2 + b^7*x^(11/2) + 7*a*b^6*x^5 + 7*a^6*b
*x^(5/2) + 21*a^5*b^2*x^3 + 35*a^3*b^4*x^4 + 35*a^4*b^3*x^(7/2) + 21*a^2*b^
5*x^(9/2)) - (1320*b^4*atanh((2*b*x^(1/2))/a + 1))/a^12
```

sympy [A] time = 30.48, size = 3077, normalized size = 14.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*x**(1/2))**8,x)
```

```
[Out] Piecewise((zoo/x**6, Eq(a, 0) & Eq(b, 0)), (-1/(2*a**8*x**2), Eq(b, 0)), (-
1/(6*b**8*x**6), Eq(a, 0)), (-21*a**11*sqrt(x)/(42*a**19*x**(5/2) + 294*a**
18*b*x**3 + 882*a**17*b**2*x**(7/2) + 1470*a**16*b**3*x**4 + 1470*a**15*b**
4*x**(9/2) + 882*a**14*b**5*x**5 + 294*a**13*b**6*x**(11/2) + 42*a**12*b**7
*x**6) + 77*a**10*b*x/(42*a**19*x**(5/2) + 294*a**18*b*x**3 + 882*a**17*b**
2*x**(7/2) + 1470*a**16*b**3*x**4 + 1470*a**15*b**4*x**(9/2) + 882*a**14*b*
*5*x**5 + 294*a**13*b**6*x**(11/2) + 42*a**12*b**7*x**6) - 385*a**9*b**2*x*
*(3/2)/(42*a**19*x**(5/2) + 294*a**18*b*x**3 + 882*a**17*b**2*x**(7/2) + 14
70*a**16*b**3*x**4 + 1470*a**15*b**4*x**(9/2) + 882*a**14*b**5*x**5 + 294*a
**13*b**6*x**(11/2) + 42*a**12*b**7*x**6) + 3465*a**8*b**3*x**2/(42*a**19*x
**(5/2) + 294*a**18*b*x**3 + 882*a**17*b**2*x**(7/2) + 1470*a**16*b**3*x**4
 + 1470*a**15*b**4*x**(9/2) + 882*a**14*b**5*x**5 + 294*a**13*b**6*x**(11/2
) + 42*a**12*b**7*x**6) + 13860*a**7*b**4*x**(5/2)*log(x)/(42*a**19*x**(5/2
) + 294*a**18*b*x**3 + 882*a**17*b**2*x**(7/2) + 1470*a**16*b**3*x**4 + 147
```

```
0*a**15*b**4*x**(9/2) + 882*a**14*b**5*x**5 + 294*a**13*b**6*x**(11/2) + 42
*a**12*b**7*x**6) - 27720*a**7*b**4*x**(5/2)*log(a/b + sqrt(x))/(42*a**19*x
**(5/2) + 294*a**18*b*x**3 + 882*a**17*b**2*x**(7/2) + 1470*a**16*b**3*x**4
 + 1470*a**15*b**4*x**(9/2) + 882*a**14*b**5*x**5 + 294*a**13*b**6*x**(11/2
) + 42*a**12*b**7*x**6) + 71874*a**7*b**4*x**(5/2)/(42*a**19*x**(5/2) + 294
*a**18*b*x**3 + 882*a**17*b**2*x**(7/2) + 1470*a**16*b**3*x**4 + 1470*a**15
*b**4*x**(9/2) + 882*a**14*b**5*x**5 + 294*a**13*b**6*x**(11/2) + 42*a**12*
b**7*x**6) + 97020*a**6*b**5*x**3*log(x)/(42*a**19*x**(5/2) + 294*a**18*b*x
**3 + 882*a**17*b**2*x**(7/2) + 1470*a**16*b**3*x**4 + 1470*a**15*b**4*x**(
9/2) + 882*a**14*b**5*x**5 + 294*a**13*b**6*x**(11/2) + 42*a**12*b**7*x**6)
 - 194040*a**6*b**5*x**3*log(a/b + sqrt(x))/(42*a**19*x**(5/2) + 294*a**18*
b*x**3 + 882*a**17*b**2*x**(7/2) + 1470*a**16*b**3*x**4 + 1470*a**15*b**4*x
**(9/2) + 882*a**14*b**5*x**5 + 294*a**13*b**6*x**(11/2) + 42*a**12*b**7*x*
*6) + 309078*a**6*b**5*x**3/(42*a**19*x**(5/2) + 294*a**18*b*x**3 + 882*a**
17*b**2*x**(7/2) + 1470*a**16*b**3*x**4 + 1470*a**15*b**4*x**(9/2) + 882*a*
*14*b**5*x**5 + 294*a**13*b**6*x**(11/2) + 42*a**12*b**7*x**6) + 291060*a**
5*b**6*x**(7/2)*log(x)/(42*a**19*x**(5/2) + 294*a**18*b*x**3 + 882*a**17*b*
*2*x**(7/2) + 1470*a**16*b**3*x**4 + 1470*a**15*b**4*x**(9/2) + 882*a**14*b
**5*x**5 + 294*a**13*b**6*x**(11/2) + 42*a**12*b**7*x**6) - 582120*a**5*b**
6*x**(7/2)*log(a/b + sqrt(x))/(42*a**19*x**(5/2) + 294*a**18*b*x**3 + 882*a
**17*b**2*x**(7/2) + 1470*a**16*b**3*x**4 + 1470*a**15*b**4*x**(9/2) + 882*
a**14*b**5*x**5 + 294*a**13*b**6*x**(11/2) + 42*a**12*b**7*x**6) + 636174*a
**5*b**6*x**(7/2)/(42*a**19*x**(5/2) + 294*a**18*b*x**3 + 882*a**17*b**2*x*
*(7/2) + 1470*a**16*b**3*x**4 + 1470*a**15*b**4*x**(9/2) + 882*a**14*b**5*x
**5 + 294*a**13*b**6*x**(11/2) + 42*a**12*b**7*x**6) + 485100*a**4*b**7*x**
4*log(x)/(42*a**19*x**(5/2) + 294*a**18*b*x**3 + 882*a**17*b**2*x**(7/2) +
1470*a**16*b**3*x**4 + 1470*a**15*b**4*x**(9/2) + 882*a**14*b**5*x**5 + 294
*a**13*b**6*x**(11/2) + 42*a**12*b**7*x**6) - 970200*a**4*b**7*x**4*log(a/b
 + sqrt(x))/(42*a**19*x**(5/2) + 294*a**18*b*x**3 + 882*a**17*b**2*x**(7/2)
 + 1470*a**16*b**3*x**4 + 1470*a**15*b**4*x**(9/2) + 882*a**14*b**5*x**5 +
294*a**13*b**6*x**(11/2) + 42*a**12*b**7*x**6) + 736890*a**4*b**7*x**4/(42*
a**19*x**(5/2) + 294*a**18*b*x**3 + 882*a**17*b**2*x**(7/2) + 1470*a**16*b*
*3*x**4 + 1470*a**15*b**4*x**(9/2) + 882*a**14*b**5*x**5 + 294*a**13*b**6*x
**(11/2) + 42*a**12*b**7*x**6) + 485100*a**3*b**8*x**(9/2)*log(x)/(42*a**19
*x**(5/2) + 294*a**18*b*x**3 + 882*a**17*b**2*x**(7/2) + 1470*a**16*b**3*x*
*4 + 1470*a**15*b**4*x**(9/2) + 882*a**14*b**5*x**5 + 294*a**13*b**6*x**(11
/2) + 42*a**12*b**7*x**6) - 970200*a**3*b**8*x**(9/2)*log(a/b + sqrt(x))/(4
2*a**19*x**(5/2) + 294*a**18*b*x**3 + 882*a**17*b**2*x**(7/2) + 1470*a**16*
b**3*x**4 + 1470*a**15*b**4*x**(9/2) + 882*a**14*b**5*x**5 + 294*a**13*b**6
*x**(11/2) + 42*a**12*b**7*x**6) + 494340*a**3*b**8*x**(9/2)/(42*a**19*x**(
5/2) + 294*a**18*b*x**3 + 882*a**17*b**2*x**(7/2) + 1470*a**16*b**3*x**4 +
1470*a**15*b**4*x**(9/2) + 882*a**14*b**5*x**5 + 294*a**13*b**6*x**(11/2) +
 42*a**12*b**7*x**6) + 291060*a**2*b**9*x**5*log(x)/(42*a**19*x**(5/2) + 29
4*a**18*b*x**3 + 882*a**17*b**2*x**(7/2) + 1470*a**16*b**3*x**4 + 1470*a**1
5*b**4*x**(9/2) + 882*a**14*b**5*x**5 + 294*a**13*b**6*x**(11/2) + 42*a**12
*b**7*x**6) - 582120*a**2*b**9*x**5*log(a/b + sqrt(x))/(42*a**19*x**(5/2) +
 294*a**18*b*x**3 + 882*a**17*b**2*x**(7/2) + 1470*a**16*b**3*x**4 + 1470*a
**15*b**4*x**(9/2) + 882*a**14*b**5*x**5 + 294*a**13*b**6*x**(11/2) + 42*a*
*12*b**7*x**6) + 180180*a**2*b**9*x**5/(42*a**19*x**(5/2) + 294*a**18*b*x**
3 + 882*a**17*b**2*x**(7/2) + 1470*a**16*b**3*x**4 + 1470*a**15*b**4*x**(9/
2) + 882*a**14*b**5*x**5 + 294*a**13*b**6*x**(11/2) + 42*a**12*b**7*x**6) +
 97020*a*b**10*x**(11/2)*log(x)/(42*a**19*x**(5/2) + 294*a**18*b*x**3 + 882
*a**17*b**2*x**(7/2) + 1470*a**16*b**3*x**4 + 1470*a**15*b**4*x**(9/2) + 88
2*a**14*b**5*x**5 + 294*a**13*b**6*x**(11/2) + 42*a**12*b**7*x**6) - 194040
*a*b**10*x**(11/2)*log(a/b + sqrt(x))/(42*a**19*x**(5/2) + 294*a**18*b*x**3
 + 882*a**17*b**2*x**(7/2) + 1470*a**16*b**3*x**4 + 1470*a**15*b**4*x**(9/2
) + 882*a**14*b**5*x**5 + 294*a**13*b**6*x**(11/2) + 42*a**12*b**7*x**6) +
27720*a*b**10*x**(11/2)/(42*a**19*x**(5/2) + 294*a**18*b*x**3 + 882*a**17*b
**2*x**(7/2) + 1470*a**16*b**3*x**4 + 1470*a**15*b**4*x**(9/2) + 882*a**14*
```

```
b**5*x**5 + 294*a**13*b**6*x**(11/2) + 42*a**12*b**7*x**6) + 13860*b**11*x*
*6*log(x)/(42*a**19*x**(5/2) + 294*a**18*b*x**3 + 882*a**17*b**2*x**(7/2) +
 1470*a**16*b**3*x**4 + 1470*a**15*b**4*x**(9/2) + 882*a**14*b**5*x**5 + 29
4*a**13*b**6*x**(11/2) + 42*a**12*b**7*x**6) - 27720*b**11*x**6*log(a/b + s
qrt(x))/(42*a**19*x**(5/2) + 294*a**18*b*x**3 + 882*a**17*b**2*x**(7/2) + 1
470*a**16*b**3*x**4 + 1470*a**15*b**4*x**(9/2) + 882*a**14*b**5*x**5 + 294*
a**13*b**6*x**(11/2) + 42*a**12*b**7*x**6), True))
```

3.2232 $\int \frac{1}{\left(2+b\sqrt{x}\right)x}\,dx$

Optimal. Leaf size=19

$$\frac{\log(x)}{2}-\log\left(b\sqrt{x}+2\right)$$

```
[Out] 1/2*ln(x)-ln(2+b*x^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 36, 29, 31}

$$\frac{\log(x)}{2}-\log\left(b\sqrt{x}+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + b*Sqrt[x])*x),x]
```

```
[Out] -Log[2 + b*Sqrt[x]] + Log[x]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(2+b\sqrt{x}\right)x}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{x(2+bx)}\,dx, x, \sqrt{x}\right)\\ &= -\left(b\,\text{Subst}\left(\int \frac{1}{2+bx}\,dx, x, \sqrt{x}\right)\right)+\text{Subst}\left(\int \frac{1}{x}\,dx, x, \sqrt{x}\right)\\ &= -\log\left(2+b\sqrt{x}\right)+\frac{\log(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{\log(x)}{2}-\log\left(b\sqrt{x}+2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2 + b*Sqrt[x])*x),x]
[Out] -Log[2 + b*Sqrt[x]] + Log[x]/2
```

fricas [A] time = 1.14, size = 15, normalized size = 0.79

$$-\log\left(b\sqrt{x}+2\right)+\log\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2+b*x^(1/2)),x, algorithm="fricas")
[Out] -log(b*sqrt(x) + 2) + log(sqrt(x))
```

giac [A] time = 0.17, size = 17, normalized size = 0.89

$$-\log\left(\left|b\sqrt{x}+2\right|\right)+\frac{1}{2}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2+b*x^(1/2)),x, algorithm="giac")
[Out] -log(abs(b*sqrt(x) + 2)) + 1/2*log(abs(x))
```

maple [A] time = 0.00, size = 16, normalized size = 0.84

$$\frac{\ln(x)}{2}-\ln\left(b\sqrt{x}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(2+b*x^(1/2)),x)
[Out] 1/2*ln(x)-ln(2+b*x^(1/2))
```

maxima [A] time = 0.89, size = 15, normalized size = 0.79

$$-\log\left(b\sqrt{x}+2\right)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2+b*x^(1/2)),x, algorithm="maxima")
[Out] -log(b*sqrt(x) + 2) + 1/2*log(x)
```

mupad [B] time = 0.05, size = 10, normalized size = 0.53

$$-2\,\operatorname{atanh}\left(b\,\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^(1/2) + 2)),x)
[Out] -2*atanh(b*x^(1/2) + 1)
```

sympy [A] time = 0.47, size = 19, normalized size = 1.00

$$\begin{cases}\frac{\log(x)}{2}-\log\left(\sqrt{x}+\frac{2}{b}\right) & \text{for } b\neq 0\\ \frac{\log(x)}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2+b*x**(1/2)),x)
[Out] Piecewise((log(x)/2 - log(sqrt(x) + 2/b), Ne(b, 0)), (log(x)/2, True))
```

3.2233 $\int \sqrt{a+b\sqrt{x}}\, x^2\, dx$

Optimal. Leaf size=132

$$-\frac{4a^5\left(a+b\sqrt{x}\right)^{3/2}}{3b^6}+\frac{4a^4\left(a+b\sqrt{x}\right)^{5/2}}{b^6}-\frac{40a^3\left(a+b\sqrt{x}\right)^{7/2}}{7b^6}+\frac{40a^2\left(a+b\sqrt{x}\right)^{9/2}}{9b^6}+\frac{4\left(a+b\sqrt{x}\right)^{13/2}}{13b^6}-\frac{20a\left(a+b\sqrt{}\right.}{11b^6}$$

```
[Out] -4/3*a^5*(a+b*x^(1/2))^(3/2)/b^6+4*a^4*(a+b*x^(1/2))^(5/2)/b^6-40/7*a^3*(a+
b*x^(1/2))^(7/2)/b^6+40/9*a^2*(a+b*x^(1/2))^(9/2)/b^6-20/11*a*(a+b*x^(1/2))
^(11/2)/b^6+4/13*(a+b*x^(1/2))^(13/2)/b^6
```

Rubi [A] time = 0.06, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$\frac{40a^2\left(a+b\sqrt{x}\right)^{9/2}}{9b^6}-\frac{40a^3\left(a+b\sqrt{x}\right)^{7/2}}{7b^6}+\frac{4a^4\left(a+b\sqrt{x}\right)^{5/2}}{b^6}-\frac{4a^5\left(a+b\sqrt{x}\right)^{3/2}}{3b^6}+\frac{4\left(a+b\sqrt{x}\right)^{13/2}}{13b^6}-\frac{20a\left(a+b\sqrt{x}\right.}{11b^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[x]]*x^2,x]
```

```
[Out] (-4*a^5*(a + b*Sqrt[x])^(3/2))/(3*b^6) + (4*a^4*(a + b*Sqrt[x])^(5/2))/b^6
- (40*a^3*(a + b*Sqrt[x])^(7/2))/(7*b^6) + (40*a^2*(a + b*Sqrt[x])^(9/2))/(
9*b^6) - (20*a*(a + b*Sqrt[x])^(11/2))/(11*b^6) + (4*(a + b*Sqrt[x])^(13/2)
)/(13*b^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\sqrt{x}}\, x^2\, dx &= 2\,\text{Subst}\left(\int x^5\sqrt{a+bx}\, dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int\left(-\frac{a^5\sqrt{a+bx}}{b^5}+\frac{5a^4(a+bx)^{3/2}}{b^5}-\frac{10a^3(a+bx)^{5/2}}{b^5}+\frac{10a^2(a+bx)^{7/2}}{b^5}-\frac{5a(a\right.\right.\\ &= -\frac{4a^5\left(a+b\sqrt{x}\right)^{3/2}}{3b^6}+\frac{4a^4\left(a+b\sqrt{x}\right)^{5/2}}{b^6}-\frac{40a^3\left(a+b\sqrt{x}\right)^{7/2}}{7b^6}+\frac{40a^2\left(a+b\sqrt{x}\right)^{9/2}}{9b^6}-\frac{20}{}\end{aligned}$$

Mathematica [A] time = 0.07, size = 78, normalized size = 0.59

$$\frac{4\left(a+b\sqrt{x}\right)^{3/2}\left(-256a^5+384a^4b\sqrt{x}-480a^3b^2x+560a^2b^3x^{3/2}-630ab^4x^2+693b^5x^{5/2}\right)}{9009b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[x]]*x^2,x]
```

```
[Out] (4*(a + b*Sqrt[x])^(3/2)*(-256*a^5 + 384*a^4*b*Sqrt[x] - 480*a^3*b^2*x + 56
0*a^2*b^3*x^(3/2) - 630*a*b^4*x^2 + 693*b^5*x^(5/2)))/(9009*b^6)
```

fricas [A] time = 1.02, size = 77, normalized size = 0.58

$$\frac{4\left(693\,b^6x^3-70\,a^2b^4x^2-96\,a^4b^2x-256\,a^6+\left(63\,ab^5x^2+80\,a^3b^3x+128\,a^5b\right)\sqrt{x}\right)\sqrt{b\sqrt{x}+a}}{9009\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 4/9009*(693*b^6*x^3 - 70*a^2*b^4*x^2 - 96*a^4*b^2*x - 256*a^6 + (63*a*b^5*x
^2 + 80*a^3*b^3*x + 128*a^5*b)*sqrt(x))*sqrt(b*sqrt(x) + a)/b^6
```

giac [A] time = 0.16, size = 191, normalized size = 1.45

$$4\left(\frac{13\left(63\left(b\sqrt{x}+a\right)^{\frac{11}{2}}-385\left(b\sqrt{x}+a\right)^{\frac{9}{2}}a+990\left(b\sqrt{x}+a\right)^{\frac{7}{2}}a^2-1386\left(b\sqrt{x}+a\right)^{\frac{5}{2}}a^3+1155\left(b\sqrt{x}+a\right)^{\frac{3}{2}}a^4-693\sqrt{b\sqrt{x}+a}\,a^5\right)a}{b^5}+\frac{3\left(231\left(b\sqrt{x}+a\right)^{\frac{13}{2}}-1638\right.}{}\right.$$

$$\left.\vphantom{\frac{1}{1}}\right)$$

$$9009\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 4/9009*(13*(63*(b*sqrt(x) + a)^(11/2) - 385*(b*sqrt(x) + a)^(9/2)*a + 990*(
b*sqrt(x) + a)^(7/2)*a^2 - 1386*(b*sqrt(x) + a)^(5/2)*a^3 + 1155*(b*sqrt(x)
 + a)^(3/2)*a^4 - 693*sqrt(b*sqrt(x) + a)*a^5)*a/b^5 + 3*(231*(b*sqrt(x) +
a)^(13/2) - 1638*(b*sqrt(x) + a)^(11/2)*a + 5005*(b*sqrt(x) + a)^(9/2)*a^2
- 8580*(b*sqrt(x) + a)^(7/2)*a^3 + 9009*(b*sqrt(x) + a)^(5/2)*a^4 - 6006*(b
*sqrt(x) + a)^(3/2)*a^5 + 3003*sqrt(b*sqrt(x) + a)*a^6)/b^5)/b
```

maple [A] time = 0.00, size = 85, normalized size = 0.64

$$\frac{-\frac{4\left(b\sqrt{x}+a\right)^{\frac{3}{2}}a^5}{3}+4\left(b\sqrt{x}+a\right)^{\frac{5}{2}}a^4-\frac{40\left(b\sqrt{x}+a\right)^{\frac{7}{2}}a^3}{7}+\frac{40\left(b\sqrt{x}+a\right)^{\frac{9}{2}}a^2}{9}-\frac{20\left(b\sqrt{x}+a\right)^{\frac{11}{2}}a}{11}+\frac{4\left(b\sqrt{x}+a\right)^{\frac{13}{2}}}{13}}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^(1/2)+a)^(1/2),x)
```

```
[Out] 4/b^6*(1/13*(b*x^(1/2)+a)^(13/2)-5/11*a*(b*x^(1/2)+a)^(11/2)+10/9*a^2*(b*x^
(1/2)+a)^(9/2)-10/7*a^3*(b*x^(1/2)+a)^(7/2)+a^4*(b*x^(1/2)+a)^(5/2)-1/3*(b*
x^(1/2)+a)^(3/2)*a^5)
```

maxima [A] time = 0.80, size = 98, normalized size = 0.74

$$\frac{4\left(b\sqrt{x}+a\right)^{\frac{13}{2}}}{13\,b^6}-\frac{20\left(b\sqrt{x}+a\right)^{\frac{11}{2}}a}{11\,b^6}+\frac{40\left(b\sqrt{x}+a\right)^{\frac{9}{2}}a^2}{9\,b^6}-\frac{40\left(b\sqrt{x}+a\right)^{\frac{7}{2}}a^3}{7\,b^6}+\frac{4\left(b\sqrt{x}+a\right)^{\frac{5}{2}}a^4}{b^6}-\frac{4\left(b\sqrt{x}+a\right)^{\frac{3}{2}}a^5}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/13*(b*sqrt(x) + a)^(13/2)/b^6 - 20/11*(b*sqrt(x) + a)^(11/2)*a/b^6 + 40/9
*(b*sqrt(x) + a)^(9/2)*a^2/b^6 - 40/7*(b*sqrt(x) + a)^(7/2)*a^3/b^6 + 4*(b*
sqrt(x) + a)^(5/2)*a^4/b^6 - 4/3*(b*sqrt(x) + a)^(3/2)*a^5/b^6
```

mupad [B] time = 1.83, size = 98, normalized size = 0.74

$$\frac{4\left(a+b\sqrt{x}\right)^{13/2}}{13\,b^6}-\frac{20\,a\left(a+b\sqrt{x}\right)^{11/2}}{11\,b^6}-\frac{4\,a^5\left(a+b\sqrt{x}\right)^{3/2}}{3\,b^6}+\frac{4\,a^4\left(a+b\sqrt{x}\right)^{5/2}}{b^6}-\frac{40\,a^3\left(a+b\sqrt{x}\right)^{7/2}}{7\,b^6}+\frac{40\,a^2\left(a+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^(1/2))^(1/2),x)
```

```
[Out] (4*(a + b*x^(1/2))^(13/2))/(13*b^6) - (20*a*(a + b*x^(1/2))^(11/2))/(11*b^6
) - (4*a^5*(a + b*x^(1/2))^(3/2))/(3*b^6) + (4*a^4*(a + b*x^(1/2))^(5/2))/b
^6 - (40*a^3*(a + b*x^(1/2))^(7/2))/(7*b^6) + (40*a^2*(a + b*x^(1/2))^(9/2)
)/(9*b^6)
```

sympy [B] time = 10.05, size = 8588, normalized size = 65.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*x**(1/2))**(1/2),x)
```

```
[Out] -1024*a**(153/2)*x**18*sqrt(1 + b*sqrt(x)/a)/(9009*a**70*b**6*x**18 + 13513
5*a**69*b**7*x**(37/2) + 945945*a**68*b**8*x**19 + 4099095*a**67*b**9*x**(3
9/2) + 12297285*a**66*b**10*x**20 + 27054027*a**65*b**11*x**(41/2) + 450900
45*a**64*b**12*x**21 + 57972915*a**63*b**13*x**(43/2) + 57972915*a**62*b**1
4*x**22 + 45090045*a**61*b**15*x**(45/2) + 27054027*a**60*b**16*x**23 + 122
97285*a**59*b**17*x**(47/2) + 4099095*a**58*b**18*x**24 + 945945*a**57*b**1
9*x**(49/2) + 135135*a**56*b**20*x**25 + 9009*a**55*b**21*x**(51/2)) + 1024
*a**(153/2)*x**18/(9009*a**70*b**6*x**18 + 135135*a**69*b**7*x**(37/2) + 94
5945*a**68*b**8*x**19 + 4099095*a**67*b**9*x**(39/2) + 12297285*a**66*b**10
*x**20 + 27054027*a**65*b**11*x**(41/2) + 45090045*a**64*b**12*x**21 + 5797
2915*a**63*b**13*x**(43/2) + 57972915*a**62*b**14*x**22 + 45090045*a**61*b*
*15*x**(45/2) + 27054027*a**60*b**16*x**23 + 12297285*a**59*b**17*x**(47/2)
 + 4099095*a**58*b**18*x**24 + 945945*a**57*b**19*x**(49/2) + 135135*a**56*
b**20*x**25 + 9009*a**55*b**21*x**(51/2)) - 14848*a**(151/2)*b*x**(37/2)*sq
rt(1 + b*sqrt(x)/a)/(9009*a**70*b**6*x**18 + 135135*a**69*b**7*x**(37/2) +
945945*a**68*b**8*x**19 + 4099095*a**67*b**9*x**(39/2) + 12297285*a**66*b**
10*x**20 + 27054027*a**65*b**11*x**(41/2) + 45090045*a**64*b**12*x**21 + 57
972915*a**63*b**13*x**(43/2) + 57972915*a**62*b**14*x**22 + 45090045*a**61*
b**15*x**(45/2) + 27054027*a**60*b**16*x**23 + 12297285*a**59*b**17*x**(47/
2) + 4099095*a**58*b**18*x**24 + 945945*a**57*b**19*x**(49/2) + 135135*a**5
6*b**20*x**25 + 9009*a**55*b**21*x**(51/2)) + 15360*a**(151/2)*b*x**(37/2)/
(9009*a**70*b**6*x**18 + 135135*a**69*b**7*x**(37/2) + 945945*a**68*b**8*x*
*19 + 4099095*a**67*b**9*x**(39/2) + 12297285*a**66*b**10*x**20 + 27054027*
a**65*b**11*x**(41/2) + 45090045*a**64*b**12*x**21 + 57972915*a**63*b**13*x
**(43/2) + 57972915*a**62*b**14*x**22 + 45090045*a**61*b**15*x**(45/2) + 27
054027*a**60*b**16*x**23 + 12297285*a**59*b**17*x**(47/2) + 4099095*a**58*b
**18*x**24 + 945945*a**57*b**19*x**(49/2) + 135135*a**56*b**20*x**25 + 9009
*a**55*b**21*x**(51/2)) - 100224*a**(149/2)*b**2*x**19*sqrt(1 + b*sqrt(x)/a
)/(9009*a**70*b**6*x**18 + 135135*a**69*b**7*x**(37/2) + 945945*a**68*b**8*
x**19 + 4099095*a**67*b**9*x**(39/2) + 12297285*a**66*b**10*x**20 + 2705402
7*a**65*b**11*x**(41/2) + 45090045*a**64*b**12*x**21 + 57972915*a**63*b**13
*x**(43/2) + 57972915*a**62*b**14*x**22 + 45090045*a**61*b**15*x**(45/2) +
27054027*a**60*b**16*x**23 + 12297285*a**59*b**17*x**(47/2) + 4099095*a**58
*b**18*x**24 + 945945*a**57*b**19*x**(49/2) + 135135*a**56*b**20*x**25 + 90
09*a**55*b**21*x**(51/2)) + 107520*a**(149/2)*b**2*x**19/(9009*a**70*b**6*x
**18 + 135135*a**69*b**7*x**(37/2) + 945945*a**68*b**8*x**19 + 4099095*a**6
7*b**9*x**(39/2) + 12297285*a**66*b**10*x**20 + 27054027*a**65*b**11*x**(41
/2) + 45090045*a**64*b**12*x**21 + 57972915*a**63*b**13*x**(43/2) + 5797291
```

```
5*a**62*b**14*x**22 + 45090045*a**61*b**15*x**(45/2) + 27054027*a**60*b**16
*x**23 + 12297285*a**59*b**17*x**(47/2) + 4099095*a**58*b**18*x**24 + 94594
5*a**57*b**19*x**(49/2) + 135135*a**56*b**20*x**25 + 9009*a**55*b**21*x**(5
1/2)) - 417600*a**(147/2)*b**3*x**(39/2)*sqrt(1 + b*sqrt(x)/a)/(9009*a**70*
b**6*x**18 + 135135*a**69*b**7*x**(37/2) + 945945*a**68*b**8*x**19 + 409909
5*a**67*b**9*x**(39/2) + 12297285*a**66*b**10*x**20 + 27054027*a**65*b**11*
x**(41/2) + 45090045*a**64*b**12*x**21 + 57972915*a**63*b**13*x**(43/2) + 5
7972915*a**62*b**14*x**22 + 45090045*a**61*b**15*x**(45/2) + 27054027*a**60
*b**16*x**23 + 12297285*a**59*b**17*x**(47/2) + 4099095*a**58*b**18*x**24 +
 945945*a**57*b**19*x**(49/2) + 135135*a**56*b**20*x**25 + 9009*a**55*b**21
*x**(51/2)) + 465920*a**(147/2)*b**3*x**(39/2)/(9009*a**70*b**6*x**18 + 135
135*a**69*b**7*x**(37/2) + 945945*a**68*b**8*x**19 + 4099095*a**67*b**9*x**
(39/2) + 12297285*a**66*b**10*x**20 + 27054027*a**65*b**11*x**(41/2) + 4509
0045*a**64*b**12*x**21 + 57972915*a**63*b**13*x**(43/2) + 57972915*a**62*b*
*14*x**22 + 45090045*a**61*b**15*x**(45/2) + 27054027*a**60*b**16*x**23 + 1
2297285*a**59*b**17*x**(47/2) + 4099095*a**58*b**18*x**24 + 945945*a**57*b*
*19*x**(49/2) + 135135*a**56*b**20*x**25 + 9009*a**55*b**21*x**(51/2)) - 12
00600*a**(145/2)*b**4*x**20*sqrt(1 + b*sqrt(x)/a)/(9009*a**70*b**6*x**18 +
135135*a**69*b**7*x**(37/2) + 945945*a**68*b**8*x**19 + 4099095*a**67*b**9*
x**(39/2) + 12297285*a**66*b**10*x**20 + 27054027*a**65*b**11*x**(41/2) + 4
5090045*a**64*b**12*x**21 + 57972915*a**63*b**13*x**(43/2) + 57972915*a**62
*b**14*x**22 + 45090045*a**61*b**15*x**(45/2) + 27054027*a**60*b**16*x**23
+ 12297285*a**59*b**17*x**(47/2) + 4099095*a**58*b**18*x**24 + 945945*a**57
*b**19*x**(49/2) + 135135*a**56*b**20*x**25 + 9009*a**55*b**21*x**(51/2)) +
 1397760*a**(145/2)*b**4*x**20/(9009*a**70*b**6*x**18 + 135135*a**69*b**7*x
**(37/2) + 945945*a**68*b**8*x**19 + 4099095*a**67*b**9*x**(39/2) + 1229728
5*a**66*b**10*x**20 + 27054027*a**65*b**11*x**(41/2) + 45090045*a**64*b**12
*x**21 + 57972915*a**63*b**13*x**(43/2) + 57972915*a**62*b**14*x**22 + 4509
0045*a**61*b**15*x**(45/2) + 27054027*a**60*b**16*x**23 + 12297285*a**59*b*
*17*x**(47/2) + 4099095*a**58*b**18*x**24 + 945945*a**57*b**19*x**(49/2) +
135135*a**56*b**20*x**25 + 9009*a**55*b**21*x**(51/2)) - 2521260*a**(143/2)
*b**5*x**(41/2)*sqrt(1 + b*sqrt(x)/a)/(9009*a**70*b**6*x**18 + 135135*a**69
*b**7*x**(37/2) + 945945*a**68*b**8*x**19 + 4099095*a**67*b**9*x**(39/2) +
12297285*a**66*b**10*x**20 + 27054027*a**65*b**11*x**(41/2) + 45090045*a**6
4*b**12*x**21 + 57972915*a**63*b**13*x**(43/2) + 57972915*a**62*b**14*x**22
 + 45090045*a**61*b**15*x**(45/2) + 27054027*a**60*b**16*x**23 + 12297285*a
**59*b**17*x**(47/2) + 4099095*a**58*b**18*x**24 + 945945*a**57*b**19*x**(4
9/2) + 135135*a**56*b**20*x**25 + 9009*a**55*b**21*x**(51/2)) + 3075072*a**
(143/2)*b**5*x**(41/2)/(9009*a**70*b**6*x**18 + 135135*a**69*b**7*x**(37/2)
 + 945945*a**68*b**8*x**19 + 4099095*a**67*b**9*x**(39/2) + 12297285*a**66*
b**10*x**20 + 27054027*a**65*b**11*x**(41/2) + 45090045*a**64*b**12*x**21 +
 57972915*a**63*b**13*x**(43/2) + 57972915*a**62*b**14*x**22 + 45090045*a**
61*b**15*x**(45/2) + 27054027*a**60*b**16*x**23 + 12297285*a**59*b**17*x**(
47/2) + 4099095*a**58*b**18*x**24 + 945945*a**57*b**19*x**(49/2) + 135135*a
**56*b**20*x**25 + 9009*a**55*b**21*x**(51/2)) - 3988992*a**(141/2)*b**6*x*
*21*sqrt(1 + b*sqrt(x)/a)/(9009*a**70*b**6*x**18 + 135135*a**69*b**7*x**(37
/2) + 945945*a**68*b**8*x**19 + 4099095*a**67*b**9*x**(39/2) + 12297285*a**
66*b**10*x**20 + 27054027*a**65*b**11*x**(41/2) + 45090045*a**64*b**12*x**2
1 + 57972915*a**63*b**13*x**(43/2) + 57972915*a**62*b**14*x**22 + 45090045*
a**61*b**15*x**(45/2) + 27054027*a**60*b**16*x**23 + 12297285*a**59*b**17*x
**(47/2) + 4099095*a**58*b**18*x**24 + 945945*a**57*b**19*x**(49/2) + 13513
5*a**56*b**20*x**25 + 9009*a**55*b**21*x**(51/2)) + 5125120*a**(141/2)*b**6
*x**21/(9009*a**70*b**6*x**18 + 135135*a**69*b**7*x**(37/2) + 945945*a**68*
b**8*x**19 + 4099095*a**67*b**9*x**(39/2) + 12297285*a**66*b**10*x**20 + 27
054027*a**65*b**11*x**(41/2) + 45090045*a**64*b**12*x**21 + 57972915*a**63*
b**13*x**(43/2) + 57972915*a**62*b**14*x**22 + 45090045*a**61*b**15*x**(45/
2) + 27054027*a**60*b**16*x**23 + 12297285*a**59*b**17*x**(47/2) + 4099095*
a**58*b**18*x**24 + 945945*a**57*b**19*x**(49/2) + 135135*a**56*b**20*x**25
 + 9009*a**55*b**21*x**(51/2)) - 4802592*a**(139/2)*b**7*x**(43/2)*sqrt(1 +
```

```
 b*sqrt(x)/a)/(9009*a**70*b**6*x**18 + 135135*a**69*b**7*x**(37/2) + 945945
*a**68*b**8*x**19 + 4099095*a**67*b**9*x**(39/2) + 12297285*a**66*b**10*x**
20 + 27054027*a**65*b**11*x**(41/2) + 45090045*a**64*b**12*x**21 + 57972915
*a**63*b**13*x**(43/2) + 57972915*a**62*b**14*x**22 + 45090045*a**61*b**15*
x**(45/2) + 27054027*a**60*b**16*x**23 + 12297285*a**59*b**17*x**(47/2) + 4
099095*a**58*b**18*x**24 + 945945*a**57*b**19*x**(49/2) + 135135*a**56*b**2
0*x**25 + 9009*a**55*b**21*x**(51/2)) + 6589440*a**(139/2)*b**7*x**(43/2)/(
9009*a**70*b**6*x**18 + 135135*a**69*b**7*x**(37/2) + 945945*a**68*b**8*x**
19 + 4099095*a**67*b**9*x**(39/2) + 12297285*a**66*b**10*x**20 + 27054027*a
**65*b**11*x**(41/2) + 45090045*a**64*b**12*x**21 + 57972915*a**63*b**13*x*
*(43/2) + 57972915*a**62*b**14*x**22 + 45090045*a**61*b**15*x**(45/2) + 270
54027*a**60*b**16*x**23 + 12297285*a**59*b**17*x**(47/2) + 4099095*a**58*b*
*18*x**24 + 945945*a**57*b**19*x**(49/2) + 135135*a**56*b**20*x**25 + 9009*
a**55*b**21*x**(51/2)) - 4232160*a**(137/2)*b**8*x**22*sqrt(1 + b*sqrt(x)/a
)/(9009*a**70*b**6*x**18 + 135135*a**69*b**7*x**(37/2) + 945945*a**68*b**8*
x**19 + 4099095*a**67*b**9*x**(39/2) + 12297285*a**66*b**10*x**20 + 2705402
7*a**65*b**11*x**(41/2) + 45090045*a**64*b**12*x**21 + 57972915*a**63*b**13
*x**(43/2) + 57972915*a**62*b**14*x**22 + 45090045*a**61*b**15*x**(45/2) +
27054027*a**60*b**16*x**23 + 12297285*a**59*b**17*x**(47/2) + 4099095*a**58
*b**18*x**24 + 945945*a**57*b**19*x**(49/2) + 135135*a**56*b**20*x**25 + 90
09*a**55*b**21*x**(51/2)) + 6589440*a**(137/2)*b**8*x**22/(9009*a**70*b**6*
x**18 + 135135*a**69*b**7*x**(37/2) + 945945*a**68*b**8*x**19 + 4099095*a**
67*b**9*x**(39/2) + 12297285*a**66*b**10*x**20 + 27054027*a**65*b**11*x**(4
1/2) + 45090045*a**64*b**12*x**21 + 57972915*a**63*b**13*x**(43/2) + 579729
15*a**62*b**14*x**22 + 45090045*a**61*b**15*x**(45/2) + 27054027*a**60*b**1
6*x**23 + 12297285*a**59*b**17*x**(47/2) + 4099095*a**58*b**18*x**24 + 9459
45*a**57*b**19*x**(49/2) + 135135*a**56*b**20*x**25 + 9009*a**55*b**21*x**(
51/2)) - 1935440*a**(135/2)*b**9*x**(45/2)*sqrt(1 + b*sqrt(x)/a)/(9009*a**7
0*b**6*x**18 + 135135*a**69*b**7*x**(37/2) + 945945*a**68*b**8*x**19 + 4099
095*a**67*b**9*x**(39/2) + 12297285*a**66*b**10*x**20 + 27054027*a**65*b**1
1*x**(41/2) + 45090045*a**64*b**12*x**21 + 57972915*a**63*b**13*x**(43/2) +
 57972915*a**62*b**14*x**22 + 45090045*a**61*b**15*x**(45/2) + 27054027*a**
60*b**16*x**23 + 12297285*a**59*b**17*x**(47/2) + 4099095*a**58*b**18*x**24
 + 945945*a**57*b**19*x**(49/2) + 135135*a**56*b**20*x**25 + 9009*a**55*b**
21*x**(51/2)) + 5125120*a**(135/2)*b**9*x**(45/2)/(9009*a**70*b**6*x**18 +
135135*a**69*b**7*x**(37/2) + 945945*a**68*b**8*x**19 + 4099095*a**67*b**9*
x**(39/2) + 12297285*a**66*b**10*x**20 + 27054027*a**65*b**11*x**(41/2) + 4
5090045*a**64*b**12*x**21 + 57972915*a**63*b**13*x**(43/2) + 57972915*a**62
*b**14*x**22 + 45090045*a**61*b**15*x**(45/2) + 27054027*a**60*b**16*x**23
+ 12297285*a**59*b**17*x**(47/2) + 4099095*a**58*b**18*x**24 + 945945*a**57
*b**19*x**(49/2) + 135135*a**56*b**20*x**25 + 9009*a**55*b**21*x**(51/2)) +
 2214784*a**(133/2)*b**10*x**23*sqrt(1 + b*sqrt(x)/a)/(9009*a**70*b**6*x**1
8 + 135135*a**69*b**7*x**(37/2) + 945945*a**68*b**8*x**19 + 4099095*a**67*b
**9*x**(39/2) + 12297285*a**66*b**10*x**20 + 27054027*a**65*b**11*x**(41/2)
 + 45090045*a**64*b**12*x**21 + 57972915*a**63*b**13*x**(43/2) + 57972915*a
**62*b**14*x**22 + 45090045*a**61*b**15*x**(45/2) + 27054027*a**60*b**16*x*
*23 + 12297285*a**59*b**17*x**(47/2) + 4099095*a**58*b**18*x**24 + 945945*a
**57*b**19*x**(49/2) + 135135*a**56*b**20*x**25 + 9009*a**55*b**21*x**(51/2
)) + 3075072*a**(133/2)*b**10*x**23/(9009*a**70*b**6*x**18 + 135135*a**69*b
**7*x**(37/2) + 945945*a**68*b**8*x**19 + 4099095*a**67*b**9*x**(39/2) + 12
297285*a**66*b**10*x**20 + 27054027*a**65*b**11*x**(41/2) + 45090045*a**64*
b**12*x**21 + 57972915*a**63*b**13*x**(43/2) + 57972915*a**62*b**14*x**22 +
 45090045*a**61*b**15*x**(45/2) + 27054027*a**60*b**16*x**23 + 12297285*a**
59*b**17*x**(47/2) + 4099095*a**58*b**18*x**24 + 945945*a**57*b**19*x**(49/
2) + 135135*a**56*b**20*x**25 + 9009*a**55*b**21*x**(51/2)) + 8060832*a**(1
31/2)*b**11*x**(47/2)*sqrt(1 + b*sqrt(x)/a)/(9009*a**70*b**6*x**18 + 135135
*a**69*b**7*x**(37/2) + 945945*a**68*b**8*x**19 + 4099095*a**67*b**9*x**(39
/2) + 12297285*a**66*b**10*x**20 + 27054027*a**65*b**11*x**(41/2) + 4509004
5*a**64*b**12*x**21 + 57972915*a**63*b**13*x**(43/2) + 57972915*a**62*b**14
```

```
*x**22 + 45090045*a**61*b**15*x**(45/2) + 27054027*a**60*b**16*x**23 + 1229
7285*a**59*b**17*x**(47/2) + 4099095*a**58*b**18*x**24 + 945945*a**57*b**19
*x**(49/2) + 135135*a**56*b**20*x**25 + 9009*a**55*b**21*x**(51/2)) + 13977
60*a**(131/2)*b**11*x**(47/2)/(9009*a**70*b**6*x**18 + 135135*a**69*b**7*x*
*(37/2) + 945945*a**68*b**8*x**19 + 4099095*a**67*b**9*x**(39/2) + 12297285
*a**66*b**10*x**20 + 27054027*a**65*b**11*x**(41/2) + 45090045*a**64*b**12*
x**21 + 57972915*a**63*b**13*x**(43/2) + 57972915*a**62*b**14*x**22 + 45090
045*a**61*b**15*x**(45/2) + 27054027*a**60*b**16*x**23 + 12297285*a**59*b**
17*x**(47/2) + 4099095*a**58*b**18*x**24 + 945945*a**57*b**19*x**(49/2) + 1
35135*a**56*b**20*x**25 + 9009*a**55*b**21*x**(51/2)) + 14375088*a**(129/2)
*b**12*x**24*sqrt(1 + b*sqrt(x)/a)/(9009*a**70*b**6*x**18 + 135135*a**69*b*
*7*x**(37/2) + 945945*a**68*b**8*x**19 + 4099095*a**67*b**9*x**(39/2) + 122
97285*a**66*b**10*x**20 + 27054027*a**65*b**11*x**(41/2) + 45090045*a**64*b
**12*x**21 + 57972915*a**63*b**13*x**(43/2) + 57972915*a**62*b**14*x**22 +
45090045*a**61*b**15*x**(45/2) + 27054027*a**60*b**16*x**23 + 12297285*a**5
9*b**17*x**(47/2) + 4099095*a**58*b**18*x**24 + 945945*a**57*b**19*x**(49/2
) + 135135*a**56*b**20*x**25 + 9009*a**55*b**21*x**(51/2)) + 465920*a**(129
/2)*b**12*x**24/(9009*a**70*b**6*x**18 + 135135*a**69*b**7*x**(37/2) + 9459
45*a**68*b**8*x**19 + 4099095*a**67*b**9*x**(39/2) + 12297285*a**66*b**10*x
**20 + 27054027*a**65*b**11*x**(41/2) + 45090045*a**64*b**12*x**21 + 579729
15*a**63*b**13*x**(43/2) + 57972915*a**62*b**14*x**22 + 45090045*a**61*b**1
5*x**(45/2) + 27054027*a**60*b**16*x**23 + 12297285*a**59*b**17*x**(47/2) +
 4099095*a**58*b**18*x**24 + 945945*a**57*b**19*x**(49/2) + 135135*a**56*b*
*20*x**25 + 9009*a**55*b**21*x**(51/2)) + 18620280*a**(127/2)*b**13*x**(49/
2)*sqrt(1 + b*sqrt(x)/a)/(9009*a**70*b**6*x**18 + 135135*a**69*b**7*x**(37/
2) + 945945*a**68*b**8*x**19 + 4099095*a**67*b**9*x**(39/2) + 12297285*a**6
6*b**10*x**20 + 27054027*a**65*b**11*x**(41/2) + 45090045*a**64*b**12*x**21
 + 57972915*a**63*b**13*x**(43/2) + 57972915*a**62*b**14*x**22 + 45090045*a
**61*b**15*x**(45/2) + 27054027*a**60*b**16*x**23 + 12297285*a**59*b**17*x*
*(47/2) + 4099095*a**58*b**18*x**24 + 945945*a**57*b**19*x**(49/2) + 135135
*a**56*b**20*x**25 + 9009*a**55*b**21*x**(51/2)) + 107520*a**(127/2)*b**13*
x**(49/2)/(9009*a**70*b**6*x**18 + 135135*a**69*b**7*x**(37/2) + 945945*a**
68*b**8*x**19 + 4099095*a**67*b**9*x**(39/2) + 12297285*a**66*b**10*x**20 +
 27054027*a**65*b**11*x**(41/2) + 45090045*a**64*b**12*x**21 + 57972915*a**
63*b**13*x**(43/2) + 57972915*a**62*b**14*x**22 + 45090045*a**61*b**15*x**(
45/2) + 27054027*a**60*b**16*x**23 + 12297285*a**59*b**17*x**(47/2) + 40990
95*a**58*b**18*x**24 + 945945*a**57*b**19*x**(49/2) + 135135*a**56*b**20*x*
*25 + 9009*a**55*b**21*x**(51/2)) + 18558720*a**(125/2)*b**14*x**25*sqrt(1
+ b*sqrt(x)/a)/(9009*a**70*b**6*x**18 + 135135*a**69*b**7*x**(37/2) + 94594
5*a**68*b**8*x**19 + 4099095*a**67*b**9*x**(39/2) + 12297285*a**66*b**10*x*
*20 + 27054027*a**65*b**11*x**(41/2) + 45090045*a**64*b**12*x**21 + 5797291
5*a**63*b**13*x**(43/2) + 57972915*a**62*b**14*x**22 + 45090045*a**61*b**15
*x**(45/2) + 27054027*a**60*b**16*x**23 + 12297285*a**59*b**17*x**(47/2) +
4099095*a**58*b**18*x**24 + 945945*a**57*b**19*x**(49/2) + 135135*a**56*b**
20*x**25 + 9009*a**55*b**21*x**(51/2)) + 15360*a**(125/2)*b**14*x**25/(9009
*a**70*b**6*x**18 + 135135*a**69*b**7*x**(37/2) + 945945*a**68*b**8*x**19 +
 4099095*a**67*b**9*x**(39/2) + 12297285*a**66*b**10*x**20 + 27054027*a**65
*b**11*x**(41/2) + 45090045*a**64*b**12*x**21 + 57972915*a**63*b**13*x**(43
/2) + 57972915*a**62*b**14*x**22 + 45090045*a**61*b**15*x**(45/2) + 2705402
7*a**60*b**16*x**23 + 12297285*a**59*b**17*x**(47/2) + 4099095*a**58*b**18*
x**24 + 945945*a**57*b**19*x**(49/2) + 135135*a**56*b**20*x**25 + 9009*a**5
5*b**21*x**(51/2)) + 14360352*a**(123/2)*b**15*x**(51/2)*sqrt(1 + b*sqrt(x)
/a)/(9009*a**70*b**6*x**18 + 135135*a**69*b**7*x**(37/2) + 945945*a**68*b**
8*x**19 + 4099095*a**67*b**9*x**(39/2) + 12297285*a**66*b**10*x**20 + 27054
027*a**65*b**11*x**(41/2) + 45090045*a**64*b**12*x**21 + 57972915*a**63*b**
13*x**(43/2) + 57972915*a**62*b**14*x**22 + 45090045*a**61*b**15*x**(45/2)
+ 27054027*a**60*b**16*x**23 + 12297285*a**59*b**17*x**(47/2) + 4099095*a**
58*b**18*x**24 + 945945*a**57*b**19*x**(49/2) + 135135*a**56*b**20*x**25 +
9009*a**55*b**21*x**(51/2)) + 1024*a**(123/2)*b**15*x**(51/2)/(9009*a**70*b
```

```
**6*x**18 + 135135*a**69*b**7*x**(37/2) + 945945*a**68*b**8*x**19 + 4099095
*a**67*b**9*x**(39/2) + 12297285*a**66*b**10*x**20 + 27054027*a**65*b**11*x
**(41/2) + 45090045*a**64*b**12*x**21 + 57972915*a**63*b**13*x**(43/2) + 57
972915*a**62*b**14*x**22 + 45090045*a**61*b**15*x**(45/2) + 27054027*a**60*
b**16*x**23 + 12297285*a**59*b**17*x**(47/2) + 4099095*a**58*b**18*x**24 +
945945*a**57*b**19*x**(49/2) + 135135*a**56*b**20*x**25 + 9009*a**55*b**21*
x**(51/2)) + 8569248*a**(121/2)*b**16*x**26*sqrt(1 + b*sqrt(x)/a)/(9009*a**
70*b**6*x**18 + 135135*a**69*b**7*x**(37/2) + 945945*a**68*b**8*x**19 + 409
9095*a**67*b**9*x**(39/2) + 12297285*a**66*b**10*x**20 + 27054027*a**65*b**
11*x**(41/2) + 45090045*a**64*b**12*x**21 + 57972915*a**63*b**13*x**(43/2)
+ 57972915*a**62*b**14*x**22 + 45090045*a**61*b**15*x**(45/2) + 27054027*a*
*60*b**16*x**23 + 12297285*a**59*b**17*x**(47/2) + 4099095*a**58*b**18*x**2
4 + 945945*a**57*b**19*x**(49/2) + 135135*a**56*b**20*x**25 + 9009*a**55*b*
*21*x**(51/2)) + 3873456*a**(119/2)*b**17*x**(53/2)*sqrt(1 + b*sqrt(x)/a)/(
9009*a**70*b**6*x**18 + 135135*a**69*b**7*x**(37/2) + 945945*a**68*b**8*x**
19 + 4099095*a**67*b**9*x**(39/2) + 12297285*a**66*b**10*x**20 + 27054027*a
**65*b**11*x**(41/2) + 45090045*a**64*b**12*x**21 + 57972915*a**63*b**13*x*
*(43/2) + 57972915*a**62*b**14*x**22 + 45090045*a**61*b**15*x**(45/2) + 270
54027*a**60*b**16*x**23 + 12297285*a**59*b**17*x**(47/2) + 4099095*a**58*b*
*18*x**24 + 945945*a**57*b**19*x**(49/2) + 135135*a**56*b**20*x**25 + 9009*
a**55*b**21*x**(51/2)) + 1283840*a**(117/2)*b**18*x**27*sqrt(1 + b*sqrt(x)/
a)/(9009*a**70*b**6*x**18 + 135135*a**69*b**7*x**(37/2) + 945945*a**68*b**8
*x**19 + 4099095*a**67*b**9*x**(39/2) + 12297285*a**66*b**10*x**20 + 270540
27*a**65*b**11*x**(41/2) + 45090045*a**64*b**12*x**21 + 57972915*a**63*b**1
3*x**(43/2) + 57972915*a**62*b**14*x**22 + 45090045*a**61*b**15*x**(45/2) +
 27054027*a**60*b**16*x**23 + 12297285*a**59*b**17*x**(47/2) + 4099095*a**5
8*b**18*x**24 + 945945*a**57*b**19*x**(49/2) + 135135*a**56*b**20*x**25 + 9
009*a**55*b**21*x**(51/2)) + 294560*a**(115/2)*b**19*x**(55/2)*sqrt(1 + b*s
qrt(x)/a)/(9009*a**70*b**6*x**18 + 135135*a**69*b**7*x**(37/2) + 945945*a**
68*b**8*x**19 + 4099095*a**67*b**9*x**(39/2) + 12297285*a**66*b**10*x**20 +
 27054027*a**65*b**11*x**(41/2) + 45090045*a**64*b**12*x**21 + 57972915*a**
63*b**13*x**(43/2) + 57972915*a**62*b**14*x**22 + 45090045*a**61*b**15*x**(
45/2) + 27054027*a**60*b**16*x**23 + 12297285*a**59*b**17*x**(47/2) + 40990
95*a**58*b**18*x**24 + 945945*a**57*b**19*x**(49/2) + 135135*a**56*b**20*x*
*25 + 9009*a**55*b**21*x**(51/2)) + 41832*a**(113/2)*b**20*x**28*sqrt(1 + b
*sqrt(x)/a)/(9009*a**70*b**6*x**18 + 135135*a**69*b**7*x**(37/2) + 945945*a
**68*b**8*x**19 + 4099095*a**67*b**9*x**(39/2) + 12297285*a**66*b**10*x**20
 + 27054027*a**65*b**11*x**(41/2) + 45090045*a**64*b**12*x**21 + 57972915*a
**63*b**13*x**(43/2) + 57972915*a**62*b**14*x**22 + 45090045*a**61*b**15*x*
*(45/2) + 27054027*a**60*b**16*x**23 + 12297285*a**59*b**17*x**(47/2) + 409
9095*a**58*b**18*x**24 + 945945*a**57*b**19*x**(49/2) + 135135*a**56*b**20*
x**25 + 9009*a**55*b**21*x**(51/2)) + 2772*a**(111/2)*b**21*x**(57/2)*sqrt(
1 + b*sqrt(x)/a)/(9009*a**70*b**6*x**18 + 135135*a**69*b**7*x**(37/2) + 945
945*a**68*b**8*x**19 + 4099095*a**67*b**9*x**(39/2) + 12297285*a**66*b**10*
x**20 + 27054027*a**65*b**11*x**(41/2) + 45090045*a**64*b**12*x**21 + 57972
915*a**63*b**13*x**(43/2) + 57972915*a**62*b**14*x**22 + 45090045*a**61*b**
15*x**(45/2) + 27054027*a**60*b**16*x**23 + 12297285*a**59*b**17*x**(47/2)
+ 4099095*a**58*b**18*x**24 + 945945*a**57*b**19*x**(49/2) + 135135*a**56*b
**20*x**25 + 9009*a**55*b**21*x**(51/2))
```

3.2234 $\int \sqrt{a+b\sqrt{x}}\, x\, dx$

Optimal. Leaf size=88

$$-\frac{4a^3\left(a+b\sqrt{x}\right)^{3/2}}{3b^4}+\frac{12a^2\left(a+b\sqrt{x}\right)^{5/2}}{5b^4}+\frac{4\left(a+b\sqrt{x}\right)^{9/2}}{9b^4}-\frac{12a\left(a+b\sqrt{x}\right)^{7/2}}{7b^4}$$

```
[Out] -4/3*a^3*(a+b*x^(1/2))^(3/2)/b^4+12/5*a^2*(a+b*x^(1/2))^(5/2)/b^4-12/7*a*(a
+b*x^(1/2))^(7/2)/b^4+4/9*(a+b*x^(1/2))^(9/2)/b^4
```

Rubi [A] time = 0.04, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{12a^2\left(a+b\sqrt{x}\right)^{5/2}}{5b^4}-\frac{4a^3\left(a+b\sqrt{x}\right)^{3/2}}{3b^4}+\frac{4\left(a+b\sqrt{x}\right)^{9/2}}{9b^4}-\frac{12a\left(a+b\sqrt{x}\right)^{7/2}}{7b^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[x]]*x,x]
```

```
[Out] (-4*a^3*(a + b*Sqrt[x])^(3/2))/(3*b^4) + (12*a^2*(a + b*Sqrt[x])^(5/2))/(5*
b^4) - (12*a*(a + b*Sqrt[x])^(7/2))/(7*b^4) + (4*(a + b*Sqrt[x])^(9/2))/(9*
b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\sqrt{x}}\, x\, dx &= 2\,\text{Subst}\left(\int x^3\sqrt{a+bx}\, dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int\left(-\frac{a^3\sqrt{a+bx}}{b^3}+\frac{3a^2(a+bx)^{3/2}}{b^3}-\frac{3a(a+bx)^{5/2}}{b^3}+\frac{(a+bx)^{7/2}}{b^3}\right)dx, x, \sqrt{x}\right)\\ &= -\frac{4a^3\left(a+b\sqrt{x}\right)^{3/2}}{3b^4}+\frac{12a^2\left(a+b\sqrt{x}\right)^{5/2}}{5b^4}-\frac{12a\left(a+b\sqrt{x}\right)^{7/2}}{7b^4}+\frac{4\left(a+b\sqrt{x}\right)^{9/2}}{9b^4}\end{aligned}$$

Mathematica [A] time = 0.03, size = 54, normalized size = 0.61

$$\frac{4\left(a+b\sqrt{x}\right)^{3/2}\left(-16a^3+24a^2b\sqrt{x}-30ab^2x+35b^3x^{3/2}\right)}{315b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[x]]*x,x]
```

```
[Out] (4*(a + b*Sqrt[x])^(3/2)*(-16*a^3 + 24*a^2*b*Sqrt[x] - 30*a*b^2*x + 35*b^3*
x^(3/2)))/(315*b^4)
```

fricas [A] time = 0.96, size = 55, normalized size = 0.62

$$\frac{4\left(35\,b^4x^2-6\,a^2b^2x-16\,a^4+\left(5\,ab^3x+8\,a^3b\right)\sqrt{x}\right)\sqrt{b\sqrt{x}+a}}{315\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 4/315*(35*b^4*x^2 - 6*a^2*b^2*x - 16*a^4 + (5*a*b^3*x + 8*a^3*b)*sqrt(x))*s
qrt(b*sqrt(x) + a)/b^4
```

giac [B] time = 0.18, size = 134, normalized size = 1.52

$$4\left(\frac{9\left(5\left(b\sqrt{x}+a\right)^{\frac{7}{2}}-21\left(b\sqrt{x}+a\right)^{\frac{5}{2}}a+35\left(b\sqrt{x}+a\right)^{\frac{3}{2}}a^2-35\sqrt{b\sqrt{x}+a}\,a^3\right)a}{b^3}+\frac{35\left(b\sqrt{x}+a\right)^{\frac{9}{2}}-180\left(b\sqrt{x}+a\right)^{\frac{7}{2}}a+378\left(b\sqrt{x}+a\right)^{\frac{5}{2}}a^2-420\left(b\sqrt{x}+a\right)^{\frac{3}{2}}a}{b^3}\right.$$

$$\frac{}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 4/315*(9*(5*(b*sqrt(x) + a)^(7/2) - 21*(b*sqrt(x) + a)^(5/2)*a + 35*(b*sqrt
(x) + a)^(3/2)*a^2 - 35*sqrt(b*sqrt(x) + a)*a^3)*a/b^3 + (35*(b*sqrt(x) + a
)^(9/2) - 180*(b*sqrt(x) + a)^(7/2)*a + 378*(b*sqrt(x) + a)^(5/2)*a^2 - 420
*(b*sqrt(x) + a)^(3/2)*a^3 + 315*sqrt(b*sqrt(x) + a)*a^4)/b^3)/b
```

maple [A] time = 0.00, size = 58, normalized size = 0.66

$$\frac{-\frac{4\left(b\sqrt{x}+a\right)^{\frac{3}{2}}a^3}{3}+\frac{12\left(b\sqrt{x}+a\right)^{\frac{5}{2}}a^2}{5}-\frac{12\left(b\sqrt{x}+a\right)^{\frac{7}{2}}a}{7}+\frac{4\left(b\sqrt{x}+a\right)^{\frac{9}{2}}}{9}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^(1/2)+a)^(1/2),x)
```

```
[Out] 4/b^4*(1/9*(b*x^(1/2)+a)^(9/2)-3/7*a*(b*x^(1/2)+a)^(7/2)+3/5*(b*x^(1/2)+a)^
(5/2)*a^2-1/3*a^3*(b*x^(1/2)+a)^(3/2))
```

maxima [A] time = 0.88, size = 64, normalized size = 0.73

$$\frac{4\left(b\sqrt{x}+a\right)^{\frac{9}{2}}}{9\,b^4}-\frac{12\left(b\sqrt{x}+a\right)^{\frac{7}{2}}a}{7\,b^4}+\frac{12\left(b\sqrt{x}+a\right)^{\frac{5}{2}}a^2}{5\,b^4}-\frac{4\left(b\sqrt{x}+a\right)^{\frac{3}{2}}a^3}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/9*(b*sqrt(x) + a)^(9/2)/b^4 - 12/7*(b*sqrt(x) + a)^(7/2)*a/b^4 + 12/5*(b*
sqrt(x) + a)^(5/2)*a^2/b^4 - 4/3*(b*sqrt(x) + a)^(3/2)*a^3/b^4
```

mupad [B] time = 1.16, size = 64, normalized size = 0.73

$$\frac{4\left(a+b\sqrt{x}\right)^{9/2}}{9\,b^4}-\frac{12\,a\left(a+b\sqrt{x}\right)^{7/2}}{7\,b^4}-\frac{4\,a^3\left(a+b\sqrt{x}\right)^{3/2}}{3\,b^4}+\frac{12\,a^2\left(a+b\sqrt{x}\right)^{5/2}}{5\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^(1/2))^(1/2),x)
```

```
[Out] (4*(a + b*x^(1/2))^(9/2))/(9*b^4) - (12*a*(a + b*x^(1/2))^(7/2))/(7*b^4) -
(4*a^3*(a + b*x^(1/2))^(3/2))/(3*b^4) + (12*a^2*(a + b*x^(1/2))^(5/2))/(5*b
^4)
```

sympy [**B**] time = 2.82, size = 1987, normalized size = 22.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x**(1/2))**(1/2),x)
```

```
[Out] -64*a**(49/2)*x**8*sqrt(1 + b*sqrt(x)/a)/(315*a**20*b**4*x**8 + 1890*a**19*
b**5*x**(17/2) + 4725*a**18*b**6*x**9 + 6300*a**17*b**7*x**(19/2) + 4725*a*
*16*b**8*x**10 + 1890*a**15*b**9*x**(21/2) + 315*a**14*b**10*x**11) + 64*a*
*(49/2)*x**8/(315*a**20*b**4*x**8 + 1890*a**19*b**5*x**(17/2) + 4725*a**18*
b**6*x**9 + 6300*a**17*b**7*x**(19/2) + 4725*a**16*b**8*x**10 + 1890*a**15*
b**9*x**(21/2) + 315*a**14*b**10*x**11) - 352*a**(47/2)*b*x**(17/2)*sqrt(1
+ b*sqrt(x)/a)/(315*a**20*b**4*x**8 + 1890*a**19*b**5*x**(17/2) + 4725*a**1
8*b**6*x**9 + 6300*a**17*b**7*x**(19/2) + 4725*a**16*b**8*x**10 + 1890*a**1
5*b**9*x**(21/2) + 315*a**14*b**10*x**11) + 384*a**(47/2)*b*x**(17/2)/(315*
a**20*b**4*x**8 + 1890*a**19*b**5*x**(17/2) + 4725*a**18*b**6*x**9 + 6300*a
**17*b**7*x**(19/2) + 4725*a**16*b**8*x**10 + 1890*a**15*b**9*x**(21/2) + 3
15*a**14*b**10*x**11) - 792*a**(45/2)*b**2*x**9*sqrt(1 + b*sqrt(x)/a)/(315*
a**20*b**4*x**8 + 1890*a**19*b**5*x**(17/2) + 4725*a**18*b**6*x**9 + 6300*a
**17*b**7*x**(19/2) + 4725*a**16*b**8*x**10 + 1890*a**15*b**9*x**(21/2) + 3
15*a**14*b**10*x**11) + 960*a**(45/2)*b**2*x**9/(315*a**20*b**4*x**8 + 1890
*a**19*b**5*x**(17/2) + 4725*a**18*b**6*x**9 + 6300*a**17*b**7*x**(19/2) +
4725*a**16*b**8*x**10 + 1890*a**15*b**9*x**(21/2) + 315*a**14*b**10*x**11)
- 924*a**(43/2)*b**3*x**(19/2)*sqrt(1 + b*sqrt(x)/a)/(315*a**20*b**4*x**8 +
 1890*a**19*b**5*x**(17/2) + 4725*a**18*b**6*x**9 + 6300*a**17*b**7*x**(19/
2) + 4725*a**16*b**8*x**10 + 1890*a**15*b**9*x**(21/2) + 315*a**14*b**10*x*
*11) + 1280*a**(43/2)*b**3*x**(19/2)/(315*a**20*b**4*x**8 + 1890*a**19*b**5
*x**(17/2) + 4725*a**18*b**6*x**9 + 6300*a**17*b**7*x**(19/2) + 4725*a**16*
b**8*x**10 + 1890*a**15*b**9*x**(21/2) + 315*a**14*b**10*x**11) - 420*a**(4
1/2)*b**4*x**10*sqrt(1 + b*sqrt(x)/a)/(315*a**20*b**4*x**8 + 1890*a**19*b**
5*x**(17/2) + 4725*a**18*b**6*x**9 + 6300*a**17*b**7*x**(19/2) + 4725*a**16
*b**8*x**10 + 1890*a**15*b**9*x**(21/2) + 315*a**14*b**10*x**11) + 960*a**(
41/2)*b**4*x**10/(315*a**20*b**4*x**8 + 1890*a**19*b**5*x**(17/2) + 4725*a*
*18*b**6*x**9 + 6300*a**17*b**7*x**(19/2) + 4725*a**16*b**8*x**10 + 1890*a*
*15*b**9*x**(21/2) + 315*a**14*b**10*x**11) + 756*a**(39/2)*b**5*x**(21/2)*
sqrt(1 + b*sqrt(x)/a)/(315*a**20*b**4*x**8 + 1890*a**19*b**5*x**(17/2) + 47
25*a**18*b**6*x**9 + 6300*a**17*b**7*x**(19/2) + 4725*a**16*b**8*x**10 + 18
90*a**15*b**9*x**(21/2) + 315*a**14*b**10*x**11) + 384*a**(39/2)*b**5*x**(2
1/2)/(315*a**20*b**4*x**8 + 1890*a**19*b**5*x**(17/2) + 4725*a**18*b**6*x**
9 + 6300*a**17*b**7*x**(19/2) + 4725*a**16*b**8*x**10 + 1890*a**15*b**9*x**
(21/2) + 315*a**14*b**10*x**11) + 2268*a**(37/2)*b**6*x**11*sqrt(1 + b*sqrt
(x)/a)/(315*a**20*b**4*x**8 + 1890*a**19*b**5*x**(17/2) + 4725*a**18*b**6*x
**9 + 6300*a**17*b**7*x**(19/2) + 4725*a**16*b**8*x**10 + 1890*a**15*b**9*x
**(21/2) + 315*a**14*b**10*x**11) + 64*a**(37/2)*b**6*x**11/(315*a**20*b**4
*x**8 + 1890*a**19*b**5*x**(17/2) + 4725*a**18*b**6*x**9 + 6300*a**17*b**7*
x**(19/2) + 4725*a**16*b**8*x**10 + 1890*a**15*b**9*x**(21/2) + 315*a**14*b
**10*x**11) + 2988*a**(35/2)*b**7*x**(23/2)*sqrt(1 + b*sqrt(x)/a)/(315*a**2
0*b**4*x**8 + 1890*a**19*b**5*x**(17/2) + 4725*a**18*b**6*x**9 + 6300*a**17
*b**7*x**(19/2) + 4725*a**16*b**8*x**10 + 1890*a**15*b**9*x**(21/2) + 315*a
**14*b**10*x**11) + 2196*a**(33/2)*b**8*x**12*sqrt(1 + b*sqrt(x)/a)/(315*a*
*20*b**4*x**8 + 1890*a**19*b**5*x**(17/2) + 4725*a**18*b**6*x**9 + 6300*a**
```

```
17*b**7*x**(19/2) + 4725*a**16*b**8*x**10 + 1890*a**15*b**9*x**(21/2) + 315
*a**14*b**10*x**11) + 860*a**(31/2)*b**9*x**(25/2)*sqrt(1 + b*sqrt(x)/a)/(3
15*a**20*b**4*x**8 + 1890*a**19*b**5*x**(17/2) + 4725*a**18*b**6*x**9 + 630
0*a**17*b**7*x**(19/2) + 4725*a**16*b**8*x**10 + 1890*a**15*b**9*x**(21/2)
+ 315*a**14*b**10*x**11) + 140*a**(29/2)*b**10*x**13*sqrt(1 + b*sqrt(x)/a)/
(315*a**20*b**4*x**8 + 1890*a**19*b**5*x**(17/2) + 4725*a**18*b**6*x**9 + 6
300*a**17*b**7*x**(19/2) + 4725*a**16*b**8*x**10 + 1890*a**15*b**9*x**(21/2
) + 315*a**14*b**10*x**11)
```

3.2235 $\int \sqrt{a+b\sqrt{x}}\,dx$

Optimal. Leaf size=42

$$\frac{4\left(a+b\sqrt{x}\right)^{5/2}}{5b^2}-\frac{4a\left(a+b\sqrt{x}\right)^{3/2}}{3b^2}$$

```
[Out] -4/3*a*(a+b*x^(1/2))^(3/2)/b^2+4/5*(a+b*x^(1/2))^(5/2)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {190, 43}

$$\frac{4\left(a+b\sqrt{x}\right)^{5/2}}{5b^2}-\frac{4a\left(a+b\sqrt{x}\right)^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[x]],x]
```

```
[Out] (-4*a*(a + b*Sqrt[x])^(3/2))/(3*b^2) + (4*(a + b*Sqrt[x])^(5/2))/(5*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\sqrt{x}}\,dx &= 2\,\text{Subst}\left(\int x\sqrt{a+bx}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int\left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx, x, \sqrt{x}\right)\\ &= -\frac{4a\left(a+b\sqrt{x}\right)^{3/2}}{3b^2}+\frac{4\left(a+b\sqrt{x}\right)^{5/2}}{5b^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 0.76

$$\frac{4\left(a+b\sqrt{x}\right)^{3/2}\left(3b\sqrt{x}-2a\right)}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[x]],x]
```

```
[Out] (4*(a + b*Sqrt[x])^(3/2)*(-2*a + 3*b*Sqrt[x]))/(15*b^2)
```

fricas [A] time = 1.67, size = 32, normalized size = 0.76

$$\frac{4\left(3\,b^2x + ab\sqrt{x} - 2\,a^2\right)\sqrt{b\sqrt{x} + a}}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 4/15*(3*b^2*x + a*b*sqrt(x) - 2*a^2)*sqrt(b*sqrt(x) + a)/b^2
```

giac [B] time = 0.17, size = 76, normalized size = 1.81

$$\frac{4\left(\frac{5\left(\left(b\sqrt{x}+a\right)^{\frac{3}{2}} - 3\sqrt{b\sqrt{x}+a}\,a\right)a}{b} + \frac{3\left(b\sqrt{x}+a\right)^{\frac{5}{2}} - 10\left(b\sqrt{x}+a\right)^{\frac{3}{2}}a + 15\sqrt{b\sqrt{x}+a}\,a^2}{b}\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 4/15*(5*((b*sqrt(x) + a)^(3/2) - 3*sqrt(b*sqrt(x) + a)*a)*a/b + (3*(b*sqrt(
x) + a)^(5/2) - 10*(b*sqrt(x) + a)^(3/2)*a + 15*sqrt(b*sqrt(x) + a)*a^2)/b)
/b
```

maple [A] time = 0.00, size = 30, normalized size = 0.71

$$\frac{-\frac{4\left(b\sqrt{x}+a\right)^{\frac{3}{2}}a}{3} + \frac{4\left(b\sqrt{x}+a\right)^{\frac{5}{2}}}{5}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(1/2)+a)^(1/2),x)
```

```
[Out] 4/b^2*(1/5*(b*x^(1/2)+a)^(5/2)-1/3*a*(b*x^(1/2)+a)^(3/2))
```

maxima [A] time = 0.92, size = 30, normalized size = 0.71

$$\frac{4\left(b\sqrt{x} + a\right)^{\frac{5}{2}}}{5\,b^2} - \frac{4\left(b\sqrt{x} + a\right)^{\frac{3}{2}}a}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/5*(b*sqrt(x) + a)^(5/2)/b^2 - 4/3*(b*sqrt(x) + a)^(3/2)*a/b^2
```

mupad [B] time = 1.15, size = 37, normalized size = 0.88

$$\frac{x\sqrt{a + b\sqrt{x}}\;{}_2\mathrm{F}_1\left(-\frac{1}{2}, 2;\; 3;\; -\frac{b\sqrt{x}}{a}\right)}{\sqrt{\frac{b\sqrt{x}}{a} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^(1/2),x)
```

```
[Out] (x*(a + b*x^(1/2))^(1/2)*hypergeom([-1/2, 2], 3, -(b*x^(1/2))/a))/((b*x^(1/
2))/a + 1)^(1/2)
```

sympy [**B**] time = 1.24, size = 272, normalized size = 6.48

$$-\frac{8a^{\frac{9}{2}}x^2\sqrt{1+\frac{b\sqrt{x}}{a}}}{15a^2b^2x^2+15ab^3x^{\frac{5}{2}}}+\frac{8a^{\frac{9}{2}}x^2}{15a^2b^2x^2+15ab^3x^{\frac{5}{2}}}-\frac{4a^{\frac{7}{2}}bx^{\frac{5}{2}}\sqrt{1+\frac{b\sqrt{x}}{a}}}{15a^2b^2x^2+15ab^3x^{\frac{5}{2}}}+\frac{8a^{\frac{7}{2}}bx^{\frac{5}{2}}}{15a^2b^2x^2+15ab^3x^{\frac{5}{2}}}+\frac{16a^{\frac{5}{2}}b^2x^3\sqrt{1+\frac{b\sqrt{x}}{a}}}{15a^2b^2x^2+15ab^3x^{\frac{5}{2}}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**(1/2),x)
```

```
[Out] -8*a**(9/2)*x**2*sqrt(1 + b*sqrt(x)/a)/(15*a**2*b**2*x**2 + 15*a*b**3*x**(5
/2)) + 8*a**(9/2)*x**2/(15*a**2*b**2*x**2 + 15*a*b**3*x**(5/2)) - 4*a**(7/2
)*b*x**(5/2)*sqrt(1 + b*sqrt(x)/a)/(15*a**2*b**2*x**2 + 15*a*b**3*x**(5/2))
 + 8*a**(7/2)*b*x**(5/2)/(15*a**2*b**2*x**2 + 15*a*b**3*x**(5/2)) + 16*a**(
5/2)*b**2*x**3*sqrt(1 + b*sqrt(x)/a)/(15*a**2*b**2*x**2 + 15*a*b**3*x**(5/2
)) + 12*a**(3/2)*b**3*x**(7/2)*sqrt(1 + b*sqrt(x)/a)/(15*a**2*b**2*x**2 + 1
5*a*b**3*x**(5/2))
```

3.2236 $\int \frac{\sqrt{a+b\sqrt{x}}}{x}\,dx$

Optimal. Leaf size=43

$$4\sqrt{a+b\sqrt{x}}\;-4\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)$$

```
[Out] -4*arctanh((a+b*x^(1/2))^(1/2)/a^(1/2))*a^(1/2)+4*(a+b*x^(1/2))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {266, 50, 63, 208}

$$4\sqrt{a+b\sqrt{x}}\;-4\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[x]]/x,x]
```

```
[Out] 4*Sqrt[a + b*Sqrt[x]] - 4*Sqrt[a]*ArcTanh[Sqrt[a + b*Sqrt[x]]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+b\sqrt{x}}}{x}\,dx &= 2\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \sqrt{x}\right)\\ &= 4\sqrt{a+b\sqrt{x}} + (2a)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sqrt{x}\right)\\ &= 4\sqrt{a+b\sqrt{x}} + \frac{(4a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sqrt{x}}\right)}{b}\\ &= 4\sqrt{a+b\sqrt{x}} - 4\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 1.00

$$4\sqrt{a+b\sqrt{x}} - 4\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[x]]/x,x]
```

```
[Out] 4*Sqrt[a + b*Sqrt[x]] - 4*Sqrt[a]*ArcTanh[Sqrt[a + b*Sqrt[x]]/Sqrt[a]]
```

fricas [A] time = 0.90, size = 88, normalized size = 2.05

$$\left[2\sqrt{a}\,\log\left(\frac{bx - 2\sqrt{b\sqrt{x}+a}\,\sqrt{a}\,\sqrt{x} + 2a\sqrt{x}}{x}\right) + 4\sqrt{b\sqrt{x}+a}, 4\sqrt{-a}\,\arctan\left(\frac{\sqrt{b\sqrt{x}+a}\,\sqrt{-a}}{a}\right) + 4\sqrt{b\sqrt{x}+a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [2*sqrt(a)*log((b*x - 2*sqrt(b*sqrt(x) + a)*sqrt(a)*sqrt(x) + 2*a*sqrt(x))/
x) + 4*sqrt(b*sqrt(x) + a), 4*sqrt(-a)*arctan(sqrt(b*sqrt(x) + a)*sqrt(-a)/
a) + 4*sqrt(b*sqrt(x) + a)]
```

giac [A] time = 0.17, size = 41, normalized size = 0.95

$$\frac{4\left(\frac{ab\arctan\left(\frac{\sqrt{b\sqrt{x}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \sqrt{b\sqrt{x}+a}\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^(1/2)/x,x, algorithm="giac")
```

```
[Out] 4*(a*b*arctan(sqrt(b*sqrt(x) + a)/sqrt(-a))/sqrt(-a) + sqrt(b*sqrt(x) + a)*
b)/b
```

maple [A] time = 0.00, size = 32, normalized size = 0.74

$$-4\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{b\sqrt{x}+a}}{\sqrt{a}}\right) + 4\sqrt{b\sqrt{x}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(1/2)+a)^(1/2)/x,x)
```

```
[Out] -4*arctanh((b*x^(1/2)+a)^(1/2)/a^(1/2))*a^(1/2)+4*(b*x^(1/2)+a)^(1/2)
```

maxima [A] time = 2.04, size = 49, normalized size = 1.14

$$2\sqrt{a}\,\log\left(\frac{\sqrt{b\sqrt{x}+a}-\sqrt{a}}{\sqrt{b\sqrt{x}+a}+\sqrt{a}}\right)+4\sqrt{b\sqrt{x}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 2*sqrt(a)*log((sqrt(b*sqrt(x) + a) - sqrt(a))/(sqrt(b*sqrt(x) + a) + sqrt(a
)))  + 4*sqrt(b*sqrt(x) + a)
```

mupad [B] time = 1.89, size = 31, normalized size = 0.72

$$4\sqrt{a+b\sqrt{x}}\;-4\sqrt{a}\,\operatorname{atanh}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^(1/2)/x,x)
```

```
[Out] 4*(a + b*x^(1/2))^(1/2) - 4*a^(1/2)*atanh((a + b*x^(1/2))^(1/2)/a^(1/2))
```

sympy [B] time = 1.57, size = 75, normalized size = 1.74

$$-4\sqrt{a}\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt[4]{x}}\right)+\frac{4a}{\sqrt{b}\,\sqrt[4]{x}\,\sqrt{\frac{a}{b\sqrt{x}}+1}}+\frac{4\sqrt{b}\,\sqrt[4]{x}}{\sqrt{\frac{a}{b\sqrt{x}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**(1/2)/x,x)
```

```
[Out] -4*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x**(1/4))) + 4*a/(sqrt(b)*x**(1/4)*sqrt(a
/(b*sqrt(x)) + 1)) + 4*sqrt(b)*x**(1/4)/sqrt(a/(b*sqrt(x)) + 1)
```

3.2237 $\int \frac{\sqrt{a+b\sqrt{x}}}{x^2}\,dx$

Optimal. Leaf size=77

$$\frac{b^2 \tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)}{2a^{3/2}} - \frac{b\sqrt{a+b\sqrt{x}}}{2a\sqrt{x}} - \frac{\sqrt{a+b\sqrt{x}}}{x}$$

```
[Out] 1/2*b^2*arctanh((a+b*x^(1/2))^(1/2)/a^(1/2))/a^(3/2)-(a+b*x^(1/2))^(1/2)/x-
1/2*b*(a+b*x^(1/2))^(1/2)/a/x^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {266, 47, 51, 63, 208}

$$\frac{b^2 \tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)}{2a^{3/2}} - \frac{b\sqrt{a+b\sqrt{x}}}{2a\sqrt{x}} - \frac{\sqrt{a+b\sqrt{x}}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[x]]/x^2,x]
```

```
[Out] -(Sqrt[a + b*Sqrt[x]]/x) - (b*Sqrt[a + b*Sqrt[x]])/(2*a*Sqrt[x]) + (b^2*Arc
Tanh[Sqrt[a + b*Sqrt[x]]/Sqrt[a]])/(2*a^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+b\sqrt{x}}}{x^2}\,dx &= 2\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^3}\,dx, x, \sqrt{x}\right) \\
&= -\frac{\sqrt{a+b\sqrt{x}}}{x} + \frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \sqrt{x}\right) \\
&= -\frac{\sqrt{a+b\sqrt{x}}}{x} - \frac{b\sqrt{a+b\sqrt{x}}}{2a\sqrt{x}} - \frac{b^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sqrt{x}\right)}{4a} \\
&= -\frac{\sqrt{a+b\sqrt{x}}}{x} - \frac{b\sqrt{a+b\sqrt{x}}}{2a\sqrt{x}} - \frac{b\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sqrt{x}}\right)}{2a} \\
&= -\frac{\sqrt{a+b\sqrt{x}}}{x} - \frac{b\sqrt{a+b\sqrt{x}}}{2a\sqrt{x}} + \frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)}{2a^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 43, normalized size = 0.56

$$-\frac{4b^2\left(a+b\sqrt{x}\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{\sqrt{x}\,b}{a}+1\right)}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[x]]/x^2,x]
```

```
[Out] (-4*b^2*(a + b*Sqrt[x])^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + (b*Sqrt[x]
)/a])/(3*a^3)
```

fricas [A] time = 1.42, size = 133, normalized size = 1.73

$$\left[\frac{\sqrt{a}\,b^2x\log\left(\frac{bx+2\sqrt{b\sqrt{x}+a}\,\sqrt{a}\,\sqrt{x}+2\,a\sqrt{x}}{x}\right)-2\left(ab\sqrt{x}+2\,a^2\right)\sqrt{b\sqrt{x}+a}}{4\,a^2x}, -\frac{\sqrt{-a}\,b^2x\arctan\left(\frac{\sqrt{b\sqrt{x}+a}\,\sqrt{-a}}{a}\right)+\left(ab\sqrt{\right.}\right.}{2\,a^2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a)*b^2*x*log((b*x + 2*sqrt(b*sqrt(x) + a)*sqrt(a)*sqrt(x) + 2*a*
sqrt(x))/x) - 2*(a*b*sqrt(x) + 2*a^2)*sqrt(b*sqrt(x) + a))/(a^2*x), -1/2*(s
qrt(-a)*b^2*x*arctan(sqrt(b*sqrt(x) + a)*sqrt(-a)/a) + (a*b*sqrt(x) + 2*a^2
)*sqrt(b*sqrt(x) + a))/(a^2*x)]
```

giac [A] time = 0.16, size = 72, normalized size = 0.94

$$-\frac{\frac{b^3\arctan\left(\frac{\sqrt{b\sqrt{x}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{\left(b\sqrt{x}+a\right)^{\frac{3}{2}}b^3+\sqrt{b\sqrt{x}+a}\,ab^3}{ab^2x}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^3*arctan(sqrt(b*sqrt(x) + a)/sqrt(-a))/(sqrt(-a)*a) + ((b*sqrt(x) +
 a)^(3/2)*b^3 + sqrt(b*sqrt(x) + a)*a*b^3)/(a*b^2*x))/b
```

maple [A] time = 0.01, size = 59, normalized size = 0.77

$$4\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\sqrt{x}+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}+\frac{-\frac{\left(b\sqrt{x}+a\right)^{\frac{3}{2}}}{8a}-\frac{\sqrt{b\sqrt{x}+a}}{8}}{b^2x}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(1/2)+a)^(1/2)/x^2,x)
```

```
[Out] 4*b^2*((-1/8/a*(b*x^(1/2)+a)^(3/2)-1/8*(b*x^(1/2)+a)^(1/2))/x/b^2+1/8/a^(3/
2)*arctanh((b*x^(1/2)+a)^(1/2)/a^(1/2)))
```

maxima [A] time = 2.04, size = 100, normalized size = 1.30

$$-\frac{b^2\log\left(\frac{\sqrt{b\sqrt{x}+a}-\sqrt{a}}{\sqrt{b\sqrt{x}+a}+\sqrt{a}}\right)}{4\,a^{\frac{3}{2}}}-\frac{\left(b\sqrt{x}+a\right)^{\frac{3}{2}}b^2+\sqrt{b\sqrt{x}+a}\,ab^2}{2\left(\left(b\sqrt{x}+a\right)^2a-2\left(b\sqrt{x}+a\right)a^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -1/4*b^2*log((sqrt(b*sqrt(x) + a) - sqrt(a))/(sqrt(b*sqrt(x) + a) + sqrt(a)
))/a^(3/2) - 1/2*((b*sqrt(x) + a)^(3/2)*b^2 + sqrt(b*sqrt(x) + a)*a*b^2)/((
b*sqrt(x) + a)^2*a - 2*(b*sqrt(x) + a)*a^2 + a^3)
```

mupad [B] time = 1.57, size = 54, normalized size = 0.70

$$\frac{b^2\,\operatorname{atanh}\left(\frac{\sqrt{a+b\,\sqrt{x}}}{\sqrt{a}}\right)}{2\,a^{3/2}}-\frac{\sqrt{a+b\,\sqrt{x}}}{2\,x}-\frac{\left(a+b\,\sqrt{x}\right)^{3/2}}{2\,a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^(1/2)/x^2,x)
```

```
[Out] (b^2*atanh((a + b*x^(1/2))^(1/2)/a^(1/2)))/(2*a^(3/2)) - (a + b*x^(1/2))^(1
/2)/(2*x) - (a + b*x^(1/2))^(3/2)/(2*a*x)
```

sympy [A] time = 4.24, size = 105, normalized size = 1.36

$$-\frac{a}{\sqrt{b}\,x^{\frac{5}{4}}\sqrt{\frac{a}{b\sqrt{x}}+1}}-\frac{3\sqrt{b}}{2x^{\frac{3}{4}}\sqrt{\frac{a}{b\sqrt{x}}+1}}-\frac{b^{\frac{3}{2}}}{2a\sqrt[4]{x}\sqrt{\frac{a}{b\sqrt{x}}+1}}+\frac{b^2\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt[4]{x}}\right)}{2a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**(1/2)/x**2,x)
```

```
[Out] -a/(sqrt(b)*x**(5/4)*sqrt(a/(b*sqrt(x)) + 1)) - 3*sqrt(b)/(2*x**(3/4)*sqrt(
a/(b*sqrt(x)) + 1)) - b**(3/2)/(2*a*x**(1/4)*sqrt(a/(b*sqrt(x)) + 1)) + b**
2*asinh(sqrt(a)/(sqrt(b)*x**(1/4)))/(2*a**(3/2))
```

3.2238 $\int \frac{\sqrt{a+b\sqrt{x}}}{x^3}\,dx$

Optimal. Leaf size=133

$$\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)}{32a^{7/2}}-\frac{5b^3\sqrt{a+b\sqrt{x}}}{32a^3\sqrt{x}}+\frac{5b^2\sqrt{a+b\sqrt{x}}}{48a^2x}-\frac{b\sqrt{a+b\sqrt{x}}}{12ax^{3/2}}-\frac{\sqrt{a+b\sqrt{x}}}{2x^2}$$

```
[Out] 5/32*b^4*arctanh((a+b*x^(1/2))^(1/2)/a^(1/2))/a^(7/2)-1/2*(a+b*x^(1/2))^(1/
2)/x^2-1/12*b*(a+b*x^(1/2))^(1/2)/a/x^(3/2)+5/48*b^2*(a+b*x^(1/2))^(1/2)/a^
2/x-5/32*b^3*(a+b*x^(1/2))^(1/2)/a^3/x^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {266, 47, 51, 63, 208}

$$-\frac{5b^3\sqrt{a+b\sqrt{x}}}{32a^3\sqrt{x}}+\frac{5b^2\sqrt{a+b\sqrt{x}}}{48a^2x}+\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)}{32a^{7/2}}-\frac{b\sqrt{a+b\sqrt{x}}}{12ax^{3/2}}-\frac{\sqrt{a+b\sqrt{x}}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[x]]/x^3,x]
```

```
[Out] -Sqrt[a + b*Sqrt[x]]/(2*x^2) - (b*Sqrt[a + b*Sqrt[x]])/(12*a*x^(3/2)) + (5*
b^2*Sqrt[a + b*Sqrt[x]])/(48*a^2*x) - (5*b^3*Sqrt[a + b*Sqrt[x]])/(32*a^3*S
qrt[x]) + (5*b^4*ArcTanh[Sqrt[a + b*Sqrt[x]]/Sqrt[a]])/(32*a^(7/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+b\sqrt{x}}}{x^3}\,dx &= 2\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^5}\,dx, x, \sqrt{x}\right) \\
&= -\frac{\sqrt{a+b\sqrt{x}}}{2x^2} + \frac{1}{4}b\,\text{Subst}\left(\int \frac{1}{x^4\sqrt{a+bx}}\,dx, x, \sqrt{x}\right) \\
&= -\frac{\sqrt{a+b\sqrt{x}}}{2x^2} - \frac{b\sqrt{a+b\sqrt{x}}}{12ax^{3/2}} - \frac{\left(5b^2\right)\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx, x, \sqrt{x}\right)}{24a} \\
&= -\frac{\sqrt{a+b\sqrt{x}}}{2x^2} - \frac{b\sqrt{a+b\sqrt{x}}}{12ax^{3/2}} + \frac{5b^2\sqrt{a+b\sqrt{x}}}{48a^2x} + \frac{\left(5b^3\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \sqrt{x}\right)}{32a^2} \\
&= -\frac{\sqrt{a+b\sqrt{x}}}{2x^2} - \frac{b\sqrt{a+b\sqrt{x}}}{12ax^{3/2}} + \frac{5b^2\sqrt{a+b\sqrt{x}}}{48a^2x} - \frac{5b^3\sqrt{a+b\sqrt{x}}}{32a^3\sqrt{x}} - \frac{\left(5b^4\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx\right.}{64a^3} \\
&= -\frac{\sqrt{a+b\sqrt{x}}}{2x^2} - \frac{b\sqrt{a+b\sqrt{x}}}{12ax^{3/2}} + \frac{5b^2\sqrt{a+b\sqrt{x}}}{48a^2x} - \frac{5b^3\sqrt{a+b\sqrt{x}}}{32a^3\sqrt{x}} - \frac{\left(5b^3\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx\right.}{32a^3} \\
&= -\frac{\sqrt{a+b\sqrt{x}}}{2x^2} - \frac{b\sqrt{a+b\sqrt{x}}}{12ax^{3/2}} + \frac{5b^2\sqrt{a+b\sqrt{x}}}{48a^2x} - \frac{5b^3\sqrt{a+b\sqrt{x}}}{32a^3\sqrt{x}} + \frac{5b^4\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)}{32a^{7/2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 43, normalized size = 0.32

$$-\frac{4b^4\left(a+b\sqrt{x}\right)^{3/2}\,{}_2F_1\left(\frac{3}{2}, 5; \frac{5}{2}; \frac{\sqrt{x}\,b}{a}+1\right)}{3a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[x]]/x^3,x]
```

```
[Out] (-4*b^4*(a + b*Sqrt[x])^(3/2)*Hypergeometric2F1[3/2, 5, 5/2, 1 + (b*Sqrt[x]
)/a])/(3*a^5)
```

fricas [A] time = 1.10, size = 184, normalized size = 1.38

$$\left[\frac{15\sqrt{a}\,b^4x^2\log\left(\frac{bx+2\sqrt{b\sqrt{x}+a}\,\sqrt{a}\,\sqrt{x}+2a\sqrt{x}}{x}\right)+2\left(10a^2b^2x-48a^4-\left(15ab^3x+8a^3b\right)\sqrt{x}\right)\sqrt{b\sqrt{x}+a}}{192a^4x^2}, -\frac{15\sqrt{-a}\,b^4x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/192*(15*sqrt(a)*b^4*x^2*log((b*x + 2*sqrt(b*sqrt(x) + a)*sqrt(a)*sqrt(x)
 + 2*a*sqrt(x))/x) + 2*(10*a^2*b^2*x - 48*a^4 - (15*a*b^3*x + 8*a^3*b)*sqrt
```

```
(x))*sqrt(b*sqrt(x) + a))/(a^4*x^2), -1/96*(15*sqrt(-a)*b^4*x^2*arctan(sqrt
(b*sqrt(x) + a)*sqrt(-a)/a) - (10*a^2*b^2*x - 48*a^4 - (15*a*b^3*x + 8*a^3*
b)*sqrt(x))*sqrt(b*sqrt(x) + a))/(a^4*x^2)]
```

giac [A] time = 0.17, size = 109, normalized size = 0.82

$$-\frac{\frac{15\,b^5\arctan\left(\frac{\sqrt{b\sqrt{x}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^3}+\frac{15\left(b\sqrt{x}+a\right)^{\frac{7}{2}}b^5-55\left(b\sqrt{x}+a\right)^{\frac{5}{2}}ab^5+73\left(b\sqrt{x}+a\right)^{\frac{3}{2}}a^2b^5+15\sqrt{b\sqrt{x}+a}\,a^3b^5}{a^3b^4x^2}}{96\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/96*(15*b^5*arctan(sqrt(b*sqrt(x) + a)/sqrt(-a))/(sqrt(-a)*a^3) + (15*(b*
sqrt(x) + a)^(7/2)*b^5 - 55*(b*sqrt(x) + a)^(5/2)*a*b^5 + 73*(b*sqrt(x) + a
)^(3/2)*a^2*b^5 + 15*sqrt(b*sqrt(x) + a)*a^3*b^5)/(a^3*b^4*x^2))/b
```

maple [A] time = 0.01, size = 87, normalized size = 0.65

$$4\left(\frac{5\,\operatorname{arctanh}\left(\frac{\sqrt{b\sqrt{x}+a}}{\sqrt{a}}\right)}{128\,a^{\frac{7}{2}}}+\frac{-\frac{73\left(b\sqrt{x}+a\right)^{\frac{3}{2}}}{384a}+\frac{55\left(b\sqrt{x}+a\right)^{\frac{5}{2}}}{384a^2}-\frac{5\left(b\sqrt{x}+a\right)^{\frac{7}{2}}}{128a^3}-\frac{5\sqrt{b\sqrt{x}+a}}{128}}{b^4x^2}\right)b^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(1/2)+a)^(1/2)/x^3,x)
```

```
[Out] 4*b^4*((-5/128/a^3*(b*x^(1/2)+a)^(7/2)+55/384/a^2*(b*x^(1/2)+a)^(5/2)-73/38
4*(b*x^(1/2)+a)^(3/2)/a-5/128*(b*x^(1/2)+a)^(1/2))/x^2/b^4+5/128/a^(7/2)*ar
ctanh((b*x^(1/2)+a)^(1/2)/a^(1/2)))
```

maxima [A] time = 1.90, size = 166, normalized size = 1.25

$$-\frac{5\,b^4\log\left(\frac{\sqrt{b\sqrt{x}+a}-\sqrt{a}}{\sqrt{b\sqrt{x}+a}+\sqrt{a}}\right)}{64\,a^{\frac{7}{2}}}-\frac{15\left(b\sqrt{x}+a\right)^{\frac{7}{2}}b^4-55\left(b\sqrt{x}+a\right)^{\frac{5}{2}}ab^4+73\left(b\sqrt{x}+a\right)^{\frac{3}{2}}a^2b^4+15\sqrt{b\sqrt{x}+a}\,a^3b^4}{96\left(\left(b\sqrt{x}+a\right)^4a^3-4\left(b\sqrt{x}+a\right)^3a^4+6\left(b\sqrt{x}+a\right)^2a^5-4\left(b\sqrt{x}+a\right)a^6+a^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -5/64*b^4*log((sqrt(b*sqrt(x) + a) - sqrt(a))/(sqrt(b*sqrt(x) + a) + sqrt(a
)))/a^(7/2) - 1/96*(15*(b*sqrt(x) + a)^(7/2)*b^4 - 55*(b*sqrt(x) + a)^(5/2)
*a*b^4 + 73*(b*sqrt(x) + a)^(3/2)*a^2*b^4 + 15*sqrt(b*sqrt(x) + a)*a^3*b^4)
/((b*sqrt(x) + a)^4*a^3 - 4*(b*sqrt(x) + a)^3*a^4 + 6*(b*sqrt(x) + a)^2*a^5
 - 4*(b*sqrt(x) + a)*a^6 + a^7)
```

mupad [B] time = 1.42, size = 91, normalized size = 0.68

$$\frac{55\left(a+b\sqrt{x}\right)^{5/2}}{96\,a^2\,x^2}-\frac{73\left(a+b\sqrt{x}\right)^{3/2}}{96\,a\,x^2}-\frac{5\sqrt{a+b\sqrt{x}}}{32\,x^2}-\frac{5\left(a+b\sqrt{x}\right)^{7/2}}{32\,a^3\,x^2}-\frac{b^4\,\operatorname{atan}\left(\frac{\sqrt{a+b\sqrt{x}}\,1\mathrm{i}}{\sqrt{a}}\right)5\mathrm{i}}{32\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^(1/2)/x^3,x)
```

```
[Out] (55*(a + b*x^(1/2))^(5/2))/(96*a^2*x^2) - (b^4*atan(((a + b*x^(1/2))^(1/2)*
1i)/a^(1/2))*5i)/(32*a^(7/2)) - (73*(a + b*x^(1/2))^(3/2))/(96*a*x^2) - (5*
(a + b*x^(1/2))^(1/2))/(32*x^2) - (5*(a + b*x^(1/2))^(7/2))/(32*a^3*x^2)
```

sympy [**A**] time = 10.26, size = 170, normalized size = 1.28

$$-\frac{a}{2\sqrt{b}\,x^{\frac{9}{4}}\sqrt{\frac{a}{b\sqrt{x}}+1}}-\frac{7\sqrt{b}}{12x^{\frac{7}{4}}\sqrt{\frac{a}{b\sqrt{x}}+1}}+\frac{b^{\frac{3}{2}}}{48ax^{\frac{5}{4}}\sqrt{\frac{a}{b\sqrt{x}}+1}}-\frac{5b^{\frac{5}{2}}}{96a^2x^{\frac{3}{4}}\sqrt{\frac{a}{b\sqrt{x}}+1}}-\frac{5b^{\frac{7}{2}}}{32a^3\sqrt[4]{x}\sqrt{\frac{a}{b\sqrt{x}}+1}}+\frac{5b^4\operatorname{asinh}\left(\frac{\sqrt{}}{\sqrt{b}}\right.}{32a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**(1/2)/x**3,x)
```

```
[Out] -a/(2*sqrt(b)*x**(9/4)*sqrt(a/(b*sqrt(x)) + 1)) - 7*sqrt(b)/(12*x**(7/4)*sq
rt(a/(b*sqrt(x)) + 1)) + b**(3/2)/(48*a*x**(5/4)*sqrt(a/(b*sqrt(x)) + 1)) -
 5*b**(5/2)/(96*a**2*x**(3/4)*sqrt(a/(b*sqrt(x)) + 1)) - 5*b**(7/2)/(32*a**
3*x**(1/4)*sqrt(a/(b*sqrt(x)) + 1)) + 5*b**4*asinh(sqrt(a)/(sqrt(b)*x**(1/4
)))/(32*a**(7/2))
```

3.2239 $\int \frac{x^2}{\sqrt{a+b\sqrt{x}}}\,dx$

Optimal. Leaf size=130

$$-\frac{4a^5\sqrt{a+b\sqrt{x}}}{b^6}+\frac{20a^4\left(a+b\sqrt{x}\right)^{3/2}}{3b^6}-\frac{8a^3\left(a+b\sqrt{x}\right)^{5/2}}{b^6}+\frac{40a^2\left(a+b\sqrt{x}\right)^{7/2}}{7b^6}+\frac{4\left(a+b\sqrt{x}\right)^{11/2}}{11b^6}-\frac{20a\left(a+b\sqrt{x}\right.}{9b^6}$$

```
[Out] 20/3*a^4*(a+b*x^(1/2))^(3/2)/b^6-8*a^3*(a+b*x^(1/2))^(5/2)/b^6+40/7*a^2*(a+
b*x^(1/2))^(7/2)/b^6-20/9*a*(a+b*x^(1/2))^(9/2)/b^6+4/11*(a+b*x^(1/2))^(11/
2)/b^6-4*a^5*(a+b*x^(1/2))^(1/2)/b^6
```

Rubi [A] time = 0.06, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$\frac{40a^2\left(a+b\sqrt{x}\right)^{7/2}}{7b^6}-\frac{8a^3\left(a+b\sqrt{x}\right)^{5/2}}{b^6}+\frac{20a^4\left(a+b\sqrt{x}\right)^{3/2}}{3b^6}-\frac{4a^5\sqrt{a+b\sqrt{x}}}{b^6}+\frac{4\left(a+b\sqrt{x}\right)^{11/2}}{11b^6}-\frac{20a\left(a+b\sqrt{x}\right)^{9}}{9b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*Sqrt[x]],x]
```

```
[Out] (-4*a^5*Sqrt[a + b*Sqrt[x]])/b^6 + (20*a^4*(a + b*Sqrt[x])^(3/2))/(3*b^6) -
 (8*a^3*(a + b*Sqrt[x])^(5/2))/b^6 + (40*a^2*(a + b*Sqrt[x])^(7/2))/(7*b^6)
 - (20*a*(a + b*Sqrt[x])^(9/2))/(9*b^6) + (4*(a + b*Sqrt[x])^(11/2))/(11*b^
6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+b\sqrt{x}}}\,dx = 2\,\text{Subst}\left(\int \frac{x^5}{\sqrt{a+bx}}\,dx, x, \sqrt{x}\right)$$

$$= 2\,\text{Subst}\left(\int\left(-\frac{a^5}{b^5\sqrt{a+bx}}+\frac{5a^4\sqrt{a+bx}}{b^5}-\frac{10a^3(a+bx)^{3/2}}{b^5}+\frac{10a^2(a+bx)^{5/2}}{b^5}-\frac{5a(a+b}{b^5}\right.\right.$$

$$= -\frac{4a^5\sqrt{a+b\sqrt{x}}}{b^6}+\frac{20a^4\left(a+b\sqrt{x}\right)^{3/2}}{3b^6}-\frac{8a^3\left(a+b\sqrt{x}\right)^{5/2}}{b^6}+\frac{40a^2\left(a+b\sqrt{x}\right)^{7/2}}{7b^6}-\frac{20a\left(a\right.}{}$$

Mathematica [A] time = 0.06, size = 78, normalized size = 0.60

$$\frac{4\sqrt{a+b\sqrt{x}}\,\left(-256a^5+128a^4b\sqrt{x}-96a^3b^2x+80a^2b^3x^{3/2}-70ab^4x^2+63b^5x^{5/2}\right)}{693b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b*Sqrt[x]],x]
```

```
[Out] (4*Sqrt[a + b*Sqrt[x]]*(-256*a^5 + 128*a^4*b*Sqrt[x] - 96*a^3*b^2*x + 80*a^
2*b^3*x^(3/2) - 70*a*b^4*x^2 + 63*b^5*x^(5/2)))/(693*b^6)
```

fricas [A] time = 0.91, size = 67, normalized size = 0.52

$$-\frac{4\left(70\,ab^4x^2+96\,a^3b^2x+256\,a^5-\left(63\,b^5x^2+80\,a^2b^3x+128\,a^4b\right)\sqrt{x}\right)\sqrt{b\sqrt{x}+a}}{693\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -4/693*(70*a*b^4*x^2 + 96*a^3*b^2*x + 256*a^5 - (63*b^5*x^2 + 80*a^2*b^3*x
+ 128*a^4*b)*sqrt(x))*sqrt(b*sqrt(x) + a)/b^6
```

giac [A] time = 0.16, size = 85, normalized size = 0.65

$$\frac{4\left(63\left(b\sqrt{x}+a\right)^{\frac{11}{2}}-385\left(b\sqrt{x}+a\right)^{\frac{9}{2}}a+990\left(b\sqrt{x}+a\right)^{\frac{7}{2}}a^2-1386\left(b\sqrt{x}+a\right)^{\frac{5}{2}}a^3+1155\left(b\sqrt{x}+a\right)^{\frac{3}{2}}a^4-693\sqrt{}\right.}{693\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 4/693*(63*(b*sqrt(x) + a)^(11/2) - 385*(b*sqrt(x) + a)^(9/2)*a + 990*(b*sqr
t(x) + a)^(7/2)*a^2 - 1386*(b*sqrt(x) + a)^(5/2)*a^3 + 1155*(b*sqrt(x) + a)
^(3/2)*a^4 - 693*sqrt(b*sqrt(x) + a)*a^5)/b^6
```

maple [A] time = 0.00, size = 86, normalized size = 0.66

$$\frac{-4\sqrt{b\sqrt{x}+a}\,a^5+\frac{20\left(b\sqrt{x}+a\right)^{\frac{3}{2}}a^4}{3}-8\left(b\sqrt{x}+a\right)^{\frac{5}{2}}a^3+\frac{40\left(b\sqrt{x}+a\right)^{\frac{7}{2}}a^2}{7}-\frac{20\left(b\sqrt{x}+a\right)^{\frac{9}{2}}a}{9}+\frac{4\left(b\sqrt{x}+a\right)^{\frac{11}{2}}}{11}}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^(1/2)+a)^(1/2),x)
```

```
[Out] 4/b^6*(1/11*(b*x^(1/2)+a)^(11/2)-5/9*a*(b*x^(1/2)+a)^(9/2)+10/7*a^2*(b*x^(1
/2)+a)^(7/2)-2*a^3*(b*x^(1/2)+a)^(5/2)+5/3*a^4*(b*x^(1/2)+a)^(3/2)-(b*x^(1/
2)+a)^(1/2)*a^5)
```

maxima [A] time = 0.88, size = 98, normalized size = 0.75

$$\frac{4\left(b\sqrt{x}+a\right)^{\frac{11}{2}}}{11\,b^6}-\frac{20\left(b\sqrt{x}+a\right)^{\frac{9}{2}}a}{9\,b^6}+\frac{40\left(b\sqrt{x}+a\right)^{\frac{7}{2}}a^2}{7\,b^6}-\frac{8\left(b\sqrt{x}+a\right)^{\frac{5}{2}}a^3}{b^6}+\frac{20\left(b\sqrt{x}+a\right)^{\frac{3}{2}}a^4}{3\,b^6}-\frac{4\sqrt{b\sqrt{x}+a}\,a^5}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/11*(b*sqrt(x) + a)^(11/2)/b^6 - 20/9*(b*sqrt(x) + a)^(9/2)*a/b^6 + 40/7*(
b*sqrt(x) + a)^(7/2)*a^2/b^6 - 8*(b*sqrt(x) + a)^(5/2)*a^3/b^6 + 20/3*(b*sq
rt(x) + a)^(3/2)*a^4/b^6 - 4*sqrt(b*sqrt(x) + a)*a^5/b^6
```

mupad [B] time = 1.46, size = 98, normalized size = 0.75

$$\frac{4\left(a+b\sqrt{x}\right)^{11/2}}{11\,b^6}-\frac{20\,a\left(a+b\sqrt{x}\right)^{9/2}}{9\,b^6}-\frac{4\,a^5\sqrt{a+b\sqrt{x}}}{b^6}+\frac{20\,a^4\left(a+b\sqrt{x}\right)^{3/2}}{3\,b^6}-\frac{8\,a^3\left(a+b\sqrt{x}\right)^{5/2}}{b^6}+\frac{40\,a^2\left(a+b\sqrt{x}\right.}{7\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^(1/2))^(1/2),x)
```

```
[Out] (4*(a + b*x^(1/2))^(11/2))/(11*b^6) - (20*a*(a + b*x^(1/2))^(9/2))/(9*b^6)
- (4*a^5*(a + b*x^(1/2))^(1/2))/b^6 + (20*a^4*(a + b*x^(1/2))^(3/2))/(3*b^6
) - (8*a^3*(a + b*x^(1/2))^(5/2))/b^6 + (40*a^2*(a + b*x^(1/2))^(7/2))/(7*b
^6)
```

sympy [**B**] time = 9.64, size = 8356, normalized size = 64.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*x**(1/2))**(1/2),x)
```

```
[Out] -1024*a**(151/2)*x**18*sqrt(1 + b*sqrt(x)/a)/(693*a**70*b**6*x**18 + 10395*
a**69*b**7*x**(37/2) + 72765*a**68*b**8*x**19 + 315315*a**67*b**9*x**(39/2)
 + 945945*a**66*b**10*x**20 + 2081079*a**65*b**11*x**(41/2) + 3468465*a**64
*b**12*x**21 + 4459455*a**63*b**13*x**(43/2) + 4459455*a**62*b**14*x**22 +
3468465*a**61*b**15*x**(45/2) + 2081079*a**60*b**16*x**23 + 945945*a**59*b*
*17*x**(47/2) + 315315*a**58*b**18*x**24 + 72765*a**57*b**19*x**(49/2) + 10
395*a**56*b**20*x**25 + 693*a**55*b**21*x**(51/2)) + 1024*a**(151/2)*x**18/
(693*a**70*b**6*x**18 + 10395*a**69*b**7*x**(37/2) + 72765*a**68*b**8*x**19
 + 315315*a**67*b**9*x**(39/2) + 945945*a**66*b**10*x**20 + 2081079*a**65*b
**11*x**(41/2) + 3468465*a**64*b**12*x**21 + 4459455*a**63*b**13*x**(43/2)
+ 4459455*a**62*b**14*x**22 + 3468465*a**61*b**15*x**(45/2) + 2081079*a**60
*b**16*x**23 + 945945*a**59*b**17*x**(47/2) + 315315*a**58*b**18*x**24 + 72
765*a**57*b**19*x**(49/2) + 10395*a**56*b**20*x**25 + 693*a**55*b**21*x**(5
1/2)) - 14848*a**(149/2)*b*x**(37/2)*sqrt(1 + b*sqrt(x)/a)/(693*a**70*b**6*
x**18 + 10395*a**69*b**7*x**(37/2) + 72765*a**68*b**8*x**19 + 315315*a**67*
b**9*x**(39/2) + 945945*a**66*b**10*x**20 + 2081079*a**65*b**11*x**(41/2) +
 3468465*a**64*b**12*x**21 + 4459455*a**63*b**13*x**(43/2) + 4459455*a**62*
b**14*x**22 + 3468465*a**61*b**15*x**(45/2) + 2081079*a**60*b**16*x**23 + 9
45945*a**59*b**17*x**(47/2) + 315315*a**58*b**18*x**24 + 72765*a**57*b**19*
x**(49/2) + 10395*a**56*b**20*x**25 + 693*a**55*b**21*x**(51/2)) + 15360*a*
*(149/2)*b*x**(37/2)/(693*a**70*b**6*x**18 + 10395*a**69*b**7*x**(37/2) + 7
2765*a**68*b**8*x**19 + 315315*a**67*b**9*x**(39/2) + 945945*a**66*b**10*x*
*20 + 2081079*a**65*b**11*x**(41/2) + 3468465*a**64*b**12*x**21 + 4459455*a
**63*b**13*x**(43/2) + 4459455*a**62*b**14*x**22 + 3468465*a**61*b**15*x**(
45/2) + 2081079*a**60*b**16*x**23 + 945945*a**59*b**17*x**(47/2) + 315315*a
**58*b**18*x**24 + 72765*a**57*b**19*x**(49/2) + 10395*a**56*b**20*x**25 +
693*a**55*b**21*x**(51/2)) - 100224*a**(147/2)*b**2*x**19*sqrt(1 + b*sqrt(x
)/a)/(693*a**70*b**6*x**18 + 10395*a**69*b**7*x**(37/2) + 72765*a**68*b**8*
x**19 + 315315*a**67*b**9*x**(39/2) + 945945*a**66*b**10*x**20 + 2081079*a*
*65*b**11*x**(41/2) + 3468465*a**64*b**12*x**21 + 4459455*a**63*b**13*x**(4
3/2) + 4459455*a**62*b**14*x**22 + 3468465*a**61*b**15*x**(45/2) + 2081079*
a**60*b**16*x**23 + 945945*a**59*b**17*x**(47/2) + 315315*a**58*b**18*x**24
 + 72765*a**57*b**19*x**(49/2) + 10395*a**56*b**20*x**25 + 693*a**55*b**21*
x**(51/2)) + 107520*a**(147/2)*b**2*x**19/(693*a**70*b**6*x**18 + 10395*a**
69*b**7*x**(37/2) + 72765*a**68*b**8*x**19 + 315315*a**67*b**9*x**(39/2) +
945945*a**66*b**10*x**20 + 2081079*a**65*b**11*x**(41/2) + 3468465*a**64*b*
*12*x**21 + 4459455*a**63*b**13*x**(43/2) + 4459455*a**62*b**14*x**22 + 346
8465*a**61*b**15*x**(45/2) + 2081079*a**60*b**16*x**23 + 945945*a**59*b**17
*x**(47/2) + 315315*a**58*b**18*x**24 + 72765*a**57*b**19*x**(49/2) + 10395
*a**56*b**20*x**25 + 693*a**55*b**21*x**(51/2)) - 417600*a**(145/2)*b**3*x*
*(39/2)*sqrt(1 + b*sqrt(x)/a)/(693*a**70*b**6*x**18 + 10395*a**69*b**7*x**(
37/2) + 72765*a**68*b**8*x**19 + 315315*a**67*b**9*x**(39/2) + 945945*a**66
*b**10*x**20 + 2081079*a**65*b**11*x**(41/2) + 3468465*a**64*b**12*x**21 +
4459455*a**63*b**13*x**(43/2) + 4459455*a**62*b**14*x**22 + 3468465*a**61*b
```

```
**15*x**(45/2) + 2081079*a**60*b**16*x**23 + 945945*a**59*b**17*x**(47/2) +
 315315*a**58*b**18*x**24 + 72765*a**57*b**19*x**(49/2) + 10395*a**56*b**20
*x**25 + 693*a**55*b**21*x**(51/2)) + 465920*a**(145/2)*b**3*x**(39/2)/(693
*a**70*b**6*x**18 + 10395*a**69*b**7*x**(37/2) + 72765*a**68*b**8*x**19 + 3
15315*a**67*b**9*x**(39/2) + 945945*a**66*b**10*x**20 + 2081079*a**65*b**11
*x**(41/2) + 3468465*a**64*b**12*x**21 + 4459455*a**63*b**13*x**(43/2) + 44
59455*a**62*b**14*x**22 + 3468465*a**61*b**15*x**(45/2) + 2081079*a**60*b**
16*x**23 + 945945*a**59*b**17*x**(47/2) + 315315*a**58*b**18*x**24 + 72765*
a**57*b**19*x**(49/2) + 10395*a**56*b**20*x**25 + 693*a**55*b**21*x**(51/2)
) - 1200600*a**(143/2)*b**4*x**20*sqrt(1 + b*sqrt(x)/a)/(693*a**70*b**6*x**
18 + 10395*a**69*b**7*x**(37/2) + 72765*a**68*b**8*x**19 + 315315*a**67*b**
9*x**(39/2) + 945945*a**66*b**10*x**20 + 2081079*a**65*b**11*x**(41/2) + 34
68465*a**64*b**12*x**21 + 4459455*a**63*b**13*x**(43/2) + 4459455*a**62*b**
14*x**22 + 3468465*a**61*b**15*x**(45/2) + 2081079*a**60*b**16*x**23 + 9459
45*a**59*b**17*x**(47/2) + 315315*a**58*b**18*x**24 + 72765*a**57*b**19*x**
(49/2) + 10395*a**56*b**20*x**25 + 693*a**55*b**21*x**(51/2)) + 1397760*a**
(143/2)*b**4*x**20/(693*a**70*b**6*x**18 + 10395*a**69*b**7*x**(37/2) + 727
65*a**68*b**8*x**19 + 315315*a**67*b**9*x**(39/2) + 945945*a**66*b**10*x**2
0 + 2081079*a**65*b**11*x**(41/2) + 3468465*a**64*b**12*x**21 + 4459455*a**
63*b**13*x**(43/2) + 4459455*a**62*b**14*x**22 + 3468465*a**61*b**15*x**(45
/2) + 2081079*a**60*b**16*x**23 + 945945*a**59*b**17*x**(47/2) + 315315*a**
58*b**18*x**24 + 72765*a**57*b**19*x**(49/2) + 10395*a**56*b**20*x**25 + 69
3*a**55*b**21*x**(51/2)) - 2521260*a**(141/2)*b**5*x**(41/2)*sqrt(1 + b*sqr
t(x)/a)/(693*a**70*b**6*x**18 + 10395*a**69*b**7*x**(37/2) + 72765*a**68*b*
*8*x**19 + 315315*a**67*b**9*x**(39/2) + 945945*a**66*b**10*x**20 + 2081079
*a**65*b**11*x**(41/2) + 3468465*a**64*b**12*x**21 + 4459455*a**63*b**13*x*
*(43/2) + 4459455*a**62*b**14*x**22 + 3468465*a**61*b**15*x**(45/2) + 20810
79*a**60*b**16*x**23 + 945945*a**59*b**17*x**(47/2) + 315315*a**58*b**18*x*
*24 + 72765*a**57*b**19*x**(49/2) + 10395*a**56*b**20*x**25 + 693*a**55*b**
21*x**(51/2)) + 3075072*a**(141/2)*b**5*x**(41/2)/(693*a**70*b**6*x**18 + 1
0395*a**69*b**7*x**(37/2) + 72765*a**68*b**8*x**19 + 315315*a**67*b**9*x**(
39/2) + 945945*a**66*b**10*x**20 + 2081079*a**65*b**11*x**(41/2) + 3468465*
a**64*b**12*x**21 + 4459455*a**63*b**13*x**(43/2) + 4459455*a**62*b**14*x**
22 + 3468465*a**61*b**15*x**(45/2) + 2081079*a**60*b**16*x**23 + 945945*a**
59*b**17*x**(47/2) + 315315*a**58*b**18*x**24 + 72765*a**57*b**19*x**(49/2)
 + 10395*a**56*b**20*x**25 + 693*a**55*b**21*x**(51/2)) - 3991764*a**(139/2
)*b**6*x**21*sqrt(1 + b*sqrt(x)/a)/(693*a**70*b**6*x**18 + 10395*a**69*b**7
*x**(37/2) + 72765*a**68*b**8*x**19 + 315315*a**67*b**9*x**(39/2) + 945945*
a**66*b**10*x**20 + 2081079*a**65*b**11*x**(41/2) + 3468465*a**64*b**12*x**
21 + 4459455*a**63*b**13*x**(43/2) + 4459455*a**62*b**14*x**22 + 3468465*a*
*61*b**15*x**(45/2) + 2081079*a**60*b**16*x**23 + 945945*a**59*b**17*x**(47
/2) + 315315*a**58*b**18*x**24 + 72765*a**57*b**19*x**(49/2) + 10395*a**56*
b**20*x**25 + 693*a**55*b**21*x**(51/2)) + 5125120*a**(139/2)*b**6*x**21/(6
93*a**70*b**6*x**18 + 10395*a**69*b**7*x**(37/2) + 72765*a**68*b**8*x**19 +
 315315*a**67*b**9*x**(39/2) + 945945*a**66*b**10*x**20 + 2081079*a**65*b**
11*x**(41/2) + 3468465*a**64*b**12*x**21 + 4459455*a**63*b**13*x**(43/2) +
4459455*a**62*b**14*x**22 + 3468465*a**61*b**15*x**(45/2) + 2081079*a**60*b
**16*x**23 + 945945*a**59*b**17*x**(47/2) + 315315*a**58*b**18*x**24 + 7276
5*a**57*b**19*x**(49/2) + 10395*a**56*b**20*x**25 + 693*a**55*b**21*x**(51/
2)) - 4844172*a**(137/2)*b**7*x**(43/2)*sqrt(1 + b*sqrt(x)/a)/(693*a**70*b*
*6*x**18 + 10395*a**69*b**7*x**(37/2) + 72765*a**68*b**8*x**19 + 315315*a**
67*b**9*x**(39/2) + 945945*a**66*b**10*x**20 + 2081079*a**65*b**11*x**(41/2
) + 3468465*a**64*b**12*x**21 + 4459455*a**63*b**13*x**(43/2) + 4459455*a**
62*b**14*x**22 + 3468465*a**61*b**15*x**(45/2) + 2081079*a**60*b**16*x**23
+ 945945*a**59*b**17*x**(47/2) + 315315*a**58*b**18*x**24 + 72765*a**57*b**
19*x**(49/2) + 10395*a**56*b**20*x**25 + 693*a**55*b**21*x**(51/2)) + 65894
40*a**(137/2)*b**7*x**(43/2)/(693*a**70*b**6*x**18 + 10395*a**69*b**7*x**(3
7/2) + 72765*a**68*b**8*x**19 + 315315*a**67*b**9*x**(39/2) + 945945*a**66*
b**10*x**20 + 2081079*a**65*b**11*x**(41/2) + 3468465*a**64*b**12*x**21 + 4
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459455*a**63*b**13*x**(43/2) + 4459455*a**62*b**14*x**22 + 3468465*a**61*b*
*15*x**(45/2) + 2081079*a**60*b**16*x**23 + 945945*a**59*b**17*x**(47/2) +
315315*a**58*b**18*x**24 + 72765*a**57*b**19*x**(49/2) + 10395*a**56*b**20*
x**25 + 693*a**55*b**21*x**(51/2)) - 4523220*a**(135/2)*b**8*x**22*sqrt(1 +
 b*sqrt(x)/a)/(693*a**70*b**6*x**18 + 10395*a**69*b**7*x**(37/2) + 72765*a*
*68*b**8*x**19 + 315315*a**67*b**9*x**(39/2) + 945945*a**66*b**10*x**20 + 2
081079*a**65*b**11*x**(41/2) + 3468465*a**64*b**12*x**21 + 4459455*a**63*b*
*13*x**(43/2) + 4459455*a**62*b**14*x**22 + 3468465*a**61*b**15*x**(45/2) +
 2081079*a**60*b**16*x**23 + 945945*a**59*b**17*x**(47/2) + 315315*a**58*b*
*18*x**24 + 72765*a**57*b**19*x**(49/2) + 10395*a**56*b**20*x**25 + 693*a**
55*b**21*x**(51/2)) + 6589440*a**(135/2)*b**8*x**22/(693*a**70*b**6*x**18 +
 10395*a**69*b**7*x**(37/2) + 72765*a**68*b**8*x**19 + 315315*a**67*b**9*x*
*(39/2) + 945945*a**66*b**10*x**20 + 2081079*a**65*b**11*x**(41/2) + 346846
5*a**64*b**12*x**21 + 4459455*a**63*b**13*x**(43/2) + 4459455*a**62*b**14*x
**22 + 3468465*a**61*b**15*x**(45/2) + 2081079*a**60*b**16*x**23 + 945945*a
**59*b**17*x**(47/2) + 315315*a**58*b**18*x**24 + 72765*a**57*b**19*x**(49/
2) + 10395*a**56*b**20*x**25 + 693*a**55*b**21*x**(51/2)) - 3196700*a**(133
/2)*b**9*x**(45/2)*sqrt(1 + b*sqrt(x)/a)/(693*a**70*b**6*x**18 + 10395*a**6
9*b**7*x**(37/2) + 72765*a**68*b**8*x**19 + 315315*a**67*b**9*x**(39/2) + 9
45945*a**66*b**10*x**20 + 2081079*a**65*b**11*x**(41/2) + 3468465*a**64*b**
12*x**21 + 4459455*a**63*b**13*x**(43/2) + 4459455*a**62*b**14*x**22 + 3468
465*a**61*b**15*x**(45/2) + 2081079*a**60*b**16*x**23 + 945945*a**59*b**17*
x**(47/2) + 315315*a**58*b**18*x**24 + 72765*a**57*b**19*x**(49/2) + 10395*
a**56*b**20*x**25 + 693*a**55*b**21*x**(51/2)) + 5125120*a**(133/2)*b**9*x*
*(45/2)/(693*a**70*b**6*x**18 + 10395*a**69*b**7*x**(37/2) + 72765*a**68*b*
*8*x**19 + 315315*a**67*b**9*x**(39/2) + 945945*a**66*b**10*x**20 + 2081079
*a**65*b**11*x**(41/2) + 3468465*a**64*b**12*x**21 + 4459455*a**63*b**13*x*
*(43/2) + 4459455*a**62*b**14*x**22 + 3468465*a**61*b**15*x**(45/2) + 20810
79*a**60*b**16*x**23 + 945945*a**59*b**17*x**(47/2) + 315315*a**58*b**18*x*
*24 + 72765*a**57*b**19*x**(49/2) + 10395*a**56*b**20*x**25 + 693*a**55*b**
21*x**(51/2)) - 1568996*a**(131/2)*b**10*x**23*sqrt(1 + b*sqrt(x)/a)/(693*a
**70*b**6*x**18 + 10395*a**69*b**7*x**(37/2) + 72765*a**68*b**8*x**19 + 315
315*a**67*b**9*x**(39/2) + 945945*a**66*b**10*x**20 + 2081079*a**65*b**11*x
**(41/2) + 3468465*a**64*b**12*x**21 + 4459455*a**63*b**13*x**(43/2) + 4459
455*a**62*b**14*x**22 + 3468465*a**61*b**15*x**(45/2) + 2081079*a**60*b**16
*x**23 + 945945*a**59*b**17*x**(47/2) + 315315*a**58*b**18*x**24 + 72765*a*
*57*b**19*x**(49/2) + 10395*a**56*b**20*x**25 + 693*a**55*b**21*x**(51/2))
+ 3075072*a**(131/2)*b**10*x**23/(693*a**70*b**6*x**18 + 10395*a**69*b**7*x
**(37/2) + 72765*a**68*b**8*x**19 + 315315*a**67*b**9*x**(39/2) + 945945*a*
*66*b**10*x**20 + 2081079*a**65*b**11*x**(41/2) + 3468465*a**64*b**12*x**21
 + 4459455*a**63*b**13*x**(43/2) + 4459455*a**62*b**14*x**22 + 3468465*a**6
1*b**15*x**(45/2) + 2081079*a**60*b**16*x**23 + 945945*a**59*b**17*x**(47/2
) + 315315*a**58*b**18*x**24 + 72765*a**57*b**19*x**(49/2) + 10395*a**56*b*
*20*x**25 + 693*a**55*b**21*x**(51/2)) - 263484*a**(129/2)*b**11*x**(47/2)*
sqrt(1 + b*sqrt(x)/a)/(693*a**70*b**6*x**18 + 10395*a**69*b**7*x**(37/2) +
72765*a**68*b**8*x**19 + 315315*a**67*b**9*x**(39/2) + 945945*a**66*b**10*x
**20 + 2081079*a**65*b**11*x**(41/2) + 3468465*a**64*b**12*x**21 + 4459455*
a**63*b**13*x**(43/2) + 4459455*a**62*b**14*x**22 + 3468465*a**61*b**15*x**
(45/2) + 2081079*a**60*b**16*x**23 + 945945*a**59*b**17*x**(47/2) + 315315*
a**58*b**18*x**24 + 72765*a**57*b**19*x**(49/2) + 10395*a**56*b**20*x**25 +
 693*a**55*b**21*x**(51/2)) + 1397760*a**(129/2)*b**11*x**(47/2)/(693*a**70
*b**6*x**18 + 10395*a**69*b**7*x**(37/2) + 72765*a**68*b**8*x**19 + 315315*
a**67*b**9*x**(39/2) + 945945*a**66*b**10*x**20 + 2081079*a**65*b**11*x**(4
1/2) + 3468465*a**64*b**12*x**21 + 4459455*a**63*b**13*x**(43/2) + 4459455*
a**62*b**14*x**22 + 3468465*a**61*b**15*x**(45/2) + 2081079*a**60*b**16*x**
23 + 945945*a**59*b**17*x**(47/2) + 315315*a**58*b**18*x**24 + 72765*a**57*
b**19*x**(49/2) + 10395*a**56*b**20*x**25 + 693*a**55*b**21*x**(51/2)) + 50
1228*a**(127/2)*b**12*x**24*sqrt(1 + b*sqrt(x)/a)/(693*a**70*b**6*x**18 + 1
0395*a**69*b**7*x**(37/2) + 72765*a**68*b**8*x**19 + 315315*a**67*b**9*x**(
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39/2) + 945945*a**66*b**10*x**20 + 2081079*a**65*b**11*x**(41/2) + 3468465*
a**64*b**12*x**21 + 4459455*a**63*b**13*x**(43/2) + 4459455*a**62*b**14*x**
22 + 3468465*a**61*b**15*x**(45/2) + 2081079*a**60*b**16*x**23 + 945945*a**
59*b**17*x**(47/2) + 315315*a**58*b**18*x**24 + 72765*a**57*b**19*x**(49/2)
 + 10395*a**56*b**20*x**25 + 693*a**55*b**21*x**(51/2)) + 465920*a**(127/2)
*b**12*x**24/(693*a**70*b**6*x**18 + 10395*a**69*b**7*x**(37/2) + 72765*a**
68*b**8*x**19 + 315315*a**67*b**9*x**(39/2) + 945945*a**66*b**10*x**20 + 20
81079*a**65*b**11*x**(41/2) + 3468465*a**64*b**12*x**21 + 4459455*a**63*b**
13*x**(43/2) + 4459455*a**62*b**14*x**22 + 3468465*a**61*b**15*x**(45/2) +
2081079*a**60*b**16*x**23 + 945945*a**59*b**17*x**(47/2) + 315315*a**58*b**
18*x**24 + 72765*a**57*b**19*x**(49/2) + 10395*a**56*b**20*x**25 + 693*a**5
5*b**21*x**(51/2)) + 782460*a**(125/2)*b**13*x**(49/2)*sqrt(1 + b*sqrt(x)/a
)/(693*a**70*b**6*x**18 + 10395*a**69*b**7*x**(37/2) + 72765*a**68*b**8*x**
19 + 315315*a**67*b**9*x**(39/2) + 945945*a**66*b**10*x**20 + 2081079*a**65
*b**11*x**(41/2) + 3468465*a**64*b**12*x**21 + 4459455*a**63*b**13*x**(43/2
) + 4459455*a**62*b**14*x**22 + 3468465*a**61*b**15*x**(45/2) + 2081079*a**
60*b**16*x**23 + 945945*a**59*b**17*x**(47/2) + 315315*a**58*b**18*x**24 +
72765*a**57*b**19*x**(49/2) + 10395*a**56*b**20*x**25 + 693*a**55*b**21*x**
(51/2)) + 107520*a**(125/2)*b**13*x**(49/2)/(693*a**70*b**6*x**18 + 10395*a
**69*b**7*x**(37/2) + 72765*a**68*b**8*x**19 + 315315*a**67*b**9*x**(39/2)
+ 945945*a**66*b**10*x**20 + 2081079*a**65*b**11*x**(41/2) + 3468465*a**64*
b**12*x**21 + 4459455*a**63*b**13*x**(43/2) + 4459455*a**62*b**14*x**22 + 3
468465*a**61*b**15*x**(45/2) + 2081079*a**60*b**16*x**23 + 945945*a**59*b**
17*x**(47/2) + 315315*a**58*b**18*x**24 + 72765*a**57*b**19*x**(49/2) + 103
95*a**56*b**20*x**25 + 693*a**55*b**21*x**(51/2)) + 720900*a**(123/2)*b**14
*x**25*sqrt(1 + b*sqrt(x)/a)/(693*a**70*b**6*x**18 + 10395*a**69*b**7*x**(3
7/2) + 72765*a**68*b**8*x**19 + 315315*a**67*b**9*x**(39/2) + 945945*a**66*
b**10*x**20 + 2081079*a**65*b**11*x**(41/2) + 3468465*a**64*b**12*x**21 + 4
459455*a**63*b**13*x**(43/2) + 4459455*a**62*b**14*x**22 + 3468465*a**61*b*
*15*x**(45/2) + 2081079*a**60*b**16*x**23 + 945945*a**59*b**17*x**(47/2) +
315315*a**58*b**18*x**24 + 72765*a**57*b**19*x**(49/2) + 10395*a**56*b**20*
x**25 + 693*a**55*b**21*x**(51/2)) + 15360*a**(123/2)*b**14*x**25/(693*a**7
0*b**6*x**18 + 10395*a**69*b**7*x**(37/2) + 72765*a**68*b**8*x**19 + 315315
*a**67*b**9*x**(39/2) + 945945*a**66*b**10*x**20 + 2081079*a**65*b**11*x**(
41/2) + 3468465*a**64*b**12*x**21 + 4459455*a**63*b**13*x**(43/2) + 4459455
*a**62*b**14*x**22 + 3468465*a**61*b**15*x**(45/2) + 2081079*a**60*b**16*x*
*23 + 945945*a**59*b**17*x**(47/2) + 315315*a**58*b**18*x**24 + 72765*a**57
*b**19*x**(49/2) + 10395*a**56*b**20*x**25 + 693*a**55*b**21*x**(51/2)) + 4
86492*a**(121/2)*b**15*x**(51/2)*sqrt(1 + b*sqrt(x)/a)/(693*a**70*b**6*x**1
8 + 10395*a**69*b**7*x**(37/2) + 72765*a**68*b**8*x**19 + 315315*a**67*b**9
*x**(39/2) + 945945*a**66*b**10*x**20 + 2081079*a**65*b**11*x**(41/2) + 346
8465*a**64*b**12*x**21 + 4459455*a**63*b**13*x**(43/2) + 4459455*a**62*b**1
4*x**22 + 3468465*a**61*b**15*x**(45/2) + 2081079*a**60*b**16*x**23 + 94594
5*a**59*b**17*x**(47/2) + 315315*a**58*b**18*x**24 + 72765*a**57*b**19*x**(
49/2) + 10395*a**56*b**20*x**25 + 693*a**55*b**21*x**(51/2)) + 1024*a**(121
/2)*b**15*x**(51/2)/(693*a**70*b**6*x**18 + 10395*a**69*b**7*x**(37/2) + 72
765*a**68*b**8*x**19 + 315315*a**67*b**9*x**(39/2) + 945945*a**66*b**10*x**
20 + 2081079*a**65*b**11*x**(41/2) + 3468465*a**64*b**12*x**21 + 4459455*a*
*63*b**13*x**(43/2) + 4459455*a**62*b**14*x**22 + 3468465*a**61*b**15*x**(4
5/2) + 2081079*a**60*b**16*x**23 + 945945*a**59*b**17*x**(47/2) + 315315*a*
*58*b**18*x**24 + 72765*a**57*b**19*x**(49/2) + 10395*a**56*b**20*x**25 + 6
93*a**55*b**21*x**(51/2)) + 244932*a**(119/2)*b**16*x**26*sqrt(1 + b*sqrt(x
)/a)/(693*a**70*b**6*x**18 + 10395*a**69*b**7*x**(37/2) + 72765*a**68*b**8*
x**19 + 315315*a**67*b**9*x**(39/2) + 945945*a**66*b**10*x**20 + 2081079*a*
*65*b**11*x**(41/2) + 3468465*a**64*b**12*x**21 + 4459455*a**63*b**13*x**(4
3/2) + 4459455*a**62*b**14*x**22 + 3468465*a**61*b**15*x**(45/2) + 2081079*
a**60*b**16*x**23 + 945945*a**59*b**17*x**(47/2) + 315315*a**58*b**18*x**24
 + 72765*a**57*b**19*x**(49/2) + 10395*a**56*b**20*x**25 + 693*a**55*b**21*
x**(51/2)) + 89676*a**(117/2)*b**17*x**(53/2)*sqrt(1 + b*sqrt(x)/a)/(693*a*
```

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*70*b**6*x**18 + 10395*a**69*b**7*x**(37/2) + 72765*a**68*b**8*x**19 + 3153
15*a**67*b**9*x**(39/2) + 945945*a**66*b**10*x**20 + 2081079*a**65*b**11*x*
*(41/2) + 3468465*a**64*b**12*x**21 + 4459455*a**63*b**13*x**(43/2) + 44594
55*a**62*b**14*x**22 + 3468465*a**61*b**15*x**(45/2) + 2081079*a**60*b**16*
x**23 + 945945*a**59*b**17*x**(47/2) + 315315*a**58*b**18*x**24 + 72765*a**
57*b**19*x**(49/2) + 10395*a**56*b**20*x**25 + 693*a**55*b**21*x**(51/2)) +
 22580*a**(115/2)*b**18*x**27*sqrt(1 + b*sqrt(x)/a)/(693*a**70*b**6*x**18 +
 10395*a**69*b**7*x**(37/2) + 72765*a**68*b**8*x**19 + 315315*a**67*b**9*x*
*(39/2) + 945945*a**66*b**10*x**20 + 2081079*a**65*b**11*x**(41/2) + 346846
5*a**64*b**12*x**21 + 4459455*a**63*b**13*x**(43/2) + 4459455*a**62*b**14*x
**22 + 3468465*a**61*b**15*x**(45/2) + 2081079*a**60*b**16*x**23 + 945945*a
**59*b**17*x**(47/2) + 315315*a**58*b**18*x**24 + 72765*a**57*b**19*x**(49/
2) + 10395*a**56*b**20*x**25 + 693*a**55*b**21*x**(51/2)) + 3500*a**(113/2)
*b**19*x**(55/2)*sqrt(1 + b*sqrt(x)/a)/(693*a**70*b**6*x**18 + 10395*a**69*
b**7*x**(37/2) + 72765*a**68*b**8*x**19 + 315315*a**67*b**9*x**(39/2) + 945
945*a**66*b**10*x**20 + 2081079*a**65*b**11*x**(41/2) + 3468465*a**64*b**12
*x**21 + 4459455*a**63*b**13*x**(43/2) + 4459455*a**62*b**14*x**22 + 346846
5*a**61*b**15*x**(45/2) + 2081079*a**60*b**16*x**23 + 945945*a**59*b**17*x*
*(47/2) + 315315*a**58*b**18*x**24 + 72765*a**57*b**19*x**(49/2) + 10395*a*
*56*b**20*x**25 + 693*a**55*b**21*x**(51/2)) + 252*a**(111/2)*b**20*x**28*s
qrt(1 + b*sqrt(x)/a)/(693*a**70*b**6*x**18 + 10395*a**69*b**7*x**(37/2) + 7
2765*a**68*b**8*x**19 + 315315*a**67*b**9*x**(39/2) + 945945*a**66*b**10*x*
*20 + 2081079*a**65*b**11*x**(41/2) + 3468465*a**64*b**12*x**21 + 4459455*a
**63*b**13*x**(43/2) + 4459455*a**62*b**14*x**22 + 3468465*a**61*b**15*x**(
45/2) + 2081079*a**60*b**16*x**23 + 945945*a**59*b**17*x**(47/2) + 315315*a
**58*b**18*x**24 + 72765*a**57*b**19*x**(49/2) + 10395*a**56*b**20*x**25 +
693*a**55*b**21*x**(51/2))
```

3.2240 $\int \frac{x}{\sqrt{a+b\sqrt{x}}}\,dx$

Optimal. Leaf size=84

$$-\frac{4a^3\sqrt{a+b\sqrt{x}}}{b^4}+\frac{4a^2\left(a+b\sqrt{x}\right)^{3/2}}{b^4}+\frac{4\left(a+b\sqrt{x}\right)^{7/2}}{7b^4}-\frac{12a\left(a+b\sqrt{x}\right)^{5/2}}{5b^4}$$

```
[Out] 4*a^2*(a+b*x^(1/2))^(3/2)/b^4-12/5*a*(a+b*x^(1/2))^(5/2)/b^4+4/7*(a+b*x^(1/
2))^(7/2)/b^4-4*a^3*(a+b*x^(1/2))^(1/2)/b^4
```

Rubi [A] time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{4a^2\left(a+b\sqrt{x}\right)^{3/2}}{b^4}-\frac{4a^3\sqrt{a+b\sqrt{x}}}{b^4}+\frac{4\left(a+b\sqrt{x}\right)^{7/2}}{7b^4}-\frac{12a\left(a+b\sqrt{x}\right)^{5/2}}{5b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*Sqrt[x]],x]
```

```
[Out] (-4*a^3*Sqrt[a + b*Sqrt[x]])/b^4 + (4*a^2*(a + b*Sqrt[x])^(3/2))/b^4 - (12*
a*(a + b*Sqrt[x])^(5/2))/(5*b^4) + (4*(a + b*Sqrt[x])^(7/2))/(7*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{a+b\sqrt{x}}}\,dx &= 2\,\text{Subst}\left(\int \frac{x^3}{\sqrt{a+bx}}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int\left(-\frac{a^3}{b^3\sqrt{a+bx}}+\frac{3a^2\sqrt{a+bx}}{b^3}-\frac{3a(a+bx)^{3/2}}{b^3}+\frac{(a+bx)^{5/2}}{b^3}\right)dx, x, \sqrt{x}\right)\\ &= -\frac{4a^3\sqrt{a+b\sqrt{x}}}{b^4}+\frac{4a^2\left(a+b\sqrt{x}\right)^{3/2}}{b^4}-\frac{12a\left(a+b\sqrt{x}\right)^{5/2}}{5b^4}+\frac{4\left(a+b\sqrt{x}\right)^{7/2}}{7b^4}\end{aligned}$$

Mathematica [A] time = 0.04, size = 54, normalized size = 0.64

$$\frac{4\sqrt{a+b\sqrt{x}}\left(-16a^3+8a^2b\sqrt{x}-6ab^2x+5b^3x^{3/2}\right)}{35b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b*Sqrt[x]],x]
```

```
[Out] (4*Sqrt[a + b*Sqrt[x]]*(-16*a^3 + 8*a^2*b*Sqrt[x] - 6*a*b^2*x + 5*b^3*x^(3/
2)))/(35*b^4)
```

fricas [A] time = 0.81, size = 45, normalized size = 0.54

$$-\frac{4\left(6\,ab^2x+16\,a^3-\left(5\,b^3x+8\,a^2b\right)\sqrt{x}\right)\sqrt{b\sqrt{x}+a}}{35\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -4/35*(6*a*b^2*x + 16*a^3 - (5*b^3*x + 8*a^2*b)*sqrt(x))*sqrt(b*sqrt(x) + a
)/b^4
```

giac [A] time = 0.16, size = 57, normalized size = 0.68

$$\frac{4\left(5\left(b\sqrt{x}+a\right)^{\frac{7}{2}}-21\left(b\sqrt{x}+a\right)^{\frac{5}{2}}a+35\left(b\sqrt{x}+a\right)^{\frac{3}{2}}a^2-35\sqrt{b\sqrt{x}+a}\,a^3\right)}{35\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 4/35*(5*(b*sqrt(x) + a)^(7/2) - 21*(b*sqrt(x) + a)^(5/2)*a + 35*(b*sqrt(x)
+ a)^(3/2)*a^2 - 35*sqrt(b*sqrt(x) + a)*a^3)/b^4
```

maple [A] time = 0.00, size = 57, normalized size = 0.68

$$\frac{-4\sqrt{b\sqrt{x}+a}\,a^3+4\left(b\sqrt{x}+a\right)^{\frac{3}{2}}a^2-\frac{12\left(b\sqrt{x}+a\right)^{\frac{5}{2}}a}{5}+\frac{4\left(b\sqrt{x}+a\right)^{\frac{7}{2}}}{7}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^(1/2)+a)^(1/2),x)
```

```
[Out] 4/b^4*(1/7*(b*x^(1/2)+a)^(7/2)-3/5*a*(b*x^(1/2)+a)^(5/2)+a^2*(b*x^(1/2)+a)^
(3/2)-(b*x^(1/2)+a)^(1/2)*a^3)
```

maxima [A] time = 0.84, size = 64, normalized size = 0.76

$$\frac{4\left(b\sqrt{x}+a\right)^{\frac{7}{2}}}{7\,b^4}-\frac{12\left(b\sqrt{x}+a\right)^{\frac{5}{2}}a}{5\,b^4}+\frac{4\left(b\sqrt{x}+a\right)^{\frac{3}{2}}a^2}{b^4}-\frac{4\sqrt{b\sqrt{x}+a}\,a^3}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/7*(b*sqrt(x) + a)^(7/2)/b^4 - 12/5*(b*sqrt(x) + a)^(5/2)*a/b^4 + 4*(b*sqr
t(x) + a)^(3/2)*a^2/b^4 - 4*sqrt(b*sqrt(x) + a)*a^3/b^4
```

mupad [B] time = 1.23, size = 64, normalized size = 0.76

$$\frac{4\left(a+b\sqrt{x}\right)^{7/2}}{7\,b^4}-\frac{12\,a\left(a+b\sqrt{x}\right)^{5/2}}{5\,b^4}-\frac{4\,a^3\sqrt{a+b\sqrt{x}}}{b^4}+\frac{4\,a^2\left(a+b\sqrt{x}\right)^{3/2}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^(1/2))^(1/2),x)
```

```
[Out] (4*(a + b*x^(1/2))^(7/2))/(7*b^4) - (12*a*(a + b*x^(1/2))^(5/2))/(5*b^4) -
(4*a^3*(a + b*x^(1/2))^(1/2))/b^4 + (4*a^2*(a + b*x^(1/2))^(3/2))/b^4
```

sympy [**B**] time = 2.95, size = 1872, normalized size = 22.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x**(1/2))**(1/2),x)
```

```
[Out] -64*a**(47/2)*x**8*sqrt(1 + b*sqrt(x)/a)/(35*a**20*b**4*x**8 + 210*a**19*b*
*5*x**(17/2) + 525*a**18*b**6*x**9 + 700*a**17*b**7*x**(19/2) + 525*a**16*b
**8*x**10 + 210*a**15*b**9*x**(21/2) + 35*a**14*b**10*x**11) + 64*a**(47/2)
*x**8/(35*a**20*b**4*x**8 + 210*a**19*b**5*x**(17/2) + 525*a**18*b**6*x**9
+ 700*a**17*b**7*x**(19/2) + 525*a**16*b**8*x**10 + 210*a**15*b**9*x**(21/2
) + 35*a**14*b**10*x**11) - 352*a**(45/2)*b*x**(17/2)*sqrt(1 + b*sqrt(x)/a)
/(35*a**20*b**4*x**8 + 210*a**19*b**5*x**(17/2) + 525*a**18*b**6*x**9 + 700
*a**17*b**7*x**(19/2) + 525*a**16*b**8*x**10 + 210*a**15*b**9*x**(21/2) + 3
5*a**14*b**10*x**11) + 384*a**(45/2)*b*x**(17/2)/(35*a**20*b**4*x**8 + 210*
a**19*b**5*x**(17/2) + 525*a**18*b**6*x**9 + 700*a**17*b**7*x**(19/2) + 525
*a**16*b**8*x**10 + 210*a**15*b**9*x**(21/2) + 35*a**14*b**10*x**11) - 792*
a**(43/2)*b**2*x**9*sqrt(1 + b*sqrt(x)/a)/(35*a**20*b**4*x**8 + 210*a**19*b
**5*x**(17/2) + 525*a**18*b**6*x**9 + 700*a**17*b**7*x**(19/2) + 525*a**16*
b**8*x**10 + 210*a**15*b**9*x**(21/2) + 35*a**14*b**10*x**11) + 960*a**(43/
2)*b**2*x**9/(35*a**20*b**4*x**8 + 210*a**19*b**5*x**(17/2) + 525*a**18*b**
6*x**9 + 700*a**17*b**7*x**(19/2) + 525*a**16*b**8*x**10 + 210*a**15*b**9*x
**(21/2) + 35*a**14*b**10*x**11) - 924*a**(41/2)*b**3*x**(19/2)*sqrt(1 + b*
sqrt(x)/a)/(35*a**20*b**4*x**8 + 210*a**19*b**5*x**(17/2) + 525*a**18*b**6*
x**9 + 700*a**17*b**7*x**(19/2) + 525*a**16*b**8*x**10 + 210*a**15*b**9*x**
(21/2) + 35*a**14*b**10*x**11) + 1280*a**(41/2)*b**3*x**(19/2)/(35*a**20*b*
*4*x**8 + 210*a**19*b**5*x**(17/2) + 525*a**18*b**6*x**9 + 700*a**17*b**7*x
**(19/2) + 525*a**16*b**8*x**10 + 210*a**15*b**9*x**(21/2) + 35*a**14*b**10
*x**11) - 560*a**(39/2)*b**4*x**10*sqrt(1 + b*sqrt(x)/a)/(35*a**20*b**4*x**
8 + 210*a**19*b**5*x**(17/2) + 525*a**18*b**6*x**9 + 700*a**17*b**7*x**(19/
2) + 525*a**16*b**8*x**10 + 210*a**15*b**9*x**(21/2) + 35*a**14*b**10*x**11
) + 960*a**(39/2)*b**4*x**10/(35*a**20*b**4*x**8 + 210*a**19*b**5*x**(17/2)
 + 525*a**18*b**6*x**9 + 700*a**17*b**7*x**(19/2) + 525*a**16*b**8*x**10 +
210*a**15*b**9*x**(21/2) + 35*a**14*b**10*x**11) - 84*a**(37/2)*b**5*x**(21
/2)*sqrt(1 + b*sqrt(x)/a)/(35*a**20*b**4*x**8 + 210*a**19*b**5*x**(17/2) +
525*a**18*b**6*x**9 + 700*a**17*b**7*x**(19/2) + 525*a**16*b**8*x**10 + 210
*a**15*b**9*x**(21/2) + 35*a**14*b**10*x**11) + 384*a**(37/2)*b**5*x**(21/2
)/(35*a**20*b**4*x**8 + 210*a**19*b**5*x**(17/2) + 525*a**18*b**6*x**9 + 70
0*a**17*b**7*x**(19/2) + 525*a**16*b**8*x**10 + 210*a**15*b**9*x**(21/2) +
35*a**14*b**10*x**11) + 168*a**(35/2)*b**6*x**11*sqrt(1 + b*sqrt(x)/a)/(35*
a**20*b**4*x**8 + 210*a**19*b**5*x**(17/2) + 525*a**18*b**6*x**9 + 700*a**1
7*b**7*x**(19/2) + 525*a**16*b**8*x**10 + 210*a**15*b**9*x**(21/2) + 35*a**
14*b**10*x**11) + 64*a**(35/2)*b**6*x**11/(35*a**20*b**4*x**8 + 210*a**19*b
**5*x**(17/2) + 525*a**18*b**6*x**9 + 700*a**17*b**7*x**(19/2) + 525*a**16*
b**8*x**10 + 210*a**15*b**9*x**(21/2) + 35*a**14*b**10*x**11) + 188*a**(33/
2)*b**7*x**(23/2)*sqrt(1 + b*sqrt(x)/a)/(35*a**20*b**4*x**8 + 210*a**19*b**
5*x**(17/2) + 525*a**18*b**6*x**9 + 700*a**17*b**7*x**(19/2) + 525*a**16*b*
*8*x**10 + 210*a**15*b**9*x**(21/2) + 35*a**14*b**10*x**11) + 96*a**(31/2)*
b**8*x**12*sqrt(1 + b*sqrt(x)/a)/(35*a**20*b**4*x**8 + 210*a**19*b**5*x**(1
7/2) + 525*a**18*b**6*x**9 + 700*a**17*b**7*x**(19/2) + 525*a**16*b**8*x**1
0 + 210*a**15*b**9*x**(21/2) + 35*a**14*b**10*x**11) + 20*a**(29/2)*b**9*x*
*(25/2)*sqrt(1 + b*sqrt(x)/a)/(35*a**20*b**4*x**8 + 210*a**19*b**5*x**(17/2
) + 525*a**18*b**6*x**9 + 700*a**17*b**7*x**(19/2) + 525*a**16*b**8*x**10 +
 210*a**15*b**9*x**(21/2) + 35*a**14*b**10*x**11)
```

3.2241 $\int \frac{1}{\sqrt{a+b\sqrt{x}}}\,dx$

Optimal. Leaf size=40

$$\frac{4\left(a+b\sqrt{x}\right)^{3/2}}{3b^2}-\frac{4a\sqrt{a+b\sqrt{x}}}{b^2}$$

```
[Out] 4/3*(a+b*x^(1/2))^(3/2)/b^2-4*a*(a+b*x^(1/2))^(1/2)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {190, 43}

$$\frac{4\left(a+b\sqrt{x}\right)^{3/2}}{3b^2}-\frac{4a\sqrt{a+b\sqrt{x}}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sqrt[x]],x]
```

```
[Out] (-4*a*Sqrt[a + b*Sqrt[x]])/b^2 + (4*(a + b*Sqrt[x])^(3/2))/(3*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+b\sqrt{x}}}\,dx &= 2\,\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int\left(-\frac{a}{b\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b}\right)dx, x, \sqrt{x}\right)\\ &= -\frac{4a\sqrt{a+b\sqrt{x}}}{b^2}+\frac{4\left(a+b\sqrt{x}\right)^{3/2}}{3b^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 31, normalized size = 0.78

$$\frac{4\left(b\sqrt{x}-2a\right)\sqrt{a+b\sqrt{x}}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sqrt[x]],x]
```

```
[Out] (4*(-2*a + b*Sqrt[x])*Sqrt[a + b*Sqrt[x]])/(3*b^2)
```

fricas [A] time = 1.18, size = 23, normalized size = 0.58

$$\frac{4\sqrt{b\sqrt{x}+a}\left(b\sqrt{x}-2a\right)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/2))^(1/2),x, algorithm="fricas")
[Out] 4/3*sqrt(b*sqrt(x) + a)*(b*sqrt(x) - 2*a)/b^2
```

giac [A] time = 0.16, size = 27, normalized size = 0.68

$$\frac{4\left(\left(b\sqrt{x}+a\right)^{\frac{3}{2}}-3\sqrt{b\sqrt{x}+a}\,a\right)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/2))^(1/2),x, algorithm="giac")
[Out] 4/3*((b*sqrt(x) + a)^(3/2) - 3*sqrt(b*sqrt(x) + a)*a)/b^2
```

maple [A] time = 0.00, size = 30, normalized size = 0.75

$$\frac{-4\sqrt{b\sqrt{x}+a}\,a+\frac{4\left(b\sqrt{x}+a\right)^{\frac{3}{2}}}{3}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^(1/2)+a)^(1/2),x)
[Out] 4/b^2*(1/3*(b*x^(1/2)+a)^(3/2)-(b*x^(1/2)+a)^(1/2)*a)
```

maxima [A] time = 0.81, size = 30, normalized size = 0.75

$$\frac{4\left(b\sqrt{x}+a\right)^{\frac{3}{2}}}{3b^2}-\frac{4\sqrt{b\sqrt{x}+a}\,a}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/2))^(1/2),x, algorithm="maxima")
[Out] 4/3*(b*sqrt(x) + a)^(3/2)/b^2 - 4*sqrt(b*sqrt(x) + a)*a/b^2
```

mupad [B] time = 1.20, size = 37, normalized size = 0.92

$$\frac{x\sqrt{\frac{b\sqrt{x}}{a}+1}\;{}_2\mathrm{F}_1\left(\frac{1}{2},2;\;3;\;-\frac{b\sqrt{x}}{a}\right)}{\sqrt{a+b\sqrt{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^(1/2))^(1/2),x)
[Out] (x*((b*x^(1/2))/a + 1)^(1/2)*hypergeom([1/2, 2], 3, -(b*x^(1/2))/a))/(a + b
*x^(1/2))^(1/2)
```

sympy [**B**] time = 1.19, size = 219, normalized size = 5.48

$$-\frac{8a^{\frac{7}{2}}x^2\sqrt{1+\frac{b\sqrt{x}}{a}}}{3a^2b^2x^2+3ab^3x^{\frac{5}{2}}}+\frac{8a^{\frac{7}{2}}x^2}{3a^2b^2x^2+3ab^3x^{\frac{5}{2}}}-\frac{4a^{\frac{5}{2}}bx^{\frac{5}{2}}\sqrt{1+\frac{b\sqrt{x}}{a}}}{3a^2b^2x^2+3ab^3x^{\frac{5}{2}}}+\frac{8a^{\frac{5}{2}}bx^{\frac{5}{2}}}{3a^2b^2x^2+3ab^3x^{\frac{5}{2}}}+\frac{4a^{\frac{3}{2}}b^2x^3\sqrt{1+\frac{b\sqrt{x}}{a}}}{3a^2b^2x^2+3ab^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**(1/2))**(1/2),x)
```

```
[Out] -8*a**(7/2)*x**2*sqrt(1 + b*sqrt(x)/a)/(3*a**2*b**2*x**2 + 3*a*b**3*x**(5/2
)) + 8*a**(7/2)*x**2/(3*a**2*b**2*x**2 + 3*a*b**3*x**(5/2)) - 4*a**(5/2)*b*
x**(5/2)*sqrt(1 + b*sqrt(x)/a)/(3*a**2*b**2*x**2 + 3*a*b**3*x**(5/2)) + 8*a
**(5/2)*b*x**(5/2)/(3*a**2*b**2*x**2 + 3*a*b**3*x**(5/2)) + 4*a**(3/2)*b**2
*x**3*sqrt(1 + b*sqrt(x)/a)/(3*a**2*b**2*x**2 + 3*a*b**3*x**(5/2))
```

3.2242 $\int \frac{1}{\sqrt{a+b\sqrt{x}}\,x}\,dx$

Optimal. Leaf size=27

$$-\frac{4\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -4*arctanh((a+b*x^(1/2))^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {266, 63, 208}

$$-\frac{4\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*Sqrt[x]]*x),x]
```

```
[Out] (-4*ArcTanh[Sqrt[a + b*Sqrt[x]]/Sqrt[a]])/Sqrt[a]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+b\sqrt{x}}\,x}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sqrt{x}\right)\\ &= \frac{4\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sqrt{x}}\right)}{b}\\ &= -\frac{4\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.00

$$-\frac{4\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*Sqrt[x]]*x),x]
```

```
[Out] (-4*ArcTanh[Sqrt[a + b*Sqrt[x]]/Sqrt[a]])/Sqrt[a]
```

fricas [A] time = 1.22, size = 67, normalized size = 2.48

$$\left[\frac{2\log\left(\frac{bx-2\sqrt{b\sqrt{x}+a}\sqrt{a}\sqrt{x}+2a\sqrt{x}}{x}\right)}{\sqrt{a}}, \frac{4\sqrt{-a}\arctan\left(\frac{\sqrt{b\sqrt{x}+a}\sqrt{-a}}{a}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] [2*log((b*x - 2*sqrt(b*sqrt(x) + a)*sqrt(a)*sqrt(x) + 2*a*sqrt(x))/x)/sqrt(
a), 4*sqrt(-a)*arctan(sqrt(b*sqrt(x) + a)*sqrt(-a)/a)/a]
```

giac [A] time = 0.16, size = 23, normalized size = 0.85

$$\frac{4\arctan\left(\frac{\sqrt{b\sqrt{x}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 4*arctan(sqrt(b*sqrt(x) + a)/sqrt(-a))/sqrt(-a)
```

maple [A] time = 0.00, size = 20, normalized size = 0.74

$$-\frac{4\operatorname{arctanh}\left(\frac{\sqrt{b\sqrt{x}+a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^(1/2)+a)^(1/2),x)
```

```
[Out] -4*arctanh((b*x^(1/2)+a)^(1/2)/a^(1/2))/a^(1/2)
```

maxima [A] time = 1.86, size = 37, normalized size = 1.37

$$\frac{2\log\left(\frac{\sqrt{b\sqrt{x}+a}-\sqrt{a}}{\sqrt{b\sqrt{x}+a}+\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*log((sqrt(b*sqrt(x) + a) - sqrt(a))/(sqrt(b*sqrt(x) + a) + sqrt(a)))/sqrt
(a)
```

mupad [B] time = 1.51, size = 19, normalized size = 0.70

$$-\frac{4\,\operatorname{atanh}\left(\frac{\sqrt{a+b\,\sqrt{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^(1/2))^(1/2)),x)
```

```
[Out] -(4*atanh((a + b*x^(1/2))^(1/2)/a^(1/2)))/a^(1/2)
```

sympy [A] time = 1.27, size = 24, normalized size = 0.89

$$-\frac{4\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt[4]{x}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x**(1/2))**(1/2),x)
```

```
[Out] -4*asinh(sqrt(a)/(sqrt(b)*x**(1/4)))/sqrt(a)
```

3.2243 $\int \frac{1}{\sqrt{a+b\sqrt{x}}\, x^2}\, dx$

Optimal. Leaf size=80

$$-\frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)}{2a^{5/2}} + \frac{3b\sqrt{a+b\sqrt{x}}}{2a^2\sqrt{x}} - \frac{\sqrt{a+b\sqrt{x}}}{ax}$$

```
[Out] -3/2*b^2*arctanh((a+b*x^(1/2))^(1/2)/a^(1/2))/a^(5/2)-(a+b*x^(1/2))^(1/2)/a
/x+3/2*b*(a+b*x^(1/2))^(1/2)/a^2/x^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {266, 51, 63, 208}

$$-\frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)}{2a^{5/2}} + \frac{3b\sqrt{a+b\sqrt{x}}}{2a^2\sqrt{x}} - \frac{\sqrt{a+b\sqrt{x}}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*Sqrt[x]]*x^2),x]
```

```
[Out] -(Sqrt[a + b*Sqrt[x]]/(a*x)) + (3*b*Sqrt[a + b*Sqrt[x]])/(2*a^2*Sqrt[x]) -
(3*b^2*ArcTanh[Sqrt[a + b*Sqrt[x]]/Sqrt[a]])/(2*a^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sqrt{x}}\,x^2}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{\sqrt{a+b\sqrt{x}}}{ax} - \frac{(3b)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \sqrt{x}\right)}{2a}$$
$$= -\frac{\sqrt{a+b\sqrt{x}}}{ax} + \frac{3b\sqrt{a+b\sqrt{x}}}{2a^2\sqrt{x}} + \frac{\left(3b^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sqrt{x}\right)}{4a^2}$$
$$= -\frac{\sqrt{a+b\sqrt{x}}}{ax} + \frac{3b\sqrt{a+b\sqrt{x}}}{2a^2\sqrt{x}} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sqrt{x}}\right)}{2a^2}$$
$$= -\frac{\sqrt{a+b\sqrt{x}}}{ax} + \frac{3b\sqrt{a+b\sqrt{x}}}{2a^2\sqrt{x}} - \frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)}{2a^{5/2}}$$

Mathematica [C] time = 0.01, size = 41, normalized size = 0.51

$$-\frac{4b^2\sqrt{a+b\sqrt{x}}\;{}_2F_1\left(\frac{1}{2}, 3; \frac{3}{2}; \frac{\sqrt{x}\,b}{a}+1\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*Sqrt[x]]*x^2),x]
```

```
[Out] (-4*b^2*Sqrt[a + b*Sqrt[x]]*Hypergeometric2F1[1/2, 3, 3/2, 1 + (b*Sqrt[x])/
a])/a^3
```

fricas [A] time = 0.96, size = 137, normalized size = 1.71

$$\left[\frac{3\sqrt{a}\,b^2x\log\left(\frac{bx-2\sqrt{b\sqrt{x}+a}\,\sqrt{a}\,\sqrt{x}+2a\sqrt{x}}{x}\right)+2\left(3ab\sqrt{x}-2a^2\right)\sqrt{b\sqrt{x}+a}}{4a^3x}, \frac{3\sqrt{-a}\,b^2x\arctan\left(\frac{\sqrt{b\sqrt{x}+a}\,\sqrt{-a}}{a}\right)+\left(3ab\right.}{2a^3x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*sqrt(a)*b^2*x*log((b*x - 2*sqrt(b*sqrt(x) + a)*sqrt(a)*sqrt(x) + 2*
a*sqrt(x))/x) + 2*(3*a*b*sqrt(x) - 2*a^2)*sqrt(b*sqrt(x) + a))/(a^3*x), 1/2
*(3*sqrt(-a)*b^2*x*arctan(sqrt(b*sqrt(x) + a)*sqrt(-a)/a) + (3*a*b*sqrt(x)
- 2*a^2)*sqrt(b*sqrt(x) + a))/(a^3*x)]
```

giac [A] time = 0.17, size = 75, normalized size = 0.94

$$\frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{b\sqrt{x}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{3\left(b\sqrt{x}+a\right)^{\frac{3}{2}}b^3-5\sqrt{b\sqrt{x}+a}\,ab^3}{a^2b^2x}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(3*b^3*arctan(sqrt(b*sqrt(x) + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*sqrt
(x) + a)^(3/2)*b^3 - 5*sqrt(b*sqrt(x) + a)*a*b^3)/(a^2*b^2*x))/b
```

maple [A] time = 0.01, size = 72, normalized size = 0.90

$$4\left(-\frac{3\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\sqrt{x}+a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}-\frac{\sqrt{b\sqrt{x}+a}}{2ab\sqrt{x}}\right)}{4a}-\frac{\sqrt{b\sqrt{x}+a}}{4a\,b^2x}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^(1/2)+a)^(1/2),x)
```

```
[Out] 4*b^2*(-1/4*(b*x^(1/2)+a)^(1/2)/a/x/b^2-3/4/a*(-1/2*(b*x^(1/2)+a)^(1/2)/a/x
^(1/2)/b+1/2/a^(3/2)*arctanh((b*x^(1/2)+a)^(1/2)/a^(1/2))))
```

maxima [A] time = 1.92, size = 104, normalized size = 1.30

$$\frac{3\,b^2\log\left(\frac{\sqrt{b\sqrt{x}+a}-\sqrt{a}}{\sqrt{b\sqrt{x}+a}+\sqrt{a}}\right)}{4\,a^{\frac{5}{2}}}+\frac{3\left(b\sqrt{x}+a\right)^{\frac{3}{2}}b^2-5\sqrt{b\sqrt{x}+a}\,ab^2}{2\left(\left(b\sqrt{x}+a\right)^2a^2-2\left(b\sqrt{x}+a\right)a^3+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 3/4*b^2*log((sqrt(b*sqrt(x) + a) - sqrt(a))/(sqrt(b*sqrt(x) + a) + sqrt(a))
)/a^(5/2) + 1/2*(3*(b*sqrt(x) + a)^(3/2)*b^2 - 5*sqrt(b*sqrt(x) + a)*a*b^2)
/((b*sqrt(x) + a)^2*a^2 - 2*(b*sqrt(x) + a)*a^3 + a^4)
```

mupad [B] time = 1.51, size = 57, normalized size = 0.71

$$\frac{3\left(a+b\sqrt{x}\right)^{3/2}}{2\,a^2\,x}-\frac{5\sqrt{a+b\sqrt{x}}}{2\,a\,x}-\frac{3\,b^2\,\operatorname{atanh}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)}{2\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^(1/2))^(1/2)),x)
```

```
[Out] (3*(a + b*x^(1/2))^(3/2))/(2*a^2*x) - (5*(a + b*x^(1/2))^(1/2))/(2*a*x) - (
3*b^2*atanh((a + b*x^(1/2))^(1/2)/a^(1/2)))/(2*a^(5/2))
```

sympy [A] time = 4.86, size = 110, normalized size = 1.38

$$-\frac{1}{\sqrt{b}\,x^{\frac{5}{4}}\sqrt{\frac{a}{b\sqrt{x}}+1}}+\frac{\sqrt{b}}{2ax^{\frac{3}{4}}\sqrt{\frac{a}{b\sqrt{x}}+1}}+\frac{3b^{\frac{3}{2}}}{2a^2\sqrt[4]{x}\sqrt{\frac{a}{b\sqrt{x}}+1}}-\frac{3b^2\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt[4]{x}}\right)}{2a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*x**(1/2))**(1/2),x)
```

```
[Out] -1/(sqrt(b)*x**(5/4)*sqrt(a/(b*sqrt(x)) + 1)) + sqrt(b)/(2*a*x**(3/4)*sqrt(
a/(b*sqrt(x)) + 1)) + 3*b**(3/2)/(2*a**2*x**(1/4)*sqrt(a/(b*sqrt(x)) + 1))
- 3*b**2*asinh(sqrt(a)/(sqrt(b)*x**(1/4)))/(2*a**(5/2))
```

3.2244 $\int \frac{1}{\sqrt{a+b\sqrt{x}}\, x^3}\, dx$

Optimal. Leaf size=136

$$-\frac{35b^4 \tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)}{32a^{9/2}} + \frac{35b^3\sqrt{a+b\sqrt{x}}}{32a^4\sqrt{x}} - \frac{35b^2\sqrt{a+b\sqrt{x}}}{48a^3x} + \frac{7b\sqrt{a+b\sqrt{x}}}{12a^2x^{3/2}} - \frac{\sqrt{a+b\sqrt{x}}}{2ax^2}$$

```
[Out] -35/32*b^4*arctanh((a+b*x^(1/2))^(1/2)/a^(1/2))/a^(9/2)-1/2*(a+b*x^(1/2))^(
1/2)/a/x^2+7/12*b*(a+b*x^(1/2))^(1/2)/a^2/x^(3/2)-35/48*b^2*(a+b*x^(1/2))^(
1/2)/a^3/x+35/32*b^3*(a+b*x^(1/2))^(1/2)/a^4/x^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {266, 51, 63, 208}

$$\frac{35b^3\sqrt{a+b\sqrt{x}}}{32a^4\sqrt{x}} - \frac{35b^2\sqrt{a+b\sqrt{x}}}{48a^3x} - \frac{35b^4 \tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)}{32a^{9/2}} + \frac{7b\sqrt{a+b\sqrt{x}}}{12a^2x^{3/2}} - \frac{\sqrt{a+b\sqrt{x}}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*Sqrt[x]]*x^3),x]
```

```
[Out] -Sqrt[a + b*Sqrt[x]]/(2*a*x^2) + (7*b*Sqrt[a + b*Sqrt[x]])/(12*a^2*x^(3/2))
 - (35*b^2*Sqrt[a + b*Sqrt[x]])/(48*a^3*x) + (35*b^3*Sqrt[a + b*Sqrt[x]])/(
32*a^4*Sqrt[x]) - (35*b^4*ArcTanh[Sqrt[a + b*Sqrt[x]]/Sqrt[a]])/(32*a^(9/2)
)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sqrt{x}}\,x^3}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^5\sqrt{a+bx}}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\sqrt{a+b\sqrt{x}}}{2ax^2} - \frac{(7b)\,\text{Subst}\left(\int \frac{1}{x^4\sqrt{a+bx}}\,dx, x, \sqrt{x}\right)}{4a}$$

$$= -\frac{\sqrt{a+b\sqrt{x}}}{2ax^2} + \frac{7b\sqrt{a+b\sqrt{x}}}{12a^2x^{3/2}} + \frac{\left(35b^2\right)\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx, x, \sqrt{x}\right)}{24a^2}$$

$$= -\frac{\sqrt{a+b\sqrt{x}}}{2ax^2} + \frac{7b\sqrt{a+b\sqrt{x}}}{12a^2x^{3/2}} - \frac{35b^2\sqrt{a+b\sqrt{x}}}{48a^3x} - \frac{\left(35b^3\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \sqrt{x}\right)}{32a^3}$$

$$= -\frac{\sqrt{a+b\sqrt{x}}}{2ax^2} + \frac{7b\sqrt{a+b\sqrt{x}}}{12a^2x^{3/2}} - \frac{35b^2\sqrt{a+b\sqrt{x}}}{48a^3x} + \frac{35b^3\sqrt{a+b\sqrt{x}}}{32a^4\sqrt{x}} + \frac{\left(35b^4\right)\text{Subst}\left(\int$$

$$= -\frac{\sqrt{a+b\sqrt{x}}}{2ax^2} + \frac{7b\sqrt{a+b\sqrt{x}}}{12a^2x^{3/2}} - \frac{35b^2\sqrt{a+b\sqrt{x}}}{48a^3x} + \frac{35b^3\sqrt{a+b\sqrt{x}}}{32a^4\sqrt{x}} + \frac{\left(35b^3\right)\text{Subst}\left(\int$$

$$= -\frac{\sqrt{a+b\sqrt{x}}}{2ax^2} + \frac{7b\sqrt{a+b\sqrt{x}}}{12a^2x^{3/2}} - \frac{35b^2\sqrt{a+b\sqrt{x}}}{48a^3x} + \frac{35b^3\sqrt{a+b\sqrt{x}}}{32a^4\sqrt{x}} - \frac{35b^4\tanh^{-1}\left(\frac{\sqrt{}\right.}{32a^{9/2}}$$

Mathematica [C] time = 0.01, size = 41, normalized size = 0.30

$$-\frac{4b^4\sqrt{a+b\sqrt{x}}\;{}_2F_1\left(\frac{1}{2}, 5; \frac{3}{2}; \frac{\sqrt{x}\,b}{a}+1\right)}{a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*Sqrt[x]]*x^3),x]
```

```
[Out] (-4*b^4*Sqrt[a + b*Sqrt[x]]*Hypergeometric2F1[1/2, 5, 3/2, 1 + (b*Sqrt[x])/
a])/a^5
```

fricas [A] time = 1.28, size = 184, normalized size = 1.35

$$\left[\frac{105\sqrt{a}\,b^4x^2\log\left(\frac{bx-2\sqrt{b\sqrt{x}+a}\,\sqrt{a}\,\sqrt{x}+2\,a\sqrt{x}}{x}\right) - 2\left(70\,a^2b^2x + 48\,a^4 - 7\left(15\,ab^3x + 8\,a^3b\right)\sqrt{x}\right)\sqrt{b\sqrt{x}+a}}{192\,a^5x^2}, \frac{105\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/192*(105*sqrt(a)*b^4*x^2*log((b*x - 2*sqrt(b*sqrt(x) + a)*sqrt(a)*sqrt(x
) + 2*a*sqrt(x))/x) - 2*(70*a^2*b^2*x + 48*a^4 - 7*(15*a*b^3*x + 8*a^3*b)*s
qrt(x))*sqrt(b*sqrt(x) + a))/(a^5*x^2), 1/96*(105*sqrt(-a)*b^4*x^2*arctan(s
qrt(b*sqrt(x) + a)*sqrt(-a)/a) - (70*a^2*b^2*x + 48*a^4 - 7*(15*a*b^3*x + 8
*a^3*b)*sqrt(x))*sqrt(b*sqrt(x) + a))/(a^5*x^2)]
```

giac [A] time = 0.16, size = 109, normalized size = 0.80

$$\frac{\frac{105\,b^5\arctan\left(\frac{\sqrt{b\sqrt{x}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^4}+\frac{105\left(b\sqrt{x}+a\right)^{\frac{7}{2}}b^5-385\left(b\sqrt{x}+a\right)^{\frac{5}{2}}ab^5+511\left(b\sqrt{x}+a\right)^{\frac{3}{2}}a^2b^5-279\sqrt{b\sqrt{x}+a}\,a^3b^5}{a^4b^4x^2}}{96\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/96*(105*b^5*arctan(sqrt(b*sqrt(x) + a)/sqrt(-a))/(sqrt(-a)*a^4) + (105*(b
*sqrt(x) + a)^(7/2)*b^5 - 385*(b*sqrt(x) + a)^(5/2)*a*b^5 + 511*(b*sqrt(x)
+ a)^(3/2)*a^2*b^5 - 279*sqrt(b*sqrt(x) + a)*a^3*b^5)/(a^4*b^4*x^2))/b
```

maple [A] time = 0.01, size = 124, normalized size = 0.91

$$\left(4\left(-\frac{\left(7\left(-\frac{\left(5\left(-\frac{\left(3\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\sqrt{x}+a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}-\frac{\sqrt{b\sqrt{x}+a}}{2ab\sqrt{x}}\right)\right)}{4a}-\frac{\sqrt{b\sqrt{x}+a}}{4ab^2x}\right)\right)}{6a}-\frac{\sqrt{b\sqrt{x}+a}}{6ab^3x^{\frac{3}{2}}}\right)\right)}{8a}-\frac{\sqrt{b\sqrt{x}+a}}{8ab^4x^2}\right)b^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^(1/2)+a)^(1/2),x)
```

```
[Out] 4*b^4*(-1/8*(b*x^(1/2)+a)^(1/2)/a/x^2/b^4-7/8/a*(-1/6*(b*x^(1/2)+a)^(1/2)/a
/x^(3/2)/b^3-5/6/a*(-1/4*(b*x^(1/2)+a)^(1/2)/a/b^2/x-3/4/a*(-1/2*(b*x^(1/2)
+a)^(1/2)/a/b/x^(1/2)+1/2/a^(3/2)*arctanh((b*x^(1/2)+a)^(1/2)/a^(1/2))))))
```

maxima [A] time = 2.00, size = 166, normalized size = 1.22

$$\frac{35\,b^4\log\left(\frac{\sqrt{b\sqrt{x}+a}-\sqrt{a}}{\sqrt{b\sqrt{x}+a}+\sqrt{a}}\right)}{64\,a^{\frac{9}{2}}}+\frac{105\left(b\sqrt{x}+a\right)^{\frac{7}{2}}b^4-385\left(b\sqrt{x}+a\right)^{\frac{5}{2}}ab^4+511\left(b\sqrt{x}+a\right)^{\frac{3}{2}}a^2b^4-279\sqrt{b\sqrt{x}+a}\,a^3}{96\left(\left(b\sqrt{x}+a\right)^4a^4-4\left(b\sqrt{x}+a\right)^3a^5+6\left(b\sqrt{x}+a\right)^2a^6-4\left(b\sqrt{x}+a\right)a^7+a^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 35/64*b^4*log((sqrt(b*sqrt(x) + a) - sqrt(a))/(sqrt(b*sqrt(x) + a) + sqrt(a
)))/a^(9/2) + 1/96*(105*(b*sqrt(x) + a)^(7/2)*b^4 - 385*(b*sqrt(x) + a)^(5/
2)*a*b^4 + 511*(b*sqrt(x) + a)^(3/2)*a^2*b^4 - 279*sqrt(b*sqrt(x) + a)*a^3*
b^4)/((b*sqrt(x) + a)^4*a^4 - 4*(b*sqrt(x) + a)^3*a^5 + 6*(b*sqrt(x) + a)^2
*a^6 - 4*(b*sqrt(x) + a)*a^7 + a^8)
```

mupad [B] time = 1.47, size = 94, normalized size = 0.69

$$\frac{511\left(a+b\sqrt{x}\right)^{3/2}}{96\,a^2\,x^2}-\frac{93\sqrt{a+b\sqrt{x}}}{32\,a\,x^2}-\frac{385\left(a+b\sqrt{x}\right)^{5/2}}{96\,a^3\,x^2}+\frac{35\left(a+b\sqrt{x}\right)^{7/2}}{32\,a^4\,x^2}+\frac{b^4\,\mathrm{atan}\left(\frac{\sqrt{a+b\sqrt{x}}\,1\mathrm{i}}{\sqrt{a}}\right)35\mathrm{i}}{32\,a^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^(1/2))^(1/2)),x)
```

```
[Out] (b^4*atan(((a + b*x^(1/2))^(1/2)*1i)/a^(1/2))*35i)/(32*a^(9/2)) - (93*(a +
b*x^(1/2))^(1/2))/(32*a*x^2) + (511*(a + b*x^(1/2))^(3/2))/(96*a^2*x^2) - (
385*(a + b*x^(1/2))^(5/2))/(96*a^3*x^2) + (35*(a + b*x^(1/2))^(7/2))/(32*a^
4*x^2)
```

sympy [A] time = 11.85, size = 173, normalized size = 1.27

$$-\frac{1}{2\sqrt{b}\,x^{\frac{9}{4}}\sqrt{\frac{a}{b\sqrt{x}}+1}}+\frac{\sqrt{b}}{12ax^{\frac{7}{4}}\sqrt{\frac{a}{b\sqrt{x}}+1}}-\frac{7b^{\frac{3}{2}}}{48a^2x^{\frac{5}{4}}\sqrt{\frac{a}{b\sqrt{x}}+1}}+\frac{35b^{\frac{5}{2}}}{96a^3x^{\frac{3}{4}}\sqrt{\frac{a}{b\sqrt{x}}+1}}+\frac{35b^{\frac{7}{2}}}{32a^4\sqrt[4]{x}\sqrt{\frac{a}{b\sqrt{x}}+1}}-\frac{35b^4\,\mathrm{asi}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*x**(1/2))**(1/2),x)
```

```
[Out] -1/(2*sqrt(b)*x**(9/4)*sqrt(a/(b*sqrt(x)) + 1)) + sqrt(b)/(12*a*x**(7/4)*sq
rt(a/(b*sqrt(x)) + 1)) - 7*b**(3/2)/(48*a**2*x**(5/4)*sqrt(a/(b*sqrt(x)) +
1)) + 35*b**(5/2)/(96*a**3*x**(3/4)*sqrt(a/(b*sqrt(x)) + 1)) + 35*b**(7/2)/
(32*a**4*x**(1/4)*sqrt(a/(b*sqrt(x)) + 1)) - 35*b**4*asinh(sqrt(a)/(sqrt(b)
*x**(1/4)))/(32*a**(9/2))
```

3.2245 $\int \left(a+b\sqrt{x}\right)^n \sqrt{x}\, dx$

Optimal. Leaf size=74

$$\frac{2a^2\left(a+b\sqrt{x}\right)^{n+1}}{b^3(n+1)} - \frac{4a\left(a+b\sqrt{x}\right)^{n+2}}{b^3(n+2)} + \frac{2\left(a+b\sqrt{x}\right)^{n+3}}{b^3(n+3)}$$

```
[Out] 2*a^2*(a+b*x^(1/2))^(1+n)/b^3/(1+n)-4*a*(a+b*x^(1/2))^(2+n)/b^3/(2+n)+2*(a+
b*x^(1/2))^(3+n)/b^3/(3+n)
```

Rubi [A] time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$\frac{2a^2\left(a+b\sqrt{x}\right)^{n+1}}{b^3(n+1)} - \frac{4a\left(a+b\sqrt{x}\right)^{n+2}}{b^3(n+2)} + \frac{2\left(a+b\sqrt{x}\right)^{n+3}}{b^3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^n*Sqrt[x],x]
```

```
[Out] (2*a^2*(a + b*Sqrt[x])^(1 + n))/(b^3*(1 + n)) - (4*a*(a + b*Sqrt[x])^(2 + n
))/(b^3*(2 + n)) + (2*(a + b*Sqrt[x])^(3 + n))/(b^3*(3 + n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\sqrt{x}\right)^n \sqrt{x}\, dx &= 2\,\text{Subst}\left(\int x^2(a+bx)^n\, dx, x, \sqrt{x}\right) \\
&= 2\,\text{Subst}\left(\int \left(\frac{a^2(a+bx)^n}{b^2} - \frac{2a(a+bx)^{1+n}}{b^2} + \frac{(a+bx)^{2+n}}{b^2}\right) dx, x, \sqrt{x}\right) \\
&= \frac{2a^2\left(a+b\sqrt{x}\right)^{1+n}}{b^3(1+n)} - \frac{4a\left(a+b\sqrt{x}\right)^{2+n}}{b^3(2+n)} + \frac{2\left(a+b\sqrt{x}\right)^{3+n}}{b^3(3+n)}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 64, normalized size = 0.86

$$\frac{2\left(a+b\sqrt{x}\right)^{n+1}\left(2a^2 - 2ab(n+1)\sqrt{x} + b^2\left(n^2+3n+2\right)x\right)}{b^3(n+1)(n+2)(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^n*Sqrt[x],x]
```

```
[Out] (2*(a + b*Sqrt[x])^(1 + n)*(2*a^2 - 2*a*b*(1 + n)*Sqrt[x] + b^2*(2 + 3*n +
n^2)*x))/(b^3*(1 + n)*(2 + n)*(3 + n))
```

fricas [A] time = 1.11, size = 98, normalized size = 1.32

$$\frac{2\left(2\,a^3+\left(ab^2n^2+ab^2n\right)x-\left(2\,a^2bn-\left(b^3n^2+3\,b^3n+2\,b^3\right)x\right)\sqrt{x}\right)\left(b\sqrt{x}+a\right)^n}{b^3n^3+6\,b^3n^2+11\,b^3n+6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(a+b*x^(1/2))^n,x, algorithm="fricas")
```

```
[Out] 2*(2*a^3 + (a*b^2*n^2 + a*b^2*n)*x - (2*a^2*b*n - (b^3*n^2 + 3*b^3*n + 2*b^
3)*x)*sqrt(x))*(b*sqrt(x) + a)^n/(b^3*n^3 + 6*b^3*n^2 + 11*b^3*n + 6*b^3)
```

giac [B] time = 0.17, size = 231, normalized size = 3.12

$$\frac{2\left(\left(b\sqrt{x}+a\right)^3\left(b\sqrt{x}+a\right)^n n^2-2\left(b\sqrt{x}+a\right)^2\left(b\sqrt{x}+a\right)^n an^2+\left(b\sqrt{x}+a\right)\left(b\sqrt{x}+a\right)^n a^2n^2+3\left(b\sqrt{x}+a\right)^3\left(b\sqrt{x}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(a+b*x^(1/2))^n,x, algorithm="giac")
```

```
[Out] 2*((b*sqrt(x) + a)^3*(b*sqrt(x) + a)^n*n^2 - 2*(b*sqrt(x) + a)^2*(b*sqrt(x)
 + a)^n*a*n^2 + (b*sqrt(x) + a)*(b*sqrt(x) + a)^n*a^2*n^2 + 3*(b*sqrt(x) +
a)^3*(b*sqrt(x) + a)^n*n - 8*(b*sqrt(x) + a)^2*(b*sqrt(x) + a)^n*a*n + 5*(b
*sqrt(x) + a)*(b*sqrt(x) + a)^n*a^2*n + 2*(b*sqrt(x) + a)^3*(b*sqrt(x) + a)
^n - 6*(b*sqrt(x) + a)^2*(b*sqrt(x) + a)^n*a + 6*(b*sqrt(x) + a)*(b*sqrt(x)
 + a)^n*a^2)/((b^2*n^3 + 6*b^2*n^2 + 11*b^2*n + 6*b^2)*b)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \sqrt{x}\,\left(b\sqrt{x}+a\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*x^(1/2)+a)^n,x)
```

```
[Out] int(x^(1/2)*(b*x^(1/2)+a)^n,x)
```

maxima [A] time = 0.93, size = 71, normalized size = 0.96

$$\frac{2\left(\left(n^2+3\,n+2\right)b^3x^{\frac{3}{2}}+\left(n^2+n\right)ab^2x-2\,a^2bn\sqrt{x}+2\,a^3\right)\left(b\sqrt{x}+a\right)^n}{\left(n^3+6\,n^2+11\,n+6\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(a+b*x^(1/2))^n,x, algorithm="maxima")
```

```
[Out] 2*((n^2 + 3*n + 2)*b^3*x^(3/2) + (n^2 + n)*a*b^2*x - 2*a^2*b*n*sqrt(x) + 2*
a^3)*(b*sqrt(x) + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,\left(a+b\,\sqrt{x}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x^(1/2))^n,x)
```

```
[Out] int(x^(1/2)*(a + b*x^(1/2))^n, x)
```

sympy [**B**] time = 4.32, size = 5039, normalized size = 68.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(a+b*x**(1/2))**n,x)
```

```
[Out] 4*a**6*a**n*x**(9/2)*(1 + b*sqrt(x)/a)**n/(a**3*b**3*n**3*x**(9/2) + 6*a**3
*b**3*n**2*x**(9/2) + 11*a**3*b**3*n*x**(9/2) + 6*a**3*b**3*x**(9/2) + 3*a*
*2*b**4*n**3*x**5 + 18*a**2*b**4*n**2*x**5 + 33*a**2*b**4*n*x**5 + 18*a**2*
b**4*x**5 + 3*a*b**5*n**3*x**(11/2) + 18*a*b**5*n**2*x**(11/2) + 33*a*b**5*
n*x**(11/2) + 18*a*b**5*x**(11/2) + b**6*n**3*x**6 + 6*b**6*n**2*x**6 + 11*
b**6*n*x**6 + 6*b**6*x**6) - 4*a**6*a**n*x**(9/2)/(a**3*b**3*n**3*x**(9/2)
+ 6*a**3*b**3*n**2*x**(9/2) + 11*a**3*b**3*n*x**(9/2) + 6*a**3*b**3*x**(9/2
) + 3*a**2*b**4*n**3*x**5 + 18*a**2*b**4*n**2*x**5 + 33*a**2*b**4*n*x**5 +
18*a**2*b**4*x**5 + 3*a*b**5*n**3*x**(11/2) + 18*a*b**5*n**2*x**(11/2) + 33
*a*b**5*n*x**(11/2) + 18*a*b**5*x**(11/2) + b**6*n**3*x**6 + 6*b**6*n**2*x*
*6 + 11*b**6*n*x**6 + 6*b**6*x**6) - 4*a**5*a**n*b*n*x**5*(1 + b*sqrt(x)/a)
**n/(a**3*b**3*n**3*x**(9/2) + 6*a**3*b**3*n**2*x**(9/2) + 11*a**3*b**3*n*x
**(9/2) + 6*a**3*b**3*x**(9/2) + 3*a**2*b**4*n**3*x**5 + 18*a**2*b**4*n**2*
x**5 + 33*a**2*b**4*n*x**5 + 18*a**2*b**4*x**5 + 3*a*b**5*n**3*x**(11/2) +
18*a*b**5*n**2*x**(11/2) + 33*a*b**5*n*x**(11/2) + 18*a*b**5*x**(11/2) + b*
*6*n**3*x**6 + 6*b**6*n**2*x**6 + 11*b**6*n*x**6 + 6*b**6*x**6) + 12*a**5*a
**n*b*x**5*(1 + b*sqrt(x)/a)**n/(a**3*b**3*n**3*x**(9/2) + 6*a**3*b**3*n**2
*x**(9/2) + 11*a**3*b**3*n*x**(9/2) + 6*a**3*b**3*x**(9/2) + 3*a**2*b**4*n*
*3*x**5 + 18*a**2*b**4*n**2*x**5 + 33*a**2*b**4*n*x**5 + 18*a**2*b**4*x**5
+ 3*a*b**5*n**3*x**(11/2) + 18*a*b**5*n**2*x**(11/2) + 33*a*b**5*n*x**(11/2
) + 18*a*b**5*x**(11/2) + b**6*n**3*x**6 + 6*b**6*n**2*x**6 + 11*b**6*n*x**
6 + 6*b**6*x**6) - 12*a**5*a**n*b*x**5/(a**3*b**3*n**3*x**(9/2) + 6*a**3*b*
*3*n**2*x**(9/2) + 11*a**3*b**3*n*x**(9/2) + 6*a**3*b**3*x**(9/2) + 3*a**2*
b**4*n**3*x**5 + 18*a**2*b**4*n**2*x**5 + 33*a**2*b**4*n*x**5 + 18*a**2*b**
4*x**5 + 3*a*b**5*n**3*x**(11/2) + 18*a*b**5*n**2*x**(11/2) + 33*a*b**5*n*x
**(11/2) + 18*a*b**5*x**(11/2) + b**6*n**3*x**6 + 6*b**6*n**2*x**6 + 11*b**
6*n*x**6 + 6*b**6*x**6) + 2*a**4*a**n*b**2*n**2*x**(11/2)*(1 + b*sqrt(x)/a)
**n/(a**3*b**3*n**3*x**(9/2) + 6*a**3*b**3*n**2*x**(9/2) + 11*a**3*b**3*n*x
**(9/2) + 6*a**3*b**3*x**(9/2) + 3*a**2*b**4*n**3*x**5 + 18*a**2*b**4*n**2*
x**5 + 33*a**2*b**4*n*x**5 + 18*a**2*b**4*x**5 + 3*a*b**5*n**3*x**(11/2) +
18*a*b**5*n**2*x**(11/2) + 33*a*b**5*n*x**(11/2) + 18*a*b**5*x**(11/2) + b*
*6*n**3*x**6 + 6*b**6*n**2*x**6 + 11*b**6*n*x**6 + 6*b**6*x**6) - 10*a**4*a
**n*b**2*n*x**(11/2)*(1 + b*sqrt(x)/a)**n/(a**3*b**3*n**3*x**(9/2) + 6*a**3
*b**3*n**2*x**(9/2) + 11*a**3*b**3*n*x**(9/2) + 6*a**3*b**3*x**(9/2) + 3*a*
*2*b**4*n**3*x**5 + 18*a**2*b**4*n**2*x**5 + 33*a**2*b**4*n*x**5 + 18*a**2*
b**4*x**5 + 3*a*b**5*n**3*x**(11/2) + 18*a*b**5*n**2*x**(11/2) + 33*a*b**5*
n*x**(11/2) + 18*a*b**5*x**(11/2) + b**6*n**3*x**6 + 6*b**6*n**2*x**6 + 11*
b**6*n*x**6 + 6*b**6*x**6) + 12*a**4*a**n*b**2*x**(11/2)*(1 + b*sqrt(x)/a)*
*n/(a**3*b**3*n**3*x**(9/2) + 6*a**3*b**3*n**2*x**(9/2) + 11*a**3*b**3*n*x*
*(9/2) + 6*a**3*b**3*x**(9/2) + 3*a**2*b**4*n**3*x**5 + 18*a**2*b**4*n**2*x
**5 + 33*a**2*b**4*n*x**5 + 18*a**2*b**4*x**5 + 3*a*b**5*n**3*x**(11/2) + 1
8*a*b**5*n**2*x**(11/2) + 33*a*b**5*n*x**(11/2) + 18*a*b**5*x**(11/2) + b**
6*n**3*x**6 + 6*b**6*n**2*x**6 + 11*b**6*n*x**6 + 6*b**6*x**6) - 12*a**4*a*
*n*b**2*x**(11/2)/(a**3*b**3*n**3*x**(9/2) + 6*a**3*b**3*n**2*x**(9/2) + 11
*a**3*b**3*n*x**(9/2) + 6*a**3*b**3*x**(9/2) + 3*a**2*b**4*n**3*x**5 + 18*a
**2*b**4*n**2*x**5 + 33*a**2*b**4*n*x**5 + 18*a**2*b**4*x**5 + 3*a*b**5*n**
3*x**(11/2) + 18*a*b**5*n**2*x**(11/2) + 33*a*b**5*n*x**(11/2) + 18*a*b**5*
x**(11/2) + b**6*n**3*x**6 + 6*b**6*n**2*x**6 + 11*b**6*n*x**6 + 6*b**6*x**
6) + 8*a**3*a**n*b**3*n**2*x**6*(1 + b*sqrt(x)/a)**n/(a**3*b**3*n**3*x**(9/
2) + 6*a**3*b**3*n**2*x**(9/2) + 11*a**3*b**3*n*x**(9/2) + 6*a**3*b**3*x**(
9/2) + 3*a**2*b**4*n**3*x**5 + 18*a**2*b**4*n**2*x**5 + 33*a**2*b**4*n*x**5
```

```
 + 18*a**2*b**4*x**5 + 3*a*b**5*n**3*x**(11/2) + 18*a*b**5*n**2*x**(11/2) +
 33*a*b**5*n*x**(11/2) + 18*a*b**5*x**(11/2) + b**6*n**3*x**6 + 6*b**6*n**2
*x**6 + 11*b**6*n*x**6 + 6*b**6*x**6) + 8*a**3*a**n*b**3*x**6*(1 + b*sqrt(x
)/a)**n/(a**3*b**3*n**3*x**(9/2) + 6*a**3*b**3*n**2*x**(9/2) + 11*a**3*b**3
*n*x**(9/2) + 6*a**3*b**3*x**(9/2) + 3*a**2*b**4*n**3*x**5 + 18*a**2*b**4*n
**2*x**5 + 33*a**2*b**4*n*x**5 + 18*a**2*b**4*x**5 + 3*a*b**5*n**3*x**(11/2
) + 18*a*b**5*n**2*x**(11/2) + 33*a*b**5*n*x**(11/2) + 18*a*b**5*x**(11/2)
+ b**6*n**3*x**6 + 6*b**6*n**2*x**6 + 11*b**6*n*x**6 + 6*b**6*x**6) - 4*a**
3*a**n*b**3*x**6/(a**3*b**3*n**3*x**(9/2) + 6*a**3*b**3*n**2*x**(9/2) + 11*
a**3*b**3*n*x**(9/2) + 6*a**3*b**3*x**(9/2) + 3*a**2*b**4*n**3*x**5 + 18*a*
*2*b**4*n**2*x**5 + 33*a**2*b**4*n*x**5 + 18*a**2*b**4*x**5 + 3*a*b**5*n**3
*x**(11/2) + 18*a*b**5*n**2*x**(11/2) + 33*a*b**5*n*x**(11/2) + 18*a*b**5*x
**(11/2) + b**6*n**3*x**6 + 6*b**6*n**2*x**6 + 11*b**6*n*x**6 + 6*b**6*x**6
) + 12*a**2*a**n*b**4*n**2*x**(13/2)*(1 + b*sqrt(x)/a)**n/(a**3*b**3*n**3*x
**(9/2) + 6*a**3*b**3*n**2*x**(9/2) + 11*a**3*b**3*n*x**(9/2) + 6*a**3*b**3
*x**(9/2) + 3*a**2*b**4*n**3*x**5 + 18*a**2*b**4*n**2*x**5 + 33*a**2*b**4*n
*x**5 + 18*a**2*b**4*x**5 + 3*a*b**5*n**3*x**(11/2) + 18*a*b**5*n**2*x**(11
/2) + 33*a*b**5*n*x**(11/2) + 18*a*b**5*x**(11/2) + b**6*n**3*x**6 + 6*b**6
*n**2*x**6 + 11*b**6*n*x**6 + 6*b**6*x**6) + 20*a**2*a**n*b**4*n*x**(13/2)*
(1 + b*sqrt(x)/a)**n/(a**3*b**3*n**3*x**(9/2) + 6*a**3*b**3*n**2*x**(9/2) +
 11*a**3*b**3*n*x**(9/2) + 6*a**3*b**3*x**(9/2) + 3*a**2*b**4*n**3*x**5 + 1
8*a**2*b**4*n**2*x**5 + 33*a**2*b**4*n*x**5 + 18*a**2*b**4*x**5 + 3*a*b**5*
n**3*x**(11/2) + 18*a*b**5*n**2*x**(11/2) + 33*a*b**5*n*x**(11/2) + 18*a*b*
*5*x**(11/2) + b**6*n**3*x**6 + 6*b**6*n**2*x**6 + 11*b**6*n*x**6 + 6*b**6*
x**6) + 12*a**2*a**n*b**4*x**(13/2)*(1 + b*sqrt(x)/a)**n/(a**3*b**3*n**3*x*
*(9/2) + 6*a**3*b**3*n**2*x**(9/2) + 11*a**3*b**3*n*x**(9/2) + 6*a**3*b**3*
x**(9/2) + 3*a**2*b**4*n**3*x**5 + 18*a**2*b**4*n**2*x**5 + 33*a**2*b**4*n*
x**5 + 18*a**2*b**4*x**5 + 3*a*b**5*n**3*x**(11/2) + 18*a*b**5*n**2*x**(11/
2) + 33*a*b**5*n*x**(11/2) + 18*a*b**5*x**(11/2) + b**6*n**3*x**6 + 6*b**6*
n**2*x**6 + 11*b**6*n*x**6 + 6*b**6*x**6) + 8*a*a**n*b**5*n**2*x**7*(1 + b*
sqrt(x)/a)**n/(a**3*b**3*n**3*x**(9/2) + 6*a**3*b**3*n**2*x**(9/2) + 11*a**
3*b**3*n*x**(9/2) + 6*a**3*b**3*x**(9/2) + 3*a**2*b**4*n**3*x**5 + 18*a**2*
b**4*n**2*x**5 + 33*a**2*b**4*n*x**5 + 18*a**2*b**4*x**5 + 3*a*b**5*n**3*x*
*(11/2) + 18*a*b**5*n**2*x**(11/2) + 33*a*b**5*n*x**(11/2) + 18*a*b**5*x**(
11/2) + b**6*n**3*x**6 + 6*b**6*n**2*x**6 + 11*b**6*n*x**6 + 6*b**6*x**6) +
 20*a*a**n*b**5*n*x**7*(1 + b*sqrt(x)/a)**n/(a**3*b**3*n**3*x**(9/2) + 6*a*
*3*b**3*n**2*x**(9/2) + 11*a**3*b**3*n*x**(9/2) + 6*a**3*b**3*x**(9/2) + 3*
a**2*b**4*n**3*x**5 + 18*a**2*b**4*n**2*x**5 + 33*a**2*b**4*n*x**5 + 18*a**
2*b**4*x**5 + 3*a*b**5*n**3*x**(11/2) + 18*a*b**5*n**2*x**(11/2) + 33*a*b**
5*n*x**(11/2) + 18*a*b**5*x**(11/2) + b**6*n**3*x**6 + 6*b**6*n**2*x**6 + 1
1*b**6*n*x**6 + 6*b**6*x**6) + 12*a*a**n*b**5*x**7*(1 + b*sqrt(x)/a)**n/(a*
*3*b**3*n**3*x**(9/2) + 6*a**3*b**3*n**2*x**(9/2) + 11*a**3*b**3*n*x**(9/2)
 + 6*a**3*b**3*x**(9/2) + 3*a**2*b**4*n**3*x**5 + 18*a**2*b**4*n**2*x**5 +
33*a**2*b**4*n*x**5 + 18*a**2*b**4*x**5 + 3*a*b**5*n**3*x**(11/2) + 18*a*b*
*5*n**2*x**(11/2) + 33*a*b**5*n*x**(11/2) + 18*a*b**5*x**(11/2) + b**6*n**3
*x**6 + 6*b**6*n**2*x**6 + 11*b**6*n*x**6 + 6*b**6*x**6) + 2*a**n*b**6*n**2
*x**(15/2)*(1 + b*sqrt(x)/a)**n/(a**3*b**3*n**3*x**(9/2) + 6*a**3*b**3*n**2
*x**(9/2) + 11*a**3*b**3*n*x**(9/2) + 6*a**3*b**3*x**(9/2) + 3*a**2*b**4*n*
*3*x**5 + 18*a**2*b**4*n**2*x**5 + 33*a**2*b**4*n*x**5 + 18*a**2*b**4*x**5
+ 3*a*b**5*n**3*x**(11/2) + 18*a*b**5*n**2*x**(11/2) + 33*a*b**5*n*x**(11/2
) + 18*a*b**5*x**(11/2) + b**6*n**3*x**6 + 6*b**6*n**2*x**6 + 11*b**6*n*x**
6 + 6*b**6*x**6) + 6*a**n*b**6*n*x**(15/2)*(1 + b*sqrt(x)/a)**n/(a**3*b**3*
n**3*x**(9/2) + 6*a**3*b**3*n**2*x**(9/2) + 11*a**3*b**3*n*x**(9/2) + 6*a**
3*b**3*x**(9/2) + 3*a**2*b**4*n**3*x**5 + 18*a**2*b**4*n**2*x**5 + 33*a**2*
b**4*n*x**5 + 18*a**2*b**4*x**5 + 3*a*b**5*n**3*x**(11/2) + 18*a*b**5*n**2*
x**(11/2) + 33*a*b**5*n*x**(11/2) + 18*a*b**5*x**(11/2) + b**6*n**3*x**6 +
6*b**6*n**2*x**6 + 11*b**6*n*x**6 + 6*b**6*x**6) + 4*a**n*b**6*x**(15/2)*(1
 + b*sqrt(x)/a)**n/(a**3*b**3*n**3*x**(9/2) + 6*a**3*b**3*n**2*x**(9/2) + 1
1*a**3*b**3*n*x**(9/2) + 6*a**3*b**3*x**(9/2) + 3*a**2*b**4*n**3*x**5 + 18*
```

```
a**2*b**4*n**2*x**5 + 33*a**2*b**4*n*x**5 + 18*a**2*b**4*x**5 + 3*a*b**5*n*
*3*x**(11/2) + 18*a*b**5*n**2*x**(11/2) + 33*a*b**5*n*x**(11/2) + 18*a*b**5
*x**(11/2) + b**6*n**3*x**6 + 6*b**6*n**2*x**6 + 11*b**6*n*x**6 + 6*b**6*x*
*6)
```

3.2246 $\int \frac{\left(a+b\sqrt{x}\right)^n}{\sqrt{x}}\,dx$

Optimal. Leaf size=23

$$\frac{2\left(a+b\sqrt{x}\right)^{n+1}}{b(n+1)}$$

```
[Out] 2*(a+b*x^(1/2))^(1+n)/b/(1+n)
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {261}

$$\frac{2\left(a+b\sqrt{x}\right)^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^n/Sqrt[x],x]
```

```
[Out] (2*(a + b*Sqrt[x])^(1 + n))/(b*(1 + n))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\sqrt{x}\right)^n}{\sqrt{x}}\,dx = \frac{2\left(a+b\sqrt{x}\right)^{1+n}}{b(1+n)}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{2\left(a+b\sqrt{x}\right)^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^n/Sqrt[x],x]
```

```
[Out] (2*(a + b*Sqrt[x])^(1 + n))/(b*(1 + n))
```

fricas [A] time = 0.81, size = 25, normalized size = 1.09

$$\frac{2\left(b\sqrt{x}+a\right)\left(b\sqrt{x}+a\right)^n}{bn+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^n/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(b*sqrt(x) + a)*(b*sqrt(x) + a)^n/(b*n + b)
```

giac [A] time = 0.16, size = 21, normalized size = 0.91

$$\frac{2\left(b\sqrt{x}+a\right)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^n/x^(1/2),x, algorithm="giac")
[Out] 2*(b*sqrt(x) + a)^(n + 1)/(b*(n + 1))
```

maple [A] time = 0.00, size = 22, normalized size = 0.96

$$\frac{2\left(b\sqrt{x}+a\right)^{n+1}}{(n+1)\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(1/2)+a)^n/x^(1/2),x)
[Out] 2*(b*x^(1/2)+a)^(1+n)/b/(1+n)
```

maxima [A] time = 0.87, size = 21, normalized size = 0.91

$$\frac{2\left(b\sqrt{x}+a\right)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^n/x^(1/2),x, algorithm="maxima")
[Out] 2*(b*sqrt(x) + a)^(n + 1)/(b*(n + 1))
```

mupad [B] time = 1.33, size = 21, normalized size = 0.91

$$\frac{2\left(a+b\sqrt{x}\right)^{n+1}}{b\ (n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^n/x^(1/2),x)
[Out] (2*(a + b*x^(1/2))^(n + 1))/(b*(n + 1))
```

sympy [A] time = 1.01, size = 182, normalized size = 7.91

$$\begin{cases} \tilde{\infty}\sqrt{x} & \text{for } a = 0 \wedge b = 0 \wedge n = -1 \\ 2\cdot 0^{n}\sqrt{x} & \text{for } a = -b\sqrt{x} \\ 2a^{n}\sqrt{x} & \text{for } b = 0 \\ \frac{2\log\left(\frac{a}{b}+\sqrt{x}\right)}{b} & \text{for } n = -1 \\ \frac{2a^{2}\left(a+b\sqrt{x}\right)^{n}}{abn+ab+b^{2}n\sqrt{x}+b^{2}\sqrt{x}} + \frac{4ab\sqrt{x}\left(a+b\sqrt{x}\right)^{n}}{abn+ab+b^{2}n\sqrt{x}+b^{2}\sqrt{x}} + \frac{2b^{2}x\left(a+b\sqrt{x}\right)^{n}}{abn+ab+b^{2}n\sqrt{x}+b^{2}\sqrt{x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**n/x**(1/2),x)
```

```
[Out] Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0) & Eq(n, -1)), (2*0**n*sqrt(x),
Eq(a, -b*sqrt(x))), (2*a**n*sqrt(x), Eq(b, 0)), (2*log(a/b + sqrt(x))/b, Eq
(n, -1)), (2*a**2*(a + b*sqrt(x))**n/(a*b*n + a*b + b**2*n*sqrt(x) + b**2*s
qrt(x)) + 4*a*b*sqrt(x)*(a + b*sqrt(x))**n/(a*b*n + a*b + b**2*n*sqrt(x) +
b**2*sqrt(x)) + 2*b**2*x*(a + b*sqrt(x))**n/(a*b*n + a*b + b**2*n*sqrt(x) +
 b**2*sqrt(x)), True))
```

3.2247 $\int \frac{1+\sqrt{x}}{\sqrt{x}}\,dx$

Optimal. Leaf size=9

$$x+2\sqrt{x}$$

```
[Out] x+2*x^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$x+2\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[x])/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x] + x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{1+\sqrt{x}}{\sqrt{x}}\,dx &= \int \left(1+\frac{1}{\sqrt{x}}\right)dx\\ &= 2\sqrt{x}+x\end{aligned}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$x+2\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[x])/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x] + x
```

fricas [A] time = 0.88, size = 7, normalized size = 0.78

$$x+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/x^(1/2),x, algorithm="fricas")
```

```
[Out] x + 2*sqrt(x)
```

giac [A] time = 0.17, size = 7, normalized size = 0.78

$$x+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] x + 2*sqrt(x)
```

maple [A] time = 0.00, size = 8, normalized size = 0.89

$$x + 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x^(1/2))/x^(1/2),x)
[Out] x+2*x^(1/2)
```

maxima [C] time = 0.86, size = 7, normalized size = 0.78

$$\left(\sqrt{x} + 1\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/x^(1/2),x, algorithm="maxima")
[Out] (sqrt(x) + 1)^2
```

mupad [B] time = 0.02, size = 7, normalized size = 0.78

$$x + 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2) + 1)/x^(1/2),x)
[Out] x + 2*x^(1/2)
```

sympy [A] time = 0.15, size = 7, normalized size = 0.78

$$2\sqrt{x} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(1/2))/x**(1/2),x)
[Out] 2*sqrt(x) + x
```

3.2248 $\int \frac{\left(1+\sqrt{x}\right)^2}{\sqrt{x}}\,dx$

Optimal. Leaf size=13

$$\frac{2}{3}\left(\sqrt{x}+1\right)^3$$

```
[Out] 2/3*(1+x^(1/2))^3
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$\frac{2}{3}\left(\sqrt{x}+1\right)^3$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[x])^2/Sqrt[x],x]
```

```
[Out] (2*(1 + Sqrt[x])^3)/3
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(1+\sqrt{x}\right)^2}{\sqrt{x}}\,dx = \frac{2}{3}\left(1+\sqrt{x}\right)^3$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.54

$$\frac{2x^{3/2}}{3} + 2x + 2\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[x])^2/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x] + 2*x + (2*x^(3/2))/3
```

fricas [A] time = 1.03, size = 12, normalized size = 0.92

$$\frac{2}{3}(x+3)\sqrt{x} + 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(x + 3)*sqrt(x) + 2*x
```

giac [A] time = 0.15, size = 14, normalized size = 1.08

$$\frac{2}{3}x^{\frac{3}{2}} + 2x + 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))^2/x^(1/2),x, algorithm="giac")
[Out] 2/3*x^(3/2) + 2*x + 2*sqrt(x)
```

maple [A] time = 0.00, size = 15, normalized size = 1.15

$$\frac{2x^{\frac{3}{2}}}{3} + 2x + 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x^(1/2))^2/x^(1/2),x)
[Out] 2/3*x^(3/2)+2*x+2*x^(1/2)
```

maxima [A] time = 0.83, size = 9, normalized size = 0.69

$$\frac{2}{3}\left(\sqrt{x}+1\right)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))^2/x^(1/2),x, algorithm="maxima")
[Out] 2/3*(sqrt(x) + 1)^3
```

mupad [B] time = 0.03, size = 13, normalized size = 1.00

$$\frac{2\sqrt{x}\,\left(x+3\sqrt{x}+3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2) + 1)^2/x^(1/2),x)
[Out] (2*x^(1/2)*(x + 3*x^(1/2) + 3))/3
```

sympy [A] time = 0.15, size = 17, normalized size = 1.31

$$\frac{2x^{\frac{3}{2}}}{3} + 2\sqrt{x} + 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(1/2))**2/x**(1/2),x)
[Out] 2*x**(3/2)/3 + 2*sqrt(x) + 2*x
```

3.2249 $\int \frac{\left(1+\sqrt{x}\right)^3}{\sqrt{x}}\,dx$

Optimal. Leaf size=13

$$\frac{1}{2}\left(\sqrt{x}+1\right)^4$$

```
[Out] 1/2*(1+x^(1/2))^4
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$\frac{1}{2}\left(\sqrt{x}+1\right)^4$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[x])^3/Sqrt[x],x]
```

```
[Out] (1 + Sqrt[x])^4/2
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(1+\sqrt{x}\right)^3}{\sqrt{x}}\,dx = \frac{1}{2}\left(1+\sqrt{x}\right)^4$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{1}{2}\left(\sqrt{x}+1\right)^4$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[x])^3/Sqrt[x],x]
```

```
[Out] (1 + Sqrt[x])^4/2
```

fricas [A] time = 0.83, size = 17, normalized size = 1.31

$$\frac{1}{2}x^2 + 2\,(x+1)\sqrt{x} + 3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 2*(x + 1)*sqrt(x) + 3*x
```

giac [B] time = 0.19, size = 19, normalized size = 1.46

$$\frac{1}{2}x^2 + 2\,x^{\frac{3}{2}} + 3\,x + 2\,\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))^3/x^(1/2),x, algorithm="giac")
[Out] 1/2*x^2 + 2*x^(3/2) + 3*x + 2*sqrt(x)
```

maple [B] time = 0.00, size = 20, normalized size = 1.54

$$\frac{x^2}{2} + 2x^{\frac{3}{2}} + 3x + 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x^(1/2))^3/x^(1/2),x)
[Out] 1/2*x^2+2*x^(3/2)+3*x+2*x^(1/2)
```

maxima [A] time = 0.75, size = 9, normalized size = 0.69

$$\frac{1}{2}\left(\sqrt{x}+1\right)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))^3/x^(1/2),x, algorithm="maxima")
[Out] 1/2*(sqrt(x) + 1)^4
```

mupad [B] time = 0.03, size = 19, normalized size = 1.46

$$3\,x + \frac{x^2}{2} + 2\,\sqrt{x} + 2\,x^{3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2) + 1)^3/x^(1/2),x)
[Out] 3*x + x^2/2 + 2*x^(1/2) + 2*x^(3/2)
```

sympy [B] time = 0.19, size = 20, normalized size = 1.54

$$2x^{\frac{3}{2}} + 2\sqrt{x} + \frac{x^2}{2} + 3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(1/2))**3/x**(1/2),x)
[Out] 2*x**(3/2) + 2*sqrt(x) + x**2/2 + 3*x
```

3.2250 $\int \frac{\sqrt{x}}{1+\sqrt{x}}\,dx$

Optimal. Leaf size=19

$$x - 2\sqrt{x} + 2\log\left(\sqrt{x}+1\right)$$

```
[Out] x+2*ln(1+x^(1/2))-2*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$x - 2\sqrt{x} + 2\log\left(\sqrt{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(1 + Sqrt[x]),x]
```

```
[Out] -2*Sqrt[x] + x + 2*Log[1 + Sqrt[x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{1+\sqrt{x}}\,dx &= 2\,\text{Subst}\left(\int \frac{x^2}{1+x}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(-1+x+\frac{1}{1+x}\right)dx, x, \sqrt{x}\right)\\ &= -2\sqrt{x} + x + 2\log\left(1+\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$x - 2\sqrt{x} + 2\log\left(\sqrt{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(1 + Sqrt[x]),x]
```

```
[Out] -2*Sqrt[x] + x + 2*Log[1 + Sqrt[x]]
```

fricas [A] time = 0.96, size = 15, normalized size = 0.79

$$x - 2\sqrt{x} + 2\log\left(\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x^(1/2)),x, algorithm="fricas")
[Out] x - 2*sqrt(x) + 2*log(sqrt(x) + 1)
```

giac [**A**] time = 0.17, size = 15, normalized size = 0.79

$$x - 2\sqrt{x} + 2\log\left(\sqrt{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x^(1/2)),x, algorithm="giac")
[Out] x - 2*sqrt(x) + 2*log(sqrt(x) + 1)
```

maple [**A**] time = 0.00, size = 16, normalized size = 0.84

$$x + 2\ln\left(\sqrt{x} + 1\right) - 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(1+x^(1/2)),x)
[Out] x+2*ln(1+x^(1/2))-2*x^(1/2)
```

maxima [**A**] time = 0.87, size = 22, normalized size = 1.16

$$\left(\sqrt{x} + 1\right)^2 - 4\sqrt{x} + 2\log\left(\sqrt{x} + 1\right) - 4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x^(1/2)),x, algorithm="maxima")
[Out] (sqrt(x) + 1)^2 - 4*sqrt(x) + 2*log(sqrt(x) + 1) - 4
```

mupad [**B**] time = 0.05, size = 15, normalized size = 0.79

$$x + 2\ln\left(\sqrt{x} + 1\right) - 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^(1/2) + 1),x)
[Out] x + 2*log(x^(1/2) + 1) - 2*x^(1/2)
```

sympy [**A**] time = 0.15, size = 17, normalized size = 0.89

$$-2\sqrt{x} + x + 2\log\left(\sqrt{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(1+x**(1/2)),x)
[Out] -2*sqrt(x) + x + 2*log(sqrt(x) + 1)
```

3.2251 $\int \frac{1}{\left(1+\sqrt{x}\right)\sqrt{x}}\,dx$

Optimal. Leaf size=10

$$2\log\left(\sqrt{x}+1\right)$$

```
[Out] 2*ln(1+x^(1/2))
```

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {260}

$$2\log\left(\sqrt{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + Sqrt[x])*Sqrt[x]),x]
```

```
[Out] 2*Log[1 + Sqrt[x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1}{\left(1+\sqrt{x}\right)\sqrt{x}}\,dx = 2\log\left(1+\sqrt{x}\right)$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$2\log\left(\sqrt{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + Sqrt[x])*Sqrt[x]),x]
```

```
[Out] 2*Log[1 + Sqrt[x]]
```

fricas [A] time = 0.78, size = 8, normalized size = 0.80

$$2\,\log\left(\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(1+x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*log(sqrt(x) + 1)
```

giac [A] time = 0.17, size = 8, normalized size = 0.80

$$2\,\log\left(\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(1+x^(1/2)),x, algorithm="giac")
```

```
[Out] 2*log(sqrt(x) + 1)
```

maple [A] time = 0.00, size = 9, normalized size = 0.90

$$2\ln\left(\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(x^(1/2)+1),x)
[Out] 2*ln(x^(1/2)+1)
```

maxima [A] time = 0.86, size = 8, normalized size = 0.80

$$2\,\log\left(\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(1+x^(1/2)),x, algorithm="maxima")
[Out] 2*log(sqrt(x) + 1)
```

mupad [B] time = 0.05, size = 8, normalized size = 0.80

$$2\,\ln\left(\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(x^(1/2) + 1)),x)
[Out] 2*log(x^(1/2) + 1)
```

sympy [A] time = 0.15, size = 8, normalized size = 0.80

$$2\log\left(\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(1+x**(1/2)),x)
[Out] 2*log(sqrt(x) + 1)
```

3.2252 $\int \frac{1}{\left(1+\sqrt{x}\right)^2 \sqrt{x}}\, dx$

Optimal. Leaf size=11

$$-\frac{2}{\sqrt{x}+1}$$

```
[Out] -2/(1+x^(1/2))
```

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$-\frac{2}{\sqrt{x}+1}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + Sqrt[x])^2*Sqrt[x]),x]
```

```
[Out] -2/(1 + Sqrt[x])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(1+\sqrt{x}\right)^2 \sqrt{x}}\, dx = -\frac{2}{1+\sqrt{x}}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$-\frac{2}{\sqrt{x}+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + Sqrt[x])^2*Sqrt[x]),x]
```

```
[Out] -2/(1 + Sqrt[x])
```

fricas [A] time = 0.96, size = 12, normalized size = 1.09

$$-\frac{2\left(\sqrt{x}-1\right)}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(1+x^(1/2))^2,x, algorithm="fricas")
```

```
[Out] -2*(sqrt(x) - 1)/(x - 1)
```

giac [A] time = 0.16, size = 9, normalized size = 0.82

$$-\frac{2}{\sqrt{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(1+x^(1/2))^2,x, algorithm="giac")
[Out] -2/(sqrt(x) + 1)
```

maple [A] time = 0.00, size = 10, normalized size = 0.91

$$-\frac{2}{\sqrt{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(x^(1/2)+1)^2,x)
[Out] -2/(x^(1/2)+1)
```

maxima [A] time = 0.81, size = 9, normalized size = 0.82

$$-\frac{2}{\sqrt{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(1+x^(1/2))^2,x, algorithm="maxima")
[Out] -2/(sqrt(x) + 1)
```

mupad [B] time = 0.03, size = 9, normalized size = 0.82

$$-\frac{2}{\sqrt{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(x^(1/2) + 1)^2),x)
[Out] -2/(x^(1/2) + 1)
```

sympy [A] time = 0.36, size = 8, normalized size = 0.73

$$-\frac{2}{\sqrt{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(1+x**(1/2))**2,x)
[Out] -2/(sqrt(x) + 1)
```

3.2253 $\int \frac{1}{\left(1+\sqrt{x}\right)^3 \sqrt{x}}\, dx$

Optimal. Leaf size=11

$$-\frac{1}{\left(\sqrt{x}+1\right)^2}$$

```
[Out] -1/(1+x^(1/2))^2
```

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$-\frac{1}{\left(\sqrt{x}+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + Sqrt[x])^3*Sqrt[x]),x]
```

```
[Out] -(1 + Sqrt[x])^(-2)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(1+\sqrt{x}\right)^3 \sqrt{x}}\, dx = -\frac{1}{\left(1+\sqrt{x}\right)^2}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$-\frac{1}{\left(\sqrt{x}+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + Sqrt[x])^3*Sqrt[x]),x]
```

```
[Out] -(1 + Sqrt[x])^(-2)
```

fricas [B] time = 0.86, size = 20, normalized size = 1.82

$$-\frac{x-2\sqrt{x}+1}{x^2-2x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(1+x^(1/2))^3,x, algorithm="fricas")
```

```
[Out] -(x - 2*sqrt(x) + 1)/(x^2 - 2*x + 1)
```

giac [A] time = 0.15, size = 9, normalized size = 0.82

$$-\frac{1}{\left(\sqrt{x}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(1+x^(1/2))^3,x, algorithm="giac")
[Out] -1/(sqrt(x) + 1)^2
```

maple [A] time = 0.00, size = 10, normalized size = 0.91

$$-\frac{1}{\left(\sqrt{x}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(x^(1/2)+1)^3,x)
[Out] -1/(x^(1/2)+1)^2
```

maxima [A] time = 0.85, size = 9, normalized size = 0.82

$$-\frac{1}{\left(\sqrt{x}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(1+x^(1/2))^3,x, algorithm="maxima")
[Out] -1/(sqrt(x) + 1)^2
```

mupad [B] time = 0.03, size = 9, normalized size = 0.82

$$-\frac{1}{\left(\sqrt{x}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(x^(1/2) + 1)^3),x)
[Out] -1/(x^(1/2) + 1)^2
```

sympy [A] time = 0.44, size = 12, normalized size = 1.09

$$-\frac{1}{2\sqrt{x}+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(1+x**(1/2))**3,x)
[Out] -1/(2*sqrt(x) + x + 1)
```

3.2254 $\int \sqrt{1+\sqrt{x}}\,\sqrt{x}\,dx$

Optimal. Leaf size=46

$$\frac{4}{7}\left(\sqrt{x}+1\right)^{7/2}-\frac{8}{5}\left(\sqrt{x}+1\right)^{5/2}+\frac{4}{3}\left(\sqrt{x}+1\right)^{3/2}$$

```
[Out] 4/3*(1+x^(1/2))^(3/2)-8/5*(1+x^(1/2))^(5/2)+4/7*(1+x^(1/2))^(7/2)
```

Rubi [A] time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$\frac{4}{7}\left(\sqrt{x}+1\right)^{7/2}-\frac{8}{5}\left(\sqrt{x}+1\right)^{5/2}+\frac{4}{3}\left(\sqrt{x}+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Sqrt[x]]*Sqrt[x],x]
```

```
[Out] (4*(1 + Sqrt[x])^(3/2))/3 - (8*(1 + Sqrt[x])^(5/2))/5 + (4*(1 + Sqrt[x])^(7
/2))/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1+\sqrt{x}}\,\sqrt{x}\,dx &= 2\,\text{Subst}\left(\int x^2\sqrt{1+x}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(\sqrt{1+x}-2(1+x)^{3/2}+(1+x)^{5/2}\right)dx, x, \sqrt{x}\right)\\ &= \frac{4}{3}\left(1+\sqrt{x}\right)^{3/2}-\frac{8}{5}\left(1+\sqrt{x}\right)^{5/2}+\frac{4}{7}\left(1+\sqrt{x}\right)^{7/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.59

$$\frac{4}{105}\left(\sqrt{x}+1\right)^{3/2}\left(15x-12\sqrt{x}+8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sqrt[x]]*Sqrt[x],x]
```

```
[Out] (4*(1 + Sqrt[x])^(3/2)*(8 - 12*Sqrt[x] + 15*x))/105
```

fricas [A] time = 0.93, size = 23, normalized size = 0.50

$$\frac{4}{105}\left((15\,x-4)\sqrt{x}+3\,x+8\right)\sqrt{\sqrt{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="fricas")
[Out] 4/105*((15*x - 4)*sqrt(x) + 3*x + 8)*sqrt(sqrt(x) + 1)
```

giac [A] time = 0.15, size = 28, normalized size = 0.61

$$\frac{4}{7}\left(\sqrt{x}+1\right)^{\frac{7}{2}}-\frac{8}{5}\left(\sqrt{x}+1\right)^{\frac{5}{2}}+\frac{4}{3}\left(\sqrt{x}+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="giac")
[Out] 4/7*(sqrt(x) + 1)^(7/2) - 8/5*(sqrt(x) + 1)^(5/2) + 4/3*(sqrt(x) + 1)^(3/2)
```

maple [A] time = 0.01, size = 29, normalized size = 0.63

$$\frac{4\left(\sqrt{x}+1\right)^{\frac{3}{2}}}{3}-\frac{8\left(\sqrt{x}+1\right)^{\frac{5}{2}}}{5}+\frac{4\left(\sqrt{x}+1\right)^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(x^(1/2)+1)^(1/2),x)
[Out] 4/3*(x^(1/2)+1)^(3/2)-8/5*(x^(1/2)+1)^(5/2)+4/7*(x^(1/2)+1)^(7/2)
```

maxima [A] time = 0.86, size = 28, normalized size = 0.61

$$\frac{4}{7}\left(\sqrt{x}+1\right)^{\frac{7}{2}}-\frac{8}{5}\left(\sqrt{x}+1\right)^{\frac{5}{2}}+\frac{4}{3}\left(\sqrt{x}+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="maxima")
[Out] 4/7*(sqrt(x) + 1)^(7/2) - 8/5*(sqrt(x) + 1)^(5/2) + 4/3*(sqrt(x) + 1)^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{x}\,\sqrt{\sqrt{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(x^(1/2) + 1)^(1/2),x)
[Out] int(x^(1/2)*(x^(1/2) + 1)^(1/2), x)
```

sympy [B] time = 1.39, size = 398, normalized size = 8.65

$$\frac{60x^{\frac{15}{2}}\sqrt{\sqrt{x}+1}}{315x^{\frac{11}{2}}+105x^{\frac{9}{2}}+105x^{6}+315x^{5}}+\frac{200x^{\frac{13}{2}}\sqrt{\sqrt{x}+1}}{315x^{\frac{11}{2}}+105x^{\frac{9}{2}}+105x^{6}+315x^{5}}+\frac{60x^{\frac{11}{2}}\sqrt{\sqrt{x}+1}}{315x^{\frac{11}{2}}+105x^{\frac{9}{2}}+105x^{6}+315x^{5}}-\frac{}{315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(1+x**(1/2))**(1/2),x)
[Out] 60*x**(15/2)*sqrt(sqrt(x) + 1)/(315*x**(11/2) + 105*x**(9/2) + 105*x**6 + 3
15*x**5) + 200*x**(13/2)*sqrt(sqrt(x) + 1)/(315*x**(11/2) + 105*x**(9/2) +
105*x**6 + 315*x**5) + 60*x**(11/2)*sqrt(sqrt(x) + 1)/(315*x**(11/2) + 105*
x**(9/2) + 105*x**6 + 315*x**5) - 96*x**(11/2)/(315*x**(11/2) + 105*x**(9/2
```

```
) + 105*x**6 + 315*x**5) + 32*x**(9/2)*sqrt(sqrt(x) + 1)/(315*x**(11/2) + 1
05*x**(9/2) + 105*x**6 + 315*x**5) - 32*x**(9/2)/(315*x**(11/2) + 105*x**(9
/2) + 105*x**6 + 315*x**5) + 192*x**7*sqrt(sqrt(x) + 1)/(315*x**(11/2) + 10
5*x**(9/2) + 105*x**6 + 315*x**5) + 80*x**6*sqrt(sqrt(x) + 1)/(315*x**(11/2
) + 105*x**(9/2) + 105*x**6 + 315*x**5) - 32*x**6/(315*x**(11/2) + 105*x**(
9/2) + 105*x**6 + 315*x**5) + 80*x**5*sqrt(sqrt(x) + 1)/(315*x**(11/2) + 10
5*x**(9/2) + 105*x**6 + 315*x**5) - 96*x**5/(315*x**(11/2) + 105*x**(9/2) +
 105*x**6 + 315*x**5)
```

3.2255 $\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx$

Optimal. Leaf size=15

$$\frac{4}{3}\left(\sqrt{x}+1\right)^{3/2}$$

```
[Out] 4/3*(1+x^(1/2))^(3/2)
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {261}

$$\frac{4}{3}\left(\sqrt{x}+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Sqrt[x]]/Sqrt[x],x]
```

```
[Out] (4*(1 + Sqrt[x])^(3/2))/3
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx = \frac{4}{3}\left(1+\sqrt{x}\right)^{3/2}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{4}{3}\left(\sqrt{x}+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sqrt[x]]/Sqrt[x],x]
```

```
[Out] (4*(1 + Sqrt[x])^(3/2))/3
```

fricas [A] time = 1.06, size = 9, normalized size = 0.60

$$\frac{4}{3}\left(\sqrt{x}+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 4/3*(sqrt(x) + 1)^(3/2)
```

giac [A] time = 0.19, size = 9, normalized size = 0.60

$$\frac{4}{3}\left(\sqrt{x}+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))^(1/2)/x^(1/2),x, algorithm="giac")
[Out] 4/3*(sqrt(x) + 1)^(3/2)
```

maple [A] time = 0.00, size = 10, normalized size = 0.67

$$\frac{4\left(\sqrt{x}+1\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)+1)^(1/2)/x^(1/2),x)
[Out] 4/3*(x^(1/2)+1)^(3/2)
```

maxima [A] time = 0.88, size = 9, normalized size = 0.60

$$\frac{4}{3}\left(\sqrt{x}+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))^(1/2)/x^(1/2),x, algorithm="maxima")
[Out] 4/3*(sqrt(x) + 1)^(3/2)
```

mupad [B] time = 1.67, size = 9, normalized size = 0.60

$$\frac{4\left(\sqrt{x}+1\right)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2) + 1)^(1/2)/x^(1/2),x)
[Out] (4*(x^(1/2) + 1)^(3/2))/3
```

sympy [B] time = 0.23, size = 31, normalized size = 2.07

$$\frac{4\sqrt{x}\sqrt{\sqrt{x}+1}}{3}+\frac{4\sqrt{\sqrt{x}+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(1/2))**(1/2)/x**(1/2),x)
[Out] 4*sqrt(x)*sqrt(sqrt(x) + 1)/3 + 4*sqrt(sqrt(x) + 1)/3
```

3.2256 $\int \frac{\sqrt[3]{x}}{1+\sqrt{x}}\,dx$

Optimal. Leaf size=58

$$\frac{6x^{5/6}}{5} - 3\sqrt[3]{x} - 3\log\left(\sqrt[6]{x}+1\right) + \log\left(\sqrt{x}+1\right) - 2\sqrt{3}\,\tan^{-1}\left(\frac{1-2\sqrt[6]{x}}{\sqrt{3}}\right)$$

```
[Out] -3*x^(1/3)+6/5*x^(5/6)-3*ln(1+x^(1/6))+ln(1+x^(1/2))-2*arctan(1/3*(1-2*x^(1
/6))*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {341, 50, 56, 618, 204, 31}

$$\frac{6x^{5/6}}{5} - 3\sqrt[3]{x} - 3\log\left(\sqrt[6]{x}+1\right) + \log\left(\sqrt{x}+1\right) - 2\sqrt{3}\,\tan^{-1}\left(\frac{1-2\sqrt[6]{x}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3)/(1 + Sqrt[x]),x]
```

```
[Out] -3*x^(1/3) + (6*x^(5/6))/5 - 2*Sqrt[3]*ArcTan[(1 - 2*x^(1/6))/Sqrt[3]] - 3*
Log[1 + x^(1/6)] + Log[1 + Sqrt[x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{x}}{1+\sqrt{x}}\,dx &= 2\,\text{Subst}\left(\int \frac{x^{5/3}}{1+x}\,dx, x, \sqrt{x}\right) \\
&= \frac{6x^{5/6}}{5} - 2\,\text{Subst}\left(\int \frac{x^{2/3}}{1+x}\,dx, x, \sqrt{x}\right) \\
&= -3\sqrt[3]{x} + \frac{6x^{5/6}}{5} + 2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{x}\,(1+x)}\,dx, x, \sqrt{x}\right) \\
&= -3\sqrt[3]{x} + \frac{6x^{5/6}}{5} + \log\left(1+\sqrt{x}\right) - 3\,\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt[6]{x}\right) + 3\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \sqrt[6]{x}\right) \\
&= -3\sqrt[3]{x} + \frac{6x^{5/6}}{5} - 3\log\left(1+\sqrt[6]{x}\right) + \log\left(1+\sqrt{x}\right) - 6\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\sqrt[6]{x}\right) \\
&= -3\sqrt[3]{x} + \frac{6x^{5/6}}{5} - 2\sqrt{3}\,\tan^{-1}\left(\frac{1-2\sqrt[6]{x}}{\sqrt{3}}\right) - 3\log\left(1+\sqrt[6]{x}\right) + \log\left(1+\sqrt{x}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.60

$$\frac{3}{5}\sqrt[3]{x}\left(5\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\sqrt{x}\right) + 2\sqrt{x} - 5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3)/(1 + Sqrt[x]),x]
```

```
[Out] (3*x^(1/3)*(-5 + 2*Sqrt[x] + 5*Hypergeometric2F1[2/3, 1, 5/3, -Sqrt[x]]))/5
```

fricas [A] time = 0.89, size = 50, normalized size = 0.86

$$2\sqrt{3}\,\arctan\left(\frac{2}{3}\sqrt{3}\,x^{\frac{1}{6}} - \frac{1}{3}\sqrt{3}\right) + \frac{6}{5}x^{\frac{5}{6}} - 3\,x^{\frac{1}{3}} + \log\left(x^{\frac{1}{3}} - x^{\frac{1}{6}} + 1\right) - 2\,\log\left(x^{\frac{1}{6}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(1+x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*sqrt(3)*arctan(2/3*sqrt(3)*x^(1/6) - 1/3*sqrt(3)) + 6/5*x^(5/6) - 3*x^(1/
3) + log(x^(1/3) - x^(1/6) + 1) - 2*log(x^(1/6) + 1)
```

giac [A] time = 0.16, size = 48, normalized size = 0.83

$$2\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^{\frac{1}{6}} - 1\right)\right) + \frac{6}{5}x^{\frac{5}{6}} - 3\,x^{\frac{1}{3}} + \log\left(x^{\frac{1}{3}} - x^{\frac{1}{6}} + 1\right) - 2\,\log\left(x^{\frac{1}{6}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(1+x^(1/2)),x, algorithm="giac")
```

```
[Out] 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/6) - 1)) + 6/5*x^(5/6) - 3*x^(1/3) + l
og(x^(1/3) - x^(1/6) + 1) - 2*log(x^(1/6) + 1)
```

maple [A] time = 0.01, size = 49, normalized size = 0.84

$$2\sqrt{3}\ \arctan\left(\frac{\left(2x^{\frac{1}{6}}-1\right)\sqrt{3}}{3}\right)-2\ln\left(x^{\frac{1}{6}}+1\right)+\ln\left(x^{\frac{1}{3}}-x^{\frac{1}{6}}+1\right)+\frac{6x^{\frac{5}{6}}}{5}-3x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)/(x^(1/2)+1),x)
```

```
[Out] 6/5*x^(5/6)-3*x^(1/3)+ln(x^(1/3)-x^(1/6)+1)+2*3^(1/2)*arctan(1/3*(2*x^(1/6)
-1)*3^(1/2))-2*ln(1+x^(1/6))
```

maxima [A] time = 1.98, size = 48, normalized size = 0.83

$$2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{6}}-1\right)\right)+\frac{6}{5}x^{\frac{5}{6}}-3x^{\frac{1}{3}}+\log\left(x^{\frac{1}{3}}-x^{\frac{1}{6}}+1\right)-2\log\left(x^{\frac{1}{6}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(1+x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/6) - 1)) + 6/5*x^(5/6) - 3*x^(1/3) + l
og(x^(1/3) - x^(1/6) + 1) - 2*log(x^(1/6) + 1)
```

mupad [B] time = 1.12, size = 78, normalized size = 1.34

$$\frac{6x^{5/6}}{5}-\ln\left(9\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2+36x^{1/6}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)+\ln\left(9\left(1+\sqrt{3}\,1\mathrm{i}\right)^2+36x^{1/6}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)-3x^{1/3}-2\ln\left(36\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)/(x^(1/2) + 1),x)
```

```
[Out] log(9*(3^(1/2)*1i + 1)^2 + 36*x^(1/6))*(3^(1/2)*1i + 1) - log(9*(3^(1/2)*1i
 - 1)^2 + 36*x^(1/6))*(3^(1/2)*1i - 1) - 2*log(36*x^(1/6) + 36) - 3*x^(1/3)
 + (6*x^(5/6))/5
```

sympy [C] time = 1.21, size = 138, normalized size = 2.38

$$\frac{16x^{\frac{5}{6}}\Gamma\left(\frac{8}{3}\right)}{5\Gamma\left(\frac{11}{3}\right)}-\frac{8\sqrt[3]{x}\,\Gamma\left(\frac{8}{3}\right)}{\Gamma\left(\frac{11}{3}\right)}-\frac{16e^{-\frac{2i\pi}{3}}\log\left(-\sqrt[6]{x}\,e^{\frac{i\pi}{3}}+1\right)\Gamma\left(\frac{8}{3}\right)}{3\Gamma\left(\frac{11}{3}\right)}-\frac{16\log\left(-\sqrt[6]{x}\,e^{i\pi}+1\right)\Gamma\left(\frac{8}{3}\right)}{3\Gamma\left(\frac{11}{3}\right)}-\frac{16e^{\frac{2i\pi}{3}}\log\left(-\sqrt[6]{x}\,e^{\frac{5i\pi}{3}}+1\right.}{3\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3)/(1+x**(1/2)),x)
```

```
[Out] 16*x**(5/6)*gamma(8/3)/(5*gamma(11/3)) - 8*x**(1/3)*gamma(8/3)/gamma(11/3)
- 16*exp(-2*I*pi/3)*log(-x**(1/6)*exp_polar(I*pi/3) + 1)*gamma(8/3)/(3*gamm
a(11/3)) - 16*log(-x**(1/6)*exp_polar(I*pi) + 1)*gamma(8/3)/(3*gamma(11/3))
 - 16*exp(2*I*pi/3)*log(-x**(1/6)*exp_polar(5*I*pi/3) + 1)*gamma(8/3)/(3*ga
mma(11/3))
```

3.2257 $\int \left(a+b\sqrt{x}\right)^4 x^m\,dx$

Optimal. Leaf size=87

$$\frac{a^4x^{m+1}}{m+1}+\frac{8a^3bx^{m+\frac{3}{2}}}{2m+3}+\frac{6a^2b^2x^{m+2}}{m+2}+\frac{8ab^3x^{m+\frac{5}{2}}}{2m+5}+\frac{b^4x^{m+3}}{m+3}$$

```
[Out] a^4*x^(1+m)/(1+m)+8*a^3*b*x^(3/2+m)/(3+2*m)+6*a^2*b^2*x^(2+m)/(2+m)+8*a*b^3
*x^(5/2+m)/(5+2*m)+b^4*x^(3+m)/(3+m)
```

Rubi [A] time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {270}

$$\frac{6a^2b^2x^{m+2}}{m+2}+\frac{8a^3bx^{m+\frac{3}{2}}}{2m+3}+\frac{a^4x^{m+1}}{m+1}+\frac{8ab^3x^{m+\frac{5}{2}}}{2m+5}+\frac{b^4x^{m+3}}{m+3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^4*x^m,x]
```

```
[Out] (a^4*x^(1 + m))/(1 + m) + (8*a^3*b*x^(3/2 + m))/(3 + 2*m) + (6*a^2*b^2*x^(2
 + m))/(2 + m) + (8*a*b^3*x^(5/2 + m))/(5 + 2*m) + (b^4*x^(3 + m))/(3 + m)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sqrt{x}\right)^4 x^m\,dx &= \int \left(a^4x^m+4a^3bx^{\frac{1}{2}+m}+6a^2b^2x^{1+m}+4ab^3x^{\frac{3}{2}+m}+b^4x^{2+m}\right)dx\\ &= \frac{a^4x^{1+m}}{1+m}+\frac{8a^3bx^{\frac{3}{2}+m}}{3+2m}+\frac{6a^2b^2x^{2+m}}{2+m}+\frac{8ab^3x^{\frac{5}{2}+m}}{5+2m}+\frac{b^4x^{3+m}}{3+m}\end{aligned}$$

Mathematica [A] time = 0.10, size = 78, normalized size = 0.90

$$x^{m+1}\left(\frac{a^4}{m+1}+\frac{8a^3b\sqrt{x}}{2m+3}+\frac{6a^2b^2x}{m+2}+\frac{8ab^3x^{3/2}}{2m+5}+\frac{b^4x^2}{m+3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^4*x^m,x]
```

```
[Out] x^(1 + m)*(a^4/(1 + m) + (8*a^3*b*Sqrt[x])/(3 + 2*m) + (6*a^2*b^2*x)/(2 + m
) + (8*a*b^3*x^(3/2))/(5 + 2*m) + (b^4*x^2)/(3 + m))
```

fricas [B] time = 1.37, size = 260, normalized size = 2.99

$$\frac{\left(\left(4\,b^4m^4+28\,b^4m^3+71\,b^4m^2+77\,b^4m+30\,b^4\right)x^3+6\left(4\,a^2b^2m^4+32\,a^2b^2m^3+91\,a^2b^2m^2+108\,a^2b^2m+45\,a^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^(1/2))^4,x, algorithm="fricas")
```

```
[Out] ((4*b^4*m^4 + 28*b^4*m^3 + 71*b^4*m^2 + 77*b^4*m + 30*b^4)*x^3 + 6*(4*a^2*b
^2*m^4 + 32*a^2*b^2*m^3 + 91*a^2*b^2*m^2 + 108*a^2*b^2*m + 45*a^2*b^2)*x^2
+ (4*a^4*m^4 + 36*a^4*m^3 + 119*a^4*m^2 + 171*a^4*m + 90*a^4)*x + 8*((2*a*b
^3*m^4 + 15*a*b^3*m^3 + 40*a*b^3*m^2 + 45*a*b^3*m + 18*a*b^3)*x^2 + (2*a^3*
b*m^4 + 17*a^3*b*m^3 + 52*a^3*b*m^2 + 67*a^3*b*m + 30*a^3*b)*x)*sqrt(x))*x^
m/(4*m^5 + 40*m^4 + 155*m^3 + 290*m^2 + 261*m + 90)
```

giac [A] time = 0.23, size = 106, normalized size = 1.22

$$\frac{b^4 x^3 \sqrt{x}^{2m}}{m+3} + \frac{8ab^3 x^{\frac{5}{2}} \sqrt{x}^{2m}}{2m+5} + \frac{6a^2 b^2 x^2 \sqrt{x}^{2m}}{m+2} + \frac{8a^3 b x^{\frac{3}{2}} \sqrt{x}^{2m}}{2m+3} + \frac{a^4 x \sqrt{x}^{2m}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^(1/2))^4,x, algorithm="giac")
```

```
[Out] b^4*x^3*sqrt(x)^(2*m)/(m + 3) + 8*a*b^3*x^(5/2)*sqrt(x)^(2*m)/(2*m + 5) + 6
*a^2*b^2*x^2*sqrt(x)^(2*m)/(m + 2) + 8*a^3*b*x^(3/2)*sqrt(x)^(2*m)/(2*m + 3
) + a^4*x*sqrt(x)^(2*m)/(m + 1)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \left(b\sqrt{x} + a\right)^4 x^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^(1/2)+a)^4,x)
```

```
[Out] int(x^m*(b*x^(1/2)+a)^4,x)
```

maxima [A] time = 0.90, size = 83, normalized size = 0.95

$$\frac{b^4 x^{m+3}}{m+3} + \frac{8ab^3 x^{m+\frac{5}{2}}}{2m+5} + \frac{6a^2 b^2 x^{m+2}}{m+2} + \frac{8a^3 b x^{m+\frac{3}{2}}}{2m+3} + \frac{a^4 x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^(1/2))^4,x, algorithm="maxima")
```

```
[Out] b^4*x^(m + 3)/(m + 3) + 8*a*b^3*x^(m + 5/2)/(2*m + 5) + 6*a^2*b^2*x^(m + 2)
/(m + 2) + 8*a^3*b*x^(m + 3/2)/(2*m + 3) + a^4*x^(m + 1)/(m + 1)
```

mupad [B] time = 1.70, size = 292, normalized size = 3.36

$$\frac{b^4 x^m x^3 \left(4m^4 + 28m^3 + 71m^2 + 77m + 30\right)}{4m^5 + 40m^4 + 155m^3 + 290m^2 + 261m + 90} + \frac{a^4 x x^m \left(4m^4 + 36m^3 + 119m^2 + 171m + 90\right)}{4m^5 + 40m^4 + 155m^3 + 290m^2 + 261m + 90} + \frac{8ab^3 x^m x^{5/}}{4m^5 + 40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^(1/2))^4,x)
```

```
[Out] (b^4*x^m*x^3*(77*m + 71*m^2 + 28*m^3 + 4*m^4 + 30))/(261*m + 290*m^2 + 155*
m^3 + 40*m^4 + 4*m^5 + 90) + (a^4*x*x^m*(171*m + 119*m^2 + 36*m^3 + 4*m^4 +
 90))/(261*m + 290*m^2 + 155*m^3 + 40*m^4 + 4*m^5 + 90) + (8*a*b^3*x^m*x^(5
/2)*(45*m + 40*m^2 + 15*m^3 + 2*m^4 + 18))/(261*m + 290*m^2 + 155*m^3 + 40*
m^4 + 4*m^5 + 90) + (8*a^3*b*x^m*x^(3/2)*(67*m + 52*m^2 + 17*m^3 + 2*m^4 +
30))/(261*m + 290*m^2 + 155*m^3 + 40*m^4 + 4*m^5 + 90) + (6*a^2*b^2*x^m*x^2
*(108*m + 91*m^2 + 32*m^3 + 4*m^4 + 45))/(261*m + 290*m^2 + 155*m^3 + 40*m^
4 + 4*m^5 + 90)
```

sympy [**A**] time = 16.33, size = 117, normalized size = 1.34

$$a^4\left(\begin{cases}\frac{x^{m+1}}{m+1} & \text{for } m \neq -1\\ \log(x) & \text{otherwise}\end{cases}\right)+8a^3b\left(\begin{cases}\frac{x^{\frac{3}{2}}x^m}{2m+3} & \text{for } m \neq -\frac{3}{2}\\ \log\left(\sqrt{x}\right) & \text{otherwise}\end{cases}\right)+6a^2b^2\left(\begin{cases}\frac{x^2x^m}{m+2} & \text{for } m \neq -2\\ \log(x) & \text{otherwise}\end{cases}\right)+4ab^3\left(\begin{cases}\frac{2x^{\frac{5}{2}}x^m}{2m+5} & \text{f}\\ \log(x) & \text{o}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*x**(1/2))**4,x)
```

```
[Out] a**4*Piecewise((x**(m + 1)/(m + 1), Ne(m, -1)), (log(x), True)) + 8*a**3*b*
Piecewise((x**(3/2)*x**m/(2*m + 3), Ne(m, -3/2)), (log(sqrt(x)), True)) + 6
*a**2*b**2*Piecewise((x**2*x**m/(m + 2), Ne(m, -2)), (log(x), True)) + 4*a*
b**3*Piecewise((2*x**(5/2)*x**m/(2*m + 5), Ne(m, -5/2)), (log(x), True)) +
b**4*Piecewise((x**3*x**m/(m + 3), Ne(m, -3)), (log(x), True))
```

3.2258 $\int \left(a+b\sqrt{x}\right)^3 x^m\,dx$

Optimal. Leaf size=70

$$\frac{a^3x^{m+1}}{m+1}+\frac{6a^2bx^{m+\frac{3}{2}}}{2m+3}+\frac{3ab^2x^{m+2}}{m+2}+\frac{2b^3x^{m+\frac{5}{2}}}{2m+5}$$

```
[Out] a^3*x^(1+m)/(1+m)+6*a^2*b*x^(3/2+m)/(3+2*m)+3*a*b^2*x^(2+m)/(2+m)+2*b^3*x^(
5/2+m)/(5+2*m)
```

Rubi [A] time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {270}

$$\frac{6a^2bx^{m+\frac{3}{2}}}{2m+3}+\frac{a^3x^{m+1}}{m+1}+\frac{3ab^2x^{m+2}}{m+2}+\frac{2b^3x^{m+\frac{5}{2}}}{2m+5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^3*x^m,x]
```

```
[Out] (a^3*x^(1 + m))/(1 + m) + (6*a^2*b*x^(3/2 + m))/(3 + 2*m) + (3*a*b^2*x^(2 +
 m))/(2 + m) + (2*b^3*x^(5/2 + m))/(5 + 2*m)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sqrt{x}\right)^3 x^m\,dx &= \int \left(a^3x^m+3a^2bx^{\frac{1}{2}+m}+3ab^2x^{1+m}+b^3x^{\frac{3}{2}+m}\right)dx\\ &= \frac{a^3x^{1+m}}{1+m}+\frac{6a^2bx^{\frac{3}{2}+m}}{3+2m}+\frac{3ab^2x^{2+m}}{2+m}+\frac{2b^3x^{\frac{5}{2}+m}}{5+2m}\end{aligned}$$

Mathematica [A] time = 0.08, size = 63, normalized size = 0.90

$$x^{m+1}\left(\frac{a^3}{m+1}+\frac{6a^2b\sqrt{x}}{2m+3}+\frac{3ab^2x}{m+2}+\frac{2b^3x^{3/2}}{2m+5}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^3*x^m,x]
```

```
[Out] x^(1 + m)*(a^3/(1 + m) + (6*a^2*b*Sqrt[x])/(3 + 2*m) + (3*a*b^2*x)/(2 + m)
 + (2*b^3*x^(3/2))/(5 + 2*m))
```

fricas [B] time = 1.41, size = 167, normalized size = 2.39

$$\frac{\left(3\left(4\,ab^2m^3+20\,ab^2m^2+31\,ab^2m+15\,ab^2\right)x^2+\left(4\,a^3m^3+24\,a^3m^2+47\,a^3m+30\,a^3\right)x+2\left(\left(2\,b^3m^3+9\,b^3m\right.\right.\right.}{4\,m^4+28\,m^3+71\,m^2+77\,m+30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^(1/2))^3,x, algorithm="fricas")
```

```
[Out] (3*(4*a*b^2*m^3 + 20*a*b^2*m^2 + 31*a*b^2*m + 15*a*b^2)*x^2 + (4*a^3*m^3 +
24*a^3*m^2 + 47*a^3*m + 30*a^3)*x + 2*((2*b^3*m^3 + 9*b^3*m^2 + 13*b^3*m +
6*b^3)*x^2 + 3*(2*a^2*b*m^3 + 11*a^2*b*m^2 + 19*a^2*b*m + 10*a^2*b)*x)*sqrt
(x))*x^m/(4*m^4 + 28*m^3 + 71*m^2 + 77*m + 30)
```

giac [A] time = 0.18, size = 84, normalized size = 1.20

$$\frac{2\,b^3 x^{\frac{5}{2}} \sqrt{x}^{\,2m}}{2m+5} + \frac{3\,ab^2 x^2 \sqrt{x}^{\,2m}}{m+2} + \frac{6\,a^2 b x^{\frac{3}{2}} \sqrt{x}^{\,2m}}{2m+3} + \frac{a^3 x \sqrt{x}^{\,2m}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^(1/2))^3,x, algorithm="giac")
```

```
[Out] 2*b^3*x^(5/2)*sqrt(x)^(2*m)/(2*m + 5) + 3*a*b^2*x^2*sqrt(x)^(2*m)/(m + 2) +
 6*a^2*b*x^(3/2)*sqrt(x)^(2*m)/(2*m + 3) + a^3*x*sqrt(x)^(2*m)/(m + 1)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \left(b\sqrt{x} + a\right)^3 x^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^(1/2)+a)^3,x)
```

```
[Out] int(x^m*(b*x^(1/2)+a)^3,x)
```

maxima [A] time = 0.90, size = 66, normalized size = 0.94

$$\frac{2\,b^3 x^{m+\frac{5}{2}}}{2m+5} + \frac{3\,ab^2 x^{m+2}}{m+2} + \frac{6\,a^2 b x^{m+\frac{3}{2}}}{2m+3} + \frac{a^3 x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^(1/2))^3,x, algorithm="maxima")
```

```
[Out] 2*b^3*x^(m + 5/2)/(2*m + 5) + 3*a*b^2*x^(m + 2)/(m + 2) + 6*a^2*b*x^(m + 3/
2)/(2*m + 3) + a^3*x^(m + 1)/(m + 1)
```

mupad [B] time = 1.43, size = 184, normalized size = 2.63

$$x^m \left(\frac{a^3 x \left(4m^3 + 24m^2 + 47m + 30\right)}{4m^4 + 28m^3 + 71m^2 + 77m + 30} + \frac{2\,b^3 x^{5/2} \left(2m^3 + 9m^2 + 13m + 6\right)}{4m^4 + 28m^3 + 71m^2 + 77m + 30} + \frac{6\,a^2 b\, x^{3/2} \left(2m^3 + 11m^2 + 19m \right.}{4m^4 + 28m^3 + 71m^2 + 77m +} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^(1/2))^3,x)
```

```
[Out] x^m*((a^3*x*(47*m + 24*m^2 + 4*m^3 + 30))/(77*m + 71*m^2 + 28*m^3 + 4*m^4 +
 30) + (2*b^3*x^(5/2)*(13*m + 9*m^2 + 2*m^3 + 6))/(77*m + 71*m^2 + 28*m^3 +
 4*m^4 + 30) + (6*a^2*b*x^(3/2)*(19*m + 11*m^2 + 2*m^3 + 10))/(77*m + 71*m^
2 + 28*m^3 + 4*m^4 + 30) + (3*a*b^2*x^2*(31*m + 20*m^2 + 4*m^3 + 15))/(77*m
 + 71*m^2 + 28*m^3 + 4*m^4 + 30))
```

sympy [A] time = 15.17, size = 94, normalized size = 1.34

$$a^3 \left(\begin{cases} \frac{x^{m+1}}{m+1} & \text{for } m \neq -1 \\ \log(x) & \text{otherwise} \end{cases} \right) + 6a^2 b \left(\begin{cases} \frac{x^{\frac{3}{2}} x^m}{2m+3} & \text{for } m \neq -\frac{3}{2} \\ \log\left(\sqrt{x}\right) & \text{otherwise} \end{cases} \right) + 3ab^2 \left(\begin{cases} \frac{x^2 x^m}{m+2} & \text{for } m \neq -2 \\ \log(x) & \text{otherwise} \end{cases} \right) + b^3 \left(\begin{cases} \frac{2x^{\frac{5}{2}} x^m}{2m+5} & \text{for } m \\ \log(x) & \text{othe} \end{cases} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*x**(1/2))**3,x)
```

```
[Out] a**3*Piecewise((x**(m + 1)/(m + 1), Ne(m, -1)), (log(x), True)) + 6*a**2*b*
Piecewise((x**(3/2)*x**m/(2*m + 3), Ne(m, -3/2)), (log(sqrt(x)), True)) + 3
*a*b**2*Piecewise((x**2*x**m/(m + 2), Ne(m, -2)), (log(x), True)) + b**3*Pi
ecewise((2*x**(5/2)*x**m/(2*m + 5), Ne(m, -5/2)), (log(x), True))
```

3.2259 $\int \left(a+b\sqrt{x}\right)^2 x^m\,dx$

Optimal. Leaf size=47

$$\frac{a^2x^{m+1}}{m+1}+\frac{4abx^{m+\frac{3}{2}}}{2m+3}+\frac{b^2x^{m+2}}{m+2}$$

```
[Out] a^2*x^(1+m)/(1+m)+4*a*b*x^(3/2+m)/(3+2*m)+b^2*x^(2+m)/(2+m)
```

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {270}

$$\frac{a^2x^{m+1}}{m+1}+\frac{4abx^{m+\frac{3}{2}}}{2m+3}+\frac{b^2x^{m+2}}{m+2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^2*x^m,x]
```

```
[Out] (a^2*x^(1 + m))/(1 + m) + (4*a*b*x^(3/2 + m))/(3 + 2*m) + (b^2*x^(2 + m))/(
2 + m)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sqrt{x}\right)^2 x^m\,dx &= \int \left(a^2x^m+2abx^{\frac{1}{2}+m}+b^2x^{1+m}\right)dx\\ &= \frac{a^2x^{1+m}}{1+m}+\frac{4abx^{\frac{3}{2}+m}}{3+2m}+\frac{b^2x^{2+m}}{2+m}\end{aligned}$$

Mathematica [A] time = 0.06, size = 42, normalized size = 0.89

$$x^{m+1}\left(\frac{a^2}{m+1}+\frac{4ab\sqrt{x}}{2m+3}+\frac{b^2x}{m+2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^2*x^m,x]
```

```
[Out] x^(1 + m)*(a^2/(1 + m) + (4*a*b*Sqrt[x])/(3 + 2*m) + (b^2*x)/(2 + m))
```

fricas [A] time = 0.86, size = 89, normalized size = 1.89

$$\frac{\left(\left(2\,b^2m^2+5\,b^2m+3\,b^2\right)x^2+4\left(abm^2+3\,abm+2\,ab\right)x^{\frac{3}{2}}+\left(2\,a^2m^2+7\,a^2m+6\,a^2\right)x\right)x^m}{2\,m^3+9\,m^2+13\,m+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^(1/2))^2,x, algorithm="fricas")
```

```
[Out] ((2*b^2*m^2 + 5*b^2*m + 3*b^2)*x^2 + 4*(a*b*m^2 + 3*a*b*m + 2*a*b)*x^(3/2)
+ (2*a^2*m^2 + 7*a^2*m + 6*a^2)*x)*x^m/(2*m^3 + 9*m^2 + 13*m + 6)
```

giac [A] time = 0.18, size = 58, normalized size = 1.23

$$\frac{b^2x^2\sqrt{x}^{2m}}{m+2}+\frac{4abx^{\frac{3}{2}}\sqrt{x}^{2m}}{2m+3}+\frac{a^2x\sqrt{x}^{2m}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^(1/2))^2,x, algorithm="giac")
```

```
[Out] b^2*x^2*sqrt(x)^(2*m)/(m + 2) + 4*a*b*x^(3/2)*sqrt(x)^(2*m)/(2*m + 3) + a^2
*x*sqrt(x)^(2*m)/(m + 1)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \left(b\sqrt{x}+a\right)^2 x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^(1/2)+a)^2,x)
```

```
[Out] int(x^m*(b*x^(1/2)+a)^2,x)
```

maxima [A] time = 0.90, size = 45, normalized size = 0.96

$$\frac{b^2x^{m+2}}{m+2}+\frac{4abx^{m+\frac{3}{2}}}{2m+3}+\frac{a^2x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^(1/2))^2,x, algorithm="maxima")
```

```
[Out] b^2*x^(m + 2)/(m + 2) + 4*a*b*x^(m + 3/2)/(2*m + 3) + a^2*x^(m + 1)/(m + 1)
```

mupad [B] time = 1.28, size = 103, normalized size = 2.19

$$x^m\left(\frac{b^2x^2\left(2m^2+5m+3\right)}{2m^3+9m^2+13m+6}+\frac{a^2x\left(2m^2+7m+6\right)}{2m^3+9m^2+13m+6}+\frac{4ab\,x^{3/2}\left(m^2+3m+2\right)}{2m^3+9m^2+13m+6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^(1/2))^2,x)
```

```
[Out] x^m*((b^2*x^2*(5*m + 2*m^2 + 3))/(13*m + 9*m^2 + 2*m^3 + 6) + (a^2*x*(7*m +
 2*m^2 + 6))/(13*m + 9*m^2 + 2*m^3 + 6) + (4*a*b*x^(3/2)*(3*m + m^2 + 2))/(
13*m + 9*m^2 + 2*m^3 + 6))
```

sympy [A] time = 3.20, size = 63, normalized size = 1.34

$$a^2\left(\begin{cases}\frac{x^{m+1}}{m+1} & \text{for } m\neq -1\\ \log(x) & \text{otherwise}\end{cases}\right)+4ab\left(\begin{cases}\frac{x^{\frac{3}{2}}x^m}{2m+3} & \text{for } m\neq -\frac{3}{2}\\ \log\left(\sqrt{x}\right) & \text{otherwise}\end{cases}\right)+b^2\left(\begin{cases}\frac{x^2x^m}{m+2} & \text{for } m\neq -2\\ \log(x) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*x**(1/2))**2,x)
```

```
[Out] a**2*Piecewise((x**(m + 1)/(m + 1), Ne(m, -1)), (log(x), True)) + 4*a*b*Pie
cewise((x**(3/2)*x**m/(2*m + 3), Ne(m, -3/2)), (log(sqrt(x)), True)) + b**2
*Piecewise((x**2*x**m/(m + 2), Ne(m, -2)), (log(x), True))
```

3.2260 $\int \left(a + b\sqrt{x}\right) x^m \, dx$

Optimal. Leaf size=30

$$\frac{ax^{m+1}}{m+1} + \frac{2bx^{m+\frac{3}{2}}}{2m+3}$$

```
[Out] a*x^(1+m)/(1+m)+2*b*x^(3/2+m)/(3+2*m)
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$\frac{ax^{m+1}}{m+1} + \frac{2bx^{m+\frac{3}{2}}}{2m+3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])*x^m,x]
```

```
[Out] (a*x^(1 + m))/(1 + m) + (2*b*x^(3/2 + m))/(3 + 2*m)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \left(a + b\sqrt{x}\right) x^m \, dx &= \int \left(ax^m + bx^{\frac{1}{2}+m}\right) dx \\ &= \frac{ax^{1+m}}{1+m} + \frac{2bx^{\frac{3}{2}+m}}{3+2m}\end{aligned}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 1.00

$$\frac{ax^{m+1}}{m+1} + \frac{2bx^{m+\frac{3}{2}}}{2m+3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])*x^m,x]
```

```
[Out] (a*x^(1 + m))/(1 + m) + (2*b*x^(3/2 + m))/(3 + 2*m)
```

fricas [A] time = 0.93, size = 37, normalized size = 1.23

$$\frac{\left(2\,(bm+b)x^{\frac{3}{2}} + (2\,am+3\,a)x\right)x^m}{2\,m^2+5\,m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^(1/2)),x, algorithm="fricas")
```

```
[Out] (2*(b*m + b)*x^(3/2) + (2*a*m + 3*a)*x)*x^m/(2*m^2 + 5*m + 3)
```

giac [A] time = 0.19, size = 36, normalized size = 1.20

$$\frac{2\,b x^{\frac{3}{2}} \sqrt{x}^{2\,m}}{2\,m+3} + \frac{a x \sqrt{x}^{2\,m}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^(1/2)),x, algorithm="giac")
[Out] 2*b*x^(3/2)*sqrt(x)^(2*m)/(2*m + 3) + a*x*sqrt(x)^(2*m)/(m + 1)
```

maple [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sqrt{x} + a\right) x^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^(1/2)+a),x)
[Out] int(x^m*(b*x^(1/2)+a),x)
```

maxima [A] time = 0.89, size = 28, normalized size = 0.93

$$\frac{2\,b x^{m+\frac{3}{2}}}{2\,m+3} + \frac{a x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^(1/2)),x, algorithm="maxima")
[Out] 2*b*x^(m + 3/2)/(2*m + 3) + a*x^(m + 1)/(m + 1)
```

mupad [B] time = 1.21, size = 39, normalized size = 1.30

$$\frac{x^{m+1}\left(3\,a + 2\,a\,m + 2\,b\,\sqrt{x} + 2\,b\,m\,\sqrt{x}\right)}{2\,m^2 + 5\,m + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^(1/2)),x)
[Out] (x^(m + 1)*(3*a + 2*a*m + 2*b*x^(1/2) + 2*b*m*x^(1/2)))/(5*m + 2*m^2 + 3)
```

sympy [A] time = 2.54, size = 41, normalized size = 1.37

$$a\left(\begin{cases} \frac{x^{m+1}}{m+1} & \text{for } m \neq -1 \\ \log(x) & \text{otherwise} \end{cases}\right) + 2b\left(\begin{cases} \frac{x^{\frac{3}{2}} x^m}{2m+3} & \text{for } m \neq -\frac{3}{2} \\ \log\left(\sqrt{x}\right) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*x**(1/2)),x)
[Out] a*Piecewise((x**(m + 1)/(m + 1), Ne(m, -1)), (log(x), True)) + 2*b*Piecewis
e((x**(3/2)*x**m/(2*m + 3), Ne(m, -3/2)), (log(sqrt(x)), True))
```

3.2261 $\int \frac{x^m}{a+b\sqrt{x}}\,dx$

Optimal. Leaf size=37

$$\frac{x^{m+1}\,{}_2F_1\left(1,2(m+1);2m+3;-\frac{b\sqrt{x}}{a}\right)}{a(m+1)}$$

```
[Out] x^(1+m)*hypergeom([1, 2+2*m],[3+2*m],-b*x^(1/2)/a)/a/(1+m)
```

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {341, 64}

$$\frac{x^{m+1}\,{}_2F_1\left(1,2(m+1);2m+3;-\frac{b\sqrt{x}}{a}\right)}{a(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*Sqrt[x]),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1, 2*(1 + m), 3 + 2*m, -((b*Sqrt[x])/a)])/(a*(
1 + m))
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{a+b\sqrt{x}}\,dx &= 2\,\text{Subst}\left(\int \frac{x^{-1+2(1+m)}}{a+bx}\,dx, x, \sqrt{x}\right)\\ &= \frac{x^{1+m}\,{}_2F_1\left(1,2(1+m);3+2m;-\frac{b\sqrt{x}}{a}\right)}{a(1+m)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 0.97

$$\frac{x^{m+1}\,{}_2F_1\left(1,2m+2;2m+3;-\frac{b\sqrt{x}}{a}\right)}{am+a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*Sqrt[x]),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1, 2 + 2*m, 3 + 2*m, -((b*Sqrt[x])/a)])/(a + a
*m)
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\sqrt{x}\,x^m - ax^m}{b^2x - a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^(1/2)),x, algorithm="fricas")
```

```
[Out] integral((b*sqrt(x)*x^m - a*x^m)/(b^2*x - a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{b\sqrt{x} + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*sqrt(x) + a), x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{x^m}{b\sqrt{x} + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^(1/2)+a),x)
```

```
[Out] int(x^m/(b*x^(1/2)+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{b\sqrt{x} + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(b*sqrt(x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{a + b\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^(1/2)),x)
```

```
[Out] int(x^m/(a + b*x^(1/2)), x)
```

sympy [C] time = 1.11, size = 82, normalized size = 2.22

$$\frac{4mxx^m\Phi\left(\frac{b\sqrt{x}\,e^{i\pi}}{a}, 1, 2m+2\right)\Gamma(2m+2)}{a\Gamma(2m+3)} + \frac{4xx^m\Phi\left(\frac{b\sqrt{x}\,e^{i\pi}}{a}, 1, 2m+2\right)\Gamma(2m+2)}{a\Gamma(2m+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a+b*x**(1/2)),x)
```

```
[Out] 4*m*x*x**m*lerchphi(b*sqrt(x)*exp_polar(I*pi)/a, 1, 2*m + 2)*gamma(2*m + 2)
/(a*gamma(2*m + 3)) + 4*x*x**m*lerchphi(b*sqrt(x)*exp_polar(I*pi)/a, 1, 2*m
 + 2)*gamma(2*m + 2)/(a*gamma(2*m + 3))
```

3.2262 $\int \frac{x^m}{\left(a+b\sqrt{x}\right)^2}\,dx$

Optimal. Leaf size=37

$$\frac{x^{m+1}\,{}_2F_1\left(2,2(m+1);2m+3;-\frac{b\sqrt{x}}{a}\right)}{a^2(m+1)}$$

```
[Out] x^(1+m)*hypergeom([2, 2+2*m],[3+2*m],-b*x^(1/2)/a)/a^2/(1+m)
```

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {341, 64}

$$\frac{x^{m+1}\,{}_2F_1\left(2,2(m+1);2m+3;-\frac{b\sqrt{x}}{a}\right)}{a^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*Sqrt[x])^2,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, 2*(1 + m), 3 + 2*m, -((b*Sqrt[x])/a)])/(a^2
*(1 + m))
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rubi steps

$$\int \frac{x^m}{\left(a+b\sqrt{x}\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^{-1+2(1+m)}}{(a+bx)^2}\,dx, x, \sqrt{x}\right)$$
$$= \frac{x^{1+m}\,{}_2F_1\left(2,2(1+m);3+2m;-\frac{b\sqrt{x}}{a}\right)}{a^2(1+m)}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 1.05

$$\frac{x^{m+1}\,{}_2F_1\left(2,2(m+1);2(m+1)+1;-\frac{b\sqrt{x}}{a}\right)}{a^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*Sqrt[x])^2,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, 2*(1 + m), 1 + 2*(1 + m), -((b*Sqrt[x])/a)]
)/(a^2*(1 + m))
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{2\,ab\sqrt{x}\,x^m - \left(b^2x + a^2\right)x^m}{b^4x^2 - 2\,a^2b^2x + a^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^(1/2))^2,x, algorithm="fricas")
```

```
[Out] integral(-(2*a*b*sqrt(x)*x^m - (b^2*x + a^2)*x^m)/(b^4*x^2 - 2*a^2*b^2*x +
a^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(b\sqrt{x} + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^(1/2))^2,x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*sqrt(x) + a)^2, x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(b\sqrt{x} + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^(1/2)+a)^2,x)
```

```
[Out] int(x^m/(b*x^(1/2)+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-(2\,m+1)\int \frac{x^m}{ab\sqrt{x} + a^2}\,dx + \frac{2\,xx^m}{ab\sqrt{x} + a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^(1/2))^2,x, algorithm="maxima")
```

```
[Out] -(2*m + 1)*integrate(x^m/(a*b*sqrt(x) + a^2), x) + 2*x*x^m/(a*b*sqrt(x) + a
^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{\left(a + b\,\sqrt{x}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^(1/2))^2,x)
```

```
[Out] int(x^m/(a + b*x^(1/2))^2, x)
```

sympy [C] time = 2.23, size = 473, normalized size = 12.78

$$-\frac{8am^2xx^m\Phi\left(\frac{b\sqrt{x}\,e^{i\pi}}{a},1,2m+2\right)\Gamma(2m+2)}{a^3\Gamma(2m+3)+a^2b\sqrt{x}\,\Gamma(2m+3)}-\frac{12amxx^m\Phi\left(\frac{b\sqrt{x}\,e^{i\pi}}{a},1,2m+2\right)\Gamma(2m+2)}{a^3\Gamma(2m+3)+a^2b\sqrt{x}\,\Gamma(2m+3)}+\frac{4amxx^m\Gamma(2m+}{a^3\Gamma(2m+3)+a^2b\sqrt{x}\,\Gamma}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a+b*x**(1/2))**2,x)
```

```
[Out] -8*a*m**2*x*x**m*lerchphi(b*sqrt(x)*exp_polar(I*pi)/a, 1, 2*m + 2)*gamma(2*
m + 2)/(a**3*gamma(2*m + 3) + a**2*b*sqrt(x)*gamma(2*m + 3)) - 12*a*m*x*x**
m*lerchphi(b*sqrt(x)*exp_polar(I*pi)/a, 1, 2*m + 2)*gamma(2*m + 2)/(a**3*ga
mma(2*m + 3) + a**2*b*sqrt(x)*gamma(2*m + 3)) + 4*a*m*x*x**m*gamma(2*m + 2)
/(a**3*gamma(2*m + 3) + a**2*b*sqrt(x)*gamma(2*m + 3)) - 4*a*x*x**m*lerchph
i(b*sqrt(x)*exp_polar(I*pi)/a, 1, 2*m + 2)*gamma(2*m + 2)/(a**3*gamma(2*m +
 3) + a**2*b*sqrt(x)*gamma(2*m + 3)) + 4*a*x*x**m*gamma(2*m + 2)/(a**3*gamm
a(2*m + 3) + a**2*b*sqrt(x)*gamma(2*m + 3)) - 8*b*m**2*x**(3/2)*x**m*lerchp
hi(b*sqrt(x)*exp_polar(I*pi)/a, 1, 2*m + 2)*gamma(2*m + 2)/(a**3*gamma(2*m
+ 3) + a**2*b*sqrt(x)*gamma(2*m + 3)) - 12*b*m*x**(3/2)*x**m*lerchphi(b*sqr
t(x)*exp_polar(I*pi)/a, 1, 2*m + 2)*gamma(2*m + 2)/(a**3*gamma(2*m + 3) + a
**2*b*sqrt(x)*gamma(2*m + 3)) - 4*b*x**(3/2)*x**m*lerchphi(b*sqrt(x)*exp_po
lar(I*pi)/a, 1, 2*m + 2)*gamma(2*m + 2)/(a**3*gamma(2*m + 3) + a**2*b*sqrt(
x)*gamma(2*m + 3))
```

3.2263 $\int \left(a+b\sqrt{x}\right)^p x^m\,dx$

Optimal. Leaf size=52

$$-\frac{2x^{m+1}\left(a+b\sqrt{x}\right)^{p+1}\,{}_2F_1\left(1,2m+p+3;p+2;\frac{a+b\sqrt{x}}{a}\right)}{a(p+1)}$$

```
[Out] -2*x^(1+m)*hypergeom([1, 3+2*m+p],[2+p],(a+b*x^(1/2))/a)*(a+b*x^(1/2))^(1+p
)/a/(1+p)
```

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.21, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {341, 66, 64}

$$\frac{x^{m+1}\left(a+b\sqrt{x}\right)^p\left(\frac{b\sqrt{x}}{a}+1\right)^{-p}\,{}_2F_1\left(2(m+1),-p;2m+3;-\frac{b\sqrt{x}}{a}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^p*x^m,x]
```

```
[Out] ((a + b*Sqrt[x])^p*x^(1 + m)*Hypergeometric2F1[2*(1 + m), -p, 3 + 2*m, -((b
*Sqrt[x])/a)])/((1 + m)*(1 + (b*Sqrt[x])/a)^p)
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ
[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] &&  !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sqrt{x}\right)^p x^m\,dx &= 2\,\text{Subst}\left(\int x^{-1+2(1+m)}(a+bx)^p\,dx,x,\sqrt{x}\right)\\ &= \left(2\left(a+b\sqrt{x}\right)^p\left(1+\frac{b\sqrt{x}}{a}\right)^{-p}\right)\text{Subst}\left(\int x^{-1+2(1+m)}\left(1+\frac{bx}{a}\right)^p\,dx,x,\sqrt{x}\right)\\ &= \frac{\left(a+b\sqrt{x}\right)^p\left(1+\frac{b\sqrt{x}}{a}\right)^{-p}x^{1+m}\,{}_2F_1\left(2(1+m),-p;3+2m;-\frac{b\sqrt{x}}{a}\right)}{1+m}\end{aligned}$$

Mathematica [A] time = 0.03, size = 65, normalized size = 1.25

$$\frac{x^{m+1}\left(a+b\sqrt{x}\right)^{p}\left(\frac{b\sqrt{x}}{a}+1\right)^{-p}\,{}_2F_1\left(2(m+1),-p;2(m+1)+1;-\frac{b\sqrt{x}}{a}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^p*x^m,x]
```

```
[Out] ((a + b*Sqrt[x])^p*x^(1 + m)*Hypergeometric2F1[2*(1 + m), -p, 1 + 2*(1 + m)
, -((b*Sqrt[x])/a)])/((1 + m)*(1 + (b*Sqrt[x])/a)^p)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^(1/2))^p,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   alglo
gextint: unimplemented
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sqrt{x}+a\right)^{p}x^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^(1/2))^p,x, algorithm="giac")
```

```
[Out] integrate((b*sqrt(x) + a)^p*x^m, x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int x^{m}\left(b\sqrt{x}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^(1/2)+a)^p,x)
```

```
[Out] int(x^m*(b*x^(1/2)+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sqrt{x}+a\right)^{p}x^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^(1/2))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sqrt(x) + a)^p*x^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^{m}\left(a+b\sqrt{x}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^(1/2))^p,x)
```

```
[Out] int(x^m*(a + b*x^(1/2))^p, x)
```

sympy [C] time = 36.19, size = 46, normalized size = 0.88

$$\frac{2a^p x x^m \Gamma\left(2m+2\right) {}_2F_1\left(\begin{matrix}-p, 2m+2 \\ 2m+3\end{matrix}\middle| \frac{b\sqrt{x}\, e^{i\pi}}{a}\right)}{\Gamma\left(2m+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*x**(1/2))**p,x)
```

```
[Out] 2*a**p*x*x**m*gamma(2*m + 2)*hyper((-p, 2*m + 2), (2*m + 3,), b*sqrt(x)*exp
_polar(I*pi)/a)/gamma(2*m + 3)
```

3.2264 $\int \left(a+b\sqrt{x}\right)^p x^3\,dx$

Optimal. Leaf size=204

$$-\frac{2a^7\left(a+b\sqrt{x}\right)^{p+1}}{b^8(p+1)}+\frac{14a^6\left(a+b\sqrt{x}\right)^{p+2}}{b^8(p+2)}-\frac{42a^5\left(a+b\sqrt{x}\right)^{p+3}}{b^8(p+3)}+\frac{70a^4\left(a+b\sqrt{x}\right)^{p+4}}{b^8(p+4)}-\frac{70a^3\left(a+b\sqrt{x}\right)^{p+5}}{b^8(p+5)}+\frac{42a^2\left(a}{b^8}$$

```
[Out] -2*a^7*(a+b*x^(1/2))^(1+p)/b^8/(1+p)+14*a^6*(a+b*x^(1/2))^(2+p)/b^8/(2+p)-4
2*a^5*(a+b*x^(1/2))^(3+p)/b^8/(3+p)+70*a^4*(a+b*x^(1/2))^(4+p)/b^8/(4+p)-70
*a^3*(a+b*x^(1/2))^(5+p)/b^8/(5+p)+42*a^2*(a+b*x^(1/2))^(6+p)/b^8/(6+p)-14*
a*(a+b*x^(1/2))^(7+p)/b^8/(7+p)+2*(a+b*x^(1/2))^(8+p)/b^8/(8+p)
```

Rubi [A] time = 0.12, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{2a^7\left(a+b\sqrt{x}\right)^{p+1}}{b^8(p+1)}+\frac{14a^6\left(a+b\sqrt{x}\right)^{p+2}}{b^8(p+2)}-\frac{42a^5\left(a+b\sqrt{x}\right)^{p+3}}{b^8(p+3)}+\frac{70a^4\left(a+b\sqrt{x}\right)^{p+4}}{b^8(p+4)}-\frac{70a^3\left(a+b\sqrt{x}\right)^{p+5}}{b^8(p+5)}+\frac{42a^2\left(a}{b^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^p*x^3,x]
```

```
[Out] (-2*a^7*(a + b*Sqrt[x])^(1 + p))/(b^8*(1 + p)) + (14*a^6*(a + b*Sqrt[x])^(2
 + p))/(b^8*(2 + p)) - (42*a^5*(a + b*Sqrt[x])^(3 + p))/(b^8*(3 + p)) + (70
*a^4*(a + b*Sqrt[x])^(4 + p))/(b^8*(4 + p)) - (70*a^3*(a + b*Sqrt[x])^(5 +
p))/(b^8*(5 + p)) + (42*a^2*(a + b*Sqrt[x])^(6 + p))/(b^8*(6 + p)) - (14*a*
(a + b*Sqrt[x])^(7 + p))/(b^8*(7 + p)) + (2*(a + b*Sqrt[x])^(8 + p))/(b^8*(
8 + p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sqrt{x}\right)^p x^3\,dx &= 2\,\text{Subst}\left(\int x^7(a+bx)^p\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int\left(-\frac{a^7(a+bx)^p}{b^7}+\frac{7a^6(a+bx)^{1+p}}{b^7}-\frac{21a^5(a+bx)^{2+p}}{b^7}+\frac{35a^4(a+bx)^{3+p}}{b^7}-\frac{35a^3}{}\right.\right.\\ &= -\frac{2a^7\left(a+b\sqrt{x}\right)^{1+p}}{b^8(1+p)}+\frac{14a^6\left(a+b\sqrt{x}\right)^{2+p}}{b^8(2+p)}-\frac{42a^5\left(a+b\sqrt{x}\right)^{3+p}}{b^8(3+p)}+\frac{70a^4\left(a+b\sqrt{x}\right)^{4+p}}{b^8(4+p)}-\frac{7}{}\end{aligned}$$

Mathematica [A] time = 0.22, size = 168, normalized size = 0.82

$$\frac{2\left(-\frac{a^7}{p+1}+\frac{7a^6\left(a+b\sqrt{x}\right)}{p+2}-\frac{21a^5\left(a+b\sqrt{x}\right)^2}{p+3}+\frac{35a^4\left(a+b\sqrt{x}\right)^3}{p+4}-\frac{35a^3\left(a+b\sqrt{x}\right)^4}{p+5}+\frac{21a^2\left(a+b\sqrt{x}\right)^5}{p+6}-\frac{7a\left(a+b\sqrt{x}\right)^6}{p+7}+\frac{\left(a+b\sqrt{x}\right)^7}{p+8}\right)\left(a+b\sqrt{}\right.}{b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^p*x^3,x]
```

```
[Out] (2*(-(a^7/(1 + p)) + (7*a^6*(a + b*Sqrt[x]))/(2 + p) - (21*a^5*(a + b*Sqrt[
x])^2)/(3 + p) + (35*a^4*(a + b*Sqrt[x])^3)/(4 + p) - (35*a^3*(a + b*Sqrt[x
])^4)/(5 + p) + (21*a^2*(a + b*Sqrt[x])^5)/(6 + p) - (7*a*(a + b*Sqrt[x])^6
)/(7 + p) + (a + b*Sqrt[x])^7/(8 + p))*(a + b*Sqrt[x])^(1 + p))/b^8
```

fricas [B] time = 1.04, size = 458, normalized size = 2.25

$$-\frac{2\left(5040\,a^8-\left(b^8p^7+28\,b^8p^6+322\,b^8p^5+1960\,b^8p^4+6769\,b^8p^3+13132\,b^8p^2+13068\,b^8p+5040\,b^8\right)x^4+7\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^(1/2))^p,x, algorithm="fricas")
```

```
[Out] -2*(5040*a^8 - (b^8*p^7 + 28*b^8*p^6 + 322*b^8*p^5 + 1960*b^8*p^4 + 6769*b^
8*p^3 + 13132*b^8*p^2 + 13068*b^8*p + 5040*b^8)*x^4 + 7*(a^2*b^6*p^6 + 15*a
^2*b^6*p^5 + 85*a^2*b^6*p^4 + 225*a^2*b^6*p^3 + 274*a^2*b^6*p^2 + 120*a^2*b
^6*p)*x^3 + 210*(a^4*b^4*p^4 + 6*a^4*b^4*p^3 + 11*a^4*b^4*p^2 + 6*a^4*b^4*p
)*x^2 + 2520*(a^6*b^2*p^2 + a^6*b^2*p)*x - (5040*a^7*b*p + (a*b^7*p^7 + 21*
a*b^7*p^6 + 175*a*b^7*p^5 + 735*a*b^7*p^4 + 1624*a*b^7*p^3 + 1764*a*b^7*p^2
 + 720*a*b^7*p)*x^3 + 42*(a^3*b^5*p^5 + 10*a^3*b^5*p^4 + 35*a^3*b^5*p^3 + 5
0*a^3*b^5*p^2 + 24*a^3*b^5*p)*x^2 + 840*(a^5*b^3*p^3 + 3*a^5*b^3*p^2 + 2*a^
5*b^3*p)*x)*sqrt(x))*(b*sqrt(x) + a)^p/(b^8*p^8 + 36*b^8*p^7 + 546*b^8*p^6
+ 4536*b^8*p^5 + 22449*b^8*p^4 + 67284*b^8*p^3 + 118124*b^8*p^2 + 109584*b^
8*p + 40320*b^8)
```

giac [B] time = 0.25, size = 1642, normalized size = 8.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^(1/2))^p,x, algorithm="giac")
```

```
[Out] 2*((b*sqrt(x) + a)^8*(b*sqrt(x) + a)^p*p^7 - 7*(b*sqrt(x) + a)^7*(b*sqrt(x)
 + a)^p*a*p^7 + 21*(b*sqrt(x) + a)^6*(b*sqrt(x) + a)^p*a^2*p^7 - 35*(b*sqrt
(x) + a)^5*(b*sqrt(x) + a)^p*a^3*p^7 + 35*(b*sqrt(x) + a)^4*(b*sqrt(x) + a)
^p*a^4*p^7 - 21*(b*sqrt(x) + a)^3*(b*sqrt(x) + a)^p*a^5*p^7 + 7*(b*sqrt(x)
 + a)^2*(b*sqrt(x) + a)^p*a^6*p^7 - (b*sqrt(x) + a)*(b*sqrt(x) + a)^p*a^7*p^
7 + 28*(b*sqrt(x) + a)^8*(b*sqrt(x) + a)^p*p^6 - 203*(b*sqrt(x) + a)^7*(b*s
qrt(x) + a)^p*a*p^6 + 630*(b*sqrt(x) + a)^6*(b*sqrt(x) + a)^p*a^2*p^6 - 108
5*(b*sqrt(x) + a)^5*(b*sqrt(x) + a)^p*a^3*p^6 + 1120*(b*sqrt(x) + a)^4*(b*s
qrt(x) + a)^p*a^4*p^6 - 693*(b*sqrt(x) + a)^3*(b*sqrt(x) + a)^p*a^5*p^6 + 2
38*(b*sqrt(x) + a)^2*(b*sqrt(x) + a)^p*a^6*p^6 - 35*(b*sqrt(x) + a)*(b*sqrt
(x) + a)^p*a^7*p^6 + 322*(b*sqrt(x) + a)^8*(b*sqrt(x) + a)^p*p^5 - 2401*(b*
sqrt(x) + a)^7*(b*sqrt(x) + a)^p*a*p^5 + 7686*(b*sqrt(x) + a)^6*(b*sqrt(x)
 + a)^p*a^2*p^5 - 13685*(b*sqrt(x) + a)^5*(b*sqrt(x) + a)^p*a^3*p^5 + 14630*
(b*sqrt(x) + a)^4*(b*sqrt(x) + a)^p*a^4*p^5 - 9387*(b*sqrt(x) + a)^3*(b*sqr
t(x) + a)^p*a^5*p^5 + 3346*(b*sqrt(x) + a)^2*(b*sqrt(x) + a)^p*a^6*p^5 - 51
1*(b*sqrt(x) + a)*(b*sqrt(x) + a)^p*a^7*p^5 + 1960*(b*sqrt(x) + a)^8*(b*sqr
t(x) + a)^p*p^4 - 14945*(b*sqrt(x) + a)^7*(b*sqrt(x) + a)^p*a*p^4 + 49140*(
b*sqrt(x) + a)^6*(b*sqrt(x) + a)^p*a^2*p^4 - 90335*(b*sqrt(x) + a)^5*(b*sqr
t(x) + a)^p*a^3*p^4 + 100240*(b*sqrt(x) + a)^4*(b*sqrt(x) + a)^p*a^4*p^4 -
67095*(b*sqrt(x) + a)^3*(b*sqrt(x) + a)^p*a^5*p^4 + 25060*(b*sqrt(x) + a)^2
*(b*sqrt(x) + a)^p*a^6*p^4 - 4025*(b*sqrt(x) + a)*(b*sqrt(x) + a)^p*a^7*p^4
 + 6769*(b*sqrt(x) + a)^8*(b*sqrt(x) + a)^p*p^3 - 52528*(b*sqrt(x) + a)^7*(
b*sqrt(x) + a)^p*a*p^3 + 176589*(b*sqrt(x) + a)^6*(b*sqrt(x) + a)^p*a^2*p^3
 - 334040*(b*sqrt(x) + a)^5*(b*sqrt(x) + a)^p*a^3*p^3 + 384755*(b*sqrt(x) +
```

```
 a)^4*(b*sqrt(x) + a)^p*a^4*p^3 - 270144*(b*sqrt(x) + a)^3*(b*sqrt(x) + a)^
p*a^5*p^3 + 107023*(b*sqrt(x) + a)^2*(b*sqrt(x) + a)^p*a^6*p^3 - 18424*(b*s
qrt(x) + a)*(b*sqrt(x) + a)^p*a^7*p^3 + 13132*(b*sqrt(x) + a)^8*(b*sqrt(x)
 + a)^p*p^2 - 103292*(b*sqrt(x) + a)^7*(b*sqrt(x) + a)^p*a*p^2 + 353430*(b*s
qrt(x) + a)^6*(b*sqrt(x) + a)^p*a^2*p^2 - 684740*(b*sqrt(x) + a)^5*(b*sqrt(
x) + a)^p*a^3*p^2 + 815920*(b*sqrt(x) + a)^4*(b*sqrt(x) + a)^p*a^4*p^2 - 60
2532*(b*sqrt(x) + a)^3*(b*sqrt(x) + a)^p*a^5*p^2 + 256942*(b*sqrt(x) + a)^2
*(b*sqrt(x) + a)^p*a^6*p^2 - 48860*(b*sqrt(x) + a)*(b*sqrt(x) + a)^p*a^7*p^
2 + 13068*(b*sqrt(x) + a)^8*(b*sqrt(x) + a)^p*p - 103824*(b*sqrt(x) + a)^7*
(b*sqrt(x) + a)^p*a*p + 360024*(b*sqrt(x) + a)^6*(b*sqrt(x) + a)^p*a^2*p -
710640*(b*sqrt(x) + a)^5*(b*sqrt(x) + a)^p*a^3*p + 870660*(b*sqrt(x) + a)^4
*(b*sqrt(x) + a)^p*a^4*p - 673008*(b*sqrt(x) + a)^3*(b*sqrt(x) + a)^p*a^5*p
 + 312984*(b*sqrt(x) + a)^2*(b*sqrt(x) + a)^p*a^6*p - 69264*(b*sqrt(x) + a)
*(b*sqrt(x) + a)^p*a^7*p + 5040*(b*sqrt(x) + a)^8*(b*sqrt(x) + a)^p - 40320
*(b*sqrt(x) + a)^7*(b*sqrt(x) + a)^p*a + 141120*(b*sqrt(x) + a)^6*(b*sqrt(x
) + a)^p*a^2 - 282240*(b*sqrt(x) + a)^5*(b*sqrt(x) + a)^p*a^3 + 352800*(b*s
qrt(x) + a)^4*(b*sqrt(x) + a)^p*a^4 - 282240*(b*sqrt(x) + a)^3*(b*sqrt(x) +
 a)^p*a^5 + 141120*(b*sqrt(x) + a)^2*(b*sqrt(x) + a)^p*a^6 - 40320*(b*sqrt(
x) + a)*(b*sqrt(x) + a)^p*a^7)/((b^7*p^8 + 36*b^7*p^7 + 546*b^7*p^6 + 4536*
b^7*p^5 + 22449*b^7*p^4 + 67284*b^7*p^3 + 118124*b^7*p^2 + 109584*b^7*p + 4
0320*b^7)*b)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int x^3 \left(b\sqrt{x} + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^(1/2)+a)^p,x)
```

```
[Out] int(x^3*(b*x^(1/2)+a)^p,x)
```

maxima [A] time = 0.91, size = 286, normalized size = 1.40

$$\frac{2\left(\left(p^7 + 28\,p^6 + 322\,p^5 + 1960\,p^4 + 6769\,p^3 + 13132\,p^2 + 13068\,p + 5040\right)b^8x^4 + \left(p^7 + 21\,p^6 + 175\,p^5 + 735\,p^4 +\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^(1/2))^p,x, algorithm="maxima")
```

```
[Out] 2*((p^7 + 28*p^6 + 322*p^5 + 1960*p^4 + 6769*p^3 + 13132*p^2 + 13068*p + 50
40)*b^8*x^4 + (p^7 + 21*p^6 + 175*p^5 + 735*p^4 + 1624*p^3 + 1764*p^2 + 720
*p)*a*b^7*x^(7/2) - 7*(p^6 + 15*p^5 + 85*p^4 + 225*p^3 + 274*p^2 + 120*p)*a
^2*b^6*x^3 + 42*(p^5 + 10*p^4 + 35*p^3 + 50*p^2 + 24*p)*a^3*b^5*x^(5/2) - 2
10*(p^4 + 6*p^3 + 11*p^2 + 6*p)*a^4*b^4*x^2 + 840*(p^3 + 3*p^2 + 2*p)*a^5*b
^3*x^(3/2) - 2520*(p^2 + p)*a^6*b^2*x + 5040*a^7*b*p*sqrt(x) - 5040*a^8)*(b
*sqrt(x) + a)^p/((p^8 + 36*p^7 + 546*p^6 + 4536*p^5 + 22449*p^4 + 67284*p^3
 + 118124*p^2 + 109584*p + 40320)*b^8)
```

mupad [B] time = 2.11, size = 590, normalized size = 2.89

$$\left(a + b\sqrt{x}\right)^p \left(\frac{2\,x^4\left(p^7 + 28\,p^6 + 322\,p^5 + 1960\,p^4 + 6769\,p^3 + 13132\,p^2 + 13068\,p + 5040\right)}{p^8 + 36\,p^7 + 546\,p^6 + 4536\,p^5 + 22449\,p^4 + 67284\,p^3 + 118124\,p^2 + 109584\,p + 40320} - \frac{}{b^8\left(p^8 +\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^(1/2))^p,x)
```

```
[Out] (a + b*x^(1/2))^p*((2*x^4*(13068*p + 13132*p^2 + 6769*p^3 + 1960*p^4 + 322*
p^5 + 28*p^6 + p^7 + 5040))/(109584*p + 118124*p^2 + 67284*p^3 + 22449*p^4
+ 4536*p^5 + 546*p^6 + 36*p^7 + p^8 + 40320) - (10080*a^8)/(b^8*(109584*p +
 118124*p^2 + 67284*p^3 + 22449*p^4 + 4536*p^5 + 546*p^6 + 36*p^7 + p^8 + 4
0320)) + (10080*a^7*p*x^(1/2))/(b^7*(109584*p + 118124*p^2 + 67284*p^3 + 22
449*p^4 + 4536*p^5 + 546*p^6 + 36*p^7 + p^8 + 40320)) - (5040*a^6*p*x*(p +
1))/(b^6*(109584*p + 118124*p^2 + 67284*p^3 + 22449*p^4 + 4536*p^5 + 546*p^
6 + 36*p^7 + p^8 + 40320)) - (14*a^2*p*x^3*(274*p + 225*p^2 + 85*p^3 + 15*p
^4 + p^5 + 120))/(b^2*(109584*p + 118124*p^2 + 67284*p^3 + 22449*p^4 + 4536
*p^5 + 546*p^6 + 36*p^7 + p^8 + 40320)) + (1680*a^5*p*x^(3/2)*(3*p + p^2 +
2))/(b^5*(109584*p + 118124*p^2 + 67284*p^3 + 22449*p^4 + 4536*p^5 + 546*p^
6 + 36*p^7 + p^8 + 40320)) - (420*a^4*p*x^2*(11*p + 6*p^2 + p^3 + 6))/(b^4*
(109584*p + 118124*p^2 + 67284*p^3 + 22449*p^4 + 4536*p^5 + 546*p^6 + 36*p^
7 + p^8 + 40320)) + (2*a*p*x^(7/2)*(1764*p + 1624*p^2 + 735*p^3 + 175*p^4 +
 21*p^5 + p^6 + 720))/(b*(109584*p + 118124*p^2 + 67284*p^3 + 22449*p^4 + 4
536*p^5 + 546*p^6 + 36*p^7 + p^8 + 40320)) + (84*a^3*p*x^(5/2)*(50*p + 35*p
^2 + 10*p^3 + p^4 + 24))/(b^3*(109584*p + 118124*p^2 + 67284*p^3 + 22449*p^
4 + 4536*p^5 + 546*p^6 + 36*p^7 + p^8 + 40320)))
```

sympy [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*x**(1/2))**p,x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.2265 $\int \left(a+b\sqrt{x}\right)^p x^2\,dx$

Optimal. Leaf size=152

$$-\frac{2a^5\left(a+b\sqrt{x}\right)^{p+1}}{b^6(p+1)}+\frac{10a^4\left(a+b\sqrt{x}\right)^{p+2}}{b^6(p+2)}-\frac{20a^3\left(a+b\sqrt{x}\right)^{p+3}}{b^6(p+3)}+\frac{20a^2\left(a+b\sqrt{x}\right)^{p+4}}{b^6(p+4)}-\frac{10a\left(a+b\sqrt{x}\right)^{p+5}}{b^6(p+5)}+\frac{2\left(a+b\sqrt{x}\right)^{p+6}}{b^6(p+6)}$$

```
[Out] -2*a^5*(a+b*x^(1/2))^(1+p)/b^6/(1+p)+10*a^4*(a+b*x^(1/2))^(2+p)/b^6/(2+p)-2
0*a^3*(a+b*x^(1/2))^(3+p)/b^6/(3+p)+20*a^2*(a+b*x^(1/2))^(4+p)/b^6/(4+p)-10
*a*(a+b*x^(1/2))^(5+p)/b^6/(5+p)+2*(a+b*x^(1/2))^(6+p)/b^6/(6+p)
```

Rubi [A] time = 0.08, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{2a^5\left(a+b\sqrt{x}\right)^{p+1}}{b^6(p+1)}+\frac{10a^4\left(a+b\sqrt{x}\right)^{p+2}}{b^6(p+2)}-\frac{20a^3\left(a+b\sqrt{x}\right)^{p+3}}{b^6(p+3)}+\frac{20a^2\left(a+b\sqrt{x}\right)^{p+4}}{b^6(p+4)}-\frac{10a\left(a+b\sqrt{x}\right)^{p+5}}{b^6(p+5)}+\frac{2\left(a+b\sqrt{x}\right)^{p+6}}{b^6(p+6)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^p*x^2,x]
```

```
[Out] (-2*a^5*(a + b*Sqrt[x])^(1 + p))/(b^6*(1 + p)) + (10*a^4*(a + b*Sqrt[x])^(2
 + p))/(b^6*(2 + p)) - (20*a^3*(a + b*Sqrt[x])^(3 + p))/(b^6*(3 + p)) + (20
*a^2*(a + b*Sqrt[x])^(4 + p))/(b^6*(4 + p)) - (10*a*(a + b*Sqrt[x])^(5 + p)
)/(b^6*(5 + p)) + (2*(a + b*Sqrt[x])^(6 + p))/(b^6*(6 + p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sqrt{x}\right)^p x^2\,dx &= 2\,\text{Subst}\left(\int x^5(a+bx)^p\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int\left(-\frac{a^5(a+bx)^p}{b^5}+\frac{5a^4(a+bx)^{1+p}}{b^5}-\frac{10a^3(a+bx)^{2+p}}{b^5}+\frac{10a^2(a+bx)^{3+p}}{b^5}-\frac{5a(a\right.\right.\\ &= -\frac{2a^5\left(a+b\sqrt{x}\right)^{1+p}}{b^6(1+p)}+\frac{10a^4\left(a+b\sqrt{x}\right)^{2+p}}{b^6(2+p)}-\frac{20a^3\left(a+b\sqrt{x}\right)^{3+p}}{b^6(3+p)}+\frac{20a^2\left(a+b\sqrt{x}\right)^{4+p}}{b^6(4+p)}-\frac{1}{}\end{aligned}$$

Mathematica [A] time = 0.13, size = 126, normalized size = 0.83

$$\frac{2\left(-\frac{a^5}{p+1}+\frac{5a^4\left(a+b\sqrt{x}\right)}{p+2}-\frac{10a^3\left(a+b\sqrt{x}\right)^2}{p+3}+\frac{10a^2\left(a+b\sqrt{x}\right)^3}{p+4}-\frac{5a\left(a+b\sqrt{x}\right)^4}{p+5}+\frac{\left(a+b\sqrt{x}\right)^5}{p+6}\right)\left(a+b\sqrt{x}\right)^{p+1}}{b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^p*x^2,x]
```

```
[Out] (2*(-(a^5/(1 + p)) + (5*a^4*(a + b*Sqrt[x]))/(2 + p) - (10*a^3*(a + b*Sqrt[
x])^2)/(3 + p) + (10*a^2*(a + b*Sqrt[x])^3)/(4 + p) - (5*a*(a + b*Sqrt[x])^
4)/(5 + p) + (a + b*Sqrt[x])^5/(6 + p))*(a + b*Sqrt[x])^(1 + p))/b^6
```

fricas [B] time = 1.00, size = 281, normalized size = 1.85

$$-\frac{2\left(120\,a^6-\left(b^6p^5+15\,b^6p^4+85\,b^6p^3+225\,b^6p^2+274\,b^6p+120\,b^6\right)x^3+5\left(a^2b^4p^4+6\,a^2b^4p^3+11\,a^2b^4p^2+\right.\right.}{b^6p^6+21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^(1/2))^p,x, algorithm="fricas")
```

```
[Out] -2*(120*a^6 - (b^6*p^5 + 15*b^6*p^4 + 85*b^6*p^3 + 225*b^6*p^2 + 274*b^6*p
+ 120*b^6)*x^3 + 5*(a^2*b^4*p^4 + 6*a^2*b^4*p^3 + 11*a^2*b^4*p^2 + 6*a^2*b^
4*p)*x^2 + 60*(a^4*b^2*p^2 + a^4*b^2*p)*x - (120*a^5*b*p + (a*b^5*p^5 + 10*
a*b^5*p^4 + 35*a*b^5*p^3 + 50*a*b^5*p^2 + 24*a*b^5*p)*x^2 + 20*(a^3*b^3*p^3
 + 3*a^3*b^3*p^2 + 2*a^3*b^3*p)*x)*sqrt(x))*(b*sqrt(x) + a)^p/(b^6*p^6 + 21
*b^6*p^5 + 175*b^6*p^4 + 735*b^6*p^3 + 1624*b^6*p^2 + 1764*b^6*p + 720*b^6)
```

giac [B] time = 0.19, size = 922, normalized size = 6.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^(1/2))^p,x, algorithm="giac")
```

```
[Out] 2*((b*sqrt(x) + a)^6*(b*sqrt(x) + a)^p*p^5 - 5*(b*sqrt(x) + a)^5*(b*sqrt(x)
 + a)^p*a*p^5 + 10*(b*sqrt(x) + a)^4*(b*sqrt(x) + a)^p*a^2*p^5 - 10*(b*sqrt
(x) + a)^3*(b*sqrt(x) + a)^p*a^3*p^5 + 5*(b*sqrt(x) + a)^2*(b*sqrt(x) + a)^
p*a^4*p^5 - (b*sqrt(x) + a)*(b*sqrt(x) + a)^p*a^5*p^5 + 15*(b*sqrt(x) + a)^
6*(b*sqrt(x) + a)^p*p^4 - 80*(b*sqrt(x) + a)^5*(b*sqrt(x) + a)^p*a*p^4 + 17
0*(b*sqrt(x) + a)^4*(b*sqrt(x) + a)^p*a^2*p^4 - 180*(b*sqrt(x) + a)^3*(b*sq
rt(x) + a)^p*a^3*p^4 + 95*(b*sqrt(x) + a)^2*(b*sqrt(x) + a)^p*a^4*p^4 - 20*
(b*sqrt(x) + a)*(b*sqrt(x) + a)^p*a^5*p^4 + 85*(b*sqrt(x) + a)^6*(b*sqrt(x)
 + a)^p*p^3 - 475*(b*sqrt(x) + a)^5*(b*sqrt(x) + a)^p*a*p^3 + 1070*(b*sqrt(
x) + a)^4*(b*sqrt(x) + a)^p*a^2*p^3 - 1210*(b*sqrt(x) + a)^3*(b*sqrt(x) + a
)^p*a^3*p^3 + 685*(b*sqrt(x) + a)^2*(b*sqrt(x) + a)^p*a^4*p^3 - 155*(b*sqrt
(x) + a)*(b*sqrt(x) + a)^p*a^5*p^3 + 225*(b*sqrt(x) + a)^6*(b*sqrt(x) + a)^
p*p^2 - 1300*(b*sqrt(x) + a)^5*(b*sqrt(x) + a)^p*a*p^2 + 3070*(b*sqrt(x) +
a)^4*(b*sqrt(x) + a)^p*a^2*p^2 - 3720*(b*sqrt(x) + a)^3*(b*sqrt(x) + a)^p*a
^3*p^2 + 2305*(b*sqrt(x) + a)^2*(b*sqrt(x) + a)^p*a^4*p^2 - 580*(b*sqrt(x)
+ a)*(b*sqrt(x) + a)^p*a^5*p^2 + 274*(b*sqrt(x) + a)^6*(b*sqrt(x) + a)^p*p
- 1620*(b*sqrt(x) + a)^5*(b*sqrt(x) + a)^p*a*p + 3960*(b*sqrt(x) + a)^4*(b*
sqrt(x) + a)^p*a^2*p - 5080*(b*sqrt(x) + a)^3*(b*sqrt(x) + a)^p*a^3*p + 351
0*(b*sqrt(x) + a)^2*(b*sqrt(x) + a)^p*a^4*p - 1044*(b*sqrt(x) + a)*(b*sqrt(
x) + a)^p*a^5*p + 120*(b*sqrt(x) + a)^6*(b*sqrt(x) + a)^p - 720*(b*sqrt(x)
+ a)^5*(b*sqrt(x) + a)^p*a + 1800*(b*sqrt(x) + a)^4*(b*sqrt(x) + a)^p*a^2 -
 2400*(b*sqrt(x) + a)^3*(b*sqrt(x) + a)^p*a^3 + 1800*(b*sqrt(x) + a)^2*(b*s
qrt(x) + a)^p*a^4 - 720*(b*sqrt(x) + a)*(b*sqrt(x) + a)^p*a^5)/((b^5*p^6 +
21*b^5*p^5 + 175*b^5*p^4 + 735*b^5*p^3 + 1624*b^5*p^2 + 1764*b^5*p + 720*b^
5)*b)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int x^2\left(b\sqrt{x}+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^(1/2)+a)^p,x)
```

```
[Out] int(x^2*(b*x^(1/2)+a)^p,x)
```

maxima [A] time = 0.97, size = 185, normalized size = 1.22

$$\frac{2\left(\left(p^5+15p^4+85p^3+225p^2+274p+120\right)b^6x^3+\left(p^5+10p^4+35p^3+50p^2+24p\right)ab^5x^{\frac{5}{2}}-5\left(p^4+6p^3+1\right.\right.}{\left(p^6+21p^5+175p^4+735p^3+16\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^(1/2))^p,x, algorithm="maxima")
```

```
[Out] 2*((p^5 + 15*p^4 + 85*p^3 + 225*p^2 + 274*p + 120)*b^6*x^3 + (p^5 + 10*p^4
+ 35*p^3 + 50*p^2 + 24*p)*a*b^5*x^(5/2) - 5*(p^4 + 6*p^3 + 11*p^2 + 6*p)*a^
2*b^4*x^2 + 20*(p^3 + 3*p^2 + 2*p)*a^3*b^3*x^(3/2) - 60*(p^2 + p)*a^4*b^2*x
 + 120*a^5*b*p*sqrt(x) - 120*a^6)*(b*sqrt(x) + a)^p/((p^6 + 21*p^5 + 175*p^
4 + 735*p^3 + 1624*p^2 + 1764*p + 720)*b^6)
```

mupad [B] time = 1.70, size = 355, normalized size = 2.34

$$\left(a+b\sqrt{x}\right)^p\left(\frac{2x^3\left(p^5+15p^4+85p^3+225p^2+274p+120\right)}{p^6+21p^5+175p^4+735p^3+1624p^2+1764p+720}-\frac{240a^6}{b^6\left(p^6+21p^5+175p^4+735p^3+1624p\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^(1/2))^p,x)
```

```
[Out] (a + b*x^(1/2))^p*((2*x^3*(274*p + 225*p^2 + 85*p^3 + 15*p^4 + p^5 + 120))/
(1764*p + 1624*p^2 + 735*p^3 + 175*p^4 + 21*p^5 + p^6 + 720) - (240*a^6)/(b
^6*(1764*p + 1624*p^2 + 735*p^3 + 175*p^4 + 21*p^5 + p^6 + 720)) + (240*a^5
*p*x^(1/2))/(b^5*(1764*p + 1624*p^2 + 735*p^3 + 175*p^4 + 21*p^5 + p^6 + 72
0)) + (2*a*p*x^(5/2)*(50*p + 35*p^2 + 10*p^3 + p^4 + 24))/(b*(1764*p + 1624
*p^2 + 735*p^3 + 175*p^4 + 21*p^5 + p^6 + 720)) + (40*a^3*p*x^(3/2)*(3*p +
p^2 + 2))/(b^3*(1764*p + 1624*p^2 + 735*p^3 + 175*p^4 + 21*p^5 + p^6 + 720)
) - (10*a^2*p*x^2*(11*p + 6*p^2 + p^3 + 6))/(b^2*(1764*p + 1624*p^2 + 735*p
^3 + 175*p^4 + 21*p^5 + p^6 + 720)) - (120*a^4*p*x*(p + 1))/(b^4*(1764*p +
1624*p^2 + 735*p^3 + 175*p^4 + 21*p^5 + p^6 + 720)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*x**(1/2))**p,x)
```

```
[Out] Timed out
```

3.2266 $\int \left(a+b\sqrt{x}\right)^p x\,dx$

Optimal. Leaf size=100

$$-\frac{2a^3\left(a+b\sqrt{x}\right)^{p+1}}{b^4(p+1)}+\frac{6a^2\left(a+b\sqrt{x}\right)^{p+2}}{b^4(p+2)}-\frac{6a\left(a+b\sqrt{x}\right)^{p+3}}{b^4(p+3)}+\frac{2\left(a+b\sqrt{x}\right)^{p+4}}{b^4(p+4)}$$

```
[Out] -2*a^3*(a+b*x^(1/2))^(1+p)/b^4/(1+p)+6*a^2*(a+b*x^(1/2))^(2+p)/b^4/(2+p)-6*
a*(a+b*x^(1/2))^(3+p)/b^4/(3+p)+2*(a+b*x^(1/2))^(4+p)/b^4/(4+p)
```

Rubi [A] time = 0.05, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$-\frac{2a^3\left(a+b\sqrt{x}\right)^{p+1}}{b^4(p+1)}+\frac{6a^2\left(a+b\sqrt{x}\right)^{p+2}}{b^4(p+2)}-\frac{6a\left(a+b\sqrt{x}\right)^{p+3}}{b^4(p+3)}+\frac{2\left(a+b\sqrt{x}\right)^{p+4}}{b^4(p+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^p*x,x]
```

```
[Out] (-2*a^3*(a + b*Sqrt[x])^(1 + p))/(b^4*(1 + p)) + (6*a^2*(a + b*Sqrt[x])^(2
+ p))/(b^4*(2 + p)) - (6*a*(a + b*Sqrt[x])^(3 + p))/(b^4*(3 + p)) + (2*(a +
 b*Sqrt[x])^(4 + p))/(b^4*(4 + p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sqrt{x}\right)^p x\,dx &= 2\,\text{Subst}\left(\int x^3(a+bx)^p\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int\left(-\frac{a^3(a+bx)^p}{b^3}+\frac{3a^2(a+bx)^{1+p}}{b^3}-\frac{3a(a+bx)^{2+p}}{b^3}+\frac{(a+bx)^{3+p}}{b^3}\right)dx, x, \sqrt{x}\right)\\ &= -\frac{2a^3\left(a+b\sqrt{x}\right)^{1+p}}{b^4(1+p)}+\frac{6a^2\left(a+b\sqrt{x}\right)^{2+p}}{b^4(2+p)}-\frac{6a\left(a+b\sqrt{x}\right)^{3+p}}{b^4(3+p)}+\frac{2\left(a+b\sqrt{x}\right)^{4+p}}{b^4(4+p)}\end{aligned}$$

Mathematica [A] time = 0.08, size = 95, normalized size = 0.95

$$\frac{2\left(a+b\sqrt{x}\right)^{p+1}\left(-6a^3+6a^2b(p+1)\sqrt{x}-3ab^2\left(p^2+3p+2\right)x+b^3\left(p^3+6p^2+11p+6\right)x^{3/2}\right)}{b^4(p+1)(p+2)(p+3)(p+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^p*x,x]
```

```
[Out] (2*(a + b*Sqrt[x])^(1 + p)*(-6*a^3 + 6*a^2*b*(1 + p)*Sqrt[x] - 3*a*b^2*(2 +
 3*p + p^2)*x + b^3*(6 + 11*p + 6*p^2 + p^3)*x^(3/2)))/(b^4*(1 + p)*(2 + p)
*(3 + p)*(4 + p))
```

fricas **[A]** time = 0.95, size = 148, normalized size = 1.48

$$-\frac{2\left(6\,a^4-\left(b^4p^3+6\,b^4p^2+11\,b^4p+6\,b^4\right)x^2+3\left(a^2b^2p^2+a^2b^2p\right)x-\left(6\,a^3bp+\left(ab^3p^3+3\,ab^3p^2+2\,ab^3p\right)x\right)\sqrt{x}\right)}{b^4p^4+10\,b^4p^3+35\,b^4p^2+50\,b^4p+24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^(1/2))^p,x, algorithm="fricas")
```

```
[Out] -2*(6*a^4 - (b^4*p^3 + 6*b^4*p^2 + 11*b^4*p + 6*b^4)*x^2 + 3*(a^2*b^2*p^2 +
 a^2*b^2*p)*x - (6*a^3*b*p + (a*b^3*p^3 + 3*a*b^3*p^2 + 2*a*b^3*p)*x)*sqrt(
x))*(b*sqrt(x) + a)^p/(b^4*p^4 + 10*b^4*p^3 + 35*b^4*p^2 + 50*b^4*p + 24*b^
4)
```

giac **[B]** time = 0.19, size = 410, normalized size = 4.10

$$\frac{2\left(\left(b\sqrt{x}+a\right)^4\left(b\sqrt{x}+a\right)^p p^3-3\left(b\sqrt{x}+a\right)^3\left(b\sqrt{x}+a\right)^p ap^3+3\left(b\sqrt{x}+a\right)^2\left(b\sqrt{x}+a\right)^p a^2p^3-\left(b\sqrt{x}+a\right)\left(b\sqrt{x}+a\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^(1/2))^p,x, algorithm="giac")
```

```
[Out] 2*((b*sqrt(x) + a)^4*(b*sqrt(x) + a)^p*p^3 - 3*(b*sqrt(x) + a)^3*(b*sqrt(x)
 + a)^p*a*p^3 + 3*(b*sqrt(x) + a)^2*(b*sqrt(x) + a)^p*a^2*p^3 - (b*sqrt(x)
+ a)*(b*sqrt(x) + a)^p*a^3*p^3 + 6*(b*sqrt(x) + a)^4*(b*sqrt(x) + a)^p*p^2
- 21*(b*sqrt(x) + a)^3*(b*sqrt(x) + a)^p*a*p^2 + 24*(b*sqrt(x) + a)^2*(b*sq
rt(x) + a)^p*a^2*p^2 - 9*(b*sqrt(x) + a)*(b*sqrt(x) + a)^p*a^3*p^2 + 11*(b*
sqrt(x) + a)^4*(b*sqrt(x) + a)^p*p - 42*(b*sqrt(x) + a)^3*(b*sqrt(x) + a)^p
*a*p + 57*(b*sqrt(x) + a)^2*(b*sqrt(x) + a)^p*a^2*p - 26*(b*sqrt(x) + a)*(b
*sqrt(x) + a)^p*a^3*p + 6*(b*sqrt(x) + a)^4*(b*sqrt(x) + a)^p - 24*(b*sqrt(
x) + a)^3*(b*sqrt(x) + a)^p*a + 36*(b*sqrt(x) + a)^2*(b*sqrt(x) + a)^p*a^2
- 24*(b*sqrt(x) + a)*(b*sqrt(x) + a)^p*a^3)/((b^3*p^4 + 10*b^3*p^3 + 35*b^3
*p^2 + 50*b^3*p + 24*b^3)*b)
```

maple **[F]** time = 0.18, size = 0, normalized size = 0.00

$$\int x\left(b\sqrt{x}+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^(1/2)+a)^p,x)
```

```
[Out] int(x*(b*x^(1/2)+a)^p,x)
```

maxima **[A]** time = 0.96, size = 104, normalized size = 1.04

$$\frac{2\left(\left(p^3+6\,p^2+11\,p+6\right)b^4x^2+\left(p^3+3\,p^2+2\,p\right)ab^3x^{\frac{3}{2}}-3\left(p^2+p\right)a^2b^2x+6\,a^3bp\sqrt{x}-6\,a^4\right)\left(b\sqrt{x}+a\right)^p}{\left(p^4+10\,p^3+35\,p^2+50\,p+24\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^(1/2))^p,x, algorithm="maxima")
```

```
[Out] 2*((p^3 + 6*p^2 + 11*p + 6)*b^4*x^2 + (p^3 + 3*p^2 + 2*p)*a*b^3*x^(3/2) - 3
*(p^2 + p)*a^2*b^2*x + 6*a^3*b*p*sqrt(x) - 6*a^4)*(b*sqrt(x) + a)^p/((p^4 +
 10*p^3 + 35*p^2 + 50*p + 24)*b^4)
```

mupad [B] time = 1.60, size = 180, normalized size = 1.80

$$\left(a+b\sqrt{x}\right)^p\left(\frac{2x^2\left(p^3+6p^2+11p+6\right)}{p^4+10p^3+35p^2+50p+24}-\frac{12a^4}{b^4\left(p^4+10p^3+35p^2+50p+24\right)}+\frac{12a^3p\sqrt{x}}{b^3\left(p^4+10p^3+35p^2+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^(1/2))^p,x)
```

```
[Out] (a + b*x^(1/2))^p*((2*x^2*(11*p + 6*p^2 + p^3 + 6))/(50*p + 35*p^2 + 10*p^3
 + p^4 + 24) - (12*a^4)/(b^4*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) + (12*a^3
*p*x^(1/2))/(b^3*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) - (6*a^2*p*x*(p + 1))
/(b^2*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) + (2*a*p*x^(3/2)*(3*p + p^2 + 2)
)/(b*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x**(1/2))**p,x)
```

```
[Out] Timed out
```

3.2267 $\int \left(a + b\sqrt{x}\right)^p \, dx$

Optimal. Leaf size=48

$$\frac{2\left(a + b\sqrt{x}\right)^{p+2}}{b^2(p+2)} - \frac{2a\left(a + b\sqrt{x}\right)^{p+1}}{b^2(p+1)}$$

```
[Out] -2*a*(a+b*x^(1/2))^(1+p)/b^2/(1+p)+2*(a+b*x^(1/2))^(2+p)/b^2/(2+p)
```

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {190, 43}

$$\frac{2\left(a + b\sqrt{x}\right)^{p+2}}{b^2(p+2)} - \frac{2a\left(a + b\sqrt{x}\right)^{p+1}}{b^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^p,x]
```

```
[Out] (-2*a*(a + b*Sqrt[x])^(1 + p))/(b^2*(1 + p)) + (2*(a + b*Sqrt[x])^(2 + p))/
(b^2*(2 + p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}
\int \left(a + b\sqrt{x}\right)^p \, dx &= 2\,\text{Subst}\left(\int x(a + bx)^p \, dx, x, \sqrt{x}\right) \\
&= 2\,\text{Subst}\left(\int \left(-\frac{a(a + bx)^p}{b} + \frac{(a + bx)^{1+p}}{b}\right) dx, x, \sqrt{x}\right) \\
&= -\frac{2a\left(a + b\sqrt{x}\right)^{1+p}}{b^2(1+p)} + \frac{2\left(a + b\sqrt{x}\right)^{2+p}}{b^2(2+p)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.88

$$\frac{2\left(a + b\sqrt{x}\right)^{p+1}\left(b(p+1)\sqrt{x} - a\right)}{b^2(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^p,x]
```

```
[Out] (2*(a + b*Sqrt[x])^(1 + p)*(-a + b*(1 + p)*Sqrt[x]))/(b^2*(1 + p)*(2 + p))
```

fricas [A] time = 1.00, size = 56, normalized size = 1.17

$$\frac{2\left(abp\sqrt{x}-a^2+\left(b^2p+b^2\right)x\right)\left(b\sqrt{x}+a\right)^p}{b^2p^2+3\,b^2p+2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^p,x, algorithm="fricas")
```

```
[Out] 2*(a*b*p*sqrt(x) - a^2 + (b^2*p + b^2)*x)*(b*sqrt(x) + a)^p/(b^2*p^2 + 3*b^
2*p + 2*b^2)
```

giac [B] time = 0.16, size = 94, normalized size = 1.96

$$\frac{2\left(\left(b\sqrt{x}+a\right)^2\left(b\sqrt{x}+a\right)^p p-\left(b\sqrt{x}+a\right)\left(b\sqrt{x}+a\right)^p ap+\left(b\sqrt{x}+a\right)^2\left(b\sqrt{x}+a\right)^p-2\left(b\sqrt{x}+a\right)\left(b\sqrt{x}+a\right)^p a\right)}{\left(p^2+3\,p+2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^p,x, algorithm="giac")
```

```
[Out] 2*((b*sqrt(x) + a)^2*(b*sqrt(x) + a)^p*p - (b*sqrt(x) + a)*(b*sqrt(x) + a)^
p*a*p + (b*sqrt(x) + a)^2*(b*sqrt(x) + a)^p - 2*(b*sqrt(x) + a)*(b*sqrt(x)
+ a)^p*a)/((p^2 + 3*p + 2)*b^2)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \left(b\sqrt{x}+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(1/2)+a)^p,x)
```

```
[Out] int((b*x^(1/2)+a)^p,x)
```

maxima [A] time = 0.94, size = 45, normalized size = 0.94

$$\frac{2\left(b^2\left(p+1\right)x+abp\sqrt{x}-a^2\right)\left(b\sqrt{x}+a\right)^p}{\left(p^2+3\,p+2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^p,x, algorithm="maxima")
```

```
[Out] 2*(b^2*(p + 1)*x + a*b*p*sqrt(x) - a^2)*(b*sqrt(x) + a)^p/((p^2 + 3*p + 2)*
b^2)
```

mupad [B] time = 1.18, size = 41, normalized size = 0.85

$$\frac{x\left(a+b\sqrt{x}\right)^p\,{}_2\mathrm{F}_1\left(2,-p;\,3;\,-\frac{b\sqrt{x}}{a}\right)}{\left(\frac{b\sqrt{x}}{a}+1\right)^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^p,x)
```

```
[Out] (x*(a + b*x^(1/2))^p*hypergeom([2, -p], 3, -(b*x^(1/2))/a))/((b*x^(1/2))/a
+ 1)^p
```

sympy [**B**] time = 1.83, size = 823, normalized size = 17.15

$$-\frac{2a^3a^px^2\left(1+\frac{b\sqrt{x}}{a}\right)^p}{ab^2p^2x^2+3ab^2px^2+2ab^2x^2+b^3p^2x^{\frac{5}{2}}+3b^3px^{\frac{5}{2}}+2b^3x^{\frac{5}{2}}}+\frac{2a^3a^px^2}{ab^2p^2x^2+3ab^2px^2+2ab^2x^2+b^3p^2x^{\frac{5}{2}}+3b^3px^{\frac{5}{2}}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**p,x)
```

```
[Out] -2*a**3*a**p*x**2*(1 + b*sqrt(x)/a)**p/(a*b**2*p**2*x**2 + 3*a*b**2*p*x**2
+ 2*a*b**2*x**2 + b**3*p**2*x**(5/2) + 3*b**3*p*x**(5/2) + 2*b**3*x**(5/2))
 + 2*a**3*a**p*x**2/(a*b**2*p**2*x**2 + 3*a*b**2*p*x**2 + 2*a*b**2*x**2 + b
**3*p**2*x**(5/2) + 3*b**3*p*x**(5/2) + 2*b**3*x**(5/2)) + 2*a**2*a**p*b*p*
x**(5/2)*(1 + b*sqrt(x)/a)**p/(a*b**2*p**2*x**2 + 3*a*b**2*p*x**2 + 2*a*b**
2*x**2 + b**3*p**2*x**(5/2) + 3*b**3*p*x**(5/2) + 2*b**3*x**(5/2)) - 2*a**2
*a**p*b*x**(5/2)*(1 + b*sqrt(x)/a)**p/(a*b**2*p**2*x**2 + 3*a*b**2*p*x**2 +
 2*a*b**2*x**2 + b**3*p**2*x**(5/2) + 3*b**3*p*x**(5/2) + 2*b**3*x**(5/2))
+ 2*a**2*a**p*b*x**(5/2)/(a*b**2*p**2*x**2 + 3*a*b**2*p*x**2 + 2*a*b**2*x**
2 + b**3*p**2*x**(5/2) + 3*b**3*p*x**(5/2) + 2*b**3*x**(5/2)) + 4*a*a**p*b*
*2*p*x**3*(1 + b*sqrt(x)/a)**p/(a*b**2*p**2*x**2 + 3*a*b**2*p*x**2 + 2*a*b*
*2*x**2 + b**3*p**2*x**(5/2) + 3*b**3*p*x**(5/2) + 2*b**3*x**(5/2)) + 2*a*a
**p*b**2*x**3*(1 + b*sqrt(x)/a)**p/(a*b**2*p**2*x**2 + 3*a*b**2*p*x**2 + 2*
a*b**2*x**2 + b**3*p**2*x**(5/2) + 3*b**3*p*x**(5/2) + 2*b**3*x**(5/2)) + 2
*a**p*b**3*p*x**(7/2)*(1 + b*sqrt(x)/a)**p/(a*b**2*p**2*x**2 + 3*a*b**2*p*x
**2 + 2*a*b**2*x**2 + b**3*p**2*x**(5/2) + 3*b**3*p*x**(5/2) + 2*b**3*x**(5
/2)) + 2*a**p*b**3*x**(7/2)*(1 + b*sqrt(x)/a)**p/(a*b**2*p**2*x**2 + 3*a*b*
*2*p*x**2 + 2*a*b**2*x**2 + b**3*p**2*x**(5/2) + 3*b**3*p*x**(5/2) + 2*b**3
*x**(5/2))
```

3.2268 $\int \frac{\left(a+b\sqrt{x}\right)^p}{x}\,dx$

Optimal. Leaf size=43

$$-\frac{2\left(a+b\sqrt{x}\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{\sqrt{x}\,b}{a}+1\right)}{a(p+1)}$$

```
[Out] -2*hypergeom([1, 1+p],[2+p],1+b*x^(1/2)/a)*(a+b*x^(1/2))^(1+p)/a/(1+p)
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 65}

$$-\frac{2\left(a+b\sqrt{x}\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{\sqrt{x}\,b}{a}+1\right)}{a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^p/x,x]
```

```
[Out] (-2*(a + b*Sqrt[x])^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sqrt[
x])/a])/(a*(1 + p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt{x}\right)^p}{x}\,dx &= 2\,\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, \sqrt{x}\right)\\ &= -\frac{2\left(a+b\sqrt{x}\right)^{1+p}\,{}_2F_1\left(1,1+p;2+p;1+\frac{b\sqrt{x}}{a}\right)}{a(1+p)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 1.00

$$-\frac{2\left(a+b\sqrt{x}\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{\sqrt{x}\,b}{a}+1\right)}{a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^p/x,x]
```

```
[Out] (-2*(a + b*Sqrt[x])^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sqrt[
x])/a])/(a*(1 + p))
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\sqrt{x}+a\right)^{p}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^p/x,x, algorithm="fricas")
```

```
[Out] integral((b*sqrt(x) + a)^p/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sqrt{x}+a\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^p/x,x, algorithm="giac")
```

```
[Out] integrate((b*sqrt(x) + a)^p/x, x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sqrt{x}+a\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(1/2)+a)^p/x,x)
```

```
[Out] int((b*x^(1/2)+a)^p/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sqrt{x}+a\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^p/x,x, algorithm="maxima")
```

```
[Out] integrate((b*sqrt(x) + a)^p/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(a+b\sqrt{x}\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^p/x,x)
```

```
[Out] int((a + b*x^(1/2))^p/x, x)
```

sympy [C] time = 2.04, size = 41, normalized size = 0.95

$$-\frac{2b^{p}x^{\frac{p}{2}}\Gamma\left(-p\right){}_{2}F_{1}\left(\begin{matrix}-p, -p\\ 1-p\end{matrix}\middle|\frac{ae^{i\pi}}{b\sqrt{x}}\right)}{\Gamma\left(1-p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**p/x,x)
```

```
[Out] -2*b**p*x**(p/2)*gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*s
qrt(x)))/gamma(1 - p)
```

3.2269 $\int \frac{\left(a+b\sqrt{x}\right)^p}{x^2}\,dx$

Optimal. Leaf size=46

$$-\frac{2b^2\left(a+b\sqrt{x}\right)^{p+1}\,{}_2F_1\left(3,p+1;p+2;\frac{\sqrt{x}\,b}{a}+1\right)}{a^3(p+1)}$$

```
[Out] -2*b^2*hypergeom([3, 1+p],[2+p],1+b*x^(1/2)/a)*(a+b*x^(1/2))^(1+p)/a^3/(1+p
)
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 65}

$$-\frac{2b^2\left(a+b\sqrt{x}\right)^{p+1}\,{}_2F_1\left(3,p+1;p+2;\frac{\sqrt{x}\,b}{a}+1\right)}{a^3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])^p/x^2,x]
```

```
[Out] (-2*b^2*(a + b*Sqrt[x])^(1 + p)*Hypergeometric2F1[3, 1 + p, 2 + p, 1 + (b*S
qrt[x])/a])/(a^3*(1 + p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt{x}\right)^p}{x^2}\,dx &= 2\,\text{Subst}\left(\int \frac{(a+bx)^p}{x^3}\,dx, x, \sqrt{x}\right)\\ &= -\frac{2b^2\left(a+b\sqrt{x}\right)^{1+p}\,{}_2F_1\left(3,1+p;2+p;1+\frac{b\sqrt{x}}{a}\right)}{a^3(1+p)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 1.00

$$-\frac{2b^2\left(a+b\sqrt{x}\right)^{p+1}\,{}_2F_1\left(3,p+1;p+2;\frac{\sqrt{x}\,b}{a}+1\right)}{a^3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])^p/x^2,x]
```

```
[Out] (-2*b^2*(a + b*Sqrt[x])^(1 + p)*Hypergeometric2F1[3, 1 + p, 2 + p, 1 + (b*S
qrt[x])/a])/(a^3*(1 + p))
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\sqrt{x}+a\right)^{p}}{x^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*sqrt(x) + a)^p/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sqrt{x}+a\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*sqrt(x) + a)^p/x^2, x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sqrt{x}+a\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(1/2)+a)^p/x^2,x)
```

```
[Out] int((b*x^(1/2)+a)^p/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sqrt{x}+a\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sqrt(x) + a)^p/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(a+b\sqrt{x}\right)^{p}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))^p/x^2,x)
```

```
[Out] int((a + b*x^(1/2))^p/x^2, x)
```

sympy [C] time = 5.35, size = 42, normalized size = 0.91

$$-\frac{2b^{p}x^{\frac{p}{2}}\Gamma\left(2-p\right){}_{2}F_{1}\left(\begin{matrix}-p, 2-p \\ 3-p\end{matrix}\middle|\,\frac{ae^{i\pi}}{b\sqrt{x}}\right)}{x\Gamma\left(3-p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))**p/x**2,x)
```

```
[Out] -2*b**p*x**(p/2)*gamma(2 - p)*hyper((-p, 2 - p), (3 - p,), a*exp_polar(I*pi
)/(b*sqrt(x)))/(x*gamma(3 - p))
```

3.2270 $\int \frac{\sqrt{x}}{1+x^{3/2}}\,dx$

Optimal. Leaf size=12

$$\frac{2}{3}\log\left(x^{3/2}+1\right)$$

```
[Out] 2/3*ln(1+x^(3/2))
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {260}

$$\frac{2}{3}\log\left(x^{3/2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(1 + x^(3/2)),x]
```

```
[Out] (2*Log[1 + x^(3/2)])/3
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\sqrt{x}}{1+x^{3/2}}\,dx = \frac{2}{3}\log\left(1+x^{3/2}\right)$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\frac{2}{3}\log\left(x^{3/2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(1 + x^(3/2)),x]
```

```
[Out] (2*Log[1 + x^(3/2)])/3
```

fricas [A] time = 0.88, size = 8, normalized size = 0.67

$$\frac{2}{3}\log\left(x^{\frac{3}{2}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x^(3/2)),x, algorithm="fricas")
```

```
[Out] 2/3*log(x^(3/2) + 1)
```

giac [B] time = 0.16, size = 20, normalized size = 1.67

$$\frac{2}{3}\log\left(x-\sqrt{x}+1\right)+\frac{2}{3}\log\left(\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x^(3/2)),x, algorithm="giac")
[Out] 2/3*log(x - sqrt(x) + 1) + 2/3*log(sqrt(x) + 1)
```

maple [A] time = 0.00, size = 9, normalized size = 0.75

$$\frac{2\ln\left(x^{\frac{3}{2}}+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(1+x^(3/2)),x)
[Out] 2/3*ln(1+x^(3/2))
```

maxima [A] time = 0.83, size = 8, normalized size = 0.67

$$\frac{2}{3}\log\left(x^{\frac{3}{2}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x^(3/2)),x, algorithm="maxima")
[Out] 2/3*log(x^(3/2) + 1)
```

mupad [B] time = 0.10, size = 8, normalized size = 0.67

$$\frac{2\,\ln\left(x^{3/2}+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^(3/2) + 1),x)
[Out] (2*log(x^(3/2) + 1))/3
```

sympy [B] time = 0.22, size = 24, normalized size = 2.00

$$\frac{2\log\left(\sqrt{x}+1\right)}{3}+\frac{2\log\left(-\sqrt{x}+x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(1+x**(3/2)),x)
[Out] 2*log(sqrt(x) + 1)/3 + 2*log(-sqrt(x) + x + 1)/3
```

3.2271 $\int \frac{x^3}{\left(a+bx^{3/2}\right)^{2/3}}\,dx$

Optimal. Leaf size=139

$$-\frac{5a^2\log\left(\sqrt[3]{b}\,\sqrt{x}-\sqrt[3]{a+bx^{3/2}}\right)}{9b^{8/3}}-\frac{10a^2\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,\sqrt{x}}{\sqrt[3]{a+bx^{3/2}}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,b^{8/3}}-\frac{5ax\sqrt[3]{a+bx^{3/2}}}{9b^2}+\frac{x^{5/2}\sqrt[3]{a+bx^{3/2}}}{3b}$$

```
[Out] -5/9*a*x*(a+b*x^(3/2))^(1/3)/b^2+1/3*x^(5/2)*(a+b*x^(3/2))^(1/3)/b-5/9*a^2*
ln(-(a+b*x^(3/2))^(1/3)+b^(1/3)*x^(1/2))/b^(8/3)-10/27*a^2*arctan(1/3*(1+2*
b^(1/3)*x^(1/2)/(a+b*x^(3/2))^(1/3))*3^(1/2))/b^(8/3)*3^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 198, normalized size of antiderivative = 1.42, number of steps used = 10, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {341, 321, 331, 292, 31, 634, 617, 204, 628}

$$-\frac{10a^2\log\left(1-\frac{\sqrt[3]{b}\,\sqrt{x}}{\sqrt[3]{a+bx^{3/2}}}\right)}{27b^{8/3}}+\frac{5a^2\log\left(\frac{b^{2/3}x}{\left(a+bx^{3/2}\right)^{2/3}}+\frac{\sqrt[3]{b}\,\sqrt{x}}{\sqrt[3]{a+bx^{3/2}}}+1\right)}{27b^{8/3}}-\frac{10a^2\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,\sqrt{x}}{\sqrt[3]{a+bx^{3/2}}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,b^{8/3}}-\frac{5ax\sqrt[3]{a+bx^{3/2}}}{9b^2}+\frac{x^{5/2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^(3/2))^(2/3),x]
```

```
[Out] (-5*a*x*(a + b*x^(3/2))^(1/3))/(9*b^2) + (x^(5/2)*(a + b*x^(3/2))^(1/3))/(3
*b) - (10*a^2*ArcTan[(1 + (2*b^(1/3)*Sqrt[x])/(a + b*x^(3/2))^(1/3))/Sqrt[3
]])/(9*Sqrt[3]*b^(8/3)) - (10*a^2*Log[1 - (b^(1/3)*Sqrt[x])/(a + b*x^(3/2))
^(1/3)])/(27*b^(8/3)) + (5*a^2*Log[1 + (b^(2/3)*x)/(a + b*x^(3/2))^(2/3) +
(b^(1/3)*Sqrt[x])/(a + b*x^(3/2))^(1/3)])/(27*b^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^{3/2}\right)^{2/3}}\,dx = 2\,\text{Subst}\left(\int \frac{x^7}{\left(a+bx^3\right)^{2/3}}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{5/2}\sqrt[3]{a+bx^{3/2}}}{3b} - \frac{(5a)\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^3\right)^{2/3}}\,dx, x, \sqrt{x}\right)}{3b}$$

$$= -\frac{5ax\sqrt[3]{a+bx^{3/2}}}{9b^2} + \frac{x^{5/2}\sqrt[3]{a+bx^{3/2}}}{3b} + \frac{\left(10a^2\right)\text{Subst}\left(\int \frac{x}{\left(a+bx^3\right)^{2/3}}\,dx, x, \sqrt{x}\right)}{9b^2}$$

$$= -\frac{5ax\sqrt[3]{a+bx^{3/2}}}{9b^2} + \frac{x^{5/2}\sqrt[3]{a+bx^{3/2}}}{3b} + \frac{\left(10a^2\right)\text{Subst}\left(\int \frac{x}{1-bx^3}\,dx, x, \frac{\sqrt{x}}{\sqrt[3]{a+bx^{3/2}}}\right)}{9b^2}$$

$$= -\frac{5ax\sqrt[3]{a+bx^{3/2}}}{9b^2} + \frac{x^{5/2}\sqrt[3]{a+bx^{3/2}}}{3b} + \frac{\left(10a^2\right)\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{b}\,x}\,dx, x, \frac{\sqrt{x}}{\sqrt[3]{a+bx^{3/2}}}\right)}{27b^{7/3}} - \frac{\left(10a^2\right)\text{S}}{}$$

$$= -\frac{5ax\sqrt[3]{a+bx^{3/2}}}{9b^2} + \frac{x^{5/2}\sqrt[3]{a+bx^{3/2}}}{3b} - \frac{10a^2\log\left(1-\frac{\sqrt[3]{b}\,\sqrt{x}}{\sqrt[3]{a+bx^{3/2}}}\right)}{27b^{8/3}} + \frac{\left(5a^2\right)\text{Subst}\left(\int \frac{\sqrt[3]{b}+2b}{1+\sqrt[3]{b}\,x+}\right.}{27b^8}$$

$$= -\frac{5ax\sqrt[3]{a+bx^{3/2}}}{9b^2} + \frac{x^{5/2}\sqrt[3]{a+bx^{3/2}}}{3b} - \frac{10a^2\log\left(1-\frac{\sqrt[3]{b}\,\sqrt{x}}{\sqrt[3]{a+bx^{3/2}}}\right)}{27b^{8/3}} + \frac{5a^2\log\left(1+\frac{b^{2/3}x}{\left(a+bx^{3/2}\right)^{2/3}}\right.}{27b^{8/3}}$$

$$= -\frac{5ax\sqrt[3]{a+bx^{3/2}}}{9b^2} + \frac{x^{5/2}\sqrt[3]{a+bx^{3/2}}}{3b} - \frac{10a^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,\sqrt{x}}{\sqrt[3]{a+bx^{3/2}}}}{\sqrt{3}}\right)}{9\sqrt{3}\,b^{8/3}} - \frac{10a^2\log\left(1-\frac{\sqrt[3]{b}\,\sqrt{x}}{\sqrt[3]{a+bx^{3/2}}}\right.}{27b^{8/3}}$$

Mathematica [C] time = 0.03, size = 75, normalized size = 0.54

$$\frac{x\left(5a^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{bx^{3/2}}{bx^{3/2}+a}\right)-5a^2-2abx^{3/2}+3b^2x^3\right)}{9b^2\left(a+bx^{3/2}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^(3/2))^(2/3),x]
```

```
[Out] (x*(-5*a^2 - 2*a*b*x^(3/2) + 3*b^2*x^3 + 5*a^2*Hypergeometric2F1[2/3, 1, 5/
3, (b*x^(3/2))/(a + b*x^(3/2))]))/(9*b^2*(a + b*x^(3/2))^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^(3/2))^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(bx^{\frac{3}{2}}+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^(3/2))^(2/3),x, algorithm="giac")
```

```
[Out] integrate(x^3/(b*x^(3/2) + a)^(2/3), x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(b\,x^{\frac{3}{2}}+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*x^(3/2))^(2/3),x)
```

```
[Out] int(x^3/(a+b*x^(3/2))^(2/3),x)
```

maxima [A] time = 1.94, size = 184, normalized size = 1.32

$$\frac{10\sqrt{3}\,a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}}{\sqrt{x}}\right)}{3\,b^{\frac{1}{3}}}\right)}{27\,b^{\frac{8}{3}}}+\frac{5\,a^2\log\left(b^{\frac{2}{3}}+\frac{\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{\sqrt{x}}+\frac{\left(bx^{\frac{3}{2}}+a\right)^{\frac{2}{3}}}{x}\right)}{27\,b^{\frac{8}{3}}}-\frac{10\,a^2\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}}{\sqrt{x}}\right)}{27\,b^{\frac{8}{3}}}+\frac{8\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}a^2}{9\left(b^4-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^(3/2))^(2/3),x, algorithm="maxima")
```

```
[Out] 10/27*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^(3/2) + a)^(1/3)/sqr
t(x))/b^(1/3))/b^(8/3) + 5/27*a^2*log(b^(2/3) + (b*x^(3/2) + a)^(1/3)*b^(1/
3)/sqrt(x) + (b*x^(3/2) + a)^(2/3)/x)/b^(8/3) - 10/27*a^2*log(-b^(1/3) + (b
*x^(3/2) + a)^(1/3)/sqrt(x))/b^(8/3) + 1/9*(8*(b*x^(3/2) + a)^(1/3)*a^2*b/s
qrt(x) - 5*(b*x^(3/2) + a)^(4/3)*a^2/x^2)/(b^4 - 2*(b*x^(3/2) + a)*b^3/x^(3
/2) + (b*x^(3/2) + a)^2*b^2/x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\left(a+b\,x^{3/2}\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^(3/2))^(2/3),x)
```

```
[Out] int(x^3/(a + b*x^(3/2))^(2/3), x)
```

sympy [C] time = 2.12, size = 41, normalized size = 0.29

$$\frac{2x^4\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{2}{3},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\right|\frac{bx^{\frac{3}{2}}e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*x**(3/2))**(2/3),x)
```

```
[Out] 2*x**4*gamma(8/3)*hyper((2/3, 8/3), (11/3,), b*x**(3/2)*exp_polar(I*pi)/a)/
(3*a**(2/3)*gamma(11/3))
```

3.2272 $\int \frac{1}{\left(a+bx^{3/2}\right)^{2/3}}\,dx$

Optimal. Leaf size=82

$$-\frac{\log\left(\sqrt[3]{b}\,\sqrt{x}-\sqrt[3]{a+bx^{3/2}}\right)}{b^{2/3}}-\frac{2\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,\sqrt{x}}{\sqrt[3]{a+bx^{3/2}}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}}$$

```
[Out] -ln(-(a+b*x^(3/2))^(1/3)+b^(1/3)*x^(1/2))/b^(2/3)-2/3*arctan(1/3*(1+2*b^(1/
3)*x^(1/2)/(a+b*x^(3/2))^(1/3))*3^(1/2))/b^(2/3)*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 140, normalized size of antiderivative = 1.71, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {243, 331, 292, 31, 634, 617, 204, 628}

$$-\frac{2\log\left(1-\frac{\sqrt[3]{b}\,\sqrt{x}}{\sqrt[3]{a+bx^{3/2}}}\right)}{3b^{2/3}}+\frac{\log\left(\frac{b^{2/3}x}{\left(a+bx^{3/2}\right)^{2/3}}+\frac{\sqrt[3]{b}\,\sqrt{x}}{\sqrt[3]{a+bx^{3/2}}}+1\right)}{3b^{2/3}}-\frac{2\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,\sqrt{x}}{\sqrt[3]{a+bx^{3/2}}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(3/2))^(-2/3),x]
```

```
[Out] (-2*ArcTan[(1 + (2*b^(1/3)*Sqrt[x])/(a + b*x^(3/2))^(1/3))/Sqrt[3]])/(Sqrt[
3]*b^(2/3)) - (2*Log[1 - (b^(1/3)*Sqrt[x])/(a + b*x^(3/2))^(1/3)])/(3*b^(2/
3)) + Log[1 + (b^(2/3)*x)/(a + b*x^(3/2))^(2/3) + (b^(1/3)*Sqrt[x])/(a + b*
x^(3/2))^(1/3)]/(3*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Free
Q[{a, b, p}, x] && FractionQ[n]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
```

```
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+bx^{3/2}\right)^{2/3}}\,dx &= 2\,\text{Subst}\left(\int \frac{x}{\left(a+bx^3\right)^{2/3}}\,dx, x, \sqrt{x}\right)\\
&= 2\,\text{Subst}\left(\int \frac{x}{1-bx^3}\,dx, x, \frac{\sqrt{x}}{\sqrt[3]{a+bx^{3/2}}}\right)\\
&= \frac{2\,\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{b}\,x}\,dx, x, \frac{\sqrt{x}}{\sqrt[3]{a+bx^{3/2}}}\right)}{3\sqrt[3]{b}} - \frac{2\,\text{Subst}\left(\int \frac{1-\sqrt[3]{b}\,x}{1+\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, \frac{\sqrt{x}}{\sqrt[3]{a+bx^{3/2}}}\right)}{3\sqrt[3]{b}}\\
&= -\frac{2\log\left(1-\frac{\sqrt[3]{b}\,\sqrt{x}}{\sqrt[3]{a+bx^{3/2}}}\right)}{3b^{2/3}} + \frac{\text{Subst}\left(\int \frac{\sqrt[3]{b}+2b^{2/3}x}{1+\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, \frac{\sqrt{x}}{\sqrt[3]{a+bx^{3/2}}}\right)}{3b^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{b}\,x+b^{2/3}x^2}\right.}{\sqrt[3]{b}}\\
&= -\frac{2\log\left(1-\frac{\sqrt[3]{b}\,\sqrt{x}}{\sqrt[3]{a+bx^{3/2}}}\right)}{3b^{2/3}} + \frac{\log\left(1+\frac{b^{2/3}x}{\left(a+bx^{3/2}\right)^{2/3}}+\frac{\sqrt[3]{b}\,\sqrt{x}}{\sqrt[3]{a+bx^{3/2}}}\right)}{3b^{2/3}} + \frac{2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2}{\sqrt[3]}\right.}{b^{2/3}}\\
&= -\frac{2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,\sqrt{x}}{\sqrt[3]{a+bx^{3/2}}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}} - \frac{2\log\left(1-\frac{\sqrt[3]{b}\,\sqrt{x}}{\sqrt[3]{a+bx^{3/2}}}\right)}{3b^{2/3}} + \frac{\log\left(1+\frac{b^{2/3}x}{\left(a+bx^{3/2}\right)^{2/3}}+\frac{\sqrt[3]{b}\,\sqrt{x}}{\sqrt[3]{a+bx^{3/2}}}\right)}{3b^{2/3}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.63

$$\frac{x\left(\frac{bx^{3/2}}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^{3/2}}{a}\right)}{\left(a+bx^{3/2}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(3/2))^(-2/3),x]
```

```
[Out] (x*(1 + (b*x^(3/2))/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, -((b*x^(3/2))
/a)])/(a + b*x^(3/2))^(2/3)
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(3/2))^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{\frac{3}{2}}+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(3/2))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^(3/2) + a)^(-2/3), x)
```

maple [**F**] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{\frac{3}{2}}+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^(3/2)+a)^(2/3),x)
```

```
[Out] int(1/(b*x^(3/2)+a)^(2/3),x)
```

maxima [**A**] time = 1.89, size = 100, normalized size = 1.22

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}}{\sqrt{x}}\right)}{3b^{\frac{1}{3}}}\right)}{3b^{\frac{2}{3}}}+\frac{\log\left(b^{\frac{2}{3}}+\frac{\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{\sqrt{x}}+\frac{\left(bx^{\frac{3}{2}}+a\right)^{\frac{2}{3}}}{x}\right)}{3b^{\frac{2}{3}}}-\frac{2\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}}{\sqrt{x}}\right)}{3b^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(3/2))^(2/3),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^(3/2) + a)^(1/3)/sqrt(x))/
b^(1/3))/b^(2/3) + 1/3*log(b^(2/3) + (b*x^(3/2) + a)^(1/3)*b^(1/3)/sqrt(x)
+ (b*x^(3/2) + a)^(2/3)/x)/b^(2/3) - 2/3*log(-b^(1/3) + (b*x^(3/2) + a)^(1/
3)/sqrt(x))/b^(2/3)
```

mupad [**B**] time = 1.21, size = 37, normalized size = 0.45

$$\frac{x\left(\frac{b\,x^{3/2}}{a}+1\right)^{2/3}\,{}_2\mathrm{F}_1\left(\frac{2}{3},\frac{2}{3};\ \frac{5}{3};\ -\frac{b\,x^{3/2}}{a}\right)}{\left(a+b\,x^{3/2}\right)^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^(3/2))^(2/3),x)
```

```
[Out] (x*((b*x^(3/2))/a + 1)^(2/3)*hypergeom([2/3, 2/3], 5/3, -(b*x^(3/2))/a))/(a
 + b*x^(3/2))^(2/3)
```

sympy [C] time = 1.13, size = 39, normalized size = 0.48

$$\frac{2x\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{2}{3}, \frac{2}{3} \\ \frac{5}{3}\end{matrix}\right| \frac{bx^{\frac{3}{2}}e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**(3/2))**(2/3),x)
```

```
[Out] 2*x*gamma(2/3)*hyper((2/3, 2/3), (5/3,), b*x**(3/2)*exp_polar(I*pi)/a)/(3*a
**(2/3)*gamma(5/3))
```

3.2273 $\int \frac{1}{x^3\left(a+bx^{3/2}\right)^{2/3}}\,dx$

Optimal. Leaf size=50

$$\frac{3b\sqrt[3]{a+bx^{3/2}}}{2a^2\sqrt{x}}-\frac{\sqrt[3]{a+bx^{3/2}}}{2ax^2}$$

```
[Out] -1/2*(a+b*x^(3/2))^(1/3)/a/x^2+3/2*b*(a+b*x^(3/2))^(1/3)/a^2/x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {271, 264}

$$\frac{3b\sqrt[3]{a+bx^{3/2}}}{2a^2\sqrt{x}}-\frac{\sqrt[3]{a+bx^{3/2}}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^(3/2))^(2/3)),x]
```

```
[Out] -(a + b*x^(3/2))^(1/3)/(2*a*x^2) + (3*b*(a + b*x^(3/2))^(1/3))/(2*a^2*Sqrt[
x])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a+bx^{3/2}\right)^{2/3}}\,dx &= -\frac{\sqrt[3]{a+bx^{3/2}}}{2ax^2}-\frac{(3b)\int \frac{1}{x^{3/2}\left(a+bx^{3/2}\right)^{2/3}}\,dx}{4a}\\ &= -\frac{\sqrt[3]{a+bx^{3/2}}}{2ax^2}+\frac{3b\sqrt[3]{a+bx^{3/2}}}{2a^2\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.66

$$-\frac{\left(a-3bx^{3/2}\right)\sqrt[3]{a+bx^{3/2}}}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^(3/2))^(2/3)),x]
```

```
[Out] -1/2*((a - 3*b*x^(3/2))*(a + b*x^(3/2))^(1/3))/(a^2*x^2)
```

fricas [A] time = 1.61, size = 27, normalized size = 0.54

$$\frac{\left(3\,bx^{\frac{3}{2}}-a\right)\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^(3/2))^(2/3),x, algorithm="fricas")
[Out] 1/2*(3*b*x^(3/2) - a)*(b*x^(3/2) + a)^(1/3)/(a^2*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{\frac{3}{2}}+a\right)^{\frac{2}{3}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^(3/2))^(2/3),x, algorithm="giac")
[Out] integrate(1/((b*x^(3/2) + a)^(2/3)*x^3), x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{\frac{3}{2}}+a\right)^{\frac{2}{3}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^(3/2)+a)^(2/3),x)
[Out] int(1/x^3/(b*x^(3/2)+a)^(2/3),x)
```

maxima [A] time = 0.82, size = 35, normalized size = 0.70

$$\frac{\frac{4\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}b}{\sqrt{x}}-\frac{\left(bx^{\frac{3}{2}}+a\right)^{\frac{4}{3}}}{x^2}}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^(3/2))^(2/3),x, algorithm="maxima")
[Out] 1/2*(4*(b*x^(3/2) + a)^(1/3)*b/sqrt(x) - (b*x^(3/2) + a)^(4/3)/x^2)/a^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^3\left(a+b\,x^{3/2}\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^(3/2))^(2/3)),x)
[Out] int(1/(x^3*(a + b*x^(3/2))^(2/3)), x)
```

sympy [**A**] time = 2.68, size = 76, normalized size = 1.52

$$-\frac{2\sqrt[3]{b}\sqrt[3]{\frac{a}{bx^{\frac{3}{2}}}+1}\,\Gamma\left(-\frac{4}{3}\right)}{9ax^{\frac{3}{2}}\Gamma\left(\frac{2}{3}\right)}+\frac{2b^{\frac{4}{3}}\sqrt[3]{\frac{a}{bx^{\frac{3}{2}}}+1}\,\Gamma\left(-\frac{4}{3}\right)}{3a^2\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*x**(3/2))**(2/3),x)
```

```
[Out] -2*b**(1/3)*(a/(b*x**(3/2)) + 1)**(1/3)*gamma(-4/3)/(9*a*x**(3/2)*gamma(2/3
)) + 2*b**(4/3)*(a/(b*x**(3/2)) + 1)**(1/3)*gamma(-4/3)/(3*a**2*gamma(2/3))
```

3.2274 $\int \frac{1}{x^6\left(a+bx^{3/2}\right)^{2/3}}\,dx$

Optimal. Leaf size=104

$$\frac{81b^3\sqrt[3]{a+bx^{3/2}}}{70a^4\sqrt{x}}-\frac{27b^2\sqrt[3]{a+bx^{3/2}}}{70a^3x^2}+\frac{9b\sqrt[3]{a+bx^{3/2}}}{35a^2x^{7/2}}-\frac{\sqrt[3]{a+bx^{3/2}}}{5ax^5}$$

```
[Out] -1/5*(a+b*x^(3/2))^(1/3)/a/x^5+9/35*b*(a+b*x^(3/2))^(1/3)/a^2/x^(7/2)-27/70
*b^2*(a+b*x^(3/2))^(1/3)/a^3/x^2+81/70*b^3*(a+b*x^(3/2))^(1/3)/a^4/x^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {271, 264}

$$\frac{81b^3\sqrt[3]{a+bx^{3/2}}}{70a^4\sqrt{x}}-\frac{27b^2\sqrt[3]{a+bx^{3/2}}}{70a^3x^2}+\frac{9b\sqrt[3]{a+bx^{3/2}}}{35a^2x^{7/2}}-\frac{\sqrt[3]{a+bx^{3/2}}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^(3/2))^(2/3)),x]
```

```
[Out] -(a + b*x^(3/2))^(1/3)/(5*a*x^5) + (9*b*(a + b*x^(3/2))^(1/3))/(35*a^2*x^(7
/2)) - (27*b^2*(a + b*x^(3/2))^(1/3))/(70*a^3*x^2) + (81*b^3*(a + b*x^(3/2))
)^(1/3))/(70*a^4*Sqrt[x])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^6\left(a+bx^{3/2}\right)^{2/3}}\,dx &= -\frac{\sqrt[3]{a+bx^{3/2}}}{5ax^5}-\frac{(9b)\int \frac{1}{x^{9/2}\left(a+bx^{3/2}\right)^{2/3}}\,dx}{10a}\\ &= -\frac{\sqrt[3]{a+bx^{3/2}}}{5ax^5}+\frac{9b\sqrt[3]{a+bx^{3/2}}}{35a^2x^{7/2}}+\frac{\left(27b^2\right)\int \frac{1}{x^3\left(a+bx^{3/2}\right)^{2/3}}\,dx}{35a^2}\\ &= -\frac{\sqrt[3]{a+bx^{3/2}}}{5ax^5}+\frac{9b\sqrt[3]{a+bx^{3/2}}}{35a^2x^{7/2}}-\frac{27b^2\sqrt[3]{a+bx^{3/2}}}{70a^3x^2}-\frac{\left(81b^3\right)\int \frac{1}{x^{3/2}\left(a+bx^{3/2}\right)^{2/3}}\,dx}{140a^3}\\ &= -\frac{\sqrt[3]{a+bx^{3/2}}}{5ax^5}+\frac{9b\sqrt[3]{a+bx^{3/2}}}{35a^2x^{7/2}}-\frac{27b^2\sqrt[3]{a+bx^{3/2}}}{70a^3x^2}+\frac{81b^3\sqrt[3]{a+bx^{3/2}}}{70a^4\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 59, normalized size = 0.57

$$\frac{\sqrt[3]{a+bx^{3/2}}\left(-14a^3+18a^2bx^{3/2}-27ab^2x^3+81b^3x^{9/2}\right)}{70a^4x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^(3/2))^(2/3)),x]
```

```
[Out] ((a + b*x^(3/2))^(1/3)*(-14*a^3 + 18*a^2*b*x^(3/2) - 27*a*b^2*x^3 + 81*b^3*
x^(9/2)))/(70*a^4*x^5)
```

fricas [A] time = 1.59, size = 53, normalized size = 0.51

$$-\frac{\left(27\,ab^2x^3+14\,a^3-9\left(9\,b^3x^4+2\,a^2bx\right)\sqrt{x}\right)\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}}{70\,a^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(a+b*x^(3/2))^(2/3),x, algorithm="fricas")
```

```
[Out] -1/70*(27*a*b^2*x^3 + 14*a^3 - 9*(9*b^3*x^4 + 2*a^2*b*x)*sqrt(x))*(b*x^(3/2
) + a)^(1/3)/(a^4*x^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{\frac{3}{2}}+a\right)^{\frac{2}{3}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(a+b*x^(3/2))^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^(3/2) + a)^(2/3)*x^6), x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{\frac{3}{2}}+a\right)^{\frac{2}{3}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^(3/2)+a)^(2/3),x)
```

```
[Out] int(1/x^6/(b*x^(3/2)+a)^(2/3),x)
```

maxima [A] time = 0.85, size = 69, normalized size = 0.66

$$\frac{\frac{140\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}b^3}{\sqrt{x}}-\frac{105\left(bx^{\frac{3}{2}}+a\right)^{\frac{4}{3}}b^2}{x^2}+\frac{60\left(bx^{\frac{3}{2}}+a\right)^{\frac{7}{3}}b}{x^{\frac{7}{2}}}-\frac{14\left(bx^{\frac{3}{2}}+a\right)^{\frac{10}{3}}}{x^5}}{70\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(a+b*x^(3/2))^(2/3),x, algorithm="maxima")
```

```
[Out] 1/70*(140*(b*x^(3/2) + a)^(1/3)*b^3/sqrt(x) - 105*(b*x^(3/2) + a)^(4/3)*b^2
/x^2 + 60*(b*x^(3/2) + a)^(7/3)*b/x^(7/2) - 14*(b*x^(3/2) + a)^(10/3)/x^5)/
a^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^6\left(a+b\,x^{3/2}\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^(3/2))^(2/3)),x)
```

```
[Out] int(1/(x^6*(a + b*x^(3/2))^(2/3)), x)
```

sympy [B] time = 14.49, size = 736, normalized size = 7.08

$$-\frac{56a^6b^{\frac{28}{3}}x^9\sqrt[3]{\frac{a}{bx^{\frac{3}{2}}}+1}\,\Gamma\left(-\frac{10}{3}\right)}{81a^7b^9x^{\frac{27}{2}}\Gamma\left(\frac{2}{3}\right)+243a^6b^{10}x^{15}\Gamma\left(\frac{2}{3}\right)+243a^5b^{11}x^{\frac{33}{2}}\Gamma\left(\frac{2}{3}\right)+81a^4b^{12}x^{18}\Gamma\left(\frac{2}{3}\right)} - \frac{96a^5b^{\frac{31}{3}}}{81a^7b^9x^{\frac{27}{2}}\Gamma\left(\frac{2}{3}\right)+243a^6b^{10}x^{15}\Gamma}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(a+b*x**(3/2))**(2/3),x)
```

```
[Out] -56*a**6*b**(28/3)*x**9*(a/(b*x**(3/2)) + 1)**(1/3)*gamma(-10/3)/(81*a**7*b
**9*x**(27/2)*gamma(2/3) + 243*a**6*b**10*x**15*gamma(2/3) + 243*a**5*b**11
*x**(33/2)*gamma(2/3) + 81*a**4*b**12*x**18*gamma(2/3)) - 96*a**5*b**(31/3)
*x**(21/2)*(a/(b*x**(3/2)) + 1)**(1/3)*gamma(-10/3)/(81*a**7*b**9*x**(27/2)
*gamma(2/3) + 243*a**6*b**10*x**15*gamma(2/3) + 243*a**5*b**11*x**(33/2)*ga
mma(2/3) + 81*a**4*b**12*x**18*gamma(2/3)) - 60*a**4*b**(34/3)*x**12*(a/(b*
x**(3/2)) + 1)**(1/3)*gamma(-10/3)/(81*a**7*b**9*x**(27/2)*gamma(2/3) + 243
*a**6*b**10*x**15*gamma(2/3) + 243*a**5*b**11*x**(33/2)*gamma(2/3) + 81*a**
4*b**12*x**18*gamma(2/3)) + 160*a**3*b**(37/3)*x**(27/2)*(a/(b*x**(3/2)) +
1)**(1/3)*gamma(-10/3)/(81*a**7*b**9*x**(27/2)*gamma(2/3) + 243*a**6*b**10*
x**15*gamma(2/3) + 243*a**5*b**11*x**(33/2)*gamma(2/3) + 81*a**4*b**12*x**1
8*gamma(2/3)) + 720*a**2*b**(40/3)*x**15*(a/(b*x**(3/2)) + 1)**(1/3)*gamma(
-10/3)/(81*a**7*b**9*x**(27/2)*gamma(2/3) + 243*a**6*b**10*x**15*gamma(2/3)
 + 243*a**5*b**11*x**(33/2)*gamma(2/3) + 81*a**4*b**12*x**18*gamma(2/3)) +
864*a*b**(43/3)*x**(33/2)*(a/(b*x**(3/2)) + 1)**(1/3)*gamma(-10/3)/(81*a**7
*b**9*x**(27/2)*gamma(2/3) + 243*a**6*b**10*x**15*gamma(2/3) + 243*a**5*b**
11*x**(33/2)*gamma(2/3) + 81*a**4*b**12*x**18*gamma(2/3)) + 324*b**(46/3)*x
**18*(a/(b*x**(3/2)) + 1)**(1/3)*gamma(-10/3)/(81*a**7*b**9*x**(27/2)*gamma
(2/3) + 243*a**6*b**10*x**15*gamma(2/3) + 243*a**5*b**11*x**(33/2)*gamma(2/
3) + 81*a**4*b**12*x**18*gamma(2/3))
```

3.2275 $\int \frac{1}{x^9\left(a+bx^{3/2}\right)^{2/3}}\,dx$

Optimal. Leaf size=158

$$\frac{729b^5\sqrt[3]{a+bx^{3/2}}}{728a^6\sqrt{x}}-\frac{243b^4\sqrt[3]{a+bx^{3/2}}}{728a^5x^2}+\frac{81b^3\sqrt[3]{a+bx^{3/2}}}{364a^4x^{7/2}}-\frac{9b^2\sqrt[3]{a+bx^{3/2}}}{52a^3x^5}+\frac{15b\sqrt[3]{a+bx^{3/2}}}{104a^2x^{13/2}}-\frac{\sqrt[3]{a+bx^{3/2}}}{8ax^8}$$

```
[Out] -1/8*(a+b*x^(3/2))^(1/3)/a/x^8+15/104*b*(a+b*x^(3/2))^(1/3)/a^2/x^(13/2)-9/
52*b^2*(a+b*x^(3/2))^(1/3)/a^3/x^5+81/364*b^3*(a+b*x^(3/2))^(1/3)/a^4/x^(7/
2)-243/728*b^4*(a+b*x^(3/2))^(1/3)/a^5/x^2+729/728*b^5*(a+b*x^(3/2))^(1/3)/
a^6/x^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {271, 264}

$$\frac{729b^5\sqrt[3]{a+bx^{3/2}}}{728a^6\sqrt{x}}-\frac{243b^4\sqrt[3]{a+bx^{3/2}}}{728a^5x^2}+\frac{81b^3\sqrt[3]{a+bx^{3/2}}}{364a^4x^{7/2}}-\frac{9b^2\sqrt[3]{a+bx^{3/2}}}{52a^3x^5}+\frac{15b\sqrt[3]{a+bx^{3/2}}}{104a^2x^{13/2}}-\frac{\sqrt[3]{a+bx^{3/2}}}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(a + b*x^(3/2))^(2/3)),x]
```

```
[Out] -(a + b*x^(3/2))^(1/3)/(8*a*x^8) + (15*b*(a + b*x^(3/2))^(1/3))/(104*a^2*x^
(13/2)) - (9*b^2*(a + b*x^(3/2))^(1/3))/(52*a^3*x^5) + (81*b^3*(a + b*x^(3/
2))^(1/3))/(364*a^4*x^(7/2)) - (243*b^4*(a + b*x^(3/2))^(1/3))/(728*a^5*x^2
) + (729*b^5*(a + b*x^(3/2))^(1/3))/(728*a^6*Sqrt[x])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^9\left(a+bx^{3/2}\right)^{2/3}}\,dx = -\frac{\sqrt[3]{a+bx^{3/2}}}{8ax^8} - \frac{(15b)\int \frac{1}{x^{15/2}\left(a+bx^{3/2}\right)^{2/3}}\,dx}{16a}$$

$$= -\frac{\sqrt[3]{a+bx^{3/2}}}{8ax^8} + \frac{15b\sqrt[3]{a+bx^{3/2}}}{104a^2x^{13/2}} + \frac{\left(45b^2\right)\int \frac{1}{x^6\left(a+bx^{3/2}\right)^{2/3}}\,dx}{52a^2}$$

$$= -\frac{\sqrt[3]{a+bx^{3/2}}}{8ax^8} + \frac{15b\sqrt[3]{a+bx^{3/2}}}{104a^2x^{13/2}} - \frac{9b^2\sqrt[3]{a+bx^{3/2}}}{52a^3x^5} - \frac{\left(81b^3\right)\int \frac{1}{x^{9/2}\left(a+bx^{3/2}\right)^{2/3}}\,dx}{104a^3}$$

$$= -\frac{\sqrt[3]{a+bx^{3/2}}}{8ax^8} + \frac{15b\sqrt[3]{a+bx^{3/2}}}{104a^2x^{13/2}} - \frac{9b^2\sqrt[3]{a+bx^{3/2}}}{52a^3x^5} + \frac{81b^3\sqrt[3]{a+bx^{3/2}}}{364a^4x^{7/2}} + \frac{\left(243b^4\right)\int \frac{}{x^3\left(a\right.}}{364a}$$

$$= -\frac{\sqrt[3]{a+bx^{3/2}}}{8ax^8} + \frac{15b\sqrt[3]{a+bx^{3/2}}}{104a^2x^{13/2}} - \frac{9b^2\sqrt[3]{a+bx^{3/2}}}{52a^3x^5} + \frac{81b^3\sqrt[3]{a+bx^{3/2}}}{364a^4x^{7/2}} - \frac{243b^4\sqrt[3]{a+bx}}{728a^5x^2}$$

$$= -\frac{\sqrt[3]{a+bx^{3/2}}}{8ax^8} + \frac{15b\sqrt[3]{a+bx^{3/2}}}{104a^2x^{13/2}} - \frac{9b^2\sqrt[3]{a+bx^{3/2}}}{52a^3x^5} + \frac{81b^3\sqrt[3]{a+bx^{3/2}}}{364a^4x^{7/2}} - \frac{243b^4\sqrt[3]{a+bx}}{728a^5x^2}$$

Mathematica [A] time = 0.04, size = 83, normalized size = 0.53

$$\frac{\sqrt[3]{a+bx^{3/2}}\left(-91a^5+105a^4bx^{3/2}-126a^3b^2x^3+162a^2b^3x^{9/2}-243ab^4x^6+729b^5x^{15/2}\right)}{728a^6x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^9*(a + b*x^(3/2))^(2/3)),x]
```

```
[Out] ((a + b*x^(3/2))^(1/3)*(-91*a^5 + 105*a^4*b*x^(3/2) - 126*a^3*b^2*x^3 + 162
*a^2*b^3*x^(9/2) - 243*a*b^4*x^6 + 729*b^5*x^(15/2)))/(728*a^6*x^8)
```

fricas [A] time = 1.33, size = 75, normalized size = 0.47

$$-\frac{\left(243\,ab^4x^6+126\,a^3b^2x^3+91\,a^5-3\left(243\,b^5x^7+54\,a^2b^3x^4+35\,a^4bx\right)\sqrt{x}\right)\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}}{728\,a^6x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(a+b*x^(3/2))^(2/3),x, algorithm="fricas")
```

```
[Out] -1/728*(243*a*b^4*x^6 + 126*a^3*b^2*x^3 + 91*a^5 - 3*(243*b^5*x^7 + 54*a^2*
b^3*x^4 + 35*a^4*b*x)*sqrt(x))*(b*x^(3/2) + a)^(1/3)/(a^6*x^8)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{\frac{3}{2}}+a\right)^{\frac{2}{3}}x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(a+b*x^(3/2))^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^(3/2) + a)^(2/3)*x^9), x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{\frac{3}{2}}+a\right)^{\frac{2}{3}}\,x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(b*x^(3/2)+a)^(2/3),x)
```

```
[Out] int(1/x^9/(b*x^(3/2)+a)^(2/3),x)
```

maxima [A] time = 0.83, size = 103, normalized size = 0.65

$$\frac{\frac{1456\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}b^5}{\sqrt{x}}-\frac{1820\left(bx^{\frac{3}{2}}+a\right)^{\frac{4}{3}}b^4}{x^2}+\frac{2080\left(bx^{\frac{3}{2}}+a\right)^{\frac{7}{3}}b^3}{x^{\frac{7}{2}}}-\frac{1456\left(bx^{\frac{3}{2}}+a\right)^{\frac{10}{3}}b^2}{x^5}+\frac{560\left(bx^{\frac{3}{2}}+a\right)^{\frac{13}{3}}b}{x^{\frac{13}{2}}}-\frac{91\left(bx^{\frac{3}{2}}+a\right)^{\frac{16}{3}}}{x^8}}{728\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(a+b*x^(3/2))^(2/3),x, algorithm="maxima")
```

```
[Out] 1/728*(1456*(b*x^(3/2) + a)^(1/3)*b^5/sqrt(x) - 1820*(b*x^(3/2) + a)^(4/3)*
b^4/x^2 + 2080*(b*x^(3/2) + a)^(7/3)*b^3/x^(7/2) - 1456*(b*x^(3/2) + a)^(10
/3)*b^2/x^5 + 560*(b*x^(3/2) + a)^(13/3)*b/x^(13/2) - 91*(b*x^(3/2) + a)^(1
6/3)/x^8)/a^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^9\left(a+b\,x^{3/2}\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(a + b*x^(3/2))^(2/3)),x)
```

```
[Out] int(1/(x^9*(a + b*x^(3/2))^(2/3)), x)
```

sympy [B] time = 55.10, size = 1554, normalized size = 9.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(a+b*x**(3/2))**(2/3),x)
```

```
[Out] -7280*a**10*b**(76/3)*x**30*(a/(b*x**(3/2)) + 1)**(1/3)*gamma(-16/3)/(729*a
**11*b**25*x**(75/2)*gamma(2/3) + 3645*a**10*b**26*x**39*gamma(2/3) + 7290*
a**9*b**27*x**(81/2)*gamma(2/3) + 7290*a**8*b**28*x**42*gamma(2/3) + 3645*a
**7*b**29*x**(87/2)*gamma(2/3) + 729*a**6*b**30*x**45*gamma(2/3)) - 28000*a
**9*b**(79/3)*x**(63/2)*(a/(b*x**(3/2)) + 1)**(1/3)*gamma(-16/3)/(729*a**11
*b**25*x**(75/2)*gamma(2/3) + 3645*a**10*b**26*x**39*gamma(2/3) + 7290*a**9
*b**27*x**(81/2)*gamma(2/3) + 7290*a**8*b**28*x**42*gamma(2/3) + 3645*a**7*
b**29*x**(87/2)*gamma(2/3) + 729*a**6*b**30*x**45*gamma(2/3)) - 40880*a**8*
b**(82/3)*x**33*(a/(b*x**(3/2)) + 1)**(1/3)*gamma(-16/3)/(729*a**11*b**25*x
**(75/2)*gamma(2/3) + 3645*a**10*b**26*x**39*gamma(2/3) + 7290*a**9*b**27*x
**(81/2)*gamma(2/3) + 7290*a**8*b**28*x**42*gamma(2/3) + 3645*a**7*b**29*x*
*(87/2)*gamma(2/3) + 729*a**6*b**30*x**45*gamma(2/3)) - 26240*a**7*b**(85/3
)*x**(69/2)*(a/(b*x**(3/2)) + 1)**(1/3)*gamma(-16/3)/(729*a**11*b**25*x**(7
5/2)*gamma(2/3) + 3645*a**10*b**26*x**39*gamma(2/3) + 7290*a**9*b**27*x**(8
```

```
1/2)*gamma(2/3) + 7290*a**8*b**28*x**42*gamma(2/3) + 3645*a**7*b**29*x**(87
/2)*gamma(2/3) + 729*a**6*b**30*x**45*gamma(2/3)) - 7840*a**6*b**(88/3)*x**
36*(a/(b*x**(3/2)) + 1)**(1/3)*gamma(-16/3)/(729*a**11*b**25*x**(75/2)*gamm
a(2/3) + 3645*a**10*b**26*x**39*gamma(2/3) + 7290*a**9*b**27*x**(81/2)*gamm
a(2/3) + 7290*a**8*b**28*x**42*gamma(2/3) + 3645*a**7*b**29*x**(87/2)*gamma
(2/3) + 729*a**6*b**30*x**45*gamma(2/3)) + 24640*a**5*b**(91/3)*x**(75/2)*(
a/(b*x**(3/2)) + 1)**(1/3)*gamma(-16/3)/(729*a**11*b**25*x**(75/2)*gamma(2/
3) + 3645*a**10*b**26*x**39*gamma(2/3) + 7290*a**9*b**27*x**(81/2)*gamma(2/
3) + 7290*a**8*b**28*x**42*gamma(2/3) + 3645*a**7*b**29*x**(87/2)*gamma(2/3
) + 729*a**6*b**30*x**45*gamma(2/3)) + 184800*a**4*b**(94/3)*x**39*(a/(b*x*
*(3/2)) + 1)**(1/3)*gamma(-16/3)/(729*a**11*b**25*x**(75/2)*gamma(2/3) + 36
45*a**10*b**26*x**39*gamma(2/3) + 7290*a**9*b**27*x**(81/2)*gamma(2/3) + 72
90*a**8*b**28*x**42*gamma(2/3) + 3645*a**7*b**29*x**(87/2)*gamma(2/3) + 729
*a**6*b**30*x**45*gamma(2/3)) + 443520*a**3*b**(97/3)*x**(81/2)*(a/(b*x**(3
/2)) + 1)**(1/3)*gamma(-16/3)/(729*a**11*b**25*x**(75/2)*gamma(2/3) + 3645*
a**10*b**26*x**39*gamma(2/3) + 7290*a**9*b**27*x**(81/2)*gamma(2/3) + 7290*
a**8*b**28*x**42*gamma(2/3) + 3645*a**7*b**29*x**(87/2)*gamma(2/3) + 729*a*
*6*b**30*x**45*gamma(2/3)) + 498960*a**2*b**(100/3)*x**42*(a/(b*x**(3/2)) +
 1)**(1/3)*gamma(-16/3)/(729*a**11*b**25*x**(75/2)*gamma(2/3) + 3645*a**10*
b**26*x**39*gamma(2/3) + 7290*a**9*b**27*x**(81/2)*gamma(2/3) + 7290*a**8*b
**28*x**42*gamma(2/3) + 3645*a**7*b**29*x**(87/2)*gamma(2/3) + 729*a**6*b**
30*x**45*gamma(2/3)) + 272160*a*b**(103/3)*x**(87/2)*(a/(b*x**(3/2)) + 1)**
(1/3)*gamma(-16/3)/(729*a**11*b**25*x**(75/2)*gamma(2/3) + 3645*a**10*b**26
*x**39*gamma(2/3) + 7290*a**9*b**27*x**(81/2)*gamma(2/3) + 7290*a**8*b**28*
x**42*gamma(2/3) + 3645*a**7*b**29*x**(87/2)*gamma(2/3) + 729*a**6*b**30*x*
*45*gamma(2/3)) + 58320*b**(106/3)*x**45*(a/(b*x**(3/2)) + 1)**(1/3)*gamma(
-16/3)/(729*a**11*b**25*x**(75/2)*gamma(2/3) + 3645*a**10*b**26*x**39*gamma
(2/3) + 7290*a**9*b**27*x**(81/2)*gamma(2/3) + 7290*a**8*b**28*x**42*gamma(
2/3) + 3645*a**7*b**29*x**(87/2)*gamma(2/3) + 729*a**6*b**30*x**45*gamma(2/
3))
```

3.2276 $\int \frac{x^8}{\left(a+bx^{3/2}\right)^{2/3}}\,dx$

Optimal. Leaf size=130

$$-\frac{2a^5\sqrt[3]{a+bx^{3/2}}}{b^6}+\frac{5a^4\left(a+bx^{3/2}\right)^{4/3}}{2b^6}-\frac{20a^3\left(a+bx^{3/2}\right)^{7/3}}{7b^6}+\frac{2a^2\left(a+bx^{3/2}\right)^{10/3}}{b^6}+\frac{\left(a+bx^{3/2}\right)^{16/3}}{8b^6}-\frac{10a\left(a+bx^{3/2}\right)^{13/3}}{13b^6}$$

```
[Out] -2*a^5*(a+b*x^(3/2))^(1/3)/b^6+5/2*a^4*(a+b*x^(3/2))^(4/3)/b^6-20/7*a^3*(a+
b*x^(3/2))^(7/3)/b^6+2*a^2*(a+b*x^(3/2))^(10/3)/b^6-10/13*a*(a+b*x^(3/2))^(
13/3)/b^6+1/8*(a+b*x^(3/2))^(16/3)/b^6
```

Rubi [A] time = 0.06, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$\frac{2a^2\left(a+bx^{3/2}\right)^{10/3}}{b^6}-\frac{20a^3\left(a+bx^{3/2}\right)^{7/3}}{7b^6}+\frac{5a^4\left(a+bx^{3/2}\right)^{4/3}}{2b^6}-\frac{2a^5\sqrt[3]{a+bx^{3/2}}}{b^6}+\frac{\left(a+bx^{3/2}\right)^{16/3}}{8b^6}-\frac{10a\left(a+bx^{3/2}\right)^{13/3}}{13b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^(3/2))^(2/3),x]
```

```
[Out] (-2*a^5*(a + b*x^(3/2))^(1/3))/b^6 + (5*a^4*(a + b*x^(3/2))^(4/3))/(2*b^6)
- (20*a^3*(a + b*x^(3/2))^(7/3))/(7*b^6) + (2*a^2*(a + b*x^(3/2))^(10/3))/b
^6 - (10*a*(a + b*x^(3/2))^(13/3))/(13*b^6) + (a + b*x^(3/2))^(16/3)/(8*b^6
)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{\left(a+bx^{3/2}\right)^{2/3}}\,dx &= \frac{2}{3}\,\text{Subst}\left(\int \frac{x^5}{(a+bx)^{2/3}}\,dx, x, x^{3/2}\right)\\ &= \frac{2}{3}\,\text{Subst}\left(\int\left(-\frac{a^5}{b^5(a+bx)^{2/3}}+\frac{5a^4\sqrt[3]{a+bx}}{b^5}-\frac{10a^3(a+bx)^{4/3}}{b^5}+\frac{10a^2(a+bx)^{7/3}}{b^5}-\frac{5a(a+\right.\right.\\ &= -\frac{2a^5\sqrt[3]{a+bx^{3/2}}}{b^6}+\frac{5a^4\left(a+bx^{3/2}\right)^{4/3}}{2b^6}-\frac{20a^3\left(a+bx^{3/2}\right)^{7/3}}{7b^6}+\frac{2a^2\left(a+bx^{3/2}\right)^{10/3}}{b^6}-\frac{10a\left(a+\right.}{1}\end{aligned}$$

Mathematica [A] time = 0.06, size = 80, normalized size = 0.62

$$\frac{\sqrt[3]{a+bx^{3/2}}\left(-729a^5+243a^4bx^{3/2}-162a^3b^2x^3+126a^2b^3x^{9/2}-105ab^4x^6+91b^5x^{15/2}\right)}{728b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^(3/2))^(2/3),x]
```

```
[Out] ((a + b*x^(3/2))^(1/3)*(-729*a^5 + 243*a^4*b*x^(3/2) - 162*a^3*b^2*x^3 + 12
6*a^2*b^3*x^(9/2) - 105*a*b^4*x^6 + 91*b^5*x^(15/2)))/(728*b^6)
```

fricas [A] time = 1.25, size = 72, normalized size = 0.55

$$-\frac{\left(105\,ab^4x^6+162\,a^3b^2x^3+729\,a^5-\left(91\,b^5x^7+126\,a^2b^3x^4+243\,a^4bx\right)\sqrt{x}\right)\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}}{728\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(a+b*x^(3/2))^(2/3),x, algorithm="fricas")
```

```
[Out] -1/728*(105*a*b^4*x^6 + 162*a^3*b^2*x^3 + 729*a^5 - (91*b^5*x^7 + 126*a^2*b
^3*x^4 + 243*a^4*b*x)*sqrt(x))*(b*x^(3/2) + a)^(1/3)/b^6
```

giac [A] time = 0.23, size = 89, normalized size = 0.68

$$-\frac{2\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}a^5}{b^6}+\frac{91\left(bx^{\frac{3}{2}}+a\right)^{\frac{16}{3}}-560\left(bx^{\frac{3}{2}}+a\right)^{\frac{13}{3}}a+1456\left(bx^{\frac{3}{2}}+a\right)^{\frac{10}{3}}a^2-2080\left(bx^{\frac{3}{2}}+a\right)^{\frac{7}{3}}a^3+1820\left(bx^{\frac{3}{2}}\right.}{728\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(a+b*x^(3/2))^(2/3),x, algorithm="giac")
```

```
[Out] -2*(b*x^(3/2) + a)^(1/3)*a^5/b^6 + 1/728*(91*(b*x^(3/2) + a)^(16/3) - 560*(
b*x^(3/2) + a)^(13/3)*a + 1456*(b*x^(3/2) + a)^(10/3)*a^2 - 2080*(b*x^(3/2)
 + a)^(7/3)*a^3 + 1820*(b*x^(3/2) + a)^(4/3)*a^4)/b^6
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(b\,x^{\frac{3}{2}}+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^(3/2)+a)^(2/3),x)
```

```
[Out] int(x^8/(b*x^(3/2)+a)^(2/3),x)
```

maxima [A] time = 0.90, size = 98, normalized size = 0.75

$$\frac{\left(bx^{\frac{3}{2}}+a\right)^{\frac{16}{3}}}{8\,b^6}-\frac{10\left(bx^{\frac{3}{2}}+a\right)^{\frac{13}{3}}a}{13\,b^6}+\frac{2\left(bx^{\frac{3}{2}}+a\right)^{\frac{10}{3}}a^2}{b^6}-\frac{20\left(bx^{\frac{3}{2}}+a\right)^{\frac{7}{3}}a^3}{7\,b^6}+\frac{5\left(bx^{\frac{3}{2}}+a\right)^{\frac{4}{3}}a^4}{2\,b^6}-\frac{2\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}a^5}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(a+b*x^(3/2))^(2/3),x, algorithm="maxima")
```

```
[Out] 1/8*(b*x^(3/2) + a)^(16/3)/b^6 - 10/13*(b*x^(3/2) + a)^(13/3)*a/b^6 + 2*(b*
x^(3/2) + a)^(10/3)*a^2/b^6 - 20/7*(b*x^(3/2) + a)^(7/3)*a^3/b^6 + 5/2*(b*x
^(3/2) + a)^(4/3)*a^4/b^6 - 2*(b*x^(3/2) + a)^(1/3)*a^5/b^6
```

mupad [B] time = 1.39, size = 98, normalized size = 0.75

$$\frac{\left(a+b\,x^{3/2}\right)^{16/3}}{8\,b^6}-\frac{10\,a\left(a+b\,x^{3/2}\right)^{13/3}}{13\,b^6}-\frac{2\,a^5\left(a+b\,x^{3/2}\right)^{1/3}}{b^6}+\frac{5\,a^4\left(a+b\,x^{3/2}\right)^{4/3}}{2\,b^6}-\frac{20\,a^3\left(a+b\,x^{3/2}\right)^{7/3}}{7\,b^6}+\frac{2\,a^2\left(a+b\right.}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^(3/2))^(2/3),x)
```

```
[Out] (a + b*x^(3/2))^(16/3)/(8*b^6) - (10*a*(a + b*x^(3/2))^(13/3))/(13*b^6) - (
2*a^5*(a + b*x^(3/2))^(1/3))/b^6 + (5*a^4*(a + b*x^(3/2))^(4/3))/(2*b^6) -
(20*a^3*(a + b*x^(3/2))^(7/3))/(7*b^6) + (2*a^2*(a + b*x^(3/2))^(10/3))/b^6
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(a+b*x**(3/2))**(2/3),x)
```

```
[Out] Timed out
```

3.2277 $\int \frac{x^5}{\left(a+bx^{3/2}\right)^{2/3}}\,dx$

Optimal. Leaf size=86

$$-\frac{2a^3\sqrt[3]{a+bx^{3/2}}}{b^4}+\frac{3a^2\left(a+bx^{3/2}\right)^{4/3}}{2b^4}+\frac{\left(a+bx^{3/2}\right)^{10/3}}{5b^4}-\frac{6a\left(a+bx^{3/2}\right)^{7/3}}{7b^4}$$

```
[Out] -2*a^3*(a+b*x^(3/2))^(1/3)/b^4+3/2*a^2*(a+b*x^(3/2))^(4/3)/b^4-6/7*a*(a+b*x
^(3/2))^(7/3)/b^4+1/5*(a+b*x^(3/2))^(10/3)/b^4
```

Rubi [A] time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$\frac{3a^2\left(a+bx^{3/2}\right)^{4/3}}{2b^4}-\frac{2a^3\sqrt[3]{a+bx^{3/2}}}{b^4}+\frac{\left(a+bx^{3/2}\right)^{10/3}}{5b^4}-\frac{6a\left(a+bx^{3/2}\right)^{7/3}}{7b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^(3/2))^(2/3),x]
```

```
[Out] (-2*a^3*(a + b*x^(3/2))^(1/3))/b^4 + (3*a^2*(a + b*x^(3/2))^(4/3))/(2*b^4)
- (6*a*(a + b*x^(3/2))^(7/3))/(7*b^4) + (a + b*x^(3/2))^(10/3)/(5*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\left(a+bx^{3/2}\right)^{2/3}}\,dx &= \frac{2}{3}\,\text{Subst}\left(\int \frac{x^3}{(a+bx)^{2/3}}\,dx, x, x^{3/2}\right)\\ &= \frac{2}{3}\,\text{Subst}\left(\int\left(-\frac{a^3}{b^3(a+bx)^{2/3}}+\frac{3a^2\sqrt[3]{a+bx}}{b^3}-\frac{3a(a+bx)^{4/3}}{b^3}+\frac{(a+bx)^{7/3}}{b^3}\right)dx, x, x^{3/2}\right)\\ &= -\frac{2a^3\sqrt[3]{a+bx^{3/2}}}{b^4}+\frac{3a^2\left(a+bx^{3/2}\right)^{4/3}}{2b^4}-\frac{6a\left(a+bx^{3/2}\right)^{7/3}}{7b^4}+\frac{\left(a+bx^{3/2}\right)^{10/3}}{5b^4}\end{aligned}$$

Mathematica [A] time = 0.03, size = 56, normalized size = 0.65

$$\frac{\sqrt[3]{a+bx^{3/2}}\left(-81a^3+27a^2bx^{3/2}-18ab^2x^3+14b^3x^{9/2}\right)}{70b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^(3/2))^(2/3),x]
```

```
[Out] ((a + b*x^(3/2))^(1/3)*(-81*a^3 + 27*a^2*b*x^(3/2) - 18*a*b^2*x^3 + 14*b^3*
x^(9/2)))/(70*b^4)
```

fricas [A] time = 1.29, size = 50, normalized size = 0.58

$$-\frac{\left(18\,ab^2x^3+81\,a^3-\left(14\,b^3x^4+27\,a^2bx\right)\sqrt{x}\right)\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}}{70\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*x^(3/2))^(2/3),x, algorithm="fricas")
```

```
[Out] -1/70*(18*a*b^2*x^3 + 81*a^3 - (14*b^3*x^4 + 27*a^2*b*x)*sqrt(x))*(b*x^(3/2
) + a)^(1/3)/b^4
```

giac [A] time = 0.21, size = 61, normalized size = 0.71

$$-\frac{2\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}a^3}{b^4}+\frac{14\left(bx^{\frac{3}{2}}+a\right)^{\frac{10}{3}}-60\left(bx^{\frac{3}{2}}+a\right)^{\frac{7}{3}}a+105\left(bx^{\frac{3}{2}}+a\right)^{\frac{4}{3}}a^2}{70\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*x^(3/2))^(2/3),x, algorithm="giac")
```

```
[Out] -2*(b*x^(3/2) + a)^(1/3)*a^3/b^4 + 1/70*(14*(b*x^(3/2) + a)^(10/3) - 60*(b*
x^(3/2) + a)^(7/3)*a + 105*(b*x^(3/2) + a)^(4/3)*a^2)/b^4
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(b\,x^{\frac{3}{2}}+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^(3/2)+a)^(2/3),x)
```

```
[Out] int(x^5/(b*x^(3/2)+a)^(2/3),x)
```

maxima [A] time = 0.90, size = 64, normalized size = 0.74

$$\frac{\left(bx^{\frac{3}{2}}+a\right)^{\frac{10}{3}}}{5\,b^4}-\frac{6\left(bx^{\frac{3}{2}}+a\right)^{\frac{7}{3}}a}{7\,b^4}+\frac{3\left(bx^{\frac{3}{2}}+a\right)^{\frac{4}{3}}a^2}{2\,b^4}-\frac{2\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}a^3}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*x^(3/2))^(2/3),x, algorithm="maxima")
```

```
[Out] 1/5*(b*x^(3/2) + a)^(10/3)/b^4 - 6/7*(b*x^(3/2) + a)^(7/3)*a/b^4 + 3/2*(b*x
^(3/2) + a)^(4/3)*a^2/b^4 - 2*(b*x^(3/2) + a)^(1/3)*a^3/b^4
```

mupad [B] time = 1.42, size = 64, normalized size = 0.74

$$\frac{\left(a+b\,x^{3/2}\right)^{10/3}}{5\,b^4}-\frac{6\,a\left(a+b\,x^{3/2}\right)^{7/3}}{7\,b^4}-\frac{2\,a^3\left(a+b\,x^{3/2}\right)^{1/3}}{b^4}+\frac{3\,a^2\left(a+b\,x^{3/2}\right)^{4/3}}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^(3/2))^(2/3),x)
```

```
[Out] (a + b*x^(3/2))^(10/3)/(5*b^4) - (6*a*(a + b*x^(3/2))^(7/3))/(7*b^4) - (2*a
^3*(a + b*x^(3/2))^(1/3))/b^4 + (3*a^2*(a + b*x^(3/2))^(4/3))/(2*b^4)
```

sympy [A] time = 39.17, size = 102, normalized size = 1.19

$$\begin{cases} -\frac{81a^3\sqrt[3]{a+bx^{\frac{3}{2}}}}{70b^4} + \frac{27a^2x^{\frac{3}{2}}\sqrt[3]{a+bx^{\frac{3}{2}}}}{70b^3} - \frac{9ax^3\sqrt[3]{a+bx^{\frac{3}{2}}}}{35b^2} + \frac{x^{\frac{9}{2}}\sqrt[3]{a+bx^{\frac{3}{2}}}}{5b} & \text{for } b \neq 0 \\ \frac{x^6}{6a^{\frac{2}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a+b*x**(3/2))**(2/3),x)
```

```
[Out] Piecewise((-81*a**3*(a + b*x**(3/2))**(1/3)/(70*b**4) + 27*a**2*x**(3/2)*(a
 + b*x**(3/2))**(1/3)/(70*b**3) - 9*a*x**3*(a + b*x**(3/2))**(1/3)/(35*b**2
) + x**(9/2)*(a + b*x**(3/2))**(1/3)/(5*b), Ne(b, 0)), (x**6/(6*a**(2/3)),
True))
```

3.2278 $\int \frac{x^2}{\left(a+bx^{3/2}\right)^{2/3}}\,dx$

Optimal. Leaf size=40

$$\frac{\left(a+bx^{3/2}\right)^{4/3}}{2b^2}-\frac{2a\sqrt[3]{a+bx^{3/2}}}{b^2}$$

```
[Out] -2*a*(a+b*x^(3/2))^(1/3)/b^2+1/2*(a+b*x^(3/2))^(4/3)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$\frac{\left(a+bx^{3/2}\right)^{4/3}}{2b^2}-\frac{2a\sqrt[3]{a+bx^{3/2}}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^(3/2))^(2/3),x]
```

```
[Out] (-2*a*(a + b*x^(3/2))^(1/3))/b^2 + (a + b*x^(3/2))^(4/3)/(2*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a+bx^{3/2}\right)^{2/3}}\,dx &= \frac{2}{3}\,\text{Subst}\left(\int \frac{x}{(a+bx)^{2/3}}\,dx, x, x^{3/2}\right)\\ &= \frac{2}{3}\,\text{Subst}\left(\int\left(-\frac{a}{b(a+bx)^{2/3}}+\frac{\sqrt[3]{a+bx}}{b}\right)dx, x, x^{3/2}\right)\\ &= -\frac{2a\sqrt[3]{a+bx^{3/2}}}{b^2}+\frac{\left(a+bx^{3/2}\right)^{4/3}}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.78

$$\frac{\left(bx^{3/2}-3a\right)\sqrt[3]{a+bx^{3/2}}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^(3/2))^(2/3),x]
```

```
[Out] ((-3*a + b*x^(3/2))*(a + b*x^(3/2))^(1/3))/(2*b^2)
```

fricas [A] time = 1.05, size = 23, normalized size = 0.58

$$\frac{\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}\left(bx^{\frac{3}{2}}-3\,a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^(3/2))^(2/3),x, algorithm="fricas")
[Out] 1/2*(b*x^(3/2) + a)^(1/3)*(b*x^(3/2) - 3*a)/b^2
```

giac [A] time = 0.20, size = 30, normalized size = 0.75

$$\frac{\left(bx^{\frac{3}{2}}+a\right)^{\frac{4}{3}}}{2\,b^2}-\frac{2\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}a}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^(3/2))^(2/3),x, algorithm="giac")
[Out] 1/2*(b*x^(3/2) + a)^(4/3)/b^2 - 2*(b*x^(3/2) + a)^(1/3)*a/b^2
```

maple [A] time = 0.00, size = 30, normalized size = 0.75

$$\frac{-2\left(b\,x^{\frac{3}{2}}+a\right)^{\frac{1}{3}}a+\frac{\left(b\,x^{\frac{3}{2}}+a\right)^{\frac{4}{3}}}{2}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^(3/2)+a)^(2/3),x)
[Out] 2/b^2*(1/4*(b*x^(3/2)+a)^(4/3)-(b*x^(3/2)+a)^(1/3)*a)
```

maxima [A] time = 0.90, size = 30, normalized size = 0.75

$$\frac{\left(bx^{\frac{3}{2}}+a\right)^{\frac{4}{3}}}{2\,b^2}-\frac{2\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}a}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^(3/2))^(2/3),x, algorithm="maxima")
[Out] 1/2*(b*x^(3/2) + a)^(4/3)/b^2 - 2*(b*x^(3/2) + a)^(1/3)*a/b^2
```

mupad [B] time = 1.39, size = 29, normalized size = 0.72

$$-\frac{4\,a\left(a+b\,x^{3/2}\right)^{1/3}-\left(a+b\,x^{3/2}\right)^{4/3}}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^(3/2))^(2/3),x)
[Out] -(4*a*(a + b*x^(3/2))^(1/3) - (a + b*x^(3/2))^(4/3))/(2*b^2)
```

sympy [**A**] time = 1.58, size = 49, normalized size = 1.22

$$\begin{cases} -\frac{3a\sqrt[3]{a+bx^{\frac{3}{2}}}}{2b^2} + \frac{x^{\frac{3}{2}}\sqrt[3]{a+bx^{\frac{3}{2}}}}{2b} & \text{for } b \neq 0 \\ \frac{x^3}{3a^{\frac{2}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*x**(3/2))**(2/3),x)
```

```
[Out] Piecewise((-3*a*(a + b*x**(3/2))**(1/3)/(2*b**2) + x**(3/2)*(a + b*x**(3/2)
)**(1/3)/(2*b), Ne(b, 0)), (x**3/(3*a**(2/3)), True))
```

3.2279 $\int \frac{1}{x\left(a+bx^{3/2}\right)^{2/3}}\,dx$

Optimal. Leaf size=85

$$\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^{3/2}}\right)}{a^{2/3}}-\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^{3/2}}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}}-\frac{\log(x)}{2a^{2/3}}$$

```
[Out] -1/2*ln(x)/a^(2/3)+ln(a^(1/3)-(a+b*x^(3/2))^(1/3))/a^(2/3)-2/3*arctan(1/3*(
a^(1/3)+2*(a+b*x^(3/2))^(1/3))/a^(1/3)*3^(1/2))/a^(2/3)*3^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {266, 57, 617, 204, 31}

$$\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^{3/2}}\right)}{a^{2/3}}-\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^{3/2}}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}}-\frac{\log(x)}{2a^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^(3/2))^(2/3)),x]
```

```
[Out] (-2*ArcTan[(a^(1/3) + 2*(a + b*x^(3/2))^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]
*a^(2/3)) - Log[x]/(2*a^(2/3)) + Log[a^(1/3) - (a + b*x^(3/2))^(1/3)]/a^(2/
3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^{3/2}\right)^{2/3}}\,dx = \frac{2}{3}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^{3/2}\right)$$

$$= -\frac{\log(x)}{2a^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^{3/2}}\right)}{a^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx, x, \sqrt[3]{a+bx^{3/2}}\right)}{\sqrt[3]{a}}$$

$$= -\frac{\log(x)}{2a^{2/3}} + \frac{\log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^{3/2}}\right)}{a^{2/3}} + \frac{2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^{3/2}}}{\sqrt[3]{a}}\right)}{a^{2/3}}$$

$$= -\frac{2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^{3/2}}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}} - \frac{\log(x)}{2a^{2/3}} + \frac{\log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^{3/2}}\right)}{a^{2/3}}$$

Mathematica [A] time = 0.09, size = 109, normalized size = 1.28

$$-\frac{\log\left(a^{2/3} + \sqrt[3]{a}\,\sqrt[3]{a+bx^{3/2}} + \left(a+bx^{3/2}\right)^{2/3}\right) - 2\log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^{3/2}}\right) + 2\sqrt{3}\,\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^{3/2}}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{3a^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^(3/2))^(2/3)),x]
```

```
[Out] -1/3*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^(3/2))^(1/3))/a^(1/3))/Sqrt[3]] - 2
*Log[a^(1/3) - (a + b*x^(3/2))^(1/3)] + Log[a^(2/3) + a^(1/3)*(a + b*x^(3/2
))^(1/3) + (a + b*x^(3/2))^(2/3)])/a^(2/3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^(3/2))^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 2.78, size = 87, normalized size = 1.02

$$-\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{2}{3}}} - \frac{\log\left(\left(bx^{\frac{3}{2}}+a\right)^{\frac{2}{3}} + \left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{2}{3}}\right)}{3\,a^{\frac{2}{3}}} + \frac{2\log\left(\left|\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}} - a^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^(3/2))^(2/3),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^(3/2) + a)^(1/3) + a^(1/3))/a^(1/3)
)/a^(2/3) - 1/3*log((b*x^(3/2) + a)^(2/3) + (b*x^(3/2) + a)^(1/3)*a^(1/3) +
 a^(2/3))/a^(2/3) + 2/3*log(abs((b*x^(3/2) + a)^(1/3) - a^(1/3)))/a^(2/3)
```

maple [A] time = 0.01, size = 85, normalized size = 1.00

$$-\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\left(b x^{\frac{3}{2}}+a\right)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{2}{3}}}+\frac{2\ln\left(-a^{\frac{1}{3}}+\left(b x^{\frac{3}{2}}+a\right)^{\frac{1}{3}}\right)}{3a^{\frac{2}{3}}}-\frac{\ln\left(a^{\frac{2}{3}}+\left(b x^{\frac{3}{2}}+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(b x^{\frac{3}{2}}+a\right)^{\frac{2}{3}}\right)}{3a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^(3/2)+a)^(2/3),x)
```

```
[Out] 2/3/a^(2/3)*ln((b*x^(3/2)+a)^(1/3)-a^(1/3))-1/3/a^(2/3)*ln((b*x^(3/2)+a)^(2
/3)+a^(1/3)*(b*x^(3/2)+a)^(1/3)+a^(2/3))-2/3/a^(2/3)*3^(1/2)*arctan(1/3*3^(
1/2)*(2/a^(1/3)*(b*x^(3/2)+a)^(1/3)+1))
```

maxima [A] time = 1.85, size = 86, normalized size = 1.01

$$-\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\left(b x^{\frac{3}{2}}+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{3a^{\frac{2}{3}}}-\frac{\log\left(\left(b x^{\frac{3}{2}}+a\right)^{\frac{2}{3}}+\left(b x^{\frac{3}{2}}+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{3a^{\frac{2}{3}}}+\frac{2\log\left(\left(b x^{\frac{3}{2}}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^(3/2))^(2/3),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^(3/2) + a)^(1/3) + a^(1/3))/a^(1/3)
)/a^(2/3) - 1/3*log((b*x^(3/2) + a)^(2/3) + (b*x^(3/2) + a)^(1/3)*a^(1/3) +
 a^(2/3))/a^(2/3) + 2/3*log((b*x^(3/2) + a)^(1/3) - a^(1/3))/a^(2/3)
```

mupad [B] time = 1.55, size = 102, normalized size = 1.20

$$\frac{2\ln\left(6\left(a+b\,x^{3/2}\right)^{1/3}-6\,a^{1/3}\right)}{3\,a^{2/3}}+\frac{\ln\left(3\,a^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)-6\left(a+b\,x^{3/2}\right)^{1/3}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{3\,a^{2/3}}-\frac{\ln\left(3\,a^{1/3}\left(1+\sqrt{3}\,1\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^(3/2))^(2/3)),x)
```

```
[Out] (2*log(6*(a + b*x^(3/2))^(1/3) - 6*a^(1/3)))/(3*a^(2/3)) + (log(3*a^(1/3)*(
3^(1/2)*1i - 1) - 6*(a + b*x^(3/2))^(1/3))*(3^(1/2)*1i - 1))/(3*a^(2/3)) -
(log(3*a^(1/3)*(3^(1/2)*1i + 1) + 6*(a + b*x^(3/2))^(1/3))*(3^(1/2)*1i + 1)
)/(3*a^(2/3))
```

sympy [C] time = 1.41, size = 41, normalized size = 0.48

$$-\frac{2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{ae^{i\pi}}{bx^{\frac{3}{2}}}\right)}{3b^{\frac{2}{3}}x\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x**(3/2))**(2/3),x)
```

```
[Out] -2*gamma(2/3)*hyper((2/3, 2/3), (5/3,), a*exp_polar(I*pi)/(b*x**(3/2)))/(3*
b**(2/3)*x*gamma(5/3))
```

3.2280 $\int \frac{1}{x^4\left(a+bx^{3/2}\right)^{2/3}}\,dx$

Optimal. Leaf size=148

$$\frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^{3/2}}\right)}{9a^{8/3}}-\frac{10b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^{3/2}}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{8/3}}-\frac{5b^2\log(x)}{18a^{8/3}}+\frac{5b\sqrt[3]{a+bx^{3/2}}}{9a^2x^{3/2}}-\frac{\sqrt[3]{a+bx^{3/2}}}{3ax^3}$$

```
[Out] -1/3*(a+b*x^(3/2))^(1/3)/a/x^3+5/9*b*(a+b*x^(3/2))^(1/3)/a^2/x^(3/2)-5/18*b
^2*ln(x)/a^(8/3)+5/9*b^2*ln(a^(1/3)-(a+b*x^(3/2))^(1/3))/a^(8/3)-10/27*b^2*
arctan(1/3*(a^(1/3)+2*(a+b*x^(3/2))^(1/3))/a^(1/3)*3^(1/2))/a^(8/3)*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {266, 51, 57, 617, 204, 31}

$$\frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^{3/2}}\right)}{9a^{8/3}}-\frac{10b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^{3/2}}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{8/3}}-\frac{5b^2\log(x)}{18a^{8/3}}+\frac{5b\sqrt[3]{a+bx^{3/2}}}{9a^2x^{3/2}}-\frac{\sqrt[3]{a+bx^{3/2}}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^(3/2))^(2/3)),x]
```

```
[Out] -(a + b*x^(3/2))^(1/3)/(3*a*x^3) + (5*b*(a + b*x^(3/2))^(1/3))/(9*a^2*x^(3/
2)) - (10*b^2*ArcTan[(a^(1/3) + 2*(a + b*x^(3/2))^(1/3))/(Sqrt[3]*a^(1/3))]
)/(9*Sqrt[3]*a^(8/3)) - (5*b^2*Log[x])/(18*a^(8/3)) + (5*b^2*Log[a^(1/3) -
(a + b*x^(3/2))^(1/3)])/(9*a^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^{3/2}\right)^{2/3}}\,dx = \frac{2}{3}\,\text{Subst}\left(\int \frac{1}{x^3(a+bx)^{2/3}}\,dx, x, x^{3/2}\right)$$

$$= -\frac{\sqrt[3]{a+bx^{3/2}}}{3ax^3} - \frac{(5b)\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{2/3}}\,dx, x, x^{3/2}\right)}{9a}$$

$$= -\frac{\sqrt[3]{a+bx^{3/2}}}{3ax^3} + \frac{5b\sqrt[3]{a+bx^{3/2}}}{9a^2x^{3/2}} + \frac{\left(10b^2\right)\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^{3/2}\right)}{27a^2}$$

$$= -\frac{\sqrt[3]{a+bx^{3/2}}}{3ax^3} + \frac{5b\sqrt[3]{a+bx^{3/2}}}{9a^2x^{3/2}} - \frac{5b^2\log(x)}{18a^{8/3}} - \frac{\left(5b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^{3/2}}\right)}{9a^{8/3}} -$$

$$= -\frac{\sqrt[3]{a+bx^{3/2}}}{3ax^3} + \frac{5b\sqrt[3]{a+bx^{3/2}}}{9a^2x^{3/2}} - \frac{5b^2\log(x)}{18a^{8/3}} + \frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^{3/2}}\right)}{9a^{8/3}} + \frac{\left(10b^2\right)\text{Sub}}{}$$

$$= -\frac{\sqrt[3]{a+bx^{3/2}}}{3ax^3} + \frac{5b\sqrt[3]{a+bx^{3/2}}}{9a^2x^{3/2}} - \frac{10b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^{3/2}}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{9\sqrt{3}\,a^{8/3}} - \frac{5b^2\log(x)}{18a^{8/3}} + \frac{5b^2\log\left(\sqrt[3]{a}-\right.}{9a^{8}}$$

Mathematica [C] time = 0.01, size = 41, normalized size = 0.28

$$-\frac{2b^2\sqrt[3]{a+bx^{3/2}}\;{}_2F_1\left(\frac{1}{3},3;\frac{4}{3};\frac{bx^{3/2}}{a}+1\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^(3/2))^(2/3)),x]
```

```
[Out] (-2*b^2*(a + b*x^(3/2))^(1/3)*Hypergeometric2F1[1/3, 3, 4/3, 1 + (b*x^(3/2)
)/a])/a^3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*x^(3/2))^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*x^(3/2))^(2/3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage3:=type(sage2):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
i,const vecteur & l) Error: Bad Argument ValueDone
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{\frac{3}{2}}+a\right)^{\frac{2}{3}}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^(3/2)+a)^(2/3),x)
```

```
[Out] int(1/x^4/(b*x^(3/2)+a)^(2/3),x)
```

maxima [A] time = 1.93, size = 158, normalized size = 1.07

$$-\frac{10\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{27\,a^{\frac{8}{3}}}-\frac{5\,b^2\log\left(\left(bx^{\frac{3}{2}}+a\right)^{\frac{2}{3}}+\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{27\,a^{\frac{8}{3}}}+\frac{10\,b^2\log\left(\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}-a\right.}{27\,a^{\frac{8}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*x^(3/2))^(2/3),x, algorithm="maxima")
```

```
[Out] -10/27*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^(3/2) + a)^(1/3) + a^(1/3))/a
^(1/3))/a^(8/3) - 5/27*b^2*log((b*x^(3/2) + a)^(2/3) + (b*x^(3/2) + a)^(1/3
)*a^(1/3) + a^(2/3))/a^(8/3) + 10/27*b^2*log((b*x^(3/2) + a)^(1/3) - a^(1/3
))/a^(8/3) + 1/9*(5*(b*x^(3/2) + a)^(4/3)*b^2 - 8*(b*x^(3/2) + a)^(1/3)*a*b
^2)/((b*x^(3/2) + a)^2*a^2 - 2*(b*x^(3/2) + a)*a^3 + a^4)
```

mupad [B] time = 1.62, size = 205, normalized size = 1.39

$$\frac{10\,b^2\,\ln\left(\left(a+b\,x^{3/2}\right)^{1/3}-a^{1/3}\right)}{27\,a^{8/3}}-\frac{\frac{8\,b^2\left(a+b\,x^{3/2}\right)^{1/3}}{9\,a}-\frac{5\,b^2\left(a+b\,x^{3/2}\right)^{4/3}}{9\,a^2}}{\left(a+b\,x^{3/2}\right)^2-2\,a\,\left(a+b\,x^{3/2}\right)+a^2}+\frac{\ln\left(\frac{-5\,b^2+\sqrt{3}\,b^2\,5\mathrm{i}}{3\,a^{5/3}}-\frac{10\,b^2\left(a+b\,x^{3/2}\right)^{1/3}}{3\,a^2}\right)\left(-5\,b^2\right.}{27\,a^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^(3/2))^(2/3)),x)
```

```
[Out] (10*b^2*log((a + b*x^(3/2))^(1/3) - a^(1/3)))/(27*a^(8/3)) - ((8*b^2*(a + b
*x^(3/2))^(1/3))/(9*a) - (5*b^2*(a + b*x^(3/2))^(4/3))/(9*a^2))/((a + b*x^(
3/2))^2 - 2*a*(a + b*x^(3/2)) + a^2) + (log((3^(1/2)*b^2*5i - 5*b^2)/(3*a^(
5/3)) - (10*b^2*(a + b*x^(3/2))^(1/3))/(3*a^2))*(3^(1/2)*b^2*5i - 5*b^2))/(
27*a^(8/3)) - (log((3^(1/2)*b^2*5i + 5*b^2)/(3*a^(5/3)) + (10*b^2*(a + b*x^
(3/2))^(1/3))/(3*a^2))*(3^(1/2)*b^2*5i + 5*b^2))/(27*a^(8/3))
```

sympy [C] time = 5.30, size = 42, normalized size = 0.28

$$-\frac{2\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{2}{3}, \frac{8}{3} \\ \frac{11}{3}\end{matrix}\right| \frac{ae^{i\pi}}{bx^{\frac{3}{2}}}\right)}{3b^{\frac{2}{3}}x^4\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a+b*x**(3/2))**(2/3),x)
```

```
[Out] -2*gamma(8/3)*hyper((2/3, 8/3), (11/3,), a*exp_polar(I*pi)/(b*x**(3/2)))/(3
*b**(2/3)*x**4*gamma(11/3))
```

3.2281 $\int \frac{x^4}{\left(a+bx^{3/2}\right)^{2/3}}\,dx$

Optimal. Leaf size=42

$$\frac{x^5\sqrt[3]{a+bx^{3/2}}\;{}_2F_1\left(1,\frac{11}{3};\frac{13}{3};-\frac{bx^{3/2}}{a}\right)}{5a}$$

```
[Out] 1/5*x^5*(a+b*x^(3/2))^(1/3)*hypergeom([1, 11/3],[13/3],-b*x^(3/2)/a)/a
```

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.36, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {341, 365, 364}

$$\frac{x^5\left(\frac{bx^{3/2}}{a}+1\right)^{2/3}\;{}_2F_1\left(\frac{2}{3},\frac{10}{3};\frac{13}{3};-\frac{bx^{3/2}}{a}\right)}{5\left(a+bx^{3/2}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^(3/2))^(2/3),x]
```

```
[Out] (x^5*(1 + (b*x^(3/2))/a)^(2/3)*Hypergeometric2F1[2/3, 10/3, 13/3, -((b*x^(3
/2))/a)])/(5*(a + b*x^(3/2))^(2/3))
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^{3/2}\right)^{2/3}}\,dx = 2\,\text{Subst}\left(\int \frac{x^9}{\left(a+bx^3\right)^{2/3}}\,dx, x, \sqrt{x}\right)$$
$$= \frac{\left(2\left(1+\frac{bx^{3/2}}{a}\right)^{2/3}\right)\text{Subst}\left(\int \frac{x^9}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx, x, \sqrt{x}\right)}{\left(a+bx^{3/2}\right)^{2/3}}$$
$$= \frac{x^5\left(1+\frac{bx^{3/2}}{a}\right)^{2/3}\,{}_2F_1\left(\frac{2}{3}, \frac{10}{3}; \frac{13}{3}; -\frac{bx^{3/2}}{a}\right)}{5\left(a+bx^{3/2}\right)^{2/3}}$$

Mathematica [A] time = 0.01, size = 57, normalized size = 1.36

$$\frac{x^5\left(\frac{bx^{3/2}}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{2}{3}, \frac{10}{3}; \frac{13}{3}; -\frac{bx^{3/2}}{a}\right)}{5\left(a+bx^{3/2}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^(3/2))^(2/3),x]
```

```
[Out] (x^5*(1 + (b*x^(3/2))/a)^(2/3)*Hypergeometric2F1[2/3, 10/3, 13/3, -((b*x^(3
/2))/a)])/(5*(a + b*x^(3/2))^(2/3))
```

fricas [F] time = 2.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^{\frac{11}{2}} - ax^4\right)\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}}{b^2x^3 - a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b*x^(3/2))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^(11/2) - a*x^4)*(b*x^(3/2) + a)^(1/3)/(b^2*x^3 - a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^{\frac{3}{2}}+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b*x^(3/2))^(2/3),x, algorithm="giac")
```

```
[Out] integrate(x^4/(b*x^(3/2) + a)^(2/3), x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(b\,x^{\frac{3}{2}}+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^(3/2)+a)^(2/3),x)
```

```
[Out] int(x^4/(b*x^(3/2)+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^{\frac{3}{2}}+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b*x^(3/2))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(b*x^(3/2) + a)^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{\left(a+b\,x^{3/2}\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^(3/2))^(2/3),x)
```

```
[Out] int(x^4/(a + b*x^(3/2))^(2/3), x)
```

sympy [C] time = 2.80, size = 41, normalized size = 0.98

$$\frac{2x^5\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{2}{3},\frac{10}{3}\\ \frac{13}{3}\end{matrix}\right|\frac{bx^{\frac{3}{2}}e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{13}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a+b*x**(3/2))**(2/3),x)
```

```
[Out] 2*x**5*gamma(10/3)*hyper((2/3, 10/3), (13/3,), b*x**(3/2)*exp_polar(I*pi)/a
)/(3*a**(2/3)*gamma(13/3))
```

3.2282 $\int \frac{x}{\left(a+bx^{3/2}\right)^{2/3}}\,dx$

Optimal. Leaf size=42

$$\frac{x^2\sqrt[3]{a+bx^{3/2}}\;{}_2F_1\left(1,\frac{5}{3};\frac{7}{3};-\frac{bx^{3/2}}{a}\right)}{2a}$$

```
[Out] 1/2*x^2*(a+b*x^(3/2))^(1/3)*hypergeom([1, 5/3],[7/3],-b*x^(3/2)/a)/a
```

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.36, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {341, 365, 364}

$$\frac{x^2\left(\frac{bx^{3/2}}{a}+1\right)^{2/3}\;{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{7}{3};-\frac{bx^{3/2}}{a}\right)}{2\left(a+bx^{3/2}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^(3/2))^(2/3),x]
```

```
[Out] (x^2*(1 + (b*x^(3/2))/a)^(2/3)*Hypergeometric2F1[2/3, 4/3, 7/3, -((b*x^(3/2
))/a)])/(2*(a + b*x^(3/2))^(2/3))
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x}{\left(a+bx^{3/2}\right)^{2/3}}\,dx = 2\,\text{Subst}\left(\int \frac{x^3}{\left(a+bx^3\right)^{2/3}}\,dx, x, \sqrt{x}\right)$$
$$= \frac{\left(2\left(1+\frac{bx^{3/2}}{a}\right)^{2/3}\right)\text{Subst}\left(\int \frac{x^3}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx, x, \sqrt{x}\right)}{\left(a+bx^{3/2}\right)^{2/3}}$$
$$= \frac{x^2\left(1+\frac{bx^{3/2}}{a}\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{7}{3};-\frac{bx^{3/2}}{a}\right)}{2\left(a+bx^{3/2}\right)^{2/3}}$$

Mathematica [A] time = 0.01, size = 57, normalized size = 1.36

$$\frac{x^2\left(\frac{bx^{3/2}}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{7}{3};-\frac{bx^{3/2}}{a}\right)}{2\left(a+bx^{3/2}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^(3/2))^(2/3),x]
```

```
[Out] (x^2*(1 + (b*x^(3/2))/a)^(2/3)*Hypergeometric2F1[2/3, 4/3, 7/3, -((b*x^(3/2
))/a)])/(2*(a + b*x^(3/2))^(2/3))
```

fricas [F] time = 3.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^{\frac{5}{2}}-ax\right)\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}}{b^2x^3-a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^(3/2))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^(5/2) - a*x)*(b*x^(3/2) + a)^(1/3)/(b^2*x^3 - a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^{\frac{3}{2}}+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^(3/2))^(2/3),x, algorithm="giac")
```

```
[Out] integrate(x/(b*x^(3/2) + a)^(2/3), x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(b\,x^{\frac{3}{2}}+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^(3/2)+a)^(2/3),x)
```

```
[Out] int(x/(b*x^(3/2)+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^{\frac{3}{2}}+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^(3/2))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*x^(3/2) + a)^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\left(a+b\,x^{3/2}\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^(3/2))^(2/3),x)
```

```
[Out] int(x/(a + b*x^(3/2))^(2/3), x)
```

sympy [C] time = 0.88, size = 41, normalized size = 0.98

$$\frac{2x^2\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^{\frac{3}{2}}e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x**(3/2))**(2/3),x)
```

```
[Out] 2*x**2*gamma(4/3)*hyper((2/3, 4/3), (7/3,), b*x**(3/2)*exp_polar(I*pi)/a)/(
3*a**(2/3)*gamma(7/3))
```

3.2283 $\int \frac{1}{x^2\left(a+bx^{3/2}\right)^{2/3}}\,dx$

Optimal. Leaf size=40

$$-\frac{\sqrt[3]{a+bx^{3/2}}\ {}_2F_1\left(-\frac{1}{3},1;\frac{1}{3};-\frac{bx^{3/2}}{a}\right)}{ax}$$

```
[Out] -(a+b*x^(3/2))^(1/3)*hypergeom([-1/3, 1],[1/3],-b*x^(3/2)/a)/a/x
```

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.38, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {341, 365, 364}

$$-\frac{\left(\frac{bx^{3/2}}{a}+1\right)^{2/3}\ {}_2F_1\left(-\frac{2}{3},\frac{2}{3};\frac{1}{3};-\frac{bx^{3/2}}{a}\right)}{x\left(a+bx^{3/2}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^(3/2))^(2/3)),x]
```

```
[Out] -(((1 + (b*x^(3/2))/a)^(2/3)*Hypergeometric2F1[-2/3, 2/3, 1/3, -((b*x^(3/2)
)/a)])/(x*(a + b*x^(3/2))^(2/3)))
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^{3/2}\right)^{2/3}}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^3\left(a+bx^3\right)^{2/3}}\,dx, x, \sqrt{x}\right)$$
$$= \frac{\left(2\left(1+\frac{bx^{3/2}}{a}\right)^{2/3}\right)\text{Subst}\left(\int \frac{1}{x^3\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx, x, \sqrt{x}\right)}{\left(a+bx^{3/2}\right)^{2/3}}$$
$$= -\frac{\left(1+\frac{bx^{3/2}}{a}\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},\frac{2}{3};\frac{1}{3};-\frac{bx^{3/2}}{a}\right)}{x\left(a+bx^{3/2}\right)^{2/3}}$$

Mathematica [A] time = 0.01, size = 55, normalized size = 1.38

$$-\frac{\left(\frac{bx^{3/2}}{a}+1\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},\frac{2}{3};\frac{1}{3};-\frac{bx^{3/2}}{a}\right)}{x\left(a+bx^{3/2}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^(3/2))^(2/3)),x]
```

```
[Out] -(((1 + (b*x^(3/2))/a)^(2/3)*Hypergeometric2F1[-2/3, 2/3, 1/3, -((b*x^(3/2)
)/a)])/(x*(a + b*x^(3/2))^(2/3)))
```

fricas [F] time = 2.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}\left(bx^{\frac{3}{2}}-a\right)}{b^2x^5-a^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^(3/2))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^(3/2) + a)^(1/3)*(b*x^(3/2) - a)/(b^2*x^5 - a^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{\frac{3}{2}}+a\right)^{\frac{2}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^(3/2))^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^(3/2) + a)^(2/3)*x^2), x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{\frac{3}{2}}+a\right)^{\frac{2}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^(3/2)+a)^(2/3),x)
```

```
[Out] int(1/x^2/(b*x^(3/2)+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{\frac{3}{2}}+a\right)^{\frac{2}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^(3/2))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^(3/2) + a)^(2/3)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2\left(a+b\,x^{3/2}\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^(3/2))^(2/3)),x)
```

```
[Out] int(1/(x^2*(a + b*x^(3/2))^(2/3)), x)
```

sympy [C] time = 1.73, size = 42, normalized size = 1.05

$$\frac{2\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{2}{3}\\ \frac{1}{3}\end{matrix}\right|\frac{bx^{\frac{3}{2}}e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}x\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*x**(3/2))**(2/3),x)
```

```
[Out] 2*gamma(-2/3)*hyper((-2/3, 2/3), (1/3,), b*x**(3/2)*exp_polar(I*pi)/a)/(3*a
**(2/3)*x*gamma(1/3))
```

3.2284 $\int \frac{1}{x^5\left(a+bx^{3/2}\right)^{2/3}}\,dx$

Optimal. Leaf size=57

$$-\frac{\left(\frac{bx^{3/2}}{a}+1\right)^{2/3}\,{}_2F_1\left(-\frac{8}{3},\frac{2}{3};-\frac{5}{3};-\frac{bx^{3/2}}{a}\right)}{4x^4\left(a+bx^{3/2}\right)^{2/3}}$$

```
[Out] -1/4*(1+b*x^(3/2)/a)^(2/3)*hypergeom([-8/3, 2/3],[-5/3],-b*x^(3/2)/a)/x^4/(
a+b*x^(3/2))^(2/3)
```

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {341, 365, 364}

$$-\frac{\left(\frac{bx^{3/2}}{a}+1\right)^{2/3}\,{}_2F_1\left(-\frac{8}{3},\frac{2}{3};-\frac{5}{3};-\frac{bx^{3/2}}{a}\right)}{4x^4\left(a+bx^{3/2}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^(3/2))^(2/3)),x]
```

```
[Out] -((1 + (b*x^(3/2))/a)^(2/3)*Hypergeometric2F1[-8/3, 2/3, -5/3, -((b*x^(3/2)
)/a)])/(4*x^4*(a + b*x^(3/2))^(2/3))
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^5\left(a+bx^{3/2}\right)^{2/3}}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^9\left(a+bx^3\right)^{2/3}}\,dx, x, \sqrt{x}\right)$$
$$= \frac{\left(2\left(1+\frac{bx^{3/2}}{a}\right)^{2/3}\right)\text{Subst}\left(\int \frac{1}{x^9\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx, x, \sqrt{x}\right)}{\left(a+bx^{3/2}\right)^{2/3}}$$
$$= -\frac{\left(1+\frac{bx^{3/2}}{a}\right)^{2/3}\,{}_2F_1\left(-\frac{8}{3},\frac{2}{3};-\frac{5}{3};-\frac{bx^{3/2}}{a}\right)}{4x^4\left(a+bx^{3/2}\right)^{2/3}}$$

Mathematica [A] time = 0.01, size = 57, normalized size = 1.00

$$-\frac{\left(\frac{bx^{3/2}}{a}+1\right)^{2/3}\,{}_2F_1\left(-\frac{8}{3},\frac{2}{3};-\frac{5}{3};-\frac{bx^{3/2}}{a}\right)}{4x^4\left(a+bx^{3/2}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^(3/2))^(2/3)),x]
```

```
[Out] -1/4*((1 + (b*x^(3/2))/a)^(2/3)*Hypergeometric2F1[-8/3, 2/3, -5/3, -((b*x^(
3/2))/a)])/(x^4*(a + b*x^(3/2))^(2/3))
```

fricas [F] time = 2.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}\left(bx^{\frac{3}{2}}-a\right)}{b^2x^8-a^2x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a+b*x^(3/2))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^(3/2) + a)^(1/3)*(b*x^(3/2) - a)/(b^2*x^8 - a^2*x^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{\frac{3}{2}}+a\right)^{\frac{2}{3}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a+b*x^(3/2))^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^(3/2) + a)^(2/3)*x^5), x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{\frac{3}{2}}+a\right)^{\frac{2}{3}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^(3/2)+a)^(2/3),x)
```

```
[Out] int(1/x^5/(b*x^(3/2)+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{\frac{3}{2}}+a\right)^{\frac{2}{3}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a+b*x^(3/2))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^(3/2) + a)^(2/3)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^5\left(a+b\,x^{3/2}\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^(3/2))^(2/3)),x)
```

```
[Out] int(1/(x^5*(a + b*x^(3/2))^(2/3)), x)
```

sympy [C] time = 8.19, size = 48, normalized size = 0.84

$$\frac{2\Gamma\left(-\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{8}{3}, \frac{2}{3} \\ -\frac{5}{3}\end{matrix}\middle| \frac{bx^{\frac{3}{2}}e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}x^4\Gamma\left(-\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(a+b*x**(3/2))**(2/3),x)
```

```
[Out] 2*gamma(-8/3)*hyper((-8/3, 2/3), (-5/3,), b*x**(3/2)*exp_polar(I*pi)/a)/(3*
a**(2/3)*x**4*gamma(-5/3))
```

3.2285 $\int \left(a + b\sqrt[3]{x}\right) x^4\, dx$

Optimal. Leaf size=19

$$\frac{ax^5}{5} + \frac{3}{16}bx^{16/3}$$

```
[Out] 1/5*a*x^5+3/16*b*x^(16/3)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$\frac{ax^5}{5} + \frac{3}{16}bx^{16/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))*x^4,x]
```

```
[Out] (a*x^5)/5 + (3*b*x^(16/3))/16
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \left(a + b\sqrt[3]{x}\right) x^4\, dx &= \int \left(ax^4 + bx^{13/3}\right) dx \\ &= \frac{ax^5}{5} + \frac{3}{16}bx^{16/3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{ax^5}{5} + \frac{3}{16}bx^{16/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))*x^4,x]
```

```
[Out] (a*x^5)/5 + (3*b*x^(16/3))/16
```

fricas [A] time = 0.49, size = 13, normalized size = 0.68

$$\frac{3}{16}\, bx^{\frac{16}{3}} + \frac{1}{5}\, ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))*x^4,x, algorithm="fricas")
```

```
[Out] 3/16*b*x^(16/3) + 1/5*a*x^5
```

giac [A] time = 0.15, size = 13, normalized size = 0.68

$$\frac{3}{16}\, bx^{\frac{16}{3}} + \frac{1}{5}\, ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))*x^4,x, algorithm="giac")
```

```
[Out] 3/16*b*x^(16/3) + 1/5*a*x^5
```

maple [A] time = 0.00, size = 14, normalized size = 0.74

$$\frac{3b\,x^{\frac{16}{3}}}{16}+\frac{a\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))*x^4,x)
```

```
[Out] 1/5*a*x^5+3/16*b*x^(16/3)
```

maxima [B] time = 0.82, size = 251, normalized size = 13.21

$$\frac{3\left(bx^{\frac{1}{3}}+a\right)^{16}}{16\,b^{15}}-\frac{14\left(bx^{\frac{1}{3}}+a\right)^{15}a}{5\,b^{15}}+\frac{39\left(bx^{\frac{1}{3}}+a\right)^{14}a^2}{2\,b^{15}}-\frac{84\left(bx^{\frac{1}{3}}+a\right)^{13}a^3}{b^{15}}+\frac{1001\left(bx^{\frac{1}{3}}+a\right)^{12}a^4}{4\,b^{15}}-\frac{546\left(bx^{\frac{1}{3}}+a\right)^{11}a^5}{b^{15}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))*x^4,x, algorithm="maxima")
```

```
[Out] 3/16*(b*x^(1/3) + a)^16/b^15 - 14/5*(b*x^(1/3) + a)^15*a/b^15 + 39/2*(b*x^(
1/3) + a)^14*a^2/b^15 - 84*(b*x^(1/3) + a)^13*a^3/b^15 + 1001/4*(b*x^(1/3)
+ a)^12*a^4/b^15 - 546*(b*x^(1/3) + a)^11*a^5/b^15 + 9009/10*(b*x^(1/3) + a
)^10*a^6/b^15 - 1144*(b*x^(1/3) + a)^9*a^7/b^15 + 9009/8*(b*x^(1/3) + a)^8*
a^8/b^15 - 858*(b*x^(1/3) + a)^7*a^9/b^15 + 1001/2*(b*x^(1/3) + a)^6*a^10/b
^15 - 1092/5*(b*x^(1/3) + a)^5*a^11/b^15 + 273/4*(b*x^(1/3) + a)^4*a^12/b^1
5 - 14*(b*x^(1/3) + a)^3*a^13/b^15 + 3/2*(b*x^(1/3) + a)^2*a^14/b^15
```

mupad [B] time = 0.03, size = 13, normalized size = 0.68

$$\frac{a\,x^5}{5}+\frac{3\,b\,x^{16/3}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^(1/3)),x)
```

```
[Out] (a*x^5)/5 + (3*b*x^(16/3))/16
```

sympy [A] time = 1.75, size = 15, normalized size = 0.79

$$\frac{ax^5}{5}+\frac{3bx^{\frac{16}{3}}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))*x**4,x)
```

```
[Out] a*x**5/5 + 3*b*x**(16/3)/16
```

3.2286 $\int \left(a + b\sqrt[3]{x}\right) x^3\, dx$

Optimal. Leaf size=19

$$\frac{ax^4}{4} + \frac{3}{13}bx^{13/3}$$

```
[Out] 1/4*a*x^4+3/13*b*x^(13/3)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$\frac{ax^4}{4} + \frac{3}{13}bx^{13/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))*x^3,x]
```

```
[Out] (a*x^4)/4 + (3*b*x^(13/3))/13
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \left(a + b\sqrt[3]{x}\right) x^3\, dx &= \int \left(ax^3 + bx^{10/3}\right) dx \\ &= \frac{ax^4}{4} + \frac{3}{13}bx^{13/3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{ax^4}{4} + \frac{3}{13}bx^{13/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))*x^3,x]
```

```
[Out] (a*x^4)/4 + (3*b*x^(13/3))/13
```

fricas [A] time = 0.67, size = 13, normalized size = 0.68

$$\frac{3}{13}\, bx^{\frac{13}{3}} + \frac{1}{4}\, ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))*x^3,x, algorithm="fricas")
```

```
[Out] 3/13*b*x^(13/3) + 1/4*a*x^4
```

giac [A] time = 0.15, size = 13, normalized size = 0.68

$$\frac{3}{13}\, bx^{\frac{13}{3}} + \frac{1}{4}\, ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))*x^3,x, algorithm="giac")
```

```
[Out] 3/13*b*x^(13/3) + 1/4*a*x^4
```

maple [A] time = 0.00, size = 14, normalized size = 0.74

$$\frac{3b\,x^{\frac{13}{3}}}{13}+\frac{a\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))*x^3,x)
```

```
[Out] 1/4*a*x^4+3/13*b*x^(13/3)
```

maxima [B] time = 0.94, size = 200, normalized size = 10.53

$$\frac{3\left(bx^{\frac{1}{3}}+a\right)^{13}}{13\,b^{12}}-\frac{11\left(bx^{\frac{1}{3}}+a\right)^{12}a}{4\,b^{12}}+\frac{15\left(bx^{\frac{1}{3}}+a\right)^{11}a^2}{b^{12}}-\frac{99\left(bx^{\frac{1}{3}}+a\right)^{10}a^3}{2\,b^{12}}+\frac{110\left(bx^{\frac{1}{3}}+a\right)^{9}a^4}{b^{12}}-\frac{693\left(bx^{\frac{1}{3}}+a\right)^{8}a^5}{4\,b^{12}}+\frac{198}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))*x^3,x, algorithm="maxima")
```

```
[Out] 3/13*(b*x^(1/3) + a)^13/b^12 - 11/4*(b*x^(1/3) + a)^12*a/b^12 + 15*(b*x^(1/
3) + a)^11*a^2/b^12 - 99/2*(b*x^(1/3) + a)^10*a^3/b^12 + 110*(b*x^(1/3) + a
)^9*a^4/b^12 - 693/4*(b*x^(1/3) + a)^8*a^5/b^12 + 198*(b*x^(1/3) + a)^7*a^6
/b^12 - 165*(b*x^(1/3) + a)^6*a^7/b^12 + 99*(b*x^(1/3) + a)^5*a^8/b^12 - 16
5/4*(b*x^(1/3) + a)^4*a^9/b^12 + 11*(b*x^(1/3) + a)^3*a^10/b^12 - 3/2*(b*x^
(1/3) + a)^2*a^11/b^12
```

mupad [B] time = 0.03, size = 13, normalized size = 0.68

$$\frac{a\,x^4}{4}+\frac{3\,b\,x^{13/3}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^(1/3)),x)
```

```
[Out] (a*x^4)/4 + (3*b*x^(13/3))/13
```

sympy [A] time = 1.52, size = 15, normalized size = 0.79

$$\frac{ax^4}{4}+\frac{3bx^{\frac{13}{3}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))*x**3,x)
```

```
[Out] a*x**4/4 + 3*b*x**(13/3)/13
```

3.2287 $\int \left(a + b\sqrt[3]{x}\right) x^2\, dx$

Optimal. Leaf size=19

$$\frac{ax^3}{3} + \frac{3}{10}bx^{10/3}$$

```
[Out] 1/3*a*x^3+3/10*b*x^(10/3)
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$\frac{ax^3}{3} + \frac{3}{10}bx^{10/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))*x^2,x]
```

```
[Out] (a*x^3)/3 + (3*b*x^(10/3))/10
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \left(a + b\sqrt[3]{x}\right) x^2\, dx &= \int \left(ax^2 + bx^{7/3}\right) dx \\ &= \frac{ax^3}{3} + \frac{3}{10}bx^{10/3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{ax^3}{3} + \frac{3}{10}bx^{10/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))*x^2,x]
```

```
[Out] (a*x^3)/3 + (3*b*x^(10/3))/10
```

fricas [A] time = 0.52, size = 13, normalized size = 0.68

$$\frac{3}{10}\, bx^{\frac{10}{3}} + \frac{1}{3}\, ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))*x^2,x, algorithm="fricas")
```

```
[Out] 3/10*b*x^(10/3) + 1/3*a*x^3
```

giac [A] time = 0.15, size = 13, normalized size = 0.68

$$\frac{3}{10}\, bx^{\frac{10}{3}} + \frac{1}{3}\, ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))*x^2,x, algorithm="giac")
```

```
[Out] 3/10*b*x^(10/3) + 1/3*a*x^3
```

maple [A] time = 0.00, size = 14, normalized size = 0.74

$$\frac{3b\,x^{\frac{10}{3}}}{10}+\frac{a\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))*x^2,x)
```

```
[Out] 1/3*a*x^3+3/10*b*x^(10/3)
```

maxima [B] time = 0.90, size = 149, normalized size = 7.84

$$\frac{3\left(bx^{\frac{1}{3}}+a\right)^{10}}{10\,b^9}-\frac{8\left(bx^{\frac{1}{3}}+a\right)^{9}a}{3\,b^9}+\frac{21\left(bx^{\frac{1}{3}}+a\right)^{8}a^2}{2\,b^9}-\frac{24\left(bx^{\frac{1}{3}}+a\right)^{7}a^3}{b^9}+\frac{35\left(bx^{\frac{1}{3}}+a\right)^{6}a^4}{b^9}-\frac{168\left(bx^{\frac{1}{3}}+a\right)^{5}a^5}{5\,b^9}+\frac{21\left(bx^{\frac{1}{3}}\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))*x^2,x, algorithm="maxima")
```

```
[Out] 3/10*(b*x^(1/3) + a)^10/b^9 - 8/3*(b*x^(1/3) + a)^9*a/b^9 + 21/2*(b*x^(1/3)
 + a)^8*a^2/b^9 - 24*(b*x^(1/3) + a)^7*a^3/b^9 + 35*(b*x^(1/3) + a)^6*a^4/b
^9 - 168/5*(b*x^(1/3) + a)^5*a^5/b^9 + 21*(b*x^(1/3) + a)^4*a^6/b^9 - 8*(b*
x^(1/3) + a)^3*a^7/b^9 + 3/2*(b*x^(1/3) + a)^2*a^8/b^9
```

mupad [B] time = 0.02, size = 13, normalized size = 0.68

$$\frac{a\,x^3}{3}+\frac{3\,b\,x^{10/3}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^(1/3)),x)
```

```
[Out] (a*x^3)/3 + (3*b*x^(10/3))/10
```

sympy [A] time = 1.35, size = 15, normalized size = 0.79

$$\frac{ax^3}{3}+\frac{3bx^{\frac{10}{3}}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))*x**2,x)
```

```
[Out] a*x**3/3 + 3*b*x**(10/3)/10
```

3.2288 $\int \left(a + b\sqrt[3]{x}\right) x\, dx$

Optimal. Leaf size=19

$$\frac{ax^2}{2} + \frac{3}{7}bx^{7/3}$$

```
[Out] 1/2*a*x^2+3/7*b*x^(7/3)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$\frac{ax^2}{2} + \frac{3}{7}bx^{7/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))*x,x]
```

```
[Out] (a*x^2)/2 + (3*b*x^(7/3))/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \left(a + b\sqrt[3]{x}\right) x\, dx &= \int \left(ax + bx^{4/3}\right) dx \\ &= \frac{ax^2}{2} + \frac{3}{7}bx^{7/3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{ax^2}{2} + \frac{3}{7}bx^{7/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))*x,x]
```

```
[Out] (a*x^2)/2 + (3*b*x^(7/3))/7
```

fricas [A] time = 0.70, size = 13, normalized size = 0.68

$$\frac{3}{7}\, bx^{\frac{7}{3}} + \frac{1}{2}\, ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))*x,x, algorithm="fricas")
```

```
[Out] 3/7*b*x^(7/3) + 1/2*a*x^2
```

giac [A] time = 0.17, size = 13, normalized size = 0.68

$$\frac{3}{7}\, bx^{\frac{7}{3}} + \frac{1}{2}\, ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))*x,x, algorithm="giac")
```

```
[Out] 3/7*b*x^(7/3) + 1/2*a*x^2
```

maple [A] time = 0.00, size = 14, normalized size = 0.74

$$\frac{3b\,x^{\frac{7}{3}}}{7}+\frac{a\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))*x,x)
```

```
[Out] 1/2*a*x^2+3/7*b*x^(7/3)
```

maxima [B] time = 0.82, size = 98, normalized size = 5.16

$$\frac{3\left(bx^{\frac{1}{3}}+a\right)^7}{7\,b^6}-\frac{5\left(bx^{\frac{1}{3}}+a\right)^6 a}{2\,b^6}+\frac{6\left(bx^{\frac{1}{3}}+a\right)^5 a^2}{b^6}-\frac{15\left(bx^{\frac{1}{3}}+a\right)^4 a^3}{2\,b^6}+\frac{5\left(bx^{\frac{1}{3}}+a\right)^3 a^4}{b^6}-\frac{3\left(bx^{\frac{1}{3}}+a\right)^2 a^5}{2\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))*x,x, algorithm="maxima")
```

```
[Out] 3/7*(b*x^(1/3) + a)^7/b^6 - 5/2*(b*x^(1/3) + a)^6*a/b^6 + 6*(b*x^(1/3) + a)
^5*a^2/b^6 - 15/2*(b*x^(1/3) + a)^4*a^3/b^6 + 5*(b*x^(1/3) + a)^3*a^4/b^6 -
 3/2*(b*x^(1/3) + a)^2*a^5/b^6
```

mupad [B] time = 0.03, size = 13, normalized size = 0.68

$$\frac{a\,x^2}{2}+\frac{3\,b\,x^{7/3}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^(1/3)),x)
```

```
[Out] (a*x^2)/2 + (3*b*x^(7/3))/7
```

sympy [A] time = 1.24, size = 15, normalized size = 0.79

$$\frac{ax^2}{2}+\frac{3bx^{\frac{7}{3}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))*x,x)
```

```
[Out] a*x**2/2 + 3*b*x**(7/3)/7
```

3.2289 $\int \left(a + b\sqrt[3]{x}\right)\, dx$

Optimal. Leaf size=14

$$ax + \frac{3}{4}bx^{4/3}$$

```
[Out] a*x+3/4*b*x^(4/3)
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$ax + \frac{3}{4}bx^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*x^(1/3),x]
```

```
[Out] a*x + (3*b*x^(4/3))/4
```

Rubi steps

$$\int \left(a + b\sqrt[3]{x}\right)\, dx = ax + \frac{3}{4}bx^{4/3}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$ax + \frac{3}{4}bx^{4/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*x^(1/3),x]
```

```
[Out] a*x + (3*b*x^(4/3))/4
```

fricas [A] time = 0.62, size = 10, normalized size = 0.71

$$\frac{3}{4}\, bx^{\frac{4}{3}} + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*x^(1/3),x, algorithm="fricas")
```

```
[Out] 3/4*b*x^(4/3) + a*x
```

giac [A] time = 0.16, size = 10, normalized size = 0.71

$$\frac{3}{4}\, bx^{\frac{4}{3}} + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*x^(1/3),x, algorithm="giac")
```

```
[Out] 3/4*b*x^(4/3) + a*x
```

maple [A] time = 0.00, size = 11, normalized size = 0.79

$$\frac{3b\, x^{\frac{4}{3}}}{4} + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*x^(1/3),x)
[Out] a*x+3/4*b*x^(4/3)
```

maxima [**A**] time = 0.87, size = 10, normalized size = 0.71

$$\frac{3}{4} b x^{\frac{4}{3}} + a x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*x^(1/3),x, algorithm="maxima")
[Out] 3/4*b*x^(4/3) + a*x
```

mupad [**B**] time = 0.03, size = 10, normalized size = 0.71

$$a\,x + \frac{3\,b\,x^{4/3}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*x^(1/3),x)
[Out] a*x + (3*b*x^(4/3))/4
```

sympy [**A**] time = 0.06, size = 12, normalized size = 0.86

$$ax + \frac{3bx^{\frac{4}{3}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*x**(1/3),x)
[Out] a*x + 3*b*x**(4/3)/4
```

3.2290 $\int \frac{a+b\sqrt[3]{x}}{x}\,dx$

Optimal. Leaf size=13

$$a\log(x)+3b\sqrt[3]{x}$$

```
[Out] 3*b*x^(1/3)+a*ln(x)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$a\log(x)+3b\sqrt[3]{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))/x,x]
```

```
[Out] 3*b*x^(1/3) + a*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sqrt[3]{x}}{x}\,dx &= \int \left(\frac{a}{x}+\frac{b}{x^{2/3}}\right)dx\\ &= 3b\sqrt[3]{x}+a\log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$a\log(x)+3b\sqrt[3]{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))/x,x]
```

```
[Out] 3*b*x^(1/3) + a*Log[x]
```

fricas [A] time = 0.68, size = 14, normalized size = 1.08

$$3\,a\log\left(x^{\frac{1}{3}}\right)+3\,bx^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))/x,x, algorithm="fricas")
```

```
[Out] 3*a*log(x^(1/3)) + 3*b*x^(1/3)
```

giac [A] time = 0.15, size = 12, normalized size = 0.92

$$a\log\left(|x|\right)+3\,bx^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))/x,x, algorithm="giac")
[Out] a*log(abs(x)) + 3*b*x^(1/3)
```

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$a\ln(x) + 3b\,x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))/x,x)
[Out] 3*b*x^(1/3)+a*ln(x)
```

maxima [A] time = 0.90, size = 11, normalized size = 0.85

$$a\log(x) + 3\,bx^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))/x,x, algorithm="maxima")
[Out] a*log(x) + 3*b*x^(1/3)
```

mupad [B] time = 1.10, size = 11, normalized size = 0.85

$$3\,b\,x^{1/3} + a\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))/x,x)
[Out] 3*b*x^(1/3) + a*log(x)
```

sympy [A] time = 0.27, size = 12, normalized size = 0.92

$$a\log(x) + 3b\sqrt[3]{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))/x,x)
[Out] a*log(x) + 3*b*x**(1/3)
```

3.2291 $\int \frac{a+b\sqrt[3]{x}}{x^2}\,dx$

Optimal. Leaf size=17

$$-\frac{a}{x}-\frac{3b}{2x^{2/3}}$$

```
[Out] -a/x-3/2*b/x^(2/3)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$-\frac{a}{x}-\frac{3b}{2x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))/x^2,x]
```

```
[Out] -(a/x) - (3*b)/(2*x^(2/3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sqrt[3]{x}}{x^2}\,dx &= \int \left(\frac{a}{x^2}+\frac{b}{x^{5/3}}\right)dx\\ &= -\frac{a}{x}-\frac{3b}{2x^{2/3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$-\frac{a}{x}-\frac{3b}{2x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))/x^2,x]
```

```
[Out] -(a/x) - (3*b)/(2*x^(2/3))
```

fricas [A] time = 0.62, size = 15, normalized size = 0.88

$$-\frac{3\,bx^{\frac{1}{3}}+2\,a}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(3*b*x^(1/3) + 2*a)/x
```

giac [A] time = 0.19, size = 15, normalized size = 0.88

$$-\frac{3\,bx^{\frac{1}{3}}+2\,a}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))/x^2,x, algorithm="giac")
[Out] -1/2*(3*b*x^(1/3) + 2*a)/x
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$-\frac{3b}{2x^{\frac{2}{3}}}-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))/x^2,x)
[Out] -a/x-3/2*b/x^(2/3)
```

maxima [A] time = 0.91, size = 15, normalized size = 0.88

$$-\frac{3\,bx^{\frac{1}{3}}+2\,a}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))/x^2,x, algorithm="maxima")
[Out] -1/2*(3*b*x^(1/3) + 2*a)/x
```

mupad [B] time = 0.03, size = 13, normalized size = 0.76

$$-\frac{a}{x}-\frac{3\,b}{2\,x^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))/x^2,x)
[Out] - a/x - (3*b)/(2*x^(2/3))
```

sympy [A] time = 0.73, size = 14, normalized size = 0.82

$$-\frac{a}{x}-\frac{3b}{2x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))/x**2,x)
[Out] -a/x - 3*b/(2*x**(2/3))
```

3.2292 $\int \frac{a+b\sqrt[3]{x}}{x^3}\,dx$

Optimal. Leaf size=19

$$-\frac{a}{2x^2}-\frac{3b}{5x^{5/3}}$$

```
[Out] -1/2*a/x^2-3/5*b/x^(5/3)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$-\frac{a}{2x^2}-\frac{3b}{5x^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))/x^3,x]
```

```
[Out] -a/(2*x^2) - (3*b)/(5*x^(5/3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sqrt[3]{x}}{x^3}\,dx &= \int \left(\frac{a}{x^3}+\frac{b}{x^{8/3}}\right)dx\\ &= -\frac{a}{2x^2}-\frac{3b}{5x^{5/3}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 19, normalized size = 1.00

$$-\frac{a}{2x^2}-\frac{3b}{5x^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))/x^3,x]
```

```
[Out] -1/2*a/x^2 - (3*b)/(5*x^(5/3))
```

fricas [A] time = 0.73, size = 15, normalized size = 0.79

$$-\frac{6\,bx^{\frac{1}{3}}+5\,a}{10\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))/x^3,x, algorithm="fricas")
```

```
[Out] -1/10*(6*b*x^(1/3) + 5*a)/x^2
```

giac [A] time = 0.19, size = 15, normalized size = 0.79

$$-\frac{6\,bx^{\frac{1}{3}}+5\,a}{10\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))/x^3,x, algorithm="giac")
[Out] -1/10*(6*b*x^(1/3) + 5*a)/x^2
```

maple [A] time = 0.01, size = 14, normalized size = 0.74

$$-\frac{3b}{5x^{\frac{5}{3}}}-\frac{a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))/x^3,x)
[Out] -1/2*a/x^2-3/5*b/x^(5/3)
```

maxima [A] time = 0.84, size = 15, normalized size = 0.79

$$-\frac{6\,bx^{\frac{1}{3}}+5\,a}{10\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))/x^3,x, algorithm="maxima")
[Out] -1/10*(6*b*x^(1/3) + 5*a)/x^2
```

mupad [B] time = 0.03, size = 15, normalized size = 0.79

$$-\frac{5\,a+6\,b\,x^{1/3}}{10\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))/x^3,x)
[Out] -(5*a + 6*b*x^(1/3))/(10*x^2)
```

sympy [A] time = 1.27, size = 17, normalized size = 0.89

$$-\frac{a}{2x^2}-\frac{3b}{5x^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))/x**3,x)
[Out] -a/(2*x**2) - 3*b/(5*x**(5/3))
```

3.2293 $\int \frac{a+b\sqrt[3]{x}}{x^4}\,dx$

Optimal. Leaf size=19

$$-\frac{a}{3x^3}-\frac{3b}{8x^{8/3}}$$

```
[Out] -1/3*a/x^3-3/8*b/x^(8/3)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$-\frac{a}{3x^3}-\frac{3b}{8x^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))/x^4,x]
```

```
[Out] -a/(3*x^3) - (3*b)/(8*x^(8/3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sqrt[3]{x}}{x^4}\,dx &= \int \left(\frac{a}{x^4}+\frac{b}{x^{11/3}}\right)dx\\ &= -\frac{a}{3x^3}-\frac{3b}{8x^{8/3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$-\frac{a}{3x^3}-\frac{3b}{8x^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))/x^4,x]
```

```
[Out] -1/3*a/x^3 - (3*b)/(8*x^(8/3))
```

fricas [A] time = 0.66, size = 15, normalized size = 0.79

$$-\frac{9\,bx^{\frac{1}{3}}+8\,a}{24\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))/x^4,x, algorithm="fricas")
```

```
[Out] -1/24*(9*b*x^(1/3) + 8*a)/x^3
```

giac [A] time = 0.16, size = 15, normalized size = 0.79

$$-\frac{9\,bx^{\frac{1}{3}}+8\,a}{24\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))/x^4,x, algorithm="giac")
[Out] -1/24*(9*b*x^(1/3) + 8*a)/x^3
```

maple [A] time = 0.00, size = 14, normalized size = 0.74

$$-\frac{3b}{8x^{\frac{8}{3}}} - \frac{a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))/x^4,x)
[Out] -1/3*a/x^3-3/8*b/x^(8/3)
```

maxima [A] time = 0.87, size = 15, normalized size = 0.79

$$-\frac{9\,bx^{\frac{1}{3}} + 8\,a}{24\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))/x^4,x, algorithm="maxima")
[Out] -1/24*(9*b*x^(1/3) + 8*a)/x^3
```

mupad [B] time = 0.03, size = 15, normalized size = 0.79

$$-\frac{8\,a + 9\,b\,x^{1/3}}{24\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))/x^4,x)
[Out] -(8*a + 9*b*x^(1/3))/(24*x^3)
```

sympy [A] time = 2.15, size = 17, normalized size = 0.89

$$-\frac{a}{3x^3} - \frac{3b}{8x^{\frac{8}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))/x**4,x)
[Out] -a/(3*x**3) - 3*b/(8*x**(8/3))
```

3.2294 $\int \left(a + b\sqrt[3]{x}\right)^2 x^4\, dx$

Optimal. Leaf size=34

$$\frac{a^2x^5}{5} + \frac{3}{8}abx^{16/3} + \frac{3}{17}b^2x^{17/3}$$

```
[Out] 1/5*a^2*x^5+3/8*a*b*x^(16/3)+3/17*b^2*x^(17/3)
```

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{a^2x^5}{5} + \frac{3}{8}abx^{16/3} + \frac{3}{17}b^2x^{17/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^2*x^4,x]
```

```
[Out] (a^2*x^5)/5 + (3*a*b*x^(16/3))/8 + (3*b^2*x^(17/3))/17
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a + b\sqrt[3]{x}\right)^2 x^4\, dx &= 3\,\text{Subst}\left(\int x^{14}(a + bx)^2\, dx, x, \sqrt[3]{x}\right) \\
&= 3\,\text{Subst}\left(\int \left(a^2x^{14} + 2abx^{15} + b^2x^{16}\right) dx, x, \sqrt[3]{x}\right) \\
&= \frac{a^2x^5}{5} + \frac{3}{8}abx^{16/3} + \frac{3}{17}b^2x^{17/3}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 34, normalized size = 1.00

$$\frac{a^2x^5}{5} + \frac{3}{8}abx^{16/3} + \frac{3}{17}b^2x^{17/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^2*x^4,x]
```

```
[Out] (a^2*x^5)/5 + (3*a*b*x^(16/3))/8 + (3*b^2*x^(17/3))/17
```

fricas [A] time = 0.68, size = 24, normalized size = 0.71

$$\frac{3}{17}\,b^2x^{\frac{17}{3}} + \frac{3}{8}\,abx^{\frac{16}{3}} + \frac{1}{5}\,a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^2*x^4,x, algorithm="fricas")
```

```
[Out] 3/17*b^2*x^(17/3) + 3/8*a*b*x^(16/3) + 1/5*a^2*x^5
```

giac [A] time = 0.20, size = 24, normalized size = 0.71

$$\frac{3}{17}b^2x^{\frac{17}{3}}+\frac{3}{8}abx^{\frac{16}{3}}+\frac{1}{5}a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^2*x^4,x, algorithm="giac")
```

```
[Out] 3/17*b^2*x^(17/3) + 3/8*a*b*x^(16/3) + 1/5*a^2*x^5
```

maple [A] time = 0.00, size = 25, normalized size = 0.74

$$\frac{3b^2x^{\frac{17}{3}}}{17}+\frac{3abx^{\frac{16}{3}}}{8}+\frac{a^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^2*x^4,x)
```

```
[Out] 1/5*a^2*x^5+3/8*a*b*x^(16/3)+3/17*b^2*x^(17/3)
```

maxima [B] time = 0.92, size = 250, normalized size = 7.35

$$\frac{3\left(bx^{\frac{1}{3}}+a\right)^{17}}{17b^{15}}-\frac{21\left(bx^{\frac{1}{3}}+a\right)^{16}a}{8b^{15}}+\frac{91\left(bx^{\frac{1}{3}}+a\right)^{15}a^2}{5b^{15}}-\frac{78\left(bx^{\frac{1}{3}}+a\right)^{14}a^3}{b^{15}}+\frac{231\left(bx^{\frac{1}{3}}+a\right)^{13}a^4}{b^{15}}-\frac{1001\left(bx^{\frac{1}{3}}+a\right)^{12}a^5}{2b^{15}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^2*x^4,x, algorithm="maxima")
```

```
[Out] 3/17*(b*x^(1/3) + a)^17/b^15 - 21/8*(b*x^(1/3) + a)^16*a/b^15 + 91/5*(b*x^(
1/3) + a)^15*a^2/b^15 - 78*(b*x^(1/3) + a)^14*a^3/b^15 + 231*(b*x^(1/3) + a
)^13*a^4/b^15 - 1001/2*(b*x^(1/3) + a)^12*a^5/b^15 + 819*(b*x^(1/3) + a)^11
*a^6/b^15 - 5148/5*(b*x^(1/3) + a)^10*a^7/b^15 + 1001*(b*x^(1/3) + a)^9*a^8
/b^15 - 3003/4*(b*x^(1/3) + a)^8*a^9/b^15 + 429*(b*x^(1/3) + a)^7*a^10/b^15
 - 182*(b*x^(1/3) + a)^6*a^11/b^15 + 273/5*(b*x^(1/3) + a)^5*a^12/b^15 - 21
/2*(b*x^(1/3) + a)^4*a^13/b^15 + (b*x^(1/3) + a)^3*a^14/b^15
```

mupad [B] time = 0.04, size = 24, normalized size = 0.71

$$\frac{a^2\,x^5}{5}+\frac{3\,b^2\,x^{17/3}}{17}+\frac{3\,a\,b\,x^{16/3}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^(1/3))^2,x)
```

```
[Out] (a^2*x^5)/5 + (3*b^2*x^(17/3))/17 + (3*a*b*x^(16/3))/8
```

sympy [A] time = 2.02, size = 31, normalized size = 0.91

$$\frac{a^2x^5}{5}+\frac{3abx^{\frac{16}{3}}}{8}+\frac{3b^2x^{\frac{17}{3}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**2*x**4,x)
```

```
[Out] a**2*x**5/5 + 3*a*b*x**(16/3)/8 + 3*b**2*x**(17/3)/17
```

3.2295 $\int \left(a + b\sqrt[3]{x}\right)^2 x^3\, dx$

Optimal. Leaf size=34

$$\frac{a^2x^4}{4} + \frac{6}{13}abx^{13/3} + \frac{3}{14}b^2x^{14/3}$$

```
[Out] 1/4*a^2*x^4+6/13*a*b*x^(13/3)+3/14*b^2*x^(14/3)
```

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{a^2x^4}{4} + \frac{6}{13}abx^{13/3} + \frac{3}{14}b^2x^{14/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^2*x^3,x]
```

```
[Out] (a^2*x^4)/4 + (6*a*b*x^(13/3))/13 + (3*b^2*x^(14/3))/14
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a + b\sqrt[3]{x}\right)^2 x^3\, dx &= 3\,\text{Subst}\left(\int x^{11}(a + bx)^2\, dx, x, \sqrt[3]{x}\right) \\
&= 3\,\text{Subst}\left(\int \left(a^2x^{11} + 2abx^{12} + b^2x^{13}\right) dx, x, \sqrt[3]{x}\right) \\
&= \frac{a^2x^4}{4} + \frac{6}{13}abx^{13/3} + \frac{3}{14}b^2x^{14/3}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 34, normalized size = 1.00

$$\frac{a^2x^4}{4} + \frac{6}{13}abx^{13/3} + \frac{3}{14}b^2x^{14/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^2*x^3,x]
```

```
[Out] (a^2*x^4)/4 + (6*a*b*x^(13/3))/13 + (3*b^2*x^(14/3))/14
```

fricas [A] time = 0.52, size = 24, normalized size = 0.71

$$\frac{3}{14}\,b^2x^{\frac{14}{3}} + \frac{6}{13}\,abx^{\frac{13}{3}} + \frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^2*x^3,x, algorithm="fricas")
```

```
[Out] 3/14*b^2*x^(14/3) + 6/13*a*b*x^(13/3) + 1/4*a^2*x^4
```

giac [A] time = 0.15, size = 24, normalized size = 0.71

$$\frac{3}{14}\,b^2x^{\frac{14}{3}} + \frac{6}{13}\,abx^{\frac{13}{3}} + \frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^2*x^3,x, algorithm="giac")
```

```
[Out] 3/14*b^2*x^(14/3) + 6/13*a*b*x^(13/3) + 1/4*a^2*x^4
```

maple [A] time = 0.00, size = 25, normalized size = 0.74

$$\frac{3b^2x^{\frac{14}{3}}}{14} + \frac{6ab\,x^{\frac{13}{3}}}{13} + \frac{a^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^2*x^3,x)
```

```
[Out] 1/4*a^2*x^4+6/13*a*b*x^(13/3)+3/14*b^2*x^(14/3)
```

maxima [B] time = 0.92, size = 200, normalized size = 5.88

$$\frac{3\left(bx^{\frac{1}{3}}+a\right)^{14}}{14\,b^{12}} - \frac{33\left(bx^{\frac{1}{3}}+a\right)^{13}a}{13\,b^{12}} + \frac{55\left(bx^{\frac{1}{3}}+a\right)^{12}a^2}{4\,b^{12}} - \frac{45\left(bx^{\frac{1}{3}}+a\right)^{11}a^3}{b^{12}} + \frac{99\left(bx^{\frac{1}{3}}+a\right)^{10}a^4}{b^{12}} - \frac{154\left(bx^{\frac{1}{3}}+a\right)^{9}a^5}{b^{12}} + \frac{693}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^2*x^3,x, algorithm="maxima")
```

```
[Out] 3/14*(b*x^(1/3) + a)^14/b^12 - 33/13*(b*x^(1/3) + a)^13*a/b^12 + 55/4*(b*x^
(1/3) + a)^12*a^2/b^12 - 45*(b*x^(1/3) + a)^11*a^3/b^12 + 99*(b*x^(1/3) + a
)^10*a^4/b^12 - 154*(b*x^(1/3) + a)^9*a^5/b^12 + 693/4*(b*x^(1/3) + a)^8*a^
6/b^12 - 990/7*(b*x^(1/3) + a)^7*a^7/b^12 + 165/2*(b*x^(1/3) + a)^6*a^8/b^1
2 - 33*(b*x^(1/3) + a)^5*a^9/b^12 + 33/4*(b*x^(1/3) + a)^4*a^10/b^12 - (b*x
^(1/3) + a)^3*a^11/b^12
```

mupad [B] time = 0.04, size = 24, normalized size = 0.71

$$\frac{a^2\,x^4}{4} + \frac{3\,b^2\,x^{14/3}}{14} + \frac{6\,a\,b\,x^{13/3}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^(1/3))^2,x)
```

```
[Out] (a^2*x^4)/4 + (3*b^2*x^(14/3))/14 + (6*a*b*x^(13/3))/13
```

sympy [A] time = 1.73, size = 31, normalized size = 0.91

$$\frac{a^2x^4}{4} + \frac{6abx^{\frac{13}{3}}}{13} + \frac{3b^2x^{\frac{14}{3}}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**2*x**3,x)
```

```
[Out] a**2*x**4/4 + 6*a*b*x**(13/3)/13 + 3*b**2*x**(14/3)/14
```

3.2296 $\int \left(a + b\sqrt[3]{x}\right)^2 x^2\, dx$

Optimal. Leaf size=34

$$\frac{a^2x^3}{3} + \frac{3}{5}abx^{10/3} + \frac{3}{11}b^2x^{11/3}$$

```
[Out] 1/3*a^2*x^3+3/5*a*b*x^(10/3)+3/11*b^2*x^(11/3)
```

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{a^2x^3}{3} + \frac{3}{5}abx^{10/3} + \frac{3}{11}b^2x^{11/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^2*x^2,x]
```

```
[Out] (a^2*x^3)/3 + (3*a*b*x^(10/3))/5 + (3*b^2*x^(11/3))/11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a + b\sqrt[3]{x}\right)^2 x^2\, dx &= 3\,\text{Subst}\left(\int x^8(a + bx)^2\, dx, x, \sqrt[3]{x}\right) \\
&= 3\,\text{Subst}\left(\int \left(a^2x^8 + 2abx^9 + b^2x^{10}\right) dx, x, \sqrt[3]{x}\right) \\
&= \frac{a^2x^3}{3} + \frac{3}{5}abx^{10/3} + \frac{3}{11}b^2x^{11/3}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 34, normalized size = 1.00

$$\frac{a^2x^3}{3} + \frac{3}{5}abx^{10/3} + \frac{3}{11}b^2x^{11/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^2*x^2,x]
```

```
[Out] (a^2*x^3)/3 + (3*a*b*x^(10/3))/5 + (3*b^2*x^(11/3))/11
```

fricas [A] time = 0.54, size = 24, normalized size = 0.71

$$\frac{3}{11}\,b^2x^{\frac{11}{3}} + \frac{3}{5}\,abx^{\frac{10}{3}} + \frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^2*x^2,x, algorithm="fricas")
[Out] 3/11*b^2*x^(11/3) + 3/5*a*b*x^(10/3) + 1/3*a^2*x^3
```

giac [A] time = 0.15, size = 24, normalized size = 0.71

$$\frac{3}{11}b^2x^{\frac{11}{3}}+\frac{3}{5}abx^{\frac{10}{3}}+\frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^2*x^2,x, algorithm="giac")
[Out] 3/11*b^2*x^(11/3) + 3/5*a*b*x^(10/3) + 1/3*a^2*x^3
```

maple [A] time = 0.00, size = 25, normalized size = 0.74

$$\frac{3b^2x^{\frac{11}{3}}}{11}+\frac{3abx^{\frac{10}{3}}}{5}+\frac{a^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^2*x^2,x)
[Out] 1/3*a^2*x^3+3/5*a*b*x^(10/3)+3/11*b^2*x^(11/3)
```

maxima [B] time = 0.89, size = 148, normalized size = 4.35

$$\frac{3\left(bx^{\frac{1}{3}}+a\right)^{11}}{11\,b^9}-\frac{12\left(bx^{\frac{1}{3}}+a\right)^{10}a}{5\,b^9}+\frac{28\left(bx^{\frac{1}{3}}+a\right)^{9}a^2}{3\,b^9}-\frac{21\left(bx^{\frac{1}{3}}+a\right)^{8}a^3}{b^9}+\frac{30\left(bx^{\frac{1}{3}}+a\right)^{7}a^4}{b^9}-\frac{28\left(bx^{\frac{1}{3}}+a\right)^{6}a^5}{b^9}+\frac{84\left(bx^{\frac{1}{3}}\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^2*x^2,x, algorithm="maxima")
[Out] 3/11*(b*x^(1/3) + a)^11/b^9 - 12/5*(b*x^(1/3) + a)^10*a/b^9 + 28/3*(b*x^(1/
3) + a)^9*a^2/b^9 - 21*(b*x^(1/3) + a)^8*a^3/b^9 + 30*(b*x^(1/3) + a)^7*a^4
/b^9 - 28*(b*x^(1/3) + a)^6*a^5/b^9 + 84/5*(b*x^(1/3) + a)^5*a^6/b^9 - 6*(b
*x^(1/3) + a)^4*a^7/b^9 + (b*x^(1/3) + a)^3*a^8/b^9
```

mupad [B] time = 0.04, size = 24, normalized size = 0.71

$$\frac{a^2\,x^3}{3}+\frac{3\,b^2\,x^{11/3}}{11}+\frac{3\,a\,b\,x^{10/3}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^(1/3))^2,x)
[Out] (a^2*x^3)/3 + (3*b^2*x^(11/3))/11 + (3*a*b*x^(10/3))/5
```

sympy [A] time = 1.61, size = 31, normalized size = 0.91

$$\frac{a^2x^3}{3}+\frac{3abx^{\frac{10}{3}}}{5}+\frac{3b^2x^{\frac{11}{3}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**2*x**2,x)
[Out] a**2*x**3/3 + 3*a*b*x**(10/3)/5 + 3*b**2*x**(11/3)/11
```

3.2297 $\int \left(a + b\sqrt[3]{x}\right)^2 x\,dx$

Optimal. Leaf size=34

$$\frac{a^2x^2}{2} + \frac{6}{7}abx^{7/3} + \frac{3}{8}b^2x^{8/3}$$

```
[Out] 1/2*a^2*x^2+6/7*a*b*x^(7/3)+3/8*b^2*x^(8/3)
```

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{a^2x^2}{2} + \frac{6}{7}abx^{7/3} + \frac{3}{8}b^2x^{8/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^2*x,x]
```

```
[Out] (a^2*x^2)/2 + (6*a*b*x^(7/3))/7 + (3*b^2*x^(8/3))/8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \left(a + b\sqrt[3]{x}\right)^2 x\,dx &= 3\,\text{Subst}\left(\int x^5(a + bx)^2\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(a^2x^5 + 2abx^6 + b^2x^7\right)\,dx, x, \sqrt[3]{x}\right)\\ &= \frac{a^2x^2}{2} + \frac{6}{7}abx^{7/3} + \frac{3}{8}b^2x^{8/3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 34, normalized size = 1.00

$$\frac{a^2x^2}{2} + \frac{6}{7}abx^{7/3} + \frac{3}{8}b^2x^{8/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^2*x,x]
```

```
[Out] (a^2*x^2)/2 + (6*a*b*x^(7/3))/7 + (3*b^2*x^(8/3))/8
```

fricas [A] time = 0.62, size = 24, normalized size = 0.71

$$\frac{3}{8}\,b^2x^{\frac{8}{3}} + \frac{6}{7}\,abx^{\frac{7}{3}} + \frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^2*x,x, algorithm="fricas")
```

```
[Out] 3/8*b^2*x^(8/3) + 6/7*a*b*x^(7/3) + 1/2*a^2*x^2
```

giac [A] time = 0.16, size = 24, normalized size = 0.71

$$\frac{3}{8}b^2x^{\frac{8}{3}} + \frac{6}{7}abx^{\frac{7}{3}} + \frac{1}{2}a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^2*x,x, algorithm="giac")
```

```
[Out] 3/8*b^2*x^(8/3) + 6/7*a*b*x^(7/3) + 1/2*a^2*x^2
```

maple [A] time = 0.00, size = 25, normalized size = 0.74

$$\frac{3b^2x^{\frac{8}{3}}}{8} + \frac{6ab\,x^{\frac{7}{3}}}{7} + \frac{a^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^2*x,x)
```

```
[Out] 1/2*a^2*x^2+6/7*a*b*x^(7/3)+3/8*b^2*x^(8/3)
```

maxima [B] time = 0.92, size = 98, normalized size = 2.88

$$\frac{3\left(bx^{\frac{1}{3}}+a\right)^8}{8\,b^6} - \frac{15\left(bx^{\frac{1}{3}}+a\right)^7 a}{7\,b^6} + \frac{5\left(bx^{\frac{1}{3}}+a\right)^6 a^2}{b^6} - \frac{6\left(bx^{\frac{1}{3}}+a\right)^5 a^3}{b^6} + \frac{15\left(bx^{\frac{1}{3}}+a\right)^4 a^4}{4\,b^6} - \frac{\left(bx^{\frac{1}{3}}+a\right)^3 a^5}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^2*x,x, algorithm="maxima")
```

```
[Out] 3/8*(b*x^(1/3) + a)^8/b^6 - 15/7*(b*x^(1/3) + a)^7*a/b^6 + 5*(b*x^(1/3) + a
)^6*a^2/b^6 - 6*(b*x^(1/3) + a)^5*a^3/b^6 + 15/4*(b*x^(1/3) + a)^4*a^4/b^6
- (b*x^(1/3) + a)^3*a^5/b^6
```

mupad [B] time = 0.04, size = 24, normalized size = 0.71

$$\frac{a^2\,x^2}{2} + \frac{3\,b^2\,x^{8/3}}{8} + \frac{6\,a\,b\,x^{7/3}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^(1/3))^2,x)
```

```
[Out] (a^2*x^2)/2 + (3*b^2*x^(8/3))/8 + (6*a*b*x^(7/3))/7
```

sympy [A] time = 1.51, size = 31, normalized size = 0.91

$$\frac{a^2x^2}{2} + \frac{6abx^{\frac{7}{3}}}{7} + \frac{3b^2x^{\frac{8}{3}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**2*x,x)
```

```
[Out] a**2*x**2/2 + 6*a*b*x**(7/3)/7 + 3*b**2*x**(8/3)/8
```

3.2298 $\int \left(a + b\sqrt[3]{x}\right)^2 dx$

Optimal. Leaf size=29

$$a^2x + \frac{3}{2}abx^{4/3} + \frac{3}{5}b^2x^{5/3}$$

```
[Out] a^2*x+3/2*a*b*x^(4/3)+3/5*b^2*x^(5/3)
```

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {190, 43}

$$a^2x + \frac{3}{2}abx^{4/3} + \frac{3}{5}b^2x^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^2,x]
```

```
[Out] a^2*x + (3*a*b*x^(4/3))/2 + (3*b^2*x^(5/3))/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\int \left(a + b\sqrt[3]{x}\right)^2 dx &= 3\,\text{Subst}\left(\int x^2(a + bx)^2\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(a^2x^2 + 2abx^3 + b^2x^4\right)\,dx, x, \sqrt[3]{x}\right)\\ &= a^2x + \frac{3}{2}abx^{4/3} + \frac{3}{5}b^2x^{5/3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 1.00

$$a^2x + \frac{3}{2}abx^{4/3} + \frac{3}{5}b^2x^{5/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^2,x]
```

```
[Out] a^2*x + (3*a*b*x^(4/3))/2 + (3*b^2*x^(5/3))/5
```

fricas [A] time = 0.55, size = 21, normalized size = 0.72

$$\frac{3}{5}\,b^2x^{\frac{5}{3}} + \frac{3}{2}\,abx^{\frac{4}{3}} + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^2,x, algorithm="fricas")
[Out] 3/5*b^2*x^(5/3) + 3/2*a*b*x^(4/3) + a^2*x
```

giac [A] time = 0.15, size = 21, normalized size = 0.72

$$\frac{3}{5}\,b^2x^{\frac{5}{3}} + \frac{3}{2}\,abx^{\frac{4}{3}} + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^2,x, algorithm="giac")
[Out] 3/5*b^2*x^(5/3) + 3/2*a*b*x^(4/3) + a^2*x
```

maple [A] time = 0.00, size = 22, normalized size = 0.76

$$\frac{3b^2x^{\frac{5}{3}}}{5} + \frac{3ab\,x^{\frac{4}{3}}}{2} + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^2,x)
[Out] a^2*x+3/2*a*b*x^(4/3)+3/5*b^2*x^(5/3)
```

maxima [A] time = 0.88, size = 21, normalized size = 0.72

$$\frac{3}{5}\,b^2x^{\frac{5}{3}} + \frac{3}{2}\,abx^{\frac{4}{3}} + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^2,x, algorithm="maxima")
[Out] 3/5*b^2*x^(5/3) + 3/2*a*b*x^(4/3) + a^2*x
```

mupad [B] time = 1.12, size = 21, normalized size = 0.72

$$a^2\,x + \frac{3\,b^2\,x^{5/3}}{5} + \frac{3\,a\,b\,x^{4/3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))^2,x)
[Out] a^2*x + (3*b^2*x^(5/3))/5 + (3*a*b*x^(4/3))/2
```

sympy [A] time = 1.39, size = 27, normalized size = 0.93

$$a^2x + \frac{3abx^{\frac{4}{3}}}{2} + \frac{3b^2x^{\frac{5}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**2,x)
[Out] a**2*x + 3*a*b*x**(4/3)/2 + 3*b**2*x**(5/3)/5
```

3.2299 $\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x}\,dx$

Optimal. Leaf size=28

$$a^2\log(x)+6ab\sqrt[3]{x}+\frac{3}{2}b^2x^{2/3}$$

```
[Out] 6*a*b*x^(1/3)+3/2*b^2*x^(2/3)+a^2*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$a^2\log(x)+6ab\sqrt[3]{x}+\frac{3}{2}b^2x^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^2/x,x]
```

```
[Out] 6*a*b*x^(1/3) + (3*b^2*x^(2/3))/2 + a^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x}\,dx &= 3\,\text{Subst}\left(\int \frac{(a+bx)^2}{x}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(2ab+\frac{a^2}{x}+b^2x\right)\,dx, x, \sqrt[3]{x}\right)\\ &= 6ab\sqrt[3]{x}+\frac{3}{2}b^2x^{2/3}+a^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$a^2\log(x)+6ab\sqrt[3]{x}+\frac{3}{2}b^2x^{2/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^2/x,x]
```

```
[Out] 6*a*b*x^(1/3) + (3*b^2*x^(2/3))/2 + a^2*Log[x]
```

fricas [A] time = 0.64, size = 25, normalized size = 0.89

$$3\,a^2\log\left(x^{\frac{1}{3}}\right)+\frac{3}{2}\,b^2x^{\frac{2}{3}}+6\,abx^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^2/x,x, algorithm="fricas")
[Out] 3*a^2*log(x^(1/3)) + 3/2*b^2*x^(2/3) + 6*a*b*x^(1/3)
```

giac [A] time = 0.17, size = 23, normalized size = 0.82

$$a^2 \log(|x|) + \frac{3}{2} b^2 x^{\frac{2}{3}} + 6\, abx^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^2/x,x, algorithm="giac")
[Out] a^2*log(abs(x)) + 3/2*b^2*x^(2/3) + 6*a*b*x^(1/3)
```

maple [A] time = 0.00, size = 23, normalized size = 0.82

$$a^2 \ln(x) + \frac{3b^2 x^{\frac{2}{3}}}{2} + 6ab\, x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^2/x,x)
[Out] 6*a*b*x^(1/3)+3/2*b^2*x^(2/3)+a^2*ln(x)
```

maxima [A] time = 0.89, size = 22, normalized size = 0.79

$$a^2 \log(x) + \frac{3}{2} b^2 x^{\frac{2}{3}} + 6\, abx^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^2/x,x, algorithm="maxima")
[Out] a^2*log(x) + 3/2*b^2*x^(2/3) + 6*a*b*x^(1/3)
```

mupad [B] time = 1.11, size = 25, normalized size = 0.89

$$3\, a^2\, \ln\left(x^{1/3}\right) + \frac{3\, b^2\, x^{2/3}}{2} + 6\, a\, b\, x^{1/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))^2/x,x)
[Out] 3*a^2*log(x^(1/3)) + (3*b^2*x^(2/3))/2 + 6*a*b*x^(1/3)
```

sympy [A] time = 0.24, size = 27, normalized size = 0.96

$$a^2 \log(x) + 6ab\sqrt[3]{x} + \frac{3b^2 x^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**2/x,x)
[Out] a**2*log(x) + 6*a*b*x**(1/3) + 3*b**2*x**(2/3)/2
```

3.2300 $\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x^2}\,dx$

Optimal. Leaf size=19

$$-\frac{\left(a+b\sqrt[3]{x}\right)^3}{ax}$$

```
[Out] -(a+b*x^(1/3))^3/a/x
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {264}

$$-\frac{\left(a+b\sqrt[3]{x}\right)^3}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^2/x^2,x]
```

```
[Out] -((a + b*x^(1/3))^3/(a*x))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x^2}\,dx = -\frac{\left(a+b\sqrt[3]{x}\right)^3}{ax}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$-\frac{\left(a+b\sqrt[3]{x}\right)^3}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^2/x^2,x]
```

```
[Out] -((a + b*x^(1/3))^3/(a*x))
```

fricas [A] time = 0.58, size = 24, normalized size = 1.26

$$-\frac{3\,b^2x^{\frac{2}{3}} + 3\,abx^{\frac{1}{3}} + a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^2/x^2,x, algorithm="fricas")
```

```
[Out] -(3*b^2*x^(2/3) + 3*a*b*x^(1/3) + a^2)/x
```

giac [A] time = 0.17, size = 24, normalized size = 1.26

$$-\frac{3\,b^2x^{\frac{2}{3}} + 3\,abx^{\frac{1}{3}} + a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^2/x^2,x, algorithm="giac")
[Out] -(3*b^2*x^(2/3) + 3*a*b*x^(1/3) + a^2)/x
```

maple [A] time = 0.00, size = 25, normalized size = 1.32

$$-\frac{3b^2}{x^{\frac{1}{3}}}-\frac{3ab}{x^{\frac{2}{3}}}-\frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^2/x^2,x)
[Out] -a^2/x-3*a*b/x^(2/3)-3*b^2/x^(1/3)
```

maxima [A] time = 0.93, size = 24, normalized size = 1.26

$$-\frac{3\,b^2x^{\frac{2}{3}}+3\,abx^{\frac{1}{3}}+a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^2/x^2,x, algorithm="maxima")
[Out] -(3*b^2*x^(2/3) + 3*a*b*x^(1/3) + a^2)/x
```

mupad [B] time = 0.03, size = 24, normalized size = 1.26

$$-\frac{a^2}{x}-\frac{3\,b^2}{x^{1/3}}-\frac{3\,a\,b}{x^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))^2/x^2,x)
[Out] - a^2/x - (3*b^2)/x^(1/3) - (3*a*b)/x^(2/3)
```

sympy [A] time = 0.63, size = 26, normalized size = 1.37

$$-\frac{a^2}{x}-\frac{3ab}{x^{\frac{2}{3}}}-\frac{3b^2}{\sqrt[3]{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**2/x**2,x)
[Out] -a**2/x - 3*a*b/x**(2/3) - 3*b**2/x**(1/3)
```

3.2301 $\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x^3}\,dx$

Optimal. Leaf size=34

$$-\frac{a^2}{2x^2}-\frac{6ab}{5x^{5/3}}-\frac{3b^2}{4x^{4/3}}$$

```
[Out] -1/2*a^2/x^2-6/5*a*b/x^(5/3)-3/4*b^2/x^(4/3)
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{a^2}{2x^2}-\frac{6ab}{5x^{5/3}}-\frac{3b^2}{4x^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^2/x^3,x]
```

```
[Out] -a^2/(2*x^2) - (6*a*b)/(5*x^(5/3)) - (3*b^2)/(4*x^(4/3))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x^3}\,dx &= 3\,\text{Subst}\left(\int \frac{(a+bx)^2}{x^7}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(\frac{a^2}{x^7}+\frac{2ab}{x^6}+\frac{b^2}{x^5}\right)dx, x, \sqrt[3]{x}\right)\\ &= -\frac{a^2}{2x^2}-\frac{6ab}{5x^{5/3}}-\frac{3b^2}{4x^{4/3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.00

$$-\frac{a^2}{2x^2}-\frac{6ab}{5x^{5/3}}-\frac{3b^2}{4x^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^2/x^3,x]
```

```
[Out] -1/2*a^2/x^2 - (6*a*b)/(5*x^(5/3)) - (3*b^2)/(4*x^(4/3))
```

fricas [A] time = 0.52, size = 26, normalized size = 0.76

$$-\frac{15\,b^2x^{\frac{2}{3}}+24\,abx^{\frac{1}{3}}+10\,a^2}{20\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^2/x^3,x, algorithm="fricas")
[Out] -1/20*(15*b^2*x^(2/3) + 24*a*b*x^(1/3) + 10*a^2)/x^2
```

giac [A] time = 0.17, size = 26, normalized size = 0.76

$$-\frac{15\,b^2x^{\frac{2}{3}}+24\,abx^{\frac{1}{3}}+10\,a^2}{20\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^2/x^3,x, algorithm="giac")
[Out] -1/20*(15*b^2*x^(2/3) + 24*a*b*x^(1/3) + 10*a^2)/x^2
```

maple [A] time = 0.01, size = 25, normalized size = 0.74

$$-\frac{3b^2}{4x^{\frac{4}{3}}}-\frac{6ab}{5x^{\frac{5}{3}}}-\frac{a^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^2/x^3,x)
[Out] -1/2*a^2/x^2-6/5*a*b/x^(5/3)-3/4*b^2/x^(4/3)
```

maxima [A] time = 0.92, size = 26, normalized size = 0.76

$$-\frac{15\,b^2x^{\frac{2}{3}}+24\,abx^{\frac{1}{3}}+10\,a^2}{20\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^2/x^3,x, algorithm="maxima")
[Out] -1/20*(15*b^2*x^(2/3) + 24*a*b*x^(1/3) + 10*a^2)/x^2
```

mupad [B] time = 0.03, size = 26, normalized size = 0.76

$$-\frac{10\,a^2+15\,b^2\,x^{2/3}+24\,a\,b\,x^{1/3}}{20\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))^2/x^3,x)
[Out] -(10*a^2 + 15*b^2*x^(2/3) + 24*a*b*x^(1/3))/(20*x^2)
```

sympy [A] time = 1.07, size = 32, normalized size = 0.94

$$-\frac{a^2}{2x^2}-\frac{6ab}{5x^{\frac{5}{3}}}-\frac{3b^2}{4x^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**2/x**3,x)
[Out] -a**2/(2*x**2) - 6*a*b/(5*x**(5/3)) - 3*b**2/(4*x**(4/3))
```

3.2302 $\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x^4}\,dx$

Optimal. Leaf size=34

$$-\frac{a^2}{3x^3}-\frac{3ab}{4x^{8/3}}-\frac{3b^2}{7x^{7/3}}$$

```
[Out] -1/3*a^2/x^3-3/4*a*b/x^(8/3)-3/7*b^2/x^(7/3)
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{a^2}{3x^3}-\frac{3ab}{4x^{8/3}}-\frac{3b^2}{7x^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^2/x^4,x]
```

```
[Out] -a^2/(3*x^3) - (3*a*b)/(4*x^(8/3)) - (3*b^2)/(7*x^(7/3))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x^4}\,dx &= 3\,\text{Subst}\left(\int \frac{(a+bx)^2}{x^{10}}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(\frac{a^2}{x^{10}}+\frac{2ab}{x^9}+\frac{b^2}{x^8}\right)dx, x, \sqrt[3]{x}\right)\\ &= -\frac{a^2}{3x^3}-\frac{3ab}{4x^{8/3}}-\frac{3b^2}{7x^{7/3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.00

$$-\frac{a^2}{3x^3}-\frac{3ab}{4x^{8/3}}-\frac{3b^2}{7x^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^2/x^4,x]
```

```
[Out] -1/3*a^2/x^3 - (3*a*b)/(4*x^(8/3)) - (3*b^2)/(7*x^(7/3))
```

fricas [A] time = 0.65, size = 26, normalized size = 0.76

$$-\frac{36\,b^2x^{\frac{2}{3}}+63\,abx^{\frac{1}{3}}+28\,a^2}{84\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^2/x^4,x, algorithm="fricas")
[Out] -1/84*(36*b^2*x^(2/3) + 63*a*b*x^(1/3) + 28*a^2)/x^3
```

giac [A] time = 0.15, size = 26, normalized size = 0.76

$$-\frac{36\,b^2x^{\frac{2}{3}}+63\,abx^{\frac{1}{3}}+28\,a^2}{84\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^2/x^4,x, algorithm="giac")
[Out] -1/84*(36*b^2*x^(2/3) + 63*a*b*x^(1/3) + 28*a^2)/x^3
```

maple [A] time = 0.01, size = 25, normalized size = 0.74

$$-\frac{3b^2}{7x^{\frac{7}{3}}}-\frac{3ab}{4x^{\frac{8}{3}}}-\frac{a^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^2/x^4,x)
[Out] -1/3*a^2/x^3-3/4*a*b/x^(8/3)-3/7*b^2/x^(7/3)
```

maxima [A] time = 0.92, size = 26, normalized size = 0.76

$$-\frac{36\,b^2x^{\frac{2}{3}}+63\,abx^{\frac{1}{3}}+28\,a^2}{84\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^2/x^4,x, algorithm="maxima")
[Out] -1/84*(36*b^2*x^(2/3) + 63*a*b*x^(1/3) + 28*a^2)/x^3
```

mupad [B] time = 0.03, size = 26, normalized size = 0.76

$$-\frac{28\,a^2+36\,b^2\,x^{2/3}+63\,a\,b\,x^{1/3}}{84\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))^2/x^4,x)
[Out] -(28*a^2 + 36*b^2*x^(2/3) + 63*a*b*x^(1/3))/(84*x^3)
```

sympy [A] time = 1.88, size = 32, normalized size = 0.94

$$-\frac{a^2}{3x^3}-\frac{3ab}{4x^{\frac{8}{3}}}-\frac{3b^2}{7x^{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**2/x**4,x)
[Out] -a**2/(3*x**3) - 3*a*b/(4*x**(8/3)) - 3*b**2/(7*x**(7/3))
```

3.2303 $\int \left(a + b\sqrt[3]{x}\right)^3 x^4\, dx$

Optimal. Leaf size=47

$$\frac{a^3x^5}{5} + \frac{9}{16}a^2bx^{16/3} + \frac{9}{17}ab^2x^{17/3} + \frac{b^3x^6}{6}$$

```
[Out] 1/5*a^3*x^5+9/16*a^2*b*x^(16/3)+9/17*a*b^2*x^(17/3)+1/6*b^3*x^6
```

Rubi [A] time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{9}{16}a^2bx^{16/3} + \frac{a^3x^5}{5} + \frac{9}{17}ab^2x^{17/3} + \frac{b^3x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^3*x^4,x]
```

```
[Out] (a^3*x^5)/5 + (9*a^2*b*x^(16/3))/16 + (9*a*b^2*x^(17/3))/17 + (b^3*x^6)/6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a + b\sqrt[3]{x}\right)^3 x^4\, dx &= 3\,\text{Subst}\left(\int x^{14}(a + bx)^3\, dx, x, \sqrt[3]{x}\right) \\
&= 3\,\text{Subst}\left(\int \left(a^3x^{14} + 3a^2bx^{15} + 3ab^2x^{16} + b^3x^{17}\right)\, dx, x, \sqrt[3]{x}\right) \\
&= \frac{a^3x^5}{5} + \frac{9}{16}a^2bx^{16/3} + \frac{9}{17}ab^2x^{17/3} + \frac{b^3x^6}{6}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 47, normalized size = 1.00

$$\frac{a^3x^5}{5} + \frac{9}{16}a^2bx^{16/3} + \frac{9}{17}ab^2x^{17/3} + \frac{b^3x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^3*x^4,x]
```

```
[Out] (a^3*x^5)/5 + (9*a^2*b*x^(16/3))/16 + (9*a*b^2*x^(17/3))/17 + (b^3*x^6)/6
```

fricas [A] time = 0.55, size = 35, normalized size = 0.74

$$\frac{1}{6}\,b^3x^6 + \frac{9}{17}\,ab^2x^{\frac{17}{3}} + \frac{9}{16}\,a^2bx^{\frac{16}{3}} + \frac{1}{5}\,a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^3*x^4,x, algorithm="fricas")
```

```
[Out] 1/6*b^3*x^6 + 9/17*a*b^2*x^(17/3) + 9/16*a^2*b*x^(16/3) + 1/5*a^3*x^5
```

giac [A] time = 0.15, size = 35, normalized size = 0.74

$$\frac{1}{6}b^3x^6+\frac{9}{17}ab^2x^{\frac{17}{3}}+\frac{9}{16}a^2bx^{\frac{16}{3}}+\frac{1}{5}a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^3*x^4,x, algorithm="giac")
```

```
[Out] 1/6*b^3*x^6 + 9/17*a*b^2*x^(17/3) + 9/16*a^2*b*x^(16/3) + 1/5*a^3*x^5
```

maple [A] time = 0.00, size = 36, normalized size = 0.77

$$\frac{b^3x^6}{6}+\frac{9a\,b^2x^{\frac{17}{3}}}{17}+\frac{9a^2b\,x^{\frac{16}{3}}}{16}+\frac{a^3x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^3*x^4,x)
```

```
[Out] 1/5*a^3*x^5+9/16*a^2*b*x^(16/3)+9/17*a*b^2*x^(17/3)+1/6*b^3*x^6
```

maxima [B] time = 0.85, size = 251, normalized size = 5.34

$$\frac{\left(bx^{\frac{1}{3}}+a\right)^{18}}{6\,b^{15}}-\frac{42\left(bx^{\frac{1}{3}}+a\right)^{17}a}{17\,b^{15}}+\frac{273\left(bx^{\frac{1}{3}}+a\right)^{16}a^2}{16\,b^{15}}-\frac{364\left(bx^{\frac{1}{3}}+a\right)^{15}a^3}{5\,b^{15}}+\frac{429\left(bx^{\frac{1}{3}}+a\right)^{14}a^4}{2\,b^{15}}-\frac{462\left(bx^{\frac{1}{3}}+a\right)^{13}a^5}{b^{15}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^3*x^4,x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^(1/3) + a)^18/b^15 - 42/17*(b*x^(1/3) + a)^17*a/b^15 + 273/16*(b*x
^(1/3) + a)^16*a^2/b^15 - 364/5*(b*x^(1/3) + a)^15*a^3/b^15 + 429/2*(b*x^(1
/3) + a)^14*a^4/b^15 - 462*(b*x^(1/3) + a)^13*a^5/b^15 + 3003/4*(b*x^(1/3)
 + a)^12*a^6/b^15 - 936*(b*x^(1/3) + a)^11*a^7/b^15 + 9009/10*(b*x^(1/3) + a
)^10*a^8/b^15 - 2002/3*(b*x^(1/3) + a)^9*a^9/b^15 + 3003/8*(b*x^(1/3) + a)^
8*a^10/b^15 - 156*(b*x^(1/3) + a)^7*a^11/b^15 + 91/2*(b*x^(1/3) + a)^6*a^12
/b^15 - 42/5*(b*x^(1/3) + a)^5*a^13/b^15 + 3/4*(b*x^(1/3) + a)^4*a^14/b^15
```

mupad [B] time = 0.04, size = 35, normalized size = 0.74

$$\frac{a^3\,x^5}{5}+\frac{b^3\,x^6}{6}+\frac{9\,a^2\,b\,x^{16/3}}{16}+\frac{9\,a\,b^2\,x^{17/3}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^(1/3))^3,x)
```

```
[Out] (a^3*x^5)/5 + (b^3*x^6)/6 + (9*a^2*b*x^(16/3))/16 + (9*a*b^2*x^(17/3))/17
```

sympy [A] time = 2.33, size = 42, normalized size = 0.89

$$\frac{a^3x^5}{5}+\frac{9a^2bx^{\frac{16}{3}}}{16}+\frac{9ab^2x^{\frac{17}{3}}}{17}+\frac{b^3x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**3*x**4,x)
```

```
[Out] a**3*x**5/5 + 9*a**2*b*x**(16/3)/16 + 9*a*b**2*x**(17/3)/17 + b**3*x**6/6
```

3.2304 $\int \left(a + b\sqrt[3]{x}\right)^3 x^3\, dx$

Optimal. Leaf size=47

$$\frac{a^3x^4}{4} + \frac{9}{13}a^2bx^{13/3} + \frac{9}{14}ab^2x^{14/3} + \frac{b^3x^5}{5}$$

```
[Out] 1/4*a^3*x^4+9/13*a^2*b*x^(13/3)+9/14*a*b^2*x^(14/3)+1/5*b^3*x^5
```

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{9}{13}a^2bx^{13/3} + \frac{a^3x^4}{4} + \frac{9}{14}ab^2x^{14/3} + \frac{b^3x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^3*x^3,x]
```

```
[Out] (a^3*x^4)/4 + (9*a^2*b*x^(13/3))/13 + (9*a*b^2*x^(14/3))/14 + (b^3*x^5)/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \left(a + b\sqrt[3]{x}\right)^3 x^3\, dx &= 3\,\text{Subst}\left(\int x^{11}(a + bx)^3\, dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(a^3x^{11} + 3a^2bx^{12} + 3ab^2x^{13} + b^3x^{14}\right)\, dx, x, \sqrt[3]{x}\right)\\ &= \frac{a^3x^4}{4} + \frac{9}{13}a^2bx^{13/3} + \frac{9}{14}ab^2x^{14/3} + \frac{b^3x^5}{5}\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 1.00

$$\frac{a^3x^4}{4} + \frac{9}{13}a^2bx^{13/3} + \frac{9}{14}ab^2x^{14/3} + \frac{b^3x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^3*x^3,x]
```

```
[Out] (a^3*x^4)/4 + (9*a^2*b*x^(13/3))/13 + (9*a*b^2*x^(14/3))/14 + (b^3*x^5)/5
```

fricas [A] time = 0.57, size = 35, normalized size = 0.74

$$\frac{1}{5}\,b^3x^5 + \frac{9}{14}\,ab^2x^{\frac{14}{3}} + \frac{9}{13}\,a^2bx^{\frac{13}{3}} + \frac{1}{4}\,a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^3*x^3,x, algorithm="fricas")
```

```
[Out] 1/5*b^3*x^5 + 9/14*a*b^2*x^(14/3) + 9/13*a^2*b*x^(13/3) + 1/4*a^3*x^4
```

giac [A] time = 0.16, size = 35, normalized size = 0.74

$$\frac{1}{5}\,b^3x^5+\frac{9}{14}\,ab^2x^{\frac{14}{3}}+\frac{9}{13}\,a^2bx^{\frac{13}{3}}+\frac{1}{4}\,a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^3*x^3,x, algorithm="giac")
```

```
[Out] 1/5*b^3*x^5 + 9/14*a*b^2*x^(14/3) + 9/13*a^2*b*x^(13/3) + 1/4*a^3*x^4
```

maple [A] time = 0.00, size = 36, normalized size = 0.77

$$\frac{b^3x^5}{5}+\frac{9a\,b^2x^{\frac{14}{3}}}{14}+\frac{9a^2b\,x^{\frac{13}{3}}}{13}+\frac{a^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^3*x^3,x)
```

```
[Out] 1/4*a^3*x^4+9/13*a^2*b*x^(13/3)+9/14*a*b^2*x^(14/3)+1/5*b^3*x^5
```

maxima [B] time = 0.90, size = 200, normalized size = 4.26

$$\frac{\left(bx^{\frac{1}{3}}+a\right)^{15}}{5\,b^{12}}-\frac{33\left(bx^{\frac{1}{3}}+a\right)^{14}a}{14\,b^{12}}+\frac{165\left(bx^{\frac{1}{3}}+a\right)^{13}a^2}{13\,b^{12}}-\frac{165\left(bx^{\frac{1}{3}}+a\right)^{12}a^3}{4\,b^{12}}+\frac{90\left(bx^{\frac{1}{3}}+a\right)^{11}a^4}{b^{12}}-\frac{693\left(bx^{\frac{1}{3}}+a\right)^{10}a^5}{5\,b^{12}}+\frac{1}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^3*x^3,x, algorithm="maxima")
```

```
[Out] 1/5*(b*x^(1/3) + a)^15/b^12 - 33/14*(b*x^(1/3) + a)^14*a/b^12 + 165/13*(b*x
^(1/3) + a)^13*a^2/b^12 - 165/4*(b*x^(1/3) + a)^12*a^3/b^12 + 90*(b*x^(1/3)
 + a)^11*a^4/b^12 - 693/5*(b*x^(1/3) + a)^10*a^5/b^12 + 154*(b*x^(1/3) + a)
^9*a^6/b^12 - 495/4*(b*x^(1/3) + a)^8*a^7/b^12 + 495/7*(b*x^(1/3) + a)^7*a^
8/b^12 - 55/2*(b*x^(1/3) + a)^6*a^9/b^12 + 33/5*(b*x^(1/3) + a)^5*a^10/b^12
 - 3/4*(b*x^(1/3) + a)^4*a^11/b^12
```

mupad [B] time = 0.04, size = 35, normalized size = 0.74

$$\frac{a^3\,x^4}{4}+\frac{b^3\,x^5}{5}+\frac{9\,a^2\,b\,x^{13/3}}{13}+\frac{9\,a\,b^2\,x^{14/3}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^(1/3))^3,x)
```

```
[Out] (a^3*x^4)/4 + (b^3*x^5)/5 + (9*a^2*b*x^(13/3))/13 + (9*a*b^2*x^(14/3))/14
```

sympy [A] time = 2.09, size = 42, normalized size = 0.89

$$\frac{a^3x^4}{4}+\frac{9a^2bx^{\frac{13}{3}}}{13}+\frac{9ab^2x^{\frac{14}{3}}}{14}+\frac{b^3x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**3*x**3,x)
```

```
[Out] a**3*x**4/4 + 9*a**2*b*x**(13/3)/13 + 9*a*b**2*x**(14/3)/14 + b**3*x**5/5
```

3.2305 $\int \left(a + b\sqrt[3]{x}\right)^3 x^2\, dx$

Optimal. Leaf size=47

$$\frac{a^3x^3}{3} + \frac{9}{10}a^2bx^{10/3} + \frac{9}{11}ab^2x^{11/3} + \frac{b^3x^4}{4}$$

```
[Out] 1/3*a^3*x^3+9/10*a^2*b*x^(10/3)+9/11*a*b^2*x^(11/3)+1/4*b^3*x^4
```

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{9}{10}a^2bx^{10/3} + \frac{a^3x^3}{3} + \frac{9}{11}ab^2x^{11/3} + \frac{b^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^3*x^2,x]
```

```
[Out] (a^3*x^3)/3 + (9*a^2*b*x^(10/3))/10 + (9*a*b^2*x^(11/3))/11 + (b^3*x^4)/4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \left(a + b\sqrt[3]{x}\right)^3 x^2\, dx &= 3\,\text{Subst}\left(\int x^8(a + bx)^3\, dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(a^3x^8 + 3a^2bx^9 + 3ab^2x^{10} + b^3x^{11}\right)\, dx, x, \sqrt[3]{x}\right)\\ &= \frac{a^3x^3}{3} + \frac{9}{10}a^2bx^{10/3} + \frac{9}{11}ab^2x^{11/3} + \frac{b^3x^4}{4}\end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 0.87

$$\frac{1}{660}x^3\left(220a^3 + 594a^2b\sqrt[3]{x} + 540ab^2x^{2/3} + 165b^3x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^3*x^2,x]
```

```
[Out] (x^3*(220*a^3 + 594*a^2*b*x^(1/3) + 540*a*b^2*x^(2/3) + 165*b^3*x))/660
```

fricas [A] time = 0.68, size = 35, normalized size = 0.74

$$\frac{1}{4}\,b^3x^4 + \frac{9}{11}\,ab^2x^{\frac{11}{3}} + \frac{9}{10}\,a^2bx^{\frac{10}{3}} + \frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^3*x^2,x, algorithm="fricas")
```

```
[Out] 1/4*b^3*x^4 + 9/11*a*b^2*x^(11/3) + 9/10*a^2*b*x^(10/3) + 1/3*a^3*x^3
```

giac [A] time = 0.16, size = 35, normalized size = 0.74

$$\frac{1}{4}\,b^3x^4+\frac{9}{11}\,ab^2x^{\frac{11}{3}}+\frac{9}{10}\,a^2bx^{\frac{10}{3}}+\frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^3*x^2,x, algorithm="giac")
```

```
[Out] 1/4*b^3*x^4 + 9/11*a*b^2*x^(11/3) + 9/10*a^2*b*x^(10/3) + 1/3*a^3*x^3
```

maple [A] time = 0.00, size = 36, normalized size = 0.77

$$\frac{b^3x^4}{4}+\frac{9a\,b^2x^{\frac{11}{3}}}{11}+\frac{9a^2b\,x^{\frac{10}{3}}}{10}+\frac{a^3x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^3*x^2,x)
```

```
[Out] 1/3*a^3*x^3+9/10*a^2*b*x^(10/3)+9/11*a*b^2*x^(11/3)+1/4*b^3*x^4
```

maxima [B] time = 0.90, size = 149, normalized size = 3.17

$$\frac{\left(bx^{\frac{1}{3}}+a\right)^{12}}{4\,b^9}-\frac{24\left(bx^{\frac{1}{3}}+a\right)^{11}a}{11\,b^9}+\frac{42\left(bx^{\frac{1}{3}}+a\right)^{10}a^2}{5\,b^9}-\frac{56\left(bx^{\frac{1}{3}}+a\right)^{9}a^3}{3\,b^9}+\frac{105\left(bx^{\frac{1}{3}}+a\right)^{8}a^4}{4\,b^9}-\frac{24\left(bx^{\frac{1}{3}}+a\right)^{7}a^5}{b^9}+\frac{14\left(bx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^3*x^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^(1/3) + a)^12/b^9 - 24/11*(b*x^(1/3) + a)^11*a/b^9 + 42/5*(b*x^(1/
3) + a)^10*a^2/b^9 - 56/3*(b*x^(1/3) + a)^9*a^3/b^9 + 105/4*(b*x^(1/3) + a)
^8*a^4/b^9 - 24*(b*x^(1/3) + a)^7*a^5/b^9 + 14*(b*x^(1/3) + a)^6*a^6/b^9 -
24/5*(b*x^(1/3) + a)^5*a^7/b^9 + 3/4*(b*x^(1/3) + a)^4*a^8/b^9
```

mupad [B] time = 0.04, size = 35, normalized size = 0.74

$$\frac{a^3\,x^3}{3}+\frac{b^3\,x^4}{4}+\frac{9\,a^2\,b\,x^{10/3}}{10}+\frac{9\,a\,b^2\,x^{11/3}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^(1/3))^3,x)
```

```
[Out] (a^3*x^3)/3 + (b^3*x^4)/4 + (9*a^2*b*x^(10/3))/10 + (9*a*b^2*x^(11/3))/11
```

sympy [A] time = 1.88, size = 42, normalized size = 0.89

$$\frac{a^3x^3}{3}+\frac{9a^2bx^{\frac{10}{3}}}{10}+\frac{9ab^2x^{\frac{11}{3}}}{11}+\frac{b^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**3*x**2,x)
```

```
[Out] a**3*x**3/3 + 9*a**2*b*x**(10/3)/10 + 9*a*b**2*x**(11/3)/11 + b**3*x**4/4
```

3.2306 $\int \left(a + b\sqrt[3]{x}\right)^3 x\,dx$

Optimal. Leaf size=47

$$\frac{a^3x^2}{2} + \frac{9}{7}a^2bx^{7/3} + \frac{9}{8}ab^2x^{8/3} + \frac{b^3x^3}{3}$$

```
[Out] 1/2*a^3*x^2+9/7*a^2*b*x^(7/3)+9/8*a*b^2*x^(8/3)+1/3*b^3*x^3
```

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{9}{7}a^2bx^{7/3} + \frac{a^3x^2}{2} + \frac{9}{8}ab^2x^{8/3} + \frac{b^3x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^3*x,x]
```

```
[Out] (a^3*x^2)/2 + (9*a^2*b*x^(7/3))/7 + (9*a*b^2*x^(8/3))/8 + (b^3*x^3)/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \left(a + b\sqrt[3]{x}\right)^3 x\,dx &= 3\,\text{Subst}\left(\int x^5(a + bx)^3\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(a^3x^5 + 3a^2bx^6 + 3ab^2x^7 + b^3x^8\right)\,dx, x, \sqrt[3]{x}\right)\\ &= \frac{a^3x^2}{2} + \frac{9}{7}a^2bx^{7/3} + \frac{9}{8}ab^2x^{8/3} + \frac{b^3x^3}{3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 1.00

$$\frac{a^3x^2}{2} + \frac{9}{7}a^2bx^{7/3} + \frac{9}{8}ab^2x^{8/3} + \frac{b^3x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^3*x,x]
```

```
[Out] (a^3*x^2)/2 + (9*a^2*b*x^(7/3))/7 + (9*a*b^2*x^(8/3))/8 + (b^3*x^3)/3
```

fricas [A] time = 0.54, size = 35, normalized size = 0.74

$$\frac{1}{3}\,b^3x^3 + \frac{9}{8}\,ab^2x^{\frac{8}{3}} + \frac{9}{7}\,a^2bx^{\frac{7}{3}} + \frac{1}{2}\,a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^3*x,x, algorithm="fricas")
```

```
[Out] 1/3*b^3*x^3 + 9/8*a*b^2*x^(8/3) + 9/7*a^2*b*x^(7/3) + 1/2*a^3*x^2
```

giac [A] time = 0.16, size = 35, normalized size = 0.74

$$\frac{1}{3}b^3x^3+\frac{9}{8}ab^2x^{\frac{8}{3}}+\frac{9}{7}a^2bx^{\frac{7}{3}}+\frac{1}{2}a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^3*x,x, algorithm="giac")
```

```
[Out] 1/3*b^3*x^3 + 9/8*a*b^2*x^(8/3) + 9/7*a^2*b*x^(7/3) + 1/2*a^3*x^2
```

maple [A] time = 0.00, size = 36, normalized size = 0.77

$$\frac{b^3x^3}{3}+\frac{9a\,b^2x^{\frac{8}{3}}}{8}+\frac{9a^2b\,x^{\frac{7}{3}}}{7}+\frac{a^3x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^3*x,x)
```

```
[Out] 1/2*a^3*x^2+9/7*a^2*b*x^(7/3)+9/8*a*b^2*x^(8/3)+1/3*b^3*x^3
```

maxima [B] time = 0.90, size = 98, normalized size = 2.09

$$\frac{\left(bx^{\frac{1}{3}}+a\right)^9}{3\,b^6}-\frac{15\left(bx^{\frac{1}{3}}+a\right)^8a}{8\,b^6}+\frac{30\left(bx^{\frac{1}{3}}+a\right)^7a^2}{7\,b^6}-\frac{5\left(bx^{\frac{1}{3}}+a\right)^6a^3}{b^6}+\frac{3\left(bx^{\frac{1}{3}}+a\right)^5a^4}{b^6}-\frac{3\left(bx^{\frac{1}{3}}+a\right)^4a^5}{4\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^3*x,x, algorithm="maxima")
```

```
[Out] 1/3*(b*x^(1/3) + a)^9/b^6 - 15/8*(b*x^(1/3) + a)^8*a/b^6 + 30/7*(b*x^(1/3)
+ a)^7*a^2/b^6 - 5*(b*x^(1/3) + a)^6*a^3/b^6 + 3*(b*x^(1/3) + a)^5*a^4/b^6
- 3/4*(b*x^(1/3) + a)^4*a^5/b^6
```

mupad [B] time = 0.04, size = 35, normalized size = 0.74

$$\frac{a^3\,x^2}{2}+\frac{b^3\,x^3}{3}+\frac{9\,a^2\,b\,x^{7/3}}{7}+\frac{9\,a\,b^2\,x^{8/3}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^(1/3))^3,x)
```

```
[Out] (a^3*x^2)/2 + (b^3*x^3)/3 + (9*a^2*b*x^(7/3))/7 + (9*a*b^2*x^(8/3))/8
```

sympy [A] time = 1.73, size = 42, normalized size = 0.89

$$\frac{a^3x^2}{2}+\frac{9a^2bx^{\frac{7}{3}}}{7}+\frac{9ab^2x^{\frac{8}{3}}}{8}+\frac{b^3x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**3*x,x)
```

```
[Out] a**3*x**2/2 + 9*a**2*b*x**(7/3)/7 + 9*a*b**2*x**(8/3)/8 + b**3*x**3/3
```

3.2307 $\int \left(a + b\sqrt[3]{x}\right)^3 \, dx$

Optimal. Leaf size=42

$$a^3x + \frac{9}{4}a^2bx^{4/3} + \frac{9}{5}ab^2x^{5/3} + \frac{b^3x^2}{2}$$

```
[Out] a^3*x+9/4*a^2*b*x^(4/3)+9/5*a*b^2*x^(5/3)+1/2*b^3*x^2
```

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {190, 43}

$$\frac{9}{4}a^2bx^{4/3} + a^3x + \frac{9}{5}ab^2x^{5/3} + \frac{b^3x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^3,x]
```

```
[Out] a^3*x + (9*a^2*b*x^(4/3))/4 + (9*a*b^2*x^(5/3))/5 + (b^3*x^2)/2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}
\int \left(a + b\sqrt[3]{x}\right)^3 \, dx &= 3\,\text{Subst}\left(\int x^2(a + bx)^3 \, dx, x, \sqrt[3]{x}\right) \\
&= 3\,\text{Subst}\left(\int \left(a^3x^2 + 3a^2bx^3 + 3ab^2x^4 + b^3x^5\right) \, dx, x, \sqrt[3]{x}\right) \\
&= a^3x + \frac{9}{4}a^2bx^{4/3} + \frac{9}{5}ab^2x^{5/3} + \frac{b^3x^2}{2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 1.00

$$a^3x + \frac{9}{4}a^2bx^{4/3} + \frac{9}{5}ab^2x^{5/3} + \frac{b^3x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^3,x]
```

```
[Out] a^3*x + (9*a^2*b*x^(4/3))/4 + (9*a*b^2*x^(5/3))/5 + (b^3*x^2)/2
```

fricas [A] time = 0.77, size = 32, normalized size = 0.76

$$\frac{1}{2}\,b^3x^2 + \frac{9}{5}\,ab^2x^{\frac{5}{3}} + \frac{9}{4}\,a^2bx^{\frac{4}{3}} + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^3,x, algorithm="fricas")
[Out] 1/2*b^3*x^2 + 9/5*a*b^2*x^(5/3) + 9/4*a^2*b*x^(4/3) + a^3*x
```

giac [A] time = 0.16, size = 32, normalized size = 0.76

$$\frac{1}{2}b^3x^2 + \frac{9}{5}ab^2x^{\frac{5}{3}} + \frac{9}{4}a^2bx^{\frac{4}{3}} + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^3,x, algorithm="giac")
[Out] 1/2*b^3*x^2 + 9/5*a*b^2*x^(5/3) + 9/4*a^2*b*x^(4/3) + a^3*x
```

maple [A] time = 0.00, size = 33, normalized size = 0.79

$$\frac{b^3x^2}{2} + \frac{9a\,b^2x^{\frac{5}{3}}}{5} + \frac{9a^2b\,x^{\frac{4}{3}}}{4} + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^3,x)
[Out] a^3*x+9/4*a^2*b*x^(4/3)+9/5*a*b^2*x^(5/3)+1/2*b^3*x^2
```

maxima [A] time = 0.82, size = 32, normalized size = 0.76

$$\frac{1}{2}b^3x^2 + \frac{9}{5}ab^2x^{\frac{5}{3}} + \frac{9}{4}a^2bx^{\frac{4}{3}} + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^3,x, algorithm="maxima")
[Out] 1/2*b^3*x^2 + 9/5*a*b^2*x^(5/3) + 9/4*a^2*b*x^(4/3) + a^3*x
```

mupad [B] time = 0.04, size = 32, normalized size = 0.76

$$a^3\,x + \frac{b^3\,x^2}{2} + \frac{9\,a^2\,b\,x^{4/3}}{4} + \frac{9\,a\,b^2\,x^{5/3}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))^3,x)
[Out] a^3*x + (b^3*x^2)/2 + (9*a^2*b*x^(4/3))/4 + (9*a*b^2*x^(5/3))/5
```

sympy [A] time = 1.60, size = 39, normalized size = 0.93

$$a^3x + \frac{9a^2bx^{\frac{4}{3}}}{4} + \frac{9ab^2x^{\frac{5}{3}}}{5} + \frac{b^3x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**3,x)
[Out] a**3*x + 9*a**2*b*x**(4/3)/4 + 9*a*b**2*x**(5/3)/5 + b**3*x**2/2
```

3.2308 $\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x}\,dx$

Optimal. Leaf size=36

$$a^3\log(x)+9a^2b\sqrt[3]{x}+\frac{9}{2}ab^2x^{2/3}+b^3x$$

```
[Out] 9*a^2*b*x^(1/3)+9/2*a*b^2*x^(2/3)+b^3*x+a^3*ln(x)
```

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$9a^2b\sqrt[3]{x}+a^3\log(x)+\frac{9}{2}ab^2x^{2/3}+b^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^3/x,x]
```

```
[Out] 9*a^2*b*x^(1/3) + (9*a*b^2*x^(2/3))/2 + b^3*x + a^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x}\,dx &= 3\,\text{Subst}\left(\int \frac{(a+bx)^3}{x}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(3a^2b+\frac{a^3}{x}+3ab^2x+b^3x^2\right)dx, x, \sqrt[3]{x}\right)\\ &= 9a^2b\sqrt[3]{x}+\frac{9}{2}ab^2x^{2/3}+b^3x+a^3\log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 1.00

$$a^3\log(x)+9a^2b\sqrt[3]{x}+\frac{9}{2}ab^2x^{2/3}+b^3x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^3/x,x]
```

```
[Out] 9*a^2*b*x^(1/3) + (9*a*b^2*x^(2/3))/2 + b^3*x + a^3*Log[x]
```

fricas [A] time = 0.45, size = 33, normalized size = 0.92

$$b^3x+3\,a^3\log\left(x^{\frac{1}{3}}\right)+\frac{9}{2}\,ab^2x^{\frac{2}{3}}+9\,a^2bx^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^3/x,x, algorithm="fricas")
[Out] b^3*x + 3*a^3*log(x^(1/3)) + 9/2*a*b^2*x^(2/3) + 9*a^2*b*x^(1/3)
```

giac [A] time = 0.15, size = 31, normalized size = 0.86

$$b^3x + a^3\log(|x|) + \frac{9}{2}ab^2x^{\frac{2}{3}} + 9a^2bx^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^3/x,x, algorithm="giac")
[Out] b^3*x + a^3*log(abs(x)) + 9/2*a*b^2*x^(2/3) + 9*a^2*b*x^(1/3)
```

maple [A] time = 0.00, size = 31, normalized size = 0.86

$$a^3\ln(x) + b^3x + \frac{9a\,b^2x^{\frac{2}{3}}}{2} + 9a^2b\,x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^3/x,x)
[Out] 9*a^2*b*x^(1/3)+9/2*a*b^2*x^(2/3)+b^3*x+a^3*ln(x)
```

maxima [A] time = 0.88, size = 30, normalized size = 0.83

$$b^3x + a^3\log(x) + \frac{9}{2}ab^2x^{\frac{2}{3}} + 9a^2bx^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^3/x,x, algorithm="maxima")
[Out] b^3*x + a^3*log(x) + 9/2*a*b^2*x^(2/3) + 9*a^2*b*x^(1/3)
```

mupad [B] time = 0.04, size = 33, normalized size = 0.92

$$3\,a^3\,\ln\left(x^{1/3}\right) + b^3\,x + 9\,a^2\,b\,x^{1/3} + \frac{9\,a\,b^2\,x^{2/3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))^3/x,x)
[Out] 3*a^3*log(x^(1/3)) + b^3*x + 9*a^2*b*x^(1/3) + (9*a*b^2*x^(2/3))/2
```

sympy [A] time = 0.25, size = 36, normalized size = 1.00

$$a^3\log(x) + 9a^2b\sqrt[3]{x} + \frac{9ab^2x^{\frac{2}{3}}}{2} + b^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**3/x,x)
[Out] a**3*log(x) + 9*a**2*b*x**(1/3) + 9*a*b**2*x**(2/3)/2 + b**3*x
```

3.2309 $\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^2}\,dx$

Optimal. Leaf size=39

$$-\frac{a^3}{x}-\frac{9a^2b}{2x^{2/3}}-\frac{9ab^2}{\sqrt[3]{x}}+b^3\log(x)$$

```
[Out] -a^3/x-9/2*a^2*b/x^(2/3)-9*a*b^2/x^(1/3)+b^3*ln(x)
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{9a^2b}{2x^{2/3}}-\frac{a^3}{x}-\frac{9ab^2}{\sqrt[3]{x}}+b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^3/x^2,x]
```

```
[Out] -(a^3/x) - (9*a^2*b)/(2*x^(2/3)) - (9*a*b^2)/x^(1/3) + b^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^2}\,dx &= 3\,\text{Subst}\left(\int \frac{(a+bx)^3}{x^4}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(\frac{a^3}{x^4}+\frac{3a^2b}{x^3}+\frac{3ab^2}{x^2}+\frac{b^3}{x}\right)dx, x, \sqrt[3]{x}\right)\\ &= -\frac{a^3}{x}-\frac{9a^2b}{2x^{2/3}}-\frac{9ab^2}{\sqrt[3]{x}}+b^3\log(x)\end{aligned}$$

Mathematica [A] time = 0.03, size = 39, normalized size = 1.00

$$-\frac{a^3}{x}-\frac{9a^2b}{2x^{2/3}}-\frac{9ab^2}{\sqrt[3]{x}}+b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^3/x^2,x]
```

```
[Out] -(a^3/x) - (9*a^2*b)/(2*x^(2/3)) - (9*a*b^2)/x^(1/3) + b^3*Log[x]
```

fricas [A] time = 0.64, size = 39, normalized size = 1.00

$$\frac{6\,b^3 x \log\left(x^{\frac{1}{3}}\right) - 18\,ab^2 x^{\frac{2}{3}} - 9\,a^2 b x^{\frac{1}{3}} - 2\,a^3}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^3/x^2,x, algorithm="fricas")
[Out] 1/2*(6*b^3*x*log(x^(1/3)) - 18*a*b^2*x^(2/3) - 9*a^2*b*x^(1/3) - 2*a^3)/x
```

giac [A] time = 0.19, size = 37, normalized size = 0.95

$$b^3 \log\left(|x|\right) - \frac{18\,ab^2 x^{\frac{2}{3}} + 9\,a^2 b x^{\frac{1}{3}} + 2\,a^3}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^3/x^2,x, algorithm="giac")
[Out] b^3*log(abs(x)) - 1/2*(18*a*b^2*x^(2/3) + 9*a^2*b*x^(1/3) + 2*a^3)/x
```

maple [A] time = 0.01, size = 34, normalized size = 0.87

$$b^3 \ln(x) - \frac{9a\,b^2}{x^{\frac{1}{3}}} - \frac{9a^2 b}{2x^{\frac{2}{3}}} - \frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^3/x^2,x)
[Out] -a^3/x-9/2*a^2*b/x^(2/3)-9*a*b^2/x^(1/3)+b^3*ln(x)
```

maxima [A] time = 0.90, size = 36, normalized size = 0.92

$$b^3 \log(x) - \frac{18\,ab^2 x^{\frac{2}{3}} + 9\,a^2 b x^{\frac{1}{3}} + 2\,a^3}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^3/x^2,x, algorithm="maxima")
[Out] b^3*log(x) - 1/2*(18*a*b^2*x^(2/3) + 9*a^2*b*x^(1/3) + 2*a^3)/x
```

mupad [B] time = 0.04, size = 37, normalized size = 0.95

$$3\,b^3\,\ln\left(x^{1/3}\right) - \frac{a^3 + \frac{9\,a^2\,b\,x^{1/3}}{2} + 9\,a\,b^2\,x^{2/3}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))^3/x^2,x)
[Out] 3*b^3*log(x^(1/3)) - (a^3 + (9*a^2*b*x^(1/3))/2 + 9*a*b^2*x^(2/3))/x
```

sympy [A] time = 0.65, size = 36, normalized size = 0.92

$$-\frac{a^3}{x} - \frac{9a^2 b}{2x^{\frac{2}{3}}} - \frac{9ab^2}{\sqrt[3]{x}} + b^3 \log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**3/x**2,x)
[Out] -a**3/x - 9*a**2*b/(2*x**(2/3)) - 9*a*b**2/x**(1/3) + b**3*log(x)
```

3.2310 $\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^3}\,dx$

Optimal. Leaf size=45

$$-\frac{a^3}{2x^2}-\frac{9a^2b}{5x^{5/3}}-\frac{9ab^2}{4x^{4/3}}-\frac{b^3}{x}$$

```
[Out] -1/2*a^3/x^2-9/5*a^2*b/x^(5/3)-9/4*a*b^2/x^(4/3)-b^3/x
```

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{9a^2b}{5x^{5/3}}-\frac{a^3}{2x^2}-\frac{9ab^2}{4x^{4/3}}-\frac{b^3}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^3/x^3,x]
```

```
[Out] -a^3/(2*x^2) - (9*a^2*b)/(5*x^(5/3)) - (9*a*b^2)/(4*x^(4/3)) - b^3/x
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^3}\,dx &= 3\,\text{Subst}\left(\int \frac{(a+bx)^3}{x^7}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(\frac{a^3}{x^7}+\frac{3a^2b}{x^6}+\frac{3ab^2}{x^5}+\frac{b^3}{x^4}\right)dx, x, \sqrt[3]{x}\right)\\ &= -\frac{a^3}{2x^2}-\frac{9a^2b}{5x^{5/3}}-\frac{9ab^2}{4x^{4/3}}-\frac{b^3}{x}\end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 0.91

$$-\frac{10a^3+36a^2b\sqrt[3]{x}+45ab^2x^{2/3}+20b^3x}{20x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^3/x^3,x]
```

```
[Out] -1/20*(10*a^3 + 36*a^2*b*x^(1/3) + 45*a*b^2*x^(2/3) + 20*b^3*x)/x^2
```

fricas [A] time = 0.58, size = 35, normalized size = 0.78

$$-\frac{20\,b^3x+45\,ab^2x^{\frac{2}{3}}+36\,a^2bx^{\frac{1}{3}}+10\,a^3}{20\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^3/x^3,x, algorithm="fricas")
[Out] -1/20*(20*b^3*x + 45*a*b^2*x^(2/3) + 36*a^2*b*x^(1/3) + 10*a^3)/x^2
```

giac [A] time = 0.21, size = 35, normalized size = 0.78

$$-\frac{20\,b^3x+45\,ab^2x^{\frac{2}{3}}+36\,a^2bx^{\frac{1}{3}}+10\,a^3}{20\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^3/x^3,x, algorithm="giac")
[Out] -1/20*(20*b^3*x + 45*a*b^2*x^(2/3) + 36*a^2*b*x^(1/3) + 10*a^3)/x^2
```

maple [A] time = 0.01, size = 36, normalized size = 0.80

$$-\frac{b^3}{x}-\frac{9a\,b^2}{4x^{\frac{4}{3}}}-\frac{9a^2b}{5x^{\frac{5}{3}}}-\frac{a^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^3/x^3,x)
[Out] -1/2*a^3/x^2-9/5*a^2*b/x^(5/3)-9/4*a*b^2/x^(4/3)-b^3/x
```

maxima [A] time = 0.86, size = 35, normalized size = 0.78

$$-\frac{20\,b^3x+45\,ab^2x^{\frac{2}{3}}+36\,a^2bx^{\frac{1}{3}}+10\,a^3}{20\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^3/x^3,x, algorithm="maxima")
[Out] -1/20*(20*b^3*x + 45*a*b^2*x^(2/3) + 36*a^2*b*x^(1/3) + 10*a^3)/x^2
```

mupad [B] time = 1.11, size = 35, normalized size = 0.78

$$-\frac{20\,b^3\,x+10\,a^3+36\,a^2\,b\,x^{1/3}+45\,a\,b^2\,x^{2/3}}{20\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))^3/x^3,x)
[Out] -(20*b^3*x + 10*a^3 + 36*a^2*b*x^(1/3) + 45*a*b^2*x^(2/3))/(20*x^2)
```

sympy [A] time = 1.16, size = 41, normalized size = 0.91

$$-\frac{a^3}{2x^2}-\frac{9a^2b}{5x^{\frac{5}{3}}}-\frac{9ab^2}{4x^{\frac{4}{3}}}-\frac{b^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**3/x**3,x)
[Out] -a**3/(2*x**2) - 9*a**2*b/(5*x**(5/3)) - 9*a*b**2/(4*x**(4/3)) - b**3/x
```

3.2311 $\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^4}\,dx$

Optimal. Leaf size=47

$$-\frac{a^3}{3x^3}-\frac{9a^2b}{8x^{8/3}}-\frac{9ab^2}{7x^{7/3}}-\frac{b^3}{2x^2}$$

```
[Out] -1/3*a^3/x^3-9/8*a^2*b/x^(8/3)-9/7*a*b^2/x^(7/3)-1/2*b^3/x^2
```

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{9a^2b}{8x^{8/3}}-\frac{a^3}{3x^3}-\frac{9ab^2}{7x^{7/3}}-\frac{b^3}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^3/x^4,x]
```

```
[Out] -a^3/(3*x^3) - (9*a^2*b)/(8*x^(8/3)) - (9*a*b^2)/(7*x^(7/3)) - b^3/(2*x^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^4}\,dx &= 3\,\text{Subst}\left(\int \frac{(a+bx)^3}{x^{10}}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(\frac{a^3}{x^{10}}+\frac{3a^2b}{x^9}+\frac{3ab^2}{x^8}+\frac{b^3}{x^7}\right)dx, x, \sqrt[3]{x}\right)\\ &= -\frac{a^3}{3x^3}-\frac{9a^2b}{8x^{8/3}}-\frac{9ab^2}{7x^{7/3}}-\frac{b^3}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 0.87

$$-\frac{56a^3+189a^2b\sqrt[3]{x}+216ab^2x^{2/3}+84b^3x}{168x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^3/x^4,x]
```

```
[Out] -1/168*(56*a^3 + 189*a^2*b*x^(1/3) + 216*a*b^2*x^(2/3) + 84*b^3*x)/x^3
```

fricas [A] time = 0.71, size = 35, normalized size = 0.74

$$-\frac{84\,b^3x+216\,ab^2x^{\frac{2}{3}}+189\,a^2bx^{\frac{1}{3}}+56\,a^3}{168\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^3/x^4,x, algorithm="fricas")
[Out] -1/168*(84*b^3*x + 216*a*b^2*x^(2/3) + 189*a^2*b*x^(1/3) + 56*a^3)/x^3
```

giac [A] time = 0.16, size = 35, normalized size = 0.74

$$-\frac{84\,b^3x+216\,ab^2x^{\frac{2}{3}}+189\,a^2bx^{\frac{1}{3}}+56\,a^3}{168\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^3/x^4,x, algorithm="giac")
[Out] -1/168*(84*b^3*x + 216*a*b^2*x^(2/3) + 189*a^2*b*x^(1/3) + 56*a^3)/x^3
```

maple [A] time = 0.01, size = 36, normalized size = 0.77

$$-\frac{b^3}{2x^2}-\frac{9a\,b^2}{7x^{\frac{7}{3}}}-\frac{9a^2b}{8x^{\frac{8}{3}}}-\frac{a^3}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^3/x^4,x)
[Out] -1/3*a^3/x^3-9/8*a^2*b/x^(8/3)-9/7*a*b^2/x^(7/3)-1/2*b^3/x^2
```

maxima [A] time = 0.81, size = 35, normalized size = 0.74

$$-\frac{84\,b^3x+216\,ab^2x^{\frac{2}{3}}+189\,a^2bx^{\frac{1}{3}}+56\,a^3}{168\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^3/x^4,x, algorithm="maxima")
[Out] -1/168*(84*b^3*x + 216*a*b^2*x^(2/3) + 189*a^2*b*x^(1/3) + 56*a^3)/x^3
```

mupad [B] time = 1.11, size = 35, normalized size = 0.74

$$-\frac{84\,b^3\,x+56\,a^3+189\,a^2\,b\,x^{1/3}+216\,a\,b^2\,x^{2/3}}{168\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))^3/x^4,x)
[Out] -(84*b^3*x + 56*a^3 + 189*a^2*b*x^(1/3) + 216*a*b^2*x^(2/3))/(168*x^3)
```

sympy [A] time = 1.95, size = 44, normalized size = 0.94

$$-\frac{a^3}{3x^3}-\frac{9a^2b}{8x^{\frac{8}{3}}}-\frac{9ab^2}{7x^{\frac{7}{3}}}-\frac{b^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**3/x**4,x)
[Out] -a**3/(3*x**3) - 9*a**2*b/(8*x**(8/3)) - 9*a*b**2/(7*x**(7/3)) - b**3/(2*x*
*2)
```

3.2312 $\int \left(a + b\sqrt[3]{x}\right)^5 x^4\, dx$

Optimal. Leaf size=77

$$\frac{a^5x^5}{5} + \frac{15}{16}a^4bx^{16/3} + \frac{30}{17}a^3b^2x^{17/3} + \frac{5}{3}a^2b^3x^6 + \frac{15}{19}ab^4x^{19/3} + \frac{3}{20}b^5x^{20/3}$$

```
[Out] 1/5*a^5*x^5+15/16*a^4*b*x^(16/3)+30/17*a^3*b^2*x^(17/3)+5/3*a^2*b^3*x^6+15/
19*a*b^4*x^(19/3)+3/20*b^5*x^(20/3)
```

Rubi [A] time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{5}{3}a^2b^3x^6 + \frac{30}{17}a^3b^2x^{17/3} + \frac{15}{16}a^4bx^{16/3} + \frac{a^5x^5}{5} + \frac{15}{19}ab^4x^{19/3} + \frac{3}{20}b^5x^{20/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^5*x^4,x]
```

```
[Out] (a^5*x^5)/5 + (15*a^4*b*x^(16/3))/16 + (30*a^3*b^2*x^(17/3))/17 + (5*a^2*b^
3*x^6)/3 + (15*a*b^4*x^(19/3))/19 + (3*b^5*x^(20/3))/20
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a + b\sqrt[3]{x}\right)^5 x^4\, dx &= 3\,\text{Subst}\left(\int x^{14}(a + bx)^5\, dx, x, \sqrt[3]{x}\right) \\
&= 3\,\text{Subst}\left(\int \left(a^5x^{14} + 5a^4bx^{15} + 10a^3b^2x^{16} + 10a^2b^3x^{17} + 5ab^4x^{18} + b^5x^{19}\right) dx, x, \sqrt[3]{x}\right) \\
&= \frac{a^5x^5}{5} + \frac{15}{16}a^4bx^{16/3} + \frac{30}{17}a^3b^2x^{17/3} + \frac{5}{3}a^2b^3x^6 + \frac{15}{19}ab^4x^{19/3} + \frac{3}{20}b^5x^{20/3}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 77, normalized size = 1.00

$$\frac{a^5x^5}{5} + \frac{15}{16}a^4bx^{16/3} + \frac{30}{17}a^3b^2x^{17/3} + \frac{5}{3}a^2b^3x^6 + \frac{15}{19}ab^4x^{19/3} + \frac{3}{20}b^5x^{20/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^5*x^4,x]
```

```
[Out] (a^5*x^5)/5 + (15*a^4*b*x^(16/3))/16 + (30*a^3*b^2*x^(17/3))/17 + (5*a^2*b^
3*x^6)/3 + (15*a*b^4*x^(19/3))/19 + (3*b^5*x^(20/3))/20
```

fricas [A] time = 0.52, size = 69, normalized size = 0.90

$$\frac{5}{3}a^2b^3x^6+\frac{1}{5}a^5x^5+\frac{3}{340}\left(17b^5x^6+200a^3b^2x^5\right)x^{\frac{2}{3}}+\frac{15}{304}\left(16ab^4x^6+19a^4bx^5\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^5*x^4,x, algorithm="fricas")
```

```
[Out] 5/3*a^2*b^3*x^6 + 1/5*a^5*x^5 + 3/340*(17*b^5*x^6 + 200*a^3*b^2*x^5)*x^(2/3
) + 15/304*(16*a*b^4*x^6 + 19*a^4*b*x^5)*x^(1/3)
```

giac [A] time = 0.15, size = 57, normalized size = 0.74

$$\frac{3}{20}b^5x^{\frac{20}{3}}+\frac{15}{19}ab^4x^{\frac{19}{3}}+\frac{5}{3}a^2b^3x^6+\frac{30}{17}a^3b^2x^{\frac{17}{3}}+\frac{15}{16}a^4bx^{\frac{16}{3}}+\frac{1}{5}a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^5*x^4,x, algorithm="giac")
```

```
[Out] 3/20*b^5*x^(20/3) + 15/19*a*b^4*x^(19/3) + 5/3*a^2*b^3*x^6 + 30/17*a^3*b^2*
x^(17/3) + 15/16*a^4*b*x^(16/3) + 1/5*a^5*x^5
```

maple [A] time = 0.00, size = 58, normalized size = 0.75

$$\frac{3b^5x^{\frac{20}{3}}}{20}+\frac{15ab^4x^{\frac{19}{3}}}{19}+\frac{5a^2b^3x^6}{3}+\frac{30a^3b^2x^{\frac{17}{3}}}{17}+\frac{15a^4bx^{\frac{16}{3}}}{16}+\frac{a^5x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^5*x^4,x)
```

```
[Out] 1/5*a^5*x^5+15/16*a^4*b*x^(16/3)+30/17*a^3*b^2*x^(17/3)+5/3*a^2*b^3*x^6+15/
19*a*b^4*x^(19/3)+3/20*b^5*x^(20/3)
```

maxima [B] time = 0.88, size = 251, normalized size = 3.26

$$\frac{3\left(bx^{\frac{1}{3}}+a\right)^{20}}{20b^{15}}-\frac{42\left(bx^{\frac{1}{3}}+a\right)^{19}a}{19b^{15}}+\frac{91\left(bx^{\frac{1}{3}}+a\right)^{18}a^2}{6b^{15}}-\frac{1092\left(bx^{\frac{1}{3}}+a\right)^{17}a^3}{17b^{15}}+\frac{3003\left(bx^{\frac{1}{3}}+a\right)^{16}a^4}{16b^{15}}-\frac{2002\left(bx^{\frac{1}{3}}+a\right)^{15}}{5b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^5*x^4,x, algorithm="maxima")
```

```
[Out] 3/20*(b*x^(1/3) + a)^20/b^15 - 42/19*(b*x^(1/3) + a)^19*a/b^15 + 91/6*(b*x^
(1/3) + a)^18*a^2/b^15 - 1092/17*(b*x^(1/3) + a)^17*a^3/b^15 + 3003/16*(b*x
^(1/3) + a)^16*a^4/b^15 - 2002/5*(b*x^(1/3) + a)^15*a^5/b^15 + 1287/2*(b*x^
(1/3) + a)^14*a^6/b^15 - 792*(b*x^(1/3) + a)^13*a^7/b^15 + 3003/4*(b*x^(1/3
) + a)^12*a^8/b^15 - 546*(b*x^(1/3) + a)^11*a^9/b^15 + 3003/10*(b*x^(1/3) +
 a)^10*a^10/b^15 - 364/3*(b*x^(1/3) + a)^9*a^11/b^15 + 273/8*(b*x^(1/3) + a
)^8*a^12/b^15 - 6*(b*x^(1/3) + a)^7*a^13/b^15 + 1/2*(b*x^(1/3) + a)^6*a^14/
b^15
```

mupad [B] time = 0.03, size = 57, normalized size = 0.74

$$\frac{a^5x^5}{5}+\frac{3b^5x^{20/3}}{20}+\frac{15a^4bx^{16/3}}{16}+\frac{15ab^4x^{19/3}}{19}+\frac{5a^2b^3x^6}{3}+\frac{30a^3b^2x^{17/3}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^(1/3))^5,x)
```

```
[Out] (a^5*x^5)/5 + (3*b^5*x^(20/3))/20 + (15*a^4*b*x^(16/3))/16 + (15*a*b^4*x^(1
9/3))/19 + (5*a^2*b^3*x^6)/3 + (30*a^3*b^2*x^(17/3))/17
```

sympy [**A**] time = 2.90, size = 75, normalized size = 0.97

$$\frac{a^5x^5}{5} + \frac{15a^4bx^{\frac{16}{3}}}{16} + \frac{30a^3b^2x^{\frac{17}{3}}}{17} + \frac{5a^2b^3x^6}{3} + \frac{15ab^4x^{\frac{19}{3}}}{19} + \frac{3b^5x^{\frac{20}{3}}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**5*x**4,x)
```

```
[Out] a**5*x**5/5 + 15*a**4*b*x**(16/3)/16 + 30*a**3*b**2*x**(17/3)/17 + 5*a**2*b
**3*x**6/3 + 15*a*b**4*x**(19/3)/19 + 3*b**5*x**(20/3)/20
```

3.2313 $\int \left(a + b\sqrt[3]{x}\right)^5 x^3\, dx$

Optimal. Leaf size=75

$$\frac{a^5x^4}{4} + \frac{15}{13}a^4bx^{13/3} + \frac{15}{7}a^3b^2x^{14/3} + 2a^2b^3x^5 + \frac{15}{16}ab^4x^{16/3} + \frac{3}{17}b^5x^{17/3}$$

```
[Out] 1/4*a^5*x^4+15/13*a^4*b*x^(13/3)+15/7*a^3*b^2*x^(14/3)+2*a^2*b^3*x^5+15/16*
a*b^4*x^(16/3)+3/17*b^5*x^(17/3)
```

Rubi [A] time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$2a^2b^3x^5 + \frac{15}{7}a^3b^2x^{14/3} + \frac{15}{13}a^4bx^{13/3} + \frac{a^5x^4}{4} + \frac{15}{16}ab^4x^{16/3} + \frac{3}{17}b^5x^{17/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^5*x^3,x]
```

```
[Out] (a^5*x^4)/4 + (15*a^4*b*x^(13/3))/13 + (15*a^3*b^2*x^(14/3))/7 + 2*a^2*b^3*
x^5 + (15*a*b^4*x^(16/3))/16 + (3*b^5*x^(17/3))/17
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a + b\sqrt[3]{x}\right)^5 x^3\, dx &= 3\,\text{Subst}\left(\int x^{11}(a + bx)^5\, dx, x, \sqrt[3]{x}\right) \\
&= 3\,\text{Subst}\left(\int \left(a^5x^{11} + 5a^4bx^{12} + 10a^3b^2x^{13} + 10a^2b^3x^{14} + 5ab^4x^{15} + b^5x^{16}\right) dx, x, \sqrt[3]{x}\right) \\
&= \frac{a^5x^4}{4} + \frac{15}{13}a^4bx^{13/3} + \frac{15}{7}a^3b^2x^{14/3} + 2a^2b^3x^5 + \frac{15}{16}ab^4x^{16/3} + \frac{3}{17}b^5x^{17/3}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 75, normalized size = 1.00

$$\frac{a^5x^4}{4} + \frac{15}{13}a^4bx^{13/3} + \frac{15}{7}a^3b^2x^{14/3} + 2a^2b^3x^5 + \frac{15}{16}ab^4x^{16/3} + \frac{3}{17}b^5x^{17/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^5*x^3,x]
```

```
[Out] (a^5*x^4)/4 + (15*a^4*b*x^(13/3))/13 + (15*a^3*b^2*x^(14/3))/7 + 2*a^2*b^3*
x^5 + (15*a*b^4*x^(16/3))/16 + (3*b^5*x^(17/3))/17
```

fricas [A] time = 0.56, size = 69, normalized size = 0.92

$$2a^2b^3x^5 + \frac{1}{4}a^5x^4 + \frac{3}{119}\left(7b^5x^5 + 85a^3b^2x^4\right)x^{\frac{2}{3}} + \frac{15}{208}\left(13ab^4x^5 + 16a^4bx^4\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^5*x^3,x, algorithm="fricas")
```

```
[Out] 2*a^2*b^3*x^5 + 1/4*a^5*x^4 + 3/119*(7*b^5*x^5 + 85*a^3*b^2*x^4)*x^(2/3) +
15/208*(13*a*b^4*x^5 + 16*a^4*b*x^4)*x^(1/3)
```

giac [A] time = 0.17, size = 57, normalized size = 0.76

$$\frac{3}{17}b^5x^{\frac{17}{3}} + \frac{15}{16}ab^4x^{\frac{16}{3}} + 2a^2b^3x^5 + \frac{15}{7}a^3b^2x^{\frac{14}{3}} + \frac{15}{13}a^4bx^{\frac{13}{3}} + \frac{1}{4}a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^5*x^3,x, algorithm="giac")
```

```
[Out] 3/17*b^5*x^(17/3) + 15/16*a*b^4*x^(16/3) + 2*a^2*b^3*x^5 + 15/7*a^3*b^2*x^(
14/3) + 15/13*a^4*b*x^(13/3) + 1/4*a^5*x^4
```

maple [A] time = 0.00, size = 58, normalized size = 0.77

$$\frac{3b^5x^{\frac{17}{3}}}{17} + \frac{15a\,b^4x^{\frac{16}{3}}}{16} + 2a^2b^3x^5 + \frac{15a^3b^2x^{\frac{14}{3}}}{7} + \frac{15a^4b\,x^{\frac{13}{3}}}{13} + \frac{a^5x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^5*x^3,x)
```

```
[Out] 1/4*a^5*x^4+15/13*a^4*b*x^(13/3)+15/7*a^3*b^2*x^(14/3)+2*a^2*b^3*x^5+15/16*
a*b^4*x^(16/3)+3/17*b^5*x^(17/3)
```

maxima [B] time = 0.89, size = 200, normalized size = 2.67

$$\frac{3\left(bx^{\frac{1}{3}}+a\right)^{17}}{17\,b^{12}} - \frac{33\left(bx^{\frac{1}{3}}+a\right)^{16}a}{16\,b^{12}} + \frac{11\left(bx^{\frac{1}{3}}+a\right)^{15}a^2}{b^{12}} - \frac{495\left(bx^{\frac{1}{3}}+a\right)^{14}a^3}{14\,b^{12}} + \frac{990\left(bx^{\frac{1}{3}}+a\right)^{13}a^4}{13\,b^{12}} - \frac{231\left(bx^{\frac{1}{3}}+a\right)^{12}a}{2\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^5*x^3,x, algorithm="maxima")
```

```
[Out] 3/17*(b*x^(1/3) + a)^17/b^12 - 33/16*(b*x^(1/3) + a)^16*a/b^12 + 11*(b*x^(1
/3) + a)^15*a^2/b^12 - 495/14*(b*x^(1/3) + a)^14*a^3/b^12 + 990/13*(b*x^(1/
3) + a)^13*a^4/b^12 - 231/2*(b*x^(1/3) + a)^12*a^5/b^12 + 126*(b*x^(1/3) +
a)^11*a^6/b^12 - 99*(b*x^(1/3) + a)^10*a^7/b^12 + 55*(b*x^(1/3) + a)^9*a^8/
b^12 - 165/8*(b*x^(1/3) + a)^8*a^9/b^12 + 33/7*(b*x^(1/3) + a)^7*a^10/b^12
- 1/2*(b*x^(1/3) + a)^6*a^11/b^12
```

mupad [B] time = 0.03, size = 57, normalized size = 0.76

$$\frac{a^5x^4}{4} + \frac{3b^5x^{17/3}}{17} + \frac{15a^4b\,x^{13/3}}{13} + \frac{15a\,b^4x^{16/3}}{16} + 2a^2b^3x^5 + \frac{15a^3b^2x^{14/3}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^(1/3))^5,x)
```

```
[Out] (a^5*x^4)/4 + (3*b^5*x^(17/3))/17 + (15*a^4*b*x^(13/3))/13 + (15*a*b^4*x^(1
6/3))/16 + 2*a^2*b^3*x^5 + (15*a^3*b^2*x^(14/3))/7
```

sympy [**A**] time = 2.61, size = 73, normalized size = 0.97

$$\frac{a^5x^4}{4} + \frac{15a^4bx^{\frac{13}{3}}}{13} + \frac{15a^3b^2x^{\frac{14}{3}}}{7} + 2a^2b^3x^5 + \frac{15ab^4x^{\frac{16}{3}}}{16} + \frac{3b^5x^{\frac{17}{3}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**5*x**3,x)
```

```
[Out] a**5*x**4/4 + 15*a**4*b*x**(13/3)/13 + 15*a**3*b**2*x**(14/3)/7 + 2*a**2*b*
**3*x**5 + 15*a*b**4*x**(16/3)/16 + 3*b**5*x**(17/3)/17
```

3.2314 $\int \left(a + b\sqrt[3]{x}\right)^5 x^2\, dx$

Optimal. Leaf size=77

$$\frac{a^5x^3}{3} + \frac{3}{2}a^4bx^{10/3} + \frac{30}{11}a^3b^2x^{11/3} + \frac{5}{2}a^2b^3x^4 + \frac{15}{13}ab^4x^{13/3} + \frac{3}{14}b^5x^{14/3}$$

```
[Out] 1/3*a^5*x^3+3/2*a^4*b*x^(10/3)+30/11*a^3*b^2*x^(11/3)+5/2*a^2*b^3*x^4+15/13
*a*b^4*x^(13/3)+3/14*b^5*x^(14/3)
```

Rubi [A] time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{30}{11}a^3b^2x^{11/3} + \frac{5}{2}a^2b^3x^4 + \frac{3}{2}a^4bx^{10/3} + \frac{a^5x^3}{3} + \frac{15}{13}ab^4x^{13/3} + \frac{3}{14}b^5x^{14/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^5*x^2,x]
```

```
[Out] (a^5*x^3)/3 + (3*a^4*b*x^(10/3))/2 + (30*a^3*b^2*x^(11/3))/11 + (5*a^2*b^3*
x^4)/2 + (15*a*b^4*x^(13/3))/13 + (3*b^5*x^(14/3))/14
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a + b\sqrt[3]{x}\right)^5 x^2\, dx &= 3\,\text{Subst}\left(\int x^8(a + bx)^5\, dx, x, \sqrt[3]{x}\right) \\
&= 3\,\text{Subst}\left(\int \left(a^5x^8 + 5a^4bx^9 + 10a^3b^2x^{10} + 10a^2b^3x^{11} + 5ab^4x^{12} + b^5x^{13}\right) dx, x, \sqrt[3]{x}\right) \\
&= \frac{a^5x^3}{3} + \frac{3}{2}a^4bx^{10/3} + \frac{30}{11}a^3b^2x^{11/3} + \frac{5}{2}a^2b^3x^4 + \frac{15}{13}ab^4x^{13/3} + \frac{3}{14}b^5x^{14/3}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 77, normalized size = 1.00

$$\frac{a^5x^3}{3} + \frac{3}{2}a^4bx^{10/3} + \frac{30}{11}a^3b^2x^{11/3} + \frac{5}{2}a^2b^3x^4 + \frac{15}{13}ab^4x^{13/3} + \frac{3}{14}b^5x^{14/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^5*x^2,x]
```

```
[Out] (a^5*x^3)/3 + (3*a^4*b*x^(10/3))/2 + (30*a^3*b^2*x^(11/3))/11 + (5*a^2*b^3*
x^4)/2 + (15*a*b^4*x^(13/3))/13 + (3*b^5*x^(14/3))/14
```

fricas [A] time = 0.68, size = 69, normalized size = 0.90

$$\frac{5}{2}a^2b^3x^4+\frac{1}{3}a^5x^3+\frac{3}{154}\left(11b^5x^4+140a^3b^2x^3\right)x^{\frac{2}{3}}+\frac{3}{26}\left(10ab^4x^4+13a^4bx^3\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^5*x^2,x, algorithm="fricas")
```

```
[Out] 5/2*a^2*b^3*x^4 + 1/3*a^5*x^3 + 3/154*(11*b^5*x^4 + 140*a^3*b^2*x^3)*x^(2/3
) + 3/26*(10*a*b^4*x^4 + 13*a^4*b*x^3)*x^(1/3)
```

giac [A] time = 0.16, size = 57, normalized size = 0.74

$$\frac{3}{14}b^5x^{\frac{14}{3}}+\frac{15}{13}ab^4x^{\frac{13}{3}}+\frac{5}{2}a^2b^3x^4+\frac{30}{11}a^3b^2x^{\frac{11}{3}}+\frac{3}{2}a^4bx^{\frac{10}{3}}+\frac{1}{3}a^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^5*x^2,x, algorithm="giac")
```

```
[Out] 3/14*b^5*x^(14/3) + 15/13*a*b^4*x^(13/3) + 5/2*a^2*b^3*x^4 + 30/11*a^3*b^2*
x^(11/3) + 3/2*a^4*b*x^(10/3) + 1/3*a^5*x^3
```

maple [A] time = 0.00, size = 58, normalized size = 0.75

$$\frac{3b^5x^{\frac{14}{3}}}{14}+\frac{15ab^4x^{\frac{13}{3}}}{13}+\frac{5a^2b^3x^4}{2}+\frac{30a^3b^2x^{\frac{11}{3}}}{11}+\frac{3a^4bx^{\frac{10}{3}}}{2}+\frac{a^5x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^5*x^2,x)
```

```
[Out] 1/3*a^5*x^3+3/2*a^4*b*x^(10/3)+30/11*a^3*b^2*x^(11/3)+5/2*a^2*b^3*x^4+15/13
*a*b^4*x^(13/3)+3/14*b^5*x^(14/3)
```

maxima [B] time = 0.89, size = 149, normalized size = 1.94

$$\frac{3\left(bx^{\frac{1}{3}}+a\right)^{14}}{14b^9}-\frac{24\left(bx^{\frac{1}{3}}+a\right)^{13}a}{13b^9}+\frac{7\left(bx^{\frac{1}{3}}+a\right)^{12}a^2}{b^9}-\frac{168\left(bx^{\frac{1}{3}}+a\right)^{11}a^3}{11b^9}+\frac{21\left(bx^{\frac{1}{3}}+a\right)^{10}a^4}{b^9}-\frac{56\left(bx^{\frac{1}{3}}+a\right)^{9}a^5}{3b^9}+\frac{21\left(b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^5*x^2,x, algorithm="maxima")
```

```
[Out] 3/14*(b*x^(1/3) + a)^14/b^9 - 24/13*(b*x^(1/3) + a)^13*a/b^9 + 7*(b*x^(1/3)
 + a)^12*a^2/b^9 - 168/11*(b*x^(1/3) + a)^11*a^3/b^9 + 21*(b*x^(1/3) + a)^1
0*a^4/b^9 - 56/3*(b*x^(1/3) + a)^9*a^5/b^9 + 21/2*(b*x^(1/3) + a)^8*a^6/b^9
 - 24/7*(b*x^(1/3) + a)^7*a^7/b^9 + 1/2*(b*x^(1/3) + a)^6*a^8/b^9
```

mupad [B] time = 0.03, size = 57, normalized size = 0.74

$$\frac{a^5x^3}{3}+\frac{3b^5x^{14/3}}{14}+\frac{3a^4bx^{10/3}}{2}+\frac{15ab^4x^{13/3}}{13}+\frac{5a^2b^3x^4}{2}+\frac{30a^3b^2x^{11/3}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^(1/3))^5,x)
```

```
[Out] (a^5*x^3)/3 + (3*b^5*x^(14/3))/14 + (3*a^4*b*x^(10/3))/2 + (15*a*b^4*x^(13/
3))/13 + (5*a^2*b^3*x^4)/2 + (30*a^3*b^2*x^(11/3))/11
```

sympy [**A**] time = 2.38, size = 75, normalized size = 0.97

$$\frac{a^5x^3}{3}+\frac{3a^4bx^{\frac{10}{3}}}{2}+\frac{30a^3b^2x^{\frac{11}{3}}}{11}+\frac{5a^2b^3x^4}{2}+\frac{15ab^4x^{\frac{13}{3}}}{13}+\frac{3b^5x^{\frac{14}{3}}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**5*x**2,x)
```

```
[Out] a**5*x**3/3 + 3*a**4*b*x**(10/3)/2 + 30*a**3*b**2*x**(11/3)/11 + 5*a**2*b**
3*x**4/2 + 15*a*b**4*x**(13/3)/13 + 3*b**5*x**(14/3)/14
```

3.2315 $\int \left(a + b\sqrt[3]{x}\right)^5 x\,dx$

Optimal. Leaf size=77

$$\frac{a^5x^2}{2} + \frac{15}{7}a^4bx^{7/3} + \frac{15}{4}a^3b^2x^{8/3} + \frac{10}{3}a^2b^3x^3 + \frac{3}{2}ab^4x^{10/3} + \frac{3}{11}b^5x^{11/3}$$

```
[Out] 1/2*a^5*x^2+15/7*a^4*b*x^(7/3)+15/4*a^3*b^2*x^(8/3)+10/3*a^2*b^3*x^3+3/2*a*
b^4*x^(10/3)+3/11*b^5*x^(11/3)
```

Rubi [A] time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{15}{4}a^3b^2x^{8/3} + \frac{10}{3}a^2b^3x^3 + \frac{15}{7}a^4bx^{7/3} + \frac{a^5x^2}{2} + \frac{3}{2}ab^4x^{10/3} + \frac{3}{11}b^5x^{11/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^5*x,x]
```

```
[Out] (a^5*x^2)/2 + (15*a^4*b*x^(7/3))/7 + (15*a^3*b^2*x^(8/3))/4 + (10*a^2*b^3*x
^3)/3 + (3*a*b^4*x^(10/3))/2 + (3*b^5*x^(11/3))/11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a + b\sqrt[3]{x}\right)^5 x\,dx &= 3\,\text{Subst}\left(\int x^5(a + bx)^5\,dx, x, \sqrt[3]{x}\right) \\
&= 3\,\text{Subst}\left(\int \left(a^5x^5 + 5a^4bx^6 + 10a^3b^2x^7 + 10a^2b^3x^8 + 5ab^4x^9 + b^5x^{10}\right)\,dx, x, \sqrt[3]{x}\right) \\
&= \frac{a^5x^2}{2} + \frac{15}{7}a^4bx^{7/3} + \frac{15}{4}a^3b^2x^{8/3} + \frac{10}{3}a^2b^3x^3 + \frac{3}{2}ab^4x^{10/3} + \frac{3}{11}b^5x^{11/3}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 77, normalized size = 1.00

$$\frac{a^5x^2}{2} + \frac{15}{7}a^4bx^{7/3} + \frac{15}{4}a^3b^2x^{8/3} + \frac{10}{3}a^2b^3x^3 + \frac{3}{2}ab^4x^{10/3} + \frac{3}{11}b^5x^{11/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^5*x,x]
```

```
[Out] (a^5*x^2)/2 + (15*a^4*b*x^(7/3))/7 + (15*a^3*b^2*x^(8/3))/4 + (10*a^2*b^3*x
^3)/3 + (3*a*b^4*x^(10/3))/2 + (3*b^5*x^(11/3))/11
```

fricas [A] time = 0.52, size = 69, normalized size = 0.90

$$\frac{10}{3}a^2b^3x^3+\frac{1}{2}a^5x^2+\frac{3}{44}\left(4b^5x^3+55a^3b^2x^2\right)x^{\frac{2}{3}}+\frac{3}{14}\left(7ab^4x^3+10a^4bx^2\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^5*x,x, algorithm="fricas")
```

```
[Out] 10/3*a^2*b^3*x^3 + 1/2*a^5*x^2 + 3/44*(4*b^5*x^3 + 55*a^3*b^2*x^2)*x^(2/3)
+ 3/14*(7*a*b^4*x^3 + 10*a^4*b*x^2)*x^(1/3)
```

giac [A] time = 0.16, size = 57, normalized size = 0.74

$$\frac{3}{11}b^5x^{\frac{11}{3}}+\frac{3}{2}ab^4x^{\frac{10}{3}}+\frac{10}{3}a^2b^3x^3+\frac{15}{4}a^3b^2x^{\frac{8}{3}}+\frac{15}{7}a^4bx^{\frac{7}{3}}+\frac{1}{2}a^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^5*x,x, algorithm="giac")
```

```
[Out] 3/11*b^5*x^(11/3) + 3/2*a*b^4*x^(10/3) + 10/3*a^2*b^3*x^3 + 15/4*a^3*b^2*x^
(8/3) + 15/7*a^4*b*x^(7/3) + 1/2*a^5*x^2
```

maple [A] time = 0.00, size = 58, normalized size = 0.75

$$\frac{3b^5x^{\frac{11}{3}}}{11}+\frac{3a\,b^4x^{\frac{10}{3}}}{2}+\frac{10a^2b^3x^3}{3}+\frac{15a^3b^2x^{\frac{8}{3}}}{4}+\frac{15a^4b\,x^{\frac{7}{3}}}{7}+\frac{a^5x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^5*x,x)
```

```
[Out] 1/2*a^5*x^2+15/7*a^4*b*x^(7/3)+15/4*a^3*b^2*x^(8/3)+10/3*a^2*b^3*x^3+3/2*a*
b^4*x^(10/3)+3/11*b^5*x^(11/3)
```

maxima [A] time = 0.80, size = 98, normalized size = 1.27

$$\frac{3\left(bx^{\frac{1}{3}}+a\right)^{11}}{11\,b^6}-\frac{3\left(bx^{\frac{1}{3}}+a\right)^{10}a}{2\,b^6}+\frac{10\left(bx^{\frac{1}{3}}+a\right)^{9}a^2}{3\,b^6}-\frac{15\left(bx^{\frac{1}{3}}+a\right)^{8}a^3}{4\,b^6}+\frac{15\left(bx^{\frac{1}{3}}+a\right)^{7}a^4}{7\,b^6}-\frac{\left(bx^{\frac{1}{3}}+a\right)^{6}a^5}{2\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^5*x,x, algorithm="maxima")
```

```
[Out] 3/11*(b*x^(1/3) + a)^11/b^6 - 3/2*(b*x^(1/3) + a)^10*a/b^6 + 10/3*(b*x^(1/3
) + a)^9*a^2/b^6 - 15/4*(b*x^(1/3) + a)^8*a^3/b^6 + 15/7*(b*x^(1/3) + a)^7*
a^4/b^6 - 1/2*(b*x^(1/3) + a)^6*a^5/b^6
```

mupad [B] time = 0.03, size = 57, normalized size = 0.74

$$\frac{a^5\,x^2}{2}+\frac{3\,b^5\,x^{11/3}}{11}+\frac{15\,a^4\,b\,x^{7/3}}{7}+\frac{3\,a\,b^4\,x^{10/3}}{2}+\frac{10\,a^2\,b^3\,x^3}{3}+\frac{15\,a^3\,b^2\,x^{8/3}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^(1/3))^5,x)
```

```
[Out] (a^5*x^2)/2 + (3*b^5*x^(11/3))/11 + (15*a^4*b*x^(7/3))/7 + (3*a*b^4*x^(10/3
))/2 + (10*a^2*b^3*x^3)/3 + (15*a^3*b^2*x^(8/3))/4
```

sympy [A] time = 2.24, size = 75, normalized size = 0.97

$$\frac{a^5x^2}{2}+\frac{15a^4bx^{\frac{7}{3}}}{7}+\frac{15a^3b^2x^{\frac{8}{3}}}{4}+\frac{10a^2b^3x^3}{3}+\frac{3ab^4x^{\frac{10}{3}}}{2}+\frac{3b^5x^{\frac{11}{3}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**5*x,x)
```

```
[Out] a**5*x**2/2 + 15*a**4*b*x**(7/3)/7 + 15*a**3*b**2*x**(8/3)/4 + 10*a**2*b**3
*x**3/3 + 3*a*b**4*x**(10/3)/2 + 3*b**5*x**(11/3)/11
```

3.2316 $\int \left(a + b\sqrt[3]{x}\right)^5 \, dx$

Optimal. Leaf size=59

$$\frac{a^2\left(a+b\sqrt[3]{x}\right)^6}{2b^3} + \frac{3\left(a+b\sqrt[3]{x}\right)^8}{8b^3} - \frac{6a\left(a+b\sqrt[3]{x}\right)^7}{7b^3}$$

```
[Out] 1/2*a^2*(a+b*x^(1/3))^6/b^3-6/7*a*(a+b*x^(1/3))^7/b^3+3/8*(a+b*x^(1/3))^8/b
^3
```

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {190, 43}

$$\frac{a^2\left(a+b\sqrt[3]{x}\right)^6}{2b^3} + \frac{3\left(a+b\sqrt[3]{x}\right)^8}{8b^3} - \frac{6a\left(a+b\sqrt[3]{x}\right)^7}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^5,x]
```

```
[Out] (a^2*(a + b*x^(1/3))^6)/(2*b^3) - (6*a*(a + b*x^(1/3))^7)/(7*b^3) + (3*(a +
 b*x^(1/3))^8)/(8*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}
\int \left(a + b\sqrt[3]{x}\right)^5 \, dx &= 3\,\text{Subst}\left(\int x^2(a+bx)^5 \, dx, x, \sqrt[3]{x}\right) \\
&= 3\,\text{Subst}\left(\int \left(\frac{a^2(a+bx)^5}{b^2} - \frac{2a(a+bx)^6}{b^2} + \frac{(a+bx)^7}{b^2}\right) dx, x, \sqrt[3]{x}\right) \\
&= \frac{a^2\left(a+b\sqrt[3]{x}\right)^6}{2b^3} - \frac{6a\left(a+b\sqrt[3]{x}\right)^7}{7b^3} + \frac{3\left(a+b\sqrt[3]{x}\right)^8}{8b^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 0.69

$$\frac{\left(a+b\sqrt[3]{x}\right)^6\left(a^2 - 6ab\sqrt[3]{x} + 21b^2x^{2/3}\right)}{56b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^5,x]
```

```
[Out] ((a + b*x^(1/3))^6*(a^2 - 6*a*b*x^(1/3) + 21*b^2*x^(2/3)))/(56*b^3)
```

fricas [A] time = 0.60, size = 61, normalized size = 1.03

$$5\,a^2b^3x^2 + a^5x + \frac{3}{8}\left(b^5x^2 + 16\,a^3b^2x\right)x^{\frac{2}{3}} + \frac{15}{28}\left(4\,ab^4x^2 + 7\,a^4bx\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^5,x, algorithm="fricas")
```

```
[Out] 5*a^2*b^3*x^2 + a^5*x + 3/8*(b^5*x^2 + 16*a^3*b^2*x)*x^(2/3) + 15/28*(4*a*b
^4*x^2 + 7*a^4*b*x)*x^(1/3)
```

giac [A] time = 0.17, size = 54, normalized size = 0.92

$$\frac{3}{8}\,b^5x^{\frac{8}{3}} + \frac{15}{7}\,ab^4x^{\frac{7}{3}} + 5\,a^2b^3x^2 + 6\,a^3b^2x^{\frac{5}{3}} + \frac{15}{4}\,a^4bx^{\frac{4}{3}} + a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^5,x, algorithm="giac")
```

```
[Out] 3/8*b^5*x^(8/3) + 15/7*a*b^4*x^(7/3) + 5*a^2*b^3*x^2 + 6*a^3*b^2*x^(5/3) +
15/4*a^4*b*x^(4/3) + a^5*x
```

maple [A] time = 0.00, size = 55, normalized size = 0.93

$$\frac{3b^5x^{\frac{8}{3}}}{8} + \frac{15a\,b^4x^{\frac{7}{3}}}{7} + 5a^2b^3x^2 + 6a^3b^2x^{\frac{5}{3}} + \frac{15a^4b\,x^{\frac{4}{3}}}{4} + a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^5,x)
```

```
[Out] a^5*x+3/8*b^5*x^(8/3)+15/7*a*b^4*x^(7/3)+5*a^2*b^3*x^2+6*a^3*b^2*x^(5/3)+15
/4*a^4*b*x^(4/3)
```

maxima [A] time = 0.88, size = 54, normalized size = 0.92

$$\frac{3}{8}\,b^5x^{\frac{8}{3}} + \frac{15}{7}\,ab^4x^{\frac{7}{3}} + 5\,a^2b^3x^2 + 6\,a^3b^2x^{\frac{5}{3}} + \frac{15}{4}\,a^4bx^{\frac{4}{3}} + a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^5,x, algorithm="maxima")
```

```
[Out] 3/8*b^5*x^(8/3) + 15/7*a*b^4*x^(7/3) + 5*a^2*b^3*x^2 + 6*a^3*b^2*x^(5/3) +
15/4*a^4*b*x^(4/3) + a^5*x
```

mupad [B] time = 0.03, size = 54, normalized size = 0.92

$$a^5\,x + \frac{3\,b^5\,x^{8/3}}{8} + \frac{15\,a^4\,b\,x^{4/3}}{4} + \frac{15\,a\,b^4\,x^{7/3}}{7} + 5\,a^2\,b^3\,x^2 + 6\,a^3\,b^2\,x^{5/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))^5,x)
```

```
[Out] a^5*x + (3*b^5*x^(8/3))/8 + (15*a^4*b*x^(4/3))/4 + (15*a*b^4*x^(7/3))/7 + 5
*a^2*b^3*x^2 + 6*a^3*b^2*x^(5/3)
```

sympy [A] time = 2.08, size = 68, normalized size = 1.15

$$a^5x + \frac{15a^4bx^{\frac{4}{3}}}{4} + 6a^3b^2x^{\frac{5}{3}} + 5a^2b^3x^2 + \frac{15ab^4x^{\frac{7}{3}}}{7} + \frac{3b^5x^{\frac{8}{3}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**5,x)
```

```
[Out] a**5*x + 15*a**4*b*x**(4/3)/4 + 6*a**3*b**2*x**(5/3) + 5*a**2*b**3*x**2 + 1
5*a*b**4*x**(7/3)/7 + 3*b**5*x**(8/3)/8
```

3.2317 $\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x}\,dx$

Optimal. Leaf size=65

$$a^5\log(x)+15a^4b\sqrt[3]{x}+15a^3b^2x^{2/3}+10a^2b^3x+\frac{15}{4}ab^4x^{4/3}+\frac{3}{5}b^5x^{5/3}$$

```
[Out] 15*a^4*b*x^(1/3)+15*a^3*b^2*x^(2/3)+10*a^2*b^3*x+15/4*a*b^4*x^(4/3)+3/5*b^5
*x^(5/3)+a^5*ln(x)
```

Rubi [A] time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$15a^3b^2x^{2/3}+10a^2b^3x+15a^4b\sqrt[3]{x}+a^5\log(x)+\frac{15}{4}ab^4x^{4/3}+\frac{3}{5}b^5x^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^5/x,x]
```

```
[Out] 15*a^4*b*x^(1/3) + 15*a^3*b^2*x^(2/3) + 10*a^2*b^3*x + (15*a*b^4*x^(4/3))/4
 + (3*b^5*x^(5/3))/5 + a^5*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x}\,dx &= 3\,\text{Subst}\left(\int \frac{(a+bx)^5}{x}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(5a^4b+\frac{a^5}{x}+10a^3b^2x+10a^2b^3x^2+5ab^4x^3+b^5x^4\right)\,dx, x, \sqrt[3]{x}\right)\\ &= 15a^4b\sqrt[3]{x}+15a^3b^2x^{2/3}+10a^2b^3x+\frac{15}{4}ab^4x^{4/3}+\frac{3}{5}b^5x^{5/3}+a^5\log(x)\end{aligned}$$

Mathematica [A] time = 0.05, size = 65, normalized size = 1.00

$$a^5\log(x)+15a^4b\sqrt[3]{x}+15a^3b^2x^{2/3}+10a^2b^3x+\frac{15}{4}ab^4x^{4/3}+\frac{3}{5}b^5x^{5/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^5/x,x]
```

```
[Out] 15*a^4*b*x^(1/3) + 15*a^3*b^2*x^(2/3) + 10*a^2*b^3*x + (15*a*b^4*x^(4/3))/4
 + (3*b^5*x^(5/3))/5 + a^5*Log[x]
```

fricas [A] time = 0.51, size = 56, normalized size = 0.86

$$10\,a^2b^3x + 3\,a^5\log\left(x^{\frac{1}{3}}\right) + \frac{3}{5}\left(b^5x + 25\,a^3b^2\right)x^{\frac{2}{3}} + \frac{15}{4}\left(ab^4x + 4\,a^4b\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^5/x,x, algorithm="fricas")
```

```
[Out] 10*a^2*b^3*x + 3*a^5*log(x^(1/3)) + 3/5*(b^5*x + 25*a^3*b^2)*x^(2/3) + 15/4
*(a*b^4*x + 4*a^4*b)*x^(1/3)
```

giac [A] time = 0.15, size = 54, normalized size = 0.83

$$\frac{3}{5}\,b^5x^{\frac{5}{3}} + \frac{15}{4}\,ab^4x^{\frac{4}{3}} + 10\,a^2b^3x + a^5\log\left(|x|\right) + 15\,a^3b^2x^{\frac{2}{3}} + 15\,a^4bx^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^5/x,x, algorithm="giac")
```

```
[Out] 3/5*b^5*x^(5/3) + 15/4*a*b^4*x^(4/3) + 10*a^2*b^3*x + a^5*log(abs(x)) + 15*
a^3*b^2*x^(2/3) + 15*a^4*b*x^(1/3)
```

maple [A] time = 0.00, size = 54, normalized size = 0.83

$$\frac{3b^5x^{\frac{5}{3}}}{5} + \frac{15a\,b^4x^{\frac{4}{3}}}{4} + a^5\ln(x) + 10a^2b^3x + 15a^3b^2x^{\frac{2}{3}} + 15a^4b\,x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^5/x,x)
```

```
[Out] 15*a^4*b*x^(1/3)+15*a^3*b^2*x^(2/3)+10*a^2*b^3*x+15/4*a*b^4*x^(4/3)+3/5*b^5
*x^(5/3)+a^5*ln(x)
```

maxima [A] time = 0.91, size = 53, normalized size = 0.82

$$\frac{3}{5}\,b^5x^{\frac{5}{3}} + \frac{15}{4}\,ab^4x^{\frac{4}{3}} + 10\,a^2b^3x + a^5\log(x) + 15\,a^3b^2x^{\frac{2}{3}} + 15\,a^4bx^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^5/x,x, algorithm="maxima")
```

```
[Out] 3/5*b^5*x^(5/3) + 15/4*a*b^4*x^(4/3) + 10*a^2*b^3*x + a^5*log(x) + 15*a^3*b
^2*x^(2/3) + 15*a^4*b*x^(1/3)
```

mupad [B] time = 0.03, size = 56, normalized size = 0.86

$$3\,a^5\,\ln\left(x^{1/3}\right) + \frac{3\,b^5\,x^{5/3}}{5} + 10\,a^2\,b^3\,x + 15\,a^4\,b\,x^{1/3} + \frac{15\,a\,b^4\,x^{4/3}}{4} + 15\,a^3\,b^2\,x^{2/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))^5/x,x)
```

```
[Out] 3*a^5*log(x^(1/3)) + (3*b^5*x^(5/3))/5 + 10*a^2*b^3*x + 15*a^4*b*x^(1/3) +
(15*a*b^4*x^(4/3))/4 + 15*a^3*b^2*x^(2/3)
```

sympy [A] time = 29.95, size = 66, normalized size = 1.02

$$a^5\log(x) + 15a^4b\sqrt[3]{x} + 15a^3b^2x^{\frac{2}{3}} + 10a^2b^3x + \frac{15ab^4x^{\frac{4}{3}}}{4} + \frac{3b^5x^{\frac{5}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**5/x,x)
```

```
[Out] a**5*log(x) + 15*a**4*b*x**(1/3) + 15*a**3*b**2*x**(2/3) + 10*a**2*b**3*x +
 15*a*b**4*x**(4/3)/4 + 3*b**5*x**(5/3)/5
```

3.2318 $\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^2}\,dx$

Optimal. Leaf size=68

$$-\frac{a^5}{x}-\frac{15a^4b}{2x^{2/3}}-\frac{30a^3b^2}{\sqrt[3]{x}}+10a^2b^3\log(x)+15ab^4\sqrt[3]{x}+\frac{3}{2}b^5x^{2/3}$$

```
[Out] -a^5/x-15/2*a^4*b/x^(2/3)-30*a^3*b^2/x^(1/3)+15*a*b^4*x^(1/3)+3/2*b^5*x^(2/
3)+10*a^2*b^3*ln(x)
```

Rubi [A] time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{30a^3b^2}{\sqrt[3]{x}}+10a^2b^3\log(x)-\frac{15a^4b}{2x^{2/3}}-\frac{a^5}{x}+15ab^4\sqrt[3]{x}+\frac{3}{2}b^5x^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^5/x^2,x]
```

```
[Out] -(a^5/x) - (15*a^4*b)/(2*x^(2/3)) - (30*a^3*b^2)/x^(1/3) + 15*a*b^4*x^(1/3)
 + (3*b^5*x^(2/3))/2 + 10*a^2*b^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^2}\,dx &= 3\,\text{Subst}\left(\int \frac{(a+bx)^5}{x^4}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(5ab^4+\frac{a^5}{x^4}+\frac{5a^4b}{x^3}+\frac{10a^3b^2}{x^2}+\frac{10a^2b^3}{x}+b^5x\right)dx, x, \sqrt[3]{x}\right)\\ &= -\frac{a^5}{x}-\frac{15a^4b}{2x^{2/3}}-\frac{30a^3b^2}{\sqrt[3]{x}}+15ab^4\sqrt[3]{x}+\frac{3}{2}b^5x^{2/3}+10a^2b^3\log(x)\end{aligned}$$

Mathematica [A] time = 0.03, size = 68, normalized size = 1.00

$$-\frac{a^5}{x}-\frac{15a^4b}{2x^{2/3}}-\frac{30a^3b^2}{\sqrt[3]{x}}+10a^2b^3\log(x)+15ab^4\sqrt[3]{x}+\frac{3}{2}b^5x^{2/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^5/x^2,x]
```

```
[Out] -(a^5/x) - (15*a^4*b)/(2*x^(2/3)) - (30*a^3*b^2)/x^(1/3) + 15*a*b^4*x^(1/3)
 + (3*b^5*x^(2/3))/2 + 10*a^2*b^3*Log[x]
```

fricas [A] time = 0.69, size = 62, normalized size = 0.91

$$\frac{60\,a^2b^3x\log\left(x^{\frac{1}{3}}\right)-2\,a^5+3\left(b^5x-20\,a^3b^2\right)x^{\frac{2}{3}}+15\left(2\,ab^4x-a^4b\right)x^{\frac{1}{3}}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^5/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(60*a^2*b^3*x*log(x^(1/3)) - 2*a^5 + 3*(b^5*x - 20*a^3*b^2)*x^(2/3) + 1
5*(2*a*b^4*x - a^4*b)*x^(1/3))/x
```

giac [A] time = 0.17, size = 60, normalized size = 0.88

$$10\,a^2b^3\log\left(|x|\right)+\frac{3}{2}\,b^5x^{\frac{2}{3}}+15\,ab^4x^{\frac{1}{3}}-\frac{60\,a^3b^2x^{\frac{2}{3}}+15\,a^4bx^{\frac{1}{3}}+2\,a^5}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^5/x^2,x, algorithm="giac")
```

```
[Out] 10*a^2*b^3*log(abs(x)) + 3/2*b^5*x^(2/3) + 15*a*b^4*x^(1/3) - 1/2*(60*a^3*b
^2*x^(2/3) + 15*a^4*b*x^(1/3) + 2*a^5)/x
```

maple [A] time = 0.01, size = 57, normalized size = 0.84

$$10a^2b^3\ln(x)+\frac{3b^5x^{\frac{2}{3}}}{2}+15a\,b^4x^{\frac{1}{3}}-\frac{30a^3b^2}{x^{\frac{1}{3}}}-\frac{15a^4b}{2x^{\frac{2}{3}}}-\frac{a^5}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^5/x^2,x)
```

```
[Out] -a^5/x-15/2*a^4*b/x^(2/3)-30*a^3*b^2/x^(1/3)+15*a*b^4*x^(1/3)+3/2*b^5*x^(2/
3)+10*a^2*b^3*ln(x)
```

maxima [A] time = 0.88, size = 59, normalized size = 0.87

$$10\,a^2b^3\log(x)+\frac{3}{2}\,b^5x^{\frac{2}{3}}+15\,ab^4x^{\frac{1}{3}}-\frac{60\,a^3b^2x^{\frac{2}{3}}+15\,a^4bx^{\frac{1}{3}}+2\,a^5}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^5/x^2,x, algorithm="maxima")
```

```
[Out] 10*a^2*b^3*log(x) + 3/2*b^5*x^(2/3) + 15*a*b^4*x^(1/3) - 1/2*(60*a^3*b^2*x^
(2/3) + 15*a^4*b*x^(1/3) + 2*a^5)/x
```

mupad [B] time = 0.04, size = 59, normalized size = 0.87

$$\frac{3\,b^5\,x^{2/3}}{2}-\frac{a^5+\frac{15\,a^4\,b\,x^{1/3}}{2}+30\,a^3\,b^2\,x^{2/3}}{x}+30\,a^2\,b^3\,\ln\left(x^{1/3}\right)+15\,a\,b^4\,x^{1/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))^5/x^2,x)
```

```
[Out] (3*b^5*x^(2/3))/2 - (a^5 + (15*a^4*b*x^(1/3))/2 + 30*a^3*b^2*x^(2/3))/x + 3
0*a^2*b^3*log(x^(1/3)) + 15*a*b^4*x^(1/3)
```

sympy [**A**] time = 0.71, size = 66, normalized size = 0.97

$$-\frac{a^5}{x} - \frac{15a^4b}{2x^{\frac{2}{3}}} - \frac{30a^3b^2}{\sqrt[3]{x}} + 10a^2b^3\log(x) + 15ab^4\sqrt[3]{x} + \frac{3b^5x^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**5/x**2,x)
```

```
[Out] -a**5/x - 15*a**4*b/(2*x**(2/3)) - 30*a**3*b**2/x**(1/3) + 10*a**2*b**3*log
(x) + 15*a*b**4*x**(1/3) + 3*b**5*x**(2/3)/2
```

3.2319 $\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^3}\,dx$

Optimal. Leaf size=21

$$-\frac{\left(a+b\sqrt[3]{x}\right)^6}{2ax^2}$$

```
[Out] -1/2*(a+b*x^(1/3))^6/a/x^2
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {264}

$$-\frac{\left(a+b\sqrt[3]{x}\right)^6}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^5/x^3,x]
```

```
[Out] -(a + b*x^(1/3))^6/(2*a*x^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^3}\,dx = -\frac{\left(a+b\sqrt[3]{x}\right)^6}{2ax^2}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$-\frac{\left(a+b\sqrt[3]{x}\right)^6}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^5/x^3,x]
```

```
[Out] -1/2*(a + b*x^(1/3))^6/(a*x^2)
```

fricas [B] time = 0.70, size = 57, normalized size = 2.71

$$-\frac{20\,a^2b^3x + a^5 + 3\left(2\,b^5x + 5\,a^3b^2\right)x^{\frac{2}{3}} + 3\left(5\,ab^4x + 2\,a^4b\right)x^{\frac{1}{3}}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^5/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(20*a^2*b^3*x + a^5 + 3*(2*b^5*x + 5*a^3*b^2)*x^(2/3) + 3*(5*a*b^4*x +
 2*a^4*b)*x^(1/3))/x^2
```

giac [B] time = 0.20, size = 55, normalized size = 2.62

$$-\frac{6\,b^5x^{\frac{5}{3}}+15\,ab^4x^{\frac{4}{3}}+20\,a^2b^3x+15\,a^3b^2x^{\frac{2}{3}}+6\,a^4bx^{\frac{1}{3}}+a^5}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^5/x^3,x, algorithm="giac")
```

```
[Out] -1/2*(6*b^5*x^(5/3) + 15*a*b^4*x^(4/3) + 20*a^2*b^3*x + 15*a^3*b^2*x^(2/3)
+ 6*a^4*b*x^(1/3) + a^5)/x^2
```

maple [B] time = 0.01, size = 58, normalized size = 2.76

$$-\frac{3b^5}{x^{\frac{1}{3}}}-\frac{15a\,b^4}{2x^{\frac{2}{3}}}-\frac{10a^2b^3}{x}-\frac{15a^3b^2}{2x^{\frac{4}{3}}}-\frac{3a^4b}{x^{\frac{5}{3}}}-\frac{a^5}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^5/x^3,x)
```

```
[Out] -3*a^4*b/x^(5/3)-15/2*a^3*b^2/x^(4/3)-10*a^2*b^3/x-15/2*a*b^4/x^(2/3)-1/2*a
^5/x^2-3*b^5/x^(1/3)
```

maxima [B] time = 0.93, size = 55, normalized size = 2.62

$$-\frac{6\,b^5x^{\frac{5}{3}}+15\,ab^4x^{\frac{4}{3}}+20\,a^2b^3x+15\,a^3b^2x^{\frac{2}{3}}+6\,a^4bx^{\frac{1}{3}}+a^5}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^5/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(6*b^5*x^(5/3) + 15*a*b^4*x^(4/3) + 20*a^2*b^3*x + 15*a^3*b^2*x^(2/3)
+ 6*a^4*b*x^(1/3) + a^5)/x^2
```

mupad [B] time = 1.12, size = 55, normalized size = 2.62

$$-\frac{a^5+6\,b^5\,x^{5/3}+20\,a^2\,b^3\,x+6\,a^4\,b\,x^{1/3}+15\,a\,b^4\,x^{4/3}+15\,a^3\,b^2\,x^{2/3}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))^5/x^3,x)
```

```
[Out] -(a^5 + 6*b^5*x^(5/3) + 20*a^2*b^3*x + 6*a^4*b*x^(1/3) + 15*a*b^4*x^(4/3) +
 15*a^3*b^2*x^(2/3))/(2*x^2)
```

sympy [B] time = 1.24, size = 70, normalized size = 3.33

$$-\frac{a^5}{2x^2}-\frac{3a^4b}{x^{\frac{5}{3}}}-\frac{15a^3b^2}{2x^{\frac{4}{3}}}-\frac{10a^2b^3}{x}-\frac{15ab^4}{2x^{\frac{2}{3}}}-\frac{3b^5}{\sqrt[3]{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**5/x**3,x)
```

```
[Out] -a**5/(2*x**2) - 3*a**4*b/x**(5/3) - 15*a**3*b**2/(2*x**(4/3)) - 10*a**2*b*
*3/x - 15*a*b**4/(2*x**(2/3)) - 3*b**5/x**(1/3)
```

3.2320 $\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^4}\,dx$

Optimal. Leaf size=73

$$-\frac{a^5}{3x^3}-\frac{15a^4b}{8x^{8/3}}-\frac{30a^3b^2}{7x^{7/3}}-\frac{5a^2b^3}{x^2}-\frac{3ab^4}{x^{5/3}}-\frac{3b^5}{4x^{4/3}}$$

```
[Out] -1/3*a^5/x^3-15/8*a^4*b/x^(8/3)-30/7*a^3*b^2/x^(7/3)-5*a^2*b^3/x^2-3*a*b^4/
x^(5/3)-3/4*b^5/x^(4/3)
```

Rubi [A] time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{30a^3b^2}{7x^{7/3}}-\frac{5a^2b^3}{x^2}-\frac{15a^4b}{8x^{8/3}}-\frac{a^5}{3x^3}-\frac{3ab^4}{x^{5/3}}-\frac{3b^5}{4x^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^5/x^4,x]
```

```
[Out] -a^5/(3*x^3) - (15*a^4*b)/(8*x^(8/3)) - (30*a^3*b^2)/(7*x^(7/3)) - (5*a^2*b
^3)/x^2 - (3*a*b^4)/x^(5/3) - (3*b^5)/(4*x^(4/3))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^4}\,dx &= 3\,\text{Subst}\left(\int \frac{(a+bx)^5}{x^{10}}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(\frac{a^5}{x^{10}}+\frac{5a^4b}{x^9}+\frac{10a^3b^2}{x^8}+\frac{10a^2b^3}{x^7}+\frac{5ab^4}{x^6}+\frac{b^5}{x^5}\right)dx, x, \sqrt[3]{x}\right)\\ &= -\frac{a^5}{3x^3}-\frac{15a^4b}{8x^{8/3}}-\frac{30a^3b^2}{7x^{7/3}}-\frac{5a^2b^3}{x^2}-\frac{3ab^4}{x^{5/3}}-\frac{3b^5}{4x^{4/3}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 67, normalized size = 0.92

$$-\frac{56a^5+315a^4b\sqrt[3]{x}+720a^3b^2x^{2/3}+840a^2b^3x+504ab^4x^{4/3}+126b^5x^{5/3}}{168x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^5/x^4,x]
```

```
[Out] -1/168*(56*a^5 + 315*a^4*b*x^(1/3) + 720*a^3*b^2*x^(2/3) + 840*a^2*b^3*x +
504*a*b^4*x^(4/3) + 126*b^5*x^(5/3))/x^3
```

fricas [A] time = 0.68, size = 59, normalized size = 0.81

$$-\frac{840\,a^2b^3x + 56\,a^5 + 18\left(7\,b^5x + 40\,a^3b^2\right)x^{\frac{2}{3}} + 63\left(8\,ab^4x + 5\,a^4b\right)x^{\frac{1}{3}}}{168\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^5/x^4,x, algorithm="fricas")
```

```
[Out] -1/168*(840*a^2*b^3*x + 56*a^5 + 18*(7*b^5*x + 40*a^3*b^2)*x^(2/3) + 63*(8*
a*b^4*x + 5*a^4*b)*x^(1/3))/x^3
```

giac [A] time = 0.16, size = 57, normalized size = 0.78

$$-\frac{126\,b^5x^{\frac{5}{3}} + 504\,ab^4x^{\frac{4}{3}} + 840\,a^2b^3x + 720\,a^3b^2x^{\frac{2}{3}} + 315\,a^4bx^{\frac{1}{3}} + 56\,a^5}{168\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^5/x^4,x, algorithm="giac")
```

```
[Out] -1/168*(126*b^5*x^(5/3) + 504*a*b^4*x^(4/3) + 840*a^2*b^3*x + 720*a^3*b^2*x
^(2/3) + 315*a^4*b*x^(1/3) + 56*a^5)/x^3
```

maple [A] time = 0.00, size = 58, normalized size = 0.79

$$-\frac{3b^5}{4x^{\frac{4}{3}}} - \frac{3a\,b^4}{x^{\frac{5}{3}}} - \frac{5a^2b^3}{x^2} - \frac{30a^3b^2}{7x^{\frac{7}{3}}} - \frac{15a^4b}{8x^{\frac{8}{3}}} - \frac{a^5}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^5/x^4,x)
```

```
[Out] -1/3*a^5/x^3-15/8*a^4*b/x^(8/3)-30/7*a^3*b^2/x^(7/3)-5*a^2*b^3/x^2-3*a*b^4/
x^(5/3)-3/4*b^5/x^(4/3)
```

maxima [A] time = 0.94, size = 57, normalized size = 0.78

$$-\frac{126\,b^5x^{\frac{5}{3}} + 504\,ab^4x^{\frac{4}{3}} + 840\,a^2b^3x + 720\,a^3b^2x^{\frac{2}{3}} + 315\,a^4bx^{\frac{1}{3}} + 56\,a^5}{168\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^5/x^4,x, algorithm="maxima")
```

```
[Out] -1/168*(126*b^5*x^(5/3) + 504*a*b^4*x^(4/3) + 840*a^2*b^3*x + 720*a^3*b^2*x
^(2/3) + 315*a^4*b*x^(1/3) + 56*a^5)/x^3
```

mupad [B] time = 0.04, size = 57, normalized size = 0.78

$$-\frac{\frac{a^5}{3} + \frac{3\,b^5\,x^{5/3}}{4} + 5\,a^2\,b^3\,x + \frac{15\,a^4\,b\,x^{1/3}}{8} + 3\,a\,b^4\,x^{4/3} + \frac{30\,a^3\,b^2\,x^{2/3}}{7}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))^5/x^4,x)
```

```
[Out] -(a^5/3 + (3*b^5*x^(5/3))/4 + 5*a^2*b^3*x + (15*a^4*b*x^(1/3))/8 + 3*a*b^4*
x^(4/3) + (30*a^3*b^2*x^(2/3))/7)/x^3
```

sympy [**A**] time = 2.15, size = 73, normalized size = 1.00

$$-\frac{a^5}{3x^3}-\frac{15a^4b}{8x^{\frac{8}{3}}}-\frac{30a^3b^2}{7x^{\frac{7}{3}}}-\frac{5a^2b^3}{x^2}-\frac{3ab^4}{x^{\frac{5}{3}}}-\frac{3b^5}{4x^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**5/x**4,x)
```

```
[Out] -a**5/(3*x**3) - 15*a**4*b/(8*x**(8/3)) - 30*a**3*b**2/(7*x**(7/3)) - 5*a**
2*b**3/x**2 - 3*a*b**4/x**(5/3) - 3*b**5/(4*x**(4/3))
```

3.2321 $\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^5}\,dx$

Optimal. Leaf size=75

$$-\frac{a^5}{4x^4}-\frac{15a^4b}{11x^{11/3}}-\frac{3a^3b^2}{x^{10/3}}-\frac{10a^2b^3}{3x^3}-\frac{15ab^4}{8x^{8/3}}-\frac{3b^5}{7x^{7/3}}$$

```
[Out] -1/4*a^5/x^4-15/11*a^4*b/x^(11/3)-3*a^3*b^2/x^(10/3)-10/3*a^2*b^3/x^3-15/8*
a*b^4/x^(8/3)-3/7*b^5/x^(7/3)
```

Rubi [A] time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{3a^3b^2}{x^{10/3}}-\frac{10a^2b^3}{3x^3}-\frac{15a^4b}{11x^{11/3}}-\frac{a^5}{4x^4}-\frac{15ab^4}{8x^{8/3}}-\frac{3b^5}{7x^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^5/x^5,x]
```

```
[Out] -a^5/(4*x^4) - (15*a^4*b)/(11*x^(11/3)) - (3*a^3*b^2)/x^(10/3) - (10*a^2*b^
3)/(3*x^3) - (15*a*b^4)/(8*x^(8/3)) - (3*b^5)/(7*x^(7/3))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^5}\,dx &= 3\,\text{Subst}\left(\int \frac{(a+bx)^5}{x^{13}}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(\frac{a^5}{x^{13}}+\frac{5a^4b}{x^{12}}+\frac{10a^3b^2}{x^{11}}+\frac{10a^2b^3}{x^{10}}+\frac{5ab^4}{x^9}+\frac{b^5}{x^8}\right)dx, x, \sqrt[3]{x}\right)\\ &= -\frac{a^5}{4x^4}-\frac{15a^4b}{11x^{11/3}}-\frac{3a^3b^2}{x^{10/3}}-\frac{10a^2b^3}{3x^3}-\frac{15ab^4}{8x^{8/3}}-\frac{3b^5}{7x^{7/3}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 67, normalized size = 0.89

$$-\frac{462a^5+2520a^4b\sqrt[3]{x}+5544a^3b^2x^{2/3}+6160a^2b^3x+3465ab^4x^{4/3}+792b^5x^{5/3}}{1848x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^5/x^5,x]
```

```
[Out] -1/1848*(462*a^5 + 2520*a^4*b*x^(1/3) + 5544*a^3*b^2*x^(2/3) + 6160*a^2*b^3
*x + 3465*a*b^4*x^(4/3) + 792*b^5*x^(5/3))/x^4
```

fricas [A] time = 0.83, size = 58, normalized size = 0.77

$$-\frac{6160\,a^2b^3x + 462\,a^5 + 792\left(b^5x + 7\,a^3b^2\right)x^{\frac{2}{3}} + 315\left(11\,ab^4x + 8\,a^4b\right)x^{\frac{1}{3}}}{1848\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^5/x^5,x, algorithm="fricas")
```

```
[Out] -1/1848*(6160*a^2*b^3*x + 462*a^5 + 792*(b^5*x + 7*a^3*b^2)*x^(2/3) + 315*(
11*a*b^4*x + 8*a^4*b)*x^(1/3))/x^4
```

giac [A] time = 0.16, size = 57, normalized size = 0.76

$$-\frac{792\,b^5x^{\frac{5}{3}} + 3465\,ab^4x^{\frac{4}{3}} + 6160\,a^2b^3x + 5544\,a^3b^2x^{\frac{2}{3}} + 2520\,a^4bx^{\frac{1}{3}} + 462\,a^5}{1848\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^5/x^5,x, algorithm="giac")
```

```
[Out] -1/1848*(792*b^5*x^(5/3) + 3465*a*b^4*x^(4/3) + 6160*a^2*b^3*x + 5544*a^3*b
^2*x^(2/3) + 2520*a^4*b*x^(1/3) + 462*a^5)/x^4
```

maple [A] time = 0.01, size = 58, normalized size = 0.77

$$-\frac{3b^5}{7x^{\frac{7}{3}}} - \frac{15a\,b^4}{8x^{\frac{8}{3}}} - \frac{10a^2b^3}{3x^3} - \frac{3a^3b^2}{x^{\frac{10}{3}}} - \frac{15a^4b}{11x^{\frac{11}{3}}} - \frac{a^5}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^5/x^5,x)
```

```
[Out] -1/4*a^5/x^4-15/11*a^4*b/x^(11/3)-3*a^3*b^2/x^(10/3)-10/3*a^2*b^3/x^3-15/8*
a*b^4/x^(8/3)-3/7*b^5/x^(7/3)
```

maxima [A] time = 0.87, size = 57, normalized size = 0.76

$$-\frac{792\,b^5x^{\frac{5}{3}} + 3465\,ab^4x^{\frac{4}{3}} + 6160\,a^2b^3x + 5544\,a^3b^2x^{\frac{2}{3}} + 2520\,a^4bx^{\frac{1}{3}} + 462\,a^5}{1848\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^5/x^5,x, algorithm="maxima")
```

```
[Out] -1/1848*(792*b^5*x^(5/3) + 3465*a*b^4*x^(4/3) + 6160*a^2*b^3*x + 5544*a^3*b
^2*x^(2/3) + 2520*a^4*b*x^(1/3) + 462*a^5)/x^4
```

mupad [B] time = 1.13, size = 57, normalized size = 0.76

$$-\frac{\frac{a^5}{4} + \frac{3\,b^5\,x^{5/3}}{7} + \frac{10\,a^2\,b^3\,x}{3} + \frac{15\,a^4\,b\,x^{1/3}}{11} + \frac{15\,a\,b^4\,x^{4/3}}{8} + 3\,a^3\,b^2\,x^{2/3}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))^5/x^5,x)
```

```
[Out] -(a^5/4 + (3*b^5*x^(5/3))/7 + (10*a^2*b^3*x)/3 + (15*a^4*b*x^(1/3))/11 + (1
5*a*b^4*x^(4/3))/8 + 3*a^3*b^2*x^(2/3))/x^4
```

sympy [**A**] time = 3.53, size = 75, normalized size = 1.00

$$-\frac{a^5}{4x^4}-\frac{15a^4b}{11x^{\frac{11}{3}}}-\frac{3a^3b^2}{x^{\frac{10}{3}}}-\frac{10a^2b^3}{3x^3}-\frac{15ab^4}{8x^{\frac{8}{3}}}-\frac{3b^5}{7x^{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**5/x**5,x)
```

```
[Out] -a**5/(4*x**4) - 15*a**4*b/(11*x**(11/3)) - 3*a**3*b**2/x**(10/3) - 10*a**2
*b**3/(3*x**3) - 15*a*b**4/(8*x**(8/3)) - 3*b**5/(7*x**(7/3))
```

3.2322 $\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^6}\,dx$

Optimal. Leaf size=77

$$-\frac{a^5}{5x^5}-\frac{15a^4b}{14x^{14/3}}-\frac{30a^3b^2}{13x^{13/3}}-\frac{5a^2b^3}{2x^4}-\frac{15ab^4}{11x^{11/3}}-\frac{3b^5}{10x^{10/3}}$$

```
[Out] -1/5*a^5/x^5-15/14*a^4*b/x^(14/3)-30/13*a^3*b^2/x^(13/3)-5/2*a^2*b^3/x^4-15
/11*a*b^4/x^(11/3)-3/10*b^5/x^(10/3)
```

Rubi [A] time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{30a^3b^2}{13x^{13/3}}-\frac{5a^2b^3}{2x^4}-\frac{15a^4b}{14x^{14/3}}-\frac{a^5}{5x^5}-\frac{15ab^4}{11x^{11/3}}-\frac{3b^5}{10x^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^5/x^6,x]
```

```
[Out] -a^5/(5*x^5) - (15*a^4*b)/(14*x^(14/3)) - (30*a^3*b^2)/(13*x^(13/3)) - (5*a
^2*b^3)/(2*x^4) - (15*a*b^4)/(11*x^(11/3)) - (3*b^5)/(10*x^(10/3))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^6}\,dx &= 3\,\text{Subst}\left(\int \frac{(a+bx)^5}{x^{16}}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(\frac{a^5}{x^{16}}+\frac{5a^4b}{x^{15}}+\frac{10a^3b^2}{x^{14}}+\frac{10a^2b^3}{x^{13}}+\frac{5ab^4}{x^{12}}+\frac{b^5}{x^{11}}\right)dx, x, \sqrt[3]{x}\right)\\ &= -\frac{a^5}{5x^5}-\frac{15a^4b}{14x^{14/3}}-\frac{30a^3b^2}{13x^{13/3}}-\frac{5a^2b^3}{2x^4}-\frac{15ab^4}{11x^{11/3}}-\frac{3b^5}{10x^{10/3}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 67, normalized size = 0.87

$$-\frac{2002a^5+10725a^4b\sqrt[3]{x}+23100a^3b^2x^{2/3}+25025a^2b^3x+13650ab^4x^{4/3}+3003b^5x^{5/3}}{10010x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^5/x^6,x]
```

```
[Out] -1/10010*(2002*a^5 + 10725*a^4*b*x^(1/3) + 23100*a^3*b^2*x^(2/3) + 25025*a^
2*b^3*x + 13650*a*b^4*x^(4/3) + 3003*b^5*x^(5/3))/x^5
```

fricas [A] time = 0.67, size = 59, normalized size = 0.77

$$-\frac{25025\,a^2b^3x + 2002\,a^5 + 231\left(13\,b^5x + 100\,a^3b^2\right)x^{\frac{2}{3}} + 975\left(14\,ab^4x + 11\,a^4b\right)x^{\frac{1}{3}}}{10010\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^5/x^6,x, algorithm="fricas")
```

```
[Out] -1/10010*(25025*a^2*b^3*x + 2002*a^5 + 231*(13*b^5*x + 100*a^3*b^2)*x^(2/3)
 + 975*(14*a*b^4*x + 11*a^4*b)*x^(1/3))/x^5
```

giac [A] time = 0.16, size = 57, normalized size = 0.74

$$-\frac{3003\,b^5x^{\frac{5}{3}} + 13650\,ab^4x^{\frac{4}{3}} + 25025\,a^2b^3x + 23100\,a^3b^2x^{\frac{2}{3}} + 10725\,a^4bx^{\frac{1}{3}} + 2002\,a^5}{10010\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^5/x^6,x, algorithm="giac")
```

```
[Out] -1/10010*(3003*b^5*x^(5/3) + 13650*a*b^4*x^(4/3) + 25025*a^2*b^3*x + 23100*
a^3*b^2*x^(2/3) + 10725*a^4*b*x^(1/3) + 2002*a^5)/x^5
```

maple [A] time = 0.01, size = 58, normalized size = 0.75

$$-\frac{3b^5}{10x^{\frac{10}{3}}} - \frac{15a\,b^4}{11x^{\frac{11}{3}}} - \frac{5a^2b^3}{2x^4} - \frac{30a^3b^2}{13x^{\frac{13}{3}}} - \frac{15a^4b}{14x^{\frac{14}{3}}} - \frac{a^5}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^5/x^6,x)
```

```
[Out] -1/5*a^5/x^5-15/14*a^4*b/x^(14/3)-30/13*a^3*b^2/x^(13/3)-5/2*a^2*b^3/x^4-15
/11*a*b^4/x^(11/3)-3/10*b^5/x^(10/3)
```

maxima [A] time = 0.82, size = 57, normalized size = 0.74

$$-\frac{3003\,b^5x^{\frac{5}{3}} + 13650\,ab^4x^{\frac{4}{3}} + 25025\,a^2b^3x + 23100\,a^3b^2x^{\frac{2}{3}} + 10725\,a^4bx^{\frac{1}{3}} + 2002\,a^5}{10010\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^5/x^6,x, algorithm="maxima")
```

```
[Out] -1/10010*(3003*b^5*x^(5/3) + 13650*a*b^4*x^(4/3) + 25025*a^2*b^3*x + 23100*
a^3*b^2*x^(2/3) + 10725*a^4*b*x^(1/3) + 2002*a^5)/x^5
```

mupad [B] time = 0.04, size = 57, normalized size = 0.74

$$-\frac{\frac{a^5}{5} + \frac{3\,b^5\,x^{5/3}}{10} + \frac{5\,a^2\,b^3\,x}{2} + \frac{15\,a^4\,b\,x^{1/3}}{14} + \frac{15\,a\,b^4\,x^{4/3}}{11} + \frac{30\,a^3\,b^2\,x^{2/3}}{13}}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))^5/x^6,x)
```

```
[Out] -(a^5/5 + (3*b^5*x^(5/3))/10 + (5*a^2*b^3*x)/2 + (15*a^4*b*x^(1/3))/14 + (1
5*a*b^4*x^(4/3))/11 + (30*a^3*b^2*x^(2/3))/13)/x^5
```

sympy [**A**] time = 5.35, size = 76, normalized size = 0.99

$$-\frac{a^5}{5x^5}-\frac{15a^4b}{14x^{\frac{14}{3}}}-\frac{30a^3b^2}{13x^{\frac{13}{3}}}-\frac{5a^2b^3}{2x^4}-\frac{15ab^4}{11x^{\frac{11}{3}}}-\frac{3b^5}{10x^{\frac{10}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**5/x**6,x)
```

```
[Out] -a**5/(5*x**5) - 15*a**4*b/(14*x**(14/3)) - 30*a**3*b**2/(13*x**(13/3)) - 5
*a**2*b**3/(2*x**4) - 15*a*b**4/(11*x**(11/3)) - 3*b**5/(10*x**(10/3))
```

3.2323 $\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^7}\,dx$

Optimal. Leaf size=75

$$-\frac{a^5}{6x^6}-\frac{15a^4b}{17x^{17/3}}-\frac{15a^3b^2}{8x^{16/3}}-\frac{2a^2b^3}{x^5}-\frac{15ab^4}{14x^{14/3}}-\frac{3b^5}{13x^{13/3}}$$

```
[Out] -1/6*a^5/x^6-15/17*a^4*b/x^(17/3)-15/8*a^3*b^2/x^(16/3)-2*a^2*b^3/x^5-15/14
*a*b^4/x^(14/3)-3/13*b^5/x^(13/3)
```

Rubi [A] time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{15a^3b^2}{8x^{16/3}}-\frac{2a^2b^3}{x^5}-\frac{15a^4b}{17x^{17/3}}-\frac{a^5}{6x^6}-\frac{15ab^4}{14x^{14/3}}-\frac{3b^5}{13x^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^5/x^7,x]
```

```
[Out] -a^5/(6*x^6) - (15*a^4*b)/(17*x^(17/3)) - (15*a^3*b^2)/(8*x^(16/3)) - (2*a^
2*b^3)/x^5 - (15*a*b^4)/(14*x^(14/3)) - (3*b^5)/(13*x^(13/3))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^7}\,dx &= 3\,\text{Subst}\left(\int \frac{(a+bx)^5}{x^{19}}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(\frac{a^5}{x^{19}}+\frac{5a^4b}{x^{18}}+\frac{10a^3b^2}{x^{17}}+\frac{10a^2b^3}{x^{16}}+\frac{5ab^4}{x^{15}}+\frac{b^5}{x^{14}}\right)dx, x, \sqrt[3]{x}\right)\\ &= -\frac{a^5}{6x^6}-\frac{15a^4b}{17x^{17/3}}-\frac{15a^3b^2}{8x^{16/3}}-\frac{2a^2b^3}{x^5}-\frac{15ab^4}{14x^{14/3}}-\frac{3b^5}{13x^{13/3}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 67, normalized size = 0.89

$$-\frac{6188a^5+32760a^4b\sqrt[3]{x}+69615a^3b^2x^{2/3}+74256a^2b^3x+39780ab^4x^{4/3}+8568b^5x^{5/3}}{37128x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^5/x^7,x]
```

```
[Out] -1/37128*(6188*a^5 + 32760*a^4*b*x^(1/3) + 69615*a^3*b^2*x^(2/3) + 74256*a^
2*b^3*x + 39780*a*b^4*x^(4/3) + 8568*b^5*x^(5/3))/x^6
```

fricas [A] time = 0.83, size = 59, normalized size = 0.79

$$-\frac{74256\,a^2b^3x + 6188\,a^5 + 1071\left(8\,b^5x + 65\,a^3b^2\right)x^{\frac{2}{3}} + 2340\left(17\,ab^4x + 14\,a^4b\right)x^{\frac{1}{3}}}{37128\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^5/x^7,x, algorithm="fricas")
```

```
[Out] -1/37128*(74256*a^2*b^3*x + 6188*a^5 + 1071*(8*b^5*x + 65*a^3*b^2)*x^(2/3)
+ 2340*(17*a*b^4*x + 14*a^4*b)*x^(1/3))/x^6
```

giac [A] time = 0.16, size = 57, normalized size = 0.76

$$-\frac{8568\,b^5x^{\frac{5}{3}} + 39780\,ab^4x^{\frac{4}{3}} + 74256\,a^2b^3x + 69615\,a^3b^2x^{\frac{2}{3}} + 32760\,a^4bx^{\frac{1}{3}} + 6188\,a^5}{37128\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^5/x^7,x, algorithm="giac")
```

```
[Out] -1/37128*(8568*b^5*x^(5/3) + 39780*a*b^4*x^(4/3) + 74256*a^2*b^3*x + 69615*
a^3*b^2*x^(2/3) + 32760*a^4*b*x^(1/3) + 6188*a^5)/x^6
```

maple [A] time = 0.01, size = 58, normalized size = 0.77

$$-\frac{3b^5}{13x^{\frac{13}{3}}} - \frac{15a\,b^4}{14x^{\frac{14}{3}}} - \frac{2a^2b^3}{x^5} - \frac{15a^3b^2}{8x^{\frac{16}{3}}} - \frac{15a^4b}{17x^{\frac{17}{3}}} - \frac{a^5}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^5/x^7,x)
```

```
[Out] -1/6*a^5/x^6-15/17*a^4*b/x^(17/3)-15/8*a^3*b^2/x^(16/3)-2*a^2*b^3/x^5-15/14
*a*b^4/x^(14/3)-3/13*b^5/x^(13/3)
```

maxima [A] time = 0.88, size = 57, normalized size = 0.76

$$-\frac{8568\,b^5x^{\frac{5}{3}} + 39780\,ab^4x^{\frac{4}{3}} + 74256\,a^2b^3x + 69615\,a^3b^2x^{\frac{2}{3}} + 32760\,a^4bx^{\frac{1}{3}} + 6188\,a^5}{37128\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^5/x^7,x, algorithm="maxima")
```

```
[Out] -1/37128*(8568*b^5*x^(5/3) + 39780*a*b^4*x^(4/3) + 74256*a^2*b^3*x + 69615*
a^3*b^2*x^(2/3) + 32760*a^4*b*x^(1/3) + 6188*a^5)/x^6
```

mupad [B] time = 0.04, size = 57, normalized size = 0.76

$$-\frac{\frac{a^5}{6} + \frac{3\,b^5\,x^{5/3}}{13} + 2\,a^2\,b^3\,x + \frac{15\,a^4\,b\,x^{1/3}}{17} + \frac{15\,a\,b^4\,x^{4/3}}{14} + \frac{15\,a^3\,b^2\,x^{2/3}}{8}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))^5/x^7,x)
```

```
[Out] -(a^5/6 + (3*b^5*x^(5/3))/13 + 2*a^2*b^3*x + (15*a^4*b*x^(1/3))/17 + (15*a*
b^4*x^(4/3))/14 + (15*a^3*b^2*x^(2/3))/8)/x^6
```

sympy [**A**] time = 8.51, size = 75, normalized size = 1.00

$$-\frac{a^5}{6x^6}-\frac{15a^4b}{17x^{\frac{17}{3}}}-\frac{15a^3b^2}{8x^{\frac{16}{3}}}-\frac{2a^2b^3}{x^5}-\frac{15ab^4}{14x^{\frac{14}{3}}}-\frac{3b^5}{13x^{\frac{13}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**5/x**7,x)
```

```
[Out] -a**5/(6*x**6) - 15*a**4*b/(17*x**(17/3)) - 15*a**3*b**2/(8*x**(16/3)) - 2*
a**2*b**3/x**5 - 15*a*b**4/(14*x**(14/3)) - 3*b**5/(13*x**(13/3))
```

3.2324 $\int \left(a + b\sqrt[3]{x}\right)^{10} x^4\, dx$

Optimal. Leaf size=144

$$\frac{a^{10}x^5}{5}+\frac{15}{8}a^9bx^{16/3}+\frac{135}{17}a^8b^2x^{17/3}+20a^7b^3x^6+\frac{630}{19}a^6b^4x^{19/3}+\frac{189}{5}a^5b^5x^{20/3}+30a^4b^6x^7+\frac{180}{11}a^3b^7x^{22/3}+\frac{135}{23}a^2b^8x^{23/}$$

```
[Out] 1/5*a^10*x^5+15/8*a^9*b*x^(16/3)+135/17*a^8*b^2*x^(17/3)+20*a^7*b^3*x^6+630
/19*a^6*b^4*x^(19/3)+189/5*a^5*b^5*x^(20/3)+30*a^4*b^6*x^7+180/11*a^3*b^7*x
^(22/3)+135/23*a^2*b^8*x^(23/3)+5/4*a*b^9*x^8+3/25*b^10*x^(25/3)
```

Rubi [A] time = 0.10, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{135}{17}a^8b^2x^{17/3}+20a^7b^3x^6+\frac{630}{19}a^6b^4x^{19/3}+\frac{189}{5}a^5b^5x^{20/3}+30a^4b^6x^7+\frac{180}{11}a^3b^7x^{22/3}+\frac{135}{23}a^2b^8x^{23/3}+\frac{15}{8}a^9bx^{16/3}+\frac{a^{10}x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^10*x^4,x]
```

```
[Out] (a^10*x^5)/5 + (15*a^9*b*x^(16/3))/8 + (135*a^8*b^2*x^(17/3))/17 + 20*a^7*b
^3*x^6 + (630*a^6*b^4*x^(19/3))/19 + (189*a^5*b^5*x^(20/3))/5 + 30*a^4*b^6*
x^7 + (180*a^3*b^7*x^(22/3))/11 + (135*a^2*b^8*x^(23/3))/23 + (5*a*b^9*x^8)
/4 + (3*b^10*x^(25/3))/25
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \left(a + b\sqrt[3]{x}\right)^{10} x^4\, dx &= 3\,\text{Subst}\left(\int x^{14}(a + bx)^{10}\, dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(a^{10}x^{14} + 10a^9bx^{15} + 45a^8b^2x^{16} + 120a^7b^3x^{17} + 210a^6b^4x^{18} + 252a^5b^5x^{19} + \right.\right.\\ &= \frac{a^{10}x^5}{5} + \frac{15}{8}a^9bx^{16/3} + \frac{135}{17}a^8b^2x^{17/3} + 20a^7b^3x^6 + \frac{630}{19}a^6b^4x^{19/3} + \frac{189}{5}a^5b^5x^{20/3} + 30a^4b^6\end{aligned}$$

Mathematica [A] time = 0.11, size = 144, normalized size = 1.00

$$\frac{a^{10}x^5}{5}+\frac{15}{8}a^9bx^{16/3}+\frac{135}{17}a^8b^2x^{17/3}+20a^7b^3x^6+\frac{630}{19}a^6b^4x^{19/3}+\frac{189}{5}a^5b^5x^{20/3}+30a^4b^6x^7+\frac{180}{11}a^3b^7x^{22/3}+\frac{135}{23}a^2b^8x^{23/}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^10*x^4,x]
```

```
[Out] (a^10*x^5)/5 + (15*a^9*b*x^(16/3))/8 + (135*a^8*b^2*x^(17/3))/17 + 20*a^7*b
^3*x^6 + (630*a^6*b^4*x^(19/3))/19 + (189*a^5*b^5*x^(20/3))/5 + 30*a^4*b^6*
x^7 + (180*a^3*b^7*x^(22/3))/11 + (135*a^2*b^8*x^(23/3))/23 + (5*a*b^9*x^8)
/4 + (3*b^10*x^(25/3))/25
```

fricas [A] time = 0.81, size = 124, normalized size = 0.86

$$\frac{5}{4}ab^9x^8+30a^4b^6x^7+20a^7b^3x^6+\frac{1}{5}a^{10}x^5+\frac{27}{1955}\left(425a^2b^8x^7+2737a^5b^5x^6+575a^8b^2x^5\right)x^{\frac{2}{3}}+\frac{3}{41800}\left(1672b^{10}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10*x^4,x, algorithm="fricas")
```

```
[Out] 5/4*a*b^9*x^8 + 30*a^4*b^6*x^7 + 20*a^7*b^3*x^6 + 1/5*a^10*x^5 + 27/1955*(4
25*a^2*b^8*x^7 + 2737*a^5*b^5*x^6 + 575*a^8*b^2*x^5)*x^(2/3) + 3/41800*(167
2*b^10*x^8 + 228000*a^3*b^7*x^7 + 462000*a^6*b^4*x^6 + 26125*a^9*b*x^5)*x^(
1/3)
```

giac [A] time = 0.16, size = 112, normalized size = 0.78

$$\frac{3}{25}b^{10}x^{\frac{25}{3}}+\frac{5}{4}ab^9x^8+\frac{135}{23}a^2b^8x^{\frac{23}{3}}+\frac{180}{11}a^3b^7x^{\frac{22}{3}}+30a^4b^6x^7+\frac{189}{5}a^5b^5x^{\frac{20}{3}}+\frac{630}{19}a^6b^4x^{\frac{19}{3}}+20a^7b^3x^6+\frac{135}{17}a^8b^2x^{\frac{1}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10*x^4,x, algorithm="giac")
```

```
[Out] 3/25*b^10*x^(25/3) + 5/4*a*b^9*x^8 + 135/23*a^2*b^8*x^(23/3) + 180/11*a^3*b
^7*x^(22/3) + 30*a^4*b^6*x^7 + 189/5*a^5*b^5*x^(20/3) + 630/19*a^6*b^4*x^(1
9/3) + 20*a^7*b^3*x^6 + 135/17*a^8*b^2*x^(17/3) + 15/8*a^9*b*x^(16/3) + 1/5
*a^10*x^5
```

maple [A] time = 0.00, size = 113, normalized size = 0.78

$$\frac{3b^{10}x^{\frac{25}{3}}}{25}+\frac{5a\,b^9x^8}{4}+\frac{135a^2b^8x^{\frac{23}{3}}}{23}+\frac{180a^3b^7x^{\frac{22}{3}}}{11}+30a^4b^6x^7+\frac{189a^5b^5x^{\frac{20}{3}}}{5}+\frac{630a^6b^4x^{\frac{19}{3}}}{19}+20a^7b^3x^6+\frac{135a^8b^2x^{\frac{17}{3}}}{17}+\frac{1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^10*x^4,x)
```

```
[Out] 1/5*a^10*x^5+15/8*a^9*b*x^(16/3)+135/17*a^8*b^2*x^(17/3)+20*a^7*b^3*x^6+630
/19*a^6*b^4*x^(19/3)+189/5*a^5*b^5*x^(20/3)+30*a^4*b^6*x^7+180/11*a^3*b^7*x
^(22/3)+135/23*a^2*b^8*x^(23/3)+5/4*a*b^9*x^8+3/25*b^10*x^(25/3)
```

maxima [B] time = 0.92, size = 251, normalized size = 1.74

$$\frac{3\left(bx^{\frac{1}{3}}+a\right)^{25}}{25\,b^{15}}-\frac{7\left(bx^{\frac{1}{3}}+a\right)^{24}a}{4\,b^{15}}+\frac{273\left(bx^{\frac{1}{3}}+a\right)^{23}a^2}{23\,b^{15}}-\frac{546\left(bx^{\frac{1}{3}}+a\right)^{22}a^3}{11\,b^{15}}+\frac{143\left(bx^{\frac{1}{3}}+a\right)^{21}a^4}{b^{15}}-\frac{3003\left(bx^{\frac{1}{3}}+a\right)^{20}}{10\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10*x^4,x, algorithm="maxima")
```

```
[Out] 3/25*(b*x^(1/3) + a)^25/b^15 - 7/4*(b*x^(1/3) + a)^24*a/b^15 + 273/23*(b*x^
(1/3) + a)^23*a^2/b^15 - 546/11*(b*x^(1/3) + a)^22*a^3/b^15 + 143*(b*x^(1/3
) + a)^21*a^4/b^15 - 3003/10*(b*x^(1/3) + a)^20*a^5/b^15 + 9009/19*(b*x^(1/
3) + a)^19*a^6/b^15 - 572*(b*x^(1/3) + a)^18*a^7/b^15 + 9009/17*(b*x^(1/3)
+ a)^17*a^8/b^15 - 3003/8*(b*x^(1/3) + a)^16*a^9/b^15 + 1001/5*(b*x^(1/3) +
 a)^15*a^10/b^15 - 78*(b*x^(1/3) + a)^14*a^11/b^15 + 21*(b*x^(1/3) + a)^13*
a^12/b^15 - 7/2*(b*x^(1/3) + a)^12*a^13/b^15 + 3/11*(b*x^(1/3) + a)^11*a^14
/b^15
```

mupad [B] time = 1.15, size = 112, normalized size = 0.78

$$\frac{a^{10}x^5}{5}+\frac{3b^{10}x^{25/3}}{25}+\frac{5ab^9x^8}{4}+\frac{15a^9bx^{16/3}}{8}+20a^7b^3x^6+30a^4b^6x^7+\frac{135a^8b^2x^{17/3}}{17}+\frac{630a^6b^4x^{19/3}}{19}+\frac{189a^5b^5x^{20/}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^(1/3))^10,x)
```

```
[Out] (a^10*x^5)/5 + (3*b^10*x^(25/3))/25 + (5*a*b^9*x^8)/4 + (15*a^9*b*x^(16/3))
/8 + 20*a^7*b^3*x^6 + 30*a^4*b^6*x^7 + (135*a^8*b^2*x^(17/3))/17 + (630*a^6
*b^4*x^(19/3))/19 + (189*a^5*b^5*x^(20/3))/5 + (180*a^3*b^7*x^(22/3))/11 +
(135*a^2*b^8*x^(23/3))/23
```

sympy [A] time = 4.67, size = 144, normalized size = 1.00

$$\frac{a^{10}x^5}{5}+\frac{15a^9bx^{\frac{16}{3}}}{8}+\frac{135a^8b^2x^{\frac{17}{3}}}{17}+20a^7b^3x^6+\frac{630a^6b^4x^{\frac{19}{3}}}{19}+\frac{189a^5b^5x^{\frac{20}{3}}}{5}+30a^4b^6x^7+\frac{180a^3b^7x^{\frac{22}{3}}}{11}+\frac{135a^2b^8x^{\frac{23}{3}}}{23}+\frac{5ab^9}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**10*x**4,x)
```

```
[Out] a**10*x**5/5 + 15*a**9*b*x**(16/3)/8 + 135*a**8*b**2*x**(17/3)/17 + 20*a**7
*b**3*x**6 + 630*a**6*b**4*x**(19/3)/19 + 189*a**5*b**5*x**(20/3)/5 + 30*a*
*4*b**6*x**7 + 180*a**3*b**7*x**(22/3)/11 + 135*a**2*b**8*x**(23/3)/23 + 5*
a*b**9*x**8/4 + 3*b**10*x**(25/3)/25
```

3.2325 $\int \left(a + b\sqrt[3]{x}\right)^{10} x^3\, dx$

Optimal. Leaf size=144

$$\frac{a^{10}x^4}{4}+\frac{30}{13}a^9bx^{13/3}+\frac{135}{14}a^8b^2x^{14/3}+24a^7b^3x^5+\frac{315}{8}a^6b^4x^{16/3}+\frac{756}{17}a^5b^5x^{17/3}+35a^4b^6x^6+\frac{360}{19}a^3b^7x^{19/3}+\frac{27}{4}a^2b^8x^{20/3}$$

```
[Out] 1/4*a^10*x^4+30/13*a^9*b*x^(13/3)+135/14*a^8*b^2*x^(14/3)+24*a^7*b^3*x^5+31
5/8*a^6*b^4*x^(16/3)+756/17*a^5*b^5*x^(17/3)+35*a^4*b^6*x^6+360/19*a^3*b^7*
x^(19/3)+27/4*a^2*b^8*x^(20/3)+10/7*a*b^9*x^7+3/22*b^10*x^(22/3)
```

Rubi [A] time = 0.09, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{135}{14}a^8b^2x^{14/3}+24a^7b^3x^5+\frac{315}{8}a^6b^4x^{16/3}+\frac{756}{17}a^5b^5x^{17/3}+35a^4b^6x^6+\frac{360}{19}a^3b^7x^{19/3}+\frac{27}{4}a^2b^8x^{20/3}+\frac{30}{13}a^9bx^{13/3}+\frac{a^{10}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^10*x^3,x]
```

```
[Out] (a^10*x^4)/4 + (30*a^9*b*x^(13/3))/13 + (135*a^8*b^2*x^(14/3))/14 + 24*a^7*
b^3*x^5 + (315*a^6*b^4*x^(16/3))/8 + (756*a^5*b^5*x^(17/3))/17 + 35*a^4*b^6
*x^6 + (360*a^3*b^7*x^(19/3))/19 + (27*a^2*b^8*x^(20/3))/4 + (10*a*b^9*x^7)
/7 + (3*b^10*x^(22/3))/22
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \left(a + b\sqrt[3]{x}\right)^{10} x^3\, dx &= 3\,\text{Subst}\left(\int x^{11}(a + bx)^{10}\, dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(a^{10}x^{11} + 10a^9bx^{12} + 45a^8b^2x^{13} + 120a^7b^3x^{14} + 210a^6b^4x^{15} + 252a^5b^5x^{16}\right.\right.\\ &= \frac{a^{10}x^4}{4} + \frac{30}{13}a^9bx^{13/3} + \frac{135}{14}a^8b^2x^{14/3} + 24a^7b^3x^5 + \frac{315}{8}a^6b^4x^{16/3} + \frac{756}{17}a^5b^5x^{17/3} + 35a\end{aligned}$$

Mathematica [A] time = 0.08, size = 144, normalized size = 1.00

$$\frac{a^{10}x^4}{4}+\frac{30}{13}a^9bx^{13/3}+\frac{135}{14}a^8b^2x^{14/3}+24a^7b^3x^5+\frac{315}{8}a^6b^4x^{16/3}+\frac{756}{17}a^5b^5x^{17/3}+35a^4b^6x^6+\frac{360}{19}a^3b^7x^{19/3}+\frac{27}{4}a^2b^8x^{20/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^10*x^3,x]
```

```
[Out] (a^10*x^4)/4 + (30*a^9*b*x^(13/3))/13 + (135*a^8*b^2*x^(14/3))/14 + 24*a^7*
b^3*x^5 + (315*a^6*b^4*x^(16/3))/8 + (756*a^5*b^5*x^(17/3))/17 + 35*a^4*b^6
*x^6 + (360*a^3*b^7*x^(19/3))/19 + (27*a^2*b^8*x^(20/3))/4 + (10*a*b^9*x^7)
/7 + (3*b^10*x^(22/3))/22
```

fricas [A] time = 0.66, size = 124, normalized size = 0.86

$$\frac{10}{7}ab^9x^7+35a^4b^6x^6+24a^7b^3x^5+\frac{1}{4}a^{10}x^4+\frac{27}{476}\left(119a^2b^8x^6+784a^5b^5x^5+170a^8b^2x^4\right)x^{\frac{2}{3}}+\frac{3}{21736}\left(988b^{10}x^7+13\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10*x^3,x, algorithm="fricas")
```

```
[Out] 10/7*a*b^9*x^7 + 35*a^4*b^6*x^6 + 24*a^7*b^3*x^5 + 1/4*a^10*x^4 + 27/476*(1
19*a^2*b^8*x^6 + 784*a^5*b^5*x^5 + 170*a^8*b^2*x^4)*x^(2/3) + 3/21736*(988*
b^10*x^7 + 137280*a^3*b^7*x^6 + 285285*a^6*b^4*x^5 + 16720*a^9*b*x^4)*x^(1/
3)
```

giac [A] time = 0.16, size = 112, normalized size = 0.78

$$\frac{3}{22}b^{10}x^{\frac{22}{3}}+\frac{10}{7}ab^9x^7+\frac{27}{4}a^2b^8x^{\frac{20}{3}}+\frac{360}{19}a^3b^7x^{\frac{19}{3}}+35a^4b^6x^6+\frac{756}{17}a^5b^5x^{\frac{17}{3}}+\frac{315}{8}a^6b^4x^{\frac{16}{3}}+24a^7b^3x^5+\frac{135}{14}a^8b^2x^{\frac{14}{3}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10*x^3,x, algorithm="giac")
```

```
[Out] 3/22*b^10*x^(22/3) + 10/7*a*b^9*x^7 + 27/4*a^2*b^8*x^(20/3) + 360/19*a^3*b^
7*x^(19/3) + 35*a^4*b^6*x^6 + 756/17*a^5*b^5*x^(17/3) + 315/8*a^6*b^4*x^(16
/3) + 24*a^7*b^3*x^5 + 135/14*a^8*b^2*x^(14/3) + 30/13*a^9*b*x^(13/3) + 1/4
*a^10*x^4
```

maple [A] time = 0.00, size = 113, normalized size = 0.78

$$\frac{3b^{10}x^{\frac{22}{3}}}{22}+\frac{10ab^9x^7}{7}+\frac{27a^2b^8x^{\frac{20}{3}}}{4}+\frac{360a^3b^7x^{\frac{19}{3}}}{19}+35a^4b^6x^6+\frac{756a^5b^5x^{\frac{17}{3}}}{17}+\frac{315a^6b^4x^{\frac{16}{3}}}{8}+24a^7b^3x^5+\frac{135a^8b^2x^{\frac{14}{3}}}{14}+\frac{30a^9}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^10*x^3,x)
```

```
[Out] 1/4*a^10*x^4+30/13*a^9*b*x^(13/3)+135/14*a^8*b^2*x^(14/3)+24*a^7*b^3*x^5+31
5/8*a^6*b^4*x^(16/3)+756/17*a^5*b^5*x^(17/3)+35*a^4*b^6*x^6+360/19*a^3*b^7*
x^(19/3)+27/4*a^2*b^8*x^(20/3)+10/7*a*b^9*x^7+3/22*b^10*x^(22/3)
```

maxima [A] time = 0.90, size = 200, normalized size = 1.39

$$\frac{3\left(bx^{\frac{1}{3}}+a\right)^{22}}{22b^{12}}-\frac{11\left(bx^{\frac{1}{3}}+a\right)^{21}a}{7b^{12}}+\frac{33\left(bx^{\frac{1}{3}}+a\right)^{20}a^2}{4b^{12}}-\frac{495\left(bx^{\frac{1}{3}}+a\right)^{19}a^3}{19b^{12}}+\frac{55\left(bx^{\frac{1}{3}}+a\right)^{18}a^4}{b^{12}}-\frac{1386\left(bx^{\frac{1}{3}}+a\right)^{17}a^5}{17b^{12}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10*x^3,x, algorithm="maxima")
```

```
[Out] 3/22*(b*x^(1/3) + a)^22/b^12 - 11/7*(b*x^(1/3) + a)^21*a/b^12 + 33/4*(b*x^(
1/3) + a)^20*a^2/b^12 - 495/19*(b*x^(1/3) + a)^19*a^3/b^12 + 55*(b*x^(1/3)
+ a)^18*a^4/b^12 - 1386/17*(b*x^(1/3) + a)^17*a^5/b^12 + 693/8*(b*x^(1/3) +
 a)^16*a^6/b^12 - 66*(b*x^(1/3) + a)^15*a^7/b^12 + 495/14*(b*x^(1/3) + a)^1
4*a^8/b^12 - 165/13*(b*x^(1/3) + a)^13*a^9/b^12 + 11/4*(b*x^(1/3) + a)^12*a
^10/b^12 - 3/11*(b*x^(1/3) + a)^11*a^11/b^12
```

mupad [B] time = 0.06, size = 112, normalized size = 0.78

$$\frac{a^{10}\,x^4}{4}+\frac{3\,b^{10}\,x^{22/3}}{22}+\frac{10\,a\,b^9\,x^7}{7}+\frac{30\,a^9\,b\,x^{13/3}}{13}+24\,a^7\,b^3\,x^5+35\,a^4\,b^6\,x^6+\frac{135\,a^8\,b^2\,x^{14/3}}{14}+\frac{315\,a^6\,b^4\,x^{16/3}}{8}+\frac{756\,a^5\,b^5}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^(1/3))^10,x)
```

```
[Out] (a^10*x^4)/4 + (3*b^10*x^(22/3))/22 + (10*a*b^9*x^7)/7 + (30*a^9*b*x^(13/3)
)/13 + 24*a^7*b^3*x^5 + 35*a^4*b^6*x^6 + (135*a^8*b^2*x^(14/3))/14 + (315*a
^6*b^4*x^(16/3))/8 + (756*a^5*b^5*x^(17/3))/17 + (360*a^3*b^7*x^(19/3))/19
+ (27*a^2*b^8*x^(20/3))/4
```

sympy [A] time = 4.20, size = 144, normalized size = 1.00

$$\frac{a^{10}x^4}{4}+\frac{30a^9bx^{\frac{13}{3}}}{13}+\frac{135a^8b^2x^{\frac{14}{3}}}{14}+24a^7b^3x^5+\frac{315a^6b^4x^{\frac{16}{3}}}{8}+\frac{756a^5b^5x^{\frac{17}{3}}}{17}+35a^4b^6x^6+\frac{360a^3b^7x^{\frac{19}{3}}}{19}+\frac{27a^2b^8x^{\frac{20}{3}}}{4}+\frac{10}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**10*x**3,x)
```

```
[Out] a**10*x**4/4 + 30*a**9*b*x**(13/3)/13 + 135*a**8*b**2*x**(14/3)/14 + 24*a**
7*b**3*x**5 + 315*a**6*b**4*x**(16/3)/8 + 756*a**5*b**5*x**(17/3)/17 + 35*a
**4*b**6*x**6 + 360*a**3*b**7*x**(19/3)/19 + 27*a**2*b**8*x**(20/3)/4 + 10*
a*b**9*x**7/7 + 3*b**10*x**(22/3)/22
```

3.2326 $\int \left(a + b\sqrt[3]{x}\right)^{10} x^2\, dx$

Optimal. Leaf size=179

$$\frac{3a^8\left(a+b\sqrt[3]{x}\right)^{11}}{11b^9}-\frac{2a^7\left(a+b\sqrt[3]{x}\right)^{12}}{b^9}+\frac{84a^6\left(a+b\sqrt[3]{x}\right)^{13}}{13b^9}-\frac{12a^5\left(a+b\sqrt[3]{x}\right)^{14}}{b^9}+\frac{14a^4\left(a+b\sqrt[3]{x}\right)^{15}}{b^9}-\frac{21a^3\left(a+b\sqrt[3]{x}\right)^{1}}{2b^9}$$

```
[Out] 3/11*a^8*(a+b*x^(1/3))^11/b^9-2*a^7*(a+b*x^(1/3))^12/b^9+84/13*a^6*(a+b*x^(
1/3))^13/b^9-12*a^5*(a+b*x^(1/3))^14/b^9+14*a^4*(a+b*x^(1/3))^15/b^9-21/2*a
^3*(a+b*x^(1/3))^16/b^9+84/17*a^2*(a+b*x^(1/3))^17/b^9-4/3*a*(a+b*x^(1/3))^
18/b^9+3/19*(a+b*x^(1/3))^19/b^9
```

Rubi [A] time = 0.09, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{84a^2\left(a+b\sqrt[3]{x}\right)^{17}}{17b^9}-\frac{21a^3\left(a+b\sqrt[3]{x}\right)^{16}}{2b^9}+\frac{14a^4\left(a+b\sqrt[3]{x}\right)^{15}}{b^9}-\frac{12a^5\left(a+b\sqrt[3]{x}\right)^{14}}{b^9}+\frac{84a^6\left(a+b\sqrt[3]{x}\right)^{13}}{13b^9}-\frac{2a^7\left(a+b\sqrt[3]{x}\right)^{}}{b^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^10*x^2,x]
```

```
[Out] (3*a^8*(a + b*x^(1/3))^11)/(11*b^9) - (2*a^7*(a + b*x^(1/3))^12)/b^9 + (84*
a^6*(a + b*x^(1/3))^13)/(13*b^9) - (12*a^5*(a + b*x^(1/3))^14)/b^9 + (14*a^
4*(a + b*x^(1/3))^15)/b^9 - (21*a^3*(a + b*x^(1/3))^16)/(2*b^9) + (84*a^2*(
a + b*x^(1/3))^17)/(17*b^9) - (4*a*(a + b*x^(1/3))^18)/(3*b^9) + (3*(a + b*
x^(1/3))^19)/(19*b^9)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \left(a + b\sqrt[3]{x}\right)^{10} x^2\, dx &= 3\,\text{Subst}\left(\int x^8(a+bx)^{10}\, dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int\left(\frac{a^8(a+bx)^{10}}{b^8}-\frac{8a^7(a+bx)^{11}}{b^8}+\frac{28a^6(a+bx)^{12}}{b^8}-\frac{56a^5(a+bx)^{13}}{b^8}+\frac{70a^4(a+\right.\right.\\ &= \frac{3a^8\left(a+b\sqrt[3]{x}\right)^{11}}{11b^9}-\frac{2a^7\left(a+b\sqrt[3]{x}\right)^{12}}{b^9}+\frac{84a^6\left(a+b\sqrt[3]{x}\right)^{13}}{13b^9}-\frac{12a^5\left(a+b\sqrt[3]{x}\right)^{14}}{b^9}+\frac{14a^4\left(a\right.}\end{aligned}$$

Mathematica [A] time = 0.09, size = 140, normalized size = 0.78

$$\frac{a^{10}x^3}{3}+3a^9bx^{10/3}+\frac{135}{11}a^8b^2x^{11/3}+30a^7b^3x^4+\frac{630}{13}a^6b^4x^{13/3}+54a^5b^5x^{14/3}+42a^4b^6x^5+\frac{45}{2}a^3b^7x^{16/3}+\frac{135}{17}a^2b^8x^{17/3}+\frac{5}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^10*x^2,x]
```

```
[Out] (a^10*x^3)/3 + 3*a^9*b*x^(10/3) + (135*a^8*b^2*x^(11/3))/11 + 30*a^7*b^3*x^
4 + (630*a^6*b^4*x^(13/3))/13 + 54*a^5*b^5*x^(14/3) + 42*a^4*b^6*x^5 + (45*
a^3*b^7*x^(16/3))/2 + (135*a^2*b^8*x^(17/3))/17 + (5*a*b^9*x^6)/3 + (3*b^10
*x^(19/3))/19
```

fricas [A] time = 0.83, size = 124, normalized size = 0.69

$$\frac{5}{3}ab^9x^6+42a^4b^6x^5+30a^7b^3x^4+\frac{1}{3}a^{10}x^3+\frac{27}{187}\left(55a^2b^8x^5+374a^5b^5x^4+85a^8b^2x^3\right)x^{\frac{2}{3}}+\frac{3}{494}\left(26b^{10}x^6+3705a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10*x^2,x, algorithm="fricas")
```

```
[Out] 5/3*a*b^9*x^6 + 42*a^4*b^6*x^5 + 30*a^7*b^3*x^4 + 1/3*a^10*x^3 + 27/187*(55
*a^2*b^8*x^5 + 374*a^5*b^5*x^4 + 85*a^8*b^2*x^3)*x^(2/3) + 3/494*(26*b^10*x
^6 + 3705*a^3*b^7*x^5 + 7980*a^6*b^4*x^4 + 494*a^9*b*x^3)*x^(1/3)
```

giac [A] time = 0.16, size = 112, normalized size = 0.63

$$\frac{3}{19}b^{10}x^{\frac{19}{3}}+\frac{5}{3}ab^9x^6+\frac{135}{17}a^2b^8x^{\frac{17}{3}}+\frac{45}{2}a^3b^7x^{\frac{16}{3}}+42a^4b^6x^5+54a^5b^5x^{\frac{14}{3}}+\frac{630}{13}a^6b^4x^{\frac{13}{3}}+30a^7b^3x^4+\frac{135}{11}a^8b^2x^{\frac{11}{3}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10*x^2,x, algorithm="giac")
```

```
[Out] 3/19*b^10*x^(19/3) + 5/3*a*b^9*x^6 + 135/17*a^2*b^8*x^(17/3) + 45/2*a^3*b^7
*x^(16/3) + 42*a^4*b^6*x^5 + 54*a^5*b^5*x^(14/3) + 630/13*a^6*b^4*x^(13/3)
+ 30*a^7*b^3*x^4 + 135/11*a^8*b^2*x^(11/3) + 3*a^9*b*x^(10/3) + 1/3*a^10*x^
3
```

maple [A] time = 0.00, size = 113, normalized size = 0.63

$$\frac{3b^{10}x^{\frac{19}{3}}}{19}+\frac{5ab^9x^6}{3}+\frac{135a^2b^8x^{\frac{17}{3}}}{17}+\frac{45a^3b^7x^{\frac{16}{3}}}{2}+42a^4b^6x^5+54a^5b^5x^{\frac{14}{3}}+\frac{630a^6b^4x^{\frac{13}{3}}}{13}+30a^7b^3x^4+\frac{135a^8b^2x^{\frac{11}{3}}}{11}+3a^9b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^10*x^2,x)
```

```
[Out] 3/19*b^10*x^(19/3)+5/3*a*b^9*x^6+135/17*a^2*b^8*x^(17/3)+45/2*a^3*b^7*x^(16
/3)+42*a^4*b^6*x^5+54*a^5*b^5*x^(14/3)+630/13*a^6*b^4*x^(13/3)+30*a^7*b^3*x
^4+135/11*a^8*b^2*x^(11/3)+3*a^9*b*x^(10/3)+1/3*x^3*a^10
```

maxima [A] time = 0.90, size = 149, normalized size = 0.83

$$\frac{3\left(bx^{\frac{1}{3}}+a\right)^{19}}{19b^9}-\frac{4\left(bx^{\frac{1}{3}}+a\right)^{18}a}{3b^9}+\frac{84\left(bx^{\frac{1}{3}}+a\right)^{17}a^2}{17b^9}-\frac{21\left(bx^{\frac{1}{3}}+a\right)^{16}a^3}{2b^9}+\frac{14\left(bx^{\frac{1}{3}}+a\right)^{15}a^4}{b^9}-\frac{12\left(bx^{\frac{1}{3}}+a\right)^{14}a^5}{b^9}+\frac{84}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10*x^2,x, algorithm="maxima")
```

```
[Out] 3/19*(b*x^(1/3) + a)^19/b^9 - 4/3*(b*x^(1/3) + a)^18*a/b^9 + 84/17*(b*x^(1/
3) + a)^17*a^2/b^9 - 21/2*(b*x^(1/3) + a)^16*a^3/b^9 + 14*(b*x^(1/3) + a)^1
5*a^4/b^9 - 12*(b*x^(1/3) + a)^14*a^5/b^9 + 84/13*(b*x^(1/3) + a)^13*a^6/b^
9 - 2*(b*x^(1/3) + a)^12*a^7/b^9 + 3/11*(b*x^(1/3) + a)^11*a^8/b^9
```

mupad [B] time = 0.06, size = 112, normalized size = 0.63

$$\frac{a^{10}\,x^3}{3}+\frac{3\,b^{10}\,x^{19/3}}{19}+\frac{5\,a\,b^9\,x^6}{3}+3\,a^9\,b\,x^{10/3}+30\,a^7\,b^3\,x^4+42\,a^4\,b^6\,x^5+\frac{135\,a^8\,b^2\,x^{11/3}}{11}+\frac{630\,a^6\,b^4\,x^{13/3}}{13}+54\,a^5\,b^5\,x^{14/3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^(1/3))^10,x)
```

```
[Out] (a^10*x^3)/3 + (3*b^10*x^(19/3))/19 + (5*a*b^9*x^6)/3 + 3*a^9*b*x^(10/3) +
30*a^7*b^3*x^4 + 42*a^4*b^6*x^5 + (135*a^8*b^2*x^(11/3))/11 + (630*a^6*b^4*
x^(13/3))/13 + 54*a^5*b^5*x^(14/3) + (45*a^3*b^7*x^(16/3))/2 + (135*a^2*b^8
*x^(17/3))/17
```

sympy [A] time = 3.87, size = 141, normalized size = 0.79

$$\frac{a^{10}x^3}{3}+3a^9bx^{\frac{10}{3}}+\frac{135a^8b^2x^{\frac{11}{3}}}{11}+30a^7b^3x^4+\frac{630a^6b^4x^{\frac{13}{3}}}{13}+54a^5b^5x^{\frac{14}{3}}+42a^4b^6x^5+\frac{45a^3b^7x^{\frac{16}{3}}}{2}+\frac{135a^2b^8x^{\frac{17}{3}}}{17}+\frac{5ab^9x^6}{3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**10*x**2,x)
```

```
[Out] a**10*x**3/3 + 3*a**9*b*x**(10/3) + 135*a**8*b**2*x**(11/3)/11 + 30*a**7*b*
*3*x**4 + 630*a**6*b**4*x**(13/3)/13 + 54*a**5*b**5*x**(14/3) + 42*a**4*b**
6*x**5 + 45*a**3*b**7*x**(16/3)/2 + 135*a**2*b**8*x**(17/3)/17 + 5*a*b**9*x
**6/3 + 3*b**10*x**(19/3)/19
```

3.2327 $\int \left(a+b\sqrt[3]{x}\right)^{10} x\,dx$

Optimal. Leaf size=120

$$-\frac{3a^5\left(a+b\sqrt[3]{x}\right)^{11}}{11b^6}+\frac{5a^4\left(a+b\sqrt[3]{x}\right)^{12}}{4b^6}-\frac{30a^3\left(a+b\sqrt[3]{x}\right)^{13}}{13b^6}+\frac{15a^2\left(a+b\sqrt[3]{x}\right)^{14}}{7b^6}+\frac{3\left(a+b\sqrt[3]{x}\right)^{16}}{16b^6}-\frac{a\left(a+b\sqrt[3]{x}\right)^{15}}{b^6}$$

```
[Out] -3/11*a^5*(a+b*x^(1/3))^11/b^6+5/4*a^4*(a+b*x^(1/3))^12/b^6-30/13*a^3*(a+b*
x^(1/3))^13/b^6+15/7*a^2*(a+b*x^(1/3))^14/b^6-a*(a+b*x^(1/3))^15/b^6+3/16*(
a+b*x^(1/3))^16/b^6
```

Rubi [A] time = 0.07, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{15a^2\left(a+b\sqrt[3]{x}\right)^{14}}{7b^6}-\frac{30a^3\left(a+b\sqrt[3]{x}\right)^{13}}{13b^6}+\frac{5a^4\left(a+b\sqrt[3]{x}\right)^{12}}{4b^6}-\frac{3a^5\left(a+b\sqrt[3]{x}\right)^{11}}{11b^6}+\frac{3\left(a+b\sqrt[3]{x}\right)^{16}}{16b^6}-\frac{a\left(a+b\sqrt[3]{x}\right)^{15}}{b^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^10*x,x]
```

```
[Out] (-3*a^5*(a + b*x^(1/3))^11)/(11*b^6) + (5*a^4*(a + b*x^(1/3))^12)/(4*b^6) -
 (30*a^3*(a + b*x^(1/3))^13)/(13*b^6) + (15*a^2*(a + b*x^(1/3))^14)/(7*b^6)
 - (a*(a + b*x^(1/3))^15)/b^6 + (3*(a + b*x^(1/3))^16)/(16*b^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sqrt[3]{x}\right)^{10} x\,dx &= 3\,\text{Subst}\left(\int x^5(a+bx)^{10}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int\left(-\frac{a^5(a+bx)^{10}}{b^5}+\frac{5a^4(a+bx)^{11}}{b^5}-\frac{10a^3(a+bx)^{12}}{b^5}+\frac{10a^2(a+bx)^{13}}{b^5}-\frac{5a(a\right.\right.\\ &= -\frac{3a^5\left(a+b\sqrt[3]{x}\right)^{11}}{11b^6}+\frac{5a^4\left(a+b\sqrt[3]{x}\right)^{12}}{4b^6}-\frac{30a^3\left(a+b\sqrt[3]{x}\right)^{13}}{13b^6}+\frac{15a^2\left(a+b\sqrt[3]{x}\right)^{14}}{7b^6}-\frac{a\left(a\right.}{}\end{aligned}$$

Mathematica [A] time = 0.09, size = 76, normalized size = 0.63

$$-\frac{\left(a+b\sqrt[3]{x}\right)^{11}\left(a^5-11a^4b\sqrt[3]{x}+66a^3b^2x^{2/3}-286a^2b^3x+1001ab^4x^{4/3}-3003b^5x^{5/3}\right)}{16016b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^10*x,x]
```

```
[Out] -1/16016*((a + b*x^(1/3))^11*(a^5 - 11*a^4*b*x^(1/3) + 66*a^3*b^2*x^(2/3) -
 286*a^2*b^3*x + 1001*a*b^4*x^(4/3) - 3003*b^5*x^(5/3)))/b^6
```

fricas [A] time = 0.89, size = 124, normalized size = 1.03

$$2\,ab^9x^5+\frac{105}{2}\,a^4b^6x^4+40\,a^7b^3x^3+\frac{1}{2}\,a^{10}x^2+\frac{27}{616}\left(220\,a^2b^8x^4+1568\,a^5b^5x^3+385\,a^8b^2x^2\right)x^{\frac{2}{3}}+\frac{3}{1456}\left(91\,b^{10}x^5+134\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10*x,x, algorithm="fricas")
```

```
[Out] 2*a*b^9*x^5 + 105/2*a^4*b^6*x^4 + 40*a^7*b^3*x^3 + 1/2*a^10*x^2 + 27/616*(2
20*a^2*b^8*x^4 + 1568*a^5*b^5*x^3 + 385*a^8*b^2*x^2)*x^(2/3) + 3/1456*(91*b
^10*x^5 + 13440*a^3*b^7*x^4 + 30576*a^6*b^4*x^3 + 2080*a^9*b*x^2)*x^(1/3)
```

giac [A] time = 0.19, size = 112, normalized size = 0.93

$$\frac{3}{16}\,b^{10}x^{\frac{16}{3}}+2\,ab^9x^5+\frac{135}{14}\,a^2b^8x^{\frac{14}{3}}+\frac{360}{13}\,a^3b^7x^{\frac{13}{3}}+\frac{105}{2}\,a^4b^6x^4+\frac{756}{11}\,a^5b^5x^{\frac{11}{3}}+63\,a^6b^4x^{\frac{10}{3}}+40\,a^7b^3x^3+\frac{135}{8}\,a^8b^2x^{\frac{8}{3}}+\frac{3}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10*x,x, algorithm="giac")
```

```
[Out] 3/16*b^10*x^(16/3) + 2*a*b^9*x^5 + 135/14*a^2*b^8*x^(14/3) + 360/13*a^3*b^7
*x^(13/3) + 105/2*a^4*b^6*x^4 + 756/11*a^5*b^5*x^(11/3) + 63*a^6*b^4*x^(10/
3) + 40*a^7*b^3*x^3 + 135/8*a^8*b^2*x^(8/3) + 30/7*a^9*b*x^(7/3) + 1/2*a^10
*x^2
```

maple [A] time = 0.00, size = 113, normalized size = 0.94

$$\frac{3b^{10}x^{\frac{16}{3}}}{16}+2a\,b^9x^5+\frac{135a^2b^8x^{\frac{14}{3}}}{14}+\frac{360a^3b^7x^{\frac{13}{3}}}{13}+\frac{105a^4b^6x^4}{2}+\frac{756a^5b^5x^{\frac{11}{3}}}{11}+63a^6b^4x^{\frac{10}{3}}+40a^7b^3x^3+\frac{135a^8b^2x^{\frac{8}{3}}}{8}+\frac{30a^9b}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^10*x,x)
```

```
[Out] 3/16*b^10*x^(16/3)+2*a*b^9*x^5+135/14*a^2*b^8*x^(14/3)+360/13*a^3*b^7*x^(13
/3)+105/2*a^4*b^6*x^4+756/11*a^5*b^5*x^(11/3)+63*a^6*b^4*x^(10/3)+40*a^7*b^
3*x^3+135/8*a^8*b^2*x^(8/3)+30/7*a^9*b*x^(7/3)+1/2*x^2*a^10
```

maxima [A] time = 0.99, size = 98, normalized size = 0.82

$$\frac{3\left(bx^{\frac{1}{3}}+a\right)^{16}}{16\,b^6}-\frac{\left(bx^{\frac{1}{3}}+a\right)^{15}a}{b^6}+\frac{15\left(bx^{\frac{1}{3}}+a\right)^{14}a^2}{7\,b^6}-\frac{30\left(bx^{\frac{1}{3}}+a\right)^{13}a^3}{13\,b^6}+\frac{5\left(bx^{\frac{1}{3}}+a\right)^{12}a^4}{4\,b^6}-\frac{3\left(bx^{\frac{1}{3}}+a\right)^{11}a^5}{11\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10*x,x, algorithm="maxima")
```

```
[Out] 3/16*(b*x^(1/3) + a)^16/b^6 - (b*x^(1/3) + a)^15*a/b^6 + 15/7*(b*x^(1/3) +
a)^14*a^2/b^6 - 30/13*(b*x^(1/3) + a)^13*a^3/b^6 + 5/4*(b*x^(1/3) + a)^12*a
^4/b^6 - 3/11*(b*x^(1/3) + a)^11*a^5/b^6
```

mupad [B] time = 0.06, size = 112, normalized size = 0.93

$$\frac{a^{10}\,x^2}{2}+\frac{3\,b^{10}\,x^{16/3}}{16}+2\,a\,b^9\,x^5+\frac{30\,a^9\,b\,x^{7/3}}{7}+40\,a^7\,b^3\,x^3+\frac{105\,a^4\,b^6\,x^4}{2}+\frac{135\,a^8\,b^2\,x^{8/3}}{8}+63\,a^6\,b^4\,x^{10/3}+\frac{756\,a^5\,b^5\,x^{11/3}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^(1/3))^10,x)
```

```
[Out] (a^10*x^2)/2 + (3*b^10*x^(16/3))/16 + 2*a*b^9*x^5 + (30*a^9*b*x^(7/3))/7 +
40*a^7*b^3*x^3 + (105*a^4*b^6*x^4)/2 + (135*a^8*b^2*x^(8/3))/8 + 63*a^6*b^4
*x^(10/3) + (756*a^5*b^5*x^(11/3))/11 + (360*a^3*b^7*x^(13/3))/13 + (135*a^
2*b^8*x^(14/3))/14
```

sympy [**A**] time = 3.56, size = 143, normalized size = 1.19

$$\frac{a^{10}x^2}{2}+\frac{30a^9bx^{\frac{7}{3}}}{7}+\frac{135a^8b^2x^{\frac{8}{3}}}{8}+40a^7b^3x^3+63a^6b^4x^{\frac{10}{3}}+\frac{756a^5b^5x^{\frac{11}{3}}}{11}+\frac{105a^4b^6x^4}{2}+\frac{360a^3b^7x^{\frac{13}{3}}}{13}+\frac{135a^2b^8x^{\frac{14}{3}}}{14}+2ab$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**10*x,x)
```

```
[Out] a**10*x**2/2 + 30*a**9*b*x**(7/3)/7 + 135*a**8*b**2*x**(8/3)/8 + 40*a**7*b*
*3*x**3 + 63*a**6*b**4*x**(10/3) + 756*a**5*b**5*x**(11/3)/11 + 105*a**4*b*
*6*x**4/2 + 360*a**3*b**7*x**(13/3)/13 + 135*a**2*b**8*x**(14/3)/14 + 2*a*b
**9*x**5 + 3*b**10*x**(16/3)/16
```

3.2328 $\int \left(a + b\sqrt[3]{x}\right)^{10}\, dx$

Optimal. Leaf size=59

$$\frac{3a^2\left(a + b\sqrt[3]{x}\right)^{11}}{11b^3} + \frac{3\left(a + b\sqrt[3]{x}\right)^{13}}{13b^3} - \frac{a\left(a + b\sqrt[3]{x}\right)^{12}}{2b^3}$$

```
[Out] 3/11*a^2*(a+b*x^(1/3))^11/b^3-1/2*a*(a+b*x^(1/3))^12/b^3+3/13*(a+b*x^(1/3))
^13/b^3
```

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {190, 43}

$$\frac{3a^2\left(a + b\sqrt[3]{x}\right)^{11}}{11b^3} + \frac{3\left(a + b\sqrt[3]{x}\right)^{13}}{13b^3} - \frac{a\left(a + b\sqrt[3]{x}\right)^{12}}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^10,x]
```

```
[Out] (3*a^2*(a + b*x^(1/3))^11)/(11*b^3) - (a*(a + b*x^(1/3))^12)/(2*b^3) + (3*(
a + b*x^(1/3))^13)/(13*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}
\int \left(a + b\sqrt[3]{x}\right)^{10}\, dx &= 3\,\text{Subst}\left(\int x^2(a + bx)^{10}\, dx, x, \sqrt[3]{x}\right) \\
&= 3\,\text{Subst}\left(\int \left(\frac{a^2(a + bx)^{10}}{b^2} - \frac{2a(a + bx)^{11}}{b^2} + \frac{(a + bx)^{12}}{b^2}\right) dx, x, \sqrt[3]{x}\right) \\
&= \frac{3a^2\left(a + b\sqrt[3]{x}\right)^{11}}{11b^3} - \frac{a\left(a + b\sqrt[3]{x}\right)^{12}}{2b^3} + \frac{3\left(a + b\sqrt[3]{x}\right)^{13}}{13b^3}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 41, normalized size = 0.69

$$\frac{\left(a + b\sqrt[3]{x}\right)^{11}\left(a^2 - 11ab\sqrt[3]{x} + 66b^2x^{2/3}\right)}{286b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^10,x]
```

```
[Out] ((a + b*x^(1/3))^11*(a^2 - 11*a*b*x^(1/3) + 66*b^2*x^(2/3)))/(286*b^3)
```

fricas [B] time = 0.83, size = 117, normalized size = 1.98

$$\frac{5}{2}ab^9x^4+70\,a^4b^6x^3+60\,a^7b^3x^2+a^{10}x+\frac{27}{22}\left(10\,a^2b^8x^3+77\,a^5b^5x^2+22\,a^8b^2x\right)x^{\frac{2}{3}}+\frac{3}{26}\left(2\,b^{10}x^4+312\,a^3b^7x^3+78$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10,x, algorithm="fricas")
```

```
[Out] 5/2*a*b^9*x^4 + 70*a^4*b^6*x^3 + 60*a^7*b^3*x^2 + a^10*x + 27/22*(10*a^2*b^
8*x^3 + 77*a^5*b^5*x^2 + 22*a^8*b^2*x)*x^(2/3) + 3/26*(2*b^10*x^4 + 312*a^3
*b^7*x^3 + 780*a^6*b^4*x^2 + 65*a^9*b*x)*x^(1/3)
```

giac [B] time = 0.16, size = 109, normalized size = 1.85

$$\frac{3}{13}b^{10}x^{\frac{13}{3}}+\frac{5}{2}ab^9x^4+\frac{135}{11}a^2b^8x^{\frac{11}{3}}+36\,a^3b^7x^{\frac{10}{3}}+70\,a^4b^6x^3+\frac{189}{2}a^5b^5x^{\frac{8}{3}}+90\,a^6b^4x^{\frac{7}{3}}+60\,a^7b^3x^2+27\,a^8b^2x^{\frac{5}{3}}+\frac{15}{2}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10,x, algorithm="giac")
```

```
[Out] 3/13*b^10*x^(13/3) + 5/2*a*b^9*x^4 + 135/11*a^2*b^8*x^(11/3) + 36*a^3*b^7*x
^(10/3) + 70*a^4*b^6*x^3 + 189/2*a^5*b^5*x^(8/3) + 90*a^6*b^4*x^(7/3) + 60*
a^7*b^3*x^2 + 27*a^8*b^2*x^(5/3) + 15/2*a^9*b*x^(4/3) + a^10*x
```

maple [B] time = 0.00, size = 110, normalized size = 1.86

$$\frac{3b^{10}x^{\frac{13}{3}}}{13}+\frac{5a\,b^9x^4}{2}+\frac{135a^2b^8x^{\frac{11}{3}}}{11}+36a^3b^7x^{\frac{10}{3}}+70a^4b^6x^3+\frac{189a^5b^5x^{\frac{8}{3}}}{2}+90a^6b^4x^{\frac{7}{3}}+60a^7b^3x^2+27a^8b^2x^{\frac{5}{3}}+\frac{15a^9b\,x^{\frac{4}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^10,x)
```

```
[Out] a^10*x+3/13*b^10*x^(13/3)+5/2*a*b^9*x^4+135/11*a^2*b^8*x^(11/3)+36*a^3*b^7*
x^(10/3)+70*a^4*b^6*x^3+189/2*a^5*b^5*x^(8/3)+90*a^6*b^4*x^(7/3)+60*a^7*b^3
*x^2+27*a^8*b^2*x^(5/3)+15/2*a^9*b*x^(4/3)
```

maxima [A] time = 0.83, size = 47, normalized size = 0.80

$$\frac{3\left(bx^{\frac{1}{3}}+a\right)^{13}}{13\,b^3}-\frac{\left(bx^{\frac{1}{3}}+a\right)^{12}a}{2\,b^3}+\frac{3\left(bx^{\frac{1}{3}}+a\right)^{11}a^2}{11\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10,x, algorithm="maxima")
```

```
[Out] 3/13*(b*x^(1/3) + a)^13/b^3 - 1/2*(b*x^(1/3) + a)^12*a/b^3 + 3/11*(b*x^(1/3
) + a)^11*a^2/b^3
```

mupad [B] time = 0.06, size = 109, normalized size = 1.85

$$a^{10}\,x+\frac{3\,b^{10}\,x^{13/3}}{13}+\frac{5\,a\,b^9\,x^4}{2}+\frac{15\,a^9\,b\,x^{4/3}}{2}+60\,a^7\,b^3\,x^2+70\,a^4\,b^6\,x^3+27\,a^8\,b^2\,x^{5/3}+90\,a^6\,b^4\,x^{7/3}+\frac{189\,a^5\,b^5\,x^{8/3}}{2}+36$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))^10,x)
```

```
[Out] a^10*x + (3*b^10*x^(13/3))/13 + (5*a*b^9*x^4)/2 + (15*a^9*b*x^(4/3))/2 + 60
*a^7*b^3*x^2 + 70*a^4*b^6*x^3 + 27*a^8*b^2*x^(5/3) + 90*a^6*b^4*x^(7/3) + (
189*a^5*b^5*x^(8/3))/2 + 36*a^3*b^7*x^(10/3) + (135*a^2*b^8*x^(11/3))/11
```

sympy [**B**] time = 3.42, size = 136, normalized size = 2.31

$$a^{10}x+\frac{15a^9bx^{\frac{4}{3}}}{2}+27a^8b^2x^{\frac{5}{3}}+60a^7b^3x^2+90a^6b^4x^{\frac{7}{3}}+\frac{189a^5b^5x^{\frac{8}{3}}}{2}+70a^4b^6x^3+36a^3b^7x^{\frac{10}{3}}+\frac{135a^2b^8x^{\frac{11}{3}}}{11}+\frac{5ab^9x^4}{2}+\frac{3b^{10}x}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**10,x)
```

```
[Out] a**10*x + 15*a**9*b*x**(4/3)/2 + 27*a**8*b**2*x**(5/3) + 60*a**7*b**3*x**2
+ 90*a**6*b**4*x**(7/3) + 189*a**5*b**5*x**(8/3)/2 + 70*a**4*b**6*x**3 + 36
*a**3*b**7*x**(10/3) + 135*a**2*b**8*x**(11/3)/11 + 5*a*b**9*x**4/2 + 3*b**
10*x**(13/3)/13
```

3.2329 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x}\,dx$

Optimal. Leaf size=136

$$a^{10}\log(x)+30a^9b\sqrt[3]{x}+\frac{135}{2}a^8b^2x^{2/3}+120a^7b^3x+\frac{315}{2}a^6b^4x^{4/3}+\frac{756}{5}a^5b^5x^{5/3}+105a^4b^6x^2+\frac{360}{7}a^3b^7x^{7/3}+\frac{135}{8}a^2b^8$$

```
[Out] 30*a^9*b*x^(1/3)+135/2*a^8*b^2*x^(2/3)+120*a^7*b^3*x+315/2*a^6*b^4*x^(4/3)+
756/5*a^5*b^5*x^(5/3)+105*a^4*b^6*x^2+360/7*a^3*b^7*x^(7/3)+135/8*a^2*b^8*x
^(8/3)+10/3*a*b^9*x^3+3/10*b^10*x^(10/3)+a^10*ln(x)
```

Rubi [A] time = 0.07, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{135}{2}a^8b^2x^{2/3}+\frac{315}{2}a^6b^4x^{4/3}+\frac{756}{5}a^5b^5x^{5/3}+105a^4b^6x^2+\frac{360}{7}a^3b^7x^{7/3}+\frac{135}{8}a^2b^8x^{8/3}+120a^7b^3x+30a^9b\sqrt[3]{x}+a^{10}\log$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^10/x,x]
```

```
[Out] 30*a^9*b*x^(1/3) + (135*a^8*b^2*x^(2/3))/2 + 120*a^7*b^3*x + (315*a^6*b^4*x
^(4/3))/2 + (756*a^5*b^5*x^(5/3))/5 + 105*a^4*b^6*x^2 + (360*a^3*b^7*x^(7/3
))/7 + (135*a^2*b^8*x^(8/3))/8 + (10*a*b^9*x^3)/3 + (3*b^10*x^(10/3))/10 +
a^10*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x}\,dx &= 3\,\text{Subst}\left(\int \frac{(a+bx)^{10}}{x}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(10a^9b+\frac{a^{10}}{x}+45a^8b^2x+120a^7b^3x^2+210a^6b^4x^3+252a^5b^5x^4+210a^4b^6x^5\right.\right.\\ &= 30a^9b\sqrt[3]{x}+\frac{135}{2}a^8b^2x^{2/3}+120a^7b^3x+\frac{315}{2}a^6b^4x^{4/3}+\frac{756}{5}a^5b^5x^{5/3}+105a^4b^6x^2+\frac{360}{7}a^7\end{aligned}$$

Mathematica [A] time = 0.08, size = 136, normalized size = 1.00

$$a^{10}\log(x)+30a^9b\sqrt[3]{x}+\frac{135}{2}a^8b^2x^{2/3}+120a^7b^3x+\frac{315}{2}a^6b^4x^{4/3}+\frac{756}{5}a^5b^5x^{5/3}+105a^4b^6x^2+\frac{360}{7}a^3b^7x^{7/3}+\frac{135}{8}a^2b^8$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^10/x,x]
```

```
[Out] 30*a^9*b*x^(1/3) + (135*a^8*b^2*x^(2/3))/2 + 120*a^7*b^3*x + (315*a^6*b^4*x
^(4/3))/2 + (756*a^5*b^5*x^(5/3))/5 + 105*a^4*b^6*x^2 + (360*a^3*b^7*x^(7/3
))/7 + (135*a^2*b^8*x^(8/3))/8 + (10*a*b^9*x^3)/3 + (3*b^10*x^(10/3))/10 +
a^10*Log[x]
```

fricas [A] time = 0.93, size = 113, normalized size = 0.83

$$\frac{10}{3}ab^9x^3+105\,a^4b^6x^2+120\,a^7b^3x+3\,a^{10}\log\left(x^{\frac{1}{3}}\right)+\frac{27}{40}\left(25\,a^2b^8x^2+224\,a^5b^5x+100\,a^8b^2\right)x^{\frac{2}{3}}+\frac{3}{70}\left(7\,b^{10}x^3+1200\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10/x,x, algorithm="fricas")
```

```
[Out] 10/3*a*b^9*x^3 + 105*a^4*b^6*x^2 + 120*a^7*b^3*x + 3*a^10*log(x^(1/3)) + 27
/40*(25*a^2*b^8*x^2 + 224*a^5*b^5*x + 100*a^8*b^2)*x^(2/3) + 3/70*(7*b^10*x
^3 + 1200*a^3*b^7*x^2 + 3675*a^6*b^4*x + 700*a^9*b)*x^(1/3)
```

giac [A] time = 0.19, size = 109, normalized size = 0.80

$$\frac{3}{10}b^{10}x^{\frac{10}{3}}+\frac{10}{3}ab^9x^3+\frac{135}{8}a^2b^8x^{\frac{8}{3}}+\frac{360}{7}a^3b^7x^{\frac{7}{3}}+105\,a^4b^6x^2+\frac{756}{5}a^5b^5x^{\frac{5}{3}}+\frac{315}{2}a^6b^4x^{\frac{4}{3}}+120\,a^7b^3x+a^{10}\log\left(|x|\right)+\frac{1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10/x,x, algorithm="giac")
```

```
[Out] 3/10*b^10*x^(10/3) + 10/3*a*b^9*x^3 + 135/8*a^2*b^8*x^(8/3) + 360/7*a^3*b^7
*x^(7/3) + 105*a^4*b^6*x^2 + 756/5*a^5*b^5*x^(5/3) + 315/2*a^6*b^4*x^(4/3)
+ 120*a^7*b^3*x + a^10*log(abs(x)) + 135/2*a^8*b^2*x^(2/3) + 30*a^9*b*x^(1/
3)
```

maple [A] time = 0.00, size = 109, normalized size = 0.80

$$\frac{3b^{10}x^{\frac{10}{3}}}{10}+\frac{10a\,b^9x^3}{3}+\frac{135a^2b^8x^{\frac{8}{3}}}{8}+\frac{360a^3b^7x^{\frac{7}{3}}}{7}+105a^4b^6x^2+\frac{756a^5b^5x^{\frac{5}{3}}}{5}+\frac{315a^6b^4x^{\frac{4}{3}}}{2}+a^{10}\ln(x)+120a^7b^3x+\frac{135a^8b^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^10/x,x)
```

```
[Out] 30*a^9*b*x^(1/3)+135/2*a^8*b^2*x^(2/3)+120*a^7*b^3*x+315/2*a^6*b^4*x^(4/3)+
756/5*a^5*b^5*x^(5/3)+105*a^4*b^6*x^2+360/7*a^3*b^7*x^(7/3)+135/8*a^2*b^8*x
^(8/3)+10/3*a*b^9*x^3+3/10*b^10*x^(10/3)+a^10*ln(x)
```

maxima [A] time = 0.90, size = 108, normalized size = 0.79

$$\frac{3}{10}b^{10}x^{\frac{10}{3}}+\frac{10}{3}ab^9x^3+\frac{135}{8}a^2b^8x^{\frac{8}{3}}+\frac{360}{7}a^3b^7x^{\frac{7}{3}}+105\,a^4b^6x^2+\frac{756}{5}a^5b^5x^{\frac{5}{3}}+\frac{315}{2}a^6b^4x^{\frac{4}{3}}+120\,a^7b^3x+a^{10}\log(x)+\frac{135}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10/x,x, algorithm="maxima")
```

```
[Out] 3/10*b^10*x^(10/3) + 10/3*a*b^9*x^3 + 135/8*a^2*b^8*x^(8/3) + 360/7*a^3*b^7
*x^(7/3) + 105*a^4*b^6*x^2 + 756/5*a^5*b^5*x^(5/3) + 315/2*a^6*b^4*x^(4/3)
+ 120*a^7*b^3*x + a^10*log(x) + 135/2*a^8*b^2*x^(2/3) + 30*a^9*b*x^(1/3)
```

mupad [B] time = 0.06, size = 111, normalized size = 0.82

$$3\,a^{10}\,\ln\left(x^{1/3}\right)+\frac{3\,b^{10}\,x^{10/3}}{10}+120\,a^7\,b^3\,x+\frac{10\,a\,b^9\,x^3}{3}+30\,a^9\,b\,x^{1/3}+105\,a^4\,b^6\,x^2+\frac{135\,a^8\,b^2\,x^{2/3}}{2}+\frac{315\,a^6\,b^4\,x^{4/3}}{2}+\frac{756\,a^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))^10/x,x)
```

```
[Out] 3*a^10*log(x^(1/3)) + (3*b^10*x^(10/3))/10 + 120*a^7*b^3*x + (10*a*b^9*x^3)
/3 + 30*a^9*b*x^(1/3) + 105*a^4*b^6*x^2 + (135*a^8*b^2*x^(2/3))/2 + (315*a^
6*b^4*x^(4/3))/2 + (756*a^5*b^5*x^(5/3))/5 + (360*a^3*b^7*x^(7/3))/7 + (135
*a^2*b^8*x^(8/3))/8
```

sympy [**A**] time = 66.36, size = 139, normalized size = 1.02

$$a^{10}\log(x)+30a^9b\sqrt[3]{x}+\frac{135a^8b^2x^{\frac{2}{3}}}{2}+120a^7b^3x+\frac{315a^6b^4x^{\frac{4}{3}}}{2}+\frac{756a^5b^5x^{\frac{5}{3}}}{5}+105a^4b^6x^2+\frac{360a^3b^7x^{\frac{7}{3}}}{7}+\frac{135a^2b^8x^{\frac{8}{3}}}{8}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**10/x,x)
```

```
[Out] a**10*log(x) + 30*a**9*b*x**(1/3) + 135*a**8*b**2*x**(2/3)/2 + 120*a**7*b**
3*x + 315*a**6*b**4*x**(4/3)/2 + 756*a**5*b**5*x**(5/3)/5 + 105*a**4*b**6*x
**2 + 360*a**3*b**7*x**(7/3)/7 + 135*a**2*b**8*x**(8/3)/8 + 10*a*b**9*x**3/
3 + 3*b**10*x**(10/3)/10
```

3.2330 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^2}\,dx$

Optimal. Leaf size=125

$$-\frac{a^{10}}{x}-\frac{15a^9b}{x^{2/3}}-\frac{135a^8b^2}{\sqrt[3]{x}}+120a^7b^3\log(x)+630a^6b^4\sqrt[3]{x}+378a^5b^5x^{2/3}+210a^4b^6x+90a^3b^7x^{4/3}+27a^2b^8x^{5/3}+5ab^9x^2+\frac{3}{7}$$

```
[Out] -a^10/x-15*a^9*b/x^(2/3)-135*a^8*b^2/x^(1/3)+630*a^6*b^4*x^(1/3)+378*a^5*b^
5*x^(2/3)+210*a^4*b^6*x+90*a^3*b^7*x^(4/3)+27*a^2*b^8*x^(5/3)+5*a*b^9*x^2+3
/7*b^10*x^(7/3)+120*a^7*b^3*ln(x)
```

Rubi [A] time = 0.07, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$378a^5b^5x^{2/3}+90a^3b^7x^{4/3}+27a^2b^8x^{5/3}-\frac{135a^8b^2}{\sqrt[3]{x}}+630a^6b^4\sqrt[3]{x}+210a^4b^6x+120a^7b^3\log(x)-\frac{15a^9b}{x^{2/3}}-\frac{a^{10}}{x}+5ab^9x^2+\frac{3}{7}b$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^10/x^2,x]
```

```
[Out] -(a^10/x) - (15*a^9*b)/x^(2/3) - (135*a^8*b^2)/x^(1/3) + 630*a^6*b^4*x^(1/3
) + 378*a^5*b^5*x^(2/3) + 210*a^4*b^6*x + 90*a^3*b^7*x^(4/3) + 27*a^2*b^8*x
^(5/3) + 5*a*b^9*x^2 + (3*b^10*x^(7/3))/7 + 120*a^7*b^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^2}\,dx &= 3\,\text{Subst}\left(\int \frac{(a+bx)^{10}}{x^4}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(210a^6b^4+\frac{a^{10}}{x^4}+\frac{10a^9b}{x^3}+\frac{45a^8b^2}{x^2}+\frac{120a^7b^3}{x}+252a^5b^5x+210a^4b^6x^2+120a^7\right.\right.\\ &= -\frac{a^{10}}{x}-\frac{15a^9b}{x^{2/3}}-\frac{135a^8b^2}{\sqrt[3]{x}}+630a^6b^4\sqrt[3]{x}+378a^5b^5x^{2/3}+210a^4b^6x+90a^3b^7x^{4/3}+27a^2b^8x^{5}\end{aligned}$$

Mathematica [A] time = 0.08, size = 125, normalized size = 1.00

$$-\frac{a^{10}}{x}-\frac{15a^9b}{x^{2/3}}-\frac{135a^8b^2}{\sqrt[3]{x}}+120a^7b^3\log(x)+630a^6b^4\sqrt[3]{x}+378a^5b^5x^{2/3}+210a^4b^6x+90a^3b^7x^{4/3}+27a^2b^8x^{5/3}+5ab^9x^2+\frac{3}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^10/x^2,x]
```

```
[Out] -(a^10/x) - (15*a^9*b)/x^(2/3) - (135*a^8*b^2)/x^(1/3) + 630*a^6*b^4*x^(1/3
) + 378*a^5*b^5*x^(2/3) + 210*a^4*b^6*x + 90*a^3*b^7*x^(4/3) + 27*a^2*b^8*x
^(5/3) + 5*a*b^9*x^2 + (3*b^10*x^(7/3))/7 + 120*a^7*b^3*Log[x]
```

fricas [A] time = 0.82, size = 116, normalized size = 0.93

$$\frac{35\,ab^9x^3+1470\,a^4b^6x^2+2520\,a^7b^3x\log\left(x^{\frac{1}{3}}\right)-7\,a^{10}+189\left(a^2b^8x^2+14\,a^5b^5x-5\,a^8b^2\right)x^{\frac{2}{3}}+3\left(b^{10}x^3+210\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10/x^2,x, algorithm="fricas")
```

```
[Out] 1/7*(35*a*b^9*x^3 + 1470*a^4*b^6*x^2 + 2520*a^7*b^3*x*log(x^(1/3)) - 7*a^10
 + 189*(a^2*b^8*x^2 + 14*a^5*b^5*x - 5*a^8*b^2)*x^(2/3) + 3*(b^10*x^3 + 210
*a^3*b^7*x^2 + 1470*a^6*b^4*x - 35*a^9*b)*x^(1/3))/x
```

giac [A] time = 0.16, size = 111, normalized size = 0.89

$$\frac{3}{7}\,b^{10}x^{\frac{7}{3}}+5\,ab^9x^2+27\,a^2b^8x^{\frac{5}{3}}+90\,a^3b^7x^{\frac{4}{3}}+210\,a^4b^6x+120\,a^7b^3\log\left(|x|\right)+378\,a^5b^5x^{\frac{2}{3}}+630\,a^6b^4x^{\frac{1}{3}}-\frac{135\,a^8b^2x^{\frac{2}{3}}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10/x^2,x, algorithm="giac")
```

```
[Out] 3/7*b^10*x^(7/3) + 5*a*b^9*x^2 + 27*a^2*b^8*x^(5/3) + 90*a^3*b^7*x^(4/3) +
210*a^4*b^6*x + 120*a^7*b^3*log(abs(x)) + 378*a^5*b^5*x^(2/3) + 630*a^6*b^4
*x^(1/3) - (135*a^8*b^2*x^(2/3) + 15*a^9*b*x^(1/3) + a^10)/x
```

maple [A] time = 0.01, size = 110, normalized size = 0.88

$$\frac{3b^{10}x^{\frac{7}{3}}}{7}+5a\,b^9x^2+27a^2b^8x^{\frac{5}{3}}+90a^3b^7x^{\frac{4}{3}}+120a^7b^3\ln(x)+210a^4b^6x+378a^5b^5x^{\frac{2}{3}}+630a^6b^4x^{\frac{1}{3}}-\frac{135a^8b^2}{x^{\frac{1}{3}}}-\frac{15a^9b}{x^{\frac{2}{3}}}-\frac{a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^10/x^2,x)
```

```
[Out] -a^10/x-15*a^9*b/x^(2/3)-135*a^8*b^2/x^(1/3)+630*a^6*b^4*x^(1/3)+378*a^5*b^
5*x^(2/3)+210*a^4*b^6*x+90*a^3*b^7*x^(4/3)+27*a^2*b^8*x^(5/3)+5*a*b^9*x^2+3
/7*b^10*x^(7/3)+120*a^7*b^3*ln(x)
```

maxima [A] time = 0.93, size = 110, normalized size = 0.88

$$\frac{3}{7}\,b^{10}x^{\frac{7}{3}}+5\,ab^9x^2+27\,a^2b^8x^{\frac{5}{3}}+90\,a^3b^7x^{\frac{4}{3}}+210\,a^4b^6x+120\,a^7b^3\log(x)+378\,a^5b^5x^{\frac{2}{3}}+630\,a^6b^4x^{\frac{1}{3}}-\frac{135\,a^8b^2x^{\frac{2}{3}}+1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10/x^2,x, algorithm="maxima")
```

```
[Out] 3/7*b^10*x^(7/3) + 5*a*b^9*x^2 + 27*a^2*b^8*x^(5/3) + 90*a^3*b^7*x^(4/3) +
210*a^4*b^6*x + 120*a^7*b^3*log(x) + 378*a^5*b^5*x^(2/3) + 630*a^6*b^4*x^(1
/3) - (135*a^8*b^2*x^(2/3) + 15*a^9*b*x^(1/3) + a^10)/x
```

mupad [B] time = 0.05, size = 112, normalized size = 0.90

$$\frac{3\,b^{10}\,x^{7/3}}{7}-\frac{a^{10}+15\,a^9\,b\,x^{1/3}+135\,a^8\,b^2\,x^{2/3}}{x}+360\,a^7\,b^3\,\ln\left(x^{1/3}\right)+210\,a^4\,b^6\,x+5\,a\,b^9\,x^2+630\,a^6\,b^4\,x^{1/3}+378\,a^5\,b^$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))^10/x^2,x)
```

```
[Out] (3*b^10*x^(7/3))/7 - (a^10 + 15*a^9*b*x^(1/3) + 135*a^8*b^2*x^(2/3))/x + 36
0*a^7*b^3*log(x^(1/3)) + 210*a^4*b^6*x + 5*a*b^9*x^2 + 630*a^6*b^4*x^(1/3)
+ 378*a^5*b^5*x^(2/3) + 90*a^3*b^7*x^(4/3) + 27*a^2*b^8*x^(5/3)
```

sympy [A] time = 15.71, size = 131, normalized size = 1.05

$$-\frac{a^{10}}{x}-\frac{15a^9b}{x^{\frac{2}{3}}}-\frac{135a^8b^2}{\sqrt[3]{x}}+360a^7b^3\log\left(\sqrt[3]{x}\right)+630a^6b^4\sqrt[3]{x}+378a^5b^5x^{\frac{2}{3}}+210a^4b^6x+90a^3b^7x^{\frac{4}{3}}+27a^2b^8x^{\frac{5}{3}}+5ab^9x^2+\frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**10/x**2,x)
```

```
[Out] -a**10/x - 15*a**9*b/x**(2/3) - 135*a**8*b**2/x**(1/3) + 360*a**7*b**3*log(
x**(1/3)) + 630*a**6*b**4*x**(1/3) + 378*a**5*b**5*x**(2/3) + 210*a**4*b**6
*x + 90*a**3*b**7*x**(4/3) + 27*a**2*b**8*x**(5/3) + 5*a*b**9*x**2 + 3*b**1
0*x**(7/3)/7
```

3.2331 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^3}\,dx$

Optimal. Leaf size=131

$$-\frac{a^{10}}{2x^2}-\frac{6a^9b}{x^{5/3}}-\frac{135a^8b^2}{4x^{4/3}}-\frac{120a^7b^3}{x}-\frac{315a^6b^4}{x^{2/3}}-\frac{756a^5b^5}{\sqrt[3]{x}}+210a^4b^6\log(x)+360a^3b^7\sqrt[3]{x}+\frac{135}{2}a^2b^8x^{2/3}+10ab^9x+\frac{3}{4}b^1$$

```
[Out] -1/2*a^10/x^2-6*a^9*b/x^(5/3)-135/4*a^8*b^2/x^(4/3)-120*a^7*b^3/x-315*a^6*b
^4/x^(2/3)-756*a^5*b^5/x^(1/3)+360*a^3*b^7*x^(1/3)+135/2*a^2*b^8*x^(2/3)+10
*a*b^9*x+3/4*b^10*x^(4/3)+210*a^4*b^6*ln(x)
```

Rubi [A] time = 0.07, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{135a^8b^2}{4x^{4/3}}-\frac{315a^6b^4}{x^{2/3}}+\frac{135}{2}a^2b^8x^{2/3}-\frac{120a^7b^3}{x}-\frac{756a^5b^5}{\sqrt[3]{x}}+360a^3b^7\sqrt[3]{x}+210a^4b^6\log(x)-\frac{6a^9b}{x^{5/3}}-\frac{a^{10}}{2x^2}+10ab^9x+\frac{3}{4}b^1$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^10/x^3,x]
```

```
[Out] -a^10/(2*x^2) - (6*a^9*b)/x^(5/3) - (135*a^8*b^2)/(4*x^(4/3)) - (120*a^7*b^
3)/x - (315*a^6*b^4)/x^(2/3) - (756*a^5*b^5)/x^(1/3) + 360*a^3*b^7*x^(1/3)
+ (135*a^2*b^8*x^(2/3))/2 + 10*a*b^9*x + (3*b^10*x^(4/3))/4 + 210*a^4*b^6*L
og[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^3}\,dx &= 3\,\text{Subst}\left(\int \frac{(a+bx)^{10}}{x^7}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(120a^3b^7+\frac{a^{10}}{x^7}+\frac{10a^9b}{x^6}+\frac{45a^8b^2}{x^5}+\frac{120a^7b^3}{x^4}+\frac{210a^6b^4}{x^3}+\frac{252a^5b^5}{x^2}+\frac{210a}{x}\right.\right.\\ &= -\frac{a^{10}}{2x^2}-\frac{6a^9b}{x^{5/3}}-\frac{135a^8b^2}{4x^{4/3}}-\frac{120a^7b^3}{x}-\frac{315a^6b^4}{x^{2/3}}-\frac{756a^5b^5}{\sqrt[3]{x}}+360a^3b^7\sqrt[3]{x}+\frac{135}{2}a^2b^8x^{2/3}\end{aligned}$$

Mathematica [A] time = 0.06, size = 131, normalized size = 1.00

$$-\frac{a^{10}}{2x^2}-\frac{6a^9b}{x^{5/3}}-\frac{135a^8b^2}{4x^{4/3}}-\frac{120a^7b^3}{x}-\frac{315a^6b^4}{x^{2/3}}-\frac{756a^5b^5}{\sqrt[3]{x}}+210a^4b^6\log(x)+360a^3b^7\sqrt[3]{x}+\frac{135}{2}a^2b^8x^{2/3}+10ab^9x+\frac{3}{4}b^1$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^10/x^3,x]
```

```
[Out] -1/2*a^10/x^2 - (6*a^9*b)/x^(5/3) - (135*a^8*b^2)/(4*x^(4/3)) - (120*a^7*b^
3)/x - (315*a^6*b^4)/x^(2/3) - (756*a^5*b^5)/x^(1/3) + 360*a^3*b^7*x^(1/3)
+ (135*a^2*b^8*x^(2/3))/2 + 10*a*b^9*x + (3*b^10*x^(4/3))/4 + 210*a^4*b^6*L
og[x]
```

fricas [A] time = 0.79, size = 117, normalized size = 0.89

$$\frac{40\,ab^9x^3 + 2520\,a^4b^6x^2\log\left(x^{\frac{1}{3}}\right) - 480\,a^7b^3x - 2\,a^{10} + 27\left(10\,a^2b^8x^2 - 112\,a^5b^5x - 5\,a^8b^2\right)x^{\frac{2}{3}} + 3\left(b^{10}x^3 + 480\,a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(40*a*b^9*x^3 + 2520*a^4*b^6*x^2*log(x^(1/3)) - 480*a^7*b^3*x - 2*a^10
+ 27*(10*a^2*b^8*x^2 - 112*a^5*b^5*x - 5*a^8*b^2)*x^(2/3) + 3*(b^10*x^3 + 4
80*a^3*b^7*x^2 - 420*a^6*b^4*x - 8*a^9*b)*x^(1/3))/x^2
```

giac [A] time = 0.20, size = 111, normalized size = 0.85

$$\frac{3}{4}\,b^{10}x^{\frac{4}{3}} + 10\,ab^9x + 210\,a^4b^6\log\left(|x|\right) + \frac{135}{2}\,a^2b^8x^{\frac{2}{3}} + 360\,a^3b^7x^{\frac{1}{3}} - \frac{3024\,a^5b^5x^{\frac{5}{3}} + 1260\,a^6b^4x^{\frac{4}{3}} + 480\,a^7b^3x + 135\,a^8b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10/x^3,x, algorithm="giac")
```

```
[Out] 3/4*b^10*x^(4/3) + 10*a*b^9*x + 210*a^4*b^6*log(abs(x)) + 135/2*a^2*b^8*x^(
2/3) + 360*a^3*b^7*x^(1/3) - 1/4*(3024*a^5*b^5*x^(5/3) + 1260*a^6*b^4*x^(4/
3) + 480*a^7*b^3*x + 135*a^8*b^2*x^(2/3) + 24*a^9*b*x^(1/3) + 2*a^10)/x^2
```

maple [A] time = 0.01, size = 110, normalized size = 0.84

$$\frac{3b^{10}x^{\frac{4}{3}}}{4} + 210a^4b^6\ln(x) + 10a\,b^9x + \frac{135a^2b^8x^{\frac{2}{3}}}{2} + 360a^3b^7x^{\frac{1}{3}} - \frac{756a^5b^5}{x^{\frac{1}{3}}} - \frac{315a^6b^4}{x^{\frac{2}{3}}} - \frac{120a^7b^3}{x} - \frac{135a^8b^2}{4x^{\frac{4}{3}}} - \frac{6a^9b}{x^{\frac{5}{3}}} - \frac{a^{10}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^10/x^3,x)
```

```
[Out] -1/2*a^10/x^2-6*a^9*b/x^(5/3)-135/4*a^8*b^2/x^(4/3)-120*a^7*b^3/x-315*a^6*b
^4/x^(2/3)-756*a^5*b^5/x^(1/3)+360*a^3*b^7*x^(1/3)+135/2*a^2*b^8*x^(2/3)+10
*a*b^9*x+3/4*b^10*x^(4/3)+210*a^4*b^6*ln(x)
```

maxima [A] time = 0.91, size = 110, normalized size = 0.84

$$\frac{3}{4}\,b^{10}x^{\frac{4}{3}} + 10\,ab^9x + 210\,a^4b^6\log(x) + \frac{135}{2}\,a^2b^8x^{\frac{2}{3}} + 360\,a^3b^7x^{\frac{1}{3}} - \frac{3024\,a^5b^5x^{\frac{5}{3}} + 1260\,a^6b^4x^{\frac{4}{3}} + 480\,a^7b^3x + 135\,a^8b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10/x^3,x, algorithm="maxima")
```

```
[Out] 3/4*b^10*x^(4/3) + 10*a*b^9*x + 210*a^4*b^6*log(x) + 135/2*a^2*b^8*x^(2/3)
+ 360*a^3*b^7*x^(1/3) - 1/4*(3024*a^5*b^5*x^(5/3) + 1260*a^6*b^4*x^(4/3) +
480*a^7*b^3*x + 135*a^8*b^2*x^(2/3) + 24*a^9*b*x^(1/3) + 2*a^10)/x^2
```

mupad [B] time = 1.13, size = 112, normalized size = 0.85

$$\frac{3\,b^{10}\,x^{4/3}}{4} - \frac{\frac{a^{10}}{2} + 120\,a^7\,b^3\,x + 6\,a^9\,b\,x^{1/3} + \frac{135\,a^8\,b^2\,x^{2/3}}{4} + 315\,a^6\,b^4\,x^{4/3} + 756\,a^5\,b^5\,x^{5/3}}{x^2} + 630\,a^4\,b^6\,\ln\left(x^{1/3}\right) + 360\,a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))^10/x^3,x)
```

```
[Out] (3*b^10*x^(4/3))/4 - (a^10/2 + 120*a^7*b^3*x + 6*a^9*b*x^(1/3) + (135*a^8*b
^2*x^(2/3))/4 + 315*a^6*b^4*x^(4/3) + 756*a^5*b^5*x^(5/3))/x^2 + 630*a^4*b^
6*log(x^(1/3)) + 360*a^3*b^7*x^(1/3) + (135*a^2*b^8*x^(2/3))/2 + 10*a*b^9*x
```

sympy [**A**] time = 15.85, size = 136, normalized size = 1.04

$$-\frac{a^{10}}{2x^2}-\frac{6a^9b}{x^{\frac{5}{3}}}-\frac{135a^8b^2}{4x^{\frac{4}{3}}}-\frac{120a^7b^3}{x}-\frac{315a^6b^4}{x^{\frac{2}{3}}}-\frac{756a^5b^5}{\sqrt[3]{x}}+630a^4b^6\log\left(\sqrt[3]{x}\right)+360a^3b^7\sqrt[3]{x}+\frac{135a^2b^8x^{\frac{2}{3}}}{2}+10ab^9x+\frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**10/x**3,x)
```

```
[Out] -a**10/(2*x**2) - 6*a**9*b/x**(5/3) - 135*a**8*b**2/(4*x**(4/3)) - 120*a**7
*b**3/x - 315*a**6*b**4/x**(2/3) - 756*a**5*b**5/x**(1/3) + 630*a**4*b**6*l
og(x**(1/3)) + 360*a**3*b**7*x**(1/3) + 135*a**2*b**8*x**(2/3)/2 + 10*a*b**
9*x + 3*b**10*x**(4/3)/4
```

3.2332 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^4}\,dx$

Optimal. Leaf size=131

$$-\frac{a^{10}}{3x^3}-\frac{15a^9b}{4x^{8/3}}-\frac{135a^8b^2}{7x^{7/3}}-\frac{60a^7b^3}{x^2}-\frac{126a^6b^4}{x^{5/3}}-\frac{189a^5b^5}{x^{4/3}}-\frac{210a^4b^6}{x}-\frac{180a^3b^7}{x^{2/3}}-\frac{135a^2b^8}{\sqrt[3]{x}}+10ab^9\log(x)+3b^{10}\sqrt[3]{x}$$

```
[Out] -1/3*a^10/x^3-15/4*a^9*b/x^(8/3)-135/7*a^8*b^2/x^(7/3)-60*a^7*b^3/x^2-126*a
^6*b^4/x^(5/3)-189*a^5*b^5/x^(4/3)-210*a^4*b^6/x-180*a^3*b^7/x^(2/3)-135*a^
2*b^8/x^(1/3)+3*b^10*x^(1/3)+10*a*b^9*ln(x)
```

Rubi [A] time = 0.07, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{135a^8b^2}{7x^{7/3}}-\frac{60a^7b^3}{x^2}-\frac{126a^6b^4}{x^{5/3}}-\frac{189a^5b^5}{x^{4/3}}-\frac{180a^3b^7}{x^{2/3}}-\frac{210a^4b^6}{x}-\frac{135a^2b^8}{\sqrt[3]{x}}-\frac{15a^9b}{4x^{8/3}}-\frac{a^{10}}{3x^3}+10ab^9\log(x)+3b^{10}\sqrt[3]{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^10/x^4,x]
```

```
[Out] -a^10/(3*x^3) - (15*a^9*b)/(4*x^(8/3)) - (135*a^8*b^2)/(7*x^(7/3)) - (60*a^
7*b^3)/x^2 - (126*a^6*b^4)/x^(5/3) - (189*a^5*b^5)/x^(4/3) - (210*a^4*b^6)/
x - (180*a^3*b^7)/x^(2/3) - (135*a^2*b^8)/x^(1/3) + 3*b^10*x^(1/3) + 10*a*b
^9*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^4}\,dx &= 3\,\text{Subst}\left(\int \frac{(a+bx)^{10}}{x^{10}}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(b^{10}+\frac{a^{10}}{x^{10}}+\frac{10a^9b}{x^9}+\frac{45a^8b^2}{x^8}+\frac{120a^7b^3}{x^7}+\frac{210a^6b^4}{x^6}+\frac{252a^5b^5}{x^5}+\frac{210a^4b^6}{x^4}+\right.\right.\\ &= -\frac{a^{10}}{3x^3}-\frac{15a^9b}{4x^{8/3}}-\frac{135a^8b^2}{7x^{7/3}}-\frac{60a^7b^3}{x^2}-\frac{126a^6b^4}{x^{5/3}}-\frac{189a^5b^5}{x^{4/3}}-\frac{210a^4b^6}{x}-\frac{180a^3b^7}{x^{2/3}}-\frac{135a^2b^8}{\sqrt[3]{x}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 131, normalized size = 1.00

$$-\frac{a^{10}}{3x^3}-\frac{15a^9b}{4x^{8/3}}-\frac{135a^8b^2}{7x^{7/3}}-\frac{60a^7b^3}{x^2}-\frac{126a^6b^4}{x^{5/3}}-\frac{189a^5b^5}{x^{4/3}}-\frac{210a^4b^6}{x}-\frac{180a^3b^7}{x^{2/3}}-\frac{135a^2b^8}{\sqrt[3]{x}}+10ab^9\log(x)+3b^{10}\sqrt[3]{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^10/x^4,x]
```

```
[Out] -1/3*a^10/x^3 - (15*a^9*b)/(4*x^(8/3)) - (135*a^8*b^2)/(7*x^(7/3)) - (60*a^
7*b^3)/x^2 - (126*a^6*b^4)/x^(5/3) - (189*a^5*b^5)/x^(4/3) - (210*a^4*b^6)/
x - (180*a^3*b^7)/x^(2/3) - (135*a^2*b^8)/x^(1/3) + 3*b^10*x^(1/3) + 10*a*b
^9*Log[x]
```

fricas [A] time = 0.87, size = 118, normalized size = 0.90

$$\frac{2520\,ab^9x^3\log\left(x^{\frac{1}{3}}\right) - 17640\,a^4b^6x^2 - 5040\,a^7b^3x - 28\,a^{10} - 324\left(35\,a^2b^8x^2 + 49\,a^5b^5x + 5\,a^8b^2\right)x^{\frac{2}{3}} + 63\left(4\,b^1}{84\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10/x^4,x, algorithm="fricas")
```

```
[Out] 1/84*(2520*a*b^9*x^3*log(x^(1/3)) - 17640*a^4*b^6*x^2 - 5040*a^7*b^3*x - 28
*a^10 - 324*(35*a^2*b^8*x^2 + 49*a^5*b^5*x + 5*a^8*b^2)*x^(2/3) + 63*(4*b^1
0*x^3 - 240*a^3*b^7*x^2 - 168*a^6*b^4*x - 5*a^9*b)*x^(1/3))/x^3
```

giac [A] time = 0.17, size = 113, normalized size = 0.86

$$10\,ab^9\log\left(|x|\right)+3\,b^{10}x^{\frac{1}{3}}-\frac{11340\,a^2b^8x^{\frac{8}{3}} + 15120\,a^3b^7x^{\frac{7}{3}} + 17640\,a^4b^6x^2 + 15876\,a^5b^5x^{\frac{5}{3}} + 10584\,a^6b^4x^{\frac{4}{3}} + 5040}{84\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10/x^4,x, algorithm="giac")
```

```
[Out] 10*a*b^9*log(abs(x)) + 3*b^10*x^(1/3) - 1/84*(11340*a^2*b^8*x^(8/3) + 15120
*a^3*b^7*x^(7/3) + 17640*a^4*b^6*x^2 + 15876*a^5*b^5*x^(5/3) + 10584*a^6*b^
4*x^(4/3) + 5040*a^7*b^3*x + 1620*a^8*b^2*x^(2/3) + 315*a^9*b*x^(1/3) + 28*
a^10)/x^3
```

maple [A] time = 0.01, size = 112, normalized size = 0.85

$$10a\,b^9\ln(x)+3b^{10}x^{\frac{1}{3}}-\frac{135a^2b^8}{x^{\frac{1}{3}}}-\frac{180a^3b^7}{x^{\frac{2}{3}}}-\frac{210a^4b^6}{x}-\frac{189a^5b^5}{x^{\frac{4}{3}}}-\frac{126a^6b^4}{x^{\frac{5}{3}}}-\frac{60a^7b^3}{x^2}-\frac{135a^8b^2}{7x^{\frac{7}{3}}}-\frac{15a^9b}{4x^{\frac{8}{3}}}-\frac{a^{10}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^10/x^4,x)
```

```
[Out] -1/3*a^10/x^3-15/4*a^9*b/x^(8/3)-135/7*a^8*b^2/x^(7/3)-60*a^7*b^3/x^2-126*a
^6*b^4/x^(5/3)-189*a^5*b^5/x^(4/3)-210*a^4*b^6/x-180*a^3*b^7/x^(2/3)-135*a^
2*b^8/x^(1/3)+3*b^10*x^(1/3)+10*a*b^9*ln(x)
```

maxima [A] time = 0.92, size = 112, normalized size = 0.85

$$10\,ab^9\log(x)+3\,b^{10}x^{\frac{1}{3}}-\frac{11340\,a^2b^8x^{\frac{8}{3}} + 15120\,a^3b^7x^{\frac{7}{3}} + 17640\,a^4b^6x^2 + 15876\,a^5b^5x^{\frac{5}{3}} + 10584\,a^6b^4x^{\frac{4}{3}} + 5040}{84\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10/x^4,x, algorithm="maxima")
```

```
[Out] 10*a*b^9*log(x) + 3*b^10*x^(1/3) - 1/84*(11340*a^2*b^8*x^(8/3) + 15120*a^3*
b^7*x^(7/3) + 17640*a^4*b^6*x^2 + 15876*a^5*b^5*x^(5/3) + 10584*a^6*b^4*x^(
4/3) + 5040*a^7*b^3*x + 1620*a^8*b^2*x^(2/3) + 315*a^9*b*x^(1/3) + 28*a^10)
/x^3
```

mupad [B] time = 1.16, size = 113, normalized size = 0.86

$$3\,b^{10}\,x^{1/3}-\frac{a^{10}}{3\,x^3}-\frac{15\,a^9\,b}{4\,x^{8/3}}-\frac{210\,a^4\,b^6}{x}-\frac{60\,a^7\,b^3}{x^2}-\frac{135\,a^2\,b^8}{x^{1/3}}-\frac{180\,a^3\,b^7}{x^{2/3}}-\frac{189\,a^5\,b^5}{x^{4/3}}-\frac{126\,a^6\,b^4}{x^{5/3}}-\frac{135\,a^8\,b^2}{7\,x^{7/3}}+30\,a\,b^9\,\ln\left(x^{1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))^10/x^4,x)
```

```
[Out] 3*b^10*x^(1/3) - a^10/(3*x^3) - (15*a^9*b)/(4*x^(8/3)) - (210*a^4*b^6)/x -
(60*a^7*b^3)/x^2 - (135*a^2*b^8)/x^(1/3) - (180*a^3*b^7)/x^(2/3) - (189*a^5
*b^5)/x^(4/3) - (126*a^6*b^4)/x^(5/3) - (135*a^8*b^2)/(7*x^(7/3)) + 30*a*b^
9*log(x^(1/3))
```

sympy [A] time = 2.42, size = 133, normalized size = 1.02

$$-\frac{a^{10}}{3x^3}-\frac{15a^9b}{4x^{\frac{8}{3}}}-\frac{135a^8b^2}{7x^{\frac{7}{3}}}-\frac{60a^7b^3}{x^2}-\frac{126a^6b^4}{x^{\frac{5}{3}}}-\frac{189a^5b^5}{x^{\frac{4}{3}}}-\frac{210a^4b^6}{x}-\frac{180a^3b^7}{x^{\frac{2}{3}}}-\frac{135a^2b^8}{\sqrt[3]{x}}+10ab^9\log(x)+3b^{10}\sqrt[3]{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**10/x**4,x)
```

```
[Out] -a**10/(3*x**3) - 15*a**9*b/(4*x**(8/3)) - 135*a**8*b**2/(7*x**(7/3)) - 60*
a**7*b**3/x**2 - 126*a**6*b**4/x**(5/3) - 189*a**5*b**5/x**(4/3) - 210*a**4
*b**6/x - 180*a**3*b**7/x**(2/3) - 135*a**2*b**8/x**(1/3) + 10*a*b**9*log(x
) + 3*b**10*x**(1/3)
```

3.2333 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^5}\,dx$

Optimal. Leaf size=46

$$\frac{b\left(a+b\sqrt[3]{x}\right)^{11}}{44a^2x^{11/3}}-\frac{\left(a+b\sqrt[3]{x}\right)^{11}}{4ax^4}$$

```
[Out] -1/4*(a+b*x^(1/3))^11/a/x^4+1/44*b*(a+b*x^(1/3))^11/a^2/x^(11/3)
```

Rubi [A] time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {266, 45, 37}

$$\frac{b\left(a+b\sqrt[3]{x}\right)^{11}}{44a^2x^{11/3}}-\frac{\left(a+b\sqrt[3]{x}\right)^{11}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^10/x^5,x]
```

```
[Out] -(a + b*x^(1/3))^11/(4*a*x^4) + (b*(a + b*x^(1/3))^11)/(44*a^2*x^(11/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^5}\,dx &= 3\,\text{Subst}\left(\int \frac{(a+bx)^{10}}{x^{13}}\,dx, x, \sqrt[3]{x}\right)\\ &= -\frac{\left(a+b\sqrt[3]{x}\right)^{11}}{4ax^4}-\frac{b\,\text{Subst}\left(\int \frac{(a+bx)^{10}}{x^{12}}\,dx, x, \sqrt[3]{x}\right)}{4a}\\ &= -\frac{\left(a+b\sqrt[3]{x}\right)^{11}}{4ax^4}+\frac{b\left(a+b\sqrt[3]{x}\right)^{11}}{44a^2x^{11/3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 0.70

$$\frac{\left(b\sqrt[3]{x}-11a\right)\left(a+b\sqrt[3]{x}\right)^{11}}{44a^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^10/x^5,x]
```

```
[Out] ((-11*a + b*x^(1/3))*(a + b*x^(1/3))^11)/(44*a^2*x^4)
```

fricas [B] time = 0.75, size = 114, normalized size = 2.48

$$-\frac{440\,ab^9x^3+4620\,a^4b^6x^2+1760\,a^7b^3x+11\,a^{10}+297\left(5\,a^2b^8x^2+16\,a^5b^5x+2\,a^8b^2\right)x^{\frac{2}{3}}+3\left(22\,b^{10}x^3+1056\,a^3b^7x^2+1155\,a^6b^4x+40\,a^9b\right)x^{\frac{1}{3}}}{44\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10/x^5,x, algorithm="fricas")
```

```
[Out] -1/44*(440*a*b^9*x^3 + 4620*a^4*b^6*x^2 + 1760*a^7*b^3*x + 11*a^10 + 297*(5
*a^2*b^8*x^2 + 16*a^5*b^5*x + 2*a^8*b^2)*x^(2/3) + 3*(22*b^10*x^3 + 1056*a^
3*b^7*x^2 + 1155*a^6*b^4*x + 40*a^9*b)*x^(1/3))/x^4
```

giac [B] time = 0.18, size = 112, normalized size = 2.43

$$-\frac{66\,b^{10}x^{\frac{10}{3}}+440\,ab^9x^3+1485\,a^2b^8x^{\frac{8}{3}}+3168\,a^3b^7x^{\frac{7}{3}}+4620\,a^4b^6x^2+4752\,a^5b^5x^{\frac{5}{3}}+3465\,a^6b^4x^{\frac{4}{3}}+1760\,a^7b^3x}{44\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10/x^5,x, algorithm="giac")
```

```
[Out] -1/44*(66*b^10*x^(10/3) + 440*a*b^9*x^3 + 1485*a^2*b^8*x^(8/3) + 3168*a^3*b
^7*x^(7/3) + 4620*a^4*b^6*x^2 + 4752*a^5*b^5*x^(5/3) + 3465*a^6*b^4*x^(4/3)
 + 1760*a^7*b^3*x + 594*a^8*b^2*x^(2/3) + 120*a^9*b*x^(1/3) + 11*a^10)/x^4
```

maple [B] time = 0.01, size = 113, normalized size = 2.46

$$-\frac{3b^{10}}{2x^{\frac{2}{3}}}-\frac{10a\,b^9}{x}-\frac{135a^2b^8}{4x^{\frac{4}{3}}}-\frac{72a^3b^7}{x^{\frac{5}{3}}}-\frac{105a^4b^6}{x^2}-\frac{108a^5b^5}{x^{\frac{7}{3}}}-\frac{315a^6b^4}{4x^{\frac{8}{3}}}-\frac{40a^7b^3}{x^3}-\frac{27a^8b^2}{2x^{\frac{10}{3}}}-\frac{30a^9b}{11x^{\frac{11}{3}}}-\frac{a^{10}}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^10/x^5,x)
```

```
[Out] -72*a^3*b^7/x^(5/3)-135/4*a^2*b^8/x^(4/3)-10*a*b^9/x-315/4*a^6*b^4/x^(8/3)-
3/2*b^10/x^(2/3)-40*a^7*b^3/x^3-108*a^5*b^5/x^(7/3)-105*a^4*b^6/x^2-1/4*a^1
0/x^4-27/2*a^8*b^2/x^(10/3)-30/11*a^9*b/x^(11/3)
```

maxima [B] time = 0.91, size = 112, normalized size = 2.43

$$-\frac{66\,b^{10}x^{\frac{10}{3}}+440\,ab^9x^3+1485\,a^2b^8x^{\frac{8}{3}}+3168\,a^3b^7x^{\frac{7}{3}}+4620\,a^4b^6x^2+4752\,a^5b^5x^{\frac{5}{3}}+3465\,a^6b^4x^{\frac{4}{3}}+1760\,a^7b^3x}{44\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10/x^5,x, algorithm="maxima")
```

```
[Out] -1/44*(66*b^10*x^(10/3) + 440*a*b^9*x^3 + 1485*a^2*b^8*x^(8/3) + 3168*a^3*b
^7*x^(7/3) + 4620*a^4*b^6*x^2 + 4752*a^5*b^5*x^(5/3) + 3465*a^6*b^4*x^(4/3)
 + 1760*a^7*b^3*x + 594*a^8*b^2*x^(2/3) + 120*a^9*b*x^(1/3) + 11*a^10)/x^4
```

mupad [B] time = 1.18, size = 112, normalized size = 2.43

$$-\frac{\frac{a^{10}}{4}+\frac{3\,b^{10}\,x^{10/3}}{2}+40\,a^7\,b^3\,x+10\,a\,b^9\,x^3+\frac{30\,a^9\,b\,x^{1/3}}{11}+105\,a^4\,b^6\,x^2+\frac{27\,a^8\,b^2\,x^{2/3}}{2}+\frac{315\,a^6\,b^4\,x^{4/3}}{4}+108\,a^5\,b^5\,x^{5/3}+7}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))^10/x^5,x)
```

```
[Out] -(a^10/4 + (3*b^10*x^(10/3))/2 + 40*a^7*b^3*x + 10*a*b^9*x^3 + (30*a^9*b*x^
(1/3))/11 + 105*a^4*b^6*x^2 + (27*a^8*b^2*x^(2/3))/2 + (315*a^6*b^4*x^(4/3)
)/4 + 108*a^5*b^5*x^(5/3) + 72*a^3*b^7*x^(7/3) + (135*a^2*b^8*x^(8/3))/4)/x
^4
```

sympy [B] time = 3.88, size = 139, normalized size = 3.02

$$-\frac{a^{10}}{4x^4}-\frac{30a^9b}{11x^{\frac{11}{3}}}-\frac{27a^8b^2}{2x^{\frac{10}{3}}}-\frac{40a^7b^3}{x^3}-\frac{315a^6b^4}{4x^{\frac{8}{3}}}-\frac{108a^5b^5}{x^{\frac{7}{3}}}-\frac{105a^4b^6}{x^2}-\frac{72a^3b^7}{x^{\frac{5}{3}}}-\frac{135a^2b^8}{4x^{\frac{4}{3}}}-\frac{10ab^9}{x}-\frac{3b^{10}}{2x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**10/x**5,x)
```

```
[Out] -a**10/(4*x**4) - 30*a**9*b/(11*x**(11/3)) - 27*a**8*b**2/(2*x**(10/3)) - 4
0*a**7*b**3/x**3 - 315*a**6*b**4/(4*x**(8/3)) - 108*a**5*b**5/x**(7/3) - 10
5*a**4*b**6/x**2 - 72*a**3*b**7/x**(5/3) - 135*a**2*b**8/(4*x**(4/3)) - 10*
a*b**9/x - 3*b**10/(2*x**(2/3))
```

3.2334 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^6}\,dx$

Optimal. Leaf size=122

$$-\frac{b^4\left(a+b\sqrt[3]{x}\right)^{11}}{5005a^5x^{11/3}}+\frac{b^3\left(a+b\sqrt[3]{x}\right)^{11}}{455a^4x^4}-\frac{6b^2\left(a+b\sqrt[3]{x}\right)^{11}}{455a^3x^{13/3}}+\frac{2b\left(a+b\sqrt[3]{x}\right)^{11}}{35a^2x^{14/3}}-\frac{\left(a+b\sqrt[3]{x}\right)^{11}}{5ax^5}$$

```
[Out] -1/5*(a+b*x^(1/3))^11/a/x^5+2/35*b*(a+b*x^(1/3))^11/a^2/x^(14/3)-6/455*b^2*
(a+b*x^(1/3))^11/a^3/x^(13/3)+1/455*b^3*(a+b*x^(1/3))^11/a^4/x^4-1/5005*b^4
*(a+b*x^(1/3))^11/a^5/x^(11/3)
```

Rubi [A] time = 0.05, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {266, 45, 37}

$$-\frac{b^4\left(a+b\sqrt[3]{x}\right)^{11}}{5005a^5x^{11/3}}+\frac{b^3\left(a+b\sqrt[3]{x}\right)^{11}}{455a^4x^4}-\frac{6b^2\left(a+b\sqrt[3]{x}\right)^{11}}{455a^3x^{13/3}}+\frac{2b\left(a+b\sqrt[3]{x}\right)^{11}}{35a^2x^{14/3}}-\frac{\left(a+b\sqrt[3]{x}\right)^{11}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^10/x^6,x]
```

```
[Out] -(a + b*x^(1/3))^11/(5*a*x^5) + (2*b*(a + b*x^(1/3))^11)/(35*a^2*x^(14/3))
- (6*b^2*(a + b*x^(1/3))^11)/(455*a^3*x^(13/3)) + (b^3*(a + b*x^(1/3))^11)/
(455*a^4*x^4) - (b^4*(a + b*x^(1/3))^11)/(5005*a^5*x^(11/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^6}\,dx = 3\,\text{Subst}\left(\int \frac{(a+bx)^{10}}{x^{16}}\,dx, x, \sqrt[3]{x}\right)$$
$$= -\frac{\left(a+b\sqrt[3]{x}\right)^{11}}{5ax^5} - \frac{(4b)\,\text{Subst}\left(\int \frac{(a+bx)^{10}}{x^{15}}\,dx, x, \sqrt[3]{x}\right)}{5a}$$
$$= -\frac{\left(a+b\sqrt[3]{x}\right)^{11}}{5ax^5} + \frac{2b\left(a+b\sqrt[3]{x}\right)^{11}}{35a^2x^{14/3}} + \frac{\left(6b^2\right)\text{Subst}\left(\int \frac{(a+bx)^{10}}{x^{14}}\,dx, x, \sqrt[3]{x}\right)}{35a^2}$$
$$= -\frac{\left(a+b\sqrt[3]{x}\right)^{11}}{5ax^5} + \frac{2b\left(a+b\sqrt[3]{x}\right)^{11}}{35a^2x^{14/3}} - \frac{6b^2\left(a+b\sqrt[3]{x}\right)^{11}}{455a^3x^{13/3}} - \frac{\left(12b^3\right)\text{Subst}\left(\int \frac{(a+bx)^{10}}{x^{13}}\,dx, x, \sqrt[3]{}\right.}{455a^3}$$
$$= -\frac{\left(a+b\sqrt[3]{x}\right)^{11}}{5ax^5} + \frac{2b\left(a+b\sqrt[3]{x}\right)^{11}}{35a^2x^{14/3}} - \frac{6b^2\left(a+b\sqrt[3]{x}\right)^{11}}{455a^3x^{13/3}} + \frac{b^3\left(a+b\sqrt[3]{x}\right)^{11}}{455a^4x^4} + \frac{b^4\,\text{Subst}\left(\int \frac{(}{}\right.}{4}$$
$$= -\frac{\left(a+b\sqrt[3]{x}\right)^{11}}{5ax^5} + \frac{2b\left(a+b\sqrt[3]{x}\right)^{11}}{35a^2x^{14/3}} - \frac{6b^2\left(a+b\sqrt[3]{x}\right)^{11}}{455a^3x^{13/3}} + \frac{b^3\left(a+b\sqrt[3]{x}\right)^{11}}{455a^4x^4} - \frac{b^4\left(a+b\sqrt[3]{x}\right)}{5005a^5x^{11/3}}$$

Mathematica [A] time = 0.01, size = 67, normalized size = 0.55

$$-\frac{\left(a+b\sqrt[3]{x}\right)^{11}\left(1001a^4 - 286a^3b\sqrt[3]{x} + 66a^2b^2x^{2/3} - 11ab^3x + b^4x^{4/3}\right)}{5005a^5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^10/x^6,x]
```

```
[Out] -1/5005*((a + b*x^(1/3))^11*(1001*a^4 - 286*a^3*b*x^(1/3) + 66*a^2*b^2*x^(2
/3) - 11*a*b^3*x + b^4*x^(4/3)))/(a^5*x^5)
```

fricas [A] time = 0.76, size = 114, normalized size = 0.93

$$-\frac{25025\,ab^9x^3 + 350350\,a^4b^6x^2 + 150150\,a^7b^3x + 1001\,a^{10} + 297\left(325\,a^2b^8x^2 + 1274\,a^5b^5x + 175\,a^8b^2\right)x^{\frac{2}{3}} + 39}{5005\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10/x^6,x, algorithm="fricas")
```

```
[Out] -1/5005*(25025*a*b^9*x^3 + 350350*a^4*b^6*x^2 + 150150*a^7*b^3*x + 1001*a^1
0 + 297*(325*a^2*b^8*x^2 + 1274*a^5*b^5*x + 175*a^8*b^2)*x^(2/3) + 39*(77*b
^10*x^3 + 5775*a^3*b^7*x^2 + 7350*a^6*b^4*x + 275*a^9*b)*x^(1/3))/x^5
```

giac [A] time = 0.19, size = 112, normalized size = 0.92

$$-\frac{3003\,b^{10}x^{\frac{10}{3}} + 25025\,ab^9x^3 + 96525\,a^2b^8x^{\frac{8}{3}} + 225225\,a^3b^7x^{\frac{7}{3}} + 350350\,a^4b^6x^2 + 378378\,a^5b^5x^{\frac{5}{3}} + 286650\,a^6}{5005\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10/x^6,x, algorithm="giac")
```

```
[Out] -1/5005*(3003*b^10*x^(10/3) + 25025*a*b^9*x^3 + 96525*a^2*b^8*x^(8/3) + 225
225*a^3*b^7*x^(7/3) + 350350*a^4*b^6*x^2 + 378378*a^5*b^5*x^(5/3) + 286650*
a^6*b^4*x^(4/3) + 150150*a^7*b^3*x + 51975*a^8*b^2*x^(2/3) + 10725*a^9*b*x^
(1/3) + 1001*a^10)/x^5
```

maple [A] time = 0.01, size = 113, normalized size = 0.93

$$-\frac{3b^{10}}{5x^{\frac{5}{3}}}-\frac{5a\,b^9}{x^2}-\frac{135a^2b^8}{7x^{\frac{7}{3}}}-\frac{45a^3b^7}{x^{\frac{8}{3}}}-\frac{70a^4b^6}{x^3}-\frac{378a^5b^5}{5x^{\frac{10}{3}}}-\frac{630a^6b^4}{11x^{\frac{11}{3}}}-\frac{30a^7b^3}{x^4}-\frac{135a^8b^2}{13x^{\frac{13}{3}}}-\frac{15a^9b}{7x^{\frac{14}{3}}}-\frac{a^{10}}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^10/x^6,x)
```

```
[Out] -3/5*b^10/x^(5/3)-15/7*a^9*b/x^(14/3)-45*a^3*b^7/x^(8/3)-1/5*a^10/x^5-135/1
3*a^8*b^2/x^(13/3)-70*a^4*b^6/x^3-135/7*a^2*b^8/x^(7/3)-5*a*b^9/x^2-30*a^7*
b^3/x^4-378/5*a^5*b^5/x^(10/3)-630/11*a^6*b^4/x^(11/3)
```

maxima [A] time = 0.91, size = 112, normalized size = 0.92

$$-\frac{3003\,b^{10}x^{\frac{10}{3}}+25025\,ab^9x^3+96525\,a^2b^8x^{\frac{8}{3}}+225225\,a^3b^7x^{\frac{7}{3}}+350350\,a^4b^6x^2+378378\,a^5b^5x^{\frac{5}{3}}+286650\,a^6b^4}{5005\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10/x^6,x, algorithm="maxima")
```

```
[Out] -1/5005*(3003*b^10*x^(10/3) + 25025*a*b^9*x^3 + 96525*a^2*b^8*x^(8/3) + 225
225*a^3*b^7*x^(7/3) + 350350*a^4*b^6*x^2 + 378378*a^5*b^5*x^(5/3) + 286650*
a^6*b^4*x^(4/3) + 150150*a^7*b^3*x + 51975*a^8*b^2*x^(2/3) + 10725*a^9*b*x^
(1/3) + 1001*a^10)/x^5
```

mupad [B] time = 1.17, size = 112, normalized size = 0.92

$$-\frac{\frac{a^{10}}{5}+\frac{3\,b^{10}\,x^{10/3}}{5}+30\,a^7\,b^3\,x+5\,a\,b^9\,x^3+\frac{15\,a^9\,b\,x^{1/3}}{7}+70\,a^4\,b^6\,x^2+\frac{135\,a^8\,b^2\,x^{2/3}}{13}+\frac{630\,a^6\,b^4\,x^{4/3}}{11}+\frac{378\,a^5\,b^5\,x^{5/3}}{5}+45\,a^3\,b^7}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))^10/x^6,x)
```

```
[Out] -(a^10/5 + (3*b^10*x^(10/3))/5 + 30*a^7*b^3*x + 5*a*b^9*x^3 + (15*a^9*b*x^(
1/3))/7 + 70*a^4*b^6*x^2 + (135*a^8*b^2*x^(2/3))/13 + (630*a^6*b^4*x^(4/3))
/11 + (378*a^5*b^5*x^(5/3))/5 + 45*a^3*b^7*x^(7/3) + (135*a^2*b^8*x^(8/3))/
7)/x^5
```

sympy [A] time = 6.12, size = 143, normalized size = 1.17

$$-\frac{a^{10}}{5x^5}-\frac{15a^9b}{7x^{\frac{14}{3}}}-\frac{135a^8b^2}{13x^{\frac{13}{3}}}-\frac{30a^7b^3}{x^4}-\frac{630a^6b^4}{11x^{\frac{11}{3}}}-\frac{378a^5b^5}{5x^{\frac{10}{3}}}-\frac{70a^4b^6}{x^3}-\frac{45a^3b^7}{x^{\frac{8}{3}}}-\frac{135a^2b^8}{7x^{\frac{7}{3}}}-\frac{5ab^9}{x^2}-\frac{3b^{10}}{5x^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**10/x**6,x)
```

```
[Out] -a**10/(5*x**5) - 15*a**9*b/(7*x**(14/3)) - 135*a**8*b**2/(13*x**(13/3)) -
30*a**7*b**3/x**4 - 630*a**6*b**4/(11*x**(11/3)) - 378*a**5*b**5/(5*x**(10/
3)) - 70*a**4*b**6/x**3 - 45*a**3*b**7/x**(8/3) - 135*a**2*b**8/(7*x**(7/3)
) - 5*a*b**9/x**2 - 3*b**10/(5*x**(5/3))
```

3.2335 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^7}\,dx$

Optimal. Leaf size=144

$$-\frac{a^{10}}{6x^6}-\frac{30a^9b}{17x^{17/3}}-\frac{135a^8b^2}{16x^{16/3}}-\frac{24a^7b^3}{x^5}-\frac{45a^6b^4}{x^{14/3}}-\frac{756a^5b^5}{13x^{13/3}}-\frac{105a^4b^6}{2x^4}-\frac{360a^3b^7}{11x^{11/3}}-\frac{27a^2b^8}{2x^{10/3}}-\frac{10ab^9}{3x^3}-\frac{3b^{10}}{8x^{8/3}}$$

```
[Out] -1/6*a^10/x^6-30/17*a^9*b/x^(17/3)-135/16*a^8*b^2/x^(16/3)-24*a^7*b^3/x^5-4
5*a^6*b^4/x^(14/3)-756/13*a^5*b^5/x^(13/3)-105/2*a^4*b^6/x^4-360/11*a^3*b^7
/x^(11/3)-27/2*a^2*b^8/x^(10/3)-10/3*a*b^9/x^3-3/8*b^10/x^(8/3)
```

Rubi [A] time = 0.07, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{135a^8b^2}{16x^{16/3}}-\frac{24a^7b^3}{x^5}-\frac{45a^6b^4}{x^{14/3}}-\frac{756a^5b^5}{13x^{13/3}}-\frac{105a^4b^6}{2x^4}-\frac{360a^3b^7}{11x^{11/3}}-\frac{27a^2b^8}{2x^{10/3}}-\frac{30a^9b}{17x^{17/3}}-\frac{a^{10}}{6x^6}-\frac{10ab^9}{3x^3}-\frac{3b^{10}}{8x^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^10/x^7,x]
```

```
[Out] -a^10/(6*x^6) - (30*a^9*b)/(17*x^(17/3)) - (135*a^8*b^2)/(16*x^(16/3)) - (2
4*a^7*b^3)/x^5 - (45*a^6*b^4)/x^(14/3) - (756*a^5*b^5)/(13*x^(13/3)) - (105
*a^4*b^6)/(2*x^4) - (360*a^3*b^7)/(11*x^(11/3)) - (27*a^2*b^8)/(2*x^(10/3))
 - (10*a*b^9)/(3*x^3) - (3*b^10)/(8*x^(8/3))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^7}\,dx &= 3\,\text{Subst}\left(\int \frac{(a+bx)^{10}}{x^{19}}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(\frac{a^{10}}{x^{19}}+\frac{10a^9b}{x^{18}}+\frac{45a^8b^2}{x^{17}}+\frac{120a^7b^3}{x^{16}}+\frac{210a^6b^4}{x^{15}}+\frac{252a^5b^5}{x^{14}}+\frac{210a^4b^6}{x^{13}}+\frac{120}{x}\right.\right.\\ &= -\frac{a^{10}}{6x^6}-\frac{30a^9b}{17x^{17/3}}-\frac{135a^8b^2}{16x^{16/3}}-\frac{24a^7b^3}{x^5}-\frac{45a^6b^4}{x^{14/3}}-\frac{756a^5b^5}{13x^{13/3}}-\frac{105a^4b^6}{2x^4}-\frac{360a^3b^7}{11x^{11/3}}-\frac{27a^2}{2x^{10}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 144, normalized size = 1.00

$$-\frac{a^{10}}{6x^6}-\frac{30a^9b}{17x^{17/3}}-\frac{135a^8b^2}{16x^{16/3}}-\frac{24a^7b^3}{x^5}-\frac{45a^6b^4}{x^{14/3}}-\frac{756a^5b^5}{13x^{13/3}}-\frac{105a^4b^6}{2x^4}-\frac{360a^3b^7}{11x^{11/3}}-\frac{27a^2b^8}{2x^{10/3}}-\frac{10ab^9}{3x^3}-\frac{3b^{10}}{8x^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^10/x^7,x]
```

```
[Out] -1/6*a^10/x^6 - (30*a^9*b)/(17*x^(17/3)) - (135*a^8*b^2)/(16*x^(16/3)) - (2
4*a^7*b^3)/x^5 - (45*a^6*b^4)/x^(14/3) - (756*a^5*b^5)/(13*x^(13/3)) - (105
*a^4*b^6)/(2*x^4) - (360*a^3*b^7)/(11*x^(11/3)) - (27*a^2*b^8)/(2*x^(10/3))
 - (10*a*b^9)/(3*x^3) - (3*b^10)/(8*x^(8/3))
```

fricas [A] time = 0.71, size = 114, normalized size = 0.79

$$-\frac{388960\,ab^9x^3+6126120\,a^4b^6x^2+2800512\,a^7b^3x+19448\,a^{10}+15147\left(104\,a^2b^8x^2+448\,a^5b^5x+65\,a^8b^2\right)x^{\frac{2}{3}}+}{116688\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10/x^7,x, algorithm="fricas")
```

```
[Out] -1/116688*(388960*a*b^9*x^3 + 6126120*a^4*b^6*x^2 + 2800512*a^7*b^3*x + 194
48*a^10 + 15147*(104*a^2*b^8*x^2 + 448*a^5*b^5*x + 65*a^8*b^2)*x^(2/3) + 23
4*(187*b^10*x^3 + 16320*a^3*b^7*x^2 + 22440*a^6*b^4*x + 880*a^9*b)*x^(1/3))
/x^6
```

giac [A] time = 0.17, size = 112, normalized size = 0.78

$$-\frac{43758\,b^{10}x^{\frac{10}{3}}+388960\,ab^9x^3+1575288\,a^2b^8x^{\frac{8}{3}}+3818880\,a^3b^7x^{\frac{7}{3}}+6126120\,a^4b^6x^2+6785856\,a^5b^5x^{\frac{5}{3}}+5250}{116688\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10/x^7,x, algorithm="giac")
```

```
[Out] -1/116688*(43758*b^10*x^(10/3) + 388960*a*b^9*x^3 + 1575288*a^2*b^8*x^(8/3)
 + 3818880*a^3*b^7*x^(7/3) + 6126120*a^4*b^6*x^2 + 6785856*a^5*b^5*x^(5/3)
+ 5250960*a^6*b^4*x^(4/3) + 2800512*a^7*b^3*x + 984555*a^8*b^2*x^(2/3) + 20
5920*a^9*b*x^(1/3) + 19448*a^10)/x^6
```

maple [A] time = 0.01, size = 113, normalized size = 0.78

$$-\frac{3b^{10}}{8x^{\frac{8}{3}}}-\frac{10a\,b^9}{3x^3}-\frac{27a^2b^8}{2x^{\frac{10}{3}}}-\frac{360a^3b^7}{11x^{\frac{11}{3}}}-\frac{105a^4b^6}{2x^4}-\frac{756a^5b^5}{13x^{\frac{13}{3}}}-\frac{45a^6b^4}{x^{\frac{14}{3}}}-\frac{24a^7b^3}{x^5}-\frac{135a^8b^2}{16x^{\frac{16}{3}}}-\frac{30a^9b}{17x^{\frac{17}{3}}}-\frac{a^{10}}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^10/x^7,x)
```

```
[Out] -1/6*a^10/x^6-30/17*a^9*b/x^(17/3)-135/16*a^8*b^2/x^(16/3)-24*a^7*b^3/x^5-4
5*a^6*b^4/x^(14/3)-756/13*a^5*b^5/x^(13/3)-105/2*a^4*b^6/x^4-360/11*a^3*b^7
/x^(11/3)-27/2*a^2*b^8/x^(10/3)-10/3*a*b^9/x^3-3/8*b^10/x^(8/3)
```

maxima [A] time = 0.85, size = 112, normalized size = 0.78

$$-\frac{43758\,b^{10}x^{\frac{10}{3}}+388960\,ab^9x^3+1575288\,a^2b^8x^{\frac{8}{3}}+3818880\,a^3b^7x^{\frac{7}{3}}+6126120\,a^4b^6x^2+6785856\,a^5b^5x^{\frac{5}{3}}+5250}{116688\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10/x^7,x, algorithm="maxima")
```

```
[Out] -1/116688*(43758*b^10*x^(10/3) + 388960*a*b^9*x^3 + 1575288*a^2*b^8*x^(8/3)
 + 3818880*a^3*b^7*x^(7/3) + 6126120*a^4*b^6*x^2 + 6785856*a^5*b^5*x^(5/3)
+ 5250960*a^6*b^4*x^(4/3) + 2800512*a^7*b^3*x + 984555*a^8*b^2*x^(2/3) + 20
5920*a^9*b*x^(1/3) + 19448*a^10)/x^6
```

mupad [B] time = 1.16, size = 112, normalized size = 0.78

$$-\frac{\frac{a^{10}}{6}+\frac{3\,b^{10}\,x^{10/3}}{8}+24\,a^7\,b^3\,x+\frac{10\,a\,b^9\,x^3}{3}+\frac{30\,a^9\,b\,x^{1/3}}{17}+\frac{105\,a^4\,b^6\,x^2}{2}+\frac{135\,a^8\,b^2\,x^{2/3}}{16}+45\,a^6\,b^4\,x^{4/3}+\frac{756\,a^5\,b^5\,x^{5/3}}{13}+\frac{360\,a^3}{}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))^10/x^7,x)
```

```
[Out] -(a^10/6 + (3*b^10*x^(10/3))/8 + 24*a^7*b^3*x + (10*a*b^9*x^3)/3 + (30*a^9*
b*x^(1/3))/17 + (105*a^4*b^6*x^2)/2 + (135*a^8*b^2*x^(2/3))/16 + 45*a^6*b^4
*x^(4/3) + (756*a^5*b^5*x^(5/3))/13 + (360*a^3*b^7*x^(7/3))/11 + (27*a^2*b^
8*x^(8/3))/2)/x^6
```

sympy [A] time = 9.50, size = 146, normalized size = 1.01

$$-\frac{a^{10}}{6x^6}-\frac{30a^9b}{17x^{\frac{17}{3}}}-\frac{135a^8b^2}{16x^{\frac{16}{3}}}-\frac{24a^7b^3}{x^5}-\frac{45a^6b^4}{x^{\frac{14}{3}}}-\frac{756a^5b^5}{13x^{\frac{13}{3}}}-\frac{105a^4b^6}{2x^4}-\frac{360a^3b^7}{11x^{\frac{11}{3}}}-\frac{27a^2b^8}{2x^{\frac{10}{3}}}-\frac{10ab^9}{3x^3}-\frac{3b^{10}}{8x^{\frac{8}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**10/x**7,x)
```

```
[Out] -a**10/(6*x**6) - 30*a**9*b/(17*x**(17/3)) - 135*a**8*b**2/(16*x**(16/3)) -
 24*a**7*b**3/x**5 - 45*a**6*b**4/x**(14/3) - 756*a**5*b**5/(13*x**(13/3))
- 105*a**4*b**6/(2*x**4) - 360*a**3*b**7/(11*x**(11/3)) - 27*a**2*b**8/(2*x
**(10/3)) - 10*a*b**9/(3*x**3) - 3*b**10/(8*x**(8/3))
```

3.2336 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^8}\,dx$

Optimal. Leaf size=144

$$-\frac{a^{10}}{7x^7}-\frac{3a^9b}{2x^{20/3}}-\frac{135a^8b^2}{19x^{19/3}}-\frac{20a^7b^3}{x^6}-\frac{630a^6b^4}{17x^{17/3}}-\frac{189a^5b^5}{4x^{16/3}}-\frac{42a^4b^6}{x^5}-\frac{180a^3b^7}{7x^{14/3}}-\frac{135a^2b^8}{13x^{13/3}}-\frac{5ab^9}{2x^4}-\frac{3b^{10}}{11x^{11/3}}$$

```
[Out] -1/7*a^10/x^7-3/2*a^9*b/x^(20/3)-135/19*a^8*b^2/x^(19/3)-20*a^7*b^3/x^6-630
/17*a^6*b^4/x^(17/3)-189/4*a^5*b^5/x^(16/3)-42*a^4*b^6/x^5-180/7*a^3*b^7/x^
(14/3)-135/13*a^2*b^8/x^(13/3)-5/2*a*b^9/x^4-3/11*b^10/x^(11/3)
```

Rubi [A] time = 0.07, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{135a^8b^2}{19x^{19/3}}-\frac{20a^7b^3}{x^6}-\frac{630a^6b^4}{17x^{17/3}}-\frac{189a^5b^5}{4x^{16/3}}-\frac{42a^4b^6}{x^5}-\frac{180a^3b^7}{7x^{14/3}}-\frac{135a^2b^8}{13x^{13/3}}-\frac{3a^9b}{2x^{20/3}}-\frac{a^{10}}{7x^7}-\frac{5ab^9}{2x^4}-\frac{3b^{10}}{11x^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^10/x^8,x]
```

```
[Out] -a^10/(7*x^7) - (3*a^9*b)/(2*x^(20/3)) - (135*a^8*b^2)/(19*x^(19/3)) - (20*
a^7*b^3)/x^6 - (630*a^6*b^4)/(17*x^(17/3)) - (189*a^5*b^5)/(4*x^(16/3)) - (
42*a^4*b^6)/x^5 - (180*a^3*b^7)/(7*x^(14/3)) - (135*a^2*b^8)/(13*x^(13/3))
- (5*a*b^9)/(2*x^4) - (3*b^10)/(11*x^(11/3))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^8}\,dx &= 3\,\text{Subst}\left(\int \frac{(a+bx)^{10}}{x^{22}}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(\frac{a^{10}}{x^{22}}+\frac{10a^9b}{x^{21}}+\frac{45a^8b^2}{x^{20}}+\frac{120a^7b^3}{x^{19}}+\frac{210a^6b^4}{x^{18}}+\frac{252a^5b^5}{x^{17}}+\frac{210a^4b^6}{x^{16}}+\frac{120a^3b}{x^{15}}\right.\right.\\ &= -\frac{a^{10}}{7x^7}-\frac{3a^9b}{2x^{20/3}}-\frac{135a^8b^2}{19x^{19/3}}-\frac{20a^7b^3}{x^6}-\frac{630a^6b^4}{17x^{17/3}}-\frac{189a^5b^5}{4x^{16/3}}-\frac{42a^4b^6}{x^5}-\frac{180a^3b^7}{7x^{14/3}}-\frac{135a^2b^8}{13x^{13/3}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 144, normalized size = 1.00

$$-\frac{a^{10}}{7x^7}-\frac{3a^9b}{2x^{20/3}}-\frac{135a^8b^2}{19x^{19/3}}-\frac{20a^7b^3}{x^6}-\frac{630a^6b^4}{17x^{17/3}}-\frac{189a^5b^5}{4x^{16/3}}-\frac{42a^4b^6}{x^5}-\frac{180a^3b^7}{7x^{14/3}}-\frac{135a^2b^8}{13x^{13/3}}-\frac{5ab^9}{2x^4}-\frac{3b^{10}}{11x^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^10/x^8,x]
```

```
[Out] -1/7*a^10/x^7 - (3*a^9*b)/(2*x^(20/3)) - (135*a^8*b^2)/(19*x^(19/3)) - (20*
a^7*b^3)/x^6 - (630*a^6*b^4)/(17*x^(17/3)) - (189*a^5*b^5)/(4*x^(16/3)) - (
42*a^4*b^6)/x^5 - (180*a^3*b^7)/(7*x^(14/3)) - (135*a^2*b^8)/(13*x^(13/3))
- (5*a*b^9)/(2*x^4) - (3*b^10)/(11*x^(11/3))
```

fricas [A] time = 0.94, size = 114, normalized size = 0.79

$$-\frac{3233230\,ab^9x^3 + 54318264\,a^4b^6x^2 + 25865840\,a^7b^3x + 184756\,a^{10} + 35343\left(380\,a^2b^8x^2 + 1729\,a^5b^5x + 260\right.}{1293292\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10/x^8,x, algorithm="fricas")
```

```
[Out] -1/1293292*(3233230*a*b^9*x^3 + 54318264*a^4*b^6*x^2 + 25865840*a^7*b^3*x +
 184756*a^10 + 35343*(380*a^2*b^8*x^2 + 1729*a^5*b^5*x + 260*a^8*b^2)*x^(2/
3) + 1482*(238*b^10*x^3 + 22440*a^3*b^7*x^2 + 32340*a^6*b^4*x + 1309*a^9*b)
*x^(1/3))/x^7
```

giac [A] time = 0.17, size = 112, normalized size = 0.78

$$-\frac{352716\,b^{10}x^{\frac{10}{3}} + 3233230\,ab^9x^3 + 13430340\,a^2b^8x^{\frac{8}{3}} + 33256080\,a^3b^7x^{\frac{7}{3}} + 54318264\,a^4b^6x^2 + 61108047\,a^5b^5}{1293292\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10/x^8,x, algorithm="giac")
```

```
[Out] -1/1293292*(352716*b^10*x^(10/3) + 3233230*a*b^9*x^3 + 13430340*a^2*b^8*x^(
8/3) + 33256080*a^3*b^7*x^(7/3) + 54318264*a^4*b^6*x^2 + 61108047*a^5*b^5*x
^(5/3) + 47927880*a^6*b^4*x^(4/3) + 25865840*a^7*b^3*x + 9189180*a^8*b^2*x^
(2/3) + 1939938*a^9*b*x^(1/3) + 184756*a^10)/x^7
```

maple [A] time = 0.01, size = 113, normalized size = 0.78

$$-\frac{3b^{10}}{11x^{\frac{11}{3}}} - \frac{5a\,b^9}{2x^4} - \frac{135a^2b^8}{13x^{\frac{13}{3}}} - \frac{180a^3b^7}{7x^{\frac{14}{3}}} - \frac{42a^4b^6}{x^5} - \frac{189a^5b^5}{4x^{\frac{16}{3}}} - \frac{630a^6b^4}{17x^{\frac{17}{3}}} - \frac{20a^7b^3}{x^6} - \frac{135a^8b^2}{19x^{\frac{19}{3}}} - \frac{3a^9b}{2x^{\frac{20}{3}}} - \frac{a^{10}}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^10/x^8,x)
```

```
[Out] -1/7*a^10/x^7-3/2*a^9*b/x^(20/3)-135/19*a^8*b^2/x^(19/3)-20*a^7*b^3/x^6-630
/17*a^6*b^4/x^(17/3)-189/4*a^5*b^5/x^(16/3)-42*a^4*b^6/x^5-180/7*a^3*b^7/x^
(14/3)-135/13*a^2*b^8/x^(13/3)-5/2*a*b^9/x^4-3/11*b^10/x^(11/3)
```

maxima [A] time = 0.91, size = 112, normalized size = 0.78

$$-\frac{352716\,b^{10}x^{\frac{10}{3}} + 3233230\,ab^9x^3 + 13430340\,a^2b^8x^{\frac{8}{3}} + 33256080\,a^3b^7x^{\frac{7}{3}} + 54318264\,a^4b^6x^2 + 61108047\,a^5b^5}{1293292\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10/x^8,x, algorithm="maxima")
```

```
[Out] -1/1293292*(352716*b^10*x^(10/3) + 3233230*a*b^9*x^3 + 13430340*a^2*b^8*x^(
8/3) + 33256080*a^3*b^7*x^(7/3) + 54318264*a^4*b^6*x^2 + 61108047*a^5*b^5*x
^(5/3) + 47927880*a^6*b^4*x^(4/3) + 25865840*a^7*b^3*x + 9189180*a^8*b^2*x^
(2/3) + 1939938*a^9*b*x^(1/3) + 184756*a^10)/x^7
```

mupad [B] time = 0.09, size = 112, normalized size = 0.78

$$-\frac{\frac{a^{10}}{7}+\frac{3\,b^{10}\,x^{10/3}}{11}+20\,a^7\,b^3\,x+\frac{5\,a\,b^9\,x^3}{2}+\frac{3\,a^9\,b\,x^{1/3}}{2}+42\,a^4\,b^6\,x^2+\frac{135\,a^8\,b^2\,x^{2/3}}{19}+\frac{630\,a^6\,b^4\,x^{4/3}}{17}+\frac{189\,a^5\,b^5\,x^{5/3}}{4}+\frac{180\,a^3\,b^7\,x^{7/3}}{7}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))^10/x^8,x)
```

```
[Out] -(a^10/7 + (3*b^10*x^(10/3))/11 + 20*a^7*b^3*x + (5*a*b^9*x^3)/2 + (3*a^9*b
*x^(1/3))/2 + 42*a^4*b^6*x^2 + (135*a^8*b^2*x^(2/3))/19 + (630*a^6*b^4*x^(4
/3))/17 + (189*a^5*b^5*x^(5/3))/4 + (180*a^3*b^7*x^(7/3))/7 + (135*a^2*b^8*
x^(8/3))/13)/x^7
```

sympy [A] time = 13.72, size = 146, normalized size = 1.01

$$-\frac{a^{10}}{7x^7}-\frac{3a^9b}{2x^{\frac{20}{3}}}-\frac{135a^8b^2}{19x^{\frac{19}{3}}}-\frac{20a^7b^3}{x^6}-\frac{630a^6b^4}{17x^{\frac{17}{3}}}-\frac{189a^5b^5}{4x^{\frac{16}{3}}}-\frac{42a^4b^6}{x^5}-\frac{180a^3b^7}{7x^{\frac{14}{3}}}-\frac{135a^2b^8}{13x^{\frac{13}{3}}}-\frac{5ab^9}{2x^4}-\frac{3b^{10}}{11x^{\frac{11}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**10/x**8,x)
```

```
[Out] -a**10/(7*x**7) - 3*a**9*b/(2*x**(20/3)) - 135*a**8*b**2/(19*x**(19/3)) - 2
0*a**7*b**3/x**6 - 630*a**6*b**4/(17*x**(17/3)) - 189*a**5*b**5/(4*x**(16/3
)) - 42*a**4*b**6/x**5 - 180*a**3*b**7/(7*x**(14/3)) - 135*a**2*b**8/(13*x*
*(13/3)) - 5*a*b**9/(2*x**4) - 3*b**10/(11*x**(11/3))
```

3.2337 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^9}\,dx$

Optimal. Leaf size=144

$$-\frac{a^{10}}{8x^8}-\frac{30a^9b}{23x^{23/3}}-\frac{135a^8b^2}{22x^{22/3}}-\frac{120a^7b^3}{7x^7}-\frac{63a^6b^4}{2x^{20/3}}-\frac{756a^5b^5}{19x^{19/3}}-\frac{35a^4b^6}{x^6}-\frac{360a^3b^7}{17x^{17/3}}-\frac{135a^2b^8}{16x^{16/3}}-\frac{2ab^9}{x^5}-\frac{3b^{10}}{14x^{14/3}}$$

```
[Out] -1/8*a^10/x^8-30/23*a^9*b/x^(23/3)-135/22*a^8*b^2/x^(22/3)-120/7*a^7*b^3/x^
7-63/2*a^6*b^4/x^(20/3)-756/19*a^5*b^5/x^(19/3)-35*a^4*b^6/x^6-360/17*a^3*b
^7/x^(17/3)-135/16*a^2*b^8/x^(16/3)-2*a*b^9/x^5-3/14*b^10/x^(14/3)
```

Rubi [A] time = 0.07, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{135a^8b^2}{22x^{22/3}}-\frac{120a^7b^3}{7x^7}-\frac{63a^6b^4}{2x^{20/3}}-\frac{756a^5b^5}{19x^{19/3}}-\frac{35a^4b^6}{x^6}-\frac{360a^3b^7}{17x^{17/3}}-\frac{135a^2b^8}{16x^{16/3}}-\frac{30a^9b}{23x^{23/3}}-\frac{a^{10}}{8x^8}-\frac{2ab^9}{x^5}-\frac{3b^{10}}{14x^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^10/x^9,x]
```

```
[Out] -a^10/(8*x^8) - (30*a^9*b)/(23*x^(23/3)) - (135*a^8*b^2)/(22*x^(22/3)) - (1
20*a^7*b^3)/(7*x^7) - (63*a^6*b^4)/(2*x^(20/3)) - (756*a^5*b^5)/(19*x^(19/3
)) - (35*a^4*b^6)/x^6 - (360*a^3*b^7)/(17*x^(17/3)) - (135*a^2*b^8)/(16*x^(
16/3)) - (2*a*b^9)/x^5 - (3*b^10)/(14*x^(14/3))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^9}\,dx &= 3\,\text{Subst}\left(\int \frac{(a+bx)^{10}}{x^{25}}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(\frac{a^{10}}{x^{25}}+\frac{10a^9b}{x^{24}}+\frac{45a^8b^2}{x^{23}}+\frac{120a^7b^3}{x^{22}}+\frac{210a^6b^4}{x^{21}}+\frac{252a^5b^5}{x^{20}}+\frac{210a^4b^6}{x^{19}}+\frac{120}{x}\right.\right.\\ &= -\frac{a^{10}}{8x^8}-\frac{30a^9b}{23x^{23/3}}-\frac{135a^8b^2}{22x^{22/3}}-\frac{120a^7b^3}{7x^7}-\frac{63a^6b^4}{2x^{20/3}}-\frac{756a^5b^5}{19x^{19/3}}-\frac{35a^4b^6}{x^6}-\frac{360a^3b^7}{17x^{17/3}}-\frac{135a}{16x^{1}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 144, normalized size = 1.00

$$-\frac{a^{10}}{8x^8}-\frac{30a^9b}{23x^{23/3}}-\frac{135a^8b^2}{22x^{22/3}}-\frac{120a^7b^3}{7x^7}-\frac{63a^6b^4}{2x^{20/3}}-\frac{756a^5b^5}{19x^{19/3}}-\frac{35a^4b^6}{x^6}-\frac{360a^3b^7}{17x^{17/3}}-\frac{135a^2b^8}{16x^{16/3}}-\frac{2ab^9}{x^5}-\frac{3b^{10}}{14x^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^10/x^9,x]
```

```
[Out] -1/8*a^10/x^8 - (30*a^9*b)/(23*x^(23/3)) - (135*a^8*b^2)/(22*x^(22/3)) - (1
20*a^7*b^3)/(7*x^7) - (63*a^6*b^4)/(2*x^(20/3)) - (756*a^5*b^5)/(19*x^(19/3
)) - (35*a^4*b^6)/x^6 - (360*a^3*b^7)/(17*x^(17/3)) - (135*a^2*b^8)/(16*x^(
16/3)) - (2*a*b^9)/x^5 - (3*b^10)/(14*x^(14/3))
```

fricas [A] time = 0.75, size = 114, normalized size = 0.79

$$-\frac{18305056\,ab^9x^3+320338480\,a^4b^6x^2+156900480\,a^7b^3x+1144066\,a^{10}+73899\left(1045\,a^2b^8x^2+4928\,a^5b^5x+7\right.}{9152528\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10/x^9,x, algorithm="fricas")
```

```
[Out] -1/9152528*(18305056*a*b^9*x^3 + 320338480*a^4*b^6*x^2 + 156900480*a^7*b^3*
x + 1144066*a^10 + 73899*(1045*a^2*b^8*x^2 + 4928*a^5*b^5*x + 760*a^8*b^2)*
x^(2/3) + 5016*(391*b^10*x^3 + 38640*a^3*b^7*x^2 + 57477*a^6*b^4*x + 2380*a
^9*b)*x^(1/3))/x^8
```

giac [A] time = 0.17, size = 112, normalized size = 0.78

$$-\frac{1961256\,b^{10}x^{\frac{10}{3}}+18305056\,ab^9x^3+77224455\,a^2b^8x^{\frac{8}{3}}+193818240\,a^3b^7x^{\frac{7}{3}}+320338480\,a^4b^6x^2+364174272\,a}{9152528\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10/x^9,x, algorithm="giac")
```

```
[Out] -1/9152528*(1961256*b^10*x^(10/3) + 18305056*a*b^9*x^3 + 77224455*a^2*b^8*x
^(8/3) + 193818240*a^3*b^7*x^(7/3) + 320338480*a^4*b^6*x^2 + 364174272*a^5*
b^5*x^(5/3) + 288304632*a^6*b^4*x^(4/3) + 156900480*a^7*b^3*x + 56163240*a^
8*b^2*x^(2/3) + 11938080*a^9*b*x^(1/3) + 1144066*a^10)/x^8
```

maple [A] time = 0.01, size = 113, normalized size = 0.78

$$-\frac{3b^{10}}{14x^{\frac{14}{3}}}-\frac{2a\,b^9}{x^5}-\frac{135a^2b^8}{16x^{\frac{16}{3}}}-\frac{360a^3b^7}{17x^{\frac{17}{3}}}-\frac{35a^4b^6}{x^6}-\frac{756a^5b^5}{19x^{\frac{19}{3}}}-\frac{63a^6b^4}{2x^{\frac{20}{3}}}-\frac{120a^7b^3}{7x^7}-\frac{135a^8b^2}{22x^{\frac{22}{3}}}-\frac{30a^9b}{23x^{\frac{23}{3}}}-\frac{a^{10}}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^10/x^9,x)
```

```
[Out] -1/8*a^10/x^8-30/23*a^9*b/x^(23/3)-135/22*a^8*b^2/x^(22/3)-120/7*a^7*b^3/x^
7-63/2*a^6*b^4/x^(20/3)-756/19*a^5*b^5/x^(19/3)-35*a^4*b^6/x^6-360/17*a^3*b
^7/x^(17/3)-135/16*a^2*b^8/x^(16/3)-2*a*b^9/x^5-3/14*b^10/x^(14/3)
```

maxima [A] time = 0.94, size = 112, normalized size = 0.78

$$-\frac{1961256\,b^{10}x^{\frac{10}{3}}+18305056\,ab^9x^3+77224455\,a^2b^8x^{\frac{8}{3}}+193818240\,a^3b^7x^{\frac{7}{3}}+320338480\,a^4b^6x^2+364174272\,a}{9152528\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10/x^9,x, algorithm="maxima")
```

```
[Out] -1/9152528*(1961256*b^10*x^(10/3) + 18305056*a*b^9*x^3 + 77224455*a^2*b^8*x
^(8/3) + 193818240*a^3*b^7*x^(7/3) + 320338480*a^4*b^6*x^2 + 364174272*a^5*
b^5*x^(5/3) + 288304632*a^6*b^4*x^(4/3) + 156900480*a^7*b^3*x + 56163240*a^
8*b^2*x^(2/3) + 11938080*a^9*b*x^(1/3) + 1144066*a^10)/x^8
```

mupad [B] time = 0.09, size = 112, normalized size = 0.78

$$-\frac{\frac{a^{10}}{8}+\frac{3\,b^{10}\,x^{10/3}}{14}+\frac{120\,a^7\,b^3\,x}{7}+2\,a\,b^9\,x^3+\frac{30\,a^9\,b\,x^{1/3}}{23}+35\,a^4\,b^6\,x^2+\frac{135\,a^8\,b^2\,x^{2/3}}{22}+\frac{63\,a^6\,b^4\,x^{4/3}}{2}+\frac{756\,a^5\,b^5\,x^{5/3}}{19}+\frac{360\,a^3\,b}{17}}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))^10/x^9,x)
```

```
[Out] -(a^10/8 + (3*b^10*x^(10/3))/14 + (120*a^7*b^3*x)/7 + 2*a*b^9*x^3 + (30*a^9
*b*x^(1/3))/23 + 35*a^4*b^6*x^2 + (135*a^8*b^2*x^(2/3))/22 + (63*a^6*b^4*x^
(4/3))/2 + (756*a^5*b^5*x^(5/3))/19 + (360*a^3*b^7*x^(7/3))/17 + (135*a^2*b
^8*x^(8/3))/16)/x^8
```

sympy [A] time = 19.83, size = 146, normalized size = 1.01

$$-\frac{a^{10}}{8x^8}-\frac{30a^9b}{23x^{\frac{23}{3}}}-\frac{135a^8b^2}{22x^{\frac{22}{3}}}-\frac{120a^7b^3}{7x^7}-\frac{63a^6b^4}{2x^{\frac{20}{3}}}-\frac{756a^5b^5}{19x^{\frac{19}{3}}}-\frac{35a^4b^6}{x^6}-\frac{360a^3b^7}{17x^{\frac{17}{3}}}-\frac{135a^2b^8}{16x^{\frac{16}{3}}}-\frac{2ab^9}{x^5}-\frac{3b^{10}}{14x^{\frac{14}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**10/x**9,x)
```

```
[Out] -a**10/(8*x**8) - 30*a**9*b/(23*x**(23/3)) - 135*a**8*b**2/(22*x**(22/3)) -
 120*a**7*b**3/(7*x**7) - 63*a**6*b**4/(2*x**(20/3)) - 756*a**5*b**5/(19*x*
*(19/3)) - 35*a**4*b**6/x**6 - 360*a**3*b**7/(17*x**(17/3)) - 135*a**2*b**8
/(16*x**(16/3)) - 2*a*b**9/x**5 - 3*b**10/(14*x**(14/3))
```

3.2338 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^{10}}\,dx$

Optimal. Leaf size=142

$$-\frac{a^{10}}{9x^9}-\frac{15a^9b}{13x^{26/3}}-\frac{27a^8b^2}{5x^{25/3}}-\frac{15a^7b^3}{x^8}-\frac{630a^6b^4}{23x^{23/3}}-\frac{378a^5b^5}{11x^{22/3}}-\frac{30a^4b^6}{x^7}-\frac{18a^3b^7}{x^{20/3}}-\frac{135a^2b^8}{19x^{19/3}}-\frac{5ab^9}{3x^6}-\frac{3b^{10}}{17x^{17/3}}$$

```
[Out] -1/9*a^10/x^9-15/13*a^9*b/x^(26/3)-27/5*a^8*b^2/x^(25/3)-15*a^7*b^3/x^8-630
/23*a^6*b^4/x^(23/3)-378/11*a^5*b^5/x^(22/3)-30*a^4*b^6/x^7-18*a^3*b^7/x^(2
0/3)-135/19*a^2*b^8/x^(19/3)-5/3*a*b^9/x^6-3/17*b^10/x^(17/3)
```

Rubi [A] time = 0.07, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{27a^8b^2}{5x^{25/3}}-\frac{15a^7b^3}{x^8}-\frac{630a^6b^4}{23x^{23/3}}-\frac{378a^5b^5}{11x^{22/3}}-\frac{30a^4b^6}{x^7}-\frac{18a^3b^7}{x^{20/3}}-\frac{135a^2b^8}{19x^{19/3}}-\frac{15a^9b}{13x^{26/3}}-\frac{a^{10}}{9x^9}-\frac{5ab^9}{3x^6}-\frac{3b^{10}}{17x^{17/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^10/x^10,x]
```

```
[Out] -a^10/(9*x^9) - (15*a^9*b)/(13*x^(26/3)) - (27*a^8*b^2)/(5*x^(25/3)) - (15*
a^7*b^3)/x^8 - (630*a^6*b^4)/(23*x^(23/3)) - (378*a^5*b^5)/(11*x^(22/3)) -
(30*a^4*b^6)/x^7 - (18*a^3*b^7)/x^(20/3) - (135*a^2*b^8)/(19*x^(19/3)) - (5
*a*b^9)/(3*x^6) - (3*b^10)/(17*x^(17/3))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^{10}}\,dx &= 3\,\text{Subst}\left(\int \frac{(a+bx)^{10}}{x^{28}}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(\frac{a^{10}}{x^{28}}+\frac{10a^9b}{x^{27}}+\frac{45a^8b^2}{x^{26}}+\frac{120a^7b^3}{x^{25}}+\frac{210a^6b^4}{x^{24}}+\frac{252a^5b^5}{x^{23}}+\frac{210a^4b^6}{x^{22}}+\frac{120a^3b}{x^{21}}\right.\right.\\ &= -\frac{a^{10}}{9x^9}-\frac{15a^9b}{13x^{26/3}}-\frac{27a^8b^2}{5x^{25/3}}-\frac{15a^7b^3}{x^8}-\frac{630a^6b^4}{23x^{23/3}}-\frac{378a^5b^5}{11x^{22/3}}-\frac{30a^4b^6}{x^7}-\frac{18a^3b^7}{x^{20/3}}-\frac{135a^2b^8}{19x^{19/3}}-\end{aligned}$$

Mathematica [A] time = 0.06, size = 142, normalized size = 1.00

$$-\frac{a^{10}}{9x^9}-\frac{15a^9b}{13x^{26/3}}-\frac{27a^8b^2}{5x^{25/3}}-\frac{15a^7b^3}{x^8}-\frac{630a^6b^4}{23x^{23/3}}-\frac{378a^5b^5}{11x^{22/3}}-\frac{30a^4b^6}{x^7}-\frac{18a^3b^7}{x^{20/3}}-\frac{135a^2b^8}{19x^{19/3}}-\frac{5ab^9}{3x^6}-\frac{3b^{10}}{17x^{17/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^10/x^10,x]
```

```
[Out] -1/9*a^10/x^9 - (15*a^9*b)/(13*x^(26/3)) - (27*a^8*b^2)/(5*x^(25/3)) - (15*
a^7*b^3)/x^8 - (630*a^6*b^4)/(23*x^(23/3)) - (378*a^5*b^5)/(11*x^(22/3)) -
(30*a^4*b^6)/x^7 - (18*a^3*b^7)/x^(20/3) - (135*a^2*b^8)/(19*x^(19/3)) - (5
*a*b^9)/(3*x^6) - (3*b^10)/(17*x^(17/3))
```

fricas [A] time = 0.93, size = 114, normalized size = 0.80

$$-\frac{79676025\,ab^9x^3+1434168450\,a^4b^6x^2+717084225\,a^7b^3x+5311735\,a^{10}+1235169\left(275\,a^2b^8x^2+1330\,a^5b^5\right.}{47805615\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10/x^10,x, algorithm="fricas")
```

```
[Out] -1/47805615*(79676025*a*b^9*x^3 + 1434168450*a^4*b^6*x^2 + 717084225*a^7*b^
3*x + 5311735*a^10 + 1235169*(275*a^2*b^8*x^2 + 1330*a^5*b^5*x + 209*a^8*b^
2)*x^(2/3) + 28215*(299*b^10*x^3 + 30498*a^3*b^7*x^2 + 46410*a^6*b^4*x + 19
55*a^9*b)*x^(1/3))/x^9
```

giac [A] time = 0.20, size = 112, normalized size = 0.79

$$-\frac{8436285\,b^{10}x^{\frac{10}{3}}+79676025\,ab^9x^3+339671475\,a^2b^8x^{\frac{8}{3}}+860501070\,a^3b^7x^{\frac{7}{3}}+1434168450\,a^4b^6x^2+16427}{4780}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10/x^10,x, algorithm="giac")
```

```
[Out] -1/47805615*(8436285*b^10*x^(10/3) + 79676025*a*b^9*x^3 + 339671475*a^2*b^8
*x^(8/3) + 860501070*a^3*b^7*x^(7/3) + 1434168450*a^4*b^6*x^2 + 1642774770*
a^5*b^5*x^(5/3) + 1309458150*a^6*b^4*x^(4/3) + 717084225*a^7*b^3*x + 258150
321*a^8*b^2*x^(2/3) + 55160325*a^9*b*x^(1/3) + 5311735*a^10)/x^9
```

maple [A] time = 0.01, size = 113, normalized size = 0.80

$$-\frac{3b^{10}}{17x^{\frac{17}{3}}}-\frac{5a\,b^9}{3x^6}-\frac{135a^2b^8}{19x^{\frac{19}{3}}}-\frac{18a^3b^7}{x^{\frac{20}{3}}}-\frac{30a^4b^6}{x^7}-\frac{378a^5b^5}{11x^{\frac{22}{3}}}-\frac{630a^6b^4}{23x^{\frac{23}{3}}}-\frac{15a^7b^3}{x^8}-\frac{27a^8b^2}{5x^{\frac{25}{3}}}-\frac{15a^9b}{13x^{\frac{26}{3}}}-\frac{a^{10}}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^10/x^10,x)
```

```
[Out] -1/9*a^10/x^9-15/13*a^9*b/x^(26/3)-27/5*a^8*b^2/x^(25/3)-15*a^7*b^3/x^8-630
/23*a^6*b^4/x^(23/3)-378/11*a^5*b^5/x^(22/3)-30*a^4*b^6/x^7-18*a^3*b^7/x^(2
0/3)-135/19*a^2*b^8/x^(19/3)-5/3*a*b^9/x^6-3/17*b^10/x^(17/3)
```

maxima [A] time = 0.84, size = 112, normalized size = 0.79

$$-\frac{8436285\,b^{10}x^{\frac{10}{3}}+79676025\,ab^9x^3+339671475\,a^2b^8x^{\frac{8}{3}}+860501070\,a^3b^7x^{\frac{7}{3}}+1434168450\,a^4b^6x^2+16427}{4780}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^10/x^10,x, algorithm="maxima")
```

```
[Out] -1/47805615*(8436285*b^10*x^(10/3) + 79676025*a*b^9*x^3 + 339671475*a^2*b^8
*x^(8/3) + 860501070*a^3*b^7*x^(7/3) + 1434168450*a^4*b^6*x^2 + 1642774770*
a^5*b^5*x^(5/3) + 1309458150*a^6*b^4*x^(4/3) + 717084225*a^7*b^3*x + 258150
321*a^8*b^2*x^(2/3) + 55160325*a^9*b*x^(1/3) + 5311735*a^10)/x^9
```

mupad [B] time = 1.19, size = 112, normalized size = 0.79

$$-\frac{\frac{a^{10}}{9}+\frac{3\,b^{10}\,x^{10/3}}{17}+15\,a^7\,b^3\,x+\frac{5\,a\,b^9\,x^3}{3}+\frac{15\,a^9\,b\,x^{1/3}}{13}+30\,a^4\,b^6\,x^2+\frac{27\,a^8\,b^2\,x^{2/3}}{5}+\frac{630\,a^6\,b^4\,x^{4/3}}{23}+\frac{378\,a^5\,b^5\,x^{5/3}}{11}+18\,a^3\,b^7\,x^7}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))^10/x^10,x)
```

```
[Out] -(a^10/9 + (3*b^10*x^(10/3))/17 + 15*a^7*b^3*x + (5*a*b^9*x^3)/3 + (15*a^9*
b*x^(1/3))/13 + 30*a^4*b^6*x^2 + (27*a^8*b^2*x^(2/3))/5 + (630*a^6*b^4*x^(4
/3))/23 + (378*a^5*b^5*x^(5/3))/11 + 18*a^3*b^7*x^(7/3) + (135*a^2*b^8*x^(8
/3))/19)/x^9
```

sympy [A] time = 27.84, size = 144, normalized size = 1.01

$$-\frac{a^{10}}{9x^9}-\frac{15a^9b}{13x^{\frac{26}{3}}}-\frac{27a^8b^2}{5x^{\frac{25}{3}}}-\frac{15a^7b^3}{x^8}-\frac{630a^6b^4}{23x^{\frac{23}{3}}}-\frac{378a^5b^5}{11x^{\frac{22}{3}}}-\frac{30a^4b^6}{x^7}-\frac{18a^3b^7}{x^{\frac{20}{3}}}-\frac{135a^2b^8}{19x^{\frac{19}{3}}}-\frac{5ab^9}{3x^6}-\frac{3b^{10}}{17x^{\frac{17}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**10/x**10,x)
```

```
[Out] -a**10/(9*x**9) - 15*a**9*b/(13*x**(26/3)) - 27*a**8*b**2/(5*x**(25/3)) - 1
5*a**7*b**3/x**8 - 630*a**6*b**4/(23*x**(23/3)) - 378*a**5*b**5/(11*x**(22/
3)) - 30*a**4*b**6/x**7 - 18*a**3*b**7/x**(20/3) - 135*a**2*b**8/(19*x**(19
/3)) - 5*a*b**9/(3*x**6) - 3*b**10/(17*x**(17/3))
```

3.2339 $\int \left(a + b\sqrt[3]{x}\right)^{15} x^5\, dx$

Optimal. Leaf size=217

$$\frac{a^{15}x^6}{6}+\frac{45}{19}a^{14}bx^{19/3}+\frac{63}{4}a^{13}b^2x^{20/3}+65a^{12}b^3x^7+\frac{4095}{22}a^{11}b^4x^{22/3}+\frac{9009}{23}a^{10}b^5x^{23/3}+\frac{5005}{8}a^9b^6x^8+\frac{3861}{5}a^8b^7x^{25/3}$$

```
[Out] 1/6*a^15*x^6+45/19*a^14*b*x^(19/3)+63/4*a^13*b^2*x^(20/3)+65*a^12*b^3*x^7+4
095/22*a^11*b^4*x^(22/3)+9009/23*a^10*b^5*x^(23/3)+5005/8*a^9*b^6*x^8+3861/
5*a^8*b^7*x^(25/3)+1485/2*a^7*b^8*x^(26/3)+5005/9*a^6*b^9*x^9+1287/4*a^5*b^
10*x^(28/3)+4095/29*a^4*b^11*x^(29/3)+91/2*a^3*b^12*x^10+315/31*a^2*b^13*x^
(31/3)+45/32*a*b^14*x^(32/3)+1/11*b^15*x^11
```

Rubi [A] time = 0.17, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{63}{4}a^{13}b^2x^{20/3}+65a^{12}b^3x^7+\frac{4095}{22}a^{11}b^4x^{22/3}+\frac{9009}{23}a^{10}b^5x^{23/3}+\frac{5005}{8}a^9b^6x^8+\frac{3861}{5}a^8b^7x^{25/3}+\frac{1485}{2}a^7b^8x^{26/3}+\frac{500}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^15*x^5,x]
```

```
[Out] (a^15*x^6)/6 + (45*a^14*b*x^(19/3))/19 + (63*a^13*b^2*x^(20/3))/4 + 65*a^12
*b^3*x^7 + (4095*a^11*b^4*x^(22/3))/22 + (9009*a^10*b^5*x^(23/3))/23 + (500
5*a^9*b^6*x^8)/8 + (3861*a^8*b^7*x^(25/3))/5 + (1485*a^7*b^8*x^(26/3))/2 +
(5005*a^6*b^9*x^9)/9 + (1287*a^5*b^10*x^(28/3))/4 + (4095*a^4*b^11*x^(29/3)
)/29 + (91*a^3*b^12*x^10)/2 + (315*a^2*b^13*x^(31/3))/31 + (45*a*b^14*x^(32
/3))/32 + (b^15*x^11)/11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \left(a + b\sqrt[3]{x}\right)^{15} x^5\, dx &= 3\,\text{Subst}\left(\int x^{17}(a + bx)^{15}\, dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(a^{15}x^{17} + 15a^{14}bx^{18} + 105a^{13}b^2x^{19} + 455a^{12}b^3x^{20} + 1365a^{11}b^4x^{21} + 3003\right.\right.\\ &= \frac{a^{15}x^6}{6} + \frac{45}{19}a^{14}bx^{19/3} + \frac{63}{4}a^{13}b^2x^{20/3} + 65a^{12}b^3x^7 + \frac{4095}{22}a^{11}b^4x^{22/3} + \frac{9009}{23}a^{10}b^5x^{23/3}\end{aligned}$$

Mathematica [A] time = 0.13, size = 217, normalized size = 1.00

$$\frac{a^{15}x^6}{6}+\frac{45}{19}a^{14}bx^{19/3}+\frac{63}{4}a^{13}b^2x^{20/3}+65a^{12}b^3x^7+\frac{4095}{22}a^{11}b^4x^{22/3}+\frac{9009}{23}a^{10}b^5x^{23/3}+\frac{5005}{8}a^9b^6x^8+\frac{3861}{5}a^8b^7x^{25/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^15*x^5,x]
```

```
[Out] (a^15*x^6)/6 + (45*a^14*b*x^(19/3))/19 + (63*a^13*b^2*x^(20/3))/4 + 65*a^12
*b^3*x^7 + (4095*a^11*b^4*x^(22/3))/22 + (9009*a^10*b^5*x^(23/3))/23 + (500
5*a^9*b^6*x^8)/8 + (3861*a^8*b^7*x^(25/3))/5 + (1485*a^7*b^8*x^(26/3))/2 +
(5005*a^6*b^9*x^9)/9 + (1287*a^5*b^10*x^(28/3))/4 + (4095*a^4*b^11*x^(29/3)
)/29 + (91*a^3*b^12*x^10)/2 + (315*a^2*b^13*x^(31/3))/31 + (45*a*b^14*x^(32
/3))/32 + (b^15*x^11)/11
```

fricas [A] time = 0.80, size = 179, normalized size = 0.82

$$\frac{1}{11}b^{15}x^{11}+\frac{91}{2}a^3b^{12}x^{10}+\frac{5005}{9}a^6b^9x^9+\frac{5005}{8}a^9b^6x^8+65a^{12}b^3x^7+\frac{1}{6}a^{15}x^6+\frac{9}{21344}\left(3335ab^{14}x^{10}+334880a^4b^{11}x^9\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15*x^5,x, algorithm="fricas")
```

```
[Out] 1/11*b^15*x^11 + 91/2*a^3*b^12*x^10 + 5005/9*a^6*b^9*x^9 + 5005/8*a^9*b^6*x
^8 + 65*a^12*b^3*x^7 + 1/6*a^15*x^6 + 9/21344*(3335*a*b^14*x^10 + 334880*a^
4*b^11*x^9 + 1760880*a^7*b^8*x^8 + 928928*a^10*b^5*x^7 + 37352*a^13*b^2*x^6
)*x^(2/3) + 9/129580*(146300*a^2*b^13*x^10 + 4632485*a^5*b^10*x^9 + 1111796
4*a^8*b^7*x^8 + 2679950*a^11*b^4*x^7 + 34100*a^14*b*x^6)*x^(1/3)
```

giac [A] time = 0.17, size = 167, normalized size = 0.77

$$\frac{1}{11}b^{15}x^{11}+\frac{45}{32}ab^{14}x^{\frac{32}{3}}+\frac{315}{31}a^2b^{13}x^{\frac{31}{3}}+\frac{91}{2}a^3b^{12}x^{10}+\frac{4095}{29}a^4b^{11}x^{\frac{29}{3}}+\frac{1287}{4}a^5b^{10}x^{\frac{28}{3}}+\frac{5005}{9}a^6b^9x^9+\frac{1485}{2}a^7b^8x^{\frac{26}{3}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15*x^5,x, algorithm="giac")
```

```
[Out] 1/11*b^15*x^11 + 45/32*a*b^14*x^(32/3) + 315/31*a^2*b^13*x^(31/3) + 91/2*a^
3*b^12*x^10 + 4095/29*a^4*b^11*x^(29/3) + 1287/4*a^5*b^10*x^(28/3) + 5005/9
*a^6*b^9*x^9 + 1485/2*a^7*b^8*x^(26/3) + 3861/5*a^8*b^7*x^(25/3) + 5005/8*a
^9*b^6*x^8 + 9009/23*a^10*b^5*x^(23/3) + 4095/22*a^11*b^4*x^(22/3) + 65*a^1
2*b^3*x^7 + 63/4*a^13*b^2*x^(20/3) + 45/19*a^14*b*x^(19/3) + 1/6*a^15*x^6
```

maple [A] time = 0.00, size = 168, normalized size = 0.77

$$\frac{b^{15}x^{11}}{11}+\frac{45ab^{14}x^{\frac{32}{3}}}{32}+\frac{315a^2b^{13}x^{\frac{31}{3}}}{31}+\frac{91a^3b^{12}x^{10}}{2}+\frac{4095a^4b^{11}x^{\frac{29}{3}}}{29}+\frac{1287a^5b^{10}x^{\frac{28}{3}}}{4}+\frac{5005a^6b^9x^9}{9}+\frac{1485a^7b^8x^{\frac{26}{3}}}{2}+\frac{3861}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^15*x^5,x)
```

```
[Out] 1/6*a^15*x^6+45/19*a^14*b*x^(19/3)+63/4*a^13*b^2*x^(20/3)+65*a^12*b^3*x^7+4
095/22*a^11*b^4*x^(22/3)+9009/23*a^10*b^5*x^(23/3)+5005/8*a^9*b^6*x^8+3861/
5*a^8*b^7*x^(25/3)+1485/2*a^7*b^8*x^(26/3)+5005/9*a^6*b^9*x^9+1287/4*a^5*b^
10*x^(28/3)+4095/29*a^4*b^11*x^(29/3)+91/2*a^3*b^12*x^10+315/31*a^2*b^13*x^
(31/3)+45/32*a*b^14*x^(32/3)+1/11*b^15*x^11
```

maxima [A] time = 0.94, size = 302, normalized size = 1.39

$$\frac{\left(bx^{\frac{1}{3}}+a\right)^{33}}{11\,b^{18}}-\frac{51\left(bx^{\frac{1}{3}}+a\right)^{32}a}{32\,b^{18}}+\frac{408\left(bx^{\frac{1}{3}}+a\right)^{31}a^2}{31\,b^{18}}-\frac{68\left(bx^{\frac{1}{3}}+a\right)^{30}a^3}{b^{18}}+\frac{7140\left(bx^{\frac{1}{3}}+a\right)^{29}a^4}{29\,b^{18}}-\frac{663\left(bx^{\frac{1}{3}}+a\right)^{28}a^5}{b^{18}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15*x^5,x, algorithm="maxima")
```

```
[Out] 1/11*(b*x^(1/3) + a)^33/b^18 - 51/32*(b*x^(1/3) + a)^32*a/b^18 + 408/31*(b*
x^(1/3) + a)^31*a^2/b^18 - 68*(b*x^(1/3) + a)^30*a^3/b^18 + 7140/29*(b*x^(1
/3) + a)^29*a^4/b^18 - 663*(b*x^(1/3) + a)^28*a^5/b^18 + 12376/9*(b*x^(1/3)
 + a)^27*a^6/b^18 - 2244*(b*x^(1/3) + a)^26*a^7/b^18 + 14586/5*(b*x^(1/3) +
 a)^25*a^8/b^18 - 12155/4*(b*x^(1/3) + a)^24*a^9/b^18 + 58344/23*(b*x^(1/3)
 + a)^23*a^10/b^18 - 18564/11*(b*x^(1/3) + a)^22*a^11/b^18 + 884*(b*x^(1/3)
 + a)^21*a^12/b^18 - 357*(b*x^(1/3) + a)^20*a^13/b^18 + 2040/19*(b*x^(1/3)
+ a)^19*a^14/b^18 - 68/3*(b*x^(1/3) + a)^18*a^15/b^18 + 3*(b*x^(1/3) + a)^1
7*a^16/b^18 - 3/16*(b*x^(1/3) + a)^16*a^17/b^18
```

mupad [B] time = 0.16, size = 167, normalized size = 0.77

$$\frac{a^{15}\,x^6}{6}+\frac{b^{15}\,x^{11}}{11}+\frac{45\,a^{14}\,b\,x^{19/3}}{19}+\frac{45\,a\,b^{14}\,x^{32/3}}{32}+65\,a^{12}\,b^3\,x^7+\frac{5005\,a^9\,b^6\,x^8}{8}+\frac{5005\,a^6\,b^9\,x^9}{9}+\frac{91\,a^3\,b^{12}\,x^{10}}{2}+\frac{63\,a^{13}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^(1/3))^15,x)
```

```
[Out] (a^15*x^6)/6 + (b^15*x^11)/11 + (45*a^14*b*x^(19/3))/19 + (45*a*b^14*x^(32/
3))/32 + 65*a^12*b^3*x^7 + (5005*a^9*b^6*x^8)/8 + (5005*a^6*b^9*x^9)/9 + (9
1*a^3*b^12*x^10)/2 + (63*a^13*b^2*x^(20/3))/4 + (4095*a^11*b^4*x^(22/3))/22
 + (9009*a^10*b^5*x^(23/3))/23 + (3861*a^8*b^7*x^(25/3))/5 + (1485*a^7*b^8*
x^(26/3))/2 + (1287*a^5*b^10*x^(28/3))/4 + (4095*a^4*b^11*x^(29/3))/29 + (3
15*a^2*b^13*x^(31/3))/31
```

sympy [A] time = 7.38, size = 218, normalized size = 1.00

$$\frac{a^{15}x^6}{6}+\frac{45a^{14}bx^{\frac{19}{3}}}{19}+\frac{63a^{13}b^2x^{\frac{20}{3}}}{4}+65a^{12}b^3x^7+\frac{4095a^{11}b^4x^{\frac{22}{3}}}{22}+\frac{9009a^{10}b^5x^{\frac{23}{3}}}{23}+\frac{5005a^9b^6x^8}{8}+\frac{3861a^8b^7x^{\frac{25}{3}}}{5}+\frac{1485}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**15*x**5,x)
```

```
[Out] a**15*x**6/6 + 45*a**14*b*x**(19/3)/19 + 63*a**13*b**2*x**(20/3)/4 + 65*a**
12*b**3*x**7 + 4095*a**11*b**4*x**(22/3)/22 + 9009*a**10*b**5*x**(23/3)/23
+ 5005*a**9*b**6*x**8/8 + 3861*a**8*b**7*x**(25/3)/5 + 1485*a**7*b**8*x**(2
6/3)/2 + 5005*a**6*b**9*x**9/9 + 1287*a**5*b**10*x**(28/3)/4 + 4095*a**4*b*
*11*x**(29/3)/29 + 91*a**3*b**12*x**10/2 + 315*a**2*b**13*x**(31/3)/31 + 45
*a*b**14*x**(32/3)/32 + b**15*x**11/11
```

3.2340 $\int \left(a + b\sqrt[3]{x}\right)^{15} x^4\, dx$

Optimal. Leaf size=217

$$\frac{a^{15}x^5}{5}+\frac{45}{16}a^{14}bx^{16/3}+\frac{315}{17}a^{13}b^2x^{17/3}+\frac{455}{6}a^{12}b^3x^6+\frac{4095}{19}a^{11}b^4x^{19/3}+\frac{9009}{20}a^{10}b^5x^{20/3}+715a^9b^6x^7+\frac{1755}{2}a^8b^7x^{22/3}+$$

```
[Out] 1/5*a^15*x^5+45/16*a^14*b*x^(16/3)+315/17*a^13*b^2*x^(17/3)+455/6*a^12*b^3*
x^6+4095/19*a^11*b^4*x^(19/3)+9009/20*a^10*b^5*x^(20/3)+715*a^9*b^6*x^7+175
5/2*a^8*b^7*x^(22/3)+19305/23*a^7*b^8*x^(23/3)+5005/8*a^6*b^9*x^8+9009/25*a
^5*b^10*x^(25/3)+315/2*a^4*b^11*x^(26/3)+455/9*a^3*b^12*x^9+45/4*a^2*b^13*x
^(28/3)+45/29*a*b^14*x^(29/3)+1/10*b^15*x^10
```

Rubi [A] time = 0.13, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{315}{17}a^{13}b^2x^{17/3}+\frac{455}{6}a^{12}b^3x^6+\frac{4095}{19}a^{11}b^4x^{19/3}+\frac{9009}{20}a^{10}b^5x^{20/3}+715a^9b^6x^7+\frac{1755}{2}a^8b^7x^{22/3}+\frac{19305}{23}a^7b^8x^{23/3}+\frac{5005}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^15*x^4,x]
```

```
[Out] (a^15*x^5)/5 + (45*a^14*b*x^(16/3))/16 + (315*a^13*b^2*x^(17/3))/17 + (455*
a^12*b^3*x^6)/6 + (4095*a^11*b^4*x^(19/3))/19 + (9009*a^10*b^5*x^(20/3))/20
 + 715*a^9*b^6*x^7 + (1755*a^8*b^7*x^(22/3))/2 + (19305*a^7*b^8*x^(23/3))/2
3 + (5005*a^6*b^9*x^8)/8 + (9009*a^5*b^10*x^(25/3))/25 + (315*a^4*b^11*x^(2
6/3))/2 + (455*a^3*b^12*x^9)/9 + (45*a^2*b^13*x^(28/3))/4 + (45*a*b^14*x^(2
9/3))/29 + (b^15*x^10)/10
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \left(a + b\sqrt[3]{x}\right)^{15} x^4\, dx &= 3\,\text{Subst}\left(\int x^{14}(a + bx)^{15}\, dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(a^{15}x^{14} + 15a^{14}bx^{15} + 105a^{13}b^2x^{16} + 455a^{12}b^3x^{17} + 1365a^{11}b^4x^{18} + 3003a^{10}\right.\right.\\ &= \frac{a^{15}x^5}{5} + \frac{45}{16}a^{14}bx^{16/3} + \frac{315}{17}a^{13}b^2x^{17/3} + \frac{455}{6}a^{12}b^3x^6 + \frac{4095}{19}a^{11}b^4x^{19/3} + \frac{9009}{20}a^{10}b^5x^{20/3}\end{aligned}$$

Mathematica [A] time = 0.10, size = 217, normalized size = 1.00

$$\frac{a^{15}x^5}{5}+\frac{45}{16}a^{14}bx^{16/3}+\frac{315}{17}a^{13}b^2x^{17/3}+\frac{455}{6}a^{12}b^3x^6+\frac{4095}{19}a^{11}b^4x^{19/3}+\frac{9009}{20}a^{10}b^5x^{20/3}+715a^9b^6x^7+\frac{1755}{2}a^8b^7x^{22/3}+$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^15*x^4,x]
```

```
[Out] (a^15*x^5)/5 + (45*a^14*b*x^(16/3))/16 + (315*a^13*b^2*x^(17/3))/17 + (455*
a^12*b^3*x^6)/6 + (4095*a^11*b^4*x^(19/3))/19 + (9009*a^10*b^5*x^(20/3))/20
 + 715*a^9*b^6*x^7 + (1755*a^8*b^7*x^(22/3))/2 + (19305*a^7*b^8*x^(23/3))/2
3 + (5005*a^6*b^9*x^8)/8 + (9009*a^5*b^10*x^(25/3))/25 + (315*a^4*b^11*x^(2
6/3))/2 + (455*a^3*b^12*x^9)/9 + (45*a^2*b^13*x^(28/3))/4 + (45*a*b^14*x^(2
9/3))/29 + (b^15*x^10)/10
```

fricas [A] time = 0.82, size = 179, normalized size = 0.82

$$\frac{1}{10}b^{15}x^{10}+\frac{455}{9}a^3b^{12}x^9+\frac{5005}{8}a^6b^9x^8+715a^9b^6x^7+\frac{455}{6}a^{12}b^3x^6+\frac{1}{5}a^{15}x^5+\frac{9}{226780}\left(39100ab^{14}x^9+3968650a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15*x^4,x, algorithm="fricas")
```

```
[Out] 1/10*b^15*x^10 + 455/9*a^3*b^12*x^9 + 5005/8*a^6*b^9*x^8 + 715*a^9*b^6*x^7
+ 455/6*a^12*b^3*x^6 + 1/5*a^15*x^5 + 9/226780*(39100*a*b^14*x^9 + 3968650*
a^4*b^11*x^8 + 21149700*a^7*b^8*x^7 + 11350339*a^10*b^5*x^6 + 466900*a^13*b
^2*x^5)*x^(2/3) + 9/7600*(9500*a^2*b^13*x^9 + 304304*a^5*b^10*x^8 + 741000*
a^8*b^7*x^7 + 182000*a^11*b^4*x^6 + 2375*a^14*b*x^5)*x^(1/3)
```

giac [A] time = 0.46, size = 167, normalized size = 0.77

$$\frac{1}{10}b^{15}x^{10}+\frac{45}{29}ab^{14}x^{\frac{29}{3}}+\frac{45}{4}a^2b^{13}x^{\frac{28}{3}}+\frac{455}{9}a^3b^{12}x^9+\frac{315}{2}a^4b^{11}x^{\frac{26}{3}}+\frac{9009}{25}a^5b^{10}x^{\frac{25}{3}}+\frac{5005}{8}a^6b^9x^8+\frac{19305}{23}a^7b^8x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15*x^4,x, algorithm="giac")
```

```
[Out] 1/10*b^15*x^10 + 45/29*a*b^14*x^(29/3) + 45/4*a^2*b^13*x^(28/3) + 455/9*a^3
*b^12*x^9 + 315/2*a^4*b^11*x^(26/3) + 9009/25*a^5*b^10*x^(25/3) + 5005/8*a^
6*b^9*x^8 + 19305/23*a^7*b^8*x^(23/3) + 1755/2*a^8*b^7*x^(22/3) + 715*a^9*b
^6*x^7 + 9009/20*a^10*b^5*x^(20/3) + 4095/19*a^11*b^4*x^(19/3) + 455/6*a^12
*b^3*x^6 + 315/17*a^13*b^2*x^(17/3) + 45/16*a^14*b*x^(16/3) + 1/5*a^15*x^5
```

maple [A] time = 0.00, size = 168, normalized size = 0.77

$$\frac{b^{15}x^{10}}{10}+\frac{45a\,b^{14}x^{\frac{29}{3}}}{29}+\frac{45a^2b^{13}x^{\frac{28}{3}}}{4}+\frac{455a^3b^{12}x^9}{9}+\frac{315a^4b^{11}x^{\frac{26}{3}}}{2}+\frac{9009a^5b^{10}x^{\frac{25}{3}}}{25}+\frac{5005a^6b^9x^8}{8}+\frac{19305a^7b^8x^{\frac{23}{3}}}{23}+\frac{17}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^15*x^4,x)
```

```
[Out] 1/5*a^15*x^5+45/16*a^14*b*x^(16/3)+315/17*a^13*b^2*x^(17/3)+455/6*a^12*b^3*
x^6+4095/19*a^11*b^4*x^(19/3)+9009/20*a^10*b^5*x^(20/3)+715*a^9*b^6*x^7+175
5/2*a^8*b^7*x^(22/3)+19305/23*a^7*b^8*x^(23/3)+5005/8*a^6*b^9*x^8+9009/25*a
^5*b^10*x^(25/3)+315/2*a^4*b^11*x^(26/3)+455/9*a^3*b^12*x^9+45/4*a^2*b^13*x
^(28/3)+45/29*a*b^14*x^(29/3)+1/10*b^15*x^10
```

maxima [A] time = 0.91, size = 251, normalized size = 1.16

$$\frac{\left(bx^{\frac{1}{3}}+a\right)^{30}}{10\,b^{15}}-\frac{42\left(bx^{\frac{1}{3}}+a\right)^{29}a}{29\,b^{15}}+\frac{39\left(bx^{\frac{1}{3}}+a\right)^{28}a^2}{4\,b^{15}}-\frac{364\left(bx^{\frac{1}{3}}+a\right)^{27}a^3}{9\,b^{15}}+\frac{231\left(bx^{\frac{1}{3}}+a\right)^{26}a^4}{2\,b^{15}}-\frac{6006\left(bx^{\frac{1}{3}}+a\right)^{25}a^{}}{25\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15*x^4,x, algorithm="maxima")
```

```
[Out] 1/10*(b*x^(1/3) + a)^30/b^15 - 42/29*(b*x^(1/3) + a)^29*a/b^15 + 39/4*(b*x^
(1/3) + a)^28*a^2/b^15 - 364/9*(b*x^(1/3) + a)^27*a^3/b^15 + 231/2*(b*x^(1/
3) + a)^26*a^4/b^15 - 6006/25*(b*x^(1/3) + a)^25*a^5/b^15 + 3003/8*(b*x^(1/
3) + a)^24*a^6/b^15 - 10296/23*(b*x^(1/3) + a)^23*a^7/b^15 + 819/2*(b*x^(1/
3) + a)^22*a^8/b^15 - 286*(b*x^(1/3) + a)^21*a^9/b^15 + 3003/20*(b*x^(1/3)
+ a)^20*a^10/b^15 - 1092/19*(b*x^(1/3) + a)^19*a^11/b^15 + 91/6*(b*x^(1/3)
+ a)^18*a^12/b^15 - 42/17*(b*x^(1/3) + a)^17*a^13/b^15 + 3/16*(b*x^(1/3) +
a)^16*a^14/b^15
```

mupad [B] time = 0.16, size = 167, normalized size = 0.77

$$\frac{a^{15}x^5}{5}+\frac{b^{15}x^{10}}{10}+\frac{45a^{14}bx^{16/3}}{16}+\frac{45ab^{14}x^{29/3}}{29}+\frac{455a^{12}b^3x^6}{6}+715a^9b^6x^7+\frac{5005a^6b^9x^8}{8}+\frac{455a^3b^{12}x^9}{9}+\frac{315a^{13}b}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^(1/3))^15,x)
```

```
[Out] (a^15*x^5)/5 + (b^15*x^10)/10 + (45*a^14*b*x^(16/3))/16 + (45*a*b^14*x^(29/
3))/29 + (455*a^12*b^3*x^6)/6 + 715*a^9*b^6*x^7 + (5005*a^6*b^9*x^8)/8 + (4
55*a^3*b^12*x^9)/9 + (315*a^13*b^2*x^(17/3))/17 + (4095*a^11*b^4*x^(19/3))/
19 + (9009*a^10*b^5*x^(20/3))/20 + (1755*a^8*b^7*x^(22/3))/2 + (19305*a^7*b
^8*x^(23/3))/23 + (9009*a^5*b^10*x^(25/3))/25 + (315*a^4*b^11*x^(26/3))/2 +
 (45*a^2*b^13*x^(28/3))/4
```

sympy [A] time = 6.80, size = 218, normalized size = 1.00

$$\frac{a^{15}x^5}{5}+\frac{45a^{14}bx^{\frac{16}{3}}}{16}+\frac{315a^{13}b^2x^{\frac{17}{3}}}{17}+\frac{455a^{12}b^3x^6}{6}+\frac{4095a^{11}b^4x^{\frac{19}{3}}}{19}+\frac{9009a^{10}b^5x^{\frac{20}{3}}}{20}+715a^9b^6x^7+\frac{1755a^8b^7x^{\frac{22}{3}}}{2}+\frac{19305a}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**15*x**4,x)
```

```
[Out] a**15*x**5/5 + 45*a**14*b*x**(16/3)/16 + 315*a**13*b**2*x**(17/3)/17 + 455*
a**12*b**3*x**6/6 + 4095*a**11*b**4*x**(19/3)/19 + 9009*a**10*b**5*x**(20/3
)/20 + 715*a**9*b**6*x**7 + 1755*a**8*b**7*x**(22/3)/2 + 19305*a**7*b**8*x*
*(23/3)/23 + 5005*a**6*b**9*x**8/8 + 9009*a**5*b**10*x**(25/3)/25 + 315*a**
4*b**11*x**(26/3)/2 + 455*a**3*b**12*x**9/9 + 45*a**2*b**13*x**(28/3)/4 + 4
5*a*b**14*x**(29/3)/29 + b**15*x**10/10
```

3.2341 $\int \left(a+b\sqrt[3]{x}\right)^{15} x^3\,dx$

Optimal. Leaf size=244

$$-\frac{3a^{11}\left(a+b\sqrt[3]{x}\right)^{16}}{16b^{12}}+\frac{33a^{10}\left(a+b\sqrt[3]{x}\right)^{17}}{17b^{12}}-\frac{55a^{9}\left(a+b\sqrt[3]{x}\right)^{18}}{6b^{12}}+\frac{495a^{8}\left(a+b\sqrt[3]{x}\right)^{19}}{19b^{12}}-\frac{99a^{7}\left(a+b\sqrt[3]{x}\right)^{20}}{2b^{12}}+\frac{66a^{6}\left(a\right.}{}$$

```
[Out] -3/16*a^11*(a+b*x^(1/3))^16/b^12+33/17*a^10*(a+b*x^(1/3))^17/b^12-55/6*a^9*
(a+b*x^(1/3))^18/b^12+495/19*a^8*(a+b*x^(1/3))^19/b^12-99/2*a^7*(a+b*x^(1/3
))^20/b^12+66*a^6*(a+b*x^(1/3))^21/b^12-63*a^5*(a+b*x^(1/3))^22/b^12+990/23
*a^4*(a+b*x^(1/3))^23/b^12-165/8*a^3*(a+b*x^(1/3))^24/b^12+33/5*a^2*(a+b*x^
(1/3))^25/b^12-33/26*a*(a+b*x^(1/3))^26/b^12+1/9*(a+b*x^(1/3))^27/b^12
```

Rubi [A] time = 0.14, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{33a^{2}\left(a+b\sqrt[3]{x}\right)^{25}}{5b^{12}}-\frac{165a^{3}\left(a+b\sqrt[3]{x}\right)^{24}}{8b^{12}}+\frac{990a^{4}\left(a+b\sqrt[3]{x}\right)^{23}}{23b^{12}}-\frac{63a^{5}\left(a+b\sqrt[3]{x}\right)^{22}}{b^{12}}+\frac{66a^{6}\left(a+b\sqrt[3]{x}\right)^{21}}{b^{12}}-\frac{99a^{7}\left(a\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^15*x^3,x]
```

```
[Out] (-3*a^11*(a + b*x^(1/3))^16)/(16*b^12) + (33*a^10*(a + b*x^(1/3))^17)/(17*b
^12) - (55*a^9*(a + b*x^(1/3))^18)/(6*b^12) + (495*a^8*(a + b*x^(1/3))^19)/
(19*b^12) - (99*a^7*(a + b*x^(1/3))^20)/(2*b^12) + (66*a^6*(a + b*x^(1/3))^
21)/b^12 - (63*a^5*(a + b*x^(1/3))^22)/b^12 + (990*a^4*(a + b*x^(1/3))^23)/
(23*b^12) - (165*a^3*(a + b*x^(1/3))^24)/(8*b^12) + (33*a^2*(a + b*x^(1/3))
^25)/(5*b^12) - (33*a*(a + b*x^(1/3))^26)/(26*b^12) + (a + b*x^(1/3))^27/(9
*b^12)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sqrt[3]{x}\right)^{15} x^3\,dx &= 3\,\text{Subst}\left(\int x^{11}(a+bx)^{15}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int\left(-\frac{a^{11}(a+bx)^{15}}{b^{11}}+\frac{11a^{10}(a+bx)^{16}}{b^{11}}-\frac{55a^{9}(a+bx)^{17}}{b^{11}}+\frac{165a^{8}(a+bx)^{18}}{b^{11}}-\right.\right.\\ &= -\frac{3a^{11}\left(a+b\sqrt[3]{x}\right)^{16}}{16b^{12}}+\frac{33a^{10}\left(a+b\sqrt[3]{x}\right)^{17}}{17b^{12}}-\frac{55a^{9}\left(a+b\sqrt[3]{x}\right)^{18}}{6b^{12}}+\frac{495a^{8}\left(a+b\sqrt[3]{x}\right)^{19}}{19b^{12}}-\end{aligned}$$

Mathematica [A] time = 0.09, size = 215, normalized size = 0.88

$$\frac{a^{15}x^{4}}{4}+\frac{45}{13}a^{14}bx^{13/3}+\frac{45}{2}a^{13}b^{2}x^{14/3}+91a^{12}b^{3}x^{5}+\frac{4095}{16}a^{11}b^{4}x^{16/3}+\frac{9009}{17}a^{10}b^{5}x^{17/3}+\frac{5005}{6}a^{9}b^{6}x^{6}+\frac{19305}{19}a^{8}b^{7}x^{19/}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^15*x^3,x]
```

```
[Out] (a^15*x^4)/4 + (45*a^14*b*x^(13/3))/13 + (45*a^13*b^2*x^(14/3))/2 + 91*a^12
*b^3*x^5 + (4095*a^11*b^4*x^(16/3))/16 + (9009*a^10*b^5*x^(17/3))/17 + (500
5*a^9*b^6*x^6)/6 + (19305*a^8*b^7*x^(19/3))/19 + (3861*a^7*b^8*x^(20/3))/4
+ 715*a^6*b^9*x^7 + (819*a^5*b^10*x^(22/3))/2 + (4095*a^4*b^11*x^(23/3))/23
 + (455*a^3*b^12*x^8)/8 + (63*a^2*b^13*x^(25/3))/5 + (45*a*b^14*x^(26/3))/2
6 + (b^15*x^9)/9
```

fricas [A] time = 0.88, size = 179, normalized size = 0.73

$$\frac{1}{9}b^{15}x^9+\frac{455}{8}a^3b^{12}x^8+715a^6b^9x^7+\frac{5005}{6}a^9b^6x^6+91a^{12}b^3x^5+\frac{1}{4}a^{15}x^4+\frac{9}{20332}\left(3910ab^{14}x^8+402220a^4b^{11}x^7+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15*x^3,x, algorithm="fricas")
```

```
[Out] 1/9*b^15*x^9 + 455/8*a^3*b^12*x^8 + 715*a^6*b^9*x^7 + 5005/6*a^9*b^6*x^6 +
91*a^12*b^3*x^5 + 1/4*a^15*x^4 + 9/20332*(3910*a*b^14*x^8 + 402220*a^4*b^11
*x^7 + 2180607*a^7*b^8*x^6 + 1197196*a^10*b^5*x^5 + 50830*a^13*b^2*x^4)*x^(
2/3) + 9/19760*(27664*a^2*b^13*x^8 + 899080*a^5*b^10*x^7 + 2230800*a^8*b^7*
x^6 + 561925*a^11*b^4*x^5 + 7600*a^14*b*x^4)*x^(1/3)
```

giac [A] time = 0.19, size = 167, normalized size = 0.68

$$\frac{1}{9}b^{15}x^9+\frac{45}{26}ab^{14}x^{\frac{26}{3}}+\frac{63}{5}a^2b^{13}x^{\frac{25}{3}}+\frac{455}{8}a^3b^{12}x^8+\frac{4095}{23}a^4b^{11}x^{\frac{23}{3}}+\frac{819}{2}a^5b^{10}x^{\frac{22}{3}}+715a^6b^9x^7+\frac{3861}{4}a^7b^8x^{\frac{20}{3}}+\frac{1930}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15*x^3,x, algorithm="giac")
```

```
[Out] 1/9*b^15*x^9 + 45/26*a*b^14*x^(26/3) + 63/5*a^2*b^13*x^(25/3) + 455/8*a^3*b
^12*x^8 + 4095/23*a^4*b^11*x^(23/3) + 819/2*a^5*b^10*x^(22/3) + 715*a^6*b^9
*x^7 + 3861/4*a^7*b^8*x^(20/3) + 19305/19*a^8*b^7*x^(19/3) + 5005/6*a^9*b^6
*x^6 + 9009/17*a^10*b^5*x^(17/3) + 4095/16*a^11*b^4*x^(16/3) + 91*a^12*b^3*
x^5 + 45/2*a^13*b^2*x^(14/3) + 45/13*a^14*b*x^(13/3) + 1/4*a^15*x^4
```

maple [A] time = 0.00, size = 168, normalized size = 0.69

$$\frac{b^{15}x^9}{9}+\frac{45ab^{14}x^{\frac{26}{3}}}{26}+\frac{63a^2b^{13}x^{\frac{25}{3}}}{5}+\frac{455a^3b^{12}x^8}{8}+\frac{4095a^4b^{11}x^{\frac{23}{3}}}{23}+\frac{819a^5b^{10}x^{\frac{22}{3}}}{2}+715a^6b^9x^7+\frac{3861a^7b^8x^{\frac{20}{3}}}{4}+\frac{19305a^8b}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^15*x^3,x)
```

```
[Out] 1/9*b^15*x^9+45/26*a*b^14*x^(26/3)+63/5*a^2*b^13*x^(25/3)+455/8*a^3*b^12*x^
8+4095/23*a^4*b^11*x^(23/3)+819/2*a^5*b^10*x^(22/3)+715*a^6*b^9*x^7+3861/4*
a^7*b^8*x^(20/3)+19305/19*a^8*b^7*x^(19/3)+5005/6*a^9*b^6*x^6+9009/17*a^10*
b^5*x^(17/3)+4095/16*a^11*b^4*x^(16/3)+91*a^12*b^3*x^5+45/2*a^13*b^2*x^(14/
3)+45/13*a^14*b*x^(13/3)+1/4*a^15*x^4
```

maxima [A] time = 0.88, size = 200, normalized size = 0.82

$$\frac{\left(bx^{\frac{1}{3}}+a\right)^{27}}{9b^{12}}-\frac{33\left(bx^{\frac{1}{3}}+a\right)^{26}a}{26b^{12}}+\frac{33\left(bx^{\frac{1}{3}}+a\right)^{25}a^2}{5b^{12}}-\frac{165\left(bx^{\frac{1}{3}}+a\right)^{24}a^3}{8b^{12}}+\frac{990\left(bx^{\frac{1}{3}}+a\right)^{23}a^4}{23b^{12}}-\frac{63\left(bx^{\frac{1}{3}}+a\right)^{22}a^5}{b^{12}}+\frac{66}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15*x^3,x, algorithm="maxima")
```

```
[Out] 1/9*(b*x^(1/3) + a)^27/b^12 - 33/26*(b*x^(1/3) + a)^26*a/b^12 + 33/5*(b*x^(
1/3) + a)^25*a^2/b^12 - 165/8*(b*x^(1/3) + a)^24*a^3/b^12 + 990/23*(b*x^(1/
3) + a)^23*a^4/b^12 - 63*(b*x^(1/3) + a)^22*a^5/b^12 + 66*(b*x^(1/3) + a)^2
1*a^6/b^12 - 99/2*(b*x^(1/3) + a)^20*a^7/b^12 + 495/19*(b*x^(1/3) + a)^19*a
^8/b^12 - 55/6*(b*x^(1/3) + a)^18*a^9/b^12 + 33/17*(b*x^(1/3) + a)^17*a^10/
b^12 - 3/16*(b*x^(1/3) + a)^16*a^11/b^12
```

mupad **[B]** time = 0.16, size = 167, normalized size = 0.68

$$\frac{a^{15}\,x^4}{4}+\frac{b^{15}\,x^9}{9}+\frac{45\,a^{14}\,b\,x^{13/3}}{13}+\frac{45\,a\,b^{14}\,x^{26/3}}{26}+91\,a^{12}\,b^3\,x^5+\frac{5005\,a^9\,b^6\,x^6}{6}+715\,a^6\,b^9\,x^7+\frac{455\,a^3\,b^{12}\,x^8}{8}+\frac{45\,a^{13}\,b^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^(1/3))^15,x)
```

```
[Out] (a^15*x^4)/4 + (b^15*x^9)/9 + (45*a^14*b*x^(13/3))/13 + (45*a*b^14*x^(26/3)
)/26 + 91*a^12*b^3*x^5 + (5005*a^9*b^6*x^6)/6 + 715*a^6*b^9*x^7 + (455*a^3*
b^12*x^8)/8 + (45*a^13*b^2*x^(14/3))/2 + (4095*a^11*b^4*x^(16/3))/16 + (900
9*a^10*b^5*x^(17/3))/17 + (19305*a^8*b^7*x^(19/3))/19 + (3861*a^7*b^8*x^(20
/3))/4 + (819*a^5*b^10*x^(22/3))/2 + (4095*a^4*b^11*x^(23/3))/23 + (63*a^2*
b^13*x^(25/3))/5
```

sympy **[A]** time = 6.21, size = 216, normalized size = 0.89

$$\frac{a^{15}x^4}{4}+\frac{45a^{14}bx^{\frac{13}{3}}}{13}+\frac{45a^{13}b^2x^{\frac{14}{3}}}{2}+91a^{12}b^3x^5+\frac{4095a^{11}b^4x^{\frac{16}{3}}}{16}+\frac{9009a^{10}b^5x^{\frac{17}{3}}}{17}+\frac{5005a^9b^6x^6}{6}+\frac{19305a^8b^7x^{\frac{19}{3}}}{19}+\frac{386}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**15*x**3,x)
```

```
[Out] a**15*x**4/4 + 45*a**14*b*x**(13/3)/13 + 45*a**13*b**2*x**(14/3)/2 + 91*a**
12*b**3*x**5 + 4095*a**11*b**4*x**(16/3)/16 + 9009*a**10*b**5*x**(17/3)/17
+ 5005*a**9*b**6*x**6/6 + 19305*a**8*b**7*x**(19/3)/19 + 3861*a**7*b**8*x**
(20/3)/4 + 715*a**6*b**9*x**7 + 819*a**5*b**10*x**(22/3)/2 + 4095*a**4*b**1
1*x**(23/3)/23 + 455*a**3*b**12*x**8/8 + 63*a**2*b**13*x**(25/3)/5 + 45*a*b
**14*x**(26/3)/26 + b**15*x**9/9
```

3.2342 $\int \left(a + b\sqrt[3]{x}\right)^{15} x^2\, dx$

Optimal. Leaf size=183

$$\frac{3a^8\left(a+b\sqrt[3]{x}\right)^{16}}{16b^9}-\frac{24a^7\left(a+b\sqrt[3]{x}\right)^{17}}{17b^9}+\frac{14a^6\left(a+b\sqrt[3]{x}\right)^{18}}{3b^9}-\frac{168a^5\left(a+b\sqrt[3]{x}\right)^{19}}{19b^9}+\frac{21a^4\left(a+b\sqrt[3]{x}\right)^{20}}{2b^9}-\frac{8a^3\left(a+b\sqrt[3]{x}\right)^{}}{b^9}$$

```
[Out] 3/16*a^8*(a+b*x^(1/3))^16/b^9-24/17*a^7*(a+b*x^(1/3))^17/b^9+14/3*a^6*(a+b*
x^(1/3))^18/b^9-168/19*a^5*(a+b*x^(1/3))^19/b^9+21/2*a^4*(a+b*x^(1/3))^20/b
^9-8*a^3*(a+b*x^(1/3))^21/b^9+42/11*a^2*(a+b*x^(1/3))^22/b^9-24/23*a*(a+b*x
^(1/3))^23/b^9+1/8*(a+b*x^(1/3))^24/b^9
```

Rubi [A] time = 0.10, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{42a^2\left(a+b\sqrt[3]{x}\right)^{22}}{11b^9}-\frac{8a^3\left(a+b\sqrt[3]{x}\right)^{21}}{b^9}+\frac{21a^4\left(a+b\sqrt[3]{x}\right)^{20}}{2b^9}-\frac{168a^5\left(a+b\sqrt[3]{x}\right)^{19}}{19b^9}+\frac{14a^6\left(a+b\sqrt[3]{x}\right)^{18}}{3b^9}-\frac{24a^7\left(a+b\sqrt[3]{x}\right)^{}}{17b^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^15*x^2,x]
```

```
[Out] (3*a^8*(a + b*x^(1/3))^16)/(16*b^9) - (24*a^7*(a + b*x^(1/3))^17)/(17*b^9)
+ (14*a^6*(a + b*x^(1/3))^18)/(3*b^9) - (168*a^5*(a + b*x^(1/3))^19)/(19*b^
9) + (21*a^4*(a + b*x^(1/3))^20)/(2*b^9) - (8*a^3*(a + b*x^(1/3))^21)/b^9 +
 (42*a^2*(a + b*x^(1/3))^22)/(11*b^9) - (24*a*(a + b*x^(1/3))^23)/(23*b^9)
+ (a + b*x^(1/3))^24/(8*b^9)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \left(a + b\sqrt[3]{x}\right)^{15} x^2\, dx &= 3\,\text{Subst}\left(\int x^8(a+bx)^{15}\, dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int\left(\frac{a^8(a+bx)^{15}}{b^8}-\frac{8a^7(a+bx)^{16}}{b^8}+\frac{28a^6(a+bx)^{17}}{b^8}-\frac{56a^5(a+bx)^{18}}{b^8}+\frac{70a^4(a+\right.\right.\\ &= \frac{3a^8\left(a+b\sqrt[3]{x}\right)^{16}}{16b^9}-\frac{24a^7\left(a+b\sqrt[3]{x}\right)^{17}}{17b^9}+\frac{14a^6\left(a+b\sqrt[3]{x}\right)^{18}}{3b^9}-\frac{168a^5\left(a+b\sqrt[3]{x}\right)^{19}}{19b^9}+\frac{21a^4}{}\end{aligned}$$

Mathematica [A] time = 0.08, size = 113, normalized size = 0.62

$$\frac{\left(a+b\sqrt[3]{x}\right)^{16}\left(a^8-16a^7b\sqrt[3]{x}+136a^6b^2x^{2/3}-816a^5b^3x+3876a^4b^4x^{4/3}-15504a^3b^5x^{5/3}+54264a^2b^6x^2-170544\right.}{3922512b^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^15*x^2,x]
```

```
[Out] ((a + b*x^(1/3))^16*(a^8 - 16*a^7*b*x^(1/3) + 136*a^6*b^2*x^(2/3) - 816*a^5
*b^3*x + 3876*a^4*b^4*x^(4/3) - 15504*a^3*b^5*x^(5/3) + 54264*a^2*b^6*x^2 -
 170544*a*b^7*x^(7/3) + 490314*b^8*x^(8/3)))/(3922512*b^9)
```

fricas [A] time = 0.54, size = 179, normalized size = 0.98

$$\frac{1}{8}b^{15}x^8+65a^3b^{12}x^7+\frac{5005}{6}a^6b^9x^6+1001a^9b^6x^5+\frac{455}{4}a^{12}b^3x^4+\frac{1}{3}a^{15}x^3+\frac{9}{17204}\left(3740ab^{14}x^7+391391a^4b^{11}x^6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15*x^2,x, algorithm="fricas")
```

```
[Out] 1/8*b^15*x^8 + 65*a^3*b^12*x^7 + 5005/6*a^6*b^9*x^6 + 1001*a^9*b^6*x^5 + 45
5/4*a^12*b^3*x^4 + 1/3*a^15*x^3 + 9/17204*(3740*a*b^14*x^7 + 391391*a^4*b^1
1*x^6 + 2170740*a^7*b^8*x^5 + 1230086*a^10*b^5*x^4 + 54740*a^13*b^2*x^3)*x^
(2/3) + 9/3344*(5320*a^2*b^13*x^7 + 176176*a^5*b^10*x^6 + 448305*a^8*b^7*x^
5 + 117040*a^11*b^4*x^4 + 1672*a^14*b*x^3)*x^(1/3)
```

giac [A] time = 0.17, size = 167, normalized size = 0.91

$$\frac{1}{8}b^{15}x^8+\frac{45}{23}ab^{14}x^{\frac{23}{3}}+\frac{315}{22}a^2b^{13}x^{\frac{22}{3}}+65a^3b^{12}x^7+\frac{819}{4}a^4b^{11}x^{\frac{20}{3}}+\frac{9009}{19}a^5b^{10}x^{\frac{19}{3}}+\frac{5005}{6}a^6b^9x^6+\frac{19305}{17}a^7b^8x^{\frac{17}{3}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15*x^2,x, algorithm="giac")
```

```
[Out] 1/8*b^15*x^8 + 45/23*a*b^14*x^(23/3) + 315/22*a^2*b^13*x^(22/3) + 65*a^3*b^
12*x^7 + 819/4*a^4*b^11*x^(20/3) + 9009/19*a^5*b^10*x^(19/3) + 5005/6*a^6*b
^9*x^6 + 19305/17*a^7*b^8*x^(17/3) + 19305/16*a^8*b^7*x^(16/3) + 1001*a^9*b
^6*x^5 + 1287/2*a^10*b^5*x^(14/3) + 315*a^11*b^4*x^(13/3) + 455/4*a^12*b^3*
x^4 + 315/11*a^13*b^2*x^(11/3) + 9/2*a^14*b*x^(10/3) + 1/3*a^15*x^3
```

maple [A] time = 0.00, size = 168, normalized size = 0.92

$$\frac{b^{15}x^8}{8}+\frac{45ab^{14}x^{\frac{23}{3}}}{23}+\frac{315a^2b^{13}x^{\frac{22}{3}}}{22}+65a^3b^{12}x^7+\frac{819a^4b^{11}x^{\frac{20}{3}}}{4}+\frac{9009a^5b^{10}x^{\frac{19}{3}}}{19}+\frac{5005a^6b^9x^6}{6}+\frac{19305a^7b^8x^{\frac{17}{3}}}{17}+\frac{193}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^15*x^2,x)
```

```
[Out] 1/8*b^15*x^8+45/23*a*b^14*x^(23/3)+315/22*a^2*b^13*x^(22/3)+65*a^3*b^12*x^7
+819/4*a^4*b^11*x^(20/3)+9009/19*a^5*b^10*x^(19/3)+5005/6*a^6*b^9*x^6+19305
/17*a^7*b^8*x^(17/3)+19305/16*a^8*b^7*x^(16/3)+1001*a^9*b^6*x^5+1287/2*a^10
*b^5*x^(14/3)+315*a^11*b^4*x^(13/3)+455/4*a^12*b^3*x^4+315/11*a^13*b^2*x^(1
1/3)+9/2*a^14*b*x^(10/3)+1/3*a^15*x^3
```

maxima [A] time = 0.92, size = 149, normalized size = 0.81

$$\frac{\left(bx^{\frac{1}{3}}+a\right)^{24}}{8b^9}-\frac{24\left(bx^{\frac{1}{3}}+a\right)^{23}a}{23b^9}+\frac{42\left(bx^{\frac{1}{3}}+a\right)^{22}a^2}{11b^9}-\frac{8\left(bx^{\frac{1}{3}}+a\right)^{21}a^3}{b^9}+\frac{21\left(bx^{\frac{1}{3}}+a\right)^{20}a^4}{2b^9}-\frac{168\left(bx^{\frac{1}{3}}+a\right)^{19}a^5}{19b^9}+\frac{14}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15*x^2,x, algorithm="maxima")
```

```
[Out] 1/8*(b*x^(1/3) + a)^24/b^9 - 24/23*(b*x^(1/3) + a)^23*a/b^9 + 42/11*(b*x^(1
/3) + a)^22*a^2/b^9 - 8*(b*x^(1/3) + a)^21*a^3/b^9 + 21/2*(b*x^(1/3) + a)^2
0*a^4/b^9 - 168/19*(b*x^(1/3) + a)^19*a^5/b^9 + 14/3*(b*x^(1/3) + a)^18*a^6
/b^9 - 24/17*(b*x^(1/3) + a)^17*a^7/b^9 + 3/16*(b*x^(1/3) + a)^16*a^8/b^9
```

mupad [B] time = 0.16, size = 167, normalized size = 0.91

$$\frac{a^{15}x^3}{3}+\frac{b^{15}x^8}{8}+\frac{9a^{14}bx^{10/3}}{2}+\frac{45ab^{14}x^{23/3}}{23}+\frac{455a^{12}b^3x^4}{4}+1001a^9b^6x^5+\frac{5005a^6b^9x^6}{6}+65a^3b^{12}x^7+\frac{315a^{13}b^2x}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^(1/3))^15,x)
```

```
[Out] (a^15*x^3)/3 + (b^15*x^8)/8 + (9*a^14*b*x^(10/3))/2 + (45*a*b^14*x^(23/3))/
23 + (455*a^12*b^3*x^4)/4 + 1001*a^9*b^6*x^5 + (5005*a^6*b^9*x^6)/6 + 65*a^
3*b^12*x^7 + (315*a^13*b^2*x^(11/3))/11 + 315*a^11*b^4*x^(13/3) + (1287*a^1
0*b^5*x^(14/3))/2 + (19305*a^8*b^7*x^(16/3))/16 + (19305*a^7*b^8*x^(17/3))/
17 + (9009*a^5*b^10*x^(19/3))/19 + (819*a^4*b^11*x^(20/3))/4 + (315*a^2*b^1
3*x^(22/3))/22
```

sympy [A] time = 5.63, size = 214, normalized size = 1.17

$$\frac{a^{15}x^3}{3}+\frac{9a^{14}bx^{\frac{10}{3}}}{2}+\frac{315a^{13}b^2x^{\frac{11}{3}}}{11}+\frac{455a^{12}b^3x^4}{4}+315a^{11}b^4x^{\frac{13}{3}}+\frac{1287a^{10}b^5x^{\frac{14}{3}}}{2}+1001a^9b^6x^5+\frac{19305a^8b^7x^{\frac{16}{3}}}{16}+\frac{19305a^7}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**15*x**2,x)
```

```
[Out] a**15*x**3/3 + 9*a**14*b*x**(10/3)/2 + 315*a**13*b**2*x**(11/3)/11 + 455*a*
*12*b**3*x**4/4 + 315*a**11*b**4*x**(13/3) + 1287*a**10*b**5*x**(14/3)/2 +
1001*a**9*b**6*x**5 + 19305*a**8*b**7*x**(16/3)/16 + 19305*a**7*b**8*x**(17
/3)/17 + 5005*a**6*b**9*x**6/6 + 9009*a**5*b**10*x**(19/3)/19 + 819*a**4*b*
*11*x**(20/3)/4 + 65*a**3*b**12*x**7 + 315*a**2*b**13*x**(22/3)/22 + 45*a*b
**14*x**(23/3)/23 + b**15*x**8/8
```

3.2343 $\int \left(a+b\sqrt[3]{x}\right)^{15} x\,dx$

Optimal. Leaf size=122

$$-\frac{3a^5\left(a+b\sqrt[3]{x}\right)^{16}}{16b^6}+\frac{15a^4\left(a+b\sqrt[3]{x}\right)^{17}}{17b^6}-\frac{5a^3\left(a+b\sqrt[3]{x}\right)^{18}}{3b^6}+\frac{30a^2\left(a+b\sqrt[3]{x}\right)^{19}}{19b^6}+\frac{\left(a+b\sqrt[3]{x}\right)^{21}}{7b^6}-\frac{3a\left(a+b\sqrt[3]{x}\right)^{20}}{4b^6}$$

```
[Out] -3/16*a^5*(a+b*x^(1/3))^16/b^6+15/17*a^4*(a+b*x^(1/3))^17/b^6-5/3*a^3*(a+b*
x^(1/3))^18/b^6+30/19*a^2*(a+b*x^(1/3))^19/b^6-3/4*a*(a+b*x^(1/3))^20/b^6+1
/7*(a+b*x^(1/3))^21/b^6
```

Rubi [A] time = 0.08, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{30a^2\left(a+b\sqrt[3]{x}\right)^{19}}{19b^6}-\frac{5a^3\left(a+b\sqrt[3]{x}\right)^{18}}{3b^6}+\frac{15a^4\left(a+b\sqrt[3]{x}\right)^{17}}{17b^6}-\frac{3a^5\left(a+b\sqrt[3]{x}\right)^{16}}{16b^6}+\frac{\left(a+b\sqrt[3]{x}\right)^{21}}{7b^6}-\frac{3a\left(a+b\sqrt[3]{x}\right)^{20}}{4b^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^15*x,x]
```

```
[Out] (-3*a^5*(a + b*x^(1/3))^16)/(16*b^6) + (15*a^4*(a + b*x^(1/3))^17)/(17*b^6)
 - (5*a^3*(a + b*x^(1/3))^18)/(3*b^6) + (30*a^2*(a + b*x^(1/3))^19)/(19*b^6
) - (3*a*(a + b*x^(1/3))^20)/(4*b^6) + (a + b*x^(1/3))^21/(7*b^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sqrt[3]{x}\right)^{15} x\,dx &= 3\,\text{Subst}\left(\int x^5(a+bx)^{15}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int\left(-\frac{a^5(a+bx)^{15}}{b^5}+\frac{5a^4(a+bx)^{16}}{b^5}-\frac{10a^3(a+bx)^{17}}{b^5}+\frac{10a^2(a+bx)^{18}}{b^5}-\frac{5a(a\right.\right.\\ &= -\frac{3a^5\left(a+b\sqrt[3]{x}\right)^{16}}{16b^6}+\frac{15a^4\left(a+b\sqrt[3]{x}\right)^{17}}{17b^6}-\frac{5a^3\left(a+b\sqrt[3]{x}\right)^{18}}{3b^6}+\frac{30a^2\left(a+b\sqrt[3]{x}\right)^{19}}{19b^6}-\frac{3a\left(\right.}{}\end{aligned}$$

Mathematica [A] time = 0.06, size = 76, normalized size = 0.62

$$-\frac{\left(a+b\sqrt[3]{x}\right)^{16}\left(a^5-16a^4b\sqrt[3]{x}+136a^3b^2x^{2/3}-816a^2b^3x+3876ab^4x^{4/3}-15504b^5x^{5/3}\right)}{108528b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^15*x,x]
```

```
[Out] -1/108528*((a + b*x^(1/3))^16*(a^5 - 16*a^4*b*x^(1/3) + 136*a^3*b^2*x^(2/3)
 - 816*a^2*b^3*x + 3876*a*b^4*x^(4/3) - 15504*b^5*x^(5/3)))/b^6
```

fricas [A] time = 0.77, size = 179, normalized size = 1.47

$$\frac{1}{7}b^{15}x^7+\frac{455}{6}a^3b^{12}x^6+1001\,a^6b^9x^5+\frac{5005}{4}a^9b^6x^4+\frac{455}{3}a^{12}b^3x^3+\frac{1}{2}a^{15}x^2+\frac{9}{952}\left(238\,ab^{14}x^6+25480\,a^4b^{11}x^5+145\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15*x,x, algorithm="fricas")
```

```
[Out] 1/7*b^15*x^7 + 455/6*a^3*b^12*x^6 + 1001*a^6*b^9*x^5 + 5005/4*a^9*b^6*x^4 +
 455/3*a^12*b^3*x^3 + 1/2*a^15*x^2 + 9/952*(238*a*b^14*x^6 + 25480*a^4*b^11
*x^5 + 145860*a^7*b^8*x^4 + 86632*a^10*b^5*x^3 + 4165*a^13*b^2*x^2)*x^(2/3)
 + 9/2128*(3920*a^2*b^13*x^6 + 133133*a^5*b^10*x^5 + 351120*a^8*b^7*x^4 + 9
6824*a^11*b^4*x^3 + 1520*a^14*b*x^2)*x^(1/3)
```

giac [A] time = 0.18, size = 167, normalized size = 1.37

$$\frac{1}{7}b^{15}x^7+\frac{9}{4}ab^{14}x^{\frac{20}{3}}+\frac{315}{19}a^2b^{13}x^{\frac{19}{3}}+\frac{455}{6}a^3b^{12}x^6+\frac{4095}{17}a^4b^{11}x^{\frac{17}{3}}+\frac{9009}{16}a^5b^{10}x^{\frac{16}{3}}+1001\,a^6b^9x^5+\frac{19305}{14}a^7b^8x^{\frac{14}{3}}+14$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15*x,x, algorithm="giac")
```

```
[Out] 1/7*b^15*x^7 + 9/4*a*b^14*x^(20/3) + 315/19*a^2*b^13*x^(19/3) + 455/6*a^3*b
^12*x^6 + 4095/17*a^4*b^11*x^(17/3) + 9009/16*a^5*b^10*x^(16/3) + 1001*a^6*
b^9*x^5 + 19305/14*a^7*b^8*x^(14/3) + 1485*a^8*b^7*x^(13/3) + 5005/4*a^9*b^
6*x^4 + 819*a^10*b^5*x^(11/3) + 819/2*a^11*b^4*x^(10/3) + 455/3*a^12*b^3*x^
3 + 315/8*a^13*b^2*x^(8/3) + 45/7*a^14*b*x^(7/3) + 1/2*a^15*x^2
```

maple [A] time = 0.00, size = 168, normalized size = 1.38

$$\frac{b^{15}x^7}{7}+\frac{9a\,b^{14}x^{\frac{20}{3}}}{4}+\frac{315a^2b^{13}x^{\frac{19}{3}}}{19}+\frac{455a^3b^{12}x^6}{6}+\frac{4095a^4b^{11}x^{\frac{17}{3}}}{17}+\frac{9009a^5b^{10}x^{\frac{16}{3}}}{16}+1001a^6b^9x^5+\frac{19305a^7b^8x^{\frac{14}{3}}}{14}+1485a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^15*x,x)
```

```
[Out] 1/7*b^15*x^7+9/4*a*b^14*x^(20/3)+315/19*a^2*b^13*x^(19/3)+455/6*a^3*b^12*x^
6+4095/17*a^4*b^11*x^(17/3)+9009/16*a^5*b^10*x^(16/3)+1001*a^6*b^9*x^5+1930
5/14*a^7*b^8*x^(14/3)+1485*a^8*b^7*x^(13/3)+5005/4*x^4*a^9*b^6+819*a^10*b^5
*x^(11/3)+819/2*a^11*b^4*x^(10/3)+455/3*a^12*b^3*x^3+315/8*a^13*b^2*x^(8/3)
+45/7*a^14*b*x^(7/3)+1/2*a^15*x^2
```

maxima [A] time = 0.88, size = 98, normalized size = 0.80

$$\frac{\left(bx^{\frac{1}{3}}+a\right)^{21}}{7\,b^6}-\frac{3\left(bx^{\frac{1}{3}}+a\right)^{20}a}{4\,b^6}+\frac{30\left(bx^{\frac{1}{3}}+a\right)^{19}a^2}{19\,b^6}-\frac{5\left(bx^{\frac{1}{3}}+a\right)^{18}a^3}{3\,b^6}+\frac{15\left(bx^{\frac{1}{3}}+a\right)^{17}a^4}{17\,b^6}-\frac{3\left(bx^{\frac{1}{3}}+a\right)^{16}a^5}{16\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15*x,x, algorithm="maxima")
```

```
[Out] 1/7*(b*x^(1/3) + a)^21/b^6 - 3/4*(b*x^(1/3) + a)^20*a/b^6 + 30/19*(b*x^(1/3
) + a)^19*a^2/b^6 - 5/3*(b*x^(1/3) + a)^18*a^3/b^6 + 15/17*(b*x^(1/3) + a)^
17*a^4/b^6 - 3/16*(b*x^(1/3) + a)^16*a^5/b^6
```

mupad [B] time = 0.16, size = 167, normalized size = 1.37

$$\frac{a^{15}x^2}{2}+\frac{b^{15}x^7}{7}+\frac{45a^{14}bx^{7/3}}{7}+\frac{9ab^{14}x^{20/3}}{4}+\frac{455a^{12}b^3x^3}{3}+\frac{5005a^9b^6x^4}{4}+1001a^6b^9x^5+\frac{455a^3b^{12}x^6}{6}+\frac{315a^{13}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^(1/3))^15,x)
```

```
[Out] (a^15*x^2)/2 + (b^15*x^7)/7 + (45*a^14*b*x^(7/3))/7 + (9*a*b^14*x^(20/3))/4
 + (455*a^12*b^3*x^3)/3 + (5005*a^9*b^6*x^4)/4 + 1001*a^6*b^9*x^5 + (455*a^
3*b^12*x^6)/6 + (315*a^13*b^2*x^(8/3))/8 + (819*a^11*b^4*x^(10/3))/2 + 819*
a^10*b^5*x^(11/3) + 1485*a^8*b^7*x^(13/3) + (19305*a^7*b^8*x^(14/3))/14 + (
9009*a^5*b^10*x^(16/3))/16 + (4095*a^4*b^11*x^(17/3))/17 + (315*a^2*b^13*x^
(19/3))/19
```

sympy [A] time = 5.10, size = 214, normalized size = 1.75

$$\frac{a^{15}x^2}{2}+\frac{45a^{14}bx^{\frac{7}{3}}}{7}+\frac{315a^{13}b^2x^{\frac{8}{3}}}{8}+\frac{455a^{12}b^3x^3}{3}+\frac{819a^{11}b^4x^{\frac{10}{3}}}{2}+819a^{10}b^5x^{\frac{11}{3}}+\frac{5005a^9b^6x^4}{4}+1485a^8b^7x^{\frac{13}{3}}+\frac{19305a}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**15*x,x)
```

```
[Out] a**15*x**2/2 + 45*a**14*b*x**(7/3)/7 + 315*a**13*b**2*x**(8/3)/8 + 455*a**1
2*b**3*x**3/3 + 819*a**11*b**4*x**(10/3)/2 + 819*a**10*b**5*x**(11/3) + 500
5*a**9*b**6*x**4/4 + 1485*a**8*b**7*x**(13/3) + 19305*a**7*b**8*x**(14/3)/1
4 + 1001*a**6*b**9*x**5 + 9009*a**5*b**10*x**(16/3)/16 + 4095*a**4*b**11*x*
*(17/3)/17 + 455*a**3*b**12*x**6/6 + 315*a**2*b**13*x**(19/3)/19 + 9*a*b**1
4*x**(20/3)/4 + b**15*x**7/7
```

3.2344 $\int \left(a + b\sqrt[3]{x}\right)^{15}\, dx$

Optimal. Leaf size=59

$$\frac{3a^2\left(a+b\sqrt[3]{x}\right)^{16}}{16b^3} + \frac{\left(a+b\sqrt[3]{x}\right)^{18}}{6b^3} - \frac{6a\left(a+b\sqrt[3]{x}\right)^{17}}{17b^3}$$

```
[Out] 3/16*a^2*(a+b*x^(1/3))^16/b^3-6/17*a*(a+b*x^(1/3))^17/b^3+1/6*(a+b*x^(1/3))
^18/b^3
```

Rubi [A] time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {190, 43}

$$\frac{3a^2\left(a+b\sqrt[3]{x}\right)^{16}}{16b^3} + \frac{\left(a+b\sqrt[3]{x}\right)^{18}}{6b^3} - \frac{6a\left(a+b\sqrt[3]{x}\right)^{17}}{17b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^15,x]
```

```
[Out] (3*a^2*(a + b*x^(1/3))^16)/(16*b^3) - (6*a*(a + b*x^(1/3))^17)/(17*b^3) + (
a + b*x^(1/3))^18/(6*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}
\int \left(a + b\sqrt[3]{x}\right)^{15}\, dx &= 3\,\text{Subst}\left(\int x^2(a+bx)^{15}\, dx, x, \sqrt[3]{x}\right) \\
&= 3\,\text{Subst}\left(\int \left(\frac{a^2(a+bx)^{15}}{b^2} - \frac{2a(a+bx)^{16}}{b^2} + \frac{(a+bx)^{17}}{b^2}\right) dx, x, \sqrt[3]{x}\right) \\
&= \frac{3a^2\left(a+b\sqrt[3]{x}\right)^{16}}{16b^3} - \frac{6a\left(a+b\sqrt[3]{x}\right)^{17}}{17b^3} + \frac{\left(a+b\sqrt[3]{x}\right)^{18}}{6b^3}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 41, normalized size = 0.69

$$\frac{\left(a+b\sqrt[3]{x}\right)^{16}\left(a^2 - 16ab\sqrt[3]{x} + 136b^2x^{2/3}\right)}{816b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^15,x]
```

```
[Out] ((a + b*x^(1/3))^16*(a^2 - 16*a*b*x^(1/3) + 136*b^2*x^(2/3)))/(816*b^3)
```

fricas [B] time = 0.55, size = 172, normalized size = 2.92

$$\frac{1}{6}b^{15}x^6+91a^3b^{12}x^5+\frac{5005}{4}a^6b^9x^4+\frac{5005}{3}a^9b^6x^3+\frac{455}{2}a^{12}b^3x^2+a^{15}x+\frac{9}{136}\left(40ab^{14}x^5+4420a^4b^{11}x^4+26520a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15,x, algorithm="fricas")
```

```
[Out] 1/6*b^15*x^6 + 91*a^3*b^12*x^5 + 5005/4*a^6*b^9*x^4 + 5005/3*a^9*b^6*x^3 +
455/2*a^12*b^3*x^2 + a^15*x + 9/136*(40*a*b^14*x^5 + 4420*a^4*b^11*x^4 + 26
520*a^7*b^8*x^3 + 17017*a^10*b^5*x^2 + 952*a^13*b^2*x)*x^(2/3) + 9/16*(35*a
^2*b^13*x^5 + 1232*a^5*b^10*x^4 + 3432*a^8*b^7*x^3 + 1040*a^11*b^4*x^2 + 20
*a^14*b*x)*x^(1/3)
```

giac [B] time = 0.17, size = 164, normalized size = 2.78

$$\frac{1}{6}b^{15}x^6+\frac{45}{17}ab^{14}x^{\frac{17}{3}}+\frac{315}{16}a^2b^{13}x^{\frac{16}{3}}+91a^3b^{12}x^5+\frac{585}{2}a^4b^{11}x^{\frac{14}{3}}+693a^5b^{10}x^{\frac{13}{3}}+\frac{5005}{4}a^6b^9x^4+1755a^7b^8x^{\frac{11}{3}}+\frac{38}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15,x, algorithm="giac")
```

```
[Out] 1/6*b^15*x^6 + 45/17*a*b^14*x^(17/3) + 315/16*a^2*b^13*x^(16/3) + 91*a^3*b^
12*x^5 + 585/2*a^4*b^11*x^(14/3) + 693*a^5*b^10*x^(13/3) + 5005/4*a^6*b^9*x
^4 + 1755*a^7*b^8*x^(11/3) + 3861/2*a^8*b^7*x^(10/3) + 5005/3*a^9*b^6*x^3 +
 9009/8*a^10*b^5*x^(8/3) + 585*a^11*b^4*x^(7/3) + 455/2*a^12*b^3*x^2 + 63*a
^13*b^2*x^(5/3) + 45/4*a^14*b*x^(4/3) + a^15*x
```

maple [B] time = 0.01, size = 165, normalized size = 2.80

$$\frac{b^{15}x^6}{6}+\frac{45ab^{14}x^{\frac{17}{3}}}{17}+\frac{315a^2b^{13}x^{\frac{16}{3}}}{16}+91a^3b^{12}x^5+\frac{585a^4b^{11}x^{\frac{14}{3}}}{2}+693a^5b^{10}x^{\frac{13}{3}}+\frac{5005a^6b^9x^4}{4}+1755a^7b^8x^{\frac{11}{3}}+\frac{3861a^8}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^15,x)
```

```
[Out] a^15*x+1/6*b^15*x^6+91*a^3*b^12*x^5+5005/3*a^9*b^6*x^3+45/4*b*a^14*x^(4/3)+
63*b^2*a^13*x^(5/3)+455/2*b^3*x^2*a^12+585*b^4*a^11*x^(7/3)+9009/8*b^5*a^10
*x^(8/3)+3861/2*b^7*a^8*x^(10/3)+1755*b^8*a^7*x^(11/3)+5005/4*b^9*x^4*a^6+6
93*b^10*a^5*x^(13/3)+585/2*b^11*a^4*x^(14/3)+315/16*b^13*a^2*x^(16/3)+45/17
*b^14*a*x^(17/3)
```

maxima [A] time = 0.85, size = 47, normalized size = 0.80

$$\frac{\left(bx^{\frac{1}{3}}+a\right)^{18}}{6b^3}-\frac{6\left(bx^{\frac{1}{3}}+a\right)^{17}a}{17b^3}+\frac{3\left(bx^{\frac{1}{3}}+a\right)^{16}a^2}{16b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15,x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^(1/3) + a)^18/b^3 - 6/17*(b*x^(1/3) + a)^17*a/b^3 + 3/16*(b*x^(1/3
) + a)^16*a^2/b^3
```

mupad [B] time = 0.15, size = 164, normalized size = 2.78

$$a^{15}x+\frac{b^{15}x^6}{6}+\frac{45a^{14}bx^{4/3}}{4}+\frac{45ab^{14}x^{17/3}}{17}+\frac{455a^{12}b^3x^2}{2}+\frac{5005a^9b^6x^3}{3}+\frac{5005a^6b^9x^4}{4}+91a^3b^{12}x^5+63a^{13}b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))^15,x)
```

```
[Out] a^15*x + (b^15*x^6)/6 + (45*a^14*b*x^(4/3))/4 + (45*a*b^14*x^(17/3))/17 + (
455*a^12*b^3*x^2)/2 + (5005*a^9*b^6*x^3)/3 + (5005*a^6*b^9*x^4)/4 + 91*a^3*
b^12*x^5 + 63*a^13*b^2*x^(5/3) + 585*a^11*b^4*x^(7/3) + (9009*a^10*b^5*x^(8
/3))/8 + (3861*a^8*b^7*x^(10/3))/2 + 1755*a^7*b^8*x^(11/3) + 693*a^5*b^10*x
^(13/3) + (585*a^4*b^11*x^(14/3))/2 + (315*a^2*b^13*x^(16/3))/16
```

sympy [**B**] time = 4.79, size = 207, normalized size = 3.51

$$a^{15}x+\frac{45a^{14}bx^{\frac{4}{3}}}{4}+63a^{13}b^{2}x^{\frac{5}{3}}+\frac{455a^{12}b^{3}x^{2}}{2}+585a^{11}b^{4}x^{\frac{7}{3}}+\frac{9009a^{10}b^{5}x^{\frac{8}{3}}}{8}+\frac{5005a^{9}b^{6}x^{3}}{3}+\frac{3861a^{8}b^{7}x^{\frac{10}{3}}}{2}+1755a^{7}b^{8}x^{\frac{11}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**15,x)
```

```
[Out] a**15*x + 45*a**14*b*x**(4/3)/4 + 63*a**13*b**2*x**(5/3) + 455*a**12*b**3*x
**2/2 + 585*a**11*b**4*x**(7/3) + 9009*a**10*b**5*x**(8/3)/8 + 5005*a**9*b*
*6*x**3/3 + 3861*a**8*b**7*x**(10/3)/2 + 1755*a**7*b**8*x**(11/3) + 5005*a*
*6*b**9*x**4/4 + 693*a**5*b**10*x**(13/3) + 585*a**4*b**11*x**(14/3)/2 + 91
*a**3*b**12*x**5 + 315*a**2*b**13*x**(16/3)/16 + 45*a*b**14*x**(17/3)/17 +
b**15*x**6/6
```

3.2345 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x}\,dx$

Optimal. Leaf size=209

$$a^{15}\log(x)+45a^{14}b\sqrt[3]{x}+\frac{315}{2}a^{13}b^2x^{2/3}+455a^{12}b^3x+\frac{4095}{4}a^{11}b^4x^{4/3}+\frac{9009}{5}a^{10}b^5x^{5/3}+\frac{5005}{2}a^9b^6x^2+\frac{19305}{7}a^8b^7x^{7/}$$

```
[Out] 45*a^14*b*x^(1/3)+315/2*a^13*b^2*x^(2/3)+455*a^12*b^3*x+4095/4*a^11*b^4*x^(
4/3)+9009/5*a^10*b^5*x^(5/3)+5005/2*a^9*b^6*x^2+19305/7*a^8*b^7*x^(7/3)+193
05/8*a^7*b^8*x^(8/3)+5005/3*a^6*b^9*x^3+9009/10*a^5*b^10*x^(10/3)+4095/11*a
^4*b^11*x^(11/3)+455/4*a^3*b^12*x^4+315/13*a^2*b^13*x^(13/3)+45/14*a*b^14*x
^(14/3)+1/5*b^15*x^5+a^15*ln(x)
```

Rubi [A] time = 0.11, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{315}{2}a^{13}b^2x^{2/3}+\frac{4095}{4}a^{11}b^4x^{4/3}+\frac{9009}{5}a^{10}b^5x^{5/3}+\frac{5005}{2}a^9b^6x^2+\frac{19305}{7}a^8b^7x^{7/3}+\frac{19305}{8}a^7b^8x^{8/3}+\frac{5005}{3}a^6b^9x^3+\frac{90}{1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^15/x,x]
```

```
[Out] 45*a^14*b*x^(1/3) + (315*a^13*b^2*x^(2/3))/2 + 455*a^12*b^3*x + (4095*a^11*
b^4*x^(4/3))/4 + (9009*a^10*b^5*x^(5/3))/5 + (5005*a^9*b^6*x^2)/2 + (19305*
a^8*b^7*x^(7/3))/7 + (19305*a^7*b^8*x^(8/3))/8 + (5005*a^6*b^9*x^3)/3 + (90
09*a^5*b^10*x^(10/3))/10 + (4095*a^4*b^11*x^(11/3))/11 + (455*a^3*b^12*x^4)
/4 + (315*a^2*b^13*x^(13/3))/13 + (45*a*b^14*x^(14/3))/14 + (b^15*x^5)/5 +
a^15*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x}\,dx &= 3\,\text{Subst}\left(\int \frac{(a+bx)^{15}}{x}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(15a^{14}b+\frac{a^{15}}{x}+105a^{13}b^2x+455a^{12}b^3x^2+1365a^{11}b^4x^3+3003a^{10}b^5x^4+5\right.\right.\\ &= 45a^{14}b\sqrt[3]{x}+\frac{315}{2}a^{13}b^2x^{2/3}+455a^{12}b^3x+\frac{4095}{4}a^{11}b^4x^{4/3}+\frac{9009}{5}a^{10}b^5x^{5/3}+\frac{5005}{2}a^9b^6x^{\end{aligned}$$

Mathematica [A] time = 0.07, size = 209, normalized size = 1.00

$$a^{15}\log(x)+45a^{14}b\sqrt[3]{x}+\frac{315}{2}a^{13}b^2x^{2/3}+455a^{12}b^3x+\frac{4095}{4}a^{11}b^4x^{4/3}+\frac{9009}{5}a^{10}b^5x^{5/3}+\frac{5005}{2}a^9b^6x^2+\frac{19305}{7}a^8b^7x^{7/}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^15/x,x]
```

```
[Out] 45*a^14*b*x^(1/3) + (315*a^13*b^2*x^(2/3))/2 + 455*a^12*b^3*x + (4095*a^11*
b^4*x^(4/3))/4 + (9009*a^10*b^5*x^(5/3))/5 + (5005*a^9*b^6*x^2)/2 + (19305*
a^8*b^7*x^(7/3))/7 + (19305*a^7*b^8*x^(8/3))/8 + (5005*a^6*b^9*x^3)/3 + (90
09*a^5*b^10*x^(10/3))/10 + (4095*a^4*b^11*x^(11/3))/11 + (455*a^3*b^12*x^4)
/4 + (315*a^2*b^13*x^(13/3))/13 + (45*a*b^14*x^(14/3))/14 + (b^15*x^5)/5 +
a^15*Log[x]
```

fricas [A] time = 0.94, size = 168, normalized size = 0.80

$$\frac{1}{5}b^{15}x^5+\frac{455}{4}a^3b^{12}x^4+\frac{5005}{3}a^6b^9x^3+\frac{5005}{2}a^9b^6x^2+455a^{12}b^3x+3a^{15}\log\left(x^{\frac{1}{3}}\right)+\frac{9}{3080}\left(1100ab^{14}x^4+127400a^4b^1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15/x,x, algorithm="fricas")
```

```
[Out] 1/5*b^15*x^5 + 455/4*a^3*b^12*x^4 + 5005/3*a^6*b^9*x^3 + 5005/2*a^9*b^6*x^2
 + 455*a^12*b^3*x + 3*a^15*log(x^(1/3)) + 9/3080*(1100*a*b^14*x^4 + 127400*
a^4*b^11*x^3 + 825825*a^7*b^8*x^2 + 616616*a^10*b^5*x + 53900*a^13*b^2)*x^(
2/3) + 9/1820*(4900*a^2*b^13*x^4 + 182182*a^5*b^10*x^3 + 557700*a^8*b^7*x^2
 + 207025*a^11*b^4*x + 9100*a^14*b)*x^(1/3)
```

giac [A] time = 0.21, size = 164, normalized size = 0.78

$$\frac{1}{5}b^{15}x^5+\frac{45}{14}ab^{14}x^{\frac{14}{3}}+\frac{315}{13}a^2b^{13}x^{\frac{13}{3}}+\frac{455}{4}a^3b^{12}x^4+\frac{4095}{11}a^4b^{11}x^{\frac{11}{3}}+\frac{9009}{10}a^5b^{10}x^{\frac{10}{3}}+\frac{5005}{3}a^6b^9x^3+\frac{19305}{8}a^7b^8x^{\frac{8}{3}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15/x,x, algorithm="giac")
```

```
[Out] 1/5*b^15*x^5 + 45/14*a*b^14*x^(14/3) + 315/13*a^2*b^13*x^(13/3) + 455/4*a^3
*b^12*x^4 + 4095/11*a^4*b^11*x^(11/3) + 9009/10*a^5*b^10*x^(10/3) + 5005/3*
a^6*b^9*x^3 + 19305/8*a^7*b^8*x^(8/3) + 19305/7*a^8*b^7*x^(7/3) + 5005/2*a^
9*b^6*x^2 + 9009/5*a^10*b^5*x^(5/3) + 4095/4*a^11*b^4*x^(4/3) + 455*a^12*b^
3*x + a^15*log(abs(x)) + 315/2*a^13*b^2*x^(2/3) + 45*a^14*b*x^(1/3)
```

maple [A] time = 0.01, size = 164, normalized size = 0.78

$$\frac{b^{15}x^5}{5}+\frac{45ab^{14}x^{\frac{14}{3}}}{14}+\frac{315a^2b^{13}x^{\frac{13}{3}}}{13}+\frac{455a^3b^{12}x^4}{4}+\frac{4095a^4b^{11}x^{\frac{11}{3}}}{11}+\frac{9009a^5b^{10}x^{\frac{10}{3}}}{10}+\frac{5005a^6b^9x^3}{3}+\frac{19305a^7b^8x^{\frac{8}{3}}}{8}+\frac{193}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^15/x,x)
```

```
[Out] 45*a^14*b*x^(1/3)+315/2*a^13*b^2*x^(2/3)+455*a^12*b^3*x+4095/4*a^11*b^4*x^(
4/3)+9009/5*a^10*b^5*x^(5/3)+5005/2*a^9*b^6*x^2+19305/7*a^8*b^7*x^(7/3)+193
05/8*a^7*b^8*x^(8/3)+5005/3*a^6*b^9*x^3+9009/10*a^5*b^10*x^(10/3)+4095/11*a
^4*b^11*x^(11/3)+455/4*a^3*b^12*x^4+315/13*a^2*b^13*x^(13/3)+45/14*a*b^14*x
^(14/3)+1/5*b^15*x^5+a^15*ln(x)
```

maxima [A] time = 0.88, size = 163, normalized size = 0.78

$$\frac{1}{5}b^{15}x^5+\frac{45}{14}ab^{14}x^{\frac{14}{3}}+\frac{315}{13}a^2b^{13}x^{\frac{13}{3}}+\frac{455}{4}a^3b^{12}x^4+\frac{4095}{11}a^4b^{11}x^{\frac{11}{3}}+\frac{9009}{10}a^5b^{10}x^{\frac{10}{3}}+\frac{5005}{3}a^6b^9x^3+\frac{19305}{8}a^7b^8x^{\frac{8}{3}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15/x,x, algorithm="maxima")
```

```
[Out] 1/5*b^15*x^5 + 45/14*a*b^14*x^(14/3) + 315/13*a^2*b^13*x^(13/3) + 455/4*a^3
*b^12*x^4 + 4095/11*a^4*b^11*x^(11/3) + 9009/10*a^5*b^10*x^(10/3) + 5005/3*
a^6*b^9*x^3 + 19305/8*a^7*b^8*x^(8/3) + 19305/7*a^8*b^7*x^(7/3) + 5005/2*a^
9*b^6*x^2 + 9009/5*a^10*b^5*x^(5/3) + 4095/4*a^11*b^4*x^(4/3) + 455*a^12*b^
3*x + a^15*log(x) + 315/2*a^13*b^2*x^(2/3) + 45*a^14*b*x^(1/3)
```

mupad **[B]** time = 0.16, size = 166, normalized size = 0.79

$$3\,a^{15}\,\ln\left(x^{1/3}\right)+\frac{b^{15}\,x^5}{5}+455\,a^{12}\,b^3\,x+45\,a^{14}\,b\,x^{1/3}+\frac{45\,a\,b^{14}\,x^{14/3}}{14}+\frac{5005\,a^9\,b^6\,x^2}{2}+\frac{5005\,a^6\,b^9\,x^3}{3}+\frac{455\,a^3\,b^{12}\,x^4}{4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))^15/x,x)
```

```
[Out] 3*a^15*log(x^(1/3)) + (b^15*x^5)/5 + 455*a^12*b^3*x + 45*a^14*b*x^(1/3) + (
45*a*b^14*x^(14/3))/14 + (5005*a^9*b^6*x^2)/2 + (5005*a^6*b^9*x^3)/3 + (455
*a^3*b^12*x^4)/4 + (315*a^13*b^2*x^(2/3))/2 + (4095*a^11*b^4*x^(4/3))/4 + (
9009*a^10*b^5*x^(5/3))/5 + (19305*a^8*b^7*x^(7/3))/7 + (19305*a^7*b^8*x^(8/
3))/8 + (9009*a^5*b^10*x^(10/3))/10 + (4095*a^4*b^11*x^(11/3))/11 + (315*a^
2*b^13*x^(13/3))/13
```

sympy **[A]** time = 4.72, size = 212, normalized size = 1.01

$$a^{15}\log(x)+45a^{14}b\sqrt[3]{x}+\frac{315a^{13}b^2x^{\frac{2}{3}}}{2}+455a^{12}b^3x+\frac{4095a^{11}b^4x^{\frac{4}{3}}}{4}+\frac{9009a^{10}b^5x^{\frac{5}{3}}}{5}+\frac{5005a^9b^6x^2}{2}+\frac{19305a^8b^7x^{\frac{7}{3}}}{7}+\frac{1}{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**15/x,x)
```

```
[Out] a**15*log(x) + 45*a**14*b*x**(1/3) + 315*a**13*b**2*x**(2/3)/2 + 455*a**12*
b**3*x + 4095*a**11*b**4*x**(4/3)/4 + 9009*a**10*b**5*x**(5/3)/5 + 5005*a**
9*b**6*x**2/2 + 19305*a**8*b**7*x**(7/3)/7 + 19305*a**7*b**8*x**(8/3)/8 + 5
005*a**6*b**9*x**3/3 + 9009*a**5*b**10*x**(10/3)/10 + 4095*a**4*b**11*x**(1
1/3)/11 + 455*a**3*b**12*x**4/4 + 315*a**2*b**13*x**(13/3)/13 + 45*a*b**14*
x**(14/3)/14 + b**15*x**5/5
```

3.2346 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^2}\,dx$

Optimal. Leaf size=202

$$-\frac{a^{15}}{x}-\frac{45a^{14}b}{2x^{2/3}}-\frac{315a^{13}b^2}{\sqrt[3]{x}}+455a^{12}b^3\log(x)+4095a^{11}b^4\sqrt[3]{x}+\frac{9009}{2}a^{10}b^5x^{2/3}+5005a^9b^6x+\frac{19305}{4}a^8b^7x^{4/3}+3861a^7b^8$$

```
[Out] -a^15/x-45/2*a^14*b/x^(2/3)-315*a^13*b^2/x^(1/3)+4095*a^11*b^4*x^(1/3)+9009
/2*a^10*b^5*x^(2/3)+5005*a^9*b^6*x+19305/4*a^8*b^7*x^(4/3)+3861*a^7*b^8*x^(
5/3)+5005/2*a^6*b^9*x^2+1287*a^5*b^10*x^(7/3)+4095/8*a^4*b^11*x^(8/3)+455/3
*a^3*b^12*x^3+63/2*a^2*b^13*x^(10/3)+45/11*a*b^14*x^(11/3)+1/4*b^15*x^4+455
*a^12*b^3*ln(x)
```

Rubi [A] time = 0.12, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{9009}{2}a^{10}b^5x^{2/3}+\frac{19305}{4}a^8b^7x^{4/3}+3861a^7b^8x^{5/3}+\frac{5005}{2}a^6b^9x^2+1287a^5b^{10}x^{7/3}+\frac{4095}{8}a^4b^{11}x^{8/3}+\frac{455}{3}a^3b^{12}x^3+\frac{63}{2}a^2b^1$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^15/x^2,x]
```

```
[Out] -(a^15/x) - (45*a^14*b)/(2*x^(2/3)) - (315*a^13*b^2)/x^(1/3) + 4095*a^11*b^
4*x^(1/3) + (9009*a^10*b^5*x^(2/3))/2 + 5005*a^9*b^6*x + (19305*a^8*b^7*x^(
4/3))/4 + 3861*a^7*b^8*x^(5/3) + (5005*a^6*b^9*x^2)/2 + 1287*a^5*b^10*x^(7/
3) + (4095*a^4*b^11*x^(8/3))/8 + (455*a^3*b^12*x^3)/3 + (63*a^2*b^13*x^(10/
3))/2 + (45*a*b^14*x^(11/3))/11 + (b^15*x^4)/4 + 455*a^12*b^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^2}\,dx = 3\,\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^4}\,dx, x, \sqrt[3]{x}\right)$$

$$= 3\,\text{Subst}\left(\int \left(1365a^{11}b^4+\frac{a^{15}}{x^4}+\frac{15a^{14}b}{x^3}+\frac{105a^{13}b^2}{x^2}+\frac{455a^{12}b^3}{x}+3003a^{10}b^5x+5005a^9b^6\right.\right.$$

$$= -\frac{a^{15}}{x}-\frac{45a^{14}b}{2x^{2/3}}-\frac{315a^{13}b^2}{\sqrt[3]{x}}+4095a^{11}b^4\sqrt[3]{x}+\frac{9009}{2}a^{10}b^5x^{2/3}+5005a^9b^6x+\frac{19305}{4}a^8b^7x^{4/}$$

Mathematica [A] time = 0.10, size = 202, normalized size = 1.00

$$-\frac{a^{15}}{x}-\frac{45a^{14}b}{2x^{2/3}}-\frac{315a^{13}b^2}{\sqrt[3]{x}}+455a^{12}b^3\log(x)+4095a^{11}b^4\sqrt[3]{x}+\frac{9009}{2}a^{10}b^5x^{2/3}+5005a^9b^6x+\frac{19305}{4}a^8b^7x^{4/3}+3861a^7b^8$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^15/x^2,x]
```

```
[Out] -(a^15/x) - (45*a^14*b)/(2*x^(2/3)) - (315*a^13*b^2)/x^(1/3) + 4095*a^11*b^
4*x^(1/3) + (9009*a^10*b^5*x^(2/3))/2 + 5005*a^9*b^6*x + (19305*a^8*b^7*x^(
4/3))/4 + 3861*a^7*b^8*x^(5/3) + (5005*a^6*b^9*x^2)/2 + 1287*a^5*b^10*x^(7/
3) + (4095*a^4*b^11*x^(8/3))/8 + (455*a^3*b^12*x^3)/3 + (63*a^2*b^13*x^(10/
3))/2 + (45*a*b^14*x^(11/3))/11 + (b^15*x^4)/4 + 455*a^12*b^3*Log[x]
```

fricas [A] time = 0.84, size = 173, normalized size = 0.86

$$\frac{66\,b^{15}x^5 + 40040\,a^3b^{12}x^4 + 660660\,a^6b^9x^3 + 1321320\,a^9b^6x^2 + 360360\,a^{12}b^3x\log\left(x^{\frac{1}{3}}\right) - 264\,a^{15} + 27\left(40\,ab^1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15/x^2,x, algorithm="fricas")
```

```
[Out] 1/264*(66*b^15*x^5 + 40040*a^3*b^12*x^4 + 660660*a^6*b^9*x^3 + 1321320*a^9*
b^6*x^2 + 360360*a^12*b^3*x*log(x^(1/3)) - 264*a^15 + 27*(40*a*b^14*x^4 + 5
005*a^4*b^11*x^3 + 37752*a^7*b^8*x^2 + 44044*a^10*b^5*x - 3080*a^13*b^2)*x^
(2/3) + 594*(14*a^2*b^13*x^4 + 572*a^5*b^10*x^3 + 2145*a^8*b^7*x^2 + 1820*a
^11*b^4*x - 10*a^14*b)*x^(1/3))/x
```

giac [A] time = 0.18, size = 168, normalized size = 0.83

$$\frac{1}{4}\,b^{15}x^4+\frac{45}{11}\,ab^{14}x^{\frac{11}{3}}+\frac{63}{2}\,a^2b^{13}x^{\frac{10}{3}}+\frac{455}{3}\,a^3b^{12}x^3+\frac{4095}{8}\,a^4b^{11}x^{\frac{8}{3}}+1287\,a^5b^{10}x^{\frac{7}{3}}+\frac{5005}{2}\,a^6b^9x^2+3861\,a^7b^8x^{\frac{5}{3}}+\frac{19}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15/x^2,x, algorithm="giac")
```

```
[Out] 1/4*b^15*x^4 + 45/11*a*b^14*x^(11/3) + 63/2*a^2*b^13*x^(10/3) + 455/3*a^3*b
^12*x^3 + 4095/8*a^4*b^11*x^(8/3) + 1287*a^5*b^10*x^(7/3) + 5005/2*a^6*b^9*
x^2 + 3861*a^7*b^8*x^(5/3) + 19305/4*a^8*b^7*x^(4/3) + 5005*a^9*b^6*x + 455
*a^12*b^3*log(abs(x)) + 9009/2*a^10*b^5*x^(2/3) + 4095*a^11*b^4*x^(1/3) - 1
/2*(630*a^13*b^2*x^(2/3) + 45*a^14*b*x^(1/3) + 2*a^15)/x
```

maple [A] time = 0.01, size = 165, normalized size = 0.82

$$\frac{b^{15}x^4}{4}+\frac{45a\,b^{14}x^{\frac{11}{3}}}{11}+\frac{63a^2b^{13}x^{\frac{10}{3}}}{2}+\frac{455a^3b^{12}x^3}{3}+\frac{4095a^4b^{11}x^{\frac{8}{3}}}{8}+1287a^5b^{10}x^{\frac{7}{3}}+\frac{5005a^6b^9x^2}{2}+3861a^7b^8x^{\frac{5}{3}}+\frac{19305}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^15/x^2,x)
```

```
[Out] -a^15/x-45/2*a^14*b/x^(2/3)-315*a^13*b^2/x^(1/3)+4095*a^11*b^4*x^(1/3)+9009
/2*a^10*b^5*x^(2/3)+5005*a^9*b^6*x+19305/4*a^8*b^7*x^(4/3)+3861*a^7*b^8*x^(
5/3)+5005/2*a^6*b^9*x^2+1287*a^5*b^10*x^(7/3)+4095/8*a^4*b^11*x^(8/3)+455/3
*a^3*b^12*x^3+63/2*a^2*b^13*x^(10/3)+45/11*a*b^14*x^(11/3)+1/4*b^15*x^4+455
*a^12*b^3*ln(x)
```

maxima [A] time = 0.91, size = 167, normalized size = 0.83

$$\frac{1}{4}\,b^{15}x^4+\frac{45}{11}\,ab^{14}x^{\frac{11}{3}}+\frac{63}{2}\,a^2b^{13}x^{\frac{10}{3}}+\frac{455}{3}\,a^3b^{12}x^3+\frac{4095}{8}\,a^4b^{11}x^{\frac{8}{3}}+1287\,a^5b^{10}x^{\frac{7}{3}}+\frac{5005}{2}\,a^6b^9x^2+3861\,a^7b^8x^{\frac{5}{3}}+\frac{19}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^15*x^4 + 45/11*a*b^14*x^(11/3) + 63/2*a^2*b^13*x^(10/3) + 455/3*a^3*b
^12*x^3 + 4095/8*a^4*b^11*x^(8/3) + 1287*a^5*b^10*x^(7/3) + 5005/2*a^6*b^9*
x^2 + 3861*a^7*b^8*x^(5/3) + 19305/4*a^8*b^7*x^(4/3) + 5005*a^9*b^6*x + 455
*a^12*b^3*log(x) + 9009/2*a^10*b^5*x^(2/3) + 4095*a^11*b^4*x^(1/3) - 1/2*(6
30*a^13*b^2*x^(2/3) + 45*a^14*b*x^(1/3) + 2*a^15)/x
```

mupad [B] time = 1.19, size = 167, normalized size = 0.83

$$\frac{b^{15}\,x^4}{4}-\frac{a^{15}+\frac{45\,a^{14}\,b\,x^{1/3}}{2}+315\,a^{13}\,b^2\,x^{2/3}}{x}+1365\,a^{12}\,b^3\,\ln\left(x^{1/3}\right)+5005\,a^9\,b^6\,x+\frac{45\,a\,b^{14}\,x^{11/3}}{11}+\frac{5005\,a^6\,b^9\,x^2}{2}+\frac{455\,a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))^15/x^2,x)
```

```
[Out] (b^15*x^4)/4 - (a^15 + (45*a^14*b*x^(1/3))/2 + 315*a^13*b^2*x^(2/3))/x + 13
65*a^12*b^3*log(x^(1/3)) + 5005*a^9*b^6*x + (45*a*b^14*x^(11/3))/11 + (5005
*a^6*b^9*x^2)/2 + (455*a^3*b^12*x^3)/3 + 4095*a^11*b^4*x^(1/3) + (9009*a^10
*b^5*x^(2/3))/2 + (19305*a^8*b^7*x^(4/3))/4 + 3861*a^7*b^8*x^(5/3) + 1287*a
^5*b^10*x^(7/3) + (4095*a^4*b^11*x^(8/3))/8 + (63*a^2*b^13*x^(10/3))/2
```

sympy [A] time = 4.85, size = 204, normalized size = 1.01

$$-\frac{a^{15}}{x}-\frac{45a^{14}b}{2x^{\frac{2}{3}}}-\frac{315a^{13}b^2}{\sqrt[3]{x}}+455a^{12}b^3\log(x)+4095a^{11}b^4\sqrt[3]{x}+\frac{9009a^{10}b^5x^{\frac{2}{3}}}{2}+5005a^9b^6x+\frac{19305a^8b^7x^{\frac{4}{3}}}{4}+3861a^7b^8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**15/x**2,x)
```

```
[Out] -a**15/x - 45*a**14*b/(2*x**(2/3)) - 315*a**13*b**2/x**(1/3) + 455*a**12*b*
*3*log(x) + 4095*a**11*b**4*x**(1/3) + 9009*a**10*b**5*x**(2/3)/2 + 5005*a*
*9*b**6*x + 19305*a**8*b**7*x**(4/3)/4 + 3861*a**7*b**8*x**(5/3) + 5005*a**
6*b**9*x**2/2 + 1287*a**5*b**10*x**(7/3) + 4095*a**4*b**11*x**(8/3)/8 + 455
*a**3*b**12*x**3/3 + 63*a**2*b**13*x**(10/3)/2 + 45*a*b**14*x**(11/3)/11 +
b**15*x**4/4
```

3.2347 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^3}\,dx$

Optimal. Leaf size=200

$$-\frac{a^{15}}{2x^2}-\frac{9a^{14}b}{x^{5/3}}-\frac{315a^{13}b^2}{4x^{4/3}}-\frac{455a^{12}b^3}{x}-\frac{4095a^{11}b^4}{2x^{2/3}}-\frac{9009a^{10}b^5}{\sqrt[3]{x}}+5005a^9b^6\log(x)+19305a^8b^7\sqrt[3]{x}+\frac{19305}{2}a^7b^8x^{2/3}$$

```
[Out] -1/2*a^15/x^2-9*a^14*b/x^(5/3)-315/4*a^13*b^2/x^(4/3)-455*a^12*b^3/x-4095/2
*a^11*b^4/x^(2/3)-9009*a^10*b^5/x^(1/3)+19305*a^8*b^7*x^(1/3)+19305/2*a^7*b
^8*x^(2/3)+5005*a^6*b^9*x+9009/4*a^5*b^10*x^(4/3)+819*a^4*b^11*x^(5/3)+455/
2*a^3*b^12*x^2+45*a^2*b^13*x^(7/3)+45/8*a*b^14*x^(8/3)+1/3*b^15*x^3+5005*a^
9*b^6*ln(x)
```

Rubi [A] time = 0.12, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{315a^{13}b^2}{4x^{4/3}}-\frac{4095a^{11}b^4}{2x^{2/3}}+\frac{19305}{2}a^7b^8x^{2/3}+\frac{9009}{4}a^5b^{10}x^{4/3}+819a^4b^{11}x^{5/3}+\frac{455}{2}a^3b^{12}x^2+45a^2b^{13}x^{7/3}-\frac{455a^{12}b^3}{x}-\frac{9}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^15/x^3,x]
```

```
[Out] -a^15/(2*x^2) - (9*a^14*b)/x^(5/3) - (315*a^13*b^2)/(4*x^(4/3)) - (455*a^12
*b^3)/x - (4095*a^11*b^4)/(2*x^(2/3)) - (9009*a^10*b^5)/x^(1/3) + 19305*a^8
*b^7*x^(1/3) + (19305*a^7*b^8*x^(2/3))/2 + 5005*a^6*b^9*x + (9009*a^5*b^10*
x^(4/3))/4 + 819*a^4*b^11*x^(5/3) + (455*a^3*b^12*x^2)/2 + 45*a^2*b^13*x^(7
/3) + (45*a*b^14*x^(8/3))/8 + (b^15*x^3)/3 + 5005*a^9*b^6*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^3}\,dx &= 3\,\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^7}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(6435a^8b^7+\frac{a^{15}}{x^7}+\frac{15a^{14}b}{x^6}+\frac{105a^{13}b^2}{x^5}+\frac{455a^{12}b^3}{x^4}+\frac{1365a^{11}b^4}{x^3}+\frac{3003a^{10}}{x^2}\right.\right.\\ &= -\frac{a^{15}}{2x^2}-\frac{9a^{14}b}{x^{5/3}}-\frac{315a^{13}b^2}{4x^{4/3}}-\frac{455a^{12}b^3}{x}-\frac{4095a^{11}b^4}{2x^{2/3}}-\frac{9009a^{10}b^5}{\sqrt[3]{x}}+19305a^8b^7\sqrt[3]{x}+\frac{19}{}\end{aligned}$$

Mathematica [A] time = 0.11, size = 200, normalized size = 1.00

$$-\frac{a^{15}}{2x^2}-\frac{9a^{14}b}{x^{5/3}}-\frac{315a^{13}b^2}{4x^{4/3}}-\frac{455a^{12}b^3}{x}-\frac{4095a^{11}b^4}{2x^{2/3}}-\frac{9009a^{10}b^5}{\sqrt[3]{x}}+5005a^9b^6\log(x)+19305a^8b^7\sqrt[3]{x}+\frac{19305}{2}a^7b^8x^{2/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^15/x^3,x]
```

```
[Out] -1/2*a^15/x^2 - (9*a^14*b)/x^(5/3) - (315*a^13*b^2)/(4*x^(4/3)) - (455*a^12
*b^3)/x - (4095*a^11*b^4)/(2*x^(2/3)) - (9009*a^10*b^5)/x^(1/3) + 19305*a^8
*b^7*x^(1/3) + (19305*a^7*b^8*x^(2/3))/2 + 5005*a^6*b^9*x + (9009*a^5*b^10*
x^(4/3))/4 + 819*a^4*b^11*x^(5/3) + (455*a^3*b^12*x^2)/2 + 45*a^2*b^13*x^(7
/3) + (45*a*b^14*x^(8/3))/8 + (b^15*x^3)/3 + 5005*a^9*b^6*Log[x]
```

fricas [A] time = 1.06, size = 173, normalized size = 0.86

$$\frac{8\,b^{15}x^5 + 5460\,a^3b^{12}x^4 + 120120\,a^6b^9x^3 + 360360\,a^9b^6x^2\log\left(x^{\frac{1}{3}}\right) - 10920\,a^{12}b^3x - 12\,a^{15} + 27\left(5\,ab^{14}x^4 + 728\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15/x^3,x, algorithm="fricas")
```

```
[Out] 1/24*(8*b^15*x^5 + 5460*a^3*b^12*x^4 + 120120*a^6*b^9*x^3 + 360360*a^9*b^6*
x^2*log(x^(1/3)) - 10920*a^12*b^3*x - 12*a^15 + 27*(5*a*b^14*x^4 + 728*a^4*
b^11*x^3 + 8580*a^7*b^8*x^2 - 8008*a^10*b^5*x - 70*a^13*b^2)*x^(2/3) + 54*(
20*a^2*b^13*x^4 + 1001*a^5*b^10*x^3 + 8580*a^8*b^7*x^2 - 910*a^11*b^4*x - 4
*a^14*b)*x^(1/3))/x^2
```

giac [A] time = 0.19, size = 166, normalized size = 0.83

$$\frac{1}{3}\,b^{15}x^3+\frac{45}{8}\,ab^{14}x^{\frac{8}{3}}+45\,a^2b^{13}x^{\frac{7}{3}}+\frac{455}{2}\,a^3b^{12}x^2+819\,a^4b^{11}x^{\frac{5}{3}}+\frac{9009}{4}\,a^5b^{10}x^{\frac{4}{3}}+5005\,a^6b^9x+5005\,a^9b^6\log\left(|x|\right)+\frac{193}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15/x^3,x, algorithm="giac")
```

```
[Out] 1/3*b^15*x^3 + 45/8*a*b^14*x^(8/3) + 45*a^2*b^13*x^(7/3) + 455/2*a^3*b^12*x
^2 + 819*a^4*b^11*x^(5/3) + 9009/4*a^5*b^10*x^(4/3) + 5005*a^6*b^9*x + 5005
*a^9*b^6*log(abs(x)) + 19305/2*a^7*b^8*x^(2/3) + 19305*a^8*b^7*x^(1/3) - 1/
4*(36036*a^10*b^5*x^(5/3) + 8190*a^11*b^4*x^(4/3) + 1820*a^12*b^3*x + 315*a
^13*b^2*x^(2/3) + 36*a^14*b*x^(1/3) + 2*a^15)/x^2
```

maple [A] time = 0.01, size = 165, normalized size = 0.82

$$\frac{b^{15}x^3}{3}+\frac{45a\,b^{14}x^{\frac{8}{3}}}{8}+45a^2b^{13}x^{\frac{7}{3}}+\frac{455a^3b^{12}x^2}{2}+819a^4b^{11}x^{\frac{5}{3}}+\frac{9009a^5b^{10}x^{\frac{4}{3}}}{4}+5005a^9b^6\ln(x)+5005a^6b^9x+\frac{19305a^7b^8}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^15/x^3,x)
```

```
[Out] -1/2*a^15/x^2-9*a^14*b/x^(5/3)-315/4*a^13*b^2/x^(4/3)-455*a^12*b^3/x-4095/2
*a^11*b^4/x^(2/3)-9009*a^10*b^5/x^(1/3)+19305*a^8*b^7*x^(1/3)+19305/2*a^7*b
^8*x^(2/3)+5005*a^6*b^9*x+9009/4*a^5*b^10*x^(4/3)+819*a^4*b^11*x^(5/3)+455/
2*a^3*b^12*x^2+45*a^2*b^13*x^(7/3)+45/8*a*b^14*x^(8/3)+1/3*b^15*x^3+5005*a^
9*b^6*ln(x)
```

maxima [A] time = 0.91, size = 165, normalized size = 0.82

$$\frac{1}{3}\,b^{15}x^3+\frac{45}{8}\,ab^{14}x^{\frac{8}{3}}+45\,a^2b^{13}x^{\frac{7}{3}}+\frac{455}{2}\,a^3b^{12}x^2+819\,a^4b^{11}x^{\frac{5}{3}}+\frac{9009}{4}\,a^5b^{10}x^{\frac{4}{3}}+5005\,a^6b^9x+5005\,a^9b^6\log(x)+\frac{19305}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15/x^3,x, algorithm="maxima")
```

```
[Out] 1/3*b^15*x^3 + 45/8*a*b^14*x^(8/3) + 45*a^2*b^13*x^(7/3) + 455/2*a^3*b^12*x
^2 + 819*a^4*b^11*x^(5/3) + 9009/4*a^5*b^10*x^(4/3) + 5005*a^6*b^9*x + 5005
*a^9*b^6*log(x) + 19305/2*a^7*b^8*x^(2/3) + 19305*a^8*b^7*x^(1/3) - 1/4*(36
036*a^10*b^5*x^(5/3) + 8190*a^11*b^4*x^(4/3) + 1820*a^12*b^3*x + 315*a^13*b
^2*x^(2/3) + 36*a^14*b*x^(1/3) + 2*a^15)/x^2
```

mupad **[B]** time = 0.08, size = 167, normalized size = 0.84

$$\frac{b^{15}\,x^3}{3}-\frac{\frac{a^{15}}{2}+455\,a^{12}\,b^3\,x+9\,a^{14}\,b\,x^{1/3}+\frac{315\,a^{13}\,b^2\,x^{2/3}}{4}+\frac{4095\,a^{11}\,b^4\,x^{4/3}}{2}+9009\,a^{10}\,b^5\,x^{5/3}}{x^2}+15015\,a^9\,b^6\,\ln\left(x^{1/3}\right)+5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))^15/x^3,x)
```

```
[Out] (b^15*x^3)/3 - (a^15/2 + 455*a^12*b^3*x + 9*a^14*b*x^(1/3) + (315*a^13*b^2*
x^(2/3))/4 + (4095*a^11*b^4*x^(4/3))/2 + 9009*a^10*b^5*x^(5/3))/x^2 + 15015
*a^9*b^6*log(x^(1/3)) + 5005*a^6*b^9*x + (45*a*b^14*x^(8/3))/8 + (455*a^3*b
^12*x^2)/2 + 19305*a^8*b^7*x^(1/3) + (19305*a^7*b^8*x^(2/3))/2 + (9009*a^5*
b^10*x^(4/3))/4 + 819*a^4*b^11*x^(5/3) + 45*a^2*b^13*x^(7/3)
```

sympy **[A]** time = 4.63, size = 202, normalized size = 1.01

$$-\frac{a^{15}}{2x^2}-\frac{9a^{14}b}{x^{\frac{5}{3}}}-\frac{315a^{13}b^2}{4x^{\frac{4}{3}}}-\frac{455a^{12}b^3}{x}-\frac{4095a^{11}b^4}{2x^{\frac{2}{3}}}-\frac{9009a^{10}b^5}{\sqrt[3]{x}}+5005a^9b^6\log(x)+19305a^8b^7\sqrt[3]{x}+\frac{19305a^7b^8x^{\frac{2}{3}}}{2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**15/x**3,x)
```

```
[Out] -a**15/(2*x**2) - 9*a**14*b/x**(5/3) - 315*a**13*b**2/(4*x**(4/3)) - 455*a*
*12*b**3/x - 4095*a**11*b**4/(2*x**(2/3)) - 9009*a**10*b**5/x**(1/3) + 5005
*a**9*b**6*log(x) + 19305*a**8*b**7*x**(1/3) + 19305*a**7*b**8*x**(2/3)/2 +
 5005*a**6*b**9*x + 9009*a**5*b**10*x**(4/3)/4 + 819*a**4*b**11*x**(5/3) +
455*a**3*b**12*x**2/2 + 45*a**2*b**13*x**(7/3) + 45*a*b**14*x**(8/3)/8 + b*
*15*x**3/3
```

3.2348 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^4}\,dx$

Optimal. Leaf size=200

$$-\frac{a^{15}}{3x^3}-\frac{45a^{14}b}{8x^{8/3}}-\frac{45a^{13}b^2}{x^{7/3}}-\frac{455a^{12}b^3}{2x^2}-\frac{819a^{11}b^4}{x^{5/3}}-\frac{9009a^{10}b^5}{4x^{4/3}}-\frac{5005a^9b^6}{x}-\frac{19305a^8b^7}{2x^{2/3}}-\frac{19305a^7b^8}{\sqrt[3]{x}}+5005a^6b^9\log(x)$$

```
[Out] -1/3*a^15/x^3-45/8*a^14*b/x^(8/3)-45*a^13*b^2/x^(7/3)-455/2*a^12*b^3/x^2-81
9*a^11*b^4/x^(5/3)-9009/4*a^10*b^5/x^(4/3)-5005*a^9*b^6/x-19305/2*a^8*b^7/x
^(2/3)-19305*a^7*b^8/x^(1/3)+9009*a^5*b^10*x^(1/3)+4095/2*a^4*b^11*x^(2/3)+
455*a^3*b^12*x+315/4*a^2*b^13*x^(4/3)+9*a*b^14*x^(5/3)+1/2*b^15*x^2+5005*a^
6*b^9*ln(x)
```

Rubi [A] time = 0.12, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{45a^{13}b^2}{x^{7/3}}-\frac{455a^{12}b^3}{2x^2}-\frac{819a^{11}b^4}{x^{5/3}}-\frac{9009a^{10}b^5}{4x^{4/3}}-\frac{19305a^8b^7}{2x^{2/3}}+\frac{4095}{2}a^4b^{11}x^{2/3}+\frac{315}{4}a^2b^{13}x^{4/3}-\frac{5005a^9b^6}{x}-\frac{19305a^7b^8}{\sqrt[3]{x}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^15/x^4,x]
```

```
[Out] -a^15/(3*x^3) - (45*a^14*b)/(8*x^(8/3)) - (45*a^13*b^2)/x^(7/3) - (455*a^12
*b^3)/(2*x^2) - (819*a^11*b^4)/x^(5/3) - (9009*a^10*b^5)/(4*x^(4/3)) - (500
5*a^9*b^6)/x - (19305*a^8*b^7)/(2*x^(2/3)) - (19305*a^7*b^8)/x^(1/3) + 9009
*a^5*b^10*x^(1/3) + (4095*a^4*b^11*x^(2/3))/2 + 455*a^3*b^12*x + (315*a^2*b
^13*x^(4/3))/4 + 9*a*b^14*x^(5/3) + (b^15*x^2)/2 + 5005*a^6*b^9*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^4}\,dx &= 3\,\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{10}}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(3003a^5b^{10}+\frac{a^{15}}{x^{10}}+\frac{15a^{14}b}{x^9}+\frac{105a^{13}b^2}{x^8}+\frac{455a^{12}b^3}{x^7}+\frac{1365a^{11}b^4}{x^6}+\frac{3003a^{10}b^5}{x^5}\right.\right.\\ &= -\frac{a^{15}}{3x^3}-\frac{45a^{14}b}{8x^{8/3}}-\frac{45a^{13}b^2}{x^{7/3}}-\frac{455a^{12}b^3}{2x^2}-\frac{819a^{11}b^4}{x^{5/3}}-\frac{9009a^{10}b^5}{4x^{4/3}}-\frac{5005a^9b^6}{x}-\frac{19305a^8b^7}{2x^{2/3}}-\end{aligned}$$

Mathematica [A] time = 0.11, size = 200, normalized size = 1.00

$$-\frac{a^{15}}{3x^3}-\frac{45a^{14}b}{8x^{8/3}}-\frac{45a^{13}b^2}{x^{7/3}}-\frac{455a^{12}b^3}{2x^2}-\frac{819a^{11}b^4}{x^{5/3}}-\frac{9009a^{10}b^5}{4x^{4/3}}-\frac{5005a^9b^6}{x}-\frac{19305a^8b^7}{2x^{2/3}}-\frac{19305a^7b^8}{\sqrt[3]{x}}+5005a^6b^9\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^15/x^4,x]
```

```
[Out] -1/3*a^15/x^3 - (45*a^14*b)/(8*x^(8/3)) - (45*a^13*b^2)/x^(7/3) - (455*a^12
*b^3)/(2*x^2) - (819*a^11*b^4)/x^(5/3) - (9009*a^10*b^5)/(4*x^(4/3)) - (500
5*a^9*b^6)/x - (19305*a^8*b^7)/(2*x^(2/3)) - (19305*a^7*b^8)/x^(1/3) + 9009
*a^5*b^10*x^(1/3) + (4095*a^4*b^11*x^(2/3))/2 + 455*a^3*b^12*x + (315*a^2*b
^13*x^(4/3))/4 + 9*a*b^14*x^(5/3) + (b^15*x^2)/2 + 5005*a^6*b^9*Log[x]
```

fricas [A] time = 0.77, size = 173, normalized size = 0.86

$$\frac{12\,b^{15}x^5 + 10920\,a^3b^{12}x^4 + 360360\,a^6b^9x^3\log\left(x^{\frac{1}{3}}\right) - 120120\,a^9b^6x^2 - 5460\,a^{12}b^3x - 8\,a^{15} + 54\left(4\,ab^{14}x^4 + 91\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15/x^4,x, algorithm="fricas")
```

```
[Out] 1/24*(12*b^15*x^5 + 10920*a^3*b^12*x^4 + 360360*a^6*b^9*x^3*log(x^(1/3)) -
120120*a^9*b^6*x^2 - 5460*a^12*b^3*x - 8*a^15 + 54*(4*a*b^14*x^4 + 910*a^4*
b^11*x^3 - 8580*a^7*b^8*x^2 - 1001*a^10*b^5*x - 20*a^13*b^2)*x^(2/3) + 27*(
70*a^2*b^13*x^4 + 8008*a^5*b^10*x^3 - 8580*a^8*b^7*x^2 - 728*a^11*b^4*x - 5
*a^14*b)*x^(1/3))/x^3
```

giac [A] time = 0.20, size = 166, normalized size = 0.83

$$\frac{1}{2}\,b^{15}x^2+9\,ab^{14}x^{\frac{5}{3}}+\frac{315}{4}\,a^2b^{13}x^{\frac{4}{3}}+455\,a^3b^{12}x+5005\,a^6b^9\log\left(|x|\right)+\frac{4095}{2}\,a^4b^{11}x^{\frac{2}{3}}+9009\,a^5b^{10}x^{\frac{1}{3}}-\frac{463320\,a^7b^8x^{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15/x^4,x, algorithm="giac")
```

```
[Out] 1/2*b^15*x^2 + 9*a*b^14*x^(5/3) + 315/4*a^2*b^13*x^(4/3) + 455*a^3*b^12*x +
 5005*a^6*b^9*log(abs(x)) + 4095/2*a^4*b^11*x^(2/3) + 9009*a^5*b^10*x^(1/3)
 - 1/24*(463320*a^7*b^8*x^(8/3) + 231660*a^8*b^7*x^(7/3) + 120120*a^9*b^6*x
^2 + 54054*a^10*b^5*x^(5/3) + 19656*a^11*b^4*x^(4/3) + 5460*a^12*b^3*x + 10
80*a^13*b^2*x^(2/3) + 135*a^14*b*x^(1/3) + 8*a^15)/x^3
```

maple [A] time = 0.01, size = 165, normalized size = 0.82

$$\frac{b^{15}x^2}{2}+9a\,b^{14}x^{\frac{5}{3}}+\frac{315a^2b^{13}x^{\frac{4}{3}}}{4}+5005a^6b^9\ln(x)+455a^3b^{12}x+\frac{4095a^4b^{11}x^{\frac{2}{3}}}{2}+9009a^5b^{10}x^{\frac{1}{3}}-\frac{19305a^7b^8}{x^{\frac{1}{3}}}-\frac{19305a^8}{2x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^15/x^4,x)
```

```
[Out] -1/3*a^15/x^3-45/8*a^14*b/x^(8/3)-45*a^13*b^2/x^(7/3)-455/2*a^12*b^3/x^2-81
9*a^11*b^4/x^(5/3)-9009/4*a^10*b^5/x^(4/3)-5005*a^9*b^6/x-19305/2*a^8*b^7/x
^(2/3)-19305*a^7*b^8/x^(1/3)+9009*a^5*b^10*x^(1/3)+4095/2*a^4*b^11*x^(2/3)+
455*a^3*b^12*x+315/4*a^2*b^13*x^(4/3)+9*a*b^14*x^(5/3)+1/2*b^15*x^2+5005*a^
6*b^9*ln(x)
```

maxima [A] time = 0.90, size = 165, normalized size = 0.82

$$\frac{1}{2}\,b^{15}x^2+9\,ab^{14}x^{\frac{5}{3}}+\frac{315}{4}\,a^2b^{13}x^{\frac{4}{3}}+455\,a^3b^{12}x+5005\,a^6b^9\log(x)+\frac{4095}{2}\,a^4b^{11}x^{\frac{2}{3}}+9009\,a^5b^{10}x^{\frac{1}{3}}-\frac{463320\,a^7b^8x^{\frac{8}{3}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15/x^4,x, algorithm="maxima")
```

```
[Out] 1/2*b^15*x^2 + 9*a*b^14*x^(5/3) + 315/4*a^2*b^13*x^(4/3) + 455*a^3*b^12*x +
 5005*a^6*b^9*log(x) + 4095/2*a^4*b^11*x^(2/3) + 9009*a^5*b^10*x^(1/3) - 1/
24*(463320*a^7*b^8*x^(8/3) + 231660*a^8*b^7*x^(7/3) + 120120*a^9*b^6*x^2 +
54054*a^10*b^5*x^(5/3) + 19656*a^11*b^4*x^(4/3) + 5460*a^12*b^3*x + 1080*a^
13*b^2*x^(2/3) + 135*a^14*b*x^(1/3) + 8*a^15)/x^3
```

mupad [B] time = 1.18, size = 167, normalized size = 0.84

$$\frac{b^{15}\,x^2}{2}-\frac{\frac{a^{15}}{3}+\frac{455\,a^{12}\,b^3\,x}{2}+\frac{45\,a^{14}\,b\,x^{1/3}}{8}+5005\,a^9\,b^6\,x^2+45\,a^{13}\,b^2\,x^{2/3}+819\,a^{11}\,b^4\,x^{4/3}+\frac{9009\,a^{10}\,b^5\,x^{5/3}}{4}+\frac{19305\,a^8\,b^7\,x^{7/3}}{2}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))^15/x^4,x)
```

```
[Out] (b^15*x^2)/2 - (a^15/3 + (455*a^12*b^3*x)/2 + (45*a^14*b*x^(1/3))/8 + 5005*
a^9*b^6*x^2 + 45*a^13*b^2*x^(2/3) + 819*a^11*b^4*x^(4/3) + (9009*a^10*b^5*x
^(5/3))/4 + (19305*a^8*b^7*x^(7/3))/2 + 19305*a^7*b^8*x^(8/3))/x^3 + 15015*
a^6*b^9*log(x^(1/3)) + 455*a^3*b^12*x + 9*a*b^14*x^(5/3) + 9009*a^5*b^10*x^
(1/3) + (4095*a^4*b^11*x^(2/3))/2 + (315*a^2*b^13*x^(4/3))/4
```

sympy [A] time = 4.61, size = 202, normalized size = 1.01

$$-\frac{a^{15}}{3x^3}-\frac{45a^{14}b}{8x^{\frac{8}{3}}}-\frac{45a^{13}b^2}{x^{\frac{7}{3}}}-\frac{455a^{12}b^3}{2x^2}-\frac{819a^{11}b^4}{x^{\frac{5}{3}}}-\frac{9009a^{10}b^5}{4x^{\frac{4}{3}}}-\frac{5005a^9b^6}{x}-\frac{19305a^8b^7}{2x^{\frac{2}{3}}}-\frac{19305a^7b^8}{\sqrt[3]{x}}+5005a^6b^9\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**15/x**4,x)
```

```
[Out] -a**15/(3*x**3) - 45*a**14*b/(8*x**(8/3)) - 45*a**13*b**2/x**(7/3) - 455*a*
*12*b**3/(2*x**2) - 819*a**11*b**4/x**(5/3) - 9009*a**10*b**5/(4*x**(4/3))
- 5005*a**9*b**6/x - 19305*a**8*b**7/(2*x**(2/3)) - 19305*a**7*b**8/x**(1/3
) + 5005*a**6*b**9*log(x) + 9009*a**5*b**10*x**(1/3) + 4095*a**4*b**11*x**(
2/3)/2 + 455*a**3*b**12*x + 315*a**2*b**13*x**(4/3)/4 + 9*a*b**14*x**(5/3)
+ b**15*x**2/2
```

3.2349 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^6}\,dx$

Optimal. Leaf size=211

$$-\frac{a^{15}}{5x^5}-\frac{45a^{14}b}{14x^{14/3}}-\frac{315a^{13}b^2}{13x^{13/3}}-\frac{455a^{12}b^3}{4x^4}-\frac{4095a^{11}b^4}{11x^{11/3}}-\frac{9009a^{10}b^5}{10x^{10/3}}-\frac{5005a^9b^6}{3x^3}-\frac{19305a^8b^7}{8x^{8/3}}-\frac{19305a^7b^8}{7x^{7/3}}-\frac{5005a^6b^9}{2x^2}-$$

```
[Out] -1/5*a^15/x^5-45/14*a^14*b/x^(14/3)-315/13*a^13*b^2/x^(13/3)-455/4*a^12*b^3
/x^4-4095/11*a^11*b^4/x^(11/3)-9009/10*a^10*b^5/x^(10/3)-5005/3*a^9*b^6/x^3
-19305/8*a^8*b^7/x^(8/3)-19305/7*a^7*b^8/x^(7/3)-5005/2*a^6*b^9/x^2-9009/5*
a^5*b^10/x^(5/3)-4095/4*a^4*b^11/x^(4/3)-455*a^3*b^12/x-315/2*a^2*b^13/x^(2
/3)-45*a*b^14/x^(1/3)+b^15*ln(x)
```

Rubi [A] time = 0.12, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{315a^{13}b^2}{13x^{13/3}}-\frac{455a^{12}b^3}{4x^4}-\frac{4095a^{11}b^4}{11x^{11/3}}-\frac{9009a^{10}b^5}{10x^{10/3}}-\frac{5005a^9b^6}{3x^3}-\frac{19305a^8b^7}{8x^{8/3}}-\frac{19305a^7b^8}{7x^{7/3}}-\frac{5005a^6b^9}{2x^2}-\frac{9009a^5b^{10}}{5x^{5/3}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^15/x^6,x]
```

```
[Out] -a^15/(5*x^5) - (45*a^14*b)/(14*x^(14/3)) - (315*a^13*b^2)/(13*x^(13/3)) -
(455*a^12*b^3)/(4*x^4) - (4095*a^11*b^4)/(11*x^(11/3)) - (9009*a^10*b^5)/(1
0*x^(10/3)) - (5005*a^9*b^6)/(3*x^3) - (19305*a^8*b^7)/(8*x^(8/3)) - (19305
*a^7*b^8)/(7*x^(7/3)) - (5005*a^6*b^9)/(2*x^2) - (9009*a^5*b^10)/(5*x^(5/3)
) - (4095*a^4*b^11)/(4*x^(4/3)) - (455*a^3*b^12)/x - (315*a^2*b^13)/(2*x^(2
/3)) - (45*a*b^14)/x^(1/3) + b^15*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^6}\,dx &= 3\,\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{16}}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(\frac{a^{15}}{x^{16}}+\frac{15a^{14}b}{x^{15}}+\frac{105a^{13}b^2}{x^{14}}+\frac{455a^{12}b^3}{x^{13}}+\frac{1365a^{11}b^4}{x^{12}}+\frac{3003a^{10}b^5}{x^{11}}+\frac{5005a^{9}}{x^{10}}\right.\right.\\ &= -\frac{a^{15}}{5x^5}-\frac{45a^{14}b}{14x^{14/3}}-\frac{315a^{13}b^2}{13x^{13/3}}-\frac{455a^{12}b^3}{4x^4}-\frac{4095a^{11}b^4}{11x^{11/3}}-\frac{9009a^{10}b^5}{10x^{10/3}}-\frac{5005a^9b^6}{3x^3}-\frac{19305}{8x^{8}}\end{aligned}$$

Mathematica [A] time = 0.14, size = 211, normalized size = 1.00

$$-\frac{a^{15}}{5x^5}-\frac{45a^{14}b}{14x^{14/3}}-\frac{315a^{13}b^2}{13x^{13/3}}-\frac{455a^{12}b^3}{4x^4}-\frac{4095a^{11}b^4}{11x^{11/3}}-\frac{9009a^{10}b^5}{10x^{10/3}}-\frac{5005a^9b^6}{3x^3}-\frac{19305a^8b^7}{8x^{8/3}}-\frac{19305a^7b^8}{7x^{7/3}}-\frac{5005a^6b^9}{2x^2}-$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^15/x^6,x]
```

```
[Out] -1/5*a^15/x^5 - (45*a^14*b)/(14*x^(14/3)) - (315*a^13*b^2)/(13*x^(13/3)) -
(455*a^12*b^3)/(4*x^4) - (4095*a^11*b^4)/(11*x^(11/3)) - (9009*a^10*b^5)/(1
0*x^(10/3)) - (5005*a^9*b^6)/(3*x^3) - (19305*a^8*b^7)/(8*x^(8/3)) - (19305
*a^7*b^8)/(7*x^(7/3)) - (5005*a^6*b^9)/(2*x^2) - (9009*a^5*b^10)/(5*x^(5/3)
) - (4095*a^4*b^11)/(4*x^(4/3)) - (455*a^3*b^12)/x - (315*a^2*b^13)/(2*x^(2
/3)) - (45*a*b^14)/x^(1/3) + b^15*Log[x]
```

fricas [A] time = 0.75, size = 173, normalized size = 0.82

$$\frac{360360\,b^{15}x^5\log\left(x^{\frac{1}{3}}\right) - 54654600\,a^3b^{12}x^4 - 300600300\,a^6b^9x^3 - 200400200\,a^9b^6x^2 - 13663650\,a^{12}b^3x - 24024}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15/x^6,x, algorithm="fricas")
```

```
[Out] 1/120120*(360360*b^15*x^5*log(x^(1/3)) - 54654600*a^3*b^12*x^4 - 300600300*
a^6*b^9*x^3 - 200400200*a^9*b^6*x^2 - 13663650*a^12*b^3*x - 24024*a^15 - 59
4*(9100*a*b^14*x^4 + 207025*a^4*b^11*x^3 + 557700*a^7*b^8*x^2 + 182182*a^10
*b^5*x + 4900*a^13*b^2)*x^(2/3) - 351*(53900*a^2*b^13*x^4 + 616616*a^5*b^10
*x^3 + 825825*a^8*b^7*x^2 + 127400*a^11*b^4*x + 1100*a^14*b)*x^(1/3))/x^5
```

giac [A] time = 0.18, size = 167, normalized size = 0.79

$$b^{15}\log(|x|) - \frac{5405400\,ab^{14}x^{\frac{14}{3}} + 18918900\,a^2b^{13}x^{\frac{13}{3}} + 54654600\,a^3b^{12}x^4 + 122972850\,a^4b^{11}x^{\frac{11}{3}} + 216432216\,a^5b^{1}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15/x^6,x, algorithm="giac")
```

```
[Out] b^15*log(abs(x)) - 1/120120*(5405400*a*b^14*x^(14/3) + 18918900*a^2*b^13*x^
(13/3) + 54654600*a^3*b^12*x^4 + 122972850*a^4*b^11*x^(11/3) + 216432216*a^
5*b^10*x^(10/3) + 300600300*a^6*b^9*x^3 + 331273800*a^7*b^8*x^(8/3) + 28986
4575*a^8*b^7*x^(7/3) + 200400200*a^9*b^6*x^2 + 108216108*a^10*b^5*x^(5/3) +
 44717400*a^11*b^4*x^(4/3) + 13663650*a^12*b^3*x + 2910600*a^13*b^2*x^(2/3)
 + 386100*a^14*b*x^(1/3) + 24024*a^15)/x^5
```

maple [A] time = 0.01, size = 166, normalized size = 0.79

$$b^{15}\ln(x) - \frac{45a\,b^{14}}{x^{\frac{1}{3}}} - \frac{315a^2b^{13}}{2x^{\frac{2}{3}}} - \frac{455a^3b^{12}}{x} - \frac{4095a^4b^{11}}{4x^{\frac{4}{3}}} - \frac{9009a^5b^{10}}{5x^{\frac{5}{3}}} - \frac{5005a^6b^9}{2x^2} - \frac{19305a^7b^8}{7x^{\frac{7}{3}}} - \frac{19305a^8b^7}{8x^{\frac{8}{3}}} - \frac{5005a^9b^6}{3x^3} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^15/x^6,x)
```

```
[Out] -1/5*a^15/x^5-45/14*a^14*b/x^(14/3)-315/13*a^13*b^2/x^(13/3)-455/4*a^12*b^3
/x^4-4095/11*a^11*b^4/x^(11/3)-9009/10*a^10*b^5/x^(10/3)-5005/3*a^9*b^6/x^3
-19305/8*a^8*b^7/x^(8/3)-19305/7*a^7*b^8/x^(7/3)-5005/2*a^6*b^9/x^2-9009/5*
a^5*b^10/x^(5/3)-4095/4*a^4*b^11/x^(4/3)-455*a^3*b^12/x-315/2*a^2*b^13/x^(2
/3)-45*a*b^14/x^(1/3)+b^15*ln(x)
```

maxima [A] time = 0.90, size = 166, normalized size = 0.79

$$b^{15}\log(x) - \frac{5405400\,ab^{14}x^{\frac{14}{3}} + 18918900\,a^2b^{13}x^{\frac{13}{3}} + 54654600\,a^3b^{12}x^4 + 122972850\,a^4b^{11}x^{\frac{11}{3}} + 216432216\,a^5b^{10}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15/x^6,x, algorithm="maxima")
```

```
[Out] b^15*log(x) - 1/120120*(5405400*a*b^14*x^(14/3) + 18918900*a^2*b^13*x^(13/3
) + 54654600*a^3*b^12*x^4 + 122972850*a^4*b^11*x^(11/3) + 216432216*a^5*b^1
0*x^(10/3) + 300600300*a^6*b^9*x^3 + 331273800*a^7*b^8*x^(8/3) + 289864575*
a^8*b^7*x^(7/3) + 200400200*a^9*b^6*x^2 + 108216108*a^10*b^5*x^(5/3) + 4471
7400*a^11*b^4*x^(4/3) + 13663650*a^12*b^3*x + 2910600*a^13*b^2*x^(2/3) + 38
6100*a^14*b*x^(1/3) + 24024*a^15)/x^5
```

mupad [B] time = 1.28, size = 168, normalized size = 0.80

$$3\,b^{15}\,\ln\left(x^{1/3}\right)-\frac{a^{15}}{5\,x^5}-\frac{45\,a\,b^{14}}{x^{1/3}}-\frac{45\,a^{14}\,b}{14\,x^{14/3}}-\frac{455\,a^3\,b^{12}}{x}-\frac{5005\,a^6\,b^9}{2\,x^2}-\frac{5005\,a^9\,b^6}{3\,x^3}-\frac{455\,a^{12}\,b^3}{4\,x^4}-\frac{315\,a^2\,b^{13}}{2\,x^{2/3}}-\frac{4095\,a^4\,b}{4\,x^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))^15/x^6,x)
```

```
[Out] 3*b^15*log(x^(1/3)) - a^15/(5*x^5) - (45*a*b^14)/x^(1/3) - (45*a^14*b)/(14*
x^(14/3)) - (455*a^3*b^12)/x - (5005*a^6*b^9)/(2*x^2) - (5005*a^9*b^6)/(3*x
^3) - (455*a^12*b^3)/(4*x^4) - (315*a^2*b^13)/(2*x^(2/3)) - (4095*a^4*b^11)
/(4*x^(4/3)) - (9009*a^5*b^10)/(5*x^(5/3)) - (19305*a^7*b^8)/(7*x^(7/3)) -
(19305*a^8*b^7)/(8*x^(8/3)) - (9009*a^10*b^5)/(10*x^(10/3)) - (4095*a^11*b^
4)/(11*x^(11/3)) - (315*a^13*b^2)/(13*x^(13/3))
```

sympy [A] time = 7.90, size = 212, normalized size = 1.00

$$-\frac{a^{15}}{5x^5}-\frac{45a^{14}b}{14x^{\frac{14}{3}}}-\frac{315a^{13}b^2}{13x^{\frac{13}{3}}}-\frac{455a^{12}b^3}{4x^4}-\frac{4095a^{11}b^4}{11x^{\frac{11}{3}}}-\frac{9009a^{10}b^5}{10x^{\frac{10}{3}}}-\frac{5005a^9b^6}{3x^3}-\frac{19305a^8b^7}{8x^{\frac{8}{3}}}-\frac{19305a^7b^8}{7x^{\frac{7}{3}}}-\frac{5005a^6b^9}{2x^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**15/x**6,x)
```

```
[Out] -a**15/(5*x**5) - 45*a**14*b/(14*x**(14/3)) - 315*a**13*b**2/(13*x**(13/3))
 - 455*a**12*b**3/(4*x**4) - 4095*a**11*b**4/(11*x**(11/3)) - 9009*a**10*b*
*5/(10*x**(10/3)) - 5005*a**9*b**6/(3*x**3) - 19305*a**8*b**7/(8*x**(8/3))
- 19305*a**7*b**8/(7*x**(7/3)) - 5005*a**6*b**9/(2*x**2) - 9009*a**5*b**10/
(5*x**(5/3)) - 4095*a**4*b**11/(4*x**(4/3)) - 455*a**3*b**12/x - 315*a**2*b
**13/(2*x**(2/3)) - 45*a*b**14/x**(1/3) + b**15*log(x)
```

3.2350 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^7}\,dx$

Optimal. Leaf size=72

$$-\frac{b^2\left(a+b\sqrt[3]{x}\right)^{16}}{816a^3x^{16/3}}+\frac{b\left(a+b\sqrt[3]{x}\right)^{16}}{51a^2x^{17/3}}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{6ax^6}$$

```
[Out] -1/6*(a+b*x^(1/3))^16/a/x^6+1/51*b*(a+b*x^(1/3))^16/a^2/x^(17/3)-1/816*b^2*
(a+b*x^(1/3))^16/a^3/x^(16/3)
```

Rubi [A] time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {266, 45, 37}

$$-\frac{b^2\left(a+b\sqrt[3]{x}\right)^{16}}{816a^3x^{16/3}}+\frac{b\left(a+b\sqrt[3]{x}\right)^{16}}{51a^2x^{17/3}}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^15/x^7,x]
```

```
[Out] -(a + b*x^(1/3))^16/(6*a*x^6) + (b*(a + b*x^(1/3))^16)/(51*a^2*x^(17/3)) -
(b^2*(a + b*x^(1/3))^16)/(816*a^3*x^(16/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^7}\,dx &= 3\,\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{19}}\,dx, x, \sqrt[3]{x}\right)\\ &= -\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{6ax^6} - \frac{b\,\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{18}}\,dx, x, \sqrt[3]{x}\right)}{3a}\\ &= -\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{6ax^6} + \frac{b\left(a+b\sqrt[3]{x}\right)^{16}}{51a^2x^{17/3}} + \frac{b^2\,\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{17}}\,dx, x, \sqrt[3]{x}\right)}{51a^2}\\ &= -\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{6ax^6} + \frac{b\left(a+b\sqrt[3]{x}\right)^{16}}{51a^2x^{17/3}} - \frac{b^2\left(a+b\sqrt[3]{x}\right)^{16}}{816a^3x^{16/3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 45, normalized size = 0.62

$$-\frac{\left(a+b\sqrt[3]{x}\right)^{16}\left(136a^2-16ab\sqrt[3]{x}+b^2x^{2/3}\right)}{816a^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^15/x^7,x]
```

```
[Out] -1/816*((a + b*x^(1/3))^16*(136*a^2 - 16*a*b*x^(1/3) + b^2*x^(2/3)))/(a^3*x
^6)
```

fricas [B] time = 0.82, size = 169, normalized size = 2.35

$$-\frac{816\,b^{15}x^5 + 185640\,a^3b^{12}x^4 + 1361360\,a^6b^9x^3 + 1021020\,a^9b^6x^2 + 74256\,a^{12}b^3x + 136\,a^{15} + 459\left(20\,ab^{14}x^4 + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15/x^7,x, algorithm="fricas")
```

```
[Out] -1/816*(816*b^15*x^5 + 185640*a^3*b^12*x^4 + 1361360*a^6*b^9*x^3 + 1021020*
a^9*b^6*x^2 + 74256*a^12*b^3*x + 136*a^15 + 459*(20*a*b^14*x^4 + 1040*a^4*b
^11*x^3 + 3432*a^7*b^8*x^2 + 1232*a^10*b^5*x + 35*a^13*b^2)*x^(2/3) + 54*(9
52*a^2*b^13*x^4 + 17017*a^5*b^10*x^3 + 26520*a^8*b^7*x^2 + 4420*a^11*b^4*x
+ 40*a^14*b)*x^(1/3))/x^6
```

giac [B] time = 0.18, size = 167, normalized size = 2.32

$$-\frac{816\,b^{15}x^5 + 9180\,ab^{14}x^{\frac{14}{3}} + 51408\,a^2b^{13}x^{\frac{13}{3}} + 185640\,a^3b^{12}x^4 + 477360\,a^4b^{11}x^{\frac{11}{3}} + 918918\,a^5b^{10}x^{\frac{10}{3}} + 13613}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15/x^7,x, algorithm="giac")
```

```
[Out] -1/816*(816*b^15*x^5 + 9180*a*b^14*x^(14/3) + 51408*a^2*b^13*x^(13/3) + 185
640*a^3*b^12*x^4 + 477360*a^4*b^11*x^(11/3) + 918918*a^5*b^10*x^(10/3) + 13
61360*a^6*b^9*x^3 + 1575288*a^7*b^8*x^(8/3) + 1432080*a^8*b^7*x^(7/3) + 102
1020*a^9*b^6*x^2 + 565488*a^10*b^5*x^(5/3) + 238680*a^11*b^4*x^(4/3) + 7425
6*a^12*b^3*x + 16065*a^13*b^2*x^(2/3) + 2160*a^14*b*x^(1/3) + 136*a^15)/x^6
```

maple [B] time = 0.01, size = 168, normalized size = 2.33

$$-\frac{b^{15}}{x} - \frac{45a\,b^{14}}{4x^{\frac{4}{3}}} - \frac{63a^2b^{13}}{x^{\frac{5}{3}}} - \frac{455a^3b^{12}}{2x^2} - \frac{585a^4b^{11}}{x^{\frac{7}{3}}} - \frac{9009a^5b^{10}}{8x^{\frac{8}{3}}} - \frac{5005a^6b^9}{3x^3} - \frac{3861a^7b^8}{2x^{\frac{10}{3}}} - \frac{1755a^8b^7}{x^{\frac{11}{3}}} - \frac{5005a^9b^6}{4x^4} - \frac{693a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^15/x^7,x)
```

```
[Out] -63*a^2*b^13/x^(5/3)-45/4*a*b^14/x^(4/3)-585/2*a^11*b^4/x^(14/3)-45/17*a^14
*b/x^(17/3)-315/16*a^13*b^2/x^(16/3)-b^15/x-91*a^12*b^3/x^5-9009/8*a^5*b^10
/x^(8/3)-693*a^10*b^5/x^(13/3)-5005/3*a^6*b^9/x^3-1/6*a^15/x^6-585*a^4*b^11
/x^(7/3)-455/2*a^3*b^12/x^2-5005/4*a^9*b^6/x^4-3861/2*a^7*b^8/x^(10/3)-1755
*a^8*b^7/x^(11/3)
```

maxima [B] time = 0.92, size = 167, normalized size = 2.32

$$-\frac{816\,b^{15}x^5+9180\,ab^{14}x^{\frac{14}{3}}+51408\,a^2b^{13}x^{\frac{13}{3}}+185640\,a^3b^{12}x^4+477360\,a^4b^{11}x^{\frac{11}{3}}+918918\,a^5b^{10}x^{\frac{10}{3}}+1361360\,$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15/x^7,x, algorithm="maxima")
```

```
[Out] -1/816*(816*b^15*x^5 + 9180*a*b^14*x^(14/3) + 51408*a^2*b^13*x^(13/3) + 185
640*a^3*b^12*x^4 + 477360*a^4*b^11*x^(11/3) + 918918*a^5*b^10*x^(10/3) + 13
61360*a^6*b^9*x^3 + 1575288*a^7*b^8*x^(8/3) + 1432080*a^8*b^7*x^(7/3) + 102
1020*a^9*b^6*x^2 + 565488*a^10*b^5*x^(5/3) + 238680*a^11*b^4*x^(4/3) + 7425
6*a^12*b^3*x + 16065*a^13*b^2*x^(2/3) + 2160*a^14*b*x^(1/3) + 136*a^15)/x^6
```

mupad [B] time = 0.19, size = 166, normalized size = 2.31

$$-\frac{\frac{a^{15}}{6}+b^{15}\,x^5+91\,a^{12}\,b^3\,x+\frac{45\,a^{14}\,b\,x^{1/3}}{17}+\frac{45\,a\,b^{14}\,x^{14/3}}{4}+\frac{5005\,a^9\,b^6\,x^2}{4}+\frac{5005\,a^6\,b^9\,x^3}{3}+\frac{455\,a^3\,b^{12}\,x^4}{2}+\frac{315\,a^{13}\,b^2\,x^{2/3}}{16}+\frac{585\,a^{11}\,b}{2}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))^15/x^7,x)
```

```
[Out] -(a^15/6 + b^15*x^5 + 91*a^12*b^3*x + (45*a^14*b*x^(1/3))/17 + (45*a*b^14*x
^(14/3))/4 + (5005*a^9*b^6*x^2)/4 + (5005*a^6*b^9*x^3)/3 + (455*a^3*b^12*x^
4)/2 + (315*a^13*b^2*x^(2/3))/16 + (585*a^11*b^4*x^(4/3))/2 + 693*a^10*b^5*
x^(5/3) + 1755*a^8*b^7*x^(7/3) + (3861*a^7*b^8*x^(8/3))/2 + (9009*a^5*b^10*
x^(10/3))/8 + 585*a^4*b^11*x^(11/3) + 63*a^2*b^13*x^(13/3))/x^6
```

sympy [B] time = 10.49, size = 209, normalized size = 2.90

$$-\frac{a^{15}}{6x^6}-\frac{45a^{14}b}{17x^{\frac{17}{3}}}-\frac{315a^{13}b^2}{16x^{\frac{16}{3}}}-\frac{91a^{12}b^3}{x^5}-\frac{585a^{11}b^4}{2x^{\frac{14}{3}}}-\frac{693a^{10}b^5}{x^{\frac{13}{3}}}-\frac{5005a^9b^6}{4x^4}-\frac{1755a^8b^7}{x^{\frac{11}{3}}}-\frac{3861a^7b^8}{2x^{\frac{10}{3}}}-\frac{5005a^6b^9}{3x^3}-\frac{9009a^5b^1}{8x^{\frac{8}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**15/x**7,x)
```

```
[Out] -a**15/(6*x**6) - 45*a**14*b/(17*x**(17/3)) - 315*a**13*b**2/(16*x**(16/3))
 - 91*a**12*b**3/x**5 - 585*a**11*b**4/(2*x**(14/3)) - 693*a**10*b**5/x**(1
3/3) - 5005*a**9*b**6/(4*x**4) - 1755*a**8*b**7/x**(11/3) - 3861*a**7*b**8/
(2*x**(10/3)) - 5005*a**6*b**9/(3*x**3) - 9009*a**5*b**10/(8*x**(8/3)) - 58
5*a**4*b**11/x**(7/3) - 455*a**3*b**12/(2*x**2) - 63*a**2*b**13/x**(5/3) -
45*a*b**14/(4*x**(4/3)) - b**15/x
```

3.2351 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^8}\,dx$

Optimal. Leaf size=148

$$\frac{b^5\left(a+b\sqrt[3]{x}\right)^{16}}{108528a^6x^{16/3}}-\frac{b^4\left(a+b\sqrt[3]{x}\right)^{16}}{6783a^5x^{17/3}}+\frac{b^3\left(a+b\sqrt[3]{x}\right)^{16}}{798a^4x^6}-\frac{b^2\left(a+b\sqrt[3]{x}\right)^{16}}{133a^3x^{19/3}}+\frac{b\left(a+b\sqrt[3]{x}\right)^{16}}{28a^2x^{20/3}}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{7ax^7}$$

```
[Out] -1/7*(a+b*x^(1/3))^16/a/x^7+1/28*b*(a+b*x^(1/3))^16/a^2/x^(20/3)-1/133*b^2*
(a+b*x^(1/3))^16/a^3/x^(19/3)+1/798*b^3*(a+b*x^(1/3))^16/a^4/x^6-1/6783*b^4
*(a+b*x^(1/3))^16/a^5/x^(17/3)+1/108528*b^5*(a+b*x^(1/3))^16/a^6/x^(16/3)
```

Rubi [A] time = 0.06, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {266, 45, 37}

$$\frac{b^5\left(a+b\sqrt[3]{x}\right)^{16}}{108528a^6x^{16/3}}-\frac{b^4\left(a+b\sqrt[3]{x}\right)^{16}}{6783a^5x^{17/3}}+\frac{b^3\left(a+b\sqrt[3]{x}\right)^{16}}{798a^4x^6}-\frac{b^2\left(a+b\sqrt[3]{x}\right)^{16}}{133a^3x^{19/3}}+\frac{b\left(a+b\sqrt[3]{x}\right)^{16}}{28a^2x^{20/3}}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^15/x^8,x]
```

```
[Out] -(a + b*x^(1/3))^16/(7*a*x^7) + (b*(a + b*x^(1/3))^16)/(28*a^2*x^(20/3)) -
(b^2*(a + b*x^(1/3))^16)/(133*a^3*x^(19/3)) + (b^3*(a + b*x^(1/3))^16)/(798
*a^4*x^6) - (b^4*(a + b*x^(1/3))^16)/(6783*a^5*x^(17/3)) + (b^5*(a + b*x^(1
/3))^16)/(108528*a^6*x^(16/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^8}\,dx = 3\,\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{22}}\,dx, x, \sqrt[3]{x}\right)$$
$$= -\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{7ax^7} - \frac{(5b)\,\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{21}}\,dx, x, \sqrt[3]{x}\right)}{7a}$$
$$= -\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{7ax^7} + \frac{b\left(a+b\sqrt[3]{x}\right)^{16}}{28a^2x^{20/3}} + \frac{b^2\,\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{20}}\,dx, x, \sqrt[3]{x}\right)}{7a^2}$$
$$= -\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{7ax^7} + \frac{b\left(a+b\sqrt[3]{x}\right)^{16}}{28a^2x^{20/3}} - \frac{b^2\left(a+b\sqrt[3]{x}\right)^{16}}{133a^3x^{19/3}} - \frac{\left(3b^3\right)\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{19}}\,dx, x, \sqrt[3]{x}\right)}{133a^3}$$
$$= -\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{7ax^7} + \frac{b\left(a+b\sqrt[3]{x}\right)^{16}}{28a^2x^{20/3}} - \frac{b^2\left(a+b\sqrt[3]{x}\right)^{16}}{133a^3x^{19/3}} + \frac{b^3\left(a+b\sqrt[3]{x}\right)^{16}}{798a^4x^6} + \frac{b^4\,\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{18}}\right.}{399a^4}$$
$$= -\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{7ax^7} + \frac{b\left(a+b\sqrt[3]{x}\right)^{16}}{28a^2x^{20/3}} - \frac{b^2\left(a+b\sqrt[3]{x}\right)^{16}}{133a^3x^{19/3}} + \frac{b^3\left(a+b\sqrt[3]{x}\right)^{16}}{798a^4x^6} - \frac{b^4\left(a+b\sqrt[3]{x}\right)^{16}}{6783a^5x^{17/3}} -$$
$$= -\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{7ax^7} + \frac{b\left(a+b\sqrt[3]{x}\right)^{16}}{28a^2x^{20/3}} - \frac{b^2\left(a+b\sqrt[3]{x}\right)^{16}}{133a^3x^{19/3}} + \frac{b^3\left(a+b\sqrt[3]{x}\right)^{16}}{798a^4x^6} - \frac{b^4\left(a+b\sqrt[3]{x}\right)^{16}}{6783a^5x^{17/3}} +$$

Mathematica [A] time = 0.02, size = 80, normalized size = 0.54

$$\frac{\left(a+b\sqrt[3]{x}\right)^{16}\left(-15504a^5+3876a^4b\sqrt[3]{x}-816a^3b^2x^{2/3}+136a^2b^3x-16ab^4x^{4/3}+b^5x^{5/3}\right)}{108528a^6x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^15/x^8,x]
```

```
[Out] ((a + b*x^(1/3))^16*(-15504*a^5 + 3876*a^4*b*x^(1/3) - 816*a^3*b^2*x^(2/3)
+ 136*a^2*b^3*x - 16*a*b^4*x^(4/3) + b^5*x^(5/3)))/(108528*a^6*x^7)
```

fricas [A] time = 0.72, size = 169, normalized size = 1.14

$$-\frac{54264\,b^{15}x^5+16460080\,a^3b^{12}x^4+135795660\,a^6b^9x^3+108636528\,a^9b^6x^2+8230040\,a^{12}b^3x+15504\,a^{15}+459}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15/x^8,x, algorithm="fricas")
```

```
[Out] -1/108528*(54264*b^15*x^5 + 16460080*a^3*b^12*x^4 + 135795660*a^6*b^9*x^3 +
 108636528*a^9*b^6*x^2 + 8230040*a^12*b^3*x + 15504*a^15 + 459*(1520*a*b^14
*x^4 + 96824*a^4*b^11*x^3 + 351120*a^7*b^8*x^2 + 133133*a^10*b^5*x + 3920*a
^13*b^2)*x^(2/3) + 1026*(4165*a^2*b^13*x^4 + 86632*a^5*b^10*x^3 + 145860*a^
8*b^7*x^2 + 25480*a^11*b^4*x + 238*a^14*b)*x^(1/3))/x^7
```

giac [A] time = 0.18, size = 167, normalized size = 1.13

$$-\frac{54264\,b^{15}x^5+697680\,ab^{14}x^{\frac{14}{3}}+4273290\,a^2b^{13}x^{\frac{13}{3}}+16460080\,a^3b^{12}x^4+44442216\,a^4b^{11}x^{\frac{11}{3}}+88884432\,a^5b^{10}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15/x^8,x, algorithm="giac")
```

```
[Out] -1/108528*(54264*b^15*x^5 + 697680*a*b^14*x^(14/3) + 4273290*a^2*b^13*x^(13
/3) + 16460080*a^3*b^12*x^4 + 44442216*a^4*b^11*x^(11/3) + 88884432*a^5*b^1
0*x^(10/3) + 135795660*a^6*b^9*x^3 + 161164080*a^7*b^8*x^(8/3) + 149652360*
a^8*b^7*x^(7/3) + 108636528*a^9*b^6*x^2 + 61108047*a^10*b^5*x^(5/3) + 26142
480*a^11*b^4*x^(4/3) + 8230040*a^12*b^3*x + 1799280*a^13*b^2*x^(2/3) + 2441
88*a^14*b*x^(1/3) + 15504*a^15)/x^7
```

maple [A] time = 0.01, size = 168, normalized size = 1.14

$$-\frac{b^{15}}{2x^2}-\frac{45a\,b^{14}}{7x^{\frac{7}{3}}}-\frac{315a^2b^{13}}{8x^{\frac{8}{3}}}-\frac{455a^3b^{12}}{3x^3}-\frac{819a^4b^{11}}{2x^{\frac{10}{3}}}-\frac{819a^5b^{10}}{x^{\frac{11}{3}}}-\frac{5005a^6b^9}{4x^4}-\frac{1485a^7b^8}{x^{\frac{13}{3}}}-\frac{19305a^8b^7}{14x^{\frac{14}{3}}}-\frac{1001a^9b^6}{x^5}-\frac{900}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^15/x^8,x)
```

```
[Out] -19305/14*a^8*b^7/x^(14/3)-9009/16*a^10*b^5/x^(16/3)-315/8*a^2*b^13/x^(8/3)
-4095/17*a^11*b^4/x^(17/3)-1485*a^7*b^8/x^(13/3)-455/6*a^12*b^3/x^6-9/4*a^1
4*b/x^(20/3)-455/3*a^3*b^12/x^3-45/7*a*b^14/x^(7/3)-1001*a^9*b^6/x^5-1/2*b^
15/x^2-5005/4*a^6*b^9/x^4-1/7*a^15/x^7-315/19*a^13*b^2/x^(19/3)-819/2*a^4*b
^11/x^(10/3)-819*a^5*b^10/x^(11/3)
```

maxima [A] time = 0.87, size = 167, normalized size = 1.13

$$-\frac{54264\,b^{15}x^5+697680\,ab^{14}x^{\frac{14}{3}}+4273290\,a^2b^{13}x^{\frac{13}{3}}+16460080\,a^3b^{12}x^4+44442216\,a^4b^{11}x^{\frac{11}{3}}+88884432\,a^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15/x^8,x, algorithm="maxima")
```

```
[Out] -1/108528*(54264*b^15*x^5 + 697680*a*b^14*x^(14/3) + 4273290*a^2*b^13*x^(13
/3) + 16460080*a^3*b^12*x^4 + 44442216*a^4*b^11*x^(11/3) + 88884432*a^5*b^1
0*x^(10/3) + 135795660*a^6*b^9*x^3 + 161164080*a^7*b^8*x^(8/3) + 149652360*
a^8*b^7*x^(7/3) + 108636528*a^9*b^6*x^2 + 61108047*a^10*b^5*x^(5/3) + 26142
480*a^11*b^4*x^(4/3) + 8230040*a^12*b^3*x + 1799280*a^13*b^2*x^(2/3) + 2441
88*a^14*b*x^(1/3) + 15504*a^15)/x^7
```

mupad [B] time = 1.34, size = 167, normalized size = 1.13

$$-\frac{\frac{a^{15}}{7}+\frac{b^{15}x^5}{2}+\frac{455\,a^{12}b^3x}{6}+\frac{9\,a^{14}b\,x^{1/3}}{4}+\frac{45\,a\,b^{14}x^{14/3}}{7}+1001\,a^9b^6x^2+\frac{5005\,a^6b^9x^3}{4}+\frac{455\,a^3b^{12}x^4}{3}+\frac{315\,a^{13}b^2x^{2/3}}{19}+\frac{4095}{}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))^15/x^8,x)
```

```
[Out] -(a^15/7 + (b^15*x^5)/2 + (455*a^12*b^3*x)/6 + (9*a^14*b*x^(1/3))/4 + (45*a
*b^14*x^(14/3))/7 + 1001*a^9*b^6*x^2 + (5005*a^6*b^9*x^3)/4 + (455*a^3*b^12
*x^4)/3 + (315*a^13*b^2*x^(2/3))/19 + (4095*a^11*b^4*x^(4/3))/17 + (9009*a^
10*b^5*x^(5/3))/16 + (19305*a^8*b^7*x^(7/3))/14 + 1485*a^7*b^8*x^(8/3) + 81
9*a^5*b^10*x^(10/3) + (819*a^4*b^11*x^(11/3))/2 + (315*a^2*b^13*x^(13/3))/8
)/x^7
```

sympy [A] time = 15.18, size = 216, normalized size = 1.46

$$-\frac{a^{15}}{7x^7}-\frac{9a^{14}b}{4x^{\frac{20}{3}}}-\frac{315a^{13}b^2}{19x^{\frac{19}{3}}}-\frac{455a^{12}b^3}{6x^6}-\frac{4095a^{11}b^4}{17x^{\frac{17}{3}}}-\frac{9009a^{10}b^5}{16x^{\frac{16}{3}}}-\frac{1001a^9b^6}{x^5}-\frac{19305a^8b^7}{14x^{\frac{14}{3}}}-\frac{1485a^7b^8}{x^{\frac{13}{3}}}-\frac{5005a^6b^9}{4x^4}-\frac{81}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**15/x**8,x)

[Out] -a**15/(7*x**7) - 9*a**14*b/(4*x**(20/3)) - 315*a**13*b**2/(19*x**(19/3)) -
 455*a**12*b**3/(6*x**6) - 4095*a**11*b**4/(17*x**(17/3)) - 9009*a**10*b**5
/(16*x**(16/3)) - 1001*a**9*b**6/x**5 - 19305*a**8*b**7/(14*x**(14/3)) - 14
85*a**7*b**8/x**(13/3) - 5005*a**6*b**9/(4*x**4) - 819*a**5*b**10/x**(11/3)
 - 819*a**4*b**11/(2*x**(10/3)) - 455*a**3*b**12/(3*x**3) - 315*a**2*b**13/
(8*x**(8/3)) - 45*a*b**14/(7*x**(7/3)) - b**15/(2*x**2)
```

3.2352 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^9}\,dx$

Optimal. Leaf size=224

$$-\frac{b^8\left(a+b\sqrt[3]{x}\right)^{16}}{3922512a^9x^{16/3}}+\frac{b^7\left(a+b\sqrt[3]{x}\right)^{16}}{245157a^8x^{17/3}}-\frac{b^6\left(a+b\sqrt[3]{x}\right)^{16}}{28842a^7x^6}+\frac{b^5\left(a+b\sqrt[3]{x}\right)^{16}}{4807a^6x^{19/3}}-\frac{b^4\left(a+b\sqrt[3]{x}\right)^{16}}{1012a^5x^{20/3}}+\frac{b^3\left(a+b\sqrt[3]{x}\right)^{16}}{253a^4x^7}-\frac{7b^2}{5}$$

```
[Out] -1/8*(a+b*x^(1/3))^16/a/x^8+1/23*b*(a+b*x^(1/3))^16/a^2/x^(23/3)-7/506*b^2*
(a+b*x^(1/3))^16/a^3/x^(22/3)+1/253*b^3*(a+b*x^(1/3))^16/a^4/x^7-1/1012*b^4
*(a+b*x^(1/3))^16/a^5/x^(20/3)+1/4807*b^5*(a+b*x^(1/3))^16/a^6/x^(19/3)-1/2
8842*b^6*(a+b*x^(1/3))^16/a^7/x^6+1/245157*b^7*(a+b*x^(1/3))^16/a^8/x^(17/3
)-1/3922512*b^8*(a+b*x^(1/3))^16/a^9/x^(16/3)
```

Rubi [A] time = 0.11, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {266, 45, 37}

$$-\frac{b^8\left(a+b\sqrt[3]{x}\right)^{16}}{3922512a^9x^{16/3}}+\frac{b^7\left(a+b\sqrt[3]{x}\right)^{16}}{245157a^8x^{17/3}}-\frac{b^6\left(a+b\sqrt[3]{x}\right)^{16}}{28842a^7x^6}+\frac{b^5\left(a+b\sqrt[3]{x}\right)^{16}}{4807a^6x^{19/3}}-\frac{b^4\left(a+b\sqrt[3]{x}\right)^{16}}{1012a^5x^{20/3}}+\frac{b^3\left(a+b\sqrt[3]{x}\right)^{16}}{253a^4x^7}-\frac{7b^2}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^15/x^9,x]
```

```
[Out] -(a + b*x^(1/3))^16/(8*a*x^8) + (b*(a + b*x^(1/3))^16)/(23*a^2*x^(23/3)) -
(7*b^2*(a + b*x^(1/3))^16)/(506*a^3*x^(22/3)) + (b^3*(a + b*x^(1/3))^16)/(2
53*a^4*x^7) - (b^4*(a + b*x^(1/3))^16)/(1012*a^5*x^(20/3)) + (b^5*(a + b*x^
(1/3))^16)/(4807*a^6*x^(19/3)) - (b^6*(a + b*x^(1/3))^16)/(28842*a^7*x^6) +
 (b^7*(a + b*x^(1/3))^16)/(245157*a^8*x^(17/3)) - (b^8*(a + b*x^(1/3))^16)/
(3922512*a^9*x^(16/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^9}\,dx = 3\,\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{25}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{8ax^8} - \frac{b\,\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{24}}\,dx, x, \sqrt[3]{x}\right)}{a}$$

$$= -\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{8ax^8} + \frac{b\left(a+b\sqrt[3]{x}\right)^{16}}{23a^2x^{23/3}} + \frac{\left(7b^2\right)\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{23}}\,dx, x, \sqrt[3]{x}\right)}{23a^2}$$

$$= -\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{8ax^8} + \frac{b\left(a+b\sqrt[3]{x}\right)^{16}}{23a^2x^{23/3}} - \frac{7b^2\left(a+b\sqrt[3]{x}\right)^{16}}{506a^3x^{22/3}} - \frac{\left(21b^3\right)\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{22}}\,dx, x, \sqrt[3]{x}\right)}{253a^3}$$

$$= -\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{8ax^8} + \frac{b\left(a+b\sqrt[3]{x}\right)^{16}}{23a^2x^{23/3}} - \frac{7b^2\left(a+b\sqrt[3]{x}\right)^{16}}{506a^3x^{22/3}} + \frac{b^3\left(a+b\sqrt[3]{x}\right)^{16}}{253a^4x^7} + \frac{\left(5b^4\right)\text{Subst}\left(\int \frac{(a+$$

$$= -\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{8ax^8} + \frac{b\left(a+b\sqrt[3]{x}\right)^{16}}{23a^2x^{23/3}} - \frac{7b^2\left(a+b\sqrt[3]{x}\right)^{16}}{506a^3x^{22/3}} + \frac{b^3\left(a+b\sqrt[3]{x}\right)^{16}}{253a^4x^7} - \frac{b^4\left(a+b\sqrt[3]{x}\right)^{16}}{1012a^5x^{20/3}} -$$

$$= -\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{8ax^8} + \frac{b\left(a+b\sqrt[3]{x}\right)^{16}}{23a^2x^{23/3}} - \frac{7b^2\left(a+b\sqrt[3]{x}\right)^{16}}{506a^3x^{22/3}} + \frac{b^3\left(a+b\sqrt[3]{x}\right)^{16}}{253a^4x^7} - \frac{b^4\left(a+b\sqrt[3]{x}\right)^{16}}{1012a^5x^{20/3}} +$$

$$= -\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{8ax^8} + \frac{b\left(a+b\sqrt[3]{x}\right)^{16}}{23a^2x^{23/3}} - \frac{7b^2\left(a+b\sqrt[3]{x}\right)^{16}}{506a^3x^{22/3}} + \frac{b^3\left(a+b\sqrt[3]{x}\right)^{16}}{253a^4x^7} - \frac{b^4\left(a+b\sqrt[3]{x}\right)^{16}}{1012a^5x^{20/3}} +$$

$$= -\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{8ax^8} + \frac{b\left(a+b\sqrt[3]{x}\right)^{16}}{23a^2x^{23/3}} - \frac{7b^2\left(a+b\sqrt[3]{x}\right)^{16}}{506a^3x^{22/3}} + \frac{b^3\left(a+b\sqrt[3]{x}\right)^{16}}{253a^4x^7} - \frac{b^4\left(a+b\sqrt[3]{x}\right)^{16}}{1012a^5x^{20/3}} +$$

Mathematica [A] time = 0.05, size = 213, normalized size = 0.95

$$-\frac{a^{15}}{8x^8} - \frac{45a^{14}b}{23x^{23/3}} - \frac{315a^{13}b^2}{22x^{22/3}} - \frac{65a^{12}b^3}{x^7} - \frac{819a^{11}b^4}{4x^{20/3}} - \frac{9009a^{10}b^5}{19x^{19/3}} - \frac{5005a^9b^6}{6x^6} - \frac{19305a^8b^7}{17x^{17/3}} - \frac{19305a^7b^8}{16x^{16/3}} - \frac{1001a^6b^9}{x^5} - \frac{1287}{2x^1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^15/x^9,x]
```

```
[Out] -1/8*a^15/x^8 - (45*a^14*b)/(23*x^(23/3)) - (315*a^13*b^2)/(22*x^(22/3)) -
(65*a^12*b^3)/x^7 - (819*a^11*b^4)/(4*x^(20/3)) - (9009*a^10*b^5)/(19*x^(19
/3)) - (5005*a^9*b^6)/(6*x^6) - (19305*a^8*b^7)/(17*x^(17/3)) - (19305*a^7*
b^8)/(16*x^(16/3)) - (1001*a^6*b^9)/x^5 - (1287*a^5*b^10)/(2*x^(14/3)) - (3
15*a^4*b^11)/x^(13/3) - (455*a^3*b^12)/(4*x^4) - (315*a^2*b^13)/(11*x^(11/3
)) - (9*a*b^14)/(2*x^(10/3)) - b^15/(3*x^3)
```

fricas [A] time = 0.83, size = 169, normalized size = 0.75

$$-\frac{1307504\,b^{15}x^5 + 446185740\,a^3b^{12}x^4 + 3926434512\,a^6b^9x^3 + 3272028760\,a^9b^6x^2 + 254963280\,a^{12}b^3x + 490314}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15/x^9,x, algorithm="fricas")
```

```
[Out] -1/3922512*(1307504*b^15*x^5 + 446185740*a^3*b^12*x^4 + 3926434512*a^6*b^9*
x^3 + 3272028760*a^9*b^6*x^2 + 254963280*a^12*b^3*x + 490314*a^15 + 10557*(
1672*a*b^14*x^4 + 117040*a^4*b^11*x^3 + 448305*a^7*b^8*x^2 + 176176*a^10*b^
```

```
5*x + 5320*a^13*b^2)*x^(2/3) + 2052*(54740*a^2*b^13*x^4 + 1230086*a^5*b^10*
x^3 + 2170740*a^8*b^7*x^2 + 391391*a^11*b^4*x + 3740*a^14*b)*x^(1/3))/x^8
```

giac [A] time = 0.18, size = 167, normalized size = 0.75

$$-\frac{1307504\,b^{15}x^5+17651304\,ab^{14}x^{\frac{14}{3}}+112326480\,a^2b^{13}x^{\frac{13}{3}}+446185740\,a^3b^{12}x^4+1235591280\,a^4b^{11}x^{\frac{11}{3}}+25}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15/x^9,x, algorithm="giac")
```

```
[Out] -1/3922512*(1307504*b^15*x^5 + 17651304*a*b^14*x^(14/3) + 112326480*a^2*b^1
3*x^(13/3) + 446185740*a^3*b^12*x^4 + 1235591280*a^4*b^11*x^(11/3) + 252413
6472*a^5*b^10*x^(10/3) + 3926434512*a^6*b^9*x^3 + 4732755885*a^7*b^8*x^(8/3
) + 4454358480*a^8*b^7*x^(7/3) + 3272028760*a^9*b^6*x^2 + 1859890032*a^10*b
^5*x^(5/3) + 803134332*a^11*b^4*x^(4/3) + 254963280*a^12*b^3*x + 56163240*a
^13*b^2*x^(2/3) + 7674480*a^14*b*x^(1/3) + 490314*a^15)/x^8
```

maple [A] time = 0.01, size = 168, normalized size = 0.75

$$-\frac{b^{15}}{3x^3}-\frac{9a\,b^{14}}{2x^{\frac{10}{3}}}-\frac{315a^2b^{13}}{11x^{\frac{11}{3}}}-\frac{455a^3b^{12}}{4x^4}-\frac{315a^4b^{11}}{x^{\frac{13}{3}}}-\frac{1287a^5b^{10}}{2x^{\frac{14}{3}}}-\frac{1001a^6b^9}{x^5}-\frac{19305a^7b^8}{16x^{\frac{16}{3}}}-\frac{19305a^8b^7}{17x^{\frac{17}{3}}}-\frac{5005a^9b^6}{6x^6}-\frac{90}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^15/x^9,x)
```

```
[Out] -1287/2*a^5*b^10/x^(14/3)-19305/16*a^7*b^8/x^(16/3)-315/22*a^13*b^2/x^(22/3
)-9009/19*a^10*b^5/x^(19/3)-5005/6*a^9*b^6/x^6-45/23*a^14*b/x^(23/3)-315*a^
4*b^11/x^(13/3)-819/4*a^11*b^4/x^(20/3)-1/8*a^15/x^8-1/3*b^15/x^3-19305/17*
a^8*b^7/x^(17/3)-65*a^12*b^3/x^7-455/4*a^3*b^12/x^4-9/2*a*b^14/x^(10/3)-315
/11*a^2*b^13/x^(11/3)-1001*a^6*b^9/x^5
```

maxima [A] time = 0.95, size = 167, normalized size = 0.75

$$-\frac{1307504\,b^{15}x^5+17651304\,ab^{14}x^{\frac{14}{3}}+112326480\,a^2b^{13}x^{\frac{13}{3}}+446185740\,a^3b^{12}x^4+1235591280\,a^4b^{11}x^{\frac{11}{3}}+25}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15/x^9,x, algorithm="maxima")
```

```
[Out] -1/3922512*(1307504*b^15*x^5 + 17651304*a*b^14*x^(14/3) + 112326480*a^2*b^1
3*x^(13/3) + 446185740*a^3*b^12*x^4 + 1235591280*a^4*b^11*x^(11/3) + 252413
6472*a^5*b^10*x^(10/3) + 3926434512*a^6*b^9*x^3 + 4732755885*a^7*b^8*x^(8/3
) + 4454358480*a^8*b^7*x^(7/3) + 3272028760*a^9*b^6*x^2 + 1859890032*a^10*b
^5*x^(5/3) + 803134332*a^11*b^4*x^(4/3) + 254963280*a^12*b^3*x + 56163240*a
^13*b^2*x^(2/3) + 7674480*a^14*b*x^(1/3) + 490314*a^15)/x^8
```

mupad [B] time = 0.20, size = 167, normalized size = 0.75

$$-\frac{\frac{a^{15}}{8}+\frac{b^{15}\,x^5}{3}+65\,a^{12}\,b^3\,x+\frac{45\,a^{14}\,b\,x^{1/3}}{23}+\frac{9\,a\,b^{14}\,x^{14/3}}{2}+\frac{5005\,a^9\,b^6\,x^2}{6}+1001\,a^6\,b^9\,x^3+\frac{455\,a^3\,b^{12}\,x^4}{4}+\frac{315\,a^{13}\,b^2\,x^{2/3}}{22}+\frac{81}{}}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))^15/x^9,x)
```

```
[Out] -(a^15/8 + (b^15*x^5)/3 + 65*a^12*b^3*x + (45*a^14*b*x^(1/3))/23 + (9*a*b^1
4*x^(14/3))/2 + (5005*a^9*b^6*x^2)/6 + 1001*a^6*b^9*x^3 + (455*a^3*b^12*x^4
```

```
)/4 + (315*a^13*b^2*x^(2/3))/22 + (819*a^11*b^4*x^(4/3))/4 + (9009*a^10*b^5
*x^(5/3))/19 + (19305*a^8*b^7*x^(7/3))/17 + (19305*a^7*b^8*x^(8/3))/16 + (1
287*a^5*b^10*x^(10/3))/2 + 315*a^4*b^11*x^(11/3) + (315*a^2*b^13*x^(13/3))/
11)/x^8
```

sympy [**A**] time = 21.80, size = 216, normalized size = 0.96

$$-\frac{a^{15}}{8x^8}-\frac{45a^{14}b}{23x^{\frac{23}{3}}}-\frac{315a^{13}b^2}{22x^{\frac{22}{3}}}-\frac{65a^{12}b^3}{x^7}-\frac{819a^{11}b^4}{4x^{\frac{20}{3}}}-\frac{9009a^{10}b^5}{19x^{\frac{19}{3}}}-\frac{5005a^9b^6}{6x^6}-\frac{19305a^8b^7}{17x^{\frac{17}{3}}}-\frac{19305a^7b^8}{16x^{\frac{16}{3}}}-\frac{1001a^6b^9}{x^5}-\frac{1287a}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**15/x**9,x)
```

```
[Out] -a**15/(8*x**8) - 45*a**14*b/(23*x**(23/3)) - 315*a**13*b**2/(22*x**(22/3))
 - 65*a**12*b**3/x**7 - 819*a**11*b**4/(4*x**(20/3)) - 9009*a**10*b**5/(19*
x**(19/3)) - 5005*a**9*b**6/(6*x**6) - 19305*a**8*b**7/(17*x**(17/3)) - 193
05*a**7*b**8/(16*x**(16/3)) - 1001*a**6*b**9/x**5 - 1287*a**5*b**10/(2*x**(
14/3)) - 315*a**4*b**11/x**(13/3) - 455*a**3*b**12/(4*x**4) - 315*a**2*b**1
3/(11*x**(11/3)) - 9*a*b**14/(2*x**(10/3)) - b**15/(3*x**3)
```

3.2353 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{10}}\,dx$

Optimal. Leaf size=215

$$-\frac{a^{15}}{9x^9}-\frac{45a^{14}b}{26x^{26/3}}-\frac{63a^{13}b^2}{5x^{25/3}}-\frac{455a^{12}b^3}{8x^8}-\frac{4095a^{11}b^4}{23x^{23/3}}-\frac{819a^{10}b^5}{2x^{22/3}}-\frac{715a^9b^6}{x^7}-\frac{3861a^8b^7}{4x^{20/3}}-\frac{19305a^7b^8}{19x^{19/3}}-\frac{5005a^6b^9}{6x^6}-\frac{9009}{17}$$

```
[Out] -1/9*a^15/x^9-45/26*a^14*b/x^(26/3)-63/5*a^13*b^2/x^(25/3)-455/8*a^12*b^3/x
^8-4095/23*a^11*b^4/x^(23/3)-819/2*a^10*b^5/x^(22/3)-715*a^9*b^6/x^7-3861/4
*a^8*b^7/x^(20/3)-19305/19*a^7*b^8/x^(19/3)-5005/6*a^6*b^9/x^6-9009/17*a^5*
b^10/x^(17/3)-4095/16*a^4*b^11/x^(16/3)-91*a^3*b^12/x^5-45/2*a^2*b^13/x^(14
/3)-45/13*a*b^14/x^(13/3)-1/4*b^15/x^4
```

Rubi [A] time = 0.11, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{63a^{13}b^2}{5x^{25/3}}-\frac{455a^{12}b^3}{8x^8}-\frac{4095a^{11}b^4}{23x^{23/3}}-\frac{819a^{10}b^5}{2x^{22/3}}-\frac{715a^9b^6}{x^7}-\frac{3861a^8b^7}{4x^{20/3}}-\frac{19305a^7b^8}{19x^{19/3}}-\frac{5005a^6b^9}{6x^6}-\frac{9009a^5b^{10}}{17x^{17/3}}-\frac{4095a^4}{16x^{16}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^15/x^10,x]
```

```
[Out] -a^15/(9*x^9) - (45*a^14*b)/(26*x^(26/3)) - (63*a^13*b^2)/(5*x^(25/3)) - (4
55*a^12*b^3)/(8*x^8) - (4095*a^11*b^4)/(23*x^(23/3)) - (819*a^10*b^5)/(2*x^
(22/3)) - (715*a^9*b^6)/x^7 - (3861*a^8*b^7)/(4*x^(20/3)) - (19305*a^7*b^8)
/(19*x^(19/3)) - (5005*a^6*b^9)/(6*x^6) - (9009*a^5*b^10)/(17*x^(17/3)) - (
4095*a^4*b^11)/(16*x^(16/3)) - (91*a^3*b^12)/x^5 - (45*a^2*b^13)/(2*x^(14/3
)) - (45*a*b^14)/(13*x^(13/3)) - b^15/(4*x^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{10}}\,dx &= 3\,\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{28}}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(\frac{a^{15}}{x^{28}}+\frac{15a^{14}b}{x^{27}}+\frac{105a^{13}b^2}{x^{26}}+\frac{455a^{12}b^3}{x^{25}}+\frac{1365a^{11}b^4}{x^{24}}+\frac{3003a^{10}b^5}{x^{23}}+\frac{5005a^9}{x^{22}}\right.\right.\\ &= -\frac{a^{15}}{9x^9}-\frac{45a^{14}b}{26x^{26/3}}-\frac{63a^{13}b^2}{5x^{25/3}}-\frac{455a^{12}b^3}{8x^8}-\frac{4095a^{11}b^4}{23x^{23/3}}-\frac{819a^{10}b^5}{2x^{22/3}}-\frac{715a^9b^6}{x^7}-\frac{3861a^8b^7}{4x^{20/3}}\end{aligned}$$

Mathematica [A] time = 0.11, size = 215, normalized size = 1.00

$$-\frac{a^{15}}{9x^9}-\frac{45a^{14}b}{26x^{26/3}}-\frac{63a^{13}b^2}{5x^{25/3}}-\frac{455a^{12}b^3}{8x^8}-\frac{4095a^{11}b^4}{23x^{23/3}}-\frac{819a^{10}b^5}{2x^{22/3}}-\frac{715a^9b^6}{x^7}-\frac{3861a^8b^7}{4x^{20/3}}-\frac{19305a^7b^8}{19x^{19/3}}-\frac{5005a^6b^9}{6x^6}-\frac{9009}{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^15/x^10,x]
```

```
[Out] -1/9*a^15/x^9 - (45*a^14*b)/(26*x^(26/3)) - (63*a^13*b^2)/(5*x^(25/3)) - (4
55*a^12*b^3)/(8*x^8) - (4095*a^11*b^4)/(23*x^(23/3)) - (819*a^10*b^5)/(2*x^
(22/3)) - (715*a^9*b^6)/x^7 - (3861*a^8*b^7)/(4*x^(20/3)) - (19305*a^7*b^8)
/(19*x^(19/3)) - (5005*a^6*b^9)/(6*x^6) - (9009*a^5*b^10)/(17*x^(17/3)) - (
4095*a^4*b^11)/(16*x^(16/3)) - (91*a^3*b^12)/x^5 - (45*a^2*b^13)/(2*x^(14/3
)) - (45*a*b^14)/(13*x^(13/3)) - b^15/(4*x^4)
```

fricas [A] time = 0.62, size = 169, normalized size = 0.79

$$-\frac{17383860\,b^{15}x^5 + 6327725040\,a^3b^{12}x^4 + 58004146200\,a^6b^9x^3 + 49717839600\,a^9b^6x^2 + 3954828150\,a^{12}b^3x + 7}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15/x^10,x, algorithm="fricas")
```

```
[Out] -1/69535440*(17383860*b^15*x^5 + 6327725040*a^3*b^12*x^4 + 58004146200*a^6*
b^9*x^3 + 49717839600*a^9*b^6*x^2 + 3954828150*a^12*b^3*x + 7726160*a^15 +
31671*(7600*a*b^14*x^4 + 561925*a^4*b^11*x^3 + 2230800*a^7*b^8*x^2 + 899080
*a^10*b^5*x + 27664*a^13*b^2)*x^(2/3) + 30780*(50830*a^2*b^13*x^4 + 1197196
*a^5*b^10*x^3 + 2180607*a^8*b^7*x^2 + 402220*a^11*b^4*x + 3910*a^14*b)*x^(1
/3))/x^9
```

giac [A] time = 0.18, size = 167, normalized size = 0.78

$$-\frac{17383860\,b^{15}x^5 + 240699600\,ab^{14}x^{\frac{14}{3}} + 1564547400\,a^2b^{13}x^{\frac{13}{3}} + 6327725040\,a^3b^{12}x^4 + 17796726675\,a^4b^{11}x^{\frac{11}{3}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15/x^10,x, algorithm="giac")
```

```
[Out] -1/69535440*(17383860*b^15*x^5 + 240699600*a*b^14*x^(14/3) + 1564547400*a^2
*b^13*x^(13/3) + 6327725040*a^3*b^12*x^4 + 17796726675*a^4*b^11*x^(11/3) +
36849692880*a^5*b^10*x^(10/3) + 58004146200*a^6*b^9*x^3 + 70651666800*a^7*b
^8*x^(8/3) + 67119083460*a^8*b^7*x^(7/3) + 49717839600*a^9*b^6*x^2 + 284747
62680*a^10*b^5*x^(5/3) + 12380331600*a^11*b^4*x^(4/3) + 3954828150*a^12*b^3
*x + 876146544*a^13*b^2*x^(2/3) + 120349800*a^14*b*x^(1/3) + 7726160*a^15)/
x^9
```

maple [A] time = 0.01, size = 168, normalized size = 0.78

$$-\frac{b^{15}}{4x^4} - \frac{45a\,b^{14}}{13x^{\frac{13}{3}}} - \frac{45a^2b^{13}}{2x^{\frac{14}{3}}} - \frac{91a^3b^{12}}{x^5} - \frac{4095a^4b^{11}}{16x^{\frac{16}{3}}} - \frac{9009a^5b^{10}}{17x^{\frac{17}{3}}} - \frac{5005a^6b^9}{6x^6} - \frac{19305a^7b^8}{19x^{\frac{19}{3}}} - \frac{3861a^8b^7}{4x^{\frac{20}{3}}} - \frac{715a^9b^6}{x^7} - \frac{819a^{10}b}{2x^{\frac{22}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^15/x^10,x)
```

```
[Out] -1/9*a^15/x^9-45/26*a^14*b/x^(26/3)-63/5*a^13*b^2/x^(25/3)-455/8*a^12*b^3/x
^8-4095/23*a^11*b^4/x^(23/3)-819/2*a^10*b^5/x^(22/3)-715*a^9*b^6/x^7-3861/4
*a^8*b^7/x^(20/3)-19305/19*a^7*b^8/x^(19/3)-5005/6*a^6*b^9/x^6-9009/17*a^5*
b^10/x^(17/3)-4095/16*a^4*b^11/x^(16/3)-91*a^3*b^12/x^5-45/2*a^2*b^13/x^(14
/3)-45/13*a*b^14/x^(13/3)-1/4*b^15/x^4
```

maxima [A] time = 0.79, size = 167, normalized size = 0.78

$$-\frac{17383860\,b^{15}x^5 + 240699600\,ab^{14}x^{\frac{14}{3}} + 1564547400\,a^2b^{13}x^{\frac{13}{3}} + 6327725040\,a^3b^{12}x^4 + 17796726675\,a^4b^{11}x^{\frac{11}{3}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15/x^10,x, algorithm="maxima")
```

```
[Out] -1/69535440*(17383860*b^15*x^5 + 240699600*a*b^14*x^(14/3) + 1564547400*a^2
*b^13*x^(13/3) + 6327725040*a^3*b^12*x^4 + 17796726675*a^4*b^11*x^(11/3) +
36849692880*a^5*b^10*x^(10/3) + 58004146200*a^6*b^9*x^3 + 70651666800*a^7*b
^8*x^(8/3) + 67119083460*a^8*b^7*x^(7/3) + 49717839600*a^9*b^6*x^2 + 284747
62680*a^10*b^5*x^(5/3) + 12380331600*a^11*b^4*x^(4/3) + 3954828150*a^12*b^3
*x + 876146544*a^13*b^2*x^(2/3) + 120349800*a^14*b*x^(1/3) + 7726160*a^15)/
x^9
```

mupad [B] time = 0.20, size = 167, normalized size = 0.78

$$-\frac{\frac{a^{15}}{9}+\frac{b^{15}x^5}{4}+\frac{455a^{12}b^3x}{8}+\frac{45a^{14}bx^{1/3}}{26}+\frac{45ab^{14}x^{14/3}}{13}+715a^9b^6x^2+\frac{5005a^6b^9x^3}{6}+91a^3b^{12}x^4+\frac{63a^{13}b^2x^{2/3}}{5}+\frac{4095}{}}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))^15/x^10,x)
```

```
[Out] -(a^15/9 + (b^15*x^5)/4 + (455*a^12*b^3*x)/8 + (45*a^14*b*x^(1/3))/26 + (45
*a*b^14*x^(14/3))/13 + 715*a^9*b^6*x^2 + (5005*a^6*b^9*x^3)/6 + 91*a^3*b^12
*x^4 + (63*a^13*b^2*x^(2/3))/5 + (4095*a^11*b^4*x^(4/3))/23 + (819*a^10*b^5
*x^(5/3))/2 + (3861*a^8*b^7*x^(7/3))/4 + (19305*a^7*b^8*x^(8/3))/19 + (9009
*a^5*b^10*x^(10/3))/17 + (4095*a^4*b^11*x^(11/3))/16 + (45*a^2*b^13*x^(13/3
))/2)/x^9
```

sympy [A] time = 30.92, size = 218, normalized size = 1.01

$$-\frac{a^{15}}{9x^9}-\frac{45a^{14}b}{26x^{\frac{26}{3}}}-\frac{63a^{13}b^2}{5x^{\frac{25}{3}}}-\frac{455a^{12}b^3}{8x^8}-\frac{4095a^{11}b^4}{23x^{\frac{23}{3}}}-\frac{819a^{10}b^5}{2x^{\frac{22}{3}}}-\frac{715a^9b^6}{x^7}-\frac{3861a^8b^7}{4x^{\frac{20}{3}}}-\frac{19305a^7b^8}{19x^{\frac{19}{3}}}-\frac{5005a^6b^9}{6x^6}-\frac{9009}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**15/x**10,x)
```

```
[Out] -a**15/(9*x**9) - 45*a**14*b/(26*x**(26/3)) - 63*a**13*b**2/(5*x**(25/3)) -
 455*a**12*b**3/(8*x**8) - 4095*a**11*b**4/(23*x**(23/3)) - 819*a**10*b**5/
(2*x**(22/3)) - 715*a**9*b**6/x**7 - 3861*a**8*b**7/(4*x**(20/3)) - 19305*a
**7*b**8/(19*x**(19/3)) - 5005*a**6*b**9/(6*x**6) - 9009*a**5*b**10/(17*x**
(17/3)) - 4095*a**4*b**11/(16*x**(16/3)) - 91*a**3*b**12/x**5 - 45*a**2*b**
13/(2*x**(14/3)) - 45*a*b**14/(13*x**(13/3)) - b**15/(4*x**4)
```

3.2354 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{11}}\,dx$

Optimal. Leaf size=217

$$-\frac{a^{15}}{10x^{10}}-\frac{45a^{14}b}{29x^{29/3}}-\frac{45a^{13}b^2}{4x^{28/3}}-\frac{455a^{12}b^3}{9x^9}-\frac{315a^{11}b^4}{2x^{26/3}}-\frac{9009a^{10}b^5}{25x^{25/3}}-\frac{5005a^9b^6}{8x^8}-\frac{19305a^8b^7}{23x^{23/3}}-\frac{1755a^7b^8}{2x^{22/3}}-\frac{715a^6b^9}{x^7}-\frac{9009a}{20x}$$

```
[Out] -1/10*a^15/x^10-45/29*a^14*b/x^(29/3)-45/4*a^13*b^2/x^(28/3)-455/9*a^12*b^3
/x^9-315/2*a^11*b^4/x^(26/3)-9009/25*a^10*b^5/x^(25/3)-5005/8*a^9*b^6/x^8-1
9305/23*a^8*b^7/x^(23/3)-1755/2*a^7*b^8/x^(22/3)-715*a^6*b^9/x^7-9009/20*a^
5*b^10/x^(20/3)-4095/19*a^4*b^11/x^(19/3)-455/6*a^3*b^12/x^6-315/17*a^2*b^1
3/x^(17/3)-45/16*a*b^14/x^(16/3)-1/5*b^15/x^5
```

Rubi [A] time = 0.11, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{45a^{13}b^2}{4x^{28/3}}-\frac{455a^{12}b^3}{9x^9}-\frac{315a^{11}b^4}{2x^{26/3}}-\frac{9009a^{10}b^5}{25x^{25/3}}-\frac{5005a^9b^6}{8x^8}-\frac{19305a^8b^7}{23x^{23/3}}-\frac{1755a^7b^8}{2x^{22/3}}-\frac{715a^6b^9}{x^7}-\frac{9009a^5b^{10}}{20x^{20/3}}-\frac{4095a^4b^{11}}{19x^{19/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^15/x^11,x]
```

```
[Out] -a^15/(10*x^10) - (45*a^14*b)/(29*x^(29/3)) - (45*a^13*b^2)/(4*x^(28/3)) -
(455*a^12*b^3)/(9*x^9) - (315*a^11*b^4)/(2*x^(26/3)) - (9009*a^10*b^5)/(25*
x^(25/3)) - (5005*a^9*b^6)/(8*x^8) - (19305*a^8*b^7)/(23*x^(23/3)) - (1755*
a^7*b^8)/(2*x^(22/3)) - (715*a^6*b^9)/x^7 - (9009*a^5*b^10)/(20*x^(20/3)) -
 (4095*a^4*b^11)/(19*x^(19/3)) - (455*a^3*b^12)/(6*x^6) - (315*a^2*b^13)/(1
7*x^(17/3)) - (45*a*b^14)/(16*x^(16/3)) - b^15/(5*x^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{11}}\,dx &= 3\,\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{31}}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(\frac{a^{15}}{x^{31}}+\frac{15a^{14}b}{x^{30}}+\frac{105a^{13}b^2}{x^{29}}+\frac{455a^{12}b^3}{x^{28}}+\frac{1365a^{11}b^4}{x^{27}}+\frac{3003a^{10}b^5}{x^{26}}+\frac{5005a^9b^6}{x^{25}}\right.\right.\\ &= -\frac{a^{15}}{10x^{10}}-\frac{45a^{14}b}{29x^{29/3}}-\frac{45a^{13}b^2}{4x^{28/3}}-\frac{455a^{12}b^3}{9x^9}-\frac{315a^{11}b^4}{2x^{26/3}}-\frac{9009a^{10}b^5}{25x^{25/3}}-\frac{5005a^9b^6}{8x^8}-\frac{19305a^8b}{23x^{23/3}}\end{aligned}$$

Mathematica [A] time = 0.10, size = 217, normalized size = 1.00

$$-\frac{a^{15}}{10x^{10}}-\frac{45a^{14}b}{29x^{29/3}}-\frac{45a^{13}b^2}{4x^{28/3}}-\frac{455a^{12}b^3}{9x^9}-\frac{315a^{11}b^4}{2x^{26/3}}-\frac{9009a^{10}b^5}{25x^{25/3}}-\frac{5005a^9b^6}{8x^8}-\frac{19305a^8b^7}{23x^{23/3}}-\frac{1755a^7b^8}{2x^{22/3}}-\frac{715a^6b^9}{x^7}-\frac{9009a}{20x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^15/x^11,x]
```

```
[Out] -1/10*a^15/x^10 - (45*a^14*b)/(29*x^(29/3)) - (45*a^13*b^2)/(4*x^(28/3)) -
(455*a^12*b^3)/(9*x^9) - (315*a^11*b^4)/(2*x^(26/3)) - (9009*a^10*b^5)/(25*
x^(25/3)) - (5005*a^9*b^6)/(8*x^8) - (19305*a^8*b^7)/(23*x^(23/3)) - (1755*
a^7*b^8)/(2*x^(22/3)) - (715*a^6*b^9)/x^7 - (9009*a^5*b^10)/(20*x^(20/3)) -
 (4095*a^4*b^11)/(19*x^(19/3)) - (455*a^3*b^12)/(6*x^6) - (315*a^2*b^13)/(1
7*x^(17/3)) - (45*a*b^14)/(16*x^(16/3)) - b^15/(5*x^5)
```

fricas [A] time = 0.79, size = 169, normalized size = 0.78

$$-\frac{155117520\,b^{15}x^5 + 58815393000\,a^3b^{12}x^4 + 554545134000\,a^6b^9x^3 + 485226992250\,a^9b^6x^2 + 39210262000\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15/x^11,x, algorithm="fricas")
```

```
[Out] -1/775587600*(155117520*b^15*x^5 + 58815393000*a^3*b^12*x^4 + 554545134000*
a^6*b^9*x^3 + 485226992250*a^9*b^6*x^2 + 39210262000*a^12*b^3*x + 77558760*
a^15 + 918459*(2375*a*b^14*x^4 + 182000*a^4*b^11*x^3 + 741000*a^7*b^8*x^2 +
 304304*a^10*b^5*x + 9500*a^13*b^2)*x^(2/3) + 30780*(466900*a^2*b^13*x^4 +
11350339*a^5*b^10*x^3 + 21149700*a^8*b^7*x^2 + 3968650*a^11*b^4*x + 39100*a
^14*b)*x^(1/3))/x^10
```

giac [A] time = 0.18, size = 167, normalized size = 0.77

$$-\frac{155117520\,b^{15}x^5 + 2181340125\,ab^{14}x^{\frac{14}{3}} + 14371182000\,a^2b^{13}x^{\frac{13}{3}} + 58815393000\,a^3b^{12}x^4 + 167159538000\,a^$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15/x^11,x, algorithm="giac")
```

```
[Out] -1/775587600*(155117520*b^15*x^5 + 2181340125*a*b^14*x^(14/3) + 14371182000
*a^2*b^13*x^(13/3) + 58815393000*a^3*b^12*x^4 + 167159538000*a^4*b^11*x^(11
/3) + 349363434420*a^5*b^10*x^(10/3) + 554545134000*a^6*b^9*x^3 + 680578119
000*a^7*b^8*x^(8/3) + 650987766000*a^8*b^7*x^(7/3) + 485226992250*a^9*b^6*x
^2 + 279490747536*a^10*b^5*x^(5/3) + 122155047000*a^11*b^4*x^(4/3) + 392102
62000*a^12*b^3*x + 8725360500*a^13*b^2*x^(2/3) + 1203498000*a^14*b*x^(1/3)
+ 77558760*a^15)/x^10
```

maple [A] time = 0.01, size = 168, normalized size = 0.77

$$-\frac{b^{15}}{5x^5} - \frac{45a\,b^{14}}{16x^{\frac{16}{3}}} - \frac{315a^2b^{13}}{17x^{\frac{17}{3}}} - \frac{455a^3b^{12}}{6x^6} - \frac{4095a^4b^{11}}{19x^{\frac{19}{3}}} - \frac{9009a^5b^{10}}{20x^{\frac{20}{3}}} - \frac{715a^6b^9}{x^7} - \frac{1755a^7b^8}{2x^{\frac{22}{3}}} - \frac{19305a^8b^7}{23x^{\frac{23}{3}}} - \frac{5005a^9b^6}{8x^8} - \frac{9}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^15/x^11,x)
```

```
[Out] -1/10*a^15/x^10-45/29*a^14*b/x^(29/3)-45/4*a^13*b^2/x^(28/3)-455/9*a^12*b^3
/x^9-315/2*a^11*b^4/x^(26/3)-9009/25*a^10*b^5/x^(25/3)-5005/8*a^9*b^6/x^8-1
9305/23*a^8*b^7/x^(23/3)-1755/2*a^7*b^8/x^(22/3)-715*a^6*b^9/x^7-9009/20*a^
5*b^10/x^(20/3)-4095/19*a^4*b^11/x^(19/3)-455/6*a^3*b^12/x^6-315/17*a^2*b^1
3/x^(17/3)-45/16*a*b^14/x^(16/3)-1/5*b^15/x^5
```

maxima [A] time = 0.95, size = 167, normalized size = 0.77

$$-\frac{155117520\,b^{15}x^5 + 2181340125\,ab^{14}x^{\frac{14}{3}} + 14371182000\,a^2b^{13}x^{\frac{13}{3}} + 58815393000\,a^3b^{12}x^4 + 167159538000\,a^$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15/x^11,x, algorithm="maxima")
```

```
[Out] -1/775587600*(155117520*b^15*x^5 + 2181340125*a*b^14*x^(14/3) + 14371182000
*a^2*b^13*x^(13/3) + 58815393000*a^3*b^12*x^4 + 167159538000*a^4*b^11*x^(11
/3) + 349363434420*a^5*b^10*x^(10/3) + 554545134000*a^6*b^9*x^3 + 680578119
000*a^7*b^8*x^(8/3) + 650987766000*a^8*b^7*x^(7/3) + 485226992250*a^9*b^6*x
^2 + 279490747536*a^10*b^5*x^(5/3) + 122155047000*a^11*b^4*x^(4/3) + 392102
62000*a^12*b^3*x + 8725360500*a^13*b^2*x^(2/3) + 1203498000*a^14*b*x^(1/3)
+ 77558760*a^15)/x^10
```

mupad [B] time = 0.19, size = 167, normalized size = 0.77

$$-\frac{\frac{a^{15}}{10}+\frac{b^{15}x^5}{5}+\frac{455a^{12}b^3x}{9}+\frac{45a^{14}bx^{1/3}}{29}+\frac{45ab^{14}x^{14/3}}{16}+\frac{5005a^9b^6x^2}{8}+715a^6b^9x^3+\frac{455a^3b^{12}x^4}{6}+\frac{45a^{13}b^2x^{2/3}}{4}+\frac{315a^{11}b^4}{2}}{x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))^15/x^11,x)
```

```
[Out] -(a^15/10 + (b^15*x^5)/5 + (455*a^12*b^3*x)/9 + (45*a^14*b*x^(1/3))/29 + (4
5*a*b^14*x^(14/3))/16 + (5005*a^9*b^6*x^2)/8 + 715*a^6*b^9*x^3 + (455*a^3*b
^12*x^4)/6 + (45*a^13*b^2*x^(2/3))/4 + (315*a^11*b^4*x^(4/3))/2 + (9009*a^1
0*b^5*x^(5/3))/25 + (19305*a^8*b^7*x^(7/3))/23 + (1755*a^7*b^8*x^(8/3))/2 +
 (9009*a^5*b^10*x^(10/3))/20 + (4095*a^4*b^11*x^(11/3))/19 + (315*a^2*b^13*
x^(13/3))/17)/x^10
```

sympy [A] time = 42.99, size = 219, normalized size = 1.01

$$-\frac{a^{15}}{10x^{10}}-\frac{45a^{14}b}{29x^{\frac{29}{3}}}-\frac{45a^{13}b^2}{4x^{\frac{28}{3}}}-\frac{455a^{12}b^3}{9x^9}-\frac{315a^{11}b^4}{2x^{\frac{26}{3}}}-\frac{9009a^{10}b^5}{25x^{\frac{25}{3}}}-\frac{5005a^9b^6}{8x^8}-\frac{19305a^8b^7}{23x^{\frac{23}{3}}}-\frac{1755a^7b^8}{2x^{\frac{22}{3}}}-\frac{715a^6b^9}{x^7}-\frac{9009a}{20x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**15/x**11,x)
```

```
[Out] -a**15/(10*x**10) - 45*a**14*b/(29*x**(29/3)) - 45*a**13*b**2/(4*x**(28/3))
 - 455*a**12*b**3/(9*x**9) - 315*a**11*b**4/(2*x**(26/3)) - 9009*a**10*b**5
/(25*x**(25/3)) - 5005*a**9*b**6/(8*x**8) - 19305*a**8*b**7/(23*x**(23/3))
- 1755*a**7*b**8/(2*x**(22/3)) - 715*a**6*b**9/x**7 - 9009*a**5*b**10/(20*x
**(20/3)) - 4095*a**4*b**11/(19*x**(19/3)) - 455*a**3*b**12/(6*x**6) - 315*
a**2*b**13/(17*x**(17/3)) - 45*a*b**14/(16*x**(16/3)) - b**15/(5*x**5)
```

3.2355 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{12}}\,dx$

Optimal. Leaf size=217

$$-\frac{a^{15}}{11x^{11}}-\frac{45a^{14}b}{32x^{32/3}}-\frac{315a^{13}b^2}{31x^{31/3}}-\frac{91a^{12}b^3}{2x^{10}}-\frac{4095a^{11}b^4}{29x^{29/3}}-\frac{1287a^{10}b^5}{4x^{28/3}}-\frac{5005a^9b^6}{9x^9}-\frac{1485a^8b^7}{2x^{26/3}}-\frac{3861a^7b^8}{5x^{25/3}}-\frac{5005a^6b^9}{8x^8}-\frac{9}{}$$

```
[Out] -1/11*a^15/x^11-45/32*a^14*b/x^(32/3)-315/31*a^13*b^2/x^(31/3)-91/2*a^12*b^
3/x^10-4095/29*a^11*b^4/x^(29/3)-1287/4*a^10*b^5/x^(28/3)-5005/9*a^9*b^6/x^
9-1485/2*a^8*b^7/x^(26/3)-3861/5*a^7*b^8/x^(25/3)-5005/8*a^6*b^9/x^8-9009/2
3*a^5*b^10/x^(23/3)-4095/22*a^4*b^11/x^(22/3)-65*a^3*b^12/x^7-63/4*a^2*b^13
/x^(20/3)-45/19*a*b^14/x^(19/3)-1/6*b^15/x^6
```

Rubi [A] time = 0.12, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{315a^{13}b^2}{31x^{31/3}}-\frac{91a^{12}b^3}{2x^{10}}-\frac{4095a^{11}b^4}{29x^{29/3}}-\frac{1287a^{10}b^5}{4x^{28/3}}-\frac{5005a^9b^6}{9x^9}-\frac{1485a^8b^7}{2x^{26/3}}-\frac{3861a^7b^8}{5x^{25/3}}-\frac{5005a^6b^9}{8x^8}-\frac{9009a^5b^{10}}{23x^{23/3}}-\frac{4095a}{22x^{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^15/x^12,x]
```

```
[Out] -a^15/(11*x^11) - (45*a^14*b)/(32*x^(32/3)) - (315*a^13*b^2)/(31*x^(31/3))
- (91*a^12*b^3)/(2*x^10) - (4095*a^11*b^4)/(29*x^(29/3)) - (1287*a^10*b^5)/
(4*x^(28/3)) - (5005*a^9*b^6)/(9*x^9) - (1485*a^8*b^7)/(2*x^(26/3)) - (3861
*a^7*b^8)/(5*x^(25/3)) - (5005*a^6*b^9)/(8*x^8) - (9009*a^5*b^10)/(23*x^(23
/3)) - (4095*a^4*b^11)/(22*x^(22/3)) - (65*a^3*b^12)/x^7 - (63*a^2*b^13)/(4
*x^(20/3)) - (45*a*b^14)/(19*x^(19/3)) - b^15/(6*x^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{12}}\,dx &= 3\,\text{Subst}\left(\int \frac{(a+bx)^{15}}{x^{34}}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(\frac{a^{15}}{x^{34}}+\frac{15a^{14}b}{x^{33}}+\frac{105a^{13}b^2}{x^{32}}+\frac{455a^{12}b^3}{x^{31}}+\frac{1365a^{11}b^4}{x^{30}}+\frac{3003a^{10}b^5}{x^{29}}+\frac{5005a^{9}}{x^{28}}\right.\right.\\ &= -\frac{a^{15}}{11x^{11}}-\frac{45a^{14}b}{32x^{32/3}}-\frac{315a^{13}b^2}{31x^{31/3}}-\frac{91a^{12}b^3}{2x^{10}}-\frac{4095a^{11}b^4}{29x^{29/3}}-\frac{1287a^{10}b^5}{4x^{28/3}}-\frac{5005a^9b^6}{9x^9}-\frac{1485a}{2x^{2}}\end{aligned}$$

Mathematica [A] time = 0.10, size = 217, normalized size = 1.00

$$-\frac{a^{15}}{11x^{11}}-\frac{45a^{14}b}{32x^{32/3}}-\frac{315a^{13}b^2}{31x^{31/3}}-\frac{91a^{12}b^3}{2x^{10}}-\frac{4095a^{11}b^4}{29x^{29/3}}-\frac{1287a^{10}b^5}{4x^{28/3}}-\frac{5005a^9b^6}{9x^9}-\frac{1485a^8b^7}{2x^{26/3}}-\frac{3861a^7b^8}{5x^{25/3}}-\frac{5005a^6b^9}{8x^8}-\frac{9}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^15/x^12,x]
```

```
[Out] -1/11*a^15/x^11 - (45*a^14*b)/(32*x^(32/3)) - (315*a^13*b^2)/(31*x^(31/3))
- (91*a^12*b^3)/(2*x^10) - (4095*a^11*b^4)/(29*x^(29/3)) - (1287*a^10*b^5)/
(4*x^(28/3)) - (5005*a^9*b^6)/(9*x^9) - (1485*a^8*b^7)/(2*x^(26/3)) - (3861
*a^7*b^8)/(5*x^(25/3)) - (5005*a^6*b^9)/(8*x^8) - (9009*a^5*b^10)/(23*x^(23
/3)) - (4095*a^4*b^11)/(22*x^(22/3)) - (65*a^3*b^12)/x^7 - (63*a^2*b^13)/(4
*x^(20/3)) - (45*a*b^14)/(19*x^(19/3)) - b^15/(6*x^6)
```

fricas [A] time = 0.86, size = 169, normalized size = 0.78

$$-\frac{1037158320\,b^{15}x^5 + 404491744800\,a^3b^{12}x^4 + 3893233043700\,a^6b^9x^3 + 3460651594400\,a^9b^6x^2 + 2831442213}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15/x^12,x, algorithm="fricas")
```

```
[Out] -1/6222949920*(1037158320*b^15*x^5 + 404491744800*a^3*b^12*x^4 + 3893233043
700*a^6*b^9*x^3 + 3460651594400*a^9*b^6*x^2 + 283144221360*a^12*b^3*x + 565
722720*a^15 + 432216*(34100*a*b^14*x^4 + 2679950*a^4*b^11*x^3 + 11117964*a^
7*b^8*x^2 + 4632485*a^10*b^5*x + 146300*a^13*b^2)*x^(2/3) + 2623995*(37352*
a^2*b^13*x^4 + 928928*a^5*b^10*x^3 + 1760880*a^8*b^7*x^2 + 334880*a^11*b^4*
x + 3335*a^14*b)*x^(1/3))/x^11
```

giac [A] time = 0.20, size = 167, normalized size = 0.77

$$-\frac{1037158320\,b^{15}x^5 + 14738565600\,ab^{14}x^{\frac{14}{3}} + 98011461240\,a^2b^{13}x^{\frac{13}{3}} + 404491744800\,a^3b^{12}x^4 + 1158317269200}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15/x^12,x, algorithm="giac")
```

```
[Out] -1/6222949920*(1037158320*b^15*x^5 + 14738565600*a*b^14*x^(14/3) + 98011461
240*a^2*b^13*x^(13/3) + 404491744800*a^3*b^12*x^4 + 1158317269200*a^4*b^11*
x^(11/3) + 2437502427360*a^5*b^10*x^(10/3) + 3893233043700*a^6*b^9*x^3 + 48
05361928224*a^7*b^8*x^(8/3) + 4620540315600*a^8*b^7*x^(7/3) + 3460651594400
*a^9*b^6*x^2 + 2002234136760*a^10*b^5*x^(5/3) + 878723445600*a^11*b^4*x^(4/
3) + 283144221360*a^12*b^3*x + 63233200800*a^13*b^2*x^(2/3) + 8751023325*a^
14*b*x^(1/3) + 565722720*a^15)/x^11
```

maple [A] time = 0.01, size = 168, normalized size = 0.77

$$-\frac{b^{15}}{6x^6} - \frac{45a\,b^{14}}{19x^{\frac{19}{3}}} - \frac{63a^2b^{13}}{4x^{\frac{20}{3}}} - \frac{65a^3b^{12}}{x^7} - \frac{4095a^4b^{11}}{22x^{\frac{22}{3}}} - \frac{9009a^5b^{10}}{23x^{\frac{23}{3}}} - \frac{5005a^6b^9}{8x^8} - \frac{3861a^7b^8}{5x^{\frac{25}{3}}} - \frac{1485a^8b^7}{2x^{\frac{26}{3}}} - \frac{5005a^9b^6}{9x^9} - \frac{1287a^{1}}{4x^{\frac{28}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/3))^15/x^12,x)
```

```
[Out] -1/11*a^15/x^11-45/32*a^14*b/x^(32/3)-315/31*a^13*b^2/x^(31/3)-91/2*a^12*b^
3/x^10-4095/29*a^11*b^4/x^(29/3)-1287/4*a^10*b^5/x^(28/3)-5005/9*a^9*b^6/x^
9-1485/2*a^8*b^7/x^(26/3)-3861/5*a^7*b^8/x^(25/3)-5005/8*a^6*b^9/x^8-9009/2
3*a^5*b^10/x^(23/3)-4095/22*a^4*b^11/x^(22/3)-65*a^3*b^12/x^7-63/4*a^2*b^13
/x^(20/3)-45/19*a*b^14/x^(19/3)-1/6*b^15/x^6
```

maxima [A] time = 0.88, size = 167, normalized size = 0.77

$$-\frac{1037158320\,b^{15}x^5 + 14738565600\,ab^{14}x^{\frac{14}{3}} + 98011461240\,a^2b^{13}x^{\frac{13}{3}} + 404491744800\,a^3b^{12}x^4 + 1158317269200}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/3))^15/x^12,x, algorithm="maxima")
```

```
[Out] -1/6222949920*(1037158320*b^15*x^5 + 14738565600*a*b^14*x^(14/3) + 98011461
240*a^2*b^13*x^(13/3) + 404491744800*a^3*b^12*x^4 + 1158317269200*a^4*b^11*
x^(11/3) + 2437502427360*a^5*b^10*x^(10/3) + 3893233043700*a^6*b^9*x^3 + 48
05361928224*a^7*b^8*x^(8/3) + 4620540315600*a^8*b^7*x^(7/3) + 3460651594400
*a^9*b^6*x^2 + 2002234136760*a^10*b^5*x^(5/3) + 878723445600*a^11*b^4*x^(4/
3) + 283144221360*a^12*b^3*x + 63233200800*a^13*b^2*x^(2/3) + 8751023325*a^
14*b*x^(1/3) + 565722720*a^15)/x^11
```

mupad [B] time = 1.28, size = 167, normalized size = 0.77

$$-\frac{\frac{a^{15}}{11}+\frac{b^{15}x^5}{6}+\frac{91a^{12}b^3x}{2}+\frac{45a^{14}bx^{1/3}}{32}+\frac{45ab^{14}x^{14/3}}{19}+\frac{5005a^9b^6x^2}{9}+\frac{5005a^6b^9x^3}{8}+65a^3b^{12}x^4+\frac{315a^{13}b^2x^{2/3}}{31}+\frac{4095a}{}}{x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/3))^15/x^12,x)
```

```
[Out] -(a^15/11 + (b^15*x^5)/6 + (91*a^12*b^3*x)/2 + (45*a^14*b*x^(1/3))/32 + (45
*a*b^14*x^(14/3))/19 + (5005*a^9*b^6*x^2)/9 + (5005*a^6*b^9*x^3)/8 + 65*a^3
*b^12*x^4 + (315*a^13*b^2*x^(2/3))/31 + (4095*a^11*b^4*x^(4/3))/29 + (1287*
a^10*b^5*x^(5/3))/4 + (1485*a^8*b^7*x^(7/3))/2 + (3861*a^7*b^8*x^(8/3))/5 +
 (9009*a^5*b^10*x^(10/3))/23 + (4095*a^4*b^11*x^(11/3))/22 + (63*a^2*b^13*x
^(13/3))/4)/x^11
```

sympy [A] time = 57.16, size = 219, normalized size = 1.01

$$-\frac{a^{15}}{11x^{11}}-\frac{45a^{14}b}{32x^{\frac{32}{3}}}-\frac{315a^{13}b^2}{31x^{\frac{31}{3}}}-\frac{91a^{12}b^3}{2x^{10}}-\frac{4095a^{11}b^4}{29x^{\frac{29}{3}}}-\frac{1287a^{10}b^5}{4x^{\frac{28}{3}}}-\frac{5005a^9b^6}{9x^9}-\frac{1485a^8b^7}{2x^{\frac{26}{3}}}-\frac{3861a^7b^8}{5x^{\frac{25}{3}}}-\frac{5005a^6b^9}{8x^8}-\frac{9}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/3))**15/x**12,x)
```

```
[Out] -a**15/(11*x**11) - 45*a**14*b/(32*x**(32/3)) - 315*a**13*b**2/(31*x**(31/3
)) - 91*a**12*b**3/(2*x**10) - 4095*a**11*b**4/(29*x**(29/3)) - 1287*a**10*
b**5/(4*x**(28/3)) - 5005*a**9*b**6/(9*x**9) - 1485*a**8*b**7/(2*x**(26/3))
 - 3861*a**7*b**8/(5*x**(25/3)) - 5005*a**6*b**9/(8*x**8) - 9009*a**5*b**10
/(23*x**(23/3)) - 4095*a**4*b**11/(22*x**(22/3)) - 65*a**3*b**12/x**7 - 63*
a**2*b**13/(4*x**(20/3)) - 45*a*b**14/(19*x**(19/3)) - b**15/(6*x**6)
```

3.2356 $\int \frac{x^3}{a+b\sqrt[3]{x}}\,dx$

Optimal. Leaf size=166

$$-\frac{3a^{11}\log\left(a+b\sqrt[3]{x}\right)}{b^{12}}+\frac{3a^{10}\sqrt[3]{x}}{b^{11}}-\frac{3a^9x^{2/3}}{2b^{10}}+\frac{a^8x}{b^9}-\frac{3a^7x^{4/3}}{4b^8}+\frac{3a^6x^{5/3}}{5b^7}-\frac{a^5x^2}{2b^6}+\frac{3a^4x^{7/3}}{7b^5}-\frac{3a^3x^{8/3}}{8b^4}+\frac{a^2x^3}{3b^3}-\frac{3ax^{10/3}}{10b^2}+\frac{3x^{11/3}}{11b}$$

```
[Out] 3*a^10*x^(1/3)/b^11-3/2*a^9*x^(2/3)/b^10+a^8*x/b^9-3/4*a^7*x^(4/3)/b^8+3/5*
a^6*x^(5/3)/b^7-1/2*a^5*x^2/b^6+3/7*a^4*x^(7/3)/b^5-3/8*a^3*x^(8/3)/b^4+1/3
*a^2*x^3/b^3-3/10*a*x^(10/3)/b^2+3/11*x^(11/3)/b-3*a^11*ln(a+b*x^(1/3))/b^1
2
```

Rubi [A] time = 0.11, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{3a^9x^{2/3}}{2b^{10}}-\frac{3a^7x^{4/3}}{4b^8}+\frac{3a^6x^{5/3}}{5b^7}-\frac{a^5x^2}{2b^6}+\frac{3a^4x^{7/3}}{7b^5}-\frac{3a^3x^{8/3}}{8b^4}+\frac{a^2x^3}{3b^3}+\frac{3a^{10}\sqrt[3]{x}}{b^{11}}+\frac{a^8x}{b^9}-\frac{3a^{11}\log\left(a+b\sqrt[3]{x}\right)}{b^{12}}-\frac{3ax^{10/3}}{10b^2}+\frac{3x^{11/3}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^(1/3)),x]
```

```
[Out] (3*a^10*x^(1/3))/b^11 - (3*a^9*x^(2/3))/(2*b^10) + (a^8*x)/b^9 - (3*a^7*x^(
4/3))/(4*b^8) + (3*a^6*x^(5/3))/(5*b^7) - (a^5*x^2)/(2*b^6) + (3*a^4*x^(7/3
))/(7*b^5) - (3*a^3*x^(8/3))/(8*b^4) + (a^2*x^3)/(3*b^3) - (3*a*x^(10/3))/(
10*b^2) + (3*x^(11/3))/(11*b) - (3*a^11*Log[a + b*x^(1/3)])/b^12
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{a+b\sqrt[3]{x}}\,dx &= 3\,\text{Subst}\left(\int \frac{x^{11}}{a+bx}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(\frac{a^{10}}{b^{11}}-\frac{a^9x}{b^{10}}+\frac{a^8x^2}{b^9}-\frac{a^7x^3}{b^8}+\frac{a^6x^4}{b^7}-\frac{a^5x^5}{b^6}+\frac{a^4x^6}{b^5}-\frac{a^3x^7}{b^4}+\frac{a^2x^8}{b^3}-\frac{ax^9}{b^2}+\frac{x^{10}}{b}-\frac{}{b^1}\right.\right.\\ &= \frac{3a^{10}\sqrt[3]{x}}{b^{11}}-\frac{3a^9x^{2/3}}{2b^{10}}+\frac{a^8x}{b^9}-\frac{3a^7x^{4/3}}{4b^8}+\frac{3a^6x^{5/3}}{5b^7}-\frac{a^5x^2}{2b^6}+\frac{3a^4x^{7/3}}{7b^5}-\frac{3a^3x^{8/3}}{8b^4}+\frac{a^2x^3}{3b^3}-\frac{3ax^{10/3}}{10b^2}+\end{aligned}$$

Mathematica [A] time = 0.14, size = 166, normalized size = 1.00

$$-\frac{3a^{11}\log\left(a+b\sqrt[3]{x}\right)}{b^{12}}+\frac{3a^{10}\sqrt[3]{x}}{b^{11}}-\frac{3a^9x^{2/3}}{2b^{10}}+\frac{a^8x}{b^9}-\frac{3a^7x^{4/3}}{4b^8}+\frac{3a^6x^{5/3}}{5b^7}-\frac{a^5x^2}{2b^6}+\frac{3a^4x^{7/3}}{7b^5}-\frac{3a^3x^{8/3}}{8b^4}+\frac{a^2x^3}{3b^3}-\frac{3ax^{10/3}}{10b^2}+\frac{3x^{11/3}}{11b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^(1/3)),x]
```

```
[Out] (3*a^10*x^(1/3))/b^11 - (3*a^9*x^(2/3))/(2*b^10) + (a^8*x)/b^9 - (3*a^7*x^(
4/3))/(4*b^8) + (3*a^6*x^(5/3))/(5*b^7) - (a^5*x^2)/(2*b^6) + (3*a^4*x^(7/3
))/(7*b^5) - (3*a^3*x^(8/3))/(8*b^4) + (a^2*x^3)/(3*b^3) - (3*a*x^(10/3))/(
10*b^2) + (3*x^(11/3))/(11*b) - (3*a^11*Log[a + b*x^(1/3)])/b^12
```

fricas [A] time = 0.82, size = 133, normalized size = 0.80

$$\frac{3080\,a^2b^9x^3 - 4620\,a^5b^6x^2 + 9240\,a^8b^3x - 27720\,a^{11}\log\left(bx^{\frac{1}{3}} + a\right) + 63\left(40\,b^{11}x^3 - 55\,a^3b^8x^2 + 88\,a^6b^5x - 22\right.}{9240\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^(1/3)),x, algorithm="fricas")
```

```
[Out] 1/9240*(3080*a^2*b^9*x^3 - 4620*a^5*b^6*x^2 + 9240*a^8*b^3*x - 27720*a^11*l
og(b*x^(1/3) + a) + 63*(40*b^11*x^3 - 55*a^3*b^8*x^2 + 88*a^6*b^5*x - 220*a
^9*b^2)*x^(2/3) - 198*(14*a*b^10*x^3 - 20*a^4*b^7*x^2 + 35*a^7*b^4*x - 140*
a^10*b)*x^(1/3))/b^12
```

giac [A] time = 0.19, size = 133, normalized size = 0.80

$$-\frac{3\,a^{11}\log\left(\left|bx^{\frac{1}{3}} + a\right|\right)}{b^{12}} + \frac{2520\,b^{10}x^{\frac{11}{3}} - 2772\,ab^9x^{\frac{10}{3}} + 3080\,a^2b^8x^3 - 3465\,a^3b^7x^{\frac{8}{3}} + 3960\,a^4b^6x^{\frac{7}{3}} - 4620\,a^5b^5x^2}{9240\,b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^(1/3)),x, algorithm="giac")
```

```
[Out] -3*a^11*log(abs(b*x^(1/3) + a))/b^12 + 1/9240*(2520*b^10*x^(11/3) - 2772*a*
b^9*x^(10/3) + 3080*a^2*b^8*x^3 - 3465*a^3*b^7*x^(8/3) + 3960*a^4*b^6*x^(7/
3) - 4620*a^5*b^5*x^2 + 5544*a^6*b^4*x^(5/3) - 6930*a^7*b^3*x^(4/3) + 9240*
a^8*b^2*x - 13860*a^9*b*x^(2/3) + 27720*a^10*x^(1/3))/b^11
```

maple [A] time = 0.00, size = 131, normalized size = 0.79

$$\frac{3x^{\frac{11}{3}}}{11b} - \frac{3a\,x^{\frac{10}{3}}}{10b^2} + \frac{a^2x^3}{3b^3} - \frac{3a^3x^{\frac{8}{3}}}{8b^4} + \frac{3a^4x^{\frac{7}{3}}}{7b^5} - \frac{a^5x^2}{2b^6} + \frac{3a^6x^{\frac{5}{3}}}{5b^7} - \frac{3a^7x^{\frac{4}{3}}}{4b^8} - \frac{3a^{11}\ln\left(b\,x^{\frac{1}{3}} + a\right)}{b^{12}} + \frac{a^8x}{b^9} - \frac{3a^9x^{\frac{2}{3}}}{2b^{10}} + \frac{3a^{10}x^{\frac{1}{3}}}{b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*x^(1/3)),x)
```

```
[Out] 3*a^10*x^(1/3)/b^11-3/2*a^9*x^(2/3)/b^10+a^8*x/b^9-3/4*a^7*x^(4/3)/b^8+3/5*
a^6*x^(5/3)/b^7-1/2*a^5*x^2/b^6+3/7*a^4*x^(7/3)/b^5-3/8*a^3*x^(8/3)/b^4+1/3
*a^2*x^3/b^3-3/10*a*x^(10/3)/b^2+3/11*x^(11/3)/b-3*a^11*ln(a+b*x^(1/3))/b^1
2
```

maxima [A] time = 0.92, size = 197, normalized size = 1.19

$$-\frac{3\,a^{11}\log\left(bx^{\frac{1}{3}} + a\right)}{b^{12}} + \frac{3\left(bx^{\frac{1}{3}} + a\right)^{11}}{11\,b^{12}} - \frac{33\left(bx^{\frac{1}{3}} + a\right)^{10}a}{10\,b^{12}} + \frac{55\left(bx^{\frac{1}{3}} + a\right)^{9}a^2}{3\,b^{12}} - \frac{495\left(bx^{\frac{1}{3}} + a\right)^{8}a^3}{8\,b^{12}} + \frac{990\left(bx^{\frac{1}{3}} + a\right)^{7}a}{7\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^(1/3)),x, algorithm="maxima")
```

```
[Out] -3*a^11*log(b*x^(1/3) + a)/b^12 + 3/11*(b*x^(1/3) + a)^11/b^12 - 33/10*(b*x
^(1/3) + a)^10*a/b^12 + 55/3*(b*x^(1/3) + a)^9*a^2/b^12 - 495/8*(b*x^(1/3)
```

```
+ a)^8*a^3/b^12 + 990/7*(b*x^(1/3) + a)^7*a^4/b^12 - 231*(b*x^(1/3) + a)^6*
a^5/b^12 + 1386/5*(b*x^(1/3) + a)^5*a^6/b^12 - 495/2*(b*x^(1/3) + a)^4*a^7/
b^12 + 165*(b*x^(1/3) + a)^3*a^8/b^12 - 165/2*(b*x^(1/3) + a)^2*a^9/b^12 +
33*(b*x^(1/3) + a)*a^10/b^12
```

mupad [**B**] time = 0.07, size = 130, normalized size = 0.78

$$\frac{3x^{11/3}}{11b}-\frac{3ax^{10/3}}{10b^2}+\frac{a^8x}{b^9}-\frac{3a^{11}\ln\left(a+bx^{1/3}\right)}{b^{12}}+\frac{a^2x^3}{3b^3}-\frac{a^5x^2}{2b^6}-\frac{3a^3x^{8/3}}{8b^4}+\frac{3a^4x^{7/3}}{7b^5}+\frac{3a^6x^{5/3}}{5b^7}-\frac{3a^7x^{4/3}}{4b^8}-\frac{3a^9x^{2/3}}{2b^{10}}+\frac{3a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^(1/3)),x)
```

```
[Out] (3*x^(11/3))/(11*b) - (3*a*x^(10/3))/(10*b^2) + (a^8*x)/b^9 - (3*a^11*log(a
 + b*x^(1/3)))/b^12 + (a^2*x^3)/(3*b^3) - (a^5*x^2)/(2*b^6) - (3*a^3*x^(8/3
))/(8*b^4) + (3*a^4*x^(7/3))/(7*b^5) + (3*a^6*x^(5/3))/(5*b^7) - (3*a^7*x^(
4/3))/(4*b^8) - (3*a^9*x^(2/3))/(2*b^10) + (3*a^10*x^(1/3))/b^11
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*x**(1/3)),x)
```

```
[Out] Timed out
```

3.2357 $\int \frac{x^2}{a+b\sqrt[3]{x}}\,dx$

Optimal. Leaf size=124

$$\frac{3a^8\log\left(a+b\sqrt[3]{x}\right)}{b^9}-\frac{3a^7\sqrt[3]{x}}{b^8}+\frac{3a^6x^{2/3}}{2b^7}-\frac{a^5x}{b^6}+\frac{3a^4x^{4/3}}{4b^5}-\frac{3a^3x^{5/3}}{5b^4}+\frac{a^2x^2}{2b^3}-\frac{3ax^{7/3}}{7b^2}+\frac{3x^{8/3}}{8b}$$

```
[Out] -3*a^7*x^(1/3)/b^8+3/2*a^6*x^(2/3)/b^7-a^5*x/b^6+3/4*a^4*x^(4/3)/b^5-3/5*a^
3*x^(5/3)/b^4+1/2*a^2*x^2/b^3-3/7*a*x^(7/3)/b^2+3/8*x^(8/3)/b+3*a^8*ln(a+b*
x^(1/3))/b^9
```

Rubi [A] time = 0.07, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{3a^6x^{2/3}}{2b^7}+\frac{3a^4x^{4/3}}{4b^5}-\frac{3a^3x^{5/3}}{5b^4}+\frac{a^2x^2}{2b^3}-\frac{3a^7\sqrt[3]{x}}{b^8}-\frac{a^5x}{b^6}+\frac{3a^8\log\left(a+b\sqrt[3]{x}\right)}{b^9}-\frac{3ax^{7/3}}{7b^2}+\frac{3x^{8/3}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^(1/3)),x]
```

```
[Out] (-3*a^7*x^(1/3))/b^8 + (3*a^6*x^(2/3))/(2*b^7) - (a^5*x)/b^6 + (3*a^4*x^(4/
3))/(4*b^5) - (3*a^3*x^(5/3))/(5*b^4) + (a^2*x^2)/(2*b^3) - (3*a*x^(7/3))/(
7*b^2) + (3*x^(8/3))/(8*b) + (3*a^8*Log[a + b*x^(1/3)])/b^9
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{a+b\sqrt[3]{x}}\,dx &= 3\,\text{Subst}\left(\int \frac{x^8}{a+bx}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int\left(-\frac{a^7}{b^8}+\frac{a^6x}{b^7}-\frac{a^5x^2}{b^6}+\frac{a^4x^3}{b^5}-\frac{a^3x^4}{b^4}+\frac{a^2x^5}{b^3}-\frac{ax^6}{b^2}+\frac{x^7}{b}+\frac{a^8}{b^8(a+bx)}\right)dx, x, \sqrt[3]{x}\right.\\ &= -\frac{3a^7\sqrt[3]{x}}{b^8}+\frac{3a^6x^{2/3}}{2b^7}-\frac{a^5x}{b^6}+\frac{3a^4x^{4/3}}{4b^5}-\frac{3a^3x^{5/3}}{5b^4}+\frac{a^2x^2}{2b^3}-\frac{3ax^{7/3}}{7b^2}+\frac{3x^{8/3}}{8b}+\frac{3a^8\log\left(a+b\sqrt[3]{x}\right.}{b^9}\end{aligned}$$

Mathematica [A] time = 0.08, size = 124, normalized size = 1.00

$$\frac{3a^8\log\left(a+b\sqrt[3]{x}\right)}{b^9}-\frac{3a^7\sqrt[3]{x}}{b^8}+\frac{3a^6x^{2/3}}{2b^7}-\frac{a^5x}{b^6}+\frac{3a^4x^{4/3}}{4b^5}-\frac{3a^3x^{5/3}}{5b^4}+\frac{a^2x^2}{2b^3}-\frac{3ax^{7/3}}{7b^2}+\frac{3x^{8/3}}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^(1/3)),x]
```

```
[Out] (-3*a^7*x^(1/3))/b^8 + (3*a^6*x^(2/3))/(2*b^7) - (a^5*x)/b^6 + (3*a^4*x^(4/
3))/(4*b^5) - (3*a^3*x^(5/3))/(5*b^4) + (a^2*x^2)/(2*b^3) - (3*a*x^(7/3))/(
7*b^2) + (3*x^(8/3))/(8*b) + (3*a^8*Log[a + b*x^(1/3)])/b^9
```

fricas [A] time = 0.79, size = 100, normalized size = 0.81

$$\frac{140\,a^2b^6x^2 - 280\,a^5b^3x + 840\,a^8\log\left(bx^{\frac{1}{3}} + a\right) + 21\left(5\,b^8x^2 - 8\,a^3b^5x + 20\,a^6b^2\right)x^{\frac{2}{3}} - 30\left(4\,ab^7x^2 - 7\,a^4b^4x + 28\,a\right.}{280\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^(1/3)),x, algorithm="fricas")
```

```
[Out] 1/280*(140*a^2*b^6*x^2 - 280*a^5*b^3*x + 840*a^8*log(b*x^(1/3) + a) + 21*(5
*b^8*x^2 - 8*a^3*b^5*x + 20*a^6*b^2)*x^(2/3) - 30*(4*a*b^7*x^2 - 7*a^4*b^4*
x + 28*a^7*b)*x^(1/3))/b^9
```

giac [A] time = 0.16, size = 100, normalized size = 0.81

$$\frac{3\,a^8\log\left(\left|bx^{\frac{1}{3}} + a\right|\right)}{b^9} + \frac{105\,b^7x^{\frac{8}{3}} - 120\,ab^6x^{\frac{7}{3}} + 140\,a^2b^5x^2 - 168\,a^3b^4x^{\frac{5}{3}} + 210\,a^4b^3x^{\frac{4}{3}} - 280\,a^5b^2x + 420\,a^6bx^{\frac{2}{3}} - 8}{280\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^(1/3)),x, algorithm="giac")
```

```
[Out] 3*a^8*log(abs(b*x^(1/3) + a))/b^9 + 1/280*(105*b^7*x^(8/3) - 120*a*b^6*x^(7
/3) + 140*a^2*b^5*x^2 - 168*a^3*b^4*x^(5/3) + 210*a^4*b^3*x^(4/3) - 280*a^5
*b^2*x + 420*a^6*b*x^(2/3) - 840*a^7*x^(1/3))/b^8
```

maple [A] time = 0.00, size = 99, normalized size = 0.80

$$\frac{3x^{\frac{8}{3}}}{8b} - \frac{3a\,x^{\frac{7}{3}}}{7b^2} + \frac{a^2x^2}{2b^3} - \frac{3a^3x^{\frac{5}{3}}}{5b^4} + \frac{3a^4x^{\frac{4}{3}}}{4b^5} + \frac{3a^8\ln\left(b\,x^{\frac{1}{3}} + a\right)}{b^9} - \frac{a^5x}{b^6} + \frac{3a^6x^{\frac{2}{3}}}{2b^7} - \frac{3a^7x^{\frac{1}{3}}}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^(1/3)+a),x)
```

```
[Out] -3*a^7*x^(1/3)/b^8+3/2*a^6*x^(2/3)/b^7-a^5/b^6*x+3/4*a^4*x^(4/3)/b^5-3/5*a^
3*x^(5/3)/b^4+1/2*a^2*x^2/b^3-3/7*a*x^(7/3)/b^2+3/8*x^(8/3)/b+3*a^8*ln(b*x^
(1/3)+a)/b^9
```

maxima [A] time = 0.88, size = 146, normalized size = 1.18

$$\frac{3\,a^8\log\left(bx^{\frac{1}{3}} + a\right)}{b^9} + \frac{3\left(bx^{\frac{1}{3}} + a\right)^8}{8\,b^9} - \frac{24\left(bx^{\frac{1}{3}} + a\right)^7a}{7\,b^9} + \frac{14\left(bx^{\frac{1}{3}} + a\right)^6a^2}{b^9} - \frac{168\left(bx^{\frac{1}{3}} + a\right)^5a^3}{5\,b^9} + \frac{105\left(bx^{\frac{1}{3}} + a\right)^4a^4}{2\,b^9} - \frac{56\left(b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^(1/3)),x, algorithm="maxima")
```

```
[Out] 3*a^8*log(b*x^(1/3) + a)/b^9 + 3/8*(b*x^(1/3) + a)^8/b^9 - 24/7*(b*x^(1/3)
+ a)^7*a/b^9 + 14*(b*x^(1/3) + a)^6*a^2/b^9 - 168/5*(b*x^(1/3) + a)^5*a^3/b
^9 + 105/2*(b*x^(1/3) + a)^4*a^4/b^9 - 56*(b*x^(1/3) + a)^3*a^5/b^9 + 42*(b
*x^(1/3) + a)^2*a^6/b^9 - 24*(b*x^(1/3) + a)*a^7/b^9
```

mupad [B] time = 0.04, size = 98, normalized size = 0.79

$$\frac{3\,x^{8/3}}{8\,b}-\frac{a^5\,x}{b^6}-\frac{3\,a\,x^{7/3}}{7\,b^2}+\frac{3\,a^8\,\ln\left(a+b\,x^{1/3}\right)}{b^9}+\frac{a^2\,x^2}{2\,b^3}-\frac{3\,a^3\,x^{5/3}}{5\,b^4}+\frac{3\,a^4\,x^{4/3}}{4\,b^5}+\frac{3\,a^6\,x^{2/3}}{2\,b^7}-\frac{3\,a^7\,x^{1/3}}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^(1/3)),x)
```

```
[Out] (3*x^(8/3))/(8*b) - (a^5*x)/b^6 - (3*a*x^(7/3))/(7*b^2) + (3*a^8*log(a + b*
x^(1/3)))/b^9 + (a^2*x^2)/(2*b^3) - (3*a^3*x^(5/3))/(5*b^4) + (3*a^4*x^(4/3
))/(4*b^5) + (3*a^6*x^(2/3))/(2*b^7) - (3*a^7*x^(1/3))/b^8
```

sympy [A] time = 103.58, size = 122, normalized size = 0.98

$$\frac{3a^8\log\left(1+\frac{b\sqrt[3]{x}}{a}\right)}{b^9}-\frac{3a^7\sqrt[3]{x}}{b^8}+\frac{3a^6x^{\frac{2}{3}}}{2b^7}-\frac{a^5x}{b^6}+\frac{3a^4x^{\frac{4}{3}}}{4b^5}-\frac{3a^3x^{\frac{5}{3}}}{5b^4}+\frac{a^2x^2}{2b^3}-\frac{3ax^{\frac{7}{3}}}{7b^2}+\frac{3x^{\frac{8}{3}}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*x**(1/3)),x)
```

```
[Out] 3*a**8*log(1 + b*x**(1/3)/a)/b**9 - 3*a**7*x**(1/3)/b**8 + 3*a**6*x**(2/3)/
(2*b**7) - a**5*x/b**6 + 3*a**4*x**(4/3)/(4*b**5) - 3*a**3*x**(5/3)/(5*b**4
) + a**2*x**2/(2*b**3) - 3*a*x**(7/3)/(7*b**2) + 3*x**(8/3)/(8*b)
```

3.2358 $\int \frac{x}{a+b\sqrt[3]{x}}\,dx$

Optimal. Leaf size=80

$$-\frac{3a^5\log\left(a+b\sqrt[3]{x}\right)}{b^6}+\frac{3a^4\sqrt[3]{x}}{b^5}-\frac{3a^3x^{2/3}}{2b^4}+\frac{a^2x}{b^3}-\frac{3ax^{4/3}}{4b^2}+\frac{3x^{5/3}}{5b}$$

```
[Out] 3*a^4*x^(1/3)/b^5-3/2*a^3*x^(2/3)/b^4+a^2*x/b^3-3/4*a*x^(4/3)/b^2+3/5*x^(5/
3)/b-3*a^5*ln(a+b*x^(1/3))/b^6
```

Rubi [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$-\frac{3a^3x^{2/3}}{2b^4}+\frac{3a^4\sqrt[3]{x}}{b^5}+\frac{a^2x}{b^3}-\frac{3a^5\log\left(a+b\sqrt[3]{x}\right)}{b^6}-\frac{3ax^{4/3}}{4b^2}+\frac{3x^{5/3}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^(1/3)),x]
```

```
[Out] (3*a^4*x^(1/3))/b^5 - (3*a^3*x^(2/3))/(2*b^4) + (a^2*x)/b^3 - (3*a*x^(4/3))
/(4*b^2) + (3*x^(5/3))/(5*b) - (3*a^5*Log[a + b*x^(1/3)])/b^6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{a+b\sqrt[3]{x}}\,dx &= 3\,\text{Subst}\left(\int \frac{x^5}{a+bx}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(\frac{a^4}{b^5}-\frac{a^3x}{b^4}+\frac{a^2x^2}{b^3}-\frac{ax^3}{b^2}+\frac{x^4}{b}-\frac{a^5}{b^5(a+bx)}\right)dx, x, \sqrt[3]{x}\right)\\ &= \frac{3a^4\sqrt[3]{x}}{b^5}-\frac{3a^3x^{2/3}}{2b^4}+\frac{a^2x}{b^3}-\frac{3ax^{4/3}}{4b^2}+\frac{3x^{5/3}}{5b}-\frac{3a^5\log\left(a+b\sqrt[3]{x}\right)}{b^6}\end{aligned}$$

Mathematica [A] time = 0.04, size = 77, normalized size = 0.96

$$\frac{-60a^5\log\left(a+b\sqrt[3]{x}\right)+60a^4b\sqrt[3]{x}-30a^3b^2x^{2/3}+20a^2b^3x-15ab^4x^{4/3}+12b^5x^{5/3}}{20b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^(1/3)),x]
```

```
[Out] (60*a^4*b*x^(1/3) - 30*a^3*b^2*x^(2/3) + 20*a^2*b^3*x - 15*a*b^4*x^(4/3) +
12*b^5*x^(5/3) - 60*a^5*Log[a + b*x^(1/3)])/(20*b^6)
```

fricas [A] time = 0.63, size = 66, normalized size = 0.82

$$\frac{20\,a^2b^3x - 60\,a^5\log\left(bx^{\frac{1}{3}} + a\right) + 6\left(2\,b^5x - 5\,a^3b^2\right)x^{\frac{2}{3}} - 15\left(ab^4x - 4\,a^4b\right)x^{\frac{1}{3}}}{20\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^(1/3)),x, algorithm="fricas")
```

```
[Out] 1/20*(20*a^2*b^3*x - 60*a^5*log(b*x^(1/3) + a) + 6*(2*b^5*x - 5*a^3*b^2)*x^
(2/3) - 15*(a*b^4*x - 4*a^4*b)*x^(1/3))/b^6
```

giac [A] time = 0.21, size = 67, normalized size = 0.84

$$-\frac{3\,a^5\log\left(\left|bx^{\frac{1}{3}} + a\right|\right)}{b^6} + \frac{12\,b^4x^{\frac{5}{3}} - 15\,ab^3x^{\frac{4}{3}} + 20\,a^2b^2x - 30\,a^3bx^{\frac{2}{3}} + 60\,a^4x^{\frac{1}{3}}}{20\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^(1/3)),x, algorithm="giac")
```

```
[Out] -3*a^5*log(abs(b*x^(1/3) + a))/b^6 + 1/20*(12*b^4*x^(5/3) - 15*a*b^3*x^(4/3
) + 20*a^2*b^2*x - 30*a^3*b*x^(2/3) + 60*a^4*x^(1/3))/b^5
```

maple [A] time = 0.00, size = 65, normalized size = 0.81

$$\frac{3x^{\frac{5}{3}}}{5b} - \frac{3a\,x^{\frac{4}{3}}}{4b^2} - \frac{3a^5\ln\left(b\,x^{\frac{1}{3}} + a\right)}{b^6} + \frac{a^2x}{b^3} - \frac{3a^3x^{\frac{2}{3}}}{2b^4} + \frac{3a^4x^{\frac{1}{3}}}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^(1/3)+a),x)
```

```
[Out] 3*a^4*x^(1/3)/b^5-3/2*a^3*x^(2/3)/b^4+a^2*x/b^3-3/4*a*x^(4/3)/b^2+3/5*x^(5/
3)/b-3*a^5*ln(b*x^(1/3)+a)/b^6
```

maxima [A] time = 0.81, size = 95, normalized size = 1.19

$$-\frac{3\,a^5\log\left(bx^{\frac{1}{3}} + a\right)}{b^6} + \frac{3\left(bx^{\frac{1}{3}} + a\right)^5}{5\,b^6} - \frac{15\left(bx^{\frac{1}{3}} + a\right)^4a}{4\,b^6} + \frac{10\left(bx^{\frac{1}{3}} + a\right)^3a^2}{b^6} - \frac{15\left(bx^{\frac{1}{3}} + a\right)^2a^3}{b^6} + \frac{15\left(bx^{\frac{1}{3}} + a\right)a^4}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^(1/3)),x, algorithm="maxima")
```

```
[Out] -3*a^5*log(b*x^(1/3) + a)/b^6 + 3/5*(b*x^(1/3) + a)^5/b^6 - 15/4*(b*x^(1/3)
 + a)^4*a/b^6 + 10*(b*x^(1/3) + a)^3*a^2/b^6 - 15*(b*x^(1/3) + a)^2*a^3/b^6
 + 15*(b*x^(1/3) + a)*a^4/b^6
```

mupad [B] time = 0.03, size = 64, normalized size = 0.80

$$\frac{3\,x^{5/3}}{5\,b} + \frac{a^2\,x}{b^3} - \frac{3\,a\,x^{4/3}}{4\,b^2} - \frac{3\,a^5\,\ln\left(a + b\,x^{1/3}\right)}{b^6} - \frac{3\,a^3\,x^{2/3}}{2\,b^4} + \frac{3\,a^4\,x^{1/3}}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^(1/3)),x)
```

```
[Out] (3*x^(5/3))/(5*b) + (a^2*x)/b^3 - (3*a*x^(4/3))/(4*b^2) - (3*a^5*log(a + b*
x^(1/3)))/b^6 - (3*a^3*x^(2/3))/(2*b^4) + (3*a^4*x^(1/3))/b^5
```

sympy [**A**] time = 8.89, size = 80, normalized size = 1.00

$$-\frac{3a^5\log\left(1+\frac{b\sqrt[3]{x}}{a}\right)}{b^6}+\frac{3a^4\sqrt[3]{x}}{b^5}-\frac{3a^3x^{\frac{2}{3}}}{2b^4}+\frac{a^2x}{b^3}-\frac{3ax^{\frac{4}{3}}}{4b^2}+\frac{3x^{\frac{5}{3}}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x**(1/3)),x)
```

```
[Out] -3*a**5*log(1 + b*x**(1/3)/a)/b**6 + 3*a**4*x**(1/3)/b**5 - 3*a**3*x**(2/3)
/(2*b**4) + a**2*x/b**3 - 3*a*x**(4/3)/(4*b**2) + 3*x**(5/3)/(5*b)
```

3.2359 $\int \frac{1}{a+b\sqrt[3]{x}}\,dx$

Optimal. Leaf size=42

$$\frac{3a^2\log\left(a+b\sqrt[3]{x}\right)}{b^3}-\frac{3a\sqrt[3]{x}}{b^2}+\frac{3x^{2/3}}{2b}$$

```
[Out] -3*a*x^(1/3)/b^2+3/2*x^(2/3)/b+3*a^2*ln(a+b*x^(1/3))/b^3
```

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {190, 43}

$$\frac{3a^2\log\left(a+b\sqrt[3]{x}\right)}{b^3}-\frac{3a\sqrt[3]{x}}{b^2}+\frac{3x^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^(-1),x]
```

```
[Out] (-3*a*x^(1/3))/b^2 + (3*x^(2/3))/(2*b) + (3*a^2*Log[a + b*x^(1/3)])/b^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\sqrt[3]{x}}\,dx &= 3\,\text{Subst}\left(\int \frac{x^2}{a+bx}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx, x, \sqrt[3]{x}\right)\\ &= -\frac{3a\sqrt[3]{x}}{b^2}+\frac{3x^{2/3}}{2b}+\frac{3a^2\log\left(a+b\sqrt[3]{x}\right)}{b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 1.00

$$\frac{3a^2\log\left(a+b\sqrt[3]{x}\right)}{b^3}-\frac{3a\sqrt[3]{x}}{b^2}+\frac{3x^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^(-1),x]
```

```
[Out] (-3*a*x^(1/3))/b^2 + (3*x^(2/3))/(2*b) + (3*a^2*Log[a + b*x^(1/3)])/b^3
```

fricas [A] time = 0.53, size = 33, normalized size = 0.79

$$\frac{3\left(2\,a^2\log\left(bx^{\frac{1}{3}}+a\right)+b^2x^{\frac{2}{3}}-2\,abx^{\frac{1}{3}}\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3)),x, algorithm="fricas")
```

```
[Out] 3/2*(2*a^2*log(b*x^(1/3) + a) + b^2*x^(2/3) - 2*a*b*x^(1/3))/b^3
```

giac [A] time = 0.16, size = 35, normalized size = 0.83

$$\frac{3\,a^2\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{b^3}+\frac{3\left(bx^{\frac{2}{3}}-2\,ax^{\frac{1}{3}}\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3)),x, algorithm="giac")
```

```
[Out] 3*a^2*log(abs(b*x^(1/3) + a))/b^3 + 3/2*(b*x^(2/3) - 2*a*x^(1/3))/b^2
```

maple [B] time = 0.02, size = 79, normalized size = 1.88

$$\frac{2a^2\ln\left(b\,x^{\frac{1}{3}}+a\right)}{b^3}-\frac{a^2\ln\left(b^2x^{\frac{2}{3}}-ab\,x^{\frac{1}{3}}+a^2\right)}{b^3}+\frac{a^2\ln\left(b^3x+a^3\right)}{b^3}+\frac{3x^{\frac{2}{3}}}{2b}-\frac{3a\,x^{\frac{1}{3}}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^(1/3)+a),x)
```

```
[Out] a^2*ln(b^3*x+a^3)/b^3+3/2*x^(2/3)/b+2*a^2*ln(b*x^(1/3)+a)/b^3-1/b^3*a^2*ln(
b^2*x^(2/3)-a*b*x^(1/3)+a^2)-3*a*x^(1/3)/b^2
```

maxima [A] time = 0.45, size = 44, normalized size = 1.05

$$\frac{3\,a^2\log\left(bx^{\frac{1}{3}}+a\right)}{b^3}+\frac{3\left(bx^{\frac{1}{3}}+a\right)^2}{2\,b^3}-\frac{6\left(bx^{\frac{1}{3}}+a\right)a}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3)),x, algorithm="maxima")
```

```
[Out] 3*a^2*log(b*x^(1/3) + a)/b^3 + 3/2*(b*x^(1/3) + a)^2/b^3 - 6*(b*x^(1/3) + a
)*a/b^3
```

mupad [B] time = 0.04, size = 34, normalized size = 0.81

$$\frac{6\,a^2\,\ln\left(a+b\,x^{1/3}\right)+3\,b^2\,x^{2/3}-6\,a\,b\,x^{1/3}}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^(1/3)),x)
```

```
[Out] (6*a^2*log(a + b*x^(1/3)) + 3*b^2*x^(2/3) - 6*a*b*x^(1/3))/(2*b^3)
```

sympy [A] time = 0.19, size = 42, normalized size = 1.00

$$\begin{cases}\frac{3a^2\log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{b^3}-\frac{3a\sqrt[3]{x}}{b^2}+\frac{3x^{\frac{2}{3}}}{2b} & \text{for } b\neq 0\\ \frac{x}{a} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**(1/3)),x)
```

```
[Out] Piecewise((3*a**2*log(a/b + x**(1/3))/b**3 - 3*a*x**(1/3)/b**2 + 3*x**(2/3)
/(2*b), Ne(b, 0)), (x/a, True))
```

3.2360 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x}\,dx$

Optimal. Leaf size=22

$$\frac{\log(x)}{a}-\frac{3\log\left(a+b\sqrt[3]{x}\right)}{a}$$

```
[Out] -3*ln(a+b*x^(1/3))/a+ln(x)/a
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 36, 29, 31}

$$\frac{\log(x)}{a}-\frac{3\log\left(a+b\sqrt[3]{x}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^(1/3))*x),x]
```

```
[Out] (-3*Log[a + b*x^(1/3)])/a + Log[x]/a
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x}\,dx &= 3\,\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx, x, \sqrt[3]{x}\right)\\ &= \frac{3\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, \sqrt[3]{x}\right)}{a}-\frac{(3b)\,\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, \sqrt[3]{x}\right)}{a}\\ &= -\frac{3\log\left(a+b\sqrt[3]{x}\right)}{a}+\frac{\log(x)}{a}\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$\frac{\log(x)}{a}-\frac{3\log\left(a+b\sqrt[3]{x}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^(1/3))*x),x]
[Out] (-3*Log[a + b*x^(1/3)])/a + Log[x]/a
```

fricas [A] time = 0.73, size = 20, normalized size = 0.91

$$-\frac{3\left(\log\left(bx^{\frac{1}{3}}+a\right)-\log\left(x^{\frac{1}{3}}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))/x,x, algorithm="fricas")
[Out] -3*(log(b*x^(1/3) + a) - log(x^(1/3)))/a
```

giac [A] time = 0.20, size = 22, normalized size = 1.00

$$-\frac{3\,\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{a}+\frac{\log\left(|x|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))/x,x, algorithm="giac")
[Out] -3*log(abs(b*x^(1/3) + a))/a + log(abs(x))/a
```

maple [A] time = 0.00, size = 21, normalized size = 0.95

$$\frac{\ln(x)}{a}-\frac{3\ln\left(b\,x^{\frac{1}{3}}+a\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^(1/3)+a)/x,x)
[Out] -3*ln(b*x^(1/3)+a)/a+1/a*ln(x)
```

maxima [A] time = 0.63, size = 20, normalized size = 0.91

$$-\frac{3\,\log\left(bx^{\frac{1}{3}}+a\right)}{a}+\frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))/x,x, algorithm="maxima")
[Out] -3*log(b*x^(1/3) + a)/a + log(x)/a
```

mupad [B] time = 1.21, size = 17, normalized size = 0.77

$$-\frac{6\,\mathrm{atanh}\left(\frac{2\,b\,x^{1/3}}{a}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^(1/3))),x)
[Out] -(6*atanh((2*b*x^(1/3))/a + 1))/a
```

sympy [**A**] time = 0.63, size = 37, normalized size = 1.68

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt[3]{x}} & \text{for } a = 0 \wedge b = 0 \\ \frac{\log(x)}{a} & \text{for } b = 0 \\ -\frac{3}{b\sqrt[3]{x}} & \text{for } a = 0 \\ \frac{\log(x)}{a} - \frac{3\log\left(\frac{a}{b} + \sqrt[3]{x}\right)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**(1/3))/x,x)
```

```
[Out] Piecewise((zoo/x**(1/3), Eq(a, 0) & Eq(b, 0)), (log(x)/a, Eq(b, 0)), (-3/(b
*x**(1/3)), Eq(a, 0)), (log(x)/a - 3*log(a/b + x**(1/3))/a, True))
```

3.2361 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x^2}\,dx$

Optimal. Leaf size=63

$$\frac{3b^3\log\left(a+b\sqrt[3]{x}\right)}{a^4}-\frac{b^3\log(x)}{a^4}-\frac{3b^2}{a^3\sqrt[3]{x}}+\frac{3b}{2a^2x^{2/3}}-\frac{1}{ax}$$

```
[Out] -1/a/x+3/2*b/a^2/x^(2/3)-3*b^2/a^3/x^(1/3)+3*b^3*ln(a+b*x^(1/3))/a^4-b^3*ln
(x)/a^4
```

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 44}

$$-\frac{3b^2}{a^3\sqrt[3]{x}}+\frac{3b^3\log\left(a+b\sqrt[3]{x}\right)}{a^4}-\frac{b^3\log(x)}{a^4}+\frac{3b}{2a^2x^{2/3}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^(1/3))*x^2),x]
```

```
[Out] -(1/(a*x)) + (3*b)/(2*a^2*x^(2/3)) - (3*b^2)/(a^3*x^(1/3)) + (3*b^3*Log[a +
 b*x^(1/3)])/a^4 - (b^3*Log[x])/a^4
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x^2}\,dx &= 3\,\text{Subst}\left(\int \frac{1}{x^4(a+bx)}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int\left(\frac{1}{ax^4}-\frac{b}{a^2x^3}+\frac{b^2}{a^3x^2}-\frac{b^3}{a^4x}+\frac{b^4}{a^4(a+bx)}\right)dx, x, \sqrt[3]{x}\right)\\ &= -\frac{1}{ax}+\frac{3b}{2a^2x^{2/3}}-\frac{3b^2}{a^3\sqrt[3]{x}}+\frac{3b^3\log\left(a+b\sqrt[3]{x}\right)}{a^4}-\frac{b^3\log(x)}{a^4}\end{aligned}$$

Mathematica [A] time = 0.03, size = 63, normalized size = 1.00

$$\frac{3b^3\log\left(a+b\sqrt[3]{x}\right)}{a^4}-\frac{b^3\log(x)}{a^4}-\frac{3b^2}{a^3\sqrt[3]{x}}+\frac{3b}{2a^2x^{2/3}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^(1/3))*x^2),x]
```

```
[Out] -(1/(a*x)) + (3*b)/(2*a^2*x^(2/3)) - (3*b^2)/(a^3*x^(1/3)) + (3*b^3*Log[a +
 b*x^(1/3)])/a^4 - (b^3*Log[x])/a^4
```

fricas [A] time = 0.53, size = 56, normalized size = 0.89

$$\frac{6\,b^3 x \log\left(b x^{\frac{1}{3}} + a\right) - 6\,b^3 x \log\left(x^{\frac{1}{3}}\right) - 6\,a b^2 x^{\frac{2}{3}} + 3\,a^2 b x^{\frac{1}{3}} - 2\,a^3}{2\,a^4 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*b^3*x*log(b*x^(1/3) + a) - 6*b^3*x*log(x^(1/3)) - 6*a*b^2*x^(2/3) +
3*a^2*b*x^(1/3) - 2*a^3)/(a^4*x)
```

giac [A] time = 0.16, size = 61, normalized size = 0.97

$$\frac{3\,b^3 \log\left(\left|b x^{\frac{1}{3}} + a\right|\right)}{a^4} - \frac{b^3 \log\left(|x|\right)}{a^4} - \frac{6\,a b^2 x^{\frac{2}{3}} - 3\,a^2 b x^{\frac{1}{3}} + 2\,a^3}{2\,a^4 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))/x^2,x, algorithm="giac")
```

```
[Out] 3*b^3*log(abs(b*x^(1/3) + a))/a^4 - b^3*log(abs(x))/a^4 - 1/2*(6*a*b^2*x^(2
/3) - 3*a^2*b*x^(1/3) + 2*a^3)/(a^4*x)
```

maple [A] time = 0.01, size = 56, normalized size = 0.89

$$-\frac{b^3 \ln(x)}{a^4} + \frac{3b^3 \ln\left(b\,x^{\frac{1}{3}} + a\right)}{a^4} - \frac{3b^2}{a^3 x^{\frac{1}{3}}} + \frac{3b}{2a^2 x^{\frac{2}{3}}} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^(1/3)+a)/x^2,x)
```

```
[Out] -1/a/x+3/2*b/a^2/x^(2/3)-3*b^2/a^3/x^(1/3)+3*b^3*ln(b*x^(1/3)+a)/a^4-1/a^4*
b^3*ln(x)
```

maxima [A] time = 0.56, size = 56, normalized size = 0.89

$$\frac{3\,b^3 \log\left(b x^{\frac{1}{3}} + a\right)}{a^4} - \frac{b^3 \log(x)}{a^4} - \frac{6\,b^2 x^{\frac{2}{3}} - 3\,a b x^{\frac{1}{3}} + 2\,a^2}{2\,a^3 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))/x^2,x, algorithm="maxima")
```

```
[Out] 3*b^3*log(b*x^(1/3) + a)/a^4 - b^3*log(x)/a^4 - 1/2*(6*b^2*x^(2/3) - 3*a*b*
x^(1/3) + 2*a^2)/(a^3*x)
```

mupad [B] time = 0.07, size = 50, normalized size = 0.79

$$\frac{6\,b^3\,\operatorname{atanh}\left(\frac{2\,b\,x^{1/3}}{a} + 1\right)}{a^4} - \frac{\frac{1}{a} - \frac{3\,b\,x^{1/3}}{2\,a^2} + \frac{3\,b^2\,x^{2/3}}{a^3}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^(1/3))),x)
```

```
[Out] (6*b^3*atanh((2*b*x^(1/3))/a + 1))/a^4 - (1/a - (3*b*x^(1/3))/(2*a^2) + (3*
b^2*x^(2/3))/a^3)/x
```

sympy [A] time = 2.79, size = 83, normalized size = 1.32

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{4}{3}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{3}{4bx^{\frac{4}{3}}} & \text{for } a = 0 \\ -\frac{1}{ax} & \text{for } b = 0 \\ -\frac{1}{ax} + \frac{3b}{2a^2x^{\frac{2}{3}}} - \frac{3b^2}{a^3\sqrt[3]{x}} - \frac{b^3\log(x)}{a^4} + \frac{3b^3\log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{a^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**(1/3))/x**2,x)
```

```
[Out] Piecewise((zoo/x**(4/3), Eq(a, 0) & Eq(b, 0)), (-3/(4*b*x**(4/3)), Eq(a, 0)
), (-1/(a*x), Eq(b, 0)), (-1/(a*x) + 3*b/(2*a**2*x**(2/3)) - 3*b**2/(a**3*x
**(1/3)) - b**3*log(x)/a**4 + 3*b**3*log(a/b + x**(1/3))/a**4, True))
```

3.2362 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x^3}\,dx$

Optimal. Leaf size=104

$$-\frac{3b^6\log\left(a+b\sqrt[3]{x}\right)}{a^7}+\frac{b^6\log(x)}{a^7}+\frac{3b^5}{a^6\sqrt[3]{x}}-\frac{3b^4}{2a^5x^{2/3}}+\frac{b^3}{a^4x}-\frac{3b^2}{4a^3x^{4/3}}+\frac{3b}{5a^2x^{5/3}}-\frac{1}{2ax^2}$$

```
[Out] -1/2/a/x^2+3/5*b/a^2/x^(5/3)-3/4*b^2/a^3/x^(4/3)+b^3/a^4/x-3/2*b^4/a^5/x^(2
/3)+3*b^5/a^6/x^(1/3)-3*b^6*ln(a+b*x^(1/3))/a^7+b^6*ln(x)/a^7
```

Rubi [A] time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 44}

$$-\frac{3b^4}{2a^5x^{2/3}}-\frac{3b^2}{4a^3x^{4/3}}+\frac{3b^5}{a^6\sqrt[3]{x}}+\frac{b^3}{a^4x}-\frac{3b^6\log\left(a+b\sqrt[3]{x}\right)}{a^7}+\frac{b^6\log(x)}{a^7}+\frac{3b}{5a^2x^{5/3}}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^(1/3))*x^3),x]
```

```
[Out] -1/(2*a*x^2) + (3*b)/(5*a^2*x^(5/3)) - (3*b^2)/(4*a^3*x^(4/3)) + b^3/(a^4*x
) - (3*b^4)/(2*a^5*x^(2/3)) + (3*b^5)/(a^6*x^(1/3)) - (3*b^6*Log[a + b*x^(1
/3)])/a^7 + (b^6*Log[x])/a^7
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x^3}\,dx &= 3\,\text{Subst}\left(\int \frac{1}{x^7(a+bx)}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int\left(\frac{1}{ax^7}-\frac{b}{a^2x^6}+\frac{b^2}{a^3x^5}-\frac{b^3}{a^4x^4}+\frac{b^4}{a^5x^3}-\frac{b^5}{a^6x^2}+\frac{b^6}{a^7x}-\frac{b^7}{a^7(a+bx)}\right)dx, x, \sqrt[3]{x}\right)\\ &= -\frac{1}{2ax^2}+\frac{3b}{5a^2x^{5/3}}-\frac{3b^2}{4a^3x^{4/3}}+\frac{b^3}{a^4x}-\frac{3b^4}{2a^5x^{2/3}}+\frac{3b^5}{a^6\sqrt[3]{x}}-\frac{3b^6\log\left(a+b\sqrt[3]{x}\right)}{a^7}+\frac{b^6\log(x)}{a^7}\end{aligned}$$

Mathematica [A] time = 0.07, size = 95, normalized size = 0.91

$$\frac{\frac{a\left(-10a^5+12a^4b\sqrt[3]{x}-15a^3b^2x^{2/3}+20a^2b^3x-30ab^4x^{4/3}+60b^5x^{5/3}\right)}{x^2}-60b^6\log\left(a+b\sqrt[3]{x}\right)+20b^6\log(x)}{20a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^(1/3))*x^3),x]
```

```
[Out] ((a*(-10*a^5 + 12*a^4*b*x^(1/3) - 15*a^3*b^2*x^(2/3) + 20*a^2*b^3*x - 30*a*
b^4*x^(4/3) + 60*b^5*x^(5/3)))/x^2 - 60*b^6*Log[a + b*x^(1/3)] + 20*b^6*Log
[x])/(20*a^7)
```

fricas [A] time = 0.91, size = 93, normalized size = 0.89

$$-\frac{60\,b^6x^2\log\left(bx^{\frac{1}{3}}+a\right)-60\,b^6x^2\log\left(x^{\frac{1}{3}}\right)-20\,a^3b^3x+10\,a^6-15\left(4\,ab^5x-a^4b^2\right)x^{\frac{2}{3}}+6\left(5\,a^2b^4x-2\,a^5b\right)x^{\frac{1}{3}}}{20\,a^7x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))/x^3,x, algorithm="fricas")
```

```
[Out] -1/20*(60*b^6*x^2*log(b*x^(1/3) + a) - 60*b^6*x^2*log(x^(1/3)) - 20*a^3*b^3
*x + 10*a^6 - 15*(4*a*b^5*x - a^4*b^2)*x^(2/3) + 6*(5*a^2*b^4*x - 2*a^5*b)*
x^(1/3))/(a^7*x^2)
```

giac [A] time = 0.16, size = 91, normalized size = 0.88

$$-\frac{3\,b^6\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{a^7}+\frac{b^6\log\left(|x|\right)}{a^7}+\frac{60\,ab^5x^{\frac{5}{3}}-30\,a^2b^4x^{\frac{4}{3}}+20\,a^3b^3x-15\,a^4b^2x^{\frac{2}{3}}+12\,a^5bx^{\frac{1}{3}}-10\,a^6}{20\,a^7x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))/x^3,x, algorithm="giac")
```

```
[Out] -3*b^6*log(abs(b*x^(1/3) + a))/a^7 + b^6*log(abs(x))/a^7 + 1/20*(60*a*b^5*x
^(5/3) - 30*a^2*b^4*x^(4/3) + 20*a^3*b^3*x - 15*a^4*b^2*x^(2/3) + 12*a^5*b*
x^(1/3) - 10*a^6)/(a^7*x^2)
```

maple [A] time = 0.01, size = 87, normalized size = 0.84

$$\frac{b^6\ln(x)}{a^7}-\frac{3b^6\ln\left(b\,x^{\frac{1}{3}}+a\right)}{a^7}+\frac{3b^5}{a^6x^{\frac{1}{3}}}-\frac{3b^4}{2a^5x^{\frac{2}{3}}}+\frac{b^3}{a^4x}-\frac{3b^2}{4a^3x^{\frac{4}{3}}}+\frac{3b}{5a^2x^{\frac{5}{3}}}-\frac{1}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^(1/3)+a)/x^3,x)
```

```
[Out] -1/2/a/x^2+3/5*b/a^2/x^(5/3)-3/4*b^2/a^3/x^(4/3)+b^3/a^4/x-3/2*b^4/a^5/x^(2
/3)+3*b^5/a^6/x^(1/3)-3*b^6*ln(b*x^(1/3)+a)/a^7+1/a^7*b^6*ln(x)
```

maxima [A] time = 0.54, size = 86, normalized size = 0.83

$$-\frac{3\,b^6\log\left(bx^{\frac{1}{3}}+a\right)}{a^7}+\frac{b^6\log(x)}{a^7}+\frac{60\,b^5x^{\frac{5}{3}}-30\,ab^4x^{\frac{4}{3}}+20\,a^2b^3x-15\,a^3b^2x^{\frac{2}{3}}+12\,a^4bx^{\frac{1}{3}}-10\,a^5}{20\,a^6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))/x^3,x, algorithm="maxima")
```

```
[Out] -3*b^6*log(b*x^(1/3) + a)/a^7 + b^6*log(x)/a^7 + 1/20*(60*b^5*x^(5/3) - 30*
a*b^4*x^(4/3) + 20*a^2*b^3*x - 15*a^3*b^2*x^(2/3) + 12*a^4*b*x^(1/3) - 10*a
^5)/(a^6*x^2)
```

mupad [B] time = 0.06, size = 83, normalized size = 0.80

$$-\frac{\frac{1}{2\,a}-\frac{3\,b\,x^{1/3}}{5\,a^2}-\frac{b^3\,x}{a^4}+\frac{3\,b^2\,x^{2/3}}{4\,a^3}+\frac{3\,b^4\,x^{4/3}}{2\,a^5}-\frac{3\,b^5\,x^{5/3}}{a^6}}{x^2}-\frac{6\,b^6\,\mathrm{atanh}\left(\frac{2\,b\,x^{1/3}}{a}+1\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^(1/3))),x)
```

```
[Out] - (1/(2*a) - (3*b*x^(1/3))/(5*a^2) - (b^3*x)/a^4 + (3*b^2*x^(2/3))/(4*a^3)
+ (3*b^4*x^(4/3))/(2*a^5) - (3*b^5*x^(5/3))/a^6)/x^2 - (6*b^6*atanh((2*b*x^
(1/3))/a + 1))/a^7
```

sympy [A] time = 8.70, size = 129, normalized size = 1.24

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{3}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{3}{7bx^{\frac{7}{3}}} & \text{for } a = 0 \\ -\frac{1}{2ax^2} & \text{for } b = 0 \\ -\frac{1}{2ax^2} + \frac{3b}{5a^2x^{\frac{5}{3}}} - \frac{3b^2}{4a^3x^{\frac{4}{3}}} + \frac{b^3}{a^4x} - \frac{3b^4}{2a^5x^{\frac{2}{3}}} + \frac{3b^5}{a^6\sqrt[3]{x}} + \frac{b^6\log(x)}{a^7} - \frac{3b^6\log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{a^7} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**(1/3))/x**3,x)
```

```
[Out] Piecewise((zoo/x**(7/3), Eq(a, 0) & Eq(b, 0)), (-3/(7*b*x**(7/3)), Eq(a, 0)
), (-1/(2*a*x**2), Eq(b, 0)), (-1/(2*a*x**2) + 3*b/(5*a**2*x**(5/3)) - 3*b*
*2/(4*a**3*x**(4/3)) + b**3/(a**4*x) - 3*b**4/(2*a**5*x**(2/3)) + 3*b**5/(a
**6*x**(1/3)) + b**6*log(x)/a**7 - 3*b**6*log(a/b + x**(1/3))/a**7, True))
```

3.2363 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x^4}\,dx$

Optimal. Leaf size=149

$$\frac{3b^9\log\left(a+b\sqrt[3]{x}\right)}{a^{10}}-\frac{b^9\log(x)}{a^{10}}-\frac{3b^8}{a^9\sqrt[3]{x}}+\frac{3b^7}{2a^8x^{2/3}}-\frac{b^6}{a^7x}+\frac{3b^5}{4a^6x^{4/3}}-\frac{3b^4}{5a^5x^{5/3}}+\frac{b^3}{2a^4x^2}-\frac{3b^2}{7a^3x^{7/3}}+\frac{3b}{8a^2x^{8/3}}-\frac{1}{3ax^3}$$

```
[Out] -1/3/a/x^3+3/8*b/a^2/x^(8/3)-3/7*b^2/a^3/x^(7/3)+1/2*b^3/a^4/x^2-3/5*b^4/a^
5/x^(5/3)+3/4*b^5/a^6/x^(4/3)-b^6/a^7/x+3/2*b^7/a^8/x^(2/3)-3*b^8/a^9/x^(1/
3)+3*b^9*ln(a+b*x^(1/3))/a^10-b^9*ln(x)/a^10
```

Rubi [A] time = 0.08, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 44}

$$\frac{3b^7}{2a^8x^{2/3}}+\frac{3b^5}{4a^6x^{4/3}}-\frac{3b^4}{5a^5x^{5/3}}+\frac{b^3}{2a^4x^2}-\frac{3b^2}{7a^3x^{7/3}}-\frac{3b^8}{a^9\sqrt[3]{x}}-\frac{b^6}{a^7x}+\frac{3b^9\log\left(a+b\sqrt[3]{x}\right)}{a^{10}}-\frac{b^9\log(x)}{a^{10}}+\frac{3b}{8a^2x^{8/3}}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^(1/3))*x^4),x]
```

```
[Out] -1/(3*a*x^3) + (3*b)/(8*a^2*x^(8/3)) - (3*b^2)/(7*a^3*x^(7/3)) + b^3/(2*a^4
*x^2) - (3*b^4)/(5*a^5*x^(5/3)) + (3*b^5)/(4*a^6*x^(4/3)) - b^6/(a^7*x) + (
3*b^7)/(2*a^8*x^(2/3)) - (3*b^8)/(a^9*x^(1/3)) + (3*b^9*Log[a + b*x^(1/3)])
/a^10 - (b^9*Log[x])/a^10
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x^4}\,dx &= 3\,\text{Subst}\left(\int \frac{1}{x^{10}(a+bx)}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int\left(\frac{1}{ax^{10}}-\frac{b}{a^2x^9}+\frac{b^2}{a^3x^8}-\frac{b^3}{a^4x^7}+\frac{b^4}{a^5x^6}-\frac{b^5}{a^6x^5}+\frac{b^6}{a^7x^4}-\frac{b^7}{a^8x^3}+\frac{b^8}{a^9x^2}-\frac{b^9}{a^{10}x}\right.\right.\\ &= -\frac{1}{3ax^3}+\frac{3b}{8a^2x^{8/3}}-\frac{3b^2}{7a^3x^{7/3}}+\frac{b^3}{2a^4x^2}-\frac{3b^4}{5a^5x^{5/3}}+\frac{3b^5}{4a^6x^{4/3}}-\frac{b^6}{a^7x}+\frac{3b^7}{2a^8x^{2/3}}-\frac{3b^8}{a^9\sqrt[3]{x}}+\frac{3}{-}\end{aligned}$$

Mathematica [A] time = 0.07, size = 138, normalized size = 0.93

$$-\frac{280a^9-315a^8b\sqrt[3]{x}+360a^7b^2x^{2/3}-420a^6b^3x+504a^5b^4x^{4/3}-630a^4b^5x^{5/3}+840a^3b^6x^2-1260a^2b^7x^{7/3}-25}{840a^{10}x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^(1/3))*x^4),x]
```

```
[Out] -1/840*(280*a^9 - 315*a^8*b*x^(1/3) + 360*a^7*b^2*x^(2/3) - 420*a^6*b^3*x +
 504*a^5*b^4*x^(4/3) - 630*a^4*b^5*x^(5/3) + 840*a^3*b^6*x^2 - 1260*a^2*b^7
*x^(7/3) + 2520*a*b^8*x^(8/3) - 2520*b^9*x^3*Log[a + b*x^(1/3)] + 840*b^9*x
^3*Log[x])/(a^10*x^3)
```

fricas [A] time = 0.70, size = 126, normalized size = 0.85

$$\frac{2520\,b^9x^3\log\left(bx^{\frac{1}{3}}+a\right)-2520\,b^9x^3\log\left(x^{\frac{1}{3}}\right)-840\,a^3b^6x^2+420\,a^6b^3x-280\,a^9-90\left(28\,ab^8x^2-7\,a^4b^5x+4\,a^7b\right.}{840\,a^{10}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))/x^4,x, algorithm="fricas")
```

```
[Out] 1/840*(2520*b^9*x^3*log(b*x^(1/3) + a) - 2520*b^9*x^3*log(x^(1/3)) - 840*a^
3*b^6*x^2 + 420*a^6*b^3*x - 280*a^9 - 90*(28*a*b^8*x^2 - 7*a^4*b^5*x + 4*a^
7*b^2)*x^(2/3) + 63*(20*a^2*b^7*x^2 - 8*a^5*b^4*x + 5*a^8*b)*x^(1/3))/(a^10
*x^3)
```

giac [A] time = 0.17, size = 125, normalized size = 0.84

$$\frac{3\,b^9\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{a^{10}}-\frac{b^9\log\left(|x|\right)}{a^{10}}-\frac{2520\,ab^8x^{\frac{8}{3}}-1260\,a^2b^7x^{\frac{7}{3}}+840\,a^3b^6x^2-630\,a^4b^5x^{\frac{5}{3}}+504\,a^5b^4x^{\frac{4}{3}}-420\,a^6b^3}{840\,a^{10}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))/x^4,x, algorithm="giac")
```

```
[Out] 3*b^9*log(abs(b*x^(1/3) + a))/a^10 - b^9*log(abs(x))/a^10 - 1/840*(2520*a*b
^8*x^(8/3) - 1260*a^2*b^7*x^(7/3) + 840*a^3*b^6*x^2 - 630*a^4*b^5*x^(5/3) +
 504*a^5*b^4*x^(4/3) - 420*a^6*b^3*x + 360*a^7*b^2*x^(2/3) - 315*a^8*b*x^(1
/3) + 280*a^9)/(a^10*x^3)
```

maple [A] time = 0.01, size = 122, normalized size = 0.82

$$-\frac{b^9\ln(x)}{a^{10}}+\frac{3b^9\ln\left(b\,x^{\frac{1}{3}}+a\right)}{a^{10}}-\frac{3b^8}{a^9x^{\frac{1}{3}}}+\frac{3b^7}{2a^8x^{\frac{2}{3}}}-\frac{b^6}{a^7x}+\frac{3b^5}{4a^6x^{\frac{4}{3}}}-\frac{3b^4}{5a^5x^{\frac{5}{3}}}+\frac{b^3}{2a^4x^2}-\frac{3b^2}{7a^3x^{\frac{7}{3}}}+\frac{3b}{8a^2x^{\frac{8}{3}}}-\frac{1}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^(1/3)+a)/x^4,x)
```

```
[Out] -1/3/a/x^3+3/8*b/a^2/x^(8/3)-3/7*b^2/a^3/x^(7/3)+1/2*b^3/a^4/x^2-3/5*b^4/a^
5/x^(5/3)+3/4*b^5/a^6/x^(4/3)-b^6/a^7/x+3/2*b^7/a^8/x^(2/3)-3*b^8/a^9/x^(1/
3)+3*b^9*ln(b*x^(1/3)+a)/a^10-b^9*ln(x)/a^10
```

maxima [A] time = 0.55, size = 120, normalized size = 0.81

$$\frac{3\,b^9\log\left(bx^{\frac{1}{3}}+a\right)}{a^{10}}-\frac{b^9\log(x)}{a^{10}}-\frac{2520\,b^8x^{\frac{8}{3}}-1260\,ab^7x^{\frac{7}{3}}+840\,a^2b^6x^2-630\,a^3b^5x^{\frac{5}{3}}+504\,a^4b^4x^{\frac{4}{3}}-420\,a^5b^3x+3}{840\,a^9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))/x^4,x, algorithm="maxima")
```

```
[Out] 3*b^9*log(b*x^(1/3) + a)/a^10 - b^9*log(x)/a^10 - 1/840*(2520*b^8*x^(8/3) -
 1260*a*b^7*x^(7/3) + 840*a^2*b^6*x^2 - 630*a^3*b^5*x^(5/3) + 504*a^4*b^4*x
```

```
^(4/3) - 420*a^5*b^3*x + 360*a^6*b^2*x^(2/3) - 315*a^7*b*x^(1/3) + 280*a^8)
/(a^9*x^3)
```

mupad [B] time = 1.16, size = 116, normalized size = 0.78

$$-\frac{280\,a^9-5040\,b^9\,x^3\,\operatorname{atanh}\left(\frac{2\,b\,x^{1/3}}{a}+1\right)-420\,a^6\,b^3\,x-315\,a^8\,b\,x^{1/3}+2520\,a\,b^8\,x^{8/3}+840\,a^3\,b^6\,x^2+360\,a^7\,b^2}{840\,a^{10}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^(1/3))),x)
```

```
[Out] -(280*a^9 - 5040*b^9*x^3*atanh((2*b*x^(1/3))/a + 1) - 420*a^6*b^3*x - 315*a
^8*b*x^(1/3) + 2520*a*b^8*x^(8/3) + 840*a^3*b^6*x^2 + 360*a^7*b^2*x^(2/3) +
 504*a^5*b^4*x^(4/3) - 630*a^4*b^5*x^(5/3) - 1260*a^2*b^7*x^(7/3))/(840*a^1
0*x^3)
```

sympy [A] time = 22.37, size = 172, normalized size = 1.15

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{10}{3}}} & \text{for } a = 0 \wedge b \\ -\frac{3}{10bx^{\frac{10}{3}}} & \text{for } a = 0 \\ -\frac{1}{3ax^3} & \text{for } b = 0 \\ -\frac{1}{3ax^3}+\frac{3b}{8a^2x^{\frac{8}{3}}}-\frac{3b^2}{7a^3x^{\frac{7}{3}}}+\frac{b^3}{2a^4x^2}-\frac{3b^4}{5a^5x^{\frac{5}{3}}}+\frac{3b^5}{4a^6x^{\frac{4}{3}}}-\frac{b^6}{a^7x}+\frac{3b^7}{2a^8x^{\frac{2}{3}}}-\frac{3b^8}{a^9\sqrt[3]{x}}-\frac{b^9\log(x)}{a^{10}}+\frac{3b^9\log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{a^{10}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**(1/3))/x**4,x)
```

```
[Out] Piecewise((zoo/x**(10/3), Eq(a, 0) & Eq(b, 0)), (-3/(10*b*x**(10/3)), Eq(a,
 0)), (-1/(3*a*x**3), Eq(b, 0)), (-1/(3*a*x**3) + 3*b/(8*a**2*x**(8/3)) - 3
*b**2/(7*a**3*x**(7/3)) + b**3/(2*a**4*x**2) - 3*b**4/(5*a**5*x**(5/3)) + 3
*b**5/(4*a**6*x**(4/3)) - b**6/(a**7*x) + 3*b**7/(2*a**8*x**(2/3)) - 3*b**8
/(a**9*x**(1/3)) - b**9*log(x)/a**10 + 3*b**9*log(a/b + x**(1/3))/a**10, Tr
ue))
```

3.2364 $\int \frac{1}{\left(2+b\sqrt[3]{x}\right)x}\,dx$

Optimal. Leaf size=21

$$\frac{\log(x)}{2}-\frac{3}{2}\log\left(b\sqrt[3]{x}+2\right)$$

```
[Out] -3/2*ln(2+b*x^(1/3))+1/2*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 36, 29, 31}

$$\frac{\log(x)}{2}-\frac{3}{2}\log\left(b\sqrt[3]{x}+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + b*x^(1/3))*x),x]
```

```
[Out] (-3*Log[2 + b*x^(1/3)])/2 + Log[x]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(2+b\sqrt[3]{x}\right)x}\,dx &= 3\,\text{Subst}\left(\int \frac{1}{x(2+bx)}\,dx, x, \sqrt[3]{x}\right)\\ &= \frac{3}{2}\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, \sqrt[3]{x}\right)-\frac{1}{2}(3b)\,\text{Subst}\left(\int \frac{1}{2+bx}\,dx, x, \sqrt[3]{x}\right)\\ &= -\frac{3}{2}\log\left(2+b\sqrt[3]{x}\right)+\frac{\log(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{\log(x)}{2}-\frac{3}{2}\log\left(b\sqrt[3]{x}+2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2 + b*x^(1/3))*x),x]
[Out] (-3*Log[2 + b*x^(1/3)])/2 + Log[x]/2
```

fricas [A] time = 0.66, size = 17, normalized size = 0.81

$$-\frac{3}{2}\log\left(bx^{\frac{1}{3}}+2\right)+\frac{3}{2}\log\left(x^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+b*x^(1/3))/x,x, algorithm="fricas")
[Out] -3/2*log(b*x^(1/3) + 2) + 3/2*log(x^(1/3))
```

giac [A] time = 0.16, size = 17, normalized size = 0.81

$$-\frac{3}{2}\log\left(\left|bx^{\frac{1}{3}}+2\right|\right)+\frac{1}{2}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+b*x^(1/3))/x,x, algorithm="giac")
[Out] -3/2*log(abs(b*x^(1/3) + 2)) + 1/2*log(abs(x))
```

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$\frac{\ln(x)}{2}-\frac{3\ln\left(b\,x^{\frac{1}{3}}+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+b*x^(1/3))/x,x)
[Out] -3/2*ln(2+b*x^(1/3))+1/2*ln(x)
```

maxima [A] time = 0.55, size = 15, normalized size = 0.71

$$-\frac{3}{2}\log\left(bx^{\frac{1}{3}}+2\right)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+b*x^(1/3))/x,x, algorithm="maxima")
[Out] -3/2*log(b*x^(1/3) + 2) + 1/2*log(x)
```

mupad [B] time = 0.05, size = 10, normalized size = 0.48

$$-3\,\operatorname{atanh}\left(b\,x^{1/3}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^(1/3) + 2)),x)
[Out] -3*atanh(b*x^(1/3) + 1)
```

sympy [A] time = 0.63, size = 22, normalized size = 1.05

$$\begin{cases}\frac{\log(x)}{2}-\frac{3\log\left(\sqrt[3]{x}+\frac{2}{b}\right)}{2} & \text{for } b\neq 0\\ \frac{\log(x)}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+b*x**(1/3))/x,x)
[Out] Piecewise((log(x)/2 - 3*log(x**(1/3) + 2/b)/2, Ne(b, 0)), (log(x)/2, True))
```

3.2365 $\int \frac{x^3}{\left(a+b\sqrt[3]{x}\right)^2}\,dx$

Optimal. Leaf size=171

$$\frac{3a^{11}}{b^{12}\left(a+b\sqrt[3]{x}\right)}+\frac{33a^{10}\log\left(a+b\sqrt[3]{x}\right)}{b^{12}}-\frac{30a^9\sqrt[3]{x}}{b^{11}}+\frac{27a^8x^{2/3}}{2b^{10}}-\frac{8a^7x}{b^9}+\frac{21a^6x^{4/3}}{4b^8}-\frac{18a^5x^{5/3}}{5b^7}+\frac{5a^4x^2}{2b^6}-\frac{12a^3x^{7/3}}{7b^5}+\frac{9a^2x}{8b}$$

```
[Out] 3*a^11/b^12/(a+b*x^(1/3))-30*a^9*x^(1/3)/b^11+27/2*a^8*x^(2/3)/b^10-8*a^7*x
/b^9+21/4*a^6*x^(4/3)/b^8-18/5*a^5*x^(5/3)/b^7+5/2*a^4*x^2/b^6-12/7*a^3*x^(
7/3)/b^5+9/8*a^2*x^(8/3)/b^4-2/3*a*x^3/b^3+3/10*x^(10/3)/b^2+33*a^10*ln(a+b
*x^(1/3))/b^12
```

Rubi [A] time = 0.14, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{27a^8x^{2/3}}{2b^{10}}+\frac{21a^6x^{4/3}}{4b^8}-\frac{18a^5x^{5/3}}{5b^7}+\frac{5a^4x^2}{2b^6}-\frac{12a^3x^{7/3}}{7b^5}+\frac{9a^2x^{8/3}}{8b^4}+\frac{3a^{11}}{b^{12}\left(a+b\sqrt[3]{x}\right)}-\frac{30a^9\sqrt[3]{x}}{b^{11}}-\frac{8a^7x}{b^9}+\frac{33a^{10}\log\left(a+b\sqrt[3]{x}\right)}{b^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^(1/3))^2,x]
```

```
[Out] (3*a^11)/(b^12*(a + b*x^(1/3))) - (30*a^9*x^(1/3))/b^11 + (27*a^8*x^(2/3))/
(2*b^10) - (8*a^7*x)/b^9 + (21*a^6*x^(4/3))/(4*b^8) - (18*a^5*x^(5/3))/(5*b
^7) + (5*a^4*x^2)/(2*b^6) - (12*a^3*x^(7/3))/(7*b^5) + (9*a^2*x^(8/3))/(8*b
^4) - (2*a*x^3)/(3*b^3) + (3*x^(10/3))/(10*b^2) + (33*a^10*Log[a + b*x^(1/3
)])/b^12
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(a+b\sqrt[3]{x}\right)^2}\,dx &= 3\,\text{Subst}\left(\int \frac{x^{11}}{(a+bx)^2}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int\left(-\frac{10a^9}{b^{11}}+\frac{9a^8x}{b^{10}}-\frac{8a^7x^2}{b^9}+\frac{7a^6x^3}{b^8}-\frac{6a^5x^4}{b^7}+\frac{5a^4x^5}{b^6}-\frac{4a^3x^6}{b^5}+\frac{3a^2x^7}{b^4}-\frac{2ax^8}{b^3}\right.\right.\\ &= \frac{3a^{11}}{b^{12}\left(a+b\sqrt[3]{x}\right)}-\frac{30a^9\sqrt[3]{x}}{b^{11}}+\frac{27a^8x^{2/3}}{2b^{10}}-\frac{8a^7x}{b^9}+\frac{21a^6x^{4/3}}{4b^8}-\frac{18a^5x^{5/3}}{5b^7}+\frac{5a^4x^2}{2b^6}-\frac{12a^3x^{7/3}}{7b^5}\end{aligned}$$

Mathematica [A] time = 0.16, size = 171, normalized size = 1.00

$$\frac{3a^{11}}{b^{12}\left(a+b\sqrt[3]{x}\right)}+\frac{33a^{10}\log\left(a+b\sqrt[3]{x}\right)}{b^{12}}-\frac{30a^9\sqrt[3]{x}}{b^{11}}+\frac{27a^8x^{2/3}}{2b^{10}}-\frac{8a^7x}{b^9}+\frac{21a^6x^{4/3}}{4b^8}-\frac{18a^5x^{5/3}}{5b^7}+\frac{5a^4x^2}{2b^6}-\frac{12a^3x^{7/3}}{7b^5}+\frac{9a^2x}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^(1/3))^2,x]
```

```
[Out] (3*a^11)/(b^12*(a + b*x^(1/3))) - (30*a^9*x^(1/3))/b^11 + (27*a^8*x^(2/3))/
(2*b^10) - (8*a^7*x)/b^9 + (21*a^6*x^(4/3))/(4*b^8) - (18*a^5*x^(5/3))/(5*b
^7) + (5*a^4*x^2)/(2*b^6) - (12*a^3*x^(7/3))/(7*b^5) + (9*a^2*x^(8/3))/(8*b
^4) - (2*a*x^3)/(3*b^3) + (3*x^(10/3))/(10*b^2) + (33*a^10*Log[a + b*x^(1/3
)])/b^12
```

fricas [A] time = 0.60, size = 181, normalized size = 1.06

$$-\frac{560\,ab^{12}x^4 - 1540\,a^4b^9x^3 + 4620\,a^7b^6x^2 + 6720\,a^{10}b^3x - 2520\,a^{13} - 27720\left(a^{10}b^3x + a^{13}\right)\log\left(bx^{\frac{1}{3}} + a\right) - 63\,(1}{840\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^(1/3))^2,x, algorithm="fricas")
```

```
[Out] -1/840*(560*a*b^12*x^4 - 1540*a^4*b^9*x^3 + 4620*a^7*b^6*x^2 + 6720*a^10*b^
3*x - 2520*a^13 - 27720*(a^10*b^3*x + a^13)*log(b*x^(1/3) + a) - 63*(15*a^2
*b^11*x^3 - 33*a^5*b^8*x^2 + 132*a^8*b^5*x + 220*a^11*b^2)*x^(2/3) - 18*(14
*b^13*x^4 - 66*a^3*b^10*x^3 + 165*a^6*b^7*x^2 - 1155*a^9*b^4*x - 1540*a^12*
b)*x^(1/3))/(b^15*x + a^3*b^12)
```

giac [A] time = 0.17, size = 144, normalized size = 0.84

$$\frac{33\,a^{10}\log\left(\left|bx^{\frac{1}{3}} + a\right|\right)}{b^{12}} + \frac{3\,a^{11}}{\left(bx^{\frac{1}{3}} + a\right)b^{12}} + \frac{252\,b^{18}x^{\frac{10}{3}} - 560\,ab^{17}x^3 + 945\,a^2b^{16}x^{\frac{8}{3}} - 1440\,a^3b^{15}x^{\frac{7}{3}} + 2100\,a^4b^{14}x^2 - 3}{84}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^(1/3))^2,x, algorithm="giac")
```

```
[Out] 33*a^10*log(abs(b*x^(1/3) + a))/b^12 + 3*a^11/((b*x^(1/3) + a)*b^12) + 1/84
0*(252*b^18*x^(10/3) - 560*a*b^17*x^3 + 945*a^2*b^16*x^(8/3) - 1440*a^3*b^1
5*x^(7/3) + 2100*a^4*b^14*x^2 - 3024*a^5*b^13*x^(5/3) + 4410*a^6*b^12*x^(4/
3) - 6720*a^7*b^11*x + 11340*a^8*b^10*x^(2/3) - 25200*a^9*b^9*x^(1/3))/b^20
```

maple [A] time = 0.01, size = 138, normalized size = 0.81

$$\frac{3x^{\frac{10}{3}}}{10b^2} - \frac{2a\,x^3}{3b^3} + \frac{9a^2x^{\frac{8}{3}}}{8b^4} - \frac{12a^3x^{\frac{7}{3}}}{7b^5} + \frac{5a^4x^2}{2b^6} - \frac{18a^5x^{\frac{5}{3}}}{5b^7} + \frac{21a^6x^{\frac{4}{3}}}{4b^8} + \frac{3a^{11}}{\left(b\,x^{\frac{1}{3}} + a\right)b^{12}} + \frac{33a^{10}\ln\left(b\,x^{\frac{1}{3}} + a\right)}{b^{12}} - \frac{8a^7x}{b^9} + \frac{27a^8x^{\frac{2}{3}}}{2b^{10}} - \frac{30}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^(1/3)+a)^2,x)
```

```
[Out] 3*a^11/b^12/(b*x^(1/3)+a)-30*a^9*x^(1/3)/b^11+27/2*a^8*x^(2/3)/b^10-8*a^7*x
/b^9+21/4*a^6*x^(4/3)/b^8-18/5*a^5*x^(5/3)/b^7+5/2*a^4*x^2/b^6-12/7*a^3*x^(
7/3)/b^5+9/8*a^2*x^(8/3)/b^4-2/3*a*x^3/b^3+3/10*x^(10/3)/b^2+33*a^10*ln(b*x
^(1/3)+a)/b^12
```

maxima [A] time = 0.49, size = 197, normalized size = 1.15

$$\frac{33\,a^{10}\log\left(bx^{\frac{1}{3}} + a\right)}{b^{12}} + \frac{3\left(bx^{\frac{1}{3}} + a\right)^{10}}{10\,b^{12}} - \frac{11\left(bx^{\frac{1}{3}} + a\right)^9 a}{3\,b^{12}} + \frac{165\left(bx^{\frac{1}{3}} + a\right)^8 a^2}{8\,b^{12}} - \frac{495\left(bx^{\frac{1}{3}} + a\right)^7 a^3}{7\,b^{12}} + \frac{165\left(bx^{\frac{1}{3}} + a\right)^6 a^4}{b^{12}} - \frac{1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^(1/3))^2,x, algorithm="maxima")
```

```
[Out] 33*a^10*log(b*x^(1/3) + a)/b^12 + 3/10*(b*x^(1/3) + a)^10/b^12 - 11/3*(b*x^
(1/3) + a)^9*a/b^12 + 165/8*(b*x^(1/3) + a)^8*a^2/b^12 - 495/7*(b*x^(1/3) +
 a)^7*a^3/b^12 + 165*(b*x^(1/3) + a)^6*a^4/b^12 - 1386/5*(b*x^(1/3) + a)^5*
a^5/b^12 + 693/2*(b*x^(1/3) + a)^4*a^6/b^12 - 330*(b*x^(1/3) + a)^3*a^7/b^1
2 + 495/2*(b*x^(1/3) + a)^2*a^8/b^12 - 165*(b*x^(1/3) + a)*a^9/b^12 + 3*a^1
1/((b*x^(1/3) + a)*b^12)
```

mupad **[B]** time = 0.08, size = 143, normalized size = 0.84

$$\frac{3\,x^{10/3}}{10\,b^2}+\frac{3\,a^{11}}{b\left(a\,b^{11}+b^{12}\,x^{1/3}\right)}-\frac{2\,a\,x^3}{3\,b^3}-\frac{8\,a^7\,x}{b^9}+\frac{33\,a^{10}\,\ln\left(a+b\,x^{1/3}\right)}{b^{12}}+\frac{5\,a^4\,x^2}{2\,b^6}+\frac{9\,a^2\,x^{8/3}}{8\,b^4}-\frac{12\,a^3\,x^{7/3}}{7\,b^5}-\frac{18\,a^5\,x^{5/3}}{5\,b^7}+\frac{2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^(1/3))^2,x)
```

```
[Out] (3*x^(10/3))/(10*b^2) + (3*a^11)/(b*(a*b^11 + b^12*x^(1/3))) - (2*a*x^3)/(3
*b^3) - (8*a^7*x)/b^9 + (33*a^10*log(a + b*x^(1/3)))/b^12 + (5*a^4*x^2)/(2*
b^6) + (9*a^2*x^(8/3))/(8*b^4) - (12*a^3*x^(7/3))/(7*b^5) - (18*a^5*x^(5/3)
)/(5*b^7) + (21*a^6*x^(4/3))/(4*b^8) + (27*a^8*x^(2/3))/(2*b^10) - (30*a^9*
x^(1/3))/b^11
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*x**(1/3))**2,x)
```

```
[Out] Timed out
```

3.2366 $\int \frac{x^2}{\left(a+b\sqrt[3]{x}\right)^2}\,dx$

Optimal. Leaf size=122

$$-\frac{3a^8}{b^9\left(a+b\sqrt[3]{x}\right)}-\frac{24a^7\log\left(a+b\sqrt[3]{x}\right)}{b^9}+\frac{21a^6\sqrt[3]{x}}{b^8}-\frac{9a^5x^{2/3}}{b^7}+\frac{5a^4x}{b^6}-\frac{3a^3x^{4/3}}{b^5}+\frac{9a^2x^{5/3}}{5b^4}-\frac{ax^2}{b^3}+\frac{3x^{7/3}}{7b^2}$$

```
[Out] -3*a^8/b^9/(a+b*x^(1/3))+21*a^6*x^(1/3)/b^8-9*a^5*x^(2/3)/b^7+5*a^4*x/b^6-3
*a^3*x^(4/3)/b^5+9/5*a^2*x^(5/3)/b^4-a*x^2/b^3+3/7*x^(7/3)/b^2-24*a^7*ln(a+
b*x^(1/3))/b^9
```

Rubi [A] time = 0.09, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{9a^5x^{2/3}}{b^7}-\frac{3a^3x^{4/3}}{b^5}+\frac{9a^2x^{5/3}}{5b^4}-\frac{3a^8}{b^9\left(a+b\sqrt[3]{x}\right)}+\frac{21a^6\sqrt[3]{x}}{b^8}+\frac{5a^4x}{b^6}-\frac{24a^7\log\left(a+b\sqrt[3]{x}\right)}{b^9}-\frac{ax^2}{b^3}+\frac{3x^{7/3}}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^(1/3))^2,x]
```

```
[Out] (-3*a^8)/(b^9*(a + b*x^(1/3))) + (21*a^6*x^(1/3))/b^8 - (9*a^5*x^(2/3))/b^7
 + (5*a^4*x)/b^6 - (3*a^3*x^(4/3))/b^5 + (9*a^2*x^(5/3))/(5*b^4) - (a*x^2)/
b^3 + (3*x^(7/3))/(7*b^2) - (24*a^7*Log[a + b*x^(1/3)])/b^9
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a+b\sqrt[3]{x}\right)^2}\,dx &= 3\,\text{Subst}\left(\int \frac{x^8}{(a+bx)^2}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int\left(\frac{7a^6}{b^8}-\frac{6a^5x}{b^7}+\frac{5a^4x^2}{b^6}-\frac{4a^3x^3}{b^5}+\frac{3a^2x^4}{b^4}-\frac{2ax^5}{b^3}+\frac{x^6}{b^2}+\frac{a^8}{b^8(a+bx)^2}-\frac{8a^7}{b^8(a+bx}\right.\right.\\ &= -\frac{3a^8}{b^9\left(a+b\sqrt[3]{x}\right)}+\frac{21a^6\sqrt[3]{x}}{b^8}-\frac{9a^5x^{2/3}}{b^7}+\frac{5a^4x}{b^6}-\frac{3a^3x^{4/3}}{b^5}+\frac{9a^2x^{5/3}}{5b^4}-\frac{ax^2}{b^3}+\frac{3x^{7/3}}{7b^2}-\frac{24a^7\log}{}\end{aligned}$$

Mathematica [A] time = 0.11, size = 122, normalized size = 1.00

$$-\frac{3a^8}{b^9\left(a+b\sqrt[3]{x}\right)}-\frac{24a^7\log\left(a+b\sqrt[3]{x}\right)}{b^9}+\frac{21a^6\sqrt[3]{x}}{b^8}-\frac{9a^5x^{2/3}}{b^7}+\frac{5a^4x}{b^6}-\frac{3a^3x^{4/3}}{b^5}+\frac{9a^2x^{5/3}}{5b^4}-\frac{ax^2}{b^3}+\frac{3x^{7/3}}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^(1/3))^2,x]
```

```
[Out] (-3*a^8)/(b^9*(a + b*x^(1/3))) + (21*a^6*x^(1/3))/b^8 - (9*a^5*x^(2/3))/b^7
 + (5*a^4*x)/b^6 - (3*a^3*x^(4/3))/b^5 + (9*a^2*x^(5/3))/(5*b^4) - (a*x^2)/
b^3 + (3*x^(7/3))/(7*b^2) - (24*a^7*Log[a + b*x^(1/3)])/b^9
```

fricas [A] time = 0.70, size = 147, normalized size = 1.20

$$-\frac{35\,ab^9x^3-140\,a^4b^6x^2-175\,a^7b^3x+105\,a^{10}+840\left(a^7b^3x+a^{10}\right)\log\left(bx^{\frac{1}{3}}+a\right)-21\left(3\,a^2b^8x^2-12\,a^5b^5x-2\right.}{35\left(b^{12}x+a^3b^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^(1/3))^2,x, algorithm="fricas")
```

```
[Out] -1/35*(35*a*b^9*x^3 - 140*a^4*b^6*x^2 - 175*a^7*b^3*x + 105*a^10 + 840*(a^7
*b^3*x + a^10)*log(b*x^(1/3) + a) - 21*(3*a^2*b^8*x^2 - 12*a^5*b^5*x - 20*a
^8*b^2)*x^(2/3) - 15*(b^10*x^3 - 6*a^3*b^7*x^2 + 42*a^6*b^4*x + 56*a^9*b)*x
^(1/3))/(b^12*x + a^3*b^9)
```

giac [A] time = 0.17, size = 111, normalized size = 0.91

$$-\frac{24\,a^7\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{b^9}-\frac{3\,a^8}{\left(bx^{\frac{1}{3}}+a\right)b^9}+\frac{15\,b^{12}x^{\frac{7}{3}}-35\,ab^{11}x^2+63\,a^2b^{10}x^{\frac{5}{3}}-105\,a^3b^9x^{\frac{4}{3}}+175\,a^4b^8x-315\,a^5b^7}{35\,b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^(1/3))^2,x, algorithm="giac")
```

```
[Out] -24*a^7*log(abs(b*x^(1/3) + a))/b^9 - 3*a^8/((b*x^(1/3) + a)*b^9) + 1/35*(1
5*b^12*x^(7/3) - 35*a*b^11*x^2 + 63*a^2*b^10*x^(5/3) - 105*a^3*b^9*x^(4/3)
+ 175*a^4*b^8*x - 315*a^5*b^7*x^(2/3) + 735*a^6*b^6*x^(1/3))/b^14
```

maple [A] time = 0.01, size = 105, normalized size = 0.86

$$\frac{3x^{\frac{7}{3}}}{7b^2}-\frac{a\,x^2}{b^3}+\frac{9a^2x^{\frac{5}{3}}}{5b^4}-\frac{3a^3x^{\frac{4}{3}}}{b^5}-\frac{3a^8}{\left(b\,x^{\frac{1}{3}}+a\right)b^9}-\frac{24a^7\ln\left(b\,x^{\frac{1}{3}}+a\right)}{b^9}+\frac{5a^4x}{b^6}-\frac{9a^5x^{\frac{2}{3}}}{b^7}+\frac{21a^6x^{\frac{1}{3}}}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^(1/3)+a)^2,x)
```

```
[Out] -3*a^8/b^9/(b*x^(1/3)+a)+21*a^6*x^(1/3)/b^8-9*a^5*x^(2/3)/b^7+5*a^4/b^6*x-3
*a^3*x^(4/3)/b^5+9/5*a^2*x^(5/3)/b^4-a*x^2/b^3+3/7*x^(7/3)/b^2-24*a^7*ln(b*
x^(1/3)+a)/b^9
```

maxima [A] time = 0.56, size = 146, normalized size = 1.20

$$-\frac{24\,a^7\log\left(bx^{\frac{1}{3}}+a\right)}{b^9}+\frac{3\left(bx^{\frac{1}{3}}+a\right)^7}{7\,b^9}-\frac{4\left(bx^{\frac{1}{3}}+a\right)^6a}{b^9}+\frac{84\left(bx^{\frac{1}{3}}+a\right)^5a^2}{5\,b^9}-\frac{42\left(bx^{\frac{1}{3}}+a\right)^4a^3}{b^9}+\frac{70\left(bx^{\frac{1}{3}}+a\right)^3a^4}{b^9}-\frac{84}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^(1/3))^2,x, algorithm="maxima")
```

```
[Out] -24*a^7*log(b*x^(1/3) + a)/b^9 + 3/7*(b*x^(1/3) + a)^7/b^9 - 4*(b*x^(1/3) +
 a)^6*a/b^9 + 84/5*(b*x^(1/3) + a)^5*a^2/b^9 - 42*(b*x^(1/3) + a)^4*a^3/b^9
 + 70*(b*x^(1/3) + a)^3*a^4/b^9 - 84*(b*x^(1/3) + a)^2*a^5/b^9 + 84*(b*x^(1
/3) + a)*a^6/b^9 - 3*a^8/((b*x^(1/3) + a)*b^9)
```

mupad [B] time = 0.05, size = 110, normalized size = 0.90

$$\frac{3\,x^{7/3}}{7\,b^2}-\frac{3\,a^8}{b\,\left(a\,b^8+b^9\,x^{1/3}\right)}-\frac{a\,x^2}{b^3}+\frac{5\,a^4\,x}{b^6}-\frac{24\,a^7\,\ln\left(a+b\,x^{1/3}\right)}{b^9}+\frac{9\,a^2\,x^{5/3}}{5\,b^4}-\frac{3\,a^3\,x^{4/3}}{b^5}-\frac{9\,a^5\,x^{2/3}}{b^7}+\frac{21\,a^6\,x^{1/3}}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^(1/3))^2,x)
```

```
[Out] (3*x^(7/3))/(7*b^2) - (3*a^8)/(b*(a*b^8 + b^9*x^(1/3))) - (a*x^2)/b^3 + (5*
a^4*x)/b^6 - (24*a^7*log(a + b*x^(1/3)))/b^9 + (9*a^2*x^(5/3))/(5*b^4) - (3
*a^3*x^(4/3))/b^5 - (9*a^5*x^(2/3))/b^7 + (21*a^6*x^(1/3))/b^8
```

sympy [B] time = 56.23, size = 343, normalized size = 2.81

$$-\frac{840a^8x^{\frac{176}{3}}\log\left(1+\frac{b\sqrt[3]{x}}{a}\right)}{35ab^9x^{\frac{176}{3}}+35b^{10}x^{59}}-\frac{840a^7bx^{59}\log\left(1+\frac{b\sqrt[3]{x}}{a}\right)}{35ab^9x^{\frac{176}{3}}+35b^{10}x^{59}}+\frac{840a^7bx^{59}}{35ab^9x^{\frac{176}{3}}+35b^{10}x^{59}}+\frac{420a^6b^2x^{\frac{178}{3}}}{35ab^9x^{\frac{176}{3}}+35b^{10}x^{59}}-\frac{140a^5}{35ab^9x^{\frac{176}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*x**(1/3))**2,x)
```

```
[Out] -840*a**8*x**(176/3)*log(1 + b*x**(1/3)/a)/(35*a*b**9*x**(176/3) + 35*b**10
*x**59) - 840*a**7*b*x**59*log(1 + b*x**(1/3)/a)/(35*a*b**9*x**(176/3) + 35
*b**10*x**59) + 840*a**7*b*x**59/(35*a*b**9*x**(176/3) + 35*b**10*x**59) +
420*a**6*b**2*x**(178/3)/(35*a*b**9*x**(176/3) + 35*b**10*x**59) - 140*a**5
*b**3*x**(179/3)/(35*a*b**9*x**(176/3) + 35*b**10*x**59) + 70*a**4*b**4*x**
60/(35*a*b**9*x**(176/3) + 35*b**10*x**59) - 42*a**3*b**5*x**(181/3)/(35*a*
b**9*x**(176/3) + 35*b**10*x**59) + 28*a**2*b**6*x**(182/3)/(35*a*b**9*x**(
176/3) + 35*b**10*x**59) - 20*a*b**7*x**61/(35*a*b**9*x**(176/3) + 35*b**10
*x**59) + 15*b**8*x**(184/3)/(35*a*b**9*x**(176/3) + 35*b**10*x**59)
```

3.2367 $\int \frac{x}{\left(a+b\sqrt[3]{x}\right)^2}\,dx$

Optimal. Leaf size=85

$$\frac{3a^5}{b^6\left(a+b\sqrt[3]{x}\right)}+\frac{15a^4\log\left(a+b\sqrt[3]{x}\right)}{b^6}-\frac{12a^3\sqrt[3]{x}}{b^5}+\frac{9a^2x^{2/3}}{2b^4}-\frac{2ax}{b^3}+\frac{3x^{4/3}}{4b^2}$$

```
[Out] 3*a^5/b^6/(a+b*x^(1/3))-12*a^3*x^(1/3)/b^5+9/2*a^2*x^(2/3)/b^4-2*a*x/b^3+3/
4*x^(4/3)/b^2+15*a^4*ln(a+b*x^(1/3))/b^6
```

Rubi [A] time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{9a^2x^{2/3}}{2b^4}+\frac{3a^5}{b^6\left(a+b\sqrt[3]{x}\right)}-\frac{12a^3\sqrt[3]{x}}{b^5}+\frac{15a^4\log\left(a+b\sqrt[3]{x}\right)}{b^6}-\frac{2ax}{b^3}+\frac{3x^{4/3}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^(1/3))^2,x]
```

```
[Out] (3*a^5)/(b^6*(a + b*x^(1/3))) - (12*a^3*x^(1/3))/b^5 + (9*a^2*x^(2/3))/(2*b
^4) - (2*a*x)/b^3 + (3*x^(4/3))/(4*b^2) + (15*a^4*Log[a + b*x^(1/3)])/b^6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a+b\sqrt[3]{x}\right)^2}\,dx &= 3\,\text{Subst}\left(\int \frac{x^5}{(a+bx)^2}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int\left(-\frac{4a^3}{b^5}+\frac{3a^2x}{b^4}-\frac{2ax^2}{b^3}+\frac{x^3}{b^2}-\frac{a^5}{b^5(a+bx)^2}+\frac{5a^4}{b^5(a+bx)}\right)dx, x, \sqrt[3]{x}\right)\\ &= \frac{3a^5}{b^6\left(a+b\sqrt[3]{x}\right)}-\frac{12a^3\sqrt[3]{x}}{b^5}+\frac{9a^2x^{2/3}}{2b^4}-\frac{2ax}{b^3}+\frac{3x^{4/3}}{4b^2}+\frac{15a^4\log\left(a+b\sqrt[3]{x}\right)}{b^6}\end{aligned}$$

Mathematica [A] time = 0.06, size = 80, normalized size = 0.94

$$\frac{\frac{12a^5}{a+b\sqrt[3]{x}}+60a^4\log\left(a+b\sqrt[3]{x}\right)-48a^3b\sqrt[3]{x}+18a^2b^2x^{2/3}-8ab^3x+3b^4x^{4/3}}{4b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^(1/3))^2,x]
```

```
[Out] ((12*a^5)/(a + b*x^(1/3)) - 48*a^3*b*x^(1/3) + 18*a^2*b^2*x^(2/3) - 8*a*b^3
*x + 3*b^4*x^(4/3) + 60*a^4*Log[a + b*x^(1/3)])/(4*b^6)
```

fricas [A] time = 0.53, size = 114, normalized size = 1.34

$$-\frac{8\,ab^6x^2+8\,a^4b^3x-12\,a^7-60\left(a^4b^3x+a^7\right)\log\left(bx^{\frac{1}{3}}+a\right)-6\left(3\,a^2b^5x+5\,a^5b^2\right)x^{\frac{2}{3}}-3\left(b^7x^2-15\,a^3b^4x-20\,a^6b\right.}{4\left(b^9x+a^3b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^(1/3))^2,x, algorithm="fricas")
```

```
[Out] -1/4*(8*a*b^6*x^2 + 8*a^4*b^3*x - 12*a^7 - 60*(a^4*b^3*x + a^7)*log(b*x^(1/
3) + a) - 6*(3*a^2*b^5*x + 5*a^5*b^2)*x^(2/3) - 3*(b^7*x^2 - 15*a^3*b^4*x -
 20*a^6*b)*x^(1/3))/(b^9*x + a^3*b^6)
```

giac [A] time = 0.16, size = 78, normalized size = 0.92

$$\frac{15\,a^4\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{b^6}+\frac{3\,a^5}{\left(bx^{\frac{1}{3}}+a\right)b^6}+\frac{3\,b^6x^{\frac{4}{3}}-8\,ab^5x+18\,a^2b^4x^{\frac{2}{3}}-48\,a^3b^3x^{\frac{1}{3}}}{4\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^(1/3))^2,x, algorithm="giac")
```

```
[Out] 15*a^4*log(abs(b*x^(1/3) + a))/b^6 + 3*a^5/((b*x^(1/3) + a)*b^6) + 1/4*(3*b
^6*x^(4/3) - 8*a*b^5*x + 18*a^2*b^4*x^(2/3) - 48*a^3*b^3*x^(1/3))/b^8
```

maple [A] time = 0.01, size = 72, normalized size = 0.85

$$\frac{3x^{\frac{4}{3}}}{4b^2}+\frac{3a^5}{\left(b\,x^{\frac{1}{3}}+a\right)b^6}+\frac{15a^4\ln\left(b\,x^{\frac{1}{3}}+a\right)}{b^6}-\frac{2ax}{b^3}+\frac{9a^2x^{\frac{2}{3}}}{2b^4}-\frac{12a^3x^{\frac{1}{3}}}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^(1/3)+a)^2,x)
```

```
[Out] 3*a^5/b^6/(b*x^(1/3)+a)-12*a^3*x^(1/3)/b^5+9/2*a^2*x^(2/3)/b^4-2*a*x/b^3+3/
4*x^(4/3)/b^2+15*a^4*ln(b*x^(1/3)+a)/b^6
```

maxima [A] time = 0.49, size = 95, normalized size = 1.12

$$\frac{15\,a^4\log\left(bx^{\frac{1}{3}}+a\right)}{b^6}+\frac{3\left(bx^{\frac{1}{3}}+a\right)^4}{4\,b^6}-\frac{5\left(bx^{\frac{1}{3}}+a\right)^3a}{b^6}+\frac{15\left(bx^{\frac{1}{3}}+a\right)^2a^2}{b^6}-\frac{30\left(bx^{\frac{1}{3}}+a\right)a^3}{b^6}+\frac{3\,a^5}{\left(bx^{\frac{1}{3}}+a\right)b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^(1/3))^2,x, algorithm="maxima")
```

```
[Out] 15*a^4*log(b*x^(1/3) + a)/b^6 + 3/4*(b*x^(1/3) + a)^4/b^6 - 5*(b*x^(1/3) +
a)^3*a/b^6 + 15*(b*x^(1/3) + a)^2*a^2/b^6 - 30*(b*x^(1/3) + a)*a^3/b^6 + 3*
a^5/((b*x^(1/3) + a)*b^6)
```

mupad [B] time = 1.12, size = 77, normalized size = 0.91

$$\frac{3\,x^{4/3}}{4\,b^2}+\frac{3\,a^5}{b\left(a\,b^5+b^6\,x^{1/3}\right)}+\frac{15\,a^4\,\ln\left(a+b\,x^{1/3}\right)}{b^6}+\frac{9\,a^2\,x^{2/3}}{2\,b^4}-\frac{12\,a^3\,x^{1/3}}{b^5}-\frac{2\,a\,x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^(1/3))^2,x)
```

```
[Out] (3*x^(4/3))/(4*b^2) + (3*a^5)/(b*(a*b^5 + b^6*x^(1/3))) + (15*a^4*log(a + b
*x^(1/3)))/b^6 + (9*a^2*x^(2/3))/(2*b^4) - (12*a^3*x^(1/3))/b^5 - (2*a*x)/b
^3
```

sympy [B] time = 8.52, size = 243, normalized size = 2.86

$$\frac{60a^5x^{\frac{80}{3}}\log\left(1+\frac{b\sqrt[3]{x}}{a}\right)}{4ab^6x^{\frac{80}{3}}+4b^7x^{27}}+\frac{60a^4bx^{27}\log\left(1+\frac{b\sqrt[3]{x}}{a}\right)}{4ab^6x^{\frac{80}{3}}+4b^7x^{27}}-\frac{60a^4bx^{27}}{4ab^6x^{\frac{80}{3}}+4b^7x^{27}}-\frac{30a^3b^2x^{\frac{82}{3}}}{4ab^6x^{\frac{80}{3}}+4b^7x^{27}}+\frac{10a^2b^3x^{\frac{83}{3}}}{4ab^6x^{\frac{80}{3}}+4b^7x^{27}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x**(1/3))**2,x)
```

```
[Out] 60*a**5*x**(80/3)*log(1 + b*x**(1/3)/a)/(4*a*b**6*x**(80/3) + 4*b**7*x**27)
 + 60*a**4*b*x**27*log(1 + b*x**(1/3)/a)/(4*a*b**6*x**(80/3) + 4*b**7*x**27
) - 60*a**4*b*x**27/(4*a*b**6*x**(80/3) + 4*b**7*x**27) - 30*a**3*b**2*x**(
82/3)/(4*a*b**6*x**(80/3) + 4*b**7*x**27) + 10*a**2*b**3*x**(83/3)/(4*a*b**
6*x**(80/3) + 4*b**7*x**27) - 5*a*b**4*x**28/(4*a*b**6*x**(80/3) + 4*b**7*x
**27) + 3*b**5*x**(85/3)/(4*a*b**6*x**(80/3) + 4*b**7*x**27)
```

3.2368 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2}\,dx$

Optimal. Leaf size=46

$$-\frac{3a^2}{b^3\left(a+b\sqrt[3]{x}\right)}-\frac{6a\log\left(a+b\sqrt[3]{x}\right)}{b^3}+\frac{3\sqrt[3]{x}}{b^2}$$

```
[Out] -3*a^2/b^3/(a+b*x^(1/3))+3*x^(1/3)/b^2-6*a*ln(a+b*x^(1/3))/b^3
```

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {190, 43}

$$-\frac{3a^2}{b^3\left(a+b\sqrt[3]{x}\right)}-\frac{6a\log\left(a+b\sqrt[3]{x}\right)}{b^3}+\frac{3\sqrt[3]{x}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^(-2),x]
```

```
[Out] (-3*a^2)/(b^3*(a + b*x^(1/3))) + (3*x^(1/3))/b^2 - (6*a*Log[a + b*x^(1/3)])
/b^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2}\,dx &= 3\,\text{Subst}\left(\int \frac{x^2}{(a+bx)^2}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(\frac{1}{b^2}+\frac{a^2}{b^2(a+bx)^2}-\frac{2a}{b^2(a+bx)}\right)dx, x, \sqrt[3]{x}\right)\\ &= -\frac{3a^2}{b^3\left(a+b\sqrt[3]{x}\right)}+\frac{3\sqrt[3]{x}}{b^2}-\frac{6a\log\left(a+b\sqrt[3]{x}\right)}{b^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 42, normalized size = 0.91

$$\frac{3\left(-\frac{a^2}{a+b\sqrt[3]{x}}-2a\log\left(a+b\sqrt[3]{x}\right)+b\sqrt[3]{x}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^(-2),x]
```

```
[Out] (3*(-(a^2/(a + b*x^(1/3))) + b*x^(1/3) - 2*a*Log[a + b*x^(1/3)]))/b^3
```

fricas [A] time = 0.60, size = 68, normalized size = 1.48

$$-\frac{3\left(a^2b^2x^{\frac{2}{3}}+a^4+2\left(ab^3x+a^4\right)\log\left(bx^{\frac{1}{3}}+a\right)-\left(b^4x+2\,a^3b\right)x^{\frac{1}{3}}\right)}{b^6x+a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))^2,x, algorithm="fricas")
```

```
[Out] -3*(a^2*b^2*x^(2/3) + a^4 + 2*(a*b^3*x + a^4)*log(b*x^(1/3) + a) - (b^4*x +
 2*a^3*b)*x^(1/3))/(b^6*x + a^3*b^3)
```

giac [A] time = 0.15, size = 41, normalized size = 0.89

$$-\frac{6\,a\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{b^3}+\frac{3\,x^{\frac{1}{3}}}{b^2}-\frac{3\,a^2}{\left(bx^{\frac{1}{3}}+a\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))^2,x, algorithm="giac")
```

```
[Out] -6*a*log(abs(b*x^(1/3) + a))/b^3 + 3*x^(1/3)/b^2 - 3*a^2/((b*x^(1/3) + a)*b
^3)
```

maple [B] time = 0.05, size = 160, normalized size = 3.48

$$-\frac{3a^4}{\left(b^3x+a^3\right)b^3}-\frac{a^2x^{\frac{1}{3}}}{\left(b^2x^{\frac{2}{3}}-ab\,x^{\frac{1}{3}}+a^2\right)b^2}+\frac{2a^3}{\left(b^2x^{\frac{2}{3}}-ab\,x^{\frac{1}{3}}+a^2\right)b^3}-\frac{2a^2}{\left(b\,x^{\frac{1}{3}}+a\right)b^3}-\frac{4a\ln\left(b\,x^{\frac{1}{3}}+a\right)}{b^3}+\frac{2a\ln\left(b^2x^{\frac{2}{3}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^(1/3)+a)^2,x)
```

```
[Out] -3*a^4/(b^3*x+a^3)/b^3+3*x^(1/3)/b^2-4*a*ln(b*x^(1/3)+a)/b^3-2*a^2/b^3/(b*x
^(1/3)+a)-1/b^2*a^2/(b^2*x^(2/3)-a*b*x^(1/3)+a^2)*x^(1/3)+2/b^3*a^3/(b^2*x^
(2/3)-a*b*x^(1/3)+a^2)+2/b^3*a*ln(b^2*x^(2/3)-a*b*x^(1/3)+a^2)-2*a/b^3*ln(b
^3*x+a^3)
```

maxima [A] time = 0.53, size = 44, normalized size = 0.96

$$-\frac{6\,a\log\left(bx^{\frac{1}{3}}+a\right)}{b^3}+\frac{3\left(bx^{\frac{1}{3}}+a\right)}{b^3}-\frac{3\,a^2}{\left(bx^{\frac{1}{3}}+a\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))^2,x, algorithm="maxima")
```

```
[Out] -6*a*log(b*x^(1/3) + a)/b^3 + 3*(b*x^(1/3) + a)/b^3 - 3*a^2/((b*x^(1/3) + a
)*b^3)
```

mupad [B] time = 0.04, size = 43, normalized size = 0.93

$$\frac{3\,x^{1/3}}{b^2}-\frac{3\,a^2}{a\,b^3+b^4\,x^{1/3}}-\frac{6\,a\,\ln\left(a+b\,x^{1/3}\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^(1/3))^2,x)
```

```
[Out] (3*x^(1/3))/b^2 - (3*a^2)/(a*b^3 + b^4*x^(1/3)) - (6*a*log(a + b*x^(1/3)))/
b^3
```

sympy [**A**] time = 0.45, size = 109, normalized size = 2.37

$$\begin{cases} -\frac{6a^2 \log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{ab^3+b^4\sqrt[3]{x}} - \frac{6a^2}{ab^3+b^4\sqrt[3]{x}} - \frac{6ab\sqrt[3]{x}\log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{ab^3+b^4\sqrt[3]{x}} + \frac{3b^2x^{\frac{2}{3}}}{ab^3+b^4\sqrt[3]{x}} & \text{for } b \neq 0 \\ \frac{x}{a^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**(1/3))**2,x)
```

```
[Out] Piecewise((-6*a**2*log(a/b + x**(1/3))/(a*b**3 + b**4*x**(1/3)) - 6*a**2/(a
*b**3 + b**4*x**(1/3)) - 6*a*b*x**(1/3)*log(a/b + x**(1/3))/(a*b**3 + b**4*
x**(1/3)) + 3*b**2*x**(2/3)/(a*b**3 + b**4*x**(1/3)), Ne(b, 0)), (x/a**2, T
rue))
```

3.2369 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x}\,dx$

Optimal. Leaf size=38

$$-\frac{3\log\left(a+b\sqrt[3]{x}\right)}{a^2}+\frac{\log(x)}{a^2}+\frac{3}{a\left(a+b\sqrt[3]{x}\right)}$$

```
[Out] 3/a/(a+b*x^(1/3))-3*ln(a+b*x^(1/3))/a^2+ln(x)/a^2
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 44}

$$-\frac{3\log\left(a+b\sqrt[3]{x}\right)}{a^2}+\frac{\log(x)}{a^2}+\frac{3}{a\left(a+b\sqrt[3]{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^(1/3))^2*x),x]
```

```
[Out] 3/(a*(a + b*x^(1/3))) - (3*Log[a + b*x^(1/3)])/a^2 + Log[x]/a^2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x}\,dx &= 3\,\text{Subst}\left(\int \frac{1}{x(a+bx)^2}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(\frac{1}{a^2 x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right)dx, x, \sqrt[3]{x}\right)\\ &= \frac{3}{a\left(a+b\sqrt[3]{x}\right)}-\frac{3\log\left(a+b\sqrt[3]{x}\right)}{a^2}+\frac{\log(x)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 33, normalized size = 0.87

$$\frac{\frac{3a}{a+b\sqrt[3]{x}}-3\log\left(a+b\sqrt[3]{x}\right)+\log(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^(1/3))^2*x),x]
```

```
[Out] ((3*a)/(a + b*x^(1/3)) - 3*Log[a + b*x^(1/3)] + Log[x])/a^2
```

fricas [B] time = 0.63, size = 70, normalized size = 1.84

$$\frac{3\left(ab^2x^{\frac{2}{3}} - a^2bx^{\frac{1}{3}} + a^3 - \left(b^3x + a^3\right)\log\left(bx^{\frac{1}{3}} + a\right) + \left(b^3x + a^3\right)\log\left(x^{\frac{1}{3}}\right)\right)}{a^2b^3x + a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))^2/x,x, algorithm="fricas")
```

```
[Out] 3*(a*b^2*x^(2/3) - a^2*b*x^(1/3) + a^3 - (b^3*x + a^3)*log(b*x^(1/3) + a) +
 (b^3*x + a^3)*log(x^(1/3)))/(a^2*b^3*x + a^5)
```

giac [A] time = 0.16, size = 36, normalized size = 0.95

$$-\frac{3\log\left(\left|bx^{\frac{1}{3}} + a\right|\right)}{a^2} + \frac{\log\left(|x|\right)}{a^2} + \frac{3}{\left(bx^{\frac{1}{3}} + a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))^2/x,x, algorithm="giac")
```

```
[Out] -3*log(abs(b*x^(1/3) + a))/a^2 + log(abs(x))/a^2 + 3/((b*x^(1/3) + a)*a)
```

maple [A] time = 0.01, size = 35, normalized size = 0.92

$$\frac{3}{\left(b\,x^{\frac{1}{3}} + a\right)a} + \frac{\ln(x)}{a^2} - \frac{3\ln\left(b\,x^{\frac{1}{3}} + a\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^(1/3)+a)^2/x,x)
```

```
[Out] 3/a/(b*x^(1/3)+a)-3*ln(b*x^(1/3)+a)/a^2+1/a^2*ln(x)
```

maxima [A] time = 0.54, size = 34, normalized size = 0.89

$$\frac{3}{abx^{\frac{1}{3}} + a^2} - \frac{3\log\left(bx^{\frac{1}{3}} + a\right)}{a^2} + \frac{\log(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))^2/x,x, algorithm="maxima")
```

```
[Out] 3/(a*b*x^(1/3) + a^2) - 3*log(b*x^(1/3) + a)/a^2 + log(x)/a^2
```

mupad [B] time = 0.05, size = 32, normalized size = 0.84

$$\frac{3}{a\left(a + b\,x^{1/3}\right)} - \frac{6\,\mathrm{atanh}\left(\frac{2\,b\,x^{1/3}}{a} + 1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^(1/3))^2),x)
```

```
[Out] 3/(a*(a + b*x^(1/3))) - (6*atanh((2*b*x^(1/3))/a + 1))/a^2
```

sympy [**A**] time = 1.38, size = 160, normalized size = 4.21

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{2}{3}}} & \text{for } a = 0 \wedge b = 0 \\ \frac{\log(x)}{a^2} & \text{for } b = 0 \\ -\frac{3}{2b^2 x^{\frac{2}{3}}} & \text{for } a = 0 \\ \frac{a x^{\frac{2}{3}} \log(x)}{a^3 x^{\frac{2}{3}} + a^2 b x} - \frac{3 a x^{\frac{2}{3}} \log\left(\frac{a}{b} + \sqrt[3]{x}\right)}{a^3 x^{\frac{2}{3}} + a^2 b x} + \frac{3 a x^{\frac{2}{3}}}{a^3 x^{\frac{2}{3}} + a^2 b x} + \frac{b x \log(x)}{a^3 x^{\frac{2}{3}} + a^2 b x} - \frac{3 b x \log\left(\frac{a}{b} + \sqrt[3]{x}\right)}{a^3 x^{\frac{2}{3}} + a^2 b x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**(1/3))**2/x,x)
```

```
[Out] Piecewise((zoo/x**(2/3), Eq(a, 0) & Eq(b, 0)), (log(x)/a**2, Eq(b, 0)), (-3
/(2*b**2*x**(2/3)), Eq(a, 0)), (a*x**(2/3)*log(x)/(a**3*x**(2/3) + a**2*b*x
) - 3*a*x**(2/3)*log(a/b + x**(1/3))/(a**3*x**(2/3) + a**2*b*x) + 3*a*x**(2
/3)/(a**3*x**(2/3) + a**2*b*x) + b*x*log(x)/(a**3*x**(2/3) + a**2*b*x) - 3*
b*x*log(a/b + x**(1/3))/(a**3*x**(2/3) + a**2*b*x), True))
```

3.2370 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x^2}\,dx$

Optimal. Leaf size=80

$$\frac{12b^3\log\left(a+b\sqrt[3]{x}\right)}{a^5}-\frac{4b^3\log(x)}{a^5}-\frac{3b^3}{a^4\left(a+b\sqrt[3]{x}\right)}-\frac{9b^2}{a^4\sqrt[3]{x}}+\frac{3b}{a^3x^{2/3}}-\frac{1}{a^2x}$$

```
[Out] -3*b^3/a^4/(a+b*x^(1/3))-1/a^2/x+3*b/a^3/x^(2/3)-9*b^2/a^4/x^(1/3)+12*b^3*l
n(a+b*x^(1/3))/a^5-4*b^3*ln(x)/a^5
```

Rubi [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 44}

$$-\frac{3b^3}{a^4\left(a+b\sqrt[3]{x}\right)}-\frac{9b^2}{a^4\sqrt[3]{x}}+\frac{12b^3\log\left(a+b\sqrt[3]{x}\right)}{a^5}-\frac{4b^3\log(x)}{a^5}+\frac{3b}{a^3x^{2/3}}-\frac{1}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^(1/3))^2*x^2),x]
```

```
[Out] (-3*b^3)/(a^4*(a + b*x^(1/3))) - 1/(a^2*x) + (3*b)/(a^3*x^(2/3)) - (9*b^2)/
(a^4*x^(1/3)) + (12*b^3*Log[a + b*x^(1/3)])/a^5 - (4*b^3*Log[x])/a^5
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x^2}\,dx &= 3\,\text{Subst}\left(\int \frac{1}{x^4(a+bx)^2}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int\left(\frac{1}{a^2x^4}-\frac{2b}{a^3x^3}+\frac{3b^2}{a^4x^2}-\frac{4b^3}{a^5x}+\frac{b^4}{a^4(a+bx)^2}+\frac{4b^4}{a^5(a+bx)}\right)dx, x, \sqrt[3]{x}\right)\\ &= -\frac{3b^3}{a^4\left(a+b\sqrt[3]{x}\right)}-\frac{1}{a^2x}+\frac{3b}{a^3x^{2/3}}-\frac{9b^2}{a^4\sqrt[3]{x}}+\frac{12b^3\log\left(a+b\sqrt[3]{x}\right)}{a^5}-\frac{4b^3\log(x)}{a^5}\end{aligned}$$

Mathematica [A] time = 0.10, size = 77, normalized size = 0.96

$$\frac{-\frac{a^4-2a^3b\sqrt[3]{x}+6a^2b^2x^{2/3}+12ab^3x}{ax+bx^{4/3}}+12b^3\log\left(a+b\sqrt[3]{x}\right)-4b^3\log(x)}{a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^(1/3))^2*x^2),x]
```

```
[Out] (-((a^4 - 2*a^3*b*x^(1/3) + 6*a^2*b^2*x^(2/3) + 12*a*b^3*x)/(a*x + b*x^(4/3
))) + 12*b^3*Log[a + b*x^(1/3)] - 4*b^3*Log[x])/a^5
```

fricas [A] time = 0.74, size = 122, normalized size = 1.52

$$-\frac{4\,a^3b^3x + a^6 - 12\left(b^6x^2 + a^3b^3x\right)\log\left(bx^{\frac{1}{3}} + a\right) + 12\left(b^6x^2 + a^3b^3x\right)\log\left(x^{\frac{1}{3}}\right) + 3\left(4\,ab^5x + 3\,a^4b^2\right)x^{\frac{2}{3}} - 3\left(2\,a\right.}{a^5b^3x^2 + a^8x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))^2/x^2,x, algorithm="fricas")
```

```
[Out] -(4*a^3*b^3*x + a^6 - 12*(b^6*x^2 + a^3*b^3*x)*log(b*x^(1/3) + a) + 12*(b^6
*x^2 + a^3*b^3*x)*log(x^(1/3)) + 3*(4*a*b^5*x + 3*a^4*b^2)*x^(2/3) - 3*(2*a
^2*b^4*x + a^5*b)*x^(1/3))/(a^5*b^3*x^2 + a^8*x)
```

giac [A] time = 0.17, size = 77, normalized size = 0.96

$$\frac{12\,b^3\log\left(\left|bx^{\frac{1}{3}} + a\right|\right)}{a^5} - \frac{4\,b^3\log\left(|x|\right)}{a^5} - \frac{12\,ab^3x + 6\,a^2b^2x^{\frac{2}{3}} - 2\,a^3bx^{\frac{1}{3}} + a^4}{\left(bx^{\frac{1}{3}} + a\right)a^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))^2/x^2,x, algorithm="giac")
```

```
[Out] 12*b^3*log(abs(b*x^(1/3) + a))/a^5 - 4*b^3*log(abs(x))/a^5 - (12*a*b^3*x +
6*a^2*b^2*x^(2/3) - 2*a^3*b*x^(1/3) + a^4)/((b*x^(1/3) + a)*a^5*x)
```

maple [A] time = 0.01, size = 73, normalized size = 0.91

$$-\frac{3b^3}{\left(b\,x^{\frac{1}{3}} + a\right)a^4} - \frac{4b^3\ln(x)}{a^5} + \frac{12b^3\ln\left(b\,x^{\frac{1}{3}} + a\right)}{a^5} - \frac{9b^2}{a^4x^{\frac{1}{3}}} + \frac{3b}{a^3x^{\frac{2}{3}}} - \frac{1}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^(1/3)+a)^2/x^2,x)
```

```
[Out] -3*b^3/a^4/(b*x^(1/3)+a)-1/a^2/x+3*b/a^3/x^(2/3)-9*b^2/a^4/x^(1/3)+12*b^3*l
n(b*x^(1/3)+a)/a^5-4*b^3*ln(x)/a^5
```

maxima [A] time = 0.57, size = 73, normalized size = 0.91

$$-\frac{12\,b^3x + 6\,ab^2x^{\frac{2}{3}} - 2\,a^2bx^{\frac{1}{3}} + a^3}{a^4bx^{\frac{4}{3}} + a^5x} + \frac{12\,b^3\log\left(bx^{\frac{1}{3}} + a\right)}{a^5} - \frac{4\,b^3\log(x)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))^2/x^2,x, algorithm="maxima")
```

```
[Out] -(12*b^3*x + 6*a*b^2*x^(2/3) - 2*a^2*b*x^(1/3) + a^3)/(a^4*b*x^(4/3) + a^5*
x) + 12*b^3*log(b*x^(1/3) + a)/a^5 - 4*b^3*log(x)/a^5
```

mupad [B] time = 1.16, size = 67, normalized size = 0.84

$$\frac{24\,b^3\,\mathrm{atanh}\left(\frac{2\,b\,x^{1/3}}{a} + 1\right)}{a^5} - \frac{\frac{1}{a} - \frac{2\,b\,x^{1/3}}{a^2} + \frac{12\,b^3\,x}{a^4} + \frac{6\,b^2\,x^{2/3}}{a^3}}{a\,x + b\,x^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^(1/3))^2),x)
```

```
[Out] (24*b^3*atanh((2*b*x^(1/3))/a + 1))/a^5 - (1/a - (2*b*x^(1/3))/a^2 + (12*b^
3*x)/a^4 + (6*b^2*x^(2/3))/a^3)/(a*x + b*x^(4/3))
```

sympy [A] time = 5.57, size = 272, normalized size = 3.40

$$\begin{cases} \frac{\infty}{x^{\frac{5}{3}}} \\ -\frac{1}{a^2 x} \\ -\frac{3}{5b^2 x^{\frac{5}{3}}} \\ -\frac{a^4 x^{\frac{2}{3}}}{a^6 x^{\frac{5}{3}} + a^5 b x^2} + \frac{2a^3 bx}{a^6 x^{\frac{5}{3}} + a^5 b x^2} - \frac{6a^2 b^2 x^{\frac{4}{3}}}{a^6 x^{\frac{5}{3}} + a^5 b x^2} - \frac{4ab^3 x^{\frac{5}{3}} \log(x)}{a^6 x^{\frac{5}{3}} + a^5 b x^2} + \frac{12ab^3 x^{\frac{5}{3}} \log\left(\frac{a}{b} + \sqrt[3]{x}\right)}{a^6 x^{\frac{5}{3}} + a^5 b x^2} - \frac{12ab^3 x^{\frac{5}{3}}}{a^6 x^{\frac{5}{3}} + a^5 b x^2} - \frac{4b^4 x^2 \log(x)}{a^6 x^{\frac{5}{3}} + a^5 b x^2} + \frac{12b^4 x^2 \log\left(\frac{a}{b} + \sqrt[3]{x}\right)}{a^6 x^{\frac{5}{3}} + a^5 b x^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**(1/3))**2/x**2,x)
```

```
[Out] Piecewise((zoo/x**(5/3), Eq(a, 0) & Eq(b, 0)), (-1/(a**2*x), Eq(b, 0)), (-3
/(5*b**2*x**(5/3)), Eq(a, 0)), (-a**4*x**(2/3)/(a**6*x**(5/3) + a**5*b*x**2
) + 2*a**3*b*x/(a**6*x**(5/3) + a**5*b*x**2) - 6*a**2*b**2*x**(4/3)/(a**6*x
**(5/3) + a**5*b*x**2) - 4*a*b**3*x**(5/3)*log(x)/(a**6*x**(5/3) + a**5*b*x
**2) + 12*a*b**3*x**(5/3)*log(a/b + x**(1/3))/(a**6*x**(5/3) + a**5*b*x**2)
 - 12*a*b**3*x**(5/3)/(a**6*x**(5/3) + a**5*b*x**2) - 4*b**4*x**2*log(x)/(a
**6*x**(5/3) + a**5*b*x**2) + 12*b**4*x**2*log(a/b + x**(1/3))/(a**6*x**(5/
3) + a**5*b*x**2), True))
```

3.2371 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x^3}\,dx$

Optimal. Leaf size=125

$$-\frac{21b^6\log\left(a+b\sqrt[3]{x}\right)}{a^8}+\frac{7b^6\log(x)}{a^8}+\frac{3b^6}{a^7\left(a+b\sqrt[3]{x}\right)}+\frac{18b^5}{a^7\sqrt[3]{x}}-\frac{15b^4}{2a^6x^{2/3}}+\frac{4b^3}{a^5x}-\frac{9b^2}{4a^4x^{4/3}}+\frac{6b}{5a^3x^{5/3}}-\frac{1}{2a^2x^2}$$

```
[Out] 3*b^6/a^7/(a+b*x^(1/3))-1/2/a^2/x^2+6/5*b/a^3/x^(5/3)-9/4*b^2/a^4/x^(4/3)+4
*b^3/a^5/x-15/2*b^4/a^6/x^(2/3)+18*b^5/a^7/x^(1/3)-21*b^6*ln(a+b*x^(1/3))/a
^8+7*b^6*ln(x)/a^8
```

Rubi [A] time = 0.08, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 44}

$$-\frac{15b^4}{2a^6x^{2/3}}-\frac{9b^2}{4a^4x^{4/3}}+\frac{3b^6}{a^7\left(a+b\sqrt[3]{x}\right)}+\frac{18b^5}{a^7\sqrt[3]{x}}+\frac{4b^3}{a^5x}-\frac{21b^6\log\left(a+b\sqrt[3]{x}\right)}{a^8}+\frac{7b^6\log(x)}{a^8}+\frac{6b}{5a^3x^{5/3}}-\frac{1}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^(1/3))^2*x^3),x]
```

```
[Out] (3*b^6)/(a^7*(a + b*x^(1/3))) - 1/(2*a^2*x^2) + (6*b)/(5*a^3*x^(5/3)) - (9*
b^2)/(4*a^4*x^(4/3)) + (4*b^3)/(a^5*x) - (15*b^4)/(2*a^6*x^(2/3)) + (18*b^5
)/(a^7*x^(1/3)) - (21*b^6*Log[a + b*x^(1/3)])/a^8 + (7*b^6*Log[x])/a^8
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x^3}\,dx &= 3\,\text{Subst}\left(\int \frac{1}{x^7(a+bx)^2}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int\left(\frac{1}{a^2x^7}-\frac{2b}{a^3x^6}+\frac{3b^2}{a^4x^5}-\frac{4b^3}{a^5x^4}+\frac{5b^4}{a^6x^3}-\frac{6b^5}{a^7x^2}+\frac{7b^6}{a^8x}-\frac{b^7}{a^7(a+bx)^2}-\frac{7b\right.\right.\\ &= \frac{3b^6}{a^7\left(a+b\sqrt[3]{x}\right)}-\frac{1}{2a^2x^2}+\frac{6b}{5a^3x^{5/3}}-\frac{9b^2}{4a^4x^{4/3}}+\frac{4b^3}{a^5x}-\frac{15b^4}{2a^6x^{2/3}}+\frac{18b^5}{a^7\sqrt[3]{x}}-\frac{21b^6\log\left(a+\right.}{a^8}\end{aligned}$$

Mathematica [A] time = 0.14, size = 117, normalized size = 0.94

$$\frac{\frac{a\left(-10a^6+14a^5b\sqrt[3]{x}-21a^4b^2x^{2/3}+35a^3b^3x-70a^2b^4x^{4/3}+210ab^5x^{5/3}+420b^6x^2\right)}{x^2\left(a+b\sqrt[3]{x}\right)}-420b^6\log\left(a+b\sqrt[3]{x}\right)+140b^6\log(x)}{20a^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^(1/3))^2*x^3),x]
```

```
[Out] ((a*(-10*a^6 + 14*a^5*b*x^(1/3) - 21*a^4*b^2*x^(2/3) + 35*a^3*b^3*x - 70*a^
2*b^4*x^(4/3) + 210*a*b^5*x^(5/3) + 420*b^6*x^2))/((a + b*x^(1/3))*x^2) - 4
20*b^6*Log[a + b*x^(1/3)] + 140*b^6*Log[x])/(20*a^8)
```

fricas [A] time = 0.61, size = 164, normalized size = 1.31

$$\frac{140\,a^3b^6x^2+70\,a^6b^3x-10\,a^9-420\left(b^9x^3+a^3b^6x^2\right)\log\left(bx^{\frac{1}{3}}+a\right)+420\left(b^9x^3+a^3b^6x^2\right)\log\left(x^{\frac{1}{3}}\right)+15\left(28\,ab^8x^2\right.}{20\left(a^8b^3x^3+a^{11}x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/20*(140*a^3*b^6*x^2 + 70*a^6*b^3*x - 10*a^9 - 420*(b^9*x^3 + a^3*b^6*x^2)
*log(b*x^(1/3) + a) + 420*(b^9*x^3 + a^3*b^6*x^2)*log(x^(1/3)) + 15*(28*a*b
^8*x^2 + 21*a^4*b^5*x - 3*a^7*b^2)*x^(2/3) - 6*(35*a^2*b^7*x^2 + 21*a^5*b^4
*x - 4*a^8*b)*x^(1/3))/(a^8*b^3*x^3 + a^11*x^2)
```

giac [A] time = 0.16, size = 112, normalized size = 0.90

$$-\frac{21\,b^6\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{a^8}+\frac{7\,b^6\log\left(|x|\right)}{a^8}+\frac{420\,ab^6x^2+210\,a^2b^5x^{\frac{5}{3}}-70\,a^3b^4x^{\frac{4}{3}}+35\,a^4b^3x-21\,a^5b^2x^{\frac{2}{3}}+14\,a^6bx^{\frac{1}{3}}-1}{20\left(bx^{\frac{1}{3}}+a\right)a^8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))^2/x^3,x, algorithm="giac")
```

```
[Out] -21*b^6*log(abs(b*x^(1/3) + a))/a^8 + 7*b^6*log(abs(x))/a^8 + 1/20*(420*a*b
^6*x^2 + 210*a^2*b^5*x^(5/3) - 70*a^3*b^4*x^(4/3) + 35*a^4*b^3*x - 21*a^5*b
^2*x^(2/3) + 14*a^6*b*x^(1/3) - 10*a^7)/((b*x^(1/3) + a)*a^8*x^2)
```

maple [A] time = 0.01, size = 106, normalized size = 0.85

$$\frac{3b^6}{\left(b\,x^{\frac{1}{3}}+a\right)a^7}+\frac{7b^6\ln(x)}{a^8}-\frac{21b^6\ln\left(b\,x^{\frac{1}{3}}+a\right)}{a^8}+\frac{18b^5}{a^7x^{\frac{1}{3}}}-\frac{15b^4}{2a^6x^{\frac{2}{3}}}+\frac{4b^3}{a^5x}-\frac{9b^2}{4a^4x^{\frac{4}{3}}}+\frac{6b}{5a^3x^{\frac{5}{3}}}-\frac{1}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^(1/3)+a)^2/x^3,x)
```

```
[Out] 3*b^6/a^7/(b*x^(1/3)+a)-1/2/a^2/x^2+6/5*b/a^3/x^(5/3)-9/4*b^2/a^4/x^(4/3)+4
*b^3/a^5/x-15/2*b^4/a^6/x^(2/3)+18*b^5/a^7/x^(1/3)-21*b^6*ln(b*x^(1/3)+a)/a
^8+7/a^8*b^6*ln(x)
```

maxima [A] time = 0.53, size = 110, normalized size = 0.88

$$\frac{420\,b^6x^2+210\,ab^5x^{\frac{5}{3}}-70\,a^2b^4x^{\frac{4}{3}}+35\,a^3b^3x-21\,a^4b^2x^{\frac{2}{3}}+14\,a^5bx^{\frac{1}{3}}-10\,a^6}{20\left(a^7bx^{\frac{7}{3}}+a^8x^2\right)}-\frac{21\,b^6\log\left(bx^{\frac{1}{3}}+a\right)}{a^8}+\frac{7\,b^6\log(x)}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/20*(420*b^6*x^2 + 210*a*b^5*x^(5/3) - 70*a^2*b^4*x^(4/3) + 35*a^3*b^3*x -
 21*a^4*b^2*x^(2/3) + 14*a^5*b*x^(1/3) - 10*a^6)/(a^7*b*x^(7/3) + a^8*x^2)
- 21*b^6*log(b*x^(1/3) + a)/a^8 + 7*b^6*log(x)/a^8
```

mupad [B] time = 1.16, size = 103, normalized size = 0.82

$$\frac{\frac{7\,b\,x^{1/3}}{10\,a^2}-\frac{1}{2\,a}+\frac{7\,b^3\,x}{4\,a^4}-\frac{21\,b^2\,x^{2/3}}{20\,a^3}+\frac{21\,b^6\,x^2}{a^7}-\frac{7\,b^4\,x^{4/3}}{2\,a^5}+\frac{21\,b^5\,x^{5/3}}{2\,a^6}}{a\,x^2+b\,x^{7/3}}-\frac{42\,b^6\,\mathrm{atanh}\left(\frac{2\,b\,x^{1/3}}{a}+1\right)}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^(1/3))^2),x)
```

```
[Out] ((7*b*x^(1/3))/(10*a^2) - 1/(2*a) + (7*b^3*x)/(4*a^4) - (21*b^2*x^(2/3))/(2
0*a^3) + (21*b^6*x^2)/a^7 - (7*b^4*x^(4/3))/(2*a^5) + (21*b^5*x^(5/3))/(2*a
^6))/(a*x^2 + b*x^(7/3)) - (42*b^6*atanh((2*b*x^(1/3))/a + 1))/a^8
```

sympy [A] time = 14.13, size = 405, normalized size = 3.24

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{8}{3}}} \\ -\frac{1}{2a^2x^2} \\ -\frac{3}{8b^2x^{\frac{8}{3}}} \\ -\frac{10a^7x^{\frac{2}{3}}}{20a^9x^{\frac{8}{3}}+20a^8bx^3}+\frac{14a^6bx}{20a^9x^{\frac{8}{3}}+20a^8bx^3}-\frac{21a^5b^2x^{\frac{4}{3}}}{20a^9x^{\frac{8}{3}}+20a^8bx^3}+\frac{35a^4b^3x^{\frac{5}{3}}}{20a^9x^{\frac{8}{3}}+20a^8bx^3}-\frac{70a^3b^4x^2}{20a^9x^{\frac{8}{3}}+20a^8bx^3}+\frac{210a^2b^5x^{\frac{7}{3}}}{20a^9x^{\frac{8}{3}}+20a^8bx^3}+\frac{140ab^6x^{\frac{8}{3}}\,\mathrm{lo}}{20a^9x^{\frac{8}{3}}+20} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**(1/3))**2/x**3,x)
```

```
[Out] Piecewise((zoo/x**(8/3), Eq(a, 0) & Eq(b, 0)), (-1/(2*a**2*x**2), Eq(b, 0))
, (-3/(8*b**2*x**(8/3)), Eq(a, 0)), (-10*a**7*x**(2/3)/(20*a**9*x**(8/3) +
20*a**8*b*x**3) + 14*a**6*b*x/(20*a**9*x**(8/3) + 20*a**8*b*x**3) - 21*a**5
*b**2*x**(4/3)/(20*a**9*x**(8/3) + 20*a**8*b*x**3) + 35*a**4*b**3*x**(5/3)/
(20*a**9*x**(8/3) + 20*a**8*b*x**3) - 70*a**3*b**4*x**2/(20*a**9*x**(8/3) +
 20*a**8*b*x**3) + 210*a**2*b**5*x**(7/3)/(20*a**9*x**(8/3) + 20*a**8*b*x**
3) + 140*a*b**6*x**(8/3)*log(x)/(20*a**9*x**(8/3) + 20*a**8*b*x**3) - 420*a
*b**6*x**(8/3)*log(a/b + x**(1/3))/(20*a**9*x**(8/3) + 20*a**8*b*x**3) + 42
0*a*b**6*x**(8/3)/(20*a**9*x**(8/3) + 20*a**8*b*x**3) + 140*b**7*x**3*log(x
)/(20*a**9*x**(8/3) + 20*a**8*b*x**3) - 420*b**7*x**3*log(a/b + x**(1/3))/(
20*a**9*x**(8/3) + 20*a**8*b*x**3), True))
```

3.2372 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x^4}\,dx$

Optimal. Leaf size=162

$$\frac{30b^9\log\left(a+b\sqrt[3]{x}\right)}{a^{11}}-\frac{10b^9\log(x)}{a^{11}}-\frac{3b^9}{a^{10}\left(a+b\sqrt[3]{x}\right)}-\frac{27b^8}{a^{10}\sqrt[3]{x}}+\frac{12b^7}{a^9x^{2/3}}-\frac{7b^6}{a^8x}+\frac{9b^5}{2a^7x^{4/3}}-\frac{3b^4}{a^6x^{5/3}}+\frac{2b^3}{a^5x^2}-\frac{9b^2}{7a^4x^{7/3}}+\frac{3b}{4a^3x^{8/}}$$

```
[Out] -3*b^9/a^10/(a+b*x^(1/3))-1/3/a^2/x^3+3/4*b/a^3/x^(8/3)-9/7*b^2/a^4/x^(7/3)
+2*b^3/a^5/x^2-3*b^4/a^6/x^(5/3)+9/2*b^5/a^7/x^(4/3)-7*b^6/a^8/x+12*b^7/a^9
/x^(2/3)-27*b^8/a^10/x^(1/3)+30*b^9*ln(a+b*x^(1/3))/a^11-10*b^9*ln(x)/a^11
```

Rubi [A] time = 0.11, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 44}

$$\frac{12b^7}{a^9x^{2/3}}+\frac{9b^5}{2a^7x^{4/3}}-\frac{3b^4}{a^6x^{5/3}}+\frac{2b^3}{a^5x^2}-\frac{9b^2}{7a^4x^{7/3}}-\frac{3b^9}{a^{10}\left(a+b\sqrt[3]{x}\right)}-\frac{27b^8}{a^{10}\sqrt[3]{x}}-\frac{7b^6}{a^8x}+\frac{30b^9\log\left(a+b\sqrt[3]{x}\right)}{a^{11}}-\frac{10b^9\log(x)}{a^{11}}+\frac{3b}{4a^3x^{8/}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^(1/3))^2*x^4),x]
```

```
[Out] (-3*b^9)/(a^10*(a + b*x^(1/3))) - 1/(3*a^2*x^3) + (3*b)/(4*a^3*x^(8/3)) - (
9*b^2)/(7*a^4*x^(7/3)) + (2*b^3)/(a^5*x^2) - (3*b^4)/(a^6*x^(5/3)) + (9*b^5
)/(2*a^7*x^(4/3)) - (7*b^6)/(a^8*x) + (12*b^7)/(a^9*x^(2/3)) - (27*b^8)/(a^
10*x^(1/3)) + (30*b^9*Log[a + b*x^(1/3)])/a^11 - (10*b^9*Log[x])/a^11
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x^4}\,dx = 3\,\text{Subst}\left(\int \frac{1}{x^{10}(a+bx)^2}\,dx, x, \sqrt[3]{x}\right)$$

$$= 3\,\text{Subst}\left(\int \left(\frac{1}{a^2x^{10}}-\frac{2b}{a^3x^9}+\frac{3b^2}{a^4x^8}-\frac{4b^3}{a^5x^7}+\frac{5b^4}{a^6x^6}-\frac{6b^5}{a^7x^5}+\frac{7b^6}{a^8x^4}-\frac{8b^7}{a^9x^3}+\frac{9b^8}{a^{10}x^2}-\frac{10b^9}{a^{11}x}\right.\right.$$

$$= -\frac{3b^9}{a^{10}\left(a+b\sqrt[3]{x}\right)}-\frac{1}{3a^2x^3}+\frac{3b}{4a^3x^{8/3}}-\frac{9b^2}{7a^4x^{7/3}}+\frac{2b^3}{a^5x^2}-\frac{3b^4}{a^6x^{5/3}}+\frac{9b^5}{2a^7x^{4/3}}-\frac{7b^6}{a^8x}+\frac{12b^7}{a^9x^{2/3}}$$

Mathematica [A] time = 0.23, size = 154, normalized size = 0.95

$$-\frac{\frac{a\left(28a^9-35a^8b\sqrt[3]{x}+45a^7b^2x^{2/3}-60a^6b^3x+84a^5b^4x^{4/3}-126a^4b^5x^{5/3}+210a^3b^6x^2-420a^2b^7x^{7/3}+1260ab^8x^{8/3}+2520b^9x^3\right)}{x^3\left(a+b\sqrt[3]{x}\right)}-2520b^9\log\left(a+b\sqrt[3]{x}\right.}{84a^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^(1/3))^2*x^4),x]
```

```
[Out] -1/84*((a*(28*a^9 - 35*a^8*b*x^(1/3) + 45*a^7*b^2*x^(2/3) - 60*a^6*b^3*x +
84*a^5*b^4*x^(4/3) - 126*a^4*b^5*x^(5/3) + 210*a^3*b^6*x^2 - 420*a^2*b^7*x^
(7/3) + 1260*a*b^8*x^(8/3) + 2520*b^9*x^3))/((a + b*x^(1/3))*x^3) - 2520*b^
9*Log[a + b*x^(1/3)] + 840*b^9*Log[x])/a^11
```

fricas [A] time = 0.60, size = 196, normalized size = 1.21

$$-\frac{840\,a^3b^9x^3+420\,a^6b^6x^2-140\,a^9b^3x+28\,a^{12}-2520\left(b^{12}x^4+a^3b^9x^3\right)\log\left(bx^{\frac{1}{3}}+a\right)+2520\left(b^{12}x^4+a^3b^9x^3\right.}{84\left(a^{11}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))^2/x^4,x, algorithm="fricas")
```

```
[Out] -1/84*(840*a^3*b^9*x^3 + 420*a^6*b^6*x^2 - 140*a^9*b^3*x + 28*a^12 - 2520*(
b^12*x^4 + a^3*b^9*x^3)*log(b*x^(1/3) + a) + 2520*(b^12*x^4 + a^3*b^9*x^3)*
log(x^(1/3)) + 18*(140*a*b^11*x^3 + 105*a^4*b^8*x^2 - 15*a^7*b^5*x + 6*a^10
*b^2)*x^(2/3) - 63*(20*a^2*b^10*x^3 + 12*a^5*b^7*x^2 - 3*a^8*b^4*x + a^11*b
)*x^(1/3))/(a^11*b^3*x^4 + a^14*x^3)
```

giac [A] time = 0.19, size = 145, normalized size = 0.90

$$\frac{30\,b^9\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{a^{11}}-\frac{10\,b^9\log\left(|x|\right)}{a^{11}}-\frac{2520\,ab^9x^3+1260\,a^2b^8x^{\frac{8}{3}}-420\,a^3b^7x^{\frac{7}{3}}+210\,a^4b^6x^2-126\,a^5b^5x^{\frac{5}{3}}+84}{84\left(bx^{\frac{1}{3}}+a\right)a^{11}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))^2/x^4,x, algorithm="giac")
```

```
[Out] 30*b^9*log(abs(b*x^(1/3) + a))/a^11 - 10*b^9*log(abs(x))/a^11 - 1/84*(2520*
a*b^9*x^3 + 1260*a^2*b^8*x^(8/3) - 420*a^3*b^7*x^(7/3) + 210*a^4*b^6*x^2 -
126*a^5*b^5*x^(5/3) + 84*a^6*b^4*x^(4/3) - 60*a^7*b^3*x + 45*a^8*b^2*x^(2/3
) - 35*a^9*b*x^(1/3) + 28*a^10)/((b*x^(1/3) + a)*a^11*x^3)
```

maple [A] time = 0.01, size = 139, normalized size = 0.86

$$-\frac{3b^9}{\left(b\,x^{\frac{1}{3}}+a\right)a^{10}}-\frac{10b^9\ln(x)}{a^{11}}+\frac{30b^9\ln\left(b\,x^{\frac{1}{3}}+a\right)}{a^{11}}-\frac{27b^8}{a^{10}x^{\frac{1}{3}}}+\frac{12b^7}{a^9x^{\frac{2}{3}}}-\frac{7b^6}{a^8x}+\frac{9b^5}{2a^7x^{\frac{4}{3}}}-\frac{3b^4}{a^6x^{\frac{5}{3}}}+\frac{2b^3}{a^5x^2}-\frac{9b^2}{7a^4x^{\frac{7}{3}}}+\frac{3b}{4a^3x^{\frac{8}{3}}}-\frac{3a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^(1/3)+a)^2/x^4,x)
```

```
[Out] -3*b^9/a^10/(b*x^(1/3)+a)-1/3/a^2/x^3+3/4*b/a^3/x^(8/3)-9/7*b^2/a^4/x^(7/3)
+2*b^3/a^5/x^2-3*b^4/a^6/x^(5/3)+9/2*b^5/a^7/x^(4/3)-7*b^6/a^8/x+12*b^7/a^9
/x^(2/3)-27*b^8/a^10/x^(1/3)+30*b^9*ln(b*x^(1/3)+a)/a^11-10*b^9*ln(x)/a^11
```

maxima [A] time = 0.53, size = 143, normalized size = 0.88

$$-\frac{2520\,b^9x^3+1260\,ab^8x^{\frac{8}{3}}-420\,a^2b^7x^{\frac{7}{3}}+210\,a^3b^6x^2-126\,a^4b^5x^{\frac{5}{3}}+84\,a^5b^4x^{\frac{4}{3}}-60\,a^6b^3x+45\,a^7b^2x^{\frac{2}{3}}-35\,a^{8}}{84\left(a^{10}bx^{\frac{10}{3}}+a^{11}x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/84*(2520*b^9*x^3 + 1260*a*b^8*x^(8/3) - 420*a^2*b^7*x^(7/3) + 210*a^3*b^
6*x^2 - 126*a^4*b^5*x^(5/3) + 84*a^5*b^4*x^(4/3) - 60*a^6*b^3*x + 45*a^7*b^
2*x^(2/3) - 35*a^8*b*x^(1/3) + 28*a^9)/(a^10*b*x^(10/3) + a^11*x^3) + 30*b^
9*log(b*x^(1/3) + a)/a^11 - 10*b^9*log(x)/a^11
```

mupad [B] time = 0.09, size = 136, normalized size = 0.84

$$\frac{60\,b^9\,\operatorname{atanh}\left(\frac{2\,b\,x^{1/3}}{a}+1\right)}{a^{11}}-\frac{\frac{1}{3\,a}-\frac{5\,b\,x^{1/3}}{12\,a^2}-\frac{5\,b^3\,x}{7\,a^4}+\frac{15\,b^2\,x^{2/3}}{28\,a^3}+\frac{5\,b^6\,x^2}{2\,a^7}+\frac{b^4\,x^{4/3}}{a^5}-\frac{3\,b^5\,x^{5/3}}{2\,a^6}+\frac{30\,b^9\,x^3}{a^{10}}-\frac{5\,b^7\,x^{7/3}}{a^8}+\frac{15\,b^8\,x^{8/3}}{a^9}}{a\,x^3+b\,x^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^(1/3))^2),x)
```

```
[Out] (60*b^9*atanh((2*b*x^(1/3))/a + 1))/a^11 - (1/(3*a) - (5*b*x^(1/3))/(12*a^2
) - (5*b^3*x)/(7*a^4) + (15*b^2*x^(2/3))/(28*a^3) + (5*b^6*x^2)/(2*a^7) + (
b^4*x^(4/3))/a^5 - (3*b^5*x^(5/3))/(2*a^6) + (30*b^9*x^3)/a^10 - (5*b^7*x^(
7/3))/a^8 + (15*b^8*x^(8/3))/a^9)/(a*x^3 + b*x^(10/3))
```

sympy [A] time = 33.66, size = 505, normalized size = 3.12

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{11}{3}}} \\ -\frac{3}{11b^2x^{\frac{11}{3}}} \\ -\frac{1}{3a^2x^3} \\ -\frac{28a^{10}x^{\frac{2}{3}}}{84a^{12}x^{\frac{11}{3}}+84a^{11}bx^4}+\frac{35a^9bx}{84a^{12}x^{\frac{11}{3}}+84a^{11}bx^4}-\frac{45a^8b^2x^{\frac{4}{3}}}{84a^{12}x^{\frac{11}{3}}+84a^{11}bx^4}+\frac{60a^7b^3x^{\frac{5}{3}}}{84a^{12}x^{\frac{11}{3}}+84a^{11}bx^4}-\frac{84a^6b^4x^2}{84a^{12}x^{\frac{11}{3}}+84a^{11}bx^4}+\frac{126a^5b^5x^{\frac{7}{3}}}{84a^{12}x^{\frac{11}{3}}+84a^{11}bx^4}- \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**(1/3))**2/x**4,x)
```

```
[Out] Piecewise((zoo/x**(11/3), Eq(a, 0) & Eq(b, 0)), (-3/(11*b**2*x**(11/3)), Eq
(a, 0)), (-1/(3*a**2*x**3), Eq(b, 0)), (-28*a**10*x**(2/3)/(84*a**12*x**(11
/3) + 84*a**11*b*x**4) + 35*a**9*b*x/(84*a**12*x**(11/3) + 84*a**11*b*x**4)
 - 45*a**8*b**2*x**(4/3)/(84*a**12*x**(11/3) + 84*a**11*b*x**4) + 60*a**7*b
**3*x**(5/3)/(84*a**12*x**(11/3) + 84*a**11*b*x**4) - 84*a**6*b**4*x**2/(84
*a**12*x**(11/3) + 84*a**11*b*x**4) + 126*a**5*b**5*x**(7/3)/(84*a**12*x**(
11/3) + 84*a**11*b*x**4) - 210*a**4*b**6*x**(8/3)/(84*a**12*x**(11/3) + 84*
a**11*b*x**4) + 420*a**3*b**7*x**3/(84*a**12*x**(11/3) + 84*a**11*b*x**4) -
 1260*a**2*b**8*x**(10/3)/(84*a**12*x**(11/3) + 84*a**11*b*x**4) - 840*a*b*
*9*x**(11/3)*log(x)/(84*a**12*x**(11/3) + 84*a**11*b*x**4) + 2520*a*b**9*x*
*(11/3)*log(a/b + x**(1/3))/(84*a**12*x**(11/3) + 84*a**11*b*x**4) - 2520*a
*b**9*x**(11/3)/(84*a**12*x**(11/3) + 84*a**11*b*x**4) - 840*b**10*x**4*log
(x)/(84*a**12*x**(11/3) + 84*a**11*b*x**4) + 2520*b**10*x**4*log(a/b + x**(
1/3))/(84*a**12*x**(11/3) + 84*a**11*b*x**4), True))
```

3.2373 $\int \frac{x^3}{\left(a+b\sqrt[3]{x}\right)^3}\,dx$

Optimal. Leaf size=171

$$\frac{3a^{11}}{2b^{12}\left(a+b\sqrt[3]{x}\right)^2}-\frac{33a^{10}}{b^{12}\left(a+b\sqrt[3]{x}\right)}-\frac{165a^9\log\left(a+b\sqrt[3]{x}\right)}{b^{12}}+\frac{135a^8\sqrt[3]{x}}{b^{11}}-\frac{54a^7x^{2/3}}{b^{10}}+\frac{28a^6x}{b^9}-\frac{63a^5x^{4/3}}{4b^8}+\frac{9a^4x^{5/3}}{b^7}-\frac{5a}{b}$$

```
[Out] 3/2*a^11/b^12/(a+b*x^(1/3))^2-33*a^10/b^12/(a+b*x^(1/3))+135*a^8*x^(1/3)/b^
11-54*a^7*x^(2/3)/b^10+28*a^6*x/b^9-63/4*a^5*x^(4/3)/b^8+9*a^4*x^(5/3)/b^7-
5*a^3*x^2/b^6+18/7*a^2*x^(7/3)/b^5-9/8*a*x^(8/3)/b^4+1/3*x^3/b^3-165*a^9*ln
(a+b*x^(1/3))/b^12
```

Rubi [A] time = 0.15, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$-\frac{54a^7x^{2/3}}{b^{10}}-\frac{63a^5x^{4/3}}{4b^8}+\frac{9a^4x^{5/3}}{b^7}-\frac{5a^3x^2}{b^6}+\frac{18a^2x^{7/3}}{7b^5}+\frac{3a^{11}}{2b^{12}\left(a+b\sqrt[3]{x}\right)^2}-\frac{33a^{10}}{b^{12}\left(a+b\sqrt[3]{x}\right)}+\frac{135a^8\sqrt[3]{x}}{b^{11}}+\frac{28a^6x}{b^9}-\frac{165a^9}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^(1/3))^3,x]
```

```
[Out] (3*a^11)/(2*b^12*(a + b*x^(1/3))^2) - (33*a^10)/(b^12*(a + b*x^(1/3))) + (1
35*a^8*x^(1/3))/b^11 - (54*a^7*x^(2/3))/b^10 + (28*a^6*x)/b^9 - (63*a^5*x^(
4/3))/(4*b^8) + (9*a^4*x^(5/3))/b^7 - (5*a^3*x^2)/b^6 + (18*a^2*x^(7/3))/(7
*b^5) - (9*a*x^(8/3))/(8*b^4) + x^3/(3*b^3) - (165*a^9*Log[a + b*x^(1/3)])/
b^12
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(a+b\sqrt[3]{x}\right)^3}\,dx &= 3\,\text{Subst}\left(\int \frac{x^{11}}{(a+bx)^3}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int\left(\frac{45a^8}{b^{11}}-\frac{36a^7x}{b^{10}}+\frac{28a^6x^2}{b^9}-\frac{21a^5x^3}{b^8}+\frac{15a^4x^4}{b^7}-\frac{10a^3x^5}{b^6}+\frac{6a^2x^6}{b^5}-\frac{3ax^7}{b^4}+\frac{x}{b}\right.\right.\\ &= \frac{3a^{11}}{2b^{12}\left(a+b\sqrt[3]{x}\right)^2}-\frac{33a^{10}}{b^{12}\left(a+b\sqrt[3]{x}\right)}+\frac{135a^8\sqrt[3]{x}}{b^{11}}-\frac{54a^7x^{2/3}}{b^{10}}+\frac{28a^6x}{b^9}-\frac{63a^5x^{4/3}}{4b^8}+\frac{9a^4x^{5/}}{b^7}\end{aligned}$$

Mathematica [A] time = 0.17, size = 157, normalized size = 0.92

$$\frac{\frac{252a^{11}}{\left(a+b\sqrt[3]{x}\right)^2}-\frac{5544a^{10}}{a+b\sqrt[3]{x}}-27720a^9\log\left(a+b\sqrt[3]{x}\right)+22680a^8b\sqrt[3]{x}-9072a^7b^2x^{2/3}+4704a^6b^3x-2646a^5b^4x^{4/3}+1512a}{168b^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^(1/3))^3,x]
```

```
[Out] ((252*a^11)/(a + b*x^(1/3))^2 - (5544*a^10)/(a + b*x^(1/3)) + 22680*a^8*b*x
^(1/3) - 9072*a^7*b^2*x^(2/3) + 4704*a^6*b^3*x - 2646*a^5*b^4*x^(4/3) + 151
2*a^4*b^5*x^(5/3) - 840*a^3*b^6*x^2 + 432*a^2*b^7*x^(7/3) - 189*a*b^8*x^(8/
3) + 56*b^9*x^3 - 27720*a^9*Log[a + b*x^(1/3)])/(168*b^12)
```

fricas [A] time = 0.76, size = 225, normalized size = 1.32

$$\frac{56\,b^{15}x^5-728\,a^3b^{12}x^4+3080\,a^6b^9x^3+8568\,a^9b^6x^2-1344\,a^{12}b^3x-5292\,a^{15}-27720\left(a^9b^6x^2+2\,a^{12}b^3x+a^{15}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^(1/3))^3,x, algorithm="fricas")
```

```
[Out] 1/168*(56*b^15*x^5 - 728*a^3*b^12*x^4 + 3080*a^6*b^9*x^3 + 8568*a^9*b^6*x^2
 - 1344*a^12*b^3*x - 5292*a^15 - 27720*(a^9*b^6*x^2 + 2*a^12*b^3*x + a^15)*
log(b*x^(1/3) + a) - 63*(3*a*b^14*x^4 - 18*a^4*b^11*x^3 + 99*a^7*b^8*x^2 +
352*a^10*b^5*x + 220*a^13*b^2)*x^(2/3) + 18*(24*a^2*b^13*x^4 - 99*a^5*b^10*
x^3 + 990*a^8*b^7*x^2 + 2695*a^11*b^4*x + 1540*a^14*b)*x^(1/3))/(b^18*x^2 +
 2*a^3*b^15*x + a^6*b^12)
```

giac [A] time = 0.17, size = 145, normalized size = 0.85

$$-\frac{165\,a^9\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{b^{12}}-\frac{3\left(22\,a^{10}bx^{\frac{1}{3}}+21\,a^{11}\right)}{2\left(bx^{\frac{1}{3}}+a\right)^2b^{12}}+\frac{56\,b^{24}x^3-189\,ab^{23}x^{\frac{8}{3}}+432\,a^2b^{22}x^{\frac{7}{3}}-840\,a^3b^{21}x^2+1512\,a^4b^2}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^(1/3))^3,x, algorithm="giac")
```

```
[Out] -165*a^9*log(abs(b*x^(1/3) + a))/b^12 - 3/2*(22*a^10*b*x^(1/3) + 21*a^11)/(
(b*x^(1/3) + a)^2*b^12) + 1/168*(56*b^24*x^3 - 189*a*b^23*x^(8/3) + 432*a^2
*b^22*x^(7/3) - 840*a^3*b^21*x^2 + 1512*a^4*b^20*x^(5/3) - 2646*a^5*b^19*x^
(4/3) + 4704*a^6*b^18*x - 9072*a^7*b^17*x^(2/3) + 22680*a^8*b^16*x^(1/3))/b
^27
```

maple [A] time = 0.01, size = 144, normalized size = 0.84

$$\frac{x^3}{3b^3}-\frac{9a\,x^{\frac{8}{3}}}{8b^4}+\frac{18a^2x^{\frac{7}{3}}}{7b^5}+\frac{3a^{11}}{2\left(b\,x^{\frac{1}{3}}+a\right)^2b^{12}}-\frac{5a^3x^2}{b^6}+\frac{9a^4x^{\frac{5}{3}}}{b^7}-\frac{63a^5x^{\frac{4}{3}}}{4b^8}-\frac{33a^{10}}{\left(b\,x^{\frac{1}{3}}+a\right)b^{12}}-\frac{165a^9\ln\left(b\,x^{\frac{1}{3}}+a\right)}{b^{12}}+\frac{28a^6x}{b^9}-\frac{5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^(1/3)+a)^3,x)
```

```
[Out] 3/2*a^11/b^12/(b*x^(1/3)+a)^2-33*a^10/b^12/(b*x^(1/3)+a)+135*a^8*x^(1/3)/b^
11-54*a^7*x^(2/3)/b^10+28*a^6*x/b^9-63/4*a^5*x^(4/3)/b^8+9*a^4*x^(5/3)/b^7-
```

```
5*a^3*x^2/b^6+18/7*a^2*x^(7/3)/b^5-9/8*a*x^(8/3)/b^4+1/3/b^3*x^3-165*a^9*ln
(b*x^(1/3)+a)/b^12
```

maxima [A] time = 0.51, size = 197, normalized size = 1.15

$$-\frac{165\,a^9\log\left(bx^{\frac{1}{3}}+a\right)}{b^{12}}+\frac{\left(bx^{\frac{1}{3}}+a\right)^9}{3\,b^{12}}-\frac{33\left(bx^{\frac{1}{3}}+a\right)^8 a}{8\,b^{12}}+\frac{165\left(bx^{\frac{1}{3}}+a\right)^7 a^2}{7\,b^{12}}-\frac{165\left(bx^{\frac{1}{3}}+a\right)^6 a^3}{2\,b^{12}}+\frac{198\left(bx^{\frac{1}{3}}+a\right)^5 a^4}{b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^(1/3))^3,x, algorithm="maxima")
```

```
[Out] -165*a^9*log(b*x^(1/3) + a)/b^12 + 1/3*(b*x^(1/3) + a)^9/b^12 - 33/8*(b*x^(
1/3) + a)^8*a/b^12 + 165/7*(b*x^(1/3) + a)^7*a^2/b^12 - 165/2*(b*x^(1/3) +
a)^6*a^3/b^12 + 198*(b*x^(1/3) + a)^5*a^4/b^12 - 693/2*(b*x^(1/3) + a)^4*a^
5/b^12 + 462*(b*x^(1/3) + a)^3*a^6/b^12 - 495*(b*x^(1/3) + a)^2*a^7/b^12 +
495*(b*x^(1/3) + a)*a^8/b^12 - 33*a^10/((b*x^(1/3) + a)*b^12) + 3/2*a^11/((
b*x^(1/3) + a)^2*b^12)
```

mupad [B] time = 0.07, size = 154, normalized size = 0.90

$$\frac{x^3}{3\,b^3}-\frac{\frac{63\,a^{11}}{2\,b}+33\,a^{10}\,x^{1/3}}{a^2\,b^{11}+b^{13}\,x^{2/3}+2\,a\,b^{12}\,x^{1/3}}-\frac{9\,a\,x^{8/3}}{8\,b^4}+\frac{28\,a^6\,x}{b^9}-\frac{165\,a^9\,\ln\left(a+b\,x^{1/3}\right)}{b^{12}}-\frac{5\,a^3\,x^2}{b^6}+\frac{18\,a^2\,x^{7/3}}{7\,b^5}+\frac{9\,a^4\,x^{5/3}}{b^7}-\frac{63}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^(1/3))^3,x)
```

```
[Out] x^3/(3*b^3) - ((63*a^11)/(2*b) + 33*a^10*x^(1/3))/(a^2*b^11 + b^13*x^(2/3)
+ 2*a*b^12*x^(1/3)) - (9*a*x^(8/3))/(8*b^4) + (28*a^6*x)/b^9 - (165*a^9*log
(a + b*x^(1/3)))/b^12 - (5*a^3*x^2)/b^6 + (18*a^2*x^(7/3))/(7*b^5) + (9*a^4
*x^(5/3))/b^7 - (63*a^5*x^(4/3))/(4*b^8) - (54*a^7*x^(2/3))/b^10 + (135*a^8
*x^(1/3))/b^11
```

sympy [A] time = 5.87, size = 624, normalized size = 3.65

$$\begin{cases} -\frac{27720a^{11}\log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{168a^2b^{12}+336ab^{13}\sqrt[3]{x}+168b^{14}x^{\frac{2}{3}}}-\frac{41580a^{11}}{168a^2b^{12}+336ab^{13}\sqrt[3]{x}+168b^{14}x^{\frac{2}{3}}}-\frac{55440a^{10}b\sqrt[3]{x}\log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{168a^2b^{12}+336ab^{13}\sqrt[3]{x}+168b^{14}x^{\frac{2}{3}}}-\frac{55440a^{10}b\sqrt[3]{x}}{168a^2b^{12}+336ab^{13}\sqrt[3]{x}+168} \\ \frac{x^4}{4a^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*x**(1/3))**3,x)
```

```
[Out] Piecewise((-27720*a**11*log(a/b + x**(1/3))/(168*a**2*b**12 + 336*a*b**13*x
**(1/3) + 168*b**14*x**(2/3)) - 41580*a**11/(168*a**2*b**12 + 336*a*b**13*x
**(1/3) + 168*b**14*x**(2/3)) - 55440*a**10*b*x**(1/3)*log(a/b + x**(1/3))/
(168*a**2*b**12 + 336*a*b**13*x**(1/3) + 168*b**14*x**(2/3)) - 55440*a**10*
b*x**(1/3)/(168*a**2*b**12 + 336*a*b**13*x**(1/3) + 168*b**14*x**(2/3)) - 2
7720*a**9*b**2*x**(2/3)*log(a/b + x**(1/3))/(168*a**2*b**12 + 336*a*b**13*x
**(1/3) + 168*b**14*x**(2/3)) + 9240*a**8*b**3*x/(168*a**2*b**12 + 336*a*b*
*13*x**(1/3) + 168*b**14*x**(2/3)) - 2310*a**7*b**4*x**(4/3)/(168*a**2*b**1
2 + 336*a*b**13*x**(1/3) + 168*b**14*x**(2/3)) + 924*a**6*b**5*x**(5/3)/(16
8*a**2*b**12 + 336*a*b**13*x**(1/3) + 168*b**14*x**(2/3)) - 462*a**5*b**6*x
**2/(168*a**2*b**12 + 336*a*b**13*x**(1/3) + 168*b**14*x**(2/3)) + 264*a**4
*b**7*x**(7/3)/(168*a**2*b**12 + 336*a*b**13*x**(1/3) + 168*b**14*x**(2/3))
 - 165*a**3*b**8*x**(8/3)/(168*a**2*b**12 + 336*a*b**13*x**(1/3) + 168*b**1
```

```
4*x**(2/3)) + 110*a**2*b**9*x**3/(168*a**2*b**12 + 336*a*b**13*x**(1/3) + 1
68*b**14*x**(2/3)) - 77*a*b**10*x**(10/3)/(168*a**2*b**12 + 336*a*b**13*x**
(1/3) + 168*b**14*x**(2/3)) + 56*b**11*x**(11/3)/(168*a**2*b**12 + 336*a*b*
*13*x**(1/3) + 168*b**14*x**(2/3)), Ne(b, 0)), (x**4/(4*a**3), True))
```

3.2374 $\int \frac{x^2}{\left(a+b\sqrt[3]{x}\right)^3}\,dx$

Optimal. Leaf size=134

$$-\frac{3a^8}{2b^9\left(a+b\sqrt[3]{x}\right)^2}+\frac{24a^7}{b^9\left(a+b\sqrt[3]{x}\right)}+\frac{84a^6\log\left(a+b\sqrt[3]{x}\right)}{b^9}-\frac{63a^5\sqrt[3]{x}}{b^8}+\frac{45a^4x^{2/3}}{2b^7}-\frac{10a^3x}{b^6}+\frac{9a^2x^{4/3}}{2b^5}-\frac{9ax^{5/3}}{5b^4}+\frac{x^2}{2b^3}$$

```
[Out] -3/2*a^8/b^9/(a+b*x^(1/3))^2+24*a^7/b^9/(a+b*x^(1/3))-63*a^5*x^(1/3)/b^8+45
/2*a^4*x^(2/3)/b^7-10*a^3*x/b^6+9/2*a^2*x^(4/3)/b^5-9/5*a*x^(5/3)/b^4+1/2*x
^2/b^3+84*a^6*ln(a+b*x^(1/3))/b^9
```

Rubi [A] time = 0.10, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{45a^4x^{2/3}}{2b^7}+\frac{9a^2x^{4/3}}{2b^5}-\frac{3a^8}{2b^9\left(a+b\sqrt[3]{x}\right)^2}+\frac{24a^7}{b^9\left(a+b\sqrt[3]{x}\right)}-\frac{63a^5\sqrt[3]{x}}{b^8}-\frac{10a^3x}{b^6}+\frac{84a^6\log\left(a+b\sqrt[3]{x}\right)}{b^9}-\frac{9ax^{5/3}}{5b^4}+\frac{x^2}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^(1/3))^3,x]
```

```
[Out] (-3*a^8)/(2*b^9*(a + b*x^(1/3))^2) + (24*a^7)/(b^9*(a + b*x^(1/3))) - (63*a
^5*x^(1/3))/b^8 + (45*a^4*x^(2/3))/(2*b^7) - (10*a^3*x)/b^6 + (9*a^2*x^(4/3
))/(2*b^5) - (9*a*x^(5/3))/(5*b^4) + x^2/(2*b^3) + (84*a^6*Log[a + b*x^(1/3
)])/b^9
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a+b\sqrt[3]{x}\right)^3}\,dx &= 3\,\text{Subst}\left(\int \frac{x^8}{(a+bx)^3}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int\left(-\frac{21a^5}{b^8}+\frac{15a^4x}{b^7}-\frac{10a^3x^2}{b^6}+\frac{6a^2x^3}{b^5}-\frac{3ax^4}{b^4}+\frac{x^5}{b^3}+\frac{a^8}{b^8(a+bx)^3}-\frac{8a^7}{b^8(a+bx)}\right.\right.\\ &= -\frac{3a^8}{2b^9\left(a+b\sqrt[3]{x}\right)^2}+\frac{24a^7}{b^9\left(a+b\sqrt[3]{x}\right)}-\frac{63a^5\sqrt[3]{x}}{b^8}+\frac{45a^4x^{2/3}}{2b^7}-\frac{10a^3x}{b^6}+\frac{9a^2x^{4/3}}{2b^5}-\frac{9ax^{5/3}}{5b^4}+\end{aligned}$$

Mathematica [A] time = 0.11, size = 120, normalized size = 0.90

$$\frac{-\frac{15a^8}{\left(a+b\sqrt[3]{x}\right)^2}+\frac{240a^7}{a+b\sqrt[3]{x}}+840a^6\log\left(a+b\sqrt[3]{x}\right)-630a^5b\sqrt[3]{x}+225a^4b^2x^{2/3}-100a^3b^3x+45a^2b^4x^{4/3}-18ab^5x^{5/3}+}{10b^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^(1/3))^3,x]
```

```
[Out] ((-15*a^8)/(a + b*x^(1/3))^2 + (240*a^7)/(a + b*x^(1/3)) - 630*a^5*b*x^(1/3
) + 225*a^4*b^2*x^(2/3) - 100*a^3*b^3*x + 45*a^2*b^4*x^(4/3) - 18*a*b^5*x^(
5/3) + 5*b^6*x^2 + 840*a^6*Log[a + b*x^(1/3)])/(10*b^9)
```

fricas [A] time = 0.65, size = 192, normalized size = 1.43

$$\frac{5\,b^{12}x^4 - 90\,a^3b^9x^3 - 195\,a^6b^6x^2 + 170\,a^9b^3x + 225\,a^{12} + 840\left(a^6b^6x^2 + 2\,a^9b^3x + a^{12}\right)\log\left(bx^{\frac{1}{3}} + a\right) - 3\left(6\,ab^{11}x\right.}{10\left(b^{15}x^2 + 2\,a^3b^{12}x + a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^(1/3))^3,x, algorithm="fricas")
```

```
[Out] 1/10*(5*b^12*x^4 - 90*a^3*b^9*x^3 - 195*a^6*b^6*x^2 + 170*a^9*b^3*x + 225*a
^12 + 840*(a^6*b^6*x^2 + 2*a^9*b^3*x + a^12)*log(b*x^(1/3) + a) - 3*(6*a*b^
11*x^3 - 63*a^4*b^8*x^2 - 224*a^7*b^5*x - 140*a^10*b^2)*x^(2/3) + 15*(3*a^2
*b^10*x^3 - 36*a^5*b^7*x^2 - 98*a^8*b^4*x - 56*a^11*b)*x^(1/3))/(b^15*x^2 +
 2*a^3*b^12*x + a^6*b^9)
```

giac [A] time = 0.21, size = 112, normalized size = 0.84

$$\frac{84\,a^6\log\left(\left|bx^{\frac{1}{3}} + a\right|\right)}{b^9} + \frac{3\left(16\,a^7bx^{\frac{1}{3}} + 15\,a^8\right)}{2\left(bx^{\frac{1}{3}} + a\right)^2b^9} + \frac{5\,b^{15}x^2 - 18\,ab^{14}x^{\frac{5}{3}} + 45\,a^2b^{13}x^{\frac{4}{3}} - 100\,a^3b^{12}x + 225\,a^4b^{11}x^{\frac{2}{3}} - 630}{10\,b^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^(1/3))^3,x, algorithm="giac")
```

```
[Out] 84*a^6*log(abs(b*x^(1/3) + a))/b^9 + 3/2*(16*a^7*b*x^(1/3) + 15*a^8)/((b*x^
(1/3) + a)^2*b^9) + 1/10*(5*b^15*x^2 - 18*a*b^14*x^(5/3) + 45*a^2*b^13*x^(4
/3) - 100*a^3*b^12*x + 225*a^4*b^11*x^(2/3) - 630*a^5*b^10*x^(1/3))/b^18
```

maple [A] time = 0.01, size = 111, normalized size = 0.83

$$-\frac{3a^8}{2\left(b\,x^{\frac{1}{3}} + a\right)^2b^9} + \frac{x^2}{2b^3} - \frac{9a\,x^{\frac{5}{3}}}{5b^4} + \frac{9a^2x^{\frac{4}{3}}}{2b^5} + \frac{24a^7}{\left(b\,x^{\frac{1}{3}} + a\right)b^9} + \frac{84a^6\ln\left(b\,x^{\frac{1}{3}} + a\right)}{b^9} - \frac{10a^3x}{b^6} + \frac{45a^4x^{\frac{2}{3}}}{2b^7} - \frac{63a^5x^{\frac{1}{3}}}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^(1/3)+a)^3,x)
```

```
[Out] -3/2*a^8/b^9/(b*x^(1/3)+a)^2+24*a^7/b^9/(b*x^(1/3)+a)-63*a^5*x^(1/3)/b^8+45
/2*a^4*x^(2/3)/b^7-10*a^3/b^6*x+9/2*a^2*x^(4/3)/b^5-9/5*a*x^(5/3)/b^4+1/2*x
^2/b^3+84*a^6*ln(b*x^(1/3)+a)/b^9
```

maxima [A] time = 0.51, size = 146, normalized size = 1.09

$$\frac{84\,a^6\log\left(bx^{\frac{1}{3}} + a\right)}{b^9} + \frac{\left(bx^{\frac{1}{3}} + a\right)^6}{2\,b^9} - \frac{24\left(bx^{\frac{1}{3}} + a\right)^5a}{5\,b^9} + \frac{21\left(bx^{\frac{1}{3}} + a\right)^4a^2}{b^9} - \frac{56\left(bx^{\frac{1}{3}} + a\right)^3a^3}{b^9} + \frac{105\left(bx^{\frac{1}{3}} + a\right)^2a^4}{b^9} - \frac{168\left(b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^(1/3))^3,x, algorithm="maxima")
```

```
[Out] 84*a^6*log(b*x^(1/3) + a)/b^9 + 1/2*(b*x^(1/3) + a)^6/b^9 - 24/5*(b*x^(1/3)
 + a)^5*a/b^9 + 21*(b*x^(1/3) + a)^4*a^2/b^9 - 56*(b*x^(1/3) + a)^3*a^3/b^9
 + 105*(b*x^(1/3) + a)^2*a^4/b^9 - 168*(b*x^(1/3) + a)*a^5/b^9 + 24*a^7/((b
*x^(1/3) + a)*b^9) - 3/2*a^8/((b*x^(1/3) + a)^2*b^9)
```

mupad [B] time = 0.05, size = 120, normalized size = 0.90

$$\frac{\frac{45\,a^8}{2\,b}+24\,a^7\,x^{1/3}}{a^2\,b^8+b^{10}\,x^{2/3}+2\,a\,b^9\,x^{1/3}}+\frac{x^2}{2\,b^3}-\frac{10\,a^3\,x}{b^6}-\frac{9\,a\,x^{5/3}}{5\,b^4}+\frac{84\,a^6\,\ln\left(a+b\,x^{1/3}\right)}{b^9}+\frac{9\,a^2\,x^{4/3}}{2\,b^5}+\frac{45\,a^4\,x^{2/3}}{2\,b^7}-\frac{63\,a^5\,x^{1/3}}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^(1/3))^3,x)
```

```
[Out] ((45*a^8)/(2*b) + 24*a^7*x^(1/3))/(a^2*b^8 + b^10*x^(2/3) + 2*a*b^9*x^(1/3)
) + x^2/(2*b^3) - (10*a^3*x)/b^6 - (9*a*x^(5/3))/(5*b^4) + (84*a^6*log(a +
b*x^(1/3)))/b^9 + (9*a^2*x^(4/3))/(2*b^5) + (45*a^4*x^(2/3))/(2*b^7) - (63*
a^5*x^(1/3))/b^8
```

sympy [A] time = 2.25, size = 493, normalized size = 3.68

$$\begin{cases} \frac{840a^8\log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{10a^2b^9+20ab^{10}\sqrt[3]{x}+10b^{11}x^{\frac{2}{3}}}+\frac{1260a^8}{10a^2b^9+20ab^{10}\sqrt[3]{x}+10b^{11}x^{\frac{2}{3}}}+\frac{1680a^7b\sqrt[3]{x}\log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{10a^2b^9+20ab^{10}\sqrt[3]{x}+10b^{11}x^{\frac{2}{3}}}+\frac{1680a^7b\sqrt[3]{x}}{10a^2b^9+20ab^{10}\sqrt[3]{x}+10b^{11}x^{\frac{2}{3}}}+\frac{840a^6b}{10a^2b^9+2} \\ \frac{x^3}{3a^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*x**(1/3))**3,x)
```

```
[Out] Piecewise((840*a**8*log(a/b + x**(1/3))/(10*a**2*b**9 + 20*a*b**10*x**(1/3)
 + 10*b**11*x**(2/3)) + 1260*a**8/(10*a**2*b**9 + 20*a*b**10*x**(1/3) + 10*
b**11*x**(2/3)) + 1680*a**7*b*x**(1/3)*log(a/b + x**(1/3))/(10*a**2*b**9 +
20*a*b**10*x**(1/3) + 10*b**11*x**(2/3)) + 1680*a**7*b*x**(1/3)/(10*a**2*b*
*9 + 20*a*b**10*x**(1/3) + 10*b**11*x**(2/3)) + 840*a**6*b**2*x**(2/3)*log(
a/b + x**(1/3))/(10*a**2*b**9 + 20*a*b**10*x**(1/3) + 10*b**11*x**(2/3)) -
280*a**5*b**3*x/(10*a**2*b**9 + 20*a*b**10*x**(1/3) + 10*b**11*x**(2/3)) +
70*a**4*b**4*x**(4/3)/(10*a**2*b**9 + 20*a*b**10*x**(1/3) + 10*b**11*x**(2/
3)) - 28*a**3*b**5*x**(5/3)/(10*a**2*b**9 + 20*a*b**10*x**(1/3) + 10*b**11*
x**(2/3)) + 14*a**2*b**6*x**2/(10*a**2*b**9 + 20*a*b**10*x**(1/3) + 10*b**1
1*x**(2/3)) - 8*a*b**7*x**(7/3)/(10*a**2*b**9 + 20*a*b**10*x**(1/3) + 10*b*
*11*x**(2/3)) + 5*b**8*x**(8/3)/(10*a**2*b**9 + 20*a*b**10*x**(1/3) + 10*b*
*11*x**(2/3)), Ne(b, 0)), (x**3/(3*a**3), True))
```

3.2375 $\int \frac{x}{\left(a+b\sqrt[3]{x}\right)^3}\,dx$

Optimal. Leaf size=90

$$\frac{3a^5}{2b^6\left(a+b\sqrt[3]{x}\right)^2}-\frac{15a^4}{b^6\left(a+b\sqrt[3]{x}\right)}-\frac{30a^3\log\left(a+b\sqrt[3]{x}\right)}{b^6}+\frac{18a^2\sqrt[3]{x}}{b^5}-\frac{9ax^{2/3}}{2b^4}+\frac{x}{b^3}$$

```
[Out] 3/2*a^5/b^6/(a+b*x^(1/3))^2-15*a^4/b^6/(a+b*x^(1/3))+18*a^2*x^(1/3)/b^5-9/2
*a*x^(2/3)/b^4+x/b^3-30*a^3*ln(a+b*x^(1/3))/b^6
```

Rubi [A] time = 0.06, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{3a^5}{2b^6\left(a+b\sqrt[3]{x}\right)^2}-\frac{15a^4}{b^6\left(a+b\sqrt[3]{x}\right)}+\frac{18a^2\sqrt[3]{x}}{b^5}-\frac{30a^3\log\left(a+b\sqrt[3]{x}\right)}{b^6}-\frac{9ax^{2/3}}{2b^4}+\frac{x}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^(1/3))^3,x]
```

```
[Out] (3*a^5)/(2*b^6*(a + b*x^(1/3))^2) - (15*a^4)/(b^6*(a + b*x^(1/3))) + (18*a^
2*x^(1/3))/b^5 - (9*a*x^(2/3))/(2*b^4) + x/b^3 - (30*a^3*Log[a + b*x^(1/3)]
)/b^6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a+b\sqrt[3]{x}\right)^3}\,dx &= 3\,\text{Subst}\left(\int \frac{x^5}{(a+bx)^3}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int\left(\frac{6a^2}{b^5}-\frac{3ax}{b^4}+\frac{x^2}{b^3}-\frac{a^5}{b^5(a+bx)^3}+\frac{5a^4}{b^5(a+bx)^2}-\frac{10a^3}{b^5(a+bx)}\right)dx, x, \sqrt[3]{x}\right)\\ &= \frac{3a^5}{2b^6\left(a+b\sqrt[3]{x}\right)^2}-\frac{15a^4}{b^6\left(a+b\sqrt[3]{x}\right)}+\frac{18a^2\sqrt[3]{x}}{b^5}-\frac{9ax^{2/3}}{2b^4}+\frac{x}{b^3}-\frac{30a^3\log\left(a+b\sqrt[3]{x}\right)}{b^6}\end{aligned}$$

Mathematica [A] time = 0.07, size = 83, normalized size = 0.92

$$\frac{\frac{3a^5}{\left(a+b\sqrt[3]{x}\right)^2}-\frac{30a^4}{a+b\sqrt[3]{x}}-60a^3\log\left(a+b\sqrt[3]{x}\right)+36a^2b\sqrt[3]{x}-9ab^2x^{2/3}+2b^3x}{2b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^(1/3))^3,x]
```

```
[Out] ((3*a^5)/(a + b*x^(1/3))^2 - (30*a^4)/(a + b*x^(1/3)) + 36*a^2*b*x^(1/3) -
9*a*b^2*x^(2/3) + 2*b^3*x - 60*a^3*Log[a + b*x^(1/3)])/(2*b^6)
```

fricas [B] time = 0.75, size = 159, normalized size = 1.77

$$\frac{2\,b^9x^3+4\,a^3b^6x^2-34\,a^6b^3x-27\,a^9-60\left(a^3b^6x^2+2\,a^6b^3x+a^9\right)\log\left(bx^{\frac{1}{3}}+a\right)-3\left(3\,ab^8x^2+16\,a^4b^5x+10\,a}{2\left(b^{12}x^2+2\,a^3b^9x+a^6b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^(1/3))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^9*x^3 + 4*a^3*b^6*x^2 - 34*a^6*b^3*x - 27*a^9 - 60*(a^3*b^6*x^2 +
2*a^6*b^3*x + a^9)*log(b*x^(1/3) + a) - 3*(3*a*b^8*x^2 + 16*a^4*b^5*x + 10*
a^7*b^2)*x^(2/3) + 3*(12*a^2*b^7*x^2 + 35*a^5*b^4*x + 20*a^8*b)*x^(1/3))/(b
^12*x^2 + 2*a^3*b^9*x + a^6*b^6)
```

giac [A] time = 0.15, size = 79, normalized size = 0.88

$$-\frac{30\,a^3\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{b^6}-\frac{3\left(10\,a^4bx^{\frac{1}{3}}+9\,a^5\right)}{2\left(bx^{\frac{1}{3}}+a\right)^2b^6}+\frac{2\,b^6x-9\,ab^5x^{\frac{2}{3}}+36\,a^2b^4x^{\frac{1}{3}}}{2\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^(1/3))^3,x, algorithm="giac")
```

```
[Out] -30*a^3*log(abs(b*x^(1/3) + a))/b^6 - 3/2*(10*a^4*b*x^(1/3) + 9*a^5)/((b*x^
(1/3) + a)^2*b^6) + 1/2*(2*b^6*x - 9*a*b^5*x^(2/3) + 36*a^2*b^4*x^(1/3))/b^
9
```

maple [A] time = 0.00, size = 77, normalized size = 0.86

$$\frac{3a^5}{2\left(b\,x^{\frac{1}{3}}+a\right)^2b^6}-\frac{15a^4}{\left(b\,x^{\frac{1}{3}}+a\right)b^6}-\frac{30a^3\ln\left(b\,x^{\frac{1}{3}}+a\right)}{b^6}+\frac{x}{b^3}-\frac{9a\,x^{\frac{2}{3}}}{2b^4}+\frac{18a^2x^{\frac{1}{3}}}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^(1/3)+a)^3,x)
```

```
[Out] 3/2*a^5/b^6/(b*x^(1/3)+a)^2-15*a^4/b^6/(b*x^(1/3)+a)+18*a^2*x^(1/3)/b^5-9/2
*a*x^(2/3)/b^4+x/b^3-30*a^3*ln(b*x^(1/3)+a)/b^6
```

maxima [A] time = 0.63, size = 94, normalized size = 1.04

$$-\frac{30\,a^3\log\left(bx^{\frac{1}{3}}+a\right)}{b^6}+\frac{\left(bx^{\frac{1}{3}}+a\right)^3}{b^6}-\frac{15\left(bx^{\frac{1}{3}}+a\right)^2a}{2\,b^6}+\frac{30\left(bx^{\frac{1}{3}}+a\right)a^2}{b^6}-\frac{15\,a^4}{\left(bx^{\frac{1}{3}}+a\right)b^6}+\frac{3\,a^5}{2\left(bx^{\frac{1}{3}}+a\right)^2b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^(1/3))^3,x, algorithm="maxima")
```

```
[Out] -30*a^3*log(b*x^(1/3) + a)/b^6 + (b*x^(1/3) + a)^3/b^6 - 15/2*(b*x^(1/3) +
a)^2*a/b^6 + 30*(b*x^(1/3) + a)*a^2/b^6 - 15*a^4/((b*x^(1/3) + a)*b^6) + 3/
2*a^5/((b*x^(1/3) + a)^2*b^6)
```

mupad [**B**] time = 0.04, size = 87, normalized size = 0.97

$$\frac{x}{b^3}-\frac{\frac{27\,a^5}{2\,b}+15\,a^4\,x^{1/3}}{a^2\,b^5+b^7\,x^{2/3}+2\,a\,b^6\,x^{1/3}}-\frac{9\,a\,x^{2/3}}{2\,b^4}-\frac{30\,a^3\,\ln\left(a+b\,x^{1/3}\right)}{b^6}+\frac{18\,a^2\,x^{1/3}}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^(1/3))^3,x)
```

```
[Out] x/b^3 - ((27*a^5)/(2*b) + 15*a^4*x^(1/3))/(a^2*b^5 + b^7*x^(2/3) + 2*a*b^6*
x^(1/3)) - (9*a*x^(2/3))/(2*b^4) - (30*a^3*log(a + b*x^(1/3)))/b^6 + (18*a^
2*x^(1/3))/b^5
```

sympy [**A**] time = 0.87, size = 362, normalized size = 4.02

$$\begin{cases}-\frac{60a^5\log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{2a^2b^6+4ab^7\sqrt[3]{x}+2b^8x^{\frac{2}{3}}}-\frac{90a^5}{2a^2b^6+4ab^7\sqrt[3]{x}+2b^8x^{\frac{2}{3}}}-\frac{120a^4b\sqrt[3]{x}\log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{2a^2b^6+4ab^7\sqrt[3]{x}+2b^8x^{\frac{2}{3}}}-\frac{120a^4b\sqrt[3]{x}}{2a^2b^6+4ab^7\sqrt[3]{x}+2b^8x^{\frac{2}{3}}}-\frac{60a^3b^2x^{\frac{2}{3}}\log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{2a^2b^6+4ab^7\sqrt[3]{x}+2b^8x^{\frac{2}{3}}}+\frac{}{2a^2b^6-} \\ \frac{x^2}{2a^3}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x**(1/3))**3,x)
```

```
[Out] Piecewise((-60*a**5*log(a/b + x**(1/3))/(2*a**2*b**6 + 4*a*b**7*x**(1/3) +
2*b**8*x**(2/3)) - 90*a**5/(2*a**2*b**6 + 4*a*b**7*x**(1/3) + 2*b**8*x**(2/
3)) - 120*a**4*b*x**(1/3)*log(a/b + x**(1/3))/(2*a**2*b**6 + 4*a*b**7*x**(1
/3) + 2*b**8*x**(2/3)) - 120*a**4*b*x**(1/3)/(2*a**2*b**6 + 4*a*b**7*x**(1/
3) + 2*b**8*x**(2/3)) - 60*a**3*b**2*x**(2/3)*log(a/b + x**(1/3))/(2*a**2*b
**6 + 4*a*b**7*x**(1/3) + 2*b**8*x**(2/3)) + 20*a**2*b**3*x/(2*a**2*b**6 +
4*a*b**7*x**(1/3) + 2*b**8*x**(2/3)) - 5*a*b**4*x**(4/3)/(2*a**2*b**6 + 4*a
*b**7*x**(1/3) + 2*b**8*x**(2/3)) + 2*b**5*x**(5/3)/(2*a**2*b**6 + 4*a*b**7
*x**(1/3) + 2*b**8*x**(2/3)), Ne(b, 0)), (x**2/(2*a**3), True))
```

3.2376 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3}\,dx$

Optimal. Leaf size=54

$$-\frac{3a^2}{2b^3\left(a+b\sqrt[3]{x}\right)^2}+\frac{6a}{b^3\left(a+b\sqrt[3]{x}\right)}+\frac{3\log\left(a+b\sqrt[3]{x}\right)}{b^3}$$

```
[Out] -3/2*a^2/b^3/(a+b*x^(1/3))^2+6*a/b^3/(a+b*x^(1/3))+3*ln(a+b*x^(1/3))/b^3
```

Rubi [A] time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {190, 43}

$$-\frac{3a^2}{2b^3\left(a+b\sqrt[3]{x}\right)^2}+\frac{6a}{b^3\left(a+b\sqrt[3]{x}\right)}+\frac{3\log\left(a+b\sqrt[3]{x}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(1/3))^(-3),x]
```

```
[Out] (-3*a^2)/(2*b^3*(a + b*x^(1/3))^2) + (6*a)/(b^3*(a + b*x^(1/3))) + (3*Log[a
 + b*x^(1/3)])/b^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
 - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
  IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3}\,dx &= 3\,\text{Subst}\left(\int \frac{x^2}{(a+bx)^3}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(\frac{a^2}{b^2(a+bx)^3}-\frac{2a}{b^2(a+bx)^2}+\frac{1}{b^2(a+bx)}\right)dx, x, \sqrt[3]{x}\right)\\ &= -\frac{3a^2}{2b^3\left(a+b\sqrt[3]{x}\right)^2}+\frac{6a}{b^3\left(a+b\sqrt[3]{x}\right)}+\frac{3\log\left(a+b\sqrt[3]{x}\right)}{b^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 45, normalized size = 0.83

$$\frac{3\left(\frac{a\left(3a+4b\sqrt[3]{x}\right)}{\left(a+b\sqrt[3]{x}\right)^2}+2\log\left(a+b\sqrt[3]{x}\right)\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(1/3))^(-3),x]
```

```
[Out] (3*((a*(3*a + 4*b*x^(1/3)))/(a + b*x^(1/3))^2 + 2*Log[a + b*x^(1/3)]))/(2*b
^3)
```

fricas [B] time = 0.55, size = 113, normalized size = 2.09

$$\frac{3\left(6\,a^3b^3x+3\,a^6+2\left(b^6x^2+2\,a^3b^3x+a^6\right)\log\left(bx^{\frac{1}{3}}+a\right)+\left(4\,ab^5x+a^4b^2\right)x^{\frac{2}{3}}-\left(5\,a^2b^4x+2\,a^5b\right)x^{\frac{1}{3}}\right)}{2\left(b^9x^2+2\,a^3b^6x+a^6b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))^3,x, algorithm="fricas")
```

```
[Out] 3/2*(6*a^3*b^3*x + 3*a^6 + 2*(b^6*x^2 + 2*a^3*b^3*x + a^6)*log(b*x^(1/3) +
a) + (4*a*b^5*x + a^4*b^2)*x^(2/3) - (5*a^2*b^4*x + 2*a^5*b)*x^(1/3))/(b^9*
x^2 + 2*a^3*b^6*x + a^6*b^3)
```

giac [A] time = 0.16, size = 44, normalized size = 0.81

$$\frac{3\,\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{b^3}+\frac{3\left(4\,ax^{\frac{1}{3}}+\frac{3\,a^2}{b}\right)}{2\left(bx^{\frac{1}{3}}+a\right)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))^3,x, algorithm="giac")
```

```
[Out] 3*log(abs(b*x^(1/3) + a))/b^3 + 3/2*(4*a*x^(1/3) + 3*a^2/b)/((b*x^(1/3) + a
)^2*b^2)
```

maple [B] time = 0.08, size = 237, normalized size = 4.39

$$-\frac{9a^6}{2\left(b^3x+a^3\right)^2b^3}+\frac{2ax}{\left(b^2x^{\frac{2}{3}}-ab\,x^{\frac{1}{3}}+a^2\right)^2}-\frac{13a^2x^{\frac{2}{3}}}{2\left(b^2x^{\frac{2}{3}}-ab\,x^{\frac{1}{3}}+a^2\right)^2b}+\frac{5a^3x^{\frac{1}{3}}}{\left(b^2x^{\frac{2}{3}}-ab\,x^{\frac{1}{3}}+a^2\right)^2b^2}-\frac{3a^4}{\left(b^2x^{\frac{2}{3}}-ab\,x^{\frac{1}{3}}+a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^(1/3)+a)^3,x)
```

```
[Out] -9/2*a^6/(b^3*x+a^3)^2/b^3+9/b^3*a^3/(b^3*x+a^3)+ln(b^3*x+a^3)/b^3+2*ln(b*x
^(1/3)+a)/b^3-a^2/b^3/(b*x^(1/3)+a)^2-13/2*a^2/b/(b^2*x^(2/3)-a*b*x^(1/3)+a
^2)^2*x^(2/3)+5*a^3/b^2/(b^2*x^(2/3)-a*b*x^(1/3)+a^2)^2*x^(1/3)-3*a^4/b^3/(
b^2*x^(2/3)-a*b*x^(1/3)+a^2)^2-1/b^3*ln(b^2*x^(2/3)-a*b*x^(1/3)+a^2)+4*a/b^
3/(b*x^(1/3)+a)+2*a/(b^2*x^(2/3)-a*b*x^(1/3)+a^2)^2*x
```

maxima [A] time = 0.54, size = 46, normalized size = 0.85

$$\frac{3\,\log\left(bx^{\frac{1}{3}}+a\right)}{b^3}+\frac{6\,a}{\left(bx^{\frac{1}{3}}+a\right)b^3}-\frac{3\,a^2}{2\left(bx^{\frac{1}{3}}+a\right)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))^3,x, algorithm="maxima")
```

```
[Out] 3*log(b*x^(1/3) + a)/b^3 + 6*a/((b*x^(1/3) + a)*b^3) - 3/2*a^2/((b*x^(1/3)
+ a)^2*b^3)
```

mupad [B] time = 1.14, size = 53, normalized size = 0.98

$$\frac{\frac{9\,a^2}{2\,b^3}+\frac{6\,a\,x^{1/3}}{b^2}}{a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}}+\frac{3\,\ln\left(a+b\,x^{1/3}\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^(1/3))^3,x)
```

```
[Out] ((9*a^2)/(2*b^3) + (6*a*x^(1/3))/b^2)/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3)) +
 (3*log(a + b*x^(1/3)))/b^3
```

sympy [A] time = 0.67, size = 228, normalized size = 4.22

$$\begin{cases} \frac{6a^2\log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{2a^2b^3+4ab^4\sqrt[3]{x}+2b^5x^{\frac{2}{3}}}+\frac{9a^2}{2a^2b^3+4ab^4\sqrt[3]{x}+2b^5x^{\frac{2}{3}}}+\frac{12ab\sqrt[3]{x}\log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{2a^2b^3+4ab^4\sqrt[3]{x}+2b^5x^{\frac{2}{3}}}+\frac{12ab\sqrt[3]{x}}{2a^2b^3+4ab^4\sqrt[3]{x}+2b^5x^{\frac{2}{3}}}+\frac{6b^2x^{\frac{2}{3}}\log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{2a^2b^3+4ab^4\sqrt[3]{x}+2b^5x^{\frac{2}{3}}} & \text{for} \\ \frac{x}{a^3} & \text{oth} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**(1/3))**3,x)
```

```
[Out] Piecewise((6*a**2*log(a/b + x**(1/3))/(2*a**2*b**3 + 4*a*b**4*x**(1/3) + 2*
b**5*x**(2/3)) + 9*a**2/(2*a**2*b**3 + 4*a*b**4*x**(1/3) + 2*b**5*x**(2/3))
 + 12*a*b*x**(1/3)*log(a/b + x**(1/3))/(2*a**2*b**3 + 4*a*b**4*x**(1/3) + 2
*b**5*x**(2/3)) + 12*a*b*x**(1/3)/(2*a**2*b**3 + 4*a*b**4*x**(1/3) + 2*b**5
*x**(2/3)) + 6*b**2*x**(2/3)*log(a/b + x**(1/3))/(2*a**2*b**3 + 4*a*b**4*x*
*(1/3) + 2*b**5*x**(2/3)), Ne(b, 0)), (x/a**3, True))
```

3.2377 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x}\,dx$

Optimal. Leaf size=56

$$-\frac{3\log\left(a+b\sqrt[3]{x}\right)}{a^3}+\frac{\log(x)}{a^3}+\frac{3}{a^2\left(a+b\sqrt[3]{x}\right)}+\frac{3}{2a\left(a+b\sqrt[3]{x}\right)^2}$$

```
[Out] 3/2/a/(a+b*x^(1/3))^2+3/a^2/(a+b*x^(1/3))-3*ln(a+b*x^(1/3))/a^3+ln(x)/a^3
```

Rubi [A] time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 44}

$$\frac{3}{a^2\left(a+b\sqrt[3]{x}\right)}-\frac{3\log\left(a+b\sqrt[3]{x}\right)}{a^3}+\frac{\log(x)}{a^3}+\frac{3}{2a\left(a+b\sqrt[3]{x}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^(1/3))^3*x),x]
```

```
[Out] 3/(2*a*(a + b*x^(1/3))^2) + 3/(a^2*(a + b*x^(1/3))) - (3*Log[a + b*x^(1/3)]
)/a^3 + Log[x]/a^3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x}\,dx &= 3\,\text{Subst}\left(\int \frac{1}{x(a+bx)^3}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(\frac{1}{a^3 x}-\frac{b}{a(a+bx)^3}-\frac{b}{a^2(a+bx)^2}-\frac{b}{a^3(a+bx)}\right)dx, x, \sqrt[3]{x}\right)\\ &= \frac{3}{2a\left(a+b\sqrt[3]{x}\right)^2}+\frac{3}{a^2\left(a+b\sqrt[3]{x}\right)}-\frac{3\log\left(a+b\sqrt[3]{x}\right)}{a^3}+\frac{\log(x)}{a^3}\end{aligned}$$

Mathematica [A] time = 0.06, size = 50, normalized size = 0.89

$$\frac{\frac{3a\left(3a+2b\sqrt[3]{x}\right)}{\left(a+b\sqrt[3]{x}\right)^2}-6\log\left(a+b\sqrt[3]{x}\right)+2\log(x)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^(1/3))^3*x),x]
```

```
[Out] ((3*a*(3*a + 2*b*x^(1/3)))/(a + b*x^(1/3))^2 - 6*Log[a + b*x^(1/3)] + 2*Log
[x])/(2*a^3)
```

fricas [B] time = 0.63, size = 129, normalized size = 2.30

$$\frac{3\left(3\,a^6-2\left(b^6x^2+2\,a^3b^3x+a^6\right)\log\left(bx^{\frac{1}{3}}+a\right)+2\left(b^6x^2+2\,a^3b^3x+a^6\right)\log\left(x^{\frac{1}{3}}\right)+\left(2\,ab^5x+5\,a^4b^2\right)x^{\frac{2}{3}}-\left(a^2\right.\right.}{2\left(a^3b^6x^2+2\,a^6b^3x+a^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))^3/x,x, algorithm="fricas")
```

```
[Out] 3/2*(3*a^6 - 2*(b^6*x^2 + 2*a^3*b^3*x + a^6)*log(b*x^(1/3) + a) + 2*(b^6*x^
2 + 2*a^3*b^3*x + a^6)*log(x^(1/3)) + (2*a*b^5*x + 5*a^4*b^2)*x^(2/3) - (a^
2*b^4*x + 4*a^5*b)*x^(1/3))/(a^3*b^6*x^2 + 2*a^6*b^3*x + a^9)
```

giac [A] time = 0.17, size = 49, normalized size = 0.88

$$-\frac{3\,\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{a^3}+\frac{\log\left(|x|\right)}{a^3}+\frac{3\left(2\,abx^{\frac{1}{3}}+3\,a^2\right)}{2\left(bx^{\frac{1}{3}}+a\right)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))^3/x,x, algorithm="giac")
```

```
[Out] -3*log(abs(b*x^(1/3) + a))/a^3 + log(abs(x))/a^3 + 3/2*(2*a*b*x^(1/3) + 3*a
^2)/((b*x^(1/3) + a)^2*a^3)
```

maple [A] time = 0.01, size = 49, normalized size = 0.88

$$\frac{3}{2\left(b\,x^{\frac{1}{3}}+a\right)^2a}+\frac{3}{\left(b\,x^{\frac{1}{3}}+a\right)a^2}+\frac{\ln(x)}{a^3}-\frac{3\ln\left(b\,x^{\frac{1}{3}}+a\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^(1/3)+a)^3/x,x)
```

```
[Out] 3/2/a/(b*x^(1/3)+a)^2+3/a^2/(b*x^(1/3)+a)-3*ln(b*x^(1/3)+a)/a^3+1/a^3*ln(x)
```

maxima [A] time = 0.49, size = 57, normalized size = 1.02

$$\frac{3\left(2\,bx^{\frac{1}{3}}+3\,a\right)}{2\left(a^2b^2x^{\frac{2}{3}}+2\,a^3bx^{\frac{1}{3}}+a^4\right)}-\frac{3\,\log\left(bx^{\frac{1}{3}}+a\right)}{a^3}+\frac{\log(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))^3/x,x, algorithm="maxima")
```

```
[Out] 3/2*(2*b*x^(1/3) + 3*a)/(a^2*b^2*x^(2/3) + 2*a^3*b*x^(1/3) + a^4) - 3*log(b
*x^(1/3) + a)/a^3 + log(x)/a^3
```

mupad [B] time = 1.14, size = 54, normalized size = 0.96

$$\frac{\frac{9}{2\,a}+\frac{3\,b\,x^{1/3}}{a^2}}{a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}}-\frac{6\,\mathrm{atanh}\left(\frac{2\,b\,x^{1/3}}{a}+1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^(1/3))^3),x)
```

```
[Out] (9/(2*a) + (3*b*x^(1/3))/a^2)/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3)) - (6*atan
h((2*b*x^(1/3))/a + 1))/a^3
```

sympy [A] time = 1.91, size = 386, normalized size = 6.89

$$\begin{cases} \frac{\tilde{\infty}}{x} \\ \frac{\log(x)}{a^3} \\ -\frac{1}{b^3 x} \\ \frac{2a^2 x^{\frac{2}{3}} \log(x)}{2a^5 x^{\frac{2}{3}} + 4a^4 bx + 2a^3 b^2 x^{\frac{4}{3}}} - \frac{6a^2 x^{\frac{2}{3}} \log\left(\frac{a}{b} + \sqrt[3]{x}\right)}{2a^5 x^{\frac{2}{3}} + 4a^4 bx + 2a^3 b^2 x^{\frac{4}{3}}} + \frac{9a^2 x^{\frac{2}{3}}}{2a^5 x^{\frac{2}{3}} + 4a^4 bx + 2a^3 b^2 x^{\frac{4}{3}}} + \frac{4abx \log(x)}{2a^5 x^{\frac{2}{3}} + 4a^4 bx + 2a^3 b^2 x^{\frac{4}{3}}} - \frac{12abx \log\left(\frac{a}{b} + \sqrt[3]{x}\right)}{2a^5 x^{\frac{2}{3}} + 4a^4 bx + 2a^3 b^2 x^{\frac{4}{3}}} + \frac{}{2a^5 x^{\frac{2}{3}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**(1/3))**3/x,x)
```

```
[Out] Piecewise((zoo/x, Eq(a, 0) & Eq(b, 0)), (log(x)/a**3, Eq(b, 0)), (-1/(b**3*
x), Eq(a, 0)), (2*a**2*x**(2/3)*log(x)/(2*a**5*x**(2/3) + 4*a**4*b*x + 2*a*
*3*b**2*x**(4/3)) - 6*a**2*x**(2/3)*log(a/b + x**(1/3))/(2*a**5*x**(2/3) +
4*a**4*b*x + 2*a**3*b**2*x**(4/3)) + 9*a**2*x**(2/3)/(2*a**5*x**(2/3) + 4*a
**4*b*x + 2*a**3*b**2*x**(4/3)) + 4*a*b*x*log(x)/(2*a**5*x**(2/3) + 4*a**4*
b*x + 2*a**3*b**2*x**(4/3)) - 12*a*b*x*log(a/b + x**(1/3))/(2*a**5*x**(2/3)
 + 4*a**4*b*x + 2*a**3*b**2*x**(4/3)) + 6*a*b*x/(2*a**5*x**(2/3) + 4*a**4*b
*x + 2*a**3*b**2*x**(4/3)) + 2*b**2*x**(4/3)*log(x)/(2*a**5*x**(2/3) + 4*a*
*4*b*x + 2*a**3*b**2*x**(4/3)) - 6*b**2*x**(4/3)*log(a/b + x**(1/3))/(2*a**
5*x**(2/3) + 4*a**4*b*x + 2*a**3*b**2*x**(4/3)), True))
```

3.2378 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x^2}\,dx$

Optimal. Leaf size=103

$$\frac{30b^3\log\left(a+b\sqrt[3]{x}\right)}{a^6}-\frac{10b^3\log(x)}{a^6}-\frac{12b^3}{a^5\left(a+b\sqrt[3]{x}\right)}-\frac{18b^2}{a^5\sqrt[3]{x}}-\frac{3b^3}{2a^4\left(a+b\sqrt[3]{x}\right)^2}+\frac{9b}{2a^4x^{2/3}}-\frac{1}{a^3x}$$

```
[Out] -3/2*b^3/a^4/(a+b*x^(1/3))^2-12*b^3/a^5/(a+b*x^(1/3))-1/a^3/x+9/2*b/a^4/x^(
2/3)-18*b^2/a^5/x^(1/3)+30*b^3*ln(a+b*x^(1/3))/a^6-10*b^3*ln(x)/a^6
```

Rubi [A] time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 44}

$$-\frac{12b^3}{a^5\left(a+b\sqrt[3]{x}\right)}-\frac{3b^3}{2a^4\left(a+b\sqrt[3]{x}\right)^2}-\frac{18b^2}{a^5\sqrt[3]{x}}+\frac{30b^3\log\left(a+b\sqrt[3]{x}\right)}{a^6}-\frac{10b^3\log(x)}{a^6}+\frac{9b}{2a^4x^{2/3}}-\frac{1}{a^3x}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^(1/3))^3*x^2),x]
```

```
[Out] (-3*b^3)/(2*a^4*(a + b*x^(1/3))^2) - (12*b^3)/(a^5*(a + b*x^(1/3))) - 1/(a^
3*x) + (9*b)/(2*a^4*x^(2/3)) - (18*b^2)/(a^5*x^(1/3)) + (30*b^3*Log[a + b*x
^(1/3)])/a^6 - (10*b^3*Log[x])/a^6
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x^2}\,dx &= 3\,\text{Subst}\left(\int \frac{1}{x^4(a+bx)^3}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int\left(\frac{1}{a^3x^4}-\frac{3b}{a^4x^3}+\frac{6b^2}{a^5x^2}-\frac{10b^3}{a^6x}+\frac{b^4}{a^4(a+bx)^3}+\frac{4b^4}{a^5(a+bx)^2}+\frac{10b^4}{a^6(a+bx)}\right)d\\ &= -\frac{3b^3}{2a^4\left(a+b\sqrt[3]{x}\right)^2}-\frac{12b^3}{a^5\left(a+b\sqrt[3]{x}\right)}-\frac{1}{a^3x}+\frac{9b}{2a^4x^{2/3}}-\frac{18b^2}{a^5\sqrt[3]{x}}+\frac{30b^3\log\left(a+b\sqrt[3]{x}\right)}{a^6}-\end{aligned}$$

Mathematica [A] time = 0.15, size = 93, normalized size = 0.90

$$-\frac{\frac{a\left(2a^4-5a^3b\sqrt[3]{x}+20a^2b^2x^{2/3}+90ab^3x+60b^4x^{4/3}\right)}{x\left(a+b\sqrt[3]{x}\right)^2}-60b^3\log\left(a+b\sqrt[3]{x}\right)+20b^3\log(x)}{2a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^(1/3))^3*x^2),x]
```

```
[Out] -1/2*((a*(2*a^4 - 5*a^3*b*x^(1/3) + 20*a^2*b^2*x^(2/3) + 90*a*b^3*x + 60*b^
4*x^(4/3)))/((a + b*x^(1/3))^2*x) - 60*b^3*Log[a + b*x^(1/3)] + 20*b^3*Log[
x])/a^6
```

fricas [B] time = 0.67, size = 191, normalized size = 1.85

$$-\frac{20\,a^3b^6x^2+31\,a^6b^3x+2\,a^9-60\left(b^9x^3+2\,a^3b^6x^2+a^6b^3x\right)\log\left(bx^{\frac{1}{3}}+a\right)+60\left(b^9x^3+2\,a^3b^6x^2+a^6b^3x\right)\log\left(x\right.}{2\left(a^6b^6x^3+2\,a^9b^3x^2+a^{12}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))^3/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(20*a^3*b^6*x^2 + 31*a^6*b^3*x + 2*a^9 - 60*(b^9*x^3 + 2*a^3*b^6*x^2 +
 a^6*b^3*x)*log(b*x^(1/3) + a) + 60*(b^9*x^3 + 2*a^3*b^6*x^2 + a^6*b^3*x)*l
og(x^(1/3)) + 3*(20*a*b^8*x^2 + 35*a^4*b^5*x + 12*a^7*b^2)*x^(2/3) - 3*(10*
a^2*b^7*x^2 + 16*a^5*b^4*x + 3*a^8*b)*x^(1/3))/(a^6*b^6*x^3 + 2*a^9*b^3*x^2
 + a^12*x)
```

giac [A] time = 0.17, size = 90, normalized size = 0.87

$$\frac{30\,b^3\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{a^6}-\frac{10\,b^3\log\left(|x|\right)}{a^6}-\frac{60\,ab^4x^{\frac{4}{3}}+90\,a^2b^3x+20\,a^3b^2x^{\frac{2}{3}}-5\,a^4bx^{\frac{1}{3}}+2\,a^5}{2\left(bx^{\frac{1}{3}}+a\right)^2a^6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))^3/x^2,x, algorithm="giac")
```

```
[Out] 30*b^3*log(abs(b*x^(1/3) + a))/a^6 - 10*b^3*log(abs(x))/a^6 - 1/2*(60*a*b^4
*x^(4/3) + 90*a^2*b^3*x + 20*a^3*b^2*x^(2/3) - 5*a^4*b*x^(1/3) + 2*a^5)/((b
*x^(1/3) + a)^2*a^6*x)
```

maple [A] time = 0.01, size = 90, normalized size = 0.87

$$-\frac{3b^3}{2\left(b\,x^{\frac{1}{3}}+a\right)^2a^4}-\frac{12b^3}{\left(b\,x^{\frac{1}{3}}+a\right)a^5}-\frac{10b^3\ln(x)}{a^6}+\frac{30b^3\ln\left(b\,x^{\frac{1}{3}}+a\right)}{a^6}-\frac{18b^2}{a^5x^{\frac{1}{3}}}+\frac{9b}{2a^4x^{\frac{2}{3}}}-\frac{1}{a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^(1/3)+a)^3/x^2,x)
```

```
[Out] -3/2*b^3/a^4/(b*x^(1/3)+a)^2-12*b^3/a^5/(b*x^(1/3)+a)-1/a^3/x+9/2*b/a^4/x^(
2/3)-18*b^2/a^5/x^(1/3)+30*b^3*ln(b*x^(1/3)+a)/a^6-10*b^3*ln(x)/a^6
```

maxima [A] time = 0.64, size = 97, normalized size = 0.94

$$-\frac{60\,b^4x^{\frac{4}{3}}+90\,ab^3x+20\,a^2b^2x^{\frac{2}{3}}-5\,a^3bx^{\frac{1}{3}}+2\,a^4}{2\left(a^5b^2x^{\frac{5}{3}}+2\,a^6bx^{\frac{4}{3}}+a^7x\right)}+\frac{30\,b^3\log\left(bx^{\frac{1}{3}}+a\right)}{a^6}-\frac{10\,b^3\log(x)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))^3/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*(60*b^4*x^(4/3) + 90*a*b^3*x + 20*a^2*b^2*x^(2/3) - 5*a^3*b*x^(1/3) +
2*a^4)/(a^5*b^2*x^(5/3) + 2*a^6*b*x^(4/3) + a^7*x) + 30*b^3*log(b*x^(1/3) +
 a)/a^6 - 10*b^3*log(x)/a^6
```

mupad **[B]** time = 1.16, size = 89, normalized size = 0.86

$$\frac{60\,b^3\,\operatorname{atanh}\left(\frac{2\,b\,x^{1/3}}{a}+1\right)}{a^6}-\frac{\frac{1}{a}-\frac{5\,b\,x^{1/3}}{2\,a^2}+\frac{45\,b^3\,x}{a^4}+\frac{10\,b^2\,x^{2/3}}{a^3}+\frac{30\,b^4\,x^{4/3}}{a^5}}{a^2\,x+b^2\,x^{5/3}+2\,a\,b\,x^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^(1/3))^3),x)
```

```
[Out] (60*b^3*atanh((2*b*x^(1/3))/a + 1))/a^6 - (1/a - (5*b*x^(1/3))/(2*a^2) + (4
5*b^3*x)/a^4 + (10*b^2*x^(2/3))/a^3 + (30*b^4*x^(4/3))/a^5)/(a^2*x + b^2*x^
(5/3) + 2*a*b*x^(4/3))
```

sympy **[A]** time = 4.87, size = 561, normalized size = 5.45

$$\begin{cases} \frac{\tilde{\infty}}{x^2} \\ -\frac{1}{a^3 x} \\ -\frac{1}{2b^3x^2} \\ -\frac{2a^5x^{\frac{2}{3}}}{2a^8x^{\frac{5}{3}}+4a^7bx^2+2a^6b^2x^{\frac{7}{3}}}+\frac{5a^4bx}{2a^8x^{\frac{5}{3}}+4a^7bx^2+2a^6b^2x^{\frac{7}{3}}}-\frac{20a^3b^2x^{\frac{4}{3}}}{2a^8x^{\frac{5}{3}}+4a^7bx^2+2a^6b^2x^{\frac{7}{3}}}-\frac{20a^2b^3x^{\frac{5}{3}}\log(x)}{2a^8x^{\frac{5}{3}}+4a^7bx^2+2a^6b^2x^{\frac{7}{3}}}+\frac{60a^2b^3x^{\frac{5}{3}}\log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{2a^8x^{\frac{5}{3}}+4a^7bx^2+2a^6b^2x^{\frac{7}{3}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**(1/3))**3/x**2,x)
```

```
[Out] Piecewise((zoo/x**2, Eq(a, 0) & Eq(b, 0)), (-1/(a**3*x), Eq(b, 0)), (-1/(2*
b**3*x**2), Eq(a, 0)), (-2*a**5*x**(2/3)/(2*a**8*x**(5/3) + 4*a**7*b*x**2 +
 2*a**6*b**2*x**(7/3)) + 5*a**4*b*x/(2*a**8*x**(5/3) + 4*a**7*b*x**2 + 2*a*
*6*b**2*x**(7/3)) - 20*a**3*b**2*x**(4/3)/(2*a**8*x**(5/3) + 4*a**7*b*x**2
+ 2*a**6*b**2*x**(7/3)) - 20*a**2*b**3*x**(5/3)*log(x)/(2*a**8*x**(5/3) + 4
*a**7*b*x**2 + 2*a**6*b**2*x**(7/3)) + 60*a**2*b**3*x**(5/3)*log(a/b + x**(
1/3))/(2*a**8*x**(5/3) + 4*a**7*b*x**2 + 2*a**6*b**2*x**(7/3)) - 90*a**2*b*
*3*x**(5/3)/(2*a**8*x**(5/3) + 4*a**7*b*x**2 + 2*a**6*b**2*x**(7/3)) - 40*a
*b**4*x**2*log(x)/(2*a**8*x**(5/3) + 4*a**7*b*x**2 + 2*a**6*b**2*x**(7/3))
+ 120*a*b**4*x**2*log(a/b + x**(1/3))/(2*a**8*x**(5/3) + 4*a**7*b*x**2 + 2*
a**6*b**2*x**(7/3)) - 60*a*b**4*x**2/(2*a**8*x**(5/3) + 4*a**7*b*x**2 + 2*a
**6*b**2*x**(7/3)) - 20*b**5*x**(7/3)*log(x)/(2*a**8*x**(5/3) + 4*a**7*b*x*
*2 + 2*a**6*b**2*x**(7/3)) + 60*b**5*x**(7/3)*log(a/b + x**(1/3))/(2*a**8*x
**(5/3) + 4*a**7*b*x**2 + 2*a**6*b**2*x**(7/3)), True))
```

3.2379 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x^3}\,dx$

Optimal. Leaf size=146

$$-\frac{84b^6\log\left(a+b\sqrt[3]{x}\right)}{a^9}+\frac{28b^6\log(x)}{a^9}+\frac{21b^6}{a^8\left(a+b\sqrt[3]{x}\right)}+\frac{63b^5}{a^8\sqrt[3]{x}}+\frac{3b^6}{2a^7\left(a+b\sqrt[3]{x}\right)^2}-\frac{45b^4}{2a^7x^{2/3}}+\frac{10b^3}{a^6x}-\frac{9b^2}{2a^5x^{4/3}}+\frac{9b}{5a^4x^{5/3}}-$$

```
[Out] 3/2*b^6/a^7/(a+b*x^(1/3))^2+21*b^6/a^8/(a+b*x^(1/3))-1/2/a^3/x^2+9/5*b/a^4/
x^(5/3)-9/2*b^2/a^5/x^(4/3)+10*b^3/a^6/x-45/2*b^4/a^7/x^(2/3)+63*b^5/a^8/x^
(1/3)-84*b^6*ln(a+b*x^(1/3))/a^9+28*b^6*ln(x)/a^9
```

Rubi [A] time = 0.10, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 44}

$$-\frac{45b^4}{2a^7x^{2/3}}-\frac{9b^2}{2a^5x^{4/3}}+\frac{21b^6}{a^8\left(a+b\sqrt[3]{x}\right)}+\frac{3b^6}{2a^7\left(a+b\sqrt[3]{x}\right)^2}+\frac{63b^5}{a^8\sqrt[3]{x}}+\frac{10b^3}{a^6x}-\frac{84b^6\log\left(a+b\sqrt[3]{x}\right)}{a^9}+\frac{28b^6\log(x)}{a^9}+\frac{9b}{5a^4x^{5/3}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^(1/3))^3*x^3),x]
```

```
[Out] (3*b^6)/(2*a^7*(a + b*x^(1/3))^2) + (21*b^6)/(a^8*(a + b*x^(1/3))) - 1/(2*a
^3*x^2) + (9*b)/(5*a^4*x^(5/3)) - (9*b^2)/(2*a^5*x^(4/3)) + (10*b^3)/(a^6*x
) - (45*b^4)/(2*a^7*x^(2/3)) + (63*b^5)/(a^8*x^(1/3)) - (84*b^6*Log[a + b*x
^(1/3)])/a^9 + (28*b^6*Log[x])/a^9
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x^3}\,dx = 3\,\text{Subst}\left(\int \frac{1}{x^7(a+bx)^3}\,dx, x, \sqrt[3]{x}\right)$$

$$= 3\,\text{Subst}\left(\int \left(\frac{1}{a^3x^7}-\frac{3b}{a^4x^6}+\frac{6b^2}{a^5x^5}-\frac{10b^3}{a^6x^4}+\frac{15b^4}{a^7x^3}-\frac{21b^5}{a^8x^2}+\frac{28b^6}{a^9x}-\frac{b^7}{a^7(a+bx)^3}-\frac{7b^7}{a^8(a+b$$

$$= \frac{3b^6}{2a^7\left(a+b\sqrt[3]{x}\right)^2}+\frac{21b^6}{a^8\left(a+b\sqrt[3]{x}\right)}-\frac{1}{2a^3x^2}+\frac{9b}{5a^4x^{5/3}}-\frac{9b^2}{2a^5x^{4/3}}+\frac{10b^3}{a^6x}-\frac{45b^4}{2a^7x^{2/3}}+\frac{63b^5}{a^8\sqrt[3]{x}}$$

Mathematica [A] time = 0.18, size = 130, normalized size = 0.89

$$\frac{\frac{a\left(-5a^7+8a^6b\sqrt[3]{x}-14a^5b^2x^{2/3}+28a^4b^3x-70a^3b^4x^{4/3}+280a^2b^5x^{5/3}+1260ab^6x^2+840b^7x^{7/3}\right)}{x^2\left(a+b\sqrt[3]{x}\right)^2}-840b^6\log\left(a+b\sqrt[3]{x}\right)+280b^6\log(x)}{10a^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^(1/3))^3*x^3),x]
```

```
[Out] ((a*(-5*a^7 + 8*a^6*b*x^(1/3) - 14*a^5*b^2*x^(2/3) + 28*a^4*b^3*x - 70*a^3*
b^4*x^(4/3) + 280*a^2*b^5*x^(5/3) + 1260*a*b^6*x^2 + 840*b^7*x^(7/3)))/((a
+ b*x^(1/3))^2*x^2) - 840*b^6*Log[a + b*x^(1/3)] + 280*b^6*Log[x])/(10*a^9)
```

fricas [A] time = 0.51, size = 230, normalized size = 1.58

$$\frac{280\,a^3b^9x^3+420\,a^6b^6x^2+90\,a^9b^3x-5\,a^{12}-840\left(b^{12}x^4+2\,a^3b^9x^3+a^6b^6x^2\right)\log\left(bx^{\frac{1}{3}}+a\right)+840\left(b^{12}x^4+2\right.}{10\left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/10*(280*a^3*b^9*x^3 + 420*a^6*b^6*x^2 + 90*a^9*b^3*x - 5*a^12 - 840*(b^12
*x^4 + 2*a^3*b^9*x^3 + a^6*b^6*x^2)*log(b*x^(1/3) + a) + 840*(b^12*x^4 + 2*
a^3*b^9*x^3 + a^6*b^6*x^2)*log(x^(1/3)) + 15*(56*a*b^11*x^3 + 98*a^4*b^8*x^
2 + 36*a^7*b^5*x - 3*a^10*b^2)*x^(2/3) - 3*(140*a^2*b^10*x^3 + 224*a^5*b^7*
x^2 + 63*a^8*b^4*x - 6*a^11*b)*x^(1/3))/(a^9*b^6*x^4 + 2*a^12*b^3*x^3 + a^1
5*x^2)
```

giac [A] time = 0.17, size = 123, normalized size = 0.84

$$-\frac{84\,b^6\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{a^9}+\frac{28\,b^6\log\left(|x|\right)}{a^9}+\frac{840\,ab^7x^{\frac{7}{3}}+1260\,a^2b^6x^2+280\,a^3b^5x^{\frac{5}{3}}-70\,a^4b^4x^{\frac{4}{3}}+28\,a^5b^3x-14\,a^6}{10\left(bx^{\frac{1}{3}}+a\right)^2a^9x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))^3/x^3,x, algorithm="giac")
```

```
[Out] -84*b^6*log(abs(b*x^(1/3) + a))/a^9 + 28*b^6*log(abs(x))/a^9 + 1/10*(840*a*
b^7*x^(7/3) + 1260*a^2*b^6*x^2 + 280*a^3*b^5*x^(5/3) - 70*a^4*b^4*x^(4/3) +
 28*a^5*b^3*x - 14*a^6*b^2*x^(2/3) + 8*a^7*b*x^(1/3) - 5*a^8)/((b*x^(1/3) +
 a)^2*a^9*x^2)
```

maple [A] time = 0.00, size = 123, normalized size = 0.84

$$\frac{3b^6}{2\left(b\,x^{\frac{1}{3}}+a\right)^2a^7}+\frac{21b^6}{\left(b\,x^{\frac{1}{3}}+a\right)a^8}+\frac{28b^6\ln(x)}{a^9}-\frac{84b^6\ln\left(b\,x^{\frac{1}{3}}+a\right)}{a^9}+\frac{63b^5}{a^8x^{\frac{1}{3}}}-\frac{45b^4}{2a^7x^{\frac{2}{3}}}+\frac{10b^3}{a^6x}-\frac{9b^2}{2a^5x^{\frac{4}{3}}}+\frac{9b}{5a^4x^{\frac{5}{3}}}-\frac{1}{2a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^(1/3)+a)^3/x^3,x)
```

```
[Out] 3/2*b^6/a^7/(b*x^(1/3)+a)^2+21*b^6/a^8/(b*x^(1/3)+a)-1/2/a^3/x^2+9/5*b/a^4/
x^(5/3)-9/2*b^2/a^5/x^(4/3)+10*b^3/a^6/x-45/2*b^4/a^7/x^(2/3)+63*b^5/a^8/x^
(1/3)-84*b^6*ln(b*x^(1/3)+a)/a^9+28/a^9*b^6*ln(x)
```

maxima [A] time = 0.53, size = 132, normalized size = 0.90

$$\frac{840\,b^7x^{\frac{7}{3}}+1260\,ab^6x^2+280\,a^2b^5x^{\frac{5}{3}}-70\,a^3b^4x^{\frac{4}{3}}+28\,a^4b^3x-14\,a^5b^2x^{\frac{2}{3}}+8\,a^6bx^{\frac{1}{3}}-5\,a^7}{10\left(a^8b^2x^{\frac{8}{3}}+2\,a^9bx^{\frac{7}{3}}+a^{10}x^2\right)}-\frac{84\,b^6\log\left(bx^{\frac{1}{3}}+a\right.}{a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/10*(840*b^7*x^(7/3) + 1260*a*b^6*x^2 + 280*a^2*b^5*x^(5/3) - 70*a^3*b^4*x
^(4/3) + 28*a^4*b^3*x - 14*a^5*b^2*x^(2/3) + 8*a^6*b*x^(1/3) - 5*a^7)/(a^8*
b^2*x^(8/3) + 2*a^9*b*x^(7/3) + a^10*x^2) - 84*b^6*log(b*x^(1/3) + a)/a^9 +
 28*b^6*log(x)/a^9
```

mupad [B] time = 1.21, size = 125, normalized size = 0.86

$$\frac{\frac{4\,b\,x^{1/3}}{5\,a^2}-\frac{1}{2\,a}+\frac{14\,b^3\,x}{5\,a^4}-\frac{7\,b^2\,x^{2/3}}{5\,a^3}+\frac{126\,b^6\,x^2}{a^7}-\frac{7\,b^4\,x^{4/3}}{a^5}+\frac{28\,b^5\,x^{5/3}}{a^6}+\frac{84\,b^7\,x^{7/3}}{a^8}}{a^2\,x^2+b^2\,x^{8/3}+2\,a\,b\,x^{7/3}}-\frac{168\,b^6\,\operatorname{atanh}\left(\frac{2\,b\,x^{1/3}}{a}+1\right)}{a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^(1/3))^3),x)
```

```
[Out] ((4*b*x^(1/3))/(5*a^2) - 1/(2*a) + (14*b^3*x)/(5*a^4) - (7*b^2*x^(2/3))/(5*
a^3) + (126*b^6*x^2)/a^7 - (7*b^4*x^(4/3))/a^5 + (28*b^5*x^(5/3))/a^6 + (84
*b^7*x^(7/3))/a^8)/(a^2*x^2 + b^2*x^(8/3) + 2*a*b*x^(7/3)) - (168*b^6*atanh
((2*b*x^(1/3))/a + 1))/a^9
```

sympy [A] time = 11.46, size = 706, normalized size = 4.84

$$\begin{cases} \frac{\tilde{\infty}}{x^3} \\ -\frac{1}{2a^3x^2} \\ -\frac{1}{3b^3x^3} \\ -\frac{5a^8x^{\frac{2}{3}}}{10a^{11}x^{\frac{8}{3}}+20a^{10}bx^3+10a^9b^2x^{\frac{10}{3}}}+\frac{8a^7bx}{10a^{11}x^{\frac{8}{3}}+20a^{10}bx^3+10a^9b^2x^{\frac{10}{3}}}-\frac{14a^6b^2x^{\frac{4}{3}}}{10a^{11}x^{\frac{8}{3}}+20a^{10}bx^3+10a^9b^2x^{\frac{10}{3}}}+\frac{28a^5b^3x^{\frac{5}{3}}}{10a^{11}x^{\frac{8}{3}}+20a^{10}bx^3+10a^9b^2x^{\frac{10}{3}}}- \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**(1/3))**3/x**3,x)
```

```
[Out] Piecewise((zoo/x**3, Eq(a, 0) & Eq(b, 0)), (-1/(2*a**3*x**2), Eq(b, 0)), (-
1/(3*b**3*x**3), Eq(a, 0)), (-5*a**8*x**(2/3)/(10*a**11*x**(8/3) + 20*a**10
*b*x**3 + 10*a**9*b**2*x**(10/3)) + 8*a**7*b*x/(10*a**11*x**(8/3) + 20*a**1
0*b*x**3 + 10*a**9*b**2*x**(10/3)) - 14*a**6*b**2*x**(4/3)/(10*a**11*x**(8/
3) + 20*a**10*b*x**3 + 10*a**9*b**2*x**(10/3)) + 28*a**5*b**3*x**(5/3)/(10*
a**11*x**(8/3) + 20*a**10*b*x**3 + 10*a**9*b**2*x**(10/3)) - 70*a**4*b**4*x
**2/(10*a**11*x**(8/3) + 20*a**10*b*x**3 + 10*a**9*b**2*x**(10/3)) + 280*a*
*3*b**5*x**(7/3)/(10*a**11*x**(8/3) + 20*a**10*b*x**3 + 10*a**9*b**2*x**(10
/3)) + 280*a**2*b**6*x**(8/3)*log(x)/(10*a**11*x**(8/3) + 20*a**10*b*x**3 +
 10*a**9*b**2*x**(10/3)) - 840*a**2*b**6*x**(8/3)*log(a/b + x**(1/3))/(10*a
**11*x**(8/3) + 20*a**10*b*x**3 + 10*a**9*b**2*x**(10/3)) + 1260*a**2*b**6*
x**(8/3)/(10*a**11*x**(8/3) + 20*a**10*b*x**3 + 10*a**9*b**2*x**(10/3)) + 5
60*a*b**7*x**3*log(x)/(10*a**11*x**(8/3) + 20*a**10*b*x**3 + 10*a**9*b**2*x
**(10/3)) - 1680*a*b**7*x**3*log(a/b + x**(1/3))/(10*a**11*x**(8/3) + 20*a*
*10*b*x**3 + 10*a**9*b**2*x**(10/3)) + 840*a*b**7*x**3/(10*a**11*x**(8/3) +
 20*a**10*b*x**3 + 10*a**9*b**2*x**(10/3)) + 280*b**8*x**(10/3)*log(x)/(10*
a**11*x**(8/3) + 20*a**10*b*x**3 + 10*a**9*b**2*x**(10/3)) - 840*b**8*x**(1
0/3)*log(a/b + x**(1/3))/(10*a**11*x**(8/3) + 20*a**10*b*x**3 + 10*a**9*b**
2*x**(10/3)), True))
```

3.2380 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x^4}\,dx$

Optimal. Leaf size=183

$$\frac{165b^9\log\left(a+b\sqrt[3]{x}\right)}{a^{12}}-\frac{55b^9\log(x)}{a^{12}}-\frac{30b^9}{a^{11}\left(a+b\sqrt[3]{x}\right)}-\frac{135b^8}{a^{11}\sqrt[3]{x}}-\frac{3b^9}{2a^{10}\left(a+b\sqrt[3]{x}\right)^2}+\frac{54b^7}{a^{10}x^{2/3}}-\frac{28b^6}{a^9x}+\frac{63b^5}{4a^8x^{4/3}}-\frac{9b^4}{a^7x^{5/}}$$

```
[Out] -3/2*b^9/a^10/(a+b*x^(1/3))^2-30*b^9/a^11/(a+b*x^(1/3))-1/3/a^3/x^3+9/8*b/a
^4/x^(8/3)-18/7*b^2/a^5/x^(7/3)+5*b^3/a^6/x^2-9*b^4/a^7/x^(5/3)+63/4*b^5/a^
8/x^(4/3)-28*b^6/a^9/x+54*b^7/a^10/x^(2/3)-135*b^8/a^11/x^(1/3)+165*b^9*ln(
a+b*x^(1/3))/a^12-55*b^9*ln(x)/a^12
```

Rubi [A] time = 0.14, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 44}

$$\frac{54b^7}{a^{10}x^{2/3}}+\frac{63b^5}{4a^8x^{4/3}}-\frac{9b^4}{a^7x^{5/3}}+\frac{5b^3}{a^6x^2}-\frac{18b^2}{7a^5x^{7/3}}-\frac{30b^9}{a^{11}\left(a+b\sqrt[3]{x}\right)}-\frac{3b^9}{2a^{10}\left(a+b\sqrt[3]{x}\right)^2}-\frac{135b^8}{a^{11}\sqrt[3]{x}}-\frac{28b^6}{a^9x}+\frac{165b^9\log\left(a+b\sqrt[3]{}\right.}{a^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^(1/3))^3*x^4),x]
```

```
[Out] (-3*b^9)/(2*a^10*(a + b*x^(1/3))^2) - (30*b^9)/(a^11*(a + b*x^(1/3))) - 1/(
3*a^3*x^3) + (9*b)/(8*a^4*x^(8/3)) - (18*b^2)/(7*a^5*x^(7/3)) + (5*b^3)/(a^
6*x^2) - (9*b^4)/(a^7*x^(5/3)) + (63*b^5)/(4*a^8*x^(4/3)) - (28*b^6)/(a^9*x
) + (54*b^7)/(a^10*x^(2/3)) - (135*b^8)/(a^11*x^(1/3)) + (165*b^9*Log[a + b
*x^(1/3)])/a^12 - (55*b^9*Log[x])/a^12
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x^4}\,dx = 3\,\text{Subst}\left(\int \frac{1}{x^{10}(a+bx)^3}\,dx, x, \sqrt[3]{x}\right)$$

$$= 3\,\text{Subst}\left(\int \left(\frac{1}{a^3x^{10}}-\frac{3b}{a^4x^9}+\frac{6b^2}{a^5x^8}-\frac{10b^3}{a^6x^7}+\frac{15b^4}{a^7x^6}-\frac{21b^5}{a^8x^5}+\frac{28b^6}{a^9x^4}-\frac{36b^7}{a^{10}x^3}+\frac{45b^8}{a^{11}x^2}-\frac{5}{a}\right.\right.$$

$$= -\frac{3b^9}{2a^{10}\left(a+b\sqrt[3]{x}\right)^2}-\frac{30b^9}{a^{11}\left(a+b\sqrt[3]{x}\right)}-\frac{1}{3a^3x^3}+\frac{9b}{8a^4x^{8/3}}-\frac{18b^2}{7a^5x^{7/3}}+\frac{5b^3}{a^6x^2}-\frac{9b^4}{a^7x^{5/3}}+\frac{}{4}$$

Mathematica [A] time = 0.28, size = 167, normalized size = 0.91

$$-\frac{\frac{a\left(56a^{10}-77a^9b\sqrt[3]{x}+110a^8b^2x^{2/3}-165a^7b^3x+264a^6b^4x^{4/3}-462a^5b^5x^{5/3}+924a^4b^6x^2-2310a^3b^7x^{7/3}+9240a^2b^8x^{8/3}+41580ab^9x^3+27720b^{10}x^{10/3}\right)}{x^3\left(a+b\sqrt[3]{x}\right)^2}-2}{168a^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^(1/3))^3*x^4),x]
```

```
[Out] -1/168*((a*(56*a^10 - 77*a^9*b*x^(1/3) + 110*a^8*b^2*x^(2/3) - 165*a^7*b^3*
x + 264*a^6*b^4*x^(4/3) - 462*a^5*b^5*x^(5/3) + 924*a^4*b^6*x^2 - 2310*a^3*
b^7*x^(7/3) + 9240*a^2*b^8*x^(8/3) + 41580*a*b^9*x^3 + 27720*b^10*x^(10/3))
)/((a + b*x^(1/3))^2*x^3) - 27720*b^9*Log[a + b*x^(1/3)] + 9240*b^9*Log[x])
/a^12
```

fricas [A] time = 0.67, size = 263, normalized size = 1.44

$$-\frac{9240\,a^3b^{12}x^4+13860\,a^6b^9x^3+3080\,a^9b^6x^2-728\,a^{12}b^3x+56\,a^{15}-27720\left(b^{15}x^5+2\,a^3b^{12}x^4+a^6b^9x^3\right)\log\left(bx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))^3/x^4,x, algorithm="fricas")
```

```
[Out] -1/168*(9240*a^3*b^12*x^4 + 13860*a^6*b^9*x^3 + 3080*a^9*b^6*x^2 - 728*a^12
*b^3*x + 56*a^15 - 27720*(b^15*x^5 + 2*a^3*b^12*x^4 + a^6*b^9*x^3)*log(b*x^
(1/3) + a) + 27720*(b^15*x^5 + 2*a^3*b^12*x^4 + a^6*b^9*x^3)*log(x^(1/3)) +
 18*(1540*a*b^14*x^4 + 2695*a^4*b^11*x^3 + 990*a^7*b^8*x^2 - 99*a^10*b^5*x
+ 24*a^13*b^2)*x^(2/3) - 63*(220*a^2*b^13*x^4 + 352*a^5*b^10*x^3 + 99*a^8*b
^7*x^2 - 18*a^11*b^4*x + 3*a^14*b)*x^(1/3))/(a^12*b^6*x^5 + 2*a^15*b^3*x^4
+ a^18*x^3)
```

giac [A] time = 0.17, size = 156, normalized size = 0.85

$$\frac{165\,b^9\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{a^{12}}-\frac{55\,b^9\log\left(|x|\right)}{a^{12}}-\frac{27720\,ab^{10}x^{\frac{10}{3}}+41580\,a^2b^9x^3+9240\,a^3b^8x^{\frac{8}{3}}-2310\,a^4b^7x^{\frac{7}{3}}+924\,a^5b^6x^2}{168\left(bx^{\frac{1}{3}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))^3/x^4,x, algorithm="giac")
```

```
[Out] 165*b^9*log(abs(b*x^(1/3) + a))/a^12 - 55*b^9*log(abs(x))/a^12 - 1/168*(277
20*a*b^10*x^(10/3) + 41580*a^2*b^9*x^3 + 9240*a^3*b^8*x^(8/3) - 2310*a^4*b^
7*x^(7/3) + 924*a^5*b^6*x^2 - 462*a^6*b^5*x^(5/3) + 264*a^7*b^4*x^(4/3) - 1
65*a^8*b^3*x + 110*a^9*b^2*x^(2/3) - 77*a^10*b*x^(1/3) + 56*a^11)/((b*x^(1/
3) + a)^2*a^12*x^3)
```

maple [A] time = 0.02, size = 156, normalized size = 0.85

$$-\frac{3b^9}{2\left(b\,x^{\frac{1}{3}}+a\right)^2a^{10}}-\frac{30b^9}{\left(b\,x^{\frac{1}{3}}+a\right)a^{11}}-\frac{55b^9\ln(x)}{a^{12}}+\frac{165b^9\ln\left(b\,x^{\frac{1}{3}}+a\right)}{a^{12}}-\frac{135b^8}{a^{11}x^{\frac{1}{3}}}+\frac{54b^7}{a^{10}x^{\frac{2}{3}}}-\frac{28b^6}{a^9x}+\frac{63b^5}{4a^8x^{\frac{4}{3}}}-\frac{9b^4}{a^7x^{\frac{5}{3}}}+\frac{5b^3}{a^6x^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^(1/3)+a)^3/x^4,x)
```

```
[Out] -3/2*b^9/a^10/(b*x^(1/3)+a)^2-30*b^9/a^11/(b*x^(1/3)+a)-1/3/a^3/x^3+9/8*b/a
^4/x^(8/3)-18/7*b^2/a^5/x^(7/3)+5*b^3/a^6/x^2-9*b^4/a^7/x^(5/3)+63/4*b^5/a^
8/x^(4/3)-28*b^6/a^9/x+54*b^7/a^10/x^(2/3)-135*b^8/a^11/x^(1/3)+165*b^9*ln(
b*x^(1/3)+a)/a^12-55*b^9*ln(x)/a^12
```

maxima [A] time = 0.56, size = 165, normalized size = 0.90

$$-\frac{27720\,b^{10}x^{\frac{10}{3}}+41580\,ab^9x^3+9240\,a^2b^8x^{\frac{8}{3}}-2310\,a^3b^7x^{\frac{7}{3}}+924\,a^4b^6x^2-462\,a^5b^5x^{\frac{5}{3}}+264\,a^6b^4x^{\frac{4}{3}}-165\,a^7}{168\left(a^{11}b^2x^{\frac{11}{3}}+2\,a^{12}bx^{\frac{10}{3}}+a^{13}x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^(1/3))^3/x^4,x, algorithm="maxima")
```

```
[Out] -1/168*(27720*b^10*x^(10/3) + 41580*a*b^9*x^3 + 9240*a^2*b^8*x^(8/3) - 2310
*a^3*b^7*x^(7/3) + 924*a^4*b^6*x^2 - 462*a^5*b^5*x^(5/3) + 264*a^6*b^4*x^(4
/3) - 165*a^7*b^3*x + 110*a^8*b^2*x^(2/3) - 77*a^9*b*x^(1/3) + 56*a^10)/(a^
11*b^2*x^(11/3) + 2*a^12*b*x^(10/3) + a^13*x^3) + 165*b^9*log(b*x^(1/3) + a
)/a^12 - 55*b^9*log(x)/a^12
```

mupad [B] time = 1.29, size = 159, normalized size = 0.87

$$\frac{330\,b^9\,\mathrm{atanh}\left(\frac{2\,b\,x^{1/3}}{a}+1\right)}{a^{12}}-\frac{\frac{1}{3\,a}-\frac{11\,b\,x^{1/3}}{24\,a^2}-\frac{55\,b^3\,x}{56\,a^4}+\frac{55\,b^2\,x^{2/3}}{84\,a^3}+\frac{11\,b^6\,x^2}{2\,a^7}+\frac{11\,b^4\,x^{4/3}}{7\,a^5}-\frac{11\,b^5\,x^{5/3}}{4\,a^6}+\frac{495\,b^9\,x^3}{2\,a^{10}}-\frac{55\,b^7\,x^{7/3}}{4\,a^8}+}{a^2\,x^3+b^2\,x^{11/3}+2\,a\,b\,x^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^(1/3))^3),x)
```

```
[Out] (330*b^9*atanh((2*b*x^(1/3))/a + 1))/a^12 - (1/(3*a) - (11*b*x^(1/3))/(24*a
^2) - (55*b^3*x)/(56*a^4) + (55*b^2*x^(2/3))/(84*a^3) + (11*b^6*x^2)/(2*a^7
) + (11*b^4*x^(4/3))/(7*a^5) - (11*b^5*x^(5/3))/(4*a^6) + (495*b^9*x^3)/(2*
a^10) - (55*b^7*x^(7/3))/(4*a^8) + (55*b^8*x^(8/3))/a^9 + (165*b^10*x^(10/3
))/a^11)/(a^2*x^3 + b^2*x^(11/3) + 2*a*b*x^(10/3))
```

sympy [A] time = 22.57, size = 847, normalized size = 4.63

$$\begin{cases}\frac{\tilde{\infty}}{x^4} \\ -\frac{1}{3a^3x^3} \\ -\frac{1}{4b^3x^4} \\ -\frac{56a^{11}x^{\frac{2}{3}}}{168a^{14}x^{\frac{11}{3}}+336a^{13}bx^4+168a^{12}b^2x^{\frac{13}{3}}}+\frac{77a^{10}bx}{168a^{14}x^{\frac{11}{3}}+336a^{13}bx^4+168a^{12}b^2x^{\frac{13}{3}}}-\frac{110a^9b^2x^{\frac{4}{3}}}{168a^{14}x^{\frac{11}{3}}+336a^{13}bx^4+168a^{12}b^2x^{\frac{13}{3}}}+\frac{165}{168a^{14}x^{\frac{11}{3}}+336a^{13}bx^4+168a^{12}b^2x^{\frac{13}{3}}}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**(1/3))**3/x**4,x)
```

```
[Out] Piecewise((zoo/x**4, Eq(a, 0) & Eq(b, 0)), (-1/(3*a**3*x**3), Eq(b, 0)), (-
1/(4*b**3*x**4), Eq(a, 0)), (-56*a**11*x**(2/3)/(168*a**14*x**(11/3) + 336*
a**13*b*x**4 + 168*a**12*b**2*x**(13/3)) + 77*a**10*b*x/(168*a**14*x**(11/3
) + 336*a**13*b*x**4 + 168*a**12*b**2*x**(13/3)) - 110*a**9*b**2*x**(4/3)/(
168*a**14*x**(11/3) + 336*a**13*b*x**4 + 168*a**12*b**2*x**(13/3)) + 165*a*
*8*b**3*x**(5/3)/(168*a**14*x**(11/3) + 336*a**13*b*x**4 + 168*a**12*b**2*x
**(13/3)) - 264*a**7*b**4*x**2/(168*a**14*x**(11/3) + 336*a**13*b*x**4 + 16
8*a**12*b**2*x**(13/3)) + 462*a**6*b**5*x**(7/3)/(168*a**14*x**(11/3) + 336
```

```
*a**13*b*x**4 + 168*a**12*b**2*x**(13/3)) - 924*a**5*b**6*x**(8/3)/(168*a**
14*x**(11/3) + 336*a**13*b*x**4 + 168*a**12*b**2*x**(13/3)) + 2310*a**4*b**
7*x**3/(168*a**14*x**(11/3) + 336*a**13*b*x**4 + 168*a**12*b**2*x**(13/3))
- 9240*a**3*b**8*x**(10/3)/(168*a**14*x**(11/3) + 336*a**13*b*x**4 + 168*a*
*12*b**2*x**(13/3)) - 9240*a**2*b**9*x**(11/3)*log(x)/(168*a**14*x**(11/3)
+ 336*a**13*b*x**4 + 168*a**12*b**2*x**(13/3)) + 27720*a**2*b**9*x**(11/3)*
log(a/b + x**(1/3))/(168*a**14*x**(11/3) + 336*a**13*b*x**4 + 168*a**12*b**
2*x**(13/3)) - 41580*a**2*b**9*x**(11/3)/(168*a**14*x**(11/3) + 336*a**13*b
*x**4 + 168*a**12*b**2*x**(13/3)) - 18480*a*b**10*x**4*log(x)/(168*a**14*x*
*(11/3) + 336*a**13*b*x**4 + 168*a**12*b**2*x**(13/3)) + 55440*a*b**10*x**4
*log(a/b + x**(1/3))/(168*a**14*x**(11/3) + 336*a**13*b*x**4 + 168*a**12*b*
*2*x**(13/3)) - 27720*a*b**10*x**4/(168*a**14*x**(11/3) + 336*a**13*b*x**4
+ 168*a**12*b**2*x**(13/3)) - 9240*b**11*x**(13/3)*log(x)/(168*a**14*x**(11
/3) + 336*a**13*b*x**4 + 168*a**12*b**2*x**(13/3)) + 27720*b**11*x**(13/3)*
log(a/b + x**(1/3))/(168*a**14*x**(11/3) + 336*a**13*b*x**4 + 168*a**12*b**
2*x**(13/3)), True))
```

3.2381 $\int \frac{1}{\sqrt{1+\sqrt[3]{x}}}\,dx$

Optimal. Leaf size=42

$$\frac{6}{5}\left(\sqrt[3]{x}+1\right)^{5/2}-4\left(\sqrt[3]{x}+1\right)^{3/2}+6\sqrt{\sqrt[3]{x}+1}$$

```
[Out] -4*(x^(1/3)+1)^(3/2)+6/5*(x^(1/3)+1)^(5/2)+6*(x^(1/3)+1)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {190, 43}

$$\frac{6}{5}\left(\sqrt[3]{x}+1\right)^{5/2}-4\left(\sqrt[3]{x}+1\right)^{3/2}+6\sqrt{\sqrt[3]{x}+1}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + x^(1/3)],x]
```

```
[Out] 6*Sqrt[1 + x^(1/3)] - 4*(1 + x^(1/3))^(3/2) + (6*(1 + x^(1/3))^(5/2))/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1+\sqrt[3]{x}}}\,dx &= 3\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x}}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(\frac{1}{\sqrt{1+x}}-2\sqrt{1+x}+(1+x)^{3/2}\right)dx, x, \sqrt[3]{x}\right)\\ &= 6\sqrt{1+\sqrt[3]{x}}-4\left(1+\sqrt[3]{x}\right)^{3/2}+\frac{6}{5}\left(1+\sqrt[3]{x}\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.74

$$\frac{2}{5}\sqrt{\sqrt[3]{x}+1}\left(3x^{2/3}-4\sqrt[3]{x}+8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + x^(1/3)],x]
```

```
[Out] (2*Sqrt[1 + x^(1/3)]*(8 - 4*x^(1/3) + 3*x^(2/3)))/5
```

fricas [A] time = 0.62, size = 21, normalized size = 0.50

$$\frac{2}{5}\left(3x^{\frac{2}{3}}-4x^{\frac{1}{3}}+8\right)\sqrt{x^{\frac{1}{3}}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(1/3))^(1/2),x, algorithm="fricas")
[Out] 2/5*(3*x^(2/3) - 4*x^(1/3) + 8)*sqrt(x^(1/3) + 1)
```

giac [A] time = 0.15, size = 28, normalized size = 0.67

$$\frac{6}{5}\left(x^{\frac{1}{3}}+1\right)^{\frac{5}{2}}-4\left(x^{\frac{1}{3}}+1\right)^{\frac{3}{2}}+6\sqrt{x^{\frac{1}{3}}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(1/3))^(1/2),x, algorithm="giac")
[Out] 6/5*(x^(1/3) + 1)^(5/2) - 4*(x^(1/3) + 1)^(3/2) + 6*sqrt(x^(1/3) + 1)
```

maple [A] time = 0.01, size = 29, normalized size = 0.69

$$-4\left(x^{\frac{1}{3}}+1\right)^{\frac{3}{2}}+\frac{6\left(x^{\frac{1}{3}}+1\right)^{\frac{5}{2}}}{5}+6\sqrt{x^{\frac{1}{3}}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/3)+1)^(1/2),x)
[Out] -4*(x^(1/3)+1)^(3/2)+6/5*(x^(1/3)+1)^(5/2)+6*(x^(1/3)+1)^(1/2)
```

maxima [A] time = 0.59, size = 28, normalized size = 0.67

$$\frac{6}{5}\left(x^{\frac{1}{3}}+1\right)^{\frac{5}{2}}-4\left(x^{\frac{1}{3}}+1\right)^{\frac{3}{2}}+6\sqrt{x^{\frac{1}{3}}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(1/3))^(1/2),x, algorithm="maxima")
[Out] 6/5*(x^(1/3) + 1)^(5/2) - 4*(x^(1/3) + 1)^(3/2) + 6*sqrt(x^(1/3) + 1)
```

mupad [B] time = 1.21, size = 12, normalized size = 0.29

$$x\,{}_2\mathrm{F}_1\left(\frac{1}{2},3;\ 4;\ -x^{1/3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/3) + 1)^(1/2),x)
[Out] x*hypergeom([1/2, 3], 4, -x^(1/3))
```

sympy [B] time = 1.36, size = 359, normalized size = 8.55

$$\frac{6x^{\frac{14}{3}}\sqrt{\sqrt[3]{x}+1}}{15x^{\frac{11}{3}}+15x^{\frac{10}{3}}+5x^4+5x^3}+\frac{10x^{\frac{13}{3}}\sqrt{\sqrt[3]{x}+1}}{15x^{\frac{11}{3}}+15x^{\frac{10}{3}}+5x^4+5x^3}+\frac{30x^{\frac{11}{3}}\sqrt{\sqrt[3]{x}+1}}{15x^{\frac{11}{3}}+15x^{\frac{10}{3}}+5x^4+5x^3}-\frac{48x^{\frac{11}{3}}}{15x^{\frac{11}{3}}+15x^{\frac{10}{3}}+5x^4+5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x**(1/3))**(1/2),x)
```

```
[Out] 6*x**(14/3)*sqrt(x**(1/3) + 1)/(15*x**(11/3) + 15*x**(10/3) + 5*x**4 + 5*x**
*3) + 10*x**(13/3)*sqrt(x**(1/3) + 1)/(15*x**(11/3) + 15*x**(10/3) + 5*x**4
 + 5*x**3) + 30*x**(11/3)*sqrt(x**(1/3) + 1)/(15*x**(11/3) + 15*x**(10/3) +
 5*x**4 + 5*x**3) - 48*x**(11/3)/(15*x**(11/3) + 15*x**(10/3) + 5*x**4 + 5*
x**3) + 40*x**(10/3)*sqrt(x**(1/3) + 1)/(15*x**(11/3) + 15*x**(10/3) + 5*x**
*4 + 5*x**3) - 48*x**(10/3)/(15*x**(11/3) + 15*x**(10/3) + 5*x**4 + 5*x**3)
 + 10*x**4*sqrt(x**(1/3) + 1)/(15*x**(11/3) + 15*x**(10/3) + 5*x**4 + 5*x**
3) - 16*x**4/(15*x**(11/3) + 15*x**(10/3) + 5*x**4 + 5*x**3) + 16*x**3*sqrt
(x**(1/3) + 1)/(15*x**(11/3) + 15*x**(10/3) + 5*x**4 + 5*x**3) - 16*x**3/(1
5*x**(11/3) + 15*x**(10/3) + 5*x**4 + 5*x**3)
```

3.2382 $\int \frac{1}{\left(1+\sqrt[3]{x}\right)x^{3/2}}\,dx$

Optimal. Leaf size=23

$$\frac{6}{\sqrt[6]{x}}-\frac{2}{\sqrt{x}}+6\tan^{-1}\left(\sqrt[6]{x}\right)$$

```
[Out] 6/x^(1/6)+6*arctan(x^(1/6))-2/x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {341, 51, 63, 203}

$$\frac{6}{\sqrt[6]{x}}-\frac{2}{\sqrt{x}}+6\tan^{-1}\left(\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^(1/3))*x^(3/2)),x]
```

```
[Out] -2/Sqrt[x] + 6/x^(1/6) + 6*ArcTan[x^(1/6)]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(1+\sqrt[3]{x}\right)x^{3/2}}\,dx &= 3\,\text{Subst}\left(\int \frac{1}{x^{5/2}(1+x)}\,dx, x, \sqrt[3]{x}\right)\\ &= -\frac{2}{\sqrt{x}} - 3\,\text{Subst}\left(\int \frac{1}{x^{3/2}(1+x)}\,dx, x, \sqrt[3]{x}\right)\\ &= -\frac{2}{\sqrt{x}} + \frac{6}{\sqrt[6]{x}} + 3\,\text{Subst}\left(\int \frac{1}{\sqrt{x}(1+x)}\,dx, x, \sqrt[3]{x}\right)\\ &= -\frac{2}{\sqrt{x}} + \frac{6}{\sqrt[6]{x}} + 6\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[6]{x}\right)\\ &= -\frac{2}{\sqrt{x}} + \frac{6}{\sqrt[6]{x}} + 6\tan^{-1}\left(\sqrt[6]{x}\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 22, normalized size = 0.96

$$-\frac{2\,{}_2F_1\left(-\frac{3}{2}, 1; -\frac{1}{2}; -\sqrt[3]{x}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^(1/3))*x^(3/2)),x]
[Out] (-2*Hypergeometric2F1[-3/2, 1, -1/2, -x^(1/3)])/Sqrt[x]
```

fricas [A] time = 0.70, size = 23, normalized size = 1.00

$$\frac{2\left(3\,x\arctan\left(x^{\frac{1}{6}}\right) + 3\,x^{\frac{5}{6}} - \sqrt{x}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(1/3))/x^(3/2),x, algorithm="fricas")
[Out] 2*(3*x*arctan(x^(1/6)) + 3*x^(5/6) - sqrt(x))/x
```

giac [A] time = 0.15, size = 19, normalized size = 0.83

$$\frac{2\left(3\,x^{\frac{1}{3}} - 1\right)}{\sqrt{x}} + 6\,\arctan\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(1/3))/x^(3/2),x, algorithm="giac")
[Out] 2*(3*x^(1/3) - 1)/sqrt(x) + 6*arctan(x^(1/6))
```

maple [A] time = 0.01, size = 18, normalized size = 0.78

$$6\arctan\left(x^{\frac{1}{6}}\right) + \frac{6}{x^{\frac{1}{6}}} - \frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/3)+1)/x^(3/2),x)
[Out] 6/x^(1/6)+6*arctan(x^(1/6))-2/x^(1/2)
```

maxima [A] time = 1.22, size = 19, normalized size = 0.83

$$\frac{2\left(3\,x^{\frac{1}{3}}-1\right)}{\sqrt{x}}+6\,\arctan\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(1/3))/x^(3/2),x, algorithm="maxima")
[Out] 2*(3*x^(1/3) - 1)/sqrt(x) + 6*arctan(x^(1/6))
```

mupad [B] time = 0.04, size = 18, normalized size = 0.78

$$6\,\mathrm{atan}\left(x^{1/6}\right)+\frac{6\,x^{1/3}-2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(x^(1/3) + 1)),x)
[Out] 6*atan(x^(1/6)) + (6*x^(1/3) - 2)/x^(1/2)
```

sympy [A] time = 1.37, size = 20, normalized size = 0.87

$$6\,\mathrm{atan}\left(\sqrt[6]{x}\right)-\frac{2}{\sqrt{x}}+\frac{6}{\sqrt[6]{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x**(1/3))/x**(3/2),x)
[Out] 6*atan(x**(1/6)) - 2/sqrt(x) + 6/x**(1/6)
```

3.2383 $\int \frac{x^{2/3}}{1+\sqrt[3]{x}}\,dx$

Optimal. Leaf size=39

$$\frac{3x^{4/3}}{4}+\frac{3x^{2/3}}{2}-x-3\sqrt[3]{x}+3\log\left(\sqrt[3]{x}+1\right)$$

```
[Out] -3*x^(1/3)+3/2*x^(2/3)-x+3/4*x^(4/3)+3*ln(x^(1/3)+1)
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {266, 43}

$$\frac{3x^{4/3}}{4}+\frac{3x^{2/3}}{2}-x-3\sqrt[3]{x}+3\log\left(\sqrt[3]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^(2/3)/(1 + x^(1/3)),x]
```

```
[Out] -3*x^(1/3) + (3*x^(2/3))/2 - x + (3*x^(4/3))/4 + 3*Log[1 + x^(1/3)]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{2/3}}{1+\sqrt[3]{x}}\,dx &= 3\,\text{Subst}\left(\int \frac{x^4}{1+x}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(-1+x-x^2+x^3+\frac{1}{1+x}\right)\,dx, x, \sqrt[3]{x}\right)\\ &= -3\sqrt[3]{x}+\frac{3x^{2/3}}{2}-x+\frac{3x^{4/3}}{4}+3\log\left(1+\sqrt[3]{x}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 1.00

$$\frac{3x^{4/3}}{4}+\frac{3x^{2/3}}{2}-x-3\sqrt[3]{x}+3\log\left(\sqrt[3]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2/3)/(1 + x^(1/3)),x]
```

```
[Out] -3*x^(1/3) + (3*x^(2/3))/2 - x + (3*x^(4/3))/4 + 3*Log[1 + x^(1/3)]
```

fricas [A] time = 0.65, size = 25, normalized size = 0.64

$$\frac{3}{4}(x-4)x^{\frac{1}{3}}-x+\frac{3}{2}x^{\frac{2}{3}}+3\log\left(x^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(1+x^(1/3)),x, algorithm="fricas")
[Out] 3/4*(x - 4)*x^(1/3) - x + 3/2*x^(2/3) + 3*log(x^(1/3) + 1)
```

giac [A] time = 0.16, size = 27, normalized size = 0.69

$$\frac{3}{4}x^{\frac{4}{3}} - x + \frac{3}{2}x^{\frac{2}{3}} - 3x^{\frac{1}{3}} + 3\log\left(x^{\frac{1}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(1+x^(1/3)),x, algorithm="giac")
[Out] 3/4*x^(4/3) - x + 3/2*x^(2/3) - 3*x^(1/3) + 3*log(x^(1/3) + 1)
```

maple [A] time = 0.00, size = 28, normalized size = 0.72

$$\frac{3x^{\frac{4}{3}}}{4} - x + 3\ln\left(x^{\frac{1}{3}} + 1\right) + \frac{3x^{\frac{2}{3}}}{2} - 3x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)/(x^(1/3)+1),x)
[Out] -3*x^(1/3)+3/2*x^(2/3)-x+3/4*x^(4/3)+3*ln(x^(1/3)+1)
```

maxima [A] time = 0.56, size = 42, normalized size = 1.08

$$\frac{3}{4}\left(x^{\frac{1}{3}} + 1\right)^4 - 4\left(x^{\frac{1}{3}} + 1\right)^3 + 9\left(x^{\frac{1}{3}} + 1\right)^2 - 12x^{\frac{1}{3}} + 3\log\left(x^{\frac{1}{3}} + 1\right) - 12$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(1+x^(1/3)),x, algorithm="maxima")
[Out] 3/4*(x^(1/3) + 1)^4 - 4*(x^(1/3) + 1)^3 + 9*(x^(1/3) + 1)^2 - 12*x^(1/3) +
3*log(x^(1/3) + 1) - 12
```

mupad [B] time = 0.02, size = 27, normalized size = 0.69

$$3\ln\left(x^{1/3} + 1\right) - x - 3x^{1/3} + \frac{3x^{2/3}}{2} + \frac{3x^{4/3}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)/(x^(1/3) + 1),x)
[Out] 3*log(x^(1/3) + 1) - x - 3*x^(1/3) + (3*x^(2/3))/2 + (3*x^(4/3))/4
```

sympy [A] time = 0.25, size = 34, normalized size = 0.87

$$\frac{3x^{\frac{4}{3}}}{4} + \frac{3x^{\frac{2}{3}}}{2} - 3\sqrt[3]{x} - x + 3\log\left(\sqrt[3]{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2/3)/(1+x**(1/3)),x)
[Out] 3*x**(4/3)/4 + 3*x**(2/3)/2 - 3*x**(1/3) - x + 3*log(x**(1/3) + 1)
```

3.2384 $\int \frac{1}{1+x^{2/3}}\,dx$

Optimal. Leaf size=16

$$3\sqrt[3]{x} - 3\tan^{-1}\left(\sqrt[3]{x}\right)$$

```
[Out] 3*x^(1/3)-3*arctan(x^(1/3))
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {243, 321, 203}

$$3\sqrt[3]{x} - 3\tan^{-1}\left(\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^(2/3))^(-1),x]
```

```
[Out] 3*x^(1/3) - 3*ArcTan[x^(1/3)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Free
Q[{a, b, p}, x] && FractionQ[n]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+x^{2/3}}\,dx &= 3\,\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\sqrt[3]{x} - 3\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\sqrt[3]{x} - 3\tan^{-1}\left(\sqrt[3]{x}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$3\sqrt[3]{x} - 3\tan^{-1}\left(\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^(2/3))^(-1),x]
```

```
[Out] 3*x^(1/3) - 3*ArcTan[x^(1/3)]
```

fricas [A] time = 0.62, size = 12, normalized size = 0.75

$$3\,x^{\frac{1}{3}} - 3\,\arctan\left(x^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(2/3)),x, algorithm="fricas")
```

```
[Out] 3*x^(1/3) - 3*arctan(x^(1/3))
```

giac [A] time = 0.15, size = 12, normalized size = 0.75

$$3\,x^{\frac{1}{3}} - 3\,\arctan\left(x^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(2/3)),x, algorithm="giac")
```

```
[Out] 3*x^(1/3) - 3*arctan(x^(1/3))
```

maple [B] time = 0.04, size = 41, normalized size = 2.56

$$\arctan(x) - 2\arctan\left(x^{\frac{1}{3}}\right) - \arctan\left(2x^{\frac{1}{3}} - \sqrt{3}\right) - \arctan\left(2x^{\frac{1}{3}} + \sqrt{3}\right) + 3x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x^(2/3)),x)
```

```
[Out] arctan(x)+3*x^(1/3)-2*arctan(x^(1/3))-arctan(2*x^(1/3)-3^(1/2))-arctan(2*x^
(1/3)+3^(1/2))
```

maxima [A] time = 1.20, size = 12, normalized size = 0.75

$$3\,x^{\frac{1}{3}} - 3\,\arctan\left(x^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(2/3)),x, algorithm="maxima")
```

```
[Out] 3*x^(1/3) - 3*arctan(x^(1/3))
```

mupad [B] time = 1.12, size = 12, normalized size = 0.75

$$3\,x^{1/3} - 3\,\mathrm{atan}\left(x^{1/3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(2/3) + 1),x)
```

```
[Out] 3*x^(1/3) - 3*atan(x^(1/3))
```

sympy [A] time = 0.17, size = 14, normalized size = 0.88

$$3\sqrt[3]{x} - 3\,\mathrm{atan}\left(\sqrt[3]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x**(2/3)),x)
```

```
[Out] 3*x**(1/3) - 3*atan(x**(1/3))
```

3.2385 $\int \frac{1}{\left(1+x^{2/3}\right)\sqrt[3]{x}}\,dx$

Optimal. Leaf size=12

$$\frac{3}{2}\log\left(x^{2/3}+1\right)$$

```
[Out] 3/2*ln(1+x^(2/3))
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {260}

$$\frac{3}{2}\log\left(x^{2/3}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^(2/3))*x^(1/3)),x]
```

```
[Out] (3*Log[1 + x^(2/3)])/2
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1}{\left(1+x^{2/3}\right)\sqrt[3]{x}}\,dx = \frac{3}{2}\log\left(1+x^{2/3}\right)$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\frac{3}{2}\log\left(x^{2/3}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^(2/3))*x^(1/3)),x]
```

```
[Out] (3*Log[1 + x^(2/3)])/2
```

fricas [A] time = 0.65, size = 8, normalized size = 0.67

$$\frac{3}{2}\log\left(x^{\frac{2}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(2/3))/x^(1/3),x, algorithm="fricas")
```

```
[Out] 3/2*log(x^(2/3) + 1)
```

giac [A] time = 0.15, size = 8, normalized size = 0.67

$$\frac{3}{2}\log\left(x^{\frac{2}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(2/3))/x^(1/3),x, algorithm="giac")
[Out] 3/2*log(x^(2/3) + 1)
```

maple [A] time = 0.00, size = 9, normalized size = 0.75

$$\frac{3\ln\left(x^{\frac{2}{3}}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x^(2/3))/x^(1/3),x)
[Out] 3/2*ln(1+x^(2/3))
```

maxima [A] time = 0.51, size = 8, normalized size = 0.67

$$\frac{3}{2}\log\left(x^{\frac{2}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(2/3))/x^(1/3),x, algorithm="maxima")
[Out] 3/2*log(x^(2/3) + 1)
```

mupad [B] time = 0.15, size = 8, normalized size = 0.67

$$\frac{3\,\ln\left(x^{2/3}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/3)*(x^(2/3) + 1)),x)
[Out] (3*log(x^(2/3) + 1))/2
```

sympy [A] time = 0.20, size = 10, normalized size = 0.83

$$\frac{3\log\left(x^{\frac{2}{3}}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x**(2/3))/x**(1/3),x)
[Out] 3*log(x**(2/3) + 1)/2
```

3.2386 $\int \frac{1}{\left(1+x^{2/3}\right)x^{2/3}}\,dx$

Optimal. Leaf size=8

$$3\tan^{-1}\left(\sqrt[3]{x}\right)$$

```
[Out] 3*arctan(x^(1/3))
```

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {341, 203}

$$3\tan^{-1}\left(\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^(2/3))*x^(2/3)),x]
```

```
[Out] 3*ArcTan[x^(1/3)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(1+x^{2/3}\right)x^{2/3}}\,dx &= 3\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\tan^{-1}\left(\sqrt[3]{x}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$3\tan^{-1}\left(\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^(2/3))*x^(2/3)),x]
```

```
[Out] 3*ArcTan[x^(1/3)]
```

fricas [A] time = 0.72, size = 6, normalized size = 0.75

$$3\arctan\left(x^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(2/3))/x^(2/3),x, algorithm="fricas")
```

```
[Out] 3*arctan(x^(1/3))
```

giac [A] time = 0.17, size = 6, normalized size = 0.75

$$3 \arctan\left(x^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(2/3))/x^(2/3),x, algorithm="giac")
[Out] 3*arctan(x^(1/3))
```

maple [A] time = 0.00, size = 7, normalized size = 0.88

$$3 \arctan\left(x^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(2/3)+1)/x^(2/3),x)
[Out] 3*arctan(x^(1/3))
```

maxima [A] time = 1.11, size = 6, normalized size = 0.75

$$3 \arctan\left(x^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(2/3))/x^(2/3),x, algorithm="maxima")
[Out] 3*arctan(x^(1/3))
```

mupad [B] time = 0.15, size = 6, normalized size = 0.75

$$3 \operatorname{atan}\left(x^{1/3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(2/3)*(x^(2/3) + 1)),x)
[Out] 3*atan(x^(1/3))
```

sympy [A] time = 0.29, size = 7, normalized size = 0.88

$$3 \operatorname{atan}\left(\sqrt[3]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x**(2/3))/x**(2/3),x)
[Out] 3*atan(x**(1/3))
```

3.2387 $\int \frac{\sqrt{-1+x^{2/3}}}{\sqrt[3]{x}}\,dx$

Optimal. Leaf size=11

$$\left(x^{2/3}-1\right)^{3/2}$$

```
[Out] (-1+x^(2/3))^(3/2)
```

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {261}

$$\left(x^{2/3}-1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x^(2/3)]/x^(1/3),x]
```

```
[Out] (-1 + x^(2/3))^(3/2)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{-1+x^{2/3}}}{\sqrt[3]{x}}\,dx = \left(-1+x^{2/3}\right)^{3/2}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\left(x^{2/3}-1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x^(2/3)]/x^(1/3),x]
```

```
[Out] (-1 + x^(2/3))^(3/2)
```

fricas [A] time = 1.91, size = 7, normalized size = 0.64

$$\left(x^{\frac{2}{3}}-1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(2/3))^(1/2)/x^(1/3),x, algorithm="fricas")
```

```
[Out] (x^(2/3) - 1)^(3/2)
```

giac [A] time = 0.18, size = 7, normalized size = 0.64

$$\left(x^{\frac{2}{3}}-1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(2/3))^(1/2)/x^(1/3),x, algorithm="giac")
[Out] (x^(2/3) - 1)^(3/2)
```

maple [A] time = 0.01, size = 8, normalized size = 0.73

$$\left(x^{\frac{2}{3}}-1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x^(2/3))^(1/2)/x^(1/3),x)
[Out] (-1+x^(2/3))^(3/2)
```

maxima [A] time = 0.52, size = 7, normalized size = 0.64

$$\left(x^{\frac{2}{3}}-1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(2/3))^(1/2)/x^(1/3),x, algorithm="maxima")
[Out] (x^(2/3) - 1)^(3/2)
```

mupad [B] time = 2.19, size = 7, normalized size = 0.64

$$\left(x^{2/3}-1\right)^{3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(2/3) - 1)^(1/2)/x^(1/3),x)
[Out] (x^(2/3) - 1)^(3/2)
```

sympy [B] time = 0.24, size = 24, normalized size = 2.18

$$x^{\frac{2}{3}}\sqrt{x^{\frac{2}{3}}-1}-\sqrt{x^{\frac{2}{3}}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x**(2/3))**(1/2)/x**(1/3),x)
[Out] x**(2/3)*sqrt(x**(2/3) - 1) - sqrt(x**(2/3) - 1)
```

3.2388 $\int \frac{\left(1+x^{2/3}\right)^{3/2}}{\sqrt[3]{x}}\,dx$

Optimal. Leaf size=15

$$\frac{3}{5}\left(x^{2/3}+1\right)^{5/2}$$

```
[Out] 3/5*(1+x^(2/3))^(5/2)
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {261}

$$\frac{3}{5}\left(x^{2/3}+1\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^(2/3))^(3/2)/x^(1/3),x]
```

```
[Out] (3*(1 + x^(2/3))^(5/2))/5
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(1+x^{2/3}\right)^{3/2}}{\sqrt[3]{x}}\,dx = \frac{3}{5}\left(1+x^{2/3}\right)^{5/2}$$

Mathematica [A] time = 0.02, size = 15, normalized size = 1.00

$$\frac{3}{5}\left(x^{2/3}+1\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^(2/3))^(3/2)/x^(1/3),x]
```

```
[Out] (3*(1 + x^(2/3))^(5/2))/5
```

fricas [B] time = 2.20, size = 19, normalized size = 1.27

$$\frac{3}{5}\left(x^{\frac{4}{3}}+2\,x^{\frac{2}{3}}+1\right)\sqrt{x^{\frac{2}{3}}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(2/3))^(3/2)/x^(1/3),x, algorithm="fricas")
```

```
[Out] 3/5*(x^(4/3) + 2*x^(2/3) + 1)*sqrt(x^(2/3) + 1)
```

giac [A] time = 0.15, size = 9, normalized size = 0.60

$$\frac{3}{5}\left(x^{\frac{2}{3}}+1\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(2/3))^(3/2)/x^(1/3),x, algorithm="giac")
[Out] 3/5*(x^(2/3) + 1)^(5/2)
```

maple [A] time = 0.01, size = 10, normalized size = 0.67

$$\frac{3\left(x^{\frac{2}{3}}+1\right)^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(2/3)+1)^(3/2)/x^(1/3),x)
[Out] 3/5*(x^(2/3)+1)^(5/2)
```

maxima [A] time = 0.45, size = 9, normalized size = 0.60

$$\frac{3}{5}\left(x^{\frac{2}{3}}+1\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(2/3))^(3/2)/x^(1/3),x, algorithm="maxima")
[Out] 3/5*(x^(2/3) + 1)^(5/2)
```

mupad [B] time = 2.09, size = 9, normalized size = 0.60

$$\frac{3\left(x^{2/3}+1\right)^{5/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(2/3) + 1)^(3/2)/x^(1/3),x)
[Out] (3*(x^(2/3) + 1)^(5/2))/5
```

sympy [B] time = 1.45, size = 49, normalized size = 3.27

$$\frac{3x^{\frac{4}{3}}\sqrt{x^{\frac{2}{3}}+1}}{5}+\frac{6x^{\frac{2}{3}}\sqrt{x^{\frac{2}{3}}+1}}{5}+\frac{3\sqrt{x^{\frac{2}{3}}+1}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(2/3))**(3/2)/x**(1/3),x)
[Out] 3*x**(4/3)*sqrt(x**(2/3) + 1)/5 + 6*x**(2/3)*sqrt(x**(2/3) + 1)/5 + 3*sqrt(
x**(2/3) + 1)/5
```

3.2389 $\int \frac{\sqrt{x}}{1+x^{2/3}}\,dx$

Optimal. Leaf size=117

$$\frac{6x^{5/6}}{5}-6\sqrt[6]{x}-\frac{3\log\left(\sqrt[3]{x}-\sqrt{2}\sqrt[6]{x}+1\right)}{2\sqrt{2}}+\frac{3\log\left(\sqrt[3]{x}+\sqrt{2}\sqrt[6]{x}+1\right)}{2\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt[6]{x}\right)}{\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}\sqrt[6]{x}+1\right.}{\sqrt{2}}$$

```
[Out] -6*x^(1/6)+6/5*x^(5/6)+3/2*arctan(-1+x^(1/6)*2^(1/2))*2^(1/2)+3/2*arctan(1+
x^(1/6)*2^(1/2))*2^(1/2)-3/4*ln(1+x^(1/3)-x^(1/6)*2^(1/2))*2^(1/2)+3/4*ln(1
+x^(1/3)+x^(1/6)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {341, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{6x^{5/6}}{5}-6\sqrt[6]{x}-\frac{3\log\left(\sqrt[3]{x}-\sqrt{2}\sqrt[6]{x}+1\right)}{2\sqrt{2}}+\frac{3\log\left(\sqrt[3]{x}+\sqrt{2}\sqrt[6]{x}+1\right)}{2\sqrt{2}}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt[6]{x}\right)}{\sqrt{2}}+\frac{3\tan^{-1}\left(\sqrt{2}\sqrt[6]{x}+1\right.}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(1 + x^(2/3)),x]
```

```
[Out] -6*x^(1/6) + (6*x^(5/6))/5 - (3*ArcTan[1 - Sqrt[2]*x^(1/6)])/Sqrt[2] + (3*A
rcTan[1 + Sqrt[2]*x^(1/6)])/Sqrt[2] - (3*Log[1 - Sqrt[2]*x^(1/6) + x^(1/3)]
)/(2*Sqrt[2]) + (3*Log[1 + Sqrt[2]*x^(1/6) + x^(1/3)])/(2*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
```

```
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{1+x^{2/3}}\,dx &= 3\,\text{Subst}\left(\int \frac{x^{7/2}}{1+x^2}\,dx, x, \sqrt[3]{x}\right)\\
&= \frac{6x^{5/6}}{5} - 3\,\text{Subst}\left(\int \frac{x^{3/2}}{1+x^2}\,dx, x, \sqrt[3]{x}\right)\\
&= -6\sqrt[6]{x} + \frac{6x^{5/6}}{5} + 3\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1+x^2\right)}\,dx, x, \sqrt[3]{x}\right)\\
&= -6\sqrt[6]{x} + \frac{6x^{5/6}}{5} + 6\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt[6]{x}\right)\\
&= -6\sqrt[6]{x} + \frac{6x^{5/6}}{5} + 3\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt[6]{x}\right) + 3\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt[6]{x}\right)\\
&= -6\sqrt[6]{x} + \frac{6x^{5/6}}{5} + \frac{3}{2}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt[6]{x}\right) + \frac{3}{2}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x, \sqrt[6]{x}\right)\\
&= -6\sqrt[6]{x} + \frac{6x^{5/6}}{5} - \frac{3\log\left(1-\sqrt{2}\,\sqrt[6]{x}+\sqrt[3]{x}\right)}{2\sqrt{2}} + \frac{3\log\left(1+\sqrt{2}\,\sqrt[6]{x}+\sqrt[3]{x}\right)}{2\sqrt{2}} + \frac{3\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x\right.}{\sqrt{2}}\\
&= -6\sqrt[6]{x} + \frac{6x^{5/6}}{5} - \frac{3\tan^{-1}\left(1-\sqrt{2}\,\sqrt[6]{x}\right)}{\sqrt{2}} + \frac{3\tan^{-1}\left(1+\sqrt{2}\,\sqrt[6]{x}\right)}{\sqrt{2}} - \frac{3\log\left(1-\sqrt{2}\,\sqrt[6]{x}+\sqrt[3]{x}\right)}{2\sqrt{2}} + \frac{3}{}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 117, normalized size = 1.00

$$\frac{6x^{5/6}}{5} - 6\sqrt[6]{x} - \frac{3\log\left(\sqrt[3]{x}-\sqrt{2}\,\sqrt[6]{x}+1\right)}{2\sqrt{2}} + \frac{3\log\left(\sqrt[3]{x}+\sqrt{2}\,\sqrt[6]{x}+1\right)}{2\sqrt{2}} - \frac{3\tan^{-1}\left(1-\sqrt{2}\,\sqrt[6]{x}\right)}{\sqrt{2}} + \frac{3\tan^{-1}\left(\sqrt{2}\,\sqrt[6]{x}+1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(1 + x^(2/3)),x]
```

```
[Out] -6*x^(1/6) + (6*x^(5/6))/5 - (3*ArcTan[1 - Sqrt[2]*x^(1/6)])/Sqrt[2] + (3*A
rcTan[1 + Sqrt[2]*x^(1/6)])/Sqrt[2] - (3*Log[1 - Sqrt[2]*x^(1/6) + x^(1/3)]
)/(2*Sqrt[2]) + (3*Log[1 + Sqrt[2]*x^(1/6) + x^(1/3)])/(2*Sqrt[2])
```

fricas [A] time = 0.57, size = 125, normalized size = 1.07

$$-3\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}x^{\frac{1}{6}}+x^{\frac{1}{3}}+1}-\sqrt{2}x^{\frac{1}{6}}-1\right)-3\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}x^{\frac{1}{6}}+4x^{\frac{1}{3}}+4}-\sqrt{2}x^{\frac{1}{6}}+1\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x^(2/3)),x, algorithm="fricas")
```

```
[Out] -3*sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*x^(1/6) + x^(1/3) + 1) - sqrt(2)*x^(
1/6) - 1) - 3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*x^(1/6) + 4*x^(1/3
) + 4) - sqrt(2)*x^(1/6) + 1) + 3/4*sqrt(2)*log(4*sqrt(2)*x^(1/6) + 4*x^(1/
3) + 4) - 3/4*sqrt(2)*log(-4*sqrt(2)*x^(1/6) + 4*x^(1/3) + 4) + 6/5*x^(5/6)
 - 6*x^(1/6)
```

giac [A] time = 0.17, size = 88, normalized size = 0.75

$$\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2x^{\frac{1}{6}}\right)\right)+\frac{3}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2x^{\frac{1}{6}}\right)\right)+\frac{3}{4}\sqrt{2}\log\left(\sqrt{2}x^{\frac{1}{6}}+x^{\frac{1}{3}}+1\right)-\frac{3}{4}\sqrt{2}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x^(2/3)),x, algorithm="giac")
```

```
[Out] 3/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*x^(1/6))) + 3/2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*x^(1/6))) + 3/4*sqrt(2)*log(sqrt(2)*x^(1/6) + x^(
1/3) + 1) - 3/4*sqrt(2)*log(-sqrt(2)*x^(1/6) + x^(1/3) + 1) + 6/5*x^(5/6) -
 6*x^(1/6)
```

maple [A] time = 0.01, size = 76, normalized size = 0.65

$$\frac{3\sqrt{2}\,\arctan\left(\sqrt{2}\,x^{\frac{1}{6}}-1\right)}{2}+\frac{3\sqrt{2}\,\arctan\left(\sqrt{2}\,x^{\frac{1}{6}}+1\right)}{2}+\frac{3\sqrt{2}\,\ln\left(\frac{x^{\frac{1}{3}}+\sqrt{2}\,x^{\frac{1}{6}}+1}{x^{\frac{1}{3}}-\sqrt{2}\,x^{\frac{1}{6}}+1}\right)}{4}+\frac{6x^{\frac{5}{6}}}{5}-6x^{\frac{1}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^(2/3)+1),x)
```

```
[Out] 6/5*x^(5/6)-6*x^(1/6)+3/2*arctan(-1+x^(1/6)*2^(1/2))*2^(1/2)+3/4*2^(1/2)*ln
((1+x^(1/3)+x^(1/6)*2^(1/2))/(1+x^(1/3)-x^(1/6)*2^(1/2)))+3/2*arctan(1+x^(1
/6)*2^(1/2))*2^(1/2)
```

maxima [A] time = 1.25, size = 88, normalized size = 0.75

$$\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2x^{\frac{1}{6}}\right)\right)+\frac{3}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2x^{\frac{1}{6}}\right)\right)+\frac{3}{4}\sqrt{2}\log\left(\sqrt{2}x^{\frac{1}{6}}+x^{\frac{1}{3}}+1\right)-\frac{3}{4}\sqrt{2}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x^(2/3)),x, algorithm="maxima")
```

```
[Out] 3/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*x^(1/6))) + 3/2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2*x^(1/6))) + 3/4*sqrt(2)*log(sqrt(2)*x^(1/6) + x^(
```

```
1/3) + 1) - 3/4*sqrt(2)*log(-sqrt(2)*x^(1/6) + x^(1/3) + 1) + 6/5*x^(5/6) -
 6*x^(1/6)
```

mupad [B] time = 1.19, size = 47, normalized size = 0.40

$$\frac{6\,x^{5/6}}{5}-6\,x^{1/6}+\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\,x^{1/6}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{3}{2}+\frac{3}{2}\mathrm{i}\right)+\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\,x^{1/6}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{3}{2}-\frac{3}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^(2/3) + 1),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x^(1/6)*(1/2 - 1i/2))*(3/2 + 3i/2) + 2^(1/2)*atan(2^(1
/2)*x^(1/6)*(1/2 + 1i/2))*(3/2 - 3i/2) - 6*x^(1/6) + (6*x^(5/6))/5
```

sympy [C] time = 1.38, size = 187, normalized size = 1.60

$$\frac{27x^{\frac{5}{6}}\Gamma\left(\frac{9}{4}\right)}{10\Gamma\left(\frac{13}{4}\right)}-\frac{27\sqrt[6]{x}\,\Gamma\left(\frac{9}{4}\right)}{2\Gamma\left(\frac{13}{4}\right)}-\frac{27e^{-\frac{i\pi}{4}}\log\left(-\sqrt[6]{x}\,e^{\frac{i\pi}{4}}+1\right)\Gamma\left(\frac{9}{4}\right)}{8\Gamma\left(\frac{13}{4}\right)}+\frac{27ie^{-\frac{i\pi}{4}}\log\left(-\sqrt[6]{x}\,e^{\frac{3i\pi}{4}}+1\right)\Gamma\left(\frac{9}{4}\right)}{8\Gamma\left(\frac{13}{4}\right)}+\frac{27e^{-\frac{i\pi}{4}}\log\left(-\sqrt[6]{x}\,e^{\frac{5i\pi}{4}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(1+x**(2/3)),x)
```

```
[Out] 27*x**(5/6)*gamma(9/4)/(10*gamma(13/4)) - 27*x**(1/6)*gamma(9/4)/(2*gamma(1
3/4)) - 27*exp(-I*pi/4)*log(-x**(1/6)*exp_polar(I*pi/4) + 1)*gamma(9/4)/(8*
gamma(13/4)) + 27*I*exp(-I*pi/4)*log(-x**(1/6)*exp_polar(3*I*pi/4) + 1)*gam
ma(9/4)/(8*gamma(13/4)) + 27*exp(-I*pi/4)*log(-x**(1/6)*exp_polar(5*I*pi/4)
 + 1)*gamma(9/4)/(8*gamma(13/4)) - 27*I*exp(-I*pi/4)*log(-x**(1/6)*exp_pola
r(7*I*pi/4) + 1)*gamma(9/4)/(8*gamma(13/4))
```

3.2390 $\int \frac{\sqrt[3]{x}}{-1+x^{5/6}}\,dx$

Optimal. Leaf size=188

$$2\sqrt{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2\sqrt[3]{x}+\left(1-\sqrt{5}\right)\sqrt[6]{x}+2\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2\sqrt[3]{x}+\left(1+\sqrt{5}\right)\sqrt[6]{x}+\right.$$

```
[Out] 6/5*ln(1-x^(1/6))-3/10*ln(2+2*x^(1/3)+x^(1/6)*(5^(1/2)+1))*(-5^(1/2)+1)-3/1
0*ln(2+2*x^(1/3)+x^(1/6)*(-5^(1/2)+1))*(5^(1/2)+1)+2*x^(1/2)+3/5*arctan((1+
4*x^(1/6)-5^(1/2))/(10+2*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)-3/5*arctan((1
+4*x^(1/6)+5^(1/2))/(10-2*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 200, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {341, 321, 294, 634, 618, 204, 628, 31}

$$2\sqrt{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2\sqrt[3]{x}-\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2\sqrt[3]{x}+\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3)/(-1 + x^(5/6)),x]
```

```
[Out] 2*Sqrt[x] + (3*Sqrt[2*(5 - Sqrt[5])]*ArcTan[(1 - Sqrt[5] + 4*x^(1/6))/Sqrt[
2*(5 + Sqrt[5])]])/5 - (3*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(Sqrt[(5 + Sqrt[5])/
10]*(1 + Sqrt[5] + 4*x^(1/6)))/2])/5 + (6*Log[1 - x^(1/6)])/5 - (3*(1 + Sqr
t[5])*Log[2 + x^(1/6) - Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10 - (3*(1 - Sqrt[5])
*Log[2 + x^(1/6) + Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[
((2*k - 1)*Pi)/n]*x + s^2*x^2), x]; (r^(m + 1)*Int[1/(r - s*x), x])/(a*n*s^
m) - Dist[(2*(-r)^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 1)/2}], x], x]] /;
 FreeQ[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && N
egQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt[3]{x}}{-1+x^{5/6}}\,dx = 6\,\text{Subst}\left(\int \frac{x^7}{-1+x^5}\,dx, x, \sqrt[6]{x}\right)$$

$$= 2\sqrt{x} + 6\,\text{Subst}\left(\int \frac{x^2}{-1+x^5}\,dx, x, \sqrt[6]{x}\right)$$

$$= 2\sqrt{x} - \frac{6}{5}\,\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[6]{x}\right) - \frac{12}{5}\,\text{Subst}\left(\int \frac{\frac{1}{4}\left(-1-\sqrt{5}\right)+\frac{1}{4}\left(1+\sqrt{5}\right)x}{1+\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx, x, \sqrt[6]{x}\right) -$$

$$= 2\sqrt{x} + \frac{6}{5}\log\left(1-\sqrt[6]{x}\right) + \frac{3\,\text{Subst}\left(\int \frac{1}{1+\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx, x, \sqrt[6]{x}\right)}{\sqrt{5}} - \frac{3\,\text{Subst}\left(\int \frac{1}{1+\frac{1}{2}\left(1+\sqrt{5}\right)x+x^2}\,dx,\right.}{\sqrt{5}}$$

$$= 2\sqrt{x} + \frac{6}{5}\log\left(1-\sqrt[6]{x}\right) - \frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2+\sqrt[6]{x}-\sqrt{5}\,\sqrt[6]{x}+2\sqrt[3]{x}\right) - \frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2+\right.$$

$$= 2\sqrt{x} + 6\sqrt{\frac{2}{5\left(5+\sqrt{5}\right)}}\tan^{-1}\left(\frac{1-\sqrt{5}+4\sqrt[6]{x}}{\sqrt{2\left(5+\sqrt{5}\right)}}\right) - \frac{3}{5}\sqrt{2\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(1+\right.\right.$$

Mathematica [C] time = 0.01, size = 22, normalized size = 0.12

$$-2\sqrt{x}\left({}_2F_1\left(\frac{3}{5},1;\frac{8}{5};x^{5/6}\right)-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3)/(-1 + x^(5/6)),x]
```

```
[Out] -2*Sqrt[x]*(-1 + Hypergeometric2F1[3/5, 1, 8/5, x^(5/6)])
```

fricas [B] time = 2.76, size = 638, normalized size = 3.39

$$\frac{1}{10}\left(3\sqrt{5}-\sqrt{-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^2+\frac{9}{2}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}-3\right)\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)-\frac{27}{4}\left($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(-1+x^(5/6)),x, algorithm="fricas")
```

```
[Out] 1/10*(3*sqrt(5) - sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 +
9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) -
sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2
)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90) - 3)*log(9/4*(sqrt(2)*sqrt(sqrt(5) -
 5) + sqrt(5) + 1)^2 + 9/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 3*
sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(
sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/
4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5
) + 18*sqrt(5) - 90)*(sqrt(5) - 1) + 72*x^(1/6) + 36) + 1/10*(3*sqrt(5) + s
qrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(s
qrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4
*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5)
 + 18*sqrt(5) - 90) - 3)*log(9/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^
2 + 9/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 - 3*sqrt(-27/4*(sqrt(2)
*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt
(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqr
t(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90
)*(sqrt(5) - 1) + 72*x^(1/6) + 36) - 3/10*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt
(5) + 1)*log(-9/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 36*x^(1/6))
 + 3/10*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)*log(-9/4*(sqrt(2)*sqrt(sq
rt(5) - 5) - sqrt(5) - 1)^2 + 36*x^(1/6)) + 2*sqrt(x) + 6/5*log(x^(1/6) - 1
)
```

giac [A] time = 0.47, size = 139, normalized size = 0.74

$$\frac{3}{5}\sqrt{-2\sqrt{5}+10}\arctan\left(-\frac{\sqrt{5}-4x^{\frac{1}{6}}-1}{\sqrt{2\sqrt{5}+10}}\right)-\frac{3}{5}\sqrt{2\sqrt{5}+10}\arctan\left(\frac{\sqrt{5}+4x^{\frac{1}{6}}+1}{\sqrt{-2\sqrt{5}+10}}\right)+\frac{3}{10}\sqrt{5}\log\left(\frac{1}{2}x^{\frac{1}{6}}\left(\sqrt{5}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(-1+x^(5/6)),x, algorithm="giac")
```

```
[Out] 3/5*sqrt(-2*sqrt(5) + 10)*arctan(-(sqrt(5) - 4*x^(1/6) - 1)/sqrt(2*sqrt(5)
+ 10)) - 3/5*sqrt(2*sqrt(5) + 10)*arctan((sqrt(5) + 4*x^(1/6) + 1)/sqrt(-2*
sqrt(5) + 10)) + 3/10*sqrt(5)*log(1/2*x^(1/6)*(sqrt(5) + 1) + x^(1/3) + 1)
- 3/10*sqrt(5)*log(-1/2*x^(1/6)*(sqrt(5) - 1) + x^(1/3) + 1) + 2*sqrt(x) -
3/10*log(x^(2/3) + sqrt(x) + x^(1/3) + x^(1/6) + 1) + 6/5*log(abs(x^(1/6) -
 1))
```

maple [A] time = 0.03, size = 175, normalized size = 0.93

$$\frac{12\sqrt{5}\ \arctan\left(\frac{4x^{\frac{1}{6}}+1-\sqrt{5}}{\sqrt{10+2\sqrt{5}}}\right)}{5\sqrt{10+2\sqrt{5}}}-\frac{12\sqrt{5}\ \arctan\left(\frac{4x^{\frac{1}{6}}+1+\sqrt{5}}{\sqrt{10-2\sqrt{5}}}\right)}{5\sqrt{10-2\sqrt{5}}}+\frac{6\ln\left(x^{\frac{1}{6}}-1\right)}{5}-\frac{3\sqrt{5}\ \ln\left(2x^{\frac{1}{3}}+x^{\frac{1}{6}}-\sqrt{5}\ x^{\frac{1}{6}}+2\right)}{10}-\frac{3\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)/(-1+x^(5/6)),x)
```

```
[Out] 2*x^(1/2)+6/5*ln(x^(1/6)-1)-3/10*ln(2+x^(1/6)+2*x^(1/3)-x^(1/6)*5^(1/2))*5^
(1/2)-3/10*ln(2+x^(1/6)+2*x^(1/3)-x^(1/6)*5^(1/2))+12/5/(10+2*5^(1/2))^(1/2
)*arctan((1+4*x^(1/6)-5^(1/2))/(10+2*5^(1/2))^(1/2))*5^(1/2)+3/10*ln(2+x^(1
/6)+2*x^(1/3)+x^(1/6)*5^(1/2))*5^(1/2)-3/10*ln(2+x^(1/6)+2*x^(1/3)+x^(1/6)*
5^(1/2))-12/5/(10-2*5^(1/2))^(1/2)*arctan((1+4*x^(1/6)+5^(1/2))/(10-2*5^(1/
2))^(1/2))*5^(1/2)
```

maxima [B] time = 1.19, size = 272, normalized size = 1.45

$$-\frac{6}{5}(-1)^{\frac{3}{5}}\log\left((-1)^{\frac{1}{5}}+x^{\frac{1}{6}}\right)-\frac{6\sqrt{5}\,(-1)^{\frac{3}{5}}\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4x^{\frac{1}{6}}}{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4x^{\frac{1}{6}}}\right)}{5\sqrt{2\sqrt{5}-10}}+\frac{6\sqrt{5}\,(-1)^{\frac{3}{5}}\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{}}{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{}}\right.}{5\sqrt{-2\sqrt{5}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(-1+x^(5/6)),x, algorithm="maxima")
```

```
[Out] -6/5*(-1)^(3/5)*log((-1)^(1/5) + x^(1/6)) - 6/5*sqrt(5)*(-1)^(3/5)*log((sqr
t(5)*(-1)^(1/5) + (-1)^(1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^(1/6))
/(sqrt(5)*(-1)^(1/5) - (-1)^(1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^(
1/6)))/sqrt(2*sqrt(5) - 10) + 6/5*sqrt(5)*(-1)^(3/5)*log((sqrt(5)*(-1)^(1/5
) - (-1)^(1/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6))/(sqrt(5)*(-1
)^(1/5) + (-1)^(1/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6)))/sqrt(
-2*sqrt(5) - 10) + 2*sqrt(x) + 6/5*log(-x^(1/6)*(sqrt(5)*(-1)^(1/5) + (-1)^
(1/5)) + 2*(-1)^(2/5) + 2*x^(1/3))/(sqrt(5)*(-1)^(2/5) + (-1)^(2/5)) - 6/5*
log(x^(1/6)*(sqrt(5)*(-1)^(1/5) - (-1)^(1/5)) + 2*(-1)^(2/5) + 2*x^(1/3))/(
sqrt(5)*(-1)^(2/5) - (-1)^(2/5))
```

mupad [B] time = 0.12, size = 223, normalized size = 1.19

$$\frac{6\,\ln\left(1296\,x^{1/6}-1296\right)}{5}-\ln\left(-750\,x^{1/6}\left(\frac{3\sqrt{2}\,\sqrt{-\sqrt{5}-5}}{10}-\frac{3\sqrt{5}}{10}+\frac{3}{10}\right)^3-1296\right)\left(\frac{3\sqrt{2}\,\sqrt{-\sqrt{5}-5}}{10}-\frac{3\sqrt{5}}{10}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)/(x^(5/6) - 1),x)
```

```
[Out] (6*log(1296*x^(1/6) - 1296))/5 - log(- 750*x^(1/6)*((3*2^(1/2)*(- 5^(1/2) -
 5)^(1/2))/10 - (3*5^(1/2))/10 + 3/10)^3 - 1296)*((3*2^(1/2)*(- 5^(1/2) - 5
)^(1/2))/10 - (3*5^(1/2))/10 + 3/10) + log(750*x^(1/6)*((3*2^(1/2)*(- 5^(1/
2) - 5)^(1/2))/10 + (3*5^(1/2))/10 - 3/10)^3 - 1296)*((3*2^(1/2)*(- 5^(1/2)
 - 5)^(1/2))/10 + (3*5^(1/2))/10 - 3/10) - log(- 750*x^(1/6)*((3*5^(1/2))/1
0 - (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10)^3 - 1296)*((3*5^(1/2))/10 -
(3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10) - log(- 750*x^(1/6)*((3*5^(1/2))
/10 + (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10)^3 - 1296)*((3*5^(1/2))/10
+ (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10) + 2*x^(1/2)
```

sympy [B] time = 135.83, size = 462, normalized size = 2.46

$$2\sqrt{x}+\frac{6\log\left(\sqrt[6]{x}-1\right)}{5}-\frac{3\log\left(8\sqrt[6]{x}+8\sqrt{5}\,\sqrt[6]{x}+16\sqrt[3]{x}+16\right)}{10}+\frac{3\sqrt{5}\log\left(8\sqrt[6]{x}+8\sqrt{5}\,\sqrt[6]{x}+16\sqrt[3]{x}+16\right)}{10}-\frac{3\sqrt{5}\,\lo}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3)/(-1+x**(5/6)),x)
```

```
[Out] 2*sqrt(x) + 6*log(x**(1/6) - 1)/5 - 3*log(8*x**(1/6) + 8*sqrt(5)*x**(1/6) +
 16*x**(1/3) + 16)/10 + 3*sqrt(5)*log(8*x**(1/6) + 8*sqrt(5)*x**(1/6) + 16*
```

```
x**(1/3) + 16)/10 - 3*sqrt(5)*log(-8*sqrt(5)*x**(1/6) + 8*x**(1/6) + 16*x**
(1/3) + 16)/10 - 3*log(-8*sqrt(5)*x**(1/6) + 8*x**(1/6) + 16*x**(1/3) + 16)
/10 - 3*sqrt(10)*sqrt(5 - sqrt(5))*atan(2*sqrt(2)*x**(1/6)/sqrt(5 - sqrt(5)
) + sqrt(2)/(2*sqrt(5 - sqrt(5))) + sqrt(10)/(2*sqrt(5 - sqrt(5))))/10 - 3*
sqrt(2)*sqrt(5 - sqrt(5))*atan(2*sqrt(2)*x**(1/6)/sqrt(5 - sqrt(5)) + sqrt(
2)/(2*sqrt(5 - sqrt(5))) + sqrt(10)/(2*sqrt(5 - sqrt(5))))/10 - 3*sqrt(2)*s
qrt(sqrt(5) + 5)*atan(2*sqrt(2)*x**(1/6)/sqrt(sqrt(5) + 5) - sqrt(10)/(2*sq
rt(sqrt(5) + 5)) + sqrt(2)/(2*sqrt(sqrt(5) + 5)))/10 + 3*sqrt(10)*sqrt(sqrt
(5) + 5)*atan(2*sqrt(2)*x**(1/6)/sqrt(sqrt(5) + 5) - sqrt(10)/(2*sqrt(sqrt(
5) + 5)) + sqrt(2)/(2*sqrt(sqrt(5) + 5)))/10
```

3.2391 $\int \sqrt{3-\frac{1}{\sqrt{x}}}\,dx$

Optimal. Leaf size=67

$$\sqrt{3-\frac{1}{\sqrt{x}}}\,x-\frac{1}{6}\sqrt{3-\frac{1}{\sqrt{x}}}\,\sqrt{x}-\frac{\tanh^{-1}\left(\frac{\sqrt{3-\frac{1}{\sqrt{x}}}}{\sqrt{3}}\right)}{6\sqrt{3}}$$

```
[Out] -1/18*arctanh(1/3*(3-1/x^(1/2))^(1/2)*3^(1/2))*3^(1/2)+x*(3-1/x^(1/2))^(1/2
)-1/6*x^(1/2)*(3-1/x^(1/2))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {190, 47, 51, 63, 206}

$$\sqrt{3-\frac{1}{\sqrt{x}}}\,x-\frac{1}{6}\sqrt{3-\frac{1}{\sqrt{x}}}\,\sqrt{x}-\frac{\tanh^{-1}\left(\frac{\sqrt{3-\frac{1}{\sqrt{x}}}}{\sqrt{3}}\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - 1/Sqrt[x]],x]
```

```
[Out] -(Sqrt[3 - 1/Sqrt[x]]*Sqrt[x])/6 + Sqrt[3 - 1/Sqrt[x]]*x - ArcTanh[Sqrt[3 -
 1/Sqrt[x]]/Sqrt[3]]/(6*Sqrt[3])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
```

```
IntegerQ[1/n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{3-\frac{1}{\sqrt{x}}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{\sqrt{3-x}}{x^3}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)\\ &= \sqrt{3-\frac{1}{\sqrt{x}}}\,x + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{3-x}\,x^2}\,dx, x, \frac{1}{\sqrt{x}}\right)\\ &= -\frac{1}{6}\sqrt{3-\frac{1}{\sqrt{x}}}\,\sqrt{x} + \sqrt{3-\frac{1}{\sqrt{x}}}\,x + \frac{1}{12}\,\text{Subst}\left(\int \frac{1}{\sqrt{3-x}\,x}\,dx, x, \frac{1}{\sqrt{x}}\right)\\ &= -\frac{1}{6}\sqrt{3-\frac{1}{\sqrt{x}}}\,\sqrt{x} + \sqrt{3-\frac{1}{\sqrt{x}}}\,x - \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{3-x^2}\,dx, x, \sqrt{3-\frac{1}{\sqrt{x}}}\right)\\ &= -\frac{1}{6}\sqrt{3-\frac{1}{\sqrt{x}}}\,\sqrt{x} + \sqrt{3-\frac{1}{\sqrt{x}}}\,x - \frac{\tanh^{-1}\left(\frac{\sqrt{3-\frac{1}{\sqrt{x}}}}{\sqrt{3}}\right)}{6\sqrt{3}}\end{aligned}$$

Mathematica [C] time = 0.03, size = 36, normalized size = 0.54

$$\frac{4}{81}\left(3-\frac{1}{\sqrt{x}}\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};1-\frac{1}{3\sqrt{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - 1/Sqrt[x]],x]
```

```
[Out] (4*(3 - 1/Sqrt[x])^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 - 1/(3*Sqrt[x])])
/81
```

fricas [A] time = 0.56, size = 63, normalized size = 0.94

$$\frac{1}{6}\left(6\,x-\sqrt{x}\right)\sqrt{\frac{3\,x-\sqrt{x}}{x}} + \frac{1}{36}\sqrt{3}\,\log\left(2\sqrt{3}\,\sqrt{x}\sqrt{\frac{3\,x-\sqrt{x}}{x}} - 6\sqrt{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-1/x^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(6*x - sqrt(x))*sqrt((3*x - sqrt(x))/x) + 1/36*sqrt(3)*log(2*sqrt(3)*sq
rt(x)*sqrt((3*x - sqrt(x))/x) - 6*sqrt(x) + 1)
```

giac [A] time = 0.84, size = 59, normalized size = 0.88

$$\frac{1}{36}\left(6\sqrt{3\,x-\sqrt{x}}\left(6\sqrt{x}-1\right)+\sqrt{3}\,\log\left(\left|-2\sqrt{3}\left(\sqrt{3}\sqrt{x}-\sqrt{3\,x-\sqrt{x}}\right)+1\right|\right)\right)\text{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-1/x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/36*(6*sqrt(3*x - sqrt(x))*(6*sqrt(x) - 1) + sqrt(3)*log(abs(-2*sqrt(3)*(s
qrt(3)*sqrt(x) - sqrt(3*x - sqrt(x))) + 1)))*sgn(x)
```

maple [A] time = 0.02, size = 91, normalized size = 1.36

$$-\frac{\sqrt{\frac{3\sqrt{x}-1}{\sqrt{x}}}\left(\sqrt{3}\ \ln\left(\sqrt{3}\ \sqrt{x}-\frac{\sqrt{3}}{6}+\sqrt{3x-\sqrt{x}}\right)-36\sqrt{3x-\sqrt{x}}\ \sqrt{x}+6\sqrt{3x-\sqrt{x}}\right)\sqrt{x}}{36\sqrt{\left(3\sqrt{x}-1\right)\sqrt{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-1/x^(1/2))^(1/2),x)
```

```
[Out] -1/36*((3*x^(1/2)-1)/x^(1/2))^(1/2)*x^(1/2)*(ln(-1/6*3^(1/2)+3^(1/2)*x^(1/2
)+(3*x-x^(1/2))^(1/2))*3^(1/2)-36*(3*x-x^(1/2))^(1/2)*x^(1/2)+6*(3*x-x^(1/2
))^(1/2))/((3*x^(1/2)-1)*x^(1/2))^(1/2)
```

maxima [A] time = 1.21, size = 78, normalized size = 1.16

$$\frac{1}{36}\sqrt{3}\log\left(-\frac{\sqrt{3}-\sqrt{-\frac{1}{\sqrt{x}}+3}}{\sqrt{3}+\sqrt{-\frac{1}{\sqrt{x}}+3}}\right)+\frac{\left(-\frac{1}{\sqrt{x}}+3\right)^{\frac{3}{2}}+3\sqrt{-\frac{1}{\sqrt{x}}+3}}{6\left(\left(\frac{1}{\sqrt{x}}-3\right)^2+\frac{6}{\sqrt{x}}-9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-1/x^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/36*sqrt(3)*log(-(sqrt(3) - sqrt(-1/sqrt(x) + 3))/(sqrt(3) + sqrt(-1/sqrt(
x) + 3))) + 1/6*((-1/sqrt(x) + 3)^(3/2) + 3*sqrt(-1/sqrt(x) + 3))/((1/sqrt(
x) - 3)^2 + 6/sqrt(x) - 9)
```

mupad [B] time = 1.27, size = 31, normalized size = 0.46

$$\frac{4\,x\,\sqrt{3-\frac{1}{\sqrt{x}}}\ {}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{3}{2};\ \frac{5}{2};\ 3\sqrt{x}\right)}{3\sqrt{1-3\sqrt{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3 - 1/x^(1/2))^(1/2),x)
```

```
[Out] (4*x*(3 - 1/x^(1/2))^(1/2)*hypergeom([-1/2, 3/2], 5/2, 3*x^(1/2)))/(3*(1 -
3*x^(1/2))^(1/2))
```

sympy [A] time = 2.96, size = 165, normalized size = 2.46

$$\begin{cases}\frac{3x^{\frac{5}{4}}}{\sqrt{3\sqrt{x}-1}}-\frac{3x^{\frac{3}{4}}}{2\sqrt{3\sqrt{x}-1}}+\frac{\sqrt[4]{x}}{6\sqrt{3\sqrt{x}-1}}-\frac{\sqrt{3}\ \operatorname{acosh}\left(\sqrt{3}\ \sqrt[4]{x}\right)}{18} & \text{for } 3\left|\sqrt{x}\right|>1\\ -\frac{3ix^{\frac{5}{4}}}{\sqrt{1-3\sqrt{x}}}+\frac{3ix^{\frac{3}{4}}}{2\sqrt{1-3\sqrt{x}}}-\frac{i\sqrt[4]{x}}{6\sqrt{1-3\sqrt{x}}}+\frac{\sqrt{3}\,i\ \operatorname{asin}\left(\sqrt{3}\ \sqrt[4]{x}\right)}{18} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-1/x**(1/2))**(1/2),x)
```

```
[Out] Piecewise((3*x**(5/4)/sqrt(3*sqrt(x) - 1) - 3*x**(3/4)/(2*sqrt(3*sqrt(x) -
1)) + x**(1/4)/(6*sqrt(3*sqrt(x) - 1)) - sqrt(3)*acosh(sqrt(3)*x**(1/4))/18
, 3*Abs(sqrt(x)) > 1), (-3*I*x**(5/4)/sqrt(1 - 3*sqrt(x)) + 3*I*x**(3/4)/(2
*sqrt(1 - 3*sqrt(x))) - I*x**(1/4)/(6*sqrt(1 - 3*sqrt(x))) + sqrt(3)*I*asin
(sqrt(3)*x**(1/4))/18, True))
```

3.2392 $\int \frac{1}{\sqrt{1+\frac{1}{\sqrt{x}}}}\,dx$

Optimal. Leaf size=50

$$\sqrt{\frac{1}{\sqrt{x}}+1}\,x-\frac{3}{2}\sqrt{\frac{1}{\sqrt{x}}+1}\,\sqrt{x}+\frac{3}{2}\tanh^{-1}\left(\sqrt{\frac{1}{\sqrt{x}}+1}\right)$$

```
[Out] 3/2*arctanh((1+1/x^(1/2))^(1/2))+x*(1+1/x^(1/2))^(1/2)-3/2*x^(1/2)*(1+1/x^(
1/2))^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {190, 51, 63, 207}

$$\sqrt{\frac{1}{\sqrt{x}}+1}\,x-\frac{3}{2}\sqrt{\frac{1}{\sqrt{x}}+1}\,\sqrt{x}+\frac{3}{2}\tanh^{-1}\left(\sqrt{\frac{1}{\sqrt{x}}+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + 1/Sqrt[x]],x]
```

```
[Out] (-3*Sqrt[1 + 1/Sqrt[x]]*Sqrt[x])/2 + Sqrt[1 + 1/Sqrt[x]]*x + (3*ArcTanh[Sqr
t[1 + 1/Sqrt[x]]])/2
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{1+\frac{1}{\sqrt{x}}}}\,dx = -\left(2\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{1+x}}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= \sqrt{1+\frac{1}{\sqrt{x}}}\,x + \frac{3}{2}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x}}\,dx, x, \frac{1}{\sqrt{x}}\right)$$

$$= -\frac{3}{2}\sqrt{1+\frac{1}{\sqrt{x}}}\,\sqrt{x} + \sqrt{1+\frac{1}{\sqrt{x}}}\,x - \frac{3}{4}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \frac{1}{\sqrt{x}}\right)$$

$$= -\frac{3}{2}\sqrt{1+\frac{1}{\sqrt{x}}}\,\sqrt{x} + \sqrt{1+\frac{1}{\sqrt{x}}}\,x - \frac{3}{2}\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\frac{1}{\sqrt{x}}}\right)$$

$$= -\frac{3}{2}\sqrt{1+\frac{1}{\sqrt{x}}}\,\sqrt{x} + \sqrt{1+\frac{1}{\sqrt{x}}}\,x + \frac{3}{2}\tanh^{-1}\left(\sqrt{1+\frac{1}{\sqrt{x}}}\right)$$

Mathematica [C] time = 0.01, size = 28, normalized size = 0.56

$$4\sqrt{\frac{1}{\sqrt{x}}+1}\;{}_2F_1\left(\frac{1}{2}, 3; \frac{3}{2}; 1+\frac{1}{\sqrt{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + 1/Sqrt[x]],x]
```

```
[Out] 4*Sqrt[1 + 1/Sqrt[x]]*Hypergeometric2F1[1/2, 3, 3/2, 1 + 1/Sqrt[x]]
```

fricas [A] time = 0.80, size = 55, normalized size = 1.10

$$\frac{1}{2}\left(2\,x - 3\,\sqrt{x}\right)\sqrt{\frac{x+\sqrt{x}}{x}} + \frac{3}{4}\,\log\left(\sqrt{\frac{x+\sqrt{x}}{x}}+1\right) - \frac{3}{4}\,\log\left(\sqrt{\frac{x+\sqrt{x}}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+1/x^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*x - 3*sqrt(x))*sqrt((x + sqrt(x))/x) + 3/4*log(sqrt((x + sqrt(x))/x)
 + 1) - 3/4*log(sqrt((x + sqrt(x))/x) - 1)
```

giac [A] time = 0.31, size = 36, normalized size = 0.72

$$\frac{1}{2}\,\sqrt{x+\sqrt{x}}\,\left(2\,\sqrt{x}-3\right) - \frac{3}{4}\,\log\left(-2\,\sqrt{x+\sqrt{x}}+2\,\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+1/x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x + sqrt(x))*(2*sqrt(x) - 3) - 3/4*log(-2*sqrt(x + sqrt(x)) + 2*sq
rt(x) + 1)
```

maple [A] time = 0.02, size = 65, normalized size = 1.30

$$\frac{\sqrt{\frac{\sqrt{x}+1}{\sqrt{x}}}\,\left(3\ln\left(\sqrt{x}+\frac{1}{2}+\sqrt{x+\sqrt{x}}\right)+4\sqrt{x+\sqrt{x}}\,\sqrt{x}-6\sqrt{x+\sqrt{x}}\right)\sqrt{x}}{4\sqrt{\left(\sqrt{x}+1\right)\sqrt{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+1/x^(1/2))^(1/2),x)
```

```
[Out] 1/4*((x^(1/2)+1)/x^(1/2))^(1/2)*x^(1/2)*(4*(x+x^(1/2))^(1/2)*x^(1/2)+3*ln(1
/2+x^(1/2)+(x+x^(1/2))^(1/2))-6*(x+x^(1/2))^(1/2))/((x^(1/2)+1)*x^(1/2))^(1
/2)
```

maxima [A] time = 0.48, size = 62, normalized size = 1.24

$$-\frac{3\left(\frac{1}{\sqrt{x}}+1\right)^{\frac{3}{2}}-5\sqrt{\frac{1}{\sqrt{x}}+1}}{2\left(\left(\frac{1}{\sqrt{x}}+1\right)^{2}-\frac{2}{\sqrt{x}}-1\right)}+\frac{3}{4}\log\left(\sqrt{\frac{1}{\sqrt{x}}+1}+1\right)-\frac{3}{4}\log\left(\sqrt{\frac{1}{\sqrt{x}}+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+1/x^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(3*(1/sqrt(x) + 1)^(3/2) - 5*sqrt(1/sqrt(x) + 1))/((1/sqrt(x) + 1)^2 -
 2/sqrt(x) - 1) + 3/4*log(sqrt(1/sqrt(x) + 1) + 1) - 3/4*log(sqrt(1/sqrt(x)
 + 1) - 1)
```

mupad [B] time = 1.19, size = 27, normalized size = 0.54

$$\frac{4x\sqrt{\sqrt{x}+1}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{5}{2};\,\frac{7}{2};\,-\sqrt{x}\right)}{5\sqrt{\frac{1}{\sqrt{x}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/x^(1/2) + 1)^(1/2),x)
```

```
[Out] (4*x*(x^(1/2) + 1)^(1/2)*hypergeom([1/2, 5/2], 7/2, -x^(1/2)))/(5*(1/x^(1/2
) + 1)^(1/2))
```

sympy [A] time = 3.24, size = 60, normalized size = 1.20

$$\frac{x^{\frac{5}{4}}}{\sqrt{\sqrt{x}+1}}-\frac{x^{\frac{3}{4}}}{2\sqrt{\sqrt{x}+1}}-\frac{3\sqrt[4]{x}}{2\sqrt{\sqrt{x}+1}}+\frac{3\,\mathrm{asinh}\left(\sqrt[4]{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+1/x**(1/2))**(1/2),x)
```

```
[Out] x**(5/4)/sqrt(sqrt(x) + 1) - x**(3/4)/(2*sqrt(sqrt(x) + 1)) - 3*x**(1/4)/(2
*sqrt(sqrt(x) + 1)) + 3*asinh(x**(1/4))/2
```

3.2393 $\int \left(a+\frac{b}{x^{3/2}}\right)^{2/3} dx$

Optimal. Leaf size=95

$$b^{2/3}\log\left(\sqrt[3]{a+\frac{b}{x^{3/2}}}-\frac{\sqrt[3]{b}}{\sqrt{x}}\right)-\frac{2b^{2/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}}{\sqrt{x}\sqrt[3]{a+\frac{b}{x^{3/2}}}}+1}{\sqrt{3}}\right)}{\sqrt{3}}+x\left(a+\frac{b}{x^{3/2}}\right)^{2/3}$$

```
[Out] (a+b/x^(3/2))^(2/3)*x+b^(2/3)*ln((a+b/x^(3/2))^(1/3)-b^(1/3)/x^(1/2))-2/3*b
^(2/3)*arctan(1/3*(1+2*b^(1/3)/(a+b/x^(3/2))^(1/3)/x^(1/2))*3^(1/2))*3^(1/2
)
```

Rubi [A] time = 0.09, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {243, 335, 277, 239}

$$b^{2/3}\log\left(\sqrt[3]{a+\frac{b}{x^{3/2}}}-\frac{\sqrt[3]{b}}{\sqrt{x}}\right)-\frac{2b^{2/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}}{\sqrt{x}\sqrt[3]{a+\frac{b}{x^{3/2}}}}+1}{\sqrt{3}}\right)}{\sqrt{3}}+x\left(a+\frac{b}{x^{3/2}}\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^(3/2))^(2/3),x]
```

```
[Out] (a + b/x^(3/2))^(2/3)*x - (2*b^(2/3)*ArcTan[(1 + (2*b^(1/3))/((a + b/x^(3/2
))^(1/3)*Sqrt[x]))/Sqrt[3]])/Sqrt[3] + b^(2/3)*Log[(a + b/x^(3/2))^(1/3) -
b^(1/3)/Sqrt[x]]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Free
Q[{a, b, p}, x] && FractionQ[n]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x^{3/2}}\right)^{2/3} dx &= 2\,\text{Subst}\left(\int \left(a+\frac{b}{x^3}\right)^{2/3} x\,dx, x, \sqrt{x}\right)\\ &= -\left(2\,\text{Subst}\left(\int \frac{\left(a+bx^3\right)^{2/3}}{x^3}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)\\ &= \left(a+\frac{b}{x^{3/2}}\right)^{2/3} x - (2b)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx, x, \frac{1}{\sqrt{x}}\right)\\ &= \left(a+\frac{b}{x^{3/2}}\right)^{2/3} x - \frac{2b^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}}{\sqrt[3]{a+\frac{b}{x^{3/2}}}\sqrt{x}}}{\sqrt{3}}\right)}{\sqrt{3}} + b^{2/3}\log\left(\sqrt[3]{a+\frac{b}{x^{3/2}}} - \frac{\sqrt[3]{b}}{\sqrt{x}}\right)\end{aligned}$$

Mathematica [C] time = 0.03, size = 52, normalized size = 0.55

$$\frac{x\left(a+\frac{b}{x^{3/2}}\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},-\frac{2}{3};\frac{1}{3};-\frac{b}{ax^{3/2}}\right)}{\left(\frac{b}{ax^{3/2}}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^(3/2))^(2/3),x]
```

```
[Out] ((a + b/x^(3/2))^(2/3)*x*Hypergeometric2F1[-2/3, -2/3, 1/3, -(b/(a*x^(3/2))
)])/(1 + b/(a*x^(3/2)))^(2/3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(3/2))^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+\frac{b}{x^{\frac{3}{2}}}\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(3/2))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((a + b/x^(3/2))^(2/3), x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \left(a+\frac{b}{x^{\frac{3}{2}}}\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^(3/2))^(2/3),x)
```

```
[Out] int((a+b/x^(3/2))^(2/3),x)
```

maxima [A] time = 1.28, size = 109, normalized size = 1.15

$$\frac{2}{3}\sqrt{3}\,b^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(a+\frac{b}{x^{\frac{3}{2}}}\right)^{\frac{1}{3}}\sqrt{x}+b^{\frac{1}{3}}\right)}{3\,b^{\frac{1}{3}}}\right)+\left(a+\frac{b}{x^{\frac{3}{2}}}\right)^{\frac{2}{3}}x-\frac{1}{3}\,b^{\frac{2}{3}}\log\left(\left(a+\frac{b}{x^{\frac{3}{2}}}\right)^{\frac{2}{3}}x+\left(a+\frac{b}{x^{\frac{3}{2}}}\right)^{\frac{1}{3}}b^{\frac{1}{3}}\sqrt{x}+b^{\frac{2}{3}}\right)+\frac{2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(3/2))^(2/3),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*b^(2/3)*arctan(1/3*sqrt(3)*(2*(a + b/x^(3/2))^(1/3)*sqrt(x) + b
^(1/3))/b^(1/3)) + (a + b/x^(3/2))^(2/3)*x - 1/3*b^(2/3)*log((a + b/x^(3/2)
)^(2/3)*x + (a + b/x^(3/2))^(1/3)*b^(1/3)*sqrt(x) + b^(2/3)) + 2/3*b^(2/3)*
log((a + b/x^(3/2))^(1/3)*sqrt(x) - b^(1/3))
```

mupad [B] time = 1.31, size = 37, normalized size = 0.39

$$\frac{x\left(a+\frac{b}{x^{3/2}}\right)^{2/3}\,{}_2\mathrm{F}_1\left(-\frac{2}{3},-\frac{2}{3};\ \frac{1}{3};\ -\frac{b}{a\,x^{3/2}}\right)}{\left(\frac{b}{a\,x^{3/2}}+1\right)^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^(3/2))^(2/3),x)
```

```
[Out] (x*(a + b/x^(3/2))^(2/3)*hypergeom([-2/3, -2/3], 1/3, -b/(a*x^(3/2))))/(b/(
a*x^(3/2)) + 1)^(2/3)
```

sympy [C] time = 4.32, size = 46, normalized size = 0.48

$$-\frac{2a^{\frac{2}{3}}x\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},-\frac{2}{3}\\ \frac{1}{3}\end{matrix}\right|\frac{be^{i\pi}}{ax^{\frac{3}{2}}}\right)}{3\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**(3/2))**(2/3),x)
```

```
[Out] -2*a**(2/3)*x*gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), b*exp_polar(I*pi)/(a*
x**(3/2)))/(3*gamma(1/3))
```

3.2394 $\int \left(a + \frac{b}{\sqrt[3]{x}}\right) x^4 \, dx$

Optimal. Leaf size=19

$$\frac{ax^5}{5} + \frac{3}{14} bx^{14/3}$$

```
[Out] 3/14*b*x^(14/3)+1/5*a*x^5
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$\frac{ax^5}{5} + \frac{3}{14} bx^{14/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^(1/3))*x^4,x]
```

```
[Out] (3*b*x^(14/3))/14 + (a*x^5)/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \left(a + \frac{b}{\sqrt[3]{x}}\right) x^4 \, dx &= \int \left(bx^{11/3} + ax^4\right) dx \\ &= \frac{3}{14} bx^{14/3} + \frac{ax^5}{5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{ax^5}{5} + \frac{3}{14} bx^{14/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^(1/3))*x^4,x]
```

```
[Out] (3*b*x^(14/3))/14 + (a*x^5)/5
```

fricas [A] time = 0.96, size = 13, normalized size = 0.68

$$\frac{1}{5} ax^5 + \frac{3}{14} bx^{\frac{14}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))*x^4,x, algorithm="fricas")
```

```
[Out] 1/5*a*x^5 + 3/14*b*x^(14/3)
```

giac [A] time = 0.16, size = 13, normalized size = 0.68

$$\frac{1}{5} ax^5 + \frac{3}{14} bx^{\frac{14}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))*x^4,x, algorithm="giac")
[Out] 1/5*a*x^5 + 3/14*b*x^(14/3)
```

maple [A] time = 0.00, size = 14, normalized size = 0.74

$$\frac{a\,x^5}{5}+\frac{3b\,x^{\frac{14}{3}}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^(1/3))*x^4,x)
[Out] 3/14*b*x^(14/3)+1/5*a*x^5
```

maxima [A] time = 0.47, size = 15, normalized size = 0.79

$$\frac{1}{70}\left(14\,a+\frac{15\,b}{x^{\frac{1}{3}}}\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))*x^4,x, algorithm="maxima")
[Out] 1/70*(14*a + 15*b/x^(1/3))*x^5
```

mupad [B] time = 0.03, size = 13, normalized size = 0.68

$$\frac{a\,x^5}{5}+\frac{3\,b\,x^{14/3}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b/x^(1/3)),x)
[Out] (a*x^5)/5 + (3*b*x^(14/3))/14
```

sympy [A] time = 2.27, size = 15, normalized size = 0.79

$$\frac{ax^5}{5}+\frac{3bx^{\frac{14}{3}}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**(1/3))*x**4,x)
[Out] a*x**5/5 + 3*b*x**(14/3)/14
```

3.2395 $\int \left(a + \frac{b}{\sqrt[3]{x}}\right) x^3 \, dx$

Optimal. Leaf size=19

$$\frac{ax^4}{4} + \frac{3}{11} bx^{11/3}$$

```
[Out] 3/11*b*x^(11/3)+1/4*a*x^4
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$\frac{ax^4}{4} + \frac{3}{11} bx^{11/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^(1/3))*x^3,x]
```

```
[Out] (3*b*x^(11/3))/11 + (a*x^4)/4
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \left(a + \frac{b}{\sqrt[3]{x}}\right) x^3 \, dx &= \int \left(bx^{8/3} + ax^3\right) dx \\ &= \frac{3}{11} bx^{11/3} + \frac{ax^4}{4}\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{ax^4}{4} + \frac{3}{11} bx^{11/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^(1/3))*x^3,x]
```

```
[Out] (3*b*x^(11/3))/11 + (a*x^4)/4
```

fricas [A] time = 1.04, size = 13, normalized size = 0.68

$$\frac{1}{4} ax^4 + \frac{3}{11} bx^{\frac{11}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))*x^3,x, algorithm="fricas")
```

```
[Out] 1/4*a*x^4 + 3/11*b*x^(11/3)
```

giac [A] time = 0.17, size = 13, normalized size = 0.68

$$\frac{1}{4} ax^4 + \frac{3}{11} bx^{\frac{11}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))*x^3,x, algorithm="giac")
[Out] 1/4*a*x^4 + 3/11*b*x^(11/3)
```

maple [A] time = 0.00, size = 14, normalized size = 0.74

$$\frac{a\,x^4}{4}+\frac{3b\,x^{\frac{11}{3}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^(1/3))*x^3,x)
[Out] 3/11*b*x^(11/3)+1/4*a*x^4
```

maxima [A] time = 0.56, size = 15, normalized size = 0.79

$$\frac{1}{44}\left(11\,a+\frac{12\,b}{x^{\frac{1}{3}}}\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))*x^3,x, algorithm="maxima")
[Out] 1/44*(11*a + 12*b/x^(1/3))*x^4
```

mupad [B] time = 0.03, size = 13, normalized size = 0.68

$$\frac{a\,x^4}{4}+\frac{3\,b\,x^{11/3}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b/x^(1/3)),x)
[Out] (a*x^4)/4 + (3*b*x^(11/3))/11
```

sympy [A] time = 1.24, size = 15, normalized size = 0.79

$$\frac{ax^4}{4}+\frac{3bx^{\frac{11}{3}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**(1/3))*x**3,x)
[Out] a*x**4/4 + 3*b*x**(11/3)/11
```

3.2396 $\int \left(a + \frac{b}{\sqrt[3]{x}}\right) x^2 \, dx$

Optimal. Leaf size=19

$$\frac{ax^3}{3} + \frac{3}{8} b x^{8/3}$$

```
[Out] 3/8*b*x^(8/3)+1/3*a*x^3
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$\frac{ax^3}{3} + \frac{3}{8} b x^{8/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^(1/3))*x^2,x]
```

```
[Out] (3*b*x^(8/3))/8 + (a*x^3)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \left(a + \frac{b}{\sqrt[3]{x}}\right) x^2 \, dx &= \int \left(b x^{5/3} + a x^2\right) dx \\ &= \frac{3}{8} b x^{8/3} + \frac{ax^3}{3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 19, normalized size = 1.00

$$\frac{ax^3}{3} + \frac{3}{8} b x^{8/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^(1/3))*x^2,x]
```

```
[Out] (3*b*x^(8/3))/8 + (a*x^3)/3
```

fricas [A] time = 0.84, size = 13, normalized size = 0.68

$$\frac{1}{3} a x^3 + \frac{3}{8} b x^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))*x^2,x, algorithm="fricas")
```

```
[Out] 1/3*a*x^3 + 3/8*b*x^(8/3)
```

giac [A] time = 0.15, size = 13, normalized size = 0.68

$$\frac{1}{3} a x^3 + \frac{3}{8} b x^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))*x^2,x, algorithm="giac")
[Out] 1/3*a*x^3 + 3/8*b*x^(8/3)
```

maple [A] time = 0.00, size = 14, normalized size = 0.74

$$\frac{a\,x^3}{3}+\frac{3b\,x^{\frac{8}{3}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^(1/3))*x^2,x)
[Out] 3/8*b*x^(8/3)+1/3*a*x^3
```

maxima [A] time = 0.50, size = 15, normalized size = 0.79

$$\frac{1}{24}\left(8\,a+\frac{9\,b}{x^{\frac{1}{3}}}\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))*x^2,x, algorithm="maxima")
[Out] 1/24*(8*a + 9*b/x^(1/3))*x^3
```

mupad [B] time = 0.03, size = 13, normalized size = 0.68

$$\frac{a\,x^3}{3}+\frac{3\,b\,x^{8/3}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b/x^(1/3)),x)
[Out] (a*x^3)/3 + (3*b*x^(8/3))/8
```

sympy [A] time = 0.55, size = 15, normalized size = 0.79

$$\frac{ax^3}{3}+\frac{3bx^{\frac{8}{3}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**(1/3))*x**2,x)
[Out] a*x**3/3 + 3*b*x**(8/3)/8
```

3.2397 $\int \left(a + \frac{b}{\sqrt[3]{x}}\right) x\, dx$

Optimal. Leaf size=19

$$\frac{ax^2}{2} + \frac{3}{5}bx^{5/3}$$

```
[Out] 3/5*b*x^(5/3)+1/2*a*x^2
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$\frac{ax^2}{2} + \frac{3}{5}bx^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^(1/3))*x,x]
```

```
[Out] (3*b*x^(5/3))/5 + (a*x^2)/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \left(a + \frac{b}{\sqrt[3]{x}}\right) x\, dx &= \int \left(bx^{2/3} + ax\right)\, dx \\ &= \frac{3}{5}bx^{5/3} + \frac{ax^2}{2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{ax^2}{2} + \frac{3}{5}bx^{5/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^(1/3))*x,x]
```

```
[Out] (3*b*x^(5/3))/5 + (a*x^2)/2
```

fricas [A] time = 0.76, size = 13, normalized size = 0.68

$$\frac{1}{2}\, ax^2 + \frac{3}{5}\, bx^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))*x,x, algorithm="fricas")
```

```
[Out] 1/2*a*x^2 + 3/5*b*x^(5/3)
```

giac [A] time = 0.19, size = 13, normalized size = 0.68

$$\frac{1}{2}\, ax^2 + \frac{3}{5}\, bx^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))*x,x, algorithm="giac")
[Out] 1/2*a*x^2 + 3/5*b*x^(5/3)
```

maple [A] time = 0.00, size = 14, normalized size = 0.74

$$\frac{a\,x^2}{2}+\frac{3b\,x^{\frac{5}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^(1/3))*x,x)
[Out] 3/5*b*x^(5/3)+1/2*a*x^2
```

maxima [A] time = 0.60, size = 15, normalized size = 0.79

$$\frac{1}{10}\left(5\,a+\frac{6\,b}{x^{\frac{1}{3}}}\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))*x,x, algorithm="maxima")
[Out] 1/10*(5*a + 6*b/x^(1/3))*x^2
```

mupad [B] time = 0.03, size = 13, normalized size = 0.68

$$\frac{a\,x^2}{2}+\frac{3\,b\,x^{5/3}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b/x^(1/3)),x)
[Out] (a*x^2)/2 + (3*b*x^(5/3))/5
```

sympy [A] time = 0.24, size = 15, normalized size = 0.79

$$\frac{ax^2}{2}+\frac{3bx^{\frac{5}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**(1/3))*x,x)
[Out] a*x**2/2 + 3*b*x**(5/3)/5
```

3.2398 $\int \left(a + \frac{b}{\sqrt[3]{x}}\right) dx$

Optimal. Leaf size=14

$$ax + \frac{3}{2}bx^{2/3}$$

```
[Out] 3/2*b*x^(2/3)+a*x
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$ax + \frac{3}{2}bx^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[a + b/x^(1/3),x]
```

```
[Out] (3*b*x^(2/3))/2 + a*x
```

Rubi steps

$$\int \left(a + \frac{b}{\sqrt[3]{x}}\right) dx = \frac{3}{2}bx^{2/3} + ax$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$ax + \frac{3}{2}bx^{2/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b/x^(1/3),x]
```

```
[Out] (3*b*x^(2/3))/2 + a*x
```

fricas [A] time = 0.87, size = 10, normalized size = 0.71

$$ax + \frac{3}{2}\,bx^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b/x^(1/3),x, algorithm="fricas")
```

```
[Out] a*x + 3/2*b*x^(2/3)
```

giac [A] time = 0.15, size = 10, normalized size = 0.71

$$ax + \frac{3}{2}\,bx^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b/x^(1/3),x, algorithm="giac")
```

```
[Out] a*x + 3/2*b*x^(2/3)
```

maple [A] time = 0.00, size = 11, normalized size = 0.79

$$ax + \frac{3b\,x^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b/x^(1/3),x)
```

```
[Out] 3/2*b*x^(2/3)+a*x
```

maxima [A] time = 0.66, size = 10, normalized size = 0.71

$$ax + \frac{3}{2} b x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b/x^(1/3),x, algorithm="maxima")
```

```
[Out] a*x + 3/2*b*x^(2/3)
```

mupad [B] time = 0.03, size = 10, normalized size = 0.71

$$a\,x + \frac{3\,b\,x^{2/3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b/x^(1/3),x)
```

```
[Out] a*x + (3*b*x^(2/3))/2
```

sympy [A] time = 0.08, size = 12, normalized size = 0.86

$$ax + \frac{3bx^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b/x**(1/3),x)
```

```
[Out] a*x + 3*b*x**(2/3)/2
```

3.2399 $\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x}\,dx$

Optimal. Leaf size=13

$$a\log(x)-\frac{3b}{\sqrt[3]{x}}$$

```
[Out] -3*b/x^(1/3)+a*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$a\log(x)-\frac{3b}{\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^(1/3))/x,x]
```

```
[Out] (-3*b)/x^(1/3) + a*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x}\,dx &= \int \left(\frac{b}{x^{4/3}}+\frac{a}{x}\right)dx\\ &= -\frac{3b}{\sqrt[3]{x}}+a\log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$a\log(x)-\frac{3b}{\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^(1/3))/x,x]
```

```
[Out] (-3*b)/x^(1/3) + a*Log[x]
```

fricas [A] time = 0.79, size = 19, normalized size = 1.46

$$\frac{3\left(ax\log\left(x^{\frac{1}{3}}\right)-bx^{\frac{2}{3}}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))/x,x, algorithm="fricas")
```

```
[Out] 3*(a*x*log(x^(1/3)) - b*x^(2/3))/x
```

giac [A] time = 0.15, size = 12, normalized size = 0.92

$$a \log(|x|) - \frac{3\,b}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))/x,x, algorithm="giac")
[Out] a*log(abs(x)) - 3*b/x^(1/3)
```

maple [A] time = 0.01, size = 12, normalized size = 0.92

$$a \ln(x) - \frac{3b}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^(1/3))/x,x)
[Out] -3*b/x^(1/3)+a*ln(x)
```

maxima [A] time = 0.60, size = 11, normalized size = 0.85

$$a \log(x) - \frac{3\,b}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))/x,x, algorithm="maxima")
[Out] a*log(x) - 3*b/x^(1/3)
```

mupad [B] time = 0.04, size = 14, normalized size = 1.08

$$3\,a\,\ln\left(x^{1/3}\right) - \frac{3\,b}{x^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^(1/3))/x,x)
[Out] 3*a*log(x^(1/3)) - (3*b)/x^(1/3)
```

sympy [A] time = 0.40, size = 12, normalized size = 0.92

$$a \log(x) - \frac{3b}{\sqrt[3]{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**(1/3))/x,x)
[Out] a*log(x) - 3*b/x**(1/3)
```

3.2400 $\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x^2}\,dx$

Optimal. Leaf size=17

$$-\frac{a}{x}-\frac{3b}{4x^{4/3}}$$

```
[Out] -3/4*b/x^(4/3)-a/x
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$-\frac{a}{x}-\frac{3b}{4x^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^(1/3))/x^2,x]
```

```
[Out] (-3*b)/(4*x^(4/3)) - a/x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x^2}\,dx &= \int \left(\frac{b}{x^{7/3}}+\frac{a}{x^2}\right)dx\\ &= -\frac{3b}{4x^{4/3}}-\frac{a}{x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$-\frac{a}{x}-\frac{3b}{4x^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^(1/3))/x^2,x]
```

```
[Out] (-3*b)/(4*x^(4/3)) - a/x
```

fricas [A] time = 0.67, size = 16, normalized size = 0.94

$$-\frac{4\,ax+3\,bx^{\frac{2}{3}}}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*a*x + 3*b*x^(2/3))/x^2
```

giac [A] time = 0.17, size = 15, normalized size = 0.88

$$-\frac{4\,a x^{\frac{1}{3}}+3\,b}{4\,x^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))/x^2,x, algorithm="giac")
```

```
[Out] -1/4*(4*a*x^(1/3) + 3*b)/x^(4/3)
```

maple [A] time = 0.01, size = 14, normalized size = 0.82

$$-\frac{a}{x}-\frac{3b}{4x^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^(1/3))/x^2,x)
```

```
[Out] -3/4*b/x^(4/3)-a/x
```

maxima [B] time = 0.52, size = 47, normalized size = 2.76

$$-\frac{3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^4}{4\,b^3}+\frac{2\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^3 a}{b^3}-\frac{3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^2 a^2}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))/x^2,x, algorithm="maxima")
```

```
[Out] -3/4*(a + b/x^(1/3))^4/b^3 + 2*(a + b/x^(1/3))^3*a/b^3 - 3/2*(a + b/x^(1/3)
)^2*a^2/b^3
```

mupad [B] time = 1.13, size = 13, normalized size = 0.76

$$-\frac{a}{x}-\frac{3\,b}{4\,x^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^(1/3))/x^2,x)
```

```
[Out] - a/x - (3*b)/(4*x^(4/3))
```

sympy [A] time = 0.74, size = 14, normalized size = 0.82

$$-\frac{a}{x}-\frac{3b}{4x^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**(1/3))/x**2,x)
```

```
[Out] -a/x - 3*b/(4*x**(4/3))
```

3.2401 $\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x^3}\,dx$

Optimal. Leaf size=19

$$-\frac{a}{2x^2}-\frac{3b}{7x^{7/3}}$$

```
[Out] -3/7*b/x^(7/3)-1/2*a/x^2
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$-\frac{a}{2x^2}-\frac{3b}{7x^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^(1/3))/x^3,x]
```

```
[Out] (-3*b)/(7*x^(7/3)) - a/(2*x^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x^3}\,dx &= \int \left(\frac{b}{x^{10/3}}+\frac{a}{x^3}\right)dx\\ &= -\frac{3b}{7x^{7/3}}-\frac{a}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$-\frac{a}{2x^2}-\frac{3b}{7x^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^(1/3))/x^3,x]
```

```
[Out] (-3*b)/(7*x^(7/3)) - a/(2*x^2)
```

fricas [A] time = 0.79, size = 16, normalized size = 0.84

$$-\frac{7\,ax+6\,bx^{\frac{2}{3}}}{14\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))/x^3,x, algorithm="fricas")
```

```
[Out] -1/14*(7*a*x + 6*b*x^(2/3))/x^3
```

giac [A] time = 0.15, size = 15, normalized size = 0.79

$$-\frac{7ax^{\frac{1}{3}}+6b}{14x^{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))/x^3,x, algorithm="giac")
[Out] -1/14*(7*a*x^(1/3) + 6*b)/x^(7/3)
```

maple [A] time = 0.00, size = 14, normalized size = 0.74

$$-\frac{a}{2x^2}-\frac{3b}{7x^{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^(1/3))/x^3,x)
[Out] -3/7*b/x^(7/3)-1/2*a/x^2
```

maxima [B] time = 0.52, size = 98, normalized size = 5.16

$$-\frac{3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^7}{7b^6}+\frac{5\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^6 a}{2b^6}-\frac{6\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^5 a^2}{b^6}+\frac{15\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^4 a^3}{2b^6}-\frac{5\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^3 a^4}{b^6}+\frac{3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^2 a^5}{2b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))/x^3,x, algorithm="maxima")
[Out] -3/7*(a + b/x^(1/3))^7/b^6 + 5/2*(a + b/x^(1/3))^6*a/b^6 - 6*(a + b/x^(1/3)
)^5*a^2/b^6 + 15/2*(a + b/x^(1/3))^4*a^3/b^6 - 5*(a + b/x^(1/3))^3*a^4/b^6
+ 3/2*(a + b/x^(1/3))^2*a^5/b^6
```

mupad [B] time = 1.13, size = 13, normalized size = 0.68

$$-\frac{a}{2\,x^2}-\frac{3\,b}{7\,x^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^(1/3))/x^3,x)
[Out] - a/(2*x^2) - (3*b)/(7*x^(7/3))
```

sympy [A] time = 1.32, size = 17, normalized size = 0.89

$$-\frac{a}{2x^2}-\frac{3b}{7x^{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**(1/3))/x**3,x)
[Out] -a/(2*x**2) - 3*b/(7*x**(7/3))
```

3.2402 $\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x^4}\,dx$

Optimal. Leaf size=19

$$-\frac{a}{3x^3}-\frac{3b}{10x^{10/3}}$$

```
[Out] -3/10*b/x^(10/3)-1/3*a/x^3
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$-\frac{a}{3x^3}-\frac{3b}{10x^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^(1/3))/x^4,x]
```

```
[Out] (-3*b)/(10*x^(10/3)) - a/(3*x^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x^4}\,dx &= \int \left(\frac{b}{x^{13/3}}+\frac{a}{x^4}\right)dx\\ &= -\frac{3b}{10x^{10/3}}-\frac{a}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$-\frac{a}{3x^3}-\frac{3b}{10x^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^(1/3))/x^4,x]
```

```
[Out] (-3*b)/(10*x^(10/3)) - a/(3*x^3)
```

fricas [A] time = 0.77, size = 16, normalized size = 0.84

$$-\frac{10\,ax+9\,bx^{\frac{2}{3}}}{30\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))/x^4,x, algorithm="fricas")
```

```
[Out] -1/30*(10*a*x + 9*b*x^(2/3))/x^4
```

giac [A] time = 0.19, size = 15, normalized size = 0.79

$$-\frac{10\,a x^{\frac{1}{3}}+9\,b}{30\,x^{\frac{10}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))/x^4,x, algorithm="giac")
[Out] -1/30*(10*a*x^(1/3) + 9*b)/x^(10/3)
```

maple [A] time = 0.01, size = 14, normalized size = 0.74

$$-\frac{a}{3x^3}-\frac{3b}{10x^{\frac{10}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^(1/3))/x^4,x)
[Out] -3/10*b/x^(10/3)-1/3*a/x^3
```

maxima [B] time = 0.67, size = 149, normalized size = 7.84

$$-\frac{3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{10}}{10\,b^9}+\frac{8\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{9}a}{3\,b^9}-\frac{21\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{8}a^2}{2\,b^9}+\frac{24\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{7}a^3}{b^9}-\frac{35\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{6}a^4}{b^9}+\frac{168\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{5}a^5}{5\,b^9}-\frac{21\left(a+\frac{b}{x}\right.}{b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))/x^4,x, algorithm="maxima")
[Out] -3/10*(a + b/x^(1/3))^10/b^9 + 8/3*(a + b/x^(1/3))^9*a/b^9 - 21/2*(a + b/x^
(1/3))^8*a^2/b^9 + 24*(a + b/x^(1/3))^7*a^3/b^9 - 35*(a + b/x^(1/3))^6*a^4/
b^9 + 168/5*(a + b/x^(1/3))^5*a^5/b^9 - 21*(a + b/x^(1/3))^4*a^6/b^9 + 8*(a
 + b/x^(1/3))^3*a^7/b^9 - 3/2*(a + b/x^(1/3))^2*a^8/b^9
```

mupad [B] time = 1.11, size = 13, normalized size = 0.68

$$-\frac{a}{3\,x^3}-\frac{3\,b}{10\,x^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^(1/3))/x^4,x)
[Out] - a/(3*x^3) - (3*b)/(10*x^(10/3))
```

sympy [A] time = 2.40, size = 17, normalized size = 0.89

$$-\frac{a}{3x^3}-\frac{3b}{10x^{\frac{10}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**(1/3))/x**4,x)
[Out] -a/(3*x**3) - 3*b/(10*x**(10/3))
```

3.2403 $\int \left(a + \frac{b}{\sqrt[3]{x}}\right)^2 x^4\, dx$

Optimal. Leaf size=34

$$\frac{a^2x^5}{5} + \frac{3}{7}abx^{14/3} + \frac{3}{13}b^2x^{13/3}$$

```
[Out] 3/13*b^2*x^(13/3)+3/7*a*b*x^(14/3)+1/5*a^2*x^5
```

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {263, 266, 43}

$$\frac{a^2x^5}{5} + \frac{3}{7}abx^{14/3} + \frac{3}{13}b^2x^{13/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^(1/3))^2*x^4,x]
```

```
[Out] (3*b^2*x^(13/3))/13 + (3*a*b*x^(14/3))/7 + (a^2*x^5)/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a + \frac{b}{\sqrt[3]{x}}\right)^2 x^4\, dx &= \int \left(b + a\sqrt[3]{x}\right)^2 x^{10/3}\, dx \\
&= 3\,\text{Subst}\left(\int x^{12}(b + ax)^2\, dx, x, \sqrt[3]{x}\right) \\
&= 3\,\text{Subst}\left(\int \left(b^2x^{12} + 2abx^{13} + a^2x^{14}\right) dx, x, \sqrt[3]{x}\right) \\
&= \frac{3}{13}b^2x^{13/3} + \frac{3}{7}abx^{14/3} + \frac{a^2x^5}{5}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 34, normalized size = 1.00

$$\frac{a^2x^5}{5} + \frac{3}{7}abx^{14/3} + \frac{3}{13}b^2x^{13/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^(1/3))^2*x^4,x]
[Out] (3*b^2*x^(13/3))/13 + (3*a*b*x^(14/3))/7 + (a^2*x^5)/5
```

fricas [A] time = 0.96, size = 24, normalized size = 0.71

$$\frac{1}{5}a^2x^5 + \frac{3}{7}abx^{\frac{14}{3}} + \frac{3}{13}b^2x^{\frac{13}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^2*x^4,x, algorithm="fricas")
[Out] 1/5*a^2*x^5 + 3/7*a*b*x^(14/3) + 3/13*b^2*x^(13/3)
```

giac [A] time = 0.15, size = 24, normalized size = 0.71

$$\frac{1}{5}a^2x^5 + \frac{3}{7}abx^{\frac{14}{3}} + \frac{3}{13}b^2x^{\frac{13}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^2*x^4,x, algorithm="giac")
[Out] 1/5*a^2*x^5 + 3/7*a*b*x^(14/3) + 3/13*b^2*x^(13/3)
```

maple [A] time = 0.00, size = 25, normalized size = 0.74

$$\frac{a^2x^5}{5} + \frac{3ab\,x^{\frac{14}{3}}}{7} + \frac{3b^2x^{\frac{13}{3}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^(1/3))^2*x^4,x)
[Out] 3/13*b^2*x^(13/3)+3/7*a*b*x^(14/3)+1/5*a^2*x^5
```

maxima [A] time = 0.54, size = 26, normalized size = 0.76

$$\frac{1}{455}\left(91\,a^2 + \frac{195\,ab}{x^{\frac{1}{3}}} + \frac{105\,b^2}{x^{\frac{2}{3}}}\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^2*x^4,x, algorithm="maxima")
[Out] 1/455*(91*a^2 + 195*a*b/x^(1/3) + 105*b^2/x^(2/3))*x^5
```

mupad [B] time = 0.05, size = 24, normalized size = 0.71

$$\frac{a^2\,x^5}{5} + \frac{3\,b^2\,x^{13/3}}{13} + \frac{3\,a\,b\,x^{14/3}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b/x^(1/3))^2,x)
[Out] (a^2*x^5)/5 + (3*b^2*x^(13/3))/13 + (3*a*b*x^(14/3))/7
```

sympy [A] time = 3.17, size = 31, normalized size = 0.91

$$\frac{a^2x^5}{5} + \frac{3abx^{\frac{14}{3}}}{7} + \frac{3b^2x^{\frac{13}{3}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**(1/3))**2*x**4,x)
[Out] a**2*x**5/5 + 3*a*b*x**(14/3)/7 + 3*b**2*x**(13/3)/13
```

3.2404 $\int \left(a + \frac{b}{\sqrt[3]{x}}\right)^2 x^3\, dx$

Optimal. Leaf size=34

$$\frac{a^2x^4}{4} + \frac{6}{11}abx^{11/3} + \frac{3}{10}b^2x^{10/3}$$

```
[Out] 3/10*b^2*x^(10/3)+6/11*a*b*x^(11/3)+1/4*a^2*x^4
```

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {263, 266, 43}

$$\frac{a^2x^4}{4} + \frac{6}{11}abx^{11/3} + \frac{3}{10}b^2x^{10/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^(1/3))^2*x^3,x]
```

```
[Out] (3*b^2*x^(10/3))/10 + (6*a*b*x^(11/3))/11 + (a^2*x^4)/4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a + \frac{b}{\sqrt[3]{x}}\right)^2 x^3\, dx &= \int \left(b + a\sqrt[3]{x}\right)^2 x^{7/3}\, dx \\
&= 3\,\text{Subst}\left(\int x^9 (b + ax)^2\, dx, x, \sqrt[3]{x}\right) \\
&= 3\,\text{Subst}\left(\int \left(b^2x^9 + 2abx^{10} + a^2x^{11}\right)\, dx, x, \sqrt[3]{x}\right) \\
&= \frac{3}{10}b^2x^{10/3} + \frac{6}{11}abx^{11/3} + \frac{a^2x^4}{4}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.00

$$\frac{a^2x^4}{4} + \frac{6}{11}abx^{11/3} + \frac{3}{10}b^2x^{10/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^(1/3))^2*x^3,x]
[Out] (3*b^2*x^(10/3))/10 + (6*a*b*x^(11/3))/11 + (a^2*x^4)/4
```

fricas [A] time = 0.77, size = 24, normalized size = 0.71

$$\frac{1}{4}a^2x^4 + \frac{6}{11}abx^{\frac{11}{3}} + \frac{3}{10}b^2x^{\frac{10}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^2*x^3,x, algorithm="fricas")
[Out] 1/4*a^2*x^4 + 6/11*a*b*x^(11/3) + 3/10*b^2*x^(10/3)
```

giac [A] time = 0.15, size = 24, normalized size = 0.71

$$\frac{1}{4}a^2x^4 + \frac{6}{11}abx^{\frac{11}{3}} + \frac{3}{10}b^2x^{\frac{10}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^2*x^3,x, algorithm="giac")
[Out] 1/4*a^2*x^4 + 6/11*a*b*x^(11/3) + 3/10*b^2*x^(10/3)
```

maple [A] time = 0.00, size = 25, normalized size = 0.74

$$\frac{a^2x^4}{4} + \frac{6ab\,x^{\frac{11}{3}}}{11} + \frac{3b^2x^{\frac{10}{3}}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^(1/3))^2*x^3,x)
[Out] 3/10*b^2*x^(10/3)+6/11*a*b*x^(11/3)+1/4*a^2*x^4
```

maxima [A] time = 0.59, size = 26, normalized size = 0.76

$$\frac{1}{220}\left(55\,a^2 + \frac{120\,ab}{x^{\frac{1}{3}}} + \frac{66\,b^2}{x^{\frac{2}{3}}}\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^2*x^3,x, algorithm="maxima")
[Out] 1/220*(55*a^2 + 120*a*b/x^(1/3) + 66*b^2/x^(2/3))*x^4
```

mupad [B] time = 1.14, size = 24, normalized size = 0.71

$$\frac{a^2\,x^4}{4} + \frac{3\,b^2\,x^{10/3}}{10} + \frac{6\,a\,b\,x^{11/3}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b/x^(1/3))^2,x)
[Out] (a^2*x^4)/4 + (3*b^2*x^(10/3))/10 + (6*a*b*x^(11/3))/11
```

sympy [A] time = 1.79, size = 31, normalized size = 0.91

$$\frac{a^2x^4}{4} + \frac{6abx^{\frac{11}{3}}}{11} + \frac{3b^2x^{\frac{10}{3}}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**(1/3))**2*x**3,x)
[Out] a**2*x**4/4 + 6*a*b*x**(11/3)/11 + 3*b**2*x**(10/3)/10
```

3.2405 $\int \left(a + \frac{b}{\sqrt[3]{x}}\right)^2 x^2\, dx$

Optimal. Leaf size=34

$$\frac{a^2x^3}{3} + \frac{3}{4}abx^{8/3} + \frac{3}{7}b^2x^{7/3}$$

```
[Out] 3/7*b^2*x^(7/3)+3/4*a*b*x^(8/3)+1/3*a^2*x^3
```

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {263, 266, 43}

$$\frac{a^2x^3}{3} + \frac{3}{4}abx^{8/3} + \frac{3}{7}b^2x^{7/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^(1/3))^2*x^2,x]
```

```
[Out] (3*b^2*x^(7/3))/7 + (3*a*b*x^(8/3))/4 + (a^2*x^3)/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a + \frac{b}{\sqrt[3]{x}}\right)^2 x^2\, dx &= \int \left(b + a\sqrt[3]{x}\right)^2 x^{4/3}\, dx \\
&= 3\,\text{Subst}\left(\int x^6 (b + ax)^2\, dx, x, \sqrt[3]{x}\right) \\
&= 3\,\text{Subst}\left(\int \left(b^2x^6 + 2abx^7 + a^2x^8\right)\, dx, x, \sqrt[3]{x}\right) \\
&= \frac{3}{7}b^2x^{7/3} + \frac{3}{4}abx^{8/3} + \frac{a^2x^3}{3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 1.00

$$\frac{a^2x^3}{3} + \frac{3}{4}abx^{8/3} + \frac{3}{7}b^2x^{7/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^(1/3))^2*x^2,x]
[Out] (3*b^2*x^(7/3))/7 + (3*a*b*x^(8/3))/4 + (a^2*x^3)/3
```

fricas [A] time = 0.53, size = 24, normalized size = 0.71

$$\frac{1}{3}a^2x^3+\frac{3}{4}abx^{\frac{8}{3}}+\frac{3}{7}b^2x^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^2*x^2,x, algorithm="fricas")
[Out] 1/3*a^2*x^3 + 3/4*a*b*x^(8/3) + 3/7*b^2*x^(7/3)
```

giac [A] time = 0.16, size = 24, normalized size = 0.71

$$\frac{1}{3}a^2x^3+\frac{3}{4}abx^{\frac{8}{3}}+\frac{3}{7}b^2x^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^2*x^2,x, algorithm="giac")
[Out] 1/3*a^2*x^3 + 3/4*a*b*x^(8/3) + 3/7*b^2*x^(7/3)
```

maple [A] time = 0.00, size = 25, normalized size = 0.74

$$\frac{a^2x^3}{3}+\frac{3ab\,x^{\frac{8}{3}}}{4}+\frac{3b^2x^{\frac{7}{3}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^(1/3))^2*x^2,x)
[Out] 3/7*b^2*x^(7/3)+3/4*a*b*x^(8/3)+1/3*a^2*x^3
```

maxima [A] time = 0.44, size = 26, normalized size = 0.76

$$\frac{1}{84}\left(28\,a^2+\frac{63\,ab}{x^{\frac{1}{3}}}+\frac{36\,b^2}{x^{\frac{2}{3}}}\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^2*x^2,x, algorithm="maxima")
[Out] 1/84*(28*a^2 + 63*a*b/x^(1/3) + 36*b^2/x^(2/3))*x^3
```

mupad [B] time = 0.04, size = 24, normalized size = 0.71

$$\frac{a^2\,x^3}{3}+\frac{3\,b^2\,x^{7/3}}{7}+\frac{3\,a\,b\,x^{8/3}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b/x^(1/3))^2,x)
[Out] (a^2*x^3)/3 + (3*b^2*x^(7/3))/7 + (3*a*b*x^(8/3))/4
```

sympy [A] time = 0.89, size = 31, normalized size = 0.91

$$\frac{a^2x^3}{3}+\frac{3abx^{\frac{8}{3}}}{4}+\frac{3b^2x^{\frac{7}{3}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**(1/3))**2*x**2,x)
[Out] a**2*x**3/3 + 3*a*b*x**(8/3)/4 + 3*b**2*x**(7/3)/7
```

3.2406 $\int \left(a + \frac{b}{\sqrt[3]{x}}\right)^2 x\,dx$

Optimal. Leaf size=34

$$\frac{a^2x^2}{2} + \frac{6}{5}abx^{5/3} + \frac{3}{4}b^2x^{4/3}$$

```
[Out] 3/4*b^2*x^(4/3)+6/5*a*b*x^(5/3)+1/2*a^2*x^2
```

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 266, 43}

$$\frac{a^2x^2}{2} + \frac{6}{5}abx^{5/3} + \frac{3}{4}b^2x^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^(1/3))^2*x,x]
```

```
[Out] (3*b^2*x^(4/3))/4 + (6*a*b*x^(5/3))/5 + (a^2*x^2)/2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a + \frac{b}{\sqrt[3]{x}}\right)^2 x\,dx &= \int \left(b + a\sqrt[3]{x}\right)^2 \sqrt[3]{x}\,dx \\
&= 3\,\text{Subst}\left(\int x^3(b + ax)^2\,dx, x, \sqrt[3]{x}\right) \\
&= 3\,\text{Subst}\left(\int \left(b^2x^3 + 2abx^4 + a^2x^5\right)\,dx, x, \sqrt[3]{x}\right) \\
&= \frac{3}{4}b^2x^{4/3} + \frac{6}{5}abx^{5/3} + \frac{a^2x^2}{2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 1.00

$$\frac{a^2x^2}{2} + \frac{6}{5}abx^{5/3} + \frac{3}{4}b^2x^{4/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^(1/3))^2*x,x]
[Out] (3*b^2*x^(4/3))/4 + (6*a*b*x^(5/3))/5 + (a^2*x^2)/2
```

fricas [A] time = 0.69, size = 24, normalized size = 0.71

$$\frac{1}{2}a^2x^2 + \frac{6}{5}abx^{\frac{5}{3}} + \frac{3}{4}b^2x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^2*x,x, algorithm="fricas")
[Out] 1/2*a^2*x^2 + 6/5*a*b*x^(5/3) + 3/4*b^2*x^(4/3)
```

giac [A] time = 0.18, size = 24, normalized size = 0.71

$$\frac{1}{2}a^2x^2 + \frac{6}{5}abx^{\frac{5}{3}} + \frac{3}{4}b^2x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^2*x,x, algorithm="giac")
[Out] 1/2*a^2*x^2 + 6/5*a*b*x^(5/3) + 3/4*b^2*x^(4/3)
```

maple [A] time = 0.00, size = 25, normalized size = 0.74

$$\frac{a^2x^2}{2} + \frac{6ab\,x^{\frac{5}{3}}}{5} + \frac{3b^2x^{\frac{4}{3}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^(1/3))^2*x,x)
[Out] 3/4*b^2*x^(4/3)+6/5*a*b*x^(5/3)+1/2*a^2*x^2
```

maxima [A] time = 0.52, size = 26, normalized size = 0.76

$$\frac{1}{20}\left(10\,a^2 + \frac{24\,ab}{x^{\frac{1}{3}}} + \frac{15\,b^2}{x^{\frac{2}{3}}}\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^2*x,x, algorithm="maxima")
[Out] 1/20*(10*a^2 + 24*a*b/x^(1/3) + 15*b^2/x^(2/3))*x^2
```

mupad [B] time = 0.04, size = 24, normalized size = 0.71

$$\frac{a^2\,x^2}{2} + \frac{3\,b^2\,x^{4/3}}{4} + \frac{6\,a\,b\,x^{5/3}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b/x^(1/3))^2,x)
[Out] (a^2*x^2)/2 + (3*b^2*x^(4/3))/4 + (6*a*b*x^(5/3))/5
```

sympy [A] time = 0.40, size = 31, normalized size = 0.91

$$\frac{a^2x^2}{2} + \frac{6abx^{\frac{5}{3}}}{5} + \frac{3b^2x^{\frac{4}{3}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**(1/3))**2*x,x)
[Out] a**2*x**2/2 + 6*a*b*x**(5/3)/5 + 3*b**2*x**(4/3)/4
```

3.2407 $\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 dx$

Optimal. Leaf size=16

$$\frac{x\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{a}$$

```
[Out] (a+b/x^(1/3))^3*x/a
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {190, 37}

$$\frac{x\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^(1/3))^2,x]
```

```
[Out] ((a + b/x^(1/3))^3*x)/a
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rubi steps

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 dx = -\left(3\,\text{Subst}\left(\int \frac{(a+bx)^2}{x^4}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$
$$= \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x}{a}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.56

$$a^2x + 3abx^{2/3} + 3b^2\sqrt[3]{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^(1/3))^2,x]
```

```
[Out] 3*b^2*x^(1/3) + 3*a*b*x^(2/3) + a^2*x
```

fricas [A] time = 0.91, size = 21, normalized size = 1.31

$$a^2x + 3\,abx^{\frac{2}{3}} + 3\,b^2x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^2,x, algorithm="fricas")
[Out] a^2*x + 3*a*b*x^(2/3) + 3*b^2*x^(1/3)
```

giac [A] time = 0.16, size = 21, normalized size = 1.31

$$a^2x + 3\,abx^{\frac{2}{3}} + 3\,b^2x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^2,x, algorithm="giac")
[Out] a^2*x + 3*a*b*x^(2/3) + 3*b^2*x^(1/3)
```

maple [A] time = 0.00, size = 14, normalized size = 0.88

$$\frac{\left(a\,x^{\frac{1}{3}} + b\right)^3}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^(1/3))^2,x)
[Out] (b+a*x^(1/3))^3/a
```

maxima [A] time = 0.61, size = 21, normalized size = 1.31

$$a^2x + 3\,abx^{\frac{2}{3}} + 3\,b^2x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^2,x, algorithm="maxima")
[Out] a^2*x + 3*a*b*x^(2/3) + 3*b^2*x^(1/3)
```

mupad [B] time = 0.04, size = 21, normalized size = 1.31

$$a^2\,x + 3\,b^2\,x^{1/3} + 3\,a\,b\,x^{2/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^(1/3))^2,x)
[Out] a^2*x + 3*b^2*x^(1/3) + 3*a*b*x^(2/3)
```

sympy [A] time = 0.19, size = 24, normalized size = 1.50

$$a^2x + 3abx^{\frac{2}{3}} + 3b^2\sqrt[3]{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**(1/3))**2,x)
[Out] a**2*x + 3*a*b*x**(2/3) + 3*b**2*x**(1/3)
```

3.2408 $\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x}\,dx$

Optimal. Leaf size=28

$$a^2\log(x)-\frac{6ab}{\sqrt[3]{x}}-\frac{3b^2}{2x^{2/3}}$$

```
[Out] -3/2*b^2/x^(2/3)-6*a*b/x^(1/3)+a^2*ln(x)
```

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {263, 266, 43}

$$a^2\log(x)-\frac{6ab}{\sqrt[3]{x}}-\frac{3b^2}{2x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^(1/3))^2/x,x]
```

```
[Out] (-3*b^2)/(2*x^(2/3)) - (6*a*b)/x^(1/3) + a^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x}\,dx &= \int \frac{\left(b+a\sqrt[3]{x}\right)^2}{x^{5/3}}\,dx\\ &= 3\,\text{Subst}\left(\int \frac{(b+ax)^2}{x^3}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(\frac{b^2}{x^3}+\frac{2ab}{x^2}+\frac{a^2}{x}\right)dx, x, \sqrt[3]{x}\right)\\ &= -\frac{3b^2}{2x^{2/3}}-\frac{6ab}{\sqrt[3]{x}}+a^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 1.00

$$a^2\log(x)-\frac{6ab}{\sqrt[3]{x}}-\frac{3b^2}{2x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^(1/3))^2/x,x]
[Out] (-3*b^2)/(2*x^(2/3)) - (6*a*b)/x^(1/3) + a^2*Log[x]
```

fricas [A] time = 0.68, size = 31, normalized size = 1.11

$$\frac{3\left(2\,a^2x\log\left(x^{\frac{1}{3}}\right)-4\,abx^{\frac{2}{3}}-b^2x^{\frac{1}{3}}\right)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^2/x,x, algorithm="fricas")
[Out] 3/2*(2*a^2*x*log(x^(1/3)) - 4*a*b*x^(2/3) - b^2*x^(1/3))/x
```

giac [A] time = 0.15, size = 24, normalized size = 0.86

$$a^2\log\left(|x|\right)-\frac{3\left(4\,abx^{\frac{1}{3}}+b^2\right)}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^2/x,x, algorithm="giac")
[Out] a^2*log(abs(x)) - 3/2*(4*a*b*x^(1/3) + b^2)/x^(2/3)
```

maple [A] time = 0.01, size = 23, normalized size = 0.82

$$a^2\ln(x)-\frac{6ab}{x^{\frac{1}{3}}}-\frac{3b^2}{2x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^(1/3))^2/x,x)
[Out] -3/2*b^2/x^(2/3)-6*a*b/x^(1/3)+a^2*ln(x)
```

maxima [A] time = 0.57, size = 22, normalized size = 0.79

$$a^2\log(x)-\frac{6\,ab}{x^{\frac{1}{3}}}-\frac{3\,b^2}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^2/x,x, algorithm="maxima")
[Out] a^2*log(x) - 6*a*b/x^(1/3) - 3/2*b^2/x^(2/3)
```

mupad [B] time = 0.04, size = 28, normalized size = 1.00

$$3\,a^2\,\ln\left(x^{1/3}\right)-\frac{\frac{3\,b^2}{2}+6\,a\,b\,x^{1/3}}{x^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^(1/3))^2/x,x)
[Out] 3*a^2*log(x^(1/3)) - ((3*b^2)/2 + 6*a*b*x^(1/3))/x^(2/3)
```

sympy [**A**] time = 0.51, size = 27, normalized size = 0.96

$$a^2 \log(x) - \frac{6ab}{\sqrt[3]{x}} - \frac{3b^2}{2x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**(1/3))**2/x,x)
```

```
[Out] a**2*log(x) - 6*a*b/x**(1/3) - 3*b**2/(2*x**(2/3))
```

3.2409 $\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x^2}\,dx$

Optimal. Leaf size=32

$$-\frac{a^2}{x}-\frac{3ab}{2x^{4/3}}-\frac{3b^2}{5x^{5/3}}$$

```
[Out] -3/5*b^2/x^(5/3)-3/2*a*b/x^(4/3)-a^2/x
```

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {263, 266, 43}

$$-\frac{a^2}{x}-\frac{3ab}{2x^{4/3}}-\frac{3b^2}{5x^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^(1/3))^2/x^2,x]
```

```
[Out] (-3*b^2)/(5*x^(5/3)) - (3*a*b)/(2*x^(4/3)) - a^2/x
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x^2}\,dx &= \int \frac{\left(b+a\sqrt[3]{x}\right)^2}{x^{8/3}}\,dx\\ &= 3\,\text{Subst}\left(\int \frac{(b+ax)^2}{x^6}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(\frac{b^2}{x^6}+\frac{2ab}{x^5}+\frac{a^2}{x^4}\right)dx, x, \sqrt[3]{x}\right)\\ &= -\frac{3b^2}{5x^{5/3}}-\frac{3ab}{2x^{4/3}}-\frac{a^2}{x}\end{aligned}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 1.00

$$-\frac{a^2}{x}-\frac{3ab}{2x^{4/3}}-\frac{3b^2}{5x^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^(1/3))^2/x^2,x]
[Out] (-3*b^2)/(5*x^(5/3)) - (3*a*b)/(2*x^(4/3)) - a^2/x
```

fricas [A] time = 0.67, size = 27, normalized size = 0.84

$$-\frac{10\,a^2x+15\,abx^{\frac{2}{3}}+6\,b^2x^{\frac{1}{3}}}{10\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^2/x^2,x, algorithm="fricas")
[Out] -1/10*(10*a^2*x + 15*a*b*x^(2/3) + 6*b^2*x^(1/3))/x^2
```

giac [A] time = 0.17, size = 26, normalized size = 0.81

$$-\frac{10\,a^2x^{\frac{2}{3}}+15\,abx^{\frac{1}{3}}+6\,b^2}{10\,x^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^2/x^2,x, algorithm="giac")
[Out] -1/10*(10*a^2*x^(2/3) + 15*a*b*x^(1/3) + 6*b^2)/x^(5/3)
```

maple [A] time = 0.00, size = 25, normalized size = 0.78

$$-\frac{a^2}{x}-\frac{3ab}{2x^{\frac{4}{3}}}-\frac{3b^2}{5x^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^(1/3))^2/x^2,x)
[Out] -3/5*b^2/x^(5/3)-3/2*a*b/x^(4/3)-a^2/x
```

maxima [A] time = 0.47, size = 47, normalized size = 1.47

$$-\frac{3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^5}{5\,b^3}+\frac{3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^4a}{2\,b^3}-\frac{\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^3a^2}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^2/x^2,x, algorithm="maxima")
[Out] -3/5*(a + b/x^(1/3))^5/b^3 + 3/2*(a + b/x^(1/3))^4*a/b^3 - (a + b/x^(1/3))^
3*a^2/b^3
```

mupad [B] time = 0.05, size = 24, normalized size = 0.75

$$-\frac{a^2}{x}-\frac{3\,b^2}{5\,x^{5/3}}-\frac{3\,a\,b}{2\,x^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^(1/3))^2/x^2,x)
[Out] - a^2/x - (3*b^2)/(5*x^(5/3)) - (3*a*b)/(2*x^(4/3))
```

sympy [**A**] time = 0.96, size = 29, normalized size = 0.91

$$-\frac{a^2}{x} - \frac{3ab}{2x^{\frac{4}{3}}} - \frac{3b^2}{5x^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**(1/3))**2/x**2,x)
```

```
[Out] -a**2/x - 3*a*b/(2*x**(4/3)) - 3*b**2/(5*x**(5/3))
```

3.2410 $\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x^3}\,dx$

Optimal. Leaf size=34

$$-\frac{a^2}{2x^2}-\frac{6ab}{7x^{7/3}}-\frac{3b^2}{8x^{8/3}}$$

```
[Out] -3/8*b^2/x^(8/3)-6/7*a*b/x^(7/3)-1/2*a^2/x^2
```

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {263, 266, 43}

$$-\frac{a^2}{2x^2}-\frac{6ab}{7x^{7/3}}-\frac{3b^2}{8x^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^(1/3))^2/x^3,x]
```

```
[Out] (-3*b^2)/(8*x^(8/3)) - (6*a*b)/(7*x^(7/3)) - a^2/(2*x^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x^3}\,dx &= \int \frac{\left(b+a\sqrt[3]{x}\right)^2}{x^{11/3}}\,dx \\
&= 3\,\text{Subst}\left(\int \frac{(b+ax)^2}{x^9}\,dx, x, \sqrt[3]{x}\right) \\
&= 3\,\text{Subst}\left(\int \left(\frac{b^2}{x^9}+\frac{2ab}{x^8}+\frac{a^2}{x^7}\right)dx, x, \sqrt[3]{x}\right) \\
&= -\frac{3b^2}{8x^{8/3}}-\frac{6ab}{7x^{7/3}}-\frac{a^2}{2x^2}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 34, normalized size = 1.00

$$-\frac{a^2}{2x^2}-\frac{6ab}{7x^{7/3}}-\frac{3b^2}{8x^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^(1/3))^2/x^3,x]
[Out] (-3*b^2)/(8*x^(8/3)) - (6*a*b)/(7*x^(7/3)) - a^2/(2*x^2)
```

fricas [A] time = 0.97, size = 27, normalized size = 0.79

$$-\frac{28\,a^2x + 48\,abx^{\frac{2}{3}} + 21\,b^2x^{\frac{1}{3}}}{56\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^2/x^3,x, algorithm="fricas")
[Out] -1/56*(28*a^2*x + 48*a*b*x^(2/3) + 21*b^2*x^(1/3))/x^3
```

giac [A] time = 0.19, size = 26, normalized size = 0.76

$$-\frac{28\,a^2x^{\frac{2}{3}} + 48\,abx^{\frac{1}{3}} + 21\,b^2}{56\,x^{\frac{8}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^2/x^3,x, algorithm="giac")
[Out] -1/56*(28*a^2*x^(2/3) + 48*a*b*x^(1/3) + 21*b^2)/x^(8/3)
```

maple [A] time = 0.00, size = 25, normalized size = 0.74

$$-\frac{a^2}{2x^2} - \frac{6ab}{7x^{\frac{7}{3}}} - \frac{3b^2}{8x^{\frac{8}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^(1/3))^2/x^3,x)
[Out] -3/8*b^2/x^(8/3)-6/7*a*b/x^(7/3)-1/2*a^2/x^2
```

maxima [B] time = 0.54, size = 97, normalized size = 2.85

$$-\frac{3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^8}{8\,b^6} + \frac{15\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^7 a}{7\,b^6} - \frac{5\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^6 a^2}{b^6} + \frac{6\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^5 a^3}{b^6} - \frac{15\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^4 a^4}{4\,b^6} + \frac{\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^3 a^5}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^2/x^3,x, algorithm="maxima")
[Out] -3/8*(a + b/x^(1/3))^8/b^6 + 15/7*(a + b/x^(1/3))^7*a/b^6 - 5*(a + b/x^(1/3
))^6*a^2/b^6 + 6*(a + b/x^(1/3))^5*a^3/b^6 - 15/4*(a + b/x^(1/3))^4*a^4/b^6
 + (a + b/x^(1/3))^3*a^5/b^6
```

mupad [B] time = 1.14, size = 24, normalized size = 0.71

$$-\frac{a^2}{2\,x^2} - \frac{3\,b^2}{8\,x^{8/3}} - \frac{6\,a\,b}{7\,x^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^(1/3))^2/x^3,x)
[Out] - a^2/(2*x^2) - (3*b^2)/(8*x^(8/3)) - (6*a*b)/(7*x^(7/3))
```

sympy [**A**] time = 1.85, size = 32, normalized size = 0.94

$$-\frac{a^2}{2x^2} - \frac{6ab}{7x^{\frac{7}{3}}} - \frac{3b^2}{8x^{\frac{8}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**(1/3))**2/x**3,x)
[Out] -a**2/(2*x**2) - 6*a*b/(7*x**(7/3)) - 3*b**2/(8*x**(8/3))
```

3.2411 $\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x^4}\,dx$

Optimal. Leaf size=34

$$-\frac{a^2}{3x^3}-\frac{3ab}{5x^{10/3}}-\frac{3b^2}{11x^{11/3}}$$

```
[Out] -3/11*b^2/x^(11/3)-3/5*a*b/x^(10/3)-1/3*a^2/x^3
```

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {263, 266, 43}

$$-\frac{a^2}{3x^3}-\frac{3ab}{5x^{10/3}}-\frac{3b^2}{11x^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^(1/3))^2/x^4,x]
```

```
[Out] (-3*b^2)/(11*x^(11/3)) - (3*a*b)/(5*x^(10/3)) - a^2/(3*x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x^4}\,dx &= \int \frac{\left(b+a\sqrt[3]{x}\right)^2}{x^{14/3}}\,dx \\
&= 3\,\text{Subst}\left(\int \frac{(b+ax)^2}{x^{12}}\,dx, x, \sqrt[3]{x}\right) \\
&= 3\,\text{Subst}\left(\int \left(\frac{b^2}{x^{12}}+\frac{2ab}{x^{11}}+\frac{a^2}{x^{10}}\right)dx, x, \sqrt[3]{x}\right) \\
&= -\frac{3b^2}{11x^{11/3}}-\frac{3ab}{5x^{10/3}}-\frac{a^2}{3x^3}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 34, normalized size = 1.00

$$-\frac{a^2}{3x^3}-\frac{3ab}{5x^{10/3}}-\frac{3b^2}{11x^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^(1/3))^2/x^4,x]
[Out] (-3*b^2)/(11*x^(11/3)) - (3*a*b)/(5*x^(10/3)) - a^2/(3*x^3)
```

fricas [A] time = 0.79, size = 27, normalized size = 0.79

$$-\frac{55\,a^2x + 99\,abx^{\frac{2}{3}} + 45\,b^2x^{\frac{1}{3}}}{165\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^2/x^4,x, algorithm="fricas")
[Out] -1/165*(55*a^2*x + 99*a*b*x^(2/3) + 45*b^2*x^(1/3))/x^4
```

giac [A] time = 0.20, size = 26, normalized size = 0.76

$$-\frac{55\,a^2x^{\frac{2}{3}} + 99\,abx^{\frac{1}{3}} + 45\,b^2}{165\,x^{\frac{11}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^2/x^4,x, algorithm="giac")
[Out] -1/165*(55*a^2*x^(2/3) + 99*a*b*x^(1/3) + 45*b^2)/x^(11/3)
```

maple [A] time = 0.01, size = 25, normalized size = 0.74

$$-\frac{a^2}{3x^3} - \frac{3ab}{5x^{\frac{10}{3}}} - \frac{3b^2}{11x^{\frac{11}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^(1/3))^2/x^4,x)
[Out] -3/11*b^2/x^(11/3)-3/5*a*b/x^(10/3)-1/3*a^2/x^3
```

maxima [B] time = 0.53, size = 149, normalized size = 4.38

$$-\frac{3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{11}}{11\,b^9} + \frac{12\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{10}a}{5\,b^9} - \frac{28\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{9}a^2}{3\,b^9} + \frac{21\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{8}a^3}{b^9} - \frac{30\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{7}a^4}{b^9} + \frac{28\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{6}a^5}{b^9} - \frac{84\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{}}{5\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^2/x^4,x, algorithm="maxima")
[Out] -3/11*(a + b/x^(1/3))^11/b^9 + 12/5*(a + b/x^(1/3))^10*a/b^9 - 28/3*(a + b/
x^(1/3))^9*a^2/b^9 + 21*(a + b/x^(1/3))^8*a^3/b^9 - 30*(a + b/x^(1/3))^7*a^
4/b^9 + 28*(a + b/x^(1/3))^6*a^5/b^9 - 84/5*(a + b/x^(1/3))^5*a^6/b^9 + 6*(
a + b/x^(1/3))^4*a^7/b^9 - (a + b/x^(1/3))^3*a^8/b^9
```

mupad [B] time = 1.13, size = 24, normalized size = 0.71

$$-\frac{a^2}{3\,x^3} - \frac{3\,b^2}{11\,x^{11/3}} - \frac{3\,a\,b}{5\,x^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^(1/3))^2/x^4,x)
```

```
[Out] - a^2/(3*x^3) - (3*b^2)/(11*x^(11/3)) - (3*a*b)/(5*x^(10/3))
```

sympy [**A**] time = 2.98, size = 32, normalized size = 0.94

$$-\frac{a^2}{3x^3} - \frac{3ab}{5x^{\frac{10}{3}}} - \frac{3b^2}{11x^{\frac{11}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**(1/3))**2/x**4,x)
[Out] -a**2/(3*x**3) - 3*a*b/(5*x**(10/3)) - 3*b**2/(11*x**(11/3))
```

3.2412 $\int \left(a + \frac{b}{\sqrt[3]{x}}\right)^3 x^4\, dx$

Optimal. Leaf size=47

$$\frac{a^3x^5}{5} + \frac{9}{14}a^2bx^{14/3} + \frac{9}{13}ab^2x^{13/3} + \frac{b^3x^4}{4}$$

```
[Out] 1/4*b^3*x^4+9/13*a*b^2*x^(13/3)+9/14*a^2*b*x^(14/3)+1/5*a^3*x^5
```

Rubi [A] time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {263, 266, 43}

$$\frac{9}{14}a^2bx^{14/3} + \frac{a^3x^5}{5} + \frac{9}{13}ab^2x^{13/3} + \frac{b^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^(1/3))^3*x^4,x]
```

```
[Out] (b^3*x^4)/4 + (9*a*b^2*x^(13/3))/13 + (9*a^2*b*x^(14/3))/14 + (a^3*x^5)/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a + \frac{b}{\sqrt[3]{x}}\right)^3 x^4\, dx &= \int \left(b + a\sqrt[3]{x}\right)^3 x^3\, dx \\
&= 3\,\text{Subst}\left(\int x^{11}(b + ax)^3\, dx, x, \sqrt[3]{x}\right) \\
&= 3\,\text{Subst}\left(\int \left(b^3x^{11} + 3ab^2x^{12} + 3a^2bx^{13} + a^3x^{14}\right)\, dx, x, \sqrt[3]{x}\right) \\
&= \frac{b^3x^4}{4} + \frac{9}{13}ab^2x^{13/3} + \frac{9}{14}a^2bx^{14/3} + \frac{a^3x^5}{5}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 47, normalized size = 1.00

$$\frac{a^3x^5}{5} + \frac{9}{14}a^2bx^{14/3} + \frac{9}{13}ab^2x^{13/3} + \frac{b^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^(1/3))^3*x^4,x]
```

```
[Out] (b^3*x^4)/4 + (9*a*b^2*x^(13/3))/13 + (9*a^2*b*x^(14/3))/14 + (a^3*x^5)/5
```

fricas [A] time = 0.68, size = 35, normalized size = 0.74

$$\frac{1}{5}a^3x^5 + \frac{9}{14}a^2bx^{\frac{14}{3}} + \frac{9}{13}ab^2x^{\frac{13}{3}} + \frac{1}{4}b^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^3*x^4,x, algorithm="fricas")
```

```
[Out] 1/5*a^3*x^5 + 9/14*a^2*b*x^(14/3) + 9/13*a*b^2*x^(13/3) + 1/4*b^3*x^4
```

giac [A] time = 0.16, size = 35, normalized size = 0.74

$$\frac{1}{5}a^3x^5 + \frac{9}{14}a^2bx^{\frac{14}{3}} + \frac{9}{13}ab^2x^{\frac{13}{3}} + \frac{1}{4}b^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^3*x^4,x, algorithm="giac")
```

```
[Out] 1/5*a^3*x^5 + 9/14*a^2*b*x^(14/3) + 9/13*a*b^2*x^(13/3) + 1/4*b^3*x^4
```

maple [A] time = 0.00, size = 36, normalized size = 0.77

$$\frac{a^3x^5}{5} + \frac{9a^2b\,x^{\frac{14}{3}}}{14} + \frac{9a\,b^2x^{\frac{13}{3}}}{13} + \frac{b^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^(1/3))^3*x^4,x)
```

```
[Out] 1/4*b^3*x^4+9/13*a*b^2*x^(13/3)+9/14*a^2*b*x^(14/3)+1/5*a^3*x^5
```

maxima [A] time = 0.70, size = 37, normalized size = 0.79

$$\frac{1}{1820}\left(364\,a^3 + \frac{1170\,a^2b}{x^{\frac{1}{3}}} + \frac{1260\,ab^2}{x^{\frac{2}{3}}} + \frac{455\,b^3}{x}\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^3*x^4,x, algorithm="maxima")
```

```
[Out] 1/1820*(364*a^3 + 1170*a^2*b/x^(1/3) + 1260*a*b^2/x^(2/3) + 455*b^3/x)*x^5
```

mupad [B] time = 0.05, size = 35, normalized size = 0.74

$$\frac{a^3\,x^5}{5} + \frac{b^3\,x^4}{4} + \frac{9\,a\,b^2\,x^{13/3}}{13} + \frac{9\,a^2\,b\,x^{14/3}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b/x^(1/3))^3,x)
```

```
[Out] (a^3*x^5)/5 + (b^3*x^4)/4 + (9*a*b^2*x^(13/3))/13 + (9*a^2*b*x^(14/3))/14
```

sympy [A] time = 3.96, size = 42, normalized size = 0.89

$$\frac{a^3x^5}{5} + \frac{9a^2bx^{\frac{14}{3}}}{14} + \frac{9ab^2x^{\frac{13}{3}}}{13} + \frac{b^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**(1/3))**3*x**4,x)
```

```
[Out] a**3*x**5/5 + 9*a**2*b*x**(14/3)/14 + 9*a*b**2*x**(13/3)/13 + b**3*x**4/4
```

3.2413 $\int \left(a + \frac{b}{\sqrt[3]{x}}\right)^3 x^3\, dx$

Optimal. Leaf size=47

$$\frac{a^3x^4}{4} + \frac{9}{11}a^2bx^{11/3} + \frac{9}{10}ab^2x^{10/3} + \frac{b^3x^3}{3}$$

```
[Out] 1/3*b^3*x^3+9/10*a*b^2*x^(10/3)+9/11*a^2*b*x^(11/3)+1/4*a^3*x^4
```

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {263, 266, 43}

$$\frac{9}{11}a^2bx^{11/3} + \frac{a^3x^4}{4} + \frac{9}{10}ab^2x^{10/3} + \frac{b^3x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^(1/3))^3*x^3,x]
```

```
[Out] (b^3*x^3)/3 + (9*a*b^2*x^(10/3))/10 + (9*a^2*b*x^(11/3))/11 + (a^3*x^4)/4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a + \frac{b}{\sqrt[3]{x}}\right)^3 x^3\, dx &= \int \left(b + a\sqrt[3]{x}\right)^3 x^2\, dx \\
&= 3\,\text{Subst}\left(\int x^8(b + ax)^3\, dx, x, \sqrt[3]{x}\right) \\
&= 3\,\text{Subst}\left(\int \left(b^3x^8 + 3ab^2x^9 + 3a^2bx^{10} + a^3x^{11}\right) dx, x, \sqrt[3]{x}\right) \\
&= \frac{b^3x^3}{3} + \frac{9}{10}ab^2x^{10/3} + \frac{9}{11}a^2bx^{11/3} + \frac{a^3x^4}{4}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 0.87

$$\frac{1}{660}x^3\left(165a^3x + 540a^2bx^{2/3} + 594ab^2\sqrt[3]{x} + 220b^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^(1/3))^3*x^3,x]
```

```
[Out] (x^3*(220*b^3 + 594*a*b^2*x^(1/3) + 540*a^2*b*x^(2/3) + 165*a^3*x))/660
```

fricas [A] time = 0.62, size = 35, normalized size = 0.74

$$\frac{1}{4}a^3x^4 + \frac{9}{11}a^2bx^{\frac{11}{3}} + \frac{9}{10}ab^2x^{\frac{10}{3}} + \frac{1}{3}b^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^3*x^3,x, algorithm="fricas")
```

```
[Out] 1/4*a^3*x^4 + 9/11*a^2*b*x^(11/3) + 9/10*a*b^2*x^(10/3) + 1/3*b^3*x^3
```

giac [A] time = 0.15, size = 35, normalized size = 0.74

$$\frac{1}{4}a^3x^4 + \frac{9}{11}a^2bx^{\frac{11}{3}} + \frac{9}{10}ab^2x^{\frac{10}{3}} + \frac{1}{3}b^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^3*x^3,x, algorithm="giac")
```

```
[Out] 1/4*a^3*x^4 + 9/11*a^2*b*x^(11/3) + 9/10*a*b^2*x^(10/3) + 1/3*b^3*x^3
```

maple [A] time = 0.00, size = 36, normalized size = 0.77

$$\frac{a^3x^4}{4} + \frac{9a^2b\,x^{\frac{11}{3}}}{11} + \frac{9a\,b^2x^{\frac{10}{3}}}{10} + \frac{b^3x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^(1/3))^3*x^3,x)
```

```
[Out] 1/3*b^3*x^3+9/10*a*b^2*x^(10/3)+9/11*a^2*b*x^(11/3)+1/4*a^3*x^4
```

maxima [A] time = 0.57, size = 37, normalized size = 0.79

$$\frac{1}{660}\left(165\,a^3 + \frac{540\,a^2b}{x^{\frac{1}{3}}} + \frac{594\,ab^2}{x^{\frac{2}{3}}} + \frac{220\,b^3}{x}\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^3*x^3,x, algorithm="maxima")
```

```
[Out] 1/660*(165*a^3 + 540*a^2*b/x^(1/3) + 594*a*b^2/x^(2/3) + 220*b^3/x)*x^4
```

mupad [B] time = 0.04, size = 35, normalized size = 0.74

$$\frac{a^3\,x^4}{4} + \frac{b^3\,x^3}{3} + \frac{9\,a\,b^2\,x^{10/3}}{10} + \frac{9\,a^2\,b\,x^{11/3}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b/x^(1/3))^3,x)
```

```
[Out] (a^3*x^4)/4 + (b^3*x^3)/3 + (9*a*b^2*x^(10/3))/10 + (9*a^2*b*x^(11/3))/11
```

sympy [A] time = 2.28, size = 42, normalized size = 0.89

$$\frac{a^3x^4}{4} + \frac{9a^2bx^{\frac{11}{3}}}{11} + \frac{9ab^2x^{\frac{10}{3}}}{10} + \frac{b^3x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**(1/3))**3*x**3,x)
```

```
[Out] a**3*x**4/4 + 9*a**2*b*x**(11/3)/11 + 9*a*b**2*x**(10/3)/10 + b**3*x**3/3
```

3.2414 $\int \left(a + \frac{b}{\sqrt[3]{x}}\right)^3 x^2\, dx$

Optimal. Leaf size=47

$$\frac{a^3x^3}{3} + \frac{9}{8}a^2bx^{8/3} + \frac{9}{7}ab^2x^{7/3} + \frac{b^3x^2}{2}$$

```
[Out] 1/2*b^3*x^2+9/7*a*b^2*x^(7/3)+9/8*a^2*b*x^(8/3)+1/3*a^3*x^3
```

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {263, 266, 43}

$$\frac{9}{8}a^2bx^{8/3} + \frac{a^3x^3}{3} + \frac{9}{7}ab^2x^{7/3} + \frac{b^3x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^(1/3))^3*x^2,x]
```

```
[Out] (b^3*x^2)/2 + (9*a*b^2*x^(7/3))/7 + (9*a^2*b*x^(8/3))/8 + (a^3*x^3)/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \left(a + \frac{b}{\sqrt[3]{x}}\right)^3 x^2\, dx &= \int \left(b + a\sqrt[3]{x}\right)^3 x\, dx \\
&= 3\,\text{Subst}\left(\int x^5(b + ax)^3\, dx, x, \sqrt[3]{x}\right) \\
&= 3\,\text{Subst}\left(\int \left(b^3x^5 + 3ab^2x^6 + 3a^2bx^7 + a^3x^8\right)\, dx, x, \sqrt[3]{x}\right) \\
&= \frac{b^3x^2}{2} + \frac{9}{7}ab^2x^{7/3} + \frac{9}{8}a^2bx^{8/3} + \frac{a^3x^3}{3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 1.00

$$\frac{a^3x^3}{3} + \frac{9}{8}a^2bx^{8/3} + \frac{9}{7}ab^2x^{7/3} + \frac{b^3x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^(1/3))^3*x^2,x]
```

```
[Out] (b^3*x^2)/2 + (9*a*b^2*x^(7/3))/7 + (9*a^2*b*x^(8/3))/8 + (a^3*x^3)/3
```

fricas [A] time = 0.89, size = 35, normalized size = 0.74

$$\frac{1}{3}a^3x^3 + \frac{9}{8}a^2bx^{\frac{8}{3}} + \frac{9}{7}ab^2x^{\frac{7}{3}} + \frac{1}{2}b^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^3*x^2,x, algorithm="fricas")
```

```
[Out] 1/3*a^3*x^3 + 9/8*a^2*b*x^(8/3) + 9/7*a*b^2*x^(7/3) + 1/2*b^3*x^2
```

giac [A] time = 0.19, size = 35, normalized size = 0.74

$$\frac{1}{3}a^3x^3 + \frac{9}{8}a^2bx^{\frac{8}{3}} + \frac{9}{7}ab^2x^{\frac{7}{3}} + \frac{1}{2}b^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^3*x^2,x, algorithm="giac")
```

```
[Out] 1/3*a^3*x^3 + 9/8*a^2*b*x^(8/3) + 9/7*a*b^2*x^(7/3) + 1/2*b^3*x^2
```

maple [A] time = 0.00, size = 36, normalized size = 0.77

$$\frac{a^3x^3}{3} + \frac{9a^2b\,x^{\frac{8}{3}}}{8} + \frac{9a\,b^2x^{\frac{7}{3}}}{7} + \frac{b^3x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^(1/3))^3*x^2,x)
```

```
[Out] 1/2*b^3*x^2+9/7*a*b^2*x^(7/3)+9/8*a^2*b*x^(8/3)+1/3*a^3*x^3
```

maxima [A] time = 0.57, size = 37, normalized size = 0.79

$$\frac{1}{168}\left(56\,a^3 + \frac{189\,a^2b}{x^{\frac{1}{3}}} + \frac{216\,ab^2}{x^{\frac{2}{3}}} + \frac{84\,b^3}{x}\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^3*x^2,x, algorithm="maxima")
```

```
[Out] 1/168*(56*a^3 + 189*a^2*b/x^(1/3) + 216*a*b^2/x^(2/3) + 84*b^3/x)*x^3
```

mupad [B] time = 0.05, size = 35, normalized size = 0.74

$$\frac{a^3\,x^3}{3} + \frac{b^3\,x^2}{2} + \frac{9\,a\,b^2\,x^{7/3}}{7} + \frac{9\,a^2\,b\,x^{8/3}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b/x^(1/3))^3,x)
```

```
[Out] (a^3*x^3)/3 + (b^3*x^2)/2 + (9*a*b^2*x^(7/3))/7 + (9*a^2*b*x^(8/3))/8
```

sympy [A] time = 1.22, size = 42, normalized size = 0.89

$$\frac{a^3x^3}{3} + \frac{9a^2bx^{\frac{8}{3}}}{8} + \frac{9ab^2x^{\frac{7}{3}}}{7} + \frac{b^3x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**(1/3))**3*x**2,x)
```

```
[Out] a**3*x**3/3 + 9*a**2*b*x**(8/3)/8 + 9*a*b**2*x**(7/3)/7 + b**3*x**2/2
```

3.2415 $\int \left(a + \frac{b}{\sqrt[3]{x}}\right)^3 x\,dx$

Optimal. Leaf size=42

$$\frac{a^3x^2}{2} + \frac{9}{5}a^2bx^{5/3} + \frac{9}{4}ab^2x^{4/3} + b^3x$$

```
[Out] b^3*x+9/4*a*b^2*x^(4/3)+9/5*a^2*b*x^(5/3)+1/2*a^3*x^2
```

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 190, 43}

$$\frac{9}{5}a^2bx^{5/3} + \frac{a^3x^2}{2} + \frac{9}{4}ab^2x^{4/3} + b^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^(1/3))^3*x,x]
```

```
[Out] b^3*x + (9*a*b^2*x^(4/3))/4 + (9*a^2*b*x^(5/3))/5 + (a^3*x^2)/2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\int \left(a + \frac{b}{\sqrt[3]{x}}\right)^3 x\,dx &= \int \left(b + a\sqrt[3]{x}\right)^3 dx \\
&= 3\,\text{Subst}\left(\int x^2(b + ax)^3\,dx, x, \sqrt[3]{x}\right) \\
&= 3\,\text{Subst}\left(\int \left(b^3x^2 + 3ab^2x^3 + 3a^2bx^4 + a^3x^5\right)\,dx, x, \sqrt[3]{x}\right) \\
&= b^3x + \frac{9}{4}ab^2x^{4/3} + \frac{9}{5}a^2bx^{5/3} + \frac{a^3x^2}{2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 1.00

$$\frac{a^3x^2}{2} + \frac{9}{5}a^2bx^{5/3} + \frac{9}{4}ab^2x^{4/3} + b^3x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^(1/3))^3*x,x]
```

```
[Out] b^3*x + (9*a*b^2*x^(4/3))/4 + (9*a^2*b*x^(5/3))/5 + (a^3*x^2)/2
```

fricas [A] time = 0.76, size = 32, normalized size = 0.76

$$\frac{1}{2}a^3x^2 + \frac{9}{5}a^2bx^{\frac{5}{3}} + \frac{9}{4}ab^2x^{\frac{4}{3}} + b^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^3*x,x, algorithm="fricas")
```

```
[Out] 1/2*a^3*x^2 + 9/5*a^2*b*x^(5/3) + 9/4*a*b^2*x^(4/3) + b^3*x
```

giac [A] time = 0.16, size = 32, normalized size = 0.76

$$\frac{1}{2}a^3x^2 + \frac{9}{5}a^2bx^{\frac{5}{3}} + \frac{9}{4}ab^2x^{\frac{4}{3}} + b^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^3*x,x, algorithm="giac")
```

```
[Out] 1/2*a^3*x^2 + 9/5*a^2*b*x^(5/3) + 9/4*a*b^2*x^(4/3) + b^3*x
```

maple [A] time = 0.00, size = 33, normalized size = 0.79

$$\frac{a^3x^2}{2} + \frac{9a^2b\,x^{\frac{5}{3}}}{5} + \frac{9a\,b^2x^{\frac{4}{3}}}{4} + b^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^(1/3))^3*x,x)
```

```
[Out] b^3*x+9/4*a*b^2*x^(4/3)+9/5*a^2*b*x^(5/3)+1/2*a^3*x^2
```

maxima [A] time = 0.55, size = 37, normalized size = 0.88

$$\frac{1}{20}\left(10\,a^3 + \frac{36\,a^2b}{x^{\frac{1}{3}}} + \frac{45\,ab^2}{x^{\frac{2}{3}}} + \frac{20\,b^3}{x}\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^3*x,x, algorithm="maxima")
```

```
[Out] 1/20*(10*a^3 + 36*a^2*b/x^(1/3) + 45*a*b^2/x^(2/3) + 20*b^3/x)*x^2
```

mupad [B] time = 0.05, size = 32, normalized size = 0.76

$$b^3\,x + \frac{a^3\,x^2}{2} + \frac{9\,a\,b^2\,x^{4/3}}{4} + \frac{9\,a^2\,b\,x^{5/3}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b/x^(1/3))^3,x)
```

```
[Out] b^3*x + (a^3*x^2)/2 + (9*a*b^2*x^(4/3))/4 + (9*a^2*b*x^(5/3))/5
```

sympy [A] time = 0.56, size = 39, normalized size = 0.93

$$\frac{a^3x^2}{2} + \frac{9a^2bx^{\frac{5}{3}}}{5} + \frac{9ab^2x^{\frac{4}{3}}}{4} + b^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**(1/3))**3*x,x)
```

```
[Out] a**3*x**2/2 + 9*a**2*b*x**(5/3)/5 + 9*a*b**2*x**(4/3)/4 + b**3*x
```

3.2416 $\int \left(a + \frac{b}{\sqrt[3]{x}}\right)^3 dx$

Optimal. Leaf size=36

$$a^3 x + \frac{9}{2} a^2 b x^{2/3} + 9ab^2 \sqrt[3]{x} + b^3 \log(x)$$

```
[Out] 9*a*b^2*x^(1/3)+9/2*a^2*b*x^(2/3)+a^3*x+b^3*ln(x)
```

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {190, 43}

$$\frac{9}{2} a^2 b x^{2/3} + a^3 x + 9ab^2 \sqrt[3]{x} + b^3 \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^(1/3))^3,x]
```

```
[Out] 9*a*b^2*x^(1/3) + (9*a^2*b*x^(2/3))/2 + a^3*x + b^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
 - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
  IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}
\int \left(a + \frac{b}{\sqrt[3]{x}}\right)^3 dx &= -\left(3\,\text{Subst}\left(\int \frac{(a+bx)^3}{x^4}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right) \\
&= -\left(3\,\text{Subst}\left(\int \left(\frac{a^3}{x^4} + \frac{3a^2 b}{x^3} + \frac{3ab^2}{x^2} + \frac{b^3}{x}\right) dx, x, \frac{1}{\sqrt[3]{x}}\right)\right) \\
&= 9ab^2 \sqrt[3]{x} + \frac{9}{2} a^2 b x^{2/3} + a^3 x + b^3 \log(x)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 36, normalized size = 1.00

$$a^3 x + \frac{9}{2} a^2 b x^{2/3} + 9ab^2 \sqrt[3]{x} + b^3 \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^(1/3))^3,x]
```

```
[Out] 9*a*b^2*x^(1/3) + (9*a^2*b*x^(2/3))/2 + a^3*x + b^3*Log[x]
```

fricas [A] time = 1.13, size = 33, normalized size = 0.92

$$a^3 x + 3\,b^3 \log\left(x^{\frac{1}{3}}\right) + \frac{9}{2}\,a^2 b x^{\frac{2}{3}} + 9\,ab^2 x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^3,x, algorithm="fricas")
[Out] a^3*x + 3*b^3*log(x^(1/3)) + 9/2*a^2*b*x^(2/3) + 9*a*b^2*x^(1/3)
```

giac [A] time = 0.17, size = 31, normalized size = 0.86

$$a^3x + b^3\log(|x|) + \frac{9}{2}a^2bx^{\frac{2}{3}} + 9ab^2x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^3,x, algorithm="giac")
[Out] a^3*x + b^3*log(abs(x)) + 9/2*a^2*b*x^(2/3) + 9*a*b^2*x^(1/3)
```

maple [A] time = 0.00, size = 31, normalized size = 0.86

$$a^3x + b^3\ln(x) + \frac{9a^2b\,x^{\frac{2}{3}}}{2} + 9a\,b^2x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^(1/3))^3,x)
[Out] 9*a*b^2*x^(1/3)+9/2*a^2*b*x^(2/3)+a^3*x+b^3*ln(x)
```

maxima [A] time = 0.49, size = 30, normalized size = 0.83

$$a^3x + b^3\log(x) + \frac{9}{2}a^2bx^{\frac{2}{3}} + 9ab^2x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^3,x, algorithm="maxima")
[Out] a^3*x + b^3*log(x) + 9/2*a^2*b*x^(2/3) + 9*a*b^2*x^(1/3)
```

mupad [B] time = 0.04, size = 33, normalized size = 0.92

$$3\,b^3\,\ln\left(x^{1/3}\right) + a^3\,x + 9\,a\,b^2\,x^{1/3} + \frac{9\,a^2\,b\,x^{2/3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^(1/3))^3,x)
[Out] 3*b^3*log(x^(1/3)) + a^3*x + 9*a*b^2*x^(1/3) + (9*a^2*b*x^(2/3))/2
```

sympy [A] time = 0.26, size = 36, normalized size = 1.00

$$a^3x + \frac{9a^2bx^{\frac{2}{3}}}{2} + 9ab^2\sqrt[3]{x} + b^3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**(1/3))**3,x)
[Out] a**3*x + 9*a**2*b*x**(2/3)/2 + 9*a*b**2*x**(1/3) + b**3*log(x)
```

3.2417 $\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x}\,dx$

Optimal. Leaf size=39

$$a^3\log(x)-\frac{9a^2b}{\sqrt[3]{x}}-\frac{9ab^2}{2x^{2/3}}-\frac{b^3}{x}$$

```
[Out] -b^3/x-9/2*a*b^2/x^(2/3)-9*a^2*b/x^(1/3)+a^3*ln(x)
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {263, 266, 43}

$$-\frac{9a^2b}{\sqrt[3]{x}}+a^3\log(x)-\frac{9ab^2}{2x^{2/3}}-\frac{b^3}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^(1/3))^3/x,x]
```

```
[Out] -(b^3/x) - (9*a*b^2)/(2*x^(2/3)) - (9*a^2*b)/x^(1/3) + a^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x}\,dx &= \int \frac{\left(b+a\sqrt[3]{x}\right)^3}{x^2}\,dx \\
&= 3\,\text{Subst}\left(\int \frac{(b+ax)^3}{x^4}\,dx, x, \sqrt[3]{x}\right) \\
&= 3\,\text{Subst}\left(\int \left(\frac{b^3}{x^4}+\frac{3ab^2}{x^3}+\frac{3a^2b}{x^2}+\frac{a^3}{x}\right)dx, x, \sqrt[3]{x}\right) \\
&= -\frac{b^3}{x}-\frac{9ab^2}{2x^{2/3}}-\frac{9a^2b}{\sqrt[3]{x}}+a^3\log(x)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 39, normalized size = 1.00

$$a^3\log(x)-\frac{9a^2b}{\sqrt[3]{x}}-\frac{9ab^2}{2x^{2/3}}-\frac{b^3}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^(1/3))^3/x,x]
[Out] -(b^3/x) - (9*a*b^2)/(2*x^(2/3)) - (9*a^2*b)/x^(1/3) + a^3*Log[x]
```

fricas [A] time = 0.69, size = 39, normalized size = 1.00

$$\frac{6\,a^3 x \log\left(x^{\frac{1}{3}}\right) - 18\,a^2 b x^{\frac{2}{3}} - 9\,a b^2 x^{\frac{1}{3}} - 2\,b^3}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^3/x,x, algorithm="fricas")
[Out] 1/2*(6*a^3*x*log(x^(1/3)) - 18*a^2*b*x^(2/3) - 9*a*b^2*x^(1/3) - 2*b^3)/x
```

giac [A] time = 0.16, size = 37, normalized size = 0.95

$$a^3 \log\left(|x|\right) - \frac{18\,a^2 b x^{\frac{2}{3}} + 9\,a b^2 x^{\frac{1}{3}} + 2\,b^3}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^3/x,x, algorithm="giac")
[Out] a^3*log(abs(x)) - 1/2*(18*a^2*b*x^(2/3) + 9*a*b^2*x^(1/3) + 2*b^3)/x
```

maple [A] time = 0.01, size = 34, normalized size = 0.87

$$a^3 \ln(x) - \frac{9a^2 b}{x^{\frac{1}{3}}} - \frac{9a\,b^2}{2x^{\frac{2}{3}}} - \frac{b^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^(1/3))^3/x,x)
[Out] -b^3/x-9/2*a*b^2/x^(2/3)-9*a^2*b/x^(1/3)+a^3*ln(x)
```

maxima [A] time = 0.53, size = 33, normalized size = 0.85

$$a^3 \log(x) - \frac{9\,a^2 b}{x^{\frac{1}{3}}} - \frac{9\,a b^2}{2\,x^{\frac{2}{3}}} - \frac{b^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^3/x,x, algorithm="maxima")
[Out] a^3*log(x) - 9*a^2*b/x^(1/3) - 9/2*a*b^2/x^(2/3) - b^3/x
```

mupad [B] time = 0.04, size = 37, normalized size = 0.95

$$3\,a^3\,\ln\left(x^{1/3}\right) - \frac{b^3 + \frac{9\,a\,b^2\,x^{1/3}}{2} + 9\,a^2\,b\,x^{2/3}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^(1/3))^3/x,x)
[Out] 3*a^3*log(x^(1/3)) - (b^3 + (9*a*b^2*x^(1/3))/2 + 9*a^2*b*x^(2/3))/x
```

sympy [**A**] time = 0.66, size = 36, normalized size = 0.92

$$a^3 \log(x) - \frac{9a^2 b}{\sqrt[3]{x}} - \frac{9ab^2}{2x^{\frac{2}{3}}} - \frac{b^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**(1/3))**3/x,x)
```

```
[Out] a**3*log(x) - 9*a**2*b/x**(1/3) - 9*a*b**2/(2*x**(2/3)) - b**3/x
```

3.2418 $\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x^2}\,dx$

Optimal. Leaf size=45

$$-\frac{a^3}{x}-\frac{9a^2b}{4x^{4/3}}-\frac{9ab^2}{5x^{5/3}}-\frac{b^3}{2x^2}$$

```
[Out] -1/2*b^3/x^2-9/5*a*b^2/x^(5/3)-9/4*a^2*b/x^(4/3)-a^3/x
```

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {263, 266, 43}

$$-\frac{9a^2b}{4x^{4/3}}-\frac{a^3}{x}-\frac{9ab^2}{5x^{5/3}}-\frac{b^3}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^(1/3))^3/x^2,x]
```

```
[Out] -b^3/(2*x^2) - (9*a*b^2)/(5*x^(5/3)) - (9*a^2*b)/(4*x^(4/3)) - a^3/x
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x^2}\,dx &= \int \frac{\left(b+a\sqrt[3]{x}\right)^3}{x^3}\,dx \\
&= 3\,\text{Subst}\left(\int \frac{(b+ax)^3}{x^7}\,dx, x, \sqrt[3]{x}\right) \\
&= 3\,\text{Subst}\left(\int \left(\frac{b^3}{x^7}+\frac{3ab^2}{x^6}+\frac{3a^2b}{x^5}+\frac{a^3}{x^4}\right)dx, x, \sqrt[3]{x}\right) \\
&= -\frac{b^3}{2x^2}-\frac{9ab^2}{5x^{5/3}}-\frac{9a^2b}{4x^{4/3}}-\frac{a^3}{x}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 0.91

$$-\frac{20a^3x+45a^2bx^{2/3}+36ab^2\sqrt[3]{x}+10b^3}{20x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^(1/3))^3/x^2,x]
[Out] -1/20*(10*b^3 + 36*a*b^2*x^(1/3) + 45*a^2*b*x^(2/3) + 20*a^3*x)/x^2
```

fricas [A] time = 0.72, size = 35, normalized size = 0.78

$$-\frac{20\,a^3x+45\,a^2bx^{\frac{2}{3}}+36\,ab^2x^{\frac{1}{3}}+10\,b^3}{20\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^3/x^2,x, algorithm="fricas")
[Out] -1/20*(20*a^3*x + 45*a^2*b*x^(2/3) + 36*a*b^2*x^(1/3) + 10*b^3)/x^2
```

giac [A] time = 0.16, size = 35, normalized size = 0.78

$$-\frac{20\,a^3x+45\,a^2bx^{\frac{2}{3}}+36\,ab^2x^{\frac{1}{3}}+10\,b^3}{20\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^3/x^2,x, algorithm="giac")
[Out] -1/20*(20*a^3*x + 45*a^2*b*x^(2/3) + 36*a*b^2*x^(1/3) + 10*b^3)/x^2
```

maple [A] time = 0.01, size = 36, normalized size = 0.80

$$-\frac{a^3}{x}-\frac{9a^2b}{4x^{\frac{4}{3}}}-\frac{9a\,b^2}{5x^{\frac{5}{3}}}-\frac{b^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^(1/3))^3/x^2,x)
[Out] -1/2*b^3/x^2-9/5*a*b^2/x^(5/3)-9/4*a^2*b/x^(4/3)-a^3/x
```

maxima [A] time = 0.52, size = 47, normalized size = 1.04

$$-\frac{\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^6}{2\,b^3}+\frac{6\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^5a}{5\,b^3}-\frac{3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^4a^2}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^3/x^2,x, algorithm="maxima")
[Out] -1/2*(a + b/x^(1/3))^6/b^3 + 6/5*(a + b/x^(1/3))^5*a/b^3 - 3/4*(a + b/x^(1/
3))^4*a^2/b^3
```

mupad [B] time = 1.12, size = 35, normalized size = 0.78

$$-\frac{20\,a^3\,x+10\,b^3+36\,a\,b^2\,x^{1/3}+45\,a^2\,b\,x^{2/3}}{20\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^(1/3))^3/x^2,x)
[Out] -(20*a^3*x + 10*b^3 + 36*a*b^2*x^(1/3) + 45*a^2*b*x^(2/3))/(20*x^2)
```

sympy [**A**] time = 1.18, size = 41, normalized size = 0.91

$$-\frac{a^3}{x} - \frac{9a^2b}{4x^{\frac{4}{3}}} - \frac{9ab^2}{5x^{\frac{5}{3}}} - \frac{b^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**(1/3))**3/x**2,x)
```

```
[Out] -a**3/x - 9*a**2*b/(4*x**(4/3)) - 9*a*b**2/(5*x**(5/3)) - b**3/(2*x**2)
```

3.2419 $\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x^3}\,dx$

Optimal. Leaf size=47

$$-\frac{a^3}{2x^2}-\frac{9a^2b}{7x^{7/3}}-\frac{9ab^2}{8x^{8/3}}-\frac{b^3}{3x^3}$$

```
[Out] -1/3*b^3/x^3-9/8*a*b^2/x^(8/3)-9/7*a^2*b/x^(7/3)-1/2*a^3/x^2
```

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {263, 266, 43}

$$-\frac{9a^2b}{7x^{7/3}}-\frac{a^3}{2x^2}-\frac{9ab^2}{8x^{8/3}}-\frac{b^3}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^(1/3))^3/x^3,x]
```

```
[Out] -b^3/(3*x^3) - (9*a*b^2)/(8*x^(8/3)) - (9*a^2*b)/(7*x^(7/3)) - a^3/(2*x^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x^3}\,dx &= \int \frac{\left(b+a\sqrt[3]{x}\right)^3}{x^4}\,dx \\
&= 3\,\text{Subst}\left(\int \frac{(b+ax)^3}{x^{10}}\,dx, x, \sqrt[3]{x}\right) \\
&= 3\,\text{Subst}\left(\int \left(\frac{b^3}{x^{10}}+\frac{3ab^2}{x^9}+\frac{3a^2b}{x^8}+\frac{a^3}{x^7}\right)dx, x, \sqrt[3]{x}\right) \\
&= -\frac{b^3}{3x^3}-\frac{9ab^2}{8x^{8/3}}-\frac{9a^2b}{7x^{7/3}}-\frac{a^3}{2x^2}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 41, normalized size = 0.87

$$-\frac{84a^3x+216a^2bx^{2/3}+189ab^2\sqrt[3]{x}+56b^3}{168x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^(1/3))^3/x^3,x]
[Out] -1/168*(56*b^3 + 189*a*b^2*x^(1/3) + 216*a^2*b*x^(2/3) + 84*a^3*x)/x^3
```

fricas [A] time = 0.72, size = 35, normalized size = 0.74

$$-\frac{84\,a^3x + 216\,a^2bx^{\frac{2}{3}} + 189\,ab^2x^{\frac{1}{3}} + 56\,b^3}{168\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^3/x^3,x, algorithm="fricas")
[Out] -1/168*(84*a^3*x + 216*a^2*b*x^(2/3) + 189*a*b^2*x^(1/3) + 56*b^3)/x^3
```

giac [A] time = 0.18, size = 35, normalized size = 0.74

$$-\frac{84\,a^3x + 216\,a^2bx^{\frac{2}{3}} + 189\,ab^2x^{\frac{1}{3}} + 56\,b^3}{168\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^3/x^3,x, algorithm="giac")
[Out] -1/168*(84*a^3*x + 216*a^2*b*x^(2/3) + 189*a*b^2*x^(1/3) + 56*b^3)/x^3
```

maple [A] time = 0.01, size = 36, normalized size = 0.77

$$-\frac{a^3}{2x^2} - \frac{9a^2b}{7x^{\frac{7}{3}}} - \frac{9a\,b^2}{8x^{\frac{8}{3}}} - \frac{b^3}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^(1/3))^3/x^3,x)
[Out] -1/3*b^3/x^3-9/8*a*b^2/x^(8/3)-9/7*a^2*b/x^(7/3)-1/2*a^3/x^2
```

maxima [B] time = 0.46, size = 98, normalized size = 2.09

$$-\frac{\left(a + \frac{b}{x^{\frac{1}{3}}}\right)^9}{3\,b^6} + \frac{15\left(a + \frac{b}{x^{\frac{1}{3}}}\right)^8 a}{8\,b^6} - \frac{30\left(a + \frac{b}{x^{\frac{1}{3}}}\right)^7 a^2}{7\,b^6} + \frac{5\left(a + \frac{b}{x^{\frac{1}{3}}}\right)^6 a^3}{b^6} - \frac{3\left(a + \frac{b}{x^{\frac{1}{3}}}\right)^5 a^4}{b^6} + \frac{3\left(a + \frac{b}{x^{\frac{1}{3}}}\right)^4 a^5}{4\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^3/x^3,x, algorithm="maxima")
[Out] -1/3*(a + b/x^(1/3))^9/b^6 + 15/8*(a + b/x^(1/3))^8*a/b^6 - 30/7*(a + b/x^(
1/3))^7*a^2/b^6 + 5*(a + b/x^(1/3))^6*a^3/b^6 - 3*(a + b/x^(1/3))^5*a^4/b^6
 + 3/4*(a + b/x^(1/3))^4*a^5/b^6
```

mupad [B] time = 1.12, size = 35, normalized size = 0.74

$$-\frac{84\,a^3\,x + 56\,b^3 + 189\,a\,b^2\,x^{1/3} + 216\,a^2\,b\,x^{2/3}}{168\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^(1/3))^3/x^3,x)
[Out] -(84*a^3*x + 56*b^3 + 189*a*b^2*x^(1/3) + 216*a^2*b*x^(2/3))/(168*x^3)
```

sympy [**A**] time = 2.04, size = 44, normalized size = 0.94

$$-\frac{a^3}{2x^2}-\frac{9a^2b}{7x^{\frac{7}{3}}}-\frac{9ab^2}{8x^{\frac{8}{3}}}-\frac{b^3}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**(1/3))**3/x**3,x)
```

```
[Out] -a**3/(2*x**2) - 9*a**2*b/(7*x**(7/3)) - 9*a*b**2/(8*x**(8/3)) - b**3/(3*x*
**3)
```

3.2420 $\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x^4}\,dx$

Optimal. Leaf size=47

$$-\frac{a^3}{3x^3}-\frac{9a^2b}{10x^{10/3}}-\frac{9ab^2}{11x^{11/3}}-\frac{b^3}{4x^4}$$

```
[Out] -1/4*b^3/x^4-9/11*a*b^2/x^(11/3)-9/10*a^2*b/x^(10/3)-1/3*a^3/x^3
```

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {263, 266, 43}

$$-\frac{9a^2b}{10x^{10/3}}-\frac{a^3}{3x^3}-\frac{9ab^2}{11x^{11/3}}-\frac{b^3}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^(1/3))^3/x^4,x]
```

```
[Out] -b^3/(4*x^4) - (9*a*b^2)/(11*x^(11/3)) - (9*a^2*b)/(10*x^(10/3)) - a^3/(3*x
^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x^4}\,dx &= \int \frac{\left(b+a\sqrt[3]{x}\right)^3}{x^5}\,dx \\
&= 3\,\text{Subst}\left(\int \frac{(b+ax)^3}{x^{13}}\,dx, x, \sqrt[3]{x}\right) \\
&= 3\,\text{Subst}\left(\int \left(\frac{b^3}{x^{13}}+\frac{3ab^2}{x^{12}}+\frac{3a^2b}{x^{11}}+\frac{a^3}{x^{10}}\right)dx, x, \sqrt[3]{x}\right) \\
&= -\frac{b^3}{4x^4}-\frac{9ab^2}{11x^{11/3}}-\frac{9a^2b}{10x^{10/3}}-\frac{a^3}{3x^3}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 41, normalized size = 0.87

$$-\frac{220a^3x+594a^2bx^{2/3}+540ab^2\sqrt[3]{x}+165b^3}{660x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^(1/3))^3/x^4,x]
[Out] -1/660*(165*b^3 + 540*a*b^2*x^(1/3) + 594*a^2*b*x^(2/3) + 220*a^3*x)/x^4
```

fricas [A] time = 1.14, size = 35, normalized size = 0.74

$$-\frac{220\,a^3x + 594\,a^2bx^{\frac{2}{3}} + 540\,ab^2x^{\frac{1}{3}} + 165\,b^3}{660\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^3/x^4,x, algorithm="fricas")
[Out] -1/660*(220*a^3*x + 594*a^2*b*x^(2/3) + 540*a*b^2*x^(1/3) + 165*b^3)/x^4
```

giac [A] time = 0.16, size = 35, normalized size = 0.74

$$-\frac{220\,a^3x + 594\,a^2bx^{\frac{2}{3}} + 540\,ab^2x^{\frac{1}{3}} + 165\,b^3}{660\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^3/x^4,x, algorithm="giac")
[Out] -1/660*(220*a^3*x + 594*a^2*b*x^(2/3) + 540*a*b^2*x^(1/3) + 165*b^3)/x^4
```

maple [A] time = 0.00, size = 36, normalized size = 0.77

$$-\frac{a^3}{3x^3} - \frac{9a^2b}{10x^{\frac{10}{3}}} - \frac{9a\,b^2}{11x^{\frac{11}{3}}} - \frac{b^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^(1/3))^3/x^4,x)
[Out] -1/4*b^3/x^4-9/11*a*b^2/x^(11/3)-9/10*a^2*b/x^(10/3)-1/3*a^3/x^3
```

maxima [B] time = 0.49, size = 149, normalized size = 3.17

$$-\frac{\left(a + \frac{b}{x^{\frac{1}{3}}}\right)^{12}}{4\,b^9} + \frac{24\left(a + \frac{b}{x^{\frac{1}{3}}}\right)^{11}a}{11\,b^9} - \frac{42\left(a + \frac{b}{x^{\frac{1}{3}}}\right)^{10}a^2}{5\,b^9} + \frac{56\left(a + \frac{b}{x^{\frac{1}{3}}}\right)^{9}a^3}{3\,b^9} - \frac{105\left(a + \frac{b}{x^{\frac{1}{3}}}\right)^{8}a^4}{4\,b^9} + \frac{24\left(a + \frac{b}{x^{\frac{1}{3}}}\right)^{7}a^5}{b^9} - \frac{14\left(a + \frac{b}{x^{\frac{1}{3}}}\right)}{b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(1/3))^3/x^4,x, algorithm="maxima")
[Out] -1/4*(a + b/x^(1/3))^12/b^9 + 24/11*(a + b/x^(1/3))^11*a/b^9 - 42/5*(a + b/
x^(1/3))^10*a^2/b^9 + 56/3*(a + b/x^(1/3))^9*a^3/b^9 - 105/4*(a + b/x^(1/3)
)^8*a^4/b^9 + 24*(a + b/x^(1/3))^7*a^5/b^9 - 14*(a + b/x^(1/3))^6*a^6/b^9 +
 24/5*(a + b/x^(1/3))^5*a^7/b^9 - 3/4*(a + b/x^(1/3))^4*a^8/b^9
```

mupad [B] time = 0.04, size = 35, normalized size = 0.74

$$-\frac{220\,a^3\,x + 165\,b^3 + 540\,a\,b^2\,x^{1/3} + 594\,a^2\,b\,x^{2/3}}{660\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^(1/3))^3/x^4,x)
```

```
[Out] -(220*a^3*x + 165*b^3 + 540*a*b^2*x^(1/3) + 594*a^2*b*x^(2/3))/(660*x^4)
```

sympy [**A**] time = 3.47, size = 44, normalized size = 0.94

$$-\frac{a^3}{3x^3}-\frac{9a^2b}{10x^{\frac{10}{3}}}-\frac{9ab^2}{11x^{\frac{11}{3}}}-\frac{b^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**(1/3))**3/x**4,x)
```

```
[Out] -a**3/(3*x**3) - 9*a**2*b/(10*x**(10/3)) - 9*a*b**2/(11*x**(11/3)) - b**3/(
4*x**4)
```

3.2421 $\int \frac{x^2}{a+\frac{b}{\sqrt[3]{x}}}\,dx$

Optimal. Leaf size=136

$$-\frac{3b^9\log\left(a\sqrt[3]{x}+b\right)}{a^{10}}+\frac{3b^8\sqrt[3]{x}}{a^9}-\frac{3b^7x^{2/3}}{2a^8}+\frac{b^6x}{a^7}-\frac{3b^5x^{4/3}}{4a^6}+\frac{3b^4x^{5/3}}{5a^5}-\frac{b^3x^2}{2a^4}+\frac{3b^2x^{7/3}}{7a^3}-\frac{3bx^{8/3}}{8a^2}+\frac{x^3}{3a}$$

```
[Out] 3*b^8*x^(1/3)/a^9-3/2*b^7*x^(2/3)/a^8+b^6*x/a^7-3/4*b^5*x^(4/3)/a^6+3/5*b^4
*x^(5/3)/a^5-1/2*b^3*x^2/a^4+3/7*b^2*x^(7/3)/a^3-3/8*b*x^(8/3)/a^2+1/3*x^3/
a-3*b^9*ln(b+a*x^(1/3))/a^10
```

Rubi [A] time = 0.09, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {263, 266, 43}

$$-\frac{3b^7x^{2/3}}{2a^8}-\frac{3b^5x^{4/3}}{4a^6}+\frac{3b^4x^{5/3}}{5a^5}-\frac{b^3x^2}{2a^4}+\frac{3b^2x^{7/3}}{7a^3}+\frac{3b^8\sqrt[3]{x}}{a^9}+\frac{b^6x}{a^7}-\frac{3b^9\log\left(a\sqrt[3]{x}+b\right)}{a^{10}}-\frac{3bx^{8/3}}{8a^2}+\frac{x^3}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b/x^(1/3)),x]
```

```
[Out] (3*b^8*x^(1/3))/a^9 - (3*b^7*x^(2/3))/(2*a^8) + (b^6*x)/a^7 - (3*b^5*x^(4/3
))/(4*a^6) + (3*b^4*x^(5/3))/(5*a^5) - (b^3*x^2)/(2*a^4) + (3*b^2*x^(7/3))/
(7*a^3) - (3*b*x^(8/3))/(8*a^2) + x^3/(3*a) - (3*b^9*Log[b + a*x^(1/3)])/a^
10
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{a+\frac{b}{\sqrt[3]{x}}}\,dx &= \int \frac{x^{7/3}}{b+a\sqrt[3]{x}}\,dx \\
&= 3\,\text{Subst}\left(\int \frac{x^9}{b+ax}\,dx, x, \sqrt[3]{x}\right) \\
&= 3\,\text{Subst}\left(\int \left(\frac{b^8}{a^9}-\frac{b^7x}{a^8}+\frac{b^6x^2}{a^7}-\frac{b^5x^3}{a^6}+\frac{b^4x^4}{a^5}-\frac{b^3x^5}{a^4}+\frac{b^2x^6}{a^3}-\frac{bx^7}{a^2}+\frac{x^8}{a}-\frac{b^9}{a^9(b+ax)}\right)dx, x, \sqrt[3]{x}\right) \\
&= \frac{3b^8\sqrt[3]{x}}{a^9}-\frac{3b^7x^{2/3}}{2a^8}+\frac{b^6x}{a^7}-\frac{3b^5x^{4/3}}{4a^6}+\frac{3b^4x^{5/3}}{5a^5}-\frac{b^3x^2}{2a^4}+\frac{3b^2x^{7/3}}{7a^3}-\frac{3bx^{8/3}}{8a^2}+\frac{x^3}{3a}-\frac{3b^9\log\left(b+a\sqrt[3]{x}\right.}{a^{10}}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 146, normalized size = 1.07

$$-\frac{3b^9\log\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^{10}}-\frac{b^9\log(x)}{a^{10}}+\frac{3b^8\sqrt[3]{x}}{a^9}-\frac{3b^7x^{2/3}}{2a^8}+\frac{b^6x}{a^7}-\frac{3b^5x^{4/3}}{4a^6}+\frac{3b^4x^{5/3}}{5a^5}-\frac{b^3x^2}{2a^4}+\frac{3b^2x^{7/3}}{7a^3}-\frac{3bx^{8/3}}{8a^2}+\frac{x^3}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b/x^(1/3)),x]
```

```
[Out] (3*b^8*x^(1/3))/a^9 - (3*b^7*x^(2/3))/(2*a^8) + (b^6*x)/a^7 - (3*b^5*x^(4/3
))/(4*a^6) + (3*b^4*x^(5/3))/(5*a^5) - (b^3*x^2)/(2*a^4) + (3*b^2*x^(7/3))/
(7*a^3) - (3*b*x^(8/3))/(8*a^2) + x^3/(3*a) - (3*b^9*Log[a + b/x^(1/3)])/a^
10 - (b^9*Log[x])/a^10
```

fricas [A] time = 1.03, size = 111, normalized size = 0.82

$$\frac{280\,a^9x^3-420\,a^6b^3x^2+840\,a^3b^6x-2520\,b^9\log\left(ax^{\frac{1}{3}}+b\right)-63\left(5\,a^8bx^2-8\,a^5b^4x+20\,a^2b^7\right)x^{\frac{2}{3}}+90\left(4\,a^7b^2x\right.}{840\,a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^(1/3)),x, algorithm="fricas")
```

```
[Out] 1/840*(280*a^9*x^3 - 420*a^6*b^3*x^2 + 840*a^3*b^6*x - 2520*b^9*log(a*x^(1/
3) + b) - 63*(5*a^8*b*x^2 - 8*a^5*b^4*x + 20*a^2*b^7)*x^(2/3) + 90*(4*a^7*b
^2*x^2 - 7*a^4*b^5*x + 28*a*b^8)*x^(1/3))/a^10
```

giac [A] time = 0.15, size = 111, normalized size = 0.82

$$-\frac{3\,b^9\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{a^{10}}+\frac{280\,a^8x^3-315\,a^7bx^{\frac{8}{3}}+360\,a^6b^2x^{\frac{7}{3}}-420\,a^5b^3x^2+504\,a^4b^4x^{\frac{5}{3}}-630\,a^3b^5x^{\frac{4}{3}}+840\,a^2b}{840\,a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^(1/3)),x, algorithm="giac")
```

```
[Out] -3*b^9*log(abs(a*x^(1/3) + b))/a^10 + 1/840*(280*a^8*x^3 - 315*a^7*b*x^(8/3
) + 360*a^6*b^2*x^(7/3) - 420*a^5*b^3*x^2 + 504*a^4*b^4*x^(5/3) - 630*a^3*b
^5*x^(4/3) + 840*a^2*b^6*x - 1260*a*b^7*x^(2/3) + 2520*b^8*x^(1/3))/a^9
```

maple [A] time = 0.01, size = 109, normalized size = 0.80

$$\frac{x^3}{3a}-\frac{3b\,x^{\frac{8}{3}}}{8a^2}+\frac{3b^2x^{\frac{7}{3}}}{7a^3}-\frac{b^3x^2}{2a^4}+\frac{3b^4x^{\frac{5}{3}}}{5a^5}-\frac{3b^5x^{\frac{4}{3}}}{4a^6}+\frac{b^6x}{a^7}-\frac{3b^9\ln\left(a\,x^{\frac{1}{3}}+b\right)}{a^{10}}-\frac{3b^7x^{\frac{2}{3}}}{2a^8}+\frac{3b^8x^{\frac{1}{3}}}{a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b/x^(1/3)),x)
```

```
[Out] 3*b^8*x^(1/3)/a^9-3/2*b^7*x^(2/3)/a^8+b^6*x/a^7-3/4*b^5*x^(4/3)/a^6+3/5*b^4
*x^(5/3)/a^5-1/2/a^4*b^3*x^2+3/7*b^2*x^(7/3)/a^3-3/8*b*x^(8/3)/a^2+1/3/a*x^
3-3*b^9*ln(a*x^(1/3)+b)/a^10
```

maxima [A] time = 0.49, size = 122, normalized size = 0.90

$$-\frac{3\,b^9\log\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{a^{10}}-\frac{b^9\log(x)}{a^{10}}+\frac{\left(280\,a^8-\frac{315\,a^7b}{x^{\frac{1}{3}}}+\frac{360\,a^6b^2}{x^{\frac{2}{3}}}-\frac{420\,a^5b^3}{x}+\frac{504\,a^4b^4}{x^{\frac{4}{3}}}-\frac{630\,a^3b^5}{x^{\frac{5}{3}}}+\frac{840\,a^2b^6}{x^2}-\frac{1260\,ab^7}{x^{\frac{7}{3}}}+\right.}{840\,a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^(1/3)),x, algorithm="maxima")
```

```
[Out] -3*b^9*log(a + b/x^(1/3))/a^10 - b^9*log(x)/a^10 + 1/840*(280*a^8 - 315*a^7
*b/x^(1/3) + 360*a^6*b^2/x^(2/3) - 420*a^5*b^3/x + 504*a^4*b^4/x^(4/3) - 63
0*a^3*b^5/x^(5/3) + 840*a^2*b^6/x^2 - 1260*a*b^7/x^(7/3) + 2520*b^8/x^(8/3)
)*x^3/a^9
```

mupad [B] time = 0.05, size = 108, normalized size = 0.79

$$\frac{x^3}{3\,a}-\frac{3\,b\,x^{8/3}}{8\,a^2}+\frac{b^6\,x}{a^7}-\frac{3\,b^9\,\ln\left(b+a\,x^{1/3}\right)}{a^{10}}-\frac{b^3\,x^2}{2\,a^4}+\frac{3\,b^2\,x^{7/3}}{7\,a^3}+\frac{3\,b^4\,x^{5/3}}{5\,a^5}-\frac{3\,b^5\,x^{4/3}}{4\,a^6}-\frac{3\,b^7\,x^{2/3}}{2\,a^8}+\frac{3\,b^8\,x^{1/3}}{a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b/x^(1/3)),x)
```

```
[Out] x^3/(3*a) - (3*b*x^(8/3))/(8*a^2) + (b^6*x)/a^7 - (3*b^9*log(b + a*x^(1/3))
)/a^10 - (b^3*x^2)/(2*a^4) + (3*b^2*x^(7/3))/(7*a^3) + (3*b^4*x^(5/3))/(5*a
^5) - (3*b^5*x^(4/3))/(4*a^6) - (3*b^7*x^(2/3))/(2*a^8) + (3*b^8*x^(1/3))/a
^9
```

sympy [A] time = 4.14, size = 143, normalized size = 1.05

$$\begin{cases} \frac{x^3}{3a} - \frac{3bx^{\frac{8}{3}}}{8a^2} + \frac{3b^2x^{\frac{7}{3}}}{7a^3} - \frac{b^3x^2}{2a^4} + \frac{3b^4x^{\frac{5}{3}}}{5a^5} - \frac{3b^5x^{\frac{4}{3}}}{4a^6} + \frac{b^6x}{a^7} - \frac{3b^7x^{\frac{2}{3}}}{2a^8} + \frac{3b^8\sqrt[3]{x}}{a^9} - \frac{3b^9\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{a^{10}} & \text{for } a \neq 0 \\ \frac{3x^{\frac{10}{3}}}{10b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b/x**(1/3)),x)
```

```
[Out] Piecewise((x**3/(3*a) - 3*b*x**(8/3)/(8*a**2) + 3*b**2*x**(7/3)/(7*a**3) -
b**3*x**2/(2*a**4) + 3*b**4*x**(5/3)/(5*a**5) - 3*b**5*x**(4/3)/(4*a**6) +
b**6*x/a**7 - 3*b**7*x**(2/3)/(2*a**8) + 3*b**8*x**(1/3)/a**9 - 3*b**9*log(
x**(1/3) + b/a)/a**10, Ne(a, 0)), (3*x**(10/3)/(10*b), True))
```

3.2422 $\int \frac{x}{a+\frac{b}{\sqrt[3]{x}}}\,dx$

Optimal. Leaf size=94

$$\frac{3b^6\log\left(a\sqrt[3]{x}+b\right)}{a^7}-\frac{3b^5\sqrt[3]{x}}{a^6}+\frac{3b^4x^{2/3}}{2a^5}-\frac{b^3x}{a^4}+\frac{3b^2x^{4/3}}{4a^3}-\frac{3bx^{5/3}}{5a^2}+\frac{x^2}{2a}$$

```
[Out] -3*b^5*x^(1/3)/a^6+3/2*b^4*x^(2/3)/a^5-b^3*x/a^4+3/4*b^2*x^(4/3)/a^3-3/5*b*
x^(5/3)/a^2+1/2*x^2/a+3*b^6*ln(b+a*x^(1/3))/a^7
```

Rubi [A] time = 0.05, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 266, 43}

$$\frac{3b^4x^{2/3}}{2a^5}+\frac{3b^2x^{4/3}}{4a^3}-\frac{3b^5\sqrt[3]{x}}{a^6}-\frac{b^3x}{a^4}+\frac{3b^6\log\left(a\sqrt[3]{x}+b\right)}{a^7}-\frac{3bx^{5/3}}{5a^2}+\frac{x^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b/x^(1/3)),x]
```

```
[Out] (-3*b^5*x^(1/3))/a^6 + (3*b^4*x^(2/3))/(2*a^5) - (b^3*x)/a^4 + (3*b^2*x^(4/
3))/(4*a^3) - (3*b*x^(5/3))/(5*a^2) + x^2/(2*a) + (3*b^6*Log[b + a*x^(1/3)]
)/a^7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{a+\frac{b}{\sqrt[3]{x}}}\,dx &= \int \frac{x^{4/3}}{b+a\sqrt[3]{x}}\,dx\\ &= 3\,\text{Subst}\left(\int \frac{x^6}{b+ax}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(-\frac{b^5}{a^6}+\frac{b^4x}{a^5}-\frac{b^3x^2}{a^4}+\frac{b^2x^3}{a^3}-\frac{bx^4}{a^2}+\frac{x^5}{a}+\frac{b^6}{a^6(b+ax)}\right)dx, x, \sqrt[3]{x}\right)\\ &= -\frac{3b^5\sqrt[3]{x}}{a^6}+\frac{3b^4x^{2/3}}{2a^5}-\frac{b^3x}{a^4}+\frac{3b^2x^{4/3}}{4a^3}-\frac{3bx^{5/3}}{5a^2}+\frac{x^2}{2a}+\frac{3b^6\log\left(b+a\sqrt[3]{x}\right)}{a^7}\end{aligned}$$

Mathematica [A] time = 0.04, size = 95, normalized size = 1.01

$$\frac{10a^6x^2 - 12a^5bx^{5/3} + 15a^4b^2x^{4/3} - 20a^3b^3x + 30a^2b^4x^{2/3} + 60b^6\log\left(a + \frac{b}{\sqrt[3]{x}}\right) - 60ab^5\sqrt[3]{x} + 20b^6\log(x)}{20a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b/x^(1/3)),x]
```

```
[Out] (-60*a*b^5*x^(1/3) + 30*a^2*b^4*x^(2/3) - 20*a^3*b^3*x + 15*a^4*b^2*x^(4/3)
 - 12*a^5*b*x^(5/3) + 10*a^6*x^2 + 60*b^6*Log[a + b/x^(1/3)] + 20*b^6*Log[x
])/(20*a^7)
```

fricas [A] time = 0.85, size = 77, normalized size = 0.82

$$\frac{10\,a^6x^2 - 20\,a^3b^3x + 60\,b^6\log\left(ax^{\frac{1}{3}} + b\right) - 6\left(2\,a^5bx - 5\,a^2b^4\right)x^{\frac{2}{3}} + 15\left(a^4b^2x - 4\,ab^5\right)x^{\frac{1}{3}}}{20\,a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^(1/3)),x, algorithm="fricas")
```

```
[Out] 1/20*(10*a^6*x^2 - 20*a^3*b^3*x + 60*b^6*log(a*x^(1/3) + b) - 6*(2*a^5*b*x
 - 5*a^2*b^4)*x^(2/3) + 15*(a^4*b^2*x - 4*a*b^5)*x^(1/3))/a^7
```

giac [A] time = 0.21, size = 78, normalized size = 0.83

$$\frac{3\,b^6\log\left(\left|ax^{\frac{1}{3}} + b\right|\right)}{a^7} + \frac{10\,a^5x^2 - 12\,a^4bx^{\frac{5}{3}} + 15\,a^3b^2x^{\frac{4}{3}} - 20\,a^2b^3x + 30\,ab^4x^{\frac{2}{3}} - 60\,b^5x^{\frac{1}{3}}}{20\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^(1/3)),x, algorithm="giac")
```

```
[Out] 3*b^6*log(abs(a*x^(1/3) + b))/a^7 + 1/20*(10*a^5*x^2 - 12*a^4*b*x^(5/3) + 1
5*a^3*b^2*x^(4/3) - 20*a^2*b^3*x + 30*a*b^4*x^(2/3) - 60*b^5*x^(1/3))/a^6
```

maple [A] time = 0.00, size = 77, normalized size = 0.82

$$\frac{x^2}{2a} - \frac{3b\,x^{\frac{5}{3}}}{5a^2} + \frac{3b^2x^{\frac{4}{3}}}{4a^3} - \frac{b^3x}{a^4} + \frac{3b^6\ln\left(a\,x^{\frac{1}{3}} + b\right)}{a^7} + \frac{3b^4x^{\frac{2}{3}}}{2a^5} - \frac{3b^5x^{\frac{1}{3}}}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b/x^(1/3)),x)
```

```
[Out] -3*b^5*x^(1/3)/a^6+3/2*b^4*x^(2/3)/a^5-1/a^4*b^3*x+3/4*b^2*x^(4/3)/a^3-3/5*
b*x^(5/3)/a^2+1/2/a*x^2+3*b^6*ln(a*x^(1/3)+b)/a^7
```

maxima [A] time = 0.56, size = 88, normalized size = 0.94

$$\frac{3\,b^6\log\left(a + \frac{b}{x^{\frac{1}{3}}}\right)}{a^7} + \frac{b^6\log(x)}{a^7} + \frac{\left(10\,a^5 - \frac{12\,a^4b}{x^{\frac{1}{3}}} + \frac{15\,a^3b^2}{x^{\frac{2}{3}}} - \frac{20\,a^2b^3}{x} + \frac{30\,ab^4}{x^{\frac{4}{3}}} - \frac{60\,b^5}{x^{\frac{5}{3}}}\right)x^2}{20\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^(1/3)),x, algorithm="maxima")
```

```
[Out] 3*b^6*log(a + b/x^(1/3))/a^7 + b^6*log(x)/a^7 + 1/20*(10*a^5 - 12*a^4*b/x^(
1/3) + 15*a^3*b^2/x^(2/3) - 20*a^2*b^3/x + 30*a*b^4/x^(4/3) - 60*b^5/x^(5/3
))*x^2/a^6
```

mupad [B] time = 0.03, size = 76, normalized size = 0.81

$$\frac{x^2}{2\,a}-\frac{b^3\,x}{a^4}-\frac{3\,b\,x^{5/3}}{5\,a^2}+\frac{3\,b^6\,\ln\left(b+a\,x^{1/3}\right)}{a^7}+\frac{3\,b^2\,x^{4/3}}{4\,a^3}+\frac{3\,b^4\,x^{2/3}}{2\,a^5}-\frac{3\,b^5\,x^{1/3}}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b/x^(1/3)),x)
```

```
[Out] x^2/(2*a) - (b^3*x)/a^4 - (3*b*x^(5/3))/(5*a^2) + (3*b^6*log(b + a*x^(1/3))
)/a^7 + (3*b^2*x^(4/3))/(4*a^3) + (3*b^4*x^(2/3))/(2*a^5) - (3*b^5*x^(1/3))
/a^6
```

sympy [A] time = 1.26, size = 100, normalized size = 1.06

$$\begin{cases}\frac{x^2}{2a}-\frac{3bx^{\frac{5}{3}}}{5a^2}+\frac{3b^2x^{\frac{4}{3}}}{4a^3}-\frac{b^3x}{a^4}+\frac{3b^4x^{\frac{2}{3}}}{2a^5}-\frac{3b^5\sqrt[3]{x}}{a^6}+\frac{3b^6\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{a^7} & \text{for } a\neq 0\\ \frac{3x^{\frac{7}{3}}}{7b} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x**(1/3)),x)
```

```
[Out] Piecewise((x**2/(2*a) - 3*b*x**(5/3)/(5*a**2) + 3*b**2*x**(4/3)/(4*a**3) -
b**3*x/a**4 + 3*b**4*x**(2/3)/(2*a**5) - 3*b**5*x**(1/3)/a**6 + 3*b**6*log(
x**(1/3) + b/a)/a**7, Ne(a, 0)), (3*x**(7/3)/(7*b), True))
```

3.2423 $\int \frac{1}{a+\frac{b}{\sqrt[3]{x}}}\,dx$

Optimal. Leaf size=60

$$-\frac{3b^3\log\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^4}-\frac{b^3\log(x)}{a^4}+\frac{3b^2\sqrt[3]{x}}{a^3}-\frac{3bx^{2/3}}{2a^2}+\frac{x}{a}$$

```
[Out] 3*b^2*x^(1/3)/a^3-3/2*b*x^(2/3)/a^2+x/a-3*b^3*ln(a+b/x^(1/3))/a^4-b^3*ln(x)
/a^4
```

Rubi [A] time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {190, 44}

$$\frac{3b^2\sqrt[3]{x}}{a^3}-\frac{3b^3\log\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^4}-\frac{b^3\log(x)}{a^4}-\frac{3bx^{2/3}}{2a^2}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^(1/3))^(-1),x]
```

```
[Out] (3*b^2*x^(1/3))/a^3 - (3*b*x^(2/3))/(2*a^2) + x/a - (3*b^3*Log[a + b/x^(1/3
)])/a^4 - (b^3*Log[x])/a^4
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
 - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+\frac{b}{\sqrt[3]{x}}}\,dx &= -\left(3\,\text{Subst}\left(\int \frac{1}{x^4(a+bx)}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)\\ &= -\left(3\,\text{Subst}\left(\int \left(\frac{1}{ax^4}-\frac{b}{a^2x^3}+\frac{b^2}{a^3x^2}-\frac{b^3}{a^4x}+\frac{b^4}{a^4(a+bx)}\right)dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)\\ &= \frac{3b^2\sqrt[3]{x}}{a^3}-\frac{3bx^{2/3}}{2a^2}+\frac{x}{a}-\frac{3b^3\log\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^4}-\frac{b^3\log(x)}{a^4}\end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.83

$$-\frac{3b^3\log\left(a\sqrt[3]{x}+b\right)}{a^4}+\frac{3b^2\sqrt[3]{x}}{a^3}-\frac{3bx^{2/3}}{2a^2}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^(1/3))^(-1),x]
```

```
[Out] (3*b^2*x^(1/3))/a^3 - (3*b*x^(2/3))/(2*a^2) + x/a - (3*b^3*Log[b + a*x^(1/3
)])/a^4
```

fricas [A] time = 0.69, size = 43, normalized size = 0.72

$$\frac{2\,a^3x - 6\,b^3\log\left(ax^{\frac{1}{3}} + b\right) - 3\,a^2bx^{\frac{2}{3}} + 6\,ab^2x^{\frac{1}{3}}}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^3*x - 6*b^3*log(a*x^(1/3) + b) - 3*a^2*b*x^(2/3) + 6*a*b^2*x^(1/3)
)/a^4
```

giac [A] time = 0.16, size = 45, normalized size = 0.75

$$-\frac{3\,b^3\log\left(\left|ax^{\frac{1}{3}} + b\right|\right)}{a^4} + \frac{2\,a^2x - 3\,abx^{\frac{2}{3}} + 6\,b^2x^{\frac{1}{3}}}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3)),x, algorithm="giac")
```

```
[Out] -3*b^3*log(abs(a*x^(1/3) + b))/a^4 + 1/2*(2*a^2*x - 3*a*b*x^(2/3) + 6*b^2*x
^(1/3))/a^3
```

maple [A] time = 0.00, size = 43, normalized size = 0.72

$$\frac{x}{a} - \frac{3b^3\ln\left(a\,x^{\frac{1}{3}} + b\right)}{a^4} - \frac{3b\,x^{\frac{2}{3}}}{2a^2} + \frac{3b^2x^{\frac{1}{3}}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^(1/3)),x)
```

```
[Out] 1/a*x-3/2*b*x^(2/3)/a^2+3*b^2*x^(1/3)/a^3-3/a^4*b^3*ln(a*x^(1/3)+b)
```

maxima [A] time = 0.45, size = 54, normalized size = 0.90

$$-\frac{3\,b^3\log\left(a + \frac{b}{x^{\frac{1}{3}}}\right)}{a^4} - \frac{b^3\log(x)}{a^4} + \frac{\left(2\,a^2 - \frac{3\,ab}{x^{\frac{1}{3}}} + \frac{6\,b^2}{x^{\frac{2}{3}}}\right)x}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3)),x, algorithm="maxima")
```

```
[Out] -3*b^3*log(a + b/x^(1/3))/a^4 - b^3*log(x)/a^4 + 1/2*(2*a^2 - 3*a*b/x^(1/3)
 + 6*b^2/x^(2/3))*x/a^3
```

mupad [B] time = 0.04, size = 42, normalized size = 0.70

$$\frac{x}{a} - \frac{3\,b\,x^{2/3}}{2\,a^2} - \frac{3\,b^3\,\ln\left(b + a\,x^{1/3}\right)}{a^4} + \frac{3\,b^2\,x^{1/3}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x^(1/3)),x)
```

```
[Out] x/a - (3*b*x^(2/3))/(2*a^2) - (3*b^3*log(b + a*x^(1/3)))/a^4 + (3*b^2*x^(1/
3))/a^3
```

sympy [**A**] time = 0.41, size = 58, normalized size = 0.97

$$\begin{cases} \frac{x}{a} - \frac{3bx^{\frac{2}{3}}}{2a^2} + \frac{3b^2\sqrt[3]{x}}{a^3} - \frac{3b^3\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{a^4} & \text{for } a \neq 0 \\ \frac{3x^{\frac{4}{3}}}{4b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**(1/3)),x)
```

```
[Out] Piecewise((x/a - 3*b*x**(2/3)/(2*a**2) + 3*b**2*x**(1/3)/a**3 - 3*b**3*log(
x**(1/3) + b/a)/a**4, Ne(a, 0)), (3*x**(4/3)/(4*b), True))
```

3.2424 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x}\,dx$

Optimal. Leaf size=15

$$\frac{3\log\left(a\sqrt[3]{x}+b\right)}{a}$$

```
[Out] 3*ln(b+a*x^(1/3))/a
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {263, 260}

$$\frac{3\log\left(a\sqrt[3]{x}+b\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^(1/3))*x),x]
```

```
[Out] (3*Log[b + a*x^(1/3)])/a
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x}\,dx &= \int \frac{1}{\left(b+a\sqrt[3]{x}\right)x^{2/3}}\,dx\\ &= \frac{3\log\left(b+a\sqrt[3]{x}\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{3\log\left(a\sqrt[3]{x}+b\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^(1/3))*x),x]
```

```
[Out] (3*Log[b + a*x^(1/3)])/a
```

fricas [A] time = 0.89, size = 13, normalized size = 0.87

$$\frac{3\,\log\left(ax^{\frac{1}{3}}+b\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))/x,x, algorithm="fricas")
[Out] 3*log(a*x^(1/3) + b)/a
```

giac [A] time = 0.16, size = 14, normalized size = 0.93

$$\frac{3\,\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))/x,x, algorithm="giac")
[Out] 3*log(abs(a*x^(1/3) + b))/a
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{3\ln\left(a\,x^{\frac{1}{3}}+b\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^(1/3))/x,x)
[Out] 3*ln(a*x^(1/3)+b)/a
```

maxima [A] time = 0.50, size = 20, normalized size = 1.33

$$\frac{3\,\log\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{a}+\frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))/x,x, algorithm="maxima")
[Out] 3*log(a + b/x^(1/3))/a + log(x)/a
```

mupad [B] time = 0.04, size = 13, normalized size = 0.87

$$\frac{3\,\ln\left(b+a\,x^{1/3}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b/x^(1/3))),x)
[Out] (3*log(b + a*x^(1/3)))/a
```

sympy [A] time = 0.63, size = 20, normalized size = 1.33

$$\begin{cases} \frac{3\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{a} & \text{for } a \neq 0 \\ \frac{3\sqrt[3]{x}}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**(1/3))/x,x)
[Out] Piecewise((3*log(x**(1/3) + b/a)/a, Ne(a, 0)), (3*x**(1/3)/b, True))
```

3.2425 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x^2}\,dx$

Optimal. Leaf size=51

$$-\frac{3a^2\log\left(a\sqrt[3]{x}+b\right)}{b^3}+\frac{a^2\log(x)}{b^3}+\frac{3a}{b^2\sqrt[3]{x}}-\frac{3}{2bx^{2/3}}$$

```
[Out] -3/2/b/x^(2/3)+3*a/b^2/x^(1/3)-3*a^2*ln(b+a*x^(1/3))/b^3+a^2*ln(x)/b^3
```

Rubi [A] time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {263, 266, 44}

$$-\frac{3a^2\log\left(a\sqrt[3]{x}+b\right)}{b^3}+\frac{a^2\log(x)}{b^3}+\frac{3a}{b^2\sqrt[3]{x}}-\frac{3}{2bx^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^(1/3))*x^2),x]
```

```
[Out] -3/(2*b*x^(2/3)) + (3*a)/(b^2*x^(1/3)) - (3*a^2*Log[b + a*x^(1/3)])/b^3 + (
a^2*Log[x])/b^3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x^2}\,dx &= \int \frac{1}{\left(b+a\sqrt[3]{x}\right)x^{5/3}}\,dx \\
&= 3\,\text{Subst}\left(\int \frac{1}{x^3(b+ax)}\,dx, x, \sqrt[3]{x}\right) \\
&= 3\,\text{Subst}\left(\int \left(\frac{1}{bx^3}-\frac{a}{b^2x^2}+\frac{a^2}{b^3x}-\frac{a^3}{b^3(b+ax)}\right)dx, x, \sqrt[3]{x}\right) \\
&= -\frac{3}{2bx^{2/3}}+\frac{3a}{b^2\sqrt[3]{x}}-\frac{3a^2\log\left(b+a\sqrt[3]{x}\right)}{b^3}+\frac{a^2\log(x)}{b^3}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 48, normalized size = 0.94

$$\frac{-6a^2\log\left(a\sqrt[3]{x}+b\right)+2a^2\log(x)-\frac{3b\left(b-2a\sqrt[3]{x}\right)}{x^{2/3}}}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^(1/3))*x^2),x]
```

```
[Out] ((-3*b*(b - 2*a*x^(1/3)))/x^(2/3) - 6*a^2*Log[b + a*x^(1/3)] + 2*a^2*Log[x]
)/(2*b^3)
```

fricas [A] time = 0.87, size = 47, normalized size = 0.92

$$-\frac{3\left(2\,a^2x\log\left(ax^{\frac{1}{3}}+b\right)-2\,a^2x\log\left(x^{\frac{1}{3}}\right)-2\,abx^{\frac{2}{3}}+b^2x^{\frac{1}{3}}\right)}{2\,b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))/x^2,x, algorithm="fricas")
```

```
[Out] -3/2*(2*a^2*x*log(a*x^(1/3) + b) - 2*a^2*x*log(x^(1/3)) - 2*a*b*x^(2/3) + b
^2*x^(1/3))/(b^3*x)
```

giac [A] time = 0.16, size = 49, normalized size = 0.96

$$-\frac{3\,a^2\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{b^3}+\frac{a^2\log\left(|x|\right)}{b^3}+\frac{3\left(2\,abx^{\frac{1}{3}}-b^2\right)}{2\,b^3x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))/x^2,x, algorithm="giac")
```

```
[Out] -3*a^2*log(abs(a*x^(1/3) + b))/b^3 + a^2*log(abs(x))/b^3 + 3/2*(2*a*b*x^(1/
3) - b^2)/(b^3*x^(2/3))
```

maple [A] time = 0.01, size = 44, normalized size = 0.86

$$\frac{a^2\ln(x)}{b^3}-\frac{3a^2\ln\left(a\,x^{\frac{1}{3}}+b\right)}{b^3}+\frac{3a}{b^2x^{\frac{1}{3}}}-\frac{3}{2b\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^(1/3))/x^2,x)
```

```
[Out] -3/2/b/x^(2/3)+3*a/b^2/x^(1/3)-3*a^2*ln(a*x^(1/3)+b)/b^3+a^2/b^3*ln(x)
```

maxima [A] time = 0.57, size = 44, normalized size = 0.86

$$-\frac{3\,a^2\log\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{b^3}-\frac{3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^2}{2\,b^3}+\frac{6\left(a+\frac{b}{x^{\frac{1}{3}}}\right)a}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))/x^2,x, algorithm="maxima")
```

```
[Out] -3*a^2*log(a + b/x^(1/3))/b^3 - 3/2*(a + b/x^(1/3))^2/b^3 + 6*(a + b/x^(1/3
))*a/b^3
```

mupad [B] time = 1.16, size = 41, normalized size = 0.80

$$-\frac{\frac{3}{2\,b}-\frac{3\,a\,x^{1/3}}{b^2}}{x^{2/3}}-\frac{6\,a^2\,\mathrm{atanh}\left(\frac{2\,a\,x^{1/3}}{b}+1\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b/x^(1/3))),x)
```

```
[Out] - (3/(2*b) - (3*a*x^(1/3))/b^2)/x^(2/3) - (6*a^2*atanh((2*a*x^(1/3))/b + 1)
)/b^3
```

sympy [A] time = 1.82, size = 73, normalized size = 1.43

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{2}{3}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{1}{ax} & \text{for } b = 0 \\ -\frac{3}{2bx^{\frac{2}{3}}} & \text{for } a = 0 \\ \frac{a^2\log(x)}{b^3} - \frac{3a^2\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{b^3} + \frac{3a}{b^2\sqrt[3]{x}} - \frac{3}{2bx^{\frac{2}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**(1/3))/x**2,x)
```

```
[Out] Piecewise((zoo/x**(2/3), Eq(a, 0) & Eq(b, 0)), (-1/(a*x), Eq(b, 0)), (-3/(2
*b*x**(2/3)), Eq(a, 0)), (a**2*log(x)/b**3 - 3*a**2*log(x**(1/3) + b/a)/b**
3 + 3*a/(b**2*x**(1/3)) - 3/(2*b*x**(2/3)), True))
```

3.2426 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x^3}\,dx$

Optimal. Leaf size=93

$$\frac{3a^5\log\left(a\sqrt[3]{x}+b\right)}{b^6}-\frac{a^5\log(x)}{b^6}-\frac{3a^4}{b^5\sqrt[3]{x}}+\frac{3a^3}{2b^4x^{2/3}}-\frac{a^2}{b^3x}+\frac{3a}{4b^2x^{4/3}}-\frac{3}{5bx^{5/3}}$$

```
[Out] -3/5/b/x^(5/3)+3/4*a/b^2/x^(4/3)-a^2/b^3/x+3/2*a^3/b^4/x^(2/3)-3*a^4/b^5/x^
(1/3)+3*a^5*ln(b+a*x^(1/3))/b^6-a^5*ln(x)/b^6
```

Rubi [A] time = 0.05, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {263, 266, 44}

$$\frac{3a^3}{2b^4x^{2/3}}-\frac{3a^4}{b^5\sqrt[3]{x}}-\frac{a^2}{b^3x}+\frac{3a^5\log\left(a\sqrt[3]{x}+b\right)}{b^6}-\frac{a^5\log(x)}{b^6}+\frac{3a}{4b^2x^{4/3}}-\frac{3}{5bx^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^(1/3))*x^3),x]
```

```
[Out] -3/(5*b*x^(5/3)) + (3*a)/(4*b^2*x^(4/3)) - a^2/(b^3*x) + (3*a^3)/(2*b^4*x^(
2/3)) - (3*a^4)/(b^5*x^(1/3)) + (3*a^5*Log[b + a*x^(1/3)])/b^6 - (a^5*Log[x
])/b^6
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x^3}\,dx &= \int \frac{1}{\left(b+a\sqrt[3]{x}\right)x^{8/3}}\,dx\\ &= 3\,\text{Subst}\left(\int \frac{1}{x^6(b+ax)}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(\frac{1}{bx^6}-\frac{a}{b^2x^5}+\frac{a^2}{b^3x^4}-\frac{a^3}{b^4x^3}+\frac{a^4}{b^5x^2}-\frac{a^5}{b^6x}+\frac{a^6}{b^6(b+ax)}\right)dx, x, \sqrt[3]{x}\right)\\ &= -\frac{3}{5bx^{5/3}}+\frac{3a}{4b^2x^{4/3}}-\frac{a^2}{b^3x}+\frac{3a^3}{2b^4x^{2/3}}-\frac{3a^4}{b^5\sqrt[3]{x}}+\frac{3a^5\log\left(b+a\sqrt[3]{x}\right)}{b^6}-\frac{a^5\log(x)}{b^6}\end{aligned}$$

Mathematica [A] time = 0.09, size = 84, normalized size = 0.90

$$-\frac{-60a^5\log\left(a\sqrt[3]{x}+b\right)+20a^5\log(x)+\frac{b\left(60a^4x^{4/3}-30a^3bx+20a^2b^2x^{2/3}-15ab^3\sqrt[3]{x}+12b^4\right)}{x^{5/3}}}{20b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^(1/3))*x^3),x]
```

```
[Out] -1/20*((b*(12*b^4 - 15*a*b^3*x^(1/3) + 20*a^2*b^2*x^(2/3) - 30*a^3*b*x + 60
*a^4*x^(4/3)))/x^(5/3) - 60*a^5*Log[b + a*x^(1/3)] + 20*a^5*Log[x])/b^6
```

fricas [A] time = 1.06, size = 85, normalized size = 0.91

$$\frac{60\,a^5x^2\log\left(ax^{\frac{1}{3}}+b\right)-60\,a^5x^2\log\left(x^{\frac{1}{3}}\right)-20\,a^2b^3x-15\left(4\,a^4bx-ab^4\right)x^{\frac{2}{3}}+6\left(5\,a^3b^2x-2\,b^5\right)x^{\frac{1}{3}}}{20\,b^6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))/x^3,x, algorithm="fricas")
```

```
[Out] 1/20*(60*a^5*x^2*log(a*x^(1/3) + b) - 60*a^5*x^2*log(x^(1/3)) - 20*a^2*b^3*
x - 15*(4*a^4*b*x - a*b^4)*x^(2/3) + 6*(5*a^3*b^2*x - 2*b^5)*x^(1/3))/(b^6*
x^2)
```

giac [A] time = 0.19, size = 81, normalized size = 0.87

$$\frac{3\,a^5\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{b^6}-\frac{a^5\log\left(|x|\right)}{b^6}-\frac{60\,a^4bx^{\frac{4}{3}}-30\,a^3b^2x+20\,a^2b^3x^{\frac{2}{3}}-15\,ab^4x^{\frac{1}{3}}+12\,b^5}{20\,b^6x^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))/x^3,x, algorithm="giac")
```

```
[Out] 3*a^5*log(abs(a*x^(1/3) + b))/b^6 - a^5*log(abs(x))/b^6 - 1/20*(60*a^4*b*x^
(4/3) - 30*a^3*b^2*x + 20*a^2*b^3*x^(2/3) - 15*a*b^4*x^(1/3) + 12*b^5)/(b^6
*x^(5/3))
```

maple [A] time = 0.01, size = 78, normalized size = 0.84

$$-\frac{a^5\ln(x)}{b^6}+\frac{3a^5\ln\left(a\,x^{\frac{1}{3}}+b\right)}{b^6}-\frac{3a^4}{b^5x^{\frac{1}{3}}}+\frac{3a^3}{2b^4x^{\frac{2}{3}}}-\frac{a^2}{b^3x}+\frac{3a}{4b^2x^{\frac{4}{3}}}-\frac{3}{5b\,x^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^(1/3))/x^3,x)
```

```
[Out] -3/5/b/x^(5/3)+3/4*a/b^2/x^(4/3)-a^2/b^3/x+3/2*a^3/b^4/x^(2/3)-3*a^4/b^5/x^
(1/3)+3*a^5*ln(a*x^(1/3)+b)/b^6-a^5/b^6*ln(x)
```

maxima [A] time = 0.56, size = 95, normalized size = 1.02

$$\frac{3\,a^5\log\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{b^6}-\frac{3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^5}{5\,b^6}+\frac{15\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^4a}{4\,b^6}-\frac{10\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^3a^2}{b^6}+\frac{15\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^2a^3}{b^6}-\frac{15\left(a+\frac{b}{x^{\frac{1}{3}}}\right)a^4}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))/x^3,x, algorithm="maxima")
```

```
[Out] 3*a^5*log(a + b/x^(1/3))/b^6 - 3/5*(a + b/x^(1/3))^5/b^6 + 15/4*(a + b/x^(1
/3))^4*a/b^6 - 10*(a + b/x^(1/3))^3*a^2/b^6 + 15*(a + b/x^(1/3))^2*a^3/b^6
- 15*(a + b/x^(1/3))*a^4/b^6
```

mupad [B] time = 1.18, size = 71, normalized size = 0.76

$$\frac{6\,a^5\,\operatorname{atanh}\left(\frac{2\,a\,x^{1/3}}{b}+1\right)}{b^6}-\frac{\frac{3}{5\,b}-\frac{3\,a\,x^{1/3}}{4\,b^2}-\frac{3\,a^3\,x}{2\,b^4}+\frac{a^2\,x^{2/3}}{b^3}+\frac{3\,a^4\,x^{4/3}}{b^5}}{x^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b/x^(1/3))),x)
```

```
[Out] (6*a^5*atanh((2*a*x^(1/3))/b + 1))/b^6 - (3/(5*b) - (3*a*x^(1/3))/(4*b^2) -
 (3*a^3*x)/(2*b^4) + (a^2*x^(2/3))/b^3 + (3*a^4*x^(4/3))/b^5)/x^(5/3)
```

sympy [A] time = 5.50, size = 116, normalized size = 1.25

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{3}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{1}{2ax^2} & \text{for } b = 0 \\ -\frac{3}{5bx^{\frac{5}{3}}} & \text{for } a = 0 \\ -\frac{a^5\log(x)}{b^6}+\frac{3a^5\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{b^6}-\frac{3a^4}{b^5\sqrt[3]{x}}+\frac{3a^3}{2b^4x^{\frac{2}{3}}}-\frac{a^2}{b^3x}+\frac{3a}{4b^2x^{\frac{4}{3}}}-\frac{3}{5bx^{\frac{5}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**(1/3))/x**3,x)
```

```
[Out] Piecewise((zoo/x**(5/3), Eq(a, 0) & Eq(b, 0)), (-1/(2*a*x**2), Eq(b, 0)), (
-3/(5*b*x**(5/3)), Eq(a, 0)), (-a**5*log(x)/b**6 + 3*a**5*log(x**(1/3) + b/
a)/b**6 - 3*a**4/(b**5*x**(1/3)) + 3*a**3/(2*b**4*x**(2/3)) - a**2/(b**3*x)
 + 3*a/(4*b**2*x**(4/3)) - 3/(5*b*x**(5/3)), True))
```

3.2427 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x^4}\,dx$

Optimal. Leaf size=134

$$-\frac{3a^8\log\left(a\sqrt[3]{x}+b\right)}{b^9}+\frac{a^8\log(x)}{b^9}+\frac{3a^7}{b^8\sqrt[3]{x}}-\frac{3a^6}{2b^7x^{2/3}}+\frac{a^5}{b^6x}-\frac{3a^4}{4b^5x^{4/3}}+\frac{3a^3}{5b^4x^{5/3}}-\frac{a^2}{2b^3x^2}+\frac{3a}{7b^2x^{7/3}}-\frac{3}{8bx^{8/3}}$$

```
[Out] -3/8/b/x^(8/3)+3/7*a/b^2/x^(7/3)-1/2*a^2/b^3/x^2+3/5*a^3/b^4/x^(5/3)-3/4*a^
4/b^5/x^(4/3)+a^5/b^6/x-3/2*a^6/b^7/x^(2/3)+3*a^7/b^8/x^(1/3)-3*a^8*ln(b+a*
x^(1/3))/b^9+a^8*ln(x)/b^9
```

Rubi [A] time = 0.08, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {263, 266, 44}

$$-\frac{3a^6}{2b^7x^{2/3}}-\frac{3a^4}{4b^5x^{4/3}}+\frac{3a^3}{5b^4x^{5/3}}-\frac{a^2}{2b^3x^2}+\frac{3a^7}{b^8\sqrt[3]{x}}+\frac{a^5}{b^6x}-\frac{3a^8\log\left(a\sqrt[3]{x}+b\right)}{b^9}+\frac{a^8\log(x)}{b^9}+\frac{3a}{7b^2x^{7/3}}-\frac{3}{8bx^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^(1/3))*x^4),x]
```

```
[Out] -3/(8*b*x^(8/3)) + (3*a)/(7*b^2*x^(7/3)) - a^2/(2*b^3*x^2) + (3*a^3)/(5*b^4
*x^(5/3)) - (3*a^4)/(4*b^5*x^(4/3)) + a^5/(b^6*x) - (3*a^6)/(2*b^7*x^(2/3))
 + (3*a^7)/(b^8*x^(1/3)) - (3*a^8*Log[b + a*x^(1/3)])/b^9 + (a^8*Log[x])/b^
9
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x^4}\,dx &= \int \frac{1}{\left(b+a\sqrt[3]{x}\right)x^{11/3}}\,dx \\
&= 3\,\text{Subst}\left(\int \frac{1}{x^9(b+ax)}\,dx, x, \sqrt[3]{x}\right) \\
&= 3\,\text{Subst}\left(\int \left(\frac{1}{bx^9}-\frac{a}{b^2x^8}+\frac{a^2}{b^3x^7}-\frac{a^3}{b^4x^6}+\frac{a^4}{b^5x^5}-\frac{a^5}{b^6x^4}+\frac{a^6}{b^7x^3}-\frac{a^7}{b^8x^2}+\frac{a^8}{b^9x}-\frac{a^9}{b^9(b+a\right.\right. \\
&= -\frac{3}{8bx^{8/3}}+\frac{3a}{7b^2x^{7/3}}-\frac{a^2}{2b^3x^2}+\frac{3a^3}{5b^4x^{5/3}}-\frac{3a^4}{4b^5x^{4/3}}+\frac{a^5}{b^6x}-\frac{3a^6}{2b^7x^{2/3}}+\frac{3a^7}{b^8\sqrt[3]{x}}-\frac{3a^8\log\left(b+\right.}{b^9}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 121, normalized size = 0.90

$$\frac{-840a^8\log\left(a\sqrt[3]{x}+b\right)+280a^8\log(x)+\frac{b\left(840a^7x^{7/3}-420a^6bx^2+280a^5b^2x^{5/3}-210a^4b^3x^{4/3}+168a^3b^4x-140a^2b^5x^{2/3}+120ab^6\sqrt[3]{x}-105b^7\right)}{x^{8/3}}}{280b^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^(1/3))*x^4),x]
```

```
[Out] ((b*(-105*b^7 + 120*a*b^6*x^(1/3) - 140*a^2*b^5*x^(2/3) + 168*a^3*b^4*x - 2
10*a^4*b^3*x^(4/3) + 280*a^5*b^2*x^(5/3) - 420*a^6*b*x^2 + 840*a^7*x^(7/3))
)/x^(8/3) - 840*a^8*Log[b + a*x^(1/3)] + 280*a^8*Log[x])/(280*b^9)
```

fricas [A] time = 0.60, size = 118, normalized size = 0.88

$$-\frac{840\,a^8x^3\log\left(ax^{\frac{1}{3}}+b\right)-840\,a^8x^3\log\left(x^{\frac{1}{3}}\right)-280\,a^5b^3x^2+140\,a^2b^6x-30\left(28\,a^7bx^2-7\,a^4b^4x+4\,ab^7\right)x^{\frac{2}{3}}+21(}{280\,b^9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))/x^4,x, algorithm="fricas")
```

```
[Out] -1/280*(840*a^8*x^3*log(a*x^(1/3) + b) - 840*a^8*x^3*log(x^(1/3)) - 280*a^5
*b^3*x^2 + 140*a^2*b^6*x - 30*(28*a^7*b*x^2 - 7*a^4*b^4*x + 4*a*b^7)*x^(2/3
) + 21*(20*a^6*b^2*x^2 - 8*a^3*b^5*x + 5*b^8)*x^(1/3))/(b^9*x^3)
```

giac [A] time = 0.18, size = 113, normalized size = 0.84

$$-\frac{3\,a^8\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{b^9}+\frac{a^8\log(|x|)}{b^9}+\frac{840\,a^7bx^{\frac{7}{3}}-420\,a^6b^2x^2+280\,a^5b^3x^{\frac{5}{3}}-210\,a^4b^4x^{\frac{4}{3}}+168\,a^3b^5x-140\,a^2b^6x^{\frac{2}{3}}}{280\,b^9x^{\frac{8}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))/x^4,x, algorithm="giac")
```

```
[Out] -3*a^8*log(abs(a*x^(1/3) + b))/b^9 + a^8*log(abs(x))/b^9 + 1/280*(840*a^7*b
*x^(7/3) - 420*a^6*b^2*x^2 + 280*a^5*b^3*x^(5/3) - 210*a^4*b^4*x^(4/3) + 16
8*a^3*b^5*x - 140*a^2*b^6*x^(2/3) + 120*a*b^7*x^(1/3) - 105*b^8)/(b^9*x^(8/
3))
```

maple [A] time = 0.01, size = 109, normalized size = 0.81

$$\frac{a^8\ln(x)}{b^9}-\frac{3a^8\ln\left(a\,x^{\frac{1}{3}}+b\right)}{b^9}+\frac{3a^7}{b^8x^{\frac{1}{3}}}-\frac{3a^6}{2b^7x^{\frac{2}{3}}}+\frac{a^5}{b^6x}-\frac{3a^4}{4b^5x^{\frac{4}{3}}}+\frac{3a^3}{5b^4x^{\frac{5}{3}}}-\frac{a^2}{2b^3x^2}+\frac{3a}{7b^2x^{\frac{7}{3}}}-\frac{3}{8b\,x^{\frac{8}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^(1/3))/x^4,x)
```

```
[Out] -3/8/b/x^(8/3)+3/7*a/b^2/x^(7/3)-1/2*a^2/b^3/x^2+3/5*a^3/b^4/x^(5/3)-3/4*a^
4/b^5/x^(4/3)+a^5/b^6/x-3/2*a^6/b^7/x^(2/3)+3*a^7/b^8/x^(1/3)-3*a^8*ln(a*x^
(1/3)+b)/b^9+a^8*ln(x)/b^9
```

maxima [A] time = 0.64, size = 146, normalized size = 1.09

$$-\frac{3\,a^8\log\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{b^9}-\frac{3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^8}{8\,b^9}+\frac{24\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^7a}{7\,b^9}-\frac{14\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^6a^2}{b^9}+\frac{168\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^5a^3}{5\,b^9}-\frac{105\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^4a^4}{2\,b^9}+\frac{56\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{}}{b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))/x^4,x, algorithm="maxima")
```

```
[Out] -3*a^8*log(a + b/x^(1/3))/b^9 - 3/8*(a + b/x^(1/3))^8/b^9 + 24/7*(a + b/x^(
1/3))^7*a/b^9 - 14*(a + b/x^(1/3))^6*a^2/b^9 + 168/5*(a + b/x^(1/3))^5*a^3/
b^9 - 105/2*(a + b/x^(1/3))^4*a^4/b^9 + 56*(a + b/x^(1/3))^3*a^5/b^9 - 42*(
a + b/x^(1/3))^2*a^6/b^9 + 24*(a + b/x^(1/3))*a^7/b^9
```

mupad [B] time = 0.07, size = 105, normalized size = 0.78

$$-\frac{\frac{3}{8\,b}-\frac{3\,a\,x^{1/3}}{7\,b^2}-\frac{3\,a^3\,x}{5\,b^4}+\frac{a^2\,x^{2/3}}{2\,b^3}+\frac{3\,a^6\,x^2}{2\,b^7}+\frac{3\,a^4\,x^{4/3}}{4\,b^5}-\frac{a^5\,x^{5/3}}{b^6}-\frac{3\,a^7\,x^{7/3}}{b^8}}{x^{8/3}}-\frac{6\,a^8\,\operatorname{atanh}\left(\frac{2\,a\,x^{1/3}}{b}+1\right)}{b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b/x^(1/3))),x)
```

```
[Out] - (3/(8*b) - (3*a*x^(1/3))/(7*b^2) - (3*a^3*x)/(5*b^4) + (a^2*x^(2/3))/(2*b
^3) + (3*a^6*x^2)/(2*b^7) + (3*a^4*x^(4/3))/(4*b^5) - (a^5*x^(5/3))/b^6 - (
3*a^7*x^(7/3))/b^8)/x^(8/3) - (6*a^8*atanh((2*a*x^(1/3))/b + 1))/b^9
```

sympy [A] time = 13.98, size = 158, normalized size = 1.18

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{8}{3}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{1}{3ax^3} & \text{for } b = 0 \\ -\frac{3}{8bx^{\frac{8}{3}}} & \text{for } a = 0 \\ \frac{a^8\log(x)}{b^9} - \frac{3a^8\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{b^9} + \frac{3a^7}{b^8\sqrt[3]{x}} - \frac{3a^6}{2b^7x^{\frac{2}{3}}} + \frac{a^5}{b^6x} - \frac{3a^4}{4b^5x^{\frac{4}{3}}} + \frac{3a^3}{5b^4x^{\frac{5}{3}}} - \frac{a^2}{2b^3x^2} + \frac{3a}{7b^2x^{\frac{7}{3}}} - \frac{3}{8bx^{\frac{8}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**(1/3))/x**4,x)
```

```
[Out] Piecewise((zoo/x**(8/3), Eq(a, 0) & Eq(b, 0)), (-1/(3*a*x**3), Eq(b, 0)), (
-3/(8*b*x**(8/3)), Eq(a, 0)), (a**8*log(x)/b**9 - 3*a**8*log(x**(1/3) + b/a
)/b**9 + 3*a**7/(b**8*x**(1/3)) - 3*a**6/(2*b**7*x**(2/3)) + a**5/(b**6*x)
- 3*a**4/(4*b**5*x**(4/3)) + 3*a**3/(5*b**4*x**(5/3)) - a**2/(2*b**3*x**2)
+ 3*a/(7*b**2*x**(7/3)) - 3/(8*b*x**(8/3)), True))
```

3.2428 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x^5}\,dx$

Optimal. Leaf size=179

$$\frac{3a^{11}\log\left(a\sqrt[3]{x}+b\right)}{b^{12}}-\frac{a^{11}\log(x)}{b^{12}}-\frac{3a^{10}}{b^{11}\sqrt[3]{x}}+\frac{3a^9}{2b^{10}x^{2/3}}-\frac{a^8}{b^9x}+\frac{3a^7}{4b^8x^{4/3}}-\frac{3a^6}{5b^7x^{5/3}}+\frac{a^5}{2b^6x^2}-\frac{3a^4}{7b^5x^{7/3}}+\frac{3a^3}{8b^4x^{8/3}}-\frac{a^2}{3b^3x^3}+\frac{10}{}$$

```
[Out] -3/11/b/x^(11/3)+3/10*a/b^2/x^(10/3)-1/3*a^2/b^3/x^3+3/8*a^3/b^4/x^(8/3)-3/
7*a^4/b^5/x^(7/3)+1/2*a^5/b^6/x^2-3/5*a^6/b^7/x^(5/3)+3/4*a^7/b^8/x^(4/3)-a
^8/b^9/x+3/2*a^9/b^10/x^(2/3)-3*a^10/b^11/x^(1/3)+3*a^11*ln(b+a*x^(1/3))/b^
12-a^11*ln(x)/b^12
```

Rubi [A] time = 0.09, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {263, 266, 44}

$$\frac{3a^9}{2b^{10}x^{2/3}}+\frac{3a^7}{4b^8x^{4/3}}-\frac{3a^6}{5b^7x^{5/3}}+\frac{a^5}{2b^6x^2}-\frac{3a^4}{7b^5x^{7/3}}+\frac{3a^3}{8b^4x^{8/3}}-\frac{a^2}{3b^3x^3}-\frac{3a^{10}}{b^{11}\sqrt[3]{x}}-\frac{a^8}{b^9x}+\frac{3a^{11}\log\left(a\sqrt[3]{x}+b\right)}{b^{12}}-\frac{a^{11}\log(x)}{b^{12}}+\frac{10}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^(1/3))*x^5),x]
```

```
[Out] -3/(11*b*x^(11/3)) + (3*a)/(10*b^2*x^(10/3)) - a^2/(3*b^3*x^3) + (3*a^3)/(8
*b^4*x^(8/3)) - (3*a^4)/(7*b^5*x^(7/3)) + a^5/(2*b^6*x^2) - (3*a^6)/(5*b^7*
x^(5/3)) + (3*a^7)/(4*b^8*x^(4/3)) - a^8/(b^9*x) + (3*a^9)/(2*b^10*x^(2/3))
 - (3*a^10)/(b^11*x^(1/3)) + (3*a^11*Log[b + a*x^(1/3)])/b^12 - (a^11*Log[x
])/b^12
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x^5}\,dx = \int \frac{1}{\left(b+a\sqrt[3]{x}\right)x^{14/3}}\,dx$$
$$= 3\,\text{Subst}\left(\int \frac{1}{x^{12}(b+ax)}\,dx, x, \sqrt[3]{x}\right)$$
$$= 3\,\text{Subst}\left(\int \left(\frac{1}{bx^{12}} - \frac{a}{b^2x^{11}} + \frac{a^2}{b^3x^{10}} - \frac{a^3}{b^4x^9} + \frac{a^4}{b^5x^8} - \frac{a^5}{b^6x^7} + \frac{a^6}{b^7x^6} - \frac{a^7}{b^8x^5} + \frac{a^8}{b^9x^4} - \frac{a^9}{b^{10}x^3}\right.\right.$$
$$= -\frac{3}{11bx^{11/3}} + \frac{3a}{10b^2x^{10/3}} - \frac{a^2}{3b^3x^3} + \frac{3a^3}{8b^4x^{8/3}} - \frac{3a^4}{7b^5x^{7/3}} + \frac{a^5}{2b^6x^2} - \frac{3a^6}{5b^7x^{5/3}} + \frac{3a^7}{4b^8x^{4/3}} - \frac{a^8}{b^9x}$$

Mathematica [A] time = 0.20, size = 158, normalized size = 0.88

$$\frac{27720a^{11}\log\left(a\sqrt[3]{x}+b\right) - 9240a^{11}\log(x) + \frac{b\left(-27720a^{10}x^{10/3}+13860a^9bx^3-9240a^8b^2x^{8/3}+6930a^7b^3x^{7/3}-5544a^6b^4x^2+4620a^5b^5x^{5}\right.}{x^{11/3}}}{9240b^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^(1/3))*x^5),x]
```

```
[Out] ((b*(-2520*b^10 + 2772*a*b^9*x^(1/3) - 3080*a^2*b^8*x^(2/3) + 3465*a^3*b^7*
x - 3960*a^4*b^6*x^(4/3) + 4620*a^5*b^5*x^(5/3) - 5544*a^6*b^4*x^2 + 6930*a
^7*b^3*x^(7/3) - 9240*a^8*b^2*x^(8/3) + 13860*a^9*b*x^3 - 27720*a^10*x^(10/
3)))/x^(11/3) + 27720*a^11*Log[b + a*x^(1/3)] - 9240*a^11*Log[x])/(9240*b^1
2)
```

fricas [A] time = 0.90, size = 151, normalized size = 0.84

$$\frac{27720\,a^{11}x^4\log\left(ax^{\frac{1}{3}}+b\right) - 27720\,a^{11}x^4\log\left(x^{\frac{1}{3}}\right) - 9240\,a^8b^3x^3 + 4620\,a^5b^6x^2 - 3080\,a^2b^9x - 198\left(140\,a^{10}bx\right.}{9240\,b^{12}x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))/x^5,x, algorithm="fricas")
```

```
[Out] 1/9240*(27720*a^11*x^4*log(a*x^(1/3) + b) - 27720*a^11*x^4*log(x^(1/3)) - 9
240*a^8*b^3*x^3 + 4620*a^5*b^6*x^2 - 3080*a^2*b^9*x - 198*(140*a^10*b*x^3 -
 35*a^7*b^4*x^2 + 20*a^4*b^7*x - 14*a*b^10)*x^(2/3) + 63*(220*a^9*b^2*x^3 -
 88*a^6*b^5*x^2 + 55*a^3*b^8*x - 40*b^11)*x^(1/3))/(b^12*x^4)
```

giac [A] time = 0.16, size = 147, normalized size = 0.82

$$\frac{3\,a^{11}\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{b^{12}} - \frac{a^{11}\log\left(|x|\right)}{b^{12}} - \frac{27720\,a^{10}bx^{\frac{10}{3}} - 13860\,a^9b^2x^3 + 9240\,a^8b^3x^{\frac{8}{3}} - 6930\,a^7b^4x^{\frac{7}{3}} + 5544\,a^6b^5x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))/x^5,x, algorithm="giac")
```

```
[Out] 3*a^11*log(abs(a*x^(1/3) + b))/b^12 - a^11*log(abs(x))/b^12 - 1/9240*(27720
*a^10*b*x^(10/3) - 13860*a^9*b^2*x^3 + 9240*a^8*b^3*x^(8/3) - 6930*a^7*b^4*
x^(7/3) + 5544*a^6*b^5*x^2 - 4620*a^5*b^6*x^(5/3) + 3960*a^4*b^7*x^(4/3) -
3465*a^3*b^8*x + 3080*a^2*b^9*x^(2/3) - 2772*a*b^10*x^(1/3) + 2520*b^11)/(b
^12*x^(11/3))
```

maple [A] time = 0.01, size = 144, normalized size = 0.80

$$-\frac{a^{11}\ln(x)}{b^{12}}+\frac{3a^{11}\ln\left(a\,x^{\frac{1}{3}}+b\right)}{b^{12}}-\frac{3a^{10}}{b^{11}x^{\frac{1}{3}}}+\frac{3a^{9}}{2b^{10}x^{\frac{2}{3}}}-\frac{a^{8}}{b^{9}x}+\frac{3a^{7}}{4b^{8}x^{\frac{4}{3}}}-\frac{3a^{6}}{5b^{7}x^{\frac{5}{3}}}+\frac{a^{5}}{2b^{6}x^{2}}-\frac{3a^{4}}{7b^{5}x^{\frac{7}{3}}}+\frac{3a^{3}}{8b^{4}x^{\frac{8}{3}}}-\frac{a^{2}}{3b^{3}x^{3}}+\frac{3a}{10b^{2}x^{\frac{10}{3}}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^(1/3))/x^5,x)
```

```
[Out] -3/11/b/x^(11/3)+3/10*a/b^2/x^(10/3)-1/3*a^2/b^3/x^3+3/8*a^3/b^4/x^(8/3)-3/
7*a^4/b^5/x^(7/3)+1/2*a^5/b^6/x^2-3/5*a^6/b^7/x^(5/3)+3/4*a^7/b^8/x^(4/3)-a
^8/b^9/x+3/2*a^9/b^10/x^(2/3)-3*a^10/b^11/x^(1/3)+3*a^11*ln(a*x^(1/3)+b)/b^
12-a^11*ln(x)/b^12
```

maxima [A] time = 0.54, size = 197, normalized size = 1.10

$$\frac{3\,a^{11}\log\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{b^{12}}-\frac{3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{11}}{11\,b^{12}}+\frac{33\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{10}a}{10\,b^{12}}-\frac{55\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{9}a^{2}}{3\,b^{12}}+\frac{495\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{8}a^{3}}{8\,b^{12}}-\frac{990\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{7}a^{4}}{7\,b^{12}}+\frac{231\left(a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))/x^5,x, algorithm="maxima")
```

```
[Out] 3*a^11*log(a + b/x^(1/3))/b^12 - 3/11*(a + b/x^(1/3))^11/b^12 + 33/10*(a +
b/x^(1/3))^10*a/b^12 - 55/3*(a + b/x^(1/3))^9*a^2/b^12 + 495/8*(a + b/x^(1/
3))^8*a^3/b^12 - 990/7*(a + b/x^(1/3))^7*a^4/b^12 + 231*(a + b/x^(1/3))^6*a
^5/b^12 - 1386/5*(a + b/x^(1/3))^5*a^6/b^12 + 495/2*(a + b/x^(1/3))^4*a^7/b
^12 - 165*(a + b/x^(1/3))^3*a^8/b^12 + 165/2*(a + b/x^(1/3))^2*a^9/b^12 - 3
3*(a + b/x^(1/3))*a^10/b^12
```

mupad [B] time = 0.08, size = 138, normalized size = 0.77

$$-\frac{2520\,b^{11}-55440\,a^{11}\,x^{11/3}\,\mathrm{atanh}\left(\frac{2\,a\,x^{1/3}}{b}+1\right)-3465\,a^{3}\,b^{8}\,x-2772\,a\,b^{10}\,x^{1/3}+27720\,a^{10}\,b\,x^{10/3}+5544\,a^{6}\,b^{5}\,x^{2}}{9240\,b^{12}\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b/x^(1/3))),x)
```

```
[Out] -(2520*b^11 - 55440*a^11*x^(11/3)*atanh((2*a*x^(1/3))/b + 1) - 3465*a^3*b^8
*x - 2772*a*b^10*x^(1/3) + 27720*a^10*b*x^(10/3) + 5544*a^6*b^5*x^2 - 13860
*a^9*b^2*x^3 + 3080*a^2*b^9*x^(2/3) + 3960*a^4*b^7*x^(4/3) - 4620*a^5*b^6*x
^(5/3) - 6930*a^7*b^4*x^(7/3) + 9240*a^8*b^3*x^(8/3))/(9240*b^12*x^(11/3))
```

sympy [A] time = 31.46, size = 201, normalized size = 1.12

$$\begin{cases}\frac{\tilde{\infty}}{x^{\frac{11}{3}}} \\ -\frac{1}{4ax^{4}} \\ -\frac{3}{11bx^{\frac{11}{3}}} \\ -\frac{a^{11}\log(x)}{b^{12}}+\frac{3a^{11}\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{b^{12}}-\frac{3a^{10}}{b^{11}\sqrt[3]{x}}+\frac{3a^{9}}{2b^{10}x^{\frac{2}{3}}}-\frac{a^{8}}{b^{9}x}+\frac{3a^{7}}{4b^{8}x^{\frac{4}{3}}}-\frac{3a^{6}}{5b^{7}x^{\frac{5}{3}}}+\frac{a^{5}}{2b^{6}x^{2}}-\frac{3a^{4}}{7b^{5}x^{\frac{7}{3}}}+\frac{3a^{3}}{8b^{4}x^{\frac{8}{3}}}-\frac{a^{2}}{3b^{3}x^{3}}+\frac{3a}{10b^{2}x^{\frac{10}{3}}}-\frac{}{11b}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**(1/3))/x**5,x)
```

```
[Out] Piecewise((zoo/x**(11/3), Eq(a, 0) & Eq(b, 0)), (-1/(4*a*x**4), Eq(b, 0)),
(-3/(11*b*x**(11/3)), Eq(a, 0)), (-a**11*log(x)/b**12 + 3*a**11*log(x**(1/3
) + b/a)/b**12 - 3*a**10/(b**11*x**(1/3)) + 3*a**9/(2*b**10*x**(2/3)) - a**
8/(b**9*x) + 3*a**7/(4*b**8*x**(4/3)) - 3*a**6/(5*b**7*x**(5/3)) + a**5/(2*
b**6*x**2) - 3*a**4/(7*b**5*x**(7/3)) + 3*a**3/(8*b**4*x**(8/3)) - a**2/(3*
b**3*x**3) + 3*a/(10*b**2*x**(10/3)) - 3/(11*b*x**(11/3)), True))
```

3.2429 $\int \frac{x^2}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}\,dx$

Optimal. Leaf size=150

$$-\frac{3b^{10}}{a^{11}\left(a\sqrt[3]{x}+b\right)}-\frac{30b^9\log\left(a\sqrt[3]{x}+b\right)}{a^{11}}+\frac{27b^8\sqrt[3]{x}}{a^{10}}-\frac{12b^7x^{2/3}}{a^9}+\frac{7b^6x}{a^8}-\frac{9b^5x^{4/3}}{2a^7}+\frac{3b^4x^{5/3}}{a^6}-\frac{2b^3x^2}{a^5}+\frac{9b^2x^{7/3}}{7a^4}-\frac{3bx^{8/3}}{4a^3}+\frac{x^3}{3a}$$

```
[Out] -3*b^10/a^11/(b+a*x^(1/3))+27*b^8*x^(1/3)/a^10-12*b^7*x^(2/3)/a^9+7*b^6*x/a
^8-9/2*b^5*x^(4/3)/a^7+3*b^4*x^(5/3)/a^6-2*b^3*x^2/a^5+9/7*b^2*x^(7/3)/a^4-
3/4*b*x^(8/3)/a^3+1/3*x^3/a^2-30*b^9*ln(b+a*x^(1/3))/a^11
```

Rubi [A] time = 0.13, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {263, 266, 43}

$$-\frac{12b^7x^{2/3}}{a^9}-\frac{9b^5x^{4/3}}{2a^7}+\frac{3b^4x^{5/3}}{a^6}-\frac{2b^3x^2}{a^5}+\frac{9b^2x^{7/3}}{7a^4}-\frac{3b^{10}}{a^{11}\left(a\sqrt[3]{x}+b\right)}+\frac{27b^8\sqrt[3]{x}}{a^{10}}+\frac{7b^6x}{a^8}-\frac{30b^9\log\left(a\sqrt[3]{x}+b\right)}{a^{11}}-\frac{3bx^{8/3}}{4a^3}+\frac{x^3}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b/x^(1/3))^2,x]
```

```
[Out] (-3*b^10)/(a^11*(b + a*x^(1/3))) + (27*b^8*x^(1/3))/a^10 - (12*b^7*x^(2/3))
/a^9 + (7*b^6*x)/a^8 - (9*b^5*x^(4/3))/(2*a^7) + (3*b^4*x^(5/3))/a^6 - (2*b
^3*x^2)/a^5 + (9*b^2*x^(7/3))/(7*a^4) - (3*b*x^(8/3))/(4*a^3) + x^3/(3*a^2)
 - (30*b^9*Log[b + a*x^(1/3)])/a^11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^2}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}\,dx = \int \frac{x^{8/3}}{\left(b+a\sqrt[3]{x}\right)^2}\,dx$$
$$= 3\,\text{Subst}\left(\int \frac{x^{10}}{(b+ax)^2}\,dx, x, \sqrt[3]{x}\right)$$
$$= 3\,\text{Subst}\left(\int \left(\frac{9b^8}{a^{10}} - \frac{8b^7x}{a^9} + \frac{7b^6x^2}{a^8} - \frac{6b^5x^3}{a^7} + \frac{5b^4x^4}{a^6} - \frac{4b^3x^5}{a^5} + \frac{3b^2x^6}{a^4} - \frac{2bx^7}{a^3} + \frac{x^8}{a^2} + \frac{}{a^{10}(b\right.\right.$$
$$= -\frac{3b^{10}}{a^{11}\left(b+a\sqrt[3]{x}\right)} + \frac{27b^8\sqrt[3]{x}}{a^{10}} - \frac{12b^7x^{2/3}}{a^9} + \frac{7b^6x}{a^8} - \frac{9b^5x^{4/3}}{2a^7} + \frac{3b^4x^{5/3}}{a^6} - \frac{2b^3x^2}{a^5} + \frac{9b^2x^{7/3}}{7a^4} - \frac{3}{}$$

Mathematica [A] time = 0.20, size = 148, normalized size = 0.99

$$\frac{a\left(28a^8x^3 - 63a^7bx^{8/3} + 108a^6b^2x^{7/3} - 168a^5b^3x^2 + 252a^4b^4x^{5/3} - 378a^3b^5x^{4/3} + 588a^2b^6x + \frac{252b^9}{a+\frac{b}{\sqrt[3]{x}}} - 1008ab^7\right.}{84a^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b/x^(1/3))^2,x]
```

```
[Out] (a*((252*b^9)/(a + b/x^(1/3)) + 2268*b^8*x^(1/3) - 1008*a*b^7*x^(2/3) + 588
*a^2*b^6*x - 378*a^3*b^5*x^(4/3) + 252*a^4*b^4*x^(5/3) - 168*a^5*b^3*x^2 +
108*a^6*b^2*x^(7/3) - 63*a^7*b*x^(8/3) + 28*a^8*x^3) - 2520*b^9*Log[a + b/x
^(1/3)] - 840*b^9*Log[x])/(84*a^11)
```

fricas [A] time = 0.66, size = 169, normalized size = 1.13

$$\frac{28\,a^{12}x^4 - 140\,a^9b^3x^3 + 420\,a^6b^6x^2 + 588\,a^3b^9x - 252\,b^{12} - 2520\left(a^3b^9x + b^{12}\right)\log\left(ax^{\frac{1}{3}} + b\right) - 63\left(a^{11}bx^3 - 3\right.}{84\left(a^{14}x + a^{11}b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^(1/3))^2,x, algorithm="fricas")
```

```
[Out] 1/84*(28*a^12*x^4 - 140*a^9*b^3*x^3 + 420*a^6*b^6*x^2 + 588*a^3*b^9*x - 252
*b^12 - 2520*(a^3*b^9*x + b^12)*log(a*x^(1/3) + b) - 63*(a^11*b*x^3 - 3*a^8
*b^4*x^2 + 12*a^5*b^7*x + 20*a^2*b^10)*x^(2/3) + 18*(6*a^10*b^2*x^3 - 15*a^
7*b^5*x^2 + 105*a^4*b^8*x + 140*a*b^11)*x^(1/3))/(a^14*x + a^11*b^3)
```

giac [A] time = 0.17, size = 133, normalized size = 0.89

$$-\frac{30\,b^9\log\left(\left|ax^{\frac{1}{3}} + b\right|\right)}{a^{11}} - \frac{3\,b^{10}}{\left(ax^{\frac{1}{3}} + b\right)a^{11}} + \frac{28\,a^{16}x^3 - 63\,a^{15}bx^{\frac{8}{3}} + 108\,a^{14}b^2x^{\frac{7}{3}} - 168\,a^{13}b^3x^2 + 252\,a^{12}b^4x^{\frac{5}{3}} - 378}{84\,a^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^(1/3))^2,x, algorithm="giac")
```

```
[Out] -30*b^9*log(abs(a*x^(1/3) + b))/a^11 - 3*b^10/((a*x^(1/3) + b)*a^11) + 1/84
*(28*a^16*x^3 - 63*a^15*b*x^(8/3) + 108*a^14*b^2*x^(7/3) - 168*a^13*b^3*x^2
 + 252*a^12*b^4*x^(5/3) - 378*a^11*b^5*x^(4/3) + 588*a^10*b^6*x - 1008*a^9*
b^7*x^(2/3) + 2268*a^8*b^8*x^(1/3))/a^18
```

maple [A] time = 0.01, size = 127, normalized size = 0.85

$$\frac{x^3}{3a^2}-\frac{3b\,x^{\frac{8}{3}}}{4a^3}+\frac{9b^2x^{\frac{7}{3}}}{7a^4}-\frac{2b^3x^2}{a^5}+\frac{3b^4x^{\frac{5}{3}}}{a^6}-\frac{9b^5x^{\frac{4}{3}}}{2a^7}+\frac{7b^6x}{a^8}-\frac{3b^{10}}{\left(a\,x^{\frac{1}{3}}+b\right)a^{11}}-\frac{30b^9\ln\left(a\,x^{\frac{1}{3}}+b\right)}{a^{11}}-\frac{12b^7x^{\frac{2}{3}}}{a^9}+\frac{27b^8x^{\frac{1}{3}}}{a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b/x^(1/3))^2,x)
```

```
[Out] -3*b^10/a^11/(a*x^(1/3)+b)+27*b^8*x^(1/3)/a^10-12*b^7*x^(2/3)/a^9+7*b^6*x/a
^8-9/2*b^5*x^(4/3)/a^7+3*b^4*x^(5/3)/a^6-2/a^5*b^3*x^2+9/7*b^2*x^(7/3)/a^4-
3/4*b*x^(8/3)/a^3+1/3/a^2*x^3-30*b^9*ln(a*x^(1/3)+b)/a^11
```

maxima [A] time = 0.48, size = 145, normalized size = 0.97

$$\frac{28\,a^9-\frac{35\,a^8b}{x^{\frac{1}{3}}}+\frac{45\,a^7b^2}{x^{\frac{2}{3}}}-\frac{60\,a^6b^3}{x}+\frac{84\,a^5b^4}{x^{\frac{4}{3}}}-\frac{126\,a^4b^5}{x^{\frac{5}{3}}}+\frac{210\,a^3b^6}{x^2}-\frac{420\,a^2b^7}{x^{\frac{7}{3}}}+\frac{1260\,ab^8}{x^{\frac{8}{3}}}+\frac{2520\,b^9}{x^3}}{84\left(\frac{a^{11}}{x^3}+\frac{a^{10}b}{x^{\frac{10}{3}}}\right)}-\frac{30\,b^9\log\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{a^{11}}-\frac{10\,b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^(1/3))^2,x, algorithm="maxima")
```

```
[Out] 1/84*(28*a^9 - 35*a^8*b/x^(1/3) + 45*a^7*b^2/x^(2/3) - 60*a^6*b^3/x + 84*a^
5*b^4/x^(4/3) - 126*a^4*b^5/x^(5/3) + 210*a^3*b^6/x^2 - 420*a^2*b^7/x^(7/3)
 + 1260*a*b^8/x^(8/3) + 2520*b^9/x^3)/(a^11/x^3 + a^10*b/x^(10/3)) - 30*b^9
*log(a + b/x^(1/3))/a^11 - 10*b^9*log(x)/a^11
```

mupad [B] time = 0.07, size = 132, normalized size = 0.88

$$\frac{x^3}{3\,a^2}-\frac{3\,b^{10}}{a\,\left(a^{10}\,b+a^{11}\,x^{1/3}\right)}-\frac{3\,b\,x^{8/3}}{4\,a^3}+\frac{7\,b^6\,x}{a^8}-\frac{30\,b^9\,\ln\left(b+a\,x^{1/3}\right)}{a^{11}}-\frac{2\,b^3\,x^2}{a^5}+\frac{9\,b^2\,x^{7/3}}{7\,a^4}+\frac{3\,b^4\,x^{5/3}}{a^6}-\frac{9\,b^5\,x^{4/3}}{2\,a^7}-\frac{12\,b^7\,x^{2/3}}{a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b/x^(1/3))^2,x)
```

```
[Out] x^3/(3*a^2) - (3*b^10)/(a*(a^10*b + a^11*x^(1/3))) - (3*b*x^(8/3))/(4*a^3)
+ (7*b^6*x)/a^8 - (30*b^9*log(b + a*x^(1/3)))/a^11 - (2*b^3*x^2)/a^5 + (9*b
^2*x^(7/3))/(7*a^4) + (3*b^4*x^(5/3))/a^6 - (9*b^5*x^(4/3))/(2*a^7) - (12*b
^7*x^(2/3))/a^9 + (27*b^8*x^(1/3))/a^10
```

sympy [A] time = 7.05, size = 367, normalized size = 2.45

$$\begin{cases}\frac{28a^{10}x^{\frac{10}{3}}}{84a^{12}\sqrt[3]{x}+84a^{11}b}-\frac{35a^9bx^3}{84a^{12}\sqrt[3]{x}+84a^{11}b}+\frac{45a^8b^2x^{\frac{8}{3}}}{84a^{12}\sqrt[3]{x}+84a^{11}b}-\frac{60a^7b^3x^{\frac{7}{3}}}{84a^{12}\sqrt[3]{x}+84a^{11}b}+\frac{84a^6b^4x^2}{84a^{12}\sqrt[3]{x}+84a^{11}b}-\frac{126a^5b^5x^{\frac{5}{3}}}{84a^{12}\sqrt[3]{x}+84a^{11}b}+\frac{210a^4b^6x^{\frac{4}{3}}}{84a^{12}\sqrt[3]{x}+84a^{1} \\ \frac{3x^{\frac{11}{3}}}{11b^2}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b/x**(1/3))**2,x)
```

```
[Out] Piecewise((28*a**10*x**(10/3)/(84*a**12*x**(1/3) + 84*a**11*b) - 35*a**9*b*
x**3/(84*a**12*x**(1/3) + 84*a**11*b) + 45*a**8*b**2*x**(8/3)/(84*a**12*x**
(1/3) + 84*a**11*b) - 60*a**7*b**3*x**(7/3)/(84*a**12*x**(1/3) + 84*a**11*b
) + 84*a**6*b**4*x**2/(84*a**12*x**(1/3) + 84*a**11*b) - 126*a**5*b**5*x**(
```

```
5/3)/(84*a**12*x**(1/3) + 84*a**11*b) + 210*a**4*b**6*x**(4/3)/(84*a**12*x*
*(1/3) + 84*a**11*b) - 420*a**3*b**7*x/(84*a**12*x**(1/3) + 84*a**11*b) + 1
260*a**2*b**8*x**(2/3)/(84*a**12*x**(1/3) + 84*a**11*b) - 2520*a*b**9*x**(1
/3)*log(x**(1/3) + b/a)/(84*a**12*x**(1/3) + 84*a**11*b) - 2520*b**10*log(x
**(1/3) + b/a)/(84*a**12*x**(1/3) + 84*a**11*b) - 2520*b**10/(84*a**12*x**(
1/3) + 84*a**11*b), Ne(a, 0)), (3*x**(11/3)/(11*b**2), True))
```

3.2430 $\int \frac{x}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}\,dx$

Optimal. Leaf size=113

$$\frac{3b^7}{a^8\left(a\sqrt[3]{x}+b\right)}+\frac{21b^6\log\left(a\sqrt[3]{x}+b\right)}{a^8}-\frac{18b^5\sqrt[3]{x}}{a^7}+\frac{15b^4x^{2/3}}{2a^6}-\frac{4b^3x}{a^5}+\frac{9b^2x^{4/3}}{4a^4}-\frac{6bx^{5/3}}{5a^3}+\frac{x^2}{2a^2}$$

```
[Out] 3*b^7/a^8/(b+a*x^(1/3))-18*b^5*x^(1/3)/a^7+15/2*b^4*x^(2/3)/a^6-4*b^3*x/a^5
+9/4*b^2*x^(4/3)/a^4-6/5*b*x^(5/3)/a^3+1/2*x^2/a^2+21*b^6*ln(b+a*x^(1/3))/a
^8
```

Rubi [A] time = 0.09, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 266, 43}

$$\frac{15b^4x^{2/3}}{2a^6}+\frac{9b^2x^{4/3}}{4a^4}+\frac{3b^7}{a^8\left(a\sqrt[3]{x}+b\right)}-\frac{18b^5\sqrt[3]{x}}{a^7}-\frac{4b^3x}{a^5}+\frac{21b^6\log\left(a\sqrt[3]{x}+b\right)}{a^8}-\frac{6bx^{5/3}}{5a^3}+\frac{x^2}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b/x^(1/3))^2,x]
```

```
[Out] (3*b^7)/(a^8*(b + a*x^(1/3))) - (18*b^5*x^(1/3))/a^7 + (15*b^4*x^(2/3))/(2*
a^6) - (4*b^3*x)/a^5 + (9*b^2*x^(4/3))/(4*a^4) - (6*b*x^(5/3))/(5*a^3) + x^
2/(2*a^2) + (21*b^6*Log[b + a*x^(1/3)])/a^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}\,dx = \int \frac{x^{5/3}}{\left(b+a\sqrt[3]{x}\right)^2}\,dx$$

$$= 3\,\text{Subst}\left(\int \frac{x^7}{(b+ax)^2}\,dx, x, \sqrt[3]{x}\right)$$

$$= 3\,\text{Subst}\left(\int \left(-\frac{6b^5}{a^7}+\frac{5b^4x}{a^6}-\frac{4b^3x^2}{a^5}+\frac{3b^2x^3}{a^4}-\frac{2bx^4}{a^3}+\frac{x^5}{a^2}-\frac{b^7}{a^7(b+ax)^2}+\frac{7b^6}{a^7(b+ax)}\right)dx\right.$$

$$= \frac{3b^7}{a^8\left(b+a\sqrt[3]{x}\right)}-\frac{18b^5\sqrt[3]{x}}{a^7}+\frac{15b^4x^{2/3}}{2a^6}-\frac{4b^3x}{a^5}+\frac{9b^2x^{4/3}}{4a^4}-\frac{6bx^{5/3}}{5a^3}+\frac{x^2}{2a^2}+\frac{21b^6\log\left(b+a\sqrt[3]{}\right.}{a^8}$$

Mathematica [A] time = 0.14, size = 111, normalized size = 0.98

$$\frac{a\left(10a^5x^2-24a^4bx^{5/3}+45a^3b^2x^{4/3}-80a^2b^3x-\frac{60b^6}{a+\frac{b}{\sqrt[3]{x}}}+150ab^4x^{2/3}-360b^5\sqrt[3]{x}\right)+420b^6\log\left(a+\frac{b}{\sqrt[3]{x}}\right)+140}{20a^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b/x^(1/3))^2,x]
```

```
[Out] (a*((-60*b^6)/(a + b/x^(1/3)) - 360*b^5*x^(1/3) + 150*a*b^4*x^(2/3) - 80*a^
2*b^3*x + 45*a^3*b^2*x^(4/3) - 24*a^4*b*x^(5/3) + 10*a^5*x^2) + 420*b^6*Log
[a + b/x^(1/3)] + 140*b^6*Log[x])/(20*a^8)
```

fricas [A] time = 0.91, size = 137, normalized size = 1.21

$$\frac{10\,a^9x^3-70\,a^6b^3x^2-80\,a^3b^6x+60\,b^9+420\left(a^3b^6x+b^9\right)\log\left(ax^{\frac{1}{3}}+b\right)-6\left(4\,a^8bx^2-21\,a^5b^4x-35\,a^2b^7\right)x^{\frac{2}{3}}}{20\left(a^{11}x+a^8b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^(1/3))^2,x, algorithm="fricas")
```

```
[Out] 1/20*(10*a^9*x^3 - 70*a^6*b^3*x^2 - 80*a^3*b^6*x + 60*b^9 + 420*(a^3*b^6*x
+ b^9)*log(a*x^(1/3) + b) - 6*(4*a^8*b*x^2 - 21*a^5*b^4*x - 35*a^2*b^7)*x^(
2/3) + 15*(3*a^7*b^2*x^2 - 21*a^4*b^5*x - 28*a*b^8)*x^(1/3))/(a^11*x + a^8*
b^3)
```

giac [A] time = 0.16, size = 100, normalized size = 0.88

$$\frac{21\,b^6\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{a^8}+\frac{3\,b^7}{\left(ax^{\frac{1}{3}}+b\right)a^8}+\frac{10\,a^{10}x^2-24\,a^9bx^{\frac{5}{3}}+45\,a^8b^2x^{\frac{4}{3}}-80\,a^7b^3x+150\,a^6b^4x^{\frac{2}{3}}-360\,a^5b^5x^{\frac{1}{3}}}{20\,a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^(1/3))^2,x, algorithm="giac")
```

```
[Out] 21*b^6*log(abs(a*x^(1/3) + b))/a^8 + 3*b^7/((a*x^(1/3) + b)*a^8) + 1/20*(10
*a^10*x^2 - 24*a^9*b*x^(5/3) + 45*a^8*b^2*x^(4/3) - 80*a^7*b^3*x + 150*a^6*
b^4*x^(2/3) - 360*a^5*b^5*x^(1/3))/a^12
```

maple [A] time = 0.01, size = 94, normalized size = 0.83

$$\frac{x^2}{2a^2}-\frac{6b\,x^{\frac{5}{3}}}{5a^3}+\frac{9b^2x^{\frac{4}{3}}}{4a^4}-\frac{4b^3x}{a^5}+\frac{3b^7}{\left(a\,x^{\frac{1}{3}}+b\right)a^8}+\frac{21b^6\ln\left(a\,x^{\frac{1}{3}}+b\right)}{a^8}+\frac{15b^4x^{\frac{2}{3}}}{2a^6}-\frac{18b^5x^{\frac{1}{3}}}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b/x^(1/3))^2,x)
```

```
[Out] 3*b^7/a^8/(a*x^(1/3)+b)-18*b^5*x^(1/3)/a^7+15/2*b^4*x^(2/3)/a^6-4/a^5*b^3*x
+9/4*b^2*x^(4/3)/a^4-6/5*b*x^(5/3)/a^3+1/2/a^2*x^2+21*b^6*ln(a*x^(1/3)+b)/a
^8
```

maxima [A] time = 0.55, size = 112, normalized size = 0.99

$$\frac{10\,a^6-\frac{14\,a^5b}{x^{\frac{1}{3}}}+\frac{21\,a^4b^2}{x^{\frac{2}{3}}}-\frac{35\,a^3b^3}{x}+\frac{70\,a^2b^4}{x^{\frac{4}{3}}}-\frac{210\,ab^5}{x^{\frac{5}{3}}}-\frac{420\,b^6}{x^2}}{20\left(\frac{a^8}{x^2}+\frac{a^7b}{x^{\frac{7}{3}}}\right)}+\frac{21\,b^6\log\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{a^8}+\frac{7\,b^6\log(x)}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^(1/3))^2,x, algorithm="maxima")
```

```
[Out] 1/20*(10*a^6 - 14*a^5*b/x^(1/3) + 21*a^4*b^2/x^(2/3) - 35*a^3*b^3/x + 70*a^
2*b^4/x^(4/3) - 210*a*b^5/x^(5/3) - 420*b^6/x^2)/(a^8/x^2 + a^7*b/x^(7/3))
+ 21*b^6*log(a + b/x^(1/3))/a^8 + 7*b^6*log(x)/a^8
```

mupad [B] time = 1.13, size = 99, normalized size = 0.88

$$\frac{x^2}{2\,a^2}+\frac{3\,b^7}{a\left(a^7\,b+a^8\,x^{1/3}\right)}-\frac{4\,b^3\,x}{a^5}-\frac{6\,b\,x^{5/3}}{5\,a^3}+\frac{21\,b^6\,\ln\left(b+a\,x^{1/3}\right)}{a^8}+\frac{9\,b^2\,x^{4/3}}{4\,a^4}+\frac{15\,b^4\,x^{2/3}}{2\,a^6}-\frac{18\,b^5\,x^{1/3}}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b/x^(1/3))^2,x)
```

```
[Out] x^2/(2*a^2) + (3*b^7)/(a*(a^7*b + a^8*x^(1/3))) - (4*b^3*x)/a^5 - (6*b*x^(5
/3))/(5*a^3) + (21*b^6*log(b + a*x^(1/3)))/a^8 + (9*b^2*x^(4/3))/(4*a^4) +
(15*b^4*x^(2/3))/(2*a^6) - (18*b^5*x^(1/3))/a^7
```

sympy [A] time = 2.20, size = 277, normalized size = 2.45

$$\begin{cases}\frac{10a^7x^{\frac{7}{3}}}{20a^9\sqrt[3]{x}+20a^8b}-\frac{14a^6bx^2}{20a^9\sqrt[3]{x}+20a^8b}+\frac{21a^5b^2x^{\frac{5}{3}}}{20a^9\sqrt[3]{x}+20a^8b}-\frac{35a^4b^3x^{\frac{4}{3}}}{20a^9\sqrt[3]{x}+20a^8b}+\frac{70a^3b^4x}{20a^9\sqrt[3]{x}+20a^8b}-\frac{210a^2b^5x^{\frac{2}{3}}}{20a^9\sqrt[3]{x}+20a^8b}+\frac{420ab^6\sqrt[3]{x}\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{20a^9\sqrt[3]{x}+20a^8b}+ \\ \frac{3x^{\frac{8}{3}}}{8b^2}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x**(1/3))**2,x)
```

```
[Out] Piecewise((10*a**7*x**(7/3)/(20*a**9*x**(1/3) + 20*a**8*b) - 14*a**6*b*x**2
/(20*a**9*x**(1/3) + 20*a**8*b) + 21*a**5*b**2*x**(5/3)/(20*a**9*x**(1/3) +
 20*a**8*b) - 35*a**4*b**3*x**(4/3)/(20*a**9*x**(1/3) + 20*a**8*b) + 70*a**
3*b**4*x/(20*a**9*x**(1/3) + 20*a**8*b) - 210*a**2*b**5*x**(2/3)/(20*a**9*x
**(1/3) + 20*a**8*b) + 420*a*b**6*x**(1/3)*log(x**(1/3) + b/a)/(20*a**9*x**
(1/3) + 20*a**8*b) + 420*b**7*log(x**(1/3) + b/a)/(20*a**9*x**(1/3) + 20*a*
*8*b) + 420*b**7/(20*a**9*x**(1/3) + 20*a**8*b), Ne(a, 0)), (3*x**(8/3)/(8*
b**2), True))
```

3.2431 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}\,dx$

Optimal. Leaf size=77

$$-\frac{12b^3\log\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^5}-\frac{4b^3\log(x)}{a^5}+\frac{3b^3}{a^4\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{9b^2\sqrt[3]{x}}{a^4}-\frac{3bx^{2/3}}{a^3}+\frac{x}{a^2}$$

```
[Out] 3*b^3/a^4/(a+b/x^(1/3))+9*b^2*x^(1/3)/a^4-3*b*x^(2/3)/a^3+x/a^2-12*b^3*ln(a
+b/x^(1/3))/a^5-4*b^3*ln(x)/a^5
```

Rubi [A] time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {190, 44}

$$\frac{3b^3}{a^4\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{9b^2\sqrt[3]{x}}{a^4}-\frac{12b^3\log\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^5}-\frac{4b^3\log(x)}{a^5}-\frac{3bx^{2/3}}{a^3}+\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^(1/3))^(-2),x]
```

```
[Out] (3*b^3)/(a^4*(a + b/x^(1/3))) + (9*b^2*x^(1/3))/a^4 - (3*b*x^(2/3))/a^3 + x
/a^2 - (12*b^3*Log[a + b/x^(1/3)])/a^5 - (4*b^3*Log[x])/a^5
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}\,dx &= -\left(3\,\text{Subst}\left(\int \frac{1}{x^4(a+bx)^2}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)\\ &= -\left(3\,\text{Subst}\left(\int \left(\frac{1}{a^2x^4}-\frac{2b}{a^3x^3}+\frac{3b^2}{a^4x^2}-\frac{4b^3}{a^5x}+\frac{b^4}{a^4(a+bx)^2}+\frac{4b^4}{a^5(a+bx)}\right)dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)\\ &= \frac{3b^3}{a^4\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{9b^2\sqrt[3]{x}}{a^4}-\frac{3bx^{2/3}}{a^3}+\frac{x}{a^2}-\frac{12b^3\log\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^5}-\frac{4b^3\log(x)}{a^5}\end{aligned}$$

Mathematica [A] time = 0.05, size = 63, normalized size = 0.82

$$\frac{a^3x-3a^2bx^{2/3}-\frac{3b^4}{a\sqrt[3]{x}+b}-12b^3\log\left(a\sqrt[3]{x}+b\right)+9ab^2\sqrt[3]{x}}{a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^(1/3))^(-2),x]
```

```
[Out] ((-3*b^4)/(b + a*x^(1/3)) + 9*a*b^2*x^(1/3) - 3*a^2*b*x^(2/3) + a^3*x - 12*
b^3*Log[b + a*x^(1/3)])/a^5
```

fricas [A] time = 0.78, size = 100, normalized size = 1.30

$$\frac{a^6x^2 + a^3b^3x - 3\,b^6 - 12\left(a^3b^3x + b^6\right)\log\left(ax^{\frac{1}{3}} + b\right) - 3\left(a^5bx + 2\,a^2b^4\right)x^{\frac{2}{3}} + 3\left(3\,a^4b^2x + 4\,ab^5\right)x^{\frac{1}{3}}}{a^8x + a^5b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))^2,x, algorithm="fricas")
```

```
[Out] (a^6*x^2 + a^3*b^3*x - 3*b^6 - 12*(a^3*b^3*x + b^6)*log(a*x^(1/3) + b) - 3*
(a^5*b*x + 2*a^2*b^4)*x^(2/3) + 3*(3*a^4*b^2*x + 4*a*b^5)*x^(1/3))/(a^8*x +
 a^5*b^3)
```

giac [A] time = 0.18, size = 65, normalized size = 0.84

$$-\frac{12\,b^3\log\left(\left|ax^{\frac{1}{3}} + b\right|\right)}{a^5} - \frac{3\,b^4}{\left(ax^{\frac{1}{3}} + b\right)a^5} + \frac{a^4x - 3\,a^3bx^{\frac{2}{3}} + 9\,a^2b^2x^{\frac{1}{3}}}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))^2,x, algorithm="giac")
```

```
[Out] -12*b^3*log(abs(a*x^(1/3) + b))/a^5 - 3*b^4/((a*x^(1/3) + b)*a^5) + (a^4*x
- 3*a^3*b*x^(2/3) + 9*a^2*b^2*x^(1/3))/a^6
```

maple [A] time = 0.01, size = 60, normalized size = 0.78

$$\frac{x}{a^2} - \frac{3b^4}{\left(a\,x^{\frac{1}{3}} + b\right)a^5} - \frac{12b^3\ln\left(a\,x^{\frac{1}{3}} + b\right)}{a^5} - \frac{3b\,x^{\frac{2}{3}}}{a^3} + \frac{9b^2x^{\frac{1}{3}}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^(1/3))^2,x)
```

```
[Out] 1/a^2*x-3*b*x^(2/3)/a^3+9*b^2*x^(1/3)/a^4-12/a^5*b^3*ln(a*x^(1/3)+b)-3/a^5*
b^4/(a*x^(1/3)+b)
```

maxima [A] time = 0.46, size = 76, normalized size = 0.99

$$\frac{a^3 - \frac{2\,a^2b}{x^{\frac{1}{3}}} + \frac{6\,ab^2}{x^{\frac{2}{3}}} + \frac{12\,b^3}{x}}{\frac{a^5}{x} + \frac{a^4b}{x^{\frac{4}{3}}}} - \frac{12\,b^3\log\left(a + \frac{b}{x^{\frac{1}{3}}}\right)}{a^5} - \frac{4\,b^3\log(x)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))^2,x, algorithm="maxima")
```

```
[Out] (a^3 - 2*a^2*b/x^(1/3) + 6*a*b^2/x^(2/3) + 12*b^3/x)/(a^5/x + a^4*b/x^(4/3)
) - 12*b^3*log(a + b/x^(1/3))/a^5 - 4*b^3*log(x)/a^5
```

mupad [B] time = 1.13, size = 65, normalized size = 0.84

$$\frac{x}{a^2}-\frac{3\,b^4}{a\left(a^4\,b+a^5\,x^{1/3}\right)}-\frac{3\,b\,x^{2/3}}{a^3}-\frac{12\,b^3\,\ln\left(b+a\,x^{1/3}\right)}{a^5}+\frac{9\,b^2\,x^{1/3}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x^(1/3))^2,x)
```

```
[Out] x/a^2 - (3*b^4)/(a*(a^4*b + a^5*x^(1/3))) - (3*b*x^(2/3))/a^3 - (12*b^3*log
(b + a*x^(1/3)))/a^5 + (9*b^2*x^(1/3))/a^4
```

sympy [A] time = 0.67, size = 165, normalized size = 2.14

$$\begin{cases}\frac{a^4x^{\frac{4}{3}}}{a^6\sqrt[3]{x}+a^5b}-\frac{2a^3bx}{a^6\sqrt[3]{x}+a^5b}+\frac{6a^2b^2x^{\frac{2}{3}}}{a^6\sqrt[3]{x}+a^5b}-\frac{12ab^3\sqrt[3]{x}\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{a^6\sqrt[3]{x}+a^5b}-\frac{12b^4\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{a^6\sqrt[3]{x}+a^5b}-\frac{12b^4}{a^6\sqrt[3]{x}+a^5b} & \text{for } a\neq 0\\ \frac{3x^{\frac{5}{3}}}{5b^2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**(1/3))**2,x)
```

```
[Out] Piecewise((a**4*x**(4/3)/(a**6*x**(1/3) + a**5*b) - 2*a**3*b*x/(a**6*x**(1/
3) + a**5*b) + 6*a**2*b**2*x**(2/3)/(a**6*x**(1/3) + a**5*b) - 12*a*b**3*x*
*(1/3)*log(x**(1/3) + b/a)/(a**6*x**(1/3) + a**5*b) - 12*b**4*log(x**(1/3)
+ b/a)/(a**6*x**(1/3) + a**5*b) - 12*b**4/(a**6*x**(1/3) + a**5*b), Ne(a, 0
)), (3*x**(5/3)/(5*b**2), True))
```

3.2432 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x}\,dx$

Optimal. Leaf size=33

$$\frac{3b}{a^2\left(a\sqrt[3]{x}+b\right)}+\frac{3\log\left(a\sqrt[3]{x}+b\right)}{a^2}$$

```
[Out] 3*b/a^2/(b+a*x^(1/3))+3*ln(b+a*x^(1/3))/a^2
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {263, 266, 43}

$$\frac{3b}{a^2\left(a\sqrt[3]{x}+b\right)}+\frac{3\log\left(a\sqrt[3]{x}+b\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^(1/3))^2*x),x]
```

```
[Out] (3*b)/(a^2*(b + a*x^(1/3))) + (3*Log[b + a*x^(1/3)])/a^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x}\,dx &= \int \frac{1}{\left(b+a\sqrt[3]{x}\right)^2\sqrt[3]{x}}\,dx \\
&= 3\,\text{Subst}\left(\int \frac{x}{(b+ax)^2}\,dx, x, \sqrt[3]{x}\right) \\
&= 3\,\text{Subst}\left(\int \left(-\frac{b}{a(b+ax)^2}+\frac{1}{a(b+ax)}\right)dx, x, \sqrt[3]{x}\right) \\
&= \frac{3b}{a^2\left(b+a\sqrt[3]{x}\right)}+\frac{3\log\left(b+a\sqrt[3]{x}\right)}{a^2}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 33, normalized size = 1.00

$$\frac{-\frac{3a}{a+\frac{b}{\sqrt[3]{x}}}+3\log\left(a+\frac{b}{\sqrt[3]{x}}\right)+\log(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^(1/3))^2*x),x]
```

```
[Out] ((-3*a)/(a + b/x^(1/3)) + 3*Log[a + b/x^(1/3)] + Log[x])/a^2
```

fricas [A] time = 0.56, size = 56, normalized size = 1.70

$$\frac{3\left(a^2bx^{\frac{2}{3}}-ab^2x^{\frac{1}{3}}+b^3+\left(a^3x+b^3\right)\log\left(ax^{\frac{1}{3}}+b\right)\right)}{a^5x+a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))^2/x,x, algorithm="fricas")
```

```
[Out] 3*(a^2*b*x^(2/3) - a*b^2*x^(1/3) + b^3 + (a^3*x + b^3)*log(a*x^(1/3) + b))/
(a^5*x + a^2*b^3)
```

giac [A] time = 0.16, size = 30, normalized size = 0.91

$$\frac{3\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{a^2}+\frac{3b}{\left(ax^{\frac{1}{3}}+b\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))^2/x,x, algorithm="giac")
```

```
[Out] 3*log(abs(a*x^(1/3) + b))/a^2 + 3*b/((a*x^(1/3) + b)*a^2)
```

maple [A] time = 0.01, size = 30, normalized size = 0.91

$$\frac{3b}{\left(a\,x^{\frac{1}{3}}+b\right)a^2}+\frac{3\ln\left(a\,x^{\frac{1}{3}}+b\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^(1/3))^2/x,x)
```

```
[Out] 3*b/a^2/(a*x^(1/3)+b)+3*ln(a*x^(1/3)+b)/a^2
```

maxima [A] time = 0.58, size = 34, normalized size = 1.03

$$-\frac{3}{a^2+\frac{ab}{x^{\frac{1}{3}}}}+\frac{3\log\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{a^2}+\frac{\log(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))^2/x,x, algorithm="maxima")
```

```
[Out] -3/(a^2 + a*b/x^(1/3)) + 3*log(a + b/x^(1/3))/a^2 + log(x)/a^2
```

mupad [**B**] time = 0.04, size = 29, normalized size = 0.88

$$\frac{3\,\ln\left(b+a\,x^{1/3}\right)}{a^2}+\frac{3\,b}{a^2\,\left(b+a\,x^{1/3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b/x^(1/3))^2),x)
```

```
[Out] (3*log(b + a*x^(1/3)))/a^2 + (3*b)/(a^2*(b + a*x^(1/3)))
```

sympy [**A**] time = 1.08, size = 99, normalized size = 3.00

$$\begin{cases}\frac{3ax\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{a^3x+a^2bx^{\frac{2}{3}}}+\frac{3bx^{\frac{2}{3}}\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{a^3x+a^2bx^{\frac{2}{3}}}+\frac{3bx^{\frac{2}{3}}}{a^3x+a^2bx^{\frac{2}{3}}} & \text{for } a\neq 0\\ \frac{3x^{\frac{2}{3}}}{2b^2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**(1/3))**2/x,x)
```

```
[Out] Piecewise((3*a*x*log(x**(1/3) + b/a)/(a**3*x + a**2*b*x**(2/3)) + 3*b*x**(2
/3)*log(x**(1/3) + b/a)/(a**3*x + a**2*b*x**(2/3)) + 3*b*x**(2/3)/(a**3*x +
 a**2*b*x**(2/3)), Ne(a, 0)), (3*x**(2/3)/(2*b**2), True))
```

3.2433 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^2}\,dx$

Optimal. Leaf size=52

$$\frac{6a\log\left(a\sqrt[3]{x}+b\right)}{b^3}-\frac{2a\log(x)}{b^3}-\frac{3a}{b^2\left(a\sqrt[3]{x}+b\right)}-\frac{3}{b^2\sqrt[3]{x}}$$

```
[Out] -3*a/b^2/(b+a*x^(1/3))-3/b^2/x^(1/3)+6*a*ln(b+a*x^(1/3))/b^3-2*a*ln(x)/b^3
```

Rubi [A] time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {263, 266, 44}

$$-\frac{3a}{b^2\left(a\sqrt[3]{x}+b\right)}+\frac{6a\log\left(a\sqrt[3]{x}+b\right)}{b^3}-\frac{2a\log(x)}{b^3}-\frac{3}{b^2\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^(1/3))^2*x^2),x]
```

```
[Out] (-3*a)/(b^2*(b + a*x^(1/3))) - 3/(b^2*x^(1/3)) + (6*a*Log[b + a*x^(1/3)])/b
^3 - (2*a*Log[x])/b^3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^2}\,dx &= \int \frac{1}{\left(b+a\sqrt[3]{x}\right)^2 x^{4/3}}\,dx\\ &= 3\,\text{Subst}\left(\int \frac{1}{x^2(b+ax)^2}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(\frac{1}{b^2x^2}-\frac{2a}{b^3x}+\frac{a^2}{b^2(b+ax)^2}+\frac{2a^2}{b^3(b+ax)}\right)dx, x, \sqrt[3]{x}\right)\\ &= -\frac{3a}{b^2\left(b+a\sqrt[3]{x}\right)}-\frac{3}{b^2\sqrt[3]{x}}+\frac{6a\log\left(b+a\sqrt[3]{x}\right)}{b^3}-\frac{2a\log(x)}{b^3}\end{aligned}$$

Mathematica [A] time = 0.05, size = 46, normalized size = 0.88

$$\frac{-\frac{3ab}{a\sqrt[3]{x}+b}+6a\log\left(a\sqrt[3]{x}+b\right)-2a\log(x)-\frac{3b}{\sqrt[3]{x}}}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^(1/3))^2*x^2),x]
```

```
[Out] ((-3*a*b)/(b + a*x^(1/3)) - (3*b)/x^(1/3) + 6*a*Log[b + a*x^(1/3)] - 2*a*Lo
g[x])/b^3
```

fricas [B] time = 0.82, size = 98, normalized size = 1.88

$$\frac{3\left(a^2b^2x^{\frac{4}{3}}-ab^3x+2\left(a^4x^2+ab^3x\right)\log\left(ax^{\frac{1}{3}}+b\right)-2\left(a^4x^2+ab^3x\right)\log\left(x^{\frac{1}{3}}\right)-\left(2\,a^3bx+b^4\right)x^{\frac{2}{3}}\right)}{a^3b^3x^2+b^6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))^2/x^2,x, algorithm="fricas")
```

```
[Out] 3*(a^2*b^2*x^(4/3) - a*b^3*x + 2*(a^4*x^2 + a*b^3*x)*log(a*x^(1/3) + b) - 2
*(a^4*x^2 + a*b^3*x)*log(x^(1/3)) - (2*a^3*b*x + b^4)*x^(2/3))/(a^3*b^3*x^2
 + b^6*x)
```

giac [A] time = 0.17, size = 51, normalized size = 0.98

$$\frac{6\,a\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{b^3}-\frac{2\,a\log\left(|x|\right)}{b^3}-\frac{3\left(2\,ax^{\frac{1}{3}}+b\right)}{\left(ax^{\frac{2}{3}}+bx^{\frac{1}{3}}\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))^2/x^2,x, algorithm="giac")
```

```
[Out] 6*a*log(abs(a*x^(1/3) + b))/b^3 - 2*a*log(abs(x))/b^3 - 3*(2*a*x^(1/3) + b)
/((a*x^(2/3) + b*x^(1/3))*b^2)
```

maple [A] time = 0.01, size = 47, normalized size = 0.90

$$-\frac{3a}{\left(a\,x^{\frac{1}{3}}+b\right)b^2}-\frac{2a\ln(x)}{b^3}+\frac{6a\ln\left(a\,x^{\frac{1}{3}}+b\right)}{b^3}-\frac{3}{b^2x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^(1/3))^2/x^2,x)
```

```
[Out] -3*a/b^2/(a*x^(1/3)+b)-3/b^2/x^(1/3)+6*a*ln(a*x^(1/3)+b)/b^3-2*a/b^3*ln(x)
```

maxima [A] time = 0.48, size = 44, normalized size = 0.85

$$\frac{6\,a\log\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{b^3}-\frac{3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{b^3}+\frac{3\,a^2}{\left(a+\frac{b}{x^{\frac{1}{3}}}\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))^2/x^2,x, algorithm="maxima")
```

```
[Out] 6*a*log(a + b/x^(1/3))/b^3 - 3*(a + b/x^(1/3))/b^3 + 3*a^2/((a + b/x^(1/3))
*b^3)
```

mupad **[B]** time = 0.08, size = 49, normalized size = 0.94

$$\frac{12\,a\,\operatorname{atanh}\left(\frac{2\,a\,x^{1/3}}{b}+1\right)}{b^3}-\frac{\frac{3}{b}+\frac{6\,a\,x^{1/3}}{b^2}}{a\,x^{2/3}+b\,x^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b/x^(1/3))^2),x)
```

```
[Out] (12*a*atanh((2*a*x^(1/3))/b + 1))/b^3 - (3/b + (6*a*x^(1/3))/b^2)/(a*x^(2/3
) + b*x^(1/3))
```

sympy **[A]** time = 2.79, size = 211, normalized size = 4.06

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt[3]{x}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{1}{a^2 x} & \text{for } b = 0 \\ -\frac{3}{b^2\sqrt[3]{x}} & \text{for } a = 0 \\ -\frac{2a^2x^2\log(x)}{ab^3x^2+b^4x^{\frac{5}{3}}}+\frac{6a^2x^2\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{ab^3x^2+b^4x^{\frac{5}{3}}}-\frac{2abx^{\frac{5}{3}}\log(x)}{ab^3x^2+b^4x^{\frac{5}{3}}}+\frac{6abx^{\frac{5}{3}}\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{ab^3x^2+b^4x^{\frac{5}{3}}}-\frac{6abx^{\frac{5}{3}}}{ab^3x^2+b^4x^{\frac{5}{3}}}-\frac{3b^2x^{\frac{4}{3}}}{ab^3x^2+b^4x^{\frac{5}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**(1/3))**2/x**2,x)
```

```
[Out] Piecewise((zoo/x**(1/3), Eq(a, 0) & Eq(b, 0)), (-1/(a**2*x), Eq(b, 0)), (-3
/(b**2*x**(1/3)), Eq(a, 0)), (-2*a**2*x**2*log(x)/(a*b**3*x**2 + b**4*x**(5
/3)) + 6*a**2*x**2*log(x**(1/3) + b/a)/(a*b**3*x**2 + b**4*x**(5/3)) - 2*a*
b*x**(5/3)*log(x)/(a*b**3*x**2 + b**4*x**(5/3)) + 6*a*b*x**(5/3)*log(x**(1/
3) + b/a)/(a*b**3*x**2 + b**4*x**(5/3)) - 6*a*b*x**(5/3)/(a*b**3*x**2 + b**
4*x**(5/3)) - 3*b**2*x**(4/3)/(a*b**3*x**2 + b**4*x**(5/3)), True))
```

3.2434 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^3}\,dx$

Optimal. Leaf size=97

$$-\frac{15a^4\log\left(a\sqrt[3]{x}+b\right)}{b^6}+\frac{5a^4\log(x)}{b^6}+\frac{3a^4}{b^5\left(a\sqrt[3]{x}+b\right)}+\frac{12a^3}{b^5\sqrt[3]{x}}-\frac{9a^2}{2b^4x^{2/3}}+\frac{2a}{b^3x}-\frac{3}{4b^2x^{4/3}}$$

```
[Out] 3*a^4/b^5/(b+a*x^(1/3))-3/4/b^2/x^(4/3)+2*a/b^3/x-9/2*a^2/b^4/x^(2/3)+12*a^
3/b^5/x^(1/3)-15*a^4*ln(b+a*x^(1/3))/b^6+5*a^4*ln(x)/b^6
```

Rubi [A] time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {263, 266, 44}

$$-\frac{9a^2}{2b^4x^{2/3}}+\frac{3a^4}{b^5\left(a\sqrt[3]{x}+b\right)}+\frac{12a^3}{b^5\sqrt[3]{x}}-\frac{15a^4\log\left(a\sqrt[3]{x}+b\right)}{b^6}+\frac{5a^4\log(x)}{b^6}+\frac{2a}{b^3x}-\frac{3}{4b^2x^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^(1/3))^2*x^3),x]
```

```
[Out] (3*a^4)/(b^5*(b + a*x^(1/3))) - 3/(4*b^2*x^(4/3)) + (2*a)/(b^3*x) - (9*a^2)
/(2*b^4*x^(2/3)) + (12*a^3)/(b^5*x^(1/3)) - (15*a^4*Log[b + a*x^(1/3)])/b^6
 + (5*a^4*Log[x])/b^6
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^3}\,dx &= \int \frac{1}{\left(b+a\sqrt[3]{x}\right)^2 x^{7/3}}\,dx \\
&= 3\,\text{Subst}\left(\int \frac{1}{x^5(b+ax)^2}\,dx, x, \sqrt[3]{x}\right) \\
&= 3\,\text{Subst}\left(\int \left(\frac{1}{b^2x^5}-\frac{2a}{b^3x^4}+\frac{3a^2}{b^4x^3}-\frac{4a^3}{b^5x^2}+\frac{5a^4}{b^6x}-\frac{a^5}{b^5(b+ax)^2}-\frac{5a^5}{b^6(b+ax)}\right)dx, x, \sqrt[3]{x}\right) \\
&= \frac{3a^4}{b^5\left(b+a\sqrt[3]{x}\right)}-\frac{3}{4b^2x^{4/3}}+\frac{2a}{b^3x}-\frac{9a^2}{2b^4x^{2/3}}+\frac{12a^3}{b^5\sqrt[3]{x}}-\frac{15a^4\log\left(b+a\sqrt[3]{x}\right)}{b^6}+\frac{5a^4\log(x)}{b^6}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 82, normalized size = 0.85

$$-\frac{\frac{12a^5}{a+\frac{b}{\sqrt[3]{x}}}+60a^4\log\left(a+\frac{b}{\sqrt[3]{x}}\right)-\frac{48a^3b}{\sqrt[3]{x}}+\frac{18a^2b^2}{x^{2/3}}-\frac{8ab^3}{x}+\frac{3b^4}{x^{4/3}}}{4b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^(1/3))^2*x^3),x]
```

```
[Out] -1/4*((12*a^5)/(a + b/x^(1/3)) + (3*b^4)/x^(4/3) - (8*a*b^3)/x + (18*a^2*b^
2)/x^(2/3) - (48*a^3*b)/x^(1/3) + 60*a^4*Log[a + b/x^(1/3)])/b^6
```

fricas [A] time = 0.86, size = 148, normalized size = 1.53

$$\frac{20\,a^4b^3x^2+8\,ab^6x-60\left(a^7x^3+a^4b^3x^2\right)\log\left(ax^{\frac{1}{3}}+b\right)+60\left(a^7x^3+a^4b^3x^2\right)\log\left(x^{\frac{1}{3}}\right)+3\left(20\,a^6bx^2+15\,a^3b^4x\right.}{4\left(a^3b^6x^3+b^9x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(20*a^4*b^3*x^2 + 8*a*b^6*x - 60*(a^7*x^3 + a^4*b^3*x^2)*log(a*x^(1/3)
+ b) + 60*(a^7*x^3 + a^4*b^3*x^2)*log(x^(1/3)) + 3*(20*a^6*b*x^2 + 15*a^3*b
^4*x - b^7)*x^(2/3) - 6*(5*a^5*b^2*x^2 + 3*a^2*b^5*x)*x^(1/3))/(a^3*b^6*x^3
 + b^9*x^2)
```

giac [A] time = 0.16, size = 90, normalized size = 0.93

$$-\frac{15\,a^4\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{b^6}+\frac{5\,a^4\log\left(|x|\right)}{b^6}+\frac{60\,a^4bx^{\frac{4}{3}}+30\,a^3b^2x-10\,a^2b^3x^{\frac{2}{3}}+5\,ab^4x^{\frac{1}{3}}-3\,b^5}{4\left(ax^{\frac{1}{3}}+b\right)b^6x^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))^2/x^3,x, algorithm="giac")
```

```
[Out] -15*a^4*log(abs(a*x^(1/3) + b))/b^6 + 5*a^4*log(abs(x))/b^6 + 1/4*(60*a^4*b
*x^(4/3) + 30*a^3*b^2*x - 10*a^2*b^3*x^(2/3) + 5*a*b^4*x^(1/3) - 3*b^5)/((a
*x^(1/3) + b)*b^6*x^(4/3))
```

maple [A] time = 0.01, size = 84, normalized size = 0.87

$$\frac{3a^4}{\left(a\,x^{\frac{1}{3}}+b\right)b^5}+\frac{5a^4\ln(x)}{b^6}-\frac{15a^4\ln\left(a\,x^{\frac{1}{3}}+b\right)}{b^6}+\frac{12a^3}{b^5x^{\frac{1}{3}}}-\frac{9a^2}{2b^4x^{\frac{2}{3}}}+\frac{2a}{b^3x}-\frac{3}{4b^2x^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^(1/3))^2/x^3,x)
```

```
[Out] 3*a^4/b^5/(a*x^(1/3)+b)-3/4/b^2/x^(4/3)+2*a/b^3/x-9/2*a^2/b^4/x^(2/3)+12*a^
3/b^5/x^(1/3)-15*a^4*ln(a*x^(1/3)+b)/b^6+5*a^4/b^6*ln(x)
```

maxima [A] time = 0.65, size = 95, normalized size = 0.98

$$-\frac{15\,a^4\log\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{b^6}-\frac{3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^4}{4\,b^6}+\frac{5\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^3a}{b^6}-\frac{15\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^2a^2}{b^6}+\frac{30\left(a+\frac{b}{x^{\frac{1}{3}}}\right)a^3}{b^6}-\frac{3\,a^5}{\left(a+\frac{b}{x^{\frac{1}{3}}}\right)b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))^2/x^3,x, algorithm="maxima")
```

```
[Out] -15*a^4*log(a + b/x^(1/3))/b^6 - 3/4*(a + b/x^(1/3))^4/b^6 + 5*(a + b/x^(1/
3))^3*a/b^6 - 15*(a + b/x^(1/3))^2*a^2/b^6 + 30*(a + b/x^(1/3))*a^3/b^6 - 3
*a^5/((a + b/x^(1/3))*b^6)
```

mupad **[B]** time = 1.17, size = 81, normalized size = 0.84

$$\frac{\frac{5\,a\,x^{1/3}}{4\,b^2}-\frac{3}{4\,b}+\frac{15\,a^3\,x}{2\,b^4}-\frac{5\,a^2\,x^{2/3}}{2\,b^3}+\frac{15\,a^4\,x^{4/3}}{b^5}}{a\,x^{5/3}+b\,x^{4/3}}-\frac{30\,a^4\,\operatorname{atanh}\left(\frac{2\,a\,x^{1/3}}{b}+1\right)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b/x^(1/3))^2),x)
```

```
[Out] ((5*a*x^(1/3))/(4*b^2) - 3/(4*b) + (15*a^3*x)/(2*b^4) - (5*a^2*x^(2/3))/(2*
b^3) + (15*a^4*x^(4/3))/b^5)/(a*x^(5/3) + b*x^(4/3)) - (30*a^4*atanh((2*a*x
^(1/3))/b + 1))/b^6
```

sympy **[A]** time = 7.66, size = 340, normalized size = 3.51

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{4}{3}}} \\ -\frac{1}{2a^2x^2} \\ -\frac{3}{4b^2x^{\frac{4}{3}}} \\ \frac{20a^5x^3\log(x)}{4ab^6x^3+4b^7x^{\frac{8}{3}}}-\frac{60a^5x^3\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{4ab^6x^3+4b^7x^{\frac{8}{3}}}+\frac{20a^4bx^{\frac{8}{3}}\log(x)}{4ab^6x^3+4b^7x^{\frac{8}{3}}}-\frac{60a^4bx^{\frac{8}{3}}\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{4ab^6x^3+4b^7x^{\frac{8}{3}}}+\frac{60a^4bx^{\frac{8}{3}}}{4ab^6x^3+4b^7x^{\frac{8}{3}}}+\frac{30a^3b^2x^{\frac{7}{3}}}{4ab^6x^3+4b^7x^{\frac{8}{3}}}-\frac{10a^2b^3x^2}{4ab^6x^3+4b^7x^{\frac{8}{3}}}+ \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**(1/3))**2/x**3,x)
```

```
[Out] Piecewise((zoo/x**(4/3), Eq(a, 0) & Eq(b, 0)), (-1/(2*a**2*x**2), Eq(b, 0))
, (-3/(4*b**2*x**(4/3)), Eq(a, 0)), (20*a**5*x**3*log(x)/(4*a*b**6*x**3 + 4
*b**7*x**(8/3)) - 60*a**5*x**3*log(x**(1/3) + b/a)/(4*a*b**6*x**3 + 4*b**7*
x**(8/3)) + 20*a**4*b*x**(8/3)*log(x)/(4*a*b**6*x**3 + 4*b**7*x**(8/3)) - 6
0*a**4*b*x**(8/3)*log(x**(1/3) + b/a)/(4*a*b**6*x**3 + 4*b**7*x**(8/3)) + 6
0*a**4*b*x**(8/3)/(4*a*b**6*x**3 + 4*b**7*x**(8/3)) + 30*a**3*b**2*x**(7/3)
/(4*a*b**6*x**3 + 4*b**7*x**(8/3)) - 10*a**2*b**3*x**2/(4*a*b**6*x**3 + 4*b
**7*x**(8/3)) + 5*a*b**4*x**(5/3)/(4*a*b**6*x**3 + 4*b**7*x**(8/3)) - 3*b**
5*x**(4/3)/(4*a*b**6*x**3 + 4*b**7*x**(8/3)), True))
```

3.2435 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^4}\,dx$

Optimal. Leaf size=133

$$\frac{24a^7\log\left(a\sqrt[3]{x}+b\right)}{b^9}-\frac{8a^7\log(x)}{b^9}-\frac{3a^7}{b^8\left(a\sqrt[3]{x}+b\right)}-\frac{21a^6}{b^8\sqrt[3]{x}}+\frac{9a^5}{b^7x^{2/3}}-\frac{5a^4}{b^6x}+\frac{3a^3}{b^5x^{4/3}}-\frac{9a^2}{5b^4x^{5/3}}+\frac{a}{b^3x^2}-\frac{3}{7b^2x^{7/3}}$$

```
[Out] -3*a^7/b^8/(b+a*x^(1/3))-3/7/b^2/x^(7/3)+a/b^3/x^2-9/5*a^2/b^4/x^(5/3)+3*a^
3/b^5/x^(4/3)-5*a^4/b^6/x+9*a^5/b^7/x^(2/3)-21*a^6/b^8/x^(1/3)+24*a^7*ln(b+
a*x^(1/3))/b^9-8*a^7*ln(x)/b^9
```

Rubi [A] time = 0.10, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {263, 266, 44}

$$\frac{9a^5}{b^7x^{2/3}}+\frac{3a^3}{b^5x^{4/3}}-\frac{9a^2}{5b^4x^{5/3}}-\frac{3a^7}{b^8\left(a\sqrt[3]{x}+b\right)}-\frac{21a^6}{b^8\sqrt[3]{x}}-\frac{5a^4}{b^6x}+\frac{24a^7\log\left(a\sqrt[3]{x}+b\right)}{b^9}-\frac{8a^7\log(x)}{b^9}+\frac{a}{b^3x^2}-\frac{3}{7b^2x^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^(1/3))^2*x^4),x]
```

```
[Out] (-3*a^7)/(b^8*(b + a*x^(1/3))) - 3/(7*b^2*x^(7/3)) + a/(b^3*x^2) - (9*a^2)/
(5*b^4*x^(5/3)) + (3*a^3)/(b^5*x^(4/3)) - (5*a^4)/(b^6*x) + (9*a^5)/(b^7*x^
(2/3)) - (21*a^6)/(b^8*x^(1/3)) + (24*a^7*Log[b + a*x^(1/3)])/b^9 - (8*a^7*
Log[x])/b^9
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^4}\,dx = \int \frac{1}{\left(b+a\sqrt[3]{x}\right)^2 x^{10/3}}\,dx$$

$$= 3\,\text{Subst}\left(\int \frac{1}{x^8(b+ax)^2}\,dx, x, \sqrt[3]{x}\right)$$

$$= 3\,\text{Subst}\left(\int \left(\frac{1}{b^2x^8} - \frac{2a}{b^3x^7} + \frac{3a^2}{b^4x^6} - \frac{4a^3}{b^5x^5} + \frac{5a^4}{b^6x^4} - \frac{6a^5}{b^7x^3} + \frac{7a^6}{b^8x^2} - \frac{8a^7}{b^9x} + \frac{a^8}{b^8(b+ax)^2} + \frac{}{b^9}\right.\right.$$

$$= -\frac{3a^7}{b^8\left(b+a\sqrt[3]{x}\right)} - \frac{3}{7b^2x^{7/3}} + \frac{a}{b^3x^2} - \frac{9a^2}{5b^4x^{5/3}} + \frac{3a^3}{b^5x^{4/3}} - \frac{5a^4}{b^6x} + \frac{9a^5}{b^7x^{2/3}} - \frac{21a^6}{b^8\sqrt[3]{x}} + \frac{24a^7\log\left(}{b^}$$

Mathematica [A] time = 0.13, size = 123, normalized size = 0.92

$$\frac{3a^8}{b^9\left(a+\frac{b}{\sqrt[3]{x}}\right)} + \frac{24a^7\log\left(a+\frac{b}{\sqrt[3]{x}}\right)}{b^9} - \frac{21a^6}{b^8\sqrt[3]{x}} + \frac{9a^5}{b^7x^{2/3}} - \frac{5a^4}{b^6x} + \frac{3a^3}{b^5x^{4/3}} - \frac{9a^2}{5b^4x^{5/3}} + \frac{a}{b^3x^2} - \frac{3}{7b^2x^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^(1/3))^2*x^4),x]
```

```
[Out] (3*a^8)/(b^9*(a + b/x^(1/3))) - 3/(7*b^2*x^(7/3)) + a/(b^3*x^2) - (9*a^2)/(
5*b^4*x^(5/3)) + (3*a^3)/(b^5*x^(4/3)) - (5*a^4)/(b^6*x) + (9*a^5)/(b^7*x^(
2/3)) - (21*a^6)/(b^8*x^(1/3)) + (24*a^7*Log[a + b/x^(1/3)])/b^9
```

fricas [A] time = 1.20, size = 179, normalized size = 1.35

$$-\frac{280\,a^7b^3x^3 + 140\,a^4b^6x^2 - 35\,ab^9x - 840\left(a^{10}x^4 + a^7b^3x^3\right)\log\left(ax^{\frac{1}{3}} + b\right) + 840\left(a^{10}x^4 + a^7b^3x^3\right)\log\left(x^{\frac{1}{3}}\right) + 15\left(}{35\left(a^3b^9x^4 + b^{12}x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))^2/x^4,x, algorithm="fricas")
```

```
[Out] -1/35*(280*a^7*b^3*x^3 + 140*a^4*b^6*x^2 - 35*a*b^9*x - 840*(a^10*x^4 + a^7
*b^3*x^3)*log(a*x^(1/3) + b) + 840*(a^10*x^4 + a^7*b^3*x^3)*log(x^(1/3)) +
15*(56*a^9*b*x^3 + 42*a^6*b^4*x^2 - 6*a^3*b^7*x + b^10)*x^(2/3) - 21*(20*a^
8*b^2*x^3 + 12*a^5*b^5*x^2 - 3*a^2*b^8*x)*x^(1/3))/(a^3*b^9*x^4 + b^12*x^3)
```

giac [A] time = 0.17, size = 123, normalized size = 0.92

$$\frac{24\,a^7\log\left(\left|ax^{\frac{1}{3}} + b\right|\right)}{b^9} - \frac{8\,a^7\log\left(|x|\right)}{b^9} - \frac{840\,a^7bx^{\frac{7}{3}} + 420\,a^6b^2x^2 - 140\,a^5b^3x^{\frac{5}{3}} + 70\,a^4b^4x^{\frac{4}{3}} - 42\,a^3b^5x + 28\,a^2b^6x^{\frac{2}{3}} -}{35\left(ax^{\frac{1}{3}} + b\right)b^9x^{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))^2/x^4,x, algorithm="giac")
```

```
[Out] 24*a^7*log(abs(a*x^(1/3) + b))/b^9 - 8*a^7*log(abs(x))/b^9 - 1/35*(840*a^7*
b*x^(7/3) + 420*a^6*b^2*x^2 - 140*a^5*b^3*x^(5/3) + 70*a^4*b^4*x^(4/3) - 42
*a^3*b^5*x + 28*a^2*b^6*x^(2/3) - 20*a*b^7*x^(1/3) + 15*b^8)/((a*x^(1/3) +
b)*b^9*x^(7/3))
```

maple [A] time = 0.01, size = 116, normalized size = 0.87

$$-\frac{3a^7}{\left(a\,x^{\frac{1}{3}}+b\right)b^8}-\frac{8a^7\ln(x)}{b^9}+\frac{24a^7\ln\left(a\,x^{\frac{1}{3}}+b\right)}{b^9}-\frac{21a^6}{b^8x^{\frac{1}{3}}}+\frac{9a^5}{b^7x^{\frac{2}{3}}}-\frac{5a^4}{b^6x}+\frac{3a^3}{b^5x^{\frac{4}{3}}}-\frac{9a^2}{5b^4x^{\frac{5}{3}}}+\frac{a}{b^3x^2}-\frac{3}{7b^2x^{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^(1/3))^2/x^4,x)
```

```
[Out] -3*a^7/b^8/(a*x^(1/3)+b)-3/7/b^2/x^(7/3)+a/b^3/x^2-9/5*a^2/b^4/x^(5/3)+3*a^
3/b^5/x^(4/3)-5*a^4/b^6/x+9*a^5/b^7/x^(2/3)-21*a^6/b^8/x^(1/3)+24*a^7*ln(a*
x^(1/3)+b)/b^9-8*a^7*ln(x)/b^9
```

maxima [A] time = 0.50, size = 146, normalized size = 1.10

$$\frac{24\,a^7\log\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{b^9}-\frac{3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^7}{7\,b^9}+\frac{4\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^6a}{b^9}-\frac{84\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^5a^2}{5\,b^9}+\frac{42\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^4a^3}{b^9}-\frac{70\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^3a^4}{b^9}+\frac{84\left(a+\frac{b}{x}\right.}{b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))^2/x^4,x, algorithm="maxima")
```

```
[Out] 24*a^7*log(a + b/x^(1/3))/b^9 - 3/7*(a + b/x^(1/3))^7/b^9 + 4*(a + b/x^(1/3
))^6*a/b^9 - 84/5*(a + b/x^(1/3))^5*a^2/b^9 + 42*(a + b/x^(1/3))^4*a^3/b^9
- 70*(a + b/x^(1/3))^3*a^4/b^9 + 84*(a + b/x^(1/3))^2*a^5/b^9 - 84*(a + b/x
^(1/3))*a^6/b^9 + 3*a^8/((a + b/x^(1/3))*b^9)
```

mupad [B] time = 1.19, size = 115, normalized size = 0.86

$$\frac{48\,a^7\,\operatorname{atanh}\left(\frac{2\,a\,x^{1/3}}{b}+1\right)}{b^9}-\frac{\frac{3}{7\,b}-\frac{4\,a\,x^{1/3}}{7\,b^2}-\frac{6\,a^3\,x}{5\,b^4}+\frac{4\,a^2\,x^{2/3}}{5\,b^3}+\frac{12\,a^6\,x^2}{b^7}+\frac{2\,a^4\,x^{4/3}}{b^5}-\frac{4\,a^5\,x^{5/3}}{b^6}+\frac{24\,a^7\,x^{7/3}}{b^8}}{a\,x^{8/3}+b\,x^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b/x^(1/3))^2),x)
```

```
[Out] (48*a^7*atanh((2*a*x^(1/3))/b + 1))/b^9 - (3/(7*b) - (4*a*x^(1/3))/(7*b^2)
- (6*a^3*x)/(5*b^4) + (4*a^2*x^(2/3))/(5*b^3) + (12*a^6*x^2)/b^7 + (2*a^4*x
^(4/3))/b^5 - (4*a^5*x^(5/3))/b^6 + (24*a^7*x^(7/3))/b^8)/(a*x^(8/3) + b*x^
(7/3))
```

sympy [A] time = 18.16, size = 440, normalized size = 3.31

$$\begin{cases}\frac{\tilde{\infty}}{x^{\frac{7}{3}}} \\ -\frac{3}{7b^2x^{\frac{7}{3}}} \\ -\frac{1}{3a^2x^3} \\ -\frac{280a^8x^4\log(x)}{35ab^9x^4+35b^{10}x^{\frac{11}{3}}}+\frac{840a^8x^4\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{35ab^9x^4+35b^{10}x^{\frac{11}{3}}}-\frac{280a^7bx^{\frac{11}{3}}\log(x)}{35ab^9x^4+35b^{10}x^{\frac{11}{3}}}+\frac{840a^7bx^{\frac{11}{3}}\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{35ab^9x^4+35b^{10}x^{\frac{11}{3}}}-\frac{840a^7bx^{\frac{11}{3}}}{35ab^9x^4+35b^{10}x^{\frac{11}{3}}}-\frac{420a^6b^2x^{\frac{10}{3}}}{35ab^9x^4+35b^{10}x^{\frac{11}{3}}}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**(1/3))**2/x**4,x)
```

```
[Out] Piecewise((zoo/x**(7/3), Eq(a, 0) & Eq(b, 0)), (-3/(7*b**2*x**(7/3)), Eq(a,
 0)), (-1/(3*a**2*x**3), Eq(b, 0)), (-280*a**8*x**4*log(x)/(35*a*b**9*x**4
+ 35*b**10*x**(11/3)) + 840*a**8*x**4*log(x**(1/3) + b/a)/(35*a*b**9*x**4 +
 35*b**10*x**(11/3)) - 280*a**7*b*x**(11/3)*log(x)/(35*a*b**9*x**4 + 35*b**
10*x**(11/3)) + 840*a**7*b*x**(11/3)*log(x**(1/3) + b/a)/(35*a*b**9*x**4 +
35*b**10*x**(11/3)) - 840*a**7*b*x**(11/3)/(35*a*b**9*x**4 + 35*b**10*x**(1
1/3)) - 420*a**6*b**2*x**(10/3)/(35*a*b**9*x**4 + 35*b**10*x**(11/3)) + 140
*a**5*b**3*x**3/(35*a*b**9*x**4 + 35*b**10*x**(11/3)) - 70*a**4*b**4*x**(8/
3)/(35*a*b**9*x**4 + 35*b**10*x**(11/3)) + 42*a**3*b**5*x**(7/3)/(35*a*b**9
*x**4 + 35*b**10*x**(11/3)) - 28*a**2*b**6*x**2/(35*a*b**9*x**4 + 35*b**10*
x**(11/3)) + 20*a*b**7*x**(5/3)/(35*a*b**9*x**4 + 35*b**10*x**(11/3)) - 15*
b**8*x**(4/3)/(35*a*b**9*x**4 + 35*b**10*x**(11/3)), True))
```

3.2436 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^5}\,dx$

Optimal. Leaf size=183

$$-\frac{33a^{10}\log\left(a\sqrt[3]{x}+b\right)}{b^{12}}+\frac{11a^{10}\log(x)}{b^{12}}+\frac{3a^{10}}{b^{11}\left(a\sqrt[3]{x}+b\right)}+\frac{30a^9}{b^{11}\sqrt[3]{x}}-\frac{27a^8}{2b^{10}x^{2/3}}+\frac{8a^7}{b^9x}-\frac{21a^6}{4b^8x^{4/3}}+\frac{18a^5}{5b^7x^{5/3}}-\frac{5a^4}{2b^6x^2}+\frac{12a}{7b^5x}$$

```
[Out] 3*a^10/b^11/(b+a*x^(1/3))-3/10/b^2/x^(10/3)+2/3*a/b^3/x^3-9/8*a^2/b^4/x^(8/
3)+12/7*a^3/b^5/x^(7/3)-5/2*a^4/b^6/x^2+18/5*a^5/b^7/x^(5/3)-21/4*a^6/b^8/x
^(4/3)+8*a^7/b^9/x-27/2*a^8/b^10/x^(2/3)+30*a^9/b^11/x^(1/3)-33*a^10*ln(b+a
*x^(1/3))/b^12+11*a^10*ln(x)/b^12
```

Rubi [A] time = 0.13, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {263, 266, 44}

$$-\frac{27a^8}{2b^{10}x^{2/3}}-\frac{21a^6}{4b^8x^{4/3}}+\frac{18a^5}{5b^7x^{5/3}}-\frac{5a^4}{2b^6x^2}+\frac{12a^3}{7b^5x^{7/3}}-\frac{9a^2}{8b^4x^{8/3}}+\frac{3a^{10}}{b^{11}\left(a\sqrt[3]{x}+b\right)}+\frac{30a^9}{b^{11}\sqrt[3]{x}}+\frac{8a^7}{b^9x}-\frac{33a^{10}\log\left(a\sqrt[3]{x}+b\right)}{b^{12}}+$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^(1/3))^2*x^5),x]
```

```
[Out] (3*a^10)/(b^11*(b + a*x^(1/3))) - 3/(10*b^2*x^(10/3)) + (2*a)/(3*b^3*x^3) -
 (9*a^2)/(8*b^4*x^(8/3)) + (12*a^3)/(7*b^5*x^(7/3)) - (5*a^4)/(2*b^6*x^2) +
 (18*a^5)/(5*b^7*x^(5/3)) - (21*a^6)/(4*b^8*x^(4/3)) + (8*a^7)/(b^9*x) - (2
7*a^8)/(2*b^10*x^(2/3)) + (30*a^9)/(b^11*x^(1/3)) - (33*a^10*Log[b + a*x^(1
/3)])/b^12 + (11*a^10*Log[x])/b^12
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^5}\,dx = \int \frac{1}{\left(b+a\sqrt[3]{x}\right)^2 x^{13/3}}\,dx$$

$$= 3\,\text{Subst}\left(\int \frac{1}{x^{11}(b+ax)^2}\,dx, x, \sqrt[3]{x}\right)$$

$$= 3\,\text{Subst}\left(\int \left(\frac{1}{b^2x^{11}} - \frac{2a}{b^3x^{10}} + \frac{3a^2}{b^4x^9} - \frac{4a^3}{b^5x^8} + \frac{5a^4}{b^6x^7} - \frac{6a^5}{b^7x^6} + \frac{7a^6}{b^8x^5} - \frac{8a^7}{b^9x^4} + \frac{9a^8}{b^{10}x^3} - \frac{10a^9}{b^{11}x^2}\right.\right.$$

$$= \frac{3a^{10}}{b^{11}\left(b+a\sqrt[3]{x}\right)} - \frac{3}{10b^2x^{10/3}} + \frac{2a}{3b^3x^3} - \frac{9a^2}{8b^4x^{8/3}} + \frac{12a^3}{7b^5x^{7/3}} - \frac{5a^4}{2b^6x^2} + \frac{18a^5}{5b^7x^{5/3}} - \frac{21a^6}{4b^8x^{4/3}} + \frac{8}{b^9}$$

Mathematica [A] time = 0.16, size = 173, normalized size = 0.95

$$-\frac{3a^{11}}{b^{12}\left(a+\frac{b}{\sqrt[3]{x}}\right)} - \frac{33a^{10}\log\left(a+\frac{b}{\sqrt[3]{x}}\right)}{b^{12}} + \frac{30a^9}{b^{11}\sqrt[3]{x}} - \frac{27a^8}{2b^{10}x^{2/3}} + \frac{8a^7}{b^9x} - \frac{21a^6}{4b^8x^{4/3}} + \frac{18a^5}{5b^7x^{5/3}} - \frac{5a^4}{2b^6x^2} + \frac{12a^3}{7b^5x^{7/3}} - \frac{9a^2}{8b^4x^{8/3}} + \frac{2a}{3b^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^(1/3))^2*x^5),x]
```

```
[Out] (-3*a^11)/(b^12*(a + b/x^(1/3))) - 3/(10*b^2*x^(10/3)) + (2*a)/(3*b^3*x^3)
- (9*a^2)/(8*b^4*x^(8/3)) + (12*a^3)/(7*b^5*x^(7/3)) - (5*a^4)/(2*b^6*x^2)
+ (18*a^5)/(5*b^7*x^(5/3)) - (21*a^6)/(4*b^8*x^(4/3)) + (8*a^7)/(b^9*x) - (
27*a^8)/(2*b^10*x^(2/3)) + (30*a^9)/(b^11*x^(1/3)) - (33*a^10*Log[a + b/x^(
1/3)])/b^12
```

fricas [A] time = 0.73, size = 214, normalized size = 1.17

$$\frac{9240\,a^{10}b^3x^4 + 4620\,a^7b^6x^3 - 1540\,a^4b^9x^2 + 560\,ab^{12}x - 27720\left(a^{13}x^5 + a^{10}b^3x^4\right)\log\left(ax^{\frac{1}{3}} + b\right) + 27720\left(a^{13}x^5 + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/840*(9240*a^10*b^3*x^4 + 4620*a^7*b^6*x^3 - 1540*a^4*b^9*x^2 + 560*a*b^12
*x - 27720*(a^13*x^5 + a^10*b^3*x^4)*log(a*x^(1/3) + b) + 27720*(a^13*x^5 +
 a^10*b^3*x^4)*log(x^(1/3)) + 18*(1540*a^12*b*x^4 + 1155*a^9*b^4*x^3 - 165*
a^6*b^7*x^2 + 66*a^3*b^10*x - 14*b^13)*x^(2/3) - 63*(220*a^11*b^2*x^4 + 132
*a^8*b^5*x^3 - 33*a^5*b^8*x^2 + 15*a^2*b^11*x)*x^(1/3))/(a^3*b^12*x^5 + b^1
5*x^4)
```

giac [A] time = 0.19, size = 156, normalized size = 0.85

$$-\frac{33\,a^{10}\log\left(\left|ax^{\frac{1}{3}} + b\right|\right)}{b^{12}} + \frac{11\,a^{10}\log\left(|x|\right)}{b^{12}} + \frac{27720\,a^{10}bx^{\frac{10}{3}} + 13860\,a^9b^2x^3 - 4620\,a^8b^3x^{\frac{8}{3}} + 2310\,a^7b^4x^{\frac{7}{3}} - 1386\,a^6b}{840\left(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))^2/x^5,x, algorithm="giac")
```

```
[Out] -33*a^10*log(abs(a*x^(1/3) + b))/b^12 + 11*a^10*log(abs(x))/b^12 + 1/840*(2
7720*a^10*b*x^(10/3) + 13860*a^9*b^2*x^3 - 4620*a^8*b^3*x^(8/3) + 2310*a^7*
```

```
b^4*x^(7/3) - 1386*a^6*b^5*x^2 + 924*a^5*b^6*x^(5/3) - 660*a^4*b^7*x^(4/3)
+ 495*a^3*b^8*x - 385*a^2*b^9*x^(2/3) + 308*a*b^10*x^(1/3) - 252*b^11)/((a*
x^(1/3) + b)*b^12*x^(10/3))
```

maple [A] time = 0.02, size = 150, normalized size = 0.82

$$\frac{3a^{10}}{\left(a\,x^{\frac{1}{3}}+b\right)b^{11}}+\frac{11a^{10}\ln(x)}{b^{12}}-\frac{33a^{10}\ln\left(a\,x^{\frac{1}{3}}+b\right)}{b^{12}}+\frac{30a^9}{b^{11}x^{\frac{1}{3}}}-\frac{27a^8}{2b^{10}x^{\frac{2}{3}}}+\frac{8a^7}{b^9x}-\frac{21a^6}{4b^8x^{\frac{4}{3}}}+\frac{18a^5}{5b^7x^{\frac{5}{3}}}-\frac{5a^4}{2b^6x^2}+\frac{12a^3}{7b^5x^{\frac{7}{3}}}-\frac{9a^2}{8b^4x^{\frac{8}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^(1/3))^2/x^5,x)
```

```
[Out] 3*a^10/b^11/(a*x^(1/3)+b)-3/10/b^2/x^(10/3)+2/3*a/b^3/x^3-9/8*a^2/b^4/x^(8/
3)+12/7*a^3/b^5/x^(7/3)-5/2*a^4/b^6/x^2+18/5*a^5/b^7/x^(5/3)-21/4*a^6/b^8/x
^(4/3)+8*a^7/b^9/x-27/2*a^8/b^10/x^(2/3)+30*a^9/b^11/x^(1/3)-33*a^10*ln(a*x
^(1/3)+b)/b^12+11*a^10*ln(x)/b^12
```

maxima [A] time = 0.49, size = 197, normalized size = 1.08

$$-\frac{33\,a^{10}\log\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{b^{12}}-\frac{3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{10}}{10\,b^{12}}+\frac{11\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{9}a}{3\,b^{12}}-\frac{165\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{8}a^2}{8\,b^{12}}+\frac{495\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{7}a^3}{7\,b^{12}}-\frac{165\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{6}a^4}{b^{12}}+\frac{13}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))^2/x^5,x, algorithm="maxima")
```

```
[Out] -33*a^10*log(a + b/x^(1/3))/b^12 - 3/10*(a + b/x^(1/3))^10/b^12 + 11/3*(a +
 b/x^(1/3))^9*a/b^12 - 165/8*(a + b/x^(1/3))^8*a^2/b^12 + 495/7*(a + b/x^(1
/3))^7*a^3/b^12 - 165*(a + b/x^(1/3))^6*a^4/b^12 + 1386/5*(a + b/x^(1/3))^5
*a^5/b^12 - 693/2*(a + b/x^(1/3))^4*a^6/b^12 + 330*(a + b/x^(1/3))^3*a^7/b^
12 - 495/2*(a + b/x^(1/3))^2*a^8/b^12 + 165*(a + b/x^(1/3))*a^9/b^12 - 3*a^
11/((a + b/x^(1/3))*b^12)
```

mupad [B] time = 0.10, size = 147, normalized size = 0.80

$$\frac{\frac{11\,a\,x^{1/3}}{30\,b^2}-\frac{3}{10\,b}+\frac{33\,a^3\,x}{56\,b^4}-\frac{11\,a^2\,x^{2/3}}{24\,b^3}-\frac{33\,a^6\,x^2}{20\,b^7}-\frac{11\,a^4\,x^{4/3}}{14\,b^5}+\frac{11\,a^5\,x^{5/3}}{10\,b^6}+\frac{33\,a^9\,x^3}{2\,b^{10}}+\frac{11\,a^7\,x^{7/3}}{4\,b^8}-\frac{11\,a^8\,x^{8/3}}{2\,b^9}+\frac{33\,a^{10}\,x^{10/3}}{b^{11}}}{a\,x^{11/3}+b\,x^{10/3}}-\frac{66\,a^{10}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b/x^(1/3))^2),x)
```

```
[Out] ((11*a*x^(1/3))/(30*b^2) - 3/(10*b) + (33*a^3*x)/(56*b^4) - (11*a^2*x^(2/3)
)/(24*b^3) - (33*a^6*x^2)/(20*b^7) - (11*a^4*x^(4/3))/(14*b^5) + (11*a^5*x^
(5/3))/(10*b^6) + (33*a^9*x^3)/(2*b^10) + (11*a^7*x^(7/3))/(4*b^8) - (11*a^
8*x^(8/3))/(2*b^9) + (33*a^10*x^(10/3))/b^11)/(a*x^(11/3) + b*x^(10/3)) - (
66*a^10*atanh((2*a*x^(1/3))/b + 1))/b^12
```

sympy [A] time = 39.25, size = 541, normalized size = 2.96

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{10}{3}}} \\ -\frac{3}{10b^2x^{\frac{10}{3}}} \\ -\frac{1}{4a^2x^4} \\ \frac{9240a^{11}x^5\log(x)}{840ab^{12}x^5+840b^{13}x^{\frac{14}{3}}} - \frac{27720a^{11}x^5\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{840ab^{12}x^5+840b^{13}x^{\frac{14}{3}}} + \frac{9240a^{10}bx^{\frac{14}{3}}\log(x)}{840ab^{12}x^5+840b^{13}x^{\frac{14}{3}}} - \frac{27720a^{10}bx^{\frac{14}{3}}\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{840ab^{12}x^5+840b^{13}x^{\frac{14}{3}}} + \frac{27720a^{10}bx^{\frac{14}{3}}}{840ab^{12}x^5+840b^{13}x^{\frac{14}{3}}} + \frac{1} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**(1/3))**2/x**5,x)
```

```
[Out] Piecewise((zoo/x**(10/3), Eq(a, 0) & Eq(b, 0)), (-3/(10*b**2*x**(10/3)), Eq
(a, 0)), (-1/(4*a**2*x**4), Eq(b, 0)), (9240*a**11*x**5*log(x)/(840*a*b**12
*x**5 + 840*b**13*x**(14/3)) - 27720*a**11*x**5*log(x**(1/3) + b/a)/(840*a*
b**12*x**5 + 840*b**13*x**(14/3)) + 9240*a**10*b*x**(14/3)*log(x)/(840*a*b*
*12*x**5 + 840*b**13*x**(14/3)) - 27720*a**10*b*x**(14/3)*log(x**(1/3) + b/
a)/(840*a*b**12*x**5 + 840*b**13*x**(14/3)) + 27720*a**10*b*x**(14/3)/(840*
a*b**12*x**5 + 840*b**13*x**(14/3)) + 13860*a**9*b**2*x**(13/3)/(840*a*b**1
2*x**5 + 840*b**13*x**(14/3)) - 4620*a**8*b**3*x**4/(840*a*b**12*x**5 + 840
*b**13*x**(14/3)) + 2310*a**7*b**4*x**(11/3)/(840*a*b**12*x**5 + 840*b**13*
x**(14/3)) - 1386*a**6*b**5*x**(10/3)/(840*a*b**12*x**5 + 840*b**13*x**(14/
3)) + 924*a**5*b**6*x**3/(840*a*b**12*x**5 + 840*b**13*x**(14/3)) - 660*a**
4*b**7*x**(8/3)/(840*a*b**12*x**5 + 840*b**13*x**(14/3)) + 495*a**3*b**8*x*
*(7/3)/(840*a*b**12*x**5 + 840*b**13*x**(14/3)) - 385*a**2*b**9*x**2/(840*a
*b**12*x**5 + 840*b**13*x**(14/3)) + 308*a*b**10*x**(5/3)/(840*a*b**12*x**5
 + 840*b**13*x**(14/3)) - 252*b**11*x**(4/3)/(840*a*b**12*x**5 + 840*b**13*
x**(14/3)), True))
```

3.2437 $\int \frac{x^2}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}\,dx$

Optimal. Leaf size=171

$$\frac{3b^{11}}{2a^{12}\left(a\sqrt[3]{x}+b\right)^2}-\frac{33b^{10}}{a^{12}\left(a\sqrt[3]{x}+b\right)}-\frac{165b^9\log\left(a\sqrt[3]{x}+b\right)}{a^{12}}+\frac{135b^8\sqrt[3]{x}}{a^{11}}-\frac{54b^7x^{2/3}}{a^{10}}+\frac{28b^6x}{a^9}-\frac{63b^5x^{4/3}}{4a^8}+\frac{9b^4x^{5/3}}{a^7}-\frac{5b}{a}$$

```
[Out] 3/2*b^11/a^12/(b+a*x^(1/3))^2-33*b^10/a^12/(b+a*x^(1/3))+135*b^8*x^(1/3)/a^
11-54*b^7*x^(2/3)/a^10+28*b^6*x/a^9-63/4*b^5*x^(4/3)/a^8+9*b^4*x^(5/3)/a^7-
5*b^3*x^2/a^6+18/7*b^2*x^(7/3)/a^5-9/8*b*x^(8/3)/a^4+1/3*x^3/a^3-165*b^9*ln
(b+a*x^(1/3))/a^12
```

Rubi [A] time = 0.15, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {263, 266, 43}

$$-\frac{54b^7x^{2/3}}{a^{10}}-\frac{63b^5x^{4/3}}{4a^8}+\frac{9b^4x^{5/3}}{a^7}-\frac{5b^3x^2}{a^6}+\frac{18b^2x^{7/3}}{7a^5}+\frac{3b^{11}}{2a^{12}\left(a\sqrt[3]{x}+b\right)^2}-\frac{33b^{10}}{a^{12}\left(a\sqrt[3]{x}+b\right)}+\frac{135b^8\sqrt[3]{x}}{a^{11}}+\frac{28b^6x}{a^9}-\frac{165b^9}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b/x^(1/3))^3,x]
```

```
[Out] (3*b^11)/(2*a^12*(b + a*x^(1/3))^2) - (33*b^10)/(a^12*(b + a*x^(1/3))) + (1
35*b^8*x^(1/3))/a^11 - (54*b^7*x^(2/3))/a^10 + (28*b^6*x)/a^9 - (63*b^5*x^(
4/3))/(4*a^8) + (9*b^4*x^(5/3))/a^7 - (5*b^3*x^2)/a^6 + (18*b^2*x^(7/3))/(7
*a^5) - (9*b*x^(8/3))/(8*a^4) + x^3/(3*a^3) - (165*b^9*Log[b + a*x^(1/3)])/
a^12
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^2}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}\,dx = \int \frac{x^3}{\left(b+a\sqrt[3]{x}\right)^3}\,dx$$

$$= 3\,\text{Subst}\left(\int \frac{x^{11}}{(b+ax)^3}\,dx, x, \sqrt[3]{x}\right)$$

$$= 3\,\text{Subst}\left(\int \left(\frac{45b^8}{a^{11}} - \frac{36b^7x}{a^{10}} + \frac{28b^6x^2}{a^9} - \frac{21b^5x^3}{a^8} + \frac{15b^4x^4}{a^7} - \frac{10b^3x^5}{a^6} + \frac{6b^2x^6}{a^5} - \frac{3bx^7}{a^4} + \frac{x^8}{a^3} - \right.\right.$$

$$= \frac{3b^{11}}{2a^{12}\left(b+a\sqrt[3]{x}\right)^2} - \frac{33b^{10}}{a^{12}\left(b+a\sqrt[3]{x}\right)} + \frac{135b^8\sqrt[3]{x}}{a^{11}} - \frac{54b^7x^{2/3}}{a^{10}} + \frac{28b^6x}{a^9} - \frac{63b^5x^{4/3}}{4a^8} + \frac{9b^4x^{5/3}}{a^7} -$$

Mathematica [A] time = 0.16, size = 157, normalized size = 0.92

$$\frac{56a^9x^3 - 189a^8bx^{8/3} + 432a^7b^2x^{7/3} - 840a^6b^3x^2 + 1512a^5b^4x^{5/3} - 2646a^4b^5x^{4/3} + 4704a^3b^6x - 9072a^2b^7x^{2/3} + \frac{}{(a}}{168a^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b/x^(1/3))^3,x]
```

```
[Out] ((252*b^11)/(b + a*x^(1/3))^2 - (5544*b^10)/(b + a*x^(1/3)) + 22680*a*b^8*x
^(1/3) - 9072*a^2*b^7*x^(2/3) + 4704*a^3*b^6*x - 2646*a^4*b^5*x^(4/3) + 151
2*a^5*b^4*x^(5/3) - 840*a^6*b^3*x^2 + 432*a^7*b^2*x^(7/3) - 189*a^8*b*x^(8/
3) + 56*a^9*x^3 - 27720*b^9*Log[b + a*x^(1/3)])/(168*a^12)
```

fricas [A] time = 0.85, size = 225, normalized size = 1.32

$$\frac{56\,a^{15}x^5 - 728\,a^{12}b^3x^4 + 3080\,a^9b^6x^3 + 8568\,a^6b^9x^2 - 1344\,a^3b^{12}x - 5292\,b^{15} - 27720\left(a^6b^9x^2 + 2\,a^3b^{12}x + b^{15}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^(1/3))^3,x, algorithm="fricas")
```

```
[Out] 1/168*(56*a^15*x^5 - 728*a^12*b^3*x^4 + 3080*a^9*b^6*x^3 + 8568*a^6*b^9*x^2
 - 1344*a^3*b^12*x - 5292*b^15 - 27720*(a^6*b^9*x^2 + 2*a^3*b^12*x + b^15)*
log(a*x^(1/3) + b) - 63*(3*a^14*b*x^4 - 18*a^11*b^4*x^3 + 99*a^8*b^7*x^2 +
352*a^5*b^10*x + 220*a^2*b^13)*x^(2/3) + 18*(24*a^13*b^2*x^4 - 99*a^10*b^5*
x^3 + 990*a^7*b^8*x^2 + 2695*a^4*b^11*x + 1540*a*b^14)*x^(1/3))/(a^18*x^2 +
 2*a^15*b^3*x + a^12*b^6)
```

giac [A] time = 0.17, size = 145, normalized size = 0.85

$$-\frac{165\,b^9\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{a^{12}} - \frac{3\left(22\,ab^{10}x^{\frac{1}{3}} + 21\,b^{11}\right)}{2\left(ax^{\frac{1}{3}}+b\right)^2a^{12}} + \frac{56\,a^{24}x^3 - 189\,a^{23}bx^{\frac{8}{3}} + 432\,a^{22}b^2x^{\frac{7}{3}} - 840\,a^{21}b^3x^2 + 1512\,a^{20}b}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^(1/3))^3,x, algorithm="giac")
```

```
[Out] -165*b^9*log(abs(a*x^(1/3) + b))/a^12 - 3/2*(22*a*b^10*x^(1/3) + 21*b^11)/(
(a*x^(1/3) + b)^2*a^12) + 1/168*(56*a^24*x^3 - 189*a^23*b*x^(8/3) + 432*a^2
2*b^2*x^(7/3) - 840*a^21*b^3*x^2 + 1512*a^20*b^4*x^(5/3) - 2646*a^19*b^5*x^
```

```
(4/3) + 4704*a^18*b^6*x - 9072*a^17*b^7*x^(2/3) + 22680*a^16*b^8*x^(1/3))/a
^27
```

maple [A] time = 0.01, size = 144, normalized size = 0.84

$$\frac{x^3}{3a^3}-\frac{9b\,x^{\frac{8}{3}}}{8a^4}+\frac{18b^2x^{\frac{7}{3}}}{7a^5}-\frac{5b^3x^2}{a^6}+\frac{3b^{11}}{2\left(a\,x^{\frac{1}{3}}+b\right)^2\,a^{12}}+\frac{9b^4x^{\frac{5}{3}}}{a^7}-\frac{63b^5x^{\frac{4}{3}}}{4a^8}+\frac{28b^6x}{a^9}-\frac{33b^{10}}{\left(a\,x^{\frac{1}{3}}+b\right)a^{12}}-\frac{165b^9\ln\left(a\,x^{\frac{1}{3}}+b\right)}{a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b/x^(1/3))^3,x)
```

```
[Out] 3/2*b^11/a^12/(a*x^(1/3)+b)^2-33*b^10/a^12/(a*x^(1/3)+b)+135*b^8*x^(1/3)/a^
11-54*b^7*x^(2/3)/a^10+28*b^6*x/a^9-63/4*b^5*x^(4/3)/a^8+9*b^4*x^(5/3)/a^7-
5/a^6*b^3*x^2+18/7*b^2*x^(7/3)/a^5-9/8*b*x^(8/3)/a^4+1/3/a^3*x^3-165*b^9*ln
(a*x^(1/3)+b)/a^12
```

maxima [A] time = 0.57, size = 167, normalized size = 0.98

$$\frac{56\,a^{10}-\frac{77\,a^9b}{x^{\frac{1}{3}}}+\frac{110\,a^8b^2}{x^{\frac{2}{3}}}-\frac{165\,a^7b^3}{x}+\frac{264\,a^6b^4}{x^{\frac{4}{3}}}-\frac{462\,a^5b^5}{x^{\frac{5}{3}}}+\frac{924\,a^4b^6}{x^2}-\frac{2310\,a^3b^7}{x^{\frac{7}{3}}}+\frac{9240\,a^2b^8}{x^{\frac{8}{3}}}+\frac{41580\,ab^9}{x^3}+\frac{27720\,b^{10}}{x^{\frac{10}{3}}}-165}{168\left(\frac{a^{13}}{x^3}+\frac{2\,a^{12}b}{x^{\frac{10}{3}}}+\frac{a^{11}b^2}{x^{\frac{11}{3}}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/x^(1/3))^3,x, algorithm="maxima")
```

```
[Out] 1/168*(56*a^10 - 77*a^9*b/x^(1/3) + 110*a^8*b^2/x^(2/3) - 165*a^7*b^3/x + 2
64*a^6*b^4/x^(4/3) - 462*a^5*b^5/x^(5/3) + 924*a^4*b^6/x^2 - 2310*a^3*b^7/x
^(7/3) + 9240*a^2*b^8/x^(8/3) + 41580*a*b^9/x^3 + 27720*b^10/x^(10/3))/(a^1
3/x^3 + 2*a^12*b/x^(10/3) + a^11*b^2/x^(11/3)) - 165*b^9*log(a + b/x^(1/3))
/a^12 - 55*b^9*log(x)/a^12
```

mupad [B] time = 1.15, size = 154, normalized size = 0.90

$$\frac{x^3}{3\,a^3}-\frac{\frac{63\,b^{11}}{2\,a}+33\,b^{10}\,x^{1/3}}{a^{11}\,b^2+a^{13}\,x^{2/3}+2\,a^{12}\,b\,x^{1/3}}-\frac{9\,b\,x^{8/3}}{8\,a^4}+\frac{28\,b^6\,x}{a^9}-\frac{165\,b^9\,\ln\left(b+a\,x^{1/3}\right)}{a^{12}}-\frac{5\,b^3\,x^2}{a^6}+\frac{18\,b^2\,x^{7/3}}{7\,a^5}+\frac{9\,b^4\,x^{5/3}}{a^7}-\frac{63}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b/x^(1/3))^3,x)
```

```
[Out] x^3/(3*a^3) - ((63*b^11)/(2*a) + 33*b^10*x^(1/3))/(a^11*b^2 + a^13*x^(2/3)
+ 2*a^12*b*x^(1/3)) - (9*b*x^(8/3))/(8*a^4) + (28*b^6*x)/a^9 - (165*b^9*log
(b + a*x^(1/3)))/a^12 - (5*b^3*x^2)/a^6 + (18*b^2*x^(7/3))/(7*a^5) + (9*b^4
*x^(5/3))/a^7 - (63*b^5*x^(4/3))/(4*a^8) - (54*b^7*x^(2/3))/a^10 + (135*b^8
*x^(1/3))/a^11
```

sympy [A] time = 6.31, size = 624, normalized size = 3.65

$$\begin{cases}\frac{56a^{11}x^{\frac{11}{3}}}{168a^{14}x^{\frac{2}{3}}+336a^{13}b\sqrt[3]{x}+168a^{12}b^2}-\frac{77a^{10}bx^{\frac{10}{3}}}{168a^{14}x^{\frac{2}{3}}+336a^{13}b\sqrt[3]{x}+168a^{12}b^2}+\frac{110a^9b^2x^3}{168a^{14}x^{\frac{2}{3}}+336a^{13}b\sqrt[3]{x}+168a^{12}b^2}-\frac{165a^8b^3x^{\frac{8}{3}}}{168a^{14}x^{\frac{2}{3}}+336a^{13}b\sqrt[3]{x}+168a^{12}b^2}\\ \frac{x^4}{4b^3}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b/x**(1/3))**3,x)

[Out] Piecewise((56*a**11*x**(11/3)/(168*a**14*x**(2/3) + 336*a**13*b*x**(1/3) +
168*a**12*b**2) - 77*a**10*b*x**(10/3)/(168*a**14*x**(2/3) + 336*a**13*b*x*
*(1/3) + 168*a**12*b**2) + 110*a**9*b**2*x**3/(168*a**14*x**(2/3) + 336*a**
13*b*x**(1/3) + 168*a**12*b**2) - 165*a**8*b**3*x**(8/3)/(168*a**14*x**(2/3
) + 336*a**13*b*x**(1/3) + 168*a**12*b**2) + 264*a**7*b**4*x**(7/3)/(168*a*
*14*x**(2/3) + 336*a**13*b*x**(1/3) + 168*a**12*b**2) - 462*a**6*b**5*x**2/
(168*a**14*x**(2/3) + 336*a**13*b*x**(1/3) + 168*a**12*b**2) + 924*a**5*b**
6*x**(5/3)/(168*a**14*x**(2/3) + 336*a**13*b*x**(1/3) + 168*a**12*b**2) - 2
310*a**4*b**7*x**(4/3)/(168*a**14*x**(2/3) + 336*a**13*b*x**(1/3) + 168*a**
12*b**2) + 9240*a**3*b**8*x/(168*a**14*x**(2/3) + 336*a**13*b*x**(1/3) + 16
8*a**12*b**2) - 27720*a**2*b**9*x**(2/3)*log(x**(1/3) + b/a)/(168*a**14*x**
(2/3) + 336*a**13*b*x**(1/3) + 168*a**12*b**2) - 55440*a*b**10*x**(1/3)*log
(x**(1/3) + b/a)/(168*a**14*x**(2/3) + 336*a**13*b*x**(1/3) + 168*a**12*b**
2) - 55440*a*b**10*x**(1/3)/(168*a**14*x**(2/3) + 336*a**13*b*x**(1/3) + 16
8*a**12*b**2) - 27720*b**11*log(x**(1/3) + b/a)/(168*a**14*x**(2/3) + 336*a
**13*b*x**(1/3) + 168*a**12*b**2) - 41580*b**11/(168*a**14*x**(2/3) + 336*a
**13*b*x**(1/3) + 168*a**12*b**2), Ne(a, 0)), (x**4/(4*b**3), True))
```

3.2438 $\int \frac{x}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}\,dx$

Optimal. Leaf size=134

$$-\frac{3b^8}{2a^9\left(a\sqrt[3]{x}+b\right)^2}+\frac{24b^7}{a^9\left(a\sqrt[3]{x}+b\right)}+\frac{84b^6\log\left(a\sqrt[3]{x}+b\right)}{a^9}-\frac{63b^5\sqrt[3]{x}}{a^8}+\frac{45b^4x^{2/3}}{2a^7}-\frac{10b^3x}{a^6}+\frac{9b^2x^{4/3}}{2a^5}-\frac{9bx^{5/3}}{5a^4}+\frac{x^2}{2a^3}$$

```
[Out] -3/2*b^8/a^9/(b+a*x^(1/3))^2+24*b^7/a^9/(b+a*x^(1/3))-63*b^5*x^(1/3)/a^8+45
/2*b^4*x^(2/3)/a^7-10*b^3*x/a^6+9/2*b^2*x^(4/3)/a^5-9/5*b*x^(5/3)/a^4+1/2*x
^2/a^3+84*b^6*ln(b+a*x^(1/3))/a^9
```

Rubi [A] time = 0.11, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {263, 266, 43}

$$\frac{45b^4x^{2/3}}{2a^7}+\frac{9b^2x^{4/3}}{2a^5}-\frac{3b^8}{2a^9\left(a\sqrt[3]{x}+b\right)^2}+\frac{24b^7}{a^9\left(a\sqrt[3]{x}+b\right)}-\frac{63b^5\sqrt[3]{x}}{a^8}-\frac{10b^3x}{a^6}+\frac{84b^6\log\left(a\sqrt[3]{x}+b\right)}{a^9}-\frac{9bx^{5/3}}{5a^4}+\frac{x^2}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b/x^(1/3))^3,x]
```

```
[Out] (-3*b^8)/(2*a^9*(b + a*x^(1/3))^2) + (24*b^7)/(a^9*(b + a*x^(1/3))) - (63*b
^5*x^(1/3))/a^8 + (45*b^4*x^(2/3))/(2*a^7) - (10*b^3*x)/a^6 + (9*b^2*x^(4/3
))/(2*a^5) - (9*b*x^(5/3))/(5*a^4) + x^2/(2*a^3) + (84*b^6*Log[b + a*x^(1/3
)])/a^9
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}\,dx = \int \frac{x^2}{\left(b+a\sqrt[3]{x}\right)^3}\,dx$$

$$= 3\,\text{Subst}\left(\int \frac{x^8}{(b+ax)^3}\,dx, x, \sqrt[3]{x}\right)$$

$$= 3\,\text{Subst}\left(\int \left(-\frac{21b^5}{a^8}+\frac{15b^4x}{a^7}-\frac{10b^3x^2}{a^6}+\frac{6b^2x^3}{a^5}-\frac{3bx^4}{a^4}+\frac{x^5}{a^3}+\frac{b^8}{a^8(b+ax)^3}-\frac{8b^7}{a^8(b+ax)^2}+\right.\right.$$

$$= -\frac{3b^8}{2a^9\left(b+a\sqrt[3]{x}\right)^2}+\frac{24b^7}{a^9\left(b+a\sqrt[3]{x}\right)}-\frac{63b^5\sqrt[3]{x}}{a^8}+\frac{45b^4x^{2/3}}{2a^7}-\frac{10b^3x}{a^6}+\frac{9b^2x^{4/3}}{2a^5}-\frac{9bx^{5/3}}{5a^4}+\frac{x^2}{2a^3}$$

Mathematica [A] time = 0.16, size = 132, normalized size = 0.99

$$\frac{\frac{a\sqrt[3]{x}\left(5a^7x^{7/3}-8a^6bx^2+14a^5b^2x^{5/3}-28a^4b^3x^{4/3}+70a^3b^4x-280a^2b^5x^{2/3}-1260ab^6\sqrt[3]{x}-840b^7\right)}{\left(a\sqrt[3]{x}+b\right)^2}+840b^6\log\left(a+\frac{b}{\sqrt[3]{x}}\right)+280b^6\log(x)}{10a^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b/x^(1/3))^3,x]
```

```
[Out] ((a*x^(1/3)*(-840*b^7 - 1260*a*b^6*x^(1/3) - 280*a^2*b^5*x^(2/3) + 70*a^3*b
^4*x - 28*a^4*b^3*x^(4/3) + 14*a^5*b^2*x^(5/3) - 8*a^6*b*x^2 + 5*a^7*x^(7/3
)))/(b + a*x^(1/3))^2 + 840*b^6*Log[a + b/x^(1/3)] + 280*b^6*Log[x])/(10*a^
9)
```

fricas [A] time = 0.85, size = 192, normalized size = 1.43

$$\frac{5\,a^{12}x^4-90\,a^9b^3x^3-195\,a^6b^6x^2+170\,a^3b^9x+225\,b^{12}+840\left(a^6b^6x^2+2\,a^3b^9x+b^{12}\right)\log\left(ax^{\frac{1}{3}}+b\right)-3\left(6\,a^{11}bx\right.}{10\left(a^{15}x^2+2\,a^{12}b^3x+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^(1/3))^3,x, algorithm="fricas")
```

```
[Out] 1/10*(5*a^12*x^4 - 90*a^9*b^3*x^3 - 195*a^6*b^6*x^2 + 170*a^3*b^9*x + 225*b
^12 + 840*(a^6*b^6*x^2 + 2*a^3*b^9*x + b^12)*log(a*x^(1/3) + b) - 3*(6*a^11
*b*x^3 - 63*a^8*b^4*x^2 - 224*a^5*b^7*x - 140*a^2*b^10)*x^(2/3) + 15*(3*a^1
0*b^2*x^3 - 36*a^7*b^5*x^2 - 98*a^4*b^8*x - 56*a*b^11)*x^(1/3))/(a^15*x^2 +
 2*a^12*b^3*x + a^9*b^6)
```

giac [A] time = 0.16, size = 112, normalized size = 0.84

$$\frac{84\,b^6\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{a^9}+\frac{3\left(16\,ab^7x^{\frac{1}{3}}+15\,b^8\right)}{2\left(ax^{\frac{1}{3}}+b\right)^2a^9}+\frac{5\,a^{15}x^2-18\,a^{14}bx^{\frac{5}{3}}+45\,a^{13}b^2x^{\frac{4}{3}}-100\,a^{12}b^3x+225\,a^{11}b^4x^{\frac{2}{3}}-630}{10\,a^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^(1/3))^3,x, algorithm="giac")
```

```
[Out] 84*b^6*log(abs(a*x^(1/3) + b))/a^9 + 3/2*(16*a*b^7*x^(1/3) + 15*b^8)/((a*x^
(1/3) + b)^2*a^9) + 1/10*(5*a^15*x^2 - 18*a^14*b*x^(5/3) + 45*a^13*b^2*x^(4
/3) - 100*a^12*b^3*x + 225*a^11*b^4*x^(2/3) - 630*a^10*b^5*x^(1/3))/a^18
```

maple [A] time = 0.01, size = 111, normalized size = 0.83

$$\frac{x^2}{2a^3}-\frac{3b^8}{2\left(a\,x^{\frac{1}{3}}+b\right)^2a^9}-\frac{9b\,x^{\frac{5}{3}}}{5a^4}+\frac{9b^2x^{\frac{4}{3}}}{2a^5}-\frac{10b^3x}{a^6}+\frac{24b^7}{\left(a\,x^{\frac{1}{3}}+b\right)a^9}+\frac{84b^6\ln\left(a\,x^{\frac{1}{3}}+b\right)}{a^9}+\frac{45b^4x^{\frac{2}{3}}}{2a^7}-\frac{63b^5x^{\frac{1}{3}}}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b/x^(1/3))^3,x)
```

```
[Out] -3/2*b^8/a^9/(a*x^(1/3)+b)^2+24*b^7/a^9/(a*x^(1/3)+b)-63*b^5*x^(1/3)/a^8+45
/2*b^4*x^(2/3)/a^7-10/a^6*b^3*x+9/2*b^2*x^(4/3)/a^5-9/5*b*x^(5/3)/a^4+1/2/a
^3*x^2+84*b^6*ln(a*x^(1/3)+b)/a^9
```

maxima [A] time = 0.50, size = 134, normalized size = 1.00

$$\frac{5\,a^7-\frac{8\,a^6b}{x^{\frac{1}{3}}}+\frac{14\,a^5b^2}{x^{\frac{2}{3}}}-\frac{28\,a^4b^3}{x}+\frac{70\,a^3b^4}{x^{\frac{4}{3}}}-\frac{280\,a^2b^5}{x^{\frac{5}{3}}}-\frac{1260\,ab^6}{x^2}-\frac{840\,b^7}{x^{\frac{7}{3}}}}{10\left(\frac{a^{10}}{x^2}+\frac{2\,a^9b}{x^{\frac{7}{3}}}+\frac{a^8b^2}{x^{\frac{8}{3}}}\right)}+\frac{84\,b^6\log\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{a^9}+\frac{28\,b^6\log(x)}{a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x^(1/3))^3,x, algorithm="maxima")
```

```
[Out] 1/10*(5*a^7 - 8*a^6*b/x^(1/3) + 14*a^5*b^2/x^(2/3) - 28*a^4*b^3/x + 70*a^3*
b^4/x^(4/3) - 280*a^2*b^5/x^(5/3) - 1260*a*b^6/x^2 - 840*b^7/x^(7/3))/(a^10
/x^2 + 2*a^9*b/x^(7/3) + a^8*b^2/x^(8/3)) + 84*b^6*log(a + b/x^(1/3))/a^9 +
 28*b^6*log(x)/a^9
```

mupad [B] time = 0.05, size = 120, normalized size = 0.90

$$\frac{\frac{45\,b^8}{2\,a}+24\,b^7\,x^{1/3}}{a^8\,b^2+a^{10}\,x^{2/3}+2\,a^9\,b\,x^{1/3}}+\frac{x^2}{2\,a^3}-\frac{10\,b^3\,x}{a^6}-\frac{9\,b\,x^{5/3}}{5\,a^4}+\frac{84\,b^6\,\ln\left(b+a\,x^{1/3}\right)}{a^9}+\frac{9\,b^2\,x^{4/3}}{2\,a^5}+\frac{45\,b^4\,x^{2/3}}{2\,a^7}-\frac{63\,b^5\,x^{1/3}}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b/x^(1/3))^3,x)
```

```
[Out] ((45*b^8)/(2*a) + 24*b^7*x^(1/3))/(a^8*b^2 + a^10*x^(2/3) + 2*a^9*b*x^(1/3)
) + x^2/(2*a^3) - (10*b^3*x)/a^6 - (9*b*x^(5/3))/(5*a^4) + (84*b^6*log(b +
a*x^(1/3)))/a^9 + (9*b^2*x^(4/3))/(2*a^5) + (45*b^4*x^(2/3))/(2*a^7) - (63*
b^5*x^(1/3))/a^8
```

sympy [A] time = 2.23, size = 493, normalized size = 3.68

$$\begin{cases}\frac{5a^8x^{\frac{8}{3}}}{10a^{11}x^{\frac{2}{3}}+20a^{10}b\sqrt[3]{x}+10a^9b^2}-\frac{8a^7bx^{\frac{7}{3}}}{10a^{11}x^{\frac{2}{3}}+20a^{10}b\sqrt[3]{x}+10a^9b^2}+\frac{14a^6b^2x^2}{10a^{11}x^{\frac{2}{3}}+20a^{10}b\sqrt[3]{x}+10a^9b^2}-\frac{28a^5b^3x^{\frac{5}{3}}}{10a^{11}x^{\frac{2}{3}}+20a^{10}b\sqrt[3]{x}+10a^9b^2}+\frac{}{10a^{11}x^{\frac{2}{3}}+} \\ \frac{x^3}{3b^3}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/x**(1/3))**3,x)
```

```
[Out] Piecewise((5*a**8*x**(8/3)/(10*a**11*x**(2/3) + 20*a**10*b*x**(1/3) + 10*a*
*9*b**2) - 8*a**7*b*x**(7/3)/(10*a**11*x**(2/3) + 20*a**10*b*x**(1/3) + 10*
a**9*b**2) + 14*a**6*b**2*x**2/(10*a**11*x**(2/3) + 20*a**10*b*x**(1/3) + 1
0*a**9*b**2) - 28*a**5*b**3*x**(5/3)/(10*a**11*x**(2/3) + 20*a**10*b*x**(1/
```

```
3) + 10*a**9*b**2) + 70*a**4*b**4*x**(4/3)/(10*a**11*x**(2/3) + 20*a**10*b*
x**(1/3) + 10*a**9*b**2) - 280*a**3*b**5*x/(10*a**11*x**(2/3) + 20*a**10*b*
x**(1/3) + 10*a**9*b**2) + 840*a**2*b**6*x**(2/3)*log(x**(1/3) + b/a)/(10*a
**11*x**(2/3) + 20*a**10*b*x**(1/3) + 10*a**9*b**2) + 1680*a*b**7*x**(1/3)*
log(x**(1/3) + b/a)/(10*a**11*x**(2/3) + 20*a**10*b*x**(1/3) + 10*a**9*b**2
) + 1680*a*b**7*x**(1/3)/(10*a**11*x**(2/3) + 20*a**10*b*x**(1/3) + 10*a**9
*b**2) + 840*b**8*log(x**(1/3) + b/a)/(10*a**11*x**(2/3) + 20*a**10*b*x**(1
/3) + 10*a**9*b**2) + 1260*b**8/(10*a**11*x**(2/3) + 20*a**10*b*x**(1/3) +
10*a**9*b**2), Ne(a, 0)), (x**3/(3*b**3), True))
```

3.2439 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}\,dx$

Optimal. Leaf size=100

$$-\frac{30b^3\log\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^6}-\frac{10b^3\log(x)}{a^6}+\frac{12b^3}{a^5\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{18b^2\sqrt[3]{x}}{a^5}+\frac{3b^3}{2a^4\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}-\frac{9bx^{2/3}}{2a^4}+\frac{x}{a^3}$$

```
[Out] 3/2*b^3/a^4/(a+b/x^(1/3))^2+12*b^3/a^5/(a+b/x^(1/3))+18*b^2*x^(1/3)/a^5-9/2
*b*x^(2/3)/a^4+x/a^3-30*b^3*ln(a+b/x^(1/3))/a^6-10*b^3*ln(x)/a^6
```

Rubi [A] time = 0.07, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {190, 44}

$$\frac{12b^3}{a^5\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{3b^3}{2a^4\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}+\frac{18b^2\sqrt[3]{x}}{a^5}-\frac{30b^3\log\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^6}-\frac{10b^3\log(x)}{a^6}-\frac{9bx^{2/3}}{2a^4}+\frac{x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^(1/3))^(-3),x]
```

```
[Out] (3*b^3)/(2*a^4*(a + b/x^(1/3))^2) + (12*b^3)/(a^5*(a + b/x^(1/3))) + (18*b^
2*x^(1/3))/a^5 - (9*b*x^(2/3))/(2*a^4) + x/a^3 - (30*b^3*Log[a + b/x^(1/3)]
)/a^6 - (10*b^3*Log[x])/a^6
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}\,dx = -\left(3\,\text{Subst}\left(\int \frac{1}{x^4(a+bx)^3}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\left(3\,\text{Subst}\left(\int \left(\frac{1}{a^3x^4}-\frac{3b}{a^4x^3}+\frac{6b^2}{a^5x^2}-\frac{10b^3}{a^6x}+\frac{b^4}{a^4(a+bx)^3}+\frac{4b^4}{a^5(a+bx)^2}+\frac{10b^4}{a^6(a+bx)}\right) d\right.\right.$$

$$= \frac{3b^3}{2a^4\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}+\frac{12b^3}{a^5\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{18b^2\sqrt[3]{x}}{a^5}-\frac{9bx^{2/3}}{2a^4}+\frac{x}{a^3}-\frac{30b^3\log\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^6}-\frac{10b^3\log($$

Mathematica [A] time = 0.08, size = 83, normalized size = 0.83

$$\frac{2a^3x - 9a^2bx^{2/3} + \frac{3b^5}{\left(a\sqrt[3]{x}+b\right)^2} - \frac{30b^4}{a\sqrt[3]{x}+b} - 60b^3\log\left(a\sqrt[3]{x}+b\right) + 36ab^2\sqrt[3]{x}}{2a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^(1/3))^(-3),x]
```

```
[Out] ((3*b^5)/(b + a*x^(1/3))^2 - (30*b^4)/(b + a*x^(1/3)) + 36*a*b^2*x^(1/3) -
9*a^2*b*x^(2/3) + 2*a^3*x - 60*b^3*Log[b + a*x^(1/3)])/(2*a^6)
```

fricas [A] time = 0.81, size = 159, normalized size = 1.59

$$\frac{2\,a^9x^3 + 4\,a^6b^3x^2 - 34\,a^3b^6x - 27\,b^9 - 60\left(a^6b^3x^2 + 2\,a^3b^6x + b^9\right)\log\left(ax^{\frac{1}{3}} + b\right) - 3\left(3\,a^8bx^2 + 16\,a^5b^4x + 10\,a^2b^7\right.}{2\left(a^{12}x^2 + 2\,a^9b^3x + a^6b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^9*x^3 + 4*a^6*b^3*x^2 - 34*a^3*b^6*x - 27*b^9 - 60*(a^6*b^3*x^2 +
2*a^3*b^6*x + b^9)*log(a*x^(1/3) + b) - 3*(3*a^8*b*x^2 + 16*a^5*b^4*x + 10*
a^2*b^7)*x^(2/3) + 3*(12*a^7*b^2*x^2 + 35*a^4*b^5*x + 20*a*b^8)*x^(1/3))/(a
^12*x^2 + 2*a^9*b^3*x + a^6*b^6)
```

giac [A] time = 0.16, size = 79, normalized size = 0.79

$$-\frac{30\,b^3\log\left(\left|ax^{\frac{1}{3}} + b\right|\right)}{a^6} - \frac{3\left(10\,ab^4x^{\frac{1}{3}} + 9\,b^5\right)}{2\left(ax^{\frac{1}{3}} + b\right)^2a^6} + \frac{2\,a^6x - 9\,a^5bx^{\frac{2}{3}} + 36\,a^4b^2x^{\frac{1}{3}}}{2\,a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))^3,x, algorithm="giac")
```

```
[Out] -30*b^3*log(abs(a*x^(1/3) + b))/a^6 - 3/2*(10*a*b^4*x^(1/3) + 9*b^5)/((a*x^
(1/3) + b)^2*a^6) + 1/2*(2*a^6*x - 9*a^5*b*x^(2/3) + 36*a^4*b^2*x^(1/3))/a^
9
```

maple [A] time = 0.01, size = 77, normalized size = 0.77

$$\frac{3b^5}{2\left(a\,x^{\frac{1}{3}} + b\right)^2a^6} + \frac{x}{a^3} - \frac{15b^4}{\left(a\,x^{\frac{1}{3}} + b\right)a^6} - \frac{30b^3\ln\left(a\,x^{\frac{1}{3}} + b\right)}{a^6} - \frac{9b\,x^{\frac{2}{3}}}{2a^4} + \frac{18b^2x^{\frac{1}{3}}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^(1/3))^3,x)
```

```
[Out] 1/a^3*x-9/2*b*x^(2/3)/a^4+18*b^2*x^(1/3)/a^5-30/a^6*b^3*ln(a*x^(1/3)+b)-15/
a^6*b^4/(a*x^(1/3)+b)+3/2*b^5/a^6/(a*x^(1/3)+b)^2
```

maxima [A] time = 0.57, size = 101, normalized size = 1.01

$$\frac{2\,a^4 - \frac{5\,a^3b}{x^{\frac{1}{3}}} + \frac{20\,a^2b^2}{x^{\frac{2}{3}}} + \frac{90\,ab^3}{x} + \frac{60\,b^4}{x^{\frac{4}{3}}}}{2\left(\frac{a^7}{x} + \frac{2\,a^6b}{x^{\frac{4}{3}}} + \frac{a^5b^2}{x^{\frac{5}{3}}}\right)} - \frac{30\,b^3\log\left(a + \frac{b}{x^{\frac{1}{3}}}\right)}{a^6} - \frac{10\,b^3\log(x)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^4 - 5*a^3*b/x^(1/3) + 20*a^2*b^2/x^(2/3) + 90*a*b^3/x + 60*b^4/x^(
4/3))/(a^7/x + 2*a^6*b/x^(4/3) + a^5*b^2/x^(5/3)) - 30*b^3*log(a + b/x^(1/3
))/a^6 - 10*b^3*log(x)/a^6
```

mupad [B] time = 0.04, size = 87, normalized size = 0.87

$$\frac{x}{a^3}-\frac{\frac{27\,b^5}{2\,a}+15\,b^4\,x^{1/3}}{a^5\,b^2+a^7\,x^{2/3}+2\,a^6\,b\,x^{1/3}}-\frac{9\,b\,x^{2/3}}{2\,a^4}-\frac{30\,b^3\,\ln\left(b+a\,x^{1/3}\right)}{a^6}+\frac{18\,b^2\,x^{1/3}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x^(1/3))^3,x)
```

```
[Out] x/a^3 - ((27*b^5)/(2*a) + 15*b^4*x^(1/3))/(a^5*b^2 + a^7*x^(2/3) + 2*a^6*b*
x^(1/3)) - (9*b*x^(2/3))/(2*a^4) - (30*b^3*log(b + a*x^(1/3)))/a^6 + (18*b^
2*x^(1/3))/a^5
```

sympy [A] time = 0.90, size = 362, normalized size = 3.62

$$\begin{cases} \frac{2a^5x^{\frac{5}{3}}}{2a^8x^{\frac{2}{3}}+4a^7b\sqrt[3]{x}+2a^6b^2}-\frac{5a^4bx^{\frac{4}{3}}}{2a^8x^{\frac{2}{3}}+4a^7b\sqrt[3]{x}+2a^6b^2}+\frac{20a^3b^2x}{2a^8x^{\frac{2}{3}}+4a^7b\sqrt[3]{x}+2a^6b^2}-\frac{60a^2b^3x^{\frac{2}{3}}\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{2a^8x^{\frac{2}{3}}+4a^7b\sqrt[3]{x}+2a^6b^2}-\frac{120ab^4\sqrt[3]{x}\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{2a^8x^{\frac{2}{3}}+4a^7b\sqrt[3]{x}+2a^6b^2}- \\ \frac{x^2}{2b^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**(1/3))**3,x)
```

```
[Out] Piecewise((2*a**5*x**(5/3)/(2*a**8*x**(2/3) + 4*a**7*b*x**(1/3) + 2*a**6*b*
*2) - 5*a**4*b*x**(4/3)/(2*a**8*x**(2/3) + 4*a**7*b*x**(1/3) + 2*a**6*b**2)
 + 20*a**3*b**2*x/(2*a**8*x**(2/3) + 4*a**7*b*x**(1/3) + 2*a**6*b**2) - 60*
a**2*b**3*x**(2/3)*log(x**(1/3) + b/a)/(2*a**8*x**(2/3) + 4*a**7*b*x**(1/3)
 + 2*a**6*b**2) - 120*a*b**4*x**(1/3)*log(x**(1/3) + b/a)/(2*a**8*x**(2/3)
+ 4*a**7*b*x**(1/3) + 2*a**6*b**2) - 120*a*b**4*x**(1/3)/(2*a**8*x**(2/3) +
 4*a**7*b*x**(1/3) + 2*a**6*b**2) - 60*b**5*log(x**(1/3) + b/a)/(2*a**8*x**
(2/3) + 4*a**7*b*x**(1/3) + 2*a**6*b**2) - 90*b**5/(2*a**8*x**(2/3) + 4*a**
7*b*x**(1/3) + 2*a**6*b**2), Ne(a, 0)), (x**2/(2*b**3), True))
```

3.2440 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x}\,dx$

Optimal. Leaf size=54

$$-\frac{3b^2}{2a^3\left(a\sqrt[3]{x}+b\right)^2}+\frac{6b}{a^3\left(a\sqrt[3]{x}+b\right)}+\frac{3\log\left(a\sqrt[3]{x}+b\right)}{a^3}$$

```
[Out] -3/2*b^2/a^3/(b+a*x^(1/3))^2+6*b/a^3/(b+a*x^(1/3))+3*ln(b+a*x^(1/3))/a^3
```

Rubi [A] time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {263, 190, 43}

$$-\frac{3b^2}{2a^3\left(a\sqrt[3]{x}+b\right)^2}+\frac{6b}{a^3\left(a\sqrt[3]{x}+b\right)}+\frac{3\log\left(a\sqrt[3]{x}+b\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^(1/3))^3*x),x]
```

```
[Out] (-3*b^2)/(2*a^3*(b + a*x^(1/3))^2) + (6*b)/(a^3*(b + a*x^(1/3))) + (3*Log[b
 + a*x^(1/3)])/a^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x}\,dx &= \int \frac{1}{\left(b+a\sqrt[3]{x}\right)^3}\,dx\\ &= 3\,\text{Subst}\left(\int \frac{x^2}{(b+ax)^3}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(\frac{b^2}{a^2(b+ax)^3}-\frac{2b}{a^2(b+ax)^2}+\frac{1}{a^2(b+ax)}\right)dx, x, \sqrt[3]{x}\right)\\ &= -\frac{3b^2}{2a^3\left(b+a\sqrt[3]{x}\right)^2}+\frac{6b}{a^3\left(b+a\sqrt[3]{x}\right)}+\frac{3\log\left(b+a\sqrt[3]{x}\right)}{a^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 45, normalized size = 0.83

$$\frac{3\left(\frac{b\left(4a\sqrt[3]{x}+3b\right)}{\left(a\sqrt[3]{x}+b\right)^2}+2\log\left(a\sqrt[3]{x}+b\right)\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^(1/3))^3*x),x]
```

```
[Out] (3*((b*(3*b + 4*a*x^(1/3)))/(b + a*x^(1/3))^2 + 2*Log[b + a*x^(1/3)]))/(2*a
^3)
```

fricas [B] time = 0.58, size = 113, normalized size = 2.09

$$\frac{3\left(6\,a^3b^3x+3\,b^6+2\left(a^6x^2+2\,a^3b^3x+b^6\right)\log\left(ax^{\frac{1}{3}}+b\right)+\left(4\,a^5bx+a^2b^4\right)x^{\frac{2}{3}}-\left(5\,a^4b^2x+2\,ab^5\right)x^{\frac{1}{3}}\right)}{2\left(a^9x^2+2\,a^6b^3x+a^3b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))^3/x,x, algorithm="fricas")
```

```
[Out] 3/2*(6*a^3*b^3*x + 3*b^6 + 2*(a^6*x^2 + 2*a^3*b^3*x + b^6)*log(a*x^(1/3) +
b) + (4*a^5*b*x + a^2*b^4)*x^(2/3) - (5*a^4*b^2*x + 2*a*b^5)*x^(1/3))/(a^9*
x^2 + 2*a^6*b^3*x + a^3*b^6)
```

giac [A] time = 0.17, size = 44, normalized size = 0.81

$$\frac{3\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{a^3}+\frac{3\left(4\,bx^{\frac{1}{3}}+\frac{3b^2}{a}\right)}{2\left(ax^{\frac{1}{3}}+b\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))^3/x,x, algorithm="giac")
```

```
[Out] 3*log(abs(a*x^(1/3) + b))/a^3 + 3/2*(4*b*x^(1/3) + 3*b^2/a)/((a*x^(1/3) + b
)^2*a^2)
```

maple [B] time = 0.08, size = 237, normalized size = 4.39

$$-\frac{9b^6}{2\left(a^3x+b^3\right)^2a^3}+\frac{2bx}{\left(a^2x^{\frac{2}{3}}-ab\,x^{\frac{1}{3}}+b^2\right)^2}-\frac{13b^2x^{\frac{2}{3}}}{2\left(a^2x^{\frac{2}{3}}-ab\,x^{\frac{1}{3}}+b^2\right)^2a}+\frac{5b^3x^{\frac{1}{3}}}{\left(a^2x^{\frac{2}{3}}-ab\,x^{\frac{1}{3}}+b^2\right)^2a^2}-\frac{3b^4}{\left(a^2x^{\frac{2}{3}}-ab\,x^{\frac{1}{3}}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^(1/3))^3/x,x)
```

```
[Out] -9/2*b^6/(a^3*x+b^3)^2/a^3+1/a^3*ln(a^3*x+b^3)+9/a^3*b^3/(a^3*x+b^3)+2*ln(a
*x^(1/3)+b)/a^3-b^2/a^3/(a*x^(1/3)+b)^2-13/2/a*b^2/(a^2*x^(2/3)-a*b*x^(1/3)
+b^2)^2*x^(2/3)+5/a^2*b^3/(a^2*x^(2/3)-a*b*x^(1/3)+b^2)^2*x^(1/3)-3/a^3*b^4
/(a^2*x^(2/3)-a*b*x^(1/3)+b^2)^2-1/a^3*ln(a^2*x^(2/3)-a*b*x^(1/3)+b^2)+4*b/
a^3/(a*x^(1/3)+b)+2*b/(a^2*x^(2/3)-a*b*x^(1/3)+b^2)^2*x
```

maxima [A] time = 0.48, size = 57, normalized size = 1.06

$$-\frac{3\left(3\,a+\frac{2\,b}{x^{\frac{1}{3}}}\right)}{2\left(a^4+\frac{2\,a^3b}{x^{\frac{1}{3}}}+\frac{a^2b^2}{x^{\frac{2}{3}}}\right)}+\frac{3\log\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{a^3}+\frac{\log(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))^3/x,x, algorithm="maxima")
```

```
[Out] -3/2*(3*a + 2*b/x^(1/3))/(a^4 + 2*a^3*b/x^(1/3) + a^2*b^2/x^(2/3)) + 3*log(
a + b/x^(1/3))/a^3 + log(x)/a^3
```

mupad **[B]** time = 1.14, size = 53, normalized size = 0.98

$$\frac{\frac{9\,b^2}{2\,a^3}+\frac{6\,b\,x^{1/3}}{a^2}}{b^2+a^2\,x^{2/3}+2\,a\,b\,x^{1/3}}+\frac{3\,\ln\left(b+a\,x^{1/3}\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b/x^(1/3))^3),x)
```

```
[Out] ((9*b^2)/(2*a^3) + (6*b*x^(1/3))/a^2)/(b^2 + a^2*x^(2/3) + 2*a*b*x^(1/3)) +
 (3*log(b + a*x^(1/3)))/a^3
```

sympy **[A]** time = 1.89, size = 240, normalized size = 4.44

$$\begin{cases} \frac{6a^2x^{\frac{4}{3}}\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{2a^5x^{\frac{4}{3}}+4a^4bx+2a^3b^2x^{\frac{2}{3}}}+\frac{12abx\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{2a^5x^{\frac{4}{3}}+4a^4bx+2a^3b^2x^{\frac{2}{3}}}+\frac{12abx}{2a^5x^{\frac{4}{3}}+4a^4bx+2a^3b^2x^{\frac{2}{3}}}+\frac{6b^2x^{\frac{2}{3}}\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{2a^5x^{\frac{4}{3}}+4a^4bx+2a^3b^2x^{\frac{2}{3}}}+\frac{9b^2x^{\frac{2}{3}}}{2a^5x^{\frac{4}{3}}+4a^4bx+2a^3b^2x^{\frac{2}{3}}} & \text{for } a \\ \frac{x}{b^3} & \text{othe} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**(1/3))**3/x,x)
```

```
[Out] Piecewise((6*a**2*x**(4/3)*log(x**(1/3) + b/a)/(2*a**5*x**(4/3) + 4*a**4*b*
x + 2*a**3*b**2*x**(2/3)) + 12*a*b*x*log(x**(1/3) + b/a)/(2*a**5*x**(4/3) +
 4*a**4*b*x + 2*a**3*b**2*x**(2/3)) + 12*a*b*x/(2*a**5*x**(4/3) + 4*a**4*b*
x + 2*a**3*b**2*x**(2/3)) + 6*b**2*x**(2/3)*log(x**(1/3) + b/a)/(2*a**5*x**
(4/3) + 4*a**4*b*x + 2*a**3*b**2*x**(2/3)) + 9*b**2*x**(2/3)/(2*a**5*x**(4/
3) + 4*a**4*b*x + 2*a**3*b**2*x**(2/3)), Ne(a, 0)), (x/b**3, True))
```

3.2441 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^2}\,dx$

Optimal. Leaf size=56

$$-\frac{3\log\left(a\sqrt[3]{x}+b\right)}{b^3}+\frac{3}{b^2\left(a\sqrt[3]{x}+b\right)}+\frac{3}{2b\left(a\sqrt[3]{x}+b\right)^2}+\frac{\log(x)}{b^3}$$

```
[Out] 3/2/b/(b+a*x^(1/3))^2+3/b^2/(b+a*x^(1/3))-3*ln(b+a*x^(1/3))/b^3+ln(x)/b^3
```

Rubi [A] time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {263, 266, 44}

$$\frac{3}{b^2\left(a\sqrt[3]{x}+b\right)}-\frac{3\log\left(a\sqrt[3]{x}+b\right)}{b^3}+\frac{3}{2b\left(a\sqrt[3]{x}+b\right)^2}+\frac{\log(x)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^(1/3))^3*x^2),x]
```

```
[Out] 3/(2*b*(b + a*x^(1/3))^2) + 3/(b^2*(b + a*x^(1/3))) - (3*Log[b + a*x^(1/3)]
)/b^3 + Log[x]/b^3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^2}\,dx &= \int \frac{1}{\left(b+a\sqrt[3]{x}\right)^3 x}\,dx \\
&= 3\,\text{Subst}\left(\int \frac{1}{x(b+ax)^3}\,dx, x, \sqrt[3]{x}\right) \\
&= 3\,\text{Subst}\left(\int \left(\frac{1}{b^3 x}-\frac{a}{b(b+ax)^3}-\frac{a}{b^2(b+ax)^2}-\frac{a}{b^3(b+ax)}\right)dx, x, \sqrt[3]{x}\right) \\
&= \frac{3}{2b\left(b+a\sqrt[3]{x}\right)^2}+\frac{3}{b^2\left(b+a\sqrt[3]{x}\right)}-\frac{3\log\left(b+a\sqrt[3]{x}\right)}{b^3}+\frac{\log(x)}{b^3}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 50, normalized size = 0.89

$$\frac{\frac{3b\left(2a\sqrt[3]{x}+3b\right)}{\left(a\sqrt[3]{x}+b\right)^2}-6\log\left(a\sqrt[3]{x}+b\right)+2\log(x)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^(1/3))^3*x^2),x]
```

```
[Out] ((3*b*(3*b + 2*a*x^(1/3)))/(b + a*x^(1/3))^2 - 6*Log[b + a*x^(1/3)] + 2*Log
[x])/(2*b^3)
```

fricas [B] time = 0.84, size = 129, normalized size = 2.30

$$\frac{3\left(3b^6-2\left(a^6x^2+2a^3b^3x+b^6\right)\log\left(ax^{\frac{1}{3}}+b\right)+2\left(a^6x^2+2a^3b^3x+b^6\right)\log\left(x^{\frac{1}{3}}\right)+\left(2a^5bx+5a^2b^4\right)x^{\frac{2}{3}}-\left(a^4b^2x\right.\right.}{2\left(a^6b^3x^2+2a^3b^6x+b^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))^3/x^2,x, algorithm="fricas")
```

```
[Out] 3/2*(3*b^6 - 2*(a^6*x^2 + 2*a^3*b^3*x + b^6)*log(a*x^(1/3) + b) + 2*(a^6*x^
2 + 2*a^3*b^3*x + b^6)*log(x^(1/3)) + (2*a^5*b*x + 5*a^2*b^4)*x^(2/3) - (a^
4*b^2*x + 4*a*b^5)*x^(1/3))/(a^6*b^3*x^2 + 2*a^3*b^6*x + b^9)
```

giac [A] time = 0.17, size = 49, normalized size = 0.88

$$-\frac{3\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{b^3}+\frac{\log(|x|)}{b^3}+\frac{3\left(2abx^{\frac{1}{3}}+3b^2\right)}{2\left(ax^{\frac{1}{3}}+b\right)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))^3/x^2,x, algorithm="giac")
```

```
[Out] -3*log(abs(a*x^(1/3) + b))/b^3 + log(abs(x))/b^3 + 3/2*(2*a*b*x^(1/3) + 3*b
^2)/((a*x^(1/3) + b)^2*b^3)
```

maple [A] time = 0.01, size = 49, normalized size = 0.88

$$\frac{3}{2\left(a\,x^{\frac{1}{3}}+b\right)^2b}+\frac{3}{\left(a\,x^{\frac{1}{3}}+b\right)b^2}+\frac{\ln(x)}{b^3}-\frac{3\ln\left(a\,x^{\frac{1}{3}}+b\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^(1/3))^3/x^2,x)
```

```
[Out] 3/2/b/(a*x^(1/3)+b)^2+3/b^2/(a*x^(1/3)+b)-3*ln(a*x^(1/3)+b)/b^3+1/b^3*ln(x)
```

maxima [A] time = 0.49, size = 46, normalized size = 0.82

$$-\frac{3\log\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{b^3}-\frac{6a}{\left(a+\frac{b}{x^{\frac{1}{3}}}\right)b^3}+\frac{3a^2}{2\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))^3/x^2,x, algorithm="maxima")
```

```
[Out] -3*log(a + b/x^(1/3))/b^3 - 6*a/((a + b/x^(1/3))*b^3) + 3/2*a^2/((a + b/x^(
1/3))^2*b^3)
```

mupad [B] time = 1.14, size = 54, normalized size = 0.96

$$\frac{\frac{9}{2\,b}+\frac{3\,a\,x^{1/3}}{b^2}}{b^2+a^2\,x^{2/3}+2\,a\,b\,x^{1/3}}-\frac{6\,\mathrm{atanh}\left(\frac{2\,a\,x^{1/3}}{b}+1\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b/x^(1/3))^3),x)
```

```
[Out] (9/(2*b) + (3*a*x^(1/3))/b^2)/(b^2 + a^2*x^(2/3) + 2*a*b*x^(1/3)) - (6*atan
h((2*a*x^(1/3))/b + 1))/b^3
```

sympy [A] time = 4.63, size = 406, normalized size = 7.25

$$\begin{cases} \tilde{\infty}\log(x) \\ \frac{\log(x)}{b^3} \\ -\frac{1}{a^3x} \\ \frac{2a^2x^{\frac{7}{3}}\log(x)}{2a^2b^3x^{\frac{7}{3}}+4ab^4x^2+2b^5x^{\frac{5}{3}}}-\frac{6a^2x^{\frac{7}{3}}\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{2a^2b^3x^{\frac{7}{3}}+4ab^4x^2+2b^5x^{\frac{5}{3}}}+\frac{4abx^2\log(x)}{2a^2b^3x^{\frac{7}{3}}+4ab^4x^2+2b^5x^{\frac{5}{3}}}-\frac{12abx^2\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{2a^2b^3x^{\frac{7}{3}}+4ab^4x^2+2b^5x^{\frac{5}{3}}}+\frac{6abx^2}{2a^2b^3x^{\frac{7}{3}}+4ab^4x^2+2b^5x^{\frac{5}{3}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**(1/3))**3/x**2,x)
```

```
[Out] Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0)), (log(x)/b**3, Eq(a, 0)), (-1/(
a**3*x), Eq(b, 0)), (2*a**2*x**(7/3)*log(x)/(2*a**2*b**3*x**(7/3) + 4*a*b**
4*x**2 + 2*b**5*x**(5/3)) - 6*a**2*x**(7/3)*log(x**(1/3) + b/a)/(2*a**2*b**
3*x**(7/3) + 4*a*b**4*x**2 + 2*b**5*x**(5/3)) + 4*a*b*x**2*log(x)/(2*a**2*b
**3*x**(7/3) + 4*a*b**4*x**2 + 2*b**5*x**(5/3)) - 12*a*b*x**2*log(x**(1/3)
+ b/a)/(2*a**2*b**3*x**(7/3) + 4*a*b**4*x**2 + 2*b**5*x**(5/3)) + 6*a*b*x**
2/(2*a**2*b**3*x**(7/3) + 4*a*b**4*x**2 + 2*b**5*x**(5/3)) + 2*b**2*x**(5/3
)*log(x)/(2*a**2*b**3*x**(7/3) + 4*a*b**4*x**2 + 2*b**5*x**(5/3)) - 6*b**2*
x**(5/3)*log(x**(1/3) + b/a)/(2*a**2*b**3*x**(7/3) + 4*a*b**4*x**2 + 2*b**5
*x**(5/3)) + 9*b**2*x**(5/3)/(2*a**2*b**3*x**(7/3) + 4*a*b**4*x**2 + 2*b**5
*x**(5/3)), True))
```

3.2442 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^3}\,dx$

Optimal. Leaf size=103

$$\frac{30a^3\log\left(a\sqrt[3]{x}+b\right)}{b^6}-\frac{10a^3\log(x)}{b^6}-\frac{12a^3}{b^5\left(a\sqrt[3]{x}+b\right)}-\frac{3a^3}{2b^4\left(a\sqrt[3]{x}+b\right)^2}-\frac{18a^2}{b^5\sqrt[3]{x}}+\frac{9a}{2b^4x^{2/3}}-\frac{1}{b^3x}$$

```
[Out] -3/2*a^3/b^4/(b+a*x^(1/3))^2-12*a^3/b^5/(b+a*x^(1/3))-1/b^3/x+9/2*a/b^4/x^(
2/3)-18*a^2/b^5/x^(1/3)+30*a^3*ln(b+a*x^(1/3))/b^6-10*a^3*ln(x)/b^6
```

Rubi [A] time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {263, 266, 44}

$$-\frac{12a^3}{b^5\left(a\sqrt[3]{x}+b\right)}-\frac{3a^3}{2b^4\left(a\sqrt[3]{x}+b\right)^2}-\frac{18a^2}{b^5\sqrt[3]{x}}+\frac{30a^3\log\left(a\sqrt[3]{x}+b\right)}{b^6}-\frac{10a^3\log(x)}{b^6}+\frac{9a}{2b^4x^{2/3}}-\frac{1}{b^3x}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^(1/3))^3*x^3),x]
```

```
[Out] (-3*a^3)/(2*b^4*(b + a*x^(1/3))^2) - (12*a^3)/(b^5*(b + a*x^(1/3))) - 1/(b^
3*x) + (9*a)/(2*b^4*x^(2/3)) - (18*a^2)/(b^5*x^(1/3)) + (30*a^3*Log[b + a*x
^(1/3)])/b^6 - (10*a^3*Log[x])/b^6
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^3}\,dx &= \int \frac{1}{\left(b+a\sqrt[3]{x}\right)^3 x^2}\,dx\\ &= 3\,\text{Subst}\left(\int \frac{1}{x^4(b+ax)^3}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(\frac{1}{b^3x^4}-\frac{3a}{b^4x^3}+\frac{6a^2}{b^5x^2}-\frac{10a^3}{b^6x}+\frac{a^4}{b^4(b+ax)^3}+\frac{4a^4}{b^5(b+ax)^2}+\frac{10a^4}{b^6(b+ax)}\right)dx, x\right.\\ &= -\frac{3a^3}{2b^4\left(b+a\sqrt[3]{x}\right)^2}-\frac{12a^3}{b^5\left(b+a\sqrt[3]{x}\right)}-\frac{1}{b^3x}+\frac{9a}{2b^4x^{2/3}}-\frac{18a^2}{b^5\sqrt[3]{x}}+\frac{30a^3\log\left(b+a\sqrt[3]{x}\right)}{b^6}-\frac{10a^3}{}\end{aligned}$$

Mathematica [A] time = 0.16, size = 93, normalized size = 0.90

$$-\frac{-60a^3\log\left(a\sqrt[3]{x}+b\right)+20a^3\log(x)+\frac{b\left(60a^4x^{4/3}+90a^3bx+20a^2b^2x^{2/3}-5ab^3\sqrt[3]{x}+2b^4\right)}{x\left(a\sqrt[3]{x}+b\right)^2}}{2b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^(1/3))^3*x^3),x]
```

```
[Out] -1/2*((b*(2*b^4 - 5*a*b^3*x^(1/3) + 20*a^2*b^2*x^(2/3) + 90*a^3*b*x + 60*a^
4*x^(4/3)))/((b + a*x^(1/3))^2*x) - 60*a^3*Log[b + a*x^(1/3)] + 20*a^3*Log[
x])/b^6
```

fricas [B] time = 0.92, size = 191, normalized size = 1.85

$$-\frac{20\,a^6b^3x^2+31\,a^3b^6x+2\,b^9-60\left(a^9x^3+2\,a^6b^3x^2+a^3b^6x\right)\log\left(ax^{\frac{1}{3}}+b\right)+60\left(a^9x^3+2\,a^6b^3x^2+a^3b^6x\right)\log}{2\left(a^6b^6x^3+2\,a^3b^9x^2+b^{12}x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))^3/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(20*a^6*b^3*x^2 + 31*a^3*b^6*x + 2*b^9 - 60*(a^9*x^3 + 2*a^6*b^3*x^2 +
 a^3*b^6*x)*log(a*x^(1/3) + b) + 60*(a^9*x^3 + 2*a^6*b^3*x^2 + a^3*b^6*x)*l
og(x^(1/3)) + 3*(20*a^8*b*x^2 + 35*a^5*b^4*x + 12*a^2*b^7)*x^(2/3) - 3*(10*
a^7*b^2*x^2 + 16*a^4*b^5*x + 3*a*b^8)*x^(1/3))/(a^6*b^6*x^3 + 2*a^3*b^9*x^2
 + b^12*x)
```

giac [A] time = 0.20, size = 90, normalized size = 0.87

$$\frac{30\,a^3\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{b^6}-\frac{10\,a^3\log(|x|)}{b^6}-\frac{60\,a^4bx^{\frac{4}{3}}+90\,a^3b^2x+20\,a^2b^3x^{\frac{2}{3}}-5\,ab^4x^{\frac{1}{3}}+2\,b^5}{2\left(ax^{\frac{1}{3}}+b\right)^2b^6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))^3/x^3,x, algorithm="giac")
```

```
[Out] 30*a^3*log(abs(a*x^(1/3) + b))/b^6 - 10*a^3*log(abs(x))/b^6 - 1/2*(60*a^4*b
*x^(4/3) + 90*a^3*b^2*x + 20*a^2*b^3*x^(2/3) - 5*a*b^4*x^(1/3) + 2*b^5)/((a
*x^(1/3) + b)^2*b^6*x)
```

maple [A] time = 0.01, size = 90, normalized size = 0.87

$$-\frac{3a^3}{2\left(a\,x^{\frac{1}{3}}+b\right)^2b^4}-\frac{12a^3}{\left(a\,x^{\frac{1}{3}}+b\right)b^5}-\frac{10a^3\ln(x)}{b^6}+\frac{30a^3\ln\left(a\,x^{\frac{1}{3}}+b\right)}{b^6}-\frac{18a^2}{b^5x^{\frac{1}{3}}}+\frac{9a}{2b^4x^{\frac{2}{3}}}-\frac{1}{b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^(1/3))^3/x^3,x)
```

```
[Out] -3/2*a^3/b^4/(a*x^(1/3)+b)^2-12*a^3/b^5/(a*x^(1/3)+b)-1/b^3/x+9/2*a/b^4/x^(
2/3)-18*a^2/b^5/x^(1/3)+30*a^3*ln(a*x^(1/3)+b)/b^6-10*a^3/b^6*ln(x)
```

maxima [A] time = 0.51, size = 95, normalized size = 0.92

$$\frac{30\,a^3\log\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{b^6}-\frac{\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^3}{b^6}+\frac{15\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^2a}{2\,b^6}-\frac{30\left(a+\frac{b}{x^{\frac{1}{3}}}\right)a^2}{b^6}+\frac{15\,a^4}{\left(a+\frac{b}{x^{\frac{1}{3}}}\right)b^6}-\frac{3\,a^5}{2\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^2b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))^3/x^3,x, algorithm="maxima")
```

```
[Out] 30*a^3*log(a + b/x^(1/3))/b^6 - (a + b/x^(1/3))^3/b^6 + 15/2*(a + b/x^(1/3)
)^2*a/b^6 - 30*(a + b/x^(1/3))*a^2/b^6 + 15*a^4/((a + b/x^(1/3))*b^6) - 3/2
*a^5/((a + b/x^(1/3))^2*b^6)
```

mupad **[B]** time = 0.10, size = 89, normalized size = 0.86

$$\frac{60\,a^3\,\operatorname{atanh}\left(\frac{2\,a\,x^{1/3}}{b}+1\right)}{b^6}-\frac{\frac{1}{b}-\frac{5\,a\,x^{1/3}}{2\,b^2}+\frac{45\,a^3\,x}{b^4}+\frac{10\,a^2\,x^{2/3}}{b^3}+\frac{30\,a^4\,x^{4/3}}{b^5}}{b^2\,x+a^2\,x^{5/3}+2\,a\,b\,x^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b/x^(1/3))^3),x)
```

```
[Out] (60*a^3*atanh((2*a*x^(1/3))/b + 1))/b^6 - (1/b - (5*a*x^(1/3))/(2*b^2) + (4
5*a^3*x)/b^4 + (10*a^2*x^(2/3))/b^3 + (30*a^4*x^(4/3))/b^5)/(b^2*x + a^2*x^
(5/3) + 2*a*b*x^(4/3))
```

sympy **[A]** time = 11.10, size = 561, normalized size = 5.45

$$\begin{cases} \frac{\tilde{\infty}}{x} \\ -\frac{1}{b^3 x} \\ -\frac{1}{2a^3x^2} \\ -\frac{20a^5x^{\frac{10}{3}}\log(x)}{2a^2b^6x^{\frac{10}{3}}+4ab^7x^3+2b^8x^{\frac{8}{3}}}+\frac{60a^5x^{\frac{10}{3}}\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{2a^2b^6x^{\frac{10}{3}}+4ab^7x^3+2b^8x^{\frac{8}{3}}}-\frac{40a^4bx^3\log(x)}{2a^2b^6x^{\frac{10}{3}}+4ab^7x^3+2b^8x^{\frac{8}{3}}}+\frac{120a^4bx^3\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{2a^2b^6x^{\frac{10}{3}}+4ab^7x^3+2b^8x^{\frac{8}{3}}}-\frac{60a^4bx^3}{2a^2b^6x^{\frac{10}{3}}+4ab^7x^3+2b^8x} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**(1/3))**3/x**3,x)
```

```
[Out] Piecewise((zoo/x, Eq(a, 0) & Eq(b, 0)), (-1/(b**3*x), Eq(a, 0)), (-1/(2*a**
3*x**2), Eq(b, 0)), (-20*a**5*x**(10/3)*log(x)/(2*a**2*b**6*x**(10/3) + 4*a
*b**7*x**3 + 2*b**8*x**(8/3)) + 60*a**5*x**(10/3)*log(x**(1/3) + b/a)/(2*a*
*2*b**6*x**(10/3) + 4*a*b**7*x**3 + 2*b**8*x**(8/3)) - 40*a**4*b*x**3*log(x
)/(2*a**2*b**6*x**(10/3) + 4*a*b**7*x**3 + 2*b**8*x**(8/3)) + 120*a**4*b*x*
*3*log(x**(1/3) + b/a)/(2*a**2*b**6*x**(10/3) + 4*a*b**7*x**3 + 2*b**8*x**(
8/3)) - 60*a**4*b*x**3/(2*a**2*b**6*x**(10/3) + 4*a*b**7*x**3 + 2*b**8*x**(
8/3)) - 20*a**3*b**2*x**(8/3)*log(x)/(2*a**2*b**6*x**(10/3) + 4*a*b**7*x**3
 + 2*b**8*x**(8/3)) + 60*a**3*b**2*x**(8/3)*log(x**(1/3) + b/a)/(2*a**2*b**
6*x**(10/3) + 4*a*b**7*x**3 + 2*b**8*x**(8/3)) - 90*a**3*b**2*x**(8/3)/(2*a
**2*b**6*x**(10/3) + 4*a*b**7*x**3 + 2*b**8*x**(8/3)) - 20*a**2*b**3*x**(7/
3)/(2*a**2*b**6*x**(10/3) + 4*a*b**7*x**3 + 2*b**8*x**(8/3)) + 5*a*b**4*x**
2/(2*a**2*b**6*x**(10/3) + 4*a*b**7*x**3 + 2*b**8*x**(8/3)) - 2*b**5*x**(5/
3)/(2*a**2*b**6*x**(10/3) + 4*a*b**7*x**3 + 2*b**8*x**(8/3)), True))
```

3.2443 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^4}\,dx$

Optimal. Leaf size=146

$$-\frac{84a^6\log\left(a\sqrt[3]{x}+b\right)}{b^9}+\frac{28a^6\log(x)}{b^9}+\frac{21a^6}{b^8\left(a\sqrt[3]{x}+b\right)}+\frac{3a^6}{2b^7\left(a\sqrt[3]{x}+b\right)^2}+\frac{63a^5}{b^8\sqrt[3]{x}}-\frac{45a^4}{2b^7x^{2/3}}+\frac{10a^3}{b^6x}-\frac{9a^2}{2b^5x^{4/3}}+\frac{9a}{5b^4x^{5/3}}$$

```
[Out] 3/2*a^6/b^7/(b+a*x^(1/3))^2+21*a^6/b^8/(b+a*x^(1/3))-1/2/b^3/x^2+9/5*a/b^4/
x^(5/3)-9/2*a^2/b^5/x^(4/3)+10*a^3/b^6/x-45/2*a^4/b^7/x^(2/3)+63*a^5/b^8/x^
(1/3)-84*a^6*ln(b+a*x^(1/3))/b^9+28*a^6*ln(x)/b^9
```

Rubi [A] time = 0.11, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {263, 266, 44}

$$-\frac{45a^4}{2b^7x^{2/3}}-\frac{9a^2}{2b^5x^{4/3}}+\frac{21a^6}{b^8\left(a\sqrt[3]{x}+b\right)}+\frac{3a^6}{2b^7\left(a\sqrt[3]{x}+b\right)^2}+\frac{63a^5}{b^8\sqrt[3]{x}}+\frac{10a^3}{b^6x}-\frac{84a^6\log\left(a\sqrt[3]{x}+b\right)}{b^9}+\frac{28a^6\log(x)}{b^9}+\frac{9a}{5b^4x^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^(1/3))^3*x^4),x]
```

```
[Out] (3*a^6)/(2*b^7*(b + a*x^(1/3))^2) + (21*a^6)/(b^8*(b + a*x^(1/3))) - 1/(2*b
^3*x^2) + (9*a)/(5*b^4*x^(5/3)) - (9*a^2)/(2*b^5*x^(4/3)) + (10*a^3)/(b^6*x
) - (45*a^4)/(2*b^7*x^(2/3)) + (63*a^5)/(b^8*x^(1/3)) - (84*a^6*Log[b + a*x
^(1/3)])/b^9 + (28*a^6*Log[x])/b^9
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^4}\,dx = \int \frac{1}{\left(b+a\sqrt[3]{x}\right)^3 x^3}\,dx$$

$$= 3\,\text{Subst}\left(\int \frac{1}{x^7(b+ax)^3}\,dx, x, \sqrt[3]{x}\right)$$

$$= 3\,\text{Subst}\left(\int \left(\frac{1}{b^3x^7} - \frac{3a}{b^4x^6} + \frac{6a^2}{b^5x^5} - \frac{10a^3}{b^6x^4} + \frac{15a^4}{b^7x^3} - \frac{21a^5}{b^8x^2} + \frac{28a^6}{b^9x} - \frac{a^7}{b^7(b+ax)^3} - \frac{7a^7}{b^8(b+ax}\right.\right.$$

$$= \frac{3a^6}{2b^7\left(b+a\sqrt[3]{x}\right)^2} + \frac{21a^6}{b^8\left(b+a\sqrt[3]{x}\right)} - \frac{1}{2b^3x^2} + \frac{9a}{5b^4x^{5/3}} - \frac{9a^2}{2b^5x^{4/3}} + \frac{10a^3}{b^6x} - \frac{45a^4}{2b^7x^{2/3}} + \frac{63a^5}{b^8\sqrt[3]{x}} -$$

Mathematica [A] time = 0.17, size = 130, normalized size = 0.89

$$\frac{-840a^6\log\left(a\sqrt[3]{x}+b\right) + 280a^6\log(x) + \frac{b\left(840a^7x^{7/3}+1260a^6bx^2+280a^5b^2x^{5/3}-70a^4b^3x^{4/3}+28a^3b^4x-14a^2b^5x^{2/3}+8ab^6\sqrt[3]{x}-5b^7\right)}{x^2\left(a\sqrt[3]{x}+b\right)^2}}{10b^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^(1/3))^3*x^4),x]
```

```
[Out] ((b*(-5*b^7 + 8*a*b^6*x^(1/3) - 14*a^2*b^5*x^(2/3) + 28*a^3*b^4*x - 70*a^4*
b^3*x^(4/3) + 280*a^5*b^2*x^(5/3) + 1260*a^6*b*x^2 + 840*a^7*x^(7/3)))/((b
+ a*x^(1/3))^2*x^2) - 840*a^6*Log[b + a*x^(1/3)] + 280*a^6*Log[x])/(10*b^9)
```

fricas [A] time = 1.01, size = 230, normalized size = 1.58

$$\frac{280\,a^9b^3x^3 + 420\,a^6b^6x^2 + 90\,a^3b^9x - 5\,b^{12} - 840\left(a^{12}x^4 + 2\,a^9b^3x^3 + a^6b^6x^2\right)\log\left(ax^{\frac{1}{3}}+b\right) + 840\left(a^{12}x^4 + 2\,a^9b^3x^3 + a^6b^6x^2\right)\log\left(x^{\frac{1}{3}}\right) + 15\left(56\,a^{11}bx^3 + 98\,a^8b^4x^2 + 36\,a^5b^7x - 3\,a^2b^{10}\right)x^{\frac{2}{3}} - 3\left(140\,a^{10}b^2x^3 + 224\,a^7b^5x^2 + 63\,a^4b^8x - 6\,ab^{11}\right)x^{\frac{1}{3}}}{10\left(a^6b^9x^4 + 2\,a^3b^{12}x^3 + b^{15}x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/10*(280*a^9*b^3*x^3 + 420*a^6*b^6*x^2 + 90*a^3*b^9*x - 5*b^12 - 840*(a^12
*x^4 + 2*a^9*b^3*x^3 + a^6*b^6*x^2)*log(a*x^(1/3) + b) + 840*(a^12*x^4 + 2*
a^9*b^3*x^3 + a^6*b^6*x^2)*log(x^(1/3)) + 15*(56*a^11*b*x^3 + 98*a^8*b^4*x^
2 + 36*a^5*b^7*x - 3*a^2*b^10)*x^(2/3) - 3*(140*a^10*b^2*x^3 + 224*a^7*b^5*
x^2 + 63*a^4*b^8*x - 6*a*b^11)*x^(1/3))/(a^6*b^9*x^4 + 2*a^3*b^12*x^3 + b^1
5*x^2)
```

giac [A] time = 0.21, size = 123, normalized size = 0.84

$$-\frac{84\,a^6\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{b^9} + \frac{28\,a^6\log\left(|x|\right)}{b^9} + \frac{840\,a^7bx^{\frac{7}{3}} + 1260\,a^6b^2x^2 + 280\,a^5b^3x^{\frac{5}{3}} - 70\,a^4b^4x^{\frac{4}{3}} + 28\,a^3b^5x - 14\,a^2b^6}{10\left(ax^{\frac{1}{3}}+b\right)^2b^9x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))^3/x^4,x, algorithm="giac")
```

```
[Out] -84*a^6*log(abs(a*x^(1/3) + b))/b^9 + 28*a^6*log(abs(x))/b^9 + 1/10*(840*a^
7*b*x^(7/3) + 1260*a^6*b^2*x^2 + 280*a^5*b^3*x^(5/3) - 70*a^4*b^4*x^(4/3) +
 28*a^3*b^5*x - 14*a^2*b^6*x^(2/3) + 8*a*b^7*x^(1/3) - 5*b^8)/((a*x^(1/3) +
 b)^2*b^9*x^2)
```

maple [A] time = 0.01, size = 123, normalized size = 0.84

$$\frac{3a^6}{2\left(a\,x^{\frac{1}{3}}+b\right)^2 b^7}+\frac{21a^6}{\left(a\,x^{\frac{1}{3}}+b\right)b^8}+\frac{28a^6\ln(x)}{b^9}-\frac{84a^6\ln\left(a\,x^{\frac{1}{3}}+b\right)}{b^9}+\frac{63a^5}{b^8x^{\frac{1}{3}}}-\frac{45a^4}{2b^7x^{\frac{2}{3}}}+\frac{10a^3}{b^6x}-\frac{9a^2}{2b^5x^{\frac{4}{3}}}+\frac{9a}{5b^4x^{\frac{5}{3}}}-\frac{1}{2b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^(1/3))^3/x^4,x)
```

```
[Out] 3/2*a^6/b^7/(a*x^(1/3)+b)^2+21*a^6/b^8/(a*x^(1/3)+b)-1/2/b^3/x^2+9/5*a/b^4/
x^(5/3)-9/2*a^2/b^5/x^(4/3)+10*a^3/b^6/x-45/2*a^4/b^7/x^(2/3)+63*a^5/b^8/x^
(1/3)-84*a^6*ln(a*x^(1/3)+b)/b^9+28*a^6*ln(x)/b^9
```

maxima [A] time = 0.49, size = 146, normalized size = 1.00

$$-\frac{84\,a^6\log\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{b^9}-\frac{\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^6}{2\,b^9}+\frac{24\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^5 a}{5\,b^9}-\frac{21\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^4 a^2}{b^9}+\frac{56\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^3 a^3}{b^9}-\frac{105\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^2 a^4}{b^9}+\frac{168\left(a+\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))^3/x^4,x, algorithm="maxima")
```

```
[Out] -84*a^6*log(a + b/x^(1/3))/b^9 - 1/2*(a + b/x^(1/3))^6/b^9 + 24/5*(a + b/x^
(1/3))^5*a/b^9 - 21*(a + b/x^(1/3))^4*a^2/b^9 + 56*(a + b/x^(1/3))^3*a^3/b^
9 - 105*(a + b/x^(1/3))^2*a^4/b^9 + 168*(a + b/x^(1/3))*a^5/b^9 - 24*a^7/((
a + b/x^(1/3))*b^9) + 3/2*a^8/((a + b/x^(1/3))^2*b^9)
```

mupad [B] time = 1.21, size = 125, normalized size = 0.86

$$\frac{\frac{4\,a\,x^{1/3}}{5\,b^2}-\frac{1}{2\,b}+\frac{14\,a^3\,x}{5\,b^4}-\frac{7\,a^2\,x^{2/3}}{5\,b^3}+\frac{126\,a^6\,x^2}{b^7}-\frac{7\,a^4\,x^{4/3}}{b^5}+\frac{28\,a^5\,x^{5/3}}{b^6}+\frac{84\,a^7\,x^{7/3}}{b^8}}{a^2\,x^{8/3}+b^2\,x^2+2\,a\,b\,x^{7/3}}-\frac{168\,a^6\,\mathrm{atanh}\left(\frac{2\,a\,x^{1/3}}{b}+1\right)}{b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b/x^(1/3))^3),x)
```

```
[Out] ((4*a*x^(1/3))/(5*b^2) - 1/(2*b) + (14*a^3*x)/(5*b^4) - (7*a^2*x^(2/3))/(5*
b^3) + (126*a^6*x^2)/b^7 - (7*a^4*x^(4/3))/b^5 + (28*a^5*x^(5/3))/b^6 + (84
*a^7*x^(7/3))/b^8)/(a^2*x^(8/3) + b^2*x^2 + 2*a*b*x^(7/3)) - (168*a^6*atanh
((2*a*x^(1/3))/b + 1))/b^9
```

sympy [A] time = 22.23, size = 707, normalized size = 4.84

$$\begin{cases}\frac{\tilde{\infty}}{x^2} \\ -\frac{1}{2b^3x^2} \\ -\frac{1}{3a^3x^3} \\ \frac{280a^8x^{\frac{13}{3}}\log(x)}{10a^2b^9x^{\frac{13}{3}}+20ab^{10}x^4+10b^{11}x^{\frac{11}{3}}}-\frac{840a^8x^{\frac{13}{3}}\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{10a^2b^9x^{\frac{13}{3}}+20ab^{10}x^4+10b^{11}x^{\frac{11}{3}}}+\frac{560a^7bx^4\log(x)}{10a^2b^9x^{\frac{13}{3}}+20ab^{10}x^4+10b^{11}x^{\frac{11}{3}}}-\frac{1680a^7bx^4\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{10a^2b^9x^{\frac{13}{3}}+20ab^{10}x^4+10b^{11}x^{\frac{11}{3}}}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**(1/3))**3/x**4,x)
```

```
[Out] Piecewise((zoo/x**2, Eq(a, 0) & Eq(b, 0)), (-1/(2*b**3*x**2), Eq(a, 0)), (-
1/(3*a**3*x**3), Eq(b, 0)), (280*a**8*x**(13/3)*log(x)/(10*a**2*b**9*x**(13
/3) + 20*a*b**10*x**4 + 10*b**11*x**(11/3)) - 840*a**8*x**(13/3)*log(x**(1/
3) + b/a)/(10*a**2*b**9*x**(13/3) + 20*a*b**10*x**4 + 10*b**11*x**(11/3)) +
 560*a**7*b*x**4*log(x)/(10*a**2*b**9*x**(13/3) + 20*a*b**10*x**4 + 10*b**1
1*x**(11/3)) - 1680*a**7*b*x**4*log(x**(1/3) + b/a)/(10*a**2*b**9*x**(13/3)
 + 20*a*b**10*x**4 + 10*b**11*x**(11/3)) + 840*a**7*b*x**4/(10*a**2*b**9*x*
*(13/3) + 20*a*b**10*x**4 + 10*b**11*x**(11/3)) + 280*a**6*b**2*x**(11/3)*l
og(x)/(10*a**2*b**9*x**(13/3) + 20*a*b**10*x**4 + 10*b**11*x**(11/3)) - 840
*a**6*b**2*x**(11/3)*log(x**(1/3) + b/a)/(10*a**2*b**9*x**(13/3) + 20*a*b**
10*x**4 + 10*b**11*x**(11/3)) + 1260*a**6*b**2*x**(11/3)/(10*a**2*b**9*x**(
13/3) + 20*a*b**10*x**4 + 10*b**11*x**(11/3)) + 280*a**5*b**3*x**(10/3)/(10
*a**2*b**9*x**(13/3) + 20*a*b**10*x**4 + 10*b**11*x**(11/3)) - 70*a**4*b**4
*x**3/(10*a**2*b**9*x**(13/3) + 20*a*b**10*x**4 + 10*b**11*x**(11/3)) + 28*
a**3*b**5*x**(8/3)/(10*a**2*b**9*x**(13/3) + 20*a*b**10*x**4 + 10*b**11*x**
(11/3)) - 14*a**2*b**6*x**(7/3)/(10*a**2*b**9*x**(13/3) + 20*a*b**10*x**4 +
 10*b**11*x**(11/3)) + 8*a*b**7*x**2/(10*a**2*b**9*x**(13/3) + 20*a*b**10*x
**4 + 10*b**11*x**(11/3)) - 5*b**8*x**(5/3)/(10*a**2*b**9*x**(13/3) + 20*a*
b**10*x**4 + 10*b**11*x**(11/3)), True))
```

3.2444 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^5}\,dx$

Optimal. Leaf size=183

$$\frac{165a^9\log\left(a\sqrt[3]{x}+b\right)}{b^{12}}-\frac{55a^9\log(x)}{b^{12}}-\frac{30a^9}{b^{11}\left(a\sqrt[3]{x}+b\right)}-\frac{3a^9}{2b^{10}\left(a\sqrt[3]{x}+b\right)^2}-\frac{135a^8}{b^{11}\sqrt[3]{x}}+\frac{54a^7}{b^{10}x^{2/3}}-\frac{28a^6}{b^9x}+\frac{63a^5}{4b^8x^{4/3}}-\frac{9a^4}{b^7x^{5/}}$$

```
[Out] -3/2*a^9/b^10/(b+a*x^(1/3))^2-30*a^9/b^11/(b+a*x^(1/3))-1/3/b^3/x^3+9/8*a/b
^4/x^(8/3)-18/7*a^2/b^5/x^(7/3)+5*a^3/b^6/x^2-9*a^4/b^7/x^(5/3)+63/4*a^5/b^
8/x^(4/3)-28*a^6/b^9/x+54*a^7/b^10/x^(2/3)-135*a^8/b^11/x^(1/3)+165*a^9*ln(
b+a*x^(1/3))/b^12-55*a^9*ln(x)/b^12
```

Rubi [A] time = 0.14, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {263, 266, 44}

$$\frac{54a^7}{b^{10}x^{2/3}}+\frac{63a^5}{4b^8x^{4/3}}-\frac{9a^4}{b^7x^{5/3}}+\frac{5a^3}{b^6x^2}-\frac{18a^2}{7b^5x^{7/3}}-\frac{30a^9}{b^{11}\left(a\sqrt[3]{x}+b\right)}-\frac{3a^9}{2b^{10}\left(a\sqrt[3]{x}+b\right)^2}-\frac{135a^8}{b^{11}\sqrt[3]{x}}-\frac{28a^6}{b^9x}+\frac{165a^9\log\left(a\sqrt[3]{x}+\right.}{b^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x^(1/3))^3*x^5),x]
```

```
[Out] (-3*a^9)/(2*b^10*(b + a*x^(1/3))^2) - (30*a^9)/(b^11*(b + a*x^(1/3))) - 1/(
3*b^3*x^3) + (9*a)/(8*b^4*x^(8/3)) - (18*a^2)/(7*b^5*x^(7/3)) + (5*a^3)/(b^
6*x^2) - (9*a^4)/(b^7*x^(5/3)) + (63*a^5)/(4*b^8*x^(4/3)) - (28*a^6)/(b^9*x
) + (54*a^7)/(b^10*x^(2/3)) - (135*a^8)/(b^11*x^(1/3)) + (165*a^9*Log[b + a
*x^(1/3)])/b^12 - (55*a^9*Log[x])/b^12
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^5}\,dx = \int \frac{1}{\left(b+a\sqrt[3]{x}\right)^3 x^4}\,dx$$
$$= 3\,\text{Subst}\left(\int \frac{1}{x^{10}(b+ax)^3}\,dx, x, \sqrt[3]{x}\right)$$
$$= 3\,\text{Subst}\left(\int \left(\frac{1}{b^3x^{10}} - \frac{3a}{b^4x^9} + \frac{6a^2}{b^5x^8} - \frac{10a^3}{b^6x^7} + \frac{15a^4}{b^7x^6} - \frac{21a^5}{b^8x^5} + \frac{28a^6}{b^9x^4} - \frac{36a^7}{b^{10}x^3} + \frac{45a^8}{b^{11}x^2} - \frac{55a^9}{b^{12}x}\right.\right.$$
$$= -\frac{3a^9}{2b^{10}\left(b+a\sqrt[3]{x}\right)^2} - \frac{30a^9}{b^{11}\left(b+a\sqrt[3]{x}\right)} - \frac{1}{3b^3x^3} + \frac{9a}{8b^4x^{8/3}} - \frac{18a^2}{7b^5x^{7/3}} + \frac{5a^3}{b^6x^2} - \frac{9a^4}{b^7x^{5/3}} + \frac{63a^5}{4b^8x^4}$$

Mathematica [A] time = 0.28, size = 167, normalized size = 0.91

$$-\frac{-27720a^9\log\left(a\sqrt[3]{x}+b\right) + 9240a^9\log(x) + \frac{b\left(27720a^{10}x^{10/3}+41580a^9bx^3+9240a^8b^2x^{8/3}-2310a^7b^3x^{7/3}+924a^6b^4x^2-462a^5b^5x^{5/3}+26\right.}{x^3\left(a\sqrt[3]{x}+b\right)^2}}{168b^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x^(1/3))^3*x^5),x]
```

```
[Out] -1/168*((b*(56*b^10 - 77*a*b^9*x^(1/3) + 110*a^2*b^8*x^(2/3) - 165*a^3*b^7*
x + 264*a^4*b^6*x^(4/3) - 462*a^5*b^5*x^(5/3) + 924*a^6*b^4*x^2 - 2310*a^7*
b^3*x^(7/3) + 9240*a^8*b^2*x^(8/3) + 41580*a^9*b*x^3 + 27720*a^10*x^(10/3))
)/((b + a*x^(1/3))^2*x^3) - 27720*a^9*Log[b + a*x^(1/3)] + 9240*a^9*Log[x])
/b^12
```

fricas [A] time = 0.71, size = 263, normalized size = 1.44

$$-\frac{9240\,a^{12}b^3x^4 + 13860\,a^9b^6x^3 + 3080\,a^6b^9x^2 - 728\,a^3b^{12}x + 56\,b^{15} - 27720\left(a^{15}x^5 + 2\,a^{12}b^3x^4 + a^9b^6x^3\right)\log\left(ax\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))^3/x^5,x, algorithm="fricas")
```

```
[Out] -1/168*(9240*a^12*b^3*x^4 + 13860*a^9*b^6*x^3 + 3080*a^6*b^9*x^2 - 728*a^3*
b^12*x + 56*b^15 - 27720*(a^15*x^5 + 2*a^12*b^3*x^4 + a^9*b^6*x^3)*log(a*x^
(1/3) + b) + 27720*(a^15*x^5 + 2*a^12*b^3*x^4 + a^9*b^6*x^3)*log(x^(1/3)) +
 18*(1540*a^14*b*x^4 + 2695*a^11*b^4*x^3 + 990*a^8*b^7*x^2 - 99*a^5*b^10*x
+ 24*a^2*b^13)*x^(2/3) - 63*(220*a^13*b^2*x^4 + 352*a^10*b^5*x^3 + 99*a^7*b
^8*x^2 - 18*a^4*b^11*x + 3*a*b^14)*x^(1/3))/(a^6*b^12*x^5 + 2*a^3*b^15*x^4
+ b^18*x^3)
```

giac [A] time = 0.17, size = 156, normalized size = 0.85

$$\frac{165\,a^9\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{b^{12}} - \frac{55\,a^9\log\left(|x|\right)}{b^{12}} - \frac{27720\,a^{10}bx^{\frac{10}{3}} + 41580\,a^9b^2x^3 + 9240\,a^8b^3x^{\frac{8}{3}} - 2310\,a^7b^4x^{\frac{7}{3}} + 924\,a^6b^5x^2}{168\left(ax^{\frac{1}{3}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))^3/x^5,x, algorithm="giac")
```

```
[Out] 165*a^9*log(abs(a*x^(1/3) + b))/b^12 - 55*a^9*log(abs(x))/b^12 - 1/168*(277
20*a^10*b*x^(10/3) + 41580*a^9*b^2*x^3 + 9240*a^8*b^3*x^(8/3) - 2310*a^7*b^
```

```
4*x^(7/3) + 924*a^6*b^5*x^2 - 462*a^5*b^6*x^(5/3) + 264*a^4*b^7*x^(4/3) - 1
65*a^3*b^8*x + 110*a^2*b^9*x^(2/3) - 77*a*b^10*x^(1/3) + 56*b^11)/((a*x^(1/
3) + b)^2*b^12*x^3)
```

maple [A] time = 0.01, size = 156, normalized size = 0.85

$$-\frac{3a^9}{2\left(a\,x^{\frac{1}{3}}+b\right)^2 b^{10}}-\frac{30a^9}{\left(a\,x^{\frac{1}{3}}+b\right) b^{11}}-\frac{55a^9\ln(x)}{b^{12}}+\frac{165a^9\ln\left(a\,x^{\frac{1}{3}}+b\right)}{b^{12}}-\frac{135a^8}{b^{11}x^{\frac{1}{3}}}+\frac{54a^7}{b^{10}x^{\frac{2}{3}}}-\frac{28a^6}{b^9x}+\frac{63a^5}{4b^8x^{\frac{4}{3}}}-\frac{9a^4}{b^7x^{\frac{5}{3}}}+\frac{5a}{b^6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^(1/3))^3/x^5,x)
```

```
[Out] -3/2*a^9/b^10/(a*x^(1/3)+b)^2-30*a^9/b^11/(a*x^(1/3)+b)-1/3/b^3/x^3+9/8*a/b
^4/x^(8/3)-18/7*a^2/b^5/x^(7/3)+5*a^3/b^6/x^2-9*a^4/b^7/x^(5/3)+63/4*a^5/b^
8/x^(4/3)-28*a^6/b^9/x+54*a^7/b^10/x^(2/3)-135*a^8/b^11/x^(1/3)+165*a^9*ln(
a*x^(1/3)+b)/b^12-55*a^9*ln(x)/b^12
```

maxima [A] time = 0.49, size = 197, normalized size = 1.08

$$\frac{165\,a^9\log\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{b^{12}}-\frac{\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^9}{3\,b^{12}}+\frac{33\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^8 a}{8\,b^{12}}-\frac{165\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^7 a^2}{7\,b^{12}}+\frac{165\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^6 a^3}{2\,b^{12}}-\frac{198\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^5 a^4}{b^{12}}+\frac{693\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^(1/3))^3/x^5,x, algorithm="maxima")
```

```
[Out] 165*a^9*log(a + b/x^(1/3))/b^12 - 1/3*(a + b/x^(1/3))^9/b^12 + 33/8*(a + b/
x^(1/3))^8*a/b^12 - 165/7*(a + b/x^(1/3))^7*a^2/b^12 + 165/2*(a + b/x^(1/3)
)^6*a^3/b^12 - 198*(a + b/x^(1/3))^5*a^4/b^12 + 693/2*(a + b/x^(1/3))^4*a^5
/b^12 - 462*(a + b/x^(1/3))^3*a^6/b^12 + 495*(a + b/x^(1/3))^2*a^7/b^12 - 4
95*(a + b/x^(1/3))*a^8/b^12 + 33*a^10/((a + b/x^(1/3))*b^12) - 3/2*a^11/((a
 + b/x^(1/3))^2*b^12)
```

mupad [B] time = 1.31, size = 159, normalized size = 0.87

$$\frac{330\,a^9\,\operatorname{atanh}\left(\frac{2\,a\,x^{1/3}}{b}+1\right)}{b^{12}}-\frac{\frac{1}{3\,b}-\frac{11\,a\,x^{1/3}}{24\,b^2}-\frac{55\,a^3\,x}{56\,b^4}+\frac{55\,a^2\,x^{2/3}}{84\,b^3}+\frac{11\,a^6\,x^2}{2\,b^7}+\frac{11\,a^4\,x^{4/3}}{7\,b^5}-\frac{11\,a^5\,x^{5/3}}{4\,b^6}+\frac{495\,a^9\,x^3}{2\,b^{10}}-\frac{55\,a^7\,x^{7/3}}{4\,b^8}+}{a^2\,x^{11/3}+b^2\,x^3+2\,a\,b\,x^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b/x^(1/3))^3),x)
```

```
[Out] (330*a^9*atanh((2*a*x^(1/3))/b + 1))/b^12 - (1/(3*b) - (11*a*x^(1/3))/(24*b
^2) - (55*a^3*x)/(56*b^4) + (55*a^2*x^(2/3))/(84*b^3) + (11*a^6*x^2)/(2*b^7
) + (11*a^4*x^(4/3))/(7*b^5) - (11*a^5*x^(5/3))/(4*b^6) + (495*a^9*x^3)/(2*
b^10) - (55*a^7*x^(7/3))/(4*b^8) + (55*a^8*x^(8/3))/b^9 + (165*a^10*x^(10/3
))/b^11)/(a^2*x^(11/3) + b^2*x^3 + 2*a*b*x^(10/3))
```

sympy [A] time = 40.81, size = 848, normalized size = 4.63

$$\begin{cases} \frac{\infty}{x^3} \\ -\frac{1}{3b^3x^3} \\ -\frac{1}{4a^3x^4} \\ -\frac{9240a^{11}x^{\frac{16}{3}}\log(x)}{168a^2b^{12}x^{\frac{16}{3}}+336ab^{13}x^5+168b^{14}x^{\frac{14}{3}}} + \frac{27720a^{11}x^{\frac{16}{3}}\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{168a^2b^{12}x^{\frac{16}{3}}+336ab^{13}x^5+168b^{14}x^{\frac{14}{3}}} - \frac{18480a^{10}bx^5\log(x)}{168a^2b^{12}x^{\frac{16}{3}}+336ab^{13}x^5+168b^{14}x^{\frac{14}{3}}} + \frac{55440a^{10}bx^5\,\text{lo}}{168a^2b^{12}x^{\frac{16}{3}}+336ab} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**(1/3))**3/x**5,x)
```

```
[Out] Piecewise((zoo/x**3, Eq(a, 0) & Eq(b, 0)), (-1/(3*b**3*x**3), Eq(a, 0)), (-
1/(4*a**3*x**4), Eq(b, 0)), (-9240*a**11*x**(16/3)*log(x)/(168*a**2*b**12*x
**(16/3) + 336*a*b**13*x**5 + 168*b**14*x**(14/3)) + 27720*a**11*x**(16/3)*
log(x**(1/3) + b/a)/(168*a**2*b**12*x**(16/3) + 336*a*b**13*x**5 + 168*b**1
4*x**(14/3)) - 18480*a**10*b*x**5*log(x)/(168*a**2*b**12*x**(16/3) + 336*a*
b**13*x**5 + 168*b**14*x**(14/3)) + 55440*a**10*b*x**5*log(x**(1/3) + b/a)/
(168*a**2*b**12*x**(16/3) + 336*a*b**13*x**5 + 168*b**14*x**(14/3)) - 27720
*a**10*b*x**5/(168*a**2*b**12*x**(16/3) + 336*a*b**13*x**5 + 168*b**14*x**(
14/3)) - 9240*a**9*b**2*x**(14/3)*log(x)/(168*a**2*b**12*x**(16/3) + 336*a*
b**13*x**5 + 168*b**14*x**(14/3)) + 27720*a**9*b**2*x**(14/3)*log(x**(1/3)
+ b/a)/(168*a**2*b**12*x**(16/3) + 336*a*b**13*x**5 + 168*b**14*x**(14/3))
- 41580*a**9*b**2*x**(14/3)/(168*a**2*b**12*x**(16/3) + 336*a*b**13*x**5 +
168*b**14*x**(14/3)) - 9240*a**8*b**3*x**(13/3)/(168*a**2*b**12*x**(16/3) +
 336*a*b**13*x**5 + 168*b**14*x**(14/3)) + 2310*a**7*b**4*x**4/(168*a**2*b*
*12*x**(16/3) + 336*a*b**13*x**5 + 168*b**14*x**(14/3)) - 924*a**6*b**5*x**
(11/3)/(168*a**2*b**12*x**(16/3) + 336*a*b**13*x**5 + 168*b**14*x**(14/3))
+ 462*a**5*b**6*x**(10/3)/(168*a**2*b**12*x**(16/3) + 336*a*b**13*x**5 + 16
8*b**14*x**(14/3)) - 264*a**4*b**7*x**3/(168*a**2*b**12*x**(16/3) + 336*a*b
**13*x**5 + 168*b**14*x**(14/3)) + 165*a**3*b**8*x**(8/3)/(168*a**2*b**12*x
**(16/3) + 336*a*b**13*x**5 + 168*b**14*x**(14/3)) - 110*a**2*b**9*x**(7/3)
/(168*a**2*b**12*x**(16/3) + 336*a*b**13*x**5 + 168*b**14*x**(14/3)) + 77*a
*b**10*x**2/(168*a**2*b**12*x**(16/3) + 336*a*b**13*x**5 + 168*b**14*x**(14
/3)) - 56*b**11*x**(5/3)/(168*a**2*b**12*x**(16/3) + 336*a*b**13*x**5 + 168
*b**14*x**(14/3)), True))
```

3.2445 $\int \frac{1}{1+\frac{b}{\sqrt[3]{x}}}\,dx$

Optimal. Leaf size=44

$$-3b^3\log\left(\frac{b}{\sqrt[3]{x}}+1\right)-b^3\log(x)+3b^2\sqrt[3]{x}-\frac{3}{2}bx^{2/3}+x$$

```
[Out] 3*b^2*x^(1/3)-3/2*b*x^(2/3)+x-3*b^3*ln(1+b/x^(1/3))-b^3*ln(x)
```

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {190, 44}

$$3b^2\sqrt[3]{x}-3b^3\log\left(\frac{b}{\sqrt[3]{x}}+1\right)-b^3\log(x)-\frac{3}{2}bx^{2/3}+x$$

Antiderivative was successfully verified.

```
[In] Int[(1 + b/x^(1/3))^(-1),x]
```

```
[Out] 3*b^2*x^(1/3) - (3*b*x^(2/3))/2 + x - 3*b^3*Log[1 + b/x^(1/3)] - b^3*Log[x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+\frac{b}{\sqrt[3]{x}}}\,dx &= -\left(3\,\text{Subst}\left(\int \frac{1}{x^4(1+bx)}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)\\ &= -\left(3\,\text{Subst}\left(\int \left(\frac{1}{x^4}-\frac{b}{x^3}+\frac{b^2}{x^2}-\frac{b^3}{x}+\frac{b^4}{1+bx}\right)dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)\\ &= 3b^2\sqrt[3]{x}-\frac{3}{2}bx^{2/3}+x-3b^3\log\left(1+\frac{b}{\sqrt[3]{x}}\right)-b^3\log(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.80

$$-3b^3\log\left(b+\sqrt[3]{x}\right)+3b^2\sqrt[3]{x}-\frac{3}{2}bx^{2/3}+x$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + b/x^(1/3))^(-1),x]
```

```
[Out] 3*b^2*x^(1/3) - (3*b*x^(2/3))/2 + x - 3*b^3*Log[b + x^(1/3)]
```

fricas [A] time = 0.72, size = 27, normalized size = 0.61

$$-3\,b^3\log\left(b+x^{\frac{1}{3}}\right)+3\,b^2x^{\frac{1}{3}}-\frac{3}{2}\,bx^{\frac{2}{3}}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+b/x^(1/3)),x, algorithm="fricas")
[Out] -3*b^3*log(b + x^(1/3)) + 3*b^2*x^(1/3) - 3/2*b*x^(2/3) + x
```

giac [A] time = 0.17, size = 28, normalized size = 0.64

$$-3\,b^3\log\left(\left|b+x^{\frac{1}{3}}\right|\right)+3\,b^2x^{\frac{1}{3}}-\frac{3}{2}\,bx^{\frac{2}{3}}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+b/x^(1/3)),x, algorithm="giac")
[Out] -3*b^3*log(abs(b + x^(1/3))) + 3*b^2*x^(1/3) - 3/2*b*x^(2/3) + x
```

maple [A] time = 0.00, size = 28, normalized size = 0.64

$$-3b^3\ln\left(b+x^{\frac{1}{3}}\right)+3b^2x^{\frac{1}{3}}-\frac{3b\,x^{\frac{2}{3}}}{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+b/x^(1/3)),x)
[Out] x-3/2*b*x^(2/3)+3*b^2*x^(1/3)-3*b^3*ln(x^(1/3)+b)
```

maxima [A] time = 0.53, size = 40, normalized size = 0.91

$$-b^3\log(x)-3\,b^3\log\left(\frac{b}{x^{\frac{1}{3}}}+1\right)+\frac{1}{2}\left(\frac{6\,b^2}{x^{\frac{2}{3}}}-\frac{3\,b}{x^{\frac{1}{3}}}+2\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+b/x^(1/3)),x, algorithm="maxima")
[Out] -b^3*log(x) - 3*b^3*log(b/x^(1/3) + 1) + 1/2*(6*b^2/x^(2/3) - 3*b/x^(1/3) +
 2)*x
```

mupad [B] time = 0.04, size = 27, normalized size = 0.61

$$x-3\,b^3\,\ln\left(b+x^{1/3}\right)-\frac{3\,b\,x^{2/3}}{2}+3\,b^2\,x^{1/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x^(1/3) + 1),x)
[Out] x - 3*b^3*log(b + x^(1/3)) - (3*b*x^(2/3))/2 + 3*b^2*x^(1/3)
```

sympy [A] time = 0.15, size = 34, normalized size = 0.77

$$-3b^3\log\left(b+\sqrt[3]{x}\right)+3b^2\sqrt[3]{x}-\frac{3bx^{\frac{2}{3}}}{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+b/x**(1/3)),x)
[Out] -3*b**3*log(b + x**(1/3)) + 3*b**2*x**(1/3) - 3*b*x**(2/3)/2 + x
```

3.2446 $\int x^{2/3}\left(1+x^{5/3}\right)^{2/3}\,dx$

Optimal. Leaf size=15

$$\frac{9}{25}\left(x^{5/3}+1\right)^{5/3}$$

```
[Out] 9/25*(1+x^(5/3))^(5/3)
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {261}

$$\frac{9}{25}\left(x^{5/3}+1\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2/3)*(1 + x^(5/3))^(2/3),x]
```

```
[Out] (9*(1 + x^(5/3))^(5/3))/25
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^{2/3}\left(1+x^{5/3}\right)^{2/3}\,dx = \frac{9}{25}\left(1+x^{5/3}\right)^{5/3}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{9}{25}\left(x^{5/3}+1\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2/3)*(1 + x^(5/3))^(2/3),x]
```

```
[Out] (9*(1 + x^(5/3))^(5/3))/25
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(1+x^(5/3))^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.19, size = 9, normalized size = 0.60

$$\frac{9}{25}\left(x^{\frac{5}{3}}+1\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(1+x^(5/3))^(2/3),x, algorithm="giac")
[Out] 9/25*(x^(5/3) + 1)^(5/3)
```

maple [A] time = 0.00, size = 10, normalized size = 0.67

$$\frac{9\left(x^{\frac{5}{3}}+1\right)^{\frac{5}{3}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)*(1+x^(5/3))^(2/3),x)
[Out] 9/25*(1+x^(5/3))^(5/3)
```

maxima [A] time = 0.56, size = 9, normalized size = 0.60

$$\frac{9}{25}\left(x^{\frac{5}{3}}+1\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(1+x^(5/3))^(2/3),x, algorithm="maxima")
[Out] 9/25*(x^(5/3) + 1)^(5/3)
```

mupad [B] time = 1.54, size = 9, normalized size = 0.60

$$\frac{9\left(x^{5/3}+1\right)^{5/3}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)*(x^(5/3) + 1)^(2/3),x)
[Out] (9*(x^(5/3) + 1)^(5/3))/25
```

sympy [B] time = 1.90, size = 31, normalized size = 2.07

$$\frac{9x^{\frac{5}{3}}\left(x^{\frac{5}{3}}+1\right)^{\frac{2}{3}}}{25}+\frac{9\left(x^{\frac{5}{3}}+1\right)^{\frac{2}{3}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2/3)*(1+x**(5/3))**(2/3),x)
[Out] 9*x**(5/3)*(x**(5/3) + 1)**(2/3)/25 + 9*(x**(5/3) + 1)**(2/3)/25
```

3.2447 $\int x^{7/3}\left(a^{10/3}-x^{10/3}\right)^{19/7}\,dx$

Optimal. Leaf size=21

$$-\frac{21}{260}\left(a^{10/3}-x^{10/3}\right)^{26/7}$$

```
[Out] -21/260*(a^(10/3)-x^(10/3))^(26/7)
```

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {261}

$$-\frac{21}{260}\left(a^{10/3}-x^{10/3}\right)^{26/7}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/3)*(a^(10/3) - x^(10/3))^(19/7),x]
```

```
[Out] (-21*(a^(10/3) - x^(10/3))^(26/7))/260
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^{7/3}\left(a^{10/3}-x^{10/3}\right)^{19/7}\,dx = -\frac{21}{260}\left(a^{10/3}-x^{10/3}\right)^{26/7}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$-\frac{21}{260}\left(a^{10/3}-x^{10/3}\right)^{26/7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/3)*(a^(10/3) - x^(10/3))^(19/7),x]
```

```
[Out] (-21*(a^(10/3) - x^(10/3))^(26/7))/260
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/3)*(a^(10/3)-x^(10/3))^(19/7),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.71, size = 81, normalized size = 3.86

$$-\frac{7}{20}\left(a^{\frac{10}{3}}-x^{\frac{10}{3}}\right)^{\frac{12}{7}}a^{\frac{20}{3}}+\frac{21}{95}\left(a^{\frac{10}{3}}-x^{\frac{10}{3}}\right)^{\frac{19}{7}}a^{\frac{10}{3}}+\frac{7}{380}\left(19\left(a^{\frac{10}{3}}-x^{\frac{10}{3}}\right)^{\frac{12}{7}}a^{\frac{10}{3}}-12\left(a^{\frac{10}{3}}-x^{\frac{10}{3}}\right)^{\frac{19}{7}}\right)a^{\frac{10}{3}}-\frac{21}{260}\left(a^{\frac{10}{3}}-x^{\frac{10}{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/3)*(a^(10/3)-x^(10/3))^(19/7),x, algorithm="giac")
```

```
[Out] -7/20*(a^(10/3) - x^(10/3))^(12/7)*a^(20/3) + 21/95*(a^(10/3) - x^(10/3))^(
19/7)*a^(10/3) + 7/380*(19*(a^(10/3) - x^(10/3))^(12/7)*a^(10/3) - 12*(a^(1
0/3) - x^(10/3))^(19/7))*a^(10/3) - 21/260*(a^(10/3) - x^(10/3))^(26/7)
```

maple [**A**] time = 0.00, size = 14, normalized size = 0.67

$$-\frac{21\left(a^{\frac{10}{3}}-x^{\frac{10}{3}}\right)^{\frac{26}{7}}}{260}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/3)*(a^(10/3)-x^(10/3))^(19/7),x)
```

```
[Out] -21/260*(a^(10/3)-x^(10/3))^(26/7)
```

maxima [**A**] time = 0.47, size = 13, normalized size = 0.62

$$-\frac{21}{260}\left(a^{\frac{10}{3}}-x^{\frac{10}{3}}\right)^{\frac{26}{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/3)*(a^(10/3)-x^(10/3))^(19/7),x, algorithm="maxima")
```

```
[Out] -21/260*(a^(10/3) - x^(10/3))^(26/7)
```

mupad [**B**] time = 2.04, size = 13, normalized size = 0.62

$$-\frac{21\left(a^{10/3}-x^{10/3}\right)^{26/7}}{260}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/3)*(a^(10/3) - x^(10/3))^(19/7),x)
```

```
[Out] -(21*(a^(10/3) - x^(10/3))^(26/7))/260
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/3)*(a**(10/3)-x**(10/3))**(19/7),x)
```

```
[Out] Timed out
```

3.2448 $\int \frac{1}{1+\sqrt[5]{x}}\,dx$

Optimal. Leaf size=45

$$\frac{5x^{4/5}}{4}-\frac{5x^{3/5}}{3}+\frac{5x^{2/5}}{2}-5\sqrt[5]{x}+5\log\left(\sqrt[5]{x}+1\right)$$

```
[Out] -5*x^(1/5)+5/2*x^(2/5)-5/3*x^(3/5)+5/4*x^(4/5)+5*ln(1+x^(1/5))
```

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {190, 43}

$$\frac{5x^{4/5}}{4}-\frac{5x^{3/5}}{3}+\frac{5x^{2/5}}{2}-5\sqrt[5]{x}+5\log\left(\sqrt[5]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^(1/5))^(-1),x]
```

```
[Out] -5*x^(1/5) + (5*x^(2/5))/2 - (5*x^(3/5))/3 + (5*x^(4/5))/4 + 5*Log[1 + x^(1
/5)]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
 - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
  IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+\sqrt[5]{x}}\,dx &= 5\,\text{Subst}\left(\int \frac{x^4}{1+x}\,dx, x, \sqrt[5]{x}\right)\\ &= 5\,\text{Subst}\left(\int \left(-1+x-x^2+x^3+\frac{1}{1+x}\right)\,dx, x, \sqrt[5]{x}\right)\\ &= -5\sqrt[5]{x}+\frac{5x^{2/5}}{2}-\frac{5x^{3/5}}{3}+\frac{5x^{4/5}}{4}+5\log\left(1+\sqrt[5]{x}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 1.00

$$\frac{5x^{4/5}}{4}-\frac{5x^{3/5}}{3}+\frac{5x^{2/5}}{2}-5\sqrt[5]{x}+5\log\left(\sqrt[5]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^(1/5))^(-1),x]
```

```
[Out] -5*x^(1/5) + (5*x^(2/5))/2 - (5*x^(3/5))/3 + (5*x^(4/5))/4 + 5*Log[1 + x^(1
/5)]
```

fricas [A] time = 1.61, size = 29, normalized size = 0.64

$$\frac{5}{4}x^{\frac{4}{5}}-\frac{5}{3}x^{\frac{3}{5}}+\frac{5}{2}x^{\frac{2}{5}}-5x^{\frac{1}{5}}+5\log\left(x^{\frac{1}{5}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(1/5)),x, algorithm="fricas")
```

```
[Out] 5/4*x^(4/5) - 5/3*x^(3/5) + 5/2*x^(2/5) - 5*x^(1/5) + 5*log(x^(1/5) + 1)
```

giac [A] time = 0.19, size = 29, normalized size = 0.64

$$\frac{5}{4}x^{\frac{4}{5}}-\frac{5}{3}x^{\frac{3}{5}}+\frac{5}{2}x^{\frac{2}{5}}-5x^{\frac{1}{5}}+5\log\left(x^{\frac{1}{5}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(1/5)),x, algorithm="giac")
```

```
[Out] 5/4*x^(4/5) - 5/3*x^(3/5) + 5/2*x^(2/5) - 5*x^(1/5) + 5*log(x^(1/5) + 1)
```

maple [B] time = 0.06, size = 79, normalized size = 1.76

$$4\ln\left(x^{\frac{1}{5}}+1\right)+\ln(x+1)-\ln\left(2x^{\frac{2}{5}}-\sqrt{5}\,x^{\frac{1}{5}}-x^{\frac{1}{5}}+2\right)-\ln\left(2x^{\frac{2}{5}}+\sqrt{5}\,x^{\frac{1}{5}}-x^{\frac{1}{5}}+2\right)+\frac{5x^{\frac{4}{5}}}{4}-\frac{5x^{\frac{3}{5}}}{3}+\frac{5x^{\frac{2}{5}}}{2}-5x^{\frac{1}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x^(1/5)),x)
```

```
[Out] ln(x+1)+5/2*x^(2/5)-ln(x^(1/5)*5^(1/2)+2*x^(2/5)-x^(1/5)+2)-ln(-x^(1/5)*5^(
1/2)+2*x^(2/5)-x^(1/5)+2)+4*ln(1+x^(1/5))+5/4*x^(4/5)-5*x^(1/5)-5/3*x^(3/5)
```

maxima [A] time = 0.50, size = 42, normalized size = 0.93

$$\frac{5}{4}\left(x^{\frac{1}{5}}+1\right)^4-\frac{20}{3}\left(x^{\frac{1}{5}}+1\right)^3+15\left(x^{\frac{1}{5}}+1\right)^2-20x^{\frac{1}{5}}+5\log\left(x^{\frac{1}{5}}+1\right)-20$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(1/5)),x, algorithm="maxima")
```

```
[Out] 5/4*(x^(1/5) + 1)^4 - 20/3*(x^(1/5) + 1)^3 + 15*(x^(1/5) + 1)^2 - 20*x^(1/5
) + 5*log(x^(1/5) + 1) - 20
```

mupad [B] time = 0.02, size = 29, normalized size = 0.64

$$5\ln\left(x^{1/5}+1\right)-5x^{1/5}+\frac{5x^{2/5}}{2}-\frac{5x^{3/5}}{3}+\frac{5x^{4/5}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/5) + 1),x)
```

```
[Out] 5*log(x^(1/5) + 1) - 5*x^(1/5) + (5*x^(2/5))/2 - (5*x^(3/5))/3 + (5*x^(4/5)
)/4
```

sympy [A] time = 4.08, size = 41, normalized size = 0.91

$$\frac{5x^{\frac{4}{5}}}{4}-\frac{5x^{\frac{3}{5}}}{3}+\frac{5x^{\frac{2}{5}}}{2}-5\sqrt[5]{x}+5\log\left(\sqrt[5]{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x**(1/5)),x)
```

```
[Out] 5*x**(4/5)/4 - 5*x**(3/5)/3 + 5*x**(2/5)/2 - 5*x**(1/5) + 5*log(x**(1/5) +
1)
```

3.2449 $\int \frac{1}{\sqrt{1+x^{4/5}}\sqrt[5]{x}}\,dx$

Optimal. Leaf size=15

$$\frac{5}{2}\sqrt{x^{4/5}+1}$$

```
[Out] 5/2*(1+x^(4/5))^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {261}

$$\frac{5}{2}\sqrt{x^{4/5}+1}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + x^(4/5)]*x^(1/5)),x]
```

```
[Out] (5*Sqrt[1 + x^(4/5)])/2
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+x^{4/5}}\sqrt[5]{x}}\,dx = \frac{5}{2}\sqrt{1+x^{4/5}}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{5}{2}\sqrt{x^{4/5}+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + x^(4/5)]*x^(1/5)),x]
```

```
[Out] (5*Sqrt[1 + x^(4/5)])/2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/5)/(1+x^(4/5))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.16, size = 9, normalized size = 0.60

$$\frac{5}{2}\sqrt{x^{\frac{4}{5}}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/5)/(1+x^(4/5))^(1/2),x, algorithm="giac")
[Out] 5/2*sqrt(x^(4/5) + 1)
```

maple [A] time = 0.00, size = 10, normalized size = 0.67

$$\frac{5\sqrt{x^{\frac{4}{5}}+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/5)/(1+x^(4/5))^(1/2),x)
[Out] 5/2*(1+x^(4/5))^(1/2)
```

maxima [A] time = 0.52, size = 9, normalized size = 0.60

$$\frac{5}{2}\sqrt{x^{\frac{4}{5}}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/5)/(1+x^(4/5))^(1/2),x, algorithm="maxima")
[Out] 5/2*sqrt(x^(4/5) + 1)
```

mupad [B] time = 1.41, size = 9, normalized size = 0.60

$$\frac{5\,\sqrt{x^{4/5}+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/5)*(x^(4/5) + 1)^(1/2)),x)
[Out] (5*(x^(4/5) + 1)^(1/2))/2
```

sympy [A] time = 0.64, size = 12, normalized size = 0.80

$$\frac{5\sqrt{x^{\frac{4}{5}}+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/5)/(1+x**(4/5))**(1/2),x)
[Out] 5*sqrt(x**(4/5) + 1)/2
```

3.2450 $\int \left(a + \frac{b}{x^{3/5}}\right)^{2/3} dx$

Optimal. Leaf size=18

$$\frac{x\left(a + \frac{b}{x^{3/5}}\right)^{5/3}}{a}$$

```
[Out] (a+b/x^(3/5))^(5/3)*x/a
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {191}

$$\frac{x\left(a + \frac{b}{x^{3/5}}\right)^{5/3}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^(3/5))^(2/3),x]
```

```
[Out] ((a + b/x^(3/5))^(5/3)*x)/a
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \left(a + \frac{b}{x^{3/5}}\right)^{2/3} dx = \frac{\left(a + \frac{b}{x^{3/5}}\right)^{5/3} x}{a}$$

Mathematica [A] time = 0.02, size = 18, normalized size = 1.00

$$\frac{x\left(a + \frac{b}{x^{3/5}}\right)^{5/3}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^(3/5))^(2/3),x]
```

```
[Out] ((a + b/x^(3/5))^(5/3)*x)/a
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(3/5))^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + \frac{b}{x^{\frac{3}{5}}}\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(3/5))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((a + b/x^(3/5))^(2/3), x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \left(a + \frac{b}{x^{\frac{3}{5}}}\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^(3/5))^(2/3),x)
```

```
[Out] int((a+b/x^(3/5))^(2/3),x)
```

maxima [A] time = 0.56, size = 14, normalized size = 0.78

$$\frac{\left(a + \frac{b}{x^{\frac{3}{5}}}\right)^{\frac{5}{3}} x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^(3/5))^(2/3),x, algorithm="maxima")
```

```
[Out] (a + b/x^(3/5))^(5/3)*x/a
```

mupad [B] time = 1.29, size = 38, normalized size = 2.11

$$\frac{5\,x\left(a + \frac{b}{x^{3/5}}\right)^{2/3}\,{}_2\mathrm{F}_1\left(-\frac{2}{3},1;\ 2;\ -\frac{a\,x^{3/5}}{b}\right)}{3\left(\frac{a\,x^{3/5}}{b}+1\right)^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^(3/5))^(2/3),x)
```

```
[Out] (5*x*(a + b/x^(3/5))^(2/3)*hypergeom([-2/3, 1], 2, -(a*x^(3/5))/b))/(3*((a*
x^(3/5))/b + 1)^(2/3))
```

sympy [B] time = 8.20, size = 78, normalized size = 4.33

$$-\frac{5b^{\frac{2}{3}}x^{\frac{3}{5}}\left(\frac{ax^{\frac{3}{5}}}{b}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{3\Gamma\left(-\frac{2}{3}\right)} - \frac{5b^{\frac{5}{3}}\left(\frac{ax^{\frac{3}{5}}}{b}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{3a\Gamma\left(-\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**(3/5))**(2/3),x)
```

```
[Out] -5*b**(2/3)*x**(3/5)*(a*x**(3/5)/b + 1)**(2/3)*gamma(-5/3)/(3*gamma(-2/3))
- 5*b**(5/3)*(a*x**(3/5)/b + 1)**(2/3)*gamma(-5/3)/(3*a*gamma(-2/3))
```

3.2451 $\int x^3 (a + bx^n)\, dx$

Optimal. Leaf size=21

$$\frac{ax^4}{4} + \frac{bx^{n+4}}{n+4}$$

```
[Out] 1/4*a*x^4+b*x^(4+n)/(4+n)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$\frac{ax^4}{4} + \frac{bx^{n+4}}{n+4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^n),x]
```

```
[Out] (a*x^4)/4 + (b*x^(4 + n))/(4 + n)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^3 (a + bx^n)\, dx &= \int \left(ax^3 + bx^{3+n}\right) dx \\ &= \frac{ax^4}{4} + \frac{bx^{4+n}}{4+n}\end{aligned}$$

Mathematica [A] time = 0.02, size = 21, normalized size = 1.00

$$\frac{ax^4}{4} + \frac{bx^{n+4}}{n+4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^n),x]
```

```
[Out] (a*x^4)/4 + (b*x^(4 + n))/(4 + n)
```

fricas [A] time = 1.42, size = 28, normalized size = 1.33

$$\frac{4\, bx^4 x^n + (an + 4\, a)x^4}{4\,(n+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^n),x, algorithm="fricas")
```

```
[Out] 1/4*(4*b*x^4*x^n + (a*n + 4*a)*x^4)/(n + 4)
```

giac [A] time = 0.17, size = 29, normalized size = 1.38

$$\frac{4\, bx^4 x^n + anx^4 + 4\, ax^4}{4\,(n+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^n),x, algorithm="giac")
[Out] 1/4*(4*b*x^4*x^n + a*n*x^4 + 4*a*x^4)/(n + 4)
```

maple [A] time = 0.01, size = 23, normalized size = 1.10

$$\frac{b\,x^4 e^{n\ln(x)}}{n+4} + \frac{a\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*x^n),x)
[Out] b/(n+4)*x^4*exp(n*ln(x))+1/4*a*x^4
```

maxima [A] time = 0.74, size = 19, normalized size = 0.90

$$\frac{1}{4}\,ax^4 + \frac{bx^{n+4}}{n+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^n),x, algorithm="maxima")
[Out] 1/4*a*x^4 + b*x^(n + 4)/(n + 4)
```

mupad [B] time = 1.33, size = 20, normalized size = 0.95

$$\frac{a\,x^4}{4} + \frac{b\,x^n\,x^4}{n+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^n),x)
[Out] (a*x^4)/4 + (b*x^n*x^4)/(n + 4)
```

sympy [A] time = 0.50, size = 51, normalized size = 2.43

$$\begin{cases} \frac{anx^4}{4n+16} + \frac{4ax^4}{4n+16} + \frac{4bx^4x^n}{4n+16} & \text{for } n \neq -4 \\ \frac{ax^4}{4} + b\log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*x**n),x)
[Out] Piecewise((a*n*x**4/(4*n + 16) + 4*a*x**4/(4*n + 16) + 4*b*x**4*x**n/(4*n +
 16), Ne(n, -4)), (a*x**4/4 + b*log(x), True))
```

3.2452 $\int x^2 (a + bx^n)\, dx$

Optimal. Leaf size=21

$$\frac{ax^3}{3} + \frac{bx^{n+3}}{n+3}$$

```
[Out] 1/3*a*x^3+b*x^(3+n)/(3+n)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$\frac{ax^3}{3} + \frac{bx^{n+3}}{n+3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^n),x]
```

```
[Out] (a*x^3)/3 + (b*x^(3 + n))/(3 + n)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^2 (a + bx^n)\, dx &= \int \left(ax^2 + bx^{2+n}\right) dx \\ &= \frac{ax^3}{3} + \frac{bx^{3+n}}{3+n}\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{ax^3}{3} + \frac{bx^{n+3}}{n+3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^n),x]
```

```
[Out] (a*x^3)/3 + (b*x^(3 + n))/(3 + n)
```

fricas [A] time = 1.30, size = 28, normalized size = 1.33

$$\frac{3\, bx^3 x^n + (an + 3\, a)x^3}{3\,(n+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^n),x, algorithm="fricas")
```

```
[Out] 1/3*(3*b*x^3*x^n + (a*n + 3*a)*x^3)/(n + 3)
```

giac [A] time = 0.19, size = 29, normalized size = 1.38

$$\frac{3\, bx^3 x^n + anx^3 + 3\, ax^3}{3\,(n+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^n),x, algorithm="giac")
```

```
[Out] 1/3*(3*b*x^3*x^n + a*n*x^3 + 3*a*x^3)/(n + 3)
```

maple [A] time = 0.01, size = 23, normalized size = 1.10

$$\frac{b\,x^3 e^{n\ln(x)}}{n+3} + \frac{a\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*x^n),x)
```

```
[Out] b/(n+3)*x^3*exp(n*ln(x))+1/3*a*x^3
```

maxima [A] time = 0.48, size = 19, normalized size = 0.90

$$\frac{1}{3}\,ax^3 + \frac{bx^{n+3}}{n+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^n),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + b*x^(n + 3)/(n + 3)
```

mupad [B] time = 1.26, size = 20, normalized size = 0.95

$$\frac{a\,x^3}{3} + \frac{b\,x^n\,x^3}{n+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^n),x)
```

```
[Out] (a*x^3)/3 + (b*x^n*x^3)/(n + 3)
```

sympy [A] time = 0.37, size = 51, normalized size = 2.43

$$\begin{cases} \frac{anx^3}{3n+9} + \frac{3ax^3}{3n+9} + \frac{3bx^3x^n}{3n+9} & \text{for } n \neq -3 \\ \frac{ax^3}{3} + b\log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*x**n),x)
```

```
[Out] Piecewise((a*n*x**3/(3*n + 9) + 3*a*x**3/(3*n + 9) + 3*b*x**3*x**n/(3*n + 9
), Ne(n, -3)), (a*x**3/3 + b*log(x), True))
```

3.2453 $\int x\,(a+bx^n)\,dx$

Optimal. Leaf size=21

$$\frac{ax^2}{2}+\frac{bx^{n+2}}{n+2}$$

```
[Out] 1/2*a*x^2+b*x^(2+n)/(2+n)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {14}

$$\frac{ax^2}{2}+\frac{bx^{n+2}}{n+2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^n),x]
```

```
[Out] (a*x^2)/2 + (b*x^(2 + n))/(2 + n)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x\,(a+bx^n)\,dx &= \int \left(ax+bx^{1+n}\right)dx\\ &= \frac{ax^2}{2}+\frac{bx^{2+n}}{2+n}\end{aligned}$$

Mathematica [A] time = 0.02, size = 21, normalized size = 1.00

$$\frac{ax^2}{2}+\frac{bx^{n+2}}{n+2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^n),x]
```

```
[Out] (a*x^2)/2 + (b*x^(2 + n))/(2 + n)
```

fricas [A] time = 0.74, size = 28, normalized size = 1.33

$$\frac{2\,bx^2x^n+(an+2\,a)x^2}{2\,(n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^n),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x^2*x^n + (a*n + 2*a)*x^2)/(n + 2)
```

giac [A] time = 0.15, size = 29, normalized size = 1.38

$$\frac{2\,bx^2x^n+anx^2+2\,ax^2}{2\,(n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^n),x, algorithm="giac")
```

```
[Out] 1/2*(2*b*x^2*x^n + a*n*x^2 + 2*a*x^2)/(n + 2)
```

maple [A] time = 0.01, size = 23, normalized size = 1.10

$$\frac{b\,x^2 e^{n\ln(x)}}{n+2}+\frac{a\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*x^n),x)
```

```
[Out] b/(n+2)*x^2*exp(n*ln(x))+1/2*a*x^2
```

maxima [A] time = 0.49, size = 19, normalized size = 0.90

$$\frac{1}{2}\,ax^2+\frac{bx^{n+2}}{n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^n),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + b*x^(n + 2)/(n + 2)
```

mupad [B] time = 1.24, size = 20, normalized size = 0.95

$$\frac{a\,x^2}{2}+\frac{b\,x^n\,x^2}{n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^n),x)
```

```
[Out] (a*x^2)/2 + (b*x^n*x^2)/(n + 2)
```

sympy [A] time = 0.29, size = 51, normalized size = 2.43

$$\begin{cases}\frac{anx^2}{2n+4}+\frac{2ax^2}{2n+4}+\frac{2bx^2x^n}{2n+4} & \text{for } n\neq -2\\ \frac{ax^2}{2}+b\log(x) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x**n),x)
```

```
[Out] Piecewise((a*n*x**2/(2*n + 4) + 2*a*x**2/(2*n + 4) + 2*b*x**2*x**n/(2*n + 4
), Ne(n, -2)), (a*x**2/2 + b*log(x), True))
```

3.2454 $\int (a + bx^n)\, dx$

Optimal. Leaf size=16

$$ax + \frac{bx^{n+1}}{n+1}$$

```
[Out] a*x+b*x^(1+n)/(1+n)
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$ax + \frac{bx^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*x^n,x]
```

```
[Out] a*x + (b*x^(1 + n))/(1 + n)
```

Rubi steps

$$\int (a + bx^n)\, dx = ax + \frac{bx^{1+n}}{1+n}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$ax + \frac{bx^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*x^n,x]
```

```
[Out] a*x + (b*x^(1 + n))/(1 + n)
```

fricas [A] time = 0.77, size = 20, normalized size = 1.25

$$\frac{bxx^n + (an + a)x}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*x^n,x, algorithm="fricas")
```

```
[Out] (b*x*x^n + (a*n + a)*x)/(n + 1)
```

giac [A] time = 0.15, size = 16, normalized size = 1.00

$$ax + \frac{bx^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*x^n,x, algorithm="giac")
```

```
[Out] a*x + b*x^(n + 1)/(n + 1)
```

maple [A] time = 0.00, size = 17, normalized size = 1.06

$$ax + \frac{b\,x^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*x^n,x)
[Out] a*x+b*x^(n+1)/(n+1)
```

maxima [A] time = 0.52, size = 16, normalized size = 1.00

$$ax + \frac{bx^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*x^n,x, algorithm="maxima")
[Out] a*x + b*x^(n + 1)/(n + 1)
```

mupad [B] time = 1.28, size = 15, normalized size = 0.94

$$a\,x + \frac{b\,x\,x^{n}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*x^n,x)
[Out] a*x + (b*x*x^n)/(n + 1)
```

sympy [A] time = 0.06, size = 17, normalized size = 1.06

$$ax + b\left(\begin{cases} \frac{x^{n+1}}{n+1} & \text{for } n \neq -1 \\ \log(x) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*x**n,x)
[Out] a*x + b*Piecewise((x**(n + 1)/(n + 1), Ne(n, -1)), (log(x), True))
```

3.2455 $\int \frac{a+bx^n}{x}\,dx$

Optimal. Leaf size=13

$$a\log(x)+\frac{bx^n}{n}$$

```
[Out] b*x^n/n+a*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$a\log(x)+\frac{bx^n}{n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)/x,x]
```

```
[Out] (b*x^n)/n + a*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^n}{x}\,dx &= \int \left(\frac{a}{x}+bx^{-1+n}\right)dx\\ &= \frac{bx^n}{n}+a\log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$a\log(x)+\frac{bx^n}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)/x,x]
```

```
[Out] (b*x^n)/n + a*Log[x]
```

fricas [A] time = 0.74, size = 15, normalized size = 1.15

$$\frac{an\log(x)+bx^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/x,x, algorithm="fricas")
```

```
[Out] (a*n*log(x) + b*x^n)/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx^n+a}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/x,x, algorithm="giac")
[Out] integrate((b*x^n + a)/x, x)
```

maple [A] time = 0.01, size = 19, normalized size = 1.46

$$\frac{a \ln(x^n)}{n} + \frac{b\, x^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)/x,x)
[Out] b*x^n/n+1/n*a*ln(x^n)
```

maxima [A] time = 0.50, size = 18, normalized size = 1.38

$$\frac{bx^n}{n} + \frac{a \log(x^n)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/x,x, algorithm="maxima")
[Out] b*x^n/n + a*log(x^n)/n
```

mupad [B] time = 1.19, size = 13, normalized size = 1.00

$$a\, \ln(x) + \frac{b\, x^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)/x,x)
[Out] a*log(x) + (b*x^n)/n
```

sympy [A] time = 0.20, size = 17, normalized size = 1.31

$$\begin{cases} a \log(x) + \frac{bx^n}{n} & \text{for } n \neq 0 \\ (a+b) \log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)/x,x)
[Out] Piecewise((a*log(x) + b*x**n/n, Ne(n, 0)), ((a + b)*log(x), True))
```

3.2456 $\int \frac{a+bx^n}{x^2}\,dx$

Optimal. Leaf size=22

$$-\frac{a}{x}-\frac{bx^{n-1}}{1-n}$$

```
[Out] -a/x-b*x^(-1+n)/(1-n)
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$-\frac{a}{x}-\frac{bx^{n-1}}{1-n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)/x^2,x]
```

```
[Out] -(a/x) - (b*x^(-1 + n))/(1 - n)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^n}{x^2}\,dx &= \int \left(\frac{a}{x^2}+bx^{-2+n}\right)dx \\ &= -\frac{a}{x}-\frac{bx^{-1+n}}{1-n}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$-\frac{a}{x}-\frac{bx^{n-1}}{1-n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)/x^2,x]
```

```
[Out] -(a/x) - (b*x^(-1 + n))/(1 - n)
```

fricas [A] time = 0.61, size = 23, normalized size = 1.05

$$-\frac{an-bx^n-a}{(n-1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/x^2,x, algorithm="fricas")
```

```
[Out] -(a*n - b*x^n - a)/((n - 1)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx^n+a}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)/x^2, x)
```

maple [A] time = 0.01, size = 21, normalized size = 0.95

$$\frac{\frac{b\,e^{n\ln(x)}}{n-1}-a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)/x^2,x)
```

```
[Out] (b/(-1+n)*exp(n*ln(x))-a)/x
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(n-2>0)', see `assume?` for more det
ails)Is n-2 equal to -1?
```

mupad [B] time = 1.19, size = 20, normalized size = 0.91

$$\frac{b\,x^n}{x\,(n-1)}-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)/x^2,x)
```

```
[Out] (b*x^n)/(x*(n - 1)) - a/x
```

sympy [A] time = 0.43, size = 32, normalized size = 1.45

$$\begin{cases} -\frac{an}{nx-x}+\frac{a}{nx-x}+\frac{bx^n}{nx-x} & \text{for } n\neq 1 \\ -\frac{a}{x}+b\log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)/x**2,x)
```

```
[Out] Piecewise((-a*n/(n*x - x) + a/(n*x - x) + b*x**n/(n*x - x), Ne(n, 1)), (-a/
x + b*log(x), True))
```

3.2457 $\int \frac{a+bx^n}{x^3}\,dx$

Optimal. Leaf size=24

$$-\frac{a}{2x^2}-\frac{bx^{n-2}}{2-n}$$

```
[Out] -1/2*a/x^2-b*x^(-2+n)/(2-n)
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$-\frac{a}{2x^2}-\frac{bx^{n-2}}{2-n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)/x^3,x]
```

```
[Out] -a/(2*x^2) - (b*x^(-2 + n))/(2 - n)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^n}{x^3}\,dx &= \int \left(\frac{a}{x^3}+bx^{-3+n}\right)dx\\ &= -\frac{a}{2x^2}-\frac{bx^{-2+n}}{2-n}\end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 0.83

$$-\frac{a-\frac{2bx^n}{n-2}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)/x^3,x]
```

```
[Out] -1/2*(a - (2*b*x^n)/(-2 + n))/x^2
```

fricas [A] time = 0.70, size = 23, normalized size = 0.96

$$-\frac{an-2\,bx^n-2\,a}{2\,(n-2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a*n - 2*b*x^n - 2*a)/((n - 2)*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx^n+a}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)/x^3, x)
```

maple [A] time = 0.01, size = 21, normalized size = 0.88

$$\frac{\frac{b\,e^{n\ln(x)}}{n-2}-\frac{a}{2}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)/x^3,x)
```

```
[Out] (b/(n-2)*exp(n*ln(x))-1/2*a)/x^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(n-3>0)', see `assume?` for more det
ails)Is n-3 equal to -1?
```

mupad [B] time = 1.15, size = 20, normalized size = 0.83

$$\frac{b\,x^n}{x^2\,(n-2)}-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)/x^3,x)
```

```
[Out] (b*x^n)/(x^2*(n - 2)) - a/(2*x^2)
```

sympy [A] time = 0.57, size = 60, normalized size = 2.50

$$\begin{cases} -\frac{an}{2nx^2-4x^2}+\frac{2a}{2nx^2-4x^2}+\frac{2bx^n}{2nx^2-4x^2} & \text{for } n \neq 2 \\ -\frac{a}{2x^2}+b\log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)/x**3,x)
```

```
[Out] Piecewise((-a*n/(2*n*x**2 - 4*x**2) + 2*a/(2*n*x**2 - 4*x**2) + 2*b*x**n/(2
*n*x**2 - 4*x**2), Ne(n, 2)), (-a/(2*x**2) + b*log(x), True))
```

3.2458 $\int x^3 (a + bx^n)^2 \, dx$

Optimal. Leaf size=44

$$\frac{a^2 x^4}{4} + \frac{2abx^{n+4}}{n+4} + \frac{b^2 x^{2(n+2)}}{2(n+2)}$$

```
[Out] 1/4*a^2*x^4+1/2*b^2*x^(4+2*n)/(2+n)+2*a*b*x^(4+n)/(4+n)
```

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{a^2 x^4}{4} + \frac{2abx^{n+4}}{n+4} + \frac{b^2 x^{2(n+2)}}{2(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^n)^2,x]
```

```
[Out] (a^2*x^4)/4 + (b^2*x^(2*(2 + n)))/(2*(2 + n)) + (2*a*b*x^(4 + n))/(4 + n)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^3 (a + bx^n)^2 \, dx &= \int \left(a^2 x^3 + 2abx^{3+n} + b^2 x^{3+2n}\right) dx \\ &= \frac{a^2 x^4}{4} + \frac{b^2 x^{2(2+n)}}{2(2+n)} + \frac{2abx^{4+n}}{4+n}\end{aligned}$$

Mathematica [A] time = 0.04, size = 38, normalized size = 0.86

$$\frac{1}{4} x^4 \left(a^2 + \frac{8abx^n}{n+4} + \frac{2b^2 x^{2n}}{n+2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^n)^2,x]
```

```
[Out] (x^4*(a^2 + (8*a*b*x^n)/(4 + n) + (2*b^2*x^(2*n))/(2 + n)))/4
```

fricas [A] time = 0.60, size = 74, normalized size = 1.68

$$\frac{2\left(b^2 n + 4\,b^2\right) x^4 x^{2\,n} + 8\,(abn + 2\,ab) x^4 x^n + \left(a^2 n^2 + 6\,a^2 n + 8\,a^2\right) x^4}{4\left(n^2 + 6\,n + 8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b^2*n + 4*b^2)*x^4*x^(2*n) + 8*(a*b*n + 2*a*b)*x^4*x^n + (a^2*n^2 +
 6*a^2*n + 8*a^2)*x^4)/(n^2 + 6*n + 8)
```

giac [B] time = 0.19, size = 88, normalized size = 2.00

$$\frac{2\,b^2 n x^4 x^{2\,n} + 8\,abnx^4 x^n + a^2 n^2 x^4 + 8\,b^2 x^4 x^{2\,n} + 16\,abx^4 x^n + 6\,a^2 n x^4 + 8\,a^2 x^4}{4\left(n^2 + 6\,n + 8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*b^2*n*x^4*x^(2*n) + 8*a*b*n*x^4*x^n + a^2*n^2*x^4 + 8*b^2*x^4*x^(2*n
) + 16*a*b*x^4*x^n + 6*a^2*n*x^4 + 8*a^2*x^4)/(n^2 + 6*n + 8)
```

maple [A] time = 0.02, size = 47, normalized size = 1.07

$$\frac{2ab\,x^4 \mathrm{e}^{n\ln(x)}}{n+4} + \frac{b^2 x^4 \mathrm{e}^{2n\ln(x)}}{4+2n} + \frac{a^2 x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*x^n)^2,x)
```

```
[Out] 1/4*a^2*x^4+1/2*b^2/(n+2)*x^4*exp(n*ln(x))^2+2*a*b/(n+4)*x^4*exp(n*ln(x))
```

maxima [A] time = 0.61, size = 40, normalized size = 0.91

$$\frac{1}{4}\,a^2 x^4 + \frac{b^2 x^{2\,n+4}}{2\,(n+2)} + \frac{2\,abx^{n+4}}{n+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*x^4 + 1/2*b^2*x^(2*n + 4)/(n + 2) + 2*a*b*x^(n + 4)/(n + 4)
```

mupad [B] time = 1.31, size = 43, normalized size = 0.98

$$\frac{a^2\,x^4}{4} + \frac{b^2\,x^{2\,n}\,x^4}{2\,n+4} + \frac{2\,a\,b\,x^n\,x^4}{n+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^n)^2,x)
```

```
[Out] (a^2*x^4)/4 + (b^2*x^(2*n)*x^4)/(2*n + 4) + (2*a*b*x^n*x^4)/(n + 4)
```

sympy [A] time = 0.97, size = 202, normalized size = 4.59

$$\begin{cases} \frac{a^2x^4}{4} + 2ab\log(x) - \frac{b^2}{4x^4} & \text{for } n = -4 \\ \frac{a^2x^4}{4} + abx^2 + b^2\log(x) & \text{for } n = -2 \\ \frac{a^2n^2x^4}{4n^2+24n+32} + \frac{6a^2nx^4}{4n^2+24n+32} + \frac{8a^2x^4}{4n^2+24n+32} + \frac{8abnx^4x^n}{4n^2+24n+32} + \frac{16abx^4x^n}{4n^2+24n+32} + \frac{2b^2nx^4x^{2n}}{4n^2+24n+32} + \frac{8b^2x^4x^{2n}}{4n^2+24n+32} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*x**n)**2,x)
```

```
[Out] Piecewise((a**2*x**4/4 + 2*a*b*log(x) - b**2/(4*x**4), Eq(n, -4)), (a**2*x*
*4/4 + a*b*x**2 + b**2*log(x), Eq(n, -2)), (a**2*n**2*x**4/(4*n**2 + 24*n +
 32) + 6*a**2*n*x**4/(4*n**2 + 24*n + 32) + 8*a**2*x**4/(4*n**2 + 24*n + 32
) + 8*a*b*n*x**4*x**n/(4*n**2 + 24*n + 32) + 16*a*b*x**4*x**n/(4*n**2 + 24*
n + 32) + 2*b**2*n*x**4*x**(2*n)/(4*n**2 + 24*n + 32) + 8*b**2*x**4*x**(2*n
)/(4*n**2 + 24*n + 32), True))
```

3.2459 $\int x^2 (a + bx^n)^2 \, dx$

Optimal. Leaf size=43

$$\frac{a^2 x^3}{3} + \frac{2abx^{n+3}}{n+3} + \frac{b^2 x^{2n+3}}{2n+3}$$

```
[Out] 1/3*a^2*x^3+2*a*b*x^(3+n)/(3+n)+b^2*x^(3+2*n)/(3+2*n)
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{a^2 x^3}{3} + \frac{2abx^{n+3}}{n+3} + \frac{b^2 x^{2n+3}}{2n+3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^n)^2,x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^(3 + n))/(3 + n) + (b^2*x^(3 + 2*n))/(3 + 2*n)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^2 (a + bx^n)^2 \, dx &= \int \left(a^2 x^2 + b^2 x^{2(1+n)} + 2abx^{2+n}\right) dx \\ &= \frac{a^2 x^3}{3} + \frac{2abx^{3+n}}{3+n} + \frac{b^2 x^{3+2n}}{3+2n}\end{aligned}$$

Mathematica [A] time = 0.04, size = 40, normalized size = 0.93

$$\frac{1}{3} x^3 \left(a^2 + \frac{6abx^n}{n+3} + \frac{3b^2 x^{2n}}{2n+3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^n)^2,x]
```

```
[Out] (x^3*(a^2 + (6*a*b*x^n)/(3 + n) + (3*b^2*x^(2*n))/(3 + 2*n)))/3
```

fricas [A] time = 0.90, size = 78, normalized size = 1.81

$$\frac{3\left(b^2 n + 3\,b^2\right) x^3 x^{2\,n} + 6\,(2\,abn + 3\,ab) x^3 x^n + \left(2\,a^2 n^2 + 9\,a^2 n + 9\,a^2\right) x^3}{3\left(2\,n^2 + 9\,n + 9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(b^2*n + 3*b^2)*x^3*x^(2*n) + 6*(2*a*b*n + 3*a*b)*x^3*x^n + (2*a^2*n
^2 + 9*a^2*n + 9*a^2)*x^3)/(2*n^2 + 9*n + 9)
```

giac [B] time = 0.17, size = 91, normalized size = 2.12

$$\frac{3\,b^2 n x^3 x^{2\,n} + 12\,abnx^3 x^n + 2\,a^2 n^2 x^3 + 9\,b^2 x^3 x^{2\,n} + 18\,abx^3 x^n + 9\,a^2 n x^3 + 9\,a^2 x^3}{3\left(2\,n^2 + 9\,n + 9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*b^2*n*x^3*x^(2*n) + 12*a*b*n*x^3*x^n + 2*a^2*n^2*x^3 + 9*b^2*x^3*x^(
2*n) + 18*a*b*x^3*x^n + 9*a^2*n*x^3 + 9*a^2*x^3)/(2*n^2 + 9*n + 9)
```

maple [A] time = 0.01, size = 48, normalized size = 1.12

$$\frac{2ab\,x^3 e^{n\ln(x)}}{n+3} + \frac{b^2 x^3 e^{2n\ln(x)}}{2n+3} + \frac{a^2 x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*x^n)^2,x)
```

```
[Out] b^2/(2*n+3)*x^3*exp(n*ln(x))^2+1/3*a^2*x^3+2*a*b/(n+3)*x^3*exp(n*ln(x))
```

maxima [A] time = 0.63, size = 41, normalized size = 0.95

$$\frac{1}{3}a^2x^3 + \frac{b^2x^{2\,n+3}}{2\,n+3} + \frac{2\,abx^{n+3}}{n+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3 + b^2*x^(2*n + 3)/(2*n + 3) + 2*a*b*x^(n + 3)/(n + 3)
```

mupad [B] time = 1.31, size = 43, normalized size = 1.00

$$\frac{a^2\,x^3}{3} + \frac{b^2\,x^{2\,n}\,x^3}{2\,n+3} + \frac{2\,a\,b\,x^n\,x^3}{n+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^n)^2,x)
```

```
[Out] (a^2*x^3)/3 + (b^2*x^(2*n)*x^3)/(2*n + 3) + (2*a*b*x^n*x^3)/(n + 3)
```

sympy [A] time = 1.92, size = 211, normalized size = 4.91

$$\begin{cases} \frac{a^2x^3}{3} + 2ab\log(x) - \frac{b^2}{3x^3} & \text{for } n = -3 \\ \frac{a^2x^3}{3} + \frac{4abx^{\frac{3}{2}}}{3} + b^2\log(x) & \text{for } n = -\frac{3}{2} \\ \frac{2a^2n^2x^3}{6n^2+27n+27} + \frac{9a^2nx^3}{6n^2+27n+27} + \frac{9a^2x^3}{6n^2+27n+27} + \frac{12abnx^3x^n}{6n^2+27n+27} + \frac{18abx^3x^n}{6n^2+27n+27} + \frac{3b^2nx^3x^{2n}}{6n^2+27n+27} + \frac{9b^2x^3x^{2n}}{6n^2+27n+27} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*x**n)**2,x)
```

```
[Out] Piecewise((a**2*x**3/3 + 2*a*b*log(x) - b**2/(3*x**3), Eq(n, -3)), (a**2*x*
*3/3 + 4*a*b*x**(3/2)/3 + b**2*log(x), Eq(n, -3/2)), (2*a**2*n**2*x**3/(6*n
**2 + 27*n + 27) + 9*a**2*n*x**3/(6*n**2 + 27*n + 27) + 9*a**2*x**3/(6*n**2
 + 27*n + 27) + 12*a*b*n*x**3*x**n/(6*n**2 + 27*n + 27) + 18*a*b*x**3*x**n/
(6*n**2 + 27*n + 27) + 3*b**2*n*x**3*x**(2*n)/(6*n**2 + 27*n + 27) + 9*b**2
*x**3*x**(2*n)/(6*n**2 + 27*n + 27), True))
```

3.2460 $\int x\,(a+bx^n)^2\,dx$

Optimal. Leaf size=44

$$\frac{a^2x^2}{2}+\frac{2abx^{n+2}}{n+2}+\frac{b^2x^{2(n+1)}}{2(n+1)}$$

```
[Out] 1/2*a^2*x^2+1/2*b^2*x^(2+2*n)/(1+n)+2*a*b*x^(2+n)/(2+n)
```

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {270}

$$\frac{a^2x^2}{2}+\frac{2abx^{n+2}}{n+2}+\frac{b^2x^{2(n+1)}}{2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^n)^2,x]
```

```
[Out] (a^2*x^2)/2 + (b^2*x^(2*(1 + n)))/(2*(1 + n)) + (2*a*b*x^(2 + n))/(2 + n)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x\,(a+bx^n)^2\,dx &= \int \left(a^2x+2abx^{1+n}+b^2x^{1+2n}\right)dx\\ &= \frac{a^2x^2}{2}+\frac{b^2x^{2(1+n)}}{2(1+n)}+\frac{2abx^{2+n}}{2+n}\end{aligned}$$

Mathematica [A] time = 0.03, size = 37, normalized size = 0.84

$$\frac{1}{2}x^2\left(a^2+\frac{4abx^n}{n+2}+\frac{b^2x^{2n}}{n+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^n)^2,x]
```

```
[Out] (x^2*(a^2 + (4*a*b*x^n)/(2 + n) + (b^2*x^(2*n))/(1 + n)))/2
```

fricas [A] time = 0.84, size = 72, normalized size = 1.64

$$\frac{\left(b^2n+2\,b^2\right)x^2x^{2\,n}+4\,(abn+ab)x^2x^n+\left(a^2n^2+3\,a^2n+2\,a^2\right)x^2}{2\left(n^2+3\,n+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((b^2*n + 2*b^2)*x^2*x^(2*n) + 4*(a*b*n + a*b)*x^2*x^n + (a^2*n^2 + 3*a
^2*n + 2*a^2)*x^2)/(n^2 + 3*n + 2)
```

giac [B] time = 0.18, size = 87, normalized size = 1.98

$$\frac{b^2 n x^2 x^{2\,n} + 4\,abnx^2 x^n + a^2 n^2 x^2 + 2\,b^2 x^2 x^{2\,n} + 4\,abx^2 x^n + 3\,a^2 nx^2 + 2\,a^2 x^2}{2\left(n^2 + 3\,n + 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*n*x^2*x^(2*n) + 4*a*b*n*x^2*x^n + a^2*n^2*x^2 + 2*b^2*x^2*x^(2*n)
+ 4*a*b*x^2*x^n + 3*a^2*n*x^2 + 2*a^2*x^2)/(n^2 + 3*n + 2)
```

maple [A] time = 0.01, size = 47, normalized size = 1.07

$$\frac{2ab\,x^2 \mathrm{e}^{n\ln(x)}}{n+2} + \frac{b^2 x^2 \mathrm{e}^{2n\ln(x)}}{2+2n} + \frac{a^2 x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*x^n)^2,x)
```

```
[Out] 1/2*a^2*x^2+1/2*b^2/(n+1)*x^2*exp(n*ln(x))^2+2*a*b/(n+2)*x^2*exp(n*ln(x))
```

maxima [A] time = 0.46, size = 40, normalized size = 0.91

$$\frac{1}{2}\,a^2 x^2 + \frac{b^2 x^{2\,n+2}}{2\,(n+1)} + \frac{2\,abx^{n+2}}{n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*x^2 + 1/2*b^2*x^(2*n + 2)/(n + 1) + 2*a*b*x^(n + 2)/(n + 2)
```

mupad [B] time = 1.24, size = 43, normalized size = 0.98

$$\frac{a^2\,x^2}{2} + \frac{b^2\,x^{2\,n}\,x^2}{2\,n+2} + \frac{2\,a\,b\,x^n\,x^2}{n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^n)^2,x)
```

```
[Out] (a^2*x^2)/2 + (b^2*x^(2*n)*x^2)/(2*n + 2) + (2*a*b*x^n*x^2)/(n + 2)
```

sympy [A] time = 0.53, size = 201, normalized size = 4.57

$$\begin{cases} \frac{a^2x^2}{2} + 2ab\log(x) - \frac{b^2}{2x^2} & \text{for } n = -2 \\ \frac{a^2x^2}{2} + 2abx + b^2\log(x) & \text{for } n = -1 \\ \frac{a^2n^2x^2}{2n^2+6n+4} + \frac{3a^2nx^2}{2n^2+6n+4} + \frac{2a^2x^2}{2n^2+6n+4} + \frac{4abnx^2x^n}{2n^2+6n+4} + \frac{4abx^2x^n}{2n^2+6n+4} + \frac{b^2nx^2x^{2n}}{2n^2+6n+4} + \frac{2b^2x^2x^{2n}}{2n^2+6n+4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x**n)**2,x)
```

```
[Out] Piecewise((a**2*x**2/2 + 2*a*b*log(x) - b**2/(2*x**2), Eq(n, -2)), (a**2*x*
*2/2 + 2*a*b*x + b**2*log(x), Eq(n, -1)), (a**2*n**2*x**2/(2*n**2 + 6*n + 4
) + 3*a**2*n*x**2/(2*n**2 + 6*n + 4) + 2*a**2*x**2/(2*n**2 + 6*n + 4) + 4*a
*b*n*x**2*x**n/(2*n**2 + 6*n + 4) + 4*a*b*x**2*x**n/(2*n**2 + 6*n + 4) + b*
*2*n*x**2*x**(2*n)/(2*n**2 + 6*n + 4) + 2*b**2*x**2*x**(2*n)/(2*n**2 + 6*n
+ 4), True))
```

3.2461 $\int (a + bx^n)^2 \, dx$

Optimal. Leaf size=38

$$a^2 x + \frac{2abx^{n+1}}{n+1} + \frac{b^2 x^{2n+1}}{2n+1}$$

```
[Out] a^2*x+2*a*b*x^(1+n)/(1+n)+b^2*x^(1+2*n)/(1+2*n)
```

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {244}

$$a^2 x + \frac{2abx^{n+1}}{n+1} + \frac{b^2 x^{2n+1}}{2n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^2,x]
```

```
[Out] a^2*x + (2*a*b*x^(1 + n))/(1 + n) + (b^2*x^(1 + 2*n))/(1 + 2*n)
```

Rule 244

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (a + bx^n)^2 \, dx &= \int \left(a^2 + 2abx^n + b^2 x^{2n}\right) dx \\ &= a^2 x + \frac{2abx^{1+n}}{1+n} + \frac{b^2 x^{1+2n}}{1+2n}\end{aligned}$$

Mathematica [A] time = 0.03, size = 34, normalized size = 0.89

$$x\left(a^2 + \frac{2abx^n}{n+1} + \frac{b^2 x^{2n}}{2n+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^2,x]
```

```
[Out] x*(a^2 + (2*a*b*x^n)/(1 + n) + (b^2*x^(2*n))/(1 + 2*n))
```

fricas [A] time = 1.04, size = 65, normalized size = 1.71

$$\frac{\left(b^2 n + b^2\right) x x^{2\,n} + 2\,(2\,abn + ab) x x^n + \left(2\,a^2 n^2 + 3\,a^2 n + a^2\right) x}{2\,n^2 + 3\,n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*n + b^2)*x*x^(2*n) + 2*(2*a*b*n + a*b)*x*x^n + (2*a^2*n^2 + 3*a^2*n +
 a^2)*x)/(2*n^2 + 3*n + 1)
```

giac [A] time = 0.17, size = 73, normalized size = 1.92

$$\frac{2\,a^2 n^2 x + b^2 n x x^{2\,n} + 4\,abnx x^n + 3\,a^2 n x + b^2 x x^{2\,n} + 2\,abx x^n + a^2 x}{2\,n^2 + 3\,n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] (2*a^2*n^2*x + b^2*n*x*x^(2*n) + 4*a*b*n*x*x^n + 3*a^2*n*x + b^2*x*x^(2*n)
+ 2*a*b*x*x^n + a^2*x)/(2*n^2 + 3*n + 1)
```

maple [A] time = 0.01, size = 41, normalized size = 1.08

$$\frac{2abx\,\mathrm{e}^{n\ln(x)}}{n+1}+\frac{b^2x\,\mathrm{e}^{2n\ln(x)}}{2n+1}+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^2,x)
```

```
[Out] a^2*x+b^2/(1+2*n)*x*exp(n*ln(x))^2+2*a*b/(n+1)*x*exp(n*ln(x))
```

maxima [A] time = 0.49, size = 38, normalized size = 1.00

$$a^2x+\frac{b^2x^{2\,n+1}}{2\,n+1}+\frac{2\,abx^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] a^2*x + b^2*x^(2*n + 1)/(2*n + 1) + 2*a*b*x^(n + 1)/(n + 1)
```

mupad [B] time = 1.18, size = 36, normalized size = 0.95

$$a^2\,x+\frac{b^2\,x\,x^{2\,n}}{2\,n+1}+\frac{2\,a\,b\,x\,x^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^2,x)
```

```
[Out] a^2*x + (b^2*x*x^(2*n))/(2*n + 1) + (2*a*b*x*x^n)/(n + 1)
```

sympy [A] time = 0.47, size = 182, normalized size = 4.79

$$\begin{cases} a^2x+2ab\log(x)-\frac{b^2}{x} & \text{for } n=-1 \\ a^2x+4ab\sqrt{x}+b^2\log(x) & \text{for } n=-\frac{1}{2} \\ \frac{2a^2n^2x}{2n^2+3n+1}+\frac{3a^2nx}{2n^2+3n+1}+\frac{a^2x}{2n^2+3n+1}+\frac{4abnxx^n}{2n^2+3n+1}+\frac{2abxx^n}{2n^2+3n+1}+\frac{b^2nxx^{2n}}{2n^2+3n+1}+\frac{b^2xx^{2n}}{2n^2+3n+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*log(x) - b**2/x, Eq(n, -1)), (a**2*x + 4*a*b*sqrt
(x) + b**2*log(x), Eq(n, -1/2)), (2*a**2*n**2*x/(2*n**2 + 3*n + 1) + 3*a**2
*n*x/(2*n**2 + 3*n + 1) + a**2*x/(2*n**2 + 3*n + 1) + 4*a*b*n*x*x**n/(2*n**
2 + 3*n + 1) + 2*a*b*x*x**n/(2*n**2 + 3*n + 1) + b**2*n*x*x**(2*n)/(2*n**2
+ 3*n + 1) + b**2*x*x**(2*n)/(2*n**2 + 3*n + 1), True))
```

3.2462 $\int \frac{(a+bx^n)^2}{x}\,dx$

Optimal. Leaf size=32

$$a^2\log(x)+\frac{2abx^n}{n}+\frac{b^2x^{2n}}{2n}$$

```
[Out] 2*a*b*x^n/n+1/2*b^2*x^(2*n)/n+a^2*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$a^2\log(x)+\frac{2abx^n}{n}+\frac{b^2x^{2n}}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^2/x,x]
```

```
[Out] (2*a*b*x^n)/n + (b^2*x^(2*n))/(2*n) + a^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^n)^2}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{x}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(2ab+\frac{a^2}{x}+b^2x\right)\,dx, x, x^n\right)}{n}\\ &= \frac{2abx^n}{n}+\frac{b^2x^{2n}}{2n}+a^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 27, normalized size = 0.84

$$a^2\log(x)+\frac{bx^n\left(4a+bx^n\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^2/x,x]
```

```
[Out] (b*x^n*(4*a + b*x^n))/(2*n) + a^2*Log[x]
```

fricas [A] time = 0.83, size = 30, normalized size = 0.94

$$\frac{2\,a^2 n\log(x)+b^2x^{2\,n}+4\,abx^n}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2/x,x, algorithm="fricas")
[Out] 1/2*(2*a^2*n*log(x) + b^2*x^(2*n) + 4*a*b*x^n)/n
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2/x,x, algorithm="giac")
[Out] integrate((b*x^n + a)^2/x, x)
```

maple [**A**] time = 0.00, size = 36, normalized size = 1.12

$$\frac{a^2 \ln(x^n)}{n} + \frac{2ab\, x^n}{n} + \frac{b^2 x^{2n}}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^2/x,x)
[Out] 1/2/n*b^2*(x^n)^2+2*a*b*x^n/n+1/n*a^2*ln(x^n)
```

maxima [**A**] time = 0.51, size = 34, normalized size = 1.06

$$\frac{a^2 \log(x^n)}{n} + \frac{b^2 x^{2\,n} + 4\, abx^n}{2\, n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2/x,x, algorithm="maxima")
[Out] a^2*log(x^n)/n + 1/2*(b^2*x^(2*n) + 4*a*b*x^n)/n
```

mupad [**B**] time = 1.18, size = 30, normalized size = 0.94

$$\frac{b^2\, x^{2\,n} + 4\, a\, b\, x^n + 2\, a^2\, n\, \ln(x)}{2\, n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^2/x,x)
[Out] (b^2*x^(2*n) + 4*a*b*x^n + 2*a^2*n*log(x))/(2*n)
```

sympy [**A**] time = 0.28, size = 36, normalized size = 1.12

$$\begin{cases} a^2 \log(x) + \frac{2abx^n}{n} + \frac{b^2 x^{2n}}{2n} & \text{for } n \neq 0 \\ (a+b)^2 \log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**2/x,x)
[Out] Piecewise((a**2*log(x) + 2*a*b*x**n/n + b**2*x**(2*n)/(2*n), Ne(n, 0)), ((a
 + b)**2*log(x), True))
```

3.2463 $\int \frac{(a+bx^n)^2}{x^2}\,dx$

Optimal. Leaf size=44

$$-\frac{a^2}{x}-\frac{2abx^{n-1}}{1-n}-\frac{b^2x^{2n-1}}{1-2n}$$

```
[Out] -a^2/x-2*a*b*x^(-1+n)/(1-n)-b^2*x^(-1+2*n)/(1-2*n)
```

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$-\frac{a^2}{x}-\frac{2abx^{n-1}}{1-n}-\frac{b^2x^{2n-1}}{1-2n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^2/x^2,x]
```

```
[Out] -(a^2/x) - (2*a*b*x^(-1 + n))/(1 - n) - (b^2*x^(-1 + 2*n))/(1 - 2*n)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^n)^2}{x^2}\,dx &= \int \left(\frac{a^2}{x^2}+2abx^{-2+n}+b^2x^{2(-1+n)}\right)dx\\ &= -\frac{a^2}{x}-\frac{2abx^{-1+n}}{1-n}-\frac{b^2x^{-1+2n}}{1-2n}\end{aligned}$$

Mathematica [A] time = 0.04, size = 38, normalized size = 0.86

$$\frac{-a^2+\frac{2abx^n}{n-1}+\frac{b^2x^{2n}}{2n-1}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^2/x^2,x]
```

```
[Out] (-a^2 + (2*a*b*x^n)/(-1 + n) + (b^2*x^(2*n))/(-1 + 2*n))/x
```

fricas [A] time = 0.94, size = 68, normalized size = 1.55

$$-\frac{2\,a^2n^2-3\,a^2n+a^2-\left(b^2n-b^2\right)x^{2\,n}-2\,(2\,abn-ab)x^n}{\left(2\,n^2-3\,n+1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2/x^2,x, algorithm="fricas")
```

```
[Out] -(2*a^2*n^2 - 3*a^2*n + a^2 - (b^2*n - b^2)*x^(2*n) - 2*(2*a*b*n - a*b)*x^n
)/((2*n^2 - 3*n + 1)*x)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^2}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^2/x^2, x)
```

maple [**A**] time = 0.02, size = 43, normalized size = 0.98

$$\frac{\frac{2ab\, e^{n \ln(x)}}{n-1} + \frac{b^2 e^{2n \ln(x)}}{2n-1} - a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^2/x^2,x)
```

```
[Out] (b^2/(-1+2*n)*exp(n*ln(x))^2-a^2+2*a*b/(n-1)*exp(n*ln(x)))/x
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(n-2>0)', see `assume?` for more det
ails)Is n-2 equal to -1?
```

mupad [**B**] time = 1.16, size = 43, normalized size = 0.98

$$\frac{b^2\, x^{2\,n}}{x\,(2\,n-1)} - \frac{a^2}{x} + \frac{2\,a\,b\,x^n}{x\,(n-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^2/x^2,x)
```

```
[Out] (b^2*x^(2*n))/(x*(2*n - 1)) - a^2/x + (2*a*b*x^n)/(x*(n - 1))
```

sympy [**A**] time = 0.75, size = 190, normalized size = 4.32

$$\begin{cases} -\frac{a^2}{x} - \frac{4ab}{\sqrt{x}} + b^2 \log(x) & \text{for } n = \frac{1}{2} \\ -\frac{a^2}{x} + 2ab \log(x) + b^2 x & \text{for } n = 1 \\ -\frac{2a^2n^2}{2n^2x - 3nx + x} + \frac{3a^2n}{2n^2x - 3nx + x} - \frac{a^2}{2n^2x - 3nx + x} + \frac{4abnx^n}{2n^2x - 3nx + x} - \frac{2abx^n}{2n^2x - 3nx + x} + \frac{b^2nx^{2n}}{2n^2x - 3nx + x} - \frac{b^2x^{2n}}{2n^2x - 3nx + x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**2/x**2,x)
```

```
[Out] Piecewise((-a**2/x - 4*a*b/sqrt(x) + b**2*log(x), Eq(n, 1/2)), (-a**2/x + 2
*a*b*log(x) + b**2*x, Eq(n, 1)), (-2*a**2*n**2/(2*n**2*x - 3*n*x + x) + 3*a
**2*n/(2*n**2*x - 3*n*x + x) - a**2/(2*n**2*x - 3*n*x + x) + 4*a*b*n*x**n/(
2*n**2*x - 3*n*x + x) - 2*a*b*x**n/(2*n**2*x - 3*n*x + x) + b**2*n*x**(2*n)
/(2*n**2*x - 3*n*x + x) - b**2*x**(2*n)/(2*n**2*x - 3*n*x + x), True))
```

3.2464 $\int \frac{(a+bx^n)^2}{x^3}\,dx$

Optimal. Leaf size=50

$$-\frac{a^2}{2x^2}-\frac{2abx^{n-2}}{2-n}-\frac{b^2x^{-2(1-n)}}{2(1-n)}$$

```
[Out] -1/2*a^2/x^2-1/2*b^2/(1-n)/(x^(2-2*n))-2*a*b*x^(-2+n)/(2-n)
```

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$-\frac{a^2}{2x^2}-\frac{2abx^{n-2}}{2-n}-\frac{b^2x^{-2(1-n)}}{2(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^2/x^3,x]
```

```
[Out] -a^2/(2*x^2) - b^2/(2*(1 - n)*x^(2*(1 - n))) - (2*a*b*x^(-2 + n))/(2 - n)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^n)^2}{x^3}\,dx &= \int \left(\frac{a^2}{x^3}+2abx^{-3+n}+b^2x^{-3+2n}\right)dx\\ &= -\frac{a^2}{2x^2}-\frac{b^2x^{-2(1-n)}}{2(1-n)}-\frac{2abx^{-2+n}}{2-n}\end{aligned}$$

Mathematica [A] time = 0.04, size = 39, normalized size = 0.78

$$\frac{-a^2+\frac{4abx^n}{n-2}+\frac{b^2x^{2n}}{n-1}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^2/x^3,x]
```

```
[Out] (-a^2 + (4*a*b*x^n)/(-2 + n) + (b^2*x^(2*n))/(-1 + n))/(2*x^2)
```

fricas [A] time = 0.70, size = 66, normalized size = 1.32

$$-\frac{a^2n^2-3\,a^2n+2\,a^2-\left(b^2n-2\,b^2\right)x^{2\,n}-4\,(abn-ab)x^n}{2\left(n^2-3\,n+2\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*n^2 - 3*a^2*n + 2*a^2 - (b^2*n - 2*b^2)*x^(2*n) - 4*(a*b*n - a*b)
*x^n)/((n^2 - 3*n + 2)*x^2)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^2/x^3, x)
```

maple [**A**] time = 0.02, size = 42, normalized size = 0.84

$$\frac{\frac{2ab\,e^{n\ln(x)}}{n-2} + \frac{b^2 e^{2n\ln(x)}}{2n-2} - \frac{a^2}{2}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^2/x^3,x)
```

```
[Out] (-1/2*a^2+1/2*b^2/(n-1)*exp(n*ln(x))^2+2*a*b/(n-2)*exp(n*ln(x)))/x^2
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(n-3>0)', see `assume?` for more det
ails)Is n-3 equal to -1?
```

mupad [**B**] time = 1.15, size = 43, normalized size = 0.86

$$\frac{b^2\,x^{2\,n}}{x^2\,(2\,n-2)} - \frac{a^2}{2\,x^2} + \frac{2\,a\,b\,x^n}{x^2\,(n-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^2/x^3,x)
```

```
[Out] (b^2*x^(2*n))/(x^2*(2*n - 2)) - a^2/(2*x^2) + (2*a*b*x^n)/(x^2*(n - 2))
```

sympy [**A**] time = 0.72, size = 245, normalized size = 4.90

$$\begin{cases} -\frac{a^2}{2x^2} - \frac{2ab}{x} + b^2\log(x) \\ -\frac{a^2}{2x^2} + 2ab\log(x) + \frac{b^2x^2}{2} \\ -\frac{a^2n^2}{2n^2x^2-6nx^2+4x^2} + \frac{3a^2n}{2n^2x^2-6nx^2+4x^2} - \frac{2a^2}{2n^2x^2-6nx^2+4x^2} + \frac{4abnx^n}{2n^2x^2-6nx^2+4x^2} - \frac{4abx^n}{2n^2x^2-6nx^2+4x^2} + \frac{b^2nx^{2n}}{2n^2x^2-6nx^2+4x^2} - \frac{2b^2x^{2n}}{2n^2x^2-6nx^2+4x} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**2/x**3,x)
```

```
[Out] Piecewise((-a**2/(2*x**2) - 2*a*b/x + b**2*log(x), Eq(n, 1)), (-a**2/(2*x**
2) + 2*a*b*log(x) + b**2*x**2/2, Eq(n, 2)), (-a**2*n**2/(2*n**2*x**2 - 6*n*
x**2 + 4*x**2) + 3*a**2*n/(2*n**2*x**2 - 6*n*x**2 + 4*x**2) - 2*a**2/(2*n**
2*x**2 - 6*n*x**2 + 4*x**2) + 4*a*b*n*x**n/(2*n**2*x**2 - 6*n*x**2 + 4*x**2
) - 4*a*b*x**n/(2*n**2*x**2 - 6*n*x**2 + 4*x**2) + b**2*n*x**(2*n)/(2*n**2*
x**2 - 6*n*x**2 + 4*x**2) - 2*b**2*x**(2*n)/(2*n**2*x**2 - 6*n*x**2 + 4*x**
2), True))
```

3.2465 $\int x^3 (a + bx^n)^3 \, dx$

Optimal. Leaf size=65

$$\frac{a^3x^4}{4} + \frac{3a^2bx^{n+4}}{n+4} + \frac{3ab^2x^{2(n+2)}}{2(n+2)} + \frac{b^3x^{3n+4}}{3n+4}$$

```
[Out] 1/4*a^3*x^4+3/2*a*b^2*x^(4+2*n)/(2+n)+3*a^2*b*x^(4+n)/(4+n)+b^3*x^(4+3*n)/(
4+3*n)
```

Rubi [A] time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{3a^2bx^{n+4}}{n+4} + \frac{a^3x^4}{4} + \frac{3ab^2x^{2(n+2)}}{2(n+2)} + \frac{b^3x^{3n+4}}{3n+4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^n)^3,x]
```

```
[Out] (a^3*x^4)/4 + (3*a*b^2*x^(2*(2 + n)))/(2*(2 + n)) + (3*a^2*b*x^(4 + n))/(4
+ n) + (b^3*x^(4 + 3*n))/(4 + 3*n)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^3 (a + bx^n)^3 \, dx &= \int \left(a^3x^3 + b^3x^{3(1+n)} + 3a^2bx^{3+n} + 3ab^2x^{3+2n}\right) dx \\ &= \frac{a^3x^4}{4} + \frac{3ab^2x^{2(2+n)}}{2(2+n)} + \frac{3a^2bx^{4+n}}{4+n} + \frac{b^3x^{4+3n}}{4+3n}\end{aligned}$$

Mathematica [A] time = 0.04, size = 58, normalized size = 0.89

$$\frac{1}{4}x^4\left(a^3 + \frac{12a^2bx^n}{n+4} + \frac{6ab^2x^{2n}}{n+2} + \frac{4b^3x^{3n}}{3n+4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^n)^3,x]
```

```
[Out] (x^4*(a^3 + (12*a^2*b*x^n)/(4 + n) + (6*a*b^2*x^(2*n))/(2 + n) + (4*b^3*x^(
3*n))/(4 + 3*n)))/4
```

fricas [B] time = 0.74, size = 145, normalized size = 2.23

$$\frac{4\left(b^3n^2 + 6\,b^3n + 8\,b^3\right)x^4x^{3\,n} + 6\left(3\,ab^2n^2 + 16\,ab^2n + 16\,ab^2\right)x^4x^{2\,n} + 12\left(3\,a^2bn^2 + 10\,a^2bn + 8\,a^2b\right)x^4x^n + \left(3\right.}{4\left(3\,n^3 + 22\,n^2 + 48\,n + 32\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(b^3*n^2 + 6*b^3*n + 8*b^3)*x^4*x^(3*n) + 6*(3*a*b^2*n^2 + 16*a*b^2*
n + 16*a*b^2)*x^4*x^(2*n) + 12*(3*a^2*b*n^2 + 10*a^2*b*n + 8*a^2*b)*x^4*x^n
 + (3*a^3*n^3 + 22*a^3*n^2 + 48*a^3*n + 32*a^3)*x^4)/(3*n^3 + 22*n^2 + 48*n
 + 32)
```

giac [B] time = 0.19, size = 188, normalized size = 2.89

$$\frac{4\,b^3n^2x^4x^{3\,n} + 18\,ab^2n^2x^4x^{2\,n} + 36\,a^2bn^2x^4x^n + 3\,a^3n^3x^4 + 24\,b^3nx^4x^{3\,n} + 96\,ab^2nx^4x^{2\,n} + 120\,a^2bnx^4x^n + 22\,a^3n}{4\left(3\,n^3 + 22\,n^2 + 48\,n + 32\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] 1/4*(4*b^3*n^2*x^4*x^(3*n) + 18*a*b^2*n^2*x^4*x^(2*n) + 36*a^2*b*n^2*x^4*x^
n + 3*a^3*n^3*x^4 + 24*b^3*n*x^4*x^(3*n) + 96*a*b^2*n*x^4*x^(2*n) + 120*a^2
*b*n*x^4*x^n + 22*a^3*n^2*x^4 + 32*b^3*x^4*x^(3*n) + 96*a*b^2*x^4*x^(2*n) +
 96*a^2*b*x^4*x^n + 48*a^3*n*x^4 + 32*a^3*x^4)/(3*n^3 + 22*n^2 + 48*n + 32)
```

maple [A] time = 0.01, size = 65, normalized size = 1.00

$$\frac{3a^2b\,x^4x^n}{n+4} + \frac{3a\,b^2x^4x^{2n}}{2\,(n+2)} + \frac{b^3x^4x^{3n}}{3n+4} + \frac{a^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*x^n)^3,x)
```

```
[Out] 1/4*a^3*x^4+b^3/(3*n+4)*x^4*(x^n)^3+3/2*a*b^2*x^4/(n+2)*(x^n)^2+3*a^2*b/(n+
4)*x^4*x^n
```

maxima [A] time = 0.44, size = 61, normalized size = 0.94

$$\frac{1}{4}\,a^3x^4 + \frac{b^3x^{3\,n+4}}{3\,n+4} + \frac{3\,ab^2x^{2\,n+4}}{2\,(n+2)} + \frac{3\,a^2bx^{n+4}}{n+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] 1/4*a^3*x^4 + b^3*x^(3*n + 4)/(3*n + 4) + 3/2*a*b^2*x^(2*n + 4)/(n + 2) + 3
*a^2*b*x^(n + 4)/(n + 4)
```

mupad [B] time = 1.34, size = 66, normalized size = 1.02

$$\frac{a^3\,x^4}{4} + \frac{b^3\,x^{3\,n}\,x^4}{3\,n+4} + \frac{3\,a\,b^2\,x^{2\,n}\,x^4}{2\,n+4} + \frac{3\,a^2\,b\,x^n\,x^4}{n+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^n)^3,x)
```

```
[Out] (a^3*x^4)/4 + (b^3*x^(3*n)*x^4)/(3*n + 4) + (3*a*b^2*x^(2*n)*x^4)/(2*n + 4)
 + (3*a^2*b*x^n*x^4)/(n + 4)
```

sympy [A] time = 10.03, size = 507, normalized size = 7.80

$$\begin{cases} \frac{a^3x^4}{4} + 3a^2b\log(x) - \frac{3ab^2}{4x^4} - \frac{b^3}{8x^8} \\ \frac{a^3x^4}{4} + \frac{3a^2bx^2}{2} + 3ab^2\log(x) - \frac{b^3}{2x^2} \\ \frac{a^3x^4}{4} + \frac{9a^2bx^{\frac{8}{3}}}{8} + \frac{9ab^2x^{\frac{4}{3}}}{4} + b^3\log(x) \\ \frac{3a^3n^3x^4}{12n^3+88n^2+192n+128} + \frac{22a^3n^2x^4}{12n^3+88n^2+192n+128} + \frac{48a^3nx^4}{12n^3+88n^2+192n+128} + \frac{32a^3x^4}{12n^3+88n^2+192n+128} + \frac{36a^2bn^2x^4x^n}{12n^3+88n^2+192n+128} + \frac{120a^2}{12n^3+88n} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*x**n)**3,x)
```

```
[Out] Piecewise((a**3*x**4/4 + 3*a**2*b*log(x) - 3*a*b**2/(4*x**4) - b**3/(8*x**8
), Eq(n, -4)), (a**3*x**4/4 + 3*a**2*b*x**2/2 + 3*a*b**2*log(x) - b**3/(2*x
**2), Eq(n, -2)), (a**3*x**4/4 + 9*a**2*b*x**(8/3)/8 + 9*a*b**2*x**(4/3)/4
+ b**3*log(x), Eq(n, -4/3)), (3*a**3*n**3*x**4/(12*n**3 + 88*n**2 + 192*n +
 128) + 22*a**3*n**2*x**4/(12*n**3 + 88*n**2 + 192*n + 128) + 48*a**3*n*x**
4/(12*n**3 + 88*n**2 + 192*n + 128) + 32*a**3*x**4/(12*n**3 + 88*n**2 + 192
*n + 128) + 36*a**2*b*n**2*x**4*x**n/(12*n**3 + 88*n**2 + 192*n + 128) + 12
0*a**2*b*n*x**4*x**n/(12*n**3 + 88*n**2 + 192*n + 128) + 96*a**2*b*x**4*x**
n/(12*n**3 + 88*n**2 + 192*n + 128) + 18*a*b**2*n**2*x**4*x**(2*n)/(12*n**3
 + 88*n**2 + 192*n + 128) + 96*a*b**2*n*x**4*x**(2*n)/(12*n**3 + 88*n**2 +
192*n + 128) + 96*a*b**2*x**4*x**(2*n)/(12*n**3 + 88*n**2 + 192*n + 128) +
4*b**3*n**2*x**4*x**(3*n)/(12*n**3 + 88*n**2 + 192*n + 128) + 24*b**3*n*x**
4*x**(3*n)/(12*n**3 + 88*n**2 + 192*n + 128) + 32*b**3*x**4*x**(3*n)/(12*n*
*3 + 88*n**2 + 192*n + 128), True))
```

3.2466 $\int x^2 (a + bx^n)^3 \, dx$

Optimal. Leaf size=66

$$\frac{a^3x^3}{3} + \frac{3a^2bx^{n+3}}{n+3} + \frac{3ab^2x^{2n+3}}{2n+3} + \frac{b^3x^{3(n+1)}}{3(n+1)}$$

```
[Out] 1/3*a^3*x^3+1/3*b^3*x^(3+3*n)/(1+n)+3*a^2*b*x^(3+n)/(3+n)+3*a*b^2*x^(3+2*n)
/(3+2*n)
```

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{3a^2bx^{n+3}}{n+3} + \frac{a^3x^3}{3} + \frac{3ab^2x^{2n+3}}{2n+3} + \frac{b^3x^{3(n+1)}}{3(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^n)^3,x]
```

```
[Out] (a^3*x^3)/3 + (b^3*x^(3*(1 + n)))/(3*(1 + n)) + (3*a^2*b*x^(3 + n))/(3 + n)
 + (3*a*b^2*x^(3 + 2*n))/(3 + 2*n)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^2 (a + bx^n)^3 \, dx &= \int \left(a^3x^2 + 3ab^2x^{2(1+n)} + 3a^2bx^{2+n} + b^3x^{2+3n}\right) dx \\ &= \frac{a^3x^3}{3} + \frac{b^3x^{3(1+n)}}{3(1+n)} + \frac{3a^2bx^{3+n}}{3+n} + \frac{3ab^2x^{3+2n}}{3+2n}\end{aligned}$$

Mathematica [A] time = 0.04, size = 57, normalized size = 0.86

$$\frac{1}{3}x^3\left(a^3 + \frac{9a^2bx^n}{n+3} + \frac{9ab^2x^{2n}}{2n+3} + \frac{b^3x^{3n}}{n+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^n)^3,x]
```

```
[Out] (x^3*(a^3 + (9*a^2*b*x^n)/(3 + n) + (9*a*b^2*x^(2*n))/(3 + 2*n) + (b^3*x^(3
*n))/(1 + n)))/3
```

fricas [B] time = 1.66, size = 144, normalized size = 2.18

$$\frac{\left(2\,b^3n^2 + 9\,b^3n + 9\,b^3\right)x^3x^{3\,n} + 9\left(ab^2n^2 + 4\,ab^2n + 3\,ab^2\right)x^3x^{2\,n} + 9\left(2\,a^2bn^2 + 5\,a^2bn + 3\,a^2b\right)x^3x^n + \left(2\,a^3n^3 + 1\right.}{3\left(2\,n^3 + 11\,n^2 + 18\,n + 9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] 1/3*((2*b^3*n^2 + 9*b^3*n + 9*b^3)*x^3*x^(3*n) + 9*(a*b^2*n^2 + 4*a*b^2*n +
 3*a*b^2)*x^3*x^(2*n) + 9*(2*a^2*b*n^2 + 5*a^2*b*n + 3*a^2*b)*x^3*x^n + (2*
a^3*n^3 + 11*a^3*n^2 + 18*a^3*n + 9*a^3)*x^3)/(2*n^3 + 11*n^2 + 18*n + 9)
```

giac [B] time = 0.26, size = 188, normalized size = 2.85

$$\frac{2\,b^3n^2x^3x^{3\,n}+9\,ab^2n^2x^3x^{2\,n}+18\,a^2bn^2x^3x^n+2\,a^3n^3x^3+9\,b^3nx^3x^{3\,n}+36\,ab^2nx^3x^{2\,n}+45\,a^2bnx^3x^n+11\,a^3n^{}}{3\left(2\,n^3+11\,n^2+18\,n+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] 1/3*(2*b^3*n^2*x^3*x^(3*n) + 9*a*b^2*n^2*x^3*x^(2*n) + 18*a^2*b*n^2*x^3*x^n
 + 2*a^3*n^3*x^3 + 9*b^3*n*x^3*x^(3*n) + 36*a*b^2*n*x^3*x^(2*n) + 45*a^2*b*
n*x^3*x^n + 11*a^3*n^2*x^3 + 9*b^3*x^3*x^(3*n) + 27*a*b^2*x^3*x^(2*n) + 27*
a^2*b*x^3*x^n + 18*a^3*n*x^3 + 9*a^3*x^3)/(2*n^3 + 11*n^2 + 18*n + 9)
```

maple [A] time = 0.02, size = 72, normalized size = 1.09

$$\frac{3a^2b\,x^3\mathrm{e}^{n\ln(x)}}{n+3}+\frac{3a\,b^2x^3\mathrm{e}^{2n\ln(x)}}{2n+3}+\frac{b^3x^3\mathrm{e}^{3n\ln(x)}}{3+3n}+\frac{a^3x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*x^n)^3,x)
```

```
[Out] 1/3*a^3*x^3+1/3*b^3/(n+1)*x^3*exp(n*ln(x))^3+3*a*b^2/(2*n+3)*x^3*exp(n*ln(x
))^2+3*a^2*b/(n+3)*x^3*exp(n*ln(x))
```

maxima [A] time = 0.53, size = 62, normalized size = 0.94

$$\frac{1}{3}\,a^3x^3+\frac{b^3x^{3\,n+3}}{3\,(n+1)}+\frac{3\,ab^2x^{2\,n+3}}{2\,n+3}+\frac{3\,a^2bx^{n+3}}{n+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*x^3 + 1/3*b^3*x^(3*n + 3)/(n + 1) + 3*a*b^2*x^(2*n + 3)/(2*n + 3) +
 3*a^2*b*x^(n + 3)/(n + 3)
```

mupad [B] time = 1.32, size = 66, normalized size = 1.00

$$\frac{a^3\,x^3}{3}+\frac{b^3\,x^{3\,n}\,x^3}{3\,n+3}+\frac{3\,a\,b^2\,x^{2\,n}\,x^3}{2\,n+3}+\frac{3\,a^2\,b\,x^n\,x^3}{n+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^n)^3,x)
```

```
[Out] (a^3*x^3)/3 + (b^3*x^(3*n)*x^3)/(3*n + 3) + (3*a*b^2*x^(2*n)*x^3)/(2*n + 3)
 + (3*a^2*b*x^n*x^3)/(n + 3)
```

sympy [A] time = 3.28, size = 500, normalized size = 7.58

$$\begin{cases}\frac{a^3x^3}{3}+3a^2b\log(x)-\frac{ab^2}{x^3}-\frac{b^3}{6x^6}\\ \frac{a^3x^3}{3}+2a^2bx^{\frac{3}{2}}+3ab^2\log(x)-\frac{2b^3}{3x^{\frac{3}{2}}}\\ \frac{a^3x^3}{3}+\frac{3a^2bx^2}{2}+3ab^2x+b^3\log(x)\\ \frac{2a^3n^3x^3}{6n^3+33n^2+54n+27}+\frac{11a^3n^2x^3}{6n^3+33n^2+54n+27}+\frac{18a^3nx^3}{6n^3+33n^2+54n+27}+\frac{9a^3x^3}{6n^3+33n^2+54n+27}+\frac{18a^2bn^2x^3x^n}{6n^3+33n^2+54n+27}+\frac{45a^2bnx^3x^n}{6n^3+33n^2+54n+27}+\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*x**n)**3,x)
```

```
[Out] Piecewise((a**3*x**3/3 + 3*a**2*b*log(x) - a*b**2/x**3 - b**3/(6*x**6), Eq(
n, -3)), (a**3*x**3/3 + 2*a**2*b*x**(3/2) + 3*a*b**2*log(x) - 2*b**3/(3*x**
(3/2)), Eq(n, -3/2)), (a**3*x**3/3 + 3*a**2*b*x**2/2 + 3*a*b**2*x + b**3*lo
g(x), Eq(n, -1)), (2*a**3*n**3*x**3/(6*n**3 + 33*n**2 + 54*n + 27) + 11*a**
3*n**2*x**3/(6*n**3 + 33*n**2 + 54*n + 27) + 18*a**3*n*x**3/(6*n**3 + 33*n*
*2 + 54*n + 27) + 9*a**3*x**3/(6*n**3 + 33*n**2 + 54*n + 27) + 18*a**2*b*n*
*2*x**3*x**n/(6*n**3 + 33*n**2 + 54*n + 27) + 45*a**2*b*n*x**3*x**n/(6*n**3
 + 33*n**2 + 54*n + 27) + 27*a**2*b*x**3*x**n/(6*n**3 + 33*n**2 + 54*n + 27
) + 9*a*b**2*n**2*x**3*x**(2*n)/(6*n**3 + 33*n**2 + 54*n + 27) + 36*a*b**2*
n*x**3*x**(2*n)/(6*n**3 + 33*n**2 + 54*n + 27) + 27*a*b**2*x**3*x**(2*n)/(6
*n**3 + 33*n**2 + 54*n + 27) + 2*b**3*n**2*x**3*x**(3*n)/(6*n**3 + 33*n**2
 + 54*n + 27) + 9*b**3*n*x**3*x**(3*n)/(6*n**3 + 33*n**2 + 54*n + 27) + 9*b*
*3*x**3*x**(3*n)/(6*n**3 + 33*n**2 + 54*n + 27), True))
```

3.2467 $\int x\,(a+bx^n)^3\,dx$

Optimal. Leaf size=65

$$\frac{a^3x^2}{2}+\frac{3a^2bx^{n+2}}{n+2}+\frac{3ab^2x^{2(n+1)}}{2(n+1)}+\frac{b^3x^{3n+2}}{3n+2}$$

```
[Out] 1/2*a^3*x^2+3/2*a*b^2*x^(2+2*n)/(1+n)+3*a^2*b*x^(2+n)/(2+n)+b^3*x^(2+3*n)/(
2+3*n)
```

Rubi [A] time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {270}

$$\frac{3a^2bx^{n+2}}{n+2}+\frac{a^3x^2}{2}+\frac{3ab^2x^{2(n+1)}}{2(n+1)}+\frac{b^3x^{3n+2}}{3n+2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^n)^3,x]
```

```
[Out] (a^3*x^2)/2 + (3*a*b^2*x^(2*(1 + n)))/(2*(1 + n)) + (3*a^2*b*x^(2 + n))/(2
+ n) + (b^3*x^(2 + 3*n))/(2 + 3*n)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x\,(a+bx^n)^3\,dx &= \int \left(a^3x+3a^2bx^{1+n}+3ab^2x^{1+2n}+b^3x^{1+3n}\right)dx\\ &= \frac{a^3x^2}{2}+\frac{3ab^2x^{2(1+n)}}{2(1+n)}+\frac{3a^2bx^{2+n}}{2+n}+\frac{b^3x^{2+3n}}{2+3n}\end{aligned}$$

Mathematica [A] time = 0.04, size = 58, normalized size = 0.89

$$\frac{1}{2}x^2\left(a^3+\frac{6a^2bx^n}{n+2}+\frac{3ab^2x^{2n}}{n+1}+\frac{2b^3x^{3n}}{3n+2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^n)^3,x]
```

```
[Out] (x^2*(a^3 + (6*a^2*b*x^n)/(2 + n) + (3*a*b^2*x^(2*n))/(1 + n) + (2*b^3*x^(3
*n))/(2 + 3*n)))/2
```

fricas [B] time = 0.82, size = 145, normalized size = 2.23

$$\frac{2\left(b^3n^2+3\,b^3n+2\,b^3\right)x^2x^{3\,n}+3\left(3\,ab^2n^2+8\,ab^2n+4\,ab^2\right)x^2x^{2\,n}+6\left(3\,a^2bn^2+5\,a^2bn+2\,a^2b\right)x^2x^n+\left(3\,a^3n\right.}{2\left(3\,n^3+11\,n^2+12\,n+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(b^3*n^2 + 3*b^3*n + 2*b^3)*x^2*x^(3*n) + 3*(3*a*b^2*n^2 + 8*a*b^2*n
 + 4*a*b^2)*x^2*x^(2*n) + 6*(3*a^2*b*n^2 + 5*a^2*b*n + 2*a^2*b)*x^2*x^n + (
3*a^3*n^3 + 11*a^3*n^2 + 12*a^3*n + 4*a^3)*x^2)/(3*n^3 + 11*n^2 + 12*n + 4)
```

giac [B] time = 0.18, size = 188, normalized size = 2.89

$$\frac{2\,b^3n^2x^2x^{3\,n}+9\,ab^2n^2x^2x^{2\,n}+18\,a^2bn^2x^2x^n+3\,a^3n^3x^2+6\,b^3nx^2x^{3\,n}+24\,ab^2nx^2x^{2\,n}+30\,a^2bnx^2x^n+11\,a^3n^2x^2}{2\left(3\,n^3+11\,n^2+12\,n+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^3*n^2*x^2*x^(3*n) + 9*a*b^2*n^2*x^2*x^(2*n) + 18*a^2*b*n^2*x^2*x^n
 + 3*a^3*n^3*x^2 + 6*b^3*n*x^2*x^(3*n) + 24*a*b^2*n*x^2*x^(2*n) + 30*a^2*b*
n*x^2*x^n + 11*a^3*n^2*x^2 + 4*b^3*x^2*x^(3*n) + 12*a*b^2*x^2*x^(2*n) + 12*
a^2*b*x^2*x^n + 12*a^3*n*x^2 + 4*a^3*x^2)/(3*n^3 + 11*n^2 + 12*n + 4)
```

maple [A] time = 0.01, size = 71, normalized size = 1.09

$$\frac{3a^2b\,x^2\mathrm{e}^{n\ln(x)}}{n+2}+\frac{3a\,b^2x^2\mathrm{e}^{2n\ln(x)}}{2\,(n+1)}+\frac{b^3x^2\mathrm{e}^{3n\ln(x)}}{3n+2}+\frac{a^3x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*x^n)^3,x)
```

```
[Out] b^3/(3*n+2)*x^2*exp(n*ln(x))^3+1/2*a^3*x^2+3/2*a*b^2/(n+1)*x^2*exp(n*ln(x))
^2+3*a^2*b/(n+2)*x^2*exp(n*ln(x))
```

maxima [A] time = 0.48, size = 61, normalized size = 0.94

$$\frac{1}{2}\,a^3x^2+\frac{b^3x^{3\,n+2}}{3\,n+2}+\frac{3\,ab^2x^{2\,n+2}}{2\,(n+1)}+\frac{3\,a^2bx^{n+2}}{n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^3*x^2 + b^3*x^(3*n + 2)/(3*n + 2) + 3/2*a*b^2*x^(2*n + 2)/(n + 1) + 3
*a^2*b*x^(n + 2)/(n + 2)
```

mupad [B] time = 1.28, size = 66, normalized size = 1.02

$$\frac{a^3\,x^2}{2}+\frac{b^3\,x^{3\,n}\,x^2}{3\,n+2}+\frac{3\,a\,b^2\,x^{2\,n}\,x^2}{2\,n+2}+\frac{3\,a^2\,b\,x^n\,x^2}{n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^n)^3,x)
```

```
[Out] (a^3*x^2)/2 + (b^3*x^(3*n)*x^2)/(3*n + 2) + (3*a*b^2*x^(2*n)*x^2)/(2*n + 2)
 + (3*a^2*b*x^n*x^2)/(n + 2)
```

sympy [A] time = 1.78, size = 500, normalized size = 7.69

$$\begin{cases}\frac{a^3x^2}{2}+3a^2b\log(x)-\frac{3ab^2}{2x^2}-\frac{b^3}{4x^4}\\ \frac{a^3x^2}{2}+3a^2bx+3ab^2\log(x)-\frac{b^3}{x}\\ \frac{a^3x^2}{2}+\frac{9a^2bx^{\frac{4}{3}}}{4}+\frac{9ab^2x^{\frac{2}{3}}}{2}+b^3\log(x)\\ \frac{3a^3n^3x^2}{6n^3+22n^2+24n+8}+\frac{11a^3n^2x^2}{6n^3+22n^2+24n+8}+\frac{12a^3nx^2}{6n^3+22n^2+24n+8}+\frac{4a^3x^2}{6n^3+22n^2+24n+8}+\frac{18a^2bn^2x^2x^n}{6n^3+22n^2+24n+8}+\frac{30a^2bnx^2x^n}{6n^3+22n^2+24n+8}+\frac{12a^2bx^2}{6n^3+22n^2+}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x**n)**3,x)
```

```
[Out] Piecewise((a**3*x**2/2 + 3*a**2*b*log(x) - 3*a*b**2/(2*x**2) - b**3/(4*x**4
), Eq(n, -2)), (a**3*x**2/2 + 3*a**2*b*x + 3*a*b**2*log(x) - b**3/x, Eq(n,
-1)), (a**3*x**2/2 + 9*a**2*b*x**(4/3)/4 + 9*a*b**2*x**(2/3)/2 + b**3*log(x
), Eq(n, -2/3)), (3*a**3*n**3*x**2/(6*n**3 + 22*n**2 + 24*n + 8) + 11*a**3*
n**2*x**2/(6*n**3 + 22*n**2 + 24*n + 8) + 12*a**3*n*x**2/(6*n**3 + 22*n**2
+ 24*n + 8) + 4*a**3*x**2/(6*n**3 + 22*n**2 + 24*n + 8) + 18*a**2*b*n**2*x*
*2*x**n/(6*n**3 + 22*n**2 + 24*n + 8) + 30*a**2*b*n*x**2*x**n/(6*n**3 + 22*
n**2 + 24*n + 8) + 12*a**2*b*x**2*x**n/(6*n**3 + 22*n**2 + 24*n + 8) + 9*a*
b**2*n**2*x**2*x**(2*n)/(6*n**3 + 22*n**2 + 24*n + 8) + 24*a*b**2*n*x**2*x*
*(2*n)/(6*n**3 + 22*n**2 + 24*n + 8) + 12*a*b**2*x**2*x**(2*n)/(6*n**3 + 22
*n**2 + 24*n + 8) + 2*b**3*n**2*x**2*x**(3*n)/(6*n**3 + 22*n**2 + 24*n + 8)
 + 6*b**3*n*x**2*x**(3*n)/(6*n**3 + 22*n**2 + 24*n + 8) + 4*b**3*x**2*x**(3
*n)/(6*n**3 + 22*n**2 + 24*n + 8), True))
```

3.2468 $\int (a + bx^n)^3 \, dx$

Optimal. Leaf size=60

$$a^3 x + \frac{3a^2 b x^{n+1}}{n+1} + \frac{3ab^2 x^{2n+1}}{2n+1} + \frac{b^3 x^{3n+1}}{3n+1}$$

```
[Out] a^3*x+3*a^2*b*x^(1+n)/(1+n)+3*a*b^2*x^(1+2*n)/(1+2*n)+b^3*x^(1+3*n)/(1+3*n)
```

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {244}

$$\frac{3a^2 b x^{n+1}}{n+1} + a^3 x + \frac{3ab^2 x^{2n+1}}{2n+1} + \frac{b^3 x^{3n+1}}{3n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^3,x]
```

```
[Out] a^3*x + (3*a^2*b*x^(1 + n))/(1 + n) + (3*a*b^2*x^(1 + 2*n))/(1 + 2*n) + (b^
3*x^(1 + 3*n))/(1 + 3*n)
```

Rule 244

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (a + bx^n)^3 \, dx &= \int \left(a^3 + 3a^2 b x^n + 3ab^2 x^{2n} + b^3 x^{3n}\right) dx \\ &= a^3 x + \frac{3a^2 b x^{1+n}}{1+n} + \frac{3ab^2 x^{1+2n}}{1+2n} + \frac{b^3 x^{1+3n}}{1+3n}\end{aligned}$$

Mathematica [A] time = 0.03, size = 54, normalized size = 0.90

$$x\left(a^3 + \frac{3a^2 b x^n}{n+1} + \frac{3ab^2 x^{2n}}{2n+1} + \frac{b^3 x^{3n}}{3n+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^3,x]
```

```
[Out] x*(a^3 + (3*a^2*b*x^n)/(1 + n) + (3*a*b^2*x^(2*n))/(1 + 2*n) + (b^3*x^(3*n)
)/(1 + 3*n))
```

fricas [B] time = 0.54, size = 130, normalized size = 2.17

$$\frac{\left(2\,b^3 n^2 + 3\,b^3 n + b^3\right) x x^{3\,n} + 3\left(3\,ab^2 n^2 + 4\,ab^2 n + ab^2\right) x x^{2\,n} + 3\left(6\,a^2 b n^2 + 5\,a^2 b n + a^2 b\right) x x^n + \left(6\,a^3 n^3 + 11\,a^3 n^2\right.}{6\,n^3 + 11\,n^2 + 6\,n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] ((2*b^3*n^2 + 3*b^3*n + b^3)*x*x^(3*n) + 3*(3*a*b^2*n^2 + 4*a*b^2*n + a*b^2
)*x*x^(2*n) + 3*(6*a^2*b*n^2 + 5*a^2*b*n + a^2*b)*x*x^n + (6*a^3*n^3 + 11*a
^3*n^2 + 6*a^3*n + a^3)*x)/(6*n^3 + 11*n^2 + 6*n + 1)
```

giac [B] time = 0.16, size = 159, normalized size = 2.65

$$\frac{6\,a^3n^3x + 2\,b^3n^2xx^{3\,n} + 9\,ab^2n^2xx^{2\,n} + 18\,a^2bn^2xx^n + 11\,a^3n^2x + 3\,b^3nxx^{3\,n} + 12\,ab^2nxx^{2\,n} + 15\,a^2bnxx^n + 6\,a}{6\,n^3 + 11\,n^2 + 6\,n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] (6*a^3*n^3*x + 2*b^3*n^2*x*x^(3*n) + 9*a*b^2*n^2*x*x^(2*n) + 18*a^2*b*n^2*x
*x^n + 11*a^3*n^2*x + 3*b^3*n*x*x^(3*n) + 12*a*b^2*n*x*x^(2*n) + 15*a^2*b*n
*x*x^n + 6*a^3*n*x + b^3*x*x^(3*n) + 3*a*b^2*x*x^(2*n) + 3*a^2*b*x*x^n + a^
3*x)/(6*n^3 + 11*n^2 + 6*n + 1)
```

maple [A] time = 0.01, size = 64, normalized size = 1.07

$$\frac{3a^2bx\,\mathrm{e}^{n\ln(x)}}{n+1} + \frac{3a\,b^2x\,\mathrm{e}^{2n\ln(x)}}{2n+1} + \frac{b^3x\,\mathrm{e}^{3n\ln(x)}}{3n+1} + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^3,x)
```

```
[Out] a^3*x+b^3/(1+3*n)*x*exp(n*ln(x))^3+3*a*b^2/(2*n+1)*x*exp(n*ln(x))^2+3*a^2*b
/(n+1)*x*exp(n*ln(x))
```

maxima [A] time = 0.46, size = 60, normalized size = 1.00

$$a^3x + \frac{b^3x^{3\,n+1}}{3\,n+1} + \frac{3\,ab^2x^{2\,n+1}}{2\,n+1} + \frac{3\,a^2bx^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] a^3*x + b^3*x^(3*n + 1)/(3*n + 1) + 3*a*b^2*x^(2*n + 1)/(2*n + 1) + 3*a^2*b
*x^(n + 1)/(n + 1)
```

mupad [B] time = 1.19, size = 57, normalized size = 0.95

$$a^3\,x + \frac{b^3\,x\,x^{3\,n}}{3\,n+1} + \frac{3\,a\,b^2\,x\,x^{2\,n}}{2\,n+1} + \frac{3\,a^2\,b\,x\,x^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^3,x)
```

```
[Out] a^3*x + (b^3*x*x^(3*n))/(3*n + 1) + (3*a*b^2*x*x^(2*n))/(2*n + 1) + (3*a^2*
b*x*x^n)/(n + 1)
```

sympy [A] time = 0.82, size = 469, normalized size = 7.82

$$\begin{cases} a^3x + 3a^2b\log(x) - \frac{3ab^2}{x} - \frac{b^3}{2x^2} \\ a^3x + 6a^2b\sqrt{x} + 3ab^2\log(x) - \frac{2b^3}{\sqrt{x}} \\ a^3x + \frac{9a^2bx^{\frac{2}{3}}}{2} + 9ab^2\sqrt[3]{x} + b^3\log(x) \\ \frac{6a^3n^3x}{6n^3+11n^2+6n+1} + \frac{11a^3n^2x}{6n^3+11n^2+6n+1} + \frac{6a^3nx}{6n^3+11n^2+6n+1} + \frac{a^3x}{6n^3+11n^2+6n+1} + \frac{18a^2bn^2xx^n}{6n^3+11n^2+6n+1} + \frac{15a^2bnxx^n}{6n^3+11n^2+6n+1} + \frac{3a^2bxx^n}{6n^3+11n^2+6n} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*log(x) - 3*a*b**2/x - b**3/(2*x**2), Eq(n, -1)
), (a**3*x + 6*a**2*b*sqrt(x) + 3*a*b**2*log(x) - 2*b**3/sqrt(x), Eq(n, -1/
2)), (a**3*x + 9*a**2*b*x**(2/3)/2 + 9*a*b**2*x**(1/3) + b**3*log(x), Eq(n,
 -1/3)), (6*a**3*n**3*x/(6*n**3 + 11*n**2 + 6*n + 1) + 11*a**3*n**2*x/(6*n*
*3 + 11*n**2 + 6*n + 1) + 6*a**3*n*x/(6*n**3 + 11*n**2 + 6*n + 1) + a**3*x/
(6*n**3 + 11*n**2 + 6*n + 1) + 18*a**2*b*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*
n + 1) + 15*a**2*b*n*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 3*a**2*b*x*x**n/
(6*n**3 + 11*n**2 + 6*n + 1) + 9*a*b**2*n**2*x*x**(2*n)/(6*n**3 + 11*n**2 +
 6*n + 1) + 12*a*b**2*n*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 3*a*b**2*
x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 2*b**3*n**2*x*x**(3*n)/(6*n**3 +
11*n**2 + 6*n + 1) + 3*b**3*n*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1) + b**
3*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1), True))
```

3.2469 $\int \frac{(a+bx^n)^3}{x}\,dx$

Optimal. Leaf size=50

$$a^3\log(x)+\frac{3a^2bx^n}{n}+\frac{3ab^2x^{2n}}{2n}+\frac{b^3x^{3n}}{3n}$$

```
[Out] 3*a^2*b*x^n/n+3/2*a*b^2*x^(2*n)/n+1/3*b^3*x^(3*n)/n+a^3*ln(x)
```

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{3a^2bx^n}{n}+a^3\log(x)+\frac{3ab^2x^{2n}}{2n}+\frac{b^3x^{3n}}{3n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^3/x,x]
```

```
[Out] (3*a^2*b*x^n)/n + (3*a*b^2*x^(2*n))/(2*n) + (b^3*x^(3*n))/(3*n) + a^3*Log[x
]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^n)^3}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^3}{x}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(3a^2b+\frac{a^3}{x}+3ab^2x+b^3x^2\right)\,dx,x,x^n\right)}{n}\\ &= \frac{3a^2bx^n}{n}+\frac{3ab^2x^{2n}}{2n}+\frac{b^3x^{3n}}{3n}+a^3\log(x)\end{aligned}$$

Mathematica [A] time = 0.03, size = 41, normalized size = 0.82

$$a^3\log(x)+\frac{bx^n\left(18a^2+9abx^n+2b^2x^{2n}\right)}{6n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^3/x,x]
```

```
[Out] (b*x^n*(18*a^2 + 9*a*b*x^n + 2*b^2*x^(2*n)))/(6*n) + a^3*Log[x]
```

fricas [A] time = 0.66, size = 44, normalized size = 0.88

$$\frac{6\,a^3 n\log(x) + 2\,b^3x^{3\,n} + 9\,ab^2x^{2\,n} + 18\,a^2bx^n}{6\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^3/x,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^3*n*log(x) + 2*b^3*x^(3*n) + 9*a*b^2*x^(2*n) + 18*a^2*b*x^n)/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^3/x, x)
```

maple [A] time = 0.00, size = 52, normalized size = 1.04

$$\frac{a^3\ln(x^n)}{n} + \frac{3a^2b\,x^n}{n} + \frac{3a\,b^2x^{2n}}{2n} + \frac{b^3x^{3n}}{3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^3/x,x)
```

```
[Out] 1/3/n*b^3*(x^n)^3+3/2/n*(x^n)^2*a*b^2+3*a^2*b*x^n/n+1/n*a^3*ln(x^n)
```

maxima [A] time = 0.45, size = 48, normalized size = 0.96

$$\frac{a^3\log(x^n)}{n} + \frac{2\,b^3x^{3\,n} + 9\,ab^2x^{2\,n} + 18\,a^2bx^n}{6\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^3/x,x, algorithm="maxima")
```

```
[Out] a^3*log(x^n)/n + 1/6*(2*b^3*x^(3*n) + 9*a*b^2*x^(2*n) + 18*a^2*b*x^n)/n
```

mupad [B] time = 1.16, size = 46, normalized size = 0.92

$$a^3\,\ln(x) + \frac{b^3\,x^{3\,n}}{3\,n} + \frac{3\,a^2\,b\,x^n}{n} + \frac{3\,a\,b^2\,x^{2\,n}}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^3/x,x)
```

```
[Out] a^3*log(x) + (b^3*x^(3*n))/(3*n) + (3*a^2*b*x^n)/n + (3*a*b^2*x^(2*n))/(2*n
)
```

sympy [A] time = 0.42, size = 53, normalized size = 1.06

$$\begin{cases} a^3\log(x) + \frac{3a^2bx^n}{n} + \frac{3ab^2x^{2n}}{2n} + \frac{b^3x^{3n}}{3n} & \text{for } n \neq 0 \\ (a+b)^3\log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**3/x,x)
```

```
[Out] Piecewise((a**3*log(x) + 3*a**2*b*x**n/n + 3*a*b**2*x**(2*n)/(2*n) + b**3*x
**(3*n)/(3*n), Ne(n, 0)), ((a + b)**3*log(x), True))
```

3.2470 $\int \frac{(a+bx^n)^3}{x^2}\,dx$

Optimal. Leaf size=66

$$-\frac{a^3}{x}-\frac{3a^2bx^{n-1}}{1-n}-\frac{3ab^2x^{2n-1}}{1-2n}-\frac{b^3x^{3n-1}}{1-3n}$$

```
[Out] -a^3/x-3*a^2*b*x^(-1+n)/(1-n)-3*a*b^2*x^(-1+2*n)/(1-2*n)-b^3*x^(-1+3*n)/(1-
3*n)
```

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$-\frac{3a^2bx^{n-1}}{1-n}-\frac{a^3}{x}-\frac{3ab^2x^{2n-1}}{1-2n}-\frac{b^3x^{3n-1}}{1-3n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^3/x^2,x]
```

```
[Out] -(a^3/x) - (3*a^2*b*x^(-1 + n))/(1 - n) - (3*a*b^2*x^(-1 + 2*n))/(1 - 2*n)
- (b^3*x^(-1 + 3*n))/(1 - 3*n)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^n)^3}{x^2}\,dx &= \int \left(\frac{a^3}{x^2}+3a^2bx^{-2+n}+3ab^2x^{2(-1+n)}+b^3x^{-2+3n}\right)dx\\ &= -\frac{a^3}{x}-\frac{3a^2bx^{-1+n}}{1-n}-\frac{3ab^2x^{-1+2n}}{1-2n}-\frac{b^3x^{-1+3n}}{1-3n}\end{aligned}$$

Mathematica [A] time = 0.05, size = 58, normalized size = 0.88

$$\frac{-a^3+\frac{3a^2bx^n}{n-1}+\frac{3ab^2x^{2n}}{2n-1}+\frac{b^3x^{3n}}{3n-1}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^3/x^2,x]
```

```
[Out] (-a^3 + (3*a^2*b*x^n)/(-1 + n) + (3*a*b^2*x^(2*n))/(-1 + 2*n) + (b^3*x^(3*n
))/(-1 + 3*n))/x
```

fricas [B] time = 1.06, size = 131, normalized size = 1.98

$$-\frac{6\,a^3n^3-11\,a^3n^2+6\,a^3n-a^3-\left(2\,b^3n^2-3\,b^3n+b^3\right)x^{3\,n}-3\left(3\,ab^2n^2-4\,ab^2n+ab^2\right)x^{2\,n}-3\left(6\,a^2bn^2-5\,a^2\right.}{\left(6\,n^3-11\,n^2+6\,n-1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^3/x^2,x, algorithm="fricas")
```

```
[Out] -(6*a^3*n^3 - 11*a^3*n^2 + 6*a^3*n - a^3 - (2*b^3*n^2 - 3*b^3*n + b^3)*x^(3
*n) - 3*(3*a*b^2*n^2 - 4*a*b^2*n + a*b^2)*x^(2*n) - 3*(6*a^2*b*n^2 - 5*a^2*
b*n + a^2*b)*x^n)/((6*n^3 - 11*n^2 + 6*n - 1)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^3}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^3/x^2, x)
```

maple [A] time = 0.02, size = 65, normalized size = 0.98

$$\frac{\frac{3a^2b\,e^{n\ln(x)}}{n-1} + \frac{3a\,b^2e^{2n\ln(x)}}{2n-1} + \frac{b^3e^{3n\ln(x)}}{3n-1} - a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^3/x^2,x)
```

```
[Out] (b^3/(-1+3*n)*exp(n*ln(x))^3-a^3+3*a*b^2/(2*n-1)*exp(n*ln(x))^2+3*a^2*b/(n-
1)*exp(n*ln(x)))/x
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^3/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(n-2>0)', see `assume?` for more det
ails)Is n-2 equal to -1?
```

mupad [B] time = 1.17, size = 66, normalized size = 1.00

$$\frac{b^3\,x^{3\,n}}{x\,(3\,n-1)} - \frac{a^3}{x} + \frac{3\,a\,b^2\,x^{2\,n}}{x\,(2\,n-1)} + \frac{3\,a^2\,b\,x^n}{x\,(n-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^3/x^2,x)
```

```
[Out] (b^3*x^(3*n))/(x*(3*n - 1)) - a^3/x + (3*a*b^2*x^(2*n))/(x*(2*n - 1)) + (3*
a^2*b*x^n)/(x*(n - 1))
```

sympy [A] time = 1.29, size = 508, normalized size = 7.70

$$\begin{cases} -\frac{a^3}{x} - \frac{9a^2b}{2x^{\frac{2}{3}}} - \frac{9ab^2}{\sqrt[3]{x}} + b^3\log(x) \\ -\frac{a^3}{x} - \frac{6a^2b}{\sqrt{x}} + 3ab^2\log(x) + 2b^3\sqrt{x} \\ -\frac{a^3}{x} + 3a^2b\log(x) + 3ab^2x + \frac{b^3x^2}{2} \\ -\frac{6a^3n^3}{6n^3x-11n^2x+6nx-x} + \frac{11a^3n^2}{6n^3x-11n^2x+6nx-x} - \frac{6a^3n}{6n^3x-11n^2x+6nx-x} + \frac{a^3}{6n^3x-11n^2x+6nx-x} + \frac{18a^2bn^2x^n}{6n^3x-11n^2x+6nx-x} - \frac{15a^2bnx^n}{6n^3x-11n^2x+6nx-x} - \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**3/x**2,x)
```

```
[Out] Piecewise((-a**3/x - 9*a**2*b/(2*x**(2/3)) - 9*a*b**2/x**(1/3) + b**3*log(x
), Eq(n, 1/3)), (-a**3/x - 6*a**2*b/sqrt(x) + 3*a*b**2*log(x) + 2*b**3*sqrt
(x), Eq(n, 1/2)), (-a**3/x + 3*a**2*b*log(x) + 3*a*b**2*x + b**3*x**2/2, Eq
(n, 1)), (-6*a**3*n**3/(6*n**3*x - 11*n**2*x + 6*n*x - x) + 11*a**3*n**2/(6
*n**3*x - 11*n**2*x + 6*n*x - x) - 6*a**3*n/(6*n**3*x - 11*n**2*x + 6*n*x -
 x) + a**3/(6*n**3*x - 11*n**2*x + 6*n*x - x) + 18*a**2*b*n**2*x**n/(6*n**3
*x - 11*n**2*x + 6*n*x - x) - 15*a**2*b*n*x**n/(6*n**3*x - 11*n**2*x + 6*n*
x - x) + 3*a**2*b*x**n/(6*n**3*x - 11*n**2*x + 6*n*x - x) + 9*a*b**2*n**2*x
**(2*n)/(6*n**3*x - 11*n**2*x + 6*n*x - x) - 12*a*b**2*n*x**(2*n)/(6*n**3*x
 - 11*n**2*x + 6*n*x - x) + 3*a*b**2*x**(2*n)/(6*n**3*x - 11*n**2*x + 6*n*x
 - x) + 2*b**3*n**2*x**(3*n)/(6*n**3*x - 11*n**2*x + 6*n*x - x) - 3*b**3*n*
x**(3*n)/(6*n**3*x - 11*n**2*x + 6*n*x - x) + b**3*x**(3*n)/(6*n**3*x - 11*
n**2*x + 6*n*x - x), True))
```

3.2471 $\int \frac{(a+bx^n)^3}{x^3}\,dx$

Optimal. Leaf size=72

$$-\frac{a^3}{2x^2}-\frac{3a^2bx^{n-2}}{2-n}-\frac{3ab^2x^{-2(1-n)}}{2(1-n)}-\frac{b^3x^{3n-2}}{2-3n}$$

```
[Out] -1/2*a^3/x^2-3/2*a*b^2/(1-n)/(x^(2-2*n))-3*a^2*b*x^(-2+n)/(2-n)-b^3*x^(-2+3
*n)/(2-3*n)
```

Rubi [A] time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$-\frac{3a^2bx^{n-2}}{2-n}-\frac{a^3}{2x^2}-\frac{3ab^2x^{-2(1-n)}}{2(1-n)}-\frac{b^3x^{3n-2}}{2-3n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^3/x^3,x]
```

```
[Out] -a^3/(2*x^2) - (3*a*b^2)/(2*(1 - n)*x^(2*(1 - n))) - (3*a^2*b*x^(-2 + n))/(
2 - n) - (b^3*x^(-2 + 3*n))/(2 - 3*n)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^n)^3}{x^3}\,dx &= \int \left(\frac{a^3}{x^3}+3a^2bx^{-3+n}+b^3x^{3(-1+n)}+3ab^2x^{-3+2n}\right)dx\\ &= -\frac{a^3}{2x^2}-\frac{3ab^2x^{-2(1-n)}}{2(1-n)}-\frac{3a^2bx^{-2+n}}{2-n}-\frac{b^3x^{-2+3n}}{2-3n}\end{aligned}$$

Mathematica [A] time = 0.05, size = 60, normalized size = 0.83

$$\frac{-a^3+\frac{6a^2bx^n}{n-2}+\frac{3ab^2x^{2n}}{n-1}+\frac{2b^3x^{3n}}{3n-2}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^3/x^3,x]
```

```
[Out] (-a^3 + (6*a^2*b*x^n)/(-2 + n) + (3*a*b^2*x^(2*n))/(-1 + n) + (2*b^3*x^(3*n
))/(-2 + 3*n))/(2*x^2)
```

fricas [B] time = 0.67, size = 134, normalized size = 1.86

$$-\frac{3\,a^3n^3-11\,a^3n^2+12\,a^3n-4\,a^3-2\left(b^3n^2-3\,b^3n+2\,b^3\right)x^{3\,n}-3\left(3\,ab^2n^2-8\,ab^2n+4\,ab^2\right)x^{2\,n}-6\left(3\,a^2bn^2-5\right.}{2\left(3\,n^3-11\,n^2+12\,n-4\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^3/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(3*a^3*n^3 - 11*a^3*n^2 + 12*a^3*n - 4*a^3 - 2*(b^3*n^2 - 3*b^3*n + 2*
b^3)*x^(3*n) - 3*(3*a*b^2*n^2 - 8*a*b^2*n + 4*a*b^2)*x^(2*n) - 6*(3*a^2*b*n
^2 - 5*a^2*b*n + 2*a^2*b)*x^n)/((3*n^3 - 11*n^2 + 12*n - 4)*x^2)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^3/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^3/x^3, x)
```

maple [**A**] time = 0.02, size = 65, normalized size = 0.90

$$\frac{3a^2b\,x^n}{(n-2)\,x^2} + \frac{3a\,b^2x^{2n}}{2\,(n-1)\,x^2} + \frac{b^3x^{3n}}{(3n-2)\,x^2} - \frac{a^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^3/x^3,x)
```

```
[Out] -1/2*a^3/x^2+1/(3*n-2)*b^3/x^2*(x^n)^3+3/2/(n-1)*a*b^2/x^2*(x^n)^2+3/(n-2)*
a^2*b/x^2*x^n
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^3/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(n-3>0)', see `assume?` for more det
ails)Is n-3 equal to -1?
```

mupad [**B**] time = 1.18, size = 66, normalized size = 0.92

$$\frac{b^3\,x^{3\,n}}{x^2\,(3\,n-2)} - \frac{a^3}{2\,x^2} + \frac{3\,a\,b^2\,x^{2\,n}}{x^2\,(2\,n-2)} + \frac{3\,a^2\,b\,x^n}{x^2\,(n-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^3/x^3,x)
```

```
[Out] (b^3*x^(3*n))/(x^2*(3*n - 2)) - a^3/(2*x^2) + (3*a*b^2*x^(2*n))/(x^2*(2*n -
 2)) + (3*a^2*b*x^n)/(x^2*(n - 2))
```

sympy [**A**] time = 1.72, size = 627, normalized size = 8.71

$$\begin{cases} -\frac{a^3}{2x^2} - \frac{9a^2b}{4x^{\frac{4}{3}}} - \frac{9ab^2}{2x^{\frac{2}{3}}} + b^3\log(x) \\ -\frac{a^3}{2x^2} - \frac{3a^2b}{x} + 3ab^2\log(x) + b^3x \\ -\frac{a^3}{2x^2} + 3a^2b\log(x) + \frac{3ab^2x^2}{2} + \frac{b^3x^4}{4} \\ -\frac{3a^3n^3}{6n^3x^2-22n^2x^2+24nx^2-8x^2} + \frac{11a^3n^2}{6n^3x^2-22n^2x^2+24nx^2-8x^2} - \frac{12a^3n}{6n^3x^2-22n^2x^2+24nx^2-8x^2} + \frac{4a^3}{6n^3x^2-22n^2x^2+24nx^2-8x^2} + \frac{18a^2bn}{6n^3x^2-22n^2x^2+} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**3/x**3,x)
```

```
[Out] Piecewise((-a**3/(2*x**2) - 9*a**2*b/(4*x**(4/3)) - 9*a*b**2/(2*x**(2/3)) +
 b**3*log(x), Eq(n, 2/3)), (-a**3/(2*x**2) - 3*a**2*b/x + 3*a*b**2*log(x) +
 b**3*x, Eq(n, 1)), (-a**3/(2*x**2) + 3*a**2*b*log(x) + 3*a*b**2*x**2/2 + b
**3*x**4/4, Eq(n, 2)), (-3*a**3*n**3/(6*n**3*x**2 - 22*n**2*x**2 + 24*n*x**
2 - 8*x**2) + 11*a**3*n**2/(6*n**3*x**2 - 22*n**2*x**2 + 24*n*x**2 - 8*x**2
) - 12*a**3*n/(6*n**3*x**2 - 22*n**2*x**2 + 24*n*x**2 - 8*x**2) + 4*a**3/(6
*n**3*x**2 - 22*n**2*x**2 + 24*n*x**2 - 8*x**2) + 18*a**2*b*n**2*x**n/(6*n*
*3*x**2 - 22*n**2*x**2 + 24*n*x**2 - 8*x**2) - 30*a**2*b*n*x**n/(6*n**3*x**
2 - 22*n**2*x**2 + 24*n*x**2 - 8*x**2) + 12*a**2*b*x**n/(6*n**3*x**2 - 22*n
**2*x**2 + 24*n*x**2 - 8*x**2) + 9*a*b**2*n**2*x**(2*n)/(6*n**3*x**2 - 22*n
**2*x**2 + 24*n*x**2 - 8*x**2) - 24*a*b**2*n*x**(2*n)/(6*n**3*x**2 - 22*n**
2*x**2 + 24*n*x**2 - 8*x**2) + 12*a*b**2*x**(2*n)/(6*n**3*x**2 - 22*n**2*x*
*2 + 24*n*x**2 - 8*x**2) + 2*b**3*n**2*x**(3*n)/(6*n**3*x**2 - 22*n**2*x**2
 + 24*n*x**2 - 8*x**2) - 6*b**3*n*x**(3*n)/(6*n**3*x**2 - 22*n**2*x**2 + 24
*n*x**2 - 8*x**2) + 4*b**3*x**(3*n)/(6*n**3*x**2 - 22*n**2*x**2 + 24*n*x**2
 - 8*x**2), True))
```

3.2472 $\int \frac{x}{a+bx^n}\,dx$

Optimal. Leaf size=33

$$\frac{x^2\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right)}{2a}$$

```
[Out] 1/2*x^2*hypergeom([1, 2/n],[(2+n)/n],-b*x^n/a)/a
```

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {364}

$$\frac{x^2\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^n),x]
```

```
[Out] (x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((b*x^n)/a)])/(2*a)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x}{a+bx^n}\,dx = \frac{x^2\,{}_2F_1\left(1,\frac{2}{n};\frac{2+n}{n};-\frac{bx^n}{a}\right)}{2a}$$

Mathematica [A] time = 0.00, size = 33, normalized size = 1.00

$$\frac{x^2\,{}_2F_1\left(1,\frac{2}{n};1+\frac{2}{n};-\frac{bx^n}{a}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^n),x]
```

```
[Out] (x^2*Hypergeometric2F1[1, 2/n, 1 + 2/n, -((b*x^n)/a)])/(2*a)
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{bx^n+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral(x/(b*x^n + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x/(b*x^n + a), x)
```

maple **[F]** time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{x}{b\,x^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*x^n),x)
```

```
[Out] int(x/(a+b*x^n),x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*x^n + a), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{a + b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^n),x)
```

```
[Out] int(x/(a + b*x^n), x)
```

sympy **[C]** time = 0.83, size = 36, normalized size = 1.09

$$\frac{2x^2\Phi\left(\frac{bx^n e^{i\pi}}{a}, 1, \frac{2}{n}\right)\Gamma\left(\frac{2}{n}\right)}{an^2\Gamma\left(1 + \frac{2}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x**n),x)
```

```
[Out] 2*x**2*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2/n)*gamma(2/n)/(a*n**2*gamma(
1 + 2/n))
```

3.2473 $\int \frac{1}{a+bx^n}\,dx$

Optimal. Leaf size=24

$$\frac{x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a}$$

```
[Out] x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {245}

$$\frac{x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^(-1),x]
```

```
[Out] (x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/a
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{a+bx^n}\,dx = \frac{x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$\frac{x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^(-1),x]
```

```
[Out] (x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/a
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{bx^n+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral(1/(b*x^n + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{bx^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n),x, algorithm="giac")
[Out] integrate(1/(b*x^n + a), x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{1}{b\,x^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^n+a),x)
[Out] int(1/(b*x^n+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{bx^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n),x, algorithm="maxima")
[Out] integrate(1/(b*x^n + a), x)
```

mupad [B] time = 1.24, size = 25, normalized size = 1.04

$$\frac{x\,{}_2\mathrm{F}_1\left(1, \frac{1}{n}; \frac{1}{n} + 1; -\frac{b\,x^n}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^n),x)
[Out] (x*hypergeom([1, 1/n], 1/n + 1, -(b*x^n)/a))/a
```

sympy [C] time = 0.80, size = 32, normalized size = 1.33

$$\frac{x\Phi\left(\frac{bx^n e^{i\pi}}{a}, 1, \frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^2\Gamma\left(1 + \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n),x)
[Out] x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(a*n**2*gamma(1 + 1
/n))
```

3.2474 $\int \frac{1}{x(a+bx^n)}\,dx$

Optimal. Leaf size=23

$$\frac{\log(x)}{a} - \frac{\log(a+bx^n)}{an}$$

```
[Out] ln(x)/a-ln(a+b*x^n)/a/n
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {266, 36, 29, 31}

$$\frac{\log(x)}{a} - \frac{\log(a+bx^n)}{an}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^n)),x]
```

```
[Out] Log[x]/a - Log[a + b*x^n]/(a*n)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\,(a+bx^n)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx, x, x^n\right)}{n} \\ &= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^n\right)}{an} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, x^n\right)}{an} \\ &= \frac{\log(x)}{a} - \frac{\log(a+bx^n)}{an}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.96

$$\frac{n\log(x) - \log(a+bx^n)}{an}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^n)),x]
[Out] (n*Log[x] - Log[a + b*x^n])/(a*n)
```

fricas [A] time = 1.93, size = 22, normalized size = 0.96

$$\frac{n\log(x) - \log(bx^n + a)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n),x, algorithm="fricas")
[Out] (n*log(x) - log(b*x^n + a))/(a*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n),x, algorithm="giac")
[Out] integrate(1/((b*x^n + a)*x), x)
```

maple [A] time = 0.00, size = 29, normalized size = 1.26

$$\frac{\ln(x^n)}{an} - \frac{\ln(b\,x^n + a)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^n+a),x)
[Out] -ln(b*x^n+a)/a/n+1/n/a*ln(x^n)
```

maxima [A] time = 0.51, size = 28, normalized size = 1.22

$$-\frac{\log(bx^n + a)}{an} + \frac{\log(x^n)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n),x, algorithm="maxima")
[Out] -log(b*x^n + a)/(a*n) + log(x^n)/(a*n)
```

mupad [B] time = 1.16, size = 22, normalized size = 0.96

$$-\frac{\ln(a + b\,x^n) - n\,\ln(x)}{a\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^n)),x)
[Out] -(log(a + b*x^n) - n*log(x))/(a*n)
```

sympy [**A**] time = 0.74, size = 41, normalized size = 1.78

$$\begin{cases} \tilde{\infty}\log(x) & \text{for } a = 0 \wedge b = 0 \wedge n = 0 \\ \frac{\log(x)}{a} & \text{for } b = 0 \\ -\frac{x^{-n}}{bn} & \text{for } a = 0 \\ \frac{\log(x)}{a+b} & \text{for } n = 0 \\ \frac{\log(x)}{a} - \frac{\log\left(\frac{a}{b}+x^n\right)}{an} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x**n),x)
```

```
[Out] Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (log(x)/a, Eq(b, 0)
), (-x**(-n)/(b*n), Eq(a, 0)), (log(x)/(a + b), Eq(n, 0)), (log(x)/a - log(
a/b + x**n)/(a*n), True))
```

3.2475 $\int \frac{1}{x^2(a+bx^n)}\,dx$

Optimal. Leaf size=34

$$-\frac{{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{ax}$$

```
[Out] -hypergeom([1, -1/n],[(-1+n)/n],-b*x^n/a)/a/x
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {364}

$$-\frac{{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^n)),x]
```

```
[Out] -(Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), -((b*x^n)/a)]/(a*x))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^2\,(a+bx^n)}\,dx = -\frac{{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{ax}$$

Mathematica [A] time = 0.00, size = 31, normalized size = 0.91

$$-\frac{{}_2F_1\left(1,-\frac{1}{n};1-\frac{1}{n};-\frac{bx^n}{a}\right)}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^n)),x]
```

```
[Out] -(Hypergeometric2F1[1, -n^(-1), 1 - n^(-1), -((b*x^n)/a)]/(a*x))
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{bx^2x^n+ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral(1/(b*x^2*x^n + a*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n),x, algorithm="giac")
[Out] integrate(1/((b*x^n + a)*x^2), x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^n + a)\,x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^n+a),x)
[Out] int(1/x^2/(b*x^n+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n),x, algorithm="maxima")
[Out] integrate(1/((b*x^n + a)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x^2\ (a + b\,x^n)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^n)),x)
[Out] int(1/(x^2*(a + b*x^n)), x)
```

sympy [C] time = 0.92, size = 39, normalized size = 1.15

$$-\frac{\Phi\left(\frac{bx^n e^{i\pi}}{a}, 1, \frac{e^{i\pi}}{n}\right)\Gamma\left(-\frac{1}{n}\right)}{an^2 x\Gamma\left(1 - \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*x**n),x)
[Out] -lerchphi(b*x**n*exp_polar(I*pi)/a, 1, exp_polar(I*pi)/n)*gamma(-1/n)/(a*n*
**2*x*gamma(1 - 1/n))
```

3.2476 $\int \frac{1}{x^3(a+bx^n)}\,dx$

Optimal. Leaf size=36

$$-\frac{{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2ax^2}$$

```
[Out] -1/2*hypergeom([1, -2/n],[(-2+n)/n],-b*x^n/a)/a/x^2
```

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {364}

$$-\frac{{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^n)),x]
```

```
[Out] -Hypergeometric2F1[1, -2/n, -((2 - n)/n), -((b*x^n)/a)]/(2*a*x^2)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^3\,(a+bx^n)}\,dx = -\frac{{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2ax^2}$$

Mathematica [A] time = 0.00, size = 33, normalized size = 0.92

$$-\frac{{}_2F_1\left(1,-\frac{2}{n};1-\frac{2}{n};-\frac{bx^n}{a}\right)}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^n)),x]
```

```
[Out] -1/2*Hypergeometric2F1[1, -2/n, 1 - 2/n, -((b*x^n)/a)]/(a*x^2)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{bx^3x^n+ax^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral(1/(b*x^3*x^n + a*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n),x, algorithm="giac")
[Out] integrate(1/((b*x^n + a)*x^3), x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^n + a)\,x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^n+a),x)
[Out] int(1/x^3/(b*x^n+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n),x, algorithm="maxima")
[Out] integrate(1/((b*x^n + a)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x^3\ (a + b\,x^n)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^n)),x)
[Out] int(1/(x^3*(a + b*x^n)), x)
```

sympy [C] time = 1.00, size = 44, normalized size = 1.22

$$-\frac{2\Phi\left(\frac{bx^n e^{i\pi}}{a}, 1, \frac{2e^{i\pi}}{n}\right)\Gamma\left(-\frac{2}{n}\right)}{an^2x^2\Gamma\left(1 - \frac{2}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*x**n),x)
[Out] -2*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2*exp_polar(I*pi)/n)*gamma(-2/n)/(
a*n**2*x**2*gamma(1 - 2/n))
```

3.2477 $\int \frac{x}{(a+bx^n)^2}\,dx$

Optimal. Leaf size=33

$$\frac{x^2\,{}_2F_1\left(2,\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right)}{2a^2}$$

```
[Out] 1/2*x^2*hypergeom([2, 2/n],[(2+n)/n],-b*x^n/a)/a^2
```

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {364}

$$\frac{x^2\,{}_2F_1\left(2,\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^n)^2,x]
```

```
[Out] (x^2*Hypergeometric2F1[2, 2/n, (2 + n)/n, -((b*x^n)/a)])/(2*a^2)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x}{(a+bx^n)^2}\,dx = \frac{x^2\,{}_2F_1\left(2,\frac{2}{n};\frac{2+n}{n};-\frac{bx^n}{a}\right)}{2a^2}$$

Mathematica [A] time = 0.00, size = 33, normalized size = 1.00

$$\frac{x^2\,{}_2F_1\left(2,\frac{2}{n};1+\frac{2}{n};-\frac{bx^n}{a}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^n)^2,x]
```

```
[Out] (x^2*Hypergeometric2F1[2, 2/n, 1 + 2/n, -((b*x^n)/a)])/(2*a^2)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{b^2x^{2\,n}+2\,abx^n+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral(x/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(bx^n + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate(x/(b*x^n + a)^2, x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{x}{(b\,x^n + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^n+a)^2,x)
```

```
[Out] int(x/(b*x^n+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$(n-2)\int \frac{x}{abnx^n + a^2 n}\, dx + \frac{x^2}{abnx^n + a^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] (n - 2)*integrate(x/(a*b*n*x^n + a^2*n), x) + x^2/(a*b*n*x^n + a^2*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{(a + b\,x^n)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^n)^2,x)
```

```
[Out] int(x/(a + b*x^n)^2, x)
```

sympy [C] time = 1.22, size = 274, normalized size = 8.30

$$\frac{2nx^2\Phi\left(\frac{bx^n e^{i\pi}}{a}, 1, \frac{2}{n}\right)\Gamma\left(\frac{2}{n}\right)}{a\left(an^3\Gamma\left(1+\frac{2}{n}\right) + bn^3x^n\Gamma\left(1+\frac{2}{n}\right)\right)} + \frac{2nx^2\Gamma\left(\frac{2}{n}\right)}{a\left(an^3\Gamma\left(1+\frac{2}{n}\right) + bn^3x^n\Gamma\left(1+\frac{2}{n}\right)\right)} - \frac{4x^2\Phi\left(\frac{bx^n e^{i\pi}}{a}, 1, \frac{2}{n}\right)\Gamma\left(\frac{2}{n}\right)}{a\left(an^3\Gamma\left(1+\frac{2}{n}\right) + bn^3x^n\Gamma\left(1+\frac{2}{n}\right)\right)} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x**n)**2,x)
```

```
[Out] 2*n*x**2*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2/n)*gamma(2/n)/(a*(a*n**3*g
amma(1 + 2/n) + b*n**3*x**n*gamma(1 + 2/n))) + 2*n*x**2*gamma(2/n)/(a*(a*n*
*3*gamma(1 + 2/n) + b*n**3*x**n*gamma(1 + 2/n))) - 4*x**2*lerchphi(b*x**n*e
xp_polar(I*pi)/a, 1, 2/n)*gamma(2/n)/(a*(a*n**3*gamma(1 + 2/n) + b*n**3*x**
n*gamma(1 + 2/n))) + 2*b*n*x**2*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1,
2/n)*gamma(2/n)/(a**2*(a*n**3*gamma(1 + 2/n) + b*n**3*x**n*gamma(1 + 2/n)))
 - 4*b*x**2*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2/n)*gamma(2/n)/(a**
2*(a*n**3*gamma(1 + 2/n) + b*n**3*x**n*gamma(1 + 2/n)))
```

3.2478 $\int \frac{1}{(a+bx^n)^2}\,dx$

Optimal. Leaf size=24

$$\frac{x\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2}$$

```
[Out] x*hypergeom([2, 1/n],[1+1/n],-b*x^n/a)/a^2
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {245}

$$\frac{x\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^(-2),x]
```

```
[Out] (x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/a^2
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx^n)^2}\,dx = \frac{x\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$\frac{x\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^(-2),x]
```

```
[Out] (x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/a^2
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b^2x^{2n}+2\,abx^n+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2,x, algorithm="giac")
[Out] integrate((b*x^n + a)^(-2), x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^n + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^n+a)^2,x)
[Out] int(1/(b*x^n+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$(n-1)\int \frac{1}{abnx^n + a^2 n}\, dx + \frac{x}{abnx^n + a^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2,x, algorithm="maxima")
[Out] (n - 1)*integrate(1/(a*b*n*x^n + a^2*n), x) + x/(a*b*n*x^n + a^2*n)
```

mupad [B] time = 1.17, size = 25, normalized size = 1.04

$$\frac{x\,{}_2\mathrm{F}_1\left(2, \frac{1}{n};\ \frac{1}{n}+1;\ -\frac{b\,x^n}{a}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^n)^2,x)
[Out] (x*hypergeom([2, 1/n], 1/n + 1, -(b*x^n)/a))/a^2
```

sympy [C] time = 1.16, size = 257, normalized size = 10.71

$$\frac{nx\Phi\left(\frac{bx^n e^{i\pi}}{a}, 1, \frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{a\left(an^3\Gamma\left(1+\frac{1}{n}\right) + bn^3x^n\Gamma\left(1+\frac{1}{n}\right)\right)} + \frac{nx\Gamma\left(\frac{1}{n}\right)}{a\left(an^3\Gamma\left(1+\frac{1}{n}\right) + bn^3x^n\Gamma\left(1+\frac{1}{n}\right)\right)} - \frac{x\Phi\left(\frac{bx^n e^{i\pi}}{a}, 1, \frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{a\left(an^3\Gamma\left(1+\frac{1}{n}\right) + bn^3x^n\Gamma\left(1+\frac{1}{n}\right)\right)} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n)**2,x)
[Out] n*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(a*(a*n**3*gamma(
1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n))) + n*x*gamma(1/n)/(a*(a*n**3*gamma(1
 + 1/n) + b*n**3*x**n*gamma(1 + 1/n))) - x*lerchphi(b*x**n*exp_polar(I*pi)/
a, 1, 1/n)*gamma(1/n)/(a*(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n
))) + b*n*x*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(a**
2*(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n))) - b*x*x**n*lerchphi
(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(a**2*(a*n**3*gamma(1 + 1/n)
+ b*n**3*x**n*gamma(1 + 1/n)))
```

3.2479 $\int \frac{1}{x(a+bx^n)^2}\,dx$

Optimal. Leaf size=39

$$-\frac{\log(a+bx^n)}{a^2 n}+\frac{\log(x)}{a^2}+\frac{1}{an(a+bx^n)}$$

```
[Out] 1/a/n/(a+b*x^n)+ln(x)/a^2-ln(a+b*x^n)/a^2/n
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 44}

$$-\frac{\log(a+bx^n)}{a^2 n}+\frac{\log(x)}{a^2}+\frac{1}{an(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^n)^2),x]
```

```
[Out] 1/(a*n*(a + b*x^n)) + Log[x]/a^2 - Log[a + b*x^n]/(a^2*n)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a+bx^n\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^2}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2 x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right)\,dx, x, x^n\right)}{n}\\ &= \frac{1}{an\left(a+bx^n\right)}+\frac{\log(x)}{a^2}-\frac{\log\left(a+bx^n\right)}{a^2 n}\end{aligned}$$

Mathematica [A] time = 0.06, size = 33, normalized size = 0.85

$$\frac{\frac{a}{a+bx^n}-\log(a+bx^n)+n\log(x)}{a^2 n}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^n)^2),x]
```

```
[Out] (a/(a + b*x^n) + n*Log[x] - Log[a + b*x^n])/(a^2*n)
```

fricas [A] time = 0.66, size = 50, normalized size = 1.28

$$\frac{bnx^n \log(x) + an \log(x) - (bx^n + a) \log(bx^n + a) + a}{a^2 bnx^n + a^3 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] (b*n*x^n*log(x) + a*n*log(x) - (b*x^n + a)*log(b*x^n + a) + a)/(a^2*b*n*x^n
 + a^3*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)^2 x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^2*x), x)
```

maple [A] time = 0.00, size = 45, normalized size = 1.15

$$\frac{1}{(b\,x^n + a)\,an} + \frac{\ln(x^n)}{a^2 n} - \frac{\ln(b\,x^n + a)}{a^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^n+a)^2,x)
```

```
[Out] -ln(b*x^n+a)/a^2/n+1/a/n/(b*x^n+a)+1/n/a^2*ln(x^n)
```

maxima [A] time = 0.58, size = 43, normalized size = 1.10

$$\frac{1}{abnx^n + a^2 n} - \frac{\log(bx^n + a)}{a^2 n} + \frac{\log(x^n)}{a^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] 1/(a*b*n*x^n + a^2*n) - log(b*x^n + a)/(a^2*n) + log(x^n)/(a^2*n)
```

mupad [B] time = 1.14, size = 39, normalized size = 1.00

$$\frac{\ln(x)}{a^2} + \frac{1}{a\,n\,(a + b\,x^n)} - \frac{\ln(a + b\,x^n)}{a^2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^n)^2),x)
```

```
[Out] log(x)/a^2 + 1/(a*n*(a + b*x^n)) - log(a + b*x^n)/(a^2*n)
```

sympy [**A**] time = 1.55, size = 160, normalized size = 4.10

$$\begin{cases} \tilde{\infty}\log(x) & \text{for } a = 0 \wedge b = 0 \wedge n = 0 \\ -\frac{x^{-2n}}{2b^2 n} & \text{for } a = 0 \\ \tilde{\infty}\log(x) & \text{for } b = -ax^{-n} \\ \frac{\log(x)}{(a+b)^2} & \text{for } n = 0 \\ \frac{\log(x)}{a^2} & \text{for } b = 0 \\ \frac{an\log(x)}{a^3 n + a^2 bnx^n} - \frac{a\log\left(\frac{a}{b}+x^n\right)}{a^3 n + a^2 bnx^n} + \frac{a}{a^3 n + a^2 bnx^n} + \frac{bnx^n\log(x)}{a^3 n + a^2 bnx^n} - \frac{bx^n\log\left(\frac{a}{b}+x^n\right)}{a^3 n + a^2 bnx^n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x**n)**2,x)
```

```
[Out] Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (-x**(-2*n)/(2*b**2
*n), Eq(a, 0)), (zoo*log(x), Eq(b, -a*x**(-n))), (log(x)/(a + b)**2, Eq(n,
0)), (log(x)/a**2, Eq(b, 0)), (a*n*log(x)/(a**3*n + a**2*b*n*x**n) - a*log(
a/b + x**n)/(a**3*n + a**2*b*n*x**n) + a/(a**3*n + a**2*b*n*x**n) + b*n*x**
n*log(x)/(a**3*n + a**2*b*n*x**n) - b*x**n*log(a/b + x**n)/(a**3*n + a**2*b
*n*x**n), True))
```

3.2480 $\int \frac{1}{x^2(a+bx^n)^2}\,dx$

Optimal. Leaf size=34

$$-\frac{{}_2F_1\left(2,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{a^2x}$$

```
[Out] -hypergeom([2, -1/n],[(-1+n)/n],-b*x^n/a)/a^2/x
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {364}

$$-\frac{{}_2F_1\left(2,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^n)^2),x]
```

```
[Out] -(Hypergeometric2F1[2, -n^(-1), -((1 - n)/n), -((b*x^n)/a)]/(a^2*x))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^n\right)^2}\,dx = -\frac{{}_2F_1\left(2,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{a^2x}$$

Mathematica [A] time = 0.00, size = 31, normalized size = 0.91

$$-\frac{{}_2F_1\left(2,-\frac{1}{n};1-\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^n)^2),x]
```

```
[Out] -(Hypergeometric2F1[2, -n^(-1), 1 - n^(-1), -((b*x^n)/a)]/(a^2*x))
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b^2x^2x^{2\,n}+2\,abx^2x^n+a^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*x^2*x^(2*n) + 2*a*b*x^2*x^n + a^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)^2 x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^2*x^2), x)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^n + a)^2\, x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^n+a)^2,x)
```

```
[Out] int(1/x^2/(b*x^n+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$(n+1)\int \frac{1}{abnx^2x^n + a^2nx^2}\, dx + \frac{1}{abnxx^n + a^2nx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] (n + 1)*integrate(1/(a*b*n*x^2*x^n + a^2*n*x^2), x) + 1/(a*b*n*x*x^n + a^2*
n*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x^2\,(a + b\,x^n)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^n)^2),x)
```

```
[Out] int(1/(x^2*(a + b*x^n)^2), x)
```

sympy [C] time = 1.50, size = 289, normalized size = 8.50

$$-\frac{n\Phi\left(\frac{bx^n e^{i\pi}}{a}, 1, \frac{e^{i\pi}}{n}\right)\Gamma\left(-\frac{1}{n}\right)}{a\left(an^3x\Gamma\left(1-\frac{1}{n}\right) + bn^3xx^n\Gamma\left(1-\frac{1}{n}\right)\right)} - \frac{n\Gamma\left(-\frac{1}{n}\right)}{a\left(an^3x\Gamma\left(1-\frac{1}{n}\right) + bn^3xx^n\Gamma\left(1-\frac{1}{n}\right)\right)} - \frac{\Phi\left(\frac{bx^n e^{i\pi}}{a}, 1, \frac{e^{i\pi}}{n}\right)\Gamma\left(-\frac{1}{n}\right)}{a\left(an^3x\Gamma\left(1-\frac{1}{n}\right) + bn^3xx^n\Gamma\left(1-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*x**n)**2,x)
```

```
[Out] -n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, exp_polar(I*pi)/n)*gamma(-1/n)/(a*
(a*n**3*x*gamma(1 - 1/n) + b*n**3*x*x**n*gamma(1 - 1/n))) - n*gamma(-1/n)/(
a*(a*n**3*x*gamma(1 - 1/n) + b*n**3*x*x**n*gamma(1 - 1/n))) - lerchphi(b*x*
*n*exp_polar(I*pi)/a, 1, exp_polar(I*pi)/n)*gamma(-1/n)/(a*(a*n**3*x*gamma(
1 - 1/n) + b*n**3*x*x**n*gamma(1 - 1/n))) - b*n*x**n*lerchphi(b*x**n*exp_po
lar(I*pi)/a, 1, exp_polar(I*pi)/n)*gamma(-1/n)/(a**2*(a*n**3*x*gamma(1 - 1/
n) + b*n**3*x*x**n*gamma(1 - 1/n))) - b*x**n*lerchphi(b*x**n*exp_polar(I*pi
)/a, 1, exp_polar(I*pi)/n)*gamma(-1/n)/(a**2*(a*n**3*x*gamma(1 - 1/n) + b*n
**3*x*x**n*gamma(1 - 1/n)))
```

3.2481 $\int \frac{1}{x^3(a+bx^n)^2}\,dx$

Optimal. Leaf size=36

$$-\frac{{}_2F_1\left(2,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2a^2x^2}$$

```
[Out] -1/2*hypergeom([2, -2/n],[(-2+n)/n],-b*x^n/a)/a^2/x^2
```

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {364}

$$-\frac{{}_2F_1\left(2,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^n)^2),x]
```

```
[Out] -Hypergeometric2F1[2, -2/n, -((2 - n)/n), -((b*x^n)/a)]/(2*a^2*x^2)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^3\,(a+bx^n)^2}\,dx = -\frac{{}_2F_1\left(2,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2a^2x^2}$$

Mathematica [A] time = 0.00, size = 33, normalized size = 0.92

$$-\frac{{}_2F_1\left(2,-\frac{2}{n};1-\frac{2}{n};-\frac{bx^n}{a}\right)}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^n)^2),x]
```

```
[Out] -1/2*Hypergeometric2F1[2, -2/n, 1 - 2/n, -((b*x^n)/a)]/(a^2*x^2)
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b^2x^3x^{2\,n}+2\,abx^3x^n+a^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*x^3*x^(2*n) + 2*a*b*x^3*x^n + a^2*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)^2 x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^2*x^3), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^n + a)^2\, x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^n+a)^2,x)
```

```
[Out] int(1/x^3/(b*x^n+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$(n+2)\int \frac{1}{abnx^3x^n + a^2nx^3}\, dx + \frac{1}{abnx^2x^n + a^2nx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] (n + 2)*integrate(1/(a*b*n*x^3*x^n + a^2*n*x^3), x) + 1/(a*b*n*x^2*x^n + a^
2*n*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x^3\,(a + b\,x^n)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^n)^2),x)
```

```
[Out] int(1/(x^3*(a + b*x^n)^2), x)
```

sympy [C] time = 1.73, size = 321, normalized size = 8.92

$$-\frac{2n\Phi\left(\frac{bx^ne^{i\pi}}{a}, 1, \frac{2e^{i\pi}}{n}\right)\Gamma\left(-\frac{2}{n}\right)}{a\left(an^3x^2\Gamma\left(1-\frac{2}{n}\right) + bn^3x^2x^n\Gamma\left(1-\frac{2}{n}\right)\right)} - \frac{2n\Gamma\left(-\frac{2}{n}\right)}{a\left(an^3x^2\Gamma\left(1-\frac{2}{n}\right) + bn^3x^2x^n\Gamma\left(1-\frac{2}{n}\right)\right)} - \frac{4\Phi\left(\frac{bx^ne^{i\pi}}{a}, 1, \frac{2e^{i\pi}}{n}\right)\Gamma\left(-\right.}{a\left(an^3x^2\Gamma\left(1-\frac{2}{n}\right) + bn^3x^2x^n\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*x**n)**2,x)
```

```
[Out] -2*n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2*exp_polar(I*pi)/n)*gamma(-2/n)
/(a*(a*n**3*x**2*gamma(1 - 2/n) + b*n**3*x**2*x**n*gamma(1 - 2/n))) - 2*n*g
amma(-2/n)/(a*(a*n**3*x**2*gamma(1 - 2/n) + b*n**3*x**2*x**n*gamma(1 - 2/n)
)) - 4*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2*exp_polar(I*pi)/n)*gamma(-2/
n)/(a*(a*n**3*x**2*gamma(1 - 2/n) + b*n**3*x**2*x**n*gamma(1 - 2/n))) - 2*b
*n*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2*exp_polar(I*pi)/n)*gamma(-2
/n)/(a**2*(a*n**3*x**2*gamma(1 - 2/n) + b*n**3*x**2*x**n*gamma(1 - 2/n))) -
 4*b*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2*exp_polar(I*pi)/n)*gamma(
-2/n)/(a**2*(a*n**3*x**2*gamma(1 - 2/n) + b*n**3*x**2*x**n*gamma(1 - 2/n)))
```

3.2482 $\int \frac{x}{(a+bx^n)^3}\,dx$

Optimal. Leaf size=33

$$\frac{x^2\,{}_2F_1\left(3,\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right)}{2a^3}$$

```
[Out] 1/2*x^2*hypergeom([3, 2/n],[(2+n)/n],-b*x^n/a)/a^3
```

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {364}

$$\frac{x^2\,{}_2F_1\left(3,\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^n)^3,x]
```

```
[Out] (x^2*Hypergeometric2F1[3, 2/n, (2 + n)/n, -((b*x^n)/a)])/(2*a^3)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x}{(a+bx^n)^3}\,dx = \frac{x^2\,{}_2F_1\left(3,\frac{2}{n};\frac{2+n}{n};-\frac{bx^n}{a}\right)}{2a^3}$$

Mathematica [A] time = 0.00, size = 33, normalized size = 1.00

$$\frac{x^2\,{}_2F_1\left(3,\frac{2}{n};1+\frac{2}{n};-\frac{bx^n}{a}\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^n)^3,x]
```

```
[Out] (x^2*Hypergeometric2F1[3, 2/n, 1 + 2/n, -((b*x^n)/a)])/(2*a^3)
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{b^3x^{3n}+3ab^2x^{2n}+3a^2bx^n+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral(x/(b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b*x^n + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(bx^n + a)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate(x/(b*x^n + a)^3, x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{x}{(b\,x^n + a)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^n+a)^3,x)
```

```
[Out] int(x/(b*x^n+a)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(n^2 - 3\,n + 2\right) \int \frac{x}{a^2 b n^2 x^n + a^3 n^2}\, dx + \frac{2\,b(n-1)x^2 x^n + a(3\,n-2)x^2}{2\left(a^2 b^2 n^2 x^{2\,n} + 2\,a^3 b n^2 x^n + a^4 n^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] (n^2 - 3*n + 2)*integrate(x/(a^2*b*n^2*x^n + a^3*n^2), x) + 1/2*(2*b*(n - 1
)*x^2*x^n + a*(3*n - 2)*x^2)/(a^2*b^2*n^2*x^(2*n) + 2*a^3*b*n^2*x^n + a^4*n
^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{(a + b\,x^n)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^n)^3,x)
```

```
[Out] int(x/(a + b*x^n)^3, x)
```

sympy [C] time = 1.77, size = 1930, normalized size = 58.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x**n)**3,x)
```

```
[Out] 2*a*n**2*x**2*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2/n)*gamma(2/n)/(a**4*n
**4*gamma(1 + 2/n) + 3*a**3*b*n**4*x**n*gamma(1 + 2/n) + 3*a**2*b**2*n**4*x
**(2*n)*gamma(1 + 2/n) + a*b**3*n**4*x**(3*n)*gamma(1 + 2/n)) + 3*a*n**2*x*
*2*gamma(2/n)/(a**4*n**4*gamma(1 + 2/n) + 3*a**3*b*n**4*x**n*gamma(1 + 2/n)
 + 3*a**2*b**2*n**4*x**(2*n)*gamma(1 + 2/n) + a*b**3*n**4*x**(3*n)*gamma(1
+ 2/n)) - 6*a*n*x**2*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2/n)*gamma(2/n)/
(a**4*n**4*gamma(1 + 2/n) + 3*a**3*b*n**4*x**n*gamma(1 + 2/n) + 3*a**2*b**2
*n**4*x**(2*n)*gamma(1 + 2/n) + a*b**3*n**4*x**(3*n)*gamma(1 + 2/n)) - 2*a*
n*x**2*gamma(2/n)/(a**4*n**4*gamma(1 + 2/n) + 3*a**3*b*n**4*x**n*gamma(1 +
```

```
2/n) + 3*a**2*b**2*n**4*x**(2*n)*gamma(1 + 2/n) + a*b**3*n**4*x**(3*n)*gamm
a(1 + 2/n)) + 4*a*x**2*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2/n)*gamma(2/n
)/(a**4*n**4*gamma(1 + 2/n) + 3*a**3*b*n**4*x**n*gamma(1 + 2/n) + 3*a**2*b*
*2*n**4*x**(2*n)*gamma(1 + 2/n) + a*b**3*n**4*x**(3*n)*gamma(1 + 2/n)) + 6*
b*n**2*x**2*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2/n)*gamma(2/n)/(a**
4*n**4*gamma(1 + 2/n) + 3*a**3*b*n**4*x**n*gamma(1 + 2/n) + 3*a**2*b**2*n**
4*x**(2*n)*gamma(1 + 2/n) + a*b**3*n**4*x**(3*n)*gamma(1 + 2/n)) + 5*b*n**2
*x**2*x**n*gamma(2/n)/(a**4*n**4*gamma(1 + 2/n) + 3*a**3*b*n**4*x**n*gamma(
1 + 2/n) + 3*a**2*b**2*n**4*x**(2*n)*gamma(1 + 2/n) + a*b**3*n**4*x**(3*n)*
gamma(1 + 2/n)) - 18*b*n*x**2*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2/
n)*gamma(2/n)/(a**4*n**4*gamma(1 + 2/n) + 3*a**3*b*n**4*x**n*gamma(1 + 2/n)
 + 3*a**2*b**2*n**4*x**(2*n)*gamma(1 + 2/n) + a*b**3*n**4*x**(3*n)*gamma(1
 + 2/n)) - 4*b*n*x**2*x**n*gamma(2/n)/(a**4*n**4*gamma(1 + 2/n) + 3*a**3*b*n
**4*x**n*gamma(1 + 2/n) + 3*a**2*b**2*n**4*x**(2*n)*gamma(1 + 2/n) + a*b**3
*n**4*x**(3*n)*gamma(1 + 2/n)) + 12*b*x**2*x**n*lerchphi(b*x**n*exp_polar(I
*pi)/a, 1, 2/n)*gamma(2/n)/(a**4*n**4*gamma(1 + 2/n) + 3*a**3*b*n**4*x**n*g
amma(1 + 2/n) + 3*a**2*b**2*n**4*x**(2*n)*gamma(1 + 2/n) + a*b**3*n**4*x**(
3*n)*gamma(1 + 2/n)) + 6*b**2*n**2*x**2*x**(2*n)*lerchphi(b*x**n*exp_polar(
I*pi)/a, 1, 2/n)*gamma(2/n)/(a*(a**4*n**4*gamma(1 + 2/n) + 3*a**3*b*n**4*x*
*n*gamma(1 + 2/n) + 3*a**2*b**2*n**4*x**(2*n)*gamma(1 + 2/n) + a*b**3*n**4*
x**(3*n)*gamma(1 + 2/n))) + 2*b**2*n**2*x**2*x**(2*n)*gamma(2/n)/(a*(a**4*n
**4*gamma(1 + 2/n) + 3*a**3*b*n**4*x**n*gamma(1 + 2/n) + 3*a**2*b**2*n**4*x
**(2*n)*gamma(1 + 2/n) + a*b**3*n**4*x**(3*n)*gamma(1 + 2/n))) - 18*b**2*n*
x**2*x**(2*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2/n)*gamma(2/n)/(a*(a**
4*n**4*gamma(1 + 2/n) + 3*a**3*b*n**4*x**n*gamma(1 + 2/n) + 3*a**2*b**2*n**
4*x**(2*n)*gamma(1 + 2/n) + a*b**3*n**4*x**(3*n)*gamma(1 + 2/n))) - 2*b**2*
n*x**2*x**(2*n)*gamma(2/n)/(a*(a**4*n**4*gamma(1 + 2/n) + 3*a**3*b*n**4*x**
n*gamma(1 + 2/n) + 3*a**2*b**2*n**4*x**(2*n)*gamma(1 + 2/n) + a*b**3*n**4*x
**(3*n)*gamma(1 + 2/n))) + 12*b**2*x**2*x**(2*n)*lerchphi(b*x**n*exp_polar(
I*pi)/a, 1, 2/n)*gamma(2/n)/(a*(a**4*n**4*gamma(1 + 2/n) + 3*a**3*b*n**4*x*
*n*gamma(1 + 2/n) + 3*a**2*b**2*n**4*x**(2*n)*gamma(1 + 2/n) + a*b**3*n**4*
x**(3*n)*gamma(1 + 2/n))) + 2*b**3*n**2*x**2*x**(3*n)*lerchphi(b*x**n*exp_p
olar(I*pi)/a, 1, 2/n)*gamma(2/n)/(a**2*(a**4*n**4*gamma(1 + 2/n) + 3*a**3*b
*n**4*x**n*gamma(1 + 2/n) + 3*a**2*b**2*n**4*x**(2*n)*gamma(1 + 2/n) + a*b*
*3*n**4*x**(3*n)*gamma(1 + 2/n))) - 6*b**3*n*x**2*x**(3*n)*lerchphi(b*x**n*
exp_polar(I*pi)/a, 1, 2/n)*gamma(2/n)/(a**2*(a**4*n**4*gamma(1 + 2/n) + 3*a
**3*b*n**4*x**n*gamma(1 + 2/n) + 3*a**2*b**2*n**4*x**(2*n)*gamma(1 + 2/n) +
 a*b**3*n**4*x**(3*n)*gamma(1 + 2/n))) + 4*b**3*x**2*x**(3*n)*lerchphi(b*x*
*n*exp_polar(I*pi)/a, 1, 2/n)*gamma(2/n)/(a**2*(a**4*n**4*gamma(1 + 2/n) +
3*a**3*b*n**4*x**n*gamma(1 + 2/n) + 3*a**2*b**2*n**4*x**(2*n)*gamma(1 + 2/n
) + a*b**3*n**4*x**(3*n)*gamma(1 + 2/n)))
```

3.2483 $\int \frac{1}{(a+bx^n)^3}\,dx$

Optimal. Leaf size=24

$$\frac{x\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^3}$$

```
[Out] x*hypergeom([3, 1/n],[1+1/n],-b*x^n/a)/a^3
```

Rubi [A] time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {245}

$$\frac{x\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^(-3),x]
```

```
[Out] (x*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/a^3
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx^n)^3}\,dx = \frac{x\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^3}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$\frac{x\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^(-3),x]
```

```
[Out] (x*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/a^3
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b^3x^{3n}+3ab^2x^{2n}+3a^2bx^n+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral(1/(b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b*x^n + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^(-3), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^n + a)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^n+a)^3,x)
```

```
[Out] int(1/(b*x^n+a)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(2\,n^2 - 3\,n + 1\right) \int \frac{1}{2\left(a^2 b n^2 x^n + a^3 n^2\right)}\, dx + \frac{b(2\,n-1)x x^n + a(3\,n-1)x}{2\left(a^2 b^2 n^2 x^{2\,n} + 2\,a^3 b n^2 x^n + a^4 n^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] (2*n^2 - 3*n + 1)*integrate(1/2/(a^2*b*n^2*x^n + a^3*n^2), x) + 1/2*(b*(2*n
 - 1)*x*x^n + a*(3*n - 1)*x)/(a^2*b^2*n^2*x^(2*n) + 2*a^3*b*n^2*x^n + a^4*n
^2)
```

mupad [B] time = 1.18, size = 25, normalized size = 1.04

$$\frac{x\,{}_2\mathrm{F}_1\left(3, \frac{1}{n}; \frac{1}{n} + 1; -\frac{b\,x^n}{a}\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^n)^3,x)
```

```
[Out] (x*hypergeom([3, 1/n], 1/n + 1, -(b*x^n)/a))/a^3
```

sympy [C] time = 1.71, size = 1953, normalized size = 81.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n)**3,x)
```

```
[Out] 2*a*n**2*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(2*a**4*n*
**4*gamma(1 + 1/n) + 6*a**3*b*n**4*x**n*gamma(1 + 1/n) + 6*a**2*b**2*n**4*x*
*(2*n)*gamma(1 + 1/n) + 2*a*b**3*n**4*x**(3*n)*gamma(1 + 1/n)) + 3*a*n**2*x
*gamma(1/n)/(2*a**4*n**4*gamma(1 + 1/n) + 6*a**3*b*n**4*x**n*gamma(1 + 1/n)
 + 6*a**2*b**2*n**4*x**(2*n)*gamma(1 + 1/n) + 2*a*b**3*n**4*x**(3*n)*gamma(
1 + 1/n)) - 3*a*n*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(
2*a**4*n**4*gamma(1 + 1/n) + 6*a**3*b*n**4*x**n*gamma(1 + 1/n) + 6*a**2*b**
2*n**4*x**(2*n)*gamma(1 + 1/n) + 2*a*b**3*n**4*x**(3*n)*gamma(1 + 1/n)) - a
```

```
*n*x*gamma(1/n)/(2*a**4*n**4*gamma(1 + 1/n) + 6*a**3*b*n**4*x**n*gamma(1 +
1/n) + 6*a**2*b**2*n**4*x**(2*n)*gamma(1 + 1/n) + 2*a*b**3*n**4*x**(3*n)*ga
mma(1 + 1/n)) + a*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(
2*a**4*n**4*gamma(1 + 1/n) + 6*a**3*b*n**4*x**n*gamma(1 + 1/n) + 6*a**2*b**
2*n**4*x**(2*n)*gamma(1 + 1/n) + 2*a*b**3*n**4*x**(3*n)*gamma(1 + 1/n)) + 6
*b*n**2*x*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(2*a**
4*n**4*gamma(1 + 1/n) + 6*a**3*b*n**4*x**n*gamma(1 + 1/n) + 6*a**2*b**2*n**
4*x**(2*n)*gamma(1 + 1/n) + 2*a*b**3*n**4*x**(3*n)*gamma(1 + 1/n)) + 5*b*n*
*2*x*x**n*gamma(1/n)/(2*a**4*n**4*gamma(1 + 1/n) + 6*a**3*b*n**4*x**n*gamma
(1 + 1/n) + 6*a**2*b**2*n**4*x**(2*n)*gamma(1 + 1/n) + 2*a*b**3*n**4*x**(3*
n)*gamma(1 + 1/n)) - 9*b*n*x*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n
)*gamma(1/n)/(2*a**4*n**4*gamma(1 + 1/n) + 6*a**3*b*n**4*x**n*gamma(1 + 1/n
) + 6*a**2*b**2*n**4*x**(2*n)*gamma(1 + 1/n) + 2*a*b**3*n**4*x**(3*n)*gamma
(1 + 1/n)) - 2*b*n*x*x**n*gamma(1/n)/(2*a**4*n**4*gamma(1 + 1/n) + 6*a**3*b
*n**4*x**n*gamma(1 + 1/n) + 6*a**2*b**2*n**4*x**(2*n)*gamma(1 + 1/n) + 2*a*
b**3*n**4*x**(3*n)*gamma(1 + 1/n)) + 3*b*x*x**n*lerchphi(b*x**n*exp_polar(I
*pi)/a, 1, 1/n)*gamma(1/n)/(2*a**4*n**4*gamma(1 + 1/n) + 6*a**3*b*n**4*x**n
*gamma(1 + 1/n) + 6*a**2*b**2*n**4*x**(2*n)*gamma(1 + 1/n) + 2*a*b**3*n**4*
x**(3*n)*gamma(1 + 1/n)) + 6*b**2*n**2*x*x**(2*n)*lerchphi(b*x**n*exp_polar
(I*pi)/a, 1, 1/n)*gamma(1/n)/(a*(2*a**4*n**4*gamma(1 + 1/n) + 6*a**3*b*n**4
*x**n*gamma(1 + 1/n) + 6*a**2*b**2*n**4*x**(2*n)*gamma(1 + 1/n) + 2*a*b**3*
n**4*x**(3*n)*gamma(1 + 1/n))) + 2*b**2*n**2*x*x**(2*n)*gamma(1/n)/(a*(2*a*
*4*n**4*gamma(1 + 1/n) + 6*a**3*b*n**4*x**n*gamma(1 + 1/n) + 6*a**2*b**2*n*
*4*x**(2*n)*gamma(1 + 1/n) + 2*a*b**3*n**4*x**(3*n)*gamma(1 + 1/n))) - 9*b*
*2*n*x*x**(2*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(a*(2
*a**4*n**4*gamma(1 + 1/n) + 6*a**3*b*n**4*x**n*gamma(1 + 1/n) + 6*a**2*b**2
*n**4*x**(2*n)*gamma(1 + 1/n) + 2*a*b**3*n**4*x**(3*n)*gamma(1 + 1/n))) - b
**2*n*x*x**(2*n)*gamma(1/n)/(a*(2*a**4*n**4*gamma(1 + 1/n) + 6*a**3*b*n**4*
x**n*gamma(1 + 1/n) + 6*a**2*b**2*n**4*x**(2*n)*gamma(1 + 1/n) + 2*a*b**3*n
**4*x**(3*n)*gamma(1 + 1/n))) + 3*b**2*x*x**(2*n)*lerchphi(b*x**n*exp_polar
(I*pi)/a, 1, 1/n)*gamma(1/n)/(a*(2*a**4*n**4*gamma(1 + 1/n) + 6*a**3*b*n**4
*x**n*gamma(1 + 1/n) + 6*a**2*b**2*n**4*x**(2*n)*gamma(1 + 1/n) + 2*a*b**3*
n**4*x**(3*n)*gamma(1 + 1/n))) + 2*b**3*n**2*x*x**(3*n)*lerchphi(b*x**n*exp
_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(a**2*(2*a**4*n**4*gamma(1 + 1/n) + 6*a*
*3*b*n**4*x**n*gamma(1 + 1/n) + 6*a**2*b**2*n**4*x**(2*n)*gamma(1 + 1/n) +
2*a*b**3*n**4*x**(3*n)*gamma(1 + 1/n))) - 3*b**3*n*x*x**(3*n)*lerchphi(b*x*
*n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(a**2*(2*a**4*n**4*gamma(1 + 1/n)
+ 6*a**3*b*n**4*x**n*gamma(1 + 1/n) + 6*a**2*b**2*n**4*x**(2*n)*gamma(1 + 1
/n) + 2*a*b**3*n**4*x**(3*n)*gamma(1 + 1/n))) + b**3*x*x**(3*n)*lerchphi(b*
x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(a**2*(2*a**4*n**4*gamma(1 + 1/n
) + 6*a**3*b*n**4*x**n*gamma(1 + 1/n) + 6*a**2*b**2*n**4*x**(2*n)*gamma(1 +
 1/n) + 2*a*b**3*n**4*x**(3*n)*gamma(1 + 1/n)))
```

3.2484 $\int \frac{1}{x(a+bx^n)^3}\,dx$

Optimal. Leaf size=58

$$-\frac{\log(a+bx^n)}{a^3 n}+\frac{\log(x)}{a^3}+\frac{1}{a^2 n\,(a+bx^n)}+\frac{1}{2an\,(a+bx^n)^2}$$

```
[Out] 1/2/a/n/(a+b*x^n)^2+1/a^2/n/(a+b*x^n)+ln(x)/a^3-ln(a+b*x^n)/a^3/n
```

Rubi [A] time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 44}

$$\frac{1}{a^2 n\,(a+bx^n)}-\frac{\log(a+bx^n)}{a^3 n}+\frac{\log(x)}{a^3}+\frac{1}{2an\,(a+bx^n)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^n)^3),x]
```

```
[Out] 1/(2*a*n*(a + b*x^n)^2) + 1/(a^2*n*(a + b*x^n)) + Log[x]/a^3 - Log[a + b*x^
n]/(a^3*n)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\,(a+bx^n)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^3}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{a^3 x}-\frac{b}{a(a+bx)^3}-\frac{b}{a^2(a+bx)^2}-\frac{b}{a^3(a+bx)}\right)\,dx, x, x^n\right)}{n}\\ &= \frac{1}{2an\,(a+bx^n)^2}+\frac{1}{a^2 n\,(a+bx^n)}+\frac{\log(x)}{a^3}-\frac{\log(a+bx^n)}{a^3 n}\end{aligned}$$

Mathematica [A] time = 0.12, size = 47, normalized size = 0.81

$$\frac{\frac{a(3a+2bx^n)}{(a+bx^n)^2}-2\log(a+bx^n)+2n\log(x)}{2a^3 n}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^n)^3),x]
```

```
[Out] ((a*(3*a + 2*b*x^n))/(a + b*x^n)^2 + 2*n*Log[x] - 2*Log[a + b*x^n])/(2*a^3*
n)
```

fricas [A] time = 0.65, size = 106, normalized size = 1.83

$$\frac{2\,b^2nx^{2\,n}\log(x)+2\,a^2n\log(x)+3\,a^2+2\left(2\,abn\log(x)+ab\right)x^n-2\left(b^2x^{2\,n}+2\,abx^n+a^2\right)\log\left(bx^n+a\right)}{2\left(a^3b^2nx^{2\,n}+2\,a^4bnx^n+a^5n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*n*x^(2*n)*log(x) + 2*a^2*n*log(x) + 3*a^2 + 2*(2*a*b*n*log(x) +
a*b)*x^n - 2*(b^2*x^(2*n) + 2*a*b*x^n + a^2)*log(b*x^n + a))/(a^3*b^2*n*x^(
2*n) + 2*a^4*b*n*x^n + a^5*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^n+a\right)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^3*x), x)
```

maple [A] time = 0.00, size = 62, normalized size = 1.07

$$\frac{1}{2\left(b\,x^n+a\right)^2an}+\frac{1}{\left(b\,x^n+a\right)a^2n}+\frac{\ln\left(x^n\right)}{a^3n}-\frac{\ln\left(b\,x^n+a\right)}{a^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^n+a)^3,x)
```

```
[Out] -ln(b*x^n+a)/a^3/n+1/a^2/n/(b*x^n+a)+1/2/a/n/(b*x^n+a)^2+1/n/a^3*ln(x^n)
```

maxima [A] time = 0.51, size = 71, normalized size = 1.22

$$\frac{2\,bx^n+3\,a}{2\left(a^2b^2nx^{2\,n}+2\,a^3bnx^n+a^4n\right)}-\frac{\log\left(bx^n+a\right)}{a^3n}+\frac{\log\left(x^n\right)}{a^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x^n + 3*a)/(a^2*b^2*n*x^(2*n) + 2*a^3*b*n*x^n + a^4*n) - log(b*x^n
 + a)/(a^3*n) + log(x^n)/(a^3*n)
```

mupad [B] time = 1.15, size = 69, normalized size = 1.19

$$\frac{\ln(x)}{a^3}+\frac{1}{a^2\,n\,\left(a+b\,x^n\right)}-\frac{\ln\left(a+b\,x^n\right)}{a^3\,n}+\frac{1}{2\,a\,n\,\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^n)^3),x)
```

```
[Out] log(x)/a^3 + 1/(a^2*n*(a + b*x^n)) - log(a + b*x^n)/(a^3*n) + 1/(2*a*n*(a^2
 + b^2*x^(2*n) + 2*a*b*x^n))
```

sympy [**A**] time = 2.80, size = 406, normalized size = 7.00

$$\begin{cases} \tilde{\infty}\log(x) \\ -\frac{x^{-3n}}{3b^3n} \\ \frac{\log(x)}{(a+b)^3} \\ \frac{\log(x)}{a^3} \\ \frac{2a^2n\log(x)}{2a^5n+4a^4bnx^n+2a^3b^2nx^{2n}} - \frac{2a^2\log\left(\frac{a}{b}+x^n\right)}{2a^5n+4a^4bnx^n+2a^3b^2nx^{2n}} + \frac{3a^2}{2a^5n+4a^4bnx^n+2a^3b^2nx^{2n}} + \frac{4abnx^n\log(x)}{2a^5n+4a^4bnx^n+2a^3b^2nx^{2n}} - \frac{4abx^n\log\left(\frac{a}{b}+x\right.}{2a^5n+4a^4bnx^n+2a^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x**n)**3,x)
```

```
[Out] Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (-x**(-3*n)/(3*b**3
*n), Eq(a, 0)), (log(x)/(a + b)**3, Eq(n, 0)), (log(x)/a**3, Eq(b, 0)), (2*
a**2*n*log(x)/(2*a**5*n + 4*a**4*b*n*x**n + 2*a**3*b**2*n*x**(2*n)) - 2*a**
2*log(a/b + x**n)/(2*a**5*n + 4*a**4*b*n*x**n + 2*a**3*b**2*n*x**(2*n)) + 3
*a**2/(2*a**5*n + 4*a**4*b*n*x**n + 2*a**3*b**2*n*x**(2*n)) + 4*a*b*n*x**n*
log(x)/(2*a**5*n + 4*a**4*b*n*x**n + 2*a**3*b**2*n*x**(2*n)) - 4*a*b*x**n*l
og(a/b + x**n)/(2*a**5*n + 4*a**4*b*n*x**n + 2*a**3*b**2*n*x**(2*n)) + 2*a*
b*x**n/(2*a**5*n + 4*a**4*b*n*x**n + 2*a**3*b**2*n*x**(2*n)) + 2*b**2*n*x**
(2*n)*log(x)/(2*a**5*n + 4*a**4*b*n*x**n + 2*a**3*b**2*n*x**(2*n)) - 2*b**2
*x**(2*n)*log(a/b + x**n)/(2*a**5*n + 4*a**4*b*n*x**n + 2*a**3*b**2*n*x**(2
*n)), True))
```

3.2485 $\int \frac{1}{x^2(a+bx^n)^3}\,dx$

Optimal. Leaf size=34

$$-\frac{{}_2F_1\left(3,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{a^3x}$$

```
[Out] -hypergeom([3, -1/n],[(-1+n)/n],-b*x^n/a)/a^3/x
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {364}

$$-\frac{{}_2F_1\left(3,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{a^3x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^n)^3),x]
```

```
[Out] -(Hypergeometric2F1[3, -n^(-1), -((1 - n)/n), -((b*x^n)/a)]/(a^3*x))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^2\,(a+bx^n)^3}\,dx = -\frac{{}_2F_1\left(3,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{a^3x}$$

Mathematica [A] time = 0.00, size = 31, normalized size = 0.91

$$-\frac{{}_2F_1\left(3,-\frac{1}{n};1-\frac{1}{n};-\frac{bx^n}{a}\right)}{a^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^n)^3),x]
```

```
[Out] -(Hypergeometric2F1[3, -n^(-1), 1 - n^(-1), -((b*x^n)/a)]/(a^3*x))
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b^3x^2x^{3\,n}+3\,ab^2x^2x^{2\,n}+3\,a^2bx^2x^n+a^3x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral(1/(b^3*x^2*x^(3*n) + 3*a*b^2*x^2*x^(2*n) + 3*a^2*b*x^2*x^n + a^3*x
^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)^3 x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^3*x^2), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^n + a)^3\, x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^n+a)^3,x)
```

```
[Out] int(1/x^2/(b*x^n+a)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(2\,n^2 + 3\,n + 1\right) \int \frac{1}{2\left(a^2 b n^2 x^2 x^n + a^3 n^2 x^2\right)}\, dx + \frac{b(2\,n+1)x^n + a(3\,n+1)}{2\left(a^2 b^2 n^2 x x^{2\,n} + 2\,a^3 b n^2 x x^n + a^4 n^2 x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] (2*n^2 + 3*n + 1)*integrate(1/2/(a^2*b*n^2*x^2*x^n + a^3*n^2*x^2), x) + 1/2
*(b*(2*n + 1)*x^n + a*(3*n + 1))/(a^2*b^2*n^2*x*x^(2*n) + 2*a^3*b*n^2*x*x^n
 + a^4*n^2*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x^2\,(a + b\,x^n)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^n)^3),x)
```

```
[Out] int(1/(x^2*(a + b*x^n)^3), x)
```

sympy [C] time = 2.38, size = 2118, normalized size = 62.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*x**n)**3,x)
```

```
[Out] -2*a*n**2*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, exp_polar(I*pi)/n)*gamma(-1
/n)/(2*a**4*n**4*x*gamma(1 - 1/n) + 6*a**3*b*n**4*x*x**n*gamma(1 - 1/n) + 6
*a**2*b**2*n**4*x*x**(2*n)*gamma(1 - 1/n) + 2*a*b**3*n**4*x*x**(3*n)*gamma(
1 - 1/n)) - 3*a*n**2*gamma(-1/n)/(2*a**4*n**4*x*gamma(1 - 1/n) + 6*a**3*b*n
**4*x*x**n*gamma(1 - 1/n) + 6*a**2*b**2*n**4*x*x**(2*n)*gamma(1 - 1/n) + 2*
a*b**3*n**4*x*x**(3*n)*gamma(1 - 1/n)) - 3*a*n*lerchphi(b*x**n*exp_polar(I*
pi)/a, 1, exp_polar(I*pi)/n)*gamma(-1/n)/(2*a**4*n**4*x*gamma(1 - 1/n) + 6*
a**3*b*n**4*x*x**n*gamma(1 - 1/n) + 6*a**2*b**2*n**4*x*x**(2*n)*gamma(1 - 1
/n) + 2*a*b**3*n**4*x*x**(3*n)*gamma(1 - 1/n)) - a*n*gamma(-1/n)/(2*a**4*n*
```

```
*4*x*gamma(1 - 1/n) + 6*a**3*b*n**4*x*x**n*gamma(1 - 1/n) + 6*a**2*b**2*n**
4*x*x**(2*n)*gamma(1 - 1/n) + 2*a*b**3*n**4*x*x**(3*n)*gamma(1 - 1/n)) - a*
lerchphi(b*x**n*exp_polar(I*pi)/a, 1, exp_polar(I*pi)/n)*gamma(-1/n)/(2*a**
4*n**4*x*gamma(1 - 1/n) + 6*a**3*b*n**4*x*x**n*gamma(1 - 1/n) + 6*a**2*b**2
*n**4*x*x**(2*n)*gamma(1 - 1/n) + 2*a*b**3*n**4*x*x**(3*n)*gamma(1 - 1/n))
- 6*b*n**2*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, exp_polar(I*pi)/n)*ga
mma(-1/n)/(2*a**4*n**4*x*gamma(1 - 1/n) + 6*a**3*b*n**4*x*x**n*gamma(1 - 1/
n) + 6*a**2*b**2*n**4*x*x**(2*n)*gamma(1 - 1/n) + 2*a*b**3*n**4*x*x**(3*n)*
gamma(1 - 1/n)) - 5*b*n**2*x**n*gamma(-1/n)/(2*a**4*n**4*x*gamma(1 - 1/n) +
 6*a**3*b*n**4*x*x**n*gamma(1 - 1/n) + 6*a**2*b**2*n**4*x*x**(2*n)*gamma(1
 - 1/n) + 2*a*b**3*n**4*x*x**(3*n)*gamma(1 - 1/n)) - 9*b*n*x**n*lerchphi(b*x
**n*exp_polar(I*pi)/a, 1, exp_polar(I*pi)/n)*gamma(-1/n)/(2*a**4*n**4*x*gam
ma(1 - 1/n) + 6*a**3*b*n**4*x*x**n*gamma(1 - 1/n) + 6*a**2*b**2*n**4*x*x**(
2*n)*gamma(1 - 1/n) + 2*a*b**3*n**4*x*x**(3*n)*gamma(1 - 1/n)) - 2*b*n*x**n
*gamma(-1/n)/(2*a**4*n**4*x*gamma(1 - 1/n) + 6*a**3*b*n**4*x*x**n*gamma(1 -
 1/n) + 6*a**2*b**2*n**4*x*x**(2*n)*gamma(1 - 1/n) + 2*a*b**3*n**4*x*x**(3*
n)*gamma(1 - 1/n)) - 3*b*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, exp_pol
ar(I*pi)/n)*gamma(-1/n)/(2*a**4*n**4*x*gamma(1 - 1/n) + 6*a**3*b*n**4*x*x**
n*gamma(1 - 1/n) + 6*a**2*b**2*n**4*x*x**(2*n)*gamma(1 - 1/n) + 2*a*b**3*n*
*4*x*x**(3*n)*gamma(1 - 1/n)) - 6*b**2*n**2*x**(2*n)*lerchphi(b*x**n*exp_po
lar(I*pi)/a, 1, exp_polar(I*pi)/n)*gamma(-1/n)/(a*(2*a**4*n**4*x*gamma(1 -
1/n) + 6*a**3*b*n**4*x*x**n*gamma(1 - 1/n) + 6*a**2*b**2*n**4*x*x**(2*n)*ga
mma(1 - 1/n) + 2*a*b**3*n**4*x*x**(3*n)*gamma(1 - 1/n))) - 2*b**2*n**2*x**(
2*n)*gamma(-1/n)/(a*(2*a**4*n**4*x*gamma(1 - 1/n) + 6*a**3*b*n**4*x*x**n*ga
mma(1 - 1/n) + 6*a**2*b**2*n**4*x*x**(2*n)*gamma(1 - 1/n) + 2*a*b**3*n**4*x
*x**(3*n)*gamma(1 - 1/n))) - 9*b**2*n*x**(2*n)*lerchphi(b*x**n*exp_polar(I*
pi)/a, 1, exp_polar(I*pi)/n)*gamma(-1/n)/(a*(2*a**4*n**4*x*gamma(1 - 1/n) +
 6*a**3*b*n**4*x*x**n*gamma(1 - 1/n) + 6*a**2*b**2*n**4*x*x**(2*n)*gamma(1
 - 1/n) + 2*a*b**3*n**4*x*x**(3*n)*gamma(1 - 1/n))) - b**2*n*x**(2*n)*gamma(
-1/n)/(a*(2*a**4*n**4*x*gamma(1 - 1/n) + 6*a**3*b*n**4*x*x**n*gamma(1 - 1/n
) + 6*a**2*b**2*n**4*x*x**(2*n)*gamma(1 - 1/n) + 2*a*b**3*n**4*x*x**(3*n)*g
amma(1 - 1/n))) - 3*b**2*x**(2*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, exp
_polar(I*pi)/n)*gamma(-1/n)/(a*(2*a**4*n**4*x*gamma(1 - 1/n) + 6*a**3*b*n**
4*x*x**n*gamma(1 - 1/n) + 6*a**2*b**2*n**4*x*x**(2*n)*gamma(1 - 1/n) + 2*a*
b**3*n**4*x*x**(3*n)*gamma(1 - 1/n))) - 2*b**3*n**2*x**(3*n)*lerchphi(b*x**
n*exp_polar(I*pi)/a, 1, exp_polar(I*pi)/n)*gamma(-1/n)/(a**2*(2*a**4*n**4*x
*gamma(1 - 1/n) + 6*a**3*b*n**4*x*x**n*gamma(1 - 1/n) + 6*a**2*b**2*n**4*x*
x**(2*n)*gamma(1 - 1/n) + 2*a*b**3*n**4*x*x**(3*n)*gamma(1 - 1/n))) - 3*b**
3*n*x**(3*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, exp_polar(I*pi)/n)*gamma
(-1/n)/(a**2*(2*a**4*n**4*x*gamma(1 - 1/n) + 6*a**3*b*n**4*x*x**n*gamma(1 -
 1/n) + 6*a**2*b**2*n**4*x*x**(2*n)*gamma(1 - 1/n) + 2*a*b**3*n**4*x*x**(3*
n)*gamma(1 - 1/n))) - b**3*x**(3*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, e
xp_polar(I*pi)/n)*gamma(-1/n)/(a**2*(2*a**4*n**4*x*gamma(1 - 1/n) + 6*a**3*
b*n**4*x*x**n*gamma(1 - 1/n) + 6*a**2*b**2*n**4*x*x**(2*n)*gamma(1 - 1/n) +
 2*a*b**3*n**4*x*x**(3*n)*gamma(1 - 1/n)))
```

3.2486 $\int \frac{1}{x^3(a+bx^n)^3}\,dx$

Optimal. Leaf size=36

$$-\frac{{}_2F_1\left(3,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2a^3x^2}$$

```
[Out] -1/2*hypergeom([3, -2/n],[(-2+n)/n],-b*x^n/a)/a^3/x^2
```

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {364}

$$-\frac{{}_2F_1\left(3,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2a^3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^n)^3),x]
```

```
[Out] -Hypergeometric2F1[3, -2/n, -((2 - n)/n), -((b*x^n)/a)]/(2*a^3*x^2)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^3\,(a+bx^n)^3}\,dx = -\frac{{}_2F_1\left(3,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2a^3x^2}$$

Mathematica [A] time = 0.00, size = 33, normalized size = 0.92

$$-\frac{{}_2F_1\left(3,-\frac{2}{n};1-\frac{2}{n};-\frac{bx^n}{a}\right)}{2a^3x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^n)^3),x]
```

```
[Out] -1/2*Hypergeometric2F1[3, -2/n, 1 - 2/n, -((b*x^n)/a)]/(a^3*x^2)
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b^3x^3x^{3n}+3\,ab^2x^3x^{2n}+3\,a^2bx^3x^n+a^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral(1/(b^3*x^3*x^(3*n) + 3*a*b^2*x^3*x^(2*n) + 3*a^2*b*x^3*x^n + a^3*x
^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)^3 x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^3*x^3), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^n + a)^3\, x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^n+a)^3,x)
```

```
[Out] int(1/x^3/(b*x^n+a)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(n^2 + 3\,n + 2\right) \int \frac{1}{a^2 b n^2 x^3 x^n + a^3 n^2 x^3}\, dx + \frac{2\,b(n+1)x^n + a(3\,n+2)}{2\left(a^2 b^2 n^2 x^2 x^{2\,n} + 2\,a^3 b n^2 x^2 x^n + a^4 n^2 x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] (n^2 + 3*n + 2)*integrate(1/(a^2*b*n^2*x^3*x^n + a^3*n^2*x^3), x) + 1/2*(2*
b*(n + 1)*x^n + a*(3*n + 2))/(a^2*b^2*n^2*x^2*x^(2*n) + 2*a^3*b*n^2*x^2*x^n
 + a^4*n^2*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x^3\,(a + b\,x^n)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^n)^3),x)
```

```
[Out] int(1/(x^3*(a + b*x^n)^3), x)
```

sympy [C] time = 2.80, size = 2207, normalized size = 61.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*x**n)**3,x)
```

```
[Out] -2*a*n**2*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2*exp_polar(I*pi)/n)*gamma(
-2/n)/(a**4*n**4*x**2*gamma(1 - 2/n) + 3*a**3*b*n**4*x**2*x**n*gamma(1 - 2/
n) + 3*a**2*b**2*n**4*x**2*x**(2*n)*gamma(1 - 2/n) + a*b**3*n**4*x**2*x**(3
*n)*gamma(1 - 2/n)) - 3*a*n**2*gamma(-2/n)/(a**4*n**4*x**2*gamma(1 - 2/n) +
 3*a**3*b*n**4*x**2*x**n*gamma(1 - 2/n) + 3*a**2*b**2*n**4*x**2*x**(2*n)*ga
mma(1 - 2/n) + a*b**3*n**4*x**2*x**(3*n)*gamma(1 - 2/n)) - 6*a*n*lerchphi(b
*x**n*exp_polar(I*pi)/a, 1, 2*exp_polar(I*pi)/n)*gamma(-2/n)/(a**4*n**4*x**
2*gamma(1 - 2/n) + 3*a**3*b*n**4*x**2*x**n*gamma(1 - 2/n) + 3*a**2*b**2*n**
4*x**2*x**(2*n)*gamma(1 - 2/n) + a*b**3*n**4*x**2*x**(3*n)*gamma(1 - 2/n))
```

```
- 2*a*n*gamma(-2/n)/(a**4*n**4*x**2*gamma(1 - 2/n) + 3*a**3*b*n**4*x**2*x**
n*gamma(1 - 2/n) + 3*a**2*b**2*n**4*x**2*x**(2*n)*gamma(1 - 2/n) + a*b**3*n
**4*x**2*x**(3*n)*gamma(1 - 2/n)) - 4*a*lerchphi(b*x**n*exp_polar(I*pi)/a,
1, 2*exp_polar(I*pi)/n)*gamma(-2/n)/(a**4*n**4*x**2*gamma(1 - 2/n) + 3*a**3
*b*n**4*x**2*x**n*gamma(1 - 2/n) + 3*a**2*b**2*n**4*x**2*x**(2*n)*gamma(1 -
 2/n) + a*b**3*n**4*x**2*x**(3*n)*gamma(1 - 2/n)) - 6*b*n**2*x**n*lerchphi(
b*x**n*exp_polar(I*pi)/a, 1, 2*exp_polar(I*pi)/n)*gamma(-2/n)/(a**4*n**4*x*
*2*gamma(1 - 2/n) + 3*a**3*b*n**4*x**2*x**n*gamma(1 - 2/n) + 3*a**2*b**2*n*
*4*x**2*x**(2*n)*gamma(1 - 2/n) + a*b**3*n**4*x**2*x**(3*n)*gamma(1 - 2/n))
 - 5*b*n**2*x**n*gamma(-2/n)/(a**4*n**4*x**2*gamma(1 - 2/n) + 3*a**3*b*n**4
*x**2*x**n*gamma(1 - 2/n) + 3*a**2*b**2*n**4*x**2*x**(2*n)*gamma(1 - 2/n) +
 a*b**3*n**4*x**2*x**(3*n)*gamma(1 - 2/n)) - 18*b*n*x**n*lerchphi(b*x**n*ex
p_polar(I*pi)/a, 1, 2*exp_polar(I*pi)/n)*gamma(-2/n)/(a**4*n**4*x**2*gamma(
1 - 2/n) + 3*a**3*b*n**4*x**2*x**n*gamma(1 - 2/n) + 3*a**2*b**2*n**4*x**2*x
**(2*n)*gamma(1 - 2/n) + a*b**3*n**4*x**2*x**(3*n)*gamma(1 - 2/n)) - 4*b*n*
x**n*gamma(-2/n)/(a**4*n**4*x**2*gamma(1 - 2/n) + 3*a**3*b*n**4*x**2*x**n*g
amma(1 - 2/n) + 3*a**2*b**2*n**4*x**2*x**(2*n)*gamma(1 - 2/n) + a*b**3*n**4
*x**2*x**(3*n)*gamma(1 - 2/n)) - 12*b*x**n*lerchphi(b*x**n*exp_polar(I*pi)/
a, 1, 2*exp_polar(I*pi)/n)*gamma(-2/n)/(a**4*n**4*x**2*gamma(1 - 2/n) + 3*a
**3*b*n**4*x**2*x**n*gamma(1 - 2/n) + 3*a**2*b**2*n**4*x**2*x**(2*n)*gamma(
1 - 2/n) + a*b**3*n**4*x**2*x**(3*n)*gamma(1 - 2/n)) - 6*b**2*n**2*x**(2*n)
*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2*exp_polar(I*pi)/n)*gamma(-2/n)/(a*
(a**4*n**4*x**2*gamma(1 - 2/n) + 3*a**3*b*n**4*x**2*x**n*gamma(1 - 2/n) + 3
*a**2*b**2*n**4*x**2*x**(2*n)*gamma(1 - 2/n) + a*b**3*n**4*x**2*x**(3*n)*ga
mma(1 - 2/n))) - 2*b**2*n**2*x**(2*n)*gamma(-2/n)/(a*(a**4*n**4*x**2*gamma(
1 - 2/n) + 3*a**3*b*n**4*x**2*x**n*gamma(1 - 2/n) + 3*a**2*b**2*n**4*x**2*x
**(2*n)*gamma(1 - 2/n) + a*b**3*n**4*x**2*x**(3*n)*gamma(1 - 2/n))) - 18*b*
*2*n*x**(2*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2*exp_polar(I*pi)/n)*ga
mma(-2/n)/(a*(a**4*n**4*x**2*gamma(1 - 2/n) + 3*a**3*b*n**4*x**2*x**n*gamma
(1 - 2/n) + 3*a**2*b**2*n**4*x**2*x**(2*n)*gamma(1 - 2/n) + a*b**3*n**4*x**
2*x**(3*n)*gamma(1 - 2/n))) - 2*b**2*n*x**(2*n)*gamma(-2/n)/(a*(a**4*n**4*x
**2*gamma(1 - 2/n) + 3*a**3*b*n**4*x**2*x**n*gamma(1 - 2/n) + 3*a**2*b**2*n
**4*x**2*x**(2*n)*gamma(1 - 2/n) + a*b**3*n**4*x**2*x**(3*n)*gamma(1 - 2/n)
)) - 12*b**2*x**(2*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2*exp_polar(I*p
i)/n)*gamma(-2/n)/(a*(a**4*n**4*x**2*gamma(1 - 2/n) + 3*a**3*b*n**4*x**2*x*
*n*gamma(1 - 2/n) + 3*a**2*b**2*n**4*x**2*x**(2*n)*gamma(1 - 2/n) + a*b**3*
n**4*x**2*x**(3*n)*gamma(1 - 2/n))) - 2*b**3*n**2*x**(3*n)*lerchphi(b*x**n*
exp_polar(I*pi)/a, 1, 2*exp_polar(I*pi)/n)*gamma(-2/n)/(a**2*(a**4*n**4*x**
2*gamma(1 - 2/n) + 3*a**3*b*n**4*x**2*x**n*gamma(1 - 2/n) + 3*a**2*b**2*n**
4*x**2*x**(2*n)*gamma(1 - 2/n) + a*b**3*n**4*x**2*x**(3*n)*gamma(1 - 2/n)))
 - 6*b**3*n*x**(3*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2*exp_polar(I*pi
)/n)*gamma(-2/n)/(a**2*(a**4*n**4*x**2*gamma(1 - 2/n) + 3*a**3*b*n**4*x**2*
x**n*gamma(1 - 2/n) + 3*a**2*b**2*n**4*x**2*x**(2*n)*gamma(1 - 2/n) + a*b**
3*n**4*x**2*x**(3*n)*gamma(1 - 2/n))) - 4*b**3*x**(3*n)*lerchphi(b*x**n*exp
_polar(I*pi)/a, 1, 2*exp_polar(I*pi)/n)*gamma(-2/n)/(a**2*(a**4*n**4*x**2*g
amma(1 - 2/n) + 3*a**3*b*n**4*x**2*x**n*gamma(1 - 2/n) + 3*a**2*b**2*n**4*x
**2*x**(2*n)*gamma(1 - 2/n) + a*b**3*n**4*x**2*x**(3*n)*gamma(1 - 2/n)))
```

3.2487 $\int x\sqrt{a+bx^n}\,dx$

Optimal. Leaf size=48

$$\frac{x^2\left(a+bx^n\right)^{3/2}\,{}_2F_1\left(1,\frac{3}{2}+\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right)}{2a}$$

```
[Out] 1/2*x^2*(a+b*x^n)^(3/2)*hypergeom([1, 3/2+2/n],[(2+n)/n],-b*x^n/a)/a
```

Rubi [A] time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.19, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {365, 364}

$$\frac{x^2\sqrt{a+bx^n}\,{}_2F_1\left(-\frac{1}{2},\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right)}{2\sqrt{\frac{bx^n}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*x^n],x]
```

```
[Out] (x^2*Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, 2/n, (2 + n)/n, -((b*x^n)/a)])
/(2*Sqrt[1 + (b*x^n)/a])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt{a+bx^n}\,dx &= \frac{\sqrt{a+bx^n}\int x\sqrt{1+\frac{bx^n}{a}}\,dx}{\sqrt{1+\frac{bx^n}{a}}}\\ &= \frac{x^2\sqrt{a+bx^n}\,{}_2F_1\left(-\frac{1}{2},\frac{2}{n};\frac{2+n}{n};-\frac{bx^n}{a}\right)}{2\sqrt{1+\frac{bx^n}{a}}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 57, normalized size = 1.19

$$\frac{x^2\sqrt{a+bx^n}\,{}_2F_1\left(-\frac{1}{2},\frac{2}{n};1+\frac{2}{n};-\frac{bx^n}{a}\right)}{2\sqrt{\frac{bx^n}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + b*x^n],x]
```

```
[Out] (x^2*Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, 2/n, 1 + 2/n, -((b*x^n)/a)])/(
2*Sqrt[1 + (b*x^n)/a])
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^n + a}\, x\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n + a)*x, x)
```

maple [**F**] time = 0.21, size = 0, normalized size = 0.00

$$\int \sqrt{b\, x^n + a}\ x\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^n+a)^(1/2),x)
```

```
[Out] int(x*(b*x^n+a)^(1/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^n + a}\, x\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^n + a)*x, x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int x\, \sqrt{a + b\, x^n}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^n)^(1/2),x)
```

```
[Out] int(x*(a + b*x^n)^(1/2), x)
```

sympy [**C**] time = 1.26, size = 42, normalized size = 0.88

$$\frac{\sqrt{a}\, x^2 \Gamma\left(\frac{2}{n}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{2}{n} \\ 1 + \frac{2}{n} \end{matrix} \middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(1 + \frac{2}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x**n)**(1/2),x)
```

```
[Out] sqrt(a)*x**2*gamma(2/n)*hyper((-1/2, 2/n), (1 + 2/n,), b*x**n*exp_polar(I*p
i)/a)/(n*gamma(1 + 2/n))
```

3.2488 $\int \sqrt{a+bx^n}\,dx$

Optimal. Leaf size=39

$$\frac{x\,(a+bx^n)^{3/2}\,{}_2F_1\left(1,\frac{3}{2}+\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a}$$

```
[Out] x*(a+b*x^n)^(3/2)*hypergeom([1, 3/2+1/n],[1+1/n],-b*x^n/a)/a
```

Rubi [A] time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.23, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {246, 245}

$$\frac{x\sqrt{a+bx^n}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{\sqrt{\frac{bx^n}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^n],x]
```

```
[Out] (x*Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, n^(-1), 1 + n^(-1), -((b*x^n)/a)
])/Sqrt[1 + (b*x^n)/a]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+bx^n}\,dx &= \frac{\sqrt{a+bx^n}\int \sqrt{1+\frac{bx^n}{a}}\,dx}{\sqrt{1+\frac{bx^n}{a}}}\\ &= \frac{x\sqrt{a+bx^n}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{\sqrt{1+\frac{bx^n}{a}}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 48, normalized size = 1.23

$$\frac{x\sqrt{a+bx^n}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{\sqrt{\frac{bx^n}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^n],x]
```

```
[Out] (x*Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, n^(-1), 1 + n^(-1), -((b*x^n)/a)
])/Sqrt[1 + (b*x^n)/a]
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n + a), x)
```

maple [**F**] time = 0.21, size = 0, normalized size = 0.00

$$\int \sqrt{b\,x^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^(1/2),x)
```

```
[Out] int((b*x^n+a)^(1/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^n + a), x)
```

mupad [**B**] time = 1.46, size = 43, normalized size = 1.10

$$\frac{x\sqrt{a + b\,x^n}\;{}_2\mathrm{F}_1\left(-\frac{1}{2}, \frac{1}{n}; \frac{1}{n} + 1; -\frac{b\,x^n}{a}\right)}{\sqrt{\frac{b\,x^n}{a} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^(1/2),x)
```

```
[Out] (x*(a + b*x^n)^(1/2)*hypergeom([-1/2, 1/n], 1/n + 1, -(b*x^n)/a))/((b*x^n)/
a + 1)^(1/2)
```

sympy [C] time = 1.15, size = 41, normalized size = 1.05

$$\frac{\sqrt{a}\, x\Gamma\left(\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{n} \\ 1 + \frac{1}{n} \end{matrix} \middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(1 + \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**(1/2),x)
```

```
[Out] sqrt(a)*x*gamma(1/n)*hyper((-1/2, 1/n), (1 + 1/n,), b*x**n*exp_polar(I*pi)/
a)/(n*gamma(1 + 1/n))
```

3.2489 $\int \frac{\sqrt{a+bx^n}}{x}\,dx$

Optimal. Leaf size=45

$$\frac{2\sqrt{a+bx^n}}{n}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{n}$$

```
[Out] -2*arctanh((a+b*x^n)^(1/2)/a^(1/2))*a^(1/2)/n+2*(a+b*x^n)^(1/2)/n
```

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 50, 63, 208}

$$\frac{2\sqrt{a+bx^n}}{n}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^n]/x,x]
```

```
[Out] (2*Sqrt[a + b*x^n])/n - (2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/n
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx^n}}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^n\right)}{n} \\ &= \frac{2\sqrt{a+bx^n}}{n} + \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^n\right)}{n} \\ &= \frac{2\sqrt{a+bx^n}}{n} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^n}\right)}{bn} \\ &= \frac{2\sqrt{a+bx^n}}{n} - \frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{n}\end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 0.96

$$\frac{2\sqrt{a+bx^n} - 2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^n]/x,x]
```

```
[Out] (2*Sqrt[a + b*x^n] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/n
```

fricas [A] time = 0.58, size = 91, normalized size = 2.02

$$\left[\frac{\sqrt{a}\,\log\left(\frac{bx^n - 2\sqrt{bx^n+a}\sqrt{a}+2a}{x^n}\right) + 2\sqrt{bx^n+a}}{n}, \frac{2\left(\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx^n+a}\sqrt{-a}}{a}\right) + \sqrt{bx^n+a}\right)}{n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [(sqrt(a)*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n) + 2*sqrt(b*x^n
 + a))/n, 2*(sqrt(-a)*arctan(sqrt(b*x^n + a)*sqrt(-a)/a) + sqrt(b*x^n + a))
/n]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^n+a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n + a)/x, x)
```

maple [A] time = 0.00, size = 36, normalized size = 0.80

$$\frac{-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^n+a}}{\sqrt{a}}\right) + 2\sqrt{b\,x^n+a}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^(1/2)/x,x)
```

```
[Out] 1/n*(2*(b*x^n+a)^(1/2)-2*a^(1/2)*arctanh((b*x^n+a)^(1/2)/a^(1/2)))
```

maxima [A] time = 1.19, size = 54, normalized size = 1.20

$$\frac{\sqrt{a}\,\log\left(\frac{\sqrt{bx^n+a}-\sqrt{a}}{\sqrt{bx^n+a}+\sqrt{a}}\right)}{n}+\frac{2\,\sqrt{bx^n+a}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(a)*log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))/n + 2*
sqrt(b*x^n + a)/n
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+b\,x^n}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^(1/2)/x,x)
```

```
[Out] int((a + b*x^n)^(1/2)/x, x)
```

sympy [B] time = 1.71, size = 76, normalized size = 1.69

$$-\frac{2\sqrt{a}\,\operatorname{asinh}\left(\frac{\sqrt{a}\,x^{-\frac{n}{2}}}{\sqrt{b}}\right)}{n}+\frac{2ax^{-\frac{n}{2}}}{\sqrt{b}\,n\sqrt{\frac{ax^{-n}}{b}+1}}+\frac{2\sqrt{b}\,x^{\frac{n}{2}}}{n\sqrt{\frac{ax^{-n}}{b}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**(1/2)/x,x)
```

```
[Out] -2*sqrt(a)*asinh(sqrt(a)*x**(-n/2)/sqrt(b))/n + 2*a*x**(-n/2)/(sqrt(b)*n*sq
rt(a*x**(-n)/b + 1)) + 2*sqrt(b)*x**(n/2)/(n*sqrt(a*x**(-n)/b + 1))
```

3.2490 $\int \frac{\sqrt{a+bx^n}}{x^2}\,dx$

Optimal. Leaf size=49

$$-\frac{(a+bx^n)^{3/2}\,{}_2F_1\left(1,\frac{3}{2}-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{ax}$$

```
[Out] -(a+b*x^n)^(3/2)*hypergeom([1, 3/2-1/n],[(-1+n)/n],-b*x^n/a)/a/x
```

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {365, 364}

$$-\frac{\sqrt{a+bx^n}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{x\sqrt{\frac{bx^n}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^n]/x^2,x]
```

```
[Out] -((Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, -n^(-1), -((1 - n)/n), -((b*x^n)
/a)])/(x*Sqrt[1 + (b*x^n)/a]))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx^n}}{x^2}\,dx &= \frac{\sqrt{a+bx^n}\int \frac{\sqrt{1+\frac{bx^n}{a}}}{x^2}\,dx}{\sqrt{1+\frac{bx^n}{a}}}\\ &= -\frac{\sqrt{a+bx^n}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{x\sqrt{1+\frac{bx^n}{a}}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 55, normalized size = 1.12

$$-\frac{\sqrt{a+bx^n}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{n};1-\frac{1}{n};-\frac{bx^n}{a}\right)}{x\sqrt{\frac{bx^n}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^n]/x^2,x]
```

```
[Out] -((Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, -n^(-1), 1 - n^(-1), -((b*x^n)/a
)])/(x*Sqrt[1 + (b*x^n)/a]))
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^n + a}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n + a)/x^2, x)
```

maple [**F**] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\,x^n + a}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^(1/2)/x^2,x)
```

```
[Out] int((b*x^n+a)^(1/2)/x^2,x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^n + a}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^n + a)/x^2, x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a + b\,x^n}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^(1/2)/x^2,x)
```

```
[Out] int((a + b*x^n)^(1/2)/x^2, x)
```

sympy [**C**] time = 1.25, size = 44, normalized size = 0.90

$$\frac{\sqrt{a}\,\Gamma\left(-\frac{1}{n}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2}, -\frac{1}{n} \\ 1 - \frac{1}{n}\end{matrix}\right| \frac{bx^n e^{i\pi}}{a}\right)}{nx\Gamma\left(1 - \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**(1/2)/x**2,x)
```

```
[Out] sqrt(a)*gamma(-1/n)*hyper((-1/2, -1/n), (1 - 1/n,), b*x**n*exp_polar(I*pi)/
a)/(n*x*gamma(1 - 1/n))
```

3.2491 $\int \frac{\sqrt{a+bx^n}}{x^3}\,dx$

Optimal. Leaf size=51

$$-\frac{(a+bx^n)^{3/2}\ {}_2F_1\left(1,\frac{3}{2}-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2ax^2}$$

```
[Out] -1/2*(a+b*x^n)^(3/2)*hypergeom([1, 3/2-2/n],[(-2+n)/n],-b*x^n/a)/a/x^2
```

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {365, 364}

$$-\frac{\sqrt{a+bx^n}\ {}_2F_1\left(-\frac{1}{2},-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2x^2\sqrt{\frac{bx^n}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^n]/x^3,x]
```

```
[Out] -(Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, -2/n, -((2 - n)/n), -((b*x^n)/a)]
)/(2*x^2*Sqrt[1 + (b*x^n)/a])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx^n}}{x^3}\,dx &= \frac{\sqrt{a+bx^n}\int \frac{\sqrt{1+\frac{bx^n}{a}}}{x^3}\,dx}{\sqrt{1+\frac{bx^n}{a}}}\\ &= -\frac{\sqrt{a+bx^n}\ {}_2F_1\left(-\frac{1}{2},-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2x^2\sqrt{1+\frac{bx^n}{a}}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 57, normalized size = 1.12

$$-\frac{\sqrt{a+bx^n}\ {}_2F_1\left(-\frac{1}{2},-\frac{2}{n};1-\frac{2}{n};-\frac{bx^n}{a}\right)}{2x^2\sqrt{\frac{bx^n}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^n]/x^3,x]
```

```
[Out] -1/2*(Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, -2/n, 1 - 2/n, -((b*x^n)/a)])
/(x^2*Sqrt[1 + (b*x^n)/a])
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^n + a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n + a)/x^3, x)
```

maple [**F**] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\,x^n + a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^(1/2)/x^3,x)
```

```
[Out] int((b*x^n+a)^(1/2)/x^3,x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^n + a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^n + a)/x^3, x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a + b\,x^n}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^(1/2)/x^3,x)
```

```
[Out] int((a + b*x^n)^(1/2)/x^3, x)
```

sympy [**C**] time = 1.43, size = 46, normalized size = 0.90

$$\frac{\sqrt{a}\,\Gamma\left(-\frac{2}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2}, -\frac{2}{n} \\ 1 - \frac{2}{n}\end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{nx^2\Gamma\left(1 - \frac{2}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**(1/2)/x**3,x)
```

```
[Out] sqrt(a)*gamma(-2/n)*hyper((-1/2, -2/n), (1 - 2/n,), b*x**n*exp_polar(I*pi)/
a)/(n*x**2*gamma(1 - 2/n))
```

3.2492 $\int x\,(a+bx^n)^{3/2}\,dx$

Optimal. Leaf size=48

$$\frac{x^2\,(a+bx^n)^{5/2}\;{}_2F_1\left(1,\frac{5}{2}+\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right)}{2a}$$

```
[Out] 1/2*x^2*(a+b*x^n)^(5/2)*hypergeom([1, 5/2+2/n],[(2+n)/n],-b*x^n/a)/a
```

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {365, 364}

$$\frac{ax^2\sqrt{a+bx^n}\;{}_2F_1\left(-\frac{3}{2},\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right)}{2\sqrt{\frac{bx^n}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^n)^(3/2),x]
```

```
[Out] (a*x^2*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, 2/n, (2 + n)/n, -((b*x^n)/a)
])/(2*Sqrt[1 + (b*x^n)/a])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x\,(a+bx^n)^{3/2}\,dx &= \frac{\left(a\sqrt{a+bx^n}\right)\int x\left(1+\frac{bx^n}{a}\right)^{3/2}\,dx}{\sqrt{1+\frac{bx^n}{a}}}\\ &= \frac{ax^2\sqrt{a+bx^n}\;{}_2F_1\left(-\frac{3}{2},\frac{2}{n};\frac{2+n}{n};-\frac{bx^n}{a}\right)}{2\sqrt{1+\frac{bx^n}{a}}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 58, normalized size = 1.21

$$\frac{ax^2\sqrt{a+bx^n}\;{}_2F_1\left(-\frac{3}{2},\frac{2}{n};1+\frac{2}{n};-\frac{bx^n}{a}\right)}{2\sqrt{\frac{bx^n}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^n)^(3/2),x]
```

```
[Out] (a*x^2*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, 2/n, 1 + 2/n, -((b*x^n)/a)])
/(2*Sqrt[1 + (b*x^n)/a])
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^{\frac{3}{2}} x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^(3/2)*x, x)
```

maple [**F**] time = 0.21, size = 0, normalized size = 0.00

$$\int (b\,x^n + a)^{\frac{3}{2}}\, x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^n+a)^(3/2),x)
```

```
[Out] int(x*(b*x^n+a)^(3/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^{\frac{3}{2}} x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^(3/2)*x, x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int x\,(a + b\,x^n)^{3/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^n)^(3/2),x)
```

```
[Out] int(x*(a + b*x^n)^(3/2), x)
```

sympy [**C**] time = 2.40, size = 42, normalized size = 0.88

$$\frac{a^{\frac{3}{2}} x^2 \Gamma\left(\frac{2}{n}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{2}{n} \\ 1 + \frac{2}{n} \end{matrix} \middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(1 + \frac{2}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x**n)**(3/2),x)
```

```
[Out] a**(3/2)*x**2*gamma(2/n)*hyper((-3/2, 2/n), (1 + 2/n,), b*x**n*exp_polar(I*
pi)/a)/(n*gamma(1 + 2/n))
```

3.2493 $\int (a+bx^n)^{3/2}\,dx$

Optimal. Leaf size=39

$$\frac{x\,(a+bx^n)^{5/2}\,{}_2F_1\left(1,\frac{5}{2}+\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a}$$

```
[Out] x*(a+b*x^n)^(5/2)*hypergeom([1, 5/2+1/n],[1+1/n],-b*x^n/a)/a
```

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {246, 245}

$$\frac{ax\sqrt{a+bx^n}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{\sqrt{\frac{bx^n}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^(3/2),x]
```

```
[Out] (a*x*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, n^(-1), 1 + n^(-1), -((b*x^n)/
a)])/Sqrt[1 + (b*x^n)/a]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int (a+bx^n)^{3/2}\,dx = \frac{\left(a\sqrt{a+bx^n}\right)\int\left(1+\frac{bx^n}{a}\right)^{3/2}\,dx}{\sqrt{1+\frac{bx^n}{a}}}$$
$$= \frac{ax\sqrt{a+bx^n}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{\sqrt{1+\frac{bx^n}{a}}}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.26

$$\frac{ax\sqrt{a+bx^n}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{\sqrt{\frac{bx^n}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^(3/2),x]
```

```
[Out] (a*x*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, n^(-1), 1 + n^(-1), -((b*x^n)/
a)])/Sqrt[1 + (b*x^n)/a]
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^(3/2), x)
```

maple [**F**] time = 0.21, size = 0, normalized size = 0.00

$$\int (b\,x^n + a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^(3/2),x)
```

```
[Out] int((b*x^n+a)^(3/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^(3/2), x)
```

mupad [**B**] time = 1.19, size = 43, normalized size = 1.10

$$\frac{x\,(a + b\,x^n)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2}, \frac{1}{n};\ \frac{1}{n} + 1;\ -\frac{b\,x^n}{a}\right)}{\left(\frac{b\,x^n}{a} + 1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^(3/2),x)
```

```
[Out] (x*(a + b*x^n)^(3/2)*hypergeom([-3/2, 1/n], 1/n + 1, -(b*x^n)/a))/((b*x^n)/
a + 1)^(3/2)
```

sympy [C] time = 1.92, size = 41, normalized size = 1.05

$$\frac{a^{\frac{3}{2}} x \Gamma\left(\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{1}{n} \\ 1 + \frac{1}{n} \end{matrix} \middle| \frac{b x^n e^{i\pi}}{a}\right)}{n \Gamma\left(1 + \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**(3/2),x)
```

```
[Out] a**(3/2)*x*gamma(1/n)*hyper((-3/2, 1/n), (1 + 1/n,), b*x**n*exp_polar(I*pi)
/a)/(n*gamma(1 + 1/n))
```

3.2494 $\int \frac{(a+bx^n)^{3/2}}{x}\,dx$

Optimal. Leaf size=64

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{n}+\frac{2a\sqrt{a+bx^n}}{n}+\frac{2\left(a+bx^n\right)^{3/2}}{3n}$$

```
[Out] 2/3*(a+b*x^n)^(3/2)/n-2*a^(3/2)*arctanh((a+b*x^n)^(1/2)/a^(1/2))/n+2*a*(a+b
*x^n)^(1/2)/n
```

Rubi [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 50, 63, 208}

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{n}+\frac{2a\sqrt{a+bx^n}}{n}+\frac{2\left(a+bx^n\right)^{3/2}}{3n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^(3/2)/x,x]
```

```
[Out] (2*a*Sqrt[a + b*x^n])/n + (2*(a + b*x^n)^(3/2))/(3*n) - (2*a^(3/2)*ArcTanh[
Sqrt[a + b*x^n]/Sqrt[a]])/n
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^n)^{3/2}}{x}\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, x^n\right)}{n}$$
$$= \frac{2(a+bx^n)^{3/2}}{3n} + \frac{a\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^n\right)}{n}$$
$$= \frac{2a\sqrt{a+bx^n}}{n} + \frac{2(a+bx^n)^{3/2}}{3n} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^n\right)}{n}$$
$$= \frac{2a\sqrt{a+bx^n}}{n} + \frac{2(a+bx^n)^{3/2}}{3n} + \frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^n}\right)}{bn}$$
$$= \frac{2a\sqrt{a+bx^n}}{n} + \frac{2(a+bx^n)^{3/2}}{3n} - \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{n}$$

Mathematica [A] time = 0.02, size = 55, normalized size = 0.86

$$\frac{2\sqrt{a+bx^n}\,(4a+bx^n) - 6a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^(3/2)/x,x]
```

```
[Out] (2*Sqrt[a + b*x^n]*(4*a + b*x^n) - 6*a^(3/2)*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a
]])/(3*n)
```

fricas [A] time = 1.33, size = 114, normalized size = 1.78

$$\left[\frac{3\,a^{\frac{3}{2}}\log\left(\frac{bx^n-2\sqrt{bx^n+a}\sqrt{a}+2a}{x^n}\right)+2(bx^n+4a)\sqrt{bx^n+a}}{3n}, \frac{2\left(3\sqrt{-a}\,a\arctan\left(\frac{\sqrt{bx^n+a}\sqrt{-a}}{a}\right)+(bx^n+4a)\sqrt{bx^n+a}\right)}{3n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/3*(3*a^(3/2)*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n) + 2*(b*x
^n + 4*a)*sqrt(b*x^n + a))/n, 2/3*(3*sqrt(-a)*a*arctan(sqrt(b*x^n + a)*sqrt
(-a)/a) + (b*x^n + 4*a)*sqrt(b*x^n + a))/n]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n+a)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^(3/2)/x, x)
```

maple [A] time = 0.00, size = 48, normalized size = 0.75

$$\frac{-2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{b\,x^n+a}}{\sqrt{a}}\right)+2\sqrt{b\,x^n+a}\;a+\frac{2(b\,x^n+a)^{\frac{3}{2}}}{3}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^(3/2)/x,x)
```

```
[Out] 1/n*(2/3*(b*x^n+a)^(3/2)+2*(b*x^n+a)^(1/2)*a-2*a^(3/2)*arctanh((b*x^n+a)^(1
/2)/a^(1/2)))
```

maxima [A] time = 1.24, size = 67, normalized size = 1.05

$$\frac{a^{\frac{3}{2}} \log\left(\frac{\sqrt{bx^n+a}-\sqrt{a}}{\sqrt{bx^n+a}+\sqrt{a}}\right)}{n} + \frac{2\left((bx^n+a)^{\frac{3}{2}} + 3\sqrt{bx^n+a}\,a\right)}{3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] a^(3/2)*log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))/n + 2/
3*((b*x^n + a)^(3/2) + 3*sqrt(b*x^n + a)*a)/n
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x^n)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^(3/2)/x,x)
```

```
[Out] int((a + b*x^n)^(3/2)/x, x)
```

sympy [A] time = 3.02, size = 87, normalized size = 1.36

$$\frac{8a^{\frac{3}{2}}\sqrt{1+\frac{bx^n}{a}}}{3n} + \frac{a^{\frac{3}{2}}\log\left(\frac{bx^n}{a}\right)}{n} - \frac{2a^{\frac{3}{2}}\log\left(\sqrt{1+\frac{bx^n}{a}}+1\right)}{n} + \frac{2\sqrt{a}\,bx^n\sqrt{1+\frac{bx^n}{a}}}{3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**(3/2)/x,x)
```

```
[Out] 8*a**(3/2)*sqrt(1 + b*x**n/a)/(3*n) + a**(3/2)*log(b*x**n/a)/n - 2*a**(3/2)
*log(sqrt(1 + b*x**n/a) + 1)/n + 2*sqrt(a)*b*x**n*sqrt(1 + b*x**n/a)/(3*n)
```

3.2495 $\int \frac{(a+bx^n)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=49

$$-\frac{(a+bx^n)^{5/2}\ {}_2F_1\left(1,\frac{5}{2}-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{ax}$$

```
[Out] -(a+b*x^n)^(5/2)*hypergeom([1, 5/2-1/n],[(-1+n)/n],-b*x^n/a)/a/x
```

Rubi [A] time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.20, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {365, 364}

$$-\frac{a\sqrt{a+bx^n}\ {}_2F_1\left(-\frac{3}{2},-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{x\sqrt{\frac{bx^n}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^(3/2)/x^2,x]
```

```
[Out] -((a*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, -n^(-1), -((1 - n)/n), -((b*x^
n)/a)])/(x*Sqrt[1 + (b*x^n)/a]))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^n)^{3/2}}{x^2}\,dx &= \frac{\left(a\sqrt{a+bx^n}\right)\int \frac{\left(1+\frac{bx^n}{a}\right)^{3/2}}{x^2}\,dx}{\sqrt{1+\frac{bx^n}{a}}}\\ &= -\frac{a\sqrt{a+bx^n}\ {}_2F_1\left(-\frac{3}{2},-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{x\sqrt{1+\frac{bx^n}{a}}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 56, normalized size = 1.14

$$-\frac{a\sqrt{a+bx^n}\ {}_2F_1\left(-\frac{3}{2},-\frac{1}{n};1-\frac{1}{n};-\frac{bx^n}{a}\right)}{x\sqrt{\frac{bx^n}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^(3/2)/x^2,x]
```

```
[Out] -((a*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, -n^(-1), 1 - n^(-1), -((b*x^n)
/a)])/(x*Sqrt[1 + (b*x^n)/a]))
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^{\frac{3}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^(3/2)/x^2, x)
```

maple [**F**] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^n + a)^{\frac{3}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^(3/2)/x^2,x)
```

```
[Out] int((b*x^n+a)^(3/2)/x^2,x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^{\frac{3}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^(3/2)/x^2, x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a + b\,x^n)^{3/2}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^(3/2)/x^2,x)
```

```
[Out] int((a + b*x^n)^(3/2)/x^2, x)
```

sympy [C] time = 1.96, size = 44, normalized size = 0.90

$$\frac{a^{\frac{3}{2}}\Gamma\left(-\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-\frac{3}{2}, -\frac{1}{n} \\ 1-\frac{1}{n}\end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{nx\Gamma\left(1-\frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**(3/2)/x**2,x)
```

```
[Out] a**(3/2)*gamma(-1/n)*hyper((-3/2, -1/n), (1 - 1/n,), b*x**n*exp_polar(I*pi)
/a)/(n*x*gamma(1 - 1/n))
```

3.2496 $\int \frac{(a+bx^n)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=51

$$-\frac{(a+bx^n)^{5/2}\ {}_2F_1\left(1,\frac{5}{2}-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2ax^2}$$

```
[Out] -1/2*(a+b*x^n)^(5/2)*hypergeom([1, 5/2-2/n],[(-2+n)/n],-b*x^n/a)/a/x^2
```

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.20, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {365, 364}

$$-\frac{a\sqrt{a+bx^n}\ {}_2F_1\left(-\frac{3}{2},-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2x^2\sqrt{\frac{bx^n}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^(3/2)/x^3,x]
```

```
[Out] -(a*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, -2/n, -((2 - n)/n), -((b*x^n)/a
)])/(2*x^2*Sqrt[1 + (b*x^n)/a])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^n)^{3/2}}{x^3}\,dx &= \frac{\left(a\sqrt{a+bx^n}\right)\int \frac{\left(1+\frac{bx^n}{a}\right)^{3/2}}{x^3}\,dx}{\sqrt{1+\frac{bx^n}{a}}}\\ &= -\frac{a\sqrt{a+bx^n}\ {}_2F_1\left(-\frac{3}{2},-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2x^2\sqrt{1+\frac{bx^n}{a}}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 58, normalized size = 1.14

$$-\frac{a\sqrt{a+bx^n}\ {}_2F_1\left(-\frac{3}{2},-\frac{2}{n};1-\frac{2}{n};-\frac{bx^n}{a}\right)}{2x^2\sqrt{\frac{bx^n}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^(3/2)/x^3,x]
```

```
[Out] -1/2*(a*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, -2/n, 1 - 2/n, -((b*x^n)/a)
])/(x^2*Sqrt[1 + (b*x^n)/a])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^(3/2)/x^3, x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^n + a)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^(3/2)/x^3,x)
```

```
[Out] int((b*x^n+a)^(3/2)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^(3/2)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a + b\,x^n)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^(3/2)/x^3,x)
```

```
[Out] int((a + b*x^n)^(3/2)/x^3, x)
```

sympy [C] time = 1.99, size = 46, normalized size = 0.90

$$\frac{a^{\frac{3}{2}}\Gamma\left(-\frac{2}{n}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{2}, -\frac{2}{n} \\ 1-\frac{2}{n}\end{matrix}\right| \frac{bx^n e^{i\pi}}{a}\right)}{nx^2\Gamma\left(1-\frac{2}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**(3/2)/x**3,x)
```

```
[Out] a**(3/2)*gamma(-2/n)*hyper((-3/2, -2/n), (1 - 2/n,), b*x**n*exp_polar(I*pi)
/a)/(n*x**2*gamma(1 - 2/n))
```

3.2497 $\int x\,(a+bx^n)^{5/2}\,dx$

Optimal. Leaf size=48

$$\frac{x^2\,(a+bx^n)^{7/2}\;{}_2F_1\left(1,\frac{7}{2}+\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right)}{2a}$$

```
[Out] 1/2*x^2*(a+b*x^n)^(7/2)*hypergeom([1, 7/2+2/n],[(2+n)/n],-b*x^n/a)/a
```

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.25, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {365, 364}

$$\frac{a^2x^2\sqrt{a+bx^n}\;{}_2F_1\left(-\frac{5}{2},\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right)}{2\sqrt{\frac{bx^n}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^n)^(5/2),x]
```

```
[Out] (a^2*x^2*Sqrt[a + b*x^n]*Hypergeometric2F1[-5/2, 2/n, (2 + n)/n, -((b*x^n)/
a)])/(2*Sqrt[1 + (b*x^n)/a])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x\,(a+bx^n)^{5/2}\,dx &= \frac{\left(a^2\sqrt{a+bx^n}\right)\int x\left(1+\frac{bx^n}{a}\right)^{5/2}dx}{\sqrt{1+\frac{bx^n}{a}}}\\ &= \frac{a^2x^2\sqrt{a+bx^n}\;{}_2F_1\left(-\frac{5}{2},\frac{2}{n};\frac{2+n}{n};-\frac{bx^n}{a}\right)}{2\sqrt{1+\frac{bx^n}{a}}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 60, normalized size = 1.25

$$\frac{a^2x^2\sqrt{a+bx^n}\;{}_2F_1\left(-\frac{5}{2},\frac{2}{n};1+\frac{2}{n};-\frac{bx^n}{a}\right)}{2\sqrt{\frac{bx^n}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^n)^(5/2),x]
```

```
[Out] (a^2*x^2*Sqrt[a + b*x^n]*Hypergeometric2F1[-5/2, 2/n, 1 + 2/n, -((b*x^n)/a)
])/(2*Sqrt[1 + (b*x^n)/a])
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^n)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^{\frac{5}{2}} x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^n)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^(5/2)*x, x)
```

maple [**F**] time = 0.21, size = 0, normalized size = 0.00

$$\int (b\,x^n + a)^{\frac{5}{2}}\, x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^n+a)^(5/2),x)
```

```
[Out] int(x*(b*x^n+a)^(5/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^{\frac{5}{2}} x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^n)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^(5/2)*x, x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int x\,(a + b\,x^n)^{5/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^n)^(5/2),x)
```

```
[Out] int(x*(a + b*x^n)^(5/2), x)
```

sympy [**C**] time = 6.90, size = 42, normalized size = 0.88

$$\frac{a^{\frac{5}{2}} x^2 \Gamma\left(\frac{2}{n}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{2}, \frac{2}{n} \\ 1 + \frac{2}{n} \end{matrix} \middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(1 + \frac{2}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x**n)**(5/2),x)
```

```
[Out] a**(5/2)*x**2*gamma(2/n)*hyper((-5/2, 2/n), (1 + 2/n,), b*x**n*exp_polar(I*
pi)/a)/(n*gamma(1 + 2/n))
```

3.2498 $\int (a+bx^n)^{5/2}\,dx$

Optimal. Leaf size=39

$$\frac{x\,(a+bx^n)^{7/2}\,{}_2F_1\left(1,\frac{7}{2}+\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a}$$

```
[Out] x*(a+b*x^n)^(7/2)*hypergeom([1, 7/2+1/n],[1+1/n],-b*x^n/a)/a
```

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.31, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {246, 245}

$$\frac{a^2x\sqrt{a+bx^n}\,{}_2F_1\left(-\frac{5}{2},\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{\sqrt{\frac{bx^n}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^(5/2),x]
```

```
[Out] (a^2*x*Sqrt[a + b*x^n]*Hypergeometric2F1[-5/2, n^(-1), 1 + n^(-1), -((b*x^n
)/a)])/Sqrt[1 + (b*x^n)/a]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)^{5/2}\,dx &= \frac{\left(a^2\sqrt{a+bx^n}\right)\int\left(1+\frac{bx^n}{a}\right)^{5/2}\,dx}{\sqrt{1+\frac{bx^n}{a}}}\\ &= \frac{a^2x\sqrt{a+bx^n}\,{}_2F_1\left(-\frac{5}{2},\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{\sqrt{1+\frac{bx^n}{a}}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 1.31

$$\frac{a^2x\sqrt{a+bx^n}\,{}_2F_1\left(-\frac{5}{2},\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{\sqrt{\frac{bx^n}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^(5/2),x]
```

```
[Out] (a^2*x*Sqrt[a + b*x^n]*Hypergeometric2F1[-5/2, n^(-1), 1 + n^(-1), -((b*x^n
)/a)])/Sqrt[1 + (b*x^n)/a]
```

fricas **[F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^(5/2), x)
```

maple **[F]** time = 0.21, size = 0, normalized size = 0.00

$$\int (b\,x^n + a)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^(5/2),x)
```

```
[Out] int((b*x^n+a)^(5/2),x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^(5/2), x)
```

mupad **[B]** time = 1.23, size = 43, normalized size = 1.10

$$\frac{x\,(a + b\,x^n)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2}, \frac{1}{n};\ \frac{1}{n} + 1;\ -\frac{b\,x^n}{a}\right)}{\left(\frac{b\,x^n}{a} + 1\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^(5/2),x)
```

```
[Out] (x*(a + b*x^n)^(5/2)*hypergeom([-5/2, 1/n], 1/n + 1, -(b*x^n)/a))/((b*x^n)/
a + 1)^(5/2)
```

sympy [C] time = 5.24, size = 41, normalized size = 1.05

$$\frac{a^{\frac{5}{2}} x \Gamma\left(\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{2}, \frac{1}{n} \\ 1 + \frac{1}{n} \end{matrix} \middle| \frac{b x^n e^{i\pi}}{a}\right)}{n \Gamma\left(1 + \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**(5/2),x)
```

```
[Out] a**(5/2)*x*gamma(1/n)*hyper((-5/2, 1/n), (1 + 1/n,), b*x**n*exp_polar(I*pi)
/a)/(n*gamma(1 + 1/n))
```

3.2499 $\int \frac{(a+bx^n)^{5/2}}{x}\,dx$

Optimal. Leaf size=85

$$-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{n}+\frac{2a^2\sqrt{a+bx^n}}{n}+\frac{2a\left(a+bx^n\right)^{3/2}}{3n}+\frac{2\left(a+bx^n\right)^{5/2}}{5n}$$

```
[Out] 2/3*a*(a+b*x^n)^(3/2)/n+2/5*(a+b*x^n)^(5/2)/n-2*a^(5/2)*arctanh((a+b*x^n)^(
1/2)/a^(1/2))/n+2*a^2*(a+b*x^n)^(1/2)/n
```

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 50, 63, 208}

$$\frac{2a^2\sqrt{a+bx^n}}{n}-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{n}+\frac{2a\left(a+bx^n\right)^{3/2}}{3n}+\frac{2\left(a+bx^n\right)^{5/2}}{5n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^(5/2)/x,x]
```

```
[Out] (2*a^2*Sqrt[a + b*x^n])/n + (2*a*(a + b*x^n)^(3/2))/(3*n) + (2*(a + b*x^n)^
(5/2))/(5*n) - (2*a^(5/2)*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/n
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^n)^{5/2}}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x}\,dx, x, x^n\right)}{n} \\
&= \frac{2(a+bx^n)^{5/2}}{5n} + \frac{a\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, x^n\right)}{n} \\
&= \frac{2a(a+bx^n)^{3/2}}{3n} + \frac{2(a+bx^n)^{5/2}}{5n} + \frac{a^2\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^n\right)}{n} \\
&= \frac{2a^2\sqrt{a+bx^n}}{n} + \frac{2a(a+bx^n)^{3/2}}{3n} + \frac{2(a+bx^n)^{5/2}}{5n} + \frac{a^3\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^n\right)}{n} \\
&= \frac{2a^2\sqrt{a+bx^n}}{n} + \frac{2a(a+bx^n)^{3/2}}{3n} + \frac{2(a+bx^n)^{5/2}}{5n} + \frac{\left(2a^3\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^n}\right.}{bn} \\
&= \frac{2a^2\sqrt{a+bx^n}}{n} + \frac{2a(a+bx^n)^{3/2}}{3n} + \frac{2(a+bx^n)^{5/2}}{5n} - \frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{n}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 69, normalized size = 0.81

$$\frac{2\sqrt{a+bx^n}\left(23a^2+11abx^n+3b^2x^{2n}\right)-30a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{15n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^(5/2)/x,x]
```

```
[Out] (2*Sqrt[a + b*x^n]*(23*a^2 + 11*a*b*x^n + 3*b^2*x^(2*n)) - 30*a^(5/2)*ArcTa
nh[Sqrt[a + b*x^n]/Sqrt[a]])/(15*n)
```

fricas [A] time = 1.14, size = 144, normalized size = 1.69

$$\left[\frac{15\,a^{\frac{5}{2}}\log\left(\frac{bx^n-2\sqrt{bx^n+a}\sqrt{a}+2a}{x^n}\right)+2\left(3b^2x^{2n}+11abx^n+23a^2\right)\sqrt{bx^n+a}}{15n}, \frac{2\left(15\sqrt{-a}a^2\arctan\left(\frac{\sqrt{bx^n+a}\sqrt{-a}}{a}\right)+\right.}{1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [1/15*(15*a^(5/2)*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n) + 2*(3
*b^2*x^(2*n) + 11*a*b*x^n + 23*a^2)*sqrt(b*x^n + a))/n, 2/15*(15*sqrt(-a)*a
^2*arctan(sqrt(b*x^n + a)*sqrt(-a)/a) + (3*b^2*x^(2*n) + 11*a*b*x^n + 23*a^
2)*sqrt(b*x^n + a))/n]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n+a)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(5/2)/x,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^(5/2)/x, x)
```

maple [A] time = 0.00, size = 62, normalized size = 0.73

$$\frac{-2a^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{b\,x^n+a}}{\sqrt{a}}\right)+2\sqrt{b\,x^n+a}\;a^2+\frac{2(b\,x^n+a)^{\frac{3}{2}}a}{3}+\frac{2(b\,x^n+a)^{\frac{5}{2}}}{5}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^(5/2)/x,x)
```

```
[Out] 1/n*(2/5*(b*x^n+a)^(5/2)+2/3*a*(b*x^n+a)^(3/2)+2*(b*x^n+a)^(1/2)*a^2-2*a^(5
/2)*arctanh((b*x^n+a)^(1/2)/a^(1/2)))
```

maxima [A] time = 1.31, size = 83, normalized size = 0.98

$$\frac{a^{\frac{5}{2}}\log\left(\frac{\sqrt{bx^n+a}-\sqrt{a}}{\sqrt{bx^n+a}+\sqrt{a}}\right)}{n}+\frac{2\left(3\,(bx^n+a)^{\frac{5}{2}}+5\,(bx^n+a)^{\frac{3}{2}}a+15\,\sqrt{bx^n+a}\,a^2\right)}{15\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] a^(5/2)*log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))/n + 2/
15*(3*(b*x^n + a)^(5/2) + 5*(b*x^n + a)^(3/2)*a + 15*sqrt(b*x^n + a)*a^2)/n
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x^n)^{5/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^(5/2)/x,x)
```

```
[Out] int((a + b*x^n)^(5/2)/x, x)
```

sympy [A] time = 7.98, size = 117, normalized size = 1.38

$$\frac{46a^{\frac{5}{2}}\sqrt{1+\frac{bx^n}{a}}}{15n}+\frac{a^{\frac{5}{2}}\log\left(\frac{bx^n}{a}\right)}{n}-\frac{2a^{\frac{5}{2}}\log\left(\sqrt{1+\frac{bx^n}{a}}+1\right)}{n}+\frac{22a^{\frac{3}{2}}bx^n\sqrt{1+\frac{bx^n}{a}}}{15n}+\frac{2\sqrt{a}\,b^2x^{2n}\sqrt{1+\frac{bx^n}{a}}}{5n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**(5/2)/x,x)
```

```
[Out] 46*a**(5/2)*sqrt(1 + b*x**n/a)/(15*n) + a**(5/2)*log(b*x**n/a)/n - 2*a**(5/
2)*log(sqrt(1 + b*x**n/a) + 1)/n + 22*a**(3/2)*b*x**n*sqrt(1 + b*x**n/a)/(1
5*n) + 2*sqrt(a)*b**2*x**(2*n)*sqrt(1 + b*x**n/a)/(5*n)
```

3.2500 $\int \frac{(a+bx^n)^{5/2}}{x^2}\,dx$

Optimal. Leaf size=49

$$-\frac{(a+bx^n)^{7/2}\ {}_2F_1\left(1,\frac{7}{2}-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{ax}$$

```
[Out] -(a+b*x^n)^(7/2)*hypergeom([1, 7/2-1/n],[(-1+n)/n],-b*x^n/a)/a/x
```

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.24, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {365, 364}

$$-\frac{a^2\sqrt{a+bx^n}\ {}_2F_1\left(-\frac{5}{2},-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{x\sqrt{\frac{bx^n}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^(5/2)/x^2,x]
```

```
[Out] -((a^2*Sqrt[a + b*x^n]*Hypergeometric2F1[-5/2, -n^(-1), -((1 - n)/n), -((b*
x^n)/a)])/(x*Sqrt[1 + (b*x^n)/a]))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^n)^{5/2}}{x^2}\,dx &= \frac{\left(a^2\sqrt{a+bx^n}\right)\int \frac{\left(1+\frac{bx^n}{a}\right)^{5/2}}{x^2}\,dx}{\sqrt{1+\frac{bx^n}{a}}}\\ &= -\frac{a^2\sqrt{a+bx^n}\ {}_2F_1\left(-\frac{5}{2},-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{x\sqrt{1+\frac{bx^n}{a}}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 58, normalized size = 1.18

$$-\frac{a^2\sqrt{a+bx^n}\ {}_2F_1\left(-\frac{5}{2},-\frac{1}{n};1-\frac{1}{n};-\frac{bx^n}{a}\right)}{x\sqrt{\frac{bx^n}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^(5/2)/x^2,x]
```

```
[Out] -((a^2*Sqrt[a + b*x^n]*Hypergeometric2F1[-5/2, -n^(-1), 1 - n^(-1), -((b*x^
n)/a)])/(x*Sqrt[1 + (b*x^n)/a]))
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^{\frac{5}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^(5/2)/x^2, x)
```

maple [**F**] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^n + a)^{\frac{5}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^(5/2)/x^2,x)
```

```
[Out] int((b*x^n+a)^(5/2)/x^2,x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^{\frac{5}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^(5/2)/x^2, x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a + b\,x^n)^{5/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^(5/2)/x^2,x)
```

```
[Out] int((a + b*x^n)^(5/2)/x^2, x)
```

sympy [C] time = 5.34, size = 44, normalized size = 0.90

$$\frac{a^{\frac{5}{2}}\Gamma\left(-\frac{1}{n}\right){}_2F_1\left(\left.\begin{matrix}-\frac{5}{2}, -\frac{1}{n} \\ 1-\frac{1}{n}\end{matrix}\right| \frac{bx^n e^{i\pi}}{a}\right)}{nx\Gamma\left(1-\frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**(5/2)/x**2,x)
```

```
[Out] a**(5/2)*gamma(-1/n)*hyper((-5/2, -1/n), (1 - 1/n,), b*x**n*exp_polar(I*pi)
/a)/(n*x*gamma(1 - 1/n))
```

3.2501 $\int \frac{(a+bx^n)^{5/2}}{x^3}\,dx$

Optimal. Leaf size=51

$$-\frac{(a+bx^n)^{7/2}\ {}_2F_1\left(1,\frac{7}{2}-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2ax^2}$$

```
[Out] -1/2*(a+b*x^n)^(7/2)*hypergeom([1, 7/2-2/n],[(-2+n)/n],-b*x^n/a)/a/x^2
```

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.24, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {365, 364}

$$-\frac{a^2\sqrt{a+bx^n}\ {}_2F_1\left(-\frac{5}{2},-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2x^2\sqrt{\frac{bx^n}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^(5/2)/x^3,x]
```

```
[Out] -(a^2*Sqrt[a + b*x^n]*Hypergeometric2F1[-5/2, -2/n, -((2 - n)/n), -((b*x^n)
/a)])/(2*x^2*Sqrt[1 + (b*x^n)/a])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^n)^{5/2}}{x^3}\,dx &= \frac{\left(a^2\sqrt{a+bx^n}\right)\int \frac{\left(1+\frac{bx^n}{a}\right)^{5/2}}{x^3}\,dx}{\sqrt{1+\frac{bx^n}{a}}}\\ &= -\frac{a^2\sqrt{a+bx^n}\ {}_2F_1\left(-\frac{5}{2},-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2x^2\sqrt{1+\frac{bx^n}{a}}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 60, normalized size = 1.18

$$-\frac{a^2\sqrt{a+bx^n}\ {}_2F_1\left(-\frac{5}{2},-\frac{2}{n};1-\frac{2}{n};-\frac{bx^n}{a}\right)}{2x^2\sqrt{\frac{bx^n}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^(5/2)/x^3,x]
```

```
[Out] -1/2*(a^2*Sqrt[a + b*x^n]*Hypergeometric2F1[-5/2, -2/n, 1 - 2/n, -((b*x^n)/
a)])/(x^2*Sqrt[1 + (b*x^n)/a])
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^{\frac{5}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^(5/2)/x^3, x)
```

maple [**F**] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^n + a)^{\frac{5}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^(5/2)/x^3,x)
```

```
[Out] int((b*x^n+a)^(5/2)/x^3,x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^{\frac{5}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^(5/2)/x^3, x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a + b\,x^n)^{5/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^(5/2)/x^3,x)
```

```
[Out] int((a + b*x^n)^(5/2)/x^3, x)
```

sympy [C] time = 5.36, size = 46, normalized size = 0.90

$$\frac{a^{\frac{5}{2}}\Gamma\left(-\frac{2}{n}\right){}_2F_1\left(\begin{matrix}-\frac{5}{2}, -\frac{2}{n} \\ 1-\frac{2}{n}\end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{nx^2\Gamma\left(1-\frac{2}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**(5/2)/x**3,x)
```

```
[Out] a**(5/2)*gamma(-2/n)*hyper((-5/2, -2/n), (1 - 2/n,), b*x**n*exp_polar(I*pi)
/a)/(n*x**2*gamma(1 - 2/n))
```

3.2502 $\int \frac{x}{\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=48

$$\frac{x^2\sqrt{a+bx^n}\ {}_2F_1\left(1,\frac{1}{2}+\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right)}{2a}$$

```
[Out] 1/2*x^2*hypergeom([1, 1/2+2/n],[(2+n)/n],-b*x^n/a)*(a+b*x^n)^(1/2)/a
```

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.19, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {365, 364}

$$\frac{x^2\sqrt{\frac{bx^n}{a}+1}\ {}_2F_1\left(\frac{1}{2},\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right)}{2\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*x^n],x]
```

```
[Out] (x^2*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, 2/n, (2 + n)/n, -((b*x^n)/a
)])/(2*Sqrt[a + b*x^n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{a+bx^n}}\,dx &= \frac{\sqrt{1+\frac{bx^n}{a}}\int \frac{x}{\sqrt{1+\frac{bx^n}{a}}}\,dx}{\sqrt{a+bx^n}}\\ &= \frac{x^2\sqrt{1+\frac{bx^n}{a}}\ {}_2F_1\left(\frac{1}{2},\frac{2}{n};\frac{2+n}{n};-\frac{bx^n}{a}\right)}{2\sqrt{a+bx^n}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 57, normalized size = 1.19

$$\frac{x^2\sqrt{\frac{bx^n}{a}+1}\ {}_2F_1\left(\frac{1}{2},\frac{2}{n};1+\frac{2}{n};-\frac{bx^n}{a}\right)}{2\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b*x^n],x]
```

```
[Out] (x^2*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, 2/n, 1 + 2/n, -((b*x^n)/a)]
)/(2*Sqrt[a + b*x^n])
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{bx^n + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(b*x^n + a), x)
```

maple [**F**] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{b\,x^n + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^n+a)^(1/2),x)
```

```
[Out] int(x/(b*x^n+a)^(1/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{bx^n + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(b*x^n + a), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\sqrt{a + b\,x^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^n)^(1/2),x)
```

```
[Out] int(x/(a + b*x^n)^(1/2), x)
```

sympy [**C**] time = 1.12, size = 41, normalized size = 0.85

$$\frac{x^2 \Gamma\left(\frac{2}{n}\right) {}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{2}{n} \\ 1 + \frac{2}{n}\end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{\sqrt{a}\, n \Gamma\left(1 + \frac{2}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x**n)**(1/2),x)
```

```
[Out] x**2*gamma(2/n)*hyper((1/2, 2/n), (1 + 2/n,), b*x**n*exp_polar(I*pi)/a)/(sq
rt(a)*n*gamma(1 + 2/n))
```

3.2503 $\int \frac{1}{\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=39

$$\frac{x\sqrt{a+bx^n}\ {}_2F_1\left(1,\frac{1}{2}+\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a}$$

```
[Out] x*hypergeom([1, 1/2+1/n],[1+1/n],-b*x^n/a)*(a+b*x^n)^(1/2)/a
```

Rubi [A] time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.23, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {246, 245}

$$\frac{x\sqrt{\frac{bx^n}{a}+1}\ {}_2F_1\left(\frac{1}{2},\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*x^n],x]
```

```
[Out] (x*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, n^(-1), 1 + n^(-1), -((b*x^n)
/a)])/Sqrt[a + b*x^n]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+bx^n}}\,dx &= \frac{\sqrt{1+\frac{bx^n}{a}}\int \frac{1}{\sqrt{1+\frac{bx^n}{a}}}\,dx}{\sqrt{a+bx^n}}\\ &= \frac{x\sqrt{1+\frac{bx^n}{a}}\ {}_2F_1\left(\frac{1}{2},\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{\sqrt{a+bx^n}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 48, normalized size = 1.23

$$\frac{x\sqrt{\frac{bx^n}{a}+1}\ {}_2F_1\left(\frac{1}{2},\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*x^n],x]
```

```
[Out] (x*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, n^(-1), 1 + n^(-1), -((b*x^n)
/a)])/Sqrt[a + b*x^n]
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^n + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*x^n + a), x)
```

maple [**F**] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\,x^n + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^n+a)^(1/2),x)
```

```
[Out] int(1/(b*x^n+a)^(1/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^n + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*x^n + a), x)
```

mupad [**B**] time = 1.39, size = 43, normalized size = 1.10

$$\frac{x\sqrt{\frac{b\,x^n}{a}+1}\;{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{1}{n};\;\frac{1}{n}+1;\;-\frac{b\,x^n}{a}\right)}{\sqrt{a+b\,x^n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^n)^(1/2),x)
```

```
[Out] (x*((b*x^n)/a + 1)^(1/2)*hypergeom([1/2, 1/n], 1/n + 1, -(b*x^n)/a))/(a + b
*x^n)^(1/2)
```

sympy [**C**] time = 1.09, size = 39, normalized size = 1.00

$$\frac{x\Gamma\left(\frac{1}{n}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|\,\frac{bx^n e^{i\pi}}{a}\right)}{\sqrt{a}\,n\Gamma\left(1+\frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n)**(1/2),x)
```

```
[Out] x*gamma(1/n)*hyper((1/2, 1/n), (1 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(sqrt(
a)*n*gamma(1 + 1/n))
```

3.2504 $\int \frac{1}{x\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=28

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

```
[Out] -2*arctanh((a+b*x^n)^(1/2)/a^(1/2))/n/a^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {266, 63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x^n]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{a+bx^n}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^n\right)}{n}\\ &= \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^n}\right)}{bn}\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}\,n}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x^n]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(Sqrt[a]*n)
```

fricas [A] time = 1.15, size = 70, normalized size = 2.50

$$\left[\frac{\log\left(\frac{bx^n-2\sqrt{bx^n+a}\sqrt{a}+2a}{x^n}\right)}{\sqrt{a}\,n}, \frac{2\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx^n+a}\sqrt{-a}}{a}\right)}{an}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n)/(sqrt(a)*n), 2*sqrt(-a)
*arctan(sqrt(b*x^n + a)*sqrt(-a)/a)/(a*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^n + a}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^n + a)*x), x)
```

maple [A] time = 0.00, size = 23, normalized size = 0.82

$$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^n+a}}{\sqrt{a}}\right)}{\sqrt{a}\;n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^n+a)^(1/2),x)
```

```
[Out] -2*arctanh((b*x^n+a)^(1/2)/a^(1/2))/n/a^(1/2)
```

maxima [A] time = 1.13, size = 39, normalized size = 1.39

$$\frac{\log\left(\frac{\sqrt{bx^n+a}-\sqrt{a}}{\sqrt{bx^n+a}+\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))/(sqrt(a)*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x\sqrt{a + b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^n)^(1/2)),x)
```

```
[Out] int(1/(x*(a + b*x^n)^(1/2)), x)
```

sympy [**A**] time = 1.33, size = 26, normalized size = 0.93

$$-\frac{2\,\operatorname{asinh}\left(\frac{\sqrt{a}\,x^{-\frac{n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x**n)**(1/2),x)
[Out] -2*asinh(sqrt(a)*x**(-n/2)/sqrt(b))/(sqrt(a)*n)
```

3.2505 $\int \frac{1}{x^2\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=49

$$-\frac{\sqrt{a+bx^n}\ {}_2F_1\left(1,\frac{1}{2}-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{ax}$$

```
[Out] -hypergeom([1, 1/2-1/n],[(-1+n)/n],-b*x^n/a)*(a+b*x^n)^(1/2)/a/x
```

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {365, 364}

$$-\frac{\sqrt{\frac{bx^n}{a}+1}\ {}_2F_1\left(\frac{1}{2},-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{x\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*x^n]),x]
```

```
[Out] -((Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, -n^(-1), -((1 - n)/n), -((b*x
^n)/a)])/(x*Sqrt[a + b*x^n]))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{a+bx^n}}\,dx &= \frac{\sqrt{1+\frac{bx^n}{a}}\int \frac{1}{x^2\sqrt{1+\frac{bx^n}{a}}}\,dx}{\sqrt{a+bx^n}}\\ &= -\frac{\sqrt{1+\frac{bx^n}{a}}\ {}_2F_1\left(\frac{1}{2},-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{x\sqrt{a+bx^n}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 55, normalized size = 1.12

$$-\frac{\sqrt{\frac{bx^n}{a}+1}\ {}_2F_1\left(\frac{1}{2},-\frac{1}{n};1-\frac{1}{n};-\frac{bx^n}{a}\right)}{x\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x^n]),x]
```

```
[Out] -((Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, -n^(-1), 1 - n^(-1), -((b*x^n
)/a)])/(x*Sqrt[a + b*x^n]))
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^n + a}\, x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^n + a)*x^2), x)
```

maple [**F**] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\, x^n + a}\ x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^n+a)^(1/2),x)
```

```
[Out] int(1/x^2/(b*x^n+a)^(1/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^n + a}\, x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^n + a)*x^2), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2\, \sqrt{a + b\, x^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^n)^(1/2)),x)
```

```
[Out] int(1/(x^2*(a + b*x^n)^(1/2)), x)
```

sympy [**C**] time = 1.33, size = 42, normalized size = 0.86

$$\frac{\Gamma\left(-\frac{1}{n}\right) {}_2F_1\left(\begin{matrix}\frac{1}{2}, -\frac{1}{n} \\ 1 - \frac{1}{n}\end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{\sqrt{a}\, nx\Gamma\left(1 - \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*x**n)**(1/2),x)
```

```
[Out] gamma(-1/n)*hyper((1/2, -1/n), (1 - 1/n,), b*x**n*exp_polar(I*pi)/a)/(sqrt(
a)*n*x*gamma(1 - 1/n))
```

3.2506 $\int \frac{1}{x^3\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=51

$$-\frac{\sqrt{a+bx^n}\ {}_2F_1\left(1,\frac{1}{2}-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2ax^2}$$

```
[Out] -1/2*hypergeom([1, 1/2-2/n],[(-2+n)/n],-b*x^n/a)*(a+b*x^n)^(1/2)/a/x^2
```

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {365, 364}

$$-\frac{\sqrt{\frac{bx^n}{a}+1}\ {}_2F_1\left(\frac{1}{2},-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2x^2\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*x^n]),x]
```

```
[Out] -(Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, -2/n, -((2 - n)/n), -((b*x^n)/
a)])/(2*x^2*Sqrt[a + b*x^n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{a+bx^n}}\,dx &= \frac{\sqrt{1+\frac{bx^n}{a}}\int \frac{1}{x^3\sqrt{1+\frac{bx^n}{a}}}\,dx}{\sqrt{a+bx^n}}\\ &= -\frac{\sqrt{1+\frac{bx^n}{a}}\ {}_2F_1\left(\frac{1}{2},-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2x^2\sqrt{a+bx^n}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 57, normalized size = 1.12

$$-\frac{\sqrt{\frac{bx^n}{a}+1}\ {}_2F_1\left(\frac{1}{2},-\frac{2}{n};1-\frac{2}{n};-\frac{bx^n}{a}\right)}{2x^2\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + b*x^n]),x]
```

```
[Out] -1/2*(Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, -2/n, 1 - 2/n, -((b*x^n)/a
)])/(x^2*Sqrt[a + b*x^n])
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^n + a}\, x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^n + a)*x^3), x)
```

maple [**F**] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\, x^n + a}\ x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^n+a)^(1/2),x)
```

```
[Out] int(1/x^3/(b*x^n+a)^(1/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^n + a}\, x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^n + a)*x^3), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^3\, \sqrt{a + b\, x^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^n)^(1/2)),x)
```

```
[Out] int(1/(x^3*(a + b*x^n)^(1/2)), x)
```

sympy [**C**] time = 1.58, size = 44, normalized size = 0.86

$$\frac{\Gamma\left(-\frac{2}{n}\right) {}_2F_1\left(\begin{matrix}\frac{1}{2}, -\frac{2}{n} \\ 1 - \frac{2}{n}\end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{\sqrt{a}\, nx^2 \Gamma\left(1 - \frac{2}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*x**n)**(1/2),x)
```

```
[Out] gamma(-2/n)*hyper((1/2, -2/n), (1 - 2/n,), b*x**n*exp_polar(I*pi)/a)/(sqrt(
a)*n*x**2*gamma(1 - 2/n))
```

3.2507 $\int \frac{x}{(a+bx^n)^{3/2}}\,dx$

Optimal. Leaf size=48

$$\frac{x^2\,{}_2F_1\left(1,\frac{2}{n}-\frac{1}{2};\frac{n+2}{n};-\frac{bx^n}{a}\right)}{2a\sqrt{a+bx^n}}$$

```
[Out] 1/2*x^2*hypergeom([1, -1/2+2/n],[(2+n)/n],-b*x^n/a)/a/(a+b*x^n)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.25, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {365, 364}

$$\frac{x^2\sqrt{\frac{bx^n}{a}+1}\;{}_2F_1\left(\frac{3}{2},\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right)}{2a\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^n)^(3/2),x]
```

```
[Out] (x^2*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[3/2, 2/n, (2 + n)/n, -((b*x^n)/a
)])/(2*a*Sqrt[a + b*x^n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx^n)^{3/2}}\,dx &= \frac{\sqrt{1+\frac{bx^n}{a}}\int \frac{x}{\left(1+\frac{bx^n}{a}\right)^{3/2}}\,dx}{a\sqrt{a+bx^n}}\\ &= \frac{x^2\sqrt{1+\frac{bx^n}{a}}\;{}_2F_1\left(\frac{3}{2},\frac{2}{n};\frac{2+n}{n};-\frac{bx^n}{a}\right)}{2a\sqrt{a+bx^n}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 60, normalized size = 1.25

$$\frac{x^2\sqrt{\frac{bx^n}{a}+1}\;{}_2F_1\left(\frac{3}{2},\frac{2}{n};1+\frac{2}{n};-\frac{bx^n}{a}\right)}{2a\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^n)^(3/2),x]
```

```
[Out] (x^2*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[3/2, 2/n, 1 + 2/n, -((b*x^n)/a)]
)/(2*a*Sqrt[a + b*x^n])
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(bx^n + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/(b*x^n + a)^(3/2), x)
```

maple [**F**] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{x}{(b\,x^n + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^n+a)^(3/2),x)
```

```
[Out] int(x/(b*x^n+a)^(3/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(bx^n + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*x^n + a)^(3/2), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{(a + b\,x^n)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^n)^(3/2),x)
```

```
[Out] int(x/(a + b*x^n)^(3/2), x)
```

sympy [**C**] time = 1.41, size = 41, normalized size = 0.85

$$\frac{x^2 \Gamma\left(\frac{2}{n}\right) {}_2F_1\left(\begin{matrix}\frac{3}{2}, \frac{2}{n} \\ 1 + \frac{2}{n}\end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{a^{\frac{3}{2}} n \Gamma\left(1 + \frac{2}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x**n)**(3/2),x)
```

```
[Out] x**2*gamma(2/n)*hyper((3/2, 2/n), (1 + 2/n,), b*x**n*exp_polar(I*pi)/a)/(a*
**(3/2)*n*gamma(1 + 2/n))
```

3.2508 $\int \frac{1}{(a+bx^n)^{3/2}}\,dx$

Optimal. Leaf size=39

$$\frac{x\,{}_2F_1\left(1,\frac{1}{n}-\frac{1}{2};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a\sqrt{a+bx^n}}$$

```
[Out] x*hypergeom([1, -1/2+1/n],[1+1/n],-b*x^n/a)/a/(a+b*x^n)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.31, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {246, 245}

$$\frac{x\sqrt{\frac{bx^n}{a}+1}\;{}_2F_1\left(\frac{3}{2},\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^(-3/2),x]
```

```
[Out] (x*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[3/2, n^(-1), 1 + n^(-1), -((b*x^n)
/a)])/(a*Sqrt[a + b*x^n])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx^n)^{3/2}}\,dx &= \frac{\sqrt{1+\frac{bx^n}{a}}\int \frac{1}{\left(1+\frac{bx^n}{a}\right)^{3/2}}\,dx}{a\sqrt{a+bx^n}}\\ &= \frac{x\sqrt{1+\frac{bx^n}{a}}\;{}_2F_1\left(\frac{3}{2},\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a\sqrt{a+bx^n}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 1.31

$$\frac{x\sqrt{\frac{bx^n}{a}+1}\;{}_2F_1\left(\frac{3}{2},\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^(-3/2),x]
```

```
[Out] (x*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[3/2, n^(-1), 1 + n^(-1), -((b*x^n)
/a)])/(a*Sqrt[a + b*x^n])
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^(-3/2), x)
```

maple [**F**] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^n + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^n+a)^(3/2),x)
```

```
[Out] int(1/(b*x^n+a)^(3/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^(-3/2), x)
```

mupad [**B**] time = 1.28, size = 43, normalized size = 1.10

$$\frac{x\left(\frac{b\,x^n}{a}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{1}{n};\ \frac{1}{n}+1;\ -\frac{b\,x^n}{a}\right)}{(a+b\,x^n)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^n)^(3/2),x)
```

```
[Out] (x*((b*x^n)/a + 1)^(3/2)*hypergeom([3/2, 1/n], 1/n + 1, -(b*x^n)/a))/(a + b
*x^n)^(3/2)
```

sympy [C] time = 1.37, size = 39, normalized size = 1.00

$$\frac{x\Gamma\left(\frac{1}{n}\right){}_2F_1\left(\begin{matrix}\frac{3}{2}, \frac{1}{n} \\ 1+\frac{1}{n}\end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{a^{\frac{3}{2}} n\Gamma\left(1+\frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n)**(3/2),x)
```

```
[Out] x*gamma(1/n)*hyper((3/2, 1/n), (1 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(a**(3
/2)*n*gamma(1 + 1/n))
```

3.2509 $\int \frac{1}{x(a+bx^n)^{3/2}}\,dx$

Optimal. Leaf size=48

$$\frac{2}{an\sqrt{a+bx^n}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}n}$$

```
[Out] -2*arctanh((a+b*x^n)^(1/2)/a^(1/2))/a^(3/2)/n+2/a/n/(a+b*x^n)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 51, 63, 208}

$$\frac{2}{an\sqrt{a+bx^n}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}n}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^n)^(3/2)),x]
```

```
[Out] 2/(a*n*Sqrt[a + b*x^n]) - (2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(a^(3/2)*n)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a+bx^n\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, x^n\right)}{n}\\ &= \frac{2}{an\sqrt{a+bx^n}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^n\right)}{an}\\ &= \frac{2}{an\sqrt{a+bx^n}} + \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^n}\right)}{abn}\\ &= \frac{2}{an\sqrt{a+bx^n}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}n}\end{aligned}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.77

$$\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx^n}{a}+1\right)}{an\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^n)^(3/2)),x]
```

```
[Out] (2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^n)/a])/(a*n*Sqrt[a + b*x^n])
```

fricas [A] time = 0.68, size = 144, normalized size = 3.00

$$\left[\frac{\left(\sqrt{a}\,bx^n + a^{\frac{3}{2}}\right)\log\left(\frac{bx^n - 2\sqrt{bx^n+a}\sqrt{a}+2a}{x^n}\right) + 2\sqrt{bx^n+a}\,a}{a^2bnx^n + a^3n}, \frac{2\left(\left(\sqrt{-a}\,bx^n + \sqrt{-a}\,a\right)\arctan\left(\frac{\sqrt{bx^n+a}\sqrt{-a}}{a}\right) + \sqrt{bx^n+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] [((sqrt(a)*b*x^n + a^(3/2))*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x
^n) + 2*sqrt(b*x^n + a)*a)/(a^2*b*n*x^n + a^3*n), 2*((sqrt(-a)*b*x^n + sqrt
(-a)*a)*arctan(sqrt(b*x^n + a)*sqrt(-a)/a) + sqrt(b*x^n + a)*a)/(a^2*b*n*x^
n + a^3*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^n+a\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^(3/2)*x), x)
```

maple [A] time = 0.00, size = 39, normalized size = 0.81

$$\frac{-\frac{2\,\text{arctanh}\left(\frac{\sqrt{b\,x^n+a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}} + \frac{2}{\sqrt{b\,x^n+a}\,a}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^n+a)^(3/2),x)
```

```
[Out] 1/n*(-2/a^(3/2)*arctanh((b*x^n+a)^(1/2)/a^(1/2))+2/a/(b*x^n+a)^(1/2))
```

maxima [A] time = 1.45, size = 57, normalized size = 1.19

$$\frac{\log\left(\frac{\sqrt{bx^n+a}-\sqrt{a}}{\sqrt{bx^n+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}n}+\frac{2}{\sqrt{bx^n+a}\,an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))/(a^(3/2)*n) +
2/(sqrt(b*x^n + a)*a*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\,(a+b\,x^n)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^n)^(3/2)),x)
```

```
[Out] int(1/(x*(a + b*x^n)^(3/2)), x)
```

sympy [B] time = 2.49, size = 184, normalized size = 3.83

$$\frac{2a^3\sqrt{1+\frac{bx^n}{a}}}{a^{\frac{9}{2}}n+a^{\frac{7}{2}}bnx^n}+\frac{a^3\log\left(\frac{bx^n}{a}\right)}{a^{\frac{9}{2}}n+a^{\frac{7}{2}}bnx^n}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^n}{a}}+1\right)}{a^{\frac{9}{2}}n+a^{\frac{7}{2}}bnx^n}+\frac{a^2bx^n\log\left(\frac{bx^n}{a}\right)}{a^{\frac{9}{2}}n+a^{\frac{7}{2}}bnx^n}-\frac{2a^2bx^n\log\left(\sqrt{1+\frac{bx^n}{a}}+1\right)}{a^{\frac{9}{2}}n+a^{\frac{7}{2}}bnx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x**n)**(3/2),x)
```

```
[Out] 2*a**3*sqrt(1 + b*x**n/a)/(a**(9/2)*n + a**(7/2)*b*n*x**n) + a**3*log(b*x**
n/a)/(a**(9/2)*n + a**(7/2)*b*n*x**n) - 2*a**3*log(sqrt(1 + b*x**n/a) + 1)/
(a**(9/2)*n + a**(7/2)*b*n*x**n) + a**2*b*x**n*log(b*x**n/a)/(a**(9/2)*n +
a**(7/2)*b*n*x**n) - 2*a**2*b*x**n*log(sqrt(1 + b*x**n/a) + 1)/(a**(9/2)*n
+ a**(7/2)*b*n*x**n)
```

3.2510 $\int \frac{1}{x^2(a+bx^n)^{3/2}}\,dx$

Optimal. Leaf size=49

$$-\frac{{}_2F_1\left(1,-\frac{1}{2}-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{ax\sqrt{a+bx^n}}$$

```
[Out] -hypergeom([1, -1/2-1/n],[(-1+n)/n],-b*x^n/a)/a/x/(a+b*x^n)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.24, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {365, 364}

$$-\frac{\sqrt{\frac{bx^n}{a}+1}\;{}_2F_1\left(\frac{3}{2},-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{ax\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^n)^(3/2)),x]
```

```
[Out] -((Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[3/2, -n^(-1), -((1 - n)/n), -((b*x
^n)/a)])/(a*x*Sqrt[a + b*x^n]))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\,(a+bx^n)^{3/2}}\,dx &= \frac{\sqrt{1+\frac{bx^n}{a}}\int \frac{1}{x^2\left(1+\frac{bx^n}{a}\right)^{3/2}}\,dx}{a\sqrt{a+bx^n}}\\ &= -\frac{\sqrt{1+\frac{bx^n}{a}}\;{}_2F_1\left(\frac{3}{2},-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{ax\sqrt{a+bx^n}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 58, normalized size = 1.18

$$-\frac{\sqrt{\frac{bx^n}{a}+1}\;{}_2F_1\left(\frac{3}{2},-\frac{1}{n};1-\frac{1}{n};-\frac{bx^n}{a}\right)}{ax\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^n)^(3/2)),x]
```

```
[Out] -((Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[3/2, -n^(-1), 1 - n^(-1), -((b*x^n
)/a)])/(a*x*Sqrt[a + b*x^n]))
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)^{\frac{3}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^(3/2)*x^2), x)
```

maple [**F**] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^n + a)^{\frac{3}{2}}\, x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^n+a)^(3/2),x)
```

```
[Out] int(1/x^2/(b*x^n+a)^(3/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)^{\frac{3}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a)^(3/2)*x^2), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2\,(a + b\,x^n)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^n)^(3/2)),x)
```

```
[Out] int(1/(x^2*(a + b*x^n)^(3/2)), x)
```

sympy [**C**] time = 1.91, size = 42, normalized size = 0.86

$$\frac{\Gamma\left(-\frac{1}{n}\right) {}_2F_1\left(\begin{matrix}\frac{3}{2}, -\frac{1}{n} \\ 1 - \frac{1}{n}\end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{a^{\frac{3}{2}} n x \Gamma\left(1 - \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*x**n)**(3/2),x)
```

```
[Out] gamma(-1/n)*hyper((3/2, -1/n), (1 - 1/n,), b*x**n*exp_polar(I*pi)/a)/(a**(3
/2)*n*x*gamma(1 - 1/n))
```

3.2511 $\int \frac{1}{x^3(a+bx^n)^{3/2}}\,dx$

Optimal. Leaf size=51

$$-\frac{{}_2F_1\left(1,-\frac{1}{2}-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2ax^2\sqrt{a+bx^n}}$$

```
[Out] -1/2*hypergeom([1, -1/2-2/n],[(-2+n)/n],-b*x^n/a)/a/x^2/(a+b*x^n)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.24, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {365, 364}

$$-\frac{\sqrt{\frac{bx^n}{a}+1}\;{}_2F_1\left(\frac{3}{2},-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2ax^2\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^n)^(3/2)),x]
```

```
[Out] -(Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[3/2, -2/n, -((2 - n)/n), -((b*x^n)/
a)])/(2*a*x^2*Sqrt[a + b*x^n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\,(a+bx^n)^{3/2}}\,dx &= \frac{\sqrt{1+\frac{bx^n}{a}}\int \frac{1}{x^3\left(1+\frac{bx^n}{a}\right)^{3/2}}\,dx}{a\sqrt{a+bx^n}}\\ &= -\frac{\sqrt{1+\frac{bx^n}{a}}\;{}_2F_1\left(\frac{3}{2},-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2ax^2\sqrt{a+bx^n}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 60, normalized size = 1.18

$$-\frac{\sqrt{\frac{bx^n}{a}+1}\;{}_2F_1\left(\frac{3}{2},-\frac{2}{n};1-\frac{2}{n};-\frac{bx^n}{a}\right)}{2ax^2\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^n)^(3/2)),x]
```

```
[Out] -1/2*(Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[3/2, -2/n, 1 - 2/n, -((b*x^n)/a
)])/(a*x^2*Sqrt[a + b*x^n])
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)^{\frac{3}{2}} x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^(3/2)*x^3), x)
```

maple [**F**] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^n + a)^{\frac{3}{2}}\, x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^n+a)^(3/2),x)
```

```
[Out] int(1/x^3/(b*x^n+a)^(3/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)^{\frac{3}{2}} x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a)^(3/2)*x^3), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^3\,(a + b\,x^n)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^n)^(3/2)),x)
```

```
[Out] int(1/(x^3*(a + b*x^n)^(3/2)), x)
```

sympy [**C**] time = 2.40, size = 44, normalized size = 0.86

$$\frac{\Gamma\left(-\frac{2}{n}\right) {}_2F_1\left(\begin{matrix}\frac{3}{2}, -\frac{2}{n} \\ 1 - \frac{2}{n}\end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{a^{\frac{3}{2}} n x^2 \Gamma\left(1 - \frac{2}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*x**n)**(3/2),x)
```

```
[Out] gamma(-2/n)*hyper((3/2, -2/n), (1 - 2/n,), b*x**n*exp_polar(I*pi)/a)/(a**(3
/2)*n*x**2*gamma(1 - 2/n))
```

3.2512 $\int \frac{x}{(a+bx^n)^{5/2}}\,dx$

Optimal. Leaf size=48

$$\frac{x^2\,{}_2F_1\left(1,\frac{2}{n}-\frac{3}{2};\frac{n+2}{n};-\frac{bx^n}{a}\right)}{2a\,(a+bx^n)^{3/2}}$$

```
[Out] 1/2*x^2*hypergeom([1, -3/2+2/n],[(2+n)/n],-b*x^n/a)/a/(a+b*x^n)^(3/2)
```

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.25, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {365, 364}

$$\frac{x^2\sqrt{\frac{bx^n}{a}+1}\;{}_2F_1\left(\frac{5}{2},\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right)}{2a^2\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^n)^(5/2),x]
```

```
[Out] (x^2*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[5/2, 2/n, (2 + n)/n, -((b*x^n)/a
)])/(2*a^2*Sqrt[a + b*x^n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx^n)^{5/2}}\,dx &= \frac{\sqrt{1+\frac{bx^n}{a}}\int \frac{x}{\left(1+\frac{bx^n}{a}\right)^{5/2}}\,dx}{a^2\sqrt{a+bx^n}}\\ &= \frac{x^2\sqrt{1+\frac{bx^n}{a}}\;{}_2F_1\left(\frac{5}{2},\frac{2}{n};\frac{2+n}{n};-\frac{bx^n}{a}\right)}{2a^2\sqrt{a+bx^n}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 60, normalized size = 1.25

$$\frac{x^2\sqrt{\frac{bx^n}{a}+1}\;{}_2F_1\left(\frac{5}{2},\frac{2}{n};1+\frac{2}{n};-\frac{bx^n}{a}\right)}{2a^2\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^n)^(5/2),x]
```

```
[Out] (x^2*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[5/2, 2/n, 1 + 2/n, -((b*x^n)/a)]
)/(2*a^2*Sqrt[a + b*x^n])
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(bx^n + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x/(b*x^n + a)^(5/2), x)
```

maple [**F**] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{x}{(b\,x^n + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^n+a)^(5/2),x)
```

```
[Out] int(x/(b*x^n+a)^(5/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(bx^n + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*x^n + a)^(5/2), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{(a + b\,x^n)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^n)^(5/2),x)
```

```
[Out] int(x/(a + b*x^n)^(5/2), x)
```

sympy [**C**] time = 2.45, size = 41, normalized size = 0.85

$$\frac{x^2 \Gamma\left(\frac{2}{n}\right) {}_2F_1\left(\begin{matrix}\frac{5}{2}, \frac{2}{n} \\ 1 + \frac{2}{n}\end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{a^{\frac{5}{2}} n \Gamma\left(1 + \frac{2}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x**n)**(5/2),x)
```

```
[Out] x**2*gamma(2/n)*hyper((5/2, 2/n), (1 + 2/n,), b*x**n*exp_polar(I*pi)/a)/(a*
**(5/2)*n*gamma(1 + 2/n))
```

3.2513 $\int \frac{1}{(a+bx^n)^{5/2}}\,dx$

Optimal. Leaf size=39

$$\frac{x\,{}_2F_1\left(1,\frac{1}{n}-\frac{3}{2};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a\,(a+bx^n)^{3/2}}$$

```
[Out] x*hypergeom([1, -3/2+1/n],[1+1/n],-b*x^n/a)/a/(a+b*x^n)^(3/2)
```

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.31, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {246, 245}

$$\frac{x\sqrt{\frac{bx^n}{a}+1}\;{}_2F_1\left(\frac{5}{2},\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^(-5/2),x]
```

```
[Out] (x*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[5/2, n^(-1), 1 + n^(-1), -((b*x^n)
/a)])/(a^2*Sqrt[a + b*x^n])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx^n)^{5/2}}\,dx &= \frac{\sqrt{1+\frac{bx^n}{a}}\int \frac{1}{\left(1+\frac{bx^n}{a}\right)^{5/2}}\,dx}{a^2\sqrt{a+bx^n}}\\ &= \frac{x\sqrt{1+\frac{bx^n}{a}}\;{}_2F_1\left(\frac{5}{2},\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2\sqrt{a+bx^n}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 1.31

$$\frac{x\sqrt{\frac{bx^n}{a}+1}\;{}_2F_1\left(\frac{5}{2},\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^(-5/2),x]
```

```
[Out] (x*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[5/2, n^(-1), 1 + n^(-1), -((b*x^n)
/a)])/(a^2*Sqrt[a + b*x^n])
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^(-5/2), x)
```

maple [**F**] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^n + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^n+a)^(5/2),x)
```

```
[Out] int(1/(b*x^n+a)^(5/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^(-5/2), x)
```

mupad [**B**] time = 1.36, size = 43, normalized size = 1.10

$$\frac{x\left(\frac{b\,x^n}{a}+1\right)^{5/2}\,{}_2\mathrm{F}_1\left(\frac{5}{2},\frac{1}{n};\ \frac{1}{n}+1;\ -\frac{b\,x^n}{a}\right)}{(a+b\,x^n)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^n)^(5/2),x)
```

```
[Out] (x*((b*x^n)/a + 1)^(5/2)*hypergeom([5/2, 1/n], 1/n + 1, -(b*x^n)/a))/(a + b
*x^n)^(5/2)
```

sympy [C] time = 2.37, size = 39, normalized size = 1.00

$$\frac{x\Gamma\left(\frac{1}{n}\right){}_2F_1\left(\begin{matrix}\frac{5}{2}, \frac{1}{n} \\ 1+\frac{1}{n}\end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{a^{\frac{5}{2}} n\Gamma\left(1+\frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n)**(5/2),x)
```

```
[Out] x*gamma(1/n)*hyper((5/2, 1/n), (1 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(a**(5
/2)*n*gamma(1 + 1/n))
```

3.2514 $\int \frac{1}{x(a+bx^n)^{5/2}}\,dx$

Optimal. Leaf size=69

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{5/2}n}+\frac{2}{a^2n\sqrt{a+bx^n}}+\frac{2}{3an\left(a+bx^n\right)^{3/2}}$$

```
[Out] 2/3/a/n/(a+b*x^n)^(3/2)-2*arctanh((a+b*x^n)^(1/2)/a^(1/2))/a^(5/2)/n+2/a^2/
n/(a+b*x^n)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 51, 63, 208}

$$\frac{2}{a^2n\sqrt{a+bx^n}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{5/2}n}+\frac{2}{3an\left(a+bx^n\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^n)^(5/2)),x]
```

```
[Out] 2/(3*a*n*(a + b*x^n)^(3/2)) + 2/(a^2*n*Sqrt[a + b*x^n]) - (2*ArcTanh[Sqrt[a
 + b*x^n]/Sqrt[a]])/(a^(5/2)*n)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^n\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, x^n\right)}{n}$$
$$= \frac{2}{3an\left(a+bx^n\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, x^n\right)}{an}$$
$$= \frac{2}{3an\left(a+bx^n\right)^{3/2}} + \frac{2}{a^2n\sqrt{a+bx^n}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^n\right)}{a^2n}$$
$$= \frac{2}{3an\left(a+bx^n\right)^{3/2}} + \frac{2}{a^2n\sqrt{a+bx^n}} + \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^n}\right)}{a^2bn}$$
$$= \frac{2}{3an\left(a+bx^n\right)^{3/2}} + \frac{2}{a^2n\sqrt{a+bx^n}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{5/2}n}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.57

$$\frac{2\,{}_2F_1\left(-\frac{3}{2}, 1; -\frac{1}{2}; \frac{bx^n}{a}+1\right)}{3an\left(a+bx^n\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^n)^(5/2)),x]
```

```
[Out] (2*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*x^n)/a])/(3*a*n*(a + b*x^n)^(3/2
))
```

fricas [A] time = 1.44, size = 230, normalized size = 3.33

$$\left[\frac{3\left(\sqrt{a}\,b^2x^{2\,n} + 2\,a^{\frac{3}{2}}bx^n + a^{\frac{5}{2}}\right)\log\left(\frac{bx^n - 2\sqrt{bx^n+a}\sqrt{a}+2\,a}{x^n}\right) + 2\left(3\,abx^n + 4\,a^2\right)\sqrt{bx^n+a}}{3\left(a^3b^2nx^{2\,n} + 2\,a^4bnx^n + a^5n\right)}, \frac{2\left(3\left(\sqrt{-a}\,b^2x^{2\,n} + 2\sqrt{-a}\,ab\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(sqrt(a)*b^2*x^(2*n) + 2*a^(3/2)*b*x^n + a^(5/2))*log((b*x^n - 2*sq
rt(b*x^n + a)*sqrt(a) + 2*a)/x^n) + 2*(3*a*b*x^n + 4*a^2)*sqrt(b*x^n + a))/
(a^3*b^2*n*x^(2*n) + 2*a^4*b*n*x^n + a^5*n), 2/3*(3*(sqrt(-a)*b^2*x^(2*n) +
 2*sqrt(-a)*a*b*x^n + sqrt(-a)*a^2)*arctan(sqrt(b*x^n + a)*sqrt(-a)/a) + (3
*a*b*x^n + 4*a^2)*sqrt(b*x^n + a))/(a^3*b^2*n*x^(2*n) + 2*a^4*b*n*x^n + a^5
*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^n+a\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^(5/2)*x), x)
```

maple [A] time = 0.01, size = 53, normalized size = 0.77

$$\frac{\frac{2}{3(b\,x^n+a)^{\frac{3}{2}}\,a}-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^n+a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2}{\sqrt{b\,x^n+a}\;a^2}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^n+a)^(5/2),x)
```

```
[Out] 1/n*(-2/a^(5/2)*arctanh((b*x^n+a)^(1/2)/a^(1/2))+2/a^2/(b*x^n+a)^(1/2)+2/3/
a/(b*x^n+a)^(3/2))
```

maxima [A] time = 1.18, size = 67, normalized size = 0.97

$$\frac{\log\left(\frac{\sqrt{bx^n+a}-\sqrt{a}}{\sqrt{bx^n+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}n}+\frac{2\,(3\,bx^n+4\,a)}{3\,(bx^n+a)^{\frac{3}{2}}a^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)^(5/2),x, algorithm="maxima")
```

```
[Out] log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))/(a^(5/2)*n) +
2/3*(3*b*x^n + 4*a)/((b*x^n + a)^(3/2)*a^2*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,(a+b\,x^n)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^n)^(5/2)),x)
```

```
[Out] int(1/(x*(a + b*x^n)^(5/2)), x)
```

sympy [B] time = 5.32, size = 860, normalized size = 12.46

$$\frac{8a^7\sqrt{1+\frac{bx^n}{a}}}{3a^{\frac{19}{2}}n+9a^{\frac{17}{2}}bnx^n+9a^{\frac{15}{2}}b^2nx^{2n}+3a^{\frac{13}{2}}b^3nx^{3n}}+\frac{3a^7\log\left(\frac{bx^n}{a}\right)}{3a^{\frac{19}{2}}n+9a^{\frac{17}{2}}bnx^n+9a^{\frac{15}{2}}b^2nx^{2n}+3a^{\frac{13}{2}}b^3nx^{3n}}-\frac{6}{3a^{\frac{19}{2}}n+9a^{\frac{17}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x**n)**(5/2),x)
```

```
[Out] 8*a**7*sqrt(1 + b*x**n/a)/(3*a**(19/2)*n + 9*a**(17/2)*b*n*x**n + 9*a**(15/
2)*b**2*n*x**(2*n) + 3*a**(13/2)*b**3*n*x**(3*n)) + 3*a**7*log(b*x**n/a)/(3
*a**(19/2)*n + 9*a**(17/2)*b*n*x**n + 9*a**(15/2)*b**2*n*x**(2*n) + 3*a**(1
3/2)*b**3*n*x**(3*n)) - 6*a**7*log(sqrt(1 + b*x**n/a) + 1)/(3*a**(19/2)*n +
 9*a**(17/2)*b*n*x**n + 9*a**(15/2)*b**2*n*x**(2*n) + 3*a**(13/2)*b**3*n*x*
*(3*n)) + 14*a**6*b*x**n*sqrt(1 + b*x**n/a)/(3*a**(19/2)*n + 9*a**(17/2)*b*
n*x**n + 9*a**(15/2)*b**2*n*x**(2*n) + 3*a**(13/2)*b**3*n*x**(3*n)) + 9*a**
6*b*x**n*log(b*x**n/a)/(3*a**(19/2)*n + 9*a**(17/2)*b*n*x**n + 9*a**(15/2)*
b**2*n*x**(2*n) + 3*a**(13/2)*b**3*n*x**(3*n)) - 18*a**6*b*x**n*log(sqrt(1
+ b*x**n/a) + 1)/(3*a**(19/2)*n + 9*a**(17/2)*b*n*x**n + 9*a**(15/2)*b**2*n
*x**(2*n) + 3*a**(13/2)*b**3*n*x**(3*n)) + 6*a**5*b**2*x**(2*n)*sqrt(1 + b*
x**n/a)/(3*a**(19/2)*n + 9*a**(17/2)*b*n*x**n + 9*a**(15/2)*b**2*n*x**(2*n)
 + 3*a**(13/2)*b**3*n*x**(3*n)) + 9*a**5*b**2*x**(2*n)*log(b*x**n/a)/(3*a**
(19/2)*n + 9*a**(17/2)*b*n*x**n + 9*a**(15/2)*b**2*n*x**(2*n) + 3*a**(13/2)
```

```
*b**3*n*x**(3*n)) - 18*a**5*b**2*x**(2*n)*log(sqrt(1 + b*x**n/a) + 1)/(3*a*
*(19/2)*n + 9*a**(17/2)*b*n*x**n + 9*a**(15/2)*b**2*n*x**(2*n) + 3*a**(13/2
)*b**3*n*x**(3*n)) + 3*a**4*b**3*x**(3*n)*log(b*x**n/a)/(3*a**(19/2)*n + 9*
a**(17/2)*b*n*x**n + 9*a**(15/2)*b**2*n*x**(2*n) + 3*a**(13/2)*b**3*n*x**(3
*n)) - 6*a**4*b**3*x**(3*n)*log(sqrt(1 + b*x**n/a) + 1)/(3*a**(19/2)*n + 9*
a**(17/2)*b*n*x**n + 9*a**(15/2)*b**2*n*x**(2*n) + 3*a**(13/2)*b**3*n*x**(3
*n))
```

3.2515 $\int \frac{1}{x^2(a+bx^n)^{5/2}}\,dx$

Optimal. Leaf size=49

$$-\frac{{}_2F_1\left(1,-\frac{3}{2}-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{ax\,(a+bx^n)^{3/2}}$$

```
[Out] -hypergeom([1, -3/2-1/n],[(-1+n)/n],-b*x^n/a)/a/x/(a+b*x^n)^(3/2)
```

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.24, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {365, 364}

$$-\frac{\sqrt{\frac{bx^n}{a}+1}\;{}_2F_1\left(\frac{5}{2},-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{a^2x\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^n)^(5/2)),x]
```

```
[Out] -((Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[5/2, -n^(-1), -((1 - n)/n), -((b*x
^n)/a)])/(a^2*x*Sqrt[a + b*x^n]))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\,(a+bx^n)^{5/2}}\,dx &= \frac{\sqrt{1+\frac{bx^n}{a}}\int \frac{1}{x^2\left(1+\frac{bx^n}{a}\right)^{5/2}}\,dx}{a^2\sqrt{a+bx^n}}\\ &= -\frac{\sqrt{1+\frac{bx^n}{a}}\;{}_2F_1\left(\frac{5}{2},-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{a^2x\sqrt{a+bx^n}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 58, normalized size = 1.18

$$-\frac{\sqrt{\frac{bx^n}{a}+1}\;{}_2F_1\left(\frac{5}{2},-\frac{1}{n};1-\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2x\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^n)^(5/2)),x]
```

```
[Out] -((Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[5/2, -n^(-1), 1 - n^(-1), -((b*x^n
)/a)])/(a^2*x*Sqrt[a + b*x^n]))
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)^{\frac{5}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^(5/2)*x^2), x)
```

maple [**F**] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^n + a)^{\frac{5}{2}}\, x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^n+a)^(5/2),x)
```

```
[Out] int(1/x^2/(b*x^n+a)^(5/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)^{\frac{5}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a)^(5/2)*x^2), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2\,(a + b\,x^n)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^n)^(5/2)),x)
```

```
[Out] int(1/(x^2*(a + b*x^n)^(5/2)), x)
```

sympy [**C**] time = 4.00, size = 42, normalized size = 0.86

$$\frac{\Gamma\left(-\frac{1}{n}\right) {}_2F_1\left(\begin{matrix}\frac{5}{2}, -\frac{1}{n} \\ 1 - \frac{1}{n}\end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{a^{\frac{5}{2}} n x \Gamma\left(1 - \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*x**n)**(5/2),x)
```

```
[Out] gamma(-1/n)*hyper((5/2, -1/n), (1 - 1/n,), b*x**n*exp_polar(I*pi)/a)/(a**(5
/2)*n*x*gamma(1 - 1/n))
```

3.2516 $\int \frac{1}{x^3(a+bx^n)^{5/2}}\,dx$

Optimal. Leaf size=51

$$-\frac{{}_2F_1\left(1,-\frac{3}{2}-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2ax^2\left(a+bx^n\right)^{3/2}}$$

```
[Out] -1/2*hypergeom([1, -3/2-2/n],[(-2+n)/n],-b*x^n/a)/a/x^2/(a+b*x^n)^(3/2)
```

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.24, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {365, 364}

$$-\frac{\sqrt{\frac{bx^n}{a}+1}\;{}_2F_1\left(\frac{5}{2},-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2a^2x^2\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^n)^(5/2)),x]
```

```
[Out] -(Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[5/2, -2/n, -((2 - n)/n), -((b*x^n)/
a)])/(2*a^2*x^2*Sqrt[a + b*x^n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a+bx^n\right)^{5/2}}\,dx &= \frac{\sqrt{1+\frac{bx^n}{a}}\int \frac{1}{x^3\left(1+\frac{bx^n}{a}\right)^{5/2}}\,dx}{a^2\sqrt{a+bx^n}}\\ &= -\frac{\sqrt{1+\frac{bx^n}{a}}\;{}_2F_1\left(\frac{5}{2},-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2a^2x^2\sqrt{a+bx^n}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 60, normalized size = 1.18

$$-\frac{\sqrt{\frac{bx^n}{a}+1}\;{}_2F_1\left(\frac{5}{2},-\frac{2}{n};1-\frac{2}{n};-\frac{bx^n}{a}\right)}{2a^2x^2\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^n)^(5/2)),x]
```

```
[Out] -1/2*(Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[5/2, -2/n, 1 - 2/n, -((b*x^n)/a
)])/(a^2*x^2*Sqrt[a + b*x^n])
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)^{\frac{5}{2}} x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^(5/2)*x^3), x)
```

maple [**F**] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^n + a)^{\frac{5}{2}}\, x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^n+a)^(5/2),x)
```

```
[Out] int(1/x^3/(b*x^n+a)^(5/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)^{\frac{5}{2}} x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a)^(5/2)*x^3), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^3\,(a + b\,x^n)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^n)^(5/2)),x)
```

```
[Out] int(1/(x^3*(a + b*x^n)^(5/2)), x)
```

sympy [**C**] time = 5.23, size = 44, normalized size = 0.86

$$\frac{\Gamma\left(-\frac{2}{n}\right) {}_2F_1\left(\begin{matrix}\frac{5}{2}, -\frac{2}{n} \\ 1 - \frac{2}{n}\end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{a^{\frac{5}{2}} n x^2 \Gamma\left(1 - \frac{2}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*x**n)**(5/2),x)
```

```
[Out] gamma(-2/n)*hyper((5/2, -2/n), (1 - 2/n,), b*x**n*exp_polar(I*pi)/a)/(a**(5
/2)*n*x**2*gamma(1 - 2/n))
```

3.2517 $\int \frac{\sqrt[3]{a+bx^n}}{x}\,dx$

Optimal. Leaf size=106

$$\frac{3\sqrt[3]{a+bx^n}}{n}+\frac{3\sqrt[3]{a}\,\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^n}\right)}{2n}-\frac{\sqrt{3}\,\sqrt[3]{a}\,\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^n}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{n}-\frac{1}{2}\sqrt[3]{a}\,\log(x)$$

```
[Out] 3*(a+b*x^n)^(1/3)/n-1/2*a^(1/3)*ln(x)+3/2*a^(1/3)*ln(a^(1/3)-(a+b*x^n)^(1/3
))/n-a^(1/3)*arctan(1/3*(a^(1/3)+2*(a+b*x^n)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2
)/n
```

Rubi [A] time = 0.07, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {266, 50, 57, 617, 204, 31}

$$\frac{3\sqrt[3]{a+bx^n}}{n}+\frac{3\sqrt[3]{a}\,\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^n}\right)}{2n}-\frac{\sqrt{3}\,\sqrt[3]{a}\,\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^n}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{n}-\frac{1}{2}\sqrt[3]{a}\,\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^(1/3)/x,x]
```

```
[Out] (3*(a + b*x^n)^(1/3))/n - (Sqrt[3]*a^(1/3)*ArcTan[(a^(1/3) + 2*(a + b*x^n)^
(1/3))/(Sqrt[3]*a^(1/3))])/n - (a^(1/3)*Log[x])/2 + (3*a^(1/3)*Log[a^(1/3)
- (a + b*x^n)^(1/3)])/(2*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{a+bx^n}}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x}\,dx, x, x^n\right)}{n} \\
&= \frac{3\sqrt[3]{a+bx^n}}{n} + \frac{a\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^n\right)}{n} \\
&= \frac{3\sqrt[3]{a+bx^n}}{n} - \frac{1}{2}\sqrt[3]{a}\,\log(x) - \frac{\left(3\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^n}\right)}{2n} - \frac{\left(3a^{2/3}\right)\text{Subst}\left(\int \frac{}{a^{2/3}+}\right.}{2} \\
&= \frac{3\sqrt[3]{a+bx^n}}{n} - \frac{1}{2}\sqrt[3]{a}\,\log(x) + \frac{3\sqrt[3]{a}\,\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^n}\right)}{2n} + \frac{\left(3\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2}{}\right.}{n} \\
&= \frac{3\sqrt[3]{a+bx^n}}{n} - \frac{\sqrt{3}\,\sqrt[3]{a}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^n}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{n} - \frac{1}{2}\sqrt[3]{a}\,\log(x) + \frac{3\sqrt[3]{a}\,\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^n}\right)}{2n}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 129, normalized size = 1.22

$$\frac{-\sqrt[3]{a}\,\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx^n}+(a+bx^n)^{2/3}\right)+6\sqrt[3]{a+bx^n}+2\sqrt[3]{a}\,\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^n}\right)-2\sqrt{3}\,\sqrt[3]{a}\,\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+b}}{\sqrt[3]{a}}}{\sqrt{}}\right.}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^(1/3)/x,x]
```

```
[Out] (6*(a + b*x^n)^(1/3) - 2*Sqrt[3]*a^(1/3)*ArcTan[(1 + (2*(a + b*x^n)^(1/3))/
a^(1/3))/Sqrt[3]] + 2*a^(1/3)*Log[a^(1/3) - (a + b*x^n)^(1/3)] - a^(1/3)*Lo
g[a^(2/3) + a^(1/3)*(a + b*x^n)^(1/3) + (a + b*x^n)^(2/3)])/(2*n)
```

fricas [A] time = 0.70, size = 106, normalized size = 1.00

$$-\frac{2\sqrt{3}\,a^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,(bx^n+a)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}\,a}{3\,a}\right)+a^{\frac{1}{3}}\log\left((bx^n+a)^{\frac{2}{3}}+(bx^n+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2\,a^{\frac{1}{3}}\log\left((bx^n+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(1/3)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*a^(1/3)*arctan(1/3*(2*sqrt(3)*(b*x^n + a)^(1/3)*a^(2/3) + s
qrt(3)*a)/a) + a^(1/3)*log((b*x^n + a)^(2/3) + (b*x^n + a)^(1/3)*a^(1/3) +
a^(2/3)) - 2*a^(1/3)*log((b*x^n + a)^(1/3) - a^(1/3)) - 6*(b*x^n + a)^(1/3)
)/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^{\frac{1}{3}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(1/3)/x,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^(1/3)/x, x)
```

maple [A] time = 0.01, size = 107, normalized size = 1.01

$$-\frac{\sqrt{3}\, a^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3}\left(\frac{2\left(b x^n+a\right)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{n}+\frac{a^{\frac{1}{3}} \ln\left(-a^{\frac{1}{3}}+\left(b x^n+a\right)^{\frac{1}{3}}\right)}{n}-\frac{a^{\frac{1}{3}} \ln\left(a^{\frac{2}{3}}+\left(b x^n+a\right)^{\frac{1}{3}} a^{\frac{1}{3}}+\left(b x^n+a\right)^{\frac{2}{3}}\right)}{2n}+\frac{3\,(b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^(1/3)/x,x)
```

```
[Out] 3*(b*x^n+a)^(1/3)/n+1/n*a^(1/3)*ln((b*x^n+a)^(1/3)-a^(1/3))-1/2/n*a^(1/3)*l
n((b*x^n+a)^(2/3)+a^(1/3)*(b*x^n+a)^(1/3)+a^(2/3))-1/n*a^(1/3)*3^(1/2)*arct
an(1/3*3^(1/2)*(2/a^(1/3)*(b*x^n+a)^(1/3)+1))
```

maxima [A] time = 1.32, size = 108, normalized size = 1.02

$$-\frac{\sqrt{3}\, a^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3}\left(2\left(bx^n+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\, a^{\frac{1}{3}}}\right)}{n}-\frac{a^{\frac{1}{3}} \log\left(\left(bx^n+a\right)^{\frac{2}{3}}+\left(bx^n+a\right)^{\frac{1}{3}} a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2\,n}+\frac{a^{\frac{1}{3}} \log\left(\left(bx^n+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{n}+\frac{3\,(b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(1/3)/x,x, algorithm="maxima")
```

```
[Out] -sqrt(3)*a^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x^n + a)^(1/3) + a^(1/3))/a^(1/3)
)/n - 1/2*a^(1/3)*log((b*x^n + a)^(2/3) + (b*x^n + a)^(1/3)*a^(1/3) + a^(2/
3))/n + a^(1/3)*log((b*x^n + a)^(1/3) - a^(1/3))/n + 3*(b*x^n + a)^(1/3)/n
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\, x^n)^{1/3}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^(1/3)/x,x)
```

```
[Out] int((a + b*x^n)^(1/3)/x, x)
```

sympy [C] time = 1.34, size = 46, normalized size = 0.43

$$-\frac{\sqrt[3]{b}\, x^{\frac{n}{3}} \Gamma\left(-\frac{1}{3}\right) {}_2F_1\left(\left.\begin{matrix} -\frac{1}{3}, -\frac{1}{3} \\ \frac{2}{3} \end{matrix}\right| \frac{a x^{-n} e^{i\pi}}{b}\right)}{n \Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**(1/3)/x,x)
```

```
[Out] -b**(1/3)*x**(n/3)*gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), a*x**(-n)*exp_po
lar(I*pi)/b)/(n*gamma(2/3))
```

3.2518 $\int x^{-1+4n}\left(a+bx^n\right)\,dx$

Optimal. Leaf size=27

$$\frac{ax^{4n}}{4n}+\frac{bx^{5n}}{5n}$$

```
[Out] 1/4*a*x^(4*n)/n+1/5*b*x^(5*n)/n
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {14}

$$\frac{ax^{4n}}{4n}+\frac{bx^{5n}}{5n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 4*n)*(a + b*x^n),x]
```

```
[Out] (a*x^(4*n))/(4*n) + (b*x^(5*n))/(5*n)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^{-1+4n}\left(a+bx^n\right)\,dx &= \int\left(ax^{-1+4n}+bx^{-1+5n}\right)\,dx\\ &= \frac{ax^{4n}}{4n}+\frac{bx^{5n}}{5n}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.81

$$\frac{x^{4n}\left(5a+4bx^n\right)}{20n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 4*n)*(a + b*x^n),x]
```

```
[Out] (x^(4*n)*(5*a + 4*b*x^n))/(20*n)
```

fricas [A] time = 0.61, size = 22, normalized size = 0.81

$$\frac{4\,bx^{5\,n}+5\,ax^{4\,n}}{20\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)*(a+b*x^n),x, algorithm="fricas")
```

```
[Out] 1/20*(4*b*x^(5*n) + 5*a*x^(4*n))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n+a)x^{4\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)*(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)*x^(4*n - 1), x)
```

maple [A] time = 0.02, size = 28, normalized size = 1.04

$$\frac{a\,\mathrm{e}^{4n\ln(x)}}{4n}+\frac{b\,\mathrm{e}^{5n\ln(x)}}{5n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+4*n)*(b*x^n+a),x)
```

```
[Out] 1/4*a/n*exp(n*ln(x))^4+1/5*b/n*exp(n*ln(x))^5
```

maxima [A] time = 0.67, size = 23, normalized size = 0.85

$$\frac{b x^{5\,n}}{5\,n}+\frac{a x^{4\,n}}{4\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)*(a+b*x^n),x, algorithm="maxima")
```

```
[Out] 1/5*b*x^(5*n)/n + 1/4*a*x^(4*n)/n
```

mupad [B] time = 1.26, size = 20, normalized size = 0.74

$$\frac{x^{4\,n}\,(5\,a+4\,b\,x^n)}{20\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4*n - 1)*(a + b*x^n),x)
```

```
[Out] (x^(4*n)*(5*a + 4*b*x^n))/(20*n)
```

sympy [A] time = 4.85, size = 26, normalized size = 0.96

$$\begin{cases}\frac{ax^{4n}}{4n}+\frac{bx^{5n}}{5n} & \text{for } n\neq 0\\ (a+b)\log(x) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+4*n)*(a+b*x**n),x)
```

```
[Out] Piecewise((a*x**(4*n)/(4*n) + b*x**(5*n)/(5*n), Ne(n, 0)), ((a + b)*log(x),
 True))
```

3.2519 $\int x^{-1+3n}\left(a+bx^n\right)\,dx$

Optimal. Leaf size=27

$$\frac{ax^{3n}}{3n}+\frac{bx^{4n}}{4n}$$

```
[Out] 1/3*a*x^(3*n)/n+1/4*b*x^(4*n)/n
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {14}

$$\frac{ax^{3n}}{3n}+\frac{bx^{4n}}{4n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)*(a + b*x^n),x]
```

```
[Out] (a*x^(3*n))/(3*n) + (b*x^(4*n))/(4*n)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^{-1+3n}\left(a+bx^n\right)\,dx &= \int\left(ax^{-1+3n}+bx^{-1+4n}\right)\,dx\\ &= \frac{ax^{3n}}{3n}+\frac{bx^{4n}}{4n}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.81

$$\frac{x^{3n}\left(4a+3bx^n\right)}{12n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)*(a + b*x^n),x]
```

```
[Out] (x^(3*n)*(4*a + 3*b*x^n))/(12*n)
```

fricas [A] time = 0.71, size = 22, normalized size = 0.81

$$\frac{3\,bx^{4\,n}+4\,ax^{3\,n}}{12\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b*x^(4*n) + 4*a*x^(3*n))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n+a)x^{3\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n),x, algorithm="giac")
[Out] integrate((b*x^n + a)*x^(3*n - 1), x)
```

maple [A] time = 0.02, size = 28, normalized size = 1.04

$$\frac{a\,\mathrm{e}^{3n\ln(x)}}{3n}+\frac{b\,\mathrm{e}^{4n\ln(x)}}{4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n-1)*(b*x^n+a),x)
[Out] 1/3*a/n*exp(n*ln(x))^3+1/4*b/n*exp(n*ln(x))^4
```

maxima [A] time = 0.49, size = 23, normalized size = 0.85

$$\frac{b x^{4\,n}}{4\,n}+\frac{a x^{3\,n}}{3\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n),x, algorithm="maxima")
[Out] 1/4*b*x^(4*n)/n + 1/3*a*x^(3*n)/n
```

mupad [B] time = 1.27, size = 20, normalized size = 0.74

$$\frac{x^{3\,n}\,\left(4\,a+3\,b\,x^n\right)}{12\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n - 1)*(a + b*x^n),x)
[Out] (x^(3*n)*(4*a + 3*b*x^n))/(12*n)
```

sympy [A] time = 4.70, size = 26, normalized size = 0.96

$$\begin{cases} \frac{ax^{3n}}{3n}+\frac{bx^{4n}}{4n} & \text{for } n\neq 0 \\ (a+b)\log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)*(a+b*x**n),x)
[Out] Piecewise((a*x**(3*n)/(3*n) + b*x**(4*n)/(4*n), Ne(n, 0)), ((a + b)*log(x),
 True))
```

3.2520 $\int x^{-1+2n}\left(a+bx^n\right)\,dx$

Optimal. Leaf size=27

$$\frac{ax^{2n}}{2n}+\frac{bx^{3n}}{3n}$$

```
[Out] 1/2*a*x^(2*n)/n+1/3*b*x^(3*n)/n
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {14}

$$\frac{ax^{2n}}{2n}+\frac{bx^{3n}}{3n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)*(a + b*x^n),x]
```

```
[Out] (a*x^(2*n))/(2*n) + (b*x^(3*n))/(3*n)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^{-1+2n}\left(a+bx^n\right)\,dx &= \int\left(ax^{-1+2n}+bx^{-1+3n}\right)\,dx\\ &= \frac{ax^{2n}}{2n}+\frac{bx^{3n}}{3n}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.81

$$\frac{x^{2n}\left(3a+2bx^n\right)}{6n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)*(a + b*x^n),x]
```

```
[Out] (x^(2*n)*(3*a + 2*b*x^n))/(6*n)
```

fricas [A] time = 0.50, size = 22, normalized size = 0.81

$$\frac{2\,bx^{3\,n}+3\,ax^{2\,n}}{6\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*x^(3*n) + 3*a*x^(2*n))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n+a)x^{2\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)*x^(2*n - 1), x)
```

maple [A] time = 0.02, size = 28, normalized size = 1.04

$$\frac{a\,\mathrm{e}^{2n\ln(x)}}{2n}+\frac{b\,\mathrm{e}^{3n\ln(x)}}{3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n-1)*(b*x^n+a),x)
```

```
[Out] 1/2*a/n*exp(n*ln(x))^2+1/3*b/n*exp(n*ln(x))^3
```

maxima [A] time = 0.52, size = 23, normalized size = 0.85

$$\frac{b x^{3\,n}}{3\,n}+\frac{a x^{2\,n}}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n),x, algorithm="maxima")
```

```
[Out] 1/3*b*x^(3*n)/n + 1/2*a*x^(2*n)/n
```

mupad [B] time = 1.26, size = 20, normalized size = 0.74

$$\frac{x^{2\,n}\,(3\,a+2\,b\,x^n)}{6\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)*(a + b*x^n),x)
```

```
[Out] (x^(2*n)*(3*a + 2*b*x^n))/(6*n)
```

sympy [A] time = 4.74, size = 26, normalized size = 0.96

$$\begin{cases}\frac{ax^{2n}}{2n}+\frac{bx^{3n}}{3n} & \text{for } n\neq 0\\ (a+b)\log(x) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a+b*x**n),x)
```

```
[Out] Piecewise((a*x**(2*n)/(2*n) + b*x**(3*n)/(3*n), Ne(n, 0)), ((a + b)*log(x),
 True))
```

3.2521 $\int x^{-1+n}\left(a+bx^n\right)\,dx$

Optimal. Leaf size=22

$$\frac{ax^n}{n}+\frac{bx^{2n}}{2n}$$

```
[Out] a*x^n/n+1/2*b*x^(2*n)/n
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$\frac{ax^n}{n}+\frac{bx^{2n}}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*(a + b*x^n),x]
```

```
[Out] (a*x^n)/n + (b*x^(2*n))/(2*n)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^{-1+n}\left(a+bx^n\right)\,dx &= \int\left(ax^{-1+n}+bx^{-1+2n}\right)\,dx\\ &= \frac{ax^n}{n}+\frac{bx^{2n}}{2n}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 0.86

$$\frac{x^n\left(2a+bx^n\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*(a + b*x^n),x]
```

```
[Out] (x^n*(2*a + b*x^n))/(2*n)
```

fricas [A] time = 0.76, size = 19, normalized size = 0.86

$$\frac{bx^{2\,n}+2\,ax^n}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(a+b*x^n),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^(2*n) + 2*a*x^n)/n
```

giac [A] time = 0.15, size = 19, normalized size = 0.86

$$\frac{bx^{2\,n}+2\,ax^n}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(a+b*x^n),x, algorithm="giac")
[Out] 1/2*(b*x^(2*n) + 2*a*x^n)/n
```

maple [A] time = 0.02, size = 25, normalized size = 1.14

$$\frac{a\,\mathrm{e}^{n\ln(x)}}{n}+\frac{b\,\mathrm{e}^{2n\ln(x)}}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*(b*x^n+a),x)
[Out] a/n*exp(n*ln(x))+1/2*b/n*exp(n*ln(x))^2
```

maxima [A] time = 0.58, size = 17, normalized size = 0.77

$$\frac{(bx^n+a)^2}{2\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(a+b*x^n),x, algorithm="maxima")
[Out] 1/2*(b*x^n + a)^2/(b*n)
```

mupad [B] time = 1.24, size = 15, normalized size = 0.68

$$\frac{x^n\left(a+\frac{b\,x^n}{2}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*(a + b*x^n),x)
[Out] (x^n*(a + (b*x^n)/2))/n
```

sympy [A] time = 1.59, size = 22, normalized size = 1.00

$$\begin{cases}\frac{ax^n}{n}+\frac{bx^{2n}}{2n} & \text{for } n\neq 0\\ (a+b)\log(x) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(a+b*x**n),x)
[Out] Piecewise((a*x**n/n + b*x**(2*n)/(2*n), Ne(n, 0)), ((a + b)*log(x), True))
```

3.2522 $\int \frac{a+bx^n}{x}\,dx$

Optimal. Leaf size=13

$$a\log(x)+\frac{bx^n}{n}$$

```
[Out] b*x^n/n+a*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$a\log(x)+\frac{bx^n}{n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)/x,x]
```

```
[Out] (b*x^n)/n + a*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^n}{x}\,dx &= \int \left(\frac{a}{x}+bx^{-1+n}\right)dx\\ &= \frac{bx^n}{n}+a\log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$a\log(x)+\frac{bx^n}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)/x,x]
```

```
[Out] (b*x^n)/n + a*Log[x]
```

fricas [A] time = 0.60, size = 15, normalized size = 1.15

$$\frac{an\log(x)+bx^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/x,x, algorithm="fricas")
```

```
[Out] (a*n*log(x) + b*x^n)/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx^n+a}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/x,x, algorithm="giac")
[Out] integrate((b*x^n + a)/x, x)
```

maple [A] time = 0.00, size = 19, normalized size = 1.46

$$\frac{a \ln(x^n)}{n} + \frac{b\, x^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)/x,x)
[Out] a/n*ln(x^n)+b/n*x^n
```

maxima [A] time = 0.50, size = 18, normalized size = 1.38

$$\frac{bx^n}{n} + \frac{a \log(x^n)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/x,x, algorithm="maxima")
[Out] b*x^n/n + a*log(x^n)/n
```

mupad [B] time = 0.00, size = 13, normalized size = 1.00

$$a\, \ln(x) + \frac{b\, x^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)/x,x)
[Out] a*log(x) + (b*x^n)/n
```

sympy [A] time = 0.20, size = 17, normalized size = 1.31

$$\begin{cases} a \log(x) + \frac{bx^n}{n} & \text{for } n \neq 0 \\ (a+b) \log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)/x,x)
[Out] Piecewise((a*log(x) + b*x**n/n, Ne(n, 0)), ((a + b)*log(x), True))
```

3.2523 $\int x^{-1-n}\left(a+bx^n\right)\,dx$

Optimal. Leaf size=16

$$b\log(x)-\frac{ax^{-n}}{n}$$

```
[Out] -a/n/(x^n)+b*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {14}

$$b\log(x)-\frac{ax^{-n}}{n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n)*(a + b*x^n),x]
```

```
[Out] -(a/(n*x^n)) + b*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^{-1-n}\left(a+bx^n\right)\,dx &= \int\left(\frac{b}{x}+ax^{-1-n}\right)dx\\ &= -\frac{ax^{-n}}{n}+b\log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$b\log(x)-\frac{ax^{-n}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n)*(a + b*x^n),x]
```

```
[Out] -(a/(n*x^n)) + b*Log[x]
```

fricas [A] time = 0.57, size = 21, normalized size = 1.31

$$\frac{bnx^n\log(x)-a}{nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*(a+b*x^n),x, algorithm="fricas")
```

```
[Out] (b*n*x^n*log(x) - a)/(n*x^n)
```

giac [A] time = 0.18, size = 21, normalized size = 1.31

$$\frac{bnx^n\log(x)-a}{nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*(a+b*x^n),x, algorithm="giac")
[Out] (b*n*x^n*log(x) - a)/(n*x^n)
```

maple [A] time = 0.01, size = 25, normalized size = 1.56

$$\left(b\,\mathrm{e}^{n\ln(x)}\ln(x)-\frac{a}{n}\right)\mathrm{e}^{-n\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n)*(b*x^n+a),x)
[Out] (b*ln(x)*exp(n*ln(x))-a/n)/exp(n*ln(x))
```

maxima [A] time = 0.67, size = 16, normalized size = 1.00

$$b\log(x)-\frac{a}{nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*(a+b*x^n),x, algorithm="maxima")
[Out] b*log(x) - a/(n*x^n)
```

mupad [B] time = 1.41, size = 29, normalized size = 1.81

$$\begin{cases} \ln(x)\ (a+b) & \text{if } n=0 \\ b\ \ln(x)-\frac{a}{n\,x^n} & \text{if } n\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)/x^(n + 1),x)
[Out] piecewise(n == 0, log(x)*(a + b), n ~= 0, b*log(x) - a/(n*x^n))
```

sympy [A] time = 12.86, size = 107, normalized size = 6.69

$$\begin{cases} ax+b\log(x) & \text{for } n=-1 \\ (a+b)\log(x) & \text{for } n=0 \\ -\frac{an}{n^2x^n+nx^n}-\frac{a}{n^2x^n+nx^n}+\frac{bn^2x^n\log(x)}{n^2x^n+nx^n}+\frac{bnx^n\log(x)}{n^2x^n+nx^n}+\frac{bnx^n}{n^2x^n+nx^n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n)*(a+b*x**n),x)
[Out] Piecewise((a*x + b*log(x), Eq(n, -1)), ((a + b)*log(x), Eq(n, 0)), (-a*n/(n
**2*x**n + n*x**n) - a/(n**2*x**n + n*x**n) + b*n**2*x**n*log(x)/(n**2*x**n
 + n*x**n) + b*n*x**n*log(x)/(n**2*x**n + n*x**n) + b*n*x**n/(n**2*x**n + n
*x**n), True))
```

3.2524 $\int x^{-1-2n}\left(a+bx^n\right)\,dx$

Optimal. Leaf size=25

$$-\frac{ax^{-2n}}{2n}-\frac{bx^{-n}}{n}$$

```
[Out] -1/2*a/n/(x^(2*n))-b/n/(x^n)
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {14}

$$-\frac{ax^{-2n}}{2n}-\frac{bx^{-n}}{n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 2*n)*(a + b*x^n),x]
```

```
[Out] -a/(2*n*x^(2*n)) - b/(n*x^n)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^{-1-2n}\left(a+bx^n\right)\,dx &= \int\left(ax^{-1-2n}+bx^{-1-n}\right)\,dx\\ &= -\frac{ax^{-2n}}{2n}-\frac{bx^{-n}}{n}\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.80

$$-\frac{x^{-2n}\left(a+2bx^n\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 2*n)*(a + b*x^n),x]
```

```
[Out] -1/2*(a + 2*b*x^n)/(n*x^(2*n))
```

fricas [A] time = 0.65, size = 20, normalized size = 0.80

$$-\frac{2\,bx^n+a}{2\,nx^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*(a+b*x^n),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*x^n + a)/(n*x^(2*n))
```

giac [A] time = 0.18, size = 20, normalized size = 0.80

$$-\frac{2\,bx^n+a}{2\,nx^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*(a+b*x^n),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*x^n + a)/(n*x^(2*n))
```

maple [A] time = 0.02, size = 27, normalized size = 1.08

$$\left(-\frac{b\,\mathrm{e}^{n\ln(x)}}{n}-\frac{a}{2n}\right)\mathrm{e}^{-2n\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-2*n)*(b*x^n+a),x)
```

```
[Out] (-1/2*a/n-b/n*exp(n*ln(x)))/exp(n*ln(x))^2
```

maxima [A] time = 0.61, size = 25, normalized size = 1.00

$$-\frac{a}{2\,nx^{2\,n}}-\frac{b}{nx^{n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*(a+b*x^n),x, algorithm="maxima")
```

```
[Out] -1/2*a/(n*x^(2*n)) - b/(n*x^n)
```

mupad [B] time = 1.21, size = 20, normalized size = 0.80

$$-\frac{a+2\,b\,x^{n}}{2\,n\,x^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)/x^(2*n + 1),x)
```

```
[Out] -(a + 2*b*x^n)/(2*n*x^(2*n))
```

sympy [A] time = 4.95, size = 24, normalized size = 0.96

$$\begin{cases}-\frac{ax^{-2n}}{2n}-\frac{bx^{-n}}{n} & \text{for } n\neq 0\\ (a+b)\log(x) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-2*n)*(a+b*x**n),x)
```

```
[Out] Piecewise((-a*x**(-2*n)/(2*n) - b*x**(-n)/n, Ne(n, 0)), ((a + b)*log(x), Tr
ue))
```

3.2525 $\int x^{-1-3n}\left(a+bx^n\right)\,dx$

Optimal. Leaf size=27

$$-\frac{ax^{-3n}}{3n}-\frac{bx^{-2n}}{2n}$$

```
[Out] -1/3*a/n/(x^(3*n))-1/2*b/n/(x^(2*n))
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {14}

$$-\frac{ax^{-3n}}{3n}-\frac{bx^{-2n}}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 3*n)*(a + b*x^n),x]
```

```
[Out] -a/(3*n*x^(3*n)) - b/(2*n*x^(2*n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^{-1-3n}\left(a+bx^n\right)\,dx &= \int\left(ax^{-1-3n}+bx^{-1-2n}\right)\,dx\\ &= -\frac{ax^{-3n}}{3n}-\frac{bx^{-2n}}{2n}\end{aligned}$$

Mathematica [A] time = 0.04, size = 22, normalized size = 0.81

$$-\frac{x^{-3n}\left(2a+3bx^n\right)}{6n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 3*n)*(a + b*x^n),x]
```

```
[Out] -1/6*(2*a + 3*b*x^n)/(n*x^(3*n))
```

fricas [A] time = 0.66, size = 22, normalized size = 0.81

$$-\frac{3\,bx^n+2\,a}{6\,nx^{3\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)*(a+b*x^n),x, algorithm="fricas")
```

```
[Out] -1/6*(3*b*x^n + 2*a)/(n*x^(3*n))
```

giac [A] time = 0.19, size = 22, normalized size = 0.81

$$-\frac{3\,bx^n+2\,a}{6\,nx^{3\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)*(a+b*x^n),x, algorithm="giac")
```

```
[Out] -1/6*(3*b*x^n + 2*a)/(n*x^(3*n))
```

maple [A] time = 0.02, size = 27, normalized size = 1.00

$$\left(-\frac{b\,\mathrm{e}^{n\ln(x)}}{2n}-\frac{a}{3n}\right)\mathrm{e}^{-3n\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-3*n)*(b*x^n+a),x)
```

```
[Out] (-1/3*a/n-1/2*b/n*exp(n*ln(x)))/exp(n*ln(x))^3
```

maxima [A] time = 0.65, size = 27, normalized size = 1.00

$$-\frac{a}{3\,nx^{3\,n}}-\frac{b}{2\,nx^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)*(a+b*x^n),x, algorithm="maxima")
```

```
[Out] -1/3*a/(n*x^(3*n)) - 1/2*b/(n*x^(2*n))
```

mupad [B] time = 1.20, size = 22, normalized size = 0.81

$$-\frac{2\,a+3\,b\,x^{n}}{6\,n\,x^{3\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)/x^(3*n + 1),x)
```

```
[Out] -(2*a + 3*b*x^n)/(6*n*x^(3*n))
```

sympy [A] time = 4.94, size = 27, normalized size = 1.00

$$\begin{cases}-\frac{ax^{-3n}}{3n}-\frac{bx^{-2n}}{2n} & \text{for } n\neq 0\\ (a+b)\log(x) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-3*n)*(a+b*x**n),x)
```

```
[Out] Piecewise((-a*x**(-3*n)/(3*n) - b*x**(-2*n)/(2*n), Ne(n, 0)), ((a + b)*log(
x), True))
```

3.2526 $\int x^{-1-4n}\left(a+bx^n\right)\,dx$

Optimal. Leaf size=27

$$-\frac{ax^{-4n}}{4n}-\frac{bx^{-3n}}{3n}$$

```
[Out] -1/4*a/n/(x^(4*n))-1/3*b/n/(x^(3*n))
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {14}

$$-\frac{ax^{-4n}}{4n}-\frac{bx^{-3n}}{3n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 4*n)*(a + b*x^n),x]
```

```
[Out] -a/(4*n*x^(4*n)) - b/(3*n*x^(3*n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^{-1-4n}\left(a+bx^n\right)\,dx &= \int\left(ax^{-1-4n}+bx^{-1-3n}\right)\,dx\\ &= -\frac{ax^{-4n}}{4n}-\frac{bx^{-3n}}{3n}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.81

$$-\frac{x^{-4n}\left(3a+4bx^n\right)}{12n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 4*n)*(a + b*x^n),x]
```

```
[Out] -1/12*(3*a + 4*b*x^n)/(n*x^(4*n))
```

fricas [A] time = 0.70, size = 22, normalized size = 0.81

$$-\frac{4\,bx^n+3\,a}{12\,nx^{4\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-4*n)*(a+b*x^n),x, algorithm="fricas")
```

```
[Out] -1/12*(4*b*x^n + 3*a)/(n*x^(4*n))
```

giac [A] time = 0.16, size = 22, normalized size = 0.81

$$-\frac{4\,bx^n+3\,a}{12\,nx^{4\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-4*n)*(a+b*x^n),x, algorithm="giac")
```

```
[Out] -1/12*(4*b*x^n + 3*a)/(n*x^(4*n))
```

maple [A] time = 0.02, size = 27, normalized size = 1.00

$$\left(-\frac{b\,\mathrm{e}^{n\ln(x)}}{3n}-\frac{a}{4n}\right)\mathrm{e}^{-4n\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-4*n)*(b*x^n+a),x)
```

```
[Out] (-1/4*a/n-1/3*b/n*exp(n*ln(x)))/exp(n*ln(x))^4
```

maxima [A] time = 0.64, size = 27, normalized size = 1.00

$$-\frac{a}{4\,nx^{4\,n}}-\frac{b}{3\,nx^{3\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-4*n)*(a+b*x^n),x, algorithm="maxima")
```

```
[Out] -1/4*a/(n*x^(4*n)) - 1/3*b/(n*x^(3*n))
```

mupad [B] time = 1.23, size = 22, normalized size = 0.81

$$-\frac{3\,a+4\,b\,x^n}{12\,n\,x^{4\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)/x^(4*n + 1),x)
```

```
[Out] -(3*a + 4*b*x^n)/(12*n*x^(4*n))
```

sympy [A] time = 4.95, size = 27, normalized size = 1.00

$$\begin{cases}-\frac{ax^{-4n}}{4n}-\frac{bx^{-3n}}{3n} & \text{for } n\neq 0\\ (a+b)\log(x) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-4*n)*(a+b*x**n),x)
```

```
[Out] Piecewise((-a*x**(-4*n)/(4*n) - b*x**(-3*n)/(3*n), Ne(n, 0)), ((a + b)*log(
x), True))
```

3.2527 $\int x^{-1-5n}\left(a+bx^n\right)\,dx$

Optimal. Leaf size=27

$$-\frac{ax^{-5n}}{5n}-\frac{bx^{-4n}}{4n}$$

```
[Out] -1/5*a/n/(x^(5*n))-1/4*b/n/(x^(4*n))
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {14}

$$-\frac{ax^{-5n}}{5n}-\frac{bx^{-4n}}{4n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 5*n)*(a + b*x^n),x]
```

```
[Out] -a/(5*n*x^(5*n)) - b/(4*n*x^(4*n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^{-1-5n}\left(a+bx^n\right)\,dx &= \int\left(ax^{-1-5n}+bx^{-1-4n}\right)\,dx\\ &= -\frac{ax^{-5n}}{5n}-\frac{bx^{-4n}}{4n}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.81

$$-\frac{x^{-5n}\left(4a+5bx^n\right)}{20n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 5*n)*(a + b*x^n),x]
```

```
[Out] -1/20*(4*a + 5*b*x^n)/(n*x^(5*n))
```

fricas [A] time = 0.76, size = 22, normalized size = 0.81

$$-\frac{5\,bx^n+4\,a}{20\,nx^{5\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-5*n)*(a+b*x^n),x, algorithm="fricas")
```

```
[Out] -1/20*(5*b*x^n + 4*a)/(n*x^(5*n))
```

giac [A] time = 0.16, size = 22, normalized size = 0.81

$$-\frac{5\,bx^n+4\,a}{20\,nx^{5\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-5*n)*(a+b*x^n),x, algorithm="giac")
```

```
[Out] -1/20*(5*b*x^n + 4*a)/(n*x^(5*n))
```

maple [A] time = 0.02, size = 27, normalized size = 1.00

$$\left(-\frac{b\,\mathrm{e}^{n\ln(x)}}{4n}-\frac{a}{5n}\right)\mathrm{e}^{-5n\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-5*n)*(b*x^n+a),x)
```

```
[Out] (-1/5*a/n-1/4*b/n*exp(n*ln(x)))/exp(n*ln(x))^5
```

maxima [A] time = 0.54, size = 27, normalized size = 1.00

$$-\frac{a}{5\,nx^{5\,n}}-\frac{b}{4\,nx^{4\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-5*n)*(a+b*x^n),x, algorithm="maxima")
```

```
[Out] -1/5*a/(n*x^(5*n)) - 1/4*b/(n*x^(4*n))
```

mupad [B] time = 1.22, size = 22, normalized size = 0.81

$$-\frac{4\,a+5\,b\,x^{n}}{20\,n\,x^{5\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)/x^(5*n + 1),x)
```

```
[Out] -(4*a + 5*b*x^n)/(20*n*x^(5*n))
```

sympy [A] time = 5.04, size = 27, normalized size = 1.00

$$\begin{cases}-\frac{ax^{-5n}}{5n}-\frac{bx^{-4n}}{4n} & \text{for } n\neq 0\\ (a+b)\log(x) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-5*n)*(a+b*x**n),x)
```

```
[Out] Piecewise((-a*x**(-5*n)/(5*n) - b*x**(-4*n)/(4*n), Ne(n, 0)), ((a + b)*log(
x), True))
```

3.2528 $\int x^{-1+4n}\left(a+bx^n\right)^2\,dx$

Optimal. Leaf size=45

$$\frac{a^2x^{4n}}{4n}+\frac{2abx^{5n}}{5n}+\frac{b^2x^{6n}}{6n}$$

```
[Out] 1/4*a^2*x^(4*n)/n+2/5*a*b*x^(5*n)/n+1/6*b^2*x^(6*n)/n
```

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$\frac{a^2x^{4n}}{4n}+\frac{2abx^{5n}}{5n}+\frac{b^2x^{6n}}{6n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 4*n)*(a + b*x^n)^2,x]
```

```
[Out] (a^2*x^(4*n))/(4*n) + (2*a*b*x^(5*n))/(5*n) + (b^2*x^(6*n))/(6*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1+4n}\left(a+bx^n\right)^2\,dx &= \frac{\text{Subst}\left(\int x^3(a+bx)^2\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(a^2x^3+2abx^4+b^2x^5\right)\,dx,x,x^n\right)}{n}\\ &= \frac{a^2x^{4n}}{4n}+\frac{2abx^{5n}}{5n}+\frac{b^2x^{6n}}{6n}\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.78

$$\frac{x^{4n}\left(15a^2+24abx^n+10b^2x^{2n}\right)}{60n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 4*n)*(a + b*x^n)^2,x]
```

```
[Out] (x^(4*n)*(15*a^2 + 24*a*b*x^n + 10*b^2*x^(2*n)))/(60*n)
```

fricas [A] time = 0.52, size = 35, normalized size = 0.78

$$\frac{10\,b^2x^{6\,n}+24\,abx^{5\,n}+15\,a^2x^{4\,n}}{60\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)*(a+b*x^n)^2,x, algorithm="fricas")
[Out] 1/60*(10*b^2*x^(6*n) + 24*a*b*x^(5*n) + 15*a^2*x^(4*n))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^2 x^{4n-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)*(a+b*x^n)^2,x, algorithm="giac")
[Out] integrate((b*x^n + a)^2*x^(4*n - 1), x)
```

maple [A] time = 0.02, size = 40, normalized size = 0.89

$$\frac{a^2 x^{4n}}{4n} + \frac{2ab\, x^{5n}}{5n} + \frac{b^2 x^{6n}}{6n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+4*n)*(b*x^n+a)^2,x)
[Out] 1/6*b^2/n*(x^n)^6+2/5*a*b/n*(x^n)^5+1/4*a^2/n*(x^n)^4
```

maxima [A] time = 0.68, size = 39, normalized size = 0.87

$$\frac{b^2 x^{6\,n}}{6\,n} + \frac{2\,abx^{5\,n}}{5\,n} + \frac{a^2 x^{4\,n}}{4\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)*(a+b*x^n)^2,x, algorithm="maxima")
[Out] 1/6*b^2*x^(6*n)/n + 2/5*a*b*x^(5*n)/n + 1/4*a^2*x^(4*n)/n
```

mupad [B] time = 1.34, size = 33, normalized size = 0.73

$$\frac{x^{4\,n}\left(15\,a^2 + 10\,b^2\,x^{2\,n} + 24\,a\,b\,x^n\right)}{60\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4*n - 1)*(a + b*x^n)^2,x)
[Out] (x^(4*n)*(15*a^2 + 10*b^2*x^(2*n) + 24*a*b*x^n))/(60*n)
```

sympy [A] time = 11.97, size = 44, normalized size = 0.98

$$\begin{cases} \frac{a^2 x^{4n}}{4n} + \frac{2abx^{5n}}{5n} + \frac{b^2 x^{6n}}{6n} & \text{for } n \neq 0 \\ (a+b)^2 \log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+4*n)*(a+b*x**n)**2,x)
[Out] Piecewise((a**2*x**(4*n)/(4*n) + 2*a*b*x**(5*n)/(5*n) + b**2*x**(6*n)/(6*n)
, Ne(n, 0)), ((a + b)**2*log(x), True))
```

3.2529 $\int x^{-1+3n}\left(a+bx^n\right)^2\,dx$

Optimal. Leaf size=45

$$\frac{a^2x^{3n}}{3n}+\frac{abx^{4n}}{2n}+\frac{b^2x^{5n}}{5n}$$

```
[Out] 1/3*a^2*x^(3*n)/n+1/2*a*b*x^(4*n)/n+1/5*b^2*x^(5*n)/n
```

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$\frac{a^2x^{3n}}{3n}+\frac{abx^{4n}}{2n}+\frac{b^2x^{5n}}{5n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)*(a + b*x^n)^2,x]
```

```
[Out] (a^2*x^(3*n))/(3*n) + (a*b*x^(4*n))/(2*n) + (b^2*x^(5*n))/(5*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1+3n}\left(a+bx^n\right)^2\,dx &= \frac{\text{Subst}\left(\int x^2(a+bx)^2\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(a^2x^2+2abx^3+b^2x^4\right)\,dx,x,x^n\right)}{n}\\ &= \frac{a^2x^{3n}}{3n}+\frac{abx^{4n}}{2n}+\frac{b^2x^{5n}}{5n}\end{aligned}$$

Mathematica [A] time = 0.04, size = 35, normalized size = 0.78

$$\frac{x^{3n}\left(10a^2+15abx^n+6b^2x^{2n}\right)}{30n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)*(a + b*x^n)^2,x]
```

```
[Out] (x^(3*n)*(10*a^2 + 15*a*b*x^n + 6*b^2*x^(2*n)))/(30*n)
```

fricas [A] time = 0.96, size = 35, normalized size = 0.78

$$\frac{6\,b^2x^{5\,n}+15\,abx^{4\,n}+10\,a^2x^{3\,n}}{30\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^2,x, algorithm="fricas")
[Out] 1/30*(6*b^2*x^(5*n) + 15*a*b*x^(4*n) + 10*a^2*x^(3*n))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^2 x^{3n-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^2,x, algorithm="giac")
[Out] integrate((b*x^n + a)^2*x^(3*n - 1), x)
```

maple [A] time = 0.02, size = 46, normalized size = 1.02

$$\frac{a^2 \mathrm{e}^{3n\ln(x)}}{3n} + \frac{ab\,\mathrm{e}^{4n\ln(x)}}{2n} + \frac{b^2 \mathrm{e}^{5n\ln(x)}}{5n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n-1)*(b*x^n+a)^2,x)
[Out] 1/3*a^2/n*exp(n*ln(x))^3+1/5*b^2/n*exp(n*ln(x))^5+1/2*a*b/n*exp(n*ln(x))^4
```

maxima [A] time = 0.52, size = 39, normalized size = 0.87

$$\frac{b^2 x^{5\,n}}{5\,n} + \frac{abx^{4\,n}}{2\,n} + \frac{a^2 x^{3\,n}}{3\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^2,x, algorithm="maxima")
[Out] 1/5*b^2*x^(5*n)/n + 1/2*a*b*x^(4*n)/n + 1/3*a^2*x^(3*n)/n
```

mupad [B] time = 1.31, size = 33, normalized size = 0.73

$$\frac{x^{3\,n}\left(10\,a^2 + 6\,b^2\,x^{2\,n} + 15\,a\,b\,x^n\right)}{30\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n - 1)*(a + b*x^n)^2,x)
[Out] (x^(3*n)*(10*a^2 + 6*b^2*x^(2*n) + 15*a*b*x^n))/(30*n)
```

sympy [A] time = 11.76, size = 42, normalized size = 0.93

$$\begin{cases} \frac{a^2 x^{3n}}{3n} + \frac{abx^{4n}}{2n} + \frac{b^2 x^{5n}}{5n} & \text{for } n \neq 0 \\ (a+b)^2 \log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)*(a+b*x**n)**2,x)
[Out] Piecewise((a**2*x**(3*n)/(3*n) + a*b*x**(4*n)/(2*n) + b**2*x**(5*n)/(5*n),
Ne(n, 0)), ((a + b)**2*log(x), True))
```

3.2530 $\int x^{-1+2n}\left(a+bx^n\right)^2\,dx$

Optimal. Leaf size=45

$$\frac{a^2x^{2n}}{2n}+\frac{2abx^{3n}}{3n}+\frac{b^2x^{4n}}{4n}$$

```
[Out] 1/2*a^2*x^(2*n)/n+2/3*a*b*x^(3*n)/n+1/4*b^2*x^(4*n)/n
```

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$\frac{a^2x^{2n}}{2n}+\frac{2abx^{3n}}{3n}+\frac{b^2x^{4n}}{4n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)*(a + b*x^n)^2,x]
```

```
[Out] (a^2*x^(2*n))/(2*n) + (2*a*b*x^(3*n))/(3*n) + (b^2*x^(4*n))/(4*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1+2n}\left(a+bx^n\right)^2\,dx &= \frac{\text{Subst}\left(\int x(a+bx)^2\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(a^2x+2abx^2+b^2x^3\right)\,dx,x,x^n\right)}{n}\\ &= \frac{a^2x^{2n}}{2n}+\frac{2abx^{3n}}{3n}+\frac{b^2x^{4n}}{4n}\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.78

$$\frac{x^{2n}\left(6a^2+8abx^n+3b^2x^{2n}\right)}{12n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)*(a + b*x^n)^2,x]
```

```
[Out] (x^(2*n)*(6*a^2 + 8*a*b*x^n + 3*b^2*x^(2*n)))/(12*n)
```

fricas [A] time = 1.08, size = 35, normalized size = 0.78

$$\frac{3\,b^2x^{4\,n}+8\,abx^{3\,n}+6\,a^2x^{2\,n}}{12\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^2,x, algorithm="fricas")
[Out] 1/12*(3*b^2*x^(4*n) + 8*a*b*x^(3*n) + 6*a^2*x^(2*n))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^2 x^{2n-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^2,x, algorithm="giac")
[Out] integrate((b*x^n + a)^2*x^(2*n - 1), x)
```

maple [A] time = 0.02, size = 46, normalized size = 1.02

$$\frac{a^2 e^{2n\ln(x)}}{2n} + \frac{2ab\, e^{3n\ln(x)}}{3n} + \frac{b^2 e^{4n\ln(x)}}{4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n-1)*(b*x^n+a)^2,x)
[Out] 1/2*a^2/n*exp(n*ln(x))^2+1/4*b^2/n*exp(n*ln(x))^4+2/3*a*b/n*exp(n*ln(x))^3
```

maxima [A] time = 0.67, size = 39, normalized size = 0.87

$$\frac{b^2 x^{4n}}{4n} + \frac{2abx^{3n}}{3n} + \frac{a^2 x^{2n}}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^2,x, algorithm="maxima")
[Out] 1/4*b^2*x^(4*n)/n + 2/3*a*b*x^(3*n)/n + 1/2*a^2*x^(2*n)/n
```

mupad [B] time = 1.29, size = 33, normalized size = 0.73

$$\frac{x^{2n}\left(6a^2 + 3b^2 x^{2n} + 8abx^n\right)}{12n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)*(a + b*x^n)^2,x)
[Out] (x^(2*n)*(6*a^2 + 3*b^2*x^(2*n) + 8*a*b*x^n))/(12*n)
```

sympy [A] time = 11.70, size = 44, normalized size = 0.98

$$\begin{cases} \frac{a^2 x^{2n}}{2n} + \frac{2abx^{3n}}{3n} + \frac{b^2 x^{4n}}{4n} & \text{for } n \neq 0 \\ (a+b)^2 \log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a+b*x**n)**2,x)
[Out] Piecewise((a**2*x**(2*n)/(2*n) + 2*a*b*x**(3*n)/(3*n) + b**2*x**(4*n)/(4*n)
, Ne(n, 0)), ((a + b)**2*log(x), True))
```

3.2531 $\int x^{-1+n}\left(a+bx^n\right)^2\,dx$

Optimal. Leaf size=19

$$\frac{\left(a+bx^n\right)^3}{3bn}$$

```
[Out] 1/3*(a+b*x^n)^3/b/n
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$\frac{\left(a+bx^n\right)^3}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*(a + b*x^n)^2,x]
```

```
[Out] (a + b*x^n)^3/(3*b*n)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^{-1+n}\left(a+bx^n\right)^2\,dx = \frac{\left(a+bx^n\right)^3}{3bn}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{\left(a+bx^n\right)^3}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*(a + b*x^n)^2,x]
```

```
[Out] (a + b*x^n)^3/(3*b*n)
```

fricas [A] time = 1.43, size = 32, normalized size = 1.68

$$\frac{b^2x^{3\,n}+3\,abx^{2\,n}+3\,a^2x^n}{3\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*x^(3*n) + 3*a*b*x^(2*n) + 3*a^2*x^n)/n
```

giac [A] time = 0.15, size = 32, normalized size = 1.68

$$\frac{b^2x^{3\,n}+3\,abx^{2\,n}+3\,a^2x^n}{3\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(a+b*x^n)^2,x, algorithm="giac")
[Out] 1/3*(b^2*x^(3*n) + 3*a*b*x^(2*n) + 3*a^2*x^n)/n
```

maple [B] time = 0.02, size = 42, normalized size = 2.21

$$\frac{a^2 e^{n \ln(x)}}{n} + \frac{ab\, e^{2n \ln(x)}}{n} + \frac{b^2 e^{3n \ln(x)}}{3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*(b*x^n+a)^2,x)
[Out] a^2/n*exp(n*ln(x))+a*b/n*exp(n*ln(x))^2+1/3*b^2/n*exp(n*ln(x))^3
```

maxima [A] time = 0.67, size = 17, normalized size = 0.89

$$\frac{(bx^n + a)^3}{3\, bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(a+b*x^n)^2,x, algorithm="maxima")
[Out] 1/3*(b*x^n + a)^3/(b*n)
```

mupad [B] time = 1.27, size = 27, normalized size = 1.42

$$\frac{x^n \left(a^2 + \frac{b^2 x^{2n}}{3} + a\, b\, x^n\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*(a + b*x^n)^2,x)
[Out] (x^n*(a^2 + (b^2*x^(2*n))/3 + a*b*x^n))/n
```

sympy [A] time = 3.15, size = 37, normalized size = 1.95

$$\begin{cases} \frac{a^2 x^n}{n} + \frac{abx^{2n}}{n} + \frac{b^2 x^{3n}}{3n} & \text{for } n \neq 0 \\ (a+b)^2 \log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(a+b*x**n)**2,x)
[Out] Piecewise((a**2*x**n/n + a*b*x**(2*n)/n + b**2*x**(3*n)/(3*n), Ne(n, 0)), (
(a + b)**2*log(x), True))
```

3.2532 $\int \frac{(a+bx^n)^2}{x}\,dx$

Optimal. Leaf size=32

$$a^2\log(x)+\frac{2abx^n}{n}+\frac{b^2x^{2n}}{2n}$$

```
[Out] 2*a*b*x^n/n+1/2*b^2*x^(2*n)/n+a^2*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$a^2\log(x)+\frac{2abx^n}{n}+\frac{b^2x^{2n}}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^2/x,x]
```

```
[Out] (2*a*b*x^n)/n + (b^2*x^(2*n))/(2*n) + a^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^n)^2}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{x}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(2ab+\frac{a^2}{x}+b^2x\right)\,dx, x, x^n\right)}{n}\\ &= \frac{2abx^n}{n}+\frac{b^2x^{2n}}{2n}+a^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 27, normalized size = 0.84

$$a^2\log(x)+\frac{bx^n\,(4a+bx^n)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^2/x,x]
```

```
[Out] (b*x^n*(4*a + b*x^n))/(2*n) + a^2*Log[x]
```

fricas [A] time = 0.79, size = 30, normalized size = 0.94

$$\frac{2\,a^2 n\log(x)+b^2x^{2\,n}+4\,abx^n}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2/x,x, algorithm="fricas")
[Out] 1/2*(2*a^2*n*log(x) + b^2*x^(2*n) + 4*a*b*x^n)/n
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2/x,x, algorithm="giac")
[Out] integrate((b*x^n + a)^2/x, x)
```

maple [**A**] time = 0.00, size = 36, normalized size = 1.12

$$\frac{a^2 \ln(x^n)}{n} + \frac{2ab\, x^n}{n} + \frac{b^2 x^{2n}}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^2/x,x)
[Out] 1/2/n*b^2*(x^n)^2+2*a*b/n*x^n+a^2/n*ln(x^n)
```

maxima [**A**] time = 0.56, size = 34, normalized size = 1.06

$$\frac{a^2 \log(x^n)}{n} + \frac{b^2 x^{2\,n} + 4\, abx^n}{2\, n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2/x,x, algorithm="maxima")
[Out] a^2*log(x^n)/n + 1/2*(b^2*x^(2*n) + 4*a*b*x^n)/n
```

mupad [**B**] time = 0.00, size = 30, normalized size = 0.94

$$\frac{b^2\, x^{2\,n} + 4\, a\, b\, x^n + 2\, a^2\, n\, \ln(x)}{2\, n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^2/x,x)
[Out] (b^2*x^(2*n) + 4*a*b*x^n + 2*a^2*n*log(x))/(2*n)
```

sympy [**A**] time = 0.29, size = 36, normalized size = 1.12

$$\begin{cases} a^2 \log(x) + \frac{2abx^n}{n} + \frac{b^2 x^{2n}}{2n} & \text{for } n \neq 0 \\ (a+b)^2 \log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**2/x,x)
[Out] Piecewise((a**2*log(x) + 2*a*b*x**n/n + b**2*x**(2*n)/(2*n), Ne(n, 0)), ((a
 + b)**2*log(x), True))
```

3.2533 $\int x^{-1-n}\left(a+bx^n\right)^2\,dx$

Optimal. Leaf size=30

$$-\frac{a^2x^{-n}}{n}+2ab\log(x)+\frac{b^2x^n}{n}$$

```
[Out] -a^2/n/(x^n)+b^2*x^n/n+2*a*b*ln(x)
```

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$-\frac{a^2x^{-n}}{n}+2ab\log(x)+\frac{b^2x^n}{n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n)*(a + b*x^n)^2,x]
```

```
[Out] -(a^2/(n*x^n)) + (b^2*x^n)/n + 2*a*b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1-n}\left(a+bx^n\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{x^2}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(b^2+\frac{a^2}{x^2}+\frac{2ab}{x}\right)\,dx,x,x^n\right)}{n}\\ &= -\frac{a^2x^{-n}}{n}+\frac{b^2x^n}{n}+2ab\log(x)\end{aligned}$$

Mathematica [A] time = 0.04, size = 29, normalized size = 0.97

$$\frac{-a^2x^{-n}+2abn\log(x)+b^2x^n}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n)*(a + b*x^n)^2,x]
```

```
[Out] (-(a^2/x^n) + b^2*x^n + 2*a*b*n*Log[x])/n
```

fricas [A] time = 0.95, size = 34, normalized size = 1.13

$$\frac{2\,abnx^n\log(x)+b^2x^{2\,n}-a^2}{nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*(a+b*x^n)^2,x, algorithm="fricas")
[Out] (2*a*b*n*x^n*log(x) + b^2*x^(2*n) - a^2)/(n*x^n)
```

giac [A] time = 0.18, size = 34, normalized size = 1.13

$$\frac{2\,abnx^{n}\log(x)+b^{2}x^{2\,n}-a^{2}}{nx^{n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*(a+b*x^n)^2,x, algorithm="giac")
[Out] (2*a*b*n*x^n*log(x) + b^2*x^(2*n) - a^2)/(n*x^n)
```

maple [A] time = 0.02, size = 43, normalized size = 1.43

$$\left(2ab\,\mathrm{e}^{n\ln(x)}\ln(x)+\frac{b^{2}\mathrm{e}^{2n\ln(x)}}{n}-\frac{a^{2}}{n}\right)\mathrm{e}^{-n\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n)*(b*x^n+a)^2,x)
[Out] (b^2/n*exp(n*ln(x))^2+2*a*b*ln(x)*exp(n*ln(x))-a^2/n)/exp(n*ln(x))
```

maxima [A] time = 0.59, size = 30, normalized size = 1.00

$$2\,ab\log(x)+\frac{b^{2}x^{n}}{n}-\frac{a^{2}}{nx^{n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*(a+b*x^n)^2,x, algorithm="maxima")
[Out] 2*a*b*log(x) + b^2*x^n/n - a^2/(n*x^n)
```

mupad [B] time = 1.32, size = 30, normalized size = 1.00

$$2\,a\,b\,\ln(x)+\frac{b^{2}\,x^{n}}{n}-\frac{a^{2}}{n\,x^{n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^2/x^(n + 1),x)
[Out] 2*a*b*log(x) + (b^2*x^n)/n - a^2/(n*x^n)
```

sympy [A] time = 28.50, size = 175, normalized size = 5.83

$$\begin{cases} a^{2}x+2ab\log(x)-\frac{b^{2}}{x} & \text{for } n=-1 \\ (a+b)^{2}\log(x) & \text{for } n=0 \\ -\frac{a^{2}n}{n^{2}x^{n}+nx^{n}}-\frac{a^{2}}{n^{2}x^{n}+nx^{n}}+\frac{2abn^{2}x^{n}\log(x)}{n^{2}x^{n}+nx^{n}}+\frac{2abnx^{n}\log(x)}{n^{2}x^{n}+nx^{n}}+\frac{2abnx^{n}}{n^{2}x^{n}+nx^{n}}+\frac{b^{2}nx^{2n}}{n^{2}x^{n}+nx^{n}}+\frac{b^{2}x^{2n}}{n^{2}x^{n}+nx^{n}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n)*(a+b*x**n)**2,x)
[Out] Piecewise((a**2*x + 2*a*b*log(x) - b**2/x, Eq(n, -1)), ((a + b)**2*log(x),
Eq(n, 0)), (-a**2*n/(n**2*x**n + n*x**n) - a**2/(n**2*x**n + n*x**n) + 2*a*
b*n**2*x**n*log(x)/(n**2*x**n + n*x**n) + 2*a*b*n*x**n*log(x)/(n**2*x**n +
n*x**n) + 2*a*b*n*x**n/(n**2*x**n + n*x**n) + b**2*n*x**(2*n)/(n**2*x**n +
n*x**n) + b**2*x**(2*n)/(n**2*x**n + n*x**n), True))
```

3.2534 $\int x^{-1-2n}\left(a+bx^n\right)^2\,dx$

Optimal. Leaf size=34

$$-\frac{a^2x^{-2n}}{2n}-\frac{2abx^{-n}}{n}+b^2\log(x)$$

```
[Out] -1/2*a^2/n/(x^(2*n))-2*a*b/n/(x^n)+b^2*ln(x)
```

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$-\frac{a^2x^{-2n}}{2n}-\frac{2abx^{-n}}{n}+b^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 2*n)*(a + b*x^n)^2,x]
```

```
[Out] -a^2/(2*n*x^(2*n)) - (2*a*b)/(n*x^n) + b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1-2n}\left(a+bx^n\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{x^3}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a^2}{x^3}+\frac{2ab}{x^2}+\frac{b^2}{x}\right)\,dx,x,x^n\right)}{n}\\ &= -\frac{a^2x^{-2n}}{2n}-\frac{2abx^{-n}}{n}+b^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.04, size = 28, normalized size = 0.82

$$b^2\log(x)-\frac{ax^{-2n}\left(a+4bx^n\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 2*n)*(a + b*x^n)^2,x]
```

```
[Out] -1/2*(a*(a + 4*b*x^n))/(n*x^(2*n)) + b^2*Log[x]
```

fricas [A] time = 0.59, size = 38, normalized size = 1.12

$$\frac{2\,b^2nx^{2\,n}\log(x)-4\,abx^n-a^2}{2\,nx^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*(a+b*x^n)^2,x, algorithm="fricas")
[Out] 1/2*(2*b^2*n*x^(2*n)*log(x) - 4*a*b*x^n - a^2)/(n*x^(2*n))
```

giac [A] time = 0.18, size = 38, normalized size = 1.12

$$\frac{2\,b^2 n x^{2\,n}\log(x) - 4\,abx^n - a^2}{2\,nx^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*(a+b*x^n)^2,x, algorithm="giac")
[Out] 1/2*(2*b^2*n*x^(2*n)*log(x) - 4*a*b*x^n - a^2)/(n*x^(2*n))
```

maple [A] time = 0.02, size = 43, normalized size = 1.26

$$\left(b^2 \mathrm{e}^{2n\ln(x)}\ln(x) - \frac{2ab\,\mathrm{e}^{n\ln(x)}}{n} - \frac{a^2}{2n}\right)\mathrm{e}^{-2n\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-2*n)*(b*x^n+a)^2,x)
[Out] (b^2*ln(x)*exp(n*ln(x))^2-1/2*a^2/n-2*a*b/n*exp(n*ln(x)))/exp(n*ln(x))^2
```

maxima [A] time = 0.54, size = 34, normalized size = 1.00

$$b^2\log(x) - \frac{a^2}{2\,nx^{2\,n}} - \frac{2\,ab}{nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*(a+b*x^n)^2,x, algorithm="maxima")
[Out] b^2*log(x) - 1/2*a^2/(n*x^(2*n)) - 2*a*b/(n*x^n)
```

mupad [B] time = 1.33, size = 34, normalized size = 1.00

$$b^2\,\ln(x) - \frac{a^2}{2\,n\,x^{2\,n}} - \frac{2\,a\,b}{n\,x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^2/x^(2*n + 1),x)
[Out] b^2*log(x) - a^2/(2*n*x^(2*n)) - (2*a*b)/(n*x^n)
```

sympy [A] time = 57.24, size = 235, normalized size = 6.91

$$\begin{cases} a^2x + 4ab\sqrt{x} + b^2\log(x) & \text{for } n = \\ (a+b)^2\log(x) & \text{for } n = 0 \\ -\frac{2a^2n}{4n^2x^{2n}+2nx^{2n}} - \frac{a^2}{4n^2x^{2n}+2nx^{2n}} - \frac{8abnx^n}{4n^2x^{2n}+2nx^{2n}} - \frac{4abx^n}{4n^2x^{2n}+2nx^{2n}} + \frac{4b^2n^2x^{2n}\log(x)}{4n^2x^{2n}+2nx^{2n}} + \frac{2b^2nx^{2n}\log(x)}{4n^2x^{2n}+2nx^{2n}} + \frac{2b^2nx^{2n}}{4n^2x^{2n}+2nx^{2n}} & \text{otherwis} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-2*n)*(a+b*x**n)**2,x)
[Out] Piecewise((a**2*x + 4*a*b*sqrt(x) + b**2*log(x), Eq(n, -1/2)), ((a + b)**2*
log(x), Eq(n, 0)), (-2*a**2*n/(4*n**2*x**(2*n) + 2*n*x**(2*n)) - a**2/(4*n*
```

```
*2*x**(2*n) + 2*n*x**(2*n)) - 8*a*b*n*x**n/(4*n**2*x**(2*n) + 2*n*x**(2*n))
 - 4*a*b*x**n/(4*n**2*x**(2*n) + 2*n*x**(2*n)) + 4*b**2*n**2*x**(2*n)*log(x
)/(4*n**2*x**(2*n) + 2*n*x**(2*n)) + 2*b**2*n*x**(2*n)*log(x)/(4*n**2*x**(2
*n) + 2*n*x**(2*n)) + 2*b**2*n*x**(2*n)/(4*n**2*x**(2*n) + 2*n*x**(2*n)), T
rue))
```

3.2535 $\int x^{-1-3n}\left(a+bx^n\right)^2\,dx$

Optimal. Leaf size=24

$$-\frac{x^{-3n}\left(a+bx^n\right)^3}{3an}$$

```
[Out] -1/3*(a+b*x^n)^3/a/n/(x^(3*n))
```

Rubi [A] time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {264}

$$-\frac{x^{-3n}\left(a+bx^n\right)^3}{3an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 3*n)*(a + b*x^n)^2,x]
```

```
[Out] -(a + b*x^n)^3/(3*a*n*x^(3*n))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^{-1-3n}\left(a+bx^n\right)^2\,dx = -\frac{x^{-3n}\left(a+bx^n\right)^3}{3an}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$-\frac{x^{-3n}\left(a+bx^n\right)^3}{3an}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 3*n)*(a + b*x^n)^2,x]
```

```
[Out] -1/3*(a + b*x^n)^3/(a*n*x^(3*n))
```

fricas [A] time = 0.48, size = 33, normalized size = 1.38

$$-\frac{3\,b^2x^{2\,n}+3\,abx^n+a^2}{3\,nx^{3\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)*(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^2*x^(2*n) + 3*a*b*x^n + a^2)/(n*x^(3*n))
```

giac [A] time = 0.19, size = 33, normalized size = 1.38

$$-\frac{3\,b^2x^{2\,n}+3\,abx^n+a^2}{3\,nx^{3\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)*(a+b*x^n)^2,x, algorithm="giac")
[Out] -1/3*(3*b^2*x^(2*n) + 3*a*b*x^n + a^2)/(n*x^(3*n))
```

maple [A] time = 0.02, size = 45, normalized size = 1.88

$$\left(-\frac{ab\,e^{n\ln(x)}}{n}-\frac{b^2e^{2n\ln(x)}}{n}-\frac{a^2}{3n}\right)e^{-3n\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-3*n)*(b*x^n+a)^2,x)
[Out] (-1/3*a^2/n-b^2/n*exp(n*ln(x))^2-a*b/n*exp(n*ln(x)))/exp(n*ln(x))^3
```

maxima [A] time = 0.62, size = 43, normalized size = 1.79

$$-\frac{a^2}{3\,nx^{3\,n}}-\frac{ab}{nx^{2\,n}}-\frac{b^2}{nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)*(a+b*x^n)^2,x, algorithm="maxima")
[Out] -1/3*a^2/(n*x^(3*n)) - a*b/(n*x^(2*n)) - b^2/(n*x^n)
```

mupad [B] time = 1.34, size = 33, normalized size = 1.38

$$-\frac{a^2+3\,b^2\,x^{2\,n}+3\,a\,b\,x^n}{3\,n\,x^{3\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^2/x^(3*n + 1),x)
[Out] -(a^2 + 3*b^2*x^(2*n) + 3*a*b*x^n)/(3*n*x^(3*n))
```

sympy [A] time = 11.45, size = 39, normalized size = 1.62

$$\begin{cases}-\frac{a^2x^{-3n}}{3n}-\frac{abx^{-2n}}{n}-\frac{b^2x^{-n}}{n} & \text{for } n\neq 0\\ (a+b)^2\log(x) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-3*n)*(a+b*x**n)**2,x)
[Out] Piecewise((-a**2*x**(-3*n)/(3*n) - a*b*x**(-2*n)/n - b**2*x**(-n)/n, Ne(n,
0)), ((a + b)**2*log(x), True))
```

3.2536 $\int x^{-1-4n}\left(a+bx^n\right)^2\,dx$

Optimal. Leaf size=45

$$-\frac{a^2x^{-4n}}{4n}-\frac{2abx^{-3n}}{3n}-\frac{b^2x^{-2n}}{2n}$$

```
[Out] -1/4*a^2/n/(x^(4*n))-2/3*a*b/n/(x^(3*n))-1/2*b^2/n/(x^(2*n))
```

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$-\frac{a^2x^{-4n}}{4n}-\frac{2abx^{-3n}}{3n}-\frac{b^2x^{-2n}}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 4*n)*(a + b*x^n)^2,x]
```

```
[Out] -a^2/(4*n*x^(4*n)) - (2*a*b)/(3*n*x^(3*n)) - b^2/(2*n*x^(2*n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1-4n}\left(a+bx^n\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{x^5}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a^2}{x^5}+\frac{2ab}{x^4}+\frac{b^2}{x^3}\right)\,dx,x,x^n\right)}{n}\\ &= -\frac{a^2x^{-4n}}{4n}-\frac{2abx^{-3n}}{3n}-\frac{b^2x^{-2n}}{2n}\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.78

$$-\frac{x^{-4n}\left(3a^2+8abx^n+6b^2x^{2n}\right)}{12n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 4*n)*(a + b*x^n)^2,x]
```

```
[Out] -1/12*(3*a^2 + 8*a*b*x^n + 6*b^2*x^(2*n))/(n*x^(4*n))
```

fricas [A] time = 0.59, size = 35, normalized size = 0.78

$$-\frac{6\,b^2x^{2\,n}+8\,abx^n+3\,a^2}{12\,nx^{4\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-4*n)*(a+b*x^n)^2,x, algorithm="fricas")
[Out] -1/12*(6*b^2*x^(2*n) + 8*a*b*x^n + 3*a^2)/(n*x^(4*n))
```

giac [A] time = 0.19, size = 35, normalized size = 0.78

$$-\frac{6\,b^2x^{2\,n}+8\,abx^n+3\,a^2}{12\,nx^{4\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-4*n)*(a+b*x^n)^2,x, algorithm="giac")
[Out] -1/12*(6*b^2*x^(2*n) + 8*a*b*x^n + 3*a^2)/(n*x^(4*n))
```

maple [A] time = 0.02, size = 45, normalized size = 1.00

$$\left(-\frac{2ab\,\mathrm{e}^{n\ln(x)}}{3n}-\frac{b^2\mathrm{e}^{2n\ln(x)}}{2n}-\frac{a^2}{4n}\right)\mathrm{e}^{-4n\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-4*n)*(b*x^n+a)^2,x)
[Out] (-1/4*a^2/n-1/2*b^2/n*exp(n*ln(x))^2-2/3*a*b/n*exp(n*ln(x)))/exp(n*ln(x))^4
```

maxima [A] time = 0.53, size = 45, normalized size = 1.00

$$-\frac{a^2}{4\,nx^{4\,n}}-\frac{2\,ab}{3\,nx^{3\,n}}-\frac{b^2}{2\,nx^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-4*n)*(a+b*x^n)^2,x, algorithm="maxima")
[Out] -1/4*a^2/(n*x^(4*n)) - 2/3*a*b/(n*x^(3*n)) - 1/2*b^2/(n*x^(2*n))
```

mupad [B] time = 1.30, size = 35, normalized size = 0.78

$$-\frac{3\,a^2+6\,b^2\,x^{2\,n}+8\,a\,b\,x^n}{12\,n\,x^{4\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^2/x^(4*n + 1),x)
[Out] -(3*a^2 + 6*b^2*x^(2*n) + 8*a*b*x^n)/(12*n*x^(4*n))
```

sympy [A] time = 11.47, size = 46, normalized size = 1.02

$$\begin{cases}-\frac{a^2x^{-4n}}{4n}-\frac{2abx^{-3n}}{3n}-\frac{b^2x^{-2n}}{2n} & \text{for } n\neq 0\\ (a+b)^2\log(x) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-4*n)*(a+b*x**n)**2,x)
[Out] Piecewise((-a**2*x**(-4*n)/(4*n) - 2*a*b*x**(-3*n)/(3*n) - b**2*x**(-2*n)/(
2*n), Ne(n, 0)), ((a + b)**2*log(x), True))
```

3.2537 $\int x^{-1-5n}\left(a+bx^n\right)^2\,dx$

Optimal. Leaf size=45

$$-\frac{a^2x^{-5n}}{5n}-\frac{abx^{-4n}}{2n}-\frac{b^2x^{-3n}}{3n}$$

```
[Out] -1/5*a^2/n/(x^(5*n))-1/2*a*b/n/(x^(4*n))-1/3*b^2/n/(x^(3*n))
```

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$-\frac{a^2x^{-5n}}{5n}-\frac{abx^{-4n}}{2n}-\frac{b^2x^{-3n}}{3n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 5*n)*(a + b*x^n)^2,x]
```

```
[Out] -a^2/(5*n*x^(5*n)) - (a*b)/(2*n*x^(4*n)) - b^2/(3*n*x^(3*n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1-5n}\left(a+bx^n\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{x^6}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a^2}{x^6}+\frac{2ab}{x^5}+\frac{b^2}{x^4}\right)\,dx,x,x^n\right)}{n}\\ &= -\frac{a^2x^{-5n}}{5n}-\frac{abx^{-4n}}{2n}-\frac{b^2x^{-3n}}{3n}\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.78

$$-\frac{x^{-5n}\left(6a^2+15abx^n+10b^2x^{2n}\right)}{30n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 5*n)*(a + b*x^n)^2,x]
```

```
[Out] -1/30*(6*a^2 + 15*a*b*x^n + 10*b^2*x^(2*n))/(n*x^(5*n))
```

fricas [A] time = 0.63, size = 35, normalized size = 0.78

$$-\frac{10\,b^2x^{2\,n}+15\,abx^n+6\,a^2}{30\,nx^{5\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-5*n)*(a+b*x^n)^2,x, algorithm="fricas")
[Out] -1/30*(10*b^2*x^(2*n) + 15*a*b*x^n + 6*a^2)/(n*x^(5*n))
```

giac [A] time = 0.18, size = 35, normalized size = 0.78

$$-\frac{10\,b^2x^{2\,n}+15\,abx^n+6\,a^2}{30\,nx^{5\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-5*n)*(a+b*x^n)^2,x, algorithm="giac")
[Out] -1/30*(10*b^2*x^(2*n) + 15*a*b*x^n + 6*a^2)/(n*x^(5*n))
```

maple [A] time = 0.02, size = 45, normalized size = 1.00

$$\left(-\frac{ab\,\mathrm{e}^{n\ln(x)}}{2n}-\frac{b^2\mathrm{e}^{2n\ln(x)}}{3n}-\frac{a^2}{5n}\right)\mathrm{e}^{-5n\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-5*n)*(b*x^n+a)^2,x)
[Out] (-1/5*a^2/n-1/3*b^2/n*exp(n*ln(x))^2-1/2*a*b/n*exp(n*ln(x)))/exp(n*ln(x))^5
```

maxima [A] time = 0.46, size = 45, normalized size = 1.00

$$-\frac{a^2}{5\,nx^{5\,n}}-\frac{ab}{2\,nx^{4\,n}}-\frac{b^2}{3\,nx^{3\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-5*n)*(a+b*x^n)^2,x, algorithm="maxima")
[Out] -1/5*a^2/(n*x^(5*n)) - 1/2*a*b/(n*x^(4*n)) - 1/3*b^2/(n*x^(3*n))
```

mupad [B] time = 1.28, size = 35, normalized size = 0.78

$$-\frac{6\,a^2+10\,b^2\,x^{2\,n}+15\,a\,b\,x^n}{30\,n\,x^{5\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^2/x^(5*n + 1),x)
[Out] -(6*a^2 + 10*b^2*x^(2*n) + 15*a*b*x^n)/(30*n*x^(5*n))
```

sympy [A] time = 11.33, size = 44, normalized size = 0.98

$$\begin{cases}-\frac{a^2x^{-5n}}{5n}-\frac{abx^{-4n}}{2n}-\frac{b^2x^{-3n}}{3n} & \text{for } n\neq 0\\ (a+b)^2\log(x) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-5*n)*(a+b*x**n)**2,x)
[Out] Piecewise((-a**2*x**(-5*n)/(5*n) - a*b*x**(-4*n)/(2*n) - b**2*x**(-3*n)/(3*
n), Ne(n, 0)), ((a + b)**2*log(x), True))
```

3.2538 $\int x^{-1-6n}\left(a+bx^n\right)^2\,dx$

Optimal. Leaf size=45

$$-\frac{a^2x^{-6n}}{6n}-\frac{2abx^{-5n}}{5n}-\frac{b^2x^{-4n}}{4n}$$

```
[Out] -1/6*a^2/n/(x^(6*n))-2/5*a*b/n/(x^(5*n))-1/4*b^2/n/(x^(4*n))
```

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$-\frac{a^2x^{-6n}}{6n}-\frac{2abx^{-5n}}{5n}-\frac{b^2x^{-4n}}{4n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 6*n)*(a + b*x^n)^2,x]
```

```
[Out] -a^2/(6*n*x^(6*n)) - (2*a*b)/(5*n*x^(5*n)) - b^2/(4*n*x^(4*n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1-6n}\left(a+bx^n\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{x^7}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a^2}{x^7}+\frac{2ab}{x^6}+\frac{b^2}{x^5}\right)\,dx,x,x^n\right)}{n}\\ &= -\frac{a^2x^{-6n}}{6n}-\frac{2abx^{-5n}}{5n}-\frac{b^2x^{-4n}}{4n}\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.78

$$-\frac{x^{-6n}\left(10a^2+24abx^n+15b^2x^{2n}\right)}{60n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 6*n)*(a + b*x^n)^2,x]
```

```
[Out] -1/60*(10*a^2 + 24*a*b*x^n + 15*b^2*x^(2*n))/(n*x^(6*n))
```

fricas [A] time = 0.78, size = 35, normalized size = 0.78

$$-\frac{15\,b^2x^{2\,n}+24\,abx^n+10\,a^2}{60\,nx^{6\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-6*n)*(a+b*x^n)^2,x, algorithm="fricas")
[Out] -1/60*(15*b^2*x^(2*n) + 24*a*b*x^n + 10*a^2)/(n*x^(6*n))
```

giac [A] time = 0.19, size = 35, normalized size = 0.78

$$-\frac{15\,b^2x^{2\,n}+24\,abx^n+10\,a^2}{60\,nx^{6\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-6*n)*(a+b*x^n)^2,x, algorithm="giac")
[Out] -1/60*(15*b^2*x^(2*n) + 24*a*b*x^n + 10*a^2)/(n*x^(6*n))
```

maple [A] time = 0.02, size = 40, normalized size = 0.89

$$-\frac{a^2x^{-6n}}{6n}-\frac{2ab\,x^{-5n}}{5n}-\frac{b^2x^{-4n}}{4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-6*n)*(b*x^n+a)^2,x)
[Out] -1/4*b^2/n/(x^n)^4-2/5*a*b/n/(x^n)^5-1/6*a^2/n/(x^n)^6
```

maxima [A] time = 0.48, size = 45, normalized size = 1.00

$$-\frac{a^2}{6\,nx^{6\,n}}-\frac{2\,ab}{5\,nx^{5\,n}}-\frac{b^2}{4\,nx^{4\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-6*n)*(a+b*x^n)^2,x, algorithm="maxima")
[Out] -1/6*a^2/(n*x^(6*n)) - 2/5*a*b/(n*x^(5*n)) - 1/4*b^2/(n*x^(4*n))
```

mupad [B] time = 1.29, size = 35, normalized size = 0.78

$$-\frac{10\,a^2+15\,b^2\,x^{2\,n}+24\,a\,b\,x^n}{60\,n\,x^{6\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^2/x^(6*n + 1),x)
[Out] -(10*a^2 + 15*b^2*x^(2*n) + 24*a*b*x^n)/(60*n*x^(6*n))
```

sympy [A] time = 11.68, size = 46, normalized size = 1.02

$$\begin{cases} -\frac{a^2x^{-6n}}{6n}-\frac{2abx^{-5n}}{5n}-\frac{b^2x^{-4n}}{4n} & \text{for } n\neq 0 \\ (a+b)^2\log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-6*n)*(a+b*x**n)**2,x)
[Out] Piecewise((-a**2*x**(-6*n)/(6*n) - 2*a*b*x**(-5*n)/(5*n) - b**2*x**(-4*n)/(
4*n), Ne(n, 0)), ((a + b)**2*log(x), True))
```

3.2539 $\int x^{-1+4n}\left(a+bx^n\right)^3\,dx$

Optimal. Leaf size=63

$$\frac{a^3x^{4n}}{4n}+\frac{3a^2bx^{5n}}{5n}+\frac{ab^2x^{6n}}{2n}+\frac{b^3x^{7n}}{7n}$$

```
[Out] 1/4*a^3*x^(4*n)/n+3/5*a^2*b*x^(5*n)/n+1/2*a*b^2*x^(6*n)/n+1/7*b^3*x^(7*n)/n
```

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$\frac{3a^2bx^{5n}}{5n}+\frac{a^3x^{4n}}{4n}+\frac{ab^2x^{6n}}{2n}+\frac{b^3x^{7n}}{7n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 4*n)*(a + b*x^n)^3,x]
```

```
[Out] (a^3*x^(4*n))/(4*n) + (3*a^2*b*x^(5*n))/(5*n) + (a*b^2*x^(6*n))/(2*n) + (b^
3*x^(7*n))/(7*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1+4n}\left(a+bx^n\right)^3\,dx &= \frac{\text{Subst}\left(\int x^3(a+bx)^3\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int\left(a^3x^3+3a^2bx^4+3ab^2x^5+b^3x^6\right)\,dx,x,x^n\right)}{n}\\ &= \frac{a^3x^{4n}}{4n}+\frac{3a^2bx^{5n}}{5n}+\frac{ab^2x^{6n}}{2n}+\frac{b^3x^{7n}}{7n}\end{aligned}$$

Mathematica [A] time = 0.02, size = 48, normalized size = 0.76

$$\frac{x^{4n}\left(35a^3+84a^2bx^n+70ab^2x^{2n}+20b^3x^{3n}\right)}{140n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 4*n)*(a + b*x^n)^3,x]
```

```
[Out] (x^(4*n)*(35*a^3 + 84*a^2*b*x^n + 70*a*b^2*x^(2*n) + 20*b^3*x^(3*n)))/(140*
n)
```

fricas [A] time = 0.67, size = 48, normalized size = 0.76

$$\frac{20\,b^3x^{7\,n}+70\,ab^2x^{6\,n}+84\,a^2bx^{5\,n}+35\,a^3x^{4\,n}}{140\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)*(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] 1/140*(20*b^3*x^(7*n) + 70*a*b^2*x^(6*n) + 84*a^2*b*x^(5*n) + 35*a^3*x^(4*n
))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n+a)^3x^{4\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)*(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^3*x^(4*n - 1), x)
```

maple [A] time = 0.02, size = 56, normalized size = 0.89

$$\frac{a^3x^{4n}}{4n}+\frac{3a^2b\,x^{5n}}{5n}+\frac{a\,b^2x^{6n}}{2n}+\frac{b^3x^{7n}}{7n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+4*n)*(b*x^n+a)^3,x)
```

```
[Out] 1/7*b^3/n*(x^n)^7+1/2*a*b^2/n*(x^n)^6+3/5*a^2*b/n*(x^n)^5+1/4*a^3/n*(x^n)^4
```

maxima [A] time = 0.56, size = 55, normalized size = 0.87

$$\frac{b^3x^{7\,n}}{7\,n}+\frac{ab^2x^{6\,n}}{2\,n}+\frac{3\,a^2bx^{5\,n}}{5\,n}+\frac{a^3x^{4\,n}}{4\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)*(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] 1/7*b^3*x^(7*n)/n + 1/2*a*b^2*x^(6*n)/n + 3/5*a^2*b*x^(5*n)/n + 1/4*a^3*x^(
4*n)/n
```

mupad [B] time = 1.36, size = 55, normalized size = 0.87

$$\frac{a^3\,x^{4\,n}}{4\,n}+\frac{b^3\,x^{7\,n}}{7\,n}+\frac{3\,a^2\,b\,x^{5\,n}}{5\,n}+\frac{a\,b^2\,x^{6\,n}}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4*n - 1)*(a + b*x^n)^3,x)
```

```
[Out] (a^3*x^(4*n))/(4*n) + (b^3*x^(7*n))/(7*n) + (3*a^2*b*x^(5*n))/(5*n) + (a*b^
2*x^(6*n))/(2*n)
```

sympy [A] time = 26.85, size = 60, normalized size = 0.95

$$\begin{cases}\frac{a^3x^{4n}}{4n}+\frac{3a^2bx^{5n}}{5n}+\frac{ab^2x^{6n}}{2n}+\frac{b^3x^{7n}}{7n} & \text{for } n\neq 0\\ (a+b)^3\log(x) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+4*n)*(a+b*x**n)**3,x)
```

```
[Out] Piecewise((a**3*x**(4*n)/(4*n) + 3*a**2*b*x**(5*n)/(5*n) + a*b**2*x**(6*n)/
(2*n) + b**3*x**(7*n)/(7*n), Ne(n, 0)), ((a + b)**3*log(x), True))
```

3.2540 $\int x^{-1+3n}\left(a+bx^n\right)^3\,dx$

Optimal. Leaf size=63

$$\frac{a^3x^{3n}}{3n}+\frac{3a^2bx^{4n}}{4n}+\frac{3ab^2x^{5n}}{5n}+\frac{b^3x^{6n}}{6n}$$

```
[Out] 1/3*a^3*x^(3*n)/n+3/4*a^2*b*x^(4*n)/n+3/5*a*b^2*x^(5*n)/n+1/6*b^3*x^(6*n)/n
```

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$\frac{3a^2bx^{4n}}{4n}+\frac{a^3x^{3n}}{3n}+\frac{3ab^2x^{5n}}{5n}+\frac{b^3x^{6n}}{6n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)*(a + b*x^n)^3,x]
```

```
[Out] (a^3*x^(3*n))/(3*n) + (3*a^2*b*x^(4*n))/(4*n) + (3*a*b^2*x^(5*n))/(5*n) + (
b^3*x^(6*n))/(6*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1+3n}\left(a+bx^n\right)^3\,dx &= \frac{\text{Subst}\left(\int x^2(a+bx)^3\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int\left(a^3x^2+3a^2bx^3+3ab^2x^4+b^3x^5\right)\,dx,x,x^n\right)}{n}\\ &= \frac{a^3x^{3n}}{3n}+\frac{3a^2bx^{4n}}{4n}+\frac{3ab^2x^{5n}}{5n}+\frac{b^3x^{6n}}{6n}\end{aligned}$$

Mathematica [A] time = 0.03, size = 48, normalized size = 0.76

$$\frac{x^{3n}\left(20a^3+45a^2bx^n+36ab^2x^{2n}+10b^3x^{3n}\right)}{60n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)*(a + b*x^n)^3,x]
```

```
[Out] (x^(3*n)*(20*a^3 + 45*a^2*b*x^n + 36*a*b^2*x^(2*n) + 10*b^3*x^(3*n)))/(60*n
)
```

fricas [A] time = 0.73, size = 48, normalized size = 0.76

$$\frac{10\,b^3x^{6\,n}+36\,ab^2x^{5\,n}+45\,a^2bx^{4\,n}+20\,a^3x^{3\,n}}{60\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] 1/60*(10*b^3*x^(6*n) + 36*a*b^2*x^(5*n) + 45*a^2*b*x^(4*n) + 20*a^3*x^(3*n)
)/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n+a)^3x^{3\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^3*x^(3*n - 1), x)
```

maple [A] time = 0.02, size = 56, normalized size = 0.89

$$\frac{a^3x^{3n}}{3n}+\frac{3a^2b\,x^{4n}}{4n}+\frac{3a\,b^2x^{5n}}{5n}+\frac{b^3x^{6n}}{6n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n-1)*(b*x^n+a)^3,x)
```

```
[Out] 1/6*b^3/n*(x^n)^6+3/5*a*b^2/n*(x^n)^5+3/4*a^2*b/n*(x^n)^4+1/3*a^3/n*(x^n)^3
```

maxima [A] time = 0.54, size = 55, normalized size = 0.87

$$\frac{b^3x^{6\,n}}{6\,n}+\frac{3\,ab^2x^{5\,n}}{5\,n}+\frac{3\,a^2bx^{4\,n}}{4\,n}+\frac{a^3x^{3\,n}}{3\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] 1/6*b^3*x^(6*n)/n + 3/5*a*b^2*x^(5*n)/n + 3/4*a^2*b*x^(4*n)/n + 1/3*a^3*x^(
3*n)/n
```

mupad [B] time = 1.34, size = 55, normalized size = 0.87

$$\frac{a^3\,x^{3\,n}}{3\,n}+\frac{b^3\,x^{6\,n}}{6\,n}+\frac{3\,a^2\,b\,x^{4\,n}}{4\,n}+\frac{3\,a\,b^2\,x^{5\,n}}{5\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n - 1)*(a + b*x^n)^3,x)
```

```
[Out] (a^3*x^(3*n))/(3*n) + (b^3*x^(6*n))/(6*n) + (3*a^2*b*x^(4*n))/(4*n) + (3*a*
b^2*x^(5*n))/(5*n)
```

sympy [A] time = 26.44, size = 61, normalized size = 0.97

$$\begin{cases}\frac{a^3x^{3n}}{3n}+\frac{3a^2bx^{4n}}{4n}+\frac{3ab^2x^{5n}}{5n}+\frac{b^3x^{6n}}{6n} & \text{for } n\neq 0\\ (a+b)^3\log(x) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)*(a+b*x**n)**3,x)
```

```
[Out] Piecewise((a**3*x**(3*n)/(3*n) + 3*a**2*b*x**(4*n)/(4*n) + 3*a*b**2*x**(5*n
)/(5*n) + b**3*x**(6*n)/(6*n), Ne(n, 0)), ((a + b)**3*log(x), True))
```

3.2541 $\int x^{-1+2n}\left(a+bx^n\right)^3\,dx$

Optimal. Leaf size=40

$$\frac{\left(a+bx^n\right)^5}{5b^2n}-\frac{a\left(a+bx^n\right)^4}{4b^2n}$$

```
[Out] -1/4*a*(a+b*x^n)^4/b^2/n+1/5*(a+b*x^n)^5/b^2/n
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$\frac{\left(a+bx^n\right)^5}{5b^2n}-\frac{a\left(a+bx^n\right)^4}{4b^2n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)*(a + b*x^n)^3,x]
```

```
[Out] -(a*(a + b*x^n)^4)/(4*b^2*n) + (a + b*x^n)^5/(5*b^2*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1+2n}\left(a+bx^n\right)^3\,dx &= \frac{\text{Subst}\left(\int x(a+bx)^3\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int\left(-\frac{a(a+bx)^3}{b}+\frac{(a+bx)^4}{b}\right)dx,x,x^n\right)}{n}\\ &= -\frac{a\left(a+bx^n\right)^4}{4b^2n}+\frac{\left(a+bx^n\right)^5}{5b^2n}\end{aligned}$$

Mathematica [A] time = 0.03, size = 27, normalized size = 0.68

$$-\frac{\left(a-4bx^n\right)\left(a+bx^n\right)^4}{20b^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)*(a + b*x^n)^3,x]
```

```
[Out] -1/20*((a - 4*b*x^n)*(a + b*x^n)^4)/(b^2*n)
```

fricas [A] time = 0.51, size = 48, normalized size = 1.20

$$\frac{4\,b^3x^{5\,n}+15\,ab^2x^{4\,n}+20\,a^2bx^{3\,n}+10\,a^3x^{2\,n}}{20\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] 1/20*(4*b^3*x^(5*n) + 15*a*b^2*x^(4*n) + 20*a^2*b*x^(3*n) + 10*a^3*x^(2*n))
/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^3 x^{2n-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^3*x^(2*n - 1), x)
```

maple [A] time = 0.02, size = 63, normalized size = 1.58

$$\frac{a^3 e^{2n\ln(x)}}{2n} + \frac{a^2 b\, e^{3n\ln(x)}}{n} + \frac{3a\, b^2 e^{4n\ln(x)}}{4n} + \frac{b^3 e^{5n\ln(x)}}{5n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n-1)*(b*x^n+a)^3,x)
```

```
[Out] a^2*b/n*exp(n*ln(x))^3+1/2*a^3/n*exp(n*ln(x))^2+1/5*b^3/n*exp(n*ln(x))^5+3/
4*a*b^2/n*exp(n*ln(x))^4
```

maxima [A] time = 0.46, size = 54, normalized size = 1.35

$$\frac{b^3 x^{5n}}{5n} + \frac{3\, ab^2 x^{4n}}{4n} + \frac{a^2 b x^{3n}}{n} + \frac{a^3 x^{2n}}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] 1/5*b^3*x^(5*n)/n + 3/4*a*b^2*x^(4*n)/n + a^2*b*x^(3*n)/n + 1/2*a^3*x^(2*n)
/n
```

mupad [B] time = 1.33, size = 54, normalized size = 1.35

$$\frac{a^3\, x^{2n}}{2n} + \frac{b^3\, x^{5n}}{5n} + \frac{a^2\, b\, x^{3n}}{n} + \frac{3\, a\, b^2\, x^{4n}}{4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)*(a + b*x^n)^3,x)
```

```
[Out] (a^3*x^(2*n))/(2*n) + (b^3*x^(5*n))/(5*n) + (a^2*b*x^(3*n))/n + (3*a*b^2*x^
(4*n))/(4*n)
```

sympy [A] time = 26.60, size = 58, normalized size = 1.45

$$\begin{cases} \frac{a^3 x^{2n}}{2n} + \frac{a^2 b x^{3n}}{n} + \frac{3ab^2 x^{4n}}{4n} + \frac{b^3 x^{5n}}{5n} & \text{for } n \neq 0 \\ (a+b)^3 \log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a+b*x**n)**3,x)
```

```
[Out] Piecewise((a**3*x**(2*n)/(2*n) + a**2*b*x**(3*n)/n + 3*a*b**2*x**(4*n)/(4*n
) + b**3*x**(5*n)/(5*n), Ne(n, 0)), ((a + b)**3*log(x), True))
```

3.2542 $\int x^{-1+n} (a + bx^n)^3 \, dx$

Optimal. Leaf size=19

$$\frac{(a + bx^n)^4}{4bn}$$

```
[Out] 1/4*(a+b*x^n)^4/b/n
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$\frac{(a + bx^n)^4}{4bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*(a + b*x^n)^3,x]
```

```
[Out] (a + b*x^n)^4/(4*b*n)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^{-1+n} (a + bx^n)^3 \, dx = \frac{(a + bx^n)^4}{4bn}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{(a + bx^n)^4}{4bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*(a + b*x^n)^3,x]
```

```
[Out] (a + b*x^n)^4/(4*b*n)
```

fricas [B] time = 0.66, size = 45, normalized size = 2.37

$$\frac{b^3 x^{4\,n} + 4\,ab^2 x^{3\,n} + 6\,a^2 b x^{2\,n} + 4\,a^3 x^n}{4\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*x^(4*n) + 4*a*b^2*x^(3*n) + 6*a^2*b*x^(2*n) + 4*a^3*x^n)/n
```

giac [B] time = 0.15, size = 45, normalized size = 2.37

$$\frac{b^3 x^{4\,n} + 4\,ab^2 x^{3\,n} + 6\,a^2 b x^{2\,n} + 4\,a^3 x^n}{4\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] 1/4*(b^3*x^(4*n) + 4*a*b^2*x^(3*n) + 6*a^2*b*x^(2*n) + 4*a^3*x^n)/n
```

maple [B] time = 0.02, size = 60, normalized size = 3.16

$$\frac{a^3 e^{n\ln(x)}}{n} + \frac{3a^2 b\, e^{2n\ln(x)}}{2n} + \frac{a\, b^2 e^{3n\ln(x)}}{n} + \frac{b^3 e^{4n\ln(x)}}{4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*(b*x^n+a)^3,x)
```

```
[Out] a^3/n*exp(n*ln(x))+a*b^2/n*exp(n*ln(x))^3+1/4*b^3/n*exp(n*ln(x))^4+3/2*a^2*
b/n*exp(n*ln(x))^2
```

maxima [A] time = 0.61, size = 17, normalized size = 0.89

$$\frac{(bx^n + a)^4}{4\, bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^n + a)^4/(b*n)
```

mupad [B] time = 1.30, size = 51, normalized size = 2.68

$$\frac{a^3\, x^n}{n} + \frac{b^3\, x^{4\,n}}{4\, n} + \frac{3\, a^2\, b\, x^{2\,n}}{2\, n} + \frac{a\, b^2\, x^{3\,n}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*(a + b*x^n)^3,x)
```

```
[Out] (a^3*x^n)/n + (b^3*x^(4*n))/(4*n) + (3*a^2*b*x^(2*n))/(2*n) + (a*b^2*x^(3*n
))/n
```

sympy [A] time = 5.48, size = 54, normalized size = 2.84

$$\begin{cases} \frac{a^3 x^n}{n} + \frac{3a^2 b x^{2n}}{2n} + \frac{ab^2 x^{3n}}{n} + \frac{b^3 x^{4n}}{4n} & \text{for } n \neq 0 \\ (a+b)^3 \log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(a+b*x**n)**3,x)
```

```
[Out] Piecewise((a**3*x**n/n + 3*a**2*b*x**(2*n)/(2*n) + a*b**2*x**(3*n)/n + b**3
*x**(4*n)/(4*n), Ne(n, 0)), ((a + b)**3*log(x), True))
```

3.2543 $\int \frac{(a+bx^n)^3}{x}\,dx$

Optimal. Leaf size=50

$$a^3\log(x)+\frac{3a^2bx^n}{n}+\frac{3ab^2x^{2n}}{2n}+\frac{b^3x^{3n}}{3n}$$

```
[Out] 3*a^2*b*x^n/n+3/2*a*b^2*x^(2*n)/n+1/3*b^3*x^(3*n)/n+a^3*ln(x)
```

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{3a^2bx^n}{n}+a^3\log(x)+\frac{3ab^2x^{2n}}{2n}+\frac{b^3x^{3n}}{3n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^3/x,x]
```

```
[Out] (3*a^2*b*x^n)/n + (3*a*b^2*x^(2*n))/(2*n) + (b^3*x^(3*n))/(3*n) + a^3*Log[x
]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^n)^3}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^3}{x}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(3a^2b+\frac{a^3}{x}+3ab^2x+b^3x^2\right)\,dx,x,x^n\right)}{n}\\ &= \frac{3a^2bx^n}{n}+\frac{3ab^2x^{2n}}{2n}+\frac{b^3x^{3n}}{3n}+a^3\log(x)\end{aligned}$$

Mathematica [A] time = 0.04, size = 41, normalized size = 0.82

$$a^3\log(x)+\frac{bx^n\left(18a^2+9abx^n+2b^2x^{2n}\right)}{6n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^3/x,x]
```

```
[Out] (b*x^n*(18*a^2 + 9*a*b*x^n + 2*b^2*x^(2*n)))/(6*n) + a^3*Log[x]
```

fricas [A] time = 0.57, size = 44, normalized size = 0.88

$$\frac{6\,a^3 n\log(x) + 2\,b^3x^{3\,n} + 9\,ab^2x^{2\,n} + 18\,a^2bx^n}{6\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^3/x,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^3*n*log(x) + 2*b^3*x^(3*n) + 9*a*b^2*x^(2*n) + 18*a^2*b*x^n)/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^3/x, x)
```

maple [A] time = 0.00, size = 52, normalized size = 1.04

$$\frac{a^3\ln(x^n)}{n} + \frac{3a^2b\,x^n}{n} + \frac{3a\,b^2x^{2n}}{2n} + \frac{b^3x^{3n}}{3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^3/x,x)
```

```
[Out] 1/3/n*b^3*(x^n)^3+3/2/n*(x^n)^2*a*b^2+3*a^2*b/n*x^n+a^3/n*ln(x^n)
```

maxima [A] time = 0.60, size = 48, normalized size = 0.96

$$\frac{a^3\log(x^n)}{n} + \frac{2\,b^3x^{3\,n} + 9\,ab^2x^{2\,n} + 18\,a^2bx^n}{6\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^3/x,x, algorithm="maxima")
```

```
[Out] a^3*log(x^n)/n + 1/6*(2*b^3*x^(3*n) + 9*a*b^2*x^(2*n) + 18*a^2*b*x^n)/n
```

mupad [B] time = 0.00, size = 46, normalized size = 0.92

$$a^3\,\ln(x) + \frac{b^3\,x^{3\,n}}{3\,n} + \frac{3\,a^2\,b\,x^n}{n} + \frac{3\,a\,b^2\,x^{2\,n}}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^3/x,x)
```

```
[Out] a^3*log(x) + (b^3*x^(3*n))/(3*n) + (3*a^2*b*x^n)/n + (3*a*b^2*x^(2*n))/(2*n
)
```

sympy [A] time = 0.45, size = 53, normalized size = 1.06

$$\begin{cases} a^3\log(x) + \frac{3a^2bx^n}{n} + \frac{3ab^2x^{2n}}{2n} + \frac{b^3x^{3n}}{3n} & \text{for } n \neq 0 \\ (a+b)^3\log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**3/x,x)
```

```
[Out] Piecewise((a**3*log(x) + 3*a**2*b*x**n/n + 3*a*b**2*x**(2*n)/(2*n) + b**3*x
**(3*n)/(3*n), Ne(n, 0)), ((a + b)**3*log(x), True))
```

3.2544 $\int x^{-1-n}\left(a+bx^n\right)^3\,dx$

Optimal. Leaf size=49

$$-\frac{a^3x^{-n}}{n}+3a^2b\log(x)+\frac{3ab^2x^n}{n}+\frac{b^3x^{2n}}{2n}$$

```
[Out] -a^3/n/(x^n)+3*a*b^2*x^n/n+1/2*b^3*x^(2*n)/n+3*a^2*b*ln(x)
```

Rubi [A] time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$3a^2b\log(x)-\frac{a^3x^{-n}}{n}+\frac{3ab^2x^n}{n}+\frac{b^3x^{2n}}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n)*(a + b*x^n)^3,x]
```

```
[Out] -(a^3/(n*x^n)) + (3*a*b^2*x^n)/n + (b^3*x^(2*n))/(2*n) + 3*a^2*b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1-n}\left(a+bx^n\right)^3\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^3}{x^2}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(3ab^2+\frac{a^3}{x^2}+\frac{3a^2b}{x}+b^3x\right)\,dx,x,x^n\right)}{n}\\ &= -\frac{a^3x^{-n}}{n}+\frac{3ab^2x^n}{n}+\frac{b^3x^{2n}}{2n}+3a^2b\log(x)\end{aligned}$$

Mathematica [A] time = 0.03, size = 45, normalized size = 0.92

$$\frac{-a^3x^{-n}+3a^2bn\log(x)+3ab^2x^n+\frac{1}{2}b^3x^{2n}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n)*(a + b*x^n)^3,x]
```

```
[Out] (-(a^3/x^n) + 3*a*b^2*x^n + (b^3*x^(2*n))/2 + 3*a^2*b*n*Log[x])/n
```

fricas [A] time = 0.69, size = 48, normalized size = 0.98

$$\frac{6\,a^2bnx^n\log(x)+b^3x^{3\,n}+6\,ab^2x^{2\,n}-2\,a^3}{2\,nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a^2*b*n*x^n*log(x) + b^3*x^(3*n) + 6*a*b^2*x^(2*n) - 2*a^3)/(n*x^n)
```

giac [A] time = 0.19, size = 48, normalized size = 0.98

$$\frac{6\,a^2bnx^n\log(x) + b^3x^{3\,n} + 6\,ab^2x^{2\,n} - 2\,a^3}{2\,nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] 1/2*(6*a^2*b*n*x^n*log(x) + b^3*x^(3*n) + 6*a*b^2*x^(2*n) - 2*a^3)/(n*x^n)
```

maple [A] time = 0.02, size = 62, normalized size = 1.27

$$\left(3a^2b\,\mathrm{e}^{n\ln(x)}\ln(x) + \frac{3a\,b^2\mathrm{e}^{2n\ln(x)}}{n} + \frac{b^3\mathrm{e}^{3n\ln(x)}}{2n} - \frac{a^3}{n}\right)\mathrm{e}^{-n\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n)*(b*x^n+a)^3,x)
```

```
[Out] (3*a^2*b*ln(x)*exp(n*ln(x))-a^3/n+1/2*b^3/n*exp(n*ln(x))^3+3*a*b^2/n*exp(n*
ln(x))^2)/exp(n*ln(x))
```

maxima [A] time = 0.69, size = 47, normalized size = 0.96

$$3\,a^2b\log(x) + \frac{b^3x^{2\,n}}{2\,n} + \frac{3\,ab^2x^n}{n} - \frac{a^3}{nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] 3*a^2*b*log(x) + 1/2*b^3*x^(2*n)/n + 3*a*b^2*x^n/n - a^3/(n*x^n)
```

mupad [B] time = 1.40, size = 47, normalized size = 0.96

$$3\,a^2\,b\,\ln(x) - \frac{a^3}{n\,x^n} + \frac{b^3\,x^{2\,n}}{2\,n} + \frac{3\,a\,b^2\,x^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^3/x^(n + 1),x)
```

```
[Out] 3*a^2*b*log(x) - a^3/(n*x^n) + (b^3*x^(2*n))/(2*n) + (3*a*b^2*x^n)/n
```

sympy [A] time = 61.81, size = 277, normalized size = 5.65

$$\begin{cases} a^3x + 3a^2b\log(x) - \frac{3ab^2}{x} - \frac{b^3}{2x^2} \\ (a+b)^3\log(x) \\ -\frac{2a^3n}{2n^2x^n+2nx^n} - \frac{2a^3}{2n^2x^n+2nx^n} + \frac{6a^2bn^2x^n\log(x)}{2n^2x^n+2nx^n} + \frac{6a^2bnx^n\log(x)}{2n^2x^n+2nx^n} + \frac{6a^2bnx^n}{2n^2x^n+2nx^n} + \frac{6ab^2nx^{2n}}{2n^2x^n+2nx^n} + \frac{6ab^2x^{2n}}{2n^2x^n+2nx^n} + \frac{b^3nx^{3n}}{2n^2x^n+2nx^n} + \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n)*(a+b*x**n)**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*log(x) - 3*a*b**2/x - b**3/(2*x**2), Eq(n, -1)
), ((a + b)**3*log(x), Eq(n, 0)), (-2*a**3*n/(2*n**2*x**n + 2*n*x**n) - 2*a
**3/(2*n**2*x**n + 2*n*x**n) + 6*a**2*b*n**2*x**n*log(x)/(2*n**2*x**n + 2*n
*x**n) + 6*a**2*b*n*x**n*log(x)/(2*n**2*x**n + 2*n*x**n) + 6*a**2*b*n*x**n/
(2*n**2*x**n + 2*n*x**n) + 6*a*b**2*n*x**(2*n)/(2*n**2*x**n + 2*n*x**n) + 6
*a*b**2*x**(2*n)/(2*n**2*x**n + 2*n*x**n) + b**3*n*x**(3*n)/(2*n**2*x**n +
2*n*x**n) + b**3*x**(3*n)/(2*n**2*x**n + 2*n*x**n), True))
```

3.2545 $\int x^{-1-2n}\left(a+bx^n\right)^3\,dx$

Optimal. Leaf size=48

$$-\frac{a^3x^{-2n}}{2n}-\frac{3a^2bx^{-n}}{n}+3ab^2\log(x)+\frac{b^3x^n}{n}$$

```
[Out] -1/2*a^3/n/(x^(2*n))-3*a^2*b/n/(x^n)+b^3*x^n/n+3*a*b^2*ln(x)
```

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$-\frac{3a^2bx^{-n}}{n}-\frac{a^3x^{-2n}}{2n}+3ab^2\log(x)+\frac{b^3x^n}{n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 2*n)*(a + b*x^n)^3,x]
```

```
[Out] -a^3/(2*n*x^(2*n)) - (3*a^2*b)/(n*x^n) + (b^3*x^n)/n + 3*a*b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1-2n}\left(a+bx^n\right)^3\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^3}{x^3}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(b^3+\frac{a^3}{x^3}+\frac{3a^2b}{x^2}+\frac{3ab^2}{x}\right)\,dx,x,x^n\right)}{n}\\ &= -\frac{a^3x^{-2n}}{2n}-\frac{3a^2bx^{-n}}{n}+\frac{b^3x^n}{n}+3ab^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.05, size = 44, normalized size = 0.92

$$\frac{-\frac{1}{2}a^3x^{-2n}-3a^2bx^{-n}+3ab^2n\log(x)+b^3x^n}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 2*n)*(a + b*x^n)^3,x]
```

```
[Out] (-1/2*a^3/x^(2*n) - (3*a^2*b)/x^n + b^3*x^n + 3*a*b^2*n*Log[x])/n
```

fricas [A] time = 0.78, size = 51, normalized size = 1.06

$$\frac{6\,ab^2nx^{2\,n}\log(x)+2\,b^3x^{3\,n}-6\,a^2bx^n-a^3}{2\,nx^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*n*x^(2*n)*log(x) + 2*b^3*x^(3*n) - 6*a^2*b*x^n - a^3)/(n*x^(2*
n))
```

giac [A] time = 0.21, size = 51, normalized size = 1.06

$$\frac{6\,ab^2nx^{2\,n}\log(x)+2\,b^3x^{3\,n}-6\,a^2bx^n-a^3}{2\,nx^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] 1/2*(6*a*b^2*n*x^(2*n)*log(x) + 2*b^3*x^(3*n) - 6*a^2*b*x^n - a^3)/(n*x^(2*
n))
```

maple [A] time = 0.02, size = 61, normalized size = 1.27

$$\left(3a\,b^2\mathrm{e}^{2n\ln(x)}\ln(x)-\frac{3a^2b\,\mathrm{e}^{n\ln(x)}}{n}+\frac{b^3\mathrm{e}^{3n\ln(x)}}{n}-\frac{a^3}{2n}\right)\mathrm{e}^{-2n\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-2*n)*(b*x^n+a)^3,x)
```

```
[Out] (b^3/n*exp(n*ln(x))^3+3*a*b^2*ln(x)*exp(n*ln(x))^2-1/2*a^3/n-3*a^2*b/n*exp(
n*ln(x)))/exp(n*ln(x))^2
```

maxima [A] time = 0.65, size = 48, normalized size = 1.00

$$3\,ab^2\log(x)+\frac{b^3x^n}{n}-\frac{a^3}{2\,nx^{2\,n}}-\frac{3\,a^2b}{nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] 3*a*b^2*log(x) + b^3*x^n/n - 1/2*a^3/(n*x^(2*n)) - 3*a^2*b/(n*x^n)
```

mupad [B] time = 1.36, size = 48, normalized size = 1.00

$$\frac{b^3\,x^n}{n}+3\,a\,b^2\,\ln(x)-\frac{a^3}{2\,n\,x^{2\,n}}-\frac{3\,a^2\,b}{n\,x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^3/x^(2*n + 1),x)
```

```
[Out] (b^3*x^n)/n + 3*a*b^2*log(x) - a^3/(2*n*x^(2*n)) - (3*a^2*b)/(n*x^n)
```

sympy [A] time = 117.45, size = 318, normalized size = 6.62

$$\begin{cases} a^3x+6a^2b\sqrt{x}+3ab^2\log(x)-\frac{2b^3}{\sqrt{x}} \\ (a+b)^3\log(x) \\ -\frac{2a^3n}{4n^2x^{2n}+2nx^{2n}}-\frac{a^3}{4n^2x^{2n}+2nx^{2n}}-\frac{12a^2bnx^n}{4n^2x^{2n}+2nx^{2n}}-\frac{6a^2bx^n}{4n^2x^{2n}+2nx^{2n}}+\frac{12ab^2n^2x^{2n}\log(x)}{4n^2x^{2n}+2nx^{2n}}+\frac{6ab^2nx^{2n}\log(x)}{4n^2x^{2n}+2nx^{2n}}+\frac{6ab^2nx^{2n}}{4n^2x^{2n}+2nx^{2n}}+\frac{4b^3n}{4n^2x^{2n}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-2*n)*(a+b*x**n)**3,x)
```

```
[Out] Piecewise((a**3*x + 6*a**2*b*sqrt(x) + 3*a*b**2*log(x) - 2*b**3/sqrt(x), Eq
(n, -1/2)), ((a + b)**3*log(x), Eq(n, 0)), (-2*a**3*n/(4*n**2*x**(2*n) + 2*
n*x**(2*n)) - a**3/(4*n**2*x**(2*n) + 2*n*x**(2*n)) - 12*a**2*b*n*x**n/(4*n
**2*x**(2*n) + 2*n*x**(2*n)) - 6*a**2*b*x**n/(4*n**2*x**(2*n) + 2*n*x**(2*n
)) + 12*a*b**2*n**2*x**(2*n)*log(x)/(4*n**2*x**(2*n) + 2*n*x**(2*n)) + 6*a*
b**2*n*x**(2*n)*log(x)/(4*n**2*x**(2*n) + 2*n*x**(2*n)) + 6*a*b**2*n*x**(2*
n)/(4*n**2*x**(2*n) + 2*n*x**(2*n)) + 4*b**3*n*x**(3*n)/(4*n**2*x**(2*n) +
2*n*x**(2*n)) + 2*b**3*x**(3*n)/(4*n**2*x**(2*n) + 2*n*x**(2*n)), True))
```

3.2546 $\int x^{-1-3n}\left(a+bx^n\right)^3\,dx$

Optimal. Leaf size=52

$$-\frac{a^3x^{-3n}}{3n}-\frac{3a^2bx^{-2n}}{2n}-\frac{3ab^2x^{-n}}{n}+b^3\log(x)$$

```
[Out] -1/3*a^3/n/(x^(3*n))-3/2*a^2*b/n/(x^(2*n))-3*a*b^2/n/(x^n)+b^3*ln(x)
```

Rubi [A] time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$-\frac{3a^2bx^{-2n}}{2n}-\frac{a^3x^{-3n}}{3n}-\frac{3ab^2x^{-n}}{n}+b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 3*n)*(a + b*x^n)^3,x]
```

```
[Out] -a^3/(3*n*x^(3*n)) - (3*a^2*b)/(2*n*x^(2*n)) - (3*a*b^2)/(n*x^n) + b^3*Log[
x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1-3n}\left(a+bx^n\right)^3\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^3}{x^4}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a^3}{x^4}+\frac{3a^2b}{x^3}+\frac{3ab^2}{x^2}+\frac{b^3}{x}\right)\,dx,x,x^n\right)}{n}\\ &= -\frac{a^3x^{-3n}}{3n}-\frac{3a^2bx^{-2n}}{2n}-\frac{3ab^2x^{-n}}{n}+b^3\log(x)\end{aligned}$$

Mathematica [A] time = 0.06, size = 43, normalized size = 0.83

$$b^3\log(x)-\frac{ax^{-3n}\left(2a^2+9abx^n+18b^2x^{2n}\right)}{6n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 3*n)*(a + b*x^n)^3,x]
```

```
[Out] -1/6*(a*(2*a^2 + 9*a*b*x^n + 18*b^2*x^(2*n)))/(n*x^(3*n)) + b^3*Log[x]
```

fricas [A] time = 0.67, size = 51, normalized size = 0.98

$$\frac{6\,b^3 n x^{3\,n}\log(x) - 18\,ab^2x^{2\,n} - 9\,a^2bx^n - 2\,a^3}{6\,nx^{3\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)*(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(6*b^3*n*x^(3*n)*log(x) - 18*a*b^2*x^(2*n) - 9*a^2*b*x^n - 2*a^3)/(n*x^
(3*n))
```

giac [A] time = 0.20, size = 51, normalized size = 0.98

$$\frac{6\,b^3 n x^{3\,n}\log(x) - 18\,ab^2x^{2\,n} - 9\,a^2bx^n - 2\,a^3}{6\,nx^{3\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)*(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] 1/6*(6*b^3*n*x^(3*n)*log(x) - 18*a*b^2*x^(2*n) - 9*a^2*b*x^n - 2*a^3)/(n*x^
(3*n))
```

maple [A] time = 0.02, size = 61, normalized size = 1.17

$$\left(b^3\mathrm{e}^{3n\ln(x)}\ln(x) - \frac{3a^2b\,\mathrm{e}^{n\ln(x)}}{2n} - \frac{3a\,b^2\mathrm{e}^{2n\ln(x)}}{n} - \frac{a^3}{3n}\right)\mathrm{e}^{-3n\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-3*n)*(b*x^n+a)^3,x)
```

```
[Out] (b^3*ln(x)*exp(n*ln(x))^3-1/3*a^3/n-3*a*b^2/n*exp(n*ln(x))^2-3/2*a^2*b/n*ex
p(n*ln(x)))/exp(n*ln(x))^3
```

maxima [A] time = 0.51, size = 52, normalized size = 1.00

$$b^3\log(x) - \frac{a^3}{3\,nx^{3\,n}} - \frac{3\,a^2b}{2\,nx^{2\,n}} - \frac{3\,ab^2}{nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)*(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] b^3*log(x) - 1/3*a^3/(n*x^(3*n)) - 3/2*a^2*b/(n*x^(2*n)) - 3*a*b^2/(n*x^n)
```

mupad [B] time = 1.34, size = 52, normalized size = 1.00

$$b^3\,\ln(x) - \frac{a^3}{3\,n\,x^{3\,n}} - \frac{3\,a\,b^2}{n\,x^n} - \frac{3\,a^2\,b}{2\,n\,x^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^3/x^(3*n + 1),x)
```

```
[Out] b^3*log(x) - a^3/(3*n*x^(3*n)) - (3*a*b^2)/(n*x^n) - (3*a^2*b)/(2*n*x^(2*n)
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-3*n)*(a+b*x**n)**3,x)
```

```
[Out] Timed out
```

3.2547 $\int x^{-1-4n}\left(a+bx^n\right)^3\,dx$

Optimal. Leaf size=24

$$-\frac{x^{-4n}\left(a+bx^n\right)^4}{4an}$$

```
[Out] -1/4*(a+b*x^n)^4/a/n/(x^(4*n))
```

Rubi [A] time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {264}

$$-\frac{x^{-4n}\left(a+bx^n\right)^4}{4an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 4*n)*(a + b*x^n)^3,x]
```

```
[Out] -(a + b*x^n)^4/(4*a*n*x^(4*n))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^{-1-4n}\left(a+bx^n\right)^3\,dx = -\frac{x^{-4n}\left(a+bx^n\right)^4}{4an}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$-\frac{x^{-4n}\left(a+bx^n\right)^4}{4an}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 4*n)*(a + b*x^n)^3,x]
```

```
[Out] -1/4*(a + b*x^n)^4/(a*n*x^(4*n))
```

fricas [A] time = 0.66, size = 46, normalized size = 1.92

$$-\frac{4\,b^3x^{3\,n}+6\,ab^2x^{2\,n}+4\,a^2bx^n+a^3}{4\,nx^{4\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-4*n)*(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^3*x^(3*n) + 6*a*b^2*x^(2*n) + 4*a^2*b*x^n + a^3)/(n*x^(4*n))
```

giac [A] time = 0.21, size = 46, normalized size = 1.92

$$-\frac{4\,b^3x^{3\,n}+6\,ab^2x^{2\,n}+4\,a^2bx^n+a^3}{4\,nx^{4\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-4*n)*(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] -1/4*(4*b^3*x^(3*n) + 6*a*b^2*x^(2*n) + 4*a^2*b*x^n + a^3)/(n*x^(4*n))
```

maple [B] time = 0.02, size = 63, normalized size = 2.62

$$\left(-\frac{a^2 b\, e^{n \ln(x)}}{n} - \frac{3a\, b^2 e^{2n \ln(x)}}{2n} - \frac{b^3 e^{3n \ln(x)}}{n} - \frac{a^3}{4n}\right) e^{-4n \ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-4*n)*(b*x^n+a)^3,x)
```

```
[Out] (-1/4*a^3/n-b^3/n*exp(n*ln(x))^3-3/2*a*b^2/n*exp(n*ln(x))^2-a^2*b/n*exp(n*l
n(x)))/exp(n*ln(x))^4
```

maxima [B] time = 0.45, size = 61, normalized size = 2.54

$$-\frac{a^3}{4\, n x^{4\, n}} - \frac{a^2 b}{n x^{3\, n}} - \frac{3\, a b^2}{2\, n x^{2\, n}} - \frac{b^3}{n x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-4*n)*(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] -1/4*a^3/(n*x^(4*n)) - a^2*b/(n*x^(3*n)) - 3/2*a*b^2/(n*x^(2*n)) - b^3/(n*x
^n)
```

mupad [B] time = 1.36, size = 61, normalized size = 2.54

$$-\frac{a^3}{4\, n\, x^{4\, n}} - \frac{b^3}{n\, x^n} - \frac{3\, a\, b^2}{2\, n\, x^{2\, n}} - \frac{a^2\, b}{n\, x^{3\, n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^3/x^(4*n + 1),x)
```

```
[Out] - a^3/(4*n*x^(4*n)) - b^3/(n*x^n) - (3*a*b^2)/(2*n*x^(2*n)) - (a^2*b)/(n*x^
(3*n))
```

sympy [A] time = 26.98, size = 56, normalized size = 2.33

$$\begin{cases} -\frac{a^3 x^{-4n}}{4n} - \frac{a^2 b x^{-3n}}{n} - \frac{3ab^2 x^{-2n}}{2n} - \frac{b^3 x^{-n}}{n} & \text{for } n \neq 0 \\ (a+b)^3 \log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-4*n)*(a+b*x**n)**3,x)
```

```
[Out] Piecewise((-a**3*x**(-4*n)/(4*n) - a**2*b*x**(-3*n)/n - 3*a*b**2*x**(-2*n)/
(2*n) - b**3*x**(-n)/n, Ne(n, 0)), ((a + b)**3*log(x), True))
```

3.2548 $\int x^{-1-5n}\left(a+bx^n\right)^3\,dx$

Optimal. Leaf size=50

$$\frac{bx^{-4n}\left(a+bx^n\right)^4}{20a^2n}-\frac{x^{-5n}\left(a+bx^n\right)^4}{5an}$$

```
[Out] -1/5*(a+b*x^n)^4/a/n/(x^(5*n))+1/20*b*(a+b*x^n)^4/a^2/n/(x^(4*n))
```

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {266, 45, 37}

$$\frac{bx^{-4n}\left(a+bx^n\right)^4}{20a^2n}-\frac{x^{-5n}\left(a+bx^n\right)^4}{5an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 5*n)*(a + b*x^n)^3,x]
```

```
[Out] -(a + b*x^n)^4/(5*a*n*x^(5*n)) + (b*(a + b*x^n)^4)/(20*a^2*n*x^(4*n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1-5n}\left(a+bx^n\right)^3\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^3}{x^6}\,dx,x,x^n\right)}{n}\\ &= -\frac{x^{-5n}\left(a+bx^n\right)^4}{5an}-\frac{b\,\text{Subst}\left(\int \frac{(a+bx)^3}{x^5}\,dx,x,x^n\right)}{5an}\\ &= -\frac{x^{-5n}\left(a+bx^n\right)^4}{5an}+\frac{bx^{-4n}\left(a+bx^n\right)^4}{20a^2n}\end{aligned}$$

Mathematica [A] time = 0.02, size = 48, normalized size = 0.96

$$-\frac{x^{-5n}\left(4a^3+15a^2bx^n+20ab^2x^{2n}+10b^3x^{3n}\right)}{20n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 5*n)*(a + b*x^n)^3,x]
```

```
[Out] -1/20*(4*a^3 + 15*a^2*b*x^n + 20*a*b^2*x^(2*n) + 10*b^3*x^(3*n))/(n*x^(5*n)
)
```

fricas [A] time = 0.83, size = 48, normalized size = 0.96

$$-\frac{10\,b^3x^{3\,n}+20\,ab^2x^{2\,n}+15\,a^2bx^n+4\,a^3}{20\,nx^{5\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-5*n)*(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] -1/20*(10*b^3*x^(3*n) + 20*a*b^2*x^(2*n) + 15*a^2*b*x^n + 4*a^3)/(n*x^(5*n)
)
```

giac [A] time = 0.20, size = 48, normalized size = 0.96

$$-\frac{10\,b^3x^{3\,n}+20\,ab^2x^{2\,n}+15\,a^2bx^n+4\,a^3}{20\,nx^{5\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-5*n)*(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] -1/20*(10*b^3*x^(3*n) + 20*a*b^2*x^(2*n) + 15*a^2*b*x^n + 4*a^3)/(n*x^(5*n)
)
```

maple [A] time = 0.02, size = 63, normalized size = 1.26

$$\left(-\frac{3a^2b\,\mathrm{e}^{n\ln(x)}}{4n}-\frac{a\,b^2\mathrm{e}^{2n\ln(x)}}{n}-\frac{b^3\mathrm{e}^{3n\ln(x)}}{2n}-\frac{a^3}{5n}\right)\mathrm{e}^{-5n\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-5*n)*(b*x^n+a)^3,x)
```

```
[Out] (-1/5*a^3/n-1/2*b^3/n*exp(n*ln(x))^3-a*b^2/n*exp(n*ln(x))^2-3/4*a^2*b/n*exp
(n*ln(x)))/exp(n*ln(x))^5
```

maxima [A] time = 0.58, size = 63, normalized size = 1.26

$$-\frac{a^3}{5\,nx^{5\,n}}-\frac{3\,a^2b}{4\,nx^{4\,n}}-\frac{ab^2}{nx^{3\,n}}-\frac{b^3}{2\,nx^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-5*n)*(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] -1/5*a^3/(n*x^(5*n)) - 3/4*a^2*b/(n*x^(4*n)) - a*b^2/(n*x^(3*n)) - 1/2*b^3/
(n*x^(2*n))
```

mupad [B] time = 1.31, size = 63, normalized size = 1.26

$$-\frac{a^3}{5\,n\,x^{5\,n}}-\frac{b^3}{2\,n\,x^{2\,n}}-\frac{a\,b^2}{n\,x^{3\,n}}-\frac{3\,a^2\,b}{4\,n\,x^{4\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^3/x^(5*n + 1),x)
```

```
[Out] - a^3/(5*n*x^(5*n)) - b^3/(2*n*x^(2*n)) - (a*b^2)/(n*x^(3*n)) - (3*a^2*b)/(
4*n*x^(4*n))
```

sympy [**A**] time = 26.47, size = 60, normalized size = 1.20

$$\begin{cases} -\frac{a^3 x^{-5n}}{5n} - \frac{3a^2 b x^{-4n}}{4n} - \frac{ab^2 x^{-3n}}{n} - \frac{b^3 x^{-2n}}{2n} & \text{for } n \neq 0 \\ (a+b)^3 \log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-5*n)*(a+b*x**n)**3,x)
```

```
[Out] Piecewise((-a**3*x**(-5*n)/(5*n) - 3*a**2*b*x**(-4*n)/(4*n) - a*b**2*x**(-3
*n)/n - b**3*x**(-2*n)/(2*n), Ne(n, 0)), ((a + b)**3*log(x), True))
```

3.2549 $\int x^{-1-6n}\left(a+bx^n\right)^3\,dx$

Optimal. Leaf size=63

$$-\frac{a^3x^{-6n}}{6n}-\frac{3a^2bx^{-5n}}{5n}-\frac{3ab^2x^{-4n}}{4n}-\frac{b^3x^{-3n}}{3n}$$

```
[Out] -1/6*a^3/n/(x^(6*n))-3/5*a^2*b/n/(x^(5*n))-3/4*a*b^2/n/(x^(4*n))-1/3*b^3/n/
(x^(3*n))
```

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$-\frac{3a^2bx^{-5n}}{5n}-\frac{a^3x^{-6n}}{6n}-\frac{3ab^2x^{-4n}}{4n}-\frac{b^3x^{-3n}}{3n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 6*n)*(a + b*x^n)^3,x]
```

```
[Out] -a^3/(6*n*x^(6*n)) - (3*a^2*b)/(5*n*x^(5*n)) - (3*a*b^2)/(4*n*x^(4*n)) - b^
3/(3*n*x^(3*n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1-6n}\left(a+bx^n\right)^3\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^3}{x^7}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int\left(\frac{a^3}{x^7}+\frac{3a^2b}{x^6}+\frac{3ab^2}{x^5}+\frac{b^3}{x^4}\right)dx,x,x^n\right)}{n}\\ &= -\frac{a^3x^{-6n}}{6n}-\frac{3a^2bx^{-5n}}{5n}-\frac{3ab^2x^{-4n}}{4n}-\frac{b^3x^{-3n}}{3n}\end{aligned}$$

Mathematica [A] time = 0.02, size = 48, normalized size = 0.76

$$-\frac{x^{-6n}\left(10a^3+36a^2bx^n+45ab^2x^{2n}+20b^3x^{3n}\right)}{60n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 6*n)*(a + b*x^n)^3,x]
```

```
[Out] -1/60*(10*a^3 + 36*a^2*b*x^n + 45*a*b^2*x^(2*n) + 20*b^3*x^(3*n))/(n*x^(6*n
))
```

fricas [A] time = 0.64, size = 48, normalized size = 0.76

$$-\frac{20\,b^3x^{3\,n}+45\,ab^2x^{2\,n}+36\,a^2bx^n+10\,a^3}{60\,nx^{6\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-6*n)*(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] -1/60*(20*b^3*x^(3*n) + 45*a*b^2*x^(2*n) + 36*a^2*b*x^n + 10*a^3)/(n*x^(6*n
))
```

giac [A] time = 0.23, size = 48, normalized size = 0.76

$$-\frac{20\,b^3x^{3\,n}+45\,ab^2x^{2\,n}+36\,a^2bx^n+10\,a^3}{60\,nx^{6\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-6*n)*(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] -1/60*(20*b^3*x^(3*n) + 45*a*b^2*x^(2*n) + 36*a^2*b*x^n + 10*a^3)/(n*x^(6*n
))
```

maple [A] time = 0.02, size = 56, normalized size = 0.89

$$-\frac{a^3x^{-6n}}{6n}-\frac{3a^2b\,x^{-5n}}{5n}-\frac{3a\,b^2x^{-4n}}{4n}-\frac{b^3x^{-3n}}{3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-6*n)*(b*x^n+a)^3,x)
```

```
[Out] -1/3*b^3/n/(x^n)^3-3/4*a*b^2/n/(x^n)^4-3/5*a^2*b/n/(x^n)^5-1/6*a^3/n/(x^n)^
6
```

maxima [A] time = 0.49, size = 63, normalized size = 1.00

$$-\frac{a^3}{6\,nx^{6\,n}}-\frac{3\,a^2b}{5\,nx^{5\,n}}-\frac{3\,ab^2}{4\,nx^{4\,n}}-\frac{b^3}{3\,nx^{3\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-6*n)*(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] -1/6*a^3/(n*x^(6*n)) - 3/5*a^2*b/(n*x^(5*n)) - 3/4*a*b^2/(n*x^(4*n)) - 1/3*
b^3/(n*x^(3*n))
```

mupad [B] time = 1.37, size = 63, normalized size = 1.00

$$-\frac{a^3}{6\,n\,x^{6\,n}}-\frac{b^3}{3\,n\,x^{3\,n}}-\frac{3\,a\,b^2}{4\,n\,x^{4\,n}}-\frac{3\,a^2\,b}{5\,n\,x^{5\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^3/x^(6*n + 1),x)
```

```
[Out] - a^3/(6*n*x^(6*n)) - b^3/(3*n*x^(3*n)) - (3*a*b^2)/(4*n*x^(4*n)) - (3*a^2*
b)/(5*n*x^(5*n))
```

sympy [A] time = 26.87, size = 63, normalized size = 1.00

$$\begin{cases} -\frac{a^3x^{-6n}}{6n}-\frac{3a^2bx^{-5n}}{5n}-\frac{3ab^2x^{-4n}}{4n}-\frac{b^3x^{-3n}}{3n} & \text{for } n\neq 0 \\ (a+b)^3\log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-6*n)*(a+b*x**n)**3,x)
```

```
[Out] Piecewise((-a**3*x**(-6*n)/(6*n) - 3*a**2*b*x**(-5*n)/(5*n) - 3*a*b**2*x**(
-4*n)/(4*n) - b**3*x**(-3*n)/(3*n), Ne(n, 0)), ((a + b)**3*log(x), True))
```

3.2550 $\int x^{-1-7n}\left(a+bx^n\right)^3\,dx$

Optimal. Leaf size=63

$$-\frac{a^3x^{-7n}}{7n}-\frac{a^2bx^{-6n}}{2n}-\frac{3ab^2x^{-5n}}{5n}-\frac{b^3x^{-4n}}{4n}$$

```
[Out] -1/7*a^3/n/(x^(7*n))-1/2*a^2*b/n/(x^(6*n))-3/5*a*b^2/n/(x^(5*n))-1/4*b^3/n/
(x^(4*n))
```

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$-\frac{a^2bx^{-6n}}{2n}-\frac{a^3x^{-7n}}{7n}-\frac{3ab^2x^{-5n}}{5n}-\frac{b^3x^{-4n}}{4n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 7*n)*(a + b*x^n)^3,x]
```

```
[Out] -a^3/(7*n*x^(7*n)) - (a^2*b)/(2*n*x^(6*n)) - (3*a*b^2)/(5*n*x^(5*n)) - b^3/
(4*n*x^(4*n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1-7n}\left(a+bx^n\right)^3\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^3}{x^8}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a^3}{x^8}+\frac{3a^2b}{x^7}+\frac{3ab^2}{x^6}+\frac{b^3}{x^5}\right)\,dx,x,x^n\right)}{n}\\ &= -\frac{a^3x^{-7n}}{7n}-\frac{a^2bx^{-6n}}{2n}-\frac{3ab^2x^{-5n}}{5n}-\frac{b^3x^{-4n}}{4n}\end{aligned}$$

Mathematica [A] time = 0.04, size = 48, normalized size = 0.76

$$-\frac{x^{-7n}\left(20a^3+70a^2bx^n+84ab^2x^{2n}+35b^3x^{3n}\right)}{140n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 7*n)*(a + b*x^n)^3,x]
```

```
[Out] -1/140*(20*a^3 + 70*a^2*b*x^n + 84*a*b^2*x^(2*n) + 35*b^3*x^(3*n))/(n*x^(7*
n))
```

fricas [A] time = 0.65, size = 48, normalized size = 0.76

$$-\frac{35\,b^3x^{3\,n}+84\,ab^2x^{2\,n}+70\,a^2bx^n+20\,a^3}{140\,nx^{7\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-7*n)*(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] -1/140*(35*b^3*x^(3*n) + 84*a*b^2*x^(2*n) + 70*a^2*b*x^n + 20*a^3)/(n*x^(7*
n))
```

giac [A] time = 0.21, size = 48, normalized size = 0.76

$$-\frac{35\,b^3x^{3\,n}+84\,ab^2x^{2\,n}+70\,a^2bx^n+20\,a^3}{140\,nx^{7\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-7*n)*(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] -1/140*(35*b^3*x^(3*n) + 84*a*b^2*x^(2*n) + 70*a^2*b*x^n + 20*a^3)/(n*x^(7*
n))
```

maple [A] time = 0.02, size = 56, normalized size = 0.89

$$-\frac{a^3x^{-7n}}{7n}-\frac{a^2b\,x^{-6n}}{2n}-\frac{3a\,b^2x^{-5n}}{5n}-\frac{b^3x^{-4n}}{4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-7*n)*(b*x^n+a)^3,x)
```

```
[Out] -1/4*b^3/n/(x^n)^4-3/5*a*b^2/n/(x^n)^5-1/2*a^2*b/n/(x^n)^6-1/7*a^3/n/(x^n)^
7
```

maxima [A] time = 0.54, size = 63, normalized size = 1.00

$$-\frac{a^3}{7\,nx^{7\,n}}-\frac{a^2b}{2\,nx^{6\,n}}-\frac{3\,ab^2}{5\,nx^{5\,n}}-\frac{b^3}{4\,nx^{4\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-7*n)*(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] -1/7*a^3/(n*x^(7*n)) - 1/2*a^2*b/(n*x^(6*n)) - 3/5*a*b^2/(n*x^(5*n)) - 1/4*
b^3/(n*x^(4*n))
```

mupad [B] time = 1.37, size = 63, normalized size = 1.00

$$-\frac{a^3}{7\,n\,x^{7\,n}}-\frac{b^3}{4\,n\,x^{4\,n}}-\frac{3\,a\,b^2}{5\,n\,x^{5\,n}}-\frac{a^2\,b}{2\,n\,x^{6\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^3/x^(7*n + 1),x)
```

```
[Out] - a^3/(7*n*x^(7*n)) - b^3/(4*n*x^(4*n)) - (3*a*b^2)/(5*n*x^(5*n)) - (a^2*b)
/(2*n*x^(6*n))
```

sympy [A] time = 27.13, size = 61, normalized size = 0.97

$$\begin{cases} -\frac{a^3x^{-7n}}{7n}-\frac{a^2bx^{-6n}}{2n}-\frac{3ab^2x^{-5n}}{5n}-\frac{b^3x^{-4n}}{4n} & \text{for } n\neq 0 \\ (a+b)^3\log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-7*n)*(a+b*x**n)**3,x)
```

```
[Out] Piecewise((-a**3*x**(-7*n)/(7*n) - a**2*b*x**(-6*n)/(2*n) - 3*a*b**2*x**(-5
*n)/(5*n) - b**3*x**(-4*n)/(4*n), Ne(n, 0)), ((a + b)**3*log(x), True))
```

3.2551 $\int x^{-1+4n}\left(a+bx^n\right)^5\,dx$

Optimal. Leaf size=84

$$-\frac{a^3\left(a+bx^n\right)^6}{6b^4n}+\frac{3a^2\left(a+bx^n\right)^7}{7b^4n}+\frac{\left(a+bx^n\right)^9}{9b^4n}-\frac{3a\left(a+bx^n\right)^8}{8b^4n}$$

```
[Out] -1/6*a^3*(a+b*x^n)^6/b^4/n+3/7*a^2*(a+b*x^n)^7/b^4/n-3/8*a*(a+b*x^n)^8/b^4/
n+1/9*(a+b*x^n)^9/b^4/n
```

Rubi [A] time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$\frac{3a^2\left(a+bx^n\right)^7}{7b^4n}-\frac{a^3\left(a+bx^n\right)^6}{6b^4n}+\frac{\left(a+bx^n\right)^9}{9b^4n}-\frac{3a\left(a+bx^n\right)^8}{8b^4n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 4*n)*(a + b*x^n)^5,x]
```

```
[Out] -(a^3*(a + b*x^n)^6)/(6*b^4*n) + (3*a^2*(a + b*x^n)^7)/(7*b^4*n) - (3*a*(a
+ b*x^n)^8)/(8*b^4*n) + (a + b*x^n)^9/(9*b^4*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1+4n}\left(a+bx^n\right)^5\,dx &= \frac{\text{Subst}\left(\int x^3(a+bx)^5\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int\left(-\frac{a^3(a+bx)^5}{b^3}+\frac{3a^2(a+bx)^6}{b^3}-\frac{3a(a+bx)^7}{b^3}+\frac{(a+bx)^8}{b^3}\right)dx,x,x^n\right)}{n}\\ &= -\frac{a^3\left(a+bx^n\right)^6}{6b^4n}+\frac{3a^2\left(a+bx^n\right)^7}{7b^4n}-\frac{3a\left(a+bx^n\right)^8}{8b^4n}+\frac{\left(a+bx^n\right)^9}{9b^4n}\end{aligned}$$

Mathematica [A] time = 0.06, size = 82, normalized size = 0.98

$$\frac{\frac{1}{4}a^5x^{4n}+a^4bx^{5n}+\frac{5}{3}a^3b^2x^{6n}+\frac{10}{7}a^2b^3x^{7n}+\frac{5}{8}ab^4x^{8n}+\frac{1}{9}b^5x^{9n}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 4*n)*(a + b*x^n)^5,x]
```

```
[Out] ((a^5*x^(4*n))/4 + a^4*b*x^(5*n) + (5*a^3*b^2*x^(6*n))/3 + (10*a^2*b^3*x^(7
*n))/7 + (5*a*b^4*x^(8*n))/8 + (b^5*x^(9*n))/9)/n
```

fricas [A] time = 0.80, size = 74, normalized size = 0.88

$$\frac{56\,b^5x^{9\,n}+315\,ab^4x^{8\,n}+720\,a^2b^3x^{7\,n}+840\,a^3b^2x^{6\,n}+504\,a^4bx^{5\,n}+126\,a^5x^{4\,n}}{504\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)*(a+b*x^n)^5,x, algorithm="fricas")
```

```
[Out] 1/504*(56*b^5*x^(9*n) + 315*a*b^4*x^(8*n) + 720*a^2*b^3*x^(7*n) + 840*a^3*b
^2*x^(6*n) + 504*a^4*b*x^(5*n) + 126*a^5*x^(4*n))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n+a)^5x^{4\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)*(a+b*x^n)^5,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^5*x^(4*n - 1), x)
```

maple [A] time = 0.02, size = 87, normalized size = 1.04

$$\frac{a^5x^{4n}}{4n}+\frac{a^4b\,x^{5n}}{n}+\frac{5a^3b^2x^{6n}}{3n}+\frac{10a^2b^3x^{7n}}{7n}+\frac{5a\,b^4x^{8n}}{8n}+\frac{b^5x^{9n}}{9n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+4*n)*(b*x^n+a)^5,x)
```

```
[Out] 1/9*b^5/n*(x^n)^9+5/8*a*b^4/n*(x^n)^8+10/7*a^2*b^3/n*(x^n)^7+5/3*a^3*b^2/n*
(x^n)^6+a^4*b/n*(x^n)^5+1/4*a^5/n*(x^n)^4
```

maxima [A] time = 0.53, size = 86, normalized size = 1.02

$$\frac{b^5x^{9\,n}}{9\,n}+\frac{5\,ab^4x^{8\,n}}{8\,n}+\frac{10\,a^2b^3x^{7\,n}}{7\,n}+\frac{5\,a^3b^2x^{6\,n}}{3\,n}+\frac{a^4bx^{5\,n}}{n}+\frac{a^5x^{4\,n}}{4\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)*(a+b*x^n)^5,x, algorithm="maxima")
```

```
[Out] 1/9*b^5*x^(9*n)/n + 5/8*a*b^4*x^(8*n)/n + 10/7*a^2*b^3*x^(7*n)/n + 5/3*a^3*
b^2*x^(6*n)/n + a^4*b*x^(5*n)/n + 1/4*a^5*x^(4*n)/n
```

mupad [B] time = 1.42, size = 86, normalized size = 1.02

$$\frac{a^5\,x^{4\,n}}{4\,n}+\frac{b^5\,x^{9\,n}}{9\,n}+\frac{5\,a^3\,b^2\,x^{6\,n}}{3\,n}+\frac{10\,a^2\,b^3\,x^{7\,n}}{7\,n}+\frac{a^4\,b\,x^{5\,n}}{n}+\frac{5\,a\,b^4\,x^{8\,n}}{8\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4*n - 1)*(a + b*x^n)^5,x)
```

```
[Out] (a^5*x^(4*n))/(4*n) + (b^5*x^(9*n))/(9*n) + (5*a^3*b^2*x^(6*n))/(3*n) + (10
*a^2*b^3*x^(7*n))/(7*n) + (a^4*b*x^(5*n))/n + (5*a*b^4*x^(8*n))/(8*n)
```

sympy [A] time = 117.30, size = 92, normalized size = 1.10

$$\begin{cases}\frac{a^5x^{4n}}{4n}+\frac{a^4bx^{5n}}{n}+\frac{5a^3b^2x^{6n}}{3n}+\frac{10a^2b^3x^{7n}}{7n}+\frac{5ab^4x^{8n}}{8n}+\frac{b^5x^{9n}}{9n} & \text{for } n\neq 0\\ (a+b)^5\log(x) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+4*n)*(a+b*x**n)**5,x)
```

```
[Out] Piecewise((a**5*x**(4*n)/(4*n) + a**4*b*x**(5*n)/n + 5*a**3*b**2*x**(6*n)/(
3*n) + 10*a**2*b**3*x**(7*n)/(7*n) + 5*a*b**4*x**(8*n)/(8*n) + b**5*x**(9*n
)/(9*n), Ne(n, 0)), ((a + b)**5*log(x), True))
```

3.2552 $\int x^{-1+3n}\left(a+bx^n\right)^5\,dx$

Optimal. Leaf size=62

$$\frac{a^2\left(a+bx^n\right)^6}{6b^3n}+\frac{\left(a+bx^n\right)^8}{8b^3n}-\frac{2a\left(a+bx^n\right)^7}{7b^3n}$$

```
[Out] 1/6*a^2*(a+b*x^n)^6/b^3/n-2/7*a*(a+b*x^n)^7/b^3/n+1/8*(a+b*x^n)^8/b^3/n
```

Rubi [A] time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$\frac{a^2\left(a+bx^n\right)^6}{6b^3n}+\frac{\left(a+bx^n\right)^8}{8b^3n}-\frac{2a\left(a+bx^n\right)^7}{7b^3n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)*(a + b*x^n)^5,x]
```

```
[Out] (a^2*(a + b*x^n)^6)/(6*b^3*n) - (2*a*(a + b*x^n)^7)/(7*b^3*n) + (a + b*x^n)
^8/(8*b^3*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1+3n}\left(a+bx^n\right)^5\,dx &= \frac{\text{Subst}\left(\int x^2(a+bx)^5\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int\left(\frac{a^2(a+bx)^5}{b^2}-\frac{2a(a+bx)^6}{b^2}+\frac{(a+bx)^7}{b^2}\right)dx,x,x^n\right)}{n}\\ &= \frac{a^2\left(a+bx^n\right)^6}{6b^3n}-\frac{2a\left(a+bx^n\right)^7}{7b^3n}+\frac{\left(a+bx^n\right)^8}{8b^3n}\end{aligned}$$

Mathematica [A] time = 0.04, size = 40, normalized size = 0.65

$$\frac{\left(a+bx^n\right)^6\left(a^2-6abx^n+21b^2x^{2n}\right)}{168b^3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)*(a + b*x^n)^5,x]
```

```
[Out] ((a + b*x^n)^6*(a^2 - 6*a*b*x^n + 21*b^2*x^(2*n)))/(168*b^3*n)
```

fricas [A] time = 0.56, size = 74, normalized size = 1.19

$$\frac{21\,b^5x^{8\,n}+120\,ab^4x^{7\,n}+280\,a^2b^3x^{6\,n}+336\,a^3b^2x^{5\,n}+210\,a^4bx^{4\,n}+56\,a^5x^{3\,n}}{168\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^5,x, algorithm="fricas")
```

```
[Out] 1/168*(21*b^5*x^(8*n) + 120*a*b^4*x^(7*n) + 280*a^2*b^3*x^(6*n) + 336*a^3*b
^2*x^(5*n) + 210*a^4*b*x^(4*n) + 56*a^5*x^(3*n))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n+a)^5x^{3\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^5,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^5*x^(3*n - 1), x)
```

maple [A] time = 0.02, size = 88, normalized size = 1.42

$$\frac{a^5x^{3n}}{3n}+\frac{5a^4b\,x^{4n}}{4n}+\frac{2a^3b^2x^{5n}}{n}+\frac{5a^2b^3x^{6n}}{3n}+\frac{5a\,b^4x^{7n}}{7n}+\frac{b^5x^{8n}}{8n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n-1)*(b*x^n+a)^5,x)
```

```
[Out] 1/8*b^5/n*(x^n)^8+5/7*a*b^4/n*(x^n)^7+5/3*a^2*b^3/n*(x^n)^6+2*a^3*b^2/n*(x^
n)^5+5/4*a^4*b/n*(x^n)^4+1/3*a^5/n*(x^n)^3
```

maxima [A] time = 0.71, size = 87, normalized size = 1.40

$$\frac{b^5x^{8\,n}}{8\,n}+\frac{5\,ab^4x^{7\,n}}{7\,n}+\frac{5\,a^2b^3x^{6\,n}}{3\,n}+\frac{2\,a^3b^2x^{5\,n}}{n}+\frac{5\,a^4bx^{4\,n}}{4\,n}+\frac{a^5x^{3\,n}}{3\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^5,x, algorithm="maxima")
```

```
[Out] 1/8*b^5*x^(8*n)/n + 5/7*a*b^4*x^(7*n)/n + 5/3*a^2*b^3*x^(6*n)/n + 2*a^3*b^2
*x^(5*n)/n + 5/4*a^4*b*x^(4*n)/n + 1/3*a^5*x^(3*n)/n
```

mupad [B] time = 1.35, size = 87, normalized size = 1.40

$$\frac{a^5\,x^{3\,n}}{3\,n}+\frac{b^5\,x^{8\,n}}{8\,n}+\frac{2\,a^3\,b^2\,x^{5\,n}}{n}+\frac{5\,a^2\,b^3\,x^{6\,n}}{3\,n}+\frac{5\,a^4\,b\,x^{4\,n}}{4\,n}+\frac{5\,a\,b^4\,x^{7\,n}}{7\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n - 1)*(a + b*x^n)^5,x)
```

```
[Out] (a^5*x^(3*n))/(3*n) + (b^5*x^(8*n))/(8*n) + (2*a^3*b^2*x^(5*n))/n + (5*a^2*
b^3*x^(6*n))/(3*n) + (5*a^4*b*x^(4*n))/(4*n) + (5*a*b^4*x^(7*n))/(7*n)
```

sympy [A] time = 117.79, size = 94, normalized size = 1.52

$$\begin{cases}\frac{a^5x^{3n}}{3n}+\frac{5a^4bx^{4n}}{4n}+\frac{2a^3b^2x^{5n}}{n}+\frac{5a^2b^3x^{6n}}{3n}+\frac{5ab^4x^{7n}}{7n}+\frac{b^5x^{8n}}{8n} & \text{for } n\neq 0\\ (a+b)^5\log(x) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)*(a+b*x**n)**5,x)
```

```
[Out] Piecewise((a**5*x**(3*n)/(3*n) + 5*a**4*b*x**(4*n)/(4*n) + 2*a**3*b**2*x**(
5*n)/n + 5*a**2*b**3*x**(6*n)/(3*n) + 5*a*b**4*x**(7*n)/(7*n) + b**5*x**(8*
n)/(8*n), Ne(n, 0)), ((a + b)**5*log(x), True))
```

3.2553 $\int x^{-1+2n}\left(a+bx^n\right)^5\,dx$

Optimal. Leaf size=40

$$\frac{\left(a+bx^n\right)^7}{7b^2n}-\frac{a\left(a+bx^n\right)^6}{6b^2n}$$

```
[Out] -1/6*a*(a+b*x^n)^6/b^2/n+1/7*(a+b*x^n)^7/b^2/n
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$\frac{\left(a+bx^n\right)^7}{7b^2n}-\frac{a\left(a+bx^n\right)^6}{6b^2n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)*(a + b*x^n)^5,x]
```

```
[Out] -(a*(a + b*x^n)^6)/(6*b^2*n) + (a + b*x^n)^7/(7*b^2*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1+2n}\left(a+bx^n\right)^5\,dx &= \frac{\text{Subst}\left(\int x(a+bx)^5\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int\left(-\frac{a(a+bx)^5}{b}+\frac{(a+bx)^6}{b}\right)dx,x,x^n\right)}{n}\\ &= -\frac{a\left(a+bx^n\right)^6}{6b^2n}+\frac{\left(a+bx^n\right)^7}{7b^2n}\end{aligned}$$

Mathematica [A] time = 0.03, size = 27, normalized size = 0.68

$$-\frac{\left(a-6bx^n\right)\left(a+bx^n\right)^6}{42b^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)*(a + b*x^n)^5,x]
```

```
[Out] -1/42*((a - 6*b*x^n)*(a + b*x^n)^6)/(b^2*n)
```

fricas [B] time = 0.77, size = 74, normalized size = 1.85

$$\frac{6\,b^5x^{7\,n}+35\,ab^4x^{6\,n}+84\,a^2b^3x^{5\,n}+105\,a^3b^2x^{4\,n}+70\,a^4bx^{3\,n}+21\,a^5x^{2\,n}}{42\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^5,x, algorithm="fricas")
```

```
[Out] 1/42*(6*b^5*x^(7*n) + 35*a*b^4*x^(6*n) + 84*a^2*b^3*x^(5*n) + 105*a^3*b^2*x
^(4*n) + 70*a^4*b*x^(3*n) + 21*a^5*x^(2*n))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^5 x^{2n-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^5,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^5*x^(2*n - 1), x)
```

maple [B] time = 0.02, size = 88, normalized size = 2.20

$$\frac{a^5x^{2n}}{2n} + \frac{5a^4b\,x^{3n}}{3n} + \frac{5a^3b^2x^{4n}}{2n} + \frac{2a^2b^3x^{5n}}{n} + \frac{5a\,b^4x^{6n}}{6n} + \frac{b^5x^{7n}}{7n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n-1)*(b*x^n+a)^5,x)
```

```
[Out] 1/7*b^5/n*(x^n)^7+5/6*a*b^4/n*(x^n)^6+2*a^2*b^3/n*(x^n)^5+5/2*a^3*b^2/n*(x^
n)^4+5/3*a^4*b/n*(x^n)^3+1/2*a^5/n*(x^n)^2
```

maxima [B] time = 0.59, size = 87, normalized size = 2.18

$$\frac{b^5x^{7\,n}}{7\,n} + \frac{5\,ab^4x^{6\,n}}{6\,n} + \frac{2\,a^2b^3x^{5\,n}}{n} + \frac{5\,a^3b^2x^{4\,n}}{2\,n} + \frac{5\,a^4bx^{3\,n}}{3\,n} + \frac{a^5x^{2\,n}}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^5,x, algorithm="maxima")
```

```
[Out] 1/7*b^5*x^(7*n)/n + 5/6*a*b^4*x^(6*n)/n + 2*a^2*b^3*x^(5*n)/n + 5/2*a^3*b^2
*x^(4*n)/n + 5/3*a^4*b*x^(3*n)/n + 1/2*a^5*x^(2*n)/n
```

mupad [B] time = 1.36, size = 87, normalized size = 2.18

$$\frac{a^5\,x^{2\,n}}{2\,n} + \frac{b^5\,x^{7\,n}}{7\,n} + \frac{5\,a^3\,b^2\,x^{4\,n}}{2\,n} + \frac{2\,a^2\,b^3\,x^{5\,n}}{n} + \frac{5\,a^4\,b\,x^{3\,n}}{3\,n} + \frac{5\,a\,b^4\,x^{6\,n}}{6\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)*(a + b*x^n)^5,x)
```

```
[Out] (a^5*x^(2*n))/(2*n) + (b^5*x^(7*n))/(7*n) + (5*a^3*b^2*x^(4*n))/(2*n) + (2*
a^2*b^3*x^(5*n))/n + (5*a^4*b*x^(3*n))/(3*n) + (5*a*b^4*x^(6*n))/(6*n)
```

sympy [A] time = 117.65, size = 94, normalized size = 2.35

$$\begin{cases} \frac{a^5x^{2n}}{2n} + \frac{5a^4bx^{3n}}{3n} + \frac{5a^3b^2x^{4n}}{2n} + \frac{2a^2b^3x^{5n}}{n} + \frac{5ab^4x^{6n}}{6n} + \frac{b^5x^{7n}}{7n} & \text{for } n \neq 0 \\ (a+b)^5 \log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a+b*x**n)**5,x)
```

```
[Out] Piecewise((a**5*x**(2*n)/(2*n) + 5*a**4*b*x**(3*n)/(3*n) + 5*a**3*b**2*x**(
4*n)/(2*n) + 2*a**2*b**3*x**(5*n)/n + 5*a*b**4*x**(6*n)/(6*n) + b**5*x**(7*
n)/(7*n), Ne(n, 0)), ((a + b)**5*log(x), True))
```

3.2554 $\int x^{-1+n}\left(a+bx^n\right)^5\,dx$

Optimal. Leaf size=19

$$\frac{\left(a+bx^n\right)^6}{6bn}$$

```
[Out] 1/6*(a+b*x^n)^6/b/n
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$\frac{\left(a+bx^n\right)^6}{6bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*(a + b*x^n)^5,x]
```

```
[Out] (a + b*x^n)^6/(6*b*n)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^{-1+n}\left(a+bx^n\right)^5\,dx = \frac{\left(a+bx^n\right)^6}{6bn}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{\left(a+bx^n\right)^6}{6bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*(a + b*x^n)^5,x]
```

```
[Out] (a + b*x^n)^6/(6*b*n)
```

fricas [B] time = 0.83, size = 71, normalized size = 3.74

$$\frac{b^5x^{6\,n}+6\,ab^4x^{5\,n}+15\,a^2b^3x^{4\,n}+20\,a^3b^2x^{3\,n}+15\,a^4bx^{2\,n}+6\,a^5x^n}{6\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(a+b*x^n)^5,x, algorithm="fricas")
```

```
[Out] 1/6*(b^5*x^(6*n) + 6*a*b^4*x^(5*n) + 15*a^2*b^3*x^(4*n) + 20*a^3*b^2*x^(3*n
) + 15*a^4*b*x^(2*n) + 6*a^5*x^n)/n
```

giac [B] time = 0.17, size = 71, normalized size = 3.74

$$\frac{b^5x^{6\,n}+6\,ab^4x^{5\,n}+15\,a^2b^3x^{4\,n}+20\,a^3b^2x^{3\,n}+15\,a^4bx^{2\,n}+6\,a^5x^n}{6\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(a+b*x^n)^5,x, algorithm="giac")
```

```
[Out] 1/6*(b^5*x^(6*n) + 6*a*b^4*x^(5*n) + 15*a^2*b^3*x^(4*n) + 20*a^3*b^2*x^(3*n
) + 15*a^4*b*x^(2*n) + 6*a^5*x^n)/n
```

maple [B] time = 0.03, size = 84, normalized size = 4.42

$$\frac{a^5x^n}{n}+\frac{5a^4b\,x^{2n}}{2n}+\frac{10a^3b^2x^{3n}}{3n}+\frac{5a^2b^3x^{4n}}{2n}+\frac{a\,b^4x^{5n}}{n}+\frac{b^5x^{6n}}{6n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*(b*x^n+a)^5,x)
```

```
[Out] 1/6*b^5/n*(x^n)^6+a*b^4/n*(x^n)^5+5/2*a^2*b^3/n*(x^n)^4+10/3*a^3*b^2/n*(x^n
)^3+5/2*a^4*b/n*(x^n)^2+a^5/n*x^n
```

maxima [A] time = 0.54, size = 17, normalized size = 0.89

$$\frac{(bx^n+a)^6}{6\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(a+b*x^n)^5,x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^n + a)^6/(b*n)
```

mupad [B] time = 1.36, size = 83, normalized size = 4.37

$$\frac{a^5\,x^n}{n}+\frac{b^5\,x^{6\,n}}{6\,n}+\frac{10\,a^3\,b^2\,x^{3\,n}}{3\,n}+\frac{5\,a^2\,b^3\,x^{4\,n}}{2\,n}+\frac{5\,a^4\,b\,x^{2\,n}}{2\,n}+\frac{a\,b^4\,x^{5\,n}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*(a + b*x^n)^5,x)
```

```
[Out] (a^5*x^n)/n + (b^5*x^(6*n))/(6*n) + (10*a^3*b^2*x^(3*n))/(3*n) + (5*a^2*b^3
*x^(4*n))/(2*n) + (5*a^4*b*x^(2*n))/(2*n) + (a*b^4*x^(5*n))/n
```

sympy [A] time = 15.83, size = 88, normalized size = 4.63

$$\begin{cases}\frac{a^5x^n}{n}+\frac{5a^4bx^{2n}}{2n}+\frac{10a^3b^2x^{3n}}{3n}+\frac{5a^2b^3x^{4n}}{2n}+\frac{ab^4x^{5n}}{n}+\frac{b^5x^{6n}}{6n} & \text{for } n\neq 0\\ (a+b)^5\log(x) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(a+b*x**n)**5,x)
```

```
[Out] Piecewise((a**5*x**n/n + 5*a**4*b*x**(2*n)/(2*n) + 10*a**3*b**2*x**(3*n)/(3
*n) + 5*a**2*b**3*x**(4*n)/(2*n) + a*b**4*x**(5*n)/n + b**5*x**(6*n)/(6*n),
 Ne(n, 0)), ((a + b)**5*log(x), True))
```

3.2555 $\int \frac{(a+bx^n)^5}{x}\,dx$

Optimal. Leaf size=84

$$a^5\log(x)+\frac{5a^4bx^n}{n}+\frac{5a^3b^2x^{2n}}{n}+\frac{10a^2b^3x^{3n}}{3n}+\frac{5ab^4x^{4n}}{4n}+\frac{b^5x^{5n}}{5n}$$

```
[Out] 5*a^4*b*x^n/n+5*a^3*b^2*x^(2*n)/n+10/3*a^2*b^3*x^(3*n)/n+5/4*a*b^4*x^(4*n)/
n+1/5*b^5*x^(5*n)/n+a^5*ln(x)
```

Rubi [A] time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{5a^3b^2x^{2n}}{n}+\frac{10a^2b^3x^{3n}}{3n}+\frac{5a^4bx^n}{n}+a^5\log(x)+\frac{5ab^4x^{4n}}{4n}+\frac{b^5x^{5n}}{5n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^5/x,x]
```

```
[Out] (5*a^4*b*x^n)/n + (5*a^3*b^2*x^(2*n))/n + (10*a^2*b^3*x^(3*n))/(3*n) + (5*a
*b^4*x^(4*n))/(4*n) + (b^5*x^(5*n))/(5*n) + a^5*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^n)^5}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^5}{x}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(5a^4b+\frac{a^5}{x}+10a^3b^2x+10a^2b^3x^2+5ab^4x^3+b^5x^4\right)\,dx,x,x^n\right)}{n}\\ &= \frac{5a^4bx^n}{n}+\frac{5a^3b^2x^{2n}}{n}+\frac{10a^2b^3x^{3n}}{3n}+\frac{5ab^4x^{4n}}{4n}+\frac{b^5x^{5n}}{5n}+a^5\log(x)\end{aligned}$$

Mathematica [A] time = 0.07, size = 74, normalized size = 0.88

$$\frac{a^5n\log(x)+5a^4bx^n+5a^3b^2x^{2n}+\frac{10}{3}a^2b^3x^{3n}+\frac{5}{4}ab^4x^{4n}+\frac{1}{5}b^5x^{5n}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^5/x,x]
```

```
[Out] (5*a^4*b*x^n + 5*a^3*b^2*x^(2*n) + (10*a^2*b^3*x^(3*n))/3 + (5*a*b^4*x^(4*n
))/4 + (b^5*x^(5*n))/5 + a^5*n*Log[x])/n
```

fricas [A] time = 0.62, size = 70, normalized size = 0.83

$$\frac{60\,a^5 n\log(x) + 12\,b^5x^{5\,n} + 75\,ab^4x^{4\,n} + 200\,a^2b^3x^{3\,n} + 300\,a^3b^2x^{2\,n} + 300\,a^4bx^n}{60\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^5/x,x, algorithm="fricas")
```

```
[Out] 1/60*(60*a^5*n*log(x) + 12*b^5*x^(5*n) + 75*a*b^4*x^(4*n) + 200*a^2*b^3*x^(
3*n) + 300*a^3*b^2*x^(2*n) + 300*a^4*b*x^n)/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^5}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^5/x,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^5/x, x)
```

maple [A] time = 0.00, size = 84, normalized size = 1.00

$$\frac{a^5\ln(x^n)}{n} + \frac{5a^4b\,x^n}{n} + \frac{5a^3b^2x^{2n}}{n} + \frac{10a^2b^3x^{3n}}{3n} + \frac{5a\,b^4x^{4n}}{4n} + \frac{b^5x^{5n}}{5n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^5/x,x)
```

```
[Out] 1/5/n*b^5*(x^n)^5+5/4/n*(x^n)^4*a*b^4+10/3/n*a^2*b^3*(x^n)^3+5/n*(x^n)^2*a^
3*b^2+5*a^4*b*x^n/n+1/n*a^5*ln(x^n)
```

maxima [A] time = 0.49, size = 74, normalized size = 0.88

$$\frac{a^5\log(x^n)}{n} + \frac{12\,b^5x^{5\,n} + 75\,ab^4x^{4\,n} + 200\,a^2b^3x^{3\,n} + 300\,a^3b^2x^{2\,n} + 300\,a^4bx^n}{60\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^5/x,x, algorithm="maxima")
```

```
[Out] a^5*log(x^n)/n + 1/60*(12*b^5*x^(5*n) + 75*a*b^4*x^(4*n) + 200*a^2*b^3*x^(3
*n) + 300*a^3*b^2*x^(2*n) + 300*a^4*b*x^n)/n
```

mupad [B] time = 1.22, size = 78, normalized size = 0.93

$$a^5\,\ln(x) + \frac{b^5\,x^{5\,n}}{5\,n} + \frac{5\,a^3\,b^2\,x^{2\,n}}{n} + \frac{10\,a^2\,b^3\,x^{3\,n}}{3\,n} + \frac{5\,a^4\,b\,x^n}{n} + \frac{5\,a\,b^4\,x^{4\,n}}{4\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^5/x,x)
```

```
[Out] a^5*log(x) + (b^5*x^(5*n))/(5*n) + (5*a^3*b^2*x^(2*n))/n + (10*a^2*b^3*x^(3
*n))/(3*n) + (5*a^4*b*x^n)/n + (5*a*b^4*x^(4*n))/(4*n)
```

sympy [**A**] time = 0.94, size = 85, normalized size = 1.01

$$\begin{cases} a^5 \log(x) + \frac{5a^4bx^n}{n} + \frac{5a^3b^2x^{2n}}{n} + \frac{10a^2b^3x^{3n}}{3n} + \frac{5ab^4x^{4n}}{4n} + \frac{b^5x^{5n}}{5n} & \text{for } n \neq 0 \\ (a+b)^5 \log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**5/x,x)
```

```
[Out] Piecewise((a**5*log(x) + 5*a**4*b*x**n/n + 5*a**3*b**2*x**(2*n)/n + 10*a**2
*b**3*x**(3*n)/(3*n) + 5*a*b**4*x**(4*n)/(4*n) + b**5*x**(5*n)/(5*n), Ne(n,
 0)), ((a + b)**5*log(x), True))
```

3.2556 $\int x^{-1-n}\left(a+bx^n\right)^5\,dx$

Optimal. Leaf size=83

$$-\frac{a^5x^{-n}}{n}+5a^4b\log(x)+\frac{10a^3b^2x^n}{n}+\frac{5a^2b^3x^{2n}}{n}+\frac{5ab^4x^{3n}}{3n}+\frac{b^5x^{4n}}{4n}$$

```
[Out] -a^5/n/(x^n)+10*a^3*b^2*x^n/n+5*a^2*b^3*x^(2*n)/n+5/3*a*b^4*x^(3*n)/n+1/4*b
^5*x^(4*n)/n+5*a^4*b*ln(x)
```

Rubi [A] time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$\frac{10a^3b^2x^n}{n}+\frac{5a^2b^3x^{2n}}{n}+5a^4b\log(x)-\frac{a^5x^{-n}}{n}+\frac{5ab^4x^{3n}}{3n}+\frac{b^5x^{4n}}{4n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n)*(a + b*x^n)^5,x]
```

```
[Out] -(a^5/(n*x^n)) + (10*a^3*b^2*x^n)/n + (5*a^2*b^3*x^(2*n))/n + (5*a*b^4*x^(3
*n))/(3*n) + (b^5*x^(4*n))/(4*n) + 5*a^4*b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1-n}\left(a+bx^n\right)^5\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^5}{x^2}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(10a^3b^2+\frac{a^5}{x^2}+\frac{5a^4b}{x}+10a^2b^3x+5ab^4x^2+b^5x^3\right)\,dx,x,x^n\right)}{n}\\ &= -\frac{a^5x^{-n}}{n}+\frac{10a^3b^2x^n}{n}+\frac{5a^2b^3x^{2n}}{n}+\frac{5ab^4x^{3n}}{3n}+\frac{b^5x^{4n}}{4n}+5a^4b\log(x)\end{aligned}$$

Mathematica [A] time = 0.06, size = 73, normalized size = 0.88

$$\frac{-a^5x^{-n}+5a^4bn\log(x)+10a^3b^2x^n+5a^2b^3x^{2n}+\frac{5}{3}ab^4x^{3n}+\frac{1}{4}b^5x^{4n}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n)*(a + b*x^n)^5,x]
```

```
[Out] (-(a^5/x^n) + 10*a^3*b^2*x^n + 5*a^2*b^3*x^(2*n) + (5*a*b^4*x^(3*n))/3 + (b
^5*x^(4*n))/4 + 5*a^4*b*n*Log[x])/n
```

fricas [A] time = 0.75, size = 75, normalized size = 0.90

$$\frac{60\,a^4bnx^n\log(x)+3\,b^5x^{5\,n}+20\,ab^4x^{4\,n}+60\,a^2b^3x^{3\,n}+120\,a^3b^2x^{2\,n}-12\,a^5}{12\,nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*(a+b*x^n)^5,x, algorithm="fricas")
```

```
[Out] 1/12*(60*a^4*b*n*x^n*log(x) + 3*b^5*x^(5*n) + 20*a*b^4*x^(4*n) + 60*a^2*b^3
*x^(3*n) + 120*a^3*b^2*x^(2*n) - 12*a^5)/(n*x^n)
```

giac [A] time = 0.28, size = 75, normalized size = 0.90

$$\frac{60\,a^4bnx^n\log(x)+3\,b^5x^{5\,n}+20\,ab^4x^{4\,n}+60\,a^2b^3x^{3\,n}+120\,a^3b^2x^{2\,n}-12\,a^5}{12\,nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*(a+b*x^n)^5,x, algorithm="giac")
```

```
[Out] 1/12*(60*a^4*b*n*x^n*log(x) + 3*b^5*x^(5*n) + 20*a*b^4*x^(4*n) + 60*a^2*b^3
*x^(3*n) + 120*a^3*b^2*x^(2*n) - 12*a^5)/(n*x^n)
```

maple [A] time = 0.03, size = 98, normalized size = 1.18

$$\left(5a^4b\,\mathrm{e}^{n\ln(x)}\ln(x)+\frac{10a^3b^2\mathrm{e}^{2n\ln(x)}}{n}+\frac{5a^2b^3\mathrm{e}^{3n\ln(x)}}{n}+\frac{5a\,b^4\mathrm{e}^{4n\ln(x)}}{3n}+\frac{b^5\mathrm{e}^{5n\ln(x)}}{4n}-\frac{a^5}{n}\right)\mathrm{e}^{-n\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n)*(b*x^n+a)^5,x)
```

```
[Out] (5*a^4*b*ln(x)*exp(n*ln(x))-a^5/n+1/4*b^5/n*exp(n*ln(x))^5+5/3*a*b^4/n*exp(
n*ln(x))^4+5*a^2*b^3/n*exp(n*ln(x))^3+10*a^3*b^2/n*exp(n*ln(x))^2)/exp(n*ln
(x))
```

maxima [A] time = 0.57, size = 79, normalized size = 0.95

$$5\,a^4b\log(x)+\frac{b^5x^{4\,n}}{4\,n}+\frac{5\,ab^4x^{3\,n}}{3\,n}+\frac{5\,a^2b^3x^{2\,n}}{n}+\frac{10\,a^3b^2x^n}{n}-\frac{a^5}{nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*(a+b*x^n)^5,x, algorithm="maxima")
```

```
[Out] 5*a^4*b*log(x) + 1/4*b^5*x^(4*n)/n + 5/3*a*b^4*x^(3*n)/n + 5*a^2*b^3*x^(2*n
)/n + 10*a^3*b^2*x^n/n - a^5/(n*x^n)
```

mupad [B] time = 1.52, size = 79, normalized size = 0.95

$$5\,a^4\,b\,\ln(x)-\frac{a^5}{n\,x^n}+\frac{b^5\,x^{4\,n}}{4\,n}+\frac{5\,a^2\,b^3\,x^{2\,n}}{n}+\frac{5\,a\,b^4\,x^{3\,n}}{3\,n}+\frac{10\,a^3\,b^2\,x^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^5/x^(n + 1),x)
```

```
[Out] 5*a^4*b*log(x) - a^5/(n*x^n) + (b^5*x^(4*n))/(4*n) + (5*a^2*b^3*x^(2*n))/n
+ (5*a*b^4*x^(3*n))/(3*n) + (10*a^3*b^2*x^n)/n
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n)*(a+b*x**n)**5,x)
```

```
[Out] Timed out
```

3.2557 $\int x^{-1-2n}\left(a+bx^n\right)^5\,dx$

Optimal. Leaf size=85

$$-\frac{a^5x^{-2n}}{2n}-\frac{5a^4bx^{-n}}{n}+10a^3b^2\log(x)+\frac{10a^2b^3x^n}{n}+\frac{5ab^4x^{2n}}{2n}+\frac{b^5x^{3n}}{3n}$$

```
[Out] -1/2*a^5/n/(x^(2*n))-5*a^4*b/n/(x^n)+10*a^2*b^3*x^n/n+5/2*a*b^4*x^(2*n)/n+1
/3*b^5*x^(3*n)/n+10*a^3*b^2*ln(x)
```

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {266, 43}

$$\frac{10a^2b^3x^n}{n}+10a^3b^2\log(x)-\frac{5a^4bx^{-n}}{n}-\frac{a^5x^{-2n}}{2n}+\frac{5ab^4x^{2n}}{2n}+\frac{b^5x^{3n}}{3n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 2*n)*(a + b*x^n)^5,x]
```

```
[Out] -a^5/(2*n*x^(2*n)) - (5*a^4*b)/(n*x^n) + (10*a^2*b^3*x^n)/n + (5*a*b^4*x^(2
*n))/(2*n) + (b^5*x^(3*n))/(3*n) + 10*a^3*b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1-2n}\left(a+bx^n\right)^5\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^5}{x^3}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(10a^2b^3+\frac{a^5}{x^3}+\frac{5a^4b}{x^2}+\frac{10a^3b^2}{x}+5ab^4x+b^5x^2\right)\,dx,x,x^n\right)}{n}\\ &= -\frac{a^5x^{-2n}}{2n}-\frac{5a^4bx^{-n}}{n}+\frac{10a^2b^3x^n}{n}+\frac{5ab^4x^{2n}}{2n}+\frac{b^5x^{3n}}{3n}+10a^3b^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.08, size = 72, normalized size = 0.85

$$10a^3b^2\log(x)+\frac{x^{-2n}\left(-3a^5-30a^4bx^n+60a^2b^3x^{3n}+15ab^4x^{4n}+2b^5x^{5n}\right)}{6n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 2*n)*(a + b*x^n)^5,x]
```

```
[Out] (-3*a^5 - 30*a^4*b*x^n + 60*a^2*b^3*x^(3*n) + 15*a*b^4*x^(4*n) + 2*b^5*x^(5
*n))/(6*n*x^(2*n)) + 10*a^3*b^2*Log[x]
```

fricas [A] time = 0.62, size = 77, normalized size = 0.91

$$\frac{60\,a^3b^2nx^{2\,n}\log(x)+2\,b^5x^{5\,n}+15\,ab^4x^{4\,n}+60\,a^2b^3x^{3\,n}-30\,a^4bx^n-3\,a^5}{6\,nx^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*(a+b*x^n)^5,x, algorithm="fricas")
```

```
[Out] 1/6*(60*a^3*b^2*n*x^(2*n)*log(x) + 2*b^5*x^(5*n) + 15*a*b^4*x^(4*n) + 60*a^
2*b^3*x^(3*n) - 30*a^4*b*x^n - 3*a^5)/(n*x^(2*n))
```

giac [A] time = 0.24, size = 77, normalized size = 0.91

$$\frac{60\,a^3b^2nx^{2\,n}\log(x)+2\,b^5x^{5\,n}+15\,ab^4x^{4\,n}+60\,a^2b^3x^{3\,n}-30\,a^4bx^n-3\,a^5}{6\,nx^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*(a+b*x^n)^5,x, algorithm="giac")
```

```
[Out] 1/6*(60*a^3*b^2*n*x^(2*n)*log(x) + 2*b^5*x^(5*n) + 15*a*b^4*x^(4*n) + 60*a^
2*b^3*x^(3*n) - 30*a^4*b*x^n - 3*a^5)/(n*x^(2*n))
```

maple [A] time = 0.02, size = 98, normalized size = 1.15

$$\left(10a^3b^2\mathrm{e}^{2n\ln(x)}\ln(x)-\frac{5a^4b\,\mathrm{e}^{n\ln(x)}}{n}+\frac{10a^2b^3\mathrm{e}^{3n\ln(x)}}{n}+\frac{5a\,b^4\mathrm{e}^{4n\ln(x)}}{2n}+\frac{b^5\mathrm{e}^{5n\ln(x)}}{3n}-\frac{a^5}{2n}\right)\mathrm{e}^{-2n\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-2*n)*(b*x^n+a)^5,x)
```

```
[Out] (10*a^3*b^2*ln(x)*exp(n*ln(x))^2-1/2*a^5/n+1/3*b^5/n*exp(n*ln(x))^5+5/2*a*b
^4/n*exp(n*ln(x))^4+10*a^2*b^3/n*exp(n*ln(x))^3-5*a^4*b/n*exp(n*ln(x)))/exp
(n*ln(x))^2
```

maxima [A] time = 0.58, size = 81, normalized size = 0.95

$$10\,a^3b^2\log(x)+\frac{b^5x^{3\,n}}{3\,n}+\frac{5\,ab^4x^{2\,n}}{2\,n}+\frac{10\,a^2b^3x^n}{n}-\frac{a^5}{2\,nx^{2\,n}}-\frac{5\,a^4b}{nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*(a+b*x^n)^5,x, algorithm="maxima")
```

```
[Out] 10*a^3*b^2*log(x) + 1/3*b^5*x^(3*n)/n + 5/2*a*b^4*x^(2*n)/n + 10*a^2*b^3*x^
n/n - 1/2*a^5/(n*x^(2*n)) - 5*a^4*b/(n*x^n)
```

mupad [B] time = 1.44, size = 81, normalized size = 0.95

$$\frac{b^5\,x^{3\,n}}{3\,n}-\frac{a^5}{2\,n\,x^{2\,n}}+10\,a^3\,b^2\,\ln(x)-\frac{5\,a^4\,b}{n\,x^n}+\frac{5\,a\,b^4\,x^{2\,n}}{2\,n}+\frac{10\,a^2\,b^3\,x^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^5/x^(2*n + 1),x)
```

```
[Out] (b^5*x^(3*n))/(3*n) - a^5/(2*n*x^(2*n)) + 10*a^3*b^2*log(x) - (5*a^4*b)/(n*
x^n) + (5*a*b^4*x^(2*n))/(2*n) + (10*a^2*b^3*x^n)/n
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-2*n)*(a+b*x**n)**5,x)
```

```
[Out] Timed out
```

3.2558 $\int x^{-1-3n}\left(a+bx^n\right)^5\,dx$

Optimal. Leaf size=85

$$-\frac{a^5x^{-3n}}{3n}-\frac{5a^4bx^{-2n}}{2n}-\frac{10a^3b^2x^{-n}}{n}+10a^2b^3\log(x)+\frac{5ab^4x^n}{n}+\frac{b^5x^{2n}}{2n}$$

```
[Out] -1/3*a^5/n/(x^(3*n))-5/2*a^4*b/n/(x^(2*n))-10*a^3*b^2/n/(x^n)+5*a*b^4*x^n/n
+1/2*b^5*x^(2*n)/n+10*a^2*b^3*ln(x)
```

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$-\frac{10a^3b^2x^{-n}}{n}+10a^2b^3\log(x)-\frac{5a^4bx^{-2n}}{2n}-\frac{a^5x^{-3n}}{3n}+\frac{5ab^4x^n}{n}+\frac{b^5x^{2n}}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 3*n)*(a + b*x^n)^5,x]
```

```
[Out] -a^5/(3*n*x^(3*n)) - (5*a^4*b)/(2*n*x^(2*n)) - (10*a^3*b^2)/(n*x^n) + (5*a*
b^4*x^n)/n + (b^5*x^(2*n))/(2*n) + 10*a^2*b^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1-3n}\left(a+bx^n\right)^5\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^5}{x^4}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(5ab^4+\frac{a^5}{x^4}+\frac{5a^4b}{x^3}+\frac{10a^3b^2}{x^2}+\frac{10a^2b^3}{x}+b^5x\right)\,dx,x,x^n\right)}{n}\\ &= -\frac{a^5x^{-3n}}{3n}-\frac{5a^4bx^{-2n}}{2n}-\frac{10a^3b^2x^{-n}}{n}+\frac{5ab^4x^n}{n}+\frac{b^5x^{2n}}{2n}+10a^2b^3\log(x)\end{aligned}$$

Mathematica [A] time = 0.07, size = 72, normalized size = 0.85

$$10a^2b^3\log(x)+\frac{x^{-3n}\left(-2a^5-15a^4bx^n-60a^3b^2x^{2n}+30ab^4x^{4n}+3b^5x^{5n}\right)}{6n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 3*n)*(a + b*x^n)^5,x]
```

```
[Out] (-2*a^5 - 15*a^4*b*x^n - 60*a^3*b^2*x^(2*n) + 30*a*b^4*x^(4*n) + 3*b^5*x^(5
*n))/(6*n*x^(3*n)) + 10*a^2*b^3*Log[x]
```

fricas [A] time = 0.61, size = 77, normalized size = 0.91

$$\frac{60\,a^2b^3nx^{3\,n}\log(x)+3\,b^5x^{5\,n}+30\,ab^4x^{4\,n}-60\,a^3b^2x^{2\,n}-15\,a^4bx^n-2\,a^5}{6\,nx^{3\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)*(a+b*x^n)^5,x, algorithm="fricas")
```

```
[Out] 1/6*(60*a^2*b^3*n*x^(3*n)*log(x) + 3*b^5*x^(5*n) + 30*a*b^4*x^(4*n) - 60*a^
3*b^2*x^(2*n) - 15*a^4*b*x^n - 2*a^5)/(n*x^(3*n))
```

giac [A] time = 0.24, size = 77, normalized size = 0.91

$$\frac{60\,a^2b^3nx^{3\,n}\log(x)+3\,b^5x^{5\,n}+30\,ab^4x^{4\,n}-60\,a^3b^2x^{2\,n}-15\,a^4bx^n-2\,a^5}{6\,nx^{3\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)*(a+b*x^n)^5,x, algorithm="giac")
```

```
[Out] 1/6*(60*a^2*b^3*n*x^(3*n)*log(x) + 3*b^5*x^(5*n) + 30*a*b^4*x^(4*n) - 60*a^
3*b^2*x^(2*n) - 15*a^4*b*x^n - 2*a^5)/(n*x^(3*n))
```

maple [A] time = 0.02, size = 98, normalized size = 1.15

$$\left(10a^2b^3\mathrm{e}^{3n\ln(x)}\ln(x)-\frac{5a^4b\,\mathrm{e}^{n\ln(x)}}{2n}-\frac{10a^3b^2\mathrm{e}^{2n\ln(x)}}{n}+\frac{5a\,b^4\mathrm{e}^{4n\ln(x)}}{n}+\frac{b^5\mathrm{e}^{5n\ln(x)}}{2n}-\frac{a^5}{3n}\right)\mathrm{e}^{-3n\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-3*n)*(b*x^n+a)^5,x)
```

```
[Out] (10*a^2*b^3*ln(x)*exp(n*ln(x))^3-1/3*a^5/n+1/2*b^5/n*exp(n*ln(x))^5+5*a*b^4
/n*exp(n*ln(x))^4-10*a^3*b^2/n*exp(n*ln(x))^2-5/2*a^4*b/n*exp(n*ln(x)))/exp
(n*ln(x))^3
```

maxima [A] time = 0.51, size = 83, normalized size = 0.98

$$10\,a^2b^3\log(x)+\frac{b^5x^{2\,n}}{2\,n}+\frac{5\,ab^4x^n}{n}-\frac{a^5}{3\,nx^{3\,n}}-\frac{5\,a^4b}{2\,nx^{2\,n}}-\frac{10\,a^3b^2}{nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)*(a+b*x^n)^5,x, algorithm="maxima")
```

```
[Out] 10*a^2*b^3*log(x) + 1/2*b^5*x^(2*n)/n + 5*a*b^4*x^n/n - 1/3*a^5/(n*x^(3*n))
 - 5/2*a^4*b/(n*x^(2*n)) - 10*a^3*b^2/(n*x^n)
```

mupad [B] time = 1.43, size = 83, normalized size = 0.98

$$\frac{b^5\,x^{2\,n}}{2\,n}-\frac{a^5}{3\,n\,x^{3\,n}}+10\,a^2\,b^3\,\ln(x)-\frac{10\,a^3\,b^2}{n\,x^n}+\frac{5\,a\,b^4\,x^n}{n}-\frac{5\,a^4\,b}{2\,n\,x^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^5/x^(3*n + 1),x)
```

```
[Out] (b^5*x^(2*n))/(2*n) - a^5/(3*n*x^(3*n)) + 10*a^2*b^3*log(x) - (10*a^3*b^2)/
(n*x^n) + (5*a*b^4*x^n)/n - (5*a^4*b)/(2*n*x^(2*n))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-3*n)*(a+b*x**n)**5,x)
```

```
[Out] Timed out
```

3.2559 $\int x^{-1-4n}\left(a+bx^n\right)^5\,dx$

Optimal. Leaf size=82

$$-\frac{a^5x^{-4n}}{4n}-\frac{5a^4bx^{-3n}}{3n}-\frac{5a^3b^2x^{-2n}}{n}-\frac{10a^2b^3x^{-n}}{n}+5ab^4\log(x)+\frac{b^5x^n}{n}$$

```
[Out] -1/4*a^5/n/(x^(4*n))-5/3*a^4*b/n/(x^(3*n))-5*a^3*b^2/n/(x^(2*n))-10*a^2*b^3
/n/(x^n)+b^5*x^n/n+5*a*b^4*ln(x)
```

Rubi [A] time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$-\frac{5a^3b^2x^{-2n}}{n}-\frac{10a^2b^3x^{-n}}{n}-\frac{5a^4bx^{-3n}}{3n}-\frac{a^5x^{-4n}}{4n}+5ab^4\log(x)+\frac{b^5x^n}{n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 4*n)*(a + b*x^n)^5,x]
```

```
[Out] -a^5/(4*n*x^(4*n)) - (5*a^4*b)/(3*n*x^(3*n)) - (5*a^3*b^2)/(n*x^(2*n)) - (1
0*a^2*b^3)/(n*x^n) + (b^5*x^n)/n + 5*a*b^4*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1-4n}\left(a+bx^n\right)^5\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^5}{x^5}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int\left(b^5+\frac{a^5}{x^5}+\frac{5a^4b}{x^4}+\frac{10a^3b^2}{x^3}+\frac{10a^2b^3}{x^2}+\frac{5ab^4}{x}\right)dx,x,x^n\right)}{n}\\ &= -\frac{a^5x^{-4n}}{4n}-\frac{5a^4bx^{-3n}}{3n}-\frac{5a^3b^2x^{-2n}}{n}-\frac{10a^2b^3x^{-n}}{n}+\frac{b^5x^n}{n}+5ab^4\log(x)\end{aligned}$$

Mathematica [A] time = 0.05, size = 72, normalized size = 0.88

$$\frac{-\frac{1}{4}a^5x^{-4n}-\frac{5}{3}a^4bx^{-3n}-5a^3b^2x^{-2n}-10a^2b^3x^{-n}+5ab^4n\log(x)+b^5x^n}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 4*n)*(a + b*x^n)^5,x]
```

```
[Out] (-1/4*a^5/x^(4*n) - (5*a^4*b)/(3*x^(3*n)) - (5*a^3*b^2)/x^(2*n) - (10*a^2*b
^3)/x^n + b^5*x^n + 5*a*b^4*n*Log[x])/n
```

fricas [A] time = 0.95, size = 77, normalized size = 0.94

$$\frac{60\,ab^4nx^{4\,n}\log(x)+12\,b^5x^{5\,n}-120\,a^2b^3x^{3\,n}-60\,a^3b^2x^{2\,n}-20\,a^4bx^n-3\,a^5}{12\,nx^{4\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-4*n)*(a+b*x^n)^5,x, algorithm="fricas")
```

```
[Out] 1/12*(60*a*b^4*n*x^(4*n)*log(x) + 12*b^5*x^(5*n) - 120*a^2*b^3*x^(3*n) - 60
*a^3*b^2*x^(2*n) - 20*a^4*b*x^n - 3*a^5)/(n*x^(4*n))
```

giac [A] time = 0.24, size = 77, normalized size = 0.94

$$\frac{60\,ab^4nx^{4\,n}\log(x)+12\,b^5x^{5\,n}-120\,a^2b^3x^{3\,n}-60\,a^3b^2x^{2\,n}-20\,a^4bx^n-3\,a^5}{12\,nx^{4\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-4*n)*(a+b*x^n)^5,x, algorithm="giac")
```

```
[Out] 1/12*(60*a*b^4*n*x^(4*n)*log(x) + 12*b^5*x^(5*n) - 120*a^2*b^3*x^(3*n) - 60
*a^3*b^2*x^(2*n) - 20*a^4*b*x^n - 3*a^5)/(n*x^(4*n))
```

maple [A] time = 0.02, size = 97, normalized size = 1.18

$$\left(5a\,b^4\mathrm{e}^{4n\ln(x)}\ln(x)-\frac{5a^4b\,\mathrm{e}^{n\ln(x)}}{3n}-\frac{5a^3b^2\mathrm{e}^{2n\ln(x)}}{n}-\frac{10a^2b^3\mathrm{e}^{3n\ln(x)}}{n}+\frac{b^5\mathrm{e}^{5n\ln(x)}}{n}-\frac{a^5}{4n}\right)\mathrm{e}^{-4n\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-4*n)*(b*x^n+a)^5,x)
```

```
[Out] (b^5/n*exp(n*ln(x))^5+5*a*b^4*ln(x)*exp(n*ln(x))^4-1/4*a^5/n-10*a^2*b^3/n*e
xp(n*ln(x))^3-5*a^3*b^2/n*exp(n*ln(x))^2-5/3*a^4*b/n*exp(n*ln(x)))/exp(n*ln
(x))^4
```

maxima [A] time = 0.71, size = 84, normalized size = 1.02

$$5\,ab^4\log(x)+\frac{b^5x^n}{n}-\frac{a^5}{4\,nx^{4\,n}}-\frac{5\,a^4b}{3\,nx^{3\,n}}-\frac{5\,a^3b^2}{nx^{2\,n}}-\frac{10\,a^2b^3}{nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-4*n)*(a+b*x^n)^5,x, algorithm="maxima")
```

```
[Out] 5*a*b^4*log(x) + b^5*x^n/n - 1/4*a^5/(n*x^(4*n)) - 5/3*a^4*b/(n*x^(3*n)) -
5*a^3*b^2/(n*x^(2*n)) - 10*a^2*b^3/(n*x^n)
```

mupad [B] time = 1.41, size = 84, normalized size = 1.02

$$\frac{b^5\,x^n}{n}+5\,a\,b^4\,\ln(x)-\frac{a^5}{4\,n\,x^{4\,n}}-\frac{10\,a^2\,b^3}{n\,x^n}-\frac{5\,a^3\,b^2}{n\,x^{2\,n}}-\frac{5\,a^4\,b}{3\,n\,x^{3\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^5/x^(4*n + 1),x)
```

```
[Out] (b^5*x^n)/n + 5*a*b^4*log(x) - a^5/(4*n*x^(4*n)) - (10*a^2*b^3)/(n*x^n) - (
5*a^3*b^2)/(n*x^(2*n)) - (5*a^4*b)/(3*n*x^(3*n))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-4*n)*(a+b*x**n)**5,x)
```

```
[Out] Timed out
```

3.2560 $\int x^{-1-5n}\left(a+bx^n\right)^5\,dx$

Optimal. Leaf size=86

$$-\frac{a^5x^{-5n}}{5n}-\frac{5a^4bx^{-4n}}{4n}-\frac{10a^3b^2x^{-3n}}{3n}-\frac{5a^2b^3x^{-2n}}{n}-\frac{5ab^4x^{-n}}{n}+b^5\log(x)$$

```
[Out] -1/5*a^5/n/(x^(5*n))-5/4*a^4*b/n/(x^(4*n))-10/3*a^3*b^2/n/(x^(3*n))-5*a^2*b
^3/n/(x^(2*n))-5*a*b^4/n/(x^n)+b^5*ln(x)
```

Rubi [A] time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$-\frac{10a^3b^2x^{-3n}}{3n}-\frac{5a^2b^3x^{-2n}}{n}-\frac{5a^4bx^{-4n}}{4n}-\frac{a^5x^{-5n}}{5n}-\frac{5ab^4x^{-n}}{n}+b^5\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 5*n)*(a + b*x^n)^5,x]
```

```
[Out] -a^5/(5*n*x^(5*n)) - (5*a^4*b)/(4*n*x^(4*n)) - (10*a^3*b^2)/(3*n*x^(3*n)) -
 (5*a^2*b^3)/(n*x^(2*n)) - (5*a*b^4)/(n*x^n) + b^5*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1-5n}\left(a+bx^n\right)^5\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^5}{x^6}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a^5}{x^6}+\frac{5a^4b}{x^5}+\frac{10a^3b^2}{x^4}+\frac{10a^2b^3}{x^3}+\frac{5ab^4}{x^2}+\frac{b^5}{x}\right)dx,x,x^n\right)}{n}\\ &= -\frac{a^5x^{-5n}}{5n}-\frac{5a^4bx^{-4n}}{4n}-\frac{10a^3b^2x^{-3n}}{3n}-\frac{5a^2b^3x^{-2n}}{n}-\frac{5ab^4x^{-n}}{n}+b^5\log(x)\end{aligned}$$

Mathematica [A] time = 0.08, size = 69, normalized size = 0.80

$$b^5\log(x)-\frac{ax^{-5n}\left(12a^4+75a^3bx^n+200a^2b^2x^{2n}+300ab^3x^{3n}+300b^4x^{4n}\right)}{60n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 5*n)*(a + b*x^n)^5,x]
```

```
[Out] -1/60*(a*(12*a^4 + 75*a^3*b*x^n + 200*a^2*b^2*x^(2*n) + 300*a*b^3*x^(3*n) +
 300*b^4*x^(4*n)))/(n*x^(5*n)) + b^5*Log[x]
```

fricas [A] time = 0.57, size = 77, normalized size = 0.90

$$\frac{60\,b^5 n x^{5\,n}\log(x) - 300\,ab^4x^{4\,n} - 300\,a^2b^3x^{3\,n} - 200\,a^3b^2x^{2\,n} - 75\,a^4bx^n - 12\,a^5}{60\,nx^{5\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-5*n)*(a+b*x^n)^5,x, algorithm="fricas")
```

```
[Out] 1/60*(60*b^5*n*x^(5*n)*log(x) - 300*a*b^4*x^(4*n) - 300*a^2*b^3*x^(3*n) - 2
00*a^3*b^2*x^(2*n) - 75*a^4*b*x^n - 12*a^5)/(n*x^(5*n))
```

giac [A] time = 0.25, size = 77, normalized size = 0.90

$$\frac{60\,b^5 n x^{5\,n}\log(x) - 300\,ab^4x^{4\,n} - 300\,a^2b^3x^{3\,n} - 200\,a^3b^2x^{2\,n} - 75\,a^4bx^n - 12\,a^5}{60\,nx^{5\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-5*n)*(a+b*x^n)^5,x, algorithm="giac")
```

```
[Out] 1/60*(60*b^5*n*x^(5*n)*log(x) - 300*a*b^4*x^(4*n) - 300*a^2*b^3*x^(3*n) - 2
00*a^3*b^2*x^(2*n) - 75*a^4*b*x^n - 12*a^5)/(n*x^(5*n))
```

maple [A] time = 0.02, size = 97, normalized size = 1.13

$$\left(b^5\mathrm{e}^{5n\ln(x)}\ln(x) - \frac{5a^4b\,\mathrm{e}^{n\ln(x)}}{4n} - \frac{10a^3b^2\mathrm{e}^{2n\ln(x)}}{3n} - \frac{5a^2b^3\mathrm{e}^{3n\ln(x)}}{n} - \frac{5a\,b^4\mathrm{e}^{4n\ln(x)}}{n} - \frac{a^5}{5n}\right)\mathrm{e}^{-5n\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-5*n)*(b*x^n+a)^5,x)
```

```
[Out] (b^5*ln(x)*exp(n*ln(x))^5-1/5*a^5/n-5*a*b^4/n*exp(n*ln(x))^4-5*a^2*b^3/n*ex
p(n*ln(x))^3-10/3*a^3*b^2/n*exp(n*ln(x))^2-5/4*a^4*b/n*exp(n*ln(x)))/exp(n*
ln(x))^5
```

maxima [A] time = 0.59, size = 88, normalized size = 1.02

$$b^5\log(x) - \frac{a^5}{5\,nx^{5\,n}} - \frac{5\,a^4b}{4\,nx^{4\,n}} - \frac{10\,a^3b^2}{3\,nx^{3\,n}} - \frac{5\,a^2b^3}{nx^{2\,n}} - \frac{5\,ab^4}{nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-5*n)*(a+b*x^n)^5,x, algorithm="maxima")
```

```
[Out] b^5*log(x) - 1/5*a^5/(n*x^(5*n)) - 5/4*a^4*b/(n*x^(4*n)) - 10/3*a^3*b^2/(n*
x^(3*n)) - 5*a^2*b^3/(n*x^(2*n)) - 5*a*b^4/(n*x^n)
```

mupad [B] time = 1.43, size = 88, normalized size = 1.02

$$b^5\,\ln(x) - \frac{a^5}{5\,n\,x^{5\,n}} - \frac{5\,a^2\,b^3}{n\,x^{2\,n}} - \frac{10\,a^3\,b^2}{3\,n\,x^{3\,n}} - \frac{5\,a\,b^4}{n\,x^n} - \frac{5\,a^4\,b}{4\,n\,x^{4\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^5/x^(5*n + 1),x)
```

```
[Out] b^5*log(x) - a^5/(5*n*x^(5*n)) - (5*a^2*b^3)/(n*x^(2*n)) - (10*a^3*b^2)/(3*
n*x^(3*n)) - (5*a*b^4)/(n*x^n) - (5*a^4*b)/(4*n*x^(4*n))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-5*n)*(a+b*x**n)**5,x)
```

```
[Out] Timed out
```

3.2561 $\int x^{-1-6n}\left(a+bx^n\right)^5\,dx$

Optimal. Leaf size=24

$$-\frac{x^{-6n}\left(a+bx^n\right)^6}{6an}$$

```
[Out] -1/6*(a+b*x^n)^6/a/n/(x^(6*n))
```

Rubi [A] time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {264}

$$-\frac{x^{-6n}\left(a+bx^n\right)^6}{6an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 6*n)*(a + b*x^n)^5,x]
```

```
[Out] -(a + b*x^n)^6/(6*a*n*x^(6*n))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^{-1-6n}\left(a+bx^n\right)^5\,dx = -\frac{x^{-6n}\left(a+bx^n\right)^6}{6an}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$-\frac{x^{-6n}\left(a+bx^n\right)^6}{6an}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 6*n)*(a + b*x^n)^5,x]
```

```
[Out] -1/6*(a + b*x^n)^6/(a*n*x^(6*n))
```

fricas [B] time = 0.70, size = 72, normalized size = 3.00

$$-\frac{6\,b^5x^{5\,n}+15\,ab^4x^{4\,n}+20\,a^2b^3x^{3\,n}+15\,a^3b^2x^{2\,n}+6\,a^4bx^n+a^5}{6\,nx^{6\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-6*n)*(a+b*x^n)^5,x, algorithm="fricas")
```

```
[Out] -1/6*(6*b^5*x^(5*n) + 15*a*b^4*x^(4*n) + 20*a^2*b^3*x^(3*n) + 15*a^3*b^2*x^
(2*n) + 6*a^4*b*x^n + a^5)/(n*x^(6*n))
```

giac [B] time = 0.24, size = 72, normalized size = 3.00

$$-\frac{6\,b^5x^{5\,n}+15\,ab^4x^{4\,n}+20\,a^2b^3x^{3\,n}+15\,a^3b^2x^{2\,n}+6\,a^4bx^n+a^5}{6\,nx^{6\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-6*n)*(a+b*x^n)^5,x, algorithm="giac")
```

```
[Out] -1/6*(6*b^5*x^(5*n) + 15*a*b^4*x^(4*n) + 20*a^2*b^3*x^(3*n) + 15*a^3*b^2*x^
(2*n) + 6*a^4*b*x^n + a^5)/(n*x^(6*n))
```

maple [B] time = 0.02, size = 88, normalized size = 3.67

$$-\frac{a^5 x^{-6n}}{6n} - \frac{a^4 b\, x^{-5n}}{n} - \frac{5a^3b^2x^{-4n}}{2n} - \frac{10a^2b^3x^{-3n}}{3n} - \frac{5a\, b^4x^{-2n}}{2n} - \frac{b^5x^{-n}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-6*n)*(b*x^n+a)^5,x)
```

```
[Out] -b^5/n/(x^n)-5/2*a*b^4/n/(x^n)^2-10/3*a^2*b^3/n/(x^n)^3-5/2*a^3*b^2/n/(x^n)
^4-a^4*b/n/(x^n)^5-1/6*a^5/n/(x^n)^6
```

maxima [B] time = 0.61, size = 97, normalized size = 4.04

$$-\frac{a^5}{6\,nx^{6\,n}} - \frac{a^4b}{nx^{5\,n}} - \frac{5\,a^3b^2}{2\,nx^{4\,n}} - \frac{10\,a^2b^3}{3\,nx^{3\,n}} - \frac{5\,ab^4}{2\,nx^{2\,n}} - \frac{b^5}{nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-6*n)*(a+b*x^n)^5,x, algorithm="maxima")
```

```
[Out] -1/6*a^5/(n*x^(6*n)) - a^4*b/(n*x^(5*n)) - 5/2*a^3*b^2/(n*x^(4*n)) - 10/3*a
^2*b^3/(n*x^(3*n)) - 5/2*a*b^4/(n*x^(2*n)) - b^5/(n*x^n)
```

mupad [B] time = 1.37, size = 97, normalized size = 4.04

$$-\frac{a^5}{6\,n\,x^{6\,n}} - \frac{b^5}{n\,x^n} - \frac{10\,a^2\,b^3}{3\,n\,x^{3\,n}} - \frac{5\,a^3\,b^2}{2\,n\,x^{4\,n}} - \frac{5\,a\,b^4}{2\,n\,x^{2\,n}} - \frac{a^4\,b}{n\,x^{5\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^5/x^(6*n + 1),x)
```

```
[Out] - a^5/(6*n*x^(6*n)) - b^5/(n*x^n) - (10*a^2*b^3)/(3*n*x^(3*n)) - (5*a^3*b^2
)/(2*n*x^(4*n)) - (5*a*b^4)/(2*n*x^(2*n)) - (a^4*b)/(n*x^(5*n))
```

sympy [A] time = 124.56, size = 90, normalized size = 3.75

$$\begin{cases} -\frac{a^5x^{-6n}}{6n} - \frac{a^4bx^{-5n}}{n} - \frac{5a^3b^2x^{-4n}}{2n} - \frac{10a^2b^3x^{-3n}}{3n} - \frac{5ab^4x^{-2n}}{2n} - \frac{b^5x^{-n}}{n} & \text{for } n \neq 0 \\ (a+b)^5 \log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-6*n)*(a+b*x**n)**5,x)
```

```
[Out] Piecewise((-a**5*x**(-6*n)/(6*n) - a**4*b*x**(-5*n)/n - 5*a**3*b**2*x**(-4*
n)/(2*n) - 10*a**2*b**3*x**(-3*n)/(3*n) - 5*a*b**4*x**(-2*n)/(2*n) - b**5*x
**(-n)/n, Ne(n, 0)), ((a + b)**5*log(x), True))
```

3.2562 $\int x^{-1-7n}\left(a+bx^n\right)^5\,dx$

Optimal. Leaf size=50

$$\frac{bx^{-6n}\left(a+bx^n\right)^6}{42a^2n}-\frac{x^{-7n}\left(a+bx^n\right)^6}{7an}$$

```
[Out] -1/7*(a+b*x^n)^6/a/n/(x^(7*n))+1/42*b*(a+b*x^n)^6/a^2/n/(x^(6*n))
```

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {266, 45, 37}

$$\frac{bx^{-6n}\left(a+bx^n\right)^6}{42a^2n}-\frac{x^{-7n}\left(a+bx^n\right)^6}{7an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 7*n)*(a + b*x^n)^5,x]
```

```
[Out] -(a + b*x^n)^6/(7*a*n*x^(7*n)) + (b*(a + b*x^n)^6)/(42*a^2*n*x^(6*n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1-7n}\left(a+bx^n\right)^5\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^5}{x^8}\,dx,x,x^n\right)}{n}\\ &= -\frac{x^{-7n}\left(a+bx^n\right)^6}{7an}-\frac{b\,\text{Subst}\left(\int \frac{(a+bx)^5}{x^7}\,dx,x,x^n\right)}{7an}\\ &= -\frac{x^{-7n}\left(a+bx^n\right)^6}{7an}+\frac{bx^{-6n}\left(a+bx^n\right)^6}{42a^2n}\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.66

$$\frac{x^{-7n}\left(bx^n-6a\right)\left(a+bx^n\right)^6}{42a^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 7*n)*(a + b*x^n)^5,x]
```

```
[Out] ((-6*a + b*x^n)*(a + b*x^n)^6)/(42*a^2*n*x^(7*n))
```

fricas [A] time = 0.52, size = 74, normalized size = 1.48

$$-\frac{21\,b^5x^{5\,n}+70\,ab^4x^{4\,n}+105\,a^2b^3x^{3\,n}+84\,a^3b^2x^{2\,n}+35\,a^4bx^n+6\,a^5}{42\,nx^{7\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-7*n)*(a+b*x^n)^5,x, algorithm="fricas")
```

```
[Out] -1/42*(21*b^5*x^(5*n) + 70*a*b^4*x^(4*n) + 105*a^2*b^3*x^(3*n) + 84*a^3*b^2
*x^(2*n) + 35*a^4*b*x^n + 6*a^5)/(n*x^(7*n))
```

giac [A] time = 0.24, size = 74, normalized size = 1.48

$$-\frac{21\,b^5x^{5\,n}+70\,ab^4x^{4\,n}+105\,a^2b^3x^{3\,n}+84\,a^3b^2x^{2\,n}+35\,a^4bx^n+6\,a^5}{42\,nx^{7\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-7*n)*(a+b*x^n)^5,x, algorithm="giac")
```

```
[Out] -1/42*(21*b^5*x^(5*n) + 70*a*b^4*x^(4*n) + 105*a^2*b^3*x^(3*n) + 84*a^3*b^2
*x^(2*n) + 35*a^4*b*x^n + 6*a^5)/(n*x^(7*n))
```

maple [A] time = 0.02, size = 88, normalized size = 1.76

$$-\frac{a^5x^{-7n}}{7n}-\frac{5a^4b\,x^{-6n}}{6n}-\frac{2a^3b^2x^{-5n}}{n}-\frac{5a^2b^3x^{-4n}}{2n}-\frac{5a\,b^4x^{-3n}}{3n}-\frac{b^5x^{-2n}}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-7*n)*(b*x^n+a)^5,x)
```

```
[Out] -1/2*b^5/n/(x^n)^2-5/3*a*b^4/n/(x^n)^3-5/2*a^2*b^3/n/(x^n)^4-2*a^3*b^2/n/(x
^n)^5-5/6*a^4*b/n/(x^n)^6-1/7*a^5/n/(x^n)^7
```

maxima [A] time = 0.51, size = 99, normalized size = 1.98

$$-\frac{a^5}{7\,nx^{7\,n}}-\frac{5\,a^4b}{6\,nx^{6\,n}}-\frac{2\,a^3b^2}{nx^{5\,n}}-\frac{5\,a^2b^3}{2\,nx^{4\,n}}-\frac{5\,ab^4}{3\,nx^{3\,n}}-\frac{b^5}{2\,nx^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-7*n)*(a+b*x^n)^5,x, algorithm="maxima")
```

```
[Out] -1/7*a^5/(n*x^(7*n)) - 5/6*a^4*b/(n*x^(6*n)) - 2*a^3*b^2/(n*x^(5*n)) - 5/2*
a^2*b^3/(n*x^(4*n)) - 5/3*a*b^4/(n*x^(3*n)) - 1/2*b^5/(n*x^(2*n))
```

mupad [B] time = 1.34, size = 99, normalized size = 1.98

$$-\frac{a^5}{7\,n\,x^{7\,n}}-\frac{b^5}{2\,n\,x^{2\,n}}-\frac{5\,a^2\,b^3}{2\,n\,x^{4\,n}}-\frac{2\,a^3\,b^2}{n\,x^{5\,n}}-\frac{5\,a\,b^4}{3\,n\,x^{3\,n}}-\frac{5\,a^4\,b}{6\,n\,x^{6\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^5/x^(7*n + 1),x)
```

```
[Out] - a^5/(7*n*x^(7*n)) - b^5/(2*n*x^(2*n)) - (5*a^2*b^3)/(2*n*x^(4*n)) - (2*a^
3*b^2)/(n*x^(5*n)) - (5*a*b^4)/(3*n*x^(3*n)) - (5*a^4*b)/(6*n*x^(6*n))
```

sympy [**F**(**-1**)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-7*n)*(a+b*x**n)**5,x)
[Out] Timed out
```

3.2563 $\int x^{-1-8n}\left(a+bx^n\right)^5\,dx$

Optimal. Leaf size=77

$$-\frac{b^2x^{-6n}\left(a+bx^n\right)^6}{168a^3n}+\frac{bx^{-7n}\left(a+bx^n\right)^6}{28a^2n}-\frac{x^{-8n}\left(a+bx^n\right)^6}{8an}$$

```
[Out] -1/8*(a+b*x^n)^6/a/n/(x^(8*n))+1/28*b*(a+b*x^n)^6/a^2/n/(x^(7*n))-1/168*b^2
*(a+b*x^n)^6/a^3/n/(x^(6*n))
```

Rubi [A] time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {266, 45, 37}

$$-\frac{b^2x^{-6n}\left(a+bx^n\right)^6}{168a^3n}+\frac{bx^{-7n}\left(a+bx^n\right)^6}{28a^2n}-\frac{x^{-8n}\left(a+bx^n\right)^6}{8an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 8*n)*(a + b*x^n)^5,x]
```

```
[Out] -(a + b*x^n)^6/(8*a*n*x^(8*n)) + (b*(a + b*x^n)^6)/(28*a^2*n*x^(7*n)) - (b^
2*(a + b*x^n)^6)/(168*a^3*n*x^(6*n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^{-1-8n}\left(a+bx^n\right)^5\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^5}{x^9}\,dx,x,x^n\right)}{n}\\
&= -\frac{x^{-8n}\left(a+bx^n\right)^6}{8an}-\frac{b\,\text{Subst}\left(\int \frac{(a+bx)^5}{x^8}\,dx,x,x^n\right)}{4an}\\
&= -\frac{x^{-8n}\left(a+bx^n\right)^6}{8an}+\frac{bx^{-7n}\left(a+bx^n\right)^6}{28a^2n}+\frac{b^2\,\text{Subst}\left(\int \frac{(a+bx)^5}{x^7}\,dx,x,x^n\right)}{28a^2n}\\
&= -\frac{x^{-8n}\left(a+bx^n\right)^6}{8an}+\frac{bx^{-7n}\left(a+bx^n\right)^6}{28a^2n}-\frac{b^2x^{-6n}\left(a+bx^n\right)^6}{168a^3n}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 74, normalized size = 0.96

$$-\frac{x^{-8n}\left(21a^5+120a^4bx^n+280a^3b^2x^{2n}+336a^2b^3x^{3n}+210ab^4x^{4n}+56b^5x^{5n}\right)}{168n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 8*n)*(a + b*x^n)^5,x]
```

```
[Out] -1/168*(21*a^5 + 120*a^4*b*x^n + 280*a^3*b^2*x^(2*n) + 336*a^2*b^3*x^(3*n)
+ 210*a*b^4*x^(4*n) + 56*b^5*x^(5*n))/(n*x^(8*n))
```

fricas [A] time = 0.59, size = 74, normalized size = 0.96

$$-\frac{56\,b^5x^{5\,n}+210\,ab^4x^{4\,n}+336\,a^2b^3x^{3\,n}+280\,a^3b^2x^{2\,n}+120\,a^4bx^n+21\,a^5}{168\,nx^{8\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-8*n)*(a+b*x^n)^5,x, algorithm="fricas")
```

```
[Out] -1/168*(56*b^5*x^(5*n) + 210*a*b^4*x^(4*n) + 336*a^2*b^3*x^(3*n) + 280*a^3*
b^2*x^(2*n) + 120*a^4*b*x^n + 21*a^5)/(n*x^(8*n))
```

giac [A] time = 0.24, size = 74, normalized size = 0.96

$$-\frac{56\,b^5x^{5\,n}+210\,ab^4x^{4\,n}+336\,a^2b^3x^{3\,n}+280\,a^3b^2x^{2\,n}+120\,a^4bx^n+21\,a^5}{168\,nx^{8\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-8*n)*(a+b*x^n)^5,x, algorithm="giac")
```

```
[Out] -1/168*(56*b^5*x^(5*n) + 210*a*b^4*x^(4*n) + 336*a^2*b^3*x^(3*n) + 280*a^3*
b^2*x^(2*n) + 120*a^4*b*x^n + 21*a^5)/(n*x^(8*n))
```

maple [A] time = 0.02, size = 88, normalized size = 1.14

$$-\frac{a^5x^{-8n}}{8n}-\frac{5a^4b\,x^{-7n}}{7n}-\frac{5a^3b^2x^{-6n}}{3n}-\frac{2a^2b^3x^{-5n}}{n}-\frac{5a\,b^4x^{-4n}}{4n}-\frac{b^5x^{-3n}}{3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-8*n)*(b*x^n+a)^5,x)
```

```
[Out] -1/3*b^5/n/(x^n)^3-5/4*a*b^4/n/(x^n)^4-2*a^2*b^3/n/(x^n)^5-5/3*a^3*b^2/n/(x
^n)^6-5/7*a^4*b/n/(x^n)^7-1/8*a^5/n/(x^n)^8
```

maxima [A] time = 0.48, size = 99, normalized size = 1.29

$$-\frac{a^5}{8\,nx^{8\,n}}-\frac{5\,a^4b}{7\,nx^{7\,n}}-\frac{5\,a^3b^2}{3\,nx^{6\,n}}-\frac{2\,a^2b^3}{nx^{5\,n}}-\frac{5\,ab^4}{4\,nx^{4\,n}}-\frac{b^5}{3\,nx^{3\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-8*n)*(a+b*x^n)^5,x, algorithm="maxima")
```

```
[Out] -1/8*a^5/(n*x^(8*n)) - 5/7*a^4*b/(n*x^(7*n)) - 5/3*a^3*b^2/(n*x^(6*n)) - 2*
a^2*b^3/(n*x^(5*n)) - 5/4*a*b^4/(n*x^(4*n)) - 1/3*b^5/(n*x^(3*n))
```

mupad [B] time = 1.36, size = 99, normalized size = 1.29

$$-\frac{a^5}{8\,n\,x^{8\,n}}-\frac{b^5}{3\,n\,x^{3\,n}}-\frac{2\,a^2\,b^3}{n\,x^{5\,n}}-\frac{5\,a^3\,b^2}{3\,n\,x^{6\,n}}-\frac{5\,a\,b^4}{4\,n\,x^{4\,n}}-\frac{5\,a^4\,b}{7\,n\,x^{7\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^5/x^(8*n + 1),x)
```

```
[Out] - a^5/(8*n*x^(8*n)) - b^5/(3*n*x^(3*n)) - (2*a^2*b^3)/(n*x^(5*n)) - (5*a^3*
b^2)/(3*n*x^(6*n)) - (5*a*b^4)/(4*n*x^(4*n)) - (5*a^4*b)/(7*n*x^(7*n))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-8*n)*(a+b*x**n)**5,x)
```

```
[Out] Timed out
```

3.2564 $\int x^{-1-9n}\left(a+bx^n\right)^5\,dx$

Optimal. Leaf size=97

$$-\frac{a^5x^{-9n}}{9n}-\frac{5a^4bx^{-8n}}{8n}-\frac{10a^3b^2x^{-7n}}{7n}-\frac{5a^2b^3x^{-6n}}{3n}-\frac{ab^4x^{-5n}}{n}-\frac{b^5x^{-4n}}{4n}$$

```
[Out] -1/9*a^5/n/(x^(9*n))-5/8*a^4*b/n/(x^(8*n))-10/7*a^3*b^2/n/(x^(7*n))-5/3*a^2
*b^3/n/(x^(6*n))-a*b^4/n/(x^(5*n))-1/4*b^5/n/(x^(4*n))
```

Rubi [A] time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$-\frac{10a^3b^2x^{-7n}}{7n}-\frac{5a^2b^3x^{-6n}}{3n}-\frac{5a^4bx^{-8n}}{8n}-\frac{a^5x^{-9n}}{9n}-\frac{ab^4x^{-5n}}{n}-\frac{b^5x^{-4n}}{4n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 9*n)*(a + b*x^n)^5,x]
```

```
[Out] -a^5/(9*n*x^(9*n)) - (5*a^4*b)/(8*n*x^(8*n)) - (10*a^3*b^2)/(7*n*x^(7*n)) -
 (5*a^2*b^3)/(3*n*x^(6*n)) - (a*b^4)/(n*x^(5*n)) - b^5/(4*n*x^(4*n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1-9n}\left(a+bx^n\right)^5\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^5}{x^{10}}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a^5}{x^{10}}+\frac{5a^4b}{x^9}+\frac{10a^3b^2}{x^8}+\frac{10a^2b^3}{x^7}+\frac{5ab^4}{x^6}+\frac{b^5}{x^5}\right)dx,x,x^n\right)}{n}\\ &= -\frac{a^5x^{-9n}}{9n}-\frac{5a^4bx^{-8n}}{8n}-\frac{10a^3b^2x^{-7n}}{7n}-\frac{5a^2b^3x^{-6n}}{3n}-\frac{ab^4x^{-5n}}{n}-\frac{b^5x^{-4n}}{4n}\end{aligned}$$

Mathematica [A] time = 0.05, size = 74, normalized size = 0.76

$$-\frac{x^{-9n}\left(56a^5+315a^4bx^n+720a^3b^2x^{2n}+840a^2b^3x^{3n}+504ab^4x^{4n}+126b^5x^{5n}\right)}{504n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 9*n)*(a + b*x^n)^5,x]
```

```
[Out] -1/504*(56*a^5 + 315*a^4*b*x^n + 720*a^3*b^2*x^(2*n) + 840*a^2*b^3*x^(3*n)
+ 504*a*b^4*x^(4*n) + 126*b^5*x^(5*n))/(n*x^(9*n))
```

fricas [A] time = 0.77, size = 74, normalized size = 0.76

$$-\frac{126\,b^5x^{5\,n}+504\,ab^4x^{4\,n}+840\,a^2b^3x^{3\,n}+720\,a^3b^2x^{2\,n}+315\,a^4bx^n+56\,a^5}{504\,nx^{9\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-9*n)*(a+b*x^n)^5,x, algorithm="fricas")
```

```
[Out] -1/504*(126*b^5*x^(5*n) + 504*a*b^4*x^(4*n) + 840*a^2*b^3*x^(3*n) + 720*a^3
*b^2*x^(2*n) + 315*a^4*b*x^n + 56*a^5)/(n*x^(9*n))
```

giac [A] time = 0.32, size = 74, normalized size = 0.76

$$-\frac{126\,b^5x^{5\,n}+504\,ab^4x^{4\,n}+840\,a^2b^3x^{3\,n}+720\,a^3b^2x^{2\,n}+315\,a^4bx^n+56\,a^5}{504\,nx^{9\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-9*n)*(a+b*x^n)^5,x, algorithm="giac")
```

```
[Out] -1/504*(126*b^5*x^(5*n) + 504*a*b^4*x^(4*n) + 840*a^2*b^3*x^(3*n) + 720*a^3
*b^2*x^(2*n) + 315*a^4*b*x^n + 56*a^5)/(n*x^(9*n))
```

maple [A] time = 0.02, size = 88, normalized size = 0.91

$$-\frac{a^5x^{-9n}}{9n}-\frac{5a^4b\,x^{-8n}}{8n}-\frac{10a^3b^2x^{-7n}}{7n}-\frac{5a^2b^3x^{-6n}}{3n}-\frac{a\,b^4x^{-5n}}{n}-\frac{b^5x^{-4n}}{4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-9*n)*(b*x^n+a)^5,x)
```

```
[Out] -1/4*b^5/n/(x^n)^4-a*b^4/n/(x^n)^5-5/3*a^2*b^3/n/(x^n)^6-10/7*a^3*b^2/n/(x^
n)^7-5/8*a^4*b/n/(x^n)^8-1/9*a^5/n/(x^n)^9
```

maxima [A] time = 0.55, size = 99, normalized size = 1.02

$$-\frac{a^5}{9\,nx^{9\,n}}-\frac{5\,a^4b}{8\,nx^{8\,n}}-\frac{10\,a^3b^2}{7\,nx^{7\,n}}-\frac{5\,a^2b^3}{3\,nx^{6\,n}}-\frac{ab^4}{nx^{5\,n}}-\frac{b^5}{4\,nx^{4\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-9*n)*(a+b*x^n)^5,x, algorithm="maxima")
```

```
[Out] -1/9*a^5/(n*x^(9*n)) - 5/8*a^4*b/(n*x^(8*n)) - 10/7*a^3*b^2/(n*x^(7*n)) - 5
/3*a^2*b^3/(n*x^(6*n)) - a*b^4/(n*x^(5*n)) - 1/4*b^5/(n*x^(4*n))
```

mupad [B] time = 1.34, size = 99, normalized size = 1.02

$$-\frac{a^5}{9\,n\,x^{9\,n}}-\frac{b^5}{4\,n\,x^{4\,n}}-\frac{5\,a^2\,b^3}{3\,n\,x^{6\,n}}-\frac{10\,a^3\,b^2}{7\,n\,x^{7\,n}}-\frac{a\,b^4}{n\,x^{5\,n}}-\frac{5\,a^4\,b}{8\,n\,x^{8\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^5/x^(9*n + 1),x)
```

```
[Out] - a^5/(9*n*x^(9*n)) - b^5/(4*n*x^(4*n)) - (5*a^2*b^3)/(3*n*x^(6*n)) - (10*a
^3*b^2)/(7*n*x^(7*n)) - (a*b^4)/(n*x^(5*n)) - (5*a^4*b)/(8*n*x^(8*n))
```

sympy [A] time = 124.33, size = 94, normalized size = 0.97

$$\begin{cases}-\frac{a^5x^{-9n}}{9n}-\frac{5a^4bx^{-8n}}{8n}-\frac{10a^3b^2x^{-7n}}{7n}-\frac{5a^2b^3x^{-6n}}{3n}-\frac{ab^4x^{-5n}}{n}-\frac{b^5x^{-4n}}{4n} & \text{for } n\neq 0\\ (a+b)^5\log(x) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-9*n)*(a+b*x**n)**5,x)
```

```
[Out] Piecewise((-a**5*x**(-9*n)/(9*n) - 5*a**4*b*x**(-8*n)/(8*n) - 10*a**3*b**2*
x**(-7*n)/(7*n) - 5*a**2*b**3*x**(-6*n)/(3*n) - a*b**4*x**(-5*n)/n - b**5*x
**(-4*n)/(4*n), Ne(n, 0)), ((a + b)**5*log(x), True))
```

3.2565 $\int x^{-1-10n}\left(a+bx^n\right)^5\,dx$

Optimal. Leaf size=99

$$-\frac{a^5x^{-10n}}{10n}-\frac{5a^4bx^{-9n}}{9n}-\frac{5a^3b^2x^{-8n}}{4n}-\frac{10a^2b^3x^{-7n}}{7n}-\frac{5ab^4x^{-6n}}{6n}-\frac{b^5x^{-5n}}{5n}$$

```
[Out] -1/10*a^5/n/(x^(10*n))-5/9*a^4*b/n/(x^(9*n))-5/4*a^3*b^2/n/(x^(8*n))-10/7*a
^2*b^3/n/(x^(7*n))-5/6*a*b^4/n/(x^(6*n))-1/5*b^5/n/(x^(5*n))
```

Rubi [A] time = 0.04, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$-\frac{5a^3b^2x^{-8n}}{4n}-\frac{10a^2b^3x^{-7n}}{7n}-\frac{5a^4bx^{-9n}}{9n}-\frac{a^5x^{-10n}}{10n}-\frac{5ab^4x^{-6n}}{6n}-\frac{b^5x^{-5n}}{5n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 10*n)*(a + b*x^n)^5,x]
```

```
[Out] -a^5/(10*n*x^(10*n)) - (5*a^4*b)/(9*n*x^(9*n)) - (5*a^3*b^2)/(4*n*x^(8*n))
- (10*a^2*b^3)/(7*n*x^(7*n)) - (5*a*b^4)/(6*n*x^(6*n)) - b^5/(5*n*x^(5*n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1-10n}\left(a+bx^n\right)^5\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^5}{x^{11}}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a^5}{x^{11}}+\frac{5a^4b}{x^{10}}+\frac{10a^3b^2}{x^9}+\frac{10a^2b^3}{x^8}+\frac{5ab^4}{x^7}+\frac{b^5}{x^6}\right)\,dx,x,x^n\right)}{n}\\ &= -\frac{a^5x^{-10n}}{10n}-\frac{5a^4bx^{-9n}}{9n}-\frac{5a^3b^2x^{-8n}}{4n}-\frac{10a^2b^3x^{-7n}}{7n}-\frac{5ab^4x^{-6n}}{6n}-\frac{b^5x^{-5n}}{5n}\end{aligned}$$

Mathematica [A] time = 0.05, size = 74, normalized size = 0.75

$$-\frac{x^{-10n}\left(126a^5+700a^4bx^n+1575a^3b^2x^{2n}+1800a^2b^3x^{3n}+1050ab^4x^{4n}+252b^5x^{5n}\right)}{1260n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 10*n)*(a + b*x^n)^5,x]
```

```
[Out] -1/1260*(126*a^5 + 700*a^4*b*x^n + 1575*a^3*b^2*x^(2*n) + 1800*a^2*b^3*x^(3
*n) + 1050*a*b^4*x^(4*n) + 252*b^5*x^(5*n))/(n*x^(10*n))
```

fricas [A] time = 0.60, size = 74, normalized size = 0.75

$$-\frac{252\,b^5x^{5\,n}+1050\,ab^4x^{4\,n}+1800\,a^2b^3x^{3\,n}+1575\,a^3b^2x^{2\,n}+700\,a^4bx^n+126\,a^5}{1260\,nx^{10\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-10*n)*(a+b*x^n)^5,x, algorithm="fricas")
```

```
[Out] -1/1260*(252*b^5*x^(5*n) + 1050*a*b^4*x^(4*n) + 1800*a^2*b^3*x^(3*n) + 1575
*a^3*b^2*x^(2*n) + 700*a^4*b*x^n + 126*a^5)/(n*x^(10*n))
```

giac [A] time = 0.25, size = 74, normalized size = 0.75

$$-\frac{252\,b^5x^{5\,n}+1050\,ab^4x^{4\,n}+1800\,a^2b^3x^{3\,n}+1575\,a^3b^2x^{2\,n}+700\,a^4bx^n+126\,a^5}{1260\,nx^{10\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-10*n)*(a+b*x^n)^5,x, algorithm="giac")
```

```
[Out] -1/1260*(252*b^5*x^(5*n) + 1050*a*b^4*x^(4*n) + 1800*a^2*b^3*x^(3*n) + 1575
*a^3*b^2*x^(2*n) + 700*a^4*b*x^n + 126*a^5)/(n*x^(10*n))
```

maple [A] time = 0.02, size = 88, normalized size = 0.89

$$-\frac{a^5x^{-10n}}{10n}-\frac{5a^4b\,x^{-9n}}{9n}-\frac{5a^3b^2x^{-8n}}{4n}-\frac{10a^2b^3x^{-7n}}{7n}-\frac{5a\,b^4x^{-6n}}{6n}-\frac{b^5x^{-5n}}{5n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-10*n)*(b*x^n+a)^5,x)
```

```
[Out] -1/5*b^5/n/(x^n)^5-5/6*a*b^4/n/(x^n)^6-10/7*a^2*b^3/n/(x^n)^7-5/4*a^3*b^2/n
/(x^n)^8-5/9*a^4*b/n/(x^n)^9-1/10*a^5/n/(x^n)^10
```

maxima [A] time = 0.58, size = 99, normalized size = 1.00

$$-\frac{a^5}{10\,nx^{10\,n}}-\frac{5\,a^4b}{9\,nx^{9\,n}}-\frac{5\,a^3b^2}{4\,nx^{8\,n}}-\frac{10\,a^2b^3}{7\,nx^{7\,n}}-\frac{5\,ab^4}{6\,nx^{6\,n}}-\frac{b^5}{5\,nx^{5\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-10*n)*(a+b*x^n)^5,x, algorithm="maxima")
```

```
[Out] -1/10*a^5/(n*x^(10*n)) - 5/9*a^4*b/(n*x^(9*n)) - 5/4*a^3*b^2/(n*x^(8*n)) -
10/7*a^2*b^3/(n*x^(7*n)) - 5/6*a*b^4/(n*x^(6*n)) - 1/5*b^5/(n*x^(5*n))
```

mupad [B] time = 1.34, size = 99, normalized size = 1.00

$$-\frac{a^5}{10\,n\,x^{10\,n}}-\frac{b^5}{5\,n\,x^{5\,n}}-\frac{10\,a^2\,b^3}{7\,n\,x^{7\,n}}-\frac{5\,a^3\,b^2}{4\,n\,x^{8\,n}}-\frac{5\,a\,b^4}{6\,n\,x^{6\,n}}-\frac{5\,a^4\,b}{9\,n\,x^{9\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^5/x^(10*n + 1),x)
```

```
[Out] - a^5/(10*n*x^(10*n)) - b^5/(5*n*x^(5*n)) - (10*a^2*b^3)/(7*n*x^(7*n)) - (5
*a^3*b^2)/(4*n*x^(8*n)) - (5*a*b^4)/(6*n*x^(6*n)) - (5*a^4*b)/(9*n*x^(9*n))
```

sympy [A] time = 118.32, size = 97, normalized size = 0.98

$$\begin{cases}-\frac{a^5x^{-10n}}{10n}-\frac{5a^4bx^{-9n}}{9n}-\frac{5a^3b^2x^{-8n}}{4n}-\frac{10a^2b^3x^{-7n}}{7n}-\frac{5ab^4x^{-6n}}{6n}-\frac{b^5x^{-5n}}{5n} & \text{for } n\neq 0\\ (a+b)^5\log(x) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-10*n)*(a+b*x**n)**5,x)
```

```
[Out] Piecewise((-a**5*x**(-10*n)/(10*n) - 5*a**4*b*x**(-9*n)/(9*n) - 5*a**3*b**2
*x**(-8*n)/(4*n) - 10*a**2*b**3*x**(-7*n)/(7*n) - 5*a*b**4*x**(-6*n)/(6*n)
- b**5*x**(-5*n)/(5*n), Ne(n, 0)), ((a + b)**5*log(x), True))
```

3.2566 $\int x^{-1+9n}\left(a+bx^n\right)^8\,dx$

Optimal. Leaf size=151

$$\frac{a^8x^{9n}}{9n}+\frac{4a^7bx^{10n}}{5n}+\frac{28a^6b^2x^{11n}}{11n}+\frac{14a^5b^3x^{12n}}{3n}+\frac{70a^4b^4x^{13n}}{13n}+\frac{4a^3b^5x^{14n}}{n}+\frac{28a^2b^6x^{15n}}{15n}+\frac{ab^7x^{16n}}{2n}+\frac{b^8x^{17n}}{17n}$$

```
[Out] 1/9*a^8*x^(9*n)/n+4/5*a^7*b*x^(10*n)/n+28/11*a^6*b^2*x^(11*n)/n+14/3*a^5*b^
3*x^(12*n)/n+70/13*a^4*b^4*x^(13*n)/n+4*a^3*b^5*x^(14*n)/n+28/15*a^2*b^6*x^
(15*n)/n+1/2*a*b^7*x^(16*n)/n+1/17*b^8*x^(17*n)/n
```

Rubi [A] time = 0.08, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$\frac{28a^6b^2x^{11n}}{11n}+\frac{14a^5b^3x^{12n}}{3n}+\frac{70a^4b^4x^{13n}}{13n}+\frac{4a^3b^5x^{14n}}{n}+\frac{28a^2b^6x^{15n}}{15n}+\frac{4a^7bx^{10n}}{5n}+\frac{a^8x^{9n}}{9n}+\frac{ab^7x^{16n}}{2n}+\frac{b^8x^{17n}}{17n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 9*n)*(a + b*x^n)^8,x]
```

```
[Out] (a^8*x^(9*n))/(9*n) + (4*a^7*b*x^(10*n))/(5*n) + (28*a^6*b^2*x^(11*n))/(11*
n) + (14*a^5*b^3*x^(12*n))/(3*n) + (70*a^4*b^4*x^(13*n))/(13*n) + (4*a^3*b^
5*x^(14*n))/n + (28*a^2*b^6*x^(15*n))/(15*n) + (a*b^7*x^(16*n))/(2*n) + (b^
8*x^(17*n))/(17*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1+9n}\left(a+bx^n\right)^8\,dx &= \frac{\text{Subst}\left(\int x^8(a+bx)^8\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(a^8x^8+8a^7bx^9+28a^6b^2x^{10}+56a^5b^3x^{11}+70a^4b^4x^{12}+56a^3b^5x^{13}+28a^2\right.\right.}{n}\\ &= \frac{a^8x^{9n}}{9n}+\frac{4a^7bx^{10n}}{5n}+\frac{28a^6b^2x^{11n}}{11n}+\frac{14a^5b^3x^{12n}}{3n}+\frac{70a^4b^4x^{13n}}{13n}+\frac{4a^3b^5x^{14n}}{n}+\frac{28a^2b^6}{15n}\end{aligned}$$

Mathematica [A] time = 0.09, size = 128, normalized size = 0.85

$$\frac{\frac{1}{9}a^8x^{9n}+\frac{4}{5}a^7bx^{10n}+\frac{28}{11}a^6b^2x^{11n}+\frac{14}{3}a^5b^3x^{12n}+\frac{70}{13}a^4b^4x^{13n}+4a^3b^5x^{14n}+\frac{28}{15}a^2b^6x^{15n}+\frac{1}{2}ab^7x^{16n}+\frac{1}{17}b^8x^{17n}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 9*n)*(a + b*x^n)^8,x]
```

```
[Out] ((a^8*x^(9*n))/9 + (4*a^7*b*x^(10*n))/5 + (28*a^6*b^2*x^(11*n))/11 + (14*a^
5*b^3*x^(12*n))/3 + (70*a^4*b^4*x^(13*n))/13 + 4*a^3*b^5*x^(14*n) + (28*a^2
*b^6*x^(15*n))/15 + (a*b^7*x^(16*n))/2 + (b^8*x^(17*n))/17)/n
```

fricas [A] time = 0.52, size = 113, normalized size = 0.75

$$\frac{12870\,b^8x^{17\,n}+109395\,ab^7x^{16\,n}+408408\,a^2b^6x^{15\,n}+875160\,a^3b^5x^{14\,n}+1178100\,a^4b^4x^{13\,n}+1021020\,a^5b^3x^{12\,n}+}{218790\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+9*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

```
[Out] 1/218790*(12870*b^8*x^(17*n) + 109395*a*b^7*x^(16*n) + 408408*a^2*b^6*x^(15
*n) + 875160*a^3*b^5*x^(14*n) + 1178100*a^4*b^4*x^(13*n) + 1021020*a^5*b^3*
x^(12*n) + 556920*a^6*b^2*x^(11*n) + 175032*a^7*b*x^(10*n) + 24310*a^8*x^(9
*n))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n+a)^8 x^{9\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+9*n)*(a+b*x^n)^8,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^8*x^(9*n - 1), x)
```

maple [A] time = 0.03, size = 136, normalized size = 0.90

$$\frac{a^8x^{9n}}{9n}+\frac{4a^7b\,x^{10n}}{5n}+\frac{28a^6b^2x^{11n}}{11n}+\frac{14a^5b^3x^{12n}}{3n}+\frac{70a^4b^4x^{13n}}{13n}+\frac{4a^3b^5x^{14n}}{n}+\frac{28a^2b^6x^{15n}}{15n}+\frac{a\,b^7x^{16n}}{2n}+\frac{b^8x^{17n}}{17n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+9*n)*(b*x^n+a)^8,x)
```

```
[Out] 1/17*b^8/n*(x^n)^17+1/2*a*b^7/n*(x^n)^16+28/15*a^2*b^6/n*(x^n)^15+4*a^3*b^5
/n*(x^n)^14+70/13*a^4*b^4/n*(x^n)^13+14/3*a^5*b^3/n*(x^n)^12+28/11*a^6*b^2/
n*(x^n)^11+4/5*a^7*b/n*(x^n)^10+1/9*a^8/n*(x^n)^9
```

maxima [A] time = 0.57, size = 135, normalized size = 0.89

$$\frac{b^8x^{17\,n}}{17\,n}+\frac{ab^7x^{16\,n}}{2\,n}+\frac{28\,a^2b^6x^{15\,n}}{15\,n}+\frac{4\,a^3b^5x^{14\,n}}{n}+\frac{70\,a^4b^4x^{13\,n}}{13\,n}+\frac{14\,a^5b^3x^{12\,n}}{3\,n}+\frac{28\,a^6b^2x^{11\,n}}{11\,n}+\frac{4\,a^7bx^{10\,n}}{5\,n}+\frac{a^8x^{9\,n}}{9\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+9*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

```
[Out] 1/17*b^8*x^(17*n)/n + 1/2*a*b^7*x^(16*n)/n + 28/15*a^2*b^6*x^(15*n)/n + 4*a
^3*b^5*x^(14*n)/n + 70/13*a^4*b^4*x^(13*n)/n + 14/3*a^5*b^3*x^(12*n)/n + 28
/11*a^6*b^2*x^(11*n)/n + 4/5*a^7*b*x^(10*n)/n + 1/9*a^8*x^(9*n)/n
```

mupad [B] time = 1.50, size = 135, normalized size = 0.89

$$\frac{a^8\,x^{9\,n}}{9\,n}+\frac{b^8\,x^{17\,n}}{17\,n}+\frac{28\,a^6\,b^2\,x^{11\,n}}{11\,n}+\frac{14\,a^5\,b^3\,x^{12\,n}}{3\,n}+\frac{70\,a^4\,b^4\,x^{13\,n}}{13\,n}+\frac{4\,a^3\,b^5\,x^{14\,n}}{n}+\frac{28\,a^2\,b^6\,x^{15\,n}}{15\,n}+\frac{4\,a^7\,b\,x^{10\,n}}{5\,n}+\frac{a\,b^7\,x^{16\,n}}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9*n - 1)*(a + b*x^n)^8,x)
```

```
[Out] (a^8*x^(9*n))/(9*n) + (b^8*x^(17*n))/(17*n) + (28*a^6*b^2*x^(11*n))/(11*n)
+ (14*a^5*b^3*x^(12*n))/(3*n) + (70*a^4*b^4*x^(13*n))/(13*n) + (4*a^3*b^5*x
^(14*n))/n + (28*a^2*b^6*x^(15*n))/(15*n) + (4*a^7*b*x^(10*n))/(5*n) + (a*b
^7*x^(16*n))/(2*n)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+9*n)*(a+b*x**n)**8,x)
```

```
[Out] Timed out
```

3.2567 $\int x^{-1+8n}\left(a+bx^n\right)^8\,dx$

Optimal. Leaf size=151

$$\frac{a^8x^{8n}}{8n}+\frac{8a^7bx^{9n}}{9n}+\frac{14a^6b^2x^{10n}}{5n}+\frac{56a^5b^3x^{11n}}{11n}+\frac{35a^4b^4x^{12n}}{6n}+\frac{56a^3b^5x^{13n}}{13n}+\frac{2a^2b^6x^{14n}}{n}+\frac{8ab^7x^{15n}}{15n}+\frac{b^8x^{16n}}{16n}$$

```
[Out] 1/8*a^8*x^(8*n)/n+8/9*a^7*b*x^(9*n)/n+14/5*a^6*b^2*x^(10*n)/n+56/11*a^5*b^3
*x^(11*n)/n+35/6*a^4*b^4*x^(12*n)/n+56/13*a^3*b^5*x^(13*n)/n+2*a^2*b^6*x^(1
4*n)/n+8/15*a*b^7*x^(15*n)/n+1/16*b^8*x^(16*n)/n
```

Rubi [A] time = 0.07, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$\frac{14a^6b^2x^{10n}}{5n}+\frac{56a^5b^3x^{11n}}{11n}+\frac{35a^4b^4x^{12n}}{6n}+\frac{56a^3b^5x^{13n}}{13n}+\frac{2a^2b^6x^{14n}}{n}+\frac{8a^7bx^{9n}}{9n}+\frac{a^8x^{8n}}{8n}+\frac{8ab^7x^{15n}}{15n}+\frac{b^8x^{16n}}{16n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 8*n)*(a + b*x^n)^8,x]
```

```
[Out] (a^8*x^(8*n))/(8*n) + (8*a^7*b*x^(9*n))/(9*n) + (14*a^6*b^2*x^(10*n))/(5*n)
 + (56*a^5*b^3*x^(11*n))/(11*n) + (35*a^4*b^4*x^(12*n))/(6*n) + (56*a^3*b^5
*x^(13*n))/(13*n) + (2*a^2*b^6*x^(14*n))/n + (8*a*b^7*x^(15*n))/(15*n) + (b
^8*x^(16*n))/(16*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1+8n}\left(a+bx^n\right)^8\,dx &= \frac{\text{Subst}\left(\int x^7(a+bx)^8\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(a^8x^7+8a^7bx^8+28a^6b^2x^9+56a^5b^3x^{10}+70a^4b^4x^{11}+56a^3b^5x^{12}+28a^2b^6x^{\right.}\right.}{n}\\ &= \frac{a^8x^{8n}}{8n}+\frac{8a^7bx^{9n}}{9n}+\frac{14a^6b^2x^{10n}}{5n}+\frac{56a^5b^3x^{11n}}{11n}+\frac{35a^4b^4x^{12n}}{6n}+\frac{56a^3b^5x^{13n}}{13n}+\frac{2a^2b^6x^{14n}}{n}\end{aligned}$$

Mathematica [A] time = 0.08, size = 128, normalized size = 0.85

$$\frac{\frac{1}{8}a^8x^{8n}+\frac{8}{9}a^7bx^{9n}+\frac{14}{5}a^6b^2x^{10n}+\frac{56}{11}a^5b^3x^{11n}+\frac{35}{6}a^4b^4x^{12n}+\frac{56}{13}a^3b^5x^{13n}+2a^2b^6x^{14n}+\frac{8}{15}ab^7x^{15n}+\frac{1}{16}b^8x^{16n}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 8*n)*(a + b*x^n)^8,x]
```

```
[Out] ((a^8*x^(8*n))/8 + (8*a^7*b*x^(9*n))/9 + (14*a^6*b^2*x^(10*n))/5 + (56*a^5*
b^3*x^(11*n))/11 + (35*a^4*b^4*x^(12*n))/6 + (56*a^3*b^5*x^(13*n))/13 + 2*a
^2*b^6*x^(14*n) + (8*a*b^7*x^(15*n))/15 + (b^8*x^(16*n))/16)/n
```

fricas [A] time = 0.58, size = 113, normalized size = 0.75

$$\frac{6435\,b^8x^{16\,n}+54912\,ab^7x^{15\,n}+205920\,a^2b^6x^{14\,n}+443520\,a^3b^5x^{13\,n}+600600\,a^4b^4x^{12\,n}+524160\,a^5b^3x^{11\,n}+}{102960\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+8*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

```
[Out] 1/102960*(6435*b^8*x^(16*n) + 54912*a*b^7*x^(15*n) + 205920*a^2*b^6*x^(14*n
) + 443520*a^3*b^5*x^(13*n) + 600600*a^4*b^4*x^(12*n) + 524160*a^5*b^3*x^(1
1*n) + 288288*a^6*b^2*x^(10*n) + 91520*a^7*b*x^(9*n) + 12870*a^8*x^(8*n))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n+a)^8x^{8\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+8*n)*(a+b*x^n)^8,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^8*x^(8*n - 1), x)
```

maple [A] time = 0.02, size = 136, normalized size = 0.90

$$\frac{a^8x^{8n}}{8n}+\frac{8a^7b\,x^{9n}}{9n}+\frac{14a^6b^2x^{10n}}{5n}+\frac{56a^5b^3x^{11n}}{11n}+\frac{35a^4b^4x^{12n}}{6n}+\frac{56a^3b^5x^{13n}}{13n}+\frac{2a^2b^6x^{14n}}{n}+\frac{8a\,b^7x^{15n}}{15n}+\frac{b^8x^{16n}}{16n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+8*n)*(b*x^n+a)^8,x)
```

```
[Out] 1/16*b^8/n*(x^n)^16+8/15*a*b^7/n*(x^n)^15+2*a^2*b^6/n*(x^n)^14+56/13*a^3*b^
5/n*(x^n)^13+35/6*a^4*b^4/n*(x^n)^12+56/11*a^5*b^3/n*(x^n)^11+14/5*a^6*b^2/
n*(x^n)^10+8/9*a^7*b/n*(x^n)^9+1/8*a^8/n*(x^n)^8
```

maxima [A] time = 0.54, size = 135, normalized size = 0.89

$$\frac{b^8x^{16\,n}}{16\,n}+\frac{8\,ab^7x^{15\,n}}{15\,n}+\frac{2\,a^2b^6x^{14\,n}}{n}+\frac{56\,a^3b^5x^{13\,n}}{13\,n}+\frac{35\,a^4b^4x^{12\,n}}{6\,n}+\frac{56\,a^5b^3x^{11\,n}}{11\,n}+\frac{14\,a^6b^2x^{10\,n}}{5\,n}+\frac{8\,a^7bx^{9\,n}}{9\,n}+\frac{a^8x^{8\,n}}{8\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+8*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

```
[Out] 1/16*b^8*x^(16*n)/n + 8/15*a*b^7*x^(15*n)/n + 2*a^2*b^6*x^(14*n)/n + 56/13*
a^3*b^5*x^(13*n)/n + 35/6*a^4*b^4*x^(12*n)/n + 56/11*a^5*b^3*x^(11*n)/n + 1
4/5*a^6*b^2*x^(10*n)/n + 8/9*a^7*b*x^(9*n)/n + 1/8*a^8*x^(8*n)/n
```

mupad [B] time = 1.49, size = 135, normalized size = 0.89

$$\frac{a^8\,x^{8\,n}}{8\,n}+\frac{b^8\,x^{16\,n}}{16\,n}+\frac{14\,a^6\,b^2\,x^{10\,n}}{5\,n}+\frac{56\,a^5\,b^3\,x^{11\,n}}{11\,n}+\frac{35\,a^4\,b^4\,x^{12\,n}}{6\,n}+\frac{56\,a^3\,b^5\,x^{13\,n}}{13\,n}+\frac{2\,a^2\,b^6\,x^{14\,n}}{n}+\frac{8\,a^7\,b\,x^{9\,n}}{9\,n}+\frac{8\,a\,b^7\,x^1}{15\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(8*n - 1)*(a + b*x^n)^8,x)
```

```
[Out] (a^8*x^(8*n))/(8*n) + (b^8*x^(16*n))/(16*n) + (14*a^6*b^2*x^(10*n))/(5*n) +
 (56*a^5*b^3*x^(11*n))/(11*n) + (35*a^4*b^4*x^(12*n))/(6*n) + (56*a^3*b^5*x
```

```
^(13*n))/(13*n) + (2*a^2*b^6*x^(14*n))/n + (8*a^7*b*x^(9*n))/(9*n) + (8*a*b
^7*x^(15*n))/(15*n)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+8*n)*(a+b*x**n)**8,x)
```

```
[Out] Timed out
```

3.2568 $\int x^{-1+7n}\left(a+bx^n\right)^8\,dx$

Optimal. Leaf size=150

$$\frac{a^6\left(a+bx^n\right)^9}{9b^7n}-\frac{3a^5\left(a+bx^n\right)^{10}}{5b^7n}+\frac{15a^4\left(a+bx^n\right)^{11}}{11b^7n}-\frac{5a^3\left(a+bx^n\right)^{12}}{3b^7n}+\frac{15a^2\left(a+bx^n\right)^{13}}{13b^7n}+\frac{\left(a+bx^n\right)^{15}}{15b^7n}-\frac{3a\left(a+bx^n\right.}{7b^7n}$$

```
[Out] 1/9*a^6*(a+b*x^n)^9/b^7/n-3/5*a^5*(a+b*x^n)^10/b^7/n+15/11*a^4*(a+b*x^n)^11
/b^7/n-5/3*a^3*(a+b*x^n)^12/b^7/n+15/13*a^2*(a+b*x^n)^13/b^7/n-3/7*a*(a+b*x
^n)^14/b^7/n+1/15*(a+b*x^n)^15/b^7/n
```

Rubi [A] time = 0.08, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$\frac{15a^2\left(a+bx^n\right)^{13}}{13b^7n}-\frac{5a^3\left(a+bx^n\right)^{12}}{3b^7n}+\frac{15a^4\left(a+bx^n\right)^{11}}{11b^7n}-\frac{3a^5\left(a+bx^n\right)^{10}}{5b^7n}+\frac{a^6\left(a+bx^n\right)^9}{9b^7n}+\frac{\left(a+bx^n\right)^{15}}{15b^7n}-\frac{3a\left(a+bx^n\right.}{7b^7n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 7*n)*(a + b*x^n)^8,x]
```

```
[Out] (a^6*(a + b*x^n)^9)/(9*b^7*n) - (3*a^5*(a + b*x^n)^10)/(5*b^7*n) + (15*a^4*
(a + b*x^n)^11)/(11*b^7*n) - (5*a^3*(a + b*x^n)^12)/(3*b^7*n) + (15*a^2*(a
+ b*x^n)^13)/(13*b^7*n) - (3*a*(a + b*x^n)^14)/(7*b^7*n) + (a + b*x^n)^15/(
15*b^7*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1+7n}\left(a+bx^n\right)^8\,dx &= \frac{\text{Subst}\left(\int x^6(a+bx)^8\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int\left(\frac{a^6(a+bx)^8}{b^6}-\frac{6a^5(a+bx)^9}{b^6}+\frac{15a^4(a+bx)^{10}}{b^6}-\frac{20a^3(a+bx)^{11}}{b^6}+\frac{15a^2(a+bx)^{12}}{b^6}-\frac{6a(a+bx)^{13}}{b^6}\right.\right.}{n}\\ &= \frac{a^6\left(a+bx^n\right)^9}{9b^7n}-\frac{3a^5\left(a+bx^n\right)^{10}}{5b^7n}+\frac{15a^4\left(a+bx^n\right)^{11}}{11b^7n}-\frac{5a^3\left(a+bx^n\right)^{12}}{3b^7n}+\frac{15a^2\left(a+bx\right.}{13b^7n}\end{aligned}$$

Mathematica [A] time = 0.08, size = 127, normalized size = 0.85

$$\frac{\frac{1}{7}a^8x^{7n}+a^7bx^{8n}+\frac{28}{9}a^6b^2x^{9n}+\frac{28}{5}a^5b^3x^{10n}+\frac{70}{11}a^4b^4x^{11n}+\frac{14}{3}a^3b^5x^{12n}+\frac{28}{13}a^2b^6x^{13n}+\frac{4}{7}ab^7x^{14n}+\frac{1}{15}b^8x^{15n}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 7*n)*(a + b*x^n)^8,x]
```

```
[Out] ((a^8*x^(7*n))/7 + a^7*b*x^(8*n) + (28*a^6*b^2*x^(9*n))/9 + (28*a^5*b^3*x^(
10*n))/5 + (70*a^4*b^4*x^(11*n))/11 + (14*a^3*b^5*x^(12*n))/3 + (28*a^2*b^6
*x^(13*n))/13 + (4*a*b^7*x^(14*n))/7 + (b^8*x^(15*n))/15)/n
```

fricas [A] time = 0.58, size = 113, normalized size = 0.75

$$\frac{3003\,b^8x^{15\,n}+25740\,ab^7x^{14\,n}+97020\,a^2b^6x^{13\,n}+210210\,a^3b^5x^{12\,n}+286650\,a^4b^4x^{11\,n}+252252\,a^5b^3x^{10\,n}+140}{45045\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+7*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

```
[Out] 1/45045*(3003*b^8*x^(15*n) + 25740*a*b^7*x^(14*n) + 97020*a^2*b^6*x^(13*n)
+ 210210*a^3*b^5*x^(12*n) + 286650*a^4*b^4*x^(11*n) + 252252*a^5*b^3*x^(10*
n) + 140140*a^6*b^2*x^(9*n) + 45045*a^7*b*x^(8*n) + 6435*a^8*x^(7*n))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n+a)^8x^{7\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+7*n)*(a+b*x^n)^8,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^8*x^(7*n - 1), x)
```

maple [A] time = 0.03, size = 135, normalized size = 0.90

$$\frac{a^8x^{7n}}{7n}+\frac{a^7b\,x^{8n}}{n}+\frac{28a^6b^2x^{9n}}{9n}+\frac{28a^5b^3x^{10n}}{5n}+\frac{70a^4b^4x^{11n}}{11n}+\frac{14a^3b^5x^{12n}}{3n}+\frac{28a^2b^6x^{13n}}{13n}+\frac{4a\,b^7x^{14n}}{7n}+\frac{b^8x^{15n}}{15n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+7*n)*(b*x^n+a)^8,x)
```

```
[Out] 1/15*b^8/n*(x^n)^15+4/7*a*b^7/n*(x^n)^14+28/13*a^2*b^6/n*(x^n)^13+14/3*a^3*
b^5/n*(x^n)^12+70/11*a^4*b^4/n*(x^n)^11+28/5*a^5*b^3/n*(x^n)^10+28/9*a^6*b^
2/n*(x^n)^9+a^7*b/n*(x^n)^8+1/7*a^8/n*(x^n)^7
```

maxima [A] time = 0.67, size = 134, normalized size = 0.89

$$\frac{b^8x^{15\,n}}{15\,n}+\frac{4\,ab^7x^{14\,n}}{7\,n}+\frac{28\,a^2b^6x^{13\,n}}{13\,n}+\frac{14\,a^3b^5x^{12\,n}}{3\,n}+\frac{70\,a^4b^4x^{11\,n}}{11\,n}+\frac{28\,a^5b^3x^{10\,n}}{5\,n}+\frac{28\,a^6b^2x^{9\,n}}{9\,n}+\frac{a^7bx^{8\,n}}{n}+\frac{a^8x^{7\,n}}{7\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+7*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

```
[Out] 1/15*b^8*x^(15*n)/n + 4/7*a*b^7*x^(14*n)/n + 28/13*a^2*b^6*x^(13*n)/n + 14/
3*a^3*b^5*x^(12*n)/n + 70/11*a^4*b^4*x^(11*n)/n + 28/5*a^5*b^3*x^(10*n)/n +
 28/9*a^6*b^2*x^(9*n)/n + a^7*b*x^(8*n)/n + 1/7*a^8*x^(7*n)/n
```

mupad [B] time = 1.47, size = 134, normalized size = 0.89

$$\frac{a^8\,x^{7\,n}}{7\,n}+\frac{b^8\,x^{15\,n}}{15\,n}+\frac{28\,a^6\,b^2\,x^{9\,n}}{9\,n}+\frac{28\,a^5\,b^3\,x^{10\,n}}{5\,n}+\frac{70\,a^4\,b^4\,x^{11\,n}}{11\,n}+\frac{14\,a^3\,b^5\,x^{12\,n}}{3\,n}+\frac{28\,a^2\,b^6\,x^{13\,n}}{13\,n}+\frac{a^7\,b\,x^{8\,n}}{n}+\frac{4\,a\,b^7\,x^{14\,n}}{7\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7*n - 1)*(a + b*x^n)^8,x)
```

```
[Out] (a^8*x^(7*n))/(7*n) + (b^8*x^(15*n))/(15*n) + (28*a^6*b^2*x^(9*n))/(9*n) +
(28*a^5*b^3*x^(10*n))/(5*n) + (70*a^4*b^4*x^(11*n))/(11*n) + (14*a^3*b^5*x^
(12*n))/(3*n) + (28*a^2*b^6*x^(13*n))/(13*n) + (a^7*b*x^(8*n))/n + (4*a*b^7
*x^(14*n))/(7*n)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+7*n)*(a+b*x**n)**8,x)
```

```
[Out] Timed out
```

3.2569 $\int x^{-1+6n}\left(a+bx^n\right)^8\,dx$

Optimal. Leaf size=128

$$-\frac{a^5\left(a+bx^n\right)^9}{9b^6n}+\frac{a^4\left(a+bx^n\right)^{10}}{2b^6n}-\frac{10a^3\left(a+bx^n\right)^{11}}{11b^6n}+\frac{5a^2\left(a+bx^n\right)^{12}}{6b^6n}+\frac{\left(a+bx^n\right)^{14}}{14b^6n}-\frac{5a\left(a+bx^n\right)^{13}}{13b^6n}$$

```
[Out] -1/9*a^5*(a+b*x^n)^9/b^6/n+1/2*a^4*(a+b*x^n)^10/b^6/n-10/11*a^3*(a+b*x^n)^1
1/b^6/n+5/6*a^2*(a+b*x^n)^12/b^6/n-5/13*a*(a+b*x^n)^13/b^6/n+1/14*(a+b*x^n)
^14/b^6/n
```

Rubi [A] time = 0.07, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$\frac{5a^2\left(a+bx^n\right)^{12}}{6b^6n}-\frac{10a^3\left(a+bx^n\right)^{11}}{11b^6n}+\frac{a^4\left(a+bx^n\right)^{10}}{2b^6n}-\frac{a^5\left(a+bx^n\right)^9}{9b^6n}+\frac{\left(a+bx^n\right)^{14}}{14b^6n}-\frac{5a\left(a+bx^n\right)^{13}}{13b^6n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 6*n)*(a + b*x^n)^8,x]
```

```
[Out] -(a^5*(a + b*x^n)^9)/(9*b^6*n) + (a^4*(a + b*x^n)^10)/(2*b^6*n) - (10*a^3*(
a + b*x^n)^11)/(11*b^6*n) + (5*a^2*(a + b*x^n)^12)/(6*b^6*n) - (5*a*(a + b*
x^n)^13)/(13*b^6*n) + (a + b*x^n)^14/(14*b^6*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1+6n}\left(a+bx^n\right)^8\,dx &= \frac{\text{Subst}\left(\int x^5(a+bx)^8\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int\left(-\frac{a^5(a+bx)^8}{b^5}+\frac{5a^4(a+bx)^9}{b^5}-\frac{10a^3(a+bx)^{10}}{b^5}+\frac{10a^2(a+bx)^{11}}{b^5}-\frac{5a(a+bx)^{12}}{b^5}+\frac{(a+bx)^{13}}{b^5}\right)dx,x\right.}{n}\\ &= -\frac{a^5\left(a+bx^n\right)^9}{9b^6n}+\frac{a^4\left(a+bx^n\right)^{10}}{2b^6n}-\frac{10a^3\left(a+bx^n\right)^{11}}{11b^6n}+\frac{5a^2\left(a+bx^n\right)^{12}}{6b^6n}-\frac{5a\left(a+bx^n\right)^{13}}{13b^6n}\end{aligned}$$

Mathematica [A] time = 0.10, size = 79, normalized size = 0.62

$$-\frac{\left(a+bx^n\right)^9\left(a^5-9a^4bx^n+45a^3b^2x^{2n}-165a^2b^3x^{3n}+495ab^4x^{4n}-1287b^5x^{5n}\right)}{18018b^6n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 6*n)*(a + b*x^n)^8,x]
```

```
[Out] -1/18018*((a + b*x^n)^9*(a^5 - 9*a^4*b*x^n + 45*a^3*b^2*x^(2*n) - 165*a^2*b
^3*x^(3*n) + 495*a*b^4*x^(4*n) - 1287*b^5*x^(5*n)))/(b^6*n)
```

fricas [A] time = 0.79, size = 113, normalized size = 0.88

$$\frac{1287\,b^8x^{14\,n}+11088\,ab^7x^{13\,n}+42042\,a^2b^6x^{12\,n}+91728\,a^3b^5x^{11\,n}+126126\,a^4b^4x^{10\,n}+112112\,a^5b^3x^{9\,n}+630}{18018\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+6*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

```
[Out] 1/18018*(1287*b^8*x^(14*n) + 11088*a*b^7*x^(13*n) + 42042*a^2*b^6*x^(12*n)
+ 91728*a^3*b^5*x^(11*n) + 126126*a^4*b^4*x^(10*n) + 112112*a^5*b^3*x^(9*n)
 + 63063*a^6*b^2*x^(8*n) + 20592*a^7*b*x^(7*n) + 3003*a^8*x^(6*n))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n+a)^8x^{6\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+6*n)*(a+b*x^n)^8,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^8*x^(6*n - 1), x)
```

maple [A] time = 0.03, size = 136, normalized size = 1.06

$$\frac{a^8x^{6n}}{6n}+\frac{8a^7b\,x^{7n}}{7n}+\frac{7a^6b^2x^{8n}}{2n}+\frac{56a^5b^3x^{9n}}{9n}+\frac{7a^4b^4x^{10n}}{n}+\frac{56a^3b^5x^{11n}}{11n}+\frac{7a^2b^6x^{12n}}{3n}+\frac{8a\,b^7x^{13n}}{13n}+\frac{b^8x^{14n}}{14n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+6*n)*(b*x^n+a)^8,x)
```

```
[Out] 1/14*b^8/n*(x^n)^14+8/13*a*b^7/n*(x^n)^13+7/3*a^2*b^6/n*(x^n)^12+56/11*a^3*
b^5/n*(x^n)^11+7*a^4*b^4/n*(x^n)^10+56/9*a^5*b^3/n*(x^n)^9+7/2*a^6*b^2/n*(x
^n)^8+8/7*a^7*b/n*(x^n)^7+1/6*a^8/n*(x^n)^6
```

maxima [A] time = 0.58, size = 135, normalized size = 1.05

$$\frac{b^8x^{14\,n}}{14\,n}+\frac{8\,ab^7x^{13\,n}}{13\,n}+\frac{7\,a^2b^6x^{12\,n}}{3\,n}+\frac{56\,a^3b^5x^{11\,n}}{11\,n}+\frac{7\,a^4b^4x^{10\,n}}{n}+\frac{56\,a^5b^3x^{9\,n}}{9\,n}+\frac{7\,a^6b^2x^{8\,n}}{2\,n}+\frac{8\,a^7bx^{7\,n}}{7\,n}+\frac{a^8x^{6\,n}}{6\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+6*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

```
[Out] 1/14*b^8*x^(14*n)/n + 8/13*a*b^7*x^(13*n)/n + 7/3*a^2*b^6*x^(12*n)/n + 56/1
1*a^3*b^5*x^(11*n)/n + 7*a^4*b^4*x^(10*n)/n + 56/9*a^5*b^3*x^(9*n)/n + 7/2*
a^6*b^2*x^(8*n)/n + 8/7*a^7*b*x^(7*n)/n + 1/6*a^8*x^(6*n)/n
```

mupad [B] time = 1.45, size = 135, normalized size = 1.05

$$\frac{a^8\,x^{6\,n}}{6\,n}+\frac{b^8\,x^{14\,n}}{14\,n}+\frac{7\,a^6\,b^2\,x^{8\,n}}{2\,n}+\frac{56\,a^5\,b^3\,x^{9\,n}}{9\,n}+\frac{7\,a^4\,b^4\,x^{10\,n}}{n}+\frac{56\,a^3\,b^5\,x^{11\,n}}{11\,n}+\frac{7\,a^2\,b^6\,x^{12\,n}}{3\,n}+\frac{8\,a^7\,b\,x^{7\,n}}{7\,n}+\frac{8\,a\,b^7\,x^{13\,n}}{13\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(6*n - 1)*(a + b*x^n)^8,x)
```

```
[Out] (a^8*x^(6*n))/(6*n) + (b^8*x^(14*n))/(14*n) + (7*a^6*b^2*x^(8*n))/(2*n) + (
56*a^5*b^3*x^(9*n))/(9*n) + (7*a^4*b^4*x^(10*n))/n + (56*a^3*b^5*x^(11*n))/
```

```
(11*n) + (7*a^2*b^6*x^(12*n))/(3*n) + (8*a^7*b*x^(7*n))/(7*n) + (8*a*b^7*x^
(13*n))/(13*n)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+6*n)*(a+b*x**n)**8,x)
```

```
[Out] Timed out
```

3.2570 $\int x^{-1+5n}\left(a+bx^n\right)^8\,dx$

Optimal. Leaf size=106

$$\frac{a^4\left(a+bx^n\right)^9}{9b^5n}-\frac{2a^3\left(a+bx^n\right)^{10}}{5b^5n}+\frac{6a^2\left(a+bx^n\right)^{11}}{11b^5n}+\frac{\left(a+bx^n\right)^{13}}{13b^5n}-\frac{a\left(a+bx^n\right)^{12}}{3b^5n}$$

```
[Out] 1/9*a^4*(a+b*x^n)^9/b^5/n-2/5*a^3*(a+b*x^n)^10/b^5/n+6/11*a^2*(a+b*x^n)^11/
b^5/n-1/3*a*(a+b*x^n)^12/b^5/n+1/13*(a+b*x^n)^13/b^5/n
```

Rubi [A] time = 0.06, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$\frac{6a^2\left(a+bx^n\right)^{11}}{11b^5n}-\frac{2a^3\left(a+bx^n\right)^{10}}{5b^5n}+\frac{a^4\left(a+bx^n\right)^9}{9b^5n}+\frac{\left(a+bx^n\right)^{13}}{13b^5n}-\frac{a\left(a+bx^n\right)^{12}}{3b^5n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 5*n)*(a + b*x^n)^8,x]
```

```
[Out] (a^4*(a + b*x^n)^9)/(9*b^5*n) - (2*a^3*(a + b*x^n)^10)/(5*b^5*n) + (6*a^2*(
a + b*x^n)^11)/(11*b^5*n) - (a*(a + b*x^n)^12)/(3*b^5*n) + (a + b*x^n)^13/(
13*b^5*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1+5n}\left(a+bx^n\right)^8\,dx &= \frac{\text{Subst}\left(\int x^4(a+bx)^8\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int\left(\frac{a^4(a+bx)^8}{b^4}-\frac{4a^3(a+bx)^9}{b^4}+\frac{6a^2(a+bx)^{10}}{b^4}-\frac{4a(a+bx)^{11}}{b^4}+\frac{(a+bx)^{12}}{b^4}\right)dx,x,x^n\right)}{n}\\ &= \frac{a^4\left(a+bx^n\right)^9}{9b^5n}-\frac{2a^3\left(a+bx^n\right)^{10}}{5b^5n}+\frac{6a^2\left(a+bx^n\right)^{11}}{11b^5n}-\frac{a\left(a+bx^n\right)^{12}}{3b^5n}+\frac{\left(a+bx^n\right)^{13}}{13b^5n}\end{aligned}$$

Mathematica [A] time = 0.06, size = 66, normalized size = 0.62

$$\frac{\left(a+bx^n\right)^9\left(a^4-9a^3bx^n+45a^2b^2x^{2n}-165ab^3x^{3n}+495b^4x^{4n}\right)}{6435b^5n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 5*n)*(a + b*x^n)^8,x]
```

```
[Out] ((a + b*x^n)^9*(a^4 - 9*a^3*b*x^n + 45*a^2*b^2*x^(2*n) - 165*a*b^3*x^(3*n)
+ 495*b^4*x^(4*n)))/(6435*b^5*n)
```

fricas [A] time = 0.62, size = 113, normalized size = 1.07

$$\frac{495\,b^8x^{13\,n}+4290\,ab^7x^{12\,n}+16380\,a^2b^6x^{11\,n}+36036\,a^3b^5x^{10\,n}+50050\,a^4b^4x^{9\,n}+45045\,a^5b^3x^{8\,n}+25740\,a^6b^2x}{6435\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+5*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

```
[Out] 1/6435*(495*b^8*x^(13*n) + 4290*a*b^7*x^(12*n) + 16380*a^2*b^6*x^(11*n) + 3
6036*a^3*b^5*x^(10*n) + 50050*a^4*b^4*x^(9*n) + 45045*a^5*b^3*x^(8*n) + 257
40*a^6*b^2*x^(7*n) + 8580*a^7*b*x^(6*n) + 1287*a^8*x^(5*n))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n+a)^8x^{5\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+5*n)*(a+b*x^n)^8,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^8*x^(5*n - 1), x)
```

maple [A] time = 0.03, size = 136, normalized size = 1.28

$$\frac{a^8x^{5n}}{5n}+\frac{4a^7b\,x^{6n}}{3n}+\frac{4a^6b^2x^{7n}}{n}+\frac{7a^5b^3x^{8n}}{n}+\frac{70a^4b^4x^{9n}}{9n}+\frac{28a^3b^5x^{10n}}{5n}+\frac{28a^2b^6x^{11n}}{11n}+\frac{2a\,b^7x^{12n}}{3n}+\frac{b^8x^{13n}}{13n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+5*n)*(b*x^n+a)^8,x)
```

```
[Out] 1/13*b^8/n*(x^n)^13+2/3*a*b^7/n*(x^n)^12+28/11*a^2*b^6/n*(x^n)^11+28/5*a^3*
b^5/n*(x^n)^10+70/9*a^4*b^4/n*(x^n)^9+7*a^5*b^3/n*(x^n)^8+4*a^6*b^2/n*(x^n)
^7+4/3*a^7*b/n*(x^n)^6+1/5*a^8/n*(x^n)^5
```

maxima [A] time = 0.59, size = 135, normalized size = 1.27

$$\frac{b^8x^{13\,n}}{13\,n}+\frac{2\,ab^7x^{12\,n}}{3\,n}+\frac{28\,a^2b^6x^{11\,n}}{11\,n}+\frac{28\,a^3b^5x^{10\,n}}{5\,n}+\frac{70\,a^4b^4x^{9\,n}}{9\,n}+\frac{7\,a^5b^3x^{8\,n}}{n}+\frac{4\,a^6b^2x^{7\,n}}{n}+\frac{4\,a^7bx^{6\,n}}{3\,n}+\frac{a^8x^{5\,n}}{5\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+5*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

```
[Out] 1/13*b^8*x^(13*n)/n + 2/3*a*b^7*x^(12*n)/n + 28/11*a^2*b^6*x^(11*n)/n + 28/
5*a^3*b^5*x^(10*n)/n + 70/9*a^4*b^4*x^(9*n)/n + 7*a^5*b^3*x^(8*n)/n + 4*a^6
*b^2*x^(7*n)/n + 4/3*a^7*b*x^(6*n)/n + 1/5*a^8*x^(5*n)/n
```

mupad [B] time = 1.46, size = 135, normalized size = 1.27

$$\frac{a^8\,x^{5\,n}}{5\,n}+\frac{b^8\,x^{13\,n}}{13\,n}+\frac{4\,a^6\,b^2\,x^{7\,n}}{n}+\frac{7\,a^5\,b^3\,x^{8\,n}}{n}+\frac{70\,a^4\,b^4\,x^{9\,n}}{9\,n}+\frac{28\,a^3\,b^5\,x^{10\,n}}{5\,n}+\frac{28\,a^2\,b^6\,x^{11\,n}}{11\,n}+\frac{4\,a^7\,b\,x^{6\,n}}{3\,n}+\frac{2\,a\,b^7\,x^{12\,n}}{3\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5*n - 1)*(a + b*x^n)^8,x)
```

```
[Out] (a^8*x^(5*n))/(5*n) + (b^8*x^(13*n))/(13*n) + (4*a^6*b^2*x^(7*n))/n + (7*a^
5*b^3*x^(8*n))/n + (70*a^4*b^4*x^(9*n))/(9*n) + (28*a^3*b^5*x^(10*n))/(5*n)
```

```
 + (28*a^2*b^6*x^(11*n))/(11*n) + (4*a^7*b*x^(6*n))/(3*n) + (2*a*b^7*x^(12*
n))/(3*n)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+5*n)*(a+b*x**n)**8,x)
```

```
[Out] Timed out
```

3.2571 $\int x^{-1+4n}\left(a+bx^n\right)^8\,dx$

Optimal. Leaf size=84

$$-\frac{a^3\left(a+bx^n\right)^9}{9b^4n}+\frac{3a^2\left(a+bx^n\right)^{10}}{10b^4n}+\frac{\left(a+bx^n\right)^{12}}{12b^4n}-\frac{3a\left(a+bx^n\right)^{11}}{11b^4n}$$

```
[Out] -1/9*a^3*(a+b*x^n)^9/b^4/n+3/10*a^2*(a+b*x^n)^10/b^4/n-3/11*a*(a+b*x^n)^11/
b^4/n+1/12*(a+b*x^n)^12/b^4/n
```

Rubi [A] time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$\frac{3a^2\left(a+bx^n\right)^{10}}{10b^4n}-\frac{a^3\left(a+bx^n\right)^9}{9b^4n}+\frac{\left(a+bx^n\right)^{12}}{12b^4n}-\frac{3a\left(a+bx^n\right)^{11}}{11b^4n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 4*n)*(a + b*x^n)^8,x]
```

```
[Out] -(a^3*(a + b*x^n)^9)/(9*b^4*n) + (3*a^2*(a + b*x^n)^10)/(10*b^4*n) - (3*a*(
a + b*x^n)^11)/(11*b^4*n) + (a + b*x^n)^12/(12*b^4*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1+4n}\left(a+bx^n\right)^8\,dx &= \frac{\text{Subst}\left(\int x^3(a+bx)^8\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int\left(-\frac{a^3(a+bx)^8}{b^3}+\frac{3a^2(a+bx)^9}{b^3}-\frac{3a(a+bx)^{10}}{b^3}+\frac{(a+bx)^{11}}{b^3}\right)dx,x,x^n\right)}{n}\\ &= -\frac{a^3\left(a+bx^n\right)^9}{9b^4n}+\frac{3a^2\left(a+bx^n\right)^{10}}{10b^4n}-\frac{3a\left(a+bx^n\right)^{11}}{11b^4n}+\frac{\left(a+bx^n\right)^{12}}{12b^4n}\end{aligned}$$

Mathematica [A] time = 0.06, size = 53, normalized size = 0.63

$$-\frac{\left(a+bx^n\right)^9\left(a^3-9a^2bx^n+45ab^2x^{2n}-165b^3x^{3n}\right)}{1980b^4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 4*n)*(a + b*x^n)^8,x]
```

```
[Out] -1/1980*((a + b*x^n)^9*(a^3 - 9*a^2*b*x^n + 45*a*b^2*x^(2*n) - 165*b^3*x^(3
*n)))/(b^4*n)
```

fricas [A] time = 0.72, size = 113, normalized size = 1.35

$$\frac{165\,b^8x^{12\,n}+1440\,ab^7x^{11\,n}+5544\,a^2b^6x^{10\,n}+12320\,a^3b^5x^{9\,n}+17325\,a^4b^4x^{8\,n}+15840\,a^5b^3x^{7\,n}+9240\,a^6b^2x^{6}}{1980\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

```
[Out] 1/1980*(165*b^8*x^(12*n) + 1440*a*b^7*x^(11*n) + 5544*a^2*b^6*x^(10*n) + 12
320*a^3*b^5*x^(9*n) + 17325*a^4*b^4*x^(8*n) + 15840*a^5*b^3*x^(7*n) + 9240*
a^6*b^2*x^(6*n) + 3168*a^7*b*x^(5*n) + 495*a^8*x^(4*n))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n+a)^8x^{4\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)*(a+b*x^n)^8,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^8*x^(4*n - 1), x)
```

maple [A] time = 0.03, size = 136, normalized size = 1.62

$$\frac{a^8x^{4n}}{4n}+\frac{8a^7b\,x^{5n}}{5n}+\frac{14a^6b^2x^{6n}}{3n}+\frac{8a^5b^3x^{7n}}{n}+\frac{35a^4b^4x^{8n}}{4n}+\frac{56a^3b^5x^{9n}}{9n}+\frac{14a^2b^6x^{10n}}{5n}+\frac{8a\,b^7x^{11n}}{11n}+\frac{b^8x^{12n}}{12n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+4*n)*(b*x^n+a)^8,x)
```

```
[Out] 1/12*b^8/n*(x^n)^12+8/11*a*b^7/n*(x^n)^11+14/5*a^2*b^6/n*(x^n)^10+56/9*a^3*
b^5/n*(x^n)^9+35/4*a^4*b^4/n*(x^n)^8+8*a^5*b^3/n*(x^n)^7+14/3*a^6*b^2/n*(x^
n)^6+8/5*a^7*b/n*(x^n)^5+1/4*a^8/n*(x^n)^4
```

maxima [A] time = 0.47, size = 135, normalized size = 1.61

$$\frac{b^8x^{12\,n}}{12\,n}+\frac{8\,ab^7x^{11\,n}}{11\,n}+\frac{14\,a^2b^6x^{10\,n}}{5\,n}+\frac{56\,a^3b^5x^{9\,n}}{9\,n}+\frac{35\,a^4b^4x^{8\,n}}{4\,n}+\frac{8\,a^5b^3x^{7\,n}}{n}+\frac{14\,a^6b^2x^{6\,n}}{3\,n}+\frac{8\,a^7bx^{5\,n}}{5\,n}+\frac{a^8x^{4\,n}}{4\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

```
[Out] 1/12*b^8*x^(12*n)/n + 8/11*a*b^7*x^(11*n)/n + 14/5*a^2*b^6*x^(10*n)/n + 56/
9*a^3*b^5*x^(9*n)/n + 35/4*a^4*b^4*x^(8*n)/n + 8*a^5*b^3*x^(7*n)/n + 14/3*a
^6*b^2*x^(6*n)/n + 8/5*a^7*b*x^(5*n)/n + 1/4*a^8*x^(4*n)/n
```

mupad [B] time = 1.46, size = 135, normalized size = 1.61

$$\frac{a^8\,x^{4\,n}}{4\,n}+\frac{b^8\,x^{12\,n}}{12\,n}+\frac{14\,a^6\,b^2\,x^{6\,n}}{3\,n}+\frac{8\,a^5\,b^3\,x^{7\,n}}{n}+\frac{35\,a^4\,b^4\,x^{8\,n}}{4\,n}+\frac{56\,a^3\,b^5\,x^{9\,n}}{9\,n}+\frac{14\,a^2\,b^6\,x^{10\,n}}{5\,n}+\frac{8\,a^7\,b\,x^{5\,n}}{5\,n}+\frac{8\,a\,b^7\,x^{11\,n}}{11\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4*n - 1)*(a + b*x^n)^8,x)
```

```
[Out] (a^8*x^(4*n))/(4*n) + (b^8*x^(12*n))/(12*n) + (14*a^6*b^2*x^(6*n))/(3*n) +
(8*a^5*b^3*x^(7*n))/n + (35*a^4*b^4*x^(8*n))/(4*n) + (56*a^3*b^5*x^(9*n))/(
9*n) + (14*a^2*b^6*x^(10*n))/(5*n) + (8*a^7*b*x^(5*n))/(5*n) + (8*a*b^7*x^(
11*n))/(11*n)
```

sympy [**F**(**-1**)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+4*n)*(a+b*x**n)**8,x)
[Out] Timed out
```

3.2572 $\int x^{-1+3n}\left(a+bx^n\right)^8\,dx$

Optimal. Leaf size=62

$$\frac{a^2\left(a+bx^n\right)^9}{9b^3n}+\frac{\left(a+bx^n\right)^{11}}{11b^3n}-\frac{a\left(a+bx^n\right)^{10}}{5b^3n}$$

```
[Out] 1/9*a^2*(a+b*x^n)^9/b^3/n-1/5*a*(a+b*x^n)^10/b^3/n+1/11*(a+b*x^n)^11/b^3/n
```

Rubi [A] time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$\frac{a^2\left(a+bx^n\right)^9}{9b^3n}+\frac{\left(a+bx^n\right)^{11}}{11b^3n}-\frac{a\left(a+bx^n\right)^{10}}{5b^3n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)*(a + b*x^n)^8,x]
```

```
[Out] (a^2*(a + b*x^n)^9)/(9*b^3*n) - (a*(a + b*x^n)^10)/(5*b^3*n) + (a + b*x^n)^
11/(11*b^3*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1+3n}\left(a+bx^n\right)^8\,dx &= \frac{\text{Subst}\left(\int x^2(a+bx)^8\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int\left(\frac{a^2(a+bx)^8}{b^2}-\frac{2a(a+bx)^9}{b^2}+\frac{(a+bx)^{10}}{b^2}\right)dx,x,x^n\right)}{n}\\ &= \frac{a^2\left(a+bx^n\right)^9}{9b^3n}-\frac{a\left(a+bx^n\right)^{10}}{5b^3n}+\frac{\left(a+bx^n\right)^{11}}{11b^3n}\end{aligned}$$

Mathematica [A] time = 0.04, size = 40, normalized size = 0.65

$$\frac{\left(a+bx^n\right)^9\left(a^2-9abx^n+45b^2x^{2n}\right)}{495b^3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)*(a + b*x^n)^8,x]
```

```
[Out] ((a + b*x^n)^9*(a^2 - 9*a*b*x^n + 45*b^2*x^(2*n)))/(495*b^3*n)
```

fricas [B] time = 0.84, size = 113, normalized size = 1.82

$$\frac{45\,b^8x^{11\,n}+396\,ab^7x^{10\,n}+1540\,a^2b^6x^{9\,n}+3465\,a^3b^5x^{8\,n}+4950\,a^4b^4x^{7\,n}+4620\,a^5b^3x^{6\,n}+2772\,a^6b^2x^{5\,n}+990\,a}{495\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

```
[Out] 1/495*(45*b^8*x^(11*n) + 396*a*b^7*x^(10*n) + 1540*a^2*b^6*x^(9*n) + 3465*a
^3*b^5*x^(8*n) + 4950*a^4*b^4*x^(7*n) + 4620*a^5*b^3*x^(6*n) + 2772*a^6*b^2
*x^(5*n) + 990*a^7*b*x^(4*n) + 165*a^8*x^(3*n))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n+a)^8x^{3\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^8,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^8*x^(3*n - 1), x)
```

maple [B] time = 0.03, size = 136, normalized size = 2.19

$$\frac{a^8x^{3n}}{3n}+\frac{2a^7b\,x^{4n}}{n}+\frac{28a^6b^2x^{5n}}{5n}+\frac{28a^5b^3x^{6n}}{3n}+\frac{10a^4b^4x^{7n}}{n}+\frac{7a^3b^5x^{8n}}{n}+\frac{28a^2b^6x^{9n}}{9n}+\frac{4a\,b^7x^{10n}}{5n}+\frac{b^8x^{11n}}{11n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n-1)*(b*x^n+a)^8,x)
```

```
[Out] 1/11*b^8/n*(x^n)^11+4/5*a*b^7/n*(x^n)^10+28/9*a^2*b^6/n*(x^n)^9+7*a^3*b^5/n
*(x^n)^8+10*a^4*b^4/n*(x^n)^7+28/3*a^5*b^3/n*(x^n)^6+28/5*a^6*b^2/n*(x^n)^5
+2*a^7*b/n*(x^n)^4+1/3*a^8/n*(x^n)^3
```

maxima [B] time = 0.46, size = 135, normalized size = 2.18

$$\frac{b^8x^{11\,n}}{11\,n}+\frac{4\,ab^7x^{10\,n}}{5\,n}+\frac{28\,a^2b^6x^{9\,n}}{9\,n}+\frac{7\,a^3b^5x^{8\,n}}{n}+\frac{10\,a^4b^4x^{7\,n}}{n}+\frac{28\,a^5b^3x^{6\,n}}{3\,n}+\frac{28\,a^6b^2x^{5\,n}}{5\,n}+\frac{2\,a^7bx^{4\,n}}{n}+\frac{a^8x^{3\,n}}{3\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

```
[Out] 1/11*b^8*x^(11*n)/n + 4/5*a*b^7*x^(10*n)/n + 28/9*a^2*b^6*x^(9*n)/n + 7*a^3
*b^5*x^(8*n)/n + 10*a^4*b^4*x^(7*n)/n + 28/3*a^5*b^3*x^(6*n)/n + 28/5*a^6*b
^2*x^(5*n)/n + 2*a^7*b*x^(4*n)/n + 1/3*a^8*x^(3*n)/n
```

mupad [B] time = 1.45, size = 135, normalized size = 2.18

$$\frac{a^8\,x^{3\,n}}{3\,n}+\frac{b^8\,x^{11\,n}}{11\,n}+\frac{28\,a^6\,b^2\,x^{5\,n}}{5\,n}+\frac{28\,a^5\,b^3\,x^{6\,n}}{3\,n}+\frac{10\,a^4\,b^4\,x^{7\,n}}{n}+\frac{7\,a^3\,b^5\,x^{8\,n}}{n}+\frac{28\,a^2\,b^6\,x^{9\,n}}{9\,n}+\frac{2\,a^7\,b\,x^{4\,n}}{n}+\frac{4\,a\,b^7\,x^{10\,n}}{5\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n - 1)*(a + b*x^n)^8,x)
```

```
[Out] (a^8*x^(3*n))/(3*n) + (b^8*x^(11*n))/(11*n) + (28*a^6*b^2*x^(5*n))/(5*n) +
(28*a^5*b^3*x^(6*n))/(3*n) + (10*a^4*b^4*x^(7*n))/n + (7*a^3*b^5*x^(8*n))/n
 + (28*a^2*b^6*x^(9*n))/(9*n) + (2*a^7*b*x^(4*n))/n + (4*a*b^7*x^(10*n))/(5
*n)
```

sympy [**F**(**-1**)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)*(a+b*x**n)**8,x)
[Out] Timed out
```

3.2573 $\int x^{-1+2n}\left(a+bx^n\right)^8\,dx$

Optimal. Leaf size=40

$$\frac{\left(a+bx^n\right)^{10}}{10b^2n}-\frac{a\left(a+bx^n\right)^9}{9b^2n}$$

```
[Out] -1/9*a*(a+b*x^n)^9/b^2/n+1/10*(a+b*x^n)^10/b^2/n
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$\frac{\left(a+bx^n\right)^{10}}{10b^2n}-\frac{a\left(a+bx^n\right)^9}{9b^2n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)*(a + b*x^n)^8,x]
```

```
[Out] -(a*(a + b*x^n)^9)/(9*b^2*n) + (a + b*x^n)^10/(10*b^2*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1+2n}\left(a+bx^n\right)^8\,dx &= \frac{\text{Subst}\left(\int x(a+bx)^8\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int\left(-\frac{a(a+bx)^8}{b}+\frac{(a+bx)^9}{b}\right)dx,x,x^n\right)}{n}\\ &= -\frac{a\left(a+bx^n\right)^9}{9b^2n}+\frac{\left(a+bx^n\right)^{10}}{10b^2n}\end{aligned}$$

Mathematica [A] time = 0.04, size = 27, normalized size = 0.68

$$-\frac{\left(a-9bx^n\right)\left(a+bx^n\right)^9}{90b^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)*(a + b*x^n)^8,x]
```

```
[Out] -1/90*((a - 9*b*x^n)*(a + b*x^n)^9)/(b^2*n)
```

fricas [B] time = 0.68, size = 113, normalized size = 2.82

$$\frac{9\,b^8x^{10\,n}+80\,ab^7x^{9\,n}+315\,a^2b^6x^{8\,n}+720\,a^3b^5x^{7\,n}+1050\,a^4b^4x^{6\,n}+1008\,a^5b^3x^{5\,n}+630\,a^6b^2x^{4\,n}+240\,a^7bx^{3\,n}+}{90\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

```
[Out] 1/90*(9*b^8*x^(10*n) + 80*a*b^7*x^(9*n) + 315*a^2*b^6*x^(8*n) + 720*a^3*b^5
*x^(7*n) + 1050*a^4*b^4*x^(6*n) + 1008*a^5*b^3*x^(5*n) + 630*a^6*b^2*x^(4*n
) + 240*a^7*b*x^(3*n) + 45*a^8*x^(2*n))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^8 x^{2n-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^8,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^8*x^(2*n - 1), x)
```

maple [B] time = 0.03, size = 136, normalized size = 3.40

$$\frac{a^8x^{2n}}{2n}+\frac{8a^7b\,x^{3n}}{3n}+\frac{7a^6b^2x^{4n}}{n}+\frac{56a^5b^3x^{5n}}{5n}+\frac{35a^4b^4x^{6n}}{3n}+\frac{8a^3b^5x^{7n}}{n}+\frac{7a^2b^6x^{8n}}{2n}+\frac{8a\,b^7x^{9n}}{9n}+\frac{b^8x^{10n}}{10n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n-1)*(b*x^n+a)^8,x)
```

```
[Out] 1/10*b^8/n*(x^n)^10+8/9*a*b^7/n*(x^n)^9+7/2*a^2*b^6/n*(x^n)^8+8*a^3*b^5/n*(
x^n)^7+35/3*a^4*b^4/n*(x^n)^6+56/5*a^5*b^3/n*(x^n)^5+7*a^6*b^2/n*(x^n)^4+8/
3*a^7*b/n*(x^n)^3+1/2*a^8/n*(x^n)^2
```

maxima [B] time = 0.50, size = 135, normalized size = 3.38

$$\frac{b^8x^{10\,n}}{10\,n}+\frac{8\,ab^7x^{9\,n}}{9\,n}+\frac{7\,a^2b^6x^{8\,n}}{2\,n}+\frac{8\,a^3b^5x^{7\,n}}{n}+\frac{35\,a^4b^4x^{6\,n}}{3\,n}+\frac{56\,a^5b^3x^{5\,n}}{5\,n}+\frac{7\,a^6b^2x^{4\,n}}{n}+\frac{8\,a^7bx^{3\,n}}{3\,n}+\frac{a^8x^{2\,n}}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

```
[Out] 1/10*b^8*x^(10*n)/n + 8/9*a*b^7*x^(9*n)/n + 7/2*a^2*b^6*x^(8*n)/n + 8*a^3*b
^5*x^(7*n)/n + 35/3*a^4*b^4*x^(6*n)/n + 56/5*a^5*b^3*x^(5*n)/n + 7*a^6*b^2*
x^(4*n)/n + 8/3*a^7*b*x^(3*n)/n + 1/2*a^8*x^(2*n)/n
```

mupad [B] time = 1.45, size = 135, normalized size = 3.38

$$\frac{a^8\,x^{2\,n}}{2\,n}+\frac{b^8\,x^{10\,n}}{10\,n}+\frac{7\,a^6\,b^2\,x^{4\,n}}{n}+\frac{56\,a^5\,b^3\,x^{5\,n}}{5\,n}+\frac{35\,a^4\,b^4\,x^{6\,n}}{3\,n}+\frac{8\,a^3\,b^5\,x^{7\,n}}{n}+\frac{7\,a^2\,b^6\,x^{8\,n}}{2\,n}+\frac{8\,a^7\,b\,x^{3\,n}}{3\,n}+\frac{8\,a\,b^7\,x^{9\,n}}{9\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)*(a + b*x^n)^8,x)
```

```
[Out] (a^8*x^(2*n))/(2*n) + (b^8*x^(10*n))/(10*n) + (7*a^6*b^2*x^(4*n))/n + (56*a
^5*b^3*x^(5*n))/(5*n) + (35*a^4*b^4*x^(6*n))/(3*n) + (8*a^3*b^5*x^(7*n))/n
+ (7*a^2*b^6*x^(8*n))/(2*n) + (8*a^7*b*x^(3*n))/(3*n) + (8*a*b^7*x^(9*n))/(
9*n)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a+b*x**n)**8,x)
```

```
[Out] Timed out
```

3.2574 $\int x^{-1+n}\left(a+bx^n\right)^8\,dx$

Optimal. Leaf size=19

$$\frac{\left(a+bx^n\right)^9}{9bn}$$

```
[Out] 1/9*(a+b*x^n)^9/b/n
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$\frac{\left(a+bx^n\right)^9}{9bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*(a + b*x^n)^8,x]
```

```
[Out] (a + b*x^n)^9/(9*b*n)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^{-1+n}\left(a+bx^n\right)^8\,dx = \frac{\left(a+bx^n\right)^9}{9bn}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{\left(a+bx^n\right)^9}{9bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*(a + b*x^n)^8,x]
```

```
[Out] (a + b*x^n)^9/(9*b*n)
```

fricas [B] time = 0.66, size = 110, normalized size = 5.79

$$\frac{b^8x^{9\,n} + 9\,ab^7x^{8\,n} + 36\,a^2b^6x^{7\,n} + 84\,a^3b^5x^{6\,n} + 126\,a^4b^4x^{5\,n} + 126\,a^5b^3x^{4\,n} + 84\,a^6b^2x^{3\,n} + 36\,a^7bx^{2\,n} + 9\,a^8x^n}{9\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(a+b*x^n)^8,x, algorithm="fricas")
```

```
[Out] 1/9*(b^8*x^(9*n) + 9*a*b^7*x^(8*n) + 36*a^2*b^6*x^(7*n) + 84*a^3*b^5*x^(6*n
) + 126*a^4*b^4*x^(5*n) + 126*a^5*b^3*x^(4*n) + 84*a^6*b^2*x^(3*n) + 36*a^7
*b*x^(2*n) + 9*a^8*x^n)/n
```

giac [B] time = 0.17, size = 110, normalized size = 5.79

$$\frac{b^8x^{9\,n} + 9\,ab^7x^{8\,n} + 36\,a^2b^6x^{7\,n} + 84\,a^3b^5x^{6\,n} + 126\,a^4b^4x^{5\,n} + 126\,a^5b^3x^{4\,n} + 84\,a^6b^2x^{3\,n} + 36\,a^7bx^{2\,n} + 9\,a^8x^n}{9\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(a+b*x^n)^8,x, algorithm="giac")
```

```
[Out] 1/9*(b^8*x^(9*n) + 9*a*b^7*x^(8*n) + 36*a^2*b^6*x^(7*n) + 84*a^3*b^5*x^(6*n
) + 126*a^4*b^4*x^(5*n) + 126*a^5*b^3*x^(4*n) + 84*a^6*b^2*x^(3*n) + 36*a^7
*b*x^(2*n) + 9*a^8*x^n)/n
```

maple [B] time = 0.03, size = 132, normalized size = 6.95

$$\frac{a^8x^n}{n}+\frac{4a^7b\,x^{2n}}{n}+\frac{28a^6b^2x^{3n}}{3n}+\frac{14a^5b^3x^{4n}}{n}+\frac{14a^4b^4x^{5n}}{n}+\frac{28a^3b^5x^{6n}}{3n}+\frac{4a^2b^6x^{7n}}{n}+\frac{a\,b^7x^{8n}}{n}+\frac{b^8x^{9n}}{9n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*(b*x^n+a)^8,x)
```

```
[Out] 1/9*b^8/n*(x^n)^9+a*b^7/n*(x^n)^8+4*a^2*b^6/n*(x^n)^7+28/3*a^3*b^5/n*(x^n)^
6+14*a^4*b^4/n*(x^n)^5+14*a^5*b^3/n*(x^n)^4+28/3*a^6*b^2/n*(x^n)^3+4*a^7*b/
n*(x^n)^2+a^8/n*x^n
```

maxima [A] time = 0.46, size = 17, normalized size = 0.89

$$\frac{(bx^n+a)^9}{9\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(a+b*x^n)^8,x, algorithm="maxima")
```

```
[Out] 1/9*(b*x^n + a)^9/(b*n)
```

mupad [B] time = 1.46, size = 131, normalized size = 6.89

$$\frac{a^8\,x^n}{n}+\frac{b^8\,x^{9\,n}}{9\,n}+\frac{28\,a^6\,b^2\,x^{3\,n}}{3\,n}+\frac{14\,a^5\,b^3\,x^{4\,n}}{n}+\frac{14\,a^4\,b^4\,x^{5\,n}}{n}+\frac{28\,a^3\,b^5\,x^{6\,n}}{3\,n}+\frac{4\,a^2\,b^6\,x^{7\,n}}{n}+\frac{4\,a^7\,b\,x^{2\,n}}{n}+\frac{a\,b^7\,x^{8\,n}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*(a + b*x^n)^8,x)
```

```
[Out] (a^8*x^n)/n + (b^8*x^(9*n))/(9*n) + (28*a^6*b^2*x^(3*n))/(3*n) + (14*a^5*b^
3*x^(4*n))/n + (14*a^4*b^4*x^(5*n))/n + (28*a^3*b^5*x^(6*n))/(3*n) + (4*a^2
*b^6*x^(7*n))/n + (4*a^7*b*x^(2*n))/n + (a*b^7*x^(8*n))/n
```

sympy [A] time = 64.51, size = 133, normalized size = 7.00

$$\begin{cases} \frac{a^8x^n}{n} + \frac{4a^7bx^{2n}}{n} + \frac{28a^6b^2x^{3n}}{3n} + \frac{14a^5b^3x^{4n}}{n} + \frac{14a^4b^4x^{5n}}{n} + \frac{28a^3b^5x^{6n}}{3n} + \frac{4a^2b^6x^{7n}}{n} + \frac{ab^7x^{8n}}{n} + \frac{b^8x^{9n}}{9n} & \text{for } n \neq 0 \\ (a+b)^8\log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(a+b*x**n)**8,x)
```

```
[Out] Piecewise((a**8*x**n/n + 4*a**7*b*x**(2*n)/n + 28*a**6*b**2*x**(3*n)/(3*n)
+ 14*a**5*b**3*x**(4*n)/n + 14*a**4*b**4*x**(5*n)/n + 28*a**3*b**5*x**(6*n)
/(3*n) + 4*a**2*b**6*x**(7*n)/n + a*b**7*x**(8*n)/n + b**8*x**(9*n)/(9*n),
Ne(n, 0)), ((a + b)**8*log(x), True))
```

3.2575 $\int \frac{(a+bx^n)^8}{x}\,dx$

Optimal. Leaf size=138

$$a^8\log(x)+\frac{8a^7bx^n}{n}+\frac{14a^6b^2x^{2n}}{n}+\frac{56a^5b^3x^{3n}}{3n}+\frac{35a^4b^4x^{4n}}{2n}+\frac{56a^3b^5x^{5n}}{5n}+\frac{14a^2b^6x^{6n}}{3n}+\frac{8ab^7x^{7n}}{7n}+\frac{b^8x^{8n}}{8n}$$

```
[Out] 8*a^7*b*x^n/n+14*a^6*b^2*x^(2*n)/n+56/3*a^5*b^3*x^(3*n)/n+35/2*a^4*b^4*x^(4
*n)/n+56/5*a^3*b^5*x^(5*n)/n+14/3*a^2*b^6*x^(6*n)/n+8/7*a*b^7*x^(7*n)/n+1/8
*b^8*x^(8*n)/n+a^8*ln(x)
```

Rubi [A] time = 0.05, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 43}

$$\frac{14a^6b^2x^{2n}}{n}+\frac{56a^5b^3x^{3n}}{3n}+\frac{35a^4b^4x^{4n}}{2n}+\frac{56a^3b^5x^{5n}}{5n}+\frac{14a^2b^6x^{6n}}{3n}+\frac{8a^7bx^n}{n}+a^8\log(x)+\frac{8ab^7x^{7n}}{7n}+\frac{b^8x^{8n}}{8n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^8/x,x]
```

```
[Out] (8*a^7*b*x^n)/n + (14*a^6*b^2*x^(2*n))/n + (56*a^5*b^3*x^(3*n))/(3*n) + (35
*a^4*b^4*x^(4*n))/(2*n) + (56*a^3*b^5*x^(5*n))/(5*n) + (14*a^2*b^6*x^(6*n))
/(3*n) + (8*a*b^7*x^(7*n))/(7*n) + (b^8*x^(8*n))/(8*n) + a^8*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^n)^8}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^8}{x}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(8a^7b+\frac{a^8}{x}+28a^6b^2x+56a^5b^3x^2+70a^4b^4x^3+56a^3b^5x^4+28a^2b^6x^5+8ab^7x^6+b^8\right.\right.}{n}\\ &= \frac{8a^7bx^n}{n}+\frac{14a^6b^2x^{2n}}{n}+\frac{56a^5b^3x^{3n}}{3n}+\frac{35a^4b^4x^{4n}}{2n}+\frac{56a^3b^5x^{5n}}{5n}+\frac{14a^2b^6x^{6n}}{3n}+\frac{8ab^7x^{7n}}{7n}+\frac{b^8x^{8n}}{8n}\end{aligned}$$

Mathematica [A] time = 0.08, size = 119, normalized size = 0.86

$$\frac{a^8n\log(x)+8a^7bx^n+14a^6b^2x^{2n}+\frac{56}{3}a^5b^3x^{3n}+\frac{35}{2}a^4b^4x^{4n}+\frac{56}{5}a^3b^5x^{5n}+\frac{14}{3}a^2b^6x^{6n}+\frac{8}{7}ab^7x^{7n}+\frac{1}{8}b^8x^{8n}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^8/x,x]
```

```
[Out] (8*a^7*b*x^n + 14*a^6*b^2*x^(2*n) + (56*a^5*b^3*x^(3*n))/3 + (35*a^4*b^4*x^
(4*n))/2 + (56*a^3*b^5*x^(5*n))/5 + (14*a^2*b^6*x^(6*n))/3 + (8*a*b^7*x^(7*
n))/7 + (b^8*x^(8*n))/8 + a^8*n*Log[x])/n
```

fricas [A] time = 0.64, size = 109, normalized size = 0.79

$$\frac{840\,a^8 n\log(x) + 105\,b^8 x^{8\,n} + 960\,ab^7 x^{7\,n} + 3920\,a^2 b^6 x^{6\,n} + 9408\,a^3 b^5 x^{5\,n} + 14700\,a^4 b^4 x^{4\,n} + 15680\,a^5 b^3 x^{3\,n} +}{840\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^8/x,x, algorithm="fricas")
```

```
[Out] 1/840*(840*a^8*n*log(x) + 105*b^8*x^(8*n) + 960*a*b^7*x^(7*n) + 3920*a^2*b^
6*x^(6*n) + 9408*a^3*b^5*x^(5*n) + 14700*a^4*b^4*x^(4*n) + 15680*a^5*b^3*x^
(3*n) + 11760*a^6*b^2*x^(2*n) + 6720*a^7*b*x^n)/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^8}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^8/x,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^8/x, x)
```

maple [A] time = 0.00, size = 132, normalized size = 0.96

$$\frac{a^8\ln(x^n)}{n}+\frac{8a^7 b\,x^n}{n}+\frac{14a^6b^2x^{2n}}{n}+\frac{56a^5b^3x^{3n}}{3n}+\frac{35a^4b^4x^{4n}}{2n}+\frac{56a^3b^5x^{5n}}{5n}+\frac{14a^2b^6x^{6n}}{3n}+\frac{8a\,b^7x^{7n}}{7n}+\frac{b^8x^{8n}}{8n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^8/x,x)
```

```
[Out] 1/8/n*b^8*(x^n)^8+8/7/n*a*b^7*(x^n)^7+14/3/n*a^2*b^6*(x^n)^6+56/5/n*(x^n)^5
*b^5*a^3+35/2/n*a^4*b^4*(x^n)^4+56/3/n*b^3*(x^n)^3*a^5+14/n*a^6*(x^n)^2*b^2
+8*a^7*b*x^n/n+1/n*a^8*ln(x^n)
```

maxima [A] time = 0.54, size = 113, normalized size = 0.82

$$\frac{a^8\log(x^n)}{n}+\frac{105\,b^8x^{8\,n} + 960\,ab^7x^{7\,n} + 3920\,a^2b^6x^{6\,n} + 9408\,a^3b^5x^{5\,n} + 14700\,a^4b^4x^{4\,n} + 15680\,a^5b^3x^{3\,n} + 117}{840\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^8/x,x, algorithm="maxima")
```

```
[Out] a^8*log(x^n)/n + 1/840*(105*b^8*x^(8*n) + 960*a*b^7*x^(7*n) + 3920*a^2*b^6*
x^(6*n) + 9408*a^3*b^5*x^(5*n) + 14700*a^4*b^4*x^(4*n) + 15680*a^5*b^3*x^(3
*n) + 11760*a^6*b^2*x^(2*n) + 6720*a^7*b*x^n)/n
```

mupad [B] time = 1.29, size = 126, normalized size = 0.91

$$a^8\,\ln(x)+\frac{b^8\,x^{8\,n}}{8\,n}+\frac{14\,a^6\,b^2\,x^{2\,n}}{n}+\frac{56\,a^5\,b^3\,x^{3\,n}}{3\,n}+\frac{35\,a^4\,b^4\,x^{4\,n}}{2\,n}+\frac{56\,a^3\,b^5\,x^{5\,n}}{5\,n}+\frac{14\,a^2\,b^6\,x^{6\,n}}{3\,n}+\frac{8\,a^7\,b\,x^n}{n}+\frac{8\,a\,b^7\,x^{7\,n}}{7\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^8/x,x)
```

```
[Out] a^8*log(x) + (b^8*x^(8*n))/(8*n) + (14*a^6*b^2*x^(2*n))/n + (56*a^5*b^3*x^(
3*n))/(3*n) + (35*a^4*b^4*x^(4*n))/(2*n) + (56*a^3*b^5*x^(5*n))/(5*n) + (14
*a^2*b^6*x^(6*n))/(3*n) + (8*a^7*b*x^n)/n + (8*a*b^7*x^(7*n))/(7*n)
```

sympy [**A**] time = 4.05, size = 136, normalized size = 0.99

$$\begin{cases} a^8 \log(x) + \frac{8a^7 b x^n}{n} + \frac{14a^6 b^2 x^{2n}}{n} + \frac{56a^5 b^3 x^{3n}}{3n} + \frac{35a^4 b^4 x^{4n}}{2n} + \frac{56a^3 b^5 x^{5n}}{5n} + \frac{14a^2 b^6 x^{6n}}{3n} + \frac{8ab^7 x^{7n}}{7n} + \frac{b^8 x^{8n}}{8n} & \text{for } n \neq 0 \\ (a+b)^8 \log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**8/x,x)
```

```
[Out] Piecewise((a**8*log(x) + 8*a**7*b*x**n/n + 14*a**6*b**2*x**(2*n)/n + 56*a**
5*b**3*x**(3*n)/(3*n) + 35*a**4*b**4*x**(4*n)/(2*n) + 56*a**3*b**5*x**(5*n)
/(5*n) + 14*a**2*b**6*x**(6*n)/(3*n) + 8*a*b**7*x**(7*n)/(7*n) + b**8*x**(8
*n)/(8*n), Ne(n, 0)), ((a + b)**8*log(x), True))
```

3.2576 $\int x^{-1-n}\left(a+bx^n\right)^8\,dx$

Optimal. Leaf size=135

$$-\frac{a^8x^{-n}}{n}+8a^7b\log(x)+\frac{28a^6b^2x^n}{n}+\frac{28a^5b^3x^{2n}}{n}+\frac{70a^4b^4x^{3n}}{3n}+\frac{14a^3b^5x^{4n}}{n}+\frac{28a^2b^6x^{5n}}{5n}+\frac{4ab^7x^{6n}}{3n}+\frac{b^8x^{7n}}{7n}$$

```
[Out] -a^8/n/(x^n)+28*a^6*b^2*x^n/n+28*a^5*b^3*x^(2*n)/n+70/3*a^4*b^4*x^(3*n)/n+1
4*a^3*b^5*x^(4*n)/n+28/5*a^2*b^6*x^(5*n)/n+4/3*a*b^7*x^(6*n)/n+1/7*b^8*x^(7
*n)/n+8*a^7*b*ln(x)
```

Rubi [A] time = 0.06, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$\frac{28a^6b^2x^n}{n}+\frac{28a^5b^3x^{2n}}{n}+\frac{70a^4b^4x^{3n}}{3n}+\frac{14a^3b^5x^{4n}}{n}+\frac{28a^2b^6x^{5n}}{5n}+8a^7b\log(x)-\frac{a^8x^{-n}}{n}+\frac{4ab^7x^{6n}}{3n}+\frac{b^8x^{7n}}{7n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n)*(a + b*x^n)^8,x]
```

```
[Out] -(a^8/(n*x^n)) + (28*a^6*b^2*x^n)/n + (28*a^5*b^3*x^(2*n))/n + (70*a^4*b^4*
x^(3*n))/(3*n) + (14*a^3*b^5*x^(4*n))/n + (28*a^2*b^6*x^(5*n))/(5*n) + (4*a
*b^7*x^(6*n))/(3*n) + (b^8*x^(7*n))/(7*n) + 8*a^7*b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1-n}\left(a+bx^n\right)^8\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^8}{x^2}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(28a^6b^2+\frac{a^8}{x^2}+\frac{8a^7b}{x}+56a^5b^3x+70a^4b^4x^2+56a^3b^5x^3+28a^2b^6x^4+8ab^7x\right.\right.}{n}\\ &= -\frac{a^8x^{-n}}{n}+\frac{28a^6b^2x^n}{n}+\frac{28a^5b^3x^{2n}}{n}+\frac{70a^4b^4x^{3n}}{3n}+\frac{14a^3b^5x^{4n}}{n}+\frac{28a^2b^6x^{5n}}{5n}+\frac{4ab^7x^{6n}}{3n}\end{aligned}$$

Mathematica [A] time = 0.10, size = 116, normalized size = 0.86

$$\frac{-a^8x^{-n}+8a^7bn\log(x)+28a^6b^2x^n+28a^5b^3x^{2n}+\frac{70}{3}a^4b^4x^{3n}+14a^3b^5x^{4n}+\frac{28}{5}a^2b^6x^{5n}+\frac{4}{3}ab^7x^{6n}+\frac{1}{7}b^8x^{7n}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n)*(a + b*x^n)^8,x]
```

```
[Out] (-(a^8/x^n) + 28*a^6*b^2*x^n + 28*a^5*b^3*x^(2*n) + (70*a^4*b^4*x^(3*n))/3
+ 14*a^3*b^5*x^(4*n) + (28*a^2*b^6*x^(5*n))/5 + (4*a*b^7*x^(6*n))/3 + (b^8*
x^(7*n))/7 + 8*a^7*b*n*Log[x])/n
```

fricas [A] time = 0.61, size = 114, normalized size = 0.84

$$\frac{840\,a^7bnx^n\log(x)+15\,b^8x^{8\,n}+140\,ab^7x^{7\,n}+588\,a^2b^6x^{6\,n}+1470\,a^3b^5x^{5\,n}+2450\,a^4b^4x^{4\,n}+2940\,a^5b^3x^{3\,n}+294}{105\,nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*(a+b*x^n)^8,x, algorithm="fricas")
```

```
[Out] 1/105*(840*a^7*b*n*x^n*log(x) + 15*b^8*x^(8*n) + 140*a*b^7*x^(7*n) + 588*a^
2*b^6*x^(6*n) + 1470*a^3*b^5*x^(5*n) + 2450*a^4*b^4*x^(4*n) + 2940*a^5*b^3*
x^(3*n) + 2940*a^6*b^2*x^(2*n) - 105*a^8)/(n*x^n)
```

giac [A] time = 0.28, size = 114, normalized size = 0.84

$$\frac{840\,a^7bnx^n\log(x)+15\,b^8x^{8\,n}+140\,ab^7x^{7\,n}+588\,a^2b^6x^{6\,n}+1470\,a^3b^5x^{5\,n}+2450\,a^4b^4x^{4\,n}+2940\,a^5b^3x^{3\,n}+294}{105\,nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*(a+b*x^n)^8,x, algorithm="giac")
```

```
[Out] 1/105*(840*a^7*b*n*x^n*log(x) + 15*b^8*x^(8*n) + 140*a*b^7*x^(7*n) + 588*a^
2*b^6*x^(6*n) + 1470*a^3*b^5*x^(5*n) + 2450*a^4*b^4*x^(4*n) + 2940*a^5*b^3*
x^(3*n) + 2940*a^6*b^2*x^(2*n) - 105*a^8)/(n*x^n)
```

maple [A] time = 0.03, size = 128, normalized size = 0.95

$$8a^7b\ln(x)-\frac{a^8x^{-n}}{n}+\frac{28a^6b^2x^n}{n}+\frac{28a^5b^3x^{2n}}{n}+\frac{70a^4b^4x^{3n}}{3n}+\frac{14a^3b^5x^{4n}}{n}+\frac{28a^2b^6x^{5n}}{5n}+\frac{4a\,b^7x^{6n}}{3n}+\frac{b^8x^{7n}}{7n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n)*(b*x^n+a)^8,x)
```

```
[Out] 8*a^7*b*ln(x)+1/7*b^8/n*(x^n)^7+4/3*a*b^7/n*(x^n)^6+28/5*a^2*b^6/n*(x^n)^5+
14*a^3*b^5/n*(x^n)^4+70/3*a^4*b^4/n*(x^n)^3+28*a^5*b^3/n*(x^n)^2+28*a^6*b^2
*x^n/n-a^8/n/(x^n)
```

maxima [A] time = 0.50, size = 127, normalized size = 0.94

$$8\,a^7b\log(x)+\frac{b^8x^{7\,n}}{7\,n}+\frac{4\,ab^7x^{6\,n}}{3\,n}+\frac{28\,a^2b^6x^{5\,n}}{5\,n}+\frac{14\,a^3b^5x^{4\,n}}{n}+\frac{70\,a^4b^4x^{3\,n}}{3\,n}+\frac{28\,a^5b^3x^{2\,n}}{n}+\frac{28\,a^6b^2x^n}{n}-\frac{a^8}{nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*(a+b*x^n)^8,x, algorithm="maxima")
```

```
[Out] 8*a^7*b*log(x) + 1/7*b^8*x^(7*n)/n + 4/3*a*b^7*x^(6*n)/n + 28/5*a^2*b^6*x^(
5*n)/n + 14*a^3*b^5*x^(4*n)/n + 70/3*a^4*b^4*x^(3*n)/n + 28*a^5*b^3*x^(2*n)
/n + 28*a^6*b^2*x^n/n - a^8/(n*x^n)
```

mupad [B] time = 1.72, size = 127, normalized size = 0.94

$$8\,a^7\,b\,\ln(x)-\frac{a^8}{n\,x^n}+\frac{b^8\,x^{7\,n}}{7\,n}+\frac{28\,a^5\,b^3\,x^{2\,n}}{n}+\frac{70\,a^4\,b^4\,x^{3\,n}}{3\,n}+\frac{14\,a^3\,b^5\,x^{4\,n}}{n}+\frac{28\,a^2\,b^6\,x^{5\,n}}{5\,n}+\frac{4\,a\,b^7\,x^{6\,n}}{3\,n}+\frac{28\,a^6\,b^2\,x^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^8/x^(n + 1),x)
```

```
[Out] 8*a^7*b*log(x) - a^8/(n*x^n) + (b^8*x^(7*n))/(7*n) + (28*a^5*b^3*x^(2*n))/n
 + (70*a^4*b^4*x^(3*n))/(3*n) + (14*a^3*b^5*x^(4*n))/n + (28*a^2*b^6*x^(5*n
))/(5*n) + (4*a*b^7*x^(6*n))/(3*n) + (28*a^6*b^2*x^n)/n
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n)*(a+b*x**n)**8,x)
```

```
[Out] Timed out
```

3.2577 $\int x^{-1-2n}\left(a+bx^n\right)^8\,dx$

Optimal. Leaf size=135

$$-\frac{a^8x^{-2n}}{2n}-\frac{8a^7bx^{-n}}{n}+28a^6b^2\log(x)+\frac{56a^5b^3x^n}{n}+\frac{35a^4b^4x^{2n}}{n}+\frac{56a^3b^5x^{3n}}{3n}+\frac{7a^2b^6x^{4n}}{n}+\frac{8ab^7x^{5n}}{5n}+\frac{b^8x^{6n}}{6n}$$

```
[Out] -1/2*a^8/n/(x^(2*n))-8*a^7*b/n/(x^n)+56*a^5*b^3*x^n/n+35*a^4*b^4*x^(2*n)/n+
56/3*a^3*b^5*x^(3*n)/n+7*a^2*b^6*x^(4*n)/n+8/5*a*b^7*x^(5*n)/n+1/6*b^8*x^(6
*n)/n+28*a^6*b^2*ln(x)
```

Rubi [A] time = 0.06, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$\frac{56a^5b^3x^n}{n}+\frac{35a^4b^4x^{2n}}{n}+\frac{56a^3b^5x^{3n}}{3n}+\frac{7a^2b^6x^{4n}}{n}+28a^6b^2\log(x)-\frac{8a^7bx^{-n}}{n}-\frac{a^8x^{-2n}}{2n}+\frac{8ab^7x^{5n}}{5n}+\frac{b^8x^{6n}}{6n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 2*n)*(a + b*x^n)^8,x]
```

```
[Out] -a^8/(2*n*x^(2*n)) - (8*a^7*b)/(n*x^n) + (56*a^5*b^3*x^n)/n + (35*a^4*b^4*x
^(2*n))/n + (56*a^3*b^5*x^(3*n))/(3*n) + (7*a^2*b^6*x^(4*n))/n + (8*a*b^7*x
^(5*n))/(5*n) + (b^8*x^(6*n))/(6*n) + 28*a^6*b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1-2n}\left(a+bx^n\right)^8\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^8}{x^3}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(56a^5b^3+\frac{a^8}{x^3}+\frac{8a^7b}{x^2}+\frac{28a^6b^2}{x}+70a^4b^4x+56a^3b^5x^2+28a^2b^6x^3+8ab^7x^4+b^8\right.\right.}{n}\\ &= -\frac{a^8x^{-2n}}{2n}-\frac{8a^7bx^{-n}}{n}+\frac{56a^5b^3x^n}{n}+\frac{35a^4b^4x^{2n}}{n}+\frac{56a^3b^5x^{3n}}{3n}+\frac{7a^2b^6x^{4n}}{n}+\frac{8ab^7x^{5n}}{5n}+\frac{b}{}\end{aligned}$$

Mathematica [A] time = 0.11, size = 116, normalized size = 0.86

$$\frac{-\frac{1}{2}a^8x^{-2n}-8a^7bx^{-n}+28a^6b^2n\log(x)+56a^5b^3x^n+35a^4b^4x^{2n}+\frac{56}{3}a^3b^5x^{3n}+7a^2b^6x^{4n}+\frac{8}{5}ab^7x^{5n}+\frac{1}{6}b^8x^{6n}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 2*n)*(a + b*x^n)^8,x]
```

```
[Out] (-1/2*a^8/x^(2*n) - (8*a^7*b)/x^n + 56*a^5*b^3*x^n + 35*a^4*b^4*x^(2*n) + (
56*a^3*b^5*x^(3*n))/3 + 7*a^2*b^6*x^(4*n) + (8*a*b^7*x^(5*n))/5 + (b^8*x^(6
*n))/6 + 28*a^6*b^2*n*Log[x])/n
```

fricas [A] time = 0.87, size = 116, normalized size = 0.86

$$\frac{840\,a^6b^2nx^{2\,n}\log(x)+5\,b^8x^{8\,n}+48\,ab^7x^{7\,n}+210\,a^2b^6x^{6\,n}+560\,a^3b^5x^{5\,n}+1050\,a^4b^4x^{4\,n}+1680\,a^5b^3x^{3\,n}-24}{30\,nx^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

```
[Out] 1/30*(840*a^6*b^2*n*x^(2*n)*log(x) + 5*b^8*x^(8*n) + 48*a*b^7*x^(7*n) + 210
*a^2*b^6*x^(6*n) + 560*a^3*b^5*x^(5*n) + 1050*a^4*b^4*x^(4*n) + 1680*a^5*b^
3*x^(3*n) - 240*a^7*b*x^n - 15*a^8)/(n*x^(2*n))
```

giac [A] time = 0.29, size = 116, normalized size = 0.86

$$\frac{840\,a^6b^2nx^{2\,n}\log(x)+5\,b^8x^{8\,n}+48\,ab^7x^{7\,n}+210\,a^2b^6x^{6\,n}+560\,a^3b^5x^{5\,n}+1050\,a^4b^4x^{4\,n}+1680\,a^5b^3x^{3\,n}-24}{30\,nx^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*(a+b*x^n)^8,x, algorithm="giac")
```

```
[Out] 1/30*(840*a^6*b^2*n*x^(2*n)*log(x) + 5*b^8*x^(8*n) + 48*a*b^7*x^(7*n) + 210
*a^2*b^6*x^(6*n) + 560*a^3*b^5*x^(5*n) + 1050*a^4*b^4*x^(4*n) + 1680*a^5*b^
3*x^(3*n) - 240*a^7*b*x^n - 15*a^8)/(n*x^(2*n))
```

maple [A] time = 0.03, size = 128, normalized size = 0.95

$$28a^6b^2\ln(x)-\frac{a^8x^{-2n}}{2n}-\frac{8a^7b\,x^{-n}}{n}+\frac{56a^5b^3x^n}{n}+\frac{35a^4b^4x^{2n}}{n}+\frac{56a^3b^5x^{3n}}{3n}+\frac{7a^2b^6x^{4n}}{n}+\frac{8a\,b^7x^{5n}}{5n}+\frac{b^8x^{6n}}{6n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-2*n)*(b*x^n+a)^8,x)
```

```
[Out] 28*a^6*b^2*ln(x)+1/6*b^8/n*(x^n)^6+8/5*a*b^7/n*(x^n)^5+7*a^2*b^6/n*(x^n)^4+
56/3*a^3*b^5/n*(x^n)^3+35*a^4*b^4/n*(x^n)^2+56*a^5*b^3*x^n/n-8*a^7*b/n/(x^n
)-1/2*a^8/n/(x^n)^2
```

maxima [A] time = 0.52, size = 129, normalized size = 0.96

$$28\,a^6b^2\log(x)+\frac{b^8x^{6\,n}}{6\,n}+\frac{8\,ab^7x^{5\,n}}{5\,n}+\frac{7\,a^2b^6x^{4\,n}}{n}+\frac{56\,a^3b^5x^{3\,n}}{3\,n}+\frac{35\,a^4b^4x^{2\,n}}{n}+\frac{56\,a^5b^3x^n}{n}-\frac{a^8}{2\,nx^{2\,n}}-\frac{8\,a^7b}{nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

```
[Out] 28*a^6*b^2*log(x) + 1/6*b^8*x^(6*n)/n + 8/5*a*b^7*x^(5*n)/n + 7*a^2*b^6*x^(
4*n)/n + 56/3*a^3*b^5*x^(3*n)/n + 35*a^4*b^4*x^(2*n)/n + 56*a^5*b^3*x^n/n -
 1/2*a^8/(n*x^(2*n)) - 8*a^7*b/(n*x^n)
```

mupad [B] time = 1.60, size = 129, normalized size = 0.96

$$\frac{b^8\,x^{6\,n}}{6\,n}-\frac{a^8}{2\,n\,x^{2\,n}}+28\,a^6\,b^2\,\ln(x)+\frac{35\,a^4\,b^4\,x^{2\,n}}{n}+\frac{56\,a^3\,b^5\,x^{3\,n}}{3\,n}+\frac{7\,a^2\,b^6\,x^{4\,n}}{n}-\frac{8\,a^7\,b}{n\,x^n}+\frac{8\,a\,b^7\,x^{5\,n}}{5\,n}+\frac{56\,a^5\,b^3\,x^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^8/x^(2*n + 1),x)
```

```
[Out] (b^8*x^(6*n))/(6*n) - a^8/(2*n*x^(2*n)) + 28*a^6*b^2*log(x) + (35*a^4*b^4*x
^(2*n))/n + (56*a^3*b^5*x^(3*n))/(3*n) + (7*a^2*b^6*x^(4*n))/n - (8*a^7*b)/
(n*x^n) + (8*a*b^7*x^(5*n))/(5*n) + (56*a^5*b^3*x^n)/n
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-2*n)*(a+b*x**n)**8,x)
```

```
[Out] Timed out
```

3.2578 $\int x^{-1-3n}\left(a+bx^n\right)^8\,dx$

Optimal. Leaf size=133

$$-\frac{a^8x^{-3n}}{3n}-\frac{4a^7bx^{-2n}}{n}-\frac{28a^6b^2x^{-n}}{n}+56a^5b^3\log(x)+\frac{70a^4b^4x^n}{n}+\frac{28a^3b^5x^{2n}}{n}+\frac{28a^2b^6x^{3n}}{3n}+\frac{2ab^7x^{4n}}{n}+\frac{b^8x^{5n}}{5n}$$

```
[Out] -1/3*a^8/n/(x^(3*n))-4*a^7*b/n/(x^(2*n))-28*a^6*b^2/n/(x^n)+70*a^4*b^4*x^n/
n+28*a^3*b^5*x^(2*n)/n+28/3*a^2*b^6*x^(3*n)/n+2*a*b^7*x^(4*n)/n+1/5*b^8*x^(
5*n)/n+56*a^5*b^3*ln(x)
```

Rubi [A] time = 0.06, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$-\frac{28a^6b^2x^{-n}}{n}+\frac{70a^4b^4x^n}{n}+\frac{28a^3b^5x^{2n}}{n}+\frac{28a^2b^6x^{3n}}{3n}+56a^5b^3\log(x)-\frac{4a^7bx^{-2n}}{n}-\frac{a^8x^{-3n}}{3n}+\frac{2ab^7x^{4n}}{n}+\frac{b^8x^{5n}}{5n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 3*n)*(a + b*x^n)^8,x]
```

```
[Out] -a^8/(3*n*x^(3*n)) - (4*a^7*b)/(n*x^(2*n)) - (28*a^6*b^2)/(n*x^n) + (70*a^4
*b^4*x^n)/n + (28*a^3*b^5*x^(2*n))/n + (28*a^2*b^6*x^(3*n))/(3*n) + (2*a*b^
7*x^(4*n))/n + (b^8*x^(5*n))/(5*n) + 56*a^5*b^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1-3n}\left(a+bx^n\right)^8\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^8}{x^4}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(70a^4b^4+\frac{a^8}{x^4}+\frac{8a^7b}{x^3}+\frac{28a^6b^2}{x^2}+\frac{56a^5b^3}{x}+56a^3b^5x+28a^2b^6x^2+8ab^7x^3+b^8\right.\right.}{n}\\ &= -\frac{a^8x^{-3n}}{3n}-\frac{4a^7bx^{-2n}}{n}-\frac{28a^6b^2x^{-n}}{n}+\frac{70a^4b^4x^n}{n}+\frac{28a^3b^5x^{2n}}{n}+\frac{28a^2b^6x^{3n}}{3n}+\frac{2ab^7x^{4n}}{n}\end{aligned}$$

Mathematica [A] time = 0.08, size = 114, normalized size = 0.86

$$\frac{-\frac{1}{3}a^8x^{-3n}-4a^7bx^{-2n}-28a^6b^2x^{-n}+56a^5b^3n\log(x)+70a^4b^4x^n+28a^3b^5x^{2n}+\frac{28}{3}a^2b^6x^{3n}+2ab^7x^{4n}+\frac{1}{5}b^8x^{5n}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 3*n)*(a + b*x^n)^8,x]
```

```
[Out] (-1/3*a^8/x^(3*n) - (4*a^7*b)/x^(2*n) - (28*a^6*b^2)/x^n + 70*a^4*b^4*x^n +
 28*a^3*b^5*x^(2*n) + (28*a^2*b^6*x^(3*n))/3 + 2*a*b^7*x^(4*n) + (b^8*x^(5*
n))/5 + 56*a^5*b^3*n*Log[x])/n
```

fricas [A] time = 0.49, size = 116, normalized size = 0.87

$$\frac{840\,a^5b^3nx^{3\,n}\log(x)+3\,b^8x^{8\,n}+30\,ab^7x^{7\,n}+140\,a^2b^6x^{6\,n}+420\,a^3b^5x^{5\,n}+1050\,a^4b^4x^{4\,n}-420\,a^6b^2x^{2\,n}-60\,a^7b}{15\,nx^{3\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

```
[Out] 1/15*(840*a^5*b^3*n*x^(3*n)*log(x) + 3*b^8*x^(8*n) + 30*a*b^7*x^(7*n) + 140
*a^2*b^6*x^(6*n) + 420*a^3*b^5*x^(5*n) + 1050*a^4*b^4*x^(4*n) - 420*a^6*b^2
*x^(2*n) - 60*a^7*b*x^n - 5*a^8)/(n*x^(3*n))
```

giac [A] time = 0.29, size = 116, normalized size = 0.87

$$\frac{840\,a^5b^3nx^{3\,n}\log(x)+3\,b^8x^{8\,n}+30\,ab^7x^{7\,n}+140\,a^2b^6x^{6\,n}+420\,a^3b^5x^{5\,n}+1050\,a^4b^4x^{4\,n}-420\,a^6b^2x^{2\,n}-60\,a^7b}{15\,nx^{3\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)*(a+b*x^n)^8,x, algorithm="giac")
```

```
[Out] 1/15*(840*a^5*b^3*n*x^(3*n)*log(x) + 3*b^8*x^(8*n) + 30*a*b^7*x^(7*n) + 140
*a^2*b^6*x^(6*n) + 420*a^3*b^5*x^(5*n) + 1050*a^4*b^4*x^(4*n) - 420*a^6*b^2
*x^(2*n) - 60*a^7*b*x^n - 5*a^8)/(n*x^(3*n))
```

maple [A] time = 0.03, size = 128, normalized size = 0.96

$$56a^5b^3\ln(x)-\frac{a^8x^{-3n}}{3n}-\frac{4a^7b\,x^{-2n}}{n}-\frac{28a^6b^2x^{-n}}{n}+\frac{70a^4b^4x^n}{n}+\frac{28a^3b^5x^{2n}}{n}+\frac{28a^2b^6x^{3n}}{3n}+\frac{2a\,b^7x^{4n}}{n}+\frac{b^8x^{5n}}{5n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-3*n)*(b*x^n+a)^8,x)
```

```
[Out] 56*a^5*b^3*ln(x)+1/5*b^8/n*(x^n)^5+2*a*b^7/n*(x^n)^4+28/3*a^2*b^6/n*(x^n)^3
+28*a^3*b^5/n*(x^n)^2+70*a^4*b^4*x^n/n-28*a^6*b^2/n/(x^n)-4*a^7*b/n/(x^n)^2
-1/3*a^8/n/(x^n)^3
```

maxima [A] time = 0.61, size = 131, normalized size = 0.98

$$56\,a^5b^3\log(x)+\frac{b^8x^{5\,n}}{5\,n}+\frac{2\,ab^7x^{4\,n}}{n}+\frac{28\,a^2b^6x^{3\,n}}{3\,n}+\frac{28\,a^3b^5x^{2\,n}}{n}+\frac{70\,a^4b^4x^n}{n}-\frac{a^8}{3\,nx^{3\,n}}-\frac{4\,a^7b}{nx^{2\,n}}-\frac{28\,a^6b^2}{nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

```
[Out] 56*a^5*b^3*log(x) + 1/5*b^8*x^(5*n)/n + 2*a*b^7*x^(4*n)/n + 28/3*a^2*b^6*x^
(3*n)/n + 28*a^3*b^5*x^(2*n)/n + 70*a^4*b^4*x^n/n - 1/3*a^8/(n*x^(3*n)) - 4
*a^7*b/(n*x^(2*n)) - 28*a^6*b^2/(n*x^n)
```

mupad [B] time = 1.60, size = 131, normalized size = 0.98

$$\frac{b^8\,x^{5\,n}}{5\,n}-\frac{a^8}{3\,n\,x^{3\,n}}+56\,a^5\,b^3\,\ln(x)-\frac{28\,a^6\,b^2}{n\,x^n}+\frac{28\,a^3\,b^5\,x^{2\,n}}{n}+\frac{28\,a^2\,b^6\,x^{3\,n}}{3\,n}-\frac{4\,a^7\,b}{n\,x^{2\,n}}+\frac{2\,a\,b^7\,x^{4\,n}}{n}+\frac{70\,a^4\,b^4\,x^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^8/x^(3*n + 1),x)
```

```
[Out] (b^8*x^(5*n))/(5*n) - a^8/(3*n*x^(3*n)) + 56*a^5*b^3*log(x) - (28*a^6*b^2)/
(n*x^n) + (28*a^3*b^5*x^(2*n))/n + (28*a^2*b^6*x^(3*n))/(3*n) - (4*a^7*b)/(
n*x^(2*n)) + (2*a*b^7*x^(4*n))/n + (70*a^4*b^4*x^n)/n
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-3*n)*(a+b*x**n)**8,x)
```

```
[Out] Timed out
```

3.2579 $\int x^{-1-4n}\left(a+bx^n\right)^8\,dx$

Optimal. Leaf size=135

$$-\frac{a^8x^{-4n}}{4n}-\frac{8a^7bx^{-3n}}{3n}-\frac{14a^6b^2x^{-2n}}{n}-\frac{56a^5b^3x^{-n}}{n}+70a^4b^4\log(x)+\frac{56a^3b^5x^n}{n}+\frac{14a^2b^6x^{2n}}{n}+\frac{8ab^7x^{3n}}{3n}+\frac{b^8x^{4n}}{4n}$$

```
[Out] -1/4*a^8/n/(x^(4*n))-8/3*a^7*b/n/(x^(3*n))-14*a^6*b^2/n/(x^(2*n))-56*a^5*b^
3/n/(x^n)+56*a^3*b^5*x^n/n+14*a^2*b^6*x^(2*n)/n+8/3*a*b^7*x^(3*n)/n+1/4*b^8
*x^(4*n)/n+70*a^4*b^4*ln(x)
```

Rubi [A] time = 0.06, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$-\frac{14a^6b^2x^{-2n}}{n}-\frac{56a^5b^3x^{-n}}{n}+\frac{56a^3b^5x^n}{n}+\frac{14a^2b^6x^{2n}}{n}+70a^4b^4\log(x)-\frac{8a^7bx^{-3n}}{3n}-\frac{a^8x^{-4n}}{4n}+\frac{8ab^7x^{3n}}{3n}+\frac{b^8x^{4n}}{4n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 4*n)*(a + b*x^n)^8,x]
```

```
[Out] -a^8/(4*n*x^(4*n)) - (8*a^7*b)/(3*n*x^(3*n)) - (14*a^6*b^2)/(n*x^(2*n)) - (
56*a^5*b^3)/(n*x^n) + (56*a^3*b^5*x^n)/n + (14*a^2*b^6*x^(2*n))/n + (8*a*b^
7*x^(3*n))/(3*n) + (b^8*x^(4*n))/(4*n) + 70*a^4*b^4*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1-4n}\left(a+bx^n\right)^8\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^8}{x^5}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(56a^3b^5+\frac{a^8}{x^5}+\frac{8a^7b}{x^4}+\frac{28a^6b^2}{x^3}+\frac{56a^5b^3}{x^2}+\frac{70a^4b^4}{x}+28a^2b^6x+8ab^7x^2+b^8x^3\right)dx}{n}\\ &= -\frac{a^8x^{-4n}}{4n}-\frac{8a^7bx^{-3n}}{3n}-\frac{14a^6b^2x^{-2n}}{n}-\frac{56a^5b^3x^{-n}}{n}+\frac{56a^3b^5x^n}{n}+\frac{14a^2b^6x^{2n}}{n}+\frac{8ab^7x^{3n}}{3n}+\end{aligned}$$

Mathematica [A] time = 0.09, size = 116, normalized size = 0.86

$$\frac{-\frac{1}{4}a^8x^{-4n}-\frac{8}{3}a^7bx^{-3n}-14a^6b^2x^{-2n}-56a^5b^3x^{-n}+70a^4b^4n\log(x)+56a^3b^5x^n+14a^2b^6x^{2n}+\frac{8}{3}ab^7x^{3n}+\frac{1}{4}b^8x^{4n}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 4*n)*(a + b*x^n)^8,x]
```

```
[Out] (-1/4*a^8/x^(4*n) - (8*a^7*b)/(3*x^(3*n)) - (14*a^6*b^2)/x^(2*n) - (56*a^5*
b^3)/x^n + 56*a^3*b^5*x^n + 14*a^2*b^6*x^(2*n) + (8*a*b^7*x^(3*n))/3 + (b^8
*x^(4*n))/4 + 70*a^4*b^4*n*Log[x])/n
```

fricas [A] time = 0.52, size = 116, normalized size = 0.86

$$\frac{840\,a^4b^4nx^{4n}\log(x) + 3\,b^8x^{8n} + 32\,ab^7x^{7n} + 168\,a^2b^6x^{6n} + 672\,a^3b^5x^{5n} - 672\,a^5b^3x^{3n} - 168\,a^6b^2x^{2n} - 32\,a^7}{12\,nx^{4n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-4*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

```
[Out] 1/12*(840*a^4*b^4*n*x^(4*n)*log(x) + 3*b^8*x^(8*n) + 32*a*b^7*x^(7*n) + 168
*a^2*b^6*x^(6*n) + 672*a^3*b^5*x^(5*n) - 672*a^5*b^3*x^(3*n) - 168*a^6*b^2*
x^(2*n) - 32*a^7*b*x^n - 3*a^8)/(n*x^(4*n))
```

giac [A] time = 0.30, size = 116, normalized size = 0.86

$$\frac{840\,a^4b^4nx^{4n}\log(x) + 3\,b^8x^{8n} + 32\,ab^7x^{7n} + 168\,a^2b^6x^{6n} + 672\,a^3b^5x^{5n} - 672\,a^5b^3x^{3n} - 168\,a^6b^2x^{2n} - 32\,a^7}{12\,nx^{4n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-4*n)*(a+b*x^n)^8,x, algorithm="giac")
```

```
[Out] 1/12*(840*a^4*b^4*n*x^(4*n)*log(x) + 3*b^8*x^(8*n) + 32*a*b^7*x^(7*n) + 168
*a^2*b^6*x^(6*n) + 672*a^3*b^5*x^(5*n) - 672*a^5*b^3*x^(3*n) - 168*a^6*b^2*
x^(2*n) - 32*a^7*b*x^n - 3*a^8)/(n*x^(4*n))
```

maple [A] time = 0.03, size = 128, normalized size = 0.95

$$70a^4b^4\ln(x) - \frac{a^8x^{-4n}}{4n} - \frac{8a^7b\,x^{-3n}}{3n} - \frac{14a^6b^2x^{-2n}}{n} - \frac{56a^5b^3x^{-n}}{n} + \frac{56a^3b^5x^n}{n} + \frac{14a^2b^6x^{2n}}{n} + \frac{8a\,b^7x^{3n}}{3n} + \frac{b^8x^{4n}}{4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-4*n)*(b*x^n+a)^8,x)
```

```
[Out] 70*a^4*b^4*ln(x)+1/4*b^8/n*(x^n)^4+8/3*a*b^7/n*(x^n)^3+14*a^2*b^6/n*(x^n)^2
+56*a^3*b^5*x^n/n-56*a^5*b^3/n/(x^n)-14*a^6*b^2/n/(x^n)^2-8/3*a^7*b/n/(x^n)
^3-1/4*a^8/n/(x^n)^4
```

maxima [A] time = 0.53, size = 133, normalized size = 0.99

$$70\,a^4b^4\log(x) + \frac{b^8x^{4n}}{4\,n} + \frac{8\,ab^7x^{3n}}{3\,n} + \frac{14\,a^2b^6x^{2n}}{n} + \frac{56\,a^3b^5x^n}{n} - \frac{a^8}{4\,nx^{4n}} - \frac{8\,a^7b}{3\,nx^{3n}} - \frac{14\,a^6b^2}{nx^{2n}} - \frac{56\,a^5b^3}{nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-4*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

```
[Out] 70*a^4*b^4*log(x) + 1/4*b^8*x^(4*n)/n + 8/3*a*b^7*x^(3*n)/n + 14*a^2*b^6*x^
(2*n)/n + 56*a^3*b^5*x^n/n - 1/4*a^8/(n*x^(4*n)) - 8/3*a^7*b/(n*x^(3*n)) -
14*a^6*b^2/(n*x^(2*n)) - 56*a^5*b^3/(n*x^n)
```

mupad [B] time = 1.54, size = 133, normalized size = 0.99

$$\frac{b^8\,x^{4\,n}}{4\,n} - \frac{a^8}{4\,n\,x^{4\,n}} + 70\,a^4\,b^4\,\ln(x) - \frac{56\,a^5\,b^3}{n\,x^n} + \frac{14\,a^2\,b^6\,x^{2\,n}}{n} - \frac{14\,a^6\,b^2}{n\,x^{2\,n}} + \frac{8\,a\,b^7\,x^{3\,n}}{3\,n} - \frac{8\,a^7\,b}{3\,n\,x^{3\,n}} + \frac{56\,a^3\,b^5\,x^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^8/x^(4*n + 1),x)
```

```
[Out] (b^8*x^(4*n))/(4*n) - a^8/(4*n*x^(4*n)) + 70*a^4*b^4*log(x) - (56*a^5*b^3)/
(n*x^n) + (14*a^2*b^6*x^(2*n))/n - (14*a^6*b^2)/(n*x^(2*n)) + (8*a*b^7*x^(3
*n))/(3*n) - (8*a^7*b)/(3*n*x^(3*n)) + (56*a^3*b^5*x^n)/n
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-4*n)*(a+b*x**n)**8,x)
```

```
[Out] Timed out
```

3.2580 $\int x^{-1-5n}\left(a+bx^n\right)^8\,dx$

Optimal. Leaf size=133

$$-\frac{a^8x^{-5n}}{5n}-\frac{2a^7bx^{-4n}}{n}-\frac{28a^6b^2x^{-3n}}{3n}-\frac{28a^5b^3x^{-2n}}{n}-\frac{70a^4b^4x^{-n}}{n}+56a^3b^5\log(x)+\frac{28a^2b^6x^n}{n}+\frac{4ab^7x^{2n}}{n}+\frac{b^8x^{3n}}{3n}$$

```
[Out] -1/5*a^8/n/(x^(5*n))-2*a^7*b/n/(x^(4*n))-28/3*a^6*b^2/n/(x^(3*n))-28*a^5*b^
3/n/(x^(2*n))-70*a^4*b^4/n/(x^n)+28*a^2*b^6*x^n/n+4*a*b^7*x^(2*n)/n+1/3*b^8
*x^(3*n)/n+56*a^3*b^5*ln(x)
```

Rubi [A] time = 0.06, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$-\frac{28a^6b^2x^{-3n}}{3n}-\frac{28a^5b^3x^{-2n}}{n}-\frac{70a^4b^4x^{-n}}{n}+\frac{28a^2b^6x^n}{n}+56a^3b^5\log(x)-\frac{2a^7bx^{-4n}}{n}-\frac{a^8x^{-5n}}{5n}+\frac{4ab^7x^{2n}}{n}+\frac{b^8x^{3n}}{3n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 5*n)*(a + b*x^n)^8,x]
```

```
[Out] -a^8/(5*n*x^(5*n)) - (2*a^7*b)/(n*x^(4*n)) - (28*a^6*b^2)/(3*n*x^(3*n)) - (
28*a^5*b^3)/(n*x^(2*n)) - (70*a^4*b^4)/(n*x^n) + (28*a^2*b^6*x^n)/n + (4*a*
b^7*x^(2*n))/n + (b^8*x^(3*n))/(3*n) + 56*a^3*b^5*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1-5n}\left(a+bx^n\right)^8\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^8}{x^6}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(28a^2b^6+\frac{a^8}{x^6}+\frac{8a^7b}{x^5}+\frac{28a^6b^2}{x^4}+\frac{56a^5b^3}{x^3}+\frac{70a^4b^4}{x^2}+\frac{56a^3b^5}{x}+8ab^7x+b^8x^2\right)dx\right.}{n}\\ &= -\frac{a^8x^{-5n}}{5n}-\frac{2a^7bx^{-4n}}{n}-\frac{28a^6b^2x^{-3n}}{3n}-\frac{28a^5b^3x^{-2n}}{n}-\frac{70a^4b^4x^{-n}}{n}+\frac{28a^2b^6x^n}{n}+\frac{4ab^7x}{n}\end{aligned}$$

Mathematica [A] time = 0.10, size = 114, normalized size = 0.86

$$\frac{-\frac{1}{5}a^8x^{-5n}-2a^7bx^{-4n}-\frac{28}{3}a^6b^2x^{-3n}-28a^5b^3x^{-2n}-70a^4b^4x^{-n}+56a^3b^5n\log(x)+28a^2b^6x^n+4ab^7x^{2n}+\frac{1}{3}b^8x}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 5*n)*(a + b*x^n)^8,x]
```

```
[Out] (-1/5*a^8/x^(5*n) - (2*a^7*b)/x^(4*n) - (28*a^6*b^2)/(3*x^(3*n)) - (28*a^5*
b^3)/x^(2*n) - (70*a^4*b^4)/x^n + 28*a^2*b^6*x^n + 4*a*b^7*x^(2*n) + (b^8*x
^(3*n))/3 + 56*a^3*b^5*n*Log[x])/n
```

fricas [A] time = 0.77, size = 116, normalized size = 0.87

$$\frac{840\,a^3b^5nx^{5\,n}\log(x)+5\,b^8x^{8\,n}+60\,ab^7x^{7\,n}+420\,a^2b^6x^{6\,n}-1050\,a^4b^4x^{4\,n}-420\,a^5b^3x^{3\,n}-140\,a^6b^2x^{2\,n}-30\,a^7bx}{15\,nx^{5\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-5*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

```
[Out] 1/15*(840*a^3*b^5*n*x^(5*n)*log(x) + 5*b^8*x^(8*n) + 60*a*b^7*x^(7*n) + 420
*a^2*b^6*x^(6*n) - 1050*a^4*b^4*x^(4*n) - 420*a^5*b^3*x^(3*n) - 140*a^6*b^2
*x^(2*n) - 30*a^7*b*x^n - 3*a^8)/(n*x^(5*n))
```

giac [A] time = 0.31, size = 116, normalized size = 0.87

$$\frac{840\,a^3b^5nx^{5\,n}\log(x)+5\,b^8x^{8\,n}+60\,ab^7x^{7\,n}+420\,a^2b^6x^{6\,n}-1050\,a^4b^4x^{4\,n}-420\,a^5b^3x^{3\,n}-140\,a^6b^2x^{2\,n}-30\,a^7bx}{15\,nx^{5\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-5*n)*(a+b*x^n)^8,x, algorithm="giac")
```

```
[Out] 1/15*(840*a^3*b^5*n*x^(5*n)*log(x) + 5*b^8*x^(8*n) + 60*a*b^7*x^(7*n) + 420
*a^2*b^6*x^(6*n) - 1050*a^4*b^4*x^(4*n) - 420*a^5*b^3*x^(3*n) - 140*a^6*b^2
*x^(2*n) - 30*a^7*b*x^n - 3*a^8)/(n*x^(5*n))
```

maple [A] time = 0.03, size = 128, normalized size = 0.96

$$56a^3b^5\ln(x)-\frac{a^8x^{-5n}}{5n}-\frac{2a^7b\,x^{-4n}}{n}-\frac{28a^6b^2x^{-3n}}{3n}-\frac{28a^5b^3x^{-2n}}{n}-\frac{70a^4b^4x^{-n}}{n}+\frac{28a^2b^6x^n}{n}+\frac{4a\,b^7x^{2n}}{n}+\frac{b^8x^{3n}}{3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-5*n)*(b*x^n+a)^8,x)
```

```
[Out] 56*a^3*b^5*ln(x)+1/3*b^8/n*(x^n)^3+4*a*b^7/n*(x^n)^2+28*a^2*b^6*x^n/n-70*a^
4*b^4/n/(x^n)-28*a^5*b^3/n/(x^n)^2-28/3*a^6*b^2/n/(x^n)^3-2*a^7*b/n/(x^n)^4
-1/5*a^8/n/(x^n)^5
```

maxima [A] time = 0.65, size = 135, normalized size = 1.02

$$56\,a^3b^5\log(x)+\frac{b^8x^{3\,n}}{3\,n}+\frac{4\,ab^7x^{2\,n}}{n}+\frac{28\,a^2b^6x^n}{n}-\frac{a^8}{5\,nx^{5\,n}}-\frac{2\,a^7b}{nx^{4\,n}}-\frac{28\,a^6b^2}{3\,nx^{3\,n}}-\frac{28\,a^5b^3}{nx^{2\,n}}-\frac{70\,a^4b^4}{nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-5*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

```
[Out] 56*a^3*b^5*log(x) + 1/3*b^8*x^(3*n)/n + 4*a*b^7*x^(2*n)/n + 28*a^2*b^6*x^n/
n - 1/5*a^8/(n*x^(5*n)) - 2*a^7*b/(n*x^(4*n)) - 28/3*a^6*b^2/(n*x^(3*n)) -
28*a^5*b^3/(n*x^(2*n)) - 70*a^4*b^4/(n*x^n)
```

mupad [B] time = 1.55, size = 135, normalized size = 1.02

$$\frac{b^8\,x^{3\,n}}{3\,n}-\frac{a^8}{5\,n\,x^{5\,n}}+56\,a^3\,b^5\,\ln(x)-\frac{70\,a^4\,b^4}{n\,x^n}-\frac{28\,a^5\,b^3}{n\,x^{2\,n}}-\frac{28\,a^6\,b^2}{3\,n\,x^{3\,n}}+\frac{4\,a\,b^7\,x^{2\,n}}{n}-\frac{2\,a^7\,b}{n\,x^{4\,n}}+\frac{28\,a^2\,b^6\,x^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^8/x^(5*n + 1),x)
```

```
[Out] (b^8*x^(3*n))/(3*n) - a^8/(5*n*x^(5*n)) + 56*a^3*b^5*log(x) - (70*a^4*b^4)/
(n*x^n) - (28*a^5*b^3)/(n*x^(2*n)) - (28*a^6*b^2)/(3*n*x^(3*n)) + (4*a*b^7*
x^(2*n))/n - (2*a^7*b)/(n*x^(4*n)) + (28*a^2*b^6*x^n)/n
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-5*n)*(a+b*x**n)**8,x)
```

```
[Out] Timed out
```

3.2581 $\int x^{-1-6n}\left(a+bx^n\right)^8\,dx$

Optimal. Leaf size=135

$$-\frac{a^8x^{-6n}}{6n}-\frac{8a^7bx^{-5n}}{5n}-\frac{7a^6b^2x^{-4n}}{n}-\frac{56a^5b^3x^{-3n}}{3n}-\frac{35a^4b^4x^{-2n}}{n}-\frac{56a^3b^5x^{-n}}{n}+28a^2b^6\log(x)+\frac{8ab^7x^n}{n}+\frac{b^8x^{2n}}{2n}$$

```
[Out] -1/6*a^8/n/(x^(6*n))-8/5*a^7*b/n/(x^(5*n))-7*a^6*b^2/n/(x^(4*n))-56/3*a^5*b
^3/n/(x^(3*n))-35*a^4*b^4/n/(x^(2*n))-56*a^3*b^5/n/(x^n)+8*a*b^7*x^n/n+1/2*
b^8*x^(2*n)/n+28*a^2*b^6*ln(x)
```

Rubi [A] time = 0.06, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$-\frac{7a^6b^2x^{-4n}}{n}-\frac{56a^5b^3x^{-3n}}{3n}-\frac{35a^4b^4x^{-2n}}{n}-\frac{56a^3b^5x^{-n}}{n}+28a^2b^6\log(x)-\frac{8a^7bx^{-5n}}{5n}-\frac{a^8x^{-6n}}{6n}+\frac{8ab^7x^n}{n}+\frac{b^8x^{2n}}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 6*n)*(a + b*x^n)^8,x]
```

```
[Out] -a^8/(6*n*x^(6*n)) - (8*a^7*b)/(5*n*x^(5*n)) - (7*a^6*b^2)/(n*x^(4*n)) - (5
6*a^5*b^3)/(3*n*x^(3*n)) - (35*a^4*b^4)/(n*x^(2*n)) - (56*a^3*b^5)/(n*x^n)
+ (8*a*b^7*x^n)/n + (b^8*x^(2*n))/(2*n) + 28*a^2*b^6*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1-6n}\left(a+bx^n\right)^8\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^8}{x^7}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(8ab^7+\frac{a^8}{x^7}+\frac{8a^7b}{x^6}+\frac{28a^6b^2}{x^5}+\frac{56a^5b^3}{x^4}+\frac{70a^4b^4}{x^3}+\frac{56a^3b^5}{x^2}+\frac{28a^2b^6}{x}+b^8x\right)dx,x,x^n\right)}{n}\\ &= -\frac{a^8x^{-6n}}{6n}-\frac{8a^7bx^{-5n}}{5n}-\frac{7a^6b^2x^{-4n}}{n}-\frac{56a^5b^3x^{-3n}}{3n}-\frac{35a^4b^4x^{-2n}}{n}-\frac{56a^3b^5x^{-n}}{n}+\frac{8ab^7x^n}{n}\end{aligned}$$

Mathematica [A] time = 0.14, size = 116, normalized size = 0.86

$$\frac{-\frac{1}{6}a^8x^{-6n}-\frac{8}{5}a^7bx^{-5n}-7a^6b^2x^{-4n}-\frac{56}{3}a^5b^3x^{-3n}-35a^4b^4x^{-2n}-56a^3b^5x^{-n}+28a^2b^6n\log(x)+8ab^7x^n+\frac{1}{2}b^8x^{2n}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 6*n)*(a + b*x^n)^8,x]
```

```
[Out] (-1/6*a^8/x^(6*n) - (8*a^7*b)/(5*x^(5*n)) - (7*a^6*b^2)/x^(4*n) - (56*a^5*b
^3)/(3*x^(3*n)) - (35*a^4*b^4)/x^(2*n) - (56*a^3*b^5)/x^n + 8*a*b^7*x^n + (
b^8*x^(2*n))/2 + 28*a^2*b^6*n*Log[x])/n
```

fricas [A] time = 0.55, size = 116, normalized size = 0.86

$$\frac{840\,a^2b^6nx^{6\,n}\log(x)+15\,b^8x^{8\,n}+240\,ab^7x^{7\,n}-1680\,a^3b^5x^{5\,n}-1050\,a^4b^4x^{4\,n}-560\,a^5b^3x^{3\,n}-210\,a^6b^2x^{2\,n}-4}{30\,nx^{6\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-6*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

```
[Out] 1/30*(840*a^2*b^6*n*x^(6*n)*log(x) + 15*b^8*x^(8*n) + 240*a*b^7*x^(7*n) - 1
680*a^3*b^5*x^(5*n) - 1050*a^4*b^4*x^(4*n) - 560*a^5*b^3*x^(3*n) - 210*a^6*
b^2*x^(2*n) - 48*a^7*b*x^n - 5*a^8)/(n*x^(6*n))
```

giac [A] time = 0.30, size = 116, normalized size = 0.86

$$\frac{840\,a^2b^6nx^{6\,n}\log(x)+15\,b^8x^{8\,n}+240\,ab^7x^{7\,n}-1680\,a^3b^5x^{5\,n}-1050\,a^4b^4x^{4\,n}-560\,a^5b^3x^{3\,n}-210\,a^6b^2x^{2\,n}-4}{30\,nx^{6\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-6*n)*(a+b*x^n)^8,x, algorithm="giac")
```

```
[Out] 1/30*(840*a^2*b^6*n*x^(6*n)*log(x) + 15*b^8*x^(8*n) + 240*a*b^7*x^(7*n) - 1
680*a^3*b^5*x^(5*n) - 1050*a^4*b^4*x^(4*n) - 560*a^5*b^3*x^(3*n) - 210*a^6*
b^2*x^(2*n) - 48*a^7*b*x^n - 5*a^8)/(n*x^(6*n))
```

maple [A] time = 0.03, size = 128, normalized size = 0.95

$$28a^2b^6\ln(x)-\frac{a^8x^{-6n}}{6n}-\frac{8a^7b\,x^{-5n}}{5n}-\frac{7a^6b^2x^{-4n}}{n}-\frac{56a^5b^3x^{-3n}}{3n}-\frac{35a^4b^4x^{-2n}}{n}-\frac{56a^3b^5x^{-n}}{n}+\frac{8a\,b^7x^n}{n}+\frac{b^8x^{2n}}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-6*n)*(b*x^n+a)^8,x)
```

```
[Out] 28*a^2*b^6*ln(x)+1/2*b^8/n*(x^n)^2+8*a*b^7*x^n/n-56*a^3*b^5/n/(x^n)-35*a^4*
b^4/n/(x^n)^2-56/3*a^5*b^3/n/(x^n)^3-7*a^6*b^2/n/(x^n)^4-8/5*a^7*b/n/(x^n)^
5-1/6*a^8/n/(x^n)^6
```

maxima [A] time = 0.54, size = 137, normalized size = 1.01

$$28\,a^2b^6\log(x)+\frac{b^8x^{2\,n}}{2\,n}+\frac{8\,ab^7x^n}{n}-\frac{a^8}{6\,nx^{6\,n}}-\frac{8\,a^7b}{5\,nx^{5\,n}}-\frac{7\,a^6b^2}{nx^{4\,n}}-\frac{56\,a^5b^3}{3\,nx^{3\,n}}-\frac{35\,a^4b^4}{nx^{2\,n}}-\frac{56\,a^3b^5}{nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-6*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

```
[Out] 28*a^2*b^6*log(x) + 1/2*b^8*x^(2*n)/n + 8*a*b^7*x^n/n - 1/6*a^8/(n*x^(6*n))
 - 8/5*a^7*b/(n*x^(5*n)) - 7*a^6*b^2/(n*x^(4*n)) - 56/3*a^5*b^3/(n*x^(3*n))
 - 35*a^4*b^4/(n*x^(2*n)) - 56*a^3*b^5/(n*x^n)
```

mupad [B] time = 1.50, size = 137, normalized size = 1.01

$$\frac{b^8\,x^{2\,n}}{2\,n}-\frac{a^8}{6\,n\,x^{6\,n}}+28\,a^2\,b^6\,\ln(x)-\frac{56\,a^3\,b^5}{n\,x^n}-\frac{35\,a^4\,b^4}{n\,x^{2\,n}}-\frac{56\,a^5\,b^3}{3\,n\,x^{3\,n}}-\frac{7\,a^6\,b^2}{n\,x^{4\,n}}+\frac{8\,a\,b^7\,x^n}{n}-\frac{8\,a^7\,b}{5\,n\,x^{5\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^8/x^(6*n + 1),x)
```

```
[Out] (b^8*x^(2*n))/(2*n) - a^8/(6*n*x^(6*n)) + 28*a^2*b^6*log(x) - (56*a^3*b^5)/
(n*x^n) - (35*a^4*b^4)/(n*x^(2*n)) - (56*a^5*b^3)/(3*n*x^(3*n)) - (7*a^6*b^
2)/(n*x^(4*n)) + (8*a*b^7*x^n)/n - (8*a^7*b)/(5*n*x^(5*n))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-6*n)*(a+b*x**n)**8,x)
```

```
[Out] Timed out
```

3.2582 $\int x^{-1-7n}\left(a+bx^n\right)^8\,dx$

Optimal. Leaf size=134

$$-\frac{a^8x^{-7n}}{7n}-\frac{4a^7bx^{-6n}}{3n}-\frac{28a^6b^2x^{-5n}}{5n}-\frac{14a^5b^3x^{-4n}}{n}-\frac{70a^4b^4x^{-3n}}{3n}-\frac{28a^3b^5x^{-2n}}{n}-\frac{28a^2b^6x^{-n}}{n}+8ab^7\log(x)+\frac{b^8x^n}{n}$$

```
[Out] -1/7*a^8/n/(x^(7*n))-4/3*a^7*b/n/(x^(6*n))-28/5*a^6*b^2/n/(x^(5*n))-14*a^5*
b^3/n/(x^(4*n))-70/3*a^4*b^4/n/(x^(3*n))-28*a^3*b^5/n/(x^(2*n))-28*a^2*b^6/
n/(x^n)+b^8*x^n/n+8*a*b^7*ln(x)
```

Rubi [A] time = 0.06, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$-\frac{28a^6b^2x^{-5n}}{5n}-\frac{14a^5b^3x^{-4n}}{n}-\frac{70a^4b^4x^{-3n}}{3n}-\frac{28a^3b^5x^{-2n}}{n}-\frac{28a^2b^6x^{-n}}{n}-\frac{4a^7bx^{-6n}}{3n}-\frac{a^8x^{-7n}}{7n}+8ab^7\log(x)+\frac{b^8x^n}{n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 7*n)*(a + b*x^n)^8,x]
```

```
[Out] -a^8/(7*n*x^(7*n)) - (4*a^7*b)/(3*n*x^(6*n)) - (28*a^6*b^2)/(5*n*x^(5*n)) -
 (14*a^5*b^3)/(n*x^(4*n)) - (70*a^4*b^4)/(3*n*x^(3*n)) - (28*a^3*b^5)/(n*x^
(2*n)) - (28*a^2*b^6)/(n*x^n) + (b^8*x^n)/n + 8*a*b^7*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1-7n}\left(a+bx^n\right)^8\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^8}{x^8}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(b^8+\frac{a^8}{x^8}+\frac{8a^7b}{x^7}+\frac{28a^6b^2}{x^6}+\frac{56a^5b^3}{x^5}+\frac{70a^4b^4}{x^4}+\frac{56a^3b^5}{x^3}+\frac{28a^2b^6}{x^2}+\frac{8ab^7}{x}\right)dx, x, x^n\right.}{n}\\ &= -\frac{a^8x^{-7n}}{7n}-\frac{4a^7bx^{-6n}}{3n}-\frac{28a^6b^2x^{-5n}}{5n}-\frac{14a^5b^3x^{-4n}}{n}-\frac{70a^4b^4x^{-3n}}{3n}-\frac{28a^3b^5x^{-2n}}{n}-\frac{28}{}\end{aligned}$$

Mathematica [A] time = 0.14, size = 115, normalized size = 0.86

$$\frac{-\frac{1}{7}a^8x^{-7n}-\frac{4}{3}a^7bx^{-6n}-\frac{28}{5}a^6b^2x^{-5n}-14a^5b^3x^{-4n}-\frac{70}{3}a^4b^4x^{-3n}-28a^3b^5x^{-2n}-28a^2b^6x^{-n}+8ab^7n\log(x)+b^8}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 7*n)*(a + b*x^n)^8,x]
```

```
[Out] (-1/7*a^8/x^(7*n) - (4*a^7*b)/(3*x^(6*n)) - (28*a^6*b^2)/(5*x^(5*n)) - (14*
a^5*b^3)/x^(4*n) - (70*a^4*b^4)/(3*x^(3*n)) - (28*a^3*b^5)/x^(2*n) - (28*a^
2*b^6)/x^n + b^8*x^n + 8*a*b^7*n*Log[x])/n
```

fricas [A] time = 0.55, size = 116, normalized size = 0.87

$$\frac{840\,ab^7nx^{7\,n}\log(x)+105\,b^8x^{8\,n}-2940\,a^2b^6x^{6\,n}-2940\,a^3b^5x^{5\,n}-2450\,a^4b^4x^{4\,n}-1470\,a^5b^3x^{3\,n}-588\,a^6b^2x^{2\,n}-}{105\,nx^{7\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-7*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

```
[Out] 1/105*(840*a*b^7*n*x^(7*n)*log(x) + 105*b^8*x^(8*n) - 2940*a^2*b^6*x^(6*n)
- 2940*a^3*b^5*x^(5*n) - 2450*a^4*b^4*x^(4*n) - 1470*a^5*b^3*x^(3*n) - 588*
a^6*b^2*x^(2*n) - 140*a^7*b*x^n - 15*a^8)/(n*x^(7*n))
```

giac [A] time = 0.31, size = 116, normalized size = 0.87

$$\frac{840\,ab^7nx^{7\,n}\log(x)+105\,b^8x^{8\,n}-2940\,a^2b^6x^{6\,n}-2940\,a^3b^5x^{5\,n}-2450\,a^4b^4x^{4\,n}-1470\,a^5b^3x^{3\,n}-588\,a^6b^2x^{2\,n}-}{105\,nx^{7\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-7*n)*(a+b*x^n)^8,x, algorithm="giac")
```

```
[Out] 1/105*(840*a*b^7*n*x^(7*n)*log(x) + 105*b^8*x^(8*n) - 2940*a^2*b^6*x^(6*n)
- 2940*a^3*b^5*x^(5*n) - 2450*a^4*b^4*x^(4*n) - 1470*a^5*b^3*x^(3*n) - 588*
a^6*b^2*x^(2*n) - 140*a^7*b*x^n - 15*a^8)/(n*x^(7*n))
```

maple [A] time = 0.03, size = 127, normalized size = 0.95

$$8a\,b^7\ln(x)-\frac{a^8x^{-7n}}{7n}-\frac{4a^7b\,x^{-6n}}{3n}-\frac{28a^6b^2x^{-5n}}{5n}-\frac{14a^5b^3x^{-4n}}{n}-\frac{70a^4b^4x^{-3n}}{3n}-\frac{28a^3b^5x^{-2n}}{n}-\frac{28a^2b^6x^{-n}}{n}+\frac{b^8x^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-7*n)*(b*x^n+a)^8,x)
```

```
[Out] 8*a*b^7*ln(x)+b^8*x^n/n-28*a^2*b^6/n/(x^n)-28*a^3*b^5/n/(x^n)^2-70/3*a^4*b^
4/n/(x^n)^3-14*a^5*b^3/n/(x^n)^4-28/5*a^6*b^2/n/(x^n)^5-4/3*a^7*b/n/(x^n)^6
-1/7*a^8/n/(x^n)^7
```

maxima [A] time = 0.59, size = 138, normalized size = 1.03

$$8\,ab^7\log(x)+\frac{b^8x^n}{n}-\frac{a^8}{7\,nx^{7\,n}}-\frac{4\,a^7b}{3\,nx^{6\,n}}-\frac{28\,a^6b^2}{5\,nx^{5\,n}}-\frac{14\,a^5b^3}{nx^{4\,n}}-\frac{70\,a^4b^4}{3\,nx^{3\,n}}-\frac{28\,a^3b^5}{nx^{2\,n}}-\frac{28\,a^2b^6}{nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-7*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

```
[Out] 8*a*b^7*log(x) + b^8*x^n/n - 1/7*a^8/(n*x^(7*n)) - 4/3*a^7*b/(n*x^(6*n)) -
28/5*a^6*b^2/(n*x^(5*n)) - 14*a^5*b^3/(n*x^(4*n)) - 70/3*a^4*b^4/(n*x^(3*n)
) - 28*a^3*b^5/(n*x^(2*n)) - 28*a^2*b^6/(n*x^n)
```

mupad [B] time = 1.46, size = 138, normalized size = 1.03

$$\frac{b^8\,x^n}{n}+8\,a\,b^7\,\ln(x)-\frac{a^8}{7\,n\,x^{7\,n}}-\frac{28\,a^2\,b^6}{n\,x^n}-\frac{28\,a^3\,b^5}{n\,x^{2\,n}}-\frac{70\,a^4\,b^4}{3\,n\,x^{3\,n}}-\frac{14\,a^5\,b^3}{n\,x^{4\,n}}-\frac{28\,a^6\,b^2}{5\,n\,x^{5\,n}}-\frac{4\,a^7\,b}{3\,n\,x^{6\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^8/x^(7*n + 1),x)
```

```
[Out] (b^8*x^n)/n + 8*a*b^7*log(x) - a^8/(7*n*x^(7*n)) - (28*a^2*b^6)/(n*x^n) - (
28*a^3*b^5)/(n*x^(2*n)) - (70*a^4*b^4)/(3*n*x^(3*n)) - (14*a^5*b^3)/(n*x^(4
*n)) - (28*a^6*b^2)/(5*n*x^(5*n)) - (4*a^7*b)/(3*n*x^(6*n))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-7*n)*(a+b*x**n)**8,x)
```

```
[Out] Timed out
```

3.2583 $\int x^{-1-8n}\left(a+bx^n\right)^8\,dx$

Optimal. Leaf size=140

$$-\frac{a^8x^{-8n}}{8n}-\frac{8a^7bx^{-7n}}{7n}-\frac{14a^6b^2x^{-6n}}{3n}-\frac{56a^5b^3x^{-5n}}{5n}-\frac{35a^4b^4x^{-4n}}{2n}-\frac{56a^3b^5x^{-3n}}{3n}-\frac{14a^2b^6x^{-2n}}{n}-\frac{8ab^7x^{-n}}{n}+b^8\log(x)$$

```
[Out] -1/8*a^8/n/(x^(8*n))-8/7*a^7*b/n/(x^(7*n))-14/3*a^6*b^2/n/(x^(6*n))-56/5*a^
5*b^3/n/(x^(5*n))-35/2*a^4*b^4/n/(x^(4*n))-56/3*a^3*b^5/n/(x^(3*n))-14*a^2*
b^6/n/(x^(2*n))-8*a*b^7/n/(x^n)+b^8*ln(x)
```

Rubi [A] time = 0.06, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$-\frac{14a^6b^2x^{-6n}}{3n}-\frac{56a^5b^3x^{-5n}}{5n}-\frac{35a^4b^4x^{-4n}}{2n}-\frac{56a^3b^5x^{-3n}}{3n}-\frac{14a^2b^6x^{-2n}}{n}-\frac{8a^7bx^{-7n}}{7n}-\frac{a^8x^{-8n}}{8n}-\frac{8ab^7x^{-n}}{n}+b^8\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 8*n)*(a + b*x^n)^8,x]
```

```
[Out] -a^8/(8*n*x^(8*n)) - (8*a^7*b)/(7*n*x^(7*n)) - (14*a^6*b^2)/(3*n*x^(6*n)) -
 (56*a^5*b^3)/(5*n*x^(5*n)) - (35*a^4*b^4)/(2*n*x^(4*n)) - (56*a^3*b^5)/(3*
n*x^(3*n)) - (14*a^2*b^6)/(n*x^(2*n)) - (8*a*b^7)/(n*x^n) + b^8*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1-8n}\left(a+bx^n\right)^8\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^8}{x^9}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a^8}{x^9}+\frac{8a^7b}{x^8}+\frac{28a^6b^2}{x^7}+\frac{56a^5b^3}{x^6}+\frac{70a^4b^4}{x^5}+\frac{56a^3b^5}{x^4}+\frac{28a^2b^6}{x^3}+\frac{8ab^7}{x^2}+\frac{b^8}{x}\right)dx,x,x^n\right)}{n}\\ &= -\frac{a^8x^{-8n}}{8n}-\frac{8a^7bx^{-7n}}{7n}-\frac{14a^6b^2x^{-6n}}{3n}-\frac{56a^5b^3x^{-5n}}{5n}-\frac{35a^4b^4x^{-4n}}{2n}-\frac{56a^3b^5x^{-3n}}{3n}-\frac{14a^2b}{n}\end{aligned}$$

Mathematica [A] time = 0.14, size = 108, normalized size = 0.77

$$b^8\log(x)-\frac{ax^{-8n}\left(105a^7+960a^6bx^n+3920a^5b^2x^{2n}+9408a^4b^3x^{3n}+14700a^3b^4x^{4n}+15680a^2b^5x^{5n}+11760ab^6x\right.}{840n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 8*n)*(a + b*x^n)^8,x]
```

```
[Out] -1/840*(a*(105*a^7 + 960*a^6*b*x^n + 3920*a^5*b^2*x^(2*n) + 9408*a^4*b^3*x^
(3*n) + 14700*a^3*b^4*x^(4*n) + 15680*a^2*b^5*x^(5*n) + 11760*a*b^6*x^(6*n)
 + 6720*b^7*x^(7*n)))/(n*x^(8*n)) + b^8*Log[x]
```

fricas [A] time = 0.72, size = 116, normalized size = 0.83

$$\frac{840\,b^8 n x^{8\,n}\log(x) - 6720\,ab^7x^{7\,n} - 11760\,a^2b^6x^{6\,n} - 15680\,a^3b^5x^{5\,n} - 14700\,a^4b^4x^{4\,n} - 9408\,a^5b^3x^{3\,n} - 3920\,a^6}{840\,nx^{8\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-8*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

```
[Out] 1/840*(840*b^8*n*x^(8*n)*log(x) - 6720*a*b^7*x^(7*n) - 11760*a^2*b^6*x^(6*n
) - 15680*a^3*b^5*x^(5*n) - 14700*a^4*b^4*x^(4*n) - 9408*a^5*b^3*x^(3*n) -
3920*a^6*b^2*x^(2*n) - 960*a^7*b*x^n - 105*a^8)/(n*x^(8*n))
```

giac [A] time = 0.30, size = 116, normalized size = 0.83

$$\frac{840\,b^8 n x^{8\,n}\log(x) - 6720\,ab^7x^{7\,n} - 11760\,a^2b^6x^{6\,n} - 15680\,a^3b^5x^{5\,n} - 14700\,a^4b^4x^{4\,n} - 9408\,a^5b^3x^{3\,n} - 3920\,a^6}{840\,nx^{8\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-8*n)*(a+b*x^n)^8,x, algorithm="giac")
```

```
[Out] 1/840*(840*b^8*n*x^(8*n)*log(x) - 6720*a*b^7*x^(7*n) - 11760*a^2*b^6*x^(6*n
) - 15680*a^3*b^5*x^(5*n) - 14700*a^4*b^4*x^(4*n) - 9408*a^5*b^3*x^(3*n) -
3920*a^6*b^2*x^(2*n) - 960*a^7*b*x^n - 105*a^8)/(n*x^(8*n))
```

maple [A] time = 0.03, size = 129, normalized size = 0.92

$$b^8\ln(x) - \frac{a^8x^{-8n}}{8n} - \frac{8a^7b\,x^{-7n}}{7n} - \frac{14a^6b^2x^{-6n}}{3n} - \frac{56a^5b^3x^{-5n}}{5n} - \frac{35a^4b^4x^{-4n}}{2n} - \frac{56a^3b^5x^{-3n}}{3n} - \frac{14a^2b^6x^{-2n}}{n} - \frac{8a\,b^7x^{-n}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-8*n)*(b*x^n+a)^8,x)
```

```
[Out] b^8*ln(x)-8*a*b^7/n/(x^n)-14*a^2*b^6/n/(x^n)^2-56/3*a^3*b^5/n/(x^n)^3-35/2*
a^4*b^4/n/(x^n)^4-56/5*a^5*b^3/n/(x^n)^5-14/3*a^6*b^2/n/(x^n)^6-8/7*a^7*b/n
/(x^n)^7-1/8*a^8/n/(x^n)^8
```

maxima [A] time = 0.47, size = 142, normalized size = 1.01

$$b^8\log(x) - \frac{a^8}{8\,nx^{8\,n}} - \frac{8\,a^7b}{7\,nx^{7\,n}} - \frac{14\,a^6b^2}{3\,nx^{6\,n}} - \frac{56\,a^5b^3}{5\,nx^{5\,n}} - \frac{35\,a^4b^4}{2\,nx^{4\,n}} - \frac{56\,a^3b^5}{3\,nx^{3\,n}} - \frac{14\,a^2b^6}{nx^{2\,n}} - \frac{8\,ab^7}{nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-8*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

```
[Out] b^8*log(x) - 1/8*a^8/(n*x^(8*n)) - 8/7*a^7*b/(n*x^(7*n)) - 14/3*a^6*b^2/(n*
x^(6*n)) - 56/5*a^5*b^3/(n*x^(5*n)) - 35/2*a^4*b^4/(n*x^(4*n)) - 56/3*a^3*b
^5/(n*x^(3*n)) - 14*a^2*b^6/(n*x^(2*n)) - 8*a*b^7/(n*x^n)
```

mupad [B] time = 1.49, size = 142, normalized size = 1.01

$$b^8\,\ln(x) - \frac{a^8}{8\,n\,x^{8\,n}} - \frac{14\,a^2\,b^6}{n\,x^{2\,n}} - \frac{56\,a^3\,b^5}{3\,n\,x^{3\,n}} - \frac{35\,a^4\,b^4}{2\,n\,x^{4\,n}} - \frac{56\,a^5\,b^3}{5\,n\,x^{5\,n}} - \frac{14\,a^6\,b^2}{3\,n\,x^{6\,n}} - \frac{8\,a\,b^7}{n\,x^n} - \frac{8\,a^7\,b}{7\,n\,x^{7\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^8/x^(8*n + 1),x)
```

```
[Out] b^8*log(x) - a^8/(8*n*x^(8*n)) - (14*a^2*b^6)/(n*x^(2*n)) - (56*a^3*b^5)/(3
*n*x^(3*n)) - (35*a^4*b^4)/(2*n*x^(4*n)) - (56*a^5*b^3)/(5*n*x^(5*n)) - (14
*a^6*b^2)/(3*n*x^(6*n)) - (8*a*b^7)/(n*x^n) - (8*a^7*b)/(7*n*x^(7*n))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-8*n)*(a+b*x**n)**8,x)
```

```
[Out] Timed out
```

3.2584 $\int x^{-1-9n}\left(a+bx^n\right)^8\,dx$

Optimal. Leaf size=24

$$-\frac{x^{-9n}\left(a+bx^n\right)^9}{9an}$$

```
[Out] -1/9*(a+b*x^n)^9/a/n/(x^(9*n))
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {264}

$$-\frac{x^{-9n}\left(a+bx^n\right)^9}{9an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 9*n)*(a + b*x^n)^8,x]
```

```
[Out] -(a + b*x^n)^9/(9*a*n*x^(9*n))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^{-1-9n}\left(a+bx^n\right)^8\,dx = -\frac{x^{-9n}\left(a+bx^n\right)^9}{9an}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$-\frac{x^{-9n}\left(a+bx^n\right)^9}{9an}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 9*n)*(a + b*x^n)^8,x]
```

```
[Out] -1/9*(a + b*x^n)^9/(a*n*x^(9*n))
```

fricas [B] time = 0.56, size = 111, normalized size = 4.62

$$-\frac{9\,b^8x^{8\,n}+36\,ab^7x^{7\,n}+84\,a^2b^6x^{6\,n}+126\,a^3b^5x^{5\,n}+126\,a^4b^4x^{4\,n}+84\,a^5b^3x^{3\,n}+36\,a^6b^2x^{2\,n}+9\,a^7bx^n+a^8}{9\,nx^{9\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-9*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

```
[Out] -1/9*(9*b^8*x^(8*n) + 36*a*b^7*x^(7*n) + 84*a^2*b^6*x^(6*n) + 126*a^3*b^5*x
^(5*n) + 126*a^4*b^4*x^(4*n) + 84*a^5*b^3*x^(3*n) + 36*a^6*b^2*x^(2*n) + 9*
a^7*b*x^n + a^8)/(n*x^(9*n))
```

giac [B] time = 0.29, size = 111, normalized size = 4.62

$$-\frac{9\,b^8x^{8\,n}+36\,ab^7x^{7\,n}+84\,a^2b^6x^{6\,n}+126\,a^3b^5x^{5\,n}+126\,a^4b^4x^{4\,n}+84\,a^5b^3x^{3\,n}+36\,a^6b^2x^{2\,n}+9\,a^7bx^n+a^8}{9\,nx^{9\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-9*n)*(a+b*x^n)^8,x, algorithm="giac")
```

```
[Out] -1/9*(9*b^8*x^(8*n) + 36*a*b^7*x^(7*n) + 84*a^2*b^6*x^(6*n) + 126*a^3*b^5*x
^(5*n) + 126*a^4*b^4*x^(4*n) + 84*a^5*b^3*x^(3*n) + 36*a^6*b^2*x^(2*n) + 9*
a^7*b*x^n + a^8)/(n*x^(9*n))
```

maple [**B**] time = 0.03, size = 136, normalized size = 5.67

$$-\frac{a^8 x^{-9n}}{9n}-\frac{a^7 b\, x^{-8n}}{n}-\frac{4a^6b^2x^{-7n}}{n}-\frac{28a^5b^3x^{-6n}}{3n}-\frac{14a^4b^4x^{-5n}}{n}-\frac{14a^3b^5x^{-4n}}{n}-\frac{28a^2b^6x^{-3n}}{3n}-\frac{4a\,b^7x^{-2n}}{n}-\frac{b^8x^{-n}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-9*n)*(b*x^n+a)^8,x)
```

```
[Out] -b^8/n/(x^n)-4*a*b^7/n/(x^n)^2-28/3*a^2*b^6/n/(x^n)^3-14*a^3*b^5/n/(x^n)^4-
14*a^4*b^4/n/(x^n)^5-28/3*a^5*b^3/n/(x^n)^6-4*a^6*b^2/n/(x^n)^7-a^7*b/n/(x^
n)^8-1/9*a^8/n/(x^n)^9
```

maxima [**B**] time = 0.50, size = 151, normalized size = 6.29

$$-\frac{a^8}{9\,nx^{9\,n}}-\frac{a^7b}{nx^{8\,n}}-\frac{4\,a^6b^2}{nx^{7\,n}}-\frac{28\,a^5b^3}{3\,nx^{6\,n}}-\frac{14\,a^4b^4}{nx^{5\,n}}-\frac{14\,a^3b^5}{nx^{4\,n}}-\frac{28\,a^2b^6}{3\,nx^{3\,n}}-\frac{4\,ab^7}{nx^{2\,n}}-\frac{b^8}{nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-9*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

```
[Out] -1/9*a^8/(n*x^(9*n)) - a^7*b/(n*x^(8*n)) - 4*a^6*b^2/(n*x^(7*n)) - 28/3*a^5
*b^3/(n*x^(6*n)) - 14*a^4*b^4/(n*x^(5*n)) - 14*a^3*b^5/(n*x^(4*n)) - 28/3*a
^2*b^6/(n*x^(3*n)) - 4*a*b^7/(n*x^(2*n)) - b^8/(n*x^n)
```

mupad [**B**] time = 1.46, size = 151, normalized size = 6.29

$$-\frac{a^8}{9\,n\,x^{9\,n}}-\frac{b^8}{n\,x^n}-\frac{28\,a^2\,b^6}{3\,n\,x^{3\,n}}-\frac{14\,a^3\,b^5}{n\,x^{4\,n}}-\frac{14\,a^4\,b^4}{n\,x^{5\,n}}-\frac{28\,a^5\,b^3}{3\,n\,x^{6\,n}}-\frac{4\,a^6\,b^2}{n\,x^{7\,n}}-\frac{4\,a\,b^7}{n\,x^{2\,n}}-\frac{a^7\,b}{n\,x^{8\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^8/x^(9*n + 1),x)
```

```
[Out] - a^8/(9*n*x^(9*n)) - b^8/(n*x^n) - (28*a^2*b^6)/(3*n*x^(3*n)) - (14*a^3*b^
5)/(n*x^(4*n)) - (14*a^4*b^4)/(n*x^(5*n)) - (28*a^5*b^3)/(3*n*x^(6*n)) - (4
*a^6*b^2)/(n*x^(7*n)) - (4*a*b^7)/(n*x^(2*n)) - (a^7*b)/(n*x^(8*n))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-9*n)*(a+b*x**n)**8,x)
```

```
[Out] Timed out
```

3.2585 $\int x^{-1-10n}\left(a+bx^n\right)^8\,dx$

Optimal. Leaf size=50

$$\frac{bx^{-9n}\left(a+bx^n\right)^9}{90a^2n}-\frac{x^{-10n}\left(a+bx^n\right)^9}{10an}$$

```
[Out] -1/10*(a+b*x^n)^9/a/n/(x^(10*n))+1/90*b*(a+b*x^n)^9/a^2/n/(x^(9*n))
```

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {266, 45, 37}

$$\frac{bx^{-9n}\left(a+bx^n\right)^9}{90a^2n}-\frac{x^{-10n}\left(a+bx^n\right)^9}{10an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 10*n)*(a + b*x^n)^8,x]
```

```
[Out] -(a + b*x^n)^9/(10*a*n*x^(10*n)) + (b*(a + b*x^n)^9)/(90*a^2*n*x^(9*n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1-10n}\left(a+bx^n\right)^8\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^8}{x^{11}}\,dx,x,x^n\right)}{n}\\ &= -\frac{x^{-10n}\left(a+bx^n\right)^9}{10an}-\frac{b\,\text{Subst}\left(\int \frac{(a+bx)^8}{x^{10}}\,dx,x,x^n\right)}{10an}\\ &= -\frac{x^{-10n}\left(a+bx^n\right)^9}{10an}+\frac{bx^{-9n}\left(a+bx^n\right)^9}{90a^2n}\end{aligned}$$

Mathematica [A] time = 0.04, size = 33, normalized size = 0.66

$$\frac{x^{-10n}\left(bx^n-9a\right)\left(a+bx^n\right)^9}{90a^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 10*n)*(a + b*x^n)^8,x]
```

```
[Out] ((-9*a + b*x^n)*(a + b*x^n)^9)/(90*a^2*n*x^(10*n))
```

fricas [B] time = 0.61, size = 113, normalized size = 2.26

$$-\frac{45\,b^8x^{8\,n}+240\,ab^7x^{7\,n}+630\,a^2b^6x^{6\,n}+1008\,a^3b^5x^{5\,n}+1050\,a^4b^4x^{4\,n}+720\,a^5b^3x^{3\,n}+315\,a^6b^2x^{2\,n}+80\,a^7bx^n}{90\,nx^{10\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-10*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

```
[Out] -1/90*(45*b^8*x^(8*n) + 240*a*b^7*x^(7*n) + 630*a^2*b^6*x^(6*n) + 1008*a^3*
b^5*x^(5*n) + 1050*a^4*b^4*x^(4*n) + 720*a^5*b^3*x^(3*n) + 315*a^6*b^2*x^(2
*n) + 80*a^7*b*x^n + 9*a^8)/(n*x^(10*n))
```

giac [B] time = 0.31, size = 113, normalized size = 2.26

$$-\frac{45\,b^8x^{8\,n}+240\,ab^7x^{7\,n}+630\,a^2b^6x^{6\,n}+1008\,a^3b^5x^{5\,n}+1050\,a^4b^4x^{4\,n}+720\,a^5b^3x^{3\,n}+315\,a^6b^2x^{2\,n}+80\,a^7bx^n}{90\,nx^{10\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-10*n)*(a+b*x^n)^8,x, algorithm="giac")
```

```
[Out] -1/90*(45*b^8*x^(8*n) + 240*a*b^7*x^(7*n) + 630*a^2*b^6*x^(6*n) + 1008*a^3*
b^5*x^(5*n) + 1050*a^4*b^4*x^(4*n) + 720*a^5*b^3*x^(3*n) + 315*a^6*b^2*x^(2
*n) + 80*a^7*b*x^n + 9*a^8)/(n*x^(10*n))
```

maple [B] time = 0.03, size = 136, normalized size = 2.72

$$-\frac{a^8x^{-10n}}{10n}-\frac{8a^7b\,x^{-9n}}{9n}-\frac{7a^6b^2x^{-8n}}{2n}-\frac{8a^5b^3x^{-7n}}{n}-\frac{35a^4b^4x^{-6n}}{3n}-\frac{56a^3b^5x^{-5n}}{5n}-\frac{7a^2b^6x^{-4n}}{n}-\frac{8a\,b^7x^{-3n}}{3n}-\frac{b^8x^{-2n}}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-10*n)*(b*x^n+a)^8,x)
```

```
[Out] -1/2*b^8/n/(x^n)^2-8/3*a*b^7/n/(x^n)^3-7*a^2*b^6/n/(x^n)^4-56/5*a^3*b^5/n/(
x^n)^5-35/3*a^4*b^4/n/(x^n)^6-8*a^5*b^3/n/(x^n)^7-7/2*a^6*b^2/n/(x^n)^8-8/9
*a^7*b/n/(x^n)^9-1/10*a^8/n/(x^n)^10
```

maxima [B] time = 0.58, size = 153, normalized size = 3.06

$$-\frac{a^8}{10\,nx^{10\,n}}-\frac{8\,a^7b}{9\,nx^{9\,n}}-\frac{7\,a^6b^2}{2\,nx^{8\,n}}-\frac{8\,a^5b^3}{nx^{7\,n}}-\frac{35\,a^4b^4}{3\,nx^{6\,n}}-\frac{56\,a^3b^5}{5\,nx^{5\,n}}-\frac{7\,a^2b^6}{nx^{4\,n}}-\frac{8\,ab^7}{3\,nx^{3\,n}}-\frac{b^8}{2\,nx^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-10*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

```
[Out] -1/10*a^8/(n*x^(10*n)) - 8/9*a^7*b/(n*x^(9*n)) - 7/2*a^6*b^2/(n*x^(8*n)) -
8*a^5*b^3/(n*x^(7*n)) - 35/3*a^4*b^4/(n*x^(6*n)) - 56/5*a^3*b^5/(n*x^(5*n))
 - 7*a^2*b^6/(n*x^(4*n)) - 8/3*a*b^7/(n*x^(3*n)) - 1/2*b^8/(n*x^(2*n))
```

mupad [B] time = 1.46, size = 153, normalized size = 3.06

$$-\frac{a^8}{10\,n\,x^{10\,n}}-\frac{b^8}{2\,n\,x^{2\,n}}-\frac{7\,a^2\,b^6}{n\,x^{4\,n}}-\frac{56\,a^3\,b^5}{5\,n\,x^{5\,n}}-\frac{35\,a^4\,b^4}{3\,n\,x^{6\,n}}-\frac{8\,a^5\,b^3}{n\,x^{7\,n}}-\frac{7\,a^6\,b^2}{2\,n\,x^{8\,n}}-\frac{8\,a\,b^7}{3\,n\,x^{3\,n}}-\frac{8\,a^7\,b}{9\,n\,x^{9\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^8/x^(10*n + 1),x)
```

```
[Out] - a^8/(10*n*x^(10*n)) - b^8/(2*n*x^(2*n)) - (7*a^2*b^6)/(n*x^(4*n)) - (56*a
^3*b^5)/(5*n*x^(5*n)) - (35*a^4*b^4)/(3*n*x^(6*n)) - (8*a^5*b^3)/(n*x^(7*n)
) - (7*a^6*b^2)/(2*n*x^(8*n)) - (8*a*b^7)/(3*n*x^(3*n)) - (8*a^7*b)/(9*n*x^
(9*n))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-10*n)*(a+b*x**n)**8,x)
```

```
[Out] Timed out
```

3.2586 $\int x^{-1-11n}\left(a+bx^n\right)^8\,dx$

Optimal. Leaf size=77

$$-\frac{b^2x^{-9n}\left(a+bx^n\right)^9}{495a^3n}+\frac{bx^{-10n}\left(a+bx^n\right)^9}{55a^2n}-\frac{x^{-11n}\left(a+bx^n\right)^9}{11an}$$

```
[Out] -1/11*(a+b*x^n)^9/a/n/(x^(11*n))+1/55*b*(a+b*x^n)^9/a^2/n/(x^(10*n))-1/495*
b^2*(a+b*x^n)^9/a^3/n/(x^(9*n))
```

Rubi [A] time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {266, 45, 37}

$$-\frac{b^2x^{-9n}\left(a+bx^n\right)^9}{495a^3n}+\frac{bx^{-10n}\left(a+bx^n\right)^9}{55a^2n}-\frac{x^{-11n}\left(a+bx^n\right)^9}{11an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 11*n)*(a + b*x^n)^8,x]
```

```
[Out] -(a + b*x^n)^9/(11*a*n*x^(11*n)) + (b*(a + b*x^n)^9)/(55*a^2*n*x^(10*n)) -
(b^2*(a + b*x^n)^9)/(495*a^3*n*x^(9*n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^{-1-11n}\left(a+bx^n\right)^8\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^8}{x^{12}}\,dx,x,x^n\right)}{n}\\
&= -\frac{x^{-11n}\left(a+bx^n\right)^9}{11an}-\frac{(2b)\,\text{Subst}\left(\int \frac{(a+bx)^8}{x^{11}}\,dx,x,x^n\right)}{11an}\\
&= -\frac{x^{-11n}\left(a+bx^n\right)^9}{11an}+\frac{bx^{-10n}\left(a+bx^n\right)^9}{55a^2n}+\frac{b^2\,\text{Subst}\left(\int \frac{(a+bx)^8}{x^{10}}\,dx,x,x^n\right)}{55a^2n}\\
&= -\frac{x^{-11n}\left(a+bx^n\right)^9}{11an}+\frac{bx^{-10n}\left(a+bx^n\right)^9}{55a^2n}-\frac{b^2x^{-9n}\left(a+bx^n\right)^9}{495a^3n}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 0.60

$$-\frac{x^{-11n}(a+bx^n)^9\left(45a^2-9abx^n+b^2x^{2n}\right)}{495a^3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 11*n)*(a + b*x^n)^8,x]
```

```
[Out] -1/495*((a + b*x^n)^9*(45*a^2 - 9*a*b*x^n + b^2*x^(2*n)))/(a^3*n*x^(11*n))
```

fricas [A] time = 0.59, size = 113, normalized size = 1.47

$$-\frac{165\,b^8x^{8n}+990\,ab^7x^{7n}+2772\,a^2b^6x^{6n}+4620\,a^3b^5x^{5n}+4950\,a^4b^4x^{4n}+3465\,a^5b^3x^{3n}+1540\,a^6b^2x^{2n}+3}{495\,nx^{11n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-11*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

```
[Out] -1/495*(165*b^8*x^(8*n) + 990*a*b^7*x^(7*n) + 2772*a^2*b^6*x^(6*n) + 4620*a
^3*b^5*x^(5*n) + 4950*a^4*b^4*x^(4*n) + 3465*a^5*b^3*x^(3*n) + 1540*a^6*b^2
*x^(2*n) + 396*a^7*b*x^n + 45*a^8)/(n*x^(11*n))
```

giac [A] time = 0.33, size = 113, normalized size = 1.47

$$-\frac{165\,b^8x^{8n}+990\,ab^7x^{7n}+2772\,a^2b^6x^{6n}+4620\,a^3b^5x^{5n}+4950\,a^4b^4x^{4n}+3465\,a^5b^3x^{3n}+1540\,a^6b^2x^{2n}+3}{495\,nx^{11n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-11*n)*(a+b*x^n)^8,x, algorithm="giac")
```

```
[Out] -1/495*(165*b^8*x^(8*n) + 990*a*b^7*x^(7*n) + 2772*a^2*b^6*x^(6*n) + 4620*a
^3*b^5*x^(5*n) + 4950*a^4*b^4*x^(4*n) + 3465*a^5*b^3*x^(3*n) + 1540*a^6*b^2
*x^(2*n) + 396*a^7*b*x^n + 45*a^8)/(n*x^(11*n))
```

maple [A] time = 0.02, size = 136, normalized size = 1.77

$$-\frac{a^8x^{-11n}}{11n}-\frac{4a^7b\,x^{-10n}}{5n}-\frac{28a^6b^2x^{-9n}}{9n}-\frac{7a^5b^3x^{-8n}}{n}-\frac{10a^4b^4x^{-7n}}{n}-\frac{28a^3b^5x^{-6n}}{3n}-\frac{28a^2b^6x^{-5n}}{5n}-\frac{2a\,b^7x^{-4n}}{n}-\frac{b^8x^{-3n}}{3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-11*n)*(b*x^n+a)^8,x)
```

```
[Out] -1/3*b^8/n/(x^n)^3-2*a*b^7/n/(x^n)^4-28/5*a^2*b^6/n/(x^n)^5-28/3*a^3*b^5/n/
(x^n)^6-10*a^4*b^4/n/(x^n)^7-7*a^5*b^3/n/(x^n)^8-28/9*a^6*b^2/n/(x^n)^9-4/5
*a^7*b/n/(x^n)^10-1/11*a^8/n/(x^n)^11
```

maxima [A] time = 0.61, size = 153, normalized size = 1.99

$$-\frac{a^8}{11\,nx^{11n}}-\frac{4\,a^7b}{5\,nx^{10n}}-\frac{28\,a^6b^2}{9\,nx^{9n}}-\frac{7\,a^5b^3}{nx^{8n}}-\frac{10\,a^4b^4}{nx^{7n}}-\frac{28\,a^3b^5}{3\,nx^{6n}}-\frac{28\,a^2b^6}{5\,nx^{5n}}-\frac{2\,ab^7}{nx^{4n}}-\frac{b^8}{3\,nx^{3n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-11*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

```
[Out] -1/11*a^8/(n*x^(11*n)) - 4/5*a^7*b/(n*x^(10*n)) - 28/9*a^6*b^2/(n*x^(9*n))
- 7*a^5*b^3/(n*x^(8*n)) - 10*a^4*b^4/(n*x^(7*n)) - 28/3*a^3*b^5/(n*x^(6*n))
 - 28/5*a^2*b^6/(n*x^(5*n)) - 2*a*b^7/(n*x^(4*n)) - 1/3*b^8/(n*x^(3*n))
```

mupad [**B**] time = 1.42, size = 153, normalized size = 1.99

$$-\frac{a^8}{11\,n\,x^{11\,n}}-\frac{b^8}{3\,n\,x^{3\,n}}-\frac{28\,a^2\,b^6}{5\,n\,x^{5\,n}}-\frac{28\,a^3\,b^5}{3\,n\,x^{6\,n}}-\frac{10\,a^4\,b^4}{n\,x^{7\,n}}-\frac{7\,a^5\,b^3}{n\,x^{8\,n}}-\frac{28\,a^6\,b^2}{9\,n\,x^{9\,n}}-\frac{2\,a\,b^7}{n\,x^{4\,n}}-\frac{4\,a^7\,b}{5\,n\,x^{10\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^8/x^(11*n + 1),x)
```

```
[Out] - a^8/(11*n*x^(11*n)) - b^8/(3*n*x^(3*n)) - (28*a^2*b^6)/(5*n*x^(5*n)) - (2
8*a^3*b^5)/(3*n*x^(6*n)) - (10*a^4*b^4)/(n*x^(7*n)) - (7*a^5*b^3)/(n*x^(8*n
)) - (28*a^6*b^2)/(9*n*x^(9*n)) - (2*a*b^7)/(n*x^(4*n)) - (4*a^7*b)/(5*n*x^
(10*n))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-11*n)*(a+b*x**n)**8,x)
```

```
[Out] Timed out
```

3.2587 $\int x^{-1-12n}\left(a+bx^n\right)^8\,dx$

Optimal. Leaf size=104

$$\frac{b^3x^{-9n}\left(a+bx^n\right)^9}{1980a^4n}-\frac{b^2x^{-10n}\left(a+bx^n\right)^9}{220a^3n}+\frac{bx^{-11n}\left(a+bx^n\right)^9}{44a^2n}-\frac{x^{-12n}\left(a+bx^n\right)^9}{12an}$$

```
[Out] -1/12*(a+b*x^n)^9/a/n/(x^(12*n))+1/44*b*(a+b*x^n)^9/a^2/n/(x^(11*n))-1/220*
b^2*(a+b*x^n)^9/a^3/n/(x^(10*n))+1/1980*b^3*(a+b*x^n)^9/a^4/n/(x^(9*n))
```

Rubi [A] time = 0.04, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {266, 45, 37}

$$-\frac{b^2x^{-10n}\left(a+bx^n\right)^9}{220a^3n}+\frac{b^3x^{-9n}\left(a+bx^n\right)^9}{1980a^4n}+\frac{bx^{-11n}\left(a+bx^n\right)^9}{44a^2n}-\frac{x^{-12n}\left(a+bx^n\right)^9}{12an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 12*n)*(a + b*x^n)^8,x]
```

```
[Out] -(a + b*x^n)^9/(12*a*n*x^(12*n)) + (b*(a + b*x^n)^9)/(44*a^2*n*x^(11*n)) -
(b^2*(a + b*x^n)^9)/(220*a^3*n*x^(10*n)) + (b^3*(a + b*x^n)^9)/(1980*a^4*n*
x^(9*n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^{-1-12n}\left(a+bx^n\right)^8\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx)^8}{x^{13}}\,dx, x, x^n\right)}{n}$$
$$= -\frac{x^{-12n}\left(a+bx^n\right)^9}{12an} - \frac{b\,\text{Subst}\left(\int \frac{(a+bx)^8}{x^{12}}\,dx, x, x^n\right)}{4an}$$
$$= -\frac{x^{-12n}\left(a+bx^n\right)^9}{12an} + \frac{bx^{-11n}\left(a+bx^n\right)^9}{44a^2n} + \frac{b^2\,\text{Subst}\left(\int \frac{(a+bx)^8}{x^{11}}\,dx, x, x^n\right)}{22a^2n}$$
$$= -\frac{x^{-12n}\left(a+bx^n\right)^9}{12an} + \frac{bx^{-11n}\left(a+bx^n\right)^9}{44a^2n} - \frac{b^2x^{-10n}\left(a+bx^n\right)^9}{220a^3n} - \frac{b^3\,\text{Subst}\left(\int \frac{(a+bx)^8}{x^{10}}\,dx, x\right.}{220a^3n}$$
$$= -\frac{x^{-12n}\left(a+bx^n\right)^9}{12an} + \frac{bx^{-11n}\left(a+bx^n\right)^9}{44a^2n} - \frac{b^2x^{-10n}\left(a+bx^n\right)^9}{220a^3n} + \frac{b^3x^{-9n}\left(a+bx^n\right)^9}{1980a^4n}$$

Mathematica [A] time = 0.02, size = 59, normalized size = 0.57

$$\frac{x^{-12n}\left(a+bx^n\right)^9\left(-165a^3+45a^2bx^n-9ab^2x^{2n}+b^3x^{3n}\right)}{1980a^4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 12*n)*(a + b*x^n)^8,x]
```

```
[Out] ((a + b*x^n)^9*(-165*a^3 + 45*a^2*b*x^n - 9*a*b^2*x^(2*n) + b^3*x^(3*n)))/(
1980*a^4*n*x^(12*n))
```

fricas [A] time = 0.60, size = 113, normalized size = 1.09

$$-\frac{495\,b^8x^{8\,n}+3168\,ab^7x^{7\,n}+9240\,a^2b^6x^{6\,n}+15840\,a^3b^5x^{5\,n}+17325\,a^4b^4x^{4\,n}+12320\,a^5b^3x^{3\,n}+5544\,a^6b^2x^{2\,n}+}{1980\,nx^{12\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-12*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

```
[Out] -1/1980*(495*b^8*x^(8*n) + 3168*a*b^7*x^(7*n) + 9240*a^2*b^6*x^(6*n) + 1584
0*a^3*b^5*x^(5*n) + 17325*a^4*b^4*x^(4*n) + 12320*a^5*b^3*x^(3*n) + 5544*a^
6*b^2*x^(2*n) + 1440*a^7*b*x^n + 165*a^8)/(n*x^(12*n))
```

giac [A] time = 0.45, size = 113, normalized size = 1.09

$$-\frac{495\,b^8x^{8\,n}+3168\,ab^7x^{7\,n}+9240\,a^2b^6x^{6\,n}+15840\,a^3b^5x^{5\,n}+17325\,a^4b^4x^{4\,n}+12320\,a^5b^3x^{3\,n}+5544\,a^6b^2x^{2\,n}+}{1980\,nx^{12\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-12*n)*(a+b*x^n)^8,x, algorithm="giac")
```

```
[Out] -1/1980*(495*b^8*x^(8*n) + 3168*a*b^7*x^(7*n) + 9240*a^2*b^6*x^(6*n) + 1584
0*a^3*b^5*x^(5*n) + 17325*a^4*b^4*x^(4*n) + 12320*a^5*b^3*x^(3*n) + 5544*a^
6*b^2*x^(2*n) + 1440*a^7*b*x^n + 165*a^8)/(n*x^(12*n))
```

maple [A] time = 0.03, size = 136, normalized size = 1.31

$$-\frac{a^8x^{-12n}}{12n}-\frac{8a^7b\,x^{-11n}}{11n}-\frac{14a^6b^2x^{-10n}}{5n}-\frac{56a^5b^3x^{-9n}}{9n}-\frac{35a^4b^4x^{-8n}}{4n}-\frac{8a^3b^5x^{-7n}}{n}-\frac{14a^2b^6x^{-6n}}{3n}-\frac{8a\,b^7x^{-5n}}{5n}-\frac{b^8x^{-4n}}{4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-12*n)*(b*x^n+a)^8,x)
```

```
[Out] -1/4*b^8/n/(x^n)^4-8/5*a*b^7/n/(x^n)^5-14/3*a^2*b^6/n/(x^n)^6-8*a^3*b^5/n/(
x^n)^7-35/4*a^4*b^4/n/(x^n)^8-56/9*a^5*b^3/n/(x^n)^9-14/5*a^6*b^2/n/(x^n)^1
0-8/11*a^7*b/n/(x^n)^11-1/12*a^8/n/(x^n)^12
```

maxima [**A**] time = 0.53, size = 153, normalized size = 1.47

$$-\frac{a^8}{12\,nx^{12\,n}}-\frac{8\,a^7b}{11\,nx^{11\,n}}-\frac{14\,a^6b^2}{5\,nx^{10\,n}}-\frac{56\,a^5b^3}{9\,nx^{9\,n}}-\frac{35\,a^4b^4}{4\,nx^{8\,n}}-\frac{8\,a^3b^5}{nx^{7\,n}}-\frac{14\,a^2b^6}{3\,nx^{6\,n}}-\frac{8\,ab^7}{5\,nx^{5\,n}}-\frac{b^8}{4\,nx^{4\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-12*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

```
[Out] -1/12*a^8/(n*x^(12*n)) - 8/11*a^7*b/(n*x^(11*n)) - 14/5*a^6*b^2/(n*x^(10*n)
) - 56/9*a^5*b^3/(n*x^(9*n)) - 35/4*a^4*b^4/(n*x^(8*n)) - 8*a^3*b^5/(n*x^(7
*n)) - 14/3*a^2*b^6/(n*x^(6*n)) - 8/5*a*b^7/(n*x^(5*n)) - 1/4*b^8/(n*x^(4*n
))
```

mupad [**B**] time = 1.43, size = 153, normalized size = 1.47

$$-\frac{a^8}{12\,n\,x^{12\,n}}-\frac{b^8}{4\,n\,x^{4\,n}}-\frac{14\,a^2\,b^6}{3\,n\,x^{6\,n}}-\frac{8\,a^3\,b^5}{n\,x^{7\,n}}-\frac{35\,a^4\,b^4}{4\,n\,x^{8\,n}}-\frac{56\,a^5\,b^3}{9\,n\,x^{9\,n}}-\frac{14\,a^6\,b^2}{5\,n\,x^{10\,n}}-\frac{8\,a\,b^7}{5\,n\,x^{5\,n}}-\frac{8\,a^7\,b}{11\,n\,x^{11\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^8/x^(12*n + 1),x)
```

```
[Out] - a^8/(12*n*x^(12*n)) - b^8/(4*n*x^(4*n)) - (14*a^2*b^6)/(3*n*x^(6*n)) - (8
*a^3*b^5)/(n*x^(7*n)) - (35*a^4*b^4)/(4*n*x^(8*n)) - (56*a^5*b^3)/(9*n*x^(9
*n)) - (14*a^6*b^2)/(5*n*x^(10*n)) - (8*a*b^7)/(5*n*x^(5*n)) - (8*a^7*b)/(1
1*n*x^(11*n))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-12*n)*(a+b*x**n)**8,x)
```

```
[Out] Timed out
```

3.2588 $\int x^{-1-13n}\left(a+bx^n\right)^8\,dx$

Optimal. Leaf size=131

$$-\frac{b^4x^{-9n}\left(a+bx^n\right)^9}{6435a^5n}+\frac{b^3x^{-10n}\left(a+bx^n\right)^9}{715a^4n}-\frac{b^2x^{-11n}\left(a+bx^n\right)^9}{143a^3n}+\frac{bx^{-12n}\left(a+bx^n\right)^9}{39a^2n}-\frac{x^{-13n}\left(a+bx^n\right)^9}{13an}$$

```
[Out] -1/13*(a+b*x^n)^9/a/n/(x^(13*n))+1/39*b*(a+b*x^n)^9/a^2/n/(x^(12*n))-1/143*
b^2*(a+b*x^n)^9/a^3/n/(x^(11*n))+1/715*b^3*(a+b*x^n)^9/a^4/n/(x^(10*n))-1/6
435*b^4*(a+b*x^n)^9/a^5/n/(x^(9*n))
```

Rubi [A] time = 0.05, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {266, 45, 37}

$$-\frac{b^2x^{-11n}\left(a+bx^n\right)^9}{143a^3n}+\frac{b^3x^{-10n}\left(a+bx^n\right)^9}{715a^4n}-\frac{b^4x^{-9n}\left(a+bx^n\right)^9}{6435a^5n}+\frac{bx^{-12n}\left(a+bx^n\right)^9}{39a^2n}-\frac{x^{-13n}\left(a+bx^n\right)^9}{13an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 13*n)*(a + b*x^n)^8,x]
```

```
[Out] -(a + b*x^n)^9/(13*a*n*x^(13*n)) + (b*(a + b*x^n)^9)/(39*a^2*n*x^(12*n)) -
(b^2*(a + b*x^n)^9)/(143*a^3*n*x^(11*n)) + (b^3*(a + b*x^n)^9)/(715*a^4*n*x
^(10*n)) - (b^4*(a + b*x^n)^9)/(6435*a^5*n*x^(9*n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^{-1-13n}\left(a+bx^n\right)^8\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx)^8}{x^{14}}\,dx, x, x^n\right)}{n}$$
$$= -\frac{x^{-13n}\left(a+bx^n\right)^9}{13an} - \frac{(4b)\,\text{Subst}\left(\int \frac{(a+bx)^8}{x^{13}}\,dx, x, x^n\right)}{13an}$$
$$= -\frac{x^{-13n}\left(a+bx^n\right)^9}{13an} + \frac{bx^{-12n}\left(a+bx^n\right)^9}{39a^2n} + \frac{b^2\,\text{Subst}\left(\int \frac{(a+bx)^8}{x^{12}}\,dx, x, x^n\right)}{13a^2n}$$
$$= -\frac{x^{-13n}\left(a+bx^n\right)^9}{13an} + \frac{bx^{-12n}\left(a+bx^n\right)^9}{39a^2n} - \frac{b^2x^{-11n}\left(a+bx^n\right)^9}{143a^3n} - \frac{\left(2b^3\right)\text{Subst}\left(\int \frac{(a+bx)}{x^{11}}\right.}{143a^3n}$$
$$= -\frac{x^{-13n}\left(a+bx^n\right)^9}{13an} + \frac{bx^{-12n}\left(a+bx^n\right)^9}{39a^2n} - \frac{b^2x^{-11n}\left(a+bx^n\right)^9}{143a^3n} + \frac{b^3x^{-10n}\left(a+bx^n\right)^9}{715a^4n} +$$
$$= -\frac{x^{-13n}\left(a+bx^n\right)^9}{13an} + \frac{bx^{-12n}\left(a+bx^n\right)^9}{39a^2n} - \frac{b^2x^{-11n}\left(a+bx^n\right)^9}{143a^3n} + \frac{b^3x^{-10n}\left(a+bx^n\right)^9}{715a^4n} -$$

Mathematica [A] time = 0.06, size = 113, normalized size = 0.86

$$-\frac{x^{-13n}\left(495a^8 + 4290a^7bx^n + 16380a^6b^2x^{2n} + 36036a^5b^3x^{3n} + 50050a^4b^4x^{4n} + 45045a^3b^5x^{5n} + 25740a^2b^6x^6\right.}{6435n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 13*n)*(a + b*x^n)^8,x]
```

```
[Out] -1/6435*(495*a^8 + 4290*a^7*b*x^n + 16380*a^6*b^2*x^(2*n) + 36036*a^5*b^3*x
^(3*n) + 50050*a^4*b^4*x^(4*n) + 45045*a^3*b^5*x^(5*n) + 25740*a^2*b^6*x^(6
*n) + 8580*a*b^7*x^(7*n) + 1287*b^8*x^(8*n))/(n*x^(13*n))
```

fricas [A] time = 0.71, size = 113, normalized size = 0.86

$$-\frac{1287\,b^8x^{8\,n} + 8580\,ab^7x^{7\,n} + 25740\,a^2b^6x^{6\,n} + 45045\,a^3b^5x^{5\,n} + 50050\,a^4b^4x^{4\,n} + 36036\,a^5b^3x^{3\,n} + 16380\,a^6b}{6435\,nx^{13\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-13*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

```
[Out] -1/6435*(1287*b^8*x^(8*n) + 8580*a*b^7*x^(7*n) + 25740*a^2*b^6*x^(6*n) + 45
045*a^3*b^5*x^(5*n) + 50050*a^4*b^4*x^(4*n) + 36036*a^5*b^3*x^(3*n) + 16380
*a^6*b^2*x^(2*n) + 4290*a^7*b*x^n + 495*a^8)/(n*x^(13*n))
```

giac [A] time = 0.31, size = 113, normalized size = 0.86

$$-\frac{1287\,b^8x^{8\,n} + 8580\,ab^7x^{7\,n} + 25740\,a^2b^6x^{6\,n} + 45045\,a^3b^5x^{5\,n} + 50050\,a^4b^4x^{4\,n} + 36036\,a^5b^3x^{3\,n} + 16380\,a^6b}{6435\,nx^{13\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-13*n)*(a+b*x^n)^8,x, algorithm="giac")
```

```
[Out] -1/6435*(1287*b^8*x^(8*n) + 8580*a*b^7*x^(7*n) + 25740*a^2*b^6*x^(6*n) + 45
045*a^3*b^5*x^(5*n) + 50050*a^4*b^4*x^(4*n) + 36036*a^5*b^3*x^(3*n) + 16380
*a^6*b^2*x^(2*n) + 4290*a^7*b*x^n + 495*a^8)/(n*x^(13*n))
```

maple [A] time = 0.03, size = 136, normalized size = 1.04

$$-\frac{a^8x^{-13n}}{13n} - \frac{2a^7b\,x^{-12n}}{3n} - \frac{28a^6b^2x^{-11n}}{11n} - \frac{28a^5b^3x^{-10n}}{5n} - \frac{70a^4b^4x^{-9n}}{9n} - \frac{7a^3b^5x^{-8n}}{n} - \frac{4a^2b^6x^{-7n}}{n} - \frac{4a\,b^7x^{-6n}}{3n} - \frac{b^8x^{-5n}}{5n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-13*n)*(b*x^n+a)^8,x)
```

```
[Out] -1/5*b^8/n/(x^n)^5-4/3*a*b^7/n/(x^n)^6-4*a^2*b^6/n/(x^n)^7-7*a^3*b^5/n/(x^n
)^8-70/9*a^4*b^4/n/(x^n)^9-28/5*a^5*b^3/n/(x^n)^10-28/11*a^6*b^2/n/(x^n)^11
-2/3*a^7*b/n/(x^n)^12-1/13*a^8/n/(x^n)^13
```

maxima [A] time = 0.51, size = 153, normalized size = 1.17

$$-\frac{a^8}{13\,nx^{13\,n}}-\frac{2\,a^7b}{3\,nx^{12\,n}}-\frac{28\,a^6b^2}{11\,nx^{11\,n}}-\frac{28\,a^5b^3}{5\,nx^{10\,n}}-\frac{70\,a^4b^4}{9\,nx^{9\,n}}-\frac{7\,a^3b^5}{nx^{8\,n}}-\frac{4\,a^2b^6}{nx^{7\,n}}-\frac{4\,ab^7}{3\,nx^{6\,n}}-\frac{b^8}{5\,nx^{5\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-13*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

```
[Out] -1/13*a^8/(n*x^(13*n)) - 2/3*a^7*b/(n*x^(12*n)) - 28/11*a^6*b^2/(n*x^(11*n)
) - 28/5*a^5*b^3/(n*x^(10*n)) - 70/9*a^4*b^4/(n*x^(9*n)) - 7*a^3*b^5/(n*x^(
8*n)) - 4*a^2*b^6/(n*x^(7*n)) - 4/3*a*b^7/(n*x^(6*n)) - 1/5*b^8/(n*x^(5*n))
```

mupad [B] time = 1.44, size = 153, normalized size = 1.17

$$-\frac{a^8}{13\,n\,x^{13\,n}}-\frac{b^8}{5\,n\,x^{5\,n}}-\frac{4\,a^2\,b^6}{n\,x^{7\,n}}-\frac{7\,a^3\,b^5}{n\,x^{8\,n}}-\frac{70\,a^4\,b^4}{9\,n\,x^{9\,n}}-\frac{28\,a^5\,b^3}{5\,n\,x^{10\,n}}-\frac{28\,a^6\,b^2}{11\,n\,x^{11\,n}}-\frac{4\,a\,b^7}{3\,n\,x^{6\,n}}-\frac{2\,a^7\,b}{3\,n\,x^{12\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^8/x^(13*n + 1),x)
```

```
[Out] - a^8/(13*n*x^(13*n)) - b^8/(5*n*x^(5*n)) - (4*a^2*b^6)/(n*x^(7*n)) - (7*a^
3*b^5)/(n*x^(8*n)) - (70*a^4*b^4)/(9*n*x^(9*n)) - (28*a^5*b^3)/(5*n*x^(10*n
)) - (28*a^6*b^2)/(11*n*x^(11*n)) - (4*a*b^7)/(3*n*x^(6*n)) - (2*a^7*b)/(3*
n*x^(12*n))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-13*n)*(a+b*x**n)**8,x)
```

```
[Out] Timed out
```

3.2589 $\int x^{-1-14n}\left(a+bx^n\right)^8\,dx$

Optimal. Leaf size=151

$$-\frac{a^8x^{-14n}}{14n}-\frac{8a^7bx^{-13n}}{13n}-\frac{7a^6b^2x^{-12n}}{3n}-\frac{56a^5b^3x^{-11n}}{11n}-\frac{7a^4b^4x^{-10n}}{n}-\frac{56a^3b^5x^{-9n}}{9n}-\frac{7a^2b^6x^{-8n}}{2n}-\frac{8ab^7x^{-7n}}{7n}-\frac{b^8x^{-6n}}{6n}$$

```
[Out] -1/14*a^8/n/(x^(14*n))-8/13*a^7*b/n/(x^(13*n))-7/3*a^6*b^2/n/(x^(12*n))-56/
11*a^5*b^3/n/(x^(11*n))-7*a^4*b^4/n/(x^(10*n))-56/9*a^3*b^5/n/(x^(9*n))-7/2
*a^2*b^6/n/(x^(8*n))-8/7*a*b^7/n/(x^(7*n))-1/6*b^8/n/(x^(6*n))
```

Rubi [A] time = 0.06, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$-\frac{7a^6b^2x^{-12n}}{3n}-\frac{56a^5b^3x^{-11n}}{11n}-\frac{7a^4b^4x^{-10n}}{n}-\frac{56a^3b^5x^{-9n}}{9n}-\frac{7a^2b^6x^{-8n}}{2n}-\frac{8a^7bx^{-13n}}{13n}-\frac{a^8x^{-14n}}{14n}-\frac{8ab^7x^{-7n}}{7n}-\frac{b^8x^{-6n}}{6n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 14*n)*(a + b*x^n)^8,x]
```

```
[Out] -a^8/(14*n*x^(14*n)) - (8*a^7*b)/(13*n*x^(13*n)) - (7*a^6*b^2)/(3*n*x^(12*n
)) - (56*a^5*b^3)/(11*n*x^(11*n)) - (7*a^4*b^4)/(n*x^(10*n)) - (56*a^3*b^5)
/(9*n*x^(9*n)) - (7*a^2*b^6)/(2*n*x^(8*n)) - (8*a*b^7)/(7*n*x^(7*n)) - b^8/
(6*n*x^(6*n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1-14n}\left(a+bx^n\right)^8\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^8}{x^{15}}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a^8}{x^{15}}+\frac{8a^7b}{x^{14}}+\frac{28a^6b^2}{x^{13}}+\frac{56a^5b^3}{x^{12}}+\frac{70a^4b^4}{x^{11}}+\frac{56a^3b^5}{x^{10}}+\frac{28a^2b^6}{x^9}+\frac{8ab^7}{x^8}+\frac{b^8}{x^7}\right)dx,x,\right.}{n}\\ &= -\frac{a^8x^{-14n}}{14n}-\frac{8a^7bx^{-13n}}{13n}-\frac{7a^6b^2x^{-12n}}{3n}-\frac{56a^5b^3x^{-11n}}{11n}-\frac{7a^4b^4x^{-10n}}{n}-\frac{56a^3b^5x^{-9n}}{9n}-\end{aligned}$$

Mathematica [A] time = 0.05, size = 113, normalized size = 0.75

$$-\frac{x^{-14n}\left(1287a^8+11088a^7bx^n+42042a^6b^2x^{2n}+91728a^5b^3x^{3n}+126126a^4b^4x^{4n}+112112a^3b^5x^{5n}+63063a^2b\right.}{18018n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 14*n)*(a + b*x^n)^8,x]
```

```
[Out] -1/18018*(1287*a^8 + 11088*a^7*b*x^n + 42042*a^6*b^2*x^(2*n) + 91728*a^5*b^
3*x^(3*n) + 126126*a^4*b^4*x^(4*n) + 112112*a^3*b^5*x^(5*n) + 63063*a^2*b^6
*x^(6*n) + 20592*a*b^7*x^(7*n) + 3003*b^8*x^(8*n))/(n*x^(14*n))
```

fricas [A] time = 0.64, size = 113, normalized size = 0.75

$$-\frac{3003\,b^8x^{8\,n}+20592\,ab^7x^{7\,n}+63063\,a^2b^6x^{6\,n}+112112\,a^3b^5x^{5\,n}+126126\,a^4b^4x^{4\,n}+91728\,a^5b^3x^{3\,n}+42042\,a^6b}{18018\,nx^{14\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-14*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

```
[Out] -1/18018*(3003*b^8*x^(8*n) + 20592*a*b^7*x^(7*n) + 63063*a^2*b^6*x^(6*n) +
112112*a^3*b^5*x^(5*n) + 126126*a^4*b^4*x^(4*n) + 91728*a^5*b^3*x^(3*n) + 4
2042*a^6*b^2*x^(2*n) + 11088*a^7*b*x^n + 1287*a^8)/(n*x^(14*n))
```

giac [A] time = 0.29, size = 113, normalized size = 0.75

$$-\frac{3003\,b^8x^{8\,n}+20592\,ab^7x^{7\,n}+63063\,a^2b^6x^{6\,n}+112112\,a^3b^5x^{5\,n}+126126\,a^4b^4x^{4\,n}+91728\,a^5b^3x^{3\,n}+42042\,a^6b}{18018\,nx^{14\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-14*n)*(a+b*x^n)^8,x, algorithm="giac")
```

```
[Out] -1/18018*(3003*b^8*x^(8*n) + 20592*a*b^7*x^(7*n) + 63063*a^2*b^6*x^(6*n) +
112112*a^3*b^5*x^(5*n) + 126126*a^4*b^4*x^(4*n) + 91728*a^5*b^3*x^(3*n) + 4
2042*a^6*b^2*x^(2*n) + 11088*a^7*b*x^n + 1287*a^8)/(n*x^(14*n))
```

maple [A] time = 0.03, size = 136, normalized size = 0.90

$$-\frac{a^8x^{-14n}}{14n}-\frac{8a^7b\,x^{-13n}}{13n}-\frac{7a^6b^2x^{-12n}}{3n}-\frac{56a^5b^3x^{-11n}}{11n}-\frac{7a^4b^4x^{-10n}}{n}-\frac{56a^3b^5x^{-9n}}{9n}-\frac{7a^2b^6x^{-8n}}{2n}-\frac{8a\,b^7x^{-7n}}{7n}-\frac{b^8x^{-6n}}{6n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-14*n)*(b*x^n+a)^8,x)
```

```
[Out] -1/6*b^8/n/(x^n)^6-8/7*a*b^7/n/(x^n)^7-7/2*a^2*b^6/n/(x^n)^8-56/9*a^3*b^5/n
/(x^n)^9-7*a^4*b^4/n/(x^n)^10-56/11*a^5*b^3/n/(x^n)^11-7/3*a^6*b^2/n/(x^n)^
12-8/13*a^7*b/n/(x^n)^13-1/14*a^8/n/(x^n)^14
```

maxima [A] time = 0.56, size = 153, normalized size = 1.01

$$-\frac{a^8}{14\,nx^{14\,n}}-\frac{8\,a^7b}{13\,nx^{13\,n}}-\frac{7\,a^6b^2}{3\,nx^{12\,n}}-\frac{56\,a^5b^3}{11\,nx^{11\,n}}-\frac{7\,a^4b^4}{nx^{10\,n}}-\frac{56\,a^3b^5}{9\,nx^{9\,n}}-\frac{7\,a^2b^6}{2\,nx^{8\,n}}-\frac{8\,ab^7}{7\,nx^{7\,n}}-\frac{b^8}{6\,nx^{6\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-14*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

```
[Out] -1/14*a^8/(n*x^(14*n)) - 8/13*a^7*b/(n*x^(13*n)) - 7/3*a^6*b^2/(n*x^(12*n))
 - 56/11*a^5*b^3/(n*x^(11*n)) - 7*a^4*b^4/(n*x^(10*n)) - 56/9*a^3*b^5/(n*x^
(9*n)) - 7/2*a^2*b^6/(n*x^(8*n)) - 8/7*a*b^7/(n*x^(7*n)) - 1/6*b^8/(n*x^(6*
n))
```

mupad [B] time = 1.51, size = 153, normalized size = 1.01

$$-\frac{a^8}{14\,n\,x^{14\,n}}-\frac{b^8}{6\,n\,x^{6\,n}}-\frac{7\,a^2\,b^6}{2\,n\,x^{8\,n}}-\frac{56\,a^3\,b^5}{9\,n\,x^{9\,n}}-\frac{7\,a^4\,b^4}{n\,x^{10\,n}}-\frac{56\,a^5\,b^3}{11\,n\,x^{11\,n}}-\frac{7\,a^6\,b^2}{3\,n\,x^{12\,n}}-\frac{8\,a\,b^7}{7\,n\,x^{7\,n}}-\frac{8\,a^7\,b}{13\,n\,x^{13\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^8/x^(14*n + 1),x)
```

```
[Out] - a^8/(14*n*x^(14*n)) - b^8/(6*n*x^(6*n)) - (7*a^2*b^6)/(2*n*x^(8*n)) - (56
*a^3*b^5)/(9*n*x^(9*n)) - (7*a^4*b^4)/(n*x^(10*n)) - (56*a^5*b^3)/(11*n*x^(
11*n)) - (7*a^6*b^2)/(3*n*x^(12*n)) - (8*a*b^7)/(7*n*x^(7*n)) - (8*a^7*b)/(
13*n*x^(13*n))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-14*n)*(a+b*x**n)**8,x)
```

```
[Out] Timed out
```

3.2590 $\int x^{-1-15n}\left(a+bx^n\right)^8\,dx$

Optimal. Leaf size=151

$$-\frac{a^8x^{-15n}}{15n}-\frac{4a^7bx^{-14n}}{7n}-\frac{28a^6b^2x^{-13n}}{13n}-\frac{14a^5b^3x^{-12n}}{3n}-\frac{70a^4b^4x^{-11n}}{11n}-\frac{28a^3b^5x^{-10n}}{5n}-\frac{28a^2b^6x^{-9n}}{9n}-\frac{ab^7x^{-8n}}{n}-\frac{b^8x^{-7n}}{7n}$$

```
[Out] -1/15*a^8/n/(x^(15*n))-4/7*a^7*b/n/(x^(14*n))-28/13*a^6*b^2/n/(x^(13*n))-14
/3*a^5*b^3/n/(x^(12*n))-70/11*a^4*b^4/n/(x^(11*n))-28/5*a^3*b^5/n/(x^(10*n)
)-28/9*a^2*b^6/n/(x^(9*n))-a*b^7/n/(x^(8*n))-1/7*b^8/n/(x^(7*n))
```

Rubi [A] time = 0.06, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$-\frac{28a^6b^2x^{-13n}}{13n}-\frac{14a^5b^3x^{-12n}}{3n}-\frac{70a^4b^4x^{-11n}}{11n}-\frac{28a^3b^5x^{-10n}}{5n}-\frac{28a^2b^6x^{-9n}}{9n}-\frac{4a^7bx^{-14n}}{7n}-\frac{a^8x^{-15n}}{15n}-\frac{ab^7x^{-8n}}{n}-\frac{b^8x^{-7n}}{7n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 15*n)*(a + b*x^n)^8,x]
```

```
[Out] -a^8/(15*n*x^(15*n)) - (4*a^7*b)/(7*n*x^(14*n)) - (28*a^6*b^2)/(13*n*x^(13*
n)) - (14*a^5*b^3)/(3*n*x^(12*n)) - (70*a^4*b^4)/(11*n*x^(11*n)) - (28*a^3*
b^5)/(5*n*x^(10*n)) - (28*a^2*b^6)/(9*n*x^(9*n)) - (a*b^7)/(n*x^(8*n)) - b^
8/(7*n*x^(7*n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1-15n}\left(a+bx^n\right)^8\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^8}{x^{16}}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int\left(\frac{a^8}{x^{16}}+\frac{8a^7b}{x^{15}}+\frac{28a^6b^2}{x^{14}}+\frac{56a^5b^3}{x^{13}}+\frac{70a^4b^4}{x^{12}}+\frac{56a^3b^5}{x^{11}}+\frac{28a^2b^6}{x^{10}}+\frac{8ab^7}{x^9}+\frac{b^8}{x^8}\right)dx,x,x^n\right)}{n}\\ &= -\frac{a^8x^{-15n}}{15n}-\frac{4a^7bx^{-14n}}{7n}-\frac{28a^6b^2x^{-13n}}{13n}-\frac{14a^5b^3x^{-12n}}{3n}-\frac{70a^4b^4x^{-11n}}{11n}-\frac{28a^3b^5x^{-10n}}{5n}-\end{aligned}$$

Mathematica [A] time = 0.06, size = 113, normalized size = 0.75

$$-\frac{x^{-15n}\left(3003a^8+25740a^7bx^n+97020a^6b^2x^{2n}+210210a^5b^3x^{3n}+286650a^4b^4x^{4n}+252252a^3b^5x^{5n}+140140a^2\right.}{45045n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 15*n)*(a + b*x^n)^8,x]
```

```
[Out] -1/45045*(3003*a^8 + 25740*a^7*b*x^n + 97020*a^6*b^2*x^(2*n) + 210210*a^5*b
^3*x^(3*n) + 286650*a^4*b^4*x^(4*n) + 252252*a^3*b^5*x^(5*n) + 140140*a^2*b
^6*x^(6*n) + 45045*a*b^7*x^(7*n) + 6435*b^8*x^(8*n))/(n*x^(15*n))
```

fricas [A] time = 0.54, size = 113, normalized size = 0.75

$$-\frac{6435\,b^8x^{8\,n}+45045\,ab^7x^{7\,n}+140140\,a^2b^6x^{6\,n}+252252\,a^3b^5x^{5\,n}+286650\,a^4b^4x^{4\,n}+210210\,a^5b^3x^{3\,n}+970}{45045\,nx^{15\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-15*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

```
[Out] -1/45045*(6435*b^8*x^(8*n) + 45045*a*b^7*x^(7*n) + 140140*a^2*b^6*x^(6*n) +
 252252*a^3*b^5*x^(5*n) + 286650*a^4*b^4*x^(4*n) + 210210*a^5*b^3*x^(3*n) +
 97020*a^6*b^2*x^(2*n) + 25740*a^7*b*x^n + 3003*a^8)/(n*x^(15*n))
```

giac [A] time = 0.29, size = 113, normalized size = 0.75

$$-\frac{6435\,b^8x^{8\,n}+45045\,ab^7x^{7\,n}+140140\,a^2b^6x^{6\,n}+252252\,a^3b^5x^{5\,n}+286650\,a^4b^4x^{4\,n}+210210\,a^5b^3x^{3\,n}+970}{45045\,nx^{15\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-15*n)*(a+b*x^n)^8,x, algorithm="giac")
```

```
[Out] -1/45045*(6435*b^8*x^(8*n) + 45045*a*b^7*x^(7*n) + 140140*a^2*b^6*x^(6*n) +
 252252*a^3*b^5*x^(5*n) + 286650*a^4*b^4*x^(4*n) + 210210*a^5*b^3*x^(3*n) +
 97020*a^6*b^2*x^(2*n) + 25740*a^7*b*x^n + 3003*a^8)/(n*x^(15*n))
```

maple [A] time = 0.03, size = 136, normalized size = 0.90

$$-\frac{a^8x^{-15n}}{15n}-\frac{4a^7b\,x^{-14n}}{7n}-\frac{28a^6b^2x^{-13n}}{13n}-\frac{14a^5b^3x^{-12n}}{3n}-\frac{70a^4b^4x^{-11n}}{11n}-\frac{28a^3b^5x^{-10n}}{5n}-\frac{28a^2b^6x^{-9n}}{9n}-\frac{a\,b^7x^{-8n}}{n}-\frac{b^8x^{-7n}}{7n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-15*n)*(b*x^n+a)^8,x)
```

```
[Out] -1/7*b^8/n/(x^n)^7-a*b^7/n/(x^n)^8-28/9*a^2*b^6/n/(x^n)^9-28/5*a^3*b^5/n/(x
^n)^10-70/11*a^4*b^4/n/(x^n)^11-14/3*a^5*b^3/n/(x^n)^12-28/13*a^6*b^2/n/(x^
n)^13-4/7*a^7*b/n/(x^n)^14-1/15*a^8/n/(x^n)^15
```

maxima [A] time = 0.54, size = 153, normalized size = 1.01

$$-\frac{a^8}{15\,nx^{15\,n}}-\frac{4\,a^7b}{7\,nx^{14\,n}}-\frac{28\,a^6b^2}{13\,nx^{13\,n}}-\frac{14\,a^5b^3}{3\,nx^{12\,n}}-\frac{70\,a^4b^4}{11\,nx^{11\,n}}-\frac{28\,a^3b^5}{5\,nx^{10\,n}}-\frac{28\,a^2b^6}{9\,nx^{9\,n}}-\frac{ab^7}{nx^{8\,n}}-\frac{b^8}{7\,nx^{7\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-15*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

```
[Out] -1/15*a^8/(n*x^(15*n)) - 4/7*a^7*b/(n*x^(14*n)) - 28/13*a^6*b^2/(n*x^(13*n)
) - 14/3*a^5*b^3/(n*x^(12*n)) - 70/11*a^4*b^4/(n*x^(11*n)) - 28/5*a^3*b^5/(
n*x^(10*n)) - 28/9*a^2*b^6/(n*x^(9*n)) - a*b^7/(n*x^(8*n)) - 1/7*b^8/(n*x^(
7*n))
```

mupad [B] time = 1.43, size = 153, normalized size = 1.01

$$-\frac{a^8}{15\,n\,x^{15\,n}}-\frac{b^8}{7\,n\,x^{7\,n}}-\frac{28\,a^2\,b^6}{9\,n\,x^{9\,n}}-\frac{28\,a^3\,b^5}{5\,n\,x^{10\,n}}-\frac{70\,a^4\,b^4}{11\,n\,x^{11\,n}}-\frac{14\,a^5\,b^3}{3\,n\,x^{12\,n}}-\frac{28\,a^6\,b^2}{13\,n\,x^{13\,n}}-\frac{a\,b^7}{n\,x^{8\,n}}-\frac{4\,a^7\,b}{7\,n\,x^{14\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^8/x^(15*n + 1),x)
```

```
[Out] - a^8/(15*n*x^(15*n)) - b^8/(7*n*x^(7*n)) - (28*a^2*b^6)/(9*n*x^(9*n)) - (2
8*a^3*b^5)/(5*n*x^(10*n)) - (70*a^4*b^4)/(11*n*x^(11*n)) - (14*a^5*b^3)/(3*
n*x^(12*n)) - (28*a^6*b^2)/(13*n*x^(13*n)) - (a*b^7)/(n*x^(8*n)) - (4*a^7*b
)/(7*n*x^(14*n))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-15*n)*(a+b*x**n)**8,x)
```

```
[Out] Timed out
```

3.2591 $\int x^{-1+n}\left(a+bx^n\right)^{16}\,dx$

Optimal. Leaf size=19

$$\frac{\left(a+bx^n\right)^{17}}{17bn}$$

```
[Out] 1/17*(a+b*x^n)^17/b/n
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$\frac{\left(a+bx^n\right)^{17}}{17bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*(a + b*x^n)^16,x]
```

```
[Out] (a + b*x^n)^17/(17*b*n)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^{-1+n}\left(a+bx^n\right)^{16}\,dx = \frac{\left(a+bx^n\right)^{17}}{17bn}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{\left(a+bx^n\right)^{17}}{17bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*(a + b*x^n)^16,x]
```

```
[Out] (a + b*x^n)^17/(17*b*n)
```

fricas [B] time = 0.70, size = 214, normalized size = 11.26

$$b^{16}x^{17\,n}+17\,ab^{15}x^{16\,n}+136\,a^2b^{14}x^{15\,n}+680\,a^3b^{13}x^{14\,n}+2380\,a^4b^{12}x^{13\,n}+6188\,a^5b^{11}x^{12\,n}+12376\,a^6b^{10}x^{11\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(a+b*x^n)^16,x, algorithm="fricas")
```

```
[Out] 1/17*(b^16*x^(17*n) + 17*a*b^15*x^(16*n) + 136*a^2*b^14*x^(15*n) + 680*a^3*
b^13*x^(14*n) + 2380*a^4*b^12*x^(13*n) + 6188*a^5*b^11*x^(12*n) + 12376*a^6
*b^10*x^(11*n) + 19448*a^7*b^9*x^(10*n) + 24310*a^8*b^8*x^(9*n) + 24310*a^9
*b^7*x^(8*n) + 19448*a^10*b^6*x^(7*n) + 12376*a^11*b^5*x^(6*n) + 6188*a^12*
b^4*x^(5*n) + 2380*a^13*b^3*x^(4*n) + 680*a^14*b^2*x^(3*n) + 136*a^15*b*x^(
2*n) + 17*a^16*x^n)/n
```

giac [B] time = 0.18, size = 214, normalized size = 11.26

$$b^{16}x^{17\,n}+17\,ab^{15}x^{16\,n}+136\,a^2b^{14}x^{15\,n}+680\,a^3b^{13}x^{14\,n}+2380\,a^4b^{12}x^{13\,n}+6188\,a^5b^{11}x^{12\,n}+12376\,a^6b^{10}x^{11\,n}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(a+b*x^n)^16,x, algorithm="giac")
```

```
[Out] 1/17*(b^16*x^(17*n) + 17*a*b^15*x^(16*n) + 136*a^2*b^14*x^(15*n) + 680*a^3*
b^13*x^(14*n) + 2380*a^4*b^12*x^(13*n) + 6188*a^5*b^11*x^(12*n) + 12376*a^6
*b^10*x^(11*n) + 19448*a^7*b^9*x^(10*n) + 24310*a^8*b^8*x^(9*n) + 24310*a^9
*b^7*x^(8*n) + 19448*a^10*b^6*x^(7*n) + 12376*a^11*b^5*x^(6*n) + 6188*a^12*
b^4*x^(5*n) + 2380*a^13*b^3*x^(4*n) + 680*a^14*b^2*x^(3*n) + 136*a^15*b*x^(
2*n) + 17*a^16*x^n)/n
```

maple [B] time = 0.04, size = 260, normalized size = 13.68

$$\frac{a^{16}x^{n}}{n}+\frac{8a^{15}b\,x^{2n}}{n}+\frac{40a^{14}b^{2}x^{3n}}{n}+\frac{140a^{13}b^{3}x^{4n}}{n}+\frac{364a^{12}b^{4}x^{5n}}{n}+\frac{728a^{11}b^{5}x^{6n}}{n}+\frac{1144a^{10}b^{6}x^{7n}}{n}+\frac{1430a^{9}b^{7}x^{8n}}{n}+\frac{1430a^{8}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*(b*x^n+a)^16,x)
```

```
[Out] 1/17*b^16/n*(x^n)^17+a*b^15/n*(x^n)^16+8*a^2*b^14/n*(x^n)^15+40*a^3*b^13/n*
(x^n)^14+140*a^4*b^12/n*(x^n)^13+364*a^5*b^11/n*(x^n)^12+728*a^6*b^10/n*(x^
n)^11+1144*a^7*b^9/n*(x^n)^10+1430*a^8*b^8/n*(x^n)^9+1430*a^9*b^7/n*(x^n)^8
+1144*a^10*b^6/n*(x^n)^7+728*a^11*b^5/n*(x^n)^6+364*a^12*b^4/n*(x^n)^5+140*
a^13*b^3/n*(x^n)^4+40*a^14*b^2/n*(x^n)^3+8*a^15*b/n*(x^n)^2+a^16/n*x^n
```

maxima [A] time = 0.55, size = 17, normalized size = 0.89

$$\frac{(bx^{n}+a)^{17}}{17\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(a+b*x^n)^16,x, algorithm="maxima")
```

```
[Out] 1/17*(b*x^n + a)^17/(b*n)
```

mupad [B] time = 1.87, size = 259, normalized size = 13.63

$$\frac{a^{16}\,x^{n}}{n}+\frac{b^{16}\,x^{17\,n}}{17\,n}+\frac{40\,a^{14}\,b^{2}\,x^{3\,n}}{n}+\frac{140\,a^{13}\,b^{3}\,x^{4\,n}}{n}+\frac{364\,a^{12}\,b^{4}\,x^{5\,n}}{n}+\frac{728\,a^{11}\,b^{5}\,x^{6\,n}}{n}+\frac{1144\,a^{10}\,b^{6}\,x^{7\,n}}{n}+\frac{1430\,a^{9}\,b^{7}\,x^{8\,n}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*(a + b*x^n)^16,x)
```

```
[Out] (a^16*x^n)/n + (b^16*x^(17*n))/(17*n) + (40*a^14*b^2*x^(3*n))/n + (140*a^13
*b^3*x^(4*n))/n + (364*a^12*b^4*x^(5*n))/n + (728*a^11*b^5*x^(6*n))/n + (11
44*a^10*b^6*x^(7*n))/n + (1430*a^9*b^7*x^(8*n))/n + (1430*a^8*b^8*x^(9*n))/
n + (1144*a^7*b^9*x^(10*n))/n + (728*a^6*b^10*x^(11*n))/n + (364*a^5*b^11*x
^(12*n))/n + (140*a^4*b^12*x^(13*n))/n + (40*a^3*b^13*x^(14*n))/n + (8*a^2*
b^14*x^(15*n))/n + (8*a^15*b*x^(2*n))/n + (a*b^15*x^(16*n))/n
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(a+b*x**n)**16,x)
```

```
[Out] Timed out
```

3.2592 $\int x^{12}\left(a+bx^{13}\right)^{12}\,dx$

Optimal. Leaf size=16

$$\frac{\left(a+bx^{13}\right)^{13}}{169b}$$

```
[Out] 1/169*(b*x^13+a)^13/b
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$\frac{\left(a+bx^{13}\right)^{13}}{169b}$$

Antiderivative was successfully verified.

```
[In] Int[x^12*(a + b*x^13)^12,x]
```

```
[Out] (a + b*x^13)^13/(169*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^{12}\left(a+bx^{13}\right)^{12}\,dx = \frac{\left(a+bx^{13}\right)^{13}}{169b}$$

Mathematica [B] time = 0.01, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{13}}{13}+\frac{6}{13}a^{11}bx^{26}+\frac{22}{13}a^{10}b^2x^{39}+\frac{55}{13}a^9b^3x^{52}+\frac{99}{13}a^8b^4x^{65}+\frac{132}{13}a^7b^5x^{78}+\frac{132}{13}a^6b^6x^{91}+\frac{99}{13}a^5b^7x^{104}+\frac{55}{13}a^4b^8x^{117}+$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12*(a + b*x^13)^12,x]
```

```
[Out] (a^12*x^13)/13 + (6*a^11*b*x^26)/13 + (22*a^10*b^2*x^39)/13 + (55*a^9*b^3*x
^52)/13 + (99*a^8*b^4*x^65)/13 + (132*a^7*b^5*x^78)/13 + (132*a^6*b^6*x^91)
/13 + (99*a^5*b^7*x^104)/13 + (55*a^4*b^8*x^117)/13 + (22*a^3*b^9*x^130)/13
 + (6*a^2*b^10*x^143)/13 + (a*b^11*x^156)/13 + (b^12*x^169)/169
```

fricas [B] time = 0.53, size = 134, normalized size = 8.38

$$\frac{1}{169}x^{169}b^{12}+\frac{1}{13}x^{156}b^{11}a+\frac{6}{13}x^{143}b^{10}a^2+\frac{22}{13}x^{130}b^9a^3+\frac{55}{13}x^{117}b^8a^4+\frac{99}{13}x^{104}b^7a^5+\frac{132}{13}x^{91}b^6a^6+\frac{132}{13}x^{78}b^5a^7+\frac{99}{13}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^13+a)^12,x, algorithm="fricas")
```

```
[Out] 1/169*x^169*b^12 + 1/13*x^156*b^11*a + 6/13*x^143*b^10*a^2 + 22/13*x^130*b^
9*a^3 + 55/13*x^117*b^8*a^4 + 99/13*x^104*b^7*a^5 + 132/13*x^91*b^6*a^6 + 1
```

```
32/13*x^78*b^5*a^7 + 99/13*x^65*b^4*a^8 + 55/13*x^52*b^3*a^9 + 22/13*x^39*b
^2*a^10 + 6/13*x^26*b*a^11 + 1/13*x^13*a^12
```

giac [A] time = 0.16, size = 14, normalized size = 0.88

$$\frac{\left(bx^{13}+a\right)^{13}}{169\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^13+a)^12,x, algorithm="giac")
```

```
[Out] 1/169*(b*x^13 + a)^13/b
```

maple [B] time = 0.00, size = 135, normalized size = 8.44

$$\frac{1}{169}b^{12}x^{169}+\frac{1}{13}a\,b^{11}x^{156}+\frac{6}{13}a^2b^{10}x^{143}+\frac{22}{13}a^3b^9x^{130}+\frac{55}{13}a^4b^8x^{117}+\frac{99}{13}a^5b^7x^{104}+\frac{132}{13}a^6b^6x^{91}+\frac{132}{13}a^7b^5x^{78}+\frac{99}{13}a^8b^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(b*x^13+a)^12,x)
```

```
[Out] 1/169*b^12*x^169+1/13*a*b^11*x^156+6/13*a^2*b^10*x^143+22/13*a^3*b^9*x^130+
55/13*a^4*b^8*x^117+99/13*a^5*b^7*x^104+132/13*a^6*b^6*x^91+132/13*a^7*b^5*
x^78+99/13*a^8*b^4*x^65+55/13*a^9*b^3*x^52+22/13*a^10*b^2*x^39+6/13*a^11*b*
x^26+1/13*a^12*x^13
```

maxima [A] time = 0.52, size = 14, normalized size = 0.88

$$\frac{\left(bx^{13}+a\right)^{13}}{169\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^13+a)^12,x, algorithm="maxima")
```

```
[Out] 1/169*(b*x^13 + a)^13/b
```

mupad [B] time = 0.12, size = 14, normalized size = 0.88

$$\frac{\left(b\,x^{13}+a\right)^{13}}{169\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(a + b*x^13)^12,x)
```

```
[Out] (a + b*x^13)^13/(169*b)
```

sympy [B] time = 0.11, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{13}}{13}+\frac{6a^{11}bx^{26}}{13}+\frac{22a^{10}b^2x^{39}}{13}+\frac{55a^9b^3x^{52}}{13}+\frac{99a^8b^4x^{65}}{13}+\frac{132a^7b^5x^{78}}{13}+\frac{132a^6b^6x^{91}}{13}+\frac{99a^5b^7x^{104}}{13}+\frac{55a^4b^8x^{117}}{13}+\frac{22a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12*(b*x**13+a)**12,x)
```

```
[Out] a**12*x**13/13 + 6*a**11*b*x**26/13 + 22*a**10*b**2*x**39/13 + 55*a**9*b**3
*x**52/13 + 99*a**8*b**4*x**65/13 + 132*a**7*b**5*x**78/13 + 132*a**6*b**6*
x**91/13 + 99*a**5*b**7*x**104/13 + 55*a**4*b**8*x**117/13 + 22*a**3*b**9*x
**130/13 + 6*a**2*b**10*x**143/13 + a*b**11*x**156/13 + b**12*x**169/169
```

3.2593 $\int x^{24}\left(a+bx^{25}\right)^{12}\,dx$

Optimal. Leaf size=16

$$\frac{\left(a+bx^{25}\right)^{13}}{325b}$$

```
[Out] 1/325*(b*x^25+a)^13/b
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$\frac{\left(a+bx^{25}\right)^{13}}{325b}$$

Antiderivative was successfully verified.

```
[In] Int[x^24*(a + b*x^25)^12,x]
```

```
[Out] (a + b*x^25)^13/(325*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^{24}\left(a+bx^{25}\right)^{12}\,dx = \frac{\left(a+bx^{25}\right)^{13}}{325b}$$

Mathematica [B] time = 0.01, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{25}}{25}+\frac{6}{25}a^{11}bx^{50}+\frac{22}{25}a^{10}b^2x^{75}+\frac{11}{5}a^9b^3x^{100}+\frac{99}{25}a^8b^4x^{125}+\frac{132}{25}a^7b^5x^{150}+\frac{132}{25}a^6b^6x^{175}+\frac{99}{25}a^5b^7x^{200}+\frac{11}{5}a^4b^8x^{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^24*(a + b*x^25)^12,x]
```

```
[Out] (a^12*x^25)/25 + (6*a^11*b*x^50)/25 + (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x
^100)/5 + (99*a^8*b^4*x^125)/25 + (132*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^1
75)/25 + (99*a^5*b^7*x^200)/25 + (11*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/
25 + (6*a^2*b^10*x^275)/25 + (a*b^11*x^300)/25 + (b^12*x^325)/325
```

fricas [B] time = 0.58, size = 134, normalized size = 8.38

$$\frac{1}{325}x^{325}b^{12}+\frac{1}{25}x^{300}b^{11}a+\frac{6}{25}x^{275}b^{10}a^2+\frac{22}{25}x^{250}b^9a^3+\frac{11}{5}x^{225}b^8a^4+\frac{99}{25}x^{200}b^7a^5+\frac{132}{25}x^{175}b^6a^6+\frac{132}{25}x^{150}b^5a^7+\frac{99}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24*(b*x^25+a)^12,x, algorithm="fricas")
```

```
[Out] 1/325*x^325*b^12 + 1/25*x^300*b^11*a + 6/25*x^275*b^10*a^2 + 22/25*x^250*b^
9*a^3 + 11/5*x^225*b^8*a^4 + 99/25*x^200*b^7*a^5 + 132/25*x^175*b^6*a^6 + 1
```

```
32/25*x^150*b^5*a^7 + 99/25*x^125*b^4*a^8 + 11/5*x^100*b^3*a^9 + 22/25*x^75
*b^2*a^10 + 6/25*x^50*b*a^11 + 1/25*x^25*a^12
```

giac [A] time = 0.15, size = 14, normalized size = 0.88

$$\frac{\left(bx^{25}+a\right)^{13}}{325\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24*(b*x^25+a)^12,x, algorithm="giac")
```

```
[Out] 1/325*(b*x^25 + a)^13/b
```

maple [B] time = 0.00, size = 135, normalized size = 8.44

$$\frac{1}{325}b^{12}x^{325}+\frac{1}{25}a\,b^{11}x^{300}+\frac{6}{25}a^2b^{10}x^{275}+\frac{22}{25}a^3b^9x^{250}+\frac{11}{5}a^4b^8x^{225}+\frac{99}{25}a^5b^7x^{200}+\frac{132}{25}a^6b^6x^{175}+\frac{132}{25}a^7b^5x^{150}+\frac{99}{25}a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^24*(b*x^25+a)^12,x)
```

```
[Out] 1/325*b^12*x^325+1/25*a*b^11*x^300+6/25*a^2*b^10*x^275+22/25*a^3*b^9*x^250+
11/5*a^4*b^8*x^225+99/25*a^5*b^7*x^200+132/25*a^6*b^6*x^175+132/25*a^7*b^5*
x^150+99/25*a^8*b^4*x^125+11/5*a^9*b^3*x^100+22/25*a^10*b^2*x^75+6/25*a^11*
b*x^50+1/25*a^12*x^25
```

maxima [A] time = 0.47, size = 14, normalized size = 0.88

$$\frac{\left(bx^{25}+a\right)^{13}}{325\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24*(b*x^25+a)^12,x, algorithm="maxima")
```

```
[Out] 1/325*(b*x^25 + a)^13/b
```

mupad [B] time = 0.11, size = 14, normalized size = 0.88

$$\frac{\left(b\,x^{25}+a\right)^{13}}{325\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^24*(a + b*x^25)^12,x)
```

```
[Out] (a + b*x^25)^13/(325*b)
```

sympy [B] time = 0.11, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{25}}{25}+\frac{6a^{11}bx^{50}}{25}+\frac{22a^{10}b^2x^{75}}{25}+\frac{11a^9b^3x^{100}}{5}+\frac{99a^8b^4x^{125}}{25}+\frac{132a^7b^5x^{150}}{25}+\frac{132a^6b^6x^{175}}{25}+\frac{99a^5b^7x^{200}}{25}+\frac{11a^4b^8x^{225}}{5}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**24*(b*x**25+a)**12,x)
```

```
[Out] a**12*x**25/25 + 6*a**11*b*x**50/25 + 22*a**10*b**2*x**75/25 + 11*a**9*b**3
*x**100/5 + 99*a**8*b**4*x**125/25 + 132*a**7*b**5*x**150/25 + 132*a**6*b**
6*x**175/25 + 99*a**5*b**7*x**200/25 + 11*a**4*b**8*x**225/5 + 22*a**3*b**9
*x**250/25 + 6*a**2*b**10*x**275/25 + a*b**11*x**300/25 + b**12*x**325/325
```

3.2594 $\int x^{36}\left(a+bx^{37}\right)^{12}\,dx$

Optimal. Leaf size=16

$$\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

```
[Out] 1/481*(b*x^37+a)^13/b
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {261}

$$\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

Antiderivative was successfully verified.

```
[In] Int[x^36*(a + b*x^37)^12,x]
```

```
[Out] (a + b*x^37)^13/(481*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^{36}\left(a+bx^{37}\right)^{12}\,dx = \frac{\left(a+bx^{37}\right)^{13}}{481b}$$

Mathematica [B] time = 0.03, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{37}}{37}+\frac{6}{37}a^{11}bx^{74}+\frac{22}{37}a^{10}b^2x^{111}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^8b^4x^{185}+\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{99}{37}a^5b^7x^{296}+\frac{55}{37}a^4b^8$$

Antiderivative was successfully verified.

```
[In] Integrate[x^36*(a + b*x^37)^12,x]
```

```
[Out] (a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*
x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x
^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^37
0)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481
```

fricas [B] time = 0.50, size = 134, normalized size = 8.38

$$\frac{1}{481}x^{481}b^{12}+\frac{1}{37}x^{444}b^{11}a+\frac{6}{37}x^{407}b^{10}a^2+\frac{22}{37}x^{370}b^9a^3+\frac{55}{37}x^{333}b^8a^4+\frac{99}{37}x^{296}b^7a^5+\frac{132}{37}x^{259}b^6a^6+\frac{132}{37}x^{222}b^5a^7+\frac{99}{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^36*(b*x^37+a)^12,x, algorithm="fricas")
```

```
[Out] 1/481*x^481*b^12 + 1/37*x^444*b^11*a + 6/37*x^407*b^10*a^2 + 22/37*x^370*b^
9*a^3 + 55/37*x^333*b^8*a^4 + 99/37*x^296*b^7*a^5 + 132/37*x^259*b^6*a^6 +
```

```
132/37*x^222*b^5*a^7 + 99/37*x^185*b^4*a^8 + 55/37*x^148*b^3*a^9 + 22/37*x^
111*b^2*a^10 + 6/37*x^74*b*a^11 + 1/37*x^37*a^12
```

giac [A] time = 0.15, size = 14, normalized size = 0.88

$$\frac{\left(bx^{37}+a\right)^{13}}{481\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^36*(b*x^37+a)^12,x, algorithm="giac")
```

```
[Out] 1/481*(b*x^37 + a)^13/b
```

maple [B] time = 0.00, size = 135, normalized size = 8.44

$$\frac{1}{481}b^{12}x^{481}+\frac{1}{37}a\,b^{11}x^{444}+\frac{6}{37}a^2b^{10}x^{407}+\frac{22}{37}a^3b^9x^{370}+\frac{55}{37}a^4b^8x^{333}+\frac{99}{37}a^5b^7x^{296}+\frac{132}{37}a^6b^6x^{259}+\frac{132}{37}a^7b^5x^{222}+\frac{99}{37}a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^36*(b*x^37+a)^12,x)
```

```
[Out] 1/481*b^12*x^481+1/37*a*b^11*x^444+6/37*a^2*b^10*x^407+22/37*a^3*b^9*x^370+
55/37*a^4*b^8*x^333+99/37*a^5*b^7*x^296+132/37*a^6*b^6*x^259+132/37*a^7*b^5
*x^222+99/37*a^8*b^4*x^185+55/37*a^9*b^3*x^148+22/37*a^10*b^2*x^111+6/37*a^
11*b*x^74+1/37*a^12*x^37
```

maxima [A] time = 0.49, size = 14, normalized size = 0.88

$$\frac{\left(bx^{37}+a\right)^{13}}{481\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^36*(b*x^37+a)^12,x, algorithm="maxima")
```

```
[Out] 1/481*(b*x^37 + a)^13/b
```

mupad [B] time = 1.18, size = 14, normalized size = 0.88

$$\frac{\left(b\,x^{37}+a\right)^{13}}{481\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^36*(a + b*x^37)^12,x)
```

```
[Out] (a + b*x^37)^13/(481*b)
```

sympy [B] time = 0.11, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{37}}{37}+\frac{6a^{11}bx^{74}}{37}+\frac{22a^{10}b^2x^{111}}{37}+\frac{55a^9b^3x^{148}}{37}+\frac{99a^8b^4x^{185}}{37}+\frac{132a^7b^5x^{222}}{37}+\frac{132a^6b^6x^{259}}{37}+\frac{99a^5b^7x^{296}}{37}+\frac{55a^4b^8x^{333}}{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**36*(b*x**37+a)**12,x)
```

```
[Out] a**12*x**37/37 + 6*a**11*b*x**74/37 + 22*a**10*b**2*x**111/37 + 55*a**9*b**
3*x**148/37 + 99*a**8*b**4*x**185/37 + 132*a**7*b**5*x**222/37 + 132*a**6*b
**6*x**259/37 + 99*a**5*b**7*x**296/37 + 55*a**4*b**8*x**333/37 + 22*a**3*b
**9*x**370/37 + 6*a**2*b**10*x**407/37 + a*b**11*x**444/37 + b**12*x**481/4
81
```

3.2595 $\int x^{12m}\left(a+bx^{1+12m}\right)^{12}\,dx$

Optimal. Leaf size=27

$$\frac{\left(a+bx^{12m+1}\right)^{13}}{13b(12m+1)}$$

```
[Out] 1/13*(a+b*x^(1+12*m))^13/b/(1+12*m)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {261}

$$\frac{\left(a+bx^{12m+1}\right)^{13}}{13b(12m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(12*m)*(a + b*x^(1 + 12*m))^12,x]
```

```
[Out] (a + b*x^(1 + 12*m))^13/(13*b*(1 + 12*m))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^{12m}\left(a+bx^{1+12m}\right)^{12}\,dx = \frac{\left(a+bx^{1+12m}\right)^{13}}{13b(1+12m)}$$

Mathematica [A] time = 0.02, size = 24, normalized size = 0.89

$$\frac{\left(a+bx^{12m+1}\right)^{13}}{156bm+13b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(12*m)*(a + b*x^(1 + 12*m))^12,x]
```

```
[Out] (a + b*x^(1 + 12*m))^13/(13*b + 156*b*m)
```

fricas [B] time = 0.70, size = 194, normalized size = 7.19

$$b^{12}x^{156\,m+13}+13\,ab^{11}x^{144\,m+12}+78\,a^2b^{10}x^{132\,m+11}+286\,a^3b^9x^{120\,m+10}+715\,a^4b^8x^{108\,m+9}+1287\,a^5b^7x^{96\,m+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(12*m)*(a+b*x^(1+12*m))^12,x, algorithm="fricas")
```

```
[Out] 1/13*(b^12*x^(156*m + 13) + 13*a*b^11*x^(144*m + 12) + 78*a^2*b^10*x^(132*m
 + 11) + 286*a^3*b^9*x^(120*m + 10) + 715*a^4*b^8*x^(108*m + 9) + 1287*a^5*
b^7*x^(96*m + 8) + 1716*a^6*b^6*x^(84*m + 7) + 1716*a^7*b^5*x^(72*m + 6) +
1287*a^8*b^4*x^(60*m + 5) + 715*a^9*b^3*x^(48*m + 4) + 286*a^10*b^2*x^(36*m
 + 3) + 78*a^11*b*x^(24*m + 2) + 13*a^12*x^(12*m + 1))/(12*m + 1)
```

giac [B] time = 0.51, size = 205, normalized size = 7.59

$$b^{12}x^{13}x^{156\,m}+13\,ab^{11}x^{12}x^{144\,m}+78\,a^2b^{10}x^{11}x^{132\,m}+286\,a^3b^9x^{10}x^{120\,m}+715\,a^4b^8x^9x^{108\,m}+1287\,a^5b^7x^8x^{96\,m}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(12*m)*(a+b*x^(1+12*m))^12,x, algorithm="giac")
```

```
[Out] 1/13*(b^12*x^13*x^(156*m) + 13*a*b^11*x^12*x^(144*m) + 78*a^2*b^10*x^11*x^(
132*m) + 286*a^3*b^9*x^10*x^(120*m) + 715*a^4*b^8*x^9*x^(108*m) + 1287*a^5*
b^7*x^8*x^(96*m) + 1716*a^6*b^6*x^7*x^(84*m) + 1716*a^7*b^5*x^6*x^(72*m) +
1287*a^8*b^4*x^5*x^(60*m) + 715*a^9*b^3*x^4*x^(48*m) + 286*a^10*b^2*x^3*x^(
36*m) + 78*a^11*b*x^2*x^(24*m) + 13*a^12*x*x^(12*m))/(12*m + 1)
```

maple [B] time = 0.03, size = 311, normalized size = 11.52

$$\frac{b^{12}x^{13}x^{156m}}{13+156m}+\frac{a\,b^{11}x^{12}x^{144m}}{12m+1}+\frac{6a^2b^{10}x^{11}x^{132m}}{12m+1}+\frac{22a^3b^9x^{10}x^{120m}}{12m+1}+\frac{55a^4b^8x^9x^{108m}}{12m+1}+\frac{99a^5b^7x^8x^{96m}}{12m+1}+\frac{132a^6b^6x^7x^{84m}}{12m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(12*m)*(a+b*x^(1+12*m))^12,x)
```

```
[Out] 1/13*b^12*x^13/(1+12*m)*(x^(12*m))^13+a*b^11*x^12/(1+12*m)*(x^(12*m))^12+6*
a^2*b^10*x^11/(1+12*m)*(x^(12*m))^11+22*a^3*b^9*x^10/(1+12*m)*(x^(12*m))^10
+55*a^4*b^8*x^9/(1+12*m)*(x^(12*m))^9+99*a^5*b^7*x^8/(1+12*m)*(x^(12*m))^8+
132*a^6*b^6*x^7/(1+12*m)*(x^(12*m))^7+132*a^7*b^5*x^6/(1+12*m)*(x^(12*m))^6
+99*a^8*b^4*x^5/(1+12*m)*(x^(12*m))^5+55*a^9*b^3*x^4/(1+12*m)*(x^(12*m))^4+
22*a^10*b^2*x^3/(1+12*m)*(x^(12*m))^3+6*a^11*b*x^2/(1+12*m)*(x^(12*m))^2+a^
12/(1+12*m)*x*x^(12*m)
```

maxima [A] time = 0.44, size = 25, normalized size = 0.93

$$\frac{\left(bx^{12\,m+1}+a\right)^{13}}{13\,b(12\,m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(12*m)*(a+b*x^(1+12*m))^12,x, algorithm="maxima")
```

```
[Out] 1/13*(b*x^(12*m + 1) + a)^13/(b*(12*m + 1))
```

mupad [B] time = 2.10, size = 285, normalized size = 10.56

$$\frac{b^{12}\,x^{156\,m}\,x^{13}}{156\,m+13}+\frac{a^{12}\,x\,x^{12\,m}}{12\,m+1}+\frac{6\,a^{11}\,b\,x^{24\,m}\,x^2}{12\,m+1}+\frac{a\,b^{11}\,x^{144\,m}\,x^{12}}{12\,m+1}+\frac{22\,a^{10}\,b^2\,x^{36\,m}\,x^3}{12\,m+1}+\frac{55\,a^9\,b^3\,x^{48\,m}\,x^4}{12\,m+1}+\frac{99\,a^8\,b^4\,x^{60\,m}\,x^5}{12\,m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(12*m)*(a + b*x^(12*m + 1))^12,x)
```

```
[Out] (b^12*x^(156*m)*x^13)/(156*m + 13) + (a^12*x*x^(12*m))/(12*m + 1) + (6*a^11
*b*x^(24*m)*x^2)/(12*m + 1) + (a*b^11*x^(144*m)*x^12)/(12*m + 1) + (22*a^10
*b^2*x^(36*m)*x^3)/(12*m + 1) + (55*a^9*b^3*x^(48*m)*x^4)/(12*m + 1) + (99*
a^8*b^4*x^(60*m)*x^5)/(12*m + 1) + (132*a^7*b^5*x^(72*m)*x^6)/(12*m + 1) +
(132*a^6*b^6*x^(84*m)*x^7)/(12*m + 1) + (99*a^5*b^7*x^(96*m)*x^8)/(12*m + 1
) + (55*a^4*b^8*x^(108*m)*x^9)/(12*m + 1) + (22*a^3*b^9*x^(120*m)*x^10)/(12
*m + 1) + (6*a^2*b^10*x^(132*m)*x^11)/(12*m + 1)
```

sympy [A] time = 159.99, size = 284, normalized size = 10.52

$$\begin{cases} \frac{13a^{12}xx^{12m}}{156m+13} + \frac{78a^{11}bx^{2}x^{24m}}{156m+13} + \frac{286a^{10}b^{2}x^{3}x^{36m}}{156m+13} + \frac{715a^{9}b^{3}x^{4}x^{48m}}{156m+13} + \frac{1287a^{8}b^{4}x^{5}x^{60m}}{156m+13} + \frac{1716a^{7}b^{5}x^{6}x^{72m}}{156m+13} + \frac{1716a^{6}b^{6}x^{7}x^{84m}}{156m+13} + \frac{1287a^{5}}{156} \\ (a+b)^{12}\log(x) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(12*m)*(a+b*x**(1+12*m))**12,x)
```

```
[Out] Piecewise((13*a**12*x*x**(12*m)/(156*m + 13) + 78*a**11*b*x**2*x**(24*m)/(1
56*m + 13) + 286*a**10*b**2*x**3*x**(36*m)/(156*m + 13) + 715*a**9*b**3*x**
4*x**(48*m)/(156*m + 13) + 1287*a**8*b**4*x**5*x**(60*m)/(156*m + 13) + 171
6*a**7*b**5*x**6*x**(72*m)/(156*m + 13) + 1716*a**6*b**6*x**7*x**(84*m)/(15
6*m + 13) + 1287*a**5*b**7*x**8*x**(96*m)/(156*m + 13) + 715*a**4*b**8*x**9
*x**(108*m)/(156*m + 13) + 286*a**3*b**9*x**10*x**(120*m)/(156*m + 13) + 78
*a**2*b**10*x**11*x**(132*m)/(156*m + 13) + 13*a*b**11*x**12*x**(144*m)/(15
6*m + 13) + b**12*x**13*x**(156*m)/(156*m + 13), Ne(m, -1/12)), ((a + b)**1
2*log(x), True))
```

3.2596 $\int x^{12+12(-1+m)}\left(a+bx^{1+12m}\right)^{12}\,dx$

Optimal. Leaf size=27

$$\frac{\left(a+bx^{12m+1}\right)^{13}}{13b(12m+1)}$$

```
[Out] 1/13*(a+b*x^(1+12*m))^13/b/(1+12*m)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {261}

$$\frac{\left(a+bx^{12m+1}\right)^{13}}{13b(12m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(12 + 12*(-1 + m))*(a + b*x^(1 + 12*m))^12,x]
```

```
[Out] (a + b*x^(1 + 12*m))^13/(13*b*(1 + 12*m))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^{12+12(-1+m)}\left(a+bx^{1+12m}\right)^{12}\,dx = \frac{\left(a+bx^{1+12m}\right)^{13}}{13b(1+12m)}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 0.89

$$\frac{\left(a+bx^{12m+1}\right)^{13}}{156bm+13b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(12 + 12*(-1 + m))*(a + b*x^(1 + 12*m))^12,x]
```

```
[Out] (a + b*x^(1 + 12*m))^13/(13*b + 156*b*m)
```

fricas [B] time = 0.76, size = 194, normalized size = 7.19

$$\frac{b^{12}x^{156\,m+13}+13\,ab^{11}x^{144\,m+12}+78\,a^2b^{10}x^{132\,m+11}+286\,a^3b^9x^{120\,m+10}+715\,a^4b^8x^{108\,m+9}+1287\,a^5b^7x^{96\,m+8}+1}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(12*m)*(a+b*x^(1+12*m))^12,x, algorithm="fricas")
```

```
[Out] 1/13*(b^12*x^(156*m + 13) + 13*a*b^11*x^(144*m + 12) + 78*a^2*b^10*x^(132*m
 + 11) + 286*a^3*b^9*x^(120*m + 10) + 715*a^4*b^8*x^(108*m + 9) + 1287*a^5*
b^7*x^(96*m + 8) + 1716*a^6*b^6*x^(84*m + 7) + 1716*a^7*b^5*x^(72*m + 6) +
1287*a^8*b^4*x^(60*m + 5) + 715*a^9*b^3*x^(48*m + 4) + 286*a^10*b^2*x^(36*m
 + 3) + 78*a^11*b*x^(24*m + 2) + 13*a^12*x^(12*m + 1))/(12*m + 1)
```

giac [B] time = 0.51, size = 205, normalized size = 7.59

$$b^{12}x^{13}x^{156\,m}+13\,ab^{11}x^{12}x^{144\,m}+78\,a^2b^{10}x^{11}x^{132\,m}+286\,a^3b^9x^{10}x^{120\,m}+715\,a^4b^8x^9x^{108\,m}+1287\,a^5b^7x^8x^{96\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(12*m)*(a+b*x^(1+12*m))^12,x, algorithm="giac")
```

```
[Out] 1/13*(b^12*x^13*x^(156*m) + 13*a*b^11*x^12*x^(144*m) + 78*a^2*b^10*x^11*x^(
132*m) + 286*a^3*b^9*x^10*x^(120*m) + 715*a^4*b^8*x^9*x^(108*m) + 1287*a^5*
b^7*x^8*x^(96*m) + 1716*a^6*b^6*x^7*x^(84*m) + 1716*a^7*b^5*x^6*x^(72*m) +
1287*a^8*b^4*x^5*x^(60*m) + 715*a^9*b^3*x^4*x^(48*m) + 286*a^10*b^2*x^3*x^(
36*m) + 78*a^11*b*x^2*x^(24*m) + 13*a^12*x*x^(12*m))/(12*m + 1)
```

maple [B] time = 0.00, size = 311, normalized size = 11.52

$$\frac{b^{12}x^{13}x^{156m}}{13+156m}+\frac{a\,b^{11}x^{12}x^{144m}}{12m+1}+\frac{6a^2b^{10}x^{11}x^{132m}}{12m+1}+\frac{22a^3b^9x^{10}x^{120m}}{12m+1}+\frac{55a^4b^8x^9x^{108m}}{12m+1}+\frac{99a^5b^7x^8x^{96m}}{12m+1}+\frac{132a^6b^6x^7x^8}{12m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(12*m)*(a+b*x^(12*m+1))^12,x)
```

```
[Out] 1/13*b^12*x^13/(12*m+1)*(x^(12*m))^13+a*b^11*x^12/(12*m+1)*(x^(12*m))^12+6*
a^2*b^10*x^11/(12*m+1)*(x^(12*m))^11+22*a^3*b^9*x^10/(12*m+1)*(x^(12*m))^10
+55*a^4*b^8*x^9/(12*m+1)*(x^(12*m))^9+99*a^5*b^7*x^8/(12*m+1)*(x^(12*m))^8+
132*a^6*b^6*x^7/(12*m+1)*(x^(12*m))^7+132*a^7*b^5*x^6/(12*m+1)*(x^(12*m))^6
+99*a^8*b^4*x^5/(12*m+1)*(x^(12*m))^5+55*a^9*b^3*x^4/(12*m+1)*(x^(12*m))^4+
22*a^10*b^2*x^3/(12*m+1)*(x^(12*m))^3+6*a^11*b*x^2/(12*m+1)*(x^(12*m))^2+1/
(12*m+1)*a^12*x*x^(12*m)
```

maxima [A] time = 0.59, size = 25, normalized size = 0.93

$$\frac{\left(bx^{12\,m+1}+a\right)^{13}}{13\,b(12\,m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(12*m)*(a+b*x^(1+12*m))^12,x, algorithm="maxima")
```

```
[Out] 1/13*(b*x^(12*m + 1) + a)^13/(b*(12*m + 1))
```

mupad [B] time = 0.00, size = 285, normalized size = 10.56

$$\frac{b^{12}\,x^{156\,m}\,x^{13}}{156\,m+13}+\frac{a^{12}\,x\,x^{12\,m}}{12\,m+1}+\frac{6\,a^{11}\,b\,x^{24\,m}\,x^2}{12\,m+1}+\frac{a\,b^{11}\,x^{144\,m}\,x^{12}}{12\,m+1}+\frac{22\,a^{10}\,b^2\,x^{36\,m}\,x^3}{12\,m+1}+\frac{55\,a^9\,b^3\,x^{48\,m}\,x^4}{12\,m+1}+\frac{99\,a^8\,b^4\,x^{60\,m}}{12\,m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(12*m)*(a + b*x^(12*m + 1))^12,x)
```

```
[Out] (b^12*x^(156*m)*x^13)/(156*m + 13) + (a^12*x*x^(12*m))/(12*m + 1) + (6*a^11
*b*x^(24*m)*x^2)/(12*m + 1) + (a*b^11*x^(144*m)*x^12)/(12*m + 1) + (22*a^10
*b^2*x^(36*m)*x^3)/(12*m + 1) + (55*a^9*b^3*x^(48*m)*x^4)/(12*m + 1) + (99*
a^8*b^4*x^(60*m)*x^5)/(12*m + 1) + (132*a^7*b^5*x^(72*m)*x^6)/(12*m + 1) +
(132*a^6*b^6*x^(84*m)*x^7)/(12*m + 1) + (99*a^5*b^7*x^(96*m)*x^8)/(12*m + 1
) + (55*a^4*b^8*x^(108*m)*x^9)/(12*m + 1) + (22*a^3*b^9*x^(120*m)*x^10)/(12
*m + 1) + (6*a^2*b^10*x^(132*m)*x^11)/(12*m + 1)
```

sympy [A] time = 161.66, size = 284, normalized size = 10.52

$$\begin{cases} \frac{13a^{12}xx^{12m}}{156m+13} + \frac{78a^{11}bx^2x^{24m}}{156m+13} + \frac{286a^{10}b^2x^3x^{36m}}{156m+13} + \frac{715a^9b^3x^4x^{48m}}{156m+13} + \frac{1287a^8b^4x^5x^{60m}}{156m+13} + \frac{1716a^7b^5x^6x^{72m}}{156m+13} + \frac{1716a^6b^6x^7x^{84m}}{156m+13} + \frac{1287a^5b^7x^{\text{}}}{156m+} \\ (a+b)^{12}\log(x) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(12*m)*(a+b*x**(1+12*m))**12,x)
```

```
[Out] Piecewise((13*a**12*x*x**(12*m)/(156*m + 13) + 78*a**11*b*x**2*x**(24*m)/(1
56*m + 13) + 286*a**10*b**2*x**3*x**(36*m)/(156*m + 13) + 715*a**9*b**3*x**
4*x**(48*m)/(156*m + 13) + 1287*a**8*b**4*x**5*x**(60*m)/(156*m + 13) + 171
6*a**7*b**5*x**6*x**(72*m)/(156*m + 13) + 1716*a**6*b**6*x**7*x**(84*m)/(15
6*m + 13) + 1287*a**5*b**7*x**8*x**(96*m)/(156*m + 13) + 715*a**4*b**8*x**9
*x**(108*m)/(156*m + 13) + 286*a**3*b**9*x**10*x**(120*m)/(156*m + 13) + 78
*a**2*b**10*x**11*x**(132*m)/(156*m + 13) + 13*a*b**11*x**12*x**(144*m)/(15
6*m + 13) + b**12*x**13*x**(156*m)/(156*m + 13), Ne(m, -1/12)), ((a + b)**1
2*log(x), True))
```

3.2597 $\int \frac{x^{-1+5n}}{a+bx^n}\,dx$

Optimal. Leaf size=82

$$\frac{a^4\log(a+bx^n)}{b^5n}-\frac{a^3x^n}{b^4n}+\frac{a^2x^{2n}}{2b^3n}-\frac{ax^{3n}}{3b^2n}+\frac{x^{4n}}{4bn}$$

```
[Out] -a^3*x^n/b^4/n+1/2*a^2*x^(2*n)/b^3/n-1/3*a*x^(3*n)/b^2/n+1/4*x^(4*n)/b/n+a^
4*ln(a+b*x^n)/b^5/n
```

Rubi [A] time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$-\frac{a^3x^n}{b^4n}+\frac{a^2x^{2n}}{2b^3n}+\frac{a^4\log(a+bx^n)}{b^5n}-\frac{ax^{3n}}{3b^2n}+\frac{x^{4n}}{4bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 5*n)/(a + b*x^n),x]
```

```
[Out] -((a^3*x^n)/(b^4*n)) + (a^2*x^(2*n))/(2*b^3*n) - (a*x^(3*n))/(3*b^2*n) + x^
(4*n)/(4*b*n) + (a^4*Log[a + b*x^n])/(b^5*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+5n}}{a+bx^n}\,dx &= \frac{\text{Subst}\left(\int \frac{x^4}{a+bx}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{a^3}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{b^2}+\frac{x^3}{b}+\frac{a^4}{b^4(a+bx)}\right)dx, x, x^n\right)}{n}\\ &= -\frac{a^3x^n}{b^4n}+\frac{a^2x^{2n}}{2b^3n}-\frac{ax^{3n}}{3b^2n}+\frac{x^{4n}}{4bn}+\frac{a^4\log(a+bx^n)}{b^5n}\end{aligned}$$

Mathematica [A] time = 0.08, size = 65, normalized size = 0.79

$$\frac{12a^4\log(a+bx^n)+bx^n\left(-12a^3+6a^2bx^n-4ab^2x^{2n}+3b^3x^{3n}\right)}{12b^5n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 5*n)/(a + b*x^n),x]
```

```
[Out] (b*x^n*(-12*a^3 + 6*a^2*b*x^n - 4*a*b^2*x^(2*n) + 3*b^3*x^(3*n)) + 12*a^4*L
og[a + b*x^n])/(12*b^5*n)
```

fricas [A] time = 0.55, size = 65, normalized size = 0.79

$$\frac{3\,b^4x^{4\,n} - 4\,ab^3x^{3\,n} + 6\,a^2b^2x^{2\,n} - 12\,a^3bx^n + 12\,a^4\log\left(bx^n + a\right)}{12\,b^5n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+5*n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*x^(4*n) - 4*a*b^3*x^(3*n) + 6*a^2*b^2*x^(2*n) - 12*a^3*b*x^n +
12*a^4*log(b*x^n + a))/(b^5*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5\,n-1}}{bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+5*n)/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(5*n - 1)/(b*x^n + a), x)
```

maple [A] time = 0.04, size = 87, normalized size = 1.06

$$\frac{a^4\ln\left(b\,\mathrm{e}^{n\ln(x)} + a\right)}{b^5n} - \frac{a^3\mathrm{e}^{n\ln(x)}}{b^4n} + \frac{a^2\mathrm{e}^{2n\ln(x)}}{2b^3n} - \frac{a\,\mathrm{e}^{3n\ln(x)}}{3b^2n} + \frac{\mathrm{e}^{4n\ln(x)}}{4bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+5*n)/(b*x^n+a),x)
```

```
[Out] 1/4/b/n*exp(n*ln(x))^4-1/3*a/b^2/n*exp(n*ln(x))^3+1/2*a^2/b^3/n*exp(n*ln(x)
)^2-a^3/b^4/n*exp(n*ln(x))+a^4/b^5/n*ln(a+b*exp(n*ln(x)))
```

maxima [A] time = 0.57, size = 72, normalized size = 0.88

$$\frac{a^4\log\left(\frac{bx^n+a}{b}\right)}{b^5n} + \frac{3\,b^3x^{4\,n} - 4\,ab^2x^{3\,n} + 6\,a^2bx^{2\,n} - 12\,a^3x^n}{12\,b^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+5*n)/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] a^4*log((b*x^n + a)/b)/(b^5*n) + 1/12*(3*b^3*x^(4*n) - 4*a*b^2*x^(3*n) + 6*
a^2*b*x^(2*n) - 12*a^3*x^n)/(b^4*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5\,n-1}}{a + b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5*n - 1)/(a + b*x^n),x)
```

```
[Out] int(x^(5*n - 1)/(a + b*x^n), x)
```

sympy [**A**] time = 95.40, size = 87, normalized size = 1.06

$$\begin{cases} \frac{\log(x)}{a} & \text{for } b = 0 \wedge n = 0 \\ \frac{\log(x)}{a+b} & \text{for } n = 0 \\ \frac{x^{5n}}{5an} & \text{for } b = 0 \\ \frac{a^4 \log\left(\frac{a}{b}+x^n\right)}{b^5 n} - \frac{a^3 x^n}{b^4 n} + \frac{a^2 x^{2n}}{2b^3 n} - \frac{a x^{3n}}{3b^2 n} + \frac{x^{4n}}{4bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+5*n)/(a+b*x**n),x)
```

```
[Out] Piecewise((log(x)/a, Eq(b, 0) & Eq(n, 0)), (log(x)/(a + b), Eq(n, 0)), (x**
(5*n)/(5*a*n), Eq(b, 0)), (a**4*log(a/b + x**n)/(b**5*n) - a**3*x**n/(b**4*
n) + a**2*x**(2*n)/(2*b**3*n) - a*x**(3*n)/(3*b**2*n) + x**(4*n)/(4*b*n), T
rue))
```

3.2598 $\int \frac{x^{-1+4n}}{a+bx^n}\,dx$

Optimal. Leaf size=64

$$-\frac{a^3\log(a+bx^n)}{b^4n}+\frac{a^2x^n}{b^3n}-\frac{ax^{2n}}{2b^2n}+\frac{x^{3n}}{3bn}$$

```
[Out] a^2*x^n/b^3/n-1/2*a*x^(2*n)/b^2/n+1/3*x^(3*n)/b/n-a^3*ln(a+b*x^n)/b^4/n
```

Rubi [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$\frac{a^2x^n}{b^3n}-\frac{a^3\log(a+bx^n)}{b^4n}-\frac{ax^{2n}}{2b^2n}+\frac{x^{3n}}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 4*n)/(a + b*x^n),x]
```

```
[Out] (a^2*x^n)/(b^3*n) - (a^3*Log[a + b*x^n])/(b^4*n) - (a*x^(2*n))/(2*b^2*n) + x^(3*n)/(3*b*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+4n}}{a+bx^n}\,dx &= \frac{\text{Subst}\left(\int \frac{x^3}{a+bx}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right)dx, x, x^n\right)}{n}\\ &= \frac{a^2x^n}{b^3n}-\frac{ax^{2n}}{2b^2n}+\frac{x^{3n}}{3bn}-\frac{a^3\log(a+bx^n)}{b^4n}\end{aligned}$$

Mathematica [A] time = 0.04, size = 52, normalized size = 0.81

$$\frac{bx^n\left(6a^2-3abx^n+2b^2x^{2n}\right)-6a^3\log(a+bx^n)}{6b^4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 4*n)/(a + b*x^n),x]
```

```
[Out] (b*x^n*(6*a^2 - 3*a*b*x^n + 2*b^2*x^(2*n)) - 6*a^3*Log[a + b*x^n])/(6*b^4*n
)
```

fricas [A] time = 0.79, size = 52, normalized size = 0.81

$$\frac{2\,b^3x^{3\,n}-3\,ab^2x^{2\,n}+6\,a^2bx^n-6\,a^3\log\left(bx^n+a\right)}{6\,b^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^(3*n) - 3*a*b^2*x^(2*n) + 6*a^2*b*x^n - 6*a^3*log(b*x^n + a))/
(b^4*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4\,n-1}}{bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(4*n - 1)/(b*x^n + a), x)
```

maple [A] time = 0.03, size = 69, normalized size = 1.08

$$-\frac{a^3\ln\left(b\,\mathrm{e}^{n\ln(x)}+a\right)}{b^4n}+\frac{a^2\mathrm{e}^{n\ln(x)}}{b^3n}-\frac{a\,\mathrm{e}^{2n\ln(x)}}{2b^2n}+\frac{\mathrm{e}^{3n\ln(x)}}{3bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+4*n)/(b*x^n+a),x)
```

```
[Out] a^2/b^3/n*exp(n*ln(x))+1/3/b/n*exp(n*ln(x))^3-1/2*a/b^2/n*exp(n*ln(x))^2-a^
3/b^4/n*ln(b*exp(n*ln(x))+a)
```

maxima [A] time = 0.50, size = 60, normalized size = 0.94

$$-\frac{a^3\log\left(\frac{bx^n+a}{b}\right)}{b^4n}+\frac{2\,b^2x^{3\,n}-3\,abx^{2\,n}+6\,a^2x^n}{6\,b^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] -a^3*log((b*x^n + a)/b)/(b^4*n) + 1/6*(2*b^2*x^(3*n) - 3*a*b*x^(2*n) + 6*a^
2*x^n)/(b^3*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{4\,n-1}}{a+b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4*n - 1)/(a + b*x^n),x)
```

```
[Out] int(x^(4*n - 1)/(a + b*x^n), x)
```

sympy [**A**] time = 45.92, size = 71, normalized size = 1.11

$$\begin{cases} \frac{\log(x)}{a} & \text{for } b = 0 \wedge n = 0 \\ \frac{\log(x)}{a+b} & \text{for } n = 0 \\ \frac{x^{4n}}{4an} & \text{for } b = 0 \\ -\frac{a^3 \log\left(\frac{a}{b}+x^n\right)}{b^4 n} + \frac{a^2 x^n}{b^3 n} - \frac{a x^{2n}}{2b^2 n} + \frac{x^{3n}}{3bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+4*n)/(a+b*x**n),x)
```

```
[Out] Piecewise((log(x)/a, Eq(b, 0) & Eq(n, 0)), (log(x)/(a + b), Eq(n, 0)), (x**
(4*n)/(4*a*n), Eq(b, 0)), (-a**3*log(a/b + x**n)/(b**4*n) + a**2*x**n/(b**3
*n) - a*x**(2*n)/(2*b**2*n) + x**(3*n)/(3*b*n), True))
```

3.2599 $\int \frac{x^{-1+3n}}{a+bx^n}\,dx$

Optimal. Leaf size=46

$$\frac{a^2\log(a+bx^n)}{b^3n}-\frac{ax^n}{b^2n}+\frac{x^{2n}}{2bn}$$

```
[Out] -a*x^n/b^2/n+1/2*x^(2*n)/b/n+a^2*ln(a+b*x^n)/b^3/n
```

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$\frac{a^2\log(a+bx^n)}{b^3n}-\frac{ax^n}{b^2n}+\frac{x^{2n}}{2bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)/(a + b*x^n),x]
```

```
[Out] -((a*x^n)/(b^2*n)) + x^(2*n)/(2*b*n) + (a^2*Log[a + b*x^n])/(b^3*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+3n}}{a+bx^n}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{a+bx}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)\,dx, x, x^n\right)}{n}\\ &= -\frac{ax^n}{b^2n}+\frac{x^{2n}}{2bn}+\frac{a^2\log(a+bx^n)}{b^3n}\end{aligned}$$

Mathematica [A] time = 0.05, size = 38, normalized size = 0.83

$$\frac{2a^2\log(a+bx^n)+bx^n(bx^n-2a)}{2b^3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)/(a + b*x^n),x]
```

```
[Out] (b*x^n*(-2*a + b*x^n) + 2*a^2*Log[a + b*x^n])/(2*b^3*n)
```

fricas [A] time = 0.83, size = 38, normalized size = 0.83

$$\frac{b^2x^{2\,n}-2\,abx^n+2\,a^2\log(bx^n+a)}{2\,b^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n),x, algorithm="fricas")
[Out] 1/2*(b^2*x^(2*n) - 2*a*b*x^n + 2*a^2*log(b*x^n + a))/(b^3*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3n-1}}{bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n),x, algorithm="giac")
[Out] integrate(x^(3*n - 1)/(b*x^n + a), x)
```

maple [A] time = 0.02, size = 51, normalized size = 1.11

$$\frac{a^2 \ln\left(b\,\mathrm{e}^{n\ln(x)} + a\right)}{b^3 n} - \frac{a\,\mathrm{e}^{n\ln(x)}}{b^2 n} + \frac{\mathrm{e}^{2n\ln(x)}}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n-1)/(b*x^n+a),x)
[Out] 1/2/b/n*exp(n*ln(x))^2-a/b^2/n*exp(n*ln(x))+a^2/b^3/n*ln(b*exp(n*ln(x))+a)
```

maxima [A] time = 0.53, size = 45, normalized size = 0.98

$$\frac{a^2 \log\left(\frac{bx^n+a}{b}\right)}{b^3 n} + \frac{bx^{2n} - 2\,ax^n}{2\,b^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n),x, algorithm="maxima")
[Out] a^2*log((b*x^n + a)/b)/(b^3*n) + 1/2*(b*x^(2*n) - 2*a*x^n)/(b^2*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{3n-1}}{a + b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n - 1)/(a + b*x^n),x)
[Out] int(x^(3*n - 1)/(a + b*x^n), x)
```

sympy [A] time = 25.36, size = 56, normalized size = 1.22

$$\begin{cases} \frac{\log(x)}{a} & \text{for } b = 0 \wedge n = 0 \\ \frac{\log(x)}{a+b} & \text{for } n = 0 \\ \frac{x^{3n}}{3an} & \text{for } b = 0 \\ \frac{a^2 \log\left(\frac{a}{b}+x^n\right)}{b^3 n} - \frac{ax^n}{b^2 n} + \frac{x^{2n}}{2bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)/(a+b*x**n),x)
```

```
[Out] Piecewise((log(x)/a, Eq(b, 0) & Eq(n, 0)), (log(x)/(a + b), Eq(n, 0)), (x**
(3*n)/(3*a*n), Eq(b, 0)), (a**2*log(a/b + x**n)/(b**3*n) - a*x**n/(b**2*n)
+ x**(2*n)/(2*b*n), True))
```

3.2600 $\int \frac{x^{-1+2n}}{a+bx^n}\,dx$

Optimal. Leaf size=28

$$\frac{x^n}{bn} - \frac{a\log(a+bx^n)}{b^2 n}$$

```
[Out] x^n/b/n-a*ln(a+b*x^n)/b^2/n
```

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$\frac{x^n}{bn} - \frac{a\log(a+bx^n)}{b^2 n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/(a + b*x^n),x]
```

```
[Out] x^n/(b*n) - (a*Log[a + b*x^n])/(b^2*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+2n}}{a+bx^n}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{a+bx}\,dx, x, x^n\right)}{n} \\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{b} - \frac{a}{b(a+bx)}\right)\,dx, x, x^n\right)}{n} \\ &= \frac{x^n}{bn} - \frac{a\log(a+bx^n)}{b^2 n}\end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 0.93

$$\frac{\frac{x^n}{b} - \frac{a\log(a+bx^n)}{b^2}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/(a + b*x^n),x]
```

```
[Out] (x^n/b - (a*Log[a + b*x^n])/b^2)/n
```

fricas [A] time = 0.66, size = 24, normalized size = 0.86

$$\frac{bx^n - a\log(bx^n + a)}{b^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] (b*x^n - a*log(b*x^n + a))/(b^2*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/(b*x^n + a), x)
```

maple [A] time = 0.02, size = 33, normalized size = 1.18

$$-\frac{a\ln\left(b\,\mathrm{e}^{n\ln(x)} + a\right)}{b^2 n} + \frac{\mathrm{e}^{n\ln(x)}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n-1)/(b*x^n+a),x)
```

```
[Out] 1/b/n*exp(n*ln(x))-a/b^2/n*ln(b*exp(n*ln(x))+a)
```

maxima [A] time = 0.60, size = 32, normalized size = 1.14

$$\frac{x^n}{bn} - \frac{a\log\left(\frac{bx^n+a}{b}\right)}{b^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] x^n/(b*n) - a*log((b*x^n + a)/b)/(b^2*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^{2n-1}}{a + b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/(a + b*x^n),x)
```

```
[Out] int(x^(2*n - 1)/(a + b*x^n), x)
```

sympy [A] time = 12.23, size = 41, normalized size = 1.46

$$\begin{cases} \frac{\log(x)}{a} & \text{for } b = 0 \wedge n = 0 \\ \frac{\log(x)}{a+b} & \text{for } n = 0 \\ \frac{x^{2n}}{2an} & \text{for } b = 0 \\ -\frac{a\log\left(\frac{a}{b}+x^n\right)}{b^2 n} + \frac{x^n}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a+b*x**n),x)
```

```
[Out] Piecewise((log(x)/a, Eq(b, 0) & Eq(n, 0)), (log(x)/(a + b), Eq(n, 0)), (x**
(2*n)/(2*a*n), Eq(b, 0)), (-a*log(a/b + x**n)/(b**2*n) + x**n/(b*n), True))
```

3.2601 $\int \frac{x^{-1+n}}{a+bx^n}\,dx$

Optimal. Leaf size=15

$$\frac{\log(a+bx^n)}{bn}$$

```
[Out] ln(a+b*x^n)/b/n
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {260}

$$\frac{\log(a+bx^n)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)/(a + b*x^n),x]
```

```
[Out] Log[a + b*x^n]/(b*n)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^{-1+n}}{a+bx^n}\,dx = \frac{\log(a+bx^n)}{bn}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log(a+bx^n)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)/(a + b*x^n),x]
```

```
[Out] Log[a + b*x^n]/(b*n)
```

fricas [A] time = 0.71, size = 15, normalized size = 1.00

$$\frac{\log(bx^n+a)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] log(b*x^n + a)/(b*n)
```

giac [A] time = 0.15, size = 16, normalized size = 1.07

$$\frac{\log(|bx^n+a|)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(a+b*x^n),x, algorithm="giac")
[Out] log(abs(b*x^n + a))/(b*n)
```

maple [A] time = 0.02, size = 18, normalized size = 1.20

$$\frac{\ln\left(b\,e^{n\ln(x)}+a\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)/(b*x^n+a),x)
[Out] 1/b/n*ln(b*exp(n*ln(x))+a)
```

maxima [A] time = 0.64, size = 15, normalized size = 1.00

$$\frac{\log\left(bx^n+a\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(a+b*x^n),x, algorithm="maxima")
[Out] log(b*x^n + a)/(b*n)
```

mupad [B] time = 1.22, size = 15, normalized size = 1.00

$$\frac{\ln\left(a+b\,x^n\right)}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)/(a + b*x^n),x)
[Out] log(a + b*x^n)/(b*n)
```

sympy [A] time = 2.54, size = 27, normalized size = 1.80

$$\begin{cases} \frac{\log(x)}{a} & \text{for } b = 0 \wedge n = 0 \\ \frac{\log(x)}{a+b} & \text{for } n = 0 \\ \frac{x^n}{an} & \text{for } b = 0 \\ \frac{\log\left(\frac{a}{b}+x^n\right)}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)/(a+b*x**n),x)
[Out] Piecewise((log(x)/a, Eq(b, 0) & Eq(n, 0)), (log(x)/(a + b), Eq(n, 0)), (x**
n/(a*n), Eq(b, 0)), (log(a/b + x**n)/(b*n), True))
```

3.2602 $\int \frac{1}{x(a+bx^n)}\,dx$

Optimal. Leaf size=23

$$\frac{\log(x)}{a} - \frac{\log(a+bx^n)}{an}$$

```
[Out] ln(x)/a-ln(a+b*x^n)/a/n
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {266, 36, 29, 31}

$$\frac{\log(x)}{a} - \frac{\log(a+bx^n)}{an}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^n)),x]
```

```
[Out] Log[x]/a - Log[a + b*x^n]/(a*n)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\,(a+bx^n)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx, x, x^n\right)}{n} \\ &= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^n\right)}{an} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, x^n\right)}{an} \\ &= \frac{\log(x)}{a} - \frac{\log(a+bx^n)}{an}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.96

$$\frac{n\log(x) - \log(a+bx^n)}{an}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^n)),x]
[Out] (n*Log[x] - Log[a + b*x^n])/(a*n)
```

fricas [A] time = 0.69, size = 22, normalized size = 0.96

$$\frac{n\log(x) - \log\left(bx^n + a\right)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n),x, algorithm="fricas")
[Out] (n*log(x) - log(b*x^n + a))/(a*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n),x, algorithm="giac")
[Out] integrate(1/((b*x^n + a)*x), x)
```

maple [A] time = 0.00, size = 29, normalized size = 1.26

$$\frac{\ln\left(x^n\right)}{an} - \frac{\ln\left(b\,x^n + a\right)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^n+a),x)
[Out] 1/a/n*ln(x^n)-1/a/n*ln(b*x^n+a)
```

maxima [A] time = 0.60, size = 28, normalized size = 1.22

$$-\frac{\log\left(bx^n + a\right)}{an} + \frac{\log\left(x^n\right)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n),x, algorithm="maxima")
[Out] -log(b*x^n + a)/(a*n) + log(x^n)/(a*n)
```

mupad [B] time = 1.18, size = 22, normalized size = 0.96

$$-\frac{\ln\left(a + b\,x^n\right) - n\,\ln(x)}{a\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^n)),x)
[Out] -(log(a + b*x^n) - n*log(x))/(a*n)
```

sympy [**A**] time = 0.74, size = 41, normalized size = 1.78

$$\begin{cases} \tilde{\infty} \log(x) & \text{for } a = 0 \wedge b = 0 \wedge n = 0 \\ \frac{\log(x)}{a} & \text{for } b = 0 \\ -\frac{x^{-n}}{bn} & \text{for } a = 0 \\ \frac{\log(x)}{a+b} & \text{for } n = 0 \\ \frac{\log(x)}{a} - \frac{\log\left(\frac{a}{b}+x^n\right)}{an} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x**n),x)
```

```
[Out] Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (log(x)/a, Eq(b, 0)
), (-x**(-n)/(b*n), Eq(a, 0)), (log(x)/(a + b), Eq(n, 0)), (log(x)/a - log(
a/b + x**n)/(a*n), True))
```

3.2603 $\int \frac{x^{-1-n}}{a+bx^n}\,dx$

Optimal. Leaf size=38

$$\frac{b\log(a+bx^n)}{a^2n} - \frac{b\log(x)}{a^2} - \frac{x^{-n}}{an}$$

```
[Out] -1/a/n/(x^n)-b*ln(x)/a^2+b*ln(a+b*x^n)/a^2/n
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 44}

$$\frac{b\log(a+bx^n)}{a^2n} - \frac{b\log(x)}{a^2} - \frac{x^{-n}}{an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n)/(a + b*x^n),x]
```

```
[Out] -(1/(a*n*x^n)) - (b*Log[x])/a^2 + (b*Log[a + b*x^n])/(a^2*n)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1-n}}{a+bx^n}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx)}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^2} - \frac{b}{a^2x} + \frac{b^2}{a^2(a+bx)}\right)\,dx, x, x^n\right)}{n}\\ &= -\frac{x^{-n}}{an} - \frac{b\log(x)}{a^2} + \frac{b\log(a+bx^n)}{a^2n}\end{aligned}$$

Mathematica [A] time = 0.05, size = 32, normalized size = 0.84

$$-\frac{-b\log(a+bx^n) + ax^{-n} + bn\log(x)}{a^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n)/(a + b*x^n),x]
```

```
[Out] -((a/x^n + b*n*Log[x] - b*Log[a + b*x^n])/(a^2*n))
```

fricas [A] time = 0.81, size = 37, normalized size = 0.97

$$-\frac{bnx^n\log(x) - bx^n\log(bx^n+a) + a}{a^2nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(a+b*x^n),x, algorithm="fricas")
[Out] -(b*n*x^n*log(x) - b*x^n*log(b*x^n + a) + a)/(a^2*n*x^n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-n-1}}{bx^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(a+b*x^n),x, algorithm="giac")
[Out] integrate(x^(-n - 1)/(b*x^n + a), x)
```

maple [A] time = 0.02, size = 50, normalized size = 1.32

$$\left(-\frac{b\,\mathrm{e}^{n\ln(x)}\ln(x)}{a^2} - \frac{1}{an}\right)\mathrm{e}^{-n\ln(x)} + \frac{b\ln\left(b\,\mathrm{e}^{n\ln(x)} + a\right)}{a^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n)/(b*x^n+a),x)
[Out] (-1/a/n-b/a^2*ln(x)*exp(n*ln(x)))/exp(n*ln(x))+b/a^2/n*ln(b*exp(n*ln(x))+a)
```

maxima [A] time = 0.48, size = 42, normalized size = 1.11

$$-\frac{b\log(x)}{a^2} + \frac{b\log\left(\frac{bx^n+a}{b}\right)}{a^2 n} - \frac{1}{anx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(a+b*x^n),x, algorithm="maxima")
[Out] -b*log(x)/a^2 + b*log((b*x^n + a)/b)/(a^2*n) - 1/(a*n*x^n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x^{n+1}\,(a + b\,x^n)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(n + 1)*(a + b*x^n)),x)
[Out] int(1/(x^(n + 1)*(a + b*x^n)), x)
```

sympy [A] time = 21.08, size = 48, normalized size = 1.26

$$\begin{cases} \frac{\log(x)}{b} & \text{for } a = 0 \wedge n = 0 \\ -\frac{x^{-2n}}{2bn} & \text{for } a = 0 \\ \frac{\log(x)}{a+b} & \text{for } n = 0 \\ -\frac{x^{-n}}{an} + \frac{b\log\left(x^{-n}+\frac{b}{a}\right)}{a^2 n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n)/(a+b*x**n),x)
```

```
[Out] Piecewise((log(x)/b, Eq(a, 0) & Eq(n, 0)), (-x**(-2*n)/(2*b*n), Eq(a, 0)),
(log(x)/(a + b), Eq(n, 0)), (-x**(-n)/(a*n) + b*log(x**(-n) + b/a)/(a**2*n)
, True))
```

3.2604 $\int \frac{x^{-1-2n}}{a+bx^n}\,dx$

Optimal. Leaf size=57

$$-\frac{b^2\log(a+bx^n)}{a^3n}+\frac{b^2\log(x)}{a^3}+\frac{bx^{-n}}{a^2n}-\frac{x^{-2n}}{2an}$$

```
[Out] -1/2/a/n/(x^(2*n))+b/a^2/n/(x^n)+b^2*ln(x)/a^3-b^2*ln(a+b*x^n)/a^3/n
```

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 44}

$$-\frac{b^2\log(a+bx^n)}{a^3n}+\frac{b^2\log(x)}{a^3}+\frac{bx^{-n}}{a^2n}-\frac{x^{-2n}}{2an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 2*n)/(a + b*x^n),x]
```

```
[Out] -1/(2*a*n*x^(2*n)) + b/(a^2*n*x^n) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x^n]
)/(a^3*n)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1-2n}}{a+bx^n}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^3(a+bx)}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)dx, x, x^n\right)}{n}\\ &= -\frac{x^{-2n}}{2an}+\frac{bx^{-n}}{a^2n}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx^n)}{a^3n}\end{aligned}$$

Mathematica [A] time = 0.07, size = 49, normalized size = 0.86

$$\frac{-2b^2\log(a+bx^n)+ax^{-2n}(2bx^n-a)+2b^2n\log(x)}{2a^3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 2*n)/(a + b*x^n),x]
```

```
[Out] ((a*(-a + 2*b*x^n))/x^(2*n) + 2*b^2*n*Log[x] - 2*b^2*Log[a + b*x^n])/(2*a^3
*n)
```

fricas [A] time = 0.80, size = 59, normalized size = 1.04

$$\frac{2\,b^2 n x^{2\,n}\log(x) - 2\,b^2 x^{2\,n}\log\left(bx^n + a\right) + 2\,abx^n - a^2}{2\,a^3 n x^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*n*x^(2*n)*log(x) - 2*b^2*x^(2*n)*log(b*x^n + a) + 2*a*b*x^n - a^
2)/(a^3*n*x^(2*n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-2\,n-1}}{bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(-2*n - 1)/(b*x^n + a), x)
```

maple [A] time = 0.02, size = 69, normalized size = 1.21

$$\left(\frac{b^2 \mathrm{e}^{2n\ln(x)}\ln(x)}{a^3} + \frac{b\,\mathrm{e}^{n\ln(x)}}{a^2 n} - \frac{1}{2an}\right)\mathrm{e}^{-2n\ln(x)} - \frac{b^2\ln\left(b\,\mathrm{e}^{n\ln(x)} + a\right)}{a^3 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-2*n)/(b*x^n+a),x)
```

```
[Out] (b/a^2/n*exp(n*ln(x))-1/2/a/n+b^2/a^3*ln(x)*exp(n*ln(x))^2)/exp(n*ln(x))^2-
b^2/a^3/n*ln(b*exp(n*ln(x))+a)
```

maxima [A] time = 0.67, size = 58, normalized size = 1.02

$$\frac{b^2\log(x)}{a^3} - \frac{b^2\log\left(\frac{bx^n+a}{b}\right)}{a^3 n} + \frac{2\,bx^n - a}{2\,a^2 n x^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] b^2*log(x)/a^3 - b^2*log((b*x^n + a)/b)/(a^3*n) + 1/2*(2*b*x^n - a)/(a^2*n*
x^(2*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^{2\,n+1}\,\left(a + b\,x^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(2*n + 1)*(a + b*x^n)),x)
```

```
[Out] int(1/(x^(2*n + 1)*(a + b*x^n)), x)
```

sympy [**A**] time = 141.58, size = 85, normalized size = 1.49

$$\begin{cases} \tilde{\infty}\log(x) & \text{for } a = 0 \wedge b = 0 \wedge n = 0 \\ -\frac{x^{-3n}}{3bn} & \text{for } a = 0 \\ \frac{\log(x)}{a+b} & \text{for } n = 0 \\ -\frac{x^{-2n}}{2an} & \text{for } b = 0 \\ -\frac{x^{-2n}}{2an} + \frac{bx^{-n}}{a^2 n} + \frac{b^2\log(x)}{a^3} - \frac{b^2\log\left(\frac{a}{b}+x^n\right)}{a^3 n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-2*n)/(a+b*x**n),x)
```

```
[Out] Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (-x**(-3*n)/(3*b*n)
, Eq(a, 0)), (log(x)/(a + b), Eq(n, 0)), (-x**(-2*n)/(2*a*n), Eq(b, 0)), (-
x**(-2*n)/(2*a*n) + b*x**(-n)/(a**2*n) + b**2*log(x)/a**3 - b**2*log(a/b +
x**n)/(a**3*n), True))
```

3.2605 $\int \frac{x^{-1-3n}}{a+bx^n}\,dx$

Optimal. Leaf size=76

$$\frac{b^3\log\left(a+bx^n\right)}{a^4n}-\frac{b^3\log(x)}{a^4}-\frac{b^2x^{-n}}{a^3n}+\frac{bx^{-2n}}{2a^2n}-\frac{x^{-3n}}{3an}$$

```
[Out] -1/3/a/n/(x^(3*n))+1/2*b/a^2/n/(x^(2*n))-b^2/a^3/n/(x^n)-b^3*ln(x)/a^4+b^3*
ln(a+b*x^n)/a^4/n
```

Rubi [A] time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 44}

$$-\frac{b^2x^{-n}}{a^3n}+\frac{b^3\log\left(a+bx^n\right)}{a^4n}-\frac{b^3\log(x)}{a^4}+\frac{bx^{-2n}}{2a^2n}-\frac{x^{-3n}}{3an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 3*n)/(a + b*x^n),x]
```

```
[Out] -1/(3*a*n*x^(3*n)) + b/(2*a^2*n*x^(2*n)) - b^2/(a^3*n*x^n) - (b^3*Log[x])/a
^4 + (b^3*Log[a + b*x^n])/(a^4*n)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1-3n}}{a+bx^n}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^4(a+bx)}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^4}-\frac{b}{a^2x^3}+\frac{b^2}{a^3x^2}-\frac{b^3}{a^4x}+\frac{b^4}{a^4(a+bx)}\right)dx,x,x^n\right)}{n}\\ &= -\frac{x^{-3n}}{3an}+\frac{bx^{-2n}}{2a^2n}-\frac{b^2x^{-n}}{a^3n}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log\left(a+bx^n\right)}{a^4n}\end{aligned}$$

Mathematica [A] time = 0.11, size = 62, normalized size = 0.82

$$-\frac{ax^{-3n}\left(2a^2-3abx^n+6b^2x^{2n}\right)-6b^3\log\left(a+bx^n\right)+6b^3n\log(x)}{6a^4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 3*n)/(a + b*x^n),x]
```

```
[Out] -1/6*((a*(2*a^2 - 3*a*b*x^n + 6*b^2*x^(2*n)))/x^(3*n) + 6*b^3*n*Log[x] - 6*
b^3*Log[a + b*x^n])/(a^4*n)
```

fricas [A] time = 0.71, size = 72, normalized size = 0.95

$$-\frac{6\,b^3 n x^{3\,n}\log(x) - 6\,b^3 x^{3\,n}\log\left(bx^n + a\right) + 6\,ab^2 x^{2\,n} - 3\,a^2 bx^n + 2\,a^3}{6\,a^4 n x^{3\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] -1/6*(6*b^3*n*x^(3*n)*log(x) - 6*b^3*x^(3*n)*log(b*x^n + a) + 6*a*b^2*x^(2*
n) - 3*a^2*b*x^n + 2*a^3)/(a^4*n*x^(3*n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-3\,n-1}}{bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(-3*n - 1)/(b*x^n + a), x)
```

maple [A] time = 0.03, size = 88, normalized size = 1.16

$$\left(-\frac{b^3 \mathrm{e}^{3n\ln(x)}\ln(x)}{a^4} + \frac{b\,\mathrm{e}^{n\ln(x)}}{2a^2 n} - \frac{b^2 \mathrm{e}^{2n\ln(x)}}{a^3 n} - \frac{1}{3an}\right)\mathrm{e}^{-3n\ln(x)} + \frac{b^3\ln\left(b\,\mathrm{e}^{n\ln(x)} + a\right)}{a^4 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-3*n)/(b*x^n+a),x)
```

```
[Out] (-1/3/a/n+1/2/a^2*b/n*exp(n*ln(x))-b^2/a^3/n*exp(n*ln(x))^2-b^3/a^4*ln(x)*e
xp(n*ln(x))^3)/exp(n*ln(x))^3+b^3/a^4/n*ln(b*exp(n*ln(x))+a)
```

maxima [A] time = 0.51, size = 71, normalized size = 0.93

$$-\frac{b^3\log(x)}{a^4} + \frac{b^3\log\left(\frac{bx^n+a}{b}\right)}{a^4 n} - \frac{6\,b^2 x^{2\,n} - 3\,abx^n + 2\,a^2}{6\,a^3 n x^{3\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] -b^3*log(x)/a^4 + b^3*log((b*x^n + a)/b)/(a^4*n) - 1/6*(6*b^2*x^(2*n) - 3*a
*b*x^n + 2*a^2)/(a^3*n*x^(3*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{3\,n+1}\,\left(a + b\,x^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3*n + 1)*(a + b*x^n)),x)
```

```
[Out] int(1/(x^(3*n + 1)*(a + b*x^n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-3*n)/(a+b*x**n),x)
```

```
[Out] Timed out
```

3.2606 $\int \frac{x^{4+5(-1+n)}}{a+bx^n}\,dx$

Optimal. Leaf size=82

$$\frac{a^4 \log(a+bx^n)}{b^5 n} - \frac{a^3 x^n}{b^4 n} + \frac{a^2 x^{2n}}{2b^3 n} - \frac{a x^{3n}}{3b^2 n} + \frac{x^{4n}}{4bn}$$

```
[Out] -a^3*x^n/b^4/n+1/2*a^2*x^(2*n)/b^3/n-1/3*a*x^(3*n)/b^2/n+1/4*x^(4*n)/b/n+a^
4*ln(a+b*x^n)/b^5/n
```

Rubi [A] time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {266, 43}

$$-\frac{a^3 x^n}{b^4 n} + \frac{a^2 x^{2n}}{2b^3 n} + \frac{a^4 \log(a+bx^n)}{b^5 n} - \frac{a x^{3n}}{3b^2 n} + \frac{x^{4n}}{4bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(4 + 5*(-1 + n))/(a + b*x^n),x]
```

```
[Out] -((a^3*x^n)/(b^4*n)) + (a^2*x^(2*n))/(2*b^3*n) - (a*x^(3*n))/(3*b^2*n) + x^
(4*n)/(4*b*n) + (a^4*Log[a + b*x^n])/(b^5*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{4+5(-1+n)}}{a+bx^n}\,dx &= \frac{\text{Subst}\left(\int \frac{x^4}{a+bx}\,dx, x, x^n\right)}{n} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{a^3}{b^4} + \frac{a^2 x}{b^3} - \frac{a x^2}{b^2} + \frac{x^3}{b} + \frac{a^4}{b^4(a+bx)}\right)\,dx, x, x^n\right)}{n} \\
&= -\frac{a^3 x^n}{b^4 n} + \frac{a^2 x^{2n}}{2b^3 n} - \frac{a x^{3n}}{3b^2 n} + \frac{x^{4n}}{4bn} + \frac{a^4 \log(a+bx^n)}{b^5 n}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 65, normalized size = 0.79

$$\frac{12a^4 \log(a+bx^n) + bx^n\left(-12a^3 + 6a^2 bx^n - 4ab^2 x^{2n} + 3b^3 x^{3n}\right)}{12b^5 n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(4 + 5*(-1 + n))/(a + b*x^n),x]
```

```
[Out] (b*x^n*(-12*a^3 + 6*a^2*b*x^n - 4*a*b^2*x^(2*n) + 3*b^3*x^(3*n)) + 12*a^4*L
og[a + b*x^n])/(12*b^5*n)
```

fricas [A] time = 0.82, size = 65, normalized size = 0.79

$$\frac{3\,b^4x^{4\,n} - 4\,ab^3x^{3\,n} + 6\,a^2b^2x^{2\,n} - 12\,a^3bx^n + 12\,a^4\log\left(bx^n + a\right)}{12\,b^5n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+5*n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*x^(4*n) - 4*a*b^3*x^(3*n) + 6*a^2*b^2*x^(2*n) - 12*a^3*b*x^n +
12*a^4*log(b*x^n + a))/(b^5*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5\,n-1}}{bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+5*n)/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(5*n - 1)/(b*x^n + a), x)
```

maple [A] time = 0.00, size = 87, normalized size = 1.06

$$\frac{a^4\ln\left(b\,\mathrm{e}^{n\ln(x)} + a\right)}{b^5n} - \frac{a^3\mathrm{e}^{n\ln(x)}}{b^4n} + \frac{a^2\mathrm{e}^{2n\ln(x)}}{2b^3n} - \frac{a\,\mathrm{e}^{3n\ln(x)}}{3b^2n} + \frac{\mathrm{e}^{4n\ln(x)}}{4bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+5*n)/(b*x^n+a),x)
```

```
[Out] 1/4/b/n*exp(n*ln(x))^4-1/3*a/b^2/n*exp(n*ln(x))^3+1/2*a^2/b^3/n*exp(n*ln(x)
)^2-a^3/b^4/n*exp(n*ln(x))+a^4/b^5/n*ln(b*exp(n*ln(x))+a)
```

maxima [A] time = 0.57, size = 72, normalized size = 0.88

$$\frac{a^4\log\left(\frac{bx^n+a}{b}\right)}{b^5n} + \frac{3\,b^3x^{4\,n} - 4\,ab^2x^{3\,n} + 6\,a^2bx^{2\,n} - 12\,a^3x^n}{12\,b^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+5*n)/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] a^4*log((b*x^n + a)/b)/(b^5*n) + 1/12*(3*b^3*x^(4*n) - 4*a*b^2*x^(3*n) + 6*
a^2*b*x^(2*n) - 12*a^3*x^n)/(b^4*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5\,n-1}}{a + b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5*n - 1)/(a + b*x^n),x)
```

```
[Out] int(x^(5*n - 1)/(a + b*x^n), x)
```

sympy [**A**] time = 95.82, size = 87, normalized size = 1.06

$$\begin{cases} \frac{\log(x)}{a} & \text{for } b = 0 \wedge n = 0 \\ \frac{\log(x)}{a+b} & \text{for } n = 0 \\ \frac{x^{5n}}{5an} & \text{for } b = 0 \\ \frac{a^4 \log\left(\frac{a}{b}+x^n\right)}{b^5 n} - \frac{a^3 x^n}{b^4 n} + \frac{a^2 x^{2n}}{2b^3 n} - \frac{a x^{3n}}{3b^2 n} + \frac{x^{4n}}{4bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+5*n)/(a+b*x**n),x)
```

```
[Out] Piecewise((log(x)/a, Eq(b, 0) & Eq(n, 0)), (log(x)/(a + b), Eq(n, 0)), (x**
(5*n)/(5*a*n), Eq(b, 0)), (a**4*log(a/b + x**n)/(b**5*n) - a**3*x**n/(b**4*
n) + a**2*x**(2*n)/(2*b**3*n) - a*x**(3*n)/(3*b**2*n) + x**(4*n)/(4*b*n), T
rue))
```

3.2607 $\int \frac{x^{3+4(-1+n)}}{a+bx^n}\,dx$

Optimal. Leaf size=64

$$-\frac{a^3\log(a+bx^n)}{b^4n}+\frac{a^2x^n}{b^3n}-\frac{ax^{2n}}{2b^2n}+\frac{x^{3n}}{3bn}$$

```
[Out] a^2*x^n/b^3/n-1/2*a*x^(2*n)/b^2/n+1/3*x^(3*n)/b/n-a^3*ln(a+b*x^n)/b^4/n
```

Rubi [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {266, 43}

$$\frac{a^2x^n}{b^3n}-\frac{a^3\log(a+bx^n)}{b^4n}-\frac{ax^{2n}}{2b^2n}+\frac{x^{3n}}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3 + 4*(-1 + n))/(a + b*x^n),x]
```

```
[Out] (a^2*x^n)/(b^3*n) - (a*x^(2*n))/(2*b^2*n) + x^(3*n)/(3*b*n) - (a^3*Log[a +
b*x^n])/(b^4*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{3+4(-1+n)}}{a+bx^n}\,dx &= \frac{\text{Subst}\left(\int \frac{x^3}{a+bx}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right)dx, x, x^n\right)}{n}\\ &= \frac{a^2x^n}{b^3n}-\frac{ax^{2n}}{2b^2n}+\frac{x^{3n}}{3bn}-\frac{a^3\log(a+bx^n)}{b^4n}\end{aligned}$$

Mathematica [A] time = 0.04, size = 52, normalized size = 0.81

$$\frac{bx^n\left(6a^2-3abx^n+2b^2x^{2n}\right)-6a^3\log(a+bx^n)}{6b^4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3 + 4*(-1 + n))/(a + b*x^n),x]
```

```
[Out] (b*x^n*(6*a^2 - 3*a*b*x^n + 2*b^2*x^(2*n)) - 6*a^3*Log[a + b*x^n])/(6*b^4*n
)
```

fricas [A] time = 0.57, size = 52, normalized size = 0.81

$$\frac{2\,b^3x^{3\,n} - 3\,ab^2x^{2\,n} + 6\,a^2bx^n - 6\,a^3\log\left(bx^n + a\right)}{6\,b^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^(3*n) - 3*a*b^2*x^(2*n) + 6*a^2*b*x^n - 6*a^3*log(b*x^n + a))/
(b^4*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4\,n-1}}{bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(4*n - 1)/(b*x^n + a), x)
```

maple [A] time = 0.00, size = 69, normalized size = 1.08

$$-\frac{a^3\ln\left(b\,\mathrm{e}^{n\ln(x)} + a\right)}{b^4n} + \frac{a^2\mathrm{e}^{n\ln(x)}}{b^3n} - \frac{a\,\mathrm{e}^{2n\ln(x)}}{2b^2n} + \frac{\mathrm{e}^{3n\ln(x)}}{3bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+4*n)/(b*x^n+a),x)
```

```
[Out] a^2/b^3/n*exp(n*ln(x))+1/3/b/n*exp(n*ln(x))^3-1/2*a/b^2/n*exp(n*ln(x))^2-a^
3/b^4/n*ln(b*exp(n*ln(x))+a)
```

maxima [A] time = 0.63, size = 60, normalized size = 0.94

$$-\frac{a^3\log\left(\frac{bx^n+a}{b}\right)}{b^4n} + \frac{2\,b^2x^{3\,n} - 3\,abx^{2\,n} + 6\,a^2x^n}{6\,b^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] -a^3*log((b*x^n + a)/b)/(b^4*n) + 1/6*(2*b^2*x^(3*n) - 3*a*b*x^(2*n) + 6*a^
2*x^n)/(b^3*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{4\,n-1}}{a + b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4*n - 1)/(a + b*x^n),x)
```

```
[Out] int(x^(4*n - 1)/(a + b*x^n), x)
```

sympy [**A**] time = 48.26, size = 71, normalized size = 1.11

$$\begin{cases} \frac{\log(x)}{a} & \text{for } b = 0 \wedge n = 0 \\ \frac{\log(x)}{a+b} & \text{for } n = 0 \\ \frac{x^{4n}}{4an} & \text{for } b = 0 \\ -\frac{a^3 \log\left(\frac{a}{b}+x^n\right)}{b^4 n} + \frac{a^2 x^n}{b^3 n} - \frac{a x^{2n}}{2b^2 n} + \frac{x^{3n}}{3bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+4*n)/(a+b*x**n),x)
```

```
[Out] Piecewise((log(x)/a, Eq(b, 0) & Eq(n, 0)), (log(x)/(a + b), Eq(n, 0)), (x**
(4*n)/(4*a*n), Eq(b, 0)), (-a**3*log(a/b + x**n)/(b**4*n) + a**2*x**n/(b**3
*n) - a*x**(2*n)/(2*b**2*n) + x**(3*n)/(3*b*n), True))
```

3.2608 $\int \frac{x^{2+3(-1+n)}}{a+bx^n}\,dx$

Optimal. Leaf size=46

$$\frac{a^2\log(a+bx^n)}{b^3n}-\frac{ax^n}{b^2n}+\frac{x^{2n}}{2bn}$$

```
[Out] -a*x^n/b^2/n+1/2*x^(2*n)/b/n+a^2*ln(a+b*x^n)/b^3/n
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {266, 43}

$$\frac{a^2\log(a+bx^n)}{b^3n}-\frac{ax^n}{b^2n}+\frac{x^{2n}}{2bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2 + 3*(-1 + n))/(a + b*x^n),x]
```

```
[Out] -((a*x^n)/(b^2*n)) + x^(2*n)/(2*b*n) + (a^2*Log[a + b*x^n])/(b^3*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{2+3(-1+n)}}{a+bx^n}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{a+bx}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)\,dx, x, x^n\right)}{n}\\ &= -\frac{ax^n}{b^2n}+\frac{x^{2n}}{2bn}+\frac{a^2\log(a+bx^n)}{b^3n}\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 0.83

$$\frac{2a^2\log(a+bx^n)+bx^n(bx^n-2a)}{2b^3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2 + 3*(-1 + n))/(a + b*x^n),x]
```

```
[Out] (b*x^n*(-2*a + b*x^n) + 2*a^2*Log[a + b*x^n])/(2*b^3*n)
```

fricas [A] time = 0.68, size = 38, normalized size = 0.83

$$\frac{b^2x^{2n}-2abx^n+2a^2\log(bx^n+a)}{2b^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n),x, algorithm="fricas")
[Out] 1/2*(b^2*x^(2*n) - 2*a*b*x^n + 2*a^2*log(b*x^n + a))/(b^3*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3\,n-1}}{bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n),x, algorithm="giac")
[Out] integrate(x^(3*n - 1)/(b*x^n + a), x)
```

maple [A] time = 0.00, size = 51, normalized size = 1.11

$$\frac{a^2 \ln\left(b\,\mathrm{e}^{n\ln(x)} + a\right)}{b^3 n} - \frac{a\,\mathrm{e}^{n\ln(x)}}{b^2 n} + \frac{\mathrm{e}^{2n\ln(x)}}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n-1)/(b*x^n+a),x)
[Out] 1/2/b/n*exp(n*ln(x))^2-a/b^2/n*exp(n*ln(x))+a^2/b^3/n*ln(b*exp(n*ln(x))+a)
```

maxima [A] time = 0.50, size = 45, normalized size = 0.98

$$\frac{a^2 \log\left(\frac{bx^n+a}{b}\right)}{b^3 n} + \frac{bx^{2\,n} - 2\,ax^n}{2\,b^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n),x, algorithm="maxima")
[Out] a^2*log((b*x^n + a)/b)/(b^3*n) + 1/2*(b*x^(2*n) - 2*a*x^n)/(b^2*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{3\,n-1}}{a + b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n - 1)/(a + b*x^n),x)
[Out] int(x^(3*n - 1)/(a + b*x^n), x)
```

sympy [A] time = 26.72, size = 56, normalized size = 1.22

$$\begin{cases} \frac{\log(x)}{a} & \text{for } b = 0 \wedge n = 0 \\ \frac{\log(x)}{a+b} & \text{for } n = 0 \\ \frac{x^{3n}}{3an} & \text{for } b = 0 \\ \frac{a^2 \log\left(\frac{a}{b}+x^n\right)}{b^3 n} - \frac{ax^n}{b^2 n} + \frac{x^{2n}}{2bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)/(a+b*x**n),x)
```

```
[Out] Piecewise((log(x)/a, Eq(b, 0) & Eq(n, 0)), (log(x)/(a + b), Eq(n, 0)), (x**
(3*n)/(3*a*n), Eq(b, 0)), (a**2*log(a/b + x**n)/(b**3*n) - a*x**n/(b**2*n)
+ x**(2*n)/(2*b*n), True))
```

3.2609 $\int \frac{x^{1+2(-1+n)}}{a+bx^n}\,dx$

Optimal. Leaf size=28

$$\frac{x^n}{bn} - \frac{a\log(a+bx^n)}{b^2 n}$$

```
[Out] x^n/b/n-a*ln(a+b*x^n)/b^2/n
```

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {266, 43}

$$\frac{x^n}{bn} - \frac{a\log(a+bx^n)}{b^2 n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1 + 2*(-1 + n))/(a + b*x^n),x]
```

```
[Out] x^n/(b*n) - (a*Log[a + b*x^n])/(b^2*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{1+2(-1+n)}}{a+bx^n}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{a+bx}\,dx, x, x^n\right)}{n} \\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{b} - \frac{a}{b(a+bx)}\right)\,dx, x, x^n\right)}{n} \\ &= \frac{x^n}{bn} - \frac{a\log(a+bx^n)}{b^2 n}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.93

$$\frac{\frac{x^n}{b} - \frac{a\log(a+bx^n)}{b^2}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1 + 2*(-1 + n))/(a + b*x^n),x]
```

```
[Out] (x^n/b - (a*Log[a + b*x^n])/b^2)/n
```

fricas [A] time = 0.80, size = 24, normalized size = 0.86

$$\frac{bx^n - a\log(bx^n + a)}{b^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n),x, algorithm="fricas")
[Out] (b*x^n - a*log(b*x^n + a))/(b^2*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2\,n-1}}{bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n),x, algorithm="giac")
[Out] integrate(x^(2*n - 1)/(b*x^n + a), x)
```

maple [A] time = 0.00, size = 33, normalized size = 1.18

$$-\frac{a\ln\left(b\,\mathrm{e}^{n\ln(x)} + a\right)}{b^2 n} + \frac{\mathrm{e}^{n\ln(x)}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n-1)/(b*x^n+a),x)
[Out] -a/b^2/n*ln(b*exp(n*ln(x))+a)+1/b/n*exp(n*ln(x))
```

maxima [A] time = 0.63, size = 32, normalized size = 1.14

$$\frac{x^n}{bn} - \frac{a\log\left(\frac{bx^n+a}{b}\right)}{b^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n),x, algorithm="maxima")
[Out] x^n/(b*n) - a*log((b*x^n + a)/b)/(b^2*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^{2\,n-1}}{a + b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/(a + b*x^n),x)
[Out] int(x^(2*n - 1)/(a + b*x^n), x)
```

sympy [A] time = 13.29, size = 41, normalized size = 1.46

$$\begin{cases} \frac{\log(x)}{a} & \text{for } b = 0 \wedge n = 0 \\ \frac{\log(x)}{a+b} & \text{for } n = 0 \\ \frac{x^{2n}}{2an} & \text{for } b = 0 \\ -\frac{a\log\left(\frac{a}{b}+x^n\right)}{b^2 n} + \frac{x^n}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a+b*x**n),x)
[Out] Piecewise((log(x)/a, Eq(b, 0) & Eq(n, 0)), (log(x)/(a + b), Eq(n, 0)), (x**
(2*n)/(2*a*n), Eq(b, 0)), (-a*log(a/b + x**n)/(b**2*n) + x**n/(b*n), True))
```

3.2610 $\int \frac{x^{-1+n}}{a+bx^n}\,dx$

Optimal. Leaf size=15

$$\frac{\log(a+bx^n)}{bn}$$

```
[Out] ln(a+b*x^n)/b/n
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {260}

$$\frac{\log(a+bx^n)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)/(a + b*x^n),x]
```

```
[Out] Log[a + b*x^n]/(b*n)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^{-1+n}}{a+bx^n}\,dx = \frac{\log(a+bx^n)}{bn}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log(a+bx^n)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)/(a + b*x^n),x]
```

```
[Out] Log[a + b*x^n]/(b*n)
```

fricas [A] time = 0.62, size = 15, normalized size = 1.00

$$\frac{\log(bx^n+a)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] log(b*x^n + a)/(b*n)
```

giac [A] time = 0.15, size = 16, normalized size = 1.07

$$\frac{\log(|bx^n+a|)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(a+b*x^n),x, algorithm="giac")
```

```
[Out] log(abs(b*x^n + a))/(b*n)
```

maple [A] time = 0.00, size = 18, normalized size = 1.20

$$\frac{\ln\left(b\,e^{n\ln(x)}+a\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)/(b*x^n+a),x)
```

```
[Out] 1/b/n*ln(b*exp(n*ln(x))+a)
```

maxima [A] time = 0.53, size = 15, normalized size = 1.00

$$\frac{\log\left(bx^n+a\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] log(b*x^n + a)/(b*n)
```

mupad [B] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\ln\left(a+b\,x^n\right)}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)/(a + b*x^n),x)
```

```
[Out] log(a + b*x^n)/(b*n)
```

sympy [A] time = 2.70, size = 27, normalized size = 1.80

$$\begin{cases} \frac{\log(x)}{a} & \text{for } b = 0 \wedge n = 0 \\ \frac{\log(x)}{a+b} & \text{for } n = 0 \\ \frac{x^n}{an} & \text{for } b = 0 \\ \frac{\log\left(\frac{a}{b}+x^n\right)}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)/(a+b*x**n),x)
```

```
[Out] Piecewise((log(x)/a, Eq(b, 0) & Eq(n, 0)), (log(x)/(a + b), Eq(n, 0)), (x**
n/(a*n), Eq(b, 0)), (log(a/b + x**n)/(b*n), True))
```

3.2611 $\int \frac{1}{x(a+bx^n)}\,dx$

Optimal. Leaf size=23

$$\frac{\log(x)}{a} - \frac{\log(a+bx^n)}{an}$$

```
[Out] ln(x)/a-ln(a+b*x^n)/a/n
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {266, 36, 29, 31}

$$\frac{\log(x)}{a} - \frac{\log(a+bx^n)}{an}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^n)),x]
```

```
[Out] Log[x]/a - Log[a + b*x^n]/(a*n)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\,(a+bx^n)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx, x, x^n\right)}{n} \\ &= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^n\right)}{an} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, x^n\right)}{an} \\ &= \frac{\log(x)}{a} - \frac{\log(a+bx^n)}{an}\end{aligned}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 0.96

$$\frac{n\log(x) - \log(a+bx^n)}{an}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^n)),x]
[Out] (n*Log[x] - Log[a + b*x^n])/(a*n)
```

fricas [A] time = 0.69, size = 22, normalized size = 0.96

$$\frac{n\log(x) - \log(bx^n + a)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n),x, algorithm="fricas")
[Out] (n*log(x) - log(b*x^n + a))/(a*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n),x, algorithm="giac")
[Out] integrate(1/((b*x^n + a)*x), x)
```

maple [A] time = 0.00, size = 29, normalized size = 1.26

$$\frac{\ln(x^n)}{an} - \frac{\ln(b\,x^n + a)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^n+a),x)
[Out] 1/a/n*ln(x^n)-1/a/n*ln(b*x^n+a)
```

maxima [A] time = 0.74, size = 28, normalized size = 1.22

$$-\frac{\log(bx^n + a)}{an} + \frac{\log(x^n)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n),x, algorithm="maxima")
[Out] -log(b*x^n + a)/(a*n) + log(x^n)/(a*n)
```

mupad [B] time = 0.00, size = 22, normalized size = 0.96

$$-\frac{\ln(a + b\,x^n) - n\,\ln(x)}{a\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^n)),x)
[Out] -(log(a + b*x^n) - n*log(x))/(a*n)
```

sympy [**A**] time = 0.74, size = 41, normalized size = 1.78

$$\begin{cases} \tilde{\infty}\log(x) & \text{for } a = 0 \wedge b = 0 \wedge n = 0 \\ \frac{\log(x)}{a} & \text{for } b = 0 \\ -\frac{x^{-n}}{bn} & \text{for } a = 0 \\ \frac{\log(x)}{a+b} & \text{for } n = 0 \\ \frac{\log(x)}{a} - \frac{\log\left(\frac{a}{b}+x^n\right)}{an} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x**n),x)
```

```
[Out] Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (log(x)/a, Eq(b, 0)
), (-x**(-n)/(b*n), Eq(a, 0)), (log(x)/(a + b), Eq(n, 0)), (log(x)/a - log(
a/b + x**n)/(a*n), True))
```

3.2612 $\int \frac{x^{-1-n}}{a+bx^n}\,dx$

Optimal. Leaf size=38

$$\frac{b\log(a+bx^n)}{a^2n} - \frac{b\log(x)}{a^2} - \frac{x^{-n}}{an}$$

```
[Out] -1/a/n/(x^n)-b*ln(x)/a^2+b*ln(a+b*x^n)/a^2/n
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 44}

$$\frac{b\log(a+bx^n)}{a^2n} - \frac{b\log(x)}{a^2} - \frac{x^{-n}}{an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n)/(a + b*x^n),x]
```

```
[Out] -(1/(a*n*x^n)) - (b*Log[x])/a^2 + (b*Log[a + b*x^n])/(a^2*n)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1-n}}{a+bx^n}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx)}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^2} - \frac{b}{a^2x} + \frac{b^2}{a^2(a+bx)}\right)\,dx, x, x^n\right)}{n}\\ &= -\frac{x^{-n}}{an} - \frac{b\log(x)}{a^2} + \frac{b\log(a+bx^n)}{a^2n}\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 0.84

$$-\frac{-b\log(a+bx^n) + ax^{-n} + bn\log(x)}{a^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n)/(a + b*x^n),x]
```

```
[Out] -((a/x^n + b*n*Log[x] - b*Log[a + b*x^n])/(a^2*n))
```

fricas [A] time = 0.84, size = 37, normalized size = 0.97

$$-\frac{bnx^n\log(x) - bx^n\log(bx^n+a) + a}{a^2nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] -(b*n*x^n*log(x) - b*x^n*log(b*x^n + a) + a)/(a^2*n*x^n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-n-1}}{bx^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(-n - 1)/(b*x^n + a), x)
```

maple [A] time = 0.00, size = 50, normalized size = 1.32

$$\left(-\frac{b\,\mathrm{e}^{n\ln(x)}\ln(x)}{a^2} - \frac{1}{an}\right)\mathrm{e}^{-n\ln(x)} + \frac{b\ln\left(b\,\mathrm{e}^{n\ln(x)} + a\right)}{a^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n)/(b*x^n+a),x)
```

```
[Out] (-1/a^2*b*exp(n*ln(x))*ln(x)-1/a/n)/exp(n*ln(x))+1/a^2*b/n*ln(b*exp(n*ln(x)
)+a)
```

maxima [A] time = 0.69, size = 42, normalized size = 1.11

$$-\frac{b\log(x)}{a^2} + \frac{b\log\left(\frac{bx^n+a}{b}\right)}{a^2 n} - \frac{1}{anx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] -b*log(x)/a^2 + b*log((b*x^n + a)/b)/(a^2*n) - 1/(a*n*x^n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x^{n+1}\,(a + b\,x^n)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(n + 1)*(a + b*x^n)),x)
```

```
[Out] int(1/(x^(n + 1)*(a + b*x^n)), x)
```

sympy [A] time = 22.38, size = 48, normalized size = 1.26

$$\begin{cases} \frac{\log(x)}{b} & \text{for } a = 0 \wedge n = 0 \\ -\frac{x^{-2n}}{2bn} & \text{for } a = 0 \\ \frac{\log(x)}{a+b} & \text{for } n = 0 \\ -\frac{x^{-n}}{an} + \frac{b\log\left(x^{-n}+\frac{b}{a}\right)}{a^2 n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n)/(a+b*x**n),x)
```

```
[Out] Piecewise((log(x)/b, Eq(a, 0) & Eq(n, 0)), (-x**(-2*n)/(2*b*n), Eq(a, 0)),
(log(x)/(a + b), Eq(n, 0)), (-x**(-n)/(a*n) + b*log(x**(-n) + b/a)/(a**2*n)
, True))
```

3.2613 $\int \frac{x^{-3-2(-1+n)}}{a+bx^n}\,dx$

Optimal. Leaf size=57

$$-\frac{b^2\log\left(a+bx^n\right)}{a^3n}+\frac{b^2\log(x)}{a^3}+\frac{bx^{-n}}{a^2n}-\frac{x^{-2n}}{2an}$$

```
[Out] -1/2/a/n/(x^(2*n))+b/a^2/n/(x^n)+b^2*ln(x)/a^3-b^2*ln(a+b*x^n)/a^3/n
```

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {266, 44}

$$-\frac{b^2\log\left(a+bx^n\right)}{a^3n}+\frac{b^2\log(x)}{a^3}+\frac{bx^{-n}}{a^2n}-\frac{x^{-2n}}{2an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3 - 2*(-1 + n))/(a + b*x^n),x]
```

```
[Out] -1/(2*a*n*x^(2*n)) + b/(a^2*n*x^n) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x^n]
)/(a^3*n)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-3-2(-1+n)}}{a+bx^n}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^3(a+bx)}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)\,dx, x, x^n\right)}{n}\\ &= -\frac{x^{-2n}}{2an}+\frac{bx^{-n}}{a^2n}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log\left(a+bx^n\right)}{a^3n}\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.86

$$\frac{-2b^2\log\left(a+bx^n\right)+ax^{-2n}\left(2bx^n-a\right)+2b^2n\log(x)}{2a^3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3 - 2*(-1 + n))/(a + b*x^n),x]
```

```
[Out] ((a*(-a + 2*b*x^n))/x^(2*n) + 2*b^2*n*Log[x] - 2*b^2*Log[a + b*x^n])/(2*a^3
*n)
```

fricas [A] time = 0.80, size = 59, normalized size = 1.04

$$\frac{2\,b^2 n x^{2\,n}\log(x) - 2\,b^2 x^{2\,n}\log\left(bx^n + a\right) + 2\,abx^n - a^2}{2\,a^3 n x^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*n*x^(2*n)*log(x) - 2*b^2*x^(2*n)*log(b*x^n + a) + 2*a*b*x^n - a^
2)/(a^3*n*x^(2*n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-2\,n-1}}{bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(-2*n - 1)/(b*x^n + a), x)
```

maple [A] time = 0.00, size = 69, normalized size = 1.21

$$\left(\frac{b^2 \mathrm{e}^{2n\ln(x)}\ln(x)}{a^3} + \frac{b\,\mathrm{e}^{n\ln(x)}}{a^2 n} - \frac{1}{2an}\right)\mathrm{e}^{-2n\ln(x)} - \frac{b^2\ln\left(b\,\mathrm{e}^{n\ln(x)} + a\right)}{a^3 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-2*n)/(b*x^n+a),x)
```

```
[Out] (1/a^2*b/n*exp(n*ln(x))-1/2/a/n+b^2/a^3*ln(x)*exp(n*ln(x))^2)/exp(n*ln(x))^
2-1/a^3*b^2/n*ln(b*exp(n*ln(x))+a)
```

maxima [A] time = 0.66, size = 58, normalized size = 1.02

$$\frac{b^2\log(x)}{a^3} - \frac{b^2\log\left(\frac{bx^n+a}{b}\right)}{a^3 n} + \frac{2\,bx^n - a}{2\,a^2 n x^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] b^2*log(x)/a^3 - b^2*log((b*x^n + a)/b)/(a^3*n) + 1/2*(2*b*x^n - a)/(a^2*n*
x^(2*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^{2\,n+1}\,\left(a + b\,x^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(2*n + 1)*(a + b*x^n)),x)
```

```
[Out] int(1/(x^(2*n + 1)*(a + b*x^n)), x)
```

sympy [**A**] time = 145.96, size = 85, normalized size = 1.49

$$\begin{cases} \tilde{\infty}\log(x) & \text{for } a = 0 \wedge b = 0 \wedge n = 0 \\ -\frac{x^{-3n}}{3bn} & \text{for } a = 0 \\ \frac{\log(x)}{a+b} & \text{for } n = 0 \\ -\frac{x^{-2n}}{2an} & \text{for } b = 0 \\ -\frac{x^{-2n}}{2an} + \frac{bx^{-n}}{a^2 n} + \frac{b^2\log(x)}{a^3} - \frac{b^2\log\left(\frac{a}{b}+x^n\right)}{a^3 n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-2*n)/(a+b*x**n),x)
```

```
[Out] Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (-x**(-3*n)/(3*b*n)
, Eq(a, 0)), (log(x)/(a + b), Eq(n, 0)), (-x**(-2*n)/(2*a*n), Eq(b, 0)), (-
x**(-2*n)/(2*a*n) + b*x**(-n)/(a**2*n) + b**2*log(x)/a**3 - b**2*log(a/b +
x**n)/(a**3*n), True))
```

3.2614 $\int \frac{x^{-4-3(-1+n)}}{a+bx^n}\,dx$

Optimal. Leaf size=76

$$\frac{b^3\log\left(a+bx^n\right)}{a^4n}-\frac{b^3\log(x)}{a^4}-\frac{b^2x^{-n}}{a^3n}+\frac{bx^{-2n}}{2a^2n}-\frac{x^{-3n}}{3an}$$

```
[Out] -1/3/a/n/(x^(3*n))+1/2*b/a^2/n/(x^(2*n))-b^2/a^3/n/(x^n)-b^3*ln(x)/a^4+b^3*
ln(a+b*x^n)/a^4/n
```

Rubi [A] time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {266, 44}

$$-\frac{b^2x^{-n}}{a^3n}+\frac{b^3\log\left(a+bx^n\right)}{a^4n}-\frac{b^3\log(x)}{a^4}+\frac{bx^{-2n}}{2a^2n}-\frac{x^{-3n}}{3an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-4 - 3*(-1 + n))/(a + b*x^n),x]
```

```
[Out] -1/(3*a*n*x^(3*n)) + b/(2*a^2*n*x^(2*n)) - b^2/(a^3*n*x^n) - (b^3*Log[x])/a
^4 + (b^3*Log[a + b*x^n])/(a^4*n)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-4-3(-1+n)}}{a+bx^n}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^4(a+bx)}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^4}-\frac{b}{a^2x^3}+\frac{b^2}{a^3x^2}-\frac{b^3}{a^4x}+\frac{b^4}{a^4(a+bx)}\right)dx, x, x^n\right)}{n}\\ &= -\frac{x^{-3n}}{3an}+\frac{bx^{-2n}}{2a^2n}-\frac{b^2x^{-n}}{a^3n}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log\left(a+bx^n\right)}{a^4n}\end{aligned}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 0.82

$$-\frac{ax^{-3n}\left(2a^2-3abx^n+6b^2x^{2n}\right)-6b^3\log\left(a+bx^n\right)+6b^3n\log(x)}{6a^4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-4 - 3*(-1 + n))/(a + b*x^n),x]
```

```
[Out] -1/6*((a*(2*a^2 - 3*a*b*x^n + 6*b^2*x^(2*n)))/x^(3*n) + 6*b^3*n*Log[x] - 6*
b^3*Log[a + b*x^n])/(a^4*n)
```

fricas [A] time = 0.58, size = 72, normalized size = 0.95

$$-\frac{6\,b^3nx^{3\,n}\log(x)-6\,b^3x^{3\,n}\log\left(bx^n+a\right)+6\,ab^2x^{2\,n}-3\,a^2bx^n+2\,a^3}{6\,a^4nx^{3\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] -1/6*(6*b^3*n*x^(3*n)*log(x) - 6*b^3*x^(3*n)*log(b*x^n + a) + 6*a*b^2*x^(2*
n) - 3*a^2*b*x^n + 2*a^3)/(a^4*n*x^(3*n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-3\,n-1}}{bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(-3*n - 1)/(b*x^n + a), x)
```

maple [A] time = 0.00, size = 88, normalized size = 1.16

$$\left(-\frac{b^3\mathrm{e}^{3n\ln(x)}\ln(x)}{a^4}+\frac{b\,\mathrm{e}^{n\ln(x)}}{2a^2n}-\frac{b^2\mathrm{e}^{2n\ln(x)}}{a^3n}-\frac{1}{3an}\right)\mathrm{e}^{-3n\ln(x)}+\frac{b^3\ln\left(b\,\mathrm{e}^{n\ln(x)}+a\right)}{a^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-3*n)/(b*x^n+a),x)
```

```
[Out] (-1/3/a/n+1/2/a^2*b/n*exp(n*ln(x))-b^2/a^3/n*exp(n*ln(x))^2-b^3/a^4*ln(x)*e
xp(n*ln(x))^3)/exp(n*ln(x))^3+1/a^4*b^3/n*ln(b*exp(n*ln(x))+a)
```

maxima [A] time = 0.53, size = 71, normalized size = 0.93

$$-\frac{b^3\log(x)}{a^4}+\frac{b^3\log\left(\frac{bx^n+a}{b}\right)}{a^4n}-\frac{6\,b^2x^{2\,n}-3\,abx^n+2\,a^2}{6\,a^3nx^{3\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] -b^3*log(x)/a^4 + b^3*log((b*x^n + a)/b)/(a^4*n) - 1/6*(6*b^2*x^(2*n) - 3*a
*b*x^n + 2*a^2)/(a^3*n*x^(3*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{3\,n+1}\,\left(a+b\,x^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3*n + 1)*(a + b*x^n)),x)
```

```
[Out] int(1/(x^(3*n + 1)*(a + b*x^n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-3*n)/(a+b*x**n),x)
```

```
[Out] Timed out
```

3.2615 $\int \frac{x^{-1+5n}}{2+bx^n}\,dx$

Optimal. Leaf size=71

$$\frac{16\log(bx^n+2)}{b^5n}-\frac{8x^n}{b^4n}+\frac{2x^{2n}}{b^3n}-\frac{2x^{3n}}{3b^2n}+\frac{x^{4n}}{4bn}$$

```
[Out] -8*x^n/b^4/n+2*x^(2*n)/b^3/n-2/3*x^(3*n)/b^2/n+1/4*x^(4*n)/b/n+16*ln(2+b*x^
n)/b^5/n
```

Rubi [A] time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$-\frac{8x^n}{b^4n}+\frac{2x^{2n}}{b^3n}-\frac{2x^{3n}}{3b^2n}+\frac{16\log(bx^n+2)}{b^5n}+\frac{x^{4n}}{4bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 5*n)/(2 + b*x^n),x]
```

```
[Out] (-8*x^n)/(b^4*n) + (2*x^(2*n))/(b^3*n) - (2*x^(3*n))/(3*b^2*n) + x^(4*n)/(4
*b*n) + (16*Log[2 + b*x^n])/(b^5*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+5n}}{2+bx^n}\,dx &= \frac{\text{Subst}\left(\int \frac{x^4}{2+bx}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{8}{b^4}+\frac{4x}{b^3}-\frac{2x^2}{b^2}+\frac{x^3}{b}+\frac{16}{b^4(2+bx)}\right)dx, x, x^n\right)}{n}\\ &= -\frac{8x^n}{b^4n}+\frac{2x^{2n}}{b^3n}-\frac{2x^{3n}}{3b^2n}+\frac{x^{4n}}{4bn}+\frac{16\log(2+bx^n)}{b^5n}\end{aligned}$$

Mathematica [A] time = 0.05, size = 54, normalized size = 0.76

$$\frac{bx^n\left(3b^3x^{3n}-8b^2x^{2n}+24bx^n-96\right)+192\log(bx^n+2)}{12b^5n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 5*n)/(2 + b*x^n),x]
```

```
[Out] (b*x^n*(-96 + 24*b*x^n - 8*b^2*x^(2*n) + 3*b^3*x^(3*n)) + 192*Log[2 + b*x^n
])/(12*b^5*n)
```

fricas [A] time = 0.47, size = 55, normalized size = 0.77

$$\frac{3\,b^4x^{4\,n} - 8\,b^3x^{3\,n} + 24\,b^2x^{2\,n} - 96\,bx^n + 192\,\log\left(bx^n + 2\right)}{12\,b^5n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+5*n)/(2+b*x^n),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*x^(4*n) - 8*b^3*x^(3*n) + 24*b^2*x^(2*n) - 96*b*x^n + 192*log(b
*x^n + 2))/(b^5*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5\,n-1}}{bx^n + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+5*n)/(2+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(5*n - 1)/(b*x^n + 2), x)
```

maple [A] time = 0.03, size = 78, normalized size = 1.10

$$\frac{\mathrm{e}^{4n\ln(x)}}{4bn} - \frac{2\,\mathrm{e}^{3n\ln(x)}}{3b^2n} + \frac{2\,\mathrm{e}^{2n\ln(x)}}{b^3n} - \frac{8\,\mathrm{e}^{n\ln(x)}}{b^4n} + \frac{16\ln\left(b\,\mathrm{e}^{n\ln(x)} + 2\right)}{b^5n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+5*n)/(2+b*x^n),x)
```

```
[Out] -8/b^4/n*exp(n*ln(x))+2/b^3/n*exp(n*ln(x))^2-2/3/b^2/n*exp(n*ln(x))^3+1/4/b
/n*exp(n*ln(x))^4+16/b^5/n*ln(2+b*exp(n*ln(x)))
```

maxima [A] time = 0.49, size = 63, normalized size = 0.89

$$\frac{3\,b^3x^{4\,n} - 8\,b^2x^{3\,n} + 24\,bx^{2\,n} - 96\,x^n}{12\,b^4n} + \frac{16\,\log\left(\frac{bx^n+2}{b}\right)}{b^5n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+5*n)/(2+b*x^n),x, algorithm="maxima")
```

```
[Out] 1/12*(3*b^3*x^(4*n) - 8*b^2*x^(3*n) + 24*b*x^(2*n) - 96*x^n)/(b^4*n) + 16*l
og((b*x^n + 2)/b)/(b^5*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5\,n-1}}{b\,x^n + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5*n - 1)/(b*x^n + 2),x)
```

```
[Out] int(x^(5*n - 1)/(b*x^n + 2), x)
```

sympy [**A**] time = 76.52, size = 78, normalized size = 1.10

$$\begin{cases} \frac{\log(x)}{2} & \text{for } b = 0 \wedge n = 0 \\ \frac{\log(x)}{b+2} & \text{for } n = 0 \\ \frac{x^{5n}}{10n} & \text{for } b = 0 \\ \frac{x^{4n}}{4bn} - \frac{2x^{3n}}{3b^2n} + \frac{2x^{2n}}{b^3n} - \frac{8x^n}{b^4n} + \frac{16\log\left(x^n+\frac{2}{b}\right)}{b^5n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+5*n)/(2+b*x**n),x)
```

```
[Out] Piecewise((log(x)/2, Eq(b, 0) & Eq(n, 0)), (log(x)/(b + 2), Eq(n, 0)), (x**
(5*n)/(10*n), Eq(b, 0)), (x**(4*n)/(4*b*n) - 2*x**(3*n)/(3*b**2*n) + 2*x**(
2*n)/(b**3*n) - 8*x**n/(b**4*n) + 16*log(x**n + 2/b)/(b**5*n), True))
```

3.2616 $\int \frac{x^{-1+4n}}{2+bx^n}\,dx$

Optimal. Leaf size=56

$$-\frac{8\log(bx^n+2)}{b^4n}+\frac{4x^n}{b^3n}-\frac{x^{2n}}{b^2n}+\frac{x^{3n}}{3bn}$$

```
[Out] 4*x^n/b^3/n-x^(2*n)/b^2/n+1/3*x^(3*n)/b/n-8*ln(2+b*x^n)/b^4/n
```

Rubi [A] time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$\frac{4x^n}{b^3n}-\frac{x^{2n}}{b^2n}-\frac{8\log(bx^n+2)}{b^4n}+\frac{x^{3n}}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 4*n)/(2 + b*x^n),x]
```

```
[Out] (4*x^n)/(b^3*n) - x^(2*n)/(b^2*n) + x^(3*n)/(3*b*n) - (8*Log[2 + b*x^n])/(b
^4*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+4n}}{2+bx^n}\,dx &= \frac{\text{Subst}\left(\int \frac{x^3}{2+bx}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{4}{b^3}-\frac{2x}{b^2}+\frac{x^2}{b}-\frac{8}{b^3(2+bx)}\right)dx, x, x^n\right)}{n}\\ &= \frac{4x^n}{b^3n}-\frac{x^{2n}}{b^2n}+\frac{x^{3n}}{3bn}-\frac{8\log(2+bx^n)}{b^4n}\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 0.77

$$\frac{bx^n\left(b^2x^{2n}-3bx^n+12\right)-24\log(bx^n+2)}{3b^4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 4*n)/(2 + b*x^n),x]
```

```
[Out] (b*x^n*(12 - 3*b*x^n + b^2*x^(2*n)) - 24*Log[2 + b*x^n])/(3*b^4*n)
```

fricas [A] time = 0.61, size = 44, normalized size = 0.79

$$\frac{b^3x^{3\,n} - 3\,b^2x^{2\,n} + 12\,bx^n - 24\,\log\left(bx^n + 2\right)}{3\,b^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(2+b*x^n),x, algorithm="fricas")
```

```
[Out] 1/3*(b^3*x^(3*n) - 3*b^2*x^(2*n) + 12*b*x^n - 24*log(b*x^n + 2))/(b^4*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4\,n-1}}{bx^n + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(2+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(4*n - 1)/(b*x^n + 2), x)
```

maple [A] time = 0.03, size = 63, normalized size = 1.12

$$\frac{\mathrm{e}^{3n\ln(x)}}{3bn} - \frac{\mathrm{e}^{2n\ln(x)}}{b^2n} + \frac{4\,\mathrm{e}^{n\ln(x)}}{b^3n} - \frac{8\ln\left(b\,\mathrm{e}^{n\ln(x)} + 2\right)}{b^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+4*n)/(2+b*x^n),x)
```

```
[Out] 4/b^3/n*exp(n*ln(x))-1/b^2/n*exp(n*ln(x))^2+1/3/b/n*exp(n*ln(x))^3-8/b^4/n*
ln(b*exp(n*ln(x))+2)
```

maxima [A] time = 0.62, size = 52, normalized size = 0.93

$$\frac{b^2x^{3\,n} - 3\,bx^{2\,n} + 12\,x^n}{3\,b^3n} - \frac{8\,\log\left(\frac{bx^n+2}{b}\right)}{b^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(2+b*x^n),x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*x^(3*n) - 3*b*x^(2*n) + 12*x^n)/(b^3*n) - 8*log((b*x^n + 2)/b)/(b^
4*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{4\,n-1}}{b\,x^n + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4*n - 1)/(b*x^n + 2),x)
```

```
[Out] int(x^(4*n - 1)/(b*x^n + 2), x)
```

sympy [A] time = 36.36, size = 63, normalized size = 1.12

$$\begin{cases} \frac{\log(x)}{2} & \text{for } b = 0 \wedge n = 0 \\ \frac{\log(x)}{b+2} & \text{for } n = 0 \\ \frac{x^{4n}}{8n} & \text{for } b = 0 \\ \frac{x^{3n}}{3bn} - \frac{x^{2n}}{b^2n} + \frac{4x^n}{b^3n} - \frac{8\log\left(x^n+\frac{2}{b}\right)}{b^4n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+4*n)/(2+b*x**n),x)
```

```
[Out] Piecewise((log(x)/2, Eq(b, 0) & Eq(n, 0)), (log(x)/(b + 2), Eq(n, 0)), (x**
(4*n)/(8*n), Eq(b, 0)), (x**(3*n)/(3*b*n) - x**(2*n)/(b**2*n) + 4*x**n/(b**
3*n) - 8*log(x**n + 2/b)/(b**4*n), True))
```

3.2617 $\int \frac{x^{-1+3n}}{2+bx^n}\,dx$

Optimal. Leaf size=43

$$\frac{4\log(bx^n+2)}{b^3n}-\frac{2x^n}{b^2n}+\frac{x^{2n}}{2bn}$$

```
[Out] -2*x^n/b^2/n+1/2*x^(2*n)/b/n+4*ln(2+b*x^n)/b^3/n
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$-\frac{2x^n}{b^2n}+\frac{4\log(bx^n+2)}{b^3n}+\frac{x^{2n}}{2bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)/(2 + b*x^n),x]
```

```
[Out] (-2*x^n)/(b^2*n) + x^(2*n)/(2*b*n) + (4*Log[2 + b*x^n])/(b^3*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+3n}}{2+bx^n}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{2+bx}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{2}{b^2}+\frac{x}{b}+\frac{4}{b^2(2+bx)}\right)\,dx, x, x^n\right)}{n}\\ &= -\frac{2x^n}{b^2n}+\frac{x^{2n}}{2bn}+\frac{4\log(2+bx^n)}{b^3n}\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.77

$$\frac{bx^n(bx^n-4)+8\log(bx^n+2)}{2b^3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)/(2 + b*x^n),x]
```

```
[Out] (b*x^n*(-4 + b*x^n) + 8*Log[2 + b*x^n])/(2*b^3*n)
```

fricas [A] time = 0.58, size = 34, normalized size = 0.79

$$\frac{b^2x^{2\,n}-4\,bx^n+8\,\log(bx^n+2)}{2\,b^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(2+b*x^n),x, algorithm="fricas")
[Out] 1/2*(b^2*x^(2*n) - 4*b*x^n + 8*log(b*x^n + 2))/(b^3*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3n-1}}{bx^n+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(2+b*x^n),x, algorithm="giac")
[Out] integrate(x^(3*n - 1)/(b*x^n + 2), x)
```

maple [A] time = 0.02, size = 48, normalized size = 1.12

$$\frac{\mathrm{e}^{2n\ln(x)}}{2bn}-\frac{2\,\mathrm{e}^{n\ln(x)}}{b^2n}+\frac{4\ln\left(b\,\mathrm{e}^{n\ln(x)}+2\right)}{b^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n-1)/(2+b*x^n),x)
[Out] -2/b^2/n*exp(n*ln(x))+1/2/b/n*exp(n*ln(x))^2+4/b^3/n*ln(b*exp(n*ln(x))+2)
```

maxima [A] time = 0.54, size = 42, normalized size = 0.98

$$\frac{bx^{2n}-4\,x^n}{2\,b^2n}+\frac{4\,\log\left(\frac{bx^n+2}{b}\right)}{b^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(2+b*x^n),x, algorithm="maxima")
[Out] 1/2*(b*x^(2*n) - 4*x^n)/(b^2*n) + 4*log((b*x^n + 2)/b)/(b^3*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{3n-1}}{b\,x^n+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n - 1)/(b*x^n + 2),x)
[Out] int(x^(3*n - 1)/(b*x^n + 2), x)
```

sympy [A] time = 19.57, size = 53, normalized size = 1.23

$$\begin{cases} \frac{\log(x)}{2} & \text{for } b=0\wedge n=0 \\ \frac{\log(x)}{b+2} & \text{for } n=0 \\ \frac{x^{3n}}{6n} & \text{for } b=0 \\ \frac{x^{2n}}{2bn}-\frac{2x^n}{b^2n}+\frac{4\log\left(x^n+\frac{2}{b}\right)}{b^3n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)/(2+b*x**n),x)
```

```
[Out] Piecewise((log(x)/2, Eq(b, 0) & Eq(n, 0)), (log(x)/(b + 2), Eq(n, 0)), (x**
(3*n)/(6*n), Eq(b, 0)), (x**(2*n)/(2*b*n) - 2*x**n/(b**2*n) + 4*log(x**n +
2/b)/(b**3*n), True))
```

3.2618 $\int \frac{x^{-1+2n}}{2+bx^n}\,dx$

Optimal. Leaf size=27

$$\frac{x^n}{bn} - \frac{2\log(bx^n+2)}{b^2 n}$$

```
[Out] x^n/b/n-2*ln(2+b*x^n)/b^2/n
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$\frac{x^n}{bn} - \frac{2\log(bx^n+2)}{b^2 n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/(2 + b*x^n),x]
```

```
[Out] x^n/(b*n) - (2*Log[2 + b*x^n])/(b^2*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+2n}}{2+bx^n}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{2+bx}\,dx, x, x^n\right)}{n} \\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{b} - \frac{2}{b(2+bx)}\right)\,dx, x, x^n\right)}{n} \\ &= \frac{x^n}{bn} - \frac{2\log(2+bx^n)}{b^2 n}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.93

$$\frac{\frac{x^n}{b} - \frac{2\log(bx^n+2)}{b^2}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/(2 + b*x^n),x]
```

```
[Out] (x^n/b - (2*Log[2 + b*x^n])/b^2)/n
```

fricas [A] time = 0.66, size = 23, normalized size = 0.85

$$\frac{bx^n - 2\log(bx^n+2)}{b^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(2+b*x^n),x, algorithm="fricas")
[Out] (b*x^n - 2*log(b*x^n + 2))/(b^2*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{bx^n+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(2+b*x^n),x, algorithm="giac")
[Out] integrate(x^(2*n - 1)/(b*x^n + 2), x)
```

maple [A] time = 0.02, size = 32, normalized size = 1.19

$$\frac{e^{n\ln(x)}}{bn} - \frac{2\ln\left(b\,e^{n\ln(x)}+2\right)}{b^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n-1)/(2+b*x^n),x)
[Out] 1/b/n*exp(n*ln(x))-2/b^2/n*ln(b*exp(n*ln(x))+2)
```

maxima [A] time = 0.66, size = 31, normalized size = 1.15

$$\frac{x^n}{bn} - \frac{2\,\log\left(\frac{bx^n+2}{b}\right)}{b^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(2+b*x^n),x, algorithm="maxima")
[Out] x^n/(b*n) - 2*log((b*x^n + 2)/b)/(b^2*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^{2n-1}}{b\,x^n+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/(b*x^n + 2),x)
[Out] int(x^(2*n - 1)/(b*x^n + 2), x)
```

sympy [A] time = 9.72, size = 39, normalized size = 1.44

$$\begin{cases} \frac{\log(x)}{2} & \text{for } b = 0 \wedge n = 0 \\ \frac{\log(x)}{b+2} & \text{for } n = 0 \\ \frac{x^{2n}}{4n} & \text{for } b = 0 \\ \frac{x^n}{bn} - \frac{2\log\left(x^n+\frac{2}{b}\right)}{b^2 n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(2+b*x**n),x)

[Out] Piecewise((log(x)/2, Eq(b, 0) & Eq(n, 0)), (log(x)/(b + 2), Eq(n, 0)), (x**
(2*n)/(4*n), Eq(b, 0)), (x**n/(b*n) - 2*log(x**n + 2/b)/(b**2*n), True))
```

3.2619 $\int \frac{x^{-1+n}}{2+bx^n}\,dx$

Optimal. Leaf size=15

$$\frac{\log(bx^n+2)}{bn}$$

```
[Out] ln(2+b*x^n)/b/n
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {260}

$$\frac{\log(bx^n+2)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)/(2 + b*x^n),x]
```

```
[Out] Log[2 + b*x^n]/(b*n)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^{-1+n}}{2+bx^n}\,dx = \frac{\log(2+bx^n)}{bn}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log(bx^n+2)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)/(2 + b*x^n),x]
```

```
[Out] Log[2 + b*x^n]/(b*n)
```

fricas [A] time = 0.53, size = 15, normalized size = 1.00

$$\frac{\log(bx^n+2)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(2+b*x^n),x, algorithm="fricas")
```

```
[Out] log(b*x^n + 2)/(b*n)
```

giac [A] time = 0.16, size = 16, normalized size = 1.07

$$\frac{\log(|bx^n+2|)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(2+b*x^n),x, algorithm="giac")
```

```
[Out] log(abs(b*x^n + 2))/(b*n)
```

maple [A] time = 0.02, size = 18, normalized size = 1.20

$$\frac{\ln\left(b\,e^{n\ln(x)}+2\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)/(2+b*x^n),x)
```

```
[Out] 1/b/n*ln(b*exp(n*ln(x))+2)
```

maxima [A] time = 0.70, size = 15, normalized size = 1.00

$$\frac{\log\left(bx^n+2\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(2+b*x^n),x, algorithm="maxima")
```

```
[Out] log(b*x^n + 2)/(b*n)
```

mupad [B] time = 1.21, size = 15, normalized size = 1.00

$$\frac{\ln\left(b\,x^n+2\right)}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)/(b*x^n + 2),x)
```

```
[Out] log(b*x^n + 2)/(b*n)
```

sympy [A] time = 2.38, size = 27, normalized size = 1.80

$$\begin{cases} \frac{\log(x)}{2} & \text{for } b = 0 \wedge n = 0 \\ \frac{\log(x)}{b+2} & \text{for } n = 0 \\ \frac{x^n}{2n} & \text{for } b = 0 \\ \frac{\log\left(x^n+\frac{2}{b}\right)}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)/(2+b*x**n),x)
```

```
[Out] Piecewise((log(x)/2, Eq(b, 0) & Eq(n, 0)), (log(x)/(b + 2), Eq(n, 0)), (x**
n/(2*n), Eq(b, 0)), (log(x**n + 2/b)/(b*n), True))
```

3.2620 $\int \frac{1}{x(2+bx^n)}\,dx$

Optimal. Leaf size=22

$$\frac{\log(x)}{2} - \frac{\log(bx^n+2)}{2n}$$

```
[Out] 1/2*ln(x)-1/2*ln(2+b*x^n)/n
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {266, 36, 29, 31}

$$\frac{\log(x)}{2} - \frac{\log(bx^n+2)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(2 + b*x^n)),x]
```

```
[Out] Log[x]/2 - Log[2 + b*x^n]/(2*n)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\,(2+bx^n)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(2+bx)}\,dx, x, x^n\right)}{n} \\ &= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^n\right)}{2n} - \frac{b\,\text{Subst}\left(\int \frac{1}{2+bx}\,dx, x, x^n\right)}{2n} \\ &= \frac{\log(x)}{2} - \frac{\log(2+bx^n)}{2n}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{n\log(x) - \log(bx^n+2)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(2 + b*x^n)),x]
[Out] (n*Log[x] - Log[2 + b*x^n])/(2*n)
```

fricas [A] time = 0.70, size = 20, normalized size = 0.91

$$\frac{n\log(x) - \log\left(bx^n + 2\right)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2+b*x^n),x, algorithm="fricas")
[Out] 1/2*(n*log(x) - log(b*x^n + 2))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + 2)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2+b*x^n),x, algorithm="giac")
[Out] integrate(1/((b*x^n + 2)*x), x)
```

maple [A] time = 0.00, size = 24, normalized size = 1.09

$$\frac{\ln\left(x^n\right)}{2n} - \frac{\ln\left(b\,x^n + 2\right)}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(2+b*x^n),x)
[Out] -1/2*ln(2+b*x^n)/n+1/2/n*ln(x^n)
```

maxima [A] time = 0.69, size = 23, normalized size = 1.05

$$-\frac{\log\left(bx^n + 2\right)}{2\,n} + \frac{\log\left(x^n\right)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2+b*x^n),x, algorithm="maxima")
[Out] -1/2*log(b*x^n + 2)/n + 1/2*log(x^n)/n
```

mupad [B] time = 1.17, size = 18, normalized size = 0.82

$$\frac{\ln(x)}{2} - \frac{\ln\left(b\,x^n + 2\right)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^n + 2)),x)
[Out] log(x)/2 - log(b*x^n + 2)/(2*n)
```

sympy [A] time = 0.59, size = 29, normalized size = 1.32

$$\begin{cases} \frac{\log(x)}{2} & \text{for } b = 0 \wedge (b = 0 \vee n = 0) \\ \frac{\log(x)}{b+2} & \text{for } n = 0 \\ \frac{\log(x)}{2} - \frac{\log\left(x^n + \frac{2}{b}\right)}{2n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2+b*x**n),x)
```

```
[Out] Piecewise((log(x)/2, Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)/(b + 2), Eq
(n, 0)), (log(x)/2 - log(x**n + 2/b)/(2*n), True))
```

3.2621 $\int \frac{x^{-1-n}}{2+bx^n}\,dx$

Optimal. Leaf size=36

$$\frac{b\log(bx^n+2)}{4n}-\frac{1}{4}b\log(x)-\frac{x^{-n}}{2n}$$

```
[Out] -1/2/n/(x^n)-1/4*b*ln(x)+1/4*b*ln(2+b*x^n)/n
```

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 44}

$$\frac{b\log(bx^n+2)}{4n}-\frac{1}{4}b\log(x)-\frac{x^{-n}}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n)/(2 + b*x^n),x]
```

```
[Out] -1/(2*n*x^n) - (b*Log[x])/4 + (b*Log[2 + b*x^n])/(4*n)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1-n}}{2+bx^n}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2(2+bx)}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{2x^2}-\frac{b}{4x}+\frac{b^2}{4(2+bx)}\right)\,dx, x, x^n\right)}{n}\\ &= -\frac{x^{-n}}{2n}-\frac{1}{4}b\log(x)+\frac{b\log(2+bx^n)}{4n}\end{aligned}$$

Mathematica [A] time = 0.05, size = 31, normalized size = 0.86

$$-\frac{-b\log(bx^n+2)+bn\log(x)+2x^{-n}}{4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n)/(2 + b*x^n),x]
```

```
[Out] -1/4*(2/x^n + b*n*Log[x] - b*Log[2 + b*x^n])/n
```

fricas [A] time = 0.55, size = 34, normalized size = 0.94

$$-\frac{bnx^n\log(x)-bx^n\log(bx^n+2)+2}{4\,nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(2+b*x^n),x, algorithm="fricas")
[Out] -1/4*(b*n*x^n*log(x) - b*x^n*log(b*x^n + 2) + 2)/(n*x^n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-n-1}}{bx^n+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(2+b*x^n),x, algorithm="giac")
[Out] integrate(x^(-n - 1)/(b*x^n + 2), x)
```

maple [A] time = 0.02, size = 42, normalized size = 1.17

$$\frac{b\ln\left(b\,\mathrm{e}^{n\ln(x)}+2\right)}{4n}+\left(-\frac{b\,\mathrm{e}^{n\ln(x)}\ln(x)}{4}-\frac{1}{2n}\right)\mathrm{e}^{-n\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n)/(b*x^n+2),x)
[Out] (-1/4*b*ln(x)*exp(n*ln(x))-1/2/n)/exp(n*ln(x))+1/4*b/n*ln(b*exp(n*ln(x))+2)
```

maxima [A] time = 0.48, size = 34, normalized size = 0.94

$$-\frac{1}{4}\,b\log(x)+\frac{b\log\left(\frac{bx^n+2}{b}\right)}{4\,n}-\frac{1}{2\,nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(2+b*x^n),x, algorithm="maxima")
[Out] -1/4*b*log(x) + 1/4*b*log((b*x^n + 2)/b)/n - 1/2/(n*x^n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x^{n+1}\,(b\,x^n+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(n + 1)*(b*x^n + 2)),x)
[Out] int(1/(x^(n + 1)*(b*x^n + 2)), x)
```

sympy [A] time = 18.26, size = 29, normalized size = 0.81

$$\begin{cases} \frac{b\log\left(\frac{b}{2}+x^{-n}\right)}{4n}-\frac{x^{-n}}{2n} & \text{for } n\neq 0 \\ \frac{\log(x)}{b+2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n)/(2+b*x**n),x)
[Out] Piecewise((b*log(b/2 + x**(-n))/(4*n) - x**(-n)/(2*n), Ne(n, 0)), (log(x)/(
b + 2), True))
```

3.2622 $\int \frac{x^{-1-2n}}{2+bx^n}\,dx$

Optimal. Leaf size=53

$$-\frac{b^2\log\left(bx^n+2\right)}{8n}+\frac{1}{8}b^2\log(x)+\frac{bx^{-n}}{4n}-\frac{x^{-2n}}{4n}$$

```
[Out] -1/4/n/(x^(2*n))+1/4*b/n/(x^n)+1/8*b^2*ln(x)-1/8*b^2*ln(2+b*x^n)/n
```

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 44}

$$-\frac{b^2\log\left(bx^n+2\right)}{8n}+\frac{1}{8}b^2\log(x)+\frac{bx^{-n}}{4n}-\frac{x^{-2n}}{4n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 2*n)/(2 + b*x^n),x]
```

```
[Out] -1/(4*n*x^(2*n)) + b/(4*n*x^n) + (b^2*Log[x])/8 - (b^2*Log[2 + b*x^n])/(8*n
)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1-2n}}{2+bx^n}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^3(2+bx)}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{2x^3}-\frac{b}{4x^2}+\frac{b^2}{8x}-\frac{b^3}{8(2+bx)}\right)dx, x, x^n\right)}{n}\\ &= -\frac{x^{-2n}}{4n}+\frac{bx^{-n}}{4n}+\frac{1}{8}b^2\log(x)-\frac{b^2\log\left(2+bx^n\right)}{8n}\end{aligned}$$

Mathematica [A] time = 0.05, size = 42, normalized size = 0.79

$$-\frac{b^2\log\left(bx^n+2\right)-b^2n\log(x)+x^{-2n}\left(2-2bx^n\right)}{8n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 2*n)/(2 + b*x^n),x]
```

```
[Out] -1/8*((2 - 2*b*x^n)/x^(2*n) - b^2*n*Log[x] + b^2*Log[2 + b*x^n])/n
```

fricas [A] time = 0.62, size = 50, normalized size = 0.94

$$\frac{b^2 n x^{2\,n} \log(x) - b^2 x^{2\,n} \log\left(b x^n + 2\right) + 2\,b x^n - 2}{8\,n x^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(2+b*x^n),x, algorithm="fricas")
```

```
[Out] 1/8*(b^2*n*x^(2*n)*log(x) - b^2*x^(2*n)*log(b*x^n + 2) + 2*b*x^n - 2)/(n*x^
(2*n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-2\,n-1}}{b x^n + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(2+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(-2*n - 1)/(b*x^n + 2), x)
```

maple [A] time = 0.02, size = 59, normalized size = 1.11

$$-\frac{b^2 \ln\left(b\,\mathrm{e}^{n \ln(x)} + 2\right)}{8n} + \left(\frac{b^2 \mathrm{e}^{2n \ln(x)} \ln(x)}{8} + \frac{b\,\mathrm{e}^{n \ln(x)}}{4n} - \frac{1}{4n}\right) \mathrm{e}^{-2n \ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-2*n)/(b*x^n+2),x)
```

```
[Out] (1/8*b^2*ln(x)*exp(n*ln(x))^2-1/4/n+1/4*b/n*exp(n*ln(x)))/exp(n*ln(x))^2-1/
8*b^2/n*ln(b*exp(n*ln(x))+2)
```

maxima [A] time = 0.55, size = 47, normalized size = 0.89

$$\frac{1}{8}\,b^2 \log(x) - \frac{b^2 \log\left(\frac{b x^n + 2}{b}\right)}{8\,n} + \frac{b x^n - 1}{4\,n x^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(2+b*x^n),x, algorithm="maxima")
```

```
[Out] 1/8*b^2*log(x) - 1/8*b^2*log((b*x^n + 2)/b)/n + 1/4*(b*x^n - 1)/(n*x^(2*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^{2\,n+1}\,\left(b\,x^n + 2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(2*n + 1)*(b*x^n + 2)),x)
```

```
[Out] int(1/(x^(2*n + 1)*(b*x^n + 2)), x)
```

sympy [**A**] time = 132.46, size = 63, normalized size = 1.19

$$\begin{cases} \frac{\log(x)}{2} & \text{for } b = 0 \wedge n = 0 \\ \frac{\log(x)}{b+2} & \text{for } n = 0 \\ -\frac{x^{-2n}}{4n} & \text{for } b = 0 \\ \frac{b^2 \log(x)}{8} - \frac{b^2 \log\left(x^n + \frac{2}{b}\right)}{8n} + \frac{b x^{-n}}{4n} - \frac{x^{-2n}}{4n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-2*n)/(2+b*x**n),x)
```

```
[Out] Piecewise((log(x)/2, Eq(b, 0) & Eq(n, 0)), (log(x)/(b + 2), Eq(n, 0)), (-x*
*(-2*n)/(4*n), Eq(b, 0)), (b**2*log(x)/8 - b**2*log(x**n + 2/b)/(8*n) + b*x
**(-n)/(4*n) - x**(-2*n)/(4*n), True))
```

3.2623 $\int \frac{x^{-1-3n}}{2+bx^n}\,dx$

Optimal. Leaf size=68

$$\frac{b^3\log\left(bx^n+2\right)}{16n}-\frac{1}{16}b^3\log(x)-\frac{b^2x^{-n}}{8n}+\frac{bx^{-2n}}{8n}-\frac{x^{-3n}}{6n}$$

```
[Out] -1/6/n/(x^(3*n))+1/8*b/n/(x^(2*n))-1/8*b^2/n/(x^n)-1/16*b^3*ln(x)+1/16*b^3*
ln(2+b*x^n)/n
```

Rubi [A] time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 44}

$$-\frac{b^2x^{-n}}{8n}+\frac{b^3\log\left(bx^n+2\right)}{16n}-\frac{1}{16}b^3\log(x)+\frac{bx^{-2n}}{8n}-\frac{x^{-3n}}{6n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 3*n)/(2 + b*x^n),x]
```

```
[Out] -1/(6*n*x^(3*n)) + b/(8*n*x^(2*n)) - b^2/(8*n*x^n) - (b^3*Log[x])/16 + (b^3
*Log[2 + b*x^n])/(16*n)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1-3n}}{2+bx^n}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^4(2+bx)}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{2x^4}-\frac{b}{4x^3}+\frac{b^2}{8x^2}-\frac{b^3}{16x}+\frac{b^4}{16(2+bx)}\right)dx,x,x^n\right)}{n}\\ &= -\frac{x^{-3n}}{6n}+\frac{bx^{-2n}}{8n}-\frac{b^2x^{-n}}{8n}-\frac{1}{16}b^3\log(x)+\frac{b^3\log\left(2+bx^n\right)}{16n}\end{aligned}$$

Mathematica [A] time = 0.08, size = 53, normalized size = 0.78

$$-\frac{-3b^3\log\left(bx^n+2\right)+3b^3n\log(x)+x^{-3n}\left(6b^2x^{2n}-6bx^n+8\right)}{48n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 3*n)/(2 + b*x^n),x]
```

```
[Out] -1/48*((8 - 6*b*x^n + 6*b^2*x^(2*n))/x^(3*n) + 3*b^3*n*Log[x] - 3*b^3*Log[2
 + b*x^n])/n
```

fricas [A] time = 0.78, size = 61, normalized size = 0.90

$$-\frac{3\,b^3 n x^{3\,n}\log(x) - 3\,b^3 x^{3\,n}\log\left(bx^n+2\right) + 6\,b^2 x^{2\,n} - 6\,bx^n + 8}{48\,n x^{3\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)/(2+b*x^n),x, algorithm="fricas")
```

```
[Out] -1/48*(3*b^3*n*x^(3*n)*log(x) - 3*b^3*x^(3*n)*log(b*x^n + 2) + 6*b^2*x^(2*n
) - 6*b*x^n + 8)/(n*x^(3*n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-3\,n-1}}{bx^n+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)/(2+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(-3*n - 1)/(b*x^n + 2), x)
```

maple [A] time = 0.03, size = 74, normalized size = 1.09

$$\frac{b^3\ln\left(b\,\mathrm{e}^{n\ln(x)}+2\right)}{16n} + \left(-\frac{b^3\mathrm{e}^{3n\ln(x)}\ln(x)}{16} - \frac{b^2\mathrm{e}^{2n\ln(x)}}{8n} + \frac{b\,\mathrm{e}^{n\ln(x)}}{8n} - \frac{1}{6n}\right)\mathrm{e}^{-3n\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-3*n)/(b*x^n+2),x)
```

```
[Out] (-1/16*b^3*ln(x)*exp(n*ln(x))^3-1/6/n+1/8*b/n*exp(n*ln(x))-1/8*b^2/n*exp(n*
ln(x))^2)/exp(n*ln(x))^3+1/16*b^3/n*ln(b*exp(n*ln(x))+2)
```

maxima [A] time = 0.51, size = 58, normalized size = 0.85

$$-\frac{1}{16}\,b^3\log(x) + \frac{b^3\log\left(\frac{bx^n+2}{b}\right)}{16\,n} - \frac{3\,b^2x^{2\,n} - 3\,bx^n + 4}{24\,nx^{3\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)/(2+b*x^n),x, algorithm="maxima")
```

```
[Out] -1/16*b^3*log(x) + 1/16*b^3*log((b*x^n + 2)/b)/n - 1/24*(3*b^2*x^(2*n) - 3*
b*x^n + 4)/(n*x^(3*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{3\,n+1}\,\left(b\,x^n+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3*n + 1)*(b*x^n + 2)),x)
```

```
[Out] int(1/(x^(3*n + 1)*(b*x^n + 2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-3*n)/(2+b*x**n),x)
```

```
[Out] Timed out
```

3.2624 $\int \frac{x^{-1+4n}}{(a+bx^n)^2}\,dx$

Optimal. Leaf size=66

$$\frac{a^3}{b^4 n\,(a+bx^n)}+\frac{3a^2\log(a+bx^n)}{b^4 n}-\frac{2ax^n}{b^3 n}+\frac{x^{2n}}{2b^2 n}$$

```
[Out] -2*a*x^n/b^3/n+1/2*x^(2*n)/b^2/n+a^3/b^4/n/(a+b*x^n)+3*a^2*ln(a+b*x^n)/b^4/
n
```

Rubi [A] time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$\frac{a^3}{b^4 n\,(a+bx^n)}+\frac{3a^2\log(a+bx^n)}{b^4 n}-\frac{2ax^n}{b^3 n}+\frac{x^{2n}}{2b^2 n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 4*n)/(a + b*x^n)^2,x]
```

```
[Out] (-2*a*x^n)/(b^3*n) + x^(2*n)/(2*b^2*n) + a^3/(b^4*n*(a + b*x^n)) + (3*a^2*L
og[a + b*x^n])/(b^4*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+4n}}{(a+bx^n)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{x^3}{(a+bx)^2}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{2a}{b^3}+\frac{x}{b^2}-\frac{a^3}{b^3(a+bx)^2}+\frac{3a^2}{b^3(a+bx)}\right)dx, x, x^n\right)}{n}\\ &= -\frac{2ax^n}{b^3 n}+\frac{x^{2n}}{2b^2 n}+\frac{a^3}{b^4 n\,(a+bx^n)}+\frac{3a^2\log(a+bx^n)}{b^4 n}\end{aligned}$$

Mathematica [A] time = 0.07, size = 54, normalized size = 0.82

$$\frac{\frac{2a^3}{a+bx^n}+6a^2\log(a+bx^n)-4abx^n+b^2x^{2n}}{2b^4 n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 4*n)/(a + b*x^n)^2,x]
```

```
[Out] (-4*a*b*x^n + b^2*x^(2*n) + (2*a^3)/(a + b*x^n) + 6*a^2*Log[a + b*x^n])/(2*
b^4*n)
```

fricas [A] time = 0.67, size = 76, normalized size = 1.15

$$\frac{b^3x^{3\,n} - 3\,ab^2x^{2\,n} - 4\,a^2bx^n + 2\,a^3 + 6\left(a^2bx^n + a^3\right)\log\left(bx^n + a\right)}{2\left(b^5nx^n + ab^4n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*x^(3*n) - 3*a*b^2*x^(2*n) - 4*a^2*b*x^n + 2*a^3 + 6*(a^2*b*x^n + a
^3)*log(b*x^n + a))/(b^5*n*x^n + a*b^4*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4\,n-1}}{\left(bx^n + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(4*n - 1)/(b*x^n + a)^2, x)
```

maple [A] time = 0.03, size = 78, normalized size = 1.18

$$\frac{3a^2\ln\left(b\,\mathrm{e}^{n\ln(x)} + a\right)}{b^4n} + \frac{-\frac{3a\,\mathrm{e}^{2n\ln(x)}}{2b^2n} + \frac{\mathrm{e}^{3n\ln(x)}}{2bn} + \frac{3a^3}{b^4n}}{b\,\mathrm{e}^{n\ln(x)} + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+4*n)/(b*x^n+a)^2,x)
```

```
[Out] (3*a^3/b^4/n+1/2/b/n*exp(n*ln(x))^3-3/2*a/b^2/n*exp(n*ln(x))^2)/(b*exp(n*ln
(x))+a)+3*a^2/b^4/n*ln(b*exp(n*ln(x))+a)
```

maxima [A] time = 0.60, size = 78, normalized size = 1.18

$$\frac{b^3x^{3\,n} - 3\,ab^2x^{2\,n} - 4\,a^2bx^n + 2\,a^3}{2\left(b^5nx^n + ab^4n\right)} + \frac{3\,a^2\log\left(\frac{bx^n+a}{b}\right)}{b^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^3*x^(3*n) - 3*a*b^2*x^(2*n) - 4*a^2*b*x^n + 2*a^3)/(b^5*n*x^n + a*b^
4*n) + 3*a^2*log((b*x^n + a)/b)/(b^4*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{4\,n-1}}{\left(a + b\,x^n\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4*n - 1)/(a + b*x^n)^2,x)
```

```
[Out] int(x^(4*n - 1)/(a + b*x^n)^2, x)
```

sympy [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+4*n)/(a+b*x**n)**2,x)
[Out] Exception raised: HeuristicGCDFailed
```

3.2625 $\int \frac{x^{-1+3n}}{(a+bx^n)^2}\,dx$

Optimal. Leaf size=48

$$-\frac{a^2}{b^3 n\,(a+bx^n)}-\frac{2a\log(a+bx^n)}{b^3 n}+\frac{x^n}{b^2 n}$$

```
[Out] x^n/b^2/n-a^2/b^3/n/(a+b*x^n)-2*a*ln(a+b*x^n)/b^3/n
```

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$-\frac{a^2}{b^3 n\,(a+bx^n)}-\frac{2a\log(a+bx^n)}{b^3 n}+\frac{x^n}{b^2 n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)/(a + b*x^n)^2,x]
```

```
[Out] x^n/(b^2*n) - a^2/(b^3*n*(a + b*x^n)) - (2*a*Log[a + b*x^n])/(b^3*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+3n}}{(a+bx^n)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{(a+bx)^2}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{b^2}+\frac{a^2}{b^2(a+bx)^2}-\frac{2a}{b^2(a+bx)}\right)dx, x, x^n\right)}{n}\\ &= \frac{x^n}{b^2 n}-\frac{a^2}{b^3 n\,(a+bx^n)}-\frac{2a\log(a+bx^n)}{b^3 n}\end{aligned}$$

Mathematica [A] time = 0.05, size = 38, normalized size = 0.79

$$\frac{-\frac{a^2}{a+bx^n}-2a\log(a+bx^n)+bx^n}{b^3 n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)/(a + b*x^n)^2,x]
```

```
[Out] (b*x^n - a^2/(a + b*x^n) - 2*a*Log[a + b*x^n])/(b^3*n)
```

fricas [A] time = 0.86, size = 59, normalized size = 1.23

$$\frac{b^2x^{2\,n}+abx^n-a^2-2\left(abx^n+a^2\right)\log\left(bx^n+a\right)}{b^4nx^n+ab^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] (b^2*x^(2*n) + a*b*x^n - a^2 - 2*(a*b*x^n + a^2)*log(b*x^n + a))/(b^4*n*x^n
 + a*b^3*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3\,n-1}}{\left(bx^n+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(3*n - 1)/(b*x^n + a)^2, x)
```

maple [A] time = 0.03, size = 59, normalized size = 1.23

$$-\frac{2a\ln\left(b\,\mathrm{e}^{n\ln(x)}+a\right)}{b^3n}+\frac{\frac{\mathrm{e}^{2n\ln(x)}}{bn}-\frac{2a^2}{b^3n}}{b\,\mathrm{e}^{n\ln(x)}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n-1)/(b*x^n+a)^2,x)
```

```
[Out] (1/b/n*exp(n*ln(x))^2-2*a^2/b^3/n)/(b*exp(n*ln(x))+a)-2*a/b^3/n*ln(b*exp(n*
ln(x))+a)
```

maxima [A] time = 0.51, size = 61, normalized size = 1.27

$$\frac{b^2x^{2\,n}+abx^n-a^2}{b^4nx^n+ab^3n}-\frac{2\,a\log\left(\frac{bx^n+a}{b}\right)}{b^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] (b^2*x^(2*n) + a*b*x^n - a^2)/(b^4*n*x^n + a*b^3*n) - 2*a*log((b*x^n + a)/b
)/(b^3*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{3\,n-1}}{\left(a+b\,x^n\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n - 1)/(a + b*x^n)^2,x)
```

```
[Out] int(x^(3*n - 1)/(a + b*x^n)^2, x)
```

sympy [**A**] time = 157.50, size = 129, normalized size = 2.69

$$\begin{cases} \frac{\log(x)}{a^2} & \text{for } b = 0 \wedge n = 0 \\ \frac{x^{3n}}{3a^2 n} & \text{for } b = 0 \\ \frac{\log(x)}{(a+b)^2} & \text{for } n = 0 \\ -\frac{2a^2 \log\left(\frac{a}{b}+x^n\right)}{ab^3 n+b^4 n x^n} - \frac{2a^2}{ab^3 n+b^4 n x^n} - \frac{2abx^n \log\left(\frac{a}{b}+x^n\right)}{ab^3 n+b^4 n x^n} + \frac{b^2 x^{2n}}{ab^3 n+b^4 n x^n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)/(a+b*x**n)**2,x)
```

```
[Out] Piecewise((log(x)/a**2, Eq(b, 0) & Eq(n, 0)), (x**(3*n)/(3*a**2*n), Eq(b, 0
)), (log(x)/(a + b)**2, Eq(n, 0)), (-2*a**2*log(a/b + x**n)/(a*b**3*n + b**
4*n*x**n) - 2*a**2/(a*b**3*n + b**4*n*x**n) - 2*a*b*x**n*log(a/b + x**n)/(a
*b**3*n + b**4*n*x**n) + b**2*x**(2*n)/(a*b**3*n + b**4*n*x**n), True))
```

3.2626 $\int \frac{x^{-1+2n}}{(a+bx^n)^2}\,dx$

Optimal. Leaf size=33

$$\frac{a}{b^2 n\,(a+bx^n)}+\frac{\log(a+bx^n)}{b^2 n}$$

```
[Out] a/b^2/n/(a+b*x^n)+ln(a+b*x^n)/b^2/n
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$\frac{a}{b^2 n\,(a+bx^n)}+\frac{\log(a+bx^n)}{b^2 n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/(a + b*x^n)^2,x]
```

```
[Out] a/(b^2*n*(a + b*x^n)) + Log[a + b*x^n]/(b^2*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+2n}}{(a+bx^n)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{(a+bx)^2}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^2}+\frac{1}{b(a+bx)}\right)dx, x, x^n\right)}{n}\\ &= \frac{a}{b^2 n\,(a+bx^n)}+\frac{\log(a+bx^n)}{b^2 n}\end{aligned}$$

Mathematica [A] time = 0.03, size = 27, normalized size = 0.82

$$\frac{\frac{a}{a+bx^n}+\log(a+bx^n)}{b^2 n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/(a + b*x^n)^2,x]
```

```
[Out] (a/(a + b*x^n) + Log[a + b*x^n])/(b^2*n)
```

fricas [A] time = 0.89, size = 36, normalized size = 1.09

$$\frac{(bx^n + a)\log(bx^n + a) + a}{b^3 n x^n + ab^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^2,x, algorithm="fricas")
[Out] ((b*x^n + a)*log(b*x^n + a) + a)/(b^3*n*x^n + a*b^2*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{(bx^n + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^2,x, algorithm="giac")
[Out] integrate(x^(2*n - 1)/(b*x^n + a)^2, x)
```

maple [A] time = 0.02, size = 38, normalized size = 1.15

$$\frac{a}{\left(b\,\mathrm{e}^{n\ln(x)} + a\right) b^2 n} + \frac{\ln\left(b\,\mathrm{e}^{n\ln(x)} + a\right)}{b^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n-1)/(b*x^n+a)^2,x)
[Out] a/b^2/n/(b*exp(n*ln(x))+a)+1/b^2/n*ln(b*exp(n*ln(x))+a)
```

maxima [A] time = 0.49, size = 39, normalized size = 1.18

$$\frac{a}{b^3 n x^n + ab^2 n} + \frac{\log\left(\frac{bx^n + a}{b}\right)}{b^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^2,x, algorithm="maxima")
[Out] a/(b^3*n*x^n + a*b^2*n) + log((b*x^n + a)/b)/(b^2*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^{2n-1}}{(a + b\,x^n)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/(a + b*x^n)^2,x)
[Out] int(x^(2*n - 1)/(a + b*x^n)^2, x)
```

sympy [A] time = 99.17, size = 95, normalized size = 2.88

$$\begin{cases} \frac{\log(x)}{a^2} & \text{for } b = 0 \wedge n = 0 \\ \frac{x^{2n}}{2a^2 n} & \text{for } b = 0 \\ \frac{\log(x)}{(a+b)^2} & \text{for } n = 0 \\ \frac{a\log\left(\frac{a}{b}+x^n\right)}{ab^2 n + b^3 n x^n} + \frac{a}{ab^2 n + b^3 n x^n} + \frac{bx^n \log\left(\frac{a}{b}+x^n\right)}{ab^2 n + b^3 n x^n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a+b*x**n)**2,x)
```

```
[Out] Piecewise((log(x)/a**2, Eq(b, 0) & Eq(n, 0)), (x**(2*n)/(2*a**2*n), Eq(b, 0
)), (log(x)/(a + b)**2, Eq(n, 0)), (a*log(a/b + x**n)/(a*b**2*n + b**3*n*x*
*n) + a/(a*b**2*n + b**3*n*x**n) + b*x**n*log(a/b + x**n)/(a*b**2*n + b**3*
n*x**n), True))
```

3.2627 $\int \frac{x^{-1+n}}{(a+bx^n)^2}\,dx$

Optimal. Leaf size=17

$$-\frac{1}{bn\,(a+bx^n)}$$

```
[Out] -1/b/n/(a+b*x^n)
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$-\frac{1}{bn\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)/(a + b*x^n)^2,x]
```

```
[Out] -(1/(b*n*(a + b*x^n)))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^{-1+n}}{(a+bx^n)^2}\,dx = -\frac{1}{bn\,(a+bx^n)}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$-\frac{1}{bn\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)/(a + b*x^n)^2,x]
```

```
[Out] -(1/(b*n*(a + b*x^n)))
```

fricas [A] time = 0.63, size = 17, normalized size = 1.00

$$-\frac{1}{b^2nx^n + abn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] -1/(b^2*n*x^n + a*b*n)
```

giac [A] time = 0.15, size = 17, normalized size = 1.00

$$-\frac{1}{(bx^n+a)bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] -1/((b*x^n + a)*b*n)
```

maple [A] time = 0.02, size = 24, normalized size = 1.41

$$\frac{e^{n \ln(x)}}{\left(b\, e^{n \ln(x)} + a\right) an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)/(b*x^n+a)^2,x)
```

```
[Out] 1/a/n*exp(n*ln(x))/(b*exp(n*ln(x))+a)
```

maxima [A] time = 0.50, size = 17, normalized size = 1.00

$$-\frac{1}{(bx^n + a)bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] -1/((b*x^n + a)*b*n)
```

mupad [B] time = 1.30, size = 21, normalized size = 1.24

$$-\frac{a}{b\left(a^2 n + a\, b\, n\, x^n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)/(a + b*x^n)^2,x)
```

```
[Out] -a/(b*(a^2*n + a*b*n*x^n))
```

sympy [A] time = 9.90, size = 37, normalized size = 2.18

$$\begin{cases} \frac{\log(x)}{a^2} & \text{for } b = 0 \wedge n = 0 \\ \frac{x^n}{a^2 n} & \text{for } b = 0 \\ \frac{\log(x)}{(a+b)^2} & \text{for } n = 0 \\ -\frac{1}{abn + b^2 n x^n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)/(a+b*x**n)**2,x)
```

```
[Out] Piecewise((log(x)/a**2, Eq(b, 0) & Eq(n, 0)), (x**n/(a**2*n), Eq(b, 0)), (l
og(x)/(a + b)**2, Eq(n, 0)), (-1/(a*b*n + b**2*n*x**n), True))
```

3.2628 $\int \frac{1}{x(a+bx^n)^2}\,dx$

Optimal. Leaf size=39

$$-\frac{\log(a+bx^n)}{a^2 n}+\frac{\log(x)}{a^2}+\frac{1}{an(a+bx^n)}$$

```
[Out] 1/a/n/(a+b*x^n)+ln(x)/a^2-ln(a+b*x^n)/a^2/n
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 44}

$$-\frac{\log(a+bx^n)}{a^2 n}+\frac{\log(x)}{a^2}+\frac{1}{an(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^n)^2),x]
```

```
[Out] 1/(a*n*(a + b*x^n)) + Log[x]/a^2 - Log[a + b*x^n]/(a^2*n)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+bx^n)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^2}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2 x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right)\,dx, x, x^n\right)}{n}\\ &= \frac{1}{an(a+bx^n)}+\frac{\log(x)}{a^2}-\frac{\log(a+bx^n)}{a^2 n}\end{aligned}$$

Mathematica [A] time = 0.07, size = 33, normalized size = 0.85

$$\frac{\frac{a}{a+bx^n}-\log(a+bx^n)+n\log(x)}{a^2 n}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^n)^2),x]
```

```
[Out] (a/(a + b*x^n) + n*Log[x] - Log[a + b*x^n])/(a^2*n)
```

fricas [A] time = 0.53, size = 50, normalized size = 1.28

$$\frac{bnx^n \log(x) + an \log(x) - (bx^n + a) \log(bx^n + a) + a}{a^2 bnx^n + a^3 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] (b*n*x^n*log(x) + a*n*log(x) - (b*x^n + a)*log(b*x^n + a) + a)/(a^2*b*n*x^n
 + a^3*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)^2 x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^2*x), x)
```

maple [A] time = 0.00, size = 45, normalized size = 1.15

$$\frac{1}{(b\,x^n + a)\,an} + \frac{\ln(x^n)}{a^2 n} - \frac{\ln(b\,x^n + a)}{a^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^n+a)^2,x)
```

```
[Out] 1/(b*x^n+a)/a/n+1/a^2/n*ln(x^n)-1/a^2/n*ln(b*x^n+a)
```

maxima [A] time = 0.55, size = 43, normalized size = 1.10

$$\frac{1}{abnx^n + a^2 n} - \frac{\log(bx^n + a)}{a^2 n} + \frac{\log(x^n)}{a^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] 1/(a*b*n*x^n + a^2*n) - log(b*x^n + a)/(a^2*n) + log(x^n)/(a^2*n)
```

mupad [B] time = 1.19, size = 39, normalized size = 1.00

$$\frac{\ln(x)}{a^2} + \frac{1}{a\,n\,(a + b\,x^n)} - \frac{\ln(a + b\,x^n)}{a^2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^n)^2),x)
```

```
[Out] log(x)/a^2 + 1/(a*n*(a + b*x^n)) - log(a + b*x^n)/(a^2*n)
```

sympy [**A**] time = 1.45, size = 160, normalized size = 4.10

$$\begin{cases} \tilde{\infty}\log(x) & \text{for } a = 0 \wedge b = 0 \wedge n = 0 \\ -\frac{x^{-2n}}{2b^2 n} & \text{for } a = 0 \\ \tilde{\infty}\log(x) & \text{for } b = -ax^{-n} \\ \frac{\log(x)}{(a+b)^2} & \text{for } n = 0 \\ \frac{\log(x)}{a^2} & \text{for } b = 0 \\ \frac{an\log(x)}{a^3 n + a^2 bnx^n} - \frac{a\log\left(\frac{a}{b}+x^n\right)}{a^3 n + a^2 bnx^n} + \frac{a}{a^3 n + a^2 bnx^n} + \frac{bnx^n\log(x)}{a^3 n + a^2 bnx^n} - \frac{bx^n\log\left(\frac{a}{b}+x^n\right)}{a^3 n + a^2 bnx^n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x**n)**2,x)
```

```
[Out] Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (-x**(-2*n)/(2*b**2
*n), Eq(a, 0)), (zoo*log(x), Eq(b, -a*x**(-n))), (log(x)/(a + b)**2, Eq(n,
0)), (log(x)/a**2, Eq(b, 0)), (a*n*log(x)/(a**3*n + a**2*b*n*x**n) - a*log(
a/b + x**n)/(a**3*n + a**2*b*n*x**n) + a/(a**3*n + a**2*b*n*x**n) + b*n*x**
n*log(x)/(a**3*n + a**2*b*n*x**n) - b*x**n*log(a/b + x**n)/(a**3*n + a**2*b
*n*x**n), True))
```

3.2629 $\int \frac{x^{-1-n}}{(a+bx^n)^2}\,dx$

Optimal. Leaf size=57

$$\frac{2b\log(a+bx^n)}{a^3n}-\frac{2b\log(x)}{a^3}-\frac{b}{a^2n(a+bx^n)}-\frac{x^{-n}}{a^2n}$$

```
[Out] -1/a^2/n/(x^n)-b/a^2/n/(a+b*x^n)-2*b*ln(x)/a^3+2*b*ln(a+b*x^n)/a^3/n
```

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 44}

$$-\frac{b}{a^2n(a+bx^n)}+\frac{2b\log(a+bx^n)}{a^3n}-\frac{2b\log(x)}{a^3}-\frac{x^{-n}}{a^2n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n)/(a + b*x^n)^2,x]
```

```
[Out] -(1/(a^2*n*x^n)) - b/(a^2*n*(a + b*x^n)) - (2*b*Log[x])/a^3 + (2*b*Log[a +
b*x^n])/(a^3*n)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1-n}}{(a+bx^n)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x^2}-\frac{2b}{a^3x}+\frac{b^2}{a^2(a+bx)^2}+\frac{2b^2}{a^3(a+bx)}\right)dx,x,x^n\right)}{n}\\ &= -\frac{x^{-n}}{a^2n}-\frac{b}{a^2n(a+bx^n)}-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx^n)}{a^3n}\end{aligned}$$

Mathematica [A] time = 0.15, size = 45, normalized size = 0.79

$$-\frac{-2b\log(a+bx^n)+a\left(\frac{b}{a+bx^n}+x^{-n}\right)+2bn\log(x)}{a^3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n)/(a + b*x^n)^2,x]
```

```
[Out] -((a*(x^(-n) + b/(a + b*x^n)) + 2*b*n*Log[x] - 2*b*Log[a + b*x^n])/(a^3*n))
```

fricas [A] time = 0.74, size = 82, normalized size = 1.44

$$-\frac{2\,b^2nx^{2\,n}\log(x)+a^2+2\left(abn\log(x)+ab\right)x^n-2\left(b^2x^{2\,n}+abx^n\right)\log\left(bx^n+a\right)}{a^3bnx^{2\,n}+a^4nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] -(2*b^2*n*x^(2*n)*log(x) + a^2 + 2*(a*b*n*log(x) + a*b)*x^n - 2*(b^2*x^(2*n
) + a*b*x^n)*log(b*x^n + a))/(a^3*b*n*x^(2*n) + a^4*n*x^n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-n-1}}{\left(bx^n+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(-n - 1)/(b*x^n + a)^2, x)
```

maple [A] time = 0.03, size = 97, normalized size = 1.70

$$\frac{\left(-\frac{2b\,\mathrm{e}^{n\ln(x)}\ln(x)}{a^2}-\frac{2b^2\mathrm{e}^{2n\ln(x)}\ln(x)}{a^3}+\frac{2b^2\mathrm{e}^{2n\ln(x)}}{a^3n}-\frac{1}{an}\right)\mathrm{e}^{-n\ln(x)}}{b\,\mathrm{e}^{n\ln(x)}+a}+\frac{2b\ln\left(b\,\mathrm{e}^{n\ln(x)}+a\right)}{a^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n)/(b*x^n+a)^2,x)
```

```
[Out] (2*b^2/a^3/n*exp(n*ln(x))^2-1/a/n-2/a^2*b*exp(n*ln(x))*ln(x)-2*b^2/a^3*ln(x
)*exp(n*ln(x))^2)/exp(n*ln(x))/(b*exp(n*ln(x))+a)+2*b/a^3/n*ln(b*exp(n*ln(x
))+a)
```

maxima [A] time = 0.62, size = 62, normalized size = 1.09

$$-\frac{2\,bx^n+a}{a^2bnx^{2\,n}+a^3nx^n}-\frac{2\,b\log(x)}{a^3}+\frac{2\,b\log\left(\frac{bx^n+a}{b}\right)}{a^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] -(2*b*x^n + a)/(a^2*b*n*x^(2*n) + a^3*n*x^n) - 2*b*log(x)/a^3 + 2*b*log((b*
x^n + a)/b)/(a^3*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^{n+1}\left(a+b\,x^n\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(n + 1)*(a + b*x^n)^2),x)
```

```
[Out] int(1/(x^(n + 1)*(a + b*x^n)^2), x)
```

sympy [**A**] time = 125.74, size = 172, normalized size = 3.02

$$\begin{cases} \tilde{\infty}\log(x) & \text{for } a = 0 \wedge b = 0 \wedge n = 0 \\ -\frac{x^{-3n}}{3b^2 n} & \text{for } a = 0 \\ \frac{\tilde{\infty}x^{-n}}{n} & \text{for } b = -ax^{-n} \\ \frac{\log(x)}{(a+b)^2} & \text{for } n = 0 \\ -\frac{a^2}{a^4 n x^n + a^3 b n x^{2n}} + \frac{2abx^n \log\left(x^{-n} + \frac{b}{a}\right)}{a^4 n x^n + a^3 b n x^{2n}} + \frac{2b^2 x^{2n} \log\left(x^{-n} + \frac{b}{a}\right)}{a^4 n x^n + a^3 b n x^{2n}} + \frac{2b^2 x^{2n}}{a^4 n x^n + a^3 b n x^{2n}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n)/(a+b*x**n)**2,x)
```

```
[Out] Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (-x**(-3*n)/(3*b**2
*n), Eq(a, 0)), (zoo*x**(-n)/n, Eq(b, -a*x**(-n))), (log(x)/(a + b)**2, Eq(
n, 0)), (-a**2/(a**4*n*x**n + a**3*b*n*x**(2*n)) + 2*a*b*x**n*log(x**(-n) +
 b/a)/(a**4*n*x**n + a**3*b*n*x**(2*n)) + 2*b**2*x**(2*n)*log(x**(-n) + b/a
)/(a**4*n*x**n + a**3*b*n*x**(2*n)) + 2*b**2*x**(2*n)/(a**4*n*x**n + a**3*b
*n*x**(2*n)), True))
```

3.2630 $\int \frac{x^{-1-2n}}{(a+bx^n)^2}\,dx$

Optimal. Leaf size=78

$$-\frac{3b^2\log(a+bx^n)}{a^4n}+\frac{3b^2\log(x)}{a^4}+\frac{b^2}{a^3n(a+bx^n)}+\frac{2bx^{-n}}{a^3n}-\frac{x^{-2n}}{2a^2n}$$

```
[Out] -1/2/a^2/n/(x^(2*n))+2*b/a^3/n/(x^n)+b^2/a^3/n/(a+b*x^n)+3*b^2*ln(x)/a^4-3*
b^2*ln(a+b*x^n)/a^4/n
```

Rubi [A] time = 0.04, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 44}

$$\frac{b^2}{a^3n(a+bx^n)}-\frac{3b^2\log(a+bx^n)}{a^4n}+\frac{3b^2\log(x)}{a^4}+\frac{2bx^{-n}}{a^3n}-\frac{x^{-2n}}{2a^2n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 2*n)/(a + b*x^n)^2,x]
```

```
[Out] -1/(2*a^2*n*x^(2*n)) + (2*b)/(a^3*n*x^n) + b^2/(a^3*n*(a + b*x^n)) + (3*b^2
*Log[x])/a^4 - (3*b^2*Log[a + b*x^n])/(a^4*n)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1-2n}}{(a+bx^n)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^3(a+bx)^2}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x^3}-\frac{2b}{a^3x^2}+\frac{3b^2}{a^4x}-\frac{b^3}{a^3(a+bx)^2}-\frac{3b^3}{a^4(a+bx)}\right)dx, x, x^n\right)}{n}\\ &= -\frac{x^{-2n}}{2a^2n}+\frac{2bx^{-n}}{a^3n}+\frac{b^2}{a^3n(a+bx^n)}+\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log(a+bx^n)}{a^4n}\end{aligned}$$

Mathematica [A] time = 0.21, size = 65, normalized size = 0.83

$$\frac{-6b^2\log(a+bx^n)+a\left(\frac{2b^2}{a+bx^n}-ax^{-2n}+4bx^{-n}\right)+6b^2n\log(x)}{2a^4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 2*n)/(a + b*x^n)^2,x]
```

```
[Out] (a*(-(a/x^(2*n)) + (4*b)/x^n + (2*b^2)/(a + b*x^n)) + 6*b^2*n*Log[x] - 6*b^
2*Log[a + b*x^n])/(2*a^4*n)
```

fricas [A] time = 0.67, size = 105, normalized size = 1.35

$$\frac{6\,b^3 n x^{3\,n}\log(x) + 3\,a^2 b x^n - a^3 + 6\left(ab^2 n\log(x) + ab^2\right)x^{2\,n} - 6\left(b^3 x^{3\,n} + ab^2 x^{2\,n}\right)\log\left(bx^n + a\right)}{2\left(a^4 b n x^{3\,n} + a^5 n x^{2\,n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*b^3*n*x^(3*n)*log(x) + 3*a^2*b*x^n - a^3 + 6*(a*b^2*n*log(x) + a*b^2
)*x^(2*n) - 6*(b^3*x^(3*n) + a*b^2*x^(2*n))*log(b*x^n + a))/(a^4*b*n*x^(3*n
) + a^5*n*x^(2*n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-2\,n-1}}{\left(bx^n + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(-2*n - 1)/(b*x^n + a)^2, x)
```

maple [A] time = 0.03, size = 117, normalized size = 1.50

$$\frac{\left(\frac{3b^2 \mathrm{e}^{2n\ln(x)}\ln(x)}{a^3} + \frac{3b^3 \mathrm{e}^{3n\ln(x)}\ln(x)}{a^4} + \frac{3b\,\mathrm{e}^{n\ln(x)}}{2a^2 n} - \frac{3b^3 \mathrm{e}^{3n\ln(x)}}{a^4 n} - \frac{1}{2an}\right)\mathrm{e}^{-2n\ln(x)}}{b\,\mathrm{e}^{n\ln(x)} + a} - \frac{3b^2\ln\left(b\,\mathrm{e}^{n\ln(x)} + a\right)}{a^4 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-2*n)/(b*x^n+a)^2,x)
```

```
[Out] (-3*b^3/a^4/n*exp(n*ln(x))^3-1/2/a/n+3/2/a^2*b/n*exp(n*ln(x))+3*b^2/a^3*ln(
x)*exp(n*ln(x))^2+3*b^3/a^4*ln(x)*exp(n*ln(x))^3)/exp(n*ln(x))^2/(b*exp(n*l
n(x))+a)-3*b^2/a^4/n*ln(b*exp(n*ln(x))+a)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{2\,n+1}\left(a + b\,x^n\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(2*n + 1)*(a + b*x^n)^2),x)
```

```
[Out] int(1/(x^(2*n + 1)*(a + b*x^n)^2), x)
```

sympy [**F**(**-1**)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-2*n)/(a+b*x**n)**2,x)
[Out] Timed out
```

3.2631 $\int \frac{x^{-1-3n}}{(a+bx^n)^2}\,dx$

Optimal. Leaf size=94

$$\frac{4b^3\log(a+bx^n)}{a^5n}-\frac{4b^3\log(x)}{a^5}-\frac{b^3}{a^4n(a+bx^n)}-\frac{3b^2x^{-n}}{a^4n}+\frac{bx^{-2n}}{a^3n}-\frac{x^{-3n}}{3a^2n}$$

```
[Out] -1/3/a^2/n/(x^(3*n))+b/a^3/n/(x^(2*n))-3*b^2/a^4/n/(x^n)-b^3/a^4/n/(a+b*x^n
)-4*b^3*ln(x)/a^5+4*b^3*ln(a+b*x^n)/a^5/n
```

Rubi [A] time = 0.06, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 44}

$$-\frac{3b^2x^{-n}}{a^4n}-\frac{b^3}{a^4n(a+bx^n)}+\frac{4b^3\log(a+bx^n)}{a^5n}-\frac{4b^3\log(x)}{a^5}+\frac{bx^{-2n}}{a^3n}-\frac{x^{-3n}}{3a^2n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 3*n)/(a + b*x^n)^2,x]
```

```
[Out] -1/(3*a^2*n*x^(3*n)) + b/(a^3*n*x^(2*n)) - (3*b^2)/(a^4*n*x^n) - b^3/(a^4*n
*(a + b*x^n)) - (4*b^3*Log[x])/a^5 + (4*b^3*Log[a + b*x^n])/(a^5*n)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1-3n}}{(a+bx^n)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^4(a+bx)^2}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x^4}-\frac{2b}{a^3x^3}+\frac{3b^2}{a^4x^2}-\frac{4b^3}{a^5x}+\frac{b^4}{a^4(a+bx)^2}+\frac{4b^4}{a^5(a+bx)}\right)dx, x, x^n\right)}{n}\\ &= -\frac{x^{-3n}}{3a^2n}+\frac{bx^{-2n}}{a^3n}-\frac{3b^2x^{-n}}{a^4n}-\frac{b^3}{a^4n(a+bx^n)}-\frac{4b^3\log(x)}{a^5}+\frac{4b^3\log(a+bx^n)}{a^5n}\end{aligned}$$

Mathematica [A] time = 0.22, size = 78, normalized size = 0.83

$$\frac{a\left(-a^2x^{-3n}-\frac{3b^3}{a+bx^n}+3abx^{-2n}-9b^2x^{-n}\right)+12b^3\log(a+bx^n)-12b^3n\log(x)}{3a^5n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 3*n)/(a + b*x^n)^2,x]
```

```
[Out] (a*(-(a^2/x^(3*n)) + (3*a*b)/x^(2*n) - (9*b^2)/x^n - (3*b^3)/(a + b*x^n)) -
 12*b^3*n*Log[x] + 12*b^3*Log[a + b*x^n])/(3*a^5*n)
```

fricas [A] time = 0.63, size = 116, normalized size = 1.23

$$-\frac{12\,b^4 n x^{4\,n}\log(x) + 6\,a^2b^2x^{2\,n} - 2\,a^3bx^n + a^4 + 12\left(ab^3n\log(x) + ab^3\right)x^{3\,n} - 12\left(b^4x^{4\,n} + ab^3x^{3\,n}\right)\log\left(bx^n + a\right.}{3\left(a^5bnx^{4\,n} + a^6nx^{3\,n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)/(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(12*b^4*n*x^(4*n)*log(x) + 6*a^2*b^2*x^(2*n) - 2*a^3*b*x^n + a^4 + 12*
(a*b^3*n*log(x) + a*b^3)*x^(3*n) - 12*(b^4*x^(4*n) + a*b^3*x^(3*n))*log(b*x
^n + a))/(a^5*b*n*x^(4*n) + a^6*n*x^(3*n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-3\,n-1}}{\left(bx^n + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)/(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(-3*n - 1)/(b*x^n + a)^2, x)
```

maple [A] time = 0.03, size = 135, normalized size = 1.44

$$\frac{\left(-\frac{4b^3\mathrm{e}^{3n\ln(x)}\ln(x)}{a^4} - \frac{4b^4\mathrm{e}^{4n\ln(x)}\ln(x)}{a^5} + \frac{2b\,\mathrm{e}^{n\ln(x)}}{3a^2n} - \frac{2b^2\mathrm{e}^{2n\ln(x)}}{a^3n} + \frac{4b^4\mathrm{e}^{4n\ln(x)}}{a^5n} - \frac{1}{3an}\right)\mathrm{e}^{-3n\ln(x)}}{b\,\mathrm{e}^{n\ln(x)} + a} + \frac{4b^3\ln\left(b\,\mathrm{e}^{n\ln(x)} + a\right)}{a^5n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-3*n)/(b*x^n+a)^2,x)
```

```
[Out] (4*b^4/a^5/n*exp(n*ln(x))^4-1/3/a/n+2/3/a^2*b/n*exp(n*ln(x))-2*b^2/a^3/n*ex
p(n*ln(x))^2-4*b^3/a^4*ln(x)*exp(n*ln(x))^3-4*b^4/a^5*ln(x)*exp(n*ln(x))^4)
/exp(n*ln(x))^3/(b*exp(n*ln(x))+a)+4*b^3/a^5/n*ln(b*exp(n*ln(x))+a)
```

maxima [A] time = 0.55, size = 94, normalized size = 1.00

$$-\frac{12\,b^3x^{3\,n} + 6\,ab^2x^{2\,n} - 2\,a^2bx^n + a^3}{3\left(a^4bnx^{4\,n} + a^5nx^{3\,n}\right)} - \frac{4\,b^3\log(x)}{a^5} + \frac{4\,b^3\log\left(\frac{bx^n+a}{b}\right)}{a^5n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)/(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(12*b^3*x^(3*n) + 6*a*b^2*x^(2*n) - 2*a^2*b*x^n + a^3)/(a^4*b*n*x^(4*n
) + a^5*n*x^(3*n)) - 4*b^3*log(x)/a^5 + 4*b^3*log((b*x^n + a)/b)/(a^5*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{3\,n+1}\left(a + b\,x^n\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3*n + 1)*(a + b*x^n)^2),x)
```

```
[Out] int(1/(x^(3*n + 1)*(a + b*x^n)^2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-3*n)/(a+b*x**n)**2,x)
[Out] Timed out
```

3.2632 $\int \frac{x^{-1+4n}}{(a+bx^n)^3}\,dx$

Optimal. Leaf size=70

$$\frac{a^3}{2b^4n\,(a+bx^n)^2}-\frac{3a^2}{b^4n\,(a+bx^n)}-\frac{3a\log(a+bx^n)}{b^4n}+\frac{x^n}{b^3n}$$

```
[Out] x^n/b^3/n+1/2*a^3/b^4/n/(a+b*x^n)^2-3*a^2/b^4/n/(a+b*x^n)-3*a*ln(a+b*x^n)/b
^4/n
```

Rubi [A] time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$-\frac{3a^2}{b^4n\,(a+bx^n)}+\frac{a^3}{2b^4n\,(a+bx^n)^2}-\frac{3a\log(a+bx^n)}{b^4n}+\frac{x^n}{b^3n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 4*n)/(a + b*x^n)^3,x]
```

```
[Out] x^n/(b^3*n) + a^3/(2*b^4*n*(a + b*x^n)^2) - (3*a^2)/(b^4*n*(a + b*x^n)) - (
3*a*Log[a + b*x^n])/(b^4*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+4n}}{(a+bx^n)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{x^3}{(a+bx)^3}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{b^3}-\frac{a^3}{b^3(a+bx)^3}+\frac{3a^2}{b^3(a+bx)^2}-\frac{3a}{b^3(a+bx)}\right)dx, x, x^n\right)}{n}\\ &= \frac{x^n}{b^3n}+\frac{a^3}{2b^4n\,(a+bx^n)^2}-\frac{3a^2}{b^4n\,(a+bx^n)}-\frac{3a\log(a+bx^n)}{b^4n}\end{aligned}$$

Mathematica [A] time = 0.18, size = 51, normalized size = 0.73

$$-\frac{\frac{a^2(5a+6bx^n)}{(a+bx^n)^2}+6a\log(a+bx^n)-2bx^n}{2b^4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 4*n)/(a + b*x^n)^3,x]
```

```
[Out] -1/2*(-2*b*x^n + (a^2*(5*a + 6*b*x^n))/(a + b*x^n)^2 + 6*a*Log[a + b*x^n])/
(b^4*n)
```

fricas [A] time = 0.60, size = 102, normalized size = 1.46

$$\frac{2\,b^3x^{3\,n}+4\,ab^2x^{2\,n}-4\,a^2bx^n-5\,a^3-6\left(ab^2x^{2\,n}+2\,a^2bx^n+a^3\right)\log\left(bx^n+a\right)}{2\left(b^6nx^{2\,n}+2\,ab^5nx^n+a^2b^4n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*x^(3*n) + 4*a*b^2*x^(2*n) - 4*a^2*b*x^n - 5*a^3 - 6*(a*b^2*x^(2*
n) + 2*a^2*b*x^n + a^3)*log(b*x^n + a))/(b^6*n*x^(2*n) + 2*a*b^5*n*x^n + a^
2*b^4*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4\,n-1}}{\left(bx^n+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate(x^(4*n - 1)/(b*x^n + a)^3, x)
```

maple [A] time = 0.04, size = 75, normalized size = 1.07

$$-\frac{3a\ln\left(b\,\mathrm{e}^{n\ln(x)}+a\right)}{b^4n}+\frac{-\frac{6a^2\mathrm{e}^{n\ln(x)}}{b^3n}+\frac{\mathrm{e}^{3n\ln(x)}}{bn}-\frac{9a^3}{2b^4n}}{\left(b\,\mathrm{e}^{n\ln(x)}+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+4*n)/(b*x^n+a)^3,x)
```

```
[Out] (1/b/n*exp(n*ln(x))^3-9/2*a^3/b^4/n-6*a^2/b^3/n*exp(n*ln(x)))/(b*exp(n*ln(x
))+a)^2-3*a/b^4/n*ln(b*exp(n*ln(x))+a)
```

maxima [A] time = 0.54, size = 91, normalized size = 1.30

$$\frac{2\,b^3x^{3\,n}+4\,ab^2x^{2\,n}-4\,a^2bx^n-5\,a^3}{2\left(b^6nx^{2\,n}+2\,ab^5nx^n+a^2b^4n\right)}-\frac{3\,a\log\left(\frac{bx^n+a}{b}\right)}{b^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^3*x^(3*n) + 4*a*b^2*x^(2*n) - 4*a^2*b*x^n - 5*a^3)/(b^6*n*x^(2*n)
+ 2*a*b^5*n*x^n + a^2*b^4*n) - 3*a*log((b*x^n + a)/b)/(b^4*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{4\,n-1}}{\left(a+b\,x^n\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4*n - 1)/(a + b*x^n)^3,x)
```

```
[Out] int(x^(4*n - 1)/(a + b*x^n)^3, x)
```

sympy [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+4*n)/(a+b*x**n)**3,x)
[Out] Exception raised: HeuristicGCDFailed
```

3.2633 $\int \frac{x^{-1+3n}}{(a+bx^n)^3}\,dx$

Optimal. Leaf size=56

$$-\frac{a^2}{2b^3n\,(a+bx^n)^2}+\frac{2a}{b^3n\,(a+bx^n)}+\frac{\log(a+bx^n)}{b^3n}$$

```
[Out] -1/2*a^2/b^3/n/(a+b*x^n)^2+2*a/b^3/n/(a+b*x^n)+ln(a+b*x^n)/b^3/n
```

Rubi [A] time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$-\frac{a^2}{2b^3n\,(a+bx^n)^2}+\frac{2a}{b^3n\,(a+bx^n)}+\frac{\log(a+bx^n)}{b^3n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)/(a + b*x^n)^3,x]
```

```
[Out] -a^2/(2*b^3*n*(a + b*x^n)^2) + (2*a)/(b^3*n*(a + b*x^n)) + Log[a + b*x^n]/(
b^3*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+3n}}{(a+bx^n)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{(a+bx)^3}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a^2}{b^2(a+bx)^3}-\frac{2a}{b^2(a+bx)^2}+\frac{1}{b^2(a+bx)}\right)dx, x, x^n\right)}{n}\\ &= -\frac{a^2}{2b^3n\,(a+bx^n)^2}+\frac{2a}{b^3n\,(a+bx^n)}+\frac{\log(a+bx^n)}{b^3n}\end{aligned}$$

Mathematica [A] time = 0.05, size = 42, normalized size = 0.75

$$\frac{\frac{a(3a+4bx^n)}{(a+bx^n)^2}+2\log(a+bx^n)}{2b^3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)/(a + b*x^n)^3,x]
```

```
[Out] ((a*(3*a + 4*b*x^n))/(a + b*x^n)^2 + 2*Log[a + b*x^n])/(2*b^3*n)
```

fricas [A] time = 0.72, size = 76, normalized size = 1.36

$$\frac{4\,abx^n + 3\,a^2 + 2\left(b^2x^{2\,n} + 2\,abx^n + a^2\right)\log\left(bx^n + a\right)}{2\left(b^5nx^{2\,n} + 2\,ab^4nx^n + a^2b^3n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*b*x^n + 3*a^2 + 2*(b^2*x^(2*n) + 2*a*b*x^n + a^2)*log(b*x^n + a))/
(b^5*n*x^(2*n) + 2*a*b^4*n*x^n + a^2*b^3*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3\,n-1}}{\left(bx^n + a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate(x^(3*n - 1)/(b*x^n + a)^3, x)
```

maple [A] time = 0.03, size = 57, normalized size = 1.02

$$\frac{\ln\left(b\,\mathrm{e}^{n\ln(x)} + a\right)}{b^3n} + \frac{\frac{2a\,\mathrm{e}^{n\ln(x)}}{b^2n} + \frac{3a^2}{2b^3n}}{\left(b\,\mathrm{e}^{n\ln(x)} + a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n-1)/(b*x^n+a)^3,x)
```

```
[Out] (3/2*a^2/b^3/n+2*a/b^2/n*exp(n*ln(x)))/(b*exp(n*ln(x))+a)^2+1/b^3/n*ln(b*ex
p(n*ln(x))+a)
```

maxima [A] time = 0.58, size = 66, normalized size = 1.18

$$\frac{4\,abx^n + 3\,a^2}{2\left(b^5nx^{2\,n} + 2\,ab^4nx^n + a^2b^3n\right)} + \frac{\log\left(\frac{bx^n+a}{b}\right)}{b^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(4*a*b*x^n + 3*a^2)/(b^5*n*x^(2*n) + 2*a*b^4*n*x^n + a^2*b^3*n) + log((
b*x^n + a)/b)/(b^3*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{3\,n-1}}{\left(a + b\,x^n\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n - 1)/(a + b*x^n)^3,x)
```

```
[Out] int(x^(3*n - 1)/(a + b*x^n)^3, x)
```

sympy [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)/(a+b*x**n)**3,x)
[Out] Exception raised: HeuristicGCDFailed
```

3.2634 $\int \frac{x^{-1+2n}}{(a+bx^n)^3}\,dx$

Optimal. Leaf size=24

$$\frac{x^{2n}}{2an\,(a+bx^n)^2}$$

```
[Out] 1/2*x^(2*n)/a/n/(a+b*x^n)^2
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {264}

$$\frac{x^{2n}}{2an\,(a+bx^n)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/(a + b*x^n)^3,x]
```

```
[Out] x^(2*n)/(2*a*n*(a + b*x^n)^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^{-1+2n}}{(a+bx^n)^3}\,dx = \frac{x^{2n}}{2an\,(a+bx^n)^2}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$\frac{x^{2n}}{2an\,(a+bx^n)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/(a + b*x^n)^3,x]
```

```
[Out] x^(2*n)/(2*a*n*(a + b*x^n)^2)
```

fricas [A] time = 0.75, size = 41, normalized size = 1.71

$$-\frac{2\,bx^n + a}{2\left(b^4 n x^{2n} + 2\,ab^3 n x^n + a^2 b^2 n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*x^n + a)/(b^4*n*x^(2*n) + 2*a*b^3*n*x^n + a^2*b^2*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{(bx^n+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^3,x, algorithm="giac")
[Out] integrate(x^(2*n - 1)/(b*x^n + a)^3, x)
```

maple [A] time = 0.03, size = 36, normalized size = 1.50

$$\frac{-\frac{e^{n\ln(x)}}{bn}-\frac{a}{2b^2n}}{\left(b\,e^{n\ln(x)}+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n-1)/(b*x^n+a)^3,x)
[Out] (-1/b/n*exp(n*ln(x))-1/2*a/b^2/n)/(b*exp(n*ln(x))+a)^2
```

maxima [A] time = 0.50, size = 41, normalized size = 1.71

$$-\frac{2\,bx^n+a}{2\left(b^4nx^{2\,n}+2\,ab^3nx^n+a^2b^2n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^3,x, algorithm="maxima")
[Out] -1/2*(2*b*x^n + a)/(b^4*n*x^(2*n) + 2*a*b^3*n*x^n + a^2*b^2*n)
```

mupad [B] time = 1.38, size = 37, normalized size = 1.54

$$\frac{x^{2\,n}}{2\left(a^3\,n+2\,a^2\,b\,n\,x^n+a\,b^2\,n\,x^{2\,n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/(a + b*x^n)^3,x)
[Out] x^(2*n)/(2*(a^3*n + 2*a^2*b*n*x^n + a*b^2*n*x^(2*n)))
```

sympy [A] time = 158.69, size = 61, normalized size = 2.54

$$\begin{cases}\frac{\log(x)}{b^3} & \text{for } a=0\wedge n=0\\ -\frac{x^{-n}}{b^3n} & \text{for } a=0\\ \frac{\log(x)}{(a+b)^3} & \text{for } n=0\\ \frac{x^{2n}}{2a^3n+4a^2bnx^n+2ab^2nx^{2n}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a+b*x**n)**3,x)
[Out] Piecewise((log(x)/b**3, Eq(a, 0) & Eq(n, 0)), (-x**(-n)/(b**3*n), Eq(a, 0))
, (log(x)/(a + b)**3, Eq(n, 0)), (x**(2*n)/(2*a**3*n + 4*a**2*b*n*x**n + 2*
a*b**2*n*x**(2*n)), True))
```

3.2635 $\int \frac{x^{-1+n}}{(a+bx^n)^3}\,dx$

Optimal. Leaf size=19

$$-\frac{1}{2bn\,(a+bx^n)^2}$$

```
[Out] -1/2/b/n/(a+b*x^n)^2
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$-\frac{1}{2bn\,(a+bx^n)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)/(a + b*x^n)^3,x]
```

```
[Out] -1/(2*b*n*(a + b*x^n)^2)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^{-1+n}}{(a+bx^n)^3}\,dx = -\frac{1}{2bn\,(a+bx^n)^2}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$-\frac{1}{2bn\,(a+bx^n)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)/(a + b*x^n)^3,x]
```

```
[Out] -1/2*1/(b*n*(a + b*x^n)^2)
```

fricas [A] time = 0.75, size = 31, normalized size = 1.63

$$-\frac{1}{2\left(b^3nx^{2\,n}+2\,ab^2nx^n+a^2bn\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] -1/2/(b^3*n*x^(2*n) + 2*a*b^2*n*x^n + a^2*b*n)
```

giac [A] time = 0.15, size = 17, normalized size = 0.89

$$-\frac{1}{2\,(bx^n+a)^2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] -1/2/((b*x^n + a)^2*b*n)
```

maple [A] time = 0.03, size = 20, normalized size = 1.05

$$-\frac{1}{2\left(b\,\mathrm{e}^{n\ln(x)}+a\right)^2 bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)/(b*x^n+a)^3,x)
```

```
[Out] -1/2/b/n/(b*exp(n*ln(x))+a)^2
```

maxima [A] time = 0.58, size = 17, normalized size = 0.89

$$-\frac{1}{2\,(bx^n+a)^2 bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] -1/2/((b*x^n + a)^2*b*n)
```

mupad [B] time = 1.38, size = 33, normalized size = 1.74

$$-\frac{1}{2\,b^3\,n\,x^{2\,n}+2\,a^2\,b\,n+4\,a\,b^2\,n\,x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)/(a + b*x^n)^3,x)
```

```
[Out] -1/(2*b^3*n*x^(2*n) + 2*a^2*b*n + 4*a*b^2*n*x^n)
```

sympy [A] time = 30.22, size = 56, normalized size = 2.95

$$\begin{cases} \frac{\log(x)}{a^3} & \text{for } b = 0 \wedge n = 0 \\ \frac{x^n}{a^3 n} & \text{for } b = 0 \\ \frac{\log(x)}{(a+b)^3} & \text{for } n = 0 \\ -\frac{1}{2a^2bn+4ab^2nx^n+2b^3nx^{2n}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)/(a+b*x**n)**3,x)
```

```
[Out] Piecewise((log(x)/a**3, Eq(b, 0) & Eq(n, 0)), (x**n/(a**3*n), Eq(b, 0)), (l
og(x)/(a + b)**3, Eq(n, 0)), (-1/(2*a**2*b*n + 4*a*b**2*n*x**n + 2*b**3*n*x
**(2*n)), True))
```

3.2636 $\int \frac{1}{x(a+bx^n)^3}\,dx$

Optimal. Leaf size=58

$$-\frac{\log(a+bx^n)}{a^3 n}+\frac{\log(x)}{a^3}+\frac{1}{a^2 n\,(a+bx^n)}+\frac{1}{2an\,(a+bx^n)^2}$$

```
[Out] 1/2/a/n/(a+b*x^n)^2+1/a^2/n/(a+b*x^n)+ln(x)/a^3-ln(a+b*x^n)/a^3/n
```

Rubi [A] time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {266, 44}

$$\frac{1}{a^2 n\,(a+bx^n)}-\frac{\log(a+bx^n)}{a^3 n}+\frac{\log(x)}{a^3}+\frac{1}{2an\,(a+bx^n)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^n)^3),x]
```

```
[Out] 1/(2*a*n*(a + b*x^n)^2) + 1/(a^2*n*(a + b*x^n)) + Log[x]/a^3 - Log[a + b*x^
n]/(a^3*n)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\,(a+bx^n)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^3}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{a^3 x}-\frac{b}{a(a+bx)^3}-\frac{b}{a^2(a+bx)^2}-\frac{b}{a^3(a+bx)}\right)\,dx, x, x^n\right)}{n}\\ &= \frac{1}{2an\,(a+bx^n)^2}+\frac{1}{a^2 n\,(a+bx^n)}+\frac{\log(x)}{a^3}-\frac{\log(a+bx^n)}{a^3 n}\end{aligned}$$

Mathematica [A] time = 0.12, size = 47, normalized size = 0.81

$$\frac{\frac{a(3a+2bx^n)}{(a+bx^n)^2}-2\log(a+bx^n)+2n\log(x)}{2a^3 n}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^n)^3),x]
```

```
[Out] ((a*(3*a + 2*b*x^n))/(a + b*x^n)^2 + 2*n*Log[x] - 2*Log[a + b*x^n])/(2*a^3*
n)
```

fricas [A] time = 0.56, size = 106, normalized size = 1.83

$$\frac{2\,b^2nx^{2\,n}\log(x)+2\,a^2n\log(x)+3\,a^2+2\left(2\,abn\log(x)+ab\right)x^n-2\left(b^2x^{2\,n}+2\,abx^n+a^2\right)\log\left(bx^n+a\right)}{2\left(a^3b^2nx^{2\,n}+2\,a^4bnx^n+a^5n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*n*x^(2*n)*log(x) + 2*a^2*n*log(x) + 3*a^2 + 2*(2*a*b*n*log(x) +
a*b)*x^n - 2*(b^2*x^(2*n) + 2*a*b*x^n + a^2)*log(b*x^n + a))/(a^3*b^2*n*x^(
2*n) + 2*a^4*b*n*x^n + a^5*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^n+a\right)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^3*x), x)
```

maple [A] time = 0.00, size = 62, normalized size = 1.07

$$\frac{1}{2\left(b\,x^n+a\right)^2an}+\frac{1}{\left(b\,x^n+a\right)a^2n}+\frac{\ln\left(x^n\right)}{a^3n}-\frac{\ln\left(b\,x^n+a\right)}{a^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^n+a)^3,x)
```

```
[Out] 1/2/(b*x^n+a)^2/a/n+1/(b*x^n+a)/a^2/n+1/a^3/n*ln(x^n)-1/a^3/n*ln(b*x^n+a)
```

maxima [A] time = 0.60, size = 71, normalized size = 1.22

$$\frac{2\,bx^n+3\,a}{2\left(a^2b^2nx^{2\,n}+2\,a^3bnx^n+a^4n\right)}-\frac{\log\left(bx^n+a\right)}{a^3n}+\frac{\log\left(x^n\right)}{a^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x^n + 3*a)/(a^2*b^2*n*x^(2*n) + 2*a^3*b*n*x^n + a^4*n) - log(b*x^n
 + a)/(a^3*n) + log(x^n)/(a^3*n)
```

mupad [B] time = 1.21, size = 69, normalized size = 1.19

$$\frac{\ln(x)}{a^3}+\frac{1}{a^2\,n\,\left(a+b\,x^n\right)}-\frac{\ln\left(a+b\,x^n\right)}{a^3\,n}+\frac{1}{2\,a\,n\,\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^n)^3),x)
```

```
[Out] log(x)/a^3 + 1/(a^2*n*(a + b*x^n)) - log(a + b*x^n)/(a^3*n) + 1/(2*a*n*(a^2
 + b^2*x^(2*n) + 2*a*b*x^n))
```

sympy [**A**] time = 2.67, size = 406, normalized size = 7.00

$$\begin{cases} \tilde{\infty}\log(x) \\ -\frac{x^{-3n}}{3b^3n} \\ \frac{\log(x)}{(a+b)^3} \\ \frac{\log(x)}{a^3} \\ \frac{2a^2n\log(x)}{2a^5n+4a^4bnx^n+2a^3b^2nx^{2n}} - \frac{2a^2\log\left(\frac{a}{b}+x^n\right)}{2a^5n+4a^4bnx^n+2a^3b^2nx^{2n}} + \frac{3a^2}{2a^5n+4a^4bnx^n+2a^3b^2nx^{2n}} + \frac{4abnx^n\log(x)}{2a^5n+4a^4bnx^n+2a^3b^2nx^{2n}} - \frac{4abx^n\log\left(\frac{a}{b}+x\right.}{2a^5n+4a^4bnx^n+2a^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x**n)**3,x)
```

```
[Out] Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (-x**(-3*n)/(3*b**3
*n), Eq(a, 0)), (log(x)/(a + b)**3, Eq(n, 0)), (log(x)/a**3, Eq(b, 0)), (2*
a**2*n*log(x)/(2*a**5*n + 4*a**4*b*n*x**n + 2*a**3*b**2*n*x**(2*n)) - 2*a**
2*log(a/b + x**n)/(2*a**5*n + 4*a**4*b*n*x**n + 2*a**3*b**2*n*x**(2*n)) + 3
*a**2/(2*a**5*n + 4*a**4*b*n*x**n + 2*a**3*b**2*n*x**(2*n)) + 4*a*b*n*x**n*
log(x)/(2*a**5*n + 4*a**4*b*n*x**n + 2*a**3*b**2*n*x**(2*n)) - 4*a*b*x**n*l
og(a/b + x**n)/(2*a**5*n + 4*a**4*b*n*x**n + 2*a**3*b**2*n*x**(2*n)) + 2*a*
b*x**n/(2*a**5*n + 4*a**4*b*n*x**n + 2*a**3*b**2*n*x**(2*n)) + 2*b**2*n*x**
(2*n)*log(x)/(2*a**5*n + 4*a**4*b*n*x**n + 2*a**3*b**2*n*x**(2*n)) - 2*b**2
*x**(2*n)*log(a/b + x**n)/(2*a**5*n + 4*a**4*b*n*x**n + 2*a**3*b**2*n*x**(2
*n)), True))
```

3.2637 $\int \frac{x^{-1-n}}{(a+bx^n)^3}\,dx$

Optimal. Leaf size=77

$$\frac{3b\log(a+bx^n)}{a^4n}-\frac{3b\log(x)}{a^4}-\frac{2b}{a^3n(a+bx^n)}-\frac{x^{-n}}{a^3n}-\frac{b}{2a^2n(a+bx^n)^2}$$

```
[Out] -1/a^3/n/(x^n)-1/2*b/a^2/n/(a+b*x^n)^2-2*b/a^3/n/(a+b*x^n)-3*b*ln(x)/a^4+3*
b*ln(a+b*x^n)/a^4/n
```

Rubi [A] time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 44}

$$-\frac{2b}{a^3n(a+bx^n)}-\frac{b}{2a^2n(a+bx^n)^2}+\frac{3b\log(a+bx^n)}{a^4n}-\frac{3b\log(x)}{a^4}-\frac{x^{-n}}{a^3n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n)/(a + b*x^n)^3,x]
```

```
[Out] -(1/(a^3*n*x^n)) - b/(2*a^2*n*(a + b*x^n)^2) - (2*b)/(a^3*n*(a + b*x^n)) -
(3*b*Log[x])/a^4 + (3*b*Log[a + b*x^n])/(a^4*n)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1-n}}{(a+bx^n)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx)^3}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{a^3x^2}-\frac{3b}{a^4x}+\frac{b^2}{a^2(a+bx)^3}+\frac{2b^2}{a^3(a+bx)^2}+\frac{3b^2}{a^4(a+bx)}\right)dx,x,x^n\right)}{n}\\ &= -\frac{x^{-n}}{a^3n}-\frac{b}{2a^2n(a+bx^n)^2}-\frac{2b}{a^3n(a+bx^n)}-\frac{3b\log(x)}{a^4}+\frac{3b\log(a+bx^n)}{a^4n}\end{aligned}$$

Mathematica [A] time = 0.22, size = 58, normalized size = 0.75

$$-\frac{\frac{ab(5a+4bx^n)}{(a+bx^n)^2}-6b\log(a+bx^n)+2ax^{-n}+6bn\log(x)}{2a^4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n)/(a + b*x^n)^3,x]
```

```
[Out] -1/2*((2*a)/x^n + (a*b*(5*a + 4*b*x^n))/(a + b*x^n)^2 + 6*b*n*Log[x] - 6*b*
Log[a + b*x^n])/(a^4*n)
```

fricas [A] time = 1.17, size = 139, normalized size = 1.81

$$-\frac{6\,b^3nx^{3\,n}\log(x)+2\,a^3+6\left(2\,ab^2n\log(x)+ab^2\right)x^{2\,n}+3\left(2\,a^2bn\log(x)+3\,a^2b\right)x^n-6\left(b^3x^{3\,n}+2\,ab^2x^{2\,n}+a^2\right.}{2\left(a^4b^2nx^{3\,n}+2\,a^5bnx^{2\,n}+a^6nx^n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(6*b^3*n*x^(3*n)*log(x) + 2*a^3 + 6*(2*a*b^2*n*log(x) + a*b^2)*x^(2*n)
 + 3*(2*a^2*b*n*log(x) + 3*a^2*b)*x^n - 6*(b^3*x^(3*n) + 2*a*b^2*x^(2*n) +
a^2*b*x^n)*log(b*x^n + a))/(a^4*b^2*n*x^(3*n) + 2*a^5*b*n*x^(2*n) + a^6*n*x
^n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-n-1}}{\left(bx^n+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate(x^(-n - 1)/(b*x^n + a)^3, x)
```

maple [A] time = 0.04, size = 132, normalized size = 1.71

$$\frac{\left(-\frac{3b\,\mathrm{e}^{n\ln(x)}\ln(x)}{a^2}-\frac{6b^2\mathrm{e}^{2n\ln(x)}\ln(x)}{a^3}-\frac{3b^3\mathrm{e}^{3n\ln(x)}\ln(x)}{a^4}+\frac{6b^2\mathrm{e}^{2n\ln(x)}}{a^3n}+\frac{9b^3\mathrm{e}^{3n\ln(x)}}{2a^4n}-\frac{1}{an}\right)\mathrm{e}^{-n\ln(x)}}{\left(b\,\mathrm{e}^{n\ln(x)}+a\right)^2}+\frac{3b\ln\left(b\,\mathrm{e}^{n\ln(x)}+a\right)}{a^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n)/(b*x^n+a)^3,x)
```

```
[Out] (-1/a/n-3/a^2*b*exp(n*ln(x))*ln(x)-6*b^2/a^3*ln(x)*exp(n*ln(x))^2+6*b^2/a^3
/n*exp(n*ln(x))^2-3*b^3/a^4*ln(x)*exp(n*ln(x))^3+9/2*b^3/a^4/n*exp(n*ln(x))
^3)/exp(n*ln(x))/(b*exp(n*ln(x))+a)^2+3*b/a^4/n*ln(b*exp(n*ln(x))+a)
```

maxima [A] time = 0.56, size = 91, normalized size = 1.18

$$-\frac{6\,b^2x^{2\,n}+9\,abx^n+2\,a^2}{2\left(a^3b^2nx^{3\,n}+2\,a^4bnx^{2\,n}+a^5nx^n\right)}-\frac{3\,b\log(x)}{a^4}+\frac{3\,b\log\left(\frac{bx^n+a}{b}\right)}{a^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(6*b^2*x^(2*n) + 9*a*b*x^n + 2*a^2)/(a^3*b^2*n*x^(3*n) + 2*a^4*b*n*x^(
2*n) + a^5*n*x^n) - 3*b*log(x)/a^4 + 3*b*log((b*x^n + a)/b)/(a^4*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{n+1}\left(a+b\,x^n\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(n + 1)*(a + b*x^n)^3),x)
```

```
[Out] int(1/(x^(n + 1)*(a + b*x^n)^3), x)
```

sympy [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n)/(a+b*x**n)**3,x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.2638 $\int \frac{x^{-1-2n}}{(a+bx^n)^3}\,dx$

Optimal. Leaf size=101

$$-\frac{6b^2\log(a+bx^n)}{a^5n}+\frac{6b^2\log(x)}{a^5}+\frac{3b^2}{a^4n(a+bx^n)}+\frac{3bx^{-n}}{a^4n}+\frac{b^2}{2a^3n(a+bx^n)^2}-\frac{x^{-2n}}{2a^3n}$$

```
[Out] -1/2/a^3/n/(x^(2*n))+3*b/a^4/n/(x^n)+1/2*b^2/a^3/n/(a+b*x^n)^2+3*b^2/a^4/n/
(a+b*x^n)+6*b^2*ln(x)/a^5-6*b^2*ln(a+b*x^n)/a^5/n
```

Rubi [A] time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 44}

$$\frac{3b^2}{a^4n(a+bx^n)}+\frac{b^2}{2a^3n(a+bx^n)^2}-\frac{6b^2\log(a+bx^n)}{a^5n}+\frac{6b^2\log(x)}{a^5}+\frac{3bx^{-n}}{a^4n}-\frac{x^{-2n}}{2a^3n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 2*n)/(a + b*x^n)^3,x]
```

```
[Out] -1/(2*a^3*n*x^(2*n)) + (3*b)/(a^4*n*x^n) + b^2/(2*a^3*n*(a + b*x^n)^2) + (3
*b^2)/(a^4*n*(a + b*x^n)) + (6*b^2*Log[x])/a^5 - (6*b^2*Log[a + b*x^n])/(a^
5*n)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1-2n}}{(a+bx^n)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^3(a+bx)^3}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{a^3x^3}-\frac{3b}{a^4x^2}+\frac{6b^2}{a^5x}-\frac{b^3}{a^3(a+bx)^3}-\frac{3b^3}{a^4(a+bx)^2}-\frac{6b^3}{a^5(a+bx)}\right)dx, x, x^n\right)}{n}\\ &= -\frac{x^{-2n}}{2a^3n}+\frac{3bx^{-n}}{a^4n}+\frac{b^2}{2a^3n(a+bx^n)^2}+\frac{3b^2}{a^4n(a+bx^n)}+\frac{6b^2\log(x)}{a^5}-\frac{6b^2\log(a+bx^n)}{a^5n}\end{aligned}$$

Mathematica [A] time = 0.27, size = 79, normalized size = 0.78

$$\frac{\frac{ax^{-2n}(a+2bx^n)\left(-a^2+6abx^n+6b^2x^{2n}\right)}{(a+bx^n)^2}-12b^2\log(a+bx^n)+12b^2n\log(x)}{2a^5n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 2*n)/(a + b*x^n)^3,x]
```

```
[Out] ((a*(a + 2*b*x^n)*(-a^2 + 6*a*b*x^n + 6*b^2*x^(2*n)))/(x^(2*n)*(a + b*x^n)^
2) + 12*b^2*n*Log[x] - 12*b^2*Log[a + b*x^n])/(2*a^5*n)
```

fricas [A] time = 0.56, size = 160, normalized size = 1.58

$$\frac{12\,b^4 n x^{4\,n}\log(x) + 4\,a^3 b x^n - a^4 + 12\left(2\,ab^3 n\log(x) + ab^3\right)x^{3\,n} + 6\left(2\,a^2b^2 n\log(x) + 3\,a^2b^2\right)x^{2\,n} - 12\left(b^4x^{4\,n} + 2\,a\right.}{2\left(a^5b^2 n x^{4\,n} + 2\,a^6 b n x^{3\,n} + a^7 n x^{2\,n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(12*b^4*n*x^(4*n)*log(x) + 4*a^3*b*x^n - a^4 + 12*(2*a*b^3*n*log(x) + a
*b^3)*x^(3*n) + 6*(2*a^2*b^2*n*log(x) + 3*a^2*b^2)*x^(2*n) - 12*(b^4*x^(4*n
) + 2*a*b^3*x^(3*n) + a^2*b^2*x^(2*n))*log(b*x^n + a))/(a^5*b^2*n*x^(4*n) +
 2*a^6*b*n*x^(3*n) + a^7*n*x^(2*n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-2\,n-1}}{\left(bx^n + a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate(x^(-2*n - 1)/(b*x^n + a)^3, x)
```

maple [A] time = 0.05, size = 152, normalized size = 1.50

$$\frac{\left(\frac{6b^2\mathrm{e}^{2n\ln(x)}\ln(x)}{a^3} + \frac{12b^3\mathrm{e}^{3n\ln(x)}\ln(x)}{a^4} + \frac{6b^4\mathrm{e}^{4n\ln(x)}\ln(x)}{a^5} + \frac{2b\,\mathrm{e}^{n\ln(x)}}{a^2 n} + \frac{9b^2\mathrm{e}^{2n\ln(x)}}{a^3 n} + \frac{6b^3\mathrm{e}^{3n\ln(x)}}{a^4 n} - \frac{1}{2an}\right)\mathrm{e}^{-2n\ln(x)}}{\left(b\,\mathrm{e}^{n\ln(x)} + a\right)^2} - \frac{6b^2\ln\left(b\,\mathrm{e}^{n\ln(x)}\right.}{a^5 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-2*n)/(b*x^n+a)^3,x)
```

```
[Out] (9*b^2/a^3/n*exp(n*ln(x))^2-1/2/a/n+6*b^2/a^3*ln(x)*exp(n*ln(x))^2+2/a^2*b/
n*exp(n*ln(x))+12*b^3/a^4*ln(x)*exp(n*ln(x))^3+6*b^4/a^5*ln(x)*exp(n*ln(x))
^4+6*b^3/a^4/n*exp(n*ln(x))^3)/exp(n*ln(x))^2/(b*exp(n*ln(x))+a)^2-6*b^2/a^
5/n*ln(b*exp(n*ln(x))+a)
```

maxima [A] time = 0.49, size = 110, normalized size = 1.09

$$\frac{12\,b^3x^{3\,n} + 18\,ab^2x^{2\,n} + 4\,a^2bx^n - a^3}{2\left(a^4b^2 n x^{4\,n} + 2\,a^5 b n x^{3\,n} + a^6 n x^{2\,n}\right)} + \frac{6\,b^2\log(x)}{a^5} - \frac{6\,b^2\log\left(\frac{bx^n+a}{b}\right)}{a^5 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(12*b^3*x^(3*n) + 18*a*b^2*x^(2*n) + 4*a^2*b*x^n - a^3)/(a^4*b^2*n*x^(4
*n) + 2*a^5*b*n*x^(3*n) + a^6*n*x^(2*n)) + 6*b^2*log(x)/a^5 - 6*b^2*log((b*
x^n + a)/b)/(a^5*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{2\,n+1}\left(a + b\,x^n\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(2*n + 1)*(a + b*x^n)^3),x)
[Out] int(1/(x^(2*n + 1)*(a + b*x^n)^3), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-2*n)/(a+b*x**n)**3,x)
[Out] Timed out
```

3.2639 $\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n}\,dx$

Optimal. Leaf size=50

$$\frac{2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,x^{-n/2}}{\sqrt{b}}\right)}{a^{3/2}n}-\frac{2x^{-n/2}}{an}$$

```
[Out] -2/a/n/(x^(1/2*n))+2*arctan(a^(1/2)/(x^(1/2*n))/b^(1/2))*b^(1/2)/a^(3/2)/n
```

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {345, 193, 321, 205}

$$\frac{2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,x^{-n/2}}{\sqrt{b}}\right)}{a^{3/2}n}-\frac{2x^{-n/2}}{an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n/2)/(a + b*x^n),x]
```

```
[Out] -2/(a*n*x^(n/2)) + (2*Sqrt[b]*ArcTan[Sqrt[a]/(Sqrt[b]*x^(n/2))])/(a^(3/2)*n
)
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{1}{a+\frac{b}{x^2}}\,dx, x, x^{-n/2}\right)}{n}\\ &= -\frac{2\,\text{Subst}\left(\int \frac{x^2}{b+ax^2}\,dx, x, x^{-n/2}\right)}{n}\\ &= -\frac{2x^{-n/2}}{an} + \frac{(2b)\,\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, x^{-n/2}\right)}{an}\\ &= -\frac{2x^{-n/2}}{an} + \frac{2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,x^{-n/2}}{\sqrt{b}}\right)}{a^{3/2}n}\end{aligned}$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.64

$$-\frac{2x^{-n/2}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{bx^n}{a}\right)}{an}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n/2)/(a + b*x^n),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/2, 1, 1/2, -((b*x^n)/a)])/(a*n*x^(n/2))
```

fricas [A] time = 0.78, size = 130, normalized size = 2.60

$$\left[-\frac{2\,xx^{-\frac{1}{2}n-1} - \sqrt{-\frac{b}{a}}\,\log\left(\frac{ax^2x^{-n-2}+2\,axx^{-\frac{1}{2}n-1}\sqrt{-\frac{b}{a}}-b}{ax^2x^{-n-2}+b}\right)}{an}, -\frac{2\left(xx^{-\frac{1}{2}n-1} + \sqrt{\frac{b}{a}}\,\arctan\left(\frac{\sqrt{\frac{b}{a}}}{xx^{-\frac{1}{2}n-1}}\right)\right)}{an}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] [-(2*x*x^(-1/2*n - 1) - sqrt(-b/a)*log((a*x^2*x^(-n - 2) + 2*a*x*x^(-1/2*n
- 1)*sqrt(-b/a) - b)/(a*x^2*x^(-n - 2) + b)))/(a*n), -2*(x*x^(-1/2*n - 1) +
 sqrt(b/a)*arctan(sqrt(b/a)/(x*x^(-1/2*n - 1))))/(a*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{1}{2}n-1}}{bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*n)/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(-1/2*n - 1)/(b*x^n + a), x)
```

maple [A] time = 0.09, size = 79, normalized size = 1.58

$$-\frac{2x^{-\frac{n}{2}}}{an} + \frac{\sqrt{-ab}\,\ln\left(x^{\frac{n}{2}} - \frac{\sqrt{-ab}}{b}\right)}{a^2n} - \frac{\sqrt{-ab}\,\ln\left(x^{\frac{n}{2}} + \frac{\sqrt{-ab}}{b}\right)}{a^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-1/2*n)/(b*x^n+a),x)
```

```
[Out] -2/a/n/(x^(1/2*n))+(-a*b)^(1/2)/a^2/n*ln(x^(1/2*n)-(-a*b)^(1/2)/b)-(-a*b)^(
1/2)/a^2/n*ln(x^(1/2*n)+(-a*b)^(1/2)/b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-b\int \frac{x^{\frac{1}{2}n}}{abxx^n + a^2x}\,dx - \frac{2}{anx^{\frac{1}{2}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*n)/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] -b*integrate(x^(1/2*n)/(a*b*x*x^n + a^2*x), x) - 2/(a*n*x^(1/2*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^{\frac{n}{2}+1}\,(a + b\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(n/2 + 1)*(a + b*x^n)),x)
```

```
[Out] int(1/(x^(n/2 + 1)*(a + b*x^n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-1/2*n)/(a+b*x**n),x)
```

```
[Out] Timed out
```

3.2640 $\int \frac{x^{-1-\frac{2n}{3}}}{a+bx^n}\,dx$

Optimal. Leaf size=160

$$-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^{n/3}\right)}{a^{5/3}n}+\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^{n/3}+b^{2/3}x^{2n/3}\right)}{2a^{5/3}n}+\frac{\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x^{n/3}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{a^{5/3}n}-\frac{3x^{-2n/3}}{2an}$$

```
[Out] -3/2/a/n/(x^(2/3*n))-b^(2/3)*ln(a^(1/3)+b^(1/3)*x^(1/3*n))/a^(5/3)/n+1/2*b^
(2/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x^(1/3*n)+b^(2/3)*x^(2/3*n))/a^(5/3)/n+b^(
2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(1/3*n))/a^(1/3)*3^(1/2))*3^(1/2)/a^(5
/3)/n
```

Rubi [A] time = 0.11, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {362, 345, 200, 31, 634, 617, 204, 628}

$$-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^{n/3}\right)}{a^{5/3}n}+\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^{n/3}+b^{2/3}x^{2n/3}\right)}{2a^{5/3}n}+\frac{\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x^{n/3}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{a^{5/3}n}-\frac{3x^{-2n/3}}{2an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - (2*n)/3)/(a + b*x^n),x]
```

```
[Out] -3/(2*a*n*x^((2*n)/3)) + (Sqrt[3]*b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(n/
3))/(Sqrt[3]*a^(1/3))])/(a^(5/3)*n) - (b^(2/3)*Log[a^(1/3) + b^(1/3)*x^(n/3
)])/(a^(5/3)*n) + (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(n/3) + b^(2/3)*
x^((2*n)/3)])/(2*a^(5/3)*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 362

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Dist[b/a, Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a,
 b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1-\frac{2n}{3}}}{a+bx^n}\,dx &= -\frac{3x^{-2n/3}}{2an} - \frac{b\int \frac{x^{\frac{1}{3}(-3+n)}}{a+bx^n}\,dx}{a} \\
&= -\frac{3x^{-2n/3}}{2an} - \frac{(3b)\,\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, x^{1+\frac{1}{3}(-3+n)}\right)}{an} \\
&= -\frac{3x^{-2n/3}}{2an} - \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, x^{1+\frac{1}{3}(-3+n)}\right)}{a^{5/3}n} - \frac{b\,\text{Subst}\left(\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, x^{1+\frac{1}{3}(-3+n)}\right.}{a^{5/3}n} \\
&= -\frac{3x^{-2n/3}}{2an} - \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^{n/3}\right)}{a^{5/3}n} + \frac{b^{2/3}\,\text{Subst}\left(\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, x^{1+\frac{1}{3}(-3+n)}\right)}{2a^{5/3}n} - \frac{(3b)\,\text{S}}{} \\
&= -\frac{3x^{-2n/3}}{2an} - \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^{n/3}\right)}{a^{5/3}n} + \frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^{n/3}+b^{2/3}x^{2n/3}\right)}{2a^{5/3}n} - \frac{\left(3b^{2/3}\right)\text{Subst}\left(\int \right.}{} \\
&= -\frac{3x^{-2n/3}}{2an} + \frac{\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x^{n/3}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{a^{5/3}n} - \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^{n/3}\right)}{a^{5/3}n} + \frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^{n/3}\right.}{2a^{5/3}n}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 34, normalized size = 0.21

$$-\frac{3x^{-2n/3}\,{}_2F_1\left(-\frac{2}{3},1;\frac{1}{3};-\frac{bx^n}{a}\right)}{2an}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - (2*n)/3)/(a + b*x^n),x]
```

```
[Out] (-3*Hypergeometric2F1[-2/3, 1, 1/3, -((b*x^n)/a)])/(2*a*n*x^((2*n)/3))
```

fricas [A] time = 0.92, size = 178, normalized size = 1.11

$$-\frac{3\,x x^{-\frac{2}{3}n-1} - 2\sqrt{3}\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,a\sqrt{x}\,x^{-\frac{1}{3}n-\frac{1}{2}}\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+\sqrt{3}\,b}{3\,b}\right)+\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(-\frac{a\sqrt{x}\,x^{-\frac{1}{3}n-\frac{1}{2}}\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-bx x^{-\frac{2}{3}n-1}+b\left(-\frac{b^2}{a^2}\right.\right.}{x}}{2\,an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2/3*n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] -1/2*(3*x*x^(-2/3*n - 1) - 2*sqrt(3)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)
*a*sqrt(x)*x^(-1/3*n - 1/2)*(-b^2/a^2)^(1/3) + sqrt(3)*b)/b) + (-b^2/a^2)^(
1/3)*log(-(a*sqrt(x)*x^(-1/3*n - 1/2)*(-b^2/a^2)^(2/3) - b*x*x^(-2/3*n - 1)
 + b*(-b^2/a^2)^(1/3))/x) - 2*(-b^2/a^2)^(1/3)*log((b*x*x^(-1/3*n - 1/2) +
a*sqrt(x)*(-b^2/a^2)^(2/3))/x))/(a*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{2}{3}n-1}}{bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2/3*n)/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(-2/3*n - 1)/(b*x^n + a), x)
```

maple [C] time = 0.16, size = 54, normalized size = 0.34

$$\mathrm{RootOf}\left(a^5n^3_Z^3 + b^2\right)\ln\left(-\frac{\mathrm{RootOf}\left(a^5n^3_Z^3 + b^2\right)a^2n}{b} + x^{\frac{n}{3}}\right) - \frac{3x^{-\frac{2n}{3}}}{2an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-2/3*n)/(b*x^n+a),x)
```

```
[Out] -3/2/a/n/(x^(1/3*n))^2+sum(_R*ln(x^(1/3*n)-a^2*n/b*_R),_R=RootOf(_Z^3*a^5*n
^3+b^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-b\int \frac{x^{\frac{1}{3}n}}{abxx^n + a^2x}\,dx - \frac{3}{2\,anx^{\frac{2}{3}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2/3*n)/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] -b*integrate(x^(1/3*n)/(a*b*x*x^n + a^2*x), x) - 3/2/(a*n*x^(2/3*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{\frac{2n}{3}+1}\,(a + b\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^((2*n)/3 + 1)*(a + b*x^n)),x)
```

```
[Out] int(1/(x^((2*n)/3 + 1)*(a + b*x^n)), x)
```

sympy [C] time = 5.47, size = 192, normalized size = 1.20

$$\frac{x^{-\frac{2n}{3}}\Gamma\left(-\frac{2}{3}\right)}{an\Gamma\left(\frac{1}{3}\right)}-\frac{2b^{\frac{2}{3}}e^{-\frac{i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{n}{3}}e^{\frac{i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{2}{3}\right)}{3a^{\frac{5}{3}}n\Gamma\left(\frac{1}{3}\right)}+\frac{2b^{\frac{2}{3}}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{n}{3}}e^{i\pi}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{2}{3}\right)}{3a^{\frac{5}{3}}n\Gamma\left(\frac{1}{3}\right)}-\frac{2b^{\frac{2}{3}}e^{\frac{i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{n}{3}}e^{\frac{5i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{2}{3}\right)}{3a^{\frac{5}{3}}n\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-2/3*n)/(a+b*x**n),x)
```

```
[Out] x**(-2*n/3)*gamma(-2/3)/(a*n*gamma(1/3)) - 2*b**(2/3)*exp(-I*pi/3)*log(1 -
b**(1/3)*x**(n/3)*exp_polar(I*pi/3)/a**(1/3))*gamma(-2/3)/(3*a**(5/3)*n*gam
ma(1/3)) + 2*b**(2/3)*log(1 - b**(1/3)*x**(n/3)*exp_polar(I*pi)/a**(1/3))*g
amma(-2/3)/(3*a**(5/3)*n*gamma(1/3)) - 2*b**(2/3)*exp(I*pi/3)*log(1 - b**(1
/3)*x**(n/3)*exp_polar(5*I*pi/3)/a**(1/3))*gamma(-2/3)/(3*a**(5/3)*n*gamma(
1/3))
```

3.2641 $\int \frac{x^{-1-\frac{3n}{4}}}{a+bx^n}\,dx$

Optimal. Leaf size=236

$$\frac{b^{3/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^{n/4}+\sqrt{a}+\sqrt{b}\,x^{n/2}\right)}{\sqrt{2}\,a^{7/4}n}-\frac{b^{3/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^{n/4}+\sqrt{a}+\sqrt{b}\,x^{n/2}\right)}{\sqrt{2}\,a^{7/4}n}+\frac{\sqrt{2}\,b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{}}{\sqrt[4]{}}\right.}{a^{7/4}n}$$

```
[Out] -4/3/a/n/(x^(3/4*n))+1/2*b^(3/4)*ln(-a^(1/4)*b^(1/4)*x^(1/4*n)*2^(1/2)+a^(1
/2)+x^(1/2*n)*b^(1/2))/a^(7/4)/n*2^(1/2)-1/2*b^(3/4)*ln(a^(1/4)*b^(1/4)*x^(
1/4*n)*2^(1/2)+a^(1/2)+x^(1/2*n)*b^(1/2))/a^(7/4)/n*2^(1/2)-b^(3/4)*arctan(
-1+b^(1/4)*x^(1/4*n)*2^(1/2)/a^(1/4))*2^(1/2)/a^(7/4)/n-b^(3/4)*arctan(1+b^
(1/4)*x^(1/4*n)*2^(1/2)/a^(1/4))*2^(1/2)/a^(7/4)/n
```

Rubi [A] time = 0.20, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {362, 345, 211, 1165, 628, 1162, 617, 204}

$$\frac{b^{3/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^{n/4}+\sqrt{a}+\sqrt{b}\,x^{n/2}\right)}{\sqrt{2}\,a^{7/4}n}-\frac{b^{3/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^{n/4}+\sqrt{a}+\sqrt{b}\,x^{n/2}\right)}{\sqrt{2}\,a^{7/4}n}+\frac{\sqrt{2}\,b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{}}{\sqrt[4]{}}\right.}{a^{7/4}n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - (3*n)/4)/(a + b*x^n),x]
```

```
[Out] -4/(3*a*n*x^((3*n)/4)) + (Sqrt[2]*b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x^(n/
4))/a^(1/4)])/(a^(7/4)*n) - (Sqrt[2]*b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x^
(n/4))/a^(1/4)])/(a^(7/4)*n) + (b^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/
4)*x^(n/4) + Sqrt[b]*x^(n/2)])/(Sqrt[2]*a^(7/4)*n) - (b^(3/4)*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*b^(1/4)*x^(n/4) + Sqrt[b]*x^(n/2)])/(Sqrt[2]*a^(7/4)*n)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 362

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Dist[b/a, Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a,
 b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1-\frac{3n}{4}}}{a+bx^n}\,dx &= -\frac{4x^{-3n/4}}{3an} - \frac{b\int \frac{x^{\frac{1}{4}(-4+n)}}{a+bx^n}\,dx}{a} \\
&= -\frac{4x^{-3n/4}}{3an} - \frac{(4b)\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, x^{1+\frac{1}{4}(-4+n)}\right)}{an} \\
&= -\frac{4x^{-3n/4}}{3an} - \frac{(2b)\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, x^{1+\frac{1}{4}(-4+n)}\right)}{a^{3/2}n} - \frac{(2b)\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, x^{1+\frac{1}{4}(-4+n)}\right)}{a^{3/2}n} \\
&= -\frac{4x^{-3n/4}}{3an} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, x^{1+\frac{1}{4}(-4+n)}\right)}{a^{3/2}n} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, x^{1+}\right.}{a^{3/2}n} \\
&= -\frac{4x^{-3n/4}}{3an} + \frac{b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^{n/4}+\sqrt{b}\,x^{n/2}\right)}{\sqrt{2}\,a^{7/4}n} - \frac{b^{3/4}\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^{n/4}+\sqrt{b}\,x^{n/2}\right)}{\sqrt{2}\,a^{7/4}n} \\
&= -\frac{4x^{-3n/4}}{3an} + \frac{\sqrt{2}\,b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x^{n/4}}{\sqrt[4]{a}}\right)}{a^{7/4}n} - \frac{\sqrt{2}\,b^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,x^{n/4}}{\sqrt[4]{a}}\right)}{a^{7/4}n} + \frac{b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{}\right.}{\sqrt{2}\,a}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.14

$$-\frac{4x^{-3n/4}\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\frac{bx^n}{a}\right)}{3an}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - (3*n)/4)/(a + b*x^n),x]
```

```
[Out] (-4*Hypergeometric2F1[-3/4, 1, 1/4, -((b*x^n)/a)])/(3*a*n*x^((3*n)/4))
```

fricas [A] time = 1.09, size = 261, normalized size = 1.11

$$\frac{12\,an\left(-\frac{b^3}{a^7n^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{a^2b^2nx^{\frac{1}{3}}x^{-\frac{1}{4}n-\frac{1}{3}}\left(-\frac{b^3}{a^7n^4}\right)^{\frac{1}{4}}-a^2nx^{\frac{1}{3}}\sqrt{-\frac{a^3b^3n^2x^{\frac{1}{3}}\sqrt{-\frac{b^3}{a^7n^4}}-b^4xx^{-\frac{1}{2}n-\frac{2}{3}}}{x}}\left(-\frac{b^3}{a^7n^4}\right)^{\frac{1}{4}}}{b^3}\right)-3\,an\left(-\frac{b^3}{a^7n^4}\right)^{\frac{1}{4}}\log$$

$$-\frac{}{3\,an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3/4*n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] -1/3*(12*a*n*(-b^3/(a^7*n^4))^(1/4)*arctan(-(a^2*b^2*n*x^(1/3)*x^(-1/4*n -
1/3)*(-b^3/(a^7*n^4))^(1/4) - a^2*n*x^(1/3)*sqrt(-(a^3*b^3*n^2*x^(1/3)*sqrt
(-b^3/(a^7*n^4)) - b^4*x*x^(-1/2*n - 2/3))/x)*(-b^3/(a^7*n^4))^(1/4))/b^3)
- 3*a*n*(-b^3/(a^7*n^4))^(1/4)*log((a^5*n^3*x^(2/3)*(-b^3/(a^7*n^4))^(3/4)
+ b^2*x*x^(-1/4*n - 1/3))/x) + 3*a*n*(-b^3/(a^7*n^4))^(1/4)*log(-(a^5*n^3*x
^(2/3)*(-b^3/(a^7*n^4))^(3/4) - b^2*x*x^(-1/4*n - 1/3))/x) + 4*x*x^(-3/4*n
- 1))/(a*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{3}{4}n-1}}{bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3/4*n)/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(-3/4*n - 1)/(b*x^n + a), x)
```

maple [C] time = 0.08, size = 54, normalized size = 0.23

$$\text{RootOf}\left(a^7n^4_Z^4+b^3\right)\ln\left(-\frac{\text{RootOf}\left(a^7n^4_Z^4+b^3\right)a^2n}{b}+x^{\frac{n}{4}}\right)-\frac{4x^{-\frac{3n}{4}}}{3an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-3/4*n)/(b*x^n+a),x)
```

```
[Out] -4/3/a/n/(x^(1/4*n))^3+sum(_R*ln(x^(1/4*n)-_R*a^2/b*n),_R=RootOf(_Z^4*a^7*n
^4+b^3))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-b\int \frac{x^{\frac{1}{4}n}}{abxx^n+a^2x}\,dx-\frac{4}{3\,anx^{\frac{3}{4}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3/4*n)/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] -b*integrate(x^(1/4*n)/(a*b*x*x^n + a^2*x), x) - 4/3/(a*n*x^(3/4*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{\frac{3n}{4}+1}\,(a+b\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^((3*n)/4 + 1)*(a + b*x^n)),x)
```

```
[Out] int(1/(x^((3*n)/4 + 1)*(a + b*x^n)), x)
```

sympy [C] time = 5.73, size = 267, normalized size = 1.13

$$\frac{x^{-\frac{3n}{4}}\Gamma\left(-\frac{3}{4}\right)}{an\Gamma\left(\frac{1}{4}\right)} - \frac{3b^{\frac{3}{4}}e^{-\frac{i\pi}{4}}\log\left(1-\frac{\sqrt[4]{b}x^{\frac{n}{4}}e^{\frac{i\pi}{4}}}{\sqrt[4]{a}}\right)\Gamma\left(-\frac{3}{4}\right)}{4a^{\frac{7}{4}}n\Gamma\left(\frac{1}{4}\right)} + \frac{3ib^{\frac{3}{4}}e^{-\frac{i\pi}{4}}\log\left(1-\frac{\sqrt[4]{b}x^{\frac{n}{4}}e^{\frac{3i\pi}{4}}}{\sqrt[4]{a}}\right)\Gamma\left(-\frac{3}{4}\right)}{4a^{\frac{7}{4}}n\Gamma\left(\frac{1}{4}\right)} + \frac{3b^{\frac{3}{4}}e^{-\frac{i\pi}{4}}\log\left(1-\frac{\sqrt[4]{b}x^{\frac{n}{4}}e^{\frac{5i\pi}{4}}}{\sqrt[4]{a}}\right)}{4a^{\frac{7}{4}}n\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-3/4*n)/(a+b*x**n),x)
```

```
[Out] x**(-3*n/4)*gamma(-3/4)/(a*n*gamma(1/4)) - 3*b**(3/4)*exp(-I*pi/4)*log(1 -
b**(1/4)*x**(n/4)*exp_polar(I*pi/4)/a**(1/4))*gamma(-3/4)/(4*a**(7/4)*n*gam
ma(1/4)) + 3*I*b**(3/4)*exp(-I*pi/4)*log(1 - b**(1/4)*x**(n/4)*exp_polar(3*
I*pi/4)/a**(1/4))*gamma(-3/4)/(4*a**(7/4)*n*gamma(1/4)) + 3*b**(3/4)*exp(-I
*pi/4)*log(1 - b**(1/4)*x**(n/4)*exp_polar(5*I*pi/4)/a**(1/4))*gamma(-3/4)/
(4*a**(7/4)*n*gamma(1/4)) - 3*I*b**(3/4)*exp(-I*pi/4)*log(1 - b**(1/4)*x**(
n/4)*exp_polar(7*I*pi/4)/a**(1/4))*gamma(-3/4)/(4*a**(7/4)*n*gamma(1/4))
```

3.2642 $\int \frac{x^{-1-n}}{a+bx^n}\,dx$

Optimal. Leaf size=38

$$\frac{b\log(a+bx^n)}{a^2n} - \frac{b\log(x)}{a^2} - \frac{x^{-n}}{an}$$

```
[Out] -1/a/n/(x^n)-b*ln(x)/a^2+b*ln(a+b*x^n)/a^2/n
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 44}

$$\frac{b\log(a+bx^n)}{a^2n} - \frac{b\log(x)}{a^2} - \frac{x^{-n}}{an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n)/(a + b*x^n),x]
```

```
[Out] -(1/(a*n*x^n)) - (b*Log[x])/a^2 + (b*Log[a + b*x^n])/(a^2*n)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1-n}}{a+bx^n}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx)}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^2} - \frac{b}{a^2x} + \frac{b^2}{a^2(a+bx)}\right)\,dx, x, x^n\right)}{n}\\ &= -\frac{x^{-n}}{an} - \frac{b\log(x)}{a^2} + \frac{b\log(a+bx^n)}{a^2n}\end{aligned}$$

Mathematica [A] time = 0.03, size = 32, normalized size = 0.84

$$-\frac{-b\log(a+bx^n) + ax^{-n} + bn\log(x)}{a^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n)/(a + b*x^n),x]
```

```
[Out] -((a/x^n + b*n*Log[x] - b*Log[a + b*x^n])/(a^2*n))
```

fricas [A] time = 0.63, size = 37, normalized size = 0.97

$$-\frac{bnx^n\log(x) - bx^n\log(bx^n+a) + a}{a^2nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] -(b*n*x^n*log(x) - b*x^n*log(b*x^n + a) + a)/(a^2*n*x^n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-n-1}}{bx^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(-n - 1)/(b*x^n + a), x)
```

maple [A] time = 0.00, size = 50, normalized size = 1.32

$$\left(-\frac{b\,\mathrm{e}^{n\ln(x)}\ln(x)}{a^2} - \frac{1}{an}\right)\mathrm{e}^{-n\ln(x)} + \frac{b\ln\left(b\,\mathrm{e}^{n\ln(x)} + a\right)}{a^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n)/(b*x^n+a),x)
```

```
[Out] (-1/a^2*b*exp(n*ln(x))*ln(x)-1/a/n)/exp(n*ln(x))+1/a^2*b/n*ln(b*exp(n*ln(x)
)+a)
```

maxima [A] time = 0.61, size = 42, normalized size = 1.11

$$-\frac{b\log(x)}{a^2} + \frac{b\log\left(\frac{bx^n+a}{b}\right)}{a^2 n} - \frac{1}{anx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] -b*log(x)/a^2 + b*log((b*x^n + a)/b)/(a^2*n) - 1/(a*n*x^n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x^{n+1}\,(a + b\,x^n)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(n + 1)*(a + b*x^n)),x)
```

```
[Out] int(1/(x^(n + 1)*(a + b*x^n)), x)
```

sympy [A] time = 25.36, size = 48, normalized size = 1.26

$$\begin{cases} \frac{\log(x)}{b} & \text{for } a = 0 \wedge n = 0 \\ -\frac{x^{-2n}}{2bn} & \text{for } a = 0 \\ \frac{\log(x)}{a+b} & \text{for } n = 0 \\ -\frac{x^{-n}}{an} + \frac{b\log\left(x^{-n}+\frac{b}{a}\right)}{a^2 n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n)/(a+b*x**n),x)
```

```
[Out] Piecewise((log(x)/b, Eq(a, 0) & Eq(n, 0)), (-x**(-2*n)/(2*b*n), Eq(a, 0)),
(log(x)/(a + b), Eq(n, 0)), (-x**(-n)/(a*n) + b*log(x**(-n) + b/a)/(a**2*n)
, True))
```

3.2643 $\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n}\,dx$

Optimal. Leaf size=50

$$\frac{2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,x^{-n/2}}{\sqrt{b}}\right)}{a^{3/2}n}-\frac{2x^{-n/2}}{an}$$

```
[Out] -2/a/n/(x^(1/2*n))+2*arctan(a^(1/2)/(x^(1/2*n))/b^(1/2))*b^(1/2)/a^(3/2)/n
```

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {345, 193, 321, 205}

$$\frac{2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,x^{-n/2}}{\sqrt{b}}\right)}{a^{3/2}n}-\frac{2x^{-n/2}}{an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n/2)/(a + b*x^n),x]
```

```
[Out] -2/(a*n*x^(n/2)) + (2*Sqrt[b]*ArcTan[Sqrt[a]/(Sqrt[b]*x^(n/2))])/(a^(3/2)*n
)
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{1}{a+\frac{b}{x^2}}\,dx, x, x^{-n/2}\right)}{n}\\ &= -\frac{2\,\text{Subst}\left(\int \frac{x^2}{b+ax^2}\,dx, x, x^{-n/2}\right)}{n}\\ &= -\frac{2x^{-n/2}}{an} + \frac{(2b)\,\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, x^{-n/2}\right)}{an}\\ &= -\frac{2x^{-n/2}}{an} + \frac{2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,x^{-n/2}}{\sqrt{b}}\right)}{a^{3/2}n}\end{aligned}$$

Mathematica [C] time = 0.00, size = 32, normalized size = 0.64

$$-\frac{2x^{-n/2}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{bx^n}{a}\right)}{an}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n/2)/(a + b*x^n),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/2, 1, 1/2, -((b*x^n)/a)])/(a*n*x^(n/2))
```

fricas [A] time = 0.68, size = 130, normalized size = 2.60

$$\left[-\frac{2\,xx^{-\frac{1}{2}n-1} - \sqrt{-\frac{b}{a}}\,\log\left(\frac{ax^2x^{-n-2}+2\,axx^{-\frac{1}{2}n-1}\sqrt{-\frac{b}{a}}-b}{ax^2x^{-n-2}+b}\right)}{an}, -\frac{2\left(xx^{-\frac{1}{2}n-1} + \sqrt{\frac{b}{a}}\,\arctan\left(\frac{\sqrt{\frac{b}{a}}}{xx^{-\frac{1}{2}n-1}}\right)\right)}{an}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] [-(2*x*x^(-1/2*n - 1) - sqrt(-b/a)*log((a*x^2*x^(-n - 2) + 2*a*x*x^(-1/2*n
- 1)*sqrt(-b/a) - b)/(a*x^2*x^(-n - 2) + b)))/(a*n), -2*(x*x^(-1/2*n - 1) +
 sqrt(b/a)*arctan(sqrt(b/a)/(x*x^(-1/2*n - 1))))/(a*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{1}{2}n-1}}{bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*n)/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(-1/2*n - 1)/(b*x^n + a), x)
```

maple [A] time = 0.00, size = 79, normalized size = 1.58

$$-\frac{2x^{-\frac{n}{2}}}{an} + \frac{\sqrt{-ab}\,\ln\left(x^{\frac{n}{2}} - \frac{\sqrt{-ab}}{b}\right)}{a^2n} - \frac{\sqrt{-ab}\,\ln\left(x^{\frac{n}{2}} + \frac{\sqrt{-ab}}{b}\right)}{a^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-1/2*n)/(b*x^n+a),x)
```

```
[Out] -2/a/n/(x^(1/2*n))+(-a*b)^(1/2)/a^2/n*ln(x^(1/2*n)-(-a*b)^(1/2)/b)-(-a*b)^(
1/2)/a^2/n*ln(x^(1/2*n)+(-a*b)^(1/2)/b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-b\int \frac{x^{\frac{1}{2}n}}{abxx^n + a^2x}\,dx - \frac{2}{anx^{\frac{1}{2}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*n)/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] -b*integrate(x^(1/2*n)/(a*b*x*x^n + a^2*x), x) - 2/(a*n*x^(1/2*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^{\frac{n}{2}+1}\,(a + b\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(n/2 + 1)*(a + b*x^n)),x)
```

```
[Out] int(1/(x^(n/2 + 1)*(a + b*x^n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-1/2*n)/(a+b*x**n),x)
```

```
[Out] Timed out
```

3.2644 $\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n}\,dx$

Optimal. Leaf size=158

$$-\frac{\sqrt[3]{b}\,\log\left(a^{2/3}x^{-2n/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^{-n/3}+b^{2/3}\right)}{2a^{4/3}n}+\frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{a}\,x^{-n/3}+\sqrt[3]{b}\right)}{a^{4/3}n}-\frac{\sqrt{3}\,\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x^{-n/3}}{\sqrt{3}\,\sqrt[3]{b}}\right)}{a^{4/3}n}-\frac{3x^{-n/3}}{an}$$

```
[Out] -3/a/n/(x^(1/3*n))+b^(1/3)*ln(b^(1/3)+a^(1/3)/(x^(1/3*n)))/a^(4/3)/n-1/2*b^
(1/3)*ln(b^(2/3)+a^(2/3)/(x^(2/3*n))-a^(1/3)*b^(1/3)/(x^(1/3*n)))/a^(4/3)/n
-b^(1/3)*arctan(1/3*(b^(1/3)-2*a^(1/3)/(x^(1/3*n)))/b^(1/3)*3^(1/2))*3^(1/2
)/a^(4/3)/n
```

Rubi [A] time = 0.11, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {345, 193, 321, 200, 31, 634, 617, 204, 628}

$$-\frac{\sqrt[3]{b}\,\log\left(a^{2/3}x^{-2n/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^{-n/3}+b^{2/3}\right)}{2a^{4/3}n}+\frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{a}\,x^{-n/3}+\sqrt[3]{b}\right)}{a^{4/3}n}-\frac{\sqrt{3}\,\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x^{-n/3}}{\sqrt{3}\,\sqrt[3]{b}}\right)}{a^{4/3}n}-\frac{3x^{-n/3}}{an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n/3)/(a + b*x^n),x]
```

```
[Out] -3/(a*n*x^(n/3)) - (Sqrt[3]*b^(1/3)*ArcTan[(b^(1/3) - (2*a^(1/3))/x^(n/3))/
(Sqrt[3]*b^(1/3))])/(a^(4/3)*n) + (b^(1/3)*Log[b^(1/3) + a^(1/3)/x^(n/3)])/
(a^(4/3)*n) - (b^(1/3)*Log[b^(2/3) + a^(2/3)/x^((2*n)/3) - (a^(1/3)*b^(1/3)
)/x^(n/3)])/(2*a^(4/3)*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n}\,dx &= -\frac{3\,\text{Subst}\left(\int \frac{1}{a+\frac{b}{x^3}}\,dx, x, x^{-n/3}\right)}{n} \\
&= -\frac{3\,\text{Subst}\left(\int \frac{x^3}{b+ax^3}\,dx, x, x^{-n/3}\right)}{n} \\
&= -\frac{3x^{-n/3}}{an} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{b+ax^3}\,dx, x, x^{-n/3}\right)}{an} \\
&= -\frac{3x^{-n/3}}{an} + \frac{\sqrt[3]{b}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{a}\,x}\,dx, x, x^{-n/3}\right)}{an} + \frac{\sqrt[3]{b}\,\text{Subst}\left(\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}\,x}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx, x, x^{-n/3}\right)}{an} \\
&= -\frac{3x^{-n/3}}{an} + \frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x^{-n/3}\right)}{a^{4/3}n} - \frac{\sqrt[3]{b}\,\text{Subst}\left(\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2a^{2/3}x}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx, x, x^{-n/3}\right)}{2a^{4/3}n} + \frac{\left(3b^{2/3}\right)\text{Sub} \\
&= -\frac{3x^{-n/3}}{an} + \frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x^{-n/3}\right)}{a^{4/3}n} - \frac{\sqrt[3]{b}\,\log\left(b^{2/3}+a^{2/3}x^{-2n/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^{-n/3}\right)}{2a^{4/3}n} + \frac{\left(3\sqrt[3]{b}\right)\text{Subst} \\
&= -\frac{3x^{-n/3}}{an} - \frac{\sqrt{3}\,\sqrt[3]{b}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}\,x^{-n/3}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{a^{4/3}n} + \frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x^{-n/3}\right)}{a^{4/3}n} - \frac{\sqrt[3]{b}\,\log\left(b^{2/3}+a^{2/3}x^{-2n/3}-\sqrt[3]{}\right.}{2a^{4/3}n}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.20

$$-\frac{3x^{-n/3}\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};-\frac{bx^n}{a}\right)}{an}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n/3)/(a + b*x^n),x]
```

```
[Out] (-3*Hypergeometric2F1[-1/3, 1, 2/3, -((b*x^n)/a)])/(a*n*x^(n/3))
```

fricas [A] time = 0.79, size = 145, normalized size = 0.92

$$-\frac{6\,xx^{-\frac{1}{3}n-1} - 2\sqrt{3}\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,axx^{-\frac{1}{3}n-1}\left(\frac{b}{a}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3\,b}\right) - 2\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(\frac{xx^{-\frac{1}{3}n-1}+\left(\frac{b}{a}\right)^{\frac{1}{3}}}{x}\right) + \left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(\frac{x^2x^{-\frac{2}{3}n-2}-xx^{-\frac{1}{3}}}{x}\right.}{2\,an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/3*n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] -1/2*(6*x*x^(-1/3*n - 1) - 2*sqrt(3)*(b/a)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*
x^(-1/3*n - 1)*(b/a)^(2/3) - sqrt(3)*b)/b) - 2*(b/a)^(1/3)*log((x*x^(-1/3*n
 - 1) + (b/a)^(1/3))/x) + (b/a)^(1/3)*log((x^2*x^(-2/3*n - 2) - x*x^(-1/3*n
 - 1)*(b/a)^(1/3) + (b/a)^(2/3))/x^2))/(a*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{1}{3}n-1}}{bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/3*n)/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(-1/3*n - 1)/(b*x^n + a), x)
```

maple [C] time = 0.07, size = 57, normalized size = 0.36

$$\text{RootOf}\left(a^4n^3_Z^3 - b\right)\ln\left(\frac{\text{RootOf}\left(a^4n^3_Z^3 - b\right)^2 a^3n^2}{b} + x^{\frac{n}{3}}\right) - \frac{3x^{-\frac{n}{3}}}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-1/3*n)/(b*x^n+a),x)
```

```
[Out] -3/a/n/(x^(1/3*n))+sum(_R*ln(x^(1/3*n)+a^3*n^2/b*_R^2),_R=RootOf(_Z^3*a^4*n
^3-b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-b\int \frac{x^{\frac{2}{3}n}}{abxx^n + a^2x}\,dx - \frac{3}{anx^{\frac{1}{3}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/3*n)/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] -b*integrate(x^(2/3*n)/(a*b*x*x^n + a^2*x), x) - 3/(a*n*x^(1/3*n))
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{\frac{n}{3}+1}\,(a+b\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(n/3 + 1)*(a + b*x^n)),x)
```

```
[Out] int(1/(x^(n/3 + 1)*(a + b*x^n)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-1/3*n)/(a+b*x**n),x)
```

```
[Out] Timed out
```

3.2645 $\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n}\,dx$

Optimal. Leaf size=234

$$-\frac{\sqrt[4]{b}\,\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^{-n/4}+\sqrt{a}\,x^{-n/2}+\sqrt{b}\right)}{\sqrt{2}\,a^{5/4}n}+\frac{\sqrt[4]{b}\,\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^{-n/4}+\sqrt{a}\,x^{-n/2}+\sqrt{b}\right)}{\sqrt{2}\,a^{5/4}n}-\frac{\sqrt{2}\,\sqrt[4]{b}\,\tan^{-1}\left(1-\right.}{a^{5/4}n}$$

```
[Out] -4/a/n/(x^(1/4*n))-1/2*b^(1/4)*ln(-a^(1/4)*b^(1/4)*2^(1/2)/(x^(1/4*n))+a^(1
/2)/(x^(1/2*n))+b^(1/2))/a^(5/4)/n*2^(1/2)+1/2*b^(1/4)*ln(a^(1/4)*b^(1/4)*2
^(1/2)/(x^(1/4*n))+a^(1/2)/(x^(1/2*n))+b^(1/2))/a^(5/4)/n*2^(1/2)-b^(1/4)*a
rctan(1-a^(1/4)*2^(1/2)/b^(1/4)/(x^(1/4*n)))*2^(1/2)/a^(5/4)/n+b^(1/4)*arct
an(1+a^(1/4)*2^(1/2)/b^(1/4)/(x^(1/4*n)))*2^(1/2)/a^(5/4)/n
```

Rubi [A] time = 0.18, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {345, 193, 321, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\sqrt[4]{b}\,\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^{-n/4}+\sqrt{a}\,x^{-n/2}+\sqrt{b}\right)}{\sqrt{2}\,a^{5/4}n}+\frac{\sqrt[4]{b}\,\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^{-n/4}+\sqrt{a}\,x^{-n/2}+\sqrt{b}\right)}{\sqrt{2}\,a^{5/4}n}-\frac{\sqrt{2}\,\sqrt[4]{b}\,\tan^{-1}\left(1-\right.}{a^{5/4}n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n/4)/(a + b*x^n),x]
```

```
[Out] -4/(a*n*x^(n/4)) - (Sqrt[2]*b^(1/4)*ArcTan[1 - (Sqrt[2]*a^(1/4))/(b^(1/4)*x
^(n/4))])/(a^(5/4)*n) + (Sqrt[2]*b^(1/4)*ArcTan[1 + (Sqrt[2]*a^(1/4))/(b^(1
/4)*x^(n/4))])/(a^(5/4)*n) - (b^(1/4)*Log[Sqrt[b] + Sqrt[a]/x^(n/2) - (Sqrt
[2]*a^(1/4)*b^(1/4))/x^(n/4)])/(Sqrt[2]*a^(5/4)*n) + (b^(1/4)*Log[Sqrt[b] +
 Sqrt[a]/x^(n/2) + (Sqrt[2]*a^(1/4)*b^(1/4))/x^(n/4)])/(Sqrt[2]*a^(5/4)*n)
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n}\,dx = -\frac{4\,\text{Subst}\left(\int \frac{1}{a+\frac{b}{x^4}}\,dx, x, x^{-n/4}\right)}{n}$$
$$= -\frac{4\,\text{Subst}\left(\int \frac{x^4}{b+ax^4}\,dx, x, x^{-n/4}\right)}{n}$$
$$= -\frac{4x^{-n/4}}{an} + \frac{(4b)\,\text{Subst}\left(\int \frac{1}{b+ax^4}\,dx, x, x^{-n/4}\right)}{an}$$
$$= -\frac{4x^{-n/4}}{an} + \frac{\left(2\sqrt{b}\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{a}\,x^2}{b+ax^4}\,dx, x, x^{-n/4}\right)}{an} + \frac{\left(2\sqrt{b}\right)\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{a}\,x^2}{b+ax^4}\,dx, x, x^{-n/4}\right)}{an}$$
$$= -\frac{4x^{-n/4}}{an} - \frac{\sqrt[4]{b}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\,\sqrt[4]{b}}{\sqrt[4]{a}}+2x}{-\frac{\sqrt{b}}{\sqrt{a}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}-x^2}\,dx, x, x^{-n/4}\right)}{\sqrt{2}\,a^{5/4}n} - \frac{\sqrt[4]{b}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\,\sqrt[4]{b}}{\sqrt[4]{a}}-2x}{-\frac{\sqrt{b}}{\sqrt{a}}+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}-x^2}\,dx, x, x^{-n/4}\right)}{\sqrt{2}\,a^{5/4}n}$$
$$= -\frac{4x^{-n/4}}{an} - \frac{\sqrt[4]{b}\,\log\left(\sqrt{b}+\sqrt{a}\,x^{-n/2}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^{-n/4}\right)}{\sqrt{2}\,a^{5/4}n} + \frac{\sqrt[4]{b}\,\log\left(\sqrt{b}+\sqrt{a}\,x^{-n/2}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\right.}{\sqrt{2}\,a^{5/4}n}$$
$$= -\frac{4x^{-n/4}}{an} - \frac{\sqrt{2}\,\sqrt[4]{b}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{a}\,x^{-n/4}}{\sqrt[4]{b}}\right)}{a^{5/4}n} + \frac{\sqrt{2}\,\sqrt[4]{b}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{a}\,x^{-n/4}}{\sqrt[4]{b}}\right)}{a^{5/4}n} - \frac{\sqrt[4]{b}\,\log\left(\sqrt{b}+\sqrt{\right.}}{}$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.14

$$-\frac{4x^{-n/4}\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\frac{bx^n}{a}\right)}{an}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n/4)/(a + b*x^n),x]
```

```
[Out] (-4*Hypergeometric2F1[-1/4, 1, 3/4, -((b*x^n)/a)])/(a*n*x^(n/4))
```

fricas [A] time = 0.65, size = 211, normalized size = 0.90

$$\frac{4\,an\left(-\frac{b}{a^5n^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{a^4n^3xx^{-\frac{1}{4}n-1}\left(-\frac{b}{a^5n^4}\right)^{\frac{3}{4}}-a^4n^3x\sqrt{\frac{a^2n^2\sqrt{-\frac{b}{a^5n^4}}+x^2x^{-\frac{1}{2}n-2}}{x^2}}\left(-\frac{b}{a^5n^4}\right)^{\frac{3}{4}}}{b}\right)+an\left(-\frac{b}{a^5n^4}\right)^{\frac{1}{4}}\log\left(\frac{an\left(-\frac{b}{a^5n^4}\right)^{\frac{1}{4}}+x}{x}\right.}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/4*n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] (4*a*n*(-b/(a^5*n^4))^(1/4)*arctan(-(a^4*n^3*x*x^(-1/4*n - 1)*(-b/(a^5*n^4)
)^(3/4) - a^4*n^3*x*sqrt((a^2*n^2*sqrt(-b/(a^5*n^4)) + x^2*x^(-1/2*n - 2))/
x^2)*(-b/(a^5*n^4))^(3/4))/b) + a*n*(-b/(a^5*n^4))^(1/4)*log((a*n*(-b/(a^5*
n^4))^(1/4) + x*x^(-1/4*n - 1))/x) - a*n*(-b/(a^5*n^4))^(1/4)*log(-(a*n*(-b
/(a^5*n^4))^(1/4) - x*x^(-1/4*n - 1))/x) - 4*x*x^(-1/4*n - 1))/(a*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{1}{4}n-1}}{bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/4*n)/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(-1/4*n - 1)/(b*x^n + a), x)
```

maple [C] time = 0.08, size = 56, normalized size = 0.24

$$\text{RootOf}\left(a^5n^4_Z^4+b\right)\ln\left(-\frac{\text{RootOf}\left(a^5n^4_Z^4+b\right)^3a^4n^3}{b}+x^{\frac{n}{4}}\right)-\frac{4x^{-\frac{n}{4}}}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-1/4*n)/(b*x^n+a),x)
```

```
[Out] -4/a/n/(x^(1/4*n))+sum(_R*ln(x^(1/4*n)-a^4*n^3/b*_R^3),_R=RootOf(_Z^4*a^5*n
^4+b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-b\int\frac{x^{\frac{3}{4}n}}{abxx^n+a^2x}\,dx-\frac{4}{anx^{\frac{1}{4}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/4*n)/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] -b*integrate(x^(3/4*n)/(a*b*x*x^n + a^2*x), x) - 4/(a*n*x^(1/4*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{1}{x^{\frac{n}{4}+1}\,(a+b\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(n/4 + 1)*(a + b*x^n)),x)
```

```
[Out] int(1/(x^(n/4 + 1)*(a + b*x^n)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-1/4*n)/(a+b*x**n),x)
```

```
[Out] Timed out
```

3.2646 $\int \frac{x^{-1-\frac{3n}{2}}}{a+bx^n}\,dx$

Optimal. Leaf size=68

$$-\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,x^{-n/2}}{\sqrt{b}}\right)}{a^{5/2}n}+\frac{2bx^{-n/2}}{a^2n}-\frac{2x^{-3n/2}}{3an}$$

```
[Out] -2/3/a/n/(x^(3/2*n))+2*b/a^2/n/(x^(1/2*n))-2*b^(3/2)*arctan(a^(1/2)/(x^(1/2
*n))/b^(1/2))/a^(5/2)/n
```

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {362, 345, 193, 321, 205}

$$-\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,x^{-n/2}}{\sqrt{b}}\right)}{a^{5/2}n}+\frac{2bx^{-n/2}}{a^2n}-\frac{2x^{-3n/2}}{3an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - (3*n)/2)/(a + b*x^n),x]
```

```
[Out] -2/(3*a*n*x^((3*n)/2)) + (2*b)/(a^2*n*x^(n/2)) - (2*b^(3/2)*ArcTan[Sqrt[a]/
(Sqrt[b]*x^(n/2))])/(a^(5/2)*n)
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 362

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Dist[b/a, Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a,
 b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1-\frac{3n}{2}}}{a+bx^n}\,dx &= -\frac{2x^{-3n/2}}{3an} - \frac{b\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n}\,dx}{a}\\ &= -\frac{2x^{-3n/2}}{3an} + \frac{(2b)\,\text{Subst}\left(\int \frac{1}{a+\frac{b}{x^2}}\,dx, x, x^{-n/2}\right)}{an}\\ &= -\frac{2x^{-3n/2}}{3an} + \frac{(2b)\,\text{Subst}\left(\int \frac{x^2}{b+ax^2}\,dx, x, x^{-n/2}\right)}{an}\\ &= -\frac{2x^{-3n/2}}{3an} + \frac{2bx^{-n/2}}{a^2n} - \frac{\left(2b^2\right)\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, x^{-n/2}\right)}{a^2n}\\ &= -\frac{2x^{-3n/2}}{3an} + \frac{2bx^{-n/2}}{a^2n} - \frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,x^{-n/2}}{\sqrt{b}}\right)}{a^{5/2}n}\end{aligned}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.50

$$-\frac{2x^{-3n/2}\,{}_2F_1\left(-\frac{3}{2}, 1; -\frac{1}{2}; -\frac{bx^n}{a}\right)}{3an}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - (3*n)/2)/(a + b*x^n),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/2, 1, -1/2, -((b*x^n)/a)])/(3*a*n*x^((3*n)/2))
```

fricas [A] time = 0.73, size = 251, normalized size = 3.69

$$\left[-\frac{2\,axx^{-\frac{3}{2}n-1} - 3\,b\sqrt{-\frac{b}{a}}\,\log\left(-\frac{2\,a^3x^{\frac{5}{3}}x^{-\frac{5}{2}n-\frac{5}{3}}\sqrt{-\frac{b}{a}} - a^3x^2x^{-3n-2} - 2\,a^2bxx^{-\frac{3}{2}n-1}\sqrt{-\frac{b}{a}} + 2\,a^2bx^{\frac{4}{3}}x^{-2n-\frac{4}{3}} + 2\,ab^2x^{\frac{1}{3}}x^{-\frac{1}{2}n-\frac{1}{3}}\sqrt{-\frac{b}{a}} - 2\,ab^2x^{\frac{2}{3}}x^{\right.}}{3\,a^2n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3/2*n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] [-1/3*(2*a*x*x^(-3/2*n - 1) - 3*b*sqrt(-b/a)*log(-(2*a^3*x^(5/3)*x^(-5/2*n
- 5/3)*sqrt(-b/a) - a^3*x^2*x^(-3*n - 2) - 2*a^2*b*x*x^(-3/2*n - 1)*sqrt(-b
/a) + 2*a^2*b*x^(4/3)*x^(-2*n - 4/3) + 2*a*b^2*x^(1/3)*x^(-1/2*n - 1/3)*sqr
t(-b/a) - 2*a*b^2*x^(2/3)*x^(-n - 2/3) + b^3)/(a^3*x^2*x^(-3*n - 2) + b^3))
 - 6*b*x^(1/3)*x^(-1/2*n - 1/3))/(a^2*n), -2/3*(a*x*x^(-3/2*n - 1) - 3*b*sq
rt(b/a)*arctan(sqrt(b/a)/(x^(1/3)*x^(-1/2*n - 1/3))) - 3*b*x^(1/3)*x^(-1/2*
n - 1/3))/(a^2*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{3}{2}n-1}}{bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3/2*n)/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(-3/2*n - 1)/(b*x^n + a), x)
```

maple [A] time = 0.09, size = 97, normalized size = 1.43

$$-\frac{2x^{-\frac{3n}{2}}}{3an}+\frac{2b\,x^{-\frac{n}{2}}}{a^2n}-\frac{\sqrt{-ab}\;b\ln\left(x^{\frac{n}{2}}-\frac{\sqrt{-ab}}{b}\right)}{a^3n}+\frac{\sqrt{-ab}\;b\ln\left(x^{\frac{n}{2}}+\frac{\sqrt{-ab}}{b}\right)}{a^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-3/2*n)/(b*x^n+a),x)
```

```
[Out] 2*b/a^2/n/(x^(1/2*n))-2/3/a/n/(x^(1/2*n))^3+(-a*b)^(1/2)/a^3*b/n*ln(x^(1/2*
n)+(-a*b)^(1/2)/b)-(-a*b)^(1/2)/a^3*b/n*ln(x^(1/2*n)-(-a*b)^(1/2)/b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^2\int\frac{x^{\frac{1}{2}n}}{a^2bxx^n+a^3x}\,dx+\frac{2\,(3\,bx^n-a)}{3\,a^2nx^{\frac{3}{2}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3/2*n)/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] b^2*integrate(x^(1/2*n)/(a^2*b*x*x^n + a^3*x), x) + 2/3*(3*b*x^n - a)/(a^2*
n*x^(3/2*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{x^{\frac{3n}{2}+1}\,(a+b\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^((3*n)/2 + 1)*(a + b*x^n)),x)
```

```
[Out] int(1/(x^((3*n)/2 + 1)*(a + b*x^n)), x)
```

sympy [A] time = 4.66, size = 56, normalized size = 0.82

$$-\frac{2x^{-\frac{3n}{2}}}{3an}+\frac{2bx^{-\frac{n}{2}}}{a^2n}+\frac{2b^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{b}\,x^{\frac{n}{2}}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-3/2*n)/(a+b*x**n),x)
```

```
[Out] -2*x**(-3*n/2)/(3*a*n) + 2*b*x**(-n/2)/(a**2*n) + 2*b**(3/2)*atan(sqrt(b)*x
**(n/2)/sqrt(a))/(a**(5/2)*n)
```

3.2647 $\int \frac{x^{-1-\frac{4n}{3}}}{a+bx^n}\,dx$

Optimal. Leaf size=176

$$-\frac{b^{4/3}\log\left(\sqrt[3]{a}\,x^{-n/3}+\sqrt[3]{b}\right)}{a^{7/3}n}+\frac{b^{4/3}\log\left(a^{2/3}x^{-2n/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^{-n/3}+b^{2/3}\right)}{2a^{7/3}n}+\frac{\sqrt{3}\,b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x^{-n/3}}{\sqrt{3}\,\sqrt[3]{b}}\right)}{a^{7/3}n}+\frac{3bx^{-n/3}}{a^2n}-\frac{3x}{4}$$

```
[Out] -3/4/a/n/(x^(4/3*n))+3*b/a^2/n/(x^(1/3*n))-b^(4/3)*ln(b^(1/3)+a^(1/3)/(x^(1
/3*n)))/a^(7/3)/n+1/2*b^(4/3)*ln(b^(2/3)+a^(2/3)/(x^(2/3*n))-a^(1/3)*b^(1/3
)/(x^(1/3*n)))/a^(7/3)/n+b^(4/3)*arctan(1/3*(1-2*a^(1/3)/b^(1/3)/(x^(1/3*n)
))*3^(1/2))*3^(1/2)/a^(7/3)/n
```

Rubi [A] time = 0.11, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {362, 345, 193, 321, 200, 31, 634, 617, 204, 628}

$$-\frac{b^{4/3}\log\left(\sqrt[3]{a}\,x^{-n/3}+\sqrt[3]{b}\right)}{a^{7/3}n}+\frac{b^{4/3}\log\left(a^{2/3}x^{-2n/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^{-n/3}+b^{2/3}\right)}{2a^{7/3}n}+\frac{\sqrt{3}\,b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\,x^{-n/3}}{\sqrt{3}\,\sqrt[3]{b}}\right)}{a^{7/3}n}+\frac{3bx^{-n/3}}{a^2n}-\frac{3x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - (4*n)/3)/(a + b*x^n),x]
```

```
[Out] -3/(4*a*n*x^((4*n)/3)) + (3*b)/(a^2*n*x^(n/3)) + (Sqrt[3]*b^(4/3)*ArcTan[(b
^(1/3) - (2*a^(1/3))/x^(n/3))/(Sqrt[3]*b^(1/3))])/(a^(7/3)*n) - (b^(4/3)*Lo
g[b^(1/3) + a^(1/3)/x^(n/3)])/(a^(7/3)*n) + (b^(4/3)*Log[b^(2/3) + a^(2/3)/
x^((2*n)/3) - (a^(1/3)*b^(1/3))/x^(n/3)])/(2*a^(7/3)*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 362

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Dist[b/a, Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a,
 b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^{-1-\frac{4n}{3}}}{a+bx^n}\,dx = -\frac{3x^{-4n/3}}{4an} - \frac{b\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n}\,dx}{a}$$

$$= -\frac{3x^{-4n/3}}{4an} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{a+\frac{b}{x^3}}\,dx, x, x^{-n/3}\right)}{an}$$

$$= -\frac{3x^{-4n/3}}{4an} + \frac{(3b)\,\text{Subst}\left(\int \frac{x^3}{b+ax^3}\,dx, x, x^{-n/3}\right)}{an}$$

$$= -\frac{3x^{-4n/3}}{4an} + \frac{3bx^{-n/3}}{a^2n} - \frac{\left(3b^2\right)\text{Subst}\left(\int \frac{1}{b+ax^3}\,dx, x, x^{-n/3}\right)}{a^2n}$$

$$= -\frac{3x^{-4n/3}}{4an} + \frac{3bx^{-n/3}}{a^2n} - \frac{b^{4/3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{a}\,x}\,dx, x, x^{-n/3}\right)}{a^2n} - \frac{b^{4/3}\,\text{Subst}\left(\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}\,x}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx, x,\right.}{a^2n}$$

$$= -\frac{3x^{-4n/3}}{4an} + \frac{3bx^{-n/3}}{a^2n} - \frac{b^{4/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x^{-n/3}\right)}{a^{7/3}n} + \frac{b^{4/3}\,\text{Subst}\left(\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2a^{2/3}x}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx, x, x^{-n/3}\right)}{2a^{7/3}n} -$$

$$= -\frac{3x^{-4n/3}}{4an} + \frac{3bx^{-n/3}}{a^2n} - \frac{b^{4/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x^{-n/3}\right)}{a^{7/3}n} + \frac{b^{4/3}\log\left(b^{2/3}+a^{2/3}x^{-2n/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^{-n/3}\right)}{2a^{7/3}n} - \frac{\left(3\right.}{}$$

$$= -\frac{3x^{-4n/3}}{4an} + \frac{3bx^{-n/3}}{a^2n} + \frac{\sqrt{3}\,b^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}\,x^{-n/3}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{a^{7/3}n} - \frac{b^{4/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x^{-n/3}\right)}{a^{7/3}n} + \frac{b^{4/3}\log\left(b^{2/3}+a\right.}{}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.19

$$-\frac{3x^{-4n/3}\,{}_2F_1\left(-\frac{4}{3},1;-\frac{1}{3};-\frac{bx^n}{a}\right)}{4an}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - (4*n)/3)/(a + b*x^n),x]
```

```
[Out] (-3*Hypergeometric2F1[-4/3, 1, -1/3, -((b*x^n)/a)])/(4*a*n*x^((4*n)/3))
```

fricas [A] time = 0.76, size = 180, normalized size = 1.02

$$-\frac{3\,axx^{-\frac{4}{3}n-1} - 4\sqrt{3}\,b\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,ax^{\frac{1}{4}}x^{-\frac{1}{3}n-\frac{1}{4}}\left(-\frac{b}{a}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3\,b}\right) + 2\,b\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(\frac{x^{\frac{3}{4}}x^{-\frac{1}{3}n-\frac{1}{4}}\left(-\frac{b}{a}\right)^{\frac{1}{3}}+xx^{-\frac{2}{3}n-\frac{1}{2}}+\sqrt{x}\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right.}{x}}{4\,a^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-4/3*n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] -1/4*(3*a*x*x^(-4/3*n - 1) - 4*sqrt(3)*b*(-b/a)^(1/3)*arctan(1/3*(2*sqrt(3)
*a*x^(1/4)*x^(-1/3*n - 1/4)*(-b/a)^(2/3) - sqrt(3)*b)/b) + 2*b*(-b/a)^(1/3)
*log((x^(3/4)*x^(-1/3*n - 1/4)*(-b/a)^(1/3) + x*x^(-2/3*n - 1/2) + sqrt(x)*
(-b/a)^(2/3))/x) - 4*b*(-b/a)^(1/3)*log((x*x^(-1/3*n - 1/4) - x^(3/4)*(-b/a
)^(1/3))/x) - 12*b*x^(1/4)*x^(-1/3*n - 1/4))/(a^2*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{4}{3}n-1}}{bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-4/3*n)/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(-4/3*n - 1)/(b*x^n + a), x)
```

maple [C] time = 0.08, size = 73, normalized size = 0.41

$$\text{RootOf}\left(a^7n^3_Z^3+b^4\right)\ln\left(\frac{\text{RootOf}\left(a^7n^3_Z^3+b^4\right)^2a^5n^2}{b^3}+x^{\frac{n}{3}}\right)-\frac{3x^{-\frac{4n}{3}}}{4an}+\frac{3bx^{-\frac{n}{3}}}{a^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-4/3*n)/(b*x^n+a),x)
```

```
[Out] 3*b/a^2/n/(x^(1/3*n))-3/4/a/n/(x^(1/3*n))^4+sum(_R*ln(x^(1/3*n)+a^5*n^2/b^3
*_R^2),_R=RootOf(_Z^3*a^7*n^3+b^4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^2\int\frac{x^{\frac{2}{3}n}}{a^2bxx^n+a^3x}\,dx+\frac{3\left(4\,bx^n-a\right)}{4\,a^2nx^{\frac{4}{3}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-4/3*n)/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] b^2*integrate(x^(2/3*n)/(a^2*b*x*x^n + a^3*x), x) + 3/4*(4*b*x^n - a)/(a^2*
n*x^(4/3*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{x^{\frac{4n}{3}+1}\left(a+b\,x^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^((4*n)/3 + 1)*(a + b*x^n)),x)
```

```
[Out] int(1/(x^((4*n)/3 + 1)*(a + b*x^n)), x)
```

sympy [C] time = 4.34, size = 230, normalized size = 1.31

$$\frac{x^{-\frac{4n}{3}}\Gamma\left(-\frac{4}{3}\right)}{an\Gamma\left(-\frac{1}{3}\right)}-\frac{4bx^{-\frac{n}{3}}\Gamma\left(-\frac{4}{3}\right)}{a^2n\Gamma\left(-\frac{1}{3}\right)}+\frac{4b^{\frac{4}{3}}e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{n}{3}}e^{\frac{i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{4}{3}\right)}{3a^{\frac{7}{3}}n\Gamma\left(-\frac{1}{3}\right)}+\frac{4b^{\frac{4}{3}}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{n}{3}}e^{i\pi}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{4}{3}\right)}{3a^{\frac{7}{3}}n\Gamma\left(-\frac{1}{3}\right)}+\frac{4b^{\frac{4}{3}}e^{\frac{2i\pi}{3}}\log\left(1\right.}{3a^{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-4/3*n)/(a+b*x**n),x)
```

```
[Out] x**(-4*n/3)*gamma(-4/3)/(a*n*gamma(-1/3)) - 4*b*x**(-n/3)*gamma(-4/3)/(a**2
*n*gamma(-1/3)) + 4*b**(4/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*x**(n/3)*exp_p
olar(I*pi/3)/a**(1/3))*gamma(-4/3)/(3*a**(7/3)*n*gamma(-1/3)) + 4*b**(4/3)*
log(1 - b**(1/3)*x**(n/3)*exp_polar(I*pi)/a**(1/3))*gamma(-4/3)/(3*a**(7/3)
*n*gamma(-1/3)) + 4*b**(4/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*x**(n/3)*exp_po
lar(5*I*pi/3)/a**(1/3))*gamma(-4/3)/(3*a**(7/3)*n*gamma(-1/3))
```

3.2648 $\int \frac{x^{-1-\frac{5n}{4}}}{a+bx^n}\,dx$

Optimal. Leaf size=252

$$\frac{b^{5/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^{-n/4}+\sqrt{a}\,x^{-n/2}+\sqrt{b}\right)}{\sqrt{2}\,a^{9/4}n}-\frac{b^{5/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^{-n/4}+\sqrt{a}\,x^{-n/2}+\sqrt{b}\right)}{\sqrt{2}\,a^{9/4}n}+\frac{\sqrt{2}\,b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{a^{9/4}n}$$

```
[Out] -4/5/a/n/(x^(5/4*n))+4*b/a^2/n/(x^(1/4*n))+1/2*b^(5/4)*ln(-a^(1/4)*b^(1/4)*
2^(1/2)/(x^(1/4*n))+a^(1/2)/(x^(1/2*n))+b^(1/2))/a^(9/4)/n*2^(1/2)-1/2*b^(5
/4)*ln(a^(1/4)*b^(1/4)*2^(1/2)/(x^(1/4*n))+a^(1/2)/(x^(1/2*n))+b^(1/2))/a^(
9/4)/n*2^(1/2)+b^(5/4)*arctan(1-a^(1/4)*2^(1/2)/b^(1/4)/(x^(1/4*n)))*2^(1/2
)/a^(9/4)/n-b^(5/4)*arctan(1+a^(1/4)*2^(1/2)/b^(1/4)/(x^(1/4*n)))*2^(1/2)/a
^(9/4)/n
```

Rubi [A] time = 0.20, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {362, 345, 193, 321, 211, 1165, 628, 1162, 617, 204}

$$\frac{b^{5/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^{-n/4}+\sqrt{a}\,x^{-n/2}+\sqrt{b}\right)}{\sqrt{2}\,a^{9/4}n}-\frac{b^{5/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^{-n/4}+\sqrt{a}\,x^{-n/2}+\sqrt{b}\right)}{\sqrt{2}\,a^{9/4}n}+\frac{\sqrt{2}\,b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{a^{9/4}n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - (5*n)/4)/(a + b*x^n),x]
```

```
[Out] -4/(5*a*n*x^((5*n)/4)) + (4*b)/(a^2*n*x^(n/4)) + (Sqrt[2]*b^(5/4)*ArcTan[1
- (Sqrt[2]*a^(1/4))/(b^(1/4)*x^(n/4))])/(a^(9/4)*n) - (Sqrt[2]*b^(5/4)*ArcT
an[1 + (Sqrt[2]*a^(1/4))/(b^(1/4)*x^(n/4))])/(a^(9/4)*n) + (b^(5/4)*Log[Sqr
t[b] + Sqrt[a]/x^(n/2) - (Sqrt[2]*a^(1/4)*b^(1/4))/x^(n/4)])/(Sqrt[2]*a^(9/
4)*n) - (b^(5/4)*Log[Sqrt[b] + Sqrt[a]/x^(n/2) + (Sqrt[2]*a^(1/4)*b^(1/4))/
x^(n/4)])/(Sqrt[2]*a^(9/4)*n)
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 362

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Dist[b/a, Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a,
 b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{-1-\frac{5n}{4}}}{a+bx^n}\,dx = -\frac{4x^{-5n/4}}{5an} - \frac{b\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n}\,dx}{a}$$

$$= -\frac{4x^{-5n/4}}{5an} + \frac{(4b)\,\text{Subst}\left(\int \frac{1}{a+\frac{b}{x^4}}\,dx, x, x^{-n/4}\right)}{an}$$

$$= -\frac{4x^{-5n/4}}{5an} + \frac{(4b)\,\text{Subst}\left(\int \frac{x^4}{b+ax^4}\,dx, x, x^{-n/4}\right)}{an}$$

$$= -\frac{4x^{-5n/4}}{5an} + \frac{4bx^{-n/4}}{a^2n} - \frac{\left(4b^2\right)\text{Subst}\left(\int \frac{1}{b+ax^4}\,dx, x, x^{-n/4}\right)}{a^2n}$$

$$= -\frac{4x^{-5n/4}}{5an} + \frac{4bx^{-n/4}}{a^2n} - \frac{\left(2b^{3/2}\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{a}\,x^2}{b+ax^4}\,dx, x, x^{-n/4}\right)}{a^2n} - \frac{\left(2b^{3/2}\right)\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{a}\,x^2}{b+ax^4}\,dx, x, x\right.}{a^2n}$$

$$= -\frac{4x^{-5n/4}}{5an} + \frac{4bx^{-n/4}}{a^2n} + \frac{b^{5/4}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\,\sqrt[4]{b}}{\sqrt[4]{a}}+2x}{-\frac{\sqrt{b}}{\sqrt{a}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}-x^2}\,dx, x, x^{-n/4}\right)}{\sqrt{2}\,a^{9/4}n} + \frac{b^{5/4}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\,\sqrt[4]{b}}{\sqrt[4]{a}}-2x}{-\frac{\sqrt{b}}{\sqrt{a}}+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}-x^2}\,d\right.}{\sqrt{2}\,a^{9/4}n}$$

$$= -\frac{4x^{-5n/4}}{5an} + \frac{4bx^{-n/4}}{a^2n} + \frac{b^{5/4}\log\left(\sqrt{b}+\sqrt{a}\,x^{-n/2}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^{-n/4}\right)}{\sqrt{2}\,a^{9/4}n} - \frac{b^{5/4}\log\left(\sqrt{b}+\sqrt{a}\,x^{-n/2}+\sqrt{\right.}}{\sqrt{2}\,a^{9/4}n}$$

$$= -\frac{4x^{-5n/4}}{5an} + \frac{4bx^{-n/4}}{a^2n} + \frac{\sqrt{2}\,b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{a}\,x^{-n/4}}{\sqrt[4]{b}}\right)}{a^{9/4}n} - \frac{\sqrt{2}\,b^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{a}\,x^{-n/4}}{\sqrt[4]{b}}\right)}{a^{9/4}n} + \frac{b^{5/4}\log}{}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.13

$$-\frac{4x^{-5n/4}\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\frac{bx^n}{a}\right)}{5an}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - (5*n)/4)/(a + b*x^n),x]
```

```
[Out] (-4*Hypergeometric2F1[-5/4, 1, -1/4, -((b*x^n)/a)])/(5*a*n*x^((5*n)/4))
```

fricas [A] time = 1.15, size = 270, normalized size = 1.07

$$-\frac{20\,a^2n\left(-\frac{b^5}{a^9n^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{a^7bn^3x^{\frac{1}{5}}x^{-\frac{1}{4}n-\frac{1}{5}}\left(-\frac{b^5}{a^9n^4}\right)^{\frac{3}{4}}-a^7n^3x^{\frac{1}{5}}\sqrt{\frac{a^4n^2x^{\frac{3}{5}}\sqrt{-\frac{b^5}{a^9n^4}}+b^2xx^{-\frac{1}{2}n-\frac{2}{5}}}{x}}\left(-\frac{b^5}{a^9n^4}\right)^{\frac{3}{4}}}{b^5}\right)+5\,a^2n\left(-\frac{b^5}{a^9n^4}\right)^{\frac{1}{4}}\log\left(\right.}{5\,a^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-5/4*n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] -1/5*(20*a^2*n*(-b^5/(a^9*n^4))^(1/4)*arctan(-(a^7*b*n^3*x^(1/5)*x^(-1/4*n
- 1/5)*(-b^5/(a^9*n^4))^(3/4) - a^7*n^3*x^(1/5)*sqrt((a^4*n^2*x^(3/5)*sqrt(
-b^5/(a^9*n^4)) + b^2*x*x^(-1/2*n - 2/5))/x)*(-b^5/(a^9*n^4))^(3/4))/b^5) +
 5*a^2*n*(-b^5/(a^9*n^4))^(1/4)*log((a^2*n*x^(4/5)*(-b^5/(a^9*n^4))^(1/4) +
 b*x*x^(-1/4*n - 1/5))/x) - 5*a^2*n*(-b^5/(a^9*n^4))^(1/4)*log(-(a^2*n*x^(4
```

```
/5)*(-b^5/(a^9*n^4))^(1/4) - b*x*x^(-1/4*n - 1/5))/x) + 4*a*x*x^(-5/4*n - 1
) - 20*b*x^(1/5)*x^(-1/4*n - 1/5))/(a^2*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{5}{4}n-1}}{bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-5/4*n)/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(-5/4*n - 1)/(b*x^n + a), x)
```

maple [C] time = 0.09, size = 73, normalized size = 0.29

$$\text{RootOf}\left(a^9n^4_Z^4+b^5\right)\ln\left(\frac{\text{RootOf}\left(a^9n^4_Z^4+b^5\right)^3a^7n^3}{b^4}+x^{\frac{n}{4}}\right)-\frac{4x^{-\frac{5n}{4}}}{5an}+\frac{4b\,x^{-\frac{n}{4}}}{a^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-5/4*n)/(b*x^n+a),x)
```

```
[Out] 4*b/a^2/n/(x^(1/4*n))-4/5/a/n/(x^(1/4*n))^5+sum(_R*ln(x^(1/4*n)+a^7*n^3/b^4
*_R^3),_R=RootOf(_Z^4*a^9*n^4+b^5))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^2\int\frac{x^{\frac{3}{4}n}}{a^2bxx^n+a^3x}\,dx+\frac{4\left(5\,bx^n-a\right)}{5\,a^2nx^{\frac{5}{4}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-5/4*n)/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] b^2*integrate(x^(3/4*n)/(a^2*b*x*x^n + a^3*x), x) + 4/5*(5*b*x^n - a)/(a^2*
n*x^(5/4*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{1}{x^{\frac{5n}{4}+1}\left(a+b\,x^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^((5*n)/4 + 1)*(a + b*x^n)),x)
```

```
[Out] int(1/(x^((5*n)/4 + 1)*(a + b*x^n)), x)
```

sympy [C] time = 4.75, size = 309, normalized size = 1.23

$$\frac{x^{-\frac{5n}{4}}\Gamma\left(-\frac{5}{4}\right)}{an\Gamma\left(-\frac{1}{4}\right)}-\frac{5bx^{-\frac{n}{4}}\Gamma\left(-\frac{5}{4}\right)}{a^2n\Gamma\left(-\frac{1}{4}\right)}+\frac{5b^{\frac{5}{4}}e^{-\frac{3i\pi}{4}}\log\left(1-\frac{\sqrt[4]{b}x^{\frac{n}{4}}e^{\frac{i\pi}{4}}}{\sqrt[4]{a}}\right)\Gamma\left(-\frac{5}{4}\right)}{4a^{\frac{9}{4}}n\Gamma\left(-\frac{1}{4}\right)}+\frac{5ib^{\frac{5}{4}}e^{-\frac{3i\pi}{4}}\log\left(1-\frac{\sqrt[4]{b}x^{\frac{n}{4}}e^{\frac{3i\pi}{4}}}{\sqrt[4]{a}}\right)\Gamma\left(-\frac{5}{4}\right)}{4a^{\frac{9}{4}}n\Gamma\left(-\frac{1}{4}\right)}-\frac{5b^{\frac{5}{4}}e^{-\frac{3i}{4}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-5/4*n)/(a+b*x**n),x)
```

```
[Out] x**(-5*n/4)*gamma(-5/4)/(a*n*gamma(-1/4)) - 5*b*x**(-n/4)*gamma(-5/4)/(a**2
*n*gamma(-1/4)) + 5*b**(5/4)*exp(-3*I*pi/4)*log(1 - b**(1/4)*x**(n/4)*exp_p
olar(I*pi/4)/a**(1/4))*gamma(-5/4)/(4*a**(9/4)*n*gamma(-1/4)) + 5*I*b**(5/4
)*exp(-3*I*pi/4)*log(1 - b**(1/4)*x**(n/4)*exp_polar(3*I*pi/4)/a**(1/4))*ga
mma(-5/4)/(4*a**(9/4)*n*gamma(-1/4)) - 5*b**(5/4)*exp(-3*I*pi/4)*log(1 - b*
*(1/4)*x**(n/4)*exp_polar(5*I*pi/4)/a**(1/4))*gamma(-5/4)/(4*a**(9/4)*n*gam
ma(-1/4)) - 5*I*b**(5/4)*exp(-3*I*pi/4)*log(1 - b**(1/4)*x**(n/4)*exp_polar
(7*I*pi/4)/a**(1/4))*gamma(-5/4)/(4*a**(9/4)*n*gamma(-1/4))
```

3.2649 $\int x^{-1+4n}\sqrt{a+bx^n}\,dx$

Optimal. Leaf size=92

$$-\frac{2a^3\left(a+bx^n\right)^{3/2}}{3b^4n}+\frac{6a^2\left(a+bx^n\right)^{5/2}}{5b^4n}+\frac{2\left(a+bx^n\right)^{9/2}}{9b^4n}-\frac{6a\left(a+bx^n\right)^{7/2}}{7b^4n}$$

```
[Out] -2/3*a^3*(a+b*x^n)^(3/2)/b^4/n+6/5*a^2*(a+b*x^n)^(5/2)/b^4/n-6/7*a*(a+b*x^n
)^(7/2)/b^4/n+2/9*(a+b*x^n)^(9/2)/b^4/n
```

Rubi [A] time = 0.04, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {266, 43}

$$\frac{6a^2\left(a+bx^n\right)^{5/2}}{5b^4n}-\frac{2a^3\left(a+bx^n\right)^{3/2}}{3b^4n}+\frac{2\left(a+bx^n\right)^{9/2}}{9b^4n}-\frac{6a\left(a+bx^n\right)^{7/2}}{7b^4n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 4*n)*Sqrt[a + b*x^n],x]
```

```
[Out] (-2*a^3*(a + b*x^n)^(3/2))/(3*b^4*n) + (6*a^2*(a + b*x^n)^(5/2))/(5*b^4*n)
- (6*a*(a + b*x^n)^(7/2))/(7*b^4*n) + (2*(a + b*x^n)^(9/2))/(9*b^4*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1+4n}\sqrt{a+bx^n}\,dx &= \frac{\text{Subst}\left(\int x^3\sqrt{a+bx}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int\left(-\frac{a^3\sqrt{a+bx}}{b^3}+\frac{3a^2(a+bx)^{3/2}}{b^3}-\frac{3a(a+bx)^{5/2}}{b^3}+\frac{(a+bx)^{7/2}}{b^3}\right)dx,x,x^n\right)}{n}\\ &= -\frac{2a^3\left(a+bx^n\right)^{3/2}}{3b^4n}+\frac{6a^2\left(a+bx^n\right)^{5/2}}{5b^4n}-\frac{6a\left(a+bx^n\right)^{7/2}}{7b^4n}+\frac{2\left(a+bx^n\right)^{9/2}}{9b^4n}\end{aligned}$$

Mathematica [A] time = 0.05, size = 57, normalized size = 0.62

$$\frac{2\left(a+bx^n\right)^{3/2}\left(-16a^3+24a^2bx^n-30ab^2x^{2n}+35b^3x^{3n}\right)}{315b^4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 4*n)*Sqrt[a + b*x^n],x]
```

```
[Out] (2*(a + b*x^n)^(3/2)*(-16*a^3 + 24*a^2*b*x^n - 30*a*b^2*x^(2*n) + 35*b^3*x^
(3*n)))/(315*b^4*n)
```

fricas [A] time = 0.58, size = 66, normalized size = 0.72

$$\frac{2\left(35\,b^4x^{4\,n}+5\,ab^3x^{3\,n}-6\,a^2b^2x^{2\,n}+8\,a^3bx^n-16\,a^4\right)\sqrt{bx^n+a}}{315\,b^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)*(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*x^(4*n) + 5*a*b^3*x^(3*n) - 6*a^2*b^2*x^(2*n) + 8*a^3*b*x^n -
 16*a^4)*sqrt(b*x^n + a)/(b^4*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^n+a}\,x^{4\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)*(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n + a)*x^(4*n - 1), x)
```

maple [A] time = 0.02, size = 67, normalized size = 0.73

$$-\frac{2\left(-8a^3b\,x^n+6a^2b^2x^{2n}-5a\,b^3x^{3n}-35b^4x^{4n}+16a^4\right)\sqrt{b\,x^n+a}}{315b^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+4*n)*(b*x^n+a)^(1/2),x)
```

```
[Out] -2/315*(-35*b^4*(x^n)^4-5*a*(x^n)^3*b^3+6*a^2*(x^n)^2*b^2-8*a^3*x^n*b+16*a^
4)*(b*x^n+a)^(1/2)/b^4/n
```

maxima [A] time = 0.57, size = 66, normalized size = 0.72

$$\frac{2\left(35\,b^4x^{4\,n}+5\,ab^3x^{3\,n}-6\,a^2b^2x^{2\,n}+8\,a^3bx^n-16\,a^4\right)\sqrt{bx^n+a}}{315\,b^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)*(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*b^4*x^(4*n) + 5*a*b^3*x^(3*n) - 6*a^2*b^2*x^(2*n) + 8*a^3*b*x^n -
 16*a^4)*sqrt(b*x^n + a)/(b^4*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{4\,n-1}\,\sqrt{a+b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4*n - 1)*(a + b*x^n)^(1/2),x)
```

```
[Out] int(x^(4*n - 1)*(a + b*x^n)^(1/2), x)
```

sympy [B] time = 36.97, size = 2572, normalized size = 27.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+4*n)*(a+b*x**n)**(1/2),x)

[Out] -32*a**(29/2)*b**(23/2)*x**(23*n/2)*sqrt(a*x**(-n)/b + 1)/(315*a**(21/2)*b*
*15*n*x**(11*n) + 1890*a**(19/2)*b**16*n*x**(12*n) + 4725*a**(17/2)*b**17*n
*x**(13*n) + 6300*a**(15/2)*b**18*n*x**(14*n) + 4725*a**(13/2)*b**19*n*x**(
15*n) + 1890*a**(11/2)*b**20*n*x**(16*n) + 315*a**(9/2)*b**21*n*x**(17*n))
 - 176*a**(27/2)*b**(25/2)*x**(25*n/2)*sqrt(a*x**(-n)/b + 1)/(315*a**(21/2)*
b**15*n*x**(11*n) + 1890*a**(19/2)*b**16*n*x**(12*n) + 4725*a**(17/2)*b**17
*n*x**(13*n) + 6300*a**(15/2)*b**18*n*x**(14*n) + 4725*a**(13/2)*b**19*n*x*
*(15*n) + 1890*a**(11/2)*b**20*n*x**(16*n) + 315*a**(9/2)*b**21*n*x**(17*n)
) - 396*a**(25/2)*b**(27/2)*x**(27*n/2)*sqrt(a*x**(-n)/b + 1)/(315*a**(21/2
)*b**15*n*x**(11*n) + 1890*a**(19/2)*b**16*n*x**(12*n) + 4725*a**(17/2)*b**
17*n*x**(13*n) + 6300*a**(15/2)*b**18*n*x**(14*n) + 4725*a**(13/2)*b**19*n*
x**(15*n) + 1890*a**(11/2)*b**20*n*x**(16*n) + 315*a**(9/2)*b**21*n*x**(17*
n)) - 462*a**(23/2)*b**(29/2)*x**(29*n/2)*sqrt(a*x**(-n)/b + 1)/(315*a**(21
/2)*b**15*n*x**(11*n) + 1890*a**(19/2)*b**16*n*x**(12*n) + 4725*a**(17/2)*b
**17*n*x**(13*n) + 6300*a**(15/2)*b**18*n*x**(14*n) + 4725*a**(13/2)*b**19*
n*x**(15*n) + 1890*a**(11/2)*b**20*n*x**(16*n) + 315*a**(9/2)*b**21*n*x**(1
7*n)) - 210*a**(21/2)*b**(31/2)*x**(31*n/2)*sqrt(a*x**(-n)/b + 1)/(315*a**(
21/2)*b**15*n*x**(11*n) + 1890*a**(19/2)*b**16*n*x**(12*n) + 4725*a**(17/2)
*b**17*n*x**(13*n) + 6300*a**(15/2)*b**18*n*x**(14*n) + 4725*a**(13/2)*b**1
9*n*x**(15*n) + 1890*a**(11/2)*b**20*n*x**(16*n) + 315*a**(9/2)*b**21*n*x**
(17*n)) + 378*a**(19/2)*b**(33/2)*x**(33*n/2)*sqrt(a*x**(-n)/b + 1)/(315*a*
*(21/2)*b**15*n*x**(11*n) + 1890*a**(19/2)*b**16*n*x**(12*n) + 4725*a**(17/
2)*b**17*n*x**(13*n) + 6300*a**(15/2)*b**18*n*x**(14*n) + 4725*a**(13/2)*b*
*19*n*x**(15*n) + 1890*a**(11/2)*b**20*n*x**(16*n) + 315*a**(9/2)*b**21*n*x
**(17*n)) + 1134*a**(17/2)*b**(35/2)*x**(35*n/2)*sqrt(a*x**(-n)/b + 1)/(315
*a**(21/2)*b**15*n*x**(11*n) + 1890*a**(19/2)*b**16*n*x**(12*n) + 4725*a**(
17/2)*b**17*n*x**(13*n) + 6300*a**(15/2)*b**18*n*x**(14*n) + 4725*a**(13/2)
*b**19*n*x**(15*n) + 1890*a**(11/2)*b**20*n*x**(16*n) + 315*a**(9/2)*b**21*
n*x**(17*n)) + 1494*a**(15/2)*b**(37/2)*x**(37*n/2)*sqrt(a*x**(-n)/b + 1)/(
315*a**(21/2)*b**15*n*x**(11*n) + 1890*a**(19/2)*b**16*n*x**(12*n) + 4725*a
**(17/2)*b**17*n*x**(13*n) + 6300*a**(15/2)*b**18*n*x**(14*n) + 4725*a**(13
/2)*b**19*n*x**(15*n) + 1890*a**(11/2)*b**20*n*x**(16*n) + 315*a**(9/2)*b**
21*n*x**(17*n)) + 1098*a**(13/2)*b**(39/2)*x**(39*n/2)*sqrt(a*x**(-n)/b + 1
)/(315*a**(21/2)*b**15*n*x**(11*n) + 1890*a**(19/2)*b**16*n*x**(12*n) + 472
5*a**(17/2)*b**17*n*x**(13*n) + 6300*a**(15/2)*b**18*n*x**(14*n) + 4725*a**
(13/2)*b**19*n*x**(15*n) + 1890*a**(11/2)*b**20*n*x**(16*n) + 315*a**(9/2)*
b**21*n*x**(17*n)) + 430*a**(11/2)*b**(41/2)*x**(41*n/2)*sqrt(a*x**(-n)/b +
 1)/(315*a**(21/2)*b**15*n*x**(11*n) + 1890*a**(19/2)*b**16*n*x**(12*n) + 4
725*a**(17/2)*b**17*n*x**(13*n) + 6300*a**(15/2)*b**18*n*x**(14*n) + 4725*a
**(13/2)*b**19*n*x**(15*n) + 1890*a**(11/2)*b**20*n*x**(16*n) + 315*a**(9/2
)*b**21*n*x**(17*n)) + 70*a**(9/2)*b**(43/2)*x**(43*n/2)*sqrt(a*x**(-n)/b +
 1)/(315*a**(21/2)*b**15*n*x**(11*n) + 1890*a**(19/2)*b**16*n*x**(12*n) + 4
725*a**(17/2)*b**17*n*x**(13*n) + 6300*a**(15/2)*b**18*n*x**(14*n) + 4725*a
**(13/2)*b**19*n*x**(15*n) + 1890*a**(11/2)*b**20*n*x**(16*n) + 315*a**(9/2
)*b**21*n*x**(17*n)) + 32*a**15*b**11*x**(11*n)/(315*a**(21/2)*b**15*n*x**(
11*n) + 1890*a**(19/2)*b**16*n*x**(12*n) + 4725*a**(17/2)*b**17*n*x**(13*n)
 + 6300*a**(15/2)*b**18*n*x**(14*n) + 4725*a**(13/2)*b**19*n*x**(15*n) + 18
90*a**(11/2)*b**20*n*x**(16*n) + 315*a**(9/2)*b**21*n*x**(17*n)) + 192*a**1
4*b**12*x**(12*n)/(315*a**(21/2)*b**15*n*x**(11*n) + 1890*a**(19/2)*b**16*n
*x**(12*n) + 4725*a**(17/2)*b**17*n*x**(13*n) + 6300*a**(15/2)*b**18*n*x**(
14*n) + 4725*a**(13/2)*b**19*n*x**(15*n) + 1890*a**(11/2)*b**20*n*x**(16*n)
 + 315*a**(9/2)*b**21*n*x**(17*n)) + 480*a**13*b**13*x**(13*n)/(315*a**(21/
2)*b**15*n*x**(11*n) + 1890*a**(19/2)*b**16*n*x**(12*n) + 4725*a**(17/2)*b*
*17*n*x**(13*n) + 6300*a**(15/2)*b**18*n*x**(14*n) + 4725*a**(13/2)*b**19*n
*x**(15*n) + 1890*a**(11/2)*b**20*n*x**(16*n) + 315*a**(9/2)*b**21*n*x**(17
*n)) + 640*a**12*b**14*x**(14*n)/(315*a**(21/2)*b**15*n*x**(11*n) + 1890*a*
*(19/2)*b**16*n*x**(12*n) + 4725*a**(17/2)*b**17*n*x**(13*n) + 6300*a**(15/
2)*b**18*n*x**(14*n) + 4725*a**(13/2)*b**19*n*x**(15*n) + 1890*a**(11/2)*b*
```

```
*20*n*x**(16*n) + 315*a**(9/2)*b**21*n*x**(17*n)) + 480*a**11*b**15*x**(15*
n)/(315*a**(21/2)*b**15*n*x**(11*n) + 1890*a**(19/2)*b**16*n*x**(12*n) + 47
25*a**(17/2)*b**17*n*x**(13*n) + 6300*a**(15/2)*b**18*n*x**(14*n) + 4725*a*
*(13/2)*b**19*n*x**(15*n) + 1890*a**(11/2)*b**20*n*x**(16*n) + 315*a**(9/2)
*b**21*n*x**(17*n)) + 192*a**10*b**16*x**(16*n)/(315*a**(21/2)*b**15*n*x**(
11*n) + 1890*a**(19/2)*b**16*n*x**(12*n) + 4725*a**(17/2)*b**17*n*x**(13*n)
 + 6300*a**(15/2)*b**18*n*x**(14*n) + 4725*a**(13/2)*b**19*n*x**(15*n) + 18
90*a**(11/2)*b**20*n*x**(16*n) + 315*a**(9/2)*b**21*n*x**(17*n)) + 32*a**9*
b**17*x**(17*n)/(315*a**(21/2)*b**15*n*x**(11*n) + 1890*a**(19/2)*b**16*n*x
**(12*n) + 4725*a**(17/2)*b**17*n*x**(13*n) + 6300*a**(15/2)*b**18*n*x**(14
*n) + 4725*a**(13/2)*b**19*n*x**(15*n) + 1890*a**(11/2)*b**20*n*x**(16*n) +
 315*a**(9/2)*b**21*n*x**(17*n))
```

3.2650 $\int x^{-1+3n}\sqrt{a+bx^n}\,dx$

Optimal. Leaf size=68

$$\frac{2a^2\left(a+bx^n\right)^{3/2}}{3b^3n}+\frac{2\left(a+bx^n\right)^{7/2}}{7b^3n}-\frac{4a\left(a+bx^n\right)^{5/2}}{5b^3n}$$

```
[Out] 2/3*a^2*(a+b*x^n)^(3/2)/b^3/n-4/5*a*(a+b*x^n)^(5/2)/b^3/n+2/7*(a+b*x^n)^(7/
2)/b^3/n
```

Rubi [A] time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {266, 43}

$$\frac{2a^2\left(a+bx^n\right)^{3/2}}{3b^3n}+\frac{2\left(a+bx^n\right)^{7/2}}{7b^3n}-\frac{4a\left(a+bx^n\right)^{5/2}}{5b^3n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)*Sqrt[a + b*x^n],x]
```

```
[Out] (2*a^2*(a + b*x^n)^(3/2))/(3*b^3*n) - (4*a*(a + b*x^n)^(5/2))/(5*b^3*n) + (
2*(a + b*x^n)^(7/2))/(7*b^3*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1+3n}\sqrt{a+bx^n}\,dx &= \frac{\text{Subst}\left(\int x^2\sqrt{a+bx}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int\left(\frac{a^2\sqrt{a+bx}}{b^2}-\frac{2a(a+bx)^{3/2}}{b^2}+\frac{(a+bx)^{5/2}}{b^2}\right)dx,x,x^n\right)}{n}\\ &= \frac{2a^2\left(a+bx^n\right)^{3/2}}{3b^3n}-\frac{4a\left(a+bx^n\right)^{5/2}}{5b^3n}+\frac{2\left(a+bx^n\right)^{7/2}}{7b^3n}\end{aligned}$$

Mathematica [A] time = 0.04, size = 44, normalized size = 0.65

$$\frac{2\left(a+bx^n\right)^{3/2}\left(8a^2-12abx^n+15b^2x^{2n}\right)}{105b^3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)*Sqrt[a + b*x^n],x]
```

```
[Out] (2*(a + b*x^n)^(3/2)*(8*a^2 - 12*a*b*x^n + 15*b^2*x^(2*n)))/(105*b^3*n)
```

fricas [A] time = 0.81, size = 53, normalized size = 0.78

$$\frac{2\left(15\,b^3x^{3\,n}+3\,ab^2x^{2\,n}-4\,a^2bx^n+8\,a^3\right)\sqrt{bx^n+a}}{105\,b^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*b^3*x^(3*n) + 3*a*b^2*x^(2*n) - 4*a^2*b*x^n + 8*a^3)*sqrt(b*x^n +
 a)/(b^3*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^n+a}\,x^{3\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n + a)*x^(3*n - 1), x)
```

maple [A] time = 0.02, size = 54, normalized size = 0.79

$$\frac{2\left(-4a^2b\,x^n+3a\,b^2x^{2n}+15b^3x^{3n}+8a^3\right)\sqrt{b\,x^n+a}}{105b^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n-1)*(b*x^n+a)^(1/2),x)
```

```
[Out] 2/105*(15*(x^n)^3*b^3+3*a*(x^n)^2*b^2-4*a^2*x^n*b+8*a^3)*(b*x^n+a)^(1/2)/b^
3/n
```

maxima [A] time = 0.60, size = 53, normalized size = 0.78

$$\frac{2\left(15\,b^3x^{3\,n}+3\,ab^2x^{2\,n}-4\,a^2bx^n+8\,a^3\right)\sqrt{bx^n+a}}{105\,b^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/105*(15*b^3*x^(3*n) + 3*a*b^2*x^(2*n) - 4*a^2*b*x^n + 8*a^3)*sqrt(b*x^n +
 a)/(b^3*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3\,n-1}\,\sqrt{a+b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n - 1)*(a + b*x^n)^(1/2),x)
```

```
[Out] int(x^(3*n - 1)*(a + b*x^n)^(1/2), x)
```

sympy [B] time = 21.96, size = 1015, normalized size = 14.93

$$\frac{16a^{\frac{19}{2}}b^{\frac{9}{2}}x^{\frac{9n}{2}}\sqrt{\frac{ax^{-n}}{b}+1}}{105a^{\frac{13}{2}}b^7nx^{4n}+315a^{\frac{11}{2}}b^8nx^{5n}+315a^{\frac{9}{2}}b^9nx^{6n}+105a^{\frac{7}{2}}b^{10}nx^{7n}}+\frac{40a^{\frac{17}{2}}b^{\frac{11}{2}}x^{\frac{11n}{2}}\sqrt{\frac{ax^{-n}}{b}+1}}{105a^{\frac{13}{2}}b^7nx^{4n}+315a^{\frac{11}{2}}b^8nx^{5n}+315a^{\frac{9}{2}}b^9nx^{6n}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)*(a+b*x**n)**(1/2),x)
```

```
[Out] 16*a**(19/2)*b**(9/2)*x**(9*n/2)*sqrt(a*x**(-n)/b + 1)/(105*a**(13/2)*b**7*
n*x**(4*n) + 315*a**(11/2)*b**8*n*x**(5*n) + 315*a**(9/2)*b**9*n*x**(6*n) +
 105*a**(7/2)*b**10*n*x**(7*n)) + 40*a**(17/2)*b**(11/2)*x**(11*n/2)*sqrt(a
*x**(-n)/b + 1)/(105*a**(13/2)*b**7*n*x**(4*n) + 315*a**(11/2)*b**8*n*x**(5
*n) + 315*a**(9/2)*b**9*n*x**(6*n) + 105*a**(7/2)*b**10*n*x**(7*n)) + 30*a*
*(15/2)*b**(13/2)*x**(13*n/2)*sqrt(a*x**(-n)/b + 1)/(105*a**(13/2)*b**7*n*x
**(4*n) + 315*a**(11/2)*b**8*n*x**(5*n) + 315*a**(9/2)*b**9*n*x**(6*n) + 10
5*a**(7/2)*b**10*n*x**(7*n)) + 40*a**(13/2)*b**(15/2)*x**(15*n/2)*sqrt(a*x*
*(-n)/b + 1)/(105*a**(13/2)*b**7*n*x**(4*n) + 315*a**(11/2)*b**8*n*x**(5*n)
 + 315*a**(9/2)*b**9*n*x**(6*n) + 105*a**(7/2)*b**10*n*x**(7*n)) + 100*a**(
11/2)*b**(17/2)*x**(17*n/2)*sqrt(a*x**(-n)/b + 1)/(105*a**(13/2)*b**7*n*x**
(4*n) + 315*a**(11/2)*b**8*n*x**(5*n) + 315*a**(9/2)*b**9*n*x**(6*n) + 105*
a**(7/2)*b**10*n*x**(7*n)) + 96*a**(9/2)*b**(19/2)*x**(19*n/2)*sqrt(a*x**(-
n)/b + 1)/(105*a**(13/2)*b**7*n*x**(4*n) + 315*a**(11/2)*b**8*n*x**(5*n) +
315*a**(9/2)*b**9*n*x**(6*n) + 105*a**(7/2)*b**10*n*x**(7*n)) + 30*a**(7/2)
*b**(21/2)*x**(21*n/2)*sqrt(a*x**(-n)/b + 1)/(105*a**(13/2)*b**7*n*x**(4*n)
 + 315*a**(11/2)*b**8*n*x**(5*n) + 315*a**(9/2)*b**9*n*x**(6*n) + 105*a**(7
/2)*b**10*n*x**(7*n)) - 16*a**10*b**4*x**(4*n)/(105*a**(13/2)*b**7*n*x**(4*
n) + 315*a**(11/2)*b**8*n*x**(5*n) + 315*a**(9/2)*b**9*n*x**(6*n) + 105*a**
(7/2)*b**10*n*x**(7*n)) - 48*a**9*b**5*x**(5*n)/(105*a**(13/2)*b**7*n*x**(4
*n) + 315*a**(11/2)*b**8*n*x**(5*n) + 315*a**(9/2)*b**9*n*x**(6*n) + 105*a*
*(7/2)*b**10*n*x**(7*n)) - 48*a**8*b**6*x**(6*n)/(105*a**(13/2)*b**7*n*x**(
4*n) + 315*a**(11/2)*b**8*n*x**(5*n) + 315*a**(9/2)*b**9*n*x**(6*n) + 105*a
**(7/2)*b**10*n*x**(7*n)) - 16*a**7*b**7*x**(7*n)/(105*a**(13/2)*b**7*n*x**
(4*n) + 315*a**(11/2)*b**8*n*x**(5*n) + 315*a**(9/2)*b**9*n*x**(6*n) + 105*
a**(7/2)*b**10*n*x**(7*n))
```

3.2651 $\int x^{-1+2n}\sqrt{a+bx^n}\,dx$

Optimal. Leaf size=44

$$\frac{2\,(a+bx^n)^{5/2}}{5b^2n}-\frac{2a\,(a+bx^n)^{3/2}}{3b^2n}$$

```
[Out] -2/3*a*(a+b*x^n)^(3/2)/b^2/n+2/5*(a+b*x^n)^(5/2)/b^2/n
```

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {266, 43}

$$\frac{2\,(a+bx^n)^{5/2}}{5b^2n}-\frac{2a\,(a+bx^n)^{3/2}}{3b^2n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)*Sqrt[a + b*x^n],x]
```

```
[Out] (-2*a*(a + b*x^n)^(3/2))/(3*b^2*n) + (2*(a + b*x^n)^(5/2))/(5*b^2*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1+2n}\sqrt{a+bx^n}\,dx &= \frac{\text{Subst}\left(\int x\sqrt{a+bx}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int\left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx, x, x^n\right)}{n}\\ &= -\frac{2a\,(a+bx^n)^{3/2}}{3b^2n}+\frac{2\,(a+bx^n)^{5/2}}{5b^2n}\end{aligned}$$

Mathematica [A] time = 0.03, size = 31, normalized size = 0.70

$$\frac{2\,(a+bx^n)^{3/2}\,(3bx^n-2a)}{15b^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)*Sqrt[a + b*x^n],x]
```

```
[Out] (2*(a + b*x^n)^(3/2)*(-2*a + 3*b*x^n))/(15*b^2*n)
```

fricas [A] time = 0.74, size = 39, normalized size = 0.89

$$\frac{2\left(3\,b^2x^{2\,n}+abx^n-2\,a^2\right)\sqrt{bx^n+a}}{15\,b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*x^(2*n) + a*b*x^n - 2*a^2)*sqrt(b*x^n + a)/(b^2*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^n + a}\, x^{2n-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n + a)*x^(2*n - 1), x)
```

maple [A] time = 0.02, size = 41, normalized size = 0.93

$$-\frac{2\left(-ab\,x^n - 3b^2x^{2n} + 2a^2\right)\sqrt{b\,x^n + a}}{15b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n-1)*(b*x^n+a)^(1/2),x)
```

```
[Out] -2/15*(-3*b^2*(x^n)^2-a*x^n*b+2*a^2)*(b*x^n+a)^(1/2)/b^2/n
```

maxima [A] time = 0.62, size = 39, normalized size = 0.89

$$\frac{2\left(3\,b^2x^{2\,n} + abx^n - 2\,a^2\right)\sqrt{bx^n + a}}{15\,b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*b^2*x^(2*n) + a*b*x^n - 2*a^2)*sqrt(b*x^n + a)/(b^2*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^{2n-1}\,\sqrt{a + b\,x^n}\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)*(a + b*x^n)^(1/2),x)
```

```
[Out] int(x^(2*n - 1)*(a + b*x^n)^(1/2), x)
```

sympy [B] time = 13.99, size = 338, normalized size = 7.68

$$-\frac{4a^{\frac{11}{2}}b^{\frac{3}{2}}x^{\frac{3n}{2}}\sqrt{\frac{ax^{-n}}{b}+1}}{15a^{\frac{7}{2}}b^3nx^n + 15a^{\frac{5}{2}}b^4nx^{2n}} - \frac{2a^{\frac{9}{2}}b^{\frac{5}{2}}x^{\frac{5n}{2}}\sqrt{\frac{ax^{-n}}{b}+1}}{15a^{\frac{7}{2}}b^3nx^n + 15a^{\frac{5}{2}}b^4nx^{2n}} + \frac{8a^{\frac{7}{2}}b^{\frac{7}{2}}x^{\frac{7n}{2}}\sqrt{\frac{ax^{-n}}{b}+1}}{15a^{\frac{7}{2}}b^3nx^n + 15a^{\frac{5}{2}}b^4nx^{2n}} + \frac{6a^{\frac{5}{2}}b^{\frac{9}{2}}x^{\frac{9n}{2}}\sqrt{\frac{ax^{-n}}{b}+1}}{15a^{\frac{7}{2}}b^3nx^n + 15a^{\frac{5}{2}}b^4nx^{2n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a+b*x**n)**(1/2),x)
```

```
[Out] -4*a**(11/2)*b**(3/2)*x**(3*n/2)*sqrt(a*x**(-n)/b + 1)/(15*a**(7/2)*b**3*n*
x**n + 15*a**(5/2)*b**4*n*x**(2*n)) - 2*a**(9/2)*b**(5/2)*x**(5*n/2)*sqrt(a
*x**(-n)/b + 1)/(15*a**(7/2)*b**3*n*x**n + 15*a**(5/2)*b**4*n*x**(2*n)) + 8
*a**(7/2)*b**(7/2)*x**(7*n/2)*sqrt(a*x**(-n)/b + 1)/(15*a**(7/2)*b**3*n*x**
n + 15*a**(5/2)*b**4*n*x**(2*n)) + 6*a**(5/2)*b**(9/2)*x**(9*n/2)*sqrt(a*x*
*(-n)/b + 1)/(15*a**(7/2)*b**3*n*x**n + 15*a**(5/2)*b**4*n*x**(2*n)) + 4*a*
*6*b*x**n/(15*a**(7/2)*b**3*n*x**n + 15*a**(5/2)*b**4*n*x**(2*n)) + 4*a**5*
b**2*x**(2*n)/(15*a**(7/2)*b**3*n*x**n + 15*a**(5/2)*b**4*n*x**(2*n))
```

3.2652 $\int x^{-1+n}\sqrt{a+bx^n}\,dx$

Optimal. Leaf size=21

$$\frac{2\,(a+bx^n)^{3/2}}{3bn}$$

```
[Out] 2/3*(a+b*x^n)^(3/2)/b/n
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {261}

$$\frac{2\,(a+bx^n)^{3/2}}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*Sqrt[a + b*x^n],x]
```

```
[Out] (2*(a + b*x^n)^(3/2))/(3*b*n)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^{-1+n}\sqrt{a+bx^n}\,dx = \frac{2\,(a+bx^n)^{3/2}}{3bn}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{2\,(a+bx^n)^{3/2}}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*Sqrt[a + b*x^n],x]
```

```
[Out] (2*(a + b*x^n)^(3/2))/(3*b*n)
```

fricas [A] time = 0.67, size = 17, normalized size = 0.81

$$\frac{2\,(bx^n+a)^{\frac{3}{2}}}{3\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b*x^n + a)^(3/2)/(b*n)
```

giac [A] time = 0.16, size = 17, normalized size = 0.81

$$\frac{2\,(bx^n+a)^{\frac{3}{2}}}{3\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(a+b*x^n)^(1/2),x, algorithm="giac")
[Out] 2/3*(b*x^n + a)^(3/2)/(b*n)
```

maple [A] time = 0.02, size = 18, normalized size = 0.86

$$\frac{2\left(b\,x^{n}+a\right)^{\frac{3}{2}}}{3bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*(b*x^n+a)^(1/2),x)
[Out] 2/3*(b*x^n+a)^(3/2)/b/n
```

maxima [A] time = 0.51, size = 17, normalized size = 0.81

$$\frac{2\left(bx^{n}+a\right)^{\frac{3}{2}}}{3\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(a+b*x^n)^(1/2),x, algorithm="maxima")
[Out] 2/3*(b*x^n + a)^(3/2)/(b*n)
```

mupad [B] time = 1.31, size = 17, normalized size = 0.81

$$\frac{2\left(a+b\,x^{n}\right)^{3/2}}{3\,b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*(a + b*x^n)^(1/2),x)
[Out] (2*(a + b*x^n)^(3/2))/(3*b*n)
```

sympy [B] time = 2.66, size = 48, normalized size = 2.29

$$\frac{2a^{\frac{3}{2}}\sqrt{1+\frac{bx^{n}}{a}}}{3bn}+\frac{2\sqrt{a}\,x^{n}\sqrt{1+\frac{bx^{n}}{a}}}{3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(a+b*x**n)**(1/2),x)
[Out] 2*a**(3/2)*sqrt(1 + b*x**n/a)/(3*b*n) + 2*sqrt(a)*x**n*sqrt(1 + b*x**n/a)/(
3*n)
```

3.2653 $\int \frac{\sqrt{a+bx^n}}{x}\,dx$

Optimal. Leaf size=45

$$\frac{2\sqrt{a+bx^n}}{n}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{n}$$

```
[Out] -2*arctanh((a+b*x^n)^(1/2)/a^(1/2))*a^(1/2)/n+2*(a+b*x^n)^(1/2)/n
```

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {266, 50, 63, 208}

$$\frac{2\sqrt{a+bx^n}}{n}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^n]/x,x]
```

```
[Out] (2*Sqrt[a + b*x^n])/n - (2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/n
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx^n}}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^n\right)}{n} \\ &= \frac{2\sqrt{a+bx^n}}{n} + \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^n\right)}{n} \\ &= \frac{2\sqrt{a+bx^n}}{n} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^n}\right)}{bn} \\ &= \frac{2\sqrt{a+bx^n}}{n} - \frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{n}\end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 0.96

$$\frac{2\sqrt{a+bx^n} - 2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^n]/x,x]
```

```
[Out] (2*Sqrt[a + b*x^n] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/n
```

fricas [A] time = 0.73, size = 91, normalized size = 2.02

$$\left[\frac{\sqrt{a}\,\log\left(\frac{bx^n - 2\sqrt{bx^n+a}\sqrt{a} + 2a}{x^n}\right) + 2\sqrt{bx^n+a}}{n}, \frac{2\left(\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx^n+a}\sqrt{-a}}{a}\right) + \sqrt{bx^n+a}\right)}{n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [(sqrt(a)*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n) + 2*sqrt(b*x^n
 + a))/n, 2*(sqrt(-a)*arctan(sqrt(b*x^n + a)*sqrt(-a)/a) + sqrt(b*x^n + a))
/n]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^n+a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n + a)/x, x)
```

maple [A] time = 0.00, size = 36, normalized size = 0.80

$$\frac{-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^n+a}}{\sqrt{a}}\right) + 2\sqrt{b\,x^n+a}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^(1/2)/x,x)
```

```
[Out] 1/n*(2*(b*x^n+a)^(1/2)-2*a^(1/2)*arctanh((b*x^n+a)^(1/2)/a^(1/2)))
```

maxima [A] time = 1.26, size = 54, normalized size = 1.20

$$\frac{\sqrt{a}\,\log\left(\frac{\sqrt{bx^n+a}-\sqrt{a}}{\sqrt{bx^n+a}+\sqrt{a}}\right)}{n}+\frac{2\,\sqrt{bx^n+a}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(a)*log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))/n + 2*
sqrt(b*x^n + a)/n
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+b\,x^n}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^(1/2)/x,x)
```

```
[Out] int((a + b*x^n)^(1/2)/x, x)
```

sympy [B] time = 1.66, size = 76, normalized size = 1.69

$$-\frac{2\sqrt{a}\,\operatorname{asinh}\left(\frac{\sqrt{a}\,x^{-\frac{n}{2}}}{\sqrt{b}}\right)}{n}+\frac{2ax^{-\frac{n}{2}}}{\sqrt{b}\,n\sqrt{\frac{ax^{-n}}{b}+1}}+\frac{2\sqrt{b}\,x^{\frac{n}{2}}}{n\sqrt{\frac{ax^{-n}}{b}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**(1/2)/x,x)
```

```
[Out] -2*sqrt(a)*asinh(sqrt(a)*x**(-n/2)/sqrt(b))/n + 2*a*x**(-n/2)/(sqrt(b)*n*sq
rt(a*x**(-n)/b + 1)) + 2*sqrt(b)*x**(n/2)/(n*sqrt(a*x**(-n)/b + 1))
```

3.2654 $\int x^{-1-n}\sqrt{a+bx^n}\,dx$

Optimal. Leaf size=51

$$-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}\,n}-\frac{x^{-n}\sqrt{a+bx^n}}{n}$$

```
[Out] -b*arctanh((a+b*x^n)^(1/2)/a^(1/2))/n/a^(1/2)-(a+b*x^n)^(1/2)/n/(x^n)
```

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {266, 47, 63, 208}

$$-\frac{x^{-n}\sqrt{a+bx^n}}{n}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n)*Sqrt[a + b*x^n],x]
```

```
[Out] -(Sqrt[a + b*x^n]/(n*x^n)) - (b*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(Sqrt[a]*
n)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1-n}\sqrt{a+bx^n}\,dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx, x, x^n\right)}{n} \\ &= -\frac{x^{-n}\sqrt{a+bx^n}}{n} + \frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^n\right)}{2n} \\ &= -\frac{x^{-n}\sqrt{a+bx^n}}{n} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^n}\right)}{n} \\ &= -\frac{x^{-n}\sqrt{a+bx^n}}{n} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}\,n}\end{aligned}$$

Mathematica [A] time = 0.06, size = 62, normalized size = 1.22

$$-\frac{x^{-n}\left(bx^n\sqrt{\frac{bx^n}{a}+1}\,\tanh^{-1}\left(\sqrt{\frac{bx^n}{a}+1}\right)+a+bx^n\right)}{n\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n)*Sqrt[a + b*x^n],x]
```

```
[Out] -((a + b*x^n + b*x^n*Sqrt[1 + (b*x^n)/a]*ArcTanh[Sqrt[1 + (b*x^n)/a]])/(n*x
^n*Sqrt[a + b*x^n]))
```

fricas [A] time = 0.88, size = 119, normalized size = 2.33

$$\left[\frac{\sqrt{a}\,bx^n\log\left(\frac{bx^n-2\sqrt{bx^n+a}\sqrt{a}+2a}{x^n}\right)-2\sqrt{bx^n+a}\,a}{2anx^n}, \frac{\sqrt{-a}\,bx^n\arctan\left(\frac{\sqrt{bx^n+a}\sqrt{-a}}{a}\right)-\sqrt{bx^n+a}\,a}{anx^n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*b*x^n*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n) - 2*
sqrt(b*x^n + a)*a)/(a*n*x^n), (sqrt(-a)*b*x^n*arctan(sqrt(b*x^n + a)*sqrt(-
a)/a) - sqrt(b*x^n + a)*a)/(a*n*x^n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^n+a}\,x^{-n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n + a)*x^(-n - 1), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \sqrt{b\,x^n+a}\;x^{-n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n)*(b*x^n+a)^(1/2),x)
```

```
[Out] int(x^(-1-n)*(b*x^n+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^n + a}\, x^{-n-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^n + a)*x^(-n - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a + b\,x^n}}{x^{n+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^(1/2)/x^(n + 1),x)
```

```
[Out] int((a + b*x^n)^(1/2)/x^(n + 1), x)
```

sympy [A] time = 25.08, size = 49, normalized size = 0.96

$$-\frac{\sqrt{b}\, x^{-\frac{n}{2}} \sqrt{\frac{ax^{-n}}{b} + 1}}{n} - \frac{b \operatorname{asinh}\left(\frac{\sqrt{a}\, x^{-\frac{n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}\, n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n)*(a+b*x**n)**(1/2),x)
```

```
[Out] -sqrt(b)*x**(-n/2)*sqrt(a*x**(-n)/b + 1)/n - b*asinh(sqrt(a)*x**(-n/2)/sqrt
(b))/(sqrt(a)*n)
```

3.2655 $\int x^{-1-2n}\sqrt{a+bx^n}\,dx$

Optimal. Leaf size=84

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{4a^{3/2}n}-\frac{x^{-2n}\sqrt{a+bx^n}}{2n}-\frac{bx^{-n}\sqrt{a+bx^n}}{4an}$$

```
[Out] 1/4*b^2*arctanh((a+b*x^n)^(1/2)/a^(1/2))/a^(3/2)/n-1/2*(a+b*x^n)^(1/2)/n/(x
^(2*n))-1/4*b*(a+b*x^n)^(1/2)/a/n/(x^n)
```

Rubi [A] time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {266, 47, 51, 63, 208}

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{4a^{3/2}n}-\frac{x^{-2n}\sqrt{a+bx^n}}{2n}-\frac{bx^{-n}\sqrt{a+bx^n}}{4an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 2*n)*Sqrt[a + b*x^n],x]
```

```
[Out] -Sqrt[a + b*x^n]/(2*n*x^(2*n)) - (b*Sqrt[a + b*x^n])/(4*a*n*x^n) + (b^2*Arc
Tanh[Sqrt[a + b*x^n]/Sqrt[a]])/(4*a^(3/2)*n)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1-2n}\sqrt{a+bx^n}\,dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^3}\,dx, x, x^n\right)}{n} \\ &= -\frac{x^{-2n}\sqrt{a+bx^n}}{2n} + \frac{b\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^n\right)}{4n} \\ &= -\frac{x^{-2n}\sqrt{a+bx^n}}{2n} - \frac{bx^{-n}\sqrt{a+bx^n}}{4an} - \frac{b^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^n\right)}{8an} \\ &= -\frac{x^{-2n}\sqrt{a+bx^n}}{2n} - \frac{bx^{-n}\sqrt{a+bx^n}}{4an} - \frac{b\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^n}\right)}{4an} \\ &= -\frac{x^{-2n}\sqrt{a+bx^n}}{2n} - \frac{bx^{-n}\sqrt{a+bx^n}}{4an} + \frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{4a^{3/2}n}\end{aligned}$$

Mathematica [C] time = 0.01, size = 42, normalized size = 0.50

$$-\frac{2b^2\,(a+bx^n)^{3/2}\;{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{bx^n}{a}+1\right)}{3a^3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 2*n)*Sqrt[a + b*x^n],x]
```

```
[Out] (-2*b^2*(a + b*x^n)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + (b*x^n)/a])/(3
*a^3*n)
```

fricas [A] time = 0.79, size = 153, normalized size = 1.82

$$\left[\frac{\sqrt{a}\,b^2x^{2\,n}\log\left(\frac{bx^n+2\,\sqrt{bx^n+a}\,\sqrt{a}+2\,a}{x^n}\right)-2\left(abx^n+2\,a^2\right)\sqrt{bx^n+a}}{8\,a^2nx^{2\,n}}, -\frac{\sqrt{-a}\,b^2x^{2\,n}\arctan\left(\frac{\sqrt{bx^n+a}\,\sqrt{-a}}{a}\right)+\left(abx^n+2\right.}{4\,a^2nx^{2\,n}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a)*b^2*x^(2*n)*log((b*x^n + 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n
) - 2*(a*b*x^n + 2*a^2)*sqrt(b*x^n + a))/(a^2*n*x^(2*n)), -1/4*(sqrt(-a)*b^
2*x^(2*n)*arctan(sqrt(b*x^n + a)*sqrt(-a)/a) + (a*b*x^n + 2*a^2)*sqrt(b*x^n
 + a))/(a^2*n*x^(2*n))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^n+a}\,x^{-2\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n + a)*x^(-2*n - 1), x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \sqrt{b\,x^n + a}\; x^{-2n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-2*n)*(b*x^n+a)^(1/2),x)
```

```
[Out] int(x^(-1-2*n)*(b*x^n+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b x^n + a}\, x^{-2\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^n + a)*x^(-2*n - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + b\,x^n}}{x^{2\,n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^(1/2)/x^(2*n + 1),x)
```

```
[Out] int((a + b*x^n)^(1/2)/x^(2*n + 1), x)
```

sympy [A] time = 54.20, size = 112, normalized size = 1.33

$$-\frac{a x^{-\frac{5n}{2}}}{2\sqrt{b}\,n\sqrt{\frac{a x^{-n}}{b}+1}} - \frac{3\sqrt{b}\,x^{-\frac{3n}{2}}}{4n\sqrt{\frac{a x^{-n}}{b}+1}} - \frac{b^{\frac{3}{2}} x^{-\frac{n}{2}}}{4an\sqrt{\frac{a x^{-n}}{b}+1}} + \frac{b^2 \operatorname{asinh}\left(\frac{\sqrt{a}\,x^{-\frac{n}{2}}}{\sqrt{b}}\right)}{4a^{\frac{3}{2}}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-2*n)*(a+b*x**n)**(1/2),x)
```

```
[Out] -a*x**(-5*n/2)/(2*sqrt(b)*n*sqrt(a*x**(-n)/b + 1)) - 3*sqrt(b)*x**(-3*n/2)/
(4*n*sqrt(a*x**(-n)/b + 1)) - b**(3/2)*x**(-n/2)/(4*a*n*sqrt(a*x**(-n)/b +
1)) + b**2*asinh(sqrt(a)*x**(-n/2)/sqrt(b))/(4*a**(3/2)*n)
```

3.2656 $\int x^{-1-3n}\sqrt{a+bx^n}\,dx$

Optimal. Leaf size=113

$$-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{8a^{5/2}n}+\frac{b^2x^{-n}\sqrt{a+bx^n}}{8a^2n}-\frac{x^{-3n}\sqrt{a+bx^n}}{3n}-\frac{bx^{-2n}\sqrt{a+bx^n}}{12an}$$

```
[Out] -1/8*b^3*arctanh((a+b*x^n)^(1/2)/a^(1/2))/a^(5/2)/n-1/3*(a+b*x^n)^(1/2)/n/(
x^(3*n))-1/12*b*(a+b*x^n)^(1/2)/a/n/(x^(2*n))+1/8*b^2*(a+b*x^n)^(1/2)/a^2/n
/(x^n)
```

Rubi [A] time = 0.05, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {266, 47, 51, 63, 208}

$$\frac{b^2x^{-n}\sqrt{a+bx^n}}{8a^2n}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{8a^{5/2}n}-\frac{x^{-3n}\sqrt{a+bx^n}}{3n}-\frac{bx^{-2n}\sqrt{a+bx^n}}{12an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 3*n)*Sqrt[a + b*x^n],x]
```

```
[Out] -Sqrt[a + b*x^n]/(3*n*x^(3*n)) - (b*Sqrt[a + b*x^n])/(12*a*n*x^(2*n)) + (b^
2*Sqrt[a + b*x^n])/(8*a^2*n*x^n) - (b^3*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(
8*a^(5/2)*n)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^{-1-3n}\sqrt{a+bx^n}\,dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^4}\,dx, x, x^n\right)}{n} \\
&= -\frac{x^{-3n}\sqrt{a+bx^n}}{3n} + \frac{b\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx, x, x^n\right)}{6n} \\
&= -\frac{x^{-3n}\sqrt{a+bx^n}}{3n} - \frac{bx^{-2n}\sqrt{a+bx^n}}{12an} - \frac{b^2\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^n\right)}{8an} \\
&= -\frac{x^{-3n}\sqrt{a+bx^n}}{3n} - \frac{bx^{-2n}\sqrt{a+bx^n}}{12an} + \frac{b^2x^{-n}\sqrt{a+bx^n}}{8a^2n} + \frac{b^3\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^n\right)}{16a^2n} \\
&= -\frac{x^{-3n}\sqrt{a+bx^n}}{3n} - \frac{bx^{-2n}\sqrt{a+bx^n}}{12an} + \frac{b^2x^{-n}\sqrt{a+bx^n}}{8a^2n} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b}\right.}{8a^2n} \\
&= -\frac{x^{-3n}\sqrt{a+bx^n}}{3n} - \frac{bx^{-2n}\sqrt{a+bx^n}}{12an} + \frac{b^2x^{-n}\sqrt{a+bx^n}}{8a^2n} - \frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{8a^{5/2}n}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 42, normalized size = 0.37

$$\frac{2b^3\,(a+bx^n)^{3/2}\;{}_2F_1\left(\frac{3}{2}, 4; \frac{5}{2}; \frac{bx^n}{a}+1\right)}{3a^4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 3*n)*Sqrt[a + b*x^n],x]
```

```
[Out] (2*b^3*(a + b*x^n)^(3/2)*Hypergeometric2F1[3/2, 4, 5/2, 1 + (b*x^n)/a])/(3*
a^4*n)
```

fricas [A] time = 0.72, size = 183, normalized size = 1.62

$$\left[\frac{3\sqrt{a}\,b^3x^{3n}\log\left(\frac{bx^n-2\sqrt{bx^n+a}\sqrt{a}+2a}{x^n}\right)+2\left(3ab^2x^{2n}-2a^2bx^n-8a^3\right)\sqrt{bx^n+a}}{48a^3nx^{3n}}, \frac{3\sqrt{-a}\,b^3x^{3n}\arctan\left(\frac{\sqrt{bx^n+a}\sqrt{-a}}{a}\right)}{24}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)*(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*sqrt(a)*b^3*x^(3*n)*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/
x^n) + 2*(3*a*b^2*x^(2*n) - 2*a^2*b*x^n - 8*a^3)*sqrt(b*x^n + a))/(a^3*n*x^
(3*n)), 1/24*(3*sqrt(-a)*b^3*x^(3*n)*arctan(sqrt(b*x^n + a)*sqrt(-a)/a) + (
3*a*b^2*x^(2*n) - 2*a^2*b*x^n - 8*a^3)*sqrt(b*x^n + a))/(a^3*n*x^(3*n))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^n+a}\,x^{-3n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)*(a+b*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^n + a)*x^(-3*n - 1), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \sqrt{b\,x^n + a}\; x^{-3n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-3*n)*(b*x^n+a)^(1/2),x)
[Out] int(x^(-1-3*n)*(b*x^n+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b x^n + a}\, x^{-3\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)*(a+b*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*x^n + a)*x^(-3*n - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + b\,x^n}}{x^{3\,n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^(1/2)/x^(3*n + 1),x)
[Out] int((a + b*x^n)^(1/2)/x^(3*n + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-3*n)*(a+b*x**n)**(1/2),x)
[Out] Timed out
```

3.2657 $\int x^{-1-4n}\sqrt{a+bx^n}\,dx$

Optimal. Leaf size=142

$$\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{64a^{7/2}n}-\frac{5b^3x^{-n}\sqrt{a+bx^n}}{64a^3n}+\frac{5b^2x^{-2n}\sqrt{a+bx^n}}{96a^2n}-\frac{x^{-4n}\sqrt{a+bx^n}}{4n}-\frac{bx^{-3n}\sqrt{a+bx^n}}{24an}$$

```
[Out] 5/64*b^4*arctanh((a+b*x^n)^(1/2)/a^(1/2))/a^(7/2)/n-1/4*(a+b*x^n)^(1/2)/n/(
x^(4*n))-1/24*b*(a+b*x^n)^(1/2)/a/n/(x^(3*n))+5/96*b^2*(a+b*x^n)^(1/2)/a^2/
n/(x^(2*n))-5/64*b^3*(a+b*x^n)^(1/2)/a^3/n/(x^n)
```

Rubi [A] time = 0.07, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {266, 47, 51, 63, 208}

$$\frac{5b^2x^{-2n}\sqrt{a+bx^n}}{96a^2n}-\frac{5b^3x^{-n}\sqrt{a+bx^n}}{64a^3n}+\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{64a^{7/2}n}-\frac{x^{-4n}\sqrt{a+bx^n}}{4n}-\frac{bx^{-3n}\sqrt{a+bx^n}}{24an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 4*n)*Sqrt[a + b*x^n],x]
```

```
[Out] -Sqrt[a + b*x^n]/(4*n*x^(4*n)) - (b*Sqrt[a + b*x^n])/(24*a*n*x^(3*n)) + (5*
b^2*Sqrt[a + b*x^n])/(96*a^2*n*x^(2*n)) - (5*b^3*Sqrt[a + b*x^n])/(64*a^3*n
*x^n) + (5*b^4*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(64*a^(7/2)*n)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^{-1-4n}\sqrt{a+bx^n}\,dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^5}\,dx, x, x^n\right)}{n} \\
&= -\frac{x^{-4n}\sqrt{a+bx^n}}{4n} + \frac{b\,\text{Subst}\left(\int \frac{1}{x^4\sqrt{a+bx}}\,dx, x, x^n\right)}{8n} \\
&= -\frac{x^{-4n}\sqrt{a+bx^n}}{4n} - \frac{bx^{-3n}\sqrt{a+bx^n}}{24an} - \frac{\left(5b^2\right)\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx, x, x^n\right)}{48an} \\
&= -\frac{x^{-4n}\sqrt{a+bx^n}}{4n} - \frac{bx^{-3n}\sqrt{a+bx^n}}{24an} + \frac{5b^2x^{-2n}\sqrt{a+bx^n}}{96a^2n} + \frac{\left(5b^3\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx\right.}{64a^2n} \\
&= -\frac{x^{-4n}\sqrt{a+bx^n}}{4n} - \frac{bx^{-3n}\sqrt{a+bx^n}}{24an} + \frac{5b^2x^{-2n}\sqrt{a+bx^n}}{96a^2n} - \frac{5b^3x^{-n}\sqrt{a+bx^n}}{64a^3n} - \frac{\left(5b^4\right)\text{S}}{} \\
&= -\frac{x^{-4n}\sqrt{a+bx^n}}{4n} - \frac{bx^{-3n}\sqrt{a+bx^n}}{24an} + \frac{5b^2x^{-2n}\sqrt{a+bx^n}}{96a^2n} - \frac{5b^3x^{-n}\sqrt{a+bx^n}}{64a^3n} - \frac{\left(5b^3\right)\text{S}}{} \\
&= -\frac{x^{-4n}\sqrt{a+bx^n}}{4n} - \frac{bx^{-3n}\sqrt{a+bx^n}}{24an} + \frac{5b^2x^{-2n}\sqrt{a+bx^n}}{96a^2n} - \frac{5b^3x^{-n}\sqrt{a+bx^n}}{64a^3n} + \frac{5b^4\tan}{}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 42, normalized size = 0.30

$$-\frac{2b^4\left(a+bx^n\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},5;\frac{5}{2};\frac{bx^n}{a}+1\right)}{3a^5n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 4*n)*Sqrt[a + b*x^n],x]
```

```
[Out] (-2*b^4*(a + b*x^n)^(3/2)*Hypergeometric2F1[3/2, 5, 5/2, 1 + (b*x^n)/a])/(3
*a^5*n)
```

fricas [A] time = 0.59, size = 209, normalized size = 1.47

$$\left[\frac{15\sqrt{a}\,b^4x^{4\,n}\log\left(\frac{bx^n+2\sqrt{bx^n+a}\sqrt{a}+2a}{x^n}\right)-2\left(15\,ab^3x^{3\,n}-10\,a^2b^2x^{2\,n}+8\,a^3bx^n+48\,a^4\right)\sqrt{bx^n+a}}{384\,a^4nx^{4\,n}}, -\frac{15\sqrt{-a}\,b^4x^4}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-4*n)*(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/384*(15*sqrt(a)*b^4*x^(4*n)*log((b*x^n + 2*sqrt(b*x^n + a)*sqrt(a) + 2*a
)/x^n) - 2*(15*a*b^3*x^(3*n) - 10*a^2*b^2*x^(2*n) + 8*a^3*b*x^n + 48*a^4)*s
qrt(b*x^n + a))/(a^4*n*x^(4*n)), -1/192*(15*sqrt(-a)*b^4*x^(4*n)*arctan(sqr
t(b*x^n + a)*sqrt(-a)/a) + (15*a*b^3*x^(3*n) - 10*a^2*b^2*x^(2*n) + 8*a^3*b
*x^n + 48*a^4)*sqrt(b*x^n + a))/(a^4*n*x^(4*n))]
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^n + a}\, x^{-4n-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-4*n)*(a+b*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^n + a)*x^(-4*n - 1), x)
```

maple [**F**] time = 0.21, size = 0, normalized size = 0.00

$$\int \sqrt{b\,x^n + a}\; x^{-4n-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-4*n)*(b*x^n+a)^(1/2),x)
[Out] int(x^(-1-4*n)*(b*x^n+a)^(1/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^n + a}\, x^{-4n-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-4*n)*(a+b*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*x^n + a)*x^(-4*n - 1), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + b\,x^n}}{x^{4n+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^(1/2)/x^(4*n + 1),x)
[Out] int((a + b*x^n)^(1/2)/x^(4*n + 1), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-4*n)*(a+b*x**n)**(1/2),x)
[Out] Timed out
```

3.2658 $\int \frac{x^{-1+4n}}{\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=88

$$-\frac{2a^3\sqrt{a+bx^n}}{b^4n}+\frac{2a^2\left(a+bx^n\right)^{3/2}}{b^4n}+\frac{2\left(a+bx^n\right)^{7/2}}{7b^4n}-\frac{6a\left(a+bx^n\right)^{5/2}}{5b^4n}$$

```
[Out] 2*a^2*(a+b*x^n)^(3/2)/b^4/n-6/5*a*(a+b*x^n)^(5/2)/b^4/n+2/7*(a+b*x^n)^(7/2)
/b^4/n-2*a^3*(a+b*x^n)^(1/2)/b^4/n
```

Rubi [A] time = 0.04, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {266, 43}

$$\frac{2a^2\left(a+bx^n\right)^{3/2}}{b^4n}-\frac{2a^3\sqrt{a+bx^n}}{b^4n}+\frac{2\left(a+bx^n\right)^{7/2}}{7b^4n}-\frac{6a\left(a+bx^n\right)^{5/2}}{5b^4n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 4*n)/Sqrt[a + b*x^n],x]
```

```
[Out] (-2*a^3*Sqrt[a + b*x^n])/(b^4*n) + (2*a^2*(a + b*x^n)^(3/2))/(b^4*n) - (6*a
*(a + b*x^n)^(5/2))/(5*b^4*n) + (2*(a + b*x^n)^(7/2))/(7*b^4*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+4n}}{\sqrt{a+bx^n}}\,dx &= \frac{\text{Subst}\left(\int \frac{x^3}{\sqrt{a+bx}}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{a^3}{b^3\sqrt{a+bx}}+\frac{3a^2\sqrt{a+bx}}{b^3}-\frac{3a(a+bx)^{3/2}}{b^3}+\frac{(a+bx)^{5/2}}{b^3}\right)dx, x, x^n\right)}{n}\\ &= -\frac{2a^3\sqrt{a+bx^n}}{b^4n}+\frac{2a^2\left(a+bx^n\right)^{3/2}}{b^4n}-\frac{6a\left(a+bx^n\right)^{5/2}}{5b^4n}+\frac{2\left(a+bx^n\right)^{7/2}}{7b^4n}\end{aligned}$$

Mathematica [A] time = 0.05, size = 57, normalized size = 0.65

$$\frac{2\sqrt{a+bx^n}\left(-16a^3+8a^2bx^n-6ab^2x^{2n}+5b^3x^{3n}\right)}{35b^4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 4*n)/Sqrt[a + b*x^n],x]
```

```
[Out] (2*Sqrt[a + b*x^n]*(-16*a^3 + 8*a^2*b*x^n - 6*a*b^2*x^(2*n) + 5*b^3*x^(3*n)
))/(35*b^4*n)
```

fricas [A] time = 0.56, size = 53, normalized size = 0.60

$$\frac{2\left(5\,b^3x^{3\,n}-6\,ab^2x^{2\,n}+8\,a^2bx^n-16\,a^3\right)\sqrt{bx^n+a}}{35\,b^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^3*x^(3*n) - 6*a*b^2*x^(2*n) + 8*a^2*b*x^n - 16*a^3)*sqrt(b*x^n +
a)/(b^4*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4\,n-1}}{\sqrt{bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(4*n - 1)/sqrt(b*x^n + a), x)
```

maple [A] time = 0.03, size = 54, normalized size = 0.61

$$-\frac{2\left(-8a^2b\,x^n+6a\,b^2x^{2n}-5b^3x^{3n}+16a^3\right)\sqrt{b\,x^n+a}}{35b^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+4*n)/(b*x^n+a)^(1/2),x)
```

```
[Out] -2/35*(-5*(x^n)^3*b^3+6*a*(x^n)^2*b^2-8*a^2*b*x^n+16*a^3)*(b*x^n+a)^(1/2)/b
^4/n
```

maxima [A] time = 0.78, size = 66, normalized size = 0.75

$$\frac{2\left(5\,b^4x^{4\,n}-ab^3x^{3\,n}+2\,a^2b^2x^{2\,n}-8\,a^3bx^n-16\,a^4\right)}{35\,\sqrt{bx^n+a}\,b^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/35*(5*b^4*x^(4*n) - a*b^3*x^(3*n) + 2*a^2*b^2*x^(2*n) - 8*a^3*b*x^n - 16*
a^4)/(sqrt(b*x^n + a)*b^4*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{4\,n-1}}{\sqrt{a+b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4*n - 1)/(a + b*x^n)^(1/2),x)
```

```
[Out] int(x^(4*n - 1)/(a + b*x^n)^(1/2), x)
```

sympy [B] time = 28.40, size = 2422, normalized size = 27.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+4*n)/(a+b*x**n)**(1/2),x)
```

```
[Out] -32*a**(25/2)*b**(23/2)*x**(23*n/2)*sqrt(a*x**(-n)/b + 1)/(35*a**(19/2)*b**
15*n*x**(11*n) + 210*a**(17/2)*b**16*n*x**(12*n) + 525*a**(15/2)*b**17*n*x*
*(13*n) + 700*a**(13/2)*b**18*n*x**(14*n) + 525*a**(11/2)*b**19*n*x**(15*n)
 + 210*a**(9/2)*b**20*n*x**(16*n) + 35*a**(7/2)*b**21*n*x**(17*n)) - 176*a*
*(23/2)*b**(25/2)*x**(25*n/2)*sqrt(a*x**(-n)/b + 1)/(35*a**(19/2)*b**15*n*x
**(11*n) + 210*a**(17/2)*b**16*n*x**(12*n) + 525*a**(15/2)*b**17*n*x**(13*n
) + 700*a**(13/2)*b**18*n*x**(14*n) + 525*a**(11/2)*b**19*n*x**(15*n) + 210
*a**(9/2)*b**20*n*x**(16*n) + 35*a**(7/2)*b**21*n*x**(17*n)) - 396*a**(21/2
)*b**(27/2)*x**(27*n/2)*sqrt(a*x**(-n)/b + 1)/(35*a**(19/2)*b**15*n*x**(11*
n) + 210*a**(17/2)*b**16*n*x**(12*n) + 525*a**(15/2)*b**17*n*x**(13*n) + 70
0*a**(13/2)*b**18*n*x**(14*n) + 525*a**(11/2)*b**19*n*x**(15*n) + 210*a**(9
/2)*b**20*n*x**(16*n) + 35*a**(7/2)*b**21*n*x**(17*n)) - 462*a**(19/2)*b**(
29/2)*x**(29*n/2)*sqrt(a*x**(-n)/b + 1)/(35*a**(19/2)*b**15*n*x**(11*n) + 2
10*a**(17/2)*b**16*n*x**(12*n) + 525*a**(15/2)*b**17*n*x**(13*n) + 700*a**(
13/2)*b**18*n*x**(14*n) + 525*a**(11/2)*b**19*n*x**(15*n) + 210*a**(9/2)*b*
*20*n*x**(16*n) + 35*a**(7/2)*b**21*n*x**(17*n)) - 280*a**(17/2)*b**(31/2)*
x**(31*n/2)*sqrt(a*x**(-n)/b + 1)/(35*a**(19/2)*b**15*n*x**(11*n) + 210*a**
(17/2)*b**16*n*x**(12*n) + 525*a**(15/2)*b**17*n*x**(13*n) + 700*a**(13/2)*
b**18*n*x**(14*n) + 525*a**(11/2)*b**19*n*x**(15*n) + 210*a**(9/2)*b**20*n*
x**(16*n) + 35*a**(7/2)*b**21*n*x**(17*n)) - 42*a**(15/2)*b**(33/2)*x**(33*
n/2)*sqrt(a*x**(-n)/b + 1)/(35*a**(19/2)*b**15*n*x**(11*n) + 210*a**(17/2)*
b**16*n*x**(12*n) + 525*a**(15/2)*b**17*n*x**(13*n) + 700*a**(13/2)*b**18*n
*x**(14*n) + 525*a**(11/2)*b**19*n*x**(15*n) + 210*a**(9/2)*b**20*n*x**(16*
n) + 35*a**(7/2)*b**21*n*x**(17*n)) + 84*a**(13/2)*b**(35/2)*x**(35*n/2)*sq
rt(a*x**(-n)/b + 1)/(35*a**(19/2)*b**15*n*x**(11*n) + 210*a**(17/2)*b**16*n
*x**(12*n) + 525*a**(15/2)*b**17*n*x**(13*n) + 700*a**(13/2)*b**18*n*x**(14
*n) + 525*a**(11/2)*b**19*n*x**(15*n) + 210*a**(9/2)*b**20*n*x**(16*n) + 35
*a**(7/2)*b**21*n*x**(17*n)) + 94*a**(11/2)*b**(37/2)*x**(37*n/2)*sqrt(a*x*
*(-n)/b + 1)/(35*a**(19/2)*b**15*n*x**(11*n) + 210*a**(17/2)*b**16*n*x**(12
*n) + 525*a**(15/2)*b**17*n*x**(13*n) + 700*a**(13/2)*b**18*n*x**(14*n) + 5
25*a**(11/2)*b**19*n*x**(15*n) + 210*a**(9/2)*b**20*n*x**(16*n) + 35*a**(7/
2)*b**21*n*x**(17*n)) + 48*a**(9/2)*b**(39/2)*x**(39*n/2)*sqrt(a*x**(-n)/b
 + 1)/(35*a**(19/2)*b**15*n*x**(11*n) + 210*a**(17/2)*b**16*n*x**(12*n) + 52
5*a**(15/2)*b**17*n*x**(13*n) + 700*a**(13/2)*b**18*n*x**(14*n) + 525*a**(1
1/2)*b**19*n*x**(15*n) + 210*a**(9/2)*b**20*n*x**(16*n) + 35*a**(7/2)*b**21
*n*x**(17*n)) + 10*a**(7/2)*b**(41/2)*x**(41*n/2)*sqrt(a*x**(-n)/b + 1)/(35
*a**(19/2)*b**15*n*x**(11*n) + 210*a**(17/2)*b**16*n*x**(12*n) + 525*a**(15
/2)*b**17*n*x**(13*n) + 700*a**(13/2)*b**18*n*x**(14*n) + 525*a**(11/2)*b**
19*n*x**(15*n) + 210*a**(9/2)*b**20*n*x**(16*n) + 35*a**(7/2)*b**21*n*x**(1
7*n)) + 32*a**13*b**11*x**(11*n)/(35*a**(19/2)*b**15*n*x**(11*n) + 210*a**(
17/2)*b**16*n*x**(12*n) + 525*a**(15/2)*b**17*n*x**(13*n) + 700*a**(13/2)*b
**18*n*x**(14*n) + 525*a**(11/2)*b**19*n*x**(15*n) + 210*a**(9/2)*b**20*n*x
**(16*n) + 35*a**(7/2)*b**21*n*x**(17*n)) + 192*a**12*b**12*x**(12*n)/(35*a
**(19/2)*b**15*n*x**(11*n) + 210*a**(17/2)*b**16*n*x**(12*n) + 525*a**(15/2
)*b**17*n*x**(13*n) + 700*a**(13/2)*b**18*n*x**(14*n) + 525*a**(11/2)*b**19
*n*x**(15*n) + 210*a**(9/2)*b**20*n*x**(16*n) + 35*a**(7/2)*b**21*n*x**(17*
n)) + 480*a**11*b**13*x**(13*n)/(35*a**(19/2)*b**15*n*x**(11*n) + 210*a**(1
7/2)*b**16*n*x**(12*n) + 525*a**(15/2)*b**17*n*x**(13*n) + 700*a**(13/2)*b*
*18*n*x**(14*n) + 525*a**(11/2)*b**19*n*x**(15*n) + 210*a**(9/2)*b**20*n*x*
*(16*n) + 35*a**(7/2)*b**21*n*x**(17*n)) + 640*a**10*b**14*x**(14*n)/(35*a*
*(19/2)*b**15*n*x**(11*n) + 210*a**(17/2)*b**16*n*x**(12*n) + 525*a**(15/2)
*b**17*n*x**(13*n) + 700*a**(13/2)*b**18*n*x**(14*n) + 525*a**(11/2)*b**19*
n*x**(15*n) + 210*a**(9/2)*b**20*n*x**(16*n) + 35*a**(7/2)*b**21*n*x**(17*n
)) + 480*a**9*b**15*x**(15*n)/(35*a**(19/2)*b**15*n*x**(11*n) + 210*a**(17/
2)*b**16*n*x**(12*n) + 525*a**(15/2)*b**17*n*x**(13*n) + 700*a**(13/2)*b**1
8*n*x**(14*n) + 525*a**(11/2)*b**19*n*x**(15*n) + 210*a**(9/2)*b**20*n*x**(
```

```
16*n) + 35*a**(7/2)*b**21*n*x**(17*n)) + 192*a**8*b**16*x**(16*n)/(35*a**(1
9/2)*b**15*n*x**(11*n) + 210*a**(17/2)*b**16*n*x**(12*n) + 525*a**(15/2)*b*
*17*n*x**(13*n) + 700*a**(13/2)*b**18*n*x**(14*n) + 525*a**(11/2)*b**19*n*x
**(15*n) + 210*a**(9/2)*b**20*n*x**(16*n) + 35*a**(7/2)*b**21*n*x**(17*n))
+ 32*a**7*b**17*x**(17*n)/(35*a**(19/2)*b**15*n*x**(11*n) + 210*a**(17/2)*b
**16*n*x**(12*n) + 525*a**(15/2)*b**17*n*x**(13*n) + 700*a**(13/2)*b**18*n*
x**(14*n) + 525*a**(11/2)*b**19*n*x**(15*n) + 210*a**(9/2)*b**20*n*x**(16*n
) + 35*a**(7/2)*b**21*n*x**(17*n))
```

3.2659 $\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=66

$$\frac{2a^2\sqrt{a+bx^n}}{b^3n}+\frac{2\left(a+bx^n\right)^{5/2}}{5b^3n}-\frac{4a\left(a+bx^n\right)^{3/2}}{3b^3n}$$

```
[Out] -4/3*a*(a+b*x^n)^(3/2)/b^3/n+2/5*(a+b*x^n)^(5/2)/b^3/n+2*a^2*(a+b*x^n)^(1/2
)/b^3/n
```

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {266, 43}

$$\frac{2a^2\sqrt{a+bx^n}}{b^3n}+\frac{2\left(a+bx^n\right)^{5/2}}{5b^3n}-\frac{4a\left(a+bx^n\right)^{3/2}}{3b^3n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)/Sqrt[a + b*x^n],x]
```

```
[Out] (2*a^2*Sqrt[a + b*x^n])/(b^3*n) - (4*a*(a + b*x^n)^(3/2))/(3*b^3*n) + (2*(a
 + b*x^n)^(5/2))/(5*b^3*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx}}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a^2}{b^2\sqrt{a+bx}}-\frac{2a\sqrt{a+bx}}{b^2}+\frac{(a+bx)^{3/2}}{b^2}\right)dx, x, x^n\right)}{n}\\ &= \frac{2a^2\sqrt{a+bx^n}}{b^3n}-\frac{4a\left(a+bx^n\right)^{3/2}}{3b^3n}+\frac{2\left(a+bx^n\right)^{5/2}}{5b^3n}\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.67

$$\frac{2\sqrt{a+bx^n}\left(8a^2-4abx^n+3b^2x^{2n}\right)}{15b^3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)/Sqrt[a + b*x^n],x]
```

```
[Out] (2*Sqrt[a + b*x^n]*(8*a^2 - 4*a*b*x^n + 3*b^2*x^(2*n)))/(15*b^3*n)
```

fricas [A] time = 0.56, size = 40, normalized size = 0.61

$$\frac{2\left(3\,b^2x^{2\,n}-4\,abx^n+8\,a^2\right)\sqrt{bx^n+a}}{15\,b^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
[Out] 2/15*(3*b^2*x^(2*n) - 4*a*b*x^n + 8*a^2)*sqrt(b*x^n + a)/(b^3*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3\,n-1}}{\sqrt{bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(x^(3*n - 1)/sqrt(b*x^n + a), x)
```

maple [A] time = 0.02, size = 41, normalized size = 0.62

$$\frac{2\left(-4ab\,x^n+3b^2x^{2n}+8a^2\right)\sqrt{b\,x^n+a}}{15b^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n-1)/(b*x^n+a)^(1/2),x)
[Out] 2/15*(3*b^2*(x^n)^2-4*a*b*x^n+8*a^2)*(b*x^n+a)^(1/2)/b^3/n
```

maxima [A] time = 0.73, size = 53, normalized size = 0.80

$$\frac{2\left(3\,b^3x^{3\,n}-ab^2x^{2\,n}+4\,a^2bx^n+8\,a^3\right)}{15\,\sqrt{bx^n+a}\,b^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
[Out] 2/15*(3*b^3*x^(3*n) - a*b^2*x^(2*n) + 4*a^2*b*x^n + 8*a^3)/(sqrt(b*x^n + a)
*b^3*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{3\,n-1}}{\sqrt{a+b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n - 1)/(a + b*x^n)^(1/2),x)
[Out] int(x^(3*n - 1)/(a + b*x^n)^(1/2), x)
```

sympy [B] time = 16.53, size = 916, normalized size = 13.88

$$\frac{16a^{\frac{15}{2}}b^{\frac{9}{2}}x^{\frac{9n}{2}}\sqrt{\frac{ax^{-n}}{b}+1}}{15a^{\frac{11}{2}}b^7nx^{4n}+45a^{\frac{9}{2}}b^8nx^{5n}+45a^{\frac{7}{2}}b^9nx^{6n}+15a^{\frac{5}{2}}b^{10}nx^{7n}}+\frac{40a^{\frac{13}{2}}b^{\frac{11}{2}}x^{\frac{11n}{2}}\sqrt{\frac{ax^{-n}}{b}+1}}{15a^{\frac{11}{2}}b^7nx^{4n}+45a^{\frac{9}{2}}b^8nx^{5n}+45a^{\frac{7}{2}}b^9nx^{6n}+15a^{\frac{5}{2}}b^{10}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)/(a+b*x**n)**(1/2),x)
```

```
[Out] 16*a**(15/2)*b**(9/2)*x**(9*n/2)*sqrt(a*x**(-n)/b + 1)/(15*a**(11/2)*b**7*n
*x**(4*n) + 45*a**(9/2)*b**8*n*x**(5*n) + 45*a**(7/2)*b**9*n*x**(6*n) + 15*
a**(5/2)*b**10*n*x**(7*n)) + 40*a**(13/2)*b**(11/2)*x**(11*n/2)*sqrt(a*x**(
-n)/b + 1)/(15*a**(11/2)*b**7*n*x**(4*n) + 45*a**(9/2)*b**8*n*x**(5*n) + 45
*a**(7/2)*b**9*n*x**(6*n) + 15*a**(5/2)*b**10*n*x**(7*n)) + 30*a**(11/2)*b*
*(13/2)*x**(13*n/2)*sqrt(a*x**(-n)/b + 1)/(15*a**(11/2)*b**7*n*x**(4*n) + 4
5*a**(9/2)*b**8*n*x**(5*n) + 45*a**(7/2)*b**9*n*x**(6*n) + 15*a**(5/2)*b**1
0*n*x**(7*n)) + 10*a**(9/2)*b**(15/2)*x**(15*n/2)*sqrt(a*x**(-n)/b + 1)/(15
*a**(11/2)*b**7*n*x**(4*n) + 45*a**(9/2)*b**8*n*x**(5*n) + 45*a**(7/2)*b**9
*n*x**(6*n) + 15*a**(5/2)*b**10*n*x**(7*n)) + 10*a**(7/2)*b**(17/2)*x**(17*
n/2)*sqrt(a*x**(-n)/b + 1)/(15*a**(11/2)*b**7*n*x**(4*n) + 45*a**(9/2)*b**8
*n*x**(5*n) + 45*a**(7/2)*b**9*n*x**(6*n) + 15*a**(5/2)*b**10*n*x**(7*n)) +
 6*a**(5/2)*b**(19/2)*x**(19*n/2)*sqrt(a*x**(-n)/b + 1)/(15*a**(11/2)*b**7*
n*x**(4*n) + 45*a**(9/2)*b**8*n*x**(5*n) + 45*a**(7/2)*b**9*n*x**(6*n) + 15
*a**(5/2)*b**10*n*x**(7*n)) - 16*a**8*b**4*x**(4*n)/(15*a**(11/2)*b**7*n*x*
*(4*n) + 45*a**(9/2)*b**8*n*x**(5*n) + 45*a**(7/2)*b**9*n*x**(6*n) + 15*a**
(5/2)*b**10*n*x**(7*n)) - 48*a**7*b**5*x**(5*n)/(15*a**(11/2)*b**7*n*x**(4*
n) + 45*a**(9/2)*b**8*n*x**(5*n) + 45*a**(7/2)*b**9*n*x**(6*n) + 15*a**(5/2
)*b**10*n*x**(7*n)) - 48*a**6*b**6*x**(6*n)/(15*a**(11/2)*b**7*n*x**(4*n) +
 45*a**(9/2)*b**8*n*x**(5*n) + 45*a**(7/2)*b**9*n*x**(6*n) + 15*a**(5/2)*b*
*10*n*x**(7*n)) - 16*a**5*b**7*x**(7*n)/(15*a**(11/2)*b**7*n*x**(4*n) + 45*
a**(9/2)*b**8*n*x**(5*n) + 45*a**(7/2)*b**9*n*x**(6*n) + 15*a**(5/2)*b**10*
n*x**(7*n))
```

3.2660 $\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=42

$$\frac{2\,(a+bx^n)^{3/2}}{3b^2n}-\frac{2a\sqrt{a+bx^n}}{b^2n}$$

```
[Out] 2/3*(a+b*x^n)^(3/2)/b^2/n-2*a*(a+b*x^n)^(1/2)/b^2/n
```

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {266, 43}

$$\frac{2\,(a+bx^n)^{3/2}}{3b^2n}-\frac{2a\sqrt{a+bx^n}}{b^2n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/Sqrt[a + b*x^n],x]
```

```
[Out] (-2*a*Sqrt[a + b*x^n])/(b^2*n) + (2*(a + b*x^n)^(3/2))/(3*b^2*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{a}{b\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b}\right)\,dx, x, x^n\right)}{n}\\ &= -\frac{2a\sqrt{a+bx^n}}{b^2n}+\frac{2\,(a+bx^n)^{3/2}}{3b^2n}\end{aligned}$$

Mathematica [A] time = 0.04, size = 30, normalized size = 0.71

$$\frac{2\,(bx^n-2a)\,\sqrt{a+bx^n}}{3b^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/Sqrt[a + b*x^n],x]
```

```
[Out] (2*(-2*a + b*x^n)*Sqrt[a + b*x^n])/(3*b^2*n)
```

fricas [A] time = 0.53, size = 26, normalized size = 0.62

$$\frac{2\sqrt{bx^n+a}\,(bx^n-2a)}{3b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(b*x^n + a)*(b*x^n - 2*a)/(b^2*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{\sqrt{bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(x^(2*n - 1)/sqrt(b*x^n + a), x)
```

maple [A] time = 0.02, size = 28, normalized size = 0.67

$$-\frac{2\,(-b\,x^n+2a)\,\sqrt{b\,x^n+a}}{3b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n-1)/(b*x^n+a)^(1/2),x)
[Out] -2/3*(-b*x^n+2*a)*(b*x^n+a)^(1/2)/b^2/n
```

maxima [A] time = 0.60, size = 39, normalized size = 0.93

$$\frac{2\left(b^2x^{2n}-abx^n-2a^2\right)}{3\sqrt{bx^n+a}\,b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
[Out] 2/3*(b^2*x^(2*n) - a*b*x^n - 2*a^2)/(sqrt(b*x^n + a)*b^2*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{2n-1}}{\sqrt{a+b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/(a + b*x^n)^(1/2),x)
[Out] int(x^(2*n - 1)/(a + b*x^n)^(1/2), x)
```

sympy [B] time = 9.48, size = 275, normalized size = 6.55

$$-\frac{4a^{\frac{7}{2}}b^{\frac{3}{2}}x^{\frac{3n}{2}}\sqrt{\frac{ax^{-n}}{b}+1}}{3a^{\frac{5}{2}}b^3nx^n+3a^{\frac{3}{2}}b^4nx^{2n}}-\frac{2a^{\frac{5}{2}}b^{\frac{5}{2}}x^{\frac{5n}{2}}\sqrt{\frac{ax^{-n}}{b}+1}}{3a^{\frac{5}{2}}b^3nx^n+3a^{\frac{3}{2}}b^4nx^{2n}}+\frac{2a^{\frac{3}{2}}b^{\frac{7}{2}}x^{\frac{7n}{2}}\sqrt{\frac{ax^{-n}}{b}+1}}{3a^{\frac{5}{2}}b^3nx^n+3a^{\frac{3}{2}}b^4nx^{2n}}+\frac{4a^4bx^n}{3a^{\frac{5}{2}}b^3nx^n+3a^{\frac{3}{2}}b^4nx^{2n}}+\frac{}{3a^{\frac{5}{2}}b^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a+b*x**n)**(1/2),x)

[Out] -4*a**(7/2)*b**(3/2)*x**(3*n/2)*sqrt(a*x**(-n)/b + 1)/(3*a**(5/2)*b**3*n*x*
*n + 3*a**(3/2)*b**4*n*x**(2*n)) - 2*a**(5/2)*b**(5/2)*x**(5*n/2)*sqrt(a*x*
*(-n)/b + 1)/(3*a**(5/2)*b**3*n*x**n + 3*a**(3/2)*b**4*n*x**(2*n)) + 2*a**(
3/2)*b**(7/2)*x**(7*n/2)*sqrt(a*x**(-n)/b + 1)/(3*a**(5/2)*b**3*n*x**n + 3*
a**(3/2)*b**4*n*x**(2*n)) + 4*a**4*b*x**n/(3*a**(5/2)*b**3*n*x**n + 3*a**(3
/2)*b**4*n*x**(2*n)) + 4*a**3*b**2*x**(2*n)/(3*a**(5/2)*b**3*n*x**n + 3*a**
(3/2)*b**4*n*x**(2*n))
```

3.2661 $\int \frac{x^{-1+n}}{\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=19

$$\frac{2\sqrt{a+bx^n}}{bn}$$

```
[Out] 2*(a+b*x^n)^(1/2)/b/n
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {261}

$$\frac{2\sqrt{a+bx^n}}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)/Sqrt[a + b*x^n],x]
```

```
[Out] (2*Sqrt[a + b*x^n])/(b*n)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^{-1+n}}{\sqrt{a+bx^n}}\,dx = \frac{2\sqrt{a+bx^n}}{bn}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{2\sqrt{a+bx^n}}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)/Sqrt[a + b*x^n],x]
```

```
[Out] (2*Sqrt[a + b*x^n])/(b*n)
```

fricas [A] time = 0.49, size = 17, normalized size = 0.89

$$\frac{2\sqrt{bx^n+a}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(b*x^n + a)/(b*n)
```

giac [A] time = 0.16, size = 17, normalized size = 0.89

$$\frac{2\sqrt{bx^n+a}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b*x^n + a)/(b*n)
```

maple [A] time = 0.02, size = 18, normalized size = 0.95

$$\frac{2\sqrt{b\,x^n+a}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)/(b*x^n+a)^(1/2),x)
```

```
[Out] 2*(b*x^n+a)^(1/2)/b/n
```

maxima [A] time = 0.64, size = 17, normalized size = 0.89

$$\frac{2\,\sqrt{bx^n+a}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b*x^n + a)/(b*n)
```

mupad [B] time = 1.27, size = 17, normalized size = 0.89

$$\frac{2\,\sqrt{a+b\,x^n}}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)/(a + b*x^n)^(1/2),x)
```

```
[Out] (2*(a + b*x^n)^(1/2))/(b*n)
```

sympy [A] time = 7.71, size = 41, normalized size = 2.16

$$\begin{cases} \frac{\log(x)}{\sqrt{a}} & \text{for } b = 0 \wedge n = 0 \\ \frac{\log(x)}{\sqrt{a+b}} & \text{for } n = 0 \\ \frac{x^n}{\sqrt{a}\,n} & \text{for } b = 0 \\ \frac{2\sqrt{a+bx^n}}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)/(a+b*x**n)**(1/2),x)
```

```
[Out] Piecewise((log(x)/sqrt(a), Eq(b, 0) & Eq(n, 0)), (log(x)/sqrt(a + b), Eq(n,
 0)), (x**n/(sqrt(a)*n), Eq(b, 0)), (2*sqrt(a + b*x**n)/(b*n), True))
```

3.2662 $\int \frac{1}{x\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=28

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

```
[Out] -2*arctanh((a+b*x^n)^(1/2)/a^(1/2))/n/a^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {266, 63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x^n]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{a+bx^n}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^n\right)}{n}\\ &= \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^n}\right)}{bn}\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}\,n}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x^n]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(Sqrt[a]*n)
```

fricas [A] time = 0.64, size = 70, normalized size = 2.50

$$\left[\frac{\log\left(\frac{bx^n-2\sqrt{bx^n+a}\sqrt{a}+2a}{x^n}\right)}{\sqrt{a}\,n}, \frac{2\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx^n+a}\sqrt{-a}}{a}\right)}{an}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n)/(sqrt(a)*n), 2*sqrt(-a)
*arctan(sqrt(b*x^n + a)*sqrt(-a)/a)/(a*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^n + a}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^n + a)*x), x)
```

maple [A] time = 0.00, size = 23, normalized size = 0.82

$$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^n+a}}{\sqrt{a}}\right)}{\sqrt{a}\;n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^n+a)^(1/2),x)
```

```
[Out] -2*arctanh((b*x^n+a)^(1/2)/a^(1/2))/n/a^(1/2)
```

maxima [A] time = 1.38, size = 39, normalized size = 1.39

$$\frac{\log\left(\frac{\sqrt{bx^n+a}-\sqrt{a}}{\sqrt{bx^n+a}+\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))/(sqrt(a)*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x\sqrt{a + b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^n)^(1/2)),x)
```

```
[Out] int(1/(x*(a + b*x^n)^(1/2)), x)
```

sympy [**A**] time = 1.70, size = 26, normalized size = 0.93

$$-\frac{2\,\operatorname{asinh}\left(\frac{\sqrt{a}\,x^{-\frac{n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x**n)**(1/2),x)
[Out] -2*asinh(sqrt(a)*x**(-n/2)/sqrt(b))/(sqrt(a)*n)
```

3.2663 $\int \frac{x^{-1-n}}{\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=53

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}n}-\frac{x^{-n}\sqrt{a+bx^n}}{an}$$

```
[Out] b*arctanh((a+b*x^n)^(1/2)/a^(1/2))/a^(3/2)/n-(a+b*x^n)^(1/2)/a/n/(x^n)
```

Rubi [A] time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {266, 51, 63, 208}

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}n}-\frac{x^{-n}\sqrt{a+bx^n}}{an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n)/Sqrt[a + b*x^n],x]
```

```
[Out] -(Sqrt[a + b*x^n]/(a*n*x^n)) + (b*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(a^(3/2
)*n)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1-n}}{\sqrt{a+bx^n}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^n\right)}{n} \\ &= -\frac{x^{-n}\sqrt{a+bx^n}}{an} - \frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^n\right)}{2an} \\ &= -\frac{x^{-n}\sqrt{a+bx^n}}{an} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^n}\right)}{an} \\ &= -\frac{x^{-n}\sqrt{a+bx^n}}{an} + \frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}n}\end{aligned}$$

Mathematica [A] time = 0.09, size = 67, normalized size = 1.26

$$\frac{2b\sqrt{a+bx^n}\left(\frac{\tanh^{-1}\left(\sqrt{\frac{bx^n}{a}+1}\right)}{2\sqrt{\frac{bx^n}{a}+1}} - \frac{ax^{-n}}{2b}\right)}{a^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n)/Sqrt[a + b*x^n],x]
```

```
[Out] (2*b*Sqrt[a + b*x^n]*(-1/2*a/(b*x^n) + ArcTanh[Sqrt[1 + (b*x^n)/a]]/(2*Sqrt
[1 + (b*x^n)/a])))/(a^2*n)
```

fricas [A] time = 0.74, size = 119, normalized size = 2.25

$$\left[\frac{\sqrt{a}\,bx^n\log\left(\frac{bx^n+2\sqrt{bx^n+a}\sqrt{a}+2a}{x^n}\right) - 2\sqrt{bx^n+a}\,a}{2a^2nx^n}, -\frac{\sqrt{-a}\,bx^n\arctan\left(\frac{\sqrt{bx^n+a}\sqrt{-a}}{a}\right) + \sqrt{bx^n+a}\,a}{a^2nx^n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*b*x^n*log((b*x^n + 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n) - 2*
sqrt(b*x^n + a)*a)/(a^2*n*x^n), -(sqrt(-a)*b*x^n*arctan(sqrt(b*x^n + a)*sqr
t(-a)/a) + sqrt(b*x^n + a)*a)/(a^2*n*x^n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-n-1}}{\sqrt{bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(-n - 1)/sqrt(b*x^n + a), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{x^{-n-1}}{\sqrt{b\,x^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-n-1)/(b*x^n+a)^(1/2),x)
```

```
[Out] int(x^(-n-1)/(b*x^n+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-n-1}}{\sqrt{bx^n + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(-n - 1)/sqrt(b*x^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^{n+1}\sqrt{a + b\,x^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(n + 1)*(a + b*x^n)^(1/2)),x)
```

```
[Out] int(1/(x^(n + 1)*(a + b*x^n)^(1/2)), x)
```

sympy [A] time = 39.24, size = 49, normalized size = 0.92

$$-\frac{\sqrt{b}\,x^{-\frac{n}{2}}\sqrt{\frac{ax^{-n}}{b} + 1}}{an} + \frac{b\,\operatorname{asinh}\left(\frac{\sqrt{a}\,x^{-\frac{n}{2}}}{\sqrt{b}}\right)}{a^{\frac{3}{2}}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n)/(a+b*x**n)**(1/2),x)
```

```
[Out] -sqrt(b)*x**(-n/2)*sqrt(a*x**(-n)/b + 1)/(a*n) + b*asinh(sqrt(a)*x**(-n/2)/
sqrt(b))/(a**(3/2)*n)
```

3.2664 $\int \frac{x^{-1-2n}}{\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=87

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{4a^{5/2}n}+\frac{3bx^{-n}\sqrt{a+bx^n}}{4a^2n}-\frac{x^{-2n}\sqrt{a+bx^n}}{2an}$$

```
[Out] -3/4*b^2*arctanh((a+b*x^n)^(1/2)/a^(1/2))/a^(5/2)/n-1/2*(a+b*x^n)^(1/2)/a/n
/(x^(2*n))+3/4*b*(a+b*x^n)^(1/2)/a^2/n/(x^n)
```

Rubi [A] time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {266, 51, 63, 208}

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{4a^{5/2}n}+\frac{3bx^{-n}\sqrt{a+bx^n}}{4a^2n}-\frac{x^{-2n}\sqrt{a+bx^n}}{2an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 2*n)/Sqrt[a + b*x^n],x]
```

```
[Out] -Sqrt[a + b*x^n]/(2*a*n*x^(2*n)) + (3*b*Sqrt[a + b*x^n])/(4*a^2*n*x^n) - (3
*b^2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(4*a^(5/2)*n)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1-2n}}{\sqrt{a+bx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx, x, x^n\right)}{n}$$
$$= -\frac{x^{-2n}\sqrt{a+bx^n}}{2an} - \frac{(3b)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^n\right)}{4an}$$
$$= -\frac{x^{-2n}\sqrt{a+bx^n}}{2an} + \frac{3bx^{-n}\sqrt{a+bx^n}}{4a^2n} + \frac{\left(3b^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^n\right)}{8a^2n}$$
$$= -\frac{x^{-2n}\sqrt{a+bx^n}}{2an} + \frac{3bx^{-n}\sqrt{a+bx^n}}{4a^2n} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^n}\right)}{4a^2n}$$
$$= -\frac{x^{-2n}\sqrt{a+bx^n}}{2an} + \frac{3bx^{-n}\sqrt{a+bx^n}}{4a^2n} - \frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{4a^{5/2}n}$$

Mathematica [C] time = 0.02, size = 40, normalized size = 0.46

$$-\frac{2b^2\sqrt{a+bx^n}\;{}_2F_1\left(\frac{1}{2}, 3; \frac{3}{2}; \frac{bx^n}{a}+1\right)}{a^3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 2*n)/Sqrt[a + b*x^n],x]
```

```
[Out] (-2*b^2*Sqrt[a + b*x^n]*Hypergeometric2F1[1/2, 3, 3/2, 1 + (b*x^n)/a])/(a^3
*n)
```

fricas [A] time = 0.68, size = 157, normalized size = 1.80

$$\left[\frac{3\sqrt{a}\,b^2x^{2\,n}\log\left(\frac{bx^n-2\sqrt{bx^n+a}\sqrt{a}+2a}{x^n}\right)+2\left(3abx^n-2a^2\right)\sqrt{bx^n+a}}{8a^3nx^{2\,n}}, \frac{3\sqrt{-a}\,b^2x^{2\,n}\arctan\left(\frac{\sqrt{bx^n+a}\sqrt{-a}}{a}\right)+\left(3abx^n-\right.}{4a^3nx^{2\,n}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(a)*b^2*x^(2*n)*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x
^n) + 2*(3*a*b*x^n - 2*a^2)*sqrt(b*x^n + a))/(a^3*n*x^(2*n)), 1/4*(3*sqrt(-
a)*b^2*x^(2*n)*arctan(sqrt(b*x^n + a)*sqrt(-a)/a) + (3*a*b*x^n - 2*a^2)*sqr
t(b*x^n + a))/(a^3*n*x^(2*n))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-2\,n-1}}{\sqrt{bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(-2*n - 1)/sqrt(b*x^n + a), x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{x^{-2n-1}}{\sqrt{b\,x^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-2*n-1)/(b*x^n+a)^(1/2),x)
```

```
[Out] int(x^(-2*n-1)/(b*x^n+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-2n-1}}{\sqrt{bx^n + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(-2*n - 1)/sqrt(b*x^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{2n+1}\sqrt{a + b\,x^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(2*n + 1)*(a + b*x^n)^(1/2)),x)
```

```
[Out] int(1/(x^(2*n + 1)*(a + b*x^n)^(1/2)), x)
```

sympy [A] time = 85.04, size = 117, normalized size = 1.34

$$-\frac{x^{-\frac{5n}{2}}}{2\sqrt{b}\,n\sqrt{\frac{ax^{-n}}{b}+1}} + \frac{\sqrt{b}\,x^{-\frac{3n}{2}}}{4an\sqrt{\frac{ax^{-n}}{b}+1}} + \frac{3b^{\frac{3}{2}}x^{-\frac{n}{2}}}{4a^2n\sqrt{\frac{ax^{-n}}{b}+1}} - \frac{3b^2\,\operatorname{asinh}\left(\frac{\sqrt{a}\,x^{-\frac{n}{2}}}{\sqrt{b}}\right)}{4a^{\frac{5}{2}}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-2*n)/(a+b*x**n)**(1/2),x)
```

```
[Out] -x**(-5*n/2)/(2*sqrt(b)*n*sqrt(a*x**(-n)/b + 1)) + sqrt(b)*x**(-3*n/2)/(4*a
*n*sqrt(a*x**(-n)/b + 1)) + 3*b**(3/2)*x**(-n/2)/(4*a**2*n*sqrt(a*x**(-n)/b
 + 1)) - 3*b**2*asinh(sqrt(a)*x**(-n/2)/sqrt(b))/(4*a**(5/2)*n)
```

3.2665 $\int \frac{x^{-1-3n}}{\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=116

$$\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{8a^{7/2}n}-\frac{5b^2x^{-n}\sqrt{a+bx^n}}{8a^3n}+\frac{5bx^{-2n}\sqrt{a+bx^n}}{12a^2n}-\frac{x^{-3n}\sqrt{a+bx^n}}{3an}$$

```
[Out] 5/8*b^3*arctanh((a+b*x^n)^(1/2)/a^(1/2))/a^(7/2)/n-1/3*(a+b*x^n)^(1/2)/a/n/
(x^(3*n))+5/12*b*(a+b*x^n)^(1/2)/a^2/n/(x^(2*n))-5/8*b^2*(a+b*x^n)^(1/2)/a^
3/n/(x^n)
```

Rubi [A] time = 0.05, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {266, 51, 63, 208}

$$-\frac{5b^2x^{-n}\sqrt{a+bx^n}}{8a^3n}+\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{8a^{7/2}n}+\frac{5bx^{-2n}\sqrt{a+bx^n}}{12a^2n}-\frac{x^{-3n}\sqrt{a+bx^n}}{3an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 3*n)/Sqrt[a + b*x^n],x]
```

```
[Out] -Sqrt[a + b*x^n]/(3*a*n*x^(3*n)) + (5*b*Sqrt[a + b*x^n])/(12*a^2*n*x^(2*n))
 - (5*b^2*Sqrt[a + b*x^n])/(8*a^3*n*x^n) + (5*b^3*ArcTanh[Sqrt[a + b*x^n]/S
qrt[a]])/(8*a^(7/2)*n)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1-3n}}{\sqrt{a+bx^n}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^4\sqrt{a+bx}}\,dx, x, x^n\right)}{n} \\
&= -\frac{x^{-3n}\sqrt{a+bx^n}}{3an} - \frac{(5b)\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx, x, x^n\right)}{6an} \\
&= -\frac{x^{-3n}\sqrt{a+bx^n}}{3an} + \frac{5bx^{-2n}\sqrt{a+bx^n}}{12a^2n} + \frac{\left(5b^2\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^n\right)}{8a^2n} \\
&= -\frac{x^{-3n}\sqrt{a+bx^n}}{3an} + \frac{5bx^{-2n}\sqrt{a+bx^n}}{12a^2n} - \frac{5b^2x^{-n}\sqrt{a+bx^n}}{8a^3n} - \frac{\left(5b^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^n\right)}{16a^3n} \\
&= -\frac{x^{-3n}\sqrt{a+bx^n}}{3an} + \frac{5bx^{-2n}\sqrt{a+bx^n}}{12a^2n} - \frac{5b^2x^{-n}\sqrt{a+bx^n}}{8a^3n} - \frac{\left(5b^2\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+}\right.}{8a^3n} \\
&= -\frac{x^{-3n}\sqrt{a+bx^n}}{3an} + \frac{5bx^{-2n}\sqrt{a+bx^n}}{12a^2n} - \frac{5b^2x^{-n}\sqrt{a+bx^n}}{8a^3n} + \frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{8a^{7/2}n}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.34

$$\frac{2b^3\sqrt{a+bx^n}\;{}_2F_1\left(\frac{1}{2}, 4; \frac{3}{2}; \frac{bx^n}{a}+1\right)}{a^4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 3*n)/Sqrt[a + b*x^n],x]
```

```
[Out] (2*b^3*Sqrt[a + b*x^n]*Hypergeometric2F1[1/2, 4, 3/2, 1 + (b*x^n)/a])/(a^4*
n)
```

fricas [A] time = 0.64, size = 183, normalized size = 1.58

$$\left[\frac{15\sqrt{a}\,b^3x^{3n}\log\left(\frac{bx^n+2\sqrt{bx^n+a}\sqrt{a}+2a}{x^n}\right) - 2\left(15ab^2x^{2n} - 10a^2bx^n + 8a^3\right)\sqrt{bx^n+a}}{48a^4nx^{3n}}, -\frac{15\sqrt{-a}\,b^3x^{3n}\arctan\left(\frac{\sqrt{bx}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*sqrt(a)*b^3*x^(3*n)*log((b*x^n + 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)
/x^n) - 2*(15*a*b^2*x^(2*n) - 10*a^2*b*x^n + 8*a^3)*sqrt(b*x^n + a))/(a^4*n
*x^(3*n)), -1/24*(15*sqrt(-a)*b^3*x^(3*n)*arctan(sqrt(b*x^n + a)*sqrt(-a)/a
) + (15*a*b^2*x^(2*n) - 10*a^2*b*x^n + 8*a^3)*sqrt(b*x^n + a))/(a^4*n*x^(3*
n))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-3n-1}}{\sqrt{bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(-3*n - 1)/sqrt(b*x^n + a), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{x^{-3n-1}}{\sqrt{b\,x^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3*n-1)/(b*x^n+a)^(1/2),x)
[Out] int(x^(-3*n-1)/(b*x^n+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-3\,n-1}}{\sqrt{bx^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(x^(-3*n - 1)/sqrt(b*x^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{3\,n+1}\,\sqrt{a + b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3*n + 1)*(a + b*x^n)^(1/2)),x)
[Out] int(1/(x^(3*n + 1)*(a + b*x^n)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-3*n)/(a+b*x**n)**(1/2),x)
[Out] Timed out
```

3.2666 $\int \frac{x^{-1-4n}}{\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=145

$$-\frac{35b^4\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{64a^{9/2}n}+\frac{35b^3x^{-n}\sqrt{a+bx^n}}{64a^4n}-\frac{35b^2x^{-2n}\sqrt{a+bx^n}}{96a^3n}+\frac{7bx^{-3n}\sqrt{a+bx^n}}{24a^2n}-\frac{x^{-4n}\sqrt{a+bx^n}}{4an}$$

```
[Out] -35/64*b^4*arctanh((a+b*x^n)^(1/2)/a^(1/2))/a^(9/2)/n-1/4*(a+b*x^n)^(1/2)/a
/n/(x^(4*n))+7/24*b*(a+b*x^n)^(1/2)/a^2/n/(x^(3*n))-35/96*b^2*(a+b*x^n)^(1/
2)/a^3/n/(x^(2*n))+35/64*b^3*(a+b*x^n)^(1/2)/a^4/n/(x^n)
```

Rubi [A] time = 0.07, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {266, 51, 63, 208}

$$-\frac{35b^2x^{-2n}\sqrt{a+bx^n}}{96a^3n}+\frac{35b^3x^{-n}\sqrt{a+bx^n}}{64a^4n}-\frac{35b^4\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{64a^{9/2}n}+\frac{7bx^{-3n}\sqrt{a+bx^n}}{24a^2n}-\frac{x^{-4n}\sqrt{a+bx^n}}{4an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 4*n)/Sqrt[a + b*x^n],x]
```

```
[Out] -Sqrt[a + b*x^n]/(4*a*n*x^(4*n)) + (7*b*Sqrt[a + b*x^n])/(24*a^2*n*x^(3*n))
 - (35*b^2*Sqrt[a + b*x^n])/(96*a^3*n*x^(2*n)) + (35*b^3*Sqrt[a + b*x^n])/(
64*a^4*n*x^n) - (35*b^4*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(64*a^(9/2)*n)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1-4n}}{\sqrt{a+bx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^5\sqrt{a+bx}}\,dx, x, x^n\right)}{n}$$

$$= -\frac{x^{-4n}\sqrt{a+bx^n}}{4an} - \frac{(7b)\,\text{Subst}\left(\int \frac{1}{x^4\sqrt{a+bx}}\,dx, x, x^n\right)}{8an}$$

$$= -\frac{x^{-4n}\sqrt{a+bx^n}}{4an} + \frac{7bx^{-3n}\sqrt{a+bx^n}}{24a^2n} + \frac{\left(35b^2\right)\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx, x, x^n\right)}{48a^2n}$$

$$= -\frac{x^{-4n}\sqrt{a+bx^n}}{4an} + \frac{7bx^{-3n}\sqrt{a+bx^n}}{24a^2n} - \frac{35b^2x^{-2n}\sqrt{a+bx^n}}{96a^3n} - \frac{\left(35b^3\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^n\right)}{64a^3n}$$

$$= -\frac{x^{-4n}\sqrt{a+bx^n}}{4an} + \frac{7bx^{-3n}\sqrt{a+bx^n}}{24a^2n} - \frac{35b^2x^{-2n}\sqrt{a+bx^n}}{96a^3n} + \frac{35b^3x^{-n}\sqrt{a+bx^n}}{64a^4n} + \frac{\left(35b^4\right)\text{Subs}}{}$$

$$= -\frac{x^{-4n}\sqrt{a+bx^n}}{4an} + \frac{7bx^{-3n}\sqrt{a+bx^n}}{24a^2n} - \frac{35b^2x^{-2n}\sqrt{a+bx^n}}{96a^3n} + \frac{35b^3x^{-n}\sqrt{a+bx^n}}{64a^4n} + \frac{\left(35b^3\right)\text{Subs}}{}$$

$$= -\frac{x^{-4n}\sqrt{a+bx^n}}{4an} + \frac{7bx^{-3n}\sqrt{a+bx^n}}{24a^2n} - \frac{35b^2x^{-2n}\sqrt{a+bx^n}}{96a^3n} + \frac{35b^3x^{-n}\sqrt{a+bx^n}}{64a^4n} - \frac{35b^4\tanh^{-1}}{64a^9}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.28

$$-\frac{2b^4\sqrt{a+bx^n}\;{}_2F_1\left(\frac{1}{2}, 5; \frac{3}{2}; \frac{bx^n}{a}+1\right)}{a^5n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 4*n)/Sqrt[a + b*x^n],x]
```

```
[Out] (-2*b^4*Sqrt[a + b*x^n]*Hypergeometric2F1[1/2, 5, 3/2, 1 + (b*x^n)/a])/(a^5
*n)
```

fricas [A] time = 0.65, size = 209, normalized size = 1.44

$$\left[\frac{105\sqrt{a}\,b^4x^{4n}\log\left(\frac{bx^n-2\sqrt{bx^n+a}\sqrt{a}+2a}{x^n}\right) + 2\left(105\,ab^3x^{3n} - 70\,a^2b^2x^{2n} + 56\,a^3bx^n - 48\,a^4\right)\sqrt{bx^n+a}}{384\,a^5nx^{4n}}, \frac{105\sqrt{-a}\,b^4x}{}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-4*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/384*(105*sqrt(a)*b^4*x^(4*n)*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*
a)/x^n) + 2*(105*a*b^3*x^(3*n) - 70*a^2*b^2*x^(2*n) + 56*a^3*b*x^n - 48*a^4
)*sqrt(b*x^n + a))/(a^5*n*x^(4*n)), 1/192*(105*sqrt(-a)*b^4*x^(4*n)*arctan(
sqrt(b*x^n + a)*sqrt(-a)/a) + (105*a*b^3*x^(3*n) - 70*a^2*b^2*x^(2*n) + 56*
a^3*b*x^n - 48*a^4)*sqrt(b*x^n + a))/(a^5*n*x^(4*n))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-4n-1}}{\sqrt{bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-4*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(x^(-4*n - 1)/sqrt(b*x^n + a), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{x^{-4n-1}}{\sqrt{b\,x^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-4*n-1)/(b*x^n+a)^(1/2),x)
[Out] int(x^(-4*n-1)/(b*x^n+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-4\,n-1}}{\sqrt{b x^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-4*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(x^(-4*n - 1)/sqrt(b*x^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{4\,n+1}\,\sqrt{a + b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(4*n + 1)*(a + b*x^n)^(1/2)),x)
[Out] int(1/(x^(4*n + 1)*(a + b*x^n)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-4*n)/(a+b*x**n)**(1/2),x)
[Out] Timed out
```

3.2667 $\int x^m (a+bx^n)^3\,dx$

Optimal. Leaf size=75

$$\frac{a^3x^{m+1}}{m+1}+\frac{3a^2bx^{m+n+1}}{m+n+1}+\frac{3ab^2x^{m+2n+1}}{m+2n+1}+\frac{b^3x^{m+3n+1}}{m+3n+1}$$

```
[Out] a^3*x^(1+m)/(1+m)+3*a^2*b*x^(1+m+n)/(1+m+n)+3*a*b^2*x^(1+m+2*n)/(1+m+2*n)+b
^3*x^(1+m+3*n)/(1+m+3*n)
```

Rubi [A] time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{3a^2bx^{m+n+1}}{m+n+1}+\frac{a^3x^{m+1}}{m+1}+\frac{3ab^2x^{m+2n+1}}{m+2n+1}+\frac{b^3x^{m+3n+1}}{m+3n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^n)^3,x]
```

```
[Out] (a^3*x^(1 + m))/(1 + m) + (3*a^2*b*x^(1 + m + n))/(1 + m + n) + (3*a*b^2*x^
(1 + m + 2*n))/(1 + m + 2*n) + (b^3*x^(1 + m + 3*n))/(1 + m + 3*n)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^m (a+bx^n)^3\,dx &= \int \left(a^3x^m+3a^2bx^{m+n}+3ab^2x^{m+2n}+b^3x^{m+3n}\right)dx\\ &= \frac{a^3x^{1+m}}{1+m}+\frac{3a^2bx^{1+m+n}}{1+m+n}+\frac{3ab^2x^{1+m+2n}}{1+m+2n}+\frac{b^3x^{1+m+3n}}{1+m+3n}\end{aligned}$$

Mathematica [A] time = 0.06, size = 67, normalized size = 0.89

$$x^{m+1}\left(\frac{a^3}{m+1}+\frac{3a^2bx^n}{m+n+1}+\frac{3ab^2x^{2n}}{m+2n+1}+\frac{b^3x^{3n}}{m+3n+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^n)^3,x]
```

```
[Out] x^(1 + m)*(a^3/(1 + m) + (3*a^2*b*x^n)/(1 + m + n) + (3*a*b^2*x^(2*n))/(1 +
 m + 2*n) + (b^3*x^(3*n))/(1 + m + 3*n))
```

fricas [B] time = 0.55, size = 362, normalized size = 4.83

$$\left(b^3m^3+3\,b^3m^2+3\,b^3m+b^3+2\left(b^3m+b^3\right)n^2+3\left(b^3m^2+2\,b^3m+b^3\right)n\right)xx^mx^{3\,n}+3\left(ab^2m^3+3\,ab^2m^2+3\,ab^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] ((b^3*m^3 + 3*b^3*m^2 + 3*b^3*m + b^3 + 2*(b^3*m + b^3)*n^2 + 3*(b^3*m^2 +
2*b^3*m + b^3)*n)*x*x^m*x^(3*n) + 3*(a*b^2*m^3 + 3*a*b^2*m^2 + 3*a*b^2*m +
a*b^2 + 3*(a*b^2*m + a*b^2)*n^2 + 4*(a*b^2*m^2 + 2*a*b^2*m + a*b^2)*n)*x*x^
m*x^(2*n) + 3*(a^2*b*m^3 + 3*a^2*b*m^2 + 3*a^2*b*m + a^2*b + 6*(a^2*b*m + a
^2*b)*n^2 + 5*(a^2*b*m^2 + 2*a^2*b*m + a^2*b)*n)*x*x^m*x^n + (a^3*m^3 + 6*a
^3*n^3 + 3*a^3*m^2 + 3*a^3*m + a^3 + 11*(a^3*m + a^3)*n^2 + 6*(a^3*m^2 + 2*
a^3*m + a^3)*n)*x*x^m)/(m^4 + 6*(m + 1)*n^3 + 4*m^3 + 11*(m^2 + 2*m + 1)*n^
2 + 6*m^2 + 6*(m^3 + 3*m^2 + 3*m + 1)*n + 4*m + 1)
```

giac [B] time = 0.21, size = 622, normalized size = 8.29

$$\frac{b^3m^3xx^mx^{3n} + 3\,b^3m^2nxx^mx^{3n} + 2\,b^3mn^2xx^mx^{3n} + 3\,ab^2m^3xx^mx^{2n} + 12\,ab^2m^2nxx^mx^{2n} + 9\,ab^2mn^2xx^mx^{2n}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] (b^3*m^3*x*x^m*x^(3*n) + 3*b^3*m^2*n*x*x^m*x^(3*n) + 2*b^3*m*n^2*x*x^m*x^(3
*n) + 3*a*b^2*m^3*x*x^m*x^(2*n) + 12*a*b^2*m^2*n*x*x^m*x^(2*n) + 9*a*b^2*m*
n^2*x*x^m*x^(2*n) + 3*a^2*b*m^3*x*x^m*x^n + 15*a^2*b*m^2*n*x*x^m*x^n + 18*a
^2*b*m*n^2*x*x^m*x^n + a^3*m^3*x*x^m + 6*a^3*m^2*n*x*x^m + 11*a^3*m*n^2*x*x
^m + 6*a^3*n^3*x*x^m + 3*b^3*m^2*x*x^m*x^(3*n) + 6*b^3*m*n*x*x^m*x^(3*n) +
2*b^3*n^2*x*x^m*x^(3*n) + 9*a*b^2*m^2*x*x^m*x^(2*n) + 24*a*b^2*m*n*x*x^m*x^
(2*n) + 9*a*b^2*n^2*x*x^m*x^(2*n) + 9*a^2*b*m^2*x*x^m*x^n + 30*a^2*b*m*n*x*
x^m*x^n + 18*a^2*b*n^2*x*x^m*x^n + 3*a^3*m^2*x*x^m + 12*a^3*m*n*x*x^m + 11*
a^3*n^2*x*x^m + 3*b^3*m*x*x^m*x^(3*n) + 3*b^3*n*x*x^m*x^(3*n) + 9*a*b^2*m*x
*x^m*x^(2*n) + 12*a*b^2*n*x*x^m*x^(2*n) + 9*a^2*b*m*x*x^m*x^n + 15*a^2*b*n*
x*x^m*x^n + 3*a^3*m*x*x^m + 6*a^3*n*x*x^m + b^3*x*x^m*x^(3*n) + 3*a*b^2*x*x
^m*x^(2*n) + 3*a^2*b*x*x^m*x^n + a^3*x*x^m)/(m^4 + 6*m^3*n + 11*m^2*n^2 + 6
*m*n^3 + 4*m^3 + 18*m^2*n + 22*m*n^2 + 6*n^3 + 6*m^2 + 18*m*n + 11*n^2 + 4*
m + 6*n + 1)
```

maple [A] time = 0.02, size = 92, normalized size = 1.23

$$\frac{3a^2bx\,\mathrm{e}^{m\ln(x)}\mathrm{e}^{n\ln(x)}}{m+n+1} + \frac{3a\,b^2x\,\mathrm{e}^{m\ln(x)}\mathrm{e}^{2n\ln(x)}}{m+2n+1} + \frac{b^3x\,\mathrm{e}^{m\ln(x)}\mathrm{e}^{3n\ln(x)}}{m+3n+1} + \frac{a^3x\,\mathrm{e}^{m\ln(x)}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^n+a)^3,x)
```

```
[Out] a^3/(m+1)*x*exp(m*ln(x))+b^3/(1+m+3*n)*x*exp(m*ln(x))*exp(n*ln(x))^3+3*a*b^
2/(1+m+2*n)*x*exp(m*ln(x))*exp(n*ln(x))^2+3*a^2*b/(1+m+n)*x*exp(m*ln(x))*ex
p(n*ln(x))
```

maxima [A] time = 0.49, size = 75, normalized size = 1.00

$$\frac{b^3x^{m+3n+1}}{m+3n+1} + \frac{3\,ab^2x^{m+2n+1}}{m+2n+1} + \frac{3\,a^2bx^{m+n+1}}{m+n+1} + \frac{a^3x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] b^3*x^(m + 3*n + 1)/(m + 3*n + 1) + 3*a*b^2*x^(m + 2*n + 1)/(m + 2*n + 1) +
 3*a^2*b*x^(m + n + 1)/(m + n + 1) + a^3*x^(m + 1)/(m + 1)
```

mupad [B] time = 1.53, size = 77, normalized size = 1.03

$$\frac{a^3\,x\,x^m}{m+1} + \frac{b^3\,x\,x^m\,x^{3n}}{m+3n+1} + \frac{3\,a^2\,b\,x\,x^m\,x^n}{m+n+1} + \frac{3\,a\,b^2\,x\,x^m\,x^{2n}}{m+2n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^n)^3,x)
```

```
[Out] (a^3*x*x^m)/(m + 1) + (b^3*x*x^m*x^(3*n))/(m + 3*n + 1) + (3*a^2*b*x*x^m*x^
n)/(m + n + 1) + (3*a*b^2*x*x^m*x^(2*n))/(m + 2*n + 1)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*x**n)**3,x)
```

```
[Out] Timed out
```

3.2668 $\int x^m (a+bx^n)^2\,dx$

Optimal. Leaf size=51

$$\frac{a^2x^{m+1}}{m+1}+\frac{2abx^{m+n+1}}{m+n+1}+\frac{b^2x^{m+2n+1}}{m+2n+1}$$

```
[Out] a^2*x^(1+m)/(1+m)+2*a*b*x^(1+m+n)/(1+m+n)+b^2*x^(1+m+2*n)/(1+m+2*n)
```

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{a^2x^{m+1}}{m+1}+\frac{2abx^{m+n+1}}{m+n+1}+\frac{b^2x^{m+2n+1}}{m+2n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^n)^2,x]
```

```
[Out] (a^2*x^(1 + m))/(1 + m) + (2*a*b*x^(1 + m + n))/(1 + m + n) + (b^2*x^(1 + m
 + 2*n))/(1 + m + 2*n)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^m (a+bx^n)^2\,dx &= \int \left(a^2x^m+2abx^{m+n}+b^2x^{m+2n}\right)dx\\ &= \frac{a^2x^{1+m}}{1+m}+\frac{2abx^{1+m+n}}{1+m+n}+\frac{b^2x^{1+m+2n}}{1+m+2n}\end{aligned}$$

Mathematica [A] time = 0.04, size = 46, normalized size = 0.90

$$x^{m+1}\left(\frac{a^2}{m+1}+\frac{2abx^n}{m+n+1}+\frac{b^2x^{2n}}{m+2n+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^n)^2,x]
```

```
[Out] x^(1 + m)*(a^2/(1 + m) + (2*a*b*x^n)/(1 + m + n) + (b^2*x^(2*n))/(1 + m + 2
*n))
```

fricas [B] time = 0.61, size = 151, normalized size = 2.96

$$\frac{\left(b^2m^2+2\,b^2m+b^2+\left(b^2m+b^2\right)n\right)xx^mx^{2\,n}+2\left(abm^2+2\,abm+ab+2\,(abm+ab)n\right)xx^mx^n+\left(a^2m^2+2\,a^2n^2\right.}{m^3+2\,(m+1)n^2+3\,m^2+3\left(m^2+2\,m+1\right)n+3\,m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*m^2 + 2*b^2*m + b^2 + (b^2*m + b^2)*n)*x*x^m*x^(2*n) + 2*(a*b*m^2 + 2
*a*b*m + a*b + 2*(a*b*m + a*b)*n)*x*x^m*x^n + (a^2*m^2 + 2*a^2*n^2 + 2*a^2*
```

```
m + a^2 + 3*(a^2*m + a^2)*n)*x*x^m)/(m^3 + 2*(m + 1)*n^2 + 3*m^2 + 3*(m^2 +
 2*m + 1)*n + 3*m + 1)
```

giac [B] time = 0.30, size = 238, normalized size = 4.67

$$\frac{b^2m^2xx^mx^{2\,n} + b^2mnxx^mx^{2\,n} + 2\,abm^2xx^mx^n + 4\,abmnxx^mx^n + a^2m^2xx^m + 3\,a^2mnxx^m + 2\,a^2n^2xx^m + 2\,b^2mxx^m}{m^3 + 3\,m^2n + 2\,mn^2 + 3\,m^2 + 6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] (b^2*m^2*x*x^m*x^(2*n) + b^2*m*n*x*x^m*x^(2*n) + 2*a*b*m^2*x*x^m*x^n + 4*a*
b*m*n*x*x^m*x^n + a^2*m^2*x*x^m + 3*a^2*m*n*x*x^m + 2*a^2*n^2*x*x^m + 2*b^2
*m*x*x^m*x^(2*n) + b^2*n*x*x^m*x^(2*n) + 4*a*b*m*x*x^m*x^n + 4*a*b*n*x*x^m*
x^n + 2*a^2*m*x*x^m + 3*a^2*n*x*x^m + b^2*x*x^m*x^(2*n) + 2*a*b*x*x^m*x^n +
 a^2*x*x^m)/(m^3 + 3*m^2*n + 2*m*n^2 + 3*m^2 + 6*m*n + 2*n^2 + 3*m + 3*n +
1)
```

maple [A] time = 0.02, size = 63, normalized size = 1.24

$$\frac{2abx\,\mathrm{e}^{m\ln(x)}\mathrm{e}^{n\ln(x)}}{m+n+1} + \frac{b^2x\,\mathrm{e}^{m\ln(x)}\mathrm{e}^{2n\ln(x)}}{m+2n+1} + \frac{a^2x\,\mathrm{e}^{m\ln(x)}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^n+a)^2,x)
```

```
[Out] a^2/(m+1)*x*exp(m*ln(x))+b^2/(m+2*n+1)*x*exp(m*ln(x))*exp(n*ln(x))^2+2*a*b/
(m+n+1)*x*exp(m*ln(x))*exp(n*ln(x))
```

maxima [A] time = 0.58, size = 51, normalized size = 1.00

$$\frac{b^2x^{m+2\,n+1}}{m+2\,n+1} + \frac{2\,abx^{m+n+1}}{m+n+1} + \frac{a^2x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] b^2*x^(m + 2*n + 1)/(m + 2*n + 1) + 2*a*b*x^(m + n + 1)/(m + n + 1) + a^2*x
^(m + 1)/(m + 1)
```

mupad [B] time = 1.46, size = 52, normalized size = 1.02

$$\frac{a^2\,x\,x^m}{m+1} + \frac{b^2\,x\,x^m\,x^{2\,n}}{m+2\,n+1} + \frac{2\,a\,b\,x\,x^m\,x^n}{m+n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^n)^2,x)
```

```
[Out] (a^2*x*x^m)/(m + 1) + (b^2*x*x^m*x^(2*n))/(m + 2*n + 1) + (2*a*b*x*x^m*x^n)
/(m + n + 1)
```

sympy [A] time = 59.34, size = 1280, normalized size = 25.10

$$\begin{cases} (a+b)^2\log(x) \\ a^2\log(x) + \frac{2abx^n}{n} + \frac{b^2x^{2n}}{2n} \\ -\frac{2a^2n}{4n^2x^{2n}+2nx^{2n}} - \frac{a^2}{4n^2x^{2n}+2nx^{2n}} - \frac{8abnx^n}{4n^2x^{2n}+2nx^{2n}} - \frac{4abx^n}{4n^2x^{2n}+2nx^{2n}} + \frac{4b^2n^2x^{2n}\log(x)}{4n^2x^{2n}+2nx^{2n}} + \frac{2b^2nx^{2n}\log(x)}{4n^2x^{2n}+2nx^{2n}} + \frac{2b^2nx^{2n}}{4n^2x^{2n}+2nx^{2n}} \\ -\frac{a^2n}{n^2x^n+nx^n} - \frac{a^2}{n^2x^n+nx^n} + \frac{2abn^2x^n\log(x)}{n^2x^n+nx^n} + \frac{2abnx^n\log(x)}{n^2x^n+nx^n} + \frac{2abnx^n}{n^2x^n+nx^n} + \frac{b^2nx^{2n}}{n^2x^n+nx^n} + \frac{b^2x^{2n}}{n^2x^n+nx^n} \\ \frac{a^2m^2xx^m}{m^3+3m^2n+3m^2+2mn^2+6mn+3m+2n^2+3n+1} + \frac{3a^2mnxx^m}{m^3+3m^2n+3m^2+2mn^2+6mn+3m+2n^2+3n+1} + \frac{2a^2mxx^m}{m^3+3m^2n+3m^2+2mn^2+6mn+3m+2n^2+3n+1} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*x**n)**2,x)
```

```
[Out] Piecewise(((a + b)**2*log(x), Eq(m, -1) & Eq(n, 0)), (a**2*log(x) + 2*a*b*x
**n/n + b**2*x**(2*n)/(2*n), Eq(m, -1)), (-2*a**2*n/(4*n**2*x**(2*n) + 2*n*
x**(2*n)) - a**2/(4*n**2*x**(2*n) + 2*n*x**(2*n)) - 8*a*b*n*x**n/(4*n**2*x*
*(2*n) + 2*n*x**(2*n)) - 4*a*b*x**n/(4*n**2*x**(2*n) + 2*n*x**(2*n)) + 4*b*
*2*n**2*x**(2*n)*log(x)/(4*n**2*x**(2*n) + 2*n*x**(2*n)) + 2*b**2*n*x**(2*n
)*log(x)/(4*n**2*x**(2*n) + 2*n*x**(2*n)) + 2*b**2*n*x**(2*n)/(4*n**2*x**(2
*n) + 2*n*x**(2*n)), Eq(m, -2*n - 1)), (-a**2*n/(n**2*x**n + n*x**n) - a**2
/(n**2*x**n + n*x**n) + 2*a*b*n**2*x**n*log(x)/(n**2*x**n + n*x**n) + 2*a*b
*n*x**n*log(x)/(n**2*x**n + n*x**n) + 2*a*b*n*x**n/(n**2*x**n + n*x**n) + b
**2*n*x**(2*n)/(n**2*x**n + n*x**n) + b**2*x**(2*n)/(n**2*x**n + n*x**n), E
q(m, -n - 1)), (a**2*m**2*x*x**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m
*n + 3*m + 2*n**2 + 3*n + 1) + 3*a**2*m*n*x*x**m/(m**3 + 3*m**2*n + 3*m**2
 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 2*a**2*m*x*x**m/(m**3 + 3*m*
*2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 2*a**2*n**2*x*
x**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1)
 + 3*a**2*n*x*x**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n
**2 + 3*n + 1) + a**2*x*x**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n +
 3*m + 2*n**2 + 3*n + 1) + 2*a*b*m**2*x*x**m*x**n/(m**3 + 3*m**2*n + 3*m**2
 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 4*a*b*m*n*x*x**m*x**n/(m**3
 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 4*a*b*m
*x*x**m*x**n/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 +
3*n + 1) + 4*a*b*n*x*x**m*x**n/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n
 + 3*m + 2*n**2 + 3*n + 1) + 2*a*b*x*x**m*x**n/(m**3 + 3*m**2*n + 3*m**2 +
2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + b**2*m**2*x*x**m*x**(2*n)/(m**
3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + b**2*m
*n*x*x**m*x**(2*n)/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n
**2 + 3*n + 1) + 2*b**2*m*x*x**m*x**(2*n)/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n
**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + b**2*n*x*x**m*x**(2*n)/(m**3 + 3*m*
*2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + b**2*x*x**m*x*
*(2*n)/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n +
1), True))
```

3.2669 $\int x^m (a + bx^n)\, dx$

Optimal. Leaf size=27

$$\frac{ax^{m+1}}{m+1} + \frac{bx^{m+n+1}}{m+n+1}$$

```
[Out] a*x^(1+m)/(1+m)+b*x^(1+m+n)/(1+m+n)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$\frac{ax^{m+1}}{m+1} + \frac{bx^{m+n+1}}{m+n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^n),x]
```

```
[Out] (a*x^(1 + m))/(1 + m) + (b*x^(1 + m + n))/(1 + m + n)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^m (a + bx^n)\, dx &= \int (ax^m + bx^{m+n})\, dx \\ &= \frac{ax^{1+m}}{1+m} + \frac{bx^{1+m+n}}{1+m+n}\end{aligned}$$

Mathematica [A] time = 0.02, size = 27, normalized size = 1.00

$$\frac{ax^{m+1}}{m+1} + \frac{bx^{m+n+1}}{m+n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^n),x]
```

```
[Out] (a*x^(1 + m))/(1 + m) + (b*x^(1 + m + n))/(1 + m + n)
```

fricas [A] time = 0.72, size = 43, normalized size = 1.59

$$\frac{(bm+b)xx^m x^n + (am+an+a)xx^m}{m^2 + (m+1)n + 2\,m + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^n),x, algorithm="fricas")
```

```
[Out] ((b*m + b)*x*x^m*x^n + (a*m + a*n + a)*x*x^m)/(m^2 + (m + 1)*n + 2*m + 1)
```

giac [B] time = 0.16, size = 55, normalized size = 2.04

$$\frac{bmxx^m x^n + amxx^m + anxx^m + bxx^m x^n + axx^m}{m^2 + mn + 2\,m + n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^n),x, algorithm="giac")
```

```
[Out] (b*m*x*x^m*x^n + a*m*x*x^m + a*n*x*x^m + b*x*x^m*x^n + a*x*x^m)/(m^2 + m*n
+ 2*m + n + 1)
```

maple [A] time = 0.01, size = 34, normalized size = 1.26

$$\frac{bx\,e^{m\ln(x)}e^{n\ln(x)}}{m+n+1}+\frac{ax\,e^{m\ln(x)}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^n+a),x)
```

```
[Out] a/(m+1)*x*exp(m*ln(x))+b/(m+n+1)*x*exp(m*ln(x))*exp(n*ln(x))
```

maxima [A] time = 0.61, size = 27, normalized size = 1.00

$$\frac{bx^{m+n+1}}{m+n+1}+\frac{ax^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^n),x, algorithm="maxima")
```

```
[Out] b*x^(m + n + 1)/(m + n + 1) + a*x^(m + 1)/(m + 1)
```

mupad [B] time = 1.40, size = 27, normalized size = 1.00

$$\frac{a\,x\,x^m}{m+1}+\frac{b\,x\,x^m\,x^n}{m+n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^n),x)
```

```
[Out] (a*x*x^m)/(m + 1) + (b*x*x^m*x^n)/(m + n + 1)
```

sympy [A] time = 3.42, size = 175, normalized size = 6.48

$$\begin{cases}(a+b)\log(x) & \text{for } m=-1\wedge n=0\\ a\log(x)+\frac{bx^n}{n} & \text{for } m=-1\\ \frac{amxx^m}{m^2+m}+\frac{bm^2\log(x)}{m^2+m}+\frac{bm\log(x)}{m^2+m} & \text{for } n=-m-1\\ \frac{amxx^m}{m^2+mn+2m+n+1}+\frac{anxx^m}{m^2+mn+2m+n+1}+\frac{axx^m}{m^2+mn+2m+n+1}+\frac{bmxx^mx^n}{m^2+mn+2m+n+1}+\frac{bxx^mx^n}{m^2+mn+2m+n+1} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*x**n),x)
```

```
[Out] Piecewise(((a + b)*log(x), Eq(m, -1) & Eq(n, 0)), (a*log(x) + b*x**n/n, Eq(
m, -1)), (a*m*x*x**m/(m**2 + m) + b*m**2*log(x)/(m**2 + m) + b*m*log(x)/(m*
*2 + m), Eq(n, -m - 1)), (a*m*x*x**m/(m**2 + m*n + 2*m + n + 1) + a*n*x*x**
m/(m**2 + m*n + 2*m + n + 1) + a*x*x**m/(m**2 + m*n + 2*m + n + 1) + b*m*x*
x**m*x**n/(m**2 + m*n + 2*m + n + 1) + b*x*x**m*x**n/(m**2 + m*n + 2*m + n
+ 1), True))
```

3.2670 $\int \frac{x^m}{a+bx^n}\,dx$

Optimal. Leaf size=40

$$\frac{x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{a(m+1)}$$

```
[Out] x^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/a/(1+m)
```

Rubi [A] time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {364}

$$\frac{x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{a(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x^n),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)])/(a
*(1 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^m}{a+bx^n}\,dx = \frac{x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{a(1+m)}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.02

$$\frac{x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+1}{n}+1;-\frac{bx^n}{a}\right)}{a(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x^n),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, 1 + (1 + m)/n, -((b*x^n)/a)])/(a
*(1 + m))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m}{bx^n+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral(x^m/(b*x^n + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{bx^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*x^n + a), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{x^m}{b\,x^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^n+a),x)
```

```
[Out] int(x^m/(b*x^n+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{bx^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(b*x^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{a + b\,x^n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^n),x)
```

```
[Out] int(x^m/(a + b*x^n), x)
```

sympy [C] time = 1.17, size = 95, normalized size = 2.38

$$\frac{mxx^m\Phi\left(\frac{bx^n e^{i\pi}}{a}, 1, \frac{m}{n} + \frac{1}{n}\right)\Gamma\left(\frac{m}{n} + \frac{1}{n}\right)}{an^2\Gamma\left(\frac{m}{n} + 1 + \frac{1}{n}\right)} + \frac{xx^m\Phi\left(\frac{bx^n e^{i\pi}}{a}, 1, \frac{m}{n} + \frac{1}{n}\right)\Gamma\left(\frac{m}{n} + \frac{1}{n}\right)}{an^2\Gamma\left(\frac{m}{n} + 1 + \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a+b*x**n),x)
```

```
[Out] m*x*x**m*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/
(a*n**2*gamma(m/n + 1 + 1/n)) + x*x**m*lerchphi(b*x**n*exp_polar(I*pi)/a, 1
, m/n + 1/n)*gamma(m/n + 1/n)/(a*n**2*gamma(m/n + 1 + 1/n))
```

3.2671 $\int \frac{x^m}{(a+bx^n)^2}\,dx$

Optimal. Leaf size=40

$$\frac{x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{a^2(m+1)}$$

```
[Out] x^(1+m)*hypergeom([2, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/a^2/(1+m)
```

Rubi [A] time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {364}

$$\frac{x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{a^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x^n)^2,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)])/(a
^2*(1 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^m}{(a+bx^n)^2}\,dx = \frac{x^{1+m}\,{}_2F_1\left(2,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{a^2(1+m)}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 1.02

$$\frac{x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{n};\frac{m+1}{n}+1;-\frac{bx^n}{a}\right)}{a^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x^n)^2,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/n, 1 + (1 + m)/n, -((b*x^n)/a)])/(a
^2*(1 + m))
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m}{b^2x^{2\,n}+2\,abx^n+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(bx^n + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*x^n + a)^2, x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(b\,x^n + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^n+a)^2,x)
```

```
[Out] int(x^m/(b*x^n+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-(m - n + 1) \int \frac{x^m}{abnx^n + a^2 n}\, dx + \frac{xx^m}{abnx^n + a^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] -(m - n + 1)*integrate(x^m/(a*b*n*x^n + a^2*n), x) + x*x^m/(a*b*n*x^n + a^2
*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{(a + b\,x^n)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^n)^2,x)
```

```
[Out] int(x^m/(a + b*x^n)^2, x)
```

sympy [C] time = 2.15, size = 840, normalized size = 21.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a+b*x**n)**2,x)
```

```
[Out] -m**2*x*x**m*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1
/n)/(a*(a*n**3*gamma(m/n + 1 + 1/n) + b*n**3*x**n*gamma(m/n + 1 + 1/n))) +
m*n*x*x**m*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n
)/(a*(a*n**3*gamma(m/n + 1 + 1/n) + b*n**3*x**n*gamma(m/n + 1 + 1/n))) + m*
n*x*x**m*gamma(m/n + 1/n)/(a*(a*n**3*gamma(m/n + 1 + 1/n) + b*n**3*x**n*gam
ma(m/n + 1 + 1/n))) - 2*m*x*x**m*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n
+ 1/n)*gamma(m/n + 1/n)/(a*(a*n**3*gamma(m/n + 1 + 1/n) + b*n**3*x**n*gamma
(m/n + 1 + 1/n))) + n*x*x**m*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/
n)*gamma(m/n + 1/n)/(a*(a*n**3*gamma(m/n + 1 + 1/n) + b*n**3*x**n*gamma(m/n
```

```
 + 1 + 1/n))) + n*x*x**m*gamma(m/n + 1/n)/(a*(a*n**3*gamma(m/n + 1 + 1/n) +
 b*n**3*x**n*gamma(m/n + 1 + 1/n))) - x*x**m*lerchphi(b*x**n*exp_polar(I*pi
)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(a*(a*n**3*gamma(m/n + 1 + 1/n) + b*n**
3*x**n*gamma(m/n + 1 + 1/n))) - b*m**2*x*x**m*x**n*lerchphi(b*x**n*exp_pola
r(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(a**2*(a*n**3*gamma(m/n + 1 + 1/n
) + b*n**3*x**n*gamma(m/n + 1 + 1/n))) + b*m*n*x*x**m*x**n*lerchphi(b*x**n*
exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(a**2*(a*n**3*gamma(m/n +
 1 + 1/n) + b*n**3*x**n*gamma(m/n + 1 + 1/n))) - 2*b*m*x*x**m*x**n*lerchphi
(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(a**2*(a*n**3*gam
ma(m/n + 1 + 1/n) + b*n**3*x**n*gamma(m/n + 1 + 1/n))) + b*n*x*x**m*x**n*le
rchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(a**2*(a*n*
*3*gamma(m/n + 1 + 1/n) + b*n**3*x**n*gamma(m/n + 1 + 1/n))) - b*x*x**m*x**
n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(a**2*(
a*n**3*gamma(m/n + 1 + 1/n) + b*n**3*x**n*gamma(m/n + 1 + 1/n)))
```

3.2672 $\int \frac{x^m}{(a+bx^n)^3}\,dx$

Optimal. Leaf size=40

$$\frac{x^{m+1}\,{}_2F_1\left(3,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{a^3(m+1)}$$

```
[Out] x^(1+m)*hypergeom([3, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/a^3/(1+m)
```

Rubi [A] time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {364}

$$\frac{x^{m+1}\,{}_2F_1\left(3,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{a^3(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x^n)^3,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[3, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)])/(a
^3*(1 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^m}{(a+bx^n)^3}\,dx = \frac{x^{1+m}\,{}_2F_1\left(3,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{a^3(1+m)}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.02

$$\frac{x^{m+1}\,{}_2F_1\left(3,\frac{m+1}{n};\frac{m+1}{n}+1;-\frac{bx^n}{a}\right)}{a^3(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x^n)^3,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[3, (1 + m)/n, 1 + (1 + m)/n, -((b*x^n)/a)])/(a
^3*(1 + m))
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m}{b^3x^{3\,n}+3\,ab^2x^{2\,n}+3\,a^2bx^n+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m/(b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b*x^n + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(bx^n + a)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*x^n + a)^3, x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(b\,x^n + a)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^n+a)^3,x)
```

```
[Out] int(x^m/(b*x^n+a)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(m^2 - m(3\,n-2) + 2\,n^2 - 3\,n + 1\right) \int \frac{x^m}{2\left(a^2 b n^2 x^n + a^3 n^2\right)}\, dx - \frac{a(m - 3\,n + 1)x x^m + b(m - 2\,n + 1)x e^{\left(m \log(x) + n \log(\right.}}{2\left(a^2 b^2 n^2 x^{2\,n} + 2\,a^3 b n^2 x^n + a^4 n^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] (m^2 - m*(3*n - 2) + 2*n^2 - 3*n + 1)*integrate(1/2*x^m/(a^2*b*n^2*x^n + a^
3*n^2), x) - 1/2*(a*(m - 3*n + 1)*x*x^m + b*(m - 2*n + 1)*x*e^(m*log(x) + n
*log(x)))/(a^2*b^2*n^2*x^(2*n) + 2*a^3*b*n^2*x^n + a^4*n^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{(a + b\,x^n)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^n)^3,x)
```

```
[Out] int(x^m/(a + b*x^n)^3, x)
```

sympy [C] time = 4.75, size = 6803, normalized size = 170.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a+b*x**n)**3,x)
```

```
[Out] a*m**3*x*x**m*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n +
1/n)/(2*a**4*n**4*gamma(m/n + 1 + 1/n) + 6*a**3*b*n**4*x**n*gamma(m/n + 1 +
 1/n) + 6*a**2*b**2*n**4*x**(2*n)*gamma(m/n + 1 + 1/n) + 2*a*b**3*n**4*x**(
3*n)*gamma(m/n + 1 + 1/n)) - 3*a*m**2*n*x*x**m*lerchphi(b*x**n*exp_polar(I*
pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(2*a**4*n**4*gamma(m/n + 1 + 1/n) + 6
*a**3*b*n**4*x**n*gamma(m/n + 1 + 1/n) + 6*a**2*b**2*n**4*x**(2*n)*gamma(m/
n + 1 + 1/n) + 2*a*b**3*n**4*x**(3*n)*gamma(m/n + 1 + 1/n)) - a*m**2*n*x*x*
```

```
*m*gamma(m/n + 1/n)/(2*a**4*n**4*gamma(m/n + 1 + 1/n) + 6*a**3*b*n**4*x**n*
gamma(m/n + 1 + 1/n) + 6*a**2*b**2*n**4*x**(2*n)*gamma(m/n + 1 + 1/n) + 2*a
*b**3*n**4*x**(3*n)*gamma(m/n + 1 + 1/n)) + 3*a*m**2*x*x**m*lerchphi(b*x**n
*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(2*a**4*n**4*gamma(m/n +
 1 + 1/n) + 6*a**3*b*n**4*x**n*gamma(m/n + 1 + 1/n) + 6*a**2*b**2*n**4*x**(
2*n)*gamma(m/n + 1 + 1/n) + 2*a*b**3*n**4*x**(3*n)*gamma(m/n + 1 + 1/n)) +
2*a*m*n**2*x*x**m*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/
n + 1/n)/(2*a**4*n**4*gamma(m/n + 1 + 1/n) + 6*a**3*b*n**4*x**n*gamma(m/n +
 1 + 1/n) + 6*a**2*b**2*n**4*x**(2*n)*gamma(m/n + 1 + 1/n) + 2*a*b**3*n**4*
x**(3*n)*gamma(m/n + 1 + 1/n)) + 3*a*m*n**2*x*x**m*gamma(m/n + 1/n)/(2*a**4
*n**4*gamma(m/n + 1 + 1/n) + 6*a**3*b*n**4*x**n*gamma(m/n + 1 + 1/n) + 6*a*
*2*b**2*n**4*x**(2*n)*gamma(m/n + 1 + 1/n) + 2*a*b**3*n**4*x**(3*n)*gamma(m
/n + 1 + 1/n)) - 6*a*m*n*x*x**m*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n +
 1/n)*gamma(m/n + 1/n)/(2*a**4*n**4*gamma(m/n + 1 + 1/n) + 6*a**3*b*n**4*x*
*n*gamma(m/n + 1 + 1/n) + 6*a**2*b**2*n**4*x**(2*n)*gamma(m/n + 1 + 1/n) +
2*a*b**3*n**4*x**(3*n)*gamma(m/n + 1 + 1/n)) - 2*a*m*n*x*x**m*gamma(m/n + 1
/n)/(2*a**4*n**4*gamma(m/n + 1 + 1/n) + 6*a**3*b*n**4*x**n*gamma(m/n + 1 +
1/n) + 6*a**2*b**2*n**4*x**(2*n)*gamma(m/n + 1 + 1/n) + 2*a*b**3*n**4*x**(3
*n)*gamma(m/n + 1 + 1/n)) + 3*a*m*x*x**m*lerchphi(b*x**n*exp_polar(I*pi)/a,
 1, m/n + 1/n)*gamma(m/n + 1/n)/(2*a**4*n**4*gamma(m/n + 1 + 1/n) + 6*a**3*
b*n**4*x**n*gamma(m/n + 1 + 1/n) + 6*a**2*b**2*n**4*x**(2*n)*gamma(m/n + 1
+ 1/n) + 2*a*b**3*n**4*x**(3*n)*gamma(m/n + 1 + 1/n)) + 2*a*n**2*x*x**m*ler
chphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(2*a**4*n**4
*gamma(m/n + 1 + 1/n) + 6*a**3*b*n**4*x**n*gamma(m/n + 1 + 1/n) + 6*a**2*b*
*2*n**4*x**(2*n)*gamma(m/n + 1 + 1/n) + 2*a*b**3*n**4*x**(3*n)*gamma(m/n +
1 + 1/n)) + 3*a*n**2*x*x**m*gamma(m/n + 1/n)/(2*a**4*n**4*gamma(m/n + 1 + 1
/n) + 6*a**3*b*n**4*x**n*gamma(m/n + 1 + 1/n) + 6*a**2*b**2*n**4*x**(2*n)*g
amma(m/n + 1 + 1/n) + 2*a*b**3*n**4*x**(3*n)*gamma(m/n + 1 + 1/n)) - 3*a*n*
x*x**m*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(2
*a**4*n**4*gamma(m/n + 1 + 1/n) + 6*a**3*b*n**4*x**n*gamma(m/n + 1 + 1/n) +
 6*a**2*b**2*n**4*x**(2*n)*gamma(m/n + 1 + 1/n) + 2*a*b**3*n**4*x**(3*n)*ga
mma(m/n + 1 + 1/n)) - a*n*x*x**m*gamma(m/n + 1/n)/(2*a**4*n**4*gamma(m/n +
1 + 1/n) + 6*a**3*b*n**4*x**n*gamma(m/n + 1 + 1/n) + 6*a**2*b**2*n**4*x**(2
*n)*gamma(m/n + 1 + 1/n) + 2*a*b**3*n**4*x**(3*n)*gamma(m/n + 1 + 1/n)) + a
*x*x**m*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(
2*a**4*n**4*gamma(m/n + 1 + 1/n) + 6*a**3*b*n**4*x**n*gamma(m/n + 1 + 1/n)
+ 6*a**2*b**2*n**4*x**(2*n)*gamma(m/n + 1 + 1/n) + 2*a*b**3*n**4*x**(3*n)*g
amma(m/n + 1 + 1/n)) + 3*b*m**3*x*x**m*x**n*lerchphi(b*x**n*exp_polar(I*pi)
/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(2*a**4*n**4*gamma(m/n + 1 + 1/n) + 6*a*
*3*b*n**4*x**n*gamma(m/n + 1 + 1/n) + 6*a**2*b**2*n**4*x**(2*n)*gamma(m/n +
 1 + 1/n) + 2*a*b**3*n**4*x**(3*n)*gamma(m/n + 1 + 1/n)) - 9*b*m**2*n*x*x**
m*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(2
*a**4*n**4*gamma(m/n + 1 + 1/n) + 6*a**3*b*n**4*x**n*gamma(m/n + 1 + 1/n) +
 6*a**2*b**2*n**4*x**(2*n)*gamma(m/n + 1 + 1/n) + 2*a*b**3*n**4*x**(3*n)*ga
mma(m/n + 1 + 1/n)) - 2*b*m**2*n*x*x**m*x**n*gamma(m/n + 1/n)/(2*a**4*n**4*
gamma(m/n + 1 + 1/n) + 6*a**3*b*n**4*x**n*gamma(m/n + 1 + 1/n) + 6*a**2*b**
2*n**4*x**(2*n)*gamma(m/n + 1 + 1/n) + 2*a*b**3*n**4*x**(3*n)*gamma(m/n + 1
 + 1/n)) + 9*b*m**2*x*x**m*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n +
 1/n)*gamma(m/n + 1/n)/(2*a**4*n**4*gamma(m/n + 1 + 1/n) + 6*a**3*b*n**4*x*
*n*gamma(m/n + 1 + 1/n) + 6*a**2*b**2*n**4*x**(2*n)*gamma(m/n + 1 + 1/n) +
2*a*b**3*n**4*x**(3*n)*gamma(m/n + 1 + 1/n)) + 6*b*m*n**2*x*x**m*x**n*lerch
phi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(2*a**4*n**4*g
amma(m/n + 1 + 1/n) + 6*a**3*b*n**4*x**n*gamma(m/n + 1 + 1/n) + 6*a**2*b**2
*n**4*x**(2*n)*gamma(m/n + 1 + 1/n) + 2*a*b**3*n**4*x**(3*n)*gamma(m/n + 1
+ 1/n)) + 5*b*m*n**2*x*x**m*x**n*gamma(m/n + 1/n)/(2*a**4*n**4*gamma(m/n +
1 + 1/n) + 6*a**3*b*n**4*x**n*gamma(m/n + 1 + 1/n) + 6*a**2*b**2*n**4*x**(2
*n)*gamma(m/n + 1 + 1/n) + 2*a*b**3*n**4*x**(3*n)*gamma(m/n + 1 + 1/n)) - 1
8*b*m*n*x*x**m*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(
m/n + 1/n)/(2*a**4*n**4*gamma(m/n + 1 + 1/n) + 6*a**3*b*n**4*x**n*gamma(m/n
```

```
 + 1 + 1/n) + 6*a**2*b**2*n**4*x**(2*n)*gamma(m/n + 1 + 1/n) + 2*a*b**3*n**
4*x**(3*n)*gamma(m/n + 1 + 1/n)) - 4*b*m*n*x*x**m*x**n*gamma(m/n + 1/n)/(2*
a**4*n**4*gamma(m/n + 1 + 1/n) + 6*a**3*b*n**4*x**n*gamma(m/n + 1 + 1/n) +
6*a**2*b**2*n**4*x**(2*n)*gamma(m/n + 1 + 1/n) + 2*a*b**3*n**4*x**(3*n)*gam
ma(m/n + 1 + 1/n)) + 9*b*m*x*x**m*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1
, m/n + 1/n)*gamma(m/n + 1/n)/(2*a**4*n**4*gamma(m/n + 1 + 1/n) + 6*a**3*b*
n**4*x**n*gamma(m/n + 1 + 1/n) + 6*a**2*b**2*n**4*x**(2*n)*gamma(m/n + 1 +
1/n) + 2*a*b**3*n**4*x**(3*n)*gamma(m/n + 1 + 1/n)) + 6*b*n**2*x*x**m*x**n*
lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(2*a**4*n
**4*gamma(m/n + 1 + 1/n) + 6*a**3*b*n**4*x**n*gamma(m/n + 1 + 1/n) + 6*a**2
*b**2*n**4*x**(2*n)*gamma(m/n + 1 + 1/n) + 2*a*b**3*n**4*x**(3*n)*gamma(m/n
 + 1 + 1/n)) + 5*b*n**2*x*x**m*x**n*gamma(m/n + 1/n)/(2*a**4*n**4*gamma(m/n
 + 1 + 1/n) + 6*a**3*b*n**4*x**n*gamma(m/n + 1 + 1/n) + 6*a**2*b**2*n**4*x*
*(2*n)*gamma(m/n + 1 + 1/n) + 2*a*b**3*n**4*x**(3*n)*gamma(m/n + 1 + 1/n))
- 9*b*n*x*x**m*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(
m/n + 1/n)/(2*a**4*n**4*gamma(m/n + 1 + 1/n) + 6*a**3*b*n**4*x**n*gamma(m/n
 + 1 + 1/n) + 6*a**2*b**2*n**4*x**(2*n)*gamma(m/n + 1 + 1/n) + 2*a*b**3*n**
4*x**(3*n)*gamma(m/n + 1 + 1/n)) - 2*b*n*x*x**m*x**n*gamma(m/n + 1/n)/(2*a*
*4*n**4*gamma(m/n + 1 + 1/n) + 6*a**3*b*n**4*x**n*gamma(m/n + 1 + 1/n) + 6*
a**2*b**2*n**4*x**(2*n)*gamma(m/n + 1 + 1/n) + 2*a*b**3*n**4*x**(3*n)*gamma
(m/n + 1 + 1/n)) + 3*b*x*x**m*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/
n + 1/n)*gamma(m/n + 1/n)/(2*a**4*n**4*gamma(m/n + 1 + 1/n) + 6*a**3*b*n**4
*x**n*gamma(m/n + 1 + 1/n) + 6*a**2*b**2*n**4*x**(2*n)*gamma(m/n + 1 + 1/n)
 + 2*a*b**3*n**4*x**(3*n)*gamma(m/n + 1 + 1/n)) + 3*b**2*m**3*x*x**m*x**(2*
n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(a*(2*
a**4*n**4*gamma(m/n + 1 + 1/n) + 6*a**3*b*n**4*x**n*gamma(m/n + 1 + 1/n) +
6*a**2*b**2*n**4*x**(2*n)*gamma(m/n + 1 + 1/n) + 2*a*b**3*n**4*x**(3*n)*gam
ma(m/n + 1 + 1/n))) - 9*b**2*m**2*n*x*x**m*x**(2*n)*lerchphi(b*x**n*exp_pol
ar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(a*(2*a**4*n**4*gamma(m/n + 1 +
1/n) + 6*a**3*b*n**4*x**n*gamma(m/n + 1 + 1/n) + 6*a**2*b**2*n**4*x**(2*n)*
gamma(m/n + 1 + 1/n) + 2*a*b**3*n**4*x**(3*n)*gamma(m/n + 1 + 1/n))) - b**2
*m**2*n*x*x**m*x**(2*n)*gamma(m/n + 1/n)/(a*(2*a**4*n**4*gamma(m/n + 1 + 1/
n) + 6*a**3*b*n**4*x**n*gamma(m/n + 1 + 1/n) + 6*a**2*b**2*n**4*x**(2*n)*ga
mma(m/n + 1 + 1/n) + 2*a*b**3*n**4*x**(3*n)*gamma(m/n + 1 + 1/n))) + 9*b**2
*m**2*x*x**m*x**(2*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamm
a(m/n + 1/n)/(a*(2*a**4*n**4*gamma(m/n + 1 + 1/n) + 6*a**3*b*n**4*x**n*gamm
a(m/n + 1 + 1/n) + 6*a**2*b**2*n**4*x**(2*n)*gamma(m/n + 1 + 1/n) + 2*a*b**
3*n**4*x**(3*n)*gamma(m/n + 1 + 1/n))) + 6*b**2*m*n**2*x*x**m*x**(2*n)*lerc
hphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(a*(2*a**4*n*
*4*gamma(m/n + 1 + 1/n) + 6*a**3*b*n**4*x**n*gamma(m/n + 1 + 1/n) + 6*a**2*
b**2*n**4*x**(2*n)*gamma(m/n + 1 + 1/n) + 2*a*b**3*n**4*x**(3*n)*gamma(m/n
+ 1 + 1/n))) + 2*b**2*m*n**2*x*x**m*x**(2*n)*gamma(m/n + 1/n)/(a*(2*a**4*n*
*4*gamma(m/n + 1 + 1/n) + 6*a**3*b*n**4*x**n*gamma(m/n + 1 + 1/n) + 6*a**2*
b**2*n**4*x**(2*n)*gamma(m/n + 1 + 1/n) + 2*a*b**3*n**4*x**(3*n)*gamma(m/n
+ 1 + 1/n))) - 18*b**2*m*n*x*x**m*x**(2*n)*lerchphi(b*x**n*exp_polar(I*pi)/
a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(a*(2*a**4*n**4*gamma(m/n + 1 + 1/n) + 6*
a**3*b*n**4*x**n*gamma(m/n + 1 + 1/n) + 6*a**2*b**2*n**4*x**(2*n)*gamma(m/n
 + 1 + 1/n) + 2*a*b**3*n**4*x**(3*n)*gamma(m/n + 1 + 1/n))) - 2*b**2*m*n*x*
x**m*x**(2*n)*gamma(m/n + 1/n)/(a*(2*a**4*n**4*gamma(m/n + 1 + 1/n) + 6*a**
3*b*n**4*x**n*gamma(m/n + 1 + 1/n) + 6*a**2*b**2*n**4*x**(2*n)*gamma(m/n +
1 + 1/n) + 2*a*b**3*n**4*x**(3*n)*gamma(m/n + 1 + 1/n))) + 9*b**2*m*x*x**m*
x**(2*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/
(a*(2*a**4*n**4*gamma(m/n + 1 + 1/n) + 6*a**3*b*n**4*x**n*gamma(m/n + 1 + 1
/n) + 6*a**2*b**2*n**4*x**(2*n)*gamma(m/n + 1 + 1/n) + 2*a*b**3*n**4*x**(3*
n)*gamma(m/n + 1 + 1/n))) + 6*b**2*n**2*x*x**m*x**(2*n)*lerchphi(b*x**n*exp
_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(a*(2*a**4*n**4*gamma(m/n +
1 + 1/n) + 6*a**3*b*n**4*x**n*gamma(m/n + 1 + 1/n) + 6*a**2*b**2*n**4*x**(2
*n)*gamma(m/n + 1 + 1/n) + 2*a*b**3*n**4*x**(3*n)*gamma(m/n + 1 + 1/n))) +
2*b**2*n**2*x*x**m*x**(2*n)*gamma(m/n + 1/n)/(a*(2*a**4*n**4*gamma(m/n + 1
```

```
+ 1/n) + 6*a**3*b*n**4*x**n*gamma(m/n + 1 + 1/n) + 6*a**2*b**2*n**4*x**(2*n
)*gamma(m/n + 1 + 1/n) + 2*a*b**3*n**4*x**(3*n)*gamma(m/n + 1 + 1/n))) - 9*
b**2*n*x*x**m*x**(2*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gam
ma(m/n + 1/n)/(a*(2*a**4*n**4*gamma(m/n + 1 + 1/n) + 6*a**3*b*n**4*x**n*gam
ma(m/n + 1 + 1/n) + 6*a**2*b**2*n**4*x**(2*n)*gamma(m/n + 1 + 1/n) + 2*a*b*
*3*n**4*x**(3*n)*gamma(m/n + 1 + 1/n))) - b**2*n*x*x**m*x**(2*n)*gamma(m/n
+ 1/n)/(a*(2*a**4*n**4*gamma(m/n + 1 + 1/n) + 6*a**3*b*n**4*x**n*gamma(m/n
+ 1 + 1/n) + 6*a**2*b**2*n**4*x**(2*n)*gamma(m/n + 1 + 1/n) + 2*a*b**3*n**4
*x**(3*n)*gamma(m/n + 1 + 1/n))) + 3*b**2*x*x**m*x**(2*n)*lerchphi(b*x**n*e
xp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(a*(2*a**4*n**4*gamma(m/n
+ 1 + 1/n) + 6*a**3*b*n**4*x**n*gamma(m/n + 1 + 1/n) + 6*a**2*b**2*n**4*x**
(2*n)*gamma(m/n + 1 + 1/n) + 2*a*b**3*n**4*x**(3*n)*gamma(m/n + 1 + 1/n)))
+ b**3*m**3*x*x**m*x**(3*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n
)*gamma(m/n + 1/n)/(a**2*(2*a**4*n**4*gamma(m/n + 1 + 1/n) + 6*a**3*b*n**4*
x**n*gamma(m/n + 1 + 1/n) + 6*a**2*b**2*n**4*x**(2*n)*gamma(m/n + 1 + 1/n)
+ 2*a*b**3*n**4*x**(3*n)*gamma(m/n + 1 + 1/n))) - 3*b**3*m**2*n*x*x**m*x**(
3*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(a**
2*(2*a**4*n**4*gamma(m/n + 1 + 1/n) + 6*a**3*b*n**4*x**n*gamma(m/n + 1 + 1/
n) + 6*a**2*b**2*n**4*x**(2*n)*gamma(m/n + 1 + 1/n) + 2*a*b**3*n**4*x**(3*n
)*gamma(m/n + 1 + 1/n))) + 3*b**3*m**2*x*x**m*x**(3*n)*lerchphi(b*x**n*exp_
polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(a**2*(2*a**4*n**4*gamma(m/n
+ 1 + 1/n) + 6*a**3*b*n**4*x**n*gamma(m/n + 1 + 1/n) + 6*a**2*b**2*n**4*x**
(2*n)*gamma(m/n + 1 + 1/n) + 2*a*b**3*n**4*x**(3*n)*gamma(m/n + 1 + 1/n)))
+ 2*b**3*m*n**2*x*x**m*x**(3*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n +
 1/n)*gamma(m/n + 1/n)/(a**2*(2*a**4*n**4*gamma(m/n + 1 + 1/n) + 6*a**3*b*n
**4*x**n*gamma(m/n + 1 + 1/n) + 6*a**2*b**2*n**4*x**(2*n)*gamma(m/n + 1 + 1
/n) + 2*a*b**3*n**4*x**(3*n)*gamma(m/n + 1 + 1/n))) - 6*b**3*m*n*x*x**m*x**
(3*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(a*
*2*(2*a**4*n**4*gamma(m/n + 1 + 1/n) + 6*a**3*b*n**4*x**n*gamma(m/n + 1 + 1
/n) + 6*a**2*b**2*n**4*x**(2*n)*gamma(m/n + 1 + 1/n) + 2*a*b**3*n**4*x**(3*
n)*gamma(m/n + 1 + 1/n))) + 3*b**3*m*x*x**m*x**(3*n)*lerchphi(b*x**n*exp_po
lar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(a**2*(2*a**4*n**4*gamma(m/n +
1 + 1/n) + 6*a**3*b*n**4*x**n*gamma(m/n + 1 + 1/n) + 6*a**2*b**2*n**4*x**(2
*n)*gamma(m/n + 1 + 1/n) + 2*a*b**3*n**4*x**(3*n)*gamma(m/n + 1 + 1/n))) +
2*b**3*n**2*x*x**m*x**(3*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n
)*gamma(m/n + 1/n)/(a**2*(2*a**4*n**4*gamma(m/n + 1 + 1/n) + 6*a**3*b*n**4*
x**n*gamma(m/n + 1 + 1/n) + 6*a**2*b**2*n**4*x**(2*n)*gamma(m/n + 1 + 1/n)
+ 2*a*b**3*n**4*x**(3*n)*gamma(m/n + 1 + 1/n))) - 3*b**3*n*x*x**m*x**(3*n)*
lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(a**2*(2*
a**4*n**4*gamma(m/n + 1 + 1/n) + 6*a**3*b*n**4*x**n*gamma(m/n + 1 + 1/n) +
6*a**2*b**2*n**4*x**(2*n)*gamma(m/n + 1 + 1/n) + 2*a*b**3*n**4*x**(3*n)*gam
ma(m/n + 1 + 1/n))) + b**3*x*x**m*x**(3*n)*lerchphi(b*x**n*exp_polar(I*pi)/
a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(a**2*(2*a**4*n**4*gamma(m/n + 1 + 1/n) +
 6*a**3*b*n**4*x**n*gamma(m/n + 1 + 1/n) + 6*a**2*b**2*n**4*x**(2*n)*gamma(
m/n + 1 + 1/n) + 2*a*b**3*n**4*x**(3*n)*gamma(m/n + 1 + 1/n)))
```

3.2673 $\int x^m (a+bx^n)^{3/2}\,dx$

Optimal. Leaf size=55

$$\frac{x^{m+1}(a+bx^n)^{5/2}\,{}_2F_1\left(1,\frac{m+1}{n}+\frac{5}{2};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{a(m+1)}$$

```
[Out] x^(1+m)*(a+b*x^n)^(5/2)*hypergeom([1, 5/2+(1+m)/n],[(1+m+n)/n],-b*x^n/a)/a/
(1+m)
```

Rubi [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {365, 364}

$$\frac{ax^{m+1}\sqrt{a+bx^n}\,{}_2F_1\left(-\frac{3}{2},\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{(m+1)\sqrt{\frac{bx^n}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^n)^(3/2),x]
```

```
[Out] (a*x^(1 + m)*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, (1 + m)/n, (1 + m + n)
/n, -((b*x^n)/a)])/((1 + m)*Sqrt[1 + (b*x^n)/a])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^m (a+bx^n)^{3/2}\,dx &= \frac{\left(a\sqrt{a+bx^n}\right)\int x^m\left(1+\frac{bx^n}{a}\right)^{3/2}dx}{\sqrt{1+\frac{bx^n}{a}}}\\ &= \frac{ax^{1+m}\sqrt{a+bx^n}\,{}_2F_1\left(-\frac{3}{2},\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{(1+m)\sqrt{1+\frac{bx^n}{a}}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 66, normalized size = 1.20

$$\frac{ax^{m+1}\sqrt{a+bx^n}\,{}_2F_1\left(-\frac{3}{2},\frac{m+1}{n};\frac{m+1}{n}+1;-\frac{bx^n}{a}\right)}{(m+1)\sqrt{\frac{bx^n}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^n)^(3/2),x]
```

```
[Out] (a*x^(1 + m)*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, (1 + m)/n, 1 + (1 + m)
/n, -((b*x^n)/a)])/((1 + m)*Sqrt[1 + (b*x^n)/a])
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^{\frac{3}{2}} x^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^(3/2)*x^m, x)
```

maple [**F**] time = 0.21, size = 0, normalized size = 0.00

$$\int (b\,x^n + a)^{\frac{3}{2}}\, x^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^n+a)^(3/2),x)
```

```
[Out] int(x^m*(b*x^n+a)^(3/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^{\frac{3}{2}} x^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^(3/2)*x^m, x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int x^m \, (a + b\,x^n)^{3/2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^n)^(3/2),x)
```

```
[Out] int(x^m*(a + b*x^n)^(3/2), x)
```

sympy [**C**] time = 15.81, size = 58, normalized size = 1.05

$$\frac{a^{\frac{3}{2}} x x^m \Gamma\left(\frac{m}{n} + \frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{m}{n} + \frac{1}{n} \\ \frac{m}{n} + 1 + \frac{1}{n} \end{matrix} \middle| \frac{b x^n e^{i\pi}}{a}\right)}{n \Gamma\left(\frac{m}{n} + 1 + \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*x**n)**(3/2),x)
```

```
[Out] a**(3/2)*x*x**m*gamma(m/n + 1/n)*hyper((-3/2, m/n + 1/n), (m/n + 1 + 1/n,),
 b*x**n*exp_polar(I*pi)/a)/(n*gamma(m/n + 1 + 1/n))
```

3.2674 $\int x^m \sqrt{a+bx^n}\, dx$

Optimal. Leaf size=55

$$\frac{x^{m+1}\,(a+bx^n)^{3/2}\;{}_2F_1\left(1,\frac{m+1}{n}+\frac{3}{2};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{a(m+1)}$$

```
[Out] x^(1+m)*(a+b*x^n)^(3/2)*hypergeom([1, 3/2+(1+m)/n],[(1+m+n)/n],-b*x^n/a)/a/
(1+m)
```

Rubi [A] time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.16, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {365, 364}

$$\frac{x^{m+1}\sqrt{a+bx^n}\;{}_2F_1\left(-\frac{1}{2},\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{(m+1)\sqrt{\frac{bx^n}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sqrt[a + b*x^n],x]
```

```
[Out] (x^(1 + m)*Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, (1 + m)/n, (1 + m + n)/n
, -((b*x^n)/a)])/((1 + m)*Sqrt[1 + (b*x^n)/a])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^m\sqrt{a+bx^n}\,dx &= \frac{\sqrt{a+bx^n}\int x^m\sqrt{1+\frac{bx^n}{a}}\,dx}{\sqrt{1+\frac{bx^n}{a}}}\\ &= \frac{x^{1+m}\sqrt{a+bx^n}\;{}_2F_1\left(-\frac{1}{2},\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{(1+m)\sqrt{1+\frac{bx^n}{a}}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 65, normalized size = 1.18

$$\frac{x^{m+1}\sqrt{a+bx^n}\;{}_2F_1\left(-\frac{1}{2},\frac{m+1}{n};\frac{m+1}{n}+1;-\frac{bx^n}{a}\right)}{(m+1)\sqrt{\frac{bx^n}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sqrt[a + b*x^n],x]
```

```
[Out] (x^(1 + m)*Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, (1 + m)/n, 1 + (1 + m)/n
, -((b*x^n)/a)])/((1 + m)*Sqrt[1 + (b*x^n)/a])
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^n + a}\, x^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n + a)*x^m, x)
```

maple [**F**] time = 0.21, size = 0, normalized size = 0.00

$$\int \sqrt{b\,x^n + a}\ x^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^n+a)^(1/2),x)
```

```
[Out] int(x^m*(b*x^n+a)^(1/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^n + a}\, x^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^n + a)*x^m, x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int x^m \sqrt{a + b\,x^n}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^n)^(1/2),x)
```

```
[Out] int(x^m*(a + b*x^n)^(1/2), x)
```

sympy [**C**] time = 2.04, size = 58, normalized size = 1.05

$$\frac{\sqrt{a}\, x x^m \Gamma\left(\frac{m}{n} + \frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{n} + \frac{1}{n} \\ \frac{m}{n} + 1 + \frac{1}{n} \end{matrix} \middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n} + 1 + \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*x**n)**(1/2),x)
```

```
[Out] sqrt(a)*x*x**m*gamma(m/n + 1/n)*hyper((-1/2, m/n + 1/n), (m/n + 1 + 1/n,),
b*x**n*exp_polar(I*pi)/a)/(n*gamma(m/n + 1 + 1/n))
```

3.2675 $\int \frac{x^m}{\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=55

$$\frac{x^{m+1}\sqrt{a+bx^n}\;{}_2F_1\left(1,\frac{m+1}{n}+\frac{1}{2};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{a(m+1)}$$

```
[Out] x^(1+m)*hypergeom([1, 1/2+(1+m)/n],[(1+m+n)/n],-b*x^n/a)*(a+b*x^n)^(1/2)/a/
(1+m)
```

Rubi [A] time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.16, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {365, 364}

$$\frac{x^{m+1}\sqrt{\frac{bx^n}{a}+1}\;{}_2F_1\left(\frac{1}{2},\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{(m+1)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/Sqrt[a + b*x^n],x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m)/n, (1 + m + n
)/n, -((b*x^n)/a)])/((1 + m)*Sqrt[a + b*x^n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{\sqrt{a+bx^n}}\,dx &= \frac{\sqrt{1+\frac{bx^n}{a}}\int \frac{x^m}{\sqrt{1+\frac{bx^n}{a}}}\,dx}{\sqrt{a+bx^n}}\\ &= \frac{x^{1+m}\sqrt{1+\frac{bx^n}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{(1+m)\sqrt{a+bx^n}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 65, normalized size = 1.18

$$\frac{x^{m+1}\sqrt{\frac{bx^n}{a}+1}\;{}_2F_1\left(\frac{1}{2},\frac{m+1}{n};\frac{m+1}{n}+1;-\frac{bx^n}{a}\right)}{(m+1)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/Sqrt[a + b*x^n],x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m)/n, 1 + (1 + m
)/n, -((b*x^n)/a)])/((1 + m)*Sqrt[a + b*x^n])
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{bx^n + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/sqrt(b*x^n + a), x)
```

maple [**F**] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{b\,x^n + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^n+a)^(1/2),x)
```

```
[Out] int(x^m/(b*x^n+a)^(1/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{bx^n + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/sqrt(b*x^n + a), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{\sqrt{a + b\,x^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^n)^(1/2),x)
```

```
[Out] int(x^m/(a + b*x^n)^(1/2), x)
```

sympy [**C**] time = 1.56, size = 56, normalized size = 1.02

$$\frac{x x^m \Gamma\left(\frac{m}{n} + \frac{1}{n}\right) {}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{m}{n} + \frac{1}{n} \\ \frac{m}{n} + 1 + \frac{1}{n}\end{matrix}\middle| \frac{b x^n e^{i\pi}}{a}\right)}{\sqrt{a}\, n \Gamma\left(\frac{m}{n} + 1 + \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a+b*x**n)**(1/2),x)
```

```
[Out] x*x**m*gamma(m/n + 1/n)*hyper((1/2, m/n + 1/n), (m/n + 1 + 1/n,), b*x**n*ex
p_polar(I*pi)/a)/(sqrt(a)*n*gamma(m/n + 1 + 1/n))
```

3.2676 $\int \frac{x^m}{(a+bx^n)^{3/2}}\,dx$

Optimal. Leaf size=55

$$\frac{x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{n}-\frac{1}{2};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{a(m+1)\sqrt{a+bx^n}}$$

```
[Out] x^(1+m)*hypergeom([1, -1/2+(1+m)/n],[(1+m+n)/n],-b*x^n/a)/a/(1+m)/(a+b*x^n)
^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {365, 364}

$$\frac{x^{m+1}\sqrt{\frac{bx^n}{a}+1}\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{a(m+1)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x^n)^(3/2),x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[3/2, (1 + m)/n, (1 + m + n
)/n, -((b*x^n)/a)])/(a*(1 + m)*Sqrt[a + b*x^n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{(a+bx^n)^{3/2}}\,dx &= \frac{\sqrt{1+\frac{bx^n}{a}}\int \frac{x^m}{\left(1+\frac{bx^n}{a}\right)^{3/2}}\,dx}{a\sqrt{a+bx^n}}\\ &= \frac{x^{1+m}\sqrt{1+\frac{bx^n}{a}}\,{}_2F_1\left(\frac{3}{2},\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{a(1+m)\sqrt{a+bx^n}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 68, normalized size = 1.24

$$\frac{x^{m+1}\sqrt{\frac{bx^n}{a}+1}\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{n};\frac{m+1}{n}+1;-\frac{bx^n}{a}\right)}{a(m+1)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x^n)^(3/2),x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[3/2, (1 + m)/n, 1 + (1 + m
)/n, -((b*x^n)/a)])/(a*(1 + m)*Sqrt[a + b*x^n])
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(bx^n + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*x^n + a)^(3/2), x)
```

maple [**F**] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(b\,x^n + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^n+a)^(3/2),x)
```

```
[Out] int(x^m/(b*x^n+a)^(3/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(bx^n + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(b*x^n + a)^(3/2), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{(a + b\,x^n)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^n)^(3/2),x)
```

```
[Out] int(x^m/(a + b*x^n)^(3/2), x)
```

sympy [C] time = 2.96, size = 56, normalized size = 1.02

$$\frac{xx^m\Gamma\left(\frac{m}{n}+\frac{1}{n}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{m}{n}+\frac{1}{n}\\ \frac{m}{n}+1+\frac{1}{n}\end{matrix}\right|\frac{bx^ne^{i\pi}}{a}\right)}{a^{\frac{3}{2}}n\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a+b*x**n)**(3/2),x)
```

```
[Out] x*x**m*gamma(m/n + 1/n)*hyper((3/2, m/n + 1/n), (m/n + 1 + 1/n,), b*x**n*ex
p_polar(I*pi)/a)/(a**(3/2)*n*gamma(m/n + 1 + 1/n))
```

3.2677 $\int \frac{x^m}{(a+bx^n)^{5/2}}\,dx$

Optimal. Leaf size=55

$$\frac{x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{n}-\frac{3}{2};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{a(m+1)\,(a+bx^n)^{3/2}}$$

```
[Out] x^(1+m)*hypergeom([1, -3/2+(1+m)/n],[(1+m+n)/n],-b*x^n/a)/a/(1+m)/(a+b*x^n)
^(3/2)
```

Rubi [A] time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {365, 364}

$$\frac{x^{m+1}\sqrt{\frac{bx^n}{a}+1}\;{}_2F_1\left(\frac{5}{2},\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{a^2(m+1)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x^n)^(5/2),x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[5/2, (1 + m)/n, (1 + m + n
)/n, -((b*x^n)/a)])/(a^2*(1 + m)*Sqrt[a + b*x^n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{(a+bx^n)^{5/2}}\,dx &= \frac{\sqrt{1+\frac{bx^n}{a}}\int \frac{x^m}{\left(1+\frac{bx^n}{a}\right)^{5/2}}\,dx}{a^2\sqrt{a+bx^n}}\\ &= \frac{x^{1+m}\sqrt{1+\frac{bx^n}{a}}\;{}_2F_1\left(\frac{5}{2},\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{a^2(1+m)\sqrt{a+bx^n}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 68, normalized size = 1.24

$$\frac{x^{m+1}\sqrt{\frac{bx^n}{a}+1}\;{}_2F_1\left(\frac{5}{2},\frac{m+1}{n};\frac{m+1}{n}+1;-\frac{bx^n}{a}\right)}{a^2(m+1)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x^n)^(5/2),x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[5/2, (1 + m)/n, 1 + (1 + m
)/n, -((b*x^n)/a)])/(a^2*(1 + m)*Sqrt[a + b*x^n])
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^n)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(bx^n + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^n)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*x^n + a)^(5/2), x)
```

maple [**F**] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(b\,x^n + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^n+a)^(5/2),x)
```

```
[Out] int(x^m/(b*x^n+a)^(5/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(bx^n + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^n)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(b*x^n + a)^(5/2), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{(a + b\,x^n)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^n)^(5/2),x)
```

```
[Out] int(x^m/(a + b*x^n)^(5/2), x)
```

sympy [C] time = 15.54, size = 56, normalized size = 1.02

$$\frac{xx^m\Gamma\left(\frac{m}{n}+\frac{1}{n}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{2}, \frac{m}{n}+\frac{1}{n}\\ \frac{m}{n}+1+\frac{1}{n}\end{matrix}\right|\frac{bx^ne^{i\pi}}{a}\right)}{a^{\frac{5}{2}}n\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a+b*x**n)**(5/2),x)
```

```
[Out] x*x**m*gamma(m/n + 1/n)*hyper((5/2, m/n + 1/n), (m/n + 1 + 1/n,), b*x**n*ex
p_polar(I*pi)/a)/(a**(5/2)*n*gamma(m/n + 1 + 1/n))
```

3.2678 $\int \frac{x^{3+2n}}{\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=59

$$\frac{x^{2(n+2)}\sqrt{a+bx^n}\;{}_2F_1\left(1,\frac{1}{2}\left(5+\frac{8}{n}\right);3+\frac{4}{n};-\frac{bx^n}{a}\right)}{2a(n+2)}$$

```
[Out] 1/2*x^(4+2*n)*hypergeom([1, 5/2+4/n],[3+4/n],-b*x^n/a)*(a+b*x^n)^(1/2)/a/(2
+n)
```

Rubi [A] time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.19, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {365, 364}

$$\frac{x^{2(n+2)}\sqrt{\frac{bx^n}{a}+1}\;{}_2F_1\left(\frac{1}{2},2\left(1+\frac{2}{n}\right);3+\frac{4}{n};-\frac{bx^n}{a}\right)}{2(n+2)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3 + 2*n)/Sqrt[a + b*x^n],x]
```

```
[Out] (x^(2*(2 + n))*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, 2*(1 + 2/n), 3 +
4/n, -((b*x^n)/a)])/(2*(2 + n)*Sqrt[a + b*x^n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{3+2n}}{\sqrt{a+bx^n}}\,dx &= \frac{\sqrt{1+\frac{bx^n}{a}}\int \frac{x^{3+2n}}{\sqrt{1+\frac{bx^n}{a}}}\,dx}{\sqrt{a+bx^n}}\\ &= \frac{x^{2(2+n)}\sqrt{1+\frac{bx^n}{a}}\;{}_2F_1\left(\frac{1}{2},2\left(1+\frac{2}{n}\right);3+\frac{4}{n};-\frac{bx^n}{a}\right)}{2(2+n)\sqrt{a+bx^n}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 68, normalized size = 1.15

$$\frac{x^{2n+4}\sqrt{\frac{bx^n}{a}+1}\;{}_2F_1\left(\frac{1}{2},2+\frac{4}{n};3+\frac{4}{n};-\frac{bx^n}{a}\right)}{2(n+2)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3 + 2*n)/Sqrt[a + b*x^n],x]
```

```
[Out] (x^(4 + 2*n)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, 2 + 4/n, 3 + 4/n, -
((b*x^n)/a)])/(2*(2 + n)*Sqrt[a + b*x^n])
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n+3}}{\sqrt{bx^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n + 3)/sqrt(b*x^n + a), x)
```

maple [**F**] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{x^{2n+3}}{\sqrt{b\,x^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n+3)/(b*x^n+a)^(1/2),x)
```

```
[Out] int(x^(2*n+3)/(b*x^n+a)^(1/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n+3}}{\sqrt{bx^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(2*n + 3)/sqrt(b*x^n + a), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{2n+3}}{\sqrt{a + b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n + 3)/(a + b*x^n)^(1/2),x)
```

```
[Out] int(x^(2*n + 3)/(a + b*x^n)^(1/2), x)
```

sympy [**C**] time = 29.26, size = 49, normalized size = 0.83

$$\frac{x^4 x^{2n}\Gamma\left(2+\frac{4}{n}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, 2+\frac{4}{n}\\ 3+\frac{4}{n}\end{matrix}\right| \frac{bx^n e^{i\pi}}{a}\right)}{\sqrt{a}\,n\Gamma\left(3+\frac{4}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3+2*n)/(a+b*x**n)**(1/2),x)
```

```
[Out] x**4*x**(2*n)*gamma(2 + 4/n)*hyper((1/2, 2 + 4/n), (3 + 4/n,), b*x**n*exp_p
olar(I*pi)/a)/(sqrt(a)*n*gamma(3 + 4/n))
```

3.2679 $\int \frac{x^{3+n}}{\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=56

$$\frac{x^{n+4}\sqrt{a+bx^n}\;{}_2F_1\left(1,\frac{1}{2}\left(3+\frac{8}{n}\right);2\left(1+\frac{2}{n}\right);-\frac{bx^n}{a}\right)}{a(n+4)}$$

```
[Out] x^(4+n)*hypergeom([1, 3/2+4/n],[2+4/n],-b*x^n/a)*(a+b*x^n)^(1/2)/a/(4+n)
```

Rubi [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.16, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {365, 364}

$$\frac{x^{n+4}\sqrt{\frac{bx^n}{a}+1}\;{}_2F_1\left(\frac{1}{2},\frac{n+4}{n};2\left(1+\frac{2}{n}\right);-\frac{bx^n}{a}\right)}{(n+4)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3 + n)/Sqrt[a + b*x^n],x]
```

```
[Out] (x^(4 + n)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (4 + n)/n, 2*(1 + 2/n
), -((b*x^n)/a)])/((4 + n)*Sqrt[a + b*x^n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{3+n}}{\sqrt{a+bx^n}}\,dx &= \frac{\sqrt{1+\frac{bx^n}{a}}\int \frac{x^{3+n}}{\sqrt{1+\frac{bx^n}{a}}}\,dx}{\sqrt{a+bx^n}}\\ &= \frac{x^{4+n}\sqrt{1+\frac{bx^n}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{4+n}{n};2\left(1+\frac{2}{n}\right);-\frac{bx^n}{a}\right)}{(4+n)\sqrt{a+bx^n}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 65, normalized size = 1.16

$$\frac{x^{n+4}\sqrt{\frac{bx^n}{a}+1}\;{}_2F_1\left(\frac{1}{2},\frac{n+4}{n};\frac{n+4}{n}+1;-\frac{bx^n}{a}\right)}{(n+4)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3 + n)/Sqrt[a + b*x^n],x]
```

```
[Out] (x^(4 + n)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (4 + n)/n, 1 + (4 + n
)/n, -((b*x^n)/a)])/((4 + n)*Sqrt[a + b*x^n])
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{n+3}}{\sqrt{bx^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(n + 3)/sqrt(b*x^n + a), x)
```

maple [**F**] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{x^{n+3}}{\sqrt{b\,x^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n+3)/(b*x^n+a)^(1/2),x)
```

```
[Out] int(x^(n+3)/(b*x^n+a)^(1/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{n+3}}{\sqrt{bx^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(n + 3)/sqrt(b*x^n + a), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{n+3}}{\sqrt{a + b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n + 3)/(a + b*x^n)^(1/2),x)
```

```
[Out] int(x^(n + 3)/(a + b*x^n)^(1/2), x)
```

sympy [**C**] time = 4.29, size = 48, normalized size = 0.86

$$\frac{x^4 x^n \Gamma\left(1 + \frac{4}{n}\right) {}_2F_1\left(\begin{matrix}\frac{1}{2}, 1 + \frac{4}{n} \\ 2 + \frac{4}{n}\end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{\sqrt{a}\, n \Gamma\left(2 + \frac{4}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3+n)/(a+b*x**n)**(1/2),x)
```

```
[Out] x**4*x**n*gamma(1 + 4/n)*hyper((1/2, 1 + 4/n), (2 + 4/n,), b*x**n*exp_polar
(I*pi)/a)/(sqrt(a)*n*gamma(2 + 4/n))
```

3.2680 $\int \frac{x^{3-n}}{\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=56

$$\frac{x^{4-n}\sqrt{a+bx^n}\;{}_2F_1\left(1,\frac{1}{2}\left(\frac{8}{n}-1\right);\frac{4}{n};-\frac{bx^n}{a}\right)}{a(4-n)}$$

```
[Out] x^(4-n)*hypergeom([1, -1/2+4/n],[4/n],-b*x^n/a)*(a+b*x^n)^(1/2)/a/(4-n)
```

Rubi [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.16, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {365, 364}

$$\frac{x^{4-n}\sqrt{\frac{bx^n}{a}+1}\;{}_2F_1\left(\frac{1}{2},\frac{4}{n}-1;\frac{4}{n};-\frac{bx^n}{a}\right)}{(4-n)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3 - n)/Sqrt[a + b*x^n],x]
```

```
[Out] (x^(4 - n)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, -1 + 4/n, 4/n, -((b*x
^n)/a)])/((4 - n)*Sqrt[a + b*x^n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{3-n}}{\sqrt{a+bx^n}}\,dx &= \frac{\sqrt{1+\frac{bx^n}{a}}\int \frac{x^{3-n}}{\sqrt{1+\frac{bx^n}{a}}}\,dx}{\sqrt{a+bx^n}}\\ &= \frac{x^{4-n}\sqrt{1+\frac{bx^n}{a}}\;{}_2F_1\left(\frac{1}{2},-1+\frac{4}{n};\frac{4}{n};-\frac{bx^n}{a}\right)}{(4-n)\sqrt{a+bx^n}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 64, normalized size = 1.14

$$-\frac{x^{4-n}\sqrt{\frac{bx^n}{a}+1}\;{}_2F_1\left(\frac{1}{2},\frac{4}{n}-1;\frac{4}{n};-\frac{bx^n}{a}\right)}{(n-4)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3 - n)/Sqrt[a + b*x^n],x]
```

```
[Out] -((x^(4 - n)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, -1 + 4/n, 4/n, -((b
*x^n)/a)])/((-4 + n)*Sqrt[a + b*x^n]))
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3-n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-n+3}}{\sqrt{bx^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3-n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(-n + 3)/sqrt(b*x^n + a), x)
```

maple [**F**] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{x^{-n+3}}{\sqrt{b\,x^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3-n)/(b*x^n+a)^(1/2),x)
```

```
[Out] int(x^(3-n)/(b*x^n+a)^(1/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-n+3}}{\sqrt{bx^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3-n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(-n + 3)/sqrt(b*x^n + a), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{3-n}}{\sqrt{a + b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3 - n)/(a + b*x^n)^(1/2),x)
```

```
[Out] int(x^(3 - n)/(a + b*x^n)^(1/2), x)
```

sympy [**C**] time = 40.98, size = 44, normalized size = 0.79

$$\frac{x^4 x^{-n}\,\Gamma\left(-1 + \frac{4}{n}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2}, -1 + \frac{4}{n} \\ \frac{4}{n}\end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{\sqrt{a}\,n\,\Gamma\left(\frac{4}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3-n)/(a+b*x**n)**(1/2),x)
```

```
[Out] x**4*x**(-n)*gamma(-1 + 4/n)*hyper((1/2, -1 + 4/n), (4/n,), b*x**n*exp_pola
r(I*pi)/a)/(sqrt(a)*n*gamma(4/n))
```

3.2681 $\int \frac{x^{3-2n}}{\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=61

$$\frac{x^{4-2n}\sqrt{a+bx^n}\;{}_2F_1\left(1,\frac{1}{2}\left(\frac{8}{n}-3\right);\frac{4}{n}-1;-\frac{bx^n}{a}\right)}{2a(2-n)}$$

```
[Out] 1/2*x^(4-2*n)*hypergeom([1, -3/2+4/n],[-1+4/n],-b*x^n/a)*(a+b*x^n)^(1/2)/a/
(2-n)
```

Rubi [A] time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {365, 364}

$$\frac{x^{4-2n}\sqrt{\frac{bx^n}{a}+1}\;{}_2F_1\left(\frac{1}{2},-2\left(1-\frac{2}{n}\right);\frac{4}{n}-1;-\frac{bx^n}{a}\right)}{2(2-n)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3 - 2*n)/Sqrt[a + b*x^n],x]
```

```
[Out] (x^(4 - 2*n)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, -2*(1 - 2/n), -1 +
4/n, -((b*x^n)/a)])/(2*(2 - n)*Sqrt[a + b*x^n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{3-2n}}{\sqrt{a+bx^n}}\,dx &= \frac{\sqrt{1+\frac{bx^n}{a}}\int \frac{x^{3-2n}}{\sqrt{1+\frac{bx^n}{a}}}\,dx}{\sqrt{a+bx^n}}\\ &= \frac{x^{4-2n}\sqrt{1+\frac{bx^n}{a}}\;{}_2F_1\left(\frac{1}{2},-2\left(1-\frac{2}{n}\right);-1+\frac{4}{n};-\frac{bx^n}{a}\right)}{2(2-n)\sqrt{a+bx^n}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 68, normalized size = 1.11

$$-\frac{x^{4-2n}\sqrt{\frac{bx^n}{a}+1}\;{}_2F_1\left(\frac{1}{2},\frac{4}{n}-2;\frac{4}{n}-1;-\frac{bx^n}{a}\right)}{2(n-2)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3 - 2*n)/Sqrt[a + b*x^n],x]
```

```
[Out] -1/2*(x^(4 - 2*n)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, -2 + 4/n, -1 +
 4/n, -((b*x^n)/a)])/((-2 + n)*Sqrt[a + b*x^n])
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3-2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-2n+3}}{\sqrt{bx^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3-2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(-2*n + 3)/sqrt(b*x^n + a), x)
```

maple [**F**] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{x^{-2n+3}}{\sqrt{b\,x^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3-2*n)/(b*x^n+a)^(1/2),x)
```

```
[Out] int(x^(3-2*n)/(b*x^n+a)^(1/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-2n+3}}{\sqrt{bx^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3-2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(-2*n + 3)/sqrt(b*x^n + a), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{3-2n}}{\sqrt{a + b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3 - 2*n)/(a + b*x^n)^(1/2),x)
```

```
[Out] int(x^(3 - 2*n)/(a + b*x^n)^(1/2), x)
```

sympy [**C**] time = 37.27, size = 49, normalized size = 0.80

$$\frac{x^4 x^{-2n}\Gamma\left(-2+\frac{4}{n}\right){}_2F_1\left(\begin{matrix}\frac{1}{2}, -2+\frac{4}{n}\\ -1+\frac{4}{n}\end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{\sqrt{a}\,n\Gamma\left(-1+\frac{4}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3-2*n)/(a+b*x**n)**(1/2),x)
```

```
[Out] x**4*x**(-2*n)*gamma(-2 + 4/n)*hyper((1/2, -2 + 4/n), (-1 + 4/n,), b*x**n*e
xp_polar(I*pi)/a)/(sqrt(a)*n*gamma(-1 + 4/n))
```

3.2682 $\int \frac{x^{m+2n}}{\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=66

$$\frac{x^{m+2n+1}\sqrt{a+bx^n}\;{}_2F_1\left(1,\frac{1}{2}\left(\frac{2(m+1)}{n}+5\right);\frac{m+3n+1}{n};-\frac{bx^n}{a}\right)}{a(m+2n+1)}$$

```
[Out] x^(1+m+2*n)*hypergeom([1, 5/2+(1+m)/n],[(1+m+3*n)/n],-b*x^n/a)*(a+b*x^n)^(1
/2)/a/(1+m+2*n)
```

Rubi [A] time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {365, 364}

$$\frac{x^{m+2n+1}\sqrt{\frac{bx^n}{a}+1}\;{}_2F_1\left(\frac{1}{2},\frac{m+2n+1}{n};\frac{m+3n+1}{n};-\frac{bx^n}{a}\right)}{(m+2n+1)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(m + 2*n)/Sqrt[a + b*x^n],x]
```

```
[Out] (x^(1 + m + 2*n)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m + 2*n)/n
, (1 + m + 3*n)/n, -((b*x^n)/a)])/((1 + m + 2*n)*Sqrt[a + b*x^n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{m+2n}}{\sqrt{a+bx^n}}\,dx &= \frac{\sqrt{1+\frac{bx^n}{a}}\int \frac{x^{m+2n}}{\sqrt{1+\frac{bx^n}{a}}}\,dx}{\sqrt{a+bx^n}}\\ &= \frac{x^{1+m+2n}\sqrt{1+\frac{bx^n}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{1+m+2n}{n};\frac{1+m+3n}{n};-\frac{bx^n}{a}\right)}{(1+m+2n)\sqrt{a+bx^n}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 77, normalized size = 1.17

$$\frac{x^{m+2n+1}\sqrt{\frac{bx^n}{a}+1}\;{}_2F_1\left(\frac{1}{2},\frac{m+2n+1}{n};\frac{m+2n+1}{n}+1;-\frac{bx^n}{a}\right)}{(m+2n+1)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(m + 2*n)/Sqrt[a + b*x^n],x]
```

```
[Out] (x^(1 + m + 2*n)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m + 2*n)/n
, 1 + (1 + m + 2*n)/n, -((b*x^n)/a)])/((1 + m + 2*n)*Sqrt[a + b*x^n])
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(m+2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m+2\,n}}{\sqrt{b x^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(m+2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(m + 2*n)/sqrt(b*x^n + a), x)
```

maple [**F**] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{x^{m+2n}}{\sqrt{b\,x^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m+2*n)/(b*x^n+a)^(1/2),x)
```

```
[Out] int(x^(m+2*n)/(b*x^n+a)^(1/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m+2\,n}}{\sqrt{b x^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(m+2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(m + 2*n)/sqrt(b*x^n + a), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{m+2\,n}}{\sqrt{a + b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m + 2*n)/(a + b*x^n)^(1/2),x)
```

```
[Out] int(x^(m + 2*n)/(a + b*x^n)^(1/2), x)
```

sympy [**C**] time = 3.29, size = 65, normalized size = 0.98

$$\frac{x x^m x^{2n} \Gamma\left(\frac{m}{n} + 2 + \frac{1}{n}\right) {}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{m}{n} + 2 + \frac{1}{n} \\ \frac{m}{n} + 3 + \frac{1}{n}\end{matrix}\middle| \frac{b x^n e^{i\pi}}{a}\right)}{\sqrt{a}\, n \Gamma\left(\frac{m}{n} + 3 + \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(m+2*n)/(a+b*x**n)**(1/2),x)
```

```
[Out] x*x**m*x**(2*n)*gamma(m/n + 2 + 1/n)*hyper((1/2, m/n + 2 + 1/n), (m/n + 3 +
 1/n,), b*x**n*exp_polar(I*pi)/a)/(sqrt(a)*n*gamma(m/n + 3 + 1/n))
```

3.2683 $\int \frac{x^{m+n}}{\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=60

$$\frac{x^{m+n+1}\sqrt{a+bx^n}\;{}_2F_1\left(1,\frac{m+n+1}{n}+\frac{1}{2};\frac{m+2n+1}{n};-\frac{bx^n}{a}\right)}{a(m+n+1)}$$

```
[Out] x^(1+m+n)*hypergeom([1, 1/2+(1+m+n)/n],[(1+m+2*n)/n],-b*x^n/a)*(a+b*x^n)^(1
/2)/a/(1+m+n)
```

Rubi [A] time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.15, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {365, 364}

$$\frac{x^{m+n+1}\sqrt{\frac{bx^n}{a}+1}\;{}_2F_1\left(\frac{1}{2},\frac{m+n+1}{n};\frac{m+2n+1}{n};-\frac{bx^n}{a}\right)}{(m+n+1)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(m + n)/Sqrt[a + b*x^n],x]
```

```
[Out] (x^(1 + m + n)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m + n)/n, (1
 + m + 2*n)/n, -((b*x^n)/a)])/((1 + m + n)*Sqrt[a + b*x^n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{m+n}}{\sqrt{a+bx^n}}\,dx &= \frac{\sqrt{1+\frac{bx^n}{a}}\int \frac{x^{m+n}}{\sqrt{1+\frac{bx^n}{a}}}\,dx}{\sqrt{a+bx^n}}\\ &= \frac{x^{1+m+n}\sqrt{1+\frac{bx^n}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{1+m+n}{n};\frac{1+m+2n}{n};-\frac{bx^n}{a}\right)}{(1+m+n)\sqrt{a+bx^n}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 69, normalized size = 1.15

$$\frac{x^{m+n+1}\sqrt{\frac{bx^n}{a}+1}\;{}_2F_1\left(\frac{1}{2},\frac{m+n+1}{n};\frac{m+n+1}{n}+1;-\frac{bx^n}{a}\right)}{(m+n+1)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(m + n)/Sqrt[a + b*x^n],x]
```

```
[Out] (x^(1 + m + n)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m + n)/n, 1
+ (1 + m + n)/n, -((b*x^n)/a)])/((1 + m + n)*Sqrt[a + b*x^n])
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(m+n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m+n}}{\sqrt{bx^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(m+n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(m + n)/sqrt(b*x^n + a), x)
```

maple [**F**] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{x^{m+n}}{\sqrt{b\,x^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m+n)/(b*x^n+a)^(1/2),x)
```

```
[Out] int(x^(m+n)/(b*x^n+a)^(1/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m+n}}{\sqrt{bx^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(m+n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(m + n)/sqrt(b*x^n + a), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{m+n}}{\sqrt{a + b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m + n)/(a + b*x^n)^(1/2),x)
```

```
[Out] int(x^(m + n)/(a + b*x^n)^(1/2), x)
```

sympy [**C**] time = 4.50, size = 63, normalized size = 1.05

$$\frac{x x^m x^n \Gamma\left(\frac{m}{n} + 1 + \frac{1}{n}\right) {}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{m}{n} + 1 + \frac{1}{n} \\ \frac{m}{n} + 2 + \frac{1}{n}\end{matrix}\middle| \frac{b x^n e^{i\pi}}{a}\right)}{\sqrt{a}\, n \Gamma\left(\frac{m}{n} + 2 + \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(m+n)/(a+b*x**n)**(1/2),x)
```

```
[Out] x*x**m*x**n*gamma(m/n + 1 + 1/n)*hyper((1/2, m/n + 1 + 1/n), (m/n + 2 + 1/n
,), b*x**n*exp_polar(I*pi)/a)/(sqrt(a)*n*gamma(m/n + 2 + 1/n))
```

3.2684 $\int \frac{x^{m-n}}{\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=63

$$\frac{x^{m-n+1}\sqrt{a+bx^n}\;{}_2F_1\left(1,\frac{1}{2}\left(\frac{2(m+1)}{n}-1\right);\frac{m+1}{n};-\frac{bx^n}{a}\right)}{a(m-n+1)}$$

```
[Out] x^(1+m-n)*hypergeom([1, -1/2+(1+m)/n],[(1+m)/n],-b*x^n/a)*(a+b*x^n)^(1/2)/a
/(1+m-n)
```

Rubi [A] time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {365, 364}

$$\frac{x^{m-n+1}\sqrt{\frac{bx^n}{a}+1}\;{}_2F_1\left(\frac{1}{2},\frac{m-n+1}{n};\frac{m+1}{n};-\frac{bx^n}{a}\right)}{(m-n+1)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(m - n)/Sqrt[a + b*x^n],x]
```

```
[Out] (x^(1 + m - n)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m - n)/n, (1
 + m)/n, -((b*x^n)/a)])/((1 + m - n)*Sqrt[a + b*x^n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{m-n}}{\sqrt{a+bx^n}}\,dx &= \frac{\sqrt{1+\frac{bx^n}{a}}\int \frac{x^{m-n}}{\sqrt{1+\frac{bx^n}{a}}}\,dx}{\sqrt{a+bx^n}}\\ &= \frac{x^{1+m-n}\sqrt{1+\frac{bx^n}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{1+m-n}{n};\frac{1+m}{n};-\frac{bx^n}{a}\right)}{(1+m-n)\sqrt{a+bx^n}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 72, normalized size = 1.14

$$\frac{x^{m-n+1}\sqrt{\frac{bx^n}{a}+1}\;{}_2F_1\left(\frac{1}{2},\frac{m-n+1}{n};\frac{m+1}{n};-\frac{bx^n}{a}\right)}{(m-n+1)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(m - n)/Sqrt[a + b*x^n],x]
```

```
[Out] (x^(1 + m - n)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m - n)/n, (1
 + m)/n, -((b*x^n)/a)])/((1 + m - n)*Sqrt[a + b*x^n])
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(m-n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m-n}}{\sqrt{bx^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(m-n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(m - n)/sqrt(b*x^n + a), x)
```

maple [**F**] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{x^{m-n}}{\sqrt{b\,x^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m-n)/(b*x^n+a)^(1/2),x)
```

```
[Out] int(x^(m-n)/(b*x^n+a)^(1/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m-n}}{\sqrt{bx^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(m-n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(m - n)/sqrt(b*x^n + a), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{m-n}}{\sqrt{a + b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m - n)/(a + b*x^n)^(1/2),x)
```

```
[Out] int(x^(m - n)/(a + b*x^n)^(1/2), x)
```

sympy [**C**] time = 9.56, size = 60, normalized size = 0.95

$$\frac{x x^m x^{-n} \Gamma\left(\frac{m}{n} - 1 + \frac{1}{n}\right) {}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{m}{n} - 1 + \frac{1}{n} \\ \frac{m}{n} + \frac{1}{n}\end{matrix}\middle| \frac{b x^n e^{i\pi}}{a}\right)}{\sqrt{a}\, n \Gamma\left(\frac{m}{n} + \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(m-n)/(a+b*x**n)**(1/2),x)
```

```
[Out] x*x**m*x**(-n)*gamma(m/n - 1 + 1/n)*hyper((1/2, m/n - 1 + 1/n), (m/n + 1/n,
), b*x**n*exp_polar(I*pi)/a)/(sqrt(a)*n*gamma(m/n + 1/n))
```

3.2685 $\int \frac{x^{m-2n}}{\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=66

$$\frac{x^{m-2n+1}\sqrt{a+bx^n}\;{}_2F_1\left(1,\frac{1}{2}\left(\frac{2(m+1)}{n}-3\right);\frac{m-n+1}{n};-\frac{bx^n}{a}\right)}{a(m-2n+1)}$$

```
[Out] x^(1+m-2*n)*hypergeom([1, -3/2+(1+m)/n],[(1+m-n)/n],-b*x^n/a)*(a+b*x^n)^(1/
2)/a/(1+m-2*n)
```

Rubi [A] time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {365, 364}

$$\frac{x^{m-2n+1}\sqrt{\frac{bx^n}{a}+1}\;{}_2F_1\left(\frac{1}{2},\frac{m-2n+1}{n};\frac{m-n+1}{n};-\frac{bx^n}{a}\right)}{(m-2n+1)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(m - 2*n)/Sqrt[a + b*x^n],x]
```

```
[Out] (x^(1 + m - 2*n)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m - 2*n)/n
, (1 + m - n)/n, -((b*x^n)/a)])/((1 + m - 2*n)*Sqrt[a + b*x^n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{m-2n}}{\sqrt{a+bx^n}}\,dx &= \frac{\sqrt{1+\frac{bx^n}{a}}\int \frac{x^{m-2n}}{\sqrt{1+\frac{bx^n}{a}}}\,dx}{\sqrt{a+bx^n}}\\ &= \frac{x^{1+m-2n}\sqrt{1+\frac{bx^n}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{1+m-2n}{n};\frac{1+m-n}{n};-\frac{bx^n}{a}\right)}{(1+m-2n)\sqrt{a+bx^n}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 77, normalized size = 1.17

$$\frac{x^{m-2n+1}\sqrt{\frac{bx^n}{a}+1}\;{}_2F_1\left(\frac{1}{2},\frac{m-2n+1}{n};\frac{m-2n+1}{n}+1;-\frac{bx^n}{a}\right)}{(m-2n+1)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(m - 2*n)/Sqrt[a + b*x^n],x]
```

```
[Out] (x^(1 + m - 2*n)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m - 2*n)/n
, 1 + (1 + m - 2*n)/n, -((b*x^n)/a)])/((1 + m - 2*n)*Sqrt[a + b*x^n])
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(m-2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m-2\,n}}{\sqrt{b x^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(m-2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(m - 2*n)/sqrt(b*x^n + a), x)
```

maple [**F**] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{x^{m-2n}}{\sqrt{b\,x^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m-2*n)/(b*x^n+a)^(1/2),x)
```

```
[Out] int(x^(m-2*n)/(b*x^n+a)^(1/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m-2\,n}}{\sqrt{b x^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(m-2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(m - 2*n)/sqrt(b*x^n + a), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{m-2\,n}}{\sqrt{a + b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m - 2*n)/(a + b*x^n)^(1/2),x)
```

```
[Out] int(x^(m - 2*n)/(a + b*x^n)^(1/2), x)
```

sympy [**C**] time = 16.60, size = 65, normalized size = 0.98

$$\frac{x x^m x^{-2n} \Gamma\left(\frac{m}{n} - 2 + \frac{1}{n}\right) {}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{m}{n} - 2 + \frac{1}{n} \\ \frac{m}{n} - 1 + \frac{1}{n}\end{matrix}\middle| \frac{b x^n e^{i\pi}}{a}\right)}{\sqrt{a}\, n \Gamma\left(\frac{m}{n} - 1 + \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(m-2*n)/(a+b*x**n)**(1/2),x)
```

```
[Out] x*x**m*x**(-2*n)*gamma(m/n - 2 + 1/n)*hyper((1/2, m/n - 2 + 1/n), (m/n - 1
+ 1/n,), b*x**n*exp_polar(I*pi)/a)/(sqrt(a)*n*gamma(m/n - 1 + 1/n))
```

3.2686 $\int \left(-\frac{bnx^{-1+m+n}}{2(a+bx^n)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^n}}\right)dx$

Optimal. Leaf size=15

$$\frac{x^m}{\sqrt{a+bx^n}}$$

```
[Out] x^m/(a+b*x^n)^(1/2)
```

Rubi [C] time = 0.08, antiderivative size = 126, normalized size of antiderivative = 8.40, number of steps used = 5, number of rules used = 2, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {365, 364}

$$\frac{x^m\sqrt{\frac{bx^n}{a}+1}\ {}_2F_1\left(\frac{1}{2},\frac{m}{n};\frac{m+n}{n};-\frac{bx^n}{a}\right)}{\sqrt{a+bx^n}}-\frac{bnx^{m+n}\sqrt{\frac{bx^n}{a}+1}\ {}_2F_1\left(\frac{3}{2},\frac{m+n}{n};\frac{m}{n}+2;-\frac{bx^n}{a}\right)}{2a(m+n)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[-(b*n*x^(-1 + m + n))/(2*(a + b*x^n)^(3/2)) + (m*x^(-1 + m))/Sqrt[a + b
*x^n],x]
```

```
[Out] (x^m*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, m/n, (m + n)/n, -((b*x^n)/a
)])/Sqrt[a + b*x^n] - (b*n*x^(m + n)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[
3/2, (m + n)/n, 2 + m/n, -((b*x^n)/a)])/(2*a*(m + n)*Sqrt[a + b*x^n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int\left(-\frac{bnx^{-1+m+n}}{2\left(a+bx^n\right)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^n}}\right)dx = m\int\frac{x^{-1+m}}{\sqrt{a+bx^n}}\,dx-\frac{1}{2}(bn)\int\frac{x^{-1+m+n}}{\left(a+bx^n\right)^{3/2}}\,dx$$

$$= \frac{\left(m\sqrt{1+\frac{bx^n}{a}}\right)\int\frac{x^{-1+m}}{\sqrt{1+\frac{bx^n}{a}}}\,dx}{\sqrt{a+bx^n}}-\frac{\left(bn\sqrt{1+\frac{bx^n}{a}}\right)\int\frac{x^{-1+m+n}}{\left(1+\frac{bx^n}{a}\right)^{3/2}}\,dx}{2a\sqrt{a+bx^n}}$$

$$= \frac{x^m\sqrt{1+\frac{bx^n}{a}}\ {}_2F_1\left(\frac{1}{2},\frac{m}{n};\frac{m+n}{n};-\frac{bx^n}{a}\right)}{\sqrt{a+bx^n}}-\frac{bnx^{m+n}\sqrt{1+\frac{bx^n}{a}}\ {}_2F_1\left(\frac{3}{2},\frac{m+n}{n};2\right.}{2a(m+n)\sqrt{a+bx^n}}$$

Mathematica [C] time = 0.23, size = 111, normalized size = 7.40

$$\frac{x^m\sqrt{\frac{bx^n}{a}+1}\left(b(2m-n)x^n\ {}_2F_1\left(\frac{3}{2},\frac{m+n}{n};\frac{m}{n}+2;-\frac{bx^n}{a}\right)+2a(m+n)\ {}_2F_1\left(\frac{3}{2},\frac{m}{n};\frac{m+n}{n};-\frac{bx^n}{a}\right)\right)}{2a(m+n)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[-1/2*(b*n*x^(-1 + m + n))/(a + b*x^n)^(3/2) + (m*x^(-1 + m))/Sqrt
[a + b*x^n],x]
```

```
[Out] (x^m*Sqrt[1 + (b*x^n)/a]*(2*a*(m + n)*Hypergeometric2F1[3/2, m/n, (m + n)/n
, -((b*x^n)/a)] + b*(2*m - n)*x^n*Hypergeometric2F1[3/2, (m + n)/n, 2 + m/n
, -((b*x^n)/a)]))/(2*a*(m + n)*Sqrt[a + b*x^n])
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*b*n*x^(-1+m+n)/(a+b*x^n)^(3/2)+m*x^(-1+m)/(a+b*x^n)^(1/2),x,
 algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{bnx^{m+n-1}}{2\left(bx^n+a\right)^{\frac{3}{2}}}+\frac{mx^{m-1}}{\sqrt{bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*b*n*x^(-1+m+n)/(a+b*x^n)^(3/2)+m*x^(-1+m)/(a+b*x^n)^(1/2),x,
 algorithm="giac")
```

```
[Out] integrate(-1/2*b*n*x^(m + n - 1)/(b*x^n + a)^(3/2) + m*x^(m - 1)/sqrt(b*x^n
 + a), x)
```

maple [**F**] time = 0.23, size = 0, normalized size = 0.00

$$\int -\frac{bn\,x^{m+n-1}}{2\left(b\,x^n+a\right)^{\frac{3}{2}}}+\frac{m\,x^{m-1}}{\sqrt{b\,x^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/2*b*n*x^(-1+m+n)/(b*x^n+a)^(3/2)+m*x^(-1+m)/(b*x^n+a)^(1/2),x)
```

```
[Out] int(-1/2*b*n*x^(-1+m+n)/(b*x^n+a)^(3/2)+m*x^(-1+m)/(b*x^n+a)^(1/2),x)
```

maxima [**A**] time = 0.89, size = 13, normalized size = 0.87

$$\frac{x^m}{\sqrt{bx^n+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*b*n*x^(-1+m+n)/(a+b*x^n)^(3/2)+m*x^(-1+m)/(a+b*x^n)^(1/2),x,
 algorithm="maxima")
```

```
[Out] x^m/sqrt(b*x^n + a)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{m\,x^{m-1}}{\sqrt{a+b\,x^n}}-\frac{b\,n\,x^{m+n-1}}{2\left(a+b\,x^n\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((m*x^(m - 1))/(a + b*x^n)^(1/2) - (b*n*x^(m + n - 1))/(2*(a + b*x^n)^(3
/2)),x)
```

```
[Out] int((m*x^(m - 1))/(a + b*x^n)^(1/2) - (b*n*x^(m + n - 1))/(2*(a + b*x^n)^(3
/2)), x)
```

sympy [C] time = 64.20, size = 94, normalized size = 6.27

$$\frac{m x^{m} \Gamma\left(\frac{m}{n}\right) {}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{m}{n} \\ \frac{m}{n}+1\end{matrix}\middle| \frac{b x^{n} e^{i\pi}}{a}\right)}{\sqrt{a}\, n \Gamma\left(\frac{m}{n}+1\right)} - \frac{b x^{m} x^{n} \Gamma\left(\frac{m}{n}+1\right) {}_2F_1\left(\begin{matrix}\frac{3}{2}, \frac{m}{n}+1 \\ \frac{m}{n}+2\end{matrix}\middle| \frac{b x^{n} e^{i\pi}}{a}\right)}{2 a^{\frac{3}{2}} \Gamma\left(\frac{m}{n}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*b*n*x**(-1+m+n)/(a+b*x**n)**(3/2)+m*x**(-1+m)/(a+b*x**n)**(1
/2),x)
```

```
[Out] m*x**m*gamma(m/n)*hyper((1/2, m/n), (m/n + 1,), b*x**n*exp_polar(I*pi)/a)/(
sqrt(a)*n*gamma(m/n + 1)) - b*x**m*x**n*gamma(m/n + 1)*hyper((3/2, m/n + 1)
, (m/n + 2,), b*x**n*exp_polar(I*pi)/a)/(2*a**(3/2)*gamma(m/n + 2))
```

3.2687 $\int \frac{x^{-1+\frac{7n}{2}}}{\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=129

$$-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x^{n/2}}{\sqrt{a+bx^n}}\right)}{8b^{7/2}n}+\frac{5a^2x^{n/2}\sqrt{a+bx^n}}{8b^3n}-\frac{5ax^{3n/2}\sqrt{a+bx^n}}{12b^2n}+\frac{x^{5n/2}\sqrt{a+bx^n}}{3bn}$$

```
[Out] -5/8*a^3*arctanh(x^(1/2*n)*b^(1/2)/(a+b*x^n)^(1/2))/b^(7/2)/n+5/8*a^2*x^(1/
2*n)*(a+b*x^n)^(1/2)/b^3/n-5/12*a*x^(3/2*n)*(a+b*x^n)^(1/2)/b^2/n+1/3*x^(5/
2*n)*(a+b*x^n)^(1/2)/b/n
```

Rubi [A] time = 0.06, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {355, 288, 206}

$$\frac{5a^2x^{n/2}\sqrt{a+bx^n}}{8b^3n}-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x^{n/2}}{\sqrt{a+bx^n}}\right)}{8b^{7/2}n}-\frac{5ax^{3n/2}\sqrt{a+bx^n}}{12b^2n}+\frac{x^{5n/2}\sqrt{a+bx^n}}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + (7*n)/2)/Sqrt[a + b*x^n],x]
```

```
[Out] (5*a^2*x^(n/2)*Sqrt[a + b*x^n])/(8*b^3*n) - (5*a*x^((3*n)/2)*Sqrt[a + b*x^n
])/(12*b^2*n) + (x^((5*n)/2)*Sqrt[a + b*x^n])/(3*b*n) - (5*a^3*ArcTanh[(Sqr
t[b]*x^(n/2))/Sqrt[a + b*x^n]])/(8*b^(7/2)*n)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 355

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[p]}, Dist[(k*a^(p + Simplify[(m + 1)/n]))/n, Subst[Int[x^(k*Simplify[
(m + 1)/n] - 1)/(1 - b*x^k)^(p + Simplify[(m + 1)/n] + 1), x], x, x^(n/k)/(
a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p + Simplify[(
m + 1)/n]] && LtQ[-1, p, 0]
```

Rubi steps

$$\int \frac{x^{-1+\frac{7n}{2}}}{\sqrt{a+bx^n}}\,dx = \frac{\left(2a^3\right)\text{Subst}\left(\int \frac{x^6}{\left(1-bx^2\right)^4}\,dx, x, \frac{x^{n/2}}{\sqrt{a+bx^n}}\right)}{n}$$

$$= \frac{x^{5n/2}\sqrt{a+bx^n}}{3bn} - \frac{\left(5a^3\right)\text{Subst}\left(\int \frac{x^4}{\left(1-bx^2\right)^3}\,dx, x, \frac{x^{n/2}}{\sqrt{a+bx^n}}\right)}{3bn}$$

$$= -\frac{5ax^{3n/2}\sqrt{a+bx^n}}{12b^2n} + \frac{x^{5n/2}\sqrt{a+bx^n}}{3bn} + \frac{\left(5a^3\right)\text{Subst}\left(\int \frac{x^2}{\left(1-bx^2\right)^2}\,dx, x, \frac{x^{n/2}}{\sqrt{a+bx^n}}\right)}{4b^2n}$$

$$= \frac{5a^2x^{n/2}\sqrt{a+bx^n}}{8b^3n} - \frac{5ax^{3n/2}\sqrt{a+bx^n}}{12b^2n} + \frac{x^{5n/2}\sqrt{a+bx^n}}{3bn} - \frac{\left(5a^3\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^{n/2}}{\sqrt{a+bx}}\right.}{8b^3n}$$

$$= \frac{5a^2x^{n/2}\sqrt{a+bx^n}}{8b^3n} - \frac{5ax^{3n/2}\sqrt{a+bx^n}}{12b^2n} + \frac{x^{5n/2}\sqrt{a+bx^n}}{3bn} - \frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x^{n/2}}{\sqrt{a+bx^n}}\right)}{8b^{7/2}n}$$

Mathematica [A] time = 0.23, size = 100, normalized size = 0.78

$$\frac{\sqrt{a+bx^n}\left(\sqrt{b}\,x^{n/2}\left(15a^2-10abx^n+8b^2x^{2n}\right) - \frac{15a^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}\,x^{n/2}}{\sqrt{a}}\right)}{\sqrt{\frac{bx^n}{a}+1}}\right)}{24b^{7/2}n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + (7*n)/2)/Sqrt[a + b*x^n],x]
```

```
[Out] (Sqrt[a + b*x^n]*(Sqrt[b]*x^(n/2)*(15*a^2 - 10*a*b*x^n + 8*b^2*x^(2*n)) - (
15*a^(5/2)*ArcSinh[(Sqrt[b]*x^(n/2))/Sqrt[a]])/Sqrt[1 + (b*x^n)/a]))/(24*b^
(7/2)*n)
```

fricas [A] time = 0.63, size = 175, normalized size = 1.36

$$\left[\frac{15\,a^3\sqrt{b}\,\log\left(2\,\sqrt{bx^n+a}\,\sqrt{b}\,x^{\frac{1}{2}n} - 2\,bx^n - a\right) + 2\left(8\,b^3x^{\frac{5}{2}n} - 10\,ab^2x^{\frac{3}{2}n} + 15\,a^2bx^{\frac{1}{2}n}\right)\sqrt{bx^n+a}}{48\,b^4n}, \frac{15\,a^3\sqrt{-b}\ \text{ar}}{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+7/2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*a^3*sqrt(b)*log(2*sqrt(b*x^n + a)*sqrt(b)*x^(1/2*n) - 2*b*x^n - a
) + 2*(8*b^3*x^(5/2*n) - 10*a*b^2*x^(3/2*n) + 15*a^2*b*x^(1/2*n))*sqrt(b*x^
n + a))/(b^4*n), 1/24*(15*a^3*sqrt(-b)*arctan(sqrt(-b)*x^(1/2*n)/sqrt(b*x^n
 + a)) + (8*b^3*x^(5/2*n) - 10*a*b^2*x^(3/2*n) + 15*a^2*b*x^(1/2*n))*sqrt(b
*x^n + a))/(b^4*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{7}{2}n-1}}{\sqrt{bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+7/2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(7/2*n - 1)/sqrt(b*x^n + a), x)
```

maple [A] time = 0.11, size = 98, normalized size = 0.76

$$-\frac{5a^3\ln\left(\sqrt{b}\,e^{\frac{n\ln(x)}{2}}+\sqrt{b\,e^{n\ln(x)}+a}\right)}{8b^{\frac{7}{2}}n}+\frac{\left(-10ab\,e^{n\ln(x)}+8b^2e^{2n\ln(x)}+15a^2\right)\sqrt{b\,e^{n\ln(x)}+a}\,e^{\frac{n\ln(x)}{2}}}{24b^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+7/2*n)/(b*x^n+a)^(1/2),x)
```

```
[Out] 1/24*exp(1/2*n*ln(x))*(8*exp(1/2*n*ln(x))^4*b^2-10*a*exp(1/2*n*ln(x))^2*b+1
5*a^2)*(b*exp(1/2*n*ln(x))^2+a)^(1/2)/b^3/n-5/8*a^3/b^(7/2)/n*ln(b^(1/2)*ex
p(1/2*n*ln(x))+(b*exp(1/2*n*ln(x))^2+a)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{7}{2}n-1}}{\sqrt{bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+7/2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(7/2*n - 1)/sqrt(b*x^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{\frac{7n}{2}-1}}{\sqrt{a+b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^((7*n)/2 - 1)/(a + b*x^n)^(1/2),x)
```

```
[Out] int(x^((7*n)/2 - 1)/(a + b*x^n)^(1/2), x)
```

sympy [A] time = 16.51, size = 148, normalized size = 1.15

$$\frac{5a^{\frac{5}{2}}x^{\frac{n}{2}}}{8b^3n\sqrt{1+\frac{bx^n}{a}}}+\frac{5a^{\frac{3}{2}}x^{\frac{3n}{2}}}{24b^2n\sqrt{1+\frac{bx^n}{a}}}-\frac{\sqrt{a}\,x^{\frac{5n}{2}}}{12bn\sqrt{1+\frac{bx^n}{a}}}-\frac{5a^3\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{\frac{n}{2}}}{\sqrt{a}}\right)}{8b^{\frac{7}{2}}n}+\frac{x^{\frac{7n}{2}}}{3\sqrt{a}\,n\sqrt{1+\frac{bx^n}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+7/2*n)/(a+b*x**n)**(1/2),x)
```

```
[Out] 5*a**(5/2)*x**(n/2)/(8*b**3*n*sqrt(1 + b*x**n/a)) + 5*a**(3/2)*x**(3*n/2)/(
24*b**2*n*sqrt(1 + b*x**n/a)) - sqrt(a)*x**(5*n/2)/(12*b*n*sqrt(1 + b*x**n/
a)) - 5*a**3*asinh(sqrt(b)*x**(n/2)/sqrt(a))/(8*b**(7/2)*n) + x**(7*n/2)/(3
*sqrt(a)*n*sqrt(1 + b*x**n/a))
```

3.2688 $\int \frac{x^{-1+\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=98

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x^{n/2}}{\sqrt{a+bx^n}}\right)}{4b^{5/2}n}-\frac{3ax^{n/2}\sqrt{a+bx^n}}{4b^2n}+\frac{x^{3n/2}\sqrt{a+bx^n}}{2bn}$$

```
[Out] 3/4*a^2*arctanh(x^(1/2*n)*b^(1/2)/(a+b*x^n)^(1/2))/b^(5/2)/n-3/4*a*x^(1/2*n
)*(a+b*x^n)^(1/2)/b^2/n+1/2*x^(3/2*n)*(a+b*x^n)^(1/2)/b/n
```

Rubi [A] time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {355, 288, 206}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x^{n/2}}{\sqrt{a+bx^n}}\right)}{4b^{5/2}n}-\frac{3ax^{n/2}\sqrt{a+bx^n}}{4b^2n}+\frac{x^{3n/2}\sqrt{a+bx^n}}{2bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + (5*n)/2)/Sqrt[a + b*x^n],x]
```

```
[Out] (-3*a*x^(n/2)*Sqrt[a + b*x^n])/(4*b^2*n) + (x^((3*n)/2)*Sqrt[a + b*x^n])/(2
*b*n) + (3*a^2*ArcTanh[(Sqrt[b]*x^(n/2))/Sqrt[a + b*x^n]])/(4*b^(5/2)*n)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 355

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[p]}, Dist[(k*a^(p + Simplify[(m + 1)/n]))/n, Subst[Int[x^(k*Simplify[
(m + 1)/n] - 1)/(1 - b*x^k)^(p + Simplify[(m + 1)/n] + 1), x], x, x^(n/k)/(
a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p + Simplify[(
m + 1)/n]] && LtQ[-1, p, 0]
```

Rubi steps

$$\int \frac{x^{-1+\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx = \frac{\left(2a^2\right)\text{Subst}\left(\int \frac{x^4}{\left(1-bx^2\right)^3}\,dx, x, \frac{x^{n/2}}{\sqrt{a+bx^n}}\right)}{n}$$
$$= \frac{x^{3n/2}\sqrt{a+bx^n}}{2bn} - \frac{\left(3a^2\right)\text{Subst}\left(\int \frac{x^2}{\left(1-bx^2\right)^2}\,dx, x, \frac{x^{n/2}}{\sqrt{a+bx^n}}\right)}{2bn}$$
$$= -\frac{3ax^{n/2}\sqrt{a+bx^n}}{4b^2n} + \frac{x^{3n/2}\sqrt{a+bx^n}}{2bn} + \frac{\left(3a^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^{n/2}}{\sqrt{a+bx^n}}\right)}{4b^2n}$$
$$= -\frac{3ax^{n/2}\sqrt{a+bx^n}}{4b^2n} + \frac{x^{3n/2}\sqrt{a+bx^n}}{2bn} + \frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x^{n/2}}{\sqrt{a+bx^n}}\right)}{4b^{5/2}n}$$

Mathematica [A] time = 0.07, size = 100, normalized size = 1.02

$$\frac{3a^{5/2}\sqrt{\frac{bx^n}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x^{n/2}}{\sqrt{a}}\right) + \sqrt{b}\,x^{n/2}\left(-3a^2 - abx^n + 2b^2x^{2n}\right)}{4b^{5/2}n\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + (5*n)/2)/Sqrt[a + b*x^n],x]
```

```
[Out] (Sqrt[b]*x^(n/2)*(-3*a^2 - a*b*x^n + 2*b^2*x^(2*n)) + 3*a^(5/2)*Sqrt[1 + (b
*x^n)/a]*ArcSinh[(Sqrt[b]*x^(n/2))/Sqrt[a]])/(4*b^(5/2)*n*Sqrt[a + b*x^n])
```

fricas [A] time = 0.84, size = 150, normalized size = 1.53

$$\left[\frac{3\,a^2\sqrt{b}\,\log\left(-2\,\sqrt{bx^n+a}\,\sqrt{b}\,x^{\frac{1}{2}n} - 2\,bx^n - a\right) + 2\left(2\,b^2x^{\frac{3}{2}n} - 3\,abx^{\frac{1}{2}n}\right)\sqrt{bx^n+a}}{8\,b^3n}, -\frac{3\,a^2\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x^{\frac{1}{2}n}}{\sqrt{bx^n+a}}\right) -}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+5/2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*a^2*sqrt(b)*log(-2*sqrt(b*x^n + a)*sqrt(b)*x^(1/2*n) - 2*b*x^n - a)
 + 2*(2*b^2*x^(3/2*n) - 3*a*b*x^(1/2*n))*sqrt(b*x^n + a))/(b^3*n), -1/4*(3*
a^2*sqrt(-b)*arctan(sqrt(-b)*x^(1/2*n)/sqrt(b*x^n + a)) - (2*b^2*x^(3/2*n)
- 3*a*b*x^(1/2*n))*sqrt(b*x^n + a))/(b^3*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}n-1}}{\sqrt{bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+5/2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(5/2*n - 1)/sqrt(b*x^n + a), x)
```

maple [A] time = 0.04, size = 82, normalized size = 0.84

$$\frac{3a^2\ln\left(\sqrt{b}\,e^{\frac{n\ln(x)}{2}} + \sqrt{b\,e^{n\ln(x)}+a}\right)}{4b^{\frac{5}{2}}n} - \frac{\left(-2b\,e^{n\ln(x)} + 3a\right)\sqrt{b\,e^{n\ln(x)}+a}\,e^{\frac{n\ln(x)}{2}}}{4b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+5/2*n)/(b*x^n+a)^(1/2),x)
```

```
[Out] -1/4*exp(1/2*n*ln(x))*(-2*b*exp(1/2*n*ln(x))^2+3*a)*(b*exp(1/2*n*ln(x))^2+a
)^(1/2)/b^2/n+3/4*a^2/b^(5/2)/n*ln(b^(1/2)*exp(1/2*n*ln(x))+(b*exp(1/2*n*ln
(x))^2+a)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}n-1}}{\sqrt{bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+5/2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(5/2*n - 1)/sqrt(b*x^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{\frac{5n}{2}-1}}{\sqrt{a+b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^((5*n)/2 - 1)/(a + b*x^n)^(1/2),x)
```

```
[Out] int(x^((5*n)/2 - 1)/(a + b*x^n)^(1/2), x)
```

sympy [A] time = 12.74, size = 116, normalized size = 1.18

$$-\frac{3a^{\frac{3}{2}}x^{\frac{n}{2}}}{4b^2n\sqrt{1+\frac{bx^n}{a}}}-\frac{\sqrt{a}\,x^{\frac{3n}{2}}}{4bn\sqrt{1+\frac{bx^n}{a}}}+\frac{3a^2\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{n}{2}}}{\sqrt{a}}\right)}{4b^{\frac{5}{2}}n}+\frac{x^{\frac{5n}{2}}}{2\sqrt{a}\,n\sqrt{1+\frac{bx^n}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+5/2*n)/(a+b*x**n)**(1/2),x)
```

```
[Out] -3*a**(3/2)*x**(n/2)/(4*b**2*n*sqrt(1 + b*x**n/a)) - sqrt(a)*x**(3*n/2)/(4*
b*n*sqrt(1 + b*x**n/a)) + 3*a**2*asinh(sqrt(b)*x**(n/2)/sqrt(a))/(4*b**(5/2
)*n) + x**(5*n/2)/(2*sqrt(a)*n*sqrt(1 + b*x**n/a))
```

3.2689 $\int \frac{x^{-1+\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=62

$$\frac{x^{n/2}\sqrt{a+bx^n}}{bn} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^{n/2}}{\sqrt{a+bx^n}}\right)}{b^{3/2}n}$$

```
[Out] -a*arctanh(x^(1/2*n)*b^(1/2)/(a+b*x^n)^(1/2))/b^(3/2)/n+x^(1/2*n)*(a+b*x^n)
^(1/2)/b/n
```

Rubi [A] time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {355, 288, 206}

$$\frac{x^{n/2}\sqrt{a+bx^n}}{bn} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^{n/2}}{\sqrt{a+bx^n}}\right)}{b^{3/2}n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + (3*n)/2)/Sqrt[a + b*x^n],x]
```

```
[Out] (x^(n/2)*Sqrt[a + b*x^n])/(b*n) - (a*ArcTanh[(Sqrt[b]*x^(n/2))/Sqrt[a + b*x
^n]])/(b^(3/2)*n)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 355

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[p]}, Dist[(k*a^(p + Simplify[(m + 1)/n]))/n, Subst[Int[x^(k*Simplify[
(m + 1)/n] - 1)/(1 - b*x^k)^(p + Simplify[(m + 1)/n] + 1), x], x, x^(n/k)/(
a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p + Simplify[(
m + 1)/n]] && LtQ[-1, p, 0]
```

Rubi steps

$$\int \frac{x^{-1+\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx = \frac{(2a)\,\text{Subst}\left(\int \frac{x^2}{\left(1-bx^2\right)^2}\,dx, x, \frac{x^{n/2}}{\sqrt{a+bx^n}}\right)}{n}$$
$$= \frac{x^{n/2}\sqrt{a+bx^n}}{bn} - \frac{a\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^{n/2}}{\sqrt{a+bx^n}}\right)}{bn}$$
$$= \frac{x^{n/2}\sqrt{a+bx^n}}{bn} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^{n/2}}{\sqrt{a+bx^n}}\right)}{b^{3/2}n}$$

Mathematica [A] time = 0.05, size = 81, normalized size = 1.31

$$\frac{\sqrt{b}\,x^{n/2}\left(a+bx^n\right) - a^{3/2}\sqrt{\frac{bx^n}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x^{n/2}}{\sqrt{a}}\right)}{b^{3/2}n\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + (3*n)/2)/Sqrt[a + b*x^n],x]
```

```
[Out] (Sqrt[b]*x^(n/2)*(a + b*x^n) - a^(3/2)*Sqrt[1 + (b*x^n)/a]*ArcSinh[(Sqrt[b]
*x^(n/2))/Sqrt[a]])/(b^(3/2)*n*Sqrt[a + b*x^n])
```

fricas [A] time = 0.72, size = 114, normalized size = 1.84

$$\left[\frac{2\sqrt{bx^n+a}\,bx^{\frac{1}{2}n} + a\sqrt{b}\,\log\left(2\sqrt{bx^n+a}\,\sqrt{b}\,x^{\frac{1}{2}n} - 2bx^n - a\right)}{2b^2n}, \frac{\sqrt{bx^n+a}\,bx^{\frac{1}{2}n} + a\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x^{\frac{1}{2}n}}{\sqrt{bx^n+a}}\right)}{b^2n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3/2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*sqrt(b*x^n + a)*b*x^(1/2*n) + a*sqrt(b)*log(2*sqrt(b*x^n + a)*sqrt(
b)*x^(1/2*n) - 2*b*x^n - a))/(b^2*n), (sqrt(b*x^n + a)*b*x^(1/2*n) + a*sqrt
(-b)*arctan(sqrt(-b)*x^(1/2*n)/sqrt(b*x^n + a)))/(b^2*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}n-1}}{\sqrt{bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3/2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(3/2*n - 1)/sqrt(b*x^n + a), x)
```

maple [A] time = 0.03, size = 64, normalized size = 1.03

$$-\frac{a\ln\left(\sqrt{b}\,e^{\frac{n\ln(x)}{2}} + \sqrt{b\,e^{n\ln(x)}+a}\right)}{b^{\frac{3}{2}}n} + \frac{\sqrt{b\,e^{n\ln(x)}+a}\,e^{\frac{n\ln(x)}{2}}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+3/2*n)/(b*x^n+a)^(1/2),x)
```

```
[Out] 1/b/n*exp(1/2*n*ln(x))*(b*exp(1/2*n*ln(x))^2+a)^(1/2)-a/b^(3/2)/n*ln(b^(1/2
)*exp(1/2*n*ln(x))+(b*exp(1/2*n*ln(x))^2+a)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}n-1}}{\sqrt{bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3/2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2*n - 1)/sqrt(b*x^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{\frac{3n}{2}-1}}{\sqrt{a+b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^((3*n)/2 - 1)/(a + b*x^n)^(1/2),x)
```

```
[Out] int(x^((3*n)/2 - 1)/(a + b*x^n)^(1/2), x)
```

sympy [A] time = 9.86, size = 49, normalized size = 0.79

$$\frac{\sqrt{a}\,x^{\frac{n}{2}}\sqrt{1+\frac{bx^n}{a}}}{bn}-\frac{a\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{\frac{n}{2}}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3/2*n)/(a+b*x**n)**(1/2),x)
```

```
[Out] sqrt(a)*x**(n/2)*sqrt(1 + b*x**n/a)/(b*n) - a*asinh(sqrt(b)*x**(n/2)/sqrt(a
))/(b**(3/2)*n)
```

3.2690 $\int \frac{x^{-1+\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=35

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x^{n/2}}{\sqrt{a+bx^n}}\right)}{\sqrt{b}\,n}$$

```
[Out] 2*arctanh(x^(1/2*n)*b^(1/2)/(a+b*x^n)^(1/2))/n/b^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {345, 217, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x^{n/2}}{\sqrt{a+bx^n}}\right)}{\sqrt{b}\,n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n/2)/Sqrt[a + b*x^n],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x^(n/2))/Sqrt[a + b*x^n]])/(Sqrt[b]*n)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, x^{n/2}\right)}{n}\\ &= \frac{2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^{n/2}}{\sqrt{a+bx^n}}\right)}{n}\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x^{n/2}}{\sqrt{a+bx^n}}\right)}{\sqrt{b}\,n}\end{aligned}$$

Mathematica [A] time = 0.02, size = 59, normalized size = 1.69

$$\frac{2\sqrt{a}\,\sqrt{\frac{bx^n}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x^{n/2}}{\sqrt{a}}\right)}{\sqrt{b}\,n\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n/2)/Sqrt[a + b*x^n],x]
```

```
[Out] (2*Sqrt[a]*Sqrt[1 + (b*x^n)/a]*ArcSinh[(Sqrt[b]*x^(n/2))/Sqrt[a]])/(Sqrt[b]
*n*Sqrt[a + b*x^n])
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument ValueUnable to isolate n in n*sign(x)-n+4*floo
r(-(n*sign(x)-n-2)/4)-4*n_0, switching to approx. solutionsUndef/Unsigned I
nf encountered in limitUndef/Unsigned Inf encountered in limitUnable to iso
late n in n*sign(x)-n+4*floor(-(n*sign(x)-n-2)/4)-4*n_1, switching to appro
x. solutionsUndef/Unsigned Inf encountered in limitUndef/Unsigned Inf encou
ntered in limitEvaluation time: 1.41-2/n/sqrt(b)*ln(sqrt(a+b*abs(x)^n+b*x^n
-2*abs(x)^(1/2*n)*sqrt(b)*sqrt(a+b*x^n)*cos(1/2*((1-sign(x))*pi/2*n-2*pi*fl
oor((1-sign(x))*pi/2*n/(2*pi)+1/2)))))
```

maple [**F**] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{n}{2}-1}}{\sqrt{b\,x^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+1/2*n)/(b*x^n+a)^(1/2),x)
```

```
[Out] int(x^(-1+1/2*n)/(b*x^n+a)^(1/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{1}{2}n-1}}{\sqrt{bx^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(1/2*n - 1)/sqrt(b*x^n + a), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^{\frac{n}{2}-1}}{\sqrt{a + b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n/2 - 1)/(a + b*x^n)^(1/2),x)
[Out] int(x^(n/2 - 1)/(a + b*x^n)^(1/2), x)
```

sympy [**A**] time = 8.14, size = 24, normalized size = 0.69

$$\frac{2\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+1/2*n)/(a+b*x**n)**(1/2),x)
[Out] 2*asinh(sqrt(b)*x**(n/2)/sqrt(a))/(sqrt(b)*n)
```

3.2691 $\int \frac{x^{-1-\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=26

$$-\frac{2x^{-n/2}\sqrt{a+bx^n}}{an}$$

```
[Out] -2*(a+b*x^n)^(1/2)/a/n/(x^(1/2*n))
```

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {264}

$$-\frac{2x^{-n/2}\sqrt{a+bx^n}}{an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n/2)/Sqrt[a + b*x^n],x]
```

```
[Out] (-2*Sqrt[a + b*x^n])/(a*n*x^(n/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^{-1-\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx = -\frac{2x^{-n/2}\sqrt{a+bx^n}}{an}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$-\frac{2x^{-n/2}\sqrt{a+bx^n}}{an}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n/2)/Sqrt[a + b*x^n],x]
```

```
[Out] (-2*Sqrt[a + b*x^n])/(a*n*x^(n/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{1}{2}n-1}}{\sqrt{bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(x^(-1/2*n - 1)/sqrt(b*x^n + a), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{n}{2}-1}}{\sqrt{b\,x^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-1/2*n)/(b*x^n+a)^(1/2),x)
[Out] int(x^(-1-1/2*n)/(b*x^n+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{1}{2}n-1}}{\sqrt{bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(x^(-1/2*n - 1)/sqrt(b*x^n + a), x)
```

mupad [B] time = 1.31, size = 24, normalized size = 0.92

$$-\frac{2\sqrt{a+b\,x^n}}{a\,n\,x^{n/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(n/2 + 1)*(a + b*x^n)^(1/2)),x)
[Out] -(2*(a + b*x^n)^(1/2))/(a*n*x^(n/2))
```

sympy [A] time = 34.93, size = 22, normalized size = 0.85

$$-\frac{2\sqrt{b}\sqrt{\frac{ax^{-n}}{b}+1}}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-1/2*n)/(a+b*x**n)**(1/2),x)
[Out] -2*sqrt(b)*sqrt(a*x**(-n)/b + 1)/(a*n)
```

3.2692 $\int \frac{x^{-1-\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=58

$$\frac{4bx^{-n/2}\sqrt{a+bx^n}}{3a^2n}-\frac{2x^{-3n/2}\sqrt{a+bx^n}}{3an}$$

```
[Out] -2/3*(a+b*x^n)^(1/2)/a/n/(x^(3/2*n))+4/3*b*(a+b*x^n)^(1/2)/a^2/n/(x^(1/2*n)
)
```

Rubi [A] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {271, 264}

$$\frac{4bx^{-n/2}\sqrt{a+bx^n}}{3a^2n}-\frac{2x^{-3n/2}\sqrt{a+bx^n}}{3an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - (3*n)/2)/Sqrt[a + b*x^n],x]
```

```
[Out] (-2*Sqrt[a + b*x^n])/(3*a*n*x^((3*n)/2)) + (4*b*Sqrt[a + b*x^n])/(3*a^2*n*x
^(n/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1-\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx &= -\frac{2x^{-3n/2}\sqrt{a+bx^n}}{3an}-\frac{(2b)\int \frac{x^{-1-\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx}{3a}\\ &= -\frac{2x^{-3n/2}\sqrt{a+bx^n}}{3an}+\frac{4bx^{-n/2}\sqrt{a+bx^n}}{3a^2n}\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 0.62

$$-\frac{2x^{-3n/2}\left(a-2bx^n\right)\sqrt{a+bx^n}}{3a^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - (3*n)/2)/Sqrt[a + b*x^n],x]
```

```
[Out] (-2*(a - 2*b*x^n)*Sqrt[a + b*x^n])/(3*a^2*n*x^((3*n)/2))
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3/2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{3}{2}n-1}}{\sqrt{bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3/2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(-3/2*n - 1)/sqrt(b*x^n + a), x)
```

maple [**F**] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{3n}{2}-1}}{\sqrt{b\,x^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-3/2*n)/(b*x^n+a)^(1/2),x)
```

```
[Out] int(x^(-1-3/2*n)/(b*x^n+a)^(1/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{3}{2}n-1}}{\sqrt{bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3/2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(-3/2*n - 1)/sqrt(b*x^n + a), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^{\frac{3n}{2}+1}\sqrt{a+b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^((3*n)/2 + 1)*(a + b*x^n)^(1/2)),x)
```

```
[Out] int(1/(x^((3*n)/2 + 1)*(a + b*x^n)^(1/2)), x)
```

sympy [**A**] time = 5.96, size = 51, normalized size = 0.88

$$-\frac{2\sqrt{b}\,x^{-n}\sqrt{\frac{ax^{-n}}{b}+1}}{3an}+\frac{4b^{\frac{3}{2}}\sqrt{\frac{ax^{-n}}{b}+1}}{3a^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-3/2*n)/(a+b*x**n)**(1/2),x)
```

```
[Out] -2*sqrt(b)*x**(-n)*sqrt(a*x**(-n)/b + 1)/(3*a*n) + 4*b**(3/2)*sqrt(a*x**(-n
)/b + 1)/(3*a**2*n)
```

3.2693 $\int \frac{x^{-1-\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=89

$$-\frac{16b^2x^{-n/2}\sqrt{a+bx^n}}{15a^3n}+\frac{8bx^{-3n/2}\sqrt{a+bx^n}}{15a^2n}-\frac{2x^{-5n/2}\sqrt{a+bx^n}}{5an}$$

```
[Out] -2/5*(a+b*x^n)^(1/2)/a/n/(x^(5/2*n))+8/15*b*(a+b*x^n)^(1/2)/a^2/n/(x^(3/2*n
))-16/15*b^2*(a+b*x^n)^(1/2)/a^3/n/(x^(1/2*n))
```

Rubi [A] time = 0.03, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {271, 264}

$$-\frac{16b^2x^{-n/2}\sqrt{a+bx^n}}{15a^3n}+\frac{8bx^{-3n/2}\sqrt{a+bx^n}}{15a^2n}-\frac{2x^{-5n/2}\sqrt{a+bx^n}}{5an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - (5*n)/2)/Sqrt[a + b*x^n],x]
```

```
[Out] (-2*Sqrt[a + b*x^n])/(5*a*n*x^((5*n)/2)) + (8*b*Sqrt[a + b*x^n])/(15*a^2*n*
x^((3*n)/2)) - (16*b^2*Sqrt[a + b*x^n])/(15*a^3*n*x^(n/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1-\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx &= -\frac{2x^{-5n/2}\sqrt{a+bx^n}}{5an}-\frac{(4b)\int \frac{x^{-1-\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx}{5a}\\ &= -\frac{2x^{-5n/2}\sqrt{a+bx^n}}{5an}+\frac{8bx^{-3n/2}\sqrt{a+bx^n}}{15a^2n}+\frac{\left(8b^2\right)\int \frac{x^{-1-\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx}{15a^2}\\ &= -\frac{2x^{-5n/2}\sqrt{a+bx^n}}{5an}+\frac{8bx^{-3n/2}\sqrt{a+bx^n}}{15a^2n}-\frac{16b^2x^{-n/2}\sqrt{a+bx^n}}{15a^3n}\end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 0.57

$$-\frac{2x^{-5n/2}\sqrt{a+bx^n}\left(3a^2-4abx^n+8b^2x^{2n}\right)}{15a^3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - (5*n)/2)/Sqrt[a + b*x^n],x]
```

```
[Out] (-2*Sqrt[a + b*x^n]*(3*a^2 - 4*a*b*x^n + 8*b^2*x^(2*n)))/(15*a^3*n*x^((5*n)
/2))
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-5/2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{5}{2}n-1}}{\sqrt{bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-5/2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(-5/2*n - 1)/sqrt(b*x^n + a), x)
```

maple [**F**] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{5n}{2}-1}}{\sqrt{b\,x^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-5/2*n)/(b*x^n+a)^(1/2),x)
```

```
[Out] int(x^(-1-5/2*n)/(b*x^n+a)^(1/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{5}{2}n-1}}{\sqrt{bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-5/2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(-5/2*n - 1)/sqrt(b*x^n + a), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{\frac{5n}{2}+1}\sqrt{a+b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^((5*n)/2 + 1)*(a + b*x^n)^(1/2)),x)
```

```
[Out] int(1/(x^((5*n)/2 + 1)*(a + b*x^n)^(1/2)), x)
```

sympy [**B**] time = 6.25, size = 354, normalized size = 3.98

$$-\frac{6a^4b^{\frac{9}{2}}\sqrt{\frac{ax^{-n}}{b}+1}}{15a^5b^4nx^{2n}+30a^4b^5nx^{3n}+15a^3b^6nx^{4n}}-\frac{4a^3b^{\frac{11}{2}}x^n\sqrt{\frac{ax^{-n}}{b}+1}}{15a^5b^4nx^{2n}+30a^4b^5nx^{3n}+15a^3b^6nx^{4n}}-\frac{6a^2b^{\frac{13}{2}}x^{2n}\sqrt{\frac{ax^{-}}{b}}}{15a^5b^4nx^{2n}+30a^4b^5nx^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-5/2*n)/(a+b*x**n)**(1/2),x)
```

```
[Out] -6*a**4*b**(9/2)*sqrt(a*x**(-n)/b + 1)/(15*a**5*b**4*n*x**(2*n) + 30*a**4*b
**5*n*x**(3*n) + 15*a**3*b**6*n*x**(4*n)) - 4*a**3*b**(11/2)*x**n*sqrt(a*x*
*(-n)/b + 1)/(15*a**5*b**4*n*x**(2*n) + 30*a**4*b**5*n*x**(3*n) + 15*a**3*b
**6*n*x**(4*n)) - 6*a**2*b**(13/2)*x**(2*n)*sqrt(a*x**(-n)/b + 1)/(15*a**5*
b**4*n*x**(2*n) + 30*a**4*b**5*n*x**(3*n) + 15*a**3*b**6*n*x**(4*n)) - 24*a
*b**(15/2)*x**(3*n)*sqrt(a*x**(-n)/b + 1)/(15*a**5*b**4*n*x**(2*n) + 30*a**
4*b**5*n*x**(3*n) + 15*a**3*b**6*n*x**(4*n)) - 16*b**(17/2)*x**(4*n)*sqrt(a
*x**(-n)/b + 1)/(15*a**5*b**4*n*x**(2*n) + 30*a**4*b**5*n*x**(3*n) + 15*a**
3*b**6*n*x**(4*n))
```

3.2694 $\int \frac{x^{-1-\frac{7n}{2}}}{\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=120

$$\frac{32b^3x^{-n/2}\sqrt{a+bx^n}}{35a^4n}-\frac{16b^2x^{-3n/2}\sqrt{a+bx^n}}{35a^3n}+\frac{12bx^{-5n/2}\sqrt{a+bx^n}}{35a^2n}-\frac{2x^{-7n/2}\sqrt{a+bx^n}}{7an}$$

```
[Out] -2/7*(a+b*x^n)^(1/2)/a/n/(x^(7/2*n))+12/35*b*(a+b*x^n)^(1/2)/a^2/n/(x^(5/2*
n))-16/35*b^2*(a+b*x^n)^(1/2)/a^3/n/(x^(3/2*n))+32/35*b^3*(a+b*x^n)^(1/2)/a
^4/n/(x^(1/2*n))
```

Rubi [A] time = 0.04, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {271, 264}

$$-\frac{16b^2x^{-3n/2}\sqrt{a+bx^n}}{35a^3n}+\frac{32b^3x^{-n/2}\sqrt{a+bx^n}}{35a^4n}+\frac{12bx^{-5n/2}\sqrt{a+bx^n}}{35a^2n}-\frac{2x^{-7n/2}\sqrt{a+bx^n}}{7an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - (7*n)/2)/Sqrt[a + b*x^n],x]
```

```
[Out] (-2*Sqrt[a + b*x^n])/(7*a*n*x^((7*n)/2)) + (12*b*Sqrt[a + b*x^n])/(35*a^2*n
*x^((5*n)/2)) - (16*b^2*Sqrt[a + b*x^n])/(35*a^3*n*x^((3*n)/2)) + (32*b^3*S
qrt[a + b*x^n])/(35*a^4*n*x^(n/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1-\frac{7n}{2}}}{\sqrt{a+bx^n}}\,dx &= -\frac{2x^{-7n/2}\sqrt{a+bx^n}}{7an}-\frac{(6b)\int \frac{x^{-1-\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx}{7a}\\ &= -\frac{2x^{-7n/2}\sqrt{a+bx^n}}{7an}+\frac{12bx^{-5n/2}\sqrt{a+bx^n}}{35a^2n}+\frac{\left(24b^2\right)\int \frac{x^{-1-\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx}{35a^2}\\ &= -\frac{2x^{-7n/2}\sqrt{a+bx^n}}{7an}+\frac{12bx^{-5n/2}\sqrt{a+bx^n}}{35a^2n}-\frac{16b^2x^{-3n/2}\sqrt{a+bx^n}}{35a^3n}-\frac{\left(16b^3\right)\int \frac{x^{-1-\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx}{35a^3}\\ &= -\frac{2x^{-7n/2}\sqrt{a+bx^n}}{7an}+\frac{12bx^{-5n/2}\sqrt{a+bx^n}}{35a^2n}-\frac{16b^2x^{-3n/2}\sqrt{a+bx^n}}{35a^3n}+\frac{32b^3x^{-n/2}\sqrt{a+bx^n}}{35a^4n}\end{aligned}$$

Mathematica [A] time = 0.02, size = 64, normalized size = 0.53

$$-\frac{2x^{-7n/2}\sqrt{a+bx^n}\left(5a^3-6a^2bx^n+8ab^2x^{2n}-16b^3x^{3n}\right)}{35a^4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - (7*n)/2)/Sqrt[a + b*x^n],x]
```

```
[Out] (-2*Sqrt[a + b*x^n]*(5*a^3 - 6*a^2*b*x^n + 8*a*b^2*x^(2*n) - 16*b^3*x^(3*n)
))/(35*a^4*n*x^((7*n)/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-7/2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{7}{2}n-1}}{\sqrt{bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-7/2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(-7/2*n - 1)/sqrt(b*x^n + a), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{7n}{2}-1}}{\sqrt{b\,x^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-7/2*n)/(b*x^n+a)^(1/2),x)
```

```
[Out] int(x^(-1-7/2*n)/(b*x^n+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{7}{2}n-1}}{\sqrt{bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-7/2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(-7/2*n - 1)/sqrt(b*x^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{\frac{7n}{2}+1}\sqrt{a+b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^((7*n)/2 + 1)*(a + b*x^n)^(1/2)),x)
```

```
[Out] int(1/(x^((7*n)/2 + 1)*(a + b*x^n)^(1/2)), x)
```

sympy [**B**] time = 6.51, size = 605, normalized size = 5.04

$$-\frac{10a^6b^{\frac{19}{2}}\sqrt{\frac{ax^{-n}}{b}+1}}{35a^7b^9nx^{3n}+105a^6b^{10}nx^{4n}+105a^5b^{11}nx^{5n}+35a^4b^{12}nx^{6n}}-\frac{18a^5b^{\frac{21}{2}}x^n\sqrt{\frac{ax^{-n}}{b}+1}}{35a^7b^9nx^{3n}+105a^6b^{10}nx^{4n}+105a^5b^{11}nx^{5n}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-7/2*n)/(a+b*x**n)**(1/2),x)
```

```
[Out] -10*a**6*b**(19/2)*sqrt(a*x**(-n)/b + 1)/(35*a**7*b**9*n*x**(3*n) + 105*a**
6*b**10*n*x**(4*n) + 105*a**5*b**11*n*x**(5*n) + 35*a**4*b**12*n*x**(6*n))
- 18*a**5*b**(21/2)*x**n*sqrt(a*x**(-n)/b + 1)/(35*a**7*b**9*n*x**(3*n) + 1
05*a**6*b**10*n*x**(4*n) + 105*a**5*b**11*n*x**(5*n) + 35*a**4*b**12*n*x**(
6*n)) - 10*a**4*b**(23/2)*x**(2*n)*sqrt(a*x**(-n)/b + 1)/(35*a**7*b**9*n*x*
*(3*n) + 105*a**6*b**10*n*x**(4*n) + 105*a**5*b**11*n*x**(5*n) + 35*a**4*b*
*12*n*x**(6*n)) + 10*a**3*b**(25/2)*x**(3*n)*sqrt(a*x**(-n)/b + 1)/(35*a**7
*b**9*n*x**(3*n) + 105*a**6*b**10*n*x**(4*n) + 105*a**5*b**11*n*x**(5*n) +
35*a**4*b**12*n*x**(6*n)) + 60*a**2*b**(27/2)*x**(4*n)*sqrt(a*x**(-n)/b + 1
)/(35*a**7*b**9*n*x**(3*n) + 105*a**6*b**10*n*x**(4*n) + 105*a**5*b**11*n*x
**(5*n) + 35*a**4*b**12*n*x**(6*n)) + 80*a*b**(29/2)*x**(5*n)*sqrt(a*x**(-n
)/b + 1)/(35*a**7*b**9*n*x**(3*n) + 105*a**6*b**10*n*x**(4*n) + 105*a**5*b*
*11*n*x**(5*n) + 35*a**4*b**12*n*x**(6*n)) + 32*b**(31/2)*x**(6*n)*sqrt(a*x
**(-n)/b + 1)/(35*a**7*b**9*n*x**(3*n) + 105*a**6*b**10*n*x**(4*n) + 105*a*
*5*b**11*n*x**(5*n) + 35*a**4*b**12*n*x**(6*n))
```

3.2695 $\int \frac{x^m}{\sqrt{a+bx^{-2+m}}}\,dx$

Optimal. Leaf size=65

$$\frac{x^{m+1}\sqrt{a+bx^{m-2}}\;{}_2F_1\left(1,-\frac{3m}{2(2-m)};\frac{1-2m}{2-m};-\frac{bx^{m-2}}{a}\right)}{a(m+1)}$$

```
[Out] x^(1+m)*hypergeom([1, -3/2*m/(2-m)],[(1-2*m)/(2-m)],-b*x^(-2+m)/a)*(a+b*x^(
-2+m))^(1/2)/a/(1+m)
```

Rubi [A] time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.23, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {365, 364}

$$\frac{x^{m+1}\sqrt{\frac{bx^{m-2}}{a}+1}\;{}_2F_1\left(\frac{1}{2},-\frac{m+1}{2-m};\frac{1-2m}{2-m};-\frac{bx^{m-2}}{a}\right)}{(m+1)\sqrt{a+bx^{m-2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/Sqrt[a + b*x^(-2 + m)],x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + (b*x^(-2 + m))/a]*Hypergeometric2F1[1/2, -((1 + m)/(2 -
 m)), (1 - 2*m)/(2 - m), -((b*x^(-2 + m))/a)])/((1 + m)*Sqrt[a + b*x^(-2 +
m)])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{\sqrt{a+bx^{-2+m}}}\,dx &= \frac{\sqrt{1+\frac{bx^{-2+m}}{a}}\int \frac{x^m}{\sqrt{1+\frac{bx^{-2+m}}{a}}}\,dx}{\sqrt{a+bx^{-2+m}}}\\ &= \frac{x^{1+m}\sqrt{1+\frac{bx^{-2+m}}{a}}\;{}_2F_1\left(\frac{1}{2},-\frac{1+m}{2-m};\frac{1-2m}{2-m};-\frac{bx^{-2+m}}{a}\right)}{(1+m)\sqrt{a+bx^{-2+m}}}\end{aligned}$$

Mathematica [A] time = 0.18, size = 110, normalized size = 1.69

$$\frac{2x\left(6ax^2\sqrt{\frac{ax^{2-m}}{b}+1}\;{}_2F_1\left(\frac{1}{2},\frac{m-8}{2(m-2)};\frac{3(m-4)}{2(m-2)};-\frac{ax^{2-m}}{b}\right)+(m-8)\left(ax^2+bx^m\right)\right)}{b(m-8)(m+4)\sqrt{a+bx^{m-2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/Sqrt[a + b*x^(-2 + m)],x]
```

```
[Out] (2*x*((-8 + m)*(a*x^2 + b*x^m) + 6*a*x^2*Sqrt[1 + (a*x^(2 - m))/b]*Hypergeo
metric2F1[1/2, (-8 + m)/(2*(-2 + m)), (3*(-4 + m))/(2*(-2 + m)), -((a*x^(2
- m))/b)]))/(b*(-8 + m)*(4 + m)*Sqrt[a + b*x^(-2 + m)])
```

fricas **[F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^(-2+m))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{bx^{m-2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^(-2+m))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/sqrt(b*x^(m - 2) + a), x)
```

maple **[F]** time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{b\,x^{m-2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a+b*x^(m-2))^(1/2),x)
```

```
[Out] int(x^m/(a+b*x^(m-2))^(1/2),x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{bx^{m-2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^(-2+m))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/sqrt(b*x^(m - 2) + a), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{\sqrt{a+b\,x^{m-2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^(m - 2))^(1/2),x)
```

```
[Out] int(x^m/(a + b*x^(m - 2))^(1/2), x)
```

sympy [C] time = 7.84, size = 94, normalized size = 1.45

$$\frac{xx^m\Gamma\left(\frac{m}{m-2}+\frac{1}{m-2}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{m}{m-2}+\frac{1}{m-2}\\ \frac{m}{m-2}+1+\frac{1}{m-2}\end{matrix}\right|\frac{bx^me^{i\pi}}{ax^2}\right)}{\sqrt{a}\,m\Gamma\left(\frac{m}{m-2}+1+\frac{1}{m-2}\right)-2\sqrt{a}\,\Gamma\left(\frac{m}{m-2}+1+\frac{1}{m-2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a+b*x**(-2+m))**(1/2),x)
```

```
[Out] x*x**m*gamma(m/(m - 2) + 1/(m - 2))*hyper((1/2, m/(m - 2) + 1/(m - 2)), (m/
(m - 2) + 1 + 1/(m - 2),), b*x**m*exp_polar(I*pi)/(a*x**2))/(sqrt(a)*m*gamm
a(m/(m - 2) + 1 + 1/(m - 2)) - 2*sqrt(a)*gamma(m/(m - 2) + 1 + 1/(m - 2)))
```

3.2696 $\int \frac{x^m}{\sqrt{a+bx^{2-m}}}\,dx$

Optimal. Leaf size=67

$$\frac{x^{m+1}\sqrt{a+bx^{2-m}}\;{}_2F_1\left(1,\frac{m+4}{2(2-m)};\frac{3}{2-m};-\frac{bx^{2-m}}{a}\right)}{a(m+1)}$$

```
[Out] x^(1+m)*hypergeom([1, 1/2*(4+m)/(2-m)],[3/(2-m)],-b*x^(2-m)/a)*(a+b*x^(2-m)
)^(1/2)/a/(1+m)
```

Rubi [A] time = 0.03, antiderivative size = 81, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {365, 364}

$$\frac{x^{m+1}\sqrt{\frac{bx^{2-m}}{a}+1}\;{}_2F_1\left(\frac{1}{2},\frac{m+1}{2-m};\frac{3}{2-m};-\frac{bx^{2-m}}{a}\right)}{(m+1)\sqrt{a+bx^{2-m}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/Sqrt[a + b*x^(2 - m)],x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + (b*x^(2 - m))/a]*Hypergeometric2F1[1/2, (1 + m)/(2 - m)
, 3/(2 - m), -((b*x^(2 - m))/a)])/((1 + m)*Sqrt[a + b*x^(2 - m)])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{\sqrt{a+bx^{2-m}}}\,dx &= \frac{\sqrt{1+\frac{bx^{2-m}}{a}}\int \frac{x^m}{\sqrt{1+\frac{bx^{2-m}}{a}}}\,dx}{\sqrt{a+bx^{2-m}}}\\ &= \frac{x^{1+m}\sqrt{1+\frac{bx^{2-m}}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{1+m}{2-m};\frac{3}{2-m};-\frac{bx^{2-m}}{a}\right)}{(1+m)\sqrt{a+bx^{2-m}}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 79, normalized size = 1.18

$$\frac{x^{m+1}\sqrt{\frac{bx^{2-m}}{a}+1}\;{}_2F_1\left(\frac{1}{2},\frac{m+1}{2-m};-\frac{3}{m-2};-\frac{bx^{2-m}}{a}\right)}{(m+1)\sqrt{a+bx^{2-m}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/Sqrt[a + b*x^(2 - m)],x]
```

```
[Out] (x^(1 + m)*Sqrt[1 + (b*x^(2 - m))/a]*Hypergeometric2F1[1/2, (1 + m)/(2 - m)
, -3/(-2 + m), -((b*x^(2 - m))/a)])/((1 + m)*Sqrt[a + b*x^(2 - m)])
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^(2-m))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{bx^{-m+2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^(2-m))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/sqrt(b*x^(-m + 2) + a), x)
```

maple [**F**] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{b\,x^{-m+2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a+b*x^(2-m))^(1/2),x)
```

```
[Out] int(x^m/(a+b*x^(2-m))^(1/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{bx^{-m+2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^(2-m))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/sqrt(b*x^(-m + 2) + a), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m}{\sqrt{a+b\,x^{2-m}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^(2 - m))^(1/2),x)
```

```
[Out] int(x^m/(a + b*x^(2 - m))^(1/2), x)
```

sympy [**C**] time = 17.10, size = 95, normalized size = 1.42

$$-\frac{x x^m \Gamma\left(\frac{m}{2-m}+\frac{1}{2-m}\right) {}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{m}{2-m}+\frac{1}{2-m} \\ \frac{m}{2-m}+1+\frac{1}{2-m}\end{matrix}\middle| \frac{b x^2 x^{-m} e^{i\pi}}{a}\right)}{\sqrt{a}\, m \Gamma\left(\frac{m}{2-m}+1+\frac{1}{2-m}\right) - 2\sqrt{a}\,\Gamma\left(\frac{m}{2-m}+1+\frac{1}{2-m}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a+b*x**(2-m))**(1/2),x)
```

```
[Out] -x*x**m*gamma(m/(2 - m) + 1/(2 - m))*hyper((1/2, m/(2 - m) + 1/(2 - m)), (m
/(2 - m) + 1 + 1/(2 - m),), b*x**2*x**(-m)*exp_polar(I*pi)/a)/(sqrt(a)*m*ga
mma(m/(2 - m) + 1 + 1/(2 - m)) - 2*sqrt(a)*gamma(m/(2 - m) + 1 + 1/(2 - m))
)
```

3.2697 $\int \left(\frac{6ax^2}{b(4+m)\sqrt{a+bx^{-2+m}}} + \frac{x^m}{\sqrt{a+bx^{-2+m}}}\right) dx$

Optimal. Leaf size=26

$$\frac{2x^3\sqrt{a+bx^{m-2}}}{b(m+4)}$$

```
[Out] 2*x^3*(a+b*x^(-2+m))^(1/2)/b/(4+m)
```

Rubi [C] time = 0.09, antiderivative size = 160, normalized size of antiderivative = 6.15, number of steps used = 5, number of rules used = 2, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.044$, Rules used = {365, 364}

$$\frac{x^{m+1}\sqrt{\frac{bx^{m-2}}{a}+1}\ {}_2F_1\left(\frac{1}{2},-\frac{m+1}{2-m};\frac{1-2m}{2-m};-\frac{bx^{m-2}}{a}\right)}{(m+1)\sqrt{a+bx^{m-2}}} + \frac{2ax^3\sqrt{\frac{bx^{m-2}}{a}+1}\ {}_2F_1\left(\frac{1}{2},-\frac{3}{2-m};-\frac{m+1}{2-m};-\frac{bx^{m-2}}{a}\right)}{b(m+4)\sqrt{a+bx^{m-2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(6*a*x^2)/(b*(4 + m)*Sqrt[a + b*x^(-2 + m)]) + x^m/Sqrt[a + b*x^(-2 + m
)],x]
```

```
[Out] (2*a*x^3*Sqrt[1 + (b*x^(-2 + m))/a]*Hypergeometric2F1[1/2, -3/(2 - m), -((1
 + m)/(2 - m)), -((b*x^(-2 + m))/a)])/(b*(4 + m)*Sqrt[a + b*x^(-2 + m)]) +
(x^(1 + m)*Sqrt[1 + (b*x^(-2 + m))/a]*Hypergeometric2F1[1/2, -((1 + m)/(2 -
 m)), (1 - 2*m)/(2 - m), -((b*x^(-2 + m))/a)])/((1 + m)*Sqrt[a + b*x^(-2 +
m)])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \left(\frac{6ax^2}{b(4+m)\sqrt{a+bx^{-2+m}}} + \frac{x^m}{\sqrt{a+bx^{-2+m}}}\right) dx = \frac{(6a)\int \frac{x^2}{\sqrt{a+bx^{-2+m}}}\,dx}{b(4+m)} + \int \frac{x^m}{\sqrt{a+bx^{-2+m}}}\,dx$$

$$= \frac{\sqrt{1+\frac{bx^{-2+m}}{a}}\int \frac{x^m}{\sqrt{1+\frac{bx^{-2+m}}{a}}}\,dx}{\sqrt{a+bx^{-2+m}}} + \frac{\left(6a\sqrt{1+\frac{bx^{-2+m}}{a}}\right)\int \frac{x^2}{\sqrt{1+\frac{bx^{-2+m}}{a}}}}{b(4+m)\sqrt{a+bx^{-2+m}}}$$

$$= \frac{2ax^3\sqrt{1+\frac{bx^{-2+m}}{a}}\ {}_2F_1\left(\frac{1}{2},-\frac{3}{2-m};-\frac{1+m}{2-m};-\frac{bx^{-2+m}}{a}\right)}{b(4+m)\sqrt{a+bx^{-2+m}}} + \frac{x^{1+m}\sqrt{1+\frac{b}{}}}{}$$

Mathematica [A] time = 0.20, size = 26, normalized size = 1.00

$$\frac{2x^3\sqrt{a+bx^{m-2}}}{b(m+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(6*a*x^2)/(b*(4 + m)*Sqrt[a + b*x^(-2 + m)]) + x^m/Sqrt[a + b*x^(
-2 + m)],x]
```

```
[Out] (2*x^3*Sqrt[a + b*x^(-2 + m)])/(b*(4 + m))
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(6*a*x^2/b/(4+m)/(a+b*x^(-2+m))^(1/2)+x^m/(a+b*x^(-2+m))^(1/2),x,
algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{bx^{m-2}+a}} + \frac{6\,ax^2}{\sqrt{bx^{m-2}+a}\,b(m+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(6*a*x^2/b/(4+m)/(a+b*x^(-2+m))^(1/2)+x^m/(a+b*x^(-2+m))^(1/2),x,
algorithm="giac")
```

```
[Out] integrate(x^m/sqrt(b*x^(m - 2) + a) + 6*a*x^2/(sqrt(b*x^(m - 2) + a)*b*(m +
 4)), x)
```

maple [**A**] time = 0.05, size = 40, normalized size = 1.54

$$\frac{2\left(a\,x^2+b\,x^m\right)x}{(m+4)\sqrt{\frac{a\,x^2+b\,x^m}{x^2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(6*a*x^2/b/(m+4)/(b*x^(m-2)+a)^(1/2)+1/(b*x^(m-2)+a)^(1/2)*x^m,x)
```

```
[Out] 2*x*(a*x^2+b*x^m)/b/(m+4)/((a*x^2+b*x^m)/x^2)^(1/2)
```

maxima [**A**] time = 0.85, size = 37, normalized size = 1.42

$$\frac{2\left(ax^4+bx^2x^m\right)}{\sqrt{ax^2+bx^m}\,b(m+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(6*a*x^2/b/(4+m)/(a+b*x^(-2+m))^(1/2)+x^m/(a+b*x^(-2+m))^(1/2),x,
algorithm="maxima")
```

```
[Out] 2*(a*x^4 + b*x^2*x^m)/(sqrt(a*x^2 + b*x^m)*b*(m + 4))
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m}{\sqrt{a+b\,x^{m-2}}} + \frac{6\,a\,x^2}{b\,\sqrt{a+b\,x^{m-2}}\;(m+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^(m - 2))^(1/2) + (6*a*x^2)/(b*(a + b*x^(m - 2))^(1/2)*(m +
 4)),x)
```

```
[Out] int(x^m/(a + b*x^(m - 2))^(1/2) + (6*a*x^2)/(b*(a + b*x^(m - 2))^(1/2)*(m +
 4)), x)
```

sympy [C] time = 25.96, size = 170, normalized size = 6.54

$$\frac{6ax^3\Gamma\left(\frac{3}{m-2}\right){}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{3}{m-2}\\ 1+\frac{3}{m-2}\end{matrix}\middle| \frac{bx^m e^{i\pi}}{ax^2}\right)}{b(m+4)\left(\sqrt{a}\,m\Gamma\left(1+\frac{3}{m-2}\right)-2\sqrt{a}\,\Gamma\left(1+\frac{3}{m-2}\right)\right)}+\frac{xx^m\Gamma\left(\frac{m}{m-2}+\frac{1}{m-2}\right){}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{m}{m-2}+\frac{1}{m-2}\\ \frac{m}{m-2}+1+\frac{1}{m-2}\end{matrix}\middle| \frac{bx^m e^{i\pi}}{ax^2}\right)}{\sqrt{a}\,m\Gamma\left(\frac{m}{m-2}+1+\frac{1}{m-2}\right)-2\sqrt{a}\,\Gamma\left(\frac{m}{m-2}+1+\frac{1}{m-2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(6*a*x**2/b/(4+m)/(a+b*x**(-2+m))**(1/2)+x**m/(a+b*x**(-2+m))**(1/
2),x)
```

```
[Out] 6*a*x**3*gamma(3/(m - 2))*hyper((1/2, 3/(m - 2)), (1 + 3/(m - 2),), b*x**m*
exp_polar(I*pi)/(a*x**2))/(b*(m + 4)*(sqrt(a)*m*gamma(1 + 3/(m - 2)) - 2*sq
rt(a)*gamma(1 + 3/(m - 2)))) + x*x**m*gamma(m/(m - 2) + 1/(m - 2))*hyper((1
/2, m/(m - 2) + 1/(m - 2)), (m/(m - 2) + 1 + 1/(m - 2),), b*x**m*exp_polar(
I*pi)/(a*x**2))/(sqrt(a)*m*gamma(m/(m - 2) + 1 + 1/(m - 2)) - 2*sqrt(a)*gam
ma(m/(m - 2) + 1 + 1/(m - 2)))
```

3.2698 $\int \frac{x^{-1+m}(2am+b(2m-n)x^n)}{2(a+bx^n)^{3/2}}\,dx$

Optimal. Leaf size=15

$$\frac{x^m}{\sqrt{a+bx^n}}$$

```
[Out] x^m/(a+b*x^n)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {12, 449}

$$\frac{x^m}{\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + m)*(2*a*m + b*(2*m - n)*x^n))/(2*(a + b*x^n)^(3/2)),x]
```

```
[Out] x^m/Sqrt[a + b*x^n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^{-1+m}\,(2am+b(2m-n)x^n)}{2\,(a+bx^n)^{3/2}}\,dx = \frac{1}{2}\int \frac{x^{-1+m}\,(2am+b(2m-n)x^n)}{(a+bx^n)^{3/2}}\,dx$$
$$= \frac{x^m}{\sqrt{a+bx^n}}$$

Mathematica [C] time = 0.13, size = 111, normalized size = 7.40

$$\frac{x^m\sqrt{\frac{bx^n}{a}+1}\left(b(2m-n)x^n\,{}_2F_1\left(\frac{3}{2},\frac{m+n}{n};\frac{m}{n}+2;-\frac{bx^n}{a}\right)+2a(m+n)\,{}_2F_1\left(\frac{3}{2},\frac{m}{n};\frac{m+n}{n};-\frac{bx^n}{a}\right)\right)}{2a(m+n)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + m)*(2*a*m + b*(2*m - n)*x^n))/(2*(a + b*x^n)^(3/2)),x]
```

```
[Out] (x^m*Sqrt[1 + (b*x^n)/a]*(2*a*(m + n)*Hypergeometric2F1[3/2, m/n, (m + n)/n
, -((b*x^n)/a)] + b*(2*m - n)*x^n*Hypergeometric2F1[3/2, (m + n)/n, 2 + m/n
, -((b*x^n)/a)]))/(2*a*(m + n)*Sqrt[a + b*x^n])
```

fricas [A] time = 0.77, size = 16, normalized size = 1.07

$$\frac{xx^{m-1}}{\sqrt{bx^n+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(a+b*x^n)^(3/2),x, algorithm="
fricas")
```

```
[Out] x*x^(m - 1)/sqrt(b*x^n + a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b(2\,m-n)x^n+2\,am)x^{m-1}}{2\,(bx^n+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(a+b*x^n)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate(1/2*(b*(2*m - n)*x^n + 2*a*m)*x^(m - 1)/(b*x^n + a)^(3/2), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{(2am+(2m-n)\,b\,x^n)\,x^{m-1}}{2\,(b\,x^n+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*x^(m-1)*(2*a*m+b*(2*m-n)*x^n)/(b*x^n+a)^(3/2),x)
```

```
[Out] int(1/2*x^(m-1)*(2*a*m+b*(2*m-n)*x^n)/(b*x^n+a)^(3/2),x)
```

maxima [A] time = 1.02, size = 13, normalized size = 0.87

$$\frac{x^m}{\sqrt{bx^n+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(a+b*x^n)^(3/2),x, algorithm="
maxima")
```

```
[Out] x^m/sqrt(b*x^n + a)
```

mupad [B] time = 1.42, size = 13, normalized size = 0.87

$$\frac{x^m}{\sqrt{a+b\,x^n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(m - 1)*(2*a*m + b*x^n*(2*m - n)))/(2*(a + b*x^n)^(3/2)),x)
```

```
[Out] x^m/(a + b*x^n)^(1/2)
```

sympy [C] time = 101.86, size = 100, normalized size = 6.67

$$\frac{mx^m\Gamma\left(\frac{m}{n}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{m}{n}\\ \frac{m}{n}+1\end{matrix}\right|\frac{bx^ne^{i\pi}}{a}\right)}{\sqrt{a}\,n\Gamma\left(\frac{m}{n}+1\right)}+\frac{bx^mx^n\,(2m-n)\,\Gamma\left(\frac{m}{n}+1\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{m}{n}+1\\ \frac{m}{n}+2\end{matrix}\right|\frac{bx^ne^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}n\Gamma\left(\frac{m}{n}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x**(-1+m)*(2*a*m+b*(2*m-n)*x**n)/(a+b*x**n)**(3/2),x)

[Out] m*x**m*gamma(m/n)*hyper((3/2, m/n), (m/n + 1,), b*x**n*exp_polar(I*pi)/a)/(
sqrt(a)*n*gamma(m/n + 1)) + b*x**m*x**n*(2*m - n)*gamma(m/n + 1)*hyper((3/2
, m/n + 1), (m/n + 2,), b*x**n*exp_polar(I*pi)/a)/(2*a**(3/2)*n*gamma(m/n +
 2))
```

3.2699 $\int \left(-\frac{bnx^{-1+m+n}}{2(a+bx^n)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^n}}\right)dx$

Optimal. Leaf size=15

$$\frac{x^m}{\sqrt{a+bx^n}}$$

```
[Out] x^m/(a+b*x^n)^(1/2)
```

Rubi [C] time = 0.07, antiderivative size = 126, normalized size of antiderivative = 8.40, number of steps used = 5, number of rules used = 2, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {365, 364}

$$\frac{x^m\sqrt{\frac{bx^n}{a}+1}\;{}_2F_1\left(\frac{1}{2},\frac{m}{n};\frac{m+n}{n};-\frac{bx^n}{a}\right)}{\sqrt{a+bx^n}}-\frac{bnx^{m+n}\sqrt{\frac{bx^n}{a}+1}\;{}_2F_1\left(\frac{3}{2},\frac{m+n}{n};\frac{m}{n}+2;-\frac{bx^n}{a}\right)}{2a(m+n)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[-(b*n*x^(-1 + m + n))/(2*(a + b*x^n)^(3/2)) + (m*x^(-1 + m))/Sqrt[a + b
*x^n],x]
```

```
[Out] (x^m*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, m/n, (m + n)/n, -((b*x^n)/a
)])/Sqrt[a + b*x^n] - (b*n*x^(m + n)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[
3/2, (m + n)/n, 2 + m/n, -((b*x^n)/a)])/(2*a*(m + n)*Sqrt[a + b*x^n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int\left(-\frac{bnx^{-1+m+n}}{2\left(a+bx^n\right)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^n}}\right)dx = m\int\frac{x^{-1+m}}{\sqrt{a+bx^n}}\,dx-\frac{1}{2}(bn)\int\frac{x^{-1+m+n}}{\left(a+bx^n\right)^{3/2}}\,dx$$

$$=\frac{\left(m\sqrt{1+\frac{bx^n}{a}}\right)\int\frac{x^{-1+m}}{\sqrt{1+\frac{bx^n}{a}}}\,dx}{\sqrt{a+bx^n}}-\frac{\left(bn\sqrt{1+\frac{bx^n}{a}}\right)\int\frac{x^{-1+m+n}}{\left(1+\frac{bx^n}{a}\right)^{3/2}}\,dx}{2a\sqrt{a+bx^n}}$$

$$=\frac{x^m\sqrt{1+\frac{bx^n}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{m}{n};\frac{m+n}{n};-\frac{bx^n}{a}\right)}{\sqrt{a+bx^n}}-\frac{bnx^{m+n}\sqrt{1+\frac{bx^n}{a}}\;{}_2F_1\left(\frac{3}{2},\frac{m+n}{n};2+\frac{m}{n}\right.}{2a(m+n)\sqrt{a+bx^n}}$$

Mathematica [C] time = 0.10, size = 111, normalized size = 7.40

$$\frac{x^m\sqrt{\frac{bx^n}{a}+1}\left(b(2m-n)x^n\;{}_2F_1\left(\frac{3}{2},\frac{m+n}{n};\frac{m}{n}+2;-\frac{bx^n}{a}\right)+2a(m+n)\;{}_2F_1\left(\frac{3}{2},\frac{m}{n};\frac{m+n}{n};-\frac{bx^n}{a}\right)\right)}{2a(m+n)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[-1/2*(b*n*x^(-1 + m + n))/(a + b*x^n)^(3/2) + (m*x^(-1 + m))/Sqrt
[a + b*x^n],x]
```

```
[Out] (x^m*Sqrt[1 + (b*x^n)/a]*(2*a*(m + n)*Hypergeometric2F1[3/2, m/n, (m + n)/n
, -((b*x^n)/a)] + b*(2*m - n)*x^n*Hypergeometric2F1[3/2, (m + n)/n, 2 + m/n
, -((b*x^n)/a)]))/(2*a*(m + n)*Sqrt[a + b*x^n])
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*b*n*x^(-1+m+n)/(a+b*x^n)^(3/2)+m*x^(-1+m)/(a+b*x^n)^(1/2),x,
 algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{bnx^{m+n-1}}{2\left(bx^n+a\right)^{\frac{3}{2}}}+\frac{mx^{m-1}}{\sqrt{bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*b*n*x^(-1+m+n)/(a+b*x^n)^(3/2)+m*x^(-1+m)/(a+b*x^n)^(1/2),x,
 algorithm="giac")
```

```
[Out] integrate(-1/2*b*n*x^(m + n - 1)/(b*x^n + a)^(3/2) + m*x^(m - 1)/sqrt(b*x^n
 + a), x)
```

maple [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{bn\,x^{m+n-1}}{2\left(b\,x^n+a\right)^{\frac{3}{2}}}+\frac{m\,x^{m-1}}{\sqrt{b\,x^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/2/(b*x^n+a)^(3/2)*b*n*x^(m+n-1)+1/(b*x^n+a)^(1/2)*m*x^(m-1),x)
```

```
[Out] int(-1/2/(b*x^n+a)^(3/2)*b*n*x^(m+n-1)+1/(b*x^n+a)^(1/2)*m*x^(m-1),x)
```

maxima [**A**] time = 0.85, size = 13, normalized size = 0.87

$$\frac{x^m}{\sqrt{bx^n+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*b*n*x^(-1+m+n)/(a+b*x^n)^(3/2)+m*x^(-1+m)/(a+b*x^n)^(1/2),x,
 algorithm="maxima")
```

```
[Out] x^m/sqrt(b*x^n + a)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{m\,x^{m-1}}{\sqrt{a+b\,x^n}}-\frac{b\,n\,x^{m+n-1}}{2\,{\left(a+b\,x^n\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((m*x^(m - 1))/(a + b*x^n)^(1/2) - (b*n*x^(m + n - 1))/(2*(a + b*x^n)^(3
/2)),x)
```

```
[Out] int((m*x^(m - 1))/(a + b*x^n)^(1/2) - (b*n*x^(m + n - 1))/(2*(a + b*x^n)^(3
/2)), x)
```

sympy [C] time = 47.51, size = 94, normalized size = 6.27

$$\frac{m x^{m} \Gamma\left(\frac{m}{n}\right) {}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{m}{n} \\ \frac{m}{n}+1\end{matrix}\middle| \frac{b x^{n} e^{i\pi}}{a}\right)}{\sqrt{a}\, n \Gamma\left(\frac{m}{n}+1\right)} - \frac{b x^{m} x^{n} \Gamma\left(\frac{m}{n}+1\right) {}_2F_1\left(\begin{matrix}\frac{3}{2}, \frac{m}{n}+1 \\ \frac{m}{n}+2\end{matrix}\middle| \frac{b x^{n} e^{i\pi}}{a}\right)}{2 a^{\frac{3}{2}} \Gamma\left(\frac{m}{n}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*b*n*x**(-1+m+n)/(a+b*x**n)**(3/2)+m*x**(-1+m)/(a+b*x**n)**(1
/2),x)
```

```
[Out] m*x**m*gamma(m/n)*hyper((1/2, m/n), (m/n + 1,), b*x**n*exp_polar(I*pi)/a)/(
sqrt(a)*n*gamma(m/n + 1)) - b*x**m*x**n*gamma(m/n + 1)*hyper((3/2, m/n + 1)
, (m/n + 2,), b*x**n*exp_polar(I*pi)/a)/(2*a**(3/2)*gamma(m/n + 2))
```

3.2700 $\int \frac{x^m}{\sqrt[3]{a+bx^{3(1+m)}}}\,dx$

Optimal. Leaf size=97

$$\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x^{m+1}}{\sqrt[3]{a+bx^{3(m+1)}}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}\,(m+1)}-\frac{\log\left(\sqrt[3]{b}\,x^{m+1}-\sqrt[3]{a+bx^{3(m+1)}}\right)}{2\sqrt[3]{b}\,(m+1)}$$

```
[Out] -1/2*ln(b^(1/3)*x^(1+m)-(a+b*x^(3+3*m))^(1/3))/b^(1/3)/(1+m)+1/3*arctan(1/3
*(1+2*b^(1/3)*x^(1+m)/(a+b*x^(3+3*m))^(1/3))*3^(1/2))/b^(1/3)/(1+m)*3^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {345, 239}

$$\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x^{m+1}}{\sqrt[3]{a+bx^{3(m+1)}}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}\,(m+1)}-\frac{\log\left(\sqrt[3]{b}\,x^{m+1}-\sqrt[3]{a+bx^{3(m+1)}}\right)}{2\sqrt[3]{b}\,(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x^(3*(1 + m)))^(1/3),x]
```

```
[Out] ArcTan[(1 + (2*b^(1/3)*x^(1 + m))/(a + b*x^(3*(1 + m)))^(1/3))/Sqrt[3]]/(Sq
rt[3]*b^(1/3)*(1 + m)) - Log[b^(1/3)*x^(1 + m) - (a + b*x^(3*(1 + m)))^(1/3
)]/(2*b^(1/3)*(1 + m))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{\sqrt[3]{a+bx^{3(1+m)}}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx, x, x^{1+m}\right)}{1+m}\\ &= \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x^{1+m}}{\sqrt[3]{a+bx^{3(1+m)}}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}\,(1+m)}-\frac{\log\left(\sqrt[3]{b}\,x^{1+m}-\sqrt[3]{a+bx^{3(1+m)}}\right)}{2\sqrt[3]{b}\,(1+m)}\end{aligned}$$

Mathematica [C] time = 0.04, size = 67, normalized size = 0.69

$$\frac{x^{m+1}\sqrt[3]{\frac{bx^{3m+3}}{a}+1}\;{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^{3m+3}}{a}\right)}{(m+1)\sqrt[3]{a+bx^{3m+3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x^(3*(1 + m)))^(1/3),x]
```

```
[Out] (x^(1 + m)*(1 + (b*x^(3 + 3*m))/a)^(1/3)*Hypergeometric2F1[1/3, 1/3, 4/3, -
((b*x^(3 + 3*m))/a)])/((1 + m)*(a + b*x^(3 + 3*m))^(1/3))
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^(3+3*m))^(1/3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(bx^{3\,m+3}+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^(3+3*m))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*x^(3*m + 3) + a)^(1/3), x)
```

maple [**F**] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(b\,x^{3m+3}+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a+b*x^(3*m+3))^(1/3),x)
```

```
[Out] int(x^m/(a+b*x^(3*m+3))^(1/3),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(bx^{3\,m+3}+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^(3+3*m))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(b*x^(3*m + 3) + a)^(1/3), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m}{\left(a+b\,x^{3\,m+3}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^(3*m + 3))^(1/3),x)
```

```
[Out] int(x^m/(a + b*x^(3*m + 3))^(1/3), x)
```

sympy [C] time = 2.48, size = 117, normalized size = 1.21

$$\frac{xx^m\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{m}{3m+3}+1+\frac{1}{3m+3}\end{matrix}\middle|\frac{bx^3x^{3m}e^{i\pi}}{a}\right)}{3a^{\frac{m}{3m+3}}a^{\frac{1}{3m+3}}m\Gamma\left(\frac{m}{3m+3}+1+\frac{1}{3m+3}\right)+3a^{\frac{m}{3m+3}}a^{\frac{1}{3m+3}}\Gamma\left(\frac{m}{3m+3}+1+\frac{1}{3m+3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a+b*x**(3+3*m))**(1/3),x)
```

```
[Out] x*x**m*gamma(1/3)*hyper((1/3, 1/3), (m/(3*m + 3) + 1 + 1/(3*m + 3),), b*x**
3*x**(3*m)*exp_polar(I*pi)/a)/(3*a**(m/(3*m + 3))*a**(1/(3*m + 3))*m*gamma(
m/(3*m + 3) + 1 + 1/(3*m + 3)) + 3*a**(m/(3*m + 3))*a**(1/(3*m + 3))*gamma(
m/(3*m + 3) + 1 + 1/(3*m + 3)))
```

3.2701 $\int x^m \left(a + bx^{-\frac{3}{2}(1+m)}\right)^{2/3} dx$

Optimal. Leaf size=139

$$\frac{b^{2/3}\log\left(\sqrt[3]{b}\,x^{\frac{1}{2}(-m-1)} - \sqrt[3]{a+bx^{-\frac{3}{2}(m+1)}}\right)}{m+1} - \frac{2b^{2/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x^{\frac{1}{2}(-m-1)}}{\sqrt[3]{a+bx^{-\frac{3}{2}(m+1)}}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,(m+1)} + \frac{x^{m+1}\left(a+bx^{-\frac{3}{2}(m+1)}\right)^{2/3}}{m+1}$$

```
[Out] x^(1+m)*(a+b/(x^(3/2+3/2*m)))^(2/3)/(1+m)+b^(2/3)*ln(b^(1/3)*x^(-1/2-1/2*m)
-(a+b/(x^(3/2+3/2*m)))^(1/3))/(1+m)-2/3*b^(2/3)*arctan(1/3*(1+2*b^(1/3)*x^(
-1/2-1/2*m)/(a+b/(x^(3/2+3/2*m)))^(1/3))*3^(1/2))/(1+m)*3^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {349, 345, 239}

$$\frac{b^{2/3}\log\left(\sqrt[3]{b}\,x^{\frac{1}{2}(-m-1)} - \sqrt[3]{a+bx^{-\frac{3}{2}(m+1)}}\right)}{m+1} - \frac{2b^{2/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x^{\frac{1}{2}(-m-1)}}{\sqrt[3]{a+bx^{-\frac{3}{2}(m+1)}}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,(m+1)} + \frac{x^{m+1}\left(a+bx^{-\frac{3}{2}(m+1)}\right)^{2/3}}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b/x^((3*(1 + m))/2))^(2/3),x]
```

```
[Out] (x^(1 + m)*(a + b/x^((3*(1 + m))/2))^(2/3))/(1 + m) - (2*b^(2/3)*ArcTan[(1
+ (2*b^(1/3)*x^((-1 - m)/2))/(a + b/x^((3*(1 + m))/2))^(1/3))/Sqrt[3]])/(Sq
rt[3]*(1 + m)) + (b^(2/3)*Log[b^(1/3)*x^((-1 - m)/2) - (a + b/x^((3*(1 + m)
)/2))^(1/3)])/(1 + m)
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 349

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*
(a + b*x^n)^p)/(m + 1), x] - Dist[(b*n*p)/(m + 1), Int[x^(m + n)*(a + b*x^n
)^(p - 1), x], x] /; FreeQ[{a, b, m, n}, x] && EqQ[(m + 1)/n + p, 0] && GtQ
[p, 0]
```

Rubi steps

$$\int x^m \left(a+bx^{-\frac{3}{2}(1+m)}\right)^{2/3} dx = \frac{x^{1+m}\left(a+bx^{-\frac{3}{2}(1+m)}\right)^{2/3}}{1+m} + b\int \frac{x^{m-\frac{3(1+m)}{2}}}{\sqrt[3]{a+bx^{-\frac{3}{2}(1+m)}}}\,dx$$

$$= \frac{x^{1+m}\left(a+bx^{-\frac{3}{2}(1+m)}\right)^{2/3}}{1+m} - \frac{(2b)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx, x, x^{1+m-\frac{3(1+m)}{2}}\right)}{1+m}$$

$$= \frac{x^{1+m}\left(a+bx^{-\frac{3}{2}(1+m)}\right)^{2/3}}{1+m} - \frac{2b^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x^{\frac{1}{2}(-1-m)}}{\sqrt[3]{a+bx^{-\frac{3}{2}(1+m)}}}}{\sqrt{3}}\right)}{\sqrt{3}\,(1+m)} + \frac{b^{2/3}\log\left(\sqrt[3]{b}\,x^{\frac{1}{2}(-1-m)}\right.}{1+}$$

Mathematica [C] time = 0.08, size = 73, normalized size = 0.53

$$\frac{x^{m+1}\left(a+bx^{-\frac{3}{2}(m+1)}\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},-\frac{2}{3};\frac{1}{3};-\frac{bx^{-\frac{3}{2}(m+1)}}{a}\right)}{(m+1)\left(\frac{bx^{-\frac{3}{2}(m+1)}}{a}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b/x^((3*(1 + m))/2))^(2/3),x]
```

```
[Out] (x^(1 + m)*(a + b/x^((3*(1 + m))/2))^(2/3)*Hypergeometric2F1[-2/3, -2/3, 1/
3, -(b/(a*x^((3*(1 + m))/2)))])/((1 + m)*(1 + b/(a*x^((3*(1 + m))/2)))^(2/3
))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b/(x^(3/2+3/2*m)))^(2/3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+\frac{b}{x^{\frac{3}{2}m+\frac{3}{2}}}\right)^{\frac{2}{3}} x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b/(x^(3/2+3/2*m)))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((a + b/x^(3/2*m + 3/2))^(2/3)*x^m, x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \left(b\,x^{-\frac{3m}{2}-\frac{3}{2}}+a\right)^{\frac{2}{3}} x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b/(x^(3/2*m+3/2)))^(2/3),x)
[Out] int(x^m*(a+b/(x^(3/2*m+3/2)))^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^{-\frac{3}{2}m-\frac{3}{2}}+a\right)^{\frac{2}{3}} x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b/(x^(3/2+3/2*m)))^(2/3),x, algorithm="maxima")
[Out] integrate((b*x^(-3/2*m - 3/2) + a)^(2/3)*x^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^m\left(a+\frac{b}{x^{\frac{3m}{2}+\frac{3}{2}}}\right)^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b/x^((3*m)/2 + 3/2))^(2/3),x)
[Out] int(x^m*(a + b/x^((3*m)/2 + 3/2))^(2/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b/(x**(3/2+3/2*m)))**(2/3),x)
[Out] Timed out
```

3.2702 $\int \frac{x^{-1+\frac{n}{3}}}{\sqrt[3]{a+bx^n}}\,dx$

Optimal. Leaf size=89

$$\frac{\sqrt{3}\,\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x^{n/3}}{\sqrt[3]{a+bx^n}}+1}{\sqrt{3}}\right)}{\sqrt[3]{b}\,n}-\frac{3\log\left(\sqrt[3]{b}\,x^{n/3}-\sqrt[3]{a+bx^n}\right)}{2\sqrt[3]{b}\,n}$$

```
[Out] -3/2*ln(b^(1/3)*x^(1/3*n)-(a+b*x^n)^(1/3))/b^(1/3)/n+arctan(1/3*(1+2*b^(1/3
)*x^(1/3*n)/(a+b*x^n)^(1/3))*3^(1/2))*3^(1/2)/b^(1/3)/n
```

Rubi [A] time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {345, 239}

$$\frac{\sqrt{3}\,\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x^{n/3}}{\sqrt[3]{a+bx^n}}+1}{\sqrt{3}}\right)}{\sqrt[3]{b}\,n}-\frac{3\log\left(\sqrt[3]{b}\,x^{n/3}-\sqrt[3]{a+bx^n}\right)}{2\sqrt[3]{b}\,n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n/3)/(a + b*x^n)^(1/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*x^(n/3))/(a + b*x^n)^(1/3))/Sqrt[3]])/(b^(1
/3)*n) - (3*Log[b^(1/3)*x^(n/3) - (a + b*x^n)^(1/3)])/(2*b^(1/3)*n)
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+\frac{n}{3}}}{\sqrt[3]{a+bx^n}}\,dx &= \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx, x, x^{n/3}\right)}{n}\\ &= \frac{\sqrt{3}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x^{n/3}}{\sqrt[3]{a+bx^n}}}{\sqrt{3}}\right)}{\sqrt[3]{b}\,n}-\frac{3\log\left(\sqrt[3]{b}\,x^{n/3}-\sqrt[3]{a+bx^n}\right)}{2\sqrt[3]{b}\,n}\end{aligned}$$

Mathematica [C] time = 0.02, size = 56, normalized size = 0.63

$$\frac{3x^{n/3}\sqrt[3]{\frac{bx^n}{a}+1}\;{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^n}{a}\right)}{n\sqrt[3]{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n/3)/(a + b*x^n)^(1/3),x]
```

```
[Out] (3*x^(n/3)*(1 + (b*x^n)/a)^(1/3)*Hypergeometric2F1[1/3, 1/3, 4/3, -((b*x^n)
/a)])/(n*(a + b*x^n)^(1/3))
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/3*n)/(a+b*x^n)^(1/3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{1}{3}n-1}}{(bx^n+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/3*n)/(a+b*x^n)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(x^(1/3*n - 1)/(b*x^n + a)^(1/3), x)
```

maple [**F**] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{n}{3}-1}}{(b\,x^n+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+1/3*n)/(b*x^n+a)^(1/3),x)
```

```
[Out] int(x^(-1+1/3*n)/(b*x^n+a)^(1/3),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{1}{3}n-1}}{(bx^n+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/3*n)/(a+b*x^n)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(x^(1/3*n - 1)/(b*x^n + a)^(1/3), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{\frac{n}{3}-1}}{(a+b\,x^n)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n/3 - 1)/(a + b*x^n)^(1/3),x)
```

```
[Out] int(x^(n/3 - 1)/(a + b*x^n)^(1/3), x)
```

sympy [C] time = 7.81, size = 39, normalized size = 0.44

$$\frac{x^{\frac{n}{3}}\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3}, \frac{1}{3} \\ \frac{4}{3}\end{matrix}\right| \frac{bx^n e^{i\pi}}{a}\right)}{\sqrt[3]{a}\, n\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+1/3*n)/(a+b*x**n)**(1/3),x)
```

```
[Out] x**(n/3)*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**n*exp_polar(I*pi)/a)/(a*
*(1/3)*n*gamma(4/3))
```

3.2703 $\int x^{-1-\frac{2n}{3}} (a+bx^n)^{2/3}\, dx$

Optimal. Leaf size=114

$$-\frac{3b^{2/3}\log\left(\sqrt[3]{b}\,x^{n/3}-\sqrt[3]{a+bx^n}\right)}{2n}+\frac{\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x^{n/3}}{\sqrt[3]{a+bx^n}}+1}{\sqrt{3}}\right)}{n}-\frac{3x^{-2n/3}\,(a+bx^n)^{2/3}}{2n}$$

```
[Out] -3/2*(a+b*x^n)^(2/3)/n/(x^(2/3*n))-3/2*b^(2/3)*ln(b^(1/3)*x^(1/3*n)-(a+b*x^
n)^(1/3))/n+b^(2/3)*arctan(1/3*(1+2*b^(1/3)*x^(1/3*n)/(a+b*x^n)^(1/3))*3^(1
/2))*3^(1/2)/n
```

Rubi [A] time = 0.05, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {349, 345, 239}

$$-\frac{3b^{2/3}\log\left(\sqrt[3]{b}\,x^{n/3}-\sqrt[3]{a+bx^n}\right)}{2n}+\frac{\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x^{n/3}}{\sqrt[3]{a+bx^n}}+1}{\sqrt{3}}\right)}{n}-\frac{3x^{-2n/3}\,(a+bx^n)^{2/3}}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - (2*n)/3)*(a + b*x^n)^(2/3),x]
```

```
[Out] (-3*(a + b*x^n)^(2/3))/(2*n*x^((2*n)/3)) + (Sqrt[3]*b^(2/3)*ArcTan[(1 + (2*
b^(1/3)*x^(n/3))/(a + b*x^n)^(1/3))/Sqrt[3]])/n - (3*b^(2/3)*Log[b^(1/3)*x^
(n/3) - (a + b*x^n)^(1/3)])/(2*n)
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 349

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*
(a + b*x^n)^p)/(m + 1), x] - Dist[(b*n*p)/(m + 1), Int[x^(m + n)*(a + b*x^n
)^(p - 1), x], x] /; FreeQ[{a, b, m, n}, x] && EqQ[(m + 1)/n + p, 0] && GtQ
[p, 0]
```

Rubi steps

$$\int x^{-1-\frac{2n}{3}}\left(a+bx^n\right)^{2/3}\,dx = -\frac{3x^{-2n/3}\left(a+bx^n\right)^{2/3}}{2n} + b\int \frac{x^{-1+\frac{n}{3}}}{\sqrt[3]{a+bx^n}}\,dx$$

$$= -\frac{3x^{-2n/3}\left(a+bx^n\right)^{2/3}}{2n} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx, x, x^{n/3}\right)}{n}$$

$$= -\frac{3x^{-2n/3}\left(a+bx^n\right)^{2/3}}{2n} + \frac{\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x^{n/3}}{\sqrt[3]{a+bx^n}}}{\sqrt{3}}\right)}{n} - \frac{3b^{2/3}\log\left(\sqrt[3]{b}\,x^{n/3}-\sqrt[3]{a+bx^n}\right)}{2n}$$

Mathematica [C] time = 0.03, size = 58, normalized size = 0.51

$$-\frac{3x^{-2n/3}\left(a+bx^n\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},-\frac{2}{3};\frac{1}{3};-\frac{bx^n}{a}\right)}{2n\left(\frac{bx^n}{a}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - (2*n)/3)*(a + b*x^n)^(2/3),x]
```

```
[Out] (-3*(a + b*x^n)^(2/3)*Hypergeometric2F1[-2/3, -2/3, 1/3, -((b*x^n)/a)])/(2*
n*x^((2*n)/3)*(1 + (b*x^n)/a)^(2/3))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2/3*n)*(a+b*x^n)^(2/3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n+a)^{\frac{2}{3}}x^{-\frac{2}{3}n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2/3*n)*(a+b*x^n)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^(2/3)*x^(-2/3*n - 1), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int (b\,x^n+a)^{\frac{2}{3}}\,x^{-\frac{2n}{3}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-2/3*n)*(b*x^n+a)^(2/3),x)
```

```
[Out] int(x^(-1-2/3*n)*(b*x^n+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n+a)^{\frac{2}{3}}x^{-\frac{2}{3}n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2/3*n)*(a+b*x^n)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^(2/3)*x^(-2/3*n - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x^n)^{2/3}}{x^{\frac{2n}{3}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^(2/3)/x^((2*n)/3 + 1),x)
```

```
[Out] int((a + b*x^n)^(2/3)/x^((2*n)/3 + 1), x)
```

sympy [C] time = 123.91, size = 46, normalized size = 0.40

$$\frac{a^{\frac{2}{3}} x^{-\frac{2n}{3}} \Gamma\left(-\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, -\frac{2}{3} \\ \frac{1}{3} \end{matrix} \middle| \frac{b x^n e^{i\pi}}{a}\right)}{n \Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-2/3*n)*(a+b*x**n)**(2/3),x)
```

```
[Out] a**(2/3)*x**(-2*n/3)*gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), b*x**n*exp_pol
ar(I*pi)/a)/(n*gamma(1/3))
```

3.2704 $\int x^m (a+bx^n)^p \, dx$

Optimal. Leaf size=54

$$\frac{x^{m+1}(a+bx^n)^{p+1}\ {}_2F_1\left(1, \frac{m+1}{n}+p+1; \frac{m+n+1}{n}; -\frac{bx^n}{a}\right)}{a(m+1)}$$

```
[Out] x^(1+m)*(a+b*x^n)^(1+p)*hypergeom([1, 1+(1+m)/n+p],[(1+m+n)/n],-b*x^n/a)/a/
(1+m)
```

Rubi [A] time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.15, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {365, 364}

$$\frac{x^{m+1}(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p}\ {}_2F_1\left(\frac{m+1}{n}, -p; \frac{m+n+1}{n}; -\frac{bx^n}{a}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^n)^p,x]
```

```
[Out] (x^(1 + m)*(a + b*x^n)^p*Hypergeometric2F1[(1 + m)/n, -p, (1 + m + n)/n, -(
(b*x^n)/a)])/((1 + m)*(1 + (b*x^n)/a)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^m (a+bx^n)^p \, dx &= \left((a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p}\right) \int x^m \left(1+\frac{bx^n}{a}\right)^p dx \\ &= \frac{x^{1+m}(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p}\ {}_2F_1\left(\frac{1+m}{n}, -p; \frac{1+m+n}{n}; -\frac{bx^n}{a}\right)}{1+m}\end{aligned}$$

Mathematica [A] time = 0.03, size = 63, normalized size = 1.17

$$\frac{x^{m+1}(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p}\ {}_2F_1\left(\frac{m+1}{n}, -p; \frac{m+1}{n}+1; -\frac{bx^n}{a}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^n)^p,x]
```

```
[Out] (x^(1 + m)*(a + b*x^n)^p*Hypergeometric2F1[(1 + m)/n, -p, 1 + (1 + m)/n, -(
(b*x^n)/a)])/((1 + m)*(1 + (b*x^n)/a)^p)
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left((bx^n + a)^p x^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p*x^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^p x^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p*x^m, x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int x^m \left(b\,x^n + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^n+a)^p,x)
```

```
[Out] int(x^m*(b*x^n+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^p x^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p*x^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^m \left(a + b\,x^n\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^n)^p,x)
```

```
[Out] int(x^m*(a + b*x^n)^p, x)
```

sympy [C] time = 10.68, size = 54, normalized size = 1.00

$$\frac{a^p x x^m \Gamma\left(\frac{m}{n} + \frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{n} + \frac{1}{n} \\ \frac{m}{n} + 1 + \frac{1}{n} \end{matrix} \middle| \frac{b x^n e^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n} + 1 + \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*x**n)**p,x)
```

```
[Out] a**p*x*x**m*gamma(m/n + 1/n)*hyper((-p, m/n + 1/n), (m/n + 1 + 1/n,), b*x**
n*exp_polar(I*pi)/a)/(n*gamma(m/n + 1 + 1/n))
```

3.2705 $\int (a+bx^n)^{-4-\frac{1}{n}}\,dx$

Optimal. Leaf size=146

$$\frac{6n^3x\,(a+bx^n)^{-1/n}}{a^4(n+1)(2n+1)(3n+1)}+\frac{6n^2x\,(a+bx^n)^{-\frac{1}{n}-1}}{a^3(n+1)(2n+1)(3n+1)}+\frac{3nx\,(a+bx^n)^{-\frac{1}{n}-2}}{a^2\left(6n^2+5n+1\right)}+\frac{x\,(a+bx^n)^{-\frac{1}{n}-3}}{a(3n+1)}$$

```
[Out] x*(a+b*x^n)^(-3-1/n)/a/(1+3*n)+3*n*x*(a+b*x^n)^(-2-1/n)/a^2/(6*n^2+5*n+1)+6
*n^2*x*(a+b*x^n)^(-1-1/n)/a^3/(6*n^3+11*n^2+6*n+1)+6*n^3*x/a^4/(6*n^3+11*n^
2+6*n+1)/((a+b*x^n)^(1/n))
```

Rubi [A] time = 0.09, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {192, 191}

$$\frac{3nx\,(a+bx^n)^{-\frac{1}{n}-2}}{a^2\left(6n^2+5n+1\right)}+\frac{6n^2x\,(a+bx^n)^{-\frac{1}{n}-1}}{a^3(n+1)(2n+1)(3n+1)}+\frac{6n^3x\,(a+bx^n)^{-1/n}}{a^4(n+1)(2n+1)(3n+1)}+\frac{x\,(a+bx^n)^{-\frac{1}{n}-3}}{a(3n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^(-4 - n^(-1)),x]
```

```
[Out] (x*(a + b*x^n)^(-3 - n^(-1)))/(a*(1 + 3*n)) + (3*n*x*(a + b*x^n)^(-2 - n^(-
1)))/(a^2*(1 + 5*n + 6*n^2)) + (6*n^2*x*(a + b*x^n)^(-1 - n^(-1)))/(a^3*(1
 + n)*(1 + 2*n)*(1 + 3*n)) + (6*n^3*x)/(a^4*(1 + n)*(1 + 2*n)*(1 + 3*n)*(a +
 b*x^n)^n^(-1))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)^{-4-\frac{1}{n}}\,dx &= \frac{x\,(a+bx^n)^{-3-\frac{1}{n}}}{a(1+3n)}+\frac{(3n)\int (a+bx^n)^{-3-\frac{1}{n}}\,dx}{a(1+3n)}\\ &= \frac{x\,(a+bx^n)^{-3-\frac{1}{n}}}{a(1+3n)}+\frac{3nx\,(a+bx^n)^{-2-\frac{1}{n}}}{a^2\left(1+5n+6n^2\right)}+\frac{\left(6n^2\right)\int (a+bx^n)^{-2-\frac{1}{n}}\,dx}{a^2\left(1+5n+6n^2\right)}\\ &= \frac{x\,(a+bx^n)^{-3-\frac{1}{n}}}{a(1+3n)}+\frac{3nx\,(a+bx^n)^{-2-\frac{1}{n}}}{a^2\left(1+5n+6n^2\right)}+\frac{6n^2x\,(a+bx^n)^{-1-\frac{1}{n}}}{a^3(1+n)\left(1+5n+6n^2\right)}+\frac{\left(6n^3\right)\int (a+bx^n)^{-1-\frac{1}{n}}}{a^3(1+n)\left(1+5n+6\right.}\\ &= \frac{x\,(a+bx^n)^{-3-\frac{1}{n}}}{a(1+3n)}+\frac{3nx\,(a+bx^n)^{-2-\frac{1}{n}}}{a^2\left(1+5n+6n^2\right)}+\frac{6n^2x\,(a+bx^n)^{-1-\frac{1}{n}}}{a^3(1+n)\left(1+5n+6n^2\right)}+\frac{6n^3x\,(a+bx^n)^{-1/n}}{a^4(1+n)\left(1+5n+6\right.}\end{aligned}$$

Mathematica [C] time = 0.07, size = 55, normalized size = 0.38

$$\frac{x\,(a+bx^n)^{-1/n}\left(\frac{bx^n}{a}+1\right)^{\frac{1}{n}}\,{}_2F_1\left(4+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^(-4 - n^(-1)),x]
```

```
[Out] (x*(1 + (b*x^n)/a)^n^(-1)*Hypergeometric2F1[4 + n^(-1), n^(-1), 1 + n^(-1),
 -((b*x^n)/a)])/(a^4*(a + b*x^n)^n^(-1))
```

fricas [A] time = 0.64, size = 192, normalized size = 1.32

$$\frac{6\,b^4n^3xx^{4\,n}+6\left(4\,ab^3n^3+ab^3n^2\right)xx^{3\,n}+3\left(12\,a^2b^2n^3+7\,a^2b^2n^2+a^2b^2n\right)xx^{2\,n}+\left(24\,a^3bn^3+26\,a^3bn^2+9\,a^3\right.}{\left(6\,a^4n^3+11\,a^4n^2+6\,a^4n+a^4\right)(bx^n+a)^{\frac{4\,n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(-4-1/n),x, algorithm="fricas")
```

```
[Out] (6*b^4*n^3*x*x^(4*n) + 6*(4*a*b^3*n^3 + a*b^3*n^2)*x*x^(3*n) + 3*(12*a^2*b^
2*n^3 + 7*a^2*b^2*n^2 + a^2*b^2*n)*x*x^(2*n) + (24*a^3*b*n^3 + 26*a^3*b*n^2
 + 9*a^3*b*n + a^3*b)*x*x^n + (6*a^4*n^3 + 11*a^4*n^2 + 6*a^4*n + a^4)*x)/(
(6*a^4*n^3 + 11*a^4*n^2 + 6*a^4*n + a^4)*(b*x^n + a)^((4*n + 1)/n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n+a)^{-\frac{1}{n}-4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(-4-1/n),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^(-1/n - 4), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int (b\,x^n+a)^{-\frac{1}{n}-4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^(-4-1/n),x)
```

```
[Out] int((b*x^n+a)^(-4-1/n),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n+a)^{-\frac{1}{n}-4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(-4-1/n),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^(-1/n - 4), x)
```

mupad [B] time = 1.56, size = 64, normalized size = 0.44

$$-\frac{x^{1-4\,n}\left(\frac{a}{b\,x^n}+1\right)^{1/n}\,{}_2\mathrm{F}_1\left(4,\frac{1}{n}+4;\;5;\;-\frac{a}{b\,x^n}\right)}{4\,b^4\,n\,(a+b\,x^n)^{1/n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^n)^(1/n + 4),x)
```

```
[Out] -(x^(1 - 4*n)*(a/(b*x^n) + 1)^(1/n)*hypergeom([4, 1/n + 4], 5, -a/(b*x^n)))
/(4*b^4*n*(a + b*x^n)^(1/n))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**(-4-1/n),x)
```

```
[Out] Timed out
```

3.2706 $\int (a+bx^n)^{-3-\frac{1}{n}}\,dx$

Optimal. Leaf size=96

$$\frac{2n^2x\,(a+bx^n)^{-1/n}}{a^3(n+1)(2n+1)}+\frac{2nx\,(a+bx^n)^{-\frac{1}{n}-1}}{a^2(n+1)(2n+1)}+\frac{x\,(a+bx^n)^{-\frac{1}{n}-2}}{a(2n+1)}$$

```
[Out] x*(a+b*x^n)^(-2-1/n)/a/(1+2*n)+2*n*x*(a+b*x^n)^(-1-1/n)/a^2/(2*n^2+3*n+1)+2
*n^2*x/a^3/(2*n^2+3*n+1)/((a+b*x^n)^(1/n))
```

Rubi [A] time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {192, 191}

$$\frac{2n^2x\,(a+bx^n)^{-1/n}}{a^3(n+1)(2n+1)}+\frac{2nx\,(a+bx^n)^{-\frac{1}{n}-1}}{a^2(n+1)(2n+1)}+\frac{x\,(a+bx^n)^{-\frac{1}{n}-2}}{a(2n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^(-3 - n^(-1)),x]
```

```
[Out] (x*(a + b*x^n)^(-2 - n^(-1)))/(a*(1 + 2*n)) + (2*n*x*(a + b*x^n)^(-1 - n^(-
1)))/(a^2*(1 + n)*(1 + 2*n)) + (2*n^2*x)/(a^3*(1 + n)*(1 + 2*n)*(a + b*x^n)
^n^(-1))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)^{-3-\frac{1}{n}}\,dx &= \frac{x\,(a+bx^n)^{-2-\frac{1}{n}}}{a(1+2n)}+\frac{(2n)\int (a+bx^n)^{-2-\frac{1}{n}}\,dx}{a(1+2n)}\\ &= \frac{x\,(a+bx^n)^{-2-\frac{1}{n}}}{a(1+2n)}+\frac{2nx\,(a+bx^n)^{-1-\frac{1}{n}}}{a^2(1+n)(1+2n)}+\frac{\left(2n^2\right)\int (a+bx^n)^{-1-\frac{1}{n}}\,dx}{a^2(1+n)(1+2n)}\\ &= \frac{x\,(a+bx^n)^{-2-\frac{1}{n}}}{a(1+2n)}+\frac{2nx\,(a+bx^n)^{-1-\frac{1}{n}}}{a^2(1+n)(1+2n)}+\frac{2n^2x\,(a+bx^n)^{-1/n}}{a^3(1+n)(1+2n)}\end{aligned}$$

Mathematica [C] time = 0.04, size = 55, normalized size = 0.57

$$\frac{x\,(a+bx^n)^{-1/n}\left(\frac{bx^n}{a}+1\right)^{\frac{1}{n}}\,{}_2F_1\left(3+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^(-3 - n^(-1)),x]
```

```
[Out] (x*(1 + (b*x^n)/a)^n^(-1)*Hypergeometric2F1[3 + n^(-1), n^(-1), 1 + n^(-1),
 -((b*x^n)/a)])/(a^3*(a + b*x^n)^n^(-1))
```

fricas [A] time = 0.85, size = 126, normalized size = 1.31

$$\frac{2\,b^3n^2xx^{3\,n}+2\left(3\,ab^2n^2+ab^2n\right)xx^{2\,n}+\left(6\,a^2bn^2+5\,a^2bn+a^2b\right)xx^n+\left(2\,a^3n^2+3\,a^3n+a^3\right)x}{\left(2\,a^3n^2+3\,a^3n+a^3\right)(bx^n+a)^{\frac{3\,n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(-3-1/n),x, algorithm="fricas")
```

```
[Out] (2*b^3*n^2*x*x^(3*n) + 2*(3*a*b^2*n^2 + a*b^2*n)*x*x^(2*n) + (6*a^2*b*n^2 +
 5*a^2*b*n + a^2*b)*x*x^n + (2*a^3*n^2 + 3*a^3*n + a^3)*x)/((2*a^3*n^2 + 3*
a^3*n + a^3)*(b*x^n + a)^((3*n + 1)/n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n+a)^{-\frac{1}{n}-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(-3-1/n),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^(-1/n - 3), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int (b\,x^n+a)^{-\frac{1}{n}-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^(-3-1/n),x)
```

```
[Out] int((b*x^n+a)^(-3-1/n),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n+a)^{-\frac{1}{n}-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(-3-1/n),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^(-1/n - 3), x)
```

mupad [B] time = 1.43, size = 64, normalized size = 0.67

$$-\frac{x^{1-3\,n}\left(\frac{a}{b\,x^n}+1\right)^{1/n}\,{}_2\mathrm{F}_1\left(3,\frac{1}{n}+3;\;4;\;-\frac{a}{b\,x^n}\right)}{3\,b^3\,n\,(a+b\,x^n)^{1/n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^n)^(1/n + 3),x)
```

```
[Out] -(x^(1 - 3*n)*(a/(b*x^n) + 1)^(1/n)*hypergeom([3, 1/n + 3], 4, -a/(b*x^n)))
/(3*b^3*n*(a + b*x^n)^(1/n))
```

sympy [**F**(**-1**)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**(-3-1/n),x)
[Out] Timed out
```

3.2707 $\int (a+bx^n)^{-2-\frac{1}{n}}\,dx$

Optimal. Leaf size=50

$$\frac{nx\,(a+bx^n)^{-1/n}}{a^2(n+1)}+\frac{x\,(a+bx^n)^{-\frac{1}{n}-1}}{a(n+1)}$$

```
[Out] x*(a+b*x^n)^(-1-1/n)/a/(1+n)+n*x/a^2/(1+n)/((a+b*x^n)^(1/n))
```

Rubi [A] time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {192, 191}

$$\frac{nx\,(a+bx^n)^{-1/n}}{a^2(n+1)}+\frac{x\,(a+bx^n)^{-\frac{1}{n}-1}}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^(-2 - n^(-1)),x]
```

```
[Out] (x*(a + b*x^n)^(-1 - n^(-1)))/(a*(1 + n)) + (n*x)/(a^2*(1 + n)*(a + b*x^n)^
n^(-1))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)^{-2-\frac{1}{n}}\,dx &= \frac{x\,(a+bx^n)^{-1-\frac{1}{n}}}{a(1+n)}+\frac{n\int (a+bx^n)^{-1-\frac{1}{n}}\,dx}{a(1+n)}\\ &= \frac{x\,(a+bx^n)^{-1-\frac{1}{n}}}{a(1+n)}+\frac{nx\,(a+bx^n)^{-1/n}}{a^2(1+n)}\end{aligned}$$

Mathematica [C] time = 0.05, size = 55, normalized size = 1.10

$$\frac{x\,(a+bx^n)^{-1/n}\left(\frac{bx^n}{a}+1\right)^{\frac{1}{n}}\,{}_2F_1\left(2+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^(-2 - n^(-1)),x]
```

```
[Out] (x*(1 + (b*x^n)/a)^n^(-1)*Hypergeometric2F1[2 + n^(-1), n^(-1), 1 + n^(-1),
 -((b*x^n)/a)])/(a^2*(a + b*x^n)^n^(-1))
```

fricas [A] time = 0.78, size = 68, normalized size = 1.36

$$\frac{b^2 n x x^{2\,n} + (2\,abn + ab) x x^n + \left(a^2 n + a^2\right) x}{\left(a^2 n + a^2\right)(b x^n + a)^{\frac{2\,n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(-2-1/n),x, algorithm="fricas")
```

```
[Out] (b^2*n*x*x^(2*n) + (2*a*b*n + a*b)*x*x^n + (a^2*n + a^2)*x)/((a^2*n + a^2)*
(b*x^n + a)^((2*n + 1)/n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b x^n + a)^{-\frac{1}{n}-2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(-2-1/n),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^(-1/n - 2), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int (b\,x^n + a)^{-\frac{1}{n}-2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^(-2-1/n),x)
```

```
[Out] int((b*x^n+a)^(-2-1/n),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b x^n + a)^{-\frac{1}{n}-2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(-2-1/n),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^(-1/n - 2), x)
```

mupad [B] time = 1.44, size = 64, normalized size = 1.28

$$-\frac{x^{1-2\,n}\left(\frac{a}{b\,x^n}+1\right)^{1/n}\,{}_2\mathrm{F}_1\left(2,\frac{1}{n}+2;\ 3;\ -\frac{a}{b\,x^n}\right)}{2\,b^2\,n\,(a+b\,x^n)^{1/n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^n)^(1/n + 2),x)
```

```
[Out] -(x^(1 - 2*n)*(a/(b*x^n) + 1)^(1/n)*hypergeom([2, 1/n + 2], 3, -a/(b*x^n)))
/(2*b^2*n*(a + b*x^n)^(1/n))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**(-2-1/n),x)
```

```
[Out] Timed out
```

3.2708 $\int (a+bx^n)^{-1-\frac{1}{n}}\,dx$

Optimal. Leaf size=18

$$\frac{x\,(a+bx^n)^{-1/n}}{a}$$

```
[Out] x/a/((a+b*x^n)^(1/n))
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {191}

$$\frac{x\,(a+bx^n)^{-1/n}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^(-1 - n^(-1)),x]
```

```
[Out] x/(a*(a + b*x^n)^n^(-1))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int (a+bx^n)^{-1-\frac{1}{n}}\,dx = \frac{x\,(a+bx^n)^{-1/n}}{a}$$

Mathematica [A] time = 0.06, size = 18, normalized size = 1.00

$$\frac{x\,(a+bx^n)^{-1/n}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^(-1 - n^(-1)),x]
```

```
[Out] x/(a*(a + b*x^n)^n^(-1))
```

fricas [A] time = 0.53, size = 31, normalized size = 1.72

$$\frac{bxx^n + ax}{(bx^n+a)^{\frac{n+1}{n}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(-1-1/n),x, algorithm="fricas")
```

```
[Out] (b*x*x^n + a*x)/((b*x^n + a)^((n + 1)/n)*a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n+a)^{-\frac{1}{n}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(-1-1/n),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^(-1/n - 1), x)
```

maple [B] time = 0.03, size = 53, normalized size = 2.94

$$\frac{bx\,e^{n\ln(x)}e^{\left(-\frac{1}{n}-1\right)\ln\left(b\,e^{n\ln(x)}+a\right)}}{a}+x\,e^{\left(-\frac{1}{n}-1\right)\ln\left(b\,e^{n\ln(x)}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^(-1-1/n),x)
```

```
[Out] x*exp((-1-1/n)*ln(b*exp(n*ln(x))+a))+1/a*b*x*exp(n*ln(x))*exp((-1-1/n)*ln(b
*exp(n*ln(x))+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n+a)^{-\frac{1}{n}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(-1-1/n),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^(-1/n - 1), x)
```

mupad [B] time = 1.33, size = 75, normalized size = 4.17

$$\frac{b\,x^{n+1}\left(\frac{a}{b\,x^n}-\left(\frac{a}{b\,x^n}+1\right)^{\frac{n+1}{n}}+1\right)}{a\,n\left(\frac{n+1}{n}-1\right)(a+b\,x^n)^{\frac{n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^n)^(1/n + 1),x)
```

```
[Out] (b*x^(n + 1)*(a/(b*x^n) - (a/(b*x^n) + 1)^((n + 1)/n) + 1))/(a*n*((n + 1)/n
 - 1)*(a + b*x^n)^((n + 1)/n))
```

sympy [A] time = 28.87, size = 211, normalized size = 11.72

$$\begin{cases} -\frac{b^{-\frac{1}{n}}xx^{-n}(x^n)^{-\frac{1}{n}}}{bn} & \text{for } a \\ 0^{-1-\frac{1}{n}}x & \text{for } a \\ x\,(0^n)^{-1-\frac{1}{n}} & \text{for } a \\ \frac{a^2x}{a^3(a+bx^n)^{\frac{1}{n}}+2a^2bx^n(a+bx^n)^{\frac{1}{n}}+ab^2x^{2n}(a+bx^n)^{\frac{1}{n}}}+\frac{abxx^n}{a^3(a+bx^n)^{\frac{1}{n}}+2a^2bx^n(a+bx^n)^{\frac{1}{n}}+ab^2x^{2n}(a+bx^n)^{\frac{1}{n}}}+\frac{bxx^n}{a^2(a+bx^n)^{\frac{1}{n}}+abx^n(a+bx^n)^{\frac{1}{n}}} & \text{othe} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**(-1-1/n),x)
```

```
[Out] Piecewise((-b**(-1/n)*x*x**(-n)*(x**n)**(-1/n)/(b*n), Eq(a, 0)), (0**(-1 -
1/n)*x, Eq(a, -b*x**n)), (x*(0**n)**(-1 - 1/n), Eq(a, 0**n - b*x**n)), (a**
2*x/(a**3*(a + b*x**n)**(1/n) + 2*a**2*b*x**n*(a + b*x**n)**(1/n) + a*b**2*
x**(2*n)*(a + b*x**n)**(1/n)) + a*b*x*x**n/(a**3*(a + b*x**n)**(1/n) + 2*a*
*2*b*x**n*(a + b*x**n)**(1/n) + a*b**2*x**(2*n)*(a + b*x**n)**(1/n)) + b*x*
x**n/(a**2*(a + b*x**n)**(1/n) + a*b*x**n*(a + b*x**n)**(1/n)), True))
```

3.2709 $\int (a+bx^n)^{-1/n}\,dx$

Optimal. Leaf size=50

$$x\,(a+bx^n)^{-1/n}\left(\frac{bx^n}{a}+1\right)^{\frac{1}{n}}\,{}_2F_1\left(\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)$$

```
[Out] x*(1+b*x^n/a)^(1/n)*hypergeom([1/n, 1/n],[1+1/n],-b*x^n/a)/((a+b*x^n)^(1/n)
)
```

Rubi [A] time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {246, 245}

$$x\,(a+bx^n)^{-1/n}\left(\frac{bx^n}{a}+1\right)^{\frac{1}{n}}\,{}_2F_1\left(\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^(-n^(-1)),x]
```

```
[Out] (x*(1 + (b*x^n)/a)^n^(-1)*Hypergeometric2F1[n^(-1), n^(-1), 1 + n^(-1), -((
b*x^n)/a)])/(a + b*x^n)^n^(-1)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)^{-1/n}\,dx &= \left((a+bx^n)^{-1/n}\left(1+\frac{bx^n}{a}\right)^{\frac{1}{n}}\right)\int\left(1+\frac{bx^n}{a}\right)^{-1/n}dx\\ &= x\,(a+bx^n)^{-1/n}\left(1+\frac{bx^n}{a}\right)^{\frac{1}{n}}\,{}_2F_1\left(\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 50, normalized size = 1.00

$$x\,(a+bx^n)^{-1/n}\left(\frac{bx^n}{a}+1\right)^{\frac{1}{n}}\,{}_2F_1\left(\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^(-n^(-1)),x]
```

```
[Out] (x*(1 + (b*x^n)/a)^n^(-1)*Hypergeometric2F1[n^(-1), n^(-1), 1 + n^(-1), -((
b*x^n)/a)])/(a + b*x^n)^n^(-1)
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{(bx^n+a)^{\left(\frac{1}{n}\right)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a+b*x^n)^(1/n)),x, algorithm="fricas")
```

```
[Out] integral(1/((b*x^n + a)^(1/n)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n+a)^{\left(\frac{1}{n}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a+b*x^n)^(1/n)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^(1/n)), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int (b\,x^n+a)^{-\frac{1}{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^n+a)^(1/n)),x)
```

```
[Out] int(1/((b*x^n+a)^(1/n)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n+a)^{\left(\frac{1}{n}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a+b*x^n)^(1/n)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a)^(1/n)), x)
```

mupad [B] time = 1.48, size = 51, normalized size = 1.02

$$\frac{x\left(\frac{b\,x^n}{a}+1\right)^{1/n}\,{}_2\mathrm{F}_1\left(\frac{1}{n},\frac{1}{n};\ \frac{1}{n}+1;\ -\frac{b\,x^n}{a}\right)}{(a+b\,x^n)^{1/n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^n)^(1/n),x)
```

```
[Out] (x*((b*x^n)/a + 1)^(1/n)*hypergeom([1/n, 1/n], 1/n + 1, -(b*x^n)/a))/(a + b
*x^n)^(1/n)
```

sympy [C] time = 10.71, size = 39, normalized size = 0.78

$$\frac{a^{-\frac{1}{n}} x \Gamma\left(\frac{1}{n}\right) {}_2F_1\left(\begin{matrix}\frac{1}{n}, \frac{1}{n} \\ 1 + \frac{1}{n}\end{matrix}\middle| \frac{b x^n e^{i\pi}}{a}\right)}{n \Gamma\left(1 + \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a+b*x**n)**(1/n)),x)
```

```
[Out] a**(-1/n)*x*gamma(1/n)*hyper((1/n, 1/n), (1 + 1/n,), b*x**n*exp_polar(I*pi)
/a)/(n*gamma(1 + 1/n))
```

3.2710 $\int (a+bx^n)^{1-\frac{1}{n}}\,dx$

Optimal. Leaf size=53

$$ax\,(a+bx^n)^{-1/n}\left(\frac{bx^n}{a}+1\right)^{\frac{1}{n}}\,{}_2F_1\left(\frac{1}{n}-1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)$$

```
[Out] a*x*(1+b*x^n/a)^(1/n)*hypergeom([1/n, -1+1/n],[1+1/n],-b*x^n/a)/((a+b*x^n)^
(1/n))
```

Rubi [A] time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {246, 245}

$$ax\,(a+bx^n)^{-1/n}\left(\frac{bx^n}{a}+1\right)^{\frac{1}{n}}\,{}_2F_1\left(\frac{1}{n}-1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^(1 - n^(-1)),x]
```

```
[Out] (a*x*(1 + (b*x^n)/a)^n^(-1)*Hypergeometric2F1[-1 + n^(-1), n^(-1), 1 + n^(-
1), -((b*x^n)/a)])/(a + b*x^n)^n^(-1)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)^{1-\frac{1}{n}}\,dx &= \left(a\,(a+bx^n)^{-1/n}\left(1+\frac{bx^n}{a}\right)^{\frac{1}{n}}\right)\int\left(1+\frac{bx^n}{a}\right)^{1-\frac{1}{n}}\,dx\\ &= ax\,(a+bx^n)^{-1/n}\left(1+\frac{bx^n}{a}\right)^{\frac{1}{n}}\,{}_2F_1\left(-1+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 53, normalized size = 1.00

$$ax\,(a+bx^n)^{-1/n}\left(\frac{bx^n}{a}+1\right)^{\frac{1}{n}}\,{}_2F_1\left(\frac{1}{n}-1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^(1 - n^(-1)),x]
```

```
[Out] (a*x*(1 + (b*x^n)/a)^n^(-1)*Hypergeometric2F1[-1 + n^(-1), n^(-1), 1 + n^(-
1), -((b*x^n)/a)])/(a + b*x^n)^n^(-1)
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left((bx^n + a)^{\frac{n-1}{n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(1-1/n),x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^((n - 1)/n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^{-\frac{1}{n}+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(1-1/n),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^(-1/n + 1), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int (b\,x^n + a)^{-\frac{1}{n}+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^(1-1/n),x)
```

```
[Out] int((b*x^n+a)^(1-1/n),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^{-\frac{1}{n}+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(1-1/n),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^(-1/n + 1), x)
```

mupad [B] time = 1.46, size = 54, normalized size = 1.02

$$\frac{a\,x\left(\frac{b\,x^n}{a}+1\right)^{1/n}\,{}_2\mathrm{F}_1\left(\frac{1}{n}-1,\frac{1}{n};\ \frac{1}{n}+1;\ -\frac{b\,x^n}{a}\right)}{\left(a+b\,x^n\right)^{1/n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^(1 - 1/n),x)
```

```
[Out] (a*x*((b*x^n)/a + 1)^(1/n)*hypergeom([1/n - 1, 1/n], 1/n + 1, -(b*x^n)/a))/
(a + b*x^n)^(1/n)
```

sympy [C] time = 7.41, size = 42, normalized size = 0.79

$$\frac{aa^{-\frac{1}{n}}x\Gamma\left(\frac{1}{n}\right){}_2F_1\left(\begin{matrix}\frac{1}{n}, -1+\frac{1}{n}\\ 1+\frac{1}{n}\end{matrix}\middle|\,\frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**(1-1/n),x)
```

```
[Out] a*a**(-1/n)*x*gamma(1/n)*hyper((1/n, -1 + 1/n), (1 + 1/n,), b*x**n*exp_pola
r(I*pi)/a)/(n*gamma(1 + 1/n))
```

3.2711 $\int (a+bx^n)^{2-\frac{1}{n}}\,dx$

Optimal. Leaf size=55

$$a^2 x\,(a+bx^n)^{-1/n}\left(\frac{bx^n}{a}+1\right)^{\frac{1}{n}}\,{}_2F_1\left(\frac{1}{n}-2,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)$$

```
[Out] a^2*x*(1+b*x^n/a)^(1/n)*hypergeom([1/n, -2+1/n],[1+1/n],-b*x^n/a)/((a+b*x^n
)^(1/n))
```

Rubi [A] time = 0.01, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {246, 245}

$$a^2 x\,(a+bx^n)^{-1/n}\left(\frac{bx^n}{a}+1\right)^{\frac{1}{n}}\,{}_2F_1\left(\frac{1}{n}-2,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^(2 - n^(-1)),x]
```

```
[Out] (a^2*x*(1 + (b*x^n)/a)^n^(-1)*Hypergeometric2F1[-2 + n^(-1), n^(-1), 1 + n^
(-1), -((b*x^n)/a)])/(a + b*x^n)^n^(-1)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)^{2-\frac{1}{n}}\,dx &= \left(a^2\,(a+bx^n)^{-1/n}\left(1+\frac{bx^n}{a}\right)^{\frac{1}{n}}\right)\int\left(1+\frac{bx^n}{a}\right)^{2-\frac{1}{n}}\,dx\\ &= a^2 x\,(a+bx^n)^{-1/n}\left(1+\frac{bx^n}{a}\right)^{\frac{1}{n}}\,{}_2F_1\left(-2+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 55, normalized size = 1.00

$$a^2 x\,(a+bx^n)^{-1/n}\left(\frac{bx^n}{a}+1\right)^{\frac{1}{n}}\,{}_2F_1\left(\frac{1}{n}-2,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^(2 - n^(-1)),x]
```

```
[Out] (a^2*x*(1 + (b*x^n)/a)^n^(-1)*Hypergeometric2F1[-2 + n^(-1), n^(-1), 1 + n^
(-1), -((b*x^n)/a)])/(a + b*x^n)^n^(-1)
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left((bx^n + a)^{\frac{2n-1}{n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(2-1/n),x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^((2*n - 1)/n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^{-\frac{1}{n}+2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(2-1/n),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^(-1/n + 2), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int (b\,x^n + a)^{-\frac{1}{n}+2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^(2-1/n),x)
```

```
[Out] int((b*x^n+a)^(2-1/n),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^{-\frac{1}{n}+2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(2-1/n),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^(-1/n + 2), x)
```

mupad [B] time = 1.31, size = 56, normalized size = 1.02

$$\frac{a^2\, x \left(\frac{b\,x^n}{a}+1\right)^{1/n} {}_2\mathrm{F}_1\left(\frac{1}{n}-2, \frac{1}{n};\ \frac{1}{n}+1;\ -\frac{b\,x^n}{a}\right)}{(a+b\,x^n)^{1/n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^(2 - 1/n),x)
```

```
[Out] (a^2*x*((b*x^n)/a + 1)^(1/n)*hypergeom([1/n - 2, 1/n], 1/n + 1, -(b*x^n)/a)
)/(a + b*x^n)^(1/n)
```

sympy [C] time = 90.92, size = 44, normalized size = 0.80

$$\frac{a^2 a^{-\frac{1}{n}} x \Gamma\left(\frac{1}{n}\right) {}_2F_1\left(\begin{matrix}\frac{1}{n}, -2+\frac{1}{n} \\ 1+\frac{1}{n}\end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**(2-1/n),x)
```

```
[Out] a**2*a**(-1/n)*x*gamma(1/n)*hyper((1/n, -2 + 1/n), (1 + 1/n,), b*x**n*exp_p
olar(I*pi)/a)/(n*gamma(1 + 1/n))
```

3.2712 $\int x^m (bx^n)^p \, dx$

Optimal. Leaf size=21

$$\frac{x^{m+1} (bx^n)^p}{m+np+1}$$

```
[Out] x^(1+m)*(b*x^n)^p/(n*p+m+1)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 30}

$$\frac{x^{m+1} (bx^n)^p}{m+np+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(b*x^n)^p,x]
```

```
[Out] (x^(1 + m)*(b*x^n)^p)/(1 + m + n*p)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^m (bx^n)^p \, dx &= \left(x^{-np} (bx^n)^p\right) \int x^{m+np} \, dx \\ &= \frac{x^{1+m} (bx^n)^p}{1+m+np}\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{x^{m+1} (bx^n)^p}{m+np+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(b*x^n)^p,x]
```

```
[Out] (x^(1 + m)*(b*x^n)^p)/(1 + m + n*p)
```

fricas [A] time = 0.92, size = 24, normalized size = 1.14

$$\frac{x x^m e^{\left(np \log(x) + p \log(b)\right)}}{np+m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^n)^p,x, algorithm="fricas")
```

```
[Out] x*x^m*e^(n*p*log(x) + p*log(b))/(n*p + m + 1)
```

giac [A] time = 0.17, size = 24, normalized size = 1.14

$$\frac{x x^m e^{\left(np \log(x)+p \log(b)\right)}}{np+m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^n)^p,x, algorithm="giac")
```

```
[Out] x*x^m*e^(n*p*log(x) + p*log(b))/(n*p + m + 1)
```

maple [A] time = 0.00, size = 22, normalized size = 1.05

$$\frac{x^{m+1} \left(b\, x^n\right)^p}{np+m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^n)^p,x)
```

```
[Out] 1/(n*p+m+1)*x^(m+1)*(b*x^n)^p
```

maxima [A] time = 0.65, size = 25, normalized size = 1.19

$$\frac{b^p x e^{\left(m \log(x)+p \log(x^n)\right)}}{np+m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^n)^p,x, algorithm="maxima")
```

```
[Out] b^p*x*e^(m*log(x) + p*log(x^n))/(n*p + m + 1)
```

mupad [B] time = 1.30, size = 21, normalized size = 1.00

$$\frac{x^{m+1} \left(b\, x^n\right)^p}{m+n\, p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^n)^p,x)
```

```
[Out] (x^(m + 1)*(b*x^n)^p)/(m + n*p + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{b^p x x^m (x^n)^p}{m+np+1} & \text{for } m \neq -np-1 \\ \int x^{-np-1} \left(bx^n\right)^p \, dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**n)**p,x)
```

```
[Out] Piecewise((b**p*x*x**m*(x**n)**p/(m + n*p + 1), Ne(m, -n*p - 1)), (Integral
(x**(-n*p - 1)*(b*x**n)**p, x), True))
```

3.2713 $\int x^2 (bx^n)^p \, dx$

Optimal. Leaf size=18

$$\frac{x^3 (bx^n)^p}{np+3}$$

```
[Out] x^3*(b*x^n)^p/(n*p+3)
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 30}

$$\frac{x^3 (bx^n)^p}{np+3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b*x^n)^p,x]
```

```
[Out] (x^3*(b*x^n)^p)/(3 + n*p)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2 (bx^n)^p \, dx &= \left(x^{-np} (bx^n)^p\right) \int x^{2+np} \, dx \\ &= \frac{x^3 (bx^n)^p}{3+np}\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{x^3 (bx^n)^p}{np+3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b*x^n)^p,x]
```

```
[Out] (x^3*(b*x^n)^p)/(3 + n*p)
```

fricas [A] time = 0.81, size = 22, normalized size = 1.22

$$\frac{x^3 e^{\left(np \log(x) + p \log(b)\right)}}{np+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^n)^p,x, algorithm="fricas")
```

```
[Out] x^3*e^(n*p*log(x) + p*log(b))/(n*p + 3)
```

giac [A] time = 0.20, size = 22, normalized size = 1.22

$$\frac{x^3 e^{\left(np\log(x)+p\log(b)\right)}}{np+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^n)^p,x, algorithm="giac")
```

```
[Out] x^3*e^(n*p*log(x) + p*log(b))/(n*p + 3)
```

maple [A] time = 0.00, size = 19, normalized size = 1.06

$$\frac{x^3 \left(b\, x^n\right)^p}{np+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^n)^p,x)
```

```
[Out] 1/(n*p+3)*x^3*(b*x^n)^p
```

maxima [A] time = 0.57, size = 19, normalized size = 1.06

$$\frac{b^p x^3 (x^n)^p}{np+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^n)^p,x, algorithm="maxima")
```

```
[Out] b^p*x^3*(x^n)^p/(n*p + 3)
```

mupad [B] time = 1.22, size = 18, normalized size = 1.00

$$\frac{x^3 \left(b\, x^n\right)^p}{n\,p+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^n)^p,x)
```

```
[Out] (x^3*(b*x^n)^p)/(n*p + 3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{b^p x^3 (x^n)^p}{np+3} & \text{for } n \neq -\frac{3}{p} \\ \int x^2 \left(b x^{-\frac{3}{p}}\right)^p dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**n)**p,x)
```

```
[Out] Piecewise((b**p*x**3*(x**n)**p/(n*p + 3), Ne(n, -3/p)), (Integral(x**2*(b*x
**(-3/p))**p, x), True))
```

3.2714 $\int x\,(bx^n)^p\,dx$

Optimal. Leaf size=18

$$\frac{x^2\,(bx^n)^p}{np+2}$$

```
[Out] x^2*(b*x^n)^p/(n*p+2)
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {15, 30}

$$\frac{x^2\,(bx^n)^p}{np+2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b*x^n)^p,x]
```

```
[Out] (x^2*(b*x^n)^p)/(2 + n*p)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\,(bx^n)^p\,dx &= \left(x^{-np}\,(bx^n)^p\right)\int x^{1+np}\,dx\\ &= \frac{x^2\,(bx^n)^p}{2+np}\end{aligned}$$

Mathematica [A] time = 0.02, size = 18, normalized size = 1.00

$$\frac{x^2\,(bx^n)^p}{np+2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b*x^n)^p,x]
```

```
[Out] (x^2*(b*x^n)^p)/(2 + n*p)
```

fricas [A] time = 0.88, size = 22, normalized size = 1.22

$$\frac{x^2 e^{\left(np\log(x)+p\log(b)\right)}}{np+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^n)^p,x, algorithm="fricas")
```

```
[Out] x^2*e^(n*p*log(x) + p*log(b))/(n*p + 2)
```

giac [A] time = 0.17, size = 22, normalized size = 1.22

$$\frac{x^2 e^{\left(np\log(x)+p\log(b)\right)}}{np+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^n)^p,x, algorithm="giac")
[Out] x^2*e^(n*p*log(x) + p*log(b))/(n*p + 2)
```

maple [A] time = 0.00, size = 19, normalized size = 1.06

$$\frac{x^2 \left(b\, x^n\right)^p}{np+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^n)^p,x)
[Out] 1/(n*p+2)*x^2*(b*x^n)^p
```

maxima [A] time = 0.57, size = 19, normalized size = 1.06

$$\frac{b^p x^2 (x^n)^p}{np+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^n)^p,x, algorithm="maxima")
[Out] b^p*x^2*(x^n)^p/(n*p + 2)
```

mupad [B] time = 1.20, size = 18, normalized size = 1.00

$$\frac{x^2 \left(b\, x^n\right)^p}{n\, p+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^n)^p,x)
[Out] (x^2*(b*x^n)^p)/(n*p + 2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{b^p x^2 (x^n)^p}{np+2} & \text{for } n \neq -\frac{2}{p} \\ \int x \left(b x^{-\frac{2}{p}}\right)^p \, dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**n)**p,x)
[Out] Piecewise((b**p*x**2*(x**n)**p/(n*p + 2), Ne(n, -2/p)), (Integral(x*(b*x**(
-2/p))**p, x), True))
```

3.2715 $\int (bx^n)^p \, dx$

Optimal. Leaf size=16

$$\frac{x\,(bx^n)^p}{np+1}$$

```
[Out] x*(b*x^n)^p/(n*p+1)
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {15, 30}

$$\frac{x\,(bx^n)^p}{np+1}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^n)^p,x]
```

```
[Out] (x*(b*x^n)^p)/(1 + n*p)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (bx^n)^p \, dx &= \left(x^{-np}\,(bx^n)^p\right) \int x^{np}\, dx \\ &= \frac{x\,(bx^n)^p}{1+np}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{x\,(bx^n)^p}{np+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^n)^p,x]
```

```
[Out] (x*(b*x^n)^p)/(1 + n*p)
```

fricas [A] time = 1.01, size = 20, normalized size = 1.25

$$\frac{x e^{\left(np\log(x)+p\log(b)\right)}}{np+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^p,x, algorithm="fricas")
```

```
[Out] x*e^(n*p*log(x) + p*log(b))/(n*p + 1)
```

giac [**A**] time = 0.16, size = 20, normalized size = 1.25

$$\frac{xe^{\left(np\log(x)+p\log(b)\right)}}{np+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^p,x, algorithm="giac")
[Out] x*e^(n*p*log(x) + p*log(b))/(n*p + 1)
```

maple [**A**] time = 0.00, size = 17, normalized size = 1.06

$$\frac{x\,(b\,x^n)^p}{np+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^p,x)
[Out] 1/(n*p+1)*x*(b*x^n)^p
```

maxima [**A**] time = 0.60, size = 17, normalized size = 1.06

$$\frac{b^p x(x^n)^p}{np+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^p,x, algorithm="maxima")
[Out] b^p*x*(x^n)^p/(n*p + 1)
```

mupad [**B**] time = 1.76, size = 16, normalized size = 1.00

$$\frac{x\,(b\,x^n)^p}{n\,p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^p,x)
[Out] (x*(b*x^n)^p)/(n*p + 1)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{b^p x(x^n)^p}{np+1} & \text{for } n \neq -\frac{1}{p} \\ \int \left(bx^{-\frac{1}{p}}\right)^p dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**n)**p,x)
[Out] Piecewise((b**p*x*(x**n)**p/(n*p + 1), Ne(n, -1/p)), (Integral((b*x**(-1/p)
)**p, x), True))
```

3.2716 $\int \frac{(bx^n)^p}{x}\,dx$

Optimal. Leaf size=14

$$\frac{(bx^n)^p}{np}$$

```
[Out] (b*x^n)^p/n/p
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 30}

$$\frac{(bx^n)^p}{np}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^n)^p/x,x]
```

```
[Out] (b*x^n)^p/(n*p)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^n)^p}{x}\,dx &= \left(x^{-np}\,(bx^n)^p\right)\int x^{-1+np}\,dx\\ &= \frac{(bx^n)^p}{np}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{(bx^n)^p}{np}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^n)^p/x,x]
```

```
[Out] (b*x^n)^p/(n*p)
```

fricas [A] time = 0.84, size = 18, normalized size = 1.29

$$\frac{e^{\left(np\log(x)+p\log(b)\right)}}{np}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^p/x,x, algorithm="fricas")
```

```
[Out] e^(n*p*log(x) + p*log(b))/(n*p)
```

giac [A] time = 0.15, size = 14, normalized size = 1.00

$$\frac{(bx^n)^p}{np}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^p/x,x, algorithm="giac")
```

```
[Out] (b*x^n)^p/(n*p)
```

maple [A] time = 0.00, size = 15, normalized size = 1.07

$$\frac{(b\,x^n)^p}{np}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^p/x,x)
```

```
[Out] 1/n/p*(b*x^n)^p
```

maxima [A] time = 0.72, size = 15, normalized size = 1.07

$$\frac{b^p (x^n)^p}{np}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^p/x,x, algorithm="maxima")
```

```
[Out] b^p*(x^n)^p/(n*p)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{(b\,x^n)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^p/x,x)
```

```
[Out] int((b*x^n)^p/x, x)
```

sympy [A] time = 0.30, size = 22, normalized size = 1.57

$$\begin{cases} \log(x) & \text{for } p = 0 \wedge \left(n = 0 \vee p = 0\right) \\ b^p \log(x) & \text{for } n = 0 \\ \frac{b^p (x^n)^p}{np} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**n)**p/x,x)
```

```
[Out] Piecewise((log(x), Eq(p, 0) & (Eq(n, 0) | Eq(p, 0))), (b**p*log(x), Eq(n, 0
)), (b**p*(x**n)**p/(n*p), True))
```

3.2717 $\int \frac{(bx^n)^p}{x^2}\,dx$

Optimal. Leaf size=20

$$-\frac{(bx^n)^p}{x(1-np)}$$

```
[Out] -(b*x^n)^p/(-n*p+1)/x
```

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 30}

$$-\frac{(bx^n)^p}{x(1-np)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^n)^p/x^2,x]
```

```
[Out] -((b*x^n)^p/((1 - n*p)*x))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^n)^p}{x^2}\,dx &= \left(x^{-np}\,(bx^n)^p\right)\int x^{-2+np}\,dx \\ &= -\frac{(bx^n)^p}{(1-np)x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 0.90

$$\frac{(bx^n)^p}{x(np-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^n)^p/x^2,x]
```

```
[Out] (b*x^n)^p/((-1 + n*p)*x)
```

fricas [A] time = 0.59, size = 22, normalized size = 1.10

$$\frac{e^{\left(np\log(x)+p\log(b)\right)}}{\left(np-1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^p/x^2,x, algorithm="fricas")
```

```
[Out] e^(n*p*log(x) + p*log(b))/((n*p - 1)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^n)^p/x^2, x)
```

maple [A] time = 0.00, size = 19, normalized size = 0.95

$$\frac{(b\,x^n)^p}{\left(np-1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^p/x^2,x)
```

```
[Out] 1/x/(n*p-1)*(b*x^n)^p
```

maxima [A] time = 0.59, size = 19, normalized size = 0.95

$$\frac{b^p(x^n)^p}{\left(np-1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^p/x^2,x, algorithm="maxima")
```

```
[Out] b^p*(x^n)^p/((n*p - 1)*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(b\,x^n)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^p/x^2,x)
```

```
[Out] int((b*x^n)^p/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{b^p(x^n)^p}{npx-x} & \text{for } n \neq \frac{1}{p} \\ \int \frac{\left(bx^{\frac{1}{p}}\right)^p}{x^2}\,dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**n)**p/x**2,x)
```

```
[Out] Piecewise((b**p*(x**n)**p/(n*p*x - x), Ne(n, 1/p)), (Integral((b*x**(1/p))*
**p/x**2, x), True))
```

3.2718 $\int \frac{(bx^n)^p}{x^3}\,dx$

Optimal. Leaf size=20

$$-\frac{(bx^n)^p}{x^2(2-np)}$$

```
[Out] -(b*x^n)^p/(-n*p+2)/x^2
```

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 30}

$$-\frac{(bx^n)^p}{x^2(2-np)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^n)^p/x^3,x]
```

```
[Out] -((b*x^n)^p/((2 - n*p)*x^2))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^n)^p}{x^3}\,dx &= \left(x^{-np}\,(bx^n)^p\right)\int x^{-3+np}\,dx\\ &= -\frac{(bx^n)^p}{(2-np)x^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 0.90

$$\frac{(bx^n)^p}{x^2(np-2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^n)^p/x^3,x]
```

```
[Out] (b*x^n)^p/((-2 + n*p)*x^2)
```

fricas [A] time = 0.67, size = 22, normalized size = 1.10

$$\frac{e^{\left(np\log(x)+p\log(b)\right)}}{\left(np-2\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^p/x^3,x, algorithm="fricas")
```

```
[Out] e^(n*p*log(x) + p*log(b))/((n*p - 2)*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^n)^p/x^3, x)
```

maple [A] time = 0.00, size = 19, normalized size = 0.95

$$\frac{(b\,x^n)^p}{\left(np-2\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^p/x^3,x)
```

```
[Out] 1/x^2/(n*p-2)*(b*x^n)^p
```

maxima [A] time = 0.60, size = 19, normalized size = 0.95

$$\frac{b^p(x^n)^p}{\left(np-2\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^p/x^3,x, algorithm="maxima")
```

```
[Out] b^p*(x^n)^p/((n*p - 2)*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(b\,x^n)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^p/x^3,x)
```

```
[Out] int((b*x^n)^p/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{b^p(x^n)^p}{npx^2-2x^2} & \text{for } n \neq \frac{2}{p} \\ \int \frac{\left(bx^{\frac{2}{p}}\right)^p}{x^3}\,dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**n)**p/x**3,x)
```

```
[Out] Piecewise((b**p*(x**n)**p/(n*p*x**2 - 2*x**2), Ne(n, 2/p)), (Integral((b*x*
**(2/p))**p/x**3, x), True))
```

3.2719 $\int \frac{(bx^n)^p}{x^4}\,dx$

Optimal. Leaf size=20

$$-\frac{(bx^n)^p}{x^3(3-np)}$$

```
[Out] -(b*x^n)^p/(-n*p+3)/x^3
```

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 30}

$$-\frac{(bx^n)^p}{x^3(3-np)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^n)^p/x^4,x]
```

```
[Out] -((b*x^n)^p/((3 - n*p)*x^3))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^n)^p}{x^4}\,dx &= \left(x^{-np}\,(bx^n)^p\right)\int x^{-4+np}\,dx\\ &= -\frac{(bx^n)^p}{(3-np)x^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 0.90

$$\frac{(bx^n)^p}{x^3(np-3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^n)^p/x^4,x]
```

```
[Out] (b*x^n)^p/((-3 + n*p)*x^3)
```

fricas [A] time = 0.71, size = 22, normalized size = 1.10

$$\frac{e^{\left(np\log(x)+p\log(b)\right)}}{\left(np-3\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^p/x^4,x, algorithm="fricas")
```

```
[Out] e^(n*p*log(x) + p*log(b))/((n*p - 3)*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^p/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*x^n)^p/x^4, x)
```

maple [A] time = 0.00, size = 19, normalized size = 0.95

$$\frac{(b\,x^n)^p}{\left(np-3\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^p/x^4,x)
```

```
[Out] 1/x^3/(n*p-3)*(b*x^n)^p
```

maxima [A] time = 0.53, size = 19, normalized size = 0.95

$$\frac{b^p(x^n)^p}{\left(np-3\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^n)^p/x^4,x, algorithm="maxima")
```

```
[Out] b^p*(x^n)^p/((n*p - 3)*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(b\,x^n)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n)^p/x^4,x)
```

```
[Out] int((b*x^n)^p/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{b^p(x^n)^p}{npx^3-3x^3} & \text{for } n \neq \frac{3}{p} \\ \int \frac{\left(bx^{\frac{3}{p}}\right)^p}{x^4}\,dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**n)**p/x**4,x)
```

```
[Out] Piecewise((b**p*(x**n)**p/(n*p*x**3 - 3*x**3), Ne(n, 3/p)), (Integral((b*x*
**(3/p))**p/x**4, x), True))
```

3.2720 $\int x^{-1+n}\,(a+bx^n)^p\,dx$

Optimal. Leaf size=23

$$\frac{(a+bx^n)^{p+1}}{bn(p+1)}$$

```
[Out] (a+b*x^n)^(1+p)/b/n/(1+p)
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$\frac{(a+bx^n)^{p+1}}{bn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*(a + b*x^n)^p,x]
```

```
[Out] (a + b*x^n)^(1 + p)/(b*n*(1 + p))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^{-1+n}\,(a+bx^n)^p\,dx = \frac{(a+bx^n)^{1+p}}{bn(1+p)}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{(a+bx^n)^{p+1}}{bn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*(a + b*x^n)^p,x]
```

```
[Out] (a + b*x^n)^(1 + p)/(b*n*(1 + p))
```

fricas [A] time = 0.64, size = 27, normalized size = 1.17

$$\frac{(bx^n+a)(bx^n+a)^p}{bnp+bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(a+b*x^n)^p,x, algorithm="fricas")
```

```
[Out] (b*x^n + a)*(b*x^n + a)^p/(b*n*p + b*n)
```

giac [A] time = 0.16, size = 23, normalized size = 1.00

$$\frac{(bx^n+a)^{p+1}}{bn\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(a+b*x^n)^p,x, algorithm="giac")
[Out] (b*x^n + a)^(p + 1)/(b*n*(p + 1))
```

maple [A] time = 0.05, size = 29, normalized size = 1.26

$$\frac{(b\,x^n + a)\,(b\,x^n + a)^p}{\left(p+1\right)bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*(b*x^n+a)^p,x)
[Out] (b*x^n+a)/b/(p+1)/n*(b*x^n+a)^p
```

maxima [A] time = 0.49, size = 23, normalized size = 1.00

$$\frac{(bx^n + a)^{p+1}}{bn\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(a+b*x^n)^p,x, algorithm="maxima")
[Out] (b*x^n + a)^(p + 1)/(b*n*(p + 1))
```

mupad [B] time = 1.59, size = 23, normalized size = 1.00

$$\frac{(a + b\,x^n)^{p+1}}{b\,n\,\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*(a + b*x^n)^p,x)
[Out] (a + b*x^n)^(p + 1)/(b*n*(p + 1))
```

sympy [A] time = 46.61, size = 75, normalized size = 3.26

$$\begin{cases} \frac{\log(x)}{a} & \text{for } b = 0 \wedge n = 0 \wedge p = -1 \\ \frac{a^p x^n}{n} & \text{for } b = 0 \\ (a+b)^p \log(x) & \text{for } n = 0 \\ \frac{\log\left(\frac{a}{b} + x^n\right)}{bn} & \text{for } p = -1 \\ \frac{a(a+bx^n)^p}{bnp+bn} + \frac{bx^n(a+bx^n)^p}{bnp+bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(a+b*x**n)**p,x)
[Out] Piecewise((log(x)/a, Eq(b, 0) & Eq(n, 0) & Eq(p, -1)), (a**p*x**n/n, Eq(b,
0)), ((a + b)**p*log(x), Eq(n, 0)), (log(a/b + x**n)/(b*n), Eq(p, -1)), (a*
(a + b*x**n)**p/(b*n*p + b*n) + b*x**n*(a + b*x**n)**p/(b*n*p + b*n), True)
)
```

3.2721 $\int x^{-1+2n}\,(a+bx^n)^p\,dx$

Optimal. Leaf size=49

$$\frac{(a+bx^n)^{p+2}}{b^2n(p+2)}-\frac{a\,(a+bx^n)^{p+1}}{b^2n(p+1)}$$

```
[Out] -a*(a+b*x^n)^(1+p)/b^2/n/(1+p)+(a+b*x^n)^(2+p)/b^2/n/(2+p)
```

Rubi [A] time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$\frac{(a+bx^n)^{p+2}}{b^2n(p+2)}-\frac{a\,(a+bx^n)^{p+1}}{b^2n(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)*(a + b*x^n)^p,x]
```

```
[Out] -((a*(a + b*x^n)^(1 + p))/(b^2*n*(1 + p))) + (a + b*x^n)^(2 + p)/(b^2*n*(2
+ p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1+2n}\,(a+bx^n)^p\,dx &= \frac{\text{Subst}\left(\int x(a+bx)^p\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int\left(-\frac{a(a+bx)^p}{b}+\frac{(a+bx)^{1+p}}{b}\right)dx,x,x^n\right)}{n}\\ &= -\frac{a\,(a+bx^n)^{1+p}}{b^2n(1+p)}+\frac{(a+bx^n)^{2+p}}{b^2n(2+p)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 0.82

$$\frac{(a+bx^n)^{p+1}\left(b(p+1)x^n-a\right)}{b^2n(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)*(a + b*x^n)^p,x]
```

```
[Out] ((a + b*x^n)^(1 + p)*(-a + b*(1 + p)*x^n))/(b^2*n*(1 + p)*(2 + p))
```

fricas [**A**] time = 0.59, size = 62, normalized size = 1.27

$$\frac{\left(abpx^n - a^2 + \left(b^2p + b^2\right)x^{2\,n}\right)(bx^n + a)^p}{b^2np^2 + 3\,b^2np + 2\,b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^p,x, algorithm="fricas")
```

```
[Out] (a*b*p*x^n - a^2 + (b^2*p + b^2)*x^(2*n))*(b*x^n + a)^p/(b^2*n*p^2 + 3*b^2*
n*p + 2*b^2*n)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^p x^{2\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p*x^(2*n - 1), x)
```

maple [**A**] time = 0.06, size = 61, normalized size = 1.24

$$-\frac{\left(-abp\,x^n - b^2p\,x^{2n} - b^2x^{2n} + a^2\right)(b\,x^n + a)^p}{\left(p+1\right)\left(p+2\right)b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n-1)*(b*x^n+a)^p,x)
```

```
[Out] -(-b^2*p*(x^n)^2-x^n*a*p*b-b^2*(x^n)^2+a^2)/(p+1)/(p+2)/n/b^2*(b*x^n+a)^p
```

maxima [**A**] time = 0.58, size = 51, normalized size = 1.04

$$\frac{\left(b^2\left(p+1\right)x^{2\,n} + abpx^n - a^2\right)(bx^n + a)^p}{\left(p^2 + 3\,p + 2\right)b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^p,x, algorithm="maxima")
```

```
[Out] (b^2*(p + 1)*x^(2*n) + a*b*p*x^n - a^2)*(b*x^n + a)^p/((p^2 + 3*p + 2)*b^2*
n)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int x^{2\,n-1}\,(a + b\,x^n)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)*(a + b*x^n)^p,x)
```

```
[Out] int(x^(2*n - 1)*(a + b*x^n)^p, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a+b*x**n)**p,x)
```

```
[Out] Timed out
```

3.2722 $\int x^{-1+3n}\,(a+bx^n)^p\,dx$

Optimal. Leaf size=75

$$\frac{a^2\,(a+bx^n)^{p+1}}{b^3n(p+1)}-\frac{2a\,(a+bx^n)^{p+2}}{b^3n(p+2)}+\frac{(a+bx^n)^{p+3}}{b^3n(p+3)}$$

```
[Out] a^2*(a+b*x^n)^(1+p)/b^3/n/(1+p)-2*a*(a+b*x^n)^(2+p)/b^3/n/(2+p)+(a+b*x^n)^(
3+p)/b^3/n/(3+p)
```

Rubi [A] time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {266, 43}

$$\frac{a^2\,(a+bx^n)^{p+1}}{b^3n(p+1)}-\frac{2a\,(a+bx^n)^{p+2}}{b^3n(p+2)}+\frac{(a+bx^n)^{p+3}}{b^3n(p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)*(a + b*x^n)^p,x]
```

```
[Out] (a^2*(a + b*x^n)^(1 + p))/(b^3*n*(1 + p)) - (2*a*(a + b*x^n)^(2 + p))/(b^3*
n*(2 + p)) + (a + b*x^n)^(3 + p)/(b^3*n*(3 + p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1+3n}\,(a+bx^n)^p\,dx &= \frac{\text{Subst}\left(\int x^2(a+bx)^p\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int\left(\frac{a^2(a+bx)^p}{b^2}-\frac{2a(a+bx)^{1+p}}{b^2}+\frac{(a+bx)^{2+p}}{b^2}\right)dx,x,x^n\right)}{n}\\ &= \frac{a^2\,(a+bx^n)^{1+p}}{b^3n(1+p)}-\frac{2a\,(a+bx^n)^{2+p}}{b^3n(2+p)}+\frac{(a+bx^n)^{3+p}}{b^3n(3+p)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 66, normalized size = 0.88

$$\frac{(a+bx^n)^{p+1}\left(2a^2-2ab(p+1)x^n+b^2\left(p^2+3p+2\right)x^{2n}\right)}{b^3n(p+1)(p+2)(p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)*(a + b*x^n)^p,x]
```

```
[Out] ((a + b*x^n)^(1 + p)*(2*a^2 - 2*a*b*(1 + p)*x^n + b^2*(2 + 3*p + p^2)*x^(2*
n)))/(b^3*n*(1 + p)*(2 + p)*(3 + p))
```

fricas [A] time = 0.74, size = 108, normalized size = 1.44

$$-\frac{\left(2\,a^2bpx^n - 2\,a^3 - \left(b^3p^2 + 3\,b^3p + 2\,b^3\right)x^{3\,n} - \left(ab^2p^2 + ab^2p\right)x^{2\,n}\right)(bx^n + a)^p}{b^3np^3 + 6\,b^3np^2 + 11\,b^3np + 6\,b^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^p,x, algorithm="fricas")
```

```
[Out] -(2*a^2*b*p*x^n - 2*a^3 - (b^3*p^2 + 3*b^3*p + 2*b^3)*x^(3*n) - (a*b^2*p^2
+ a*b^2*p)*x^(2*n))*(b*x^n + a)^p/(b^3*n*p^3 + 6*b^3*n*p^2 + 11*b^3*n*p + 6
*b^3*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^p x^{3\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p*x^(3*n - 1), x)
```

maple [A] time = 0.07, size = 105, normalized size = 1.40

$$\frac{\left(a\,b^2p^2x^{2n} + b^3p^2x^{3n} - 2a^2bp\,x^n + a\,b^2p\,x^{2n} + 3b^3p\,x^{3n} + 2b^3x^{3n} + 2a^3\right)(b\,x^n + a)^p}{\left(p+2\right)\left(p+3\right)\left(p+1\right)b^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n-1)*(b*x^n+a)^p,x)
```

```
[Out] (b^3*p^2*(x^n)^3+a*b^2*p^2*(x^n)^2+3*b^3*p*(x^n)^3+(x^n)^2*a*p*b^2+2*(x^n)^
3*b^3-2*x^n*a^2*p*b+2*a^3)/(p+2)/(3+p)/(p+1)/n/b^3*(b*x^n+a)^p
```

maxima [A] time = 0.60, size = 79, normalized size = 1.05

$$\frac{\left(\left(p^2 + 3\,p + 2\right)b^3x^{3\,n} + \left(p^2 + p\right)ab^2x^{2\,n} - 2\,a^2bpx^n + 2\,a^3\right)(bx^n + a)^p}{\left(p^3 + 6\,p^2 + 11\,p + 6\right)b^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^p,x, algorithm="maxima")
```

```
[Out] ((p^2 + 3*p + 2)*b^3*x^(3*n) + (p^2 + p)*a*b^2*x^(2*n) - 2*a^2*b*p*x^n + 2*
a^3)*(b*x^n + a)^p/((p^3 + 6*p^2 + 11*p + 6)*b^3*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3\,n-1}\,(a + b\,x^n)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n - 1)*(a + b*x^n)^p,x)
```

```
[Out] int(x^(3*n - 1)*(a + b*x^n)^p, x)
```

sympy [**F**(**-2**)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)*(a+b*x**n)**p,x)
[Out] Exception raised: HeuristicGCDFailed
```

3.2723 $\int x^{-1+4n}\,(a+bx^n)^p\,dx$

Optimal. Leaf size=103

$$-\frac{a^3\,(a+bx^n)^{p+1}}{b^4n(p+1)}+\frac{3a^2\,(a+bx^n)^{p+2}}{b^4n(p+2)}-\frac{3a\,(a+bx^n)^{p+3}}{b^4n(p+3)}+\frac{(a+bx^n)^{p+4}}{b^4n(p+4)}$$

```
[Out] -a^3*(a+b*x^n)^(1+p)/b^4/n/(1+p)+3*a^2*(a+b*x^n)^(2+p)/b^4/n/(2+p)-3*a*(a+b
*x^n)^(3+p)/b^4/n/(3+p)+(a+b*x^n)^(4+p)/b^4/n/(4+p)
```

Rubi [A] time = 0.06, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {266, 43}

$$-\frac{a^3\,(a+bx^n)^{p+1}}{b^4n(p+1)}+\frac{3a^2\,(a+bx^n)^{p+2}}{b^4n(p+2)}-\frac{3a\,(a+bx^n)^{p+3}}{b^4n(p+3)}+\frac{(a+bx^n)^{p+4}}{b^4n(p+4)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 4*n)*(a + b*x^n)^p,x]
```

```
[Out] -((a^3*(a + b*x^n)^(1 + p))/(b^4*n*(1 + p))) + (3*a^2*(a + b*x^n)^(2 + p))/
(b^4*n*(2 + p)) - (3*a*(a + b*x^n)^(3 + p))/(b^4*n*(3 + p)) + (a + b*x^n)^(
4 + p)/(b^4*n*(4 + p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1+4n}\,(a+bx^n)^p\,dx &= \frac{\text{Subst}\left(\int x^3(a+bx)^p\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int\left(-\frac{a^3(a+bx)^p}{b^3}+\frac{3a^2(a+bx)^{1+p}}{b^3}-\frac{3a(a+bx)^{2+p}}{b^3}+\frac{(a+bx)^{3+p}}{b^3}\right)dx,x,x^n\right)}{n}\\ &= -\frac{a^3\,(a+bx^n)^{1+p}}{b^4n(1+p)}+\frac{3a^2\,(a+bx^n)^{2+p}}{b^4n(2+p)}-\frac{3a\,(a+bx^n)^{3+p}}{b^4n(3+p)}+\frac{(a+bx^n)^{4+p}}{b^4n(4+p)}\end{aligned}$$

Mathematica [A] time = 0.09, size = 78, normalized size = 0.76

$$\frac{(a+bx^n)^{p+1}\left(-\frac{a^3}{p+1}+\frac{3a^2(a+bx^n)}{p+2}-\frac{3a(a+bx^n)^2}{p+3}+\frac{(a+bx^n)^3}{p+4}\right)}{b^4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 4*n)*(a + b*x^n)^p,x]
```

```
[Out] ((a + b*x^n)^(1 + p)*(-(a^3/(1 + p)) + (3*a^2*(a + b*x^n))/(2 + p) - (3*a*(
a + b*x^n)^2)/(3 + p) + (a + b*x^n)^3/(4 + p)))/(b^4*n)
```

fricas [A] time = 0.63, size = 158, normalized size = 1.53

$$\frac{\left(6\,a^3bpx^n - 6\,a^4 + \left(b^4p^3 + 6\,b^4p^2 + 11\,b^4p + 6\,b^4\right)x^{4\,n} + \left(ab^3p^3 + 3\,ab^3p^2 + 2\,ab^3p\right)x^{3\,n} - 3\left(a^2b^2p^2 + a^2b^2p\right)x^2\right.}{b^4np^4 + 10\,b^4np^3 + 35\,b^4np^2 + 50\,b^4np + 24\,b^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)*(a+b*x^n)^p,x, algorithm="fricas")
```

```
[Out] (6*a^3*b*p*x^n - 6*a^4 + (b^4*p^3 + 6*b^4*p^2 + 11*b^4*p + 6*b^4)*x^(4*n) +
 (a*b^3*p^3 + 3*a*b^3*p^2 + 2*a*b^3*p)*x^(3*n) - 3*(a^2*b^2*p^2 + a^2*b^2*p
)*x^(2*n))*(b*x^n + a)^p/(b^4*n*p^4 + 10*b^4*n*p^3 + 35*b^4*n*p^2 + 50*b^4*
n*p + 24*b^4*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^p x^{4\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)*(a+b*x^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p*x^(4*n - 1), x)
```

maple [A] time = 0.08, size = 171, normalized size = 1.66

$$-\frac{\left(-a\,b^3p^3x^{3n} - b^4p^3x^{4n} + 3a^2b^2p^2x^{2n} - 3a\,b^3p^2x^{3n} - 6b^4p^2x^{4n} - 6a^3bp\,x^n + 3a^2b^2p\,x^{2n} - 2a\,b^3p\,x^{3n} - 11b^4p\,x^{4n}\right.}{\left(p+3\right)\left(p+4\right)\left(p+2\right)\left(p+1\right)b^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+4*n)*(b*x^n+a)^p,x)
```

```
[Out] -(-b^4*p^3*(x^n)^4-a*b^3*p^3*(x^n)^3-6*b^4*p^2*(x^n)^4-3*a*b^3*p^2*(x^n)^3-
11*b^4*p*(x^n)^4+3*a^2*b^2*p^2*(x^n)^2-2*a*p*(x^n)^3*b^3-6*b^4*(x^n)^4+3*(x
^n)^2*a^2*p*b^2-6*p*a^3*x^n*b+6*a^4)/(p+3)/(4+p)/(p+2)/(p+1)/n/b^4*(b*x^n+a
)^p
```

maxima [A] time = 0.58, size = 114, normalized size = 1.11

$$\frac{\left(\left(p^3 + 6\,p^2 + 11\,p + 6\right)b^4x^{4\,n} + \left(p^3 + 3\,p^2 + 2\,p\right)ab^3x^{3\,n} - 3\left(p^2 + p\right)a^2b^2x^{2\,n} + 6\,a^3bpx^n - 6\,a^4\right)(bx^n + a)^p}{\left(p^4 + 10\,p^3 + 35\,p^2 + 50\,p + 24\right)b^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)*(a+b*x^n)^p,x, algorithm="maxima")
```

```
[Out] ((p^3 + 6*p^2 + 11*p + 6)*b^4*x^(4*n) + (p^3 + 3*p^2 + 2*p)*a*b^3*x^(3*n) -
 3*(p^2 + p)*a^2*b^2*x^(2*n) + 6*a^3*b*p*x^n - 6*a^4)*(b*x^n + a)^p/((p^4 +
 10*p^3 + 35*p^2 + 50*p + 24)*b^4*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{4\,n-1}\,(a + b\,x^n)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4*n - 1)*(a + b*x^n)^p,x)
```

```
[Out] int(x^(4*n - 1)*(a + b*x^n)^p, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+4*n)*(a+b*x**n)**p,x)
[Out] Timed out
```

3.2724 $\int x^{-1-n-np}\,(a+bx^n)^p\,dx$

Optimal. Leaf size=32

$$-\frac{x^{-n(p+1)}\,(a+bx^n)^{p+1}}{an(p+1)}$$

```
[Out] -(a+b*x^n)^(1+p)/a/n/(1+p)/(x^(n*(1+p)))
```

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {264}

$$-\frac{x^{-n(p+1)}\,(a+bx^n)^{p+1}}{an(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n - n*p)*(a + b*x^n)^p,x]
```

```
[Out] -((a + b*x^n)^(1 + p)/(a*n*(1 + p)*x^(n*(1 + p))))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^{-1-n-np}\,(a+bx^n)^p\,dx = -\frac{x^{-n(1+p)}\,(a+bx^n)^{1+p}}{an(1+p)}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 1.00

$$-\frac{x^{-n(p+1)}\,(a+bx^n)^{p+1}}{an(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n - n*p)*(a + b*x^n)^p,x]
```

```
[Out] -((a + b*x^n)^(1 + p)/(a*n*(1 + p)*x^(n*(1 + p))))
```

fricas [A] time = 0.64, size = 53, normalized size = 1.66

$$-\frac{\left(bxx^{-np-n-1}x^n + axx^{-np-n-1}\right)(bx^n+a)^p}{anp+an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-n*p-n-1)*(a+b*x^n)^p,x, algorithm="fricas")
```

```
[Out] -(b*x*x^(-n*p - n - 1)*x^n + a*x*x^(-n*p - n - 1))*(b*x^n + a)^p/(a*n*p + a
*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n+a)^p x^{-np-n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-n*p-n-1)*(a+b*x^n)^p,x, algorithm="giac")
[Out] integrate((b*x^n + a)^p*x^(-n*p - n - 1), x)
```

maple **[F]** time = 0.21, size = 0, normalized size = 0.00

$$\int x^{-np-n-1} (b\,x^n + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-n*p-n-1)*(b*x^n+a)^p,x)
[Out] int(x^(-n*p-n-1)*(b*x^n+a)^p,x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^p x^{-np-n-1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-n*p-n-1)*(a+b*x^n)^p,x, algorithm="maxima")
[Out] integrate((b*x^n + a)^p*x^(-n*p - n - 1), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a + b\,x^n)^p}{x^{n+n\,p+1}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^p/x^(n + n*p + 1),x)
[Out] int((a + b*x^n)^p/x^(n + n*p + 1), x)
```

sympy **[F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-n*p-n-1)*(a+b*x**n)**p,x)
[Out] Exception raised: HeuristicGCDFailed
```

3.2725 $\int x^{-1-9n}\left(a+bx^n\right)^8\,dx$

Optimal. Leaf size=24

$$-\frac{x^{-9n}\left(a+bx^n\right)^9}{9an}$$

```
[Out] -1/9*(a+b*x^n)^9/a/n/(x^(9*n))
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {264}

$$-\frac{x^{-9n}\left(a+bx^n\right)^9}{9an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 9*n)*(a + b*x^n)^8,x]
```

```
[Out] -(a + b*x^n)^9/(9*a*n*x^(9*n))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^{-1-9n}\left(a+bx^n\right)^8\,dx = -\frac{x^{-9n}\left(a+bx^n\right)^9}{9an}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$-\frac{x^{-9n}\left(a+bx^n\right)^9}{9an}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 9*n)*(a + b*x^n)^8,x]
```

```
[Out] -1/9*(a + b*x^n)^9/(a*n*x^(9*n))
```

fricas [B] time = 0.60, size = 111, normalized size = 4.62

$$-\frac{9\,b^8x^{8\,n}+36\,ab^7x^{7\,n}+84\,a^2b^6x^{6\,n}+126\,a^3b^5x^{5\,n}+126\,a^4b^4x^{4\,n}+84\,a^5b^3x^{3\,n}+36\,a^6b^2x^{2\,n}+9\,a^7bx^n+a^8}{9\,nx^{9\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-9*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

```
[Out] -1/9*(9*b^8*x^(8*n) + 36*a*b^7*x^(7*n) + 84*a^2*b^6*x^(6*n) + 126*a^3*b^5*x
^(5*n) + 126*a^4*b^4*x^(4*n) + 84*a^5*b^3*x^(3*n) + 36*a^6*b^2*x^(2*n) + 9*
a^7*b*x^n + a^8)/(n*x^(9*n))
```

giac [B] time = 0.29, size = 111, normalized size = 4.62

$$-\frac{9\,b^8x^{8\,n}+36\,ab^7x^{7\,n}+84\,a^2b^6x^{6\,n}+126\,a^3b^5x^{5\,n}+126\,a^4b^4x^{4\,n}+84\,a^5b^3x^{3\,n}+36\,a^6b^2x^{2\,n}+9\,a^7bx^n+a^8}{9\,nx^{9\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-9*n)*(a+b*x^n)^8,x, algorithm="giac")
```

```
[Out] -1/9*(9*b^8*x^(8*n) + 36*a*b^7*x^(7*n) + 84*a^2*b^6*x^(6*n) + 126*a^3*b^5*x
^(5*n) + 126*a^4*b^4*x^(4*n) + 84*a^5*b^3*x^(3*n) + 36*a^6*b^2*x^(2*n) + 9*
a^7*b*x^n + a^8)/(n*x^(9*n))
```

maple [B] time = 0.00, size = 136, normalized size = 5.67

$$-\frac{a^8 x^{-9n}}{9n}-\frac{a^7 b\, x^{-8n}}{n}-\frac{4a^6b^2x^{-7n}}{n}-\frac{28a^5b^3x^{-6n}}{3n}-\frac{14a^4b^4x^{-5n}}{n}-\frac{14a^3b^5x^{-4n}}{n}-\frac{28a^2b^6x^{-3n}}{3n}-\frac{4a\,b^7x^{-2n}}{n}-\frac{b^8x^{-n}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-9*n)*(b*x^n+a)^8,x)
```

```
[Out] -b^8/n/(x^n)-4*a*b^7/n/(x^n)^2-28/3*a^2*b^6/n/(x^n)^3-14*a^3*b^5/n/(x^n)^4-
14*a^4*b^4/n/(x^n)^5-28/3*a^5*b^3/n/(x^n)^6-4*a^6*b^2/n/(x^n)^7-a^7*b/n/(x^
n)^8-1/9*a^8/n/(x^n)^9
```

maxima [B] time = 0.54, size = 151, normalized size = 6.29

$$-\frac{a^8}{9\,nx^{9\,n}}-\frac{a^7b}{nx^{8\,n}}-\frac{4\,a^6b^2}{nx^{7\,n}}-\frac{28\,a^5b^3}{3\,nx^{6\,n}}-\frac{14\,a^4b^4}{nx^{5\,n}}-\frac{14\,a^3b^5}{nx^{4\,n}}-\frac{28\,a^2b^6}{3\,nx^{3\,n}}-\frac{4\,ab^7}{nx^{2\,n}}-\frac{b^8}{nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-9*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

```
[Out] -1/9*a^8/(n*x^(9*n)) - a^7*b/(n*x^(8*n)) - 4*a^6*b^2/(n*x^(7*n)) - 28/3*a^5
*b^3/(n*x^(6*n)) - 14*a^4*b^4/(n*x^(5*n)) - 14*a^3*b^5/(n*x^(4*n)) - 28/3*a
^2*b^6/(n*x^(3*n)) - 4*a*b^7/(n*x^(2*n)) - b^8/(n*x^n)
```

mupad [B] time = 0.00, size = 151, normalized size = 6.29

$$-\frac{a^8}{9\,n\,x^{9\,n}}-\frac{b^8}{n\,x^n}-\frac{28\,a^2\,b^6}{3\,n\,x^{3\,n}}-\frac{14\,a^3\,b^5}{n\,x^{4\,n}}-\frac{14\,a^4\,b^4}{n\,x^{5\,n}}-\frac{28\,a^5\,b^3}{3\,n\,x^{6\,n}}-\frac{4\,a^6\,b^2}{n\,x^{7\,n}}-\frac{4\,a\,b^7}{n\,x^{2\,n}}-\frac{a^7\,b}{n\,x^{8\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^8/x^(9*n + 1),x)
```

```
[Out] - a^8/(9*n*x^(9*n)) - b^8/(n*x^n) - (28*a^2*b^6)/(3*n*x^(3*n)) - (14*a^3*b^
5)/(n*x^(4*n)) - (14*a^4*b^4)/(n*x^(5*n)) - (28*a^5*b^3)/(3*n*x^(6*n)) - (4
*a^6*b^2)/(n*x^(7*n)) - (4*a*b^7)/(n*x^(2*n)) - (a^7*b)/(n*x^(8*n))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-9*n)*(a+b*x**n)**8,x)
```

```
[Out] Timed out
```

3.2726 $\int x^{-4-3p}\left(a+bx^3\right)^p\,dx$

Optimal. Leaf size=30

$$-\frac{x^{-3(p+1)}\left(a+bx^3\right)^{p+1}}{3a(p+1)}$$

```
[Out] -1/3*(b*x^3+a)^(1+p)/a/(1+p)/(x^(3+3*p))
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {264}

$$-\frac{x^{-3(p+1)}\left(a+bx^3\right)^{p+1}}{3a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-4 - 3*p)*(a + b*x^3)^p,x]
```

```
[Out] -(a + b*x^3)^(1 + p)/(3*a*(1 + p)*x^(3*(1 + p)))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^{-4-3p}\left(a+bx^3\right)^p\,dx = -\frac{x^{-3(1+p)}\left(a+bx^3\right)^{1+p}}{3a(1+p)}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 0.97

$$\frac{x^{-3p-3}\left(a+bx^3\right)^{p+1}}{a(-3p-3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-4 - 3*p)*(a + b*x^3)^p,x]
```

```
[Out] (x^(-3 - 3*p)*(a + b*x^3)^(1 + p))/(a*(-3 - 3*p))
```

fricas [A] time = 0.63, size = 34, normalized size = 1.13

$$-\frac{\left(bx^4+ax\right)\left(bx^3+a\right)^p x^{-3p-4}}{3\left(ap+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4-3*p)*(b*x^3+a)^p,x, algorithm="fricas")
```

```
[Out] -1/3*(b*x^4 + a*x)*(b*x^3 + a)^p*x^(-3*p - 4)/(a*p + a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^p x^{-3p-4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4-3*p)*(b*x^3+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^p*x^(-3*p - 4), x)
```

maple [A] time = 0.00, size = 29, normalized size = 0.97

$$-\frac{x^{-3p-3}\left(b\,x^3+a\right)^{p+1}}{3\left(p+1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-4-3*p)*(b*x^3+a)^p,x)
```

```
[Out] -1/3*x^(-3-3*p)*(b*x^3+a)^(p+1)/a/(p+1)
```

maxima [A] time = 0.54, size = 37, normalized size = 1.23

$$-\frac{\left(bx^3+a\right)e^{\left(p\log\left(bx^3+a\right)-3\,p\log(x)\right)}}{3\,a\left(p+1\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4-3*p)*(b*x^3+a)^p,x, algorithm="maxima")
```

```
[Out] -1/3*(b*x^3 + a)*e^(p*log(b*x^3 + a) - 3*p*log(x))/(a*(p + 1)*x^3)
```

mupad [B] time = 1.42, size = 52, normalized size = 1.73

$$-\left(b\,x^3+a\right)^p\left(\frac{x}{3\,x^{3\,p+4}\left(p+1\right)}+\frac{b\,x^4}{3\,a\,x^{3\,p+4}\left(p+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^p/x^(3*p + 4),x)
```

```
[Out] -(a + b*x^3)^p*(x/(3*x^(3*p + 4)*(p + 1)) + (b*x^4)/(3*a*x^(3*p + 4)*(p + 1
)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-4-3*p)*(b*x**3+a)**p,x)
```

```
[Out] Timed out
```

3.2727 $\int \frac{\left(a+bx^3\right)^8}{x^{28}}\,dx$

Optimal. Leaf size=19

$$-\frac{\left(a+bx^3\right)^9}{27ax^{27}}$$

```
[Out] -1/27*(b*x^3+a)^9/a/x^27
```

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {264}

$$-\frac{\left(a+bx^3\right)^9}{27ax^{27}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^8/x^28,x]
```

```
[Out] -(a + b*x^3)^9/(27*a*x^27)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^8}{x^{28}}\,dx = -\frac{\left(a+bx^3\right)^9}{27ax^{27}}$$

Mathematica [B] time = 0.01, size = 108, normalized size = 5.68

$$-\frac{a^8}{27x^{27}} - \frac{a^7 b}{3x^{24}} - \frac{4a^6b^2}{3x^{21}} - \frac{28a^5b^3}{9x^{18}} - \frac{14a^4b^4}{3x^{15}} - \frac{14a^3b^5}{3x^{12}} - \frac{28a^2b^6}{9x^9} - \frac{4ab^7}{3x^6} - \frac{b^8}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^8/x^28,x]
```

```
[Out] -1/27*a^8/x^27 - (a^7*b)/(3*x^24) - (4*a^6*b^2)/(3*x^21) - (28*a^5*b^3)/(9*
x^18) - (14*a^4*b^4)/(3*x^15) - (14*a^3*b^5)/(3*x^12) - (28*a^2*b^6)/(9*x^9
) - (4*a*b^7)/(3*x^6) - b^8/(3*x^3)
```

fricas [B] time = 0.76, size = 90, normalized size = 4.74

$$-\frac{9\,b^8x^{24} + 36\,ab^7x^{21} + 84\,a^2b^6x^{18} + 126\,a^3b^5x^{15} + 126\,a^4b^4x^{12} + 84\,a^5b^3x^9 + 36\,a^6b^2x^6 + 9\,a^7bx^3 + a^8}{27\,x^{27}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^28,x, algorithm="fricas")
```

```
[Out] -1/27*(9*b^8*x^24 + 36*a*b^7*x^21 + 84*a^2*b^6*x^18 + 126*a^3*b^5*x^15 + 12
6*a^4*b^4*x^12 + 84*a^5*b^3*x^9 + 36*a^6*b^2*x^6 + 9*a^7*b*x^3 + a^8)/x^27
```

giac [B] time = 0.20, size = 90, normalized size = 4.74

$$-\frac{9\,b^8x^{24}+36\,ab^7x^{21}+84\,a^2b^6x^{18}+126\,a^3b^5x^{15}+126\,a^4b^4x^{12}+84\,a^5b^3x^9+36\,a^6b^2x^6+9\,a^7bx^3+a^8}{27\,x^{27}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^28,x, algorithm="giac")
```

```
[Out] -1/27*(9*b^8*x^24 + 36*a*b^7*x^21 + 84*a^2*b^6*x^18 + 126*a^3*b^5*x^15 + 12
6*a^4*b^4*x^12 + 84*a^5*b^3*x^9 + 36*a^6*b^2*x^6 + 9*a^7*b*x^3 + a^8)/x^27
```

maple [B] time = 0.01, size = 91, normalized size = 4.79

$$-\frac{b^8}{3x^3}-\frac{4a\,b^7}{3x^6}-\frac{28a^2b^6}{9x^9}-\frac{14a^3b^5}{3x^{12}}-\frac{14a^4b^4}{3x^{15}}-\frac{28a^5b^3}{9x^{18}}-\frac{4a^6b^2}{3x^{21}}-\frac{a^7b}{3x^{24}}-\frac{a^8}{27x^{27}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^8/x^28,x)
```

```
[Out] -1/27*a^8/x^27-1/3*a^7*b/x^24-1/3*b^8/x^3-4/3*a^6*b^2/x^21-28/9*a^5*b^3/x^1
8-28/9*a^2*b^6/x^9-4/3*a*b^7/x^6-14/3*a^4*b^4/x^15-14/3*a^3*b^5/x^12
```

maxima [B] time = 0.45, size = 90, normalized size = 4.74

$$-\frac{9\,b^8x^{24}+36\,ab^7x^{21}+84\,a^2b^6x^{18}+126\,a^3b^5x^{15}+126\,a^4b^4x^{12}+84\,a^5b^3x^9+36\,a^6b^2x^6+9\,a^7bx^3+a^8}{27\,x^{27}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^8/x^28,x, algorithm="maxima")
```

```
[Out] -1/27*(9*b^8*x^24 + 36*a*b^7*x^21 + 84*a^2*b^6*x^18 + 126*a^3*b^5*x^15 + 12
6*a^4*b^4*x^12 + 84*a^5*b^3*x^9 + 36*a^6*b^2*x^6 + 9*a^7*b*x^3 + a^8)/x^27
```

mupad [B] time = 1.23, size = 92, normalized size = 4.84

$$-\frac{\frac{a^8}{27}+\frac{a^7\,b\,x^3}{3}+\frac{4\,a^6\,b^2\,x^6}{3}+\frac{28\,a^5\,b^3\,x^9}{9}+\frac{14\,a^4\,b^4\,x^{12}}{3}+\frac{14\,a^3\,b^5\,x^{15}}{3}+\frac{28\,a^2\,b^6\,x^{18}}{9}+\frac{4\,a\,b^7\,x^{21}}{3}+\frac{b^8\,x^{24}}{3}}{x^{27}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^8/x^28,x)
```

```
[Out] -(a^8/27 + (b^8*x^24)/3 + (a^7*b*x^3)/3 + (4*a*b^7*x^21)/3 + (4*a^6*b^2*x^6
)/3 + (28*a^5*b^3*x^9)/9 + (14*a^4*b^4*x^12)/3 + (14*a^3*b^5*x^15)/3 + (28*
a^2*b^6*x^18)/9)/x^27
```

sympy [B] time = 1.65, size = 97, normalized size = 5.11

$$\frac{-a^8-9a^7bx^3-36a^6b^2x^6-84a^5b^3x^9-126a^4b^4x^{12}-126a^3b^5x^{15}-84a^2b^6x^{18}-36ab^7x^{21}-9b^8x^{24}}{27x^{27}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**8/x**28,x)
```

```
[Out] (-a**8 - 9*a**7*b*x**3 - 36*a**6*b**2*x**6 - 84*a**5*b**3*x**9 - 126*a**4*b
**4*x**12 - 126*a**3*b**5*x**15 - 84*a**2*b**6*x**18 - 36*a*b**7*x**21 - 9*
b**8*x**24)/(27*x**27)
```

3.2728 $\int \frac{1}{x(a+bx^n)}\,dx$

Optimal. Leaf size=23

$$\frac{\log(x)}{a} - \frac{\log(a+bx^n)}{an}$$

```
[Out] ln(x)/a-ln(a+b*x^n)/a/n
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {266, 36, 29, 31}

$$\frac{\log(x)}{a} - \frac{\log(a+bx^n)}{an}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^n)),x]
```

```
[Out] Log[x]/a - Log[a + b*x^n]/(a*n)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\,(a+bx^n)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx, x, x^n\right)}{n} \\ &= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^n\right)}{an} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, x^n\right)}{an} \\ &= \frac{\log(x)}{a} - \frac{\log(a+bx^n)}{an}\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 0.96

$$\frac{n\log(x) - \log(a+bx^n)}{an}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^n)),x]
[Out] (n*Log[x] - Log[a + b*x^n])/(a*n)
```

fricas [A] time = 0.76, size = 22, normalized size = 0.96

$$\frac{n\log(x) - \log(bx^n + a)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n),x, algorithm="fricas")
[Out] (n*log(x) - log(b*x^n + a))/(a*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n),x, algorithm="giac")
[Out] integrate(1/((b*x^n + a)*x), x)
```

maple [A] time = 0.00, size = 29, normalized size = 1.26

$$\frac{\ln(x^n)}{an} - \frac{\ln(b\,x^n + a)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^n+a),x)
[Out] 1/a/n*ln(x^n)-1/a/n*ln(b*x^n+a)
```

maxima [A] time = 0.50, size = 28, normalized size = 1.22

$$-\frac{\log(bx^n + a)}{an} + \frac{\log(x^n)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n),x, algorithm="maxima")
[Out] -log(b*x^n + a)/(a*n) + log(x^n)/(a*n)
```

mupad [B] time = 0.00, size = 22, normalized size = 0.96

$$-\frac{\ln(a + b\,x^n) - n\,\ln(x)}{a\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^n)),x)
[Out] -(log(a + b*x^n) - n*log(x))/(a*n)
```

sympy [**A**] time = 1.00, size = 41, normalized size = 1.78

$$\begin{cases} \tilde{\infty}\log(x) & \text{for } a = 0 \wedge b = 0 \wedge n = 0 \\ \frac{\log(x)}{a} & \text{for } b = 0 \\ -\frac{x^{-n}}{bn} & \text{for } a = 0 \\ \frac{\log(x)}{a+b} & \text{for } n = 0 \\ \frac{\log(x)}{a} - \frac{\log\left(\frac{a}{b}+x^n\right)}{an} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x**n),x)
```

```
[Out] Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (log(x)/a, Eq(b, 0)
), (-x**(-n)/(b*n), Eq(a, 0)), (log(x)/(a + b), Eq(n, 0)), (log(x)/a - log(
a/b + x**n)/(a*n), True))
```

3.2729 $\int \frac{1}{x\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=22

$$\frac{\log(x)}{a}-\frac{\log\left(a+bx^3\right)}{3a}$$

```
[Out] ln(x)/a-1/3*ln(b*x^3+a)/a
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {266, 36, 29, 31}

$$\frac{\log(x)}{a}-\frac{\log\left(a+bx^3\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3)),x]
```

```
[Out] Log[x]/a - Log[a + b*x^3]/(3*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a+bx^3\right)}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx, x, x^3\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^3\right)}{3a}-\frac{b\,\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, x^3\right)}{3a}\\ &= \frac{\log(x)}{a}-\frac{\log\left(a+bx^3\right)}{3a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{\log(x)}{a}-\frac{\log\left(a+bx^3\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3)),x]
[Out] Log[x]/a - Log[a + b*x^3]/(3*a)
```

fricas [A] time = 0.54, size = 18, normalized size = 0.82

$$-\frac{\log\left(bx^3+a\right)-3\,\log(x)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a),x, algorithm="fricas")
[Out] -1/3*(log(b*x^3 + a) - 3*log(x))/a
```

giac [A] time = 0.16, size = 22, normalized size = 1.00

$$-\frac{\log\left(\left|bx^3+a\right|\right)}{3\,a}+\frac{\log\left(|x|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a),x, algorithm="giac")
[Out] -1/3*log(abs(b*x^3 + a))/a + log(abs(x))/a
```

maple [A] time = 0.00, size = 21, normalized size = 0.95

$$\frac{\ln(x)}{a}-\frac{\ln\left(b\,x^3+a\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a),x)
[Out] 1/a*ln(x)-1/3/a*ln(b*x^3+a)
```

maxima [A] time = 0.49, size = 23, normalized size = 1.05

$$-\frac{\log\left(bx^3+a\right)}{3\,a}+\frac{\log\left(x^3\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a),x, algorithm="maxima")
[Out] -1/3*log(b*x^3 + a)/a + 1/3*log(x^3)/a
```

mupad [B] time = 1.18, size = 18, normalized size = 0.82

$$-\frac{\ln\left(b\,x^3+a\right)-3\,\ln(x)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3)),x)
[Out] -(log(a + b*x^3) - 3*log(x))/(3*a)
```

sympy [A] time = 0.32, size = 15, normalized size = 0.68

$$\frac{\log(x)}{a}-\frac{\log\left(\frac{a}{b}+x^3\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a),x)
[Out] log(x)/a - log(a/b + x**3)/(3*a)
```

3.2730 $\int (a+bx^n)^{-\frac{1+4n}{n}}\,dx$

Optimal. Leaf size=147

$$\frac{6n^3x\,(a+bx^n)^{-1/n}}{a^4(n+1)(2n+1)(3n+1)}+\frac{6n^2x\,(a+bx^n)^{-\frac{n+1}{n}}}{a^3(n+1)(2n+1)(3n+1)}+\frac{3nx\,(a+bx^n)^{-\frac{1}{n}-2}}{a^2\left(6n^2+5n+1\right)}+\frac{x\,(a+bx^n)^{-\frac{1}{n}-3}}{a(3n+1)}$$

```
[Out] x*(a+b*x^n)^(-3-1/n)/a/(1+3*n)+3*n*x*(a+b*x^n)^(-2-1/n)/a^2/(6*n^2+5*n+1)+6
*n^3*x/a^4/(6*n^3+11*n^2+6*n+1)/((a+b*x^n)^(1/n))+6*n^2*x/a^3/(6*n^3+11*n^2
+6*n+1)/((a+b*x^n)^((1+n)/n))
```

Rubi [A] time = 0.10, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {192, 191}

$$\frac{3nx\,(a+bx^n)^{-\frac{1}{n}-2}}{a^2\left(6n^2+5n+1\right)}+\frac{6n^3x\,(a+bx^n)^{-1/n}}{a^4(n+1)(2n+1)(3n+1)}+\frac{6n^2x\,(a+bx^n)^{-\frac{n+1}{n}}}{a^3(n+1)(2n+1)(3n+1)}+\frac{x\,(a+bx^n)^{-\frac{1}{n}-3}}{a(3n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^(-((1 + 4*n)/n)),x]
```

```
[Out] (x*(a + b*x^n)^(-3 - n^(-1)))/(a*(1 + 3*n)) + (3*n*x*(a + b*x^n)^(-2 - n^(-
1)))/(a^2*(1 + 5*n + 6*n^2)) + (6*n^3*x)/(a^4*(1 + n)*(1 + 2*n)*(1 + 3*n)*(
a + b*x^n)^n^(-1)) + (6*n^2*x)/(a^3*(1 + n)*(1 + 2*n)*(1 + 3*n)*(a + b*x^n)
^((1 + n)/n))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)^{-\frac{1+4n}{n}}\,dx &= \frac{x\,(a+bx^n)^{-3-\frac{1}{n}}}{a(1+3n)}+\frac{(3n)\int (a+bx^n)^{1-\frac{1+4n}{n}}\,dx}{a(1+3n)}\\ &= \frac{x\,(a+bx^n)^{-3-\frac{1}{n}}}{a(1+3n)}+\frac{3nx\,(a+bx^n)^{-2-\frac{1}{n}}}{a^2\left(1+5n+6n^2\right)}+\frac{\left(6n^2\right)\int (a+bx^n)^{2-\frac{1+4n}{n}}\,dx}{a^2\left(1+5n+6n^2\right)}\\ &= \frac{x\,(a+bx^n)^{-3-\frac{1}{n}}}{a(1+3n)}+\frac{3nx\,(a+bx^n)^{-2-\frac{1}{n}}}{a^2\left(1+5n+6n^2\right)}+\frac{6n^2x\,(a+bx^n)^{-\frac{1+n}{n}}}{a^3(1+n)\left(1+5n+6n^2\right)}+\frac{\left(6n^3\right)\int (a+bx^n)^{3}}{a^3(1+n)\left(1+5n\right.}\\ &= \frac{x\,(a+bx^n)^{-3-\frac{1}{n}}}{a(1+3n)}+\frac{3nx\,(a+bx^n)^{-2-\frac{1}{n}}}{a^2\left(1+5n+6n^2\right)}+\frac{6n^3x\,(a+bx^n)^{-1/n}}{a^4(1+n)\left(1+5n+6n^2\right)}+\frac{6n^2x\,(a+bx^n)^{-}}{a^3(1+n)\left(1+5n\right.}\end{aligned}$$

Mathematica [C] time = 0.04, size = 55, normalized size = 0.37

$$\frac{x\,(a+bx^n)^{-1/n}\left(\frac{bx^n}{a}+1\right)^{\frac{1}{n}}\,{}_2F_1\left(4+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^(-((1 + 4*n)/n)),x]
```

```
[Out] (x*(1 + (b*x^n)/a)^n^(-1)*Hypergeometric2F1[4 + n^(-1), n^(-1), 1 + n^(-1),
 -((b*x^n)/a)])/(a^4*(a + b*x^n)^n^(-1))
```

fricas [A] time = 0.68, size = 192, normalized size = 1.31

$$\frac{6\,b^4n^3xx^{4\,n}+6\left(4\,ab^3n^3+ab^3n^2\right)xx^{3\,n}+3\left(12\,a^2b^2n^3+7\,a^2b^2n^2+a^2b^2n\right)xx^{2\,n}+\left(24\,a^3bn^3+26\,a^3bn^2+9\,a^3bn\right.}{\left(6\,a^4n^3+11\,a^4n^2+6\,a^4n+a^4\right)(bx^n+a)^{\frac{4\,n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a+b*x^n)^((1+4*n)/n)),x, algorithm="fricas")
```

```
[Out] (6*b^4*n^3*x*x^(4*n) + 6*(4*a*b^3*n^3 + a*b^3*n^2)*x*x^(3*n) + 3*(12*a^2*b^
2*n^3 + 7*a^2*b^2*n^2 + a^2*b^2*n)*x*x^(2*n) + (24*a^3*b*n^3 + 26*a^3*b*n^2
 + 9*a^3*b*n + a^3*b)*x*x^n + (6*a^4*n^3 + 11*a^4*n^2 + 6*a^4*n + a^4)*x)/(
(6*a^4*n^3 + 11*a^4*n^2 + 6*a^4*n + a^4)*(b*x^n + a)^((4*n + 1)/n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n+a)^{\frac{4\,n+1}{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a+b*x^n)^((1+4*n)/n)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^((4*n + 1)/n)), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int (b\,x^n+a)^{-\frac{4n+1}{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^n+a)^((1+4*n)/n)),x)
```

```
[Out] int(1/((b*x^n+a)^((1+4*n)/n)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n+a)^{\frac{4\,n+1}{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a+b*x^n)^((1+4*n)/n)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a)^((4*n + 1)/n)), x)
```

mupad [B] time = 1.53, size = 64, normalized size = 0.44

$$-\frac{x^{1-4\,n}\left(\frac{a}{b\,x^n}+1\right)^{1/n}\,{}_2\mathrm{F}_1\left(4,\frac{1}{n}+4;\;5;\;-\frac{a}{b\,x^n}\right)}{4\,b^4\,n\,(a+b\,x^n)^{1/n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^n)^((4*n + 1)/n),x)
```

```
[Out] -(x^(1 - 4*n)*(a/(b*x^n) + 1)^(1/n)*hypergeom([4, 1/n + 4], 5, -a/(b*x^n)))
/(4*b^4*n*(a + b*x^n)^(1/n))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a+b*x**n)**((1+4*n)/n)),x)
```

```
[Out] Timed out
```

3.2731 $\int (a+bx^n)^{-\frac{1+3n}{n}}\,dx$

Optimal. Leaf size=97

$$\frac{2n^2x\left(a+bx^n\right)^{-1/n}}{a^3(n+1)(2n+1)}+\frac{2nx\left(a+bx^n\right)^{-\frac{n+1}{n}}}{a^2(n+1)(2n+1)}+\frac{x\left(a+bx^n\right)^{-\frac{1}{n}-2}}{a(2n+1)}$$

```
[Out] x*(a+b*x^n)^(-2-1/n)/a/(1+2*n)+2*n^2*x/a^3/(2*n^2+3*n+1)/((a+b*x^n)^(1/n))+
2*n*x/a^2/(2*n^2+3*n+1)/((a+b*x^n)^((1+n)/n))
```

Rubi [A] time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {192, 191}

$$\frac{2n^2x\left(a+bx^n\right)^{-1/n}}{a^3(n+1)(2n+1)}+\frac{2nx\left(a+bx^n\right)^{-\frac{n+1}{n}}}{a^2(n+1)(2n+1)}+\frac{x\left(a+bx^n\right)^{-\frac{1}{n}-2}}{a(2n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^(-((1 + 3*n)/n)),x]
```

```
[Out] (x*(a + b*x^n)^(-2 - n^(-1)))/(a*(1 + 2*n)) + (2*n^2*x)/(a^3*(1 + n)*(1 + 2
*n)*(a + b*x^n)^n^(-1)) + (2*n*x)/(a^2*(1 + n)*(1 + 2*n)*(a + b*x^n)^((1 +
n)/n))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)^{-\frac{1+3n}{n}}\,dx &= \frac{x\left(a+bx^n\right)^{-2-\frac{1}{n}}}{a(1+2n)}+\frac{(2n)\int \left(a+bx^n\right)^{1-\frac{1+3n}{n}}\,dx}{a(1+2n)}\\ &= \frac{x\left(a+bx^n\right)^{-2-\frac{1}{n}}}{a(1+2n)}+\frac{2nx\left(a+bx^n\right)^{-\frac{1+n}{n}}}{a^2(1+n)(1+2n)}+\frac{\left(2n^2\right)\int \left(a+bx^n\right)^{2-\frac{1+3n}{n}}\,dx}{a^2(1+n)(1+2n)}\\ &= \frac{x\left(a+bx^n\right)^{-2-\frac{1}{n}}}{a(1+2n)}+\frac{2n^2x\left(a+bx^n\right)^{-1/n}}{a^3(1+n)(1+2n)}+\frac{2nx\left(a+bx^n\right)^{-\frac{1+n}{n}}}{a^2(1+n)(1+2n)}\end{aligned}$$

Mathematica [C] time = 0.04, size = 55, normalized size = 0.57

$$\frac{x\left(a+bx^n\right)^{-1/n}\left(\frac{bx^n}{a}+1\right)^{\frac{1}{n}}\,{}_2F_1\left(3+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^(-((1 + 3*n)/n)),x]
```

```
[Out] (x*(1 + (b*x^n)/a)^n^(-1)*Hypergeometric2F1[3 + n^(-1), n^(-1), 1 + n^(-1),
 -((b*x^n)/a)])/(a^3*(a + b*x^n)^n^(-1))
```

fricas [A] time = 0.67, size = 126, normalized size = 1.30

$$\frac{2\,b^3n^2xx^{3\,n}+2\left(3\,ab^2n^2+ab^2n\right)xx^{2\,n}+\left(6\,a^2bn^2+5\,a^2bn+a^2b\right)xx^n+\left(2\,a^3n^2+3\,a^3n+a^3\right)x}{\left(2\,a^3n^2+3\,a^3n+a^3\right)(bx^n+a)^{\frac{3\,n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a+b*x^n)^((1+3*n)/n)),x, algorithm="fricas")
```

```
[Out] (2*b^3*n^2*x*x^(3*n) + 2*(3*a*b^2*n^2 + a*b^2*n)*x*x^(2*n) + (6*a^2*b*n^2 +
 5*a^2*b*n + a^2*b)*x*x^n + (2*a^3*n^2 + 3*a^3*n + a^3)*x)/((2*a^3*n^2 + 3*
a^3*n + a^3)*(b*x^n + a)^((3*n + 1)/n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n+a)^{\frac{3\,n+1}{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a+b*x^n)^((1+3*n)/n)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^((3*n + 1)/n)), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int (b\,x^n+a)^{-\frac{3n+1}{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^n+a)^((3*n+1)/n)),x)
```

```
[Out] int(1/((b*x^n+a)^((3*n+1)/n)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n+a)^{\frac{3\,n+1}{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a+b*x^n)^((1+3*n)/n)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a)^((3*n + 1)/n)), x)
```

mupad [B] time = 1.34, size = 64, normalized size = 0.66

$$-\frac{x^{1-3\,n}\left(\frac{a}{b\,x^n}+1\right)^{1/n}\,{}_2\mathrm{F}_1\left(3,\frac{1}{n}+3;\;4;\;-\frac{a}{b\,x^n}\right)}{3\,b^3\,n\,(a+b\,x^n)^{1/n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^n)^((3*n + 1)/n),x)
```

```
[Out] -(x^(1 - 3*n)*(a/(b*x^n) + 1)^(1/n)*hypergeom([3, 1/n + 3], 4, -a/(b*x^n)))
/(3*b^3*n*(a + b*x^n)^(1/n))
```

sympy [**F**(**-1**)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a+b*x**n)**((1+3*n)/n)),x)
[Out] Timed out
```

3.2732 $\int (a+bx^n)^{-\frac{1+2n}{n}}\,dx$

Optimal. Leaf size=51

$$\frac{nx\,(a+bx^n)^{-1/n}}{a^2(n+1)}+\frac{x\,(a+bx^n)^{-\frac{n+1}{n}}}{a(n+1)}$$

```
[Out] n*x/a^2/(1+n)/((a+b*x^n)^(1/n))+x/a/(1+n)/((a+b*x^n)^((1+n)/n))
```

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {192, 191}

$$\frac{nx\,(a+bx^n)^{-1/n}}{a^2(n+1)}+\frac{x\,(a+bx^n)^{-\frac{n+1}{n}}}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^(-((1 + 2*n)/n)),x]
```

```
[Out] (n*x)/(a^2*(1 + n)*(a + b*x^n)^n^(-1)) + x/(a*(1 + n)*(a + b*x^n)^((1 + n)/
n))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)^{-\frac{1+2n}{n}}\,dx &= \frac{x\,(a+bx^n)^{-\frac{1+n}{n}}}{a(1+n)}+\frac{n\int (a+bx^n)^{1-\frac{1+2n}{n}}\,dx}{a(1+n)}\\ &= \frac{nx\,(a+bx^n)^{-1/n}}{a^2(1+n)}+\frac{x\,(a+bx^n)^{-\frac{1+n}{n}}}{a(1+n)}\end{aligned}$$

Mathematica [C] time = 0.05, size = 55, normalized size = 1.08

$$\frac{x\,(a+bx^n)^{-1/n}\left(\frac{bx^n}{a}+1\right)^{\frac{1}{n}}\,{}_2F_1\left(2+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^(-((1 + 2*n)/n)),x]
```

```
[Out] (x*(1 + (b*x^n)/a)^n^(-1)*Hypergeometric2F1[2 + n^(-1), n^(-1), 1 + n^(-1),
 -((b*x^n)/a)])/(a^2*(a + b*x^n)^n^(-1))
```

fricas [A] time = 0.68, size = 68, normalized size = 1.33

$$\frac{b^2 n x x^{2\,n} + (2\,abn + ab) x x^n + \left(a^2 n + a^2\right) x}{\left(a^2 n + a^2\right)(bx^n + a)^{\frac{2\,n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a+b*x^n)^((1+2*n)/n)),x, algorithm="fricas")
```

```
[Out] (b^2*n*x*x^(2*n) + (2*a*b*n + a*b)*x*x^n + (a^2*n + a^2)*x)/((a^2*n + a^2)*
(b*x^n + a)^((2*n + 1)/n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)^{\frac{2n+1}{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a+b*x^n)^((1+2*n)/n)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^((2*n + 1)/n)), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int (b\,x^n + a)^{-\frac{2n+1}{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^n+a)^((2*n+1)/n)),x)
```

```
[Out] int(1/((b*x^n+a)^((2*n+1)/n)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)^{\frac{2n+1}{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a+b*x^n)^((1+2*n)/n)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a)^((2*n + 1)/n)), x)
```

mupad [B] time = 1.32, size = 64, normalized size = 1.25

$$-\frac{x^{1-2\,n}\left(\frac{a}{b\,x^n}+1\right)^{1/n}\,{}_2\mathrm{F}_1\left(2,\frac{1}{n}+2;\;3;\;-\frac{a}{b\,x^n}\right)}{2\,b^2\,n\,(a+b\,x^n)^{1/n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^n)^((2*n + 1)/n),x)
```

```
[Out] -(x^(1 - 2*n)*(a/(b*x^n) + 1)^(1/n)*hypergeom([2, 1/n + 2], 3, -a/(b*x^n)))
/(2*b^2*n*(a + b*x^n)^(1/n))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a+b*x**n)**((1+2*n)/n)),x)
```

```
[Out] Timed out
```

3.2733 $\int (a+bx^n)^{-\frac{1+n}{n}}\,dx$

Optimal. Leaf size=18

$$\frac{x\,(a+bx^n)^{-1/n}}{a}$$

```
[Out] x/a/((a+b*x^n)^(1/n))
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {191}

$$\frac{x\,(a+bx^n)^{-1/n}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^(-((1 + n)/n)),x]
```

```
[Out] x/(a*(a + b*x^n)^n^(-1))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int (a+bx^n)^{-\frac{1+n}{n}}\,dx = \frac{x\,(a+bx^n)^{-1/n}}{a}$$

Mathematica [A] time = 0.04, size = 18, normalized size = 1.00

$$\frac{x\,(a+bx^n)^{-1/n}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^(-((1 + n)/n)),x]
```

```
[Out] x/(a*(a + b*x^n)^n^(-1))
```

fricas [A] time = 0.69, size = 31, normalized size = 1.72

$$\frac{bxx^n + ax}{(bx^n+a)^{\frac{n+1}{n}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a+b*x^n)^((1+n)/n)),x, algorithm="fricas")
```

```
[Out] (b*x*x^n + a*x)/((b*x^n + a)^((n + 1)/n)*a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n+a)^{\frac{n+1}{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a+b*x^n)^((1+n)/n)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^((n + 1)/n)), x)
```

maple [A] time = 0.03, size = 35, normalized size = 1.94

$$\left(\frac{bx\,\mathrm{e}^{n\ln(x)}}{a}+x\right)\mathrm{e}^{\frac{(-n-1)\ln\left(b\,\mathrm{e}^{n\ln(x)}+a\right)}{n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^n+a)^((n+1)/n)),x)
```

```
[Out] (x+1/a*b*x*exp(n*ln(x)))/exp((n+1)/n*ln(b*exp(n*ln(x))+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n+a)^{\frac{n+1}{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a+b*x^n)^((1+n)/n)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a)^((n + 1)/n)), x)
```

mupad [B] time = 1.30, size = 75, normalized size = 4.17

$$\frac{b\,x^{n+1}\left(\frac{a}{b\,x^n}-\left(\frac{a}{b\,x^n}+1\right)^{\frac{n+1}{n}}+1\right)}{a\,n\left(\frac{n+1}{n}-1\right)(a+b\,x^n)^{\frac{n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^n)^((n + 1)/n),x)
```

```
[Out] (b*x^(n + 1)*(a/(b*x^n) - (a/(b*x^n) + 1)^((n + 1)/n) + 1))/(a*n*((n + 1)/n
 - 1)*(a + b*x^n)^((n + 1)/n))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a+b*x**n)**((1+n)/n)),x)
```

```
[Out] Timed out
```

3.2734 $\int (a+bx^n)^{-1/n}\,dx$

Optimal. Leaf size=50

$$x\,(a+bx^n)^{-1/n}\left(\frac{bx^n}{a}+1\right)^{\frac{1}{n}}\,{}_2F_1\left(\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)$$

```
[Out] x*(1+b*x^n/a)^(1/n)*hypergeom([1/n, 1/n],[1+1/n],-b*x^n/a)/((a+b*x^n)^(1/n)
)
```

Rubi [A] time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {246, 245}

$$x\,(a+bx^n)^{-1/n}\left(\frac{bx^n}{a}+1\right)^{\frac{1}{n}}\,{}_2F_1\left(\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^(-n^(-1)),x]
```

```
[Out] (x*(1 + (b*x^n)/a)^n^(-1)*Hypergeometric2F1[n^(-1), n^(-1), 1 + n^(-1), -((
b*x^n)/a)])/(a + b*x^n)^n^(-1)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)^{-1/n}\,dx &= \left((a+bx^n)^{-1/n}\left(1+\frac{bx^n}{a}\right)^{\frac{1}{n}}\right)\int\left(1+\frac{bx^n}{a}\right)^{-1/n}\,dx\\ &= x\,(a+bx^n)^{-1/n}\left(1+\frac{bx^n}{a}\right)^{\frac{1}{n}}\,{}_2F_1\left(\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 50, normalized size = 1.00

$$x\,(a+bx^n)^{-1/n}\left(\frac{bx^n}{a}+1\right)^{\frac{1}{n}}\,{}_2F_1\left(\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^(-n^(-1)),x]
```

```
[Out] (x*(1 + (b*x^n)/a)^n^(-1)*Hypergeometric2F1[n^(-1), n^(-1), 1 + n^(-1), -((
b*x^n)/a)])/(a + b*x^n)^n^(-1)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{(bx^n+a)^{\left(\frac{1}{n}\right)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a+b*x^n)^(1/n)),x, algorithm="fricas")
```

```
[Out] integral(1/((b*x^n + a)^(1/n)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n+a)^{\left(\frac{1}{n}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a+b*x^n)^(1/n)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^(1/n)), x)
```

maple [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,x^n+a)^{-\frac{1}{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^n+a)^(1/n)),x)
```

```
[Out] int(1/((b*x^n+a)^(1/n)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n+a)^{\left(\frac{1}{n}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a+b*x^n)^(1/n)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a)^(1/n)), x)
```

mupad [B] time = 0.00, size = 51, normalized size = 1.02

$$\frac{x\left(\frac{b\,x^n}{a}+1\right)^{1/n}\,{}_2\mathrm{F}_1\left(\frac{1}{n},\frac{1}{n};\ \frac{1}{n}+1;\ -\frac{b\,x^n}{a}\right)}{(a+b\,x^n)^{1/n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^n)^(1/n),x)
```

```
[Out] (x*((b*x^n)/a + 1)^(1/n)*hypergeom([1/n, 1/n], 1/n + 1, -(b*x^n)/a))/(a + b
*x^n)^(1/n)
```

sympy [C] time = 11.54, size = 39, normalized size = 0.78

$$\frac{a^{-\frac{1}{n}} x \Gamma\left(\frac{1}{n}\right) {}_2F_1\left(\begin{matrix}\frac{1}{n}, \frac{1}{n} \\ 1 + \frac{1}{n}\end{matrix}\middle| \frac{b x^n e^{i\pi}}{a}\right)}{n \Gamma\left(1 + \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a+b*x**n)**(1/n)),x)
```

```
[Out] a**(-1/n)*x*gamma(1/n)*hyper((1/n, 1/n), (1 + 1/n,), b*x**n*exp_polar(I*pi)
/a)/(n*gamma(1 + 1/n))
```

3.2735 $\int (a+bx^n)^{-\frac{1-n}{n}}\,dx$

Optimal. Leaf size=59

$$x\,(a+bx^n)^{-\frac{1-n}{n}}\left(\frac{bx^n}{a}+1\right)^{\frac{1}{n}-1}\,{}_2F_1\left(\frac{1}{n}-1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)$$

```
[Out] x*(1+b*x^n/a)^(-1+1/n)*hypergeom([1/n, -1+1/n],[1+1/n],-b*x^n/a)/((a+b*x^n)
^((1-n)/n))
```

Rubi [A] time = 0.01, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {246, 245}

$$x\,(a+bx^n)^{-\frac{1-n}{n}}\left(\frac{bx^n}{a}+1\right)^{\frac{1}{n}-1}\,{}_2F_1\left(\frac{1}{n}-1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^(-((1 - n)/n)),x]
```

```
[Out] (x*(1 + (b*x^n)/a)^(-1 + n^(-1))*Hypergeometric2F1[-1 + n^(-1), n^(-1), 1 +
 n^(-1), -((b*x^n)/a)])/(a + b*x^n)^((1 - n)/n)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)^{-\frac{1-n}{n}}\,dx &= \left((a+bx^n)^{-\frac{1-n}{n}}\left(1+\frac{bx^n}{a}\right)^{\frac{1-n}{n}}\right)\int\left(1+\frac{bx^n}{a}\right)^{-\frac{1-n}{n}}\,dx\\ &= x\,(a+bx^n)^{-\frac{1-n}{n}}\left(1+\frac{bx^n}{a}\right)^{-1+\frac{1}{n}}\,{}_2F_1\left(-1+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 53, normalized size = 0.90

$$ax\,(a+bx^n)^{-1/n}\left(\frac{bx^n}{a}+1\right)^{\frac{1}{n}}\,{}_2F_1\left(\frac{1}{n}-1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^(-((1 - n)/n)),x]
```

```
[Out] (a*x*(1 + (b*x^n)/a)^n^(-1)*Hypergeometric2F1[-1 + n^(-1), n^(-1), 1 + n^(-
1), -((b*x^n)/a)])/(a + b*x^n)^n^(-1)
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left((bx^n + a)^{\frac{n-1}{n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a+b*x^n)^((1-n)/n)),x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^((n - 1)/n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^{\frac{n-1}{n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a+b*x^n)^((1-n)/n)),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^((n - 1)/n), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int (b\,x^n + a)^{-\frac{-n+1}{n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^n+a)^((1-n)/n)),x)
```

```
[Out] int(1/((b*x^n+a)^((1-n)/n)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^{\frac{n-1}{n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a+b*x^n)^((1-n)/n)),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^((n - 1)/n), x)
```

mupad [B] time = 1.51, size = 54, normalized size = 0.92

$$\frac{a\,x\left(\frac{b\,x^n}{a}+1\right)^{1/n}\,{}_2\mathrm{F}_1\left(\frac{1}{n}-1,\frac{1}{n};\ \frac{1}{n}+1;\ -\frac{b\,x^n}{a}\right)}{\left(a+b\,x^n\right)^{1/n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^((n - 1)/n),x)
```

```
[Out] (a*x*((b*x^n)/a + 1)^(1/n)*hypergeom([1/n - 1, 1/n], 1/n + 1, -(b*x^n)/a))/
(a + b*x^n)^(1/n)
```

sympy [C] time = 12.22, size = 42, normalized size = 0.71

$$\frac{a a^{-\frac{1}{n}} x \Gamma\left(\frac{1}{n}\right) {}_2F_1\left(\begin{matrix}\frac{1}{n}, -1+\frac{1}{n} \\ 1+\frac{1}{n}\end{matrix}\middle| \frac{b x^n e^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a+b*x**n)**((1-n)/n)),x)
```

```
[Out] a*a**(-1/n)*x*gamma(1/n)*hyper((1/n, -1 + 1/n), (1 + 1/n,), b*x**n*exp_pola
r(I*pi)/a)/(n*gamma(1 + 1/n))
```

3.2736 $\int (a+bx^n)^{-\frac{1-2n}{n}}\,dx$

Optimal. Leaf size=56

$$x\,(a+bx^n)^{2-\frac{1}{n}}\left(\frac{bx^n}{a}+1\right)^{\frac{1}{n}-2}\,{}_2F_1\left(\frac{1}{n}-2,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)$$

```
[Out] x*(a+b*x^n)^(2-1/n)*(1+b*x^n/a)^(-2+1/n)*hypergeom([1/n, -2+1/n],[1+1/n],-b
*x^n/a)
```

Rubi [A] time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {246, 245}

$$x\,(a+bx^n)^{2-\frac{1}{n}}\left(\frac{bx^n}{a}+1\right)^{\frac{1}{n}-2}\,{}_2F_1\left(\frac{1}{n}-2,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^(-((1 - 2*n)/n)),x]
```

```
[Out] x*(a + b*x^n)^(2 - n^(-1))*(1 + (b*x^n)/a)^(-2 + n^(-1))*Hypergeometric2F1[
-2 + n^(-1), n^(-1), 1 + n^(-1), -((b*x^n)/a)]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)^{-\frac{1-2n}{n}}\,dx &= \left((a+bx^n)^{-\frac{1-2n}{n}}\left(1+\frac{bx^n}{a}\right)^{\frac{1-2n}{n}}\right)\int\left(1+\frac{bx^n}{a}\right)^{-\frac{1-2n}{n}}\,dx\\ &= x\,(a+bx^n)^{2-\frac{1}{n}}\left(1+\frac{bx^n}{a}\right)^{-2+\frac{1}{n}}\,{}_2F_1\left(-2+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 55, normalized size = 0.98

$$a^2x\,(a+bx^n)^{-1/n}\left(\frac{bx^n}{a}+1\right)^{\frac{1}{n}}\,{}_2F_1\left(\frac{1}{n}-2,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^(-((1 - 2*n)/n)),x]
```

```
[Out] (a^2*x*(1 + (b*x^n)/a)^n^(-1)*Hypergeometric2F1[-2 + n^(-1), n^(-1), 1 + n^
(-1), -((b*x^n)/a)])/(a + b*x^n)^n^(-1)
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left((bx^n + a)^{\frac{2n-1}{n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a+b*x^n)^((1-2*n)/n)),x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^((2*n - 1)/n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^{\frac{2n-1}{n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a+b*x^n)^((1-2*n)/n)),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^((2*n - 1)/n), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int (b\,x^n + a)^{-\frac{-2n+1}{n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^n+a)^((1-2*n)/n)),x)
```

```
[Out] int(1/((b*x^n+a)^((1-2*n)/n)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^{\frac{2n-1}{n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a+b*x^n)^((1-2*n)/n)),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^((2*n - 1)/n), x)
```

mupad [B] time = 1.32, size = 56, normalized size = 1.00

$$\frac{a^2\, x \left(\frac{b\,x^n}{a} + 1\right)^{1/n} {}_2\mathrm{F}_1\left(\frac{1}{n} - 2, \frac{1}{n};\ \frac{1}{n} + 1;\ -\frac{b\,x^n}{a}\right)}{(a + b\,x^n)^{1/n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^((2*n - 1)/n),x)
```

```
[Out] (a^2*x*((b*x^n)/a + 1)^(1/n)*hypergeom([1/n - 2, 1/n], 1/n + 1, -(b*x^n)/a)
)/(a + b*x^n)^(1/n)
```

sympy [C] time = 109.60, size = 44, normalized size = 0.79

$$\frac{a^2 a^{-\frac{1}{n}} x \Gamma\left(\frac{1}{n}\right) {}_2F_1\left(\begin{matrix}\frac{1}{n}, -2 + \frac{1}{n} \\ 1 + \frac{1}{n}\end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(1 + \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a+b*x**n)**((1-2*n)/n)),x)
```

```
[Out] a**2*a**(-1/n)*x*gamma(1/n)*hyper((1/n, -2 + 1/n), (1 + 1/n,), b*x**n*exp_p
olar(I*pi)/a)/(n*gamma(1 + 1/n))
```

3.2737 $\int \frac{1}{x(a+bx^{-n})}\,dx$

Optimal. Leaf size=15

$$\frac{\log(ax^n+b)}{an}$$

```
[Out] ln(b+a*x^n)/a/n
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {263, 260}

$$\frac{\log(ax^n+b)}{an}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b/x^n)),x]
```

```
[Out] Log[b + a*x^n]/(a*n)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\,(a+bx^{-n})}\,dx &= \int \frac{x^{-1+n}}{b+ax^n}\,dx \\ &= \frac{\log(b+ax^n)}{an}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log(ax^n+b)}{an}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b/x^n)),x]
```

```
[Out] Log[b + a*x^n]/(a*n)
```

fricas [A] time = 0.85, size = 15, normalized size = 1.00

$$\frac{\log(ax^n+b)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/(x^n)),x, algorithm="fricas")
```

```
[Out] log(a*x^n + b)/(a*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x^n}\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/(x^n)),x, algorithm="giac")
[Out] integrate(1/((a + b/x^n)*x), x)
```

maple [A] time = 0.00, size = 16, normalized size = 1.07

$$\frac{\ln\left(a\,x^n+b\right)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b/(x^n)),x)
[Out] ln(b+a*x^n)/a/n
```

maxima [B] time = 0.46, size = 32, normalized size = 2.13

$$\frac{\log\left(a+\frac{b}{x^n}\right)}{an}-\frac{\log\left(\frac{1}{x^n}\right)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/(x^n)),x, algorithm="maxima")
[Out] log(a + b/x^n)/(a*n) - log(1/(x^n))/(a*n)
```

mupad [B] time = 1.18, size = 22, normalized size = 1.47

$$\frac{\ln\left(a+\frac{b}{x^n}\right)+n\,\ln(x)}{a\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b/x^n)),x)
[Out] (log(a + b/x^n) + n*log(x))/(a*n)
```

sympy [A] time = 1.30, size = 39, normalized size = 2.60

$$\begin{cases} \tilde{\infty}\log(x) & \text{for } a=0\wedge b=0\wedge n=0 \\ \frac{\log(x)}{a} & \text{for } b=0 \\ \frac{x^n}{bn} & \text{for } a=0 \\ \frac{\log(x)}{a+b} & \text{for } n=0 \\ \frac{\log(x)}{a}+\frac{\log\left(\frac{a}{b}+x^{-n}\right)}{an} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/(x**n)),x)
[Out] Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (log(x)/a, Eq(b, 0)
), (x**n/(b*n), Eq(a, 0)), (log(x)/(a + b), Eq(n, 0)), (log(x)/a + log(a/b
+ x**(-n))/(a*n), True))
```

3.2738 $\int \frac{x^m}{a+bx^{1+m}}\,dx$

Optimal. Leaf size=19

$$\frac{\log\left(a+bx^{m+1}\right)}{b(m+1)}$$

```
[Out] ln(a+b*x^(1+m))/b/(1+m)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {260}

$$\frac{\log\left(a+bx^{m+1}\right)}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x^(1 + m)),x]
```

```
[Out] Log[a + b*x^(1 + m)]/(b*(1 + m))
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^m}{a+bx^{1+m}}\,dx = \frac{\log\left(a+bx^{1+m}\right)}{b(1+m)}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{\log\left(a+bx^{m+1}\right)}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x^(1 + m)),x]
```

```
[Out] Log[a + b*x^(1 + m)]/(b*(1 + m))
```

fricas [A] time = 0.67, size = 18, normalized size = 0.95

$$\frac{\log\left(bx^{m+1}+a\right)}{bm+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^(1+m)),x, algorithm="fricas")
```

```
[Out] log(b*x^(m + 1) + a)/(b*m + b)
```

giac [A] time = 0.15, size = 19, normalized size = 1.00

$$\frac{\log\left(\left|bx^{m+1}+a\right|\right)}{bm+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^(1+m)),x, algorithm="giac")
[Out] log(abs(b*x^(m + 1) + a))/(b*m + b)
```

maple [A] time = 0.02, size = 21, normalized size = 1.11

$$\frac{\ln\left(bx\,\mathrm{e}^{m\ln(x)}+a\right)}{(m+1)\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a+b*x^(m+1)),x)
[Out] 1/b/(m+1)*ln(a+b*x*exp(m*ln(x)))
```

maxima [A] time = 0.57, size = 19, normalized size = 1.00

$$\frac{\log\left(bx^{m+1}+a\right)}{b(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^(1+m)),x, algorithm="maxima")
[Out] log(b*x^(m + 1) + a)/(b*(m + 1))
```

mupad [B] time = 1.25, size = 19, normalized size = 1.00

$$\frac{\ln\left(a+b\,x^{m+1}\right)}{b\,\left(m+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^(m + 1)),x)
[Out] log(a + b*x^(m + 1))/(b*(m + 1))
```

sympy [A] time = 1.46, size = 37, normalized size = 1.95

$$\begin{cases} \frac{\log(x)}{a} & \text{for } b = 0 \wedge m = -1 \\ \frac{xx^{m}}{a(m+1)} & \text{for } b = 0 \\ \frac{\log(x)}{a+b} & \text{for } m = -1 \\ \frac{\log\left(\frac{a}{b}+xx^{m}\right)}{bm+b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a+b*x**(1+m)),x)
[Out] Piecewise((log(x)/a, Eq(b, 0) & Eq(m, -1)), (x*x**m/(a*(m + 1)), Eq(b, 0)),
 (log(x)/(a + b), Eq(m, -1)), (log(a/b + x*x**m)/(b*m + b), True))
```

3.2739 $\int x^m \left(a + bx^{1+m}\right)^n \, dx$

Optimal. Leaf size=27

$$\frac{\left(a + bx^{m+1}\right)^{n+1}}{b(m+1)(n+1)}$$

```
[Out] (a+b*x^(1+m))^(1+n)/b/(1+m)/(1+n)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$\frac{\left(a + bx^{m+1}\right)^{n+1}}{b(m+1)(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^(1 + m))^n,x]
```

```
[Out] (a + b*x^(1 + m))^(1 + n)/(b*(1 + m)*(1 + n))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^m \left(a + bx^{1+m}\right)^n \, dx = \frac{\left(a + bx^{1+m}\right)^{1+n}}{b(1+m)(1+n)}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{\left(a + bx^{m+1}\right)^{n+1}}{b(m+1)(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^(1 + m))^n,x]
```

```
[Out] (a + b*x^(1 + m))^(1 + n)/(b*(1 + m)*(1 + n))
```

fricas [A] time = 0.58, size = 35, normalized size = 1.30

$$\frac{\left(bx^{m+1} + a\right)\left(bx^{m+1} + a\right)^n}{bm + (bm + b)n + b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^(1+m))^n,x, algorithm="fricas")
```

```
[Out] (b*x^(m + 1) + a)*(b*x^(m + 1) + a)^n/(b*m + (b*m + b)*n + b)
```

giac [A] time = 0.20, size = 26, normalized size = 0.96

$$\frac{\left(bx^{m+1} + a\right)^{n+1}}{(bm + b)(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^(1+m))^n,x, algorithm="giac")
```

```
[Out] (b*x^(m + 1) + a)^(n + 1)/((b*m + b)*(n + 1))
```

maple [B] time = 0.04, size = 60, normalized size = 2.22

$$\frac{x\,\mathrm{e}^{m\ln(x)}\mathrm{e}^{n\ln\left(bx\,\mathrm{e}^{m\ln(x)}+a\right)}}{mn+m+n+1}+\frac{a\,\mathrm{e}^{n\ln\left(bx\,\mathrm{e}^{m\ln(x)}+a\right)}}{(mn+m+n+1)\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*x^(m+1))^n,x)
```

```
[Out] a/b/(m*n+m+n+1)*exp(n*ln(b*x*exp(m*ln(x))+a))+1/(m*n+m+n+1)*x*exp(m*ln(x))*
exp(n*ln(b*x*exp(m*ln(x))+a))
```

maxima [A] time = 0.50, size = 27, normalized size = 1.00

$$\frac{\left(bx^{m+1}+a\right)^{n+1}}{b(m+1)(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^(1+m))^n,x, algorithm="maxima")
```

```
[Out] (b*x^(m + 1) + a)^(n + 1)/(b*(m + 1)*(n + 1))
```

mupad [B] time = 1.49, size = 27, normalized size = 1.00

$$\frac{\left(a+b\,x^{m+1}\right)^{n+1}}{b\,(m+1)\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^(m + 1))^n,x)
```

```
[Out] (a + b*x^(m + 1))^(n + 1)/(b*(m + 1)*(n + 1))
```

sympy [A] time = 134.23, size = 100, normalized size = 3.70

$$\begin{cases}\frac{\log(x)}{a} & \text{for } b=0\wedge m=-1\wedge n=-1\\ \frac{a^n x x^m}{m+1} & \text{for } b=0\\ (a+b)^n\log(x) & \text{for } m=-1\\ \frac{\log\left(\frac{a}{b}+xx^m\right)}{bm+b} & \text{for } n=-1\\ \frac{a(a+bxx^m)^n}{bmn+bm+bn+b}+\frac{bxx^m(a+bxx^m)^n}{bmn+bm+bn+b} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*x**(1+m))**n,x)
```

```
[Out] Piecewise((log(x)/a, Eq(b, 0) & Eq(m, -1) & Eq(n, -1)), (a**n*x*x**m/(m + 1
), Eq(b, 0)), ((a + b)**n*log(x), Eq(m, -1)), (log(a/b + x*x**m)/(b*m + b),
 Eq(n, -1)), (a*(a + b*x*x**m)**n/(b*m*n + b*m + b*n + b) + b*x*x**m*(a + b
*x*x**m)**n/(b*m*n + b*m + b*n + b), True))
```

3.2740 $\int x^m \left(a + bx^{2+2m}\right)^3 dx$

Optimal. Leaf size=71

$$\frac{a^3 x^{m+1}}{m+1} + \frac{a^2 b x^{3(m+1)}}{m+1} + \frac{3ab^2 x^{5(m+1)}}{5(m+1)} + \frac{b^3 x^{7(m+1)}}{7(m+1)}$$

```
[Out] a^3*x^(1+m)/(1+m)+a^2*b*x^(3+3*m)/(1+m)+3/5*a*b^2*x^(5+5*m)/(1+m)+1/7*b^3*x
^(7+7*m)/(1+m)
```

Rubi [A] time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {270}

$$\frac{a^2 b x^{3(m+1)}}{m+1} + \frac{a^3 x^{m+1}}{m+1} + \frac{3ab^2 x^{5(m+1)}}{5(m+1)} + \frac{b^3 x^{7(m+1)}}{7(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^(2 + 2*m))^3,x]
```

```
[Out] (a^3*x^(1 + m))/(1 + m) + (a^2*b*x^(3*(1 + m)))/(1 + m) + (3*a*b^2*x^(5*(1
+ m)))/(5*(1 + m)) + (b^3*x^(7*(1 + m)))/(7*(1 + m))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^m \left(a + bx^{2+2m}\right)^3 dx &= \int \left(a^3 x^m + 3a^2 b x^{2+3m} + 3ab^2 x^{4+5m} + b^3 x^{6+7m}\right) dx \\ &= \frac{a^3 x^{1+m}}{1+m} + \frac{a^2 b x^{3(1+m)}}{1+m} + \frac{3ab^2 x^{5(1+m)}}{5(1+m)} + \frac{b^3 x^{7(1+m)}}{7(1+m)}\end{aligned}$$

Mathematica [A] time = 0.06, size = 57, normalized size = 0.80

$$\frac{35a^3 x^{m+1} + 35a^2 b x^{3m+3} + 21ab^2 x^{5m+5} + 5b^3 x^{7m+7}}{35m+35}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^(2 + 2*m))^3,x]
```

```
[Out] (35*a^3*x^(1 + m) + 35*a^2*b*x^(3 + 3*m) + 21*a*b^2*x^(5 + 5*m) + 5*b^3*x^(
7 + 7*m))/(35 + 35*m)
```

fricas [A] time = 0.66, size = 58, normalized size = 0.82

$$\frac{5\,b^3 x^7 x^{7\,m} + 21\,ab^2 x^5 x^{5\,m} + 35\,a^2 b x^3 x^{3\,m} + 35\,a^3 x x^m}{35\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^(2+2*m))^3,x, algorithm="fricas")
```

```
[Out] 1/35*(5*b^3*x^7*x^(7*m) + 21*a*b^2*x^5*x^(5*m) + 35*a^2*b*x^3*x^(3*m) + 35*
a^3*x*x^m)/(m + 1)
```

giac [A] time = 0.24, size = 58, normalized size = 0.82

$$\frac{5\,b^3x^7x^{7\,m} + 21\,ab^2x^5x^{5\,m} + 35\,a^2bx^3x^{3\,m} + 35\,a^3xx^m}{35\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^(2+2*m))^3,x, algorithm="giac")
```

```
[Out] 1/35*(5*b^3*x^7*x^(7*m) + 21*a*b^2*x^5*x^(5*m) + 35*a^2*b*x^3*x^(3*m) + 35*
a^3*x*x^m)/(m + 1)
```

maple [A] time = 0.02, size = 70, normalized size = 0.99

$$\frac{b^3x^7x^{7m}}{7m+7} + \frac{3a\,b^2x^5x^{5m}}{5\,(m+1)} + \frac{a^2b\,x^3x^{3m}}{m+1} + \frac{a^3x\,x^m}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*x^(2*m+2))^3,x)
```

```
[Out] 1/7*b^3*x^7/(m+1)*(x^m)^7+3/5*a*b^2*x^5/(m+1)*(x^m)^5+a^2*b*x^3/(m+1)*(x^m)
^3+1/(m+1)*a^3*x*x^m
```

maxima [A] time = 0.63, size = 67, normalized size = 0.94

$$\frac{b^3x^{7\,m+7}}{7\,(m+1)} + \frac{3\,ab^2x^{5\,m+5}}{5\,(m+1)} + \frac{a^2bx^{3\,m+3}}{m+1} + \frac{a^3x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^(2+2*m))^3,x, algorithm="maxima")
```

```
[Out] 1/7*b^3*x^(7*m + 7)/(m + 1) + 3/5*a*b^2*x^(5*m + 5)/(m + 1) + a^2*b*x^(3*m
+ 3)/(m + 1) + a^3*x^(m + 1)/(m + 1)
```

mupad [B] time = 1.45, size = 72, normalized size = 1.01

$$\frac{a^3\,x\,x^m}{m+1} + \frac{b^3\,x^{7\,m}\,x^7}{7\,m+7} + \frac{a^2\,b\,x^{3\,m}\,x^3}{m+1} + \frac{3\,a\,b^2\,x^{5\,m}\,x^5}{5\,m+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^(2*m + 2))^3,x)
```

```
[Out] (a^3*x*x^m)/(m + 1) + (b^3*x^(7*m)*x^7)/(7*m + 7) + (a^2*b*x^(3*m)*x^3)/(m
+ 1) + (3*a*b^2*x^(5*m)*x^5)/(5*m + 5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*x**(2+2*m))**3,x)
```

```
[Out] Timed out
```

3.2741 $\int x^m \left(a + bx^{2+2m}\right)^2 dx$

Optimal. Leaf size=52

$$\frac{a^2 x^{m+1}}{m+1} + \frac{2abx^{3(m+1)}}{3(m+1)} + \frac{b^2 x^{5(m+1)}}{5(m+1)}$$

```
[Out] a^2*x^(1+m)/(1+m)+2/3*a*b*x^(3+3*m)/(1+m)+1/5*b^2*x^(5+5*m)/(1+m)
```

Rubi [A] time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {270}

$$\frac{a^2 x^{m+1}}{m+1} + \frac{2abx^{3(m+1)}}{3(m+1)} + \frac{b^2 x^{5(m+1)}}{5(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^(2 + 2*m))^2,x]
```

```
[Out] (a^2*x^(1 + m))/(1 + m) + (2*a*b*x^(3*(1 + m)))/(3*(1 + m)) + (b^2*x^(5*(1
+ m)))/(5*(1 + m))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^m \left(a + bx^{2+2m}\right)^2 dx &= \int \left(a^2 x^m + 2abx^{2+3m} + b^2 x^{4+5m}\right) dx \\ &= \frac{a^2 x^{1+m}}{1+m} + \frac{2abx^{3(1+m)}}{3(1+m)} + \frac{b^2 x^{5(1+m)}}{5(1+m)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 42, normalized size = 0.81

$$\frac{15a^2 x^{m+1} + 10abx^{3m+3} + 3b^2 x^{5m+5}}{15m+15}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^(2 + 2*m))^2,x]
```

```
[Out] (15*a^2*x^(1 + m) + 10*a*b*x^(3 + 3*m) + 3*b^2*x^(5 + 5*m))/(15 + 15*m)
```

fricas [A] time = 0.89, size = 42, normalized size = 0.81

$$\frac{3\,b^2 x^5 x^{5\,m} + 10\,abx^3 x^{3\,m} + 15\,a^2 x x^m}{15\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^(2+2*m))^2,x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^2*x^5*x^(5*m) + 10*a*b*x^3*x^(3*m) + 15*a^2*x*x^m)/(m + 1)
```

giac [A] time = 0.18, size = 42, normalized size = 0.81

$$\frac{3\,b^2x^5x^{5\,m}+10\,abx^3x^{3\,m}+15\,a^2xx^m}{15\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^(2+2*m))^2,x, algorithm="giac")
```

```
[Out] 1/15*(3*b^2*x^5*x^(5*m) + 10*a*b*x^3*x^(3*m) + 15*a^2*x*x^m)/(m + 1)
```

maple [A] time = 0.02, size = 50, normalized size = 0.96

$$\frac{b^2x^5x^{5m}}{5m+5}+\frac{2ab\,x^3x^{3m}}{3\,(m+1)}+\frac{a^2x\,x^m}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*x^(2*m+2))^2,x)
```

```
[Out] 1/5*b^2*x^5/(m+1)*(x^m)^5+2/3*a*b*x^3/(m+1)*(x^m)^3+1/(m+1)*a^2*x*x^m
```

maxima [A] time = 0.59, size = 48, normalized size = 0.92

$$\frac{b^2x^{5\,m+5}}{5\,(m+1)}+\frac{2\,abx^{3\,m+3}}{3\,(m+1)}+\frac{a^2x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^(2+2*m))^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*x^(5*m + 5)/(m + 1) + 2/3*a*b*x^(3*m + 3)/(m + 1) + a^2*x^(m + 1)/(
m + 1)
```

mupad [B] time = 1.36, size = 38, normalized size = 0.73

$$\frac{x^{m+1}\left(\frac{b^2\,x^{4\,m+4}}{5}+a^2+\frac{2\,a\,b\,x^{2\,m+2}}{3}\right)}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^(2*m + 2))^2,x)
```

```
[Out] (x^(m + 1)*((b^2*x^(4*m + 4))/5 + a^2 + (2*a*b*x^(2*m + 2))/3))/(m + 1)
```

sympy [A] time = 113.14, size = 61, normalized size = 1.17

$$\begin{cases}\frac{15a^2xx^m}{15m+15}+\frac{10abx^3x^{3m}}{15m+15}+\frac{3b^2x^5x^{5m}}{15m+15} & \text{for } m\neq -1\\ (a+b)^2\log(x) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*x**(2+2*m))**2,x)
```

```
[Out] Piecewise((15*a**2*x*x**m/(15*m + 15) + 10*a*b*x**3*x**(3*m)/(15*m + 15) +
3*b**2*x**5*x**(5*m)/(15*m + 15), Ne(m, -1)), ((a + b)**2*log(x), True))
```

3.2742 $\int x^m \left(a + bx^{2+2m}\right) dx$

Optimal. Leaf size=30

$$\frac{ax^{m+1}}{m+1} + \frac{bx^{3(m+1)}}{3(m+1)}$$

```
[Out] a*x^(1+m)/(1+m)+1/3*b*x^(3+3*m)/(1+m)
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {14}

$$\frac{ax^{m+1}}{m+1} + \frac{bx^{3(m+1)}}{3(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^(2 + 2*m)),x]
```

```
[Out] (a*x^(1 + m))/(1 + m) + (b*x^(3*(1 + m)))/(3*(1 + m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^m \left(a + bx^{2+2m}\right) dx &= \int \left(ax^m + bx^{2+3m}\right) dx \\ &= \frac{ax^{1+m}}{1+m} + \frac{bx^{3(1+m)}}{3(1+m)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.87

$$\frac{3ax^{m+1} + bx^{3m+3}}{3m+3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^(2 + 2*m)),x]
```

```
[Out] (3*a*x^(1 + m) + b*x^(3 + 3*m))/(3 + 3*m)
```

fricas [A] time = 0.65, size = 25, normalized size = 0.83

$$\frac{bx^3 x^{3\,m} + 3\,axx^m}{3\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^(2+2*m)),x, algorithm="fricas")
```

```
[Out] 1/3*(b*x^3*x^(3*m) + 3*a*x*x^m)/(m + 1)
```

giac [A] time = 0.16, size = 25, normalized size = 0.83

$$\frac{bx^3 x^{3\,m} + 3\,axx^m}{3\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^(2+2*m)),x, algorithm="giac")
[Out] 1/3*(b*x^3*x^(3*m) + 3*a*x*x^m)/(m + 1)
```

maple [A] time = 0.02, size = 33, normalized size = 1.10

$$\frac{b\,x^3 \mathrm{e}^{3m\ln(x)}}{3m+3} + \frac{ax\,\mathrm{e}^{m\ln(x)}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*x^(2*m+2)),x)
[Out] 1/(m+1)*a*x*exp(m*ln(x))+1/3*b/(m+1)*x^3*exp(m*ln(x))^3
```

maxima [A] time = 0.65, size = 28, normalized size = 0.93

$$\frac{bx^{3\,m+3}}{3\,(m+1)} + \frac{ax^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^(2+2*m)),x, algorithm="maxima")
[Out] 1/3*b*x^(3*m + 3)/(m + 1) + a*x^(m + 1)/(m + 1)
```

mupad [B] time = 1.29, size = 23, normalized size = 0.77

$$\frac{x^{m+1}\left(a + \frac{b\,x^{2\,m+2}}{3}\right)}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^(2*m + 2)),x)
[Out] (x^(m + 1)*(a + (b*x^(2*m + 2))/3))/(m + 1)
```

sympy [A] time = 9.91, size = 36, normalized size = 1.20

$$\begin{cases} \frac{3axx^m}{3m+3} + \frac{bx^3x^{3m}}{3m+3} & \text{for } m \neq -1 \\ (a+b)\log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*x**(2+2*m)),x)
[Out] Piecewise((3*a*x*x**m/(3*m + 3) + b*x**3*x**(3*m)/(3*m + 3), Ne(m, -1)), ((
a + b)*log(x), True))
```

3.2743 $\int \frac{x^m}{a+bx^{2+2m}}\,dx$

Optimal. Leaf size=33

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x^{m+1}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,(m+1)}$$

```
[Out] arctan(x^(1+m)*b^(1/2)/a^(1/2))/(1+m)/a^(1/2)/b^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {345, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x^{m+1}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x^(2 + 2*m)),x]
```

```
[Out] ArcTan[(Sqrt[b]*x^(1 + m))/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*(1 + m))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{a+bx^{2+2m}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^{1+m}\right)}{1+m}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x^{1+m}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,(1+m)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x^{m+1}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x^(2 + 2*m)),x]
```

```
[Out] ArcTan[(Sqrt[b]*x^(1 + m))/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*(1 + m))
```

fricas [A] time = 0.72, size = 96, normalized size = 2.91

$$\left[-\frac{\sqrt{-ab}\,\log\left(\frac{bx^2x^{2\,m}-2\,\sqrt{-ab}\,xx^m-a}{bx^2x^{2\,m}+a}\right)}{2\,(abm+ab)}, -\frac{\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}}{bxx^m}\right)}{abm+ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^(2+2*m)),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b)*log((b*x^2*x^(2*m) - 2*sqrt(-a*b)*x*x^m - a)/(b*x^2*x^(2*m
) + a))/(a*b*m + a*b), -sqrt(a*b)*arctan(sqrt(a*b)/(b*x*x^m))/(a*b*m + a*b)
]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{bx^{2\,m+2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^(2+2*m)),x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*x^(2*m + 2) + a), x)
```

maple [B] time = 0.06, size = 61, normalized size = 1.85

$$-\frac{\ln\left(x^m-\frac{a}{\sqrt{-ab}\,x}\right)}{2\sqrt{-ab}\ (m+1)}+\frac{\ln\left(x^m+\frac{a}{\sqrt{-ab}\,x}\right)}{2\sqrt{-ab}\ (m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a+b*x^(2*m+2)),x)
```

```
[Out] -1/2/(-a*b)^(1/2)/(m+1)*ln(x^m-a/x/(-a*b)^(1/2))+1/2/(-a*b)^(1/2)/(m+1)*ln(
x^m+a/x/(-a*b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{bx^{2\,m+2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^(2+2*m)),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(b*x^(2*m + 2) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{a+b\,x^{2\,m+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^(2*m + 2)),x)
```

```
[Out] int(x^m/(a + b*x^(2*m + 2)), x)
```

sympy [C] time = 2.57, size = 197, normalized size = 5.97

$$\frac{i\sqrt{\pi}\,a^{-\frac{m}{2m+2}}a^{-\frac{1}{2m+2}}\log\left(1-\frac{\sqrt{b}\,x x^m e^{\frac{i\pi}{2}}}{\sqrt{a}}\right)}{4\sqrt{b}\,m\Gamma\left(\frac{m}{2m+2}+1+\frac{1}{2m+2}\right)+4\sqrt{b}\,\Gamma\left(\frac{m}{2m+2}+1+\frac{1}{2m+2}\right)}-\frac{i\sqrt{\pi}\,a^{-\frac{m}{2m+2}}a^{-\frac{1}{2m+2}}\log\left(1-\frac{\sqrt{b}\,x x^m e^{\frac{3i\pi}{2}}}{\sqrt{a}}\right)}{4\sqrt{b}\,m\Gamma\left(\frac{m}{2m+2}+1+\frac{1}{2m+2}\right)+4\sqrt{b}\,\Gamma\left(\frac{m}{2m+2}+1+\frac{1}{2m+}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a+b*x**(2+2*m)),x)
```

```
[Out] I*sqrt(pi)*a**(-m/(2*m + 2))*a**(-1/(2*m + 2))*log(1 - sqrt(b)*x*x**m*exp_p
olar(I*pi/2)/sqrt(a))/(4*sqrt(b)*m*gamma(m/(2*m + 2) + 1 + 1/(2*m + 2)) + 4
*sqrt(b)*gamma(m/(2*m + 2) + 1 + 1/(2*m + 2))) - I*sqrt(pi)*a**(-m/(2*m + 2
))*a**(-1/(2*m + 2))*log(1 - sqrt(b)*x*x**m*exp_polar(3*I*pi/2)/sqrt(a))/(4
*sqrt(b)*m*gamma(m/(2*m + 2) + 1 + 1/(2*m + 2)) + 4*sqrt(b)*gamma(m/(2*m +
2) + 1 + 1/(2*m + 2)))
```

3.2744 $\int \frac{x^m}{\left(a+bx^{2+2m}\right)^2}\,dx$

Optimal. Leaf size=67

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x^{m+1}}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\,(m+1)}+\frac{x^{m+1}}{2a(m+1)\left(a+bx^{2(m+1)}\right)}$$

```
[Out] 1/2*x^(1+m)/a/(1+m)/(a+b*x^(2+2*m))+1/2*arctan(x^(1+m)*b^(1/2)/a^(1/2))/a^(
3/2)/(1+m)/b^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {345, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x^{m+1}}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\,(m+1)}+\frac{x^{m+1}}{2a(m+1)\left(a+bx^{2(m+1)}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x^(2 + 2*m))^2,x]
```

```
[Out] x^(1 + m)/(2*a*(1 + m)*(a + b*x^(2*(1 + m)))) + ArcTan[(Sqrt[b]*x^(1 + m))/
Sqrt[a]]/(2*a^(3/2)*Sqrt[b]*(1 + m))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{\left(a+bx^{2+2m}\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^2}\,dx, x, x^{1+m}\right)}{1+m}\\ &= \frac{x^{1+m}}{2a(1+m)\left(a+bx^{2(1+m)}\right)}+\frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^{1+m}\right)}{2a(1+m)}\\ &= \frac{x^{1+m}}{2a(1+m)\left(a+bx^{2(1+m)}\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x^{1+m}}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\,(1+m)}\end{aligned}$$

Mathematica [C] time = 0.01, size = 53, normalized size = 0.79

$$\frac{x^{m+1}\,{}_2F_1\left(2, \frac{m+1}{2m+2}; \frac{m+1}{2m+2}+1; -\frac{bx^{2m+2}}{a}\right)}{a^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x^(2 + 2*m))^2,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/(2 + 2*m), 1 + (1 + m)/(2 + 2*m), -
((b*x^(2 + 2*m))/a)])/(a^2*(1 + m))
```

fricas [A] time = 0.60, size = 212, normalized size = 3.16

$$\left[\frac{2\,abxx^m - \left(\sqrt{-ab}\,bx^2x^{2\,m} + \sqrt{-ab}\,a\right)\log\left(\frac{bx^2x^{2\,m} - 2\sqrt{-ab}\,xx^m - a}{bx^2x^{2\,m}+a}\right)}{4\left(a^3bm + a^3b + \left(a^2b^2m + a^2b^2\right)x^2x^{2\,m}\right)}, \frac{abxx^m - \left(\sqrt{ab}\,bx^2x^{2\,m} + \sqrt{ab}\,a\right)\arctan\left(\frac{\sqrt{ab}}{bxx^m}\right)}{2\left(a^3bm + a^3b + \left(a^2b^2m + a^2b^2\right)x^2x^{2\,m}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^(2+2*m))^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b*x*x^m - (sqrt(-a*b)*b*x^2*x^(2*m) + sqrt(-a*b)*a)*log((b*x^2*x^
(2*m) - 2*sqrt(-a*b)*x*x^m - a)/(b*x^2*x^(2*m) + a)))/(a^3*b*m + a^3*b + (a
^2*b^2*m + a^2*b^2)*x^2*x^(2*m)), 1/2*(a*b*x*x^m - (sqrt(a*b)*b*x^2*x^(2*m)
 + sqrt(a*b)*a)*arctan(sqrt(a*b)/(b*x*x^m)))/(a^3*b*m + a^3*b + (a^2*b^2*m
+ a^2*b^2)*x^2*x^(2*m))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(bx^{2\,m+2} + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^(2+2*m))^2,x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*x^(2*m + 2) + a)^2, x)
```

maple [A] time = 0.05, size = 95, normalized size = 1.42

$$\frac{x\,x^m}{2\,(m+1)\left(b\,x^2x^{2m} + a\right)a} - \frac{\ln\left(x^m - \frac{a}{\sqrt{-ab}\;x}\right)}{4\sqrt{-ab}\;\;(m+1)\,a} + \frac{\ln\left(x^m + \frac{a}{\sqrt{-ab}\;x}\right)}{4\sqrt{-ab}\;\;(m+1)\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a+b*x^(2*m+2))^2,x)
```

```
[Out] 1/2*x/(m+1)/a*x^m/(a+b*x^2*(x^m)^2)-1/4/(-a*b)^(1/2)/(m+1)/a*ln(x^m-1/(-a*b
)^(1/2)*a/x)+1/4/(-a*b)^(1/2)/(m+1)/a*ln(x^m+1/(-a*b)^(1/2)*a/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{xx^m}{2\left(ab(m+1)x^2x^{2\,m} + a^2(m+1)\right)} + \int \frac{x^m}{2\left(abx^2x^{2\,m} + a^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^(2+2*m))^2,x, algorithm="maxima")
```

```
[Out] 1/2*x*x^m/(a*b*(m + 1)*x^2*x^(2*m) + a^2*(m + 1)) + integrate(1/2*x^m/(a*b*
x^2*x^(2*m) + a^2), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m}{\left(a + b\,x^{2\,m+2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^(2*m + 2))^2,x)
```

```
[Out] int(x^m/(a + b*x^(2*m + 2))^2, x)
```

sympy **[C]** time = 8.62, size = 865, normalized size = 12.91

$$-\frac{i\sqrt{\pi}\,a^{-\frac{m}{2m+2}}a^{-\frac{1}{2m+2}}\log\left(1-\frac{\sqrt{b}\,x x^m e^{\frac{i\pi}{2}}}{\sqrt{a}}\right)}{-8a\sqrt{b}\,m\Gamma\left(\frac{m}{2m+2}+1+\frac{1}{2m+2}\right)-8a\sqrt{b}\,\Gamma\left(\frac{m}{2m+2}+1+\frac{1}{2m+2}\right)-8b^{\frac{3}{2}}mx^2x^{2m}\Gamma\left(\frac{m}{2m+2}+1+\frac{1}{2m+2}\right)-8b^{\frac{3}{2}}x^2x^{2m}\Gamma\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a+b*x**(2+2*m))**2,x)
```

```
[Out] -I*sqrt(pi)*a**(-m/(2*m + 2))*a**(-1/(2*m + 2))*log(1 - sqrt(b)*x*x***m*exp_
polar(I*pi/2)/sqrt(a))/(-8*a*sqrt(b)*m*gamma(m/(2*m + 2) + 1 + 1/(2*m + 2))
 - 8*a*sqrt(b)*gamma(m/(2*m + 2) + 1 + 1/(2*m + 2)) - 8*b**(3/2)*m*x**2*x**
(2*m)*gamma(m/(2*m + 2) + 1 + 1/(2*m + 2)) - 8*b**(3/2)*x**2*x**(2*m)*gamma
(m/(2*m + 2) + 1 + 1/(2*m + 2))) + I*sqrt(pi)*a**(-m/(2*m + 2))*a**(-1/(2*m
 + 2))*log(1 - sqrt(b)*x*x***m*exp_polar(3*I*pi/2)/sqrt(a))/(-8*a*sqrt(b)*m*
gamma(m/(2*m + 2) + 1 + 1/(2*m + 2)) - 8*a*sqrt(b)*gamma(m/(2*m + 2) + 1 +
1/(2*m + 2)) - 8*b**(3/2)*m*x**2*x**(2*m)*gamma(m/(2*m + 2) + 1 + 1/(2*m +
2)) - 8*b**(3/2)*x**2*x**(2*m)*gamma(m/(2*m + 2) + 1 + 1/(2*m + 2))) - I*sq
rt(pi)*a**(-m/(2*m + 2))*a**(-1/(2*m + 2))*b*x**2*x**(2*m)*log(1 - sqrt(b)*
x*x***m*exp_polar(I*pi/2)/sqrt(a))/(a*(-8*a*sqrt(b)*m*gamma(m/(2*m + 2) + 1
+ 1/(2*m + 2)) - 8*a*sqrt(b)*gamma(m/(2*m + 2) + 1 + 1/(2*m + 2)) - 8*b**(3
/2)*m*x**2*x**(2*m)*gamma(m/(2*m + 2) + 1 + 1/(2*m + 2)) - 8*b**(3/2)*x**2*
x**(2*m)*gamma(m/(2*m + 2) + 1 + 1/(2*m + 2)))) + I*sqrt(pi)*a**(-m/(2*m +
2))*a**(-1/(2*m + 2))*b*x**2*x**(2*m)*log(1 - sqrt(b)*x*x***m*exp_polar(3*I*
pi/2)/sqrt(a))/(a*(-8*a*sqrt(b)*m*gamma(m/(2*m + 2) + 1 + 1/(2*m + 2)) - 8*
a*sqrt(b)*gamma(m/(2*m + 2) + 1 + 1/(2*m + 2)) - 8*b**(3/2)*m*x**2*x**(2*m)
*gamma(m/(2*m + 2) + 1 + 1/(2*m + 2)) - 8*b**(3/2)*x**2*x**(2*m)*gamma(m/(2
*m + 2) + 1 + 1/(2*m + 2)))) - 2*sqrt(pi)*a**(-m/(2*m + 2))*a**(-1/(2*m + 2
))*sqrt(b)*x*x***m/(sqrt(a)*(-8*a*sqrt(b)*m*gamma(m/(2*m + 2) + 1 + 1/(2*m +
 2)) - 8*a*sqrt(b)*gamma(m/(2*m + 2) + 1 + 1/(2*m + 2)) - 8*b**(3/2)*m*x**2
*x**(2*m)*gamma(m/(2*m + 2) + 1 + 1/(2*m + 2)) - 8*b**(3/2)*x**2*x**(2*m)*g
amma(m/(2*m + 2) + 1 + 1/(2*m + 2))))
```

3.2745 $\int \frac{x^m}{\left(a+bx^{2+2m}\right)^3}\,dx$

Optimal. Leaf size=97

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,x^{m+1}}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\,(m+1)}+\frac{3x^{m+1}}{8a^2(m+1)\left(a+bx^{2(m+1)}\right)}+\frac{x^{m+1}}{4a(m+1)\left(a+bx^{2(m+1)}\right)^2}$$

```
[Out] 1/4*x^(1+m)/a/(1+m)/(a+b*x^(2+2*m))^2+3/8*x^(1+m)/a^2/(1+m)/(a+b*x^(2+2*m))
+3/8*arctan(x^(1+m)*b^(1/2)/a^(1/2))/a^(5/2)/(1+m)/b^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {345, 199, 205}

$$\frac{3x^{m+1}}{8a^2(m+1)\left(a+bx^{2(m+1)}\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,x^{m+1}}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\,(m+1)}+\frac{x^{m+1}}{4a(m+1)\left(a+bx^{2(m+1)}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x^(2 + 2*m))^3,x]
```

```
[Out] x^(1 + m)/(4*a*(1 + m)*(a + b*x^(2*(1 + m)))^2) + (3*x^(1 + m))/(8*a^2*(1 +
 m)*(a + b*x^(2*(1 + m)))) + (3*ArcTan[(Sqrt[b]*x^(1 + m))/Sqrt[a]])/(8*a^(
5/2)*Sqrt[b]*(1 + m))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{x^m}{\left(a+bx^{2+2m}\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^3}\,dx, x, x^{1+m}\right)}{1+m}$$

$$= \frac{x^{1+m}}{4a(1+m)\left(a+bx^{2(1+m)}\right)^2} + \frac{3\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^2}\,dx, x, x^{1+m}\right)}{4a(1+m)}$$

$$= \frac{x^{1+m}}{4a(1+m)\left(a+bx^{2(1+m)}\right)^2} + \frac{3x^{1+m}}{8a^2(1+m)\left(a+bx^{2(1+m)}\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^{1+m}\right)}{8a^2(1+m)}$$

$$= \frac{x^{1+m}}{4a(1+m)\left(a+bx^{2(1+m)}\right)^2} + \frac{3x^{1+m}}{8a^2(1+m)\left(a+bx^{2(1+m)}\right)} + \frac{3\tan^{-1}\left(\frac{\sqrt{b}\,x^{1+m}}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\,(1+m)}$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.36

$$\frac{x^{m+1}\,{}_2F_1\left(\frac{1}{2}, 3; \frac{3}{2}; -\frac{bx^{2m+2}}{a}\right)}{a^3(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x^(2 + 2*m))^3,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1/2, 3, 3/2, -((b*x^(2 + 2*m))/a)])/(a^3*(1 +
m))
```

fricas [A] time = 0.63, size = 340, normalized size = 3.51

$$\left[\frac{6\,ab^2x^3x^{3\,m} + 10\,a^2bxx^m - 3\left(\sqrt{-ab}\,b^2x^4x^{4\,m} + 2\sqrt{-ab}\,abx^2x^{2\,m} + \sqrt{-ab}\,a^2\right)\log\left(\frac{bx^2x^{2\,m} - 2\sqrt{-ab}\,xx^m - a}{bx^2x^{2\,m} + a}\right)}{16\left(a^5bm + a^5b + \left(a^3b^3m + a^3b^3\right)x^4x^{4\,m} + 2\left(a^4b^2m + a^4b^2\right)x^2x^{2\,m}\right)}, \frac{3\,ab^2x^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^(2+2*m))^3,x, algorithm="fricas")
```

```
[Out] [1/16*(6*a*b^2*x^3*x^(3*m) + 10*a^2*b*x*x^m - 3*(sqrt(-a*b)*b^2*x^4*x^(4*m)
 + 2*sqrt(-a*b)*a*b*x^2*x^(2*m) + sqrt(-a*b)*a^2)*log((b*x^2*x^(2*m) - 2*sq
rt(-a*b)*x*x^m - a)/(b*x^2*x^(2*m) + a)))/(a^5*b*m + a^5*b + (a^3*b^3*m + a
^3*b^3)*x^4*x^(4*m) + 2*(a^4*b^2*m + a^4*b^2)*x^2*x^(2*m)), 1/8*(3*a*b^2*x^
3*x^(3*m) + 5*a^2*b*x*x^m - 3*(sqrt(a*b)*b^2*x^4*x^(4*m) + 2*sqrt(a*b)*a*b*
x^2*x^(2*m) + sqrt(a*b)*a^2)*arctan(sqrt(a*b)/(b*x*x^m)))/(a^5*b*m + a^5*b
+ (a^3*b^3*m + a^3*b^3)*x^4*x^(4*m) + 2*(a^4*b^2*m + a^4*b^2)*x^2*x^(2*m))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(bx^{2\,m+2} + a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^(2+2*m))^3,x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*x^(2*m + 2) + a)^3, x)
```

maple [A] time = 0.05, size = 110, normalized size = 1.13

$$\frac{\left(3b\,x^2x^{2m}+5a\right)x\,x^m}{8\,(m+1)\left(b\,x^2x^{2m}+a\right)^2a^2}-\frac{3\ln\left(x^m-\frac{a}{\sqrt{-ab}\,x}\right)}{16\sqrt{-ab}\;(m+1)\,a^2}+\frac{3\ln\left(x^m+\frac{a}{\sqrt{-ab}\,x}\right)}{16\sqrt{-ab}\;(m+1)\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a+b*x^(2*m+2))^3,x)
```

```
[Out] 1/8*x*x^m*(3*b*x^2*(x^m)^2+5*a)/(m+1)/a^2/(a+b*x^2*(x^m)^2)^2-3/16/(-a*b)^(
1/2)/(m+1)/a^2*ln(x^m-1/(-a*b)^(1/2)*a/x)+3/16/(-a*b)^(1/2)/(m+1)/a^2*ln(x^
m+1/(-a*b)^(1/2)*a/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3\,bx^3x^{3\,m}+5\,axx^m}{8\left(a^2b^2(m+1)x^4x^{4\,m}+2\,a^3b(m+1)x^2x^{2\,m}+a^4(m+1)\right)}+3\int\frac{x^m}{8\left(a^2bx^2x^{2\,m}+a^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^(2+2*m))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*b*x^3*x^(3*m) + 5*a*x*x^m)/(a^2*b^2*(m + 1)*x^4*x^(4*m) + 2*a^3*b*(m
 + 1)*x^2*x^(2*m) + a^4*(m + 1)) + 3*integrate(1/8*x^m/(a^2*b*x^2*x^(2*m) +
 a^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^m}{\left(a+b\,x^{2\,m+2}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^(2*m + 2))^3,x)
```

```
[Out] int(x^m/(a + b*x^(2*m + 2))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a+b*x**(2+2*m))**3,x)
```

```
[Out] Timed out
```

3.2746 $\int x^m \left(a + bx^{2+2m}\right)^{5/2}\, dx$

Optimal. Leaf size=136

$$\frac{5a^3 \tanh^{-1}\left(\frac{\sqrt{b}\, x^{m+1}}{\sqrt{a+bx^{2(m+1)}}}\right)}{16\sqrt{b}\,(m+1)} + \frac{5a^2 x^{m+1}\sqrt{a+bx^{2(m+1)}}}{16(m+1)} + \frac{x^{m+1}\left(a+bx^{2(m+1)}\right)^{5/2}}{6(m+1)} + \frac{5ax^{m+1}\left(a+bx^{2(m+1)}\right)^{3/2}}{24(m+1)}$$

```
[Out] 5/24*a*x^(1+m)*(a+b*x^(2+2*m))^(3/2)/(1+m)+1/6*x^(1+m)*(a+b*x^(2+2*m))^(5/2
)/(1+m)+5/16*a^3*arctanh(x^(1+m)*b^(1/2)/(a+b*x^(2+2*m))^(1/2))/(1+m)/b^(1/
2)+5/16*a^2*x^(1+m)*(a+b*x^(2+2*m))^(1/2)/(1+m)
```

Rubi [A] time = 0.05, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {345, 195, 217, 206}

$$\frac{5a^2 x^{m+1}\sqrt{a+bx^{2(m+1)}}}{16(m+1)} + \frac{5a^3 \tanh^{-1}\left(\frac{\sqrt{b}\, x^{m+1}}{\sqrt{a+bx^{2(m+1)}}}\right)}{16\sqrt{b}\,(m+1)} + \frac{x^{m+1}\left(a+bx^{2(m+1)}\right)^{5/2}}{6(m+1)} + \frac{5ax^{m+1}\left(a+bx^{2(m+1)}\right)^{3/2}}{24(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^(2 + 2*m))^(5/2),x]
```

```
[Out] (5*a^2*x^(1 + m)*Sqrt[a + b*x^(2*(1 + m))])/(16*(1 + m)) + (5*a*x^(1 + m)*(
a + b*x^(2*(1 + m)))^(3/2))/(24*(1 + m)) + (x^(1 + m)*(a + b*x^(2*(1 + m)))
^(5/2))/(6*(1 + m)) + (5*a^3*ArcTanh[(Sqrt[b]*x^(1 + m))/Sqrt[a + b*x^(2*(1
 + m))]])/(16*Sqrt[b]*(1 + m))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rubi steps

$$\int x^m \left(a+bx^{2+2m}\right)^{5/2}\,dx = \frac{\text{Subst}\left(\int \left(a+bx^2\right)^{5/2}\,dx, x, x^{1+m}\right)}{1+m}$$
$$= \frac{x^{1+m}\left(a+bx^{2(1+m)}\right)^{5/2}}{6(1+m)} + \frac{(5a)\,\text{Subst}\left(\int \left(a+bx^2\right)^{3/2}\,dx, x, x^{1+m}\right)}{6(1+m)}$$
$$= \frac{5ax^{1+m}\left(a+bx^{2(1+m)}\right)^{3/2}}{24(1+m)} + \frac{x^{1+m}\left(a+bx^{2(1+m)}\right)^{5/2}}{6(1+m)} + \frac{\left(5a^2\right)\text{Subst}\left(\int \sqrt{a+bx^2}\,dx, x, x\right.}{8(1+m)}$$
$$= \frac{5a^2x^{1+m}\sqrt{a+bx^{2(1+m)}}}{16(1+m)} + \frac{5ax^{1+m}\left(a+bx^{2(1+m)}\right)^{3/2}}{24(1+m)} + \frac{x^{1+m}\left(a+bx^{2(1+m)}\right)^{5/2}}{6(1+m)} + \frac{\left(5a^3\right.}{}$$
$$= \frac{5a^2x^{1+m}\sqrt{a+bx^{2(1+m)}}}{16(1+m)} + \frac{5ax^{1+m}\left(a+bx^{2(1+m)}\right)^{3/2}}{24(1+m)} + \frac{x^{1+m}\left(a+bx^{2(1+m)}\right)^{5/2}}{6(1+m)} + \frac{\left(5a^3\right)}{}$$
$$= \frac{5a^2x^{1+m}\sqrt{a+bx^{2(1+m)}}}{16(1+m)} + \frac{5ax^{1+m}\left(a+bx^{2(1+m)}\right)^{3/2}}{24(1+m)} + \frac{x^{1+m}\left(a+bx^{2(1+m)}\right)^{5/2}}{6(1+m)} + \frac{5a^3\,\text{ta}}{}$$

Mathematica [C] time = 0.09, size = 88, normalized size = 0.65

$$\frac{a^2x^{m+1}\sqrt{a+bx^{2m+2}}\;{}_2F_1\left(-\frac{5}{2}, \frac{m+1}{2m+2}; \frac{m+1}{2m+2}+1; -\frac{bx^{2m+2}}{a}\right)}{(m+1)\sqrt{\frac{bx^{2m+2}}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^(2 + 2*m))^(5/2),x]
```

```
[Out] (a^2*x^(1 + m)*Sqrt[a + b*x^(2 + 2*m)]*Hypergeometric2F1[-5/2, (1 + m)/(2 +
 2*m), 1 + (1 + m)/(2 + 2*m), -((b*x^(2 + 2*m))/a)])/((1 + m)*Sqrt[1 + (b*x
^(2 + 2*m))/a])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^(2+2*m))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^{2\,m+2}+a\right)^{\frac{5}{2}} x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^(2+2*m))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^(2*m + 2) + a)^(5/2)*x^m, x)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \left(b\,x^{2m+2}+a\right)^{\frac{5}{2}} x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*x^(2*m+2))^(5/2),x)
[Out] int(x^m*(a+b*x^(2*m+2))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^{2\,m+2}+a\right)^{\frac{5}{2}} x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^(2+2*m))^(5/2),x, algorithm="maxima")
[Out] integrate((b*x^(2*m + 2) + a)^(5/2)*x^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \left(a+b\,x^{2\,m+2}\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^(2*m + 2))^(5/2),x)
[Out] int(x^m*(a + b*x^(2*m + 2))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*x**(2+2*m))**(5/2),x)
[Out] Timed out
```

3.2747 $\int x^m \left(a + bx^{2+2m}\right)^{3/2}\, dx$

Optimal. Leaf size=104

$$\frac{3a^2 \tanh^{-1}\left(\frac{\sqrt{b}\, x^{m+1}}{\sqrt{a+bx^{2(m+1)}}}\right)}{8\sqrt{b}\,(m+1)} + \frac{x^{m+1}\left(a + bx^{2(m+1)}\right)^{3/2}}{4(m+1)} + \frac{3ax^{m+1}\sqrt{a + bx^{2(m+1)}}}{8(m+1)}$$

```
[Out] 1/4*x^(1+m)*(a+b*x^(2+2*m))^(3/2)/(1+m)+3/8*a^2*arctanh(x^(1+m)*b^(1/2)/(a+
b*x^(2+2*m))^(1/2))/(1+m)/b^(1/2)+3/8*a*x^(1+m)*(a+b*x^(2+2*m))^(1/2)/(1+m)
```

Rubi [A] time = 0.04, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {345, 195, 217, 206}

$$\frac{3a^2 \tanh^{-1}\left(\frac{\sqrt{b}\, x^{m+1}}{\sqrt{a+bx^{2(m+1)}}}\right)}{8\sqrt{b}\,(m+1)} + \frac{x^{m+1}\left(a + bx^{2(m+1)}\right)^{3/2}}{4(m+1)} + \frac{3ax^{m+1}\sqrt{a + bx^{2(m+1)}}}{8(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^(2 + 2*m))^(3/2),x]
```

```
[Out] (3*a*x^(1 + m)*Sqrt[a + b*x^(2*(1 + m))])/(8*(1 + m)) + (x^(1 + m)*(a + b*x
^(2*(1 + m)))^(3/2))/(4*(1 + m)) + (3*a^2*ArcTanh[(Sqrt[b]*x^(1 + m))/Sqrt[
a + b*x^(2*(1 + m))]])/(8*Sqrt[b]*(1 + m))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rubi steps

$$\int x^m \left(a+bx^{2+2m}\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \left(a+bx^2\right)^{3/2}\,dx, x, x^{1+m}\right)}{1+m}$$
$$= \frac{x^{1+m}\left(a+bx^{2(1+m)}\right)^{3/2}}{4(1+m)} + \frac{(3a)\,\text{Subst}\left(\int \sqrt{a+bx^2}\,dx, x, x^{1+m}\right)}{4(1+m)}$$
$$= \frac{3ax^{1+m}\sqrt{a+bx^{2(1+m)}}}{8(1+m)} + \frac{x^{1+m}\left(a+bx^{2(1+m)}\right)^{3/2}}{4(1+m)} + \frac{\left(3a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, x^{1+}\right.}{8(1+m)}$$
$$= \frac{3ax^{1+m}\sqrt{a+bx^{2(1+m)}}}{8(1+m)} + \frac{x^{1+m}\left(a+bx^{2(1+m)}\right)^{3/2}}{4(1+m)} + \frac{\left(3a^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^1}{\sqrt{a+b}}\right.}{8(1+m)}$$
$$= \frac{3ax^{1+m}\sqrt{a+bx^{2(1+m)}}}{8(1+m)} + \frac{x^{1+m}\left(a+bx^{2(1+m)}\right)^{3/2}}{4(1+m)} + \frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x^{1+m}}{\sqrt{a+bx^{2(1+m)}}}\right)}{8\sqrt{b}\,(1+m)}$$

Mathematica [C] time = 0.05, size = 86, normalized size = 0.83

$$\frac{ax^{m+1}\sqrt{a+bx^{2m+2}}\;{}_2F_1\left(-\frac{3}{2}, \frac{m+1}{2m+2}; \frac{m+1}{2m+2}+1; -\frac{bx^{2m+2}}{a}\right)}{(m+1)\sqrt{\frac{bx^{2m+2}}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^(2 + 2*m))^(3/2),x]
```

```
[Out] (a*x^(1 + m)*Sqrt[a + b*x^(2 + 2*m)]*Hypergeometric2F1[-3/2, (1 + m)/(2 + 2
*m), 1 + (1 + m)/(2 + 2*m), -((b*x^(2 + 2*m))/a)])/((1 + m)*Sqrt[1 + (b*x^(
2 + 2*m))/a])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^(2+2*m))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^{2\,m+2}+a\right)^{\frac{3}{2}} x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^(2+2*m))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^(2*m + 2) + a)^(3/2)*x^m, x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \left(b\,x^{2m+2}+a\right)^{\frac{3}{2}} x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^(2*m+2)+a)^(3/2),x)
[Out] int(x^m*(b*x^(2*m+2)+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^{2\,m+2}+a\right)^{\frac{3}{2}} x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^(2+2*m))^(3/2),x, algorithm="maxima")
[Out] integrate((b*x^(2*m + 2) + a)^(3/2)*x^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \left(a+b\,x^{2\,m+2}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^(2*m + 2))^(3/2),x)
[Out] int(x^m*(a + b*x^(2*m + 2))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*x**(2+2*m))**(3/2),x)
[Out] Timed out
```

3.2748 $\int x^m \sqrt{a + b x^{2+2m}}\, dx$

Optimal. Leaf size=72

$$\frac{x^{m+1}\sqrt{a+bx^{2(m+1)}}}{2(m+1)} + \frac{a \tanh^{-1}\left(\frac{\sqrt{b}\, x^{m+1}}{\sqrt{a+bx^{2(m+1)}}}\right)}{2\sqrt{b}\,(m+1)}$$

```
[Out] 1/2*a*arctanh(x^(1+m)*b^(1/2)/(a+b*x^(2+2*m))^(1/2))/(1+m)/b^(1/2)+1/2*x^(1
+m)*(a+b*x^(2+2*m))^(1/2)/(1+m)
```

Rubi [A] time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {345, 195, 217, 206}

$$\frac{x^{m+1}\sqrt{a+bx^{2(m+1)}}}{2(m+1)} + \frac{a \tanh^{-1}\left(\frac{\sqrt{b}\, x^{m+1}}{\sqrt{a+bx^{2(m+1)}}}\right)}{2\sqrt{b}\,(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sqrt[a + b*x^(2 + 2*m)],x]
```

```
[Out] (x^(1 + m)*Sqrt[a + b*x^(2*(1 + m))])/(2*(1 + m)) + (a*ArcTanh[(Sqrt[b]*x^(
1 + m))/Sqrt[a + b*x^(2*(1 + m))]])/(2*Sqrt[b]*(1 + m))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rubi steps

$$\int x^m \sqrt{a+bx^{2+2m}}\,dx = \frac{\text{Subst}\left(\int \sqrt{a+bx^2}\,dx, x, x^{1+m}\right)}{1+m}$$
$$= \frac{x^{1+m}\sqrt{a+bx^{2(1+m)}}}{2(1+m)} + \frac{a\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, x^{1+m}\right)}{2(1+m)}$$
$$= \frac{x^{1+m}\sqrt{a+bx^{2(1+m)}}}{2(1+m)} + \frac{a\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^{1+m}}{\sqrt{a+bx^{2+2m}}}\right)}{2(1+m)}$$
$$= \frac{x^{1+m}\sqrt{a+bx^{2(1+m)}}}{2(1+m)} + \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^{1+m}}{\sqrt{a+bx^{2(1+m)}}}\right)}{2\sqrt{b}\,(1+m)}$$

Mathematica [C] time = 0.03, size = 85, normalized size = 1.18

$$\frac{x^{m+1}\sqrt{a+bx^{2m+2}}\;{}_2F_1\left(-\frac{1}{2}, \frac{m+1}{2m+2}; \frac{m+1}{2m+2}+1; -\frac{bx^{2m+2}}{a}\right)}{(m+1)\sqrt{\frac{bx^{2m+2}}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sqrt[a + b*x^(2 + 2*m)],x]
```

```
[Out] (x^(1 + m)*Sqrt[a + b*x^(2 + 2*m)]*Hypergeometric2F1[-1/2, (1 + m)/(2 + 2*m
), 1 + (1 + m)/(2 + 2*m), -((b*x^(2 + 2*m))/a)])/((1 + m)*Sqrt[1 + (b*x^(2
+ 2*m))/a])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^(2+2*m))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^{2\,m+2}+a}\,x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^(2+2*m))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^(2*m + 2) + a)*x^m, x)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \sqrt{b\,x^{2m+2}+a}\;x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^(2*m+2)+a)^(1/2),x)
```

```
[Out] int(x^m*(b*x^(2*m+2)+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^{2\,m+2}+a}\,x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*x^(2+2*m))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^(2*m + 2) + a)*x^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^m\,\sqrt{a+b\,x^{2\,m+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^(2*m + 2))^(1/2),x)
```

```
[Out] int(x^m*(a + b*x^(2*m + 2))^(1/2), x)
```

sympy [C] time = 9.59, size = 121, normalized size = 1.68

$$\frac{\sqrt{\pi}\,axx^m\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{2}\\ \frac{m}{2m+2}+1+\frac{1}{2m+2}\end{matrix}\middle|\frac{bx^2x^{2m}e^{i\pi}}{a}\right)}{2a^{\frac{m}{2m+2}}a^{\frac{1}{2m+2}}m\Gamma\left(\frac{m}{2m+2}+1+\frac{1}{2m+2}\right)+2a^{\frac{m}{2m+2}}a^{\frac{1}{2m+2}}\Gamma\left(\frac{m}{2m+2}+1+\frac{1}{2m+2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*x**(2+2*m))**(1/2),x)
```

```
[Out] sqrt(pi)*a*x*x**m*hyper((-1/2, 1/2), (m/(2*m + 2) + 1 + 1/(2*m + 2),), b*x*
*2*x**(2m)*exp_polar(I*pi)/a)/(2*a**(m/(2*m + 2))*a**(1/(2*m + 2))*m*gamma
(m/(2*m + 2) + 1 + 1/(2*m + 2)) + 2*a**(m/(2*m + 2))*a**(1/(2*m + 2))*gamma
(m/(2*m + 2) + 1 + 1/(2*m + 2)))
```

3.2749 $\int \frac{x^m}{\sqrt{a+bx^{2+2m}}}\,dx$

Optimal. Leaf size=38

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x^{m+1}}{\sqrt{a+bx^{2(m+1)}}}\right)}{\sqrt{b}\,(m+1)}$$

```
[Out] arctanh(x^(1+m)*b^(1/2)/(a+b*x^(2+2*m))^(1/2))/(1+m)/b^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {345, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x^{m+1}}{\sqrt{a+bx^{2(m+1)}}}\right)}{\sqrt{b}\,(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/Sqrt[a + b*x^(2 + 2*m)],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x^(1 + m))/Sqrt[a + b*x^(2*(1 + m))]]/(Sqrt[b]*(1 + m))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{\sqrt{a+bx^{2+2m}}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, x^{1+m}\right)}{1+m}\\ &= \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^{1+m}}{\sqrt{a+bx^{2+2m}}}\right)}{1+m}\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x^{1+m}}{\sqrt{a+bx^{2(1+m)}}}\right)}{\sqrt{b}\,(1+m)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 66, normalized size = 1.74

$$\frac{\sqrt{a}\,\sqrt{\frac{bx^{2m+2}}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x^{m+1}}{\sqrt{a}}\right)}{\sqrt{b}\,(m+1)\sqrt{a+bx^{2m+2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/Sqrt[a + b*x^(2 + 2*m)],x]
```

```
[Out] (Sqrt[a]*Sqrt[1 + (b*x^(2 + 2*m))/a]*ArcSinh[(Sqrt[b]*x^(1 + m))/Sqrt[a]])/
(Sqrt[b]*(1 + m)*Sqrt[a + b*x^(2 + 2*m)])
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^(2+2*m))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{bx^{2\,m+2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^(2+2*m))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/sqrt(b*x^(2*m + 2) + a), x)
```

maple [**F**] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{b\,x^{2m+2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^(2*m+2)+a)^(1/2),x)
```

```
[Out] int(x^m/(b*x^(2*m+2)+a)^(1/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{bx^{2\,m+2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^(2+2*m))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/sqrt(b*x^(2*m + 2) + a), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{\sqrt{a+b\,x^{2\,m+2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^(2*m + 2))^(1/2),x)
```

```
[Out] int(x^m/(a + b*x^(2*m + 2))^(1/2), x)
```

sympy [C] time = 3.42, size = 117, normalized size = 3.08

$$\frac{\sqrt{\pi}\, x x^{m}\, {}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{1}{2} \\ \frac{m}{2m+2}+1+\frac{1}{2m+2}\end{matrix}\middle| \frac{b x^2 x^{2m} e^{i\pi}}{a}\right)}{2a^{\frac{m}{2m+2}} a^{\frac{1}{2m+2}} m \Gamma\left(\frac{m}{2m+2}+1+\frac{1}{2m+2}\right) + 2a^{\frac{m}{2m+2}} a^{\frac{1}{2m+2}} \Gamma\left(\frac{m}{2m+2}+1+\frac{1}{2m+2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a+b*x**(2+2*m))**(1/2),x)
```

```
[Out] sqrt(pi)*x*x**m*hyper((1/2, 1/2), (m/(2*m + 2) + 1 + 1/(2*m + 2),), b*x**2*
x**(2*m)*exp_polar(I*pi)/a)/(2*a**(m/(2*m + 2))*a**(1/(2*m + 2))*m*gamma(m/
(2*m + 2) + 1 + 1/(2*m + 2)) + 2*a**(m/(2*m + 2))*a**(1/(2*m + 2))*gamma(m/
(2*m + 2) + 1 + 1/(2*m + 2)))
```

3.2750 $\int \frac{x^m}{\left(a+bx^{2+2m}\right)^{3/2}}\,dx$

Optimal. Leaf size=29

$$\frac{x^{m+1}}{a(m+1)\sqrt{a+bx^{2(m+1)}}}$$

```
[Out] x^(1+m)/a/(1+m)/(a+b*x^(2+2*m))^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {264}

$$\frac{x^{m+1}}{a(m+1)\sqrt{a+bx^{2(m+1)}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x^(2 + 2*m))^(3/2),x]
```

```
[Out] x^(1 + m)/(a*(1 + m)*Sqrt[a + b*x^(2*(1 + m))])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^m}{\left(a+bx^{2+2m}\right)^{3/2}}\,dx = \frac{x^{1+m}}{a(1+m)\sqrt{a+bx^{2(1+m)}}}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{x^{m+1}}{a(m+1)\sqrt{a+bx^{2m+2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x^(2 + 2*m))^(3/2),x]
```

```
[Out] x^(1 + m)/(a*(1 + m)*Sqrt[a + b*x^(2 + 2*m)])
```

fricas [A] time = 0.58, size = 47, normalized size = 1.62

$$\frac{\sqrt{bx^2x^{2\,m}+a}\,xx^m}{(abm+ab)x^2x^{2\,m}+a^2m+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^(2+2*m))^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*x^2*x^(2*m) + a)*x*x^m/((a*b*m + a*b)*x^2*x^(2*m) + a^2*m + a^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(bx^{2\,m+2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^(2+2*m))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*x^(2*m + 2) + a)^(3/2), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(b\,x^{2m+2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^(2*m+2)+a)^(3/2),x)
```

```
[Out] int(x^m/(b*x^(2*m+2)+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(bx^{2\,m+2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^(2+2*m))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(b*x^(2*m + 2) + a)^(3/2), x)
```

mupad [B] time = 1.22, size = 27, normalized size = 0.93

$$\frac{x^{m+1}}{a\,\sqrt{a+b\,x^{2\,m+2}}\;(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^(2*m + 2))^(3/2),x)
```

```
[Out] x^(m + 1)/(a*(a + b*x^(2*m + 2))^(1/2)*(m + 1))
```

sympy [C] time = 21.63, size = 121, normalized size = 4.17

$$\frac{\sqrt{\pi}\,x x^m\,{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{1}{2}\\ \frac{m}{2m+2}+1+\frac{1}{2m+2}\end{matrix}\middle|\,\frac{bx^2x^{2m}e^{i\pi}}{a}\right)}{2aa^{\frac{m}{2m+2}}a^{\frac{1}{2m+2}}m\Gamma\left(\frac{m}{2m+2}+1+\frac{1}{2m+2}\right)+2aa^{\frac{m}{2m+2}}a^{\frac{1}{2m+2}}\Gamma\left(\frac{m}{2m+2}+1+\frac{1}{2m+2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a+b*x**(2+2*m))**(3/2),x)
```

```
[Out] sqrt(pi)*x*x**m*hyper((3/2, 1/2), (m/(2*m + 2) + 1 + 1/(2*m + 2),), b*x**2*
x**(2*m)*exp_polar(I*pi)/a)/(2*a*a**(m/(2*m + 2))*a**(1/(2*m + 2))*m*gamma(
m/(2*m + 2) + 1 + 1/(2*m + 2)) + 2*a*a**(m/(2*m + 2))*a**(1/(2*m + 2))*gamm
a(m/(2*m + 2) + 1 + 1/(2*m + 2)))
```

3.2751 $\int \frac{x^m}{\left(a+bx^{2+2m}\right)^{5/2}}\,dx$

Optimal. Leaf size=65

$$\frac{2bx^{3(m+1)}}{3a^2(m+1)\left(a+bx^{2(m+1)}\right)^{3/2}}+\frac{x^{m+1}}{a(m+1)\left(a+bx^{2(m+1)}\right)^{3/2}}$$

```
[Out] x^(1+m)/a/(1+m)/(a+b*x^(2+2*m))^(3/2)+2/3*b*x^(3+3*m)/a^2/(1+m)/(a+b*x^(2+2
*m))^(3/2)
```

Rubi [A] time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {271, 264}

$$\frac{2bx^{3(m+1)}}{3a^2(m+1)\left(a+bx^{2(m+1)}\right)^{3/2}}+\frac{x^{m+1}}{a(m+1)\left(a+bx^{2(m+1)}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x^(2 + 2*m))^(5/2),x]
```

```
[Out] x^(1 + m)/(a*(1 + m)*(a + b*x^(2*(1 + m)))^(3/2)) + (2*b*x^(3*(1 + m)))/(3*
a^2*(1 + m)*(a + b*x^(2*(1 + m)))^(3/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{\left(a+bx^{2+2m}\right)^{5/2}}\,dx &= \frac{x^{1+m}}{a(1+m)\left(a+bx^{2(1+m)}\right)^{3/2}}+\frac{(2b)\int \frac{x^{2+3m}}{\left(a+bx^{2+2m}\right)^{5/2}}\,dx}{a}\\ &= \frac{x^{1+m}}{a(1+m)\left(a+bx^{2(1+m)}\right)^{3/2}}+\frac{2bx^{3(1+m)}}{3a^2(1+m)\left(a+bx^{2(1+m)}\right)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 46, normalized size = 0.71

$$\frac{x^{m+1}\left(3a+2bx^{2m+2}\right)}{3a^2(m+1)\left(a+bx^{2m+2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x^(2 + 2*m))^(5/2),x]
```

```
[Out] (x^(1 + m)*(3*a + 2*b*x^(2 + 2*m)))/(3*a^2*(1 + m)*(a + b*x^(2 + 2*m))^(3/2
))
```

fricas [A] time = 0.57, size = 93, normalized size = 1.43

$$\frac{\left(2\,bx^3x^{3\,m}+3\,axx^m\right)\sqrt{bx^2x^{2\,m}+a}}{3\left(\left(a^2b^2m+a^2b^2\right)x^4x^{4\,m}+a^4m+a^4+2\left(a^3bm+a^3b\right)x^2x^{2\,m}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^(2+2*m))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*b*x^3*x^(3*m) + 3*a*x*x^m)*sqrt(b*x^2*x^(2*m) + a)/((a^2*b^2*m + a^2
*b^2)*x^4*x^(4*m) + a^4*m + a^4 + 2*(a^3*b*m + a^3*b)*x^2*x^(2*m))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(bx^{2\,m+2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^(2+2*m))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*x^(2*m + 2) + a)^(5/2), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(b\,x^{2m+2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^(2*m+2)+a)^(5/2),x)
```

```
[Out] int(x^m/(b*x^(2*m+2)+a)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(bx^{2\,m+2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^(2+2*m))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(b*x^(2*m + 2) + a)^(5/2), x)
```

mupad [B] time = 1.24, size = 39, normalized size = 0.60

$$\frac{x^{m+1}\left(a+\frac{2\,b\,x^{2\,m+2}}{3}\right)}{a^2\left(a+b\,x^{2\,m+2}\right)^{3/2}\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^(2*m + 2))^(5/2),x)
```

```
[Out] (x^(m + 1)*(a + (2*b*x^(2*m + 2))/3))/(a^2*(a + b*x^(2*m + 2))^(3/2)*(m + 1
))
```

sympy [**F**(**-1**)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a+b*x**(2+2*m))**(5/2),x)
[Out] Timed out
```

3.2752 $\int \frac{x^m}{\left(a+bx^{2+2m}\right)^{7/2}}\,dx$

Optimal. Leaf size=102

$$\frac{8b^2x^{5(m+1)}}{15a^3(m+1)\left(a+bx^{2(m+1)}\right)^{5/2}}+\frac{4bx^{3(m+1)}}{3a^2(m+1)\left(a+bx^{2(m+1)}\right)^{5/2}}+\frac{x^{m+1}}{a(m+1)\left(a+bx^{2(m+1)}\right)^{5/2}}$$

```
[Out] x^(1+m)/a/(1+m)/(a+b*x^(2+2*m))^(5/2)+4/3*b*x^(3+3*m)/a^2/(1+m)/(a+b*x^(2+2
*m))^(5/2)+8/15*b^2*x^(5+5*m)/a^3/(1+m)/(a+b*x^(2+2*m))^(5/2)
```

Rubi [A] time = 0.05, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {271, 264}

$$\frac{8b^2x^{5(m+1)}}{15a^3(m+1)\left(a+bx^{2(m+1)}\right)^{5/2}}+\frac{4bx^{3(m+1)}}{3a^2(m+1)\left(a+bx^{2(m+1)}\right)^{5/2}}+\frac{x^{m+1}}{a(m+1)\left(a+bx^{2(m+1)}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x^(2 + 2*m))^(7/2),x]
```

```
[Out] x^(1 + m)/(a*(1 + m)*(a + b*x^(2*(1 + m)))^(5/2)) + (4*b*x^(3*(1 + m)))/(3*
a^2*(1 + m)*(a + b*x^(2*(1 + m)))^(5/2)) + (8*b^2*x^(5*(1 + m)))/(15*a^3*(1
 + m)*(a + b*x^(2*(1 + m)))^(5/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{\left(a+bx^{2+2m}\right)^{7/2}}\,dx &= \frac{x^{1+m}}{a(1+m)\left(a+bx^{2(1+m)}\right)^{5/2}}+\frac{(4b)\int \frac{x^{2+3m}}{\left(a+bx^{2+2m}\right)^{7/2}}\,dx}{a}\\ &= \frac{x^{1+m}}{a(1+m)\left(a+bx^{2(1+m)}\right)^{5/2}}+\frac{4bx^{3(1+m)}}{3a^2(1+m)\left(a+bx^{2(1+m)}\right)^{5/2}}+\frac{\left(8b^2\right)\int \frac{x^{4+5m}}{\left(a+bx^{2+2m}\right)^{7/2}}\,dx}{3a^2}\\ &= \frac{x^{1+m}}{a(1+m)\left(a+bx^{2(1+m)}\right)^{5/2}}+\frac{4bx^{3(1+m)}}{3a^2(1+m)\left(a+bx^{2(1+m)}\right)^{5/2}}+\frac{8b^2x^{5(1+m)}}{15a^3(1+m)\left(a+bx^{2(1+m)}\right)^{5/}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 61, normalized size = 0.60

$$\frac{x^{m+1}\left(15a^2+20abx^{2m+2}+8b^2x^{4m+4}\right)}{15a^3(m+1)\left(a+bx^{2m+2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x^(2 + 2*m))^(7/2),x]
```

```
[Out] (x^(1 + m)*(15*a^2 + 20*a*b*x^(2 + 2*m) + 8*b^2*x^(4 + 4*m)))/(15*a^3*(1 +
m)*(a + b*x^(2 + 2*m))^(5/2))
```

fricas [A] time = 0.80, size = 135, normalized size = 1.32

$$\frac{\left(8\,b^2x^5x^{5\,m}+20\,abx^3x^{3\,m}+15\,a^2xx^m\right)\sqrt{bx^2x^{2\,m}+a}}{15\left(\left(a^3b^3m+a^3b^3\right)x^6x^{6\,m}+a^6m+a^6+3\left(a^4b^2m+a^4b^2\right)x^4x^{4\,m}+3\left(a^5bm+a^5b\right)x^2x^{2\,m}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^(2+2*m))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(8*b^2*x^5*x^(5*m) + 20*a*b*x^3*x^(3*m) + 15*a^2*x*x^m)*sqrt(b*x^2*x^(
2*m) + a)/((a^3*b^3*m + a^3*b^3)*x^6*x^(6*m) + a^6*m + a^6 + 3*(a^4*b^2*m +
 a^4*b^2)*x^4*x^(4*m) + 3*(a^5*b*m + a^5*b)*x^2*x^(2*m))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(bx^{2\,m+2}+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^(2+2*m))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*x^(2*m + 2) + a)^(7/2), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(b\,x^{2m+2}+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x^(2*m+2)+a)^(7/2),x)
```

```
[Out] int(x^m/(b*x^(2*m+2)+a)^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(bx^{2\,m+2}+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+b*x^(2+2*m))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(b*x^(2*m + 2) + a)^(7/2), x)
```

mupad [B] time = 1.28, size = 54, normalized size = 0.53

$$\frac{x^{m+1}\left(\frac{8\,b^2\,x^{4\,m+4}}{15}+a^2+\frac{4\,a\,b\,x^{2\,m+2}}{3}\right)}{a^3\left(a+b\,x^{2\,m+2}\right)^{5/2}\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^(2*m + 2))^(7/2),x)
```

```
[Out] (x^(m + 1)*((8*b^2*x^(4*m + 4))/15 + a^2 + (4*a*b*x^(2*m + 2))/3))/(a^3*(a
+ b*x^(2*m + 2))^(5/2)*(m + 1))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a+b*x**(2+2*m))**(7/2),x)
```

```
[Out] Timed out
```

3.2753 $\int x^n \sqrt{1+x^{1+n}}\, dx$

Optimal. Leaf size=20

$$\frac{2\left(x^{n+1}+1\right)^{3/2}}{3(n+1)}$$

```
[Out] 2/3*(1+x^(1+n))^(3/2)/(1+n)
```

Rubi [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {261}

$$\frac{2\left(x^{n+1}+1\right)^{3/2}}{3(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^n*Sqrt[1 + x^(1 + n)],x]
```

```
[Out] (2*(1 + x^(1 + n))^(3/2))/(3*(1 + n))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^n \sqrt{1+x^{1+n}}\, dx = \frac{2\left(1+x^{1+n}\right)^{3/2}}{3(1+n)}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 1.00

$$\frac{2\left(x^{n+1}+1\right)^{3/2}}{3(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^n*Sqrt[1 + x^(1 + n)],x]
```

```
[Out] (2*(1 + x^(1 + n))^(3/2))/(3*(1 + n))
```

fricas [A] time = 0.65, size = 16, normalized size = 0.80

$$\frac{2\left(x^{n+1}+1\right)^{\frac{3}{2}}}{3\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(1+x^(1+n))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(x^(n + 1) + 1)^(3/2)/(n + 1)
```

giac [A] time = 0.15, size = 16, normalized size = 0.80

$$\frac{2\left(x^{n+1}+1\right)^{\frac{3}{2}}}{3\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(1+x^(1+n))^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(x^(n + 1) + 1)^(3/2)/(n + 1)
```

maple [A] time = 0.03, size = 17, normalized size = 0.85

$$\frac{2\left(x\,x^{n}+1\right)^{\frac{3}{2}}}{3\left(n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*(1+x^(n+1))^(1/2),x)
```

```
[Out] 2/3*(1+x*x^n)^(3/2)/(n+1)
```

maxima [A] time = 0.55, size = 16, normalized size = 0.80

$$\frac{2\left(x^{n+1}+1\right)^{\frac{3}{2}}}{3\left(n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(1+x^(1+n))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(x^(n + 1) + 1)^(3/2)/(n + 1)
```

mupad [B] time = 1.25, size = 18, normalized size = 0.90

$$\frac{2\left(x^{n+1}+1\right)^{3/2}}{3\,\left(n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*(x^(n + 1) + 1)^(1/2),x)
```

```
[Out] (2*(x^(n + 1) + 1)^(3/2))/(3*(n + 1))
```

sympy [A] time = 4.45, size = 48, normalized size = 2.40

$$\begin{cases} \frac{2xx^{n}\sqrt{xx^{n}+1}}{3n+3}+\frac{2\sqrt{xx^{n}+1}}{3n+3} & \text{for } n\neq -1 \\ \sqrt{2}\,\log\left(x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**n*(1+x**(1+n))**(1/2),x)
```

```
[Out] Piecewise((2*x*x**n*sqrt(x*x**n + 1)/(3*n + 3) + 2*sqrt(x*x**n + 1)/(3*n +
3), Ne(n, -1)), (sqrt(2)*log(x), True))
```

3.2754 $\int x^n \sqrt{a^2 + x^{1+n}}\, dx$

Optimal. Leaf size=22

$$\frac{2\left(a^2 + x^{n+1}\right)^{3/2}}{3(n+1)}$$

```
[Out] 2/3*(a^2+x^(1+n))^(3/2)/(1+n)
```

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {261}

$$\frac{2\left(a^2 + x^{n+1}\right)^{3/2}}{3(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^n*Sqrt[a^2 + x^(1 + n)],x]
```

```
[Out] (2*(a^2 + x^(1 + n))^(3/2))/(3*(1 + n))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^n \sqrt{a^2 + x^{1+n}}\, dx = \frac{2\left(a^2 + x^{1+n}\right)^{3/2}}{3(1+n)}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{2\left(a^2 + x^{n+1}\right)^{3/2}}{3(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^n*Sqrt[a^2 + x^(1 + n)],x]
```

```
[Out] (2*(a^2 + x^(1 + n))^(3/2))/(3*(1 + n))
```

fricas [A] time = 0.62, size = 18, normalized size = 0.82

$$\frac{2\left(a^2 + x^{n+1}\right)^{\frac{3}{2}}}{3\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(a^2+x^(1+n))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(a^2 + x^(n + 1))^(3/2)/(n + 1)
```

giac [A] time = 0.16, size = 18, normalized size = 0.82

$$\frac{2\left(a^2 + x^{n+1}\right)^{\frac{3}{2}}}{3\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(a^2+x^(1+n))^(1/2),x, algorithm="giac")
[Out] 2/3*(a^2 + x^(n + 1))^(3/2)/(n + 1)
```

maple [A] time = 0.03, size = 19, normalized size = 0.86

$$\frac{2\left(a^2+x\,x^n\right)^{\frac{3}{2}}}{3\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*(a^2+x^(n+1))^(1/2),x)
[Out] 2/3*(a^2+x*x^n)^(3/2)/(n+1)
```

maxima [A] time = 0.49, size = 18, normalized size = 0.82

$$\frac{2\left(a^2+x^{n+1}\right)^{\frac{3}{2}}}{3\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(a^2+x^(1+n))^(1/2),x, algorithm="maxima")
[Out] 2/3*(a^2 + x^(n + 1))^(3/2)/(n + 1)
```

mupad [B] time = 1.23, size = 20, normalized size = 0.91

$$\frac{2\left(x^{n+1}+a^2\right)^{3/2}}{3\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*(x^(n + 1) + a^2)^(1/2),x)
[Out] (2*(x^(n + 1) + a^2)^(3/2))/(3*(n + 1))
```

sympy [A] time = 4.39, size = 58, normalized size = 2.64

$$\begin{cases} \frac{2a^2\sqrt{a^2+xx^n}}{3n+3}+\frac{2xx^n\sqrt{a^2+xx^n}}{3n+3} & \text{for } n\neq -1 \\ \sqrt{a^2+1}\,\log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**n*(a**2+x**(1+n))**(1/2),x)
[Out] Piecewise((2*a**2*sqrt(a**2 + x*x**n)/(3*n + 3) + 2*x*x**n*sqrt(a**2 + x*x*
**n)/(3*n + 3), Ne(n, -1)), (sqrt(a**2 + 1)*log(x), True))
```

3.2755 $\int (cx)^m (a+bx^n)^2\,dx$

Optimal. Leaf size=64

$$\frac{a^2(cx)^{m+1}}{c(m+1)}+\frac{2abx^{n+1}(cx)^m}{m+n+1}+\frac{b^2x^{2n+1}(cx)^m}{m+2n+1}$$

```
[Out] 2*a*b*x^(1+n)*(c*x)^m/(1+m+n)+b^2*x^(1+2*n)*(c*x)^m/(1+m+2*n)+a^2*(c*x)^(1+
m)/c/(1+m)
```

Rubi [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {270, 20, 30}

$$\frac{a^2(cx)^{m+1}}{c(m+1)}+\frac{2abx^{n+1}(cx)^m}{m+n+1}+\frac{b^2x^{2n+1}(cx)^m}{m+2n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*(a + b*x^n)^2,x]
```

```
[Out] (2*a*b*x^(1 + n)*(c*x)^m)/(1 + m + n) + (b^2*x^(1 + 2*n)*(c*x)^m)/(1 + m +
2*n) + (a^2*(c*x)^(1 + m))/(c*(1 + m))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (cx)^m (a+bx^n)^2\,dx &= \int \left(a^2(cx)^m+2abx^n(cx)^m+b^2x^{2n}(cx)^m\right)dx\\ &= \frac{a^2(cx)^{1+m}}{c(1+m)}+(2ab)\int x^n(cx)^m\,dx+b^2\int x^{2n}(cx)^m\,dx\\ &= \frac{a^2(cx)^{1+m}}{c(1+m)}+(2abx^{-m}(cx)^m)\int x^{m+n}\,dx+\left(b^2x^{-m}(cx)^m\right)\int x^{m+2n}\,dx\\ &= \frac{2abx^{1+n}(cx)^m}{1+m+n}+\frac{b^2x^{1+2n}(cx)^m}{1+m+2n}+\frac{a^2(cx)^{1+m}}{c(1+m)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 47, normalized size = 0.73

$$x(cx)^m\left(\frac{a^2}{m+1}+\frac{2abx^n}{m+n+1}+\frac{b^2x^{2n}}{m+2n+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*(a + b*x^n)^2,x]
```

```
[Out] x*(c*x)^m*(a^2/(1 + m) + (2*a*b*x^n)/(1 + m + n) + (b^2*x^(2*n))/(1 + m + 2
*n))
```

fricas [B] time = 0.64, size = 172, normalized size = 2.69

$$\frac{\left(b^2m^2+2\,b^2m+b^2+\left(b^2m+b^2\right)n\right)xx^{2\,n}e^{\left(m\log(c)+m\log(x)\right)}+2\left(abm^2+2\,abm+ab+2\,(abm+ab)n\right)xx^{n}e^{\left(m\log(c)+m\right.}}{m^3+2\,(m+1)n^2+3\,m^2+3\left(m^2+2\,m+1\right)n+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*m^2 + 2*b^2*m + b^2 + (b^2*m + b^2)*n)*x*x^(2*n)*e^(m*log(c) + m*log(
x)) + 2*(a*b*m^2 + 2*a*b*m + a*b + 2*(a*b*m + a*b)*n)*x*x^n*e^(m*log(c) + m
*log(x)) + (a^2*m^2 + 2*a^2*n^2 + 2*a^2*m + a^2 + 3*(a^2*m + a^2)*n)*x*e^(m
*log(c) + m*log(x)))/(m^3 + 2*(m + 1)*n^2 + 3*m^2 + 3*(m^2 + 2*m + 1)*n + 3
*m + 1)
```

giac [B] time = 0.20, size = 613, normalized size = 9.58

$$\frac{b^2m^2xx^{2\,n}e^{\left(m\log(c)+m\log(x)\right)}+b^2mnxx^{2\,n}e^{\left(m\log(c)+m\log(x)\right)}+2\,abm^2xx^{n}e^{\left(m\log(c)+m\log(x)\right)}+b^2m^2xx^{n}e^{\left(m\log(c)+m\log(x)\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] (b^2*m^2*x*x^(2*n)*e^(m*log(c) + m*log(x)) + b^2*m*n*x*x^(2*n)*e^(m*log(c)
 + m*log(x)) + 2*a*b*m^2*x*x^n*e^(m*log(c) + m*log(x)) + b^2*m^2*x*x^n*e^(m*
log(c) + m*log(x)) + 4*a*b*m*n*x*x^n*e^(m*log(c) + m*log(x)) + b^2*m*n*x*x^
n*e^(m*log(c) + m*log(x)) + a^2*m^2*x*e^(m*log(c) + m*log(x)) + 2*a*b*m^2*x
*e^(m*log(c) + m*log(x)) + b^2*m^2*x*e^(m*log(c) + m*log(x)) + 3*a^2*m*n*x*
e^(m*log(c) + m*log(x)) + 4*a*b*m*n*x*e^(m*log(c) + m*log(x)) + b^2*m*n*x*e
^(m*log(c) + m*log(x)) + 2*a^2*n^2*x*e^(m*log(c) + m*log(x)) + 2*b^2*m*x*x^
(2*n)*e^(m*log(c) + m*log(x)) + b^2*n*x*x^(2*n)*e^(m*log(c) + m*log(x)) + 4
*a*b*m*x*x^n*e^(m*log(c) + m*log(x)) + 2*b^2*m*x*x^n*e^(m*log(c) + m*log(x)
) + 4*a*b*n*x*x^n*e^(m*log(c) + m*log(x)) + b^2*n*x*x^n*e^(m*log(c) + m*log
(x)) + 2*a^2*m*x*e^(m*log(c) + m*log(x)) + 4*a*b*m*x*e^(m*log(c) + m*log(x)
) + 2*b^2*m*x*e^(m*log(c) + m*log(x)) + 3*a^2*n*x*e^(m*log(c) + m*log(x)) +
 4*a*b*n*x*e^(m*log(c) + m*log(x)) + b^2*n*x*e^(m*log(c) + m*log(x)) + b^2*
x*x^(2*n)*e^(m*log(c) + m*log(x)) + 2*a*b*x*x^n*e^(m*log(c) + m*log(x)) + b
^2*x*x^n*e^(m*log(c) + m*log(x)) + a^2*x*e^(m*log(c) + m*log(x)) + 2*a*b*x*
e^(m*log(c) + m*log(x)) + b^2*x*e^(m*log(c) + m*log(x)))/(m^3 + 3*m^2*n + 2
*m*n^2 + 3*m^2 + 6*m*n + 2*n^2 + 3*m + 3*n + 1)
```

maple [C] time = 0.06, size = 234, normalized size = 3.66

$$\frac{\left(2ab\,m^2x^{n}+4abmn\,x^{n}+b^2m^2x^{2n}+b^2mn\,x^{2n}+a^2m^2+3a^2mn+2a^2n^2+4abm\,x^{n}+4abn\,x^{n}+2b^2m\,x^{2n}+b^2n\right.}{(m+1)\,(m+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(b*x^n+a)^2,x)
```

```
[Out] x*(b^2*m^2*(x^n)^2+b^2*m*n*(x^n)^2+2*a*b*m^2*x^n+4*a*b*m*n*x^n+2*m*b^2*(x^n
)^2+b^2*n*(x^n)^2+a^2*m^2+3*a^2*m*n+2*a^2*n^2+4*m*a*b*x^n+4*a*b*n*x^n+b^2*(
```

```
x^n)^2+2*m*a^2+3*a^2*n+2*a*b*x^n+a^2)/(m+1)/(m+n+1)/(m+2*n+1)*exp(1/2*m*(-I
*Pi*csgn(I*c*x)^3+I*Pi*csgn(I*c*x)^2*csgn(I*c)+I*Pi*csgn(I*c*x)^2*csgn(I*x)
-I*Pi*csgn(I*c*x)*csgn(I*c)*csgn(I*x)+2*ln(x)+2*ln(c)))
```

maxima [A] time = 0.48, size = 71, normalized size = 1.11

$$\frac{b^2 c^m x e^{\left(m \log(x)+2 n \log(x)\right)}}{m+2n+1}+\frac{2 a b c^m x e^{\left(m \log(x)+n \log(x)\right)}}{m+n+1}+\frac{(c x)^{m+1} a^2}{c(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] b^2*c^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + 2*a*b*c^m*x*e^(m*log(x)
 + n*log(x))/(m + n + 1) + (c*x)^(m + 1)*a^2/(c*(m + 1))
```

mupad [B] time = 1.24, size = 89, normalized size = 1.39

$$(c\,x)^m\left(\frac{a^2\,x}{m+1}+\frac{b^2\,x\,x^{2\,n}\,(m+n+1)}{m^2+3\,m\,n+2\,m+2\,n^2+3\,n+1}+\frac{2\,a\,b\,x\,x^n\,(m+2\,n+1)}{m^2+3\,m\,n+2\,m+2\,n^2+3\,n+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(a + b*x^n)^2,x)
```

```
[Out] (c*x)^m*((a^2*x)/(m + 1) + (b^2*x*x^(2*n)*(m + n + 1))/(2*m + 3*n + 3*m*n +
 m^2 + 2*n^2 + 1) + (2*a*b*x*x^n*(m + 2*n + 1))/(2*m + 3*n + 3*m*n + m^2 +
2*n^2 + 1))
```

sympy [A] time = 25.85, size = 1290, normalized size = 20.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(a+b*x**n)**2,x)
```

```
[Out] Piecewise(((a + b)**2*log(x)/c, Eq(m, -1) & Eq(n, 0)), ((a**2*log(x) + 2*a*
b*x**n/n + b**2*x**(2*n)/(2*n))/c, Eq(m, -1)), (a**2*Piecewise((log(x), Eq(
n, 0)), (-x**(-2*n)*(0**(1/n))**(-2*n)/(2*n), Eq(c, 0**(1/n))), (-c**(-2*n)
*x**(-2*n)/(2*n), True))/c + 2*a*b*Piecewise((log(x), Eq(n, 0)), (-x**n/(2*
0**(1/n)*zoo**(1/n)*n*x**(2*n)*(0**(1/n))**(2*n) - n*x**(2*n)*(0**(1/n))**(
2*n)), Eq(c, 0**(1/n))), (-c**(-2*n)*x**(-n)/n, True))/c + b**2*Piecewise((
c**(-2*n)*log(x), Abs(x) < 1), (-c**(-2*n)*log(1/x), 1/Abs(x) < 1), (-c**(-
2*n)*meijerg(((), (1, 1)), ((0, 0), ()), x) + c**(-2*n)*meijerg(((1, 1), ()
), ((), (0, 0)), x), True))/c, Eq(m, -2*n - 1)), (a**2*Piecewise((-x**(-n)*
(0**(1/n))**(-n)/n, Eq(c, 0**(1/n))), (log(x), Eq(n, 0)), (-c**(-n)*x**(-n)
/n, True))/c + 2*a*b*Piecewise((c**(-n)*log(x), Abs(x) < 1), (-c**(-n)*log(
1/x), 1/Abs(x) < 1), (-c**(-n)*meijerg(((), (1, 1)), ((0, 0), ()), x) + c**
(-n)*meijerg(((1, 1), ()), ((), (0, 0)), x), True))/c + b**2*Piecewise((-x*
*(2*n)/(0**(1/n)*zoo**(1/n)*n*x**n*(0**(1/n))**n - 2*n*x**n*(0**(1/n))**n),
 Eq(c, 0**(1/n))), (log(x), Eq(n, 0)), (c**(-n)*x**n/n, True))/c, Eq(m, -n
- 1)), (a**2*c**m*m**2*x*x**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n
+ 3*m + 2*n**2 + 3*n + 1) + 3*a**2*c**m*m*n*x*x**m/(m**3 + 3*m**2*n + 3*m**
2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 2*a**2*c**m*m*x*x**m/(m**3
 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 2*a**2*
c**m*n**2*x*x**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**
2 + 3*n + 1) + 3*a**2*c**m*n*x*x**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 +
6*m*n + 3*m + 2*n**2 + 3*n + 1) + a**2*c**m*x*x**m/(m**3 + 3*m**2*n + 3*m**
2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 2*a*b*c**m*m**2*x*x**m*x**
n/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) +
4*a*b*c**m*m*n*x*x**m*x**n/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3
```

```
*m + 2*n**2 + 3*n + 1) + 4*a*b*c**m*m*x*x**m*x**n/(m**3 + 3*m**2*n + 3*m**2
 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 4*a*b*c**m*n*x*x**m*x**n/(m
**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 2*a*
b*c**m*x*x**m*x**n/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n
**2 + 3*n + 1) + b**2*c**m*m**2*x*x**m*x**(2*n)/(m**3 + 3*m**2*n + 3*m**2 +
 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + b**2*c**m*m*n*x*x**m*x**(2*n)
/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 2
*b**2*c**m*m*x*x**m*x**(2*n)/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n +
 3*m + 2*n**2 + 3*n + 1) + b**2*c**m*n*x*x**m*x**(2*n)/(m**3 + 3*m**2*n + 3
*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + b**2*c**m*x*x**m*x**(2
*n)/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1),
 True))
```

3.2756 $\int (cx)^m \left(a + bx^3\right)^2 \, dx$

Optimal. Leaf size=58

$$\frac{a^2 (cx)^{m+1}}{c(m+1)} + \frac{2ab(cx)^{m+4}}{c^4(m+4)} + \frac{b^2(cx)^{m+7}}{c^7(m+7)}$$

```
[Out] a^2*(c*x)^(1+m)/c/(1+m)+2*a*b*(c*x)^(4+m)/c^4/(4+m)+b^2*(c*x)^(7+m)/c^7/(7+
m)
```

Rubi [A] time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {270}

$$\frac{a^2 (cx)^{m+1}}{c(m+1)} + \frac{2ab(cx)^{m+4}}{c^4(m+4)} + \frac{b^2(cx)^{m+7}}{c^7(m+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*(a + b*x^3)^2,x]
```

```
[Out] (a^2*(c*x)^(1 + m))/(c*(1 + m)) + (2*a*b*(c*x)^(4 + m))/(c^4*(4 + m)) + (b^
2*(c*x)^(7 + m))/(c^7*(7 + m))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (cx)^m \left(a + bx^3\right)^2 \, dx &= \int \left(a^2 (cx)^m + \frac{2ab(cx)^{3+m}}{c^3} + \frac{b^2(cx)^{6+m}}{c^6}\right) dx \\ &= \frac{a^2 (cx)^{1+m}}{c(1+m)} + \frac{2ab(cx)^{4+m}}{c^4(4+m)} + \frac{b^2(cx)^{7+m}}{c^7(7+m)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 41, normalized size = 0.71

$$x(cx)^m \left(\frac{a^2}{m+1} + \frac{2abx^3}{m+4} + \frac{b^2x^6}{m+7}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*(a + b*x^3)^2,x]
```

```
[Out] x*(c*x)^m*(a^2/(1 + m) + (2*a*b*x^3)/(4 + m) + (b^2*x^6)/(7 + m))
```

fricas [A] time = 0.69, size = 87, normalized size = 1.50

$$\frac{\left(\left(b^2m^2 + 5\,b^2m + 4\,b^2\right)x^7 + 2\left(abm^2 + 8\,abm + 7\,ab\right)x^4 + \left(a^2m^2 + 11\,a^2m + 28\,a^2\right)x\right)(cx)^m}{m^3 + 12\,m^2 + 39\,m + 28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*m^2 + 5*b^2*m + 4*b^2)*x^7 + 2*(a*b*m^2 + 8*a*b*m + 7*a*b)*x^4 + (a^2
*m^2 + 11*a^2*m + 28*a^2)*x)*(c*x)^m/(m^3 + 12*m^2 + 39*m + 28)
```

giac [B] time = 0.19, size = 135, normalized size = 2.33

$$\frac{(cx)^m\, b^2m^2x^7 + 5\,(cx)^m\, b^2mx^7 + 4\,(cx)^m\, b^2x^7 + 2\,(cx)^m\, abm^2x^4 + 16\,(cx)^m\, abmx^4 + 14\,(cx)^m\, abx^4 + (cx)^m\, a^2m^2x^3}{m^3 + 12\,m^2 + 39\,m + 28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] ((c*x)^m*b^2*m^2*x^7 + 5*(c*x)^m*b^2*m*x^7 + 4*(c*x)^m*b^2*x^7 + 2*(c*x)^m*
a*b*m^2*x^4 + 16*(c*x)^m*a*b*m*x^4 + 14*(c*x)^m*a*b*x^4 + (c*x)^m*a^2*m^2*x
 + 11*(c*x)^m*a^2*m*x + 28*(c*x)^m*a^2*x)/(m^3 + 12*m^2 + 39*m + 28)
```

maple [A] time = 0.01, size = 94, normalized size = 1.62

$$\frac{\left(b^2m^2x^6 + 5b^2m\,x^6 + 4b^2x^6 + 2ab\,m^2x^3 + 16abm\,x^3 + 14ab\,x^3 + a^2m^2 + 11a^2m + 28a^2\right)x\,(cx)^m}{(m+7)\,(m+4)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(b*x^3+a)^2,x)
```

```
[Out] x*(b^2*m^2*x^6+5*b^2*m*x^6+4*b^2*x^6+2*a*b*m^2*x^3+16*a*b*m*x^3+14*a*b*x^3+
a^2*m^2+11*a^2*m+28*a^2)*(c*x)^m/(m+7)/(m+4)/(m+1)
```

maxima [A] time = 0.53, size = 56, normalized size = 0.97

$$\frac{b^2c^mx^7x^m}{m+7} + \frac{2\,abc^mx^4x^m}{m+4} + \frac{(cx)^{m+1}\,a^2}{c(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] b^2*c^m*x^7*x^m/(m + 7) + 2*a*b*c^m*x^4*x^m/(m + 4) + (c*x)^(m + 1)*a^2/(c*
(m + 1))
```

mupad [B] time = 1.22, size = 95, normalized size = 1.64

$$(c\,x)^m\left(\frac{a^2\,x\,\left(m^2+11\,m+28\right)}{m^3+12\,m^2+39\,m+28} + \frac{b^2\,x^7\,\left(m^2+5\,m+4\right)}{m^3+12\,m^2+39\,m+28} + \frac{2\,a\,b\,x^4\,\left(m^2+8\,m+7\right)}{m^3+12\,m^2+39\,m+28}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(a + b*x^3)^2,x)
```

```
[Out] (c*x)^m*((a^2*x*(11*m + m^2 + 28))/(39*m + 12*m^2 + m^3 + 28) + (b^2*x^7*(5
*m + m^2 + 4))/(39*m + 12*m^2 + m^3 + 28) + (2*a*b*x^4*(8*m + m^2 + 7))/(39
*m + 12*m^2 + m^3 + 28))
```

sympy [A] time = 1.39, size = 352, normalized size = 6.07

$$\begin{cases} \dfrac{-\frac{a^2}{6x^6} - \frac{2ab}{3x^3} + b^2\log(x)}{c^7} \\ \dfrac{-\frac{a^2}{3x^3} + 2ab\log(x) + \frac{b^2x^3}{3}}{c^4} \\ \dfrac{a^2\log(x) + \frac{2abx^3}{3} + \frac{b^2x^6}{6}}{c} \\ \dfrac{a^2c^mm^2xx^m}{m^3+12m^2+39m+28} + \dfrac{11a^2c^mmxx^m}{m^3+12m^2+39m+28} + \dfrac{28a^2c^mxx^m}{m^3+12m^2+39m+28} + \dfrac{2abc^mm^2x^4x^m}{m^3+12m^2+39m+28} + \dfrac{16abc^mmx^4x^m}{m^3+12m^2+39m+28} + \dfrac{14abc^mx^4x^m}{m^3+12m^2+39m+28} + \dfrac{}{m} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(b*x**3+a)**2,x)
```

```
[Out] Piecewise(((-a**2/(6*x**6) - 2*a*b/(3*x**3) + b**2*log(x))/c**7, Eq(m, -7))
, ((-a**2/(3*x**3) + 2*a*b*log(x) + b**2*x**3/3)/c**4, Eq(m, -4)), ((a**2*l
og(x) + 2*a*b*x**3/3 + b**2*x**6/6)/c, Eq(m, -1)), (a**2*c**m*m**2*x*x**m/(
m**3 + 12*m**2 + 39*m + 28) + 11*a**2*c**m*m*x*x**m/(m**3 + 12*m**2 + 39*m
+ 28) + 28*a**2*c**m*x*x**m/(m**3 + 12*m**2 + 39*m + 28) + 2*a*b*c**m*m**2*
x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) + 16*a*b*c**m*m*x**4*x**m/(m**3 + 12
*m**2 + 39*m + 28) + 14*a*b*c**m*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) + b
**2*c**m*m**2*x**7*x**m/(m**3 + 12*m**2 + 39*m + 28) + 5*b**2*c**m*m*x**7*x
**m/(m**3 + 12*m**2 + 39*m + 28) + 4*b**2*c**m*x**7*x**m/(m**3 + 12*m**2 +
39*m + 28), True))
```

3.2757 $\int (cx)^m \left(a + bx^2\right)^2 \, dx$

Optimal. Leaf size=58

$$\frac{a^2(cx)^{m+1}}{c(m+1)} + \frac{2ab(cx)^{m+3}}{c^3(m+3)} + \frac{b^2(cx)^{m+5}}{c^5(m+5)}$$

```
[Out] a^2*(c*x)^(1+m)/c/(1+m)+2*a*b*(c*x)^(3+m)/c^3/(3+m)+b^2*(c*x)^(5+m)/c^5/(5+
m)
```

Rubi [A] time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {270}

$$\frac{a^2(cx)^{m+1}}{c(m+1)} + \frac{2ab(cx)^{m+3}}{c^3(m+3)} + \frac{b^2(cx)^{m+5}}{c^5(m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*(a + b*x^2)^2,x]
```

```
[Out] (a^2*(c*x)^(1 + m))/(c*(1 + m)) + (2*a*b*(c*x)^(3 + m))/(c^3*(3 + m)) + (b^
2*(c*x)^(5 + m))/(c^5*(5 + m))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (cx)^m \left(a + bx^2\right)^2 \, dx &= \int \left(a^2(cx)^m + \frac{2ab(cx)^{2+m}}{c^2} + \frac{b^2(cx)^{4+m}}{c^4}\right) dx \\ &= \frac{a^2(cx)^{1+m}}{c(1+m)} + \frac{2ab(cx)^{3+m}}{c^3(3+m)} + \frac{b^2(cx)^{5+m}}{c^5(5+m)}\end{aligned}$$

Mathematica [A] time = 0.07, size = 41, normalized size = 0.71

$$x(cx)^m \left(\frac{a^2}{m+1} + \frac{2abx^2}{m+3} + \frac{b^2x^4}{m+5}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*(a + b*x^2)^2,x]
```

```
[Out] x*(c*x)^m*(a^2/(1 + m) + (2*a*b*x^2)/(3 + m) + (b^2*x^4)/(5 + m))
```

fricas [A] time = 0.63, size = 87, normalized size = 1.50

$$\frac{\left(\left(b^2m^2 + 4\,b^2m + 3\,b^2\right)x^5 + 2\left(abm^2 + 6\,abm + 5\,ab\right)x^3 + \left(a^2m^2 + 8\,a^2m + 15\,a^2\right)x\right)(cx)^m}{m^3 + 9\,m^2 + 23\,m + 15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*m^2 + 4*b^2*m + 3*b^2)*x^5 + 2*(a*b*m^2 + 6*a*b*m + 5*a*b)*x^3 + (a^2
*m^2 + 8*a^2*m + 15*a^2)*x)*(c*x)^m/(m^3 + 9*m^2 + 23*m + 15)
```

giac [B] time = 0.20, size = 135, normalized size = 2.33

$$\frac{(cx)^m b^2 m^2 x^5 + 4\,(cx)^m b^2 m x^5 + 2\,(cx)^m abm^2 x^3 + 3\,(cx)^m b^2 x^5 + 12\,(cx)^m abm x^3 + (cx)^m a^2 m^2 x + 10\,(cx)^m a}{m^3 + 9\,m^2 + 23\,m + 15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] ((c*x)^m*b^2*m^2*x^5 + 4*(c*x)^m*b^2*m*x^5 + 2*(c*x)^m*a*b*m^2*x^3 + 3*(c*x
)^m*b^2*x^5 + 12*(c*x)^m*a*b*m*x^3 + (c*x)^m*a^2*m^2*x + 10*(c*x)^m*a*b*x^3
 + 8*(c*x)^m*a^2*m*x + 15*(c*x)^m*a^2*x)/(m^3 + 9*m^2 + 23*m + 15)
```

maple [A] time = 0.00, size = 94, normalized size = 1.62

$$\frac{\left(b^2 m^2 x^4 + 4b^2 m\, x^4 + 2ab\, m^2 x^2 + 3b^2 x^4 + 12abm\, x^2 + a^2 m^2 + 10ab\, x^2 + 8a^2 m + 15a^2\right) x\,(cx)^m}{(m+5)\,(m+3)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(b*x^2+a)^2,x)
```

```
[Out] x*(b^2*m^2*x^4+4*b^2*m*x^4+2*a*b*m^2*x^2+3*b^2*x^4+12*a*b*m*x^2+a^2*m^2+10*
a*b*x^2+8*a^2*m+15*a^2)*(c*x)^m/(m+5)/(m+3)/(m+1)
```

maxima [A] time = 0.61, size = 56, normalized size = 0.97

$$\frac{b^2 c^m x^5 x^m}{m+5} + \frac{2\,abc^m x^3 x^m}{m+3} + \frac{(cx)^{m+1}\,a^2}{c(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] b^2*c^m*x^5*x^m/(m + 5) + 2*a*b*c^m*x^3*x^m/(m + 3) + (c*x)^(m + 1)*a^2/(c*
(m + 1))
```

mupad [B] time = 1.20, size = 95, normalized size = 1.64

$$(c\,x)^m \left(\frac{a^2\,x\,\left(m^2+8\,m+15\right)}{m^3+9\,m^2+23\,m+15} + \frac{b^2\,x^5\,\left(m^2+4\,m+3\right)}{m^3+9\,m^2+23\,m+15} + \frac{2\,a\,b\,x^3\,\left(m^2+6\,m+5\right)}{m^3+9\,m^2+23\,m+15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(a + b*x^2)^2,x)
```

```
[Out] (c*x)^m*((a^2*x*(8*m + m^2 + 15))/(23*m + 9*m^2 + m^3 + 15) + (b^2*x^5*(4*m
 + m^2 + 3))/(23*m + 9*m^2 + m^3 + 15) + (2*a*b*x^3*(6*m + m^2 + 5))/(23*m
+ 9*m^2 + m^3 + 15))
```

sympy [A] time = 0.90, size = 345, normalized size = 5.95

$$\begin{cases} \frac{-\frac{a^2}{4x^4}-\frac{ab}{x^2}+b^2\log(x)}{c^5} \\ \frac{-\frac{a^2}{2x^2}+2ab\log(x)+\frac{b^2x^2}{2}}{c^3} \\ \frac{a^2\log(x)+abx^2+\frac{b^2x^4}{4}}{c} \\ \frac{a^2c^m m^2 x x^m}{m^3+9m^2+23m+15} + \frac{8a^2c^m m x x^m}{m^3+9m^2+23m+15} + \frac{15a^2c^m x x^m}{m^3+9m^2+23m+15} + \frac{2abc^m m^2 x^3 x^m}{m^3+9m^2+23m+15} + \frac{12abc^m m x^3 x^m}{m^3+9m^2+23m+15} + \frac{10abc^m x^3 x^m}{m^3+9m^2+23m+15} + \frac{b^2}{m^3+9} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(b*x**2+a)**2,x)
```

```
[Out] Piecewise(((-a**2/(4*x**4) - a*b/x**2 + b**2*log(x))/c**5, Eq(m, -5)), ((-a
**2/(2*x**2) + 2*a*b*log(x) + b**2*x**2/2)/c**3, Eq(m, -3)), ((a**2*log(x)
+ a*b*x**2 + b**2*x**4/4)/c, Eq(m, -1)), (a**2*c**m*m**2*x*x**m/(m**3 + 9*m
**2 + 23*m + 15) + 8*a**2*c**m*m*x*x**m/(m**3 + 9*m**2 + 23*m + 15) + 15*a*
*2*c**m*x*x**m/(m**3 + 9*m**2 + 23*m + 15) + 2*a*b*c**m*m**2*x**3*x**m/(m**
3 + 9*m**2 + 23*m + 15) + 12*a*b*c**m*m*x**3*x**m/(m**3 + 9*m**2 + 23*m + 1
5) + 10*a*b*c**m*x**3*x**m/(m**3 + 9*m**2 + 23*m + 15) + b**2*c**m*m**2*x**
5*x**m/(m**3 + 9*m**2 + 23*m + 15) + 4*b**2*c**m*m*x**5*x**m/(m**3 + 9*m**2
 + 23*m + 15) + 3*b**2*c**m*x**5*x**m/(m**3 + 9*m**2 + 23*m + 15), True))
```

3.2758 $\int (cx)^m (a+bx)^2\,dx$

Optimal. Leaf size=58

$$\frac{a^2(cx)^{m+1}}{c(m+1)} + \frac{2ab(cx)^{m+2}}{c^2(m+2)} + \frac{b^2(cx)^{m+3}}{c^3(m+3)}$$

```
[Out] a^2*(c*x)^(1+m)/c/(1+m)+2*a*b*(c*x)^(2+m)/c^2/(2+m)+b^2*(c*x)^(3+m)/c^3/(3+
m)
```

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {43}

$$\frac{a^2(cx)^{m+1}}{c(m+1)} + \frac{2ab(cx)^{m+2}}{c^2(m+2)} + \frac{b^2(cx)^{m+3}}{c^3(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*(a + b*x)^2,x]
```

```
[Out] (a^2*(c*x)^(1 + m))/(c*(1 + m)) + (2*a*b*(c*x)^(2 + m))/(c^2*(2 + m)) + (b^
2*(c*x)^(3 + m))/(c^3*(3 + m))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (cx)^m (a+bx)^2\,dx &= \int \left(a^2(cx)^m + \frac{2ab(cx)^{1+m}}{c} + \frac{b^2(cx)^{2+m}}{c^2}\right) dx \\ &= \frac{a^2(cx)^{1+m}}{c(1+m)} + \frac{2ab(cx)^{2+m}}{c^2(2+m)} + \frac{b^2(cx)^{3+m}}{c^3(3+m)}\end{aligned}$$

Mathematica [A] time = 0.06, size = 39, normalized size = 0.67

$$x(cx)^m\left(\frac{a^2}{m+1} + \frac{2abx}{m+2} + \frac{b^2x^2}{m+3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*(a + b*x)^2,x]
```

```
[Out] x*(c*x)^m*(a^2/(1 + m) + (2*a*b*x)/(2 + m) + (b^2*x^2)/(3 + m))
```

fricas [A] time = 0.64, size = 87, normalized size = 1.50

$$\frac{\left(\left(b^2m^2 + 3\,b^2m + 2\,b^2\right)x^3 + 2\left(abm^2 + 4\,abm + 3\,ab\right)x^2 + \left(a^2m^2 + 5\,a^2m + 6\,a^2\right)x\right)(cx)^m}{m^3 + 6\,m^2 + 11\,m + 6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*m^2 + 3*b^2*m + 2*b^2)*x^3 + 2*(a*b*m^2 + 4*a*b*m + 3*a*b)*x^2 + (a^2
*m^2 + 5*a^2*m + 6*a^2)*x)*(c*x)^m/(m^3 + 6*m^2 + 11*m + 6)
```

giac [B] time = 0.16, size = 135, normalized size = 2.33

$$\frac{(cx)^m\, b^2m^2x^3 + 2\,(cx)^m\, abm^2x^2 + 3\,(cx)^m\, b^2mx^3 + (cx)^m\, a^2m^2x + 8\,(cx)^m\, abmx^2 + 2\,(cx)^m\, b^2x^3 + 5\,(cx)^m\, a^2mx}{m^3 + 6\,m^2 + 11\,m + 6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] ((c*x)^m*b^2*m^2*x^3 + 2*(c*x)^m*a*b*m^2*x^2 + 3*(c*x)^m*b^2*m*x^3 + (c*x)^
m*a^2*m^2*x + 8*(c*x)^m*a*b*m*x^2 + 2*(c*x)^m*b^2*x^3 + 5*(c*x)^m*a^2*m*x +
 6*(c*x)^m*a*b*x^2 + 6*(c*x)^m*a^2*x)/(m^3 + 6*m^2 + 11*m + 6)
```

maple [A] time = 0.00, size = 88, normalized size = 1.52

$$\frac{\left(b^2m^2x^2 + 2ab\,m^2x + 3b^2m\,x^2 + a^2m^2 + 8abmx + 2b^2x^2 + 5a^2m + 6abx + 6a^2\right)x\,(cx)^m}{(m+3)\,(m+2)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(b*x+a)^2,x)
```

```
[Out] x*(b^2*m^2*x^2+2*a*b*m^2*x+3*b^2*m*x^2+a^2*m^2+8*a*b*m*x+2*b^2*x^2+5*a^2*m+
6*a*b*x+6*a^2)*(c*x)^m/(m+3)/(m+2)/(m+1)
```

maxima [A] time = 0.51, size = 56, normalized size = 0.97

$$\frac{b^2c^mx^3x^m}{m+3} + \frac{2\,abc^mx^2x^m}{m+2} + \frac{(cx)^{m+1}\,a^2}{c(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] b^2*c^m*x^3*x^m/(m + 3) + 2*a*b*c^m*x^2*x^m/(m + 2) + (c*x)^(m + 1)*a^2/(c*
(m + 1))
```

mupad [B] time = 1.37, size = 95, normalized size = 1.64

$$(c\,x)^m\left(\frac{a^2\,x\,\left(m^2+5\,m+6\right)}{m^3+6\,m^2+11\,m+6} + \frac{b^2\,x^3\,\left(m^2+3\,m+2\right)}{m^3+6\,m^2+11\,m+6} + \frac{2\,a\,b\,x^2\,\left(m^2+4\,m+3\right)}{m^3+6\,m^2+11\,m+6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(a + b*x)^2,x)
```

```
[Out] (c*x)^m*((a^2*x*(5*m + m^2 + 6))/(11*m + 6*m^2 + m^3 + 6) + (b^2*x^3*(3*m +
 m^2 + 2))/(11*m + 6*m^2 + m^3 + 6) + (2*a*b*x^2*(4*m + m^2 + 3))/(11*m + 6
*m^2 + m^3 + 6))
```

sympy [A] time = 0.55, size = 338, normalized size = 5.83

$$\begin{cases} \frac{-\frac{a^2}{2x^2}-\frac{2ab}{x}+b^2\log(x)}{c^3} \\ \frac{-\frac{a^2}{x}+2ab\log(x)+b^2x}{c^2} \\ \frac{a^2\log(x)+2abx+\frac{b^2x^2}{2}}{c} \\ \frac{a^2c^mm^2xx^m}{m^3+6m^2+11m+6} + \frac{5a^2c^mmxx^m}{m^3+6m^2+11m+6} + \frac{6a^2c^mxx^m}{m^3+6m^2+11m+6} + \frac{2abc^mm^2x^2x^m}{m^3+6m^2+11m+6} + \frac{8abc^mmx^2x^m}{m^3+6m^2+11m+6} + \frac{6abc^mx^2x^m}{m^3+6m^2+11m+6} + \frac{b^2c^mm^2x^3x^m}{m^3+6m^2+11m+} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(b*x+a)**2,x)
```

```
[Out] Piecewise(((-a**2/(2*x**2) - 2*a*b/x + b**2*log(x))/c**3, Eq(m, -3)), ((-a*
**2/x + 2*a*b*log(x) + b**2*x)/c**2, Eq(m, -2)), ((a**2*log(x) + 2*a*b*x + b
**2*x**2/2)/c, Eq(m, -1)), (a**2*c**m*m**2*x*x**m/(m**3 + 6*m**2 + 11*m + 6
) + 5*a**2*c**m*m*x*x**m/(m**3 + 6*m**2 + 11*m + 6) + 6*a**2*c**m*x*x**m/(m
**3 + 6*m**2 + 11*m + 6) + 2*a*b*c**m*m**2*x**2*x**m/(m**3 + 6*m**2 + 11*m
+ 6) + 8*a*b*c**m*m*x**2*x**m/(m**3 + 6*m**2 + 11*m + 6) + 6*a*b*c**m*x**2*
x**m/(m**3 + 6*m**2 + 11*m + 6) + b**2*c**m*m**2*x**3*x**m/(m**3 + 6*m**2 +
 11*m + 6) + 3*b**2*c**m*m*x**3*x**m/(m**3 + 6*m**2 + 11*m + 6) + 2*b**2*c*
*m*x**3*x**m/(m**3 + 6*m**2 + 11*m + 6), True))
```

3.2759 $\int \left(a+\frac{b}{x}\right)^2 (cx)^m \, dx$

Optimal. Leaf size=52

$$\frac{a^2(cx)^{m+1}}{c(m+1)}+\frac{2ab(cx)^m}{m}-\frac{b^2c(cx)^{m-1}}{1-m}$$

```
[Out] -b^2*c*(c*x)^(-1+m)/(1-m)+2*a*b*(c*x)^m/m+a^2*(c*x)^(1+m)/c/(1+m)
```

Rubi [A] time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {270}

$$\frac{a^2(cx)^{m+1}}{c(m+1)}+\frac{2ab(cx)^m}{m}-\frac{b^2c(cx)^{m-1}}{1-m}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^2*(c*x)^m,x]
```

```
[Out] -((b^2*c*(c*x)^(-1 + m))/(1 - m)) + (2*a*b*(c*x)^m)/m + (a^2*(c*x)^(1 + m))
/(c*(1 + m))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^2 (cx)^m \, dx &= \int \left(b^2c^2(cx)^{-2+m}+2abc(cx)^{-1+m}+a^2(cx)^m\right) dx \\ &= -\frac{b^2c(cx)^{-1+m}}{1-m}+\frac{2ab(cx)^m}{m}+\frac{a^2(cx)^{1+m}}{c(1+m)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 36, normalized size = 0.69

$$(cx)^m\left(\frac{a^2x}{m+1}+\frac{2ab}{m}+\frac{b^2}{(m-1)x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^2*(c*x)^m,x]
```

```
[Out] (c*x)^m*((2*a*b)/m + b^2/((-1 + m)*x) + (a^2*x)/(1 + m))
```

fricas [A] time = 0.74, size = 63, normalized size = 1.21

$$\frac{\left(b^2m^2+b^2m+\left(a^2m^2-a^2m\right)x^2+2\left(abm^2-ab\right)x\right)(cx)^m}{\left(m^3-m\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2*(c*x)^m,x, algorithm="fricas")
```

```
[Out] (b^2*m^2 + b^2*m + (a^2*m^2 - a^2*m)*x^2 + 2*(a*b*m^2 - a*b)*x)*(c*x)^m/((m
^3 - m)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx)^m \left(a + \frac{b}{x}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2*(c*x)^m,x, algorithm="giac")
```

```
[Out] integrate((c*x)^m*(a + b/x)^2, x)
```

maple [A] time = 0.01, size = 68, normalized size = 1.31

$$\frac{\left(a^2x^2m^2 - a^2x^2m + 2ab\,m^2x + b^2m^2 - 2abx + b^2m\right)(cx)^m}{(m+1)(m-1)mx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^2*(c*x)^m,x)
```

```
[Out] (c*x)^m*(a^2*m^2*x^2-a^2*m*x^2+2*a*b*m^2*x+b^2*m^2-2*a*b*x+b^2*m)/x/(m+1)/m
/(m-1)
```

maxima [A] time = 0.58, size = 51, normalized size = 0.98

$$\frac{2\,abc^m x^m}{m} + \frac{b^2 c^m x^m}{(m-1)x} + \frac{(cx)^{m+1}\,a^2}{c(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^2*(c*x)^m,x, algorithm="maxima")
```

```
[Out] 2*a*b*c^m*x^m/m + b^2*c^m*x^m/((m - 1)*x) + (c*x)^(m + 1)*a^2/(c*(m + 1))
```

mupad [B] time = 1.21, size = 45, normalized size = 0.87

$$\frac{b^2\,(c\,x)^m}{x\,(m-1)} + \frac{a\,(c\,x)^m\,(2\,b + 2\,b\,m + a\,m\,x)}{m\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(a + b/x)^2,x)
```

```
[Out] (b^2*(c*x)^m)/(x*(m - 1)) + (a*(c*x)^m*(2*b + 2*b*m + a*m*x))/(m*(m + 1))
```

sympy [A] time = 0.57, size = 202, normalized size = 3.88

$$\begin{cases} \frac{a^2\log(x) - \frac{2ab}{x} - \frac{b^2}{2x^2}}{c} & \text{for } m = -1 \\ a^2x + 2ab\log(x) - \frac{b^2}{x} & \text{for } m = 0 \\ c\left(\frac{a^2x^2}{2} + 2abx + b^2\log(x)\right) & \text{for } m = 1 \\ \frac{a^2c^m m^2x^2x^m}{m^3x - mx} - \frac{a^2c^m mx^2x^m}{m^3x - mx} + \frac{2abc^m m^2xx^m}{m^3x - mx} - \frac{2abc^m xx^m}{m^3x - mx} + \frac{b^2c^m m^2x^m}{m^3x - mx} + \frac{b^2c^m mx^m}{m^3x - mx} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**2*(c*x)**m,x)
```

```
[Out] Piecewise(((a**2*log(x) - 2*a*b/x - b**2/(2*x**2))/c, Eq(m, -1)), (a**2*x +
 2*a*b*log(x) - b**2/x, Eq(m, 0)), (c*(a**2*x**2/2 + 2*a*b*x + b**2*log(x))
, Eq(m, 1)), (a**2*c**m*m**2*x**2*x**m/(m**3*x - m*x) - a**2*c**m*m*x**2*x*
*m/(m**3*x - m*x) + 2*a*b*c**m*m**2*x*x**m/(m**3*x - m*x) - 2*a*b*c**m*x*x*
*m/(m**3*x - m*x) + b**2*c**m*m**2*x**m/(m**3*x - m*x) + b**2*c**m*m*x**m/(
m**3*x - m*x), True))
```

3.2760 $\int \left(a+\frac{b}{x^2}\right)^2 (cx)^m\,dx$

Optimal. Leaf size=61

$$\frac{a^2(cx)^{m+1}}{c(m+1)}-\frac{2abc(cx)^{m-1}}{1-m}-\frac{b^2c^3(cx)^{m-3}}{3-m}$$

```
[Out] -b^2*c^3*(c*x)^(-3+m)/(3-m)-2*a*b*c*(c*x)^(-1+m)/(1-m)+a^2*(c*x)^(1+m)/c/(1
+m)
```

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {270}

$$\frac{a^2(cx)^{m+1}}{c(m+1)}-\frac{2abc(cx)^{m-1}}{1-m}-\frac{b^2c^3(cx)^{m-3}}{3-m}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^2*(c*x)^m,x]
```

```
[Out] -((b^2*c^3*(c*x)^(-3 + m))/(3 - m)) - (2*a*b*c*(c*x)^(-1 + m))/(1 - m) + (a
^2*(c*x)^(1 + m))/(c*(1 + m))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x^2}\right)^2 (cx)^m\,dx &= \int \left(b^2c^4(cx)^{-4+m}+2abc^2(cx)^{-2+m}+a^2(cx)^m\right)dx\\ &= -\frac{b^2c^3(cx)^{-3+m}}{3-m}-\frac{2abc(cx)^{-1+m}}{1-m}+\frac{a^2(cx)^{1+m}}{c(1+m)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 64, normalized size = 1.05

$$\frac{(cx)^m\left(a^2\left(m^2-4m+3\right)x^4+2ab\left(m^2-2m-3\right)x^2+b^2\left(m^2-1\right)\right)}{(m-3)(m-1)(m+1)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)^2*(c*x)^m,x]
```

```
[Out] ((c*x)^m*(b^2*(-1 + m^2) + 2*a*b*(-3 - 2*m + m^2)*x^2 + a^2*(3 - 4*m + m^2)
*x^4))/((-3 + m)*(-1 + m)*(1 + m)*x^3)
```

fricas [A] time = 0.78, size = 81, normalized size = 1.33

$$\frac{\left(\left(a^2m^2-4\,a^2m+3\,a^2\right)x^4+b^2m^2+2\left(abm^2-2\,abm-3\,ab\right)x^2-b^2\right)(cx)^m}{\left(m^3-3\,m^2-m+3\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2*(c*x)^m,x, algorithm="fricas")
```

```
[Out] ((a^2*m^2 - 4*a^2*m + 3*a^2)*x^4 + b^2*m^2 + 2*(a*b*m^2 - 2*a*b*m - 3*a*b)*
x^2 - b^2)*(c*x)^m/((m^3 - 3*m^2 - m + 3)*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx)^m \left(a + \frac{b}{x^2}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2*(c*x)^m,x, algorithm="giac")
```

```
[Out] integrate((c*x)^m*(a + b/x^2)^2, x)
```

maple [A] time = 0.00, size = 90, normalized size = 1.48

$$\frac{\left(a^2m^2x^4 - 4a^2m\,x^4 + 3a^2x^4 + 2ab\,m^2x^2 - 4abm\,x^2 - 6ab\,x^2 + b^2m^2 - b^2\right)(cx)^m}{(m+1)\,(m-1)\,(m-3)\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^2*(c*x)^m,x)
```

```
[Out] (c*x)^m*(a^2*m^2*x^4-4*a^2*m*x^4+3*a^2*x^4+2*a*b*m^2*x^2-4*a*b*m*x^2-6*a*b*
x^2+b^2*m^2-b^2)/x^3/(m+1)/(m-1)/(-3+m)
```

maxima [A] time = 0.50, size = 56, normalized size = 0.92

$$\frac{2\,abc^m x^m}{(m-1)x} + \frac{(cx)^{m+1}\,a^2}{c(m+1)} + \frac{b^2c^m x^m}{(m-3)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^2*(c*x)^m,x, algorithm="maxima")
```

```
[Out] 2*a*b*c^m*x^m/((m - 1)*x) + (c*x)^(m + 1)*a^2/(c*(m + 1)) + b^2*c^m*x^m/((m
 - 3)*x^3)
```

mupad [B] time = 1.23, size = 89, normalized size = 1.46

$$\frac{(c\,x)^m\left(-a^2\,m^2\,x^4 + 4\,a^2\,m\,x^4 - 3\,a^2\,x^4 - 2\,a\,b\,m^2\,x^2 + 4\,a\,b\,m\,x^2 + 6\,a\,b\,x^2 - b^2\,m^2 + b^2\right)}{x^3\left(-m^3 + 3\,m^2 + m - 3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(a + b/x^2)^2,x)
```

```
[Out] ((c*x)^m*(b^2 - b^2*m^2 - 3*a^2*x^4 + 4*a^2*m*x^4 - a^2*m^2*x^4 + 6*a*b*x^2
 + 4*a*b*m*x^2 - 2*a*b*m^2*x^2))/(x^3*(m + 3*m^2 - m^3 - 3))
```

sympy [A] time = 1.11, size = 401, normalized size = 6.57

$$\begin{cases} \frac{a^2\log(x) - \frac{ab}{x^2} - \frac{b^2}{4x^4}}{c} \\ c\left(\frac{a^2x^2}{2} + 2ab\log(x) - \frac{b^2}{2x^2}\right) \\ c^3\left(\frac{a^2x^4}{4} + abx^2 + b^2\log(x)\right) \\ \frac{a^2c^m m^2x^4x^m}{m^3x^3 - 3m^2x^3 - mx^3 + 3x^3} - \frac{4a^2c^m mx^4x^m}{m^3x^3 - 3m^2x^3 - mx^3 + 3x^3} + \frac{3a^2c^m x^4x^m}{m^3x^3 - 3m^2x^3 - mx^3 + 3x^3} + \frac{2abc^m m^2x^2x^m}{m^3x^3 - 3m^2x^3 - mx^3 + 3x^3} - \frac{4abc^m mx^2x^m}{m^3x^3 - 3m^2x^3 - mx^3 + 3x^3} - \frac{}{m^3x^3 -} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**2*(c*x)**m,x)
```

```
[Out] Piecewise(((a**2*log(x) - a*b/x**2 - b**2/(4*x**4))/c, Eq(m, -1)), (c*(a**2
*x**2/2 + 2*a*b*log(x) - b**2/(2*x**2)), Eq(m, 1)), (c**3*(a**2*x**4/4 + a*
b*x**2 + b**2*log(x)), Eq(m, 3)), (a**2*c**m*m**2*x**4*x**m/(m**3*x**3 - 3*
m**2*x**3 - m*x**3 + 3*x**3) - 4*a**2*c**m*m*x**4*x**m/(m**3*x**3 - 3*m**2*
x**3 - m*x**3 + 3*x**3) + 3*a**2*c**m*x**4*x**m/(m**3*x**3 - 3*m**2*x**3 -
m*x**3 + 3*x**3) + 2*a*b*c**m*m**2*x**2*x**m/(m**3*x**3 - 3*m**2*x**3 - m*x
**3 + 3*x**3) - 4*a*b*c**m*m*x**2*x**m/(m**3*x**3 - 3*m**2*x**3 - m*x**3 +
3*x**3) - 6*a*b*c**m*x**2*x**m/(m**3*x**3 - 3*m**2*x**3 - m*x**3 + 3*x**3)
+ b**2*c**m*m**2*x**m/(m**3*x**3 - 3*m**2*x**3 - m*x**3 + 3*x**3) - b**2*c*
*m*x**m/(m**3*x**3 - 3*m**2*x**3 - m*x**3 + 3*x**3), True))
```

3.2761 $\int \left(a+\frac{b}{x^3}\right)^2 (cx)^m\,dx$

Optimal. Leaf size=63

$$\frac{a^2(cx)^{m+1}}{c(m+1)}-\frac{2abc^2(cx)^{m-2}}{2-m}-\frac{b^2c^5(cx)^{m-5}}{5-m}$$

```
[Out] -b^2*c^5*(c*x)^(-5+m)/(5-m)-2*a*b*c^2*(c*x)^(-2+m)/(2-m)+a^2*(c*x)^(1+m)/c/
(1+m)
```

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {270}

$$\frac{a^2(cx)^{m+1}}{c(m+1)}-\frac{2abc^2(cx)^{m-2}}{2-m}-\frac{b^2c^5(cx)^{m-5}}{5-m}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^3)^2*(c*x)^m,x]
```

```
[Out] -((b^2*c^5*(c*x)^(-5 + m))/(5 - m)) - (2*a*b*c^2*(c*x)^(-2 + m))/(2 - m) +
(a^2*(c*x)^(1 + m))/(c*(1 + m))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x^3}\right)^2 (cx)^m\,dx &= \int \left(b^2c^6(cx)^{-6+m}+2abc^3(cx)^{-3+m}+a^2(cx)^m\right)dx\\ &= -\frac{b^2c^5(cx)^{-5+m}}{5-m}-\frac{2abc^2(cx)^{-2+m}}{2-m}+\frac{a^2(cx)^{1+m}}{c(1+m)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 67, normalized size = 1.06

$$\frac{(cx)^m\left(a^2\left(m^2-7m+10\right)x^6+2ab\left(m^2-4m-5\right)x^3+b^2\left(m^2-m-2\right)\right)}{(m-5)(m-2)(m+1)x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^3)^2*(c*x)^m,x]
```

```
[Out] ((c*x)^m*(b^2*(-2 - m + m^2) + 2*a*b*(-5 - 4*m + m^2)*x^3 + a^2*(10 - 7*m +
 m^2)*x^6))/((-5 + m)*(-2 + m)*(1 + m)*x^5)
```

fricas [A] time = 0.56, size = 87, normalized size = 1.38

$$\frac{\left(\left(a^2m^2-7\,a^2m+10\,a^2\right)x^6+b^2m^2+2\left(abm^2-4\,abm-5\,ab\right)x^3-b^2m-2\,b^2\right)(cx)^m}{\left(m^3-6\,m^2+3\,m+10\right)x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^2*(c*x)^m,x, algorithm="fricas")
```

```
[Out] ((a^2*m^2 - 7*a^2*m + 10*a^2)*x^6 + b^2*m^2 + 2*(a*b*m^2 - 4*a*b*m - 5*a*b)
*x^3 - b^2*m - 2*b^2)*(c*x)^m/((m^3 - 6*m^2 + 3*m + 10)*x^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx)^m \left(a + \frac{b}{x^3}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^2*(c*x)^m,x, algorithm="giac")
```

```
[Out] integrate((c*x)^m*(a + b/x^3)^2, x)
```

maple [A] time = 0.01, size = 96, normalized size = 1.52

$$\frac{\left(a^2m^2x^6 - 7a^2m\,x^6 + 10a^2x^6 + 2ab\,m^2x^3 - 8abm\,x^3 - 10ab\,x^3 + b^2m^2 - b^2m - 2b^2\right)(cx)^m}{(m+1)\,(m-2)\,(m-5)\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^3)^2*(c*x)^m,x)
```

```
[Out] (c*x)^m*(a^2*m^2*x^6-7*a^2*m*x^6+10*a^2*x^6+2*a*b*m^2*x^3-8*a*b*m*x^3-10*a*
b*x^3+b^2*m^2-b^2*m-2*b^2)/x^5/(m+1)/(m-2)/(-5+m)
```

maxima [A] time = 0.63, size = 56, normalized size = 0.89

$$\frac{(cx)^{m+1}\,a^2}{c(m+1)} + \frac{2\,abc^m x^m}{(m-2)x^2} + \frac{b^2c^m x^m}{(m-5)x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)^2*(c*x)^m,x, algorithm="maxima")
```

```
[Out] (c*x)^(m + 1)*a^2/(c*(m + 1)) + 2*a*b*c^m*x^m/((m - 2)*x^2) + b^2*c^m*x^m/(
(m - 5)*x^5)
```

mupad [B] time = 1.22, size = 97, normalized size = 1.54

$$-\frac{(c\,x)^m\left(-a^2\,m^2\,x^6 + 7\,a^2\,m\,x^6 - 10\,a^2\,x^6 - 2\,a\,b\,m^2\,x^3 + 8\,a\,b\,m\,x^3 + 10\,a\,b\,x^3 - b^2\,m^2 + b^2\,m + 2\,b^2\right)}{x^5\left(m^3 - 6\,m^2 + 3\,m + 10\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(a + b/x^3)^2,x)
```

```
[Out] -((c*x)^m*(b^2*m + 2*b^2 - b^2*m^2 - 10*a^2*x^6 + 7*a^2*m*x^6 - a^2*m^2*x^6
 + 10*a*b*x^3 + 8*a*b*m*x^3 - 2*a*b*m^2*x^3))/(x^5*(3*m - 6*m^2 + m^3 + 10)
)
```

sympy [A] time = 1.61, size = 464, normalized size = 7.37

$$\begin{cases} \frac{a^2\log(x) - \frac{2ab}{3x^3} - \frac{b^2}{6x^6}}{c} \\ c^2\left(\frac{a^2x^3}{3} + 2ab\log(x) - \frac{b^2}{3x^3}\right) \\ c^5\left(\frac{a^2x^6}{6} + \frac{2abx^3}{3} + b^2\log(x)\right) \\ \frac{a^2c^m m^2x^6x^m}{m^3x^5 - 6m^2x^5 + 3mx^5 + 10x^5} - \frac{7a^2c^m mx^6x^m}{m^3x^5 - 6m^2x^5 + 3mx^5 + 10x^5} + \frac{10a^2c^m x^6x^m}{m^3x^5 - 6m^2x^5 + 3mx^5 + 10x^5} + \frac{2abc^m m^2x^3x^m}{m^3x^5 - 6m^2x^5 + 3mx^5 + 10x^5} - \frac{8abc^m mx^3x^m}{m^3x^5 - 6m^2x^5 + 3mx^5} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**3)**2*(c*x)**m,x)
```

```
[Out] Piecewise(((a**2*log(x) - 2*a*b/(3*x**3) - b**2/(6*x**6))/c, Eq(m, -1)), (c
**2*(a**2*x**3/3 + 2*a*b*log(x) - b**2/(3*x**3)), Eq(m, 2)), (c**5*(a**2*x**
*6/6 + 2*a*b*x**3/3 + b**2*log(x)), Eq(m, 5)), (a**2*c**m*m**2*x**6*x**m/(m
**3*x**5 - 6*m**2*x**5 + 3*m*x**5 + 10*x**5) - 7*a**2*c**m*m*x**6*x**m/(m**
3*x**5 - 6*m**2*x**5 + 3*m*x**5 + 10*x**5) + 10*a**2*c**m*x**6*x**m/(m**3*x
**5 - 6*m**2*x**5 + 3*m*x**5 + 10*x**5) + 2*a*b*c**m*m**2*x**3*x**m/(m**3*x
**5 - 6*m**2*x**5 + 3*m*x**5 + 10*x**5) - 8*a*b*c**m*m*x**3*x**m/(m**3*x**5
 - 6*m**2*x**5 + 3*m*x**5 + 10*x**5) - 10*a*b*c**m*x**3*x**m/(m**3*x**5 - 6
*m**2*x**5 + 3*m*x**5 + 10*x**5) + b**2*c**m*m**2*x**m/(m**3*x**5 - 6*m**2*
x**5 + 3*m*x**5 + 10*x**5) - b**2*c**m*m*x**m/(m**3*x**5 - 6*m**2*x**5 + 3*
m*x**5 + 10*x**5) - 2*b**2*c**m*x**m/(m**3*x**5 - 6*m**2*x**5 + 3*m*x**5 +
10*x**5), True))
```

3.2762 $\int \frac{(cx)^{-1-\frac{n}{2}}}{a+bx^n}\,dx$

Optimal. Leaf size=74

$$\frac{2\sqrt{b}\,x^{n/2}(cx)^{-n/2}\tan^{-1}\left(\frac{\sqrt{a}\,x^{-n/2}}{\sqrt{b}}\right)}{a^{3/2}cn}-\frac{2(cx)^{-n/2}}{acn}$$

```
[Out] -2/a/c/n/((c*x)^(1/2*n))+2*x^(1/2*n)*arctan(a^(1/2)/(x^(1/2*n))/b^(1/2))*b^
(1/2)/a^(3/2)/c/n/((c*x)^(1/2*n))
```

Rubi [A] time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {347, 345, 193, 321, 205}

$$\frac{2\sqrt{b}\,x^{n/2}(cx)^{-n/2}\tan^{-1}\left(\frac{\sqrt{a}\,x^{-n/2}}{\sqrt{b}}\right)}{a^{3/2}cn}-\frac{2(cx)^{-n/2}}{acn}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 - n/2)/(a + b*x^n),x]
```

```
[Out] -2/(a*c*n*(c*x)^(n/2)) + (2*Sqrt[b]*x^(n/2)*ArcTan[Sqrt[a]/(Sqrt[b]*x^(n/2)
)])/(a^(3/2)*c*n*(c*x)^(n/2))
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 347

```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(c^In
tPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a + b*x^n)^p, x], x] /;
 FreeQ[{a, b, c, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !Integer
Q[n]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx)^{-1-\frac{n}{2}}}{a+bx^n}\,dx &= \frac{\left(x^{n/2}(cx)^{-n/2}\right)\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n}\,dx}{c}\\ &= -\frac{\left(2x^{n/2}(cx)^{-n/2}\right)\text{Subst}\left(\int \frac{1}{a+\frac{b}{x^2}}\,dx, x, x^{-n/2}\right)}{cn}\\ &= -\frac{\left(2x^{n/2}(cx)^{-n/2}\right)\text{Subst}\left(\int \frac{x^2}{b+ax^2}\,dx, x, x^{-n/2}\right)}{cn}\\ &= -\frac{2(cx)^{-n/2}}{acn} + \frac{\left(2bx^{n/2}(cx)^{-n/2}\right)\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, x^{-n/2}\right)}{acn}\\ &= -\frac{2(cx)^{-n/2}}{acn} + \frac{2\sqrt{b}\,x^{n/2}(cx)^{-n/2}\tan^{-1}\left(\frac{\sqrt{a}\,x^{-n/2}}{\sqrt{b}}\right)}{a^{3/2}cn}\end{aligned}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.50

$$-\frac{2x(cx)^{-\frac{n}{2}-1}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{bx^n}{a}\right)}{an}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 - n/2)/(a + b*x^n),x]
```

```
[Out] (-2*x*(c*x)^(-1 - n/2)*Hypergeometric2F1[-1/2, 1, 1/2, -((b*x^n)/a)])/(a*n)
```

fricas [A] time = 0.79, size = 225, normalized size = 3.04

$$\left[-\frac{2\,xe^{\left(-\frac{1}{2}(n+2)\log(c)-\frac{1}{2}(n+2)\log(x)\right)} - \sqrt{-\frac{bc^{-n-2}}{a}}\log\left(\frac{ax^2e^{\left(-(n+2)\log(c)-(n+2)\log(x)\right)}+2\,a\sqrt{-\frac{bc^{-n-2}}{a}}xe^{\left(-\frac{1}{2}(n+2)\log(c)-\frac{1}{2}(n+2)\log(x)\right)}-bc^{-n-2}}{ax^2e^{\left(-(n+2)\log(c)-(n+2)\log(x)\right)}+bc^{-n-2}}\right.}{an}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-1/2*n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] [-(2*x*e^(-1/2*(n + 2)*log(c) - 1/2*(n + 2)*log(x)) - sqrt(-b*c^(-n - 2)/a)
*log((a*x^2*e^(-(n + 2)*log(c) - (n + 2)*log(x)) + 2*a*sqrt(-b*c^(-n - 2)/a
)*x*e^(-1/2*(n + 2)*log(c) - 1/2*(n + 2)*log(x)) - b*c^(-n - 2))/(a*x^2*e^(
-(n + 2)*log(c) - (n + 2)*log(x)) + b*c^(-n - 2))))/(a*n), -2*(x*e^(-1/2*(n
 + 2)*log(c) - 1/2*(n + 2)*log(x)) + sqrt(b*c^(-n - 2)/a)*arctan(sqrt(b*c^(
-n - 2)/a)*e^(1/2*(n + 2)*log(c) + 1/2*(n + 2)*log(x))/x))/(a*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{-\frac{1}{2}n-1}}{bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-1/2*n)/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(-1/2*n - 1)/(b*x^n + a), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{-\frac{n}{2}-1}}{b\,x^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-1/2*n-1)/(b*x^n+a),x)
```

```
[Out] int((c*x)^(-1/2*n-1)/(b*x^n+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-b\int \frac{x^{\frac{1}{2}n}}{abc^{\frac{1}{2}n+1}xx^n + a^2c^{\frac{1}{2}n+1}x}\,dx - \frac{2\,c^{-\frac{1}{2}n-1}}{anx^{\frac{1}{2}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-1/2*n)/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] -b*integrate(x^(1/2*n)/(a*b*c^(1/2*n + 1)*x*x^n + a^2*c^(1/2*n + 1)*x), x)
- 2*c^(-1/2*n - 1)/(a*n*x^(1/2*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\,x)^{\frac{n}{2}+1}\,(a + b\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(n/2 + 1)*(a + b*x^n)),x)
```

```
[Out] int(1/((c*x)^(n/2 + 1)*(a + b*x^n)), x)
```

sympy [A] time = 3.84, size = 54, normalized size = 0.73

$$-\frac{2c^{-\frac{n}{2}}x^{-\frac{n}{2}}}{acn} - \frac{2\sqrt{b}\,c^{-\frac{n}{2}}\,\operatorname{atan}\left(\frac{\sqrt{b}\,x^{\frac{n}{2}}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-1-1/2*n)/(a+b*x**n),x)
```

```
[Out] -2*c**(-n/2)*x**(-n/2)/(a*c*n) - 2*sqrt(b)*c**(-n/2)*atan(sqrt(b)*x**(n/2)/
sqrt(a))/(a**(3/2)*c*n)
```

3.2763 $\int \frac{(cx)^{-1-\frac{2n}{3}}}{a+bx^n}\,dx$

Optimal. Leaf size=222

$$-\frac{b^{2/3}x^{2n/3}(cx)^{-2n/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^{n/3}\right)}{a^{5/3}cn}+\frac{b^{2/3}x^{2n/3}(cx)^{-2n/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^{n/3}+b^{2/3}x^{2n/3}\right)}{2a^{5/3}cn}+\frac{\sqrt{3}\,b^{2/3}x^{2n/3}(cx)^{-2n}}{a^{5}}$$

```
[Out] -3/2/a/c/n/((c*x)^(2/3*n))-b^(2/3)*x^(2/3*n)*ln(a^(1/3)+b^(1/3)*x^(1/3*n))/
a^(5/3)/c/n/((c*x)^(2/3*n))+1/2*b^(2/3)*x^(2/3*n)*ln(a^(2/3)-a^(1/3)*b^(1/3
)*x^(1/3*n)+b^(2/3)*x^(2/3*n))/a^(5/3)/c/n/((c*x)^(2/3*n))+b^(2/3)*x^(2/3*n
)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(1/3*n))/a^(1/3)*3^(1/2))*3^(1/2)/a^(5/3)
/c/n/((c*x)^(2/3*n))
```

Rubi [A] time = 0.14, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {363, 362, 345, 200, 31, 634, 617, 204, 628}

$$-\frac{b^{2/3}x^{2n/3}(cx)^{-2n/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^{n/3}\right)}{a^{5/3}cn}+\frac{b^{2/3}x^{2n/3}(cx)^{-2n/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^{n/3}+b^{2/3}x^{2n/3}\right)}{2a^{5/3}cn}+\frac{\sqrt{3}\,b^{2/3}x^{2n/3}(cx)^{-2n}}{a^{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 - (2*n)/3)/(a + b*x^n),x]
```

```
[Out] -3/(2*a*c*n*(c*x)^((2*n)/3)) + (Sqrt[3]*b^(2/3)*x^((2*n)/3)*ArcTan[(a^(1/3)
 - 2*b^(1/3)*x^(n/3))/(Sqrt[3]*a^(1/3))])/(a^(5/3)*c*n*(c*x)^((2*n)/3)) - (
b^(2/3)*x^((2*n)/3)*Log[a^(1/3) + b^(1/3)*x^(n/3)])/(a^(5/3)*c*n*(c*x)^((2*
n)/3)) + (b^(2/3)*x^((2*n)/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(n/3) + b^(2/
3)*x^((2*n)/3)])/(2*a^(5/3)*c*n*(c*x)^((2*n)/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 362

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Dist[b/a, Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a,
```

```
b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

Rule 363

```
Int[((c_)*(x_))^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Dist[(c^IntPart
[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m/(a + b*x^n), x], x] /; FreeQ[
{a, b, c, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && (SumSimplerQ[m, n]
 || SumSimplerQ[m, -n])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(cx)^{-1-\frac{2n}{3}}}{a+bx^n}\,dx = \frac{\left(x^{2n/3}(cx)^{-2n/3}\right)\int \frac{x^{-1-\frac{2n}{3}}}{a+bx^n}\,dx}{c}$$

$$= -\frac{3(cx)^{-2n/3}}{2acn} - \frac{\left(bx^{2n/3}(cx)^{-2n/3}\right)\int \frac{x^{\frac{1}{3}(-3+n)}}{a+bx^n}\,dx}{ac}$$

$$= -\frac{3(cx)^{-2n/3}}{2acn} - \frac{\left(3bx^{2n/3}(cx)^{-2n/3}\right)\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, x^{1+\frac{1}{3}(-3+n)}\right)}{acn}$$

$$= -\frac{3(cx)^{-2n/3}}{2acn} - \frac{\left(bx^{2n/3}(cx)^{-2n/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, x^{1+\frac{1}{3}(-3+n)}\right)}{a^{5/3}cn} - \frac{\left(bx^{2n/3}(cx)^{-2n/3}\right)\text{Subs}}{}$$

$$= -\frac{3(cx)^{-2n/3}}{2acn} - \frac{b^{2/3}x^{2n/3}(cx)^{-2n/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^{n/3}\right)}{a^{5/3}cn} + \frac{\left(b^{2/3}x^{2n/3}(cx)^{-2n/3}\right)\text{Subst}\left(\int \frac{-\sqrt[3]{a}\,\sqrt[3]{}}{a^{2/3}-\sqrt[3]{a}}\right.}{2a^{5/3}cn}$$

$$= -\frac{3(cx)^{-2n/3}}{2acn} - \frac{b^{2/3}x^{2n/3}(cx)^{-2n/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x^{n/3}\right)}{a^{5/3}cn} + \frac{b^{2/3}x^{2n/3}(cx)^{-2n/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x\right.}{2a^{5/3}cn}$$

$$= -\frac{3(cx)^{-2n/3}}{2acn} + \frac{\sqrt{3}\,b^{2/3}x^{2n/3}(cx)^{-2n/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x^{n/3}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{a^{5/3}cn} - \frac{b^{2/3}x^{2n/3}(cx)^{-2n/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right.}{a^{5/3}cn}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.18

$$-\frac{3x(cx)^{-\frac{2n}{3}-1}\,{}_2F_1\left(-\frac{2}{3},1;\frac{1}{3};-\frac{bx^n}{a}\right)}{2an}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 - (2*n)/3)/(a + b*x^n),x]
```

```
[Out] (-3*x*(c*x)^(-1 - (2*n)/3)*Hypergeometric2F1[-2/3, 1, 1/3, -((b*x^n)/a)])/(
2*a*n)
```

fricas [A] time = 1.04, size = 329, normalized size = 1.48

$$-\frac{3\,xe^{\left(-\frac{1}{3}(2n+3)\log(c)-\frac{1}{3}(2n+3)\log(x)\right)}-2\sqrt{3}\left(-\frac{b^2c^{-2n-3}}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,a\left(-\frac{b^2c^{-2n-3}}{a^2}\right)^{\frac{1}{3}}\sqrt{x}\,e^{\left(-\frac{1}{6}(2n+3)\log(c)-\frac{1}{6}(2n+3)\log(x)\right)}+\sqrt{3}\,bc^{-n}}{3\,bc^{-n-\frac{3}{2}}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-2/3*n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] -1/2*(3*x*e^(-1/3*(2*n + 3)*log(c) - 1/3*(2*n + 3)*log(x)) - 2*sqrt(3)*(-b^
2*c^(-2*n - 3)/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*(-b^2*c^(-2*n - 3)/a^2)^(
1/3)*sqrt(x)*e^(-1/6*(2*n + 3)*log(c) - 1/6*(2*n + 3)*log(x)) + sqrt(3)*b*c
^(-n - 3/2))/(b*c^(-n - 3/2))) - 2*(-b^2*c^(-2*n - 3)/a^2)^(1/3)*log((b*c^(
-n - 3/2)*x*e^(-1/6*(2*n + 3)*log(c) - 1/6*(2*n + 3)*log(x)) + a*(-b^2*c^(-
2*n - 3)/a^2)^(2/3)*sqrt(x))/x) + (-b^2*c^(-2*n - 3)/a^2)^(1/3)*log((b*c^(-
n - 3/2)*x*e^(-1/3*(2*n + 3)*log(c) - 1/3*(2*n + 3)*log(x)) - a*(-b^2*c^(-2
*n - 3)/a^2)^(2/3)*sqrt(x)*e^(-1/6*(2*n + 3)*log(c) - 1/6*(2*n + 3)*log(x))
 - b*c^(-n - 3/2)*(-b^2*c^(-2*n - 3)/a^2)^(1/3))/x))/(a*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{-\frac{2}{3}n-1}}{bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-2/3*n)/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(-2/3*n - 1)/(b*x^n + a), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{-\frac{2n}{3}-1}}{b\,x^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-2/3*n-1)/(b*x^n+a),x)
```

```
[Out] int((c*x)^(-2/3*n-1)/(b*x^n+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-b\int \frac{x^{\frac{1}{3}n}}{abc^{\frac{2}{3}n+1}xx^n+a^2c^{\frac{2}{3}n+1}x}\,dx-\frac{3\,c^{-\frac{2}{3}n-1}}{2\,anx^{\frac{2}{3}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-2/3*n)/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] -b*integrate(x^(1/3*n)/(a*b*c^(2/3*n + 1)*x*x^n + a^2*c^(2/3*n + 1)*x), x)
- 3/2*c^(-2/3*n - 1)/(a*n*x^(2/3*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(c\,x)^{\frac{2n}{3}+1}\,(a+b\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^((2*n)/3 + 1)*(a + b*x^n)),x)
```

```
[Out] int(1/((c*x)^((2*n)/3 + 1)*(a + b*x^n)), x)
```

sympy [C] time = 4.29, size = 226, normalized size = 1.02

$$\frac{c^{-\frac{2n}{3}}x^{-\frac{2n}{3}}\Gamma\left(-\frac{2}{3}\right)}{acn\Gamma\left(\frac{1}{3}\right)}-\frac{2b^{\frac{2}{3}}c^{-\frac{2n}{3}}e^{-\frac{i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{n}{3}}e^{\frac{i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{2}{3}\right)}{3a^{\frac{5}{3}}cn\Gamma\left(\frac{1}{3}\right)}+\frac{2b^{\frac{2}{3}}c^{-\frac{2n}{3}}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{n}{3}}e^{i\pi}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{2}{3}\right)}{3a^{\frac{5}{3}}cn\Gamma\left(\frac{1}{3}\right)}-\frac{2b^{\frac{2}{3}}c^{-\frac{2n}{3}}e^{\frac{i\pi}{3}}\log\left(1\right.}{3a^{\frac{5}{3}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-1-2/3*n)/(a+b*x**n),x)
```

```
[Out] c**(-2*n/3)*x**(-2*n/3)*gamma(-2/3)/(a*c*n*gamma(1/3)) - 2*b**(2/3)*c**(-2*
n/3)*exp(-I*pi/3)*log(1 - b**(1/3)*x**(n/3)*exp_polar(I*pi/3)/a**(1/3))*gam
ma(-2/3)/(3*a**(5/3)*c*n*gamma(1/3)) + 2*b**(2/3)*c**(-2*n/3)*log(1 - b**(1
/3)*x**(n/3)*exp_polar(I*pi)/a**(1/3))*gamma(-2/3)/(3*a**(5/3)*c*n*gamma(1/
3)) - 2*b**(2/3)*c**(-2*n/3)*exp(I*pi/3)*log(1 - b**(1/3)*x**(n/3)*exp_pola
r(5*I*pi/3)/a**(1/3))*gamma(-2/3)/(3*a**(5/3)*c*n*gamma(1/3))
```

3.2764 $\int \frac{(cx)^{-1-\frac{3n}{4}}}{a+bx^n}\,dx$

Optimal. Leaf size=317

$$\frac{b^{3/4}x^{3n/4}(cx)^{-3n/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^{n/4}+\sqrt{a}+\sqrt{b}\,x^{n/2}\right)}{\sqrt{2}\,a^{7/4}cn}-\frac{b^{3/4}x^{3n/4}(cx)^{-3n/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^{n/4}+\sqrt{a}+\sqrt{b}\,x^{n/2}\right)}{\sqrt{2}\,a^{7/4}cn}+$$

```
[Out] -4/3/a/c/n/((c*x)^(3/4*n))+1/2*b^(3/4)*x^(3/4*n)*ln(-a^(1/4)*b^(1/4)*x^(1/4
*n)*2^(1/2)+a^(1/2)+x^(1/2*n)*b^(1/2))/a^(7/4)/c/n/((c*x)^(3/4*n))*2^(1/2)-
1/2*b^(3/4)*x^(3/4*n)*ln(a^(1/4)*b^(1/4)*x^(1/4*n)*2^(1/2)+a^(1/2)+x^(1/2*n
)*b^(1/2))/a^(7/4)/c/n/((c*x)^(3/4*n))*2^(1/2)-b^(3/4)*x^(3/4*n)*arctan(-1+
b^(1/4)*x^(1/4*n)*2^(1/2)/a^(1/4))*2^(1/2)/a^(7/4)/c/n/((c*x)^(3/4*n))-b^(3
/4)*x^(3/4*n)*arctan(1+b^(1/4)*x^(1/4*n)*2^(1/2)/a^(1/4))*2^(1/2)/a^(7/4)/c
/n/((c*x)^(3/4*n))
```

Rubi [A] time = 0.24, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {363, 362, 345, 211, 1165, 628, 1162, 617, 204}

$$\frac{b^{3/4}x^{3n/4}(cx)^{-3n/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^{n/4}+\sqrt{a}+\sqrt{b}\,x^{n/2}\right)}{\sqrt{2}\,a^{7/4}cn}-\frac{b^{3/4}x^{3n/4}(cx)^{-3n/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^{n/4}+\sqrt{a}+\sqrt{b}\,x^{n/2}\right)}{\sqrt{2}\,a^{7/4}cn}+$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 - (3*n)/4)/(a + b*x^n),x]
```

```
[Out] -4/(3*a*c*n*(c*x)^((3*n)/4)) + (Sqrt[2]*b^(3/4)*x^((3*n)/4)*ArcTan[1 - (Sqr
t[2]*b^(1/4)*x^(n/4))/a^(1/4)])/(a^(7/4)*c*n*(c*x)^((3*n)/4)) - (Sqrt[2]*b^
(3/4)*x^((3*n)/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x^(n/4))/a^(1/4)])/(a^(7/4)*c
*n*(c*x)^((3*n)/4)) + (b^(3/4)*x^((3*n)/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^
(1/4)*x^(n/4) + Sqrt[b]*x^(n/2)])/(Sqrt[2]*a^(7/4)*c*n*(c*x)^((3*n)/4)) - (
b^(3/4)*x^((3*n)/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x^(n/4) + Sqrt[b]
*x^(n/2)])/(Sqrt[2]*a^(7/4)*c*n*(c*x)^((3*n)/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 362

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Dist[b/a, Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a,
 b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

Rule 363

```
Int[((c_)*(x_))^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Dist[(c^IntPart
[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m/(a + b*x^n), x], x] /; FreeQ[
{a, b, c, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && (SumSimplerQ[m, n]
 || SumSimplerQ[m, -n])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(cx)^{-1-\frac{3n}{4}}}{a+bx^n}\,dx = \frac{\left(x^{3n/4}(cx)^{-3n/4}\right)\int \frac{x^{-1-\frac{3n}{4}}}{a+bx^n}\,dx}{c}$$

$$= -\frac{4(cx)^{-3n/4}}{3acn} - \frac{\left(bx^{3n/4}(cx)^{-3n/4}\right)\int \frac{x^{\frac{1}{4}(-4+n)}}{a+bx^n}\,dx}{ac}$$

$$= -\frac{4(cx)^{-3n/4}}{3acn} - \frac{\left(4bx^{3n/4}(cx)^{-3n/4}\right)\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, x^{1+\frac{1}{4}(-4+n)}\right)}{acn}$$

$$= -\frac{4(cx)^{-3n/4}}{3acn} - \frac{\left(2bx^{3n/4}(cx)^{-3n/4}\right)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, x^{1+\frac{1}{4}(-4+n)}\right)}{a^{3/2}cn} - \frac{\left(2bx^{3n/4}(cx)^{-3n/4}\right)\text{Subs}}{}$$

$$= -\frac{4(cx)^{-3n/4}}{3acn} - \frac{\left(\sqrt{b}\,x^{3n/4}(cx)^{-3n/4}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, x^{1+\frac{1}{4}(-4+n)}\right)}{a^{3/2}cn} - \frac{\left(\sqrt{b}\,x^{3n/4}(cx)^{-3n/4}\right.}{}$$

$$= -\frac{4(cx)^{-3n/4}}{3acn} + \frac{b^{3/4}x^{3n/4}(cx)^{-3n/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x^{n/4}+\sqrt{b}\,x^{n/2}\right)}{\sqrt{2}\,a^{7/4}cn} - \frac{b^{3/4}x^{3n/4}(cx)^{-3n/4}\log\left(}{}$$

$$= -\frac{4(cx)^{-3n/4}}{3acn} + \frac{\sqrt{2}\,b^{3/4}x^{3n/4}(cx)^{-3n/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x^{n/4}}{\sqrt[4]{a}}\right)}{a^{7/4}cn} - \frac{\sqrt{2}\,b^{3/4}x^{3n/4}(cx)^{-3n/4}\tan^{-1}\left(1+\frac{\sqrt{2}\right.}{}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.12

$$-\frac{4x(cx)^{-\frac{3n}{4}-1}\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\frac{bx^n}{a}\right)}{3an}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 - (3*n)/4)/(a + b*x^n),x]
```

```
[Out] (-4*x*(c*x)^(-1 - (3*n)/4)*Hypergeometric2F1[-3/4, 1, 1/4, -((b*x^n)/a)])/(
3*a*n)
```

fricas [A] time = 1.40, size = 426, normalized size = 1.34

$$12\,an\left(-\frac{b^3c^{-3n-4}}{a^7n^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{a^2b^2c^{-2n-\frac{8}{3}}nx^{\frac{1}{3}}\left(-\frac{b^3c^{-3n-4}}{a^7n^4}\right)^{\frac{1}{4}}e^{\left(-\frac{1}{12}(3n+4)\log(c)-\frac{1}{12}(3n+4)\log(x)\right)}-a^2nx^{\frac{1}{3}}\sqrt{-\frac{a^3b^3c^{-3n-4}n^2x^{\frac{1}{3}}\sqrt{-\frac{b^3c^{-3n-4}}{a^7n^4}}-b^4}}}{b^3c^{-3n-4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-3/4*n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] -1/3*(12*a*n*(-b^3*c^(-3*n - 4)/(a^7*n^4))^(1/4)*arctan(-(a^2*b^2*c^(-2*n -
 8/3)*n*x^(1/3)*(-b^3*c^(-3*n - 4)/(a^7*n^4))^(1/4)*e^(-1/12*(3*n + 4)*log(
c) - 1/12*(3*n + 4)*log(x)) - a^2*n*x^(1/3)*sqrt(-(a^3*b^3*c^(-3*n - 4)*n^2
*x^(1/3)*sqrt(-b^3*c^(-3*n - 4)/(a^7*n^4)) - b^4*c^(-4*n - 16/3)*x*e^(-1/6*
(3*n + 4)*log(c) - 1/6*(3*n + 4)*log(x)))/x)*(-b^3*c^(-3*n - 4)/(a^7*n^4))^
(1/4))/(b^3*c^(-3*n - 4))) - 3*a*n*(-b^3*c^(-3*n - 4)/(a^7*n^4))^(1/4)*log(
```

```
(a^5*n^3*x^(2/3)*(-b^3*c^(-3*n - 4)/(a^7*n^4))^(3/4) + b^2*c^(-2*n - 8/3)*x
*e^(-1/12*(3*n + 4)*log(c) - 1/12*(3*n + 4)*log(x)))/x) + 3*a*n*(-b^3*c^(-3
*n - 4)/(a^7*n^4))^(1/4)*log(-(a^5*n^3*x^(2/3)*(-b^3*c^(-3*n - 4)/(a^7*n^4)
)^(3/4) - b^2*c^(-2*n - 8/3)*x*e^(-1/12*(3*n + 4)*log(c) - 1/12*(3*n + 4)*l
og(x)))/x) + 4*x*e^(-1/4*(3*n + 4)*log(c) - 1/4*(3*n + 4)*log(x)))/(a*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{-\frac{3}{4}n-1}}{bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-3/4*n)/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(-3/4*n - 1)/(b*x^n + a), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{-\frac{3n}{4}-1}}{b\,x^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-1-3/4*n)/(b*x^n+a),x)
```

```
[Out] int((c*x)^(-1-3/4*n)/(b*x^n+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-b\int \frac{x^{\frac{1}{4}n}}{abc^{\frac{3}{4}n+1}xx^n + a^2c^{\frac{3}{4}n+1}x}\,dx - \frac{4\,c^{-\frac{3}{4}n-1}}{3\,anx^{\frac{3}{4}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-3/4*n)/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] -b*integrate(x^(1/4*n)/(a*b*c^(3/4*n + 1)*x*x^n + a^2*c^(3/4*n + 1)*x), x)
- 4/3*c^(-3/4*n - 1)/(a*n*x^(3/4*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(c\,x)^{\frac{3n}{4}+1}\,(a + b\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^((3*n)/4 + 1)*(a + b*x^n)),x)
```

```
[Out] int(1/((c*x)^((3*n)/4 + 1)*(a + b*x^n)), x)
```

sympy [C] time = 4.55, size = 309, normalized size = 0.97

$$\frac{c^{-\frac{3n}{4}}x^{-\frac{3n}{4}}\Gamma\left(-\frac{3}{4}\right)}{acn\Gamma\left(\frac{1}{4}\right)} - \frac{3b^{\frac{3}{4}}c^{-\frac{3n}{4}}e^{-\frac{i\pi}{4}}\log\left(1 - \frac{\sqrt[4]{b}x^{\frac{n}{4}}e^{\frac{i\pi}{4}}}{\sqrt[4]{a}}\right)\Gamma\left(-\frac{3}{4}\right)}{4a^{\frac{7}{4}}cn\Gamma\left(\frac{1}{4}\right)} + \frac{3ib^{\frac{3}{4}}c^{-\frac{3n}{4}}e^{-\frac{i\pi}{4}}\log\left(1 - \frac{\sqrt[4]{b}x^{\frac{n}{4}}e^{\frac{3i\pi}{4}}}{\sqrt[4]{a}}\right)\Gamma\left(-\frac{3}{4}\right)}{4a^{\frac{7}{4}}cn\Gamma\left(\frac{1}{4}\right)} + \frac{3b^{\frac{3}{4}}c^{-\frac{3n}{4}}e^{-\frac{i\pi}{4}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-1-3/4*n)/(a+b*x**n),x)
```

```
[Out] c**(-3*n/4)*x**(-3*n/4)*gamma(-3/4)/(a*c*n*gamma(1/4)) - 3*b**(3/4)*c**(-3*
n/4)*exp(-I*pi/4)*log(1 - b**(1/4)*x**(n/4)*exp_polar(I*pi/4)/a**(1/4))*gam
ma(-3/4)/(4*a**(7/4)*c*n*gamma(1/4)) + 3*I*b**(3/4)*c**(-3*n/4)*exp(-I*pi/4
)*log(1 - b**(1/4)*x**(n/4)*exp_polar(3*I*pi/4)/a**(1/4))*gamma(-3/4)/(4*a*
*(7/4)*c*n*gamma(1/4)) + 3*b**(3/4)*c**(-3*n/4)*exp(-I*pi/4)*log(1 - b**(1/
4)*x**(n/4)*exp_polar(5*I*pi/4)/a**(1/4))*gamma(-3/4)/(4*a**(7/4)*c*n*gamma
(1/4)) - 3*I*b**(3/4)*c**(-3*n/4)*exp(-I*pi/4)*log(1 - b**(1/4)*x**(n/4)*ex
p_polar(7*I*pi/4)/a**(1/4))*gamma(-3/4)/(4*a**(7/4)*c*n*gamma(1/4))
```

3.2765 $\int \frac{(cx)^{-1-n}}{a+bx^n}\,dx$

Optimal. Leaf size=69

$$-\frac{bx^n \log(x)(cx)^{-n}}{a^2c} + \frac{bx^n(cx)^{-n}\log(a+bx^n)}{a^2cn} - \frac{(cx)^{-n}}{acn}$$

```
[Out] -1/a/c/n/((c*x)^n)-b*x^n*ln(x)/a^2/c/((c*x)^n)+b*x^n*ln(a+b*x^n)/a^2/c/n/((
c*x)^n)
```

Rubi [A] time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {268, 266, 44}

$$-\frac{bx^n \log(x)(cx)^{-n}}{a^2c} + \frac{bx^n(cx)^{-n}\log(a+bx^n)}{a^2cn} - \frac{(cx)^{-n}}{acn}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 - n)/(a + b*x^n),x]
```

```
[Out] -(1/(a*c*n*(c*x)^n)) - (b*x^n*Log[x])/(a^2*c*(c*x)^n) + (b*x^n*Log[a + b*x^
n])/(a^2*c*n*(c*x)^n)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 268

```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(c^In
tPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a + b*x^n)^p, x], x] /;
 FreeQ[{a, b, c, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx)^{-1-n}}{a+bx^n}\,dx &= \frac{(x^n(cx)^{-n})\int \frac{x^{-1-n}}{a+bx^n}\,dx}{c} \\
&= \frac{(x^n(cx)^{-n})\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)}\,dx, x, x^n\right)}{cn} \\
&= \frac{(x^n(cx)^{-n})\,\text{Subst}\left(\int \left(\frac{1}{ax^2} - \frac{b}{a^2x} + \frac{b^2}{a^2(a+bx)}\right)\,dx, x, x^n\right)}{cn} \\
&= -\frac{(cx)^{-n}}{acn} - \frac{bx^n(cx)^{-n}\log(x)}{a^2c} + \frac{bx^n(cx)^{-n}\log(a+bx^n)}{a^2cn}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 42, normalized size = 0.61

$$-\frac{(cx)^{-n}\left(-bx^n\log\left(a+bx^n\right)+a+bnx^n\log(x)\right)}{a^2cn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 - n)/(a + b*x^n),x]
```

```
[Out] -((a + b*n*x^n*Log[x] - b*x^n*Log[a + b*x^n])/(a^2*c*n*(c*x)^n))
```

fricas [A] time = 0.64, size = 59, normalized size = 0.86

$$-\frac{bc^{-n-1}nx^n\log(x)-bc^{-n-1}x^n\log\left(bx^n+a\right)+ac^{-n-1}}{a^2nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] -(b*c^(-n - 1)*n*x^n*log(x) - b*c^(-n - 1)*x^n*log(b*x^n + a) + a*c^(-n - 1
))/(a^2*n*x^n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{-n-1}}{bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-n)/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(-n - 1)/(b*x^n + a), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{-n-1}}{b\,x^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-n-1)/(b*x^n+a),x)
```

```
[Out] int((c*x)^(-n-1)/(b*x^n+a),x)
```

maxima [A] time = 0.62, size = 63, normalized size = 0.91

$$-\frac{bc^{-n-1}\log(x)}{a^2}+\frac{bc^{-n-1}\log\left(\frac{bx^n+a}{b}\right)}{a^2n}-\frac{c^{-n-1}}{anx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-n)/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] -b*c^(-n - 1)*log(x)/a^2 + b*c^(-n - 1)*log((b*x^n + a)/b)/(a^2*n) - c^(-n
 - 1)/(a*n*x^n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\,x)^{n+1}\,(a+b\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(n + 1)*(a + b*x^n)),x)
```

```
[Out] int(1/((c*x)^(n + 1)*(a + b*x^n)), x)
```

sympy [**A**] time = 4.11, size = 34, normalized size = 0.49

$$-\frac{c^{-n}x^{-n}}{acn} + \frac{bc^{-n}\log\left(\frac{ax^{-n}}{b}+1\right)}{a^2cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-1-n)/(a+b*x**n),x)
```

```
[Out] -c**(-n)*x**(-n)/(a*c*n) + b*c**(-n)*log(a*x**(-n)/b + 1)/(a**2*c*n)
```

3.2766 $\int \frac{(cx)^{-1-\frac{n}{2}}}{a+bx^n}\,dx$

Optimal. Leaf size=74

$$\frac{2\sqrt{b}\,x^{n/2}(cx)^{-n/2}\tan^{-1}\left(\frac{\sqrt{a}\,x^{-n/2}}{\sqrt{b}}\right)}{a^{3/2}cn}-\frac{2(cx)^{-n/2}}{acn}$$

```
[Out] -2/a/c/n/((c*x)^(1/2*n))+2*x^(1/2*n)*arctan(a^(1/2)/(x^(1/2*n))/b^(1/2))*b^
(1/2)/a^(3/2)/c/n/((c*x)^(1/2*n))
```

Rubi [A] time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {347, 345, 193, 321, 205}

$$\frac{2\sqrt{b}\,x^{n/2}(cx)^{-n/2}\tan^{-1}\left(\frac{\sqrt{a}\,x^{-n/2}}{\sqrt{b}}\right)}{a^{3/2}cn}-\frac{2(cx)^{-n/2}}{acn}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 - n/2)/(a + b*x^n),x]
```

```
[Out] -2/(a*c*n*(c*x)^(n/2)) + (2*Sqrt[b]*x^(n/2)*ArcTan[Sqrt[a]/(Sqrt[b]*x^(n/2)
)])/(a^(3/2)*c*n*(c*x)^(n/2))
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 347

```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(c^In
tPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a + b*x^n)^p, x], x] /;
 FreeQ[{a, b, c, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !Integer
Q[n]
```

Rubi steps

$$\int \frac{(cx)^{-1-\frac{n}{2}}}{a+bx^n}\,dx = \frac{\left(x^{n/2}(cx)^{-n/2}\right)\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n}\,dx}{c}$$
$$= -\frac{\left(2x^{n/2}(cx)^{-n/2}\right)\text{Subst}\left(\int \frac{1}{a+\frac{b}{x^2}}\,dx, x, x^{-n/2}\right)}{cn}$$
$$= -\frac{\left(2x^{n/2}(cx)^{-n/2}\right)\text{Subst}\left(\int \frac{x^2}{b+ax^2}\,dx, x, x^{-n/2}\right)}{cn}$$
$$= -\frac{2(cx)^{-n/2}}{acn} + \frac{\left(2bx^{n/2}(cx)^{-n/2}\right)\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, x^{-n/2}\right)}{acn}$$
$$= -\frac{2(cx)^{-n/2}}{acn} + \frac{2\sqrt{b}\,x^{n/2}(cx)^{-n/2}\tan^{-1}\left(\frac{\sqrt{a}\,x^{-n/2}}{\sqrt{b}}\right)}{a^{3/2}cn}$$

Mathematica [C] time = 0.00, size = 37, normalized size = 0.50

$$-\frac{2x(cx)^{-\frac{n}{2}-1}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{bx^n}{a}\right)}{an}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 - n/2)/(a + b*x^n),x]
```

```
[Out] (-2*x*(c*x)^(-1 - n/2)*Hypergeometric2F1[-1/2, 1, 1/2, -((b*x^n)/a)])/(a*n)
```

fricas [A] time = 0.65, size = 225, normalized size = 3.04

$$\left[-\frac{2\,xe^{\left(-\frac{1}{2}(n+2)\log(c)-\frac{1}{2}(n+2)\log(x)\right)} - \sqrt{-\frac{bc^{-n-2}}{a}}\log\left(\frac{ax^2e^{\left(-(n+2)\log(c)-(n+2)\log(x)\right)}+2\,a\sqrt{-\frac{bc^{-n-2}}{a}}\,xe^{\left(-\frac{1}{2}(n+2)\log(c)-\frac{1}{2}(n+2)\log(x)\right)}-bc^{-}}{ax^2e^{\left(-(n+2)\log(c)-(n+2)\log(x)\right)}+bc^{-n-2}}\right.}{an}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-1/2*n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] [-(2*x*e^(-1/2*(n + 2)*log(c) - 1/2*(n + 2)*log(x)) - sqrt(-b*c^(-n - 2)/a)
*log((a*x^2*e^(-(n + 2)*log(c) - (n + 2)*log(x)) + 2*a*sqrt(-b*c^(-n - 2)/a
)*x*e^(-1/2*(n + 2)*log(c) - 1/2*(n + 2)*log(x)) - b*c^(-n - 2))/(a*x^2*e^(
-(n + 2)*log(c) - (n + 2)*log(x)) + b*c^(-n - 2))))/(a*n), -2*(x*e^(-1/2*(n
 + 2)*log(c) - 1/2*(n + 2)*log(x)) + sqrt(b*c^(-n - 2)/a)*arctan(sqrt(b*c^(
-n - 2)/a)*e^(1/2*(n + 2)*log(c) + 1/2*(n + 2)*log(x))/x))/(a*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{-\frac{1}{2}n-1}}{bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-1/2*n)/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(-1/2*n - 1)/(b*x^n + a), x)
```

maple [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{-\frac{n}{2}-1}}{b\,x^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^n+a)*(c*x)^(-1/2*n-1),x)
```

```
[Out] int(1/(b*x^n+a)*(c*x)^(-1/2*n-1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-b\int \frac{x^{\frac{1}{2}n}}{abc^{\frac{1}{2}n+1}xx^n + a^2c^{\frac{1}{2}n+1}x}\,dx - \frac{2\,c^{-\frac{1}{2}n-1}}{anx^{\frac{1}{2}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-1/2*n)/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] -b*integrate(x^(1/2*n)/(a*b*c^(1/2*n + 1)*x*x^n + a^2*c^(1/2*n + 1)*x), x)
- 2*c^(-1/2*n - 1)/(a*n*x^(1/2*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\,x)^{\frac{n}{2}+1}\,(a + b\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(n/2 + 1)*(a + b*x^n)),x)
```

```
[Out] int(1/((c*x)^(n/2 + 1)*(a + b*x^n)), x)
```

sympy [A] time = 3.78, size = 54, normalized size = 0.73

$$-\frac{2c^{-\frac{n}{2}}x^{-\frac{n}{2}}}{acn} - \frac{2\sqrt{b}\,c^{-\frac{n}{2}}\,\operatorname{atan}\left(\frac{\sqrt{b}\,x^{\frac{n}{2}}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-1-1/2*n)/(a+b*x**n),x)
```

```
[Out] -2*c**(-n/2)*x**(-n/2)/(a*c*n) - 2*sqrt(b)*c**(-n/2)*atan(sqrt(b)*x**(n/2)/
sqrt(a))/(a**(3/2)*c*n)
```

3.2767 $\int \frac{(cx)^{-1-\frac{n}{3}}}{a+bx^n}\,dx$

Optimal. Leaf size=220

$$-\frac{\sqrt[3]{b}\,x^{n/3}(cx)^{-n/3}\log\left(a^{2/3}x^{-2n/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^{-n/3}+b^{2/3}\right)}{2a^{4/3}cn}+\frac{\sqrt[3]{b}\,x^{n/3}(cx)^{-n/3}\log\left(\sqrt[3]{a}\,x^{-n/3}+\sqrt[3]{b}\right)}{a^{4/3}cn}-\frac{\sqrt{3}\,\sqrt[3]{b}\,x^{n/3}(cx)^{-n}}{}$$

```
[Out] -3/a/c/n/((c*x)^(1/3*n))+b^(1/3)*x^(1/3*n)*ln(b^(1/3)+a^(1/3)/(x^(1/3*n)))/
a^(4/3)/c/n/((c*x)^(1/3*n))-1/2*b^(1/3)*x^(1/3*n)*ln(b^(2/3)+a^(2/3)/(x^(2/
3*n))-a^(1/3)*b^(1/3)/(x^(1/3*n)))/a^(4/3)/c/n/((c*x)^(1/3*n))-b^(1/3)*x^(1
/3*n)*arctan(1/3*(b^(1/3)-2*a^(1/3)/(x^(1/3*n)))/b^(1/3)*3^(1/2))*3^(1/2)/a
^(4/3)/c/n/((c*x)^(1/3*n))
```

Rubi [A] time = 0.14, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {347, 345, 193, 321, 200, 31, 634, 617, 204, 628}

$$-\frac{\sqrt[3]{b}\,x^{n/3}(cx)^{-n/3}\log\left(a^{2/3}x^{-2n/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^{-n/3}+b^{2/3}\right)}{2a^{4/3}cn}+\frac{\sqrt[3]{b}\,x^{n/3}(cx)^{-n/3}\log\left(\sqrt[3]{a}\,x^{-n/3}+\sqrt[3]{b}\right)}{a^{4/3}cn}-\frac{\sqrt{3}\,\sqrt[3]{b}\,x^{n/3}(cx)^{-n}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 - n/3)/(a + b*x^n),x]
```

```
[Out] -3/(a*c*n*(c*x)^(n/3)) - (Sqrt[3]*b^(1/3)*x^(n/3)*ArcTan[(b^(1/3) - (2*a^(1
/3))/x^(n/3))/(Sqrt[3]*b^(1/3))])/(a^(4/3)*c*n*(c*x)^(n/3)) + (b^(1/3)*x^(n
/3)*Log[b^(1/3) + a^(1/3)/x^(n/3)])/(a^(4/3)*c*n*(c*x)^(n/3)) - (b^(1/3)*x^
(n/3)*Log[b^(2/3) + a^(2/3)/x^((2*n)/3) - (a^(1/3)*b^(1/3))/x^(n/3)])/(2*a^
(4/3)*c*n*(c*x)^(n/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 347

```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(c^In
tPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a + b*x^n)^p, x], x] /;
 FreeQ[{a, b, c, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !Integer
Q[n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(cx)^{-1-\frac{n}{3}}}{a+bx^n}\,dx = \frac{\left(x^{n/3}(cx)^{-n/3}\right)\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n}\,dx}{c}$$

$$= -\frac{\left(3x^{n/3}(cx)^{-n/3}\right)\text{Subst}\left(\int \frac{1}{a+\frac{b}{x^3}}\,dx, x, x^{-n/3}\right)}{cn}$$

$$= -\frac{\left(3x^{n/3}(cx)^{-n/3}\right)\text{Subst}\left(\int \frac{x^3}{b+ax^3}\,dx, x, x^{-n/3}\right)}{cn}$$

$$= -\frac{3(cx)^{-n/3}}{acn} + \frac{\left(3bx^{n/3}(cx)^{-n/3}\right)\text{Subst}\left(\int \frac{1}{b+ax^3}\,dx, x, x^{-n/3}\right)}{acn}$$

$$= -\frac{3(cx)^{-n/3}}{acn} + \frac{\left(\sqrt[3]{b}\,x^{n/3}(cx)^{-n/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{a}\,x}\,dx, x, x^{-n/3}\right)}{acn} + \frac{\left(\sqrt[3]{b}\,x^{n/3}(cx)^{-n/3}\right)\text{Subst}\left(\int$$

$$= -\frac{3(cx)^{-n/3}}{acn} + \frac{\sqrt[3]{b}\,x^{n/3}(cx)^{-n/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x^{-n/3}\right)}{a^{4/3}cn} - \frac{\left(\sqrt[3]{b}\,x^{n/3}(cx)^{-n/3}\right)\text{Subst}\left(\int$$

$$= -\frac{3(cx)^{-n/3}}{acn} + \frac{\sqrt[3]{b}\,x^{n/3}(cx)^{-n/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x^{-n/3}\right)}{a^{4/3}cn} - \frac{\sqrt[3]{b}\,x^{n/3}(cx)^{-n/3}\log\left(b^{2/3}+$$

$$= -\frac{3(cx)^{-n/3}}{acn} - \frac{\sqrt{3}\,\sqrt[3]{b}\,x^{n/3}(cx)^{-n/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}\,x^{-n/3}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{a^{4/3}cn} + \frac{\sqrt[3]{b}\,x^{n/3}(cx)^{-n/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x^{-n/3}\right)}{a^{4/3}cn} -$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.17

$$-\frac{3x(cx)^{-\frac{n}{3}-1}\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};-\frac{bx^n}{a}\right)}{an}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 - n/3)/(a + b*x^n),x]
```

```
[Out] (-3*x*(c*x)^(-1 - n/3)*Hypergeometric2F1[-1/3, 1, 2/3, -((b*x^n)/a)])/(a*n)
```

fricas [A] time = 0.56, size = 255, normalized size = 1.16

$$\frac{6\,xe^{\left(-\frac{1}{3}(n+3)\log(c)-\frac{1}{3}(n+3)\log(x)\right)} - 2\sqrt{3}\left(\frac{bc^{-n-3}}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,a\left(\frac{bc^{-n-3}}{a}\right)^{\frac{2}{3}}xe^{\left(-\frac{1}{3}(n+3)\log(c)-\frac{1}{3}(n+3)\log(x)\right)} - \sqrt{3}\,bc^{-n-3}}{3\,bc^{-n-3}}\right) - 2\left($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-1/3*n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] -1/2*(6*x*e^(-1/3*(n + 3)*log(c) - 1/3*(n + 3)*log(x)) - 2*sqrt(3)*(b*c^(-n
 - 3)/a)^(1/3)*arctan(1/3*(2*sqrt(3)*a*(b*c^(-n - 3)/a)^(2/3)*x*e^(-1/3*(n
+ 3)*log(c) - 1/3*(n + 3)*log(x)) - sqrt(3)*b*c^(-n - 3))/(b*c^(-n - 3))) -
 2*(b*c^(-n - 3)/a)^(1/3)*log((x*e^(-1/3*(n + 3)*log(c) - 1/3*(n + 3)*log(x
)) + (b*c^(-n - 3)/a)^(1/3))/x) + (b*c^(-n - 3)/a)^(1/3)*log((x^2*e^(-2/3*(
n + 3)*log(c) - 2/3*(n + 3)*log(x)) - (b*c^(-n - 3)/a)^(1/3)*x*e^(-1/3*(n +
 3)*log(c) - 1/3*(n + 3)*log(x)) + (b*c^(-n - 3)/a)^(2/3))/x^2))/(a*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{-\frac{1}{3}n-1}}{bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-1/3*n)/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(-1/3*n - 1)/(b*x^n + a), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{-\frac{n}{3}-1}}{b\,x^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-1-1/3*n)/(b*x^n+a),x)
```

```
[Out] int((c*x)^(-1-1/3*n)/(b*x^n+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-b\int \frac{x^{\frac{2}{3}n}}{abc^{\frac{1}{3}n+1}xx^n+a^2c^{\frac{1}{3}n+1}x}\,dx-\frac{3\,c^{-\frac{1}{3}n-1}}{anx^{\frac{1}{3}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-1/3*n)/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] -b*integrate(x^(2/3*n)/(a*b*c^(1/3*n + 1)*x*x^n + a^2*c^(1/3*n + 1)*x), x)
- 3*c^(-1/3*n - 1)/(a*n*x^(1/3*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(c\,x)^{\frac{n}{3}+1}\,(a+b\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(n/3 + 1)*(a + b*x^n)),x)
```

```
[Out] int(1/((c*x)^(n/3 + 1)*(a + b*x^n)), x)
```

sympy [C] time = 4.23, size = 216, normalized size = 0.98

$$\frac{c^{-\frac{n}{3}}x^{-\frac{n}{3}}\Gamma\left(-\frac{1}{3}\right)}{acn\Gamma\left(\frac{2}{3}\right)}-\frac{\sqrt[3]{b}\,c^{-\frac{n}{3}}e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{n}{3}}e^{\frac{i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{1}{3}\right)}{3a^{\frac{4}{3}}cn\Gamma\left(\frac{2}{3}\right)}-\frac{\sqrt[3]{b}\,c^{-\frac{n}{3}}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{n}{3}}e^{i\pi}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{1}{3}\right)}{3a^{\frac{4}{3}}cn\Gamma\left(\frac{2}{3}\right)}-\frac{\sqrt[3]{b}\,c^{-\frac{n}{3}}e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}x}{}\right.}{3a^{\frac{4}{3}}cn\Gamma\left(\frac{2}{3}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-1-1/3*n)/(a+b*x**n),x)
```

```
[Out] c**(-n/3)*x**(-n/3)*gamma(-1/3)/(a*c*n*gamma(2/3)) - b**(1/3)*c**(-n/3)*exp
(-2*I*pi/3)*log(1 - b**(1/3)*x**(n/3)*exp_polar(I*pi/3)/a**(1/3))*gamma(-1/
3)/(3*a**(4/3)*c*n*gamma(2/3)) - b**(1/3)*c**(-n/3)*log(1 - b**(1/3)*x**(n/
3)*exp_polar(I*pi)/a**(1/3))*gamma(-1/3)/(3*a**(4/3)*c*n*gamma(2/3)) - b**(
1/3)*c**(-n/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*x**(n/3)*exp_polar(5*I*pi/3)/
a**(1/3))*gamma(-1/3)/(3*a**(4/3)*c*n*gamma(2/3))
```

3.2768 $\int \frac{(cx)^{-1-\frac{n}{4}}}{a+bx^n}\,dx$

Optimal. Leaf size=315

$$-\frac{\sqrt[4]{b}\,x^{n/4}(cx)^{-n/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^{-n/4}+\sqrt{a}\,x^{-n/2}+\sqrt{b}\right)}{\sqrt{2}\,a^{5/4}cn}+\frac{\sqrt[4]{b}\,x^{n/4}(cx)^{-n/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^{-n/4}+\sqrt{a}\,x^{-n/2}+\right.}{\sqrt{2}\,a^{5/4}cn}$$

```
[Out] -4/a/c/n/((c*x)^(1/4*n))-1/2*b^(1/4)*x^(1/4*n)*ln(-a^(1/4)*b^(1/4)*2^(1/2)/
(x^(1/4*n))+a^(1/2)/(x^(1/2*n))+b^(1/2))/a^(5/4)/c/n/((c*x)^(1/4*n))*2^(1/2
)+1/2*b^(1/4)*x^(1/4*n)*ln(a^(1/4)*b^(1/4)*2^(1/2)/(x^(1/4*n))+a^(1/2)/(x^(
1/2*n))+b^(1/2))/a^(5/4)/c/n/((c*x)^(1/4*n))*2^(1/2)-b^(1/4)*x^(1/4*n)*arct
an(1-a^(1/4)*2^(1/2)/b^(1/4)/(x^(1/4*n)))*2^(1/2)/a^(5/4)/c/n/((c*x)^(1/4*n
))+b^(1/4)*x^(1/4*n)*arctan(1+a^(1/4)*2^(1/2)/b^(1/4)/(x^(1/4*n)))*2^(1/2)/
a^(5/4)/c/n/((c*x)^(1/4*n))
```

Rubi [A] time = 0.23, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {347, 345, 193, 321, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\sqrt[4]{b}\,x^{n/4}(cx)^{-n/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^{-n/4}+\sqrt{a}\,x^{-n/2}+\sqrt{b}\right)}{\sqrt{2}\,a^{5/4}cn}+\frac{\sqrt[4]{b}\,x^{n/4}(cx)^{-n/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^{-n/4}+\sqrt{a}\,x^{-n/2}+\right.}{\sqrt{2}\,a^{5/4}cn}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 - n/4)/(a + b*x^n),x]
```

```
[Out] -4/(a*c*n*(c*x)^(n/4)) - (Sqrt[2]*b^(1/4)*x^(n/4)*ArcTan[1 - (Sqrt[2]*a^(1/
4))/(b^(1/4)*x^(n/4))])/(a^(5/4)*c*n*(c*x)^(n/4)) + (Sqrt[2]*b^(1/4)*x^(n/4
)*ArcTan[1 + (Sqrt[2]*a^(1/4))/(b^(1/4)*x^(n/4))])/(a^(5/4)*c*n*(c*x)^(n/4)
) - (b^(1/4)*x^(n/4)*Log[Sqrt[b] + Sqrt[a]/x^(n/2) - (Sqrt[2]*a^(1/4)*b^(1/
4))/x^(n/4)])/(Sqrt[2]*a^(5/4)*c*n*(c*x)^(n/4)) + (b^(1/4)*x^(n/4)*Log[Sqrt
[b] + Sqrt[a]/x^(n/2) + (Sqrt[2]*a^(1/4)*b^(1/4))/x^(n/4)])/(Sqrt[2]*a^(5/4
)*c*n*(c*x)^(n/4))
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 347

```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(c^In
tPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a + b*x^n)^p, x], x] /;
 FreeQ[{a, b, c, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !Integer
Q[n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(cx)^{-1-\frac{n}{4}}}{a+bx^n}\,dx = \frac{\left(x^{n/4}(cx)^{-n/4}\right)\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n}\,dx}{c}$$

$$= -\frac{\left(4x^{n/4}(cx)^{-n/4}\right)\text{Subst}\left(\int \frac{1}{a+\frac{b}{x^4}}\,dx, x, x^{-n/4}\right)}{cn}$$

$$= -\frac{\left(4x^{n/4}(cx)^{-n/4}\right)\text{Subst}\left(\int \frac{x^4}{b+ax^4}\,dx, x, x^{-n/4}\right)}{cn}$$

$$= -\frac{4(cx)^{-n/4}}{acn} + \frac{\left(4bx^{n/4}(cx)^{-n/4}\right)\text{Subst}\left(\int \frac{1}{b+ax^4}\,dx, x, x^{-n/4}\right)}{acn}$$

$$= -\frac{4(cx)^{-n/4}}{acn} + \frac{\left(2\sqrt{b}\,x^{n/4}(cx)^{-n/4}\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{a}\,x^2}{b+ax^4}\,dx, x, x^{-n/4}\right)}{acn} + \frac{\left(2\sqrt{b}\,x^{n/4}(cx)^{-n/4}\right)\text{Subst}\left(\right.}{acn}$$

$$= -\frac{4(cx)^{-n/4}}{acn} - \frac{\left(\sqrt[4]{b}\,x^{n/4}(cx)^{-n/4}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\,\sqrt[4]{b}}{\sqrt[4]{a}}+2x}{-\frac{\sqrt{b}}{\sqrt{a}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}-x^2}\,dx, x, x^{-n/4}\right)}{\sqrt{2}\,a^{5/4}cn} - \frac{\left(\sqrt[4]{b}\,x^{n/4}(cx)^{-n/4}\right)\text{Subs}}{}$$

$$= -\frac{4(cx)^{-n/4}}{acn} - \frac{\sqrt[4]{b}\,x^{n/4}(cx)^{-n/4}\log\left(\sqrt{b}+\sqrt{a}\,x^{-n/2}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^{-n/4}\right)}{\sqrt{2}\,a^{5/4}cn} + \frac{\sqrt[4]{b}\,x^{n/4}(cx)^{-n/4}\log\left(\right.}{}$$

$$= -\frac{4(cx)^{-n/4}}{acn} - \frac{\sqrt{2}\,\sqrt[4]{b}\,x^{n/4}(cx)^{-n/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{a}\,x^{-n/4}}{\sqrt[4]{b}}\right)}{a^{5/4}cn} + \frac{\sqrt{2}\,\sqrt[4]{b}\,x^{n/4}(cx)^{-n/4}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{}}{}\right.}{a^{5/4}cn}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.12

$$-\frac{4x(cx)^{-\frac{n}{4}-1}\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\frac{bx^n}{a}\right)}{an}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 - n/4)/(a + b*x^n),x]
```

```
[Out] (-4*x*(c*x)^(-1 - n/4)*Hypergeometric2F1[-1/4, 1, 3/4, -((b*x^n)/a)])/(a*n)
```

fricas [A] time = 0.83, size = 321, normalized size = 1.02

$$4\,an\left(-\frac{bc^{-n-4}}{a^5n^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{a^4n^3x\left(-\frac{bc^{-n-4}}{a^5n^4}\right)^{\frac{3}{4}}e^{\left(-\frac{1}{4}(n+4)\log(c)-\frac{1}{4}(n+4)\log(x)\right)}-a^4n^3x\sqrt{\frac{a^2n^2\sqrt{-\frac{bc^{-n-4}}{a^5n^4}}+x^2e^{\left(-\frac{1}{2}(n+4)\log(c)-\frac{1}{2}(n+4)\log(x)\right)}}{x^2}}\left(-\frac{bc^{-}}{a^{5}}\right.}{bc^{-n-4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-1/4*n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] (4*a*n*(-b*c^(-n - 4)/(a^5*n^4))^(1/4)*arctan(-(a^4*n^3*x*(-b*c^(-n - 4)/(a
^5*n^4))^(3/4)*e^(-1/4*(n + 4)*log(c) - 1/4*(n + 4)*log(x)) - a^4*n^3*x*sqr
t((a^2*n^2*sqrt(-b*c^(-n - 4)/(a^5*n^4)) + x^2*e^(-1/2*(n + 4)*log(c) - 1/2
*(n + 4)*log(x)))/x^2)*(-b*c^(-n - 4)/(a^5*n^4))^(3/4))/(b*c^(-n - 4))) + a
*n*(-b*c^(-n - 4)/(a^5*n^4))^(1/4)*log((a*n*(-b*c^(-n - 4)/(a^5*n^4))^(1/4)
```

```
 + x*e^(-1/4*(n + 4)*log(c) - 1/4*(n + 4)*log(x)))/x) - a*n*(-b*c^(-n - 4)/
(a^5*n^4))^(1/4)*log(-(a*n*(-b*c^(-n - 4)/(a^5*n^4))^(1/4) - x*e^(-1/4*(n +
 4)*log(c) - 1/4*(n + 4)*log(x)))/x) - 4*x*e^(-1/4*(n + 4)*log(c) - 1/4*(n
+ 4)*log(x)))/(a*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{-\frac{1}{4}n-1}}{bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-1/4*n)/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(-1/4*n - 1)/(b*x^n + a), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{-\frac{n}{4}-1}}{b\,x^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-1-1/4*n)/(b*x^n+a),x)
```

```
[Out] int((c*x)^(-1-1/4*n)/(b*x^n+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-b\int \frac{x^{\frac{3}{4}n}}{abc^{\frac{1}{4}n+1}xx^n+a^2c^{\frac{1}{4}n+1}x}\,dx-\frac{4\,c^{-\frac{1}{4}n-1}}{anx^{\frac{1}{4}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-1/4*n)/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] -b*integrate(x^(3/4*n)/(a*b*c^(1/4*n + 1)*x*x^n + a^2*c^(1/4*n + 1)*x), x)
- 4*c^(-1/4*n - 1)/(a*n*x^(1/4*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(c\,x)^{\frac{n}{4}+1}\,(a+b\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(n/4 + 1)*(a + b*x^n)),x)
```

```
[Out] int(1/((c*x)^(n/4 + 1)*(a + b*x^n)), x)
```

sympy [C] time = 4.66, size = 299, normalized size = 0.95

$$\frac{c^{-\frac{n}{4}}x^{-\frac{n}{4}}\Gamma\left(-\frac{1}{4}\right)}{acn\Gamma\left(\frac{3}{4}\right)}-\frac{\sqrt[4]{b}\,c^{-\frac{n}{4}}e^{-\frac{3i\pi}{4}}\log\left(1-\frac{\sqrt[4]{b}\,x^{\frac{n}{4}}e^{\frac{i\pi}{4}}}{\sqrt[4]{a}}\right)\Gamma\left(-\frac{1}{4}\right)}{4a^{\frac{5}{4}}cn\Gamma\left(\frac{3}{4}\right)}-\frac{i\sqrt[4]{b}\,c^{-\frac{n}{4}}e^{-\frac{3i\pi}{4}}\log\left(1-\frac{\sqrt[4]{b}\,x^{\frac{n}{4}}e^{\frac{3i\pi}{4}}}{\sqrt[4]{a}}\right)\Gamma\left(-\frac{1}{4}\right)}{4a^{\frac{5}{4}}cn\Gamma\left(\frac{3}{4}\right)}+\frac{\sqrt[4]{b}\,c^{-\frac{n}{4}}e^{-\frac{3i\pi}{4}}\log\left(\right.}{4a^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-1-1/4*n)/(a+b*x**n),x)
```

```
[Out] c**(-n/4)*x**(-n/4)*gamma(-1/4)/(a*c*n*gamma(3/4)) - b**(1/4)*c**(-n/4)*exp
(-3*I*pi/4)*log(1 - b**(1/4)*x**(n/4)*exp_polar(I*pi/4)/a**(1/4))*gamma(-1/
4)/(4*a**(5/4)*c*n*gamma(3/4)) - I*b**(1/4)*c**(-n/4)*exp(-3*I*pi/4)*log(1
- b**(1/4)*x**(n/4)*exp_polar(3*I*pi/4)/a**(1/4))*gamma(-1/4)/(4*a**(5/4)*c
*n*gamma(3/4)) + b**(1/4)*c**(-n/4)*exp(-3*I*pi/4)*log(1 - b**(1/4)*x**(n/4
)*exp_polar(5*I*pi/4)/a**(1/4))*gamma(-1/4)/(4*a**(5/4)*c*n*gamma(3/4)) + I
*b**(1/4)*c**(-n/4)*exp(-3*I*pi/4)*log(1 - b**(1/4)*x**(n/4)*exp_polar(7*I*
pi/4)/a**(1/4))*gamma(-1/4)/(4*a**(5/4)*c*n*gamma(3/4))
```

3.2769 $\int \frac{(cx)^{-1-\frac{3n}{2}}}{a+bx^n}\,dx$

Optimal. Leaf size=100

$$-\frac{2b^{3/2}x^{3n/2}(cx)^{-3n/2}\tan^{-1}\left(\frac{\sqrt{a}\,x^{-n/2}}{\sqrt{b}}\right)}{a^{5/2}cn}+\frac{2bx^n(cx)^{-3n/2}}{a^2cn}-\frac{2(cx)^{-3n/2}}{3acn}$$

```
[Out] -2/3/a/c/n/((c*x)^(3/2*n))+2*b*x^n/a^2/c/n/((c*x)^(3/2*n))-2*b^(3/2)*x^(3/2
*n)*arctan(a^(1/2)/(x^(1/2*n))/b^(1/2))/a^(5/2)/c/n/((c*x)^(3/2*n))
```

Rubi [A] time = 0.05, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {363, 362, 345, 193, 321, 205}

$$-\frac{2b^{3/2}x^{3n/2}(cx)^{-3n/2}\tan^{-1}\left(\frac{\sqrt{a}\,x^{-n/2}}{\sqrt{b}}\right)}{a^{5/2}cn}+\frac{2bx^n(cx)^{-3n/2}}{a^2cn}-\frac{2(cx)^{-3n/2}}{3acn}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 - (3*n)/2)/(a + b*x^n),x]
```

```
[Out] -2/(3*a*c*n*(c*x)^((3*n)/2)) + (2*b*x^n)/(a^2*c*n*(c*x)^((3*n)/2)) - (2*b^(
3/2)*x^((3*n)/2)*ArcTan[Sqrt[a]/(Sqrt[b]*x^(n/2))])/(a^(5/2)*c*n*(c*x)^((3*
n)/2))
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 362

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Dist[b/a, Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a,
 b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

Rule 363

```
Int[((c_)*(x_))^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Dist[(c^IntPart
[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m/(a + b*x^n), x], x] /; FreeQ[
```

```
{a, b, c, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && (SumSimplerQ[m, n]
 || SumSimplerQ[m, -n])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx)^{-1-\frac{3n}{2}}}{a+bx^n}\,dx &= \frac{\left(x^{3n/2}(cx)^{-3n/2}\right)\int \frac{x^{-1-\frac{3n}{2}}}{a+bx^n}\,dx}{c} \\
&= -\frac{2(cx)^{-3n/2}}{3acn} - \frac{\left(bx^{3n/2}(cx)^{-3n/2}\right)\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n}\,dx}{ac} \\
&= -\frac{2(cx)^{-3n/2}}{3acn} + \frac{\left(2bx^{3n/2}(cx)^{-3n/2}\right)\text{Subst}\left(\int \frac{1}{a+\frac{b}{x^2}}\,dx, x, x^{-n/2}\right)}{acn} \\
&= -\frac{2(cx)^{-3n/2}}{3acn} + \frac{\left(2bx^{3n/2}(cx)^{-3n/2}\right)\text{Subst}\left(\int \frac{x^2}{b+ax^2}\,dx, x, x^{-n/2}\right)}{acn} \\
&= -\frac{2(cx)^{-3n/2}}{3acn} + \frac{2bx^n(cx)^{-3n/2}}{a^2cn} - \frac{\left(2b^2x^{3n/2}(cx)^{-3n/2}\right)\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, x^{-n/2}\right)}{a^2cn} \\
&= -\frac{2(cx)^{-3n/2}}{3acn} + \frac{2bx^n(cx)^{-3n/2}}{a^2cn} - \frac{2b^{3/2}x^{3n/2}(cx)^{-3n/2}\tan^{-1}\left(\frac{\sqrt{a}\,x^{-n/2}}{\sqrt{b}}\right)}{a^{5/2}cn}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.39

$$-\frac{2x(cx)^{-\frac{3n}{2}-1}\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\frac{bx^n}{a}\right)}{3an}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 - (3*n)/2)/(a + b*x^n),x]
```

```
[Out] (-2*x*(c*x)^(-1 - (3*n)/2)*Hypergeometric2F1[-3/2, 1, -1/2, -((b*x^n)/a)])/
(3*a*n)
```

fricas [A] time = 1.03, size = 492, normalized size = 4.92

$$\left[\frac{3\,bc^{-n-\frac{2}{3}}\sqrt{-\frac{bc^{-n-\frac{2}{3}}}{a}}\,\log\left(-\frac{2\,a^2bc^{-n-\frac{2}{3}}x^{\frac{4}{3}}e^{\left(-\frac{2}{3}(3n+2)\log(c)-\frac{2}{3}(3n+2)\log(x)\right)}-a^3x^2e^{\left(-(3n+2)\log(c)-(3n+2)\log(x)\right)}-2\,ab^2c^{-2n-\frac{4}{3}}x^{\frac{2}{3}}e^{\left(-\frac{1}{3}(3n+2)\log\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-3/2*n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] [1/3*(3*b*c^(-n - 2/3)*sqrt(-b*c^(-n - 2/3)/a)*log(-(2*a^2*b*c^(-n - 2/3)*x
^(4/3)*e^(-2/3*(3*n + 2)*log(c) - 2/3*(3*n + 2)*log(x)) - a^3*x^2*e^(-(3*n
+ 2)*log(c) - (3*n + 2)*log(x)) - 2*a*b^2*c^(-2*n - 4/3)*x^(2/3)*e^(-1/3*(3
*n + 2)*log(c) - 1/3*(3*n + 2)*log(x)) + b^3*c^(-3*n - 2) - 2*(a^2*b*c^(-n
```

```
- 2/3)*x*e^(-1/2*(3*n + 2)*log(c) - 1/2*(3*n + 2)*log(x)) - a^3*x^(5/3)*e^(
-5/6*(3*n + 2)*log(c) - 5/6*(3*n + 2)*log(x)) - a*b^2*c^(-2*n - 4/3)*x^(1/3
)*e^(-1/6*(3*n + 2)*log(c) - 1/6*(3*n + 2)*log(x)))*sqrt(-b*c^(-n - 2/3)/a)
)/(a^3*x^2*e^(-(3*n + 2)*log(c) - (3*n + 2)*log(x)) + b^3*c^(-3*n - 2))) +
6*b*c^(-n - 2/3)*x^(1/3)*e^(-1/6*(3*n + 2)*log(c) - 1/6*(3*n + 2)*log(x)) -
 2*a*x*e^(-1/2*(3*n + 2)*log(c) - 1/2*(3*n + 2)*log(x)))/(a^2*n), 2/3*(3*b*
c^(-n - 2/3)*sqrt(b*c^(-n - 2/3)/a)*arctan(sqrt(b*c^(-n - 2/3)/a)*e^(1/6*(3
*n + 2)*log(c) + 1/6*(3*n + 2)*log(x))/x^(1/3)) + 3*b*c^(-n - 2/3)*x^(1/3)*
e^(-1/6*(3*n + 2)*log(c) - 1/6*(3*n + 2)*log(x)) - a*x*e^(-1/2*(3*n + 2)*lo
g(c) - 1/2*(3*n + 2)*log(x)))/(a^2*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{-\frac{3}{2}n-1}}{bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-3/2*n)/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(-3/2*n - 1)/(b*x^n + a), x)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{-\frac{3n}{2}-1}}{b\,x^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-3/2*n-1)/(b*x^n+a),x)
```

```
[Out] int((c*x)^(-3/2*n-1)/(b*x^n+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^2 \int \frac{x^{\frac{1}{2}n}}{a^2 b c^{\frac{3}{2}n+1} x x^n + a^3 c^{\frac{3}{2}n+1} x}\,dx + \frac{2\,(3\,bx^n - a) c^{-\frac{3}{2}n-1}}{3\,a^2 n x^{\frac{3}{2}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-3/2*n)/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] b^2*integrate(x^(1/2*n)/(a^2*b*c^(3/2*n + 1)*x*x^n + a^3*c^(3/2*n + 1)*x),
x) + 2/3*(3*b*x^n - a)*c^(-3/2*n - 1)/(a^2*n*x^(3/2*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\,x)^{\frac{3n}{2}+1}\,(a + b\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^((3*n)/2 + 1)*(a + b*x^n)),x)
```

```
[Out] int(1/((c*x)^((3*n)/2 + 1)*(a + b*x^n)), x)
```

sympy [A] time = 4.65, size = 82, normalized size = 0.82

$$-\frac{2c^{-\frac{3n}{2}}x^{-\frac{3n}{2}}}{3acn} + \frac{2bc^{-\frac{3n}{2}}x^{-\frac{n}{2}}}{a^2cn} + \frac{2b^{\frac{3}{2}}c^{-\frac{3n}{2}}\operatorname{atan}\left(\frac{\sqrt{b}\,x^{\frac{n}{2}}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-1-3/2*n)/(a+b*x**n),x)
```

```
[Out] -2*c**(-3*n/2)*x**(-3*n/2)/(3*a*c*n) + 2*b*c**(-3*n/2)*x**(-n/2)/(a**2*c*n)
 + 2*b**(3/2)*c**(-3*n/2)*atan(sqrt(b)*x**(n/2)/sqrt(a))/(a**(5/2)*c*n)
```

3.2770 $\int \frac{(cx)^{-1-\frac{4n}{3}}}{a+bx^n}\,dx$

Optimal. Leaf size=246

$$-\frac{b^{4/3}x^{4n/3}(cx)^{-4n/3}\log\left(\sqrt[3]{a}\,x^{-n/3}+\sqrt[3]{b}\right)}{a^{7/3}cn}+\frac{b^{4/3}x^{4n/3}(cx)^{-4n/3}\log\left(a^{2/3}x^{-2n/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^{-n/3}+b^{2/3}\right)}{2a^{7/3}cn}+\frac{\sqrt{3}\,b^{4/3}x^{4n/3}(cx)}{}$$

```
[Out] -3/4/a/c/n/((c*x)^(4/3*n))+3*b*x^n/a^2/c/n/((c*x)^(4/3*n))-b^(4/3)*x^(4/3*n
)*ln(b^(1/3)+a^(1/3)/(x^(1/3*n)))/a^(7/3)/c/n/((c*x)^(4/3*n))+1/2*b^(4/3)*x
^(4/3*n)*ln(b^(2/3)+a^(2/3)/(x^(2/3*n))-a^(1/3)*b^(1/3)/(x^(1/3*n)))/a^(7/3
)/c/n/((c*x)^(4/3*n))+b^(4/3)*x^(4/3*n)*arctan(1/3*(1-2*a^(1/3)/b^(1/3)/(x^
(1/3*n)))*3^(1/2))*3^(1/2)/a^(7/3)/c/n/((c*x)^(4/3*n))
```

Rubi [A] time = 0.15, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {363, 362, 345, 193, 321, 200, 31, 634, 617, 204, 628}

$$-\frac{b^{4/3}x^{4n/3}(cx)^{-4n/3}\log\left(\sqrt[3]{a}\,x^{-n/3}+\sqrt[3]{b}\right)}{a^{7/3}cn}+\frac{b^{4/3}x^{4n/3}(cx)^{-4n/3}\log\left(a^{2/3}x^{-2n/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x^{-n/3}+b^{2/3}\right)}{2a^{7/3}cn}+\frac{\sqrt{3}\,b^{4/3}x^{4n/3}(cx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 - (4*n)/3)/(a + b*x^n),x]
```

```
[Out] -3/(4*a*c*n*(c*x)^((4*n)/3)) + (3*b*x^n)/(a^2*c*n*(c*x)^((4*n)/3)) + (Sqrt[
3]*b^(4/3)*x^((4*n)/3)*ArcTan[(b^(1/3) - (2*a^(1/3))/x^(n/3))/(Sqrt[3]*b^(1
/3))])/(a^(7/3)*c*n*(c*x)^((4*n)/3)) - (b^(4/3)*x^((4*n)/3)*Log[b^(1/3) + a
^(1/3)/x^(n/3)])/(a^(7/3)*c*n*(c*x)^((4*n)/3)) + (b^(4/3)*x^((4*n)/3)*Log[b
^(2/3) + a^(2/3)/x^((2*n)/3) - (a^(1/3)*b^(1/3))/x^(n/3)])/(2*a^(7/3)*c*n*(
c*x)^((4*n)/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 362

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Dist[b/a, Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a,
 b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

Rule 363

```
Int[((c_)*(x_))^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Dist[(c^IntPart
[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m/(a + b*x^n), x], x] /; FreeQ[
{a, b, c, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && (SumSimplerQ[m, n]
 || SumSimplerQ[m, -n])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(cx)^{-1-\frac{4n}{3}}}{a+bx^n}\,dx = \frac{\left(x^{4n/3}(cx)^{-4n/3}\right)\int \frac{x^{-1-\frac{4n}{3}}}{a+bx^n}\,dx}{c}$$

$$= -\frac{3(cx)^{-4n/3}}{4acn} - \frac{\left(bx^{4n/3}(cx)^{-4n/3}\right)\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n}\,dx}{ac}$$

$$= -\frac{3(cx)^{-4n/3}}{4acn} + \frac{\left(3bx^{4n/3}(cx)^{-4n/3}\right)\text{Subst}\left(\int \frac{1}{a+\frac{b}{x^3}}\,dx, x, x^{-n/3}\right)}{acn}$$

$$= -\frac{3(cx)^{-4n/3}}{4acn} + \frac{\left(3bx^{4n/3}(cx)^{-4n/3}\right)\text{Subst}\left(\int \frac{x^3}{b+ax^3}\,dx, x, x^{-n/3}\right)}{acn}$$

$$= -\frac{3(cx)^{-4n/3}}{4acn} + \frac{3bx^n(cx)^{-4n/3}}{a^2cn} - \frac{\left(3b^2x^{4n/3}(cx)^{-4n/3}\right)\text{Subst}\left(\int \frac{1}{b+ax^3}\,dx, x, x^{-n/3}\right)}{a^2cn}$$

$$= -\frac{3(cx)^{-4n/3}}{4acn} + \frac{3bx^n(cx)^{-4n/3}}{a^2cn} - \frac{\left(b^{4/3}x^{4n/3}(cx)^{-4n/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{a}\,x}\,dx, x, x^{-n/3}\right)}{a^2cn} - \frac{\left(b^{4/3}x^{4n/3}(c\right.}{}$$

$$= -\frac{3(cx)^{-4n/3}}{4acn} + \frac{3bx^n(cx)^{-4n/3}}{a^2cn} - \frac{b^{4/3}x^{4n/3}(cx)^{-4n/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x^{-n/3}\right)}{a^{7/3}cn} + \frac{\left(b^{4/3}x^{4n/3}(cx)^{-4n/3}\right)\text{Su}}{}$$

$$= -\frac{3(cx)^{-4n/3}}{4acn} + \frac{3bx^n(cx)^{-4n/3}}{a^2cn} - \frac{b^{4/3}x^{4n/3}(cx)^{-4n/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,x^{-n/3}\right)}{a^{7/3}cn} + \frac{b^{4/3}x^{4n/3}(cx)^{-4n/3}\log\left(\right.}{}$$

$$= -\frac{3(cx)^{-4n/3}}{4acn} + \frac{3bx^n(cx)^{-4n/3}}{a^2cn} + \frac{\sqrt{3}\,b^{4/3}x^{4n/3}(cx)^{-4n/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}\,x^{-n/3}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{a^{7/3}cn} - \frac{b^{4/3}x^{4n/3}(cx)^{-4n/3}\log}{a^{7/3}cn}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.16

$$-\frac{3x(cx)^{-\frac{4n}{3}-1}\,{}_2F_1\left(-\frac{4}{3},1;-\frac{1}{3};-\frac{bx^n}{a}\right)}{4an}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 - (4*n)/3)/(a + b*x^n),x]
```

```
[Out] (-3*x*(c*x)^(-1 - (4*n)/3)*Hypergeometric2F1[-4/3, 1, -1/3, -((b*x^n)/a)])/
(4*a*n)
```

fricas [A] time = 1.26, size = 351, normalized size = 1.43

$$\frac{4\sqrt{3}\,bc^{-n-\frac{3}{4}}\left(-\frac{bc^{-n-\frac{3}{4}}}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,a\left(-\frac{bc^{-n-\frac{3}{4}}}{a}\right)^{\frac{2}{3}}x^{\frac{1}{4}}e^{\left(-\frac{1}{12}(4n+3)\log(c)-\frac{1}{12}(4n+3)\log(x)\right)}-\sqrt{3}\,bc^{-n-\frac{3}{4}}}{3\,bc^{-n-\frac{3}{4}}}\right)-2\,bc^{-n-\frac{3}{4}}\left(-\frac{bc^{-n-\frac{3}{4}}}{a}\right)^{\frac{1}{3}}\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-4/3*n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(3)*b*c^(-n - 3/4)*(-b*c^(-n - 3/4)/a)^(1/3)*arctan(1/3*(2*sqrt(
3)*a*(-b*c^(-n - 3/4)/a)^(2/3)*x^(1/4)*e^(-1/12*(4*n + 3)*log(c) - 1/12*(4*
```

```
n + 3)*log(x)) - sqrt(3)*b*c^(-n - 3/4))/(b*c^(-n - 3/4))) - 2*b*c^(-n - 3/
4)*(-b*c^(-n - 3/4)/a)^(1/3)*log(((-b*c^(-n - 3/4)/a)^(1/3)*x^(3/4)*e^(-1/1
2*(4*n + 3)*log(c) - 1/12*(4*n + 3)*log(x)) + x*e^(-1/6*(4*n + 3)*log(c) -
1/6*(4*n + 3)*log(x)) + (-b*c^(-n - 3/4)/a)^(2/3)*sqrt(x))/x) + 4*b*c^(-n -
 3/4)*(-b*c^(-n - 3/4)/a)^(1/3)*log((x*e^(-1/12*(4*n + 3)*log(c) - 1/12*(4*
n + 3)*log(x)) - (-b*c^(-n - 3/4)/a)^(1/3)*x^(3/4))/x) + 12*b*c^(-n - 3/4)*
x^(1/4)*e^(-1/12*(4*n + 3)*log(c) - 1/12*(4*n + 3)*log(x)) - 3*a*x*e^(-1/3*
(4*n + 3)*log(c) - 1/3*(4*n + 3)*log(x)))/(a^2*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{-\frac{4}{3}n-1}}{bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-4/3*n)/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(-4/3*n - 1)/(b*x^n + a), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{-\frac{4n}{3}-1}}{b\,x^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-1-4/3*n)/(b*x^n+a),x)
```

```
[Out] int((c*x)^(-1-4/3*n)/(b*x^n+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^2\int \frac{x^{\frac{2}{3}n}}{a^2bc^{\frac{4}{3}n+1}xx^n+a^3c^{\frac{4}{3}n+1}x}\,dx+\frac{3\,(4\,bx^n-a)c^{-\frac{4}{3}n-1}}{4\,a^2nx^{\frac{4}{3}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-4/3*n)/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] b^2*integrate(x^(2/3*n)/(a^2*b*c^(4/3*n + 1)*x*x^n + a^3*c^(4/3*n + 1)*x),
x) + 3/4*(4*b*x^n - a)*c^(-4/3*n - 1)/(a^2*n*x^(4/3*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(c\,x)^{\frac{4n}{3}+1}\,(a+b\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^((4*n)/3 + 1)*(a + b*x^n)),x)
```

```
[Out] int(1/((c*x)^((4*n)/3 + 1)*(a + b*x^n)), x)
```

sympy [C] time = 5.17, size = 272, normalized size = 1.11

$$\frac{c^{-\frac{4n}{3}}x^{-\frac{4n}{3}}\Gamma\left(-\frac{4}{3}\right)}{acn\Gamma\left(-\frac{1}{3}\right)}-\frac{4bc^{-\frac{4n}{3}}x^{-\frac{n}{3}}\Gamma\left(-\frac{4}{3}\right)}{a^2cn\Gamma\left(-\frac{1}{3}\right)}+\frac{4b^{\frac{4}{3}}c^{-\frac{4n}{3}}e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{n}{3}}e^{\frac{i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{4}{3}\right)}{3a^{\frac{7}{3}}cn\Gamma\left(-\frac{1}{3}\right)}+\frac{4b^{\frac{4}{3}}c^{-\frac{4n}{3}}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{n}{3}}e^{i\pi}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{4}{3}\right.}{3a^{\frac{7}{3}}cn\Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-1-4/3*n)/(a+b*x**n),x)
```

```
[Out] c**(-4*n/3)*x**(-4*n/3)*gamma(-4/3)/(a*c*n*gamma(-1/3)) - 4*b*c**(-4*n/3)*x
**(-n/3)*gamma(-4/3)/(a**2*c*n*gamma(-1/3)) + 4*b**(4/3)*c**(-4*n/3)*exp(-2
*I*pi/3)*log(1 - b**(1/3)*x**(n/3)*exp_polar(I*pi/3)/a**(1/3))*gamma(-4/3)/
(3*a**(7/3)*c*n*gamma(-1/3)) + 4*b**(4/3)*c**(-4*n/3)*log(1 - b**(1/3)*x**(
n/3)*exp_polar(I*pi)/a**(1/3))*gamma(-4/3)/(3*a**(7/3)*c*n*gamma(-1/3)) + 4
*b**(4/3)*c**(-4*n/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*x**(n/3)*exp_polar(5*I
*pi/3)/a**(1/3))*gamma(-4/3)/(3*a**(7/3)*c*n*gamma(-1/3))
```

3.2771 $\int \frac{(cx)^{-1-\frac{5n}{4}}}{a+bx^n}\,dx$

Optimal. Leaf size=341

$$\frac{b^{5/4}x^{5n/4}(cx)^{-5n/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^{-n/4}+\sqrt{a}\,x^{-n/2}+\sqrt{b}\right)}{\sqrt{2}\,a^{9/4}cn}-\frac{b^{5/4}x^{5n/4}(cx)^{-5n/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^{-n/4}+\sqrt{a}\,x^{-n/2}-\right.}{\sqrt{2}\,a^{9/4}cn}$$

```
[Out] -4/5/a/c/n/((c*x)^(5/4*n))+4*b*x^n/a^2/c/n/((c*x)^(5/4*n))+1/2*b^(5/4)*x^(5
/4*n)*ln(-a^(1/4)*b^(1/4)*2^(1/2)/(x^(1/4*n))+a^(1/2)/(x^(1/2*n))+b^(1/2))/
a^(9/4)/c/n/((c*x)^(5/4*n))*2^(1/2)-1/2*b^(5/4)*x^(5/4*n)*ln(a^(1/4)*b^(1/4
)*2^(1/2)/(x^(1/4*n))+a^(1/2)/(x^(1/2*n))+b^(1/2))/a^(9/4)/c/n/((c*x)^(5/4*
n))*2^(1/2)+b^(5/4)*x^(5/4*n)*arctan(1-a^(1/4)*2^(1/2)/b^(1/4)/(x^(1/4*n)))
*2^(1/2)/a^(9/4)/c/n/((c*x)^(5/4*n))-b^(5/4)*x^(5/4*n)*arctan(1+a^(1/4)*2^(
1/2)/b^(1/4)/(x^(1/4*n)))*2^(1/2)/a^(9/4)/c/n/((c*x)^(5/4*n))
```

Rubi [A] time = 0.24, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {363, 362, 345, 193, 321, 211, 1165, 628, 1162, 617, 204}

$$\frac{b^{5/4}x^{5n/4}(cx)^{-5n/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^{-n/4}+\sqrt{a}\,x^{-n/2}+\sqrt{b}\right)}{\sqrt{2}\,a^{9/4}cn}-\frac{b^{5/4}x^{5n/4}(cx)^{-5n/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^{-n/4}+\sqrt{a}\,x^{-n/2}-\right.}{\sqrt{2}\,a^{9/4}cn}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 - (5*n)/4)/(a + b*x^n),x]
```

```
[Out] -4/(5*a*c*n*(c*x)^((5*n)/4)) + (4*b*x^n)/(a^2*c*n*(c*x)^((5*n)/4)) + (Sqrt[
2]*b^(5/4)*x^((5*n)/4)*ArcTan[1 - (Sqrt[2]*a^(1/4))/(b^(1/4)*x^(n/4))])/(a^
(9/4)*c*n*(c*x)^((5*n)/4)) - (Sqrt[2]*b^(5/4)*x^((5*n)/4)*ArcTan[1 + (Sqrt[
2]*a^(1/4))/(b^(1/4)*x^(n/4))])/(a^(9/4)*c*n*(c*x)^((5*n)/4)) + (b^(5/4)*x^
((5*n)/4)*Log[Sqrt[b] + Sqrt[a]/x^(n/2) - (Sqrt[2]*a^(1/4)*b^(1/4))/x^(n/4)
])/(Sqrt[2]*a^(9/4)*c*n*(c*x)^((5*n)/4)) - (b^(5/4)*x^((5*n)/4)*Log[Sqrt[b]
 + Sqrt[a]/x^(n/2) + (Sqrt[2]*a^(1/4)*b^(1/4))/x^(n/4)])/(Sqrt[2]*a^(9/4)*c
*n*(c*x)^((5*n)/4))
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 362

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Dist[b/a, Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a,
 b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

Rule 363

```
Int[((c_)*(x_))^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Dist[(c^IntPart
[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m/(a + b*x^n), x], x] /; FreeQ[
{a, b, c, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && (SumSimplerQ[m, n]
 || SumSimplerQ[m, -n])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(cx)^{-1-\frac{5n}{4}}}{a+bx^n}\,dx = \frac{\left(x^{5n/4}(cx)^{-5n/4}\right)\int \frac{x^{-1-\frac{5n}{4}}}{a+bx^n}\,dx}{c}$$

$$= -\frac{4(cx)^{-5n/4}}{5acn} - \frac{\left(bx^{5n/4}(cx)^{-5n/4}\right)\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n}\,dx}{ac}$$

$$= -\frac{4(cx)^{-5n/4}}{5acn} + \frac{\left(4bx^{5n/4}(cx)^{-5n/4}\right)\text{Subst}\left(\int \frac{1}{a+\frac{b}{x^4}}\,dx, x, x^{-n/4}\right)}{acn}$$

$$= -\frac{4(cx)^{-5n/4}}{5acn} + \frac{\left(4bx^{5n/4}(cx)^{-5n/4}\right)\text{Subst}\left(\int \frac{x^4}{b+ax^4}\,dx, x, x^{-n/4}\right)}{acn}$$

$$= -\frac{4(cx)^{-5n/4}}{5acn} + \frac{4bx^n(cx)^{-5n/4}}{a^2cn} - \frac{\left(4b^2x^{5n/4}(cx)^{-5n/4}\right)\text{Subst}\left(\int \frac{1}{b+ax^4}\,dx, x, x^{-n/4}\right)}{a^2cn}$$

$$= -\frac{4(cx)^{-5n/4}}{5acn} + \frac{4bx^n(cx)^{-5n/4}}{a^2cn} - \frac{\left(2b^{3/2}x^{5n/4}(cx)^{-5n/4}\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{a}\,x^2}{b+ax^4}\,dx, x, x^{-n/4}\right)}{a^2cn} - \frac{\left(2b^{3/2}\right.}{}$$

$$= -\frac{4(cx)^{-5n/4}}{5acn} + \frac{4bx^n(cx)^{-5n/4}}{a^2cn} + \frac{\left(b^{5/4}x^{5n/4}(cx)^{-5n/4}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\,\sqrt[4]{b}}{\sqrt[4]{a}}+2x}{-\frac{\sqrt{b}}{\sqrt{a}}-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}-x^2}\,dx, x, x^{-n/4}\right)}{\sqrt{2}\,a^{9/4}cn} + \frac{(}{}$$

$$= -\frac{4(cx)^{-5n/4}}{5acn} + \frac{4bx^n(cx)^{-5n/4}}{a^2cn} + \frac{b^{5/4}x^{5n/4}(cx)^{-5n/4}\log\left(\sqrt{b}+\sqrt{a}\,x^{-n/2}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x^{-n/4}\right)}{\sqrt{2}\,a^{9/4}cn} - \frac{b}{}$$

$$= -\frac{4(cx)^{-5n/4}}{5acn} + \frac{4bx^n(cx)^{-5n/4}}{a^2cn} + \frac{\sqrt{2}\,b^{5/4}x^{5n/4}(cx)^{-5n/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{a}\,x^{-n/4}}{\sqrt[4]{b}}\right)}{a^{9/4}cn} - \frac{\sqrt{2}\,b^{5/4}x^{5n/4}(cx}{}$$

Mathematica [C] time = 0.01, size = 39, normalized size = 0.11

$$-\frac{4x(cx)^{-\frac{5n}{4}-1}\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\frac{bx^n}{a}\right)}{5an}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 - (5*n)/4)/(a + b*x^n),x]
```

```
[Out] (-4*x*(c*x)^(-1 - (5*n)/4)*Hypergeometric2F1[-5/4, 1, -1/4, -((b*x^n)/a)])/
(5*a*n)
```

fricas [A] time = 3.03, size = 448, normalized size = 1.31

$$20\,a^2n\left(-\frac{b^5c^{-5n-4}}{a^9n^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{a^7bc^{-n-\frac{4}{5}}n^3x^{\frac{1}{5}}\left(-\frac{b^5c^{-5n-4}}{a^9n^4}\right)^{\frac{3}{4}}e^{\left(-\frac{1}{20}(5n+4)\log(c)-\frac{1}{20}(5n+4)\log(x)\right)}-a^7n^3x^{\frac{1}{5}}\sqrt{\frac{a^4n^2x^{\frac{3}{5}}\sqrt{-\frac{b^5c^{-5n-4}}{a^9n^4}}+b^2c^{-2n-}}{b^5c^{-5n-4}}}\right.$$

$$-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-5/4*n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] -1/5*(20*a^2*n*(-b^5*c^(-5*n - 4)/(a^9*n^4))^(1/4)*arctan(-(a^7*b*c^(-n - 4
/5)*n^3*x^(1/5)*(-b^5*c^(-5*n - 4)/(a^9*n^4))^(3/4)*e^(-1/20*(5*n + 4)*log(
```

```
c) - 1/20*(5*n + 4)*log(x)) - a^7*n^3*x^(1/5)*sqrt((a^4*n^2*x^(3/5)*sqrt(-b
^5*c^(-5*n - 4)/(a^9*n^4)) + b^2*c^(-2*n - 8/5)*x*e^(-1/10*(5*n + 4)*log(c)
 - 1/10*(5*n + 4)*log(x)))/x)*(-b^5*c^(-5*n - 4)/(a^9*n^4))^(3/4))/(b^5*c^(
-5*n - 4))) + 5*a^2*n*(-b^5*c^(-5*n - 4)/(a^9*n^4))^(1/4)*log((a^2*n*x^(4/5
)*(-b^5*c^(-5*n - 4)/(a^9*n^4))^(1/4) + b*c^(-n - 4/5)*x*e^(-1/20*(5*n + 4)
*log(c) - 1/20*(5*n + 4)*log(x)))/x) - 5*a^2*n*(-b^5*c^(-5*n - 4)/(a^9*n^4)
)^(1/4)*log(-(a^2*n*x^(4/5)*(-b^5*c^(-5*n - 4)/(a^9*n^4))^(1/4) - b*c^(-n -
 4/5)*x*e^(-1/20*(5*n + 4)*log(c) - 1/20*(5*n + 4)*log(x)))/x) - 20*b*c^(-n
 - 4/5)*x^(1/5)*e^(-1/20*(5*n + 4)*log(c) - 1/20*(5*n + 4)*log(x)) + 4*a*x*
e^(-1/4*(5*n + 4)*log(c) - 1/4*(5*n + 4)*log(x)))/(a^2*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{-\frac{5}{4}n-1}}{bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-5/4*n)/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(-5/4*n - 1)/(b*x^n + a), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{-\frac{5n}{4}-1}}{b\,x^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-1-5/4*n)/(b*x^n+a),x)
```

```
[Out] int((c*x)^(-1-5/4*n)/(b*x^n+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^2\int \frac{x^{\frac{3}{4}n}}{a^2bc^{\frac{5}{4}n+1}xx^n+a^3c^{\frac{5}{4}n+1}x}\,dx+\frac{4\,(5\,bx^n-a)c^{-\frac{5}{4}n-1}}{5\,a^2nx^{\frac{5}{4}n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-5/4*n)/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] b^2*integrate(x^(3/4*n)/(a^2*b*c^(5/4*n + 1)*x*x^n + a^3*c^(5/4*n + 1)*x),
x) + 4/5*(5*b*x^n - a)*c^(-5/4*n - 1)/(a^2*n*x^(5/4*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(c\,x)^{\frac{5n}{4}+1}\,(a+b\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^((5*n)/4 + 1)*(a + b*x^n)),x)
```

```
[Out] int(1/((c*x)^((5*n)/4 + 1)*(a + b*x^n)), x)
```

sympy [C] time = 6.10, size = 360, normalized size = 1.06

$$\frac{c^{-\frac{5n}{4}}x^{-\frac{5n}{4}}\Gamma\left(-\frac{5}{4}\right)}{acn\Gamma\left(-\frac{1}{4}\right)}-\frac{5bc^{-\frac{5n}{4}}x^{-\frac{n}{4}}\Gamma\left(-\frac{5}{4}\right)}{a^2cn\Gamma\left(-\frac{1}{4}\right)}+\frac{5b^{\frac{5}{4}}c^{-\frac{5n}{4}}e^{-\frac{3i\pi}{4}}\log\left(1-\frac{\sqrt[4]{b}x^{\frac{n}{4}}e^{\frac{i\pi}{4}}}{\sqrt[4]{a}}\right)\Gamma\left(-\frac{5}{4}\right)}{4a^{\frac{9}{4}}cn\Gamma\left(-\frac{1}{4}\right)}+\frac{5ib^{\frac{5}{4}}c^{-\frac{5n}{4}}e^{-\frac{3i\pi}{4}}\log\left(1-\frac{\sqrt[4]{b}x^{\frac{n}{4}}e^{\frac{3i\pi}{4}}}{\sqrt[4]{a}}\right)\Gamma\left(\right.}{4a^{\frac{9}{4}}cn\Gamma\left(-\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-1-5/4*n)/(a+b*x**n),x)
```

```
[Out] c**(-5*n/4)*x**(-5*n/4)*gamma(-5/4)/(a*c*n*gamma(-1/4)) - 5*b*c**(-5*n/4)*x
**(-n/4)*gamma(-5/4)/(a**2*c*n*gamma(-1/4)) + 5*b**(5/4)*c**(-5*n/4)*exp(-3
*I*pi/4)*log(1 - b**(1/4)*x**(n/4)*exp_polar(I*pi/4)/a**(1/4))*gamma(-5/4)/
(4*a**(9/4)*c*n*gamma(-1/4)) + 5*I*b**(5/4)*c**(-5*n/4)*exp(-3*I*pi/4)*log(
1 - b**(1/4)*x**(n/4)*exp_polar(3*I*pi/4)/a**(1/4))*gamma(-5/4)/(4*a**(9/4)
*c*n*gamma(-1/4)) - 5*b**(5/4)*c**(-5*n/4)*exp(-3*I*pi/4)*log(1 - b**(1/4)*
x**(n/4)*exp_polar(5*I*pi/4)/a**(1/4))*gamma(-5/4)/(4*a**(9/4)*c*n*gamma(-1
/4)) - 5*I*b**(5/4)*c**(-5*n/4)*exp(-3*I*pi/4)*log(1 - b**(1/4)*x**(n/4)*ex
p_polar(7*I*pi/4)/a**(1/4))*gamma(-5/4)/(4*a**(9/4)*c*n*gamma(-1/4))
```

3.2772 $\int \frac{(cx)^{4+n}}{a+bx^n}\,dx$

Optimal. Leaf size=44

$$\frac{(cx)^{n+5}\,{}_2F_1\left(1,\frac{n+5}{n};2+\frac{5}{n};-\frac{bx^n}{a}\right)}{ac(n+5)}$$

```
[Out] (c*x)^(5+n)*hypergeom([1, (5+n)/n],[2+5/n],-b*x^n/a)/a/c/(5+n)
```

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {364}

$$\frac{(cx)^{n+5}\,{}_2F_1\left(1,\frac{n+5}{n};2+\frac{5}{n};-\frac{bx^n}{a}\right)}{ac(n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(4 + n)/(a + b*x^n),x]
```

```
[Out] ((c*x)^(5 + n)*Hypergeometric2F1[1, (5 + n)/n, 2 + 5/n, -((b*x^n)/a)])/(a*c
*(5 + n))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(cx)^{4+n}}{a+bx^n}\,dx = \frac{(cx)^{5+n}\,{}_2F_1\left(1,\frac{5+n}{n};2+\frac{5}{n};-\frac{bx^n}{a}\right)}{ac(5+n)}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 1.00

$$\frac{x(cx)^{n+4}\,{}_2F_1\left(1,\frac{n+5}{n};\frac{n+5}{n}+1;-\frac{bx^n}{a}\right)}{a(n+5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(4 + n)/(a + b*x^n),x]
```

```
[Out] (x*(c*x)^(4 + n)*Hypergeometric2F1[1, (5 + n)/n, 1 + (5 + n)/n, -((b*x^n)/a
)])/(a*(5 + n))
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(cx)^{n+4}}{bx^n+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(4+n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral((c*x)^(n + 4)/(b*x^n + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{n+4}}{bx^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(4+n)/(a+b*x^n),x, algorithm="giac")
[Out] integrate((c*x)^(n + 4)/(b*x^n + a), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{n+4}}{b\,x^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(n+4)/(b*x^n+a),x)
[Out] int((c*x)^(n+4)/(b*x^n+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^{n+4}x^5}{5\,b} - ac^{n+4}\int \frac{x^4}{b^2x^n + ab}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(4+n)/(a+b*x^n),x, algorithm="maxima")
[Out] 1/5*c^(n + 4)*x^5/b - a*c^(n + 4)*integrate(x^4/(b^2*x^n + a*b), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c\,x)^{n+4}}{a + b\,x^n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(n + 4)/(a + b*x^n),x)
[Out] int((c*x)^(n + 4)/(a + b*x^n), x)
```

sympy [C] time = 8.24, size = 97, normalized size = 2.20

$$\frac{c^4c^nx^5x^n\Phi\left(\frac{bx^ne^{i\pi}}{a}, 1, 1+\frac{5}{n}\right)\Gamma\left(1+\frac{5}{n}\right)}{an\Gamma\left(2+\frac{5}{n}\right)} + \frac{5c^4c^nx^5x^n\Phi\left(\frac{bx^ne^{i\pi}}{a}, 1, 1+\frac{5}{n}\right)\Gamma\left(1+\frac{5}{n}\right)}{an^2\Gamma\left(2+\frac{5}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(4+n)/(a+b*x**n),x)
[Out] c**4*c**n*x**5*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1 + 5/n)*gamma(1
+ 5/n)/(a*n*gamma(2 + 5/n)) + 5*c**4*c**n*x**5*x**n*lerchphi(b*x**n*exp_pol
ar(I*pi)/a, 1, 1 + 5/n)*gamma(1 + 5/n)/(a*n**2*gamma(2 + 5/n))
```

3.2773 $\int \frac{(cx)^{3+n}}{a+bx^n}\,dx$

Optimal. Leaf size=46

$$\frac{(cx)^{n+4}\,{}_2F_1\left(1,\frac{n+4}{n};2\left(1+\frac{2}{n}\right);-\frac{bx^n}{a}\right)}{ac(n+4)}$$

```
[Out] (c*x)^(4+n)*hypergeom([1, (4+n)/n],[2+4/n],-b*x^n/a)/a/c/(4+n)
```

Rubi [A] time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {364}

$$\frac{(cx)^{n+4}\,{}_2F_1\left(1,\frac{n+4}{n};2\left(1+\frac{2}{n}\right);-\frac{bx^n}{a}\right)}{ac(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3 + n)/(a + b*x^n),x]
```

```
[Out] ((c*x)^(4 + n)*Hypergeometric2F1[1, (4 + n)/n, 2*(1 + 2/n), -((b*x^n)/a)])/
(a*c*(4 + n))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(cx)^{3+n}}{a+bx^n}\,dx = \frac{(cx)^{4+n}\,{}_2F_1\left(1,\frac{4+n}{n};2\left(1+\frac{2}{n}\right);-\frac{bx^n}{a}\right)}{ac(4+n)}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.96

$$\frac{x(cx)^{n+3}\,{}_2F_1\left(1,\frac{n+4}{n};\frac{n+4}{n}+1;-\frac{bx^n}{a}\right)}{a(n+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3 + n)/(a + b*x^n),x]
```

```
[Out] (x*(c*x)^(3 + n)*Hypergeometric2F1[1, (4 + n)/n, 1 + (4 + n)/n, -((b*x^n)/a
)])/(a*(4 + n))
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(cx)^{n+3}}{bx^n+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3+n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral((c*x)^(n + 3)/(b*x^n + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{n+3}}{bx^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3+n)/(a+b*x^n),x, algorithm="giac")
[Out] integrate((c*x)^(n + 3)/(b*x^n + a), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{n+3}}{b\,x^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(n+3)/(b*x^n+a),x)
[Out] int((c*x)^(n+3)/(b*x^n+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^{n+3}x^4}{4\,b} - ac^{n+3}\int \frac{x^3}{b^2x^n + ab}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3+n)/(a+b*x^n),x, algorithm="maxima")
[Out] 1/4*c^(n + 3)*x^4/b - a*c^(n + 3)*integrate(x^3/(b^2*x^n + a*b), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c\,x)^{n+3}}{a + b\,x^n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(n + 3)/(a + b*x^n),x)
[Out] int((c*x)^(n + 3)/(a + b*x^n), x)
```

sympy [C] time = 5.52, size = 97, normalized size = 2.11

$$\frac{c^3c^nx^4x^n\Phi\left(\frac{bx^ne^{i\pi}}{a}, 1, 1 + \frac{4}{n}\right)\Gamma\left(1 + \frac{4}{n}\right)}{an\Gamma\left(2 + \frac{4}{n}\right)} + \frac{4c^3c^nx^4x^n\Phi\left(\frac{bx^ne^{i\pi}}{a}, 1, 1 + \frac{4}{n}\right)\Gamma\left(1 + \frac{4}{n}\right)}{an^2\Gamma\left(2 + \frac{4}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3+n)/(a+b*x**n),x)
[Out] c**3*c**n*x**4*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1 + 4/n)*gamma(1
+ 4/n)/(a*n*gamma(2 + 4/n)) + 4*c**3*c**n*x**4*x**n*lerchphi(b*x**n*exp_pol
ar(I*pi)/a, 1, 1 + 4/n)*gamma(1 + 4/n)/(a*n**2*gamma(2 + 4/n))
```

3.2774 $\int \frac{(cx)^{2+n}}{a+bx^n}\,dx$

Optimal. Leaf size=44

$$\frac{(cx)^{n+3}\,{}_2F_1\left(1,\frac{n+3}{n};2+\frac{3}{n};-\frac{bx^n}{a}\right)}{ac(n+3)}$$

```
[Out] (c*x)^(3+n)*hypergeom([1, (3+n)/n],[2+3/n],-b*x^n/a)/a/c/(3+n)
```

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {364}

$$\frac{(cx)^{n+3}\,{}_2F_1\left(1,\frac{n+3}{n};2+\frac{3}{n};-\frac{bx^n}{a}\right)}{ac(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(2 + n)/(a + b*x^n),x]
```

```
[Out] ((c*x)^(3 + n)*Hypergeometric2F1[1, (3 + n)/n, 2 + 3/n, -((b*x^n)/a)])/(a*c
*(3 + n))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(cx)^{2+n}}{a+bx^n}\,dx = \frac{(cx)^{3+n}\,{}_2F_1\left(1,\frac{3+n}{n};2+\frac{3}{n};-\frac{bx^n}{a}\right)}{ac(3+n)}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 1.00

$$\frac{x(cx)^{n+2}\,{}_2F_1\left(1,\frac{n+3}{n};\frac{n+3}{n}+1;-\frac{bx^n}{a}\right)}{a(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(2 + n)/(a + b*x^n),x]
```

```
[Out] (x*(c*x)^(2 + n)*Hypergeometric2F1[1, (3 + n)/n, 1 + (3 + n)/n, -((b*x^n)/a
)])/(a*(3 + n))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(cx)^{n+2}}{bx^n+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(2+n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral((c*x)^(n + 2)/(b*x^n + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{n+2}}{bx^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(2+n)/(a+b*x^n),x, algorithm="giac")
[Out] integrate((c*x)^(n + 2)/(b*x^n + a), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{n+2}}{b\,x^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(n+2)/(b*x^n+a),x)
[Out] int((c*x)^(n+2)/(b*x^n+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^{n+2}x^3}{3\,b} - ac^{n+2}\int \frac{x^2}{b^2x^n + ab}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(2+n)/(a+b*x^n),x, algorithm="maxima")
[Out] 1/3*c^(n + 2)*x^3/b - a*c^(n + 2)*integrate(x^2/(b^2*x^n + a*b), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c\,x)^{n+2}}{a + b\,x^n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(n + 2)/(a + b*x^n),x)
[Out] int((c*x)^(n + 2)/(a + b*x^n), x)
```

sympy [C] time = 3.63, size = 97, normalized size = 2.20

$$\frac{c^2c^nx^3x^n\Phi\left(\frac{bx^ne^{i\pi}}{a}, 1, 1 + \frac{3}{n}\right)\Gamma\left(1 + \frac{3}{n}\right)}{an\Gamma\left(2 + \frac{3}{n}\right)} + \frac{3c^2c^nx^3x^n\Phi\left(\frac{bx^ne^{i\pi}}{a}, 1, 1 + \frac{3}{n}\right)\Gamma\left(1 + \frac{3}{n}\right)}{an^2\Gamma\left(2 + \frac{3}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(2+n)/(a+b*x**n),x)
[Out] c**2*c**n*x**3*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1 + 3/n)*gamma(1
+ 3/n)/(a*n*gamma(2 + 3/n)) + 3*c**2*c**n*x**3*x**n*lerchphi(b*x**n*exp_pol
ar(I*pi)/a, 1, 1 + 3/n)*gamma(1 + 3/n)/(a*n**2*gamma(2 + 3/n))
```

3.2775 $\int \frac{(cx)^{1+n}}{a+bx^n}\,dx$

Optimal. Leaf size=44

$$\frac{(cx)^{n+2}\,{}_2F_1\left(1,\frac{n+2}{n};2\left(1+\frac{1}{n}\right);-\frac{bx^n}{a}\right)}{ac(n+2)}$$

```
[Out] (c*x)^(2+n)*hypergeom([1, (2+n)/n],[2+2/n],-b*x^n/a)/a/c/(2+n)
```

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {364}

$$\frac{(cx)^{n+2}\,{}_2F_1\left(1,\frac{n+2}{n};2\left(1+\frac{1}{n}\right);-\frac{bx^n}{a}\right)}{ac(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(1 + n)/(a + b*x^n),x]
```

```
[Out] ((c*x)^(2 + n)*Hypergeometric2F1[1, (2 + n)/n, 2*(1 + n^(-1)), -((b*x^n)/a)
])/(a*c*(2 + n))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(cx)^{1+n}}{a+bx^n}\,dx = \frac{(cx)^{2+n}\,{}_2F_1\left(1,\frac{2+n}{n};2\left(1+\frac{1}{n}\right);-\frac{bx^n}{a}\right)}{ac(2+n)}$$

Mathematica [A] time = 0.01, size = 45, normalized size = 1.02

$$\frac{cx^2(cx)^n\,{}_2F_1\left(1,\frac{n+2}{n};\frac{n+2}{n}+1;-\frac{bx^n}{a}\right)}{a(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(1 + n)/(a + b*x^n),x]
```

```
[Out] (c*x^2*(c*x)^n*Hypergeometric2F1[1, (2 + n)/n, 1 + (2 + n)/n, -((b*x^n)/a)]
)/(a*(2 + n))
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(cx)^{n+1}}{bx^n+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1+n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral((c*x)^(n + 1)/(b*x^n + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{n+1}}{bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1+n)/(a+b*x^n),x, algorithm="giac")
[Out] integrate((c*x)^(n + 1)/(b*x^n + a), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{n+1}}{b\,x^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(n+1)/(b*x^n+a),x)
[Out] int((c*x)^(n+1)/(b*x^n+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-ac^{n+1}\int \frac{x}{b^2x^n + ab}\,dx + \frac{c^{n+1}x^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1+n)/(a+b*x^n),x, algorithm="maxima")
[Out] -a*c^(n + 1)*integrate(x/(b^2*x^n + a*b), x) + 1/2*c^(n + 1)*x^2/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c\,x)^{n+1}}{a + b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(n + 1)/(a + b*x^n),x)
[Out] int((c*x)^(n + 1)/(a + b*x^n), x)
```

sympy [C] time = 2.65, size = 94, normalized size = 2.14

$$\frac{cc^n x^2 x^n \Phi\left(\frac{bx^n e^{i\pi}}{a}, 1, 1 + \frac{2}{n}\right)\Gamma\left(1 + \frac{2}{n}\right)}{an\Gamma\left(2 + \frac{2}{n}\right)} + \frac{2cc^n x^2 x^n \Phi\left(\frac{bx^n e^{i\pi}}{a}, 1, 1 + \frac{2}{n}\right)\Gamma\left(1 + \frac{2}{n}\right)}{an^2\Gamma\left(2 + \frac{2}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1+n)/(a+b*x**n),x)
[Out] c*c**n*x**2*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1 + 2/n)*gamma(1 + 2
/n)/(a*n*gamma(2 + 2/n)) + 2*c*c**n*x**2*x**n*lerchphi(b*x**n*exp_polar(I*p
i)/a, 1, 1 + 2/n)*gamma(1 + 2/n)/(a*n**2*gamma(2 + 2/n))
```

3.2776 $\int \frac{(cx)^n}{a+bx^n}\,dx$

Optimal. Leaf size=40

$$\frac{(cx)^{n+1}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{bx^n}{a}\right)}{ac(n+1)}$$

```
[Out] (c*x)^(1+n)*hypergeom([1, 1+1/n],[2+1/n],-b*x^n/a)/a/c/(1+n)
```

Rubi [A] time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {364}

$$\frac{(cx)^{n+1}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{bx^n}{a}\right)}{ac(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^n/(a + b*x^n),x]
```

```
[Out] ((c*x)^(1 + n)*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -((b*x^n)/a)])/
(a*c*(1 + n))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(cx)^n}{a+bx^n}\,dx = \frac{(cx)^{1+n}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{bx^n}{a}\right)}{ac(1+n)}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 0.90

$$\frac{x(cx)^n\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^n/(a + b*x^n),x]
```

```
[Out] (x*(c*x)^n*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -((b*x^n)/a)])/(a*(
1 + n))
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(cx)^n}{bx^n+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^n/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral((c*x)^n/(b*x^n + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^n}{bx^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^n/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate((c*x)^n/(b*x^n + a), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{(cx)^n}{b\,x^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^n/(b*x^n+a),x)
```

```
[Out] int((c*x)^n/(b*x^n+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-ac^n \int \frac{1}{b^2x^n + ab}\, dx + \frac{c^n x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^n/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] -a*c^n*integrate(1/(b^2*x^n + a*b), x) + c^n*x/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c\,x)^n}{a + b\,x^n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^n/(a + b*x^n),x)
```

```
[Out] int((c*x)^n/(a + b*x^n), x)
```

sympy [C] time = 1.22, size = 41, normalized size = 1.02

$$-\frac{c^n x \Phi\left(\frac{ax^{-n}e^{i\pi}}{b}, 1, \frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{bn^2\Gamma\left(1 + \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**n/(a+b*x**n),x)
```

```
[Out] -c**n*x*lerchphi(a*x**(-n)*exp_polar(I*pi)/b, 1, exp_polar(I*pi)/n)*gamma(1
/n)/(b*n**2*gamma(1 + 1/n))
```

3.2777 $\int \frac{(cx)^{-1+n}}{a+bx^n}\,dx$

Optimal. Leaf size=28

$$\frac{x^{-n}(cx)^n \log(a+bx^n)}{bcn}$$

```
[Out] (c*x)^n*ln(a+b*x^n)/b/c/n/(x^n)
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {268, 260}

$$\frac{x^{-n}(cx)^n \log(a+bx^n)}{bcn}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 + n)/(a + b*x^n),x]
```

```
[Out] ((c*x)^n*Log[a + b*x^n])/(b*c*n*x^n)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 268

```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(c^In
tPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a + b*x^n)^p, x], x] /;
 FreeQ[{a, b, c, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx)^{-1+n}}{a+bx^n}\,dx &= \frac{(x^{-n}(cx)^n)\int \frac{x^{-1+n}}{a+bx^n}\,dx}{c} \\ &= \frac{x^{-n}(cx)^n \log(a+bx^n)}{bcn}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.04

$$\frac{x^{1-n}(cx)^{n-1} \log(a+bx^n)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 + n)/(a + b*x^n),x]
```

```
[Out] (x^(1 - n)*(c*x)^(-1 + n)*Log[a + b*x^n])/(b*n)
```

fricas [A] time = 0.60, size = 20, normalized size = 0.71

$$\frac{c^{n-1} \log(bx^n+a)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] c^(n - 1)*log(b*x^n + a)/(b*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{n-1}}{bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+n)/(a+b*x^n),x, algorithm="giac")
[Out] integrate((c*x)^(n - 1)/(b*x^n + a), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{n-1}}{b\,x^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(n-1)/(b*x^n+a),x)
[Out] int((c*x)^(n-1)/(b*x^n+a),x)
```

maxima [A] time = 0.70, size = 24, normalized size = 0.86

$$\frac{c^{n-1}\log\left(\frac{bx^n+a}{b}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+n)/(a+b*x^n),x, algorithm="maxima")
[Out] c^(n - 1)*log((b*x^n + a)/b)/(b*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c\,x)^{n-1}}{a + b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(n - 1)/(a + b*x^n),x)
[Out] int((c*x)^(n - 1)/(a + b*x^n), x)
```

sympy [A] time = 4.80, size = 17, normalized size = 0.61

$$\frac{c^n\log\left(1 + \frac{bx^n}{a}\right)}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-1+n)/(a+b*x**n),x)
[Out] c**n*log(1 + b*x**n/a)/(b*c*n)
```

3.2778 $\int \frac{(cx)^{-2+n}}{a+bx^n}\,dx$

Optimal. Leaf size=50

$$-\frac{(cx)^{n-1}\,{}_2F_1\left(1,-\frac{1-n}{n};2-\frac{1}{n};-\frac{bx^n}{a}\right)}{ac(1-n)}$$

```
[Out] -(c*x)^(-1+n)*hypergeom([1, (-1+n)/n],[2-1/n],-b*x^n/a)/a/c/(1-n)
```

Rubi [A] time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {364}

$$-\frac{(cx)^{n-1}\,{}_2F_1\left(1,-\frac{1-n}{n};2-\frac{1}{n};-\frac{bx^n}{a}\right)}{ac(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-2 + n)/(a + b*x^n),x]
```

```
[Out] -(((c*x)^(-1 + n)*Hypergeometric2F1[1, -((1 - n)/n), 2 - n^(-1), -((b*x^n)/
a)])/(a*c*(1 - n)))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(cx)^{-2+n}}{a+bx^n}\,dx = -\frac{(cx)^{-1+n}\,{}_2F_1\left(1,-\frac{1-n}{n};2-\frac{1}{n};-\frac{bx^n}{a}\right)}{ac(1-n)}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 0.88

$$\frac{x(cx)^{n-2}\,{}_2F_1\left(1,\frac{n-1}{n};\frac{n-1}{n}+1;-\frac{bx^n}{a}\right)}{a(n-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-2 + n)/(a + b*x^n),x]
```

```
[Out] (x*(c*x)^(-2 + n)*Hypergeometric2F1[1, (-1 + n)/n, 1 + (-1 + n)/n, -((b*x^n
)/a)])/(a*(-1 + n))
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(cx)^{n-2}}{bx^n+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-2+n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral((c*x)^(n - 2)/(b*x^n + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{n-2}}{bx^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-2+n)/(a+b*x^n),x, algorithm="giac")
[Out] integrate((c*x)^(n - 2)/(b*x^n + a), x)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{n-2}}{b\,x^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(n-2)/(b*x^n+a),x)
[Out] int((c*x)^(n-2)/(b*x^n+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-ac^n \int \frac{1}{b^2c^2x^2x^n + abc^2x^2}\, dx - \frac{c^{n-2}}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-2+n)/(a+b*x^n),x, algorithm="maxima")
[Out] -a*c^n*integrate(1/(b^2*c^2*x^2*x^n + a*b*c^2*x^2), x) - c^(n - 2)/(b*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c\,x)^{n-2}}{a + b\,x^n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(n - 2)/(a + b*x^n),x)
[Out] int((c*x)^(n - 2)/(a + b*x^n), x)
```

sympy [C] time = 11.52, size = 41, normalized size = 0.82

$$\frac{c^n \Phi\left(\frac{ax^{-n}e^{i\pi}}{b}, 1, \frac{1}{n}\right)\Gamma\left(-\frac{1}{n}\right)}{bc^2n^2x\Gamma\left(1 - \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-2+n)/(a+b*x**n),x)
[Out] c**n*lerchphi(a*x**(-n)*exp_polar(I*pi)/b, 1, 1/n)*gamma(-1/n)/(b*c**2*n**2
*x*gamma(1 - 1/n))
```

3.2779 $\int \frac{(cx)^{-3+n}}{a+bx^n}\,dx$

Optimal. Leaf size=52

$$-\frac{(cx)^{n-2}\,{}_2F_1\left(1,-\frac{2-n}{n};2\left(1-\frac{1}{n}\right);-\frac{bx^n}{a}\right)}{ac(2-n)}$$

```
[Out] -(c*x)^(-2+n)*hypergeom([1, (-2+n)/n],[2-2/n],-b*x^n/a)/a/c/(2-n)
```

Rubi [A] time = 0.01, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {364}

$$-\frac{(cx)^{n-2}\,{}_2F_1\left(1,-\frac{2-n}{n};2\left(1-\frac{1}{n}\right);-\frac{bx^n}{a}\right)}{ac(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-3 + n)/(a + b*x^n),x]
```

```
[Out] -(((c*x)^(-2 + n)*Hypergeometric2F1[1, -((2 - n)/n), 2*(1 - n^(-1)), -((b*x
^n)/a)])/(a*c*(2 - n)))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(cx)^{-3+n}}{a+bx^n}\,dx = -\frac{(cx)^{-2+n}\,{}_2F_1\left(1,-\frac{2-n}{n};2\left(1-\frac{1}{n}\right);-\frac{bx^n}{a}\right)}{ac(2-n)}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 0.85

$$\frac{x(cx)^{n-3}\,{}_2F_1\left(1,\frac{n-2}{n};\frac{n-2}{n}+1;-\frac{bx^n}{a}\right)}{a(n-2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-3 + n)/(a + b*x^n),x]
```

```
[Out] (x*(c*x)^(-3 + n)*Hypergeometric2F1[1, (-2 + n)/n, 1 + (-2 + n)/n, -((b*x^n
)/a)])/(a*(-2 + n))
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(cx)^{n-3}}{bx^n+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-3+n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral((c*x)^(n - 3)/(b*x^n + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{n-3}}{bx^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-3+n)/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(n - 3)/(b*x^n + a), x)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{n-3}}{b\,x^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(n-3)/(b*x^n+a),x)
```

```
[Out] int((c*x)^(n-3)/(b*x^n+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-ac^n \int \frac{1}{b^2c^3x^3x^n + abc^3x^3}\, dx - \frac{c^{n-3}}{2\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-3+n)/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] -a*c^n*integrate(1/(b^2*c^3*x^3*x^n + a*b*c^3*x^3), x) - 1/2*c^(n - 3)/(b*x
^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c\,x)^{n-3}}{a + b\,x^n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(n - 3)/(a + b*x^n),x)
```

```
[Out] int((c*x)^(n - 3)/(a + b*x^n), x)
```

sympy [C] time = 27.44, size = 44, normalized size = 0.85

$$\frac{2c^n \Phi\left(\frac{ax^{-n}e^{i\pi}}{b}, 1, \frac{2}{n}\right)\Gamma\left(-\frac{2}{n}\right)}{bc^3n^2x^2\Gamma\left(1 - \frac{2}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-3+n)/(a+b*x**n),x)
```

```
[Out] 2*c**n*lerchphi(a*x**(-n)*exp_polar(I*pi)/b, 1, 2/n)*gamma(-2/n)/(b*c**3*n*
*2*x**2*gamma(1 - 2/n))
```

3.2780 $\int \frac{(cx)^{-1+n}}{(a+bx^n)^2}\,dx$

Optimal. Leaf size=24

$$\frac{(cx)^n}{acn\,(a+bx^n)}$$

```
[Out] (c*x)^n/a/c/n/(a+b*x^n)
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {264}

$$\frac{(cx)^n}{acn\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 + n)/(a + b*x^n)^2,x]
```

```
[Out] (c*x)^n/(a*c*n*(a + b*x^n))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(cx)^{-1+n}}{(a+bx^n)^2}\,dx = \frac{(cx)^n}{acn\,(a+bx^n)}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 1.29

$$-\frac{x^{1-n}(cx)^{n-1}}{bn\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 + n)/(a + b*x^n)^2,x]
```

```
[Out] -((x^(1 - n)*(c*x)^(-1 + n))/(b*n*(a + b*x^n)))
```

fricas [A] time = 0.74, size = 22, normalized size = 0.92

$$-\frac{c^{n-1}}{b^2nx^n+abn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+n)/(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] -c^(n - 1)/(b^2*n*x^n + a*b*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{n-1}}{(bx^n+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+n)/(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((c*x)^(n - 1)/(b*x^n + a)^2, x)
```

maple [C] time = 0.03, size = 99, normalized size = 4.12

$$\frac{x\,\mathrm{e}^{\frac{(n-1)\left(-i\pi\,\mathrm{csgn}(ic)\mathrm{csgn}(ix)\mathrm{csgn}(icx)+i\pi\,\mathrm{csgn}(ic)\mathrm{csgn}(icx)^2+i\pi\,\mathrm{csgn}(ix)\mathrm{csgn}(icx)^2-i\pi\mathrm{csgn}(icx)^3+2\ln(c)+2\ln(x)\right)}{2}}}{(b\,x^n+a)\,an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(n-1)/(b*x^n+a)^2,x)
```

```
[Out] 1/a/n*x/(b*x^n+a)*exp(1/2*(n-1)*(-I*Pi*csgn(I*c)*csgn(I*x)*csgn(I*c*x)+I*Pi
*csgn(I*c)*csgn(I*c*x)^2+I*Pi*csgn(I*x)*csgn(I*c*x)^2-I*Pi*csgn(I*c*x)^3+2*
ln(c)+2*ln(x)))
```

maxima [A] time = 0.55, size = 22, normalized size = 0.92

$$-\frac{c^n}{b^2cnx^n+abcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+n)/(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] -c^n/(b^2*c*n*x^n + a*b*c*n)
```

mupad [B] time = 1.16, size = 29, normalized size = 1.21

$$\frac{x\,(c\,x)^{n-1}}{a\,b\,n\,\left(x^n+\frac{a}{b}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(n - 1)/(a + b*x^n)^2,x)
```

```
[Out] (x*(c*x)^(n - 1))/(a*b*n*(x^n + a/b))
```

sympy [A] time = 43.30, size = 330, normalized size = 13.75

$$\begin{cases} \tilde{\infty} x & \\ -\frac{c^n x^{-n}}{b^2 cn} & \\ \frac{\tilde{\infty} c^n x^n}{cn} & \\ 0^{n-1}\left(\frac{nx\Phi\left(\frac{bx^n e^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{a\left(an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)\right)}+\frac{nx\Gamma\left(\frac{1}{n}\right)}{a\left(an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)\right)}-\frac{x\Phi\left(\frac{bx^n e^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{a\left(an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)\right)}+\frac{bnxx^n\Phi\left(\frac{bx^n e^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{a^2\left(an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)\right)}\right) & \\ \frac{\log(x)}{c(a+b)^2} & \\ \frac{c^n x^n}{a^2cn+abcnx^n} & \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-1+n)/(a+b*x**n)**2,x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(c, 0) & Eq(n, 0)), (-c**n*x**(-n
)/(b**2*c*n), Eq(a, 0)), (zoo*c**n*x**n/(c*n), Eq(b, -a*x**(-n))), (0**(n -
 1)*(n*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(a*(a*n**3*g
amma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n))) + n*x*gamma(1/n)/(a*(a*n**3*ga
mma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n))) - x*lerchphi(b*x**n*exp_polar(I
*pi)/a, 1, 1/n)*gamma(1/n)/(a*(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1
+ 1/n))) + b*n*x*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)
/(a**2*(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n))) - b*x*x**n*ler
chphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(a**2*(a*n**3*gamma(1 +
1/n) + b*n**3*x**n*gamma(1 + 1/n)))), Eq(c, 0)), (log(x)/(c*(a + b)**2), Eq
(n, 0)), (c**n*x**n/(a**2*c*n + a*b*c*n*x**n), True))
```

3.2781 $\int \frac{(cx)^{-1+\frac{7n}{2}}}{\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=178

$$-\frac{5a^3x^{-7n/2}(cx)^{7n/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x^{n/2}}{\sqrt{a+bx^n}}\right)}{8b^{7/2}cn}+\frac{5a^2x^{-3n}(cx)^{7n/2}\sqrt{a+bx^n}}{8b^3cn}-\frac{5ax^{-2n}(cx)^{7n/2}\sqrt{a+bx^n}}{12b^2cn}+\frac{x^{-n}(cx)^{7n/2}\sqrt{a+bx}}{3bcn}$$

```
[Out] -5/8*a^3*(c*x)^(7/2*n)*arctanh(x^(1/2*n)*b^(1/2)/(a+b*x^n)^(1/2))/b^(7/2)/c
/n/(x^(7/2*n))+5/8*a^2*(c*x)^(7/2*n)*(a+b*x^n)^(1/2)/b^3/c/n/(x^(3*n))-5/12
*a*(c*x)^(7/2*n)*(a+b*x^n)^(1/2)/b^2/c/n/(x^(2*n))+1/3*(c*x)^(7/2*n)*(a+b*x
^n)^(1/2)/b/c/n/(x^n)
```

Rubi [A] time = 0.08, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {357, 355, 288, 206}

$$\frac{5a^2x^{-3n}(cx)^{7n/2}\sqrt{a+bx^n}}{8b^3cn}-\frac{5a^3x^{-7n/2}(cx)^{7n/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x^{n/2}}{\sqrt{a+bx^n}}\right)}{8b^{7/2}cn}-\frac{5ax^{-2n}(cx)^{7n/2}\sqrt{a+bx^n}}{12b^2cn}+\frac{x^{-n}(cx)^{7n/2}\sqrt{a+bx^n}}{3bcn}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 + (7*n)/2)/Sqrt[a + b*x^n],x]
```

```
[Out] (5*a^2*(c*x)^((7*n)/2)*Sqrt[a + b*x^n])/(8*b^3*c*n*x^(3*n)) - (5*a*(c*x)^((
7*n)/2)*Sqrt[a + b*x^n])/(12*b^2*c*n*x^(2*n)) + ((c*x)^((7*n)/2)*Sqrt[a + b
*x^n])/(3*b*c*n*x^n) - (5*a^3*(c*x)^((7*n)/2)*ArcTanh[(Sqrt[b]*x^(n/2))/Sqr
t[a + b*x^n]])/(8*b^(7/2)*c*n*x^((7*n)/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 355

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[p]}, Dist[(k*a^(p + Simplify[(m + 1)/n]))/n, Subst[Int[x^(k*Simplify[
(m + 1)/n] - 1)/(1 - b*x^k)^(p + Simplify[(m + 1)/n] + 1), x], x, x^(n/k)/(
a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p + Simplify[(
m + 1)/n]] && LtQ[-1, p, 0]
```

Rule 357

```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(c^In
tPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a + b*x^n)^p, x], x] /;
 FreeQ[{a, b, c, m, n}, x] && IntegerQ[p + Simplify[(m + 1)/n]] && LtQ[-1,
p, 0]
```

Rubi steps

$$\int \frac{(cx)^{-1+\frac{7n}{2}}}{\sqrt{a+bx^n}}\,dx = \frac{\left(x^{-7n/2}(cx)^{7n/2}\right)\int \frac{x^{-1+\frac{7n}{2}}}{\sqrt{a+bx^n}}\,dx}{c}$$

$$= \frac{\left(2a^3x^{-7n/2}(cx)^{7n/2}\right)\text{Subst}\left(\int \frac{x^6}{\left(1-bx^2\right)^4}\,dx, x, \frac{x^{n/2}}{\sqrt{a+bx^n}}\right)}{cn}$$

$$= \frac{x^{-n}(cx)^{7n/2}\sqrt{a+bx^n}}{3bcn} - \frac{\left(5a^3x^{-7n/2}(cx)^{7n/2}\right)\text{Subst}\left(\int \frac{x^4}{\left(1-bx^2\right)^3}\,dx, x, \frac{x^{n/2}}{\sqrt{a+bx^n}}\right)}{3bcn}$$

$$= -\frac{5ax^{-2n}(cx)^{7n/2}\sqrt{a+bx^n}}{12b^2cn} + \frac{x^{-n}(cx)^{7n/2}\sqrt{a+bx^n}}{3bcn} + \frac{\left(5a^3x^{-7n/2}(cx)^{7n/2}\right)\text{Subst}\left(\int \frac{x^2}{\left(1-bx^2\right)^2}\,dx, x\right.}{4b^2cn}$$

$$= \frac{5a^2x^{-3n}(cx)^{7n/2}\sqrt{a+bx^n}}{8b^3cn} - \frac{5ax^{-2n}(cx)^{7n/2}\sqrt{a+bx^n}}{12b^2cn} + \frac{x^{-n}(cx)^{7n/2}\sqrt{a+bx^n}}{3bcn} - \frac{\left(5a^3x^{-7n/2}(cx)\right.}{}$$

$$= \frac{5a^2x^{-3n}(cx)^{7n/2}\sqrt{a+bx^n}}{8b^3cn} - \frac{5ax^{-2n}(cx)^{7n/2}\sqrt{a+bx^n}}{12b^2cn} + \frac{x^{-n}(cx)^{7n/2}\sqrt{a+bx^n}}{3bcn} - \frac{5a^3x^{-7n/2}(cx)^{7}}{8}$$

Mathematica [A] time = 0.27, size = 133, normalized size = 0.75

$$\frac{x^{-7n/2}(cx)^{7n/2}\sqrt{a+bx^n}\left(\sqrt{b}\,x^{n/2}\sqrt{\frac{bx^n}{a}+1}\left(15a^2-10abx^n+8b^2x^{2n}\right)-15a^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}\,x^{n/2}}{\sqrt{a}}\right)\right)}{24b^{7/2}cn\sqrt{\frac{bx^n}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 + (7*n)/2)/Sqrt[a + b*x^n],x]
```

```
[Out] ((c*x)^((7*n)/2)*Sqrt[a + b*x^n]*(Sqrt[b]*x^(n/2)*Sqrt[1 + (b*x^n)/a]*(15*a
^2 - 10*a*b*x^n + 8*b^2*x^(2*n)) - 15*a^(5/2)*ArcSinh[(Sqrt[b]*x^(n/2))/Sqr
t[a]]))/(24*b^(7/2)*c*n*x^((7*n)/2)*Sqrt[1 + (b*x^n)/a])
```

fricas [A] time = 0.62, size = 231, normalized size = 1.30

$$\left[\frac{15\,a^3\sqrt{b}\,c^{\frac{7}{2}n-1}\log\left(2\sqrt{bx^n+a}\,\sqrt{b}\,x^{\frac{1}{2}n}-2\,bx^n-a\right)+2\left(8\,b^3c^{\frac{7}{2}n-1}x^{\frac{5}{2}n}-10\,ab^2c^{\frac{7}{2}n-1}x^{\frac{3}{2}n}+15\,a^2bc^{\frac{7}{2}n-1}x^{\frac{1}{2}n}\right)\sqrt{bx}\right.}{48\,b^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+7/2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*a^3*sqrt(b)*c^(7/2*n - 1)*log(2*sqrt(b*x^n + a)*sqrt(b)*x^(1/2*n)
 - 2*b*x^n - a) + 2*(8*b^3*c^(7/2*n - 1)*x^(5/2*n) - 10*a*b^2*c^(7/2*n - 1)
*x^(3/2*n) + 15*a^2*b*c^(7/2*n - 1)*x^(1/2*n))*sqrt(b*x^n + a))/(b^4*n), 1/
24*(15*a^3*sqrt(-b)*c^(7/2*n - 1)*arctan(sqrt(-b)*x^(1/2*n)/sqrt(b*x^n + a)
) + (8*b^3*c^(7/2*n - 1)*x^(5/2*n) - 10*a*b^2*c^(7/2*n - 1)*x^(3/2*n) + 15*
a^2*b*c^(7/2*n - 1)*x^(1/2*n))*sqrt(b*x^n + a))/(b^4*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{7}{2}n-1}}{\sqrt{bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+7/2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(7/2*n - 1)/sqrt(b*x^n + a), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{7n}{2}-1}}{\sqrt{b\,x^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-1+7/2*n)/(b*x^n+a)^(1/2),x)
```

```
[Out] int((c*x)^(-1+7/2*n)/(b*x^n+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{7}{2}n-1}}{\sqrt{bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+7/2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(7/2*n - 1)/sqrt(b*x^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{\frac{7n}{2}-1}}{\sqrt{a+b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^((7*n)/2 - 1)/(a + b*x^n)^(1/2),x)
```

```
[Out] int((c*x)^((7*n)/2 - 1)/(a + b*x^n)^(1/2), x)
```

sympy [A] time = 15.87, size = 190, normalized size = 1.07

$$\frac{5a^{\frac{5}{2}}c^{\frac{7n}{2}}x^{\frac{n}{2}}}{8b^3cn\sqrt{1+\frac{bx^n}{a}}}+\frac{5a^{\frac{3}{2}}c^{\frac{7n}{2}}x^{\frac{3n}{2}}}{24b^2cn\sqrt{1+\frac{bx^n}{a}}}-\frac{\sqrt{a}\,c^{\frac{7n}{2}}x^{\frac{5n}{2}}}{12bcn\sqrt{1+\frac{bx^n}{a}}}-\frac{5a^3c^{\frac{7n}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{\frac{n}{2}}}{\sqrt{a}}\right)}{8b^{\frac{7}{2}}cn}+\frac{c^{\frac{7n}{2}}x^{\frac{7n}{2}}}{3\sqrt{a}\,cn\sqrt{1+\frac{bx^n}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-1+7/2*n)/(a+b*x**n)**(1/2),x)
```

```
[Out] 5*a**(5/2)*c**(7*n/2)*x**(n/2)/(8*b**3*c*n*sqrt(1 + b*x**n/a)) + 5*a**(3/2)
*c**(7*n/2)*x**(3*n/2)/(24*b**2*c*n*sqrt(1 + b*x**n/a)) - sqrt(a)*c**(7*n/2
)*x**(5*n/2)/(12*b*c*n*sqrt(1 + b*x**n/a)) - 5*a**3*c**(7*n/2)*asinh(sqrt(b
)*x**(n/2)/sqrt(a))/(8*b**(7/2)*c*n) + c**(7*n/2)*x**(7*n/2)/(3*sqrt(a)*c*n
*sqrt(1 + b*x**n/a))
```

3.2782 $\int \frac{(cx)^{-1+\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=137

$$\frac{3a^2x^{-5n/2}(cx)^{5n/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x^{n/2}}{\sqrt{a+bx^n}}\right)}{4b^{5/2}cn}-\frac{3ax^{-2n}(cx)^{5n/2}\sqrt{a+bx^n}}{4b^2cn}+\frac{x^{-n}(cx)^{5n/2}\sqrt{a+bx^n}}{2bcn}$$

```
[Out] 3/4*a^2*(c*x)^(5/2*n)*arctanh(x^(1/2*n)*b^(1/2)/(a+b*x^n)^(1/2))/b^(5/2)/c/
n/(x^(5/2*n))-3/4*a*(c*x)^(5/2*n)*(a+b*x^n)^(1/2)/b^2/c/n/(x^(2*n))+1/2*(c*
x)^(5/2*n)*(a+b*x^n)^(1/2)/b/c/n/(x^n)
```

Rubi [A] time = 0.06, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {357, 355, 288, 206}

$$\frac{3a^2x^{-5n/2}(cx)^{5n/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x^{n/2}}{\sqrt{a+bx^n}}\right)}{4b^{5/2}cn}-\frac{3ax^{-2n}(cx)^{5n/2}\sqrt{a+bx^n}}{4b^2cn}+\frac{x^{-n}(cx)^{5n/2}\sqrt{a+bx^n}}{2bcn}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 + (5*n)/2)/Sqrt[a + b*x^n],x]
```

```
[Out] (-3*a*(c*x)^((5*n)/2)*Sqrt[a + b*x^n])/(4*b^2*c*n*x^(2*n)) + ((c*x)^((5*n)/
2)*Sqrt[a + b*x^n])/(2*b*c*n*x^n) + (3*a^2*(c*x)^((5*n)/2)*ArcTanh[(Sqrt[b]
*x^(n/2))/Sqrt[a + b*x^n]])/(4*b^(5/2)*c*n*x^((5*n)/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 355

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[p]}, Dist[(k*a^(p + Simplify[(m + 1)/n]))/n, Subst[Int[x^(k*Simplify[
(m + 1)/n] - 1)/(1 - b*x^k)^(p + Simplify[(m + 1)/n] + 1), x], x, x^(n/k)/(
a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p + Simplify[(
m + 1)/n]] && LtQ[-1, p, 0]
```

Rule 357

```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(c^In
tPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a + b*x^n)^p, x], x] /;
 FreeQ[{a, b, c, m, n}, x] && IntegerQ[p + Simplify[(m + 1)/n]] && LtQ[-1,
p, 0]
```

Rubi steps

$$\int \frac{(cx)^{-1+\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx = \frac{\left(x^{-5n/2}(cx)^{5n/2}\right)\int \frac{x^{-1+\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx}{c}$$

$$= \frac{\left(2a^2x^{-5n/2}(cx)^{5n/2}\right)\text{Subst}\left(\int \frac{x^4}{\left(1-bx^2\right)^3}\,dx, x, \frac{x^{n/2}}{\sqrt{a+bx^n}}\right)}{cn}$$

$$= \frac{x^{-n}(cx)^{5n/2}\sqrt{a+bx^n}}{2bcn} - \frac{\left(3a^2x^{-5n/2}(cx)^{5n/2}\right)\text{Subst}\left(\int \frac{x^2}{\left(1-bx^2\right)^2}\,dx, x, \frac{x^{n/2}}{\sqrt{a+bx^n}}\right)}{2bcn}$$

$$= -\frac{3ax^{-2n}(cx)^{5n/2}\sqrt{a+bx^n}}{4b^2cn} + \frac{x^{-n}(cx)^{5n/2}\sqrt{a+bx^n}}{2bcn} + \frac{\left(3a^2x^{-5n/2}(cx)^{5n/2}\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,\right.}{4b^2cn}$$

$$= -\frac{3ax^{-2n}(cx)^{5n/2}\sqrt{a+bx^n}}{4b^2cn} + \frac{x^{-n}(cx)^{5n/2}\sqrt{a+bx^n}}{2bcn} + \frac{3a^2x^{-5n/2}(cx)^{5n/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x^{n/2}}{\sqrt{a+bx^n}}\right)}{4b^{5/2}cn}$$

Mathematica [A] time = 0.12, size = 121, normalized size = 0.88

$$\frac{ax^{-5n/2}(cx)^{5n/2}\sqrt{\frac{bx^n}{a}+1}\left(3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\,x^{n/2}}{\sqrt{a}}\right)+\sqrt{b}\,x^{n/2}\left(2bx^n-3a\right)\sqrt{\frac{bx^n}{a}+1}\right)}{4b^{5/2}cn\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 + (5*n)/2)/Sqrt[a + b*x^n],x]
```

```
[Out] (a*(c*x)^((5*n)/2)*Sqrt[1 + (b*x^n)/a]*(Sqrt[b]*x^(n/2)*(-3*a + 2*b*x^n)*Sq
rt[1 + (b*x^n)/a] + 3*a^(3/2)*ArcSinh[(Sqrt[b]*x^(n/2))/Sqrt[a]]))/(4*b^(5/
2)*c*n*x^((5*n)/2)*Sqrt[a + b*x^n])
```

fricas [A] time = 0.74, size = 192, normalized size = 1.40

$$\left[\frac{3\,a^2\sqrt{b}\,c^{\frac{5}{2}n-1}\log\left(-2\sqrt{bx^n+a}\,\sqrt{b}\,x^{\frac{1}{2}n}-2\,bx^n-a\right)+2\left(2\,b^2c^{\frac{5}{2}n-1}x^{\frac{3}{2}n}-3\,abc^{\frac{5}{2}n-1}x^{\frac{1}{2}n}\right)\sqrt{bx^n+a}}{8\,b^3n}, -\frac{3\,a^2\sqrt{-b}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+5/2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*a^2*sqrt(b)*c^(5/2*n - 1)*log(-2*sqrt(b*x^n + a)*sqrt(b)*x^(1/2*n)
- 2*b*x^n - a) + 2*(2*b^2*c^(5/2*n - 1)*x^(3/2*n) - 3*a*b*c^(5/2*n - 1)*x^(
1/2*n))*sqrt(b*x^n + a))/(b^3*n), -1/4*(3*a^2*sqrt(-b)*c^(5/2*n - 1)*arctan
(sqrt(-b)*x^(1/2*n)/sqrt(b*x^n + a)) - (2*b^2*c^(5/2*n - 1)*x^(3/2*n) - 3*a
*b*c^(5/2*n - 1)*x^(1/2*n))*sqrt(b*x^n + a))/(b^3*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{2}n-1}}{\sqrt{bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+5/2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(5/2*n - 1)/sqrt(b*x^n + a), x)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5n}{2}-1}}{\sqrt{b\,x^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-1+5/2*n)/(b*x^n+a)^(1/2),x)
```

```
[Out] int((c*x)^(-1+5/2*n)/(b*x^n+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{5}{2}n-1}}{\sqrt{bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+5/2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(5/2*n - 1)/sqrt(b*x^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{\frac{5\,n}{2}-1}}{\sqrt{a+b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^((5*n)/2 - 1)/(a + b*x^n)^(1/2),x)
```

```
[Out] int((c*x)^((5*n)/2 - 1)/(a + b*x^n)^(1/2), x)
```

sympy [A] time = 11.85, size = 150, normalized size = 1.09

$$-\frac{3a^{\frac{3}{2}}c^{\frac{5n}{2}}x^{\frac{n}{2}}}{4b^2cn\sqrt{1+\frac{bx^n}{a}}}-\frac{\sqrt{a}\,c^{\frac{5n}{2}}x^{\frac{3n}{2}}}{4bcn\sqrt{1+\frac{bx^n}{a}}}+\frac{3a^2c^{\frac{5n}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{\frac{n}{2}}}{\sqrt{a}}\right)}{4b^{\frac{5}{2}}cn}+\frac{c^{\frac{5n}{2}}x^{\frac{5n}{2}}}{2\sqrt{a}\,cn\sqrt{1+\frac{bx^n}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-1+5/2*n)/(a+b*x**n)**(1/2),x)
```

```
[Out] -3*a**(3/2)*c**(5*n/2)*x**(n/2)/(4*b**2*c*n*sqrt(1 + b*x**n/a)) - sqrt(a)*c
**(5*n/2)*x**(3*n/2)/(4*b*c*n*sqrt(1 + b*x**n/a)) + 3*a**2*c**(5*n/2)*asinh
(sqrt(b)*x**(n/2)/sqrt(a))/(4*b**(5/2)*c*n) + c**(5*n/2)*x**(5*n/2)/(2*sqrt
(a)*c*n*sqrt(1 + b*x**n/a))
```

3.2783 $\int \frac{(cx)^{-1+\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=91

$$\frac{x^{-n}(cx)^{3n/2}\sqrt{a+bx^n}}{bcn} - \frac{ax^{-3n/2}(cx)^{3n/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x^{n/2}}{\sqrt{a+bx^n}}\right)}{b^{3/2}cn}$$

```
[Out] -a*(c*x)^(3/2*n)*arctanh(x^(1/2*n)*b^(1/2)/(a+b*x^n)^(1/2))/b^(3/2)/c/n/(x^
(3/2*n))+(c*x)^(3/2*n)*(a+b*x^n)^(1/2)/b/c/n/(x^n)
```

Rubi [A] time = 0.04, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {357, 355, 288, 206}

$$\frac{x^{-n}(cx)^{3n/2}\sqrt{a+bx^n}}{bcn} - \frac{ax^{-3n/2}(cx)^{3n/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x^{n/2}}{\sqrt{a+bx^n}}\right)}{b^{3/2}cn}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 + (3*n)/2)/Sqrt[a + b*x^n],x]
```

```
[Out] ((c*x)^((3*n)/2)*Sqrt[a + b*x^n])/(b*c*n*x^n) - (a*(c*x)^((3*n)/2)*ArcTanh[
(Sqrt[b]*x^(n/2))/Sqrt[a + b*x^n]])/(b^(3/2)*c*n*x^((3*n)/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 355

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[p]}, Dist[(k*a^(p + Simplify[(m + 1)/n]))/n, Subst[Int[x^(k*Simplify[
(m + 1)/n] - 1)/(1 - b*x^k)^(p + Simplify[(m + 1)/n] + 1), x], x, x^(n/k)/(
a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p + Simplify[(
m + 1)/n]] && LtQ[-1, p, 0]
```

Rule 357

```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(c^In
tPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a + b*x^n)^p, x], x] /;
 FreeQ[{a, b, c, m, n}, x] && IntegerQ[p + Simplify[(m + 1)/n]] && LtQ[-1,
p, 0]
```

Rubi steps

$$\int \frac{(cx)^{-1+\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx = \frac{\left(x^{-3n/2}(cx)^{3n/2}\right)\int \frac{x^{-1+\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx}{c}$$
$$= \frac{\left(2ax^{-3n/2}(cx)^{3n/2}\right)\text{Subst}\left(\int \frac{x^2}{\left(1-bx^2\right)^2}\,dx, x, \frac{x^{n/2}}{\sqrt{a+bx^n}}\right)}{cn}$$
$$= \frac{x^{-n}(cx)^{3n/2}\sqrt{a+bx^n}}{bcn} - \frac{\left(ax^{-3n/2}(cx)^{3n/2}\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^{n/2}}{\sqrt{a+bx^n}}\right)}{bcn}$$
$$= \frac{x^{-n}(cx)^{3n/2}\sqrt{a+bx^n}}{bcn} - \frac{ax^{-3n/2}(cx)^{3n/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x^{n/2}}{\sqrt{a+bx^n}}\right)}{b^{3/2}cn}$$

Mathematica [A] time = 0.06, size = 110, normalized size = 1.21

$$\frac{ax^{1-\frac{3n}{2}}(cx)^{\frac{3n}{2}-1}\sqrt{\frac{bx^n}{a}+1}\left(\sqrt{b}\,x^{n/2}\sqrt{\frac{a+bx^n}{a}} - \sqrt{a}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x^{n/2}}{\sqrt{a}}\right)\right)}{b^{3/2}n\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 + (3*n)/2)/Sqrt[a + b*x^n],x]
```

```
[Out] (a*x^(1 - (3*n)/2)*(c*x)^(-1 + (3*n)/2)*Sqrt[1 + (b*x^n)/a]*(Sqrt[b]*x^(n/2
)*Sqrt[(a + b*x^n)/a] - Sqrt[a]*ArcSinh[(Sqrt[b]*x^(n/2))/Sqrt[a]]))/(b^(3/
2)*n*Sqrt[a + b*x^n])
```

fricas [A] time = 0.81, size = 142, normalized size = 1.56

$$\left[\frac{2\sqrt{bx^n+a}\,bc^{\frac{3}{2}n-1}x^{\frac{1}{2}n} + a\sqrt{b}\,c^{\frac{3}{2}n-1}\log\left(2\sqrt{bx^n+a}\sqrt{b}\,x^{\frac{1}{2}n} - 2bx^n - a\right)}{2b^2n}, \frac{\sqrt{bx^n+a}\,bc^{\frac{3}{2}n-1}x^{\frac{1}{2}n} + a\sqrt{-b}\,c^{\frac{3}{2}n-1}\text{ar}}{b^2n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+3/2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*sqrt(b*x^n + a)*b*c^(3/2*n - 1)*x^(1/2*n) + a*sqrt(b)*c^(3/2*n - 1)
*log(2*sqrt(b*x^n + a)*sqrt(b)*x^(1/2*n) - 2*b*x^n - a))/(b^2*n), (sqrt(b*x
^n + a)*b*c^(3/2*n - 1)*x^(1/2*n) + a*sqrt(-b)*c^(3/2*n - 1)*arctan(sqrt(-b
)*x^(1/2*n)/sqrt(b*x^n + a)))/(b^2*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}n-1}}{\sqrt{bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+3/2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(3/2*n - 1)/sqrt(b*x^n + a), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3n}{2}-1}}{\sqrt{b\,x^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-1+3/2*n)/(b*x^n+a)^(1/2),x)
```

```
[Out] int((c*x)^(-1+3/2*n)/(b*x^n+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}n-1}}{\sqrt{bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+3/2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(3/2*n - 1)/sqrt(b*x^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{\frac{3n}{2}-1}}{\sqrt{a+b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^((3*n)/2 - 1)/(a + b*x^n)^(1/2),x)
```

```
[Out] int((c*x)^((3*n)/2 - 1)/(a + b*x^n)^(1/2), x)
```

sympy [A] time = 8.79, size = 66, normalized size = 0.73

$$\frac{\sqrt{a}\,c^{\frac{3n}{2}}x^{\frac{n}{2}}\sqrt{1+\frac{bx^n}{a}}}{bcn} - \frac{ac^{\frac{3n}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{\frac{n}{2}}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-1+3/2*n)/(a+b*x**n)**(1/2),x)
```

```
[Out] sqrt(a)*c**(3*n/2)*x**(n/2)*sqrt(1 + b*x**n/a)/(b*c*n) - a*c**(3*n/2)*asinh
(sqrt(b)*x**(n/2)/sqrt(a))/(b**(3/2)*c*n)
```

3.2784 $\int \frac{(cx)^{-1+\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=54

$$\frac{2x^{-n/2}(cx)^{n/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x^{n/2}}{\sqrt{a+bx^n}}\right)}{\sqrt{b}\,cn}$$

```
[Out] 2*(c*x)^(1/2*n)*arctanh(x^(1/2*n)*b^(1/2)/(a+b*x^n)^(1/2))/c/n/(x^(1/2*n))/
b^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {347, 345, 217, 206}

$$\frac{2x^{-n/2}(cx)^{n/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x^{n/2}}{\sqrt{a+bx^n}}\right)}{\sqrt{b}\,cn}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 + n/2)/Sqrt[a + b*x^n],x]
```

```
[Out] (2*(c*x)^(n/2)*ArcTanh[(Sqrt[b]*x^(n/2))/Sqrt[a + b*x^n]])/(Sqrt[b]*c*n*x^(
n/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 347

```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(c^In
tPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a + b*x^n)^p, x], x] /;
 FreeQ[{a, b, c, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !Integer
Q[n]
```

Rubi steps

$$\int \frac{(cx)^{-1+\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx = \frac{\left(x^{-n/2}(cx)^{n/2}\right)\int \frac{x^{-1+\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx}{c}$$
$$= \frac{\left(2x^{-n/2}(cx)^{n/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, x^{n/2}\right)}{cn}$$
$$= \frac{\left(2x^{-n/2}(cx)^{n/2}\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^{n/2}}{\sqrt{a+bx^n}}\right)}{cn}$$
$$= \frac{2x^{-n/2}(cx)^{n/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x^{n/2}}{\sqrt{a+bx^n}}\right)}{\sqrt{b}\,cn}$$

Mathematica [A] time = 0.03, size = 78, normalized size = 1.44

$$\frac{2\sqrt{a}\,x^{-n/2}(cx)^{n/2}\sqrt{\frac{bx^n}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x^{n/2}}{\sqrt{a}}\right)}{\sqrt{b}\,cn\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 + n/2)/Sqrt[a + b*x^n],x]
```

```
[Out] (2*Sqrt[a]*(c*x)^(n/2)*Sqrt[1 + (b*x^n)/a]*ArcSinh[(Sqrt[b]*x^(n/2))/Sqrt[a
]])/(Sqrt[b]*c*n*x^(n/2)*Sqrt[a + b*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+1/2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{1}{2}n-1}}{\sqrt{bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+1/2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(1/2*n - 1)/sqrt(b*x^n + a), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{n}{2}-1}}{\sqrt{b\,x^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2*n-1)/(b*x^n+a)^(1/2),x)
```

```
[Out] int((c*x)^(1/2*n-1)/(b*x^n+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{1}{2}n-1}}{\sqrt{bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+1/2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate((c*x)^(1/2*n - 1)/sqrt(b*x^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c\,x)^{\frac{n}{2}-1}}{\sqrt{a+b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(n/2 - 1)/(a + b*x^n)^(1/2),x)
[Out] int((c*x)^(n/2 - 1)/(a + b*x^n)^(1/2), x)
```

sympy [A] time = 7.11, size = 31, normalized size = 0.57

$$\frac{2c^{\frac{n}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}\,cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-1+1/2*n)/(a+b*x**n)**(1/2),x)
[Out] 2*c**(n/2)*asinh(sqrt(b)*x**(n/2)/sqrt(a))/(sqrt(b)*c*n)
```

3.2785 $\int \frac{(cx)^{-1-\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=31

$$-\frac{2(cx)^{-n/2}\sqrt{a+bx^n}}{acn}$$

```
[Out] -2*(a+b*x^n)^(1/2)/a/c/n/((c*x)^(1/2*n))
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {264}

$$-\frac{2(cx)^{-n/2}\sqrt{a+bx^n}}{acn}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 - n/2)/Sqrt[a + b*x^n],x]
```

```
[Out] (-2*Sqrt[a + b*x^n])/(a*c*n*(c*x)^(n/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(cx)^{-1-\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx = -\frac{2(cx)^{-n/2}\sqrt{a+bx^n}}{acn}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$-\frac{2x(cx)^{-\frac{n}{2}-1}\sqrt{a+bx^n}}{an}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 - n/2)/Sqrt[a + b*x^n],x]
```

```
[Out] (-2*x*(c*x)^(-1 - n/2)*Sqrt[a + b*x^n])/(a*n)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-1/2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{-\frac{1}{2}n-1}}{\sqrt{bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-1/2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
[Out] integrate((c*x)^(-1/2*n - 1)/sqrt(b*x^n + a), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{-\frac{n}{2}-1}}{\sqrt{b\,x^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-1/2*n-1)/(b*x^n+a)^(1/2),x)
[Out] int((c*x)^(-1/2*n-1)/(b*x^n+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{-\frac{1}{2}n-1}}{\sqrt{bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-1/2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate((c*x)^(-1/2*n - 1)/sqrt(b*x^n + a), x)
```

mupad [B] time = 1.21, size = 29, normalized size = 0.94

$$-\frac{2\,x\,\sqrt{a+b\,x^n}}{a\,n\,(c\,x)^{\frac{n}{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(n/2 + 1)*(a + b*x^n)^(1/2)),x)
[Out] -(2*x*(a + b*x^n)^(1/2))/(a*n*(c*x)^(n/2 + 1))
```

sympy [A] time = 5.45, size = 29, normalized size = 0.94

$$-\frac{2\sqrt{b}\,c^{-\frac{n}{2}}\sqrt{\frac{ax^{-n}}{b}+1}}{acn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-1-1/2*n)/(a+b*x**n)**(1/2),x)
[Out] -2*sqrt(b)*c**(-n/2)*sqrt(a*x**(-n)/b + 1)/(a*c*n)
```

3.2786 $\int \frac{(cx)^{-1-\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=65

$$\frac{4(cx)^{-3n/2}\left(a+bx^n\right)^{3/2}}{3a^2cn}-\frac{2(cx)^{-3n/2}\sqrt{a+bx^n}}{acn}$$

```
[Out] 4/3*(a+b*x^n)^(3/2)/a^2/c/n/((c*x)^(3/2*n))-2*(a+b*x^n)^(1/2)/a/c/n/((c*x)^
(3/2*n))
```

Rubi [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {273, 264}

$$\frac{4(cx)^{-3n/2}\left(a+bx^n\right)^{3/2}}{3a^2cn}-\frac{2(cx)^{-3n/2}\sqrt{a+bx^n}}{acn}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 - (3*n)/2)/Sqrt[a + b*x^n],x]
```

```
[Out] (-2*Sqrt[a + b*x^n])/(a*c*n*(c*x)^((3*n)/2)) + (4*(a + b*x^n)^(3/2))/(3*a^2
*c*n*(c*x)^((3*n)/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx)^{-1-\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx &= -\frac{2(cx)^{-3n/2}\sqrt{a+bx^n}}{acn}-\frac{2\int (cx)^{-1-\frac{3n}{2}}\sqrt{a+bx^n}\,dx}{a}\\ &= -\frac{2(cx)^{-3n/2}\sqrt{a+bx^n}}{acn}+\frac{4(cx)^{-3n/2}\left(a+bx^n\right)^{3/2}}{3a^2cn}\end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 0.63

$$-\frac{2(cx)^{-3n/2}\left(a-2bx^n\right)\sqrt{a+bx^n}}{3a^2cn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 - (3*n)/2)/Sqrt[a + b*x^n],x]
```

```
[Out] (-2*(a - 2*b*x^n)*Sqrt[a + b*x^n])/(3*a^2*c*n*(c*x)^((3*n)/2))
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-3/2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{-\frac{3}{2}n-1}}{\sqrt{bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-3/2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(-3/2*n - 1)/sqrt(b*x^n + a), x)
```

maple [**F**] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{-\frac{3n}{2}-1}}{\sqrt{b\,x^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-3/2*n-1)/(b*x^n+a)^(1/2),x)
```

```
[Out] int((c*x)^(-3/2*n-1)/(b*x^n+a)^(1/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{-\frac{3}{2}n-1}}{\sqrt{bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-3/2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(-3/2*n - 1)/sqrt(b*x^n + a), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(c\,x)^{\frac{3n}{2}+1}\,\sqrt{a+b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^((3*n)/2 + 1)*(a + b*x^n)^(1/2)),x)
```

```
[Out] int(1/((c*x)^((3*n)/2 + 1)*(a + b*x^n)^(1/2)), x)
```

sympy [**A**] time = 5.05, size = 68, normalized size = 1.05

$$-\frac{2\sqrt{b}\,c^{-\frac{3n}{2}}x^{-n}\sqrt{\frac{ax^{-n}}{b}+1}}{3acn}+\frac{4b^{\frac{3}{2}}c^{-\frac{3n}{2}}\sqrt{\frac{ax^{-n}}{b}+1}}{3a^2cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-1-3/2*n)/(a+b*x**n)**(1/2),x)
```

```
[Out] -2*sqrt(b)*c**(-3*n/2)*x**(-n)*sqrt(a*x**(-n)/b + 1)/(3*a*c*n) + 4*b**(3/2)
*c**(-3*n/2)*sqrt(a*x**(-n)/b + 1)/(3*a**2*c*n)
```

3.2787 $\int \frac{(cx)^{-1-\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=98

$$-\frac{16(cx)^{-5n/2}\left(a+bx^n\right)^{5/2}}{15a^3cn}+\frac{8(cx)^{-5n/2}\left(a+bx^n\right)^{3/2}}{3a^2cn}-\frac{2(cx)^{-5n/2}\sqrt{a+bx^n}}{acn}$$

```
[Out] 8/3*(a+b*x^n)^(3/2)/a^2/c/n/((c*x)^(5/2*n))-16/15*(a+b*x^n)^(5/2)/a^3/c/n/(
(c*x)^(5/2*n))-2*(a+b*x^n)^(1/2)/a/c/n/((c*x)^(5/2*n))
```

Rubi [A] time = 0.03, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {273, 264}

$$-\frac{16(cx)^{-5n/2}\left(a+bx^n\right)^{5/2}}{15a^3cn}+\frac{8(cx)^{-5n/2}\left(a+bx^n\right)^{3/2}}{3a^2cn}-\frac{2(cx)^{-5n/2}\sqrt{a+bx^n}}{acn}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 - (5*n)/2)/Sqrt[a + b*x^n],x]
```

```
[Out] (-2*Sqrt[a + b*x^n])/(a*c*n*(c*x)^((5*n)/2)) + (8*(a + b*x^n)^(3/2))/(3*a^2
*c*n*(c*x)^((5*n)/2)) - (16*(a + b*x^n)^(5/2))/(15*a^3*c*n*(c*x)^((5*n)/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx)^{-1-\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx &= -\frac{2(cx)^{-5n/2}\sqrt{a+bx^n}}{acn}-\frac{4\int (cx)^{-1-\frac{5n}{2}}\sqrt{a+bx^n}\,dx}{a}\\ &= -\frac{2(cx)^{-5n/2}\sqrt{a+bx^n}}{acn}+\frac{8(cx)^{-5n/2}\left(a+bx^n\right)^{3/2}}{3a^2cn}+\frac{8\int (cx)^{-1-\frac{5n}{2}}\left(a+bx^n\right)^{3/2}\,dx}{3a^2}\\ &= -\frac{2(cx)^{-5n/2}\sqrt{a+bx^n}}{acn}+\frac{8(cx)^{-5n/2}\left(a+bx^n\right)^{3/2}}{3a^2cn}-\frac{16(cx)^{-5n/2}\left(a+bx^n\right)^{5/2}}{15a^3cn}\end{aligned}$$

Mathematica [A] time = 0.02, size = 56, normalized size = 0.57

$$-\frac{2(cx)^{-5n/2}\sqrt{a+bx^n}\left(3a^2-4abx^n+8b^2x^{2n}\right)}{15a^3cn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 - (5*n)/2)/Sqrt[a + b*x^n],x]
```

```
[Out] (-2*Sqrt[a + b*x^n]*(3*a^2 - 4*a*b*x^n + 8*b^2*x^(2*n)))/(15*a^3*c*n*(c*x)^
((5*n)/2))
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-5/2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{-\frac{5}{2}n-1}}{\sqrt{bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-5/2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(-5/2*n - 1)/sqrt(b*x^n + a), x)
```

maple [**F**] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{-\frac{5n}{2}-1}}{\sqrt{b\,x^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-5/2*n-1)/(b*x^n+a)^(1/2),x)
```

```
[Out] int((c*x)^(-5/2*n-1)/(b*x^n+a)^(1/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{-\frac{5}{2}n-1}}{\sqrt{bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-5/2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(-5/2*n - 1)/sqrt(b*x^n + a), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\,x)^{\frac{5n}{2}+1}\,\sqrt{a+b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^((5*n)/2 + 1)*(a + b*x^n)^(1/2)),x)
```

```
[Out] int(1/((c*x)^((5*n)/2 + 1)*(a + b*x^n)^(1/2)), x)
```

sympy [**B**] time = 5.38, size = 396, normalized size = 4.04

$$-\frac{6a^4b^{\frac{9}{2}}c^{-\frac{5n}{2}}\sqrt{\frac{ax^{-n}}{b}+1}}{c\left(15a^5b^4nx^{2n}+30a^4b^5nx^{3n}+15a^3b^6nx^{4n}\right)}-\frac{4a^3b^{\frac{11}{2}}c^{-\frac{5n}{2}}x^n\sqrt{\frac{ax^{-n}}{b}+1}}{c\left(15a^5b^4nx^{2n}+30a^4b^5nx^{3n}+15a^3b^6nx^{4n}\right)}-\frac{6a^2b^{\frac{13}{2}}c^{-\frac{5n}{2}}}{c\left(15a^5b^4nx^{2n}+30a^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-1-5/2*n)/(a+b*x**n)**(1/2),x)
```

```
[Out] -6*a**4*b**(9/2)*c**(-5*n/2)*sqrt(a*x**(-n)/b + 1)/(c*(15*a**5*b**4*n*x**(2
*n) + 30*a**4*b**5*n*x**(3*n) + 15*a**3*b**6*n*x**(4*n))) - 4*a**3*b**(11/2
)*c**(-5*n/2)*x**n*sqrt(a*x**(-n)/b + 1)/(c*(15*a**5*b**4*n*x**(2*n) + 30*a
**4*b**5*n*x**(3*n) + 15*a**3*b**6*n*x**(4*n))) - 6*a**2*b**(13/2)*c**(-5*n
/2)*x**(2*n)*sqrt(a*x**(-n)/b + 1)/(c*(15*a**5*b**4*n*x**(2*n) + 30*a**4*b*
*5*n*x**(3*n) + 15*a**3*b**6*n*x**(4*n))) - 24*a*b**(15/2)*c**(-5*n/2)*x**(
3*n)*sqrt(a*x**(-n)/b + 1)/(c*(15*a**5*b**4*n*x**(2*n) + 30*a**4*b**5*n*x**
(3*n) + 15*a**3*b**6*n*x**(4*n))) - 16*b**(17/2)*c**(-5*n/2)*x**(4*n)*sqrt(
a*x**(-n)/b + 1)/(c*(15*a**5*b**4*n*x**(2*n) + 30*a**4*b**5*n*x**(3*n) + 15
*a**3*b**6*n*x**(4*n)))
```

3.2788 $\int \frac{(cx)^{-1-\frac{7n}{2}}}{\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=129

$$\frac{32(cx)^{-7n/2}\left(a+bx^n\right)^{7/2}}{35a^4cn}-\frac{16(cx)^{-7n/2}\left(a+bx^n\right)^{5/2}}{5a^3cn}+\frac{4(cx)^{-7n/2}\left(a+bx^n\right)^{3/2}}{a^2cn}-\frac{2(cx)^{-7n/2}\sqrt{a+bx^n}}{acn}$$

```
[Out] 4*(a+b*x^n)^(3/2)/a^2/c/n/((c*x)^(7/2*n))-16/5*(a+b*x^n)^(5/2)/a^3/c/n/((c*
x)^(7/2*n))+32/35*(a+b*x^n)^(7/2)/a^4/c/n/((c*x)^(7/2*n))-2*(a+b*x^n)^(1/2)
/a/c/n/((c*x)^(7/2*n))
```

Rubi [A] time = 0.05, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {273, 264}

$$\frac{32(cx)^{-7n/2}\left(a+bx^n\right)^{7/2}}{35a^4cn}-\frac{16(cx)^{-7n/2}\left(a+bx^n\right)^{5/2}}{5a^3cn}+\frac{4(cx)^{-7n/2}\left(a+bx^n\right)^{3/2}}{a^2cn}-\frac{2(cx)^{-7n/2}\sqrt{a+bx^n}}{acn}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 - (7*n)/2)/Sqrt[a + b*x^n],x]
```

```
[Out] (-2*Sqrt[a + b*x^n])/(a*c*n*(c*x)^((7*n)/2)) + (4*(a + b*x^n)^(3/2))/(a^2*c
*n*(c*x)^((7*n)/2)) - (16*(a + b*x^n)^(5/2))/(5*a^3*c*n*(c*x)^((7*n)/2)) +
(32*(a + b*x^n)^(7/2))/(35*a^4*c*n*(c*x)^((7*n)/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx)^{-1-\frac{7n}{2}}}{\sqrt{a+bx^n}}\,dx &= -\frac{2(cx)^{-7n/2}\sqrt{a+bx^n}}{acn}-\frac{6\int (cx)^{-1-\frac{7n}{2}}\sqrt{a+bx^n}\,dx}{a}\\ &= -\frac{2(cx)^{-7n/2}\sqrt{a+bx^n}}{acn}+\frac{4(cx)^{-7n/2}\left(a+bx^n\right)^{3/2}}{a^2cn}+\frac{8\int (cx)^{-1-\frac{7n}{2}}\left(a+bx^n\right)^{3/2}\,dx}{a^2}\\ &= -\frac{2(cx)^{-7n/2}\sqrt{a+bx^n}}{acn}+\frac{4(cx)^{-7n/2}\left(a+bx^n\right)^{3/2}}{a^2cn}-\frac{16(cx)^{-7n/2}\left(a+bx^n\right)^{5/2}}{5a^3cn}-\frac{16\int (cx)^{-1-\frac{7n}{2}}(a+\\ &= -\frac{2(cx)^{-7n/2}\sqrt{a+bx^n}}{acn}+\frac{4(cx)^{-7n/2}\left(a+bx^n\right)^{3/2}}{a^2cn}-\frac{16(cx)^{-7n/2}\left(a+bx^n\right)^{5/2}}{5a^3cn}+\frac{32(cx)^{-7n/2}(a+bx\end{aligned}$$

Mathematica [A] time = 0.03, size = 69, normalized size = 0.53

$$-\frac{2(cx)^{-7n/2}\sqrt{a+bx^n}\left(5a^3-6a^2bx^n+8ab^2x^{2n}-16b^3x^{3n}\right)}{35a^4cn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 - (7*n)/2)/Sqrt[a + b*x^n],x]
```

```
[Out] (-2*Sqrt[a + b*x^n]*(5*a^3 - 6*a^2*b*x^n + 8*a*b^2*x^(2*n) - 16*b^3*x^(3*n)
))/(35*a^4*c*n*(c*x)^((7*n)/2))
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-7/2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{-\frac{7}{2}n-1}}{\sqrt{bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-7/2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(-7/2*n - 1)/sqrt(b*x^n + a), x)
```

maple [**F**] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{-\frac{7n}{2}-1}}{\sqrt{b\,x^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-7/2*n-1)/(b*x^n+a)^(1/2),x)
```

```
[Out] int((c*x)^(-7/2*n-1)/(b*x^n+a)^(1/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{-\frac{7}{2}n-1}}{\sqrt{bx^n+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-7/2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(-7/2*n - 1)/sqrt(b*x^n + a), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\,x)^{\frac{7n}{2}+1}\,\sqrt{a+b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^((7*n)/2 + 1)*(a + b*x^n)^(1/2)),x)
```

```
[Out] int(1/((c*x)^((7*n)/2 + 1)*(a + b*x^n)^(1/2)), x)
```

sympy [**B**] time = 5.91, size = 665, normalized size = 5.16

$$-\frac{10a^6b^{\frac{19}{2}}c^{-\frac{7n}{2}}\sqrt{\frac{ax^{-n}}{b}+1}}{c\left(35a^7b^9nx^{3n}+105a^6b^{10}nx^{4n}+105a^5b^{11}nx^{5n}+35a^4b^{12}nx^{6n}\right)}-\frac{18a^5b^{\frac{21}{2}}c^{-\frac{7n}{2}}x^n\sqrt{\frac{ax^{-n}}{b}+1}}{c\left(35a^7b^9nx^{3n}+105a^6b^{10}nx^{4n}+105a^5b^{11}nx^{5n}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-1-7/2*n)/(a+b*x**n)**(1/2),x)
```

```
[Out] -10*a**6*b**(19/2)*c**(-7*n/2)*sqrt(a*x**(-n)/b + 1)/(c*(35*a**7*b**9*n*x**
(3*n) + 105*a**6*b**10*n*x**(4*n) + 105*a**5*b**11*n*x**(5*n) + 35*a**4*b**
12*n*x**(6*n))) - 18*a**5*b**(21/2)*c**(-7*n/2)*x**n*sqrt(a*x**(-n)/b + 1)/
(c*(35*a**7*b**9*n*x**(3*n) + 105*a**6*b**10*n*x**(4*n) + 105*a**5*b**11*n*
x**(5*n) + 35*a**4*b**12*n*x**(6*n))) - 10*a**4*b**(23/2)*c**(-7*n/2)*x**(2
*n)*sqrt(a*x**(-n)/b + 1)/(c*(35*a**7*b**9*n*x**(3*n) + 105*a**6*b**10*n*x*
*(4*n) + 105*a**5*b**11*n*x**(5*n) + 35*a**4*b**12*n*x**(6*n))) + 10*a**3*b
**(25/2)*c**(-7*n/2)*x**(3*n)*sqrt(a*x**(-n)/b + 1)/(c*(35*a**7*b**9*n*x**(
3*n) + 105*a**6*b**10*n*x**(4*n) + 105*a**5*b**11*n*x**(5*n) + 35*a**4*b**1
2*n*x**(6*n))) + 60*a**2*b**(27/2)*c**(-7*n/2)*x**(4*n)*sqrt(a*x**(-n)/b +
1)/(c*(35*a**7*b**9*n*x**(3*n) + 105*a**6*b**10*n*x**(4*n) + 105*a**5*b**11
*n*x**(5*n) + 35*a**4*b**12*n*x**(6*n))) + 80*a*b**(29/2)*c**(-7*n/2)*x**(5
*n)*sqrt(a*x**(-n)/b + 1)/(c*(35*a**7*b**9*n*x**(3*n) + 105*a**6*b**10*n*x*
*(4*n) + 105*a**5*b**11*n*x**(5*n) + 35*a**4*b**12*n*x**(6*n))) + 32*b**(31
/2)*c**(-7*n/2)*x**(6*n)*sqrt(a*x**(-n)/b + 1)/(c*(35*a**7*b**9*n*x**(3*n)
+ 105*a**6*b**10*n*x**(4*n) + 105*a**5*b**11*n*x**(5*n) + 35*a**4*b**12*n*x
**(6*n)))
```

3.2789 $\int (cx)^m (a+bx^n)^p \, dx$

Optimal. Leaf size=59

$$\frac{(cx)^{m+1}(a+bx^n)^{p+1}\ {}_2F_1\left(1, \frac{m+1}{n}+p+1; \frac{m+n+1}{n}; -\frac{bx^n}{a}\right)}{ac(m+1)}$$

```
[Out] (c*x)^(1+m)*(a+b*x^n)^(1+p)*hypergeom([1, 1+(1+m)/n+p],[(1+m+n)/n],-b*x^n/a
)/a/c/(1+m)
```

Rubi [A] time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {365, 364}

$$\frac{(cx)^{m+1}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\ {}_2F_1\left(\frac{m+1}{n}, -p; \frac{m+n+1}{n}; -\frac{bx^n}{a}\right)}{c(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*(a + b*x^n)^p,x]
```

```
[Out] ((c*x)^(1 + m)*(a + b*x^n)^p*Hypergeometric2F1[(1 + m)/n, -p, (1 + m + n)/n
, -((b*x^n)/a)])/(c*(1 + m)*(1 + (b*x^n)/a)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (cx)^m (a+bx^n)^p \, dx &= \left((a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\right)\int (cx)^m \left(1+\frac{bx^n}{a}\right)^p dx \\ &= \frac{(cx)^{1+m}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\ {}_2F_1\left(\frac{1+m}{n}, -p; \frac{1+m+n}{n}; -\frac{bx^n}{a}\right)}{c(1+m)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 64, normalized size = 1.08

$$\frac{x(cx)^m(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\ {}_2F_1\left(\frac{m+1}{n}, -p; \frac{m+1}{n}+1; -\frac{bx^n}{a}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*(a + b*x^n)^p,x]
```

```
[Out] (x*(c*x)^m*(a + b*x^n)^p*Hypergeometric2F1[(1 + m)/n, -p, 1 + (1 + m)/n, -(
(b*x^n)/a)])/((1 + m)*(1 + (b*x^n)/a)^p)
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left((bx^n + a)^p (cx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(a+b*x^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p*(c*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^p (cx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(a+b*x^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p*(c*x)^m, x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int (cx)^m (b\,x^n + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(b*x^n+a)^p,x)
```

```
[Out] int((c*x)^m*(b*x^n+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^p (cx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(a+b*x^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p*(c*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (c\,x)^m (a + b\,x^n)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(a + b*x^n)^p,x)
```

```
[Out] int((c*x)^m*(a + b*x^n)^p, x)
```

sympy [C] time = 9.77, size = 58, normalized size = 0.98

$$\frac{a^p c^m x x^m \Gamma\left(\frac{m}{n} + \frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{n} + \frac{1}{n} \\ \frac{m}{n} + 1 + \frac{1}{n} \end{matrix} \middle| \frac{b x^n e^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n} + 1 + \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(a+b*x**n)**p,x)
```

```
[Out] a**p*c**m*x*x**m*gamma(m/n + 1/n)*hyper((-p, m/n + 1/n), (m/n + 1 + 1/n,),
b*x**n*exp_polar(I*pi)/a)/(n*gamma(m/n + 1 + 1/n))
```

3.2790 $\int (cx)^{3n} (a + bx^n)^p \, dx$

Optimal. Leaf size=56

$$\frac{(cx)^{3n+1} (a + bx^n)^{p+1} \, {}_2F_1\left(1, p + \frac{1}{n} + 4; 4 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{ac(3n+1)}$$

```
[Out] (c*x)^(1+3*n)*(a+b*x^n)^(1+p)*hypergeom([1, 4+1/n+p],[4+1/n],-b*x^n/a)/a/c/
(1+3*n)
```

Rubi [A] time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {365, 364}

$$\frac{(cx)^{3n+1} (a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} \, {}_2F_1\left(3 + \frac{1}{n}, -p; 4 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{c(3n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(3*n)*(a + b*x^n)^p,x]
```

```
[Out] ((c*x)^(1 + 3*n)*(a + b*x^n)^p*Hypergeometric2F1[3 + n^(-1), -p, 4 + n^(-1)
, -((b*x^n)/a)])/(c*(1 + 3*n)*(1 + (b*x^n)/a)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (cx)^{3n} (a + bx^n)^p \, dx &= \left((a + bx^n)^p \left(1 + \frac{bx^n}{a}\right)^{-p}\right) \int (cx)^{3n} \left(1 + \frac{bx^n}{a}\right)^p \, dx \\ &= \frac{(cx)^{1+3n} (a + bx^n)^p \left(1 + \frac{bx^n}{a}\right)^{-p} \, {}_2F_1\left(3 + \frac{1}{n}, -p; 4 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{c(1+3n)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 1.11

$$\frac{x(cx)^{3n} (a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} \, {}_2F_1\left(3 + \frac{1}{n}, -p; 4 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{3n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(3*n)*(a + b*x^n)^p,x]
```

```
[Out] (x*(c*x)^(3*n)*(a + b*x^n)^p*Hypergeometric2F1[3 + n^(-1), -p, 4 + n^(-1),
-((b*x^n)/a)])/((1 + 3*n)*(1 + (b*x^n)/a)^p)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left((bx^n + a)^p\,(cx)^{3\,n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3*n)*(a+b*x^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p*(c*x)^(3*n), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3*n)*(a+b*x^n)^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Simplification assuming x near 0Simplificati
on assuming c near 0Simplification assuming x near 0Simplification assuming
 c near 0Unable to divide, perhaps due to rounding error%%%{1,[2,0,6,4,2,4,
4]%%%}+%%%{4,[2,0,6,4,2,3,4]%%%}+%%%{6,[2,0,6,4,2,2,4]%%%}+%%%{4,[2,0,6,4,2
,1,4]%%%}+%%%{1,[2,0,6,4,2,0,4]%%%}+%%%{-1,[1,0,6,4,1,4,5]%%%}+%%%{-4,[1,0,
6,4,1,3,5]%%%}+%%%{-6,[1,0,6,4,1,2,5]%%%}+%%%{-4,[1,0,6,4,1,1,5]%%%}+%%%{-1
,[1,0,6,4,1,0,5]%%%}+%%%{-1,[0,0,6,3,0,3,6]%%%}+%%%{-3,[0,0,6,3,0,2,6]%%%}+
%%%{-3,[0,0,6,3,0,1,6]%%%}+%%%{-1,[0,0,6,3,0,0,6]%%%} / %%%{1,[0,0,7,4,3,4,
4]%%%}+%%%{4,[0,0,7,4,3,3,4]%%%}+%%%{6,[0,0,7,4,3,2,4]%%%}+%%%{4,[0,0,7,4,3
,1,4]%%%}+%%%{1,[0,0,7,4,3,0,4]%%%} Error: Bad Argument Value
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int (cx)^{3n}\,(b\,x^n + a)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3*n)*(b*x^n+a)^p,x)
```

```
[Out] int((c*x)^(3*n)*(b*x^n+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^p\,(cx)^{3\,n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3*n)*(a+b*x^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p*(c*x)^(3*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (c\,x)^{3\,n}\,(a + b\,x^n)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(3*n)*(a + b*x^n)^p,x)
```

```
[Out] int((c*x)^(3*n)*(a + b*x^n)^p, x)
```

sympy [C] time = 7.63, size = 51, normalized size = 0.91

$$\frac{a^p c^{3n} x x^{3n} \Gamma\left(3+\frac{1}{n}\right) {}_2F_1\left(\left.\begin{matrix}-p, 3+\frac{1}{n} \\ 4+\frac{1}{n}\end{matrix}\right| \frac{b x^n e^{i\pi}}{a}\right)}{n\Gamma\left(4+\frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(3*n)*(a+b*x**n)**p,x)
```

```
[Out] a**p*c**(3*n)*x*x**(3*n)*gamma(3 + 1/n)*hyper((-p, 3 + 1/n), (4 + 1/n,), b*
x**n*exp_polar(I*pi)/a)/(n*gamma(4 + 1/n))
```

3.2791 $\int (cx)^{2n}\,(a+bx^n)^p\,dx$

Optimal. Leaf size=56

$$\frac{(cx)^{2n+1}\,(a+bx^n)^{p+1}\;{}_2F_1\left(1,p+\frac{1}{n}+3;3+\frac{1}{n};-\frac{bx^n}{a}\right)}{ac(2n+1)}$$

```
[Out] (c*x)^(1+2*n)*(a+b*x^n)^(1+p)*hypergeom([1, 3+1/n+p],[3+1/n],-b*x^n/a)/a/c/
(1+2*n)
```

Rubi [A] time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {365, 364}

$$\frac{(cx)^{2n+1}\,(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\;{}_2F_1\left(2+\frac{1}{n},-p;3+\frac{1}{n};-\frac{bx^n}{a}\right)}{c(2n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(2*n)*(a + b*x^n)^p,x]
```

```
[Out] ((c*x)^(1 + 2*n)*(a + b*x^n)^p*Hypergeometric2F1[2 + n^(-1), -p, 3 + n^(-1)
, -((b*x^n)/a)])/(c*(1 + 2*n)*(1 + (b*x^n)/a)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (cx)^{2n}\,(a+bx^n)^p\,dx &= \left((a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\right)\int (cx)^{2n}\left(1+\frac{bx^n}{a}\right)^p dx\\ &= \frac{(cx)^{1+2n}\,(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\;{}_2F_1\left(2+\frac{1}{n},-p;3+\frac{1}{n};-\frac{bx^n}{a}\right)}{c(1+2n)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 1.11

$$\frac{x(cx)^{2n}\,(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\;{}_2F_1\left(2+\frac{1}{n},-p;3+\frac{1}{n};-\frac{bx^n}{a}\right)}{2n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(2*n)*(a + b*x^n)^p,x]
```

```
[Out] (x*(c*x)^(2*n)*(a + b*x^n)^p*Hypergeometric2F1[2 + n^(-1), -p, 3 + n^(-1),
-((b*x^n)/a)])/((1 + 2*n)*(1 + (b*x^n)/a)^p)
```

fricas [**F**] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left((bx^n + a)^p\,(cx)^{2\,n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(2*n)*(a+b*x^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p*(c*x)^(2*n), x)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(2*n)*(a+b*x^n)^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Simplification assuming x near 0Simplificati
on assuming c near 0Simplification assuming x near 0Simplification assuming
 c near 0Unable to divide, perhaps due to rounding error%%%{-1,[1,0,4,3,1,3
,3]%%%}+%%%{-3,[1,0,4,3,1,2,3]%%%}+%%%{-3,[1,0,4,3,1,1,3]%%%}+%%%{-1,[1,0,4
,3,1,0,3]%%%}+%%%{-1,[0,0,4,2,0,2,4]%%%}+%%%{-2,[0,0,4,2,0,1,4]%%%}+%%%{-1,
[0,0,4,2,0,0,4]%%%} / %%%{-1,[0,0,5,3,2,3,3]%%%}+%%%{-3,[0,0,5,3,2,2,3]%%%}
+%%%{-3,[0,0,5,3,2,1,3]%%%}+%%%{-1,[0,0,5,3,2,0,3]%%%} Error: Bad Argument
Value
```

maple [**F**] time = 0.22, size = 0, normalized size = 0.00

$$\int (cx)^{2n}\,(b\,x^n + a)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(2*n)*(b*x^n+a)^p,x)
```

```
[Out] int((c*x)^(2*n)*(b*x^n+a)^p,x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^p\,(cx)^{2\,n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(2*n)*(a+b*x^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p*(c*x)^(2*n), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int (c\,x)^{2\,n}\,(a + b\,x^n)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(2*n)*(a + b*x^n)^p,x)
```

```
[Out] int((c*x)^(2*n)*(a + b*x^n)^p, x)
```

sympy [C] time = 8.08, size = 51, normalized size = 0.91

$$\frac{a^p c^{2n} x x^{2n} \Gamma\left(2+\frac{1}{n}\right) {}_2F_1\left(\begin{matrix}-p, 2+\frac{1}{n} \\ 3+\frac{1}{n}\end{matrix}\middle| \frac{b x^n e^{i\pi}}{a}\right)}{n\Gamma\left(3+\frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(2*n)*(a+b*x**n)**p,x)
```

```
[Out] a**p*c**(2*n)*x*x**(2*n)*gamma(2 + 1/n)*hyper((-p, 2 + 1/n), (3 + 1/n,), b*
x**n*exp_polar(I*pi)/a)/(n*gamma(3 + 1/n))
```

3.2792 $\int (cx)^n (a+bx^n)^p \, dx$

Optimal. Leaf size=52

$$\frac{(cx)^{n+1} (a+bx^n)^{p+1} \, {}_2F_1\left(1, p+\frac{1}{n}+2; 2+\frac{1}{n}; -\frac{bx^n}{a}\right)}{ac(n+1)}$$

```
[Out] (c*x)^(1+n)*(a+b*x^n)^(1+p)*hypergeom([1, 2+1/n+p],[2+1/n],-b*x^n/a)/a/c/(1
+n)
```

Rubi [A] time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.19, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {365, 364}

$$\frac{(cx)^{n+1} (a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \, {}_2F_1\left(1+\frac{1}{n}, -p; 2+\frac{1}{n}; -\frac{bx^n}{a}\right)}{c(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^n*(a + b*x^n)^p,x]
```

```
[Out] ((c*x)^(1 + n)*(a + b*x^n)^p*Hypergeometric2F1[1 + n^(-1), -p, 2 + n^(-1),
-((b*x^n)/a)])/(c*(1 + n)*(1 + (b*x^n)/a)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (cx)^n (a+bx^n)^p \, dx &= \left((a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p}\right) \int (cx)^n \left(1+\frac{bx^n}{a}\right)^p \, dx \\ &= \frac{(cx)^{1+n} (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \, {}_2F_1\left(1+\frac{1}{n}, -p; 2+\frac{1}{n}; -\frac{bx^n}{a}\right)}{c(1+n)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 58, normalized size = 1.12

$$\frac{x(cx)^n (a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \, {}_2F_1\left(1+\frac{1}{n}, -p; 2+\frac{1}{n}; -\frac{bx^n}{a}\right)}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^n*(a + b*x^n)^p,x]
```

```
[Out] (x*(c*x)^n*(a + b*x^n)^p*Hypergeometric2F1[1 + n^(-1), -p, 2 + n^(-1), -((b
*x^n)/a)])/((1 + n)*(1 + (b*x^n)/a)^p)
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left((bx^n + a)^p\,(cx)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^n*(a+b*x^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p*(c*x)^n, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^n*(a+b*x^n)^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Simplification assuming x near 0Simplificati
on assuming c near 0Simplification assuming x near 0Simplification assuming
 c near 0Unable to divide, perhaps due to rounding error%%%{-1,[0,0,2,1,0,1
,2]%%%}+%%%{-1,[0,0,2,1,0,0,2]%%%} / %%%{1,[0,0,3,2,1,2,2]%%%}+%%%{2,[0,0,3
,2,1,1,2]%%%}+%%%{1,[0,0,3,2,1,0,2]%%%} Error: Bad Argument Value
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int (cx)^n\,(b\,x^n + a)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^n*(b*x^n+a)^p,x)
```

```
[Out] int((c*x)^n*(b*x^n+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^p\,(cx)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^n*(a+b*x^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p*(c*x)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (c\,x)^n\,(a + b\,x^n)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^n*(a + b*x^n)^p,x)
```

```
[Out] int((c*x)^n*(a + b*x^n)^p, x)
```

sympy [C] time = 7.32, size = 48, normalized size = 0.92

$$\frac{a^p c^n x x^n \Gamma\left(1 + \frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -p, 1 + \frac{1}{n} \\ 2 + \frac{1}{n} \end{matrix} \middle| \frac{b x^n e^{i\pi}}{a}\right)}{n\Gamma\left(2 + \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**n*(a+b*x**n)**p,x)
```

```
[Out] a**p*c**n*x*x**n*gamma(1 + 1/n)*hyper((-p, 1 + 1/n), (2 + 1/n,), b*x**n*exp
_polar(I*pi)/a)/(n*gamma(2 + 1/n))
```

3.2793 $\int (a+bx^n)^p\,dx$

Optimal. Leaf size=38

$$\frac{x\,(a+bx^n)^{p+1}\;{}_2F_1\left(1,p+\frac{1}{n}+1;1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a}$$

```
[Out] x*(a+b*x^n)^(1+p)*hypergeom([1, 1+1/n+p],[1+1/n],-b*x^n/a)/a
```

Rubi [A] time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {246, 245}

$$x\,(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\;{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^p,x]
```

```
[Out] (x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)])/(
1 + (b*x^n)/a)^p
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)^p\,dx &= \left((a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\right)\int\left(1+\frac{bx^n}{a}\right)^p dx\\ &= x\,(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\;{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 1.21

$$x\,(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\;{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^p,x]
```

```
[Out] (x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)])/(
1 + (b*x^n)/a)^p
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left((bx^n + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p,x, algorithm="fricas")
[Out] integral((b*x^n + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p,x, algorithm="giac")
[Out] integrate((b*x^n + a)^p, x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int (b\,x^n + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^p,x)
[Out] int((b*x^n+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p,x, algorithm="maxima")
[Out] integrate((b*x^n + a)^p, x)
```

mupad [B] time = 1.94, size = 47, normalized size = 1.24

$$\frac{x\,(a + b\,x^n)^p \, {}_2\mathrm{F}_1\left(\frac{1}{n}, -p;\ \frac{1}{n} + 1;\ -\frac{b\,x^n}{a}\right)}{\left(\frac{b\,x^n}{a} + 1\right)^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^p,x)
[Out] (x*(a + b*x^n)^p*hypergeom([1/n, -p], 1/n + 1, -(b*x^n)/a))/((b*x^n)/a + 1)
^p
```

sympy [C] time = 1.64, size = 37, normalized size = 0.97

$$\frac{a^p x \Gamma\left(\frac{1}{n}\right) {}_2F_1\left(\begin{matrix}\frac{1}{n}, -p \\ 1 + \frac{1}{n}\end{matrix}\middle| \frac{b x^n e^{i\pi}}{a}\right)}{n\Gamma\left(1 + \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p,x)
```

```
[Out] a**p*x*gamma(1/n)*hyper((1/n, -p), (1 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n
*gamma(1 + 1/n))
```

3.2794 $\int (cx)^{-n} (a + bx^n)^p \, dx$

Optimal. Leaf size=53

$$\frac{(cx)^{1-n} (a + bx^n)^{p+1} \, {}_2F_1\left(1, p + \frac{1}{n}; \frac{1}{n}; -\frac{bx^n}{a}\right)}{ac(1-n)}$$

```
[Out] (c*x)^(1-n)*(a+b*x^n)^(1+p)*hypergeom([1, 1/n+p],[1/n],-b*x^n/a)/a/c/(1-n)
```

Rubi [A] time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {365, 364}

$$\frac{(cx)^{1-n} (a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} \, {}_2F_1\left(\frac{1}{n} - 1, -p; \frac{1}{n}; -\frac{bx^n}{a}\right)}{c(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^p/(c*x)^n,x]
```

```
[Out] ((c*x)^(1 - n)*(a + b*x^n)^p*Hypergeometric2F1[-1 + n^(-1), -p, n^(-1), -((
b*x^n)/a)])/(c*(1 - n)*(1 + (b*x^n)/a)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (cx)^{-n} (a + bx^n)^p \, dx &= \left((a + bx^n)^p \left(1 + \frac{bx^n}{a}\right)^{-p}\right) \int (cx)^{-n} \left(1 + \frac{bx^n}{a}\right)^p \, dx \\ &= \frac{(cx)^{1-n} (a + bx^n)^p \left(1 + \frac{bx^n}{a}\right)^{-p} \, {}_2F_1\left(-1 + \frac{1}{n}, -p; \frac{1}{n}; -\frac{bx^n}{a}\right)}{c(1-n)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 59, normalized size = 1.11

$$-\frac{x(cx)^{-n} (a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} \, {}_2F_1\left(\frac{1}{n} - 1, -p; \frac{1}{n}; -\frac{bx^n}{a}\right)}{n-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^p/(c*x)^n,x]
```

```
[Out] -((x*(a + b*x^n)^p*Hypergeometric2F1[-1 + n^(-1), -p, n^(-1), -((b*x^n)/a)]
)/((-1 + n)*(c*x)^n*(1 + (b*x^n)/a)^p))
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(bx^n+a)^p}{(cx)^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p/((c*x)^n),x, algorithm="fricas")
[Out] integral((b*x^n + a)^p/(c*x)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n+a)^p}{(cx)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p/((c*x)^n),x, algorithm="giac")
[Out] integrate((b*x^n + a)^p/(c*x)^n, x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int (cx)^{-n}\,(b\,x^n+a)^p\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^p/((c*x)^n),x)
[Out] int((b*x^n+a)^p/((c*x)^n),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n+a)^p}{(cx)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p/((c*x)^n),x, algorithm="maxima")
[Out] integrate((b*x^n + a)^p/(c*x)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x^n)^p}{(c\,x)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^p/(c*x)^n,x)
[Out] int((a + b*x^n)^p/(c*x)^n, x)
```

sympy [C] time = 7.58, size = 44, normalized size = 0.83

$$\frac{a^p c^{-n} x x^{-n} \Gamma\left(-1+\frac{1}{n}\right) {}_2F_1\left(\begin{matrix}-p, -1+\frac{1}{n} \\ \frac{1}{n}\end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(\frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p/((c*x)**n),x)
```

```
[Out] a**p*c**(-n)*x*x**(-n)*gamma(-1 + 1/n)*hyper((-p, -1 + 1/n), (1/n,), b*x**n
*exp_polar(I*pi)/a)/(n*gamma(1/n))
```

3.2795 $\int (cx)^{-2n} (a + bx^n)^p \, dx$

Optimal. Leaf size=56

$$\frac{(cx)^{1-2n} (a + bx^n)^{p+1} \, {}_2F_1\left(1, p + \frac{1}{n} - 1; \frac{1}{n} - 1; -\frac{bx^n}{a}\right)}{ac(1 - 2n)}$$

```
[Out] (c*x)^(1-2*n)*(a+b*x^n)^(1+p)*hypergeom([1, -1+1/n+p],[-1+1/n],-b*x^n/a)/a/
c/(1-2*n)
```

Rubi [A] time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {365, 364}

$$\frac{(cx)^{1-2n} (a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} \, {}_2F_1\left(\frac{1}{n} - 2, -p; \frac{1}{n} - 1; -\frac{bx^n}{a}\right)}{c(1 - 2n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^p/(c*x)^(2*n),x]
```

```
[Out] ((c*x)^(1 - 2*n)*(a + b*x^n)^p*Hypergeometric2F1[-2 + n^(-1), -p, -1 + n^(-
1), -((b*x^n)/a)])/(c*(1 - 2*n)*(1 + (b*x^n)/a)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (cx)^{-2n} (a + bx^n)^p \, dx &= \left((a + bx^n)^p \left(1 + \frac{bx^n}{a}\right)^{-p}\right) \int (cx)^{-2n} \left(1 + \frac{bx^n}{a}\right)^p \, dx \\ &= \frac{(cx)^{1-2n} (a + bx^n)^p \left(1 + \frac{bx^n}{a}\right)^{-p} \, {}_2F_1\left(-2 + \frac{1}{n}, -p; -1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{c(1 - 2n)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 63, normalized size = 1.12

$$-\frac{x(cx)^{-2n} (a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} \, {}_2F_1\left(\frac{1}{n} - 2, -p; \frac{1}{n} - 1; -\frac{bx^n}{a}\right)}{2n - 1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^p/(c*x)^(2*n),x]
```

```
[Out] -((x*(a + b*x^n)^p*Hypergeometric2F1[-2 + n^(-1), -p, -1 + n^(-1), -((b*x^n
)/a)])/((-1 + 2*n)*(c*x)^(2*n)*(1 + (b*x^n)/a)^p))
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(bx^n+a)^p}{(cx)^{2n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p/((c*x)^(2*n)),x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p/(c*x)^(2*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n+a)^p}{(cx)^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p/((c*x)^(2*n)),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p/(c*x)^(2*n), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int (cx)^{-2n}\,(b\,x^n+a)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^p/((c*x)^(2*n)),x)
```

```
[Out] int((b*x^n+a)^p/((c*x)^(2*n)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n+a)^p}{(cx)^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p/((c*x)^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p/(c*x)^(2*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x^n)^p}{(c\,x)^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^p/(c*x)^(2*n),x)
```

```
[Out] int((a + b*x^n)^p/(c*x)^(2*n), x)
```

sympy [C] time = 7.67, size = 51, normalized size = 0.91

$$\frac{a^p c^{-2n} x x^{-2n} \Gamma\left(-2+\frac{1}{n}\right) {}_2F_1\left(\begin{matrix}-p, -2+\frac{1}{n} \\ -1+\frac{1}{n}\end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(-1+\frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p/((c*x)**(2*n)),x)
```

```
[Out] a**p*c**(-2*n)*x*x**(-2*n)*gamma(-2 + 1/n)*hyper((-p, -2 + 1/n), (-1 + 1/n,
), b*x**n*exp_polar(I*pi)/a)/(n*gamma(-1 + 1/n))
```

3.2796 $\int (cx)^{-3n}\,(a+bx^n)^p\,dx$

Optimal. Leaf size=56

$$\frac{(cx)^{1-3n}\,(a+bx^n)^{p+1}\;{}_2F_1\left(1,p+\frac{1}{n}-2;\frac{1}{n}-2;-\frac{bx^n}{a}\right)}{ac(1-3n)}$$

```
[Out] (c*x)^(1-3*n)*(a+b*x^n)^(1+p)*hypergeom([1, -2+1/n+p],[-2+1/n],-b*x^n/a)/a/
c/(1-3*n)
```

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {365, 364}

$$\frac{(cx)^{1-3n}\,(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\;{}_2F_1\left(\frac{1}{n}-3,-p;\frac{1}{n}-2;-\frac{bx^n}{a}\right)}{c(1-3n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^p/(c*x)^(3*n),x]
```

```
[Out] ((c*x)^(1 - 3*n)*(a + b*x^n)^p*Hypergeometric2F1[-3 + n^(-1), -p, -2 + n^(-
1), -((b*x^n)/a)])/(c*(1 - 3*n)*(1 + (b*x^n)/a)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (cx)^{-3n}\,(a+bx^n)^p\,dx &= \left((a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\right)\int (cx)^{-3n}\left(1+\frac{bx^n}{a}\right)^p\,dx\\ &= \frac{(cx)^{1-3n}\,(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\;{}_2F_1\left(-3+\frac{1}{n},-p;-2+\frac{1}{n};-\frac{bx^n}{a}\right)}{c(1-3n)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 63, normalized size = 1.12

$$-\frac{x(cx)^{-3n}\,(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\;{}_2F_1\left(\frac{1}{n}-3,-p;\frac{1}{n}-2;-\frac{bx^n}{a}\right)}{3n-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^p/(c*x)^(3*n),x]
```

```
[Out] -((x*(a + b*x^n)^p*Hypergeometric2F1[-3 + n^(-1), -p, -2 + n^(-1), -((b*x^n
)/a)])/((-1 + 3*n)*(c*x)^(3*n)*(1 + (b*x^n)/a)^p))
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(bx^n+a)^p}{(cx)^{3n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p/((c*x)^(3*n)),x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p/(c*x)^(3*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n+a)^p}{(cx)^{3n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p/((c*x)^(3*n)),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p/(c*x)^(3*n), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int (cx)^{-3n}\,(b\,x^n+a)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^p/((c*x)^(3*n)),x)
```

```
[Out] int((b*x^n+a)^p/((c*x)^(3*n)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n+a)^p}{(cx)^{3n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p/((c*x)^(3*n)),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p/(c*x)^(3*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x^n)^p}{(c\,x)^{3n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^p/(c*x)^(3*n),x)
```

```
[Out] int((a + b*x^n)^p/(c*x)^(3*n), x)
```

sympy [C] time = 7.63, size = 51, normalized size = 0.91

$$\frac{a^p c^{-3n} x x^{-3n} \Gamma\left(-3+\frac{1}{n}\right) {}_2F_1\left(\begin{matrix}-p, -3+\frac{1}{n}\\ -2+\frac{1}{n}\end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(-2+\frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p/((c*x)**(3*n)),x)
```

```
[Out] a**p*c**(-3*n)*x*x**(-3*n)*gamma(-3 + 1/n)*hyper((-p, -3 + 1/n), (-2 + 1/n,
), b*x**n*exp_polar(I*pi)/a)/(n*gamma(-2 + 1/n))
```

3.2797 $\int (cx)^{-1-np} (a+bx^n)^p \, dx$

Optimal. Leaf size=63

$$-\frac{(cx)^{-np} (a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} {}_2F_1\left(-p,-p;1-p;-\frac{bx^n}{a}\right)}{cnp}$$

```
[Out] -(a+b*x^n)^p*hypergeom([-p, -p],[-p+1],-b*x^n/a)/c/n/p/((c*x)^(n*p))/((1+b*
x^n/a)^p)
```

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {365, 364}

$$-\frac{(cx)^{-np} (a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} {}_2F_1\left(-p,-p;1-p;-\frac{bx^n}{a}\right)}{cnp}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 - n*p)*(a + b*x^n)^p,x]
```

```
[Out] -(((a + b*x^n)^p*Hypergeometric2F1[-p, -p, 1 - p, -((b*x^n)/a)])/(c*n*p*(c*
x)^(n*p)*(1 + (b*x^n)/a)^p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (cx)^{-1-np} (a+bx^n)^p \, dx &= \left((a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p}\right) \int (cx)^{-1-np} \left(1+\frac{bx^n}{a}\right)^p \, dx \\ &= -\frac{(cx)^{-np} (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} {}_2F_1\left(-p,-p;1-p;-\frac{bx^n}{a}\right)}{cnp}\end{aligned}$$

Mathematica [A] time = 0.01, size = 63, normalized size = 1.00

$$-\frac{x(cx)^{-np-1} (a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} {}_2F_1\left(-p,-p;1-p;-\frac{bx^n}{a}\right)}{np}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 - n*p)*(a + b*x^n)^p,x]
```

```
[Out] -((x*(c*x)^(-1 - n*p)*(a + b*x^n)^p*Hypergeometric2F1[-p, -p, 1 - p, -((b*x
^n)/a)])/(n*p*(1 + (b*x^n)/a)^p))
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left((bx^n + a)^p (cx)^{-np-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-n*p-1)*(a+b*x^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p*(c*x)^(-n*p - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^p (cx)^{-np-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-n*p-1)*(a+b*x^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p*(c*x)^(-n*p - 1), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int (cx)^{-np-1} (b\,x^n + a)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-n*p-1)*(b*x^n+a)^p,x)
```

```
[Out] int((c*x)^(-n*p-1)*(b*x^n+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^p (cx)^{-np-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-n*p-1)*(a+b*x^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p*(c*x)^(-n*p - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a + b\,x^n)^p}{(c\,x)^{n\,p+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^p/(c*x)^(n*p + 1),x)
```

```
[Out] int((a + b*x^n)^p/(c*x)^(n*p + 1), x)
```

sympy [C] time = 119.28, size = 44, normalized size = 0.70

$$\frac{a^p c^{-np} x^{-np} \Gamma\left(-p\right) {}_2F_1\left(\begin{matrix} -p, -p \\ 1-p \end{matrix} \middle| \frac{b x^n e^{i\pi}}{a}\right)}{cn\Gamma\left(1-p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-n*p-1)*(a+b*x**n)**p,x)
```

```
[Out] a**p*c**(-n*p)*x**(-n*p)*gamma(-p)*hyper((-p, -p), (1 - p,), b*x**n*exp_pol
ar(I*pi)/a)/(c*n*gamma(1 - p))
```

3.2798 $\int (cx)^{-1-n-np} (a+bx^n)^p \, dx$

Optimal. Leaf size=37

$$-\frac{(cx)^{-n(p+1)} (a+bx^n)^{p+1}}{acn(p+1)}$$

```
[Out] -(a+b*x^n)^(1+p)/a/c/n/(1+p)/((c*x)^(n*(1+p)))
```

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {264}

$$-\frac{(cx)^{-n(p+1)} (a+bx^n)^{p+1}}{acn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 - n - n*p)*(a + b*x^n)^p,x]
```

```
[Out] -((a + b*x^n)^(1 + p)/(a*c*n*(1 + p)*(c*x)^(n*(1 + p))))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int (cx)^{-1-n-np} (a+bx^n)^p \, dx = -\frac{(cx)^{-n(1+p)} (a+bx^n)^{1+p}}{acn(1+p)}$$

Mathematica [A] time = 0.03, size = 37, normalized size = 1.00

$$-\frac{x(cx)^{-n(p+1)-1} (a+bx^n)^{p+1}}{an(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 - n - n*p)*(a + b*x^n)^p,x]
```

```
[Out] -((x*(c*x)^(-1 - n*(1 + p))*(a + b*x^n)^(1 + p))/(a*n*(1 + p)))
```

fricas [A] time = 0.57, size = 75, normalized size = 2.03

$$-\frac{\left(bxx^n e^{\left(-\left(np+n+1\right)\log(c)-\left(np+n+1\right)\log(x)\right)} + axe^{\left(-\left(np+n+1\right)\log(c)-\left(np+n+1\right)\log(x)\right)}\right)(bx^n+a)^p}{anp+an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-n*p-n-1)*(a+b*x^n)^p,x, algorithm="fricas")
```

```
[Out] -(b*x*x^n*e^(-(n*p + n + 1)*log(c) - (n*p + n + 1)*log(x)) + a*x*e^(-(n*p +
 n + 1)*log(c) - (n*p + n + 1)*log(x)))*(b*x^n + a)^p/(a*n*p + a*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n+a)^p (cx)^{-np-n-1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-n*p-n-1)*(a+b*x^n)^p,x, algorithm="giac")
[Out] integrate((b*x^n + a)^p*(c*x)^(-n*p - n - 1), x)
```

maple **[F]** time = 0.22, size = 0, normalized size = 0.00

$$\int (cx)^{-np-n-1} (b\,x^n + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-n*p-n-1)*(b*x^n+a)^p,x)
[Out] int((c*x)^(-n*p-n-1)*(b*x^n+a)^p,x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^p \, (cx)^{-np-n-1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-n*p-n-1)*(a+b*x^n)^p,x, algorithm="maxima")
[Out] integrate((b*x^n + a)^p*(c*x)^(-n*p - n - 1), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a + b\,x^n)^p}{(c\,x)^{n+n\,p+1}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^p/(c*x)^(n + n*p + 1),x)
[Out] int((a + b*x^n)^p/(c*x)^(n + n*p + 1), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-n*p-n-1)*(a+b*x**n)**p,x)
[Out] Timed out
```

3.2799 $\int (cx)^{-1-2n-np} (a+bx^n)^p \, dx$

Optimal. Leaf size=79

$$\frac{(cx)^{-n(p+2)} (a+bx^n)^{p+2}}{a^2 cn(p+1)(p+2)} - \frac{(cx)^{-n(p+2)} (a+bx^n)^{p+1}}{acn(p+1)}$$

```
[Out] -(a+b*x^n)^(1+p)/a/c/n/(1+p)/((c*x)^(n*(2+p)))+(a+b*x^n)^(2+p)/a^2/c/n/(1+p
)/(2+p)/((c*x)^(n*(2+p)))
```

Rubi [A] time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {273, 264}

$$\frac{(cx)^{-n(p+2)} (a+bx^n)^{p+2}}{a^2 cn(p+1)(p+2)} - \frac{(cx)^{-n(p+2)} (a+bx^n)^{p+1}}{acn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 - 2*n - n*p)*(a + b*x^n)^p,x]
```

```
[Out] -((a + b*x^n)^(1 + p)/(a*c*n*(1 + p)*(c*x)^(n*(2 + p)))) + (a + b*x^n)^(2 +
 p)/(a^2*c*n*(1 + p)*(2 + p)*(c*x)^(n*(2 + p)))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int (cx)^{-1-2n-np} (a+bx^n)^p \, dx &= -\frac{(cx)^{-n(2+p)} (a+bx^n)^{1+p}}{acn(1+p)} - \frac{\int (cx)^{-1-2n-np} (a+bx^n)^{1+p} \, dx}{a(1+p)} \\ &= -\frac{(cx)^{-n(2+p)} (a+bx^n)^{1+p}}{acn(1+p)} + \frac{(cx)^{-n(2+p)} (a+bx^n)^{2+p}}{a^2 cn(1+p)(2+p)}\end{aligned}$$

Mathematica [C] time = 0.05, size = 69, normalized size = 0.87

$$-\frac{x(cx)^{-n(p+2)-1} (a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \, {}_2F_1\left(-p-2,-p;-p-1;-\frac{bx^n}{a}\right)}{n(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 - 2*n - n*p)*(a + b*x^n)^p,x]
```

```
[Out] -((x*(c*x)^(-1 - n*(2 + p))*(a + b*x^n)^p*Hypergeometric2F1[-2 - p, -p, -1
 - p, -((b*x^n)/a)])/(n*(2 + p)*(1 + (b*x^n)/a)^p))
```

fricas [A] time = 0.54, size = 144, normalized size = 1.82

$$-\frac{\left(abpxx^n e^{\left(-\left(np+2n+1\right)\log(c)-\left(np+2n+1\right)\log(x)\right)} - b^2xx^{2n}e^{\left(-\left(np+2n+1\right)\log(c)-\left(np+2n+1\right)\log(x)\right)} + \left(a^2p+a^2\right)xe^{\left(-\left(np+2n+1\right)\log}\right.}{a^2np^2+3a^2np+2a^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-n*p-2*n-1)*(a+b*x^n)^p,x, algorithm="fricas")
```

```
[Out] -(a*b*p*x*x^n*e^(-(n*p + 2*n + 1)*log(c) - (n*p + 2*n + 1)*log(x)) - b^2*x*
x^(2*n)*e^(-(n*p + 2*n + 1)*log(c) - (n*p + 2*n + 1)*log(x)) + (a^2*p + a^2
)*x*e^(-(n*p + 2*n + 1)*log(c) - (n*p + 2*n + 1)*log(x)))*(b*x^n + a)^p/(a^
2*n*p^2 + 3*a^2*n*p + 2*a^2*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^p \, (cx)^{-np-2n-1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-n*p-2*n-1)*(a+b*x^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p*(c*x)^(-n*p - 2*n - 1), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int (cx)^{-np-2n-1} \, (b\,x^n + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-n*p-2*n-1)*(b*x^n+a)^p,x)
```

```
[Out] int((c*x)^(-n*p-2*n-1)*(b*x^n+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^p \, (cx)^{-np-2n-1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-n*p-2*n-1)*(a+b*x^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p*(c*x)^(-n*p - 2*n - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,x^n)^p}{(c\,x)^{2\,n+n\,p+1}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^p/(c*x)^(2*n + n*p + 1),x)
```

```
[Out] int((a + b*x^n)^p/(c*x)^(2*n + n*p + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-n*p-2*n-1)*(a+b*x**n)**p,x)
```

```
[Out] Timed out
```

3.2800 $\int (cx)^{-1-3n-np}\,(a+bx^n)^p\,dx$

Optimal. Leaf size=127

$$-\frac{2(cx)^{-n(p+3)}\,(a+bx^n)^{p+3}}{a^3cn(p+1)(p+2)(p+3)}+\frac{2(cx)^{-n(p+3)}\,(a+bx^n)^{p+2}}{a^2cn(p+1)(p+2)}-\frac{(cx)^{-n(p+3)}\,(a+bx^n)^{p+1}}{acn(p+1)}$$

```
[Out] -(a+b*x^n)^(1+p)/a/c/n/(1+p)/((c*x)^(n*(3+p)))+2*(a+b*x^n)^(2+p)/a^2/c/n/(1
+p)/(2+p)/((c*x)^(n*(3+p)))-2*(a+b*x^n)^(3+p)/a^3/c/n/(1+p)/(2+p)/(3+p)/((c
*x)^(n*(3+p)))
```

Rubi [A] time = 0.07, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {273, 264}

$$\frac{2(cx)^{-n(p+3)}\,(a+bx^n)^{p+2}}{a^2cn(p+1)(p+2)}-\frac{2(cx)^{-n(p+3)}\,(a+bx^n)^{p+3}}{a^3cn(p+1)(p+2)(p+3)}-\frac{(cx)^{-n(p+3)}\,(a+bx^n)^{p+1}}{acn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 - 3*n - n*p)*(a + b*x^n)^p,x]
```

```
[Out] -((a + b*x^n)^(1 + p)/(a*c*n*(1 + p)*(c*x)^(n*(3 + p)))) + (2*(a + b*x^n)^(
2 + p))/(a^2*c*n*(1 + p)*(2 + p)*(c*x)^(n*(3 + p))) - (2*(a + b*x^n)^(3 + p
))/(a^3*c*n*(1 + p)*(2 + p)*(3 + p)*(c*x)^(n*(3 + p)))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int (cx)^{-1-3n-np}\,(a+bx^n)^p\,dx &= -\frac{(cx)^{-n(3+p)}\,(a+bx^n)^{1+p}}{acn(1+p)}-\frac{2\int (cx)^{-1-3n-np}\,(a+bx^n)^{1+p}\,dx}{a(1+p)}\\ &= -\frac{(cx)^{-n(3+p)}\,(a+bx^n)^{1+p}}{acn(1+p)}+\frac{2(cx)^{-n(3+p)}\,(a+bx^n)^{2+p}}{a^2cn(1+p)(2+p)}+\frac{2\int (cx)^{-1-3n-np}\,(a+bx\\ &= -\frac{(cx)^{-n(3+p)}\,(a+bx^n)^{1+p}}{acn(1+p)}+\frac{2(cx)^{-n(3+p)}\,(a+bx^n)^{2+p}}{a^2cn(1+p)(2+p)}-\frac{2(cx)^{-n(3+p)}\,(a+bx^n)^3}{a^3cn(1+p)(2+p)(3+\end{aligned}$$

Mathematica [C] time = 0.04, size = 69, normalized size = 0.54

$$-\frac{x(cx)^{-n(p+3)-1}\,(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\,{}_2F_1\left(-p-3,-p;-p-2;-\frac{bx^n}{a}\right)}{n(p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 - 3*n - n*p)*(a + b*x^n)^p,x]
```

```
[Out] -((x*(c*x)^(-1 - n*(3 + p))*(a + b*x^n)^p*Hypergeometric2F1[-3 - p, -p, -2
- p, -((b*x^n)/a)])/(n*(3 + p)*(1 + (b*x^n)/a)^p))
```

fricas [A] time = 0.59, size = 215, normalized size = 1.69

$$\frac{\left(2\,ab^2pxx^{2\,n}e^{\left(-\left(np+3\,n+1\right)\log(c)-\left(np+3\,n+1\right)\log(x)\right)}-2\,b^3xx^{3\,n}e^{\left(-\left(np+3\,n+1\right)\log(c)-\left(np+3\,n+1\right)\log(x)\right)}-\left(a^2bp^2+a^2bp\right)xx^ne^{(-}\right.}{a^3np^3+6\,a^3np^2+11\,a^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-n*p-3*n-1)*(a+b*x^n)^p,x, algorithm="fricas")
```

```
[Out] (2*a*b^2*p*x*x^(2*n)*e^(-(n*p + 3*n + 1)*log(c) - (n*p + 3*n + 1)*log(x)) -
 2*b^3*x*x^(3*n)*e^(-(n*p + 3*n + 1)*log(c) - (n*p + 3*n + 1)*log(x)) - (a^
2*b*p^2 + a^2*b*p)*x*x^n*e^(-(n*p + 3*n + 1)*log(c) - (n*p + 3*n + 1)*log(x
)) - (a^3*p^2 + 3*a^3*p + 2*a^3)*x*e^(-(n*p + 3*n + 1)*log(c) - (n*p + 3*n
 + 1)*log(x)))*(b*x^n + a)^p/(a^3*n*p^3 + 6*a^3*n*p^2 + 11*a^3*n*p + 6*a^3*n
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n+a)^p\,(cx)^{-np-3\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-n*p-3*n-1)*(a+b*x^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p*(c*x)^(-n*p - 3*n - 1), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int (cx)^{-np-3n-1}\,(b\,x^n+a)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-n*p-3*n-1)*(b*x^n+a)^p,x)
```

```
[Out] int((c*x)^(-n*p-3*n-1)*(b*x^n+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n+a)^p\,(cx)^{-np-3\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-n*p-3*n-1)*(a+b*x^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p*(c*x)^(-n*p - 3*n - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x^n)^p}{(c\,x)^{3\,n+n\,p+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^p/(c*x)^(3*n + n*p + 1),x)
```

```
[Out] int((a + b*x^n)^p/(c*x)^(3*n + n*p + 1), x)
```

sympy [**F**(**-1**)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-n*p-3*n-1)*(a+b*x**n)**p,x)
[Out] Timed out
```

3.2801 $\int (cx)^{-1-4n-np} (a+bx^n)^p \, dx$

Optimal. Leaf size=179

$$\frac{6(cx)^{-n(p+4)}(a+bx^n)^{p+4}}{a^4cn(p+1)(p+2)(p+3)(p+4)}-\frac{6(cx)^{-n(p+4)}(a+bx^n)^{p+3}}{a^3cn(p+1)(p+2)(p+3)}+\frac{3(cx)^{-n(p+4)}(a+bx^n)^{p+2}}{a^2cn(p+1)(p+2)}-\frac{(cx)^{-n(p+4)}(a+bx^n)^{p+1}}{acn(p+1)}$$

```
[Out] -(a+b*x^n)^(1+p)/a/c/n/(1+p)/((c*x)^(n*(4+p)))+3*(a+b*x^n)^(2+p)/a^2/c/n/(1
+p)/(2+p)/((c*x)^(n*(4+p)))-6*(a+b*x^n)^(3+p)/a^3/c/n/(1+p)/(2+p)/(3+p)/((c
*x)^(n*(4+p)))+6*(a+b*x^n)^(4+p)/a^4/c/n/(p^2+3*p+2)/(p^2+7*p+12)/((c*x)^(n
*(4+p)))
```

Rubi [A] time = 0.13, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {273, 264}

$$\frac{3(cx)^{-n(p+4)}(a+bx^n)^{p+2}}{a^2cn(p+1)(p+2)}-\frac{6(cx)^{-n(p+4)}(a+bx^n)^{p+3}}{a^3cn(p+1)(p+2)(p+3)}+\frac{6(cx)^{-n(p+4)}(a+bx^n)^{p+4}}{a^4cn(p+1)(p+2)(p+3)(p+4)}-\frac{(cx)^{-n(p+4)}(a+bx^n)^{p+1}}{acn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 - 4*n - n*p)*(a + b*x^n)^p,x]
```

```
[Out] -((a + b*x^n)^(1 + p)/(a*c*n*(1 + p)*(c*x)^(n*(4 + p)))) + (3*(a + b*x^n)^(
2 + p))/(a^2*c*n*(1 + p)*(2 + p)*(c*x)^(n*(4 + p))) - (6*(a + b*x^n)^(3 + p
))/(a^3*c*n*(1 + p)*(2 + p)*(3 + p)*(c*x)^(n*(4 + p))) + (6*(a + b*x^n)^(4
+ p))/(a^4*c*n*(1 + p)*(2 + p)*(3 + p)*(4 + p)*(c*x)^(n*(4 + p)))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int (cx)^{-1-4n-np}(a+bx^n)^p\,dx &= -\frac{(cx)^{-n(4+p)}(a+bx^n)^{1+p}}{acn(1+p)}-\frac{3\int (cx)^{-1-4n-np}(a+bx^n)^{1+p}\,dx}{a(1+p)}\\ &= -\frac{(cx)^{-n(4+p)}(a+bx^n)^{1+p}}{acn(1+p)}+\frac{3(cx)^{-n(4+p)}(a+bx^n)^{2+p}}{a^2cn(1+p)(2+p)}+\frac{6\int (cx)^{-1-4n-np}(a+bx^n)^{2}}{a^2(1+p)(2+p)}\\ &= -\frac{(cx)^{-n(4+p)}(a+bx^n)^{1+p}}{acn(1+p)}+\frac{3(cx)^{-n(4+p)}(a+bx^n)^{2+p}}{a^2cn(1+p)(2+p)}-\frac{6(cx)^{-n(4+p)}(a+bx^n)^{3+p}}{a^3cn(1+p)(2+p)(3+p)}\\ &= -\frac{(cx)^{-n(4+p)}(a+bx^n)^{1+p}}{acn(1+p)}+\frac{3(cx)^{-n(4+p)}(a+bx^n)^{2+p}}{a^2cn(1+p)(2+p)}-\frac{6(cx)^{-n(4+p)}(a+bx^n)^{3+p}}{a^3cn(1+p)(2+p)(3+p)}\end{aligned}$$

Mathematica [C] time = 0.03, size = 69, normalized size = 0.39

$$-\frac{x(cx)^{-n(p+4)-1}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\,{}_2F_1\left(-p-4,-p;-p-3;-\frac{bx^n}{a}\right)}{n(p+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 - 4*n - n*p)*(a + b*x^n)^p,x]
```

```
[Out] -((x*(c*x)^(-1 - n*(4 + p))*(a + b*x^n)^p*Hypergeometric2F1[-4 - p, -p, -3
- p, -((b*x^n)/a)])/(n*(4 + p)*(1 + (b*x^n)/a)^p))
```

fricas [A] time = 0.77, size = 294, normalized size = 1.64

$$-\frac{\left(6\,ab^3pxx^{3\,n}e^{\left(-\left(np+4\,n+1\right)\log(c)-\left(np+4\,n+1\right)\log(x)\right)}-6\,b^4xx^{4\,n}e^{\left(-\left(np+4\,n+1\right)\log(c)-\left(np+4\,n+1\right)\log(x)\right)}-3\left(a^2b^2p^2+a^2b^2p\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-n*p-4*n-1)*(a+b*x^n)^p,x, algorithm="fricas")
```

```
[Out] -(6*a*b^3*p*x*x^(3*n)*e^(-(n*p + 4*n + 1)*log(c) - (n*p + 4*n + 1)*log(x))
- 6*b^4*x*x^(4*n)*e^(-(n*p + 4*n + 1)*log(c) - (n*p + 4*n + 1)*log(x)) - 3*
(a^2*b^2*p^2 + a^2*b^2*p)*x*x^(2*n)*e^(-(n*p + 4*n + 1)*log(c) - (n*p + 4*n
 + 1)*log(x)) + (a^3*b*p^3 + 3*a^3*b*p^2 + 2*a^3*b*p)*x*x^n*e^(-(n*p + 4*n
+ 1)*log(c) - (n*p + 4*n + 1)*log(x)) + (a^4*p^3 + 6*a^4*p^2 + 11*a^4*p + 6
*a^4)*x*e^(-(n*p + 4*n + 1)*log(c) - (n*p + 4*n + 1)*log(x)))*(b*x^n + a)^p
/(a^4*n*p^4 + 10*a^4*n*p^3 + 35*a^4*n*p^2 + 50*a^4*n*p + 24*a^4*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^p\,(cx)^{-np-4\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-n*p-4*n-1)*(a+b*x^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p*(c*x)^(-n*p - 4*n - 1), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int (cx)^{-np-4n-1}\,(b\,x^n + a)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-n*p-4*n-1)*(b*x^n+a)^p,x)
```

```
[Out] int((c*x)^(-n*p-4*n-1)*(b*x^n+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^p\,(cx)^{-np-4\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-n*p-4*n-1)*(a+b*x^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p*(c*x)^(-n*p - 4*n - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,x^n)^p}{(c\,x)^{4\,n+n\,p+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^p/(c*x)^(4*n + n*p + 1),x)
```

```
[Out] int((a + b*x^n)^p/(c*x)^(4*n + n*p + 1), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-n*p-4*n-1)*(a+b*x**n)**p,x)
```

```
[Out] Timed out
```

3.2802 $\int \left(c(a+bx)^2\right)^{5/2}\,dx$

Optimal. Leaf size=30

$$\frac{c^2(a+bx)^5\sqrt{c(a+bx)^2}}{6b}$$

```
[Out] 1/6*c^2*(b*x+a)^5*(c*(b*x+a)^2)^(1/2)/b
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 15, 30}

$$\frac{c^2(a+bx)^5\sqrt{c(a+bx)^2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x)^2)^(5/2),x]
```

```
[Out] (c^2*(a + b*x)^5*Sqrt[c*(a + b*x)^2])/(6*b)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \left(c(a+bx)^2\right)^{5/2}\,dx &= \frac{\text{Subst}\left(\int \left(cx^2\right)^{5/2}\,dx, x, a+bx\right)}{b}\\ &= \frac{\left(c^2\sqrt{c(a+bx)^2}\right)\text{Subst}\left(\int x^5\,dx, x, a+bx\right)}{b(a+bx)}\\ &= \frac{c^2(a+bx)^5\sqrt{c(a+bx)^2}}{6b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 0.83

$$\frac{(a+bx)\left(c(a+bx)^2\right)^{5/2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x)^2)^(5/2),x]
```

```
[Out] ((a + b*x)*(c*(a + b*x)^2)^(5/2))/(6*b)
```

fricas [B] time = 0.93, size = 103, normalized size = 3.43

$$\frac{\left(b^5c^2x^6+6\,ab^4c^2x^5+15\,a^2b^3c^2x^4+20\,a^3b^2c^2x^3+15\,a^4bc^2x^2+6\,a^5c^2x\right)\sqrt{b^2cx^2+2\,abcx+a^2c}}{6\,(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x+a)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(b^5*c^2*x^6 + 6*a*b^4*c^2*x^5 + 15*a^2*b^3*c^2*x^4 + 20*a^3*b^2*c^2*x^
3 + 15*a^4*b*c^2*x^2 + 6*a^5*c^2*x)*sqrt(b^2*c*x^2 + 2*a*b*c*x + a^2*c)/(b*
x + a)
```

giac [B] time = 0.17, size = 98, normalized size = 3.27

$$\frac{1}{6}\left(3\left(bx^2+2\,ax\right)a^4c^2\mathrm{sgn}\left(bx+a\right)+\frac{a^6c^2\mathrm{sgn}\left(bx+a\right)}{b}+3\left(bx^2+2\,ax\right)^2a^2bc^2\mathrm{sgn}\left(bx+a\right)+\left(bx^2+2\,ax\right)^3b^2c^2\mathrm{sgr}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x+a)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*(3*(b*x^2 + 2*a*x)*a^4*c^2*sgn(b*x + a) + a^6*c^2*sgn(b*x + a)/b + 3*(b
*x^2 + 2*a*x)^2*a^2*b*c^2*sgn(b*x + a) + (b*x^2 + 2*a*x)^3*b^2*c^2*sgn(b*x
+ a))*sqrt(c)
```

maple [B] time = 0.01, size = 73, normalized size = 2.43

$$\frac{\left(b^5x^5+6a\,b^4x^4+15a^2b^3x^3+20a^3b^2x^2+15a^4bx+6a^5\right)\left(\left(bx+a\right)^2c\right)^{\frac{5}{2}}x}{6\,\left(bx+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(b*x+a)^2)^(5/2),x)
```

```
[Out] 1/6*x*(b^5*x^5+6*a*b^4*x^4+15*a^2*b^3*x^3+20*a^3*b^2*x^2+15*a^4*b*x+6*a^5)*
(c*(b*x+a)^2)^(5/2)/(b*x+a)^5
```

maxima [B] time = 0.59, size = 54, normalized size = 1.80

$$\frac{1}{6}\left(b^2cx^2+2\,abcx+a^2c\right)^{\frac{5}{2}}x+\frac{\left(b^2cx^2+2\,abcx+a^2c\right)^{\frac{5}{2}}a}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x+a)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(b^2*c*x^2 + 2*a*b*c*x + a^2*c)^(5/2)*x + 1/6*(b^2*c*x^2 + 2*a*b*c*x +
a^2*c)^(5/2)*a/b
```

mupad [B] time = 1.26, size = 36, normalized size = 1.20

$$\frac{\left(x\,b^2+a\,b\right)\left(c\,a^2+2\,c\,a\,b\,x+c\,b^2\,x^2\right)^{5/2}}{6\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x)^2)^(5/2),x)
```

```
[Out] ((a*b + b^2*x)*(a^2*c + b^2*c*x^2 + 2*a*b*c*x)^(5/2))/(6*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\,(a+bx)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x+a)**2)**(5/2),x)
```

```
[Out] Integral((c*(a + b*x)**2)**(5/2), x)
```

3.2803 $\int \left(c(a+bx)^2\right)^{3/2}\,dx$

Optimal. Leaf size=28

$$\frac{c(a+bx)^3\sqrt{c(a+bx)^2}}{4b}$$

```
[Out] 1/4*c*(b*x+a)^3*(c*(b*x+a)^2)^(1/2)/b
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 15, 30}

$$\frac{c(a+bx)^3\sqrt{c(a+bx)^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x)^2)^(3/2),x]
```

```
[Out] (c*(a + b*x)^3*Sqrt[c*(a + b*x)^2])/(4*b)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \left(c(a+bx)^2\right)^{3/2}\,dx &= \frac{\text{Subst}\left(\int \left(cx^2\right)^{3/2}\,dx, x, a+bx\right)}{b}\\ &= \frac{\left(c\sqrt{c(a+bx)^2}\right)\text{Subst}\left(\int x^3\,dx, x, a+bx\right)}{b(a+bx)}\\ &= \frac{c(a+bx)^3\sqrt{c(a+bx)^2}}{4b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.89

$$\frac{(a+bx)\left(c(a+bx)^2\right)^{3/2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x)^2)^(3/2),x]
```

```
[Out] ((a + b*x)*(c*(a + b*x)^2)^(3/2))/(4*b)
```

fricas [B] time = 0.81, size = 67, normalized size = 2.39

$$\frac{\left(b^3cx^4 + 4\,ab^2cx^3 + 6\,a^2bcx^2 + 4\,a^3cx\right)\sqrt{b^2cx^2 + 2\,abcx + a^2c}}{4\,(bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*c*x^4 + 4*a*b^2*c*x^3 + 6*a^2*b*c*x^2 + 4*a^3*c*x)*sqrt(b^2*c*x^2
+ 2*a*b*c*x + a^2*c)/(b*x + a)
```

giac [A] time = 0.15, size = 21, normalized size = 0.75

$$\frac{(bx + a)^4 c^{\frac{3}{2}} \operatorname{sgn}(bx + a)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(b*x + a)^4*c^(3/2)*sgn(b*x + a)/b
```

maple [B] time = 0.00, size = 51, normalized size = 1.82

$$\frac{\left(b^3x^3 + 4a\,b^2x^2 + 6a^2bx + 4a^3\right)\left((bx + a)^2\,c\right)^{\frac{3}{2}}\,x}{4\,(bx + a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2*c)^(3/2),x)
```

```
[Out] 1/4*x*(b^3*x^3+4*a*b^2*x^2+6*a^2*b*x+4*a^3)*((b*x+a)^2*c)^(3/2)/(b*x+a)^3
```

maxima [B] time = 0.53, size = 54, normalized size = 1.93

$$\frac{1}{4}\left(b^2cx^2 + 2\,abcx + a^2c\right)^{\frac{3}{2}}x + \frac{\left(b^2cx^2 + 2\,abcx + a^2c\right)^{\frac{3}{2}}a}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(b^2*c*x^2 + 2*a*b*c*x + a^2*c)^(3/2)*x + 1/4*(b^2*c*x^2 + 2*a*b*c*x +
a^2*c)^(3/2)*a/b
```

mupad [B] time = 1.20, size = 36, normalized size = 1.29

$$\frac{\left(x\,b^2 + a\,b\right)\left(c\,a^2 + 2\,c\,a\,b\,x + c\,b^2\,x^2\right)^{3/2}}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x)^2)^(3/2),x)
```

```
[Out] ((a*b + b^2*x)*(a^2*c + b^2*c*x^2 + 2*a*b*c*x)^(3/2))/(4*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\,(a + bx)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x+a)**2)**(3/2),x)
[Out] Integral((c*(a + b*x)**2)**(3/2), x)
```

3.2804 $\int \sqrt{c(a+bx)^2}\,dx$

Optimal. Leaf size=25

$$\frac{(a+bx)\sqrt{c(a+bx)^2}}{2b}$$

```
[Out] 1/2*(b*x+a)*(c*(b*x+a)^2)^(1/2)/b
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 15, 30}

$$\frac{(a+bx)\sqrt{c(a+bx)^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*(a + b*x)^2],x]
```

```
[Out] ((a + b*x)*Sqrt[c*(a + b*x)^2])/(2*b)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{c(a+bx)^2}\,dx &= \frac{\text{Subst}\left(\int \sqrt{cx^2}\,dx, x, a+bx\right)}{b}\\ &= \frac{\sqrt{c(a+bx)^2}\ \text{Subst}(\int x\,dx, x, a+bx)}{b(a+bx)}\\ &= \frac{(a+bx)\sqrt{c(a+bx)^2}}{2b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.24

$$\frac{cx(a+bx)(2a+bx)}{2\sqrt{c(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*(a + b*x)^2],x]
```

```
[Out] (c*x*(a + b*x)*(2*a + b*x))/(2*Sqrt[c*(a + b*x)^2])
```

fricas [A] time = 0.55, size = 41, normalized size = 1.64

$$\frac{\sqrt{b^2cx^2 + 2\,abcx + a^2c}\left(bx^2 + 2\,ax\right)}{2\,(bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(b^2*c*x^2 + 2*a*b*c*x + a^2*c)*(b*x^2 + 2*a*x)/(b*x + a)
```

giac [A] time = 0.17, size = 36, normalized size = 1.44

$$\frac{1}{2}\left(\left(bx^2 + 2\,ax\right)\mathrm{sgn}\left(bx + a\right) + \frac{a^2\mathrm{sgn}\left(bx + a\right)}{b}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*((b*x^2 + 2*a*x)*sgn(b*x + a) + a^2*sgn(b*x + a)/b)*sqrt(c)
```

maple [A] time = 0.00, size = 29, normalized size = 1.16

$$\frac{(bx + 2a)\sqrt{(bx + a)^2\,c}\;x}{2bx + 2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2*c)^(1/2),x)
```

```
[Out] 1/2*x*(b*x+2*a)*((b*x+a)^2*c)^(1/2)/(b*x+a)
```

maxima [B] time = 0.64, size = 54, normalized size = 2.16

$$\frac{1}{2}\sqrt{b^2cx^2 + 2\,abcx + a^2c}\,x + \frac{\sqrt{b^2cx^2 + 2\,abcx + a^2c}\,a}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b^2*c*x^2 + 2*a*b*c*x + a^2*c)*x + 1/2*sqrt(b^2*c*x^2 + 2*a*b*c*x
+ a^2*c)*a/b
```

mupad [B] time = 1.47, size = 21, normalized size = 0.84

$$\frac{\sqrt{c\,(a + b\,x)^2}\;\left(a + b\,x\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x)^2)^(1/2),x)
```

```
[Out] ((c*(a + b*x)^2)^(1/2)*(a + b*x))/(2*b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c\,(a + bx)^2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x+a)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(c*(a + b*x)**2), x)
```

3.2805 $\int \frac{1}{\sqrt{c(a+bx)^2}}\,dx$

Optimal. Leaf size=28

$$\frac{(a+bx)\log(a+bx)}{b\sqrt{c(a+bx)^2}}$$

```
[Out] (b*x+a)*ln(b*x+a)/b/(c*(b*x+a)^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 15, 29}

$$\frac{(a+bx)\log(a+bx)}{b\sqrt{c(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c*(a + b*x)^2],x]
```

```
[Out] ((a + b*x)*Log[a + b*x])/(b*Sqrt[c*(a + b*x)^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{c(a+bx)^2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{cx^2}}\,dx, x, a+bx\right)}{b}\\ &= \frac{(a+bx)\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, a+bx\right)}{b\sqrt{c(a+bx)^2}}\\ &= \frac{(a+bx)\log(a+bx)}{b\sqrt{c(a+bx)^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$\frac{(a+bx)\log(a+bx)}{b\sqrt{c(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c*(a + b*x)^2],x]
```

```
[Out] ((a + b*x)*Log[a + b*x])/(b*Sqrt[c*(a + b*x)^2])
```

fricas [A] time = 0.81, size = 42, normalized size = 1.50

$$\frac{\sqrt{b^2cx^2 + 2\,abcx + a^2c}\,\log(bx + a)}{b^2cx + abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b*x+a)^2)^(1/2),x, algorithm="fricas")
[Out] sqrt(b^2*c*x^2 + 2*a*b*c*x + a^2*c)*log(b*x + a)/(b^2*c*x + a*b*c)
```

giac [A] time = 0.15, size = 34, normalized size = 1.21

$$\frac{\log\left(|bx + a|\sqrt{|c|}\,\left|\operatorname{sgn}(bx + a)\right|\right)}{b\sqrt{c}\operatorname{sgn}(bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b*x+a)^2)^(1/2),x, algorithm="giac")
[Out] log(abs(b*x + a)*sqrt(abs(c))*abs(sgn(b*x + a)))/(b*sqrt(c)*sgn(b*x + a))
```

maple [A] time = 0.01, size = 27, normalized size = 0.96

$$\frac{(bx + a)\ln(bx + a)}{\sqrt{(bx + a)^2\,c}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x+a)^2*c)^(1/2),x)
[Out] (b*x+a)*ln(b*x+a)/b/((b*x+a)^2*c)^(1/2)
```

maxima [A] time = 0.53, size = 15, normalized size = 0.54

$$\frac{\log\left(x + \frac{a}{b}\right)}{b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b*x+a)^2)^(1/2),x, algorithm="maxima")
[Out] log(x + a/b)/(b*sqrt(c))
```

mupad [B] time = 1.33, size = 29, normalized size = 1.04

$$\frac{\ln\left(\sqrt{c}\;\;(a + b\,x) + \sqrt{c\,(a + b\,x)^2}\right)}{b\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*(a + b*x)^2)^(1/2),x)
[Out] log(c^(1/2)*(a + b*x) + (c*(a + b*x)^2)^(1/2))/(b*c^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\,(a + bx)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b*x+a)**2)**(1/2),x)
[Out] Integral(1/sqrt(c*(a + b*x)**2), x)
```

3.2806 $\int \frac{1}{\left(c(a+bx)^2\right)^{3/2}}\,dx$

Optimal. Leaf size=30

$$-\frac{1}{2bc(a+bx)\sqrt{c(a+bx)^2}}$$

```
[Out] -1/2/b/c/(b*x+a)/(c*(b*x+a)^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 15, 30}

$$-\frac{1}{2bc(a+bx)\sqrt{c(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x)^2)^(-3/2),x]
```

```
[Out] -1/(2*b*c*(a + b*x)*Sqrt[c*(a + b*x)^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(c(a+bx)^2\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\left(cx^2\right)^{3/2}}\,dx, x, a+bx\right)}{b}\\ &= \frac{(a+bx)\,\text{Subst}\left(\int \frac{1}{x^3}\,dx, x, a+bx\right)}{bc\sqrt{c(a+bx)^2}}\\ &= -\frac{1}{2bc(a+bx)\sqrt{c(a+bx)^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.83

$$-\frac{a+bx}{2b\left(c(a+bx)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x)^2)^(-3/2),x]
```

```
[Out] -1/2*(a + b*x)/(b*(c*(a + b*x)^2)^(3/2))
```

fricas [B] time = 0.84, size = 69, normalized size = 2.30

$$-\frac{\sqrt{b^2cx^2 + 2\,abcx + a^2c}}{2\left(b^4c^2x^3 + 3\,ab^3c^2x^2 + 3\,a^2b^2c^2x + a^3bc^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(b^2*c*x^2 + 2*a*b*c*x + a^2*c)/(b^4*c^2*x^3 + 3*a*b^3*c^2*x^2 + 3
*a^2*b^2*c^2*x + a^3*b*c^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b*x+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.00, size = 22, normalized size = 0.73

$$-\frac{bx + a}{2\left((bx + a)^2 c\right)^{\frac{3}{2}} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x+a)^2*c)^(3/2),x)
```

```
[Out] -1/2*(b*x+a)/b/((b*x+a)^2*c)^(3/2)
```

maxima [A] time = 0.55, size = 17, normalized size = 0.57

$$-\frac{1}{2\,b^3c^{\frac{3}{2}}\left(x + \frac{a}{b}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2/(b^3*c^(3/2)*(x + a/b)^2)
```

mupad [B] time = 1.25, size = 26, normalized size = 0.87

$$-\frac{\sqrt{c\,(a + b\,x)^2}}{2\,b\,c^2\,(a + b\,x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*(a + b*x)^2)^(3/2),x)
```

```
[Out] -(c*(a + b*x)^2)^(1/2)/(2*b*c^2*(a + b*x)^3)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\left(a+bx\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b*x+a)**2)**(3/2),x)
```

```
[Out] Integral((c*(a + b*x)**2)**(-3/2), x)
```

3.2807 $\int \frac{1}{\left(c(a+bx)^2\right)^{5/2}}\,dx$

Optimal. Leaf size=30

$$-\frac{1}{4bc^2(a+bx)^3\sqrt{c(a+bx)^2}}$$

```
[Out] -1/4/b/c^2/(b*x+a)^3/(c*(b*x+a)^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 15, 30}

$$-\frac{1}{4bc^2(a+bx)^3\sqrt{c(a+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x)^2)^(-5/2),x]
```

```
[Out] -1/(4*b*c^2*(a + b*x)^3*Sqrt[c*(a + b*x)^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(c(a+bx)^2\right)^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\left(cx^2\right)^{5/2}}\,dx, x, a+bx\right)}{b}\\ &= \frac{(a+bx)\,\text{Subst}\left(\int \frac{1}{x^5}\,dx, x, a+bx\right)}{bc^2\sqrt{c(a+bx)^2}}\\ &= -\frac{1}{4bc^2(a+bx)^3\sqrt{c(a+bx)^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.83

$$-\frac{a+bx}{4b\left(c(a+bx)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x)^2)^(-5/2),x]
```

```
[Out] -1/4*(a + b*x)/(b*(c*(a + b*x)^2)^(5/2))
```

fricas [B] time = 0.82, size = 97, normalized size = 3.23

$$-\frac{\sqrt{b^2cx^2+2\,abcx+a^2c}}{4\left(b^6c^3x^5+5\,ab^5c^3x^4+10\,a^2b^4c^3x^3+10\,a^3b^3c^3x^2+5\,a^4b^2c^3x+a^5bc^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b*x+a)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(b^2*c*x^2 + 2*a*b*c*x + a^2*c)/(b^6*c^3*x^5 + 5*a*b^5*c^3*x^4 + 1
0*a^2*b^4*c^3*x^3 + 10*a^3*b^3*c^3*x^2 + 5*a^4*b^2*c^3*x + a^5*b*c^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b*x+a)^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.00, size = 22, normalized size = 0.73

$$-\frac{bx+a}{4\left(\left(bx+a\right)^2c\right)^{\frac{5}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x+a)^2*c)^(5/2),x)
```

```
[Out] -1/4*(b*x+a)/b/((b*x+a)^2*c)^(5/2)
```

maxima [A] time = 0.56, size = 17, normalized size = 0.57

$$-\frac{1}{4\,b^5c^{\frac{5}{2}}\left(x+\frac{a}{b}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b*x+a)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4/(b^5*c^(5/2)*(x + a/b)^4)
```

mupad [B] time = 1.28, size = 26, normalized size = 0.87

$$-\frac{\sqrt{c\left(a+b\,x\right)^2}}{4\,b\,c^3\left(a+b\,x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*(a + b*x)^2)^(5/2),x)
```

```
[Out] -(c*(a + b*x)^2)^(1/2)/(4*b*c^3*(a + b*x)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\left(a+bx\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b*x+a)**2)**(5/2),x)
[Out] Integral((c*(a + b*x)**2)**(-5/2), x)
```

3.2808 $\int \sqrt{(3+5x)^2}\,dx$

Optimal. Leaf size=20

$$\frac{1}{10}(5x+3)\sqrt{(5x+3)^2}$$

```
[Out] 1/10*(3+5*x)*((3+5*x)^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {247, 15, 30}

$$\frac{1}{10}(5x+3)\sqrt{(5x+3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(3 + 5*x)^2],x]
```

```
[Out] ((3 + 5*x)*Sqrt[(3 + 5*x)^2])/10
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{(3+5x)^2}\,dx &= \frac{1}{5}\,\text{Subst}\left(\int \sqrt{x^2}\,dx, x, 3+5x\right)\\ &= \frac{\sqrt{(3+5x)^2}\,\text{Subst}(\int x\,dx, x, 3+5x)}{5(3+5x)}\\ &= \frac{1}{10}(3+5x)\sqrt{(3+5x)^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.25

$$\frac{x\sqrt{(5x+3)^2}\,(5x+6)}{10x+6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(3 + 5*x)^2],x]
```

```
[Out] (x*Sqrt[(3 + 5*x)^2]*(6 + 5*x))/(6 + 10*x)
```

fricas [A] time = 0.62, size = 9, normalized size = 0.45

$$\frac{5}{2}x^2 + 3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((3+5*x)^2)^(1/2),x, algorithm="fricas")
[Out] 5/2*x^2 + 3*x
```

giac [A] time = 0.15, size = 26, normalized size = 1.30

$$\frac{1}{2}\left(5x^2 + 6x\right)\operatorname{sgn}(5x+3) + \frac{9}{10}\operatorname{sgn}(5x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((3+5*x)^2)^(1/2),x, algorithm="giac")
[Out] 1/2*(5*x^2 + 6*x)*sgn(5*x + 3) + 9/10*sgn(5*x + 3)
```

maple [A] time = 0.01, size = 25, normalized size = 1.25

$$\frac{(5x+6)\sqrt{(5x+3)^2}\,x}{6+10x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3+5*x)^2)^(1/2),x)
[Out] 1/2*x*(5*x+6)*((3+5*x)^2)^(1/2)/(3+5*x)
```

maxima [A] time = 1.38, size = 30, normalized size = 1.50

$$\frac{1}{2}\sqrt{25x^2+30x+9}\,x + \frac{3}{10}\sqrt{25x^2+30x+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((3+5*x)^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(25*x^2 + 30*x + 9)*x + 3/10*sqrt(25*x^2 + 30*x + 9)
```

mupad [B] time = 0.11, size = 13, normalized size = 0.65

$$\frac{|5x+3|\,(5x+3)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x + 3)^2)^(1/2),x)
[Out] (abs(5*x + 3)*(5*x + 3))/10
```

sympy [A] time = 0.07, size = 8, normalized size = 0.40

$$\frac{5x^2}{2} + 3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((3+5*x)**2)**(1/2),x)
[Out] 5*x**2/2 + 3*x
```

3.2809 $\int \sqrt{(6+10x)^2}\,dx$

Optimal. Leaf size=20

$$\frac{1}{5}(5x+3)\sqrt{(5x+3)^2}$$

```
[Out] 1/5*(3+5*x)*((3+5*x)^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {247, 15, 30}

$$\frac{1}{5}(5x+3)\sqrt{(5x+3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(6 + 10*x)^2],x]
```

```
[Out] ((3 + 5*x)*Sqrt[(3 + 5*x)^2])/5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{(6+10x)^2}\,dx &= \frac{1}{10}\,\text{Subst}\left(\int \sqrt{x^2}\,dx, x, 6+10x\right)\\ &= \frac{\sqrt{(6+10x)^2}\,\text{Subst}(\int x\,dx, x, 6+10x)}{10(6+10x)}\\ &= \frac{1}{5}(3+5x)\sqrt{(3+5x)^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.25

$$\frac{x\sqrt{(5x+3)^2}\,(5x+6)}{5x+3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(6 + 10*x)^2],x]
```

```
[Out] (x*Sqrt[(3 + 5*x)^2]*(6 + 5*x))/(3 + 5*x)
```

fricas [A] time = 0.77, size = 9, normalized size = 0.45

$$5\,x^2 + 6\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((6+10*x)^2)^(1/2),x, algorithm="fricas")
[Out] 5*x^2 + 6*x
```

giac [A] time = 0.15, size = 25, normalized size = 1.25

$$\left(5\,x^2 + 6\,x\right)\operatorname{sgn}\left(5\,x + 3\right) + \frac{9}{5}\operatorname{sgn}\left(5\,x + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((6+10*x)^2)^(1/2),x, algorithm="giac")
[Out] (5*x^2 + 6*x)*sgn(5*x + 3) + 9/5*sgn(5*x + 3)
```

maple [A] time = 0.00, size = 24, normalized size = 1.20

$$\frac{(5x+6)\sqrt{(5x+3)^2}\,x}{5x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((6+10*x)^2)^(1/2),x)
[Out] (5*x+6)*((5*x+3)^2)^(1/2)/(5*x+3)*x
```

maxima [A] time = 1.23, size = 29, normalized size = 1.45

$$\sqrt{25\,x^2 + 30\,x + 9}\,x + \frac{3}{5}\sqrt{25\,x^2 + 30\,x + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((6+10*x)^2)^(1/2),x, algorithm="maxima")
[Out] sqrt(25*x^2 + 30*x + 9)*x + 3/5*sqrt(25*x^2 + 30*x + 9)
```

mupad [B] time = 0.08, size = 13, normalized size = 0.65

$$\frac{\left|10\,x + 6\right|\,\left(5\,x + 3\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((10*x + 6)^2)^(1/2),x)
[Out] (abs(10*x + 6)*(5*x + 3))/10
```

sympy [A] time = 0.07, size = 7, normalized size = 0.35

$$5x^2 + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((6+10*x)**2)**(1/2),x)
[Out] 5*x**2 + 6*x
```

3.2810 $\int \frac{1}{\sqrt{(3+5x)^2}}\,dx$

Optimal. Leaf size=26

$$\frac{(5x+3)\log(5x+3)}{5\sqrt{(5x+3)^2}}$$

```
[Out] 1/5*(3+5*x)*ln(3+5*x)/((3+5*x)^2)^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {247, 15, 29}

$$\frac{(5x+3)\log(5x+3)}{5\sqrt{(5x+3)^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(3 + 5*x)^2],x]
```

```
[Out] ((3 + 5*x)*Log[3 + 5*x])/(5*Sqrt[(3 + 5*x)^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{(3+5x)^2}}\,dx &= \frac{1}{5}\,\text{Subst}\left(\int \frac{1}{\sqrt{x^2}}\,dx, x, 3+5x\right)\\ &= \frac{(3+5x)\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, 3+5x\right)}{5\sqrt{(3+5x)^2}}\\ &= \frac{(3+5x)\log(3+5x)}{5\sqrt{(3+5x)^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$\frac{(5x+3)\log(5x+3)}{5\sqrt{(5x+3)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(3 + 5*x)^2],x]
```

```
[Out] ((3 + 5*x)*Log[3 + 5*x])/(5*Sqrt[(3 + 5*x)^2])
```

fricas [A] time = 0.84, size = 8, normalized size = 0.31

$$\frac{1}{5}\log(5x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3+5*x)^2)^(1/2),x, algorithm="fricas")
[Out] 1/5*log(5*x + 3)
```

giac [A] time = 0.15, size = 15, normalized size = 0.58

$$\frac{1}{5}\log(|5x+3|)\operatorname{sgn}(5x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3+5*x)^2)^(1/2),x, algorithm="giac")
[Out] 1/5*log(abs(5*x + 3))*sgn(5*x + 3)
```

maple [A] time = 0.01, size = 23, normalized size = 0.88

$$\frac{(5x+3)\ln(5x+3)}{5\sqrt{(5x+3)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((5*x+3)^2)^(1/2),x)
[Out] 1/5*(5*x+3)*ln(5*x+3)/((5*x+3)^2)^(1/2)
```

maxima [A] time = 1.26, size = 6, normalized size = 0.23

$$\frac{1}{5}\log\left(x+\frac{3}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3+5*x)^2)^(1/2),x, algorithm="maxima")
[Out] 1/5*log(x + 3/5)
```

mupad [B] time = 1.38, size = 14, normalized size = 0.54

$$\frac{\ln(5x+3)\ \operatorname{sign}(5x+3)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((5*x + 3)^2)^(1/2),x)
[Out] (log(5*x + 3)*sign(5*x + 3))/5
```

sympy [A] time = 0.07, size = 7, normalized size = 0.27

$$\frac{\log(5x+3)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3+5*x)**2)**(1/2),x)
[Out] log(5*x + 3)/5
```

3.2811 $\int \frac{1}{\sqrt{(6+10x)^2}}\,dx$

Optimal. Leaf size=26

$$\frac{(5x+3)\log(5x+3)}{10\sqrt{(5x+3)^2}}$$

```
[Out] 1/10*(3+5*x)*ln(3+5*x)/((3+5*x)^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {247, 15, 29}

$$\frac{(5x+3)\log(10x+6)}{10\sqrt{(5x+3)^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(6 + 10*x)^2],x]
```

```
[Out] ((3 + 5*x)*Log[6 + 10*x])/(10*Sqrt[(3 + 5*x)^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{(6+10x)^2}}\,dx &= \frac{1}{10}\,\text{Subst}\left(\int \frac{1}{\sqrt{x^2}}\,dx, x, 6+10x\right)\\ &= \frac{(6+10x)\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, 6+10x\right)}{10\sqrt{(6+10x)^2}}\\ &= \frac{(3+5x)\log(3+5x)}{10\sqrt{(3+5x)^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$\frac{(10x+6)\log(10x+6)}{10\sqrt{(10x+6)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(6 + 10*x)^2],x]
```

```
[Out] ((6 + 10*x)*Log[6 + 10*x])/(10*Sqrt[(6 + 10*x)^2])
```

fricas [A] time = 0.64, size = 8, normalized size = 0.31

$$\frac{1}{10}\log(5x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((6+10*x)^2)^(1/2),x, algorithm="fricas")
[Out] 1/10*log(5*x + 3)
```

giac [A] time = 0.17, size = 15, normalized size = 0.58

$$\frac{1}{10}\log(|5x+3|)\operatorname{sgn}(5x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((6+10*x)^2)^(1/2),x, algorithm="giac")
[Out] 1/10*log(abs(5*x + 3))*sgn(5*x + 3)
```

maple [A] time = 0.00, size = 26, normalized size = 1.00

$$\frac{(5x+3)\sqrt{4}\ \ln(5x+3)}{20\sqrt{(5x+3)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((6+10*x)^2)^(1/2),x)
[Out] 1/20/((5*x+3)^2)^(1/2)*(5*x+3)*4^(1/2)*ln(5*x+3)
```

maxima [A] time = 1.13, size = 6, normalized size = 0.23

$$\frac{1}{10}\log\left(x+\frac{3}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((6+10*x)^2)^(1/2),x, algorithm="maxima")
[Out] 1/10*log(x + 3/5)
```

mupad [B] time = 1.26, size = 14, normalized size = 0.54

$$\frac{\ln(10x+6)\ \operatorname{sign}(10x+6)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((10*x + 6)^2)^(1/2),x)
[Out] (log(10*x + 6)*sign(10*x + 6))/10
```

sympy [A] time = 0.08, size = 7, normalized size = 0.27

$$\frac{\log(10x+6)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((6+10*x)**2)**(1/2),x)
[Out] log(10*x + 6)/10
```

3.2812 $\int \frac{1}{\sqrt{-(2+3x)^2}}\,dx$

Optimal. Leaf size=28

$$\frac{(3x+2)\log(3x+2)}{3\sqrt{-(3x+2)^2}}$$

```
[Out] 1/3*(2+3*x)*ln(2+3*x)/(-(2+3*x)^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 15, 29}

$$\frac{(3x+2)\log(3x+2)}{3\sqrt{-(3x+2)^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-(2 + 3*x)^2],x]
```

```
[Out] ((2 + 3*x)*Log[2 + 3*x])/(3*Sqrt[-(2 + 3*x)^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-(2+3x)^2}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{-x^2}}\,dx, x, 2+3x\right)\\ &= \frac{(2+3x)\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, 2+3x\right)}{3\sqrt{-(2+3x)^2}}\\ &= \frac{(2+3x)\log(2+3x)}{3\sqrt{-(2+3x)^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$\frac{(3x+2)\log(3x+2)}{3\sqrt{-(3x+2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-(2 + 3*x)^2],x]
```

```
[Out] ((2 + 3*x)*Log[2 + 3*x])/(3*Sqrt[-(2 + 3*x)^2])
```

fricas [C] time = 0.81, size = 6, normalized size = 0.21

$$-\frac{1}{3}i\,\log\left(x+\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-(2+3*x)^2)^(1/2),x, algorithm="fricas")
[Out] -1/3*I*log(x + 2/3)
```

giac [C] time = 0.24, size = 23, normalized size = 0.82

$$\frac{i\,\log\left((-3i\,x-2i)\mathrm{sgn}\left(-3\,x-2\right)\right)}{3\,\mathrm{sgn}\left(-3\,x-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-(2+3*x)^2)^(1/2),x, algorithm="giac")
[Out] 1/3*I*log((-3*I*x - 2*I)*sgn(-3*x - 2))/sgn(-3*x - 2)
```

maple [A] time = 0.01, size = 25, normalized size = 0.89

$$\frac{(3x+2)\ln(3x+2)}{3\sqrt{-(3x+2)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-(2+3*x)^2)^(1/2),x)
[Out] 1/3*(2+3*x)*ln(2+3*x)/(-(2+3*x)^2)^(1/2)
```

maxima [C] time = 1.29, size = 6, normalized size = 0.21

$$\frac{1}{3}i\,\log\left(x+\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-(2+3*x)^2)^(1/2),x, algorithm="maxima")
[Out] 1/3*I*log(x + 2/3)
```

mupad [B] time = 1.36, size = 15, normalized size = 0.54

$$-\frac{\ln\left(-3\,x-2\right)\,\mathrm{sign}\left(3\,x+2\right)\,1\mathrm{i}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-(3*x + 2)^2)^(1/2),x)
[Out] -(log(- 3*x - 2)*sign(3*x + 2)*1i)/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-(3x+2)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-(2+3*x)**2)**(1/2),x)
[Out] Integral(1/sqrt(-(3*x + 2)**2), x)
```

3.2813 $\int \left(c(a+bx)^3\right)^{5/2}\,dx$

Optimal. Leaf size=30

$$\frac{2c^2(a+bx)^7\sqrt{c(a+bx)^3}}{17b}$$

```
[Out] 2/17*c^2*(b*x+a)^7*(c*(b*x+a)^3)^(1/2)/b
```

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 15, 30}

$$\frac{2c^2(a+bx)^7\sqrt{c(a+bx)^3}}{17b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x)^3)^(5/2),x]
```

```
[Out] (2*c^2*(a + b*x)^7*Sqrt[c*(a + b*x)^3])/(17*b)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \left(c(a+bx)^3\right)^{5/2}\,dx &= \frac{\text{Subst}\left(\int \left(cx^3\right)^{5/2}\,dx, x, a+bx\right)}{b}\\ &= \frac{\left(c^2\sqrt{c(a+bx)^3}\right)\text{Subst}\left(\int x^{15/2}\,dx, x, a+bx\right)}{b(a+bx)^{3/2}}\\ &= \frac{2c^2(a+bx)^7\sqrt{c(a+bx)^3}}{17b}\end{aligned}$$

Mathematica [A] time = 0.03, size = 25, normalized size = 0.83

$$\frac{2(a+bx)\left(c(a+bx)^3\right)^{5/2}}{17b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x)^3)^(5/2),x]
```

```
[Out] (2*(a + b*x)*(c*(a + b*x)^3)^(5/2))/(17*b)
```

fricas [B] time = 0.89, size = 135, normalized size = 4.50

$$\frac{2\left(b^7c^2x^7+7\,ab^6c^2x^6+21\,a^2b^5c^2x^5+35\,a^3b^4c^2x^4+35\,a^4b^3c^2x^3+21\,a^5b^2c^2x^2+7\,a^6bc^2x+a^7c^2\right)\sqrt{b^3cx^3+3\,a}}{17\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x+a)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/17*(b^7*c^2*x^7 + 7*a*b^6*c^2*x^6 + 21*a^2*b^5*c^2*x^5 + 35*a^3*b^4*c^2*x
^4 + 35*a^4*b^3*c^2*x^3 + 21*a^5*b^2*c^2*x^2 + 7*a^6*b*c^2*x + a^7*c^2)*sqr
t(b^3*c*x^3 + 3*a*b^2*c*x^2 + 3*a^2*b*c*x + a^3*c)/b
```

giac [B] time = 0.25, size = 851, normalized size = 28.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x+a)^3)^(5/2),x, algorithm="giac")
```

```
[Out] 2/109395*(109395*sqrt(b*c*x + a*c)*a^8*c^2*sgn(b*x + a) - 291720*(3*sqrt(b*
c*x + a*c)*a*c - (b*c*x + a*c)^(3/2))*a^7*c*sgn(b*x + a) + 204204*(15*sqrt(
b*c*x + a*c)*a^2*c^2 - 10*(b*c*x + a*c)^(3/2)*a*c + 3*(b*c*x + a*c)^(5/2))*
a^6*sgn(b*x + a) - 175032*(35*sqrt(b*c*x + a*c)*a^3*c^3 - 35*(b*c*x + a*c)^
(3/2)*a^2*c^2 + 21*(b*c*x + a*c)^(5/2)*a*c - 5*(b*c*x + a*c)^(7/2))*a^5*sgn
(b*x + a)/c + 24310*(315*sqrt(b*c*x + a*c)*a^4*c^4 - 420*(b*c*x + a*c)^(3/2
)*a^3*c^3 + 378*(b*c*x + a*c)^(5/2)*a^2*c^2 - 180*(b*c*x + a*c)^(7/2)*a*c +
 35*(b*c*x + a*c)^(9/2))*a^4*sgn(b*x + a)/c^2 - 8840*(693*sqrt(b*c*x + a*c)
*a^5*c^5 - 1155*(b*c*x + a*c)^(3/2)*a^4*c^4 + 1386*(b*c*x + a*c)^(5/2)*a^3*
c^3 - 990*(b*c*x + a*c)^(7/2)*a^2*c^2 + 385*(b*c*x + a*c)^(9/2)*a*c - 63*(b
*c*x + a*c)^(11/2))*a^3*sgn(b*x + a)/c^3 + 1020*(3003*sqrt(b*c*x + a*c)*a^6
*c^6 - 6006*(b*c*x + a*c)^(3/2)*a^5*c^5 + 9009*(b*c*x + a*c)^(5/2)*a^4*c^4
- 8580*(b*c*x + a*c)^(7/2)*a^3*c^3 + 5005*(b*c*x + a*c)^(9/2)*a^2*c^2 - 163
8*(b*c*x + a*c)^(11/2)*a*c + 231*(b*c*x + a*c)^(13/2))*a^2*sgn(b*x + a)/c^4
 - 136*(6435*sqrt(b*c*x + a*c)*a^7*c^7 - 15015*(b*c*x + a*c)^(3/2)*a^6*c^6
+ 27027*(b*c*x + a*c)^(5/2)*a^5*c^5 - 32175*(b*c*x + a*c)^(7/2)*a^4*c^4 + 2
5025*(b*c*x + a*c)^(9/2)*a^3*c^3 - 12285*(b*c*x + a*c)^(11/2)*a^2*c^2 + 346
5*(b*c*x + a*c)^(13/2)*a*c - 429*(b*c*x + a*c)^(15/2))*a*sgn(b*x + a)/c^5 +
 (109395*sqrt(b*c*x + a*c)*a^8*c^8 - 291720*(b*c*x + a*c)^(3/2)*a^7*c^7 + 6
12612*(b*c*x + a*c)^(5/2)*a^6*c^6 - 875160*(b*c*x + a*c)^(7/2)*a^5*c^5 + 85
0850*(b*c*x + a*c)^(9/2)*a^4*c^4 - 556920*(b*c*x + a*c)^(11/2)*a^3*c^3 + 23
5620*(b*c*x + a*c)^(13/2)*a^2*c^2 - 58344*(b*c*x + a*c)^(15/2)*a*c + 6435*(
b*c*x + a*c)^(17/2))*sgn(b*x + a)/c^6)/b
```

maple [A] time = 0.00, size = 22, normalized size = 0.73

$$\frac{2\,(bx+a)\left((bx+a)^3\,c\right)^{\frac{5}{2}}}{17b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(b*x+a)^3)^(5/2),x)
```

```
[Out] 2/17*(b*x+a)*(c*(b*x+a)^3)^(5/2)/b
```

maxima [B] time = 0.64, size = 108, normalized size = 3.60

$$\frac{2\left(b^7c^{\frac{5}{2}}x^7+7\,ab^6c^{\frac{5}{2}}x^6+21\,a^2b^5c^{\frac{5}{2}}x^5+35\,a^3b^4c^{\frac{5}{2}}x^4+35\,a^4b^3c^{\frac{5}{2}}x^3+21\,a^5b^2c^{\frac{5}{2}}x^2+7\,a^6bc^{\frac{5}{2}}x+a^7c^{\frac{5}{2}}\right)(bx+a)^{\frac{3}{2}}}{17\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x+a)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/17*(b^7*c^(5/2)*x^7 + 7*a*b^6*c^(5/2)*x^6 + 21*a^2*b^5*c^(5/2)*x^5 + 35*a
^3*b^4*c^(5/2)*x^4 + 35*a^4*b^3*c^(5/2)*x^3 + 21*a^5*b^2*c^(5/2)*x^2 + 7*a^
6*b*c^(5/2)*x + a^7*c^(5/2))*(b*x + a)^(3/2)/b
```

mupad [B] time = 1.39, size = 110, normalized size = 3.67

$$\sqrt{c\,(a+b\,x)^3}\left(\frac{14\,a^6\,c^2\,x}{17}+\frac{2\,a^7\,c^2}{17\,b}+\frac{2\,b^6\,c^2\,x^7}{17}+\frac{42\,a^5\,b\,c^2\,x^2}{17}+\frac{14\,a\,b^5\,c^2\,x^6}{17}+\frac{70\,a^4\,b^2\,c^2\,x^3}{17}+\frac{70\,a^3\,b^3\,c^2\,x^4}{17}+\frac{4}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x)^3)^(5/2),x)
```

```
[Out] (c*(a + b*x)^3)^(1/2)*((14*a^6*c^2*x)/17 + (2*a^7*c^2)/(17*b) + (2*b^6*c^2*
x^7)/17 + (42*a^5*b*c^2*x^2)/17 + (14*a*b^5*c^2*x^6)/17 + (70*a^4*b^2*c^2*x
^3)/17 + (70*a^3*b^3*c^2*x^4)/17 + (42*a^2*b^4*c^2*x^5)/17)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\,(a+bx)^3\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x+a)**3)**(5/2),x)
```

```
[Out] Integral((c*(a + b*x)**3)**(5/2), x)
```

3.2814 $\int \left(c(a+bx)^3\right)^{3/2}\,dx$

Optimal. Leaf size=28

$$\frac{2c(a+bx)^4\sqrt{c(a+bx)^3}}{11b}$$

```
[Out] 2/11*c*(b*x+a)^4*(c*(b*x+a)^3)^(1/2)/b
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 15, 30}

$$\frac{2c(a+bx)^4\sqrt{c(a+bx)^3}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x)^3)^(3/2),x]
```

```
[Out] (2*c*(a + b*x)^4*Sqrt[c*(a + b*x)^3])/(11*b)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \left(c(a+bx)^3\right)^{3/2}\,dx &= \frac{\text{Subst}\left(\int \left(cx^3\right)^{3/2}\,dx, x, a+bx\right)}{b}\\ &= \frac{\left(c\sqrt{c(a+bx)^3}\right)\text{Subst}\left(\int x^{9/2}\,dx, x, a+bx\right)}{b(a+bx)^{3/2}}\\ &= \frac{2c(a+bx)^4\sqrt{c(a+bx)^3}}{11b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 0.89

$$\frac{2(a+bx)\left(c(a+bx)^3\right)^{3/2}}{11b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x)^3)^(3/2),x]
```

```
[Out] (2*(a + b*x)*(c*(a + b*x)^3)^(3/2))/(11*b)
```

fricas [B] time = 0.60, size = 83, normalized size = 2.96

$$\frac{2\left(b^4cx^4+4\,ab^3cx^3+6\,a^2b^2cx^2+4\,a^3bcx+a^4c\right)\sqrt{b^3cx^3+3\,ab^2cx^2+3\,a^2bcx+a^3c}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x+a)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/11*(b^4*c*x^4 + 4*a*b^3*c*x^3 + 6*a^2*b^2*c*x^2 + 4*a^3*b*c*x + a^4*c)*sq
rt(b^3*c*x^3 + 3*a*b^2*c*x^2 + 3*a^2*b*c*x + a^3*c)/b
```

giac [B] time = 0.20, size = 408, normalized size = 14.57

$$2\left(693\sqrt{bcx+ac}\,a^5\mathrm{sgn}\left(bx+a\right)-\frac{1155\left(3\sqrt{bcx+ac}\,ac-(bcx+ac)^{\frac{3}{2}}\right)a^4\mathrm{sgn}(bx+a)}{c}+\frac{462\left(15\sqrt{bcx+ac}\,a^2c^2-10\,(bcx+ac)^{\frac{3}{2}}ac+3\,(bcx+ac)^{\frac{5}{2}}\right)}{c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x+a)^3)^(3/2),x, algorithm="giac")
```

```
[Out] 2/693*(693*sqrt(b*c*x + a*c)*a^5*sgn(b*x + a) - 1155*(3*sqrt(b*c*x + a*c)*a
*c - (b*c*x + a*c)^(3/2))*a^4*sgn(b*x + a)/c + 462*(15*sqrt(b*c*x + a*c)*a^
2*c^2 - 10*(b*c*x + a*c)^(3/2)*a*c + 3*(b*c*x + a*c)^(5/2))*a^3*sgn(b*x + a
)/c^2 - 198*(35*sqrt(b*c*x + a*c)*a^3*c^3 - 35*(b*c*x + a*c)^(3/2)*a^2*c^2
+ 21*(b*c*x + a*c)^(5/2)*a*c - 5*(b*c*x + a*c)^(7/2))*a^2*sgn(b*x + a)/c^3
+ 11*(315*sqrt(b*c*x + a*c)*a^4*c^4 - 420*(b*c*x + a*c)^(3/2)*a^3*c^3 + 378
*(b*c*x + a*c)^(5/2)*a^2*c^2 - 180*(b*c*x + a*c)^(7/2)*a*c + 35*(b*c*x + a*
c)^(9/2))*a*sgn(b*x + a)/c^4 - (693*sqrt(b*c*x + a*c)*a^5*c^5 - 1155*(b*c*x
 + a*c)^(3/2)*a^4*c^4 + 1386*(b*c*x + a*c)^(5/2)*a^3*c^3 - 990*(b*c*x + a*c
)^(7/2)*a^2*c^2 + 385*(b*c*x + a*c)^(9/2)*a*c - 63*(b*c*x + a*c)^(11/2))*sg
n(b*x + a)/c^5)*c/b
```

maple [A] time = 0.00, size = 22, normalized size = 0.79

$$\frac{2\left(bx+a\right)\left(\left(bx+a\right)^3c\right)^{\frac{3}{2}}}{11b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^3*c)^(3/2),x)
```

```
[Out] 2/11*(b*x+a)*((b*x+a)^3*c)^(3/2)/b
```

maxima [B] time = 0.69, size = 66, normalized size = 2.36

$$\frac{2\left(b^4c^{\frac{3}{2}}x^4+4\,ab^3c^{\frac{3}{2}}x^3+6\,a^2b^2c^{\frac{3}{2}}x^2+4\,a^3bc^{\frac{3}{2}}x+a^4c^{\frac{3}{2}}\right)(bx+a)^{\frac{3}{2}}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x+a)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/11*(b^4*c^(3/2)*x^4 + 4*a*b^3*c^(3/2)*x^3 + 6*a^2*b^2*c^(3/2)*x^2 + 4*a^3
*b*c^(3/2)*x + a^4*c^(3/2))*(b*x + a)^(3/2)/b
```

mupad [B] time = 1.25, size = 58, normalized size = 2.07

$$\sqrt{c\,(a+b\,x)^3}\,\left(\frac{2\,a^4\,c}{11\,b}+\frac{2\,b^3\,c\,x^4}{11}+\frac{8\,a^3\,c\,x}{11}+\frac{12\,a^2\,b\,c\,x^2}{11}+\frac{8\,a\,b^2\,c\,x^3}{11}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x)^3)^(3/2),x)
```

```
[Out] (c*(a + b*x)^3)^(1/2)*((2*a^4*c)/(11*b) + (2*b^3*c*x^4)/11 + (8*a^3*c*x)/11
 + (12*a^2*b*c*x^2)/11 + (8*a*b^2*c*x^3)/11)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\,(a+bx)^3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x+a)**3)**(3/2),x)
```

```
[Out] Integral((c*(a + b*x)**3)**(3/2), x)
```

3.2815 $\int \sqrt{c(a+bx)^3}\,dx$

Optimal. Leaf size=25

$$\frac{2(a+bx)\sqrt{c(a+bx)^3}}{5b}$$

```
[Out] 2/5*(b*x+a)*(c*(b*x+a)^3)^(1/2)/b
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 15, 30}

$$\frac{2(a+bx)\sqrt{c(a+bx)^3}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*(a + b*x)^3],x]
```

```
[Out] (2*(a + b*x)*Sqrt[c*(a + b*x)^3])/(5*b)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{c(a+bx)^3}\,dx &= \frac{\text{Subst}\left(\int \sqrt{cx^3}\,dx, x, a+bx\right)}{b}\\ &= \frac{\sqrt{c(a+bx)^3}\ \text{Subst}\left(\int x^{3/2}\,dx, x, a+bx\right)}{b(a+bx)^{3/2}}\\ &= \frac{2(a+bx)\sqrt{c(a+bx)^3}}{5b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{2(a+bx)\sqrt{c(a+bx)^3}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*(a + b*x)^3],x]
```

```
[Out] (2*(a + b*x)*Sqrt[c*(a + b*x)^3])/(5*b)
```

fricas [B] time = 0.57, size = 44, normalized size = 1.76

$$\frac{2\sqrt{b^3cx^3+3ab^2cx^2+3a^2bcx+a^3c}\,(bx+a)}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x+a)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*sqrt(b^3*c*x^3 + 3*a*b^2*c*x^2 + 3*a^2*b*c*x + a^3*c)*(b*x + a)/b
```

giac [B] time = 0.18, size = 121, normalized size = 4.84

$$\frac{2\left(15\sqrt{bcx+ac}\,a^2\mathrm{sgn}\,(bx+a)-\frac{10\left(3\sqrt{bcx+ac}\,ac-(bcx+ac)^{\frac{3}{2}}\right)a\mathrm{sgn}(bx+a)}{c}+\frac{\left(15\sqrt{bcx+ac}\,a^2c^2-10\,(bcx+ac)^{\frac{3}{2}}ac+3\,(bcx+ac)^{\frac{5}{2}}\right)\mathrm{sgn}(b}{c^2}\right.}{15b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x+a)^3)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(15*sqrt(b*c*x + a*c)*a^2*sgn(b*x + a) - 10*(3*sqrt(b*c*x + a*c)*a*c -
 (b*c*x + a*c)^(3/2))*a*sgn(b*x + a)/c + (15*sqrt(b*c*x + a*c)*a^2*c^2 - 10
*(b*c*x + a*c)^(3/2)*a*c + 3*(b*c*x + a*c)^(5/2))*sgn(b*x + a)/c^2)/b
```

maple [A] time = 0.00, size = 22, normalized size = 0.88

$$\frac{2\,(bx+a)\sqrt{(bx+a)^3\,c}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^3*c)^(1/2),x)
```

```
[Out] 2/5*(b*x+a)*((b*x+a)^3*c)^(1/2)/b
```

maxima [A] time = 0.82, size = 24, normalized size = 0.96

$$\frac{2\left(b\sqrt{c}\,x+a\sqrt{c}\right)(bx+a)^{\frac{3}{2}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x+a)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b*sqrt(c)*x + a*sqrt(c))*(b*x + a)^(3/2)/b
```

mupad [B] time = 1.15, size = 22, normalized size = 0.88

$$\sqrt{c\,(a+b\,x)^3}\,\left(\frac{2\,x}{5}+\frac{2\,a}{5\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x)^3)^(1/2),x)
```

```
[Out] (c*(a + b*x)^3)^(1/2)*((2*x)/5 + (2*a)/(5*b))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c\,(a+bx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x+a)**3)**(1/2),x)
[Out] Integral(sqrt(c*(a + b*x)**3), x)
```

3.2816 $\int \frac{1}{\sqrt{c(a+bx)^3}}\,dx$

Optimal. Leaf size=23

$$-\frac{2(a+bx)}{b\sqrt{c(a+bx)^3}}$$

```
[Out] -2*(b*x+a)/b/(c*(b*x+a)^3)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 15, 30}

$$-\frac{2(a+bx)}{b\sqrt{c(a+bx)^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c*(a + b*x)^3],x]
```

```
[Out] (-2*(a + b*x))/(b*Sqrt[c*(a + b*x)^3])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{c(a+bx)^3}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{cx^3}}\,dx, x, a+bx\right)}{b}\\ &= \frac{(a+bx)^{3/2}\,\text{Subst}\left(\int \frac{1}{x^{3/2}}\,dx, x, a+bx\right)}{b\sqrt{c(a+bx)^3}}\\ &= -\frac{2(a+bx)}{b\sqrt{c(a+bx)^3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$-\frac{2(a+bx)}{b\sqrt{c(a+bx)^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c*(a + b*x)^3],x]
```

```
[Out] (-2*(a + b*x))/(b*Sqrt[c*(a + b*x)^3])
```

fricas [B] time = 0.79, size = 61, normalized size = 2.65

$$-\frac{2\sqrt{b^3cx^3+3ab^2cx^2+3a^2bcx+a^3c}}{b^3cx^2+2ab^2cx+a^2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b*x+a)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(b^3*c*x^3 + 3*a*b^2*c*x^2 + 3*a^2*b*c*x + a^3*c)/(b^3*c*x^2 + 2*a*b
^2*c*x + a^2*b*c)
```

giac [A] time = 0.16, size = 23, normalized size = 1.00

$$-\frac{2}{\sqrt{bcx+ac}\,b\operatorname{sgn}(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b*x+a)^3)^(1/2),x, algorithm="giac")
```

```
[Out] -2/(sqrt(b*c*x + a*c)*b*sgn(b*x + a))
```

maple [A] time = 0.00, size = 22, normalized size = 0.96

$$-\frac{2\,(bx+a)}{\sqrt{(bx+a)^3\,c}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x+a)^3*c)^(1/2),x)
```

```
[Out] -2*(b*x+a)/b/((b*x+a)^3*c)^(1/2)
```

maxima [A] time = 0.68, size = 27, normalized size = 1.17

$$-\frac{2\left(b\sqrt{c}\,x+a\sqrt{c}\right)}{(bx+a)^{\frac{3}{2}}bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b*x+a)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*(b*sqrt(c)*x + a*sqrt(c))/((b*x + a)^(3/2)*b*c)
```

mupad [B] time = 1.17, size = 26, normalized size = 1.13

$$-\frac{2\sqrt{c\,(a+b\,x)^3}}{b\,c\,(a+b\,x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*(a + b*x)^3)^(1/2),x)
```

```
[Out] -(2*(c*(a + b*x)^3)^(1/2))/(b*c*(a + b*x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\,(a+bx)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b*x+a)**3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(c*(a + b*x)**3), x)
```

3.2817 $\int \frac{1}{\left(c(a+bx)^3\right)^{3/2}}\,dx$

Optimal. Leaf size=30

$$-\frac{2}{7bc(a+bx)^2\sqrt{c(a+bx)^3}}$$

```
[Out] -2/7/b/c/(b*x+a)^2/(c*(b*x+a)^3)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 15, 30}

$$-\frac{2}{7bc(a+bx)^2\sqrt{c(a+bx)^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x)^3)^(-3/2),x]
```

```
[Out] -2/(7*b*c*(a + b*x)^2*Sqrt[c*(a + b*x)^3])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(c(a+bx)^3\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\left(cx^3\right)^{3/2}}\,dx, x, a+bx\right)}{b}\\ &= \frac{(a+bx)^{3/2}\,\text{Subst}\left(\int \frac{1}{x^{9/2}}\,dx, x, a+bx\right)}{bc\sqrt{c(a+bx)^3}}\\ &= -\frac{2}{7bc(a+bx)^2\sqrt{c(a+bx)^3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.83

$$-\frac{2(a+bx)}{7b\left(c(a+bx)^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x)^3)^(-3/2),x]
```

```
[Out] (-2*(a + b*x))/(7*b*(c*(a + b*x)^3)^(3/2))
```

fricas [B] time = 0.89, size = 109, normalized size = 3.63

$$-\frac{2\sqrt{b^3cx^3+3ab^2cx^2+3a^2bcx+a^3c}}{7\left(b^6c^2x^5+5ab^5c^2x^4+10a^2b^4c^2x^3+10a^3b^3c^2x^2+5a^4b^2c^2x+a^5bc^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b*x+a)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/7*sqrt(b^3*c*x^3 + 3*a*b^2*c*x^2 + 3*a^2*b*c*x + a^3*c)/(b^6*c^2*x^5 + 5
*a*b^5*c^2*x^4 + 10*a^2*b^4*c^2*x^3 + 10*a^3*b^3*c^2*x^2 + 5*a^4*b^2*c^2*x
+ a^5*b*c^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b*x+a)^3)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.00, size = 22, normalized size = 0.73

$$-\frac{2(bx+a)}{7\left((bx+a)^3c\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x+a)^3*c)^(3/2),x)
```

```
[Out] -2/7*(b*x+a)/b/((b*x+a)^3*c)^(3/2)
```

maxima [A] time = 0.67, size = 43, normalized size = 1.43

$$-\frac{2\sqrt{c}}{7\left(b^3c^2x^2+2ab^2c^2x+a^2bc^2\right)(bx+a)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b*x+a)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/7*sqrt(c)/((b^3*c^2*x^2 + 2*a*b^2*c^2*x + a^2*b*c^2)*(b*x + a)^(3/2))
```

mupad [B] time = 1.24, size = 26, normalized size = 0.87

$$-\frac{2\sqrt{c(a+bx)^3}}{7bc^2(a+bx)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*(a + b*x)^3)^(3/2),x)
```

```
[Out] -(2*(c*(a + b*x)^3)^(1/2))/(7*b*c^2*(a + b*x)^5)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\,(a+bx)^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b*x+a)**3)**(3/2),x)
```

```
[Out] Integral((c*(a + b*x)**3)**(-3/2), x)
```

3.2818 $\int \frac{1}{\left(c(a+bx)^3\right)^{5/2}}\,dx$

Optimal. Leaf size=30

$$-\frac{2}{13bc^2(a+bx)^5\sqrt{c(a+bx)^3}}$$

```
[Out] -2/13/b/c^2/(b*x+a)^5/(c*(b*x+a)^3)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 15, 30}

$$-\frac{2}{13bc^2(a+bx)^5\sqrt{c(a+bx)^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x)^3)^(-5/2),x]
```

```
[Out] -2/(13*b*c^2*(a + b*x)^5*Sqrt[c*(a + b*x)^3])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(c(a+bx)^3\right)^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\left(cx^3\right)^{5/2}}\,dx, x, a+bx\right)}{b}\\ &= \frac{(a+bx)^{3/2}\,\text{Subst}\left(\int \frac{1}{x^{15/2}}\,dx, x, a+bx\right)}{bc^2\sqrt{c(a+bx)^3}}\\ &= -\frac{2}{13bc^2(a+bx)^5\sqrt{c(a+bx)^3}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 0.83

$$-\frac{2(a+bx)}{13b\left(c(a+bx)^3\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x)^3)^(-5/2),x]
```

```
[Out] (-2*(a + b*x))/(13*b*(c*(a + b*x)^3)^(5/2))
```

fricas [B] time = 0.67, size = 151, normalized size = 5.03

$$-\frac{2\sqrt{b^3cx^3+3ab^2cx^2+3a^2bcx+a^3c}}{13\left(b^9c^3x^8+8ab^8c^3x^7+28a^2b^7c^3x^6+56a^3b^6c^3x^5+70a^4b^5c^3x^4+56a^5b^4c^3x^3+28a^6b^3c^3x^2+8a^7b^2c^3x+a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b*x+a)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/13*sqrt(b^3*c*x^3 + 3*a*b^2*c*x^2 + 3*a^2*b*c*x + a^3*c)/(b^9*c^3*x^8 +
8*a*b^8*c^3*x^7 + 28*a^2*b^7*c^3*x^6 + 56*a^3*b^6*c^3*x^5 + 70*a^4*b^5*c^3*
x^4 + 56*a^5*b^4*c^3*x^3 + 28*a^6*b^3*c^3*x^2 + 8*a^7*b^2*c^3*x + a^8*b*c^3
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b*x+a)^3)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.00, size = 22, normalized size = 0.73

$$-\frac{2(bx+a)}{13\left((bx+a)^3c\right)^{\frac{5}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x+a)^3*c)^(5/2),x)
```

```
[Out] -2/13*(b*x+a)/b/((b*x+a)^3*c)^(5/2)
```

maxima [B] time = 0.79, size = 85, normalized size = 2.83

$$-\frac{2\sqrt{c}}{13\left(b^6c^3x^5+5ab^5c^3x^4+10a^2b^4c^3x^3+10a^3b^3c^3x^2+5a^4b^2c^3x+a^5bc^3\right)(bx+a)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b*x+a)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/13*sqrt(c)/((b^6*c^3*x^5 + 5*a*b^5*c^3*x^4 + 10*a^2*b^4*c^3*x^3 + 10*a^3
*b^3*c^3*x^2 + 5*a^4*b^2*c^3*x + a^5*b*c^3)*(b*x + a)^(3/2))
```

mupad [B] time = 1.32, size = 26, normalized size = 0.87

$$-\frac{2\sqrt{c(a+bx)^3}}{13bc^3(a+bx)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*(a + b*x)^3)^(5/2),x)
```

```
[Out] -(2*(c*(a + b*x)^3)^(1/2))/(13*b*c^3*(a + b*x)^8)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\left(a+bx\right)^3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b*x+a)**3)**(5/2),x)
```

```
[Out] Integral((c*(a + b*x)**3)**(-5/2), x)
```

3.2819 $\int \left(\frac{c}{a+bx}\right)^{5/2} dx$

Optimal. Leaf size=30

$$-\frac{2c^2\sqrt{\frac{c}{a+bx}}}{3b(a+bx)}$$

```
[Out] -2/3*c^2*(c/(b*x+a))^(1/2)/b/(b*x+a)
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 15, 30}

$$-\frac{2c^2\sqrt{\frac{c}{a+bx}}}{3b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(c/(a + b*x))^(5/2),x]
```

```
[Out] (-2*c^2*Sqrt[c/(a + b*x)])/(3*b*(a + b*x))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{c}{a+bx}\right)^{5/2} dx &= \frac{\text{Subst}\left(\int \left(\frac{c}{x}\right)^{5/2} dx, x, a+bx\right)}{b} \\ &= \frac{\left(c^2\sqrt{\frac{c}{a+bx}}\sqrt{a+bx}\right)\text{Subst}\left(\int \frac{1}{x^{5/2}} dx, x, a+bx\right)}{b} \\ &= -\frac{2c^2\sqrt{\frac{c}{a+bx}}}{3b(a+bx)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 0.70

$$-\frac{2c\left(\frac{c}{a+bx}\right)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c/(a + b*x))^(5/2),x]
[Out] (-2*c*(c/(a + b*x))^(3/2))/(3*b)
```

fricas [A] time = 0.75, size = 27, normalized size = 0.90

$$-\frac{2\,c^2\sqrt{\frac{c}{bx+a}}}{3\left(b^2x+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x+a))^(5/2),x, algorithm="fricas")
[Out] -2/3*c^2*sqrt(c/(b*x + a))/(b^2*x + a*b)
```

giac [A] time = 0.16, size = 24, normalized size = 0.80

$$-\frac{2\,c^4\mathrm{sgn}\left(bx+a\right)}{3\left(bcx+ac\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x+a))^(5/2),x, algorithm="giac")
[Out] -2/3*c^4*sgn(b*x + a)/((b*c*x + a*c)^(3/2)*b)
```

maple [A] time = 0.00, size = 22, normalized size = 0.73

$$-\frac{2\left(bx+a\right)\left(\frac{c}{bx+a}\right)^{\frac{5}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/(b*x+a))^(5/2),x)
[Out] -2/3*(b*x+a)*(c/(b*x+a))^(5/2)/b
```

maxima [A] time = 0.62, size = 17, normalized size = 0.57

$$-\frac{2\,c\left(\frac{c}{bx+a}\right)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x+a))^(5/2),x, algorithm="maxima")
[Out] -2/3*c*(c/(b*x + a))^(3/2)/b
```

mupad [B] time = 1.18, size = 17, normalized size = 0.57

$$-\frac{2\,c\left(\frac{c}{a+b\,x}\right)^{3/2}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/(a + b*x))^(5/2),x)
[Out] -(2*c*(c/(a + b*x))^(3/2))/(3*b)
```

sympy [**A**] time = 3.59, size = 51, normalized size = 1.70

$$\begin{cases} -\frac{2ac^{\frac{5}{2}}\left(\frac{1}{a+bx}\right)^{\frac{5}{2}}}{3b} - \frac{2c^{\frac{5}{2}}x\left(\frac{1}{a+bx}\right)^{\frac{5}{2}}}{3} & \text{for } b \neq 0 \\ x\left(\frac{c}{a}\right)^{\frac{5}{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x+a))**(5/2),x)
```

```
[Out] Piecewise((-2*a*c**(5/2)*(1/(a + b*x))**(5/2)/(3*b) - 2*c**(5/2)*x*(1/(a +
b*x))**(5/2)/3, Ne(b, 0)), (x*(c/a)**(5/2), True))
```

3.2820 $\int \left(\frac{c}{a+bx}\right)^{3/2} dx$

Optimal. Leaf size=19

$$-\frac{2c\sqrt{\frac{c}{a+bx}}}{b}$$

```
[Out] -2*c*(c/(b*x+a))^(1/2)/b
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 15, 30}

$$-\frac{2c\sqrt{\frac{c}{a+bx}}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c/(a + b*x))^(3/2),x]
```

```
[Out] (-2*c*Sqrt[c/(a + b*x)])/b
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{c}{a+bx}\right)^{3/2} dx &= \frac{\text{Subst}\left(\int \left(\frac{c}{x}\right)^{3/2} dx, x, a+bx\right)}{b} \\ &= \frac{\left(c\sqrt{\frac{c}{a+bx}}\sqrt{a+bx}\right)\text{Subst}\left(\int \frac{1}{x^{3/2}} dx, x, a+bx\right)}{b} \\ &= -\frac{2c\sqrt{\frac{c}{a+bx}}}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$-\frac{2c\sqrt{\frac{c}{a+bx}}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c/(a + b*x))^(3/2),x]
[Out] (-2*c*Sqrt[c/(a + b*x)])/b
```

fricas [A] time = 0.93, size = 17, normalized size = 0.89

$$-\frac{2\,c\sqrt{\frac{c}{bx+a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x+a))^(3/2),x, algorithm="fricas")
[Out] -2*c*sqrt(c/(b*x + a))/b
```

giac [A] time = 0.20, size = 24, normalized size = 1.26

$$-\frac{2\,c^2\mathrm{sgn}\,(bx+a)}{\sqrt{bcx+ac}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x+a))^(3/2),x, algorithm="giac")
[Out] -2*c^2*sgn(b*x + a)/(sqrt(b*c*x + a*c)*b)
```

maple [A] time = 0.00, size = 22, normalized size = 1.16

$$-\frac{2\,(bx+a)\left(\frac{c}{bx+a}\right)^{\frac{3}{2}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(b*x+a)*c)^(3/2),x)
[Out] -2*(b*x+a)*(1/(b*x+a)*c)^(3/2)/b
```

maxima [A] time = 0.54, size = 17, normalized size = 0.89

$$-\frac{2\,c\sqrt{\frac{c}{bx+a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x+a))^(3/2),x, algorithm="maxima")
[Out] -2*c*sqrt(c/(b*x + a))/b
```

mupad [B] time = 1.14, size = 17, normalized size = 0.89

$$-\frac{2\,c\,\sqrt{\frac{c}{a+b\,x}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/(a + b*x))^(3/2),x)
[Out] -(2*c*(c/(a + b*x))^(1/2))/b
```

sympy [**A**] time = 1.23, size = 48, normalized size = 2.53

$$\begin{cases} -\dfrac{2ac^{\frac{3}{2}}\left(\frac{1}{a+bx}\right)^{\frac{3}{2}}}{b} - 2c^{\frac{3}{2}}x\left(\dfrac{1}{a+bx}\right)^{\frac{3}{2}} & \text{for } b \neq 0 \\ x\left(\dfrac{c}{a}\right)^{\frac{3}{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x+a))**(3/2),x)
```

```
[Out] Piecewise((-2*a*c**(3/2)*(1/(a + b*x))**(3/2)/b - 2*c**(3/2)*x*(1/(a + b*x)
)**(3/2), Ne(b, 0)), (x*(c/a)**(3/2), True))
```

3.2821 $\int \sqrt{\frac{c}{a+bx}}\,dx$

Optimal. Leaf size=23

$$\frac{2(a+bx)\sqrt{\frac{c}{a+bx}}}{b}$$

```
[Out] 2*(b*x+a)*(c/(b*x+a))^(1/2)/b
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 15, 30}

$$\frac{2(a+bx)\sqrt{\frac{c}{a+bx}}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c/(a + b*x)],x]
```

```
[Out] (2*Sqrt[c/(a + b*x)]*(a + b*x))/b
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{c}{a+bx}}\,dx &= \frac{\text{Subst}\left(\int \sqrt{\frac{c}{x}}\,dx, x, a+bx\right)}{b}\\ &= \frac{\left(\sqrt{\frac{c}{a+bx}}\sqrt{a+bx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, a+bx\right)}{b}\\ &= \frac{2\sqrt{\frac{c}{a+bx}}\,(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 0.83

$$\frac{2c}{b\sqrt{\frac{c}{a+bx}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c/(a + b*x)],x]
[Out] (2*c)/(b*Sqrt[c/(a + b*x)])
```

fricas [A] time = 0.57, size = 21, normalized size = 0.91

$$\frac{2\,(bx+a)\sqrt{\frac{c}{bx+a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x+a))^(1/2),x, algorithm="fricas")
[Out] 2*(b*x + a)*sqrt(c/(b*x + a))/b
```

giac [A] time = 0.16, size = 21, normalized size = 0.91

$$\frac{2\sqrt{bcx+ac}\,\operatorname{sgn}(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x+a))^(1/2),x, algorithm="giac")
[Out] 2*sqrt(b*c*x + a*c)*sgn(b*x + a)/b
```

maple [A] time = 0.00, size = 22, normalized size = 0.96

$$\frac{2\,(bx+a)\sqrt{\frac{c}{bx+a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(b*x+a)*c)^(1/2),x)
[Out] 2*(b*x+a)*(1/(b*x+a)*c)^(1/2)/b
```

maxima [A] time = 0.56, size = 17, normalized size = 0.74

$$\frac{2\,c}{b\sqrt{\frac{c}{bx+a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x+a))^(1/2),x, algorithm="maxima")
[Out] 2*c/(b*sqrt(c/(b*x + a)))
```

mupad [B] time = 1.16, size = 22, normalized size = 0.96

$$\sqrt{\frac{c}{a+b\,x}}\,\left(2\,x+\frac{2\,a}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/(a + b*x))^(1/2),x)
[Out] (c/(a + b*x))^(1/2)*(2*x + (2*a)/b)
```

sympy [A] time = 0.34, size = 46, normalized size = 2.00

$$\begin{cases} \frac{2a\sqrt{c}\sqrt{\frac{1}{a+bx}}}{b} + 2\sqrt{c}\,x\sqrt{\frac{1}{a+bx}} & \text{for } b \neq 0 \\ x\sqrt{\frac{c}{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x+a))**(1/2),x)
```

```
[Out] Piecewise((2*a*sqrt(c)*sqrt(1/(a + b*x))/b + 2*sqrt(c)*x*sqrt(1/(a + b*x)),
 Ne(b, 0)), (x*sqrt(c/a), True))
```

3.2822 $\int \frac{1}{\sqrt{\frac{c}{a+bx}}}\,dx$

Optimal. Leaf size=25

$$\frac{2(a+bx)}{3b\sqrt{\frac{c}{a+bx}}}$$

```
[Out] 2/3*(b*x+a)/b/(c/(b*x+a))^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 15, 30}

$$\frac{2(a+bx)}{3b\sqrt{\frac{c}{a+bx}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c/(a + b*x)],x]
```

```
[Out] (2*(a + b*x))/(3*b*Sqrt[c/(a + b*x)])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\frac{c}{a+bx}}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\frac{c}{x}}}\,dx, x, a+bx\right)}{b}\\ &= \frac{\text{Subst}\left(\int \sqrt{x}\,dx, x, a+bx\right)}{b\sqrt{\frac{c}{a+bx}}\sqrt{a+bx}}\\ &= \frac{2(a+bx)}{3b\sqrt{\frac{c}{a+bx}}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 0.84

$$\frac{2c}{3b\left(\frac{c}{a+bx}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c/(a + b*x)],x]
[Out] (2*c)/(3*b*(c/(a + b*x))^(3/2))
```

fricas [A] time = 0.76, size = 35, normalized size = 1.40

$$\frac{2\left(b^2x^2+2\,abx+a^2\right)\sqrt{\frac{c}{bx+a}}}{3\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a))^(1/2),x, algorithm="fricas")
[Out] 2/3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(c/(b*x + a))/(b*c)
```

giac [B] time = 0.16, size = 62, normalized size = 2.48

$$\frac{2\left(3\,\sqrt{bcx+ac}\,a-\frac{3\,\sqrt{bcx+ac}\,ac-(bcx+ac)^{\frac{3}{2}}}{c}\right)}{3\,bc\mathrm{sgn}\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a))^(1/2),x, algorithm="giac")
[Out] 2/3*(3*sqrt(b*c*x + a*c)*a - (3*sqrt(b*c*x + a*c)*a*c - (b*c*x + a*c)^(3/2)
)/c)/(b*c*sgn(b*x + a))
```

maple [A] time = 0.00, size = 22, normalized size = 0.88

$$\frac{\frac{2bx}{3}+\frac{2a}{3}}{\sqrt{\frac{c}{bx+a}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/(b*x+a)*c)^(1/2),x)
[Out] 2/3*(b*x+a)/b/(1/(b*x+a)*c)^(1/2)
```

maxima [A] time = 0.63, size = 17, normalized size = 0.68

$$\frac{2\,c}{3\,b\left(\frac{c}{bx+a}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a))^(1/2),x, algorithm="maxima")
[Out] 2/3*c/(b*(c/(b*x + a))^(3/2))
```

mupad [B] time = 1.19, size = 40, normalized size = 1.60

$$\sqrt{\frac{c}{a+b\,x}}\,\left(\frac{2\,b\,x^2}{3\,c}+\frac{2\,a^2}{3\,b\,c}+\frac{4\,a\,x}{3\,c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c/(a + b*x))^(1/2),x)
```

```
[Out] (c/(a + b*x))^(1/2)*((2*b*x^2)/(3*c) + (2*a^2)/(3*b*c) + (4*a*x)/(3*c))
```

sympy [A] time = 1.62, size = 49, normalized size = 1.96

$$\begin{cases} \frac{2a}{3b\sqrt{c}\sqrt{\frac{1}{a+bx}}} + \frac{2x}{3\sqrt{c}\sqrt{\frac{1}{a+bx}}} & \text{for } b \neq 0 \\ \frac{x}{\sqrt{\frac{c}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a))**(1/2),x)
```

```
[Out] Piecewise((2*a/(3*b*sqrt(c)*sqrt(1/(a + b*x))) + 2*x/(3*sqrt(c)*sqrt(1/(a +
 b*x))), Ne(b, 0)), (x/sqrt(c/a), True))
```

3.2823 $\int \frac{1}{\left(\frac{c}{a+bx}\right)^{3/2}}\,dx$

Optimal. Leaf size=30

$$\frac{2(a+bx)^2}{5bc\sqrt{\frac{c}{a+bx}}}$$

```
[Out] 2/5*(b*x+a)^2/b/c/(c/(b*x+a))^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 15, 30}

$$\frac{2(a+bx)^2}{5bc\sqrt{\frac{c}{a+bx}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c/(a + b*x))^(-3/2),x]
```

```
[Out] (2*(a + b*x)^2)/(5*b*c*Sqrt[c/(a + b*x)])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{c}{a+bx}\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\left(\frac{c}{x}\right)^{3/2}}\,dx, x, a+bx\right)}{b}\\ &= \frac{\text{Subst}\left(\int x^{3/2}\,dx, x, a+bx\right)}{bc\sqrt{\frac{c}{a+bx}}\sqrt{a+bx}}\\ &= \frac{2(a+bx)^2}{5bc\sqrt{\frac{c}{a+bx}}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 21, normalized size = 0.70

$$\frac{2c}{5b\left(\frac{c}{a+bx}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c/(a + b*x))^(-3/2),x]
[Out] (2*c)/(5*b*(c/(a + b*x))^(5/2))
```

fricas [A] time = 0.93, size = 46, normalized size = 1.53

$$\frac{2\left(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3\right)\sqrt{\frac{c}{bx+a}}}{5\,bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a))^(3/2),x, algorithm="fricas")
[Out] 2/5*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(c/(b*x + a))/(b*c^2)
```

giac [A] time = 0.17, size = 26, normalized size = 0.87

$$\frac{2\,(bx+a)^2}{5\,bc\sqrt{\frac{c}{bx+a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a))^(3/2),x, algorithm="giac")
[Out] 2/5*(b*x + a)^2/(b*c*sqrt(c/(b*x + a)))
```

maple [A] time = 0.00, size = 22, normalized size = 0.73

$$\frac{\frac{2bx}{5}+\frac{2a}{5}}{\left(\frac{c}{bx+a}\right)^{\frac{3}{2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/(b*x+a)*c)^(3/2),x)
[Out] 2/5*(b*x+a)/b/(1/(b*x+a)*c)^(3/2)
```

maxima [A] time = 0.50, size = 17, normalized size = 0.57

$$\frac{2\,c}{5\,b\left(\frac{c}{bx+a}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a))^(3/2),x, algorithm="maxima")
[Out] 2/5*c/(b*(c/(b*x + a))^(5/2))
```

mupad [B] time = 1.23, size = 54, normalized size = 1.80

$$\sqrt{\frac{c}{a+b\,x}}\left(\frac{6\,a^2\,x}{5\,c^2}+\frac{2\,a^3}{5\,b\,c^2}+\frac{2\,b^2\,x^3}{5\,c^2}+\frac{6\,a\,b\,x^2}{5\,c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c/(a + b*x))^(3/2),x)
[Out] (c/(a + b*x))^(1/2)*((6*a^2*x)/(5*c^2) + (2*a^3)/(5*b*c^2) + (2*b^2*x^3)/(5
*c^2) + (6*a*b*x^2)/(5*c^2))
```

sympy [**A**] time = 1.69, size = 49, normalized size = 1.63

$$\begin{cases} \frac{2a}{5bc^{\frac{3}{2}}\left(\frac{1}{a+bx}\right)^{\frac{3}{2}}} + \frac{2x}{5c^{\frac{3}{2}}\left(\frac{1}{a+bx}\right)^{\frac{3}{2}}} & \text{for } b \neq 0 \\ \frac{x}{\left(\frac{c}{a}\right)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a))**(3/2),x)
```

```
[Out] Piecewise((2*a/(5*b*c**(3/2)*(1/(a + b*x))**(3/2)) + 2*x/(5*c**(3/2)*(1/(a
+ b*x))**(3/2)), Ne(b, 0)), (x/(c/a)**(3/2), True))
```

3.2824 $\int \frac{1}{\left(\frac{c}{a+bx}\right)^{5/2}}\,dx$

Optimal. Leaf size=30

$$\frac{2(a+bx)^3}{7bc^2\sqrt{\frac{c}{a+bx}}}$$

```
[Out] 2/7*(b*x+a)^3/b/c^2/(c/(b*x+a))^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 15, 30}

$$\frac{2(a+bx)^3}{7bc^2\sqrt{\frac{c}{a+bx}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c/(a + b*x))^(-5/2),x]
```

```
[Out] (2*(a + b*x)^3)/(7*b*c^2*Sqrt[c/(a + b*x)])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{c}{a+bx}\right)^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\left(\frac{c}{x}\right)^{5/2}}\,dx, x, a+bx\right)}{b}\\ &= \frac{\text{Subst}\left(\int x^{5/2}\,dx, x, a+bx\right)}{bc^2\sqrt{\frac{c}{a+bx}}\sqrt{a+bx}}\\ &= \frac{2(a+bx)^3}{7bc^2\sqrt{\frac{c}{a+bx}}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 21, normalized size = 0.70

$$\frac{2c}{7b\left(\frac{c}{a+bx}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c/(a + b*x))^(-5/2),x]
```

```
[Out] (2*c)/(7*b*(c/(a + b*x))^(7/2))
```

fricas [B] time = 0.84, size = 57, normalized size = 1.90

$$\frac{2\left(b^4x^4+4\,ab^3x^3+6\,a^2b^2x^2+4\,a^3bx+a^4\right)\sqrt{\frac{c}{bx+a}}}{7\,bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/7*(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4)*sqrt(c/(b*x +
 a))/(b*c^3)
```

giac [A] time = 0.18, size = 26, normalized size = 0.87

$$\frac{2\,(bx+a)^3}{7\,bc^2\sqrt{\frac{c}{bx+a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 2/7*(b*x + a)^3/(b*c^2*sqrt(c/(b*x + a)))
```

maple [A] time = 0.00, size = 22, normalized size = 0.73

$$\frac{\frac{2bx}{7}+\frac{2a}{7}}{\left(\frac{c}{bx+a}\right)^{\frac{5}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/(b*x+a)*c)^(5/2),x)
```

```
[Out] 2/7*(b*x+a)/b/(1/(b*x+a)*c)^(5/2)
```

maxima [A] time = 0.62, size = 17, normalized size = 0.57

$$\frac{2\,c}{7\,b\left(\frac{c}{bx+a}\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/7*c/(b*(c/(b*x + a))^(7/2))
```

mupad [B] time = 1.28, size = 26, normalized size = 0.87

$$\frac{2\sqrt{\frac{c}{a+b\,x}}\,(a+b\,x)^4}{7\,b\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c/(a + b*x))^(5/2),x)
```

```
[Out] (2*(c/(a + b*x))^(1/2)*(a + b*x)^4)/(7*b*c^3)
```

sympy [**A**] time = 2.57, size = 49, normalized size = 1.63

$$\begin{cases} \frac{2a}{7bc^{\frac{5}{2}}\left(\frac{1}{a+bx}\right)^{\frac{5}{2}}} + \frac{2x}{7c^{\frac{5}{2}}\left(\frac{1}{a+bx}\right)^{\frac{5}{2}}} & \text{for } b \neq 0 \\ \frac{x}{\left(\frac{c}{a}\right)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a))**(5/2),x)
```

```
[Out] Piecewise((2*a/(7*b*c**(5/2)*(1/(a + b*x))**(5/2)) + 2*x/(7*c**(5/2)*(1/(a
+ b*x))**(5/2)), Ne(b, 0)), (x/(c/a)**(5/2), True))
```

3.2825 $\int \left(\frac{c}{(a+bx)^2}\right)^{5/2} dx$

Optimal. Leaf size=30

$$-\frac{c^2\sqrt{\frac{c}{(a+bx)^2}}}{4b(a+bx)^3}$$

```
[Out] -1/4*c^2*(c/(b*x+a)^2)^(1/2)/b/(b*x+a)^3
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 15, 30}

$$-\frac{c^2\sqrt{\frac{c}{(a+bx)^2}}}{4b(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c/(a + b*x)^2)^(5/2),x]
```

```
[Out] -(c^2*Sqrt[c/(a + b*x)^2])/(4*b*(a + b*x)^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{c}{(a+bx)^2}\right)^{5/2} dx &= \frac{\text{Subst}\left(\int \left(\frac{c}{x^2}\right)^{5/2} dx, x, a+bx\right)}{b} \\ &= \frac{\left(c^2\sqrt{\frac{c}{(a+bx)^2}}\,(a+bx)\right)\text{Subst}\left(\int \frac{1}{x^5} dx, x, a+bx\right)}{b} \\ &= -\frac{c^2\sqrt{\frac{c}{(a+bx)^2}}}{4b(a+bx)^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.83

$$-\frac{(a+bx)\left(\frac{c}{(a+bx)^2}\right)^{5/2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c/(a + b*x)^2)^(5/2),x]
```

```
[Out] -1/4*((c/(a + b*x)^2)^(5/2)*(a + b*x))/b
```

fricas [B] time = 0.82, size = 60, normalized size = 2.00

$$-\frac{c^2\sqrt{\frac{c}{b^2x^2+2\,abx+a^2}}}{4\left(b^4x^3+3\,ab^3x^2+3\,a^2b^2x+a^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x+a)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/4*c^2*sqrt(c/(b^2*x^2 + 2*a*b*x + a^2))/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b
^2*x + a^3*b)
```

giac [A] time = 0.20, size = 21, normalized size = 0.70

$$-\frac{c^{\frac{5}{2}}\operatorname{sgn}(bx+a)}{4\,(bx+a)^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x+a)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/4*c^(5/2)*sgn(b*x + a)/((b*x + a)^4*b)
```

maple [A] time = 0.00, size = 22, normalized size = 0.73

$$-\frac{(bx+a)\left(\frac{c}{(bx+a)^2}\right)^{\frac{5}{2}}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/(b*x+a)^2)^(5/2),x)
```

```
[Out] -1/4*(b*x+a)/b*(c/(b*x+a)^2)^(5/2)
```

maxima [A] time = 0.67, size = 49, normalized size = 1.63

$$-\frac{c^{\frac{5}{2}}}{4\left(b^5x^4+4\,ab^4x^3+6\,a^2b^3x^2+4\,a^3b^2x+a^4b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x+a)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4*c^(5/2)/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b)
```

mupad [B] time = 1.23, size = 26, normalized size = 0.87

$$-\frac{c^2\sqrt{\frac{c}{(a+b\,x)^2}}}{4\,b\,(a+b\,x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/(a + b*x)^2)^(5/2),x)
```

```
[Out] -(c^2*(c/(a + b*x)^2)^(1/2))/(4*b*(a + b*x)^3)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{c}{(a+bx)^2}\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x+a)**2)**(5/2),x)
[Out] Integral((c/(a + b*x)**2)**(5/2), x)
```

3.2826 $\int \left(\frac{c}{(a+bx)^2}\right)^{3/2} dx$

Optimal. Leaf size=28

$$-\frac{c\sqrt{\frac{c}{(a+bx)^2}}}{2b(a+bx)}$$

```
[Out] -1/2*c*(c/(b*x+a)^2)^(1/2)/b/(b*x+a)
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 15, 30}

$$-\frac{c\sqrt{\frac{c}{(a+bx)^2}}}{2b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(c/(a + b*x)^2)^(3/2),x]
```

```
[Out] -(c*Sqrt[c/(a + b*x)^2])/(2*b*(a + b*x))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{c}{(a+bx)^2}\right)^{3/2} dx &= \frac{\text{Subst}\left(\int \left(\frac{c}{x^2}\right)^{3/2} dx, x, a+bx\right)}{b} \\ &= \frac{\left(c\sqrt{\frac{c}{(a+bx)^2}}\,(a+bx)\right)\text{Subst}\left(\int \frac{1}{x^3} dx, x, a+bx\right)}{b} \\ &= -\frac{c\sqrt{\frac{c}{(a+bx)^2}}}{2b(a+bx)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.89

$$-\frac{(a+bx)\left(\frac{c}{(a+bx)^2}\right)^{3/2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c/(a + b*x)^2)^(3/2),x]
[Out] -1/2*((c/(a + b*x)^2)^(3/2)*(a + b*x))/b
```

fricas [A] time = 0.54, size = 36, normalized size = 1.29

$$-\frac{c\sqrt{\frac{c}{b^2x^2+2\,abx+a^2}}}{2\left(b^2x+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x+a)^2)^(3/2),x, algorithm="fricas")
[Out] -1/2*c*sqrt(c/(b^2*x^2 + 2*a*b*x + a^2))/(b^2*x + a*b)
```

giac [A] time = 0.16, size = 21, normalized size = 0.75

$$-\frac{c^{\frac{3}{2}}\operatorname{sgn}\left(bx+a\right)}{2\left(bx+a\right)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x+a)^2)^(3/2),x, algorithm="giac")
[Out] -1/2*c^(3/2)*sgn(b*x + a)/((b*x + a)^2*b)
```

maple [A] time = 0.00, size = 22, normalized size = 0.79

$$-\frac{\left(bx+a\right)\left(\frac{c}{\left(bx+a\right)^2}\right)^{\frac{3}{2}}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(b*x+a)^2*c)^(3/2),x)
[Out] -1/2*(b*x+a)/b*(1/(b*x+a)^2*c)^(3/2)
```

maxima [A] time = 0.66, size = 27, normalized size = 0.96

$$-\frac{c^{\frac{3}{2}}}{2\left(b^3x^2+2\,ab^2x+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x+a)^2)^(3/2),x, algorithm="maxima")
[Out] -1/2*c^(3/2)/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

mupad [B] time = 1.17, size = 24, normalized size = 0.86

$$-\frac{c\sqrt{\frac{c}{\left(a+b\,x\right)^2}}}{2\,b\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/(a + b*x)^2)^(3/2),x)
[Out] -(c*(c/(a + b*x)^2)^(1/2))/(2*b*(a + b*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{c}{(a+bx)^2}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x+a)**2)**(3/2),x)
[Out] Integral((c/(a + b*x)**2)**(3/2), x)
```

3.2827 $\int \sqrt{\frac{c}{(a+bx)^2}}\,dx$

Optimal. Leaf size=28

$$\frac{(a+bx)\sqrt{\frac{c}{(a+bx)^2}}\,\log(a+bx)}{b}$$

```
[Out] (b*x+a)*ln(b*x+a)*(c/(b*x+a)^2)^(1/2)/b
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 15, 29}

$$\frac{(a+bx)\sqrt{\frac{c}{(a+bx)^2}}\,\log(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c/(a + b*x)^2],x]
```

```
[Out] (Sqrt[c/(a + b*x)^2]*(a + b*x)*Log[a + b*x])/b
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{c}{(a+bx)^2}}\,dx &= \frac{\text{Subst}\left(\int \sqrt{\frac{c}{x^2}}\,dx, x, a+bx\right)}{b}\\ &= \frac{\left(\sqrt{\frac{c}{(a+bx)^2}}\,(a+bx)\right)\text{Subst}\left(\int \frac{1}{x}\,dx, x, a+bx\right)}{b}\\ &= \frac{\sqrt{\frac{c}{(a+bx)^2}}\,(a+bx)\log(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$\frac{(a+bx)\sqrt{\frac{c}{(a+bx)^2}}\,\log(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c/(a + b*x)^2],x]
[Out] (Sqrt[c/(a + b*x)^2]*(a + b*x)*Log[a + b*x])/b
```

fricas [A] time = 0.72, size = 37, normalized size = 1.32

$$\frac{(bx+a)\sqrt{\frac{c}{b^2x^2+2\,abx+a^2}}\,\log(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x+a)^2)^(1/2),x, algorithm="fricas")
[Out] (b*x + a)*sqrt(c/(b^2*x^2 + 2*a*b*x + a^2))*log(b*x + a)/b
```

giac [A] time = 0.17, size = 20, normalized size = 0.71

$$\frac{\sqrt{c}\,\log(|bx+a|)\,\mathrm{sgn}(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x+a)^2)^(1/2),x, algorithm="giac")
[Out] sqrt(c)*log(abs(b*x + a))*sgn(b*x + a)/b
```

maple [A] time = 0.01, size = 27, normalized size = 0.96

$$\frac{(bx+a)\,\sqrt{\frac{c}{(bx+a)^2}}\,\ln(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(b*x+a)^2*c)^(1/2),x)
[Out] (b*x+a)*ln(b*x+a)*(1/(b*x+a)^2*c)^(1/2)/b
```

maxima [A] time = 0.66, size = 13, normalized size = 0.46

$$\frac{\sqrt{c}\,\log(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x+a)^2)^(1/2),x, algorithm="maxima")
[Out] sqrt(c)*log(b*x + a)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{\frac{c}{(a+b\,x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/(a + b*x)^2)^(1/2),x)
[Out] int((c/(a + b*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{c}{(a+bx)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x+a)**2)**(1/2),x)
[Out] Integral(sqrt(c/(a + b*x)**2), x)
```

3.2828 $\int \frac{1}{\sqrt{\frac{c}{(a+bx)^2}}}\,dx$

Optimal. Leaf size=25

$$\frac{a+bx}{2b\sqrt{\frac{c}{(a+bx)^2}}}$$

```
[Out] 1/2*(b*x+a)/b/(c/(b*x+a)^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 15, 30}

$$\frac{a+bx}{2b\sqrt{\frac{c}{(a+bx)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c/(a + b*x)^2],x]
```

```
[Out] (a + b*x)/(2*b*Sqrt[c/(a + b*x)^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\frac{c}{(a+bx)^2}}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\frac{c}{x^2}}}\,dx, x, a+bx\right)}{b}\\ &= \frac{\text{Subst}(\int x\,dx, x, a+bx)}{b\sqrt{\frac{c}{(a+bx)^2}}\,(a+bx)}\\ &= \frac{a+bx}{2b\sqrt{\frac{c}{(a+bx)^2}}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 1.28

$$\frac{x(2a+bx)}{2(a+bx)\sqrt{\frac{c}{(a+bx)^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c/(a + b*x)^2],x]
[Out] (x*(2*a + b*x))/(2*Sqrt[c/(a + b*x)^2]*(a + b*x))
```

fricas [B] time = 0.85, size = 48, normalized size = 1.92

$$\frac{\left(b^2x^3 + 3\,abx^2 + 2\,a^2x\right)\sqrt{\frac{c}{b^2x^2+2\,abx+a^2}}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*(b^2*x^3 + 3*a*b*x^2 + 2*a^2*x)*sqrt(c/(b^2*x^2 + 2*a*b*x + a^2))/c
```

giac [A] time = 0.25, size = 23, normalized size = 0.92

$$\frac{bx^2 + 2\,ax}{2\,\sqrt{c}\,\mathrm{sgn}\left(bx + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/2*(b*x^2 + 2*a*x)/(sqrt(c)*sgn(b*x + a))
```

maple [A] time = 0.00, size = 29, normalized size = 1.16

$$\frac{\left(bx + 2a\right)x}{2\left(bx + a\right)\sqrt{\frac{c}{\left(bx+a\right)^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/(b*x+a)^2*c)^(1/2),x)
[Out] 1/2*x*(b*x+2*a)/(b*x+a)/(1/(b*x+a)^2*c)^(1/2)
```

maxima [A] time = 0.61, size = 15, normalized size = 0.60

$$\frac{bx^2 + 2\,ax}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a)^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*(b*x^2 + 2*a*x)/sqrt(c)
```

mupad [B] time = 1.22, size = 42, normalized size = 1.68

$$\sqrt{\frac{c}{\left(a + b\,x\right)^2}}\left(\frac{a^2\,x}{c} + \frac{b^2\,x^3}{2\,c} + \frac{3\,a\,b\,x^2}{2\,c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c/(a + b*x)^2)^(1/2),x)
[Out] (c/(a + b*x)^2)^(1/2)*((a^2*x)/c + (b^2*x^3)/(2*c) + (3*a*b*x^2)/(2*c))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{c}{(a+bx)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(c/(a + b*x)**2), x)
```

3.2829 $\int \frac{1}{\left(\frac{c}{(a+bx)^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=30

$$\frac{(a+bx)^3}{4bc\sqrt{\frac{c}{(a+bx)^2}}}$$

```
[Out] 1/4*(b*x+a)^3/b/c/(c/(b*x+a)^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 15, 30}

$$\frac{(a+bx)^3}{4bc\sqrt{\frac{c}{(a+bx)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c/(a + b*x)^2)^(-3/2),x]
```

```
[Out] (a + b*x)^3/(4*b*c*Sqrt[c/(a + b*x)^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{c}{(a+bx)^2}\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\left(\frac{c}{x^2}\right)^{3/2}}\,dx, x, a+bx\right)}{b}\\ &= \frac{\text{Subst}\left(\int x^3\,dx, x, a+bx\right)}{bc\sqrt{\frac{c}{(a+bx)^2}}\,(a+bx)}\\ &= \frac{(a+bx)^3}{4bc\sqrt{\frac{c}{(a+bx)^2}}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.83

$$\frac{a+bx}{4b\left(\frac{c}{(a+bx)^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c/(a + b*x)^2)^(-3/2),x]
```

```
[Out] (a + b*x)/(4*b*(c/(a + b*x)^2)^(3/2))
```

fricas [B] time = 0.90, size = 70, normalized size = 2.33

$$\frac{\left(b^4x^5+5\,ab^3x^4+10\,a^2b^2x^3+10\,a^3bx^2+4\,a^4x\right)\sqrt{\frac{c}{b^2x^2+2\,abx+a^2}}}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(b^4*x^5 + 5*a*b^3*x^4 + 10*a^2*b^2*x^3 + 10*a^3*b*x^2 + 4*a^4*x)*sqrt(
c/(b^2*x^2 + 2*a*b*x + a^2))/c^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{c}{(bx+a)^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c/(b*x + a)^2)^(-3/2), x)
```

maple [A] time = 0.00, size = 51, normalized size = 1.70

$$\frac{\left(b^3x^3+4a\,b^2x^2+6a^2bx+4a^3\right)x}{4\,(bx+a)^3\left(\frac{c}{(bx+a)^2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/(b*x+a)^2*c)^(3/2),x)
```

```
[Out] 1/4*x*(b^3*x^3+4*a*b^2*x^2+6*a^2*b*x+4*a^3)/(b*x+a)^3/(1/(b*x+a)^2*c)^(3/2)
```

maxima [A] time = 0.58, size = 37, normalized size = 1.23

$$\frac{b^3x^4+4\,ab^2x^3+6\,a^2bx^2+4\,a^3x}{4\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(b^3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2 + 4*a^3*x)/c^(3/2)
```

mupad [B] time = 1.29, size = 70, normalized size = 2.33

$$\sqrt{\frac{c}{(a+b\,x)^2}}\,\left(\frac{a^4\,x}{c^2}+\frac{b^4\,x^5}{4\,c^2}+\frac{5\,a^3\,b\,x^2}{2\,c^2}+\frac{5\,a\,b^3\,x^4}{4\,c^2}+\frac{5\,a^2\,b^2\,x^3}{2\,c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c/(a + b*x)^2)^(3/2),x)
```

```
[Out] (c/(a + b*x)^2)^(1/2)*((a^4*x)/c^2 + (b^4*x^5)/(4*c^2) + (5*a^3*b*x^2)/(2*c
^2) + (5*a*b^3*x^4)/(4*c^2) + (5*a^2*b^2*x^3)/(2*c^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{c}{(a+bx)^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a)**2)**(3/2),x)
```

```
[Out] Integral((c/(a + b*x)**2)**(-3/2), x)
```

3.2830 $\int \frac{1}{\left(\frac{c}{(a+bx)^2}\right)^{5/2}}\,dx$

Optimal. Leaf size=30

$$\frac{(a+bx)^5}{6bc^2\sqrt{\frac{c}{(a+bx)^2}}}$$

```
[Out] 1/6*(b*x+a)^5/b/c^2/(c/(b*x+a)^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 15, 30}

$$\frac{(a+bx)^5}{6bc^2\sqrt{\frac{c}{(a+bx)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c/(a + b*x)^2)^(-5/2),x]
```

```
[Out] (a + b*x)^5/(6*b*c^2*Sqrt[c/(a + b*x)^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{c}{(a+bx)^2}\right)^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\left(\frac{c}{x^2}\right)^{5/2}}\,dx, x, a+bx\right)}{b}\\ &= \frac{\text{Subst}\left(\int x^5\,dx, x, a+bx\right)}{bc^2\sqrt{\frac{c}{(a+bx)^2}}\,(a+bx)}\\ &= \frac{(a+bx)^5}{6bc^2\sqrt{\frac{c}{(a+bx)^2}}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 0.83

$$\frac{a+bx}{6b\left(\frac{c}{(a+bx)^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c/(a + b*x)^2)^(-5/2),x]
```

```
[Out] (a + b*x)/(6*b*(c/(a + b*x)^2)^(5/2))
```

fricas [B] time = 0.82, size = 92, normalized size = 3.07

$$\frac{\left(b^6x^7+7\,ab^5x^6+21\,a^2b^4x^5+35\,a^3b^3x^4+35\,a^4b^2x^3+21\,a^5bx^2+6\,a^6x\right)\sqrt{\frac{c}{b^2x^2+2\,abx+a^2}}}{6\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(b^6*x^7 + 7*a*b^5*x^6 + 21*a^2*b^4*x^5 + 35*a^3*b^3*x^4 + 35*a^4*b^2*x
^3 + 21*a^5*b*x^2 + 6*a^6*x)*sqrt(c/(b^2*x^2 + 2*a*b*x + a^2))/c^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{c}{(bx+a)^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c/(b*x + a)^2)^(-5/2), x)
```

maple [B] time = 0.00, size = 73, normalized size = 2.43

$$\frac{\left(b^5x^5+6a\,b^4x^4+15a^2b^3x^3+20a^3b^2x^2+15a^4bx+6a^5\right)x}{6\,(bx+a)^5\left(\frac{c}{(bx+a)^2}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/(b*x+a)^2*c)^(5/2),x)
```

```
[Out] 1/6*x*(b^5*x^5+6*a*b^4*x^4+15*a^2*b^3*x^3+20*a^3*b^2*x^2+15*a^4*b*x+6*a^5)/
(b*x+a)^5/(1/(b*x+a)^2*c)^(5/2)
```

maxima [B] time = 0.71, size = 59, normalized size = 1.97

$$\frac{b^5x^6+6\,ab^4x^5+15\,a^2b^3x^4+20\,a^3b^2x^3+15\,a^4bx^2+6\,a^5x}{6\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(b^5*x^6 + 6*a*b^4*x^5 + 15*a^2*b^3*x^4 + 20*a^3*b^2*x^3 + 15*a^4*b*x^2
 + 6*a^5*x)/c^(5/2)
```

mupad [B] time = 1.33, size = 98, normalized size = 3.27

$$\sqrt{\frac{c}{(a+b\,x)^2}}\left(\frac{a^6\,x}{c^3}+\frac{b^6\,x^7}{6\,c^3}+\frac{7\,a^5\,b\,x^2}{2\,c^3}+\frac{7\,a\,b^5\,x^6}{6\,c^3}+\frac{35\,a^4\,b^2\,x^3}{6\,c^3}+\frac{35\,a^3\,b^3\,x^4}{6\,c^3}+\frac{7\,a^2\,b^4\,x^5}{2\,c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c/(a + b*x)^2)^(5/2),x)
```

```
[Out] (c/(a + b*x)^2)^(1/2)*((a^6*x)/c^3 + (b^6*x^7)/(6*c^3) + (7*a^5*b*x^2)/(2*c
^3) + (7*a*b^5*x^6)/(6*c^3) + (35*a^4*b^2*x^3)/(6*c^3) + (35*a^3*b^3*x^4)/(
6*c^3) + (7*a^2*b^4*x^5)/(2*c^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{c}{(a+bx)^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a)**2)**(5/2),x)
```

```
[Out] Integral((c/(a + b*x)**2)**(-5/2), x)
```

3.2831 $\int \left(\frac{c}{(a+bx)^3}\right)^{5/2} dx$

Optimal. Leaf size=30

$$-\frac{2c^2\sqrt{\frac{c}{(a+bx)^3}}}{13b(a+bx)^5}$$

```
[Out] -2/13*c^2*(c/(b*x+a)^3)^(1/2)/b/(b*x+a)^5
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 15, 30}

$$-\frac{2c^2\sqrt{\frac{c}{(a+bx)^3}}}{13b(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c/(a + b*x)^3)^(5/2),x]
```

```
[Out] (-2*c^2*Sqrt[c/(a + b*x)^3])/(13*b*(a + b*x)^5)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{c}{(a+bx)^3}\right)^{5/2} dx &= \frac{\text{Subst}\left(\int \left(\frac{c}{x^3}\right)^{5/2} dx, x, a+bx\right)}{b} \\ &= \frac{\left(c^2\sqrt{\frac{c}{(a+bx)^3}}(a+bx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{x^{15/2}} dx, x, a+bx\right)}{b} \\ &= -\frac{2c^2\sqrt{\frac{c}{(a+bx)^3}}}{13b(a+bx)^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.83

$$-\frac{2(a+bx)\left(\frac{c}{(a+bx)^3}\right)^{5/2}}{13b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c/(a + b*x)^3)^(5/2),x]
```

```
[Out] (-2*(c/(a + b*x)^3)^(5/2)*(a + b*x))/(13*b)
```

fricas [B] time = 0.67, size = 93, normalized size = 3.10

$$-\frac{2\,c^2\sqrt{\frac{c}{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3}}}{13\left(b^6x^5+5\,ab^5x^4+10\,a^2b^4x^3+10\,a^3b^3x^2+5\,a^4b^2x+a^5b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x+a)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/13*c^2*sqrt(c/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3))/(b^6*x^5 + 5*a*
b^5*x^4 + 10*a^2*b^4*x^3 + 10*a^3*b^3*x^2 + 5*a^4*b^2*x + a^5*b)
```

giac [A] time = 0.18, size = 52, normalized size = 1.73

$$-\frac{2\,c^9\mathrm{sgn}\left(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3\right)\mathrm{sgn}\left(bx+a\right)}{13\left(bcx+ac\right)^{\frac{13}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x+a)^3)^(5/2),x, algorithm="giac")
```

```
[Out] -2/13*c^9*sgn(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sgn(b*x + a)/((b*c*x
 + a*c)^(13/2)*b)
```

maple [A] time = 0.00, size = 22, normalized size = 0.73

$$-\frac{2\left(bx+a\right)\left(\frac{c}{\left(bx+a\right)^3}\right)^{\frac{5}{2}}}{13b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/(b*x+a)^3)^(5/2),x)
```

```
[Out] -2/13*(b*x+a)*(c/(b*x+a)^3)^(5/2)/b
```

maxima [A] time = 0.69, size = 24, normalized size = 0.80

$$-\frac{2\left(bc^{\frac{5}{2}}x+ac^{\frac{5}{2}}\right)}{13\left(bx+a\right)^{\frac{15}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x+a)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/13*(b*c^(5/2)*x + a*c^(5/2))/((b*x + a)^(15/2)*b)
```

mupad [B] time = 1.24, size = 55, normalized size = 1.83

$$-\frac{2\,b^2\,c^2\,\sqrt{\frac{c}{\left(a+b\,x\right)^3}}\,\left(x^3+\frac{a^3}{b^3}+\frac{3\,a\,x^2}{b}+\frac{3\,a^2\,x}{b^2}\right)}{13\left(a+b\,x\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/(a + b*x)^3)^(5/2),x)
```

```
[Out] -(2*b^2*c^2*(c/(a + b*x)^3)^(1/2)*(x^3 + a^3/b^3 + (3*a*x^2)/b + (3*a^2*x)/
b^2))/(13*(a + b*x)^8)
```

sympy [A] time = 113.94, size = 1294, normalized size = 43.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x+a)**3)**(5/2),x)
```

```
[Out] Piecewise((-638*a**6*c**(5/2)*(1/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*
x**3))**(5/2)/(91*a**5*b + 455*a**4*b**2*x + 910*a**3*b**3*x**2 + 910*a**2*
b**4*x**3 + 455*a*b**5*x**4 + 91*b**6*x**5) - 3828*a**5*b*c**(5/2)*x*(1/(a*
*3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))**(5/2)/(91*a**5*b + 455*a**4*
b**2*x + 910*a**3*b**3*x**2 + 910*a**2*b**4*x**3 + 455*a*b**5*x**4 + 91*b**
6*x**5) - 9570*a**4*b**2*c**(5/2)*x**2*(1/(a**3 + 3*a**2*b*x + 3*a*b**2*x**
2 + b**3*x**3))**(5/2)/(91*a**5*b + 455*a**4*b**2*x + 910*a**3*b**3*x**2 +
910*a**2*b**4*x**3 + 455*a*b**5*x**4 + 91*b**6*x**5) - 12760*a**3*b**3*c**(
5/2)*x**3*(1/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))**(5/2)/(91*a*
*5*b + 455*a**4*b**2*x + 910*a**3*b**3*x**2 + 910*a**2*b**4*x**3 + 455*a*b*
*5*x**4 + 91*b**6*x**5) - 1170*a**3*c**(5/2)*(1/(a**3 + 3*a**2*b*x + 3*a*b*
*2*x**2 + b**3*x**3))**(3/2)/(91*a**5*b + 455*a**4*b**2*x + 910*a**3*b**3*x
**2 + 910*a**2*b**4*x**3 + 455*a*b**5*x**4 + 91*b**6*x**5) - 9570*a**2*b**4
*c**(5/2)*x**4*(1/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))**(5/2)/(
91*a**5*b + 455*a**4*b**2*x + 910*a**3*b**3*x**2 + 910*a**2*b**4*x**3 + 455
*a*b**5*x**4 + 91*b**6*x**5) - 3510*a**2*b*c**(5/2)*x*(1/(a**3 + 3*a**2*b*x
 + 3*a*b**2*x**2 + b**3*x**3))**(3/2)/(91*a**5*b + 455*a**4*b**2*x + 910*a*
*3*b**3*x**2 + 910*a**2*b**4*x**3 + 455*a*b**5*x**4 + 91*b**6*x**5) - 3828*
a*b**5*c**(5/2)*x**5*(1/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))**(
5/2)/(91*a**5*b + 455*a**4*b**2*x + 910*a**3*b**3*x**2 + 910*a**2*b**4*x**3
 + 455*a*b**5*x**4 + 91*b**6*x**5) - 3510*a*b**2*c**(5/2)*x**2*(1/(a**3 + 3
*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))**(3/2)/(91*a**5*b + 455*a**4*b**2*x
 + 910*a**3*b**3*x**2 + 910*a**2*b**4*x**3 + 455*a*b**5*x**4 + 91*b**6*x**5
) - 638*b**6*c**(5/2)*x**6*(1/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**
3))**(5/2)/(91*a**5*b + 455*a**4*b**2*x + 910*a**3*b**3*x**2 + 910*a**2*b**
4*x**3 + 455*a*b**5*x**4 + 91*b**6*x**5) - 1170*b**3*c**(5/2)*x**3*(1/(a**3
 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))**(3/2)/(91*a**5*b + 455*a**4*b*
*2*x + 910*a**3*b**3*x**2 + 910*a**2*b**4*x**3 + 455*a*b**5*x**4 + 91*b**6*
x**5) + 1794*c**(5/2)*sqrt(1/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3
))/(91*a**5*b + 455*a**4*b**2*x + 910*a**3*b**3*x**2 + 910*a**2*b**4*x**3 +
 455*a*b**5*x**4 + 91*b**6*x**5), Ne(b, 0)), (x*(c/a**3)**(5/2), True))
```

3.2832 $\int \left(\frac{c}{(a+bx)^3}\right)^{3/2} dx$

Optimal. Leaf size=28

$$-\frac{2c\sqrt{\frac{c}{(a+bx)^3}}}{7b(a+bx)^2}$$

```
[Out] -2/7*c*(c/(b*x+a)^3)^(1/2)/b/(b*x+a)^2
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 15, 30}

$$-\frac{2c\sqrt{\frac{c}{(a+bx)^3}}}{7b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c/(a + b*x)^3)^(3/2),x]
```

```
[Out] (-2*c*Sqrt[c/(a + b*x)^3])/(7*b*(a + b*x)^2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{c}{(a+bx)^3}\right)^{3/2} dx &= \frac{\text{Subst}\left(\int \left(\frac{c}{x^3}\right)^{3/2} dx, x, a+bx\right)}{b} \\ &= \frac{\left(c\sqrt{\frac{c}{(a+bx)^3}}\,(a+bx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{x^{9/2}} dx, x, a+bx\right)}{b} \\ &= -\frac{2c\sqrt{\frac{c}{(a+bx)^3}}}{7b(a+bx)^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.89

$$-\frac{2(a+bx)\left(\frac{c}{(a+bx)^3}\right)^{3/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c/(a + b*x)^3)^(3/2),x]
```

```
[Out] (-2*(c/(a + b*x)^3)^(3/2)*(a + b*x))/(7*b)
```

fricas [B] time = 0.85, size = 58, normalized size = 2.07

$$-\frac{2\,c\sqrt{\frac{c}{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3}}}{7\left(b^3x^2+2\,ab^2x+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x+a)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/7*c*sqrt(c/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3))/(b^3*x^2 + 2*a*b^2
*x + a^2*b)
```

giac [B] time = 0.18, size = 52, normalized size = 1.86

$$-\frac{2\,c^5\mathrm{sgn}\left(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3\right)\mathrm{sgn}\left(bx+a\right)}{7\left(bcx+ac\right)^{\frac{7}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x+a)^3)^(3/2),x, algorithm="giac")
```

```
[Out] -2/7*c^5*sgn(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sgn(b*x + a)/((b*c*x
+ a*c)^(7/2)*b)
```

maple [A] time = 0.00, size = 22, normalized size = 0.79

$$-\frac{2\left(bx+a\right)\left(\frac{c}{\left(bx+a\right)^3}\right)^{\frac{3}{2}}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(b*x+a)^3*c)^(3/2),x)
```

```
[Out] -2/7*(b*x+a)*(1/(b*x+a)^3*c)^(3/2)/b
```

maxima [A] time = 0.60, size = 24, normalized size = 0.86

$$-\frac{2\left(bc^{\frac{3}{2}}x+ac^{\frac{3}{2}}\right)}{7\left(bx+a\right)^{\frac{9}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x+a)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/7*(b*c^(3/2)*x + a*c^(3/2))/((b*x + a)^(9/2)*b)
```

mupad [B] time = 1.20, size = 24, normalized size = 0.86

$$-\frac{2\,c\,\sqrt{\frac{c}{\left(a+b\,x\right)^3}}}{7\,b\,\left(a+b\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/(a + b*x)^3)^(3/2),x)
```

```
[Out] -(2*c*(c/(a + b*x)^3)^(1/2))/(7*b*(a + b*x)^2)
```

sympy [A] time = 8.92, size = 357, normalized size = 12.75

$$\begin{cases} -\frac{67a^3c^{\frac{3}{2}}\left(\frac{1}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\right)^{\frac{3}{2}}}{14a^2b+28ab^2x+14b^3x^2} - \frac{201a^2bc^{\frac{3}{2}}x\left(\frac{1}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\right)^{\frac{3}{2}}}{14a^2b+28ab^2x+14b^3x^2} - \frac{201ab^2c^{\frac{3}{2}}x^2\left(\frac{1}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\right)^{\frac{3}{2}}}{14a^2b+28ab^2x+14b^3x^2} - \frac{67b^3c^{\frac{3}{2}}x^3\left(\frac{1}{a^3+3a^2bx+3ab^2x^2} \right.} \\ x\left(\frac{c}{a^3}\right)^{\frac{3}{2}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x+a)**3)**(3/2),x)
```

```
[Out] Piecewise((-67*a**3*c**(3/2)*(1/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x
**3))**(3/2)/(14*a**2*b + 28*a*b**2*x + 14*b**3*x**2) - 201*a**2*b*c**(3/2)
*x*(1/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))**(3/2)/(14*a**2*b +
28*a*b**2*x + 14*b**3*x**2) - 201*a*b**2*c**(3/2)*x**2*(1/(a**3 + 3*a**2*b*
x + 3*a*b**2*x**2 + b**3*x**3))**(3/2)/(14*a**2*b + 28*a*b**2*x + 14*b**3*x
**2) - 67*b**3*c**(3/2)*x**3*(1/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x
**3))**(3/2)/(14*a**2*b + 28*a*b**2*x + 14*b**3*x**2) + 63*c**(3/2)*sqrt(1/
(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))/(14*a**2*b + 28*a*b**2*x +
 14*b**3*x**2), Ne(b, 0)), (x*(c/a**3)**(3/2), True))
```

3.2833 $\int \sqrt{\frac{c}{(a+bx)^3}}\,dx$

Optimal. Leaf size=23

$$-\frac{2(a+bx)\sqrt{\frac{c}{(a+bx)^3}}}{b}$$

```
[Out] -2*(b*x+a)*(c/(b*x+a)^3)^(1/2)/b
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 15, 30}

$$-\frac{2(a+bx)\sqrt{\frac{c}{(a+bx)^3}}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c/(a + b*x)^3],x]
```

```
[Out] (-2*Sqrt[c/(a + b*x)^3]*(a + b*x))/b
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{c}{(a+bx)^3}}\,dx &= \frac{\text{Subst}\left(\int \sqrt{\frac{c}{x^3}}\,dx, x, a+bx\right)}{b}\\ &= \frac{\left(\sqrt{\frac{c}{(a+bx)^3}}\,(a+bx)^{3/2}\right)\text{Subst}\left(\int \frac{1}{x^{3/2}}\,dx, x, a+bx\right)}{b}\\ &= -\frac{2\sqrt{\frac{c}{(a+bx)^3}}\,(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$-\frac{2(a+bx)\sqrt{\frac{c}{(a+bx)^3}}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c/(a + b*x)^3],x]
[Out] (-2*Sqrt[c/(a + b*x)^3]*(a + b*x))/b
```

fricas [B] time = 0.83, size = 43, normalized size = 1.87

$$-\frac{2\,(bx+a)\sqrt{\frac{c}{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x+a)^3)^(1/2),x, algorithm="fricas")
[Out] -2*(b*x + a)*sqrt(c/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3))/b
```

giac [B] time = 0.39, size = 50, normalized size = 2.17

$$-\frac{2\,c\mathrm{sgn}\left(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3\right)\mathrm{sgn}\left(bx+a\right)}{\sqrt{bcx+ac}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x+a)^3)^(1/2),x, algorithm="giac")
[Out] -2*c*sgn(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sgn(b*x + a)/(sqrt(b*c*x
+ a*c)*b)
```

maple [A] time = 0.00, size = 22, normalized size = 0.96

$$-\frac{2\,(bx+a)\,\sqrt{\frac{c}{(bx+a)^3}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(b*x+a)^3*c)^(1/2),x)
[Out] -2*(b*x+a)*(1/(b*x+a)^3*c)^(1/2)/b
```

maxima [A] time = 0.57, size = 24, normalized size = 1.04

$$-\frac{2\left(b\sqrt{c}\,x+a\sqrt{c}\right)}{(bx+a)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x+a)^3)^(1/2),x, algorithm="maxima")
[Out] -2*(b*sqrt(c)*x + a*sqrt(c))/((b*x + a)^(3/2)*b)
```

mupad [B] time = 1.16, size = 52, normalized size = 2.26

$$-\frac{2\,b^2\,\sqrt{\frac{c}{(a+b\,x)^3}}\,\left(x^3+\frac{a^3}{b^3}+\frac{3\,a\,x^2}{b}+\frac{3\,a^2\,x}{b^2}\right)}{(a+b\,x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/(a + b*x)^3)^(1/2),x)
[Out] -(2*b^2*(c/(a + b*x)^3)^(1/2)*(x^3 + a^3/b^3 + (3*a*x^2)/b + (3*a^2*x)/b^2)
)/(a + b*x)^2
```

sympy [**A**] time = 0.87, size = 97, normalized size = 4.22

$$\begin{cases} -\dfrac{2a\sqrt{c}\sqrt{\frac{1}{a^3+3a^2bx+3ab^2x^2+b^3x^3}}}{b} - 2\sqrt{c}\,x\sqrt{\dfrac{1}{a^3+3a^2bx+3ab^2x^2+b^3x^3}} & \text{for } b \neq 0 \\ x\sqrt{\dfrac{c}{a^3}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x+a)**3)**(1/2),x)
```

```
[Out] Piecewise((-2*a*sqrt(c)*sqrt(1/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**
**3))/b - 2*sqrt(c)*x*sqrt(1/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)
), Ne(b, 0)), (x*sqrt(c/a**3), True))
```

3.2834 $\int \frac{1}{\sqrt{\frac{c}{(a+bx)^3}}}\,dx$

Optimal. Leaf size=25

$$\frac{2(a+bx)}{5b\sqrt{\frac{c}{(a+bx)^3}}}$$

```
[Out] 2/5*(b*x+a)/b/(c/(b*x+a)^3)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 15, 30}

$$\frac{2(a+bx)}{5b\sqrt{\frac{c}{(a+bx)^3}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c/(a + b*x)^3],x]
```

```
[Out] (2*(a + b*x))/(5*b*Sqrt[c/(a + b*x)^3])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\frac{c}{(a+bx)^3}}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\frac{c}{x^3}}}\,dx, x, a+bx\right)}{b}\\ &= \frac{\text{Subst}\left(\int x^{3/2}\,dx, x, a+bx\right)}{b\sqrt{\frac{c}{(a+bx)^3}}\,(a+bx)^{3/2}}\\ &= \frac{2(a+bx)}{5b\sqrt{\frac{c}{(a+bx)^3}}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 1.00

$$\frac{2(a+bx)}{5b\sqrt{\frac{c}{(a+bx)^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c/(a + b*x)^3],x]
```

```
[Out] (2*(a + b*x))/(5*b*Sqrt[c/(a + b*x)^3])
```

fricas [B] time = 0.82, size = 79, normalized size = 3.16

$$\frac{2\left(b^4x^4+4\,ab^3x^3+6\,a^2b^2x^2+4\,a^3bx+a^4\right)\sqrt{\frac{c}{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3}}}{5\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4)*sqrt(c/(b^3*x
^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3))/(b*c)
```

giac [B] time = 0.36, size = 144, normalized size = 5.76

$$\frac{2\left(15\sqrt{bcx+ac}\,a^2-\frac{10\left(3\sqrt{bcx+ac}\,ac-(bcx+ac)^{\frac{3}{2}}\right)a}{c}+\frac{15\sqrt{bcx+ac}\,a^2c^2-10\,(bcx+ac)^{\frac{3}{2}}ac+3\,(bcx+ac)^{\frac{5}{2}}}{c^2}\right)}{15\,bc\operatorname{sgn}\left(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3\right)\operatorname{sgn}\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a)^3)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(15*sqrt(b*c*x + a*c)*a^2 - 10*(3*sqrt(b*c*x + a*c)*a*c - (b*c*x + a*c
)^(3/2))*a/c + (15*sqrt(b*c*x + a*c)*a^2*c^2 - 10*(b*c*x + a*c)^(3/2)*a*c +
 3*(b*c*x + a*c)^(5/2))/c^2)/(b*c*sgn(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a
^3)*sgn(b*x + a))
```

maple [A] time = 0.00, size = 22, normalized size = 0.88

$$\frac{\frac{2bx}{5}+\frac{2a}{5}}{\sqrt{\frac{c}{(bx+a)^3}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/(b*x+a)^3*c)^(1/2),x)
```

```
[Out] 2/5*(b*x+a)/b/(1/(b*x+a)^3*c)^(1/2)
```

maxima [A] time = 0.56, size = 27, normalized size = 1.08

$$\frac{2\left(b\sqrt{c}\,x+a\sqrt{c}\right)(bx+a)^{\frac{3}{2}}}{5\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b*sqrt(c)*x + a*sqrt(c))*(b*x + a)^(3/2)/(b*c)
```

mupad [B] time = 1.25, size = 26, normalized size = 1.04

$$\frac{2\sqrt{\frac{c}{(a+b\,x)^3}}\,(a+b\,x)^4}{5\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c/(a + b*x)^3)^(1/2),x)
[Out] (2*(c/(a + b*x)^3)^(1/2)*(a + b*x)^4)/(5*b*c)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{c}{(a+bx)^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a)**3)**(1/2),x)
[Out] Integral(1/sqrt(c/(a + b*x)**3), x)
```

3.2835 $\int \frac{1}{\left(\frac{c}{(a+bx)^3}\right)^{3/2}}\,dx$

Optimal. Leaf size=30

$$\frac{2(a+bx)^4}{11bc\sqrt{\frac{c}{(a+bx)^3}}}$$

```
[Out] 2/11*(b*x+a)^4/b/c/(c/(b*x+a)^3)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 15, 30}

$$\frac{2(a+bx)^4}{11bc\sqrt{\frac{c}{(a+bx)^3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c/(a + b*x)^3)^(-3/2),x]
```

```
[Out] (2*(a + b*x)^4)/(11*b*c*Sqrt[c/(a + b*x)^3])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{c}{(a+bx)^3}\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\left(\frac{c}{x^3}\right)^{3/2}}\,dx, x, a+bx\right)}{b}\\ &= \frac{\text{Subst}\left(\int x^{9/2}\,dx, x, a+bx\right)}{bc\sqrt{\frac{c}{(a+bx)^3}}\,(a+bx)^{3/2}}\\ &= \frac{2(a+bx)^4}{11bc\sqrt{\frac{c}{(a+bx)^3}}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 0.83

$$\frac{2(a+bx)}{11b\left(\frac{c}{(a+bx)^3}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c/(a + b*x)^3)^(-3/2),x]
[Out] (2*(a + b*x))/(11*b*(c/(a + b*x)^3)^(3/2))
```

fricas [B] time = 0.81, size = 112, normalized size = 3.73

$$\frac{2\left(b^7x^7+7\,ab^6x^6+21\,a^2b^5x^5+35\,a^3b^4x^4+35\,a^4b^3x^3+21\,a^5b^2x^2+7\,a^6bx+a^7\right)\sqrt{\frac{c}{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3}}}{11\,bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a)^3)^(3/2),x, algorithm="fricas")
[Out] 2/11*(b^7*x^7 + 7*a*b^6*x^6 + 21*a^2*b^5*x^5 + 35*a^3*b^4*x^4 + 35*a^4*b^3*
x^3 + 21*a^5*b^2*x^2 + 7*a^6*b*x + a^7)*sqrt(c/(b^3*x^3 + 3*a*b^2*x^2 + 3*a
^2*b*x + a^3))/(b*c^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{c}{(bx+a)^3}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a)^3)^(3/2),x, algorithm="giac")
[Out] integrate((c/(b*x + a)^3)^(-3/2), x)
```

maple [A] time = 0.00, size = 22, normalized size = 0.73

$$\frac{\frac{2bx}{11}+\frac{2a}{11}}{\left(\frac{c}{(bx+a)^3}\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/(b*x+a)^3*c)^(3/2),x)
[Out] 2/11*(b*x+a)/b/(1/(b*x+a)^3*c)^(3/2)
```

maxima [A] time = 0.73, size = 27, normalized size = 0.90

$$\frac{2\left(b\sqrt{c}\,x+a\sqrt{c}\right)(bx+a)^{\frac{9}{2}}}{11\,bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a)^3)^(3/2),x, algorithm="maxima")
[Out] 2/11*(b*sqrt(c)*x + a*sqrt(c))*(b*x + a)^(9/2)/(b*c^2)
```

mupad [B] time = 1.38, size = 110, normalized size = 3.67

$$\sqrt{\frac{c}{(a+b\,x)^3}}\left(\frac{14\,a^6\,x}{11\,c^2}+\frac{2\,a^7}{11\,b\,c^2}+\frac{2\,b^6\,x^7}{11\,c^2}+\frac{42\,a^5\,b\,x^2}{11\,c^2}+\frac{14\,a\,b^5\,x^6}{11\,c^2}+\frac{70\,a^4\,b^2\,x^3}{11\,c^2}+\frac{70\,a^3\,b^3\,x^4}{11\,c^2}+\frac{42\,a^2\,b^4\,x^5}{11\,c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c/(a + b*x)^3)^(3/2),x)
```

```
[Out] (c/(a + b*x)^3)^(1/2)*((14*a^6*x)/(11*c^2) + (2*a^7)/(11*b*c^2) + (2*b^6*x^
7)/(11*c^2) + (42*a^5*b*x^2)/(11*c^2) + (14*a*b^5*x^6)/(11*c^2) + (70*a^4*b
^2*x^3)/(11*c^2) + (70*a^3*b^3*x^4)/(11*c^2) + (42*a^2*b^4*x^5)/(11*c^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{c}{(a+bx)^3}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a)**3)**(3/2),x)
```

```
[Out] Integral((c/(a + b*x)**3)**(-3/2), x)
```

3.2836 $\int \frac{1}{\left(\frac{c}{(a+bx)^3}\right)^{5/2}}\,dx$

Optimal. Leaf size=30

$$\frac{2(a+bx)^7}{17bc^2\sqrt{\frac{c}{(a+bx)^3}}}$$

```
[Out] 2/17*(b*x+a)^7/b/c^2/(c/(b*x+a)^3)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 15, 30}

$$\frac{2(a+bx)^7}{17bc^2\sqrt{\frac{c}{(a+bx)^3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c/(a + b*x)^3)^(-5/2),x]
```

```
[Out] (2*(a + b*x)^7)/(17*b*c^2*Sqrt[c/(a + b*x)^3])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{c}{(a+bx)^3}\right)^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\left(\frac{c}{x^3}\right)^{5/2}}\,dx, x, a+bx\right)}{b}\\ &= \frac{\text{Subst}\left(\int x^{15/2}\,dx, x, a+bx\right)}{bc^2\sqrt{\frac{c}{(a+bx)^3}}\,(a+bx)^{3/2}}\\ &= \frac{2(a+bx)^7}{17bc^2\sqrt{\frac{c}{(a+bx)^3}}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 0.83

$$\frac{2(a+bx)}{17b\left(\frac{c}{(a+bx)^3}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c/(a + b*x)^3)^(-5/2),x]
[Out] (2*(a + b*x))/(17*b*(c/(a + b*x)^3)^(5/2))
```

fricas [B] time = 0.88, size = 145, normalized size = 4.83

$$\frac{2\left(b^{10}x^{10}+10\,ab^9x^9+45\,a^2b^8x^8+120\,a^3b^7x^7+210\,a^4b^6x^6+252\,a^5b^5x^5+210\,a^6b^4x^4+120\,a^7b^3x^3+45\,a^8b^2\right.}{17\,bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a)^3)^(5/2),x, algorithm="fricas")
[Out] 2/17*(b^10*x^10 + 10*a*b^9*x^9 + 45*a^2*b^8*x^8 + 120*a^3*b^7*x^7 + 210*a^4
*b^6*x^6 + 252*a^5*b^5*x^5 + 210*a^6*b^4*x^4 + 120*a^7*b^3*x^3 + 45*a^8*b^2
*x^2 + 10*a^9*b*x + a^10)*sqrt(c/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3))
/(b*c^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{c}{(bx+a)^3}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a)^3)^(5/2),x, algorithm="giac")
[Out] integrate((c/(b*x + a)^3)^(-5/2), x)
```

maple [A] time = 0.00, size = 22, normalized size = 0.73

$$\frac{\frac{2bx}{17}+\frac{2a}{17}}{\left(\frac{c}{(bx+a)^3}\right)^{\frac{5}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/(b*x+a)^3*c)^(5/2),x)
[Out] 2/17*(b*x+a)/b/(1/(b*x+a)^3*c)^(5/2)
```

maxima [A] time = 0.57, size = 27, normalized size = 0.90

$$\frac{2\left(b\sqrt{c}\,x+a\sqrt{c}\right)(bx+a)^{\frac{15}{2}}}{17\,bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a)^3)^(5/2),x, algorithm="maxima")
[Out] 2/17*(b*sqrt(c)*x + a*sqrt(c))*(b*x + a)^(15/2)/(b*c^3)
```

mupad [B] time = 1.49, size = 152, normalized size = 5.07

$$\sqrt{\frac{c}{(a+b\,x)^3}}\left(\frac{20\,a^9\,x}{17\,c^3}+\frac{2\,a^{10}}{17\,b\,c^3}+\frac{2\,b^9\,x^{10}}{17\,c^3}+\frac{90\,a^8\,b\,x^2}{17\,c^3}+\frac{20\,a\,b^8\,x^9}{17\,c^3}+\frac{240\,a^7\,b^2\,x^3}{17\,c^3}+\frac{420\,a^6\,b^3\,x^4}{17\,c^3}+\frac{504\,a^5\,b^4\,x^5}{17\,c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c/(a + b*x)^3)^(5/2),x)
```

```
[Out] (c/(a + b*x)^3)^(1/2)*((20*a^9*x)/(17*c^3) + (2*a^10)/(17*b*c^3) + (2*b^9*x
^10)/(17*c^3) + (90*a^8*b*x^2)/(17*c^3) + (20*a*b^8*x^9)/(17*c^3) + (240*a^
7*b^2*x^3)/(17*c^3) + (420*a^6*b^3*x^4)/(17*c^3) + (504*a^5*b^4*x^5)/(17*c^
3) + (420*a^4*b^5*x^6)/(17*c^3) + (240*a^3*b^6*x^7)/(17*c^3) + (90*a^2*b^7*
x^8)/(17*c^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{c}{(a+bx)^3}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a)**3)**(5/2),x)
```

```
[Out] Integral((c/(a + b*x)**3)**(-5/2), x)
```

3.2837 $\int \left(c(a+bx)^{3/2}\right)^{2/3}\,dx$

Optimal. Leaf size=27

$$\frac{(a+bx)\left(c(a+bx)^{3/2}\right)^{2/3}}{2b}$$

```
[Out] 1/2*(b*x+a)*(c*(b*x+a)^(3/2))^(2/3)/b
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {247, 15, 30}

$$\frac{(a+bx)\left(c(a+bx)^{3/2}\right)^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x)^(3/2))^(2/3),x]
```

```
[Out] ((a + b*x)*(c*(a + b*x)^(3/2))^(2/3))/(2*b)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \left(c(a+bx)^{3/2}\right)^{2/3}\,dx &= \frac{\text{Subst}\left(\int \left(cx^{3/2}\right)^{2/3}\,dx, x, a+bx\right)}{b}\\ &= \frac{\left(c(a+bx)^{3/2}\right)^{2/3}\,\text{Subst}(\int x\,dx, x, a+bx)}{b(a+bx)}\\ &= \frac{(a+bx)\left(c(a+bx)^{3/2}\right)^{2/3}}{2b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 1.26

$$\frac{x(2a+bx)\left(c(a+bx)^{3/2}\right)^{2/3}}{2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x)^(3/2))^(2/3),x]
[Out] (x*(c*(a + b*x)^(3/2))^(2/3)*(2*a + b*x))/(2*(a + b*x))
```

fricas [A] time = 0.86, size = 37, normalized size = 1.37

$$\frac{\left(bx^2+2\,ax\right)\left((bcx+ac)\sqrt{bx+a}\right)^{\frac{2}{3}}}{2\,(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x+a)^(3/2))^(2/3),x, algorithm="fricas")
[Out] 1/2*(b*x^2 + 2*a*x)*((b*c*x + a*c)*sqrt(b*x + a))^(2/3)/(b*x + a)
```

giac [A] time = 0.24, size = 15, normalized size = 0.56

$$\frac{(bx+a)^2 c^{\frac{2}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x+a)^(3/2))^(2/3),x, algorithm="giac")
[Out] 1/2*(b*x + a)^2*c^(2/3)/b
```

maple [A] time = 0.00, size = 29, normalized size = 1.07

$$\frac{(bx+2a)\left((bx+a)^{\frac{3}{2}}c\right)^{\frac{2}{3}}x}{2bx+2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(b*x+a)^(3/2))^(2/3),x)
[Out] 1/2*x*(b*x+2*a)*(c*(b*x+a)^(3/2))^(2/3)/(b*x+a)
```

maxima [A] time = 0.50, size = 21, normalized size = 0.78

$$\frac{\left((bx+a)^{\frac{3}{2}}c\right)^{\frac{2}{3}}(bx+a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x+a)^(3/2))^(2/3),x, algorithm="maxima")
[Out] 1/2*((b*x + a)^(3/2)*c)^(2/3)*(b*x + a)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \left(c\,(a+b\,x)^{3/2}\right)^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x)^(3/2))^(2/3),x)
[Out] int((c*(a + b*x)^(3/2))^(2/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\,(a+bx)^{\frac{3}{2}}\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x+a)**(3/2))**(2/3),x)
[Out] Integral((c*(a + b*x)**(3/2))**(2/3), x)
```

3.2838 $\int \left(c(a+bx)^{2/3}\right)^{3/2}\,dx$

Optimal. Leaf size=32

$$\frac{c(a+bx)^{5/3}\sqrt{c(a+bx)^{2/3}}}{2b}$$

```
[Out] 1/2*c*(b*x+a)^(5/3)*(c*(b*x+a)^(2/3))^(1/2)/b
```

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {247, 15, 30}

$$\frac{c(a+bx)^{5/3}\sqrt{c(a+bx)^{2/3}}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x)^(2/3))^(3/2),x]
```

```
[Out] (c*Sqrt[c*(a + b*x)^(2/3)]*(a + b*x)^(5/3))/(2*b)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \left(c(a+bx)^{2/3}\right)^{3/2}\,dx &= \frac{\text{Subst}\left(\int \left(cx^{2/3}\right)^{3/2}\,dx, x, a+bx\right)}{b}\\ &= \frac{\left(c\sqrt{c(a+bx)^{2/3}}\right)\text{Subst}(\int x\,dx, x, a+bx)}{b\sqrt[3]{a+bx}}\\ &= \frac{c\sqrt{c(a+bx)^{2/3}}\,(a+bx)^{5/3}}{2b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 1.06

$$\frac{x(2a+bx)\left(c(a+bx)^{2/3}\right)^{3/2}}{2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x)^(2/3))^(3/2),x]
```

```
[Out] (x*(c*(a + b*x)^(2/3))^(3/2)*(2*a + b*x))/(2*(a + b*x))
```

fricas [A] time = 0.80, size = 17, normalized size = 0.53

$$\frac{1}{2}\left(bcx^2+2\,acx\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x+a)^(2/3))^(3/2),x, algorithm="fricas")
[Out] 1/2*(b*c*x^2 + 2*a*c*x)*sqrt(c)
```

giac [A] time = 0.15, size = 15, normalized size = 0.47

$$\frac{1}{2}\left(bx^2+2\,ax\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x+a)^(2/3))^(3/2),x, algorithm="giac")
[Out] 1/2*(b*x^2 + 2*a*x)*c^(3/2)
```

maple [A] time = 0.00, size = 29, normalized size = 0.91

$$\frac{(bx+2a)\left((bx+a)^{\frac{2}{3}}c\right)^{\frac{3}{2}}x}{2bx+2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(b*x+a)^(2/3))^(3/2),x)
[Out] 1/2*x*(b*x+2*a)*(c*(b*x+a)^(2/3))^(3/2)/(b*x+a)
```

maxima [A] time = 0.52, size = 15, normalized size = 0.47

$$\frac{1}{2}\left(bx^2+2\,ax\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x+a)^(2/3))^(3/2),x, algorithm="maxima")
[Out] 1/2*(b*x^2 + 2*a*x)*c^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \left(c\,(a+b\,x)^{2/3}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x)^(2/3))^(3/2),x)
[Out] int((c*(a + b*x)^(2/3))^(3/2), x)
```

sympy [A] time = 4.60, size = 65, normalized size = 2.03

$$\begin{cases} \frac{2a^2c^{\frac{3}{2}}x}{2a+2bx}+\frac{3abc^{\frac{3}{2}}x^2}{2a+2bx}+\frac{b^2c^{\frac{3}{2}}x^3}{2a+2bx} & \text{for } a\neq 0\vee b\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x+a)**(2/3))**(3/2),x)
```

```
[Out] Piecewise((2*a**2*c**(3/2)*x/(2*a + 2*b*x) + 3*a*b*c**(3/2)*x**2/(2*a + 2*b
*x) + b**2*c**(3/2)*x**3/(2*a + 2*b*x), Ne(a, 0) | Ne(b, 0)), (0, True))
```

3.2839 $\int \frac{1}{\left(\frac{c}{(a+bx)^{3/2}}\right)^{2/3}}\,dx$

Optimal. Leaf size=27

$$\frac{a+bx}{2b\left(\frac{c}{(a+bx)^{3/2}}\right)^{2/3}}$$

```
[Out] 1/2*(b*x+a)/b/(c/(b*x+a)^(3/2))^(2/3)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {247, 15, 30}

$$\frac{a+bx}{2b\left(\frac{c}{(a+bx)^{3/2}}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c/(a + b*x)^(3/2))^(-2/3),x]
```

```
[Out] (a + b*x)/(2*b*(c/(a + b*x)^(3/2))^(2/3))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{c}{(a+bx)^{3/2}}\right)^{2/3}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\left(\frac{c}{x^{3/2}}\right)^{2/3}}\,dx, x, a+bx\right)}{b}\\ &= \frac{\text{Subst}(\int x\,dx, x, a+bx)}{b\left(\frac{c}{(a+bx)^{3/2}}\right)^{2/3}(a+bx)}\\ &= \frac{a+bx}{2b\left(\frac{c}{(a+bx)^{3/2}}\right)^{2/3}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 34, normalized size = 1.26

$$\frac{x(2a+bx)}{2(a+bx)\left(\frac{c}{(a+bx)^{3/2}}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c/(a + b*x)^(3/2))^(-2/3),x]
```

```
[Out] (x*(2*a + b*x))/(2*(c/(a + b*x)^(3/2))^(2/3)*(a + b*x))
```

fricas [B] time = 1.26, size = 51, normalized size = 1.89

$$\frac{\left(bx^2+2\,ax\right)\sqrt{bx+a}\,\left(\frac{\sqrt{bx+a}\,c}{b^2x^2+2\,abx+a^2}\right)^{\frac{1}{3}}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a)^(3/2))^(2/3),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)*sqrt(b*x + a)*(sqrt(b*x + a)*c/(b^2*x^2 + 2*a*b*x + a^2
))^(1/3)/c
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{c}{(bx+a)^{\frac{3}{2}}}\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a)^(3/2))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c/(b*x + a)^(3/2))^(-2/3), x)
```

maple [A] time = 0.00, size = 29, normalized size = 1.07

$$\frac{(bx+2a)\,x}{2\,(bx+a)\left(\frac{c}{(bx+a)^{\frac{3}{2}}}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c/(b*x+a)^(3/2))^(2/3),x)
```

```
[Out] 1/2*x*(b*x+2*a)/(b*x+a)/(c/(b*x+a)^(3/2))^(2/3)
```

maxima [A] time = 0.67, size = 21, normalized size = 0.78

$$\frac{bx+a}{2\,b\left(\frac{c}{(bx+a)^{\frac{3}{2}}}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a)^(3/2))^(2/3),x, algorithm="maxima")
```

```
[Out] 1/2*(b*x + a)/(b*(c/(b*x + a)^(3/2))^(2/3))
```

mupad [B] time = 1.25, size = 31, normalized size = 1.15

$$\frac{x\left(\frac{c}{(a+b\,x)^{3/2}}\right)^{1/3}\,(2\,a+b\,x)\,\sqrt{a+b\,x}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c/(a + b*x)^(3/2))^(2/3),x)
[Out] (x*(c/(a + b*x)^(3/2))^(1/3)*(2*a + b*x)*(a + b*x)^(1/2))/(2*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{c}{(a+bx)^{\frac{3}{2}}}\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a)**(3/2))**(2/3),x)
[Out] Integral((c/(a + b*x)**(3/2))**(-2/3), x)
```

3.2840 $\int \frac{1}{\left(\frac{c}{(a+bx)^{2/3}}\right)^{3/2}}\,dx$

Optimal. Leaf size=34

$$\frac{(a+bx)^{5/3}}{2bc\sqrt{\frac{c}{(a+bx)^{2/3}}}}$$

```
[Out] 1/2*(b*x+a)^(5/3)/b/c/(c/(b*x+a)^(2/3))^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {247, 15, 30}

$$\frac{(a+bx)^{5/3}}{2bc\sqrt{\frac{c}{(a+bx)^{2/3}}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c/(a + b*x)^(2/3))^(-3/2),x]
```

```
[Out] (a + b*x)^(5/3)/(2*b*c*Sqrt[c/(a + b*x)^(2/3)])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{c}{(a+bx)^{2/3}}\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\left(\frac{c}{x^{2/3}}\right)^{3/2}}\,dx, x, a+bx\right)}{b}\\ &= \frac{\text{Subst}(\int x\,dx, x, a+bx)}{bc\sqrt{\frac{c}{(a+bx)^{2/3}}}\sqrt[3]{a+bx}}\\ &= \frac{(a+bx)^{5/3}}{2bc\sqrt{\frac{c}{(a+bx)^{2/3}}}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 34, normalized size = 1.00

$$\frac{x(2a+bx)}{2(a+bx)\left(\frac{c}{(a+bx)^{2/3}}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c/(a + b*x)^(2/3))^(-3/2),x]
[Out] (x*(2*a + b*x))/(2*(c/(a + b*x)^(2/3))^(3/2)*(a + b*x))
```

fricas [A] time = 0.74, size = 15, normalized size = 0.44

$$\frac{bx^2+2\,ax}{2\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a)^(2/3))^(3/2),x, algorithm="fricas")
[Out] 1/2*(b*x^2 + 2*a*x)/c^(3/2)
```

giac [A] time = 0.15, size = 15, normalized size = 0.44

$$\frac{bx^2+2\,ax}{2\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a)^(2/3))^(3/2),x, algorithm="giac")
[Out] 1/2*(b*x^2 + 2*a*x)/c^(3/2)
```

maple [A] time = 0.00, size = 29, normalized size = 0.85

$$\frac{(bx+2a)\,x}{2\,(bx+a)\left(\frac{c}{(bx+a)^{\frac{2}{3}}}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c/(b*x+a)^(2/3))^(3/2),x)
[Out] 1/2*x*(b*x+2*a)/(b*x+a)/(c/(b*x+a)^(2/3))^(3/2)
```

maxima [A] time = 0.48, size = 15, normalized size = 0.44

$$\frac{bx^2+2\,ax}{2\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a)^(2/3))^(3/2),x, algorithm="maxima")
[Out] 1/2*(b*x^2 + 2*a*x)/c^(3/2)
```

mupad [B] time = 1.26, size = 42, normalized size = 1.24

$$\sqrt{\frac{c}{(a+b\,x)^{2/3}}}\left(\frac{a\,x\,(a+b\,x)^{1/3}}{c^2}+\frac{b\,x^2\,(a+b\,x)^{1/3}}{2\,c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c/(a + b*x)^(2/3))^(3/2),x)
```

```
[Out] (c/(a + b*x)^(2/3))^(1/2)*((a*x*(a + b*x)^(1/3))/c^2 + (b*x^2*(a + b*x)^(1/
3))/(2*c^2))
```

sympy [**A**] time = 3.15, size = 76, normalized size = 2.24

$$\begin{cases} \frac{2ax}{\frac{2ac^{\frac{3}{2}}}{a+bx}+\frac{2bc^{\frac{3}{2}}x}{a+bx}}+\frac{bx^2}{\frac{2ac^{\frac{3}{2}}}{a+bx}+\frac{2bc^{\frac{3}{2}}x}{a+bx}} & \text{for } a \neq 0 \vee b \neq 0 \\ \frac{x}{(\tilde{\infty}c)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c/(b*x+a)**(2/3))**(3/2),x)
```

```
[Out] Piecewise((2*a*x/(2*a*c**(3/2)/(a + b*x) + 2*b*c**(3/2)*x/(a + b*x)) + b*x*
*2/(2*a*c**(3/2)/(a + b*x) + 2*b*c**(3/2)*x/(a + b*x)), Ne(a, 0) | Ne(b, 0)
), (x/(zoo*c)**(3/2), True))
```

3.2841 $\int (c+dx)^3 \left(a+b(c+dx)^2\right) dx$

Optimal. Leaf size=31

$$\frac{a(c+dx)^4}{4d}+\frac{b(c+dx)^6}{6d}$$

```
[Out] 1/4*a*(d*x+c)^4/d+1/6*b*(d*x+c)^6/d
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {372, 14}

$$\frac{a(c+dx)^4}{4d}+\frac{b(c+dx)^6}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + b*(c + d*x)^2),x]
```

```
[Out] (a*(c + d*x)^4)/(4*d) + (b*(c + d*x)^6)/(6*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^3\left(a+b(c+dx)^2\right)dx &= \frac{\text{Subst}\left(\int x^3\left(a+bx^2\right)dx, x, c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(ax^3+bx^5\right)dx, x, c+dx\right)}{d}\\ &= \frac{a(c+dx)^4}{4d}+\frac{b(c+dx)^6}{6d}\end{aligned}$$

Mathematica [B] time = 0.03, size = 77, normalized size = 2.48

$$\frac{1}{12}x(2c+dx)\left(3a\left(2c^2+2cdx+d^2x^2\right)+2b\left(3c^4+6c^3dx+7c^2d^2x^2+4cd^3x^3+d^4x^4\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + b*(c + d*x)^2),x]
```

```
[Out] (x*(2*c + d*x)*(3*a*(2*c^2 + 2*c*d*x + d^2*x^2) + 2*b*(3*c^4 + 6*c^3*d*x +
7*c^2*d^2*x^2 + 4*c*d^3*x^3 + d^4*x^4)))/12
```

fricas [B] time = 0.66, size = 93, normalized size = 3.00

$$\frac{1}{6}x^6d^5b+x^5d^4cb+\frac{5}{2}x^4d^3c^2b+\frac{10}{3}x^3d^2c^3b+\frac{5}{2}x^2dc^4b+\frac{1}{4}x^4d^3a+xc^5b+x^3d^2ca+\frac{3}{2}x^2dc^2a+xc^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/6*x^6*d^5*b + x^5*d^4*c*b + 5/2*x^4*d^3*c^2*b + 10/3*x^3*d^2*c^3*b + 5/2*
x^2*d*c^4*b + 1/4*x^4*d^3*a + x*c^5*b + x^3*d^2*c*a + 3/2*x^2*d*c^2*a + x*c
^3*a
```

giac [B] time = 0.17, size = 86, normalized size = 2.77

$$\frac{1}{2}\left(dx^2+2cx\right)bc^4+\frac{1}{2}\left(dx^2+2cx\right)^2bc^2d+\frac{1}{6}\left(dx^2+2cx\right)^3bd^2+\frac{1}{2}\left(dx^2+2cx\right)ac^2+\frac{1}{4}\left(dx^2+2cx\right)^2ad$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(d*x^2 + 2*c*x)*b*c^4 + 1/2*(d*x^2 + 2*c*x)^2*b*c^2*d + 1/6*(d*x^2 + 2*
c*x)^3*b*d^2 + 1/2*(d*x^2 + 2*c*x)*a*c^2 + 1/4*(d*x^2 + 2*c*x)^2*a*d
```

maple [B] time = 0.00, size = 112, normalized size = 3.61

$$\frac{b\,d^5x^6}{6}+bc\,d^4x^5+\left(b\,c^2+a\right)c^3x+\frac{\left(9b\,c^2d^3+\left(b\,c^2+a\right)d^3\right)x^4}{4}+\frac{\left(7b\,c^3d^2+3\left(b\,c^2+a\right)c\,d^2\right)x^3}{3}+\frac{\left(2b\,c^4d+3\left(b\,c^2+a\right.\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+b*(d*x+c)^2),x)
```

```
[Out] 1/6*d^5*b*x^6+c*d^4*b*x^5+1/4*(9*c^2*d^3*b+d^3*(b*c^2+a))*x^4+1/3*(7*c^3*b*
d^2+3*c*d^2*(b*c^2+a))*x^3+1/2*(2*c^4*b*d+3*c^2*d*(b*c^2+a))*x^2+c^3*(b*c^2
+a)*x
```

maxima [B] time = 0.53, size = 86, normalized size = 2.77

$$\frac{1}{6}\,bd^5x^6+bcd^4x^5+\frac{1}{4}\left(10\,bc^2+a\right)d^3x^4+\frac{1}{3}\left(10\,bc^3+3\,ac\right)d^2x^3+\frac{1}{2}\left(5\,bc^4+3\,ac^2\right)dx^2+\left(bc^5+ac^3\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/6*b*d^5*x^6 + b*c*d^4*x^5 + 1/4*(10*b*c^2 + a)*d^3*x^4 + 1/3*(10*b*c^3 +
3*a*c)*d^2*x^3 + 1/2*(5*b*c^4 + 3*a*c^2)*d*x^2 + (b*c^5 + a*c^3)*x
```

mupad [B] time = 0.06, size = 86, normalized size = 2.77

$$x\,\left(b\,c^5+a\,c^3\right)+\frac{d^3\,x^4\,\left(10\,b\,c^2+a\right)}{4}+\frac{b\,d^5\,x^6}{6}+\frac{c^2\,d\,x^2\,\left(5\,b\,c^2+3\,a\right)}{2}+\frac{c\,d^2\,x^3\,\left(10\,b\,c^2+3\,a\right)}{3}+b\,c\,d^4\,x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^2)*(c + d*x)^3,x)
```

```
[Out] x*(a*c^3 + b*c^5) + (d^3*x^4*(a + 10*b*c^2))/4 + (b*d^5*x^6)/6 + (c^2*d*x^2
*(3*a + 5*b*c^2))/2 + (c*d^2*x^3*(3*a + 10*b*c^2))/3 + b*c*d^4*x^5
```

sympy [B] time = 0.11, size = 99, normalized size = 3.19

$$bcd^4x^5+\frac{bd^5x^6}{6}+x^4\left(\frac{ad^3}{4}+\frac{5bc^2d^3}{2}\right)+x^3\left(acd^2+\frac{10bc^3d^2}{3}\right)+x^2\left(\frac{3ac^2d}{2}+\frac{5bc^4d}{2}\right)+x\left(ac^3+bc^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+b*(d*x+c)**2),x)

[Out] b*c*d**4*x**5 + b*d**5*x**6/6 + x**4*(a*d**3/4 + 5*b*c**2*d**3/2) + x**3*(a
*c*d**2 + 10*b*c**3*d**2/3) + x**2*(3*a*c**2*d/2 + 5*b*c**4*d/2) + x*(a*c**
3 + b*c**5)
```

3.2842 $\int (c+dx)^3 \left(a+b(c+dx)^2\right)^2 dx$

Optimal. Leaf size=51

$$\frac{a^2(c+dx)^4}{4d}+\frac{ab(c+dx)^6}{3d}+\frac{b^2(c+dx)^8}{8d}$$

```
[Out] 1/4*a^2*(d*x+c)^4/d+1/3*a*b*(d*x+c)^6/d+1/8*b^2*(d*x+c)^8/d
```

Rubi [A] time = 0.07, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {372, 266, 43}

$$\frac{a^2(c+dx)^4}{4d}+\frac{ab(c+dx)^6}{3d}+\frac{b^2(c+dx)^8}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + b*(c + d*x)^2)^2,x]
```

```
[Out] (a^2*(c + d*x)^4)/(4*d) + (a*b*(c + d*x)^6)/(3*d) + (b^2*(c + d*x)^8)/(8*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^3\left(a+b(c+dx)^2\right)^2 dx &= \frac{\text{Subst}\left(\int x^3\left(a+bx^2\right)^2 dx, x, c+dx\right)}{d}\\
&= \frac{\text{Subst}\left(\int x(a+bx)^2\, dx, x, (c+dx)^2\right)}{2d}\\
&= \frac{\text{Subst}\left(\int \left(a^2x+2abx^2+b^2x^3\right) dx, x, (c+dx)^2\right)}{2d}\\
&= \frac{a^2(c+dx)^4}{4d}+\frac{ab(c+dx)^6}{3d}+\frac{b^2(c+dx)^8}{8d}
\end{aligned}$$

Mathematica [B] time = 0.03, size = 172, normalized size = 3.37

$$\frac{1}{4}d^3x^4\left(a^2+20abc^2+35b^2c^4\right)+\frac{1}{3}cd^2x^3\left(3a^2+20abc^2+21b^2c^4\right)+\frac{1}{2}c^2dx^2\left(3a^2+10abc^2+7b^2c^4\right)+\frac{1}{6}bd^5x^6\left(2a+2\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + b*(c + d*x)^2)^2,x]
```

```
[Out] c^3*(a + b*c^2)^2*x + (c^2*(3*a^2 + 10*a*b*c^2 + 7*b^2*c^4)*d*x^2)/2 + (c*(
3*a^2 + 20*a*b*c^2 + 21*b^2*c^4)*d^2*x^3)/3 + ((a^2 + 20*a*b*c^2 + 35*b^2*c
^4)*d^3*x^4)/4 + b*c*(2*a + 7*b*c^2)*d^4*x^5 + (b*(2*a + 21*b*c^2)*d^5*x^6)
/6 + b^2*c*d^6*x^7 + (b^2*d^7*x^8)/8
```

fricas [**B**] time = 0.78, size = 207, normalized size = 4.06

$$\frac{1}{8}x^8d^7b^2+x^7d^6cb^2+\frac{7}{2}x^6d^5c^2b^2+7x^5d^4c^3b^2+\frac{35}{4}x^4d^3c^4b^2+\frac{1}{3}x^6d^5ba+7x^3d^2c^5b^2+2x^5d^4cba+\frac{7}{2}x^2dc^6b^2+5x^4d^3c^2ba$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*d^7*b^2 + x^7*d^6*c*b^2 + 7/2*x^6*d^5*c^2*b^2 + 7*x^5*d^4*c^3*b^2 +
 35/4*x^4*d^3*c^4*b^2 + 1/3*x^6*d^5*b*a + 7*x^3*d^2*c^5*b^2 + 2*x^5*d^4*c*b
*a + 7/2*x^2*d*c^6*b^2 + 5*x^4*d^3*c^2*b*a + x*c^7*b^2 + 20/3*x^3*d^2*c^3*b
*a + 5*x^2*d*c^4*b*a + 1/4*x^4*d^3*a^2 + 2*x*c^5*b*a + x^3*d^2*c*a^2 + 3/2*
x^2*d*c^2*a^2 + x*c^3*a^2
```

giac [**B**] time = 0.16, size = 173, normalized size = 3.39

$$\frac{1}{2}\left(dx^2+2\,cx\right)b^2c^6+\frac{3}{4}\left(dx^2+2\,cx\right)^2b^2c^4d+\frac{1}{2}\left(dx^2+2\,cx\right)^3b^2c^2d^2+\frac{1}{8}\left(dx^2+2\,cx\right)^4b^2d^3+\left(dx^2+2\,cx\right)abc^4+\left(dx^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(d*x^2 + 2*c*x)*b^2*c^6 + 3/4*(d*x^2 + 2*c*x)^2*b^2*c^4*d + 1/2*(d*x^2
 + 2*c*x)^3*b^2*c^2*d^2 + 1/8*(d*x^2 + 2*c*x)^4*b^2*d^3 + (d*x^2 + 2*c*x)*a*
b*c^4 + (d*x^2 + 2*c*x)^2*a*b*c^2*d + 1/3*(d*x^2 + 2*c*x)^3*a*b*d^2 + 1/2*(
d*x^2 + 2*c*x)*a^2*c^2 + 1/4*(d*x^2 + 2*c*x)^2*a^2*d
```

maple [**B**] time = 0.00, size = 324, normalized size = 6.35

$$\frac{b^2d^7x^8}{8}+b^2c\,d^6x^7+\frac{\left(15b^2c^2d^5+\left(4b^2c^2d^2+2\left(b\,c^2+a\right)b\,d^2\right)d^3\right)x^6}{6}+\frac{\left(13b^2c^3d^4+4\left(b\,c^2+a\right)bc\,d^4+3\left(4b^2c^2d^2\right.\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+b*(d*x+c)^2)^2,x)
```

```
[Out] 1/8*d^7*b^2*x^8+c*d^6*b^2*x^7+1/6*(15*c^2*d^5*b^2+d^3*(2*(b*c^2+a)*b*d^2+4*
b^2*c^2*d^2))*x^6+1/5*(13*c^3*b^2*d^4+3*c*d^2*(2*(b*c^2+a)*b*d^2+4*b^2*c^2*
d^2)+4*d^4*(b*c^2+a)*b*c)*x^5+1/4*(4*c^4*b^2*d^3+3*c^2*d*(2*(b*c^2+a)*b*d^2
+4*b^2*c^2*d^2)+12*c^2*d^3*(b*c^2+a)*b+d^3*(b*c^2+a)^2)*x^4+1/3*(c^3*(2*(b*
c^2+a)*b*d^2+4*b^2*c^2*d^2)+12*c^3*d^2*(b*c^2+a)*b+3*c*d^2*(b*c^2+a)^2)*x^3
+1/2*(4*c^4*(b*c^2+a)*b*d+3*c^2*d*(b*c^2+a)^2)*x^2+c^3*(b*c^2+a)^2*x
```

maxima [**B**] time = 0.51, size = 176, normalized size = 3.45

$$\frac{1}{8}\,b^2d^7x^8+b^2cd^6x^7+\frac{1}{6}\left(21\,b^2c^2+2\,ab\right)d^5x^6+\left(7\,b^2c^3+2\,abc\right)d^4x^5+\frac{1}{4}\left(35\,b^2c^4+20\,abc^2+a^2\right)d^3x^4+\frac{1}{3}\left(21\,b^2c^5+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*d^7*x^8 + b^2*c*d^6*x^7 + 1/6*(21*b^2*c^2 + 2*a*b)*d^5*x^6 + (7*b^2
*c^3 + 2*a*b*c)*d^4*x^5 + 1/4*(35*b^2*c^4 + 20*a*b*c^2 + a^2)*d^3*x^4 + 1/3
*(21*b^2*c^5 + 20*a*b*c^3 + 3*a^2*c)*d^2*x^3 + 1/2*(7*b^2*c^6 + 10*a*b*c^4
+ 3*a^2*c^2)*d*x^2 + (b^2*c^7 + 2*a*b*c^5 + a^2*c^3)*x
```

mupad [B] time = 1.20, size = 162, normalized size = 3.18

$$c^3 x \left(b c^2 + a\right)^2 + \frac{b^2 d^7 x^8}{8} + \frac{d^3 x^4 \left(a^2 + 20 a b c^2 + 35 b^2 c^4\right)}{4} + \frac{b d^5 x^6 \left(21 b c^2 + 2 a\right)}{6} + b^2 c d^6 x^7 + \frac{c^2 d x^2 \left(3 a^2 + 10 a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^2)^2*(c + d*x)^3,x)
```

```
[Out] c^3*x*(a + b*c^2)^2 + (b^2*d^7*x^8)/8 + (d^3*x^4*(a^2 + 35*b^2*c^4 + 20*a*b
*c^2))/4 + (b*d^5*x^6*(2*a + 21*b*c^2))/6 + b^2*c*d^6*x^7 + (c^2*d*x^2*(3*a
^2 + 7*b^2*c^4 + 10*a*b*c^2))/2 + (c*d^2*x^3*(3*a^2 + 21*b^2*c^4 + 20*a*b*c
^2))/3 + b*c*d^4*x^5*(2*a + 7*b*c^2)
```

sympy [B] time = 0.16, size = 209, normalized size = 4.10

$$b^2 c d^6 x^7 + \frac{b^2 d^7 x^8}{8} + x^6 \left(\frac{a b d^5}{3} + \frac{7 b^2 c^2 d^5}{2}\right) + x^5 \left(2 a b c d^4 + 7 b^2 c^3 d^4\right) + x^4 \left(\frac{a^2 d^3}{4} + 5 a b c^2 d^3 + \frac{35 b^2 c^4 d^3}{4}\right) + x^3 \left(a^2 c d^2 + \frac{20}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+b*(d*x+c)**2)**2,x)
```

```
[Out] b**2*c*d**6*x**7 + b**2*d**7*x**8/8 + x**6*(a*b*d**5/3 + 7*b**2*c**2*d**5/2
) + x**5*(2*a*b*c*d**4 + 7*b**2*c**3*d**4) + x**4*(a**2*d**3/4 + 5*a*b*c**2
*d**3 + 35*b**2*c**4*d**3/4) + x**3*(a**2*c*d**2 + 20*a*b*c**3*d**2/3 + 7*b
**2*c**5*d**2) + x**2*(3*a**2*c**2*d/2 + 5*a*b*c**4*d + 7*b**2*c**6*d/2) +
x*(a**2*c**3 + 2*a*b*c**5 + b**2*c**7)
```

3.2843 $\int (c+dx)^3 \left(a+b(c+dx)^2\right)^3 \, dx$

Optimal. Leaf size=48

$$\frac{\left(a+b(c+dx)^2\right)^5}{10b^2d} - \frac{a\left(a+b(c+dx)^2\right)^4}{8b^2d}$$

```
[Out] -1/8*a*(a+b*(d*x+c)^2)^4/b^2/d+1/10*(a+b*(d*x+c)^2)^5/b^2/d
```

Rubi [A] time = 0.22, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {372, 266, 43}

$$\frac{\left(a+b(c+dx)^2\right)^5}{10b^2d} - \frac{a\left(a+b(c+dx)^2\right)^4}{8b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + b*(c + d*x)^2)^3,x]
```

```
[Out] -(a*(a + b*(c + d*x)^2)^4)/(8*b^2*d) + (a + b*(c + d*x)^2)^5/(10*b^2*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^3 \left(a+b(c+dx)^2\right)^3 \, dx &= \frac{\text{Subst}\left(\int x^3 \left(a+bx^2\right)^3 \, dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int x(a+bx)^3 \, dx, x, (c+dx)^2\right)}{2d} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{a(a+bx)^3}{b} + \frac{(a+bx)^4}{b}\right) dx, x, (c+dx)^2\right)}{2d} \\
&= -\frac{a\left(a+b(c+dx)^2\right)^4}{8b^2d} + \frac{\left(a+b(c+dx)^2\right)^5}{10b^2d}
\end{aligned}$$

Mathematica [B] time = 0.05, size = 249, normalized size = 5.19

$$\frac{1}{2}bd^5x^6\left(a^2+21abc^2+42b^2c^4\right)+\frac{3}{5}bcd^4x^5\left(5a^2+35abc^2+42b^2c^4\right)+\frac{1}{4}d^3x^4\left(a^3+30a^2bc^2+105ab^2c^4+84b^3c^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + b*(c + d*x)^2)^3,x]
```

```
[Out] c^3*(a + b*c^2)^3*x + (3*c^2*(a + b*c^2)^2*(a + 3*b*c^2)*d*x^2)/2 + c*(a^3
+ 10*a^2*b*c^2 + 21*a*b^2*c^4 + 12*b^3*c^6)*d^2*x^3 + ((a^3 + 30*a^2*b*c^2
+ 105*a*b^2*c^4 + 84*b^3*c^6)*d^3*x^4)/4 + (3*b*c*(5*a^2 + 35*a*b*c^2 + 42*
b^2*c^4)*d^4*x^5)/5 + (b*(a^2 + 21*a*b*c^2 + 42*b^2*c^4)*d^5*x^6)/2 + 3*b^2
*c*(a + 4*b*c^2)*d^6*x^7 + (3*b^2*(a + 12*b*c^2)*d^7*x^8)/8 + b^3*c*d^8*x^9
 + (b^3*d^9*x^10)/10
```

fricas [B] time = 0.74, size = 355, normalized size = 7.40

$$\frac{1}{10}x^{10}d^9b^3+x^9d^8cb^3+\frac{9}{2}x^8d^7c^2b^3+12x^7d^6c^3b^3+21x^6d^5c^4b^3+\frac{3}{8}x^8d^7b^2a+\frac{126}{5}x^5d^4c^5b^3+3x^7d^6cb^2a+21x^4d^3c^6b^3+\frac{21}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*d^9*b^3 + x^9*d^8*c*b^3 + 9/2*x^8*d^7*c^2*b^3 + 12*x^7*d^6*c^3*b^
3 + 21*x^6*d^5*c^4*b^3 + 3/8*x^8*d^7*b^2*a + 126/5*x^5*d^4*c^5*b^3 + 3*x^7*
d^6*c*b^2*a + 21*x^4*d^3*c^6*b^3 + 21/2*x^6*d^5*c^2*b^2*a + 12*x^3*d^2*c^7*
b^3 + 21*x^5*d^4*c^3*b^2*a + 9/2*x^2*d*c^8*b^3 + 105/4*x^4*d^3*c^4*b^2*a +
1/2*x^6*d^5*b*a^2 + x*c^9*b^3 + 21*x^3*d^2*c^5*b^2*a + 3*x^5*d^4*c*b*a^2 +
21/2*x^2*d*c^6*b^2*a + 15/2*x^4*d^3*c^2*b*a^2 + 3*x*c^7*b^2*a + 10*x^3*d^2*
c^3*b*a^2 + 15/2*x^2*d*c^4*b*a^2 + 1/4*x^4*d^3*a^3 + 3*x*c^5*b*a^2 + x^3*d^
2*c*a^3 + 3/2*x^2*d*c^2*a^3 + x*c^3*a^3
```

giac [B] time = 0.18, size = 288, normalized size = 6.00

$$\frac{1}{2}\left(dx^2+2\,cx\right)b^3c^8+\left(dx^2+2\,cx\right)^2b^3c^6d+\left(dx^2+2\,cx\right)^3b^3c^4d^2+\frac{1}{2}\left(dx^2+2\,cx\right)^4b^3c^2d^3+\frac{1}{10}\left(dx^2+2\,cx\right)^5b^3d^4+\frac{3}{2}\left(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*(d*x^2 + 2*c*x)*b^3*c^8 + (d*x^2 + 2*c*x)^2*b^3*c^6*d + (d*x^2 + 2*c*x)
^3*b^3*c^4*d^2 + 1/2*(d*x^2 + 2*c*x)^4*b^3*c^2*d^3 + 1/10*(d*x^2 + 2*c*x)^5
*b^3*d^4 + 3/2*(d*x^2 + 2*c*x)*a*b^2*c^6 + 9/4*(d*x^2 + 2*c*x)^2*a*b^2*c^4*
d + 3/2*(d*x^2 + 2*c*x)^3*a*b^2*c^2*d^2 + 3/8*(d*x^2 + 2*c*x)^4*a*b^2*d^3 +
 3/2*(d*x^2 + 2*c*x)*a^2*b*c^4 + 3/2*(d*x^2 + 2*c*x)^2*a^2*b*c^2*d + 1/2*(d
*x^2 + 2*c*x)^3*a^2*b*d^2 + 1/2*(d*x^2 + 2*c*x)*a^3*c^2 + 1/4*(d*x^2 + 2*c*
x)^2*a^3*d
```

maple [B] time = 0.00, size = 960, normalized size = 20.00

$$\frac{b^3d^9x^{10}}{10}+b^3c\,d^8x^9+\frac{\left(21b^3c^2d^7+\left(8b^3c^2d^4+\left(b\,c^2+a\right)b^2d^4+\left(4b^2c^2d^2+2\left(b\,c^2+a\right)b\,d^2\right)b\,d^2\right)d^3\right)x^8}{8}+\frac{\left(19b^3c^3d^6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+b*(d*x+c)^2)^3,x)
```

```
[Out] 1/10*d^9*b^3*x^10+c*d^8*b^3*x^9+1/8*(21*c^2*d^7*b^3+d^3*((b*c^2+a)*b^2*d^4+
8*b^3*c^2*d^4+b*d^2*(4*b^2*c^2*d^2+2*(b*c^2+a)*b*d^2)))*x^8+1/7*(19*c^3*b^3
*d^6+3*c*d^2*((b*c^2+a)*b^2*d^4+8*b^3*c^2*d^4+b*d^2*(4*b^2*c^2*d^2+2*(b*c^2
+a)*b*d^2))+d^3*(8*(b*c^2+a)*b^2*c*d^3+2*b*c*d*(4*b^2*c^2*d^2+2*(b*c^2+a)*b
*d^2)))*x^7+1/6*(6*c^4*b^3*d^5+3*c^2*d*((b*c^2+a)*b^2*d^4+8*b^3*c^2*d^4+b*d
^2*(4*b^2*c^2*d^2+2*(b*c^2+a)*b*d^2))+3*c*d^2*(8*(b*c^2+a)*b^2*c*d^3+2*b*c*
d*(4*b^2*c^2*d^2+2*(b*c^2+a)*b*d^2))+d^3*((b*c^2+a)*(4*b^2*c^2*d^2+2*(b*c^2
```

```
+a)*b*d^2)+8*b^2*c^2*d^2*(b*c^2+a)+b*d^2*(b*c^2+a)^2))*x^6+1/5*(c^3*((b*c^2
+a)*b^2*d^4+8*b^3*c^2*d^4+b*d^2*(4*b^2*c^2*d^2+2*(b*c^2+a)*b*d^2))+3*c^2*d*
(8*(b*c^2+a)*b^2*c*d^3+2*b*c*d*(4*b^2*c^2*d^2+2*(b*c^2+a)*b*d^2))+3*c*d^2*(
(b*c^2+a)*(4*b^2*c^2*d^2+2*(b*c^2+a)*b*d^2)+8*b^2*c^2*d^2*(b*c^2+a)+b*d^2*(
b*c^2+a)^2)+6*d^4*(b*c^2+a)^2*b*c)*x^5+1/4*(c^3*(8*(b*c^2+a)*b^2*c*d^3+2*b*
c*d*(4*b^2*c^2*d^2+2*(b*c^2+a)*b*d^2))+3*c^2*d*((b*c^2+a)*(4*b^2*c^2*d^2+2*
(b*c^2+a)*b*d^2)+8*b^2*c^2*d^2*(b*c^2+a)+b*d^2*(b*c^2+a)^2)+18*c^2*d^3*(b*c
^2+a)^2*b+d^3*(b*c^2+a)^3)*x^4+1/3*(c^3*((b*c^2+a)*(4*b^2*c^2*d^2+2*(b*c^2+
a)*b*d^2)+8*b^2*c^2*d^2*(b*c^2+a)+b*d^2*(b*c^2+a)^2)+18*c^3*d^2*(b*c^2+a)^2
*b+3*c*d^2*(b*c^2+a)^3)*x^3+1/2*(6*c^4*(b*c^2+a)^2*b*d+3*c^2*d*(b*c^2+a)^3)
*x^2+c^3*(b*c^2+a)^3*x
```

maxima [B] time = 0.64, size = 284, normalized size = 5.92

$$\frac{1}{10}b^3d^9x^{10}+b^3cd^8x^9+\frac{3}{8}\left(12b^3c^2+ab^2\right)d^7x^8+3\left(4b^3c^3+ab^2c\right)d^6x^7+\frac{1}{2}\left(42b^3c^4+21ab^2c^2+a^2b\right)d^5x^6+\frac{3}{5}\left(42b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/10*b^3*d^9*x^10 + b^3*c*d^8*x^9 + 3/8*(12*b^3*c^2 + a*b^2)*d^7*x^8 + 3*(4
*b^3*c^3 + a*b^2*c)*d^6*x^7 + 1/2*(42*b^3*c^4 + 21*a*b^2*c^2 + a^2*b)*d^5*x
^6 + 3/5*(42*b^3*c^5 + 35*a*b^2*c^3 + 5*a^2*b*c)*d^4*x^5 + 1/4*(84*b^3*c^6
+ 105*a*b^2*c^4 + 30*a^2*b*c^2 + a^3)*d^3*x^4 + (12*b^3*c^7 + 21*a*b^2*c^5
+ 10*a^2*b*c^3 + a^3*c)*d^2*x^3 + 3/2*(3*b^3*c^8 + 7*a*b^2*c^6 + 5*a^2*b*c^
4 + a^3*c^2)*d*x^2 + (b^3*c^9 + 3*a*b^2*c^7 + 3*a^2*b*c^5 + a^3*c^3)*x
```

mupad [B] time = 1.23, size = 237, normalized size = 4.94

$$c^3x\left(bc^2+a\right)^3+\frac{d^3x^4\left(a^3+30a^2bc^2+105ab^2c^4+84b^3c^6\right)}{4}+\frac{b^3d^9x^{10}}{10}+\frac{3b^2d^7x^8\left(12bc^2+a\right)}{8}+b^3cd^8x^9+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^2)^3*(c + d*x)^3,x)
```

```
[Out] c^3*x*(a + b*c^2)^3 + (d^3*x^4*(a^3 + 84*b^3*c^6 + 30*a^2*b*c^2 + 105*a*b^2
*c^4))/4 + (b^3*d^9*x^10)/10 + (3*b^2*d^7*x^8*(a + 12*b*c^2))/8 + b^3*c*d^8
*x^9 + (b*d^5*x^6*(a^2 + 42*b^2*c^4 + 21*a*b*c^2))/2 + c*d^2*x^3*(a^3 + 12*
b^3*c^6 + 10*a^2*b*c^2 + 21*a*b^2*c^4) + (3*b*c*d^4*x^5*(5*a^2 + 42*b^2*c^4
 + 35*a*b*c^2))/5 + (3*c^2*d*x^2*(a + b*c^2)^2*(a + 3*b*c^2))/2 + 3*b^2*c*d
^6*x^7*(a + 4*b*c^2)
```

sympy [B] time = 0.19, size = 357, normalized size = 7.44

$$b^3cd^8x^9+\frac{b^3d^9x^{10}}{10}+x^8\left(\frac{3ab^2d^7}{8}+\frac{9b^3c^2d^7}{2}\right)+x^7\left(3ab^2cd^6+12b^3c^3d^6\right)+x^6\left(\frac{a^2bd^5}{2}+\frac{21ab^2c^2d^5}{2}+21b^3c^4d^5\right)+x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+b*(d*x+c)**2)**3,x)
```

```
[Out] b**3*c*d**8*x**9 + b**3*d**9*x**10/10 + x**8*(3*a*b**2*d**7/8 + 9*b**3*c**2
*d**7/2) + x**7*(3*a*b**2*c*d**6 + 12*b**3*c**3*d**6) + x**6*(a**2*b*d**5/2
 + 21*a*b**2*c**2*d**5/2 + 21*b**3*c**4*d**5) + x**5*(3*a**2*b*c*d**4 + 21*
a*b**2*c**3*d**4 + 126*b**3*c**5*d**4/5) + x**4*(a**3*d**3/4 + 15*a**2*b*c*
*2*d**3/2 + 105*a*b**2*c**4*d**3/4 + 21*b**3*c**6*d**3) + x**3*(a**3*c*d**2
 + 10*a**2*b*c**3*d**2 + 21*a*b**2*c**5*d**2 + 12*b**3*c**7*d**2) + x**2*(3
*a**3*c**2*d/2 + 15*a**2*b*c**4*d/2 + 21*a*b**2*c**6*d/2 + 9*b**3*c**8*d/2)
 + x*(a**3*c**3 + 3*a**2*b*c**5 + 3*a*b**2*c**7 + b**3*c**9)
```

3.2844 $\int \frac{2+x}{1+(2+x)^2}\,dx$

Optimal. Leaf size=12

$$\frac{1}{2}\log\left((x+2)^2+1\right)$$

```
[Out] 1/2*ln(1+(2+x)^2)
```

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {372, 260}

$$\frac{1}{2}\log\left((x+2)^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/(1 + (2 + x)^2),x]
```

```
[Out] Log[1 + (2 + x)^2]/2
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{2+x}{1+(2+x)^2}\,dx &= \text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, 2+x\right)\\ &= \frac{1}{2}\log\left(1+(2+x)^2\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\frac{1}{2}\log\left((x+2)^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/(1 + (2 + x)^2),x]
```

```
[Out] Log[1 + (2 + x)^2]/2
```

fricas [A] time = 0.81, size = 11, normalized size = 0.92

$$\frac{1}{2}\,\log\left(x^2+4\,x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(1+(2+x)^2),x, algorithm="fricas")
```

```
[Out] 1/2*log(x^2 + 4*x + 5)
```

giac [A] time = 0.15, size = 11, normalized size = 0.92

$$\frac{1}{2}\log\left(x^2+4x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(1+(2+x)^2),x, algorithm="giac")
[Out] 1/2*log(x^2 + 4*x + 5)
```

maple [A] time = 0.00, size = 12, normalized size = 1.00

$$\frac{\ln\left(x^2+4x+5\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+2)/(1+(x+2)^2),x)
[Out] 1/2*ln(x^2+4*x+5)
```

maxima [A] time = 0.53, size = 10, normalized size = 0.83

$$\frac{1}{2}\log\left((x+2)^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(1+(2+x)^2),x, algorithm="maxima")
[Out] 1/2*log((x + 2)^2 + 1)
```

mupad [B] time = 0.06, size = 10, normalized size = 0.83

$$\frac{\ln\left((x+2)^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/((x + 2)^2 + 1),x)
[Out] log((x + 2)^2 + 1)/2
```

sympy [A] time = 0.11, size = 10, normalized size = 0.83

$$\frac{\log\left(x^2+4x+5\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(1+(2+x)**2),x)
[Out] log(x**2 + 4*x + 5)/2
```

3.2845 $\int \frac{2+x}{\left(1+(2+x)^2\right)^2}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{2\left((x+2)^2+1\right)}$$

```
[Out] -1/2/(1+(2+x)^2)
```

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {372, 261}

$$-\frac{1}{2\left((x+2)^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/(1 + (2 + x)^2)^2,x]
```

```
[Out] -1/(2*(1 + (2 + x)^2))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{2+x}{\left(1+(2+x)^2\right)^2}\,dx = \text{Subst}\left(\int \frac{x}{\left(1+x^2\right)^2}\,dx, x, 2+x\right)$$
$$= -\frac{1}{2\left(1+(2+x)^2\right)}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$-\frac{1}{2\left((x+2)^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/(1 + (2 + x)^2)^2,x]
```

```
[Out] -1/2*1/(1 + (2 + x)^2)
```

fricas [A] time = 0.65, size = 12, normalized size = 0.92

$$-\frac{1}{2\left(x^2+4x+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(1+(2+x)^2)^2,x, algorithm="fricas")
[Out] -1/2/(x^2 + 4*x + 5)
```

giac [**A**] time = 0.20, size = 12, normalized size = 0.92

$$-\frac{1}{2\left(x^2+4x+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(1+(2+x)^2)^2,x, algorithm="giac")
[Out] -1/2/(x^2 + 4*x + 5)
```

maple [**A**] time = 0.00, size = 13, normalized size = 1.00

$$-\frac{1}{2\left(x^2+4x+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+2)/(1+(x+2)^2)^2,x)
[Out] -1/2/(x^2+4*x+5)
```

maxima [**A**] time = 0.58, size = 11, normalized size = 0.85

$$-\frac{1}{2\left((x+2)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(1+(2+x)^2)^2,x, algorithm="maxima")
[Out] -1/2/((x + 2)^2 + 1)
```

mupad [**B**] time = 1.14, size = 13, normalized size = 1.00

$$-\frac{1}{2\left((x+2)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/((x + 2)^2 + 1)^2,x)
[Out] -1/(2*((x + 2)^2 + 1))
```

sympy [**A**] time = 0.12, size = 12, normalized size = 0.92

$$-\frac{1}{2x^2+8x+10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(1+(2+x)**2)**2,x)
[Out] -1/(2*x**2 + 8*x + 10)
```

3.2846 $\int \frac{2+x}{\left(1+(2+x)^2\right)^3}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{4\left((x+2)^2+1\right)^2}$$

```
[Out] -1/4/(1+(2+x)^2)^2
```

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {372, 261}

$$-\frac{1}{4\left((x+2)^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/(1 + (2 + x)^2)^3,x]
```

```
[Out] -1/(4*(1 + (2 + x)^2)^2)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{2+x}{\left(1+(2+x)^2\right)^3}\,dx &= \text{Subst}\left(\int \frac{x}{\left(1+x^2\right)^3}\,dx, x, 2+x\right)\\ &= -\frac{1}{4\left(1+(2+x)^2\right)^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$-\frac{1}{4\left((x+2)^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/(1 + (2 + x)^2)^3,x]
```

```
[Out] -1/4*1/(1 + (2 + x)^2)^2
```

fricas [A] time = 0.56, size = 22, normalized size = 1.69

$$-\frac{1}{4\left(x^4+8x^3+26x^2+40x+25\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(1+(2+x)^2)^3,x, algorithm="fricas")
[Out] -1/4/(x^4 + 8*x^3 + 26*x^2 + 40*x + 25)
```

giac [A] time = 0.17, size = 12, normalized size = 0.92

$$-\frac{1}{4\left(x^2+4\,x+5\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(1+(2+x)^2)^3,x, algorithm="giac")
[Out] -1/4/(x^2 + 4*x + 5)^2
```

maple [A] time = 0.00, size = 13, normalized size = 1.00

$$-\frac{1}{4\left(x^2+4x+5\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+2)/(1+(x+2)^2)^3,x)
[Out] -1/4/(x^2+4*x+5)^2
```

maxima [A] time = 0.47, size = 11, normalized size = 0.85

$$-\frac{1}{4\left((x+2)^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(1+(2+x)^2)^3,x, algorithm="maxima")
[Out] -1/4/((x + 2)^2 + 1)^2
```

mupad [B] time = 1.14, size = 11, normalized size = 0.85

$$-\frac{1}{4\left((x+2)^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/((x + 2)^2 + 1)^3,x)
[Out] -1/(4*((x + 2)^2 + 1)^2)
```

sympy [A] time = 0.14, size = 22, normalized size = 1.69

$$-\frac{1}{4x^4+32x^3+104x^2+160x+100}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(1+(2+x)**2)**3,x)
[Out] -1/(4*x**4 + 32*x**3 + 104*x**2 + 160*x + 100)
```

3.2847 $\int (c+dx)^5 \left(a+b(c+dx)^2\right)^p \, dx$

Optimal. Leaf size=93

$$\frac{a^2\left(a+b(c+dx)^2\right)^{p+1}}{2b^3d(p+1)} - \frac{a\left(a+b(c+dx)^2\right)^{p+2}}{b^3d(p+2)} + \frac{\left(a+b(c+dx)^2\right)^{p+3}}{2b^3d(p+3)}$$

```
[Out] 1/2*a^2*(a+b*(d*x+c)^2)^(1+p)/b^3/d/(1+p)-a*(a+b*(d*x+c)^2)^(2+p)/b^3/d/(2+
p)+1/2*(a+b*(d*x+c)^2)^(3+p)/b^3/d/(3+p)
```

Rubi [A] time = 0.10, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {372, 266, 43}

$$\frac{a^2\left(a+b(c+dx)^2\right)^{p+1}}{2b^3d(p+1)} - \frac{a\left(a+b(c+dx)^2\right)^{p+2}}{b^3d(p+2)} + \frac{\left(a+b(c+dx)^2\right)^{p+3}}{2b^3d(p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^5*(a + b*(c + d*x)^2)^p,x]
```

```
[Out] (a^2*(a + b*(c + d*x)^2)^(1 + p))/(2*b^3*d*(1 + p)) - (a*(a + b*(c + d*x)^2
)^(2 + p))/(b^3*d*(2 + p)) + (a + b*(c + d*x)^2)^(3 + p)/(2*b^3*d*(3 + p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^5 \left(a+b(c+dx)^2\right)^p \, dx &= \frac{\text{Subst}\left(\int x^5 \left(a+bx^2\right)^p \, dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int x^2 (a+bx)^p \, dx, x, (c+dx)^2\right)}{2d} \\
&= \frac{\text{Subst}\left(\int \left(\frac{a^2(a+bx)^p}{b^2} - \frac{2a(a+bx)^{1+p}}{b^2} + \frac{(a+bx)^{2+p}}{b^2}\right) dx, x, (c+dx)^2\right)}{2d} \\
&= \frac{a^2\left(a+b(c+dx)^2\right)^{1+p}}{2b^3d(1+p)} - \frac{a\left(a+b(c+dx)^2\right)^{2+p}}{b^3d(2+p)} + \frac{\left(a+b(c+dx)^2\right)^{3+p}}{2b^3d(3+p)}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 73, normalized size = 0.78

$$\frac{\left(a+b(c+dx)^2\right)^{p+1}\left(\frac{a^2}{p+1}-\frac{2a\left(a+b(c+dx)^2\right)}{p+2}+\frac{\left(a+b(c+dx)^2\right)^2}{p+3}\right)}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^5*(a + b*(c + d*x)^2)^p,x]
```

```
[Out] ((a + b*(c + d*x)^2)^(1 + p)*(a^2/(1 + p) - (2*a*(a + b*(c + d*x)^2))/(2 +
p) + (a + b*(c + d*x)^2)^2/(3 + p)))/(2*b^3*d)
```

fricas [B] time = 0.71, size = 438, normalized size = 4.71

$$\frac{\left(2b^3c^6+\left(b^3d^6p^2+3b^3d^6p+2b^3d^6\right)x^6+6\left(b^3cd^5p^2+3b^3cd^5p+2b^3cd^5\right)x^5+\left(30b^3c^2d^4+\left(15b^3c^2+ab^2\right)d^4\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^5*(a+b*(d*x+c)^2)^p,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*c^6 + (b^3*d^6*p^2 + 3*b^3*d^6*p + 2*b^3*d^6)*x^6 + 6*(b^3*c*d^5
*p^2 + 3*b^3*c*d^5*p + 2*b^3*c*d^5)*x^5 + (30*b^3*c^2*d^4 + (15*b^3*c^2 + a
*b^2)*d^4*p^2 + (45*b^3*c^2 + a*b^2)*d^4*p)*x^4 + 4*(10*b^3*c^3*d^3 + (5*b^
3*c^3 + a*b^2*c)*d^3*p^2 + (15*b^3*c^3 + a*b^2*c)*d^3*p)*x^3 + 2*a^3 + (b^3
*c^6 + a*b^2*c^4)*p^2 + (30*b^3*c^4*d^2 + 3*(5*b^3*c^4 + 2*a*b^2*c^2)*d^2*p
^2 + (45*b^3*c^4 + 6*a*b^2*c^2 - 2*a^2*b)*d^2*p)*x^2 + (3*b^3*c^6 + a*b^2*c
^4 - 2*a^2*b*c^2)*p + 2*(6*b^3*c^5*d + (3*b^3*c^5 + 2*a*b^2*c^3)*d*p^2 + (9
*b^3*c^5 + 2*a*b^2*c^3 - 2*a^2*b*c)*d*p)*x)*(b*d^2*x^2 + 2*b*c*d*x + b*c^2
+ a)^p/(b^3*d*p^3 + 6*b^3*d*p^2 + 11*b^3*d*p + 6*b^3*d)
```

giac [B] time = 0.25, size = 1290, normalized size = 13.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^5*(a+b*(d*x+c)^2)^p,x, algorithm="giac")
```

```
[Out] 1/2*((b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)^p*b^3*d^6*p^2*x^6 + 6*(b*d^2*x^2 +
 2*b*c*d*x + b*c^2 + a)^p*b^3*c*d^5*p^2*x^5 + 3*(b*d^2*x^2 + 2*b*c*d*x + b*
c^2 + a)^p*b^3*d^6*p*x^6 + 15*(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)^p*b^3*c^2
*d^4*p^2*x^4 + 18*(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)^p*b^3*c*d^5*p*x^5 + 2
*(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)^p*b^3*d^6*x^6 + 20*(b*d^2*x^2 + 2*b*c*
d*x + b*c^2 + a)^p*b^3*c^3*d^3*p^2*x^3 + 45*(b*d^2*x^2 + 2*b*c*d*x + b*c^2
+ a)^p*b^3*c^2*d^4*p*x^4 + 12*(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)^p*b^3*c*d
^5*x^5 + 15*(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)^p*b^3*c^4*d^2*p^2*x^2 + 60*
(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)^p*b^3*c^3*d^3*p*x^3 + 30*(b*d^2*x^2 + 2
*b*c*d*x + b*c^2 + a)^p*b^3*c^2*d^4*x^4 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2 +
a)^p*a*b^2*d^4*p^2*x^4 + 6*(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)^p*b^3*c^5*d*
p^2*x + 45*(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)^p*b^3*c^4*d^2*p*x^2 + 40*(b*
d^2*x^2 + 2*b*c*d*x + b*c^2 + a)^p*b^3*c^3*d^3*x^3 + 4*(b*d^2*x^2 + 2*b*c*d
*x + b*c^2 + a)^p*a*b^2*c*d^3*p^2*x^3 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)
^p*a*b^2*d^4*p*x^4 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)^p*b^3*c^6*p^2 + 18
*(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)^p*b^3*c^5*d*p*x + 30*(b*d^2*x^2 + 2*b*
c*d*x + b*c^2 + a)^p*b^3*c^4*d^2*x^2 + 6*(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a
)^p*a*b^2*c^2*d^2*p^2*x^2 + 4*(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)^p*a*b^2*c
*d^3*p*x^3 + 3*(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)^p*b^3*c^6*p + 12*(b*d^2*
x^2 + 2*b*c*d*x + b*c^2 + a)^p*b^3*c^5*d*x + 4*(b*d^2*x^2 + 2*b*c*d*x + b*c
^2 + a)^p*a*b^2*c^3*d*p^2*x + 6*(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)^p*a*b^2
```

```
*c^2*d^2*p*x^2 + 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)^p*b^3*c^6 + (b*d^2*x
^2 + 2*b*c*d*x + b*c^2 + a)^p*a*b^2*c^4*p^2 + 4*(b*d^2*x^2 + 2*b*c*d*x + b*
c^2 + a)^p*a*b^2*c^3*d*p*x + (b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)^p*a*b^2*c^
4*p - 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)^p*a^2*b*d^2*p*x^2 - 4*(b*d^2*x^
2 + 2*b*c*d*x + b*c^2 + a)^p*a^2*b*c*d*p*x - 2*(b*d^2*x^2 + 2*b*c*d*x + b*c
^2 + a)^p*a^2*b*c^2*p + 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)^p*a^3)/(b^3*d
*p^3 + 6*b^3*d*p^2 + 11*b^3*d*p + 6*b^3*d)
```

maple [B] time = 0.02, size = 289, normalized size = 3.11

$$\left(b^2d^4p^2x^4+4b^2c\,d^3p^2x^3+3b^2d^4p\,x^4+6b^2c^2d^2p^2x^2+12b^2c\,d^3p\,x^3+2d^4x^4b^2+4b^2c^3d\,p^2x+18b^2c^2d^2p\,x^2+8c\,d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^5*(a+b*(d*x+c)^2)^p,x)
```

```
[Out] 1/2*(b*d^2*x^2+2*b*c*d*x+b*c^2+a)^(p+1)*(b^2*d^4*p^2*x^4+4*b^2*c*d^3*p^2*x^
3+3*b^2*d^4*p*x^4+6*b^2*c^2*d^2*p^2*x^2+12*b^2*c*d^3*p*x^3+2*b^2*d^4*x^4+4*
b^2*c^3*d*p^2*x+18*b^2*c^2*d^2*p*x^2+8*b^2*c*d^3*x^3+b^2*c^4*p^2+12*b^2*c^3
*d*p*x+12*b^2*c^2*d^2*x^2-2*a*b*d^2*p*x^2+3*b^2*c^4*p+8*b^2*c^3*d*x-4*a*b*c
*d*p*x-2*a*b*d^2*x^2+2*b^2*c^4-2*a*b*c^2*p-4*a*b*c*d*x-2*a*b*c^2+2*a^2)/b^3
/d/(p^3+6*p^2+11*p+6)
```

maxima [B] time = 0.99, size = 300, normalized size = 3.23

$$\left(\left(p^2+3\,p+2\right)b^3d^6x^6+6\left(p^2+3\,p+2\right)b^3cd^5x^5+\left(p^2+3\,p+2\right)b^3c^6+\left(p^2+p\right)ab^2c^4-2\,a^2bc^2p+\left(15\left(p^2+3\,p+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^5*(a+b*(d*x+c)^2)^p,x, algorithm="maxima")
```

```
[Out] 1/2*((p^2 + 3*p + 2)*b^3*d^6*x^6 + 6*(p^2 + 3*p + 2)*b^3*c*d^5*x^5 + (p^2 +
 3*p + 2)*b^3*c^6 + (p^2 + p)*a*b^2*c^4 - 2*a^2*b*c^2*p + (15*(p^2 + 3*p +
2)*b^3*c^2*d^4 + (p^2 + p)*a*b^2*d^4)*x^4 + 4*(5*(p^2 + 3*p + 2)*b^3*c^3*d^
3 + (p^2 + p)*a*b^2*c*d^3)*x^3 + 2*a^3 + (15*(p^2 + 3*p + 2)*b^3*c^4*d^2 +
6*(p^2 + p)*a*b^2*c^2*d^2 - 2*a^2*b*d^2*p)*x^2 + 2*(3*(p^2 + 3*p + 2)*b^3*c
^5*d + 2*(p^2 + p)*a*b^2*c^3*d - 2*a^2*b*c*d*p)*x)*(b*d^2*x^2 + 2*b*c*d*x +
 b*c^2 + a)^p/((p^3 + 6*p^2 + 11*p + 6)*b^3*d)
```

mupad [B] time = 1.57, size = 401, normalized size = 4.31

$$\left(a+b\left(c+d\,x\right)^2\right)^p\left(\frac{d^5\,x^6\,\left(p^2+3\,p+2\right)}{2\,\left(p^3+6\,p^2+11\,p+6\right)}+\frac{\left(b\,c^2+a\right)\,\left(2\,a^2-2\,a\,b\,c^2\,p-2\,a\,b\,c^2+b^2\,c^4\,p^2+3\,b^2\,c^4\,p+2\,b^2\,c^4\right.}{2\,b^3\,d\,\left(p^3+6\,p^2+11\,p+6\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^2)^p*(c + d*x)^5,x)
```

```
[Out] (a + b*(c + d*x)^2)^p*((d^5*x^6*(3*p + p^2 + 2))/(2*(11*p + 6*p^2 + p^3 + 6
)) + ((a + b*c^2)*(2*a^2 + 2*b^2*c^4 + 3*b^2*c^4*p + b^2*c^4*p^2 - 2*a*b*c^
2 - 2*a*b*c^2*p))/(2*b^3*d*(11*p + 6*p^2 + p^3 + 6)) + (c*x*(6*b^2*c^4 - 2*
a^2*p + 9*b^2*c^4*p + 3*b^2*c^4*p^2 + 2*a*b*c^2*p + 2*a*b*c^2*p^2))/(b^2*(1
1*p + 6*p^2 + p^3 + 6)) + (d*x^2*(30*b^2*c^4 - 2*a^2*p + 45*b^2*c^4*p + 15*
b^2*c^4*p^2 + 6*a*b*c^2*p + 6*a*b*c^2*p^2))/(2*b^2*(11*p + 6*p^2 + p^3 + 6)
) + (3*c*d^4*x^5*(3*p + p^2 + 2))/(11*p + 6*p^2 + p^3 + 6) + (d^3*x^4*(p +
1)*(a*p + 30*b*c^2 + 15*b*c^2*p))/(2*b*(11*p + 6*p^2 + p^3 + 6)) + (2*c*d^2
*x^3*(p + 1)*(a*p + 10*b*c^2 + 5*b*c^2*p))/(b*(11*p + 6*p^2 + p^3 + 6)))
```

sympy [**F**(**-1**)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**5*(a+b*(d*x+c)**2)**p,x)
[Out] Timed out
```

3.2848 $\int (c+dx)^4 \left(a+b(c+dx)^2\right)^p \, dx$

Optimal. Leaf size=55

$$\frac{(c+dx)^5 \left(a+b(c+dx)^2\right)^{p+1} \, {}_2F_1\left(1, p+\frac{7}{2}; \frac{7}{2}; -\frac{b(c+dx)^2}{a}\right)}{5ad}$$

```
[Out] 1/5*(d*x+c)^5*(a+b*(d*x+c)^2)^(1+p)*hypergeom([1, 7/2+p],[7/2],-b*(d*x+c)^2
/a)/a/d
```

Rubi [A] time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.24, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {372, 365, 364}

$$\frac{(c+dx)^5 \left(a+b(c+dx)^2\right)^p \left(\frac{b(c+dx)^2}{a}+1\right)^{-p} \, {}_2F_1\left(\frac{5}{2}, -p; \frac{7}{2}; -\frac{b(c+dx)^2}{a}\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*(a + b*(c + d*x)^2)^p,x]
```

```
[Out] ((c + d*x)^5*(a + b*(c + d*x)^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -((b*(c
+ d*x)^2)/a)])/(5*d*(1 + (b*(c + d*x)^2)/a)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^4 \left(a+b(c+dx)^2\right)^p \, dx &= \frac{\text{Subst}\left(\int x^4 \left(a+bx^2\right)^p \, dx, x, c+dx\right)}{d} \\
&= \frac{\left(\left(a+b(c+dx)^2\right)^p \left(1+\frac{b(c+dx)^2}{a}\right)^{-p}\right) \text{Subst}\left(\int x^4 \left(1+\frac{bx^2}{a}\right)^p \, dx, x, c+dx\right)}{d} \\
&= \frac{(c+dx)^5 \left(a+b(c+dx)^2\right)^p \left(1+\frac{b(c+dx)^2}{a}\right)^{-p} \, {}_2F_1\left(\frac{5}{2}, -p; \frac{7}{2}; -\frac{b(c+dx)^2}{a}\right)}{5d}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 68, normalized size = 1.24

$$\frac{(c+dx)^5\left(a+b(c+dx)^2\right)^p\left(\frac{b(c+dx)^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{b(c+dx)^2}{a}\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*(a + b*(c + d*x)^2)^p,x]
```

```
[Out] ((c + d*x)^5*(a + b*(c + d*x)^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -((b*(c
+ d*x)^2)/a)])/(5*d*(1 + (b*(c + d*x)^2)/a)^p)
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(d^4x^4+4\,cd^3x^3+6\,c^2d^2x^2+4\,c^3dx+c^4\right)\left(bd^2x^2+2\,bcdx+bc^2+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*(a+b*(d*x+c)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4)*(b*d^2*x
^2 + 2*b*c*d*x + b*c^2 + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^4\left((dx+c)^2b+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*(a+b*(d*x+c)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*((d*x + c)^2*b + a)^p, x)
```

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int (dx+c)^4\left(a+(dx+c)^2\,b\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*(a+b*(d*x+c)^2)^p,x)
```

```
[Out] int((d*x+c)^4*(a+b*(d*x+c)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^4\left((dx+c)^2b+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*(a+b*(d*x+c)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^4*((d*x + c)^2*b + a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(a+b\,(c+d\,x)^2\right)^p\,(c+d\,x)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^2)^p*(c + d*x)^4,x)
```

```
[Out] int((a + b*(c + d*x)^2)^p*(c + d*x)^4, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*(a+b*(d*x+c)**2)**p,x)
[Out] Timed out
```

3.2849 $\int (c+dx)^3 \left(a+b(c+dx)^2\right)^p \, dx$

Optimal. Leaf size=62

$$\frac{\left(a+b(c+dx)^2\right)^{p+2}}{2b^2d(p+2)} - \frac{a\left(a+b(c+dx)^2\right)^{p+1}}{2b^2d(p+1)}$$

```
[Out] -1/2*a*(a+b*(d*x+c)^2)^(1+p)/b^2/d/(1+p)+1/2*(a+b*(d*x+c)^2)^(2+p)/b^2/d/(2
+p)
```

Rubi [A] time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {372, 266, 43}

$$\frac{\left(a+b(c+dx)^2\right)^{p+2}}{2b^2d(p+2)} - \frac{a\left(a+b(c+dx)^2\right)^{p+1}}{2b^2d(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + b*(c + d*x)^2)^p,x]
```

```
[Out] -(a*(a + b*(c + d*x)^2)^(1 + p))/(2*b^2*d*(1 + p)) + (a + b*(c + d*x)^2)^(2
 + p)/(2*b^2*d*(2 + p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^3 \left(a+b(c+dx)^2\right)^p \, dx &= \frac{\text{Subst}\left(\int x^3 \left(a+bx^2\right)^p \, dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int x(a+bx)^p \, dx, x, (c+dx)^2\right)}{2d} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{a(a+bx)^p}{b} + \frac{(a+bx)^{1+p}}{b}\right) dx, x, (c+dx)^2\right)}{2d} \\
&= -\frac{a\left(a+b(c+dx)^2\right)^{1+p}}{2b^2d(1+p)} + \frac{\left(a+b(c+dx)^2\right)^{2+p}}{2b^2d(2+p)}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 51, normalized size = 0.82

$$\frac{\left(a+b(c+dx)^2\right)^{p+1}\left(b(p+1)(c+dx)^2-a\right)}{2b^2d(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + b*(c + d*x)^2)^p,x]
```

```
[Out] ((a + b*(c + d*x)^2)^(1 + p)*(-a + b*(1 + p)*(c + d*x)^2))/(2*b^2*d*(1 + p)
*(2 + p))
```

fricas [B] time = 0.88, size = 183, normalized size = 2.95

$$\frac{\left(b^2c^4+\left(b^2d^4p+b^2d^4\right)x^4+4\left(b^2cd^3p+b^2cd^3\right)x^3+\left(6\,b^2c^2d^2+\left(6\,b^2c^2+ab\right)d^2p\right)x^2-a^2+\left(b^2c^4+abc^2\right)p+2\left(2\right.\right.}{2\left(b^2dp^2+3\,b^2dp+2\,b^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*(d*x+c)^2)^p,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*c^4 + (b^2*d^4*p + b^2*d^4)*x^4 + 4*(b^2*c*d^3*p + b^2*c*d^3)*x^3
+ (6*b^2*c^2*d^2 + (6*b^2*c^2 + a*b)*d^2*p)*x^2 - a^2 + (b^2*c^4 + a*b*c^2)
*p + 2*(2*b^2*c^3*d + (2*b^2*c^3 + a*b*c)*d*p)*x)*(b*d^2*x^2 + 2*b*c*d*x +
b*c^2 + a)^p/(b^2*d*p^2 + 3*b^2*d*p + 2*b^2*d)
```

giac [B] time = 0.20, size = 345, normalized size = 5.56

$$\frac{\left(bc^2+\left(dx^2+2\,cx\right)bd+a\right)^p b^2c^4p+2\left(dx^2+2\,cx\right)\left(bc^2+\left(dx^2+2\,cx\right)bd+a\right)^p b^2c^2dp+\left(dx^2+2\,cx\right)^2\left(bc^2+\left(dx^2+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*(d*x+c)^2)^p,x, algorithm="giac")
```

```
[Out] 1/2*((b*c^2 + (d*x^2 + 2*c*x)*b*d + a)^p*b^2*c^4*p + 2*(d*x^2 + 2*c*x)*(b*c
^2 + (d*x^2 + 2*c*x)*b*d + a)^p*b^2*c^2*d*p + (d*x^2 + 2*c*x)^2*(b*c^2 + (d
*x^2 + 2*c*x)*b*d + a)^p*b^2*d^2*p + (b*c^2 + (d*x^2 + 2*c*x)*b*d + a)^p*b^
2*c^4 + 2*(d*x^2 + 2*c*x)*(b*c^2 + (d*x^2 + 2*c*x)*b*d + a)^p*b^2*c^2*d + (
d*x^2 + 2*c*x)^2*(b*c^2 + (d*x^2 + 2*c*x)*b*d + a)^p*b^2*d^2 + (b*c^2 + (d*
x^2 + 2*c*x)*b*d + a)^p*a*b*c^2*p + (d*x^2 + 2*c*x)*(b*c^2 + (d*x^2 + 2*c*x
)*b*d + a)^p*a*b*d*p - (b*c^2 + (d*x^2 + 2*c*x)*b*d + a)^p*a^2)/(b^2*d*p^2
+ 3*b^2*d*p + 2*b^2*d)
```

maple [A] time = 0.01, size = 91, normalized size = 1.47

$$-\frac{\left(-b\,d^2p\,x^2-2bcdpx-b\,d^2x^2-b\,c^2p-2bcdx-b\,c^2+a\right)\left(b\,d^2x^2+2bcdx+b\,c^2+a\right)^{p+1}}{2\left(p^2+3p+2\right)b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+(d*x+c)^2*b)^p,x)
```

```
[Out] -1/2*(b*d^2*x^2+2*b*c*d*x+b*c^2+a)^(p+1)*(-b*d^2*p*x^2-2*b*c*d*p*x-b*d^2*x^
2-b*c^2*p-2*b*c*d*x-b*c^2+a)/b^2/d/(p^2+3*p+2)
```

maxima [B] time = 1.02, size = 140, normalized size = 2.26

$$\frac{\left(b^2d^4\left(p+1\right)x^4+4\,b^2cd^3\left(p+1\right)x^3+b^2c^4\left(p+1\right)+abc^2p+\left(6\,b^2c^2d^2\left(p+1\right)+abd^2p\right)x^2-a^2+2\left(2\,b^2c^3d\left(p+1\right)\right.\right.}{2\left(p^2+3\,p+2\right)b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*(d*x+c)^2)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*d^4*(p + 1)*x^4 + 4*b^2*c*d^3*(p + 1)*x^3 + b^2*c^4*(p + 1) + a*b*
c^2*p + (6*b^2*c^2*d^2*(p + 1) + a*b*d^2*p)*x^2 - a^2 + 2*(2*b^2*c^3*d*(p +
 1) + a*b*c*d*p)*x)*(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)^p/((p^2 + 3*p + 2)*
b^2*d)
```

mupad [B] time = 1.34, size = 168, normalized size = 2.71

$$\left(a+b\left(c+d\,x\right)^2\right)^p\left(\frac{d^3\,x^4\,\left(p+1\right)}{2\left(p^2+3\,p+2\right)}+\frac{\left(b\,c^2+a\right)\left(b\,c^2-a+b\,c^2\,p\right)}{2\,b^2\,d\left(p^2+3\,p+2\right)}+\frac{c\,x\left(a\,p+2\,b\,c^2+2\,b\,c^2\,p\right)}{b\left(p^2+3\,p+2\right)}+\frac{2\,c\,d^2\,x^3\,\left(\right.}{p^2+3\,p}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^2)^p*(c + d*x)^3,x)
```

```
[Out] (a + b*(c + d*x)^2)^p*((d^3*x^4*(p + 1))/(2*(3*p + p^2 + 2)) + ((a + b*c^2)
*(b*c^2 - a + b*c^2*p))/(2*b^2*d*(3*p + p^2 + 2)) + (c*x*(a*p + 2*b*c^2 + 2
*b*c^2*p))/(b*(3*p + p^2 + 2)) + (2*c*d^2*x^3*(p + 1))/(3*p + p^2 + 2) + (d
*x^2*(a*p + 6*b*c^2 + 6*b*c^2*p))/(2*b*(3*p + p^2 + 2)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+b*(d*x+c)**2)**p,x)
```

```
[Out] Timed out
```

3.2850 $\int (c+dx)^2 \left(a+b(c+dx)^2\right)^p \, dx$

Optimal. Leaf size=55

$$\frac{(c+dx)^3 \left(a+b(c+dx)^2\right)^{p+1} \, {}_2F_1\left(1, p+\frac{5}{2}; \frac{5}{2}; -\frac{b(c+dx)^2}{a}\right)}{3ad}$$

```
[Out] 1/3*(d*x+c)^3*(a+b*(d*x+c)^2)^(1+p)*hypergeom([1, 5/2+p],[5/2],-b*(d*x+c)^2
/a)/a/d
```

Rubi [A] time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.24, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {372, 365, 364}

$$\frac{(c+dx)^3 \left(a+b(c+dx)^2\right)^p \left(\frac{b(c+dx)^2}{a}+1\right)^{-p} \, {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; -\frac{b(c+dx)^2}{a}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + b*(c + d*x)^2)^p,x]
```

```
[Out] ((c + d*x)^3*(a + b*(c + d*x)^2)^p*Hypergeometric2F1[3/2, -p, 5/2, -((b*(c
+ d*x)^2)/a)])/(3*d*(1 + (b*(c + d*x)^2)/a)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^2 \left(a+b(c+dx)^2\right)^p \, dx &= \frac{\text{Subst}\left(\int x^2 \left(a+bx^2\right)^p \, dx, x, c+dx\right)}{d} \\
&= \frac{\left(\left(a+b(c+dx)^2\right)^p \left(1+\frac{b(c+dx)^2}{a}\right)^{-p}\right) \text{Subst}\left(\int x^2 \left(1+\frac{bx^2}{a}\right)^p \, dx, x, c+dx\right)}{d} \\
&= \frac{(c+dx)^3 \left(a+b(c+dx)^2\right)^p \left(1+\frac{b(c+dx)^2}{a}\right)^{-p} \, {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; -\frac{b(c+dx)^2}{a}\right)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 68, normalized size = 1.24

$$\frac{(c+dx)^3\left(a+b(c+dx)^2\right)^p\left(\frac{b(c+dx)^2}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{b(c+dx)^2}{a}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*(a + b*(c + d*x)^2)^p,x]
```

```
[Out] ((c + d*x)^3*(a + b*(c + d*x)^2)^p*Hypergeometric2F1[3/2, -p, 5/2, -((b*(c
+ d*x)^2)/a)])/(3*d*(1 + (b*(c + d*x)^2)/a)^p)
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(d^2x^2+2\,cdx+c^2\right)\left(bd^2x^2+2\,bcdx+bc^2+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*(d*x+c)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((d^2*x^2 + 2*c*d*x + c^2)*(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)^p, x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2\left((dx+c)^2b+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*(d*x+c)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*((d*x + c)^2*b + a)^p, x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int (dx+c)^2\left(a+(dx+c)^2\,b\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+(d*x+c)^2*b)^p,x)
```

```
[Out] int((d*x+c)^2*(a+(d*x+c)^2*b)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2\left((dx+c)^2b+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*(d*x+c)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^2*((d*x + c)^2*b + a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(a+b\,(c+d\,x)^2\right)^p\,(c+d\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^2)^p*(c + d*x)^2,x)
```

```
[Out] int((a + b*(c + d*x)^2)^p*(c + d*x)^2, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+b*(d*x+c)**2)**p,x)
[Out] Timed out
```

3.2851 $\int (c+dx)\left(a+b(c+dx)^2\right)^p\,dx$

Optimal. Leaf size=30

$$\frac{\left(a+b(c+dx)^2\right)^{p+1}}{2bd(p+1)}$$

```
[Out] 1/2*(a+b*(d*x+c)^2)^(1+p)/b/d/(1+p)
```

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {372, 261}

$$\frac{\left(a+b(c+dx)^2\right)^{p+1}}{2bd(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + b*(c + d*x)^2)^p,x]
```

```
[Out] (a + b*(c + d*x)^2)^(1 + p)/(2*b*d*(1 + p))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\left(a+b(c+dx)^2\right)^p\,dx &= \frac{\text{Subst}\left(\int x\left(a+bx^2\right)^p\,dx, x, c+dx\right)}{d}\\ &= \frac{\left(a+b(c+dx)^2\right)^{1+p}}{2bd(1+p)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$\frac{\left(a+b(c+dx)^2\right)^{p+1}}{2bd(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + b*(c + d*x)^2)^p,x]
```

```
[Out] (a + b*(c + d*x)^2)^(1 + p)/(2*b*d*(1 + p))
```

fricas [A] time = 0.90, size = 56, normalized size = 1.87

$$\frac{\left(bd^2x^2+2\,bcdx+bc^2+a\right)\left(bd^2x^2+2\,bcdx+bc^2+a\right)^p}{2\left(bdp+bd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*(d*x+c)^2)^p,x, algorithm="fricas")
```

```
[Out] 1/2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)*(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)
^p/(b*d*p + b*d)
```

giac [A] time = 0.17, size = 38, normalized size = 1.27

$$\frac{\left(bd^2x^2 + 2\,bcdx + bc^2 + a\right)^{p+1}}{2\,bd\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*(d*x+c)^2)^p,x, algorithm="giac")
```

```
[Out] 1/2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)^(p + 1)/(b*d*(p + 1))
```

maple [A] time = 0.00, size = 39, normalized size = 1.30

$$\frac{\left(b\,d^2x^2 + 2bcdx + b\,c^2 + a\right)^{p+1}}{2\left(p+1\right)bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(a+(d*x+c)^2*b)^p,x)
```

```
[Out] 1/2*(b*d^2*x^2+2*b*c*d*x+b*c^2+a)^(p+1)/b/d/(p+1)
```

maxima [A] time = 0.53, size = 28, normalized size = 0.93

$$\frac{\left((dx+c)^2b + a\right)^{p+1}}{2\,bd\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*(d*x+c)^2)^p,x, algorithm="maxima")
```

```
[Out] 1/2*((d*x + c)^2*b + a)^(p + 1)/(b*d*(p + 1))
```

mupad [B] time = 1.21, size = 55, normalized size = 1.83

$$\left(a + b\,(c + d\,x)^2\right)^p\left(\frac{c\,x}{p+1} + \frac{d\,x^2}{2\,\left(p+1\right)} + \frac{b\,c^2 + a}{2\,b\,d\,\left(p+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^2)^p*(c + d*x),x)
```

```
[Out] (a + b*(c + d*x)^2)^p*((c*x)/(p + 1) + (d*x^2)/(2*(p + 1)) + (a + b*c^2)/(2
*b*d*(p + 1)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*(d*x+c)**2)**p,x)
```

```
[Out] Timed out
```

3.2852 $\int \frac{\left(a+b(c+dx)^2\right)^p}{c+dx}\,dx$

Optimal. Leaf size=52

$$-\frac{\left(a+b(c+dx)^2\right)^{p+1}\ {}_2F_1\left(1,p+1;p+2;\frac{b(c+dx)^2}{a}+1\right)}{2ad(p+1)}$$

```
[Out] -1/2*(a+b*(d*x+c)^2)^(1+p)*hypergeom([1, 1+p],[2+p],1+b*(d*x+c)^2/a)/a/d/(1
+p)
```

Rubi [A] time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {372, 266, 65}

$$-\frac{\left(a+b(c+dx)^2\right)^{p+1}\ {}_2F_1\left(1,p+1;p+2;\frac{b(c+dx)^2}{a}+1\right)}{2ad(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c + d*x)^2)^p/(c + d*x),x]
```

```
[Out] -((a + b*(c + d*x)^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*(c
+ d*x)^2)/a])/(2*a*d*(1 + p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b(c+dx)^2\right)^p}{c+dx}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^p}{x}\,dx, x, c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, (c+dx)^2\right)}{2d}\\ &= -\frac{\left(a+b(c+dx)^2\right)^{1+p}\ {}_2F_1\left(1,1+p;2+p;1+\frac{b(c+dx)^2}{a}\right)}{2ad(1+p)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 52, normalized size = 1.00

$$-\frac{\left(a+b(c+dx)^2\right)^{p+1}\ {}_2F_1\left(1,p+1;p+2;\frac{b(c+dx)^2}{a}+1\right)}{2ad(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c + d*x)^2)^p/(c + d*x),x]
```

```
[Out] -1/2*((a + b*(c + d*x)^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b
*(c + d*x)^2)/a])/(a*d*(1 + p))
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bd^2x^2+2\,bcdx+bc^2+a\right)^p}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^2)^p/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)^p/(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((dx+c)^2b+a\right)^p}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^2)^p/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(((d*x + c)^2*b + a)^p/(d*x + c), x)
```

maple [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\left(a+(dx+c)^2\,b\right)^p}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(d*x+c)^2*b)^p/(d*x+c),x)
```

```
[Out] int((a+(d*x+c)^2*b)^p/(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((dx+c)^2b+a\right)^p}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^2)^p/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(((d*x + c)^2*b + a)^p/(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(a+b\,(c+d\,x)^2\right)^p}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^2)^p/(c + d*x),x)
```

```
[Out] int((a + b*(c + d*x)^2)^p/(c + d*x), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)**2)**p/(d*x+c),x)
```

```
[Out] Timed out
```

3.2853 $\int (c+dx)^3 \left(a+b(c+dx)^3\right) dx$

Optimal. Leaf size=31

$$\frac{a(c+dx)^4}{4d}+\frac{b(c+dx)^7}{7d}$$

```
[Out] 1/4*a*(d*x+c)^4/d+1/7*b*(d*x+c)^7/d
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {372, 14}

$$\frac{a(c+dx)^4}{4d}+\frac{b(c+dx)^7}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + b*(c + d*x)^3),x]
```

```
[Out] (a*(c + d*x)^4)/(4*d) + (b*(c + d*x)^7)/(7*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^3\left(a+b(c+dx)^3\right)dx &= \frac{\text{Subst}\left(\int x^3\left(a+bx^3\right)dx, x, c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(ax^3+bx^6\right)dx, x, c+dx\right)}{d}\\ &= \frac{a(c+dx)^4}{4d}+\frac{b(c+dx)^7}{7d}\end{aligned}$$

Mathematica [B] time = 0.01, size = 98, normalized size = 3.16

$$\frac{1}{4}d^3x^4\left(a+20bc^3\right)+cd^2x^3\left(a+5bc^3\right)+c^3x\left(a+bc^3\right)+\frac{3}{2}c^2dx^2\left(a+2bc^3\right)+3bc^2d^4x^5+bcd^5x^6+\frac{1}{7}bd^6x^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + b*(c + d*x)^3),x]
```

```
[Out] c^3*(a + b*c^3)*x + (3*c^2*(a + 2*b*c^3)*d*x^2)/2 + c*(a + 5*b*c^3)*d^2*x^3
 + ((a + 20*b*c^3)*d^3*x^4)/4 + 3*b*c^2*d^4*x^5 + b*c*d^5*x^6 + (b*d^6*x^7)
/7
```

fricas [B] time = 0.77, size = 105, normalized size = 3.39

$$\frac{1}{7}x^7d^6b+x^6d^5cb+3x^5d^4c^2b+5x^4d^3c^3b+5x^3d^2c^4b+3x^2dc^5b+xc^6b+\frac{1}{4}x^4d^3a+x^3d^2ca+\frac{3}{2}x^2dc^2a+xc^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*d^6*b + x^6*d^5*c*b + 3*x^5*d^4*c^2*b + 5*x^4*d^3*c^3*b + 5*x^3*d^2
*c^4*b + 3*x^2*d*c^5*b + x*c^6*b + 1/4*x^4*d^3*a + x^3*d^2*c*a + 3/2*x^2*d*
c^2*a + x*c^3*a
```

giac [B] time = 0.15, size = 105, normalized size = 3.39

$$\frac{1}{7}\,bd^6x^7+bcd^5x^6+3\,bc^2d^4x^5+5\,bc^3d^3x^4+5\,bc^4d^2x^3+3\,bc^5dx^2+bc^6x+\frac{1}{4}\,ad^3x^4+acd^2x^3+\frac{3}{2}\,ac^2dx^2+ac^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/7*b*d^6*x^7 + b*c*d^5*x^6 + 3*b*c^2*d^4*x^5 + 5*b*c^3*d^3*x^4 + 5*b*c^4*d
^2*x^3 + 3*b*c^5*d*x^2 + b*c^6*x + 1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*
d*x^2 + a*c^3*x
```

maple [B] time = 0.00, size = 124, normalized size = 4.00

$$\frac{b\,d^6x^7}{7}+bc\,d^5x^6+3b\,c^2d^4x^5+\left(b\,c^3+a\right)c^3x+\frac{\left(19b\,c^3d^3+\left(b\,c^3+a\right)d^3\right)x^4}{4}+\frac{\left(12b\,c^4d^2+3\left(b\,c^3+a\right)c\,d^2\right)x^3}{3}+\frac{\left(3b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+b*(d*x+c)^3),x)
```

```
[Out] 1/7*d^6*b*x^7+c*d^5*b*x^6+3*c^2*d^4*b*x^5+1/4*(19*c^3*b*d^3+d^3*(b*c^3+a))*
x^4+1/3*(12*c^4*b*d^2+3*c*d^2*(b*c^3+a))*x^3+1/2*(3*c^5*b*d+3*c^2*d*(b*c^3+
a))*x^2+c^3*(b*c^3+a)*x
```

maxima [B] time = 0.51, size = 95, normalized size = 3.06

$$\frac{1}{7}\,bd^6x^7+bcd^5x^6+3\,bc^2d^4x^5+\frac{1}{4}\left(20\,bc^3+a\right)d^3x^4+\left(5\,bc^4+ac\right)d^2x^3+\frac{3}{2}\left(2\,bc^5+ac^2\right)dx^2+\left(bc^6+ac^3\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/7*b*d^6*x^7 + b*c*d^5*x^6 + 3*b*c^2*d^4*x^5 + 1/4*(20*b*c^3 + a)*d^3*x^4
+ (5*b*c^4 + a*c)*d^2*x^3 + 3/2*(2*b*c^5 + a*c^2)*d*x^2 + (b*c^6 + a*c^3)*x
```

mupad [B] time = 0.05, size = 93, normalized size = 3.00

$$x\,\left(b\,c^6+a\,c^3\right)+\frac{d^3\,x^4\,\left(20\,b\,c^3+a\right)}{4}+\frac{b\,d^6\,x^7}{7}+3\,b\,c^2\,d^4\,x^5+\frac{3\,c^2\,d\,x^2\,\left(2\,b\,c^3+a\right)}{2}+c\,d^2\,x^3\,\left(5\,b\,c^3+a\right)+b\,c\,d^5\,x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^3)*(c + d*x)^3,x)
```

```
[Out] x*(a*c^3 + b*c^6) + (d^3*x^4*(a + 20*b*c^3))/4 + (b*d^6*x^7)/7 + 3*b*c^2*d^
4*x^5 + (3*c^2*d*x^2*(a + 2*b*c^3))/2 + c*d^2*x^3*(a + 5*b*c^3) + b*c*d^5*x
^6
```

sympy [B] time = 0.09, size = 107, normalized size = 3.45

$$3bc^2d^4x^5+bcd^5x^6+\frac{bd^6x^7}{7}+x^4\left(\frac{ad^3}{4}+5bc^3d^3\right)+x^3\left(acd^2+5bc^4d^2\right)+x^2\left(\frac{3ac^2d}{2}+3bc^5d\right)+x\left(ac^3+bc^6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+b*(d*x+c)**3),x)
```

```
[Out] 3*b*c**2*d**4*x**5 + b*c*d**5*x**6 + b*d**6*x**7/7 + x**4*(a*d**3/4 + 5*b*c
**3*d**3) + x**3*(a*c*d**2 + 5*b*c**4*d**2) + x**2*(3*a*c**2*d/2 + 3*b*c**5
*d) + x*(a*c**3 + b*c**6)
```

3.2854 $\int (c+dx)^3 \left(a+b(c+dx)^3\right)^2 dx$

Optimal. Leaf size=51

$$\frac{a^2(c+dx)^4}{4d}+\frac{2ab(c+dx)^7}{7d}+\frac{b^2(c+dx)^{10}}{10d}$$

```
[Out] 1/4*a^2*(d*x+c)^4/d+2/7*a*b*(d*x+c)^7/d+1/10*b^2*(d*x+c)^10/d
```

Rubi [A] time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {372, 270}

$$\frac{a^2(c+dx)^4}{4d}+\frac{2ab(c+dx)^7}{7d}+\frac{b^2(c+dx)^{10}}{10d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + b*(c + d*x)^3)^2,x]
```

```
[Out] (a^2*(c + d*x)^4)/(4*d) + (2*a*b*(c + d*x)^7)/(7*d) + (b^2*(c + d*x)^10)/(1
0*d)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^3\left(a+b(c+dx)^3\right)^2 dx &= \frac{\text{Subst}\left(\int x^3\left(a+bx^3\right)^2 dx, x, c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(a^2x^3+2abx^6+b^2x^9\right) dx, x, c+dx\right)}{d}\\ &= \frac{a^2(c+dx)^4}{4d}+\frac{2ab(c+dx)^7}{7d}+\frac{b^2(c+dx)^{10}}{10d}\end{aligned}$$

Mathematica [B] time = 0.03, size = 203, normalized size = 3.98

$$\frac{1}{4}d^3x^4\left(a^2+40abc^3+84b^2c^6\right)+cd^2x^3\left(a^2+10abc^3+12b^2c^6\right)+\frac{3}{2}c^2dx^2\left(a^2+4abc^3+3b^2c^6\right)+\frac{2}{7}bd^6x^7\left(a+42bc\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + b*(c + d*x)^3)^2,x]
```

```
[Out] c^3*(a + b*c^3)^2*x + (3*c^2*(a^2 + 4*a*b*c^3 + 3*b^2*c^6)*d*x^2)/2 + c*(a^
2 + 10*a*b*c^3 + 12*b^2*c^6)*d^2*x^3 + ((a^2 + 40*a*b*c^3 + 84*b^2*c^6)*d^3
*x^4)/4 + (6*b*c^2*(5*a + 21*b*c^3)*d^4*x^5)/5 + b*c*(2*a + 21*b*c^3)*d^5*x
```

```
^6 + (2*b*(a + 42*b*c^3)*d^6*x^7)/7 + (9*b^2*c^2*d^7*x^8)/2 + b^2*c*d^8*x^9
 + (b^2*d^9*x^10)/10
```

fricas [B] time = 0.63, size = 248, normalized size = 4.86

$$\frac{1}{10}x^{10}d^9b^2+x^9d^8cb^2+\frac{9}{2}x^8d^7c^2b^2+12x^7d^6c^3b^2+21x^6d^5c^4b^2+\frac{126}{5}x^5d^4c^5b^2+21x^4d^3c^6b^2+\frac{2}{7}x^7d^6ba+12x^3d^2c^7b^2+2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*d^9*b^2 + x^9*d^8*c*b^2 + 9/2*x^8*d^7*c^2*b^2 + 12*x^7*d^6*c^3*b^
2 + 21*x^6*d^5*c^4*b^2 + 126/5*x^5*d^4*c^5*b^2 + 21*x^4*d^3*c^6*b^2 + 2/7*x
^7*d^6*b*a + 12*x^3*d^2*c^7*b^2 + 2*x^6*d^5*c*b*a + 9/2*x^2*d*c^8*b^2 + 6*x
^5*d^4*c^2*b*a + x*c^9*b^2 + 10*x^4*d^3*c^3*b*a + 10*x^3*d^2*c^4*b*a + 6*x^
2*d*c^5*b*a + 2*x*c^6*b*a + 1/4*x^4*d^3*a^2 + x^3*d^2*c*a^2 + 3/2*x^2*d*c^2
*a^2 + x*c^3*a^2
```

giac [B] time = 0.16, size = 248, normalized size = 4.86

$$\frac{1}{10}\,b^2d^9x^{10}+b^2cd^8x^9+\frac{9}{2}\,b^2c^2d^7x^8+12\,b^2c^3d^6x^7+21\,b^2c^4d^5x^6+\frac{126}{5}\,b^2c^5d^4x^5+21\,b^2c^6d^3x^4+\frac{2}{7}\,abd^6x^7+12\,b^2c^7d^2x^3+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/10*b^2*d^9*x^10 + b^2*c*d^8*x^9 + 9/2*b^2*c^2*d^7*x^8 + 12*b^2*c^3*d^6*x^
7 + 21*b^2*c^4*d^5*x^6 + 126/5*b^2*c^5*d^4*x^5 + 21*b^2*c^6*d^3*x^4 + 2/7*a
*b*d^6*x^7 + 12*b^2*c^7*d^2*x^3 + 2*a*b*c*d^5*x^6 + 9/2*b^2*c^8*d*x^2 + 6*a
*b*c^2*d^4*x^5 + b^2*c^9*x + 10*a*b*c^3*d^3*x^4 + 10*a*b*c^4*d^2*x^3 + 6*a*
b*c^5*d*x^2 + 2*a*b*c^6*x + 1/4*a^2*d^3*x^4 + a^2*c*d^2*x^3 + 3/2*a^2*c^2*d
*x^2 + a^2*c^3*x
```

maple [B] time = 0.00, size = 470, normalized size = 9.22

$$\frac{b^2d^9x^{10}}{10}+b^2c\,d^8x^9+\frac{9b^2c^2d^7x^8}{2}+\frac{\left(64b^2c^3d^6+\left(18b^2c^3d^3+2\left(b\,c^3+a\right)b\,d^3\right)d^3\right)x^7}{7}+\frac{\left(51b^2c^4d^5+3\left(18b^2c^3d^3+2\left(b\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+b*(d*x+c)^3)^2,x)
```

```
[Out] 1/10*d^9*b^2*x^10+c*d^8*b^2*x^9+9/2*c^2*d^7*b^2*x^8+1/7*(64*c^3*b^2*d^6+d^3
*(2*(b*c^3+a)*b*d^3+18*b^2*c^3*d^3))*x^7+1/6*(51*c^4*b^2*d^5+3*c*d^2*(2*(b*
c^3+a)*b*d^3+18*b^2*c^3*d^3)+d^3*(6*(b*c^3+a)*b*c*d^2+9*b^2*c^4*d^2))*x^6+1
/5*(15*c^5*b^2*d^4+3*c^2*d*(2*(b*c^3+a)*b*d^3+18*b^2*c^3*d^3)+3*c*d^2*(6*(b
*c^3+a)*b*c*d^2+9*b^2*c^4*d^2)+6*d^4*(b*c^3+a)*b*c^2)*x^5+1/4*(c^3*(2*(b*c^
3+a)*b*d^3+18*b^2*c^3*d^3)+3*c^2*d*(6*(b*c^3+a)*b*c*d^2+9*b^2*c^4*d^2)+18*c
^3*d^3*(b*c^3+a)*b+d^3*(b*c^3+a)^2)*x^4+1/3*(c^3*(6*(b*c^3+a)*b*c*d^2+9*b^2
*c^4*d^2)+18*c^4*d^2*(b*c^3+a)*b+3*c*d^2*(b*c^3+a)^2)*x^3+1/2*(6*c^5*(b*c^3
+a)*b*d+3*c^2*d*(b*c^3+a)^2)*x^2+c^3*(b*c^3+a)^2*x
```

maxima [B] time = 0.63, size = 210, normalized size = 4.12

$$\frac{1}{10}\,b^2d^9x^{10}+b^2cd^8x^9+\frac{9}{2}\,b^2c^2d^7x^8+\frac{2}{7}\left(42\,b^2c^3+ab\right)d^6x^7+\left(21\,b^2c^4+2\,abc\right)d^5x^6+\frac{6}{5}\left(21\,b^2c^5+5\,abc^2\right)d^4x^5+\frac{1}{4}\left(84\,b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 1/10*b^2*d^9*x^10 + b^2*c*d^8*x^9 + 9/2*b^2*c^2*d^7*x^8 + 2/7*(42*b^2*c^3 +
 a*b)*d^6*x^7 + (21*b^2*c^4 + 2*a*b*c)*d^5*x^6 + 6/5*(21*b^2*c^5 + 5*a*b*c^
2)*d^4*x^5 + 1/4*(84*b^2*c^6 + 40*a*b*c^3 + a^2)*d^3*x^4 + (12*b^2*c^7 + 10
*a*b*c^4 + a^2*c)*d^2*x^3 + 3/2*(3*b^2*c^8 + 4*a*b*c^5 + a^2*c^2)*d*x^2 + (
b^2*c^9 + 2*a*b*c^6 + a^2*c^3)*x
```

mupad **[B]** time = 1.23, size = 191, normalized size = 3.75

$$c^3\,x\left(b\,c^3+a\right)^2+\frac{b^2\,d^9\,x^{10}}{10}+\frac{d^3\,x^4\,\left(a^2+40\,a\,b\,c^3+84\,b^2\,c^6\right)}{4}+b^2\,c\,d^8\,x^9+\frac{3\,c^2\,d\,x^2\,\left(a^2+4\,a\,b\,c^3+3\,b^2\,c^6\right)}{2}+c\,d^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^3)^2*(c + d*x)^3,x)
```

```
[Out] c^3*x*(a + b*c^3)^2 + (b^2*d^9*x^10)/10 + (d^3*x^4*(a^2 + 84*b^2*c^6 + 40*a
*b*c^3))/4 + b^2*c*d^8*x^9 + (3*c^2*d*x^2*(a^2 + 3*b^2*c^6 + 4*a*b*c^3))/2
+ c*d^2*x^3*(a^2 + 12*b^2*c^6 + 10*a*b*c^3) + (9*b^2*c^2*d^7*x^8)/2 + (2*b*
d^6*x^7*(a + 42*b*c^3))/7 + b*c*d^5*x^6*(2*a + 21*b*c^3) + (6*b*c^2*d^4*x^5
*(5*a + 21*b*c^3))/5
```

sympy **[B]** time = 0.13, size = 252, normalized size = 4.94

$$\frac{9b^2c^2d^7x^8}{2}+b^2cd^8x^9+\frac{b^2d^9x^{10}}{10}+x^7\left(\frac{2abd^6}{7}+12b^2c^3d^6\right)+x^6\left(2abcd^5+21b^2c^4d^5\right)+x^5\left(6abc^2d^4+\frac{126b^2c^5d^4}{5}\right)+x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+b*(d*x+c)**3)**2,x)
```

```
[Out] 9*b**2*c**2*d**7*x**8/2 + b**2*c*d**8*x**9 + b**2*d**9*x**10/10 + x**7*(2*a
*b*d**6/7 + 12*b**2*c**3*d**6) + x**6*(2*a*b*c*d**5 + 21*b**2*c**4*d**5) +
x**5*(6*a*b*c**2*d**4 + 126*b**2*c**5*d**4/5) + x**4*(a**2*d**3/4 + 10*a*b*
c**3*d**3 + 21*b**2*c**6*d**3) + x**3*(a**2*c*d**2 + 10*a*b*c**4*d**2 + 12*
b**2*c**7*d**2) + x**2*(3*a**2*c**2*d/2 + 6*a*b*c**5*d + 9*b**2*c**8*d/2) +
 x*(a**2*c**3 + 2*a*b*c**6 + b**2*c**9)
```

3.2855 $\int (c+dx)^3 \left(a+b(c+dx)^3\right)^3 \, dx$

Optimal. Leaf size=71

$$\frac{a^3(c+dx)^4}{4d}+\frac{3a^2b(c+dx)^7}{7d}+\frac{3ab^2(c+dx)^{10}}{10d}+\frac{b^3(c+dx)^{13}}{13d}$$

```
[Out] 1/4*a^3*(d*x+c)^4/d+3/7*a^2*b*(d*x+c)^7/d+3/10*a*b^2*(d*x+c)^10/d+1/13*b^3*
(d*x+c)^13/d
```

Rubi [A] time = 0.11, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {372, 270}

$$\frac{3a^2b(c+dx)^7}{7d}+\frac{a^3(c+dx)^4}{4d}+\frac{3ab^2(c+dx)^{10}}{10d}+\frac{b^3(c+dx)^{13}}{13d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + b*(c + d*x)^3)^3,x]
```

```
[Out] (a^3*(c + d*x)^4)/(4*d) + (3*a^2*b*(c + d*x)^7)/(7*d) + (3*a*b^2*(c + d*x)^
10)/(10*d) + (b^3*(c + d*x)^13)/(13*d)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^3\left(a+b(c+dx)^3\right)^3\,dx &= \frac{\text{Subst}\left(\int x^3\left(a+bx^3\right)^3\,dx, x, c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(a^3x^3+3a^2bx^6+3ab^2x^9+b^3x^{12}\right)\,dx, x, c+dx\right)}{d}\\ &= \frac{a^3(c+dx)^4}{4d}+\frac{3a^2b(c+dx)^7}{7d}+\frac{3ab^2(c+dx)^{10}}{10d}+\frac{b^3(c+dx)^{13}}{13d}\end{aligned}$$

Mathematica [B] time = 0.08, size = 323, normalized size = 4.55

$$\frac{3}{7}bd^6x^7\left(a^2+84abc^3+308b^2c^6\right)+3bcd^5x^6\left(a^2+21abc^3+44b^2c^6\right)+\frac{9}{5}bc^2d^4x^5\left(5a^2+42abc^3+55b^2c^6\right)+\frac{1}{4}d^3x^4\left(a^3\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + b*(c + d*x)^3)^3,x]
```

```
[Out] c^3*(a + b*c^3)^3*x + (3*c^2*(a + b*c^3)^2*(a + 4*b*c^3)*d*x^2)/2 + c*(a^3
+ 15*a^2*b*c^3 + 36*a*b^2*c^6 + 22*b^3*c^9)*d^2*x^3 + ((a^3 + 60*a^2*b*c^3
+ 252*a*b^2*c^6 + 220*b^3*c^9)*d^3*x^4)/4 + (9*b*c^2*(5*a^2 + 42*a*b*c^3 +
```

```
55*b^2*c^6)*d^4*x^5)/5 + 3*b*c*(a^2 + 21*a*b*c^3 + 44*b^2*c^6)*d^5*x^6 + (3
*b*(a^2 + 84*a*b*c^3 + 308*b^2*c^6)*d^6*x^7)/7 + (9*b^2*c^2*(3*a + 22*b*c^3
)*d^7*x^8)/2 + b^2*c*(3*a + 55*b*c^3)*d^8*x^9 + (b^2*(3*a + 220*b*c^3)*d^9*
x^10)/10 + 6*b^3*c^2*d^10*x^11 + b^3*c*d^11*x^12 + (b^3*d^12*x^13)/13
```

fricas [B] time = 0.75, size = 442, normalized size = 6.23

$$\frac{1}{13}x^{13}d^{12}b^3+x^{12}d^{11}cb^3+6x^{11}d^{10}c^2b^3+22x^{10}d^9c^3b^3+55x^9d^8c^4b^3+99x^8d^7c^5b^3+132x^7d^6c^6b^3+\frac{3}{10}x^{10}d^9b^2a+132x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*d^12*b^3 + x^12*d^11*c*b^3 + 6*x^11*d^10*c^2*b^3 + 22*x^10*d^9*c^
3*b^3 + 55*x^9*d^8*c^4*b^3 + 99*x^8*d^7*c^5*b^3 + 132*x^7*d^6*c^6*b^3 + 3/1
0*x^10*d^9*b^2*a + 132*x^6*d^5*c^7*b^3 + 3*x^9*d^8*c*b^2*a + 99*x^5*d^4*c^8
*b^3 + 27/2*x^8*d^7*c^2*b^2*a + 55*x^4*d^3*c^9*b^3 + 36*x^7*d^6*c^3*b^2*a +
 22*x^3*d^2*c^10*b^3 + 63*x^6*d^5*c^4*b^2*a + 6*x^2*d*c^11*b^3 + 378/5*x^5*
d^4*c^5*b^2*a + x*c^12*b^3 + 63*x^4*d^3*c^6*b^2*a + 3/7*x^7*d^6*b*a^2 + 36*
x^3*d^2*c^7*b^2*a + 3*x^6*d^5*c*b*a^2 + 27/2*x^2*d*c^8*b^2*a + 9*x^5*d^4*c^
2*b*a^2 + 3*x*c^9*b^2*a + 15*x^4*d^3*c^3*b*a^2 + 15*x^3*d^2*c^4*b*a^2 + 9*x
^2*d*c^5*b*a^2 + 3*x*c^6*b*a^2 + 1/4*x^4*d^3*a^3 + x^3*d^2*c*a^3 + 3/2*x^2*
d*c^2*a^3 + x*c^3*a^3
```

giac [B] time = 0.16, size = 442, normalized size = 6.23

$$\frac{1}{13}\,b^3d^{12}x^{13}+b^3cd^{11}x^{12}+6\,b^3c^2d^{10}x^{11}+22\,b^3c^3d^9x^{10}+55\,b^3c^4d^8x^9+99\,b^3c^5d^7x^8+132\,b^3c^6d^6x^7+\frac{3}{10}\,ab^2d^9x^{10}+13$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 1/13*b^3*d^12*x^13 + b^3*c*d^11*x^12 + 6*b^3*c^2*d^10*x^11 + 22*b^3*c^3*d^9
*x^10 + 55*b^3*c^4*d^8*x^9 + 99*b^3*c^5*d^7*x^8 + 132*b^3*c^6*d^6*x^7 + 3/1
0*a*b^2*d^9*x^10 + 132*b^3*c^7*d^5*x^6 + 3*a*b^2*c*d^8*x^9 + 99*b^3*c^8*d^4
*x^5 + 27/2*a*b^2*c^2*d^7*x^8 + 55*b^3*c^9*d^3*x^4 + 36*a*b^2*c^3*d^6*x^7 +
 22*b^3*c^10*d^2*x^3 + 63*a*b^2*c^4*d^5*x^6 + 6*b^3*c^11*d*x^2 + 378/5*a*b^
2*c^5*d^4*x^5 + b^3*c^12*x + 63*a*b^2*c^6*d^3*x^4 + 3/7*a^2*b*d^6*x^7 + 36*
a*b^2*c^7*d^2*x^3 + 3*a^2*b*c*d^5*x^6 + 27/2*a*b^2*c^8*d*x^2 + 9*a^2*b*c^2*
d^4*x^5 + 3*a*b^2*c^9*x + 15*a^2*b*c^3*d^3*x^4 + 15*a^2*b*c^4*d^2*x^3 + 9*a
^2*b*c^5*d*x^2 + 3*a^2*b*c^6*x + 1/4*a^3*d^3*x^4 + a^3*c*d^2*x^3 + 3/2*a^3*
c^2*d*x^2 + a^3*c^3*x
```

maple [B] time = 0.00, size = 1948, normalized size = 27.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+b*(d*x+c)^3)^3,x)
```

```
[Out] 1/13*d^12*b^3*x^13+c*d^11*b^3*x^12+6*c^2*d^10*b^3*x^11+1/10*(136*c^3*b^3*d^
9+d^3*((b*c^3+a)*b^2*d^6+63*b^3*c^3*d^6+b*d^3*(18*b^2*c^3*d^3+2*(b*c^3+a)*b
*d^3)))*x^10+1/9*(117*c^4*b^3*d^8+3*c*d^2*((b*c^3+a)*b^2*d^6+63*b^3*c^3*d^6
+b*d^3*(18*b^2*c^3*d^3+2*(b*c^3+a)*b*d^3))+d^3*(6*(b*c^3+a)*b^2*c*d^5+45*b^
3*c^4*d^5+3*b*c*d^2*(18*b^2*c^3*d^3+2*(b*c^3+a)*b*d^3)+b*d^3*(9*b^2*c^4*d^2
+6*(b*c^3+a)*b*c*d^2)))*x^9+1/8*(36*c^5*b^3*d^7+3*c^2*d*((b*c^3+a)*b^2*d^6+
63*b^3*c^3*d^6+b*d^3*(18*b^2*c^3*d^3+2*(b*c^3+a)*b*d^3))+3*c*d^2*(6*(b*c^3+
a)*b^2*c*d^5+45*b^3*c^4*d^5+3*b*c*d^2*(18*b^2*c^3*d^3+2*(b*c^3+a)*b*d^3)+b*
d^3*(9*b^2*c^4*d^2+6*(b*c^3+a)*b*c*d^2))+d^3*(21*(b*c^3+a)*b^2*c^2*d^4+3*b*
c^2*d*(18*b^2*c^3*d^3+2*(b*c^3+a)*b*d^3)+3*b*c*d^2*(9*b^2*c^4*d^2+6*(b*c^3+
```

```
a)*b*c*d^2)))*x^8+1/7*(c^3*((b*c^3+a)*b^2*d^6+63*b^3*c^3*d^6+b*d^3*(18*b^2*
c^3*d^3+2*(b*c^3+a)*b*d^3))+3*c^2*d*(6*(b*c^3+a)*b^2*c*d^5+45*b^3*c^4*d^5+3
*b*c*d^2*(18*b^2*c^3*d^3+2*(b*c^3+a)*b*d^3)+b*d^3*(9*b^2*c^4*d^2+6*(b*c^3+a
)*b*c*d^2))+3*c*d^2*(21*(b*c^3+a)*b^2*c^2*d^4+3*b*c^2*d*(18*b^2*c^3*d^3+2*(
b*c^3+a)*b*d^3)+3*b*c*d^2*(9*b^2*c^4*d^2+6*(b*c^3+a)*b*c*d^2))+d^3*((b*c^3+
a)*(18*b^2*c^3*d^3+2*(b*c^3+a)*b*d^3)+3*b*c^2*d*(9*b^2*c^4*d^2+6*(b*c^3+a)*
b*c*d^2)+18*b^2*c^3*d^3*(b*c^3+a)+b*d^3*(b*c^3+a)^2))*x^7+1/6*(c^3*(6*(b*c^
3+a)*b^2*c*d^5+45*b^3*c^4*d^5+3*b*c*d^2*(18*b^2*c^3*d^3+2*(b*c^3+a)*b*d^3)+
b*d^3*(9*b^2*c^4*d^2+6*(b*c^3+a)*b*c*d^2))+3*c^2*d*(21*(b*c^3+a)*b^2*c^2*d^
4+3*b*c^2*d*(18*b^2*c^3*d^3+2*(b*c^3+a)*b*d^3)+3*b*c*d^2*(9*b^2*c^4*d^2+6*(
b*c^3+a)*b*c*d^2))+3*c*d^2*((b*c^3+a)*(18*b^2*c^3*d^3+2*(b*c^3+a)*b*d^3)+3*
b*c^2*d*(9*b^2*c^4*d^2+6*(b*c^3+a)*b*c*d^2)+18*b^2*c^3*d^3*(b*c^3+a)+b*d^3*
(b*c^3+a)^2)+d^3*((b*c^3+a)*(9*b^2*c^4*d^2+6*(b*c^3+a)*b*c*d^2)+18*b^2*c^4*
d^2*(b*c^3+a)+3*b*c*d^2*(b*c^3+a)^2))*x^6+1/5*(c^3*(21*(b*c^3+a)*b^2*c^2*d^
4+3*b*c^2*d*(18*b^2*c^3*d^3+2*(b*c^3+a)*b*d^3)+3*b*c*d^2*(9*b^2*c^4*d^2+6*(
b*c^3+a)*b*c*d^2))+3*c^2*d*((b*c^3+a)*(18*b^2*c^3*d^3+2*(b*c^3+a)*b*d^3)+3*
b*c^2*d*(9*b^2*c^4*d^2+6*(b*c^3+a)*b*c*d^2)+18*b^2*c^3*d^3*(b*c^3+a)+b*d^3*
(b*c^3+a)^2)+3*c*d^2*((b*c^3+a)*(9*b^2*c^4*d^2+6*(b*c^3+a)*b*c*d^2)+18*b^2*
c^4*d^2*(b*c^3+a)+3*b*c*d^2*(b*c^3+a)^2)+9*d^4*(b*c^3+a)^2*b*c^2)*x^5+1/4*(
c^3*((b*c^3+a)*(18*b^2*c^3*d^3+2*(b*c^3+a)*b*d^3)+3*b*c^2*d*(9*b^2*c^4*d^2+
6*(b*c^3+a)*b*c*d^2)+18*b^2*c^3*d^3*(b*c^3+a)+b*d^3*(b*c^3+a)^2)+3*c^2*d*((
b*c^3+a)*(9*b^2*c^4*d^2+6*(b*c^3+a)*b*c*d^2)+18*b^2*c^4*d^2*(b*c^3+a)+3*b*c
*d^2*(b*c^3+a)^2)+27*c^3*d^3*(b*c^3+a)^2*b+d^3*(b*c^3+a)^3)*x^4+1/3*(c^3*((
b*c^3+a)*(9*b^2*c^4*d^2+6*(b*c^3+a)*b*c*d^2)+18*b^2*c^4*d^2*(b*c^3+a)+3*b*c
*d^2*(b*c^3+a)^2)+27*c^4*d^2*(b*c^3+a)^2*b+3*c*d^2*(b*c^3+a)^3)*x^3+1/2*(9*
c^5*(b*c^3+a)^2*b*d+3*c^2*d*(b*c^3+a)^3)*x^2+c^3*(b*c^3+a)^3*x
```

maxima [B] time = 0.58, size = 359, normalized size = 5.06

$$\frac{1}{13}b^3d^{12}x^{13}+b^3cd^{11}x^{12}+6\,b^3c^2d^{10}x^{11}+\frac{1}{10}\left(220\,b^3c^3+3\,ab^2\right)d^9x^{10}+\left(55\,b^3c^4+3\,ab^2c\right)d^8x^9+\frac{9}{2}\left(22\,b^3c^5+3\,ab^2c^2\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] 1/13*b^3*d^12*x^13 + b^3*c*d^11*x^12 + 6*b^3*c^2*d^10*x^11 + 1/10*(220*b^3*
c^3 + 3*a*b^2)*d^9*x^10 + (55*b^3*c^4 + 3*a*b^2*c)*d^8*x^9 + 9/2*(22*b^3*c^
5 + 3*a*b^2*c^2)*d^7*x^8 + 3/7*(308*b^3*c^6 + 84*a*b^2*c^3 + a^2*b)*d^6*x^7
 + 3*(44*b^3*c^7 + 21*a*b^2*c^4 + a^2*b*c)*d^5*x^6 + 9/5*(55*b^3*c^8 + 42*a
*b^2*c^5 + 5*a^2*b*c^2)*d^4*x^5 + 1/4*(220*b^3*c^9 + 252*a*b^2*c^6 + 60*a^2
*b*c^3 + a^3)*d^3*x^4 + (22*b^3*c^10 + 36*a*b^2*c^7 + 15*a^2*b*c^4 + a^3*c)
*d^2*x^3 + 3/2*(4*b^3*c^11 + 9*a*b^2*c^8 + 6*a^2*b*c^5 + a^3*c^2)*d*x^2 + (
b^3*c^12 + 3*a*b^2*c^9 + 3*a^2*b*c^6 + a^3*c^3)*x
```

mupad [B] time = 1.28, size = 309, normalized size = 4.35

$$c^3\,x\left(b\,c^3+a\right)^3+\frac{d^3\,x^4\left(a^3+60\,a^2\,b\,c^3+252\,a\,b^2\,c^6+220\,b^3\,c^9\right)}{4}+\frac{b^3\,d^{12}\,x^{13}}{13}+b^3\,c\,d^{11}\,x^{12}+\frac{3\,b\,d^6\,x^7\left(a^2+84\,a\,b\,c^3\right.}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^3)^3*(c + d*x)^3,x)
```

```
[Out] c^3*x*(a + b*c^3)^3 + (d^3*x^4*(a^3 + 220*b^3*c^9 + 60*a^2*b*c^3 + 252*a*b^
2*c^6))/4 + (b^3*d^12*x^13)/13 + b^3*c*d^11*x^12 + (3*b*d^6*x^7*(a^2 + 308*
b^2*c^6 + 84*a*b*c^3))/7 + (b^2*d^9*x^10*(3*a + 220*b*c^3))/10 + 6*b^3*c^2*
d^10*x^11 + c*d^2*x^3*(a^3 + 22*b^3*c^9 + 15*a^2*b*c^3 + 36*a*b^2*c^6) + (9
*b^2*c^2*d^7*x^8*(3*a + 22*b*c^3))/2 + (3*c^2*d*x^2*(a + b*c^3)^2*(a + 4*b*
c^3))/2 + (9*b*c^2*d^4*x^5*(5*a^2 + 55*b^2*c^6 + 42*a*b*c^3))/5 + 3*b*c*d^5
*x^6*(a^2 + 44*b^2*c^6 + 21*a*b*c^3) + b^2*c*d^8*x^9*(3*a + 55*b*c^3)
```

sympy [**B**] time = 0.17, size = 437, normalized size = 6.15

$$6b^3c^2d^{10}x^{11}+b^3cd^{11}x^{12}+\frac{b^3d^{12}x^{13}}{13}+x^{10}\left(\frac{3ab^2d^9}{10}+22b^3c^3d^9\right)+x^9\left(3ab^2cd^8+55b^3c^4d^8\right)+x^8\left(\frac{27ab^2c^2d^7}{2}+99b^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+b*(d*x+c)**3)**3,x)
```

```
[Out] 6*b**3*c**2*d**10*x**11 + b**3*c*d**11*x**12 + b**3*d**12*x**13/13 + x**10*
(3*a*b**2*d**9/10 + 22*b**3*c**3*d**9) + x**9*(3*a*b**2*c*d**8 + 55*b**3*c*
*4*d**8) + x**8*(27*a*b**2*c**2*d**7/2 + 99*b**3*c**5*d**7) + x**7*(3*a**2*
b*d**6/7 + 36*a*b**2*c**3*d**6 + 132*b**3*c**6*d**6) + x**6*(3*a**2*b*c*d**
5 + 63*a*b**2*c**4*d**5 + 132*b**3*c**7*d**5) + x**5*(9*a**2*b*c**2*d**4 +
378*a*b**2*c**5*d**4/5 + 99*b**3*c**8*d**4) + x**4*(a**3*d**3/4 + 15*a**2*b
*c**3*d**3 + 63*a*b**2*c**6*d**3 + 55*b**3*c**9*d**3) + x**3*(a**3*c*d**2 +
 15*a**2*b*c**4*d**2 + 36*a*b**2*c**7*d**2 + 22*b**3*c**10*d**2) + x**2*(3*
a**3*c**2*d/2 + 9*a**2*b*c**5*d + 27*a*b**2*c**8*d/2 + 6*b**3*c**11*d) + x*
(a**3*c**3 + 3*a**2*b*c**6 + 3*a*b**2*c**9 + b**3*c**12)
```

3.2856 $\int (ce+dex)^3\left(a+b(c+dx)^3\right)\,dx$

Optimal. Leaf size=37

$$\frac{ae^3(c+dx)^4}{4d}+\frac{be^3(c+dx)^7}{7d}$$

```
[Out] 1/4*a*e^3*(d*x+c)^4/d+1/7*b*e^3*(d*x+c)^7/d
```

Rubi [A] time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {372, 14}

$$\frac{ae^3(c+dx)^4}{4d}+\frac{be^3(c+dx)^7}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*(c + d*x)^3),x]
```

```
[Out] (a*e^3*(c + d*x)^4)/(4*d) + (b*e^3*(c + d*x)^7)/(7*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int (ce+dex)^3\left(a+b(c+dx)^3\right)\,dx &= \frac{e^3\,\text{Subst}\left(\int x^3\left(a+bx^3\right)\,dx,x,c+dx\right)}{d}\\ &= \frac{e^3\,\text{Subst}\left(\int \left(ax^3+bx^6\right)\,dx,x,c+dx\right)}{d}\\ &= \frac{ae^3(c+dx)^4}{4d}+\frac{be^3(c+dx)^7}{7d}\end{aligned}$$

Mathematica [B] time = 0.01, size = 102, normalized size = 2.76

$$e^3\left(\frac{1}{4}d^3x^4\left(a+20bc^3\right)+cd^2x^3\left(a+5bc^3\right)+c^3x\left(a+bc^3\right)+\frac{3}{2}c^2dx^2\left(a+2bc^3\right)+3bc^2d^4x^5+bcd^5x^6+\frac{1}{7}bd^6x^7\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*(c + d*x)^3),x]
```

```
[Out] e^3*(c^3*(a + b*c^3)*x + (3*c^2*(a + 2*b*c^3)*d*x^2)/2 + c*(a + 5*b*c^3)*d^
2*x^3 + ((a + 20*b*c^3)*d^3*x^4)/4 + 3*b*c^2*d^4*x^5 + b*c*d^5*x^6 + (b*d^6
*x^7)/7)
```

fricas [B] time = 0.64, size = 138, normalized size = 3.73

$$\frac{1}{7}x^7e^3d^6b+x^6e^3d^5cb+3x^5e^3d^4c^2b+5x^4e^3d^3c^3b+5x^3e^3d^2c^4b+3x^2e^3dc^5b+xe^3c^6b+\frac{1}{4}x^4e^3d^3a+x^3e^3d^2ca+\frac{3}{2}x^2e^3dc^2a+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*e^3*d^6*b + x^6*e^3*d^5*c*b + 3*x^5*e^3*d^4*c^2*b + 5*x^4*e^3*d^3*c
^3*b + 5*x^3*e^3*d^2*c^4*b + 3*x^2*e^3*d*c^5*b + x*e^3*c^6*b + 1/4*x^4*e^3*
d^3*a + x^3*e^3*d^2*c*a + 3/2*x^2*e^3*d*c^2*a + x*e^3*c^3*a
```

giac [B] time = 0.15, size = 127, normalized size = 3.43

$$\frac{1}{7}\,bd^6x^7e^3+bcd^5x^6e^3+3\,bc^2d^4x^5e^3+5\,bc^3d^3x^4e^3+5\,bc^4d^2x^3e^3+3\,bc^5dx^2e^3+bc^6xe^3+\frac{1}{4}\,ad^3x^4e^3+acd^2x^3e^3+\frac{3}{2}\,ac^2d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/7*b*d^6*x^7*e^3 + b*c*d^5*x^6*e^3 + 3*b*c^2*d^4*x^5*e^3 + 5*b*c^3*d^3*x^4
*e^3 + 5*b*c^4*d^2*x^3*e^3 + 3*b*c^5*d*x^2*e^3 + b*c^6*x*e^3 + 1/4*a*d^3*x^
4*e^3 + a*c*d^2*x^3*e^3 + 3/2*a*c^2*d*x^2*e^3 + a*c^3*x*e^3
```

maple [B] time = 0.00, size = 154, normalized size = 4.16

$$\frac{b\,d^6e^3x^7}{7}+bc\,d^5e^3x^6+3b\,c^2d^4e^3x^5+\left(b\,c^3+a\right)c^3e^3x+\frac{\left(19b\,c^3d^3e^3+\left(b\,c^3+a\right)d^3e^3\right)x^4}{4}+\frac{\left(12b\,c^4d^2e^3+3\left(b\,c^3+a\right)\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*(d*x+c)^3),x)
```

```
[Out] 1/7*d^6*e^3*b*x^7+c*e^3*d^5*b*x^6+3*c^2*e^3*d^4*b*x^5+1/4*(19*c^3*e^3*b*d^3
+d^3*e^3*(b*c^3+a))*x^4+1/3*(12*c^4*e^3*b*d^2+3*c*e^3*d^2*(b*c^3+a))*x^3+1/
2*(3*c^5*e^3*b*d+3*c^2*e^3*d*(b*c^3+a))*x^2+c^3*e^3*(b*c^3+a)*x
```

maxima [B] time = 0.52, size = 116, normalized size = 3.14

$$\frac{1}{7}\,bd^6e^3x^7+bcd^5e^3x^6+3\,bc^2d^4e^3x^5+\frac{1}{4}\left(20\,bc^3+a\right)d^3e^3x^4+\left(5\,bc^4+ac\right)d^2e^3x^3+\frac{3}{2}\left(2\,bc^5+ac^2\right)de^3x^2+\left(bc^6+ac^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/7*b*d^6*e^3*x^7 + b*c*d^5*e^3*x^6 + 3*b*c^2*d^4*e^3*x^5 + 1/4*(20*b*c^3 +
 a)*d^3*e^3*x^4 + (5*b*c^4 + a*c)*d^2*e^3*x^3 + 3/2*(2*b*c^5 + a*c^2)*d*e^3
*x^2 + (b*c^6 + a*c^3)*e^3*x
```

mupad [B] time = 1.19, size = 113, normalized size = 3.05

$$\frac{d^3\,e^3\,x^4\,\left(20\,b\,c^3+a\right)}{4}+\frac{b\,d^6\,e^3\,x^7}{7}+c^3\,e^3\,x\,\left(b\,c^3+a\right)+\frac{3\,c^2\,d\,e^3\,x^2\,\left(2\,b\,c^3+a\right)}{2}+c\,d^2\,e^3\,x^3\,\left(5\,b\,c^3+a\right)+b\,c\,d^5\,e^3\,x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3*(a + b*(c + d*x)^3),x)
```

```
[Out] (d^3*e^3*x^4*(a + 20*b*c^3))/4 + (b*d^6*e^3*x^7)/7 + c^3*e^3*x*(a + b*c^3)
+ (3*c^2*d*e^3*x^2*(a + 2*b*c^3))/2 + c*d^2*e^3*x^3*(a + 5*b*c^3) + b*c*d^5
*e^3*x^6 + 3*b*c^2*d^4*e^3*x^5
```

sympy [B] time = 0.11, size = 144, normalized size = 3.89

$$3bc^2d^4e^3x^5+bcd^5e^3x^6+\frac{bd^6e^3x^7}{7}+x^4\left(\frac{ad^3e^3}{4}+5bc^3d^3e^3\right)+x^3\left(acd^2e^3+5bc^4d^2e^3\right)+x^2\left(\frac{3ac^2de^3}{2}+3bc^5de^3\right)+x\left(a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*(d*x+c)**3),x)
```

```
[Out] 3*b*c**2*d**4*e**3*x**5 + b*c*d**5*e**3*x**6 + b*d**6*e**3*x**7/7 + x**4*(a
*d**3*e**3/4 + 5*b*c**3*d**3*e**3) + x**3*(a*c*d**2*e**3 + 5*b*c**4*d**2*e*
*3) + x**2*(3*a*c**2*d*e**3/2 + 3*b*c**5*d*e**3) + x*(a*c**3*e**3 + b*c**6*
e**3)
```

3.2857 $\int (ce+dex)^3 \left(a+b(c+dx)^3\right)^2 dx$

Optimal. Leaf size=60

$$\frac{a^2e^3(c+dx)^4}{4d}+\frac{2abe^3(c+dx)^7}{7d}+\frac{b^2e^3(c+dx)^{10}}{10d}$$

```
[Out] 1/4*a^2*e^3*(d*x+c)^4/d+2/7*a*b*e^3*(d*x+c)^7/d+1/10*b^2*e^3*(d*x+c)^10/d
```

Rubi [A] time = 0.06, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {372, 270}

$$\frac{a^2e^3(c+dx)^4}{4d}+\frac{2abe^3(c+dx)^7}{7d}+\frac{b^2e^3(c+dx)^{10}}{10d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*(c + d*x)^3)^2,x]
```

```
[Out] (a^2*e^3*(c + d*x)^4)/(4*d) + (2*a*b*e^3*(c + d*x)^7)/(7*d) + (b^2*e^3*(c +
 d*x)^10)/(10*d)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int (ce+dex)^3\left(a+b(c+dx)^3\right)^2 dx &= \frac{e^3\,\text{Subst}\left(\int x^3\left(a+bx^3\right)^2 dx, x, c+dx\right)}{d}\\ &= \frac{e^3\,\text{Subst}\left(\int \left(a^2x^3+2abx^6+b^2x^9\right) dx, x, c+dx\right)}{d}\\ &= \frac{a^2e^3(c+dx)^4}{4d}+\frac{2abe^3(c+dx)^7}{7d}+\frac{b^2e^3(c+dx)^{10}}{10d}\end{aligned}$$

Mathematica [B] time = 0.01, size = 207, normalized size = 3.45

$$e^3\left(\frac{1}{4}d^3x^4\left(a^2+40abc^3+84b^2c^6\right)+cd^2x^3\left(a^2+10abc^3+12b^2c^6\right)+\frac{3}{2}c^2dx^2\left(a^2+4abc^3+3b^2c^6\right)+\frac{2}{7}bd^6x^7\left(a\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*(c + d*x)^3)^2,x]
```

```
[Out] e^3*(c^3*(a + b*c^3)^2*x + (3*c^2*(a^2 + 4*a*b*c^3 + 3*b^2*c^6)*d*x^2)/2 +
c*(a^2 + 10*a*b*c^3 + 12*b^2*c^6)*d^2*x^3 + ((a^2 + 40*a*b*c^3 + 84*b^2*c^6
)*d^3*x^4)/4 + (6*b*c^2*(5*a + 21*b*c^3)*d^4*x^5)/5 + b*c*(2*a + 21*b*c^3)*
```

```
d^5*x^6 + (2*b*(a + 42*b*c^3)*d^6*x^7)/7 + (9*b^2*c^2*d^7*x^8)/2 + b^2*c*d^
8*x^9 + (b^2*d^9*x^10)/10)
```

fricas [B] time = 0.48, size = 311, normalized size = 5.18

$$\frac{1}{10}x^{10}e^3d^9b^2+x^9e^3d^8cb^2+\frac{9}{2}x^8e^3d^7c^2b^2+12x^7e^3d^6c^3b^2+21x^6e^3d^5c^4b^2+\frac{126}{5}x^5e^3d^4c^5b^2+21x^4e^3d^3c^6b^2+\frac{2}{7}x^7e^3d^6ba+1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*e^3*d^9*b^2 + x^9*e^3*d^8*c*b^2 + 9/2*x^8*e^3*d^7*c^2*b^2 + 12*x^
7*e^3*d^6*c^3*b^2 + 21*x^6*e^3*d^5*c^4*b^2 + 126/5*x^5*e^3*d^4*c^5*b^2 + 21
*x^4*e^3*d^3*c^6*b^2 + 2/7*x^7*e^3*d^6*b*a + 12*x^3*e^3*d^2*c^7*b^2 + 2*x^6
*e^3*d^5*c*b*a + 9/2*x^2*e^3*d*c^8*b^2 + 6*x^5*e^3*d^4*c^2*b*a + x*e^3*c^9*
b^2 + 10*x^4*e^3*d^3*c^3*b*a + 10*x^3*e^3*d^2*c^4*b*a + 6*x^2*e^3*d*c^5*b*a
 + 2*x*e^3*c^6*b*a + 1/4*x^4*e^3*d^3*a^2 + x^3*e^3*d^2*c*a^2 + 3/2*x^2*e^3*
d*c^2*a^2 + x*e^3*c^3*a^2
```

giac [B] time = 0.16, size = 290, normalized size = 4.83

$$\frac{1}{10}\,b^2d^9x^{10}e^3+b^2cd^8x^9e^3+\frac{9}{2}\,b^2c^2d^7x^8e^3+12\,b^2c^3d^6x^7e^3+21\,b^2c^4d^5x^6e^3+\frac{126}{5}\,b^2c^5d^4x^5e^3+21\,b^2c^6d^3x^4e^3+\frac{2}{7}\,abd^6x^7e^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/10*b^2*d^9*x^10*e^3 + b^2*c*d^8*x^9*e^3 + 9/2*b^2*c^2*d^7*x^8*e^3 + 12*b^
2*c^3*d^6*x^7*e^3 + 21*b^2*c^4*d^5*x^6*e^3 + 126/5*b^2*c^5*d^4*x^5*e^3 + 21
*b^2*c^6*d^3*x^4*e^3 + 2/7*a*b*d^6*x^7*e^3 + 12*b^2*c^7*d^2*x^3*e^3 + 2*a*b
*c*d^5*x^6*e^3 + 9/2*b^2*c^8*d*x^2*e^3 + 6*a*b*c^2*d^4*x^5*e^3 + b^2*c^9*x*
e^3 + 10*a*b*c^3*d^3*x^4*e^3 + 10*a*b*c^4*d^2*x^3*e^3 + 6*a*b*c^5*d*x^2*e^3
 + 2*a*b*c^6*x*e^3 + 1/4*a^2*d^3*x^4*e^3 + a^2*c*d^2*x^3*e^3 + 3/2*a^2*c^2*
d*x^2*e^3 + a^2*c^3*x*e^3
```

maple [B] time = 0.00, size = 536, normalized size = 8.93

$$\frac{b^2d^9e^3x^{10}}{10}+b^2c\,d^8e^3x^9+\frac{9b^2c^2d^7e^3x^8}{2}+\left(b\,c^3+a\right)^2c^3e^3x+\frac{\left(64b^2c^3d^6e^3+\left(18b^2c^3d^3+2\left(b\,c^3+a\right)b\,d^3\right)d^3e^3\right)x^7}{7}+\frac{(51$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*(d*x+c)^3)^2,x)
```

```
[Out] 1/10*d^9*e^3*b^2*x^10+c*e^3*d^8*b^2*x^9+9/2*c^2*e^3*d^7*b^2*x^8+1/7*(64*c^3
*e^3*b^2*d^6+d^3*e^3*(18*b^2*c^3*d^3+2*(b*c^3+a)*b*d^3))*x^7+1/6*(51*c^4*e^
3*b^2*d^5+3*c*e^3*d^2*(18*b^2*c^3*d^3+2*(b*c^3+a)*b*d^3)+d^3*e^3*(9*b^2*c^4
*d^2+6*(b*c^3+a)*b*c*d^2))*x^6+1/5*(15*c^5*e^3*b^2*d^4+3*c^2*e^3*d*(18*b^2*
c^3*d^3+2*(b*c^3+a)*b*d^3)+3*c*e^3*d^2*(9*b^2*c^4*d^2+6*(b*c^3+a)*b*c*d^2)+
6*d^4*e^3*(b*c^3+a)*b*c^2)*x^5+1/4*(c^3*e^3*(18*b^2*c^3*d^3+2*(b*c^3+a)*b*d
^3)+3*c^2*e^3*d*(9*b^2*c^4*d^2+6*(b*c^3+a)*b*c*d^2)+18*c^3*e^3*d^3*(b*c^3+a
)*b+d^3*e^3*(b*c^3+a)^2)*x^4+1/3*(c^3*e^3*(9*b^2*c^4*d^2+6*(b*c^3+a)*b*c*d^
2)+18*c^4*e^3*d^2*(b*c^3+a)*b+3*c*e^3*d^2*(b*c^3+a)^2)*x^3+1/2*(6*c^5*e^3*(
b*c^3+a)*b*d+3*c^2*e^3*d*(b*c^3+a)^2)*x^2+c^3*e^3*(b*c^3+a)^2*x
```

maxima [B] time = 0.55, size = 240, normalized size = 4.00

$$\frac{1}{10}\,b^2d^9e^3x^{10}+b^2cd^8e^3x^9+\frac{9}{2}\,b^2c^2d^7e^3x^8+\frac{2}{7}\left(42\,b^2c^3+ab\right)d^6e^3x^7+\left(21\,b^2c^4+2\,abc\right)d^5e^3x^6+\frac{6}{5}\left(21\,b^2c^5+5\,abc^2\right)d^4e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 1/10*b^2*d^9*e^3*x^10 + b^2*c*d^8*e^3*x^9 + 9/2*b^2*c^2*d^7*e^3*x^8 + 2/7*(
42*b^2*c^3 + a*b)*d^6*e^3*x^7 + (21*b^2*c^4 + 2*a*b*c)*d^5*e^3*x^6 + 6/5*(2
1*b^2*c^5 + 5*a*b*c^2)*d^4*e^3*x^5 + 1/4*(84*b^2*c^6 + 40*a*b*c^3 + a^2)*d^
3*e^3*x^4 + (12*b^2*c^7 + 10*a*b*c^4 + a^2*c)*d^2*e^3*x^3 + 3/2*(3*b^2*c^8
+ 4*a*b*c^5 + a^2*c^2)*d*e^3*x^2 + (b^2*c^9 + 2*a*b*c^6 + a^2*c^3)*e^3*x
```

mupad **[B]** time = 1.24, size = 221, normalized size = 3.68

$$c^3\,e^3\,x\left(b\,c^3+a\right)^2+\frac{b^2\,d^9\,e^3\,x^{10}}{10}+\frac{d^3\,e^3\,x^4\,\left(a^2+40\,a\,b\,c^3+84\,b^2\,c^6\right)}{4}+\frac{3\,c^2\,d\,e^3\,x^2\,\left(a^2+4\,a\,b\,c^3+3\,b^2\,c^6\right)}{2}+c\,d^2\,e^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3*(a + b*(c + d*x)^3)^2,x)
```

```
[Out] c^3*e^3*x*(a + b*c^3)^2 + (b^2*d^9*e^3*x^10)/10 + (d^3*e^3*x^4*(a^2 + 84*b^
2*c^6 + 40*a*b*c^3))/4 + (3*c^2*d*e^3*x^2*(a^2 + 3*b^2*c^6 + 4*a*b*c^3))/2
+ c*d^2*e^3*x^3*(a^2 + 12*b^2*c^6 + 10*a*b*c^3) + (9*b^2*c^2*d^7*e^3*x^8)/2
 + (2*b*d^6*e^3*x^7*(a + 42*b*c^3))/7 + b^2*c*d^8*e^3*x^9 + b*c*d^5*e^3*x^6
*(2*a + 21*b*c^3) + (6*b*c^2*d^4*e^3*x^5*(5*a + 21*b*c^3))/5
```

sympy **[B]** time = 0.14, size = 323, normalized size = 5.38

$$\frac{9b^2c^2d^7e^3x^8}{2}+b^2cd^8e^3x^9+\frac{b^2d^9e^3x^{10}}{10}+x^7\left(\frac{2abd^6e^3}{7}+12b^2c^3d^6e^3\right)+x^6\left(2abcd^5e^3+21b^2c^4d^5e^3\right)+x^5\left(6abc^2d^4e^3+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*(d*x+c)**3)**2,x)
```

```
[Out] 9*b**2*c**2*d**7*e**3*x**8/2 + b**2*c*d**8*e**3*x**9 + b**2*d**9*e**3*x**10
/10 + x**7*(2*a*b*d**6*e**3/7 + 12*b**2*c**3*d**6*e**3) + x**6*(2*a*b*c*d**
5*e**3 + 21*b**2*c**4*d**5*e**3) + x**5*(6*a*b*c**2*d**4*e**3 + 126*b**2*c*
*5*d**4*e**3/5) + x**4*(a**2*d**3*e**3/4 + 10*a*b*c**3*d**3*e**3 + 21*b**2*
c**6*d**3*e**3) + x**3*(a**2*c*d**2*e**3 + 10*a*b*c**4*d**2*e**3 + 12*b**2*
c**7*d**2*e**3) + x**2*(3*a**2*c**2*d*e**3/2 + 6*a*b*c**5*d*e**3 + 9*b**2*c
**8*d*e**3/2) + x*(a**2*c**3*e**3 + 2*a*b*c**6*e**3 + b**2*c**9*e**3)
```

3.2858 $\int (ce+dex)^3\left(a+b(c+dx)^3\right)^3\,dx$

Optimal. Leaf size=83

$$\frac{a^3e^3(c+dx)^4}{4d}+\frac{3a^2be^3(c+dx)^7}{7d}+\frac{3ab^2e^3(c+dx)^{10}}{10d}+\frac{b^3e^3(c+dx)^{13}}{13d}$$

```
[Out] 1/4*a^3*e^3*(d*x+c)^4/d+3/7*a^2*b*e^3*(d*x+c)^7/d+3/10*a*b^2*e^3*(d*x+c)^10
/d+1/13*b^3*e^3*(d*x+c)^13/d
```

Rubi [A] time = 0.10, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {372, 270}

$$\frac{3a^2be^3(c+dx)^7}{7d}+\frac{a^3e^3(c+dx)^4}{4d}+\frac{3ab^2e^3(c+dx)^{10}}{10d}+\frac{b^3e^3(c+dx)^{13}}{13d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*(c + d*x)^3)^3,x]
```

```
[Out] (a^3*e^3*(c + d*x)^4)/(4*d) + (3*a^2*b*e^3*(c + d*x)^7)/(7*d) + (3*a*b^2*e^
3*(c + d*x)^10)/(10*d) + (b^3*e^3*(c + d*x)^13)/(13*d)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int (ce+dex)^3\left(a+b(c+dx)^3\right)^3\,dx &= \frac{e^3\,\text{Subst}\left(\int x^3\left(a+bx^3\right)^3\,dx, x, c+dx\right)}{d}\\ &= \frac{e^3\,\text{Subst}\left(\int \left(a^3x^3+3a^2bx^6+3ab^2x^9+b^3x^{12}\right)\,dx, x, c+dx\right)}{d}\\ &= \frac{a^3e^3(c+dx)^4}{4d}+\frac{3a^2be^3(c+dx)^7}{7d}+\frac{3ab^2e^3(c+dx)^{10}}{10d}+\frac{b^3e^3(c+dx)^{13}}{13d}\end{aligned}$$

Mathematica [B] time = 0.02, size = 327, normalized size = 3.94

$$e^3\left(\frac{3}{7}bd^6x^7\left(a^2+84abc^3+308b^2c^6\right)+3bcd^5x^6\left(a^2+21abc^3+44b^2c^6\right)+\frac{9}{5}bc^2d^4x^5\left(5a^2+42abc^3+55b^2c^6\right)+\frac{1}{4}d\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*(c + d*x)^3)^3,x]
```

```
[Out] e^3*(c^3*(a + b*c^3)^3*x + (3*c^2*(a + b*c^3)^2*(a + 4*b*c^3)*d*x^2)/2 + c*
(a^3 + 15*a^2*b*c^3 + 36*a*b^2*c^6 + 22*b^3*c^9)*d^2*x^3 + ((a^3 + 60*a^2*b
```

```
*c^3 + 252*a*b^2*c^6 + 220*b^3*c^9)*d^3*x^4)/4 + (9*b*c^2*(5*a^2 + 42*a*b*c
^3 + 55*b^2*c^6)*d^4*x^5)/5 + 3*b*c*(a^2 + 21*a*b*c^3 + 44*b^2*c^6)*d^5*x^6
 + (3*b*(a^2 + 84*a*b*c^3 + 308*b^2*c^6)*d^6*x^7)/7 + (9*b^2*c^2*(3*a + 22*
b*c^3)*d^7*x^8)/2 + b^2*c*(3*a + 55*b*c^3)*d^8*x^9 + (b^2*(3*a + 220*b*c^3)
*d^9*x^10)/10 + 6*b^3*c^2*d^10*x^11 + b^3*c*d^11*x^12 + (b^3*d^12*x^13)/13)
```

fricas [B] time = 0.78, size = 544, normalized size = 6.55

$$\frac{1}{13}x^{13}e^3d^{12}b^3+x^{12}e^3d^{11}cb^3+6x^{11}e^3d^{10}c^2b^3+22x^{10}e^3d^9c^3b^3+55x^9e^3d^8c^4b^3+99x^8e^3d^7c^5b^3+132x^7e^3d^6c^6b^3+\frac{3}{10}x^1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*e^3*d^12*b^3 + x^12*e^3*d^11*c*b^3 + 6*x^11*e^3*d^10*c^2*b^3 + 22
*x^10*e^3*d^9*c^3*b^3 + 55*x^9*e^3*d^8*c^4*b^3 + 99*x^8*e^3*d^7*c^5*b^3 + 1
32*x^7*e^3*d^6*c^6*b^3 + 3/10*x^10*e^3*d^9*b^2*a + 132*x^6*e^3*d^5*c^7*b^3
+ 3*x^9*e^3*d^8*c*b^2*a + 99*x^5*e^3*d^4*c^8*b^3 + 27/2*x^8*e^3*d^7*c^2*b^2
*a + 55*x^4*e^3*d^3*c^9*b^3 + 36*x^7*e^3*d^6*c^3*b^2*a + 22*x^3*e^3*d^2*c^1
0*b^3 + 63*x^6*e^3*d^5*c^4*b^2*a + 6*x^2*e^3*d*c^11*b^3 + 378/5*x^5*e^3*d^4
*c^5*b^2*a + x*e^3*c^12*b^3 + 63*x^4*e^3*d^3*c^6*b^2*a + 3/7*x^7*e^3*d^6*b*
a^2 + 36*x^3*e^3*d^2*c^7*b^2*a + 3*x^6*e^3*d^5*c*b*a^2 + 27/2*x^2*e^3*d*c^8
*b^2*a + 9*x^5*e^3*d^4*c^2*b*a^2 + 3*x*e^3*c^9*b^2*a + 15*x^4*e^3*d^3*c^3*b
*a^2 + 15*x^3*e^3*d^2*c^4*b*a^2 + 9*x^2*e^3*d*c^5*b*a^2 + 3*x*e^3*c^6*b*a^2
 + 1/4*x^4*e^3*d^3*a^3 + x^3*e^3*d^2*c*a^3 + 3/2*x^2*e^3*d*c^2*a^3 + x*e^3*
c^3*a^3
```

giac [B] time = 0.18, size = 510, normalized size = 6.14

$$\frac{1}{13}\,b^3d^{12}x^{13}e^3+b^3cd^{11}x^{12}e^3+6\,b^3c^2d^{10}x^{11}e^3+22\,b^3c^3d^9x^{10}e^3+55\,b^3c^4d^8x^9e^3+99\,b^3c^5d^7x^8e^3+132\,b^3c^6d^6x^7e^3+\frac{3}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 1/13*b^3*d^12*x^13*e^3 + b^3*c*d^11*x^12*e^3 + 6*b^3*c^2*d^10*x^11*e^3 + 22
*b^3*c^3*d^9*x^10*e^3 + 55*b^3*c^4*d^8*x^9*e^3 + 99*b^3*c^5*d^7*x^8*e^3 + 1
32*b^3*c^6*d^6*x^7*e^3 + 3/10*a*b^2*d^9*x^10*e^3 + 132*b^3*c^7*d^5*x^6*e^3
+ 3*a*b^2*c*d^8*x^9*e^3 + 99*b^3*c^8*d^4*x^5*e^3 + 27/2*a*b^2*c^2*d^7*x^8*e
^3 + 55*b^3*c^9*d^3*x^4*e^3 + 36*a*b^2*c^3*d^6*x^7*e^3 + 22*b^3*c^10*d^2*x^
3*e^3 + 63*a*b^2*c^4*d^5*x^6*e^3 + 6*b^3*c^11*d*x^2*e^3 + 378/5*a*b^2*c^5*d
^4*x^5*e^3 + b^3*c^12*x*e^3 + 63*a*b^2*c^6*d^3*x^4*e^3 + 3/7*a^2*b*d^6*x^7*
e^3 + 36*a*b^2*c^7*d^2*x^3*e^3 + 3*a^2*b*c*d^5*x^6*e^3 + 27/2*a*b^2*c^8*d*x
^2*e^3 + 9*a^2*b*c^2*d^4*x^5*e^3 + 3*a*b^2*c^9*x*e^3 + 15*a^2*b*c^3*d^3*x^4
*e^3 + 15*a^2*b*c^4*d^2*x^3*e^3 + 9*a^2*b*c^5*d*x^2*e^3 + 3*a^2*b*c^6*x*e^3
 + 1/4*a^3*d^3*x^4*e^3 + a^3*c*d^2*x^3*e^3 + 3/2*a^3*c^2*d*x^2*e^3 + a^3*c^
3*x*e^3
```

maple [B] time = 0.00, size = 2050, normalized size = 24.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*(d*x+c)^3)^3,x)
```

```
[Out] 1/13*d^12*e^3*b^3*x^13+c*e^3*d^11*b^3*x^12+6*c^2*e^3*d^10*b^3*x^11+1/10*(13
6*c^3*e^3*b^3*d^9+d^3*e^3*(63*b^3*c^3*d^6+(b*c^3+a)*b^2*d^6+(18*b^2*c^3*d^3
+2*(b*c^3+a)*b*d^3)*b*d^3))*x^10+1/9*(117*c^4*e^3*b^3*d^8+3*c*e^3*d^2*(63*b
^3*c^3*d^6+(b*c^3+a)*b^2*d^6+(18*b^2*c^3*d^3+2*(b*c^3+a)*b*d^3)*b*d^3)+d^3*
e^3*(45*b^3*c^4*d^5+6*(b*c^3+a)*b^2*c*d^5+3*(18*b^2*c^3*d^3+2*(b*c^3+a)*b*d
```

```
^3)*b*c*d^2+(9*b^2*c^4*d^2+6*(b*c^3+a)*b*c*d^2)*b*d^3))*x^9+1/8*(36*c^5*e^3
*b^3*d^7+3*c^2*e^3*d*(63*b^3*c^3*d^6+(b*c^3+a)*b^2*d^6+(18*b^2*c^3*d^3+2*(b
*c^3+a)*b*d^3)*b*d^3)+3*c*e^3*d^2*(45*b^3*c^4*d^5+6*(b*c^3+a)*b^2*c*d^5+3*(
18*b^2*c^3*d^3+2*(b*c^3+a)*b*d^3)*b*c*d^2+(9*b^2*c^4*d^2+6*(b*c^3+a)*b*c*d^
2)*b*d^3)+d^3*e^3*(21*(b*c^3+a)*b^2*c^2*d^4+3*(18*b^2*c^3*d^3+2*(b*c^3+a)*b
*d^3)*b*c^2*d+3*(9*b^2*c^4*d^2+6*(b*c^3+a)*b*c*d^2)*b*c*d^2))*x^8+1/7*(c^3*
e^3*(63*b^3*c^3*d^6+(b*c^3+a)*b^2*d^6+(18*b^2*c^3*d^3+2*(b*c^3+a)*b*d^3)*b*
d^3)+3*c^2*e^3*d*(45*b^3*c^4*d^5+6*(b*c^3+a)*b^2*c*d^5+3*(18*b^2*c^3*d^3+2*
(b*c^3+a)*b*d^3)*b*c*d^2+(9*b^2*c^4*d^2+6*(b*c^3+a)*b*c*d^2)*b*d^3)+3*c*e^3
*d^2*(21*(b*c^3+a)*b^2*c^2*d^4+3*(18*b^2*c^3*d^3+2*(b*c^3+a)*b*d^3)*b*c^2*d
+3*(9*b^2*c^4*d^2+6*(b*c^3+a)*b*c*d^2)*b*c*d^2)+d^3*e^3*(18*(b*c^3+a)*b^2*c
^3*d^3+3*(9*b^2*c^4*d^2+6*(b*c^3+a)*b*c*d^2)*b*c^2*d+(b*c^3+a)^2*b*d^3+(b*c
^3+a)*(18*b^2*c^3*d^3+2*(b*c^3+a)*b*d^3)))*x^7+1/6*(c^3*e^3*(45*b^3*c^4*d^5
+6*(b*c^3+a)*b^2*c*d^5+3*(18*b^2*c^3*d^3+2*(b*c^3+a)*b*d^3)*b*c*d^2+(9*b^2*
c^4*d^2+6*(b*c^3+a)*b*c*d^2)*b*d^3)+3*c^2*e^3*d*(21*(b*c^3+a)*b^2*c^2*d^4+3
*(18*b^2*c^3*d^3+2*(b*c^3+a)*b*d^3)*b*c^2*d+3*(9*b^2*c^4*d^2+6*(b*c^3+a)*b*
c*d^2)*b*c*d^2)+3*c*e^3*d^2*(18*(b*c^3+a)*b^2*c^3*d^3+3*(9*b^2*c^4*d^2+6*(b
*c^3+a)*b*c*d^2)*b*c^2*d+(b*c^3+a)^2*b*d^3+(b*c^3+a)*(18*b^2*c^3*d^3+2*(b*c
^3+a)*b*d^3))+d^3*e^3*(18*(b*c^3+a)*b^2*c^4*d^2+3*(b*c^3+a)^2*b*c*d^2+(b*c^
3+a)*(9*b^2*c^4*d^2+6*(b*c^3+a)*b*c*d^2)))*x^6+1/5*(c^3*e^3*(21*(b*c^3+a)*b
^2*c^2*d^4+3*(18*b^2*c^3*d^3+2*(b*c^3+a)*b*d^3)*b*c^2*d+3*(9*b^2*c^4*d^2+6*
(b*c^3+a)*b*c*d^2)*b*c*d^2)+3*c^2*e^3*d*(18*(b*c^3+a)*b^2*c^3*d^3+3*(9*b^2*
c^4*d^2+6*(b*c^3+a)*b*c*d^2)*b*c^2*d+(b*c^3+a)^2*b*d^3+(b*c^3+a)*(18*b^2*c^
3*d^3+2*(b*c^3+a)*b*d^3))+3*c*e^3*d^2*(18*(b*c^3+a)*b^2*c^4*d^2+3*(b*c^3+a)
^2*b*c*d^2+(b*c^3+a)*(9*b^2*c^4*d^2+6*(b*c^3+a)*b*c*d^2))+9*d^4*e^3*(b*c^3+
a)^2*b*c^2)*x^5+1/4*(c^3*e^3*(18*(b*c^3+a)*b^2*c^3*d^3+3*(9*b^2*c^4*d^2+6*(
b*c^3+a)*b*c*d^2)*b*c^2*d+(b*c^3+a)^2*b*d^3+(b*c^3+a)*(18*b^2*c^3*d^3+2*(b*
c^3+a)*b*d^3))+3*c^2*e^3*d*(18*(b*c^3+a)*b^2*c^4*d^2+3*(b*c^3+a)^2*b*c*d^2+
(b*c^3+a)*(9*b^2*c^4*d^2+6*(b*c^3+a)*b*c*d^2))+27*c^3*e^3*d^3*(b*c^3+a)^2*b
+d^3*e^3*(b*c^3+a)^3)*x^4+1/3*(c^3*e^3*(18*(b*c^3+a)*b^2*c^4*d^2+3*(b*c^3+a
)^2*b*c*d^2+(b*c^3+a)*(9*b^2*c^4*d^2+6*(b*c^3+a)*b*c*d^2))+27*c^4*e^3*d^2*(
b*c^3+a)^2*b+3*c*e^3*d^2*(b*c^3+a)^3)*x^3+1/2*(9*c^5*e^3*(b*c^3+a)^2*b*d+3*
c^2*e^3*d*(b*c^3+a)^3)*x^2+c^3*e^3*(b*c^3+a)^3*x
```

maxima [B] time = 0.49, size = 398, normalized size = 4.80

$$\frac{1}{13}b^3d^{12}e^3x^{13}+b^3cd^{11}e^3x^{12}+6\,b^3c^2d^{10}e^3x^{11}+\frac{1}{10}\left(220\,b^3c^3+3\,ab^2\right)d^9e^3x^{10}+\left(55\,b^3c^4+3\,ab^2c\right)d^8e^3x^9+\frac{9}{2}\left(22\,b^3c^5+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] 1/13*b^3*d^12*e^3*x^13 + b^3*c*d^11*e^3*x^12 + 6*b^3*c^2*d^10*e^3*x^11 + 1/
10*(220*b^3*c^3 + 3*a*b^2)*d^9*e^3*x^10 + (55*b^3*c^4 + 3*a*b^2*c)*d^8*e^3*
x^9 + 9/2*(22*b^3*c^5 + 3*a*b^2*c^2)*d^7*e^3*x^8 + 3/7*(308*b^3*c^6 + 84*a*
b^2*c^3 + a^2*b)*d^6*e^3*x^7 + 3*(44*b^3*c^7 + 21*a*b^2*c^4 + a^2*b*c)*d^5*
e^3*x^6 + 9/5*(55*b^3*c^8 + 42*a*b^2*c^5 + 5*a^2*b*c^2)*d^4*e^3*x^5 + 1/4*(
220*b^3*c^9 + 252*a*b^2*c^6 + 60*a^2*b*c^3 + a^3)*d^3*e^3*x^4 + (22*b^3*c^1
0 + 36*a*b^2*c^7 + 15*a^2*b*c^4 + a^3*c)*d^2*e^3*x^3 + 3/2*(4*b^3*c^11 + 9*
a*b^2*c^8 + 6*a^2*b*c^5 + a^3*c^2)*d*e^3*x^2 + (b^3*c^12 + 3*a*b^2*c^9 + 3*
a^2*b*c^6 + a^3*c^3)*e^3*x
```

mupad [B] time = 1.25, size = 348, normalized size = 4.19

$$c^3e^3x\left(bc^3+a\right)^3+\frac{d^3e^3x^4\left(a^3+60\,a^2bc^3+252\,ab^2c^6+220\,b^3c^9\right)}{4}+\frac{b^3d^{12}e^3x^{13}}{13}+\frac{3\,bd^6e^3x^7\left(a^2+84\,abc^3+3\right.}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3*(a + b*(c + d*x)^3)^3,x)
```

```
[Out] c^3*e^3*x*(a + b*c^3)^3 + (d^3*e^3*x^4*(a^3 + 220*b^3*c^9 + 60*a^2*b*c^3 +
252*a*b^2*c^6))/4 + (b^3*d^12*e^3*x^13)/13 + (3*b*d^6*e^3*x^7*(a^2 + 308*b^
2*c^6 + 84*a*b*c^3))/7 + (b^2*d^9*e^3*x^10*(3*a + 220*b*c^3))/10 + 6*b^3*c^
2*d^10*e^3*x^11 + c*d^2*e^3*x^3*(a^3 + 22*b^3*c^9 + 15*a^2*b*c^3 + 36*a*b^2
*c^6) + b^3*c*d^11*e^3*x^12 + (3*c^2*d*e^3*x^2*(a + b*c^3)^2*(a + 4*b*c^3))
/2 + (9*b*c^2*d^4*e^3*x^5*(5*a^2 + 55*b^2*c^6 + 42*a*b*c^3))/5 + 3*b*c*d^5*
e^3*x^6*(a^2 + 44*b^2*c^6 + 21*a*b*c^3) + b^2*c*d^8*e^3*x^9*(3*a + 55*b*c^3
) + (9*b^2*c^2*d^7*e^3*x^8*(3*a + 22*b*c^3))/2
```

sympy [**B**] time = 0.19, size = 552, normalized size = 6.65

$$6b^3c^2d^{10}e^3x^{11}+b^3cd^{11}e^3x^{12}+\frac{b^3d^{12}e^3x^{13}}{13}+x^{10}\left(\frac{3ab^2d^9e^3}{10}+22b^3c^3d^9e^3\right)+x^9\left(3ab^2cd^8e^3+55b^3c^4d^8e^3\right)+x^8\left(\frac{27ab}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*(d*x+c)**3)**3,x)
```

```
[Out] 6*b**3*c**2*d**10*e**3*x**11 + b**3*c*d**11*e**3*x**12 + b**3*d**12*e**3*x*
*13/13 + x**10*(3*a*b**2*d**9*e**3/10 + 22*b**3*c**3*d**9*e**3) + x**9*(3*a
*b**2*c*d**8*e**3 + 55*b**3*c**4*d**8*e**3) + x**8*(27*a*b**2*c**2*d**7*e**
3/2 + 99*b**3*c**5*d**7*e**3) + x**7*(3*a**2*b*d**6*e**3/7 + 36*a*b**2*c**3
*d**6*e**3 + 132*b**3*c**6*d**6*e**3) + x**6*(3*a**2*b*c*d**5*e**3 + 63*a*b
**2*c**4*d**5*e**3 + 132*b**3*c**7*d**5*e**3) + x**5*(9*a**2*b*c**2*d**4*e*
*3 + 378*a*b**2*c**5*d**4*e**3/5 + 99*b**3*c**8*d**4*e**3) + x**4*(a**3*d**
3*e**3/4 + 15*a**2*b*c**3*d**3*e**3 + 63*a*b**2*c**6*d**3*e**3 + 55*b**3*c*
*9*d**3*e**3) + x**3*(a**3*c*d**2*e**3 + 15*a**2*b*c**4*d**2*e**3 + 36*a*b*
*2*c**7*d**2*e**3 + 22*b**3*c**10*d**2*e**3) + x**2*(3*a**3*c**2*d*e**3/2 +
 9*a**2*b*c**5*d*e**3 + 27*a*b**2*c**8*d*e**3/2 + 6*b**3*c**11*d*e**3) + x*
(a**3*c**3*e**3 + 3*a**2*b*c**6*e**3 + 3*a*b**2*c**9*e**3 + b**3*c**12*e**3
)
```

3.2859 $\int \frac{(c+dx)^4}{a+b(c+dx)^3}\,dx$

Optimal. Leaf size=156

$$\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3b^{5/3}d}-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6b^{5/3}d}+\frac{a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{5/3}d}+\frac{(c+dx}{2bd}$$

```
[Out] 1/2*(d*x+c)^2/b/d+1/3*a^(2/3)*ln(a^(1/3)+b^(1/3)*(d*x+c))/b^(5/3)/d-1/6*a^(
2/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/b^(5/3)/d+1/3*a^
(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/b^(5/3)/d*3^(
1/2)
```

Rubi [A] time = 0.17, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {372, 321, 292, 31, 634, 617, 204, 628}

$$\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3b^{5/3}d}-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6b^{5/3}d}+\frac{a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{5/3}d}+\frac{(c+dx}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4/(a + b*(c + d*x)^3),x]
```

```
[Out] (c + d*x)^2/(2*b*d) + (a^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt
[3]*a^(1/3))])/(Sqrt[3]*b^(5/3)*d) + (a^(2/3)*Log[a^(1/3) + b^(1/3)*(c + d*
x)])/(3*b^(5/3)*d) - (a^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(
2/3)*(c + d*x)^2])/(6*b^(5/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
```

```
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^4}{a+b(c+dx)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{x^4}{a+bx^3}\,dx, x, c+dx\right)}{d} \\
&= \frac{(c+dx)^2}{2bd} - \frac{a\,\text{Subst}\left(\int \frac{x}{a+bx^3}\,dx, x, c+dx\right)}{bd} \\
&= \frac{(c+dx)^2}{2bd} + \frac{a^{2/3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, c+dx\right)}{3b^{4/3}d} - \frac{a^{2/3}\,\text{Subst}\left(\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x\right.}{3b^{4/3}d} \\
&= \frac{(c+dx)^2}{2bd} + \frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3b^{5/3}d} - \frac{a^{2/3}\,\text{Subst}\left(\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, c+dx\right.}{6b^{5/3}d} \\
&= \frac{(c+dx)^2}{2bd} + \frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3b^{5/3}d} - \frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx\right.}{6b^{5/3}d} \\
&= \frac{(c+dx)^2}{2bd} + \frac{a^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{5/3}d} + \frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3b^{5/3}d} - \frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{}\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 159, normalized size = 1.02

$$\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3b^{5/3}d} - \frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6b^{5/3}d} - \frac{a^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{b}\,(c+dx)-\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{5/3}d} + \frac{(c+}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4/(a + b*(c + d*x)^3),x]
```

```
[Out] (c + d*x)^2/(2*b*d) - (a^(2/3)*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c + d*x))/(Sqr
t[3]*a^(1/3))])/(Sqrt[3]*b^(5/3)*d) + (a^(2/3)*Log[a^(1/3) + b^(1/3)*(c + d
```

```
*x)])/(3*b^(5/3)*d) - (a^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^
(2/3)*(c + d*x)^2])/(6*b^(5/3)*d)
```

fricas [A] time = 0.87, size = 164, normalized size = 1.05

$$\frac{3\,d^2x^2 + 6\,cdx - 2\sqrt{3}\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,(bdx+bc)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}} - \sqrt{3}\,a}{3\,a}\right) - \left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ad^2x^2 + 2\,acdx + ac^2 - (bdx+bc)\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}\right.}{6\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/6*(3*d^2*x^2 + 6*c*d*x - 2*sqrt(3)*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*
(b*d*x + b*c)*(a^2/b^2)^(1/3) - sqrt(3)*a)/a) - (a^2/b^2)^(1/3)*log(a*d^2*x
^2 + 2*a*c*d*x + a*c^2 - (b*d*x + b*c)*(a^2/b^2)^(2/3) + a*(a^2/b^2)^(1/3))
 + 2*(a^2/b^2)^(1/3)*log(a*d*x + a*c + b*(a^2/b^2)^(2/3)))/(b*d)
```

giac [A] time = 0.22, size = 160, normalized size = 1.03

$$\frac{bd^7x^2 + 2\,bcd^6x}{2\,b^2d^6} - \frac{2\sqrt{3}\left(a^2bd^{15}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,abdx+2\,abc-\left(a^2b\right)^{\frac{2}{3}}\right)}{3\left(a^2b\right)^{\frac{2}{3}}}\right) + \left(a^2bd^{15}\right)^{\frac{1}{3}}\log\left(\left(2\,abdx + 2\,abc - \left(a^2b\right)^{\frac{2}{3}}\right)^2 + 3\right.}{6\,b^2d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/2*(b*d^7*x^2 + 2*b*c*d^6*x)/(b^2*d^6) - 1/6*(2*sqrt(3)*(a^2*b*d^15)^(1/3)
*arctan(1/3*sqrt(3)*(2*a*b*d*x + 2*a*b*c - (a^2*b)^(2/3))/(a^2*b)^(2/3)) +
(a^2*b*d^15)^(1/3)*log((2*a*b*d*x + 2*a*b*c - (a^2*b)^(2/3))^2 + 3*(a^2*b)^
(4/3)) - 2*(a^2*b*d^15)^(1/3)*log(abs(a*b*d*x + a*b*c + (a^2*b)^(2/3))))/(b
^2*d^6)
```

maple [C] time = 0.01, size = 93, normalized size = 0.60

$$\frac{d\,x^2}{2b} + \frac{cx}{b} - \frac{a\left(\mathrm{RootOf}\left(b\,d^3_Z^3 + 3b\,d^2c\,_Z^2 + 3bd\,c^2_Z + b\,c^3 + a\right)d + c\right)\ln\left(-\mathrm{RootOf}\left(b\,d^3_Z^3 + 3b\,d^2c\,_Z^2 + \right.\right.}{3b^2d\left(d^2\,\mathrm{RootOf}\left(b\,d^3_Z^3 + 3b\,d^2c\,_Z^2 + 3bd\,c^2_Z + b\,c^3 + a\right)^2 + 2cd\,\mathrm{RootOf}\left(b\,d^3_Z^3 + 3b\,d^2c\,_Z^2 + \right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4/(a+b*(d*x+c)^3),x)
```

```
[Out] 1/2/b*x^2*d+1/b*c*x-1/3/b^2/d*sum((_R*d+c)/(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x
),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))*a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{-\frac{1}{6}\left(2\sqrt{3}\left(-\frac{1}{ab^2d^3}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)}{3\left(-a^2b\right)^{\frac{2}{3}}}\right) + \left(-\frac{1}{ab^2d^3}\right)^{\frac{1}{3}}\log\left(\left(2\,abdx + 2\,abc - \left(-a^2b\right)^{\frac{2}{3}}\right)^2 + 3\left(-a^2b\right)^{\frac{4}{3}}\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -a*integrate((d*x + c)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a
), x)/b + 1/2*(d*x^2 + 2*c*x)/b
```

mupad [B] time = 1.40, size = 169, normalized size = 1.08

$$\frac{d\,x^2}{2\,b}+\frac{c\,x}{b}+\frac{a^{2/3}\,\ln\left(b^{1/3}\,c+a^{1/3}+b^{1/3}\,d\,x\right)}{3\,b^{5/3}\,d}-\frac{a^{2/3}\,\ln\left(\frac{a^2\,c\,d^4}{b}+\frac{a^2\,d^5\,x}{b}+\frac{a^{7/3}\,d^4\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{b^{4/3}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,b^{5/3}\,d}+\frac{a^{2/3}\,\ln\left(\frac{a^2\,c\,d^4}{b}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^4/(a + b*(c + d*x)^3),x)
```

```
[Out] (d*x^2)/(2*b) + (c*x)/b + (a^(2/3)*log(b^(1/3)*c + a^(1/3) + b^(1/3)*d*x))/
(3*b^(5/3)*d) - (a^(2/3)*log((a^2*c*d^4)/b + (a^2*d^5*x)/b + (a^(7/3)*d^4*(
(3^(1/2)*1i)/2 + 1/2)^2)/b^(4/3))*((3^(1/2)*1i)/2 + 1/2))/(3*b^(5/3)*d) + (
a^(2/3)*log((a^2*c*d^4)/b + (a^2*d^5*x)/b + (9*a^(7/3)*d^4*((3^(1/2)*1i)/6
- 1/6)^2)/b^(4/3))*((3^(1/2)*1i)/6 - 1/6))/(b^(5/3)*d)
```

sympy [A] time = 0.37, size = 46, normalized size = 0.29

$$\frac{\mathrm{RootSum}\left(27t^3b^5-a^2,\left(t\mapsto t\log\left(x+\frac{9t^2b^3+ac}{ad}\right)\right)\right)}{d}+\frac{cx}{b}+\frac{dx^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4/(a+b*(d*x+c)**3),x)
```

```
[Out] RootSum(27*_t**3*b**5 - a**2, Lambda(_t, _t*log(x + (9*_t**2*b**3 + a*c)/(a
*d))))/d + c*x/b + d*x**2/(2*b)
```

3.2860 $\int \frac{(c+dx)^3}{a+b(c+dx)^3}\,dx$

Optimal. Leaf size=144

$$\frac{\sqrt[3]{a}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6b^{4/3}d}-\frac{\sqrt[3]{a}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3b^{4/3}d}+\frac{\sqrt[3]{a}\,\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{4/3}d}+\frac{x}{b}$$

```
[Out] x/b-1/3*a^(1/3)*ln(a^(1/3)+b^(1/3)*(d*x+c))/b^(4/3)/d+1/6*a^(1/3)*ln(a^(2/3
)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/b^(4/3)/d+1/3*a^(1/3)*arctan(1
/3*(a^(1/3)-2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/b^(4/3)/d*3^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {372, 321, 200, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{a}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6b^{4/3}d}-\frac{\sqrt[3]{a}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3b^{4/3}d}+\frac{\sqrt[3]{a}\,\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{4/3}d}+\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*(c + d*x)^3),x]
```

```
[Out] x/b + (a^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*b^(4/3)*d) - (a^(1/3)*Log[a^(1/3) + b^(1/3)*(c + d*x)])/(3*b^(4/3)*
d) + (a^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2
])/(6*b^(4/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
```

```
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^3}{a+b(c+dx)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{x^3}{a+bx^3}\,dx, x, c+dx\right)}{d} \\
&= \frac{x}{b} - \frac{a\,\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, c+dx\right)}{bd} \\
&= \frac{x}{b} - \frac{\sqrt[3]{a}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, c+dx\right)}{3bd} - \frac{\sqrt[3]{a}\,\text{Subst}\left(\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, c+dx\right.}{3bd} \\
&= \frac{x}{b} - \frac{\sqrt[3]{a}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3b^{4/3}d} + \frac{\sqrt[3]{a}\,\text{Subst}\left(\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, c+dx\right)}{6b^{4/3}d} - \frac{a^{2/3}}{} \\
&= \frac{x}{b} - \frac{\sqrt[3]{a}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3b^{4/3}d} + \frac{\sqrt[3]{a}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6b^{4/3}d} - \frac{}{} \\
&= \frac{x}{b} + \frac{\sqrt[3]{a}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{4/3}d} - \frac{\sqrt[3]{a}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3b^{4/3}d} + \frac{\sqrt[3]{a}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c-\right.}{6b^{4/3}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 142, normalized size = 0.99

$$\frac{\sqrt[3]{a}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)-2\sqrt[3]{a}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)-2\sqrt{3}\,\sqrt[3]{a}\,\tan^{-1}\left(\frac{2\sqrt[3]{b}\,(c+dx)-\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right.}{6b^{4/3}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*(c + d*x)^3),x]
```

```
[Out] (6*b^(1/3)*c + 6*b^(1/3)*d*x - 2*Sqrt[3]*a^(1/3)*ArcTan[(-a^(1/3) + 2*b^(1/
3)*(c + d*x))/(Sqrt[3]*a^(1/3))] - 2*a^(1/3)*Log[a^(1/3) + b^(1/3)*(c + d*x
```

```
)] + a^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]
)/(6*b^(4/3)*d)
```

fricas [A] time = 0.69, size = 135, normalized size = 0.94

$$\frac{6\,dx + 2\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,(bdx+bc)\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,a}{3\,a}\right)-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(d^2x^2+2\,cdx+c^2+(dx+c)\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)+2}{6\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/6*(6*d*x + 2*sqrt(3)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*(b*d*x + b*c)*(-a
/b)^(2/3) - sqrt(3)*a)/a) - (-a/b)^(1/3)*log(d^2*x^2 + 2*c*d*x + c^2 + (d*x
 + c)*(-a/b)^(1/3) + (-a/b)^(2/3)) + 2*(-a/b)^(1/3)*log(d*x + c - (-a/b)^(1
/3)))/(b*d)
```

giac [A] time = 0.25, size = 180, normalized size = 1.25

$$\frac{x}{b}+\frac{2\sqrt{3}\left(-\frac{ad^6}{b}\right)^{\frac{1}{3}}\arctan\left(-\frac{bdx+bc-\left(-ab^2\right)^{\frac{1}{3}}}{\sqrt{3}\,bdx+\sqrt{3}\,bc+\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}}\right)-\left(-\frac{ad^6}{b}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}\,bdx+\sqrt{3}\,bc+\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\right)^2+4\left(bdx+\right.\right.}{6\,bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] x/b + 1/6*(2*sqrt(3)*(-a*d^6/b)^(1/3)*arctan(-(b*d*x + b*c - (-a*b^2)^(1/3)
)/(sqrt(3)*b*d*x + sqrt(3)*b*c + sqrt(3)*(-a*b^2)^(1/3))) - (-a*d^6/b)^(1/3
)*log(4*(sqrt(3)*b*d*x + sqrt(3)*b*c + sqrt(3)*(-a*b^2)^(1/3))^2 + 4*(b*d*x
 + b*c - (-a*b^2)^(1/3))^2) + 2*(-a*d^6/b)^(1/3)*log(abs(-b*d*x - b*c + (-a
*b^2)^(1/3))))/(b*d^3)
```

maple [C] time = 0.00, size = 78, normalized size = 0.54

$$\frac{x}{b}-\frac{a\ln\left(-\operatorname{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_Z+b\,c^3+a\right)+x\right)}{3b^2d\left(d^2\operatorname{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_Z+b\,c^3+a\right)^2+2cd\operatorname{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+b*(d*x+c)^3),x)
```

```
[Out] 1/b*x-1/3/b^2/d*sum(1/(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x),_R=RootOf(_Z^3*b*d^
3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))*a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\frac{1}{6}\,a\left(\frac{2\sqrt{3}\left(\frac{1}{a^2b}\right)^{\frac{1}{3}}\arctan\left(-\frac{bdx+bc+\left(ab^2\right)^{\frac{1}{3}}}{\sqrt{3}\,bdx+\sqrt{3}\,bc-\sqrt{3}\left(ab^2\right)^{\frac{1}{3}}}\right)}{d}-\frac{\left(\frac{1}{a^2b}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}\,bdx+\sqrt{3}\,bc-\sqrt{3}\left(ab^2\right)^{\frac{1}{3}}\right)^2+4\left(bdx+bc+\left(ab^2\right)^{\frac{1}{3}}\right)^2\right)}{d}+\frac{2\left(\frac{1}{a^2b}\right)^{\frac{1}{3}}\log\left(\left|bdx+\right.\right.}{d}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -a*integrate(1/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/b
+ x/b
```

mupad [B] time = 0.24, size = 158, normalized size = 1.10

$$\frac{x}{b}+\frac{(-a)^{1/3}\,\ln\left((-a)^{4/3}+a\,b^{1/3}\,c+a\,b^{1/3}\,d\,x\right)}{3\,b^{4/3}\,d}-\frac{(-a)^{1/3}\,\ln\left(3\,a\,b\,c\,d^5-3\,(-a)^{4/3}\,b^{2/3}\,d^5\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)+3\,a\,b\,d^6\,x\right)\,\left(\right.}{3\,b^{4/3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*(c + d*x)^3),x)
```

```
[Out] x/b + ((-a)^(1/3)*log((-a)^(4/3) + a*b^(1/3)*c + a*b^(1/3)*d*x))/(3*b^(4/3)
*d) - ((-a)^(1/3)*log(3*a*b*c*d^5 - 3*(-a)^(4/3)*b^(2/3)*d^5*((3^(1/2)*1i)/
2 + 1/2) + 3*a*b*d^6*x)*((3^(1/2)*1i)/2 + 1/2))/(3*b^(4/3)*d) + ((-a)^(1/3)
*log(9*(-a)^(4/3)*b^(2/3)*d^5*((3^(1/2)*1i)/6 - 1/6) + 3*a*b*c*d^5 + 3*a*b*
d^6*x)*((3^(1/2)*1i)/6 - 1/6))/(b^(4/3)*d)
```

sympy [A] time = 0.31, size = 27, normalized size = 0.19

$$\frac{\mathrm{RootSum}\left(27t^3b^4+a,\left(t\mapsto t\log\left(x+\frac{-3tb+c}{d}\right)\right)\right)}{d}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+b*(d*x+c)**3),x)
```

```
[Out] RootSum(27*_t**3*b**4 + a, Lambda(_t, _t*log(x + (-3*_t*b + c)/d)))/d + x/b
```

3.2861 $\int \frac{(c+dx)^2}{a+b(c+dx)^3}\,dx$

Optimal. Leaf size=22

$$\frac{\log\left(a+b(c+dx)^3\right)}{3bd}$$

```
[Out] 1/3*ln(a+b*(d*x+c)^3)/b/d
```

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {372, 260}

$$\frac{\log\left(a+b(c+dx)^3\right)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*(c + d*x)^3),x]
```

```
[Out] Log[a + b*(c + d*x)^3]/(3*b*d)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^2}{a+b(c+dx)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{a+bx^3}\,dx, x, c+dx\right)}{d}\\ &= \frac{\log\left(a+b(c+dx)^3\right)}{3bd}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{\log\left(a+b(c+dx)^3\right)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*(c + d*x)^3),x]
```

```
[Out] Log[a + b*(c + d*x)^3]/(3*b*d)
```

fricas [B] time = 0.76, size = 42, normalized size = 1.91

$$\frac{\log\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*(d*x+c)^3),x, algorithm="fricas")
[Out] 1/3*log(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(b*d)
```

giac [A] time = 0.18, size = 21, normalized size = 0.95

$$\frac{\log\left(\left|(dx+c)^3 b + a\right|\right)}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*(d*x+c)^3),x, algorithm="giac")
[Out] 1/3*log(abs((d*x + c)^3*b + a))/(b*d)
```

maple [B] time = 0.00, size = 43, normalized size = 1.95

$$\frac{\ln\left(b\,d^3x^3 + 3bc\,d^2x^2 + 3b\,c^2dx + b\,c^3 + a\right)}{3bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+b*(d*x+c)^3),x)
[Out] 1/3/b/d*ln(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)
```

maxima [A] time = 0.50, size = 20, normalized size = 0.91

$$\frac{\log\left((dx+c)^3 b + a\right)}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*(d*x+c)^3),x, algorithm="maxima")
[Out] 1/3*log((d*x + c)^3*b + a)/(b*d)
```

mupad [B] time = 1.19, size = 42, normalized size = 1.91

$$\frac{\ln\left(b\,c^3 + 3\,b\,c^2\,d\,x + 3\,b\,c\,d^2\,x^2 + b\,d^3\,x^3 + a\right)}{3\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*(c + d*x)^3),x)
[Out] log(a + b*c^3 + b*d^3*x^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2)/(3*b*d)
```

sympy [B] time = 0.27, size = 42, normalized size = 1.91

$$\frac{\log\left(a + bc^3 + 3bc^2dx + 3bcd^2x^2 + bd^3x^3\right)}{3bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+b*(d*x+c)**3),x)
[Out] log(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)/(3*b*d)
```

3.2862 $\int \frac{c+dx}{a+b(c+dx)^3}\,dx$

Optimal. Leaf size=140

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6\sqrt[3]{a}\,b^{2/3}d}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3\sqrt[3]{a}\,b^{2/3}d}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{2/3}d}$$

```
[Out] -1/3*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(1/3)/b^(2/3)/d+1/6*ln(a^(2/3)-a^(1/3)*b
^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(1/3)/b^(2/3)/d-1/3*arctan(1/3*(a^(1/3)
-2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(1/3)/b^(2/3)/d*3^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {372, 292, 31, 634, 617, 204, 628}

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6\sqrt[3]{a}\,b^{2/3}d}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3\sqrt[3]{a}\,b^{2/3}d}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{2/3}d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*(c + d*x)^3),x]
```

```
[Out] -(ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(1/3
)*b^(2/3)*d)) - Log[a^(1/3) + b^(1/3)*(c + d*x)]/(3*a^(1/3)*b^(2/3)*d) + Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/(6*a^(1/3)*b^(
2/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx}{a+b(c+dx)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{a+bx^3}\,dx, x, c+dx\right)}{d} \\
&= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, c+dx\right)}{3\sqrt[3]{a}\,\sqrt[3]{b}\,d} + \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, c+dx\right)}{3\sqrt[3]{a}\,\sqrt[3]{b}\,d} \\
&= -\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3\sqrt[3]{a}\,b^{2/3}d} + \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, c+dx\right)}{6\sqrt[3]{a}\,b^{2/3}d} + \frac{\text{Subst}\left(\int \frac{}{a^{2/3}-}\right.}{} \\
&= -\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3\sqrt[3]{a}\,b^{2/3}d} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6\sqrt[3]{a}\,b^{2/3}d} + \frac{\text{Subst}\left(\int \frac{}{-3}\right.}{} \\
&= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{2/3}d} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3\sqrt[3]{a}\,b^{2/3}d} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+\right.}{6\sqrt[3]{a}\,b^{2/3}d}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 114, normalized size = 0.81

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)+2\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{b}\,(c+dx)-\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{6\sqrt[3]{a}\,b^{2/3}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*(c + d*x)^3),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))] - 2*L
og[a^(1/3) + b^(1/3)*(c + d*x)] + Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) +
 b^(2/3)*(c + d*x)^2])/(6*a^(1/3)*b^(2/3)*d)
```

fricas [A] time = 0.68, size = 452, normalized size = 3.23

$$\left[3\sqrt{\frac{1}{3}}\,ab\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\,\log\left(\frac{2\,b^2d^3x^3+6\,b^2cd^2x^2+6\,b^2c^2dx+2\,b^2c^3-ab+3\,\sqrt{\frac{1}{3}}\left(abdx+abc+2\left(d^2x^2+2\,cdx+c^2\right)\left(-ab^2\right)^{\frac{2}{3}}+\left(-ab^2\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}-3\right.}{bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*d^3*x^3 + 6*b^2*c*d
^2*x^2 + 6*b^2*c^2*d*x + 2*b^2*c^3 - a*b + 3*sqrt(1/3)*(a*b*d*x + a*b*c + 2
*(d^2*x^2 + 2*c*d*x + c^2)*(-a*b^2)^(2/3) + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)
^(1/3)/a) - 3*(-a*b^2)^(2/3)*(d*x + c))/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^
2*d*x + b*c^3 + a)) + (-a*b^2)^(2/3)*log(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^
2 + (-a*b^2)^(1/3)*(b*d*x + b*c) + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*log(b
*d*x + b*c - (-a*b^2)^(1/3)))/(a*b^2*d), 1/6*(6*sqrt(1/3)*a*b*sqrt(-(-a*b^2
)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*d*x + 2*b*c + (-a*b^2)^(1/3))*sqrt(-(-a*b^
2)^(1/3)/a)/b) + (-a*b^2)^(2/3)*log(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (
-a*b^2)^(1/3)*(b*d*x + b*c) + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*log(b*d*x
+ b*c - (-a*b^2)^(1/3)))/(a*b^2*d)]
```

giac [A] time = 0.22, size = 141, normalized size = 1.01

$$-\frac{1}{3}\sqrt{3}\left(-\frac{1}{ab^2d^3}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)}{3\left(-a^2b\right)^{\frac{2}{3}}}\right)-\frac{1}{6}\left(-\frac{1}{ab^2d^3}\right)^{\frac{1}{3}}\log\left(\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)^2+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(-1/(a*b^2*d^3))^(1/3)*arctan(1/3*sqrt(3)*(2*a*b*d*x + 2*a*b*c
 - (-a^2*b)^(2/3))/(-a^2*b)^(2/3)) - 1/6*(-1/(a*b^2*d^3))^(1/3)*log((2*a*b*
d*x + 2*a*b*c - (-a^2*b)^(2/3))^2 + 3*(-a^2*b)^(4/3)) + 1/3*(-1/(a*b^2*d^3)
)^(1/3)*log(abs(a*b*d*x + a*b*c + (-a^2*b)^(2/3)))
```

maple [C] time = 0.00, size = 76, normalized size = 0.54

$$\frac{\left(\text{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_Z+b\,c^3+a\right)d+c\right)\ln\left(-\text{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_Z+\right.\right.}{3bd\left(d^2\,\text{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_Z+b\,c^3+a\right)^2+2cd\,\text{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_Z+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+b*(d*x+c)^3),x)
```

```
[Out] 1/3/b/d*sum((_R*d+c)/(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x),_R=RootOf(_Z^3*b*d^3
+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx+c}{(dx+c)^3 b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)/((d*x + c)^3*b + a), x)
```

mupad [B] time = 1.44, size = 150, normalized size = 1.07

$$\frac{\ln\left(b^{1/3}\,c-(-a)^{1/3}+b^{1/3}\,d\,x\right)}{3\,(-a)^{1/3}\,b^{2/3}\,d}+\frac{\ln\left(b\,c\,d^4+b\,d^5\,x-\frac{(-a)^{1/3}\,b^{2/3}\,d^4\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,(-a)^{1/3}\,b^{2/3}\,d}-\frac{\ln\left(b\,c\,d^4+b\,d^5\,x-\frac{(-a)^{1/3}}{6\,(-}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*(c + d*x)^3),x)
```

```
[Out] log(b^(1/3)*c - (-a)^(1/3) + b^(1/3)*d*x)/(3*(-a)^(1/3)*b^(2/3)*d) + (log(b
*c*d^4 + b*d^5*x - ((-a)^(1/3)*b^(2/3)*d^4*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*
1i - 1))/(6*(-a)^(1/3)*b^(2/3)*d) - (log(b*c*d^4 + b*d^5*x - ((-a)^(1/3)*b^
(2/3)*d^4*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/(6*(-a)^(1/3)*b^(2/3)*d)
```

sympy [**A**] time = 0.26, size = 29, normalized size = 0.21

$$\frac{\text{RootSum}\left(27t^3ab^2+1,\left(t\mapsto t\log\left(x+\frac{9t^2ab+c}{d}\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*(d*x+c)**3),x)
```

```
[Out] RootSum(27*_t**3*a*b**2 + 1, Lambda(_t, _t*log(x + (9*_t**2*a*b + c)/d)))/d
```

3.2863 $\int \frac{1}{a+b(c+dx)^3}\,dx$

Optimal. Leaf size=140

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\sqrt[3]{b}\,d}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}\,d}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}\,d}$$

```
[Out] 1/3*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(2/3)/b^(1/3)/d-1/6*ln(a^(2/3)-a^(1/3)*b^
(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(2/3)/b^(1/3)/d-1/3*arctan(1/3*(a^(1/3)-
2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(2/3)/b^(1/3)/d*3^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {247, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\sqrt[3]{b}\,d}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}\,d}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c + d*x)^3)^(-1),x]
```

```
[Out] -(ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3
)*b^(1/3)*d)) + Log[a^(1/3) + b^(1/3)*(c + d*x)]/(3*a^(2/3)*b^(1/3)*d) - Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/(6*a^(2/3)*b^(
1/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{a+b(c+dx)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, c+dx\right)}{3a^{2/3}d} + \frac{\text{Subst}\left(\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, c+dx\right)}{3a^{2/3}d} \\
&= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}\,d} + \frac{\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, c+dx\right)}{2\sqrt[3]{a}\,d} - \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{a}}{a^{2/3}-\sqrt[3]{a}}\right.}{6} \\
&= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}\,d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\sqrt[3]{b}\,d} + \frac{\text{Subst}\left(\int \frac{1}{-3-x}\right.}{} \\
&= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}\,d} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}\,d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+\right.}{6a^{2/3}\sqrt[3]{b}\,d}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 116, normalized size = 0.83

$$\frac{-\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)+2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)+2\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{b}\,(c+dx)-\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{6a^{2/3}\sqrt[3]{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c + d*x)^3)^(-1),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))] + 2*L
og[a^(1/3) + b^(1/3)*(c + d*x)] - Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) +
 b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*b^(1/3)*d)
```

fricas [A] time = 0.99, size = 442, normalized size = 3.16

$$\left[3\sqrt{\frac{1}{3}}\,ab\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\,\log\left(\frac{2\,abd^3x^3+6\,abcd^2x^2+6\,abc^2dx+2\,abc^3-a^2+3\sqrt{\frac{1}{3}}\left(2\,abd^2x^2+4\,abcdx+2\,abc^2+\left(a^2b\right)^{\frac{2}{3}}(dx+c)-\left(a^2b\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}-\right.}{bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*d^3*x^3 + 6*a*b*c*d
^2*x^2 + 6*a*b*c^2*d*x + 2*a*b*c^3 - a^2 + 3*sqrt(1/3)*(2*a*b*d^2*x^2 + 4*a
*b*c*d*x + 2*a*b*c^2 + (a^2*b)^(2/3)*(d*x + c) - (a^2*b)^(1/3)*a)*sqrt(-(a^
2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*(a*d*x + a*c))/(b*d^3*x^3 + 3*b*c*d^2*x^2 +
 3*b*c^2*d*x + b*c^3 + a)) - (a^2*b)^(2/3)*log(a*b*d^2*x^2 + 2*a*b*c*d*x +
a*b*c^2 - (a^2*b)^(2/3)*(d*x + c) + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log(
a*b*d*x + a*b*c + (a^2*b)^(2/3)))/(a^2*b*d), 1/6*(6*sqrt(1/3)*a*b*sqrt((a^2
*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*(d*x + c) - (a^2*b)^(1/3)*a)
*sqrt((a^2*b)^(1/3)/b)/a^2) - (a^2*b)^(2/3)*log(a*b*d^2*x^2 + 2*a*b*c*d*x +
 a*b*c^2 - (a^2*b)^(2/3)*(d*x + c) + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log
(a*b*d*x + a*b*c + (a^2*b)^(2/3)))/(a^2*b*d)]
```

giac [A] time = 0.17, size = 160, normalized size = 1.14

$$\frac{1}{3}\sqrt{3}\left(\frac{1}{a^2bd^3}\right)^{\frac{1}{3}}\arctan\left(-\frac{bdx+bc+\left(ab^2\right)^{\frac{1}{3}}}{\sqrt{3}\,bdx+\sqrt{3}\,bc-\sqrt{3}\,\left(ab^2\right)^{\frac{1}{3}}}\right)-\frac{1}{6}\left(\frac{1}{a^2bd^3}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}\,bdx+\sqrt{3}\,bc-\sqrt{3}\,\left(ab^2\right)^{\frac{1}{3}}\right)^2+4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(1/(a^2*b*d^3))^(1/3)*arctan(-(b*d*x + b*c + (a*b^2)^(1/3))/(sq
rt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))) - 1/6*(1/(a^2*b*d^3))^(
1/3)*log(4*(sqrt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))^2 + 4*(b*d
*x + b*c + (a*b^2)^(1/3))^2) + 1/3*(1/(a^2*b*d^3))^(1/3)*log(abs(b*d*x + b*
c + (a*b^2)^(1/3)))
```

maple [C] time = 0.00, size = 71, normalized size = 0.51

$$\frac{\ln\left(-\operatorname{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_Z+b\,c^3+a\right)+x\right)}{3bd\left(d^2\operatorname{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_Z+b\,c^3+a\right)^2+2cd\operatorname{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_Z+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*(d*x+c)^3),x)
```

```
[Out] 1/3/b/d*sum(1/(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x),_R=RootOf(_Z^3*b*d^3+3*_Z^2
*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)^3 b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x + c)^3*b + a), x)
```

mupad [B] time = 1.40, size = 144, normalized size = 1.03

$$\frac{\ln\left(b^{1/3}\,c+a^{1/3}+b^{1/3}\,d\,x\right)}{3\,a^{2/3}\,b^{1/3}\,d}+\frac{\ln\left(3\,b^2\,c\,d^5+3\,b^2\,d^6\,x+\frac{3\,a^{1/3}\,b^{5/3}\,d^5\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}\,d}-\frac{\ln\left(3\,b^2\,c\,d^5+3\,b^2\,d^6\,x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(c + d*x)^3),x)
```

```
[Out] log(b^(1/3)*c + a^(1/3) + b^(1/3)*d*x)/(3*a^(2/3)*b^(1/3)*d) + (log(3*b^2*c
*d^5 + 3*b^2*d^6*x + (3*a^(1/3)*b^(5/3)*d^5*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1
i - 1))/(6*a^(2/3)*b^(1/3)*d) - (log(3*b^2*c*d^5 + 3*b^2*d^6*x - (3*a^(1/3)
*b^(5/3)*d^5*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(6*a^(2/3)*b^(1/3)*d)
```

sympy [**A**] time = 0.26, size = 26, normalized size = 0.19

$$\frac{\operatorname{RootSum}\left(27t^3a^2b-1,\left(t\mapsto t\log\left(x+\frac{3ta+c}{d}\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)**3),x)
```

```
[Out] RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(x + (3*_t*a + c)/d)))/d
```

3.2864 $\int \frac{1}{(c+dx)\left(a+b(c+dx)^3\right)}\,dx$

Optimal. Leaf size=36

$$\frac{\log(c+dx)}{ad} - \frac{\log\left(a+b(c+dx)^3\right)}{3ad}$$

```
[Out] ln(d*x+c)/a/d-1/3*ln(a+b*(d*x+c)^3)/a/d
```

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {372, 266, 36, 29, 31}

$$\frac{\log(c+dx)}{ad} - \frac{\log\left(a+b(c+dx)^3\right)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)*(a + b*(c + d*x)^3)),x]
```

```
[Out] Log[c + d*x]/(a*d) - Log[a + b*(c + d*x)^3]/(3*a*d)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{(c+dx)\left(a+b(c+dx)^3\right)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx^3\right)}\,dx, x, c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx, x, (c+dx)^3\right)}{3d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, (c+dx)^3\right)}{3ad} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, (c+dx)^3\right)}{3ad}$$
$$= \frac{\log(c+dx)}{ad} - \frac{\log\left(a+b(c+dx)^3\right)}{3ad}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 1.00

$$\frac{\log(c+dx)}{ad} - \frac{\log\left(a+b(c+dx)^3\right)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)*(a + b*(c + d*x)^3)),x]
```

```
[Out] Log[c + d*x]/(a*d) - Log[a + b*(c + d*x)^3]/(3*a*d)
```

fricas [A] time = 0.61, size = 51, normalized size = 1.42

$$-\frac{\log\left(bd^3x^3 + 3\,bcd^2x^2 + 3\,bc^2dx + bc^3 + a\right) - 3\,\log(dx+c)}{3\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/3*(log(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a) - 3*log(d*x
+ c))/(a*d)
```

giac [A] time = 0.21, size = 58, normalized size = 1.61

$$-\frac{\log\left(\left|bd^3x^3 + 3\,bcd^2x^2 + 3\,bc^2dx + bc^3 + a\right|\right)}{3\,ad} + \frac{\log(|dx+c|)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] -1/3*log(abs(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a))/(a*d) +
log(abs(d*x + c))/(a*d)
```

maple [A] time = 0.01, size = 57, normalized size = 1.58

$$\frac{\ln(dx+c)}{ad} - \frac{\ln\left(b\,d^3x^3 + 3bc\,d^2x^2 + 3b\,c^2dx + b\,c^3 + a\right)}{3ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+b*(d*x+c)^3),x)
```

```
[Out] ln(d*x+c)/a/d-1/3/a/d*ln(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)
```

maxima [A] time = 0.55, size = 56, normalized size = 1.56

$$-\frac{\log\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)}{3\,ad}+\frac{\log\left(dx+c\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -1/3*log(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(a*d) + log(d
*x + c)/(a*d)
```

mupad [B] time = 1.28, size = 56, normalized size = 1.56

$$\frac{\ln\left(c+d\,x\right)}{a\,d}-\frac{\ln\left(b\,c^3+3\,b\,c^2\,d\,x+3\,b\,c\,d^2\,x^2+b\,d^3\,x^3+a\right)}{3\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*(c + d*x)^3)*(c + d*x)),x)
```

```
[Out] log(c + d*x)/(a*d) - log(a + b*c^3 + b*d^3*x^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^
2)/(3*a*d)
```

sympy [A] time = 0.47, size = 49, normalized size = 1.36

$$\frac{\log\left(\frac{c}{d}+x\right)}{ad}-\frac{\log\left(\frac{3c^2x}{d^2}+\frac{3cx^2}{d}+x^3+\frac{a+bc^3}{bd^3}\right)}{3ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*(d*x+c)**3),x)
```

```
[Out] log(c/d + x)/(a*d) - log(3*c**2*x/d**2 + 3*c*x**2/d + x**3 + (a + b*c**3)/(
b*d**3))/(3*a*d)
```

3.2865 $\int \frac{1}{(c+dx)^2\left(a+b(c+dx)^3\right)}\,dx$

Optimal. Leaf size=154

$$-\frac{\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{4/3}d}+\frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{4/3}d}+\frac{\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}d}-\frac{}{ad}$$

```
[Out] -1/a/d/(d*x+c)+1/3*b^(1/3)*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(4/3)/d-1/6*b^(1/3
)*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(4/3)/d+1/3*b^(1/
3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(4/3)/d*3^(1/2
)
```

Rubi [A] time = 0.13, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {372, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{4/3}d}+\frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{4/3}d}+\frac{\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}d}-\frac{}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)^2*(a + b*(c + d*x)^3)),x]
```

```
[Out] -(1/(a*d*(c + d*x))) + (b^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqr
t[3]*a^(1/3))])/(Sqrt[3]*a^(4/3)*d) + (b^(1/3)*Log[a^(1/3) + b^(1/3)*(c + d
*x)])/(3*a^(4/3)*d) - (b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^
(2/3)*(c + d*x)^2])/(6*a^(4/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
```

```
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(c+dx)^2\left(a+b(c+dx)^3\right)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^3\right)}\,dx, x, c+dx\right)}{d} \\
&= -\frac{1}{ad(c+dx)} - \frac{b\,\text{Subst}\left(\int \frac{x}{a+bx^3}\,dx, x, c+dx\right)}{ad} \\
&= -\frac{1}{ad(c+dx)} + \frac{b^{2/3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, c+dx\right)}{3a^{4/3}d} - \frac{b^{2/3}\,\text{Subst}\left(\int \frac{\sqrt[3]{a}+\sqrt[3]{}}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x}\right.}{3a^{4/3}} \\
&= -\frac{1}{ad(c+dx)} + \frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{4/3}d} - \frac{\sqrt[3]{b}\,\text{Subst}\left(\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\right.}{6a^{4/3}d} \\
&= -\frac{1}{ad(c+dx)} + \frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{4/3}d} - \frac{\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+\right.}{6a^{4/3}d} \\
&= -\frac{1}{ad(c+dx)} + \frac{\sqrt[3]{b}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{4/3}d} + \frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{4/3}d} - \frac{\sqrt[3]{b}\,\log}{}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 140, normalized size = 0.91

$$\frac{-\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)+2\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)-2\sqrt{3}\,\sqrt[3]{b}\,\tan^{-1}\left(\frac{2\sqrt[3]{b}\,(c+dx)-\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{6a^{4/3}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)^2*(a + b*(c + d*x)^3)),x]
```

```
[Out] ((-6*a^(1/3))/(c + d*x) - 2*Sqrt[3]*b^(1/3)*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c
 + d*x))/(Sqrt[3]*a^(1/3))] + 2*b^(1/3)*Log[a^(1/3) + b^(1/3)*(c + d*x)] -
b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*
a^(4/3)*d)
```

fricas [A] time = 0.83, size = 150, normalized size = 0.97

$$-\frac{2\sqrt{3}\,(dx+c)\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\,(dx+c)\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+(dx+c)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bd^2x^2+2\,bcdx+bc^2-(adx+a\right.}{6\left(ad^2x+acd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(d*x + c)*(b/a)^(1/3)*arctan(2/3*sqrt(3)*(d*x + c)*(b/a)^(1
/3) - 1/3*sqrt(3)) + (d*x + c)*(b/a)^(1/3)*log(b*d^2*x^2 + 2*b*c*d*x + b*c^
2 - (a*d*x + a*c)*(b/a)^(2/3) + a*(b/a)^(1/3)) - 2*(d*x + c)*(b/a)^(1/3)*lo
g(b*d*x + b*c + a*(b/a)^(2/3)) + 6)/(a*d^2*x + a*c*d)
```

giac [A] time = 0.20, size = 175, normalized size = 1.14

$$\frac{\left(\frac{b}{ad^3}\right)^{\frac{1}{3}}\log\left(\left|-\left(\frac{b}{ad^3}\right)^{\frac{1}{3}}-\frac{1}{(dx+c)d}\right|\right)}{3\,a}-\frac{\sqrt{3}\,\left(a^2b\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(\frac{b}{ad^3}\right)^{\frac{1}{3}}-\frac{2}{(dx+c)d}\right)}{3\left(\frac{b}{ad^3}\right)^{\frac{1}{3}}}\right)}{3\,a^2d}-\frac{\left(a^2b\right)^{\frac{1}{3}}\log\left(\left(\frac{b}{ad^3}\right)^{\frac{2}{3}}-\frac{\left(\frac{b}{ad^3}\right)^{\frac{1}{3}}}{(dx+c)d}+\frac{1}{(dx+c)^2}\right.}{6\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/3*(b/(a*d^3))^(1/3)*log(abs(-(b/(a*d^3))^(1/3) - 1/((d*x + c)*d)))/a - 1/
3*sqrt(3)*(a^2*b)^(1/3)*arctan(1/3*sqrt(3)*((b/(a*d^3))^(1/3) - 2/((d*x + c
)*d))/(b/(a*d^3))^(1/3))/(a^2*d) - 1/6*(a^2*b)^(1/3)*log((b/(a*d^3))^(2/3)
- (b/(a*d^3))^(1/3)/((d*x + c)*d) + 1/((d*x + c)^2*d^2))/(a^2*d) - 1/((d*x
+ c)*a*d)
```

maple [C] time = 0.00, size = 92, normalized size = 0.60

$$-\frac{\left(\text{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_Z+b\,c^3+a\right)d+c\right)\ln\left(-\text{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_\right.\right.}{3ad\left(d^2\,\text{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_Z+b\,c^3+a\right)^2+2cd\,\text{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+b*(d*x+c)^3),x)
```

```
[Out] -1/a/d/(d*x+c)-1/3/d*sum((_R*d+c)/(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x),_R=Root
Of(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{-\frac{1}{6}\left(2\sqrt{3}\,\left(-\frac{1}{ab^2d^3}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)}{3\left(-a^2b\right)^{\frac{2}{3}}}\right)+\left(-\frac{1}{ab^2d^3}\right)^{\frac{1}{3}}\log\left(\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)^2+3\,\left(-a^2b\right.\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -b*integrate((d*x + c)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a
), x)/a - 1/(a*d^2*x + a*c*d)
```

mupad [B] time = 0.38, size = 138, normalized size = 0.90

$$\frac{b^{1/3}\,\ln\left(b^{1/3}\,c+a^{1/3}+b^{1/3}\,d\,x\right)}{3\,a^{4/3}\,d}-\frac{1}{a\,d\,\left(c+d\,x\right)}-\frac{b^{1/3}\,\ln\left(2\,b^{1/3}\,c-a^{1/3}+2\,b^{1/3}\,d\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,a^{4/3}\,d}+\frac{b^{1/3}\,\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*(c + d*x)^3)*(c + d*x)^2),x)
```

```
[Out] (b^(1/3)*log(b^(1/3)*c + a^(1/3) + b^(1/3)*d*x))/(3*a^(4/3)*d) - 1/(a*d*(c
+ d*x)) - (b^(1/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*c - a^(1/3) + 2*b^(1/
3)*d*x)*((3^(1/2)*1i)/2 + 1/2))/(3*a^(4/3)*d) + (b^(1/3)*log(2*b^(1/3)*c -
3^(1/2)*a^(1/3)*1i - a^(1/3) + 2*b^(1/3)*d*x)*((3^(1/2)*1i)/6 - 1/6))/(a^(4
/3)*d)
```

sympy [A] time = 0.66, size = 44, normalized size = 0.29

$$-\frac{1}{acd+ad^2x}+\frac{\operatorname{RootSum}\left(27t^3a^4-b,\left(t\mapsto t\log\left(x+\frac{9t^2a^3+bc}{bd}\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+b*(d*x+c)**3),x)
```

```
[Out] -1/(a*c*d + a*d**2*x) + RootSum(27*_t**3*a**4 - b, Lambda(_t, _t*log(x + (9
*_t**2*a**3 + b*c)/(b*d))))/d
```

3.2866 $\int \frac{1}{(c+dx)^3\left(a+b(c+dx)^3\right)}\,dx$

Optimal. Leaf size=156

$$-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{5/3}d}+\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{5/3}d}+\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}d}-\frac{2a}{}$$

```
[Out] -1/2/a/d/(d*x+c)^2-1/3*b^(2/3)*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(5/3)/d+1/6*b^
(2/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(5/3)/d+1/3*b
^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(5/3)/d*3^
(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {372, 325, 200, 31, 634, 617, 204, 628}

$$-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{5/3}d}+\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{5/3}d}+\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}d}-\frac{2a}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)^3*(a + b*(c + d*x)^3)),x]
```

```
[Out] -1/(2*a*d*(c + d*x)^2) + (b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(S
qrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)*d) - (b^(2/3)*Log[a^(1/3) + b^(1/3)*(c +
 d*x)])/(3*a^(5/3)*d) + (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) +
b^(2/3)*(c + d*x)^2])/(6*a^(5/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
```

```
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(c+dx)^3\left(a+b(c+dx)^3\right)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3\left(a+bx^3\right)}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{1}{2ad(c+dx)^2} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, c+dx\right)}{ad}$$

$$= -\frac{1}{2ad(c+dx)^2} - \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, c+dx\right)}{3a^{5/3}d} - \frac{b\,\text{Subst}\left(\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, c+dx\right)}{3a^{5/3}d}$$

$$= -\frac{1}{2ad(c+dx)^2} - \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{5/3}d} + \frac{b^{2/3}\,\text{Subst}\left(\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, c+dx\right)}{6a^{5/3}d}$$

$$= -\frac{1}{2ad(c+dx)^2} - \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{5/3}d} + \frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)\right)}{6a^{5/3}d}$$

$$= -\frac{1}{2ad(c+dx)^2} + \frac{b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{5/3}d} - \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{5/3}d} + \frac{b^{2/3}}{}$$

Mathematica [A] time = 0.04, size = 139, normalized size = 0.89

$$\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right) - \frac{3a^{2/3}}{(c+dx)^2} - 2b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right) - 2\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{b}\,(c+d}{\sqrt{3}}\right.}{6a^{5/3}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)^3*(a + b*(c + d*x)^3)),x]
```

```
[Out] ((-3*a^(2/3))/(c + d*x)^2 - 2*Sqrt[3]*b^(2/3)*ArcTan[(-a^(1/3) + 2*b^(1/3)*
(c + d*x))/(Sqrt[3]*a^(1/3))] - 2*b^(2/3)*Log[a^(1/3) + b^(1/3)*(c + d*x)]
+ b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(
6*a^(5/3)*d)
```

fricas [A] time = 0.63, size = 236, normalized size = 1.51

$$\frac{2\sqrt{3}\left(d^2x^2+2cdx+c^2\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,(adx+ac)\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3b}\right)-\left(d^2x^2+2cdx+c^2\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2d^2x^2+2\right.}{6\left(ad^3x^2+2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*(d^2*x^2 + 2*c*d*x + c^2)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqr
t(3)*(a*d*x + a*c)*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) - (d^2*x^2 + 2*c*d*x +
c^2)*(-b^2/a^2)^(1/3)*log(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + a^2*(-b^2/a
^2)^(2/3) + (a*b*d*x + a*b*c)*(-b^2/a^2)^(1/3)) + 2*(d^2*x^2 + 2*c*d*x + c^
2)*(-b^2/a^2)^(1/3)*log(b*d*x + b*c - a*(-b^2/a^2)^(1/3)) - 3)/(a*d^3*x^2 +
 2*a*c*d^2*x + a*c^2*d)
```

giac [A] time = 0.21, size = 193, normalized size = 1.24

$$\frac{2\sqrt{3}\left(-\frac{b^2}{a^2d^3}\right)^{\frac{1}{3}}\arctan\left(-\frac{bdx+bc-\left(-ab^2\right)^{\frac{1}{3}}}{\sqrt{3}\,bdx+\sqrt{3}\,bc+\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}}\right)-\left(-\frac{b^2}{a^2d^3}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}\,bdx+\sqrt{3}\,bc+\sqrt{3}\,\left(-ab^2\right)^{\frac{1}{3}}\right)^2+4\left(bdx\right.\right.}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/6*(2*sqrt(3)*(-b^2/(a^2*d^3))^(1/3)*arctan(-(b*d*x + b*c - (-a*b^2)^(1/3)
)/(sqrt(3)*b*d*x + sqrt(3)*b*c + sqrt(3)*(-a*b^2)^(1/3))) - (-b^2/(a^2*d^3)
)^(1/3)*log(4*(sqrt(3)*b*d*x + sqrt(3)*b*c + sqrt(3)*(-a*b^2)^(1/3))^2 + 4*
(b*d*x + b*c - (-a*b^2)^(1/3))^2) + 2*(-b^2/(a^2*d^3))^(1/3)*log(abs(-b*d*x
 - b*c + (-a*b^2)^(1/3))))/a - 1/2/((d*x + c)^2*a*d)
```

maple [C] time = 0.00, size = 87, normalized size = 0.56

$$-\frac{\ln\left(-\operatorname{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_Z+b\,c^3+a\right)+x\right)}{3ad\left(d^2\operatorname{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_Z+b\,c^3+a\right)^2+2cd\operatorname{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^3/(a+b*(d*x+c)^3),x)
```

```
[Out] -1/2/a/d/(d*x+c)^2-1/3/d*sum(1/(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x),_R=RootOf(
_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\frac{1}{6}\,b\left(\frac{2\sqrt{3}\left(\frac{1}{a^2b}\right)^{\frac{1}{3}}\arctan\left(-\frac{bdx+bc+\left(ab^2\right)^{\frac{1}{3}}}{\sqrt{3}\,bdx+\sqrt{3}\,bc-\sqrt{3}\left(ab^2\right)^{\frac{1}{3}}}\right)}{d}-\frac{\left(\frac{1}{a^2b}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}\,bdx+\sqrt{3}\,bc-\sqrt{3}\left(ab^2\right)^{\frac{1}{3}}\right)^2+4\left(bdx+bc+\left(ab^2\right)^{\frac{1}{3}}\right)^2\right)}{d}+\frac{2\left(\frac{1}{a^2b}\right)^{\frac{1}{3}}\log\left(\left|b\right.\right.\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -b*integrate(1/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/a
- 1/2/(a*d^3*x^2 + 2*a*c*d^2*x + a*c^2*d)
```

mupad **[B]** time = 0.33, size = 201, normalized size = 1.29

$$\frac{b^{2/3}\,\ln\left(a^2\,b^{1/3}\,c-(-a)^{7/3}+a^2\,b^{1/3}\,d\,x\right)}{3\,(-a)^{5/3}\,d}-\frac{1}{2\,a\,d\,\left(c^2+2\,c\,d\,x+d^2\,x^2\right)}-\frac{b^{2/3}\,\ln\left(3\,a^2\,b^3\,c\,d^5+3\,a^2\,b^3\,d^6\,x+3\,(-a)^{7/3}\,b\right.}{3\,(-a)^{5/3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*(c + d*x)^3)*(c + d*x)^3),x)
```

```
[Out] (b^(2/3)*log(a^2*b^(1/3)*c - (-a)^(7/3) + a^2*b^(1/3)*d*x))/(3*(-a)^(5/3)*d
) - 1/(2*a*d*(c^2 + d^2*x^2 + 2*c*d*x)) - (b^(2/3)*log(3*a^2*b^3*c*d^5 + 3*
a^2*b^3*d^6*x + 3*(-a)^(7/3)*b^(8/3)*d^5*((3^(1/2)*1i)/2 + 1/2))*((3^(1/2)*
1i)/2 + 1/2))/(3*(-a)^(5/3)*d) + (b^(2/3)*log(3*a^2*b^3*c*d^5 + 3*a^2*b^3*d
^6*x - 9*(-a)^(7/3)*b^(8/3)*d^5*((3^(1/2)*1i)/6 - 1/6))*((3^(1/2)*1i)/6 - 1
/6))/((-a)^(5/3)*d)
```

sympy **[A]** time = 0.87, size = 61, normalized size = 0.39

$$-\frac{1}{2ac^2d+4acd^2x+2ad^3x^2}+\frac{\operatorname{RootSum}\left(27t^3a^5+b^2,\left(t\mapsto t\log\left(x+\frac{-3ta^2+bc}{bd}\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**3/(a+b*(d*x+c)**3),x)
```

```
[Out] -1/(2*a*c**2*d + 4*a*c*d**2*x + 2*a*d**3*x**2) + RootSum(27*_t**3*a**5 + b*
*2, Lambda(_t, _t*log(x + (-3*_t*a**2 + b*c)/(b*d))))/d
```

3.2867 $\int \frac{1}{(c+dx)^4\left(a+b(c+dx)^3\right)}\,dx$

Optimal. Leaf size=56

$$-\frac{b\log(c+dx)}{a^2d}+\frac{b\log\left(a+b(c+dx)^3\right)}{3a^2d}-\frac{1}{3ad(c+dx)^3}$$

```
[Out] -1/3/a/d/(d*x+c)^3-b*ln(d*x+c)/a^2/d+1/3*b*ln(a+b*(d*x+c)^3)/a^2/d
```

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {372, 266, 44}

$$-\frac{b\log(c+dx)}{a^2d}+\frac{b\log\left(a+b(c+dx)^3\right)}{3a^2d}-\frac{1}{3ad(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)^4*(a + b*(c + d*x)^3)),x]
```

```
[Out] -1/(3*a*d*(c + d*x)^3) - (b*Log[c + d*x])/(a^2*d) + (b*Log[a + b*(c + d*x)^
3])/(3*a^2*d)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(c+dx)^4\left(a+b(c+dx)^3\right)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^3\right)}\,dx, x, c+dx\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx)}\,dx, x, (c+dx)^3\right)}{3d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)\,dx, x, (c+dx)^3\right)}{3d}\\
&= -\frac{1}{3ad(c+dx)^3}-\frac{b\log(c+dx)}{a^2d}+\frac{b\log\left(a+b(c+dx)^3\right)}{3a^2d}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.79

$$\frac{b\log\left(a+b(c+dx)^3\right)-\frac{a}{(c+dx)^3}-3b\log(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)^4*(a + b*(c + d*x)^3)),x]
```

```
[Out] (-(a/(c + d*x)^3) - 3*b*Log[c + d*x] + b*Log[a + b*(c + d*x)^3])/(3*a^2*d)
```

fricas [B] time = 0.82, size = 158, normalized size = 2.82

$$\frac{\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3\right)\log\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)-3\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+\right.}{3\left(a^2d^4x^3+3\,a^2cd^3x^2+3\,a^2c^2d^2x+a^2c^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^4/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/3*((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*log(b*d^3*x^3 + 3*b*
c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a) - 3*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c
^2*d*x + b*c^3)*log(d*x + c) - a)/(a^2*d^4*x^3 + 3*a^2*c*d^3*x^2 + 3*a^2*c^
2*d^2*x + a^2*c^3*d)
```

giac [A] time = 0.21, size = 41, normalized size = 0.73

$$\frac{b\log\left(\left|-b-\frac{a}{(dx+c)^3}\right|\right)}{3\,a^2d}-\frac{1}{3\,(dx+c)^3ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^4/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/3*b*log(abs(-b - a/(d*x + c)^3))/(a^2*d) - 1/3/((d*x + c)^3*a*d)
```

maple [A] time = 0.01, size = 75, normalized size = 1.34

$$-\frac{b\ln(dx+c)}{a^2d}+\frac{b\ln\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)}{3a^2d}-\frac{1}{3\,(dx+c)^3\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^4/(a+b*(d*x+c)^3),x)
```

```
[Out] -1/3/a/d/(d*x+c)^3-b*ln(d*x+c)/a^2/d+1/3/a^2*b/d*ln(b*d^3*x^3+3*b*c*d^2*x^2
+3*b*c^2*d*x+b*c^3+a)
```

maxima [A] time = 0.59, size = 98, normalized size = 1.75

$$-\frac{1}{3\left(ad^4x^3+3\,acd^3x^2+3\,ac^2d^2x+ac^3d\right)}+\frac{b\log\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)}{3\,a^2d}-\frac{b\log(dx+c)}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^4/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -1/3/(a*d^4*x^3 + 3*a*c*d^3*x^2 + 3*a*c^2*d^2*x + a*c^3*d) + 1/3*b*log(b*d^
3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(a^2*d) - b*log(d*x + c)/(
a^2*d)
```

mupad [B] time = 1.42, size = 104, normalized size = 1.86

$$\frac{b\,\ln\left(-b\,c^3-3\,b\,c^2\,d\,x-3\,b\,c\,d^2\,x^2-b\,d^3\,x^3-a\right)}{3\,a^2\,d}-\frac{b\,\ln\left(c+d\,x\right)}{a^2\,d}-\frac{1}{3\,\left(a\,c^3\,d+3\,a\,c^2\,d^2\,x+3\,a\,c\,d^3\,x^2+a\,d^4\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*(c + d*x)^3)*(c + d*x)^4),x)
```

```
[Out] (b*log(- a - b*c^3 - b*d^3*x^3 - 3*b*c^2*d*x - 3*b*c*d^2*x^2))/(3*a^2*d) -
(b*log(c + d*x))/(a^2*d) - 1/(3*(a*d^4*x^3 + a*c^3*d + 3*a*c^2*d^2*x + 3*a*
c*d^3*x^2))
```

sympy [B] time = 1.28, size = 100, normalized size = 1.79

$$-\frac{1}{3ac^3d+9ac^2d^2x+9acd^3x^2+3ad^4x^3}-\frac{b\log\left(\frac{c}{d}+x\right)}{a^2d}+\frac{b\log\left(\frac{3c^2x}{d^2}+\frac{3cx^2}{d}+x^3+\frac{a+bc^3}{bd^3}\right)}{3a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**4/(a+b*(d*x+c)**3),x)
```

```
[Out] -1/(3*a*c**3*d + 9*a*c**2*d**2*x + 9*a*c*d**3*x**2 + 3*a*d**4*x**3) - b*log
(c/d + x)/(a**2*d) + b*log(3*c**2*x/d**2 + 3*c*x**2/d + x**3 + (a + b*c**3)
/(b*d**3))/(3*a**2*d)
```

3.2868 $\int \frac{(c+dx)^4}{\left(a+b(c+dx)^3\right)^2}\,dx$

Optimal. Leaf size=172

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{9\sqrt[3]{a}\,b^{5/3}d}-\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9\sqrt[3]{a}\,b^{5/3}d}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,\sqrt[3]{a}\,b^{5/3}d}-\frac{(c+dx)^2}{3bd\left(a+b(c+d$$

```
[Out] -1/3*(d*x+c)^2/b/d/(a+b*(d*x+c)^3)-2/9*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(1/3)/
b^(5/3)/d+1/9*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(1/3)
/b^(5/3)/d-2/9*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(1
/3)/b^(5/3)/d*3^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {372, 288, 292, 31, 634, 617, 204, 628}

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{9\sqrt[3]{a}\,b^{5/3}d}-\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9\sqrt[3]{a}\,b^{5/3}d}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,\sqrt[3]{a}\,b^{5/3}d}-\frac{(c+dx)^2}{3bd\left(a+b(c+d$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4/(a + b*(c + d*x)^3)^2,x]
```

```
[Out] -(c + d*x)^2/(3*b*d*(a + b*(c + d*x)^3)) - (2*ArcTan[(a^(1/3) - 2*b^(1/3)*(
c + d*x))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(1/3)*b^(5/3)*d) - (2*Log[a^(1/3
) + b^(1/3)*(c + d*x)])/(9*a^(1/3)*b^(5/3)*d) + Log[a^(2/3) - a^(1/3)*b^(1/
3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/(9*a^(1/3)*b^(5/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(c+dx)^4}{\left(a+b(c+dx)^3\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(a+bx^3\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{(c+dx)^2}{3bd\left(a+b(c+dx)^3\right)} + \frac{2\,\text{Subst}\left(\int \frac{x}{a+bx^3}\,dx, x, c+dx\right)}{3bd}$$

$$= -\frac{(c+dx)^2}{3bd\left(a+b(c+dx)^3\right)} - \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, c+dx\right)}{9\sqrt[3]{a}\,b^{4/3}d} + \frac{2\,\text{Subst}\left(\int \frac{\sqrt[3]{a}+\sqrt[3]{b}}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x}\right.}{9\sqrt[3]{a}\,b}$$

$$= -\frac{(c+dx)^2}{3bd\left(a+b(c+dx)^3\right)} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9\sqrt[3]{a}\,b^{5/3}d} + \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx,\right.}{9\sqrt[3]{a}\,b^{5/3}d}$$

$$= -\frac{(c+dx)^2}{3bd\left(a+b(c+dx)^3\right)} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9\sqrt[3]{a}\,b^{5/3}d} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b\right.}{9\sqrt[3]{a}\,b^{5/3}d}$$

$$= -\frac{(c+dx)^2}{3bd\left(a+b(c+dx)^3\right)} - \frac{2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,\sqrt[3]{a}\,b^{5/3}d} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9\sqrt[3]{a}\,b^{5/3}d} + \frac{\log\left(a^2\right.}{9\sqrt[3]{a}\,b^{5/3}d}$$

Mathematica [A] time = 0.10, size = 152, normalized size = 0.88

$$\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{\sqrt[3]{a}} - \frac{3b^{2/3}(c+dx)^2}{a+b(c+dx)^3} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{\sqrt[3]{a}} + \frac{2\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{b}\,(c+dx)-\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt[3]{a}}}{9b^{5/3}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4/(a + b*(c + d*x)^3)^2,x]
```

```
[Out] ((-3*b^(2/3)*(c + d*x)^2)/(a + b*(c + d*x)^3) + (2*Sqrt[3]*ArcTan[(-a^(1/3)
 + 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/a^(1/3) - (2*Log[a^(1/3) + b^(1
/3)*(c + d*x)])/a^(1/3) + Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)
*(c + d*x)^2]/a^(1/3))/(9*b^(5/3)*d)
```

fricas **[B]** time = 0.74, size = 838, normalized size = 4.87

$$\left[-\frac{3\,ab^2d^2x^2 + 6\,ab^2cdx + 3\,ab^2c^2 - 3\sqrt{\frac{1}{3}}\left(ab^2d^3x^3 + 3\,ab^2cd^2x^2 + 3\,ab^2c^2dx + ab^2c^3 + a^2b\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2d^3x^3}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4/(a+b*(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] [-1/9*(3*a*b^2*d^2*x^2 + 6*a*b^2*c*d*x + 3*a*b^2*c^2 - 3*sqrt(1/3)*(a*b^2*d
^3*x^3 + 3*a*b^2*c*d^2*x^2 + 3*a*b^2*c^2*d*x + a*b^2*c^3 + a^2*b)*sqrt((-a*
b^2)^(1/3)/a)*log((2*b^2*d^3*x^3 + 6*b^2*c*d^2*x^2 + 6*b^2*c^2*d*x + 2*b^2*
c^3 - a*b + 3*sqrt(1/3)*(a*b*d*x + a*b*c + 2*(d^2*x^2 + 2*c*d*x + c^2)*(-a*
b^2)^(2/3) + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*(d
*x + c))/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)) - (b*d^3*x^
3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*(-a*b^2)^(2/3)*log(b^2*d^2*x^2
 + 2*b^2*c*d*x + b^2*c^2 + (-a*b^2)^(1/3)*(b*d*x + b*c) + (-a*b^2)^(2/3)) +
 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*(-a*b^2)^(2/3)*log
(b*d*x + b*c - (-a*b^2)^(1/3)))/(a*b^4*d^4*x^3 + 3*a*b^4*c*d^3*x^2 + 3*a*b^
4*c^2*d^2*x + (a*b^4*c^3 + a^2*b^3)*d), -1/9*(3*a*b^2*d^2*x^2 + 6*a*b^2*c*d
*x + 3*a*b^2*c^2 - 6*sqrt(1/3)*(a*b^2*d^3*x^3 + 3*a*b^2*c*d^2*x^2 + 3*a*b^2
*c^2*d*x + a*b^2*c^3 + a^2*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b
*d*x + 2*b*c + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - (b*d^3*x^3 + 3*
b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*(-a*b^2)^(2/3)*log(b^2*d^2*x^2 + 2*b
^2*c*d*x + b^2*c^2 + (-a*b^2)^(1/3)*(b*d*x + b*c) + (-a*b^2)^(2/3)) + 2*(b*
d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*(-a*b^2)^(2/3)*log(b*d*x
 + b*c - (-a*b^2)^(1/3)))/(a*b^4*d^4*x^3 + 3*a*b^4*c*d^3*x^2 + 3*a*b^4*c^2*
d^2*x + (a*b^4*c^3 + a^2*b^3)*d)]
```

giac **[A]** time = 0.22, size = 205, normalized size = 1.19

$$-\frac{2\sqrt{3}\left(-\frac{1}{ab^2d^3}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)}{3\left(-a^2b\right)^{\frac{2}{3}}}\right)+\left(-\frac{1}{ab^2d^3}\right)^{\frac{1}{3}}\log\left(\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)^2+3\left(-a^2b\right)^{\frac{4}{3}}\right)-2}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4/(a+b*(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -1/9*(2*sqrt(3)*(-1/(a*b^2*d^3))^(1/3)*arctan(1/3*sqrt(3)*(2*a*b*d*x + 2*a*
b*c - (-a^2*b)^(2/3))/(-a^2*b)^(2/3)) + (-1/(a*b^2*d^3))^(1/3)*log((2*a*b*d
*x + 2*a*b*c - (-a^2*b)^(2/3))^2 + 3*(-a^2*b)^(4/3)) - 2*(-1/(a*b^2*d^3))^(
```

```
1/3)*log(abs(a*b*d*x + a*b*c + (-a^2*b)^(2/3))))/b - 1/3*(d^2*x^2 + 2*c*d*x
 + c^2)/((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*b*d)
```

maple [C] time = 0.01, size = 141, normalized size = 0.82

$$\frac{2\left(\text{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_Z+b\,c^3+a\right)d+c\right)\ln\left(-\text{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_\right.\right.}{9b^2d\left(d^2\,\text{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_Z+b\,c^3+a\right)^2+2cd\,\text{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4/(a+b*(d*x+c)^3)^2,x)
```

```
[Out] (-1/3/b*d*x^2-2/3/b*c*x-1/3*c^2/d/b)/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b
*c^3+a)+2/9/b^2/d*sum((_R*d+c)/(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x),_R=RootOf(
_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{d^2x^2+2\,cdx+c^2}{3\left(b^2d^4x^3+3\,b^2cd^3x^2+3\,b^2c^2d^2x+\left(b^2c^3+ab\right)d\right)}+\frac{2\left(-\frac{1}{3}\sqrt{3}\left(-\frac{1}{ab^2d^3}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)}{3\left(-a^2b\right)^{\frac{2}{3}}}\right)-\frac{1}{6}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4/(a+b*(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(d^2*x^2 + 2*c*d*x + c^2)/(b^2*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d
^2*x + (b^2*c^3 + a*b)*d) + 2/3*integrate((d*x + c)/(b*d^3*x^3 + 3*b*c*d^2*
x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/b
```

mupad [B] time = 1.50, size = 240, normalized size = 1.40

$$\frac{2\,\ln\left(\frac{4\,c\,d^4}{9\,b}-\frac{4\,(-a)^{1/3}\,d^4}{9\,b^{4/3}}+\frac{4\,d^5\,x}{9\,b}\right)}{9\,(-a)^{1/3}\,b^{5/3}\,d}-\frac{\frac{d\,x^2}{3\,b}+\frac{c^2}{3\,b\,d}+\frac{2\,c\,x}{3\,b}}{b\,c^3+3\,b\,c^2\,d\,x+3\,b\,c\,d^2\,x^2+b\,d^3\,x^3+a}+\frac{\ln\left(\frac{4\,c\,d^4}{9\,b}+\frac{4\,d^5\,x}{9\,b}-\frac{(-a)^{1/3}\,d^4\left(-1+\sqrt{3}\,1\text{i}\right.}{9\,b^{4/3}}\right.}{9\,(-a)^{1/3}\,b^{5/3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^4/(a + b*(c + d*x)^3)^2,x)
```

```
[Out] (2*log((4*c*d^4)/(9*b) - (4*(-a)^(1/3)*d^4)/(9*b^(4/3)) + (4*d^5*x)/(9*b)))
/(9*(-a)^(1/3)*b^(5/3)*d) - ((d*x^2)/(3*b) + c^2/(3*b*d) + (2*c*x)/(3*b))/(
a + b*c^3 + b*d^3*x^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2) + (log((4*c*d^4)/(9*b)
 + (4*d^5*x)/(9*b) - ((-a)^(1/3)*d^4*(3^(1/2)*1i - 1)^2)/(9*b^(4/3)))*(3^(1
/2)*1i - 1))/(9*(-a)^(1/3)*b^(5/3)*d) - (log((4*c*d^4)/(9*b) + (4*d^5*x)/(9
*b) - ((-a)^(1/3)*d^4*(3^(1/2)*1i + 1)^2)/(9*b^(4/3)))*(3^(1/2)*1i + 1))/(9
*(-a)^(1/3)*b^(5/3)*d)
```

sympy [A] time = 1.11, size = 109, normalized size = 0.63

$$\frac{-c^2-2cdx-d^2x^2}{3abd+3b^2c^3d+9b^2c^2d^2x+9b^2cd^3x^2+3b^2d^4x^3}+\frac{\text{RootSum}\left(729t^3ab^5+8,\left(t\mapsto t\log\left(x+\frac{81t^2ab^3+4c}{4d}\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4/(a+b*(d*x+c)**3)**2,x)
```

```
[Out] (-c**2 - 2*c*d*x - d**2*x**2)/(3*a*b*d + 3*b**2*c**3*d + 9*b**2*c**2*d**2*x
 + 9*b**2*c*d**3*x**2 + 3*b**2*d**4*x**3) + RootSum(729*_t**3*a*b**5 + 8, L
ambda(_t, _t*log(x + (81*_t**2*a*b**3 + 4*c)/(4*d))))/d
```

3.2869 $\int \frac{(c+dx)^3}{\left(a+b(c+dx)^3\right)^2}\,dx$

Optimal. Leaf size=170

$$\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9a^{2/3}b^{4/3}d}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{18a^{2/3}b^{4/3}d}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}b^{4/3}d}-\frac{c+dx}{3bd\left(a+b(c+dx)^3\right)}$$

```
[Out] 1/3*(-d*x-c)/b/d/(a+b*(d*x+c)^3)+1/9*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(2/3)/b^
(4/3)/d-1/18*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(2/3)/
b^(4/3)/d-1/9*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(2/
3)/b^(4/3)/d*3^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {372, 288, 200, 31, 634, 617, 204, 628}

$$\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9a^{2/3}b^{4/3}d}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{18a^{2/3}b^{4/3}d}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}b^{4/3}d}-\frac{c+dx}{3bd\left(a+b(c+dx)^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*(c + d*x)^3)^2,x]
```

```
[Out] -(c + d*x)/(3*b*d*(a + b*(c + d*x)^3)) - ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d
*x))/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(2/3)*b^(4/3)*d) + Log[a^(1/3) + b^(1/
3)*(c + d*x)]/(9*a^(2/3)*b^(4/3)*d) - Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*
x) + b^(2/3)*(c + d*x)^2]/(18*a^(2/3)*b^(4/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(c+dx)^3}{\left(a+b(c+dx)^3\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{\left(a+bx^3\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{c+dx}{3bd\left(a+b(c+dx)^3\right)} + \frac{\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, c+dx\right)}{3bd}$$

$$= -\frac{c+dx}{3bd\left(a+b(c+dx)^3\right)} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, c+dx\right)}{9a^{2/3}bd} + \frac{\text{Subst}\left(\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/}}\right.}{9a^{2/3}bd}$$

$$= -\frac{c+dx}{3bd\left(a+b(c+dx)^3\right)} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9a^{2/3}b^{4/3}d} - \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x\right.}{18a^{2/3}b^{4/3}d}$$

$$= -\frac{c+dx}{3bd\left(a+b(c+dx)^3\right)} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9a^{2/3}b^{4/3}d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/}\right.}{18a^{2/3}b^{4/3}d}$$

$$= -\frac{c+dx}{3bd\left(a+b(c+dx)^3\right)} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{2/3}b^{4/3}d} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9a^{2/3}b^{4/3}d} - \frac{\log\left(a^{2/3}-\right.}{}$$

Mathematica [A] time = 0.07, size = 151, normalized size = 0.89

$$\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{a^{2/3}} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{a^{2/3}} + \frac{2\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{b}\,(c+dx)-\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{a^{2/3}} - \frac{6\sqrt[3]{b}\,(c+dx)}{a+b(c+dx)^3}}{18b^{4/3}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*(c + d*x)^3)^2,x]
```

```
[Out] ((-6*b^(1/3)*(c + d*x))/(a + b*(c + d*x)^3) + (2*Sqrt[3]*ArcTan[(-a^(1/3) +
 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/a^(2/3) + (2*Log[a^(1/3) + b^(1/3
)*(c + d*x)])/a^(2/3) - Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(
c + d*x)^2]/a^(2/3))/(18*b^(4/3)*d)
```

fricas **[B]** time = 0.71, size = 810, normalized size = 4.76

$$\left[-\frac{6\,a^2bdx + 6\,a^2bc - 3\sqrt{\frac{1}{3}}\left(ab^2d^3x^3 + 3\,ab^2cd^2x^2 + 3\,ab^2c^2dx + ab^2c^3 + a^2b\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\,\log\left(\frac{2\,abd^3x^3+6\,abcd^2x^2+6\,abc^2}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] [-1/18*(6*a^2*b*d*x + 6*a^2*b*c - 3*sqrt(1/3)*(a*b^2*d^3*x^3 + 3*a*b^2*c*d^
2*x^2 + 3*a*b^2*c^2*d*x + a*b^2*c^3 + a^2*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*
a*b*d^3*x^3 + 6*a*b*c*d^2*x^2 + 6*a*b*c^2*d*x + 2*a*b*c^3 - a^2 + 3*sqrt(1/
3)*(2*a*b*d^2*x^2 + 4*a*b*c*d*x + 2*a*b*c^2 + (a^2*b)^(2/3)*(d*x + c) - (a^
2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*(a*d*x + a*c))/(b*d^
3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)) + (b*d^3*x^3 + 3*b*c*d^2*
x^2 + 3*b*c^2*d*x + b*c^3 + a)*(a^2*b)^(2/3)*log(a*b*d^2*x^2 + 2*a*b*c*d*x
+ a*b*c^2 - (a^2*b)^(2/3)*(d*x + c) + (a^2*b)^(1/3)*a) - 2*(b*d^3*x^3 + 3*b
*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*(a^2*b)^(2/3)*log(a*b*d*x + a*b*c + (
a^2*b)^(2/3)))/(a^2*b^3*d^4*x^3 + 3*a^2*b^3*c*d^3*x^2 + 3*a^2*b^3*c^2*d^2*x
 + (a^2*b^3*c^3 + a^3*b^2)*d), -1/18*(6*a^2*b*d*x + 6*a^2*b*c - 6*sqrt(1/3)
*(a*b^2*d^3*x^3 + 3*a*b^2*c*d^2*x^2 + 3*a*b^2*c^2*d*x + a*b^2*c^3 + a^2*b)*
sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*(d*x + c) - (a^2*b)
^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2
*d*x + b*c^3 + a)*(a^2*b)^(2/3)*log(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2 - (
a^2*b)^(2/3)*(d*x + c) + (a^2*b)^(1/3)*a) - 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 +
3*b*c^2*d*x + b*c^3 + a)*(a^2*b)^(2/3)*log(a*b*d*x + a*b*c + (a^2*b)^(2/3))
)/(a^2*b^3*d^4*x^3 + 3*a^2*b^3*c*d^3*x^2 + 3*a^2*b^3*c^2*d^2*x + (a^2*b^3*c
^3 + a^3*b^2)*d)]
```

giac **[A]** time = 0.25, size = 214, normalized size = 1.26

$$\frac{2\sqrt{3}\left(\frac{1}{a^2bd^3}\right)^{\frac{1}{3}}\arctan\left(-\frac{bdx+bc+\left(ab^2\right)^{\frac{1}{3}}}{\sqrt{3}\,bdx+\sqrt{3}\,bc-\sqrt{3}\left(ab^2\right)^{\frac{1}{3}}}\right)-\left(\frac{1}{a^2bd^3}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}\,bdx+\sqrt{3}\,bc-\sqrt{3}\,\left(ab^2\right)^{\frac{1}{3}}\right)^2+4\left(bdx+bc+\right.\right.}{18\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/18*(2*sqrt(3)*(1/(a^2*b*d^3))^(1/3)*arctan(-(b*d*x + b*c + (a*b^2)^(1/3))
/(sqrt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))) - (1/(a^2*b*d^3))^(
1/3)*log(4*(sqrt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))^2 + 4*(b*d
*x + b*c + (a*b^2)^(1/3))^2) + 2*(1/(a^2*b*d^3))^(1/3)*log(abs(b*d*x + b*c
```

```
+ (a*b^2)^(1/3))))/b - 1/3*(d*x + c)/((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*
d*x + b*c^3 + a)*b*d)
```

maple [C] time = 0.01, size = 124, normalized size = 0.73

$$\frac{\ln\left(-\operatorname{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_Z+b\,c^3+a\right)+x\right)}{9b^2d\left(d^2\operatorname{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_Z+b\,c^3+a\right)^2+2cd\operatorname{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+b*(d*x+c)^3)^2,x)
```

```
[Out] (-1/3/b*x-1/3*c/d/b)/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)+1/9/b^2/
d*sum(1/(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d
^2+3*_Z*b*c^2*d+b*c^3+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{dx+c}{3\left(b^2d^4x^3+3\,b^2cd^3x^2+3\,b^2c^2d^2x+\left(b^2c^3+ab\right)d\right)}+\frac{\frac{\sqrt{3}\left(\frac{1}{a^2b}\right)^{\frac{1}{3}}\arctan\left(-\frac{bdx+bc+\left(ab^2\right)^{\frac{1}{3}}}{\sqrt{3}\,bdx+\sqrt{3}\,bc-\sqrt{3}\left(ab^2\right)^{\frac{1}{3}}}\right)}{3d}-\frac{\left(\frac{1}{a^2b}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}\,bdx+\sqrt{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(d*x + c)/(b^2*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d^2*x + (b^2*c^3
+ a*b)*d) + 1/3*integrate(1/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^
3 + a), x)/b
```

mupad [B] time = 0.29, size = 185, normalized size = 1.09

$$\frac{\ln\left(b^{1/3}\,c+a^{1/3}+b^{1/3}\,d\,x\right)}{9\,a^{2/3}\,b^{4/3}\,d}-\frac{\frac{x}{3\,b}+\frac{c}{3\,b\,d}}{b\,c^3+3\,b\,c^2\,d\,x+3\,b\,c\,d^2\,x^2+b\,d^3\,x^3+a}+\frac{\ln\left(b\,c\,d^5+b\,d^6\,x+\frac{a^{1/3}\,b^{2/3}\,d^5\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)}{18\,a^{2/3}\,b^{4/3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*(c + d*x)^3)^2,x)
```

```
[Out] log(b^(1/3)*c + a^(1/3) + b^(1/3)*d*x)/(9*a^(2/3)*b^(4/3)*d) - (x/(3*b) + c
/(3*b*d))/(a + b*c^3 + b*d^3*x^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2) + (log(b*c*
d^5 + b*d^6*x + (a^(1/3)*b^(2/3)*d^5*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))
/(18*a^(2/3)*b^(4/3)*d) - (log(b*c*d^5 + b*d^6*x - (a^(1/3)*b^(2/3)*d^5*(3^
(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(18*a^(2/3)*b^(4/3)*d)
```

sympy [A] time = 1.03, size = 92, normalized size = 0.54

$$\frac{-c-dx}{3abd+3b^2c^3d+9b^2c^2d^2x+9b^2cd^3x^2+3b^2d^4x^3}+\frac{\operatorname{RootSum}\left(729t^3a^2b^4-1,\left(t\mapsto t\log\left(x+\frac{9tab+c}{d}\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+b*(d*x+c)**3)**2,x)
```

```
[Out] (-c - d*x)/(3*a*b*d + 3*b**2*c**3*d + 9*b**2*c**2*d**2*x + 9*b**2*c*d**3*x*
*2 + 3*b**2*d**4*x**3) + RootSum(729*_t**3*a**2*b**4 - 1, Lambda(_t, _t*log
(x + (9*_t*a*b + c)/d)))/d
```

3.2870 $\int \frac{(c+dx)^2}{\left(a+b(c+dx)^3\right)^2}\,dx$

Optimal. Leaf size=23

$$-\frac{1}{3bd\left(a+b(c+dx)^3\right)}$$

```
[Out] -1/3/b/d/(a+b*(d*x+c)^3)
```

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {372, 261}

$$-\frac{1}{3bd\left(a+b(c+dx)^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*(c + d*x)^3)^2,x]
```

```
[Out] -1/(3*b*d*(a + b*(c + d*x)^3))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^2}{\left(a+b(c+dx)^3\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{\left(a+bx^3\right)^2}\,dx, x, c+dx\right)}{d}\\ &= -\frac{1}{3bd\left(a+b(c+dx)^3\right)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$-\frac{1}{3bd\left(a+b(c+dx)^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*(c + d*x)^3)^2,x]
```

```
[Out] -1/3*1/(b*d*(a + b*(c + d*x)^3))
```

fricas [B] time = 0.55, size = 52, normalized size = 2.26

$$-\frac{1}{3\left(b^2d^4x^3+3\,b^2cd^3x^2+3\,b^2c^2d^2x+\left(b^2c^3+ab\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*(d*x+c)^3)^2,x, algorithm="fricas")
[Out] -1/3/(b^2*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d^2*x + (b^2*c^3 + a*b)*d)
```

giac [A] time = 0.19, size = 21, normalized size = 0.91

$$-\frac{1}{3\left((dx+c)^3 b+a\right)bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*(d*x+c)^3)^2,x, algorithm="giac")
[Out] -1/3/(((d*x + c)^3*b + a)*b*d)
```

maple [B] time = 0.00, size = 44, normalized size = 1.91

$$-\frac{1}{3\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+b*(d*x+c)^3)^2,x)
[Out] -1/3/b/d/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)
```

maxima [A] time = 0.63, size = 21, normalized size = 0.91

$$-\frac{1}{3\left((dx+c)^3 b+a\right)bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*(d*x+c)^3)^2,x, algorithm="maxima")
[Out] -1/3/(((d*x + c)^3*b + a)*b*d)
```

mupad [B] time = 1.21, size = 43, normalized size = 1.87

$$-\frac{1}{3\,b\,d\,\left(b\,c^3+3\,b\,c^2\,d\,x+3\,b\,c\,d^2\,x^2+b\,d^3\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*(c + d*x)^3)^2,x)
[Out] -1/(3*b*d*(a + b*c^3 + b*d^3*x^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2))
```

sympy [B] time = 0.96, size = 58, normalized size = 2.52

$$-\frac{1}{3abd+3b^2c^3d+9b^2c^2d^2x+9b^2cd^3x^2+3b^2d^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+b*(d*x+c)**3)**2,x)
[Out] -1/(3*a*b*d + 3*b**2*c**3*d + 9*b**2*c**2*d**2*x + 9*b**2*c*d**3*x**2 + 3*b
**2*d**4*x**3)
```

3.2871 $\int \frac{c+dx}{\left(a+b(c+dx)^3\right)^2}\,dx$

Optimal. Leaf size=172

$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9a^{4/3}b^{2/3}d}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{18a^{4/3}b^{2/3}d}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}b^{2/3}d}+\frac{(c+dx)^2}{3ad\left(a+b(c+dx\right.}$$

```
[Out] 1/3*(d*x+c)^2/a/d/(a+b*(d*x+c)^3)-1/9*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(4/3)/b
^(2/3)/d+1/18*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(4/3)
/b^(2/3)/d-1/9*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(4
/3)/b^(2/3)/d*3^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {372, 290, 292, 31, 634, 617, 204, 628}

$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9a^{4/3}b^{2/3}d}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{18a^{4/3}b^{2/3}d}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}b^{2/3}d}+\frac{(c+dx)^2}{3ad\left(a+b(c+dx\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*(c + d*x)^3)^2,x]
```

```
[Out] (c + d*x)^2/(3*a*d*(a + b*(c + d*x)^3)) - ArcTan[(a^(1/3) - 2*b^(1/3)*(c +
d*x))/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(4/3)*b^(2/3)*d) - Log[a^(1/3) + b^(1
/3)*(c + d*x)]/(9*a^(4/3)*b^(2/3)*d) + Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d
*x) + b^(2/3)*(c + d*x)^2]/(18*a^(4/3)*b^(2/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{c+dx}{\left(a+b(c+dx)^3\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x}{\left(a+bx^3\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{(c+dx)^2}{3ad\left(a+b(c+dx)^3\right)} + \frac{\text{Subst}\left(\int \frac{x}{a+bx^3}\,dx, x, c+dx\right)}{3ad}$$

$$= \frac{(c+dx)^2}{3ad\left(a+b(c+dx)^3\right)} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, c+dx\right)}{9a^{4/3}\sqrt[3]{b}\,d} + \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x}\right.}{9a^{4/3}\sqrt[3]{b}\,d}$$

$$= \frac{(c+dx)^2}{3ad\left(a+b(c+dx)^3\right)} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9a^{4/3}b^{2/3}d} + \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x,\right.}{18a^{4/3}b^{2/3}d}$$

$$= \frac{(c+dx)^2}{3ad\left(a+b(c+dx)^3\right)} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9a^{4/3}b^{2/3}d} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}\right.}{18a^{4/3}b^{2/3}d}$$

$$= \frac{(c+dx)^2}{3ad\left(a+b(c+dx)^3\right)} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{4/3}b^{2/3}d} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9a^{4/3}b^{2/3}d} + \frac{\log\left(a^{2/3}-\right.}{}$$

Mathematica [A] time = 0.09, size = 152, normalized size = 0.88

$$\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{b^{2/3}} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{b^{2/3}} + \frac{2\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{b}\,(c+dx)-\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{b^{2/3}} + \frac{6\sqrt[3]{a}\,(c+dx)^2}{a+b(c+dx)^3}}{18a^{4/3}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*(c + d*x)^3)^2,x]
```

```
[Out] ((6*a^(1/3)*(c + d*x)^2)/(a + b*(c + d*x)^3) + (2*Sqrt[3]*ArcTan[(-a^(1/3)
+ 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/b^(2/3) - (2*Log[a^(1/3) + b^(1/
3)*(c + d*x)])/b^(2/3) + Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*
(c + d*x)^2]/b^(2/3))/(18*a^(4/3)*d)
```

fricas **[B]** time = 0.84, size = 852, normalized size = 4.95

$$\left[\frac{6\,ab^2d^2x^2+12\,ab^2cdx+6\,ab^2c^2+3\sqrt{\frac{1}{3}}\left(ab^2d^3x^3+3\,ab^2cd^2x^2+3\,ab^2c^2dx+ab^2c^3+a^2b\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\,\log\left(\frac{2\,b^2d^3x^3}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] [1/18*(6*a*b^2*d^2*x^2 + 12*a*b^2*c*d*x + 6*a*b^2*c^2 + 3*sqrt(1/3)*(a*b^2*
d^3*x^3 + 3*a*b^2*c*d^2*x^2 + 3*a*b^2*c^2*d*x + a*b^2*c^3 + a^2*b)*sqrt((-a
*b^2)^(1/3)/a)*log((2*b^2*d^3*x^3 + 6*b^2*c*d^2*x^2 + 6*b^2*c^2*d*x + 2*b^2
*c^3 - a*b + 3*sqrt(1/3)*(a*b*d*x + a*b*c + 2*(d^2*x^2 + 2*c*d*x + c^2)*(-a
*b^2)^(2/3) + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*(
d*x + c))/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)) + (b*d^3*x
^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*(-a*b^2)^(2/3)*log(b^2*d^2*x^
2 + 2*b^2*c*d*x + b^2*c^2 + (-a*b^2)^(1/3)*(b*d*x + b*c) + (-a*b^2)^(2/3))
- 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*(-a*b^2)^(2/3)*lo
g(b*d*x + b*c - (-a*b^2)^(1/3)))/(a^2*b^3*d^4*x^3 + 3*a^2*b^3*c*d^3*x^2 + 3
*a^2*b^3*c^2*d^2*x + (a^2*b^3*c^3 + a^3*b^2)*d), 1/18*(6*a*b^2*d^2*x^2 + 12
*a*b^2*c*d*x + 6*a*b^2*c^2 + 6*sqrt(1/3)*(a*b^2*d^3*x^3 + 3*a*b^2*c*d^2*x^2
 + 3*a*b^2*c^2*d*x + a*b^2*c^3 + a^2*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt
(1/3)*(2*b*d*x + 2*b*c + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (b*d^
3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*(-a*b^2)^(2/3)*log(b^2*d^2
*x^2 + 2*b^2*c*d*x + b^2*c^2 + (-a*b^2)^(1/3)*(b*d*x + b*c) + (-a*b^2)^(2/3
)) - 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*(-a*b^2)^(2/3)
*log(b*d*x + b*c - (-a*b^2)^(1/3)))/(a^2*b^3*d^4*x^3 + 3*a^2*b^3*c*d^3*x^2
+ 3*a^2*b^3*c^2*d^2*x + (a^2*b^3*c^3 + a^3*b^2)*d)]
```

giac **[A]** time = 0.29, size = 205, normalized size = 1.19

$$-\frac{2\sqrt{3}\left(-\frac{1}{ab^2d^3}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)}{3\left(-a^2b\right)^{\frac{2}{3}}}\right)+\left(-\frac{1}{ab^2d^3}\right)^{\frac{1}{3}}\log\left(\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)^2+3\left(-a^2b\right)^{\frac{4}{3}}\right)-2}{18\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -1/18*(2*sqrt(3)*(-1/(a*b^2*d^3))^(1/3)*arctan(1/3*sqrt(3)*(2*a*b*d*x + 2*a
*b*c - (-a^2*b)^(2/3))/(-a^2*b)^(2/3)) + (-1/(a*b^2*d^3))^(1/3)*log((2*a*b*
d*x + 2*a*b*c - (-a^2*b)^(2/3))^2 + 3*(-a^2*b)^(4/3)) - 2*(-1/(a*b^2*d^3))^
```

```
(1/3)*log(abs(a*b*d*x + a*b*c + (-a^2*b)^(2/3))))/a + 1/3*(d^2*x^2 + 2*c*d*
x + c^2)/((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*a*d)
```

maple [C] time = 0.01, size = 144, normalized size = 0.84

$$\frac{\left(\text{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_Z+b\,c^3+a\right)d+c\right)\ln\left(-\text{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_\right.\right.}{9abd\left(d^2\,\text{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_Z+b\,c^3+a\right)^2+2cd\,\text{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+b*(d*x+c)^3)^2,x)
```

```
[Out] (1/3*d/a*x^2+2/3*c/a*x+1/3*c^2/d/a)/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*
c^3+a)+1/9/a/b/d*sum((_R*d+c)/(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x),_R=RootOf(_
Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{d^2x^2+2\,cdx+c^2}{3\left(abd^4x^3+3\,abcd^3x^2+3\,abc^2d^2x+\left(abc^3+a^2\right)d\right)}+\frac{-\frac{1}{3}\sqrt{3}\left(-\frac{1}{ab^2d^3}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)}{3\left(-a^2b\right)^{\frac{2}{3}}}\right)-\frac{1}{6}\left(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(d^2*x^2 + 2*c*d*x + c^2)/(a*b*d^4*x^3 + 3*a*b*c*d^3*x^2 + 3*a*b*c^2*d^
2*x + (a*b*c^3 + a^2)*d) + 1/3*integrate((d*x + c)/(b*d^3*x^3 + 3*b*c*d^2*x
^2 + 3*b*c^2*d*x + b*c^3 + a), x)/a
```

mupad [B] time = 1.42, size = 216, normalized size = 1.26

$$\frac{\frac{d\,x^2}{3\,a}+\frac{c^2}{3\,a\,d}+\frac{2\,c\,x}{3\,a}}{b\,c^3+3\,b\,c^2\,d\,x+3\,b\,c\,d^2\,x^2+b\,d^3\,x^3+a}-\frac{\ln\left(b^{1/3}\,c+a^{1/3}+b^{1/3}\,d\,x\right)}{9\,a^{4/3}\,b^{2/3}\,d}-\frac{\ln\left(\frac{b^{2/3}\,d^4\left(-1+\sqrt{3}\,1\text{i}\right)^2}{36\,a^{5/3}}+\frac{b\,c\,d^4}{9\,a^2}+\frac{b\,d^5\,x}{9\,a^2}\right)\left(-1\right.}{18\,a^{4/3}\,b^{2/3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*(c + d*x)^3)^2,x)
```

```
[Out] ((d*x^2)/(3*a) + c^2/(3*a*d) + (2*c*x)/(3*a))/(a + b*c^3 + b*d^3*x^3 + 3*b*
c^2*d*x + 3*b*c*d^2*x^2) - log(b^(1/3)*c + a^(1/3) + b^(1/3)*d*x)/(9*a^(4/3
)*b^(2/3)*d) - (log((b^(2/3)*d^4*(3^(1/2)*1i - 1)^2)/(36*a^(5/3)) + (b*c*d^
4)/(9*a^2) + (b*d^5*x)/(9*a^2))*(3^(1/2)*1i - 1))/(18*a^(4/3)*b^(2/3)*d) +
(log((b^(2/3)*d^4*(3^(1/2)*1i + 1)^2)/(36*a^(5/3)) + (b*c*d^4)/(9*a^2) + (b
*d^5*x)/(9*a^2))*(3^(1/2)*1i + 1))/(18*a^(4/3)*b^(2/3)*d)
```

sympy [A] time = 1.00, size = 105, normalized size = 0.61

$$\frac{c^2+2cdx+d^2x^2}{3a^2d+3abc^3d+9abc^2d^2x+9abcd^3x^2+3abd^4x^3}+\frac{\text{RootSum}\left(729t^3a^4b^2+1,\left(t\mapsto t\log\left(x+\frac{81t^2a^3b+c}{d}\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*(d*x+c)**3)**2,x)
```

```
[Out] (c**2 + 2*c*d*x + d**2*x**2)/(3*a**2*d + 3*a*b*c**3*d + 9*a*b*c**2*d**2*x +
 9*a*b*c*d**3*x**2 + 3*a*b*d**4*x**3) + RootSum(729*_t**3*a**4*b**2 + 1, La
mbda(_t, _t*log(x + (81*_t**2*a**3*b + c)/d)))/d
```

3.2872 $\int \frac{1}{\left(a+b(c+dx)^3\right)^2}\,dx$

Optimal. Leaf size=170

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{9a^{5/3}\sqrt[3]{b}\,d}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}\,d}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}\sqrt[3]{b}\,d}+\frac{c+dx}{3ad\left(a+b(c+\right.}$$

```
[Out] 1/3*(d*x+c)/a/d/(a+b*(d*x+c)^3)+2/9*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(5/3)/b^(
1/3)/d-1/9*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(5/3)/b^
(1/3)/d-2/9*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(5/3)
/b^(1/3)/d*3^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {247, 199, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{9a^{5/3}\sqrt[3]{b}\,d}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}\,d}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}\sqrt[3]{b}\,d}+\frac{c+dx}{3ad\left(a+b(c+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c + d*x)^3)^(-2),x]
```

```
[Out] (c + d*x)/(3*a*d*(a + b*(c + d*x)^3)) - (2*ArcTan[(a^(1/3) - 2*b^(1/3)*(c +
 d*x))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(1/3)*d) + (2*Log[a^(1/3) +
 b^(1/3)*(c + d*x)])/(9*a^(5/3)*b^(1/3)*d) - Log[a^(2/3) - a^(1/3)*b^(1/3)*
(c + d*x) + b^(2/3)*(c + d*x)^2]/(9*a^(5/3)*b^(1/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(a+b(c+dx)^3\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+bx^3\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{c+dx}{3ad\left(a+b(c+dx)^3\right)} + \frac{2\,\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, c+dx\right)}{3ad}$$

$$= \frac{c+dx}{3ad\left(a+b(c+dx)^3\right)} + \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, c+dx\right)}{9a^{5/3}d} + \frac{2\,\text{Subst}\left(\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+}\right.}{9a^{5/3}}$$

$$= \frac{c+dx}{3ad\left(a+b(c+dx)^3\right)} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}\,d} + \frac{\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x\right.}{3a^{4/3}d}$$

$$= \frac{c+dx}{3ad\left(a+b(c+dx)^3\right)} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}\,d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/}\right.}{9a^{5/3}\sqrt[3]{b}\,d}$$

$$= \frac{c+dx}{3ad\left(a+b(c+dx)^3\right)} - \frac{2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{5/3}\sqrt[3]{b}\,d} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}\,d} - \frac{\log\left(a^{2/3}\right.}{}$$

Mathematica [A] time = 0.06, size = 151, normalized size = 0.89

$$\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{\sqrt[3]{b}} + \frac{3a^{2/3}(c+dx)}{a+b(c+dx)^3} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{\sqrt[3]{b}} + \frac{2\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{b}\,(c+dx)-\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{9a^{5/3}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c + d*x)^3)^(-2),x]
```

```
[Out] ((3*a^(2/3)*(c + d*x))/(a + b*(c + d*x)^3) + (2*Sqrt[3]*ArcTan[(-a^(1/3) +
2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/b^(1/3) + (2*Log[a^(1/3) + b^(1/3)
*(c + d*x)])/b^(1/3) - Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c
 + d*x)^2]/b^(1/3))/(9*a^(5/3)*d)
```

fricas **[B]** time = 0.98, size = 808, normalized size = 4.75

$$\left[\frac{3\,a^2bdx + 3\,a^2bc + 3\sqrt{\frac{1}{3}}\left(ab^2d^3x^3 + 3\,ab^2cd^2x^2 + 3\,ab^2c^2dx + ab^2c^3 + a^2b\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\,\log\left(\frac{2\,abd^3x^3+6\,abcd^2x^2+6\,abc^2d}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] [1/9*(3*a^2*b*d*x + 3*a^2*b*c + 3*sqrt(1/3)*(a*b^2*d^3*x^3 + 3*a*b^2*c*d^2*
x^2 + 3*a*b^2*c^2*d*x + a*b^2*c^3 + a^2*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*
b*d^3*x^3 + 6*a*b*c*d^2*x^2 + 6*a*b*c^2*d*x + 2*a*b*c^3 - a^2 + 3*sqrt(1/3)
*(2*a*b*d^2*x^2 + 4*a*b*c*d*x + 2*a*b*c^2 + (a^2*b)^(2/3)*(d*x + c) - (a^2*
b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*(a*d*x + a*c))/(b*d^3*
x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)) - (b*d^3*x^3 + 3*b*c*d^2*x^
2 + 3*b*c^2*d*x + b*c^3 + a)*(a^2*b)^(2/3)*log(a*b*d^2*x^2 + 2*a*b*c*d*x +
a*b*c^2 - (a^2*b)^(2/3)*(d*x + c) + (a^2*b)^(1/3)*a) + 2*(b*d^3*x^3 + 3*b*c
*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*(a^2*b)^(2/3)*log(a*b*d*x + a*b*c + (a^
2*b)^(2/3)))/(a^3*b^2*d^4*x^3 + 3*a^3*b^2*c*d^3*x^2 + 3*a^3*b^2*c^2*d^2*x +
 (a^3*b^2*c^3 + a^4*b)*d), 1/9*(3*a^2*b*d*x + 3*a^2*b*c + 6*sqrt(1/3)*(a*b^
2*d^3*x^3 + 3*a*b^2*c*d^2*x^2 + 3*a*b^2*c^2*d*x + a*b^2*c^3 + a^2*b)*sqrt((
a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*(d*x + c) - (a^2*b)^(1/3)
*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x +
 b*c^3 + a)*(a^2*b)^(2/3)*log(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2 - (a^2*b)
^(2/3)*(d*x + c) + (a^2*b)^(1/3)*a) + 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^
2*d*x + b*c^3 + a)*(a^2*b)^(2/3)*log(a*b*d*x + a*b*c + (a^2*b)^(2/3)))/(a^3
*b^2*d^4*x^3 + 3*a^3*b^2*c*d^3*x^2 + 3*a^3*b^2*c^2*d^2*x + (a^3*b^2*c^3 + a
^4*b)*d)]
```

giac **[A]** time = 0.23, size = 214, normalized size = 1.26

$$\frac{2\sqrt{3}\left(\frac{1}{a^2bd^3}\right)^{\frac{1}{3}}\arctan\left(-\frac{bdx+bc+\left(ab^2\right)^{\frac{1}{3}}}{\sqrt{3}\,bdx+\sqrt{3}\,bc-\sqrt{3}\left(ab^2\right)^{\frac{1}{3}}}\right)-\left(\frac{1}{a^2bd^3}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}\,bdx+\sqrt{3}\,bc-\sqrt{3}\,\left(ab^2\right)^{\frac{1}{3}}\right)^2+4\left(bdx+bc+\right.\right.}{9\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/9*(2*sqrt(3)*(1/(a^2*b*d^3))^(1/3)*arctan(-(b*d*x + b*c + (a*b^2)^(1/3))/
(sqrt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))) - (1/(a^2*b*d^3))^(1
/3)*log(4*(sqrt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))^2 + 4*(b*d*
x + b*c + (a*b^2)^(1/3))^2) + 2*(1/(a^2*b*d^3))^(1/3)*log(abs(b*d*x + b*c +
```

```
 (a*b^2)^(1/3))))/a + 1/3*(d*x + c)/((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d
*x + b*c^3 + a)*a*d)
```

maple [C] time = 0.01, size = 127, normalized size = 0.75

$$\frac{2\ln\left(-\operatorname{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_Z+b\,c^3+a\right)+x\right)}{9abd\left(d^2\operatorname{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_Z+b\,c^3+a\right)^2+2cd\operatorname{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*(d*x+c)^3)^2,x)
```

```
[Out] (1/3/a*x+1/3*c/d/a)/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)+2/9/a/b/d
*sum(1/(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^
2+3*_Z*b*c^2*d+b*c^3+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{dx+c}{3\left(abd^4x^3+3\,abcd^3x^2+3\,abc^2d^2x+\left(abc^3+a^2\right)d\right)}+\frac{2\left(\frac{\sqrt{3}\left(\frac{1}{a^2b}\right)^{\frac{1}{3}}\arctan\left(-\frac{bdx+bc+\left(ab^2\right)^{\frac{1}{3}}}{\sqrt{3}\,bdx+\sqrt{3}\,bc-\sqrt{3}\left(ab^2\right)^{\frac{1}{3}}}\right)}{3\,d}-\frac{\left(\frac{1}{a^2b}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}\,bdx+\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(d*x + c)/(a*b*d^4*x^3 + 3*a*b*c*d^3*x^2 + 3*a*b*c^2*d^2*x + (a*b*c^3 +
 a^2)*d) + 2/3*integrate(1/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3
 + a), x)/a
```

mupad [B] time = 1.46, size = 207, normalized size = 1.22

$$\frac{\frac{x}{3\,a}+\frac{c}{3\,a\,d}}{b\,c^3+3\,b\,c^2\,d\,x+3\,b\,c\,d^2\,x^2+b\,d^3\,x^3+a}+\frac{2\,\ln\left(b^{1/3}\,c+a^{1/3}+b^{1/3}\,d\,x\right)}{9\,a^{5/3}\,b^{1/3}\,d}+\frac{\ln\left(\frac{2\,b^2\,c\,d^5}{a}+\frac{2\,b^2\,d^6\,x}{a}+\frac{b^{5/3}\,d^5\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{a^{2/3}}\right.}{9\,a^{5/3}\,b^{1/3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(c + d*x)^3)^2,x)
```

```
[Out] (x/(3*a) + c/(3*a*d))/(a + b*c^3 + b*d^3*x^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2)
 + (2*log(b^(1/3)*c + a^(1/3) + b^(1/3)*d*x))/(9*a^(5/3)*b^(1/3)*d) + (log(
(2*b^2*c*d^5)/a + (2*b^2*d^6*x)/a + (b^(5/3)*d^5*(3^(1/2)*1i - 1))/a^(2/3))
*(3^(1/2)*1i - 1))/(9*a^(5/3)*b^(1/3)*d) - (log((2*b^2*c*d^5)/a + (2*b^2*d^
6*x)/a - (b^(5/3)*d^5*(3^(1/2)*1i + 1))/a^(2/3))*(3^(1/2)*1i + 1))/(9*a^(5/
3)*b^(1/3)*d)
```

sympy [A] time = 0.94, size = 92, normalized size = 0.54

$$\frac{c+dx}{3a^2d+3abc^3d+9abc^2d^2x+9abcd^3x^2+3abd^4x^3}+\frac{\operatorname{RootSum}\left(729t^3a^5b-8,\left(t\mapsto t\log\left(x+\frac{9ta^2+2c}{2d}\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)**3)**2,x)
```

```
[Out] (c + d*x)/(3*a**2*d + 3*a*b*c**3*d + 9*a*b*c**2*d**2*x + 9*a*b*c*d**3*x**2
 + 3*a*b*d**4*x**3) + RootSum(729*_t**3*a**5*b - 8, Lambda(_t, _t*log(x + (9
*_t*a**2 + 2*c)/(2*d))))/d
```

3.2873 $\int \frac{1}{(c+dx)\left(a+b(c+dx)^3\right)^2}\,dx$

Optimal. Leaf size=59

$$-\frac{\log\left(a+b(c+dx)^3\right)}{3a^2d}+\frac{\log(c+dx)}{a^2d}+\frac{1}{3ad\left(a+b(c+dx)^3\right)}$$

```
[Out] 1/3/a/d/(a+b*(d*x+c)^3)+ln(d*x+c)/a^2/d-1/3*ln(a+b*(d*x+c)^3)/a^2/d
```

Rubi [A] time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {372, 266, 44}

$$-\frac{\log\left(a+b(c+dx)^3\right)}{3a^2d}+\frac{\log(c+dx)}{a^2d}+\frac{1}{3ad\left(a+b(c+dx)^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)*(a + b*(c + d*x)^3)^2),x]
```

```
[Out] 1/(3*a*d*(a + b*(c + d*x)^3)) + Log[c + d*x]/(a^2*d) - Log[a + b*(c + d*x)^
3]/(3*a^2*d)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(c+dx)\left(a+b(c+dx)^3\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx^3\right)^2}\,dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^2}\,dx, x, (c+dx)^3\right)}{3d} \\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right)\,dx, x, (c+dx)^3\right)}{3d} \\
&= \frac{1}{3ad\left(a+b(c+dx)^3\right)}+\frac{\log(c+dx)}{a^2d}-\frac{\log\left(a+b(c+dx)^3\right)}{3a^2d}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 48, normalized size = 0.81

$$\frac{\frac{a}{a+b(c+dx)^3} - \log\left(a + b(c+dx)^3\right) + 3\log(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)*(a + b*(c + d*x)^3)^2),x]
```

```
[Out] (a/(a + b*(c + d*x)^3) + 3*Log[c + d*x] - Log[a + b*(c + d*x)^3])/(3*a^2*d)
```

fricas [B] time = 0.81, size = 169, normalized size = 2.86

$$-\frac{\left(bd^3x^3 + 3\,bcd^2x^2 + 3\,bc^2dx + bc^3 + a\right)\log\left(bd^3x^3 + 3\,bcd^2x^2 + 3\,bc^2dx + bc^3 + a\right) - 3\left(bd^3x^3 + 3\,bcd^2x^2 + 3\right.}{3\left(a^2bd^4x^3 + 3\,a^2bcd^3x^2 + 3\,a^2bc^2d^2x + \left(a^2bc^3 + a^3\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] -1/3*((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*log(b*d^3*x^3 +
 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a) - 3*(b*d^3*x^3 + 3*b*c*d^2*x^2 +
3*b*c^2*d*x + b*c^3 + a)*log(d*x + c) - a)/(a^2*b*d^4*x^3 + 3*a^2*b*c*d^3*x
^2 + 3*a^2*b*c^2*d^2*x + (a^2*b*c^3 + a^3)*d)
```

giac [A] time = 0.19, size = 101, normalized size = 1.71

$$-\frac{\log\left(\left|bd^3x^3 + 3\,bcd^2x^2 + 3\,bc^2dx + bc^3 + a\right|\right)}{3\,a^2d} + \frac{\log\left(|dx+c|\right)}{a^2d} + \frac{1}{3\left(bd^3x^3 + 3\,bcd^2x^2 + 3\,bc^2dx + bc^3 + a\right)ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -1/3*log(abs(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a))/(a^2*d)
+ log(abs(d*x + c))/(a^2*d) + 1/3/((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x
 + b*c^3 + a)*a*d)
```

maple [A] time = 0.02, size = 100, normalized size = 1.69

$$\frac{1}{3\left(b\,d^3x^3 + 3bc\,d^2x^2 + 3b\,c^2dx + b\,c^3 + a\right)ad} + \frac{\ln\left(dx+c\right)}{a^2d} - \frac{\ln\left(b\,d^3x^3 + 3bc\,d^2x^2 + 3b\,c^2dx + b\,c^3 + a\right)}{3a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+b*(d*x+c)^3)^2,x)
```

```
[Out] ln(d*x+c)/a^2/d+1/3/a/d/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)-1/3/a
^2/d*ln(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)
```

maxima [A] time = 0.56, size = 104, normalized size = 1.76

$$\frac{1}{3\left(abd^4x^3 + 3\,abcd^3x^2 + 3\,abc^2d^2x + \left(abc^3 + a^2\right)d\right)} - \frac{\log\left(bd^3x^3 + 3\,bcd^2x^2 + 3\,bc^2dx + bc^3 + a\right)}{3\,a^2d} + \frac{\log\left(dx+c\right)}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 1/3/(a*b*d^4*x^3 + 3*a*b*c*d^3*x^2 + 3*a*b*c^2*d^2*x + (a*b*c^3 + a^2)*d) -
 1/3*log(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(a^2*d) + log
(d*x + c)/(a^2*d)
```

mupad [B] time = 0.22, size = 105, normalized size = 1.78

$$\frac{1}{3\left(a^2 d + b a c^3 d + 3 b a c^2 d^2 x + 3 b a c d^3 x^2 + b a d^4 x^3\right)} - \frac{\ln\left(b c^3 + 3 b c^2 d x + 3 b c d^2 x^2 + b d^3 x^3 + a\right)}{3 a^2 d} + \frac{\ln(c +}{a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*(c + d*x)^3)^2*(c + d*x)),x)
```

```
[Out] 1/(3*(a^2*d + a*b*c^3*d + a*b*d^4*x^3 + 3*a*b*c^2*d^2*x + 3*a*b*c*d^3*x^2))
 - log(a + b*c^3 + b*d^3*x^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2)/(3*a^2*d) + log
(c + d*x)/(a^2*d)
```

sympy [B] time = 1.48, size = 110, normalized size = 1.86

$$\frac{1}{3a^2d + 3abc^3d + 9abc^2d^2x + 9abcd^3x^2 + 3abd^4x^3} + \frac{\log\left(\frac{c}{d} + x\right)}{a^2d} - \frac{\log\left(\frac{3c^2x}{d^2} + \frac{3cx^2}{d} + x^3 + \frac{a+bc^3}{bd^3}\right)}{3a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*(d*x+c)**3)**2,x)
```

```
[Out] 1/(3*a**2*d + 3*a*b*c**3*d + 9*a*b*c**2*d**2*x + 9*a*b*c*d**3*x**2 + 3*a*b*
d**4*x**3) + log(c/d + x)/(a**2*d) - log(3*c**2*x/d**2 + 3*c*x**2/d + x**3
+ (a + b*c**3)/(b*d**3))/(3*a**2*d)
```

3.2874 $\int \frac{1}{(c+dx)^2\left(a+b(c+dx)^3\right)^2}\,dx$

Optimal. Leaf size=189

$$-\frac{2\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{9a^{7/3}d}+\frac{4\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9a^{7/3}d}+\frac{4\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}d}$$

```
[Out] -4/3/a^2/d/(d*x+c)+1/3/a/d/(d*x+c)/(a+b*(d*x+c)^3)+4/9*b^(1/3)*ln(a^(1/3)+b
^(1/3)*(d*x+c))/a^(7/3)/d-2/9*b^(1/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^
(2/3)*(d*x+c)^2)/a^(7/3)/d+4/9*b^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c
))/a^(1/3)*3^(1/2))/a^(7/3)/d*3^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {372, 290, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{2\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{9a^{7/3}d}+\frac{4\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9a^{7/3}d}+\frac{4\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)^2*(a + b*(c + d*x)^3)^2),x]
```

```
[Out] -4/(3*a^2*d*(c + d*x)) + 1/(3*a*d*(c + d*x)*(a + b*(c + d*x)^3)) + (4*b^(1/
3)*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^
(7/3)*d) + (4*b^(1/3)*Log[a^(1/3) + b^(1/3)*(c + d*x)])/(9*a^(7/3)*d) - (2*
b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(9*
a^(7/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2\left(a+b(c+dx)^3\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^3\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{1}{3ad(c+dx)\left(a+b(c+dx)^3\right)} + \frac{4\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^3\right)}\,dx, x, c+dx\right)}{3ad}$$

$$= -\frac{4}{3a^2d(c+dx)} + \frac{1}{3ad(c+dx)\left(a+b(c+dx)^3\right)} - \frac{(4b)\,\text{Subst}\left(\int \frac{x}{a+bx^3}\,dx, x,}{3a^2d}$$

$$= -\frac{4}{3a^2d(c+dx)} + \frac{1}{3ad(c+dx)\left(a+b(c+dx)^3\right)} + \frac{\left(4b^{2/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\right.}{9a^{7/3}d}$$

$$= -\frac{4}{3a^2d(c+dx)} + \frac{1}{3ad(c+dx)\left(a+b(c+dx)^3\right)} + \frac{4\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+\right.}{9a^{7/3}d}$$

$$= -\frac{4}{3a^2d(c+dx)} + \frac{1}{3ad(c+dx)\left(a+b(c+dx)^3\right)} + \frac{4\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+\right.}{9a^{7/3}d}$$

$$= -\frac{4}{3a^2d(c+dx)} + \frac{1}{3ad(c+dx)\left(a+b(c+dx)^3\right)} + \frac{4\sqrt[3]{b}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{7/3}d}$$

Mathematica [A] time = 0.09, size = 168, normalized size = 0.89

$$\frac{-2\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)-\frac{3\sqrt[3]{a}\,b(c+dx)^2}{a+b(c+dx)^3}+4\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)-4\sqrt{3}\,\sqrt[3]{b}\,\tan}{9a^{7/3}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)^2*(a + b*(c + d*x)^3)^2),x]
```

```
[Out] ((-9*a^(1/3))/(c + d*x) - (3*a^(1/3)*b*(c + d*x)^2)/(a + b*(c + d*x)^3) - 4
*Sqrt[3]*b^(1/3)*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))]
 + 4*b^(1/3)*Log[a^(1/3) + b^(1/3)*(c + d*x)] - 2*b^(1/3)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(9*a^(7/3)*d)
```

fricas [B] time = 0.82, size = 384, normalized size = 2.03

$$-\frac{12\,bd^3x^3+36\,bcd^2x^2+36\,bc^2dx+12\,bc^3+4\sqrt{3}\left(bd^4x^4+4\,bcd^3x^3+6\,bc^2d^2x^2+bc^4+\left(4\,bc^3+a\right)dx+ac\right)\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] -1/9*(12*b*d^3*x^3 + 36*b*c*d^2*x^2 + 36*b*c^2*d*x + 12*b*c^3 + 4*sqrt(3)*(
b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + b*c^4 + (4*b*c^3 + a)*d*x + a
*c)*(b/a)^(1/3)*arctan(2/3*sqrt(3)*(d*x + c)*(b/a)^(1/3) - 1/3*sqrt(3)) + 2
```

```
*(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + b*c^4 + (4*b*c^3 + a)*d*x +
 a*c)*(b/a)^(1/3)*log(b*d^2*x^2 + 2*b*c*d*x + b*c^2 - (a*d*x + a*c)*(b/a)^(
2/3) + a*(b/a)^(1/3)) - 4*(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + b*
c^4 + (4*b*c^3 + a)*d*x + a*c)*(b/a)^(1/3)*log(b*d*x + b*c + a*(b/a)^(2/3))
 + 9*a)/(a^2*b*d^5*x^4 + 4*a^2*b*c*d^4*x^3 + 6*a^2*b*c^2*d^3*x^2 + (4*a^2*b
*c^3 + a^3)*d^2*x + (a^2*b*c^4 + a^3*c)*d)
```

giac [A] time = 0.24, size = 204, normalized size = 1.08

$$\frac{4\left(\frac{b}{ad^3}\right)^{\frac{1}{3}}\log\left(\left|-\left(\frac{b}{ad^3}\right)^{\frac{1}{3}}-\frac{1}{(dx+c)d}\right|\right)}{9a^2}-\frac{4\sqrt{3}\left(a^2b\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(\frac{b}{ad^3}\right)^{\frac{1}{3}}-\frac{2}{(dx+c)d}\right)}{3\left(\frac{b}{ad^3}\right)^{\frac{1}{3}}}\right)}{9a^3d}-\frac{2\left(a^2b\right)^{\frac{1}{3}}\log\left(\left(\frac{b}{ad^3}\right)^{\frac{2}{3}}-\frac{\left(\frac{b}{ad^3}\right)^{\frac{1}{3}}}{(dx+c)d}+\frac{}{(dx}\right.}{9a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 4/9*(b/(a*d^3))^(1/3)*log(abs(-(b/(a*d^3))^(1/3) - 1/((d*x + c)*d)))/a^2 -
4/9*sqrt(3)*(a^2*b)^(1/3)*arctan(1/3*sqrt(3)*((b/(a*d^3))^(1/3) - 2/((d*x +
 c)*d))/(b/(a*d^3))^(1/3))/(a^3*d) - 2/9*(a^2*b)^(1/3)*log((b/(a*d^3))^(2/3
) - (b/(a*d^3))^(1/3)/((d*x + c)*d) + 1/((d*x + c)^2*d^2))/(a^3*d) - 1/((d*
x + c)*a^2*d) - 1/3*b/((d*x + c)*a^2*(b + a/(d*x + c)^3)*d)
```

maple [C] time = 0.02, size = 227, normalized size = 1.20

$$-\frac{bd\,x^2}{3\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)a^2}-\frac{2bcx}{3\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)a^2}-\frac{}{3\left(b\,d^3x^3+3bc\,d^2x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+b*(d*x+c)^3)^2,x)
```

```
[Out] -1/a^2/d/(d*x+c)-1/3/a^2*b/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)*x^
2*d-2/3/a^2*b/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)*c*x-1/3/a^2*b/(
b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)*c^2/d-4/9/a^2/d*sum((_R*d+c)/(
_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b
*c^2*d+b*c^3+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{4\,bd^3x^3+12\,bcd^2x^2+12\,bc^2dx+4\,bc^3+3\,a}{3\left(a^2bd^5x^4+4\,a^2bcd^4x^3+6\,a^2bc^2d^3x^2+\left(4\,a^2bc^3+a^3\right)d^2x+\left(a^2bc^4+a^3c\right)d\right)}-\frac{-\frac{2}{3}\left(2\sqrt{3}\left(-\frac{1}{ab^2d^3}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(4*b*d^3*x^3 + 12*b*c*d^2*x^2 + 12*b*c^2*d*x + 4*b*c^3 + 3*a)/(a^2*b*d
^5*x^4 + 4*a^2*b*c*d^4*x^3 + 6*a^2*b*c^2*d^3*x^2 + (4*a^2*b*c^3 + a^3)*d^2*
x + (a^2*b*c^4 + a^3*c)*d) - 4/3*b*integrate((d*x + c)/(b*d^3*x^3 + 3*b*c*d
^2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/a^2
```

mupad [B] time = 1.81, size = 231, normalized size = 1.22

$$\frac{4\,b^{1/3}\,\ln\left(b^{1/3}\,c+a^{1/3}+b^{1/3}\,d\,x\right)}{9\,a^{7/3}\,d}-\frac{\frac{4\,b\,c^3+3\,a}{3\,a^2\,d}+\frac{4\,b\,d^2\,x^3}{3\,a^2}+\frac{4\,b\,c^2\,x}{a^2}+\frac{4\,b\,c\,d\,x^2}{a^2}}{a\,c+x\left(4\,b\,d\,c^3+a\,d\right)+b\,c^4+b\,d^4\,x^4+6\,b\,c^2\,d^2\,x^2+4\,b\,c\,d^3\,x^3}-\frac{4\,b^{1/3}\,\ln\left(2\,b^{1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*(c + d*x)^3)^2*(c + d*x)^2),x)
```

```
[Out] (4*b^(1/3)*log(b^(1/3)*c + a^(1/3) + b^(1/3)*d*x))/(9*a^(7/3)*d) - ((3*a +
4*b*c^3)/(3*a^2*d) + (4*b*d^2*x^3)/(3*a^2) + (4*b*c^2*x)/a^2 + (4*b*c*d*x^2
)/a^2)/(a*c + x*(a*d + 4*b*c^3*d) + b*c^4 + b*d^4*x^4 + 6*b*c^2*d^2*x^2 + 4
*b*c*d^3*x^3) - (4*b^(1/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*c - a^(1/3) +
 2*b^(1/3)*d*x)*((3^(1/2)*1i)/2 + 1/2))/(9*a^(7/3)*d) + (b^(1/3)*log(2*b^(1
/3)*c - 3^(1/2)*a^(1/3)*1i - a^(1/3) + 2*b^(1/3)*d*x)*((3^(1/2)*2i)/9 - 2/9
))/(a^(7/3)*d)
```

sympy [A] time = 1.95, size = 170, normalized size = 0.90

$$\frac{-3a-4bc^3-12bc^2dx-12bcd^2x^2-4bd^3x^3}{3a^3cd+3a^2bc^4d+18a^2bc^2d^3x^2+12a^2bcd^4x^3+3a^2bd^5x^4+x\left(3a^3d^2+12a^2bc^3d^2\right)}+\frac{\text{RootSum}\left(729t^3a^7-64b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+b*(d*x+c)**3)**2,x)
```

```
[Out] (-3*a - 4*b*c**3 - 12*b*c**2*d*x - 12*b*c*d**2*x**2 - 4*b*d**3*x**3)/(3*a**
3*c*d + 3*a**2*b*c**4*d + 18*a**2*b*c**2*d**3*x**2 + 12*a**2*b*c*d**4*x**3
+ 3*a**2*b*d**5*x**4 + x*(3*a**3*d**2 + 12*a**2*b*c**3*d**2)) + RootSum(729
*_t**3*a**7 - 64*b, Lambda(_t, _t*log(x + (81*_t**2*a**5 + 16*b*c)/(16*b*d)
)))/d
```

3.2875 $\int \frac{1}{(c+dx)^3\left(a+b(c+dx)^3\right)^2}\,dx$

Optimal. Leaf size=189

$$-\frac{5b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9a^{8/3}d}+\frac{5b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{18a^{8/3}d}+\frac{5b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{8/3}d}-\frac{6}{}$$

```
[Out] -5/6/a^2/d/(d*x+c)^2+1/3/a/d/(d*x+c)^2/(a+b*(d*x+c)^3)-5/9*b^(2/3)*ln(a^(1/
3)+b^(1/3)*(d*x+c))/a^(8/3)/d+5/18*b^(2/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+
c)+b^(2/3)*(d*x+c)^2)/a^(8/3)/d+5/9*b^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*(
d*x+c))/a^(1/3)*3^(1/2))/a^(8/3)/d*3^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {372, 290, 325, 200, 31, 634, 617, 204, 628}

$$-\frac{5b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9a^{8/3}d}+\frac{5b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{18a^{8/3}d}+\frac{5b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{8/3}d}-\frac{6}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)^3*(a + b*(c + d*x)^3)^2),x]
```

```
[Out] -5/(6*a^2*d*(c + d*x)^2) + 1/(3*a*d*(c + d*x)^2*(a + b*(c + d*x)^3)) + (5*b
^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3
]*a^(8/3)*d) - (5*b^(2/3)*Log[a^(1/3) + b^(1/3)*(c + d*x)])/(9*a^(8/3)*d) +
 (5*b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])
/(18*a^(8/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(c+dx)^3\left(a+b(c+dx)^3\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3\left(a+bx^3\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{1}{3ad(c+dx)^2\left(a+b(c+dx)^3\right)} + \frac{5\,\text{Subst}\left(\int \frac{1}{x^3\left(a+bx^3\right)}\,dx, x, c+dx\right)}{3ad}$$

$$= -\frac{5}{6a^2d(c+dx)^2} + \frac{1}{3ad(c+dx)^2\left(a+b(c+dx)^3\right)} - \frac{(5b)\,\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, c\right.}{3a^2d}$$

$$= -\frac{5}{6a^2d(c+dx)^2} + \frac{1}{3ad(c+dx)^2\left(a+b(c+dx)^3\right)} - \frac{(5b)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx,\right.}{9a^{8/3}d}$$

$$= -\frac{5}{6a^2d(c+dx)^2} + \frac{1}{3ad(c+dx)^2\left(a+b(c+dx)^3\right)} - \frac{5b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+d\right.}{9a^{8/3}d}$$

$$= -\frac{5}{6a^2d(c+dx)^2} + \frac{1}{3ad(c+dx)^2\left(a+b(c+dx)^3\right)} - \frac{5b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+d\right.}{9a^{8/3}d}$$

$$= -\frac{5}{6a^2d(c+dx)^2} + \frac{1}{3ad(c+dx)^2\left(a+b(c+dx)^3\right)} + \frac{5b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{8/3}d} -$$

Mathematica [A] time = 0.09, size = 166, normalized size = 0.88

$$\frac{5b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)-\frac{6a^{2/3}b(c+dx)}{a+b(c+dx)^3}-\frac{9a^{2/3}}{(c+dx)^2}-10b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)-10\sqrt{3}\,b^2}{18a^{8/3}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)^3*(a + b*(c + d*x)^3)^2),x]
```

```
[Out] ((-9*a^(2/3))/(c + d*x)^2 - (6*a^(2/3)*b*(c + d*x))/(a + b*(c + d*x)^3) - 1
0*Sqrt[3]*b^(2/3)*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))
] - 10*b^(2/3)*Log[a^(1/3) + b^(1/3)*(c + d*x)] + 5*b^(2/3)*Log[a^(2/3) - a
^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(18*a^(8/3)*d)
```

fricas [B] time = 0.88, size = 509, normalized size = 2.69

$$-\frac{15\,bd^3x^3+45\,bcd^2x^2+45\,bc^2dx+15\,bc^3-10\sqrt{3}\left(bd^5x^5+5\,bcd^4x^4+10\,bc^2d^3x^3+bc^5+\left(10\,bc^3+a\right)d^2x^2+ac\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3/(a+b*(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] -1/18*(15*b*d^3*x^3 + 45*b*c*d^2*x^2 + 45*b*c^2*d*x + 15*b*c^3 - 10*sqrt(3)
*(b*d^5*x^5 + 5*b*c*d^4*x^4 + 10*b*c^2*d^3*x^3 + b*c^5 + (10*b*c^3 + a)*d^2
```

```
*x^2 + a*c^2 + (5*b*c^4 + 2*a*c)*d*x)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3
)*(a*d*x + a*c)*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) + 5*(b*d^5*x^5 + 5*b*c*d^4
*x^4 + 10*b*c^2*d^3*x^3 + b*c^5 + (10*b*c^3 + a)*d^2*x^2 + a*c^2 + (5*b*c^4
 + 2*a*c)*d*x)*(-b^2/a^2)^(1/3)*log(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + a
^2*(-b^2/a^2)^(2/3) + (a*b*d*x + a*b*c)*(-b^2/a^2)^(1/3)) - 10*(b*d^5*x^5 +
 5*b*c*d^4*x^4 + 10*b*c^2*d^3*x^3 + b*c^5 + (10*b*c^3 + a)*d^2*x^2 + a*c^2
+ (5*b*c^4 + 2*a*c)*d*x)*(-b^2/a^2)^(1/3)*log(b*d*x + b*c - a*(-b^2/a^2)^(1
/3)) + 9*a)/(a^2*b*d^6*x^5 + 5*a^2*b*c*d^5*x^4 + 10*a^2*b*c^2*d^4*x^3 + (10
*a^2*b*c^3 + a^3)*d^3*x^2 + (5*a^2*b*c^4 + 2*a^3*c)*d^2*x + (a^2*b*c^5 + a^
3*c^2)*d)
```

giac [A] time = 0.22, size = 244, normalized size = 1.29

$$\frac{5\left(2\sqrt{3}\left(-\frac{b^2}{a^2d^3}\right)^{\frac{1}{3}}\arctan\left(-\frac{bdx+bc-\left(-ab^2\right)^{\frac{1}{3}}}{\sqrt{3}\,bdx+\sqrt{3}\,bc+\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}}\right)-\left(-\frac{b^2}{a^2d^3}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}\,bdx+\sqrt{3}\,bc+\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\right)^2+4\left(b\right.\right.\right.}{18\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3/(a+b*(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 5/18*(2*sqrt(3)*(-b^2/(a^2*d^3))^(1/3)*arctan(-(b*d*x + b*c - (-a*b^2)^(1/3
))/(sqrt(3)*b*d*x + sqrt(3)*b*c + sqrt(3)*(-a*b^2)^(1/3))) - (-b^2/(a^2*d^3
))^(1/3)*log(4*(sqrt(3)*b*d*x + sqrt(3)*b*c + sqrt(3)*(-a*b^2)^(1/3))^2 + 4
*(b*d*x + b*c - (-a*b^2)^(1/3))^2) + 2*(-b^2/(a^2*d^3))^(1/3)*log(abs(-b*d*
x - b*c + (-a*b^2)^(1/3))))/a^2 - 1/3*(b*d*x + b*c)/((b*d^3*x^3 + 3*b*c*d^2
*x^2 + 3*b*c^2*d*x + b*c^3 + a)*a^2*d) - 1/2/((d*x + c)^2*a^2*d)
```

maple [C] time = 0.02, size = 174, normalized size = 0.92

$$-\frac{bx}{3\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)a^2}-\frac{bc}{3\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)a^2d}-\frac{}{9a^2d\left(d^2\,\mathrm{RootO}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^3/(a+b*(d*x+c)^3)^2,x)
```

```
[Out] -1/2/a^2/d/(d*x+c)^2-1/3/a^2*b/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a
)*x-1/3/a^2*b/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)*c/d-5/9/a^2/d*s
um(1/(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+
3*_Z*b*c^2*d+b*c^3+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{5}{6}$$

$$-\frac{5\,bd^3x^3+15\,bcd^2x^2+15\,bc^2dx+5\,bc^3+3\,a}{6\left(a^2bd^6x^5+5\,a^2bcd^5x^4+10\,a^2bc^2d^4x^3+\left(10\,a^2bc^3+a^3\right)d^3x^2+\left(5\,a^2bc^4+2\,a^3c\right)d^2x+\left(a^2bc^5+a^3c^2\right)d\right)}-\frac{}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3/(a+b*(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(5*b*d^3*x^3 + 15*b*c*d^2*x^2 + 15*b*c^2*d*x + 5*b*c^3 + 3*a)/(a^2*b*d
^6*x^5 + 5*a^2*b*c*d^5*x^4 + 10*a^2*b*c^2*d^4*x^3 + (10*a^2*b*c^3 + a^3)*d^
3*x^2 + (5*a^2*b*c^4 + 2*a^3*c)*d^2*x + (a^2*b*c^5 + a^3*c^2)*d) - 5/3*b*in
tegrate(1/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/a^2
```

mupad [B] time = 1.98, size = 255, normalized size = 1.35

$$-\frac{\frac{5bc^3+3a}{6a^2d}+\frac{5bd^2x^3}{6a^2}+\frac{5bc^2x}{2a^2}+\frac{5bcdx^2}{2a^2}}{x^2\left(10bc^3d^2+ad^2\right)+ac^2+bc^5+x\left(5bdc^4+2adc\right)+bd^5x^5+10bc^2d^3x^3+5bcd^4x^4}-\frac{5b^{2/3}\ln\left(b^{1/3}c+\right.}{9a^{8/}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*(c + d*x)^3)^2*(c + d*x)^3),x)
```

```
[Out] (5*b^(2/3)*log(2*b^(1/3)*c - 3^(1/2)*a^(1/3)*1i - a^(1/3) + 2*b^(1/3)*d*x)*
((3^(1/2)*1i)/2 + 1/2))/(9*a^(8/3)*d) - (5*b^(2/3)*log(b^(1/3)*c + a^(1/3)
+ b^(1/3)*d*x))/(9*a^(8/3)*d) - ((3*a + 5*b*c^3)/(6*a^2*d) + (5*b*d^2*x^3)/
(6*a^2) + (5*b*c^2*x)/(2*a^2) + (5*b*c*d*x^2)/(2*a^2))/(x^2*(a*d^2 + 10*b*c
^3*d^2) + a*c^2 + b*c^5 + x*(2*a*c*d + 5*b*c^4*d) + b*d^5*x^5 + 10*b*c^2*d^
3*x^3 + 5*b*c*d^4*x^4) - (5*b^(2/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*c -
a^(1/3) + 2*b^(1/3)*d*x)*((3^(1/2)*1i)/2 - 1/2))/(9*a^(8/3)*d)
```

sympy [A] time = 2.50, size = 199, normalized size = 1.05

$$\frac{-3a-5bc^3-15bc^2dx-15bcd^2x^2-5bd^3x^3}{6a^3c^2d+6a^2bc^5d+60a^2bc^2d^4x^3+30a^2bcd^5x^4+6a^2bd^6x^5+x^2\left(6a^3d^3+60a^2bc^3d^3\right)+x\left(12a^3cd^2+30a^2bc^4d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**3/(a+b*(d*x+c)**3)**2,x)
```

```
[Out] (-3*a - 5*b*c**3 - 15*b*c**2*d*x - 15*b*c*d**2*x**2 - 5*b*d**3*x**3)/(6*a**
3*c**2*d + 6*a**2*b*c**5*d + 60*a**2*b*c**2*d**4*x**3 + 30*a**2*b*c*d**5*x*
*4 + 6*a**2*b*d**6*x**5 + x**2*(6*a**3*d**3 + 60*a**2*b*c**3*d**3) + x*(12*
a**3*c*d**2 + 30*a**2*b*c**4*d**2)) + RootSum(729*_t**3*a**8 + 125*b**2, La
mbda(_t, _t*log(x + (-9*_t*a**3 + 5*b*c)/(5*b*d))))/d
```

3.2876 $\int \frac{1}{(c+dx)^4\left(a+b(c+dx)^3\right)^2}\,dx$

Optimal. Leaf size=80

$$-\frac{2b\log(c+dx)}{a^3d}+\frac{2b\log\left(a+b(c+dx)^3\right)}{3a^3d}-\frac{b}{3a^2d\left(a+b(c+dx)^3\right)}-\frac{1}{3a^2d(c+dx)^3}$$

```
[Out] -1/3/a^2/d/(d*x+c)^3-1/3*b/a^2/d/(a+b*(d*x+c)^3)-2*b*ln(d*x+c)/a^3/d+2/3*b*
ln(a+b*(d*x+c)^3)/a^3/d
```

Rubi [A] time = 0.07, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {372, 266, 44}

$$-\frac{b}{3a^2d\left(a+b(c+dx)^3\right)}-\frac{2b\log(c+dx)}{a^3d}+\frac{2b\log\left(a+b(c+dx)^3\right)}{3a^3d}-\frac{1}{3a^2d(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)^4*(a + b*(c + d*x)^3)^2),x]
```

```
[Out] -1/(3*a^2*d*(c + d*x)^3) - b/(3*a^2*d*(a + b*(c + d*x)^3)) - (2*b*Log[c + d
*x])/(a^3*d) + (2*b*Log[a + b*(c + d*x)^3])/(3*a^3*d)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^4\left(a+b(c+dx)^3\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^3\right)^2}\,dx, x, c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2}\,dx, x, (c+dx)^3\right)}{3d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x^2} - \frac{2b}{a^3x} + \frac{b^2}{a^2(a+bx)^2} + \frac{2b^2}{a^3(a+bx)}\right)\,dx, x, (c+dx)^3\right)}{3d}$$
$$= -\frac{1}{3a^2d(c+dx)^3} - \frac{b}{3a^2d\left(a+b(c+dx)^3\right)} - \frac{2b\log(c+dx)}{a^3d} + \frac{2b\log\left(a+b(c+$$

Mathematica [A] time = 0.08, size = 60, normalized size = 0.75

$$-\frac{a\left(\frac{b}{a+b(c+dx)^3} + \frac{1}{(c+dx)^3}\right) - 2b\log\left(a+b(c+dx)^3\right) + 6b\log(c+dx)}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)^4*(a + b*(c + d*x)^3)^2),x]
```

```
[Out] -1/3*(a*((c + d*x)^(-3) + b/(a + b*(c + d*x)^3)) + 6*b*Log[c + d*x] - 2*b*L
og[a + b*(c + d*x)^3])/(a^3*d)
```

fricas [B] time = 0.81, size = 431, normalized size = 5.39

$$-\frac{2\,abd^3x^3 + 6\,abcd^2x^2 + 6\,abc^2dx + 2\,abc^3 + a^2 - 2\left(b^2d^6x^6 + 6\,b^2cd^5x^5 + 15\,b^2c^2d^4x^4 + b^2c^6 + \left(20\,b^2c^3 + ab\right)d^3x\right.}{3\left(a^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^4/(a+b*(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(2*a*b*d^3*x^3 + 6*a*b*c*d^2*x^2 + 6*a*b*c^2*d*x + 2*a*b*c^3 + a^2 - 2
*(b^2*d^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4 + b^2*c^6 + (20*b^2*c^
3 + a*b)*d^3*x^3 + a*b*c^3 + 3*(5*b^2*c^4 + a*b*c)*d^2*x^2 + 3*(2*b^2*c^5 +
 a*b*c^2)*d*x)*log(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a) + 6
*(b^2*d^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4 + b^2*c^6 + (20*b^2*c^
3 + a*b)*d^3*x^3 + a*b*c^3 + 3*(5*b^2*c^4 + a*b*c)*d^2*x^2 + 3*(2*b^2*c^5 +
 a*b*c^2)*d*x)*log(d*x + c))/(a^3*b*d^7*x^6 + 6*a^3*b*c*d^6*x^5 + 15*a^3*b*
c^2*d^5*x^4 + (20*a^3*b*c^3 + a^4)*d^4*x^3 + 3*(5*a^3*b*c^4 + a^4*c)*d^3*x^
2 + 3*(2*a^3*b*c^5 + a^4*c^2)*d^2*x + (a^3*b*c^6 + a^4*c^3)*d)
```

giac [A] time = 0.22, size = 65, normalized size = 0.81

$$\frac{2\,b\log\left(\left|-b - \frac{a}{(dx+c)^3}\right|\right)}{3\,a^3d} + \frac{b^2}{3\,a^3\left(b + \frac{a}{(dx+c)^3}\right)d} - \frac{1}{3\,(dx+c)^3a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^4/(a+b*(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 2/3*b*log(abs(-b - a/(d*x + c)^3))/(a^3*d) + 1/3*b^2/(a^3*(b + a/(d*x + c)^
3)*d) - 1/3/((d*x + c)^3*a^2*d)
```

maple [A] time = 0.02, size = 119, normalized size = 1.49

$$-\frac{b}{3\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)a^2d}-\frac{2b\ln(dx+c)}{a^3d}+\frac{2b\ln\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)}{3a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^4/(a+b*(d*x+c)^3)^2,x)
```

```
[Out] -1/3/a^2/d/(d*x+c)^3-2*b*ln(d*x+c)/a^3/d-1/3/a^2*b/d/(b*d^3*x^3+3*b*c*d^2*x
^2+3*b*c^2*d*x+b*c^3+a)+2/3/a^3*b/d*ln(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+
b*c^3+a)
```

maxima [B] time = 0.68, size = 222, normalized size = 2.78

$$-\frac{2\,bd^3x^3+6\,bcd^2x^2+6\,bc^2dx+2\,bc^3+a}{3\left(a^2bd^7x^6+6\,a^2bcd^6x^5+15\,a^2bc^2d^5x^4+\left(20\,a^2bc^3+a^3\right)d^4x^3+3\left(5\,a^2bc^4+a^3c\right)d^3x^2+3\left(2\,a^2bc^5+a^3c^2\right)d^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^4/(a+b*(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(2*b*d^3*x^3 + 6*b*c*d^2*x^2 + 6*b*c^2*d*x + 2*b*c^3 + a)/(a^2*b*d^7*x
^6 + 6*a^2*b*c*d^6*x^5 + 15*a^2*b*c^2*d^5*x^4 + (20*a^2*b*c^3 + a^3)*d^4*x^
3 + 3*(5*a^2*b*c^4 + a^3*c)*d^3*x^2 + 3*(2*a^2*b*c^5 + a^3*c^2)*d^2*x + (a^
2*b*c^6 + a^3*c^3)*d) + 2/3*b*log(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x +
 b*c^3 + a)/(a^3*d) - 2*b*log(d*x + c)/(a^3*d)
```

mupad [B] time = 1.94, size = 211, normalized size = 2.64

$$\frac{2\,b\,\ln\left(b\,c^3+3\,b\,c^2\,d\,x+3\,b\,c\,d^2\,x^2+b\,d^3\,x^3+a\right)}{3\,a^3\,d}-\frac{\frac{2\,b\,c^3+a}{3\,a^2\,d}+\frac{2\,b\,d^2\,x^3}{3\,a^2}}{x\left(6\,b\,d\,c^5+3\,a\,d\,c^2\right)+x^3\left(20\,b\,c^3\,d^3+a\,d^3\right)+a\,c^3+b\,c^6+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*(c + d*x)^3)^2*(c + d*x)^4),x)
```

```
[Out] (2*b*log(a + b*c^3 + b*d^3*x^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2))/(3*a^3*d) -
((a + 2*b*c^3)/(3*a^2*d) + (2*b*d^2*x^3)/(3*a^2) + (2*b*c^2*x)/a^2 + (2*b*c
*d*x^2)/a^2)/(x*(3*a*c^2*d + 6*b*c^5*d) + x^3*(a*d^3 + 20*b*c^3*d^3) + a*c^
3 + b*c^6 + x^2*(15*b*c^4*d^2 + 3*a*c*d^2) + b*d^6*x^6 + 15*b*c^2*d^4*x^4 +
 6*b*c*d^5*x^5) - (2*b*log(c + d*x))/(a^3*d)
```

sympy [B] time = 3.58, size = 250, normalized size = 3.12

$$\frac{-a-2bc^3-6bc^2dx-6bcd^2x^2-2bd^3x^3}{3a^3c^3d+3a^2bc^6d+45a^2bc^2d^5x^4+18a^2bcd^6x^5+3a^2bd^7x^6+x^3\left(3a^3d^4+60a^2bc^3d^4\right)+x^2\left(9a^3cd^3+45a^2bc^4d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**4/(a+b*(d*x+c)**3)**2,x)
```

```
[Out] (-a - 2*b*c**3 - 6*b*c**2*d*x - 6*b*c*d**2*x**2 - 2*b*d**3*x**3)/(3*a**3*c*
*3*d + 3*a**2*b*c**6*d + 45*a**2*b*c**2*d**5*x**4 + 18*a**2*b*c*d**6*x**5 +
 3*a**2*b*d**7*x**6 + x**3*(3*a**3*d**4 + 60*a**2*b*c**3*d**4) + x**2*(9*a*
*3*c*d**3 + 45*a**2*b*c**4*d**3) + x*(9*a**3*c**2*d**2 + 18*a**2*b*c**5*d**
2)) - 2*b*log(c/d + x)/(a**3*d) + 2*b*log(3*c**2*x/d**2 + 3*c*x**2/d + x**3
 + (a + b*c**3)/(b*d**3))/(3*a**3*d)
```

3.2877 $\int \frac{(c+dx)^4}{\left(a+b(c+dx)^3\right)^3}\,dx$

Optimal. Leaf size=205

$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{27a^{4/3}b^{5/3}d}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{54a^{4/3}b^{5/3}d}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{4/3}b^{5/3}d}+\frac{(c+dx)^2}{9abd\left(a+b(c+d\right.}$$

```
[Out] -1/6*(d*x+c)^2/b/d/(a+b*(d*x+c)^3)^2+1/9*(d*x+c)^2/a/b/d/(a+b*(d*x+c)^3)-1/
27*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(4/3)/b^(5/3)/d+1/54*ln(a^(2/3)-a^(1/3)*b^
(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(4/3)/b^(5/3)/d-1/27*arctan(1/3*(a^(1/3)
-2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(4/3)/b^(5/3)/d*3^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {372, 288, 290, 292, 31, 634, 617, 204, 628}

$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{27a^{4/3}b^{5/3}d}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{54a^{4/3}b^{5/3}d}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{4/3}b^{5/3}d}+\frac{(c+dx)^2}{9abd\left(a+b(c+d\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4/(a + b*(c + d*x)^3)^3,x]
```

```
[Out] -(c + d*x)^2/(6*b*d*(a + b*(c + d*x)^3)^2) + (c + d*x)^2/(9*a*b*d*(a + b*(c
 + d*x)^3)) - ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))]/(9*
Sqrt[3]*a^(4/3)*b^(5/3)*d) - Log[a^(1/3) + b^(1/3)*(c + d*x)]/(27*a^(4/3)*b
^(5/3)*d) + Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/
(54*a^(4/3)*b^(5/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(c+dx)^4}{\left(a+b(c+dx)^3\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(a+bx^3\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{(c+dx)^2}{6bd\left(a+b(c+dx)^3\right)^2} + \frac{\text{Subst}\left(\int \frac{x}{\left(a+bx^3\right)^2}\,dx, x, c+dx\right)}{3bd}$$

$$= -\frac{(c+dx)^2}{6bd\left(a+b(c+dx)^3\right)^2} + \frac{(c+dx)^2}{9abd\left(a+b(c+dx)^3\right)} + \frac{\text{Subst}\left(\int \frac{x}{a+bx^3}\,dx, x, c+dx\right)}{9abd}$$

$$= -\frac{(c+dx)^2}{6bd\left(a+b(c+dx)^3\right)^2} + \frac{(c+dx)^2}{9abd\left(a+b(c+dx)^3\right)} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, c+dx\right)}{27a^{4/3}b^{4/3}d} + \frac{\text{S}}{}$$

$$= -\frac{(c+dx)^2}{6bd\left(a+b(c+dx)^3\right)^2} + \frac{(c+dx)^2}{9abd\left(a+b(c+dx)^3\right)} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{27a^{4/3}b^{5/3}d} + \frac{\text{Subst}\left(\int}{}$$

$$= -\frac{(c+dx)^2}{6bd\left(a+b(c+dx)^3\right)^2} + \frac{(c+dx)^2}{9abd\left(a+b(c+dx)^3\right)} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{27a^{4/3}b^{5/3}d} + \frac{\log\left(a^{2/3}\right.}{}$$

$$= -\frac{(c+dx)^2}{6bd\left(a+b(c+dx)^3\right)^2} + \frac{(c+dx)^2}{9abd\left(a+b(c+dx)^3\right)} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{9\sqrt{3}\,a^{4/3}b^{5/3}d} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{27a^{4/3}b^{5}}$$

Mathematica [A] time = 0.18, size = 182, normalized size = 0.89

$$\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{a^{4/3}} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{a^{4/3}} + \frac{2\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{b}\,(c+dx)-\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{a^{4/3}} + \frac{6b^{2/3}(c+dx)^2}{a\left(a+b(c+dx)^3\right)} - \frac{9b^{2/3}(c+dx)^2}{\left(a+b(c+dx)^3\right)^2}}{54b^{5/3}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4/(a + b*(c + d*x)^3)^3,x]
```

```
[Out] ((-9*b^(2/3)*(c + d*x)^2)/(a + b*(c + d*x)^3)^2 + (6*b^(2/3)*(c + d*x)^2)/(
a*(a + b*(c + d*x)^3)) + (2*Sqrt[3]*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c + d*x))
/(Sqrt[3]*a^(1/3))])/a^(4/3) - (2*Log[a^(1/3) + b^(1/3)*(c + d*x)])/a^(4/3)
 + Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/a^(4/3))/
(54*b^(5/3)*d)
```

fricas [B] time = 0.82, size = 1706, normalized size = 8.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4/(a+b*(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(6*a*b^3*d^5*x^5 + 30*a*b^3*c*d^4*x^4 + 60*a*b^3*c^2*d^3*x^3 + 6*a*b^
3*c^5 - 3*a^2*b^2*c^2 + 3*(20*a*b^3*c^3 - a^2*b^2)*d^2*x^2 + 6*(5*a*b^3*c^4
 - a^2*b^2*c)*d*x + 3*sqrt(1/3)*(a*b^3*d^6*x^6 + 6*a*b^3*c*d^5*x^5 + 15*a*b
```

```
^3*c^2*d^4*x^4 + a*b^3*c^6 + 2*a^2*b^2*c^3 + 2*(10*a*b^3*c^3 + a^2*b^2)*d^3
*x^3 + 3*(5*a*b^3*c^4 + 2*a^2*b^2*c)*d^2*x^2 + a^3*b + 6*(a*b^3*c^5 + a^2*b
^2*c^2)*d*x)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*d^3*x^3 + 6*b^2*c*d^2*x^2 +
6*b^2*c^2*d*x + 2*b^2*c^3 - a*b + 3*sqrt(1/3)*(a*b*d*x + a*b*c + 2*(d^2*x^2
 + 2*c*d*x + c^2)*(-a*b^2)^(2/3) + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a)
 - 3*(-a*b^2)^(2/3)*(d*x + c))/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b
*c^3 + a)) + (b^2*d^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4 + b^2*c^6
+ 2*(10*b^2*c^3 + a*b)*d^3*x^3 + 2*a*b*c^3 + 3*(5*b^2*c^4 + 2*a*b*c)*d^2*x^
2 + 6*(b^2*c^5 + a*b*c^2)*d*x + a^2)*(-a*b^2)^(2/3)*log(b^2*d^2*x^2 + 2*b^2
*c*d*x + b^2*c^2 + (-a*b^2)^(1/3)*(b*d*x + b*c) + (-a*b^2)^(2/3)) - 2*(b^2*
d^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4 + b^2*c^6 + 2*(10*b^2*c^3 +
a*b)*d^3*x^3 + 2*a*b*c^3 + 3*(5*b^2*c^4 + 2*a*b*c)*d^2*x^2 + 6*(b^2*c^5 + a
*b*c^2)*d*x + a^2)*(-a*b^2)^(2/3)*log(b*d*x + b*c - (-a*b^2)^(1/3)))/(a^2*b
^5*d^7*x^6 + 6*a^2*b^5*c*d^6*x^5 + 15*a^2*b^5*c^2*d^5*x^4 + 2*(10*a^2*b^5*c
^3 + a^3*b^4)*d^4*x^3 + 3*(5*a^2*b^5*c^4 + 2*a^3*b^4*c)*d^3*x^2 + 6*(a^2*b^
5*c^5 + a^3*b^4*c^2)*d^2*x + (a^2*b^5*c^6 + 2*a^3*b^4*c^3 + a^4*b^3)*d), 1/
54*(6*a*b^3*d^5*x^5 + 30*a*b^3*c*d^4*x^4 + 60*a*b^3*c^2*d^3*x^3 + 6*a*b^3*c
^5 - 3*a^2*b^2*c^2 + 3*(20*a*b^3*c^3 - a^2*b^2)*d^2*x^2 + 6*(5*a*b^3*c^4 -
a^2*b^2*c)*d*x + 6*sqrt(1/3)*(a*b^3*d^6*x^6 + 6*a*b^3*c*d^5*x^5 + 15*a*b^3*
c^2*d^4*x^4 + a*b^3*c^6 + 2*a^2*b^2*c^3 + 2*(10*a*b^3*c^3 + a^2*b^2)*d^3*x^
3 + 3*(5*a*b^3*c^4 + 2*a^2*b^2*c)*d^2*x^2 + a^3*b + 6*(a*b^3*c^5 + a^2*b^2*
c^2)*d*x)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*d*x + 2*b*c + (-a*b
^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (b^2*d^6*x^6 + 6*b^2*c*d^5*x^5 + 15
*b^2*c^2*d^4*x^4 + b^2*c^6 + 2*(10*b^2*c^3 + a*b)*d^3*x^3 + 2*a*b*c^3 + 3*(
5*b^2*c^4 + 2*a*b*c)*d^2*x^2 + 6*(b^2*c^5 + a*b*c^2)*d*x + a^2)*(-a*b^2)^(2
/3)*log(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (-a*b^2)^(1/3)*(b*d*x + b*c)
+ (-a*b^2)^(2/3)) - 2*(b^2*d^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4 +
 b^2*c^6 + 2*(10*b^2*c^3 + a*b)*d^3*x^3 + 2*a*b*c^3 + 3*(5*b^2*c^4 + 2*a*b*
c)*d^2*x^2 + 6*(b^2*c^5 + a*b*c^2)*d*x + a^2)*(-a*b^2)^(2/3)*log(b*d*x + b*
c - (-a*b^2)^(1/3)))/(a^2*b^5*d^7*x^6 + 6*a^2*b^5*c*d^6*x^5 + 15*a^2*b^5*c^
2*d^5*x^4 + 2*(10*a^2*b^5*c^3 + a^3*b^4)*d^4*x^3 + 3*(5*a^2*b^5*c^4 + 2*a^3
*b^4*c)*d^3*x^2 + 6*(a^2*b^5*c^5 + a^3*b^4*c^2)*d^2*x + (a^2*b^5*c^6 + 2*a^
3*b^4*c^3 + a^4*b^3)*d)]
```

giac [A] time = 0.26, size = 274, normalized size = 1.34

$$-\frac{2\sqrt{3}\left(-\frac{1}{ab^2d^3}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)}{3\left(-a^2b\right)^{\frac{2}{3}}}\right)+\left(-\frac{1}{ab^2d^3}\right)^{\frac{1}{3}}\log\left(\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)^2+3\left(-a^2b\right)^{\frac{4}{3}}\right)}{54\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4/(a+b*(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] -1/54*(2*sqrt(3)*(-1/(a*b^2*d^3))^(1/3)*arctan(1/3*sqrt(3)*(2*a*b*d*x + 2*a
*b*c - (-a^2*b)^(2/3))/(-a^2*b)^(2/3)) + (-1/(a*b^2*d^3))^(1/3)*log((2*a*b*
d*x + 2*a*b*c - (-a^2*b)^(2/3))^2 + 3*(-a^2*b)^(4/3)) - 2*(-1/(a*b^2*d^3))^
(1/3)*log(abs(a*b*d*x + a*b*c + (-a^2*b)^(2/3))))/(a*b) + 1/18*(2*b*d^5*x^5
 + 10*b*c*d^4*x^4 + 20*b*c^2*d^3*x^3 + 20*b*c^3*d^2*x^2 + 10*b*c^4*d*x + 2*
b*c^5 - a*d^2*x^2 - 2*a*c*d*x - a*c^2)/((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^
2*d*x + b*c^3 + a)^2*a*b*d)
```

maple [C] time = 0.02, size = 214, normalized size = 1.04

$$\frac{\left(\mathrm{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_Z+b\,c^3+a\right)d+c\right)\ln\left(-\mathrm{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2\right.\right.}{27a\,b^2d\left(d^2\,\mathrm{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_Z+b\,c^3+a\right)^2+2cd\,\mathrm{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4/(a+b*(d*x+c)^3)^3,x)
```

```
[Out] (1/9*d^4/a*x^5+5/9*c*d^3/a*x^4+10/9*c^2*d^2/a*x^3-1/18/b*d*(-20*b*c^3+a)/a*
x^2-1/9/b*c*(-5*b*c^3+a)/a*x-1/18/b*c^2/d*(-2*b*c^3+a)/a)/(b*d^3*x^3+3*b*c*
d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2+1/27/b^2/a/d*sum((_R*d+c)/(_R^2*d^2+2*_R*c*d
+c^2)*ln(-_R+x),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,bd^5x^5+10\,bcd^4x^4+20\,bc^2d^3x^3+2\,bc^5+\left(20\,bc^3-a\right)d^2x^2-ac^2+2\left(5\,bc^4-ac\right.}{18\left(ab^3d^7x^6+6\,ab^3cd^6x^5+15\,ab^3c^2d^5x^4+2\left(10\,ab^3c^3+a^2b^2\right)d^4x^3+3\left(5\,ab^3c^4+2\,a^2b^2c\right)d^3x^2+6\left(ab^3c^5+a^2b^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4/(a+b*(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(2*b*d^5*x^5 + 10*b*c*d^4*x^4 + 20*b*c^2*d^3*x^3 + 2*b*c^5 + (20*b*c^3
 - a)*d^2*x^2 - a*c^2 + 2*(5*b*c^4 - a*c)*d*x)/(a*b^3*d^7*x^6 + 6*a*b^3*c*d
^6*x^5 + 15*a*b^3*c^2*d^5*x^4 + 2*(10*a*b^3*c^3 + a^2*b^2)*d^4*x^3 + 3*(5*a
*b^3*c^4 + 2*a^2*b^2*c)*d^3*x^2 + 6*(a*b^3*c^5 + a^2*b^2*c^2)*d^2*x + (a*b^
3*c^6 + 2*a^2*b^2*c^3 + a^3*b)*d) + 1/9*integrate((d*x + c)/(b*d^3*x^3 + 3*
b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/(a*b)
```

mupad [B] time = 1.81, size = 399, normalized size = 1.95

$$\frac{\frac{d^4x^5}{9a}-\frac{ac^2-2bc^5}{18abd}+\frac{5cd^3x^4}{9a}+\frac{10c^2d^2x^3}{9a}-\frac{cx\left(a-5bc^3\right)}{9ab}-\frac{dx^2\left(a-20bc^3\right)}{18ab}}{x^3\left(20b^2c^3d^3+2abd^3\right)+x^2\left(15b^2c^4d^2+6abcd^2\right)+a^2+x\left(6db^2c^5+6adbc^2\right)+b^2c^6+b^2d^6x^6+2abc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^4/(a + b*(c + d*x)^3)^3,x)
```

```
[Out] ((d^4*x^5)/(9*a) - (a*c^2 - 2*b*c^5)/(18*a*b*d) + (5*c*d^3*x^4)/(9*a) + (10
*c^2*d^2*x^3)/(9*a) - (c*x*(a - 5*b*c^3))/(9*a*b) - (d*x^2*(a - 20*b*c^3))/
(18*a*b))/(x^3*(20*b^2*c^3*d^3 + 2*a*b*d^3) + x^2*(15*b^2*c^4*d^2 + 6*a*b*c
*d^2) + a^2 + x*(6*b^2*c^5*d + 6*a*b*c^2*d) + b^2*c^6 + b^2*d^6*x^6 + 2*a*b
*c^3 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4) + log(a^(1/3)*b + (-b)^(4/3)*c
 + (-b)^(4/3)*d*x)/(27*a^(4/3)*(-b)^(5/3)*d) + (log((c*d^4)/(81*a^2*b) + (d
^5*x)/(81*a^2*b) + (d^4*(3^(1/2)*1i - 1)^2)/(324*a^(5/3)*(-b)^(4/3)))*(3^(1
/2)*1i - 1))/(54*a^(4/3)*(-b)^(5/3)*d) - (log((c*d^4)/(81*a^2*b) + (d^5*x)/
(81*a^2*b) + (d^4*(3^(1/2)*1i + 1)^2)/(324*a^(5/3)*(-b)^(4/3)))*(3^(1/2)*1i
 + 1))/(54*a^(4/3)*(-b)^(5/3)*d)
```

sympy [A] time = 3.44, size = 287, normalized size = 1.40

$$\frac{-ac^2+2bc^5+20bc^2d^3x^3+10bcd^4x^4+2bd^5x^5+x^2\left(-ad^2+20bc^3d^2\right)+}{18a^3bd+36a^2b^2c^3d+18ab^3c^6d+270ab^3c^2d^5x^4+108ab^3cd^6x^5+18ab^3d^7x^6+x^3\left(36a^2b^2d^4+360ab^3c^3d^4\right)+x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4/(a+b*(d*x+c)**3)**3,x)
```

```
[Out] (-a*c**2 + 2*b*c**5 + 20*b*c**2*d**3*x**3 + 10*b*c*d**4*x**4 + 2*b*d**5*x**
5 + x**2*(-a*d**2 + 20*b*c**3*d**2) + x*(-2*a*c*d + 10*b*c**4*d))/(18*a**3*
b*d + 36*a**2*b**2*c**3*d + 18*a*b**3*c**6*d + 270*a*b**3*c**2*d**5*x**4 +
108*a*b**3*c*d**6*x**5 + 18*a*b**3*d**7*x**6 + x**3*(36*a**2*b**2*d**4 + 36
0*a*b**3*c**3*d**4) + x**2*(108*a**2*b**2*c*d**3 + 270*a*b**3*c**4*d**3) +
x*(108*a**2*b**2*c**2*d**2 + 108*a*b**3*c**5*d**2)) + RootSum(19683*_t**3*a
**4*b**5 + 1, Lambda(_t, _t*log(x + (729*_t**2*a**3*b**3 + c)/d)))/d
```

3.2878 $\int \frac{(c+dx)^3}{\left(a+b(c+dx)^3\right)^3}\,dx$

Optimal. Leaf size=201

$$\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{27a^{5/3}b^{4/3}d}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{54a^{5/3}b^{4/3}d}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{5/3}b^{4/3}d}+\frac{c+dx}{18abd\left(a+b(c+\right.}$$

```
[Out] 1/6*(-d*x-c)/b/d/(a+b*(d*x+c)^3)^2+1/18*(d*x+c)/a/b/d/(a+b*(d*x+c)^3)+1/27*
ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(5/3)/b^(4/3)/d-1/54*ln(a^(2/3)-a^(1/3)*b^(1/
3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(5/3)/b^(4/3)/d-1/27*arctan(1/3*(a^(1/3)-2*
b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(5/3)/b^(4/3)/d*3^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {372, 288, 199, 200, 31, 634, 617, 204, 628}

$$\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{27a^{5/3}b^{4/3}d}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{54a^{5/3}b^{4/3}d}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{5/3}b^{4/3}d}+\frac{c+dx}{18abd\left(a+b(c+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*(c + d*x)^3)^3,x]
```

```
[Out] -(c + d*x)/(6*b*d*(a + b*(c + d*x)^3)^2) + (c + d*x)/(18*a*b*d*(a + b*(c +
d*x)^3)) - ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))]/(9*Sqr
t[3]*a^(5/3)*b^(4/3)*d) + Log[a^(1/3) + b^(1/3)*(c + d*x)]/(27*a^(5/3)*b^(4
/3)*d) - Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/(54
*a^(5/3)*b^(4/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(c+dx)^3}{\left(a+b(c+dx)^3\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{\left(a+bx^3\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{c+dx}{6bd\left(a+b(c+dx)^3\right)^2} + \frac{\text{Subst}\left(\int \frac{1}{\left(a+bx^3\right)^2}\,dx, x, c+dx\right)}{6bd}$$

$$= -\frac{c+dx}{6bd\left(a+b(c+dx)^3\right)^2} + \frac{c+dx}{18abd\left(a+b(c+dx)^3\right)} + \frac{\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, c+dx\right)}{9abd}$$

$$= -\frac{c+dx}{6bd\left(a+b(c+dx)^3\right)^2} + \frac{c+dx}{18abd\left(a+b(c+dx)^3\right)} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, c+dx\right)}{27a^{5/3}bd}$$

$$= -\frac{c+dx}{6bd\left(a+b(c+dx)^3\right)^2} + \frac{c+dx}{18abd\left(a+b(c+dx)^3\right)} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{27a^{5/3}b^{4/3}d} - \frac{\text{Subs}}{}$$

$$= -\frac{c+dx}{6bd\left(a+b(c+dx)^3\right)^2} + \frac{c+dx}{18abd\left(a+b(c+dx)^3\right)} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{27a^{5/3}b^{4/3}d} - \frac{\log(}{}$$

$$= -\frac{c+dx}{6bd\left(a+b(c+dx)^3\right)^2} + \frac{c+dx}{18abd\left(a+b(c+dx)^3\right)} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{9\sqrt{3}\,a^{5/3}b^{4/3}d} + \frac{\log\left(\sqrt[3]{a}+\right.}{27a}$$

Mathematica [A] time = 0.12, size = 179, normalized size = 0.89

$$\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{a^{5/3}} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{a^{5/3}} + \frac{2\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{b}\,(c+dx)-\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{a^{5/3}} + \frac{3\sqrt[3]{b}\,(c+dx)}{a\left(a+b(c+dx)^3\right)} - \frac{9\sqrt[3]{b}\,(c+dx)}{\left(a+b(c+dx)^3\right)^2}}{54b^{4/3}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*(c + d*x)^3)^3,x]
```

```
[Out] ((-9*b^(1/3)*(c + d*x))/(a + b*(c + d*x)^3)^2 + (3*b^(1/3)*(c + d*x))/(a*(a
 + b*(c + d*x)^3)) + (2*Sqrt[3]*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c + d*x))/(Sq
rt[3]*a^(1/3))])/a^(5/3) + (2*Log[a^(1/3) + b^(1/3)*(c + d*x)])/a^(5/3) - L
og[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/a^(5/3))/(54*
b^(4/3)*d)
```

fricas [B] time = 0.90, size = 1650, normalized size = 8.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(3*a^2*b^2*d^4*x^4 + 12*a^2*b^2*c*d^3*x^3 + 18*a^2*b^2*c^2*d^2*x^2 +
3*a^2*b^2*c^4 - 6*a^3*b*c + 6*(2*a^2*b^2*c^3 - a^3*b)*d*x + 3*sqrt(1/3)*(a*
b^3*d^6*x^6 + 6*a*b^3*c*d^5*x^5 + 15*a*b^3*c^2*d^4*x^4 + a*b^3*c^6 + 2*a^2*
```

```
b^2*c^3 + 2*(10*a*b^3*c^3 + a^2*b^2)*d^3*x^3 + 3*(5*a*b^3*c^4 + 2*a^2*b^2*c
)*d^2*x^2 + a^3*b + 6*(a*b^3*c^5 + a^2*b^2*c^2)*d*x)*sqrt(-(a^2*b)^(1/3)/b)
*log((2*a*b*d^3*x^3 + 6*a*b*c*d^2*x^2 + 6*a*b*c^2*d*x + 2*a*b*c^3 - a^2 + 3
*sqrt(1/3)*(2*a*b*d^2*x^2 + 4*a*b*c*d*x + 2*a*b*c^2 + (a^2*b)^(2/3)*(d*x +
c) - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*(a*d*x + a*c
))/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)) - (b^2*d^6*x^6 +
6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4 + b^2*c^6 + 2*(10*b^2*c^3 + a*b)*d^3*x
^3 + 2*a*b*c^3 + 3*(5*b^2*c^4 + 2*a*b*c)*d^2*x^2 + 6*(b^2*c^5 + a*b*c^2)*d*
x + a^2)*(a^2*b)^(2/3)*log(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2 - (a^2*b)^(2
/3)*(d*x + c) + (a^2*b)^(1/3)*a) + 2*(b^2*d^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^
2*c^2*d^4*x^4 + b^2*c^6 + 2*(10*b^2*c^3 + a*b)*d^3*x^3 + 2*a*b*c^3 + 3*(5*b
^2*c^4 + 2*a*b*c)*d^2*x^2 + 6*(b^2*c^5 + a*b*c^2)*d*x + a^2)*(a^2*b)^(2/3)*
log(a*b*d*x + a*b*c + (a^2*b)^(2/3)))/(a^3*b^4*d^7*x^6 + 6*a^3*b^4*c*d^6*x^
5 + 15*a^3*b^4*c^2*d^5*x^4 + 2*(10*a^3*b^4*c^3 + a^4*b^3)*d^4*x^3 + 3*(5*a^
3*b^4*c^4 + 2*a^4*b^3*c)*d^3*x^2 + 6*(a^3*b^4*c^5 + a^4*b^3*c^2)*d^2*x + (a
^3*b^4*c^6 + 2*a^4*b^3*c^3 + a^5*b^2)*d), 1/54*(3*a^2*b^2*d^4*x^4 + 12*a^2*
b^2*c*d^3*x^3 + 18*a^2*b^2*c^2*d^2*x^2 + 3*a^2*b^2*c^4 - 6*a^3*b*c + 6*(2*a
^2*b^2*c^3 - a^3*b)*d*x + 6*sqrt(1/3)*(a*b^3*d^6*x^6 + 6*a*b^3*c*d^5*x^5 +
15*a*b^3*c^2*d^4*x^4 + a*b^3*c^6 + 2*a^2*b^2*c^3 + 2*(10*a*b^3*c^3 + a^2*b^
2)*d^3*x^3 + 3*(5*a*b^3*c^4 + 2*a^2*b^2*c)*d^2*x^2 + a^3*b + 6*(a*b^3*c^5 +
 a^2*b^2*c^2)*d*x)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*
(d*x + c) - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - (b^2*d^6*x^6 + 6*
b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4 + b^2*c^6 + 2*(10*b^2*c^3 + a*b)*d^3*x^3
 + 2*a*b*c^3 + 3*(5*b^2*c^4 + 2*a*b*c)*d^2*x^2 + 6*(b^2*c^5 + a*b*c^2)*d*x
+ a^2)*(a^2*b)^(2/3)*log(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2 - (a^2*b)^(2/3
)*(d*x + c) + (a^2*b)^(1/3)*a) + 2*(b^2*d^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^2*
c^2*d^4*x^4 + b^2*c^6 + 2*(10*b^2*c^3 + a*b)*d^3*x^3 + 2*a*b*c^3 + 3*(5*b^2
*c^4 + 2*a*b*c)*d^2*x^2 + 6*(b^2*c^5 + a*b*c^2)*d*x + a^2)*(a^2*b)^(2/3)*lo
g(a*b*d*x + a*b*c + (a^2*b)^(2/3)))/(a^3*b^4*d^7*x^6 + 6*a^3*b^4*c*d^6*x^5
+ 15*a^3*b^4*c^2*d^5*x^4 + 2*(10*a^3*b^4*c^3 + a^4*b^3)*d^4*x^3 + 3*(5*a^3*
b^4*c^4 + 2*a^4*b^3*c)*d^3*x^2 + 6*(a^3*b^4*c^5 + a^4*b^3*c^2)*d^2*x + (a^3
*b^4*c^6 + 2*a^4*b^3*c^3 + a^5*b^2)*d)]
```

giac [A] time = 0.27, size = 268, normalized size = 1.33

$$\frac{2\sqrt{3}\left(\frac{1}{a^2bd^3}\right)^{\frac{1}{3}}\arctan\left(-\frac{bdx+bc+\left(ab^2\right)^{\frac{1}{3}}}{\sqrt{3}\,bdx+\sqrt{3}\,bc-\sqrt{3}\left(ab^2\right)^{\frac{1}{3}}}\right)-\left(\frac{1}{a^2bd^3}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}\,bdx+\sqrt{3}\,bc-\sqrt{3}\left(ab^2\right)^{\frac{1}{3}}\right)^2+4\left(bdx+bc+\right.\right.}{54\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 1/54*(2*sqrt(3)*(1/(a^2*b*d^3))^(1/3)*arctan(-(b*d*x + b*c + (a*b^2)^(1/3))
/(sqrt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))) - (1/(a^2*b*d^3))^(
1/3)*log(4*(sqrt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))^2 + 4*(b*d
*x + b*c + (a*b^2)^(1/3))^2) + 2*(1/(a^2*b*d^3))^(1/3)*log(abs(b*d*x + b*c
+ (a*b^2)^(1/3))))/(a*b) + 1/18*(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^
2 + 4*b*c^3*d*x + b*c^4 - 2*a*d*x - 2*a*c)/((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*
b*c^2*d*x + b*c^3 + a)^2*a*b*d)
```

maple [C] time = 0.02, size = 186, normalized size = 0.93

$$\frac{\ln\left(-\operatorname{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_Z+b\,c^3+a\right)+x\right)}{27a\,b^2d\left(d^2\operatorname{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_Z+b\,c^3+a\right)^2+2cd\operatorname{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+b*(d*x+c)^3)^3,x)
```

```
[Out] (1/18*d^3/a*x^4+2/9*c*d^2/a*x^3+1/3/a*c^2*d*x^2-1/9/b*(-2*b*c^3+a)/a*x-1/18
/b*c/d*(-b*c^3+2*a)/a)/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2+1/27
/b^2/a/d*sum(1/(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x),_R=RootOf(_Z^3*b*d^3+3*_Z^
2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{bd^4x^4 + 4\,bcd^3x^3 + 6\,bc^2d^2x^2 + bc^4 + 2\left(2\,bc^3 - a\right)dx - 2\,ac}{18\left(ab^3d^7x^6 + 6\,ab^3cd^6x^5 + 15\,ab^3c^2d^5x^4 + 2\left(10\,ab^3c^3 + a^2b^2\right)d^4x^3 + 3\left(5\,ab^3c^4 + 2\,a^2b^2c\right)d^3x^2 + 6\left(ab^3c^5 + a^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + b*c^4 + 2*(2*b*c^3 - a)
*d*x - 2*a*c)/(a*b^3*d^7*x^6 + 6*a*b^3*c*d^6*x^5 + 15*a*b^3*c^2*d^5*x^4 + 2
*(10*a*b^3*c^3 + a^2*b^2)*d^4*x^3 + 3*(5*a*b^3*c^4 + 2*a^2*b^2*c)*d^3*x^2 +
 6*(a*b^3*c^5 + a^2*b^2*c^2)*d^2*x + (a*b^3*c^6 + 2*a^2*b^2*c^3 + a^3*b)*d)
 + 1/9*integrate(1/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x
)/(a*b)
```

mupad [B] time = 1.66, size = 347, normalized size = 1.73

$$\frac{\frac{d^3x^4}{18a} + \frac{c^2dx^2}{3a} + \frac{2cd^2x^3}{9a} - \frac{x\left(a-2bc^3\right)}{9ab} - \frac{2ac-bc^4}{18abd}}{x^3\left(20b^2c^3d^3 + 2abd^3\right) + x^2\left(15b^2c^4d^2 + 6abcd^2\right) + a^2 + x\left(6db^2c^5 + 6adbc^2\right) + b^2c^6 + b^2d^6x^6 + 2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*(c + d*x)^3)^3,x)
```

```
[Out] ((d^3*x^4)/(18*a) + (c^2*d*x^2)/(3*a) + (2*c*d^2*x^3)/(9*a) - (x*(a - 2*b*c
^3))/(9*a*b) - (2*a*c - b*c^4)/(18*a*b*d))/(x^3*(20*b^2*c^3*d^3 + 2*a*b*d^3
) + x^2*(15*b^2*c^4*d^2 + 6*a*b*c*d^2) + a^2 + x*(6*b^2*c^5*d + 6*a*b*c^2*d
) + b^2*c^6 + b^2*d^6*x^6 + 2*a*b*c^3 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^
4) + log(b^(1/3)*c + a^(1/3) + b^(1/3)*d*x)/(27*a^(5/3)*b^(4/3)*d) + (log((
b*c*d^5)/(3*a) + (b*d^6*x)/(3*a) + (b^(2/3)*d^5*(3^(1/2)*1i - 1))/(6*a^(2/3
)))*(3^(1/2)*1i - 1))/(54*a^(5/3)*b^(4/3)*d) - (log((b*c*d^5)/(3*a) + (b*d^
6*x)/(3*a) - (b^(2/3)*d^5*(3^(1/2)*1i + 1))/(6*a^(2/3)))*(3^(1/2)*1i + 1))/
(54*a^(5/3)*b^(4/3)*d)
```

sympy [A] time = 3.31, size = 260, normalized size = 1.29

$$\frac{-2ac + bc^4 + 6bc^2d^2x^2 + 4bcd^3x^3 + bd^4x^4 + x\left(-2ad +}{18a^3bd + 36a^2b^2c^3d + 18ab^3c^6d + 270ab^3c^2d^5x^4 + 108ab^3cd^6x^5 + 18ab^3d^7x^6 + x^3\left(36a^2b^2d^4 + 360ab^3c^3d^4\right) +}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+b*(d*x+c)**3)**3,x)
```

```
[Out] (-2*a*c + b*c**4 + 6*b*c**2*d**2*x**2 + 4*b*c*d**3*x**3 + b*d**4*x**4 + x*(
-2*a*d + 4*b*c**3*d))/(18*a**3*b*d + 36*a**2*b**2*c**3*d + 18*a*b**3*c**6*d
 + 270*a*b**3*c**2*d**5*x**4 + 108*a*b**3*c*d**6*x**5 + 18*a*b**3*d**7*x**6
 + x**3*(36*a**2*b**2*d**4 + 360*a*b**3*c**3*d**4) + x**2*(108*a**2*b**2*c*
d**3 + 270*a*b**3*c**4*d**3) + x*(108*a**2*b**2*c**2*d**2 + 108*a*b**3*c**5
*d**2)) + RootSum(19683*_t**3*a**5*b**4 - 1, Lambda(_t, _t*log(x + (27*_t*a
**2*b + c)/d)))/d
```

3.2879 $\int \frac{(c+dx)^2}{\left(a+b(c+dx)^3\right)^3}\,dx$

Optimal. Leaf size=23

$$-\frac{1}{6bd\left(a+b(c+dx)^3\right)^2}$$

```
[Out] -1/6/b/d/(a+b*(d*x+c)^3)^2
```

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {372, 261}

$$-\frac{1}{6bd\left(a+b(c+dx)^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*(c + d*x)^3)^3,x]
```

```
[Out] -1/(6*b*d*(a + b*(c + d*x)^3)^2)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^2}{\left(a+b(c+dx)^3\right)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{\left(a+bx^3\right)^3}\,dx, x, c+dx\right)}{d}\\ &= -\frac{1}{6bd\left(a+b(c+dx)^3\right)^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$-\frac{1}{6bd\left(a+b(c+dx)^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*(c + d*x)^3)^3,x]
```

```
[Out] -1/6*1/(b*d*(a + b*(c + d*x)^3)^2)
```

fricas [B] time = 0.80, size = 133, normalized size = 5.78

$$-\frac{1}{6\left(b^3d^7x^6+6\,b^3cd^6x^5+15\,b^3c^2d^5x^4+2\left(10\,b^3c^3+ab^2\right)d^4x^3+3\left(5\,b^3c^4+2\,ab^2c\right)d^3x^2+6\left(b^3c^5+ab^2c^2\right)d^2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] -1/6/(b^3*d^7*x^6 + 6*b^3*c*d^6*x^5 + 15*b^3*c^2*d^5*x^4 + 2*(10*b^3*c^3 +
a*b^2)*d^4*x^3 + 3*(5*b^3*c^4 + 2*a*b^2*c)*d^3*x^2 + 6*(b^3*c^5 + a*b^2*c^2
)*d^2*x + (b^3*c^6 + 2*a*b^2*c^3 + a^2*b)*d)
```

giac [A] time = 0.23, size = 21, normalized size = 0.91

$$-\frac{1}{6\left((dx+c)^3b+a\right)^2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] -1/6/(((d*x + c)^3*b + a)^2*b*d)
```

maple [B] time = 0.00, size = 44, normalized size = 1.91

$$-\frac{1}{6\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)^2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+b*(d*x+c)^3)^3,x)
```

```
[Out] -1/6/b/d/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2
```

maxima [A] time = 0.51, size = 21, normalized size = 0.91

$$-\frac{1}{6\left((dx+c)^3b+a\right)^2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] -1/6/(((d*x + c)^3*b + a)^2*b*d)
```

mupad [B] time = 1.35, size = 131, normalized size = 5.70

$$-\frac{1}{6\,b\,d\,\left(x^3\,\left(20\,b^2\,c^3\,d^3+2\,a\,b\,d^3\right)+x^2\,\left(15\,b^2\,c^4\,d^2+6\,a\,b\,c\,d^2\right)+a^2+x\,\left(6\,d\,b^2\,c^5+6\,a\,d\,b\,c^2\right)+b^2\,c^6+b^2\,d^6\,x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*(c + d*x)^3)^3,x)
```

```
[Out] -1/(6*b*d*(x^3*(20*b^2*c^3*d^3 + 2*a*b*d^3) + x^2*(15*b^2*c^4*d^2 + 6*a*b*c
*d^2) + a^2 + x*(6*b^2*c^5*d + 6*a*b*c^2*d) + b^2*c^6 + b^2*d^6*x^6 + 2*a*b
*c^3 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4))
```

sympy [B] time = 3.05, size = 153, normalized size = 6.65

$$-\frac{1}{6a^2bd+12ab^2c^3d+6b^3c^6d+90b^3c^2d^5x^4+36b^3cd^6x^5+6b^3d^7x^6+x^3\left(12ab^2d^4+120b^3c^3d^4\right)+x^2\left(36ab^2cd^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+b*(d*x+c)**3)**3,x)
```

```
[Out] -1/(6*a**2*b*d + 12*a*b**2*c**3*d + 6*b**3*c**6*d + 90*b**3*c**2*d**5*x**4
+ 36*b**3*c*d**6*x**5 + 6*b**3*d**7*x**6 + x**3*(12*a*b**2*d**4 + 120*b**3*
c**3*d**4) + x**2*(36*a*b**2*c*d**3 + 90*b**3*c**4*d**3) + x*(36*a*b**2*c**
2*d**2 + 36*b**3*c**5*d**2))
```

3.2880 $\int \frac{c+dx}{\left(a+b(c+dx)^3\right)^3}\,dx$

Optimal. Leaf size=202

$$-\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{27a^{7/3}b^{2/3}d}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{27a^{7/3}b^{2/3}d}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{7/3}b^{2/3}d}+\frac{2(c+$$

```
[Out] 1/6*(d*x+c)^2/a/d/(a+b*(d*x+c)^3)^2+2/9*(d*x+c)^2/a^2/d/(a+b*(d*x+c)^3)-2/2
7*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(7/3)/b^(2/3)/d+1/27*ln(a^(2/3)-a^(1/3)*b^(
1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(7/3)/b^(2/3)/d-2/27*arctan(1/3*(a^(1/3)-
2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(7/3)/b^(2/3)/d*3^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {372, 290, 292, 31, 634, 617, 204, 628}

$$-\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{27a^{7/3}b^{2/3}d}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{27a^{7/3}b^{2/3}d}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{7/3}b^{2/3}d}+\frac{2(c+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*(c + d*x)^3)^3,x]
```

```
[Out] (c + d*x)^2/(6*a*d*(a + b*(c + d*x)^3)^2) + (2*(c + d*x)^2)/(9*a^2*d*(a + b
*(c + d*x)^3)) - (2*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3)
)])/(9*Sqrt[3]*a^(7/3)*b^(2/3)*d) - (2*Log[a^(1/3) + b^(1/3)*(c + d*x)])/(2
7*a^(7/3)*b^(2/3)*d) + Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c
 + d*x)^2]/(27*a^(7/3)*b^(2/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{c+dx}{\left(a+b(c+dx)^3\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x}{\left(a+bx^3\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{(c+dx)^2}{6ad\left(a+b(c+dx)^3\right)^2} + \frac{2\,\text{Subst}\left(\int \frac{x}{\left(a+bx^3\right)^2}\,dx, x, c+dx\right)}{3ad}$$

$$= \frac{(c+dx)^2}{6ad\left(a+b(c+dx)^3\right)^2} + \frac{2(c+dx)^2}{9a^2d\left(a+b(c+dx)^3\right)} + \frac{2\,\text{Subst}\left(\int \frac{x}{a+bx^3}\,dx, x, c+dx\right)}{9a^2d}$$

$$= \frac{(c+dx)^2}{6ad\left(a+b(c+dx)^3\right)^2} + \frac{2(c+dx)^2}{9a^2d\left(a+b(c+dx)^3\right)} - \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, c+dx\right)}{27a^{7/3}\sqrt[3]{b}\,d} + \frac{2}{}$$

$$= \frac{(c+dx)^2}{6ad\left(a+b(c+dx)^3\right)^2} + \frac{2(c+dx)^2}{9a^2d\left(a+b(c+dx)^3\right)} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{27a^{7/3}b^{2/3}d} + \frac{\text{Subst}\left(\int}{}$$

$$= \frac{(c+dx)^2}{6ad\left(a+b(c+dx)^3\right)^2} + \frac{2(c+dx)^2}{9a^2d\left(a+b(c+dx)^3\right)} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{27a^{7/3}b^{2/3}d} + \frac{\log\left(a^{2/3}\right.}{}$$

$$= \frac{(c+dx)^2}{6ad\left(a+b(c+dx)^3\right)^2} + \frac{2(c+dx)^2}{9a^2d\left(a+b(c+dx)^3\right)} - \frac{2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{9\sqrt{3}\,a^{7/3}b^{2/3}d} - \frac{2\log\left(\sqrt[3]{a}+\sqrt{}\right.}{27a^{7/3}b}$$

Mathematica [A] time = 0.13, size = 180, normalized size = 0.89

$$\frac{\frac{2\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{b^{2/3}}+\frac{9a^{4/3}(c+dx)^2}{\left(a+b(c+dx)^3\right)^2}-\frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{b^{2/3}}+\frac{4\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{b}\,(c+dx)-\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{b^{2/3}}+\frac{12\sqrt[3]{a}\,(c+dx)^2}{a+b(c+dx)^3}}{54a^{7/3}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*(c + d*x)^3)^3,x]
```

```
[Out] ((9*a^(4/3)*(c + d*x)^2)/(a + b*(c + d*x)^3)^2 + (12*a^(1/3)*(c + d*x)^2)/(
a + b*(c + d*x)^3) + (4*Sqrt[3]*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c + d*x))/(Sq
rt[3]*a^(1/3))])/b^(2/3) - (4*Log[a^(1/3) + b^(1/3)*(c + d*x)])/b^(2/3) + (
2*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/b^(2/3))/
(54*a^(7/3)*d)
```

fricas [B] time = 0.90, size = 1708, normalized size = 8.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(12*a*b^3*d^5*x^5 + 60*a*b^3*c*d^4*x^4 + 120*a*b^3*c^2*d^3*x^3 + 12*a
*b^3*c^5 + 21*a^2*b^2*c^2 + 3*(40*a*b^3*c^3 + 7*a^2*b^2)*d^2*x^2 + 6*(10*a*
b^3*c^4 + 7*a^2*b^2*c)*d*x + 6*sqrt(1/3)*(a*b^3*d^6*x^6 + 6*a*b^3*c*d^5*x^5
 + 15*a*b^3*c^2*d^4*x^4 + a*b^3*c^6 + 2*a^2*b^2*c^3 + 2*(10*a*b^3*c^3 + a^2
*b^2)*d^3*x^3 + 3*(5*a*b^3*c^4 + 2*a^2*b^2*c)*d^2*x^2 + a^3*b + 6*(a*b^3*c^
5 + a^2*b^2*c^2)*d*x)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*d^3*x^3 + 6*b^2*c*d
^2*x^2 + 6*b^2*c^2*d*x + 2*b^2*c^3 - a*b + 3*sqrt(1/3)*(a*b*d*x + a*b*c + 2
*(d^2*x^2 + 2*c*d*x + c^2)*(-a*b^2)^(2/3) + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)
^(1/3)/a) - 3*(-a*b^2)^(2/3)*(d*x + c))/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^
2*d*x + b*c^3 + a)) + 2*(b^2*d^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4
 + b^2*c^6 + 2*(10*b^2*c^3 + a*b)*d^3*x^3 + 2*a*b*c^3 + 3*(5*b^2*c^4 + 2*a*
b*c)*d^2*x^2 + 6*(b^2*c^5 + a*b*c^2)*d*x + a^2)*(-a*b^2)^(2/3)*log(b^2*d^2*
x^2 + 2*b^2*c*d*x + b^2*c^2 + (-a*b^2)^(1/3)*(b*d*x + b*c) + (-a*b^2)^(2/3)
) - 4*(b^2*d^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4 + b^2*c^6 + 2*(10
*b^2*c^3 + a*b)*d^3*x^3 + 2*a*b*c^3 + 3*(5*b^2*c^4 + 2*a*b*c)*d^2*x^2 + 6*(
b^2*c^5 + a*b*c^2)*d*x + a^2)*(-a*b^2)^(2/3)*log(b*d*x + b*c - (-a*b^2)^(1/
3)))/(a^3*b^4*d^7*x^6 + 6*a^3*b^4*c*d^6*x^5 + 15*a^3*b^4*c^2*d^5*x^4 + 2*(1
0*a^3*b^4*c^3 + a^4*b^3)*d^4*x^3 + 3*(5*a^3*b^4*c^4 + 2*a^4*b^3*c)*d^3*x^2
+ 6*(a^3*b^4*c^5 + a^4*b^3*c^2)*d^2*x + (a^3*b^4*c^6 + 2*a^4*b^3*c^3 + a^5*
b^2)*d), 1/54*(12*a*b^3*d^5*x^5 + 60*a*b^3*c*d^4*x^4 + 120*a*b^3*c^2*d^3*x^
3 + 12*a*b^3*c^5 + 21*a^2*b^2*c^2 + 3*(40*a*b^3*c^3 + 7*a^2*b^2)*d^2*x^2 +
6*(10*a*b^3*c^4 + 7*a^2*b^2*c)*d*x + 12*sqrt(1/3)*(a*b^3*d^6*x^6 + 6*a*b^3*
c*d^5*x^5 + 15*a*b^3*c^2*d^4*x^4 + a*b^3*c^6 + 2*a^2*b^2*c^3 + 2*(10*a*b^3*
c^3 + a^2*b^2)*d^3*x^3 + 3*(5*a*b^3*c^4 + 2*a^2*b^2*c)*d^2*x^2 + a^3*b + 6*
(a*b^3*c^5 + a^2*b^2*c^2)*d*x)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*
b*d*x + 2*b*c + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 2*(b^2*d^6*x^6
 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4 + b^2*c^6 + 2*(10*b^2*c^3 + a*b)*d^
3*x^3 + 2*a*b*c^3 + 3*(5*b^2*c^4 + 2*a*b*c)*d^2*x^2 + 6*(b^2*c^5 + a*b*c^2)
*d*x + a^2)*(-a*b^2)^(2/3)*log(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (-a*b^
2)^(1/3)*(b*d*x + b*c) + (-a*b^2)^(2/3)) - 4*(b^2*d^6*x^6 + 6*b^2*c*d^5*x^5
 + 15*b^2*c^2*d^4*x^4 + b^2*c^6 + 2*(10*b^2*c^3 + a*b)*d^3*x^3 + 2*a*b*c^3
+ 3*(5*b^2*c^4 + 2*a*b*c)*d^2*x^2 + 6*(b^2*c^5 + a*b*c^2)*d*x + a^2)*(-a*b^
2)^(2/3)*log(b*d*x + b*c - (-a*b^2)^(1/3)))/(a^3*b^4*d^7*x^6 + 6*a^3*b^4*c*
d^6*x^5 + 15*a^3*b^4*c^2*d^5*x^4 + 2*(10*a^3*b^4*c^3 + a^4*b^3)*d^4*x^3 + 3
*(5*a^3*b^4*c^4 + 2*a^4*b^3*c)*d^3*x^2 + 6*(a^3*b^4*c^5 + a^4*b^3*c^2)*d^2*
x + (a^3*b^4*c^6 + 2*a^4*b^3*c^3 + a^5*b^2)*d)]
```

giac [A] time = 0.25, size = 268, normalized size = 1.33

$$-\frac{2\sqrt{3}\left(-\frac{1}{ab^2d^3}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)}{3\left(-a^2b\right)^{\frac{2}{3}}}\right)+\left(-\frac{1}{ab^2d^3}\right)^{\frac{1}{3}}\log\left(\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)^2+3\left(-a^2b\right)^{\frac{4}{3}}\right)-2}{27\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] -1/27*(2*sqrt(3)*(-1/(a*b^2*d^3))^(1/3)*arctan(1/3*sqrt(3)*(2*a*b*d*x + 2*a
*b*c - (-a^2*b)^(2/3))/(-a^2*b)^(2/3)) + (-1/(a*b^2*d^3))^(1/3)*log((2*a*b*
d*x + 2*a*b*c - (-a^2*b)^(2/3))^2 + 3*(-a^2*b)^(4/3)) - 2*(-1/(a*b^2*d^3))^
(1/3)*log(abs(a*b*d*x + a*b*c + (-a^2*b)^(2/3))))/a^2 + 1/18*(4*b*d^5*x^5 +
 20*b*c*d^4*x^4 + 40*b*c^2*d^3*x^3 + 40*b*c^3*d^2*x^2 + 20*b*c^4*d*x + 4*b*
c^5 + 7*a*d^2*x^2 + 14*a*c*d*x + 7*a*c^2)/((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b
*c^2*d*x + b*c^3 + a)^2*a^2*d)
```

maple [C] time = 0.02, size = 214, normalized size = 1.06

$$\frac{2\left(\text{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_Z+b\,c^3+a\right)d+c\right)\ln\left(-\text{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_Z\right.\right.}{27a^2bd\left(d^2\,\text{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_Z+b\,c^3+a\right)^2+2cd\,\text{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+b*(d*x+c)^3)^3,x)
```

```
[Out] (2/9*b*d^4/a^2*x^5+10/9*b*c*d^3/a^2*x^4+20/9*b*c^2*d^2/a^2*x^3+1/18*d*(40*b
*c^3+7*a)/a^2*x^2+1/9*c*(10*b*c^3+7*a)/a^2*x+1/18*c^2/d*(4*b*c^3+7*a)/a^2)/
(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2+2/27/a^2/b/d*sum((_R*d+c)/(
_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b
*c^2*d+b*c^3+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{4\,bd^5x^5+20\,bcd^4x^4+40\,bc^2d^3x^3+4\,bc^5+\left(40\,bc^3+7\,a\right)d^2x^2+7\,ac^2+2\left(10\,bc^4+\right.}{18\left(a^2b^2d^7x^6+6\,a^2b^2cd^6x^5+15\,a^2b^2c^2d^5x^4+2\left(10\,a^2b^2c^3+a^3b\right)d^4x^3+3\left(5\,a^2b^2c^4+2\,a^3bc\right)d^3x^2+6\left(a^2b^2c^5+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(4*b*d^5*x^5 + 20*b*c*d^4*x^4 + 40*b*c^2*d^3*x^3 + 4*b*c^5 + (40*b*c^3
 + 7*a)*d^2*x^2 + 7*a*c^2 + 2*(10*b*c^4 + 7*a*c)*d*x)/(a^2*b^2*d^7*x^6 + 6*
a^2*b^2*c*d^6*x^5 + 15*a^2*b^2*c^2*d^5*x^4 + 2*(10*a^2*b^2*c^3 + a^3*b)*d^4
*x^3 + 3*(5*a^2*b^2*c^4 + 2*a^3*b*c)*d^3*x^2 + 6*(a^2*b^2*c^5 + a^3*b*c^2)*
d^2*x + (a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4)*d) + 2/9*integrate((d*x + c)/(b*d
^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/a^2
```

mupad [B] time = 0.50, size = 403, normalized size = 2.00

$$\frac{\frac{4\,bc^5+7\,ac^2}{18\,a^2d}+\frac{2\,bd^4x^5}{9\,a^2}+\frac{cx\left(10\,bc^3+7\,a\right)}{9\,a^2}+\frac{dx^2\left(40\,bc^3+7\,a\right)}{18\,a^2}+\frac{10\,bcd^3x^4}{9\,a^2}+\frac{20\,bc^2d^2x^3}{9\,a^2}}{x^3\left(20\,b^2c^3d^3+2\,ab\,d^3\right)+x^2\left(15\,b^2c^4d^2+6\,ab\,cd^2\right)+a^2+x\left(6\,db^2c^5+6\,ad\,bc^2\right)+b^2c^6+b^2d^6x^6+2\,abc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*(c + d*x)^3)^3,x)
```

```
[Out] ((7*a*c^2 + 4*b*c^5)/(18*a^2*d) + (2*b*d^4*x^5)/(9*a^2) + (c*x*(7*a + 10*b*
c^3))/(9*a^2) + (d*x^2*(7*a + 40*b*c^3))/(18*a^2) + (10*b*c*d^3*x^4)/(9*a^2
) + (20*b*c^2*d^2*x^3)/(9*a^2))/(x^3*(20*b^2*c^3*d^3 + 2*a*b*d^3) + x^2*(15
*b^2*c^4*d^2 + 6*a*b*c*d^2) + a^2 + x*(6*b^2*c^5*d + 6*a*b*c^2*d) + b^2*c^6
 + b^2*d^6*x^6 + 2*a*b*c^3 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4) + (2*log
((4*b*c*d^4)/(81*a^4) - (4*b^(2/3)*d^4)/(81*(-a)^(11/3)) + (4*b*d^5*x)/(81*
a^4)))/(27*(-a)^(7/3)*b^(2/3)*d) + (log((4*b*c*d^4)/(81*a^4) - (b^(2/3)*d^4
*(3^(1/2)*1i - 1)^2)/(81*(-a)^(11/3)) + (4*b*d^5*x)/(81*a^4))*(3^(1/2)*1i -
 1))/(27*(-a)^(7/3)*b^(2/3)*d) - (log((4*b*c*d^4)/(81*a^4) - (b^(2/3)*d^4*(
3^(1/2)*1i + 1)^2)/(81*(-a)^(11/3)) + (4*b*d^5*x)/(81*a^4))*(3^(1/2)*1i + 1
))/(27*(-a)^(7/3)*b^(2/3)*d)
```

sympy [A] time = 3.28, size = 296, normalized size = 1.47

$$\frac{7ac^2 + 4bc^5 + 40bc^2d^3x^3 + 20bcd^4x^4 + 4bd^5x^5 + x^2\left(7ad^2 + 40bc^3d^2\right.}{18a^4d + 36a^3bc^3d + 18a^2b^2c^6d + 270a^2b^2c^2d^5x^4 + 108a^2b^2cd^6x^5 + 18a^2b^2d^7x^6 + x^3\left(36a^3bd^4 + 360a^2b^2c^3d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*(d*x+c)**3)**3,x)
```

```
[Out] (7*a*c**2 + 4*b*c**5 + 40*b*c**2*d**3*x**3 + 20*b*c*d**4*x**4 + 4*b*d**5*x*
*5 + x**2*(7*a*d**2 + 40*b*c**3*d**2) + x*(14*a*c*d + 20*b*c**4*d))/(18*a**
4*d + 36*a**3*b*c**3*d + 18*a**2*b**2*c**6*d + 270*a**2*b**2*c**2*d**5*x**4
 + 108*a**2*b**2*c*d**6*x**5 + 18*a**2*b**2*d**7*x**6 + x**3*(36*a**3*b*d**
4 + 360*a**2*b**2*c**3*d**4) + x**2*(108*a**3*b*c*d**3 + 270*a**2*b**2*c**4
*d**3) + x*(108*a**3*b*c**2*d**2 + 108*a**2*b**2*c**5*d**2)) + RootSum(1968
3*_t**3*a**7*b**2 + 8, Lambda(_t, _t*log(x + (729*_t**2*a**5*b + 4*c)/(4*d)
)))/d
```

3.2881 $\int \frac{1}{\left(a+b(c+dx)^3\right)^3}\,dx$

Optimal. Leaf size=198

$$-\frac{5\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{54a^{8/3}\sqrt[3]{b}\,d}+\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{27a^{8/3}\sqrt[3]{b}\,d}-\frac{5\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{8/3}\sqrt[3]{b}\,d}+\frac{5(c+$$

```
[Out] 1/6*(d*x+c)/a/d/(a+b*(d*x+c)^3)^2+5/18*(d*x+c)/a^2/d/(a+b*(d*x+c)^3)+5/27*l
n(a^(1/3)+b^(1/3)*(d*x+c))/a^(8/3)/b^(1/3)/d-5/54*ln(a^(2/3)-a^(1/3)*b^(1/3
)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(8/3)/b^(1/3)/d-5/27*arctan(1/3*(a^(1/3)-2*b
^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(8/3)/b^(1/3)/d*3^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {247, 199, 200, 31, 634, 617, 204, 628}

$$-\frac{5\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{54a^{8/3}\sqrt[3]{b}\,d}+\frac{5(c+dx)}{18a^2d\left(a+b(c+dx)^3\right)}+\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{27a^{8/3}\sqrt[3]{b}\,d}-\frac{5\tan^{-1}\left(\frac{\sqrt[3]{a}-}{9\sqrt{3}\,a^{8/3}}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c + d*x)^3)^(-3),x]
```

```
[Out] (c + d*x)/(6*a*d*(a + b*(c + d*x)^3)^2) + (5*(c + d*x))/(18*a^2*d*(a + b*(c
 + d*x)^3)) - (5*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])
/(9*Sqrt[3]*a^(8/3)*b^(1/3)*d) + (5*Log[a^(1/3) + b^(1/3)*(c + d*x)])/(27*a
^(8/3)*b^(1/3)*d) - (5*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c
 + d*x)^2])/(54*a^(8/3)*b^(1/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(a+b(c+dx)^3\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+bx^3\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{c+dx}{6ad\left(a+b(c+dx)^3\right)^2} + \frac{5\,\text{Subst}\left(\int \frac{1}{\left(a+bx^3\right)^2}\,dx, x, c+dx\right)}{6ad}$$

$$= \frac{c+dx}{6ad\left(a+b(c+dx)^3\right)^2} + \frac{5(c+dx)}{18a^2d\left(a+b(c+dx)^3\right)} + \frac{5\,\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, c+dx\right)}{9a^2d}$$

$$= \frac{c+dx}{6ad\left(a+b(c+dx)^3\right)^2} + \frac{5(c+dx)}{18a^2d\left(a+b(c+dx)^3\right)} + \frac{5\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, c+dx\right)}{27a^{8/3}d}$$

$$= \frac{c+dx}{6ad\left(a+b(c+dx)^3\right)^2} + \frac{5(c+dx)}{18a^2d\left(a+b(c+dx)^3\right)} + \frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{27a^{8/3}\sqrt[3]{b}\,d} + \frac{5\,\text{Su}}{}$$

$$= \frac{c+dx}{6ad\left(a+b(c+dx)^3\right)^2} + \frac{5(c+dx)}{18a^2d\left(a+b(c+dx)^3\right)} + \frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{27a^{8/3}\sqrt[3]{b}\,d} - \frac{5\,\text{log}}{}$$

$$= \frac{c+dx}{6ad\left(a+b(c+dx)^3\right)^2} + \frac{5(c+dx)}{18a^2d\left(a+b(c+dx)^3\right)} - \frac{5\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{9\sqrt{3}\,a^{8/3}\sqrt[3]{b}\,d} + \frac{5\log\left(\sqrt[3]{a}\right.}{27}$$

Mathematica [A] time = 0.11, size = 176, normalized size = 0.89

$$\frac{-\frac{5\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{\sqrt[3]{b}}+\frac{9a^{5/3}(c+dx)}{\left(a+b(c+dx)^3\right)^2}+\frac{15a^{2/3}(c+dx)}{a+b(c+dx)^3}+\frac{10\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{\sqrt[3]{b}}+\frac{10\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{b}\,(c+dx)-\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{54a^{8/3}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c + d*x)^3)^(-3),x]
```

```
[Out] ((9*a^(5/3)*(c + d*x))/(a + b*(c + d*x)^3)^2 + (15*a^(2/3)*(c + d*x))/(a +
b*(c + d*x)^3) + (10*Sqrt[3]*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c + d*x))/(Sqrt[
3]*a^(1/3))])/b^(1/3) + (10*Log[a^(1/3) + b^(1/3)*(c + d*x)])/b^(1/3) - (5*
Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/b^(1/3))/(5
4*a^(8/3)*d)
```

fricas [B] time = 1.09, size = 1646, normalized size = 8.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(15*a^2*b^2*d^4*x^4 + 60*a^2*b^2*c*d^3*x^3 + 90*a^2*b^2*c^2*d^2*x^2 +
 15*a^2*b^2*c^4 + 24*a^3*b*c + 12*(5*a^2*b^2*c^3 + 2*a^3*b)*d*x + 15*sqrt(1
/3)*(a*b^3*d^6*x^6 + 6*a*b^3*c*d^5*x^5 + 15*a*b^3*c^2*d^4*x^4 + a*b^3*c^6 +
 2*a^2*b^2*c^3 + 2*(10*a*b^3*c^3 + a^2*b^2)*d^3*x^3 + 3*(5*a*b^3*c^4 + 2*a^
2*b^2*c)*d^2*x^2 + a^3*b + 6*(a*b^3*c^5 + a^2*b^2*c^2)*d*x)*sqrt(-(a^2*b)^(
1/3)/b)*log((2*a*b*d^3*x^3 + 6*a*b*c*d^2*x^2 + 6*a*b*c^2*d*x + 2*a*b*c^3 -
a^2 + 3*sqrt(1/3)*(2*a*b*d^2*x^2 + 4*a*b*c*d*x + 2*a*b*c^2 + (a^2*b)^(2/3)*
(d*x + c) - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*(a*d*
x + a*c))/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)) - 5*(b^2*d
^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4 + b^2*c^6 + 2*(10*b^2*c^3 + a
*b)*d^3*x^3 + 2*a*b*c^3 + 3*(5*b^2*c^4 + 2*a*b*c)*d^2*x^2 + 6*(b^2*c^5 + a*
b*c^2)*d*x + a^2)*(a^2*b)^(2/3)*log(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2 - (
a^2*b)^(2/3)*(d*x + c) + (a^2*b)^(1/3)*a) + 10*(b^2*d^6*x^6 + 6*b^2*c*d^5*x
^5 + 15*b^2*c^2*d^4*x^4 + b^2*c^6 + 2*(10*b^2*c^3 + a*b)*d^3*x^3 + 2*a*b*c^
3 + 3*(5*b^2*c^4 + 2*a*b*c)*d^2*x^2 + 6*(b^2*c^5 + a*b*c^2)*d*x + a^2)*(a^2
*b)^(2/3)*log(a*b*d*x + a*b*c + (a^2*b)^(2/3)))/(a^4*b^3*d^7*x^6 + 6*a^4*b^
3*c*d^6*x^5 + 15*a^4*b^3*c^2*d^5*x^4 + 2*(10*a^4*b^3*c^3 + a^5*b^2)*d^4*x^3
 + 3*(5*a^4*b^3*c^4 + 2*a^5*b^2*c)*d^3*x^2 + 6*(a^4*b^3*c^5 + a^5*b^2*c^2)*
d^2*x + (a^4*b^3*c^6 + 2*a^5*b^2*c^3 + a^6*b)*d), 1/54*(15*a^2*b^2*d^4*x^4
+ 60*a^2*b^2*c*d^3*x^3 + 90*a^2*b^2*c^2*d^2*x^2 + 15*a^2*b^2*c^4 + 24*a^3*b
*c + 12*(5*a^2*b^2*c^3 + 2*a^3*b)*d*x + 30*sqrt(1/3)*(a*b^3*d^6*x^6 + 6*a*b
^3*c*d^5*x^5 + 15*a*b^3*c^2*d^4*x^4 + a*b^3*c^6 + 2*a^2*b^2*c^3 + 2*(10*a*b
^3*c^3 + a^2*b^2)*d^3*x^3 + 3*(5*a*b^3*c^4 + 2*a^2*b^2*c)*d^2*x^2 + a^3*b +
 6*(a*b^3*c^5 + a^2*b^2*c^2)*d*x)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2
*(a^2*b)^(2/3)*(d*x + c) - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 5*
(b^2*d^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4 + b^2*c^6 + 2*(10*b^2*c
^3 + a*b)*d^3*x^3 + 2*a*b*c^3 + 3*(5*b^2*c^4 + 2*a*b*c)*d^2*x^2 + 6*(b^2*c^
5 + a*b*c^2)*d*x + a^2)*(a^2*b)^(2/3)*log(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c
^2 - (a^2*b)^(2/3)*(d*x + c) + (a^2*b)^(1/3)*a) + 10*(b^2*d^6*x^6 + 6*b^2*c
*d^5*x^5 + 15*b^2*c^2*d^4*x^4 + b^2*c^6 + 2*(10*b^2*c^3 + a*b)*d^3*x^3 + 2*
a*b*c^3 + 3*(5*b^2*c^4 + 2*a*b*c)*d^2*x^2 + 6*(b^2*c^5 + a*b*c^2)*d*x + a^2
)*(a^2*b)^(2/3)*log(a*b*d*x + a*b*c + (a^2*b)^(2/3)))/(a^4*b^3*d^7*x^6 + 6*
a^4*b^3*c*d^6*x^5 + 15*a^4*b^3*c^2*d^5*x^4 + 2*(10*a^4*b^3*c^3 + a^5*b^2)*d
^4*x^3 + 3*(5*a^4*b^3*c^4 + 2*a^5*b^2*c)*d^3*x^2 + 6*(a^4*b^3*c^5 + a^5*b^2
*c^2)*d^2*x + (a^4*b^3*c^6 + 2*a^5*b^2*c^3 + a^6*b)*d)]
```

giac [A] time = 0.19, size = 264, normalized size = 1.33

$$\frac{5\left(2\sqrt{3}\left(\frac{1}{a^2bd^3}\right)^{\frac{1}{3}}\arctan\left(-\frac{bdx+bc+\left(ab^2\right)^{\frac{1}{3}}}{\sqrt{3}\,bdx+\sqrt{3}\,bc-\sqrt{3}\left(ab^2\right)^{\frac{1}{3}}}\right)-\left(\frac{1}{a^2bd^3}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}\,bdx+\sqrt{3}\,bc-\sqrt{3}\left(ab^2\right)^{\frac{1}{3}}\right)^2+4\left(bdx+\right.\right.\right.}{54\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 5/54*(2*sqrt(3)*(1/(a^2*b*d^3))^(1/3)*arctan(-(b*d*x + b*c + (a*b^2)^(1/3))
/(sqrt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))) - (1/(a^2*b*d^3))^(
1/3)*log(4*(sqrt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))^2 + 4*(b*d
*x + b*c + (a*b^2)^(1/3))^2) + 2*(1/(a^2*b*d^3))^(1/3)*log(abs(b*d*x + b*c
+ (a*b^2)^(1/3))))/a^2 + 1/18*(5*b*d^4*x^4 + 20*b*c*d^3*x^3 + 30*b*c^2*d^2*
x^2 + 20*b*c^3*d*x + 5*b*c^4 + 8*a*d*x + 8*a*c)/((b*d^3*x^3 + 3*b*c*d^2*x^2
 + 3*b*c^2*d*x + b*c^3 + a)^2*a^2*d)
```

maple [C] time = 0.02, size = 185, normalized size = 0.93

$$\frac{5\ln\left(-\operatorname{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_Z+b\,c^3+a\right)+x\right)}{27a^2bd\left(d^2\operatorname{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_Z+b\,c^3+a\right)^2+2cd\operatorname{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*(d*x+c)^3)^3,x)
```

```
[Out] (5/18/a^2*b*d^3*x^4+10/9*b*c*d^2/a^2*x^3+5/3/a^2*b*c^2*d*x^2+2/9*(5*b*c^3+2
*a)/a^2*x+1/18*c/d*(5*b*c^3+8*a)/a^2)/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+
b*c^3+a)^2+5/27/a^2/b/d*sum(1/(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x),_R=RootOf(_
Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{5\,bd^4x^4+20\,bcd^3x^3+30\,bc^2d^2x^2+5\,bc^4+4\left(5\,bc^3+2\,a\right)dx+8\,ac}{18\left(a^2b^2d^7x^6+6\,a^2b^2cd^6x^5+15\,a^2b^2c^2d^5x^4+2\left(10\,a^2b^2c^3+a^3b\right)d^4x^3+3\left(5\,a^2b^2c^4+2\,a^3bc\right)d^3x^2+6\left(a^2b^2c^5\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(5*b*d^4*x^4 + 20*b*c*d^3*x^3 + 30*b*c^2*d^2*x^2 + 5*b*c^4 + 4*(5*b*c^
3 + 2*a)*d*x + 8*a*c)/(a^2*b^2*d^7*x^6 + 6*a^2*b^2*c*d^6*x^5 + 15*a^2*b^2*c
^2*d^5*x^4 + 2*(10*a^2*b^2*c^3 + a^3*b)*d^4*x^3 + 3*(5*a^2*b^2*c^4 + 2*a^3*
b*c)*d^3*x^2 + 6*(a^2*b^2*c^5 + a^3*b*c^2)*d^2*x + (a^2*b^2*c^6 + 2*a^3*b*c
^3 + a^4)*d) + 5/9*integrate(1/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b
*c^3 + a), x)/a^2
```

mupad [B] time = 1.61, size = 354, normalized size = 1.79

$$\frac{\frac{2x\left(5\,bc^3+2\,a\right)}{9a^2}+\frac{5\,bc^4+8\,ac}{18a^2d}+\frac{5\,bd^3x^4}{18a^2}+\frac{5\,bc^2dx^2}{3a^2}+\frac{10\,bcd^2x^3}{9a^2}}{x^3\left(20b^2c^3d^3+2\,a\,bd^3\right)+x^2\left(15b^2c^4d^2+6\,abcd^2\right)+a^2+x\left(6\,db^2c^5+6\,adbc^2\right)+b^2c^6+b^2d^6x^6+2\,a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(c + d*x)^3)^3,x)
```

```
[Out] ((2*x*(2*a + 5*b*c^3))/(9*a^2) + (8*a*c + 5*b*c^4)/(18*a^2*d) + (5*b*d^3*x^
4)/(18*a^2) + (5*b*c^2*d*x^2)/(3*a^2) + (10*b*c*d^2*x^3)/(9*a^2))/(x^3*(20*
b^2*c^3*d^3 + 2*a*b*d^3) + x^2*(15*b^2*c^4*d^2 + 6*a*b*c*d^2) + a^2 + x*(6*
b^2*c^5*d + 6*a*b*c^2*d) + b^2*c^6 + b^2*d^6*x^6 + 2*a*b*c^3 + 6*b^2*c*d^5*
x^5 + 15*b^2*c^2*d^4*x^4) + (5*log(b^(1/3)*c + a^(1/3) + b^(1/3)*d*x))/(27*
a^(8/3)*b^(1/3)*d) + (log((5*b^2*c*d^5)/(3*a^2) + (5*b^2*d^6*x)/(3*a^2) + (
b^(5/3)*d^5*(3^(1/2)*5i - 5))/(6*a^(5/3)))*(3^(1/2)*5i - 5))/(54*a^(8/3)*b^
(1/3)*d) - (log((5*b^2*c*d^5)/(3*a^2) + (5*b^2*d^6*x)/(3*a^2) - (b^(5/3)*d^
5*(3^(1/2)*5i + 5))/(6*a^(5/3)))*(3^(1/2)*5i + 5))/(54*a^(8/3)*b^(1/3)*d)
```

sympy [**A**] time = 3.27, size = 267, normalized size = 1.35

$$\frac{8ac + 5bc^4 + 30bc^2d^2x^2 + 20bcd^3x^3 + 5bd^4x^4 + x\left(8ad + 2\right.}{18a^4d + 36a^3bc^3d + 18a^2b^2c^6d + 270a^2b^2c^2d^5x^4 + 108a^2b^2cd^6x^5 + 18a^2b^2d^7x^6 + x^3\left(36a^3bd^4 + 360a^2b^2c^3d^4\right) + x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)**3)**3,x)
```

```
[Out] (8*a*c + 5*b*c**4 + 30*b*c**2*d**2*x**2 + 20*b*c*d**3*x**3 + 5*b*d**4*x**4
+ x*(8*a*d + 20*b*c**3*d))/(18*a**4*d + 36*a**3*b*c**3*d + 18*a**2*b**2*c**
6*d + 270*a**2*b**2*c**2*d**5*x**4 + 108*a**2*b**2*c*d**6*x**5 + 18*a**2*b*
*2*d**7*x**6 + x**3*(36*a**3*b*d**4 + 360*a**2*b**2*c**3*d**4) + x**2*(108*
a**3*b*c*d**3 + 270*a**2*b**2*c**4*d**3) + x*(108*a**3*b*c**2*d**2 + 108*a*
*2*b**2*c**5*d**2)) + RootSum(19683*_t**3*a**8*b - 125, Lambda(_t, _t*log(x
 + (27*_t*a**3 + 5*c)/(5*d))))/d
```

3.2882 $\int \frac{1}{(c+dx)\left(a+b(c+dx)^3\right)^3}\,dx$

Optimal. Leaf size=82

$$-\frac{\log\left(a+b(c+dx)^3\right)}{3a^3d}+\frac{\log(c+dx)}{a^3d}+\frac{1}{3a^2d\left(a+b(c+dx)^3\right)}+\frac{1}{6ad\left(a+b(c+dx)^3\right)^2}$$

```
[Out] 1/6/a/d/(a+b*(d*x+c)^3)^2+1/3/a^2/d/(a+b*(d*x+c)^3)+ln(d*x+c)/a^3/d-1/3*ln(
a+b*(d*x+c)^3)/a^3/d
```

Rubi [A] time = 0.08, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {372, 266, 44}

$$\frac{1}{3a^2d\left(a+b(c+dx)^3\right)}-\frac{\log\left(a+b(c+dx)^3\right)}{3a^3d}+\frac{\log(c+dx)}{a^3d}+\frac{1}{6ad\left(a+b(c+dx)^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)*(a + b*(c + d*x)^3)^3),x]
```

```
[Out] 1/(6*a*d*(a + b*(c + d*x)^3)^2) + 1/(3*a^2*d*(a + b*(c + d*x)^3)) + Log[c +
 d*x]/(a^3*d) - Log[a + b*(c + d*x)^3]/(3*a^3*d)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{(c+dx)\left(a+b(c+dx)^3\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx^3\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^3}\,dx, x, (c+dx)^3\right)}{3d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^3x} - \frac{b}{a(a+bx)^3} - \frac{b}{a^2(a+bx)^2} - \frac{b}{a^3(a+bx)}\right)\,dx, x, (c+dx)^3\right)}{3d}$$

$$= \frac{1}{6ad\left(a+b(c+dx)^3\right)^2} + \frac{1}{3a^2d\left(a+b(c+dx)^3\right)} + \frac{\log(c+dx)}{a^3d} - \frac{\log\left(a+b(c+$$

Mathematica [A] time = 0.06, size = 63, normalized size = 0.77

$$\frac{\frac{a\left(2\left(a+b(c+dx)^3\right)+a\right)}{\left(a+b(c+dx)^3\right)^2} - 2\log\left(a+b(c+dx)^3\right) + 6\log(c+dx)}{6a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)*(a + b*(c + d*x)^3)^3),x]
```

```
[Out] ((a*(a + 2*(a + b*(c + d*x)^3)))/(a + b*(c + d*x)^3)^2 + 6*Log[c + d*x] - 2
*Log[a + b*(c + d*x)^3])/(6*a^3*d)
```

fricas [B] time = 1.14, size = 467, normalized size = 5.70

$$\frac{2\,abd^3x^3 + 6\,abcd^2x^2 + 6\,abc^2dx + 2\,abc^3 + 3\,a^2 - 2\left(b^2d^6x^6 + 6\,b^2cd^5x^5 + 15\,b^2c^2d^4x^4 + b^2c^6 + 2\left(10\,b^2c^3 + ab\right)d\right.}{6\left(a^3b^2d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*b*d^3*x^3 + 6*a*b*c*d^2*x^2 + 6*a*b*c^2*d*x + 2*a*b*c^3 + 3*a^2 -
2*(b^2*d^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4 + b^2*c^6 + 2*(10*b^2
*c^3 + a*b)*d^3*x^3 + 2*a*b*c^3 + 3*(5*b^2*c^4 + 2*a*b*c)*d^2*x^2 + 6*(b^2*
c^5 + a*b*c^2)*d*x + a^2)*log(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c
^3 + a) + 6*(b^2*d^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4 + b^2*c^6 +
 2*(10*b^2*c^3 + a*b)*d^3*x^3 + 2*a*b*c^3 + 3*(5*b^2*c^4 + 2*a*b*c)*d^2*x^2
 + 6*(b^2*c^5 + a*b*c^2)*d*x + a^2)*log(d*x + c))/(a^3*b^2*d^7*x^6 + 6*a^3*
b^2*c*d^6*x^5 + 15*a^3*b^2*c^2*d^5*x^4 + 2*(10*a^3*b^2*c^3 + a^4*b)*d^4*x^3
 + 3*(5*a^3*b^2*c^4 + 2*a^4*b*c)*d^3*x^2 + 6*(a^3*b^2*c^5 + a^4*b*c^2)*d^2*
x + (a^3*b^2*c^6 + 2*a^4*b*c^3 + a^5)*d)
```

giac [A] time = 0.24, size = 144, normalized size = 1.76

$$-\frac{\log\left(\left|bd^3x^3 + 3\,bcd^2x^2 + 3\,bc^2dx + bc^3 + a\right|\right)}{3\,a^3d} + \frac{\log\left(|dx+c|\right)}{a^3d} + \frac{2\,abd^3x^3 + 6\,abcd^2x^2 + 6\,abc^2dx + 2\,abc^3 + 3\,a^2}{6\left(bd^3x^3 + 3\,bcd^2x^2 + 3\,bc^2dx + bc^3 + a\right)^2a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] -1/3*log(abs(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a))/(a^3*d)
+ log(abs(d*x + c))/(a^3*d) + 1/6*(2*a*b*d^3*x^3 + 6*a*b*c*d^2*x^2 + 6*a*b*
```

```
c^2*d*x + 2*a*b*c^3 + 3*a^2)/((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*
c^3 + a)^2*a^3*d)
```

maple [B] time = 0.03, size = 283, normalized size = 3.45

$$\frac{b\,d^2x^3}{3\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)^2a^2}+\frac{bcd\,x^2}{\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)^2a^2}+\frac{}{\left(b\,d^3x^3+3bc\,d^2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+b*(d*x+c)^3)^3,x)
```

```
[Out] ln(d*x+c)/a^3/d+1/3*b/a^2/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2*d
^2*x^3+b/a^2/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2*c*d*x^2+b/a^2/
(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2*c^2*x+1/3*b/a^2/(b*d^3*x^3+
3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2/d*c^3+1/2/a/(b*d^3*x^3+3*b*c*d^2*x^2+3
*b*c^2*d*x+b*c^3+a)^2/d-1/3/a^3/d*ln(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*
c^3+a)
```

maxima [B] time = 0.66, size = 245, normalized size = 2.99

$$\frac{2\,bd^3x^3+6\,bcd^2x^2+6\,bc^2dx+2\,bc^3+3\,a}{6\left(a^2b^2d^7x^6+6\,a^2b^2cd^6x^5+15\,a^2b^2c^2d^5x^4+2\left(10\,a^2b^2c^3+a^3b\right)d^4x^3+3\left(5\,a^2b^2c^4+2\,a^3bc\right)d^3x^2+6\left(a^2b^2c^5+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b*d^3*x^3 + 6*b*c*d^2*x^2 + 6*b*c^2*d*x + 2*b*c^3 + 3*a)/(a^2*b^2*d^
7*x^6 + 6*a^2*b^2*c*d^6*x^5 + 15*a^2*b^2*c^2*d^5*x^4 + 2*(10*a^2*b^2*c^3 +
a^3*b)*d^4*x^3 + 3*(5*a^2*b^2*c^4 + 2*a^3*b*c)*d^3*x^2 + 6*(a^2*b^2*c^5 + a
^3*b*c^2)*d^2*x + (a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4)*d) - 1/3*log(b*d^3*x^3
+ 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(a^3*d) + log(d*x + c)/(a^3*d)
```

mupad [B] time = 1.87, size = 230, normalized size = 2.80

$$\frac{\frac{2\,bc^3+3\,a}{6\,a^2d}+\frac{b\,d^2x^3}{3\,a^2}+\frac{b\,c^2x}{a^2}+\frac{b\,c\,d\,x^2}{a^2}}{x^3\left(20\,b^2c^3d^3+2\,a\,b\,d^3\right)+x^2\left(15\,b^2c^4d^2+6\,a\,b\,c\,d^2\right)+a^2+x\left(6\,d\,b^2c^5+6\,a\,d\,b\,c^2\right)+b^2c^6+b^2d^6x^6+2\,a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*(c + d*x)^3)^3*(c + d*x)),x)
```

```
[Out] ((3*a + 2*b*c^3)/(6*a^2*d) + (b*d^2*x^3)/(3*a^2) + (b*c^2*x)/a^2 + (b*c*d*x
^2)/a^2)/(x^3*(20*b^2*c^3*d^3 + 2*a*b*d^3) + x^2*(15*b^2*c^4*d^2 + 6*a*b*c*
d^2) + a^2 + x*(6*b^2*c^5*d + 6*a*b*c^2*d) + b^2*c^6 + b^2*d^6*x^6 + 2*a*b*
c^3 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4) - log(a + b*c^3 + b*d^3*x^3 + 3
*b*c^2*d*x + 3*b*c*d^2*x^2)/(3*a^3*d) + log(c + d*x)/(a^3*d)
```

sympy [B] time = 4.76, size = 269, normalized size = 3.28

$$\frac{3a+2bc^3+6bc^2dx+6bcd^2x^2+2bd^3x^3}{6a^4d+12a^3bc^3d+6a^2b^2c^6d+90a^2b^2c^2d^5x^4+36a^2b^2cd^6x^5+6a^2b^2d^7x^6+x^3\left(12a^3bd^4+120a^2b^2c^3d^4\right)+x^2\left(3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*(d*x+c)**3)**3,x)
```

```
[Out] (3*a + 2*b*c**3 + 6*b*c**2*d*x + 6*b*c*d**2*x**2 + 2*b*d**3*x**3)/(6*a**4*d
 + 12*a**3*b*c**3*d + 6*a**2*b**2*c**6*d + 90*a**2*b**2*c**2*d**5*x**4 + 36
```

```
*a**2*b**2*c*d**6*x**5 + 6*a**2*b**2*d**7*x**6 + x**3*(12*a**3*b*d**4 + 120
*a**2*b**2*c**3*d**4) + x**2*(36*a**3*b*c*d**3 + 90*a**2*b**2*c**4*d**3) +
x*(36*a**3*b*c**2*d**2 + 36*a**2*b**2*c**5*d**2)) + log(c/d + x)/(a**3*d) -
 log(3*c**2*x/d**2 + 3*c*x**2/d + x**3 + (a + b*c**3)/(b*d**3))/(3*a**3*d)
```

3.2883 $\int \frac{1}{(c+dx)^2\left(a+b(c+dx)^3\right)^3}\,dx$

Optimal. Leaf size=219

$$-\frac{7\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{27a^{10/3}d}+\frac{14\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{27a^{10/3}d}+\frac{14\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right.}{9\sqrt{3}\,a^{10/3}d}$$

```
[Out] -14/9/a^3/d/(d*x+c)+1/6/a/d/(d*x+c)/(a+b*(d*x+c)^3)^2+7/18/a^2/d/(d*x+c)/(a
+b*(d*x+c)^3)+14/27*b^(1/3)*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(10/3)/d-7/27*b^(
1/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(10/3)/d+14/27
*b^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(10/3)/d
*3^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {372, 290, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{7\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{27a^{10/3}d}+\frac{7}{18a^2d(c+dx)\left(a+b(c+dx)^3\right)}+\frac{14\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+\right.}{27a^{10/3}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)^2*(a + b*(c + d*x)^3)^3),x]
```

```
[Out] -14/(9*a^3*d*(c + d*x)) + 1/(6*a*d*(c + d*x)*(a + b*(c + d*x)^3)^2) + 7/(18
*a^2*d*(c + d*x)*(a + b*(c + d*x)^3)) + (14*b^(1/3)*ArcTan[(a^(1/3) - 2*b^(
1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(10/3)*d) + (14*b^(1/3)*Lo
g[a^(1/3) + b^(1/3)*(c + d*x)])/(27*a^(10/3)*d) - (7*b^(1/3)*Log[a^(2/3) -
a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(27*a^(10/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2\left(a+b(c+dx)^3\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^3\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{1}{6ad(c+dx)\left(a+b(c+dx)^3\right)^2} + \frac{7\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^3\right)^2}\,dx, x, c+dx\right)}{6ad}$$

$$= \frac{1}{6ad(c+dx)\left(a+b(c+dx)^3\right)^2} + \frac{7}{18a^2d(c+dx)\left(a+b(c+dx)^3\right)} + \frac{14\,\text{Subs}}{}$$

$$= -\frac{14}{9a^3d(c+dx)} + \frac{1}{6ad(c+dx)\left(a+b(c+dx)^3\right)^2} + \frac{7}{18a^2d(c+dx)\left(a+b(c+\right.}$$

$$= -\frac{14}{9a^3d(c+dx)} + \frac{1}{6ad(c+dx)\left(a+b(c+dx)^3\right)^2} + \frac{7}{18a^2d(c+dx)\left(a+b(c+\right.}$$

$$= -\frac{14}{9a^3d(c+dx)} + \frac{1}{6ad(c+dx)\left(a+b(c+dx)^3\right)^2} + \frac{7}{18a^2d(c+dx)\left(a+b(c+\right.}$$

$$= -\frac{14}{9a^3d(c+dx)} + \frac{1}{6ad(c+dx)\left(a+b(c+dx)^3\right)^2} + \frac{7}{18a^2d(c+dx)\left(a+b(c+\right.}$$

Mathematica [A] time = 0.16, size = 196, normalized size = 0.89

$$\frac{-14\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right) - \frac{9a^{4/3}b(c+dx)^2}{\left(a+b(c+dx)^3\right)^2} - \frac{30\sqrt[3]{a}\,b(c+dx)^2}{a+b(c+dx)^3} + 28\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx\right.}{54a^{10/3}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)^2*(a + b*(c + d*x)^3)^3),x]
```

```
[Out] ((-54*a^(1/3))/(c + d*x) - (9*a^(4/3)*b*(c + d*x)^2)/(a + b*(c + d*x)^3)^2
- (30*a^(1/3)*b*(c + d*x)^2)/(a + b*(c + d*x)^3) - 28*Sqrt[3]*b^(1/3)*ArcTa
n[(-a^(1/3) + 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))] + 28*b^(1/3)*Log[a^(1
/3) + b^(1/3)*(c + d*x)] - 14*b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*
x) + b^(2/3)*(c + d*x)^2])/(54*a^(10/3)*d)
```

fricas [B] time = 1.18, size = 855, normalized size = 3.90

$$-\frac{84\,b^2d^6x^6 + 504\,b^2cd^5x^5 + 1260\,b^2c^2d^4x^4 + 84\,b^2c^6 + 21\left(80\,b^2c^3+7\,ab\right)d^3x^3 + 147\,abc^3 + 63\left(20\,b^2c^4+7\,ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] -1/54*(84*b^2*d^6*x^6 + 504*b^2*c*d^5*x^5 + 1260*b^2*c^2*d^4*x^4 + 84*b^2*c
^6 + 21*(80*b^2*c^3 + 7*a*b)*d^3*x^3 + 147*a*b*c^3 + 63*(20*b^2*c^4 + 7*a*b
*c)*d^2*x^2 + 63*(8*b^2*c^5 + 7*a*b*c^2)*d*x + 28*sqrt(3)*(b^2*d^7*x^7 + 7*
b^2*c*d^6*x^6 + 21*b^2*c^2*d^5*x^5 + b^2*c^7 + (35*b^2*c^3 + 2*a*b)*d^4*x^4
 + (35*b^2*c^4 + 8*a*b*c)*d^3*x^3 + 2*a*b*c^4 + 3*(7*b^2*c^5 + 4*a*b*c^2)*d
^2*x^2 + a^2*c + (7*b^2*c^6 + 8*a*b*c^3 + a^2)*d*x)*(b/a)^(1/3)*arctan(2/3*
sqrt(3)*(d*x + c)*(b/a)^(1/3) - 1/3*sqrt(3)) + 14*(b^2*d^7*x^7 + 7*b^2*c*d^
6*x^6 + 21*b^2*c^2*d^5*x^5 + b^2*c^7 + (35*b^2*c^3 + 2*a*b)*d^4*x^4 + (35*b
^2*c^4 + 8*a*b*c)*d^3*x^3 + 2*a*b*c^4 + 3*(7*b^2*c^5 + 4*a*b*c^2)*d^2*x^2 +
 a^2*c + (7*b^2*c^6 + 8*a*b*c^3 + a^2)*d*x)*(b/a)^(1/3)*log(b*d^2*x^2 + 2*b
*c*d*x + b*c^2 - (a*d*x + a*c)*(b/a)^(2/3) + a*(b/a)^(1/3)) - 28*(b^2*d^7*x
^7 + 7*b^2*c*d^6*x^6 + 21*b^2*c^2*d^5*x^5 + b^2*c^7 + (35*b^2*c^3 + 2*a*b)*
d^4*x^4 + (35*b^2*c^4 + 8*a*b*c)*d^3*x^3 + 2*a*b*c^4 + 3*(7*b^2*c^5 + 4*a*b
*c^2)*d^2*x^2 + a^2*c + (7*b^2*c^6 + 8*a*b*c^3 + a^2)*d*x)*(b/a)^(1/3)*log(
b*d*x + b*c + a*(b/a)^(2/3)) + 54*a^2)/(a^3*b^2*d^8*x^7 + 7*a^3*b^2*c*d^7*x
^6 + 21*a^3*b^2*c^2*d^6*x^5 + (35*a^3*b^2*c^3 + 2*a^4*b)*d^5*x^4 + (35*a^3*
b^2*c^4 + 8*a^4*b*c)*d^4*x^3 + 3*(7*a^3*b^2*c^5 + 4*a^4*b*c^2)*d^3*x^2 + (7
*a^3*b^2*c^6 + 8*a^4*b*c^3 + a^5)*d^2*x + (a^3*b^2*c^7 + 2*a^4*b*c^4 + a^5*
c)*d)
```

giac [A] time = 0.25, size = 223, normalized size = 1.02

$$\frac{14\left(\frac{b}{ad^3}\right)^{\frac{1}{3}}\log\left(\left|-\left(\frac{b}{ad^3}\right)^{\frac{1}{3}}-\frac{1}{(dx+c)d}\right|\right)}{27a^3}-\frac{14\sqrt{3}\left(a^2b\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(\frac{b}{ad^3}\right)^{\frac{1}{3}}-\frac{2}{(dx+c)d}\right)}{3\left(\frac{b}{ad^3}\right)^{\frac{1}{3}}}\right)}{27a^4d}-\frac{7\left(a^2b\right)^{\frac{1}{3}}\log\left(\left(\frac{b}{ad^3}\right)^{\frac{2}{3}}-\frac{\left(\frac{b}{ad^3}\right)^{\frac{1}{3}}}{(dx+c)d}+\right.}{27a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 14/27*(b/(a*d^3))^(1/3)*log(abs(-(b/(a*d^3))^(1/3) - 1/((d*x + c)*d)))/a^3
- 14/27*sqrt(3)*(a^2*b)^(1/3)*arctan(1/3*sqrt(3)*((b/(a*d^3))^(1/3) - 2/((d
*x + c)*d))/(b/(a*d^3))^(1/3))/(a^4*d) - 7/27*(a^2*b)^(1/3)*log((b/(a*d^3))
^(2/3) - (b/(a*d^3))^(1/3)/((d*x + c)*d) + 1/((d*x + c)^2*d^2))/(a^4*d) - 1
/18*(10*b^2/((d*x + c)*d) + 13*a*b/((d*x + c)^4*d))/(a^3*(b + a/(d*x + c)^3
)^2) - 1/((d*x + c)*a^3*d)
```

maple [C] time = 0.03, size = 524, normalized size = 2.39

$$-\frac{5b^2d^4x^5}{9\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)^2a^3}-\frac{25b^2c\,d^3x^4}{9\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)^2a^3}-\frac{}{9\left(b\,d^3x^3+3bc\,d^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+b*(d*x+c)^3)^3,x)
```

```
[Out] -1/a^3/d/(d*x+c)-5/9*b^2/a^3/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^
2*d^4*x^5-25/9*b^2/a^3/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2*c*d^
3*x^4-50/9*b^2/a^3/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2*c^2*d^2*
x^3-50/9*b^2/a^3/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2*x^2*c^3*d-
13/18*b/a^2/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2*x^2*d-25/9*b^2/
a^3/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2*x*c^4-13/9*b/a^2/(b*d^3
*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2*c*x-5/9*b^2/a^3/(b*d^3*x^3+3*b*c*
d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2*c^5/d-13/18*b/a^2/(b*d^3*x^3+3*b*c*d^2*x^2+3
```

```
*b*c^2*d*x+b*c^3+a)^2*c^2/d-14/27/a^3/d*sum((_R*d+c)/(_R^2*d^2+2*_R*c*d+c^2
)*ln(-_R+x),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{28\,b^2d^6x^6+168\,b^2cd^5x^5+420\,b^2c^2d^4x^4+28\,b^2c^6+7\left(80\,b^2c^3+7\,ab\right)d^3x^3+49\,abc^3+}{18\left(a^3b^2d^8x^7+7\,a^3b^2cd^7x^6+21\,a^3b^2c^2d^6x^5+\left(35\,a^3b^2c^3+2\,a^4b\right)d^5x^4+\left(35\,a^3b^2c^4+8\,a^4bc\right)d^4x^3+3\left(7\,a^3b^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] -1/18*(28*b^2*d^6*x^6 + 168*b^2*c*d^5*x^5 + 420*b^2*c^2*d^4*x^4 + 28*b^2*c^
6 + 7*(80*b^2*c^3 + 7*a*b)*d^3*x^3 + 49*a*b*c^3 + 21*(20*b^2*c^4 + 7*a*b*c)
*d^2*x^2 + 21*(8*b^2*c^5 + 7*a*b*c^2)*d*x + 18*a^2)/(a^3*b^2*d^8*x^7 + 7*a^
3*b^2*c*d^7*x^6 + 21*a^3*b^2*c^2*d^6*x^5 + (35*a^3*b^2*c^3 + 2*a^4*b)*d^5*x
^4 + (35*a^3*b^2*c^4 + 8*a^4*b*c)*d^4*x^3 + 3*(7*a^3*b^2*c^5 + 4*a^4*b*c^2)
*d^3*x^2 + (7*a^3*b^2*c^6 + 8*a^4*b*c^3 + a^5)*d^2*x + (a^3*b^2*c^7 + 2*a^4
*b*c^4 + a^5*c)*d) - 14/9*b*integrate((d*x + c)/(b*d^3*x^3 + 3*b*c*d^2*x^2
+ 3*b*c^2*d*x + b*c^3 + a), x)/a^3
```

mupad [B] time = 2.58, size = 431, normalized size = 1.97

$$\frac{14\,b^{1/3}\,\ln\left(b^{1/3}\,c+a^{1/3}+b^{1/3}\,d\,x\right)}{27\,a^{10/3}\,d}-\frac{\frac{18\,a^2+49\,a\,b\,c^3+28\,b^2\,c^6}{18\,a^3\,d}+\frac{7\,x^2\left(20\,d\,b^2\,c^4+7\,a\,d\,b\,c\right)}{6\,a^3}+}{x^4\left(35\,b^2\,c^3\,d^4+2\,a\,b\,d^4\right)+x\left(d\,a^2+8\,d\,a\,b\,c^3+7\,d\,b^2\,c^6\right)+a^2\,c+x^3\left(35\,b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*(c + d*x)^3)^3*(c + d*x)^2),x)
```

```
[Out] (14*b^(1/3)*log(b^(1/3)*c + a^(1/3) + b^(1/3)*d*x))/(27*a^(10/3)*d) - ((18*
a^2 + 28*b^2*c^6 + 49*a*b*c^3)/(18*a^3*d) + (7*x^2*(20*b^2*c^4*d + 7*a*b*c*
d))/(6*a^3) + (7*x*(8*b^2*c^5 + 7*a*b*c^2))/(6*a^3) + (7*x^3*(80*b^2*c^3*d^
2 + 7*a*b*d^2))/(18*a^3) + (14*b^2*d^5*x^6)/(9*a^3) + (70*b^2*c^2*d^3*x^4)/
(3*a^3) + (28*b^2*c*d^4*x^5)/(3*a^3))/(x^4*(35*b^2*c^3*d^4 + 2*a*b*d^4) + x
*(a^2*d + 7*b^2*c^6*d + 8*a*b*c^3*d) + a^2*c + x^3*(35*b^2*c^4*d^3 + 8*a*b*
c*d^3) + b^2*c^7 + x^2*(21*b^2*c^5*d^2 + 12*a*b*c^2*d^2) + b^2*d^7*x^7 + 2*
a*b*c^4 + 7*b^2*c*d^6*x^6 + 21*b^2*c^2*d^5*x^5) + (14*b^(1/3)*log(2*b^(1/3)
*c - 3^(1/2)*a^(1/3)*1i - a^(1/3) + 2*b^(1/3)*d*x)*((3^(1/2)*1i)/2 - 1/2))/
(27*a^(10/3)*d) - (14*b^(1/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*c - a^(1/3
) + 2*b^(1/3)*d*x)*((3^(1/2)*1i)/2 + 1/2))/(27*a^(10/3)*d)
```

sympy [B] time = 5.97, size = 391, normalized size = 1.79

$$\frac{-18a^2-49abc^3-28b^2c^6-420b^2c^2d^4x^4-168b^2cd^5x^5-28b^2d^6x^6+x^3\left(-49a\right.}{18a^5cd+36a^4bc^4d+18a^3b^2c^7d+378a^3b^2c^2d^6x^5+126a^3b^2cd^7x^6+18a^3b^2d^8x^7+x^4\left(36a^4bd^5+630a^3b^2c^3d^5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+b*(d*x+c)**3)**3,x)
```

```
[Out] (-18*a**2 - 49*a*b*c**3 - 28*b**2*c**6 - 420*b**2*c**2*d**4*x**4 - 168*b**2
*c*d**5*x**5 - 28*b**2*d**6*x**6 + x**3*(-49*a*b*d**3 - 560*b**2*c**3*d**3)
 + x**2*(-147*a*b*c*d**2 - 420*b**2*c**4*d**2) + x*(-147*a*b*c**2*d - 168*b
**2*c**5*d))/(18*a**5*c*d + 36*a**4*b*c**4*d + 18*a**3*b**2*c**7*d + 378*a*
*3*b**2*c**2*d**6*x**5 + 126*a**3*b**2*c*d**7*x**6 + 18*a**3*b**2*d**8*x**7
```

```
 + x**4*(36*a**4*b*d**5 + 630*a**3*b**2*c**3*d**5) + x**3*(144*a**4*b*c*d**
4 + 630*a**3*b**2*c**4*d**4) + x**2*(216*a**4*b*c**2*d**3 + 378*a**3*b**2*c
**5*d**3) + x*(18*a**5*d**2 + 144*a**4*b*c**3*d**2 + 126*a**3*b**2*c**6*d**
2)) + RootSum(19683*_t**3*a**10 - 2744*b, Lambda(_t, _t*log(x + (729*_t**2*
a**7 + 196*b*c)/(196*b*d))))/d
```

3.2884 $\int \frac{1}{(c+dx)^3\left(a+b(c+dx)^3\right)^3}\,dx$

Optimal. Leaf size=219

$$-\frac{20b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{27a^{11/3}d}+\frac{10b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{27a^{11/3}d}+\frac{20b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+d}{\sqrt{3}\,\sqrt[3]{a}}\right.}{9\sqrt{3}\,a^{11/3}d}$$

```
[Out] -10/9/a^3/d/(d*x+c)^2+1/6/a/d/(d*x+c)^2/(a+b*(d*x+c)^3)^2+4/9/a^2/d/(d*x+c)
^2/(a+b*(d*x+c)^3)-20/27*b^(2/3)*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(11/3)/d+10/
27*b^(2/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(11/3)/d
+20/27*b^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(1
1/3)/d*3^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {372, 290, 325, 200, 31, 634, 617, 204, 628}

$$-\frac{20b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{27a^{11/3}d}+\frac{10b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{27a^{11/3}d}+\frac{20b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+d}{\sqrt{3}\,\sqrt[3]{a}}\right.}{9\sqrt{3}\,a^{11/3}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)^3*(a + b*(c + d*x)^3)^3),x]
```

```
[Out] -10/(9*a^3*d*(c + d*x)^2) + 1/(6*a*d*(c + d*x)^2*(a + b*(c + d*x)^3)^2) + 4
/(9*a^2*d*(c + d*x)^2*(a + b*(c + d*x)^3)) + (20*b^(2/3)*ArcTan[(a^(1/3) -
2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(11/3)*d) - (20*b^(2/
3)*Log[a^(1/3) + b^(1/3)*(c + d*x)])/(27*a^(11/3)*d) + (10*b^(2/3)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(27*a^(11/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(c+dx)^3\left(a+b(c+dx)^3\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3\left(a+bx^3\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{1}{6ad(c+dx)^2\left(a+b(c+dx)^3\right)^2} + \frac{4\,\text{Subst}\left(\int \frac{1}{x^3\left(a+bx^3\right)^2}\,dx, x, c+dx\right)}{3ad}$$

$$= \frac{1}{6ad(c+dx)^2\left(a+b(c+dx)^3\right)^2} + \frac{4}{9a^2d(c+dx)^2\left(a+b(c+dx)^3\right)} + \frac{20\,\text{Sub}}{}$$

$$= -\frac{10}{9a^3d(c+dx)^2} + \frac{1}{6ad(c+dx)^2\left(a+b(c+dx)^3\right)^2} + \frac{4}{9a^2d(c+dx)^2\left(a+b(c\right.}$$

$$= -\frac{10}{9a^3d(c+dx)^2} + \frac{1}{6ad(c+dx)^2\left(a+b(c+dx)^3\right)^2} + \frac{4}{9a^2d(c+dx)^2\left(a+b(c\right.}$$

$$= -\frac{10}{9a^3d(c+dx)^2} + \frac{1}{6ad(c+dx)^2\left(a+b(c+dx)^3\right)^2} + \frac{4}{9a^2d(c+dx)^2\left(a+b(c\right.}$$

$$= -\frac{10}{9a^3d(c+dx)^2} + \frac{1}{6ad(c+dx)^2\left(a+b(c+dx)^3\right)^2} + \frac{4}{9a^2d(c+dx)^2\left(a+b(c\right.}$$

$$= -\frac{10}{9a^3d(c+dx)^2} + \frac{1}{6ad(c+dx)^2\left(a+b(c+dx)^3\right)^2} + \frac{4}{9a^2d(c+dx)^2\left(a+b(c\right.}$$

Mathematica [A] time = 0.16, size = 192, normalized size = 0.88

$$\frac{20b^{2/3}\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}(c+dx) + b^{2/3}(c+dx)^2\right) - \frac{9a^{5/3}b(c+dx)}{\left(a+b(c+dx)^3\right)^2} - \frac{33a^{2/3}b(c+dx)}{a+b(c+dx)^3} - \frac{27a^{2/3}}{(c+dx)^2} - 40b^{2/3}\log\left(\sqrt[3]{a} + \sqrt[3]{b}\right.}{54a^{11/3}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)^3*(a + b*(c + d*x)^3)^3),x]
```

```
[Out] ((-27*a^(2/3))/(c + d*x)^2 - (9*a^(5/3)*b*(c + d*x))/(a + b*(c + d*x)^3)^2
- (33*a^(2/3)*b*(c + d*x))/(a + b*(c + d*x)^3) - 40*Sqrt[3]*b^(2/3)*ArcTan[
(-a^(1/3) + 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))] - 40*b^(2/3)*Log[a^(1/3
) + b^(1/3)*(c + d*x)] + 20*b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x)
 + b^(2/3)*(c + d*x)^2])/(54*a^(11/3)*d)
```

fricas [B] time = 1.28, size = 1024, normalized size = 4.68

$$\frac{60\,b^2d^6x^6 + 360\,b^2cd^5x^5 + 900\,b^2c^2d^4x^4 + 60\,b^2c^6 + 48\left(25\,b^2c^3 + 2\,ab\right)d^3x^3 + 96\,abc^3 + 36\left(25\,b^2c^4 + 8\,abc\right.}{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3/(a+b*(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] -1/54*(60*b^2*d^6*x^6 + 360*b^2*c*d^5*x^5 + 900*b^2*c^2*d^4*x^4 + 60*b^2*c^
6 + 48*(25*b^2*c^3 + 2*a*b)*d^3*x^3 + 96*a*b*c^3 + 36*(25*b^2*c^4 + 8*a*b*c
)*d^2*x^2 + 72*(5*b^2*c^5 + 4*a*b*c^2)*d*x - 40*sqrt(3)*(b^2*d^8*x^8 + 8*b^
2*c*d^7*x^7 + 28*b^2*c^2*d^6*x^6 + 2*(28*b^2*c^3 + a*b)*d^5*x^5 + b^2*c^8 +
 10*(7*b^2*c^4 + a*b*c)*d^4*x^4 + 2*a*b*c^5 + 4*(14*b^2*c^5 + 5*a*b*c^2)*d^
3*x^3 + (28*b^2*c^6 + 20*a*b*c^3 + a^2)*d^2*x^2 + a^2*c^2 + 2*(4*b^2*c^7 +
5*a*b*c^4 + a^2*c)*d*x)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*(a*d*x + a*c
)*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) + 20*(b^2*d^8*x^8 + 8*b^2*c*d^7*x^7 + 28
*b^2*c^2*d^6*x^6 + 2*(28*b^2*c^3 + a*b)*d^5*x^5 + b^2*c^8 + 10*(7*b^2*c^4 +
 a*b*c)*d^4*x^4 + 2*a*b*c^5 + 4*(14*b^2*c^5 + 5*a*b*c^2)*d^3*x^3 + (28*b^2*
c^6 + 20*a*b*c^3 + a^2)*d^2*x^2 + a^2*c^2 + 2*(4*b^2*c^7 + 5*a*b*c^4 + a^2*
c)*d*x)*(-b^2/a^2)^(1/3)*log(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + a^2*(-b^
2/a^2)^(2/3) + (a*b*d*x + a*b*c)*(-b^2/a^2)^(1/3)) - 40*(b^2*d^8*x^8 + 8*b^
2*c*d^7*x^7 + 28*b^2*c^2*d^6*x^6 + 2*(28*b^2*c^3 + a*b)*d^5*x^5 + b^2*c^8 +
 10*(7*b^2*c^4 + a*b*c)*d^4*x^4 + 2*a*b*c^5 + 4*(14*b^2*c^5 + 5*a*b*c^2)*d^
3*x^3 + (28*b^2*c^6 + 20*a*b*c^3 + a^2)*d^2*x^2 + a^2*c^2 + 2*(4*b^2*c^7 +
5*a*b*c^4 + a^2*c)*d*x)*(-b^2/a^2)^(1/3)*log(b*d*x + b*c - a*(-b^2/a^2)^(1/
3)) + 27*a^2)/(a^3*b^2*d^9*x^8 + 8*a^3*b^2*c*d^8*x^7 + 28*a^3*b^2*c^2*d^7*x
^6 + 2*(28*a^3*b^2*c^3 + a^4*b)*d^6*x^5 + 10*(7*a^3*b^2*c^4 + a^4*b*c)*d^5*
x^4 + 4*(14*a^3*b^2*c^5 + 5*a^4*b*c^2)*d^4*x^3 + (28*a^3*b^2*c^6 + 20*a^4*b
*c^3 + a^5)*d^3*x^2 + 2*(4*a^3*b^2*c^7 + 5*a^4*b*c^4 + a^5*c)*d^2*x + (a^3*
b^2*c^8 + 2*a^4*b*c^5 + a^5*c^2)*d)
```

giac [B] time = 0.27, size = 365, normalized size = 1.67

$$\frac{10\left(2\sqrt{3}\left(-\frac{b^2}{a^2d^3}\right)^{\frac{1}{3}}\arctan\left(-\frac{bdx+bc-\left(-ab^2\right)^{\frac{1}{3}}}{\sqrt{3}\,bdx+\sqrt{3}\,bc+\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}}\right)-\left(-\frac{b^2}{a^2d^3}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}\,bdx+\sqrt{3}\,bc+\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\right)^2+4\left(bdx\right.\right.\right.}{27\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3/(a+b*(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 10/27*(2*sqrt(3)*(-b^2/(a^2*d^3))^(1/3)*arctan(-(b*d*x + b*c - (-a*b^2)^(1/
3))/(sqrt(3)*b*d*x + sqrt(3)*b*c + sqrt(3)*(-a*b^2)^(1/3))) - (-b^2/(a^2*d^
3))^(1/3)*log(4*(sqrt(3)*b*d*x + sqrt(3)*b*c + sqrt(3)*(-a*b^2)^(1/3))^2 +
4*(b*d*x + b*c - (-a*b^2)^(1/3))^2) + 2*(-b^2/(a^2*d^3))^(1/3)*log(abs(-b*d
*x - b*c + (-a*b^2)^(1/3))))/a^3 - 1/18*(20*b^2*d^6*x^6 + 120*b^2*c*d^5*x^5
 + 300*b^2*c^2*d^4*x^4 + 400*b^2*c^3*d^3*x^3 + 300*b^2*c^4*d^2*x^2 + 120*b^
2*c^5*d*x + 20*b^2*c^6 + 32*a*b*d^3*x^3 + 96*a*b*c*d^2*x^2 + 96*a*b*c^2*d*x
 + 32*a*b*c^3 + 9*a^2)/((b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*
c^3*d*x + b*c^4 + a*d*x + a*c)^2*a^3*d)
```

maple [C] time = 0.03, size = 419, normalized size = 1.91

$$-\frac{11b^2d^3x^4}{18\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)^2a^3}-\frac{22b^2c\,d^2x^3}{9\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)^2a^3}-\frac{}{3\left(b\,d^3x^3+3bc\,d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^3/(a+b*(d*x+c)^3)^3,x)
```

```
[Out] -1/2/a^3/d/(d*x+c)^2-11/18*b^2/a^3/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c
^3+a)^2*d^3*x^4-22/9*b^2/a^3/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^
2*c*d^2*x^3-11/3*b^2/a^3/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2*c^
```

```
2*d*x^2-22/9*b^2/a^3/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2*x*c^3-
7/9*b/a^2/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2*x-11/18*b^2/a^3/(
b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2*c^4/d-7/9*b/a^2/(b*d^3*x^3+3
*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2*c/d-20/27/a^3/d*sum(1/(_R^2*d^2+2*_R*c*
d+c^2)*ln(-_R+x),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{20\,b^2d^6x^6+120\,b^2cd^5x^5+300\,b^2c^2d^4x^4+20\,b^2c^6+16\left(25\,b^2c^3+2\right.}{18\left(a^3b^2d^9x^8+8\,a^3b^2cd^8x^7+28\,a^3b^2c^2d^7x^6+2\left(28\,a^3b^2c^3+a^4b\right)d^6x^5+10\left(7\,a^3b^2c^4+a^4bc\right)d^5x^4+4\left(14\,a^3b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3/(a+b*(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] -1/18*(20*b^2*d^6*x^6 + 120*b^2*c*d^5*x^5 + 300*b^2*c^2*d^4*x^4 + 20*b^2*c^
6 + 16*(25*b^2*c^3 + 2*a*b)*d^3*x^3 + 32*a*b*c^3 + 12*(25*b^2*c^4 + 8*a*b*c
)*d^2*x^2 + 24*(5*b^2*c^5 + 4*a*b*c^2)*d*x + 9*a^2)/(a^3*b^2*d^9*x^8 + 8*a^
3*b^2*c*d^8*x^7 + 28*a^3*b^2*c^2*d^7*x^6 + 2*(28*a^3*b^2*c^3 + a^4*b)*d^6*x
^5 + 10*(7*a^3*b^2*c^4 + a^4*b*c)*d^5*x^4 + 4*(14*a^3*b^2*c^5 + 5*a^4*b*c^2
)*d^4*x^3 + (28*a^3*b^2*c^6 + 20*a^4*b*c^3 + a^5)*d^3*x^2 + 2*(4*a^3*b^2*c^
7 + 5*a^4*b*c^4 + a^5*c)*d^2*x + (a^3*b^2*c^8 + 2*a^4*b*c^5 + a^5*c^2)*d) -
 20/9*b*integrate(1/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a),
x)/a^3
```

mupad [B] time = 2.69, size = 520, normalized size = 2.37

$$\frac{20\,b^{2/3}\,\ln\left(a^6\,b^{1/3}\,c-(-a)^{19/3}+a^6\,b^{1/3}\,d\,x\right)}{27\,(-a)^{11/3}\,d}-\frac{\frac{9\,a^2+32\,a\,b\,c}{18\,a^}}{x^5\left(56\,b^2\,c^3\,d^5+2\,a\,b\,d^5\right)+x^4\left(70\,b^2\,c^4\,d^4+10\,a\,b\,c\,d^4\right)+x\left(2\,d\,\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*(c + d*x)^3)^3*(c + d*x)^3),x)
```

```
[Out] (20*b^(2/3)*log(a^6*b^(1/3)*c - (-a)^(19/3) + a^6*b^(1/3)*d*x))/(27*(-a)^(1
1/3)*d) - ((9*a^2 + 20*b^2*c^6 + 32*a*b*c^3)/(18*a^3*d) + (2*x^2*(25*b^2*c^
4*d + 8*a*b*c*d))/(3*a^3) + (4*x*(5*b^2*c^5 + 4*a*b*c^2))/(3*a^3) + (8*x^3*
(25*b^2*c^3*d^2 + 2*a*b*d^2))/(9*a^3) + (10*b^2*d^5*x^6)/(9*a^3) + (50*b^2*
c^2*d^3*x^4)/(3*a^3) + (20*b^2*c*d^4*x^5)/(3*a^3))/(x^5*(56*b^2*c^3*d^5 + 2
*a*b*d^5) + x^4*(70*b^2*c^4*d^4 + 10*a*b*c*d^4) + x*(8*b^2*c^7*d + 2*a^2*c*
d + 10*a*b*c^4*d) + x^2*(a^2*d^2 + 28*b^2*c^6*d^2 + 20*a*b*c^3*d^2) + a^2*c
^2 + b^2*c^8 + x^3*(56*b^2*c^5*d^3 + 20*a*b*c^2*d^3) + b^2*d^8*x^8 + 2*a*b*
c^5 + 8*b^2*c*d^7*x^7 + 28*b^2*c^2*d^6*x^6) + (20*b^(2/3)*log(4860*a^6*b^3*
c*d^5 + 4860*a^6*b^3*d^6*x - 4860*(-a)^(19/3)*b^(8/3)*d^5*((3^(1/2)*1i)/2 -
 1/2))*((3^(1/2)*1i)/2 - 1/2))/(27*(-a)^(11/3)*d) - (20*b^(2/3)*log(4860*a^
6*b^3*c*d^5 + 4860*a^6*b^3*d^6*x + 4860*(-a)^(19/3)*b^(8/3)*d^5*((3^(1/2)*1
i)/2 + 1/2))*((3^(1/2)*1i)/2 + 1/2))/(27*(-a)^(11/3)*d)
```

sympy [B] time = 6.63, size = 435, normalized size = 1.99

$$\frac{-9a^2-32abc^3-20b^2c^6-300b^2c^2d^4x^4-120b^2cd^5x}{18a^5c^2d+36a^4bc^5d+18a^3b^2c^8d+504a^3b^2c^2d^7x^6+144a^3b^2cd^8x^7+18a^3b^2d^9x^8+x^5\left(36a^4bd^6+1008a^3b^2c^3d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**3/(a+b*(d*x+c)**3)**3,x)
```

```
[Out] (-9*a**2 - 32*a*b*c**3 - 20*b**2*c**6 - 300*b**2*c**2*d**4*x**4 - 120*b**2*
c*d**5*x**5 - 20*b**2*d**6*x**6 + x**3*(-32*a*b*d**3 - 400*b**2*c**3*d**3)
+ x**2*(-96*a*b*c*d**2 - 300*b**2*c**4*d**2) + x*(-96*a*b*c**2*d - 120*b**2
*c**5*d))/(18*a**5*c**2*d + 36*a**4*b*c**5*d + 18*a**3*b**2*c**8*d + 504*a*
*3*b**2*c**2*d**7*x**6 + 144*a**3*b**2*c*d**8*x**7 + 18*a**3*b**2*d**9*x**8
 + x**5*(36*a**4*b*d**6 + 1008*a**3*b**2*c**3*d**6) + x**4*(180*a**4*b*c*d*
*5 + 1260*a**3*b**2*c**4*d**5) + x**3*(360*a**4*b*c**2*d**4 + 1008*a**3*b**
2*c**5*d**4) + x**2*(18*a**5*d**3 + 360*a**4*b*c**3*d**3 + 504*a**3*b**2*c*
*6*d**3) + x*(36*a**5*c*d**2 + 180*a**4*b*c**4*d**2 + 144*a**3*b**2*c**7*d*
*2)) + RootSum(19683*_t**3*a**11 + 8000*b**2, Lambda(_t, _t*log(x + (-27*_t
*a**4 + 20*b*c)/(20*b*d))))/d
```

3.2885 $\int \frac{1}{(c+dx)^4\left(a+b(c+dx)^3\right)^3}\,dx$

Optimal. Leaf size=101

$$-\frac{3b\log(c+dx)}{a^4d}+\frac{b\log\left(a+b(c+dx)^3\right)}{a^4d}-\frac{2b}{3a^3d\left(a+b(c+dx)^3\right)}-\frac{1}{3a^3d(c+dx)^3}-\frac{b}{6a^2d\left(a+b(c+dx)^3\right)^2}$$

```
[Out] -1/3/a^3/d/(d*x+c)^3-1/6*b/a^2/d/(a+b*(d*x+c)^3)^2-2/3*b/a^3/d/(a+b*(d*x+c)
^3)-3*b*ln(d*x+c)/a^4/d+b*ln(a+b*(d*x+c)^3)/a^4/d
```

Rubi [A] time = 0.10, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {372, 266, 44}

$$-\frac{2b}{3a^3d\left(a+b(c+dx)^3\right)}-\frac{b}{6a^2d\left(a+b(c+dx)^3\right)^2}-\frac{3b\log(c+dx)}{a^4d}+\frac{b\log\left(a+b(c+dx)^3\right)}{a^4d}-\frac{1}{3a^3d(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)^4*(a + b*(c + d*x)^3)^3),x]
```

```
[Out] -1/(3*a^3*d*(c + d*x)^3) - b/(6*a^2*d*(a + b*(c + d*x)^3)^2) - (2*b)/(3*a^3
*d*(a + b*(c + d*x)^3)) - (3*b*Log[c + d*x])/(a^4*d) + (b*Log[a + b*(c + d*
x)^3])/(a^4*d)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^4\left(a+b(c+dx)^3\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^3\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx)^3}\,dx, x, (c+dx)^3\right)}{3d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^3x^2} - \frac{3b}{a^4x} + \frac{b^2}{a^2(a+bx)^3} + \frac{2b^2}{a^3(a+bx)^2} + \frac{3b^2}{a^4(a+bx)}\right)dx, x, (c+dx)^3\right)}{3d}$$

$$= -\frac{1}{3a^3d(c+dx)^3} - \frac{b}{6a^2d\left(a+b(c+dx)^3\right)^2} - \frac{2b}{3a^3d\left(a+b(c+dx)^3\right)} - \frac{3b\log(c$$

Mathematica [A] time = 0.12, size = 80, normalized size = 0.79

$$\frac{a\left(-\frac{4b}{a+b(c+dx)^3} - \frac{ab}{\left(a+b(c+dx)^3\right)^2} - \frac{2}{(c+dx)^3}\right) + 6b\log\left(a+b(c+dx)^3\right) - 18b\log(c+dx)}{6a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)^4*(a + b*(c + d*x)^3)^3),x]
```

```
[Out] (a*(-2/(c + d*x)^3 - (a*b)/(a + b*(c + d*x)^3)^2 - (4*b)/(a + b*(c + d*x)^3
)) - 18*b*Log[c + d*x] + 6*b*Log[a + b*(c + d*x)^3])/(6*a^4*d)
```

fricas [B] time = 1.20, size = 889, normalized size = 8.80

$$-\frac{6\,ab^2d^6x^6 + 36\,ab^2cd^5x^5 + 90\,ab^2c^2d^4x^4 + 6\,ab^2c^6 + 3\left(40\,ab^2c^3 + 3\,a^2b\right)d^3x^3 + 9\,a^2bc^3 + 9\left(10\,ab^2c^4 + 3\,a^2bc\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^4/(a+b*(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] -1/6*(6*a*b^2*d^6*x^6 + 36*a*b^2*c*d^5*x^5 + 90*a*b^2*c^2*d^4*x^4 + 6*a*b^2
*c^6 + 3*(40*a*b^2*c^3 + 3*a^2*b)*d^3*x^3 + 9*a^2*b*c^3 + 9*(10*a*b^2*c^4 +
 3*a^2*b*c)*d^2*x^2 + 2*a^3 + 9*(4*a*b^2*c^5 + 3*a^2*b*c^2)*d*x - 6*(b^3*d^
9*x^9 + 9*b^3*c*d^8*x^8 + 36*b^3*c^2*d^7*x^7 + 2*(42*b^3*c^3 + a*b^2)*d^6*x
^6 + b^3*c^9 + 6*(21*b^3*c^4 + 2*a*b^2*c)*d^5*x^5 + 2*a*b^2*c^6 + 6*(21*b^3
*c^5 + 5*a*b^2*c^2)*d^4*x^4 + (84*b^3*c^6 + 40*a*b^2*c^3 + a^2*b)*d^3*x^3 +
 a^2*b*c^3 + 3*(12*b^3*c^7 + 10*a*b^2*c^4 + a^2*b*c)*d^2*x^2 + 3*(3*b^3*c^8
 + 4*a*b^2*c^5 + a^2*b*c^2)*d*x)*log(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*
x + b*c^3 + a) + 18*(b^3*d^9*x^9 + 9*b^3*c*d^8*x^8 + 36*b^3*c^2*d^7*x^7 + 2
*(42*b^3*c^3 + a*b^2)*d^6*x^6 + b^3*c^9 + 6*(21*b^3*c^4 + 2*a*b^2*c)*d^5*x^
5 + 2*a*b^2*c^6 + 6*(21*b^3*c^5 + 5*a*b^2*c^2)*d^4*x^4 + (84*b^3*c^6 + 40*a
*b^2*c^3 + a^2*b)*d^3*x^3 + a^2*b*c^3 + 3*(12*b^3*c^7 + 10*a*b^2*c^4 + a^2*
b*c)*d^2*x^2 + 3*(3*b^3*c^8 + 4*a*b^2*c^5 + a^2*b*c^2)*d*x)*log(d*x + c))/(
a^4*b^2*d^10*x^9 + 9*a^4*b^2*c*d^9*x^8 + 36*a^4*b^2*c^2*d^8*x^7 + 2*(42*a^4
*b^2*c^3 + a^5*b)*d^7*x^6 + 6*(21*a^4*b^2*c^4 + 2*a^5*b*c)*d^6*x^5 + 6*(21*
a^4*b^2*c^5 + 5*a^5*b*c^2)*d^5*x^4 + (84*a^4*b^2*c^6 + 40*a^5*b*c^3 + a^6)*
d^4*x^3 + 3*(12*a^4*b^2*c^7 + 10*a^5*b*c^4 + a^6*c)*d^3*x^2 + 3*(3*a^4*b^2*
c^8 + 4*a^5*b*c^5 + a^6*c^2)*d^2*x + (a^4*b^2*c^9 + 2*a^5*b*c^6 + a^6*c^3)*
d)
```

giac [A] time = 0.25, size = 80, normalized size = 0.79

$$\frac{b\log\left(\left|-b-\frac{a}{(dx+c)^3}\right|\right)}{a^4d}+\frac{5\,b^3+\frac{6\,ab^2}{(dx+c)^3}}{6\,a^4\left(b+\frac{a}{(dx+c)^3}\right)^2d}-\frac{1}{3\,(dx+c)^3a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^4/(a+b*(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] b*log(abs(-b - a/(d*x + c)^3))/(a^4*d) + 1/6*(5*b^3 + 6*a*b^2/(d*x + c)^3)/
(a^4*(b + a/(d*x + c)^3)^2*d) - 1/3/((d*x + c)^3*a^3*d)
```

maple [B] time = 0.03, size = 311, normalized size = 3.08

$$-\frac{2b^2d^2x^3}{3\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)^2a^3}-\frac{2b^2cd\,x^2}{\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)^2a^3}-\frac{}{\left(b\,d^3x^3+3bc\,d^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^4/(a+b*(d*x+c)^3)^3,x)
```

```
[Out] -1/3/a^3/d/(d*x+c)^3-3*b*ln(d*x+c)/a^4/d-2/3/a^3*b^2/(b*d^3*x^3+3*b*c*d^2*x
^2+3*b*c^2*d*x+b*c^3+a)^2*d^2*x^3-2/a^3*b^2/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^
2*d*x+b*c^3+a)^2*c*d*x^2-2/a^3*b^2/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c
^3+a)^2*c^2*x-2/3/a^3*b^2/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2/d
*c^3-5/6/a^2*b/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2/d+1/a^4*b/d*
ln(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)
```

maxima [B] time = 0.78, size = 438, normalized size = 4.34

$$-\frac{6\,b^2d^6x^6+36\,b^2cd^5x^5+90\,b^2c^2d^4x^4+6\,b^2}{6\left(a^3b^2d^{10}x^9+9\,a^3b^2cd^9x^8+36\,a^3b^2c^2d^8x^7+2\left(42\,a^3b^2c^3+a^4b\right)d^7x^6+6\left(21\,a^3b^2c^4+2\,a^4bc\right)d^6x^5+6\left(21\,a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^4/(a+b*(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(6*b^2*d^6*x^6 + 36*b^2*c*d^5*x^5 + 90*b^2*c^2*d^4*x^4 + 6*b^2*c^6 + 3
*(40*b^2*c^3 + 3*a*b)*d^3*x^3 + 9*a*b*c^3 + 9*(10*b^2*c^4 + 3*a*b*c)*d^2*x^
2 + 9*(4*b^2*c^5 + 3*a*b*c^2)*d*x + 2*a^2)/(a^3*b^2*d^10*x^9 + 9*a^3*b^2*c*
d^9*x^8 + 36*a^3*b^2*c^2*d^8*x^7 + 2*(42*a^3*b^2*c^3 + a^4*b)*d^7*x^6 + 6*(
21*a^3*b^2*c^4 + 2*a^4*b*c)*d^6*x^5 + 6*(21*a^3*b^2*c^5 + 5*a^4*b*c^2)*d^5*
x^4 + (84*a^3*b^2*c^6 + 40*a^4*b*c^3 + a^5)*d^4*x^3 + 3*(12*a^3*b^2*c^7 + 1
0*a^4*b*c^4 + a^5*c)*d^3*x^2 + 3*(3*a^3*b^2*c^8 + 4*a^4*b*c^5 + a^5*c^2)*d^
2*x + (a^3*b^2*c^9 + 2*a^4*b*c^6 + a^5*c^3)*d) + b*log(b*d^3*x^3 + 3*b*c*d^
2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(a^4*d) - 3*b*log(d*x + c)/(a^4*d)
```

mupad [B] time = 3.20, size = 438, normalized size = 4.34

$$\frac{b\,\ln\left(b\,c^3+3\,b\,c^2\,d\,x+3\,b\,c\,d^2\,x^2+b\,d^3\,x^3+a\right)}{a^4\,d}-\frac{}{x\left(3\,d\,a^2\,c^2+12\,d\,a\,b\,c^5+9\,d\,b^2\,c^8\right)+x^6\left(84\,b^2\,c^3\,d^6+2\,a\,b\,d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*(c + d*x)^3)^3*(c + d*x)^4),x)
```

```
[Out] (b*log(a + b*c^3 + b*d^3*x^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2))/(a^4*d) - ((2*
a^2 + 6*b^2*c^6 + 9*a*b*c^3)/(6*a^3*d) + (3*x^2*(10*b^2*c^4*d + 3*a*b*c*d))
```

```
/(2*a^3) + (3*x*(4*b^2*c^5 + 3*a*b*c^2))/(2*a^3) + (x^3*(40*b^2*c^3*d^2 + 3
*a*b*d^2))/(2*a^3) + (b^2*d^5*x^6)/a^3 + (15*b^2*c^2*d^3*x^4)/a^3 + (6*b^2*
c*d^4*x^5)/a^3)/(x*(3*a^2*c^2*d + 9*b^2*c^8*d + 12*a*b*c^5*d) + x^6*(84*b^2
*c^3*d^6 + 2*a*b*d^6) + x^2*(3*a^2*c*d^2 + 36*b^2*c^7*d^2 + 30*a*b*c^4*d^2)
 + x^5*(126*b^2*c^4*d^5 + 12*a*b*c*d^5) + x^3*(a^2*d^3 + 84*b^2*c^6*d^3 + 4
0*a*b*c^3*d^3) + a^2*c^3 + b^2*c^9 + x^4*(126*b^2*c^5*d^4 + 30*a*b*c^2*d^4)
 + b^2*d^9*x^9 + 2*a*b*c^6 + 9*b^2*c*d^8*x^8 + 36*b^2*c^2*d^7*x^7) - (3*b*l
og(c + d*x))/(a^4*d)
```

sympy [**B**] time = 7.80, size = 500, normalized size = 4.95

$$\frac{-2a^2 - 9abc^3 - 6b^2c^6 - 90b^2c^2d^4}{6a^5c^3d + 12a^4bc^6d + 6a^3b^2c^9d + 216a^3b^2c^2d^8x^7 + 54a^3b^2cd^9x^8 + 6a^3b^2d^{10}x^9 + x^6\left(12a^4bd^7 + 504a^3b^2c^3d^7\right) + x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**4/(a+b*(d*x+c)**3)**3,x)
```

```
[Out] (-2*a**2 - 9*a*b*c**3 - 6*b**2*c**6 - 90*b**2*c**2*d**4*x**4 - 36*b**2*c*d*
*5*x**5 - 6*b**2*d**6*x**6 + x**3*(-9*a*b*d**3 - 120*b**2*c**3*d**3) + x**2
*(-27*a*b*c*d**2 - 90*b**2*c**4*d**2) + x*(-27*a*b*c**2*d - 36*b**2*c**5*d)
)/(6*a**5*c**3*d + 12*a**4*b*c**6*d + 6*a**3*b**2*c**9*d + 216*a**3*b**2*c*
*2*d**8*x**7 + 54*a**3*b**2*c*d**9*x**8 + 6*a**3*b**2*d**10*x**9 + x**6*(12
*a**4*b*d**7 + 504*a**3*b**2*c**3*d**7) + x**5*(72*a**4*b*c*d**6 + 756*a**3
*b**2*c**4*d**6) + x**4*(180*a**4*b*c**2*d**5 + 756*a**3*b**2*c**5*d**5) +
x**3*(6*a**5*d**4 + 240*a**4*b*c**3*d**4 + 504*a**3*b**2*c**6*d**4) + x**2*
(18*a**5*c*d**3 + 180*a**4*b*c**4*d**3 + 216*a**3*b**2*c**7*d**3) + x*(18*a
**5*c**2*d**2 + 72*a**4*b*c**5*d**2 + 54*a**3*b**2*c**8*d**2)) - 3*b*log(c/
d + x)/(a**4*d) + b*log(3*c**2*x/d**2 + 3*c*x**2/d + x**3 + (a + b*c**3)/(b
*d**3))/(a**4*d)
```

3.2886 $\int \frac{(ce+dex)^4}{a+b(c+dx)^3}\,dx$

Optimal. Leaf size=168

$$\frac{a^{2/3}e^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3b^{5/3}d}-\frac{a^{2/3}e^4\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6b^{5/3}d}+\frac{a^{2/3}e^4\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{5/3}d}$$

```
[Out] 1/2*e^4*(d*x+c)^2/b/d+1/3*a^(2/3)*e^4*ln(a^(1/3)+b^(1/3)*(d*x+c))/b^(5/3)/d
-1/6*a^(2/3)*e^4*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/b^(5
/3)/d+1/3*a^(2/3)*e^4*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2
))/b^(5/3)/d*3^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {372, 321, 292, 31, 634, 617, 204, 628}

$$\frac{a^{2/3}e^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3b^{5/3}d}-\frac{a^{2/3}e^4\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6b^{5/3}d}+\frac{a^{2/3}e^4\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{5/3}d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*(c + d*x)^3),x]
```

```
[Out] (e^4*(c + d*x)^2)/(2*b*d) + (a^(2/3)*e^4*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d
*x))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(5/3)*d) + (a^(2/3)*e^4*Log[a^(1/3) + b
^(1/3)*(c + d*x)])/(3*b^(5/3)*d) - (a^(2/3)*e^4*Log[a^(2/3) - a^(1/3)*b^(1/
3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*b^(5/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
```

```
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{(ce+dex)^4}{a+b(c+dx)^3}\,dx &= \frac{e^4\,\text{Subst}\left(\int \frac{x^4}{a+bx^3}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^4(c+dx)^2}{2bd} - \frac{\left(ae^4\right)\text{Subst}\left(\int \frac{x}{a+bx^3}\,dx, x, c+dx\right)}{bd} \\
&= \frac{e^4(c+dx)^2}{2bd} + \frac{\left(a^{2/3}e^4\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, c+dx\right)}{3b^{4/3}d} - \frac{\left(a^{2/3}e^4\right)\text{Subst}\left(\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}}\right.}{3b^{4/3}d} \\
&= \frac{e^4(c+dx)^2}{2bd} + \frac{a^{2/3}e^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3b^{5/3}d} - \frac{\left(a^{2/3}e^4\right)\text{Subst}\left(\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, c\right.}{6b^{5/3}d} \\
&= \frac{e^4(c+dx)^2}{2bd} + \frac{a^{2/3}e^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3b^{5/3}d} - \frac{a^{2/3}e^4\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+\right.}{6b^{5/3}d} \\
&= \frac{e^4(c+dx)^2}{2bd} + \frac{a^{2/3}e^4\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{5/3}d} + \frac{a^{2/3}e^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3b^{5/3}d} - \frac{a^{2/3}e^4\log\left(a^{2/3}\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 163, normalized size = 0.97

$$e^4\left(\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3b^{5/3}d} - \frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6b^{5/3}d} - \frac{a^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{b}\,(c+dx)-\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{5/3}d} + \right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*(c + d*x)^3),x]
```

```
[Out] e^4*((c + d*x)^2/(2*b*d) - (a^(2/3)*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c + d*x))
/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(5/3)*d) + (a^(2/3)*Log[a^(1/3) + b^(1/3)*(
c + d*x)])/(3*b^(5/3)*d) - (a^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x)
 + b^(2/3)*(c + d*x)^2])/(6*b^(5/3)*d))
```

fricas [A] time = 0.99, size = 179, normalized size = 1.07

$$\frac{3\,d^2e^4x^2 + 6\,cde^4x - 2\sqrt{3}\,e^4\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,(bdx+bc)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}} - \sqrt{3}\,a}{3\,a}\right) - e^4\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ad^2x^2 + 2\,acdx + ac^2 - (bdx + \right.}{6\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/6*(3*d^2*e^4*x^2 + 6*c*d*e^4*x - 2*sqrt(3)*e^4*(a^2/b^2)^(1/3)*arctan(1/3
*(2*sqrt(3)*(b*d*x + b*c)*(a^2/b^2)^(1/3) - sqrt(3)*a)/a) - e^4*(a^2/b^2)^(
1/3)*log(a*d^2*x^2 + 2*a*c*d*x + a*c^2 - (b*d*x + b*c)*(a^2/b^2)^(2/3) + a*
(a^2/b^2)^(1/3)) + 2*e^4*(a^2/b^2)^(1/3)*log(a*d*x + a*c + b*(a^2/b^2)^(2/3
)))/(b*d)
```

giac [A] time = 0.24, size = 170, normalized size = 1.01

$$\frac{bd^7x^2e^4 + 2\,bcd^6xe^4}{2\,b^2d^6} - \frac{2\sqrt{3}\,\left(a^2bd^{15}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,abdx+2\,abc-\left(a^2b\right)^{\frac{2}{3}}\right)}{3\left(a^2b\right)^{\frac{2}{3}}}\right)e^4 + \left(a^2bd^{15}\right)^{\frac{1}{3}}e^4\log\left(\left(2\,abdx + 2\,abc - \right.\right.}{6\,b^2d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/2*(b*d^7*x^2*e^4 + 2*b*c*d^6*x*e^4)/(b^2*d^6) - 1/6*(2*sqrt(3)*(a^2*b*d^1
5)^(1/3)*arctan(1/3*sqrt(3)*(2*a*b*d*x + 2*a*b*c - (a^2*b)^(2/3))/(a^2*b)^(
2/3))*e^4 + (a^2*b*d^15)^(1/3)*e^4*log((2*a*b*d*x + 2*a*b*c - (a^2*b)^(2/3)
)^2 + 3*(a^2*b)^(4/3)) - 2*(a^2*b*d^15)^(1/3)*e^4*log(abs(a*b*d*x + a*b*c +
 (a^2*b)^(2/3))))/(b^2*d^6)
```

maple [C] time = 0.00, size = 102, normalized size = 0.61

$$\frac{d\,e^4x^2}{2b} + \frac{c\,e^4x}{b} - \frac{a\,e^4\left(\text{RootOf}\left(b\,d^3_Z^3 + 3b\,d^2c\,_Z^2 + 3bd\,c^2_Z + b\,c^3 + a\right)d + c\right)\ln\left(-\text{RootOf}\left(b\,d^3_Z^3 + 3b\,d\right.\right.}{3b^2d\left(d^2\,\text{RootOf}\left(b\,d^3_Z^3 + 3b\,d^2c\,_Z^2 + 3bd\,c^2_Z + b\,c^3 + a\right)^2 + 2cd\,\text{RootOf}\left(b\,d^3_Z^3 + 3b\,d^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*(d*x+c)^3),x)
```

```
[Out] 1/2*e^4/b*x^2*d+e^4/b*c*x-1/3*e^4/b^2/d*sum((_R*d+c)/(_R^2*d^2+2*_R*c*d+c^2
)*ln(-_R+x),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))*a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{-\frac{1}{6}\left(2\sqrt{3}\,\left(-\frac{1}{ab^2d^3}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)}{3\left(-a^2b\right)^{\frac{2}{3}}}\right) + \left(-\frac{1}{ab^2d^3}\right)^{\frac{1}{3}}\log\left(\left(2\,abdx + 2\,abc - \left(-a^2b\right)^{\frac{2}{3}}\right)^2 + 3\,\left(-a^2b\right.\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -a*e^4*integrate((d*x + c)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3
 + a), x)/b + 1/2*(d*e^4*x^2 + 2*c*e^4*x)/b
```

mupad [B] time = 1.70, size = 151, normalized size = 0.90

$$\frac{d\,e^4\,x^2}{2\,b}+\frac{c\,e^4\,x}{b}+\frac{a^{2/3}\,e^4\,\ln\left(b^{1/3}\,c+a^{1/3}+b^{1/3}\,d\,x\right)}{3\,b^{5/3}\,d}+\frac{a^{2/3}\,e^4\,\ln\left(a^{1/3}-2\,b^{1/3}\,c-2\,b^{1/3}\,d\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{b^{5/3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4/(a + b*(c + d*x)^3),x)
```

```
[Out] (d*e^4*x^2)/(2*b) + (c*e^4*x)/b + (a^(2/3)*e^4*log(b^(1/3)*c + a^(1/3) + b^
(1/3)*d*x))/(3*b^(5/3)*d) + (a^(2/3)*e^4*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)
*c + a^(1/3) - 2*b^(1/3)*d*x)*((3^(1/2)*1i)/6 - 1/6))/(b^(5/3)*d) - (a^(2/3
)*e^4*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*c - a^(1/3) + 2*b^(1/3)*d*x)*((3^(
1/2)*1i)/2 + 1/2))/(3*b^(5/3)*d)
```

sympy [A] time = 0.54, size = 66, normalized size = 0.39

$$\frac{e^4\,\mathrm{RootSum}\left(27t^3b^5-a^2,\left(t\mapsto t\log\left(x+\frac{9t^2b^3e^8+ace^8}{ade^8}\right)\right)\right)}{d}+\frac{ce^4x}{b}+\frac{de^4x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*(d*x+c)**3),x)
```

```
[Out] e**4*RootSum(27*_t**3*b**5 - a**2, Lambda(_t, _t*log(x + (9*_t**2*b**3*e**8
 + a*c*e**8)/(a*d*e**8))))/d + c*e**4*x/b + d*e**4*x**2/(2*b)
```

3.2887 $\int \frac{(ce+dex)^3}{a+b(c+dx)^3}\,dx$

Optimal. Leaf size=156

$$\frac{\sqrt[3]{a}\,e^3\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6b^{4/3}d}-\frac{\sqrt[3]{a}\,e^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3b^{4/3}d}+\frac{\sqrt[3]{a}\,e^3\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{4/3}d}$$

```
[Out] e^3*x/b-1/3*a^(1/3)*e^3*ln(a^(1/3)+b^(1/3)*(d*x+c))/b^(4/3)/d+1/6*a^(1/3)*e
^3*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/b^(4/3)/d+1/3*a^(1
/3)*e^3*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/b^(4/3)/d*3
^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {372, 321, 200, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{a}\,e^3\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6b^{4/3}d}-\frac{\sqrt[3]{a}\,e^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3b^{4/3}d}+\frac{\sqrt[3]{a}\,e^3\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{4/3}d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*(c + d*x)^3),x]
```

```
[Out] (e^3*x)/b + (a^(1/3)*e^3*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^
(1/3))])/(Sqrt[3]*b^(4/3)*d) - (a^(1/3)*e^3*Log[a^(1/3) + b^(1/3)*(c + d*x)
])/(3*b^(4/3)*d) + (a^(1/3)*e^3*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b
^(2/3)*(c + d*x)^2])/(6*b^(4/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
```

```
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{(ce+dex)^3}{a+b(c+dx)^3}\,dx &= \frac{e^3\,\text{Subst}\left(\int \frac{x^3}{a+bx^3}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^3 x}{b} - \frac{\left(ae^3\right)\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, c+dx\right)}{bd} \\
&= \frac{e^3 x}{b} - \frac{\left(\sqrt[3]{a}\,e^3\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, c+dx\right)}{3bd} - \frac{\left(\sqrt[3]{a}\,e^3\right)\text{Subst}\left(\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x,\right.}{3bd} \\
&= \frac{e^3 x}{b} - \frac{\sqrt[3]{a}\,e^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3b^{4/3}d} + \frac{\left(\sqrt[3]{a}\,e^3\right)\text{Subst}\left(\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, c+dx\right)}{6b^{4/3}d} - \\
&= \frac{e^3 x}{b} - \frac{\sqrt[3]{a}\,e^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3b^{4/3}d} + \frac{\sqrt[3]{a}\,e^3\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6b^{4/3}d} - \\
&= \frac{e^3 x}{b} + \frac{\sqrt[3]{a}\,e^3\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{4/3}d} - \frac{\sqrt[3]{a}\,e^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3b^{4/3}d} + \frac{\sqrt[3]{a}\,e^3\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{}\right.}{6}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 145, normalized size = 0.93

$$\frac{e^3\left(\sqrt[3]{a}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)-2\sqrt[3]{a}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)-2\sqrt{3}\,\sqrt[3]{a}\,\tan^{-1}\left(\frac{2\sqrt[3]{b}\,(c+dx)-\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right.\right.}{6b^{4/3}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*(c + d*x)^3),x]
```

```
[Out] (e^3*(6*b^(1/3)*c + 6*b^(1/3)*d*x - 2*Sqrt[3]*a^(1/3)*ArcTan[(-a^(1/3) + 2*
b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))] - 2*a^(1/3)*Log[a^(1/3) + b^(1/3)*(c
```

```
+ d*x)] + a^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*
x)^2]))/(6*b^(4/3)*d)
```

fricas [A] time = 1.01, size = 147, normalized size = 0.94

$$\frac{6\,de^3x + 2\sqrt{3}\,e^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,(bdx+bc)\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,a}{3\,a}\right) - e^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(d^2x^2 + 2\,cdx + c^2 + (dx+c)\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\right.\right.}{6\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/6*(6*d*e^3*x + 2*sqrt(3)*e^3*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*(b*d*x +
b*c)*(-a/b)^(2/3) - sqrt(3)*a)/a) - e^3*(-a/b)^(1/3)*log(d^2*x^2 + 2*c*d*x
+ c^2 + (d*x + c)*(-a/b)^(1/3) + (-a/b)^(2/3)) + 2*e^3*(-a/b)^(1/3)*log(d*x
 + c - (-a/b)^(1/3)))/(b*d)
```

giac [A] time = 0.22, size = 188, normalized size = 1.21

$$\frac{xe^3}{b} + \frac{2\sqrt{3}\left(-\frac{ad^6e^9}{b}\right)^{\frac{1}{3}}\arctan\left(-\frac{bdx+bc-\left(-ab^2\right)^{\frac{1}{3}}}{\sqrt{3}\,bdx+\sqrt{3}\,bc+\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}}\right) - \left(-\frac{ad^6e^9}{b}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}\,bdx + \sqrt{3}\,bc + \sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\right)^2 + \right.}{6\,bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] x*e^3/b + 1/6*(2*sqrt(3)*(-a*d^6*e^9/b)^(1/3)*arctan(-(b*d*x + b*c - (-a*b^
2)^(1/3))/(sqrt(3)*b*d*x + sqrt(3)*b*c + sqrt(3)*(-a*b^2)^(1/3))) - (-a*d^6
*e^9/b)^(1/3)*log(4*(sqrt(3)*b*d*x + sqrt(3)*b*c + sqrt(3)*(-a*b^2)^(1/3))^
2 + 4*(b*d*x + b*c - (-a*b^2)^(1/3))^2) + 2*(-a*d^6*e^9/b)^(1/3)*log(abs(-b
*d*x - b*c + (-a*b^2)^(1/3))))/(b*d^3)
```

maple [C] time = 0.00, size = 84, normalized size = 0.54

$$\frac{e^3x}{b} - \frac{a\,e^3\ln\left(-\operatorname{RootOf}\left(b\,d^3_Z^3 + 3b\,d^2c\,_Z^2 + 3bd\,c^2_Z + b\,c^3 + a\right) + x\right)}{3b^2d\left(d^2\operatorname{RootOf}\left(b\,d^3_Z^3 + 3b\,d^2c\,_Z^2 + 3bd\,c^2_Z + b\,c^3 + a\right)^2 + 2cd\operatorname{RootOf}\left(b\,d^3_Z^3 + 3b\,d^2c\,_Z^2 + 3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*(d*x+c)^3),x)
```

```
[Out] e^3*x/b-1/3*e^3/b^2/d*sum(1/(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x),_R=RootOf(_Z^
3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))*a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}\,ae^3\left(\frac{2\sqrt{3}\left(\frac{1}{a^2b}\right)^{\frac{1}{3}}\arctan\left(-\frac{bdx+bc+\left(ab^2\right)^{\frac{1}{3}}}{\sqrt{3}\,bdx+\sqrt{3}\,bc-\sqrt{3}\left(ab^2\right)^{\frac{1}{3}}}\right)}{d} - \frac{\left(\frac{1}{a^2b}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}\,bdx+\sqrt{3}\,bc-\sqrt{3}\left(ab^2\right)^{\frac{1}{3}}\right)^2+4\left(bdx+bc+\left(ab^2\right)^{\frac{1}{3}}\right)^2\right)}{d} + \frac{2\left(\frac{1}{a^2b}\right)^{\frac{1}{3}}\log\right.$$

$$-\frac{}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -a*e^3*integrate(1/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x
)/b + e^3*x/b
```

mupad [B] time = 1.52, size = 162, normalized size = 1.04

$$\frac{e^3\,x}{b}+\frac{(-a)^{1/3}\,e^3\,\ln\left((-a)^{4/3}+a\,b^{1/3}\,c+a\,b^{1/3}\,d\,x\right)}{3\,b^{4/3}\,d}-\frac{(-a)^{1/3}\,e^3\,\ln\left(2\,a\,b^{1/3}\,c-(-a)^{4/3}+2\,a\,b^{1/3}\,d\,x-\sqrt{3}\,(-a)^{4/3}\,1\mathrm{i}\right)\left($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3/(a + b*(c + d*x)^3),x)
```

```
[Out] (e^3*x)/b + ((-a)^(1/3)*e^3*log((-a)^(4/3) + a*b^(1/3)*c + a*b^(1/3)*d*x))/
(3*b^(4/3)*d) - ((-a)^(1/3)*e^3*log(2*a*b^(1/3)*c - 3^(1/2)*(-a)^(4/3)*1i -
 (-a)^(4/3) + 2*a*b^(1/3)*d*x)*((3^(1/2)*1i)/2 + 1/2))/(3*b^(4/3)*d) + ((-a
)^(1/3)*e^3*log(3^(1/2)*(-a)^(4/3)*1i - (-a)^(4/3) + 2*a*b^(1/3)*c + 2*a*b^
(1/3)*d*x)*((3^(1/2)*1i)/6 - 1/6))/(b^(4/3)*d)
```

sympy [A] time = 0.50, size = 44, normalized size = 0.28

$$\frac{e^3\,\mathrm{RootSum}\left(27t^3b^4+a,\left(t\mapsto t\log\left(x+\frac{-3tbe^3+ce^3}{de^3}\right)\right)\right)}{d}+\frac{e^3x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*(d*x+c)**3),x)
```

```
[Out] e**3*RootSum(27*_t**3*b**4 + a, Lambda(_t, _t*log(x + (-3*_t*b*e**3 + c*e**
3)/(d*e**3))))/d + e**3*x/b
```

3.2888 $\int \frac{(ce+dex)^2}{a+b(c+dx)^3}\,dx$

Optimal. Leaf size=25

$$\frac{e^2 \log\left(a+b(c+dx)^3\right)}{3bd}$$

```
[Out] 1/3*e^2*ln(a+b*(d*x+c)^3)/b/d
```

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {372, 260}

$$\frac{e^2 \log\left(a+b(c+dx)^3\right)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*(c + d*x)^3),x]
```

```
[Out] (e^2*Log[a + b*(c + d*x)^3])/(3*b*d)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(ce+dex)^2}{a+b(c+dx)^3}\,dx &= \frac{e^2\,\text{Subst}\left(\int \frac{x^2}{a+bx^3}\,dx, x, c+dx\right)}{d}\\ &= \frac{e^2 \log\left(a+b(c+dx)^3\right)}{3bd}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{e^2 \log\left(a+b(c+dx)^3\right)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*(c + d*x)^3),x]
```

```
[Out] (e^2*Log[a + b*(c + d*x)^3])/(3*b*d)
```

fricas [A] time = 0.76, size = 45, normalized size = 1.80

$$\frac{e^2 \log\left(bd^3x^3 + 3\,bcd^2x^2 + 3\,bc^2dx + bc^3 + a\right)}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*(d*x+c)^3),x, algorithm="fricas")
[Out] 1/3*e^2*log(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(b*d)
```

giac [A] time = 0.19, size = 45, normalized size = 1.80

$$\frac{e^2 \log\left(\left|bd^3x^3 + 3\,bcd^2x^2 + 3\,bc^2dx + bc^3 + a\right|\right)}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*(d*x+c)^3),x, algorithm="giac")
[Out] 1/3*e^2*log(abs(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a))/(b*d)
```

maple [A] time = 0.00, size = 46, normalized size = 1.84

$$\frac{e^2 \ln\left(b\,d^3x^3 + 3bc\,d^2x^2 + 3b\,c^2dx + b\,c^3 + a\right)}{3bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*(d*x+c)^3),x)
[Out] 1/3*e^2/b/d*ln(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)
```

maxima [A] time = 0.52, size = 45, normalized size = 1.80

$$\frac{e^2 \log\left(bd^3x^3 + 3\,bcd^2x^2 + 3\,bc^2dx + bc^3 + a\right)}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*(d*x+c)^3),x, algorithm="maxima")
[Out] 1/3*e^2*log(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(b*d)
```

mupad [B] time = 1.41, size = 45, normalized size = 1.80

$$\frac{e^2\,\ln\left(b\,c^3 + 3\,b\,c^2\,d\,x + 3\,b\,c\,d^2\,x^2 + b\,d^3\,x^3 + a\right)}{3\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2/(a + b*(c + d*x)^3),x)
[Out] (e^2*log(a + b*c^3 + b*d^3*x^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2))/(3*b*d)
```

sympy [B] time = 0.41, size = 46, normalized size = 1.84

$$\frac{e^2 \log\left(a + bc^3 + 3bc^2dx + 3bcd^2x^2 + bd^3x^3\right)}{3bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*(d*x+c)**3),x)
[Out] e**2*log(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)/(3*b*d)
```

3.2889 $\int \frac{ce+dex}{a+b(c+dx)^3}\,dx$

Optimal. Leaf size=143

$$\frac{e\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6\sqrt[3]{a}\,b^{2/3}d}-\frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3\sqrt[3]{a}\,b^{2/3}d}-\frac{e\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{2/3}d}$$

```
[Out] -1/3*e*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(1/3)/b^(2/3)/d+1/6*e*ln(a^(2/3)-a^(1/
3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(1/3)/b^(2/3)/d-1/3*e*arctan(1/3*(a
^(1/3)-2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(1/3)/b^(2/3)/d*3^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {372, 292, 31, 634, 617, 204, 628}

$$\frac{e\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6\sqrt[3]{a}\,b^{2/3}d}-\frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3\sqrt[3]{a}\,b^{2/3}d}-\frac{e\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{2/3}d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*(c + d*x)^3),x]
```

```
[Out] -((e*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^
(1/3)*b^(2/3)*d)) - (e*Log[a^(1/3) + b^(1/3)*(c + d*x)])/(3*a^(1/3)*b^(2/3)
*d) + (e*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6
*a^(1/3)*b^(2/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{ce+dex}{a+b(c+dx)^3}\,dx = \frac{e\,\text{Subst}\left(\int \frac{x}{a+bx^3}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, c+dx\right)}{3\sqrt[3]{a}\,\sqrt[3]{b}\,d} + \frac{e\,\text{Subst}\left(\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, c+dx\right)}{3\sqrt[3]{a}\,\sqrt[3]{b}\,d}$$

$$= -\frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3\sqrt[3]{a}\,b^{2/3}d} + \frac{e\,\text{Subst}\left(\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, c+dx\right)}{6\sqrt[3]{a}\,b^{2/3}d} + \frac{e\,\text{Subst}\left(\int \frac{}{a^{2/3}}\right.}{}$$

$$= -\frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3\sqrt[3]{a}\,b^{2/3}d} + \frac{e\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6\sqrt[3]{a}\,b^{2/3}d} + \frac{e\,\text{Subst}\left(\int -\right.}{}$$

$$= -\frac{e\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{2/3}d} - \frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3\sqrt[3]{a}\,b^{2/3}d} + \frac{e\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c\right.}{6\sqrt[3]{a}\,b^{2/3}d}$$

Mathematica [A] time = 0.01, size = 115, normalized size = 0.80

$$\frac{e\left(\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)+2\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{b}\,(c+dx)-\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)\right)}{6\sqrt[3]{a}\,b^{2/3}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)/(a + b*(c + d*x)^3),x]
```

```
[Out] (e*(2*Sqrt[3]*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))] -
2*Log[a^(1/3) + b^(1/3)*(c + d*x)] + Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x
) + b^(2/3)*(c + d*x)^2]))/(6*a^(1/3)*b^(2/3)*d)
```

fricas [A] time = 0.97, size = 458, normalized size = 3.20

$$\left[3\sqrt{\frac{1}{3}}\,abe\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\,\log\left(\frac{2b^2d^3x^3+6b^2cd^2x^2+6b^2c^2dx+2b^2c^3-ab+3\sqrt{\frac{1}{3}}\left(abdx+abc+2\left(d^2x^2+2cdx+c^2\right)\left(-ab^2\right)^{\frac{2}{3}}+\left(-ab^2\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}-3\left(-\right.\right.}{bd^3x^3+3bcd^2x^2+3bc^2dx+bc^3+a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*e*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*d^3*x^3 + 6*b^2*c
*d^2*x^2 + 6*b^2*c^2*d*x + 2*b^2*c^3 - a*b + 3*sqrt(1/3)*(a*b*d*x + a*b*c +
 2*(d^2*x^2 + 2*c*d*x + c^2)*(-a*b^2)^(2/3) + (-a*b^2)^(1/3)*a)*sqrt((-a*b^
2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*(d*x + c))/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*
c^2*d*x + b*c^3 + a)) + (-a*b^2)^(2/3)*e*log(b^2*d^2*x^2 + 2*b^2*c*d*x + b^
2*c^2 + (-a*b^2)^(1/3)*(b*d*x + b*c) + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*e
*log(b*d*x + b*c - (-a*b^2)^(1/3)))/(a*b^2*d), 1/6*(6*sqrt(1/3)*a*b*e*sqrt(
-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*d*x + 2*b*c + (-a*b^2)^(1/3))*sqrt
(-(-a*b^2)^(1/3)/a)/b) + (-a*b^2)^(2/3)*e*log(b^2*d^2*x^2 + 2*b^2*c*d*x + b
^2*c^2 + (-a*b^2)^(1/3)*(b*d*x + b*c) + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*
e*log(b*d*x + b*c - (-a*b^2)^(1/3)))/(a*b^2*d)]
```

giac [A] time = 0.22, size = 147, normalized size = 1.03

$$-\frac{1}{3}\sqrt{3}\left(-\frac{e^3}{ab^2d^3}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)}{3\left(-a^2b\right)^{\frac{2}{3}}}\right)-\frac{1}{6}\left(-\frac{e^3}{ab^2d^3}\right)^{\frac{1}{3}}\log\left(\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(-e^3/(a*b^2*d^3))^(1/3)*arctan(1/3*sqrt(3)*(2*a*b*d*x + 2*a*b
*c - (-a^2*b)^(2/3))/(-a^2*b)^(2/3)) - 1/6*(-e^3/(a*b^2*d^3))^(1/3)*log((2*
a*b*d*x + 2*a*b*c - (-a^2*b)^(2/3))^2 + 3*(-a^2*b)^(4/3)) + 1/3*(-e^3/(a*b^
2*d^3))^(1/3)*log(abs(a*b*d*x + a*b*c + (-a^2*b)^(2/3)))
```

maple [C] time = 0.00, size = 77, normalized size = 0.54

$$\frac{e\left(\text{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_Z+b\,c^3+a\right)d+c\right)\ln\left(-\text{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_\right.\right.}{3bd\left(d^2\,\text{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_Z+b\,c^3+a\right)^2+2cd\,\text{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*(d*x+c)^3),x)
```

```
[Out] 1/3*e/b/d*sum((_R*d+c)/(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x),_R=RootOf(_Z^3*b*d
^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dex+ce}{(dx+c)^3 b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)/((d*x + c)^3*b + a), x)
```

mupad [B] time = 0.52, size = 127, normalized size = 0.89

$$-\frac{e\,\ln\left(b^{1/3}c+a^{1/3}+b^{1/3}d\,x\right)}{3\,a^{1/3}\,b^{2/3}\,d}+\frac{\ln\left(2\,b^{1/3}c-a^{1/3}+2\,b^{1/3}d\,x-\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(e-\sqrt{3}\,e\,1\mathrm{i}\right)}{6\,a^{1/3}\,b^{2/3}\,d}+\frac{\ln\left(2\,b^{1/3}c-a^{1/3}+2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)/(a + b*(c + d*x)^3),x)
```

```
[Out] (log(2*b^(1/3)*c - 3^(1/2)*a^(1/3)*1i - a^(1/3) + 2*b^(1/3)*d*x)*(e - 3^(1/
2)*e*1i))/(6*a^(1/3)*b^(2/3)*d) - (e*log(b^(1/3)*c + a^(1/3) + b^(1/3)*d*x)
)/(3*a^(1/3)*b^(2/3)*d) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*c - a^(1/3) +
 2*b^(1/3)*d*x)*(e + 3^(1/2)*e*1i))/(6*a^(1/3)*b^(2/3)*d)
```

sympy [**A**] time = 0.41, size = 41, normalized size = 0.29

$$\frac{e\,\mathrm{RootSum}\left(27t^3ab^2+1,\left(t\mapsto t\log\left(x+\frac{9t^2abe^2+ce^2}{de^2}\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*(d*x+c)**3),x)
```

```
[Out] e*RootSum(27*_t**3*a*b**2 + 1, Lambda(_t, _t*log(x + (9*_t**2*a*b*e**2 + c*
e**2)/(d*e**2))))/d
```

3.2890 $\int \frac{1}{(ce+dex)\left(a+b(c+dx)^3\right)}\,dx$

Optimal. Leaf size=42

$$\frac{\log(c+dx)}{ade}-\frac{\log\left(a+b(c+dx)^3\right)}{3ade}$$

```
[Out] ln(d*x+c)/a/d/e-1/3*ln(a+b*(d*x+c)^3)/a/d/e
```

Rubi [A] time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {372, 266, 36, 29, 31}

$$\frac{\log(c+dx)}{ade}-\frac{\log\left(a+b(c+dx)^3\right)}{3ade}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*e + d*e*x)*(a + b*(c + d*x)^3)),x]
```

```
[Out] Log[c + d*x]/(a*d*e) - Log[a + b*(c + d*x)^3]/(3*a*d*e)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{(ce+dex)\left(a+b(c+dx)^3\right)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx^3\right)}\,dx, x, c+dx\right)}{de}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx, x, (c+dx)^3\right)}{3de}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, (c+dx)^3\right)}{3ade} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, (c+dx)^3\right)}{3ade}$$
$$= \frac{\log(c+dx)}{ade} - \frac{\log\left(a+b(c+dx)^3\right)}{3ade}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.00

$$\frac{\log(c+dx)}{ade} - \frac{\log\left(a+b(c+dx)^3\right)}{3ade}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*e + d*e*x)*(a + b*(c + d*x)^3)),x]
```

```
[Out] Log[c + d*x]/(a*d*e) - Log[a + b*(c + d*x)^3]/(3*a*d*e)
```

fricas [A] time = 1.09, size = 54, normalized size = 1.29

$$-\frac{\log\left(bd^3x^3 + 3\,bcd^2x^2 + 3\,bc^2dx + bc^3 + a\right) - 3\,\log(dx+c)}{3\,ade}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/3*(log(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a) - 3*log(d*x
+ c))/(a*d*e)
```

giac [A] time = 0.20, size = 62, normalized size = 1.48

$$-\frac{e^{(-1)}\log\left(\left|bd^3x^3 + 3\,bcd^2x^2 + 3\,bc^2dx + bc^3 + a\right|\right)}{3\,ad} + \frac{e^{(-1)}\log\left(|dx+c|\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] -1/3*e^(-1)*log(abs(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a))/(
a*d) + e^(-1)*log(abs(d*x + c))/(a*d)
```

maple [A] time = 0.00, size = 63, normalized size = 1.50

$$\frac{\ln(dx+c)}{ade} - \frac{\ln\left(b\,d^3x^3 + 3bc\,d^2x^2 + 3b\,c^2dx + b\,c^3 + a\right)}{3ade}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*(d*x+c)^3),x)
```

```
[Out] ln(d*x+c)/a/d/e-1/3/e/a/d*ln(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)
```

maxima [A] time = 0.50, size = 62, normalized size = 1.48

$$-\frac{\log\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)}{3\,ade}+\frac{\log\left(dx+c\right)}{ade}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -1/3*log(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(a*d*e) + log
(d*x + c)/(a*d*e)
```

mupad [B] time = 0.14, size = 62, normalized size = 1.48

$$\frac{\ln\left(c+d\,x\right)}{a\,d\,e}-\frac{\ln\left(b\,c^3+3\,b\,c^2\,d\,x+3\,b\,c\,d^2\,x^2+b\,d^3\,x^3+a\right)}{3\,a\,d\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)*(a + b*(c + d*x)^3)),x)
```

```
[Out] log(c + d*x)/(a*d*e) - log(a + b*c^3 + b*d^3*x^3 + 3*b*c^2*d*x + 3*b*c*d^2*
x^2)/(3*a*d*e)
```

sympy [A] time = 0.75, size = 53, normalized size = 1.26

$$\frac{\log\left(\frac{c}{d}+x\right)}{ade}-\frac{\log\left(\frac{3c^2x}{d^2}+\frac{3cx^2}{d}+x^3+\frac{a+bc^3}{bd^3}\right)}{3ade}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*(d*x+c)**3),x)
```

```
[Out] log(c/d + x)/(a*d*e) - log(3*c**2*x/d**2 + 3*c*x**2/d + x**3 + (a + b*c**3)
/(b*d**3))/(3*a*d*e)
```

3.2891 $\int \frac{1}{(ce+dex)^2\left(a+b(c+dx)^3\right)}\,dx$

Optimal. Leaf size=166

$$-\frac{\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{4/3}de^2}+\frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{4/3}de^2}+\frac{\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}de^2}-\frac{}{ade^2(}$$

```
[Out] -1/a/d/e^2/(d*x+c)+1/3*b^(1/3)*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(4/3)/d/e^2-1/
6*b^(1/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(4/3)/d/e
^2+1/3*b^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(4
/3)/d/e^2*3^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {372, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{4/3}de^2}+\frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{4/3}de^2}+\frac{\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}de^2}-\frac{}{ade^2(}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*e + d*e*x)^2*(a + b*(c + d*x)^3)),x]
```

```
[Out] -(1/(a*d*e^2*(c + d*x))) + (b^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/
(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(4/3)*d*e^2) + (b^(1/3)*Log[a^(1/3) + b^(1/3
)*(c + d*x)])/(3*a^(4/3)*d*e^2) - (b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c
 + d*x) + b^(2/3)*(c + d*x)^2])/(6*a^(4/3)*d*e^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
```

```
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(ce+dex)^2\left(a+b(c+dx)^3\right)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^3\right)}\,dx, x, c+dx\right)}{de^2}$$

$$= -\frac{1}{ade^2(c+dx)} - \frac{b\,\text{Subst}\left(\int \frac{x}{a+bx^3}\,dx, x, c+dx\right)}{ade^2}$$

$$= -\frac{1}{ade^2(c+dx)} + \frac{b^{2/3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, c+dx\right)}{3a^{4/3}de^2} - \frac{b^{2/3}\,\text{Subst}\left(\int \frac{}{a^{2/3}}\right.}{}$$

$$= -\frac{1}{ade^2(c+dx)} + \frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{4/3}de^2} - \frac{\sqrt[3]{b}\,\text{Subst}\left(\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+}\right.}{6a^{4/3}de}$$

$$= -\frac{1}{ade^2(c+dx)} + \frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{4/3}de^2} - \frac{\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+\right.}{6a^{4/3}d}$$

$$= -\frac{1}{ade^2(c+dx)} + \frac{\sqrt[3]{b}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{4/3}de^2} + \frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{4/3}de^2} -$$

Mathematica [A] time = 0.04, size = 143, normalized size = 0.86

$$\frac{-\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)+2\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)-2\sqrt{3}\,\sqrt[3]{b}\,\tan^{-1}\left(\frac{2\sqrt[3]{b}\,(c+dx)-\sqrt[3]{}}{\sqrt{3}\,\sqrt[3]{a}}\right.}{6a^{4/3}de^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*e + d*e*x)^2*(a + b*(c + d*x)^3)),x]
```

```
[Out] ((-6*a^(1/3))/(c + d*x) - 2*Sqrt[3]*b^(1/3)*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c
 + d*x))/(Sqrt[3]*a^(1/3))] + 2*b^(1/3)*Log[a^(1/3) + b^(1/3)*(c + d*x)] -
b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*
a^(4/3)*d*e^2)
```

fricas [A] time = 1.10, size = 156, normalized size = 0.94

$$-\frac{2\sqrt{3}\,(dx+c)\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\,(dx+c)\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+(dx+c)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bd^2x^2+2\,bcdx+bc^2-(adx+ac)\left(\right.\right.}{6\left(ad^2e^2x+acde^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^2/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(d*x + c)*(b/a)^(1/3)*arctan(2/3*sqrt(3)*(d*x + c)*(b/a)^(1
/3) - 1/3*sqrt(3)) + (d*x + c)*(b/a)^(1/3)*log(b*d^2*x^2 + 2*b*c*d*x + b*c^
2 - (a*d*x + a*c)*(b/a)^(2/3) + a*(b/a)^(1/3)) - 2*(d*x + c)*(b/a)^(1/3)*lo
g(b*d*x + b*c + a*(b/a)^(2/3)) + 6)/(a*d^2*e^2*x + a*c*d*e^2)
```

giac [A] time = 0.21, size = 226, normalized size = 1.36

$$\frac{\left(\frac{b}{ad^3}\right)^{\frac{1}{3}}e^{(-2)}\log\left(\left|-\left(\frac{b}{ad^3}\right)^{\frac{1}{3}}e^{(-2)}-\frac{e^{(-1)}}{(dxe+ce)d}\right|\right)}{3\,a}-\frac{\sqrt{3}\,\left(a^2b\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(\frac{b}{ad^3}\right)^{\frac{1}{3}}e^{(-2)}-\frac{2\,e^{(-1)}}{(dxe+ce)d}\right)e^2}{3\left(\frac{b}{ad^3}\right)^{\frac{1}{3}}}\right)e^{(-2)}}{3\,a^2d}-\frac{\left(a^2b\right)^{\frac{1}{3}}e^{(-2)}\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^2/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/3*(b/(a*d^3))^(1/3)*e^(-2)*log(abs(-(b/(a*d^3))^(1/3)*e^(-2) - e^(-1)/((d
*x*e + c*e)*d)))/a - 1/3*sqrt(3)*(a^2*b)^(1/3)*arctan(1/3*sqrt(3)*((b/(a*d^
3))^(1/3)*e^(-2) - 2*e^(-1)/((d*x*e + c*e)*d))*e^2/(b/(a*d^3))^(1/3))*e^(-2
)/(a^2*d) - 1/6*(a^2*b)^(1/3)*e^(-2)*log((b/(a*d^3))^(2/3)*e^(-4) - (b/(a*d
^3))^(1/3)*e^(-3)/((d*x*e + c*e)*d) + e^(-2)/((d*x*e + c*e)^2*d^2))/(a^2*d)
 - e^(-1)/((d*x*e + c*e)*a*d)
```

maple [C] time = 0.00, size = 98, normalized size = 0.59

$$-\frac{\left(\text{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_Z+b\,c^3+a\right)d+c\right)\ln\left(-\text{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_Z\right.\right.}{3ad\,e^2\left(d^2\,\text{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_Z+b\,c^3+a\right)^2+2cd\,\text{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)^2/(a+b*(d*x+c)^3),x)
```

```
[Out] -1/a/d/e^2/(d*x+c)-1/3/e^2/d*sum((_R*d+c)/(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x)
,_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{ad^2e^2x+acde^2}-\frac{-\frac{1}{6}\left(2\sqrt{3}\,\left(-\frac{1}{ab^2d^3}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)}{3\left(-a^2b\right)^{\frac{2}{3}}}\right)+\left(-\frac{1}{ab^2d^3}\right)^{\frac{1}{3}}\log\left(\left(2\,abdx+2\,abc-\left(-a^2b\right)\right.\right.}{ae^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^2/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -1/(a*d^2*e^2*x + a*c*d*e^2) - b*integrate((d*x + c)/(b*d^3*x^3 + 3*b*c*d^2
*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/(a*e^2)
```

mupad [B] time = 1.59, size = 154, normalized size = 0.93

$$\frac{b^{1/3}\,\ln\left(b^{1/3}\,c+a^{1/3}+b^{1/3}\,d\,x\right)}{3\,a^{4/3}\,d\,e^2}-\frac{1}{a\,d\,\left(c\,e^2+d\,e^2\,x\right)}-\frac{b^{1/3}\,\ln\left(2\,b^{1/3}\,c-a^{1/3}+2\,b^{1/3}\,d\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,a^{4/3}\,d\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)^2*(a + b*(c + d*x)^3)),x)
```

```
[Out] (b^(1/3)*log(b^(1/3)*c + a^(1/3) + b^(1/3)*d*x))/(3*a^(4/3)*d*e^2) - 1/(a*d
*(c*e^2 + d*e^2*x)) - (b^(1/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*c - a^(1/
3) + 2*b^(1/3)*d*x)*((3^(1/2)*1i)/2 + 1/2))/(3*a^(4/3)*d*e^2) + (b^(1/3)*lo
g(2*b^(1/3)*c - 3^(1/2)*a^(1/3)*1i - a^(1/3) + 2*b^(1/3)*d*x)*((3^(1/2)*1i)
/6 - 1/6))/(a^(4/3)*d*e^2)
```

sympy [A] time = 0.99, size = 54, normalized size = 0.33

$$-\frac{1}{acde^2+ad^2e^2x}+\frac{\operatorname{RootSum}\left(27t^3a^4-b,\left(t\mapsto t\log\left(x+\frac{9t^2a^3+bc}{bd}\right)\right)\right)}{de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)**2/(a+b*(d*x+c)**3),x)
```

```
[Out] -1/(a*c*d*e**2 + a*d**2*e**2*x) + RootSum(27*_t**3*a**4 - b, Lambda(_t, _t*
log(x + (9*_t**2*a**3 + b*c)/(b*d))))/(d*e**2)
```

3.2892 $\int \frac{1}{(ce+dex)^3\left(a+b(c+dx)^3\right)}\,dx$

Optimal. Leaf size=168

$$-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{5/3}de^3}+\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{5/3}de^3}+\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}de^3}-\frac{}{2ade^3}$$

```
[Out] -1/2/a/d/e^3/(d*x+c)^2-1/3*b^(2/3)*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(5/3)/d/e^
3+1/6*b^(2/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(5/3)
/d/e^3+1/3*b^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/
a^(5/3)/d/e^3*3^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {372, 325, 200, 31, 634, 617, 204, 628}

$$-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{5/3}de^3}+\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{5/3}de^3}+\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}de^3}-\frac{}{2ade^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*e + d*e*x)^3*(a + b*(c + d*x)^3)),x]
```

```
[Out] -1/(2*a*d*e^3*(c + d*x)^2) + (b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x)
)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)*d*e^3) - (b^(2/3)*Log[a^(1/3) + b^(1
/3)*(c + d*x)])/(3*a^(5/3)*d*e^3) + (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*
(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a^(5/3)*d*e^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
```

```
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(ce+dex)^3\left(a+b(c+dx)^3\right)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3\left(a+bx^3\right)}\,dx, x, c+dx\right)}{de^3}$$

$$= -\frac{1}{2ade^3(c+dx)^2} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, c+dx\right)}{ade^3}$$

$$= -\frac{1}{2ade^3(c+dx)^2} - \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, c+dx\right)}{3a^{5/3}de^3} - \frac{b\,\text{Subst}\left(\int \frac{2\sqrt[3]{}}{a^{2/3}-\sqrt[3]{}}\right.}{3}$$

$$= -\frac{1}{2ade^3(c+dx)^2} - \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{5/3}de^3} + \frac{b^{2/3}\,\text{Subst}\left(\int \frac{-\sqrt[3]{a}\sqrt[3]{b}}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{}}\right.}{6a^{5/3}}$$

$$= -\frac{1}{2ade^3(c+dx)^2} - \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{5/3}de^3} + \frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\right.}{6a^{5}}$$

$$= -\frac{1}{2ade^3(c+dx)^2} + \frac{b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{5/3}de^3} - \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{5/3}de^3}$$

Mathematica [A] time = 0.04, size = 142, normalized size = 0.85

$$\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right) - \frac{3a^{2/3}}{(c+dx)^2} - 2b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right) - 2\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{b}}{}\right.}{6a^{5/3}de^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*e + d*e*x)^3*(a + b*(c + d*x)^3)),x]
```

```
[Out] ((-3*a^(2/3))/(c + d*x)^2 - 2*Sqrt[3]*b^(2/3)*ArcTan[(-a^(1/3) + 2*b^(1/3)*
(c + d*x))/(Sqrt[3]*a^(1/3))] - 2*b^(2/3)*Log[a^(1/3) + b^(1/3)*(c + d*x)]
+ b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(
6*a^(5/3)*d*e^3)
```

fricas [A] time = 0.98, size = 245, normalized size = 1.46

$$\frac{2\sqrt{3}\left(d^2x^2+2cdx+c^2\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,(adx+ac)\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3b}\right)-\left(d^2x^2+2cdx+c^2\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2d^2x^2+2b^2c\right.}{6\left(ad^3e^3x^2+2acd^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^3/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*(d^2*x^2 + 2*c*d*x + c^2)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqr
t(3)*(a*d*x + a*c)*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) - (d^2*x^2 + 2*c*d*x +
c^2)*(-b^2/a^2)^(1/3)*log(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + a^2*(-b^2/a
^2)^(2/3) + (a*b*d*x + a*b*c)*(-b^2/a^2)^(1/3)) + 2*(d^2*x^2 + 2*c*d*x + c^
2)*(-b^2/a^2)^(1/3)*log(b*d*x + b*c - a*(-b^2/a^2)^(1/3)) - 3)/(a*d^3*e^3*x
^2 + 2*a*c*d^2*e^3*x + a*c^2*d*e^3)
```

giac [A] time = 0.22, size = 197, normalized size = 1.17

$$\frac{\left(2\sqrt{3}\left(-\frac{b^2}{a^2d^3}\right)^{\frac{1}{3}}\arctan\left(-\frac{bdx+bc-\left(-ab^2\right)^{\frac{1}{3}}}{\sqrt{3}\,bdx+\sqrt{3}\,bc+\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}}\right)-\left(-\frac{b^2}{a^2d^3}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}\,bdx+\sqrt{3}\,bc+\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\right)^2+4\left(bdx+\right.\right.\right.}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^3/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/6*(2*sqrt(3)*(-b^2/(a^2*d^3))^(1/3)*arctan(-(b*d*x + b*c - (-a*b^2)^(1/3)
)/(sqrt(3)*b*d*x + sqrt(3)*b*c + sqrt(3)*(-a*b^2)^(1/3))) - (-b^2/(a^2*d^3)
)^(1/3)*log(4*(sqrt(3)*b*d*x + sqrt(3)*b*c + sqrt(3)*(-a*b^2)^(1/3))^2 + 4*
(b*d*x + b*c - (-a*b^2)^(1/3))^2) + 2*(-b^2/(a^2*d^3))^(1/3)*log(abs(-b*d*x
 - b*c + (-a*b^2)^(1/3))))*e^(-3)/a - 1/2*e^(-3)/((d*x + c)^2*a*d)
```

maple [C] time = 0.01, size = 93, normalized size = 0.55

$$-\frac{\ln\left(-\operatorname{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_Z+b\,c^3+a\right)+x\right)}{3ad\,e^3\left(d^2\operatorname{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_Z+b\,c^3+a\right)^2+2cd\operatorname{RootOf}\left(b\,d^3_Z^3+3b\,d^2c\,_Z^2+3bd\,c^2_\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)^3/(a+b*(d*x+c)^3),x)
```

```
[Out] -1/2/a/d/e^3/(d*x+c)^2-1/3/e^3/d*sum(1/(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x),_R
=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2\left(ad^3e^3x^2+2acd^2e^3x+ac^2de^3\right)}-\frac{\frac{1}{6}b\left(\frac{2\sqrt{3}\left(\frac{1}{a^2b}\right)^{\frac{1}{3}}\arctan\left(-\frac{bdx+bc+\left(ab^2\right)^{\frac{1}{3}}}{\sqrt{3}\,bdx+\sqrt{3}\,bc-\sqrt{3}\left(ab^2\right)^{\frac{1}{3}}}\right)}{d}-\frac{\left(\frac{1}{a^2b}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}\,bdx+\sqrt{3}\,bc-\sqrt{3}\left(ab^2\right)^{\frac{1}{3}}\right)^2+\right.}{d}\right.}{ae^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^3/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -1/2/(a*d^3*e^3*x^2 + 2*a*c*d^2*e^3*x + a*c^2*d*e^3) - b*integrate(1/(b*d^3
*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/(a*e^3)
```

mupad [B] time = 0.42, size = 200, normalized size = 1.19

$$\frac{b^{2/3}\,\ln\left(a^2\,b^{1/3}\,c-(-a)^{7/3}+a^2\,b^{1/3}\,d\,x\right)}{3\,(-a)^{5/3}\,d\,e^3}-\frac{1}{2\,a\,d\,\left(c^2\,e^3+2\,c\,d\,e^3\,x+d^2\,e^3\,x^2\right)}-\frac{b^{2/3}\,\ln\left((-a)^{7/3}+2\,a^2\,b^{1/3}\,c+2\,a^2\right.}{3\,(-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)^3*(a + b*(c + d*x)^3)),x)
```

```
[Out] (b^(2/3)*log(a^2*b^(1/3)*c - (-a)^(7/3) + a^2*b^(1/3)*d*x))/(3*(-a)^(5/3)*d
*e^3) - 1/(2*a*d*(c^2*e^3 + d^2*e^3*x^2 + 2*c*d*e^3*x)) - (b^(2/3)*log((-a)
^(7/3) + 3^(1/2)*(-a)^(7/3)*1i + 2*a^2*b^(1/3)*c + 2*a^2*b^(1/3)*d*x)*((3^(
1/2)*1i)/2 + 1/2))/(3*(-a)^(5/3)*d*e^3) + (b^(2/3)*log((-a)^(7/3) - 3^(1/2)
*(-a)^(7/3)*1i + 2*a^2*b^(1/3)*c + 2*a^2*b^(1/3)*d*x)*((3^(1/2)*1i)/6 - 1/6
))/((-a)^(5/3)*d*e^3)
```

sympy [A] time = 1.35, size = 75, normalized size = 0.45

$$-\frac{1}{2ac^2de^3+4acd^2e^3x+2ad^3e^3x^2}+\frac{\operatorname{RootSum}\left(27t^3a^5+b^2,\left(t\mapsto t\log\left(x+\frac{-3ta^2+bc}{bd}\right)\right)\right)}{de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)**3/(a+b*(d*x+c)**3),x)
```

```
[Out] -1/(2*a*c**2*d*e**3 + 4*a*c*d**2*e**3*x + 2*a*d**3*e**3*x**2) + RootSum(27*
_t**3*a**5 + b**2, Lambda(_t, _t*log(x + (-3*_t*a**2 + b*c)/(b*d))))/(d*e**
3)
```

3.2893 $\int \frac{1}{(ce+dex)^4\left(a+b(c+dx)^3\right)}\,dx$

Optimal. Leaf size=65

$$-\frac{b\log(c+dx)}{a^2de^4}+\frac{b\log\left(a+b(c+dx)^3\right)}{3a^2de^4}-\frac{1}{3ade^4(c+dx)^3}$$

```
[Out] -1/3/a/d/e^4/(d*x+c)^3-b*ln(d*x+c)/a^2/d/e^4+1/3*b*ln(a+b*(d*x+c)^3)/a^2/d/
e^4
```

Rubi [A] time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {372, 266, 44}

$$-\frac{b\log(c+dx)}{a^2de^4}+\frac{b\log\left(a+b(c+dx)^3\right)}{3a^2de^4}-\frac{1}{3ade^4(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*e + d*e*x)^4*(a + b*(c + d*x)^3)),x]
```

```
[Out] -1/(3*a*d*e^4*(c + d*x)^3) - (b*Log[c + d*x])/(a^2*d*e^4) + (b*Log[a + b*(c
 + d*x)^3])/(3*a^2*d*e^4)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(ce+dex)^4\left(a+b(c+dx)^3\right)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^3\right)}\,dx, x, c+dx\right)}{de^4}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx)}\,dx, x, (c+dx)^3\right)}{3de^4}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)\,dx, x, (c+dx)^3\right)}{3de^4}\\
&= -\frac{1}{3ade^4(c+dx)^3}-\frac{b\log(c+dx)}{a^2de^4}+\frac{b\log\left(a+b(c+dx)^3\right)}{3a^2de^4}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 0.72

$$\frac{b\log\left(a+b(c+dx)^3\right)-\frac{a}{(c+dx)^3}-3b\log(c+dx)}{3a^2de^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*e + d*e*x)^4*(a + b*(c + d*x)^3)),x]
```

```
[Out] (-(a/(c + d*x)^3) - 3*b*Log[c + d*x] + b*Log[a + b*(c + d*x)^3])/(3*a^2*d*e
^4)
```

fricas [B] time = 1.06, size = 170, normalized size = 2.62

$$\frac{\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3\right)\log\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)-3\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx\right.}{3\left(a^2d^4e^4x^3+3\,a^2cd^3e^4x^2+3\,a^2c^2d^2e^4x+a^2c^3de^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^4/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/3*((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*log(b*d^3*x^3 + 3*b*
c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a) - 3*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c
^2*d*x + b*c^3)*log(d*x + c) - a)/(a^2*d^4*e^4*x^3 + 3*a^2*c*d^3*e^4*x^2 +
3*a^2*c^2*d^2*e^4*x + a^2*c^3*d*e^4)
```

giac [A] time = 0.19, size = 82, normalized size = 1.26

$$\frac{be^{(-4)}\log\left(\left|bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right|\right)}{3\,a^2d}-\frac{be^{(-4)}\log\left(|dx+c|\right)}{a^2d}-\frac{e^{(-4)}}{3\,(dx+c)^3ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^4/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/3*b*e^(-4)*log(abs(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a))/
(a^2*d) - b*e^(-4)*log(abs(d*x + c))/(a^2*d) - 1/3*e^(-4)/((d*x + c)^3*a*d)
```

maple [A] time = 0.01, size = 84, normalized size = 1.29

$$-\frac{b\ln(dx+c)}{a^2d\,e^4}+\frac{b\ln\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)}{3a^2d\,e^4}-\frac{1}{3\,(dx+c)^3\,ad\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)^4/(a+b*(d*x+c)^3),x)
```

```
[Out] -1/3/a/d/e^4/(d*x+c)^3-b*ln(d*x+c)/a^2/d/e^4+1/3/e^4/a^2*b/d*ln(b*d^3*x^3+3
*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)
```

maxima [A] time = 0.69, size = 116, normalized size = 1.78

$$-\frac{1}{3\left(ad^4e^4x^3+3\,acd^3e^4x^2+3\,ac^2d^2e^4x+ac^3de^4\right)}+\frac{b\log\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)}{3\,a^2de^4}-\frac{b\log(dx+c}{a^2de^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^4/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -1/3/(a*d^4*e^4*x^3 + 3*a*c*d^3*e^4*x^2 + 3*a*c^2*d^2*e^4*x + a*c^3*d*e^4)
+ 1/3*b*log(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(a^2*d*e^4
) - b*log(d*x + c)/(a^2*d*e^4)
```

mupad [B] time = 1.44, size = 118, normalized size = 1.82

$$\frac{b\,\ln\left(b\,c^3+3\,b\,c^2\,d\,x+3\,b\,c\,d^2\,x^2+b\,d^3\,x^3+a\right)}{3\,a^2\,d\,e^4}-\frac{1}{3\,\left(a\,c^3\,d\,e^4+3\,a\,c^2\,d^2\,e^4\,x+3\,a\,c\,d^3\,e^4\,x^2+a\,d^4\,e^4\,x^3\right)}-\frac{b\,\ln(c+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)^4*(a + b*(c + d*x)^3)),x)
```

```
[Out] (b*log(a + b*c^3 + b*d^3*x^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2))/(3*a^2*d*e^4)
- 1/(3*(a*d^4*e^4*x^3 + a*c^3*d*e^4 + 3*a*c^2*d^2*e^4*x + 3*a*c*d^3*e^4*x^2
)) - (b*log(c + d*x))/(a^2*d*e^4)
```

sympy [B] time = 2.10, size = 121, normalized size = 1.86

$$-\frac{1}{3ac^3de^4+9ac^2d^2e^4x+9acd^3e^4x^2+3ad^4e^4x^3}-\frac{b\log\left(\frac{c}{d}+x\right)}{a^2de^4}+\frac{b\log\left(\frac{3c^2x}{d^2}+\frac{3cx^2}{d}+x^3+\frac{a+bc^3}{bd^3}\right)}{3a^2de^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)**4/(a+b*(d*x+c)**3),x)
```

```
[Out] -1/(3*a*c**3*d*e**4 + 9*a*c**2*d**2*e**4*x + 9*a*c*d**3*e**4*x**2 + 3*a*d**
4*e**4*x**3) - b*log(c/d + x)/(a**2*d*e**4) + b*log(3*c**2*x/d**2 + 3*c*x**
2/d + x**3 + (a + b*c**3)/(b*d**3))/(3*a**2*d*e**4)
```

3.2894 $\int \frac{(ce+dex)^4}{\left(a+b(c+dx)^3\right)^2}\,dx$

Optimal. Leaf size=184

$$\frac{e^4 \log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{9\sqrt[3]{a}\,b^{5/3}d}-\frac{2e^4 \log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9\sqrt[3]{a}\,b^{5/3}d}-\frac{2e^4 \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,\sqrt[3]{a}\,b^{5/3}d}-\frac{e^4}{3bd\left(a$$

```
[Out] -1/3*e^4*(d*x+c)^2/b/d/(a+b*(d*x+c)^3)-2/9*e^4*ln(a^(1/3)+b^(1/3)*(d*x+c))/
a^(1/3)/b^(5/3)/d+1/9*e^4*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c
)^2)/a^(1/3)/b^(5/3)/d-2/9*e^4*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))/a^(1/
3)*3^(1/2))/a^(1/3)/b^(5/3)/d*3^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {372, 288, 292, 31, 634, 617, 204, 628}

$$\frac{e^4 \log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{9\sqrt[3]{a}\,b^{5/3}d}-\frac{2e^4 \log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9\sqrt[3]{a}\,b^{5/3}d}-\frac{2e^4 \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,\sqrt[3]{a}\,b^{5/3}d}-\frac{e^4}{3bd\left(a$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*(c + d*x)^3)^2,x]
```

```
[Out] -(e^4*(c + d*x)^2)/(3*b*d*(a + b*(c + d*x)^3)) - (2*e^4*ArcTan[(a^(1/3) - 2
*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(1/3)*b^(5/3)*d) - (2*
e^4*Log[a^(1/3) + b^(1/3)*(c + d*x)])/(9*a^(1/3)*b^(5/3)*d) + (e^4*Log[a^(2
/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(9*a^(1/3)*b^(5/3)*
d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\left(a+b(c+dx)^3\right)^2}\,dx = \frac{e^4\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^3\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^4(c+dx)^2}{3bd\left(a+b(c+dx)^3\right)} + \frac{\left(2e^4\right)\text{Subst}\left(\int \frac{x}{a+bx^3}\,dx, x, c+dx\right)}{3bd}$$

$$= -\frac{e^4(c+dx)^2}{3bd\left(a+b(c+dx)^3\right)} - \frac{\left(2e^4\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, c+dx\right)}{9\sqrt[3]{a}\,b^{4/3}d} + \frac{\left(2e^4\right)\text{Subst}\left(\int \frac{\sqrt[3]{}}{a^{2/3}-\sqrt[3]{}}\right.}{9\sqrt[3]{}}$$

$$= -\frac{e^4(c+dx)^2}{3bd\left(a+b(c+dx)^3\right)} - \frac{2e^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9\sqrt[3]{a}\,b^{5/3}d} + \frac{e^4\,\text{Subst}\left(\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,d\right.}{9\sqrt[3]{a}\,b^{5/3}d}$$

$$= -\frac{e^4(c+dx)^2}{3bd\left(a+b(c+dx)^3\right)} - \frac{2e^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9\sqrt[3]{a}\,b^{5/3}d} + \frac{e^4\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+\right.}{9\sqrt[3]{a}\,b^{5/3}d}$$

$$= -\frac{e^4(c+dx)^2}{3bd\left(a+b(c+dx)^3\right)} - \frac{2e^4\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,\sqrt[3]{a}\,b^{5/3}d} - \frac{2e^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9\sqrt[3]{a}\,b^{5/3}d} + \frac{e^4\log}{}$$

Mathematica [A] time = 0.09, size = 155, normalized size = 0.84

$$\frac{e^4\left(\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{\sqrt[3]{a}} - \frac{3b^{2/3}(c+dx)^2}{a+b(c+dx)^3} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{\sqrt[3]{a}} + \frac{2\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{b}\,(c+dx)-\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt[3]{a}}\right)}{9b^{5/3}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*(c + d*x)^3)^2,x]
```

```
[Out] (e^4*((-3*b^(2/3)*(c + d*x)^2)/(a + b*(c + d*x)^3) + (2*Sqrt[3]*ArcTan[(-a^
(1/3) + 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/a^(1/3) - (2*Log[a^(1/3) +
 b^(1/3)*(c + d*x)])/a^(1/3) + Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^
(2/3)*(c + d*x)^2]/a^(1/3)))/(9*b^(5/3)*d)
```

fricas [**B**] time = 0.98, size = 940, normalized size = 5.11

$$\left[-\frac{3\,ab^2d^2e^4x^2 + 6\,ab^2cde^4x + 3\,ab^2c^2e^4 - 3\sqrt{\frac{1}{3}}\left(ab^2d^3e^4x^3 + 3\,ab^2cd^2e^4x^2 + 3\,ab^2c^2de^4x + \left(ab^2c^3 + a^2b\right)e^4\right)\sqrt{} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] [-1/9*(3*a*b^2*d^2*e^4*x^2 + 6*a*b^2*c*d*e^4*x + 3*a*b^2*c^2*e^4 - 3*sqrt(1
/3)*(a*b^2*d^3*e^4*x^3 + 3*a*b^2*c*d^2*e^4*x^2 + 3*a*b^2*c^2*d*e^4*x + (a*b
^2*c^3 + a^2*b)*e^4)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*d^3*x^3 + 6*b^2*c*d^
2*x^2 + 6*b^2*c^2*d*x + 2*b^2*c^3 - a*b + 3*sqrt(1/3)*(a*b*d*x + a*b*c + 2*
(d^2*x^2 + 2*c*d*x + c^2)*(-a*b^2)^(2/3) + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^
(1/3)/a) - 3*(-a*b^2)^(2/3)*(d*x + c))/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2
*d*x + b*c^3 + a)) - (b*d^3*e^4*x^3 + 3*b*c*d^2*e^4*x^2 + 3*b*c^2*d*e^4*x +
 (b*c^3 + a)*e^4)*(-a*b^2)^(2/3)*log(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 +
(-a*b^2)^(1/3)*(b*d*x + b*c) + (-a*b^2)^(2/3)) + 2*(b*d^3*e^4*x^3 + 3*b*c*d
^2*e^4*x^2 + 3*b*c^2*d*e^4*x + (b*c^3 + a)*e^4)*(-a*b^2)^(2/3)*log(b*d*x +
b*c - (-a*b^2)^(1/3)))/(a*b^4*d^4*x^3 + 3*a*b^4*c*d^3*x^2 + 3*a*b^4*c^2*d^2
*x + (a*b^4*c^3 + a^2*b^3)*d), -1/9*(3*a*b^2*d^2*e^4*x^2 + 6*a*b^2*c*d*e^4*
x + 3*a*b^2*c^2*e^4 - 6*sqrt(1/3)*(a*b^2*d^3*e^4*x^3 + 3*a*b^2*c*d^2*e^4*x^
2 + 3*a*b^2*c^2*d*e^4*x + (a*b^2*c^3 + a^2*b)*e^4)*sqrt(-(-a*b^2)^(1/3)/a)*
arctan(sqrt(1/3)*(2*b*d*x + 2*b*c + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)
/b) - (b*d^3*e^4*x^3 + 3*b*c*d^2*e^4*x^2 + 3*b*c^2*d*e^4*x + (b*c^3 + a)*e^
4)*(-a*b^2)^(2/3)*log(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (-a*b^2)^(1/3)*
(b*d*x + b*c) + (-a*b^2)^(2/3)) + 2*(b*d^3*e^4*x^3 + 3*b*c*d^2*e^4*x^2 + 3*
b*c^2*d*e^4*x + (b*c^3 + a)*e^4)*(-a*b^2)^(2/3)*log(b*d*x + b*c - (-a*b^2)^
(1/3)))/(a*b^4*d^4*x^3 + 3*a*b^4*c*d^3*x^2 + 3*a*b^4*c^2*d^2*x + (a*b^4*c^3
 + a^2*b^3)*d)]
```

giac [**A**] time = 0.25, size = 218, normalized size = 1.18

$$-\frac{2\sqrt{3}\left(-\frac{e^{12}}{ab^2d^3}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)}{3\left(-a^2b\right)^{\frac{2}{3}}}\right)+\left(-\frac{e^{12}}{ab^2d^3}\right)^{\frac{1}{3}}\log\left(\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)^2+3\left(-a^2b\right)^{\frac{4}{3}}\right)}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -1/9*(2*sqrt(3)*(-e^12/(a*b^2*d^3))^(1/3)*arctan(1/3*sqrt(3)*(2*a*b*d*x + 2
*a*b*c - (-a^2*b)^(2/3))/(-a^2*b)^(2/3)) + (-e^12/(a*b^2*d^3))^(1/3)*log((2
```

```
*a*b*d*x + 2*a*b*c - (-a^2*b)^(2/3))^2 + 3*(-a^2*b)^(4/3)) - 2*(-e^12/(a*b^
2*d^3))^(1/3)*log(abs(a*b*d*x + a*b*c + (-a^2*b)^(2/3))))/b - 1/3*(d^2*x^2*
e^4 + 2*c*d*x*e^4 + c^2*e^4)/((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*
c^3 + a)*b*d)
```

maple [C] time = 0.01, size = 221, normalized size = 1.20

$$-\frac{d\,e^4x^2}{3\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)b}-\frac{2c\,e^4x}{3\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)b}-\frac{c}{3\left(b\,d^3x^3+3bc\,d^2x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*(d*x+c)^3)^2,x)
```

```
[Out] -1/3*e^4/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)/b*x^2*d-2/3*e^4/(b*d
^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)/b*c*x-1/3*e^4/(b*d^3*x^3+3*b*c*d^
2*x^2+3*b*c^2*d*x+b*c^3+a)/b*c^2/d+2/9*e^4/b^2/d*sum((_R*d+c)/(_R^2*d^2+2*_
R*c*d+c^2)*ln(-_R+x),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3
+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\frac{1}{3}\left(2\sqrt{3}\left(-\frac{1}{ab^2d^3}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)}{3\left(-a^2b\right)^{\frac{2}{3}}}\right)+\left(-\frac{1}{ab^2d^3}\right)^{\frac{1}{3}}\log\left(\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)^2+3\left(-a^2b\right)^{\frac{4}{3}}\right)\right.}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 2/3*e^4*integrate((d*x + c)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^
3 + a), x)/b - 1/3*(d^2*e^4*x^2 + 2*c*d*e^4*x + c^2*e^4)/(b^2*d^4*x^3 + 3*b
^2*c*d^3*x^2 + 3*b^2*c^2*d^2*x + (b^2*c^3 + a*b)*d)
```

mupad [B] time = 1.56, size = 266, normalized size = 1.45

$$\frac{2\,e^4\,\ln\left(b^{1/3}\,c-(-a)^{1/3}+b^{1/3}\,d\,x\right)}{9\,(-a)^{1/3}\,b^{5/3}\,d}-\frac{\frac{d\,e^4\,x^2}{3\,b}+\frac{c^2\,e^4}{3\,b\,d}+\frac{2\,c\,e^4\,x}{3\,b}}{b\,c^3+3\,b\,c^2\,d\,x+3\,b\,c\,d^2\,x^2+b\,d^3\,x^3+a}-\frac{\ln\left(\frac{4\,c\,d^4\,e^8}{9\,b}-\frac{(-a)^{1/3}\,d^4\left(e^4+\sqrt{3}\,e^4\,1\mathrm{i}\right)^2}{9\,b^{4/3}}+\right.}{9\,(-a)^{1/3}\,b^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4/(a + b*(c + d*x)^3)^2,x)
```

```
[Out] (2*e^4*log(b^(1/3)*c - (-a)^(1/3) + b^(1/3)*d*x))/(9*(-a)^(1/3)*b^(5/3)*d)
- ((d*e^4*x^2)/(3*b) + (c^2*e^4)/(3*b*d) + (2*c*e^4*x)/(3*b))/(a + b*c^3 +
b*d^3*x^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2) - (log((4*c*d^4*e^8)/(9*b) - ((-a)
^(1/3)*d^4*(3^(1/2)*e^4*1i + e^4)^2)/(9*b^(4/3)) + (4*d^5*e^8*x)/(9*b))*(3^
(1/2)*e^4*1i + e^4))/(9*(-a)^(1/3)*b^(5/3)*d) + (e^4*log((4*c*d^4*e^8)/(9*b
) + (4*d^5*e^8*x)/(9*b) - (9*(-a)^(1/3)*d^4*e^8*((3^(1/2)*1i)/9 - 1/9)^2)/b
^(4/3))*((3^(1/2)*1i)/9 - 1/9))/((-a)^(1/3)*b^(5/3)*d)
```

sympy [A] time = 1.67, size = 133, normalized size = 0.72

$$\frac{-c^2e^4-2cde^4x-d^2e^4x^2}{3abd+3b^2c^3d+9b^2c^2d^2x+9b^2cd^3x^2+3b^2d^4x^3}+\frac{e^4\,\mathrm{RootSum}\left(729t^3ab^5+8,\left(t\mapsto t\log\left(x+\frac{81t^2ab^3e^8+4ce^8}{4de^8}\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*(d*x+c)**3)**2,x)
```

```
[Out] (-c**2*e**4 - 2*c*d*e**4*x - d**2*e**4*x**2)/(3*a*b*d + 3*b**2*c**3*d + 9*b
**2*c**2*d**2*x + 9*b**2*c*d**3*x**2 + 3*b**2*d**4*x**3) + e**4*RootSum(729
*_t**3*a*b**5 + 8, Lambda(_t, _t*log(x + (81*_t**2*a*b**3*e**8 + 4*c*e**8)/
(4*d*e**8))))/d
```

3.2895 $\int \frac{(ce+dex)^3}{\left(a+b(c+dx)^3\right)^2}\,dx$

Optimal. Leaf size=182

$$\frac{e^3 \log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9a^{2/3}b^{4/3}d}-\frac{e^3 \log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{18a^{2/3}b^{4/3}d}-\frac{e^3 \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}b^{4/3}d}-\frac{e^3(c+$$
$$3bd\left(a+b(c$$

```
[Out] -1/3*e^3*(d*x+c)/b/d/(a+b*(d*x+c)^3)+1/9*e^3*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^
(2/3)/b^(4/3)/d-1/18*e^3*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)
^2)/a^(2/3)/b^(4/3)/d-1/9*e^3*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))/a^(1/3
)*3^(1/2))/a^(2/3)/b^(4/3)/d*3^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {372, 288, 200, 31, 634, 617, 204, 628}

$$\frac{e^3 \log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9a^{2/3}b^{4/3}d}-\frac{e^3 \log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{18a^{2/3}b^{4/3}d}-\frac{e^3 \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}b^{4/3}d}-\frac{e^3(c+$$
$$3bd\left(a+b(c$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*(c + d*x)^3)^2,x]
```

```
[Out] -(e^3*(c + d*x))/(3*b*d*(a + b*(c + d*x)^3)) - (e^3*ArcTan[(a^(1/3) - 2*b^(
1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)*b^(4/3)*d) + (e^3*Lo
g[a^(1/3) + b^(1/3)*(c + d*x)])/(9*a^(2/3)*b^(4/3)*d) - (e^3*Log[a^(2/3) -
a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(18*a^(2/3)*b^(4/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{\left(a+b(c+dx)^3\right)^2}\,dx = \frac{e^3\,\text{Subst}\left(\int \frac{x^3}{\left(a+bx^3\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^3(c+dx)}{3bd\left(a+b(c+dx)^3\right)} + \frac{e^3\,\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, c+dx\right)}{3bd}$$

$$= -\frac{e^3(c+dx)}{3bd\left(a+b(c+dx)^3\right)} + \frac{e^3\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, c+dx\right)}{9a^{2/3}bd} + \frac{e^3\,\text{Subst}\left(\int \frac{2\sqrt[3]{a}-}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{}}\right.}{9a^{2}}$$

$$= -\frac{e^3(c+dx)}{3bd\left(a+b(c+dx)^3\right)} + \frac{e^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9a^{2/3}b^{4/3}d} - \frac{e^3\,\text{Subst}\left(\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\right.}{18a^{2/3}b^{4/3}d}$$

$$= -\frac{e^3(c+dx)}{3bd\left(a+b(c+dx)^3\right)} + \frac{e^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9a^{2/3}b^{4/3}d} - \frac{e^3\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)\right.}{18a^{2/3}b^{4/3}d}$$

$$= -\frac{e^3(c+dx)}{3bd\left(a+b(c+dx)^3\right)} - \frac{e^3\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{2/3}b^{4/3}d} + \frac{e^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9a^{2/3}b^{4/3}d} - \frac{e^3\log}{}$$

Mathematica [A] time = 0.07, size = 154, normalized size = 0.85

$$\frac{e^3\left(-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{a^{2/3}} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{a^{2/3}} + \frac{2\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{b}\,(c+dx)-\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{a^{2/3}} - \frac{6\sqrt[3]{b}\,(c+dx)}{a+b(c+dx)^3}\right)}{18b^{4/3}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*(c + d*x)^3)^2,x]
```

```
[Out] (e^3*((-6*b^(1/3)*(c + d*x))/(a + b*(c + d*x)^3) + (2*Sqrt[3]*ArcTan[(-a^(1
/3) + 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/a^(2/3) + (2*Log[a^(1/3) + b
^(1/3)*(c + d*x)])/a^(2/3) - Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2
/3)*(c + d*x)^2]/a^(2/3)))/(18*b^(4/3)*d)
```

fricas **[B]** time = 1.04, size = 906, normalized size = 4.98

$$\left[-\frac{6\,a^2bde^3x + 6\,a^2bce^3 - 3\sqrt{\frac{1}{3}}\left(ab^2d^3e^3x^3 + 3\,ab^2cd^2e^3x^2 + 3\,ab^2c^2de^3x + \left(ab^2c^3 + a^2b\right)e^3\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\,\log\left(\frac{2\,abd^3x^3}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] [-1/18*(6*a^2*b*d*e^3*x + 6*a^2*b*c*e^3 - 3*sqrt(1/3)*(a*b^2*d^3*e^3*x^3 +
3*a*b^2*c*d^2*e^3*x^2 + 3*a*b^2*c^2*d*e^3*x + (a*b^2*c^3 + a^2*b)*e^3)*sqrt
(-(a^2*b)^(1/3)/b)*log((2*a*b*d^3*x^3 + 6*a*b*c*d^2*x^2 + 6*a*b*c^2*d*x + 2
*a*b*c^3 - a^2 + 3*sqrt(1/3)*(2*a*b*d^2*x^2 + 4*a*b*c*d*x + 2*a*b*c^2 + (a^
2*b)^(2/3)*(d*x + c) - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^
(1/3)*(a*d*x + a*c))/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a))
 + (b*d^3*e^3*x^3 + 3*b*c*d^2*e^3*x^2 + 3*b*c^2*d*e^3*x + (b*c^3 + a)*e^3)*
(a^2*b)^(2/3)*log(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2 - (a^2*b)^(2/3)*(d*x
+ c) + (a^2*b)^(1/3)*a) - 2*(b*d^3*e^3*x^3 + 3*b*c*d^2*e^3*x^2 + 3*b*c^2*d*
e^3*x + (b*c^3 + a)*e^3)*(a^2*b)^(2/3)*log(a*b*d*x + a*b*c + (a^2*b)^(2/3))
)/(a^2*b^3*d^4*x^3 + 3*a^2*b^3*c*d^3*x^2 + 3*a^2*b^3*c^2*d^2*x + (a^2*b^3*c
^3 + a^3*b^2)*d), -1/18*(6*a^2*b*d*e^3*x + 6*a^2*b*c*e^3 - 6*sqrt(1/3)*(a*b
^2*d^3*e^3*x^3 + 3*a*b^2*c*d^2*e^3*x^2 + 3*a*b^2*c^2*d*e^3*x + (a*b^2*c^3 +
 a^2*b)*e^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*(d*x +
 c) - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) + (b*d^3*e^3*x^3 + 3*b*c*
d^2*e^3*x^2 + 3*b*c^2*d*e^3*x + (b*c^3 + a)*e^3)*(a^2*b)^(2/3)*log(a*b*d^2*
x^2 + 2*a*b*c*d*x + a*b*c^2 - (a^2*b)^(2/3)*(d*x + c) + (a^2*b)^(1/3)*a) -
2*(b*d^3*e^3*x^3 + 3*b*c*d^2*e^3*x^2 + 3*b*c^2*d*e^3*x + (b*c^3 + a)*e^3)*(
a^2*b)^(2/3)*log(a*b*d*x + a*b*c + (a^2*b)^(2/3)))/(a^2*b^3*d^4*x^3 + 3*a^2
*b^3*c*d^3*x^2 + 3*a^2*b^3*c^2*d^2*x + (a^2*b^3*c^3 + a^3*b^2)*d)]
```

giac **[A]** time = 0.25, size = 225, normalized size = 1.24

$$\frac{2\sqrt{3}\left(\frac{1}{a^2bd^3}\right)^{\frac{1}{3}}\arctan\left(-\frac{bdx+bc+\left(ab^2\right)^{\frac{1}{3}}}{\sqrt{3}\,bdx+\sqrt{3}\,bc-\sqrt{3}\left(ab^2\right)^{\frac{1}{3}}}\right)e^3-\left(\frac{1}{a^2bd^3}\right)^{\frac{1}{3}}e^3\log\left(4\left(\sqrt{3}\,bdx+\sqrt{3}\,bc-\sqrt{3}\left(ab^2\right)^{\frac{1}{3}}\right)^2+4\left(bdx+\right.\right.}{18\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/18*(2*sqrt(3)*(1/(a^2*b*d^3))^(1/3)*arctan(-(b*d*x + b*c + (a*b^2)^(1/3))
/(sqrt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3)))*e^3 - (1/(a^2*b*d^3
))^(1/3)*e^3*log(4*(sqrt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))^2
```

```
+ 4*(b*d*x + b*c + (a*b^2)^(1/3))^2) + 2*(1/(a^2*b*d^3))^(1/3)*e^3*log(abs(
b*d*x + b*c + (a*b^2)^(1/3))))/b - 1/3*(d*x*e^3 + c*e^3)/((b*d^3*x^3 + 3*b*
c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*b*d)
```

maple [C] time = 0.01, size = 166, normalized size = 0.91

$$-\frac{e^3 x}{3\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)b}-\frac{c\,e^3}{3\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)bd}+\frac{}{9b^2d\left(d^2\,\mathrm{RootOf}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*(d*x+c)^3)^2,x)
```

```
[Out] -1/3*e^3/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)/b*x-1/3*e^3/(b*d^3*x
^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)/b*c/d+1/9*e^3/b^2/d*sum(1/(_R^2*d^2+2
*_R*c*d+c^2)*ln(-_R+x),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c
^3+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}\,e^3\left(\frac{2\sqrt{3}\left(\frac{1}{a^2b}\right)^{\frac{1}{3}}\arctan\left(-\frac{bdx+bc+\left(ab^2\right)^{\frac{1}{3}}}{\sqrt{3}\,bdx+\sqrt{3}\,bc-\sqrt{3}\left(ab^2\right)^{\frac{1}{3}}}\right)}{d}-\frac{\left(\frac{1}{a^2b}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}\,bdx+\sqrt{3}\,bc-\sqrt{3}\left(ab^2\right)^{\frac{1}{3}}\right)^2+4\left(bdx+bc+\left(ab^2\right)^{\frac{1}{3}}\right)^2\right)}{d}+\frac{2\left(\frac{1}{a^2b}\right)^{\frac{1}{3}}\log\left(\right.}{d}$$

$$\frac{}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 1/3*e^3*integrate(1/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a),
x)/b - 1/3*(d*e^3*x + c*e^3)/(b^2*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d^2
*x + (b^2*c^3 + a*b)*d)
```

mupad [B] time = 0.34, size = 194, normalized size = 1.07

$$\frac{e^3\,\ln\left(b^{1/3}\,c+a^{1/3}+b^{1/3}\,d\,x\right)}{9\,a^{2/3}\,b^{4/3}\,d}-\frac{\ln\left(2\,b^{1/3}\,c-a^{1/3}+2\,b^{1/3}\,d\,x-\sqrt{3}\;a^{1/3}\,1\mathrm{i}\right)\left(e^3+\sqrt{3}\;e^3\,1\mathrm{i}\right)}{18\,a^{2/3}\,b^{4/3}\,d}-\frac{\frac{e^3\,x}{3\,b}}{b\,c^3+3\,b\,c^2\,d\,x+3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3/(a + b*(c + d*x)^3)^2,x)
```

```
[Out] (e^3*log(b^(1/3)*c + a^(1/3) + b^(1/3)*d*x))/(9*a^(2/3)*b^(4/3)*d) - (log(2
*b^(1/3)*c - 3^(1/2)*a^(1/3)*1i - a^(1/3) + 2*b^(1/3)*d*x)*(3^(1/2)*e^3*1i
+ e^3))/(18*a^(2/3)*b^(4/3)*d) - ((e^3*x)/(3*b) + (c*e^3)/(3*b*d))/(a + b*c
^3 + b*d^3*x^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2) + (e^3*log(3^(1/2)*a^(1/3)*1i
 + 2*b^(1/3)*c - a^(1/3) + 2*b^(1/3)*d*x)*((3^(1/2)*1i)/2 - 1/2))/(9*a^(2/3
)*b^(4/3)*d)
```

sympy [A] time = 1.55, size = 112, normalized size = 0.62

$$\frac{-ce^3-de^3x}{3abd+3b^2c^3d+9b^2c^2d^2x+9b^2cd^3x^2+3b^2d^4x^3}+\frac{e^3\,\mathrm{RootSum}\left(729t^3a^2b^4-1,\left(t\mapsto t\log\left(x+\frac{9tabe^3+ce^3}{de^3}\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*(d*x+c)**3)**2,x)
```

```
[Out] (-c*e**3 - d*e**3*x)/(3*a*b*d + 3*b**2*c**3*d + 9*b**2*c**2*d**2*x + 9*b**2
*c*d**3*x**2 + 3*b**2*d**4*x**3) + e**3*RootSum(729*_t**3*a**2*b**4 - 1, La
mbda(_t, _t*log(x + (9*_t*a*b*e**3 + c*e**3)/(d*e**3))))/d
```

3.2896 $\int \frac{(ce+dex)^2}{\left(a+b(c+dx)^3\right)^2}\,dx$

Optimal. Leaf size=26

$$-\frac{e^2}{3bd\left(a+b(c+dx)^3\right)}$$

```
[Out] -1/3*e^2/b/d/(a+b*(d*x+c)^3)
```

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {372, 261}

$$-\frac{e^2}{3bd\left(a+b(c+dx)^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*(c + d*x)^3)^2,x]
```

```
[Out] -e^2/(3*b*d*(a + b*(c + d*x)^3))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(ce+dex)^2}{\left(a+b(c+dx)^3\right)^2}\,dx &= \frac{e^2\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^3\right)^2}\,dx, x, c+dx\right)}{d}\\ &= -\frac{e^2}{3bd\left(a+b(c+dx)^3\right)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$-\frac{e^2}{3bd\left(a+b(c+dx)^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*(c + d*x)^3)^2,x]
```

```
[Out] -1/3*e^2/(b*d*(a + b*(c + d*x)^3))
```

fricas [B] time = 0.95, size = 55, normalized size = 2.12

$$-\frac{e^2}{3\left(b^2d^4x^3+3\,b^2cd^3x^2+3\,b^2c^2d^2x+\left(b^2c^3+ab\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] -1/3*e^2/(b^2*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d^2*x + (b^2*c^3 + a*b)
*d)
```

giac [A] time = 0.19, size = 45, normalized size = 1.73

$$-\frac{e^2}{3\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -1/3*e^2/((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*b*d)
```

maple [A] time = 0.00, size = 47, normalized size = 1.81

$$-\frac{e^2}{3\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*(d*x+c)^3)^2,x)
```

```
[Out] -1/3*e^2/b/d/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)
```

maxima [B] time = 0.55, size = 55, normalized size = 2.12

$$-\frac{e^2}{3\left(b^2d^4x^3+3\,b^2cd^3x^2+3\,b^2c^2d^2x+\left(b^2c^3+ab\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] -1/3*e^2/(b^2*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d^2*x + (b^2*c^3 + a*b)
*d)
```

mupad [B] time = 1.25, size = 46, normalized size = 1.77

$$-\frac{e^2}{3\,b\,d\,\left(b\,c^3+3\,b\,c^2\,d\,x+3\,b\,c\,d^2\,x^2+b\,d^3\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2/(a + b*(c + d*x)^3)^2,x)
```

```
[Out] -e^2/(3*b*d*(a + b*c^3 + b*d^3*x^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2))
```

sympy [B] time = 1.37, size = 60, normalized size = 2.31

$$-\frac{e^2}{3abd+3b^2c^3d+9b^2c^2d^2x+9b^2cd^3x^2+3b^2d^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*(d*x+c)**3)**2,x)
```

```
[Out] -e**2/(3*a*b*d + 3*b**2*c**3*d + 9*b**2*c**2*d**2*x + 9*b**2*c*d**3*x**2 +
3*b**2*d**4*x**3)
```

3.2897 $\int \frac{ce+dex}{\left(a+b(c+dx)^3\right)^2}\,dx$

Optimal. Leaf size=176

$$-\frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9a^{4/3}b^{2/3}d}+\frac{e\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{18a^{4/3}b^{2/3}d}-\frac{e\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}b^{2/3}d}+\frac{e(c+$$

```
[Out] 1/3*e*(d*x+c)^2/a/d/(a+b*(d*x+c)^3)-1/9*e*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(4/
3)/b^(2/3)/d+1/18*e*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a
^(4/3)/b^(2/3)/d-1/9*e*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/
2))/a^(4/3)/b^(2/3)/d*3^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {372, 290, 292, 31, 634, 617, 204, 628}

$$-\frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9a^{4/3}b^{2/3}d}+\frac{e\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{18a^{4/3}b^{2/3}d}-\frac{e\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}b^{2/3}d}+\frac{e(c+$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*(c + d*x)^3)^2,x]
```

```
[Out] (e*(c + d*x)^2)/(3*a*d*(a + b*(c + d*x)^3)) - (e*ArcTan[(a^(1/3) - 2*b^(1/3
)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)*b^(2/3)*d) - (e*Log[a^(
1/3) + b^(1/3)*(c + d*x)])/(9*a^(4/3)*b^(2/3)*d) + (e*Log[a^(2/3) - a^(1/3)
*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(18*a^(4/3)*b^(2/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{ce+dex}{\left(a+b(c+dx)^3\right)^2}\,dx = \frac{e\,\text{Subst}\left(\int \frac{x}{\left(a+bx^3\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e(c+dx)^2}{3ad\left(a+b(c+dx)^3\right)} + \frac{e\,\text{Subst}\left(\int \frac{x}{a+bx^3}\,dx, x, c+dx\right)}{3ad}$$

$$= \frac{e(c+dx)^2}{3ad\left(a+b(c+dx)^3\right)} - \frac{e\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, c+dx\right)}{9a^{4/3}\sqrt[3]{b}\,d} + \frac{e\,\text{Subst}\left(\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x}\right.}{9a^{4/3}\sqrt[3]{b}\,d}$$

$$= \frac{e(c+dx)^2}{3ad\left(a+b(c+dx)^3\right)} - \frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9a^{4/3}b^{2/3}d} + \frac{e\,\text{Subst}\left(\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\right.}{18a^{4/3}b^{2/3}d}$$

$$= \frac{e(c+dx)^2}{3ad\left(a+b(c+dx)^3\right)} - \frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9a^{4/3}b^{2/3}d} + \frac{e\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}\right.}{18a^{4/3}b^{2/3}d}$$

$$= \frac{e(c+dx)^2}{3ad\left(a+b(c+dx)^3\right)} - \frac{e\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{4/3}b^{2/3}d} - \frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9a^{4/3}b^{2/3}d} + \frac{e\log\left(a^{2/3}-\right.}{}$$

Mathematica [A] time = 0.01, size = 153, normalized size = 0.87

$$\frac{e\left(\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{b^{2/3}} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{b^{2/3}} + \frac{2\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{b}\,(c+dx)-\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{b^{2/3}} + \frac{6\sqrt[3]{a}\,(c+dx)^2}{a+b(c+dx)^3}\right)}{18a^{4/3}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)/(a + b*(c + d*x)^3)^2,x]
```

```
[Out] (e*((6*a^(1/3)*(c + d*x)^2)/(a + b*(c + d*x)^3) + (2*Sqrt[3]*ArcTan[(-a^(1/
3) + 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/b^(2/3) - (2*Log[a^(1/3) + b^
(1/3)*(c + d*x)])/b^(2/3) + Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/
3)*(c + d*x)^2]/b^(2/3)))/(18*a^(4/3)*d)
```

fricas [B] time = 1.06, size = 894, normalized size = 5.08

$$\left[\frac{6\,ab^2d^2ex^2 + 12\,ab^2cdex + 6\,ab^2c^2e + 3\sqrt{\frac{1}{3}}\left(ab^2d^3ex^3 + 3\,ab^2cd^2ex^2 + 3\,ab^2c^2dex + \left(ab^2c^3 + a^2b\right)e\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] [1/18*(6*a*b^2*d^2*e*x^2 + 12*a*b^2*c*d*e*x + 6*a*b^2*c^2*e + 3*sqrt(1/3)*(
a*b^2*d^3*e*x^3 + 3*a*b^2*c*d^2*e*x^2 + 3*a*b^2*c^2*d*e*x + (a*b^2*c^3 + a^
2*b)*e)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*d^3*x^3 + 6*b^2*c*d^2*x^2 + 6*b^2
*c^2*d*x + 2*b^2*c^3 - a*b + 3*sqrt(1/3)*(a*b*d*x + a*b*c + 2*(d^2*x^2 + 2*
c*d*x + c^2)*(-a*b^2)^(2/3) + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*
(-a*b^2)^(2/3)*(d*x + c))/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3
 + a)) + (b*d^3*e*x^3 + 3*b*c*d^2*e*x^2 + 3*b*c^2*d*e*x + (b*c^3 + a)*e)*(-a
*b^2)^(2/3)*log(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (-a*b^2)^(1/3)*(b*d*x
 + b*c) + (-a*b^2)^(2/3)) - 2*(b*d^3*e*x^3 + 3*b*c*d^2*e*x^2 + 3*b*c^2*d*e*
x + (b*c^3 + a)*e)*(-a*b^2)^(2/3)*log(b*d*x + b*c - (-a*b^2)^(1/3)))/(a^2*b
^3*d^4*x^3 + 3*a^2*b^3*c*d^3*x^2 + 3*a^2*b^3*c^2*d^2*x + (a^2*b^3*c^3 + a^3
*b^2)*d), 1/18*(6*a*b^2*d^2*e*x^2 + 12*a*b^2*c*d*e*x + 6*a*b^2*c^2*e + 6*sq
rt(1/3)*(a*b^2*d^3*e*x^3 + 3*a*b^2*c*d^2*e*x^2 + 3*a*b^2*c^2*d*e*x + (a*b^2
*c^3 + a^2*b)*e)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*d*x + 2*b*c
+ (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (b*d^3*e*x^3 + 3*b*c*d^2*e*x
^2 + 3*b*c^2*d*e*x + (b*c^3 + a)*e)*(-a*b^2)^(2/3)*log(b^2*d^2*x^2 + 2*b^2*
c*d*x + b^2*c^2 + (-a*b^2)^(1/3)*(b*d*x + b*c) + (-a*b^2)^(2/3)) - 2*(b*d^3
*e*x^3 + 3*b*c*d^2*e*x^2 + 3*b*c^2*d*e*x + (b*c^3 + a)*e)*(-a*b^2)^(2/3)*lo
g(b*d*x + b*c - (-a*b^2)^(1/3)))/(a^2*b^3*d^4*x^3 + 3*a^2*b^3*c*d^3*x^2 + 3
*a^2*b^3*c^2*d^2*x + (a^2*b^3*c^3 + a^3*b^2)*d)]
```

giac [A] time = 0.23, size = 218, normalized size = 1.24

$$-\frac{2\sqrt{3}\left(-\frac{e^3}{ab^2d^3}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)}{3\left(-a^2b\right)^{\frac{2}{3}}}\right)+\left(-\frac{e^3}{ab^2d^3}\right)^{\frac{1}{3}}\log\left(\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)^2+3\left(-a^2b\right)^{\frac{4}{3}}\right)}{18\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -1/18*(2*sqrt(3)*(-e^3/(a*b^2*d^3))^(1/3)*arctan(1/3*sqrt(3)*(2*a*b*d*x + 2
*a*b*c - (-a^2*b)^(2/3))/(-a^2*b)^(2/3)) + (-e^3/(a*b^2*d^3))^(1/3)*log((2*
a*b*d*x + 2*a*b*c - (-a^2*b)^(2/3))^2 + 3*(-a^2*b)^(4/3)) - 2*(-e^3/(a*b^2*
```

```
d^3))^(1/3)*log(abs(a*b*d*x + a*b*c + (-a^2*b)^(2/3))))/a + 1/3*(d^2*x^2*e
+ 2*c*d*x*e + c^2*e)/((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)
*a*d)
```

maple [C] time = 0.01, size = 216, normalized size = 1.23

$$\frac{de\,x^2}{3\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)a}+\frac{2cex}{3\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)a}+\frac{c^2}{3\left(b\,d^3x^3+3bc\,d^2x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*(d*x+c)^3)^2,x)
```

```
[Out] 1/3*e/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)*d/a*x^2+2/3*e/(b*d^3*x^
3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)*c/a*x+1/3*e/(b*d^3*x^3+3*b*c*d^2*x^2+3
*b*c^2*d*x+b*c^3+a)/a*c^2/d+1/9*e/a/b/d*sum((_R*d+c)/(_R^2*d^2+2*_R*c*d+c^2
)*ln(-_R+x),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{-\frac{1}{6}\left(2\sqrt{3}\left(-\frac{1}{ab^2d^3}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)}{3\left(-a^2b\right)^{\frac{2}{3}}}\right)+\left(-\frac{1}{ab^2d^3}\right)^{\frac{1}{3}}\log\left(\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)^2+3\left(-a^2b\right)^{\frac{4}{3}}\right)\right.}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 1/3*e*integrate((d*x + c)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3
+ a), x)/a + 1/3*(d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(a*b*d^4*x^3 + 3*a*b*c*d^3
*x^2 + 3*a*b*c^2*d^2*x + (a*b*c^3 + a^2)*d)
```

mupad [B] time = 0.32, size = 192, normalized size = 1.09

$$\frac{\frac{c^2e}{3ad}+\frac{2cex}{3a}+\frac{dex^2}{3a}}{b\,c^3+3\,b\,c^2\,d\,x+3\,b\,c\,d^2\,x^2+b\,d^3\,x^3+a}-\frac{e\,\ln\left(b^{1/3}\,c+a^{1/3}+b^{1/3}\,d\,x\right)}{9\,a^{4/3}\,b^{2/3}\,d}+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,c-2\,b^{1/3}\,d\,x+\sqrt{3}\;a^{1/3}1\mathrm{i}\right.}{18\,a^{4/3}\,b^{2/3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)/(a + b*(c + d*x)^3)^2,x)
```

```
[Out] ((c^2*e)/(3*a*d) + (2*c*e*x)/(3*a) + (d*e*x^2)/(3*a))/(a + b*c^3 + b*d^3*x^
3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2) - (e*log(b^(1/3)*c + a^(1/3) + b^(1/3)*d*x
))/(9*a^(4/3)*b^(2/3)*d) + (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*c + a^(1/3)
- 2*b^(1/3)*d*x)*(e - 3^(1/2)*e*1i))/(18*a^(4/3)*b^(2/3)*d) + (log(3^(1/2)*
a^(1/3)*1i + 2*b^(1/3)*c - a^(1/3) + 2*b^(1/3)*d*x)*(e + 3^(1/2)*e*1i))/(18
*a^(4/3)*b^(2/3)*d)
```

sympy [A] time = 1.37, size = 122, normalized size = 0.69

$$\frac{c^2e+2cdex+d^2ex^2}{3a^2d+3abc^3d+9abc^2d^2x+9abcd^3x^2+3abd^4x^3}+\frac{e\,\mathrm{RootSum}\left(729t^3a^4b^2+1,\left(t\mapsto t\log\left(x+\frac{81t^2a^3be^2+ce^2}{de^2}\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*(d*x+c)**3)**2,x)
```

```
[Out] (c**2*e + 2*c*d*e*x + d**2*e*x**2)/(3*a**2*d + 3*a*b*c**3*d + 9*a*b*c**2*d*
*2*x + 9*a*b*c*d**3*x**2 + 3*a*b*d**4*x**3) + e*RootSum(729*_t**3*a**4*b**2
 + 1, Lambda(_t, _t*log(x + (81*_t**2*a**3*b*e**2 + c*e**2)/(d*e**2))))/d
```

3.2898 $\int \frac{1}{(ce+dex)\left(a+b(c+dx)^3\right)^2}\,dx$

Optimal. Leaf size=68

$$-\frac{\log\left(a+b(c+dx)^3\right)}{3a^2de}+\frac{\log(c+dx)}{a^2de}+\frac{1}{3ade\left(a+b(c+dx)^3\right)}$$

```
[Out] 1/3/a/d/e/(a+b*(d*x+c)^3)+ln(d*x+c)/a^2/d/e-1/3*ln(a+b*(d*x+c)^3)/a^2/d/e
```

Rubi [A] time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {372, 266, 44}

$$-\frac{\log\left(a+b(c+dx)^3\right)}{3a^2de}+\frac{\log(c+dx)}{a^2de}+\frac{1}{3ade\left(a+b(c+dx)^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*e + d*e*x)*(a + b*(c + d*x)^3)^2),x]
```

```
[Out] 1/(3*a*d*e*(a + b*(c + d*x)^3)) + Log[c + d*x]/(a^2*d*e) - Log[a + b*(c + d
*x)^3]/(3*a^2*d*e)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(ce+dex)\left(a+b(c+dx)^3\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx^3\right)^2}\,dx, x, c+dx\right)}{de} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^2}\,dx, x, (c+dx)^3\right)}{3de} \\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right)\,dx, x, (c+dx)^3\right)}{3de} \\
&= \frac{1}{3ade\left(a+b(c+dx)^3\right)}+\frac{\log(c+dx)}{a^2de}-\frac{\log\left(a+b(c+dx)^3\right)}{3a^2de}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 0.75

$$\frac{\frac{a}{a+b(c+dx)^3} - \log\left(a + b(c+dx)^3\right) + 3\log(c+dx)}{3a^2de}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*e + d*e*x)*(a + b*(c + d*x)^3)^2),x]
```

```
[Out] (a/(a + b*(c + d*x)^3) + 3*Log[c + d*x] - Log[a + b*(c + d*x)^3])/(3*a^2*d*
e)
```

fricas [B] time = 0.79, size = 173, normalized size = 2.54

$$-\frac{\left(bd^3x^3 + 3\,bcd^2x^2 + 3\,bc^2dx + bc^3 + a\right)\log\left(bd^3x^3 + 3\,bcd^2x^2 + 3\,bc^2dx + bc^3 + a\right) - 3\left(bd^3x^3 + 3\,bcd^2x^2 + 3\,bc^2\right.}{3\left(a^2bd^4ex^3 + 3\,a^2bcd^3ex^2 + 3\,a^2bc^2d^2ex + \left(a^2bc^3 + a^3\right)de\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] -1/3*((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*log(b*d^3*x^3 +
 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a) - 3*(b*d^3*x^3 + 3*b*c*d^2*x^2 +
3*b*c^2*d*x + b*c^3 + a)*log(d*x + c) - a)/(a^2*b*d^4*e*x^3 + 3*a^2*b*c*d^3
*e*x^2 + 3*a^2*b*c^2*d^2*e*x + (a^2*b*c^3 + a^3)*d*e)
```

giac [A] time = 0.24, size = 107, normalized size = 1.57

$$-\frac{e^{(-1)}\log\left(\left|bd^3x^3 + 3\,bcd^2x^2 + 3\,bc^2dx + bc^3 + a\right|\right)}{3\,a^2d} + \frac{e^{(-1)}\log\left(|dx+c|\right)}{a^2d} + \frac{e^{(-1)}}{3\left(bd^3x^3 + 3\,bcd^2x^2 + 3\,bc^2dx + bc^3 + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -1/3*e^(-1)*log(abs(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a))/(
a^2*d) + e^(-1)*log(abs(d*x + c))/(a^2*d) + 1/3*e^(-1)/((b*d^3*x^3 + 3*b*c*
d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*a*d)
```

maple [A] time = 0.02, size = 109, normalized size = 1.60

$$\frac{1}{3\left(b\,d^3x^3 + 3bc\,d^2x^2 + 3b\,c^2dx + b\,c^3 + a\right)ade} + \frac{\ln(dx+c)}{a^2de} - \frac{\ln\left(b\,d^3x^3 + 3bc\,d^2x^2 + 3b\,c^2dx + b\,c^3 + a\right)}{3a^2de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*(d*x+c)^3)^2,x)
```

```
[Out] ln(d*x+c)/a^2/d/e+1/3/e/a/d/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)-1
/3/e/a^2/d*ln(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)
```

maxima [A] time = 0.57, size = 114, normalized size = 1.68

$$\frac{1}{3\left(abd^4ex^3 + 3\,abcd^3ex^2 + 3\,abc^2d^2ex + \left(abc^3 + a^2\right)de\right)} - \frac{\log\left(bd^3x^3 + 3\,bcd^2x^2 + 3\,bc^2dx + bc^3 + a\right)}{3\,a^2de} + \frac{\log(dx+c)}{a^2de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 1/3/(a*b*d^4*e*x^3 + 3*a*b*c*d^3*e*x^2 + 3*a*b*c^2*d^2*e*x + (a*b*c^3 + a^2
)*d*e) - 1/3*log(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(a^2*
d*e) + log(d*x + c)/(a^2*d*e)
```

mupad **[B]** time = 1.39, size = 116, normalized size = 1.71

$$\frac{1}{3\left(e\,a^2\,d+b\,e\,a\,c^3\,d+3\,b\,e\,a\,c^2\,d^2\,x+3\,b\,e\,a\,c\,d^3\,x^2+b\,e\,a\,d^4\,x^3\right)}+\frac{\ln\left(c+d\,x\right)}{a^2\,d\,e}-\frac{\ln\left(b\,c^3+3\,b\,c^2\,d\,x+3\,b\,c\,d^2\,x^2\right.}{3\,a^2\,d\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)*(a + b*(c + d*x)^3)^2),x)
```

```
[Out] 1/(3*(a^2*d*e + a*b*d^4*e*x^3 + a*b*c^3*d*e + 3*a*b*c^2*d^2*e*x + 3*a*b*c*d
^3*e*x^2)) + log(c + d*x)/(a^2*d*e) - log(a + b*c^3 + b*d^3*x^3 + 3*b*c^2*d
*x + 3*b*c*d^2*x^2)/(3*a^2*d*e)
```

sympy **[B]** time = 2.17, size = 122, normalized size = 1.79

$$\frac{1}{3a^2de+3abc^3de+9abc^2d^2ex+9abcd^3ex^2+3abd^4ex^3}+\frac{\log\left(\frac{c}{d}+x\right)}{a^2de}-\frac{\log\left(\frac{3c^2x}{d^2}+\frac{3cx^2}{d}+x^3+\frac{a+bc^3}{bd^3}\right)}{3a^2de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*(d*x+c)**3)**2,x)
```

```
[Out] 1/(3*a**2*d*e + 3*a*b*c**3*d*e + 9*a*b*c**2*d**2*e*x + 9*a*b*c*d**3*e*x**2
+ 3*a*b*d**4*e*x**3) + log(c/d + x)/(a**2*d*e) - log(3*c**2*x/d**2 + 3*c*x*
*2/d + x**3 + (a + b*c**3)/(b*d**3))/(3*a**2*d*e)
```

3.2899 $\int \frac{1}{(ce+dex)^2\left(a+b(c+dx)^3\right)^2}\,dx$

Optimal. Leaf size=204

$$-\frac{2\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{9a^{7/3}de^2}+\frac{4\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9a^{7/3}de^2}+\frac{4\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}de^2}-\frac{3}{}$$

```
[Out] -4/3/a^2/d/e^2/(d*x+c)+1/3/a/d/e^2/(d*x+c)/(a+b*(d*x+c)^3)+4/9*b^(1/3)*ln(a
^(1/3)+b^(1/3)*(d*x+c))/a^(7/3)/d/e^2-2/9*b^(1/3)*ln(a^(2/3)-a^(1/3)*b^(1/3
)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(7/3)/d/e^2+4/9*b^(1/3)*arctan(1/3*(a^(1/3)-
2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(7/3)/d/e^2*3^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {372, 290, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{2\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{9a^{7/3}de^2}+\frac{4\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9a^{7/3}de^2}+\frac{4\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}de^2}-\frac{3}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*e + d*e*x)^2*(a + b*(c + d*x)^3)^2),x]
```

```
[Out] -4/(3*a^2*d*e^2*(c + d*x)) + 1/(3*a*d*e^2*(c + d*x)*(a + b*(c + d*x)^3)) +
(4*b^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(3*Sq
rt[3]*a^(7/3)*d*e^2) + (4*b^(1/3)*Log[a^(1/3) + b^(1/3)*(c + d*x)])/(9*a^(7
/3)*d*e^2) - (2*b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(
c + d*x)^2])/(9*a^(7/3)*d*e^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(ce+dex)^2\left(a+b(c+dx)^3\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^3\right)^2}\,dx, x, c+dx\right)}{de^2}$$

$$= \frac{1}{3ade^2(c+dx)\left(a+b(c+dx)^3\right)} + \frac{4\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^3\right)}\,dx, x, c+dx\right)}{3ade^2}$$

$$= -\frac{4}{3a^2de^2(c+dx)} + \frac{1}{3ade^2(c+dx)\left(a+b(c+dx)^3\right)} - \frac{(4b)\,\text{Subst}\left(\int \frac{x}{a+bx^3}\,dx\right.}{3a^2de^2}$$

$$= -\frac{4}{3a^2de^2(c+dx)} + \frac{1}{3ade^2(c+dx)\left(a+b(c+dx)^3\right)} + \frac{\left(4b^{2/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{}}\right.}{9a^{7/3}de}$$

$$= -\frac{4}{3a^2de^2(c+dx)} + \frac{1}{3ade^2(c+dx)\left(a+b(c+dx)^3\right)} + \frac{4\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c\right.}{9a^{7/3}de^2}$$

$$= -\frac{4}{3a^2de^2(c+dx)} + \frac{1}{3ade^2(c+dx)\left(a+b(c+dx)^3\right)} + \frac{4\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c\right.}{9a^{7/3}de^2}$$

$$= -\frac{4}{3a^2de^2(c+dx)} + \frac{1}{3ade^2(c+dx)\left(a+b(c+dx)^3\right)} + \frac{4\sqrt[3]{b}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right.}{3\sqrt{3}\,a^{7/3}de^2}$$

Mathematica [A] time = 0.09, size = 171, normalized size = 0.84

$$\frac{-2\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)-\frac{3\sqrt[3]{a}\,b(c+dx)^2}{a+b(c+dx)^3}+4\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)-4\sqrt{3}\,\sqrt[3]{b}\,\tan^{-1}\left(\right.}{9a^{7/3}de^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*e + d*e*x)^2*(a + b*(c + d*x)^3)^2),x]
```

```
[Out] ((-9*a^(1/3))/(c + d*x) - (3*a^(1/3)*b*(c + d*x)^2)/(a + b*(c + d*x)^3) - 4
*Sqrt[3]*b^(1/3)*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))]
 + 4*b^(1/3)*Log[a^(1/3) + b^(1/3)*(c + d*x)] - 2*b^(1/3)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(9*a^(7/3)*d*e^2)
```

fricas [B] time = 1.18, size = 399, normalized size = 1.96

$$-\frac{12\,bd^3x^3+36\,bcd^2x^2+36\,bc^2dx+12\,bc^3+4\sqrt{3}\left(bd^4x^4+4\,bcd^3x^3+6\,bc^2d^2x^2+bc^4+\left(4\,bc^3+a\right)dx+ac\right)\left(\frac{b}{a}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^2/(a+b*(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] -1/9*(12*b*d^3*x^3 + 36*b*c*d^2*x^2 + 36*b*c^2*d*x + 12*b*c^3 + 4*sqrt(3)*(
b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + b*c^4 + (4*b*c^3 + a)*d*x + a
*c)*(b/a)^(1/3)*arctan(2/3*sqrt(3)*(d*x + c)*(b/a)^(1/3) - 1/3*sqrt(3)) + 2
```

```
*(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + b*c^4 + (4*b*c^3 + a)*d*x +
 a*c)*(b/a)^(1/3)*log(b*d^2*x^2 + 2*b*c*d*x + b*c^2 - (a*d*x + a*c)*(b/a)^(
2/3) + a*(b/a)^(1/3)) - 4*(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + b*
c^4 + (4*b*c^3 + a)*d*x + a*c)*(b/a)^(1/3)*log(b*d*x + b*c + a*(b/a)^(2/3))
 + 9*a)/(a^2*b*d^5*e^2*x^4 + 4*a^2*b*c*d^4*e^2*x^3 + 6*a^2*b*c^2*d^3*e^2*x^
2 + (4*a^2*b*c^3 + a^3)*d^2*e^2*x + (a^2*b*c^4 + a^3*c)*d*e^2)
```

giac [A] time = 0.23, size = 269, normalized size = 1.32

$$\frac{4\left(\frac{b}{ad^3}\right)^{\frac{1}{3}}e^{(-2)}\log\left(\left|-\left(\frac{b}{ad^3}\right)^{\frac{1}{3}}e^{(-2)}-\frac{e^{(-1)}}{(dxe+ce)d}\right|\right)}{9a^2}-\frac{4\sqrt{3}\left(a^2b\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(\frac{b}{ad^3}\right)^{\frac{1}{3}}e^{(-2)}-\frac{2e^{(-1)}}{(dxe+ce)d}\right)e^2}{3\left(\frac{b}{ad^3}\right)^{\frac{1}{3}}}\right)e^{(-2)}}{9a^3d}-\frac{2\left(a^2b\right)^{\frac{1}{3}}e^{(}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^2/(a+b*(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 4/9*(b/(a*d^3))^(1/3)*e^(-2)*log(abs(-(b/(a*d^3))^(1/3)*e^(-2) - e^(-1)/((d
*x*e + c*e)*d)))/a^2 - 4/9*sqrt(3)*(a^2*b)^(1/3)*arctan(1/3*sqrt(3)*((b/(a*
d^3))^(1/3)*e^(-2) - 2*e^(-1)/((d*x*e + c*e)*d))*e^2/(b/(a*d^3))^(1/3))*e^(
-2)/(a^3*d) - 2/9*(a^2*b)^(1/3)*e^(-2)*log((b/(a*d^3))^(2/3)*e^(-4) - (b/(a
*d^3))^(1/3)*e^(-3)/((d*x*e + c*e)*d) + e^(-2)/((d*x*e + c*e)^2*d^2))/(a^3*
d) - e^(-1)/((d*x*e + c*e)*a^2*d) - 1/3*b*e^(-1)/((d*x*e + c*e)*a^2*(b + a*
e^3/(d*x*e + c*e)^3)*d)
```

maple [C] time = 0.02, size = 242, normalized size = 1.19

$$-\frac{bd\,x^2}{3\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)a^2e^2}-\frac{2bcx}{3\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)a^2e^2}-\frac{}{3\left(b\,d^3x^3+3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)^2/(a+b*(d*x+c)^3)^2,x)
```

```
[Out] -1/a^2/d/e^2/(d*x+c)-1/3/e^2/a^2*b/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c
^3+a)*x^2*d-2/3/e^2/a^2*b/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)*c*x
-1/3/e^2/a^2*b/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)*c^2/d-4/9/e^2/
a^2/d*sum((_R*d+c)/(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x),_R=RootOf(_Z^3*b*d^3+3
*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{4\,bd^3x^3+12\,bcd^2x^2+12\,bc^2dx+4\,bc^3+3\,a}{3\left(a^2bd^5e^2x^4+4\,a^2bcd^4e^2x^3+6\,a^2bc^2d^3e^2x^2+\left(4\,a^2bc^3+a^3\right)d^2e^2x+\left(a^2bc^4+a^3c\right)de^2\right)}-\frac{-\frac{2}{3}\left(2\sqrt{3}\left(-\frac{1}{ab^2d^3}\right)^{\frac{1}{}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^2/(a+b*(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(4*b*d^3*x^3 + 12*b*c*d^2*x^2 + 12*b*c^2*d*x + 4*b*c^3 + 3*a)/(a^2*b*d
^5*e^2*x^4 + 4*a^2*b*c*d^4*e^2*x^3 + 6*a^2*b*c^2*d^3*e^2*x^2 + (4*a^2*b*c^3
 + a^3)*d^2*e^2*x + (a^2*b*c^4 + a^3*c)*d*e^2) - 4/3*b*integrate((d*x + c)/
(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/(a^2*e^2)
```

mupad [B] time = 1.85, size = 261, normalized size = 1.28

$$\frac{4\,b^{1/3}\,\ln\left(b^{1/3}\,c+a^{1/3}+b^{1/3}\,d\,x\right)}{9\,a^{7/3}\,d\,e^2}-\frac{\frac{4\,b\,c^3+3\,a}{3\,a^2\,d}+\frac{4\,b\,d^2\,x^3}{3\,a^2}+\frac{4\,b\,c^2\,x}{a^2}+\frac{4\,b\,c\,d\,x^2}{a^2}}{x\left(4\,b\,d\,c^3\,e^2+a\,d\,e^2\right)+b\,c^4\,e^2+a\,c\,e^2+b\,d^4\,e^2\,x^4+4\,b\,c\,d^3\,e^2\,x^3+6\,b\,c^2\,d^2\,e^2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)^2*(a + b*(c + d*x)^3)^2),x)
```

```
[Out] (4*b^(1/3)*log(b^(1/3)*c + a^(1/3) + b^(1/3)*d*x))/(9*a^(7/3)*d*e^2) - ((3*
a + 4*b*c^3)/(3*a^2*d) + (4*b*d^2*x^3)/(3*a^2) + (4*b*c^2*x)/a^2 + (4*b*c*d
*x^2)/a^2)/(x*(a*d*e^2 + 4*b*c^3*d*e^2) + b*c^4*e^2 + a*c*e^2 + b*d^4*e^2*x
^4 + 4*b*c*d^3*e^2*x^3 + 6*b*c^2*d^2*e^2*x^2) - (4*b^(1/3)*log(3^(1/2)*a^(1
/3)*1i + 2*b^(1/3)*c - a^(1/3) + 2*b^(1/3)*d*x)*((3^(1/2)*1i)/2 + 1/2))/(9*
a^(7/3)*d*e^2) + (b^(1/3)*log(2*b^(1/3)*c - 3^(1/2)*a^(1/3)*1i - a^(1/3) +
2*b^(1/3)*d*x)*((3^(1/2)*2i)/9 - 2/9))/(a^(7/3)*d*e^2)
```

sympy [A] time = 2.77, size = 197, normalized size = 0.97

$$\frac{-3a-4bc^3-12bc^2dx-12bcd^2x^2-4bd^3x^3}{3a^3cde^2+3a^2bc^4de^2+18a^2bc^2d^3e^2x^2+12a^2bcd^4e^2x^3+3a^2bd^5e^2x^4+x\left(3a^3d^2e^2+12a^2bc^3d^2e^2\right)}+\frac{\mathrm{RootSum}\left(729$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)**2/(a+b*(d*x+c)**3)**2,x)
```

```
[Out] (-3*a - 4*b*c**3 - 12*b*c**2*d*x - 12*b*c*d**2*x**2 - 4*b*d**3*x**3)/(3*a**
3*c*d*e**2 + 3*a**2*b*c**4*d*e**2 + 18*a**2*b*c**2*d**3*e**2*x**2 + 12*a**2
*b*c*d**4*e**2*x**3 + 3*a**2*b*d**5*e**2*x**4 + x*(3*a**3*d**2*e**2 + 12*a*
*2*b*c**3*d**2*e**2)) + RootSum(729*_t**3*a**7 - 64*b, Lambda(_t, _t*log(x
+ (81*_t**2*a**5 + 16*b*c)/(16*b*d))))/(d*e**2)
```

3.2900 $\int \frac{1}{(ce+dex)^3\left(a+b(c+dx)^3\right)^2}\,dx$

Optimal. Leaf size=204

$$-\frac{5b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9a^{8/3}de^3}+\frac{5b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{18a^{8/3}de^3}+\frac{5b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{8/3}de^3}$$

```
[Out] -5/6/a^2/d/e^3/(d*x+c)^2+1/3/a/d/e^3/(d*x+c)^2/(a+b*(d*x+c)^3)-5/9*b^(2/3)*
ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(8/3)/d/e^3+5/18*b^(2/3)*ln(a^(2/3)-a^(1/3)*b
^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(8/3)/d/e^3+5/9*b^(2/3)*arctan(1/3*(a^(
1/3)-2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(8/3)/d/e^3*3^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {372, 290, 325, 200, 31, 634, 617, 204, 628}

$$-\frac{5b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{9a^{8/3}de^3}+\frac{5b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{18a^{8/3}de^3}+\frac{5b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{8/3}de^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*e + d*e*x)^3*(a + b*(c + d*x)^3)^2),x]
```

```
[Out] -5/(6*a^2*d*e^3*(c + d*x)^2) + 1/(3*a*d*e^3*(c + d*x)^2*(a + b*(c + d*x)^3)
) + (5*b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(
3*Sqrt[3]*a^(8/3)*d*e^3) - (5*b^(2/3)*Log[a^(1/3) + b^(1/3)*(c + d*x)])/(9*
a^(8/3)*d*e^3) + (5*b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/
3)*(c + d*x)^2])/(18*a^(8/3)*d*e^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(ce+dex)^3\left(a+b(c+dx)^3\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3\left(a+bx^3\right)^2}\,dx, x, c+dx\right)}{de^3}$$

$$= \frac{1}{3ade^3(c+dx)^2\left(a+b(c+dx)^3\right)} + \frac{5\,\text{Subst}\left(\int \frac{1}{x^3\left(a+bx^3\right)}\,dx, x, c+dx\right)}{3ade^3}$$

$$= -\frac{5}{6a^2de^3(c+dx)^2} + \frac{1}{3ade^3(c+dx)^2\left(a+b(c+dx)^3\right)} - \frac{(5b)\,\text{Subst}\left(\int \frac{1}{a+b}\right.}{3a^2d}$$

$$= -\frac{5}{6a^2de^3(c+dx)^2} + \frac{1}{3ade^3(c+dx)^2\left(a+b(c+dx)^3\right)} - \frac{(5b)\,\text{Subst}\left(\int \frac{}{\sqrt[3]{a}}\right.}{9a^8}$$

$$= -\frac{5}{6a^2de^3(c+dx)^2} + \frac{1}{3ade^3(c+dx)^2\left(a+b(c+dx)^3\right)} - \frac{5b^{2/3}\log\left(\sqrt[3]{a}+\right.}{9a^{8/3}de}$$

$$= -\frac{5}{6a^2de^3(c+dx)^2} + \frac{1}{3ade^3(c+dx)^2\left(a+b(c+dx)^3\right)} - \frac{5b^{2/3}\log\left(\sqrt[3]{a}+\right.}{9a^{8/3}de}$$

$$= -\frac{5}{6a^2de^3(c+dx)^2} + \frac{1}{3ade^3(c+dx)^2\left(a+b(c+dx)^3\right)} + \frac{5b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{}}{}}{\sqrt{}}\right.}{3\sqrt{3}\,a^{8/3}de^3}$$

Mathematica [A] time = 0.06, size = 169, normalized size = 0.83

$$\frac{5b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)-\frac{6a^{2/3}b(c+dx)}{a+b(c+dx)^3}-\frac{9a^{2/3}}{(c+dx)^2}-10b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)-10\sqrt{3}}{18a^{8/3}de^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*e + d*e*x)^3*(a + b*(c + d*x)^3)^2),x]
```

```
[Out] ((-9*a^(2/3))/(c + d*x)^2 - (6*a^(2/3)*b*(c + d*x))/(a + b*(c + d*x)^3) - 1
0*Sqrt[3]*b^(2/3)*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))
] - 10*b^(2/3)*Log[a^(1/3) + b^(1/3)*(c + d*x)] + 5*b^(2/3)*Log[a^(2/3) - a
^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(18*a^(8/3)*d*e^3)
```

fricas [B] time = 1.06, size = 527, normalized size = 2.58

$$-\frac{15\,bd^3x^3+45\,bcd^2x^2+45\,bc^2dx+15\,bc^3-10\sqrt{3}\left(bd^5x^5+5\,bcd^4x^4+10\,bc^2d^3x^3+bc^5+\left(10\,bc^3+a\right)d^2x^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^3/(a+b*(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] -1/18*(15*b*d^3*x^3 + 45*b*c*d^2*x^2 + 45*b*c^2*d*x + 15*b*c^3 - 10*sqrt(3)
*(b*d^5*x^5 + 5*b*c*d^4*x^4 + 10*b*c^2*d^3*x^3 + b*c^5 + (10*b*c^3 + a)*d^2
```

```
*x^2 + a*c^2 + (5*b*c^4 + 2*a*c)*d*x)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3
)*(a*d*x + a*c)*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) + 5*(b*d^5*x^5 + 5*b*c*d^4
*x^4 + 10*b*c^2*d^3*x^3 + b*c^5 + (10*b*c^3 + a)*d^2*x^2 + a*c^2 + (5*b*c^4
 + 2*a*c)*d*x)*(-b^2/a^2)^(1/3)*log(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + a
^2*(-b^2/a^2)^(2/3) + (a*b*d*x + a*b*c)*(-b^2/a^2)^(1/3)) - 10*(b*d^5*x^5 +
 5*b*c*d^4*x^4 + 10*b*c^2*d^3*x^3 + b*c^5 + (10*b*c^3 + a)*d^2*x^2 + a*c^2
+ (5*b*c^4 + 2*a*c)*d*x)*(-b^2/a^2)^(1/3)*log(b*d*x + b*c - a*(-b^2/a^2)^(1
/3)) + 9*a)/(a^2*b*d^6*e^3*x^5 + 5*a^2*b*c*d^5*e^3*x^4 + 10*a^2*b*c^2*d^4*e
^3*x^3 + (10*a^2*b*c^3 + a^3)*d^3*e^3*x^2 + (5*a^2*b*c^4 + 2*a^3*c)*d^2*e^3
*x + (a^2*b*c^5 + a^3*c^2)*d*e^3)
```

giac [A] time = 0.23, size = 250, normalized size = 1.23

$$\frac{5\left(2\sqrt{3}\left(-\frac{b^2}{a^2d^3}\right)^{\frac{1}{3}}\arctan\left(-\frac{bdx+bc-\left(-ab^2\right)^{\frac{1}{3}}}{\sqrt{3}\,bdx+\sqrt{3}\,bc+\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}}\right)-\left(-\frac{b^2}{a^2d^3}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}\,bdx+\sqrt{3}\,bc+\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\right)^2+4\left(bdx\right.\right.\right.}{18\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^3/(a+b*(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 5/18*(2*sqrt(3)*(-b^2/(a^2*d^3))^(1/3)*arctan(-(b*d*x + b*c - (-a*b^2)^(1/3
))/(sqrt(3)*b*d*x + sqrt(3)*b*c + sqrt(3)*(-a*b^2)^(1/3))) - (-b^2/(a^2*d^3
))^(1/3)*log(4*(sqrt(3)*b*d*x + sqrt(3)*b*c + sqrt(3)*(-a*b^2)^(1/3))^2 + 4
*(b*d*x + b*c - (-a*b^2)^(1/3))^2) + 2*(-b^2/(a^2*d^3))^(1/3)*log(abs(b*d*x
 + b*c - (-a*b^2)^(1/3))))*e^(-3)/a^2 - 1/3*(b*d*x + b*c)*e^(-3)/((b*d^3*x^
3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*a^2*d) - 1/2*e^(-3)/((d*x + c)
^2*a^2*d)
```

maple [C] time = 0.02, size = 186, normalized size = 0.91

$$-\frac{bx}{3\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)a^2e^3}-\frac{bc}{3\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)a^2d\,e^3}-\frac{}{9a^2d\,e^3\left(d^2\,\mathrm{Ro}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)^3/(a+b*(d*x+c)^3)^2,x)
```

```
[Out] -1/2/a^2/d/e^3/(d*x+c)^2-1/3/e^3/a^2*b/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x
+b*c^3+a)*x-1/3/e^3/a^2*b/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)*c/d
-5/9/e^3/a^2/d*sum(1/(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x),_R=RootOf(_Z^3*b*d^3
+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{5\,bd^3x^3+15\,bcd^2x^2+15\,bc^2dx+5\,bc^3+3\,a}{6\left(a^2bd^6e^3x^5+5\,a^2bcd^5e^3x^4+10\,a^2bc^2d^4e^3x^3+\left(10\,a^2bc^3+a^3\right)d^3e^3x^2+\left(5\,a^2bc^4+2\,a^3c\right)d^2e^3x+\left(a^2bc^5+a^3c^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^3/(a+b*(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(5*b*d^3*x^3 + 15*b*c*d^2*x^2 + 15*b*c^2*d*x + 5*b*c^3 + 3*a)/(a^2*b*d
^6*e^3*x^5 + 5*a^2*b*c*d^5*e^3*x^4 + 10*a^2*b*c^2*d^4*e^3*x^3 + (10*a^2*b*c
```

```
^3 + a^3)*d^3*e^3*x^2 + (5*a^2*b*c^4 + 2*a^3*c)*d^2*e^3*x + (a^2*b*c^5 + a^
3*c^2)*d*e^3) - 5/3*b*integrate(1/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x
+ b*c^3 + a), x)/(a^2*e^3)
```

mupad [B] time = 2.10, size = 291, normalized size = 1.43

$$-\frac{\frac{5\,b\,c^3+3\,a}{6\,a^2\,d}+\frac{5\,b\,d^2\,x^3}{6\,a^2}+\frac{5\,b\,c^2\,x}{2\,a^2}+\frac{5\,b\,c\,d\,x^2}{2\,a^2}}{x\left(5\,b\,d\,c^4\,e^3+2\,a\,d\,c\,e^3\right)+x^2\left(10\,b\,c^3\,d^2\,e^3+a\,d^2\,e^3\right)+a\,c^2\,e^3+b\,c^5\,e^3+b\,d^5\,e^3\,x^5+5\,b\,c\,d^4\,e^3\,x^4+10\,b\,c^2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)^3*(a + b*(c + d*x)^3)^2),x)
```

```
[Out] (5*b^(2/3)*log(2*b^(1/3)*c - 3^(1/2)*a^(1/3)*1i - a^(1/3) + 2*b^(1/3)*d*x)*
((3^(1/2)*1i)/2 + 1/2))/(9*a^(8/3)*d*e^3) - (5*b^(2/3)*log(b^(1/3)*c + a^(1
/3) + b^(1/3)*d*x))/(9*a^(8/3)*d*e^3) - ((3*a + 5*b*c^3)/(6*a^2*d) + (5*b*d
^2*x^3)/(6*a^2) + (5*b*c^2*x)/(2*a^2) + (5*b*c*d*x^2)/(2*a^2))/(x*(2*a*c*d*
e^3 + 5*b*c^4*d*e^3) + x^2*(a*d^2*e^3 + 10*b*c^3*d^2*e^3) + a*c^2*e^3 + b*c
^5*e^3 + b*d^5*e^3*x^5 + 5*b*c*d^4*e^3*x^4 + 10*b*c^2*d^3*e^3*x^3) - (5*b^(
2/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*c - a^(1/3) + 2*b^(1/3)*d*x)*((3^(1
/2)*1i)/2 - 1/2))/(9*a^(8/3)*d*e^3)
```

sympy [A] time = 4.11, size = 233, normalized size = 1.14

$$\frac{-3a-5bc^3-15bc^2dx-15bcd^2x^2-5bd^3x^3}{6a^3c^2de^3+6a^2bc^5de^3+60a^2bc^2d^4e^3x^3+30a^2bcd^5e^3x^4+6a^2bd^6e^3x^5+x^2\left(6a^3d^3e^3+60a^2bc^3d^3e^3\right)+x\left(12a^3c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)**3/(a+b*(d*x+c)**3)**2,x)
```

```
[Out] (-3*a - 5*b*c**3 - 15*b*c**2*d*x - 15*b*c*d**2*x**2 - 5*b*d**3*x**3)/(6*a**
3*c**2*d*e**3 + 6*a**2*b*c**5*d*e**3 + 60*a**2*b*c**2*d**4*e**3*x**3 + 30*a
**2*b*c*d**5*e**3*x**4 + 6*a**2*b*d**6*e**3*x**5 + x**2*(6*a**3*d**3*e**3 +
 60*a**2*b*c**3*d**3*e**3) + x*(12*a**3*c*d**2*e**3 + 30*a**2*b*c**4*d**2*e
**3)) + RootSum(729*_t**3*a**8 + 125*b**2, Lambda(_t, _t*log(x + (-9*_t*a**
3 + 5*b*c)/(5*b*d))))/(d*e**3)
```

3.2901 $\int \frac{1}{(ce+dex)^4\left(a+b(c+dx)^3\right)^2}\,dx$

Optimal. Leaf size=92

$$-\frac{2b\log(c+dx)}{a^3de^4}+\frac{2b\log\left(a+b(c+dx)^3\right)}{3a^3de^4}-\frac{b}{3a^2de^4\left(a+b(c+dx)^3\right)}-\frac{1}{3a^2de^4(c+dx)^3}$$

```
[Out] -1/3/a^2/d/e^4/(d*x+c)^3-1/3*b/a^2/d/e^4/(a+b*(d*x+c)^3)-2*b*ln(d*x+c)/a^3/
d/e^4+2/3*b*ln(a+b*(d*x+c)^3)/a^3/d/e^4
```

Rubi [A] time = 0.07, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {372, 266, 44}

$$-\frac{b}{3a^2de^4\left(a+b(c+dx)^3\right)}-\frac{2b\log(c+dx)}{a^3de^4}+\frac{2b\log\left(a+b(c+dx)^3\right)}{3a^3de^4}-\frac{1}{3a^2de^4(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*e + d*e*x)^4*(a + b*(c + d*x)^3)^2),x]
```

```
[Out] -1/(3*a^2*d*e^4*(c + d*x)^3) - b/(3*a^2*d*e^4*(a + b*(c + d*x)^3)) - (2*b*L
og[c + d*x])/(a^3*d*e^4) + (2*b*Log[a + b*(c + d*x)^3])/(3*a^3*d*e^4)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{(ce+dex)^4\left(a+b(c+dx)^3\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^3\right)^2}\,dx, x, c+dx\right)}{de^4}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2}\,dx, x, (c+dx)^3\right)}{3de^4}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x^2} - \frac{2b}{a^3x} + \frac{b^2}{a^2(a+bx)^2} + \frac{2b^2}{a^3(a+bx)}\right)\,dx, x, (c+dx)^3\right)}{3de^4}$$
$$= -\frac{1}{3a^2de^4(c+dx)^3} - \frac{b}{3a^2de^4\left(a+b(c+dx)^3\right)} - \frac{2b\log(c+dx)}{a^3de^4} + \frac{2b\log($$

Mathematica [A] time = 0.06, size = 63, normalized size = 0.68

$$-\frac{a\left(\frac{b}{a+b(c+dx)^3} + \frac{1}{(c+dx)^3}\right) - 2b\log\left(a+b(c+dx)^3\right) + 6b\log(c+dx)}{3a^3de^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*e + d*e*x)^4*(a + b*(c + d*x)^3)^2),x]
```

```
[Out] -1/3*(a*((c + d*x)^(-3) + b/(a + b*(c + d*x)^3)) + 6*b*Log[c + d*x] - 2*b*L
og[a + b*(c + d*x)^3])/(a^3*d*e^4)
```

fricas [B] time = 0.92, size = 452, normalized size = 4.91

$$-\frac{2\,abd^3x^3 + 6\,abcd^2x^2 + 6\,abc^2dx + 2\,abc^3 + a^2 - 2\left(b^2d^6x^6 + 6\,b^2cd^5x^5 + 15\,b^2c^2d^4x^4 + b^2c^6 + \left(20\,b^2c^3 + ab\right)\right.}{3\left(a^3bd^7e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^4/(a+b*(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(2*a*b*d^3*x^3 + 6*a*b*c*d^2*x^2 + 6*a*b*c^2*d*x + 2*a*b*c^3 + a^2 - 2
*(b^2*d^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4 + b^2*c^6 + (20*b^2*c^
3 + a*b)*d^3*x^3 + a*b*c^3 + 3*(5*b^2*c^4 + a*b*c)*d^2*x^2 + 3*(2*b^2*c^5 +
 a*b*c^2)*d*x)*log(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a) + 6
*(b^2*d^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4 + b^2*c^6 + (20*b^2*c^
3 + a*b)*d^3*x^3 + a*b*c^3 + 3*(5*b^2*c^4 + a*b*c)*d^2*x^2 + 3*(2*b^2*c^5 +
 a*b*c^2)*d*x)*log(d*x + c))/(a^3*b*d^7*e^4*x^6 + 6*a^3*b*c*d^6*e^4*x^5 + 1
5*a^3*b*c^2*d^5*e^4*x^4 + (20*a^3*b*c^3 + a^4)*d^4*e^4*x^3 + 3*(5*a^3*b*c^4
 + a^4*c)*d^3*e^4*x^2 + 3*(2*a^3*b*c^5 + a^4*c^2)*d^2*e^4*x + (a^3*b*c^6 +
a^4*c^3)*d*e^4)
```

giac [A] time = 0.20, size = 158, normalized size = 1.72

$$\frac{2\,be^{(-4)}\log\left(\left|bd^3x^3 + 3\,bcd^2x^2 + 3\,bc^2dx + bc^3 + a\right|\right)}{3\,a^3d} - \frac{2\,be^{(-4)}\log\left(|dx+c|\right)}{a^3d} - \frac{\left(2\,abd^3x^3 + 6\,abcd^2x^2 + 6\,abc^2dx\right.}{3\left(bd^3x^3 + 3\,bcd^2x^2 + 3\,bc^2dx + b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^4/(a+b*(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 2/3*b*e^(-4)*log(abs(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a))/
(a^3*d) - 2*b*e^(-4)*log(abs(d*x + c))/(a^3*d) - 1/3*(2*a*b*d^3*x^3 + 6*a*b
```

```
*c*d^2*x^2 + 6*a*b*c^2*d*x + 2*a*b*c^3 + a^2)*e^(-4)/((b*d^3*x^3 + 3*b*c*d^
2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*(d*x + c)^3*a^3*d)
```

maple [A] time = 0.02, size = 131, normalized size = 1.42

$$-\frac{b}{3\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)a^2d\,e^4}-\frac{2b\ln\left(dx+c\right)}{a^3d\,e^4}+\frac{2b\ln\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)}{3a^3d\,e^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)^4/(a+b*(d*x+c)^3)^2,x)
```

```
[Out] -1/3/a^2/d/e^4/(d*x+c)^3-2*b*ln(d*x+c)/a^3/d/e^4-1/3/e^4/a^2*b/d/(b*d^3*x^3
+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)+2/3/e^4/a^3*b/d*ln(b*d^3*x^3+3*b*c*d^2*
x^2+3*b*c^2*d*x+b*c^3+a)
```

maxima [B] time = 0.65, size = 249, normalized size = 2.71

$$-\frac{2\,bd^3x^3+6\,bcd^2x^2+6\,bc^2dx+2\,bc^3+a}{3\left(a^2bd^7e^4x^6+6\,a^2bcd^6e^4x^5+15\,a^2bc^2d^5e^4x^4+\left(20\,a^2bc^3+a^3\right)d^4e^4x^3+3\left(5\,a^2bc^4+a^3c\right)d^3e^4x^2+3\left(2\,a^2bc^5+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^4/(a+b*(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(2*b*d^3*x^3 + 6*b*c*d^2*x^2 + 6*b*c^2*d*x + 2*b*c^3 + a)/(a^2*b*d^7*e
^4*x^6 + 6*a^2*b*c*d^6*e^4*x^5 + 15*a^2*b*c^2*d^5*e^4*x^4 + (20*a^2*b*c^3 +
 a^3)*d^4*e^4*x^3 + 3*(5*a^2*b*c^4 + a^3*c)*d^3*e^4*x^2 + 3*(2*a^2*b*c^5 +
a^3*c^2)*d^2*e^4*x + (a^2*b*c^6 + a^3*c^3)*d*e^4) + 2/3*b*log(b*d^3*x^3 + 3
*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(a^3*d*e^4) - 2*b*log(d*x + c)/(a^3
*d*e^4)
```

mupad [B] time = 1.98, size = 250, normalized size = 2.72

$$\frac{2\,b\,\ln\left(b\,c^3+3\,b\,c^2\,d\,x+3\,b\,c\,d^2\,x^2+b\,d^3\,x^3+a\right)}{3\,a^3\,d\,e^4}-\frac{\frac{2\,b\,c^3+}{3\,a^2\,d}}{x^2\left(15\,b\,c^4\,d^2\,e^4+3\,a\,c\,d^2\,e^4\right)+x\left(6\,b\,d\,c^5\,e^4+3\,a\,d\,c^2\,e^4\right)+x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)^4*(a + b*(c + d*x)^3)^2),x)
```

```
[Out] (2*b*log(a + b*c^3 + b*d^3*x^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2))/(3*a^3*d*e^4
) - ((a + 2*b*c^3)/(3*a^2*d) + (2*b*d^2*x^3)/(3*a^2) + (2*b*c^2*x)/a^2 + (2
*b*c*d*x^2)/a^2)/(x^2*(15*b*c^4*d^2*e^4 + 3*a*c*d^2*e^4) + x*(3*a*c^2*d*e^4
 + 6*b*c^5*d*e^4) + x^3*(a*d^3*e^4 + 20*b*c^3*d^3*e^4) + a*c^3*e^4 + b*c^6*
e^4 + b*d^6*e^4*x^6 + 6*b*c*d^5*e^4*x^5 + 15*b*c^2*d^4*e^4*x^4) - (2*b*log(
c + d*x))/(a^3*d*e^4)
```

sympy [B] time = 5.10, size = 294, normalized size = 3.20

$$\frac{-a-2bc^3-6bc^2dx-6bcd^2x^2-2bd^3x^3}{3a^3c^3de^4+3a^2bc^6de^4+45a^2bc^2d^5e^4x^4+18a^2bcd^6e^4x^5+3a^2bd^7e^4x^6+x^3\left(3a^3d^4e^4+60a^2bc^3d^4e^4\right)+x^2\left(9a^3cd^3e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)**4/(a+b*(d*x+c)**3)**2,x)
```

```
[Out] (-a - 2*b*c**3 - 6*b*c**2*d*x - 6*b*c*d**2*x**2 - 2*b*d**3*x**3)/(3*a**3*c*
*3*d*e**4 + 3*a**2*b*c**6*d*e**4 + 45*a**2*b*c**2*d**5*e**4*x**4 + 18*a**2*
```

```
b*c*d**6*e**4*x**5 + 3*a**2*b*d**7*e**4*x**6 + x**3*(3*a**3*d**4*e**4 + 60*
a**2*b*c**3*d**4*e**4) + x**2*(9*a**3*c*d**3*e**4 + 45*a**2*b*c**4*d**3*e**
4) + x*(9*a**3*c**2*d**2*e**4 + 18*a**2*b*c**5*d**2*e**4)) - 2*b*log(c/d +
x)/(a**3*d*e**4) + 2*b*log(3*c**2*x/d**2 + 3*c*x**2/d + x**3 + (a + b*c**3)
/(b*d**3))/(3*a**3*d*e**4)
```

3.2902 $\int \frac{(ce+dex)^4}{\left(a+b(c+dx)^3\right)^3}\,dx$

Optimal. Leaf size=220

$$-\frac{e^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{27a^{4/3}b^{5/3}d}+\frac{e^4\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{54a^{4/3}b^{5/3}d}-\frac{e^4\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{4/3}b^{5/3}d}+\frac{e^4(c+$$

```
[Out] -1/6*e^4*(d*x+c)^2/b/d/(a+b*(d*x+c)^3)^2+1/9*e^4*(d*x+c)^2/a/b/d/(a+b*(d*x+
c)^3)-1/27*e^4*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(4/3)/b^(5/3)/d+1/54*e^4*ln(a^
(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(4/3)/b^(5/3)/d-1/27*e^4
*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(4/3)/b^(5/3)/d*
3^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {372, 288, 290, 292, 31, 634, 617, 204, 628}

$$-\frac{e^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{27a^{4/3}b^{5/3}d}+\frac{e^4\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{54a^{4/3}b^{5/3}d}-\frac{e^4\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{4/3}b^{5/3}d}+\frac{e^4(c+$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*(c + d*x)^3)^3,x]
```

```
[Out] -(e^4*(c + d*x)^2)/(6*b*d*(a + b*(c + d*x)^3)^2) + (e^4*(c + d*x)^2)/(9*a*b
*d*(a + b*(c + d*x)^3)) - (e^4*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt
[3]*a^(1/3))])/(9*Sqrt[3]*a^(4/3)*b^(5/3)*d) - (e^4*Log[a^(1/3) + b^(1/3)*(
c + d*x)])/(27*a^(4/3)*b^(5/3)*d) + (e^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c +
 d*x) + b^(2/3)*(c + d*x)^2])/(54*a^(4/3)*b^(5/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
```

```
]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\left(a+b(c+dx)^3\right)^3}\,dx = \frac{e^4\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^3\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^4(c+dx)^2}{6bd\left(a+b(c+dx)^3\right)^2} + \frac{e^4\,\text{Subst}\left(\int \frac{x}{\left(a+bx^3\right)^2}\,dx, x, c+dx\right)}{3bd}$$

$$= -\frac{e^4(c+dx)^2}{6bd\left(a+b(c+dx)^3\right)^2} + \frac{e^4(c+dx)^2}{9abd\left(a+b(c+dx)^3\right)} + \frac{e^4\,\text{Subst}\left(\int \frac{x}{a+bx^3}\,dx, x, c+dx\right)}{9abd}$$

$$= -\frac{e^4(c+dx)^2}{6bd\left(a+b(c+dx)^3\right)^2} + \frac{e^4(c+dx)^2}{9abd\left(a+b(c+dx)^3\right)} - \frac{e^4\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, c+dx\right)}{27a^{4/3}b^{4/3}d} +$$

$$= -\frac{e^4(c+dx)^2}{6bd\left(a+b(c+dx)^3\right)^2} + \frac{e^4(c+dx)^2}{9abd\left(a+b(c+dx)^3\right)} - \frac{e^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{27a^{4/3}b^{5/3}d} + \frac{e^4\,\text{Sub}}{}$$

$$= -\frac{e^4(c+dx)^2}{6bd\left(a+b(c+dx)^3\right)^2} + \frac{e^4(c+dx)^2}{9abd\left(a+b(c+dx)^3\right)} - \frac{e^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{27a^{4/3}b^{5/3}d} + \frac{e^4\log}{}$$

$$= -\frac{e^4(c+dx)^2}{6bd\left(a+b(c+dx)^3\right)^2} + \frac{e^4(c+dx)^2}{9abd\left(a+b(c+dx)^3\right)} - \frac{e^4\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{9\sqrt{3}\,a^{4/3}b^{5/3}d} - \frac{e^4\log\left(\sqrt[3]{a}\right.}{27a^4}$$

Mathematica [A] time = 0.02, size = 185, normalized size = 0.84

$$\frac{e^4\left(\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{a^{4/3}} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{a^{4/3}} + \frac{2\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{b}\,(c+dx)-\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{a^{4/3}} + \frac{6b^{2/3}(c+dx)^2}{a\left(a+b(c+dx)^3\right)} - \frac{9b^{2/3}(c+dx)^2}{\left(a+b(c+dx)^3\right)^2}\right)}{54b^{5/3}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*(c + d*x)^3)^3,x]
```

```
[Out] (e^4*((-9*b^(2/3)*(c + d*x)^2)/(a + b*(c + d*x)^3)^2 + (6*b^(2/3)*(c + d*x)
^2)/(a*(a + b*(c + d*x)^3)) + (2*Sqrt[3]*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c +
d*x))/(Sqrt[3]*a^(1/3))])/a^(4/3) - (2*Log[a^(1/3) + b^(1/3)*(c + d*x)])/a^
(4/3) + Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/a^(4
/3)))/(54*b^(5/3)*d)
```

fricas [B] time = 1.09, size = 1886, normalized size = 8.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(6*a*b^3*d^5*e^4*x^5 + 30*a*b^3*c*d^4*e^4*x^4 + 60*a*b^3*c^2*d^3*e^4*
x^3 + 3*(20*a*b^3*c^3 - a^2*b^2)*d^2*e^4*x^2 + 6*(5*a*b^3*c^4 - a^2*b^2*c)*
d*e^4*x + 3*(2*a*b^3*c^5 - a^2*b^2*c^2)*e^4 + 3*sqrt(1/3)*(a*b^3*d^6*e^4*x^
```

```
6 + 6*a*b^3*c*d^5*e^4*x^5 + 15*a*b^3*c^2*d^4*e^4*x^4 + 2*(10*a*b^3*c^3 + a^
2*b^2)*d^3*e^4*x^3 + 3*(5*a*b^3*c^4 + 2*a^2*b^2*c)*d^2*e^4*x^2 + 6*(a*b^3*c
^5 + a^2*b^2*c^2)*d*e^4*x + (a*b^3*c^6 + 2*a^2*b^2*c^3 + a^3*b)*e^4)*sqrt((
-a*b^2)^(1/3)/a)*log((2*b^2*d^3*x^3 + 6*b^2*c*d^2*x^2 + 6*b^2*c^2*d*x + 2*b
^2*c^3 - a*b + 3*sqrt(1/3)*(a*b*d*x + a*b*c + 2*(d^2*x^2 + 2*c*d*x + c^2)*(
-a*b^2)^(2/3) + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)
*(d*x + c))/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)) + (b^2*d
^6*e^4*x^6 + 6*b^2*c*d^5*e^4*x^5 + 15*b^2*c^2*d^4*e^4*x^4 + 2*(10*b^2*c^3 +
 a*b)*d^3*e^4*x^3 + 3*(5*b^2*c^4 + 2*a*b*c)*d^2*e^4*x^2 + 6*(b^2*c^5 + a*b*
c^2)*d*e^4*x + (b^2*c^6 + 2*a*b*c^3 + a^2)*e^4)*(-a*b^2)^(2/3)*log(b^2*d^2*
x^2 + 2*b^2*c*d*x + b^2*c^2 + (-a*b^2)^(1/3)*(b*d*x + b*c) + (-a*b^2)^(2/3)
) - 2*(b^2*d^6*e^4*x^6 + 6*b^2*c*d^5*e^4*x^5 + 15*b^2*c^2*d^4*e^4*x^4 + 2*(
10*b^2*c^3 + a*b)*d^3*e^4*x^3 + 3*(5*b^2*c^4 + 2*a*b*c)*d^2*e^4*x^2 + 6*(b^
2*c^5 + a*b*c^2)*d*e^4*x + (b^2*c^6 + 2*a*b*c^3 + a^2)*e^4)*(-a*b^2)^(2/3)*
log(b*d*x + b*c - (-a*b^2)^(1/3)))/(a^2*b^5*d^7*x^6 + 6*a^2*b^5*c*d^6*x^5 +
 15*a^2*b^5*c^2*d^5*x^4 + 2*(10*a^2*b^5*c^3 + a^3*b^4)*d^4*x^3 + 3*(5*a^2*b
^5*c^4 + 2*a^3*b^4*c)*d^3*x^2 + 6*(a^2*b^5*c^5 + a^3*b^4*c^2)*d^2*x + (a^2*
b^5*c^6 + 2*a^3*b^4*c^3 + a^4*b^3)*d), 1/54*(6*a*b^3*d^5*e^4*x^5 + 30*a*b^3
*c*d^4*e^4*x^4 + 60*a*b^3*c^2*d^3*e^4*x^3 + 3*(20*a*b^3*c^3 - a^2*b^2)*d^2*
e^4*x^2 + 6*(5*a*b^3*c^4 - a^2*b^2*c)*d*e^4*x + 3*(2*a*b^3*c^5 - a^2*b^2*c^
2)*e^4 + 6*sqrt(1/3)*(a*b^3*d^6*e^4*x^6 + 6*a*b^3*c*d^5*e^4*x^5 + 15*a*b^3*
c^2*d^4*e^4*x^4 + 2*(10*a*b^3*c^3 + a^2*b^2)*d^3*e^4*x^3 + 3*(5*a*b^3*c^4 +
 2*a^2*b^2*c)*d^2*e^4*x^2 + 6*(a*b^3*c^5 + a^2*b^2*c^2)*d*e^4*x + (a*b^3*c^
6 + 2*a^2*b^2*c^3 + a^3*b)*e^4)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2
*b*d*x + 2*b*c + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (b^2*d^6*e^4*
x^6 + 6*b^2*c*d^5*e^4*x^5 + 15*b^2*c^2*d^4*e^4*x^4 + 2*(10*b^2*c^3 + a*b)*d
^3*e^4*x^3 + 3*(5*b^2*c^4 + 2*a*b*c)*d^2*e^4*x^2 + 6*(b^2*c^5 + a*b*c^2)*d*
e^4*x + (b^2*c^6 + 2*a*b*c^3 + a^2)*e^4)*(-a*b^2)^(2/3)*log(b^2*d^2*x^2 + 2
*b^2*c*d*x + b^2*c^2 + (-a*b^2)^(1/3)*(b*d*x + b*c) + (-a*b^2)^(2/3)) - 2*(
b^2*d^6*e^4*x^6 + 6*b^2*c*d^5*e^4*x^5 + 15*b^2*c^2*d^4*e^4*x^4 + 2*(10*b^2*
c^3 + a*b)*d^3*e^4*x^3 + 3*(5*b^2*c^4 + 2*a*b*c)*d^2*e^4*x^2 + 6*(b^2*c^5 +
 a*b*c^2)*d*e^4*x + (b^2*c^6 + 2*a*b*c^3 + a^2)*e^4)*(-a*b^2)^(2/3)*log(b*d
*x + b*c - (-a*b^2)^(1/3)))/(a^2*b^5*d^7*x^6 + 6*a^2*b^5*c*d^6*x^5 + 15*a^2
*b^5*c^2*d^5*x^4 + 2*(10*a^2*b^5*c^3 + a^3*b^4)*d^4*x^3 + 3*(5*a^2*b^5*c^4
 + 2*a^3*b^4*c)*d^3*x^2 + 6*(a^2*b^5*c^5 + a^3*b^4*c^2)*d^2*x + (a^2*b^5*c^6
 + 2*a^3*b^4*c^3 + a^4*b^3)*d)]
```

giac [A] time = 0.29, size = 298, normalized size = 1.35

$$-\frac{2\sqrt{3}\left(-\frac{e^{12}}{ab^2d^3}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)}{3\left(-a^2b\right)^{\frac{2}{3}}}\right)+\left(-\frac{e^{12}}{ab^2d^3}\right)^{\frac{1}{3}}\log\left(\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)^2+3\left(-a^2b\right)^{\frac{4}{3}}\right)}{54\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] -1/54*(2*sqrt(3)*(-e^12/(a*b^2*d^3))^(1/3)*arctan(1/3*sqrt(3)*(2*a*b*d*x +
2*a*b*c - (-a^2*b)^(2/3))/(-a^2*b)^(2/3)) + (-e^12/(a*b^2*d^3))^(1/3)*log((
2*a*b*d*x + 2*a*b*c - (-a^2*b)^(2/3))^2 + 3*(-a^2*b)^(4/3)) - 2*(-e^12/(a*b
^2*d^3))^(1/3)*log(abs(a*b*d*x + a*b*c + (-a^2*b)^(2/3))))/(a*b) + 1/18*(2*
b*d^5*x^5*e^4 + 10*b*c*d^4*x^4*e^4 + 20*b*c^2*d^3*x^3*e^4 + 20*b*c^3*d^2*x^
2*e^4 + 10*b*c^4*d*x*e^4 + 2*b*c^5*e^4 - a*d^2*x^2*e^4 - 2*a*c*d*x*e^4 - a*
c^2*e^4)/((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)^2*a*b*d)
```

maple [C] time = 0.02, size = 521, normalized size = 2.37

$$\frac{d^4e^4x^5}{9\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)^2a}+\frac{5c\,d^3e^4x^4}{9\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)^2a}+\frac{10c^2}{9\left(b\,d^3x^3+3bc\,d^2x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*(d*x+c)^3)^3,x)
```

```
[Out] 1/9*e^4/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2*d^4/a*x^5+5/9*e^4/(
b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2*c*d^3/a*x^4+10/9*e^4/(b*d^3*
x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2*c^2*d^2/a*x^3+10/9*e^4/(b*d^3*x^3+
3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2*d/a*x^2*c^3-1/18*e^4/(b*d^3*x^3+3*b*c*
d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2/b*x^2*d+5/9*e^4/(b*d^3*x^3+3*b*c*d^2*x^2+3*b
*c^2*d*x+b*c^3+a)^2*c^4/a*x-1/9*e^4/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*
c^3+a)^2/b*c*x+1/9*e^4/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2/a*c^
5/d-1/18*e^4/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2/b*c^2/d+1/27*e
^4/b^2/a/d*sum((_R*d+c)/(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x),_R=RootOf(_Z^3*b*
d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{6}\left(2\sqrt{3}\left(-\frac{1}{ab^2d^3}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)}{3\left(-a^2b\right)^{\frac{2}{3}}}\right)+\left(-\frac{1}{ab^2d^3}\right)^{\frac{1}{3}}\log\left(\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)^2+3\left(-a^2b\right)^{\frac{4}{3}}\right.\right.$$
$$9\,ab$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] 1/9*e^4*integrate((d*x + c)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^
3 + a), x)/(a*b) + 1/18*(2*b*d^5*e^4*x^5 + 10*b*c*d^4*e^4*x^4 + 20*b*c^2*d^
3*e^4*x^3 + (20*b*c^3 - a)*d^2*e^4*x^2 + 2*(5*b*c^4 - a*c)*d*e^4*x + (2*b*c
^5 - a*c^2)*e^4)/(a*b^3*d^7*x^6 + 6*a*b^3*c*d^6*x^5 + 15*a*b^3*c^2*d^5*x^4
+ 2*(10*a*b^3*c^3 + a^2*b^2)*d^4*x^3 + 3*(5*a*b^3*c^4 + 2*a^2*b^2*c)*d^3*x^
2 + 6*(a*b^3*c^5 + a^2*b^2*c^2)*d^2*x + (a*b^3*c^6 + 2*a^2*b^2*c^3 + a^3*b)
*d)
```

mupad [B] time = 1.86, size = 430, normalized size = 1.95

$$\frac{\frac{d^4\,e^4\,x^5}{9\,a}-\frac{a\,c^2\,e^4-2\,b\,c^5\,e^4}{18\,a\,b\,d}+\frac{10\,c^2\,d^2\,e^4\,x^3}{9\,a}+\frac{5\,c\,d^3\,e^4\,x^4}{9\,a}-\frac{c\,x\left(a\,e^4-5\,b\,c^3\,e^4\right)}{9\,a\,b}-\frac{d\,e^4\,x^2\left(a-20\,b\,c^3\right)}{18\,a\,b}}{x^3\left(20\,b^2\,c^3\,d^3+2\,a\,b\,d^3\right)+x^2\left(15\,b^2\,c^4\,d^2+6\,a\,b\,c\,d^2\right)+a^2+x\left(6\,d\,b^2\,c^5+6\,a\,d\,b\,c^2\right)+b^2\,c^6+b^2\,d^6\,x^6+2\,a\,b\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4/(a + b*(c + d*x)^3)^3,x)
```

```
[Out] ((d^4*e^4*x^5)/(9*a) - (a*c^2*e^4 - 2*b*c^5*e^4)/(18*a*b*d) + (10*c^2*d^2*e
^4*x^3)/(9*a) + (5*c*d^3*e^4*x^4)/(9*a) - (c*x*(a*e^4 - 5*b*c^3*e^4))/(9*a*
b) - (d*e^4*x^2*(a - 20*b*c^3))/(18*a*b))/(x^3*(20*b^2*c^3*d^3 + 2*a*b*d^3)
 + x^2*(15*b^2*c^4*d^2 + 6*a*b*c*d^2) + a^2 + x*(6*b^2*c^5*d + 6*a*b*c^2*d)
 + b^2*c^6 + b^2*d^6*x^6 + 2*a*b*c^3 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4
) - (log(2*(-b)^(4/3)*c - a^(1/3)*b + 2*(-b)^(4/3)*d*x + 3^(1/2)*a^(1/3)*b*
1i)*(3^(1/2)*e^4*1i + e^4))/(54*a^(4/3)*(-b)^(5/3)*d) + (e^4*log(a^(1/3)*b
+ (-b)^(4/3)*c + (-b)^(4/3)*d*x))/(27*a^(4/3)*(-b)^(5/3)*d) + (e^4*log((d^4
```

```
*e^8*((3^(1/2)*1i)/2 - 1/2)^2)/(81*a^(5/3)*(-b)^(4/3)) + (c*d^4*e^8)/(81*a^
2*b) + (d^5*e^8*x)/(81*a^2*b))*((3^(1/2)*1i)/2 - 1/2))/(27*a^(4/3)*(-b)^(5/
3)*d)
```

sympy [A] time = 3.85, size = 332, normalized size = 1.51

$$\frac{-ac^2e^4 + 2bc^5e^4 + 20bc^2d^3e^4x^3 + 10bcd^4e^4x^4 + 2bd^5e^4x^5 + x^2\left(-ad^2e^4 + 20bc^3\right.}{18a^3bd + 36a^2b^2c^3d + 18ab^3c^6d + 270ab^3c^2d^5x^4 + 108ab^3cd^6x^5 + 18ab^3d^7x^6 + x^3\left(36a^2b^2d^4 + 360ab^3c^3d^4\right) +}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*(d*x+c)**3)**3,x)
```

```
[Out] (-a*c**2*e**4 + 2*b*c**5*e**4 + 20*b*c**2*d**3*e**4*x**3 + 10*b*c*d**4*e**4
*x**4 + 2*b*d**5*e**4*x**5 + x**2*(-a*d**2*e**4 + 20*b*c**3*d**2*e**4) + x*
(-2*a*c*d*e**4 + 10*b*c**4*d*e**4))/(18*a**3*b*d + 36*a**2*b**2*c**3*d + 18
*a*b**3*c**6*d + 270*a*b**3*c**2*d**5*x**4 + 108*a*b**3*c*d**6*x**5 + 18*a*
b**3*d**7*x**6 + x**3*(36*a**2*b**2*d**4 + 360*a*b**3*c**3*d**4) + x**2*(10
8*a**2*b**2*c*d**3 + 270*a*b**3*c**4*d**3) + x*(108*a**2*b**2*c**2*d**2 + 1
08*a*b**3*c**5*d**2)) + e**4*RootSum(19683*_t**3*a**4*b**5 + 1, Lambda(_t,
_t*log(x + (729*_t**2*a**3*b**3*e**8 + c*e**8)/(d*e**8))))/d
```

3.2903 $\int \frac{(ce+dex)^3}{\left(a+b(c+dx)^3\right)^3}\,dx$

Optimal. Leaf size=216

$$\frac{e^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{27a^{5/3}b^{4/3}d}-\frac{e^3\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{54a^{5/3}b^{4/3}d}-\frac{e^3\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{5/3}b^{4/3}d}+\frac{e^3(c+$$

```
[Out] -1/6*e^3*(d*x+c)/b/d/(a+b*(d*x+c)^3)^2+1/18*e^3*(d*x+c)/a/b/d/(a+b*(d*x+c)^
3)+1/27*e^3*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(5/3)/b^(4/3)/d-1/54*e^3*ln(a^(2/
3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(5/3)/b^(4/3)/d-1/27*e^3*ar
ctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(5/3)/b^(4/3)/d*3^(
1/2)
```

Rubi [A] time = 0.16, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {372, 288, 199, 200, 31, 634, 617, 204, 628}

$$\frac{e^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{27a^{5/3}b^{4/3}d}-\frac{e^3\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{54a^{5/3}b^{4/3}d}-\frac{e^3\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{5/3}b^{4/3}d}+\frac{e^3(c+$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*(c + d*x)^3)^3,x]
```

```
[Out] -(e^3*(c + d*x))/(6*b*d*(a + b*(c + d*x)^3)^2) + (e^3*(c + d*x))/(18*a*b*d*
(a + b*(c + d*x)^3)) - (e^3*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]
*a^(1/3))])/(9*Sqrt[3]*a^(5/3)*b^(4/3)*d) + (e^3*Log[a^(1/3) + b^(1/3)*(c +
 d*x)])/(27*a^(5/3)*b^(4/3)*d) - (e^3*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*
x) + b^(2/3)*(c + d*x)^2])/(54*a^(5/3)*b^(4/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{\left(a+b(c+dx)^3\right)^3}\,dx = \frac{e^3\,\text{Subst}\left(\int \frac{x^3}{\left(a+bx^3\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^3(c+dx)}{6bd\left(a+b(c+dx)^3\right)^2} + \frac{e^3\,\text{Subst}\left(\int \frac{1}{\left(a+bx^3\right)^2}\,dx, x, c+dx\right)}{6bd}$$

$$= -\frac{e^3(c+dx)}{6bd\left(a+b(c+dx)^3\right)^2} + \frac{e^3(c+dx)}{18abd\left(a+b(c+dx)^3\right)} + \frac{e^3\,\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, c+dx\right)}{9abd}$$

$$= -\frac{e^3(c+dx)}{6bd\left(a+b(c+dx)^3\right)^2} + \frac{e^3(c+dx)}{18abd\left(a+b(c+dx)^3\right)} + \frac{e^3\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, c+dx\right)}{27a^{5/3}bd}$$

$$= -\frac{e^3(c+dx)}{6bd\left(a+b(c+dx)^3\right)^2} + \frac{e^3(c+dx)}{18abd\left(a+b(c+dx)^3\right)} + \frac{e^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{27a^{5/3}b^{4/3}d} - \frac{e^3\,\text{Su}}{}$$

$$= -\frac{e^3(c+dx)}{6bd\left(a+b(c+dx)^3\right)^2} + \frac{e^3(c+dx)}{18abd\left(a+b(c+dx)^3\right)} + \frac{e^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{27a^{5/3}b^{4/3}d} - \frac{e^3\log}{}$$

$$= -\frac{e^3(c+dx)}{6bd\left(a+b(c+dx)^3\right)^2} + \frac{e^3(c+dx)}{18abd\left(a+b(c+dx)^3\right)} - \frac{e^3\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{9\sqrt{3}\,a^{5/3}b^{4/3}d} + \frac{e^3\log\left(\sqrt[3]{a}\right.}{27}$$

Mathematica [A] time = 0.02, size = 182, normalized size = 0.84

$$\frac{e^3\left(-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{a^{5/3}} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{a^{5/3}} + \frac{2\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{b}\,(c+dx)-\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{a^{5/3}} + \frac{3\sqrt[3]{b}\,(c+dx)}{a\left(a+b(c+dx)^3\right)} - \frac{9\sqrt[3]{b}\,(c+dx)}{\left(a+b(c+dx)^3\right)^2}\right)}{54b^{4/3}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*(c + d*x)^3)^3,x]
```

```
[Out] (e^3*((-9*b^(1/3)*(c + d*x))/(a + b*(c + d*x)^3)^2 + (3*b^(1/3)*(c + d*x))/
(a*(a + b*(c + d*x)^3)) + (2*Sqrt[3]*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c + d*x)
)/(Sqrt[3]*a^(1/3))])/a^(5/3) + (2*Log[a^(1/3) + b^(1/3)*(c + d*x)])/a^(5/3
) - Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/a^(5/3))
)/(54*b^(4/3)*d)
```

fricas [B] time = 1.20, size = 1822, normalized size = 8.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(3*a^2*b^2*d^4*e^3*x^4 + 12*a^2*b^2*c*d^3*e^3*x^3 + 18*a^2*b^2*c^2*d^
2*e^3*x^2 + 6*(2*a^2*b^2*c^3 - a^3*b)*d*e^3*x + 3*(a^2*b^2*c^4 - 2*a^3*b*c)
```

```
*e^3 + 3*sqrt(1/3)*(a*b^3*d^6*e^3*x^6 + 6*a*b^3*c*d^5*e^3*x^5 + 15*a*b^3*c^
2*d^4*e^3*x^4 + 2*(10*a*b^3*c^3 + a^2*b^2)*d^3*e^3*x^3 + 3*(5*a*b^3*c^4 + 2
*a^2*b^2*c)*d^2*e^3*x^2 + 6*(a*b^3*c^5 + a^2*b^2*c^2)*d*e^3*x + (a*b^3*c^6
 + 2*a^2*b^2*c^3 + a^3*b)*e^3)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*d^3*x^3 + 6
*a*b*c*d^2*x^2 + 6*a*b*c^2*d*x + 2*a*b*c^3 - a^2 + 3*sqrt(1/3)*(2*a*b*d^2*x
^2 + 4*a*b*c*d*x + 2*a*b*c^2 + (a^2*b)^(2/3)*(d*x + c) - (a^2*b)^(1/3)*a)*s
qrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*(a*d*x + a*c))/(b*d^3*x^3 + 3*b*c*d
^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)) - (b^2*d^6*e^3*x^6 + 6*b^2*c*d^5*e^3*x^5
 + 15*b^2*c^2*d^4*e^3*x^4 + 2*(10*b^2*c^3 + a*b)*d^3*e^3*x^3 + 3*(5*b^2*c^4
 + 2*a*b*c)*d^2*e^3*x^2 + 6*(b^2*c^5 + a*b*c^2)*d*e^3*x + (b^2*c^6 + 2*a*b*
c^3 + a^2)*e^3)*(a^2*b)^(2/3)*log(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2 - (a^
2*b)^(2/3)*(d*x + c) + (a^2*b)^(1/3)*a) + 2*(b^2*d^6*e^3*x^6 + 6*b^2*c*d^5*
e^3*x^5 + 15*b^2*c^2*d^4*e^3*x^4 + 2*(10*b^2*c^3 + a*b)*d^3*e^3*x^3 + 3*(5*
b^2*c^4 + 2*a*b*c)*d^2*e^3*x^2 + 6*(b^2*c^5 + a*b*c^2)*d*e^3*x + (b^2*c^6 +
 2*a*b*c^3 + a^2)*e^3)*(a^2*b)^(2/3)*log(a*b*d*x + a*b*c + (a^2*b)^(2/3)))/
(a^3*b^4*d^7*x^6 + 6*a^3*b^4*c*d^6*x^5 + 15*a^3*b^4*c^2*d^5*x^4 + 2*(10*a^3
*b^4*c^3 + a^4*b^3)*d^4*x^3 + 3*(5*a^3*b^4*c^4 + 2*a^4*b^3*c)*d^3*x^2 + 6*(
a^3*b^4*c^5 + a^4*b^3*c^2)*d^2*x + (a^3*b^4*c^6 + 2*a^4*b^3*c^3 + a^5*b^2)*
d), 1/54*(3*a^2*b^2*d^4*e^3*x^4 + 12*a^2*b^2*c*d^3*e^3*x^3 + 18*a^2*b^2*c^2
*d^2*e^3*x^2 + 6*(2*a^2*b^2*c^3 - a^3*b)*d*e^3*x + 3*(a^2*b^2*c^4 - 2*a^3*b
*c)*e^3 + 6*sqrt(1/3)*(a*b^3*d^6*e^3*x^6 + 6*a*b^3*c*d^5*e^3*x^5 + 15*a*b^3
*c^2*d^4*e^3*x^4 + 2*(10*a*b^3*c^3 + a^2*b^2)*d^3*e^3*x^3 + 3*(5*a*b^3*c^4
 + 2*a^2*b^2*c)*d^2*e^3*x^2 + 6*(a*b^3*c^5 + a^2*b^2*c^2)*d*e^3*x + (a*b^3*c
^6 + 2*a^2*b^2*c^3 + a^3*b)*e^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*
(a^2*b)^(2/3)*(d*x + c) - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - (b^
2*d^6*e^3*x^6 + 6*b^2*c*d^5*e^3*x^5 + 15*b^2*c^2*d^4*e^3*x^4 + 2*(10*b^2*c^
3 + a*b)*d^3*e^3*x^3 + 3*(5*b^2*c^4 + 2*a*b*c)*d^2*e^3*x^2 + 6*(b^2*c^5 + a
*b*c^2)*d*e^3*x + (b^2*c^6 + 2*a*b*c^3 + a^2)*e^3)*(a^2*b)^(2/3)*log(a*b*d^
2*x^2 + 2*a*b*c*d*x + a*b*c^2 - (a^2*b)^(2/3)*(d*x + c) + (a^2*b)^(1/3)*a)
 + 2*(b^2*d^6*e^3*x^6 + 6*b^2*c*d^5*e^3*x^5 + 15*b^2*c^2*d^4*e^3*x^4 + 2*(10
*b^2*c^3 + a*b)*d^3*e^3*x^3 + 3*(5*b^2*c^4 + 2*a*b*c)*d^2*e^3*x^2 + 6*(b^2*
c^5 + a*b*c^2)*d*e^3*x + (b^2*c^6 + 2*a*b*c^3 + a^2)*e^3)*(a^2*b)^(2/3)*log
(a*b*d*x + a*b*c + (a^2*b)^(2/3)))/(a^3*b^4*d^7*x^6 + 6*a^3*b^4*c*d^6*x^5 +
 15*a^3*b^4*c^2*d^5*x^4 + 2*(10*a^3*b^4*c^3 + a^4*b^3)*d^4*x^3 + 3*(5*a^3*b
^4*c^4 + 2*a^4*b^3*c)*d^3*x^2 + 6*(a^3*b^4*c^5 + a^4*b^3*c^2)*d^2*x + (a^3*
b^4*c^6 + 2*a^4*b^3*c^3 + a^5*b^2)*d)]
```

giac [A] time = 0.28, size = 288, normalized size = 1.33

$$\frac{2\sqrt{3}\left(\frac{1}{a^2bd^3}\right)^{\frac{1}{3}}\arctan\left(-\frac{bdx+bc+\left(ab^2\right)^{\frac{1}{3}}}{\sqrt{3}\,bdx+\sqrt{3}\,bc-\sqrt{3}\left(ab^2\right)^{\frac{1}{3}}}\right)e^3-\left(\frac{1}{a^2bd^3}\right)^{\frac{1}{3}}e^3\log\left(4\left(\sqrt{3}\,bdx+\sqrt{3}\,bc-\sqrt{3}\,\left(ab^2\right)^{\frac{1}{3}}\right)^2+4\left(bdx\right.\right.}{54\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 1/54*(2*sqrt(3)*(1/(a^2*b*d^3))^(1/3)*arctan(-(b*d*x + b*c + (a*b^2)^(1/3))
/(sqrt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3)))*e^3 - (1/(a^2*b*d^3
))^(1/3)*e^3*log(4*(sqrt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))^2
 + 4*(b*d*x + b*c + (a*b^2)^(1/3))^2) + 2*(1/(a^2*b*d^3))^(1/3)*e^3*log(abs(
b*d*x + b*c + (a*b^2)^(1/3))))/(a*b) + 1/18*(b*d^4*x^4*e^3 + 4*b*c*d^3*x^3*
e^3 + 6*b*c^2*d^2*x^2*e^3 + 4*b*c^3*d*x*e^3 + b*c^4*e^3 - 2*a*d*x*e^3 - 2*a
*c*e^3)/((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)^2*a*b*d)
```

maple [C] time = 0.02, size = 414, normalized size = 1.92

$$\frac{d^3e^3x^4}{18\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)^2a}+\frac{2c\,d^2e^3x^3}{9\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)^2a}+\frac{}{3\left(b\,d^3x^3+3bc\,d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*(d*x+c)^3)^3,x)
```

```
[Out] 1/18*e^3/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2*d^3/a*x^4+2/9*e^3/
(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2*c*d^2/a*x^3+1/3*e^3/(b*d^3*
x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2/a*c^2*d*x^2+2/9*e^3/(b*d^3*x^3+3*b
*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2/a*x*c^3-1/9*e^3/(b*d^3*x^3+3*b*c*d^2*x^2+
3*b*c^2*d*x+b*c^3+a)^2/b*x+1/18*e^3/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*
c^3+a)^2/a*c^4/d-1/9*e^3/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2/b*
c/d+1/27*e^3/b^2/a/d*sum(1/(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x),_R=RootOf(_Z^3
*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\frac{1}{6}e^3\left(\frac{2\sqrt{3}\left(\frac{1}{a^2b}\right)^{\frac{1}{3}}\arctan\left(-\frac{bdx+bc+\left(ab^2\right)^{\frac{1}{3}}}{\sqrt{3}\,bdx+\sqrt{3}\,bc-\sqrt{3}\left(ab^2\right)^{\frac{1}{3}}}\right)}{d}-\frac{\left(\frac{1}{a^2b}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}\,bdx+\sqrt{3}\,bc-\sqrt{3}\left(ab^2\right)^{\frac{1}{3}}\right)^2+4\left(bdx+bc+\left(ab^2\right)^{\frac{1}{3}}\right)^2\right)}{d}+\frac{2\left(\frac{1}{a^2b}\right)^{\frac{1}{3}}\log\left(\left|bdx+b\right.\right.}{d}\right)}{9\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] 1/9*e^3*integrate(1/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a),
x)/(a*b) + 1/18*(b*d^4*e^3*x^4 + 4*b*c*d^3*e^3*x^3 + 6*b*c^2*d^2*e^3*x^2 +
2*(2*b*c^3 - a)*d*e^3*x + (b*c^4 - 2*a*c)*e^3)/(a*b^3*d^7*x^6 + 6*a*b^3*c*d
^6*x^5 + 15*a*b^3*c^2*d^5*x^4 + 2*(10*a*b^3*c^3 + a^2*b^2)*d^4*x^3 + 3*(5*a
*b^3*c^4 + 2*a^2*b^2*c)*d^3*x^2 + 6*(a*b^3*c^5 + a^2*b^2*c^2)*d^2*x + (a*b^
3*c^6 + 2*a^2*b^2*c^3 + a^3*b)*d)
```

mupad [B] time = 1.67, size = 349, normalized size = 1.62

$$\frac{\frac{b\,c^4\,e^3-2\,a\,c\,e^3}{18\,a\,b\,d}+\frac{d^3\,e^3\,x^4}{18\,a}-\frac{e^3\,x\left(a-2\,b\,c^3\right)}{9\,a\,b}+\frac{c^2\,d\,e^3\,x^2}{3\,a}+\frac{2\,c\,d^2\,e^3\,x^3}{9\,a}}{x^3\left(20\,b^2\,c^3\,d^3+2\,a\,b\,d^3\right)+x^2\left(15\,b^2\,c^4\,d^2+6\,a\,b\,c\,d^2\right)+a^2+x\left(6\,d\,b^2\,c^5+6\,a\,d\,b\,c^2\right)+b^2\,c^6+b^2\,d^6\,x^6+2\,a\,b\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3/(a + b*(c + d*x)^3)^3,x)
```

```
[Out] ((b*c^4*e^3 - 2*a*c*e^3)/(18*a*b*d) + (d^3*e^3*x^4)/(18*a) - (e^3*x*(a - 2*
b*c^3))/(9*a*b) + (c^2*d*e^3*x^2)/(3*a) + (2*c*d^2*e^3*x^3)/(9*a))/(x^3*(20
*b^2*c^3*d^3 + 2*a*b*d^3) + x^2*(15*b^2*c^4*d^2 + 6*a*b*c*d^2) + a^2 + x*(6
*b^2*c^5*d + 6*a*b*c^2*d) + b^2*c^6 + b^2*d^6*x^6 + 2*a*b*c^3 + 6*b^2*c*d^5
*x^5 + 15*b^2*c^2*d^4*x^4) + (e^3*log(b^(1/3)*c + a^(1/3) + b^(1/3)*d*x))/(
27*a^(5/3)*b^(4/3)*d) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*c + a^(1/3) - 2
*b^(1/3)*d*x)*(3^(1/2)*e^3*1i + e^3))/(54*a^(5/3)*b^(4/3)*d) + (e^3*log(3^(
1/2)*a^(1/3)*1i + 2*b^(1/3)*c - a^(1/3) + 2*b^(1/3)*d*x)*((3^(1/2)*1i)/2 -
1/2))/(27*a^(5/3)*b^(4/3)*d)
```

sympy [A] time = 3.66, size = 298, normalized size = 1.38

$$\frac{-2ace^3+bc^4e^3+6bc^2d^2e^3x^2+4bcd^3e^3x^3+bd^4e^3x^4+x\left(-2ade^3\right.}{18a^3bd+36a^2b^2c^3d+18ab^3c^6d+270ab^3c^2d^5x^4+108ab^3cd^6x^5+18ab^3d^7x^6+x^3\left(36a^2b^2d^4+360ab^3c^3d^4\right)+x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*(d*x+c)**3)**3,x)
```

```
[Out] (-2*a*c*e**3 + b*c**4*e**3 + 6*b*c**2*d**2*e**3*x**2 + 4*b*c*d**3*e**3*x**3
 + b*d**4*e**3*x**4 + x*(-2*a*d*e**3 + 4*b*c**3*d*e**3))/(18*a**3*b*d + 36*
a**2*b**2*c**3*d + 18*a*b**3*c**6*d + 270*a*b**3*c**2*d**5*x**4 + 108*a*b**
3*c*d**6*x**5 + 18*a*b**3*d**7*x**6 + x**3*(36*a**2*b**2*d**4 + 360*a*b**3*
c**3*d**4) + x**2*(108*a**2*b**2*c*d**3 + 270*a*b**3*c**4*d**3) + x*(108*a*
*2*b**2*c**2*d**2 + 108*a*b**3*c**5*d**2)) + e**3*RootSum(19683*_t**3*a**5*
b**4 - 1, Lambda(_t, _t*log(x + (27*_t*a**2*b*e**3 + c*e**3)/(d*e**3))))/d
```

3.2904 $\int \frac{(ce+dex)^2}{\left(a+b(c+dx)^3\right)^3}\,dx$

Optimal. Leaf size=26

$$-\frac{e^2}{6bd\left(a+b(c+dx)^3\right)^2}$$

```
[Out] -1/6*e^2/b/d/(a+b*(d*x+c)^3)^2
```

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {372, 261}

$$-\frac{e^2}{6bd\left(a+b(c+dx)^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*(c + d*x)^3)^3,x]
```

```
[Out] -e^2/(6*b*d*(a + b*(c + d*x)^3)^2)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{(ce+dex)^2}{\left(a+b(c+dx)^3\right)^3}\,dx = \frac{e^2\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^3\right)^3}\,dx, x, c+dx\right)}{d}$$
$$= -\frac{e^2}{6bd\left(a+b(c+dx)^3\right)^2}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$-\frac{e^2}{6bd\left(a+b(c+dx)^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*(c + d*x)^3)^3,x]
```

```
[Out] -1/6*e^2/(b*d*(a + b*(c + d*x)^3)^2)
```

fricas [B] time = 0.91, size = 136, normalized size = 5.23

$$-\frac{e^2}{6\left(b^3d^7x^6+6\,b^3cd^6x^5+15\,b^3c^2d^5x^4+2\left(10\,b^3c^3+ab^2\right)d^4x^3+3\left(5\,b^3c^4+2\,ab^2c\right)d^3x^2+6\left(b^3c^5+ab^2c^2\right)d^2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] -1/6*e^2/(b^3*d^7*x^6 + 6*b^3*c*d^6*x^5 + 15*b^3*c^2*d^5*x^4 + 2*(10*b^3*c^
3 + a*b^2)*d^4*x^3 + 3*(5*b^3*c^4 + 2*a*b^2*c)*d^3*x^2 + 6*(b^3*c^5 + a*b^2
*c^2)*d^2*x + (b^3*c^6 + 2*a*b^2*c^3 + a^2*b)*d)
```

giac [A] time = 0.24, size = 45, normalized size = 1.73

$$-\frac{e^2}{6\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)^2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] -1/6*e^2/((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)^2*b*d)
```

maple [A] time = 0.00, size = 47, normalized size = 1.81

$$-\frac{e^2}{6\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)^2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*(d*x+c)^3)^3,x)
```

```
[Out] -1/6*e^2/b/d/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2
```

maxima [B] time = 0.58, size = 136, normalized size = 5.23

$$-\frac{e^2}{6\left(b^3d^7x^6+6\,b^3cd^6x^5+15\,b^3c^2d^5x^4+2\left(10\,b^3c^3+ab^2\right)d^4x^3+3\left(5\,b^3c^4+2\,ab^2c\right)d^3x^2+6\left(b^3c^5+ab^2c^2\right)d^2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] -1/6*e^2/(b^3*d^7*x^6 + 6*b^3*c*d^6*x^5 + 15*b^3*c^2*d^5*x^4 + 2*(10*b^3*c^
3 + a*b^2)*d^4*x^3 + 3*(5*b^3*c^4 + 2*a*b^2*c)*d^3*x^2 + 6*(b^3*c^5 + a*b^2
*c^2)*d^2*x + (b^3*c^6 + 2*a*b^2*c^3 + a^2*b)*d)
```

mupad [B] time = 1.36, size = 134, normalized size = 5.15

$$-\frac{e^2}{6\,b\,d\,\left(x^3\,\left(20\,b^2\,c^3\,d^3+2\,a\,b\,d^3\right)+x^2\,\left(15\,b^2\,c^4\,d^2+6\,a\,b\,c\,d^2\right)+a^2+x\,\left(6\,d\,b^2\,c^5+6\,a\,d\,b\,c^2\right)+b^2\,c^6+b^2\,d^6\,x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2/(a + b*(c + d*x)^3)^3,x)
```

```
[Out] -e^2/(6*b*d*(x^3*(20*b^2*c^3*d^3 + 2*a*b*d^3) + x^2*(15*b^2*c^4*d^2 + 6*a*b
*c*d^2) + a^2 + x*(6*b^2*c^5*d + 6*a*b*c^2*d) + b^2*c^6 + b^2*d^6*x^6 + 2*a
*b*c^3 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4))
```

sympy [**B**] time = 3.37, size = 155, normalized size = 5.96

$$-\frac{e^2}{6a^2bd + 12ab^2c^3d + 6b^3c^6d + 90b^3c^2d^5x^4 + 36b^3cd^6x^5 + 6b^3d^7x^6 + x^3\left(12ab^2d^4 + 120b^3c^3d^4\right) + x^2\left(36ab^2cd^3 +\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*(d*x+c)**3)**3,x)
```

```
[Out] -e**2/(6*a**2*b*d + 12*a*b**2*c**3*d + 6*b**3*c**6*d + 90*b**3*c**2*d**5*x*
*4 + 36*b**3*c*d**6*x**5 + 6*b**3*d**7*x**6 + x**3*(12*a*b**2*d**4 + 120*b*
*3*c**3*d**4) + x**2*(36*a*b**2*c*d**3 + 90*b**3*c**4*d**3) + x*(36*a*b**2*
c**2*d**2 + 36*b**3*c**5*d**2))
```

3.2905 $\int \frac{ce+dex}{\left(a+b(c+dx)^3\right)^3}\,dx$

Optimal. Leaf size=207

$$-\frac{2e\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{27a^{7/3}b^{2/3}d}+\frac{e\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{27a^{7/3}b^{2/3}d}-\frac{2e\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{2/3}d}+\frac{2e(}{9a^2d\left(a\right.}$$

```
[Out] 1/6*e*(d*x+c)^2/a/d/(a+b*(d*x+c)^3)^2+2/9*e*(d*x+c)^2/a^2/d/(a+b*(d*x+c)^3)
-2/27*e*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(7/3)/b^(2/3)/d+1/27*e*ln(a^(2/3)-a^(
1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(7/3)/b^(2/3)/d-2/27*e*arctan(1/3
*(a^(1/3)-2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(7/3)/b^(2/3)/d*3^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {372, 290, 292, 31, 634, 617, 204, 628}

$$-\frac{2e\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{27a^{7/3}b^{2/3}d}+\frac{e\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{27a^{7/3}b^{2/3}d}-\frac{2e\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{2/3}d}+\frac{2e(}{9a^2d\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*(c + d*x)^3)^3,x]
```

```
[Out] (e*(c + d*x)^2)/(6*a*d*(a + b*(c + d*x)^3)^2) + (2*e*(c + d*x)^2)/(9*a^2*d*
(a + b*(c + d*x)^3)) - (2*e*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]
*a^(1/3))])/(9*Sqrt[3]*a^(7/3)*b^(2/3)*d) - (2*e*Log[a^(1/3) + b^(1/3)*(c +
 d*x)])/(27*a^(7/3)*b^(2/3)*d) + (e*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x)
 + b^(2/3)*(c + d*x)^2])/(27*a^(7/3)*b^(2/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{ce+dex}{\left(a+b(c+dx)^3\right)^3}\,dx = \frac{e\,\text{Subst}\left(\int \frac{x}{\left(a+bx^3\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e(c+dx)^2}{6ad\left(a+b(c+dx)^3\right)^2} + \frac{(2e)\,\text{Subst}\left(\int \frac{x}{\left(a+bx^3\right)^2}\,dx, x, c+dx\right)}{3ad}$$

$$= \frac{e(c+dx)^2}{6ad\left(a+b(c+dx)^3\right)^2} + \frac{2e(c+dx)^2}{9a^2d\left(a+b(c+dx)^3\right)} + \frac{(2e)\,\text{Subst}\left(\int \frac{x}{a+bx^3}\,dx, x, c+dx\right)}{9a^2d}$$

$$= \frac{e(c+dx)^2}{6ad\left(a+b(c+dx)^3\right)^2} + \frac{2e(c+dx)^2}{9a^2d\left(a+b(c+dx)^3\right)} - \frac{(2e)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, c+dx\right)}{27a^{7/3}\sqrt[3]{b}\,d} -$$

$$= \frac{e(c+dx)^2}{6ad\left(a+b(c+dx)^3\right)^2} + \frac{2e(c+dx)^2}{9a^2d\left(a+b(c+dx)^3\right)} - \frac{2e\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{27a^{7/3}b^{2/3}d} + \frac{e\,\text{Subst}}{}$$

$$= \frac{e(c+dx)^2}{6ad\left(a+b(c+dx)^3\right)^2} + \frac{2e(c+dx)^2}{9a^2d\left(a+b(c+dx)^3\right)} - \frac{2e\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{27a^{7/3}b^{2/3}d} + \frac{e\log\left(a^{}\right.}{}$$

$$= \frac{e(c+dx)^2}{6ad\left(a+b(c+dx)^3\right)^2} + \frac{2e(c+dx)^2}{9a^2d\left(a+b(c+dx)^3\right)} - \frac{2e\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{9\sqrt{3}\,a^{7/3}b^{2/3}d} - \frac{2e\log\left(\sqrt[3]{a}+\right.}{27a^{7/}}$$

Mathematica [A] time = 0.01, size = 181, normalized size = 0.87

$$\frac{e\left(\frac{2\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{b^{2/3}}+\frac{9a^{4/3}(c+dx)^2}{\left(a+b(c+dx)^3\right)^2}-\frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{b^{2/3}}+\frac{4\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{b}\,(c+dx)-\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{b^{2/3}}+\frac{12\sqrt[3]{a}\,(c+dx)^2}{a+b(c+dx)^3}\right)}{54a^{7/3}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)/(a + b*(c + d*x)^3)^3,x]
```

```
[Out] (e*((9*a^(4/3)*(c + d*x)^2)/(a + b*(c + d*x)^3)^2 + (12*a^(1/3)*(c + d*x)^2
)/(a + b*(c + d*x)^3) + (4*Sqrt[3]*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c + d*x))/
(Sqrt[3]*a^(1/3))])/b^(2/3) - (4*Log[a^(1/3) + b^(1/3)*(c + d*x)])/b^(2/3)
+ (2*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/b^(2/3
)))/(54*a^(7/3)*d)
```

fricas [B] time = 1.27, size = 1780, normalized size = 8.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(12*a*b^3*d^5*e*x^5 + 60*a*b^3*c*d^4*e*x^4 + 120*a*b^3*c^2*d^3*e*x^3
+ 3*(40*a*b^3*c^3 + 7*a^2*b^2)*d^2*e*x^2 + 6*(10*a*b^3*c^4 + 7*a^2*b^2*c)*d
*e*x + 6*sqrt(1/3)*(a*b^3*d^6*e*x^6 + 6*a*b^3*c*d^5*e*x^5 + 15*a*b^3*c^2*d^
4*e*x^4 + 2*(10*a*b^3*c^3 + a^2*b^2)*d^3*e*x^3 + 3*(5*a*b^3*c^4 + 2*a^2*b^2
*c)*d^2*e*x^2 + 6*(a*b^3*c^5 + a^2*b^2*c^2)*d*e*x + (a*b^3*c^6 + 2*a^2*b^2*
c^3 + a^3*b)*e)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*d^3*x^3 + 6*b^2*c*d^2*x^2
 + 6*b^2*c^2*d*x + 2*b^2*c^3 - a*b + 3*sqrt(1/3)*(a*b*d*x + a*b*c + 2*(d^2*
x^2 + 2*c*d*x + c^2)*(-a*b^2)^(2/3) + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)
/a) - 3*(-a*b^2)^(2/3)*(d*x + c))/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x
+ b*c^3 + a)) + 2*(b^2*d^6*e*x^6 + 6*b^2*c*d^5*e*x^5 + 15*b^2*c^2*d^4*e*x^4
 + 2*(10*b^2*c^3 + a*b)*d^3*e*x^3 + 3*(5*b^2*c^4 + 2*a*b*c)*d^2*e*x^2 + 6*(
b^2*c^5 + a*b*c^2)*d*e*x + (b^2*c^6 + 2*a*b*c^3 + a^2)*e)*(-a*b^2)^(2/3)*lo
g(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (-a*b^2)^(1/3)*(b*d*x + b*c) + (-a*
b^2)^(2/3)) - 4*(b^2*d^6*e*x^6 + 6*b^2*c*d^5*e*x^5 + 15*b^2*c^2*d^4*e*x^4 +
 2*(10*b^2*c^3 + a*b)*d^3*e*x^3 + 3*(5*b^2*c^4 + 2*a*b*c)*d^2*e*x^2 + 6*(b^
2*c^5 + a*b*c^2)*d*e*x + (b^2*c^6 + 2*a*b*c^3 + a^2)*e)*(-a*b^2)^(2/3)*log(
b*d*x + b*c - (-a*b^2)^(1/3)) + 3*(4*a*b^3*c^5 + 7*a^2*b^2*c^2)*e)/(a^3*b^4
*d^7*x^6 + 6*a^3*b^4*c*d^6*x^5 + 15*a^3*b^4*c^2*d^5*x^4 + 2*(10*a^3*b^4*c^3
 + a^4*b^3)*d^4*x^3 + 3*(5*a^3*b^4*c^4 + 2*a^4*b^3*c)*d^3*x^2 + 6*(a^3*b^4*
c^5 + a^4*b^3*c^2)*d^2*x + (a^3*b^4*c^6 + 2*a^4*b^3*c^3 + a^5*b^2)*d), 1/54
*(12*a*b^3*d^5*e*x^5 + 60*a*b^3*c*d^4*e*x^4 + 120*a*b^3*c^2*d^3*e*x^3 + 3*(
40*a*b^3*c^3 + 7*a^2*b^2)*d^2*e*x^2 + 6*(10*a*b^3*c^4 + 7*a^2*b^2*c)*d*e*x
+ 12*sqrt(1/3)*(a*b^3*d^6*e*x^6 + 6*a*b^3*c*d^5*e*x^5 + 15*a*b^3*c^2*d^4*e*
x^4 + 2*(10*a*b^3*c^3 + a^2*b^2)*d^3*e*x^3 + 3*(5*a*b^3*c^4 + 2*a^2*b^2*c)*
d^2*e*x^2 + 6*(a*b^3*c^5 + a^2*b^2*c^2)*d*e*x + (a*b^3*c^6 + 2*a^2*b^2*c^3
+ a^3*b)*e)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*d*x + 2*b*c + (-a
*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 2*(b^2*d^6*e*x^6 + 6*b^2*c*d^5*e*
x^5 + 15*b^2*c^2*d^4*e*x^4 + 2*(10*b^2*c^3 + a*b)*d^3*e*x^3 + 3*(5*b^2*c^4
+ 2*a*b*c)*d^2*e*x^2 + 6*(b^2*c^5 + a*b*c^2)*d*e*x + (b^2*c^6 + 2*a*b*c^3 +
 a^2)*e)*(-a*b^2)^(2/3)*log(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (-a*b^2)^
(1/3)*(b*d*x + b*c) + (-a*b^2)^(2/3)) - 4*(b^2*d^6*e*x^6 + 6*b^2*c*d^5*e*x^
5 + 15*b^2*c^2*d^4*e*x^4 + 2*(10*b^2*c^3 + a*b)*d^3*e*x^3 + 3*(5*b^2*c^4 +
2*a*b*c)*d^2*e*x^2 + 6*(b^2*c^5 + a*b*c^2)*d*e*x + (b^2*c^6 + 2*a*b*c^3 + a
^2)*e)*(-a*b^2)^(2/3)*log(b*d*x + b*c - (-a*b^2)^(1/3)) + 3*(4*a*b^3*c^5 +
7*a^2*b^2*c^2)*e)/(a^3*b^4*d^7*x^6 + 6*a^3*b^4*c*d^6*x^5 + 15*a^3*b^4*c^2*d
^5*x^4 + 2*(10*a^3*b^4*c^3 + a^4*b^3)*d^4*x^3 + 3*(5*a^3*b^4*c^4 + 2*a^4*b^
```

```
3*c)*d^3*x^2 + 6*(a^3*b^4*c^5 + a^4*b^3*c^2)*d^2*x + (a^3*b^4*c^6 + 2*a^4*b
^3*c^3 + a^5*b^2)*d)]
```

giac [A] time = 0.28, size = 292, normalized size = 1.41

$$-\frac{2\sqrt{3}\left(-\frac{e^3}{ab^2d^3}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)}{3\left(-a^2b\right)^{\frac{2}{3}}}\right)+\left(-\frac{e^3}{ab^2d^3}\right)^{\frac{1}{3}}\log\left(\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)^2+3\left(-a^2b\right)^{\frac{4}{3}}\right)-2}{27\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] -1/27*(2*sqrt(3)*(-e^3/(a*b^2*d^3))^(1/3)*arctan(1/3*sqrt(3)*(2*a*b*d*x + 2
*a*b*c - (-a^2*b)^(2/3))/(-a^2*b)^(2/3)) + (-e^3/(a*b^2*d^3))^(1/3)*log((2*
a*b*d*x + 2*a*b*c - (-a^2*b)^(2/3))^2 + 3*(-a^2*b)^(4/3)) - 2*(-e^3/(a*b^2*
d^3))^(1/3)*log(abs(a*b*d*x + a*b*c + (-a^2*b)^(2/3))))/a^2 + 1/18*(4*b*d^5
*x^5*e + 20*b*c*d^4*x^4*e + 40*b*c^2*d^3*x^3*e + 40*b*c^3*d^2*x^2*e + 20*b*
c^4*d*x*e + 4*b*c^5*e + 7*a*d^2*x^2*e + 14*a*c*d*x*e + 7*a*c^2*e)/((b*d^3*x
^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)^2*a^2*d)
```

maple [C] time = 0.02, size = 507, normalized size = 2.45

$$\frac{2b\,d^4e\,x^5}{9\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)^2a^2}+\frac{10bc\,d^3e\,x^4}{9\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)^2a^2}+\frac{20}{9\left(b\,d^3x^3+3bc\,d^2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*(d*x+c)^3)^3,x)
```

```
[Out] 2/9*e/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2*b*d^4/a^2*x^5+10/9*e/
(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2*b*c*d^3/a^2*x^4+20/9*e/(b*d
^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2*b*c^2*d^2/a^2*x^3+20/9*e/(b*d^3
*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2*d/a^2*x^2*b*c^3+7/18*e/(b*d^3*x^3
+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2*d/a*x^2+10/9*e/(b*d^3*x^3+3*b*c*d^2*x
^2+3*b*c^2*d*x+b*c^3+a)^2*c^4/a^2*x*b+7/9*e/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^
2*d*x+b*c^3+a)^2*c/a*x+2/9*e/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^
2/a^2*c^5/d*b+7/18*e/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2/a*c^2/
d+2/27*e/a^2/b/d*sum((_R*d+c)/(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x),_R=RootOf(_
Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{4\,bd^5ex^5+20\,bcd^4ex^4+40\,bc^2d^3ex^3+\left(40\,bc^3+7\,a\right)d^2ex^2+2\left(10\,bc^4+7\,ac\right)dex+\left(4\,b\right.}{18\left(a^2b^2d^7x^6+6\,a^2b^2cd^6x^5+15\,a^2b^2c^2d^5x^4+2\left(10\,a^2b^2c^3+a^3b\right)d^4x^3+3\left(5\,a^2b^2c^4+2\,a^3bc\right)d^3x^2+6\left(a^2b^2c^5+a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(4*b*d^5*e*x^5 + 20*b*c*d^4*e*x^4 + 40*b*c^2*d^3*e*x^3 + (40*b*c^3 + 7
*a)*d^2*e*x^2 + 2*(10*b*c^4 + 7*a*c)*d*e*x + (4*b*c^5 + 7*a*c^2)*e)/(a^2*b^
2*d^7*x^6 + 6*a^2*b^2*c*d^6*x^5 + 15*a^2*b^2*c^2*d^5*x^4 + 2*(10*a^2*b^2*c^
3 + a^3*b)*d^4*x^3 + 3*(5*a^2*b^2*c^4 + 2*a^3*b*c)*d^3*x^2 + 6*(a^2*b^2*c^5
 + a^3*b*c^2)*d^2*x + (a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4)*d) + 2/9*e*integrat
e((d*x + c)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/a^2
```

mupad [B] time = 0.50, size = 357, normalized size = 1.72

$$\frac{\frac{4bec^5+7aec^2}{18a^2d}+\frac{x^2\left(40bdec^3+7ade\right)}{18a^2}+\frac{cx\left(10bec^3+7ae\right)}{9a^2}+\frac{2bd^4ex^5}{9a^2}+\frac{20bc^2d^2ex^3}{9a^2}+\frac{10bcd^3}{9a^2}}{x^3\left(20b^2c^3d^3+2abd^3\right)+x^2\left(15b^2c^4d^2+6abcd^2\right)+a^2+x\left(6db^2c^5+6adbc^2\right)+b^2c^6+b^2d^6x^6+2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)/(a + b*(c + d*x)^3)^3,x)
```

```
[Out] ((7*a*c^2*e + 4*b*c^5*e)/(18*a^2*d) + (x^2*(7*a*d*e + 40*b*c^3*d*e))/(18*a^
2) + (c*x*(7*a*e + 10*b*c^3*e))/(9*a^2) + (2*b*d^4*e*x^5)/(9*a^2) + (20*b*c
^2*d^2*e*x^3)/(9*a^2) + (10*b*c*d^3*e*x^4)/(9*a^2))/(x^3*(20*b^2*c^3*d^3 +
2*a*b*d^3) + x^2*(15*b^2*c^4*d^2 + 6*a*b*c*d^2) + a^2 + x*(6*b^2*c^5*d + 6*
a*b*c^2*d) + b^2*c^6 + b^2*d^6*x^6 + 2*a*b*c^3 + 6*b^2*c*d^5*x^5 + 15*b^2*c
^2*d^4*x^4) - (2*e*log(b^(1/3)*c + a^(1/3) + b^(1/3)*d*x))/(27*a^(7/3)*b^(2
/3)*d) + (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*c + a^(1/3) - 2*b^(1/3)*d*x)*(
e - 3^(1/2)*e*1i))/(27*a^(7/3)*b^(2/3)*d) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(
1/3)*c - a^(1/3) + 2*b^(1/3)*d*x)*(e + 3^(1/2)*e*1i))/(27*a^(7/3)*b^(2/3)*d
)
```

sympy [A] time = 3.59, size = 323, normalized size = 1.56

$$\frac{7ac^2e+4bc^5e+40bc^2d^3ex^3+20bcd^4ex^4+4bd^5ex^5+x^2\left(7ad^2e+40bc^3d\right.}{18a^4d+36a^3bc^3d+18a^2b^2c^6d+270a^2b^2c^2d^5x^4+108a^2b^2cd^6x^5+18a^2b^2d^7x^6+x^3\left(36a^3bd^4+360a^2b^2c^3d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*(d*x+c)**3)**3,x)
```

```
[Out] (7*a*c**2*e + 4*b*c**5*e + 40*b*c**2*d**3*e*x**3 + 20*b*c*d**4*e*x**4 + 4*b
*d**5*e*x**5 + x**2*(7*a*d**2*e + 40*b*c**3*d**2*e) + x*(14*a*c*d*e + 20*b*
c**4*d*e))/(18*a**4*d + 36*a**3*b*c**3*d + 18*a**2*b**2*c**6*d + 270*a**2*b
**2*c**2*d**5*x**4 + 108*a**2*b**2*c*d**6*x**5 + 18*a**2*b**2*d**7*x**6 + x
**3*(36*a**3*b*d**4 + 360*a**2*b**2*c**3*d**4) + x**2*(108*a**3*b*c*d**3 +
270*a**2*b**2*c**4*d**3) + x*(108*a**3*b*c**2*d**2 + 108*a**2*b**2*c**5*d**
2)) + e*RootSum(19683*_t**3*a**7*b**2 + 8, Lambda(_t, _t*log(x + (729*_t**2
*a**5*b*e**2 + 4*c*e**2)/(4*d*e**2))))/d
```

3.2906 $\int \frac{1}{(ce+dex)\left(a+b(c+dx)^3\right)^3}\,dx$

Optimal. Leaf size=94

$$-\frac{\log\left(a+b(c+dx)^3\right)}{3a^3de}+\frac{\log(c+dx)}{a^3de}+\frac{1}{3a^2de\left(a+b(c+dx)^3\right)}+\frac{1}{6ade\left(a+b(c+dx)^3\right)^2}$$

```
[Out] 1/6/a/d/e/(a+b*(d*x+c)^3)^2+1/3/a^2/d/e/(a+b*(d*x+c)^3)+ln(d*x+c)/a^3/d/e-1
/3*ln(a+b*(d*x+c)^3)/a^3/d/e
```

Rubi [A] time = 0.08, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {372, 266, 44}

$$\frac{1}{3a^2de\left(a+b(c+dx)^3\right)}-\frac{\log\left(a+b(c+dx)^3\right)}{3a^3de}+\frac{\log(c+dx)}{a^3de}+\frac{1}{6ade\left(a+b(c+dx)^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*e + d*e*x)*(a + b*(c + d*x)^3)^3),x]
```

```
[Out] 1/(6*a*d*e*(a + b*(c + d*x)^3)^2) + 1/(3*a^2*d*e*(a + b*(c + d*x)^3)) + Log
[c + d*x]/(a^3*d*e) - Log[a + b*(c + d*x)^3]/(3*a^3*d*e)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{(ce+dex)\left(a+b(c+dx)^3\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx^3\right)^3}\,dx, x, c+dx\right)}{de}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^3}\,dx, x, (c+dx)^3\right)}{3de}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^3x} - \frac{b}{a(a+bx)^3} - \frac{b}{a^2(a+bx)^2} - \frac{b}{a^3(a+bx)}\right)dx, x, (c+dx)^3\right)}{3de}$$
$$= \frac{1}{6ade\left(a+b(c+dx)^3\right)^2} + \frac{1}{3a^2de\left(a+b(c+dx)^3\right)} + \frac{\log(c+dx)}{a^3de} - \frac{\log\left(a+$$

Mathematica [A] time = 0.04, size = 66, normalized size = 0.70

$$\frac{\frac{a\left(2\left(a+b(c+dx)^3\right)+a\right)}{\left(a+b(c+dx)^3\right)^2} - 2\log\left(a+b(c+dx)^3\right) + 6\log(c+dx)}{6a^3de}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*e + d*e*x)*(a + b*(c + d*x)^3)^3),x]
```

```
[Out] ((a*(a + 2*(a + b*(c + d*x)^3)))/(a + b*(c + d*x)^3)^2 + 6*Log[c + d*x] - 2
*Log[a + b*(c + d*x)^3])/(6*a^3*d*e)
```

fricas [B] time = 1.10, size = 474, normalized size = 5.04

$$\frac{2\,abd^3x^3 + 6\,abcd^2x^2 + 6\,abc^2dx + 2\,abc^3 + 3\,a^2 - 2\left(b^2d^6x^6 + 6\,b^2cd^5x^5 + 15\,b^2c^2d^4x^4 + b^2c^6 + 2\left(10\,b^2c^3 + a\right.\right.}{6\left(a^3b^2d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*b*d^3*x^3 + 6*a*b*c*d^2*x^2 + 6*a*b*c^2*d*x + 2*a*b*c^3 + 3*a^2 -
2*(b^2*d^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4 + b^2*c^6 + 2*(10*b^2
*c^3 + a*b)*d^3*x^3 + 2*a*b*c^3 + 3*(5*b^2*c^4 + 2*a*b*c)*d^2*x^2 + 6*(b^2*
c^5 + a*b*c^2)*d*x + a^2)*log(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c
^3 + a) + 6*(b^2*d^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4 + b^2*c^6 +
 2*(10*b^2*c^3 + a*b)*d^3*x^3 + 2*a*b*c^3 + 3*(5*b^2*c^4 + 2*a*b*c)*d^2*x^2
 + 6*(b^2*c^5 + a*b*c^2)*d*x + a^2)*log(d*x + c))/(a^3*b^2*d^7*e*x^6 + 6*a^
3*b^2*c*d^6*e*x^5 + 15*a^3*b^2*c^2*d^5*e*x^4 + 2*(10*a^3*b^2*c^3 + a^4*b)*d
^4*e*x^3 + 3*(5*a^3*b^2*c^4 + 2*a^4*b*c)*d^3*e*x^2 + 6*(a^3*b^2*c^5 + a^4*b
*c^2)*d^2*e*x + (a^3*b^2*c^6 + 2*a^4*b*c^3 + a^5)*d*e)
```

giac [A] time = 0.24, size = 150, normalized size = 1.60

$$-\frac{e^{(-1)}\log\left(\left|bd^3x^3 + 3\,bcd^2x^2 + 3\,bc^2dx + bc^3 + a\right|\right)}{3\,a^3d} + \frac{e^{(-1)}\log\left(|dx+c|\right)}{a^3d} + \frac{\left(2\,abd^3x^3 + 6\,abcd^2x^2 + 6\,abc^2dx + 2\right.}{6\left(bd^3x^3 + 3\,bcd^2x^2 + 3\,bc^2dx + b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] -1/3*e^(-1)*log(abs(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a))/(
a^3*d) + e^(-1)*log(abs(d*x + c))/(a^3*d) + 1/6*(2*a*b*d^3*x^3 + 6*a*b*c*d^
```

```
2*x^2 + 6*a*b*c^2*d*x + 2*a*b*c^3 + 3*a^2)*e^(-1)/((b*d^3*x^3 + 3*b*c*d^2*x
^2 + 3*b*c^2*d*x + b*c^3 + a)^2*a^3*d)
```

maple [B] time = 0.03, size = 304, normalized size = 3.23

$$\frac{b\,d^2x^3}{3\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)^2a^2e}+\frac{bcd\,x^2}{\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)^2a^2e}+\frac{}{\left(b\,d^3x^3+3bc\,d^2x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*(d*x+c)^3)^3,x)
```

```
[Out] ln(d*x+c)/a^3/d/e+1/3/e*b/a^2/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)
^2*d^2*x^3+1/e*b/a^2/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2*c*d*x^
2+1/e*b/a^2/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2*c^2*x+1/3/e*b/a
^2/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2/d*c^3+1/2/e/a/(b*d^3*x^3
+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2/d-1/3/e/a^3/d*ln(b*d^3*x^3+3*b*c*d^2*
x^2+3*b*c^2*d*x+b*c^3+a)
```

maxima [B] time = 0.61, size = 258, normalized size = 2.74

$$\frac{2\,bd^3x^3+6\,bcd^2x^2+6\,bc^2dx+2\,bc^3+3\,a}{6\left(a^2b^2d^7ex^6+6\,a^2b^2cd^6ex^5+15\,a^2b^2c^2d^5ex^4+2\left(10\,a^2b^2c^3+a^3b\right)d^4ex^3+3\left(5\,a^2b^2c^4+2\,a^3bc\right)d^3ex^2+6\left(a^2b^2c^5\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b*d^3*x^3 + 6*b*c*d^2*x^2 + 6*b*c^2*d*x + 2*b*c^3 + 3*a)/(a^2*b^2*d^
7*e*x^6 + 6*a^2*b^2*c*d^6*e*x^5 + 15*a^2*b^2*c^2*d^5*e*x^4 + 2*(10*a^2*b^2*
c^3 + a^3*b)*d^4*e*x^3 + 3*(5*a^2*b^2*c^4 + 2*a^3*b*c)*d^3*e*x^2 + 6*(a^2*b
^2*c^5 + a^3*b*c^2)*d^2*e*x + (a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4)*d*e) - 1/3*
log(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(a^3*d*e) + log(d*
x + c)/(a^3*d*e)
```

mupad [B] time = 1.91, size = 249, normalized size = 2.65

$$\frac{\frac{2\,bc^3+3\,a}{6\,a^2d}+\frac{bd^2x^3}{3\,a^2}+\frac{bc^2x}{a^2}+\frac{bcdx^2}{a^2}}{x^2\left(15\,e\,b^2\,c^4\,d^2+6\,a\,e\,b\,c\,d^2\right)+a^2\,e+x\left(6\,d\,e\,b^2\,c^5+6\,a\,d\,e\,b\,c^2\right)+x^3\left(20\,e\,b^2\,c^3\,d^3+2\,a\,e\,b\,d^3\right)+b^2\,c^6\,e+b^2\,d^6\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)*(a + b*(c + d*x)^3)^3),x)
```

```
[Out] ((3*a + 2*b*c^3)/(6*a^2*d) + (b*d^2*x^3)/(3*a^2) + (b*c^2*x)/a^2 + (b*c*d*x
^2)/a^2)/(x^2*(15*b^2*c^4*d^2*e + 6*a*b*c*d^2*e) + a^2*e + x*(6*b^2*c^5*d*e
 + 6*a*b*c^2*d*e) + x^3*(20*b^2*c^3*d^3*e + 2*a*b*d^3*e) + b^2*c^6*e + b^2*
d^6*e*x^6 + 2*a*b*c^3*e + 6*b^2*c*d^5*e*x^5 + 15*b^2*c^2*d^4*e*x^4) + log(c
 + d*x)/(a^3*d*e) - log(a + b*c^3 + b*d^3*x^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2
)/(3*a^3*d*e)
```

sympy [B] time = 5.06, size = 292, normalized size = 3.11

$$\frac{3a+2bc^3+6bc^2dx+6bcd^2x^2+2bd^3x^3}{6a^4de+12a^3bc^3de+6a^2b^2c^6de+90a^2b^2c^2d^5ex^4+36a^2b^2cd^6ex^5+6a^2b^2d^7ex^6+x^3\left(12a^3bd^4e+120a^2b^2c^3d^4e\right)+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*(d*x+c)**3)**3,x)
```

```
[Out] (3*a + 2*b*c**3 + 6*b*c**2*d*x + 6*b*c*d**2*x**2 + 2*b*d**3*x**3)/(6*a**4*d
*e + 12*a**3*b*c**3*d*e + 6*a**2*b**2*c**6*d*e + 90*a**2*b**2*c**2*d**5*e*x
**4 + 36*a**2*b**2*c*d**6*e*x**5 + 6*a**2*b**2*d**7*e*x**6 + x**3*(12*a**3*
b*d**4*e + 120*a**2*b**2*c**3*d**4*e) + x**2*(36*a**3*b*c*d**3*e + 90*a**2*
b**2*c**4*d**3*e) + x*(36*a**3*b*c**2*d**2*e + 36*a**2*b**2*c**5*d**2*e)) +
 log(c/d + x)/(a**3*d*e) - log(3*c**2*x/d**2 + 3*c*x**2/d + x**3 + (a + b*c
**3)/(b*d**3))/(3*a**3*d*e)
```

3.2907 $\int \frac{1}{(ce+dex)^2\left(a+b(c+dx)^3\right)^3}\,dx$

Optimal. Leaf size=237

$$-\frac{7\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{27a^{10/3}de^2}+\frac{14\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{27a^{10/3}de^2}+\frac{14\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{10/3}de^2}$$

```
[Out] -14/9/a^3/d/e^2/(d*x+c)+1/6/a/d/e^2/(d*x+c)/(a+b*(d*x+c)^3)^2+7/18/a^2/d/e^
2/(d*x+c)/(a+b*(d*x+c)^3)+14/27*b^(1/3)*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(10/3
)/d/e^2-7/27*b^(1/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/
a^(10/3)/d/e^2+14/27*b^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))/a^(1/3)
*3^(1/2))/a^(10/3)/d/e^2*3^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {372, 290, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{7\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{27a^{10/3}de^2}+\frac{7}{18a^2de^2(c+dx)\left(a+b(c+dx)^3\right)}+\frac{14\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+d\right.}{27a^{10/3}de^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*e + d*e*x)^2*(a + b*(c + d*x)^3)^3),x]
```

```
[Out] -14/(9*a^3*d*e^2*(c + d*x)) + 1/(6*a*d*e^2*(c + d*x)*(a + b*(c + d*x)^3)^2)
 + 7/(18*a^2*d*e^2*(c + d*x)*(a + b*(c + d*x)^3)) + (14*b^(1/3)*ArcTan[(a^(
1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(10/3)*d*e^2)
+ (14*b^(1/3)*Log[a^(1/3) + b^(1/3)*(c + d*x)])/(27*a^(10/3)*d*e^2) - (7*b^
(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(27*a
^(10/3)*d*e^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
```

```
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(ce+dex)^2\left(a+b(c+dx)^3\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^3\right)^3}\,dx, x, c+dx\right)}{de^2}$$

$$= \frac{1}{6ade^2(c+dx)\left(a+b(c+dx)^3\right)^2} + \frac{7\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^3\right)^2}\,dx, x, c+dx\right)}{6ade^2}$$

$$= \frac{1}{6ade^2(c+dx)\left(a+b(c+dx)^3\right)^2} + \frac{7}{18a^2de^2(c+dx)\left(a+b(c+dx)^3\right)} + \frac{14\,\text{Su}}{}$$

$$= -\frac{14}{9a^3de^2(c+dx)} + \frac{1}{6ade^2(c+dx)\left(a+b(c+dx)^3\right)^2} + \frac{7}{18a^2de^2(c+dx)\left(a+\right.}$$

$$= -\frac{14}{9a^3de^2(c+dx)} + \frac{1}{6ade^2(c+dx)\left(a+b(c+dx)^3\right)^2} + \frac{7}{18a^2de^2(c+dx)\left(a+\right.}$$

$$= -\frac{14}{9a^3de^2(c+dx)} + \frac{1}{6ade^2(c+dx)\left(a+b(c+dx)^3\right)^2} + \frac{7}{18a^2de^2(c+dx)\left(a+\right.}$$

$$= -\frac{14}{9a^3de^2(c+dx)} + \frac{1}{6ade^2(c+dx)\left(a+b(c+dx)^3\right)^2} + \frac{7}{18a^2de^2(c+dx)\left(a+\right.}$$

Mathematica [A] time = 0.10, size = 199, normalized size = 0.84

$$\frac{-14\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right) - \frac{9a^{4/3}b(c+dx)^2}{\left(a+b(c+dx)^3\right)^2} - \frac{30\sqrt[3]{a}\,b(c+dx)^2}{a+b(c+dx)^3} + 28\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right) +}{54a^{10/3}de^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*e + d*e*x)^2*(a + b*(c + d*x)^3)^3),x]
```

```
[Out] ((-54*a^(1/3))/(c + d*x) - (9*a^(4/3)*b*(c + d*x)^2)/(a + b*(c + d*x)^3)^2
- (30*a^(1/3)*b*(c + d*x)^2)/(a + b*(c + d*x)^3) - 28*Sqrt[3]*b^(1/3)*ArcTa
n[(-a^(1/3) + 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))] + 28*b^(1/3)*Log[a^(1
/3) + b^(1/3)*(c + d*x)] - 14*b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*
x) + b^(2/3)*(c + d*x)^2])/(54*a^(10/3)*d*e^2)
```

fricas [B] time = 1.05, size = 879, normalized size = 3.71

$$-\frac{84\,b^2d^6x^6 + 504\,b^2cd^5x^5 + 1260\,b^2c^2d^4x^4 + 84\,b^2c^6 + 21\left(80\,b^2c^3+7\,ab\right)d^3x^3 + 147\,abc^3 + 63\left(20\,b^2c^4+7\,abc\right)d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^2/(a+b*(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] -1/54*(84*b^2*d^6*x^6 + 504*b^2*c*d^5*x^5 + 1260*b^2*c^2*d^4*x^4 + 84*b^2*c
^6 + 21*(80*b^2*c^3 + 7*a*b)*d^3*x^3 + 147*a*b*c^3 + 63*(20*b^2*c^4 + 7*a*b
*c)*d^2*x^2 + 63*(8*b^2*c^5 + 7*a*b*c^2)*d*x + 28*sqrt(3)*(b^2*d^7*x^7 + 7*
b^2*c*d^6*x^6 + 21*b^2*c^2*d^5*x^5 + b^2*c^7 + (35*b^2*c^3 + 2*a*b)*d^4*x^4
 + (35*b^2*c^4 + 8*a*b*c)*d^3*x^3 + 2*a*b*c^4 + 3*(7*b^2*c^5 + 4*a*b*c^2)*d
^2*x^2 + a^2*c + (7*b^2*c^6 + 8*a*b*c^3 + a^2)*d*x)*(b/a)^(1/3)*arctan(2/3*
sqrt(3)*(d*x + c)*(b/a)^(1/3) - 1/3*sqrt(3)) + 14*(b^2*d^7*x^7 + 7*b^2*c*d^
6*x^6 + 21*b^2*c^2*d^5*x^5 + b^2*c^7 + (35*b^2*c^3 + 2*a*b)*d^4*x^4 + (35*b
^2*c^4 + 8*a*b*c)*d^3*x^3 + 2*a*b*c^4 + 3*(7*b^2*c^5 + 4*a*b*c^2)*d^2*x^2 +
 a^2*c + (7*b^2*c^6 + 8*a*b*c^3 + a^2)*d*x)*(b/a)^(1/3)*log(b*d^2*x^2 + 2*b
*c*d*x + b*c^2 - (a*d*x + a*c)*(b/a)^(2/3) + a*(b/a)^(1/3)) - 28*(b^2*d^7*x
^7 + 7*b^2*c*d^6*x^6 + 21*b^2*c^2*d^5*x^5 + b^2*c^7 + (35*b^2*c^3 + 2*a*b)*
d^4*x^4 + (35*b^2*c^4 + 8*a*b*c)*d^3*x^3 + 2*a*b*c^4 + 3*(7*b^2*c^5 + 4*a*b
*c^2)*d^2*x^2 + a^2*c + (7*b^2*c^6 + 8*a*b*c^3 + a^2)*d*x)*(b/a)^(1/3)*log(
b*d*x + b*c + a*(b/a)^(2/3)) + 54*a^2)/(a^3*b^2*d^8*e^2*x^7 + 7*a^3*b^2*c*d
^7*e^2*x^6 + 21*a^3*b^2*c^2*d^6*e^2*x^5 + (35*a^3*b^2*c^3 + 2*a^4*b)*d^5*e^
2*x^4 + (35*a^3*b^2*c^4 + 8*a^4*b*c)*d^4*e^2*x^3 + 3*(7*a^3*b^2*c^5 + 4*a^4
*b*c^2)*d^3*e^2*x^2 + (7*a^3*b^2*c^6 + 8*a^4*b*c^3 + a^5)*d^2*e^2*x + (a^3*
b^2*c^7 + 2*a^4*b*c^4 + a^5*c)*d*e^2)
```

giac [A] time = 0.27, size = 295, normalized size = 1.24

$$\frac{14\left(\frac{b}{ad^3}\right)^{\frac{1}{3}}e^{(-2)}\log\left(\left|-\left(\frac{b}{ad^3}\right)^{\frac{1}{3}}e^{(-2)}-\frac{e^{(-1)}}{(dxe+ce)d}\right|\right)}{27a^3}-\frac{14\sqrt{3}\left(a^2b\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(\frac{b}{ad^3}\right)^{\frac{1}{3}}e^{(-2)}-\frac{2e^{(-1)}}{(dxe+ce)d}\right)e^2}{3\left(\frac{b}{ad^3}\right)^{\frac{1}{3}}}\right)e^{(-2)}}{27a^4d}-\frac{7\left(a^2b\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^2/(a+b*(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 14/27*(b/(a*d^3))^(1/3)*e^(-2)*log(abs(-(b/(a*d^3))^(1/3)*e^(-2) - e^(-1)/(
(d*x*e + c*e)*d)))/a^3 - 14/27*sqrt(3)*(a^2*b)^(1/3)*arctan(1/3*sqrt(3)*((b
/(a*d^3))^(1/3)*e^(-2) - 2*e^(-1)/((d*x*e + c*e)*d))*e^2/(b/(a*d^3))^(1/3))
*e^(-2)/(a^4*d) - 7/27*(a^2*b)^(1/3)*e^(-2)*log((b/(a*d^3))^(2/3)*e^(-4) -
(b/(a*d^3))^(1/3)*e^(-3)/((d*x*e + c*e)*d) + e^(-2)/((d*x*e + c*e)^2*d^2))/
(a^4*d) - 1/18*(10*b^2*e^(-1)/((d*x*e + c*e)*d) + 13*a*b*e^2/((d*x*e + c*e)
^4*d))/(a^3*(b + a*e^3/(d*x*e + c*e)^3)^2) - e^(-1)/((d*x*e + c*e)*a^3*d)
```

maple [C] time = 0.03, size = 557, normalized size = 2.35

$$-\frac{5b^2d^4x^5}{9\left(bd^3x^3+3bcd^2x^2+3bc^2dx+bc^3+a\right)^2a^3e^2}-\frac{25b^2cd^3x^4}{9\left(bd^3x^3+3bcd^2x^2+3bc^2dx+bc^3+a\right)^2a^3e^2}-\frac{}{9\left(bd^3x^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)^2/(a+b*(d*x+c)^3)^3,x)
```

```
[Out] -1/a^3/d/e^2/(d*x+c)-5/9/e^2*b^2/a^3/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b
*c^3+a)^2*d^4*x^5-25/9/e^2*b^2/a^3/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c
^3+a)^2*c*d^3*x^4-50/9/e^2*b^2/a^3/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c
^3+a)^2*c^2*d^2*x^3-50/9/e^2*b^2/a^3/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b
*c^3+a)^2*x^2*c^3*d-13/18/e^2*b/a^2/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*
c^3+a)^2*x^2*d-25/9/e^2*b^2/a^3/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+
a)^2*x*c^4-13/9/e^2*b/a^2/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2*c
*x-5/9/e^2*b^2/a^3/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2*c^5/d-13
```

```
/18/e^2*b/a^2/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2*c^2/d-14/27/e
^2/a^3/d*sum((_R*d+c)/(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x),_R=RootOf(_Z^3*b*d^
3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{28\,b^2d^6x^6+168\,b^2cd^5x^5+420\,b^2c^2d^4x^4+28\,b^2c^6+7\left(80\,b^2c^3+7\,ab\right)d^3x^3+49\,ab}{18\left(a^3b^2d^8e^2x^7+7\,a^3b^2cd^7e^2x^6+21\,a^3b^2c^2d^6e^2x^5+\left(35\,a^3b^2c^3+2\,a^4b\right)d^5e^2x^4+\left(35\,a^3b^2c^4+8\,a^4bc\right)d^4e^2x^3+3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^2/(a+b*(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] -1/18*(28*b^2*d^6*x^6 + 168*b^2*c*d^5*x^5 + 420*b^2*c^2*d^4*x^4 + 28*b^2*c^
6 + 7*(80*b^2*c^3 + 7*a*b)*d^3*x^3 + 49*a*b*c^3 + 21*(20*b^2*c^4 + 7*a*b*c)
*d^2*x^2 + 21*(8*b^2*c^5 + 7*a*b*c^2)*d*x + 18*a^2)/(a^3*b^2*d^8*e^2*x^7 +
7*a^3*b^2*c*d^7*e^2*x^6 + 21*a^3*b^2*c^2*d^6*e^2*x^5 + (35*a^3*b^2*c^3 + 2*
a^4*b)*d^5*e^2*x^4 + (35*a^3*b^2*c^4 + 8*a^4*b*c)*d^4*e^2*x^3 + 3*(7*a^3*b^
2*c^5 + 4*a^4*b*c^2)*d^3*e^2*x^2 + (7*a^3*b^2*c^6 + 8*a^4*b*c^3 + a^5)*d^2*
e^2*x + (a^3*b^2*c^7 + 2*a^4*b*c^4 + a^5*c)*d*e^2) - 14/9*b*integrate((d*x
+ c)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/(a^3*e^2)
```

mupad [B] time = 2.60, size = 485, normalized size = 2.05

$$\frac{14\,b^{1/3}\,\ln\left(b^{1/3}\,c+a^{1/3}+b^{1/3}\,d\,x\right)}{27\,a^{10/3}\,d\,e^2}-\frac{\frac{18\,a^2+49\,a\,b\,c^3+28\,b^2\,c^6}{18\,a^3\,d}+\frac{7\,x^2\left(20\right.}{x^3\,\left(35\,b^2\,c^4\,d^3\,e^2+8\,a\,b\,c\,d^3\,e^2\right)+x^2\,\left(21\,b^2\,c^5\,d^2\,e^2+12\,a\,b\,c^2\,d^2\,e^2\right)+x\,\left(d\,a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)^2*(a + b*(c + d*x)^3)^3),x)
```

```
[Out] (14*b^(1/3)*log(b^(1/3)*c + a^(1/3) + b^(1/3)*d*x))/(27*a^(10/3)*d*e^2) - (
(18*a^2 + 28*b^2*c^6 + 49*a*b*c^3)/(18*a^3*d) + (7*x^2*(20*b^2*c^4*d + 7*a*
b*c*d))/(6*a^3) + (7*x*(8*b^2*c^5 + 7*a*b*c^2))/(6*a^3) + (7*x^3*(80*b^2*c^
3*d^2 + 7*a*b*d^2))/(18*a^3) + (14*b^2*d^5*x^6)/(9*a^3) + (70*b^2*c^2*d^3*x
^4)/(3*a^3) + (28*b^2*c*d^4*x^5)/(3*a^3))/(x^3*(35*b^2*c^4*d^3*e^2 + 8*a*b*
c*d^3*e^2) + x^2*(21*b^2*c^5*d^2*e^2 + 12*a*b*c^2*d^2*e^2) + x*(a^2*d*e^2 +
 7*b^2*c^6*d*e^2 + 8*a*b*c^3*d*e^2) + x^4*(35*b^2*c^3*d^4*e^2 + 2*a*b*d^4*e
^2) + a^2*c*e^2 + b^2*c^7*e^2 + b^2*d^7*e^2*x^7 + 2*a*b*c^4*e^2 + 21*b^2*c^
2*d^5*e^2*x^5 + 7*b^2*c*d^6*e^2*x^6) + (14*b^(1/3)*log(2*b^(1/3)*c - 3^(1/2
)*a^(1/3)*1i - a^(1/3) + 2*b^(1/3)*d*x)*((3^(1/2)*1i)/2 - 1/2))/(27*a^(10/3
)*d*e^2) - (14*b^(1/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*c - a^(1/3) + 2*b
^(1/3)*d*x)*((3^(1/2)*1i)/2 + 1/2))/(27*a^(10/3)*d*e^2)
```

sympy [B] time = 6.13, size = 445, normalized size = 1.88

$$\frac{-18a^2-49abc^3-28b^2c^6-420b^2c^2d^4x^4-168b^2cd^5x^5-28b^2d^6x^6}{18a^5cde^2+36a^4bc^4de^2+18a^3b^2c^7de^2+378a^3b^2c^2d^6e^2x^5+126a^3b^2cd^7e^2x^6+18a^3b^2d^8e^2x^7+x^4\left(36a^4bd^5e^2+63\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)**2/(a+b*(d*x+c)**3)**3,x)
```

```
[Out] (-18*a**2 - 49*a*b*c**3 - 28*b**2*c**6 - 420*b**2*c**2*d**4*x**4 - 168*b**2
*c*d**5*x**5 - 28*b**2*d**6*x**6 + x**3*(-49*a*b*d**3 - 560*b**2*c**3*d**3)
 + x**2*(-147*a*b*c*d**2 - 420*b**2*c**4*d**2) + x*(-147*a*b*c**2*d - 168*b
```

```
**2*c**5*d))/(18*a**5*c*d*e**2 + 36*a**4*b*c**4*d*e**2 + 18*a**3*b**2*c**7*
d*e**2 + 378*a**3*b**2*c**2*d**6*e**2*x**5 + 126*a**3*b**2*c*d**7*e**2*x**6
 + 18*a**3*b**2*d**8*e**2*x**7 + x**4*(36*a**4*b*d**5*e**2 + 630*a**3*b**2*
c**3*d**5*e**2) + x**3*(144*a**4*b*c*d**4*e**2 + 630*a**3*b**2*c**4*d**4*e*
*2) + x**2*(216*a**4*b*c**2*d**3*e**2 + 378*a**3*b**2*c**5*d**3*e**2) + x*(
18*a**5*d**2*e**2 + 144*a**4*b*c**3*d**2*e**2 + 126*a**3*b**2*c**6*d**2*e**
2)) + RootSum(19683*_t**3*a**10 - 2744*b, Lambda(_t, _t*log(x + (729*_t**2*
a**7 + 196*b*c)/(196*b*d))))/(d*e**2)
```

3.2908 $\int \frac{1}{(ce+dex)^3\left(a+b(c+dx)^3\right)^3}\,dx$

Optimal. Leaf size=237

$$-\frac{20b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{27a^{11/3}de^3}+\frac{10b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{27a^{11/3}de^3}+\frac{20b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{11/3}de^3}$$

```
[Out] -10/9/a^3/d/e^3/(d*x+c)^2+1/6/a/d/e^3/(d*x+c)^2/(a+b*(d*x+c)^3)^2+4/9/a^2/d
/e^3/(d*x+c)^2/(a+b*(d*x+c)^3)-20/27*b^(2/3)*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^
(11/3)/d/e^3+10/27*b^(2/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+
c)^2)/a^(11/3)/d/e^3+20/27*b^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))/a
^(1/3)*3^(1/2))/a^(11/3)/d/e^3*3^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {372, 290, 325, 200, 31, 634, 617, 204, 628}

$$-\frac{20b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{27a^{11/3}de^3}+\frac{10b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{27a^{11/3}de^3}+\frac{20b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{11/3}de^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*e + d*e*x)^3*(a + b*(c + d*x)^3)^3),x]
```

```
[Out] -10/(9*a^3*d*e^3*(c + d*x)^2) + 1/(6*a*d*e^3*(c + d*x)^2*(a + b*(c + d*x)^3
)^2) + 4/(9*a^2*d*e^3*(c + d*x)^2*(a + b*(c + d*x)^3)) + (20*b^(2/3)*ArcTan
[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(11/3)*d*
e^3) - (20*b^(2/3)*Log[a^(1/3) + b^(1/3)*(c + d*x)])/(27*a^(11/3)*d*e^3) +
(10*b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])
/(27*a^(11/3)*d*e^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
```

```
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(ce+dex)^3\left(a+b(c+dx)^3\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3\left(a+bx^3\right)^3}\,dx, x, c+dx\right)}{de^3}$$

$$= \frac{1}{6ade^3(c+dx)^2\left(a+b(c+dx)^3\right)^2} + \frac{4\,\text{Subst}\left(\int \frac{1}{x^3\left(a+bx^3\right)^2}\,dx, x, c+dx\right)}{3ade^3}$$

$$= \frac{1}{6ade^3(c+dx)^2\left(a+b(c+dx)^3\right)^2} + \frac{4}{9a^2de^3(c+dx)^2\left(a+b(c+dx)^3\right)} + \frac{20\,\text{S}}{}$$

$$= -\frac{10}{9a^3de^3(c+dx)^2} + \frac{1}{6ade^3(c+dx)^2\left(a+b(c+dx)^3\right)^2} + \frac{4}{9a^2de^3(c+dx)^2\left(a$$

$$= -\frac{10}{9a^3de^3(c+dx)^2} + \frac{1}{6ade^3(c+dx)^2\left(a+b(c+dx)^3\right)^2} + \frac{4}{9a^2de^3(c+dx)^2\left(a$$

$$= -\frac{10}{9a^3de^3(c+dx)^2} + \frac{1}{6ade^3(c+dx)^2\left(a+b(c+dx)^3\right)^2} + \frac{4}{9a^2de^3(c+dx)^2\left(a$$

$$= -\frac{10}{9a^3de^3(c+dx)^2} + \frac{1}{6ade^3(c+dx)^2\left(a+b(c+dx)^3\right)^2} + \frac{4}{9a^2de^3(c+dx)^2\left(a$$

$$= -\frac{10}{9a^3de^3(c+dx)^2} + \frac{1}{6ade^3(c+dx)^2\left(a+b(c+dx)^3\right)^2} + \frac{4}{9a^2de^3(c+dx)^2\left(a$$

Mathematica [A] time = 0.06, size = 195, normalized size = 0.82

$$\frac{20b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)-\frac{9a^{5/3}b(c+dx)}{\left(a+b(c+dx)^3\right)^2}-\frac{33a^{2/3}b(c+dx)}{a+b(c+dx)^3}-\frac{27a^{2/3}}{(c+dx)^2}-40b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c-\right.}{54a^{11/3}de^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*e + d*e*x)^3*(a + b*(c + d*x)^3)^3),x]

[Out] ((-27*a^(2/3))/(c + d*x)^2 - (9*a^(5/3)*b*(c + d*x))/(a + b*(c + d*x)^3)^2
- (33*a^(2/3)*b*(c + d*x))/(a + b*(c + d*x)^3) - 40*Sqrt[3]*b^(2/3)*ArcTan[
(-a^(1/3) + 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))] - 40*b^(2/3)*Log[a^(1/3
) + b^(1/3)*(c + d*x)] + 20*b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x)
 + b^(2/3)*(c + d*x)^2])/(54*a^(11/3)*d*e^3)
```

fricas [B] time = 1.03, size = 1051, normalized size = 4.43

$$\frac{60\,b^2d^6x^6+360\,b^2cd^5x^5+900\,b^2c^2d^4x^4+60\,b^2c^6+48\left(25\,b^2c^3+2\,ab\right)d^3x^3+96\,abc^3+36\left(25\,b^2c^4+8\,abc\right)d^2x^2}{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^3/(a+b*(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] -1/54*(60*b^2*d^6*x^6 + 360*b^2*c*d^5*x^5 + 900*b^2*c^2*d^4*x^4 + 60*b^2*c^
6 + 48*(25*b^2*c^3 + 2*a*b)*d^3*x^3 + 96*a*b*c^3 + 36*(25*b^2*c^4 + 8*a*b*c
)*d^2*x^2 + 72*(5*b^2*c^5 + 4*a*b*c^2)*d*x - 40*sqrt(3)*(b^2*d^8*x^8 + 8*b^
2*c*d^7*x^7 + 28*b^2*c^2*d^6*x^6 + 2*(28*b^2*c^3 + a*b)*d^5*x^5 + b^2*c^8 +
 10*(7*b^2*c^4 + a*b*c)*d^4*x^4 + 2*a*b*c^5 + 4*(14*b^2*c^5 + 5*a*b*c^2)*d^
3*x^3 + (28*b^2*c^6 + 20*a*b*c^3 + a^2)*d^2*x^2 + a^2*c^2 + 2*(4*b^2*c^7 +
5*a*b*c^4 + a^2*c)*d*x)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*(a*d*x + a*c
)*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) + 20*(b^2*d^8*x^8 + 8*b^2*c*d^7*x^7 + 28
*b^2*c^2*d^6*x^6 + 2*(28*b^2*c^3 + a*b)*d^5*x^5 + b^2*c^8 + 10*(7*b^2*c^4 +
 a*b*c)*d^4*x^4 + 2*a*b*c^5 + 4*(14*b^2*c^5 + 5*a*b*c^2)*d^3*x^3 + (28*b^2*
c^6 + 20*a*b*c^3 + a^2)*d^2*x^2 + a^2*c^2 + 2*(4*b^2*c^7 + 5*a*b*c^4 + a^2*
c)*d*x)*(-b^2/a^2)^(1/3)*log(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + a^2*(-b^
2/a^2)^(2/3) + (a*b*d*x + a*b*c)*(-b^2/a^2)^(1/3)) - 40*(b^2*d^8*x^8 + 8*b^
2*c*d^7*x^7 + 28*b^2*c^2*d^6*x^6 + 2*(28*b^2*c^3 + a*b)*d^5*x^5 + b^2*c^8 +
 10*(7*b^2*c^4 + a*b*c)*d^4*x^4 + 2*a*b*c^5 + 4*(14*b^2*c^5 + 5*a*b*c^2)*d^
3*x^3 + (28*b^2*c^6 + 20*a*b*c^3 + a^2)*d^2*x^2 + a^2*c^2 + 2*(4*b^2*c^7 +
5*a*b*c^4 + a^2*c)*d*x)*(-b^2/a^2)^(1/3)*log(b*d*x + b*c - a*(-b^2/a^2)^(1/
3)) + 27*a^2)/(a^3*b^2*d^9*e^3*x^8 + 8*a^3*b^2*c*d^8*e^3*x^7 + 28*a^3*b^2*c
^2*d^7*e^3*x^6 + 2*(28*a^3*b^2*c^3 + a^4*b)*d^6*e^3*x^5 + 10*(7*a^3*b^2*c^4
 + a^4*b*c)*d^5*e^3*x^4 + 4*(14*a^3*b^2*c^5 + 5*a^4*b*c^2)*d^4*e^3*x^3 + (2
8*a^3*b^2*c^6 + 20*a^4*b*c^3 + a^5)*d^3*e^3*x^2 + 2*(4*a^3*b^2*c^7 + 5*a^4*
b*c^4 + a^5*c)*d^2*e^3*x + (a^3*b^2*c^8 + 2*a^4*b*c^5 + a^5*c^2)*d*e^3)
```

giac [A] time = 0.32, size = 369, normalized size = 1.56

$$\frac{10\left(2\sqrt{3}\left(-\frac{b^2}{a^2d^3}\right)^{\frac{1}{3}}\arctan\left(-\frac{bdx+bc-\left(-ab^2\right)^{\frac{1}{3}}}{\sqrt{3}\,bdx+\sqrt{3}\,bc+\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}}\right)-\left(-\frac{b^2}{a^2d^3}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}\,bdx+\sqrt{3}\,bc+\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\right)^2+4\left(\right.\right.\right.}{27\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^3/(a+b*(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 10/27*(2*sqrt(3)*(-b^2/(a^2*d^3))^(1/3)*arctan(-(b*d*x + b*c - (-a*b^2)^(1/
3))/(sqrt(3)*b*d*x + sqrt(3)*b*c + sqrt(3)*(-a*b^2)^(1/3))) - (-b^2/(a^2*d^
3))^(1/3)*log(4*(sqrt(3)*b*d*x + sqrt(3)*b*c + sqrt(3)*(-a*b^2)^(1/3))^2 +
4*(b*d*x + b*c - (-a*b^2)^(1/3))^2) + 2*(-b^2/(a^2*d^3))^(1/3)*log(abs(-b*d
*x - b*c + (-a*b^2)^(1/3))))*e^(-3)/a^3 - 1/18*(20*b^2*d^6*x^6 + 120*b^2*c*
d^5*x^5 + 300*b^2*c^2*d^4*x^4 + 400*b^2*c^3*d^3*x^3 + 300*b^2*c^4*d^2*x^2 +
 120*b^2*c^5*d*x + 20*b^2*c^6 + 32*a*b*d^3*x^3 + 96*a*b*c*d^2*x^2 + 96*a*b*
c^2*d*x + 32*a*b*c^3 + 9*a^2)*e^(-3)/((b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*
d^2*x^2 + 4*b*c^3*d*x + b*c^4 + a*d*x + a*c)^2*a^3*d)
```

maple [C] time = 0.02, size = 446, normalized size = 1.88

$$-\frac{11b^2d^3x^4}{18\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)^2a^3e^3}-\frac{22b^2c\,d^2x^3}{9\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)^2a^3e^3}-\frac{}{3\left(b\,d^3x^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)^3/(a+b*(d*x+c)^3)^3,x)
```

```
[Out] -1/2/a^3/d/e^3/(d*x+c)^2-11/18/e^3*b^2/a^3/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2
*d*x+b*c^3+a)^2*d^3*x^4-22/9/e^3*b^2/a^3/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d
*x+b*c^3+a)^2*c*d^2*x^3-11/3/e^3*b^2/a^3/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d
```

```
*x+b*c^3+a)^2*c^2*d*x^2-22/9/e^3*b^2/a^3/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d
*x+b*c^3+a)^2*x*c^3-7/9/e^3*b/a^2/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^
3+a)^2*x-11/18/e^3*b^2/a^3/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2*
c^4/d-7/9/e^3*b/a^2/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2*c/d-20/
27/e^3/a^3/d*sum(1/(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x),_R=RootOf(_Z^3*b*d^3+3
*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{20\,b^2d^6x^6+120\,b^2cd^5x^5+300\,b^2c^2d^4x^4+20\,b^2c^6+16\left(25\,b^2\right.}{18\left(a^3b^2d^9e^3x^8+8\,a^3b^2cd^8e^3x^7+28\,a^3b^2c^2d^7e^3x^6+2\left(28\,a^3b^2c^3+a^4b\right)d^6e^3x^5+10\left(7\,a^3b^2c^4+a^4bc\right)d^5e^3x^4+4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^3/(a+b*(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] -1/18*(20*b^2*d^6*x^6 + 120*b^2*c*d^5*x^5 + 300*b^2*c^2*d^4*x^4 + 20*b^2*c^
6 + 16*(25*b^2*c^3 + 2*a*b)*d^3*x^3 + 32*a*b*c^3 + 12*(25*b^2*c^4 + 8*a*b*c
)*d^2*x^2 + 24*(5*b^2*c^5 + 4*a*b*c^2)*d*x + 9*a^2)/(a^3*b^2*d^9*e^3*x^8 +
8*a^3*b^2*c*d^8*e^3*x^7 + 28*a^3*b^2*c^2*d^7*e^3*x^6 + 2*(28*a^3*b^2*c^3 +
a^4*b)*d^6*e^3*x^5 + 10*(7*a^3*b^2*c^4 + a^4*b*c)*d^5*e^3*x^4 + 4*(14*a^3*b
^2*c^5 + 5*a^4*b*c^2)*d^4*e^3*x^3 + (28*a^3*b^2*c^6 + 20*a^4*b*c^3 + a^5)*d
^3*e^3*x^2 + 2*(4*a^3*b^2*c^7 + 5*a^4*b*c^4 + a^5*c)*d^2*e^3*x + (a^3*b^2*c
^8 + 2*a^4*b*c^5 + a^5*c^2)*d*e^3) - 20/9*b*integrate(1/(b*d^3*x^3 + 3*b*c*
d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/(a^3*e^3)
```

mupad [B] time = 3.46, size = 563, normalized size = 2.38

$$\frac{20\,b^{2/3}\,\ln\left(a^6\,b^{1/3}\,c-(-a)^{19/3}+a^6\,b^{1/3}\,d\,x\right)}{27\,(-a)^{11/3}\,d\,e^3}-\frac{}{x^4\,\left(70\,b^2\,c^4\,d^4\,e^3+10\,a\,b\,c\,d^4\,e^3\right)+x^2\,\left(a^2\,d^2\,e^3+20\,a\,b\,c^3\,d^2\,e^3+28\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)^3*(a + b*(c + d*x)^3)^3),x)
```

```
[Out] (20*b^(2/3)*log(a^6*b^(1/3)*c - (-a)^(19/3) + a^6*b^(1/3)*d*x))/(27*(-a)^(1
1/3)*d*e^3) - ((9*a^2 + 20*b^2*c^6 + 32*a*b*c^3)/(18*a^3*d) + (2*x^2*(25*b^
2*c^4*d + 8*a*b*c*d))/(3*a^3) + (4*x*(5*b^2*c^5 + 4*a*b*c^2))/(3*a^3) + (8*
x^3*(25*b^2*c^3*d^2 + 2*a*b*d^2))/(9*a^3) + (10*b^2*d^5*x^6)/(9*a^3) + (50*
b^2*c^2*d^3*x^4)/(3*a^3) + (20*b^2*c*d^4*x^5)/(3*a^3))/(x^4*(70*b^2*c^4*d^4
*e^3 + 10*a*b*c*d^4*e^3) + x^2*(a^2*d^2*e^3 + 28*b^2*c^6*d^2*e^3 + 20*a*b*c
^3*d^2*e^3) + x^3*(56*b^2*c^5*d^3*e^3 + 20*a*b*c^2*d^3*e^3) + x^5*(56*b^2*c
^3*d^5*e^3 + 2*a*b*d^5*e^3) + x*(8*b^2*c^7*d*e^3 + 2*a^2*c*d*e^3 + 10*a*b*c
^4*d*e^3) + a^2*c^2*e^3 + b^2*c^8*e^3 + b^2*d^8*e^3*x^8 + 2*a*b*c^5*e^3 + 2
8*b^2*c^2*d^6*e^3*x^6 + 8*b^2*c*d^7*e^3*x^7) + (20*b^(2/3)*log((-a)^(19/3)
- 3^(1/2)*(-a)^(19/3)*1i + 2*a^6*b^(1/3)*c + 2*a^6*b^(1/3)*d*x)*((3^(1/2)*1
i)/2 - 1/2))/(27*(-a)^(11/3)*d*e^3) - (20*b^(2/3)*log((-a)^(19/3) + 3^(1/2)
*(-a)^(19/3)*1i + 2*a^6*b^(1/3)*c + 2*a^6*b^(1/3)*d*x)*((3^(1/2)*1i)/2 + 1/
2))/(27*(-a)^(11/3)*d*e^3)
```

sympy [B] time = 7.36, size = 500, normalized size = 2.11

$$\frac{-9a^2-32abc^3-20b^2c^6-300b^2c^2d^4}{18a^5c^2de^3+36a^4bc^5de^3+18a^3b^2c^8de^3+504a^3b^2c^2d^7e^3x^6+144a^3b^2cd^8e^3x^7+18a^3b^2d^9e^3x^8+x^5\left(36a^4bd^6e^3+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)**3/(a+b*(d*x+c)**3)**3,x)
```

```
[Out] (-9*a**2 - 32*a*b*c**3 - 20*b**2*c**6 - 300*b**2*c**2*d**4*x**4 - 120*b**2*
c*d**5*x**5 - 20*b**2*d**6*x**6 + x**3*(-32*a*b*d**3 - 400*b**2*c**3*d**3)
+ x**2*(-96*a*b*c*d**2 - 300*b**2*c**4*d**2) + x*(-96*a*b*c**2*d - 120*b**2
*c**5*d))/(18*a**5*c**2*d*e**3 + 36*a**4*b*c**5*d*e**3 + 18*a**3*b**2*c**8*
d*e**3 + 504*a**3*b**2*c**2*d**7*e**3*x**6 + 144*a**3*b**2*c*d**8*e**3*x**7
 + 18*a**3*b**2*d**9*e**3*x**8 + x**5*(36*a**4*b*d**6*e**3 + 1008*a**3*b**2
*c**3*d**6*e**3) + x**4*(180*a**4*b*c*d**5*e**3 + 1260*a**3*b**2*c**4*d**5*
e**3) + x**3*(360*a**4*b*c**2*d**4*e**3 + 1008*a**3*b**2*c**5*d**4*e**3) +
x**2*(18*a**5*d**3*e**3 + 360*a**4*b*c**3*d**3*e**3 + 504*a**3*b**2*c**6*d*
*3*e**3) + x*(36*a**5*c*d**2*e**3 + 180*a**4*b*c**4*d**2*e**3 + 144*a**3*b*
*2*c**7*d**2*e**3)) + RootSum(19683*_t**3*a**11 + 8000*b**2, Lambda(_t, _t*
log(x + (-27*_t*a**4 + 20*b*c)/(20*b*d))))/(d*e**3)
```

3.2909 $\int \frac{1}{(ce+dex)^4\left(a+b(c+dx)^3\right)^3}\,dx$

Optimal. Leaf size=116

$$-\frac{3b\log(c+dx)}{a^4de^4}+\frac{b\log\left(a+b(c+dx)^3\right)}{a^4de^4}-\frac{2b}{3a^3de^4\left(a+b(c+dx)^3\right)}-\frac{1}{3a^3de^4(c+dx)^3}-\frac{b}{6a^2de^4\left(a+b(c+dx)^3\right)^2}$$

```
[Out] -1/3/a^3/d/e^4/(d*x+c)^3-1/6*b/a^2/d/e^4/(a+b*(d*x+c)^3)^2-2/3*b/a^3/d/e^4/
(a+b*(d*x+c)^3)-3*b*ln(d*x+c)/a^4/d/e^4+b*ln(a+b*(d*x+c)^3)/a^4/d/e^4
```

Rubi [A] time = 0.09, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {372, 266, 44}

$$-\frac{2b}{3a^3de^4\left(a+b(c+dx)^3\right)}-\frac{b}{6a^2de^4\left(a+b(c+dx)^3\right)^2}-\frac{3b\log(c+dx)}{a^4de^4}+\frac{b\log\left(a+b(c+dx)^3\right)}{a^4de^4}-\frac{1}{3a^3de^4(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*e + d*e*x)^4*(a + b*(c + d*x)^3)^3),x]
```

```
[Out] -1/(3*a^3*d*e^4*(c + d*x)^3) - b/(6*a^2*d*e^4*(a + b*(c + d*x)^3)^2) - (2*b
)/(3*a^3*d*e^4*(a + b*(c + d*x)^3)) - (3*b*Log[c + d*x])/(a^4*d*e^4) + (b*L
og[a + b*(c + d*x)^3])/(a^4*d*e^4)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{(ce+dex)^4\left(a+b(c+dx)^3\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^3\right)^3}\,dx, x, c+dx\right)}{de^4}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx)^3}\,dx, x, (c+dx)^3\right)}{3de^4}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^3x^2} - \frac{3b}{a^4x} + \frac{b^2}{a^2(a+bx)^3} + \frac{2b^2}{a^3(a+bx)^2} + \frac{3b^2}{a^4(a+bx)}\right)dx, x, (c+dx)^3\right)}{3de^4}$$
$$= -\frac{1}{3a^3de^4(c+dx)^3} - \frac{b}{6a^2de^4\left(a+b(c+dx)^3\right)^2} - \frac{2b}{3a^3de^4\left(a+b(c+dx)^3\right)}$$

Mathematica [A] time = 0.06, size = 83, normalized size = 0.72

$$\frac{a\left(-\frac{4b}{a+b(c+dx)^3} - \frac{ab}{\left(a+b(c+dx)^3\right)^2} - \frac{2}{(c+dx)^3}\right) + 6b\log\left(a+b(c+dx)^3\right) - 18b\log(c+dx)}{6a^4de^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*e + d*e*x)^4*(a + b*(c + d*x)^3)^3),x]
```

```
[Out] (a*(-2/(c + d*x)^3 - (a*b)/(a + b*(c + d*x)^3)^2 - (4*b)/(a + b*(c + d*x)^3
)) - 18*b*Log[c + d*x] + 6*b*Log[a + b*(c + d*x)^3])/(6*a^4*d*e^4)
```

fricas [B] time = 1.16, size = 919, normalized size = 7.92

$$-\frac{6\,ab^2d^6x^6 + 36\,ab^2cd^5x^5 + 90\,ab^2c^2d^4x^4 + 6\,ab^2c^6 + 3\left(40\,ab^2c^3 + 3\,a^2b\right)d^3x^3 + 9\,a^2bc^3 + 9\left(10\,ab^2c^4 + 3\,a^2b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^4/(a+b*(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] -1/6*(6*a*b^2*d^6*x^6 + 36*a*b^2*c*d^5*x^5 + 90*a*b^2*c^2*d^4*x^4 + 6*a*b^2
*c^6 + 3*(40*a*b^2*c^3 + 3*a^2*b)*d^3*x^3 + 9*a^2*b*c^3 + 9*(10*a*b^2*c^4 +
 3*a^2*b*c)*d^2*x^2 + 2*a^3 + 9*(4*a*b^2*c^5 + 3*a^2*b*c^2)*d*x - 6*(b^3*d^
9*x^9 + 9*b^3*c*d^8*x^8 + 36*b^3*c^2*d^7*x^7 + 2*(42*b^3*c^3 + a*b^2)*d^6*x
^6 + b^3*c^9 + 6*(21*b^3*c^4 + 2*a*b^2*c)*d^5*x^5 + 2*a*b^2*c^6 + 6*(21*b^3
*c^5 + 5*a*b^2*c^2)*d^4*x^4 + (84*b^3*c^6 + 40*a*b^2*c^3 + a^2*b)*d^3*x^3 +
 a^2*b*c^3 + 3*(12*b^3*c^7 + 10*a*b^2*c^4 + a^2*b*c)*d^2*x^2 + 3*(3*b^3*c^8
 + 4*a*b^2*c^5 + a^2*b*c^2)*d*x)*log(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*
x + b*c^3 + a) + 18*(b^3*d^9*x^9 + 9*b^3*c*d^8*x^8 + 36*b^3*c^2*d^7*x^7 + 2
*(42*b^3*c^3 + a*b^2)*d^6*x^6 + b^3*c^9 + 6*(21*b^3*c^4 + 2*a*b^2*c)*d^5*x^
5 + 2*a*b^2*c^6 + 6*(21*b^3*c^5 + 5*a*b^2*c^2)*d^4*x^4 + (84*b^3*c^6 + 40*a
*b^2*c^3 + a^2*b)*d^3*x^3 + a^2*b*c^3 + 3*(12*b^3*c^7 + 10*a*b^2*c^4 + a^2*
b*c)*d^2*x^2 + 3*(3*b^3*c^8 + 4*a*b^2*c^5 + a^2*b*c^2)*d*x)*log(d*x + c))/(
a^4*b^2*d^10*e^4*x^9 + 9*a^4*b^2*c*d^9*e^4*x^8 + 36*a^4*b^2*c^2*d^8*e^4*x^7
 + 2*(42*a^4*b^2*c^3 + a^5*b)*d^7*e^4*x^6 + 6*(21*a^4*b^2*c^4 + 2*a^5*b*c)*
d^6*e^4*x^5 + 6*(21*a^4*b^2*c^5 + 5*a^5*b*c^2)*d^5*e^4*x^4 + (84*a^4*b^2*c^
6 + 40*a^5*b*c^3 + a^6)*d^4*e^4*x^3 + 3*(12*a^4*b^2*c^7 + 10*a^5*b*c^4 + a^
6*c)*d^3*e^4*x^2 + 3*(3*a^4*b^2*c^8 + 4*a^5*b*c^5 + a^6*c^2)*d^2*e^4*x + (a
^4*b^2*c^9 + 2*a^5*b*c^6 + a^6*c^3)*d*e^4)
```

giac [B] time = 0.25, size = 259, normalized size = 2.23

$$\frac{be^{(-4)}\log\left(\left|bd^3x^3+3bcd^2x^2+3bc^2dx+bc^3+a\right|\right)}{a^4d}-\frac{3be^{(-4)}\log\left(|dx+c|\right)}{a^4d}-\frac{\left(6ab^2d^6x^6+36ab^2cd^5x^5+90ab^2c^2d^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^4/(a+b*(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] b*e^(-4)*log(abs(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a))/(a^4
*d) - 3*b*e^(-4)*log(abs(d*x + c))/(a^4*d) - 1/6*(6*a*b^2*d^6*x^6 + 36*a*b^
2*c*d^5*x^5 + 90*a*b^2*c^2*d^4*x^4 + 6*a*b^2*c^6 + 9*a^2*b*c^3 + 3*(40*a*b^
2*c^3*d^3 + 3*a^2*b*d^3)*x^3 + 2*a^3 + 9*(10*a*b^2*c^4*d^2 + 3*a^2*b*c*d^2)
*x^2 + 9*(4*a*b^2*c^5*d + 3*a^2*b*c^2*d)*x)*e^(-4)/((b*d^3*x^3 + 3*b*c*d^2*
x^2 + 3*b*c^2*d*x + b*c^3 + a)^2*(d*x + c)^3*a^4*d)
```

maple [B] time = 0.03, size = 335, normalized size = 2.89

$$-\frac{2b^2d^2x^3}{3\left(bd^3x^3+3bcd^2x^2+3bc^2dx+bc^3+a\right)^2a^3e^4}-\frac{2b^2cdx^2}{\left(bd^3x^3+3bcd^2x^2+3bc^2dx+bc^3+a\right)^2a^3e^4}-\frac{\left(bd^3x^3+3bcd\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)^4/(a+b*(d*x+c)^3)^3,x)
```

```
[Out] -1/3/a^3/d/e^4/(d*x+c)^3-3*b*ln(d*x+c)/a^4/d/e^4-2/3/e^4/a^3*b^2/(b*d^3*x^3
+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2*d^2*x^3-2/e^4/a^3*b^2/(b*d^3*x^3+3*b*
c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2*c*d*x^2-2/e^4/a^3*b^2/(b*d^3*x^3+3*b*c*d^2
*x^2+3*b*c^2*d*x+b*c^3+a)^2*c^2*x-2/3/e^4/a^3*b^2/(b*d^3*x^3+3*b*c*d^2*x^2+
3*b*c^2*d*x+b*c^3+a)^2/d*c^3-5/6/e^4/a^2*b/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2
*d*x+b*c^3+a)^2/d+1/e^4/a^4*b/d*ln(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^
3+a)
```

maxima [B] time = 0.71, size = 474, normalized size = 4.09

$$-\frac{6b^2d^6x^6+36b^2cd^5x^5+90b^2c^2d^4x^4}{6\left(a^3b^2d^{10}e^4x^9+9a^3b^2cd^9e^4x^8+36a^3b^2c^2d^8e^4x^7+2\left(42a^3b^2c^3+a^4b\right)d^7e^4x^6+6\left(21a^3b^2c^4+2a^4bc\right)d^6e^4x^5+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)^4/(a+b*(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(6*b^2*d^6*x^6 + 36*b^2*c*d^5*x^5 + 90*b^2*c^2*d^4*x^4 + 6*b^2*c^6 + 3
*(40*b^2*c^3 + 3*a*b)*d^3*x^3 + 9*a*b*c^3 + 9*(10*b^2*c^4 + 3*a*b*c)*d^2*x^
2 + 9*(4*b^2*c^5 + 3*a*b*c^2)*d*x + 2*a^2)/(a^3*b^2*d^10*e^4*x^9 + 9*a^3*b^
2*c*d^9*e^4*x^8 + 36*a^3*b^2*c^2*d^8*e^4*x^7 + 2*(42*a^3*b^2*c^3 + a^4*b)*d
^7*e^4*x^6 + 6*(21*a^3*b^2*c^4 + 2*a^4*b*c)*d^6*e^4*x^5 + 6*(21*a^3*b^2*c^5
 + 5*a^4*b*c^2)*d^5*e^4*x^4 + (84*a^3*b^2*c^6 + 40*a^4*b*c^3 + a^5)*d^4*e^4
*x^3 + 3*(12*a^3*b^2*c^7 + 10*a^4*b*c^4 + a^5*c)*d^3*e^4*x^2 + 3*(3*a^3*b^2
*c^8 + 4*a^4*b*c^5 + a^5*c^2)*d^2*e^4*x + (a^3*b^2*c^9 + 2*a^4*b*c^6 + a^5*
c^3)*d*e^4) + b*log(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(a
^4*d*e^4) - 3*b*log(d*x + c)/(a^4*d*e^4)
```

mupad [B] time = 3.57, size = 507, normalized size = 4.37

$$\frac{b\,\ln\left(b\,c^3+3\,b\,c^2\,d\,x+3\,b\,c\,d^2\,x^2+b\,d^3\,x^3+a\right)}{a^4\,d\,e^4}-\frac{}{x^5\left(126\,b^2\,c^4\,d^5\,e^4+12\,a\,b\,c\,d^5\,e^4\right)+x^3\left(a^2\,d^3\,e^4+40\,a\,b\,c^3\,d^3\,e^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)^4*(a + b*(c + d*x)^3)^3),x)
```

```
[Out] (b*log(a + b*c^3 + b*d^3*x^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2))/(a^4*d*e^4) -
((2*a^2 + 6*b^2*c^6 + 9*a*b*c^3)/(6*a^3*d) + (3*x^2*(10*b^2*c^4*d + 3*a*b*c
*d))/(2*a^3) + (3*x*(4*b^2*c^5 + 3*a*b*c^2))/(2*a^3) + (x^3*(40*b^2*c^3*d^2
 + 3*a*b*d^2))/(2*a^3) + (b^2*d^5*x^6)/a^3 + (15*b^2*c^2*d^3*x^4)/a^3 + (6*
b^2*c*d^4*x^5)/a^3)/(x^5*(126*b^2*c^4*d^5*e^4 + 12*a*b*c*d^5*e^4) + x^3*(a^
2*d^3*e^4 + 84*b^2*c^6*d^3*e^4 + 40*a*b*c^3*d^3*e^4) + x*(3*a^2*c^2*d*e^4 +
 9*b^2*c^8*d*e^4 + 12*a*b*c^5*d*e^4) + x^4*(126*b^2*c^5*d^4*e^4 + 30*a*b*c^
2*d^4*e^4) + x^2*(3*a^2*c*d^2*e^4 + 36*b^2*c^7*d^2*e^4 + 30*a*b*c^4*d^2*e^4
) + x^6*(84*b^2*c^3*d^6*e^4 + 2*a*b*d^6*e^4) + a^2*c^3*e^4 + b^2*c^9*e^4 +
b^2*d^9*e^4*x^9 + 2*a*b*c^6*e^4 + 36*b^2*c^2*d^7*e^4*x^7 + 9*b^2*c*d^8*e^4*
x^8) - (3*b*log(c + d*x))/(a^4*d*e^4)
```

sympy [**B**] time = 8.76, size = 578, normalized size = 4.98

$$\frac{-2a^2 - 9}{6a^5c^3de^4 + 12a^4bc^6de^4 + 6a^3b^2c^9de^4 + 216a^3b^2c^2d^8e^4x^7 + 54a^3b^2cd^9e^4x^8 + 6a^3b^2d^{10}e^4x^9 + x^6\left(12a^4bd^7e^4 + 50\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)**4/(a+b*(d*x+c)**3)**3,x)
```

```
[Out] (-2*a**2 - 9*a*b*c**3 - 6*b**2*c**6 - 90*b**2*c**2*d**4*x**4 - 36*b**2*c*d*
*5*x**5 - 6*b**2*d**6*x**6 + x**3*(-9*a*b*d**3 - 120*b**2*c**3*d**3) + x**2
*(-27*a*b*c*d**2 - 90*b**2*c**4*d**2) + x*(-27*a*b*c**2*d - 36*b**2*c**5*d)
)/(6*a**5*c**3*d*e**4 + 12*a**4*b*c**6*d*e**4 + 6*a**3*b**2*c**9*d*e**4 + 2
16*a**3*b**2*c**2*d**8*e**4*x**7 + 54*a**3*b**2*c*d**9*e**4*x**8 + 6*a**3*b
**2*d**10*e**4*x**9 + x**6*(12*a**4*b*d**7*e**4 + 504*a**3*b**2*c**3*d**7*e
**4) + x**5*(72*a**4*b*c*d**6*e**4 + 756*a**3*b**2*c**4*d**6*e**4) + x**4*(
180*a**4*b*c**2*d**5*e**4 + 756*a**3*b**2*c**5*d**5*e**4) + x**3*(6*a**5*d*
*4*e**4 + 240*a**4*b*c**3*d**4*e**4 + 504*a**3*b**2*c**6*d**4*e**4) + x**2*
(18*a**5*c*d**3*e**4 + 180*a**4*b*c**4*d**3*e**4 + 216*a**3*b**2*c**7*d**3*
e**4) + x*(18*a**5*c**2*d**2*e**4 + 72*a**4*b*c**5*d**2*e**4 + 54*a**3*b**2
*c**8*d**2*e**4)) - 3*b*log(c/d + x)/(a**4*d*e**4) + b*log(3*c**2*x/d**2 +
3*c*x**2/d + x**3 + (a + b*c**3)/(b*d**3))/(a**4*d*e**4)
```

3.2910 $\int (c+dx)^3 \left(a+b(c+dx)^4\right)^p \, dx$

Optimal. Leaf size=30

$$\frac{\left(a+b(c+dx)^4\right)^{p+1}}{4bd(p+1)}$$

```
[Out] 1/4*(a+b*(d*x+c)^4)^(1+p)/b/d/(1+p)
```

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {372, 261}

$$\frac{\left(a+b(c+dx)^4\right)^{p+1}}{4bd(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + b*(c + d*x)^4)^p,x]
```

```
[Out] (a + b*(c + d*x)^4)^(1 + p)/(4*b*d*(1 + p))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^3 \left(a+b(c+dx)^4\right)^p \, dx &= \frac{\text{Subst}\left(\int x^3 \left(a+bx^4\right)^p \, dx, x, c+dx\right)}{d} \\ &= \frac{\left(a+b(c+dx)^4\right)^{1+p}}{4bd(1+p)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 1.00

$$\frac{\left(a+b(c+dx)^4\right)^{p+1}}{4bd(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + b*(c + d*x)^4)^p,x]
```

```
[Out] (a + b*(c + d*x)^4)^(1 + p)/(4*b*d*(1 + p))
```

fricas [B] time = 1.07, size = 104, normalized size = 3.47

$$\frac{\left(bd^4x^4+4\,bcd^3x^3+6\,bc^2d^2x^2+4\,bc^3dx+bc^4+a\right)\left(bd^4x^4+4\,bcd^3x^3+6\,bc^2d^2x^2+4\,bc^3dx+bc^4+a\right)^p}{4\left(bdp+bd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*(d*x+c)^4)^p,x, algorithm="fricas")
```

```
[Out] 1/4*(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c^4 + a)
*(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c^4 + a)^p/
(b*d*p + b*d)
```

giac [**B**] time = 0.18, size = 62, normalized size = 2.07

$$\frac{\left(bd^4x^4 + 4\,bcd^3x^3 + 6\,bc^2d^2x^2 + 4\,bc^3dx + bc^4 + a\right)^{p+1}}{4\,bd\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*(d*x+c)^4)^p,x, algorithm="giac")
```

```
[Out] 1/4*(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c^4 + a)
^(p + 1)/(b*d*(p + 1))
```

maple [**B**] time = 0.01, size = 63, normalized size = 2.10

$$\frac{\left(b\,d^4x^4 + 4bc\,d^3x^3 + 6b\,c^2d^2x^2 + 4b\,c^3dx + b\,c^4 + a\right)^{p+1}}{4\left(p+1\right)bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+b*(d*x+c)^4)^p,x)
```

```
[Out] 1/4*(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^(p+1)/b/d
/(p+1)
```

maxima [**A**] time = 0.44, size = 28, normalized size = 0.93

$$\frac{\left((dx+c)^4b + a\right)^{p+1}}{4\,bd\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*(d*x+c)^4)^p,x, algorithm="maxima")
```

```
[Out] 1/4*((d*x + c)^4*b + a)^(p + 1)/(b*d*(p + 1))
```

mupad [**B**] time = 1.41, size = 88, normalized size = 2.93

$$\left(a + b\,(c + d\,x)^4\right)^p \left(\frac{d^3\,x^4}{4\,\left(p+1\right)} + \frac{c^3\,x}{p+1} + \frac{b\,c^4 + a}{4\,b\,d\,\left(p+1\right)} + \frac{3\,c^2\,d\,x^2}{2\,\left(p+1\right)} + \frac{c\,d^2\,x^3}{p+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^4)^p*(c + d*x)^3,x)
```

```
[Out] (a + b*(c + d*x)^4)^p*((d^3*x^4)/(4*(p + 1)) + (c^3*x)/(p + 1) + (a + b*c^4
)/(4*b*d*(p + 1)) + (3*c^2*d*x^2)/(2*(p + 1)) + (c*d^2*x^3)/(p + 1))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+b*(d*x+c)**4)**p,x)
```

```
[Out] Timed out
```

3.2911 $\int (c+dx)^3 \left(a+b(c+dx)^4\right) dx$

Optimal. Leaf size=23

$$\frac{\left(a+b(c+dx)^4\right)^2}{8bd}$$

```
[Out] 1/8*(a+b*(d*x+c)^4)^2/b/d
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.35, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {372, 14}

$$\frac{a(c+dx)^4}{4d} + \frac{b(c+dx)^8}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + b*(c + d*x)^4),x]
```

```
[Out] (a*(c + d*x)^4)/(4*d) + (b*(c + d*x)^8)/(8*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^3 \left(a+b(c+dx)^4\right) dx &= \frac{\text{Subst}\left(\int x^3 \left(a+bx^4\right) dx, x, c+dx\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(ax^3+bx^7\right) dx, x, c+dx\right)}{d} \\ &= \frac{a(c+dx)^4}{4d} + \frac{b(c+dx)^8}{8d}\end{aligned}$$

Mathematica [B] time = 0.02, size = 80, normalized size = 3.48

$$\frac{1}{8}x\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right)\left(2a+b\left(2c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + b*(c + d*x)^4),x]
```

```
[Out] (x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*(2*a + b*(2*c^4 + 4*c^3*d*x
+ 6*c^2*d^2*x^2 + 4*c*d^3*x^3 + d^4*x^4)))/8
```

fricas [B] time = 0.82, size = 117, normalized size = 5.09

$$\frac{1}{8}x^8d^7b+x^7d^6cb+\frac{7}{2}x^6d^5c^2b+7x^5d^4c^3b+\frac{35}{4}x^4d^3c^4b+7x^3d^2c^5b+\frac{7}{2}x^2dc^6b+xc^7b+\frac{1}{4}x^4d^3a+x^3d^2ca+\frac{3}{2}x^2dc^2a+xc^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/8*x^8*d^7*b + x^7*d^6*c*b + 7/2*x^6*d^5*c^2*b + 7*x^5*d^4*c^3*b + 35/4*x^
4*d^3*c^4*b + 7*x^3*d^2*c^5*b + 7/2*x^2*d*c^6*b + x*c^7*b + 1/4*x^4*d^3*a +
 x^3*d^2*c*a + 3/2*x^2*d*c^2*a + x*c^3*a
```

giac [A] time = 0.16, size = 25, normalized size = 1.09

$$\frac{(dx+c)^8 b + 2\,(dx+c)^4 a}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/8*((d*x + c)^8*b + 2*(d*x + c)^4*a)/d
```

maple [B] time = 0.00, size = 136, normalized size = 5.91

$$\frac{b\,d^7x^8}{8}+bc\,d^6x^7+\frac{7b\,c^2d^5x^6}{2}+7b\,c^3d^4x^5+\left(b\,c^4+a\right)c^3x+\frac{\left(34b\,c^4d^3+\left(b\,c^4+a\right)d^3\right)x^4}{4}+\frac{\left(18b\,c^5d^2+3\left(b\,c^4+a\right)c\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+b*(d*x+c)^4),x)
```

```
[Out] 1/8*d^7*b*x^8+c*d^6*b*x^7+7/2*c^2*d^5*b*x^6+7*c^3*d^4*b*x^5+1/4*(34*c^4*b*d
^3+d^3*(b*c^4+a))*x^4+1/3*(18*c^5*d^2*b+3*c*d^2*(b*c^4+a))*x^3+1/2*(4*c^6*d
*b+3*c^2*d*(b*c^4+a))*x^2+c^3*(b*c^4+a)*x
```

maxima [A] time = 0.48, size = 21, normalized size = 0.91

$$\frac{\left((dx+c)^4 b + a\right)^2}{8\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/8*((d*x + c)^4*b + a)^2/(b*d)
```

mupad [B] time = 0.06, size = 107, normalized size = 4.65

$$x\,\left(b\,c^7+a\,c^3\right)+\frac{d^3\,x^4\,\left(35\,b\,c^4+a\right)}{4}+\frac{b\,d^7\,x^8}{8}+\frac{c^2\,d\,x^2\,\left(7\,b\,c^4+3\,a\right)}{2}+7\,b\,c^3\,d^4\,x^5+\frac{7\,b\,c^2\,d^5\,x^6}{2}+c\,d^2\,x^3\,\left(7\,b\,c^4+a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^4)*(c + d*x)^3,x)
```

```
[Out] x*(a*c^3 + b*c^7) + (d^3*x^4*(a + 35*b*c^4))/4 + (b*d^7*x^8)/8 + (c^2*d*x^2
*(3*a + 7*b*c^4))/2 + 7*b*c^3*d^4*x^5 + (7*b*c^2*d^5*x^6)/2 + c*d^2*x^3*(a
+ 7*b*c^4) + b*c*d^6*x^7
```

sympy [B] time = 0.10, size = 126, normalized size = 5.48

$$7bc^3d^4x^5+\frac{7bc^2d^5x^6}{2}+bcd^6x^7+\frac{bd^7x^8}{8}+x^4\left(\frac{ad^3}{4}+\frac{35bc^4d^3}{4}\right)+x^3\left(acd^2+7bc^5d^2\right)+x^2\left(\frac{3ac^2d}{2}+\frac{7bc^6d}{2}\right)+x\left(ac^3+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+b*(d*x+c)**4),x)

[Out] 7*b*c**3*d**4*x**5 + 7*b*c**2*d**5*x**6/2 + b*c*d**6*x**7 + b*d**7*x**8/8 +
 x**4*(a*d**3/4 + 35*b*c**4*d**3/4) + x**3*(a*c*d**2 + 7*b*c**5*d**2) + x**
2*(3*a*c**2*d/2 + 7*b*c**6*d/2) + x*(a*c**3 + b*c**7)
```

3.2912 $\int (c+dx)^3 \left(a+b(c+dx)^4\right)^2 dx$

Optimal. Leaf size=23

$$\frac{\left(a+b(c+dx)^4\right)^3}{12bd}$$

```
[Out] 1/12*(a+b*(d*x+c)^4)^3/b/d
```

Rubi [A] time = 0.09, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {372, 261}

$$\frac{\left(a+b(c+dx)^4\right)^3}{12bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + b*(c + d*x)^4)^2,x]
```

```
[Out] (a + b*(c + d*x)^4)^3/(12*b*d)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^3 \left(a+b(c+dx)^4\right)^2 dx &= \frac{\text{Subst}\left(\int x^3 \left(a+bx^4\right)^2 dx, x, c+dx\right)}{d} \\ &= \frac{\left(a+b(c+dx)^4\right)^3}{12bd}\end{aligned}$$

Mathematica [B] time = 0.04, size = 172, normalized size = 7.48

$$\frac{1}{12}x\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right)\left(3a^2+3ab\left(2c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4\right)+b^2\left(3c^8+12c^7dx+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + b*(c + d*x)^4)^2,x]
```

```
[Out] (x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*(3*a^2 + 3*a*b*(2*c^4 + 4*c^
3*d*x + 6*c^2*d^2*x^2 + 4*c*d^3*x^3 + d^4*x^4) + b^2*(3*c^8 + 12*c^7*d*x +
34*c^6*d^2*x^2 + 60*c^5*d^3*x^3 + 71*c^4*d^4*x^4 + 56*c^3*d^5*x^5 + 28*c^2*
d^6*x^6 + 8*c*d^7*x^7 + d^8*x^8)))/12
```

fricas [B] time = 0.89, size = 289, normalized size = 12.57

$$\frac{1}{12}x^{12}d^{11}b^2+x^{11}d^{10}cb^2+\frac{11}{2}x^{10}d^9c^2b^2+\frac{55}{3}x^9d^8c^3b^2+\frac{165}{4}x^8d^7c^4b^2+66x^7d^6c^5b^2+77x^6d^5c^6b^2+66x^5d^4c^7b^2+\frac{165}{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*d^11*b^2 + x^11*d^10*c*b^2 + 11/2*x^10*d^9*c^2*b^2 + 55/3*x^9*d^8
*c^3*b^2 + 165/4*x^8*d^7*c^4*b^2 + 66*x^7*d^6*c^5*b^2 + 77*x^6*d^5*c^6*b^2
+ 66*x^5*d^4*c^7*b^2 + 165/4*x^4*d^3*c^8*b^2 + 1/4*x^8*d^7*b*a + 55/3*x^3*d
^2*c^9*b^2 + 2*x^7*d^6*c*b*a + 11/2*x^2*d*c^10*b^2 + 7*x^6*d^5*c^2*b*a + x*
c^11*b^2 + 14*x^5*d^4*c^3*b*a + 35/2*x^4*d^3*c^4*b*a + 14*x^3*d^2*c^5*b*a +
 7*x^2*d*c^6*b*a + 2*x*c^7*b*a + 1/4*x^4*d^3*a^2 + x^3*d^2*c*a^2 + 3/2*x^2*
d*c^2*a^2 + x*c^3*a^2
```

giac [A] time = 0.18, size = 21, normalized size = 0.91

$$\frac{\left((dx+c)^4 b+a\right)^3}{12\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/12*((d*x + c)^4*b + a)^3/(b*d)
```

maple [B] time = 0.00, size = 622, normalized size = 27.04

$$\frac{b^2 d^{11} x^{12}}{12}+b^2 c\, d^{10} x^{11}+\frac{11 b^2 c^2 d^9 x^{10}}{2}+\frac{55 b^2 c^3 d^8 x^9}{3}+\frac{\left(260 b^2 c^4 d^7+\left(68 b^2 c^4 d^4+2\left(b\,c^4+a\right) b\,d^4\right) d^3\right) x^8}{8}+\frac{\left(196 b^2 c^5 d^6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+b*(d*x+c)^4)^2,x)
```

```
[Out] 1/12*d^11*b^2*x^12+c*d^10*b^2*x^11+11/2*c^2*d^9*b^2*x^10+55/3*c^3*b^2*d^8*x
^9+1/8*(260*c^4*b^2*d^7+d^3*(2*(b*c^4+a)*b*d^4+68*b^2*d^4*c^4))*x^8+1/7*(19
6*c^5*d^6*b^2+3*c*d^2*(2*(b*c^4+a)*b*d^4+68*b^2*d^4*c^4)+d^3*(8*(b*c^4+a)*b
*c*d^3+48*c^5*d^3*b^2))*x^7+1/6*(56*c^6*d^5*b^2+3*c^2*d*(2*(b*c^4+a)*b*d^4+
68*b^2*d^4*c^4)+3*c*d^2*(8*(b*c^4+a)*b*c*d^3+48*c^5*d^3*b^2)+d^3*(12*(b*c^4
+a)*c^2*d^2*b+16*c^6*d^2*b^2))*x^6+1/5*(c^3*(2*(b*c^4+a)*b*d^4+68*b^2*d^4*c
^4)+3*c^2*d*(8*(b*c^4+a)*b*c*d^3+48*c^5*d^3*b^2)+3*c*d^2*(12*(b*c^4+a)*c^2*
d^2*b+16*c^6*d^2*b^2)+8*d^4*(b*c^4+a)*c^3*b)*x^5+1/4*(c^3*(8*(b*c^4+a)*b*c*
d^3+48*c^5*d^3*b^2)+3*c^2*d*(12*(b*c^4+a)*c^2*d^2*b+16*c^6*d^2*b^2)+24*c^4*
d^3*(b*c^4+a)*b+d^3*(b*c^4+a)^2)*x^4+1/3*(c^3*(12*(b*c^4+a)*c^2*d^2*b+16*c^
6*d^2*b^2)+24*c^5*d^2*(b*c^4+a)*b+3*c*d^2*(b*c^4+a)^2)*x^3+1/2*(8*c^6*(b*c^
4+a)*d*b+3*c^2*d*(b*c^4+a)^2)*x^2+c^3*(b*c^4+a)^2*x
```

maxima [A] time = 0.55, size = 21, normalized size = 0.91

$$\frac{\left((dx+c)^4 b+a\right)^3}{12\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/12*((d*x + c)^4*b + a)^3/(b*d)
```

mupad [B] time = 0.15, size = 229, normalized size = 9.96

$$c^3 x\left(b\,c^4+a\right)^2+\frac{b^2 d^{11} x^{12}}{12}+\frac{d^3 x^4\left(a^2+70\,a\,b\,c^4+165\,b^2 c^8\right)}{4}+b^2 c\, d^{10} x^{11}+\frac{55\,b^2 c^3 d^8 x^9}{3}+\frac{11\,b^2 c^2 d^9 x^{10}}{2}+\frac{b\,d^7 x^8}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^4)^2*(c + d*x)^3,x)
```

```
[Out] c^3*x*(a + b*c^4)^2 + (b^2*d^11*x^12)/12 + (d^3*x^4*(a^2 + 165*b^2*c^8 + 70
*a*b*c^4))/4 + b^2*c*d^10*x^11 + (55*b^2*c^3*d^8*x^9)/3 + (11*b^2*c^2*d^9*x
^10)/2 + (b*d^7*x^8*(a + 165*b*c^4))/4 + (c^2*d*x^2*(3*a^2 + 11*b^2*c^8 + 1
4*a*b*c^4))/2 + (c*d^2*x^3*(3*a^2 + 55*b^2*c^8 + 42*a*b*c^4))/3 + 2*b*c*d^6
*x^7*(a + 33*b*c^4) + 7*b*c^2*d^5*x^6*(a + 11*b*c^4) + 2*b*c^3*d^4*x^5*(7*a
 + 33*b*c^4)
```

sympy [**B**] time = 0.14, size = 299, normalized size = 13.00

$$\frac{55b^2c^3d^8x^9}{3}+\frac{11b^2c^2d^9x^{10}}{2}+b^2cd^{10}x^{11}+\frac{b^2d^{11}x^{12}}{12}+x^8\left(\frac{abd^7}{4}+\frac{165b^2c^4d^7}{4}\right)+x^7\left(2abcd^6+66b^2c^5d^6\right)+x^6\left(7abc^2d^{\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+b*(d*x+c)**4)**2,x)
```

```
[Out] 55*b**2*c**3*d**8*x**9/3 + 11*b**2*c**2*d**9*x**10/2 + b**2*c*d**10*x**11 +
 b**2*d**11*x**12/12 + x**8*(a*b*d**7/4 + 165*b**2*c**4*d**7/4) + x**7*(2*a
*b*c*d**6 + 66*b**2*c**5*d**6) + x**6*(7*a*b*c**2*d**5 + 77*b**2*c**6*d**5)
 + x**5*(14*a*b*c**3*d**4 + 66*b**2*c**7*d**4) + x**4*(a**2*d**3/4 + 35*a*b
*c**4*d**3/2 + 165*b**2*c**8*d**3/4) + x**3*(a**2*c*d**2 + 14*a*b*c**5*d**2
 + 55*b**2*c**9*d**2/3) + x**2*(3*a**2*c**2*d/2 + 7*a*b*c**6*d + 11*b**2*c*
*10*d/2) + x*(a**2*c**3 + 2*a*b*c**7 + b**2*c**11)
```

3.2913 $\int (c+dx)^3 \left(a+b(c+dx)^4\right)^3 \, dx$

Optimal. Leaf size=23

$$\frac{\left(a+b(c+dx)^4\right)^4}{16bd}$$

```
[Out] 1/16*(a+b*(d*x+c)^4)^4/b/d
```

Rubi [A] time = 0.17, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {372, 261}

$$\frac{\left(a+b(c+dx)^4\right)^4}{16bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + b*(c + d*x)^4)^3,x]
```

```
[Out] (a + b*(c + d*x)^4)^4/(16*b*d)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^3 \left(a+b(c+dx)^4\right)^3 \, dx &= \frac{\text{Subst}\left(\int x^3 \left(a+bx^4\right)^3 \, dx, x, c+dx\right)}{d} \\ &= \frac{\left(a+b(c+dx)^4\right)^4}{16bd}\end{aligned}$$

Mathematica [B] time = 0.07, size = 308, normalized size = 13.39

$$\frac{1}{16}x\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right)\left(4a^3+6a^2b\left(2c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4\right)+4ab^2\left(3c^8+12c^7dx+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + b*(c + d*x)^4)^3,x]
```

```
[Out] (x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*(4*a^3 + 6*a^2*b*(2*c^4 + 4*
c^3*d*x + 6*c^2*d^2*x^2 + 4*c*d^3*x^3 + d^4*x^4) + 4*a*b^2*(3*c^8 + 12*c^7*
d*x + 34*c^6*d^2*x^2 + 60*c^5*d^3*x^3 + 71*c^4*d^4*x^4 + 56*c^3*d^5*x^5 + 2
8*c^2*d^6*x^6 + 8*c*d^7*x^7 + d^8*x^8) + b^3*(4*c^12 + 24*c^11*d*x + 100*c^
10*d^2*x^2 + 280*c^9*d^3*x^3 + 566*c^8*d^4*x^4 + 848*c^7*d^5*x^5 + 952*c^6*
d^6*x^6 + 800*c^5*d^7*x^7 + 496*c^4*d^8*x^8 + 220*c^3*d^9*x^9 + 66*c^2*d^10
*x^10 + 12*c*d^11*x^11 + d^12*x^12)))/16
```

fricas [B] time = 0.65, size = 529, normalized size = 23.00

$$\frac{1}{16}x^{16}d^{15}b^3+x^{15}d^{14}cb^3+\frac{15}{2}x^{14}d^{13}c^2b^3+35x^{13}d^{12}c^3b^3+\frac{455}{4}x^{12}d^{11}c^4b^3+273x^{11}d^{10}c^5b^3+\frac{1001}{2}x^{10}d^9c^6b^3+715x^9d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*d^15*b^3 + x^15*d^14*c*b^3 + 15/2*x^14*d^13*c^2*b^3 + 35*x^13*d^1
2*c^3*b^3 + 455/4*x^12*d^11*c^4*b^3 + 273*x^11*d^10*c^5*b^3 + 1001/2*x^10*d
^9*c^6*b^3 + 715*x^9*d^8*c^7*b^3 + 6435/8*x^8*d^7*c^8*b^3 + 1/4*x^12*d^11*b
^2*a + 715*x^7*d^6*c^9*b^3 + 3*x^11*d^10*c*b^2*a + 1001/2*x^6*d^5*c^10*b^3
+ 33/2*x^10*d^9*c^2*b^2*a + 273*x^5*d^4*c^11*b^3 + 55*x^9*d^8*c^3*b^2*a + 4
55/4*x^4*d^3*c^12*b^3 + 495/4*x^8*d^7*c^4*b^2*a + 35*x^3*d^2*c^13*b^3 + 198
*x^7*d^6*c^5*b^2*a + 15/2*x^2*d*c^14*b^3 + 231*x^6*d^5*c^6*b^2*a + x*c^15*b
^3 + 198*x^5*d^4*c^7*b^2*a + 495/4*x^4*d^3*c^8*b^2*a + 3/8*x^8*d^7*b*a^2 +
55*x^3*d^2*c^9*b^2*a + 3*x^7*d^6*c*b*a^2 + 33/2*x^2*d*c^10*b^2*a + 21/2*x^6
*d^5*c^2*b*a^2 + 3*x*c^11*b^2*a + 21*x^5*d^4*c^3*b*a^2 + 105/4*x^4*d^3*c^4*
b*a^2 + 21*x^3*d^2*c^5*b*a^2 + 21/2*x^2*d*c^6*b*a^2 + 3*x*c^7*b*a^2 + 1/4*x
^4*d^3*a^3 + x^3*d^2*c*a^3 + 3/2*x^2*d*c^2*a^3 + x*c^3*a^3
```

giac [A] time = 0.22, size = 21, normalized size = 0.91

$$\frac{\left((dx+c)^4b+a\right)^4}{16\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/16*((d*x + c)^4*b + a)^4/(b*d)
```

maple [B] time = 0.00, size = 3262, normalized size = 141.83

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+b*(d*x+c)^4)^3,x)
```

```
[Out] 1/16*d^15*b^3*x^16+c*d^14*b^3*x^15+15/2*c^2*d^13*b^3*x^14+35*c^3*b^3*d^12*x
^13+1/12*(870*c^4*b^3*d^11+d^3*((b*c^4+a)*b^2*d^8+424*b^3*c^4*d^8+b*d^4*(68
*b^2*c^4*d^4+2*(b*c^4+a)*b*d^4)))*x^12+1/11*(726*c^5*d^10*b^3+3*c*d^2*((b*c
^4+a)*b^2*d^8+424*b^3*c^4*d^8+b*d^4*(68*b^2*c^4*d^4+2*(b*c^4+a)*b*d^4))+d^3
*(8*(b*c^4+a)*b^2*c*d^7+448*b^3*c^5*d^7+4*b*c*d^3*(68*b^2*c^4*d^4+2*(b*c^4+
a)*b*d^4)+b*d^4*(48*b^2*c^5*d^3+8*(b*c^4+a)*b*c*d^3)))*x^11+1/10*(220*c^6*b
^3*d^9+3*c^2*d*((b*c^4+a)*b^2*d^8+424*b^3*c^4*d^8+b*d^4*(68*b^2*c^4*d^4+2*(
b*c^4+a)*b*d^4))+3*c*d^2*(8*(b*c^4+a)*b^2*c*d^7+448*b^3*c^5*d^7+4*b*c*d^3*(
68*b^2*c^4*d^4+2*(b*c^4+a)*b*d^4)+b*d^4*(48*b^2*c^5*d^3+8*(b*c^4+a)*b*c*d^3
))+d^3*(28*(b*c^4+a)*c^2*d^6*b^2+224*c^6*d^6*b^3+6*c^2*d^2*b*(68*b^2*c^4*d^
4+2*(b*c^4+a)*b*d^4)+4*b*c*d^3*(48*b^2*c^5*d^3+8*(b*c^4+a)*b*c*d^3)+b*d^4*(
16*b^2*c^6*d^2+12*(b*c^4+a)*b*c^2*d^2)))*x^10+1/9*(c^3*((b*c^4+a)*b^2*d^8+4
24*b^3*c^4*d^8+b*d^4*(68*b^2*c^4*d^4+2*(b*c^4+a)*b*d^4))+3*c^2*d*(8*(b*c^4+
a)*b^2*c*d^7+448*b^3*c^5*d^7+4*b*c*d^3*(68*b^2*c^4*d^4+2*(b*c^4+a)*b*d^4)+b
*d^4*(48*b^2*c^5*d^3+8*(b*c^4+a)*b*c*d^3))+3*c*d^2*(28*(b*c^4+a)*c^2*d^6*b^
2+224*c^6*d^6*b^3+6*c^2*d^2*b*(68*b^2*c^4*d^4+2*(b*c^4+a)*b*d^4)+4*b*c*d^3*
(48*b^2*c^5*d^3+8*(b*c^4+a)*b*c*d^3)+b*d^4*(16*b^2*c^6*d^2+12*(b*c^4+a)*b*c
^2*d^2))+d^3*(64*(b*c^4+a)*c^3*d^5*b^2+4*c^3*d*b*(68*b^2*c^4*d^4+2*(b*c^4+a
)*b*d^4)+6*c^2*d^2*b*(48*b^2*c^5*d^3+8*(b*c^4+a)*b*c*d^3)+4*b*c*d^3*(16*b^2
*c^6*d^2+12*(b*c^4+a)*b*c^2*d^2)))*x^9+1/8*(c^3*(8*(b*c^4+a)*b^2*c*d^7+448*
```

```
b^3*c^5*d^7+4*b*c*d^3*(68*b^2*c^4*d^4+2*(b*c^4+a)*b*d^4)+b*d^4*(48*b^2*c^5*
d^3+8*(b*c^4+a)*b*c*d^3))+3*c^2*d*(28*(b*c^4+a)*c^2*d^6*b^2+224*c^6*d^6*b^3
+6*c^2*d^2*b*(68*b^2*c^4*d^4+2*(b*c^4+a)*b*d^4)+4*b*c*d^3*(48*b^2*c^5*d^3+8
*(b*c^4+a)*b*c*d^3)+b*d^4*(16*b^2*c^6*d^2+12*(b*c^4+a)*b*c^2*d^2))+3*c*d^2*
(64*(b*c^4+a)*c^3*d^5*b^2+4*c^3*d*b*(68*b^2*c^4*d^4+2*(b*c^4+a)*b*d^4)+6*c^
2*d^2*b*(48*b^2*c^5*d^3+8*(b*c^4+a)*b*c*d^3)+4*b*c*d^3*(16*b^2*c^6*d^2+12*(
b*c^4+a)*b*c^2*d^2))+d^3*((b*c^4+a)*(68*b^2*c^4*d^4+2*(b*c^4+a)*b*d^4)+4*c^
3*d*b*(48*b^2*c^5*d^3+8*(b*c^4+a)*b*c*d^3)+6*c^2*d^2*b*(16*b^2*c^6*d^2+12*(
b*c^4+a)*b*c^2*d^2)+32*b^2*c^4*d^4*(b*c^4+a)+b*d^4*(b*c^4+a)^2))*x^8+1/7*(c
^3*(28*(b*c^4+a)*c^2*d^6*b^2+224*c^6*d^6*b^3+6*c^2*d^2*b*(68*b^2*c^4*d^4+2*
(b*c^4+a)*b*d^4)+4*b*c*d^3*(48*b^2*c^5*d^3+8*(b*c^4+a)*b*c*d^3)+b*d^4*(16*b
^2*c^6*d^2+12*(b*c^4+a)*b*c^2*d^2))+3*c^2*d*(64*(b*c^4+a)*c^3*d^5*b^2+4*c^3
*d*b*(68*b^2*c^4*d^4+2*(b*c^4+a)*b*d^4)+6*c^2*d^2*b*(48*b^2*c^5*d^3+8*(b*c^
4+a)*b*c*d^3)+4*b*c*d^3*(16*b^2*c^6*d^2+12*(b*c^4+a)*b*c^2*d^2))+3*c*d^2*((
b*c^4+a)*(68*b^2*c^4*d^4+2*(b*c^4+a)*b*d^4)+4*c^3*d*b*(48*b^2*c^5*d^3+8*(b*
c^4+a)*b*c*d^3)+6*c^2*d^2*b*(16*b^2*c^6*d^2+12*(b*c^4+a)*b*c^2*d^2)+32*b^2*
c^4*d^4*(b*c^4+a)+b*d^4*(b*c^4+a)^2)+d^3*((b*c^4+a)*(48*b^2*c^5*d^3+8*(b*c^
4+a)*b*c*d^3)+4*c^3*d*b*(16*b^2*c^6*d^2+12*(b*c^4+a)*b*c^2*d^2)+48*c^5*d^3*
b^2*(b*c^4+a)+4*b*c*d^3*(b*c^4+a)^2))*x^7+1/6*(c^3*(64*(b*c^4+a)*c^3*d^5*b^
2+4*c^3*d*b*(68*b^2*c^4*d^4+2*(b*c^4+a)*b*d^4)+6*c^2*d^2*b*(48*b^2*c^5*d^3+
8*(b*c^4+a)*b*c*d^3)+4*b*c*d^3*(16*b^2*c^6*d^2+12*(b*c^4+a)*b*c^2*d^2))+3*c
^2*d*((b*c^4+a)*(68*b^2*c^4*d^4+2*(b*c^4+a)*b*d^4)+4*c^3*d*b*(48*b^2*c^5*d^
3+8*(b*c^4+a)*b*c*d^3)+6*c^2*d^2*b*(16*b^2*c^6*d^2+12*(b*c^4+a)*b*c^2*d^2)+
32*b^2*c^4*d^4*(b*c^4+a)+b*d^4*(b*c^4+a)^2)+3*c*d^2*((b*c^4+a)*(48*b^2*c^5*
d^3+8*(b*c^4+a)*b*c*d^3)+4*c^3*d*b*(16*b^2*c^6*d^2+12*(b*c^4+a)*b*c^2*d^2)+
48*c^5*d^3*b^2*(b*c^4+a)+4*b*c*d^3*(b*c^4+a)^2)+d^3*((b*c^4+a)*(16*b^2*c^6*
d^2+12*(b*c^4+a)*b*c^2*d^2)+32*c^6*d^2*b^2*(b*c^4+a)+6*c^2*d^2*b*(b*c^4+a)^
2))*x^6+1/5*(c^3*((b*c^4+a)*(68*b^2*c^4*d^4+2*(b*c^4+a)*b*d^4)+4*c^3*d*b*(4
8*b^2*c^5*d^3+8*(b*c^4+a)*b*c*d^3)+6*c^2*d^2*b*(16*b^2*c^6*d^2+12*(b*c^4+a)
*b*c^2*d^2)+32*b^2*c^4*d^4*(b*c^4+a)+b*d^4*(b*c^4+a)^2)+3*c^2*d*((b*c^4+a)*
(48*b^2*c^5*d^3+8*(b*c^4+a)*b*c*d^3)+4*c^3*d*b*(16*b^2*c^6*d^2+12*(b*c^4+a)
*b*c^2*d^2)+48*c^5*d^3*b^2*(b*c^4+a)+4*b*c*d^3*(b*c^4+a)^2)+3*c*d^2*((b*c^4
+a)*(16*b^2*c^6*d^2+12*(b*c^4+a)*b*c^2*d^2)+32*c^6*d^2*b^2*(b*c^4+a)+6*c^2*
d^2*b*(b*c^4+a)^2)+12*d^4*(b*c^4+a)^2*c^3*b)*x^5+1/4*(c^3*((b*c^4+a)*(48*b^
2*c^5*d^3+8*(b*c^4+a)*b*c*d^3)+4*c^3*d*b*(16*b^2*c^6*d^2+12*(b*c^4+a)*b*c^2
*d^2)+48*c^5*d^3*b^2*(b*c^4+a)+4*b*c*d^3*(b*c^4+a)^2)+3*c^2*d*((b*c^4+a)*(1
6*b^2*c^6*d^2+12*(b*c^4+a)*b*c^2*d^2)+32*c^6*d^2*b^2*(b*c^4+a)+6*c^2*d^2*b*
(b*c^4+a)^2)+36*c^4*d^3*(b*c^4+a)^2*b+d^3*(b*c^4+a)^3)*x^4+1/3*(c^3*((b*c^4
+a)*(16*b^2*c^6*d^2+12*(b*c^4+a)*b*c^2*d^2)+32*c^6*d^2*b^2*(b*c^4+a)+6*c^2*
d^2*b*(b*c^4+a)^2)+36*c^5*d^2*(b*c^4+a)^2*b+3*c*d^2*(b*c^4+a)^3)*x^3+1/2*(1
2*c^6*(b*c^4+a)^2*d*b+3*c^2*d*(b*c^4+a)^3)*x^2+c^3*(b*c^4+a)^3*x
```

maxima [A] time = 0.64, size = 21, normalized size = 0.91

$$\frac{\left((dx+c)^4 b+a\right)^4}{16\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((d*x + c)^4*b + a)^4/(b*d)
```

mupad [B] time = 0.29, size = 376, normalized size = 16.35

$$c^3 x\left(b c^4+a\right)^3+\frac{d^3 x^4\left(a^3+105\,a^2 b c^4+495\,a b^2 c^8+455\,b^3 c^{12}\right)}{4}+\frac{b^3 d^{15} x^{16}}{16}+\frac{b^2 d^{11} x^{12}\left(455\,b c^4+a\right)}{4}+b^3 c d^{14} x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^4)^3*(c + d*x)^3,x)
```

```
[Out] c^3*x*(a + b*c^4)^3 + (d^3*x^4*(a^3 + 455*b^3*c^12 + 105*a^2*b*c^4 + 495*a*
b^2*c^8))/4 + (b^3*d^15*x^16)/16 + (b^2*d^11*x^12*(a + 455*b*c^4))/4 + b^3*
c*d^14*x^15 + (3*b*d^7*x^8*(a^2 + 2145*b^2*c^8 + 330*a*b*c^4))/8 + 35*b^3*c
^3*d^12*x^13 + (15*b^3*c^2*d^13*x^14)/2 + c*d^2*x^3*(a^3 + 35*b^3*c^12 + 21
*a^2*b*c^4 + 55*a*b^2*c^8) + b*c*d^6*x^7*(3*a^2 + 715*b^2*c^8 + 198*a*b*c^4
) + (11*b^2*c^2*d^9*x^10*(3*a + 91*b*c^4))/2 + (3*c^2*d*x^2*(a + b*c^4)^2*(
a + 5*b*c^4))/2 + 3*b^2*c*d^10*x^11*(a + 91*b*c^4) + 3*b*c^3*d^4*x^5*(7*a^2
 + 91*b^2*c^8 + 66*a*b*c^4) + (7*b*c^2*d^5*x^6*(3*a^2 + 143*b^2*c^8 + 66*a*
b*c^4))/2 + 55*b^2*c^3*d^8*x^9*(a + 13*b*c^4)
```

sympy [**B**] time = 0.23, size = 541, normalized size = 23.52

$$35b^3c^3d^{12}x^{13}+\frac{15b^3c^2d^{13}x^{14}}{2}+b^3cd^{14}x^{15}+\frac{b^3d^{15}x^{16}}{16}+x^{12}\left(\frac{ab^2d^{11}}{4}+\frac{455b^3c^4d^{11}}{4}\right)+x^{11}\left(3ab^2cd^{10}+273b^3c^5d^{10}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+b*(d*x+c)**4)**3,x)
```

```
[Out] 35*b**3*c**3*d**12*x**13 + 15*b**3*c**2*d**13*x**14/2 + b**3*c*d**14*x**15
+ b**3*d**15*x**16/16 + x**12*(a*b**2*d**11/4 + 455*b**3*c**4*d**11/4) + x*
*11*(3*a*b**2*c*d**10 + 273*b**3*c**5*d**10) + x**10*(33*a*b**2*c**2*d**9/2
 + 1001*b**3*c**6*d**9/2) + x**9*(55*a*b**2*c**3*d**8 + 715*b**3*c**7*d**8)
 + x**8*(3*a**2*b*d**7/8 + 495*a*b**2*c**4*d**7/4 + 6435*b**3*c**8*d**7/8)
+ x**7*(3*a**2*b*c*d**6 + 198*a*b**2*c**5*d**6 + 715*b**3*c**9*d**6) + x**6
*(21*a**2*b*c**2*d**5/2 + 231*a*b**2*c**6*d**5 + 1001*b**3*c**10*d**5/2) +
x**5*(21*a**2*b*c**3*d**4 + 198*a*b**2*c**7*d**4 + 273*b**3*c**11*d**4) + x
**4*(a**3*d**3/4 + 105*a**2*b*c**4*d**3/4 + 495*a*b**2*c**8*d**3/4 + 455*b*
*3*c**12*d**3/4) + x**3*(a**3*c*d**2 + 21*a**2*b*c**5*d**2 + 55*a*b**2*c**9
*d**2 + 35*b**3*c**13*d**2) + x**2*(3*a**3*c**2*d/2 + 21*a**2*b*c**6*d/2 +
33*a*b**2*c**10*d/2 + 15*b**3*c**14*d/2) + x*(a**3*c**3 + 3*a**2*b*c**7 + 3
*a*b**2*c**11 + b**3*c**15)
```

3.2914 $\int \frac{(c+dx)^3}{a+b(c+dx)^4}\,dx$

Optimal. Leaf size=22

$$\frac{\log\left(a+b(c+dx)^4\right)}{4bd}$$

```
[Out] 1/4*ln(a+b*(d*x+c)^4)/b/d
```

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {372, 260}

$$\frac{\log\left(a+b(c+dx)^4\right)}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*(c + d*x)^4),x]
```

```
[Out] Log[a + b*(c + d*x)^4]/(4*b*d)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^3}{a+b(c+dx)^4}\,dx &= \frac{\text{Subst}\left(\int \frac{x^3}{a+bx^4}\,dx, x, c+dx\right)}{d}\\ &= \frac{\log\left(a+b(c+dx)^4\right)}{4bd}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{\log\left(a+b(c+dx)^4\right)}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*(c + d*x)^4),x]
```

```
[Out] Log[a + b*(c + d*x)^4]/(4*b*d)
```

fricas [B] time = 1.00, size = 54, normalized size = 2.45

$$\frac{\log\left(bd^4x^4+4\,bcd^3x^3+6\,bc^2d^2x^2+4\,bc^3dx+bc^4+a\right)}{4\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/4*log(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c^4 +
 a)/(b*d)
```

giac [A] time = 0.19, size = 20, normalized size = 0.91

$$\frac{\log\left((dx+c)^4 b + a\right)}{4\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/4*log((d*x + c)^4*b + a)/(b*d)
```

maple [B] time = 0.00, size = 55, normalized size = 2.50

$$\frac{\ln\left(b\,d^4x^4 + 4bc\,d^3x^3 + 6b\,c^2d^2x^2 + 4b\,c^3dx + b\,c^4 + a\right)}{4bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+b*(d*x+c)^4),x)
```

```
[Out] 1/4/b/d*ln(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)
```

maxima [A] time = 0.55, size = 20, normalized size = 0.91

$$\frac{\log\left((dx+c)^4 b + a\right)}{4\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/4*log((d*x + c)^4*b + a)/(b*d)
```

mupad [B] time = 1.26, size = 54, normalized size = 2.45

$$\frac{\ln\left(b\,c^4 + 4\,b\,c^3\,d\,x + 6\,b\,c^2\,d^2\,x^2 + 4\,b\,c\,d^3\,x^3 + b\,d^4\,x^4 + a\right)}{4\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*(c + d*x)^4),x)
```

```
[Out] log(a + b*c^4 + b*d^4*x^4 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + 4*b*c*d^3*x^3)/
(4*b*d)
```

sympy [B] time = 0.44, size = 56, normalized size = 2.55

$$\frac{\log\left(a + bc^4 + 4bc^3dx + 6bc^2d^2x^2 + 4bcd^3x^3 + bd^4x^4\right)}{4bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+b*(d*x+c)**4),x)
```

```
[Out] log(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2 + 4*b*c*d**3*x**3 + b*d*
**4*x**4)/(4*b*d)
```

3.2915 $\int \frac{(c+dx)^3}{\left(a+b(c+dx)^4\right)^2}\,dx$

Optimal. Leaf size=23

$$-\frac{1}{4bd\left(a+b(c+dx)^4\right)}$$

```
[Out] -1/4/b/d/(a+b*(d*x+c)^4)
```

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {372, 261}

$$-\frac{1}{4bd\left(a+b(c+dx)^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*(c + d*x)^4)^2,x]
```

```
[Out] -1/(4*b*d*(a + b*(c + d*x)^4))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^3}{\left(a+b(c+dx)^4\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{x^3}{\left(a+bx^4\right)^2}\,dx, x, c+dx\right)}{d}\\ &= -\frac{1}{4bd\left(a+b(c+dx)^4\right)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$-\frac{1}{4bd\left(a+b(c+dx)^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*(c + d*x)^4)^2,x]
```

```
[Out] -1/4*1/(b*d*(a + b*(c + d*x)^4))
```

fricas [B] time = 0.71, size = 66, normalized size = 2.87

$$-\frac{1}{4\left(b^2d^5x^4+4\,b^2cd^4x^3+6\,b^2c^2d^3x^2+4\,b^2c^3d^2x+\left(b^2c^4+ab\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/4/(b^2*d^5*x^4 + 4*b^2*c*d^4*x^3 + 6*b^2*c^2*d^3*x^2 + 4*b^2*c^3*d^2*x +
 (b^2*c^4 + a*b)*d)
```

giac [A] time = 0.20, size = 21, normalized size = 0.91

$$-\frac{1}{4\left((dx+c)^4 b+a\right)bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] -1/4/(((d*x + c)^4*b + a)*b*d)
```

maple [B] time = 0.00, size = 56, normalized size = 2.43

$$-\frac{1}{4\left(b\,d^4x^4+4bc\,d^3x^3+6b\,c^2d^2x^2+4b\,c^3dx+b\,c^4+a\right)bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+b*(d*x+c)^4)^2,x)
```

```
[Out] -1/4/b/d/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)
```

maxima [A] time = 0.46, size = 21, normalized size = 0.91

$$-\frac{1}{4\left((dx+c)^4 b+a\right)bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/4/(((d*x + c)^4*b + a)*b*d)
```

mupad [B] time = 0.19, size = 55, normalized size = 2.39

$$-\frac{1}{4\,b\,d\,\left(b\,c^4+4\,b\,c^3\,d\,x+6\,b\,c^2\,d^2\,x^2+4\,b\,c\,d^3\,x^3+b\,d^4\,x^4+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*(c + d*x)^4)^2,x)
```

```
[Out] -1/(4*b*d*(a + b*c^4 + b*d^4*x^4 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + 4*b*c*d^
3*x^3))
```

sympy [B] time = 2.02, size = 73, normalized size = 3.17

$$-\frac{1}{4abd+4b^2c^4d+16b^2c^3d^2x+24b^2c^2d^3x^2+16b^2cd^4x^3+4b^2d^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+b*(d*x+c)**4)**2,x)
```

```
[Out] -1/(4*a*b*d + 4*b**2*c**4*d + 16*b**2*c**3*d**2*x + 24*b**2*c**2*d**3*x**2
+ 16*b**2*c*d**4*x**3 + 4*b**2*d**5*x**4)
```

3.2916 $\int \frac{(c+dx)^3}{\left(a+b(c+dx)^4\right)^3}\,dx$

Optimal. Leaf size=23

$$-\frac{1}{8bd\left(a+b(c+dx)^4\right)^2}$$

```
[Out] -1/8/b/d/(a+b*(d*x+c)^4)^2
```

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {372, 261}

$$-\frac{1}{8bd\left(a+b(c+dx)^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*(c + d*x)^4)^3,x]
```

```
[Out] -1/(8*b*d*(a + b*(c + d*x)^4)^2)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 372

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^3}{\left(a+b(c+dx)^4\right)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{x^3}{\left(a+bx^4\right)^3}\,dx, x, c+dx\right)}{d}\\ &= -\frac{1}{8bd\left(a+b(c+dx)^4\right)^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$-\frac{1}{8bd\left(a+b(c+dx)^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*(c + d*x)^4)^3,x]
```

```
[Out] -1/8*1/(b*d*(a + b*(c + d*x)^4)^2)
```

fricas [B] time = 1.01, size = 172, normalized size = 7.48

$$-\frac{1}{8\left(b^3d^9x^8+8\,b^3cd^8x^7+28\,b^3c^2d^7x^6+56\,b^3c^3d^6x^5+2\left(35\,b^3c^4+ab^2\right)d^5x^4+8\left(7\,b^3c^5+ab^2c\right)d^4x^3+4\left(7\,b^3c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] -1/8/(b^3*d^9*x^8 + 8*b^3*c*d^8*x^7 + 28*b^3*c^2*d^7*x^6 + 56*b^3*c^3*d^6*x
^5 + 2*(35*b^3*c^4 + a*b^2)*d^5*x^4 + 8*(7*b^3*c^5 + a*b^2*c)*d^4*x^3 + 4*(
7*b^3*c^6 + 3*a*b^2*c^2)*d^3*x^2 + 8*(b^3*c^7 + a*b^2*c^3)*d^2*x + (b^3*c^8
 + 2*a*b^2*c^4 + a^2*b)*d)
```

giac [A] time = 0.25, size = 21, normalized size = 0.91

$$-\frac{1}{8\left((dx+c)^4b+a\right)^2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -1/8/(((d*x + c)^4*b + a)^2*b*d)
```

maple [B] time = 0.00, size = 56, normalized size = 2.43

$$-\frac{1}{8\left(b\,d^4x^4+4bc\,d^3x^3+6b\,c^2d^2x^2+4b\,c^3dx+b\,c^4+a\right)^2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+b*(d*x+c)^4)^3,x)
```

```
[Out] -1/8/b/d/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^2
```

maxima [A] time = 0.56, size = 21, normalized size = 0.91

$$-\frac{1}{8\left((dx+c)^4b+a\right)^2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/8/(((d*x + c)^4*b + a)^2*b*d)
```

mupad [B] time = 2.35, size = 171, normalized size = 7.43

$$-\frac{1}{8\,b\,d\,\left(x^4\,\left(70\,b^2\,c^4\,d^4+2\,a\,b\,d^4\right)+x^3\,\left(56\,b^2\,c^5\,d^3+8\,a\,b\,c\,d^3\right)+a^2+x\,\left(8\,d\,b^2\,c^7+8\,a\,d\,b\,c^3\right)+b^2\,c^8+x^2\,\left(28\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*(c + d*x)^4)^3,x)
```

```
[Out] -1/(8*b*d*(x^4*(70*b^2*c^4*d^4 + 2*a*b*d^4) + x^3*(56*b^2*c^5*d^3 + 8*a*b*c
*d^3) + a^2 + x*(8*b^2*c^7*d + 8*a*b*c^3*d) + b^2*c^8 + x^2*(28*b^2*c^6*d^2
 + 12*a*b*c^2*d^2) + b^2*d^8*x^8 + 2*a*b*c^4 + 8*b^2*c*d^7*x^7 + 56*b^2*c^3
*d^5*x^5 + 28*b^2*c^2*d^6*x^6))
```

sympy [**B**] time = 5.42, size = 197, normalized size = 8.57

$$-\frac{1}{8a^2bd + 16ab^2c^4d + 8b^3c^8d + 448b^3c^3d^6x^5 + 224b^3c^2d^7x^6 + 64b^3cd^8x^7 + 8b^3d^9x^8 + x^4\left(16ab^2d^5 + 560b^3c^4d^5\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+b*(d*x+c)**4)**3,x)
```

```
[Out] -1/(8*a**2*b*d + 16*a*b**2*c**4*d + 8*b**3*c**8*d + 448*b**3*c**3*d**6*x**5
 + 224*b**3*c**2*d**7*x**6 + 64*b**3*c*d**8*x**7 + 8*b**3*d**9*x**8 + x**4*
(16*a*b**2*d**5 + 560*b**3*c**4*d**5) + x**3*(64*a*b**2*c*d**4 + 448*b**3*c
**5*d**4) + x**2*(96*a*b**2*c**2*d**3 + 224*b**3*c**6*d**3) + x*(64*a*b**2*
c**3*d**2 + 64*b**3*c**7*d**2))
```

3.2917 $\int \frac{1}{\sqrt{a+b(c+dx)^4}}\,dx$

Optimal. Leaf size=111

$$\frac{\left(\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{b}\,d\sqrt{a+b(c+dx)^4}}$$

```
[Out] 1/2*(cos(2*arctan(b^(1/4)*(d*x+c)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(
d*x+c)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*(d*x+c)/a^(1/4))),1/2*2^(1/
2))*(a^(1/2)+(d*x+c)^2*b^(1/2))*((a+b*(d*x+c)^4)/(a^(1/2)+(d*x+c)^2*b^(1/2)
)^2)^(1/2)/a^(1/4)/b^(1/4)/d/(a+b*(d*x+c)^4)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {247, 220}

$$\frac{\left(\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{b}\,d\sqrt{a+b(c+dx)^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*(c + d*x)^4],x]
```

```
[Out] ((Sqrt[a] + Sqrt[b]*(c + d*x)^2)*Sqrt[(a + b*(c + d*x)^4)/(Sqrt[a] + Sqrt[b
]*(c + d*x)^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*(c + d*x))/a^(1/4)], 1/2])/(2
*a^(1/4)*b^(1/4)*d*Sqrt[a + b*(c + d*x)^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b(c+dx)^4}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\left(\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{b}\,d\sqrt{a+b(c+dx)^4}}$$

Mathematica [C] time = 0.06, size = 90, normalized size = 0.81

$$-\frac{i\sqrt{\frac{a+b(c+dx)^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,(c+dx)\right)\middle|-1\right)}{d\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{a+b(c+dx)^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*(c + d*x)^4],x]
```

```
[Out] ((-I)*Sqrt[(a + b*(c + d*x)^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt
[a]]*(c + d*x)], -1])/(Sqrt[(I*Sqrt[b])/Sqrt[a]]*d*Sqrt[a + b*(c + d*x)^4])
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{bd^4x^4 + 4\,bcd^3x^3 + 6\,bc^2d^2x^2 + 4\,bc^3dx + bc^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x +
 b*c^4 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(dx + c)^4 b + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((d*x + c)^4*b + a), x)
```

maple [C] time = 0.44, size = 1036, normalized size = 9.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*(d*x+c)^4)^(1/2),x)
```

```
[Out] 2*((1/b*(-b^3*a)^(1/4)-c)/d-(-I/b*(-b^3*a)^(1/4)-c)/d)*(((-I/b*(-b^3*a)^(1/
4)-c)/d-(I/b*(-b^3*a)^(1/4)-c)/d)*(x-(1/b*(-b^3*a)^(1/4)-c)/d)/((-I/b*(-b^3
*a)^(1/4)-c)/d-(1/b*(-b^3*a)^(1/4)-c)/d)/(x-(I/b*(-b^3*a)^(1/4)-c)/d))^(1/2
)*(x-(I/b*(-b^3*a)^(1/4)-c)/d)^2*(((I/b*(-b^3*a)^(1/4)-c)/d-(1/b*(-b^3*a)^(
1/4)-c)/d)*(x-(-1/b*(-b^3*a)^(1/4)-c)/d)/((-1/b*(-b^3*a)^(1/4)-c)/d-(1/b*(-
b^3*a)^(1/4)-c)/d)/(x-(I/b*(-b^3*a)^(1/4)-c)/d))^(1/2)*(((I/b*(-b^3*a)^(1/4
)-c)/d-(1/b*(-b^3*a)^(1/4)-c)/d)*(x-(-I/b*(-b^3*a)^(1/4)-c)/d)/((-I/b*(-b^3
*a)^(1/4)-c)/d-(1/b*(-b^3*a)^(1/4)-c)/d)/(x-(I/b*(-b^3*a)^(1/4)-c)/d))^(1/2
)/((-I/b*(-b^3*a)^(1/4)-c)/d-(I/b*(-b^3*a)^(1/4)-c)/d)/((I/b*(-b^3*a)^(1/4)
-c)/d-(1/b*(-b^3*a)^(1/4)-c)/d)/(b*d^4*(x-(1/b*(-b^3*a)^(1/4)-c)/d)*(x-(I/b
*(-b^3*a)^(1/4)-c)/d)*(x-(-1/b*(-b^3*a)^(1/4)-c)/d)*(x-(-I/b*(-b^3*a)^(1/4)
-c)/d))^(1/2)*EllipticF(((-I/b*(-b^3*a)^(1/4)-c)/d-(I/b*(-b^3*a)^(1/4)-c)/
d)*(x-(1/b*(-b^3*a)^(1/4)-c)/d)/((-I/b*(-b^3*a)^(1/4)-c)/d-(1/b*(-b^3*a)^(1
/4)-c)/d)/(x-(I/b*(-b^3*a)^(1/4)-c)/d))^(1/2),(((I/b*(-b^3*a)^(1/4)-c)/d-(-
1/b*(-b^3*a)^(1/4)-c)/d)*((1/b*(-b^3*a)^(1/4)-c)/d-(-I/b*(-b^3*a)^(1/4)-c)/
d)/((1/b*(-b^3*a)^(1/4)-c)/d-(-1/b*(-b^3*a)^(1/4)-c)/d)/((I/b*(-b^3*a)^(1/4
)-c)/d-(-I/b*(-b^3*a)^(1/4)-c)/d))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(dx + c)^4 b + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((d*x + c)^4*b + a), x)
```

mupad **[B]** time = 1.67, size = 56, normalized size = 0.50

$$\frac{\sqrt{\frac{b\,(c+d\,x)^4}{a}+1}\;(c+d\,x)\;{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\;\frac{5}{4};\;-\frac{b\,(c+d\,x)^4}{a}\right)}{d\,\sqrt{a+b\,(c+d\,x)^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(c + d*x)^4)^(1/2),x)
```

```
[Out] (((b*(c + d*x)^4)/a + 1)^(1/2)*(c + d*x)*hypergeom([1/4, 1/2], 5/4, -(b*(c
+ d*x)^4)/a))/(d*(a + b*(c + d*x)^4)^(1/2))
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\,(c+dx)^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)**4)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*(c + d*x)**4), x)
```

3.2918 $\int \frac{x}{\sqrt{a+b(c+dx)^4}}\,dx$

Optimal. Leaf size=154

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a+b(c+dx)^4}}\right)}{2\sqrt{b}\,d^2}-\frac{c\left(\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{b}\,d^2\sqrt{a+b(c+dx)^4}}$$

```
[Out] 1/2*arctanh((d*x+c)^2*b^(1/2)/(a+b*(d*x+c)^4)^(1/2))/d^2/b^(1/2)-1/2*c*(cos
(2*arctan(b^(1/4)*(d*x+c)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(d*x+c)/a
^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*(d*x+c)/a^(1/4))),1/2*2^(1/2))*(a^(
1/2)+(d*x+c)^2*b^(1/2))*((a+b*(d*x+c)^4)/(a^(1/2)+(d*x+c)^2*b^(1/2))^2)^(1/
2)/a^(1/4)/b^(1/4)/d^2/(a+b*(d*x+c)^4)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {371, 1885, 220, 275, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a+b(c+dx)^4}}\right)}{2\sqrt{b}\,d^2}-\frac{c\left(\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{b}\,d^2\sqrt{a+b(c+dx)^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*(c + d*x)^4],x]
```

```
[Out] ArcTanh[(Sqrt[b]*(c + d*x)^2)/Sqrt[a + b*(c + d*x)^4]]/(2*Sqrt[b]*d^2) - (c
*(Sqrt[a] + Sqrt[b]*(c + d*x)^2)*Sqrt[(a + b*(c + d*x)^4)/(Sqrt[a] + Sqrt[b
]*(c + d*x)^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*(c + d*x))/a^(1/4)], 1/2])/(2
*a^(1/4)*b^(1/4)*d^2*Sqrt[a + b*(c + d*x)^4])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
```

```
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 1885

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (
2*(q - j))/n + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+b(c+dx)^4}}\,dx = \frac{\text{Subst}\left(\int \frac{-c+x}{\sqrt{a+bx^4}}\,dx, x, c+dx\right)}{d^2}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{c}{\sqrt{a+bx^4}} + \frac{x}{\sqrt{a+bx^4}}\right)\,dx, x, c+dx\right)}{d^2}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{\sqrt{a+bx^4}}\,dx, x, c+dx\right)}{d^2} - \frac{c\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, c+dx\right)}{d^2}$$

$$= -\frac{c\left(\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{b}\,d^2\sqrt{a+b(c+dx)^4}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b}}\right.}{}$$

$$= -\frac{c\left(\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{b}\,d^2\sqrt{a+b(c+dx)^4}} + \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\right.}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a+b(c+dx)^4}}\right)}{2\sqrt{b}\,d^2} - \frac{c\left(\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}\right.\right.}{2\sqrt[4]{a}\,\sqrt[4]{b}\,d^2\sqrt{a+b(c+dx)^4}}$$

Mathematica [C] time = 1.01, size = 330, normalized size = 2.14

$$\frac{\sqrt[4]{-1}\,\sqrt{2}\,\sqrt{-\frac{i\left(\sqrt[4]{-1}\,\sqrt[4]{a}+\sqrt[4]{b}\,(c+dx)\right)}{\sqrt[4]{-1}\,\sqrt[4]{a}-\sqrt[4]{b}\,(c+dx)}}\left(\sqrt{b}\,(c+dx)^2+i\sqrt{a}\right)\left(\left(\sqrt[4]{-1}\,\sqrt[4]{a}-\sqrt[4]{b}\,c\right)F\left(\sin^{-1}\left(\sqrt{-\frac{i\left(\sqrt[4]{b}\,(c+dx)+\sqrt[4]{-1}\,\sqrt[4]{a}\right)}{\sqrt[4]{-1}\,\sqrt[4]{a}-\sqrt[4]{b}\,(c+dx)}}\right)\right|-1$$

$$\sqrt[4]{a}\,\sqrt{b}\,d^2\sqrt{\frac{\sqrt{b}\,(c+dx)^2+i\sqrt{a}}{\left(\sqrt[4]{-1}\,\sqrt[4]{a}-\sqrt[4]{b}\,(c+dx)\right)^2}}\,\sqrt{a+b(c+dx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b*(c + d*x)^4],x]
```

```
[Out] ((-1)^(1/4)*Sqrt[2]*Sqrt[((-I)*((-1)^(1/4)*a^(1/4) + b^(1/4)*(c + d*x)))/((
-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))]*(I*Sqrt[a] + Sqrt[b]*(c + d*x)^2)*(
((-1)^(1/4)*a^(1/4) - b^(1/4)*c)*EllipticF[ArcSin[Sqrt[((-I)*((-1)^(1/4)*a^
(1/4) + b^(1/4)*(c + d*x)))/((-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))]], -1]
 - 2*(-1)^(1/4)*a^(1/4)*EllipticPi[-I, ArcSin[Sqrt[((-I)*((-1)^(1/4)*a^(1/4
) + b^(1/4)*(c + d*x)))/((-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))]], -1]))/(
a^(1/4)*Sqrt[b]*d^2*Sqrt[(I*Sqrt[a] + Sqrt[b]*(c + d*x)^2)/((-1)^(1/4)*a^(1
/4) - b^(1/4)*(c + d*x))^2]*Sqrt[a + b*(c + d*x)^4])
```

fricas [F] time = 1.22, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{\sqrt{bd^4x^4 + 4\,bcd^3x^3 + 6\,bc^2d^2x^2 + 4\,bc^3dx + bc^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x +
 b*c^4 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{(dx + c)^4 b + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt((d*x + c)^4*b + a), x)
```

maple [C] time = 0.03, size = 1528, normalized size = 9.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*(d*x+c)^4)^(1/2),x)
```

```
[Out] 2*((-c+(-a*b^3)^(1/4)/b)/d-(-c-I*(-a*b^3)^(1/4)/b)/d)*(((-c-I*(-a*b^3)^(1/4
)/b)/d-(-c+I*(-a*b^3)^(1/4)/b)/d)*(x-(-c+(-a*b^3)^(1/4)/b)/d)/((-c-I*(-a*b^
3)^(1/4)/b)/d-(-c+(-a*b^3)^(1/4)/b)/d)/(x-(-c+I*(-a*b^3)^(1/4)/b)/d))^(1/2)
*(x-(-c+I*(-a*b^3)^(1/4)/b)/d)^2*(((-c+I*(-a*b^3)^(1/4)/b)/d-(-c+(-a*b^3)^(
1/4)/b)/d)*(x-(-c-(-a*b^3)^(1/4)/b)/d)/((-c-(-a*b^3)^(1/4)/b)/d-(-c+(-a*b^3
)^(1/4)/b)/d)/(x-(-c+I*(-a*b^3)^(1/4)/b)/d))^(1/2)*(((-c+I*(-a*b^3)^(1/4)/b
)/d-(-c+(-a*b^3)^(1/4)/b)/d)*(x-(-c-I*(-a*b^3)^(1/4)/b)/d)/((-c-I*(-a*b^3)^
(1/4)/b)/d-(-c+(-a*b^3)^(1/4)/b)/d)/(x-(-c+I*(-a*b^3)^(1/4)/b)/d))^(1/2)/((
-c-I*(-a*b^3)^(1/4)/b)/d-(-c+I*(-a*b^3)^(1/4)/b)/d)/((-c+I*(-a*b^3)^(1/4)/b
)/d-(-c+(-a*b^3)^(1/4)/b)/d)/((x-(-c+(-a*b^3)^(1/4)/b)/d)*(x-(-c+I*(-a*b^3)
^(1/4)/b)/d)*(x-(-c-(-a*b^3)^(1/4)/b)/d)*(x-(-c-I*(-a*b^3)^(1/4)/b)/d)*b*d^
4)^(1/2)*((-c+I*(-a*b^3)^(1/4)/b)/d*EllipticF((((-c-I*(-a*b^3)^(1/4)/b)/d-(
-c+I*(-a*b^3)^(1/4)/b)/d)*(x-(-c+(-a*b^3)^(1/4)/b)/d)/((-c-I*(-a*b^3)^(1/4)
/b)/d-(-c+(-a*b^3)^(1/4)/b)/d)/(x-(-c+I*(-a*b^3)^(1/4)/b)/d))^(1/2),(((-c+I
*(-a*b^3)^(1/4)/b)/d-(-c-(-a*b^3)^(1/4)/b)/d)*((-c+(-a*b^3)^(1/4)/b)/d-(-c-
I*(-a*b^3)^(1/4)/b)/d)/((-c+(-a*b^3)^(1/4)/b)/d-(-c-(-a*b^3)^(1/4)/b)/d)/((
-c+I*(-a*b^3)^(1/4)/b)/d-(-c-I*(-a*b^3)^(1/4)/b)/d))^(1/2))+((-c+(-a*b^3)^(
1/4)/b)/d-(-c+I*(-a*b^3)^(1/4)/b)/d)*EllipticPi((((-c-I*(-a*b^3)^(1/4)/b)/d
-(-c+I*(-a*b^3)^(1/4)/b)/d)*(x-(-c+(-a*b^3)^(1/4)/b)/d)/((-c-I*(-a*b^3)^(1/
4)/b)/d-(-c+(-a*b^3)^(1/4)/b)/d)/(x-(-c+I*(-a*b^3)^(1/4)/b)/d))^(1/2),((-c-
I*(-a*b^3)^(1/4)/b)/d-(-c+(-a*b^3)^(1/4)/b)/d)/((-c-I*(-a*b^3)^(1/4)/b)/d-(
-c+I*(-a*b^3)^(1/4)/b)/d),(((-c+I*(-a*b^3)^(1/4)/b)/d-(-c-(-a*b^3)^(1/4)/b)
/d)*((-c+(-a*b^3)^(1/4)/b)/d-(-c-I*(-a*b^3)^(1/4)/b)/d)/((-c+(-a*b^3)^(1/4)
/b)/d-(-c-(-a*b^3)^(1/4)/b)/d)/((-c+I*(-a*b^3)^(1/4)/b)/d-(-c-I*(-a*b^3)^(1
/4)/b)/d))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{(dx + c)^4 b + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt((d*x + c)^4*b + a), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{a + b\,(c + d\,x)^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*(c + d*x)^4)^(1/2),x)
```

```
[Out] int(x/(a + b*(c + d*x)^4)^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a + bc^4 + 4bc^3dx + 6bc^2d^2x^2 + 4bcd^3x^3 + bd^4x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)**4)**(1/2),x)
```

```
[Out] Integral(x/sqrt(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2 + 4*b*c*d**3
*x**3 + b*d**4*x**4), x)
```

3.2919 $\int (dx)^m \sqrt{a + b(cx)^{3/2}}\, dx$

Optimal. Leaf size=78

$$\frac{x(dx)^m \sqrt{a + b(cx)^{3/2}}\ {}_2F_1\left(-\frac{1}{2}, \frac{2(m+1)}{3}; \frac{1}{3}(2m+5); -\frac{b(cx)^{3/2}}{a}\right)}{(m+1)\sqrt{\frac{b(cx)^{3/2}}{a} + 1}}$$

```
[Out] x*(d*x)^m*hypergeom([-1/2, 2/3+2/3*m],[5/3+2/3*m],-b*(c*x)^(3/2)/a)*(a+b*(c
*x)^(3/2))^(1/2)/(1+m)/(1+b*(c*x)^(3/2)/a)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {367, 343, 341, 365, 364}

$$\frac{x(dx)^m \sqrt{a + b(cx)^{3/2}}\ {}_2F_1\left(-\frac{1}{2}, \frac{2(m+1)}{3}; \frac{1}{3}(2m+5); -\frac{b(cx)^{3/2}}{a}\right)}{(m+1)\sqrt{\frac{b(cx)^{3/2}}{a} + 1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[a + b*(c*x)^(3/2)],x]
```

```
[Out] (x*(d*x)^m*Sqrt[a + b*(c*x)^(3/2)]*Hypergeometric2F1[-1/2, (2*(1 + m))/3, (
5 + 2*m)/3, -((b*(c*x)^(3/2))/a)])/((1 + m)*Sqrt[1 + (b*(c*x)^(3/2))/a])
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 343

```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(c^In
tPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a + b*x^n)^p, x], x] /;
 FreeQ[{a, b, c, m, p}, x] && FractionQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int (dx)^m \sqrt{a+b(cx)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \left(\frac{dx}{c}\right)^m \sqrt{a+bx^{3/2}}\,dx, x, cx\right)}{c} \\ &= \frac{((cx)^{-m}(dx)^m)\,\text{Subst}\left(\int x^m \sqrt{a+bx^{3/2}}\,dx, x, cx\right)}{c} \\ &= \frac{(2(cx)^{-m}(dx)^m)\,\text{Subst}\left(\int x^{-1+2(1+m)} \sqrt{a+bx^3}\,dx, x, \sqrt{cx}\right)}{c} \\ &= \frac{\left(2(cx)^{-m}(dx)^m \sqrt{a+b(cx)^{3/2}}\right)\text{Subst}\left(\int x^{-1+2(1+m)} \sqrt{1+\frac{bx^3}{a}}\,dx, x, \sqrt{cx}\right)}{c\sqrt{1+\frac{b(cx)^{3/2}}{a}}} \\ &= \frac{x(dx)^m \sqrt{a+b(cx)^{3/2}}\;{}_2F_1\left(-\frac{1}{2}, \frac{2(1+m)}{3}; \frac{1}{3}(5+2m); -\frac{b(cx)^{3/2}}{a}\right)}{(1+m)\sqrt{1+\frac{b(cx)^{3/2}}{a}}}\end{aligned}$$

Mathematica [A] time = 0.11, size = 79, normalized size = 1.01

$$\frac{x(dx)^m \sqrt{a+b(cx)^{3/2}}\;{}_2F_1\left(-\frac{1}{2}, \frac{2(m+1)}{3}; \frac{1}{3}(2m+5); -\frac{b(cx)^{3/2}}{a}\right)}{(m+1)\sqrt{\frac{a+b(cx)^{3/2}}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*Sqrt[a + b*(c*x)^(3/2)],x]
```

```
[Out] (x*(d*x)^m*Sqrt[a + b*(c*x)^(3/2)]*Hypergeometric2F1[-1/2, (2*(1 + m))/3, (
5 + 2*m)/3, -((b*(c*x)^(3/2))/a)])/((1 + m)*Sqrt[(a + b*(c*x)^(3/2))/a])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*(c*x)^(3/2))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   alglo
gextint: unimplemented
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{(cx)^{\frac{3}{2}}\,b + a}\;(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*(c*x)^(3/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((c*x)^(3/2)*b + a)*(d*x)^m, x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \sqrt{a + (cx)^{\frac{3}{2}}\,b}\;(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*(c*x)^(3/2))^(1/2),x)
[Out] int((d*x)^m*(a+b*(c*x)^(3/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{(cx)^{\frac{3}{2}}\, b + a}\ (dx)^m\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*(c*x)^(3/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt((c*x)^(3/2)*b + a)*(d*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a + b\,(c\,x)^{3/2}}\ (d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x)^(3/2))^(1/2)*(d*x)^m,x)
[Out] int((a + b*(c*x)^(3/2))^(1/2)*(d*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{a + b\,(cx)^{\frac{3}{2}}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*(c*x)**(3/2))**(1/2),x)
[Out] Integral((d*x)**m*sqrt(a + b*(c*x)**(3/2)), x)
```

3.2920 $\int (dx)^m \sqrt{a + b\sqrt{cx}}\, dx$

Optimal. Leaf size=74

$$-\frac{4a(dx)^m \left(a + b\sqrt{cx}\right)^{3/2} \left(-\frac{b\sqrt{cx}}{a}\right)^{-2m} {}_2F_1\left(\frac{3}{2}, -2m-1; \frac{5}{2}; \frac{\sqrt{cx}\,b}{a} + 1\right)}{3b^2c}$$

```
[Out] -4/3*a*(d*x)^m*hypergeom([3/2, -1-2*m],[5/2],1+b*(c*x)^(1/2)/a)*(a+b*(c*x)^
(1/2))^(3/2)/b^2/c/((-b*(c*x)^(1/2)/a)^(2*m))
```

Rubi [A] time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {367, 343, 341, 67, 65}

$$-\frac{4a(dx)^m \left(a + b\sqrt{cx}\right)^{3/2} \left(-\frac{b\sqrt{cx}}{a}\right)^{-2m} {}_2F_1\left(\frac{3}{2}, -2m-1; \frac{5}{2}; \frac{\sqrt{cx}\,b}{a} + 1\right)}{3b^2c}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[a + b*Sqrt[c*x]],x]
```

```
[Out] (-4*a*(d*x)^m*(a + b*Sqrt[c*x])^(3/2)*Hypergeometric2F1[3/2, -1 - 2*m, 5/2,
 1 + (b*Sqrt[c*x])/a])/(3*b^2*c*(-((b*Sqrt[c*x])/a))^(2*m))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 343

```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(c^In
tPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a + b*x^n)^p, x], x] /;
 FreeQ[{a, b, c, m, p}, x] && FractionQ[n]
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\int (dx)^m \sqrt{a+b\sqrt{cx}}\ dx = \frac{\text{Subst}\left(\int \sqrt{a+b\sqrt{x}}\ \left(\frac{dx}{c}\right)^m dx, x, cx\right)}{c}$$
$$= \frac{((cx)^{-m}(dx)^m)\,\text{Subst}\left(\int \sqrt{a+b\sqrt{x}}\ x^m\, dx, x, cx\right)}{c}$$
$$= \frac{(2(cx)^{-m}(dx)^m)\,\text{Subst}\left(\int x^{-1+2(1+m)}\sqrt{a+bx}\ dx, x, \sqrt{cx}\right)}{c}$$
$$= -\frac{\left(2a(dx)^m\left(-\frac{b\sqrt{cx}}{a}\right)^{-2m}\right)\text{Subst}\left(\int \left(-\frac{bx}{a}\right)^{-1+2(1+m)}\sqrt{a+bx}\ dx, x, \sqrt{cx}\right)}{bc}$$
$$= -\frac{4a(dx)^m\left(-\frac{b\sqrt{cx}}{a}\right)^{-2m}\left(a+b\sqrt{cx}\right)^{3/2}\ {}_2F_1\left(\frac{3}{2}, -1-2m; \frac{5}{2}; 1+\frac{b\sqrt{cx}}{a}\right)}{3b^2c}$$

Mathematica [A] time = 0.22, size = 112, normalized size = 1.51

$$\frac{4(dx)^m\left(a+b\sqrt{cx}\right)^{3/2}\left(-\frac{b\sqrt{cx}}{a}\right)^{-2m}\left(3\left(a+b\sqrt{cx}\right)\ {}_2F_1\left(\frac{5}{2}, -2m; \frac{7}{2}; \frac{\sqrt{cx}\,b}{a}+1\right) - 5a\ {}_2F_1\left(\frac{3}{2}, -2m; \frac{5}{2}; \frac{\sqrt{cx}\,b}{a}+1\right)\right)}{15b^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*Sqrt[a + b*Sqrt[c*x]],x]
```

```
[Out] (4*(d*x)^m*(a + b*Sqrt[c*x])^(3/2)*(-5*a*Hypergeometric2F1[3/2, -2*m, 5/2,
1 + (b*Sqrt[c*x])/a] + 3*(a + b*Sqrt[c*x])*Hypergeometric2F1[5/2, -2*m, 7/2
, 1 + (b*Sqrt[c*x])/a]))/(15*b^2*c*(-((b*Sqrt[c*x])/a))^(2*m))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*(c*x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   alglo
gextint: unimplemented
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{cx}\,b + a}\ (dx)^m\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*(c*x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sqrt(c*x)*b + a)*(d*x)^m, x)
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \sqrt{a+\sqrt{cx}\ b}\ (dx)^m\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*(c*x)^(1/2))^(1/2),x)
[Out] int((d*x)^m*(a+b*(c*x)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{cx}\, b + a}\ (dx)^m\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*(c*x)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(sqrt(c*x)*b + a)*(d*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a + b\sqrt{c\,x}}\ (d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x)^(1/2))^(1/2)*(d*x)^m,x)
[Out] int((a + b*(c*x)^(1/2))^(1/2)*(d*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{a + b\sqrt{cx}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*(c*x)**(1/2))**(1/2),x)
[Out] Integral((d*x)**m*sqrt(a + b*sqrt(c*x)), x)
```

3.2921 $\int (dx)^m \sqrt{a + \frac{b}{\sqrt{cx}}}\, dx$

Optimal. Leaf size=76

$$\frac{4b^2(dx)^m \left(a + \frac{b}{\sqrt{cx}}\right)^{3/2} \left(-\frac{b}{a\sqrt{cx}}\right)^{2m} {}_2F_1\left(\frac{3}{2}, 2m+3; \frac{5}{2}; \frac{b}{a\sqrt{cx}} + 1\right)}{3a^3c}$$

```
[Out] 4/3*b^2*(d*x)^m*hypergeom([3/2, 3+2*m],[5/2],1+b/a/(c*x)^(1/2))*(a+b/(c*x)^
(1/2))^(3/2)*(-b/a/(c*x)^(1/2))^(2*m)/a^3/c
```

Rubi [A] time = 0.09, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {367, 343, 341, 339, 67, 65}

$$\frac{4b^2(dx)^m \left(a + \frac{b}{\sqrt{cx}}\right)^{3/2} \left(-\frac{b}{a\sqrt{cx}}\right)^{2m} {}_2F_1\left(\frac{3}{2}, 2m+3; \frac{5}{2}; \frac{b}{a\sqrt{cx}} + 1\right)}{3a^3c}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[a + b/Sqrt[c*x]],x]
```

```
[Out] (4*b^2*(d*x)^m*(-(b/(a*Sqrt[c*x])))^(2*m)*(a + b/Sqrt[c*x])^(3/2)*Hypergeom
etric2F1[3/2, 3 + 2*m, 5/2, 1 + b/(a*Sqrt[c*x])])/(3*a^3*c)
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0]
```

Rule 339

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Dist[((c
*x)^(m + 1)*(1/x)^(m + 1))/c, Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x
], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 343

```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(c^In
tPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a + b*x^n)^p, x], x] /;
 FreeQ[{a, b, c, m, p}, x] && FractionQ[n]
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int (dx)^m \sqrt{a+\frac{b}{\sqrt{cx}}}\, dx &= \frac{\text{Subst}\left(\int \sqrt{a+\frac{b}{\sqrt{x}}} \left(\frac{dx}{c}\right)^m dx, x, cx\right)}{c} \\
&= \frac{((cx)^{-m}(dx)^m)\,\text{Subst}\left(\int \sqrt{a+\frac{b}{\sqrt{x}}}\, x^m\, dx, x, cx\right)}{c} \\
&= \frac{(2(cx)^{-m}(dx)^m)\,\text{Subst}\left(\int \sqrt{a+\frac{b}{x}}\, x^{-1+2(1+m)}\, dx, x, \sqrt{cx}\right)}{c} \\
&= -\frac{(2(cx)^{-m}(dx)^m)\,\text{Subst}\left(\int x^{-1-2(1+m)}\sqrt{a+bx}\, dx, x, \frac{1}{\sqrt{cx}}\right)}{c} \\
&= \frac{\left(2b^3(dx)^m\left(-\frac{b}{a\sqrt{cx}}\right)^{2m}\right)\text{Subst}\left(\int \left(-\frac{bx}{a}\right)^{-1-2(1+m)}\sqrt{a+bx}\, dx, x, \frac{1}{\sqrt{cx}}\right)}{a^3c} \\
&= \frac{4b^2(dx)^m\left(-\frac{b}{a\sqrt{cx}}\right)^{2m}\left(a+\frac{b}{\sqrt{cx}}\right)^{3/2}{}_2F_1\left(\frac{3}{2}, 3+2m; \frac{5}{2}; 1+\frac{b}{a\sqrt{cx}}\right)}{3a^3c}
\end{aligned}$$

Mathematica [A] time = 0.34, size = 135, normalized size = 1.78

$$\frac{4(dx)^m\sqrt{a+\frac{b}{\sqrt{cx}}}\left(a\sqrt{cx}+b\right)\left(-\frac{a\sqrt{cx}}{b}\right)^{\frac{1}{2}-2m}\left(3\left(a\sqrt{cx}+b\right){}_2F_1\left(\frac{5}{2}, \frac{1}{2}-2m; \frac{7}{2}; \frac{\sqrt{cx}\,a}{b}+1\right)-5b\,{}_2F_1\left(\frac{3}{2}, \frac{1}{2}-2m; \frac{5}{2}\right.\right.}{15a^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*Sqrt[a + b/Sqrt[c*x]],x]
```

```
[Out] (4*(d*x)^m*(-((a*Sqrt[c*x])/b))^(1/2 - 2*m)*Sqrt[a + b/Sqrt[c*x]]*(b + a*Sq
rt[c*x])*(-5*b*Hypergeometric2F1[3/2, 1/2 - 2*m, 5/2, 1 + (a*Sqrt[c*x])/b]
+ 3*(b + a*Sqrt[c*x])*Hypergeometric2F1[5/2, 1/2 - 2*m, 7/2, 1 + (a*Sqrt[c*
x])/b]))/(15*a^2*c)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b/(c*x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   alglo
gextint: unimplemented
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b/(c*x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, integration of abs or sign assumes constant sign by intervals (cor
rect if the argument is real):Check [abs(t_nostep)]Unable to divide, perhap
s due to rounding error%%%{1,[0,1,1,0]%%%} / %%%{1,[0,0,0,1]%%%} Error: Bad
 Argument Value
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \sqrt{a + \frac{b}{\sqrt{cx}}}\ (dx)^m\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b/(c*x)^(1/2))^(1/2),x)
```

```
[Out] int((d*x)^m*(a+b/(c*x)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{a + \frac{b}{\sqrt{cx}}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b/(c*x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m*sqrt(a + b/sqrt(c*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a + \frac{b}{\sqrt{c\,x}}}\ (d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c*x)^(1/2))^(1/2)*(d*x)^m,x)
```

```
[Out] int((a + b/(c*x)^(1/2))^(1/2)*(d*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{a + \frac{b}{\sqrt{cx}}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b/(c*x)**(1/2))**(1/2),x)
```

```
[Out] Integral((d*x)**m*sqrt(a + b/sqrt(c*x)), x)
```

3.2922 $\int (dx)^m \sqrt{a + \frac{b}{(cx)^{3/2}}}\, dx$

Optimal. Leaf size=78

$$\frac{x(dx)^m \sqrt{a + \frac{b}{(cx)^{3/2}}}\ {}_2F_1\left(-\frac{1}{2}, -\frac{2}{3}(m+1); \frac{1}{3}(1-2m); -\frac{b}{a(cx)^{3/2}}\right)}{(m+1)\sqrt{\frac{b}{a(cx)^{3/2}} + 1}}$$

```
[Out] x*(d*x)^m*hypergeom([-1/2, -2/3-2/3*m],[1/3-2/3*m],-b/a/(c*x)^(3/2))*(a+b/(
c*x)^(3/2))^(1/2)/(1+m)/(1+b/a/(c*x)^(3/2))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {367, 343, 341, 339, 365, 364}

$$\frac{x(dx)^m \sqrt{a + \frac{b}{(cx)^{3/2}}}\ {}_2F_1\left(-\frac{1}{2}, -\frac{2}{3}(m+1); \frac{1}{3}(1-2m); -\frac{b}{a(cx)^{3/2}}\right)}{(m+1)\sqrt{\frac{b}{a(cx)^{3/2}} + 1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[a + b/(c*x)^(3/2)],x]
```

```
[Out] (x*(d*x)^m*Sqrt[a + b/(c*x)^(3/2)]*Hypergeometric2F1[-1/2, (-2*(1 + m))/3,
(1 - 2*m)/3, -(b/(a*(c*x)^(3/2)))])/((1 + m)*Sqrt[1 + b/(a*(c*x)^(3/2))])
```

Rule 339

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Dist[((c
*x)^(m + 1)*(1/x)^(m + 1))/c, Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x
], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 343

```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(c^In
tPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a + b*x^n)^p, x], x] /;
 FreeQ[{a, b, c, m, p}, x] && FractionQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
```

```
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int (dx)^m \sqrt{a+\frac{b}{(cx)^{3/2}}}\,dx &= \frac{\text{Subst}\left(\int \sqrt{a+\frac{b}{x^{3/2}}}\left(\frac{dx}{c}\right)^m dx, x, cx\right)}{c} \\
&= \frac{((cx)^{-m}(dx)^m)\,\text{Subst}\left(\int \sqrt{a+\frac{b}{x^{3/2}}}\,x^m\,dx, x, cx\right)}{c} \\
&= \frac{(2(cx)^{-m}(dx)^m)\,\text{Subst}\left(\int \sqrt{a+\frac{b}{x^3}}\,x^{-1+2(1+m)}\,dx, x, \sqrt{cx}\right)}{c} \\
&= -\frac{(2(cx)^{-m}(dx)^m)\,\text{Subst}\left(\int x^{-1-2(1+m)}\sqrt{a+bx^3}\,dx, x, \frac{1}{\sqrt{cx}}\right)}{c} \\
&= -\frac{\left(2(cx)^{-m}(dx)^m\sqrt{a+\frac{b}{(cx)^{3/2}}}\right)\text{Subst}\left(\int x^{-1-2(1+m)}\sqrt{1+\frac{bx^3}{a}}\,dx, x, \frac{1}{\sqrt{cx}}\right)}{c\sqrt{1+\frac{b}{a(cx)^{3/2}}}} \\
&= \frac{x(dx)^m\sqrt{a+\frac{b}{(cx)^{3/2}}}\;{}_2F_1\left(-\frac{1}{2},-\frac{2}{3}(1+m);\frac{1}{3}(1-2m);-\frac{b}{a(cx)^{3/2}}\right)}{(1+m)\sqrt{1+\frac{b}{a(cx)^{3/2}}}}
\end{aligned}$$

Mathematica [A] time = 0.16, size = 84, normalized size = 1.08

$$\frac{4x(dx)^m\sqrt{a+\frac{b}{(cx)^{3/2}}}\;{}_2F_1\left(-\frac{1}{2},\frac{1}{6}(4m+1);\frac{1}{6}(4m+7);-\frac{a(cx)^{3/2}}{b}\right)}{(4m+1)\sqrt{\frac{a(cx)^{3/2}+b}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*Sqrt[a + b/(c*x)^(3/2)],x]
```

```
[Out] (4*x*(d*x)^m*Sqrt[a + b/(c*x)^(3/2)]*Hypergeometric2F1[-1/2, (1 + 4*m)/6, (
7 + 4*m)/6, -((a*(c*x)^(3/2))/b)])/((1 + 4*m)*Sqrt[(b + a*(c*x)^(3/2))/b])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b/(c*x)^(3/2))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   alglo
gextint: unimplemented
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{a + \frac{b}{(cx)^{\frac{3}{2}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b/(c*x)^(3/2))^(1/2),x, algorithm="giac")
[Out] integrate((d*x)^m*sqrt(a + b/(c*x)^(3/2)), x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \sqrt{a + \frac{b}{(cx)^{\frac{3}{2}}}}\, (dx)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b/(c*x)^(3/2))^(1/2),x)
[Out] int((d*x)^m*(a+b/(c*x)^(3/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{a + \frac{b}{(cx)^{\frac{3}{2}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b/(c*x)^(3/2))^(1/2),x, algorithm="maxima")
[Out] integrate((d*x)^m*sqrt(a + b/(c*x)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a + \frac{b}{(c\,x)^{3/2}}}\, (d\,x)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c*x)^(3/2))^(1/2)*(d*x)^m,x)
[Out] int((a + b/(c*x)^(3/2))^(1/2)*(d*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{a + \frac{b}{(cx)^{\frac{3}{2}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b/(c*x)**(3/2))**(1/2),x)
[Out] Integral((d*x)**m*sqrt(a + b/(c*x)**(3/2)), x)
```

3.2923 $\int (dx)^m (a + b(cx)^n)^p \, dx$

Optimal. Leaf size=73

$$\frac{(dx)^{m+1} (a + b(cx)^n)^p \left(\frac{b(cx)^n}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+1}{n}, -p; \frac{m+n+1}{n}; -\frac{b(cx)^n}{a}\right)}{d(m+1)}$$

```
[Out] (d*x)^(1+m)*(a+b*(c*x)^n)^p*hypergeom([-p, (1+m)/n],[(1+m+n)/n],-b*(c*x)^n/
a)/d/(1+m)/((1+b*(c*x)^n/a)^p)
```

Rubi [A] time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {367, 365, 364}

$$\frac{(dx)^{m+1} (a + b(cx)^n)^p \left(\frac{b(cx)^n}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+1}{n}, -p; \frac{m+n+1}{n}; -\frac{b(cx)^n}{a}\right)}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*(c*x)^n)^p,x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*(c*x)^n)^p*Hypergeometric2F1[(1 + m)/n, -p, (1 + m +
n)/n, -((b*(c*x)^n)/a)])/(d*(1 + m)*(1 + (b*(c*x)^n)/a)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int (dx)^m (a + b(cx)^n)^p \, dx &= \frac{\text{Subst}\left(\int \left(\frac{dx}{c}\right)^m (a + bx^n)^p \, dx, x, cx\right)}{c} \\
&= \frac{\left((a + b(cx)^n)^p \left(1 + \frac{b(cx)^n}{a}\right)^{-p}\right) \text{Subst}\left(\int \left(\frac{dx}{c}\right)^m \left(1 + \frac{bx^n}{a}\right)^p \, dx, x, cx\right)}{c} \\
&= \frac{(dx)^{1+m} (a + b(cx)^n)^p \left(1 + \frac{b(cx)^n}{a}\right)^{-p} {}_2F_1\left(\frac{1+m}{n}, -p; \frac{1+m+n}{n}; -\frac{b(cx)^n}{a}\right)}{d(1+m)}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 70, normalized size = 0.96

$$\frac{x(dx)^m (a+b(cx)^n)^p \left(\frac{b(cx)^n}{a}+1\right)^{-p} {}_2F_1\left(\frac{m+1}{n}, -p; \frac{m+1}{n}+1; -\frac{b(cx)^n}{a}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*(c*x)^n)^p,x]
```

```
[Out] (x*(d*x)^m*(a + b*(c*x)^n)^p*Hypergeometric2F1[(1 + m)/n, -p, 1 + (1 + m)/n
, -((b*(c*x)^n)/a)])/((1 + m)*(1 + (b*(c*x)^n)/a)^p)
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\left((cx)^n b + a\right)^p (dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*(c*x)^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*x)^n*b + a)^p*(d*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left((cx)^n b + a\right)^p (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*(c*x)^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*x)^n*b + a)^p*(d*x)^m, x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int (dx)^m \left(b\,(cx)^n + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*(c*x)^n)^p,x)
```

```
[Out] int((d*x)^m*(a+b*(c*x)^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left((cx)^n b + a\right)^p (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*(c*x)^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*x)^n*b + a)^p*(d*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m \left(a + b\,(c\,x)^n\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*(c*x)^n)^p,x)
```

```
[Out] int((d*x)^m*(a + b*(c*x)^n)^p, x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a + b\,(cx)^n\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*(c*x)**n)**p,x)
```

```
[Out] Integral((d*x)**m*(a + b*(c*x)**n)**p, x)
```

3.2924 $\int x^2 (a + b(cx)^n)^p \, dx$

Optimal. Leaf size=61

$$\frac{1}{3}x^3 (a + b(cx)^n)^p \left(\frac{b(cx)^n}{a} + 1\right)^{-p} {}_2F_1\left(\frac{3}{n}, -p; \frac{n+3}{n}; -\frac{b(cx)^n}{a}\right)$$

```
[Out] 1/3*x^3*(a+b*(c*x)^n)^p*hypergeom([-p, 3/n],[(3+n)/n],-b*(c*x)^n/a)/((1+b*(
c*x)^n/a)^p)
```

Rubi [A] time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {367, 12, 365, 364}

$$\frac{1}{3}x^3 (a + b(cx)^n)^p \left(\frac{b(cx)^n}{a} + 1\right)^{-p} {}_2F_1\left(\frac{3}{n}, -p; \frac{n+3}{n}; -\frac{b(cx)^n}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*(c*x)^n)^p,x]
```

```
[Out] (x^3*(a + b*(c*x)^n)^p*Hypergeometric2F1[3/n, -p, (3 + n)/n, -((b*(c*x)^n)/
a)])/(3*(1 + (b*(c*x)^n)/a)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int x^2 (a+b(cx)^n)^p \, dx &= \frac{\text{Subst}\left(\int \frac{x^2(a+bx^n)^p}{c^2} \, dx, x, cx\right)}{c} \\ &= \frac{\text{Subst}\left(\int x^2 (a+bx^n)^p \, dx, x, cx\right)}{c^3} \\ &= \frac{\left((a+b(cx)^n)^p \left(1+\frac{b(cx)^n}{a}\right)^{-p}\right) \text{Subst}\left(\int x^2 \left(1+\frac{bx^n}{a}\right)^p \, dx, x, cx\right)}{c^3} \\ &= \frac{1}{3}x^3 (a+b(cx)^n)^p \left(1+\frac{b(cx)^n}{a}\right)^{-p} {}_2F_1\left(\frac{3}{n}, -p; \frac{3+n}{n}; -\frac{b(cx)^n}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.06, size = 61, normalized size = 1.00

$$\frac{1}{3}x^3 (a+b(cx)^n)^p \left(\frac{b(cx)^n}{a}+1\right)^{-p} {}_2F_1\left(\frac{3}{n}, -p; 1+\frac{3}{n}; -\frac{b(cx)^n}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*(c*x)^n)^p,x]
```

```
[Out] (x^3*(a + b*(c*x)^n)^p*Hypergeometric2F1[3/n, -p, 1 + 3/n, -((b*(c*x)^n)/a)
])/(3*(1 + (b*(c*x)^n)/a)^p)
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\left((cx)^n b + a\right)^p x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(c*x)^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*x)^n*b + a)^p*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left((cx)^n b + a\right)^p x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(c*x)^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*x)^n*b + a)^p*x^2, x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int x^2 \left(b\,(cx)^n + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*(c*x)^n+a)^p,x)
```

```
[Out] int(x^2*(b*(c*x)^n+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left((cx)^n b + a\right)^p x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(c*x)^n)^p,x, algorithm="maxima")
[Out] integrate(((c*x)^n*b + a)^p*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^2 \left(a + b\,(c\,x)^n\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*(c*x)^n)^p,x)
[Out] int(x^2*(a + b*(c*x)^n)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + b\,(cx)^n\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*(c*x)**n)**p,x)
[Out] Integral(x**2*(a + b*(c*x)**n)**p, x)
```

3.2925 $\int x\,(a+b(cx)^n)^p\,dx$

Optimal. Leaf size=61

$$\frac{1}{2}x^2\,(a+b(cx)^n)^p\left(\frac{b(cx)^n}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{2}{n},-p;\frac{n+2}{n};-\frac{b(cx)^n}{a}\right)$$

```
[Out] 1/2*x^2*(a+b*(c*x)^n)^p*hypergeom([-p, 2/n],[(2+n)/n],-b*(c*x)^n/a)/((1+b*(
c*x)^n/a)^p)
```

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {367, 12, 365, 364}

$$\frac{1}{2}x^2\,(a+b(cx)^n)^p\left(\frac{b(cx)^n}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{2}{n},-p;\frac{n+2}{n};-\frac{b(cx)^n}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*(c*x)^n)^p,x]
```

```
[Out] (x^2*(a + b*(c*x)^n)^p*Hypergeometric2F1[2/n, -p, (2 + n)/n, -((b*(c*x)^n)/
a)])/(2*(1 + (b*(c*x)^n)/a)^p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int x\,(a+b(cx)^n)^p\,dx &= \frac{\text{Subst}\left(\int \frac{x(a+bx^n)^p}{c}\,dx, x, cx\right)}{c} \\ &= \frac{\text{Subst}\left(\int x\,(a+bx^n)^p\,dx, x, cx\right)}{c^2} \\ &= \frac{\left((a+b(cx)^n)^p\left(1+\frac{b(cx)^n}{a}\right)^{-p}\right)\text{Subst}\left(\int x\left(1+\frac{bx^n}{a}\right)^p dx, x, cx\right)}{c^2} \\ &= \frac{1}{2}x^2\,(a+b(cx)^n)^p\left(1+\frac{b(cx)^n}{a}\right)^{-p}\,{}_2F_1\left(\frac{2}{n}, -p; \frac{2+n}{n}; -\frac{b(cx)^n}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.04, size = 61, normalized size = 1.00

$$\frac{1}{2}x^2\,(a+b(cx)^n)^p\left(\frac{b(cx)^n}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{2}{n}, -p; 1+\frac{2}{n}; -\frac{b(cx)^n}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*(c*x)^n)^p,x]
```

```
[Out] (x^2*(a + b*(c*x)^n)^p*Hypergeometric2F1[2/n, -p, 1 + 2/n, -((b*(c*x)^n)/a)
])/(2*(1 + (b*(c*x)^n)/a)^p)
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\left((cx)^n b + a\right)^p x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(c*x)^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*x)^n*b + a)^p*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left((cx)^n b + a\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(c*x)^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*x)^n*b + a)^p*x, x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int x\left(b\,(cx)^n + a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*(c*x)^n+a)^p,x)
```

```
[Out] int(x*(b*(c*x)^n+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left((cx)^n b + a\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(c*x)^n)^p,x, algorithm="maxima")
[Out] integrate(((c*x)^n*b + a)^p*x, x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.02

$$\int x \left(a + b\,(c\,x)^n\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*(c*x)^n)^p,x)
[Out] int(x*(a + b*(c*x)^n)^p, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x \left(a + b\,(cx)^n\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(c*x)**n)**p,x)
[Out] Integral(x*(a + b*(c*x)**n)**p, x)
```

3.2926 $\int (a+b(cx)^n)^p\,dx$

Optimal. Leaf size=52

$$x\,(a+b(cx)^n)^p\left(\frac{b(cx)^n}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b(cx)^n}{a}\right)$$

```
[Out] x*(a+b*(c*x)^n)^p*hypergeom([-p, 1/n],[1+1/n],-b*(c*x)^n/a)/((1+b*(c*x)^n/a
)^p)
```

Rubi [A] time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {247, 246, 245}

$$x\,(a+b(cx)^n)^p\left(\frac{b(cx)^n}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b(cx)^n}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c*x)^n)^p,x]
```

```
[Out] (x*(a + b*(c*x)^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*(c*x)^n
)/a)])/(1 + (b*(c*x)^n)/a)^p
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int (a+b(cx)^n)^p\,dx &= \frac{\text{Subst}\left(\int (a+bx^n)^p\,dx,x,cx\right)}{c}\\ &= \frac{\left((a+b(cx)^n)^p\left(1+\frac{b(cx)^n}{a}\right)^{-p}\right)\text{Subst}\left(\int \left(1+\frac{bx^n}{a}\right)^p\,dx,x,cx\right)}{c}\\ &= x\,(a+b(cx)^n)^p\left(1+\frac{b(cx)^n}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b(cx)^n}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 52, normalized size = 1.00

$$x\,(a+b(cx)^n)^p\left(\frac{b(cx)^n}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b(cx)^n}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c*x)^n)^p,x]
```

```
[Out] (x*(a + b*(c*x)^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*(c*x)^n
)/a)])/(1 + (b*(c*x)^n)/a)^p
```

fricas **[F]** time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\left((cx)^n b + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*x)^n*b + a)^p, x)
```

giac **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left((cx)^n b + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*x)^n*b + a)^p, x)
```

maple **[F]** time = 0.26, size = 0, normalized size = 0.00

$$\int \left(b\,(cx)^n + a\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*x)^n+a)^p,x)
```

```
[Out] int((b*(c*x)^n+a)^p,x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left((cx)^n b + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*x)^n*b + a)^p, x)
```

mupad **[B]** time = 2.80, size = 53, normalized size = 1.02

$$\frac{x\left(a + b\,(c\,x)^n\right)^p\,{}_2\mathrm{F}_1\left(\frac{1}{n}, -p;\ \frac{1}{n}+1;\ -\frac{b\,(c\,x)^n}{a}\right)}{\left(\frac{b\,(c\,x)^n}{a}+1\right)^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x)^n)^p,x)
```

```
[Out] (x*(a + b*(c*x)^n)^p*hypergeom([1/n, -p], 1/n + 1, -(b*(c*x)^n)/a))/((b*(c*
x)^n)/a + 1)^p
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\,(cx)^n\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)**n)**p,x)
```

```
[Out] Integral((a + b*(c*x)**n)**p, x)
```

3.2927 $\int \frac{(a+b(cx)^n)^p}{x}\,dx$

Optimal. Leaf size=46

$$-\frac{(a+b(cx)^n)^{p+1}\ {}_2F_1\left(1,p+1;p+2;\frac{b(cx)^n}{a}+1\right)}{an(p+1)}$$

```
[Out] -(a+b*(c*x)^n)^(1+p)*hypergeom([1, 1+p],[2+p],1+b*(c*x)^n/a)/a/n/(1+p)
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {367, 12, 266, 65}

$$-\frac{(a+b(cx)^n)^{p+1}\ {}_2F_1\left(1,p+1;p+2;\frac{b(cx)^n}{a}+1\right)}{an(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c*x)^n)^p/x,x]
```

```
[Out] -(((a + b*(c*x)^n)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*(c*x)^
n)/a])/(a*n*(1 + p)))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b(cx)^n)^p}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{c(a+bx^n)^p}{x}\,dx, x, cx\right)}{c} \\ &= \text{Subst}\left(\int \frac{(a+bx^n)^p}{x}\,dx, x, cx\right) \\ &= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, (cx)^n\right)}{n} \\ &= -\frac{(a+b(cx)^n)^{1+p}\ {}_2F_1\left(1, 1+p; 2+p; 1+\frac{b(cx)^n}{a}\right)}{an(1+p)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 46, normalized size = 1.00

$$-\frac{(a+b(cx)^n)^{p+1}\ {}_2F_1\left(1, p+1; p+2; \frac{b(cx)^n}{a}+1\right)}{an(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c*x)^n)^p/x,x]
```

```
[Out] -(((a + b*(c*x)^n)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*(c*x)^
n)/a])/(a*n*(1 + p)))
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left((cx)^n b + a\right)^p}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^p/x,x, algorithm="fricas")
```

```
[Out] integral(((c*x)^n*b + a)^p/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((cx)^n b + a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^p/x,x, algorithm="giac")
```

```
[Out] integrate(((c*x)^n*b + a)^p/x, x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,(cx)^n + a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*x)^n+a)^p/x,x)
```

```
[Out] int((b*(c*x)^n+a)^p/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((cx)^n b + a\right)^p}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^p/x,x, algorithm="maxima")
[Out] integrate(((c*x)^n*b + a)^p/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(a + b\,(c\,x)^n\right)^p}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x)^n)^p/x,x)
[Out] int((a + b*(c*x)^n)^p/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\,(cx)^n\right)^p}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)**n)**p/x,x)
[Out] Integral((a + b*(c*x)**n)**p/x, x)
```

3.2928 $\int \frac{(a+b(cx)^n)^p}{x^2}\,dx$

Optimal. Leaf size=62

$$-\frac{(a+b(cx)^n)^p\left(\frac{b(cx)^n}{a}+1\right)^{-p}\,{}_2F_1\left(-\frac{1}{n},-p;-\frac{1-n}{n};-\frac{b(cx)^n}{a}\right)}{x}$$

```
[Out] -(a+b*(c*x)^n)^p*hypergeom([-p, -1/n],[(-1+n)/n],-b*(c*x)^n/a)/x/((1+b*(c*x
)^n/a)^p)
```

Rubi [A] time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {367, 12, 365, 364}

$$-\frac{(a+b(cx)^n)^p\left(\frac{b(cx)^n}{a}+1\right)^{-p}\,{}_2F_1\left(-\frac{1}{n},-p;-\frac{1-n}{n};-\frac{b(cx)^n}{a}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c*x)^n)^p/x^2,x]
```

```
[Out] -(((a + b*(c*x)^n)^p*Hypergeometric2F1[-n^(-1), -p, -((1 - n)/n), -((b*(c*x
)^n)/a)])/(x*(1 + (b*(c*x)^n)/a)^p))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b(cx)^n)^p}{x^2}\,dx &= \frac{\text{Subst}\left(\int \frac{c^2(a+bx^n)^p}{x^2}\,dx, x, cx\right)}{c} \\ &= c\,\text{Subst}\left(\int \frac{(a+bx^n)^p}{x^2}\,dx, x, cx\right) \\ &= \left(c\,(a+b(cx)^n)^p\left(1+\frac{b(cx)^n}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^n}{a}\right)^p}{x^2}\,dx, x, cx\right) \\ &= -\frac{(a+b(cx)^n)^p\left(1+\frac{b(cx)^n}{a}\right)^{-p}\,{}_2F_1\left(-\frac{1}{n}, -p; -\frac{1-n}{n}; -\frac{b(cx)^n}{a}\right)}{x}\end{aligned}$$

Mathematica [A] time = 0.05, size = 59, normalized size = 0.95

$$-\frac{(a+b(cx)^n)^p\left(\frac{b(cx)^n}{a}+1\right)^{-p}\,{}_2F_1\left(-\frac{1}{n}, -p; 1-\frac{1}{n}; -\frac{b(cx)^n}{a}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c*x)^n)^p/x^2,x]
```

```
[Out] -(((a + b*(c*x)^n)^p*Hypergeometric2F1[-n^(-1), -p, 1 - n^(-1), -((b*(c*x)^
n)/a)])/(x*(1 + (b*(c*x)^n)/a)^p))
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left((cx)^n b + a\right)^p}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^p/x^2,x, algorithm="fricas")
```

```
[Out] integral(((c*x)^n*b + a)^p/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((cx)^n b + a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^p/x^2,x, algorithm="giac")
```

```
[Out] integrate(((c*x)^n*b + a)^p/x^2, x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,(cx)^n + a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*x)^n+a)^p/x^2,x)
```

```
[Out] int((b*(c*x)^n+a)^p/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((cx)^n b + a\right)^p}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^p/x^2,x, algorithm="maxima")
[Out] integrate(((c*x)^n*b + a)^p/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(a + b\,(c\,x)^n\right)^p}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x)^n)^p/x^2,x)
[Out] int((a + b*(c*x)^n)^p/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\,(cx)^n\right)^p}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)**n)**p/x**2,x)
[Out] Integral((a + b*(c*x)**n)**p/x**2, x)
```

3.2929 $\int \frac{1}{1+\left(x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=83

$$-\frac{x\log\left(x^2-\sqrt{x^2}+1\right)}{6\sqrt{x^2}}+\frac{x\log\left(\sqrt{x^2}+1\right)}{3\sqrt{x^2}}-\frac{x\tan^{-1}\left(\frac{1-2\sqrt{x^2}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{x^2}}$$

```
[Out] -1/6*x*ln(1+x^2-(x^2)^(1/2))/(x^2)^(1/2)+1/3*x*ln(1+(x^2)^(1/2))/(x^2)^(1/2
)-1/3*x*arctan(1/3*(1-2*(x^2)^(1/2))*3^(1/2))*3^(1/2)/(x^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {254, 200, 31, 634, 618, 204, 628}

$$-\frac{x\log\left(x^2-\sqrt{x^2}+1\right)}{6\sqrt{x^2}}+\frac{x\log\left(\sqrt{x^2}+1\right)}{3\sqrt{x^2}}-\frac{x\tan^{-1}\left(\frac{1-2\sqrt{x^2}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + (x^2)^(3/2))^(-1),x]
```

```
[Out] -((x*ArcTan[(1 - 2*Sqrt[x^2])/Sqrt[3]])/(Sqrt[3]*Sqrt[x^2])) - (x*Log[1 + x
^2 - Sqrt[x^2]])/(6*Sqrt[x^2]) + (x*Log[1 + Sqrt[x^2]])/(3*Sqrt[x^2])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 254

```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_.), x_Symbol] :> Dist[x/(c*x^
q)^(1/q), Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1+\left(x^2\right)^{3/2}}\,dx &= \frac{x\,\text{Subst}\left(\int \frac{1}{1+x^3}\,dx, x, \sqrt{x^2}\right)}{\sqrt{x^2}} \\
&= \frac{x\,\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt{x^2}\right)}{3\sqrt{x^2}} + \frac{x\,\text{Subst}\left(\int \frac{2-x}{1-x+x^2}\,dx, x, \sqrt{x^2}\right)}{3\sqrt{x^2}} \\
&= \frac{x\log\left(1+\sqrt{x^2}\right)}{3\sqrt{x^2}} - \frac{x\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \sqrt{x^2}\right)}{6\sqrt{x^2}} + \frac{x\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \sqrt{x^2}\right)}{2\sqrt{x^2}} \\
&= -\frac{x\log\left(1+x^2-\sqrt{x^2}\right)}{6\sqrt{x^2}} + \frac{x\log\left(1+\sqrt{x^2}\right)}{3\sqrt{x^2}} - \frac{x\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\sqrt{x^2}\right)}{\sqrt{x^2}} \\
&= -\frac{x\tan^{-1}\left(\frac{1-2\sqrt{x^2}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt{x^2}} - \frac{x\log\left(1+x^2-\sqrt{x^2}\right)}{6\sqrt{x^2}} + \frac{x\log\left(1+\sqrt{x^2}\right)}{3\sqrt{x^2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 67, normalized size = 0.81

$$\frac{x\left(-\log\left(x^2-\sqrt{x^2}+1\right)+2\log\left(\sqrt{x^2}+1\right)+2\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt{x^2}-1}{\sqrt{3}}\right)\right)}{6\sqrt{x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + (x^2)^(3/2))^(-1),x]
```

```
[Out] (x*(2*Sqrt[3]*ArcTan[(-1 + 2*Sqrt[x^2])/Sqrt[3]] - Log[1 + x^2 - Sqrt[x^2]]
 + 2*Log[1 + Sqrt[x^2]]))/(6*Sqrt[x^2])
```

fricas [A] time = 0.92, size = 48, normalized size = 0.58

$$\frac{1}{3}\sqrt{3}\,\arctan\left(\frac{2}{3}\sqrt{3}\,\sqrt{x^2}-\frac{1}{3}\sqrt{3}\right)-\frac{1}{6}\log\left(x^2-\sqrt{x^2}+1\right)+\frac{1}{3}\log\left(\sqrt{x^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(x^2)^(3/2)),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(2/3*sqrt(3)*sqrt(x^2) - 1/3*sqrt(3)) - 1/6*log(x^2 - sqr
t(x^2) + 1) + 1/3*log(sqrt(x^2) + 1)
```

giac [C] time = 0.20, size = 108, normalized size = 1.30

$$-\frac{\sqrt{3}\left(-i\sqrt{3}-1\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{1}{\operatorname{sgn}(x)}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{1}{\operatorname{sgn}(x)}\right)^{\frac{1}{3}}}\right)}{6\operatorname{sgn}(x)^{\frac{1}{3}}}-\frac{1}{9}i\pi\operatorname{sgn}(x)-\frac{\left(-i\sqrt{3}-1\right)\log\left(x^2+x\left(-\frac{1}{\operatorname{sgn}(x)}\right)^{\frac{1}{3}}+\left(-\frac{1}{\operatorname{sgn}(x)}\right)^{\frac{2}{3}}\right)}{12\operatorname{sgn}(x)^{\frac{1}{3}}}-\frac{1}{3}\left($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(x^2)^(3/2)),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*(-I*sqrt(3) - 1)*arctan(1/3*sqrt(3)*(2*x + (-1/sgn(x))^(1/3))/
(-1/sgn(x))^(1/3))/sgn(x)^(1/3) - 1/9*I*pi*sgn(x) - 1/12*(-I*sqrt(3) - 1)*l
og(x^2 + x*(-1/sgn(x))^(1/3) + (-1/sgn(x))^(2/3))/sgn(x)^(1/3) - 1/3*(-1/sg
n(x))^(1/3)*log(abs(x - (-1/sgn(x))^(1/3)))
```

maple [A] time = 0.01, size = 108, normalized size = 1.30

$$\frac{\left(-2\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{x^3}{\left(x^2\right)^{\frac{3}{2}}}\right)^{\frac{1}{3}}\right)}{3\left(\frac{x^3}{\left(x^2\right)^{\frac{3}{2}}}\right)^{\frac{1}{3}}}\right)+2\ln\left(x+\left(\frac{x^3}{\left(x^2\right)^{\frac{3}{2}}}\right)^{\frac{1}{3}}\right)-\ln\left(x^2-\left(\frac{x^3}{\left(x^2\right)^{\frac{3}{2}}}\right)^{\frac{1}{3}}x+\left(\frac{x^3}{\left(x^2\right)^{\frac{3}{2}}}\right)^{\frac{2}{3}}\right)\right)x^3}{6\left(x^2\right)^{\frac{3}{2}}\left(\frac{x^3}{\left(x^2\right)^{\frac{3}{2}}}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+(x^2)^(3/2)),x)
```

```
[Out] 1/6*x^3*(-2*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(1/(x^2)^(3/2)*x^3)^(1/3))/(1/
(x^2)^(3/2)*x^3)^(1/3))+2*ln(x+(1/(x^2)^(3/2)*x^3)^(1/3))-ln(x^2-x*(1/(x^2)
^(3/2)*x^3)^(1/3)+(1/(x^2)^(3/2)*x^3)^(2/3)))/(x^2)^(3/2)/(1/(x^2)^(3/2)*x^
3)^(2/3)
```

maxima [A] time = 1.24, size = 34, normalized size = 0.41

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x-1\right)\right)-\frac{1}{6}\log\left(x^2-x+1\right)+\frac{1}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(x^2)^(3/2)),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*log(x^2 - x + 1) + 1/3*log(
x + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^2\right)^{3/2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2)^(3/2) + 1),x)
```

```
[Out] int(1/((x^2)^(3/2) + 1), x)
```

sympy [**A**] time = 0.19, size = 41, normalized size = 0.49

$$\frac{\log(x+1)}{3} - \frac{\log\left(x^2-x+1\right)}{6} + \frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(x**2)**(3/2)),x)
```

```
[Out] log(x + 1)/3 - log(x**2 - x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3
)/3
```

3.2930 $\int x^5\sqrt{a+b\sqrt{cx^2}}\,dx$

Optimal. Leaf size=174

$$-\frac{2a^5\left(a+b\sqrt{cx^2}\right)^{3/2}}{3b^6c^3}+\frac{2a^4\left(a+b\sqrt{cx^2}\right)^{5/2}}{b^6c^3}-\frac{20a^3\left(a+b\sqrt{cx^2}\right)^{7/2}}{7b^6c^3}+\frac{20a^2\left(a+b\sqrt{cx^2}\right)^{9/2}}{9b^6c^3}+\frac{2\left(a+b\sqrt{cx^2}\right)^{13/2}}{13b^6c^3}-\frac{10a}{}$$

```
[Out] -2/3*a^5*(a+b*(c*x^2)^(1/2))^(3/2)/b^6/c^3+2*a^4*(a+b*(c*x^2)^(1/2))^(5/2)/
b^6/c^3-20/7*a^3*(a+b*(c*x^2)^(1/2))^(7/2)/b^6/c^3+20/9*a^2*(a+b*(c*x^2)^(1
/2))^(9/2)/b^6/c^3-10/11*a*(a+b*(c*x^2)^(1/2))^(11/2)/b^6/c^3+2/13*(a+b*(c*
x^2)^(1/2))^(13/2)/b^6/c^3
```

Rubi [A] time = 0.08, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {368, 43}

$$\frac{20a^2\left(a+b\sqrt{cx^2}\right)^{9/2}}{9b^6c^3}-\frac{20a^3\left(a+b\sqrt{cx^2}\right)^{7/2}}{7b^6c^3}+\frac{2a^4\left(a+b\sqrt{cx^2}\right)^{5/2}}{b^6c^3}-\frac{2a^5\left(a+b\sqrt{cx^2}\right)^{3/2}}{3b^6c^3}+\frac{2\left(a+b\sqrt{cx^2}\right)^{13/2}}{13b^6c^3}-\frac{10a\left(}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[a + b*Sqrt[c*x^2]],x]
```

```
[Out] (-2*a^5*(a + b*Sqrt[c*x^2])^(3/2))/(3*b^6*c^3) + (2*a^4*(a + b*Sqrt[c*x^2])
^(5/2))/(b^6*c^3) - (20*a^3*(a + b*Sqrt[c*x^2])^(7/2))/(7*b^6*c^3) + (20*a^
2*(a + b*Sqrt[c*x^2])^(9/2))/(9*b^6*c^3) - (10*a*(a + b*Sqrt[c*x^2])^(11/2)
)/(11*b^6*c^3) + (2*(a + b*Sqrt[c*x^2])^(13/2))/(13*b^6*c^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\int x^5\sqrt{a+b\sqrt{cx^2}}\,dx = \frac{\text{Subst}\left(\int x^5\sqrt{a+bx}\,dx, x, \sqrt{cx^2}\right)}{c^3}$$

$$= \frac{\text{Subst}\left(\int\left(-\frac{a^5\sqrt{a+bx}}{b^5}+\frac{5a^4(a+bx)^{3/2}}{b^5}-\frac{10a^3(a+bx)^{5/2}}{b^5}+\frac{10a^2(a+bx)^{7/2}}{b^5}-\frac{5a(a+bx)^{9/2}}{b^5}+\frac{(a+bx)^{11/2}}{b^5}\right)dx\right.}{c^3}$$

$$= -\frac{2a^5\left(a+b\sqrt{cx^2}\right)^{3/2}}{3b^6c^3}+\frac{2a^4\left(a+b\sqrt{cx^2}\right)^{5/2}}{b^6c^3}-\frac{20a^3\left(a+b\sqrt{cx^2}\right)^{7/2}}{7b^6c^3}+\frac{20a^2\left(a+b\sqrt{cx^2}\right)^{9/2}}{9b^6c^3}$$

Mathematica [A] time = 0.09, size = 103, normalized size = 0.59

$$\frac{2\left(a+b\sqrt{cx^2}\right)^{3/2}\left(-256a^5+384a^4b\sqrt{cx^2}-480a^3b^2cx^2+560a^2b^3\left(cx^2\right)^{3/2}-630ab^4c^2x^4+693b^5\left(cx^2\right)^{5/2}\right)}{9009b^6c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[a + b*Sqrt[c*x^2]],x]
```

```
[Out] (2*(a + b*Sqrt[c*x^2])^(3/2)*(-256*a^5 - 480*a^3*b^2*c*x^2 - 630*a*b^4*c^2*
x^4 + 384*a^4*b*Sqrt[c*x^2] + 560*a^2*b^3*(c*x^2)^(3/2) + 693*b^5*(c*x^2)^(
5/2)))/(9009*b^6*c^3)
```

fricas [A] time = 0.83, size = 103, normalized size = 0.59

$$\frac{2\left(693\,b^6c^3x^6-70\,a^2b^4c^2x^4-96\,a^4b^2cx^2-256\,a^6+\left(63\,ab^5c^2x^4+80\,a^3b^3cx^2+128\,a^5b\right)\sqrt{cx^2}\right)\sqrt{\sqrt{cx^2}\,b+a}}{9009\,b^6c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9009*(693*b^6*c^3*x^6 - 70*a^2*b^4*c^2*x^4 - 96*a^4*b^2*c*x^2 - 256*a^6 +
 (63*a*b^5*c^2*x^4 + 80*a^3*b^3*c*x^2 + 128*a^5*b)*sqrt(c*x^2))*sqrt(sqrt(c
*x^2)*b + a)/(b^6*c^3)
```

giac [A] time = 0.36, size = 231, normalized size = 1.33

$$2\left(\frac{13\left(63\left(b\sqrt{c}\,x+a\right)^{\frac{11}{2}}\sqrt{c}-385\left(b\sqrt{c}\,x+a\right)^{\frac{9}{2}}a\sqrt{c}+990\left(b\sqrt{c}\,x+a\right)^{\frac{7}{2}}a^2\sqrt{c}-1386\left(b\sqrt{c}\,x+a\right)^{\frac{5}{2}}a^3\sqrt{c}+1155\left(b\sqrt{c}\,x+a\right)^{\frac{3}{2}}a^4\sqrt{c}-693\sqrt{b\sqrt{c}\,x+a}\,a^5\sqrt{c}\right)}{b^5c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 2/9009*(13*(63*(b*sqrt(c)*x + a)^(11/2)*sqrt(c) - 385*(b*sqrt(c)*x + a)^(9/
2)*a*sqrt(c) + 990*(b*sqrt(c)*x + a)^(7/2)*a^2*sqrt(c) - 1386*(b*sqrt(c)*x
 + a)^(5/2)*a^3*sqrt(c) + 1155*(b*sqrt(c)*x + a)^(3/2)*a^4*sqrt(c) - 693*sqr
t(b*sqrt(c)*x + a)*a^5*sqrt(c))*a/(b^5*c^3) + 3*(231*(b*sqrt(c)*x + a)^(13/
2) - 1638*(b*sqrt(c)*x + a)^(11/2)*a + 5005*(b*sqrt(c)*x + a)^(9/2)*a^2 - 8
580*(b*sqrt(c)*x + a)^(7/2)*a^3 + 9009*(b*sqrt(c)*x + a)^(5/2)*a^4 - 6006*(
b*sqrt(c)*x + a)^(3/2)*a^5 + 3003*sqrt(b*sqrt(c)*x + a)*a^6)/(b^5*c^(5/2)))
/(b*sqrt(c))
```

maple [A] time = 0.01, size = 92, normalized size = 0.53

$$\frac{2\left(a+\sqrt{c\,x^2}\,b\right)^{\frac{3}{2}}\left(-630a\,b^4c^2x^4-480a^3b^2c\,x^2-256a^5+384\sqrt{c\,x^2}\,a^4b+560\left(c\,x^2\right)^{\frac{3}{2}}a^2b^3+693\left(c\,x^2\right)^{\frac{5}{2}}b^5\right)}{9009b^6c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*(c*x^2)^(1/2))^(1/2),x)
```

```
[Out] 2/9009*(a+b*(c*x^2)^(1/2))^(3/2)*(693*(c*x^2)^(5/2)*b^5-630*c^2*x^4*a*b^4+5
60*(c*x^2)^(3/2)*a^2*b^3-480*c*x^2*a^3*b^2+384*(c*x^2)^(1/2)*a^4*b-256*a^5)
/c^3/b^6
```

maxima [A] time = 0.56, size = 127, normalized size = 0.73

$$2\left(\frac{693\left(\sqrt{cx^2}\,b+a\right)^{\frac{13}{2}}}{b^6}-\frac{4095\left(\sqrt{cx^2}\,b+a\right)^{\frac{11}{2}}a}{b^6}+\frac{10010\left(\sqrt{cx^2}\,b+a\right)^{\frac{9}{2}}a^2}{b^6}-\frac{12870\left(\sqrt{cx^2}\,b+a\right)^{\frac{7}{2}}a^3}{b^6}+\frac{9009\left(\sqrt{cx^2}\,b+a\right)^{\frac{5}{2}}a^4}{b^6}-\frac{3003\left(\sqrt{cx^2}\,b+a\right)^{\frac{3}{2}}a^5}{b^6}\right.$$

$$9009\,c^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9009*(693*(sqrt(c*x^2)*b + a)^(13/2)/b^6 - 4095*(sqrt(c*x^2)*b + a)^(11/2
)*a/b^6 + 10010*(sqrt(c*x^2)*b + a)^(9/2)*a^2/b^6 - 12870*(sqrt(c*x^2)*b +
a)^(7/2)*a^3/b^6 + 9009*(sqrt(c*x^2)*b + a)^(5/2)*a^4/b^6 - 3003*(sqrt(c*x^
2)*b + a)^(3/2)*a^5/b^6)/c^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5\sqrt{a+b\sqrt{c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*(c*x^2)^(1/2))^(1/2),x)
```

```
[Out] int(x^5*(a + b*(c*x^2)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\sqrt{a+b\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*(c*x**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(x**5*sqrt(a + b*sqrt(c*x**2)), x)
```

3.2931 $\int x^3\sqrt{a+b\sqrt{cx^2}}\,dx$

Optimal. Leaf size=116

$$-\frac{2a^3\left(a+b\sqrt{cx^2}\right)^{3/2}}{3b^4c^2}+\frac{6a^2\left(a+b\sqrt{cx^2}\right)^{5/2}}{5b^4c^2}+\frac{2\left(a+b\sqrt{cx^2}\right)^{9/2}}{9b^4c^2}-\frac{6a\left(a+b\sqrt{cx^2}\right)^{7/2}}{7b^4c^2}$$

```
[Out] -2/3*a^3*(a+b*(c*x^2)^(1/2))^(3/2)/b^4/c^2+6/5*a^2*(a+b*(c*x^2)^(1/2))^(5/2
)/b^4/c^2-6/7*a*(a+b*(c*x^2)^(1/2))^(7/2)/b^4/c^2+2/9*(a+b*(c*x^2)^(1/2))^(
9/2)/b^4/c^2
```

Rubi [A] time = 0.05, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {368, 43}

$$\frac{6a^2\left(a+b\sqrt{cx^2}\right)^{5/2}}{5b^4c^2}-\frac{2a^3\left(a+b\sqrt{cx^2}\right)^{3/2}}{3b^4c^2}+\frac{2\left(a+b\sqrt{cx^2}\right)^{9/2}}{9b^4c^2}-\frac{6a\left(a+b\sqrt{cx^2}\right)^{7/2}}{7b^4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + b*Sqrt[c*x^2]],x]
```

```
[Out] (-2*a^3*(a + b*Sqrt[c*x^2])^(3/2))/(3*b^4*c^2) + (6*a^2*(a + b*Sqrt[c*x^2])
^(5/2))/(5*b^4*c^2) - (6*a*(a + b*Sqrt[c*x^2])^(7/2))/(7*b^4*c^2) + (2*(a +
 b*Sqrt[c*x^2])^(9/2))/(9*b^4*c^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{a+b\sqrt{cx^2}}\,dx &= \frac{\text{Subst}\left(\int x^3\sqrt{a+bx}\,dx, x, \sqrt{cx^2}\right)}{c^2}\\ &= \frac{\text{Subst}\left(\int\left(-\frac{a^3\sqrt{a+bx}}{b^3}+\frac{3a^2(a+bx)^{3/2}}{b^3}-\frac{3a(a+bx)^{5/2}}{b^3}+\frac{(a+bx)^{7/2}}{b^3}\right)dx, x, \sqrt{cx^2}\right)}{c^2}\\ &= -\frac{2a^3\left(a+b\sqrt{cx^2}\right)^{3/2}}{3b^4c^2}+\frac{6a^2\left(a+b\sqrt{cx^2}\right)^{5/2}}{5b^4c^2}-\frac{6a\left(a+b\sqrt{cx^2}\right)^{7/2}}{7b^4c^2}+\frac{2\left(a+b\sqrt{cx^2}\right)^{9/2}}{9b^4c^2}\end{aligned}$$

Mathematica [A] time = 0.06, size = 72, normalized size = 0.62

$$\frac{2\left(a+b\sqrt{cx^2}\right)^{3/2}\left(-16a^3+24a^2b\sqrt{cx^2}-30ab^2cx^2+35b^3\left(cx^2\right)^{3/2}\right)}{315b^4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + b*Sqrt[c*x^2]],x]
```

```
[Out] (2*(a + b*Sqrt[c*x^2])^(3/2)*(-16*a^3 - 30*a*b^2*c*x^2 + 24*a^2*b*Sqrt[c*x^
2] + 35*b^3*(c*x^2)^(3/2)))/(315*b^4*c^2)
```

fricas [A] time = 0.93, size = 75, normalized size = 0.65

$$\frac{2\left(35\,b^4c^2x^4-6\,a^2b^2cx^2-16\,a^4+\left(5\,ab^3cx^2+8\,a^3b\right)\sqrt{cx^2}\right)\sqrt{\sqrt{cx^2}\,b+a}}{315\,b^4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*c^2*x^4 - 6*a^2*b^2*c*x^2 - 16*a^4 + (5*a*b^3*c*x^2 + 8*a^3*b
)*sqrt(c*x^2))*sqrt(sqrt(c*x^2)*b + a)/(b^4*c^2)
```

giac [A] time = 0.17, size = 164, normalized size = 1.41

$$2\left(\frac{9\left(5\left(b\sqrt{c}\,x+a\right)^{\frac{7}{2}}\sqrt{c}-21\left(b\sqrt{c}\,x+a\right)^{\frac{5}{2}}a\sqrt{c}+35\left(b\sqrt{c}\,x+a\right)^{\frac{3}{2}}a^2\sqrt{c}-35\sqrt{b\sqrt{c}\,x+a}\,a^3\sqrt{c}\right)a}{b^3c^2}+\frac{35\left(b\sqrt{c}\,x+a\right)^{\frac{9}{2}}-180\left(b\sqrt{c}\,x+a\right)^{\frac{7}{2}}a+378\left(b\sqrt{c}\,x+a\right)^{\frac{5}{2}}a^2-}{b^3c^{\frac{3}{2}}}\right)$$

$$315\,b\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(9*(5*(b*sqrt(c)*x + a)^(7/2)*sqrt(c) - 21*(b*sqrt(c)*x + a)^(5/2)*a*
sqrt(c) + 35*(b*sqrt(c)*x + a)^(3/2)*a^2*sqrt(c) - 35*sqrt(b*sqrt(c)*x + a)
*a^3*sqrt(c))*a/(b^3*c^2) + (35*(b*sqrt(c)*x + a)^(9/2) - 180*(b*sqrt(c)*x
+ a)^(7/2)*a + 378*(b*sqrt(c)*x + a)^(5/2)*a^2 - 420*(b*sqrt(c)*x + a)^(3/2
)*a^3 + 315*sqrt(b*sqrt(c)*x + a)*a^4)/(b^3*c^(3/2)))/(b*sqrt(c))
```

maple [A] time = 0.00, size = 63, normalized size = 0.54

$$\frac{2\left(a+\sqrt{c\,x^2}\;b\right)^{\frac{3}{2}}\left(-30a\,b^2c\,x^2-16a^3+24\sqrt{c\,x^2}\;a^2b+35\left(c\,x^2\right)^{\frac{3}{2}}b^3\right)}{315b^4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+(c*x^2)^(1/2)*b)^(1/2),x)
```

```
[Out] 2/315*(a+(c*x^2)^(1/2)*b)^(3/2)*(35*(c*x^2)^(3/2)*b^3-30*c*x^2*a*b^2+24*(c*
x^2)^(1/2)*a^2*b-16*a^3)/c^2/b^4
```

maxima [A] time = 0.63, size = 85, normalized size = 0.73

$$\frac{2\left(\frac{35\left(\sqrt{cx^2}\,b+a\right)^{\frac{9}{2}}}{b^4}-\frac{135\left(\sqrt{cx^2}\,b+a\right)^{\frac{7}{2}}a}{b^4}+\frac{189\left(\sqrt{cx^2}\,b+a\right)^{\frac{5}{2}}a^2}{b^4}-\frac{105\left(\sqrt{cx^2}\,b+a\right)^{\frac{3}{2}}a^3}{b^4}\right)}{315\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*(sqrt(c*x^2)*b + a)^(9/2)/b^4 - 135*(sqrt(c*x^2)*b + a)^(7/2)*a/b
^4 + 189*(sqrt(c*x^2)*b + a)^(5/2)*a^2/b^4 - 105*(sqrt(c*x^2)*b + a)^(3/2)*
a^3/b^4)/c^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \sqrt{a + b \sqrt{c\, x^2}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*(c*x^2)^(1/2))^(1/2),x)
```

```
[Out] int(x^3*(a + b*(c*x^2)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sqrt{a + b\sqrt{cx^2}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*(c*x**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a + b*sqrt(c*x**2)), x)
```

3.2932 $\int x\sqrt{a+b\sqrt{cx^2}}\,dx$

Optimal. Leaf size=56

$$\frac{2\left(a+b\sqrt{cx^2}\right)^{5/2}}{5b^2c}-\frac{2a\left(a+b\sqrt{cx^2}\right)^{3/2}}{3b^2c}$$

```
[Out] -2/3*a*(a+b*(c*x^2)^(1/2))^(3/2)/b^2/c+2/5*(a+b*(c*x^2)^(1/2))^(5/2)/b^2/c
```

Rubi [A] time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {368, 43}

$$\frac{2\left(a+b\sqrt{cx^2}\right)^{5/2}}{5b^2c}-\frac{2a\left(a+b\sqrt{cx^2}\right)^{3/2}}{3b^2c}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*Sqrt[c*x^2]],x]
```

```
[Out] (-2*a*(a + b*Sqrt[c*x^2])^(3/2))/(3*b^2*c) + (2*(a + b*Sqrt[c*x^2])^(5/2))/
(5*b^2*c)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{a+b\sqrt{cx^2}}\,dx &= \frac{\text{Subst}\left(\int x\sqrt{a+bx}\,dx, x, \sqrt{cx^2}\right)}{c}\\ &= \frac{\text{Subst}\left(\int\left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx, x, \sqrt{cx^2}\right)}{c}\\ &= -\frac{2a\left(a+b\sqrt{cx^2}\right)^{3/2}}{3b^2c}+\frac{2\left(a+b\sqrt{cx^2}\right)^{5/2}}{5b^2c}\end{aligned}$$

Mathematica [A] time = 0.03, size = 43, normalized size = 0.77

$$\frac{2\left(a+b\sqrt{cx^2}\right)^{3/2}\left(3b\sqrt{cx^2}-2a\right)}{15b^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + b*Sqrt[c*x^2]],x]
```

```
[Out] (2*(a + b*Sqrt[c*x^2])^(3/2)*(-2*a + 3*b*Sqrt[c*x^2]))/(15*b^2*c)
```

fricas [A] time = 0.78, size = 46, normalized size = 0.82

$$\frac{2\left(3\,b^2cx^2+\sqrt{cx^2}\,ab-2\,a^2\right)\sqrt{\sqrt{cx^2}\,b+a}}{15\,b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*c*x^2 + sqrt(c*x^2)*a*b - 2*a^2)*sqrt(sqrt(c*x^2)*b + a)/(b^2*c
)
```

giac [B] time = 0.16, size = 90, normalized size = 1.61

$$2\left(\frac{5\left(\left(b\sqrt{c}\,x+a\right)^{\frac{3}{2}}-3\sqrt{b\sqrt{c}\,x+a}\,a\right)a}{b\sqrt{c}}+\frac{3\left(b\sqrt{c}\,x+a\right)^{\frac{5}{2}}-10\left(b\sqrt{c}\,x+a\right)^{\frac{3}{2}}a+15\sqrt{b\sqrt{c}\,x+a}\,a^2}{b\sqrt{c}}\right)\Bigg/\left(15\,b\sqrt{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(5*((b*sqrt(c)*x + a)^(3/2) - 3*sqrt(b*sqrt(c)*x + a)*a)*a/(b*sqrt(c))
 + (3*(b*sqrt(c)*x + a)^(5/2) - 10*(b*sqrt(c)*x + a)^(3/2)*a + 15*sqrt(b*sq
rt(c)*x + a)*a^2)/(b*sqrt(c)))/(b*sqrt(c))
```

maple [A] time = 0.01, size = 36, normalized size = 0.64

$$\frac{2\left(a+\sqrt{c\,x^2}\;b\right)^{\frac{3}{2}}\left(-2a+3\sqrt{c\,x^2}\;b\right)}{15b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+(c*x^2)^(1/2)*b)^(1/2),x)
```

```
[Out] 2/15*(a+(c*x^2)^(1/2)*b)^(3/2)*(-2*a+3*(c*x^2)^(1/2)*b)/c/b^2
```

maxima [A] time = 0.58, size = 43, normalized size = 0.77

$$2\left(\frac{3\left(\sqrt{cx^2}\,b+a\right)^{\frac{5}{2}}}{b^2}-\frac{5\left(\sqrt{cx^2}\,b+a\right)^{\frac{3}{2}}a}{b^2}\right)\Bigg/\left(15\,c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*(sqrt(c*x^2)*b + a)^(5/2)/b^2 - 5*(sqrt(c*x^2)*b + a)^(3/2)*a/b^2)/
c
```

mupad [B] time = 1.42, size = 52, normalized size = 0.93

$$\frac{x^2\sqrt{a+b\sqrt{c}\;\sqrt{x^2}}\;{}_2\mathrm{F}_1\left(-\frac{1}{2},2;\;3;\;-\frac{b\sqrt{c}\;|x|}{a}\right)}{2\sqrt{\frac{b\sqrt{c}\;\sqrt{x^2}}{a}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*(c*x^2)^(1/2))^(1/2),x)
```

```
[Out] (x^2*(a + b*c^(1/2)*(x^2)^(1/2))^(1/2)*hypergeom([-1/2, 2], 3, -(b*c^(1/2)*
abs(x))/a))/(2*((b*c^(1/2)*(x^2)^(1/2))/a + 1)^(1/2))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{a+b\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(c*x**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(x*sqrt(a + b*sqrt(c*x**2)), x)
```

3.2933 $\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x}\,dx$

Optimal. Leaf size=51

$$2\sqrt{a+b\sqrt{cx^2}}\;-2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{cx^2}}}{\sqrt{a}}\right)$$

```
[Out] -2*arctanh((a+b*(c*x^2)^(1/2))^(1/2)/a^(1/2))*a^(1/2)+2*(a+b*(c*x^2)^(1/2))
^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {368, 50, 63, 208}

$$2\sqrt{a+b\sqrt{cx^2}}\;-2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{cx^2}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[c*x^2]]/x,x]
```

```
[Out] 2*Sqrt[a + b*Sqrt[c*x^2]] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sqrt[c*x^2]]/Sqrt[
a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x}\,dx = \text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \sqrt{cx^2}\right)$$
$$= 2\sqrt{a+b\sqrt{cx^2}} + a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sqrt{cx^2}\right)$$
$$= 2\sqrt{a+b\sqrt{cx^2}} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sqrt{cx^2}}\right)}{b}$$
$$= 2\sqrt{a+b\sqrt{cx^2}} - 2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{cx^2}}}{\sqrt{a}}\right)$$

Mathematica [A] time = 0.03, size = 51, normalized size = 1.00

$$2\sqrt{a+b\sqrt{cx^2}} - 2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{cx^2}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[c*x^2]]/x,x]
```

```
[Out] 2*Sqrt[a + b*Sqrt[c*x^2]] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sqrt[c*x^2]]/Sqrt[
a]]
```

fricas [A] time = 0.68, size = 114, normalized size = 2.24

$$\left[\sqrt{a}\,\log\left(\frac{bcx^2 - 2\sqrt{cx^2}\sqrt{\sqrt{cx^2}\,b+a}\sqrt{a} + 2\sqrt{cx^2}\,a}{x^2}\right) + 2\sqrt{\sqrt{cx^2}\,b+a}, 2\sqrt{-a}\,\arctan\left(\frac{\sqrt{\sqrt{cx^2}\,b+a}\sqrt{-a}}{a}\right) + 2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(1/2))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [sqrt(a)*log((b*c*x^2 - 2*sqrt(c*x^2)*sqrt(sqrt(c*x^2)*b + a)*sqrt(a) + 2*s
qrt(c*x^2)*a)/x^2) + 2*sqrt(sqrt(c*x^2)*b + a), 2*sqrt(-a)*arctan(sqrt(sqrt
(c*x^2)*b + a)*sqrt(-a)/a) + 2*sqrt(sqrt(c*x^2)*b + a)]
```

giac [A] time = 0.16, size = 38, normalized size = 0.75

$$\frac{2\,a\arctan\left(\frac{\sqrt{b\sqrt{c}\,x+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2\sqrt{b\sqrt{c}\,x+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(1/2))^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a*arctan(sqrt(b*sqrt(c)*x + a)/sqrt(-a))/sqrt(-a) + 2*sqrt(b*sqrt(c)*x +
a)
```

maple [A] time = 0.01, size = 40, normalized size = 0.78

$$-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+\sqrt{c\,x^2}\,b}}{\sqrt{a}}\right) + 2\sqrt{a+\sqrt{c\,x^2}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(c*x^2)^(1/2)*b)^(1/2)/x,x)
```

```
[Out] -2*arctanh((a+(c*x^2)^(1/2)*b)^(1/2)/a^(1/2))*a^(1/2)+2*(a+(c*x^2)^(1/2)*b)
^(1/2)
```

maxima [A] time = 1.22, size = 60, normalized size = 1.18

$$\sqrt{a}\,\log\left(\frac{\sqrt{\sqrt{cx^2}\,b+a}-\sqrt{a}}{\sqrt{\sqrt{cx^2}\,b+a}+\sqrt{a}}\right)+2\sqrt{\sqrt{cx^2}\,b+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(1/2))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(a)*log((sqrt(sqrt(c*x^2)*b + a) - sqrt(a))/(sqrt(sqrt(c*x^2)*b + a) +
sqrt(a))) + 2*sqrt(sqrt(c*x^2)*b + a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+b\sqrt{c\,x^2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x^2)^(1/2))^(1/2)/x,x)
```

```
[Out] int((a + b*(c*x^2)^(1/2))^(1/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x**2)**(1/2))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a + b*sqrt(c*x**2))/x, x)
```

3.2934 $\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^3}\,dx$

Optimal. Leaf size=97

$$\frac{b^2c\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{cx^2}}}{\sqrt{a}}\right)}{4a^{3/2}}-\frac{bc\sqrt{a+b\sqrt{cx^2}}}{4a\sqrt{cx^2}}-\frac{\sqrt{a+b\sqrt{cx^2}}}{2x^2}$$

```
[Out] 1/4*b^2*c*arctanh((a+b*(c*x^2)^(1/2))^(1/2)/a^(1/2))/a^(3/2)-1/2*(a+b*(c*x^
2)^(1/2))^(1/2)/x^2-1/4*b*c*(a+b*(c*x^2)^(1/2))^(1/2)/a/(c*x^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {368, 47, 51, 63, 208}

$$\frac{b^2c\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{cx^2}}}{\sqrt{a}}\right)}{4a^{3/2}}-\frac{bc\sqrt{a+b\sqrt{cx^2}}}{4a\sqrt{cx^2}}-\frac{\sqrt{a+b\sqrt{cx^2}}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[c*x^2]]/x^3,x]
```

```
[Out] -Sqrt[a + b*Sqrt[c*x^2]]/(2*x^2) - (b*c*Sqrt[a + b*Sqrt[c*x^2]])/(4*a*Sqrt[
c*x^2]) + (b^2*c*ArcTanh[Sqrt[a + b*Sqrt[c*x^2]]/Sqrt[a]])/(4*a^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^3}\,dx &= c\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^3}\,dx, x, \sqrt{cx^2}\right) \\
&= -\frac{\sqrt{a+b\sqrt{cx^2}}}{2x^2} + \frac{1}{4}(bc)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \sqrt{cx^2}\right) \\
&= -\frac{\sqrt{a+b\sqrt{cx^2}}}{2x^2} - \frac{bc\sqrt{a+b\sqrt{cx^2}}}{4a\sqrt{cx^2}} - \frac{\left(b^2c\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sqrt{cx^2}\right)}{8a} \\
&= -\frac{\sqrt{a+b\sqrt{cx^2}}}{2x^2} - \frac{bc\sqrt{a+b\sqrt{cx^2}}}{4a\sqrt{cx^2}} - \frac{(bc)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sqrt{cx^2}}\right)}{4a} \\
&= -\frac{\sqrt{a+b\sqrt{cx^2}}}{2x^2} - \frac{bc\sqrt{a+b\sqrt{cx^2}}}{4a\sqrt{cx^2}} + \frac{b^2c\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{cx^2}}}{\sqrt{a}}\right)}{4a^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 52, normalized size = 0.54

$$-\frac{2b^2c\left(a+b\sqrt{cx^2}\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{\sqrt{cx^2}\,b}{a}+1\right)}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[c*x^2]]/x^3,x]
```

```
[Out] (-2*b^2*c*(a + b*Sqrt[c*x^2])^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + (b*S
qrt[c*x^2])/a])/(3*a^3)
```

fricas [A] time = 1.11, size = 174, normalized size = 1.79

$$\left[\frac{\sqrt{a}\,b^2cx^2\log\left(\frac{bcx^2+2\sqrt{cx^2}\sqrt{\sqrt{cx^2}\,b+a}\sqrt{a}+2\sqrt{cx^2}\,a}{x^2}\right)-2\left(\sqrt{cx^2}\,ab+2\,a^2\right)\sqrt{\sqrt{cx^2}\,b+a}}{8\,a^2x^2}, -\frac{\sqrt{-a}\,b^2cx^2\arctan\left(\frac{\sqrt{\sqrt{cx^2}\,b}}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(1/2))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a)*b^2*c*x^2*log((b*c*x^2 + 2*sqrt(c*x^2)*sqrt(sqrt(c*x^2)*b + a
)*sqrt(a) + 2*sqrt(c*x^2)*a)/x^2) - 2*(sqrt(c*x^2)*a*b + 2*a^2)*sqrt(sqrt(c
*x^2)*b + a))/(a^2*x^2), -1/4*(sqrt(-a)*b^2*c*x^2*arctan(sqrt(sqrt(c*x^2)*b
 + a)*sqrt(-a)/a) + (sqrt(c*x^2)*a*b + 2*a^2)*sqrt(sqrt(c*x^2)*b + a))/(a^2
*x^2)]
```

giac [A] time = 0.17, size = 90, normalized size = 0.93

$$-\frac{\frac{b^3 c^{\frac{3}{2}} \arctan\left(\frac{\sqrt{b\sqrt{c}\,x+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{\left(b\sqrt{c}\,x+a\right)^{\frac{3}{2}} b^3 c^{\frac{3}{2}}+\sqrt{b\sqrt{c}\,x+a}\,a b^3 c^{\frac{3}{2}}}{a b^2 c x^2}}{4\,b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(1/2))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/4*(b^3*c^(3/2)*arctan(sqrt(b*sqrt(c)*x + a)/sqrt(-a))/(sqrt(-a)*a) + ((b
*sqrt(c)*x + a)^(3/2)*b^3*c^(3/2) + sqrt(b*sqrt(c)*x + a)*a*b^3*c^(3/2))/(a
*b^2*c*x^2))/(b*sqrt(c))
```

maple [A] time = 0.01, size = 72, normalized size = 0.74

$$-\frac{-a\,b^2 c\,x^2 \operatorname{arctanh}\left(\frac{\sqrt{a+\sqrt{c\,x^2}\,b}}{\sqrt{a}}\right)+\sqrt{a+\sqrt{c\,x^2}\,b}\;a^{\frac{5}{2}}+\left(a+\sqrt{c\,x^2}\,b\right)^{\frac{3}{2}} a^{\frac{3}{2}}}{4a^{\frac{5}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(c*x^2)^(1/2)*b)^(1/2)/x^3,x)
```

```
[Out] -1/4*(-arctanh((a+(c*x^2)^(1/2)*b)^(1/2)/a^(1/2))*c*x^2*a*b^2+(a+(c*x^2)^(1
/2)*b)^(3/2)*a^(3/2)+(a+(c*x^2)^(1/2)*b)^(1/2)*a^(5/2))/x^2/a^(5/2)
```

maxima [A] time = 1.21, size = 126, normalized size = 1.30

$$-\frac{1}{8}\left(\frac{b^2 \log\left(\frac{\sqrt{\sqrt{cx^2}\,b+a}-\sqrt{a}}{\sqrt{\sqrt{cx^2}\,b+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2\left(\left(\sqrt{cx^2}\,b+a\right)^{\frac{3}{2}} b^2+\sqrt{\sqrt{cx^2}\,b+a}\,ab^2\right)}{\left(\sqrt{cx^2}\,b+a\right)^2 a-2\left(\sqrt{cx^2}\,b+a\right)a^2+a^3}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(1/2))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/8*(b^2*log((sqrt(sqrt(c*x^2)*b + a) - sqrt(a))/(sqrt(sqrt(c*x^2)*b + a)
+ sqrt(a)))/a^(3/2) + 2*((sqrt(c*x^2)*b + a)^(3/2)*b^2 + sqrt(sqrt(c*x^2)*b
 + a)*a*b^2)/((sqrt(c*x^2)*b + a)^2*a - 2*(sqrt(c*x^2)*b + a)*a^2 + a^3))*c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+b\sqrt{c\,x^2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x^2)^(1/2))^(1/2)/x^3,x)
```

```
[Out] int((a + b*(c*x^2)^(1/2))^(1/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x**2)**(1/2))**(1/2)/x**3,x)
[Out] Integral(sqrt(a + b*sqrt(c*x**2))/x**3, x)
```

3.2935 $\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^5}\,dx$

Optimal. Leaf size=171

$$\frac{5b^4c^2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{cx^2}}}{\sqrt{a}}\right)}{64a^{7/2}}-\frac{5b^3c^2\sqrt{a+b\sqrt{cx^2}}}{64a^3\sqrt{cx^2}}+\frac{5b^2c\sqrt{a+b\sqrt{cx^2}}}{96a^2x^2}-\frac{bc^2\sqrt{a+b\sqrt{cx^2}}}{24a\left(cx^2\right)^{3/2}}-\frac{\sqrt{a+b\sqrt{cx^2}}}{4x^4}$$

```
[Out] 5/64*b^4*c^2*arctanh((a+b*(c*x^2)^(1/2))^(1/2)/a^(1/2))/a^(7/2)-1/4*(a+b*(c
*x^2)^(1/2))^(1/2)/x^4+5/96*b^2*c*(a+b*(c*x^2)^(1/2))^(1/2)/a^2/x^2-1/24*b*
c^2*(a+b*(c*x^2)^(1/2))^(1/2)/a/(c*x^2)^(3/2)-5/64*b^3*c^2*(a+b*(c*x^2)^(1/
2))^(1/2)/a^3/(c*x^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {368, 47, 51, 63, 208}

$$-\frac{5b^3c^2\sqrt{a+b\sqrt{cx^2}}}{64a^3\sqrt{cx^2}}+\frac{5b^4c^2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{cx^2}}}{\sqrt{a}}\right)}{64a^{7/2}}+\frac{5b^2c\sqrt{a+b\sqrt{cx^2}}}{96a^2x^2}-\frac{bc^2\sqrt{a+b\sqrt{cx^2}}}{24a\left(cx^2\right)^{3/2}}-\frac{\sqrt{a+b\sqrt{cx^2}}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[c*x^2]]/x^5,x]
```

```
[Out] -Sqrt[a + b*Sqrt[c*x^2]]/(4*x^4) + (5*b^2*c*Sqrt[a + b*Sqrt[c*x^2]])/(96*a^
2*x^2) - (b*c^2*Sqrt[a + b*Sqrt[c*x^2]])/(24*a*(c*x^2)^(3/2)) - (5*b^3*c^2*
Sqrt[a + b*Sqrt[c*x^2]])/(64*a^3*Sqrt[c*x^2]) + (5*b^4*c^2*ArcTanh[Sqrt[a +
 b*Sqrt[c*x^2]]/Sqrt[a]])/(64*a^(7/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^5}\,dx = c^2\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^5}\,dx, x, \sqrt{cx^2}\right)$$

$$= -\frac{\sqrt{a+b\sqrt{cx^2}}}{4x^4} + \frac{1}{8}\left(bc^2\right)\text{Subst}\left(\int \frac{1}{x^4\sqrt{a+bx}}\,dx, x, \sqrt{cx^2}\right)$$

$$= -\frac{\sqrt{a+b\sqrt{cx^2}}}{4x^4} - \frac{bc^2\sqrt{a+b\sqrt{cx^2}}}{24a\left(cx^2\right)^{3/2}} - \frac{\left(5b^2c^2\right)\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx, x, \sqrt{cx^2}\right)}{48a}$$

$$= -\frac{\sqrt{a+b\sqrt{cx^2}}}{4x^4} + \frac{5b^2c\sqrt{a+b\sqrt{cx^2}}}{96a^2x^2} - \frac{bc^2\sqrt{a+b\sqrt{cx^2}}}{24a\left(cx^2\right)^{3/2}} + \frac{\left(5b^3c^2\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx\right.}{64a^2}$$

$$= -\frac{\sqrt{a+b\sqrt{cx^2}}}{4x^4} + \frac{5b^2c\sqrt{a+b\sqrt{cx^2}}}{96a^2x^2} - \frac{bc^2\sqrt{a+b\sqrt{cx^2}}}{24a\left(cx^2\right)^{3/2}} - \frac{5b^3c^2\sqrt{a+b\sqrt{cx^2}}}{64a^3\sqrt{cx^2}} - \frac{\left(5b^4c^2\right.}{}$$

$$= -\frac{\sqrt{a+b\sqrt{cx^2}}}{4x^4} + \frac{5b^2c\sqrt{a+b\sqrt{cx^2}}}{96a^2x^2} - \frac{bc^2\sqrt{a+b\sqrt{cx^2}}}{24a\left(cx^2\right)^{3/2}} - \frac{5b^3c^2\sqrt{a+b\sqrt{cx^2}}}{64a^3\sqrt{cx^2}} - \frac{\left(5b^3c^2\right.}{}$$

$$= -\frac{\sqrt{a+b\sqrt{cx^2}}}{4x^4} + \frac{5b^2c\sqrt{a+b\sqrt{cx^2}}}{96a^2x^2} - \frac{bc^2\sqrt{a+b\sqrt{cx^2}}}{24a\left(cx^2\right)^{3/2}} - \frac{5b^3c^2\sqrt{a+b\sqrt{cx^2}}}{64a^3\sqrt{cx^2}} + \frac{5b^4c^2\,\mathrm{t}}{}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.32

$$-\frac{2b^4c^2\left(a+b\sqrt{cx^2}\right)^{3/2}\,{}_2F_1\left(\frac{3}{2}, 5; \frac{5}{2}; \frac{\sqrt{cx^2}\,b}{a}+1\right)}{3a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[c*x^2]]/x^5,x]
```

```
[Out] (-2*b^4*c^2*(a + b*Sqrt[c*x^2])^(3/2)*Hypergeometric2F1[3/2, 5, 5/2, 1 + (b
*Sqrt[c*x^2])/a])/(3*a^5)
```

fricas [A] time = 0.95, size = 237, normalized size = 1.39

$$\left[\frac{15\sqrt{a}\,b^4c^2x^4\log\left(\frac{bcx^2+2\sqrt{cx^2}\sqrt{\sqrt{cx^2}\,b+a}\sqrt{a}+2\sqrt{cx^2}\,a}{x^2}\right)+2\left(10\,a^2b^2cx^2-48\,a^4-\left(15\,ab^3cx^2+8\,a^3b\right)\sqrt{cx^2}\right)\sqrt{\sqrt{cx^2}}}{384\,a^4x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(1/2))^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/384*(15*sqrt(a)*b^4*c^2*x^4*log((b*c*x^2 + 2*sqrt(c*x^2)*sqrt(sqrt(c*x^2
)*b + a)*sqrt(a) + 2*sqrt(c*x^2)*a)/x^2) + 2*(10*a^2*b^2*c*x^2 - 48*a^4 - (
15*a*b^3*c*x^2 + 8*a^3*b)*sqrt(c*x^2))*sqrt(sqrt(c*x^2)*b + a))/(a^4*x^4),
-1/192*(15*sqrt(-a)*b^4*c^2*x^4*arctan(sqrt(sqrt(c*x^2)*b + a)*sqrt(-a)/a)
- (10*a^2*b^2*c*x^2 - 48*a^4 - (15*a*b^3*c*x^2 + 8*a^3*b)*sqrt(c*x^2))*sqrt
(sqrt(c*x^2)*b + a))/(a^4*x^4)]
```

giac [A] time = 0.21, size = 135, normalized size = 0.79

$$-\frac{\frac{15\,b^5c^{\frac{5}{2}}\arctan\left(\frac{\sqrt{b\sqrt{c}\,x+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^3}+\frac{15\left(b\sqrt{c}\,x+a\right)^{\frac{7}{2}}b^5c^{\frac{5}{2}}-55\left(b\sqrt{c}\,x+a\right)^{\frac{5}{2}}ab^5c^{\frac{5}{2}}+73\left(b\sqrt{c}\,x+a\right)^{\frac{3}{2}}a^2b^5c^{\frac{5}{2}}+15\sqrt{b\sqrt{c}\,x+a}\,a^3b^5c^{\frac{5}{2}}}{a^3b^4c^2x^4}}{192\,b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(1/2))^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/192*(15*b^5*c^(5/2)*arctan(sqrt(b*sqrt(c)*x + a)/sqrt(-a))/(sqrt(-a)*a^3
) + (15*(b*sqrt(c)*x + a)^(7/2)*b^5*c^(5/2) - 55*(b*sqrt(c)*x + a)^(5/2)*a*
b^5*c^(5/2) + 73*(b*sqrt(c)*x + a)^(3/2)*a^2*b^5*c^(5/2) + 15*sqrt(b*sqrt(c
)*x + a)*a^3*b^5*c^(5/2))/(a^3*b^4*c^2*x^4))/(b*sqrt(c))
```

maple [A] time = 0.01, size = 114, normalized size = 0.67

$$-\frac{-15a^3b^4c^2x^4\operatorname{arctanh}\left(\frac{\sqrt{a+\sqrt{c\,x^2}\,b}}{\sqrt{a}}\right)+15\sqrt{a+\sqrt{c\,x^2}\,b}\;a^{\frac{13}{2}}+73\left(a+\sqrt{c\,x^2}\,b\right)^{\frac{3}{2}}a^{\frac{11}{2}}-55\left(a+\sqrt{c\,x^2}\,b\right)^{\frac{5}{2}}a^{\frac{9}{2}}+15\left(}{192a^{\frac{13}{2}}x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(c*x^2)^(1/2)*b)^(1/2)/x^5,x)
```

```
[Out] -1/192*(15*(a+(c*x^2)^(1/2)*b)^(7/2)*a^(7/2)-15*arctanh((a+(c*x^2)^(1/2)*b)
^(1/2)/a^(1/2))*a^3*b^4*c^2*x^4-55*(a+(c*x^2)^(1/2)*b)^(5/2)*a^(9/2)+73*(a+
(c*x^2)^(1/2)*b)^(3/2)*a^(11/2)+15*(a+(c*x^2)^(1/2)*b)^(1/2)*a^(13/2))/a^(1
3/2)/x^4
```

maxima [A] time = 1.20, size = 211, normalized size = 1.23

$$-\frac{1}{384}\left(\frac{15\,b^4\log\left(\frac{\sqrt{\sqrt{cx^2}\,b+a}-\sqrt{a}}{\sqrt{\sqrt{cx^2}\,b+a}+\sqrt{a}}\right)}{a^{\frac{7}{2}}}+\frac{2\left(15\left(\sqrt{cx^2}\,b+a\right)^{\frac{7}{2}}b^4-55\left(\sqrt{cx^2}\,b+a\right)^{\frac{5}{2}}ab^4+73\left(\sqrt{cx^2}\,b+a\right)^{\frac{3}{2}}a^2b^4+15\sqrt{\right.}\right.}{\left(\sqrt{cx^2}\,b+a\right)^4a^3-4\left(\sqrt{cx^2}\,b+a\right)^3a^4+6\left(\sqrt{cx^2}\,b+a\right)^2a^5-4\left(\sqrt{cx^2}\,b+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(1/2))^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/384*(15*b^4*log((sqrt(sqrt(c*x^2)*b + a) - sqrt(a))/(sqrt(sqrt(c*x^2)*b
+ a) + sqrt(a)))/a^(7/2) + 2*(15*(sqrt(c*x^2)*b + a)^(7/2)*b^4 - 55*(sqrt(c
*x^2)*b + a)^(5/2)*a*b^4 + 73*(sqrt(c*x^2)*b + a)^(3/2)*a^2*b^4 + 15*sqrt(s
qrt(c*x^2)*b + a)*a^3*b^4)/((sqrt(c*x^2)*b + a)^4*a^3 - 4*(sqrt(c*x^2)*b +
a)^3*a^4 + 6*(sqrt(c*x^2)*b + a)^2*a^5 - 4*(sqrt(c*x^2)*b + a)*a^6 + a^7))*
c^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + b\sqrt{c\,x^2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x^2)^(1/2))^(1/2)/x^5,x)
```

```
[Out] int((a + b*(c*x^2)^(1/2))^(1/2)/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\sqrt{cx^2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x**2)**(1/2))**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(a + b*sqrt(c*x**2))/x**5, x)
```

3.2936 $\int x^4\sqrt{a+b\sqrt{cx^2}}\,dx$

Optimal. Leaf size=191

$$\frac{2a^4x^5\left(a+b\sqrt{cx^2}\right)^{3/2}}{3b^5\left(cx^2\right)^{5/2}}-\frac{8a^3x^5\left(a+b\sqrt{cx^2}\right)^{5/2}}{5b^5\left(cx^2\right)^{5/2}}+\frac{12a^2x^5\left(a+b\sqrt{cx^2}\right)^{7/2}}{7b^5\left(cx^2\right)^{5/2}}+\frac{2x^5\left(a+b\sqrt{cx^2}\right)^{11/2}}{11b^5\left(cx^2\right)^{5/2}}-\frac{8ax^5\left(a+b\sqrt{cx^2}\right)^{9}}{9b^5\left(cx^2\right)^{5/2}}$$

```
[Out] 2/3*a^4*x^5*(a+b*(c*x^2)^(1/2))^(3/2)/b^5/(c*x^2)^(5/2)-8/5*a^3*x^5*(a+b*(c
*x^2)^(1/2))^(5/2)/b^5/(c*x^2)^(5/2)+12/7*a^2*x^5*(a+b*(c*x^2)^(1/2))^(7/2)
/b^5/(c*x^2)^(5/2)-8/9*a*x^5*(a+b*(c*x^2)^(1/2))^(9/2)/b^5/(c*x^2)^(5/2)+2/
11*x^5*(a+b*(c*x^2)^(1/2))^(11/2)/b^5/(c*x^2)^(5/2)
```

Rubi [A] time = 0.07, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {368, 43}

$$\frac{12a^2x^5\left(a+b\sqrt{cx^2}\right)^{7/2}}{7b^5\left(cx^2\right)^{5/2}}-\frac{8a^3x^5\left(a+b\sqrt{cx^2}\right)^{5/2}}{5b^5\left(cx^2\right)^{5/2}}+\frac{2a^4x^5\left(a+b\sqrt{cx^2}\right)^{3/2}}{3b^5\left(cx^2\right)^{5/2}}+\frac{2x^5\left(a+b\sqrt{cx^2}\right)^{11/2}}{11b^5\left(cx^2\right)^{5/2}}-\frac{8ax^5\left(a+b\sqrt{cx^2}\right)^{9}}{9b^5\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[a + b*Sqrt[c*x^2]],x]
```

```
[Out] (2*a^4*x^5*(a + b*Sqrt[c*x^2])^(3/2))/(3*b^5*(c*x^2)^(5/2)) - (8*a^3*x^5*(a
 + b*Sqrt[c*x^2])^(5/2))/(5*b^5*(c*x^2)^(5/2)) + (12*a^2*x^5*(a + b*Sqrt[c*
x^2])^(7/2))/(7*b^5*(c*x^2)^(5/2)) - (8*a*x^5*(a + b*Sqrt[c*x^2])^(9/2))/(9
*b^5*(c*x^2)^(5/2)) + (2*x^5*(a + b*Sqrt[c*x^2])^(11/2))/(11*b^5*(c*x^2)^(5
/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\int x^4\sqrt{a+b\sqrt{cx^2}}\,dx = \frac{x^5\,\text{Subst}\left(\int x^4\sqrt{a+bx}\,dx, x, \sqrt{cx^2}\right)}{\left(cx^2\right)^{5/2}}$$

$$= \frac{x^5\,\text{Subst}\left(\int\left(\frac{a^4\sqrt{a+bx}}{b^4}-\frac{4a^3(a+bx)^{3/2}}{b^4}+\frac{6a^2(a+bx)^{5/2}}{b^4}-\frac{4a(a+bx)^{7/2}}{b^4}+\frac{(a+bx)^{9/2}}{b^4}\right)dx, x, \sqrt{cx^2}\right)}{\left(cx^2\right)^{5/2}}$$

$$= \frac{2a^4x^5\left(a+b\sqrt{cx^2}\right)^{3/2}}{3b^5\left(cx^2\right)^{5/2}}-\frac{8a^3x^5\left(a+b\sqrt{cx^2}\right)^{5/2}}{5b^5\left(cx^2\right)^{5/2}}+\frac{12a^2x^5\left(a+b\sqrt{cx^2}\right)^{7/2}}{7b^5\left(cx^2\right)^{5/2}}-\frac{8ax^5\left(a+b\sqrt{cx^2}\right)}{9b^5\left(cx^2\right)}$$

Mathematica [A] time = 0.05, size = 96, normalized size = 0.50

$$\frac{2x\left(a+b\sqrt{cx^2}\right)^{3/2}\left(128a^4-192a^3b\sqrt{cx^2}+240a^2b^2cx^2-280ab^3\left(cx^2\right)^{3/2}+315b^4c^2x^4\right)}{3465b^5c^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[a + b*Sqrt[c*x^2]],x]
```

```
[Out] (2*x*(a + b*Sqrt[c*x^2])^(3/2)*(128*a^4 + 240*a^2*b^2*c*x^2 + 315*b^4*c^2*x
^4 - 192*a^3*b*Sqrt[c*x^2] - 280*a*b^3*(c*x^2)^(3/2)))/(3465*b^5*c^2*Sqrt[c
*x^2])
```

fricas [A] time = 0.93, size = 98, normalized size = 0.51

$$\frac{2\left(315\,b^5c^3x^6-40\,a^2b^3c^2x^4-64\,a^4bcx^2+\left(35\,ab^4c^2x^4+48\,a^3b^2cx^2+128\,a^5\right)\sqrt{cx^2}\right)\sqrt{\sqrt{cx^2}\,b+a}}{3465\,b^5c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3465*(315*b^5*c^3*x^6 - 40*a^2*b^3*c^2*x^4 - 64*a^4*b*c*x^2 + (35*a*b^4*c
^2*x^4 + 48*a^3*b^2*c*x^2 + 128*a^5)*sqrt(c*x^2))*sqrt(sqrt(c*x^2)*b + a)/(
b^5*c^3*x)
```

giac [A] time = 0.22, size = 201, normalized size = 1.05

$$2\left(\frac{11\left(35\left(b\sqrt{c}\,x+a\right)^{\frac{9}{2}}-180\left(b\sqrt{c}\,x+a\right)^{\frac{7}{2}}a+378\left(b\sqrt{c}\,x+a\right)^{\frac{5}{2}}a^2-420\left(b\sqrt{c}\,x+a\right)^{\frac{3}{2}}a^3+315\sqrt{b\sqrt{c}\,x+a}\,a^4\right)a}{b^4c^2}+\frac{5\left(63\left(b\sqrt{c}\,x+a\right)^{\frac{11}{2}}\sqrt{c}-385\left(b\sqrt{c}\,x+a\right)^{\frac{9}{2}}\right.}{3465\,b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 2/3465*(11*(35*(b*sqrt(c)*x + a)^(9/2) - 180*(b*sqrt(c)*x + a)^(7/2)*a + 37
8*(b*sqrt(c)*x + a)^(5/2)*a^2 - 420*(b*sqrt(c)*x + a)^(3/2)*a^3 + 315*sqrt(
b*sqrt(c)*x + a)*a^4)*a/(b^4*c^2) + 5*(63*(b*sqrt(c)*x + a)^(11/2)*sqrt(c)
- 385*(b*sqrt(c)*x + a)^(9/2)*a*sqrt(c) + 990*(b*sqrt(c)*x + a)^(7/2)*a^2*s
qrt(c) - 1386*(b*sqrt(c)*x + a)^(5/2)*a^3*sqrt(c) + 1155*(b*sqrt(c)*x + a)^
(3/2)*a^4*sqrt(c) - 693*sqrt(b*sqrt(c)*x + a)*a^5*sqrt(c))/(b^4*c^(5/2)))/(
b*sqrt(c))
```

maple [A] time = 0.01, size = 84, normalized size = 0.44

$$-\frac{2\left(a+\sqrt{c\,x^2}\;b\right)^{\frac{3}{2}}\left(-315b^4c^2x^4-240a^2b^2c\,x^2-128a^4+192\sqrt{c\,x^2}\;a^3b+280\left(c\,x^2\right)^{\frac{3}{2}}a\,b^3\right)x^5}{3465\left(c\,x^2\right)^{\frac{5}{2}}b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+(c*x^2)^(1/2)*b)^(1/2),x)
```

```
[Out] -2/3465*x^5*(a+(c*x^2)^(1/2)*b)^(3/2)*(-315*c^2*x^4*b^4+280*(c*x^2)^(3/2)*a
*b^3-240*c*x^2*a^2*b^2+192*(c*x^2)^(1/2)*a^3*b-128*a^4)/(c*x^2)^(5/2)/b^5
```

maxima [B] time = 1.26, size = 2192, normalized size = 11.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] ((253*c^33 + 2558956*c^32 + 7217549950*c^31 + 8987703765844*c^30 + 60364683
73437617*c^29 + 244642952984981127 2*c^28 + 642455910258816305144*c^27 + 114
777366281226527056208*c^26 + 144442069312273663673308 58*c^25 + 131365425653
7258900978878920*c^24 + 880077875356516130906461851 40*c^23 + 44057110039828
78865632262198872*c^22 + 166544000020720524719921573991514*c^21 + 478943871
60644348054598418641620 48*c^20 + 105284116366548048830595983583302024*c^19
+ 177344492814615042790508221708781288 0*c^18 + 228948392597758710018299060 6
4713305625*c^17 + 226076660023411473110523953150238987500*c^16 + 1700246465
24692768615005073827382421875 0*c^15 + 96729932465482518375578962444814453 12
500*c^14 + 41230185720792035261437425937884033203125*c^13 + 129956781520382
04985093937690209960937500 0*c^12 + 29769407278591668426367728464111328125 00
00*c^11 + 48432952950241518875035730468750000000000000*c^10 + 542693518652974
49023880468750000000000000 0*c^9 + 40155973753395455088875000000000000000000
0*c^8 + 18484985390862231687500000000000000000000*c^7 + 483942549851902800
00000000000000000000000*c^6 + 62121641266800000000000000000000000000*c^5
+ 29220652800000000000000000000000000000*c^4 + 20995200000000000000000000
0000000000*c^3 + (c^33 + 31444*c^32 + 153361414*c^31 + 277761034468*c^30 +
249531421449205*c^29 + 128781547874762192*c^28 + 41710765820505500216*c^27
+ 898886882712107944 1936*c^26 + 13428407804947489477667 06*c^25 + 1432661664
24564257427917848*c^24 + 11159995340528093004218780644*c^23 + 6453082537456
84073542574566840*c^22 + 28040775985068292094392289155714*c^21 + 9238391372
57607053430155106920656*c^20 + 23217713909073497257593845317682600*c^19 + 4
46699434346765451578586034218289616*c^18 + 65879622783179986520166277831497
54125*c^17 + 74411331047450090714926280659005912500*c^16 + 6416877026113094
96545321307620461343750*c^15 + 42016270404022689571953161296266015625 00*c^1
4 + 20719469703534769824847871522785791015625*c^13 + 76087449461581872206 55
731471054687500000 0*c^12 + 20496011645616511158441810602294921875 0000*c^11
+ 39698346147974366208712478984375000000000000*c^10 + 53838717268202350494014
37500000000000000000*c^9 + 49325891800537963477587500000000000000000000*c^8 +
29046014678305553487500000000000000000000*c^7 + 10224496256242962000000000
0000000000000000*c^6 + 19203795434390400000000000000000000000000000*c^5 + 1572
81030720000000000000000000000000000*c^4 + 35411904000000000000000000000000
000000*c^3)*sqrt(c))*b^5*x^5 + (c^33 + 30448*c^32 + 143612758*c^31 + 251188
638796*c^30 + 217599624606565*c^29 + 108117268374716764*c^28 + 336549239951
01723352*c^27 + 69575239700074417949 92*c^26 + 99505169888996190826174 6*c^25
 + 10141016588189418249078235 2*c^24 + 75279409684893643201557825 96*c^23 + 4
13724159098907450302482347816*c^22 + 17037518514719295836702957925282*c^21
+ 53026343341023368793771613775911 2*c^20 + 12544173979379497042757936065180
200*c^19 + 22626975255064520894032907692796472 0*c^18 + 31144986065437202834
41564145645938125*c^17 + 32663951713046131242671682730487700000*c^16 + 2600
04289926401703985972418707308593750*c^15 + 15607098182369854763706718758482
42187500*c^14 + 69988538091335300965470365584318847656 25*c^13 + 23148367524
55021270556019589392089843750 0*c^12 + 55506870767440315469623732717285156 25
0000*c^11 + 94317102615121972546395374755859375000000*c^10 + 11014269651431
43106810405273437500000000000*c^9 + 84767987622510644717304687500000000000000
0*c^8 + 40508998607407646796875000000000000000000*c^7 + 1098952464646270125
00000000000000000000000*c^6 + 14591698370325000000000000000000000000000*c^5 +
 70871193000000000000000000000000000000*c^4 + 52488000000000000000000000000
0000000*c^3 + (249*c^32 + 2437164*c^31 + 6643098918*c^30 + 7982949210660*c^
29 + 5166069875011357*c^28 + 20139604563509442 16*c^27 + 5078362142784094117
36*c^26 + 86947270401196759876240*c^25 + 10464000135667518734283874*c^24 +
90801359300968217101574951 2*c^23 + 57896023661694155810023054756*c^22 + 275
```

```
08143675872490644223328076 08*c^21 + 98393925961843341373109742290386*c^20 +
 26683849824235000537089773131256 00*c^19 + 55107420449030060659564239322581
224*c^18 + 86836591794356959214376590937595400 0*c^17 + 10436844833600989868
063649482129553125*c^16 + 95420853171226948139837222228288187500*c^15 + 660
229305541320870206161063444589843750*c^14 + 34301539736003099320752087410 88
476562500*c^13 + 13234770484257914875249279704156494140625*c^12 + 373633114
221811990286985933264160156250 00*c^11 + 75666589716155422385182353771972656
250000*c^10 + 10706111904192729856477580566406250000000 00*c^9 + 102122732595
71724751464257812500000000000 00*c^8 + 62487787043911972019531250000000000000
000*c^7 + 22813859478991729687500000000000000000000 00*c^6 + 44361563993394750
00000000000000000000000 00*c^5 + 37548477855000000000000000000000000000000*c^4 +
 87217560000000000000000000000000000 00*c^3)*sqrt(c))*a*b^4*x^4 - 4*(c^32 +
29710*c^31 + 136568656*c^30 + 232488459946*c^29 + 195743173114099*c^28 + 94
380747307709584*c^27 + 28462469351818276960*c^26 + 569017755133857805 2424*c
^25 + 78542148564841883110 4842*c^24 + 77086958845727395767874308*c^23 + 549
76828190718643690194028 32*c^22 + 28951523348547176219358780 9036*c^21 + 1139
07125133267945031782497837 82*c^20 + 33759806814464481434699115894 1992*c^19
+ 75774016454108002107539245562813 28*c^18 + 12914410601225222885842167996 67
53000*c^17 + 16716978068232815667360615372188531 25*c^16 + 16398697473652695
739105288119068718750*c^15 + 12133000767559512121840834509406250 0000*c^14 +
 67199197069337895671786379136103515625 0*c^13 + 27563196245730109107821844 5
7415771484375*c^12 + 82509326929335662768520168981933593750 00*c^11 + 176753
307372988148751961048217773437500 00*c^10 + 26395310102345039267613256835937
500000000*c^9 + 26517130309637768395232421875000000000000*c^8 + 17053962622
098817730468750000000000000000*c^7 + 65313010745610903125000000000000000000
00*c^6 + 13296558805373250000000000000000000000000*c^5 + 1176035638500000000
000000000000000000000*c^4 + 28489320000000000000000000000000000000*c^3 + 2*(1
23*c^31 + 1174017*c^30 + 3116696475*c^29 + 3642741915411*c^28 + 22894201778
34530*c^27 + 86540910721390773 2*c^26 + 21122440311147729042 8*c^25 + 3493836
8873590512859484*c^24 + 40538678393611311204846 74*c^23 + 338376358236249991
856063294*c^22 + 20701487602239281351482423130*c^21 + 941134333565416888920
784690250*c^20 + 32110894210931478931787496469520*c^19 + 827795388994782805
334001918149812*c^18 + 16187607756398830013651232826868620*c^17 + 240466799
95340645278425043473784750 0*c^16 + 27108757065655725839277324352364968 75*c^
15 + 23112380375134430461260678935541015625*c^14 + 14811964125726775327546 8
014081201171875*c^13 + 70708903076008653096080868350268554687 5*c^12 + 24829
058052694410714513631659545898437 50*c^11 + 63052566716902500990712713159179
68750000*c^10 + 11320298752129488879797019653320312500000*c^9 + 13937594367
446090380968017578125000000000*c^8 + 11285670833401971164472656250000000000
000*c^7 + 56629495888077594140625000000000000000000*c^6 + 160997812765422937
500000000000000000000*c^5 + 22359437886375000000000000000000000000000*c^4 +
 11337043500000000000000000000000000000*c^3 + 87480000000000000000000000000
00000*c^2)*sqrt(c))*a^2*b^3*x^3 + 12*(c^31 + 29222*c^30 + 131989476*c^29 +
220549631950*c^28 + 182054403980255*c^27 + 85951284212292484*c^26 + 2534463
8059811815968*c^25 + 49466584911319161545 84*c^24 + 66545464411858444428524 2
*c^23 + 63533306064757209063076580*c^22 + 43983106103858932378474559 76*c^21
 + 22430252551805820973904794042 0*c^20 + 85233852811373597864513027844 62*c^
19 + 24324803242546833776564638420176 0*c^18 + 52392122191335423404805024204
89120*c^17 + 85350523863191078165738758341235000*c^16 + 1051232702462420068
262014831632403125*c^15 + 97601254572400856764424177046523437 50*c^14 + 6792
098800401774207913530017050781250 0*c^13 + 35119735768037891193253293571826 1
718750*c^12 + 13327529322541804346690814662780761718 75*c^11 + 3650321200872
523729722890099487304687500*c^10 + 70555888540279093978025799560546875000 0
*c^9 + 93364705099387213396354980468750000000000*c^8 + 81126348796082922299 0
234375000000000000*c^7 + 43618188069241019921875000000000000000000*c^6 + 13
267779470264606250000000000000000000000*c^5 + 19685515802625000000000000000
000000000*c^4 + 10646680500000000000000000000000000000*c^3 + 87480000000000
000000000000000000*c^2 + 2*(122*c^30 + 1144795*c^29 + 2984706999*c^28 +
3422192283461*c^27 + 2107365773854275*c^26 + 779457823001615248*c^25 + 1858
```

```
7976505166547446 0*c^24 + 29991710382458596704900*c^23 + 3388413195242546676
199432*c^22 + 27484305217149278279298671 4*c^21 + 16303176991853388113634967
154*c^20 + 71683180804735867918173674983 0*c^19 + 23587508929794119145336193
685058*c^18 + 58454735656931446756835553394 8052*c^17 + 10948395537265287673
170730406379500*c^16 + 15511627609021537461851167639661 2500*c^15 + 16596430
041031525156657176036040937 50*c^14 + 13352254917894344784818261230888671875
*c^13 + 80198653253250011196332713910693359375*c^12 + 35589167307970761902 8
27574778442382812 5*c^11 + 11501528730152606367822816996765136718 75*c^10 + 2
65493547081772636934838121643066406 2500*c^9 + 42647098981015794819944396972
65625000000*c^8 + 46011238575073690413325195312500000000000*c^7 + 3173035953
793678934570312500000000000000*c^6 + 13011307818836574218750000000000000000
00*c^5 + 28320018062776875000000000000000000000000*c^4 + 26739220837500000000
000000000000000000*c^3 + 690363000000000000000000000000000000*c^2)*sqrt(c))
*a^3*b^2*x^2 - 24*(c^30 + 28979*c^29 + 129728865*c^28 + 214709946817*c^27 +
 175424729360150*c^26 + 8191197739394 4084*c^25 + 23867634391202529556*c^24
+ 4598766595419787735220*c^23 + 61006998994908703861 0662*c^22 + 57366549664
221202749288378*c^21 + 39059910557071288750107709 26*c^20 + 1956021625900585
62386788777038*c^19 + 72853238276327009904746180618 40*c^18 + 20335833839351
280046544861489348 4*c^17 + 42734758443884262058092399674865 00*c^16 + 677272
086330489290252065374959625 00*c^15 + 80872735891503824805019801633514062 5*c
^14 + 72495668079488188931611805137792968 75*c^13 + 48466044976177871402659 9
58222509765625*c^12 + 23926609615005676094252746611938476562 5*c^11 + 860235
68224482195755505743682861328125 0*c^10 + 22102511370868244137133841369628 90
625000*c^9 + 39559690494792810728192016601562500000000*c^8 + 476301976321484
344846582031250000000000000*c^7 + 36734069278083975957031250000000000000000*c^
6 + 16891538271451417187500000000000000000000*c^5 + 41367021040428750000000 0
000000000000000*c^4 + 44125007175000000000000000000000000000000*c^3 + 12932460
000000000000000000000000000*c^2 + (243*c^29 + 2260611*c^28 + 5839685133*c
^27 + 6629674620105*c^26 + 4039306818348400*c^25 + 147700366860928641 2*c^24
 + 34789189571212841 9364*c^23 + 55384654169497405674580*c^22 + 616675640053
6006313788202*c^21 + 49231955467876436283668505 0*c^20 + 28700362927999647352
259163382*c^19 + 12380614535046587959766847226 22*c^18 + 398896940319555373
00197769308276*c^17 + 96573637474511613467126245300262 0*c^16 + 176233152301
421491405322208452725 00*c^15 + 24250534354738182021181681529726250 0*c^14 +
25105586492912667832812371908730468 75*c^13 + 19454943027839870676475341947 9
98046875*c^12 + 11193126153032215099000546959887695312 5*c^11 + 472517250009
358477114024029449462890625*c^10 + 14400700637856993160095059625244140625 00
*c^9 + 30996198045486283249833782958984375000 00*c^8 + 45734507467238778911 6
96777343750000000000*c^7 + 44392279517998946341992187500000000000000*c^6 + 26
726649797789602734375000000000000000000*c^5 + 91310773662217312500000000000
000000000*c^4 + 15273015085125000000000000000000000000000*c^3 + 9353434500000
000000000000000000000000*c^2 + 87480000000000000000000000000000000*c)*sqrt(
c))*a^4*b*x + 24*(c^29 + 28979*c^28 + 129728865*c^27 + 214709946817*c^26 +
175424729360150*c^25 + 8191197739394 4084*c^24 + 23867634391202529556*c^23 +
 4598766595419787735220*c^22 + 61006998994908703861 0662*c^21 + 573665496642
21202749288378*c^20 + 39059910557071288750107709 26*c^19 + 1956021625900585
62386788777038*c^18 + 72853238276327009904746180618 40*c^17 + 203358338393512
800465448614893484*c^16 + 42734758443884262058092399674865 00*c^15 + 6772720
863304892902520653749596 2500*c^14 + 80872735891503824805019801633514062 5*c^
13 + 72495668079488188931611805137792968 75*c^12 + 48466044976177871402659 95
8222509765625*c^11 + 23926609615005676094252746611938476562 5*c^10 + 8602356
822448219575550574368286132812 50*c^9 + 22102511370868244137133841369628 9062
5000*c^8 + 39559690494792810728192016601562500000000*c^7 + 47630197632148434
484658203125000000000000*c^6 + 36734069278083975957031250000000000000000*c^5
+ 16891538271451417187500000000000000000000*c^4 + 41367021040428750000000 0
0000000000000*c^3 + 44125007175000000000000000000000000000000*c^2 + (243*c^28
+ 2260611*c^27 + 5839685133*c^26 + 6629674620105*c^25 + 4039306818348400*c^
24 + 14770036686092864 12*c^23 + 34789189571212841 9364*c^22 + 55384654169497
405674580*c^21 + 61667564005360063137882 02*c^20 + 49231955467876436283668 50
```

```
50*c^19 + 28700362927999647352259163382*c^18 + 12380614535046587959766847226
22*c^17 + 39889694031955537300197769308276*c^16 + 96573637474511613467126245
3002620*c^15 + 17623315230142149140532220845272500*c^14 + 242505343547381820
2118168152972625000*c^13 + 25105586492912667832812371908730468 75*c^12 + 1945
4943027839870676475341947998046875*c^11 + 11193126153032215099000546959887
6953125*c^10 + 472517250009358477114024029449462890625*c^9 + 144007006378569
9316009505962524414062500*c^8 + 309961980454862832498337829589843750000000
*c^7 + 45734507467238778911696777343750000000000*c^6 + 443922795179989463419
9218750000000000000*c^5 + 26726649797789602734375000000000000000000*c^4 + 91
3107736622173125000000000000000000000000*c^3 + 15273015085125000000000000000
0000000*c^2 + 935343450000000000000000000000000000*c + 87480000000000000000
00000000000000)*sqrt(c) + 12932460000000000000000000000000000000*c)*a^5)*
sqrt(b*sqrt(c)*x + a)/((c^34 + 32709*c^33 + 166156194*c^32 + 313848784218*c
^31 + 294469940278425*c^30 + 158963889741950277*c^29 + 53942913469754556576
*c^28 + 12201148378415160967656*c^27 + 191672761190088158304 7746*c^26 + 215
48720108070108926457213 8*c^25 + 17728266623214387509113175244*c^24 + 108534
71914239421389958054925 40*c^23 + 50069331004982686422553600150074*c^22 + 17
56559137361209677029762976878226*c^21 + 47164907489395671284893054638492840
*c^20 + 97312001617950569573156595213479973 6*c^19 + 154551869190487507915420
38868588818525*c^18 + 18888552734632944572407581098257244062 5*c^17 + 17720
71002728366862097941073371656281250*c^16 + 12702859366636907387945569820995
722656250*c^15 + 690844359362760290126373527451930175781 25*c^14 + 282238378
06554204851374444439996704101562 5*c^13 + 85474402405807536083911499053344726
5625000*c^12 + 18854538254093270834055112130493164062 50000*c^11 + 29600348
20194099448691930273437500000000000*c^10 + 32067265112702520859698984375000
00000000000*c^9 + 22982588344528282893187500000000000000000000*c^8 + 1026494
23210554120437500000000000000000000*c^7 + 26117507036034180000000000000000
0000000000*c^6 + 32633630940600000000000000000000000000000*c^5 + 1496444544
00000000000000000000000000000*c^4 + 10497600000000000000000000000000000000
*c^3 + 2*(129*c^33 + 1358088*c^32 + 3992178510*c^31 + 5188254469092*c^30 +
3642062740341821*c^29 + 1545168634611811116*c^28 + 42550486968067190311 2*c^
27 + 79860855208415962132944*c^26 + 10579205416850555553082194*c^25 + 10149
92544330040094059234080*c^24 + 71903882119146039055870044180*c^23 + 3816126
13635564961667256751653 6*c^22 + 15337393997303099259594150988504 2*c^21 + 47
04317201176235036305308699382664*c^20 + 11068634295595776755928260508585751
2*c^19 + 20034710499399888428990061940896304 80*c^18 + 279173253256829321309
56522490231038125*c^17 + 29906665763033096334257767822263427 5000*c^16 + 245
43424891517375844383286381880654687 50*c^15 + 15340564224279798311767238446307226562500*c^14 + 72413767119232942192838391775906494140625*c^13 + 2551970
14414145705441862975227416992187500*c^12 + 66124732753337112109288390737792
9687500000*c^11 + 12346234184505667495929906269531250000000000*c^10 + 161731
46910315460074697617187500000000000000*c^9 + 14339271637804263623840625000000000000000*c^8 + 81857529391194999562500000000000000000000000*c^7 + 2798095
33898669190000000000000000000000000000*c^6 + 51115570649316000000000000000000000
00000000*c^5 + 40781290320000000000000000000000000000000*c^4 + 8957952000000
00000000000000000000*c^3)*sqrt(c))*b^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \sqrt{a + b \sqrt{c\, x^2}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*(c*x^2)^(1/2))^(1/2),x)
```

```
[Out] int(x^4*(a + b*(c*x^2))^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \sqrt{a + b\sqrt{cx^2}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*(c*x**2)**(1/2))**(1/2),x)
[Out] Integral(x**4*sqrt(a + b*sqrt(c*x**2)), x)
```

3.2937 $\int x^2\sqrt{a+b\sqrt{cx^2}}\,dx$

Optimal. Leaf size=113

$$\frac{2a^2x^3\left(a+b\sqrt{cx^2}\right)^{3/2}}{3b^3\left(cx^2\right)^{3/2}}+\frac{2x^3\left(a+b\sqrt{cx^2}\right)^{7/2}}{7b^3\left(cx^2\right)^{3/2}}-\frac{4ax^3\left(a+b\sqrt{cx^2}\right)^{5/2}}{5b^3\left(cx^2\right)^{3/2}}$$

```
[Out] 2/3*a^2*x^3*(a+b*(c*x^2)^(1/2))^(3/2)/b^3/(c*x^2)^(3/2)-4/5*a*x^3*(a+b*(c*x
^2)^(1/2))^(5/2)/b^3/(c*x^2)^(3/2)+2/7*x^3*(a+b*(c*x^2)^(1/2))^(7/2)/b^3/(c
*x^2)^(3/2)
```

Rubi [A] time = 0.04, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {368, 43}

$$\frac{2a^2x^3\left(a+b\sqrt{cx^2}\right)^{3/2}}{3b^3\left(cx^2\right)^{3/2}}+\frac{2x^3\left(a+b\sqrt{cx^2}\right)^{7/2}}{7b^3\left(cx^2\right)^{3/2}}-\frac{4ax^3\left(a+b\sqrt{cx^2}\right)^{5/2}}{5b^3\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*Sqrt[c*x^2]],x]
```

```
[Out] (2*a^2*x^3*(a + b*Sqrt[c*x^2])^(3/2))/(3*b^3*(c*x^2)^(3/2)) - (4*a*x^3*(a +
 b*Sqrt[c*x^2])^(5/2))/(5*b^3*(c*x^2)^(3/2)) + (2*x^3*(a + b*Sqrt[c*x^2])^(
7/2))/(7*b^3*(c*x^2)^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{a+b\sqrt{cx^2}}\,dx &= \frac{x^3\,\text{Subst}\left(\int x^2\sqrt{a+bx}\,dx,x,\sqrt{cx^2}\right)}{\left(cx^2\right)^{3/2}}\\ &= \frac{x^3\,\text{Subst}\left(\int\left(\frac{a^2\sqrt{a+bx}}{b^2}-\frac{2a(a+bx)^{3/2}}{b^2}+\frac{(a+bx)^{5/2}}{b^2}\right)dx,x,\sqrt{cx^2}\right)}{\left(cx^2\right)^{3/2}}\\ &= \frac{2a^2x^3\left(a+b\sqrt{cx^2}\right)^{3/2}}{3b^3\left(cx^2\right)^{3/2}}-\frac{4ax^3\left(a+b\sqrt{cx^2}\right)^{5/2}}{5b^3\left(cx^2\right)^{3/2}}+\frac{2x^3\left(a+b\sqrt{cx^2}\right)^{7/2}}{7b^3\left(cx^2\right)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 64, normalized size = 0.57

$$\frac{2x^3\left(a+b\sqrt{cx^2}\right)^{3/2}\left(8a^2-12ab\sqrt{cx^2}+15b^2cx^2\right)}{105b^3\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b*Sqrt[c*x^2]],x]
```

```
[Out] (2*x^3*(a + b*Sqrt[c*x^2])^(3/2)*(8*a^2 + 15*b^2*c*x^2 - 12*a*b*Sqrt[c*x^2]
))/(105*b^3*(c*x^2)^(3/2))
```

fricas [A] time = 0.91, size = 70, normalized size = 0.62

$$\frac{2\left(15\,b^3c^2x^4-4\,a^2bcx^2+\left(3\,ab^2cx^2+8\,a^3\right)\sqrt{cx^2}\right)\sqrt{\sqrt{cx^2}\,b+a}}{105\,b^3c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*b^3*c^2*x^4 - 4*a^2*b*c*x^2 + (3*a*b^2*c*x^2 + 8*a^3)*sqrt(c*x^2)
)*sqrt(sqrt(c*x^2)*b + a)/(b^3*c^2*x)
```

giac [A] time = 0.17, size = 135, normalized size = 1.19

$$\frac{2\left(\frac{7\left(3\left(b\sqrt{c}\,x+a\right)^{\frac{5}{2}}-10\left(b\sqrt{c}\,x+a\right)^{\frac{3}{2}}a+15\sqrt{b\sqrt{c}\,x+a}\,a^2\right)a}{b^2c}+\frac{3\left(5\left(b\sqrt{c}\,x+a\right)^{\frac{7}{2}}\sqrt{c}-21\left(b\sqrt{c}\,x+a\right)^{\frac{5}{2}}a\sqrt{c}+35\left(b\sqrt{c}\,x+a\right)^{\frac{3}{2}}a^2\sqrt{c}-35\sqrt{b\sqrt{c}\,x+a}\,a^3\sqrt{c}\right)}{b^2c^{\frac{3}{2}}}\right)}{105\,b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(7*(3*(b*sqrt(c)*x + a)^(5/2) - 10*(b*sqrt(c)*x + a)^(3/2)*a + 15*sqr
t(b*sqrt(c)*x + a)*a^2)*a/(b^2*c) + 3*(5*(b*sqrt(c)*x + a)^(7/2)*sqrt(c) -
21*(b*sqrt(c)*x + a)^(5/2)*a*sqrt(c) + 35*(b*sqrt(c)*x + a)^(3/2)*a^2*sqrt(
c) - 35*sqrt(b*sqrt(c)*x + a)*a^3*sqrt(c))/(b^2*c^(3/2)))/(b*sqrt(c))
```

maple [A] time = 0.00, size = 55, normalized size = 0.49

$$-\frac{2\left(a+\sqrt{c\,x^2}\,b\right)^{\frac{3}{2}}\left(-15b^2c\,x^2-8a^2+12\sqrt{c\,x^2}\,ab\right)x^3}{105\left(c\,x^2\right)^{\frac{3}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+(c*x^2)^(1/2)*b)^(1/2),x)
```

```
[Out] -2/105*x^3*(a+(c*x^2)^(1/2)*b)^(3/2)*(-15*c*x^2*b^2+12*(c*x^2)^(1/2)*a*b-8*
a^2)/(c*x^2)^(3/2)/b^3
```

maxima [B] time = 0.71, size = 387, normalized size = 3.42

$$\frac{\left(\left(31\,c^8+3784\,c^7+91078\,c^6+622632\,c^5+1266003\,c^4+635688\,c^3+34992\,c^2+\left(c^8+440\,c^7+21986\,c^6+2765\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] ((31*c^8 + 3784*c^7 + 91078*c^6 + 622632*c^5 + 1266003*c^4 + 635688*c^3 + 3
4992*c^2 + (c^8 + 440*c^7 + 21986*c^6 + 276544*c^5 + 1038501*c^4 + 1095120*
c^3 + 221616*c^2)*sqrt(c))*b^3*x^3 + (c^8 + 382*c^7 + 15946*c^6 + 158172*c^
5 + 425925*c^4 + 266814*c^3 + 17496*c^2 + (29*c^7 + 3020*c^6 + 59186*c^5 +
306288*c^4 + 414153*c^3 + 102060*c^2)*sqrt(c))*a*b^2*x^2 - 2*(c^7 + 354*c^6
 + 13280*c^5 + 112266*c^4 + 231903*c^3 + 84564*c^2 + 2*(14*c^6 + 1333*c^5 +
 22953*c^4 + 97011*c^3 + 91125*c^2 + 8748*c)*sqrt(c))*a^2*b*x + 2*(c^6 + 35
4*c^5 + 13280*c^4 + 112266*c^3 + 231903*c^2 + 2*(14*c^5 + 1333*c^4 + 22953*
c^3 + 97011*c^2 + 91125*c + 8748)*sqrt(c) + 84564*c)*a^3)*sqrt(b*sqrt(c)*x
+ a)/((c^9 + 533*c^8 + 33338*c^7 + 549778*c^6 + 2906397*c^5 + 4893129*c^4 +
 2128680*c^3 + 104976*c^2 + 2*(17*c^8 + 2552*c^7 + 78518*c^6 + 726132*c^5 +
 2190753*c^4 + 1960524*c^3 + 349920*c^2)*sqrt(c))*b^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \sqrt{a + b \sqrt{c\, x^2}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*(c*x^2)^(1/2))^(1/2),x)
```

```
[Out] int(x^2*(a + b*(c*x^2)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{a + b\sqrt{cx^2}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*(c*x**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a + b*sqrt(c*x**2)), x)
```

3.2938 $\int \sqrt{a+b\sqrt{cx^2}}\,dx$

Optimal. Leaf size=34

$$\frac{2x\left(a+b\sqrt{cx^2}\right)^{3/2}}{3b\sqrt{cx^2}}$$

```
[Out] 2/3*x*(a+b*(c*x^2)^(1/2))^(3/2)/b/(c*x^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {254, 32}

$$\frac{2x\left(a+b\sqrt{cx^2}\right)^{3/2}}{3b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[c*x^2]],x]
```

```
[Out] (2*x*(a + b*Sqrt[c*x^2])^(3/2))/(3*b*Sqrt[c*x^2])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 254

```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_.), x_Symbol] :> Dist[x/(c*x^
q)^(1/q), Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\sqrt{cx^2}}\,dx &= \frac{x\,\text{Subst}\left(\int \sqrt{a+bx}\,dx, x, \sqrt{cx^2}\right)}{\sqrt{cx^2}}\\ &= \frac{2x\left(a+b\sqrt{cx^2}\right)^{3/2}}{3b\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.00

$$\frac{2x\left(a+b\sqrt{cx^2}\right)^{3/2}}{3b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[c*x^2]],x]
```

```
[Out] (2*x*(a + b*Sqrt[c*x^2])^(3/2))/(3*b*Sqrt[c*x^2])
```

fricas [A] time = 0.96, size = 40, normalized size = 1.18

$$\frac{2\left(bcx^2+\sqrt{cx^2}\,a\right)\sqrt{\sqrt{cx^2}\,b+a}}{3\,bcx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b*c*x^2 + sqrt(c*x^2)*a)*sqrt(sqrt(c*x^2)*b + a)/(b*c*x)
```

giac [A] time = 0.16, size = 18, normalized size = 0.53

$$\frac{2\left(b\sqrt{c}\,x+a\right)^{\frac{3}{2}}}{3\,b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(b*sqrt(c)*x + a)^(3/2)/(b*sqrt(c))
```

maple [A] time = 0.00, size = 27, normalized size = 0.79

$$\frac{2\left(a+\sqrt{c\,x^2}\,b\right)^{\frac{3}{2}}x}{3\sqrt{c\,x^2}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(c*x^2)^(1/2)*b)^(1/2),x)
```

```
[Out] 2/3*x*(a+(c*x^2)^(1/2)*b)^(3/2)/b/(c*x^2)^(1/2)
```

maxima [A] time = 0.68, size = 42, normalized size = 1.24

$$\frac{\left(\left(c^{\frac{3}{2}}+c\right)bx+a\left(c+\sqrt{c}\right)\right)\sqrt{b\sqrt{c}\,x+a}}{\left(c^2+2\,c^{\frac{3}{2}}+c\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] ((c^(3/2) + c)*b*x + a*(c + sqrt(c)))*sqrt(b*sqrt(c)*x + a)/((c^2 + 2*c^(3/
2) + c)*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{a+b\,\sqrt{c\,x^2}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x^2)^(1/2))^(1/2),x)
```

```
[Out] int((a + b*(c*x^2)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sqrt{cx^2}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x**2)**(1/2))**(1/2),x)
[Out] Integral(sqrt(a + b*sqrt(c*x**2)), x)
```

3.2939 $\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^2}\,dx$

Optimal. Leaf size=67

$$-\frac{\sqrt{a+b\sqrt{cx^2}}}{x}-\frac{b\sqrt{cx^2}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{cx^2}}}{\sqrt{a}}\right)}{\sqrt{a}\,x}$$

```
[Out] -b*arctanh((a+b*(c*x^2)^(1/2))^(1/2)/a^(1/2))*(c*x^2)^(1/2)/x/a^(1/2)-(a+b*
(c*x^2)^(1/2))^(1/2)/x
```

Rubi [A] time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {368, 47, 63, 208}

$$-\frac{\sqrt{a+b\sqrt{cx^2}}}{x}-\frac{b\sqrt{cx^2}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{cx^2}}}{\sqrt{a}}\right)}{\sqrt{a}\,x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[c*x^2]]/x^2,x]
```

```
[Out] -(Sqrt[a + b*Sqrt[c*x^2]]/x) - (b*Sqrt[c*x^2]*ArcTanh[Sqrt[a + b*Sqrt[c*x^2
]]/Sqrt[a]])/(Sqrt[a]*x)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^2}\,dx &= \frac{\sqrt{cx^2}\ \text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx, x, \sqrt{cx^2}\right)}{x}\\ &= -\frac{\sqrt{a+b\sqrt{cx^2}}}{x} + \frac{\left(b\sqrt{cx^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sqrt{cx^2}\right)}{2x}\\ &= -\frac{\sqrt{a+b\sqrt{cx^2}}}{x} + \frac{\sqrt{cx^2}\ \text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sqrt{cx^2}}\right)}{x}\\ &= -\frac{\sqrt{a+b\sqrt{cx^2}}}{x} - \frac{b\sqrt{cx^2}\ \tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{cx^2}}}{\sqrt{a}}\right)}{\sqrt{a}\,x}\end{aligned}$$

Mathematica [A] time = 0.05, size = 87, normalized size = 1.30

$$-\frac{b\sqrt{cx^2}\ \sqrt{\frac{b\sqrt{cx^2}}{a}+1}\ \tanh^{-1}\left(\sqrt{\frac{b\sqrt{cx^2}}{a}+1}\right)+a+b\sqrt{cx^2}}{x\sqrt{a+b\sqrt{cx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[c*x^2]]/x^2,x]
```

```
[Out] -((a + b*Sqrt[c*x^2] + b*Sqrt[c*x^2]*Sqrt[1 + (b*Sqrt[c*x^2])/a]*ArcTanh[Sq
rt[1 + (b*Sqrt[c*x^2])/a]])/(x*Sqrt[a + b*Sqrt[c*x^2]]))
```

fricas [A] time = 1.11, size = 180, normalized size = 2.69

$$\left[\frac{bx\sqrt{\frac{c}{a}}\ \log\left(\frac{bcx^2-2\sqrt{\sqrt{cx^2}\,b+a}\,ax\sqrt{\frac{c}{a}}+2\sqrt{cx^2}\,a}{x^2}\right)-2\sqrt{\sqrt{cx^2}\,b+a}}{2x}, -\frac{bx\sqrt{-\frac{c}{a}}\ \arctan\left(-\frac{\left(abcx^2\sqrt{-\frac{c}{a}}-\sqrt{cx^2}\,a^2\sqrt{-\frac{c}{a}}\right)\sqrt{\sqrt{cx^2}\,b+a}}{b^2c^2x^3-a^2cx}\right)}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(1/2))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(b*x*sqrt(c/a)*log((b*c*x^2 - 2*sqrt(sqrt(c*x^2)*b + a)*a*x*sqrt(c/a)
+ 2*sqrt(c*x^2)*a)/x^2) - 2*sqrt(sqrt(c*x^2)*b + a))/x, -(b*x*sqrt(-c/a)*ar
ctan(-(a*b*c*x^2*sqrt(-c/a) - sqrt(c*x^2)*a^2*sqrt(-c/a))*sqrt(sqrt(c*x^2)*
b + a)/(b^2*c^2*x^3 - a^2*c*x)) + sqrt(sqrt(c*x^2)*b + a))/x]
```

giac [A] time = 0.16, size = 54, normalized size = 0.81

$$\frac{\frac{b^2c\arctan\left(\frac{\sqrt{b\sqrt{c}\,x+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\sqrt{b\sqrt{c}\,x+a}\,b\sqrt{c}}{x}}{b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(1/2))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] (b^2*c*arctan(sqrt(b*sqrt(c)*x + a)/sqrt(-a))/sqrt(-a) - sqrt(b*sqrt(c)*x +
 a)*b*sqrt(c)/x)/(b*sqrt(c))
```

maple [A] time = 0.01, size = 54, normalized size = 0.81

$$-\frac{\sqrt{c\,x^2}\;b\,\operatorname{arctanh}\left(\frac{\sqrt{a+\sqrt{c\,x^2}\;b}}{\sqrt{a}}\right)+\sqrt{a+\sqrt{c\,x^2}\;b}\;\sqrt{a}}{\sqrt{a}\;x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(c*x^2)^(1/2)*b)^(1/2)/x^2,x)
```

```
[Out] -(arctanh((a+(c*x^2)^(1/2)*b)^(1/2)/a^(1/2))*b*(c*x^2)^(1/2)+(a+(c*x^2)^(1/
2)*b)^(1/2)*a^(1/2))/x/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{cx^2}\,b+a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(1/2))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(c*x^2)*b + a)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+b\,\sqrt{c\,x^2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x^2)^(1/2))^(1/2)/x^2,x)
```

```
[Out] int((a + b*(c*x^2)^(1/2))^(1/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x**2)**(1/2))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(a + b*sqrt(c*x**2))/x**2, x)
```

3.2940 $\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^4}\,dx$

Optimal. Leaf size=144

$$-\frac{b^3\left(cx^2\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{cx^2}}}{\sqrt{a}}\right)}{8a^{5/2}x^3}+\frac{b^2c\sqrt{a+b\sqrt{cx^2}}}{8a^2x}-\frac{b\left(cx^2\right)^{3/2}\sqrt{a+b\sqrt{cx^2}}}{12acx^5}-\frac{\sqrt{a+b\sqrt{cx^2}}}{3x^3}$$

```
[Out] -1/8*b^3*(c*x^2)^(3/2)*arctanh((a+b*(c*x^2)^(1/2))^(1/2)/a^(1/2))/a^(5/2)/x
^3-1/3*(a+b*(c*x^2)^(1/2))^(1/2)/x^3+1/8*b^2*c*(a+b*(c*x^2)^(1/2))^(1/2)/a^
2/x-1/12*b*(c*x^2)^(3/2)*(a+b*(c*x^2)^(1/2))^(1/2)/a/c/x^5
```

Rubi [A] time = 0.06, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {368, 47, 51, 63, 208}

$$\frac{b^2c\sqrt{a+b\sqrt{cx^2}}}{8a^2x}-\frac{b^3\left(cx^2\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{cx^2}}}{\sqrt{a}}\right)}{8a^{5/2}x^3}-\frac{b\left(cx^2\right)^{3/2}\sqrt{a+b\sqrt{cx^2}}}{12acx^5}-\frac{\sqrt{a+b\sqrt{cx^2}}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[c*x^2]]/x^4,x]
```

```
[Out] -Sqrt[a + b*Sqrt[c*x^2]]/(3*x^3) + (b^2*c*Sqrt[a + b*Sqrt[c*x^2]])/(8*a^2*x
) - (b*(c*x^2)^(3/2)*Sqrt[a + b*Sqrt[c*x^2]])/(12*a*c*x^5) - (b^3*(c*x^2)^(
3/2)*ArcTanh[Sqrt[a + b*Sqrt[c*x^2]]/Sqrt[a]])/(8*a^(5/2)*x^3)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^4}\,dx = \frac{\left(cx^2\right)^{3/2}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^4}\,dx, x, \sqrt{cx^2}\right)}{x^3}$$

$$= -\frac{\sqrt{a+b\sqrt{cx^2}}}{3x^3} + \frac{\left(b\left(cx^2\right)^{3/2}\right)\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx, x, \sqrt{cx^2}\right)}{6x^3}$$

$$= -\frac{\sqrt{a+b\sqrt{cx^2}}}{3x^3} - \frac{b\left(cx^2\right)^{3/2}\sqrt{a+b\sqrt{cx^2}}}{12acx^5} - \frac{\left(b^2\left(cx^2\right)^{3/2}\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \sqrt{cx^2}\right)}{8ax^3}$$

$$= -\frac{\sqrt{a+b\sqrt{cx^2}}}{3x^3} + \frac{b^2c\sqrt{a+b\sqrt{cx^2}}}{8a^2x} - \frac{b\left(cx^2\right)^{3/2}\sqrt{a+b\sqrt{cx^2}}}{12acx^5} + \frac{\left(b^3\left(cx^2\right)^{3/2}\right)\text{Subst}\left(\int$$

$$= -\frac{\sqrt{a+b\sqrt{cx^2}}}{3x^3} + \frac{b^2c\sqrt{a+b\sqrt{cx^2}}}{8a^2x} - \frac{b\left(cx^2\right)^{3/2}\sqrt{a+b\sqrt{cx^2}}}{12acx^5} + \frac{\left(b^2\left(cx^2\right)^{3/2}\right)\text{Subst}\left(\int$$

$$= -\frac{\sqrt{a+b\sqrt{cx^2}}}{3x^3} + \frac{b^2c\sqrt{a+b\sqrt{cx^2}}}{8a^2x} - \frac{b\left(cx^2\right)^{3/2}\sqrt{a+b\sqrt{cx^2}}}{12acx^5} - \frac{b^3\left(cx^2\right)^{3/2}\tanh^{-1}\left($$

Mathematica [C] time = 0.01, size = 63, normalized size = 0.44

$$\frac{2b^3\left(cx^2\right)^{3/2}\left(a+b\sqrt{cx^2}\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},4;\frac{5}{2};\frac{\sqrt{cx^2}\,b}{a}+1\right)}{3a^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[c*x^2]]/x^4,x]
```

```
[Out] (2*b^3*(c*x^2)^(3/2)*(a + b*Sqrt[c*x^2])^(3/2)*Hypergeometric2F1[3/2, 4, 5/
2, 1 + (b*Sqrt[c*x^2])/a])/(3*a^4*x^3)
```

fricas [A] time = 0.74, size = 252, normalized size = 1.75

$$\left[\frac{3\,b^3cx^3\sqrt{\frac{c}{a}}\,\log\left(\frac{bcx^2-2\sqrt{cx^2}\,b+a\,ax\sqrt{\frac{c}{a}}+2\sqrt{cx^2}\,a}{x^2}\right)+2\left(3\,b^2cx^2-2\sqrt{cx^2}\,ab-8\,a^2\right)\sqrt{\sqrt{cx^2}\,b+a}}{48\,a^2x^3}, -\frac{3\,b^3cx^3\sqrt{-\frac{c}{a}}\,\text{ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(1/2))^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(3*b^3*c*x^3*sqrt(c/a)*log((b*c*x^2 - 2*sqrt(sqrt(c*x^2)*b + a)*a*x*s
qrt(c/a) + 2*sqrt(c*x^2)*a)/x^2) + 2*(3*b^2*c*x^2 - 2*sqrt(c*x^2)*a*b - 8*a
^2)*sqrt(sqrt(c*x^2)*b + a))/(a^2*x^3), -1/24*(3*b^3*c*x^3*sqrt(-c/a)*arcta
```

```
n(-(a*b*c*x^2*sqrt(-c/a) - sqrt(c*x^2)*a^2*sqrt(-c/a))*sqrt(sqrt(c*x^2)*b +
 a)/(b^2*c^2*x^3 - a^2*c*x)) - (3*b^2*c*x^2 - 2*sqrt(c*x^2)*a*b - 8*a^2)*sq
rt(sqrt(c*x^2)*b + a))/(a^2*x^3)]
```

giac [A] time = 0.17, size = 114, normalized size = 0.79

$$\frac{\frac{3\,b^4c^2\arctan\left(\frac{\sqrt{b\sqrt{c}\,x+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{3\left(b\sqrt{c}\,x+a\right)^{\frac{5}{2}}b^4c^2-8\left(b\sqrt{c}\,x+a\right)^{\frac{3}{2}}ab^4c^2-3\sqrt{b\sqrt{c}\,x+a}\,a^2b^4c^2}{a^2b^3c^{\frac{3}{2}}x^3}}{24\,b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(1/2))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/24*(3*b^4*c^2*arctan(sqrt(b*sqrt(c)*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*
(b*sqrt(c)*x + a)^(5/2)*b^4*c^2 - 8*(b*sqrt(c)*x + a)^(3/2)*a*b^4*c^2 - 3*s
qrt(b*sqrt(c)*x + a)*a^2*b^4*c^2)/(a^2*b^3*c^(3/2)*x^3))/(b*sqrt(c))
```

maple [A] time = 0.01, size = 97, normalized size = 0.67

$$-\frac{3\left(c\,x^2\right)^{\frac{3}{2}}a^2b^3\operatorname{arctanh}\left(\frac{\sqrt{a+\sqrt{c\,x^2}\,b}}{\sqrt{a}}\right)+3\sqrt{a+\sqrt{c\,x^2}\,b}\;a^{\frac{9}{2}}+8\left(a+\sqrt{c\,x^2}\,b\right)^{\frac{3}{2}}a^{\frac{7}{2}}-3\left(a+\sqrt{c\,x^2}\,b\right)^{\frac{5}{2}}a^{\frac{5}{2}}}{24a^{\frac{9}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(c*x^2)^(1/2)*b)^(1/2)/x^4,x)
```

```
[Out] -1/24*(-3*(a+(c*x^2)^(1/2)*b)^(5/2)*a^(5/2)+3*arctanh((a+(c*x^2)^(1/2)*b)^(
1/2)/a^(1/2))*a^2*b^3*(c*x^2)^(3/2)+8*(a+(c*x^2)^(1/2)*b)^(3/2)*a^(7/2)+3*(
a+(c*x^2)^(1/2)*b)^(1/2)*a^(9/2))/x^3/a^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{cx^2}\,b+a}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(1/2))^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(c*x^2)*b + a)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+b\sqrt{c\,x^2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x^2)^(1/2))^(1/2)/x^4,x)
```

```
[Out] int((a + b*(c*x^2)^(1/2))^(1/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x**2)**(1/2))**(1/2)/x**4,x)
[Out] Integral(sqrt(a + b*sqrt(c*x**2))/x**4, x)
```

3.2941 $\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^6}\,dx$

Optimal. Leaf size=219

$$-\frac{7b^5\left(cx^2\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{cx^2}}}{\sqrt{a}}\right)}{128a^{9/2}x^5}+\frac{7b^4c^2\sqrt{a+b\sqrt{cx^2}}}{128a^4x}-\frac{7b^3\left(cx^2\right)^{5/2}\sqrt{a+b\sqrt{cx^2}}}{192a^3cx^7}+\frac{7b^2c\sqrt{a+b\sqrt{cx^2}}}{240a^2x^3}-\frac{b\left(cx^2\right)^{5/2}}{40}$$

```
[Out] -7/128*b^5*(c*x^2)^(5/2)*arctanh((a+b*(c*x^2)^(1/2))^(1/2)/a^(1/2))/a^(9/2)
/x^5-1/5*(a+b*(c*x^2)^(1/2))^(1/2)/x^5+7/240*b^2*c*(a+b*(c*x^2)^(1/2))^(1/2
)/a^2/x^3+7/128*b^4*c^2*(a+b*(c*x^2)^(1/2))^(1/2)/a^4/x-1/40*b*(c*x^2)^(5/2
)*(a+b*(c*x^2)^(1/2))^(1/2)/a/c^2/x^9-7/192*b^3*(c*x^2)^(5/2)*(a+b*(c*x^2)^
(1/2))^(1/2)/a^3/c/x^7
```

Rubi [A] time = 0.09, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {368, 47, 51, 63, 208}

$$\frac{7b^4c^2\sqrt{a+b\sqrt{cx^2}}}{128a^4x}-\frac{7b^3\left(cx^2\right)^{5/2}\sqrt{a+b\sqrt{cx^2}}}{192a^3cx^7}+\frac{7b^2c\sqrt{a+b\sqrt{cx^2}}}{240a^2x^3}-\frac{7b^5\left(cx^2\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{cx^2}}}{\sqrt{a}}\right)}{128a^{9/2}x^5}-\frac{b\left(cx^2\right)^{5/2}}{40a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[c*x^2]]/x^6,x]
```

```
[Out] -Sqrt[a + b*Sqrt[c*x^2]]/(5*x^5) + (7*b^2*c*Sqrt[a + b*Sqrt[c*x^2]])/(240*a
^2*x^3) + (7*b^4*c^2*Sqrt[a + b*Sqrt[c*x^2]])/(128*a^4*x) - (b*(c*x^2)^(5/2
)*Sqrt[a + b*Sqrt[c*x^2]])/(40*a*c^2*x^9) - (7*b^3*(c*x^2)^(5/2)*Sqrt[a + b
*Sqrt[c*x^2]])/(192*a^3*c*x^7) - (7*b^5*(c*x^2)^(5/2)*ArcTanh[Sqrt[a + b*Sq
rt[c*x^2]]/Sqrt[a]])/(128*a^(9/2)*x^5)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sqrt{cx^2}}}{x^6}\,dx = \frac{\left(cx^2\right)^{5/2}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^6}\,dx, x, \sqrt{cx^2}\right)}{x^5}$$

$$= -\frac{\sqrt{a+b\sqrt{cx^2}}}{5x^5} + \frac{\left(b\left(cx^2\right)^{5/2}\right)\text{Subst}\left(\int \frac{1}{x^5\sqrt{a+bx}}\,dx, x, \sqrt{cx^2}\right)}{10x^5}$$

$$= -\frac{\sqrt{a+b\sqrt{cx^2}}}{5x^5} - \frac{b\left(cx^2\right)^{5/2}\sqrt{a+b\sqrt{cx^2}}}{40ac^2x^9} - \frac{\left(7b^2\left(cx^2\right)^{5/2}\right)\text{Subst}\left(\int \frac{1}{x^4\sqrt{a+bx}}\,dx, x, \sqrt{cx^2}\right.}{80ax^5}$$

$$= -\frac{\sqrt{a+b\sqrt{cx^2}}}{5x^5} + \frac{7b^2c\sqrt{a+b\sqrt{cx^2}}}{240a^2x^3} - \frac{b\left(cx^2\right)^{5/2}\sqrt{a+b\sqrt{cx^2}}}{40ac^2x^9} + \frac{\left(7b^3\left(cx^2\right)^{5/2}\right)\text{Subst}\left(\right.}{96a}$$

$$= -\frac{\sqrt{a+b\sqrt{cx^2}}}{5x^5} + \frac{7b^2c\sqrt{a+b\sqrt{cx^2}}}{240a^2x^3} - \frac{b\left(cx^2\right)^{5/2}\sqrt{a+b\sqrt{cx^2}}}{40ac^2x^9} - \frac{7b^3\left(cx^2\right)^{5/2}\sqrt{a+b\sqrt{c}}}{192a^3cx^7}$$

$$= -\frac{\sqrt{a+b\sqrt{cx^2}}}{5x^5} + \frac{7b^2c\sqrt{a+b\sqrt{cx^2}}}{240a^2x^3} + \frac{7b^4c^2\sqrt{a+b\sqrt{cx^2}}}{128a^4x} - \frac{b\left(cx^2\right)^{5/2}\sqrt{a+b\sqrt{cx^2}}}{40ac^2x^9} -$$

$$= -\frac{\sqrt{a+b\sqrt{cx^2}}}{5x^5} + \frac{7b^2c\sqrt{a+b\sqrt{cx^2}}}{240a^2x^3} + \frac{7b^4c^2\sqrt{a+b\sqrt{cx^2}}}{128a^4x} - \frac{b\left(cx^2\right)^{5/2}\sqrt{a+b\sqrt{cx^2}}}{40ac^2x^9} -$$

$$= -\frac{\sqrt{a+b\sqrt{cx^2}}}{5x^5} + \frac{7b^2c\sqrt{a+b\sqrt{cx^2}}}{240a^2x^3} + \frac{7b^4c^2\sqrt{a+b\sqrt{cx^2}}}{128a^4x} - \frac{b\left(cx^2\right)^{5/2}\sqrt{a+b\sqrt{cx^2}}}{40ac^2x^9} -$$

Mathematica [C] time = 0.01, size = 63, normalized size = 0.29

$$\frac{2b^5\left(cx^2\right)^{5/2}\left(a+b\sqrt{cx^2}\right)^{3/2}\,{}_2F_1\left(\frac{3}{2}, 6; \frac{5}{2}; \frac{\sqrt{cx^2}\,b}{a}+1\right)}{3a^6x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[c*x^2]]/x^6,x]
```

```
[Out] (2*b^5*(c*x^2)^(5/2)*(a + b*Sqrt[c*x^2])^(3/2)*Hypergeometric2F1[3/2, 6, 5/
2, 1 + (b*Sqrt[c*x^2])/a])/(3*a^6*x^5)
```

fricas [A] time = 1.04, size = 314, normalized size = 1.43

$$\left[\frac{105\,b^5c^2x^5\sqrt{\frac{c}{a}}\,\log\left(\frac{bcx^2-2\,\sqrt{\sqrt{cx^2}\,b+a}\,ax\sqrt{\frac{c}{a}}+2\,\sqrt{cx^2}\,a}{x^2}\right)+2\left(105\,b^4c^2x^4+56\,a^2b^2cx^2-384\,a^4-2\left(35\,ab^3cx^2+24\,a^3b\right)\right.}{3840\,a^4x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(1/2))^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/3840*(105*b^5*c^2*x^5*sqrt(c/a)*log((b*c*x^2 - 2*sqrt(sqrt(c*x^2)*b + a)
*a*x*sqrt(c/a) + 2*sqrt(c*x^2)*a)/x^2) + 2*(105*b^4*c^2*x^4 + 56*a^2*b^2*c*
x^2 - 384*a^4 - 2*(35*a*b^3*c*x^2 + 24*a^3*b)*sqrt(c*x^2))*sqrt(sqrt(c*x^2)
*b + a))/(a^4*x^5), -1/1920*(105*b^5*c^2*x^5*sqrt(-c/a)*arctan(-(a*b*c*x^2*
sqrt(-c/a) - sqrt(c*x^2)*a^2*sqrt(-c/a))*sqrt(sqrt(c*x^2)*b + a)/(b^2*c^2*x
^3 - a^2*c*x)) - (105*b^4*c^2*x^4 + 56*a^2*b^2*c*x^2 - 384*a^4 - 2*(35*a*b^
3*c*x^2 + 24*a^3*b)*sqrt(c*x^2))*sqrt(sqrt(c*x^2)*b + a))/(a^4*x^5)]
```

giac [A] time = 0.17, size = 156, normalized size = 0.71

$$\frac{105\,b^6c^3\arctan\left(\frac{\sqrt{b\sqrt{c}\,x+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^4}+\frac{105\left(b\sqrt{c}\,x+a\right)^{\frac{9}{2}}b^6c^3-490\left(b\sqrt{c}\,x+a\right)^{\frac{7}{2}}ab^6c^3+896\left(b\sqrt{c}\,x+a\right)^{\frac{5}{2}}a^2b^6c^3-790\left(b\sqrt{c}\,x+a\right)^{\frac{3}{2}}a^3b^6c^3-105\sqrt{b\sqrt{c}\,x+a}\,a^4b^6c^3}{a^4b^5c^{\frac{5}{2}}x^5}}{1920\,b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(1/2))^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/1920*(105*b^6*c^3*arctan(sqrt(b*sqrt(c)*x + a)/sqrt(-a))/(sqrt(-a)*a^4) +
 (105*(b*sqrt(c)*x + a)^(9/2)*b^6*c^3 - 490*(b*sqrt(c)*x + a)^(7/2)*a*b^6*c
^3 + 896*(b*sqrt(c)*x + a)^(5/2)*a^2*b^6*c^3 - 790*(b*sqrt(c)*x + a)^(3/2)*
a^3*b^6*c^3 - 105*sqrt(b*sqrt(c)*x + a)*a^4*b^6*c^3)/(a^4*b^5*c^(5/2)*x^5))
/(b*sqrt(c))
```

maple [A] time = 0.02, size = 133, normalized size = 0.61

$$-\frac{105\left(c\,x^2\right)^{\frac{5}{2}}a^4b^5\operatorname{arctanh}\left(\frac{\sqrt{a+\sqrt{c\,x^2}\;b}}{\sqrt{a}}\right)+105\sqrt{a+\sqrt{c\,x^2}\;b}\;a^{\frac{17}{2}}+790\left(a+\sqrt{c\,x^2}\;b\right)^{\frac{3}{2}}a^{\frac{15}{2}}-896\left(a+\sqrt{c\,x^2}\;b\right)^{\frac{5}{2}}a^{\frac{13}{2}}}{1920a^{\frac{17}{2}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(c*x^2)^(1/2)*b)^(1/2)/x^6,x)
```

```
[Out] -1/1920*(-105*(a+(c*x^2)^(1/2)*b)^(9/2)*a^(9/2)+490*(a+(c*x^2)^(1/2)*b)^(7/
2)*a^(11/2)-896*(a+(c*x^2)^(1/2)*b)^(5/2)*a^(13/2)+105*arctanh((a+(c*x^2)^(
1/2)*b)^(1/2)/a^(1/2))*a^4*b^5*(c*x^2)^(5/2)+790*(a+(c*x^2)^(1/2)*b)^(3/2)*
a^(15/2)+105*(a+(c*x^2)^(1/2)*b)^(1/2)*a^(17/2))/x^5/a^(17/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{cx^2}\,b+a}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(1/2))^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(c*x^2)*b + a)/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a + b\sqrt{c\,x^2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x^2)^(1/2))^(1/2)/x^6,x)
```

```
[Out] int((a + b*(c*x^2)^(1/2))^(1/2)/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\sqrt{cx^2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x**2)**(1/2))**(1/2)/x**6,x)
```

```
[Out] Integral(sqrt(a + b*sqrt(c*x**2))/x**6, x)
```

3.2942 $\int x^8 \sqrt{a + b\left(cx^2\right)^{3/2}}\, dx$

Optimal. Leaf size=113

$$\frac{2a^2x^9\left(a+b\left(cx^2\right)^{3/2}\right)^{3/2}}{9b^3\left(cx^2\right)^{9/2}} + \frac{2x^9\left(a+b\left(cx^2\right)^{3/2}\right)^{7/2}}{21b^3\left(cx^2\right)^{9/2}} - \frac{4ax^9\left(a+b\left(cx^2\right)^{3/2}\right)^{5/2}}{15b^3\left(cx^2\right)^{9/2}}$$

```
[Out] 2/9*a^2*x^9*(a+b*(c*x^2)^(3/2))^(3/2)/b^3/(c*x^2)^(9/2)-4/15*a*x^9*(a+b*(c*
x^2)^(3/2))^(5/2)/b^3/(c*x^2)^(9/2)+2/21*x^9*(a+b*(c*x^2)^(3/2))^(7/2)/b^3/
(c*x^2)^(9/2)
```

Rubi [A] time = 0.07, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {368, 266, 43}

$$\frac{2a^2x^9\left(a+b\left(cx^2\right)^{3/2}\right)^{3/2}}{9b^3\left(cx^2\right)^{9/2}} + \frac{2x^9\left(a+b\left(cx^2\right)^{3/2}\right)^{7/2}}{21b^3\left(cx^2\right)^{9/2}} - \frac{4ax^9\left(a+b\left(cx^2\right)^{3/2}\right)^{5/2}}{15b^3\left(cx^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*Sqrt[a + b*(c*x^2)^(3/2)],x]
```

```
[Out] (2*a^2*x^9*(a + b*(c*x^2)^(3/2))^(3/2))/(9*b^3*(c*x^2)^(9/2)) - (4*a*x^9*(a
 + b*(c*x^2)^(3/2))^(5/2))/(15*b^3*(c*x^2)^(9/2)) + (2*x^9*(a + b*(c*x^2)^(
3/2))^(7/2))/(21*b^3*(c*x^2)^(9/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\int x^8\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx = \frac{x^9\,\text{Subst}\left(\int x^8\sqrt{a+bx^3}\,dx, x, \sqrt{cx^2}\right)}{\left(cx^2\right)^{9/2}}$$
$$= \frac{x^9\,\text{Subst}\left(\int x^2\sqrt{a+bx}\,dx, x, \left(cx^2\right)^{3/2}\right)}{3\left(cx^2\right)^{9/2}}$$
$$= \frac{x^9\,\text{Subst}\left(\int \left(\frac{a^2\sqrt{a+bx}}{b^2} - \frac{2a(a+bx)^{3/2}}{b^2} + \frac{(a+bx)^{5/2}}{b^2}\right)dx, x, \left(cx^2\right)^{3/2}\right)}{3\left(cx^2\right)^{9/2}}$$
$$= \frac{2a^2x^9\left(a+b\left(cx^2\right)^{3/2}\right)^{3/2}}{9b^3\left(cx^2\right)^{9/2}} - \frac{4ax^9\left(a+b\left(cx^2\right)^{3/2}\right)^{5/2}}{15b^3\left(cx^2\right)^{9/2}} + \frac{2x^9\left(a+b\left(cx^2\right)^{3/2}\right)^{7/2}}{21b^3\left(cx^2\right)^{9/2}}$$

Mathematica [A] time = 0.04, size = 67, normalized size = 0.59

$$\frac{2x\left(a+b\left(cx^2\right)^{3/2}\right)^{3/2}\left(8a^2-12ab\left(cx^2\right)^{3/2}+15b^2c^3x^6\right)}{315b^3c^4\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*Sqrt[a + b*(c*x^2)^(3/2)],x]
```

```
[Out] (2*x*(a + b*(c*x^2)^(3/2))^(3/2)*(8*a^2 + 15*b^2*c^3*x^6 - 12*a*b*(c*x^2)^(
3/2)))/(315*b^3*c^4*Sqrt[c*x^2])
```

fricas [A] time = 0.88, size = 78, normalized size = 0.69

$$\frac{2\left(15\,b^3c^5x^{10} - 4\,a^2bc^2x^4 + \left(3\,ab^2c^3x^6 + 8\,a^3\right)\sqrt{cx^2}\right)\sqrt{\sqrt{cx^2}\,bcx^2 + a}}{315\,b^3c^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(15*b^3*c^5*x^10 - 4*a^2*b*c^2*x^4 + (3*a*b^2*c^3*x^6 + 8*a^3)*sqrt(c
*x^2))*sqrt(sqrt(c*x^2)*b*c*x^2 + a)/(b^3*c^5*x)
```

giac [A] time = 0.18, size = 64, normalized size = 0.57

$$\frac{2\left(15\left(bc^{\frac{3}{2}}x^3+a\right)^{\frac{7}{2}}\sqrt{c} - 42\left(bc^{\frac{3}{2}}x^3+a\right)^{\frac{5}{2}}a\sqrt{c} + 35\left(bc^{\frac{3}{2}}x^3+a\right)^{\frac{3}{2}}a^2\sqrt{c}\right)}{315\,b^3c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(15*(b*c^(3/2)*x^3 + a)^(7/2)*sqrt(c) - 42*(b*c^(3/2)*x^3 + a)^(5/2)*
a*sqrt(c) + 35*(b*c^(3/2)*x^3 + a)^(3/2)*a^2*sqrt(c))/(b^3*c^5)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \sqrt{a+\left(c\,x^2\right)^{\frac{3}{2}}\,b}\;x^8\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a+b*(c*x^2)^(3/2))^(1/2),x)
```

```
[Out] int(x^8*(a+b*(c*x^2)^(3/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*((c^7 + 3*c^6 + 2*c^5)*b^3*x^9 + (c^5 + c^4)*a*b^2*sqrt(c)*x^6 - 2*a^2*
b*c^3*x^3 + 2*a^3*sqrt(c))*sqrt(b*c^(3/2)*x^3 + a)/((c^8 + 6*c^7 + 11*c^6 +
 6*c^5)*b^3) + integrate(-((c^5 + 3*c^4 + 2*c^3 - (c^5 + 3*c^4 + 2*c^3)*sqr
t(c))*b^2*x^8 + 2*(c^3 + c^2 - (c^2 + c)*sqrt(c))*a*b*x^5 - 2*a^2*x^2*(sqrt
(c) - 1))*sqrt(b*c^(3/2)*x^3 + a), x)/((c^5 + 6*c^4 + 11*c^3 + 6*c^2)*b^2*s
qrt(c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^8 \sqrt{a + b\left(c\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*(c*x^2)^(3/2))^(1/2),x)
```

```
[Out] int(x^8*(a + b*(c*x^2)^(3/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^8 \sqrt{a + b\left(cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(a+b*(c*x**2)**(3/2))**(1/2),x)
```

```
[Out] Integral(x**8*sqrt(a + b*(c*x**2)**(3/2)), x)
```

3.2943 $\int x^5 \sqrt{a + b\,(cx^2)^{3/2}}\, dx$

Optimal. Leaf size=56

$$\frac{2\left(a + b\left(cx^2\right)^{3/2}\right)^{5/2}}{15b^2c^3} - \frac{2a\left(a + b\left(cx^2\right)^{3/2}\right)^{3/2}}{9b^2c^3}$$

```
[Out] -2/9*a*(a+b*(c*x^2)^(3/2))^(3/2)/b^2/c^3+2/15*(a+b*(c*x^2)^(3/2))^(5/2)/b^2
/c^3
```

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {368, 266, 43}

$$\frac{2\left(a + b\left(cx^2\right)^{3/2}\right)^{5/2}}{15b^2c^3} - \frac{2a\left(a + b\left(cx^2\right)^{3/2}\right)^{3/2}}{9b^2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[a + b*(c*x^2)^(3/2)],x]
```

```
[Out] (-2*a*(a + b*(c*x^2)^(3/2))^(3/2))/(9*b^2*c^3) + (2*(a + b*(c*x^2)^(3/2))^(
5/2))/(15*b^2*c^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\begin{aligned}\int x^5\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int x^5\sqrt{a+bx^3}\,dx, x, \sqrt{cx^2}\right)}{c^3}\\ &= \frac{\text{Subst}\left(\int x\sqrt{a+bx}\,dx, x, \left(cx^2\right)^{3/2}\right)}{3c^3}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx, x, \left(cx^2\right)^{3/2}\right)}{3c^3}\\ &= -\frac{2a\left(a+b\left(cx^2\right)^{3/2}\right)^{3/2}}{9b^2c^3}+\frac{2\left(a+b\left(cx^2\right)^{3/2}\right)^{5/2}}{15b^2c^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 0.77

$$\frac{2\left(a+b\left(cx^2\right)^{3/2}\right)^{3/2}\left(3b\left(cx^2\right)^{3/2}-2a\right)}{45b^2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[a + b*(c*x^2)^(3/2)],x]
```

```
[Out] (2*(a + b*(c*x^2)^(3/2))^(3/2)*(-2*a + 3*b*(c*x^2)^(3/2)))/(45*b^2*c^3)
```

fricas [A] time = 1.09, size = 56, normalized size = 1.00

$$\frac{2\left(3\,b^2c^3x^6+\sqrt{cx^2}\,abcx^2-2\,a^2\right)\sqrt{\sqrt{cx^2}\,bcx^2+a}}{45\,b^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*(3*b^2*c^3*x^6 + sqrt(c*x^2)*a*b*c*x^2 - 2*a^2)*sqrt(sqrt(c*x^2)*b*c*x
^2 + a)/(b^2*c^3)
```

giac [A] time = 0.18, size = 38, normalized size = 0.68

$$\frac{2\left(3\left(bc^{\frac{3}{2}}x^3+a\right)^{\frac{5}{2}}-5\left(bc^{\frac{3}{2}}x^3+a\right)^{\frac{3}{2}}a\right)}{45\,b^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="giac")
```

```
[Out] 2/45*(3*(b*c^(3/2)*x^3 + a)^(5/2) - 5*(b*c^(3/2)*x^3 + a)^(3/2)*a)/(b^2*c^3
)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \sqrt{a+\left(c\,x^2\right)^{\frac{3}{2}}\,b}\;x^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+(c*x^2)^(3/2)*b)^(1/2),x)
```

```
[Out] int(x^5*(a+(c*x^2)^(3/2)*b)^(1/2),x)
```

maxima [A] time = 0.53, size = 43, normalized size = 0.77

$$\frac{2\left(\frac{3\left(\left(cx^2\right)^{\frac{3}{2}}b+a\right)^{\frac{5}{2}}}{b^2}-\frac{5\left(\left(cx^2\right)^{\frac{3}{2}}b+a\right)^{\frac{3}{2}}a}{b^2}\right)}{45\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/45*(3*((c*x^2)^(3/2)*b + a)^(5/2)/b^2 - 5*((c*x^2)^(3/2)*b + a)^(3/2)*a/b
^2)/c^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^5\sqrt{a+b\left(c\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*(c*x^2)^(3/2))^(1/2),x)
```

```
[Out] int(x^5*(a + b*(c*x^2)^(3/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\sqrt{a+b\left(cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*(c*x**2)**(3/2))**(1/2),x)
```

```
[Out] Integral(x**5*sqrt(a + b*(c*x**2)**(3/2)), x)
```

3.2944 $\int x^2\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=36

$$\frac{2x^3\left(a+b\left(cx^2\right)^{3/2}\right)^{3/2}}{9b\left(cx^2\right)^{3/2}}$$

```
[Out] 2/9*x^3*(a+b*(c*x^2)^(3/2))^(3/2)/b/(c*x^2)^(3/2)
```

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {368, 261}

$$\frac{2x^3\left(a+b\left(cx^2\right)^{3/2}\right)^{3/2}}{9b\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*(c*x^2)^(3/2)],x]
```

```
[Out] (2*x^3*(a + b*(c*x^2)^(3/2))^(3/2))/(9*b*(c*x^2)^(3/2))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx &= \frac{x^3\,\text{Subst}\left(\int x^2\sqrt{a+bx^3}\,dx, x, \sqrt{cx^2}\right)}{\left(cx^2\right)^{3/2}}\\ &= \frac{2x^3\left(a+b\left(cx^2\right)^{3/2}\right)^{3/2}}{9b\left(cx^2\right)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 1.00

$$\frac{2x^3\left(a+b\left(cx^2\right)^{3/2}\right)^{3/2}}{9b\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b*(c*x^2)^(3/2)],x]
```

```
[Out] (2*x^3*(a + b*(c*x^2)^(3/2))^(3/2))/(9*b*(c*x^2)^(3/2))
```

fricas [A] time = 1.05, size = 46, normalized size = 1.28

$$\frac{2\left(bc^2x^4+\sqrt{cx^2}\,a\right)\sqrt{\sqrt{cx^2}\,bcx^2+a}}{9\,bc^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="fricas")
[Out] 2/9*(b*c^2*x^4 + sqrt(c*x^2)*a)*sqrt(sqrt(c*x^2)*b*c*x^2 + a)/(b*c^2*x)
```

giac [A] time = 0.16, size = 20, normalized size = 0.56

$$\frac{2\left(bc^{\frac{3}{2}}x^3+a\right)^{\frac{3}{2}}}{9\,bc^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="giac")
[Out] 2/9*(b*c^(3/2)*x^3 + a)^(3/2)/(b*c^(3/2))
```

maple [A] time = 0.00, size = 29, normalized size = 0.81

$$\frac{2\left(a+\left(c\,x^2\right)^{\frac{3}{2}}b\right)^{\frac{3}{2}}x^3}{9\left(c\,x^2\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+(c*x^2)^(3/2)*b)^(1/2),x)
[Out] 2/9*x^3*(a+(c*x^2)^(3/2)*b)^(3/2)/b/(c*x^2)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\frac{2\left(bc^{\frac{3}{2}}x^3+a\right)^{\frac{3}{2}}\left(c-\sqrt{c}\right)}{9\,bc^{\frac{3}{2}}}}{c+1}+\frac{\left(bc^{\frac{3}{2}}x^3+a\right)^{\frac{3}{2}}}{3\left(c^2+c\right)b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="maxima")
[Out] (c - sqrt(c))*integrate(sqrt(b*c^(3/2)*x^3 + a)*x^2, x)/(c + 1) + 1/3*(b*c^
(3/2)*x^3 + a)^(3/2)/((c^2 + c)*b*sqrt(c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x^2\sqrt{a+b\left(c\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*(c*x^2)^(3/2))^(1/2),x)
```

```
[Out] int(x^2*(a + b*(c*x^2)^(3/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{a + b\left(cx^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*(c*x**2)**(3/2))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a + b*(c*x**2)**(3/2)), x)
```

3.2945 $\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x}\,dx$

Optimal. Leaf size=55

$$\frac{2}{3}\sqrt{a+b\left(cx^2\right)^{3/2}}-\frac{2}{3}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{\sqrt{a}}\right)$$

```
[Out] -2/3*arctanh((a+b*(c*x^2)^(3/2))^(1/2)/a^(1/2))*a^(1/2)+2/3*(a+b*(c*x^2)^(3
/2))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {368, 266, 50, 63, 208}

$$\frac{2}{3}\sqrt{a+b\left(cx^2\right)^{3/2}}-\frac{2}{3}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*(c*x^2)^(3/2)]/x,x]
```

```
[Out] (2*Sqrt[a + b*(c*x^2)^(3/2)])/3 - (2*Sqrt[a]*ArcTanh[Sqrt[a + b*(c*x^2)^(3/
2)]/Sqrt[a]])/3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
```

```
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x}\,dx &= \text{Subst}\left(\int \frac{\sqrt{a+bx^3}}{x}\,dx, x, \sqrt{cx^2}\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \left(cx^2\right)^{3/2}\right)\\ &= \frac{2}{3}\sqrt{a+b\left(cx^2\right)^{3/2}} + \frac{1}{3}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \left(cx^2\right)^{3/2}\right)\\ &= \frac{2}{3}\sqrt{a+b\left(cx^2\right)^{3/2}} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\left(cx^2\right)^{3/2}}\right)}{3b}\\ &= \frac{2}{3}\sqrt{a+b\left(cx^2\right)^{3/2}} - \frac{2}{3}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{\sqrt{a}}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 55, normalized size = 1.00

$$\frac{2}{3}\sqrt{a+b\left(cx^2\right)^{3/2}} - \frac{2}{3}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*(c*x^2)^(3/2)]/x,x]
```

```
[Out] (2*Sqrt[a + b*(c*x^2)^(3/2)])/3 - (2*Sqrt[a]*ArcTanh[Sqrt[a + b*(c*x^2)^(3/
2)]/Sqrt[a]])/3
```

fricas [A] time = 0.80, size = 133, normalized size = 2.42

$$\left[\frac{1}{3}\sqrt{a}\,\log\left(\frac{bc^2x^4 - 2\sqrt{\sqrt{cx^2}\,bcx^2+a}\,\sqrt{cx^2}\,\sqrt{a} + 2\sqrt{cx^2}\,a}{x^4}\right) + \frac{2}{3}\sqrt{\sqrt{cx^2}\,bcx^2+a}\,,\ \frac{2}{3}\sqrt{-a}\,\arctan\left(\frac{\sqrt{\sqrt{cx^2}\,bcx^2+}}{a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(3/2))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/3*sqrt(a)*log((b*c^2*x^4 - 2*sqrt(sqrt(c*x^2)*b*c*x^2 + a)*sqrt(c*x^2)*s
qrt(a) + 2*sqrt(c*x^2)*a)/x^4) + 2/3*sqrt(sqrt(c*x^2)*b*c*x^2 + a), 2/3*sqr
t(-a)*arctan(sqrt(sqrt(c*x^2)*b*c*x^2 + a)*sqrt(-a)/a) + 2/3*sqrt(sqrt(c*x^
2)*b*c*x^2 + a)]
```

giac [A] time = 0.17, size = 42, normalized size = 0.76

$$\frac{2\,a\arctan\left(\frac{\sqrt{bc^{\frac{3}{2}}x^3+a}}{\sqrt{-a}}\right)}{3\sqrt{-a}} + \frac{2}{3}\sqrt{bc^{\frac{3}{2}}x^3+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(3/2))^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2/3*a*arctan(sqrt(b*c^(3/2)*x^3 + a)/sqrt(-a))/sqrt(-a) + 2/3*sqrt(b*c^(3/2
)*x^3 + a)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \left(c\,x^2\right)^{\frac{3}{2}} b}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(c*x^2)^(3/2)*b)^(1/2)/x,x)
```

```
[Out] int((a+(c*x^2)^(3/2)*b)^(1/2)/x,x)
```

maxima [A] time = 1.15, size = 61, normalized size = 1.11

$$\frac{1}{3}\sqrt{a}\,\log\left(\frac{\sqrt{\left(cx^2\right)^{\frac{3}{2}} b + a} - \sqrt{a}}{\sqrt{\left(cx^2\right)^{\frac{3}{2}} b + a} + \sqrt{a}}\right) + \frac{2}{3}\sqrt{\left(cx^2\right)^{\frac{3}{2}} b + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(3/2))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(a)*log((sqrt((c*x^2)^(3/2)*b + a) - sqrt(a))/(sqrt((c*x^2)^(3/2)*b
 + a) + sqrt(a))) + 2/3*sqrt((c*x^2)^(3/2)*b + a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a + b\left(c\,x^2\right)^{3/2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x^2)^(3/2))^(1/2)/x,x)
```

```
[Out] int((a + b*(c*x^2)^(3/2))^(1/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\left(cx^2\right)^{\frac{3}{2}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x**2)**(3/2))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a + b*(c*x**2)**(3/2))/x, x)
```

3.2946 $\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^4}\,dx$

Optimal. Leaf size=71

$$-\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{3x^3}-\frac{b\left(cx^2\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{\sqrt{a}}\right)}{3\sqrt{a}\,x^3}$$

```
[Out] -1/3*b*(c*x^2)^(3/2)*arctanh((a+b*(c*x^2)^(3/2))^(1/2)/a^(1/2))/x^3/a^(1/2)
-1/3*(a+b*(c*x^2)^(3/2))^(1/2)/x^3
```

Rubi [A] time = 0.06, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {368, 266, 47, 63, 208}

$$-\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{3x^3}-\frac{b\left(cx^2\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{\sqrt{a}}\right)}{3\sqrt{a}\,x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*(c*x^2)^(3/2)]/x^4,x]
```

```
[Out] -Sqrt[a + b*(c*x^2)^(3/2)]/(3*x^3) - (b*(c*x^2)^(3/2)*ArcTanh[Sqrt[a + b*(c
*x^2)^(3/2)]/Sqrt[a]])/(3*Sqrt[a]*x^3)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^4}\,dx &= \frac{\left(cx^2\right)^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{a+bx^3}}{x^4}\,dx, x, \sqrt{cx^2}\right)}{x^3} \\
&= \frac{\left(cx^2\right)^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx, x, \left(cx^2\right)^{3/2}\right)}{3x^3} \\
&= -\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{3x^3} + \frac{\left(b\left(cx^2\right)^{3/2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \left(cx^2\right)^{3/2}\right)}{6x^3} \\
&= -\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{3x^3} + \frac{\left(cx^2\right)^{3/2}\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\left(cx^2\right)^{3/2}}\right)}{3x^3} \\
&= -\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{3x^3} - \frac{b\left(cx^2\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{\sqrt{a}}\right)}{3\sqrt{a}\,x^3}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 93, normalized size = 1.31

$$\frac{-b\left(cx^2\right)^{3/2}\sqrt{\frac{b\left(cx^2\right)^{3/2}}{a}+1}\,\tanh^{-1}\left(\sqrt{\frac{b\left(cx^2\right)^{3/2}}{a}+1}\right) - a - b\left(cx^2\right)^{3/2}}{3x^3\sqrt{a+b\left(cx^2\right)^{3/2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*(c*x^2)^(3/2)]/x^4,x]
```

```
[Out] (-a - b*(c*x^2)^(3/2) - b*(c*x^2)^(3/2)*Sqrt[1 + (b*(c*x^2)^(3/2))/a]*ArcTa
nh[Sqrt[1 + (b*(c*x^2)^(3/2))/a]])/(3*x^3*Sqrt[a + b*(c*x^2)^(3/2)])
```

fricas [A] time = 1.08, size = 206, normalized size = 2.90

$$\left[\frac{bcx^3\sqrt{\frac{c}{a}}\,\log\left(\frac{bc^2x^4-2\sqrt{\sqrt{cx^2}\,bcx^2+a}\,ax\sqrt{\frac{c}{a}}+2\sqrt{cx^2}\,a}{x^4}\right) - 2\sqrt{\sqrt{cx^2}\,bcx^2+a}}{6x^3}, -\frac{bcx^3\sqrt{-\frac{c}{a}}\,\arctan\left(-\frac{\left(abc^2x^4\sqrt{-\frac{c}{a}}-\sqrt{cx^2}\,a^2\right.\right.}{b^2c^4x^7-}\right.}{3x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(3/2))^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/6*(b*c*x^3*sqrt(c/a)*log((b*c^2*x^4 - 2*sqrt(sqrt(c*x^2)*b*c*x^2 + a)*a*
x*sqrt(c/a) + 2*sqrt(c*x^2)*a)/x^4) - 2*sqrt(sqrt(c*x^2)*b*c*x^2 + a))/x^3,
 -1/3*(b*c*x^3*sqrt(-c/a)*arctan(-(a*b*c^2*x^4*sqrt(-c/a) - sqrt(c*x^2)*a^2
*sqrt(-c/a))*sqrt(sqrt(c*x^2)*b*c*x^2 + a)/(b^2*c^4*x^7 - a^2*c*x)) + sqrt(
sqrt(c*x^2)*b*c*x^2 + a))/x^3]
```

giac [A] time = 0.17, size = 61, normalized size = 0.86

$$\frac{\frac{b^2c^3\arctan\left(\frac{\sqrt{bc^{\frac{3}{2}}x^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\sqrt{bc^{\frac{3}{2}}x^3+a}\,bc^{\frac{3}{2}}}{x^3}}{3\,bc^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(3/2))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/3*(b^2*c^3*arctan(sqrt(b*c^(3/2)*x^3 + a)/sqrt(-a))/sqrt(-a) - sqrt(b*c^(
3/2)*x^3 + a)*b*c^(3/2)/x^3)/(b*c^(3/2))
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\left(c\,x^2\right)^{\frac{3}{2}}b}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(c*x^2)^(3/2)*b)^(1/2)/x^4,x)
```

```
[Out] int((a+(c*x^2)^(3/2)*b)^(1/2)/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(cx^2\right)^{\frac{3}{2}}b+a}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(3/2))^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((c*x^2)^(3/2)*b + a)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+b\left(c\,x^2\right)^{3/2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x^2)^(3/2))^(1/2)/x^4,x)
```

```
[Out] int((a + b*(c*x^2)^(3/2))^(1/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{\frac{3}{2}}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x**2)**(3/2))**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(a + b*(c*x**2)**(3/2))/x**4, x)
```

3.2947 $\int x^3\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=340

$$-\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^2\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}+b^{2/3}cx^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\sqrt{cx^2}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\sqrt{cx^2}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{55b^{4/3}c^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\sqrt{a+b\left(cx^2\right)^{3/2}}}+\frac{6a\sqrt{}}{}$$

```
[Out] 2/11*x^4*(a+b*(c*x^2)^(3/2))^(1/2)+6/55*a*(c*x^2)^(1/2)*(a+b*(c*x^2)^(3/2))
^(1/2)/b/c^2-4/55*3^(3/4)*a^2*EllipticF((a^(1/3)*(1-3^(1/2))+b^(1/3)*(c*x^2
)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*(c*x^2)^(1/2)),I*3^(1/2)+2*I)*(a^(1/3
)+b^(1/3)*(c*x^2)^(1/2))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)+b^(2/3)*c*x^2-
a^(1/3)*b^(1/3)*(c*x^2)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*(c*x^2)^(1/2))^
2)^(1/2)/b^(4/3)/c^2/(a+b*(c*x^2)^(3/2))^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*(c
*x^2)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*(c*x^2)^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {368, 279, 321, 218}

$$-\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^2\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}+b^{2/3}cx^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\sqrt{cx^2}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\sqrt{cx^2}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{55b^{4/3}c^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\sqrt{a+b\left(cx^2\right)^{3/2}}}+\frac{6a\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + b*(c*x^2)^(3/2)],x]
```

```
[Out] (2*x^4*Sqrt[a + b*(c*x^2)^(3/2)])/11 + (6*a*Sqrt[c*x^2]*Sqrt[a + b*(c*x^2)^
(3/2)])/(55*b*c^2) - (4*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*(a^(1/3) + b^(1/3)*Sq
rt[c*x^2])*Sqrt[(a^(2/3) + b^(2/3)*c*x^2 - a^(1/3)*b^(1/3)*Sqrt[c*x^2])/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])^2]*EllipticF[ArcSin[((1 - Sqrt[3
])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x
^2])], -7 - 4*Sqrt[3]])/(55*b^(4/3)*c^2*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*Sq
rt[c*x^2]))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])^2]*Sqrt[a + b*(c*
x^2)^(3/2)])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\int x^3\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int x^3\sqrt{a+bx^3}\,dx, x, \sqrt{cx^2}\right)}{c^2}$$

$$= \frac{2}{11}x^4\sqrt{a+b\left(cx^2\right)^{3/2}} + \frac{(3a)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{a+bx^3}}\,dx, x, \sqrt{cx^2}\right)}{11c^2}$$

$$= \frac{2}{11}x^4\sqrt{a+b\left(cx^2\right)^{3/2}} + \frac{6a\sqrt{cx^2}\sqrt{a+b\left(cx^2\right)^{3/2}}}{55bc^2} - \frac{\left(6a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^3}}\,dx, x, \sqrt{cx^2}\right.}{55bc^2}$$

$$= \frac{2}{11}x^4\sqrt{a+b\left(cx^2\right)^{3/2}} + \frac{6a\sqrt{cx^2}\sqrt{a+b\left(cx^2\right)^{3/2}}}{55bc^2} - \frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^2\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right.}{55b^{4/3}c}$$

Mathematica [C] time = 0.11, size = 109, normalized size = 0.32

$$\frac{2\sqrt{cx^2}\sqrt{a+b\left(cx^2\right)^{3/2}}\left(a\left(\frac{a+b\left(cx^2\right)^{3/2}}{a}\right)^{3/2} - a\,{}_2F_1\left(-\frac{1}{2},\frac{1}{3};\frac{4}{3};-\frac{b\left(cx^2\right)^{3/2}}{a}\right)\right)}{11bc^2\sqrt{\frac{a+b\left(cx^2\right)^{3/2}}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + b*(c*x^2)^(3/2)],x]
```

```
[Out] (2*Sqrt[c*x^2]*Sqrt[a + b*(c*x^2)^(3/2)]*(a*((a + b*(c*x^2)^(3/2))/a)^(3/2)
 - a*Hypergeometric2F1[-1/2, 1/3, 4/3, -((b*(c*x^2)^(3/2))/a)]))/(11*b*c^2*
Sqrt[(a + b*(c*x^2)^(3/2))/a])
```

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{\sqrt{cx^2}\,bcx^2+a}\,x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sqrt(c*x^2)*b*c*x^2 + a)*x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\left(cx^2\right)^{\frac{3}{2}} b + a}\, x^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt((c*x^2)^(3/2)*b + a)*x^3, x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \sqrt{a + \left(c\, x^2\right)^{\frac{3}{2}} b}\; x^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+(c*x^2)^(3/2)*b)^(1/2),x)
[Out] int(x^3*(a+(c*x^2)^(3/2)*b)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\left(cx^2\right)^{\frac{3}{2}} b + a}\, x^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt((c*x^2)^(3/2)*b + a)*x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \sqrt{a + b\left(c\, x^2\right)^{3/2}}\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*(c*x^2)^(3/2))^(1/2),x)
[Out] int(x^3*(a + b*(c*x^2)^(3/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sqrt{a + b\left(cx^2\right)^{\frac{3}{2}}}\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*(c*x**2)**(3/2))**(1/2),x)
[Out] Integral(x**3*sqrt(a + b*(c*x**2)**(3/2)), x)
```

3.2948 $\int \sqrt{a+b\left(cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=306

$$\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\,ax\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}+b^{2/3}cx^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\sqrt{cx^2}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\sqrt{cx^2}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{5\sqrt[3]{b}\sqrt{cx^2}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\sqrt{a+b\left(cx^2\right)^{3/2}}}+\frac{2}{5}x\sqrt{a+}$$

```
[Out] 2/5*x*(a+b*(c*x^2)^(3/2))^(1/2)+2/5*3^(3/4)*a*x*EllipticF((a^(1/3)*(1-3^(1/
2))+b^(1/3)*(c*x^2)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*(c*x^2)^(1/2)),I*3^
(1/2)+2*I)*(a^(1/3)+b^(1/3)*(c*x^2)^(1/2))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2
/3)+b^(2/3)*c*x^2-a^(1/3)*b^(1/3)*(c*x^2)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/
3)*(c*x^2)^(1/2))^2)^(1/2)/b^(1/3)/(c*x^2)^(1/2)/(a+b*(c*x^2)^(3/2))^(1/2)/
(a^(1/3)*(a^(1/3)+b^(1/3)*(c*x^2)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*(c*x^
2)^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {254, 195, 218}

$$\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\,ax\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}+b^{2/3}cx^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\sqrt{cx^2}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\sqrt{cx^2}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{5\sqrt[3]{b}\sqrt{cx^2}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\sqrt{a+b\left(cx^2\right)^{3/2}}}+\frac{2}{5}x\sqrt{a+}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*(c*x^2)^(3/2)],x]
```

```
[Out] (2*x*Sqrt[a + b*(c*x^2)^(3/2)])/5 + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*x*(a^(1/
3) + b^(1/3)*Sqrt[c*x^2])*Sqrt[(a^(2/3) + b^(2/3)*c*x^2 - a^(1/3)*b^(1/3)*S
qrt[c*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])^2]*EllipticF[ArcS
in[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*Sqrt[c*x^2])], -7 - 4*Sqrt[3]])/(5*b^(1/3)*Sqrt[c*x^2]*Sqrt[(a^(1/3)
*(a^(1/3) + b^(1/3)*Sqrt[c*x^2]))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x
^2])^2]*Sqrt[a + b*(c*x^2)^(3/2)])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 254

```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_.), x_Symbol] :> Dist[x/(c*x^
q)^(1/q), Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\int \sqrt{a+b\left(cx^2\right)^{3/2}}\,dx = \frac{x\,\text{Subst}\left(\int \sqrt{a+bx^3}\,dx, x, \sqrt{cx^2}\right)}{\sqrt{cx^2}}$$

$$= \frac{2}{5}x\sqrt{a+b\left(cx^2\right)^{3/2}} + \frac{(3ax)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^3}}\,dx, x, \sqrt{cx^2}\right)}{5\sqrt{cx^2}}$$

$$= \frac{2}{5}x\sqrt{a+b\left(cx^2\right)^{3/2}} + \frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\,ax\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}+b^{2/3}cx^2-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}F\left(}{5\sqrt[3]{b}\sqrt{cx^2}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\sqrt{a}}$$

Mathematica [C] time = 0.01, size = 64, normalized size = 0.21

$$\frac{x\sqrt{a+b\left(cx^2\right)^{3/2}}\;{}_2F_1\left(-\frac{1}{2},\frac{1}{3};\frac{4}{3};-\frac{b\left(cx^2\right)^{3/2}}{a}\right)}{\sqrt{\frac{b\left(cx^2\right)^{3/2}}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*(c*x^2)^(3/2)],x]
```

```
[Out] (x*Sqrt[a + b*(c*x^2)^(3/2)]*Hypergeometric2F1[-1/2, 1/3, 4/3, -((b*(c*x^2)
^(3/2))/a)])/Sqrt[1 + (b*(c*x^2)^(3/2))/a]
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{\sqrt{cx^2}\,bcx^2+a}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sqrt(c*x^2)*b*c*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\left(cx^2\right)^{\frac{3}{2}}b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((c*x^2)^(3/2)*b + a), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \sqrt{a+\left(c\,x^2\right)^{\frac{3}{2}}b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(c*x^2)^(3/2)*b)^(1/2),x)
[Out] int((a+(c*x^2)^(3/2)*b)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\left(cx^2\right)^{\frac{3}{2}} b + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt((c*x^2)^(3/2)*b + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a + b\left(c\,x^2\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x^2)^(3/2))^(1/2),x)
[Out] int((a + b*(c*x^2)^(3/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\left(cx^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x**2)**(3/2))**(1/2),x)
[Out] Integral(sqrt(a + b*(c*x**2)**(3/2)), x)
```

3.2949 $\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^3}\,dx$

Optimal. Leaf size=298

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{2/3}c\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}+b^{2/3}cx^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\sqrt{cx^2}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\sqrt{cx^2}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\sqrt{a+b\left(cx^2\right)^{3/2}}}-\frac{\sqrt{a+}}{}$$

```
[Out] -1/2*(a+b*(c*x^2)^(3/2))^(1/2)/x^2+1/2*3^(3/4)*b^(2/3)*c*EllipticF((a^(1/3)
*(1-3^(1/2))+b^(1/3)*(c*x^2)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*(c*x^2)^(1
/2)),I*3^(1/2)+2*I)*(a^(1/3)+b^(1/3)*(c*x^2)^(1/2))*(1/2*6^(1/2)+1/2*2^(1/2
))*((a^(2/3)+b^(2/3)*c*x^2-a^(1/3)*b^(1/3)*(c*x^2)^(1/2))/(a^(1/3)*(1+3^(1/
2))+b^(1/3)*(c*x^2)^(1/2))^2)^(1/2)/(a+b*(c*x^2)^(3/2))^(1/2)/(a^(1/3)*(a^(
1/3)+b^(1/3)*(c*x^2)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*(c*x^2)^(1/2))^2)^
(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {368, 277, 218}

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{2/3}c\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}+b^{2/3}cx^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\sqrt{cx^2}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\sqrt{cx^2}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\sqrt{a+b\left(cx^2\right)^{3/2}}}-\frac{\sqrt{a+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*(c*x^2)^(3/2)]/x^3,x]
```

```
[Out] -Sqrt[a + b*(c*x^2)^(3/2)]/(2*x^2) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(2/3)*c*(
a^(1/3) + b^(1/3)*Sqrt[c*x^2])*Sqrt[(a^(2/3) + b^(2/3)*c*x^2 - a^(1/3)*b^(1
/3)*Sqrt[c*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])^2]*EllipticF
[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*Sqrt[c*x^2])], -7 - 4*Sqrt[3]])/(2*Sqrt[(a^(1/3)*(a^(1/3) + b^(
1/3)*Sqrt[c*x^2]))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])^2]*Sqrt[a
+ b*(c*x^2)^(3/2)])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^3}\,dx = c\,\text{Subst}\left(\int \frac{\sqrt{a+bx^3}}{x^3}\,dx, x, \sqrt{cx^2}\right)$$

$$= -\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{2x^2} + \frac{1}{4}(3bc)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^3}}\,dx, x, \sqrt{cx^2}\right)$$

$$= -\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{2x^2} + \frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{2/3}c\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}+b^{2/3}cx^2-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\,F\left(\sin$$

$$2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\sqrt{a+b\left(cx^2$$

Mathematica [C] time = 0.02, size = 69, normalized size = 0.23

$$-\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}\;{}_2F_1\left(-\frac{2}{3},-\frac{1}{2};\frac{1}{3};-\frac{b\left(cx^2\right)^{3/2}}{a}\right)}{2x^2\sqrt{\frac{b\left(cx^2\right)^{3/2}}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*(c*x^2)^(3/2)]/x^3,x]
```

```
[Out] -1/2*(Sqrt[a + b*(c*x^2)^(3/2)]*Hypergeometric2F1[-2/3, -1/2, 1/3, -((b*(c*
x^2)^(3/2))/a)])/(x^2*Sqrt[1 + (b*(c*x^2)^(3/2))/a])
```

fricas [F] time = 1.06, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\sqrt{cx^2}\,bcx^2+a}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(3/2))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(sqrt(c*x^2)*b*c*x^2 + a)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(cx^2\right)^{\frac{3}{2}}b+a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(3/2))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt((c*x^2)^(3/2)*b + a)/x^3, x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \left(c\,x^2\right)^{\frac{3}{2}}\,b}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(c*x^2)^(3/2)*b)^(1/2)/x^3,x)
[Out] int((a+(c*x^2)^(3/2)*b)^(1/2)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(cx^2\right)^{\frac{3}{2}}\,b + a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(3/2))^(1/2)/x^3,x, algorithm="maxima")
[Out] integrate(sqrt((c*x^2)^(3/2)*b + a)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a + b\left(c\,x^2\right)^{3/2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x^2)^(3/2))^(1/2)/x^3,x)
[Out] int((a + b*(c*x^2)^(3/2))^(1/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\left(cx^2\right)^{\frac{3}{2}}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x**2)**(3/2))**(1/2)/x**3,x)
[Out] Integral(sqrt(a + b*(c*x**2)**(3/2))/x**3, x)
```

3.2950 $\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^6}\,dx$

Optimal. Leaf size=352

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{5/3}\left(cx^2\right)^{5/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}+b^{2/3}cx^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\sqrt{cx^2}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\sqrt{cx^2}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{20ax^5\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\sqrt{a+b\left(cx^2\right)^{3/2}}}$$

```
[Out] -1/5*(a+b*(c*x^2)^(3/2))^(1/2)/x^5-3/20*b*(c*x^2)^(5/2)*(a+b*(c*x^2)^(3/2))
^(1/2)/a/c/x^7-1/20*3^(3/4)*b^(5/3)*(c*x^2)^(5/2)*EllipticF((a^(1/3)*(1-3^(
1/2))+b^(1/3)*(c*x^2)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*(c*x^2)^(1/2)),I*
3^(1/2)+2*I)*(a^(1/3)+b^(1/3)*(c*x^2)^(1/2))*(1/2*6^(1/2)+1/2*2^(1/2))*((a^
(2/3)+b^(2/3)*c*x^2-a^(1/3)*b^(1/3)*(c*x^2)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(
1/3)*(c*x^2)^(1/2))^2)^(1/2)/a/x^5/(a+b*(c*x^2)^(3/2))^(1/2)/(a^(1/3)*(a^(1
/3)+b^(1/3)*(c*x^2)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*(c*x^2)^(1/2))^2)^(
1/2)
```

Rubi [A] time = 0.19, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {368, 277, 325, 218}

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{5/3}\left(cx^2\right)^{5/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}+b^{2/3}cx^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\sqrt{cx^2}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\sqrt{cx^2}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{20ax^5\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\sqrt{a+b\left(cx^2\right)^{3/2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*(c*x^2)^(3/2)]/x^6,x]
```

```
[Out] -Sqrt[a + b*(c*x^2)^(3/2)]/(5*x^5) - (3*b*(c*x^2)^(5/2)*Sqrt[a + b*(c*x^2)^
(3/2)])/(20*a*c*x^7) - (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(5/3)*(c*x^2)^(5/2)*(a^
(1/3) + b^(1/3)*Sqrt[c*x^2])*Sqrt[(a^(2/3) + b^(2/3)*c*x^2 - a^(1/3)*b^(1/3
)*Sqrt[c*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])^2]*EllipticF[A
rcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*Sqrt[c*x^2])], -7 - 4*Sqrt[3]])/(20*a*x^5*Sqrt[(a^(1/3)*(a^(1/3)
+ b^(1/3)*Sqrt[c*x^2]))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])^2]*Sq
rt[a + b*(c*x^2)^(3/2)])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

```
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^6}\,dx = \frac{\left(cx^2\right)^{5/2}\text{Subst}\left(\int \frac{\sqrt{a+bx^3}}{x^6}\,dx, x, \sqrt{cx^2}\right)}{x^5}$$

$$= -\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{5x^5} + \frac{\left(3b\left(cx^2\right)^{5/2}\right)\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx^3}}\,dx, x, \sqrt{cx^2}\right)}{10x^5}$$

$$= -\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{5x^5} - \frac{3b\left(cx^2\right)^{5/2}\sqrt{a+b\left(cx^2\right)^{3/2}}}{20acx^7} - \frac{\left(3b^2\left(cx^2\right)^{5/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^3}}\,dx,\right.}{40ax^5}$$

$$= -\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{5x^5} - \frac{3b\left(cx^2\right)^{5/2}\sqrt{a+b\left(cx^2\right)^{3/2}}}{20acx^7} - \frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{5/3}\left(cx^2\right)^{5/2}\left(\sqrt[3]{a}+\right.}{}$$

$$20$$

Mathematica [C] time = 0.02, size = 69, normalized size = 0.20

$$-\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}\;{}_2F_1\left(-\frac{5}{3},-\frac{1}{2};-\frac{2}{3};-\frac{b\left(cx^2\right)^{3/2}}{a}\right)}{5x^5\sqrt{\frac{b\left(cx^2\right)^{3/2}}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*(c*x^2)^(3/2)]/x^6,x]
```

```
[Out] -1/5*(Sqrt[a + b*(c*x^2)^(3/2)]*Hypergeometric2F1[-5/3, -1/2, -2/3, -((b*(c
*x^2)^(3/2))/a)])/(x^5*Sqrt[1 + (b*(c*x^2)^(3/2))/a])
```

fricas [F] time = 1.26, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\sqrt{cx^2}\,bcx^2+a}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(3/2))^(1/2)/x^6,x, algorithm="fricas")
[Out] integral(sqrt(sqrt(c*x^2)*b*c*x^2 + a)/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(cx^2\right)^{\frac{3}{2}} b + a}}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(3/2))^(1/2)/x^6,x, algorithm="giac")
[Out] integrate(sqrt((c*x^2)^(3/2)*b + a)/x^6, x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \left(c\,x^2\right)^{\frac{3}{2}} b}}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(c*x^2)^(3/2)*b)^(1/2)/x^6,x)
[Out] int((a+(c*x^2)^(3/2)*b)^(1/2)/x^6,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(cx^2\right)^{\frac{3}{2}} b + a}}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(3/2))^(1/2)/x^6,x, algorithm="maxima")
[Out] integrate(sqrt((c*x^2)^(3/2)*b + a)/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a + b\left(c\,x^2\right)^{3/2}}}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x^2)^(3/2))^(1/2)/x^6,x)
[Out] int((a + b*(c*x^2)^(3/2))^(1/2)/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\left(cx^2\right)^{\frac{3}{2}}}}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x**2)**(3/2))**(1/2)/x**6,x)
[Out] Integral(sqrt(a + b*(c*x**2)**(3/2))/x**6, x)
```

3.2951 $\int x^4 \sqrt{a + b\,(cx^2)^{3/2}}\,dx$

Optimal. Leaf size=709

$$-\frac{8\sqrt{2}\,3^{3/4}a^{7/3}x^5\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}+b^{2/3}cx^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\sqrt{cx^2}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\sqrt{cx^2}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\quad 12\sqrt[4]{3}\sqrt{}}{91b^{5/3}\left(cx^2\right)^{5/2}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\sqrt{a+b\left(cx^2\right)^{3/2}}}+$$

```
[Out] 2/13*x^5*(a+b*(c*x^2)^(3/2))^(1/2)+6/91*a*c*x^7*(a+b*(c*x^2)^(3/2))^(1/2)/b
/(c*x^2)^(5/2)-24/91*a^2*x^5*(a+b*(c*x^2)^(3/2))^(1/2)/b^(5/3)/(c*x^2)^(5/2
)/(a^(1/3)*(1+3^(1/2))+b^(1/3)*(c*x^2)^(1/2))-8/91*3^(3/4)*a^(7/3)*x^5*Elli
pticF((a^(1/3)*(1-3^(1/2))+b^(1/3)*(c*x^2)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1
/3)*(c*x^2)^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*(a^(1/3)+b^(1/3)*(c*x^2)^(1/2))*(
(a^(2/3)+b^(2/3)*c*x^2-a^(1/3)*b^(1/3)*(c*x^2)^(1/2))/(a^(1/3)*(1+3^(1/2))+
b^(1/3)*(c*x^2)^(1/2))^2)^(1/2)/b^(5/3)/(c*x^2)^(5/2)/(a+b*(c*x^2)^(3/2))^(
1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*(c*x^2)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*
(c*x^2)^(1/2))^2)^(1/2)+12/91*3^(1/4)*a^(7/3)*x^5*EllipticE((a^(1/3)*(1-3^(
1/2))+b^(1/3)*(c*x^2)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*(c*x^2)^(1/2)),I*
3^(1/2)+2*I)*(a^(1/3)+b^(1/3)*(c*x^2)^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((a^
(2/3)+b^(2/3)*c*x^2-a^(1/3)*b^(1/3)*(c*x^2)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(
1/3)*(c*x^2)^(1/2))^2)^(1/2)/b^(5/3)/(c*x^2)^(5/2)/(a+b*(c*x^2)^(3/2))^(1/2
)/(a^(1/3)*(a^(1/3)+b^(1/3)*(c*x^2)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*(c*
x^2)^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 709, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {368, 279, 321, 303, 218, 1877}

$$-\frac{24a^2x^5\sqrt{a+b\left(cx^2\right)^{3/2}}}{91b^{5/3}\left(cx^2\right)^{5/2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}-\frac{8\sqrt{2}\,3^{3/4}a^{7/3}x^5\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}+b^{2/3}cx^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\,F\left(\sin^{-1}\right.}{91b^{5/3}\left(cx^2\right)^{5/2}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[a + b*(c*x^2)^(3/2)],x]
```

```
[Out] (2*x^5*Sqrt[a + b*(c*x^2)^(3/2)])/13 + (6*a*c*x^7*Sqrt[a + b*(c*x^2)^(3/2)]
)/(91*b*(c*x^2)^(5/2)) - (24*a^2*x^5*Sqrt[a + b*(c*x^2)^(3/2)])/(91*b^(5/3)
*(c*x^2)^(5/2)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])) + (12*3^(1/4)
*Sqrt[2 - Sqrt[3]]*a^(7/3)*x^5*(a^(1/3) + b^(1/3)*Sqrt[c*x^2])*Sqrt[(a^(2/3
) + b^(2/3)*c*x^2 - a^(1/3)*b^(1/3)*Sqrt[c*x^2])/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*Sqrt[c*x^2])^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*Sq
rt[c*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])], -7 - 4*Sqrt[3]])
/(91*b^(5/3)*(c*x^2)^(5/2)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*Sqrt[c*x^2]))/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])^2]*Sqrt[a + b*(c*x^2)^(3/2)])
 - (8*Sqrt[2]*3^(3/4)*a^(7/3)*x^5*(a^(1/3) + b^(1/3)*Sqrt[c*x^2])*Sqrt[(a^(2
/3) + b^(2/3)*c*x^2 - a^(1/3)*b^(1/3)*Sqrt[c*x^2])/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*Sqrt[c*x^2])^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*
Sqrt[c*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])], -7 - 4*Sqrt[3]
])/(91*b^(5/3)*(c*x^2)^(5/2)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*Sqrt[c*x^2]))
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])^2]*Sqrt[a + b*(c*x^2)^(3/2)]
)
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int x^4\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx = \frac{x^5\,\text{Subst}\left(\int x^4\sqrt{a+bx^3}\,dx, x, \sqrt{cx^2}\right)}{\left(cx^2\right)^{5/2}}$$

$$= \frac{2}{13}x^5\sqrt{a+b\left(cx^2\right)^{3/2}} + \frac{\left(3ax^5\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^3}}\,dx, x, \sqrt{cx^2}\right)}{13\left(cx^2\right)^{5/2}}$$

$$= \frac{2}{13}x^5\sqrt{a+b\left(cx^2\right)^{3/2}} + \frac{6acx^7\sqrt{a+b\left(cx^2\right)^{3/2}}}{91b\left(cx^2\right)^{5/2}} - \frac{\left(12a^2x^5\right)\text{Subst}\left(\int \frac{x}{\sqrt{a+bx^3}}\,dx, x, \sqrt{}\right.}{91b\left(cx^2\right)^{5/2}}$$

$$= \frac{2}{13}x^5\sqrt{a+b\left(cx^2\right)^{3/2}} + \frac{6acx^7\sqrt{a+b\left(cx^2\right)^{3/2}}}{91b\left(cx^2\right)^{5/2}} - \frac{\left(12a^2x^5\right)\text{Subst}\left(\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\right.}{91b^{4/3}\left(cx^2\right)^{5/2}}$$

$$= \frac{2}{13}x^5\sqrt{a+b\left(cx^2\right)^{3/2}} + \frac{6acx^7\sqrt{a+b\left(cx^2\right)^{3/2}}}{91b\left(cx^2\right)^{5/2}} - \frac{24a^2x^5\sqrt{a+b\left(cx^2\right)^{3/2}}}{91b^{5/3}\left(cx^2\right)^{5/2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{}\right.}$$

Mathematica [C] time = 0.09, size = 109, normalized size = 0.15

$$\frac{2x^5\sqrt{a+b\left(cx^2\right)^{3/2}}\left(a\left(\frac{a+b\left(cx^2\right)^{3/2}}{a}\right)^{3/2} - a\,{}_2F_1\left(-\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; -\frac{b\left(cx^2\right)^{3/2}}{a}\right)\right)}{13b\left(cx^2\right)^{3/2}\sqrt{\frac{a+b\left(cx^2\right)^{3/2}}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[a + b*(c*x^2)^(3/2)],x]
```

```
[Out] (2*x^5*Sqrt[a + b*(c*x^2)^(3/2)]*(a*((a + b*(c*x^2)^(3/2))/a)^(3/2) - a*Hyp
ergeometric2F1[-1/2, 2/3, 5/3, -((b*(c*x^2)^(3/2))/a)]))/(13*b*(c*x^2)^(3/2
)*Sqrt[(a + b*(c*x^2)^(3/2))/a])
```

fricas [F] time = 1.30, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{\sqrt{cx^2}\,bcx^2 + a}\,x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sqrt(c*x^2)*b*c*x^2 + a)*x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\left(cx^2\right)^{\frac{3}{2}}b + a}\,x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((c*x^2)^(3/2)*b + a)*x^4, x)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \sqrt{a + \left(c\,x^2\right)^{\frac{3}{2}}\,b}\;x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+(c*x^2)^(3/2)*b)^(1/2),x)
[Out] int(x^4*(a+(c*x^2)^(3/2)*b)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\left(cx^2\right)^{\frac{3}{2}}\,b + a}\,x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt((c*x^2)^(3/2)*b + a)*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,\sqrt{a + b\left(c\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*(c*x^2)^(3/2))^(1/2),x)
[Out] int(x^4*(a + b*(c*x^2)^(3/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \sqrt{a + b\left(cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*(c*x**2)**(3/2))**(1/2),x)
[Out] Integral(x**4*sqrt(a + b*(c*x**2)**(3/2)), x)
```

3.2952 $\int x\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=642

$$\frac{2\sqrt{2}\,3^{3/4}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}+b^{2/3}cx^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\sqrt{cx^2}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\sqrt{cx^2}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\quad 3\sqrt[4]{3}\sqrt{2-}}{7b^{2/3}c\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\sqrt{a+b\left(cx^2\right)^{3/2}}}$$

```
[Out] 2/7*x^2*(a+b*(c*x^2)^(3/2))^(1/2)+6/7*a*(a+b*(c*x^2)^(3/2))^(1/2)/b^(2/3)/c
/(a^(1/3)*(1+3^(1/2))+b^(1/3)*(c*x^2)^(1/2))+2/7*3^(3/4)*a^(4/3)*EllipticF(
(a^(1/3)*(1-3^(1/2))+b^(1/3)*(c*x^2)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*(c
*x^2)^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*(a^(1/3)+b^(1/3)*(c*x^2)^(1/2))*((a^(2/
3)+b^(2/3)*c*x^2-a^(1/3)*b^(1/3)*(c*x^2)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3
)*(c*x^2)^(1/2))^2)^(1/2)/b^(2/3)/c/(a+b*(c*x^2)^(3/2))^(1/2)/(a^(1/3)*(a^(
1/3)+b^(1/3)*(c*x^2)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*(c*x^2)^(1/2))^2)^
(1/2)-3/7*3^(1/4)*a^(4/3)*EllipticE((a^(1/3)*(1-3^(1/2))+b^(1/3)*(c*x^2)^(1
/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*(c*x^2)^(1/2)),I*3^(1/2)+2*I)*(a^(1/3)+b^
(1/3)*(c*x^2)^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)+b^(2/3)*c*x^2-a^(1
/3)*b^(1/3)*(c*x^2)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*(c*x^2)^(1/2))^2)^(
1/2)/b^(2/3)/c/(a+b*(c*x^2)^(3/2))^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*(c*x^2)^
(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*(c*x^2)^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 642, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {368, 279, 303, 218, 1877}

$$\frac{2\sqrt{2}\,3^{3/4}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}+b^{2/3}cx^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\sqrt{cx^2}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\sqrt{cx^2}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\quad 3\sqrt[4]{3}\sqrt{2-}}{7b^{2/3}c\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\sqrt{a+b\left(cx^2\right)^{3/2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*(c*x^2)^(3/2)],x]
```

```
[Out] (2*x^2*Sqrt[a + b*(c*x^2)^(3/2)])/7 + (6*a*Sqrt[a + b*(c*x^2)^(3/2)])/(7*b^
(2/3)*c*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])) - (3*3^(1/4)*Sqrt[2
- Sqrt[3]]*a^(4/3)*(a^(1/3) + b^(1/3)*Sqrt[c*x^2])*Sqrt[(a^(2/3) + b^(2/3)*
c*x^2 - a^(1/3)*b^(1/3)*Sqrt[c*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[
c*x^2])^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])], -7 - 4*Sqrt[3]])/(7*b^(2/3)*
c*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*Sqrt[c*x^2]))/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*Sqrt[c*x^2])^2]*Sqrt[a + b*(c*x^2)^(3/2)]) + (2*Sqrt[2]*3^(3/4)*a^(4
/3)*(a^(1/3) + b^(1/3)*Sqrt[c*x^2])*Sqrt[(a^(2/3) + b^(2/3)*c*x^2 - a^(1/3)
*b^(1/3)*Sqrt[c*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])^2]*Elli
pticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*Sqrt[c*x^2])], -7 - 4*Sqrt[3]])/(7*b^(2/3)*c*Sqrt[(a^(1/3)
*(a^(1/3) + b^(1/3)*Sqrt[c*x^2]))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x
^2])^2]*Sqrt[a + b*(c*x^2)^(3/2)])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
```

```
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int x\sqrt{a+b\left(cx^2\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int x\sqrt{a+bx^3}\,dx, x, \sqrt{cx^2}\right)}{c}$$

$$= \frac{2}{7}x^2\sqrt{a+b\left(cx^2\right)^{3/2}} + \frac{(3a)\,\text{Subst}\left(\int \frac{x}{\sqrt{a+bx^3}}\,dx, x, \sqrt{cx^2}\right)}{7c}$$

$$= \frac{2}{7}x^2\sqrt{a+b\left(cx^2\right)^{3/2}} + \frac{(3a)\,\text{Subst}\left(\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx, x, \sqrt{cx^2}\right)}{7\sqrt[3]{b}\,c} + \frac{\left(3\sqrt{2\left(2-\sqrt{3}\right)}\,a^{4/}\right.}{}$$

$$= \frac{2}{7}x^2\sqrt{a+b\left(cx^2\right)^{3/2}} + \frac{6a\sqrt{a+b\left(cx^2\right)^{3/2}}}{7b^{2/3}c\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt{cx^2}\right)} - \frac{3\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}+\right.}{}$$

Mathematica [C] time = 0.02, size = 69, normalized size = 0.11

$$\frac{x^2\sqrt{a+b\left(cx^2\right)^{3/2}}\ {}_2F_1\left(-\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{b\left(cx^2\right)^{3/2}}{a}\right)}{2\sqrt{\frac{b\left(cx^2\right)^{3/2}}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + b*(c*x^2)^(3/2)],x]
```

```
[Out] (x^2*Sqrt[a + b*(c*x^2)^(3/2)]*Hypergeometric2F1[-1/2, 2/3, 5/3, -((b*(c*x^
2)^(3/2))/a)])/(2*Sqrt[1 + (b*(c*x^2)^(3/2))/a])
```

fricas [F] time = 1.38, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{\sqrt{cx^2}\,bcx^2+a}\,x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sqrt(c*x^2)*b*c*x^2 + a)*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\left(cx^2\right)^{\frac{3}{2}}b+a}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((c*x^2)^(3/2)*b + a)*x, x)
```

maple [A] time = 0.01, size = 495, normalized size = 0.77

$$4i\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(\frac{\left(-ab^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}}{2b}+\sqrt{cx^2}\right)\sqrt{3}\,b}{\left(-ab^2\right)^{\frac{1}{3}}}}\sqrt{\frac{-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{b}+\sqrt{cx^2}}{-\frac{3\left(-ab^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(\frac{\left(-ab^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}}{2b}+\sqrt{cx^2}\right)\sqrt{3}}{\left(-ab^2\right)^{\frac{1}{3}}}}$$

$$\frac{4\sqrt{a+\left(cx^2\right)^{\frac{3}{2}}b}\,cx^2}{7}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+(c*x^2)^(3/2)*b)^(1/2),x)
```

```
[Out] 1/2/c*(4/7*c*x^2*(a+(c*x^2)^(3/2)*b)^(1/2)-4/7*I*a*3^(1/2)*(-a*b^2)^(1/3)/b
*(I*((c*x^2)^(1/2)+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2)*(((c*x^2)^(1/2)-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^
2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*((c*x^2)^(1/2)+1/2*(-
a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2)/(a+(c*x^2)^(3/2)*b)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2
)^(1/3)/b)*EllipticE(1/3*3^(1/2)*(I*((c*x^2)^(1/2)+1/2*(-a*b^2)^(1/3)/b-1/2
*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)*(-a
*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)
)+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(I*((c*x^2)^(1/2)+1/2*(-a*b^2)^(1/
3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(
1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/
b)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\left(cx^2\right)^{\frac{3}{2}} b + a}\, x\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((c*x^2)^(3/2)*b + a)*x, x)
```

mupad [B] time = 1.54, size = 55, normalized size = 0.09

$$\frac{x^2 \sqrt{a + b\, c^{3/2} \sqrt{x^6}}\ {}_2\mathrm{F}_1\left(-\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; -\frac{b\, c^{3/2} \sqrt{x^6}}{a}\right)}{2 \sqrt{\frac{b\, c^{3/2} \sqrt{x^6}}{a} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*(c*x^2)^(3/2))^(1/2),x)
```

```
[Out] (x^2*(a + b*c^(3/2)*(x^6)^(1/2))^(1/2)*hypergeom([-1/2, 2/3], 5/3, -(b*c^(3
/2)*(x^6)^(1/2))/a))/(2*((b*c^(3/2)*(x^6)^(1/2))/a + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \sqrt{a + b \left(cx^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(c*x**2)**(3/2))**(1/2),x)
```

```
[Out] Integral(x*sqrt(a + b*(c*x**2)**(3/2)), x)
```

3.2953 $\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^2}\,dx$

Optimal. Leaf size=661

$$\frac{\sqrt{2}\,3^{3/4}\sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt{cx^2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt{cx^2}+b^{2/3}cx^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt{cx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,\sqrt{cx^2}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,\sqrt{cx^2}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)\quad 3\sqrt[4]{}}{x\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt{cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt{cx^2}\right)^2}}\,\sqrt{a+b\left(cx^2\right)^{3/2}}}$$

```
[Out] -(a+b*(c*x^2)^(3/2))^(1/2)/x+3*b^(1/3)*(c*x^2)^(1/2)*(a+b*(c*x^2)^(3/2))^(1
/2)/x/(a^(1/3)*(1+3^(1/2))+b^(1/3)*(c*x^2)^(1/2))+3^(3/4)*a^(1/3)*b^(1/3)*E
llipticF((a^(1/3)*(1-3^(1/2))+b^(1/3)*(c*x^2)^(1/2))/(a^(1/3)*(1+3^(1/2))+b
^(1/3)*(c*x^2)^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*(c*x^2)^(1/2)*(a^(1/3)+b^(1/3)
*(c*x^2)^(1/2))*((a^(2/3)+b^(2/3)*c*x^2-a^(1/3)*b^(1/3)*(c*x^2)^(1/2))/(a^(
1/3)*(1+3^(1/2))+b^(1/3)*(c*x^2)^(1/2))^2)^(1/2)/x/(a+b*(c*x^2)^(3/2))^(1/2
)/(a^(1/3)*(a^(1/3)+b^(1/3)*(c*x^2)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*(c*
x^2)^(1/2))^2)^(1/2)-3/2*3^(1/4)*a^(1/3)*b^(1/3)*EllipticE((a^(1/3)*(1-3^(1
/2))+b^(1/3)*(c*x^2)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*(c*x^2)^(1/2)),I*3
^(1/2)+2*I)*(c*x^2)^(1/2)*(a^(1/3)+b^(1/3)*(c*x^2)^(1/2))*(1/2*6^(1/2)-1/2*
2^(1/2))*((a^(2/3)+b^(2/3)*c*x^2-a^(1/3)*b^(1/3)*(c*x^2)^(1/2))/(a^(1/3)*(1
+3^(1/2))+b^(1/3)*(c*x^2)^(1/2))^2)^(1/2)/x/(a+b*(c*x^2)^(3/2))^(1/2)/(a^(1
/3)*(a^(1/3)+b^(1/3)*(c*x^2)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*(c*x^2)^(1
/2))^2)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 661, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {368, 277, 303, 218, 1877}

$$\frac{\sqrt{2}\,3^{3/4}\sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt{cx^2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt{cx^2}+b^{2/3}cx^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt{cx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,\sqrt{cx^2}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,\sqrt{cx^2}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)\quad 3\sqrt[4]{}}{x\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt{cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt{cx^2}\right)^2}}\,\sqrt{a+b\left(cx^2\right)^{3/2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*(c*x^2)^(3/2)]/x^2,x]
```

```
[Out] -(Sqrt[a + b*(c*x^2)^(3/2)]/x) + (3*b^(1/3)*Sqrt[c*x^2]*Sqrt[a + b*(c*x^2)^
(3/2)])/(x*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])) - (3*3^(1/4)*Sqrt
[2 - Sqrt[3]]*a^(1/3)*b^(1/3)*Sqrt[c*x^2]*(a^(1/3) + b^(1/3)*Sqrt[c*x^2])*S
qrt[(a^(2/3) + b^(2/3)*c*x^2 - a^(1/3)*b^(1/3)*Sqrt[c*x^2])/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*Sqrt[c*x^2])^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) +
 b^(1/3)*Sqrt[c*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])], -7 -
4*Sqrt[3]])/(2*x*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*Sqrt[c*x^2]))/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])^2]*Sqrt[a + b*(c*x^2)^(3/2)]) + (Sqrt[2]
*3^(3/4)*a^(1/3)*b^(1/3)*Sqrt[c*x^2]*(a^(1/3) + b^(1/3)*Sqrt[c*x^2])*Sqrt[(
a^(2/3) + b^(2/3)*c*x^2 - a^(1/3)*b^(1/3)*Sqrt[c*x^2])/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*Sqrt[c*x^2])^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*Sqrt[c*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])], -7 - 4*Sqr
t[3]])/(x*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*Sqrt[c*x^2]))/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*Sqrt[c*x^2])^2]*Sqrt[a + b*(c*x^2)^(3/2)])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^2}\,dx = \frac{\sqrt{cx^2}\ \text{Subst}\left(\int \frac{\sqrt{a+bx^3}}{x^2}\,dx, x, \sqrt{cx^2}\right)}{x}$$

$$= -\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x} + \frac{\left(3b\sqrt{cx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{a+bx^3}}\,dx, x, \sqrt{cx^2}\right)}{2x}$$

$$= -\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x} + \frac{\left(3b^{2/3}\sqrt{cx^2}\right)\text{Subst}\left(\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx, x, \sqrt{cx^2}\right)}{2x} + \frac{\left(3\sqrt{\frac{1}{2}\left(2-\sqrt{3}\right.\right.}$$

$$= -\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x} + \frac{3\sqrt[3]{b}\,\sqrt{cx^2}\,\sqrt{a+b\left(cx^2\right)^{3/2}}}{x\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt{cx^2}\right)} - \frac{3\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\ \sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt{cx^2}\,\left(\sqrt[3]{a}+\right.}{}$$

Mathematica [C] time = 0.02, size = 67, normalized size = 0.10

$$-\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}\ {}_2F_1\left(-\frac{1}{2},-\frac{1}{3};\frac{2}{3};-\frac{b\left(cx^2\right)^{3/2}}{a}\right)}{x\sqrt{\frac{b\left(cx^2\right)^{3/2}}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*(c*x^2)^(3/2)]/x^2,x]
```

```
[Out] -((Sqrt[a + b*(c*x^2)^(3/2)]*Hypergeometric2F1[-1/2, -1/3, 2/3, -((b*(c*x^2
)^(3/2))/a)])/(x*Sqrt[1 + (b*(c*x^2)^(3/2))/a]))
```

fricas [F] time = 8.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\sqrt{cx^2}\,bcx^2+a}}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(3/2))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(sqrt(c*x^2)*b*c*x^2 + a)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(cx^2\right)^{\frac{3}{2}}b+a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(3/2))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt((c*x^2)^(3/2)*b + a)/x^2, x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\left(c\,x^2\right)^{\frac{3}{2}}b}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(c*x^2)^(3/2)*b)^(1/2)/x^2,x)
```

```
[Out] int((a+(c*x^2)^(3/2)*b)^(1/2)/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(cx^2\right)^{\frac{3}{2}}b+a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(3/2))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt((c*x^2)^(3/2)*b + a)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a + b\left(c\,x^2\right)^{3/2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x^2)^(3/2))^(1/2)/x^2,x)
```

```
[Out] int((a + b*(c*x^2)^(3/2))^(1/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\left(cx^2\right)^{\frac{3}{2}}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x**2)**(3/2))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(a + b*(c*x**2)**(3/2))/x**2, x)
```

3.2954 $\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^5}\,dx$

Optimal. Leaf size=681

$$\frac{3^{3/4}b^{4/3}c^2\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}+b^{2/3}cx^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\sqrt{cx^2}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\sqrt{cx^2}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\quad 3\sqrt[4]{3}\sqrt{2-\sqrt{3}}}{4\sqrt{2}\,a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\sqrt{a+b\left(cx^2\right)^{3/2}}}$$

```
[Out] -1/4*(a+b*(c*x^2)^(3/2))^(1/2)/x^4-3/8*b*c^2*(a+b*(c*x^2)^(3/2))^(1/2)/a/(c
*x^2)^(1/2)+3/8*b^(4/3)*c^2*(a+b*(c*x^2)^(3/2))^(1/2)/a/(a^(1/3)*(1+3^(1/2)
)+b^(1/3)*(c*x^2)^(1/2))+1/8*3^(3/4)*b^(4/3)*c^2*EllipticF((a^(1/3)*(1-3^(1
/2))+b^(1/3)*(c*x^2)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*(c*x^2)^(1/2)),I*3
^(1/2)+2*I)*(a^(1/3)+b^(1/3)*(c*x^2)^(1/2))*((a^(2/3)+b^(2/3)*c*x^2-a^(1/3)
*b^(1/3)*(c*x^2)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*(c*x^2)^(1/2))^2)^(1/2
)/a^(2/3)*2^(1/2)/(a+b*(c*x^2)^(3/2))^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*(c*x^
2)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*(c*x^2)^(1/2))^2)^(1/2)-3/16*3^(1/4)
*b^(4/3)*c^2*EllipticE((a^(1/3)*(1-3^(1/2))+b^(1/3)*(c*x^2)^(1/2))/(a^(1/3)
*(1+3^(1/2))+b^(1/3)*(c*x^2)^(1/2)),I*3^(1/2)+2*I)*(a^(1/3)+b^(1/3)*(c*x^2)
^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)+b^(2/3)*c*x^2-a^(1/3)*b^(1/3)*(
c*x^2)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*(c*x^2)^(1/2))^2)^(1/2)/a^(2/3)/
(a+b*(c*x^2)^(3/2))^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*(c*x^2)^(1/2))/(a^(1/3)
*(1+3^(1/2))+b^(1/3)*(c*x^2)^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 681, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {368, 277, 325, 303, 218, 1877}

$$\frac{3^{3/4}b^{4/3}c^2\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx^2}+b^{2/3}cx^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\sqrt{cx^2}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\sqrt{cx^2}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\quad 3\sqrt[4]{3}\sqrt{2-\sqrt{3}}}{4\sqrt{2}\,a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\sqrt{cx^2}\right)^2}}\sqrt{a+b\left(cx^2\right)^{3/2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*(c*x^2)^(3/2)]/x^5,x]
```

```
[Out] -Sqrt[a + b*(c*x^2)^(3/2)]/(4*x^4) - (3*b*c^2*Sqrt[a + b*(c*x^2)^(3/2)])/(8
*a*Sqrt[c*x^2]) + (3*b^(4/3)*c^2*Sqrt[a + b*(c*x^2)^(3/2)])/(8*a*((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)
*c^2*(a^(1/3) + b^(1/3)*Sqrt[c*x^2])*Sqrt[(a^(2/3) + b^(2/3)*c*x^2 - a^(1/3
)*b^(1/3)*Sqrt[c*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])^2]*Ell
ipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*Sqrt[c*x^2])], -7 - 4*Sqrt[3]])/(16*a^(2/3)*Sqrt[(a^(1/3)
*(a^(1/3) + b^(1/3)*Sqrt[c*x^2]))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x
^2])^2]*Sqrt[a + b*(c*x^2)^(3/2)]) + (3^(3/4)*b^(4/3)*c^2*(a^(1/3) + b^(1/3
)*Sqrt[c*x^2])*Sqrt[(a^(2/3) + b^(2/3)*c*x^2 - a^(1/3)*b^(1/3)*Sqrt[c*x^2])
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])^2]*EllipticF[ArcSin[((1 - Sq
rt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt
[c*x^2])], -7 - 4*Sqrt[3]])/(4*Sqrt[2]*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(
1/3)*Sqrt[c*x^2]))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^2])^2]*Sqrt[a
+ b*(c*x^2)^(3/2)])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{x^5}\,dx = c^2\,\text{Subst}\left(\int \frac{\sqrt{a+bx^3}}{x^5}\,dx, x, \sqrt{cx^2}\right)$$
$$= -\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{4x^4} + \frac{1}{8}\left(3bc^2\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx, x, \sqrt{cx^2}\right)$$
$$= -\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{4x^4} - \frac{3bc^2\sqrt{a+b\left(cx^2\right)^{3/2}}}{8a\sqrt{cx^2}} + \frac{\left(3b^2c^2\right)\text{Subst}\left(\int \frac{x}{\sqrt{a+bx^3}}\,dx, x, \sqrt{cx^2}\right)}{16a}$$
$$= -\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{4x^4} - \frac{3bc^2\sqrt{a+b\left(cx^2\right)^{3/2}}}{8a\sqrt{cx^2}} + \frac{\left(3b^{5/3}c^2\right)\text{Subst}\left(\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx, x, \right.}{16a}$$
$$= -\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}}{4x^4} - \frac{3bc^2\sqrt{a+b\left(cx^2\right)^{3/2}}}{8a\sqrt{cx^2}} + \frac{3b^{4/3}c^2\sqrt{a+b\left(cx^2\right)^{3/2}}}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt{cx^2}\right)} - \frac{3\sqrt[4]{3}\sqrt{}}{}$$

Mathematica [C] time = 0.04, size = 69, normalized size = 0.10

$$-\frac{\sqrt{a+b\left(cx^2\right)^{3/2}}\;{}_2F_1\left(-\frac{4}{3},-\frac{1}{2};-\frac{1}{3};-\frac{b\left(cx^2\right)^{3/2}}{a}\right)}{4x^4\sqrt{\frac{b\left(cx^2\right)^{3/2}}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*(c*x^2)^(3/2)]/x^5,x]
```

```
[Out] -1/4*(Sqrt[a + b*(c*x^2)^(3/2)]*Hypergeometric2F1[-4/3, -1/2, -1/3, -((b*(c
*x^2)^(3/2))/a)])/(x^4*Sqrt[1 + (b*(c*x^2)^(3/2))/a])
```

fricas [F] time = 1.12, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\sqrt{cx^2}\,bcx^2+a}}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(3/2))^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] integral(sqrt(sqrt(c*x^2)*b*c*x^2 + a)/x^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(cx^2\right)^{\frac{3}{2}}b+a}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(3/2))^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(sqrt((c*x^2)^(3/2)*b + a)/x^5, x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \left(c\,x^2\right)^{\frac{3}{2}} b}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(c*x^2)^(3/2)*b)^(1/2)/x^5,x)
[Out] int((a+(c*x^2)^(3/2)*b)^(1/2)/x^5,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(cx^2\right)^{\frac{3}{2}} b + a}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^2)^(3/2))^(1/2)/x^5,x, algorithm="maxima")
[Out] integrate(sqrt((c*x^2)^(3/2)*b + a)/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a + b\left(c\,x^2\right)^{3/2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x^2)^(3/2))^(1/2)/x^5,x)
[Out] int((a + b*(c*x^2)^(3/2))^(1/2)/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\left(cx^2\right)^{\frac{3}{2}}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x**2)**(3/2))**(1/2)/x**5,x)
[Out] Integral(sqrt(a + b*(c*x**2)**(3/2))/x**5, x)
```

3.2955 $\int (dx)^m \sqrt{a + b\,(cx^2)^{3/2}}\, dx$

Optimal. Leaf size=86

$$\frac{(dx)^{m+1}\sqrt{a+b\left(cx^2\right)^{3/2}}\;{}_2F_1\left(-\frac{1}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{b\left(cx^2\right)^{3/2}}{a}\right)}{d(m+1)\sqrt{\frac{b\left(cx^2\right)^{3/2}}{a}+1}}$$

```
[Out] (d*x)^(1+m)*hypergeom([-1/2, 1/3+1/3*m],[4/3+1/3*m],-b*(c*x^2)^(3/2)/a)*(a+
b*(c*x^2)^(3/2))^(1/2)/d/(1+m)/(1+b*(c*x^2)^(3/2)/a)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {368, 365, 364}

$$\frac{(dx)^{m+1}\sqrt{a+b\left(cx^2\right)^{3/2}}\;{}_2F_1\left(-\frac{1}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{b\left(cx^2\right)^{3/2}}{a}\right)}{d(m+1)\sqrt{\frac{b\left(cx^2\right)^{3/2}}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[a + b*(c*x^2)^(3/2)],x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[a + b*(c*x^2)^(3/2)]*Hypergeometric2F1[-1/2, (1 + m)/3,
 (4 + m)/3, -((b*(c*x^2)^(3/2))/a)])/(d*(1 + m)*Sqrt[1 + (b*(c*x^2)^(3/2))/
a])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\int (dx)^m \sqrt{a+b\left(cx^2\right)^{3/2}}\,dx = \frac{\left((dx)^{1+m}\left(cx^2\right)^{\frac{1}{2}(-1-m)}\right)\text{Subst}\left(\int x^m\sqrt{a+bx^3}\,dx, x, \sqrt{cx^2}\right)}{d}$$

$$= \frac{\left((dx)^{1+m}\left(cx^2\right)^{\frac{1}{2}(-1-m)}\sqrt{a+b\left(cx^2\right)^{3/2}}\right)\text{Subst}\left(\int x^m\sqrt{1+\frac{bx^3}{a}}\,dx, x, \sqrt{cx^2}\right)}{d\sqrt{1+\frac{b\left(cx^2\right)^{3/2}}{a}}}$$

$$= \frac{(dx)^{1+m}\sqrt{a+b\left(cx^2\right)^{3/2}}\;{}_2F_1\left(-\frac{1}{2}, \frac{1+m}{3}; \frac{4+m}{3}; -\frac{b\left(cx^2\right)^{3/2}}{a}\right)}{d(1+m)\sqrt{1+\frac{b\left(cx^2\right)^{3/2}}{a}}}$$

Mathematica [F] time = 0.08, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{a+b\left(cx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*Sqrt[a + b*(c*x^2)^(3/2)],x]
[Out] Integrate[(d*x)^m*Sqrt[a + b*(c*x^2)^(3/2)], x]
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{\sqrt{cx^2}\,bcx^2+a}\,(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(sqrt(c*x^2)*b*c*x^2 + a)*(d*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\left(cx^2\right)^{\frac{3}{2}}b+a}\,(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt((c*x^2)^(3/2)*b + a)*(d*x)^m, x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \sqrt{a+\left(c\,x^2\right)^{\frac{3}{2}}b}\;(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+(c*x^2)^(3/2)*b)^(1/2),x)
[Out] int((d*x)^m*(a+(c*x^2)^(3/2)*b)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\left(cx^2\right)^{\frac{3}{2}} b + a}\ (dx)^m\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*(c*x^2)^(3/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt((c*x^2)^(3/2)*b + a)*(d*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m \sqrt{a + b\left(c\,x^2\right)^{3/2}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*(c*x^2)^(3/2))^(1/2),x)
[Out] int((d*x)^m*(a + b*(c*x^2)^(3/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{a + b\left(cx^2\right)^{\frac{3}{2}}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*(c*x**2)**(3/2))**(1/2),x)
[Out] Integral((d*x)**m*sqrt(a + b*(c*x**2)**(3/2)), x)
```

3.2956 $\int (dx)^m \sqrt{a + b\sqrt{cx^2}}\, dx$

Optimal. Leaf size=88

$$\frac{2(dx)^{m+1}\left(a+b\sqrt{cx^2}\right)^{3/2}\left(-\frac{b\sqrt{cx^2}}{a}\right)^{-m}{}_2F_1\left(\frac{3}{2},-m;\frac{5}{2};\frac{\sqrt{cx^2}\,b}{a}+1\right)}{3bd\sqrt{cx^2}}$$

```
[Out] 2/3*(d*x)^(1+m)*hypergeom([3/2, -m],[5/2],1+b*(c*x^2)^(1/2)/a)*(a+b*(c*x^2)
^(1/2))^(3/2)/b/d/(c*x^2)^(1/2)/((-b*(c*x^2)^(1/2)/a)^m)
```

Rubi [A] time = 0.04, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {368, 67, 65}

$$\frac{2(dx)^{m+1}\left(a+b\sqrt{cx^2}\right)^{3/2}\left(-\frac{b\sqrt{cx^2}}{a}\right)^{-m}{}_2F_1\left(\frac{3}{2},-m;\frac{5}{2};\frac{\sqrt{cx^2}\,b}{a}+1\right)}{3bd\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[a + b*Sqrt[c*x^2]],x]
```

```
[Out] (2*(d*x)^(1 + m)*(a + b*Sqrt[c*x^2])^(3/2)*Hypergeometric2F1[3/2, -m, 5/2,
1 + (b*Sqrt[c*x^2])/a])/(3*b*d*Sqrt[c*x^2]*(-((b*Sqrt[c*x^2])/a))^m)
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0]
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\int (dx)^m \sqrt{a+b\sqrt{cx^2}}\ dx = \frac{\left((dx)^{1+m}\left(cx^2\right)^{\frac{1}{2}(-1-m)}\right)\text{Subst}\left(\int x^m\sqrt{a+bx}\ dx, x, \sqrt{cx^2}\right)}{d}$$
$$= \frac{\left((dx)^{1+m}\left(cx^2\right)^{\frac{1}{2}(-1-m)+\frac{m}{2}}\left(-\frac{b\sqrt{cx^2}}{a}\right)^{-m}\right)\text{Subst}\left(\int \left(-\frac{bx}{a}\right)^m \sqrt{a+bx}\ dx, x, \sqrt{cx^2}\right)}{d}$$
$$= \frac{2(dx)^{1+m}\left(-\frac{b\sqrt{cx^2}}{a}\right)^{-m}\left(a+b\sqrt{cx^2}\right)^{3/2}\ {}_2F_1\left(\frac{3}{2}, -m; \frac{5}{2}; 1+\frac{b\sqrt{cx^2}}{a}\right)}{3bd\sqrt{cx^2}}$$

Mathematica [A] time = 0.08, size = 74, normalized size = 0.84

$$\frac{x(dx)^m\sqrt{a+b\sqrt{cx^2}}\ {}_2F_1\left(-\frac{1}{2}, m+1; m+2; -\frac{b\sqrt{cx^2}}{a}\right)}{(m+1)\sqrt{\frac{b\sqrt{cx^2}}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*Sqrt[a + b*Sqrt[c*x^2]],x]
```

```
[Out] (x*(d*x)^m*Sqrt[a + b*Sqrt[c*x^2]]*Hypergeometric2F1[-1/2, 1 + m, 2 + m, -(
(b*Sqrt[c*x^2])/a)])/((1 + m)*Sqrt[1 + (b*Sqrt[c*x^2])/a])
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{\sqrt{cx^2}\,b+a}\ (dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sqrt(c*x^2)*b + a)*(d*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{cx^2}\,b+a}\ (dx)^m\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sqrt(c*x^2)*b + a)*(d*x)^m, x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \sqrt{a+\sqrt{c\,x^2}\ b}\ (dx)^m\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+(c*x^2)^(1/2)*b)^(1/2),x)
```

```
[Out] int((d*x)^m*(a+(c*x^2)^(1/2)*b)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{cx^2}\, b + a}\ (dx)^m\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*(c*x^2)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(sqrt(c*x^2)*b + a)*(d*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m \sqrt{a + b\,\sqrt{c\,x^2}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*(c*x^2)^(1/2))^(1/2),x)
[Out] int((d*x)^m*(a + b*(c*x^2)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{a + b\sqrt{cx^2}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*(c*x**2)**(1/2))**(1/2),x)
[Out] Integral((d*x)**m*sqrt(a + b*sqrt(c*x**2)), x)
```

3.2957 $\int (dx)^m \sqrt{a+\frac{b}{\sqrt{cx^2}}}\,dx$

Optimal. Leaf size=87

$$-\frac{2b(dx)^{m+1}\left(a+\frac{b}{\sqrt{cx^2}}\right)^{3/2}\left(-\frac{b}{a\sqrt{cx^2}}\right)^m {}_2F_1\left(\frac{3}{2},m+2;\frac{5}{2};\frac{b}{a\sqrt{cx^2}}+1\right)}{3a^2d\sqrt{cx^2}}$$

```
[Out] -2/3*b*(d*x)^(1+m)*hypergeom([3/2, 2+m],[5/2],1+b/a/(c*x^2)^(1/2))*(a+b/(c*
x^2)^(1/2))^(3/2)*(-b/a/(c*x^2)^(1/2))^m/a^2/d/(c*x^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {368, 339, 67, 65}

$$-\frac{2b(dx)^{m+1}\left(a+\frac{b}{\sqrt{cx^2}}\right)^{3/2}\left(-\frac{b}{a\sqrt{cx^2}}\right)^m {}_2F_1\left(\frac{3}{2},m+2;\frac{5}{2};\frac{b}{a\sqrt{cx^2}}+1\right)}{3a^2d\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[a + b/Sqrt[c*x^2]],x]
```

```
[Out] (-2*b*(d*x)^(1 + m)*(-(b/(a*Sqrt[c*x^2])))^m*(a + b/Sqrt[c*x^2])^(3/2)*Hype
rgeometric2F1[3/2, 2 + m, 5/2, 1 + b/(a*Sqrt[c*x^2])])/(3*a^2*d*Sqrt[c*x^2]
)
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0]
```

Rule 339

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Dist[((c
*x)^(m + 1)*(1/x)^(m + 1))/c, Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x
], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\int (dx)^m \sqrt{a+\frac{b}{\sqrt{cx^2}}}\,dx = \frac{\left((dx)^{1+m}\left(cx^2\right)^{\frac{1}{2}(-1-m)}\right)\text{Subst}\left(\int \sqrt{a+\frac{b}{x}}\,x^m\,dx, x, \sqrt{cx^2}\right)}{d}$$

$$= -\frac{\left((dx)^{1+m}\left(cx^2\right)^{\frac{1}{2}(-1-m)}\right)\text{Subst}\left(\int x^{-2-m}\sqrt{a+bx}\,dx, x, \frac{1}{\sqrt{cx^2}}\right)}{d}$$

$$= -\frac{\left(b^2(dx)^{1+m}\left(cx^2\right)^{\frac{1}{2}(-1-m)+\frac{m}{2}}\left(-\frac{b}{a\sqrt{cx^2}}\right)^m\right)\text{Subst}\left(\int \left(-\frac{bx}{a}\right)^{-2-m}\sqrt{a+bx}\,dx, x, \frac{1}{\sqrt{cx^2}}\right)}{a^2 d}$$

$$= -\frac{2b(dx)^{1+m}\left(-\frac{b}{a\sqrt{cx^2}}\right)^m\left(a+\frac{b}{\sqrt{cx^2}}\right)^{3/2}{}_2F_1\left(\frac{3}{2}, 2+m; \frac{5}{2}; 1+\frac{b}{a\sqrt{cx^2}}\right)}{3a^2 d\sqrt{cx^2}}$$

Mathematica [A] time = 0.12, size = 81, normalized size = 0.93

$$\frac{2x(dx)^m\sqrt{a+\frac{b}{\sqrt{cx^2}}}\;{}_2F_1\left(-\frac{1}{2}, m+\frac{1}{2}; m+\frac{3}{2}; -\frac{a\sqrt{cx^2}}{b}\right)}{(2m+1)\sqrt{\frac{a\sqrt{cx^2}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*Sqrt[a + b/Sqrt[c*x^2]],x]
```

```
[Out] (2*x*(d*x)^m*Sqrt[a + b/Sqrt[c*x^2]]*Hypergeometric2F1[-1/2, 1/2 + m, 3/2 +
 m, -((a*Sqrt[c*x^2])/b)])/((1 + 2*m)*Sqrt[1 + (a*Sqrt[c*x^2])/b])
```

fricas [F] time = 1.21, size = 0, normalized size = 0.00

$$\text{integral}\left((dx)^m\sqrt{\frac{acx^2+\sqrt{cx^2}\,b}{cx^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b/(c*x^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m*sqrt((a*c*x^2 + sqrt(c*x^2)*b)/(c*x^2)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m\sqrt{a+\frac{b}{\sqrt{cx^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b/(c*x^2)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m*sqrt(a + b/sqrt(c*x^2)), x)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \sqrt{a+\frac{b}{\sqrt{c\,x^2}}}\;(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b/(c*x^2)^(1/2))^(1/2),x)
[Out] int((d*x)^m*(a+b/(c*x^2)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{a + \frac{b}{\sqrt{cx^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b/(c*x^2)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate((d*x)^m*sqrt(a + b/sqrt(c*x^2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m \sqrt{a + \frac{b}{\sqrt{c\,x^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b/(c*x^2)^(1/2))^(1/2),x)
[Out] int((d*x)^m*(a + b/(c*x^2)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{a + \frac{b}{\sqrt{cx^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b/(c*x**2)**(1/2))**(1/2),x)
[Out] Integral((d*x)**m*sqrt(a + b/sqrt(c*x**2)), x)
```

3.2958 $\int (dx)^m \sqrt{a + \frac{b}{\left(cx^2\right)^{3/2}}}\, dx$

Optimal. Leaf size=90

$$\frac{(dx)^{m+1}\sqrt{a+\frac{b}{\left(cx^2\right)^{3/2}}}\;{}_2F_1\left(-\frac{1}{2},\frac{1}{3}(-m-1);\frac{2-m}{3};-\frac{b}{a\left(cx^2\right)^{3/2}}\right)}{d(m+1)\sqrt{\frac{b}{a\left(cx^2\right)^{3/2}}+1}}$$

```
[Out] (d*x)^(1+m)*hypergeom([-1/2, -1/3-1/3*m],[2/3-1/3*m],-b/a/(c*x^2)^(3/2))*(a
+b/(c*x^2)^(3/2))^(1/2)/d/(1+m)/(1+b/a/(c*x^2)^(3/2))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {368, 339, 365, 364}

$$\frac{(dx)^{m+1}\sqrt{a+\frac{b}{\left(cx^2\right)^{3/2}}}\;{}_2F_1\left(-\frac{1}{2},\frac{1}{3}(-m-1);\frac{2-m}{3};-\frac{b}{a\left(cx^2\right)^{3/2}}\right)}{d(m+1)\sqrt{\frac{b}{a\left(cx^2\right)^{3/2}}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[a + b/(c*x^2)^(3/2)],x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[a + b/(c*x^2)^(3/2)]*Hypergeometric2F1[-1/2, (-1 - m)/3
, (2 - m)/3, -(b/(a*(c*x^2)^(3/2)))])/(d*(1 + m)*Sqrt[1 + b/(a*(c*x^2)^(3/2
))])
```

Rule 339

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Dist[((c
*x)^(m + 1)*(1/x)^(m + 1))/c, Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x
], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\int (dx)^m \sqrt{a+\frac{b}{\left(cx^2\right)^{3/2}}}\,dx = \frac{\left((dx)^{1+m}\left(cx^2\right)^{\frac{1}{2}(-1-m)}\right)\text{Subst}\left(\int \sqrt{a+\frac{b}{x^3}}\,x^m\,dx, x, \sqrt{cx^2}\right)}{d}$$

$$= -\frac{\left((dx)^{1+m}\left(cx^2\right)^{\frac{1}{2}(-1-m)}\right)\text{Subst}\left(\int x^{-2-m}\sqrt{a+bx^3}\,dx, x, \frac{1}{\sqrt{cx^2}}\right)}{d}$$

$$= -\frac{\left((dx)^{1+m}\left(cx^2\right)^{\frac{1}{2}(-1-m)}\sqrt{a+\frac{b}{\left(cx^2\right)^{3/2}}}\right)\text{Subst}\left(\int x^{-2-m}\sqrt{1+\frac{bx^3}{a}}\,dx, x, \frac{1}{\sqrt{cx^2}}\right)}{d\sqrt{1+\frac{b}{a\left(cx^2\right)^{3/2}}}}$$

$$= \frac{(dx)^{1+m}\sqrt{a+\frac{b}{\left(cx^2\right)^{3/2}}}\;{}_2F_1\left(-\frac{1}{2}, \frac{1}{3}(-1-m); \frac{2-m}{3}; -\frac{b}{a\left(cx^2\right)^{3/2}}\right)}{d(1+m)\sqrt{1+\frac{b}{a\left(cx^2\right)^{3/2}}}}$$

Mathematica [F] time = 0.16, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{a+\frac{b}{\left(cx^2\right)^{3/2}}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*Sqrt[a + b/(c*x^2)^(3/2)],x]
```

```
[Out] Integrate[(d*x)^m*Sqrt[a + b/(c*x^2)^(3/2)], x]
```

fricas [F] time = 1.13, size = 0, normalized size = 0.00

$$\text{integral}\left((dx)^m \sqrt{\frac{ac^2x^4+\sqrt{cx^2}\,b}{c^2x^4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b/(c*x^2)^(3/2))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m*sqrt((a*c^2*x^4 + sqrt(c*x^2)*b)/(c^2*x^4)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b/(c*x^2)^(3/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument Valuesym2poly/r2sym(const gen & e,const index_m & i,const vecteu
r & l) Error: Bad Argument ValueWarning, integration of abs or sign assumes
 constant sign by intervals (correct if the argument is real):Check [abs(t_
nostep)]Done
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \sqrt{a + \frac{b}{\left(c\,x^2\right)^{\frac{3}{2}}}}\ (dx)^m\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b/(c*x^2)^(3/2))^(1/2),x)
[Out] int((d*x)^m*(a+b/(c*x^2)^(3/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{a + \frac{b}{\left(cx^2\right)^{\frac{3}{2}}}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b/(c*x^2)^(3/2))^(1/2),x, algorithm="maxima")
[Out] integrate((d*x)^m*sqrt(a + b/(c*x^2)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m \sqrt{a + \frac{b}{\left(c\,x^2\right)^{3/2}}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b/(c*x^2)^(3/2))^(1/2),x)
[Out] int((d*x)^m*(a + b/(c*x^2)^(3/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{a + \frac{b}{\left(cx^2\right)^{\frac{3}{2}}}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b/(c*x**2)**(3/2))**(1/2),x)
[Out] Integral((d*x)**m*sqrt(a + b/(c*x**2)**(3/2)), x)
```

3.2959 $\int \frac{1}{1+\left(x^3\right)^{2/3}}\,dx$

Optimal. Leaf size=17

$$\frac{x\tan^{-1}\left(\sqrt[3]{x^3}\right)}{\sqrt[3]{x^3}}$$

```
[Out] x*arctan((x^3)^(1/3))/(x^3)^(1/3)
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {254, 203}

$$\frac{x\tan^{-1}\left(\sqrt[3]{x^3}\right)}{\sqrt[3]{x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + (x^3)^(2/3))^(-1),x]
```

```
[Out] (x*ArcTan[(x^3)^(1/3)])/(x^3)^(1/3)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 254

```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_.), x_Symbol] :> Dist[x/(c*x^
q)^(1/q), Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+\left(x^3\right)^{2/3}}\,dx &= \frac{x\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[3]{x^3}\right)}{\sqrt[3]{x^3}}\\ &= \frac{x\tan^{-1}\left(\sqrt[3]{x^3}\right)}{\sqrt[3]{x^3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{x\tan^{-1}\left(\sqrt[3]{x^3}\right)}{\sqrt[3]{x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + (x^3)^(2/3))^(-1),x]
```

```
[Out] (x*ArcTan[(x^3)^(1/3)])/(x^3)^(1/3)
```

fricas [A] time = 0.91, size = 6, normalized size = 0.35

$$\arctan\left(\left(x^3\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(x^3)^(2/3)),x, algorithm="fricas")
[Out] arctan((x^3)^(1/3))
```

giac [A] time = 0.16, size = 2, normalized size = 0.12

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(x^3)^(2/3)),x, algorithm="giac")
[Out] arctan(x)
```

maple [A] time = 0.22, size = 14, normalized size = 0.82

$$\frac{x \arctan\left(\left(x^3\right)^{\frac{1}{3}}\right)}{\left(x^3\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+(x^3)^(2/3)),x)
[Out] x*arctan((x^3)^(1/3))/(x^3)^(1/3)
```

maxima [A] time = 1.17, size = 2, normalized size = 0.12

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(x^3)^(2/3)),x, algorithm="maxima")
[Out] arctan(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\left(x^3\right)^{2/3}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^3)^(2/3) + 1),x)
[Out] int(1/((x^3)^(2/3) + 1), x)
```

sympy [A] time = 0.13, size = 2, normalized size = 0.12

$$\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(x**3)**(2/3)),x)
[Out] atan(x)
```

3.2960 $\int x^5\sqrt{a+b\sqrt{cx^3}}\,dx$

Optimal. Leaf size=116

$$-\frac{4a^3\left(a+b\sqrt{cx^3}\right)^{3/2}}{9b^4c^2}+\frac{4a^2\left(a+b\sqrt{cx^3}\right)^{5/2}}{5b^4c^2}+\frac{4\left(a+b\sqrt{cx^3}\right)^{9/2}}{27b^4c^2}-\frac{4a\left(a+b\sqrt{cx^3}\right)^{7/2}}{7b^4c^2}$$

```
[Out] -4/9*a^3*(a+b*(c*x^3)^(1/2))^(3/2)/b^4/c^2+4/5*a^2*(a+b*(c*x^3)^(1/2))^(5/2
)/b^4/c^2-4/7*a*(a+b*(c*x^3)^(1/2))^(7/2)/b^4/c^2+4/27*(a+b*(c*x^3)^(1/2))^
(9/2)/b^4/c^2
```

Rubi [A] time = 0.07, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {369, 266, 43}

$$\frac{4a^2\left(a+b\sqrt{cx^3}\right)^{5/2}}{5b^4c^2}-\frac{4a^3\left(a+b\sqrt{cx^3}\right)^{3/2}}{9b^4c^2}+\frac{4\left(a+b\sqrt{cx^3}\right)^{9/2}}{27b^4c^2}-\frac{4a\left(a+b\sqrt{cx^3}\right)^{7/2}}{7b^4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[a + b*Sqrt[c*x^3]],x]
```

```
[Out] (-4*a^3*(a + b*Sqrt[c*x^3])^(3/2))/(9*b^4*c^2) + (4*a^2*(a + b*Sqrt[c*x^3])
^(5/2))/(5*b^4*c^2) - (4*a*(a + b*Sqrt[c*x^3])^(7/2))/(7*b^4*c^2) + (4*(a +
 b*Sqrt[c*x^3])^(9/2))/(27*b^4*c^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Rubi steps

$$\int x^5\sqrt{a+b\sqrt{cx^3}}\,dx = \text{Subst}\left(\int x^5\sqrt{a+b\sqrt{c}\,x^{3/2}}\,dx, \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$
$$= \text{Subst}\left(\frac{2}{3}\text{Subst}\left(\int x^3\sqrt{a+b\sqrt{c}\,x}\,dx, x, x^{3/2}\right), \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$
$$= \text{Subst}\left(\frac{2}{3}\text{Subst}\left(\int\left(-\frac{a^3\sqrt{a+b\sqrt{c}\,x}}{b^3c^{3/2}} + \frac{3a^2\left(a+b\sqrt{c}\,x\right)^{3/2}}{b^3c^{3/2}} - \frac{3a\left(a+b\sqrt{c}\,x\right)^{5/2}}{b^3c^{3/2}} + \frac{\left(a+b\right.}{b^3}\right.\right.$$
$$= -\frac{4a^3\left(a+b\sqrt{cx^3}\right)^{3/2}}{9b^4c^2} + \frac{4a^2\left(a+b\sqrt{cx^3}\right)^{5/2}}{5b^4c^2} - \frac{4a\left(a+b\sqrt{cx^3}\right)^{7/2}}{7b^4c^2} + \frac{4\left(a+b\sqrt{cx^3}\right)^{9/2}}{27b^4c^2}$$

Mathematica [A] time = 0.10, size = 72, normalized size = 0.62

$$\frac{4\left(a+b\sqrt{cx^3}\right)^{3/2}\left(-16a^3+24a^2b\sqrt{cx^3}-30ab^2cx^3+35b^3\left(cx^3\right)^{3/2}\right)}{945b^4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[a + b*Sqrt[c*x^3]],x]
```

```
[Out] (4*(a + b*Sqrt[c*x^3])^(3/2)*(-16*a^3 - 30*a*b^2*c*x^3 + 24*a^2*b*Sqrt[c*x^
3] + 35*b^3*(c*x^3)^(3/2)))/(945*b^4*c^2)
```

fricas [A] time = 1.30, size = 75, normalized size = 0.65

$$\frac{4\left(35\,b^4c^2x^6-6\,a^2b^2cx^3-16\,a^4+\left(5\,ab^3cx^3+8\,a^3b\right)\sqrt{cx^3}\right)\sqrt{\sqrt{cx^3}\,b+a}}{945\,b^4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*(c*x^3)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 4/945*(35*b^4*c^2*x^6 - 6*a^2*b^2*c*x^3 - 16*a^4 + (5*a*b^3*c*x^3 + 8*a^3*b
)*sqrt(c*x^3))*sqrt(sqrt(c*x^3)*b + a)/(b^4*c^2)
```

giac [A] time = 0.20, size = 111, normalized size = 0.96

$$-\frac{4\left(105\left(\sqrt{cx}\,bc^2x+ac^2\right)^{\frac{3}{2}}a^3c^6-189\left(\sqrt{cx}\,bc^2x+ac^2\right)^{\frac{5}{2}}a^2c^4+135\left(\sqrt{cx}\,bc^2x+ac^2\right)^{\frac{7}{2}}ac^2-35\left(\sqrt{cx}\,bc^2x+ac^2\right)^{\frac{9}{2}}\right.}{945\,b^4c^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*(c*x^3)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -4/945*(105*(sqrt(c*x)*b*c^2*x + a*c^2)^(3/2)*a^3*c^6 - 189*(sqrt(c*x)*b*c^
2*x + a*c^2)^(5/2)*a^2*c^4 + 135*(sqrt(c*x)*b*c^2*x + a*c^2)^(7/2)*a*c^2 -
35*(sqrt(c*x)*b*c^2*x + a*c^2)^(9/2))*abs(c)/(b^4*c^12)
```

maple [A] time = 0.43, size = 103, normalized size = 0.89

$$\frac{4\sqrt{a+\sqrt{c\,x^3}}\;b\left(5a\,b^3c^3x^9+8a^3b\,c^2x^6+35\left(c\,x^3\right)^{\frac{3}{2}}b^4c^2x^6-6\left(c\,x^3\right)^{\frac{3}{2}}a^2b^2c\,x^3-16\left(c\,x^3\right)^{\frac{3}{2}}a^4\right)}{945\left(c\,x^3\right)^{\frac{3}{2}}b^4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*(c*x^3)^(1/2))^(1/2),x)
```

```
[Out] 4/945/c^2*(a+b*(c*x^3)^(1/2))^(1/2)*(35*c^2*x^6*b^4*(c*x^3)^(3/2)+5*a*x^9*c
^3*b^3+8*a^3*x^6*c^2*b-6*a^2*c*x^3*b^2*(c*x^3)^(3/2)-16*a^4*(c*x^3)^(3/2))/
b^4/(c*x^3)^(3/2)
```

maxima [A] time = 0.64, size = 85, normalized size = 0.73

$$\frac{4\left(\frac{35\left(\sqrt{cx^3}\,b+a\right)^{\frac{9}{2}}}{b^4}-\frac{135\left(\sqrt{cx^3}\,b+a\right)^{\frac{7}{2}}a}{b^4}+\frac{189\left(\sqrt{cx^3}\,b+a\right)^{\frac{5}{2}}a^2}{b^4}-\frac{105\left(\sqrt{cx^3}\,b+a\right)^{\frac{3}{2}}a^3}{b^4}\right)}{945\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*(c*x^3)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/945*(35*(sqrt(c*x^3)*b + a)^(9/2)/b^4 - 135*(sqrt(c*x^3)*b + a)^(7/2)*a/b
^4 + 189*(sqrt(c*x^3)*b + a)^(5/2)*a^2/b^4 - 105*(sqrt(c*x^3)*b + a)^(3/2)*
a^3/b^4)/c^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5\sqrt{a+b\sqrt{c\,x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*(c*x^3)^(1/2))^(1/2),x)
```

```
[Out] int(x^5*(a + b*(c*x^3)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\sqrt{a+b\sqrt{cx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*(c*x**3)**(1/2))**(1/2),x)
```

```
[Out] Integral(x**5*sqrt(a + b*sqrt(c*x**3)), x)
```

3.2961 $\int x^2\sqrt{a+b\sqrt{cx^3}}\,dx$

Optimal. Leaf size=56

$$\frac{4\left(a+b\sqrt{cx^3}\right)^{5/2}}{15b^2c}-\frac{4a\left(a+b\sqrt{cx^3}\right)^{3/2}}{9b^2c}$$

```
[Out] -4/9*a*(a+b*(c*x^3)^(1/2))^(3/2)/b^2/c+4/15*(a+b*(c*x^3)^(1/2))^(5/2)/b^2/c
```

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {369, 266, 43}

$$\frac{4\left(a+b\sqrt{cx^3}\right)^{5/2}}{15b^2c}-\frac{4a\left(a+b\sqrt{cx^3}\right)^{3/2}}{9b^2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*Sqrt[c*x^3]],x]
```

```
[Out] (-4*a*(a + b*Sqrt[c*x^3])^(3/2))/(9*b^2*c) + (4*(a + b*Sqrt[c*x^3])^(5/2))/
(15*b^2*c)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int x^2\sqrt{a+b\sqrt{cx^3}}\,dx &= \text{Subst}\left(\int x^2\sqrt{a+b\sqrt{c}\,x^{3/2}}\,dx,\sqrt{x},\frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)\\
&= \text{Subst}\left(\frac{2}{3}\text{Subst}\left(\int x\sqrt{a+b\sqrt{c}\,x}\,dx,x,x^{3/2}\right),\sqrt{x},\frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)\\
&= \text{Subst}\left(\frac{2}{3}\text{Subst}\left(\int\left(-\frac{a\sqrt{a+b\sqrt{c}\,x}}{b\sqrt{c}}+\frac{\left(a+b\sqrt{c}\,x\right)^{3/2}}{b\sqrt{c}}\right)dx,x,x^{3/2}\right),\sqrt{x},\frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)\\
&= -\frac{4a\left(a+b\sqrt{cx^3}\right)^{3/2}}{9b^2c}+\frac{4\left(a+b\sqrt{cx^3}\right)^{5/2}}{15b^2c}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 43, normalized size = 0.77

$$\frac{4\left(a+b\sqrt{cx^3}\right)^{3/2}\left(3b\sqrt{cx^3}-2a\right)}{45b^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b*Sqrt[c*x^3]],x]
```

```
[Out] (4*(a + b*Sqrt[c*x^3])^(3/2)*(-2*a + 3*b*Sqrt[c*x^3]))/(45*b^2*c)
```

fricas [A] time = 1.02, size = 46, normalized size = 0.82

$$\frac{4\left(3\,b^2cx^3+\sqrt{cx^3}\,ab-2\,a^2\right)\sqrt{\sqrt{cx^3}\,b+a}}{45\,b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(c*x^3)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 4/45*(3*b^2*c*x^3 + sqrt(c*x^3)*a*b - 2*a^2)*sqrt(sqrt(c*x^3)*b + a)/(b^2*c
)
```

giac [A] time = 0.18, size = 57, normalized size = 1.02

$$-\frac{4\left(5\left(\sqrt{cx}\,bc^2x+ac^2\right)^{\frac{3}{2}}ac^2-3\left(\sqrt{cx}\,bc^2x+ac^2\right)^{\frac{5}{2}}\right)|c|}{45\,b^2c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(c*x^3)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -4/45*(5*(sqrt(c*x)*b*c^2*x + a*c^2)^(3/2)*a*c^2 - 3*(sqrt(c*x)*b*c^2*x + a
*c^2)^(5/2))*abs(c)/(b^2*c^7)
```

maple [A] time = 0.29, size = 65, normalized size = 1.16

$$\frac{4\sqrt{a+\sqrt{c\,x^3}\;b}\;\left(abc\,x^3+3\sqrt{c\,x^3}\;b^2c\,x^3-2\sqrt{c\,x^3}\;a^2\right)}{45\sqrt{c\,x^3}\;b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+(c*x^3)^(1/2)*b)^(1/2),x)
```

```
[Out] 4/45/c*(a+(c*x^3)^(1/2)*b)^(1/2)*(3*c*x^3*(c*x^3)^(1/2)*b^2+a*x^3*c*b-2*a^2
*(c*x^3)^(1/2))/(c*x^3)^(1/2)/b^2
```

maxima [A] time = 0.45, size = 43, normalized size = 0.77

$$\frac{4\left(\frac{3\left(\sqrt{cx^3}\,b+a\right)^{\frac{5}{2}}}{b^2}-\frac{5\left(\sqrt{cx^3}\,b+a\right)^{\frac{3}{2}}a}{b^2}\right)}{45\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(c*x^3)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/45*(3*(sqrt(c*x^3)*b + a)^(5/2)/b^2 - 5*(sqrt(c*x^3)*b + a)^(3/2)*a/b^2)/
c
```

mupad **[B]** time = 1.38, size = 52, normalized size = 0.93

$$\frac{x^3\sqrt{a+b\sqrt{c\,x^3}}\;{}_2\mathrm{F}_1\left(-\frac{1}{2},2;\;3;\;-\frac{b\sqrt{c\,x^3}}{a}\right)}{3\sqrt{\frac{b\sqrt{c\,x^3}}{a}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*(c*x^3)^(1/2))^(1/2),x)
```

```
[Out] (x^3*(a + b*(c*x^3)^(1/2))^(1/2)*hypergeom([-1/2, 2], 3, -(b*(c*x^3)^(1/2))
/a))/(3*((b*(c*x^3)^(1/2))/a + 1)^(1/2))
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{a+b\sqrt{cx^3}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*(c*x**3)**(1/2))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a + b*sqrt(c*x**3)), x)
```

3.2962 $\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x}\,dx$

Optimal. Leaf size=55

$$\frac{4}{3}\sqrt{a+b\sqrt{cx^3}}\;-\frac{4}{3}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{cx^3}}}{\sqrt{a}}\right)$$

```
[Out] -4/3*arctanh((a+b*(c*x^3)^(1/2))^(1/2)/a^(1/2))*a^(1/2)+4/3*(a+b*(c*x^3)^(1
/2))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {369, 266, 50, 63, 208}

$$\frac{4}{3}\sqrt{a+b\sqrt{cx^3}}\;-\frac{4}{3}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{cx^3}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[c*x^3]]/x,x]
```

```
[Out] (4*Sqrt[a + b*Sqrt[c*x^3]])/3 - (4*Sqrt[a]*ArcTanh[Sqrt[a + b*Sqrt[c*x^3]]/
Sqrt[a]])/3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
```

```
d, m, p, q}, x] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x}\,dx &= \text{Subst}\left(\int \frac{\sqrt{a+b\sqrt{c}\,x^{3/2}}}{x}\,dx, \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right) \\
&= \text{Subst}\left(\frac{2}{3}\,\text{Subst}\left(\int \frac{\sqrt{a+b\sqrt{c}\,x}}{x}\,dx, x, x^{3/2}\right), \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right) \\
&= \frac{4}{3}\sqrt{a+b\sqrt{cx^3}} + \text{Subst}\left(\frac{1}{3}(2a)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+b\sqrt{c}\,x}}\,dx, x, x^{3/2}\right), \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right) \\
&= \frac{4}{3}\sqrt{a+b\sqrt{cx^3}} + \text{Subst}\left(\frac{(4a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b\sqrt{c}}+\frac{x^2}{b\sqrt{c}}}\,dx, x, \sqrt{a+b\sqrt{c}\,x^{3/2}}\right)}{3b\sqrt{c}}, \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right) \\
&= \frac{4}{3}\sqrt{a+b\sqrt{cx^3}} - \frac{4}{3}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{cx^3}}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 55, normalized size = 1.00

$$\frac{4}{3}\sqrt{a+b\sqrt{cx^3}} - \frac{4}{3}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{cx^3}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[c*x^3]]/x,x]
```

```
[Out] (4*Sqrt[a + b*Sqrt[c*x^3]])/3 - (4*Sqrt[a]*ArcTanh[Sqrt[a + b*Sqrt[c*x^3]]/
Sqrt[a]])/3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^3)^(1/2))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.21, size = 67, normalized size = 1.22

$$\frac{4\left(\frac{a\arctan\left(\frac{\sqrt{\sqrt{c}x}\,bc^2x+ac^2}{\sqrt{-a}\,c}\right)}{\sqrt{-a}} + \frac{\sqrt{\sqrt{c}x}\,bc^2x+ac^2}{c}\right)|c|}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^3)^(1/2))^(1/2)/x,x, algorithm="giac")
```

```
[Out] 4/3*(a*arctan(sqrt(sqrt(c*x)*b*c^2*x + a*c^2)/(sqrt(-a)*c))/sqrt(-a) + sqrt
(sqrt(c*x)*b*c^2*x + a*c^2)/c)*abs(c)/c
```

maple [A] time = 0.29, size = 40, normalized size = 0.73

$$-\frac{4\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{a+\sqrt{c\,x^3}\ b}}{\sqrt{a}}\right)}{3}+\frac{4\sqrt{a+\sqrt{c\,x^3}\ b}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(c*x^3)^(1/2)*b)^(1/2)/x,x)
```

```
[Out] -4/3*arctanh((a+(c*x^3)^(1/2)*b)^(1/2)/a^(1/2))*a^(1/2)+4/3*(a+(c*x^3)^(1/2
)*b)^(1/2)
```

maxima [A] time = 1.26, size = 61, normalized size = 1.11

$$\frac{2}{3}\sqrt{a}\,\log\left(\frac{\sqrt{\sqrt{cx^3}\,b+a}-\sqrt{a}}{\sqrt{\sqrt{cx^3}\,b+a}+\sqrt{a}}\right)+\frac{4}{3}\sqrt{\sqrt{cx^3}\,b+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^3)^(1/2))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(a)*log((sqrt(sqrt(c*x^3)*b + a) - sqrt(a))/(sqrt(sqrt(c*x^3)*b + a
) + sqrt(a))) + 4/3*sqrt(sqrt(c*x^3)*b + a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+b\sqrt{c\,x^3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x^3)^(1/2))^(1/2)/x,x)
```

```
[Out] int((a + b*(c*x^3)^(1/2))^(1/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x**3)**(1/2))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a + b*sqrt(c*x**3))/x, x)
```

3.2963 $\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^4}\,dx$

Optimal. Leaf size=97

$$\frac{b^2c\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{cx^3}}}{\sqrt{a}}\right)}{6a^{3/2}}-\frac{bc\sqrt{a+b\sqrt{cx^3}}}{6a\sqrt{cx^3}}-\frac{\sqrt{a+b\sqrt{cx^3}}}{3x^3}$$

```
[Out] 1/6*b^2*c*arctanh((a+b*(c*x^3)^(1/2))^(1/2)/a^(1/2))/a^(3/2)-1/3*(a+b*(c*x^
3)^(1/2))^(1/2)/x^3-1/6*b*c*(a+b*(c*x^3)^(1/2))^(1/2)/a/(c*x^3)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {369, 266, 47, 51, 63, 208}

$$\frac{b^2c\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{cx^3}}}{\sqrt{a}}\right)}{6a^{3/2}}-\frac{bc\sqrt{a+b\sqrt{cx^3}}}{6a\sqrt{cx^3}}-\frac{\sqrt{a+b\sqrt{cx^3}}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[c*x^3]]/x^4,x]
```

```
[Out] -Sqrt[a + b*Sqrt[c*x^3]]/(3*x^3) - (b*c*Sqrt[a + b*Sqrt[c*x^3]])/(6*a*Sqrt[
c*x^3]) + (b^2*c*ArcTanh[Sqrt[a + b*Sqrt[c*x^3]]/Sqrt[a]])/(6*a^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^4}\,dx = \text{Subst}\left(\int \frac{\sqrt{a+b\sqrt{c}\,x^{3/2}}}{x^4}\,dx, \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= \text{Subst}\left(\frac{2}{3}\,\text{Subst}\left(\int \frac{\sqrt{a+b\sqrt{c}\,x}}{x^3}\,dx, x, x^{3/2}\right), \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= -\frac{\sqrt{a+b\sqrt{cx^3}}}{3x^3} + \text{Subst}\left(\frac{1}{6}\left(b\sqrt{c}\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+b\sqrt{c}\,x}}\,dx, x, x^{3/2}\right), \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= -\frac{\sqrt{a+b\sqrt{cx^3}}}{3x^3} - \frac{bc\sqrt{a+b\sqrt{cx^3}}}{6a\sqrt{cx^3}} - \text{Subst}\left(\frac{\left(b^2c\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+b\sqrt{c}\,x}}\,dx, x, x^{3/2}\right)}{12a}, \sqrt{x}, \right.$$

$$= -\frac{\sqrt{a+b\sqrt{cx^3}}}{3x^3} - \frac{bc\sqrt{a+b\sqrt{cx^3}}}{6a\sqrt{cx^3}} - \text{Subst}\left(\frac{\left(b\sqrt{c}\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b\sqrt{c}}+\frac{x^2}{b\sqrt{c}}}\,dx, x, \sqrt{a+b\sqrt{c}\right.}\right.}{6a}\right.$$

$$= -\frac{\sqrt{a+b\sqrt{cx^3}}}{3x^3} - \frac{bc\sqrt{a+b\sqrt{cx^3}}}{6a\sqrt{cx^3}} + \frac{b^2c\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{cx^3}}}{\sqrt{a}}\right)}{6a^{3/2}}$$

Mathematica [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sqrt[c*x^3]]/x^4,x]
```

```
[Out] Integrate[Sqrt[a + b*Sqrt[c*x^3]]/x^4, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^3)^(1/2))^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.21, size = 122, normalized size = 1.26

$$-\frac{\left(\frac{b^3c^3\arctan\left(\frac{\sqrt{\sqrt{cx}\,bc^2x+ac^2}}{\sqrt{-a}\,c}\right)}{\sqrt{-a}\,a}+\frac{\sqrt{\sqrt{cx}\,bc^2x+ac^2}\,ab^3c^6+\left(\sqrt{cx}\,bc^2x+ac^2\right)^{\frac{3}{2}}b^3c^4}{ab^2c^5x^3}\right)|c|}{6\,bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^3)^(1/2))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/6*(b^3*c^3*arctan(sqrt(sqrt(c*x)*b*c^2*x + a*c^2)/(sqrt(-a)*c))/(sqrt(-a
)*a) + (sqrt(sqrt(c*x)*b*c^2*x + a*c^2)*a*b^3*c^6 + (sqrt(c*x)*b*c^2*x + a*
c^2)^(3/2)*b^3*c^4)/(a*b^2*c^5*x^3))*abs(c)/(b*c^3)
```

maple [A] time = 0.30, size = 81, normalized size = 0.84

$$-\frac{-a\,b^2c\,x^3\operatorname{arctanh}\left(\frac{\sqrt{a+\sqrt{c\,x^3}\,b}}{\sqrt{a}}\right)+2\sqrt{a+\sqrt{c\,x^3}\,b}\;a^{\frac{5}{2}}+\sqrt{c\,x^3}\;\sqrt{a+\sqrt{c\,x^3}\,b}\;a^{\frac{3}{2}}b}{6a^{\frac{5}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(c*x^3)^(1/2)*b)^(1/2)/x^4,x)
```

```
[Out] -1/6*(-b^2*arctanh((a+(c*x^3)^(1/2)*b)^(1/2)/a^(1/2))*c*x^3*a+(c*x^3)^(1/2)
*b*(a+(c*x^3)^(1/2)*b)^(1/2)*a^(3/2)+2*(a+(c*x^3)^(1/2)*b)^(1/2)*a^(5/2))/x
^3/a^(5/2)
```

maxima [A] time = 1.16, size = 126, normalized size = 1.30

$$-\frac{1}{12}\left(\frac{b^2\log\left(\frac{\sqrt{\sqrt{cx^3}\,b+a}-\sqrt{a}}{\sqrt{\sqrt{cx^3}\,b+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2\left(\left(\sqrt{cx^3}\,b+a\right)^{\frac{3}{2}}b^2+\sqrt{\sqrt{cx^3}\,b+a}\,ab^2\right)}{\left(\sqrt{cx^3}\,b+a\right)^2a-2\left(\sqrt{cx^3}\,b+a\right)a^2+a^3}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^3)^(1/2))^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/12*(b^2*log((sqrt(sqrt(c*x^3)*b + a) - sqrt(a))/(sqrt(sqrt(c*x^3)*b + a)
 + sqrt(a)))/a^(3/2) + 2*((sqrt(c*x^3)*b + a)^(3/2)*b^2 + sqrt(sqrt(c*x^3)*
b + a)*a*b^2)/((sqrt(c*x^3)*b + a)^2*a - 2*(sqrt(c*x^3)*b + a)*a^2 + a^3))*
c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+b\sqrt{c\,x^3}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x^3)^(1/2))^(1/2)/x^4,x)
```

```
[Out] int((a + b*(c*x^3)^(1/2))^(1/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\sqrt{cx^3}}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x**3)**(1/2))**(1/2)/x**4,x)
[Out] Integral(sqrt(a + b*sqrt(c*x**3))/x**4, x)
```

3.2964 $\int x\sqrt{a+b\sqrt{cx^3}}\,dx$

Optimal. Leaf size=400

$$-\frac{8\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^2\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\,\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}+b^{2/3}\sqrt[3]{c}\,x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{55b^{4/3}c^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}\right)^2}}\sqrt{a+b\sqrt{cx^3}}}+\frac{12ax^2\sqrt{}}{55}$$

```
[Out] -8/55*3^(3/4)*a^2*EllipticF((a^(1/3)*(1-3^(1/2))+b^(1/3)*c^(2/3)*x^2/(c*x^3
)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2)),I*3^(1/2)+
2*I)*(a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))*(1/2*6^(1/2)+1/2*2^(1/2))*
((a^(2/3)+b^(2/3)*c^(1/3)*x-a^(1/3)*b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))/(a^(
1/3)*(1+3^(1/2))+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))^2)^(1/2)/b^(4/3)/c^(2/3
)/(a^(1/3)*(a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))/(a^(1/3)*(1+3^(1/2))
+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))^2)^(1/2)/(a+b*(c*x^3)^(1/2))^(1/2)+4/11
*x^2*(a+b*(c*x^3)^(1/2))^(1/2)+12/55*a*x^2*(a+b*(c*x^3)^(1/2))^(1/2)/b/(c*x
^3)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 400, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {369, 341, 279, 321, 218}

$$-\frac{8\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^2\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\,\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}+b^{2/3}\sqrt[3]{c}\,x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{55b^{4/3}c^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}\right)^2}}\sqrt{a+b\sqrt{cx^3}}}+\frac{12ax^2\sqrt{}}{55}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*Sqrt[c*x^3]],x]
```

```
[Out] (4*x^2*Sqrt[a + b*Sqrt[c*x^3]])/11 + (12*a*x^2*Sqrt[a + b*Sqrt[c*x^3]])/(55
*b*Sqrt[c*x^3]) - (8*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*(a^(1/3) + (b^(1/3)*c^(2
/3)*x^2)/Sqrt[c*x^3])*Sqrt[(a^(2/3) + b^(2/3)*c^(1/3)*x - (a^(1/3)*b^(1/3)*
c^(2/3)*x^2)/Sqrt[c*x^3])/((1 + Sqrt[3])*a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sq
rt[c*x^3])^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + (b^(1/3)*c^(2/3)*x^
2)/Sqrt[c*x^3])/((1 + Sqrt[3])*a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3])
], -7 - 4*Sqrt[3]])/(55*b^(4/3)*c^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + (b^(1/3)*c
^(2/3)*x^2)/Sqrt[c*x^3]))/((1 + Sqrt[3])*a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sq
rt[c*x^3])^2]*Sqrt[a + b*Sqrt[c*x^3]])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Rubi steps

$$\int x\sqrt{a+b\sqrt{cx^3}}\,dx = \text{Subst}\left(\int x\sqrt{a+b\sqrt{c}\,x^{3/2}}\,dx, \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= \text{Subst}\left(2\,\text{Subst}\left(\int x^3\sqrt{a+b\sqrt{c}\,x^3}\,dx, x, \sqrt{x}\right), \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= \frac{4}{11}x^2\sqrt{a+b\sqrt{cx^3}} + \text{Subst}\left(\frac{1}{11}(6a)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{a+b\sqrt{c}\,x^3}}\,dx, x, \sqrt{x}\right), \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= \frac{4}{11}x^2\sqrt{a+b\sqrt{cx^3}} + \frac{12ax^2\sqrt{a+b\sqrt{cx^3}}}{55b\sqrt{cx^3}} - \text{Subst}\left(\frac{\left(12a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+b\sqrt{c}\,x^3}}\,dx, x, \sqrt{x}\right.}{55b\sqrt{c}}\right.$$

$$= \frac{4}{11}x^2\sqrt{a+b\sqrt{cx^3}} + \frac{12ax^2\sqrt{a+b\sqrt{cx^3}}}{55b\sqrt{cx^3}} - \frac{8\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^2\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}\right)\sqrt{\frac{a^{2/3}+b^{}}{\left((1+\sqrt{}\right.}}}{55b^{4/3}c^{2/3}\sqrt{\frac{\sqrt[3]{}}{\left((1-\right.}}}$$

Mathematica [F] time = 0.04, size = 0, normalized size = 0.00

$$\int x\sqrt{a+b\sqrt{cx^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*Sqrt[a + b*Sqrt[c*x^3]],x]
```

```
[Out] Integrate[x*Sqrt[a + b*Sqrt[c*x^3]], x]
```

fricas [F] time = 6.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{\sqrt{cx^3}\,b + a}\,x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(c*x^3)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sqrt(c*x^3)*b + a)*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{cx^3}\,b + a}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(c*x^3)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sqrt(c*x^3)*b + a)*x, x)
```

maple [A] time = 0.43, size = 350, normalized size = 0.88

$$4i\sqrt{3}\left(-ab^2c\right)^{\frac{1}{3}}\sqrt{2}\sqrt{-\frac{i\left(-2\sqrt{cx^3}\,b+i\sqrt{3}\left(-ab^2c\right)^{\frac{1}{3}}x-\left(-ab^2c\right)^{\frac{1}{3}}x\right)\sqrt{3}}{\left(-ab^2c\right)^{\frac{1}{3}}x}}\sqrt{\frac{\sqrt{cx^3}\,b-\left(-ab^2c\right)^{\frac{1}{3}}x}{\left(-ab^2c\right)^{\frac{1}{3}}\left(i\sqrt{3}-3\right)x}}\sqrt{-\frac{i\left(2\sqrt{cx^3}\,b+i\sqrt{3}\left(-ab^2c\right)^{\frac{1}{3}}x-\left(-ab^2c\right)^{\frac{1}{3}}x\right)\sqrt{3}}{\left(-ab^2c\right)^{\frac{1}{3}}x}}$$

$$\frac{4b^3c^2x^5}{11}+\frac{12a^2bcx^2}{55}+\frac{32\sqrt{cx^3}\,ab^2cx^2}{55}+\frac{}{\sqrt{cx^3}\,\sqrt{a+\sqrt{cx^3}\,b}\,b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+(c*x^3)^(1/2)*b)^(1/2),x)
```

```
[Out] 4/55*(I*a^2*3^(1/2)*(-a*c*b^2)^(1/3)*2^(1/2)*(-I*(I*3^(1/2)*x*(-a*c*b^2)^(1
/3)-2*(c*x^3)^(1/2)*b-x*(-a*c*b^2)^(1/3))*3^(1/2)/x/(-a*c*b^2)^(1/3))^(1/2)
*(((c*x^3)^(1/2)*b-x*(-a*c*b^2)^(1/3))/x/(-a*c*b^2)^(1/3)/(I*3^(1/2)-3))^(1
/2)*(-I*(I*3^(1/2)*x*(-a*c*b^2)^(1/3)+2*(c*x^3)^(1/2)*b+x*(-a*c*b^2)^(1/3))
*3^(1/2)/x/(-a*c*b^2)^(1/3))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*(-I*(I*3^(
1/2)*x*(-a*c*b^2)^(1/3)-2*(c*x^3)^(1/2)*b-x*(-a*c*b^2)^(1/3))*3^(1/2)/x/(-a
*c*b^2)^(1/3))^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))*(c*x^3)^(1/2)
+5*c^2*x^5*b^3+8*(c*x^3)^(1/2)*x^2*a*b^2*c+3*x^2*a^2*b*c)/c/(c*x^3)^(1/2)/b
^2/(a+(c*x^3)^(1/2)*b)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{cx^3}\,b + a}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(c*x^3)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(c*x^3)*b + a)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x \sqrt{a + b \sqrt{c\, x^3}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*(c*x^3)^(1/2))^(1/2),x)
```

```
[Out] int(x*(a + b*(c*x^3)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{a + b\sqrt{cx^3}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(c*x**3)**(1/2))**(1/2),x)
```

```
[Out] Integral(x*sqrt(a + b*sqrt(c*x**3)), x)
```

3.2965 $\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^2}\,dx$

Optimal. Leaf size=355

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{2/3}\sqrt[3]{c}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}+b^{2/3}\sqrt[3]{c}\,x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}\right)^2}}\,\sqrt{a+b\sqrt{cx^3}}}-\frac{\sqrt{a+}}{}$$

```
[Out] 3^(3/4)*b^(2/3)*c^(1/3)*EllipticF((a^(1/3)*(1-3^(1/2))+b^(1/3)*c^(2/3)*x^2/
(c*x^3)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2)),I*3^
(1/2)+2*I)*(a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))*(1/2*6^(1/2)+1/2*2^(
1/2))*((a^(2/3)+b^(2/3)*c^(1/3)*x-a^(1/3)*b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2)
)/(a^(1/3)*(1+3^(1/2))+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))^2)^(1/2)/(a^(1/3)
*(a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*c
^(2/3)*x^2/(c*x^3)^(1/2))^2)^(1/2)/(a+b*(c*x^3)^(1/2))^(1/2)-(a+b*(c*x^3)^(
1/2))^(1/2)/x
```

Rubi [A] time = 0.15, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {369, 341, 277, 218}

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{2/3}\sqrt[3]{c}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}+b^{2/3}\sqrt[3]{c}\,x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}\right)^2}}\,\sqrt{a+b\sqrt{cx^3}}}-\frac{\sqrt{a+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[c*x^3]]/x^2,x]
```

```
[Out] -(Sqrt[a + b*Sqrt[c*x^3]]/x) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(2/3)*c^(1/3)*(
a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3])*Sqrt[(a^(2/3) + b^(2/3)*c^(1/3
)*x - (a^(1/3)*b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3])/((1 + Sqrt[3])*a^(1/3) + (
b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3])^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3
) + (b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3])/((1 + Sqrt[3])*a^(1/3) + (b^(1/3)*c^
(2/3)*x^2)/Sqrt[c*x^3])], -7 - 4*Sqrt[3]])/(Sqrt[(a^(1/3)*(a^(1/3) + (b^(1/
3)*c^(2/3)*x^2)/Sqrt[c*x^3]))/((1 + Sqrt[3])*a^(1/3) + (b^(1/3)*c^(2/3)*x^2
)/Sqrt[c*x^3])^2]*Sqrt[a + b*Sqrt[c*x^3]])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
```

```
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^2}\,dx = \text{Subst}\left(\int \frac{\sqrt{a+b\sqrt{c}\,x^{3/2}}}{x^2}\,dx, \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= \text{Subst}\left(2\,\text{Subst}\left(\int \frac{\sqrt{a+b\sqrt{c}\,x^3}}{x^3}\,dx, x, \sqrt{x}\right), \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= -\frac{\sqrt{a+b\sqrt{cx^3}}}{x} + \text{Subst}\left(\frac{1}{2}\left(3b\sqrt{c}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+b\sqrt{c}\,x^3}}\,dx, x, \sqrt{x}\right), \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= -\frac{\sqrt{a+b\sqrt{cx^3}}}{x} + \frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{2/3}\sqrt[3]{c}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}\right)\sqrt{\frac{a^{2/3}+b^{2/3}\sqrt[3]{c}\,x-\frac{\sqrt[3]{a}\,\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}\right)^2}}\,F\left(\sin$$

$$\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}\right)^2}}\sqrt{a+b\sqrt{cx^3}}$$

Mathematica [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sqrt[c*x^3]]/x^2,x]
```

```
[Out] Integrate[Sqrt[a + b*Sqrt[c*x^3]]/x^2, x]
```

fricas [F] time = 6.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\sqrt{cx^3}\,b+a}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^3)^(1/2))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(sqrt(c*x^3)*b + a)/x^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^3)^(1/2))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sy
m(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Va
lueEvaluation time: 22.1-2*abs(c)/c^2/c*4*c^2/8*sqrt(a*c^2/(c*x)^2+b*c/sqrt
(c*x))+integrate(2*abs(c)/c^2/c*6*c^3*b/8/2*c*c/(sqrt(c*x)*sqrt(b*c^2*x*sqr
t(c*x)+a*c^2)*abs(c)*sign(x)),x)
```

maple [A] time = 0.29, size = 306, normalized size = 0.86

$$-\frac{i\sqrt{3}\,\left(-a\,b^2c\right)^{\frac{1}{3}}\,\sqrt{2}\,\sqrt{-\frac{i\left(-2\sqrt{c\,x^3}\,b+i\sqrt{3}\,\left(-a\,b^2c\right)^{\frac{1}{3}}x-\left(-a\,b^2c\right)^{\frac{1}{3}}x\right)\sqrt{3}}{\left(-a\,b^2c\right)^{\frac{1}{3}}x}}\,\sqrt{\frac{\sqrt{c\,x^3}\,b-\left(-a\,b^2c\right)^{\frac{1}{3}}x}{\left(-a\,b^2c\right)^{\frac{1}{3}}\left(i\sqrt{3}-3\right)x}}\,\sqrt{-\frac{i\left(2\sqrt{c\,x^3}\,b+i\sqrt{3}\,\left(-a\,b^2c\right)^{\frac{1}{3}}x+\left(-a\,b^2c\right)^{\frac{1}{3}}\right.\right.}{\left(-a\,b^2c\right)^{\frac{1}{3}}x}}}{2\sqrt{a+\sqrt{c\,x^3}}\,b\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(c*x^3)^(1/2)*b)^(1/2)/x^2,x)
```

```
[Out] -1/2*(I*3^(1/2)*(-a*b^2*c)^(1/3)*2^(1/2)*(-I*(-2*(c*x^3)^(1/2)*b+I*3^(1/2)*
(-a*b^2*c)^(1/3)*x-(-a*b^2*c)^(1/3)*x)*3^(1/2)/(-a*b^2*c)^(1/3)/x)^(1/2)*((
(c*x^3)^(1/2)*b-(-a*b^2*c)^(1/3)*x)/(-a*b^2*c)^(1/3)/(I*3^(1/2)-3)/x)^(1/2)
*(-I*(2*(c*x^3)^(1/2)*b+I*3^(1/2)*(-a*b^2*c)^(1/3)*x+(-a*b^2*c)^(1/3)*x)*3^
(1/2)/(-a*b^2*c)^(1/3)/x)^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*(-I*(-2*(c*x^
3)^(1/2)*b+I*3^(1/2)*(-a*b^2*c)^(1/3)*x-(-a*b^2*c)^(1/3)*x)*3^(1/2)/(-a*b^2
*c)^(1/3)/x)^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))*x+2*a+2*(c*x^3)
^(1/2)*b)/x/(a+(c*x^3)^(1/2)*b)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{cx^3}\,b+a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^3)^(1/2))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(c*x^3)*b + a)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\,\sqrt{c\,x^3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x^3)^(1/2))^(1/2)/x^2,x)
```

```
[Out] int((a + b*(c*x^3)^(1/2))^(1/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\sqrt{cx^3}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x**3)**(1/2))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(a + b*sqrt(c*x**3))/x**2, x)
```

3.2966 $\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^5}\,dx$

Optimal. Leaf size=434

$$\frac{21b^2c\sqrt{a+b\sqrt{cx^3}}}{160a^2x}+\frac{7\,3^{3/4}\sqrt{2+\sqrt{3}}\,b^{8/3}c^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\,\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}+b^{2/3}\sqrt[3]{c}\,x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\right.}{160a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}\right)^2}}\,\sqrt{a+b\sqrt{cx^3}}}$$

```
[Out] 7/160*3^(3/4)*b^(8/3)*c^(4/3)*EllipticF((a^(1/3)*(1-3^(1/2))+b^(1/3)*c^(2/3
)*x^2/(c*x^3)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2)
),I*3^(1/2)+2*I)*(a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))*(1/2*6^(1/2)+1
/2*2^(1/2))*((a^(2/3)+b^(2/3)*c^(1/3)*x-a^(1/3)*b^(1/3)*c^(2/3)*x^2/(c*x^3)
^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))^2)^(1/2)/a^
2/(a^(1/3)*(a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))/(a^(1/3)*(1+3^(1/2))
+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))^2)^(1/2)/(a+b*(c*x^3)^(1/2))^(1/2)-1/4*
(a+b*(c*x^3)^(1/2))^(1/2)/x^4+21/160*b^2*c*(a+b*(c*x^3)^(1/2))^(1/2)/a^2/x-
3/40*b*c^3*x^5*(a+b*(c*x^3)^(1/2))^(1/2)/a/(c*x^3)^(5/2)
```

Rubi [A] time = 0.27, antiderivative size = 434, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {369, 341, 277, 325, 218}

$$\frac{7\,3^{3/4}\sqrt{2+\sqrt{3}}\,b^{8/3}c^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\,\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}+b^{2/3}\sqrt[3]{c}\,x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{160a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}\right)^2}}\,\sqrt{a+b\sqrt{cx^3}}}+\frac{21b^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[c*x^3]]/x^5,x]
```

```
[Out] -Sqrt[a + b*Sqrt[c*x^3]]/(4*x^4) + (21*b^2*c*Sqrt[a + b*Sqrt[c*x^3]])/(160*
a^2*x) - (3*b*c^3*x^5*Sqrt[a + b*Sqrt[c*x^3]])/(40*a*(c*x^3)^(5/2)) + (7*3^
(3/4)*Sqrt[2 + Sqrt[3]]*b^(8/3)*c^(4/3)*(a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sq
rt[c*x^3])*Sqrt[(a^(2/3) + b^(2/3)*c^(1/3)*x - (a^(1/3)*b^(1/3)*c^(2/3)*x^2
)/Sqrt[c*x^3])/((1 + Sqrt[3])*a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3])^
2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x
^3])/((1 + Sqrt[3])*a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3])], -7 - 4*S
qrt[3]])/(160*a^2*Sqrt[(a^(1/3)*(a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3
]))/((1 + Sqrt[3])*a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3])^2]*Sqrt[a +
 b*Sqrt[c*x^3]])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^5}\,dx = \text{Subst}\left(\int \frac{\sqrt{a+b\sqrt{c}\,x^{3/2}}}{x^5}\,dx, \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= \text{Subst}\left(2\,\text{Subst}\left(\int \frac{\sqrt{a+b\sqrt{c}\,x^3}}{x^9}\,dx, x, \sqrt{x}\right), \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= -\frac{\sqrt{a+b\sqrt{cx^3}}}{4x^4} + \text{Subst}\left(\frac{1}{8}\left(3b\sqrt{c}\right)\text{Subst}\left(\int \frac{1}{x^6\sqrt{a+b\sqrt{c}\,x^3}}\,dx, x, \sqrt{x}\right), \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= -\frac{\sqrt{a+b\sqrt{cx^3}}}{4x^4} - \frac{3bc^3x^5\sqrt{a+b\sqrt{cx^3}}}{40a\left(cx^3\right)^{5/2}} - \text{Subst}\left(\frac{\left(21b^2c\right)\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+b\sqrt{c}\,x^3}}\,dx, x, \sqrt{x}\right)}{80a},\right.$$

$$= -\frac{\sqrt{a+b\sqrt{cx^3}}}{4x^4} + \frac{21b^2c\sqrt{a+b\sqrt{cx^3}}}{160a^2x} - \frac{3bc^3x^5\sqrt{a+b\sqrt{cx^3}}}{40a\left(cx^3\right)^{5/2}} + \text{Subst}\left(\frac{\left(21b^3c^{3/2}\right)\text{Subst}\left(\int\right.\right.$$

$$= -\frac{\sqrt{a+b\sqrt{cx^3}}}{4x^4} + \frac{21b^2c\sqrt{a+b\sqrt{cx^3}}}{160a^2x} - \frac{3bc^3x^5\sqrt{a+b\sqrt{cx^3}}}{40a\left(cx^3\right)^{5/2}} + \frac{7\,3^{3/4}\sqrt{2+\sqrt{3}}\,b^{8/3}c^{4/3}\left(\sqrt[3]{}\right.}{}$$

Mathematica [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^5}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sqrt[c*x^3]]/x^5,x]
[Out] Integrate[Sqrt[a + b*Sqrt[c*x^3]]/x^5, x]
```

fricas [F] time = 4.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\sqrt{cx^3}\,b+a}}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^3)^(1/2))^(1/2)/x^5,x, algorithm="fricas")
[Out] integral(sqrt(sqrt(c*x^3)*b + a)/x^5, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^3)^(1/2))^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sy
m(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Va
lueEvaluation time: 22.112*abs(c)/c^2/c*2*((-5120*a^4*c^9/81920/a^4/c^4/sqr
t(c*x)/sqrt(c*x)/sqrt(c*x)-1536*a^3*c^8*b/81920/a^4/c^4)/sqrt(c*x)/sqrt(c*x
)/sqrt(c*x)+2688*a^2*c^7*b^2/81920/a^4/c^4)*sqrt(a*c^2/(c*x)^2+b*c/sqrt(c*x
))+integrate(2*abs(c)/c^2/c*2688*a^2*c^8*b^3/81920/a^4/c^4/2*c*c/(sqrt(c*x)
*sqrt(b*c^2*x*sqrt(c*x)+a*c^2)*abs(c)*sign(x)),x)
```

maple [A] time = 0.30, size = 346, normalized size = 0.80

$$7i\sqrt{3}\,\left(-a\,b^2c\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{-\frac{i\left(-2\sqrt{c\,x^3}\,b+i\sqrt{3}\,\left(-a\,b^2c\right)^{\frac{1}{3}}x-\left(-a\,b^2c\right)^{\frac{1}{3}}x\right)\sqrt{3}}{\left(-a\,b^2c\right)^{\frac{1}{3}}x}}\,\sqrt{\frac{\sqrt{c\,x^3}\,b-\left(-a\,b^2c\right)^{\frac{1}{3}}x}{\left(-a\,b^2c\right)^{\frac{1}{3}}\left(i\sqrt{3}-3\right)x}}\,\sqrt{-\frac{i\left(2\sqrt{c\,x^3}\,b+i\sqrt{3}\,\left(-a\,b^2c\right)^{\frac{1}{3}}x+\left(-\right.\right.}$$

$$-\frac{}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(c*x^3)^(1/2)*b)^(1/2)/x^5,x)
```

```
[Out] -1/320*(7*I*b^2*3^(1/2)*(-a*b^2*c)^(1/3)*2^(1/2)*(-I*(-2*(c*x^3)^(1/2)*b+I*
3^(1/2)*(-a*b^2*c)^(1/3)*x-(-a*b^2*c)^(1/3)*x)*3^(1/2)/(-a*b^2*c)^(1/3)/x)^
(1/2)*(((c*x^3)^(1/2)*b-(-a*b^2*c)^(1/3)*x)/(-a*b^2*c)^(1/3)/(I*3^(1/2)-3)/
x)^(1/2)*(-I*(2*(c*x^3)^(1/2)*b+I*3^(1/2)*(-a*b^2*c)^(1/3)*x+(-a*b^2*c)^(1/
3)*x)*3^(1/2)/(-a*b^2*c)^(1/3)/x)^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*(-I*(
-2*(c*x^3)^(1/2)*b+I*3^(1/2)*(-a*b^2*c)^(1/3)*x-(-a*b^2*c)^(1/3)*x)*3^(1/2)
/(-a*b^2*c)^(1/3)/x)^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))*c*x^4-4
2*(c*x^3)^(1/2)*x^3*b^3*c-18*x^3*a*b^2*c+104*(c*x^3)^(1/2)*a^2*b+80*a^3)/x^
4/a^2/(a+(c*x^3)^(1/2)*b)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{cx^3}\,b+a}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^3)^(1/2))^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(c*x^3)*b + a)/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\,\sqrt{c\,x^3}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x^3)^(1/2))^(1/2)/x^5,x)
```

```
[Out] int((a + b*(c*x^3)^(1/2))^(1/2)/x^5, x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x**3)**(1/2))**(1/2)/x**5,x)
[Out] Integral(sqrt(a + b*sqrt(c*x**3))/x**5, x)
```

3.2967 $\int x^3\sqrt{a+b\sqrt{cx^3}}\,dx$

Optimal. Leaf size=843

$$\frac{4}{19}\sqrt{a+b\sqrt{cx^3}}\,x^4+\frac{12a\sqrt{cx^3}\,\sqrt{a+b\sqrt{cx^3}}\,x}{247bc}-\frac{120a^2\sqrt{a+b\sqrt{cx^3}}\,x}{1729b^2c}-\frac{240\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,a^{10/3}\left(\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}+\sqrt[3]{a}\right)\sqrt{}}{1729b^{8/3}c^{4/3}}$$

```
[Out] 160/1729*3^(3/4)*a^(10/3)*EllipticF((a^(1/3)*(1-3^(1/2))+b^(1/3)*c^(2/3)*x^
2/(c*x^3)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2)),I*
3^(1/2)+2*I)*2^(1/2)*(a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))*((a^(2/3)+
b^(2/3)*c^(1/3)*x-a^(1/3)*b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))/(a^(1/3)*(1+3^
(1/2))+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))^2)^(1/2)/b^(8/3)/c^(4/3)/(a^(1/3)
*(a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*c
^(2/3)*x^2/(c*x^3)^(1/2))^2)^(1/2)/(a+b*(c*x^3)^(1/2))^(1/2)-240/1729*3^(1/
4)*a^(10/3)*EllipticE((a^(1/3)*(1-3^(1/2))+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2
))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2)),I*3^(1/2)+2*I)*(
a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2
/3)+b^(2/3)*c^(1/3)*x-a^(1/3)*b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))/(a^(1/3)*(
1+3^(1/2))+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))^2)^(1/2)/b^(8/3)/c^(4/3)/(a^(
1/3)*(a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/
3)*c^(2/3)*x^2/(c*x^3)^(1/2))^2)^(1/2)/(a+b*(c*x^3)^(1/2))^(1/2)-120/1729*a
^2*x*(a+b*(c*x^3)^(1/2))^(1/2)/b^2/c+4/19*x^4*(a+b*(c*x^3)^(1/2))^(1/2)+480
/1729*a^3*(a+b*(c*x^3)^(1/2))^(1/2)/b^(8/3)/c^(4/3)/(a^(1/3)*(1+3^(1/2))+b^
(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))+12/247*a*x*(c*x^3)^(1/2)*(a+b*(c*x^3)^(1/2
))^(1/2)/b/c
```

Rubi [A] time = 0.64, antiderivative size = 843, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {369, 341, 279, 321, 303, 218, 1877}

$$\frac{4}{19}\sqrt{a+b\sqrt{cx^3}}\,x^4+\frac{12a\sqrt{cx^3}\,\sqrt{a+b\sqrt{cx^3}}\,x}{247bc}-\frac{120a^2\sqrt{a+b\sqrt{cx^3}}\,x}{1729b^2c}-\frac{240\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,a^{10/3}\left(\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}+\sqrt[3]{a}\right)\sqrt{}}{1729b^{8/3}c^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + b*Sqrt[c*x^3]],x]
```

```
[Out] (-120*a^2*x*Sqrt[a + b*Sqrt[c*x^3]])/(1729*b^2*c) + (4*x^4*Sqrt[a + b*Sqrt[
c*x^3]])/19 + (12*a*x*Sqrt[c*x^3]*Sqrt[a + b*Sqrt[c*x^3]])/(247*b*c) + (480
*a^3*Sqrt[a + b*Sqrt[c*x^3]])/(1729*b^(8/3)*c^(4/3)*((1 + Sqrt[3])*a^(1/3)
 + (b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3])) - (240*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1
0/3)*(a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3])*Sqrt[(a^(2/3) + b^(2/3)*
c^(1/3)*x - (a^(1/3)*b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3])/((1 + Sqrt[3])*a^(1/
3) + (b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3])^2]*EllipticE[ArcSin[((1 - Sqrt[3])*
a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3])/((1 + Sqrt[3])*a^(1/3) + (b^(1
/3)*c^(2/3)*x^2)/Sqrt[c*x^3])], -7 - 4*Sqrt[3]])/(1729*b^(8/3)*c^(4/3)*Sqrt
[(a^(1/3)*(a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3]))/((1 + Sqrt[3])*a^(
```

```
1/3) + (b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3])^2]*Sqrt[a + b*Sqrt[c*x^3]]) + (16
0*Sqrt[2]*3^(3/4)*a^(10/3)*(a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3])*Sq
rt[(a^(2/3) + b^(2/3)*c^(1/3)*x - (a^(1/3)*b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3]
)/((1 + Sqrt[3])*a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3])^2]*EllipticF[
ArcSin[((1 - Sqrt[3])*a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3])/((1 + Sq
rt[3])*a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3])], -7 - 4*Sqrt[3]])/(172
9*b^(8/3)*c^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3
]))/((1 + Sqrt[3])*a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3])^2]*Sqrt[a +
 b*Sqrt[c*x^3]])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
 + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
```

```
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int x^3\sqrt{a+b\sqrt{cx^3}}\,dx = \text{Subst}\left(\int x^3\sqrt{a+b\sqrt{c}\,x^{3/2}}\,dx, \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= \text{Subst}\left(2\,\text{Subst}\left(\int x^7\sqrt{a+b\sqrt{c}\,x^3}\,dx, x, \sqrt{x}\right), \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= \frac{4}{19}x^4\sqrt{a+b\sqrt{cx^3}} + \text{Subst}\left(\frac{1}{19}(6a)\,\text{Subst}\left(\int \frac{x^7}{\sqrt{a+b\sqrt{c}\,x^3}}\,dx, x, \sqrt{x}\right), \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= \frac{4}{19}x^4\sqrt{a+b\sqrt{cx^3}} + \frac{12ax\sqrt{cx^3}\sqrt{a+b\sqrt{cx^3}}}{247bc} - \text{Subst}\left(\frac{\left(60a^2\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{a+b\sqrt{c}\,x^3}}\,dx,\right.}{247b\sqrt{c}}\right.$$

$$= -\frac{120a^2x\sqrt{a+b\sqrt{cx^3}}}{1729b^2c} + \frac{4}{19}x^4\sqrt{a+b\sqrt{cx^3}} + \frac{12ax\sqrt{cx^3}\sqrt{a+b\sqrt{cx^3}}}{247bc} + \text{Subst}\left(\frac{(240}{}\right.$$

$$= -\frac{120a^2x\sqrt{a+b\sqrt{cx^3}}}{1729b^2c} + \frac{4}{19}x^4\sqrt{a+b\sqrt{cx^3}} + \frac{12ax\sqrt{cx^3}\sqrt{a+b\sqrt{cx^3}}}{247bc} + \text{Subst}\left(\frac{(240}{}\right.$$

$$= -\frac{120a^2x\sqrt{a+b\sqrt{cx^3}}}{1729b^2c} + \frac{4}{19}x^4\sqrt{a+b\sqrt{cx^3}} + \frac{12ax\sqrt{cx^3}\sqrt{a+b\sqrt{cx^3}}}{247bc} + \frac{48}{1729b^{8/3}c^{4/3}}$$

Mathematica [F] time = 0.06, size = 0, normalized size = 0.00

$$\int x^3\sqrt{a+b\sqrt{cx^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3*Sqrt[a + b*Sqrt[c*x^3]],x]
```

```
[Out] Integrate[x^3*Sqrt[a + b*Sqrt[c*x^3]], x]
```

fricas [F] time = 5.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{\sqrt{cx^3}\,b + a}\,x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(c*x^3)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sqrt(c*x^3)*b + a)*x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{cx^3}\,b + a}\,x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(c*x^3)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sqrt(c*x^3)*b + a)*x^3, x)
```

maple [A] time = 0.29, size = 932, normalized size = 1.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+(c*x^3)^(1/2)*b)^(1/2),x)
```

```
[Out] 4/1729/c^2/x^2*(30*I*2^(1/2)*(-I*(-2*(c*x^3)^(1/2)*b+I*3^(1/2)*(-a*b^2*c)^(
1/3)*x-(-a*b^2*c)^(1/3)*x)*3^(1/2)/(-a*b^2*c)^(1/3)/x)^(1/2)*(((c*x^3)^(1/2
)*b-(-a*b^2*c)^(1/3)*x)/(-a*b^2*c)^(1/3)/(I*3^(1/2)-3)/x)^(1/2)*(-I*(2*(c*x
^3)^(1/2)*b+I*3^(1/2)*(-a*b^2*c)^(1/3)*x+(-a*b^2*c)^(1/3)*x)*3^(1/2)/(-a*b^
2*c)^(1/3)/x)^(1/2)*EllipticE(1/6*3^(1/2)*2^(1/2)*(-I*(-2*(c*x^3)^(1/2)*b+I
*3^(1/2)*(-a*b^2*c)^(1/3)*x-(-a*b^2*c)^(1/3)*x)*3^(1/2)/(-a*b^2*c)^(1/3)/x)
^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))*x^2*3^(1/2)*(-a*b^2*c)^(2/3
)*a^3-20*I*2^(1/2)*(-I*(-2*(c*x^3)^(1/2)*b+I*3^(1/2)*(-a*b^2*c)^(1/3)*x-(-a
*b^2*c)^(1/3)*x)*3^(1/2)/(-a*b^2*c)^(1/3)/x)^(1/2)*(((c*x^3)^(1/2)*b-(-a*b^
2*c)^(1/3)*x)/(-a*b^2*c)^(1/3)/(I*3^(1/2)-3)/x)^(1/2)*(-I*(2*(c*x^3)^(1/2)*
b+I*3^(1/2)*(-a*b^2*c)^(1/3)*x+(-a*b^2*c)^(1/3)*x)*3^(1/2)/(-a*b^2*c)^(1/3)
/x)^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*(-I*(-2*(c*x^3)^(1/2)*b+I*3^(1/2)*(
-a*b^2*c)^(1/3)*x-(-a*b^2*c)^(1/3)*x)*3^(1/2)/(-a*b^2*c)^(1/3)/x)^(1/2),2^(
1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))*x^2*3^(1/2)*(-a*b^2*c)^(2/3)*a^3+91*(
c*x^3)^(1/2)*x^6*b^5*c^2+112*x^6*a*b^4*c^2+30*2^(1/2)*(-I*(-2*(c*x^3)^(1/2)
*b+I*3^(1/2)*(-a*b^2*c)^(1/3)*x-(-a*b^2*c)^(1/3)*x)*3^(1/2)/(-a*b^2*c)^(1/3
)/x)^(1/2)*(((c*x^3)^(1/2)*b-(-a*b^2*c)^(1/3)*x)/(-a*b^2*c)^(1/3)/(I*3^(1/2
)-3)/x)^(1/2)*(-I*(2*(c*x^3)^(1/2)*b+I*3^(1/2)*(-a*b^2*c)^(1/3)*x+(-a*b^2*c
)^(1/3)*x)*3^(1/2)/(-a*b^2*c)^(1/3)/x)^(1/2)*EllipticE(1/6*3^(1/2)*2^(1/2)*
(-I*(-2*(c*x^3)^(1/2)*b+I*3^(1/2)*(-a*b^2*c)^(1/3)*x-(-a*b^2*c)^(1/3)*x)*3^
(1/2)/(-a*b^2*c)^(1/3)/x)^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))*x^
2*(-a*b^2*c)^(2/3)*a^3-30*(c*x^3)^(1/2)*x^3*a^2*b^3*c-30*x^3*a^3*b^2*c+21*(
c*x^3)^(3/2)*a^2*b^3)/b^4/(a+(c*x^3)^(1/2)*b)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{cx^3}\,b + a}\,x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(c*x^3)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(c*x^3)*b + a)*x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \sqrt{a + b\sqrt{c\,x^3}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*(c*x^3)^(1/2))^(1/2),x)
[Out] int(x^3*(a + b*(c*x^3)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sqrt{a + b\sqrt{cx^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*(c*x**3)**(1/2))**(1/2),x)
[Out] Integral(x**3*sqrt(a + b*sqrt(c*x**3)), x)
```

3.2968 $\int \sqrt{a+b\sqrt{cx^3}}\,dx$

Optimal. Leaf size=770

$$\frac{4\sqrt{2}\,3^{3/4}a^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\,\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}+b^{2/3}\sqrt[3]{c}\,x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)\quad 6\sqrt[4]{3}\sqrt{2-\sqrt{3}}}{7b^{2/3}\sqrt[3]{c}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}\right)^2}}\sqrt{a+b\sqrt{cx^3}}}$$

```
[Out] 4/7*3^(3/4)*a^(4/3)*EllipticF((a^(1/3)*(1-3^(1/2))+b^(1/3)*c^(2/3)*x^2/(c*x
^3)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2)),I*3^(1/2
)+2*I)*2^(1/2)*(a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))*((a^(2/3)+b^(2/3
)*c^(1/3)*x-a^(1/3)*b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))/(a^(1/3)*(1+3^(1/2))
+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))^2)^(1/2)/b^(2/3)/c^(1/3)/(a^(1/3)*(a^(1
/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*c^(2/3)
*x^2/(c*x^3)^(1/2))^2)^(1/2)/(a+b*(c*x^3)^(1/2))^(1/2)-6/7*3^(1/4)*a^(4/3)*
EllipticE((a^(1/3)*(1-3^(1/2))+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))/(a^(1/3)*
(1+3^(1/2))+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2)),I*3^(1/2)+2*I)*(a^(1/3)+b^(1
/3)*c^(2/3)*x^2/(c*x^3)^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)+b^(2/3)*
c^(1/3)*x-a^(1/3)*b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))/(a^(1/3)*(1+3^(1/2))+b
^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))^2)^(1/2)/b^(2/3)/c^(1/3)/(a^(1/3)*(a^(1/3
)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*c^(2/3)*x
^2/(c*x^3)^(1/2))^2)^(1/2)/(a+b*(c*x^3)^(1/2))^(1/2)+4/7*x*(a+b*(c*x^3)^(1/
2))^(1/2)+12/7*a*(a+b*(c*x^3)^(1/2))^(1/2)/b^(2/3)/c^(1/3)/(a^(1/3)*(1+3^(1
/2))+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))
```

Rubi [A] time = 0.39, antiderivative size = 770, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {255, 243, 279, 303, 218, 1877}

$$\frac{4\sqrt{2}\,3^{3/4}a^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\,\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}+b^{2/3}\sqrt[3]{c}\,x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)\quad 6\sqrt[4]{3}\sqrt{2-\sqrt{3}}}{7b^{2/3}\sqrt[3]{c}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}\right)^2}}\sqrt{a+b\sqrt{cx^3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[c*x^3]],x]
```

```
[Out] (4*x*Sqrt[a + b*Sqrt[c*x^3]])/7 + (12*a*Sqrt[a + b*Sqrt[c*x^3]])/(7*b^(2/3)
*c^(1/3)*((1 + Sqrt[3])*a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3])) - (6*
3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x
^3])*Sqrt[(a^(2/3) + b^(2/3)*c^(1/3)*x - (a^(1/3)*b^(1/3)*c^(2/3)*x^2)/Sqrt
[c*x^3])/((1 + Sqrt[3])*a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3])^2]*Ell
ipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3])/(
(1 + Sqrt[3])*a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3])], -7 - 4*Sqrt[3]
])/(7*b^(2/3)*c^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sqrt[c
*x^3]))/((1 + Sqrt[3])*a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3])^2]*Sqrt
[a + b*Sqrt[c*x^3]]) + (4*Sqrt[2]*3^(3/4)*a^(4/3)*(a^(1/3) + (b^(1/3)*c^(2/
3)*x^2)/Sqrt[c*x^3])*Sqrt[(a^(2/3) + b^(2/3)*c^(1/3)*x - (a^(1/3)*b^(1/3)*c
^(2/3)*x^2)/Sqrt[c*x^3])/((1 + Sqrt[3])*a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sqr
```

```
t[c*x^3])^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + (b^(1/3)*c^(2/3)*x^2
)/Sqrt[c*x^3])/((1 + Sqrt[3])*a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3])]
, -7 - 4*Sqrt[3]])/(7*b^(2/3)*c^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + (b^(1/3)*c^(
2/3)*x^2)/Sqrt[c*x^3]))/((1 + Sqrt[3])*a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sqrt
[c*x^3])^2]*Sqrt[a + b*Sqrt[c*x^3]])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Free
Q[{a, b, p}, x] && FractionQ[n]
```

Rule 255

```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_.), x_Symbol] :> With[{k = De
nominator[n]}, Subst[Int[(a + b*c^n*x^(n*q))^p, x], x^(1/k), (c*x^q)^(1/k)/
(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c, p, q}, x] && FractionQ[n]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \sqrt{a+b\sqrt{cx^3}}\,dx = \text{Subst}\left(\int \sqrt{a+b\sqrt{c}\,x^{3/2}}\,dx, \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= \text{Subst}\left(2\,\text{Subst}\left(\int x\sqrt{a+b\sqrt{c}\,x^3}\,dx, x, \sqrt{x}\right), \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= \frac{4}{7}x\sqrt{a+b\sqrt{cx^3}} + \text{Subst}\left(\frac{1}{7}(6a)\,\text{Subst}\left(\int \frac{x}{\sqrt{a+b\sqrt{c}\,x^3}}\,dx, x, \sqrt{x}\right), \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= \frac{4}{7}x\sqrt{a+b\sqrt{cx^3}} + \text{Subst}\left(\frac{(6a)\,\text{Subst}\left(\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[6]{c}\,x}{\sqrt{a+b\sqrt{c}\,x^3}}\,dx, x, \sqrt{x}\right)}{7\sqrt[3]{b}\,\sqrt[6]{c}}, \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right) + \text{Subst}$$

$$= \frac{4}{7}x\sqrt{a+b\sqrt{cx^3}} + \frac{12a\sqrt{a+b\sqrt{cx^3}}}{7b^{2/3}\sqrt[3]{c}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}\right)} - \frac{6\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}\right.}{7b^{2/3}}$$

Mathematica [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sqrt{cx^3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sqrt[c*x^3]],x]
[Out] Integrate[Sqrt[a + b*Sqrt[c*x^3]], x]
```

fricas [F] time = 5.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{\sqrt{cx^3}\,b+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^3)^(1/2))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(sqrt(c*x^3)*b + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{cx^3}\,b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^3)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(sqrt(c*x^3)*b + a), x)
```

maple [A] time = 0.35, size = 854, normalized size = 1.11

$$4ab^2cx+4\sqrt{cx^3}\,b^3cx+3i\sqrt{\frac{\sqrt{cx^3}\,b-\left(-ab^2c\right)^{\frac{1}{3}}x}{\left(-ab^2c\right)^{\frac{1}{3}}\left(i\sqrt{3}-3\right)x}}\sqrt{-\frac{i\left(2\sqrt{cx^3}\,b+i\sqrt{3}\left(-ab^2c\right)^{\frac{1}{3}}x+\left(-ab^2c\right)^{\frac{1}{3}}x\right)\sqrt{3}}{\left(-ab^2c\right)^{\frac{1}{3}}x}}\left(-ab^2c\right)^{\frac{2}{3}}\sqrt{3}\sqrt{\frac{i\left(2\sqrt{cx^3}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(c*x^3)^(1/2)*b)^(1/2),x)
```

```
[Out] 1/7/c*(3*I*(((c*x^3)^(1/2)*b-(-a*b^2*c)^(1/3)*x)/(-a*b^2*c)^(1/3)/(I*3^(1/2
)-3)/x)^(1/2)*(-I*(2*(c*x^3)^(1/2)*b+I*3^(1/2)*(-a*b^2*c)^(1/3)*x+(-a*b^2*c
)^(1/3)*x)*3^(1/2)/(-a*b^2*c)^(1/3)/x)^(1/2)*EllipticE(1/6*3^(1/2)*2^(1/2)*
(I*(-I*3^(1/2)*x*(-a*b^2*c)^(1/3)+2*(c*x^3)^(1/2)*b+(-a*b^2*c)^(1/3)*x)*3^(
1/2)/x/(-a*b^2*c)^(1/3))^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))*(-a
*b^2*c)^(2/3)*3^(1/2)*(I*(-I*3^(1/2)*x*(-a*b^2*c)^(1/3)+2*(c*x^3)^(1/2)*b+(
-a*b^2*c)^(1/3)*x)*3^(1/2)/x/(-a*b^2*c)^(1/3))^(1/2)*2^(1/2)*a-2*I*(((c*x^3
)^(1/2)*b-(-a*b^2*c)^(1/3)*x)/(-a*b^2*c)^(1/3)/(I*3^(1/2)-3)/x)^(1/2)*(-I*(
2*(c*x^3)^(1/2)*b+I*3^(1/2)*(-a*b^2*c)^(1/3)*x+(-a*b^2*c)^(1/3)*x)*3^(1/2)/
(-a*b^2*c)^(1/3)/x)^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*(I*(-I*3^(1/2)*x*(-
a*b^2*c)^(1/3)+2*(c*x^3)^(1/2)*b+(-a*b^2*c)^(1/3)*x)*3^(1/2)/x/(-a*b^2*c)^(
1/3))^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))*(-a*b^2*c)^(2/3)*3^(1/
2)*(I*(-I*3^(1/2)*x*(-a*b^2*c)^(1/3)+2*(c*x^3)^(1/2)*b+(-a*b^2*c)^(1/3)*x)*
3^(1/2)/x/(-a*b^2*c)^(1/3))^(1/2)*2^(1/2)*a+3*(((c*x^3)^(1/2)*b-(-a*b^2*c)^
(1/3)*x)/(-a*b^2*c)^(1/3)/(I*3^(1/2)-3)/x)^(1/2)*(-I*(2*(c*x^3)^(1/2)*b+I*3
^(1/2)*(-a*b^2*c)^(1/3)*x+(-a*b^2*c)^(1/3)*x)*3^(1/2)/(-a*b^2*c)^(1/3)/x)^(
1/2)*EllipticE(1/6*3^(1/2)*2^(1/2)*(I*(-I*3^(1/2)*x*(-a*b^2*c)^(1/3)+2*(c*x
^3)^(1/2)*b+(-a*b^2*c)^(1/3)*x)*3^(1/2)/x/(-a*b^2*c)^(1/3))^(1/2),2^(1/2)*(
I*3^(1/2)/(I*3^(1/2)-3))^(1/2))*(-a*b^2*c)^(2/3)*(I*(-I*3^(1/2)*x*(-a*b^2*c
)^(1/3)+2*(c*x^3)^(1/2)*b+(-a*b^2*c)^(1/3)*x)*3^(1/2)/x/(-a*b^2*c)^(1/3))^(
1/2)*2^(1/2)*a+4*(c*x^3)^(1/2)*x*b^3*c+4*x*a*b^2*c)/b^2/(a+(c*x^3)^(1/2)*b)
^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{cx^3}\,b+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^3)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(c*x^3)*b + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+b\sqrt{c\,x^3}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x^3)^(1/2))^(1/2),x)
```

```
[Out] int((a + b*(c*x^3)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sqrt{cx^3}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x**3)**(1/2))**(1/2),x)
[Out] Integral(sqrt(a + b*sqrt(c*x**3)), x)
```

3.2969 $\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^3}\,dx$

Optimal. Leaf size=810

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,c^{2/3}\left(\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}+\sqrt[3]{a}\right)\sqrt{\frac{-\frac{\sqrt[3]{a}\,\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}+b^{2/3}\sqrt[3]{c}\,x+a^{2/3}}{\left(\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)^2}}\,E\left(\sin^{-1}\left(\frac{\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)b^{4/3}}{8a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}+\sqrt[3]{a}\right)}{\left(\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)^2}}\sqrt{a+b\sqrt{cx^3}}}$$

```
[Out] 1/4*3^(3/4)*b^(4/3)*c^(2/3)*EllipticF((a^(1/3)*(1-3^(1/2))+b^(1/3)*c^(2/3)*
x^2/(c*x^3)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2)),
I*3^(1/2)+2*I)*(a^(1/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))*((a^(2/3)+b^(2/3
)*c^(1/3)*x-a^(1/3)*b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))/(a^(1/3)*(1+3^(1/2))
+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))^2)^(1/2)/a^(2/3)*2^(1/2)/(a^(1/3)*(a^(1
/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*c^(2/3)
*x^2/(c*x^3)^(1/2))^2)^(1/2)/(a+b*(c*x^3)^(1/2))^(1/2)-3/8*3^(1/4)*b^(4/3)*
c^(2/3)*EllipticE((a^(1/3)*(1-3^(1/2))+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))/(
a^(1/3)*(1+3^(1/2))+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2)),I*3^(1/2)+2*I)*(a^(1
/3)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)+
b^(2/3)*c^(1/3)*x-a^(1/3)*b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))/(a^(1/3)*(1+3^
(1/2))+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))^2)^(1/2)/a^(2/3)/(a^(1/3)*(a^(1/3
)+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*c^(2/3)*x
^2/(c*x^3)^(1/2))^2)^(1/2)/(a+b*(c*x^3)^(1/2))^(1/2)-1/2*(a+b*(c*x^3)^(1/2)
)^(1/2)/x^2+3/4*b^(4/3)*c^(2/3)*(a+b*(c*x^3)^(1/2))^(1/2)/a/(a^(1/3)*(1+3^(
1/2))+b^(1/3)*c^(2/3)*x^2/(c*x^3)^(1/2))-3/4*b*c*x*(a+b*(c*x^3)^(1/2))^(1/2
)/a/(c*x^3)^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 810, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {369, 341, 277, 325, 303, 218, 1877}

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,c^{2/3}\left(\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}+\sqrt[3]{a}\right)\sqrt{\frac{-\frac{\sqrt[3]{a}\,\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}+b^{2/3}\sqrt[3]{c}\,x+a^{2/3}}{\left(\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)^2}}\,E\left(\sin^{-1}\left(\frac{\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)b^{4/3}}{8a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}+\sqrt[3]{a}\right)}{\left(\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)^2}}\sqrt{a+b\sqrt{cx^3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[c*x^3]]/x^3,x]
```

```
[Out] -Sqrt[a + b*Sqrt[c*x^3]]/(2*x^2) - (3*b*c*x*Sqrt[a + b*Sqrt[c*x^3]])/(4*a*S
qrt[c*x^3]) + (3*b^(4/3)*c^(2/3)*Sqrt[a + b*Sqrt[c*x^3]])/(4*a*((1 + Sqrt[3
])*a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3])) - (3*3^(1/4)*Sqrt[2 - Sqrt
[3]]*b^(4/3)*c^(2/3)*(a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3])*Sqrt[(a^
(2/3) + b^(2/3)*c^(1/3)*x - (a^(1/3)*b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3])/((1
+ Sqrt[3])*a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3])^2]*EllipticE[ArcSin
[((1 - Sqrt[3])*a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3])/((1 + Sqrt[3])
*a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3])], -7 - 4*Sqrt[3]])/(8*a^(2/3)
*Sqrt[(a^(1/3)*(a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3]))/((1 + Sqrt[3]
)*a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3])^2]*Sqrt[a + b*Sqrt[c*x^3]])
+ (3^(3/4)*b^(4/3)*c^(2/3)*(a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3])*Sq
```

```
rt[(a^(2/3) + b^(2/3)*c^(1/3)*x - (a^(1/3)*b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3]
)/((1 + Sqrt[3])*a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3])^2]*EllipticF[
ArcSin[((1 - Sqrt[3])*a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3])/((1 + Sq
rt[3])*a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3])], -7 - 4*Sqrt[3]])/(2*S
qrt[2]*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3]))
/((1 + Sqrt[3])*a^(1/3) + (b^(1/3)*c^(2/3)*x^2)/Sqrt[c*x^3])^2]*Sqrt[a + b*
Sqrt[c*x^3]])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
```

```
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^3}\,dx = \text{Subst}\left(\int \frac{\sqrt{a+b\sqrt{c}\,x^{3/2}}}{x^3}\,dx, \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= \text{Subst}\left(2\,\text{Subst}\left(\int \frac{\sqrt{a+b\sqrt{c}\,x^3}}{x^5}\,dx, x, \sqrt{x}\right), \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= -\frac{\sqrt{a+b\sqrt{cx^3}}}{2x^2} + \text{Subst}\left(\frac{1}{4}\left(3b\sqrt{c}\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+b\sqrt{c}\,x^3}}\,dx, x, \sqrt{x}\right), \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= -\frac{\sqrt{a+b\sqrt{cx^3}}}{2x^2} - \frac{3bcx\sqrt{a+b\sqrt{cx^3}}}{4a\sqrt{cx^3}} + \text{Subst}\left(\frac{\left(3b^2c\right)\text{Subst}\left(\int \frac{x}{\sqrt{a+b\sqrt{c}\,x^3}}\,dx, x, \sqrt{x}\right)}{8a}, \sqrt{}\right.$$

$$= -\frac{\sqrt{a+b\sqrt{cx^3}}}{2x^2} - \frac{3bcx\sqrt{a+b\sqrt{cx^3}}}{4a\sqrt{cx^3}} + \text{Subst}\left(\frac{\left(3b^{5/3}c^{5/6}\right)\text{Subst}\left(\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[6]{c}\,x}{\sqrt{a+b\sqrt{c}\,x^3}}\,dx\right.}{8a}\right.$$

$$= -\frac{\sqrt{a+b\sqrt{cx^3}}}{2x^2} - \frac{3bcx\sqrt{a+b\sqrt{cx^3}}}{4a\sqrt{cx^3}} + \frac{3b^{4/3}c^{2/3}\sqrt{a+b\sqrt{cx^3}}}{4a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}\,c^{2/3}x^2}{\sqrt{cx^3}}\right)} - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b}{}$$

Mathematica [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sqrt{cx^3}}}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sqrt[c*x^3]]/x^3,x]
```

```
[Out] Integrate[Sqrt[a + b*Sqrt[c*x^3]]/x^3, x]
```

fricas [F] time = 6.10, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\sqrt{cx^3}\,b+a}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^3)^(1/2))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(sqrt(c*x^3)*b + a)/x^3, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^3)^(1/2))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sy
m(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Va
lueEvaluation time: 22.11-2*abs(c)/c^2/c*16*c^3/64/sqrt(c*x)/sqrt(c*x)*sqrt
(a*c^2/(c*x)^2+b*c/sqrt(c*x))+integrate(2*abs(c)/c^2/c*24*c^4*b/64/2*c*c/(s
qrt(c*x)*sqrt(b*c^2*x*sqrt(c*x)+a*c^2)*c*x*abs(c)*sign(x)),x)
```

maple [A] time = 0.30, size = 863, normalized size = 1.07

$$-12b^2c\,x^3+3i\sqrt{2}\,\sqrt{\frac{i\left(2\sqrt{c\,x^3}\;b-i\sqrt{3}\,\left(-a\,b^2c\right)^{\frac{1}{3}}x+\left(-a\,b^2c\right)^{\frac{1}{3}}x\right)\sqrt{3}}{\left(-a\,b^2c\right)^{\frac{1}{3}}x}}\,\sqrt{\frac{\sqrt{c\,x^3}\;b-\left(-a\,b^2c\right)^{\frac{1}{3}}x}{\left(-a\,b^2c\right)^{\frac{1}{3}}\left(i\sqrt{3}-3\right)x}}\,\sqrt{-\frac{i\left(2\sqrt{c\,x^3}\;b+i\sqrt{3}\,\left(-a\,b^2c\right)^{\frac{1}{3}}x+\left(-a\,b^2c\right)^{\frac{1}{3}}x\right)\sqrt{3}}{\left(-a\,b^2c\right)^{\frac{1}{3}}x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(c*x^3)^(1/2)*b)^(1/2)/x^3,x)
```

```
[Out] 1/16*(3*I*2^(1/2)*(I*(2*(c*x^3)^(1/2)*b-I*3^(1/2)*(-a*b^2*c)^(1/3)*x+(-a*b^
2*c)^(1/3)*x)*3^(1/2)/(-a*b^2*c)^(1/3)/x)^(1/2)*(((c*x^3)^(1/2)*b-(-a*b^2*c
)^(1/3)*x)/(-a*b^2*c)^(1/3)/(I*3^(1/2)-3)/x)^(1/2)*(-I*(2*(c*x^3)^(1/2)*b+I
*3^(1/2)*(-a*b^2*c)^(1/3)*x+(-a*b^2*c)^(1/3)*x)*3^(1/2)/(-a*b^2*c)^(1/3)/x)
^(1/2)*EllipticE(1/6*3^(1/2)*2^(1/2)*(I*(2*(c*x^3)^(1/2)*b-I*3^(1/2)*(-a*b^
2*c)^(1/3)*x+(-a*b^2*c)^(1/3)*x)*3^(1/2)/(-a*b^2*c)^(1/3)/x)^(1/2),2^(1/2)*
(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))*(-a*b^2*c)^(2/3)*3^(1/2)*x^2-2*I*2^(1/2)*(
I*(2*(c*x^3)^(1/2)*b-I*3^(1/2)*(-a*b^2*c)^(1/3)*x+(-a*b^2*c)^(1/3)*x)*3^(1/
2)/(-a*b^2*c)^(1/3)/x)^(1/2)*(((c*x^3)^(1/2)*b-(-a*b^2*c)^(1/3)*x)/(-a*b^2*
c)^(1/3)/(I*3^(1/2)-3)/x)^(1/2)*(-I*(2*(c*x^3)^(1/2)*b+I*3^(1/2)*(-a*b^2*c)
^(1/3)*x+(-a*b^2*c)^(1/3)*x)*3^(1/2)/(-a*b^2*c)^(1/3)/x)^(1/2)*EllipticF(1/
6*3^(1/2)*2^(1/2)*(I*(2*(c*x^3)^(1/2)*b-I*3^(1/2)*(-a*b^2*c)^(1/3)*x+(-a*b^
2*c)^(1/3)*x)*3^(1/2)/(-a*b^2*c)^(1/3)/x)^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/
2)-3))^(1/2))*(-a*b^2*c)^(2/3)*3^(1/2)*x^2+3*2^(1/2)*(I*(2*(c*x^3)^(1/2)*b-
I*3^(1/2)*(-a*b^2*c)^(1/3)*x+(-a*b^2*c)^(1/3)*x)*3^(1/2)/(-a*b^2*c)^(1/3)/x
)^(1/2)*(((c*x^3)^(1/2)*b-(-a*b^2*c)^(1/3)*x)/(-a*b^2*c)^(1/3)/(I*3^(1/2)-3
)/x)^(1/2)*(-I*(2*(c*x^3)^(1/2)*b+I*3^(1/2)*(-a*b^2*c)^(1/3)*x+(-a*b^2*c)^(
1/3)*x)*3^(1/2)/(-a*b^2*c)^(1/3)/x)^(1/2)*EllipticE(1/6*3^(1/2)*2^(1/2)*(I*
(2*(c*x^3)^(1/2)*b-I*3^(1/2)*(-a*b^2*c)^(1/3)*x+(-a*b^2*c)^(1/3)*x)*3^(1/2)
/(-a*b^2*c)^(1/3)/x)^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))*(-a*b^2
*c)^(2/3)*x^2-12*c*x^3*b^2-20*(c*x^3)^(1/2)*a*b-8*a^2)/x^2/a/(a+(c*x^3)^(1/
2)*b)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{cx^3}\,b+a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^3)^(1/2))^(1/2)/x^3,x, algorithm="maxima")
[Out] integrate(sqrt(sqrt(c*x^3)*b + a)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a + b\sqrt{c\,x^3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x^3)^(1/2))^(1/2)/x^3,x)
[Out] int((a + b*(c*x^3)^(1/2))^(1/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\sqrt{cx^3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x**3)**(1/2))**(1/2)/x**3,x)
[Out] Integral(sqrt(a + b*sqrt(c*x**3))/x**3, x)
```

3.2970 $\int x^{17}\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=116

$$-\frac{4a^3\left(a+b\left(cx^3\right)^{3/2}\right)^{3/2}}{27b^4c^6}+\frac{4a^2\left(a+b\left(cx^3\right)^{3/2}\right)^{5/2}}{15b^4c^6}+\frac{4\left(a+b\left(cx^3\right)^{3/2}\right)^{9/2}}{81b^4c^6}-\frac{4a\left(a+b\left(cx^3\right)^{3/2}\right)^{7/2}}{21b^4c^6}$$

```
[Out] -4/27*a^3*(a+b*(c*x^3)^(3/2))^(3/2)/b^4/c^6+4/15*a^2*(a+b*(c*x^3)^(3/2))^(5
/2)/b^4/c^6-4/21*a*(a+b*(c*x^3)^(3/2))^(7/2)/b^4/c^6+4/81*(a+b*(c*x^3)^(3/2
))^(9/2)/b^4/c^6
```

Rubi [A] time = 0.08, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {369, 266, 43}

$$\frac{4a^2\left(a+b\left(cx^3\right)^{3/2}\right)^{5/2}}{15b^4c^6}-\frac{4a^3\left(a+b\left(cx^3\right)^{3/2}\right)^{3/2}}{27b^4c^6}+\frac{4\left(a+b\left(cx^3\right)^{3/2}\right)^{9/2}}{81b^4c^6}-\frac{4a\left(a+b\left(cx^3\right)^{3/2}\right)^{7/2}}{21b^4c^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^17*Sqrt[a + b*(c*x^3)^(3/2)],x]
```

```
[Out] (-4*a^3*(a + b*(c*x^3)^(3/2))^(3/2))/(27*b^4*c^6) + (4*a^2*(a + b*(c*x^3)^(
3/2))^(5/2))/(15*b^4*c^6) - (4*a*(a + b*(c*x^3)^(3/2))^(7/2))/(21*b^4*c^6)
+ (4*(a + b*(c*x^3)^(3/2))^(9/2))/(81*b^4*c^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Rubi steps

$$\int x^{17}\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx = \text{Subst}\left(\int x^{17}\sqrt{a+bc^{3/2}x^{9/2}}\,dx, \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= \text{Subst}\left(\frac{2}{9}\,\text{Subst}\left(\int x^3\sqrt{a+bc^{3/2}x}\,dx, x, x^{9/2}\right), \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= \text{Subst}\left(\frac{2}{9}\,\text{Subst}\left(\int\left(-\frac{a^3\sqrt{a+bc^{3/2}x}}{b^3c^{9/2}} + \frac{3a^2\left(a+bc^{3/2}x\right)^{3/2}}{b^3c^{9/2}} - \frac{3a\left(a+bc^{3/2}x\right)^{5/2}}{b^3c^{9/2}} +\right.\right.\right.$$

$$= -\frac{4a^3\left(a+b\left(cx^3\right)^{3/2}\right)^{3/2}}{27b^4c^6} + \frac{4a^2\left(a+b\left(cx^3\right)^{3/2}\right)^{5/2}}{15b^4c^6} - \frac{4a\left(a+b\left(cx^3\right)^{3/2}\right)^{7/2}}{21b^4c^6} + \frac{4\left(a+\right.}{}$$

Mathematica [A] time = 0.15, size = 80, normalized size = 0.69

$$\frac{4\left(a+b\left(cx^3\right)^{3/2}\right)^{3/2}\left(-16a^3+24a^2b\left(cx^3\right)^{3/2}-30ab^2c^3x^9+35b^3c^3x^9\left(cx^3\right)^{3/2}\right)}{2835b^4c^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^17*Sqrt[a + b*(c*x^3)^(3/2)],x]
```

```
[Out] (4*(a + b*(c*x^3)^(3/2))^(3/2)*(-16*a^3 - 30*a*b^2*c^3*x^9 + 24*a^2*b*(c*x^
3)^(3/2) + 35*b^3*c^3*x^9*(c*x^3)^(3/2)))/(2835*b^4*c^6)
```

fricas [A] time = 5.27, size = 87, normalized size = 0.75

$$\frac{4\left(35\,b^4c^6x^{18}-6\,a^2b^2c^3x^9-16\,a^4+\left(5\,ab^3c^4x^{12}+8\,a^3bcx^3\right)\sqrt{cx^3}\right)\sqrt{\sqrt{cx^3}\,bcx^3+a}}{2835\,b^4c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17*(a+b*(c*x^3)^(3/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 4/2835*(35*b^4*c^6*x^18 - 6*a^2*b^2*c^3*x^9 - 16*a^4 + (5*a*b^3*c^4*x^12 +
8*a^3*b*c*x^3)*sqrt(c*x^3))*sqrt(sqrt(c*x^3)*b*c*x^3 + a)/(b^4*c^6)
```

giac [A] time = 0.28, size = 121, normalized size = 1.04

$$-\frac{4\left(105\left(\sqrt{cx}\,bc^5x^4+ac^4\right)^{\frac{3}{2}}a^3c^{12}-189\left(\sqrt{cx}\,bc^5x^4+ac^4\right)^{\frac{5}{2}}a^2c^8+135\left(\sqrt{cx}\,bc^5x^4+ac^4\right)^{\frac{7}{2}}ac^4-35\left(\sqrt{cx}\,bc^5x^4\right.\right.}{2835\,b^4c^{26}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17*(a+b*(c*x^3)^(3/2))^(1/2),x, algorithm="giac")
```

```
[Out] -4/2835*(105*(sqrt(c*x)*b*c^5*x^4 + a*c^4)^(3/2)*a^3*c^12 - 189*(sqrt(c*x)*
b*c^5*x^4 + a*c^4)^(5/2)*a^2*c^8 + 135*(sqrt(c*x)*b*c^5*x^4 + a*c^4)^(7/2)*
a*c^4 - 35*(sqrt(c*x)*b*c^5*x^4 + a*c^4)^(9/2))*abs(c)^2/(b^4*c^26)
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int\sqrt{a+\left(c\,x^3\right)^{\frac{3}{2}}b}\;x^{17}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^17*(a+b*(c*x^3)^(3/2))^(1/2),x)
```

```
[Out] int(x^17*(a+b*(c*x^3)^(3/2))^(1/2),x)
```

maxima [A] time = 0.60, size = 85, normalized size = 0.73

$$\frac{4\left(\frac{35\left(\left(cx^3\right)^{\frac{3}{2}}b+a\right)^{\frac{9}{2}}}{b^4}-\frac{135\left(\left(cx^3\right)^{\frac{3}{2}}b+a\right)^{\frac{7}{2}}a}{b^4}+\frac{189\left(\left(cx^3\right)^{\frac{3}{2}}b+a\right)^{\frac{5}{2}}a^2}{b^4}-\frac{105\left(\left(cx^3\right)^{\frac{3}{2}}b+a\right)^{\frac{3}{2}}a^3}{b^4}\right)}{2835\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17*(a+b*(c*x^3)^(3/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/2835*(35*((c*x^3)^(3/2)*b + a)^(9/2)/b^4 - 135*((c*x^3)^(3/2)*b + a)^(7/2
)*a/b^4 + 189*((c*x^3)^(3/2)*b + a)^(5/2)*a^2/b^4 - 105*((c*x^3)^(3/2)*b +
a)^(3/2)*a^3/b^4)/c^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{17}\sqrt{a+b\left(c\,x^3\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^17*(a + b*(c*x^3)^(3/2))^(1/2),x)
```

```
[Out] int(x^17*(a + b*(c*x^3)^(3/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{17}\sqrt{a+b\left(cx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**17*(a+b*(c*x**3)**(3/2))**(1/2),x)
```

```
[Out] Integral(x**17*sqrt(a + b*(c*x**3)**(3/2)), x)
```

3.2971 $\int x^8 \sqrt{a + b\left(cx^3\right)^{3/2}}\, dx$

Optimal. Leaf size=56

$$\frac{4\left(a + b\left(cx^3\right)^{3/2}\right)^{5/2}}{45b^2c^3} - \frac{4a\left(a + b\left(cx^3\right)^{3/2}\right)^{3/2}}{27b^2c^3}$$

```
[Out] -4/27*a*(a+b*(c*x^3)^(3/2))^(3/2)/b^2/c^3+4/45*(a+b*(c*x^3)^(3/2))^(5/2)/b^
2/c^3
```

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {369, 266, 43}

$$\frac{4\left(a + b\left(cx^3\right)^{3/2}\right)^{5/2}}{45b^2c^3} - \frac{4a\left(a + b\left(cx^3\right)^{3/2}\right)^{3/2}}{27b^2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*Sqrt[a + b*(c*x^3)^(3/2)],x]
```

```
[Out] (-4*a*(a + b*(c*x^3)^(3/2))^(3/2))/(27*b^2*c^3) + (4*(a + b*(c*x^3)^(3/2))^
(5/2))/(45*b^2*c^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Rubi steps

$$\int x^8\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx = \text{Subst}\left(\int x^8\sqrt{a+bc^{3/2}x^{9/2}}\,dx, \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$
$$= \text{Subst}\left(\frac{2}{9}\text{Subst}\left(\int x\sqrt{a+bc^{3/2}x}\,dx, x, x^{9/2}\right), \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$
$$= \text{Subst}\left(\frac{2}{9}\text{Subst}\left(\int\left(-\frac{a\sqrt{a+bc^{3/2}x}}{bc^{3/2}} + \frac{\left(a+bc^{3/2}x\right)^{3/2}}{bc^{3/2}}\right)dx, x, x^{9/2}\right), \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$
$$= -\frac{4a\left(a+b\left(cx^3\right)^{3/2}\right)^{3/2}}{27b^2c^3} + \frac{4\left(a+b\left(cx^3\right)^{3/2}\right)^{5/2}}{45b^2c^3}$$

Mathematica [A] time = 0.08, size = 43, normalized size = 0.77

$$\frac{4\left(a+b\left(cx^3\right)^{3/2}\right)^{3/2}\left(3b\left(cx^3\right)^{3/2}-2a\right)}{135b^2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*Sqrt[a + b*(c*x^3)^(3/2)],x]
```

```
[Out] (4*(a + b*(c*x^3)^(3/2))^(3/2)*(-2*a + 3*b*(c*x^3)^(3/2)))/(135*b^2*c^3)
```

fricas [A] time = 2.86, size = 56, normalized size = 1.00

$$\frac{4\left(3\,b^2c^3x^9+\sqrt{cx^3}\,abcx^3-2\,a^2\right)\sqrt{\sqrt{cx^3}\,bcx^3+a}}{135\,b^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*(c*x^3)^(3/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 4/135*(3*b^2*c^3*x^9 + sqrt(c*x^3)*a*b*c*x^3 - 2*a^2)*sqrt(sqrt(c*x^3)*b*c*
x^3 + a)/(b^2*c^3)
```

giac [A] time = 0.20, size = 63, normalized size = 1.12

$$-\frac{4\left(5\left(\sqrt{cx}\,bc^5x^4+ac^4\right)^{\frac{3}{2}}ac^4-3\left(\sqrt{cx}\,bc^5x^4+ac^4\right)^{\frac{5}{2}}\right)|c|^2}{135\,b^2c^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*(c*x^3)^(3/2))^(1/2),x, algorithm="giac")
```

```
[Out] -4/135*(5*(sqrt(c*x)*b*c^5*x^4 + a*c^4)^(3/2)*a*c^4 - 3*(sqrt(c*x)*b*c^5*x^
4 + a*c^4)^(5/2))*abs(c)^2/(b^2*c^15)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int\sqrt{a+\left(c\,x^3\right)^{\frac{3}{2}}b}\;x^8\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a+(c*x^3)^(3/2)*b)^(1/2),x)
```

```
[Out] int(x^8*(a+(c*x^3)^(3/2)*b)^(1/2),x)
```

maxima [A] time = 0.62, size = 43, normalized size = 0.77

$$\frac{4\left(\frac{3\left(\left(cx^3\right)^{\frac{3}{2}}b+a\right)^{\frac{5}{2}}}{b^2}-\frac{5\left(\left(cx^3\right)^{\frac{3}{2}}b+a\right)^{\frac{3}{2}}a}{b^2}\right)}{135\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*(c*x^3)^(3/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/135*(3*((c*x^3)^(3/2)*b + a)^(5/2)/b^2 - 5*((c*x^3)^(3/2)*b + a)^(3/2)*a/
b^2)/c^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^8\sqrt{a+b\left(c\,x^3\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*(c*x^3)^(3/2))^(1/2),x)
```

```
[Out] int(x^8*(a + b*(c*x^3)^(3/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^8\sqrt{a+b\left(cx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(a+b*(c*x**3)**(3/2))**(1/2),x)
```

```
[Out] Integral(x**8*sqrt(a + b*(c*x**3)**(3/2)), x)
```

3.2972 $\int \frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{x}\,dx$

Optimal. Leaf size=55

$$\frac{4}{9}\sqrt{a+b\left(cx^3\right)^{3/2}}-\frac{4}{9}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{\sqrt{a}}\right)$$

```
[Out] -4/9*arctanh((a+b*(c*x^3)^(3/2))^(1/2)/a^(1/2))*a^(1/2)+4/9*(a+b*(c*x^3)^(3
/2))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {369, 266, 50, 63, 208}

$$\frac{4}{9}\sqrt{a+b\left(cx^3\right)^{3/2}}-\frac{4}{9}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*(c*x^3)^(3/2)]/x,x]
```

```
[Out] (4*Sqrt[a + b*(c*x^3)^(3/2)])/9 - (4*Sqrt[a]*ArcTanh[Sqrt[a + b*(c*x^3)^(3/
2)]/Sqrt[a]])/9
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
```

```
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{x}\,dx &= \text{Subst}\left(\int \frac{\sqrt{a+bc^{3/2}x^{9/2}}}{x}\,dx, \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right) \\
&= \text{Subst}\left(\frac{2}{9}\,\text{Subst}\left(\int \frac{\sqrt{a+bc^{3/2}x}}{x}\,dx, x, x^{9/2}\right), \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right) \\
&= \frac{4}{9}\sqrt{a+b\left(cx^3\right)^{3/2}} + \text{Subst}\left(\frac{1}{9}(2a)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bc^{3/2}x}}\,dx, x, x^{9/2}\right), \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right) \\
&= \frac{4}{9}\sqrt{a+b\left(cx^3\right)^{3/2}} + \text{Subst}\left(\frac{(4a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{bc^{3/2}}+\frac{x^2}{bc^{3/2}}}\,dx, x, \sqrt{a+bc^{3/2}x^{9/2}}\right)}{9bc^{3/2}}, \sqrt{x}, \frac{\sqrt{cx}}{\sqrt{c}}\right. \\
&= \frac{4}{9}\sqrt{a+b\left(cx^3\right)^{3/2}} - \frac{4}{9}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [A] time = 0.07, size = 55, normalized size = 1.00

$$\frac{4}{9}\sqrt{a+b\left(cx^3\right)^{3/2}} - \frac{4}{9}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*(c*x^3)^(3/2)]/x,x]
```

```
[Out] (4*Sqrt[a + b*(c*x^3)^(3/2)])/9 - (4*Sqrt[a]*ArcTanh[Sqrt[a + b*(c*x^3)^(3/
2)]/Sqrt[a]])/9
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^3)^(3/2))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.22, size = 74, normalized size = 1.35

$$\frac{4\left(\frac{ac\arctan\left(\frac{\sqrt{\sqrt{cx}\,bc^5x^4+ac^4}}{\sqrt{-a}\,c^2}\right)}{\sqrt{-a}} + \frac{\sqrt{\sqrt{cx}\,bc^5x^4+ac^4}}{c}\right)|c|^2}{9\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^3)^(3/2))^(1/2)/x,x, algorithm="giac")
```

```
[Out] 4/9*(a*c*arctan(sqrt(sqrt(c*x)*b*c^5*x^4 + a*c^4)/(sqrt(-a)*c^2))/sqrt(-a)
+ sqrt(sqrt(c*x)*b*c^5*x^4 + a*c^4)/c)*abs(c)^2/c^3
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \left(c\,x^3\right)^{\frac{3}{2}} b}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(c*x^3)^(3/2)*b)^(1/2)/x,x)
```

```
[Out] int((a+(c*x^3)^(3/2)*b)^(1/2)/x,x)
```

maxima [A] time = 1.24, size = 61, normalized size = 1.11

$$\frac{2}{9}\sqrt{a}\,\log\left(\frac{\sqrt{\left(cx^3\right)^{\frac{3}{2}} b + a} - \sqrt{a}}{\sqrt{\left(cx^3\right)^{\frac{3}{2}} b + a} + \sqrt{a}}\right) + \frac{4}{9}\sqrt{\left(cx^3\right)^{\frac{3}{2}} b + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^3)^(3/2))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(a)*log((sqrt((c*x^3)^(3/2)*b + a) - sqrt(a))/(sqrt((c*x^3)^(3/2)*b
 + a) + sqrt(a))) + 4/9*sqrt((c*x^3)^(3/2)*b + a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a + b\left(c\,x^3\right)^{3/2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x^3)^(3/2))^(1/2)/x,x)
```

```
[Out] int((a + b*(c*x^3)^(3/2))^(1/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\left(cx^3\right)^{\frac{3}{2}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x**3)**(3/2))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a + b*(c*x**3)**(3/2))/x, x)
```

3.2973 $\int \frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{x^{10}}\,dx$

Optimal. Leaf size=101

$$\frac{b^2c^3\tanh^{-1}\left(\frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{\sqrt{a}}\right)}{18a^{3/2}}-\frac{bc^3\sqrt{a+b\left(cx^3\right)^{3/2}}}{18a\left(cx^3\right)^{3/2}}-\frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{9x^9}$$

```
[Out] 1/18*b^2*c^3*arctanh((a+b*(c*x^3)^(3/2))^(1/2)/a^(1/2))/a^(3/2)-1/9*(a+b*(c
*x^3)^(3/2))^(1/2)/x^9-1/18*b*c^3*(a+b*(c*x^3)^(3/2))^(1/2)/a/(c*x^3)^(3/2)
```

Rubi [A] time = 0.07, antiderivative size = 104, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {369, 266, 47, 51, 63, 208}

$$\frac{b^2c^3\tanh^{-1}\left(\frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{\sqrt{a}}\right)}{18a^{3/2}}-\frac{bc^6x^9\sqrt{a+b\left(cx^3\right)^{3/2}}}{18a\left(cx^3\right)^{9/2}}-\frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*(c*x^3)^(3/2)]/x^10,x]
```

```
[Out] -Sqrt[a + b*(c*x^3)^(3/2)]/(9*x^9) - (b*c^6*x^9*Sqrt[a + b*(c*x^3)^(3/2)])/
(18*a*(c*x^3)^(9/2)) + (b^2*c^3*ArcTanh[Sqrt[a + b*(c*x^3)^(3/2)]/Sqrt[a]])
/(18*a^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Rubi steps

$$\int \frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{x^{10}}\,dx = \text{Subst}\left(\int \frac{\sqrt{a+bc^{3/2}x^{9/2}}}{x^{10}}\,dx, \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= \text{Subst}\left(\frac{2}{9}\,\text{Subst}\left(\int \frac{\sqrt{a+bc^{3/2}x}}{x^3}\,dx, x, x^{9/2}\right), \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= -\frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{9x^9} + \text{Subst}\left(\frac{1}{18}\left(bc^{3/2}\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bc^{3/2}x}}\,dx, x, x^{9/2}\right), \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= -\frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{9x^9} - \frac{bc^6x^9\sqrt{a+b\left(cx^3\right)^{3/2}}}{18a\left(cx^3\right)^{9/2}} - \text{Subst}\left(\frac{\left(b^2c^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bc^{3/2}x}}\,dx, x, x^{9/}\right.}{36a}\right.$$

$$= -\frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{9x^9} - \frac{bc^6x^9\sqrt{a+b\left(cx^3\right)^{3/2}}}{18a\left(cx^3\right)^{9/2}} - \text{Subst}\left(\frac{\left(bc^{3/2}\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{bc^{3/2}}+\frac{x^2}{bc^{3/2}}}\,dx, x, \sqrt{\right.}\right.}{18a}\right.$$

$$= -\frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{9x^9} - \frac{bc^6x^9\sqrt{a+b\left(cx^3\right)^{3/2}}}{18a\left(cx^3\right)^{9/2}} + \frac{b^2c^3\tanh^{-1}\left(\frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{\sqrt{a}}\right)}{18a^{3/2}}$$

Mathematica [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{x^{10}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*(c*x^3)^(3/2)]/x^10,x]
```

```
[Out] Integrate[Sqrt[a + b*(c*x^3)^(3/2)]/x^10, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^3)^(3/2))^(1/2)/x^10,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.25, size = 130, normalized size = 1.29

$$-\frac{\left(\frac{b^3c^6\arctan\left(\frac{\sqrt{\sqrt{cx}\,bc^5x^4+ac^4}}{\sqrt{-a}\,c^2}\right)}{\sqrt{-a}\,a}+\frac{\sqrt{\sqrt{cx}\,bc^5x^4+ac^4}\,ab^3c^{12}+\left(\sqrt{cx}\,bc^5x^4+ac^4\right)^{\frac{3}{2}}b^3c^8}{ab^2c^{11}x^9}\right)|c|^2}{18\,bc^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^3)^(3/2))^(1/2)/x^10,x, algorithm="giac")
```

```
[Out] -1/18*(b^3*c^6*arctan(sqrt(sqrt(c*x)*b*c^5*x^4 + a*c^4)/(sqrt(-a)*c^2))/(sq
rt(-a)*a) + (sqrt(sqrt(c*x)*b*c^5*x^4 + a*c^4)*a*b^3*c^12 + (sqrt(c*x)*b*c^
5*x^4 + a*c^4)^(3/2)*b^3*c^8)/(a*b^2*c^11*x^9))*abs(c)^2/(b*c^5)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\left(c\,x^3\right)^{\frac{3}{2}}b}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(c*x^3)^(3/2)*b)^(1/2)/x^10,x)
```

```
[Out] int((a+(c*x^3)^(3/2)*b)^(1/2)/x^10,x)
```

maxima [A] time = 1.21, size = 128, normalized size = 1.27

$$-\frac{1}{36}\left(\frac{b^2\log\left(\frac{\sqrt{\left(cx^3\right)^{\frac{3}{2}}b+a}-\sqrt{a}}{\sqrt{\left(cx^3\right)^{\frac{3}{2}}b+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2\left(\left(\left(cx^3\right)^{\frac{3}{2}}b+a\right)^{\frac{3}{2}}b^2+\sqrt{\left(cx^3\right)^{\frac{3}{2}}b+a}\,ab^2\right)}{\left(\left(cx^3\right)^{\frac{3}{2}}b+a\right)^2a-2\left(\left(cx^3\right)^{\frac{3}{2}}b+a\right)a^2+a^3}\right)c^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^3)^(3/2))^(1/2)/x^10,x, algorithm="maxima")
```

```
[Out] -1/36*(b^2*log((sqrt((c*x^3)^(3/2)*b + a) - sqrt(a))/(sqrt((c*x^3)^(3/2)*b
+ a) + sqrt(a)))/a^(3/2) + 2*(((c*x^3)^(3/2)*b + a)^(3/2)*b^2 + sqrt((c*x^3
)^(3/2)*b + a)*a*b^2)/(((c*x^3)^(3/2)*b + a)^2*a - 2*((c*x^3)^(3/2)*b + a)*
a^2 + a^3))*c^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+b\left(c\,x^3\right)^{3/2}}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x^3)^(3/2))^(1/2)/x^10,x)
```

```
[Out] int((a + b*(c*x^3)^(3/2))^(1/2)/x^10, x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\left(cx^3\right)^{\frac{3}{2}}}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x**3)**(3/2))**(1/2)/x**10,x)
[Out] Integral(sqrt(a + b*(c*x**3)**(3/2))/x**10, x)
```

3.2974 $\int x^2\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=642

$$\frac{4\sqrt{2}\,a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt{cx^3}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt{cx^3}+b^{2/3}cx^3}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt{cx^3}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,\sqrt{cx^3}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,\sqrt{cx^3}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\quad 2\sqrt[4]{3}\sqrt{2-\sqrt{3}}}{7\sqrt[4]{3}\,b^{2/3}c\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt{cx^3}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt{cx^3}\right)^2}}\sqrt{a+b\left(cx^3\right)^{3/2}}}$$

```
[Out] 4/21*x^3*(a+b*(c*x^3)^(3/2))^(1/2)+4/7*a*(a+b*(c*x^3)^(3/2))^(1/2)/b^(2/3)/
c/(a^(1/3)*(1+3^(1/2))+b^(1/3)*(c*x^3)^(1/2))+4/21*a^(4/3)*EllipticF((a^(1/
3)*(1-3^(1/2))+b^(1/3)*(c*x^3)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*(c*x^3)^
(1/2)),I*3^(1/2)+2*I)*2^(1/2)*(a^(1/3)+b^(1/3)*(c*x^3)^(1/2))*((a^(2/3)+b^(
2/3)*c*x^3-a^(1/3)*b^(1/3)*(c*x^3)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*(c*x
^3)^(1/2))^2)^(1/2)*3^(3/4)/b^(2/3)/c/(a+b*(c*x^3)^(3/2))^(1/2)/(a^(1/3)*(a
^(1/3)+b^(1/3)*(c*x^3)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*(c*x^3)^(1/2))^2
)^(1/2)-2/7*3^(1/4)*a^(4/3)*EllipticE((a^(1/3)*(1-3^(1/2))+b^(1/3)*(c*x^3)^
(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*(c*x^3)^(1/2)),I*3^(1/2)+2*I)*(a^(1/3)+
b^(1/3)*(c*x^3)^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)+b^(2/3)*c*x^3-a^
(1/3)*b^(1/3)*(c*x^3)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*(c*x^3)^(1/2))^2)
^(1/2)/b^(2/3)/c/(a+b*(c*x^3)^(3/2))^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*(c*x^3
)^(1/2))/(a^(1/3)*(1+3^(1/2))+b^(1/3)*(c*x^3)^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.73, antiderivative size = 642, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {369, 341, 275, 279, 303, 218, 1877}

$$\frac{4\sqrt{2}\,a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt{cx^3}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt{cx^3}+b^{2/3}cx^3}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt{cx^3}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,\sqrt{cx^3}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,\sqrt{cx^3}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)\quad 2\sqrt[4]{3}\sqrt{2-\sqrt{3}}}{7\sqrt[4]{3}\,b^{2/3}c\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt{cx^3}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt{cx^3}\right)^2}}\sqrt{a+b\left(cx^3\right)^{3/2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*(c*x^3)^(3/2)],x]
```

```
[Out] (4*x^3*Sqrt[a + b*(c*x^3)^(3/2)])/21 + (4*a*Sqrt[a + b*(c*x^3)^(3/2)])/(7*b
^(2/3)*c*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^3])) - (2*3^(1/4)*Sqrt[2
 - Sqrt[3]]*a^(4/3)*(a^(1/3) + b^(1/3)*Sqrt[c*x^3])*Sqrt[(a^(2/3) + b^(2/3)
*c*x^3 - a^(1/3)*b^(1/3)*Sqrt[c*x^3])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt
[c*x^3])^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^3])/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^3])], -7 - 4*Sqrt[3]])/(7*b^(2/3)
*c*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*Sqrt[c*x^3]))/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*Sqrt[c*x^3])^2]*Sqrt[a + b*(c*x^3)^(3/2)]) + (4*Sqrt[2]*a^(4/3)*(a^
(1/3) + b^(1/3)*Sqrt[c*x^3])*Sqrt[(a^(2/3) + b^(2/3)*c*x^3 - a^(1/3)*b^(1/3
)*Sqrt[c*x^3])/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^3])^2]*EllipticF[A
rcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*x^3])/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*Sqrt[c*x^3])], -7 - 4*Sqrt[3]])/(7*3^(1/4)*b^(2/3)*c*Sqrt[(a^(1/3
)*(a^(1/3) + b^(1/3)*Sqrt[c*x^3]))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*Sqrt[c*
x^3])^2]*Sqrt[a + b*(c*x^3)^(3/2)])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
```

```
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int x^2\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx = \text{Subst}\left(\int x^2\sqrt{a+bc^{3/2}x^{9/2}}\,dx, \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= \text{Subst}\left(2\,\text{Subst}\left(\int x^5\sqrt{a+bc^{3/2}x^9}\,dx, x, \sqrt{x}\right), \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= \text{Subst}\left(\frac{2}{3}\,\text{Subst}\left(\int x\sqrt{a+bc^{3/2}x^3}\,dx, x, x^{3/2}\right), \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= \frac{4}{21}x^3\sqrt{a+b\left(cx^3\right)^{3/2}} + \text{Subst}\left(\frac{1}{7}(2a)\,\text{Subst}\left(\int \frac{x}{\sqrt{a+bc^{3/2}x^3}}\,dx, x, x^{3/2}\right), \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= \frac{4}{21}x^3\sqrt{a+b\left(cx^3\right)^{3/2}} + \text{Subst}\left(\frac{(2a)\,\text{Subst}\left(\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt{c}\,x}{\sqrt{a+bc^{3/2}x^3}}\,dx, x, x^{3/2}\right)}{7\sqrt[3]{b}\,\sqrt{c}}, \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= \frac{4}{21}x^3\sqrt{a+b\left(cx^3\right)^{3/2}} + \frac{4a\sqrt{a+b\left(cx^3\right)^{3/2}}}{7b^{2/3}c\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt{cx^3}\right)} - \frac{2\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{}\right.}{}$$

Mathematica [C] time = 0.05, size = 69, normalized size = 0.11

$$\frac{x^3\sqrt{a+b\left(cx^3\right)^{3/2}}\;{}_2F_1\left(-\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; -\frac{b\left(cx^3\right)^{3/2}}{a}\right)}{3\sqrt{\frac{b\left(cx^3\right)^{3/2}}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b*(c*x^3)^(3/2)],x]
```

```
[Out] (x^3*Sqrt[a + b*(c*x^3)^(3/2)]*Hypergeometric2F1[-1/2, 2/3, 5/3, -((b*(c*x^
3)^(3/2))/a)])/(3*Sqrt[1 + (b*(c*x^3)^(3/2))/a])
```

fricas [F] time = 2.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{\sqrt{cx^3}\,bcx^3+a}\,x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(c*x^3)^(3/2))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sqrt(c*x^3)*b*c*x^3 + a)*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\left(cx^3\right)^{\frac{3}{2}}b+a}\,x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(c*x^3)^(3/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((c*x^3)^(3/2)*b + a)*x^2, x)
```

maple [A] time = 0.01, size = 495, normalized size = 0.77

$$\frac{4\sqrt{a+\left(cx^3\right)^{\frac{3}{2}}b}\,cx^3}{7}-\frac{4i\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(\frac{\left(-ab^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}}{2b}+\sqrt{cx^3}\right)\sqrt{3}\,b}{\left(-ab^2\right)^{\frac{1}{3}}}}\sqrt{\frac{-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{b}+\sqrt{cx^3}}{-\frac{3\left(-ab^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(\frac{\left(-ab^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}}{2b}+\sqrt{cx^3}\right)\sqrt{3}\,b}{\left(-ab^2\right)^{\frac{1}{3}}}}\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+(c*x^3)^(3/2)*b)^(1/2),x)
```

```
[Out] 1/3/c*(4/7*c*x^3*(a+(c*x^3)^(3/2)*b)^(1/2)-4/7*I*a*3^(1/2)*(-a*b^2)^(1/3)/b
*(I*((c*x^3)^(1/2)+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2)*(((c*x^3)^(1/2)-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^
2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*((c*x^3)^(1/2)+1/2*(-
a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2)/(a+(c*x^3)^(3/2)*b)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2
)^(1/3)/b)*EllipticE(1/3*3^(1/2)*(I*((c*x^3)^(1/2)+1/2*(-a*b^2)^(1/3)/b-1/2
*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)*(-a
*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)
)+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(I*((c*x^3)^(1/2)+1/2*(-a*b^2)^(1/
3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(
1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/
b)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\left(cx^3\right)^{\frac{3}{2}}b+a}\,x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(c*x^3)^(3/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((c*x^3)^(3/2)*b + a)*x^2, x)
```

mupad [B] time = 1.48, size = 52, normalized size = 0.08

$$\frac{x^3\sqrt{a+b\left(cx^3\right)^{3/2}}\;{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{2}{3};\;\frac{5}{3};\;-\frac{b\left(cx^3\right)^{3/2}}{a}\right)}{3\sqrt{\frac{b\left(cx^3\right)^{3/2}}{a}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*(c*x^3)^(3/2))^(1/2),x)
```

```
[Out] (x^3*(a + b*(c*x^3)^(3/2))^(1/2)*hypergeom([-1/2, 2/3], 5/3, -(b*(c*x^3)^(3
/2))/a))/(3*((b*(c*x^3)^(3/2))/a + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{a + b\left(cx^3\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*(c*x**3)**(3/2))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a + b*(c*x**3)**(3/2)), x)
```

3.2975 $\int x^9 \sqrt{a + b\left(cx^3\right)^{3/2}}\, dx$

Optimal. Leaf size=170

$$\frac{792a^3x\sqrt{\frac{b\left(cx^3\right)^{3/2}}{a}+1}\ {}_2F_1\left(\frac{2}{9},\frac{1}{2};\frac{11}{9};-\frac{b\left(cx^3\right)^{3/2}}{a}\right)}{19747b^2c^3\sqrt{a+b\left(cx^3\right)^{3/2}}}-\frac{792a^2x\sqrt{a+b\left(cx^3\right)^{3/2}}}{19747b^2c^3}+\frac{36ax\left(cx^3\right)^{3/2}\sqrt{a+b\left(cx^3\right)^{3/2}}}{1519bc^3}+\frac{4}{49}x^{10}\sqrt{a}$$

```
[Out] -792/19747*a^2*x*(a+b*(c*x^3)^(3/2))^(1/2)/b^2/c^3+4/49*x^10*(a+b*(c*x^3)^(
3/2))^(1/2)+36/1519*a*x*(c*x^3)^(3/2)*(a+b*(c*x^3)^(3/2))^(1/2)/b/c^3+792/1
9747*a^3*x*hypergeom([2/9, 1/2],[11/9],-b*(c*x^3)^(3/2)/a)*(1+b*(c*x^3)^(3/
2)/a)^(1/2)/b^2/c^3/(a+b*(c*x^3)^(3/2))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {369, 341, 279, 321, 365, 364}

$$\frac{792a^3x\sqrt{\frac{b\left(cx^3\right)^{3/2}}{a}+1}\ {}_2F_1\left(\frac{2}{9},\frac{1}{2};\frac{11}{9};-\frac{b\left(cx^3\right)^{3/2}}{a}\right)}{19747b^2c^3\sqrt{a+b\left(cx^3\right)^{3/2}}}-\frac{792a^2x\sqrt{a+b\left(cx^3\right)^{3/2}}}{19747b^2c^3}+\frac{36ax\left(cx^3\right)^{3/2}\sqrt{a+b\left(cx^3\right)^{3/2}}}{1519bc^3}+\frac{4}{49}x^{10}\sqrt{a}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*Sqrt[a + b*(c*x^3)^(3/2)],x]
```

```
[Out] (-792*a^2*x*Sqrt[a + b*(c*x^3)^(3/2)])/(19747*b^2*c^3) + (4*x^10*Sqrt[a + b
*(c*x^3)^(3/2)])/49 + (36*a*x*(c*x^3)^(3/2)*Sqrt[a + b*(c*x^3)^(3/2)])/(151
9*b*c^3) + (792*a^3*x*Sqrt[1 + (b*(c*x^3)^(3/2))/a]*Hypergeometric2F1[2/9,
1/2, 11/9, -((b*(c*x^3)^(3/2))/a)])/(19747*b^2*c^3*Sqrt[a + b*(c*x^3)^(3/2)
])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
```

```
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Rubi steps

$$\int x^9\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx = \text{Subst}\left(\int x^9\sqrt{a+bc^{3/2}x^{9/2}}\,dx, \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= \text{Subst}\left(2\,\text{Subst}\left(\int x^{19}\sqrt{a+bc^{3/2}x^9}\,dx, x, \sqrt{x}\right), \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= \frac{4}{49}x^{10}\sqrt{a+b\left(cx^3\right)^{3/2}} + \text{Subst}\left(\frac{1}{49}(18a)\,\text{Subst}\left(\int \frac{x^{19}}{\sqrt{a+bc^{3/2}x^9}}\,dx, x, \sqrt{x}\right), \sqrt{x},\right.$$

$$= \frac{4}{49}x^{10}\sqrt{a+b\left(cx^3\right)^{3/2}} + \frac{36ax\left(cx^3\right)^{3/2}\sqrt{a+b\left(cx^3\right)^{3/2}}}{1519bc^3} - \text{Subst}\left(\frac{\left(396a^2\right)\text{Subst}\left(\int\right.\right.$$

$$= -\frac{792a^2x\sqrt{a+b\left(cx^3\right)^{3/2}}}{19747b^2c^3} + \frac{4}{49}x^{10}\sqrt{a+b\left(cx^3\right)^{3/2}} + \frac{36ax\left(cx^3\right)^{3/2}\sqrt{a+b\left(cx^3\right)^{3/2}}}{1519bc^3}$$

$$= -\frac{792a^2x\sqrt{a+b\left(cx^3\right)^{3/2}}}{19747b^2c^3} + \frac{4}{49}x^{10}\sqrt{a+b\left(cx^3\right)^{3/2}} + \frac{36ax\left(cx^3\right)^{3/2}\sqrt{a+b\left(cx^3\right)^{3/2}}}{1519bc^3}$$

$$= -\frac{792a^2x\sqrt{a+b\left(cx^3\right)^{3/2}}}{19747b^2c^3} + \frac{4}{49}x^{10}\sqrt{a+b\left(cx^3\right)^{3/2}} + \frac{36ax\left(cx^3\right)^{3/2}\sqrt{a+b\left(cx^3\right)^{3/2}}}{1519bc^3}$$

Mathematica [F] time = 0.07, size = 0, normalized size = 0.00

$$\int x^9\sqrt{a+b\left(cx^3\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^9*Sqrt[a + b*(c*x^3)^(3/2)],x]
```

```
[Out] Integrate[x^9*Sqrt[a + b*(c*x^3)^(3/2)], x]
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(a+b*(c*x^3)^(3/2))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\left(cx^3\right)^{\frac{3}{2}} b + a}\, x^9\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(a+b*(c*x^3)^(3/2))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt((c*x^3)^(3/2)*b + a)*x^9, x)
```

maple [**F**] time = 0.23, size = 0, normalized size = 0.00

$$\int \sqrt{a + \left(c\, x^3\right)^{\frac{3}{2}} b}\; x^9\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(a+(c*x^3)^(3/2)*b)^(1/2),x)
[Out] int(x^9*(a+(c*x^3)^(3/2)*b)^(1/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\left(cx^3\right)^{\frac{3}{2}} b + a}\, x^9\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(a+b*(c*x^3)^(3/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt((c*x^3)^(3/2)*b + a)*x^9, x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int x^9 \sqrt{a + b\left(c\, x^3\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(a + b*(c*x^3)^(3/2))^(1/2),x)
[Out] int(x^9*(a + b*(c*x^3)^(3/2))^(1/2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int x^9 \sqrt{a + b\left(cx^3\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(a+b*(c*x**3)**(3/2))**(1/2),x)
[Out] Integral(x**9*sqrt(a + b*(c*x**3)**(3/2)), x)
```

3.2976 $\int \sqrt{a+b\left(cx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=91

$$\frac{9ax\sqrt{\frac{b\left(cx^3\right)^{3/2}}{a}+1}\;{}_2F_1\left(\frac{2}{9},\frac{1}{2};\frac{11}{9};-\frac{b\left(cx^3\right)^{3/2}}{a}\right)}{13\sqrt{a+b\left(cx^3\right)^{3/2}}}+\frac{4}{13}x\sqrt{a+b\left(cx^3\right)^{3/2}}$$

```
[Out] 4/13*x*(a+b*(c*x^3)^(3/2))^(1/2)+9/13*a*x*hypergeom([2/9, 1/2],[11/9],-b*(c
*x^3)^(3/2)/a)*(1+b*(c*x^3)^(3/2)/a)^(1/2)/(a+b*(c*x^3)^(3/2))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {255, 243, 279, 365, 364}

$$\frac{9ax\sqrt{\frac{b\left(cx^3\right)^{3/2}}{a}+1}\;{}_2F_1\left(\frac{2}{9},\frac{1}{2};\frac{11}{9};-\frac{b\left(cx^3\right)^{3/2}}{a}\right)}{13\sqrt{a+b\left(cx^3\right)^{3/2}}}+\frac{4}{13}x\sqrt{a+b\left(cx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*(c*x^3)^(3/2)],x]
```

```
[Out] (4*x*Sqrt[a + b*(c*x^3)^(3/2)])/13 + (9*a*x*Sqrt[1 + (b*(c*x^3)^(3/2))/a]*H
ypergeometric2F1[2/9, 1/2, 11/9, -((b*(c*x^3)^(3/2))/a)])/(13*Sqrt[a + b*(c
*x^3)^(3/2)])
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Free
Q[{a, b, p}, x] && FractionQ[n]
```

Rule 255

```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_.), x_Symbol] :> With[{k = De
nominator[n]}, Subst[Int[(a + b*c^n*x^(n*q))^p, x], x^(1/k), (c*x^q)^(1/k)/
(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c, p, q}, x] && FractionQ[n]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
```

```
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \sqrt{a+b\left(cx^3\right)^{3/2}}\,dx = \text{Subst}\left(\int \sqrt{a+bc^{3/2}x^{9/2}}\,dx, \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= \text{Subst}\left(2\,\text{Subst}\left(\int x\sqrt{a+bc^{3/2}x^9}\,dx, x, \sqrt{x}\right), \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= \frac{4}{13}x\sqrt{a+b\left(cx^3\right)^{3/2}} + \text{Subst}\left(\frac{1}{13}(18a)\,\text{Subst}\left(\int \frac{x}{\sqrt{a+bc^{3/2}x^9}}\,dx, x, \sqrt{x}\right), \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= \frac{4}{13}x\sqrt{a+b\left(cx^3\right)^{3/2}} + \text{Subst}\left(\frac{\left(18a\sqrt{1+\frac{bc^{3/2}x^{9/2}}{a}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{1+\frac{bc^{3/2}x^9}{a}}}\,dx, x, \sqrt{x}\right)}{13\sqrt{a+bc^{3/2}x^{9/2}}}, \sqrt{x}, \right.$$

$$= \frac{4}{13}x\sqrt{a+b\left(cx^3\right)^{3/2}} + \frac{9ax\sqrt{1+\frac{b\left(cx^3\right)^{3/2}}{a}}\;{}_2F_1\left(\frac{2}{9}, \frac{1}{2}; \frac{11}{9}; -\frac{b\left(cx^3\right)^{3/2}}{a}\right)}{13\sqrt{a+b\left(cx^3\right)^{3/2}}}$$

Mathematica [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\left(cx^3\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*(c*x^3)^(3/2)],x]
```

```
[Out] Integrate[Sqrt[a + b*(c*x^3)^(3/2)], x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^3)^(3/2))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\left(cx^3\right)^{\frac{3}{2}}b + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^3)^(3/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((c*x^3)^(3/2)*b + a), x)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \sqrt{a + \left(c\,x^3\right)^{\frac{3}{2}}\,b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(c*x^3)^(3/2)*b)^(1/2),x)
[Out] int((a+(c*x^3)^(3/2)*b)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\left(cx^3\right)^{\frac{3}{2}}\,b + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^3)^(3/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt((c*x^3)^(3/2)*b + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a + b\left(c\,x^3\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x^3)^(3/2))^(1/2),x)
[Out] int((a + b*(c*x^3)^(3/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\left(cx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x**3)**(3/2))**(1/2),x)
[Out] Integral(sqrt(a + b*(c*x**3)**(3/2)), x)
```

3.2977 $\int \frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{x^9}\,dx$

Optimal. Leaf size=139

$$-\frac{45b^2c^3x\sqrt{\frac{b\left(cx^3\right)^{3/2}}{a}+1}\;{}_2F_1\left(\frac{2}{9},\frac{1}{2};\frac{11}{9};-\frac{b\left(cx^3\right)^{3/2}}{a}\right)}{448a\sqrt{a+b\left(cx^3\right)^{3/2}}}-\frac{9bc^3x\sqrt{a+b\left(cx^3\right)^{3/2}}}{112a\left(cx^3\right)^{3/2}}-\frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{8x^8}$$

```
[Out] -1/8*(a+b*(c*x^3)^(3/2))^(1/2)/x^8-9/112*b*c^3*x*(a+b*(c*x^3)^(3/2))^(1/2)/
a/(c*x^3)^(3/2)-45/448*b^2*c^3*x*hypergeom([2/9, 1/2],[11/9],-b*(c*x^3)^(3/
2)/a)*(1+b*(c*x^3)^(3/2)/a)^(1/2)/a/(a+b*(c*x^3)^(3/2))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 141, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {369, 341, 277, 325, 365, 364}

$$-\frac{45b^2c^3x\sqrt{\frac{b\left(cx^3\right)^{3/2}}{a}+1}\;{}_2F_1\left(\frac{2}{9},\frac{1}{2};\frac{11}{9};-\frac{b\left(cx^3\right)^{3/2}}{a}\right)}{448a\sqrt{a+b\left(cx^3\right)^{3/2}}}-\frac{9bc^5x^7\sqrt{a+b\left(cx^3\right)^{3/2}}}{112a\left(cx^3\right)^{7/2}}-\frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*(c*x^3)^(3/2)]/x^9,x]
```

```
[Out] -Sqrt[a + b*(c*x^3)^(3/2)]/(8*x^8) - (9*b*c^5*x^7*Sqrt[a + b*(c*x^3)^(3/2)]
)/(112*a*(c*x^3)^(7/2)) - (45*b^2*c^3*x*Sqrt[1 + (b*(c*x^3)^(3/2))/a]*Hyper
geometric2F1[2/9, 1/2, 11/9, -((b*(c*x^3)^(3/2))/a)])/(448*a*Sqrt[a + b*(c*
x^3)^(3/2)])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Rubi steps

$$\int \frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{x^9}\,dx = \text{Subst}\left(\int \frac{\sqrt{a+bc^{3/2}x^{9/2}}}{x^9}\,dx, \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= \text{Subst}\left(2\,\text{Subst}\left(\int \frac{\sqrt{a+bc^{3/2}x^9}}{x^{17}}\,dx, x, \sqrt{x}\right), \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= -\frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{8x^8} + \text{Subst}\left(\frac{1}{16}\left(9bc^{3/2}\right)\text{Subst}\left(\int \frac{1}{x^8\sqrt{a+bc^{3/2}x^9}}\,dx, x, \sqrt{x}\right), \sqrt{x}, \frac{\sqrt{}}{\sqrt{}}\right.$$

$$= -\frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{8x^8} - \frac{9bc^5x^7\sqrt{a+b\left(cx^3\right)^{3/2}}}{112a\left(cx^3\right)^{7/2}} - \text{Subst}\left(\frac{\left(45b^2c^3\right)\text{Subst}\left(\int \frac{x}{\sqrt{a+bc^{3/2}x^9}}\,dx\right.\right.}{224a}$$

$$= -\frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{8x^8} - \frac{9bc^5x^7\sqrt{a+b\left(cx^3\right)^{3/2}}}{112a\left(cx^3\right)^{7/2}} - \text{Subst}\left(\frac{\left(45b^2c^3\sqrt{1+\frac{bc^{3/2}x^{9/2}}{a}}\right)\text{Subst}\left(\right.\right.}{224a\sqrt{a+bc^3}}$$

$$= -\frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{8x^8} - \frac{9bc^5x^7\sqrt{a+b\left(cx^3\right)^{3/2}}}{112a\left(cx^3\right)^{7/2}} - \frac{45b^2c^3x\sqrt{1+\frac{b\left(cx^3\right)^{3/2}}{a}}\;{}_2F_1\left(\frac{2}{9},\frac{1}{2};\frac{11}{9};-\right.}{448a\sqrt{a+b\left(cx^3\right)^{3/2}}}$$

Mathematica [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\left(cx^3\right)^{3/2}}}{x^9}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*(c*x^3)^(3/2)]/x^9,x]
```

```
[Out] Integrate[Sqrt[a + b*(c*x^3)^(3/2)]/x^9, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^3)^(3/2))^(1/2)/x^9,x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(cx^3\right)^{\frac{3}{2}} b + a}}{x^9}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^3)^(3/2))^(1/2)/x^9,x, algorithm="giac")
[Out] integrate(sqrt((c*x^3)^(3/2)*b + a)/x^9, x)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \left(c\, x^3\right)^{\frac{3}{2}} b}}{x^9}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(c*x^3)^(3/2)*b)^(1/2)/x^9,x)
[Out] int((a+(c*x^3)^(3/2)*b)^(1/2)/x^9,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(cx^3\right)^{\frac{3}{2}} b + a}}{x^9}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^3)^(3/2))^(1/2)/x^9,x, algorithm="maxima")
[Out] integrate(sqrt((c*x^3)^(3/2)*b + a)/x^9, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + b\left(c\, x^3\right)^{3/2}}}{x^9}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x^3)^(3/2))^(1/2)/x^9,x)
[Out] int((a + b*(c*x^3)^(3/2))^(1/2)/x^9, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\left(cx^3\right)^{\frac{3}{2}}}}{x^9}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x**3)**(3/2))**(1/2)/x**9,x)
[Out] Integral(sqrt(a + b*(c*x**3)**(3/2))/x**9, x)
```

3.2978 $\int (dx)^m \sqrt{a + b\,(cx^3)^{3/2}}\,dx$

Optimal. Leaf size=84

$$\frac{x(dx)^m \sqrt{a + b\left(cx^3\right)^{3/2}}\ {}_2F_1\left(-\frac{1}{2}, \frac{2(m+1)}{9}; \frac{2(m+1)}{9} + 1; -\frac{b\left(cx^3\right)^{3/2}}{a}\right)}{(m+1)\sqrt{\frac{b\left(cx^3\right)^{3/2}}{a} + 1}}$$

```
[Out] x*(d*x)^m*hypergeom([-1/2, 2/9+2/9*m],[11/9+2/9*m],-b*(c*x^3)^(3/2)/a)*(a+b
*(c*x^3)^(3/2))^(1/2)/(1+m)/(1+b*(c*x^3)^(3/2)/a)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {369, 343, 341, 365, 364}

$$\frac{x(dx)^m \sqrt{a + b\left(cx^3\right)^{3/2}}\ {}_2F_1\left(-\frac{1}{2}, \frac{2(m+1)}{9}; \frac{2(m+1)}{9} + 1; -\frac{b\left(cx^3\right)^{3/2}}{a}\right)}{(m+1)\sqrt{\frac{b\left(cx^3\right)^{3/2}}{a} + 1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[a + b*(c*x^3)^(3/2)],x]
```

```
[Out] (x*(d*x)^m*Sqrt[a + b*(c*x^3)^(3/2)]*Hypergeometric2F1[-1/2, (2*(1 + m))/9,
 1 + (2*(1 + m))/9, -((b*(c*x^3)^(3/2))/a)])/((1 + m)*Sqrt[1 + (b*(c*x^3)^(
3/2))/a])
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 343

```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(c^In
tPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a + b*x^n)^p, x], x] /;
 FreeQ[{a, b, c, m, p}, x] && FractionQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
```

```
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int (dx)^m \sqrt{a + b\left(cx^3\right)^{3/2}}\, dx &= \text{Subst}\left(\int (dx)^m \sqrt{a + bc^{3/2}x^{9/2}}\, dx, \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right) \\
&= \text{Subst}\left((x^{-m}(dx)^m) \int x^m \sqrt{a + bc^{3/2}x^{9/2}}\, dx, \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right) \\
&= \text{Subst}\left((2x^{-m}(dx)^m)\,\text{Subst}\left(\int x^{-1+2(1+m)} \sqrt{a + bc^{3/2}x^9}\, dx, x, \sqrt{x}\right), \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right) \\
&= \text{Subst}\left(\frac{\left(2x^{-m}(dx)^m \sqrt{a + bc^{3/2}x^{9/2}}\right)\text{Subst}\left(\int x^{-1+2(1+m)} \sqrt{1 + \frac{bc^{3/2}x^9}{a}}\, dx, x, \sqrt{x}\right)}{\sqrt{1 + \frac{bc^{3/2}x^{9/2}}{a}}}, \sqrt{x}\right. \\
&= \frac{x(dx)^m \sqrt{a + b\left(cx^3\right)^{3/2}}\; {}_2F_1\left(-\frac{1}{2}, \frac{2(1+m)}{9}; 1 + \frac{2(1+m)}{9}; -\frac{b\left(cx^3\right)^{3/2}}{a}\right)}{(1+m)\sqrt{1 + \frac{b\left(cx^3\right)^{3/2}}{a}}}
\end{aligned}$$

Mathematica [F] time = 0.09, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{a + b\left(cx^3\right)^{3/2}}\, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*Sqrt[a + b*(c*x^3)^(3/2)],x]
```

```
[Out] Integrate[(d*x)^m*Sqrt[a + b*(c*x^3)^(3/2)], x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*(c*x^3)^(3/2))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   alglo
gextint: unimplemented
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\left(cx^3\right)^{\frac{3}{2}} b + a}\; (dx)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*(c*x^3)^(3/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((c*x^3)^(3/2)*b + a)*(d*x)^m, x)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \sqrt{a + \left(c\,x^3\right)^{\frac{3}{2}} b}\; (dx)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+(c*x^3)^(3/2)*b)^(1/2),x)
[Out] int((d*x)^m*(a+(c*x^3)^(3/2)*b)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\left(cx^3\right)^{\frac{3}{2}} b + a}\ (dx)^m\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*(c*x^3)^(3/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt((c*x^3)^(3/2)*b + a)*(d*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m \sqrt{a + b\left(c\,x^3\right)^{3/2}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*(c*x^3)^(3/2))^(1/2),x)
[Out] int((d*x)^m*(a + b*(c*x^3)^(3/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{a + b\left(cx^3\right)^{\frac{3}{2}}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*(c*x**3)**(3/2))**(1/2),x)
[Out] Integral((d*x)**m*sqrt(a + b*(c*x**3)**(3/2)), x)
```

3.2979 $\int (dx)^m \sqrt{a + b\sqrt{cx^3}}\, dx$

Optimal. Leaf size=84

$$\frac{x(dx)^m \sqrt{a + b\sqrt{cx^3}}\ {}_2F_1\left(-\frac{1}{2}, \frac{2(m+1)}{3}; \frac{1}{3}(2m+5); -\frac{b\sqrt{cx^3}}{a}\right)}{(m+1)\sqrt{\frac{b\sqrt{cx^3}}{a}+1}}$$

```
[Out] x*(d*x)^m*hypergeom([-1/2, 2/3+2/3*m],[5/3+2/3*m],-b*(c*x^3)^(1/2)/a)*(a+b*
(c*x^3)^(1/2))^(1/2)/(1+m)/(1+b*(c*x^3)^(1/2)/a)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {369, 343, 341, 365, 364}

$$\frac{x(dx)^m \sqrt{a + b\sqrt{cx^3}}\ {}_2F_1\left(-\frac{1}{2}, \frac{2(m+1)}{3}; \frac{1}{3}(2m+5); -\frac{b\sqrt{cx^3}}{a}\right)}{(m+1)\sqrt{\frac{b\sqrt{cx^3}}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[a + b*Sqrt[c*x^3]],x]
```

```
[Out] (x*(d*x)^m*Sqrt[a + b*Sqrt[c*x^3]]*Hypergeometric2F1[-1/2, (2*(1 + m))/3, (
5 + 2*m)/3, -((b*Sqrt[c*x^3])/a)])/((1 + m)*Sqrt[1 + (b*Sqrt[c*x^3])/a])
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 343

```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(c^In
tPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a + b*x^n)^p, x], x] /;
 FreeQ[{a, b, c, m, p}, x] && FractionQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
```

```
d, m, p, q}, x] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int (dx)^m \sqrt{a+b\sqrt{cx^3}}\, dx &= \text{Subst}\left(\int (dx)^m \sqrt{a+b\sqrt{c}\,x^{3/2}}\, dx, \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right) \\
&= \text{Subst}\left((x^{-m}(dx)^m)\int x^m \sqrt{a+b\sqrt{c}\,x^{3/2}}\, dx, \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right) \\
&= \text{Subst}\left((2x^{-m}(dx)^m)\,\text{Subst}\left(\int x^{-1+2(1+m)}\sqrt{a+b\sqrt{c}\,x^3}\, dx, x, \sqrt{x}\right), \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right) \\
&= \text{Subst}\left(\frac{\left(2x^{-m}(dx)^m\sqrt{a+b\sqrt{c}\,x^{3/2}}\right)\text{Subst}\left(\int x^{-1+2(1+m)}\sqrt{1+\frac{b\sqrt{c}\,x^3}{a}}\, dx, x, \sqrt{x}\right)}{\sqrt{1+\frac{b\sqrt{c}\,x^{3/2}}{a}}}, \sqrt{}\right. \\
&= \frac{x(dx)^m\sqrt{a+b\sqrt{cx^3}}\;{}_2F_1\left(-\frac{1}{2}, \frac{2(1+m)}{3}; \frac{1}{3}(5+2m); -\frac{b\sqrt{cx^3}}{a}\right)}{(1+m)\sqrt{1+\frac{b\sqrt{cx^3}}{a}}}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 84, normalized size = 1.00

$$\frac{x(dx)^m\sqrt{a+b\sqrt{cx^3}}\;{}_2F_1\left(-\frac{1}{2}, \frac{2(m+1)}{3}; \frac{1}{3}(2m+5); -\frac{b\sqrt{cx^3}}{a}\right)}{(m+1)\sqrt{\frac{b\sqrt{cx^3}}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*Sqrt[a + b*Sqrt[c*x^3]],x]
```

```
[Out] (x*(d*x)^m*Sqrt[a + b*Sqrt[c*x^3]]*Hypergeometric2F1[-1/2, (2*(1 + m))/3, (
5 + 2*m)/3, -((b*Sqrt[c*x^3])/a)])/((1 + m)*Sqrt[1 + (b*Sqrt[c*x^3])/a])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*(c*x^3)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   alglo
gextint: unimplemented
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{cx^3}\,b+a}\;(dx)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*(c*x^3)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sqrt(c*x^3)*b + a)*(d*x)^m, x)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \sqrt{a + \sqrt{c\,x^3}\ b}\ (dx)^m\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+(c*x^3)^(1/2)*b)^(1/2),x)
[Out] int((d*x)^m*(a+(c*x^3)^(1/2)*b)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{cx^3}\,b + a}\ (dx)^m\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*(c*x^3)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(sqrt(c*x^3)*b + a)*(d*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m \sqrt{a + b\,\sqrt{c\,x^3}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*(c*x^3)^(1/2))^(1/2),x)
[Out] int((d*x)^m*(a + b*(c*x^3)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{a + b\sqrt{cx^3}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*(c*x**3)**(1/2))**(1/2),x)
[Out] Integral((d*x)**m*sqrt(a + b*sqrt(c*x**3)), x)
```

3.2980 $\int (dx)^m \sqrt{a+\frac{b}{\sqrt{cx^3}}}\,dx$

Optimal. Leaf size=84

$$\frac{x(dx)^m\sqrt{a+\frac{b}{\sqrt{cx^3}}}\;{}_2F_1\left(-\frac{1}{2},-\frac{2}{3}(m+1);\frac{1}{3}(1-2m);-\frac{b}{a\sqrt{cx^3}}\right)}{(m+1)\sqrt{\frac{b}{a\sqrt{cx^3}}+1}}$$

```
[Out] x*(d*x)^m*hypergeom([-1/2, -2/3-2/3*m],[1/3-2/3*m],-b/a/(c*x^3)^(1/2))*(a+b
/(c*x^3)^(1/2))^(1/2)/(1+m)/(1+b/a/(c*x^3)^(1/2))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {369, 343, 341, 339, 365, 364}

$$\frac{x(dx)^m\sqrt{a+\frac{b}{\sqrt{cx^3}}}\;{}_2F_1\left(-\frac{1}{2},-\frac{2}{3}(m+1);\frac{1}{3}(1-2m);-\frac{b}{a\sqrt{cx^3}}\right)}{(m+1)\sqrt{\frac{b}{a\sqrt{cx^3}}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[a + b/Sqrt[c*x^3]],x]
```

```
[Out] (x*(d*x)^m*Sqrt[a + b/Sqrt[c*x^3]]*Hypergeometric2F1[-1/2, (-2*(1 + m))/3,
(1 - 2*m)/3, -(b/(a*Sqrt[c*x^3]))])/((1 + m)*Sqrt[1 + b/(a*Sqrt[c*x^3])])
```

Rule 339

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Dist[((c
*x)^(m + 1)*(1/x)^(m + 1))/c, Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x
], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 343

```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(c^In
tPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a + b*x^n)^p, x], x] /;
 FreeQ[{a, b, c, m, p}, x] && FractionQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
```

```
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int (dx)^m \sqrt{a+\frac{b}{\sqrt{cx^3}}}\,dx &= \text{Subst}\left(\int \sqrt{a+\frac{b}{\sqrt{c}\,x^{3/2}}}\,(dx)^m\,dx, \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)\\
&= \text{Subst}\left((x^{-m}(dx)^m)\int \sqrt{a+\frac{b}{\sqrt{c}\,x^{3/2}}}\,x^m\,dx, \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)\\
&= \text{Subst}\left((2x^{-m}(dx)^m)\,\text{Subst}\left(\int \sqrt{a+\frac{b}{\sqrt{c}\,x^3}}\,x^{-1+2(1+m)}\,dx, x, \sqrt{x}\right), \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)\\
&= -\text{Subst}\left((2x^{-m}(dx)^m)\,\text{Subst}\left(\int x^{-1-2(1+m)}\sqrt{a+\frac{bx^3}{\sqrt{c}}}\,dx, x, \frac{1}{\sqrt{x}}\right), \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)\\
&= -\text{Subst}\left(\frac{\left(2\sqrt{a+\frac{b}{\sqrt{c}\,x^{3/2}}}\,x^{-m}(dx)^m\right)\text{Subst}\left(\int x^{-1-2(1+m)}\sqrt{1+\frac{bx^3}{a\sqrt{c}}}\,dx, x, \frac{1}{\sqrt{x}}\right)}{\sqrt{1+\frac{b}{a\sqrt{c}\,x^{3/2}}}}, \sqrt{x}, \frac{\sqrt{c}}{\sqrt{c}}\right.\\
&= \frac{x(dx)^m\sqrt{a+\frac{b}{\sqrt{cx^3}}}\;{}_2F_1\left(-\frac{1}{2}, -\frac{2}{3}(1+m); \frac{1}{3}(1-2m); -\frac{b}{a\sqrt{cx^3}}\right)}{(1+m)\sqrt{1+\frac{b}{a\sqrt{cx^3}}}}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 89, normalized size = 1.06

$$\frac{4x(dx)^m\sqrt{a+\frac{b}{\sqrt{cx^3}}}\;{}_2F_1\left(-\frac{1}{2}, \frac{2m}{3}+\frac{1}{6}; \frac{2m}{3}+\frac{7}{6}; -\frac{a\sqrt{cx^3}}{b}\right)}{(4m+1)\sqrt{\frac{a\sqrt{cx^3}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*Sqrt[a + b/Sqrt[c*x^3]],x]
```

```
[Out] (4*x*(d*x)^m*Sqrt[a + b/Sqrt[c*x^3]]*Hypergeometric2F1[-1/2, 1/6 + (2*m)/3,
 7/6 + (2*m)/3, -((a*Sqrt[c*x^3])/b)])/((1 + 4*m)*Sqrt[1 + (a*Sqrt[c*x^3])/
b])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b/(c*x^3)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   alglo
gextint: unimplemented
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{a + \frac{b}{\sqrt{cx^3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b/(c*x^3)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate((d*x)^m*sqrt(a + b/sqrt(c*x^3)), x)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \sqrt{a + \frac{b}{\sqrt{c\,x^3}}}\, (dx)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b/(c*x^3)^(1/2))^(1/2),x)
[Out] int((d*x)^m*(a+b/(c*x^3)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{a + \frac{b}{\sqrt{cx^3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b/(c*x^3)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate((d*x)^m*sqrt(a + b/sqrt(c*x^3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m \sqrt{a + \frac{b}{\sqrt{c\,x^3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b/(c*x^3)^(1/2))^(1/2),x)
[Out] int((d*x)^m*(a + b/(c*x^3)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{a + \frac{b}{\sqrt{cx^3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b/(c*x**3)**(1/2))**(1/2),x)
[Out] Integral((d*x)**m*sqrt(a + b/sqrt(c*x**3)), x)
```

3.2981 $\int (dx)^m \sqrt{a + \frac{b}{\left(cx^3\right)^{3/2}}}\, dx$

Optimal. Leaf size=102

$$\frac{x(dx)^m \sqrt{a + \frac{bc^3x^9}{\left(cx^3\right)^{9/2}}}\ {}_2F_1\left(-\frac{1}{2}, -\frac{2}{9}(m+1); \frac{1}{9}(7-2m); -\frac{bc^3x^9}{a\left(cx^3\right)^{9/2}}\right)}{(m+1)\sqrt{\frac{bc^3x^9}{a\left(cx^3\right)^{9/2}} + 1}}$$

```
[Out] x*(d*x)^m*hypergeom([-1/2, -2/9-2/9*m],[7/9-2/9*m],-b*c^3*x^9/a/(c*x^3)^(9/
2))*(a+b*c^3*x^9/(c*x^3)^(9/2))^(1/2)/(1+m)/(1+b*c^3*x^9/a/(c*x^3)^(9/2))^(
1/2)
```

Rubi [A] time = 0.12, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {369, 343, 341, 339, 365, 364}

$$\frac{x(dx)^m \sqrt{a + \frac{bc^3x^9}{\left(cx^3\right)^{9/2}}}\ {}_2F_1\left(-\frac{1}{2}, -\frac{2}{9}(m+1); \frac{1}{9}(7-2m); -\frac{bc^3x^9}{a\left(cx^3\right)^{9/2}}\right)}{(m+1)\sqrt{\frac{bc^3x^9}{a\left(cx^3\right)^{9/2}} + 1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[a + b/(c*x^3)^(3/2)],x]
```

```
[Out] (x*(d*x)^m*Sqrt[a + (b*c^3*x^9)/(c*x^3)^(9/2)]*Hypergeometric2F1[-1/2, (-2*
(1 + m))/9, (7 - 2*m)/9, -((b*c^3*x^9)/(a*(c*x^3)^(9/2)))])/((1 + m)*Sqrt[1
 + (b*c^3*x^9)/(a*(c*x^3)^(9/2))])
```

Rule 339

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Dist[((c
*x)^(m + 1)*(1/x)^(m + 1))/c, Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x
], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 343

```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(c^In
tPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a + b*x^n)^p, x], x] /;
 FreeQ[{a, b, c, m, p}, x] && FractionQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Rubi steps

$$\int (dx)^m \sqrt{a+\frac{b}{\left(cx^3\right)^{3/2}}}\,dx = \text{Subst}\left(\int \sqrt{a+\frac{b}{c^{3/2}x^{9/2}}}\,(dx)^m\,dx, \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= \text{Subst}\left((x^{-m}(dx)^m)\int \sqrt{a+\frac{b}{c^{3/2}x^{9/2}}}\,x^m\,dx, \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= \text{Subst}\left((2x^{-m}(dx)^m)\,\text{Subst}\left(\int \sqrt{a+\frac{b}{c^{3/2}x^9}}\,x^{-1+2(1+m)}\,dx, x, \sqrt{x}\right), \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= -\text{Subst}\left((2x^{-m}(dx)^m)\,\text{Subst}\left(\int x^{-1-2(1+m)}\sqrt{a+\frac{bx^9}{c^{3/2}}}\,dx, x, \frac{1}{\sqrt{x}}\right), \sqrt{x}, \frac{\sqrt{cx^3}}{\sqrt{c}\,x}\right)$$

$$= -\text{Subst}\left(\frac{\left(2\sqrt{a+\frac{b}{c^{3/2}x^{9/2}}}\,x^{-m}(dx)^m\right)\text{Subst}\left(\int x^{-1-2(1+m)}\sqrt{1+\frac{bx^9}{ac^{3/2}}}\,dx, x, \frac{1}{\sqrt{x}}\right)}{\sqrt{1+\frac{b}{ac^{3/2}x^{9/2}}}}, \sqrt{x}\right.$$

$$= \frac{x(dx)^m\sqrt{a+\frac{bc^3x^9}{\left(cx^3\right)^{9/2}}}\;{}_2F_1\left(-\frac{1}{2}, -\frac{2}{9}(1+m); \frac{1}{9}(7-2m); -\frac{bc^3x^9}{a\left(cx^3\right)^{9/2}}\right)}{(1+m)\sqrt{1+\frac{bc^3x^9}{a\left(cx^3\right)^{9/2}}}}$$

Mathematica [F] time = 0.19, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{a+\frac{b}{\left(cx^3\right)^{3/2}}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m*Sqrt[a + b/(c*x^3)^(3/2)],x]
```

```
[Out] Integrate[(d*x)^m*Sqrt[a + b/(c*x^3)^(3/2)], x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b/(c*x^3)^(3/2))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   alglo
gextint: unimplemented
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{a + \frac{b}{\left(cx^3\right)^{\frac{3}{2}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b/(c*x^3)^(3/2))^(1/2),x, algorithm="giac")
[Out] integrate((d*x)^m*sqrt(a + b/(c*x^3)^(3/2)), x)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \sqrt{a + \frac{b}{\left(c\,x^3\right)^{\frac{3}{2}}}}\, (dx)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b/(c*x^3)^(3/2))^(1/2),x)
[Out] int((d*x)^m*(a+b/(c*x^3)^(3/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{a + \frac{b}{\left(cx^3\right)^{\frac{3}{2}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b/(c*x^3)^(3/2))^(1/2),x, algorithm="maxima")
[Out] integrate((d*x)^m*sqrt(a + b/(c*x^3)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m \sqrt{a + \frac{b}{\left(c\,x^3\right)^{3/2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b/(c*x^3)^(3/2))^(1/2),x)
[Out] int((d*x)^m*(a + b/(c*x^3)^(3/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{a + \frac{b}{\left(cx^3\right)^{\frac{3}{2}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b/(c*x**3)**(3/2))**(1/2),x)
[Out] Integral((d*x)**m*sqrt(a + b/(c*x**3)**(3/2)), x)
```

3.2982 $\int \sqrt{a+b\sqrt{\frac{c}{x}}}\, x\, dx$

Optimal. Leaf size=169

$$-\frac{5b^4c^2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{\sqrt{a}}\right)}{32a^{7/2}}+\frac{5b^3c^2\sqrt{a+b\sqrt{\frac{c}{x}}}}{32a^3\sqrt{\frac{c}{x}}}-\frac{5b^2cx\sqrt{a+b\sqrt{\frac{c}{x}}}}{48a^2}+\frac{bc^2\sqrt{a+b\sqrt{\frac{c}{x}}}}{12a\left(\frac{c}{x}\right)^{3/2}}+\frac{1}{2}x^2\sqrt{a+b\sqrt{\frac{c}{x}}}$$

```
[Out] -5/32*b^4*c^2*arctanh((a+b*(c/x)^(1/2))^(1/2)/a^(1/2))/a^(7/2)+1/12*b*c^2*(
a+b*(c/x)^(1/2))^(1/2)/a/(c/x)^(3/2)-5/48*b^2*c*x*(a+b*(c/x)^(1/2))^(1/2)/a
^2+1/2*x^2*(a+b*(c/x)^(1/2))^(1/2)+5/32*b^3*c^2*(a+b*(c/x)^(1/2))^(1/2)/a^3
/(c/x)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 172, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {369, 266, 47, 51, 63, 208}

$$\frac{5b^3c^2\sqrt{a+b\sqrt{\frac{c}{x}}}}{32a^3\sqrt{\frac{c}{x}}}-\frac{5b^4c^2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{\sqrt{a}}\right)}{32a^{7/2}}-\frac{5b^2cx\sqrt{a+b\sqrt{\frac{c}{x}}}}{48a^2}+\frac{bx^3\left(\frac{c}{x}\right)^{3/2}\sqrt{a+b\sqrt{\frac{c}{x}}}}{12ac}+\frac{1}{2}x^2\sqrt{a+b\sqrt{\frac{c}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[c/x]]*x,x]
```

```
[Out] (5*b^3*c^2*Sqrt[a + b*Sqrt[c/x]])/(32*a^3*Sqrt[c/x]) - (5*b^2*c*Sqrt[a + b*
Sqrt[c/x]]*x)/(48*a^2) + (Sqrt[a + b*Sqrt[c/x]]*x^2)/2 + (b*Sqrt[a + b*Sqrt
[c/x]]*(c/x)^(3/2)*x^3)/(12*a*c) - (5*b^4*c^2*ArcTanh[Sqrt[a + b*Sqrt[c/x]]
/Sqrt[a]])/(32*a^(7/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Rubi steps

$$\int \sqrt{a+b\sqrt{\frac{c}{x}}}\,x\,dx = \text{Subst}\left(\int \sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}\,x\,dx, \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= -\text{Subst}\left(2\,\text{Subst}\left(\int \frac{\sqrt{a+b\sqrt{c}\,x}}{x^5}\,dx, x, \frac{1}{\sqrt{x}}\right), \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= \frac{1}{2}\sqrt{a+b\sqrt{\frac{c}{x}}}\,x^2 - \text{Subst}\left(\frac{1}{4}\left(b\sqrt{c}\right)\text{Subst}\left(\int \frac{1}{x^4\sqrt{a+b\sqrt{c}\,x}}\,dx, x, \frac{1}{\sqrt{x}}\right), \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= \frac{1}{2}\sqrt{a+b\sqrt{\frac{c}{x}}}\,x^2 + \frac{b\sqrt{a+b\sqrt{\frac{c}{x}}}\,\left(\frac{c}{x}\right)^{3/2}x^3}{12ac} + \text{Subst}\left(\frac{\left(5b^2c\right)\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+b\sqrt{c}\,x}}\,dx, x, \right.}{24a}\right.$$

$$= -\frac{5b^2c\sqrt{a+b\sqrt{\frac{c}{x}}}\,x}{48a^2} + \frac{1}{2}\sqrt{a+b\sqrt{\frac{c}{x}}}\,x^2 + \frac{b\sqrt{a+b\sqrt{\frac{c}{x}}}\,\left(\frac{c}{x}\right)^{3/2}x^3}{12ac} - \text{Subst}\left(\frac{\left(5b^3c^{3/2}\right)\text{Su}\right.}{}$$

$$= \frac{5b^3c^2\sqrt{a+b\sqrt{\frac{c}{x}}}}{32a^3\sqrt{\frac{c}{x}}} - \frac{5b^2c\sqrt{a+b\sqrt{\frac{c}{x}}}\,x}{48a^2} + \frac{1}{2}\sqrt{a+b\sqrt{\frac{c}{x}}}\,x^2 + \frac{b\sqrt{a+b\sqrt{\frac{c}{x}}}\,\left(\frac{c}{x}\right)^{3/2}x^3}{12ac} + \text{S}$$

$$= \frac{5b^3c^2\sqrt{a+b\sqrt{\frac{c}{x}}}}{32a^3\sqrt{\frac{c}{x}}} - \frac{5b^2c\sqrt{a+b\sqrt{\frac{c}{x}}}\,x}{48a^2} + \frac{1}{2}\sqrt{a+b\sqrt{\frac{c}{x}}}\,x^2 + \frac{b\sqrt{a+b\sqrt{\frac{c}{x}}}\,\left(\frac{c}{x}\right)^{3/2}x^3}{12ac} + \text{S}$$

$$= \frac{5b^3c^2\sqrt{a+b\sqrt{\frac{c}{x}}}}{32a^3\sqrt{\frac{c}{x}}} - \frac{5b^2c\sqrt{a+b\sqrt{\frac{c}{x}}}\,x}{48a^2} + \frac{1}{2}\sqrt{a+b\sqrt{\frac{c}{x}}}\,x^2 + \frac{b\sqrt{a+b\sqrt{\frac{c}{x}}}\,\left(\frac{c}{x}\right)^{3/2}x^3}{12ac} - -$$

Mathematica [A] time = 0.16, size = 111, normalized size = 0.66

$$\frac{\sqrt{a}\,x\sqrt{a+b\sqrt{\frac{c}{x}}}\,\left(48a^3x+8a^2bx\sqrt{\frac{c}{x}}-10ab^2c+15b^3c\sqrt{\frac{c}{x}}\right)-15b^4c^2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{\sqrt{a}}\right)}{96a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[c/x]]*x,x]
```

```
[Out] (Sqrt[a]*Sqrt[a + b*Sqrt[c/x]]*x*(-10*a*b^2*c + 15*b^3*c*Sqrt[c/x] + 48*a^3
*x + 8*a^2*b*Sqrt[c/x]*x) - 15*b^4*c^2*ArcTanh[Sqrt[a + b*Sqrt[c/x]]/Sqrt[a
]])/(96*a^(7/2))
```

fricas [**A**] time = 1.30, size = 224, normalized size = 1.33

$$\left[\frac{15\sqrt{a}\,b^4c^2\log\left(-2\sqrt{b\sqrt{\frac{c}{x}}+a}\,\sqrt{a}\,x\sqrt{\frac{c}{x}}+2\,ax\sqrt{\frac{c}{x}}+bc\right)-2\left(10\,a^2b^2cx-48\,a^4x^2-\left(15\,ab^3cx+8\,a^3bx^2\right)\sqrt{\frac{c}{x}}\right)\sqrt{}\right.$$
$$\left.192\,a^4\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(c/x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/192*(15*sqrt(a)*b^4*c^2*log(-2*sqrt(b*sqrt(c/x) + a)*sqrt(a)*x*sqrt(c/x)
 + 2*a*x*sqrt(c/x) + b*c) - 2*(10*a^2*b^2*c*x - 48*a^4*x^2 - (15*a*b^3*c*x
+ 8*a^3*b*x^2)*sqrt(c/x))*sqrt(b*sqrt(c/x) + a))/a^4, 1/96*(15*sqrt(-a)*b^4
*c^2*arctan(sqrt(b*sqrt(c/x) + a)*sqrt(-a)/a) - (10*a^2*b^2*c*x - 48*a^4*x^
2 - (15*a*b^3*c*x + 8*a^3*b*x^2)*sqrt(c/x))*sqrt(b*sqrt(c/x) + a))/a^4]
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(c/x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Ba
d Argument ValueWarning, choosing root of [1,0,0,0,%%%{-1,[1]%%%}] at param
eters values [-33]Warning, choosing root of [1,0,0,0,%%%{-1,[1]%%%}] at par
ameters values [-70]Warning, choosing root of [1,0,0,0,%%%{-1,[1]%%%}] at p
arameters values [8]Warning, choosing root of [1,0,0,0,%%%{-1,[1]%%%}] at p
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maple [A] time = 0.03, size = 211, normalized size = 1.25

$$\frac{\sqrt{a+\sqrt{\frac{c}{x}}\,b}\left(-15ab^4c^2\ln\left(\frac{2a\sqrt{x}+\sqrt{\frac{c}{x}}\,b\sqrt{x}+2\sqrt{ax+\sqrt{\frac{c}{x}}\,bx}\,\sqrt{a}}{2\sqrt{a}}\right)+60\sqrt{ax+\sqrt{\frac{c}{x}}\,bx}\,a^{\frac{5}{2}}b^2c\sqrt{x}+30\sqrt{ax+\sqrt{\frac{c}{x}}\,bx}\,\left(\frac{c}{x}\right)\right.}{192\sqrt{\left(a+\sqrt{\frac{c}{x}}\,b\right)x}\,a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*(c/x)^(1/2))^(1/2),x)
```

```
[Out] 1/192*(a+b*(c/x)^(1/2))^(1/2)*x^(1/2)*(30*a^(3/2)*(a*x+b*(c/x)^(1/2)*x)^(1/
2)*(c/x)^(3/2)*x^(3/2)*b^3-15*c^2*ln(1/2*(b*(c/x)^(1/2)*x^(1/2)+2*(a*x+b*(c
/x)^(1/2)*x)^(1/2)*a^(1/2)+2*a*x^(1/2))/a^(1/2))*a*b^4+60*c*a^(5/2)*(a*x+b*
(c/x)^(1/2)*x)^(1/2)*x^(1/2)*b^2-80*a^(5/2)*(a*x+b*(c/x)^(1/2)*x)^(3/2)*(c/
x)^(1/2)*x^(1/2)*b+96*x^(1/2)*(a*x+b*(c/x)^(1/2)*x)^(3/2)*a^(7/2))/(x*(a+b*
(c/x)^(1/2)))^(1/2)/a^(9/2)
```

maxima [A] time = 1.35, size = 211, normalized size = 1.25

$$\frac{1}{192}\left(\frac{15b^4\log\left(\frac{\sqrt{b\sqrt{\frac{c}{x}}+a}-\sqrt{a}}{\sqrt{b\sqrt{\frac{c}{x}}+a}+\sqrt{a}}\right)}{a^{\frac{7}{2}}}+\frac{2\left(15\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{7}{2}}b^4-55\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{5}{2}}ab^4+73\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{3}{2}}a^2b^4+15\sqrt{b\sqrt{\frac{c}{x}}+}\right.}{\left(b\sqrt{\frac{c}{x}}+a\right)^4a^3-4\left(b\sqrt{\frac{c}{x}}+a\right)^3a^4+6\left(b\sqrt{\frac{c}{x}}+a\right)^2a^5-4\left(b\sqrt{\frac{c}{x}}+a\right)a^6+a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(c/x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/192*(15*b^4*log((sqrt(b*sqrt(c/x) + a) - sqrt(a))/(sqrt(b*sqrt(c/x) + a)
+ sqrt(a)))/a^(7/2) + 2*(15*(b*sqrt(c/x) + a)^(7/2)*b^4 - 55*(b*sqrt(c/x) +
 a)^(5/2)*a*b^4 + 73*(b*sqrt(c/x) + a)^(3/2)*a^2*b^4 + 15*sqrt(b*sqrt(c/x)
+ a)*a^3*b^4)/((b*sqrt(c/x) + a)^4*a^3 - 4*(b*sqrt(c/x) + a)^3*a^4 + 6*(b*s
qrt(c/x) + a)^2*a^5 - 4*(b*sqrt(c/x) + a)*a^6 + a^7))*c^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\sqrt{a+b\sqrt{\frac{c}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*(c/x)^(1/2))^(1/2),x)
```

```
[Out] int(x*(a + b*(c/x)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{a+b\sqrt{\frac{c}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(c/x)**(1/2))**(1/2),x)
[Out] Integral(x*sqrt(a + b*sqrt(c/x)), x)
```

3.2983 $\int \sqrt{a+b\sqrt{\frac{c}{x}}}\,dx$

Optimal. Leaf size=92

$$-\frac{b^2 c \tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{bc\sqrt{a+b\sqrt{\frac{c}{x}}}}{2a\sqrt{\frac{c}{x}}}+x\sqrt{a+b\sqrt{\frac{c}{x}}}$$

```
[Out] -1/2*b^2*c*arctanh((a+b*(c/x)^(1/2))^(1/2)/a^(1/2))/a^(3/2)+x*(a+b*(c/x)^(1
/2))^(1/2)+1/2*b*c*(a+b*(c/x)^(1/2))^(1/2)/a/(c/x)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {255, 190, 47, 51, 63, 208}

$$-\frac{b^2 c \tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{bc\sqrt{a+b\sqrt{\frac{c}{x}}}}{2a\sqrt{\frac{c}{x}}}+x\sqrt{a+b\sqrt{\frac{c}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[c/x]],x]
```

```
[Out] (b*c*Sqrt[a + b*Sqrt[c/x]])/(2*a*Sqrt[c/x]) + Sqrt[a + b*Sqrt[c/x]]*x - (b^
2*c*ArcTanh[Sqrt[a + b*Sqrt[c/x]]/Sqrt[a]])/(2*a^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 255

```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_.), x_Symbol] :> With[{k = De
nominator[n]}, Subst[Int[(a + b*c^n*x^(n*q))^p, x], x^(1/k), (c*x^q)^(1/k)/
(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c, p, q}, x] && FractionQ[n]
```

Rubi steps

$$\int \sqrt{a+b\sqrt{\frac{c}{x}}}\,dx = \text{Subst}\left(\int \sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}\,dx, \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= -\text{Subst}\left(2\,\text{Subst}\left(\int \frac{\sqrt{a+b\sqrt{c}\,x}}{x^3}\,dx, x, \frac{1}{\sqrt{x}}\right), \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= \sqrt{a+b\sqrt{\frac{c}{x}}}\,x - \text{Subst}\left(\frac{1}{2}\left(b\sqrt{c}\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+b\sqrt{c}\,x}}\,dx, x, \frac{1}{\sqrt{x}}\right), \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= \frac{bc\sqrt{a+b\sqrt{\frac{c}{x}}}}{2a\sqrt{\frac{c}{x}}} + \sqrt{a+b\sqrt{\frac{c}{x}}}\,x + \text{Subst}\left(\frac{\left(b^2c\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+b\sqrt{c}\,x}}\,dx, x, \frac{1}{\sqrt{x}}\right)}{4a}, \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right.$$

$$= \frac{bc\sqrt{a+b\sqrt{\frac{c}{x}}}}{2a\sqrt{\frac{c}{x}}} + \sqrt{a+b\sqrt{\frac{c}{x}}}\,x + \text{Subst}\left(\frac{\left(b\sqrt{c}\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b\sqrt{c}}+\frac{x^2}{b\sqrt{c}}}\,dx, x, \sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}\right)}{2a}, \right.$$

$$= \frac{bc\sqrt{a+b\sqrt{\frac{c}{x}}}}{2a\sqrt{\frac{c}{x}}} + \sqrt{a+b\sqrt{\frac{c}{x}}}\,x - \frac{b^2c\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{\sqrt{a}}\right)}{2a^{3/2}}$$

Mathematica [A] time = 0.07, size = 79, normalized size = 0.86

$$\frac{\sqrt{a}\,x\sqrt{a+b\sqrt{\frac{c}{x}}}\left(2a+b\sqrt{\frac{c}{x}}\right) - b^2c\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{\sqrt{a}}\right)}{2a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[c/x]],x]
```

```
[Out] (Sqrt[a]*Sqrt[a + b*Sqrt[c/x]]*(2*a + b*Sqrt[c/x])*x - b^2*c*ArcTanh[Sqrt[a
 + b*Sqrt[c/x]]/Sqrt[a]])/(2*a^(3/2))
```

fricas [A] time = 1.36, size = 161, normalized size = 1.75

$$\left[\frac{\sqrt{a}\,b^2c\log\left(-2\sqrt{b\sqrt{\frac{c}{x}}+a}\,\sqrt{a}\,x\sqrt{\frac{c}{x}}+2\,ax\sqrt{\frac{c}{x}}+bc\right)+2\left(abx\sqrt{\frac{c}{x}}+2\,a^2x\right)\sqrt{b\sqrt{\frac{c}{x}}+a}}{4\,a^2},\ \frac{\sqrt{-a}\,b^2c\arctan\left(\frac{\sqrt{b\sqrt{\frac{c}{x}}}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c/x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a)*b^2*c*log(-2*sqrt(b*sqrt(c/x) + a)*sqrt(a)*x*sqrt(c/x) + 2*a*
x*sqrt(c/x) + b*c) + 2*(a*b*x*sqrt(c/x) + 2*a^2*x)*sqrt(b*sqrt(c/x) + a))/a
^2, 1/2*(sqrt(-a)*b^2*c*arctan(sqrt(b*sqrt(c/x) + a)*sqrt(-a)/a) + (a*b*x*s
qrt(c/x) + 2*a^2*x)*sqrt(b*sqrt(c/x) + a))/a^2]
```

giac [A] time = 0.36, size = 130, normalized size = 1.41

$$-\frac{\left(\frac{b^2c^3\log\left(c^2|b|\right)}{\sqrt{ac}\,a}-\frac{b^2c^3\log\left(\left|-bc^2-2\sqrt{ac}\left(\sqrt{ac}\,\sqrt{cx}-\sqrt{ac^2x+\sqrt{cx}\,bc^2}\right)\right|\right)}{\sqrt{ac}\,a}-2\sqrt{ac^2x+\sqrt{cx}\,bc^2}\left(\frac{bc}{a}+2\sqrt{cx}\right)\right)\mathrm{sgn}(x)}{4\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c/x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(b^2*c^3*log(c^2*abs(b))/(sqrt(a*c)*a) - b^2*c^3*log(abs(-b*c^2 - 2*sq
rt(a*c)*(sqrt(a*c)*sqrt(c*x) - sqrt(a*c^2*x + sqrt(c*x)*b*c^2))))/(sqrt(a*c
)*a) - 2*sqrt(a*c^2*x + sqrt(c*x)*b*c^2)*(b*c/a + 2*sqrt(c*x)))*sgn(x)/c^(3
/2)
```

maple [B] time = 0.02, size = 147, normalized size = 1.60

$$\frac{\sqrt{a+\sqrt{\frac{c}{x}}\,b}\left(-a\,b^2c\ln\left(\frac{2a\sqrt{x}+\sqrt{\frac{c}{x}}\,b\sqrt{x}+2\sqrt{ax+\sqrt{\frac{c}{x}}\,bx}\,\sqrt{a}}{2\sqrt{a}}\right)+4\sqrt{ax+\sqrt{\frac{c}{x}}\,bx}\,a^{\frac{5}{2}}\sqrt{x}+2\sqrt{ax+\sqrt{\frac{c}{x}}\,bx}\,\sqrt{\frac{c}{x}}\,a^{\frac{3}{2}}b\sqrt{x}\right)}{4\sqrt{\left(a+\sqrt{\frac{c}{x}}\,b\right)x}\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(c/x)^(1/2)*b)^(1/2),x)
```

```
[Out] 1/4*(a+(c/x)^(1/2)*b)^(1/2)*x^(1/2)*(2*a^(3/2)*(a*x+(c/x)^(1/2)*b*x)^(1/2)*
(c/x)^(1/2)*x^(1/2)*b-b^2*c*ln(1/2*(2*a*x^(1/2)+(c/x)^(1/2)*b*x^(1/2)+2*(a*
x+(c/x)^(1/2)*b*x)^(1/2)*a^(1/2))/a^(1/2))*a+4*a^(5/2)*(a*x+(c/x)^(1/2)*b*x
)^(1/2)*x^(1/2))/((a+(c/x)^(1/2)*b)*x)^(1/2)/a^(5/2)
```

maxima [A] time = 1.32, size = 126, normalized size = 1.37

$$\frac{1}{4}\left(\frac{b^2\log\left(\frac{\sqrt{b\sqrt{\frac{c}{x}}+a}-\sqrt{a}}{\sqrt{b\sqrt{\frac{c}{x}}+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2\left(\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{3}{2}}b^2+\sqrt{b\sqrt{\frac{c}{x}}+a}\,ab^2\right)}{\left(b\sqrt{\frac{c}{x}}+a\right)^2a-2\left(b\sqrt{\frac{c}{x}}+a\right)a^2+a^3}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c/x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(b^2*log((sqrt(b*sqrt(c/x) + a) - sqrt(a))/(sqrt(b*sqrt(c/x) + a) + sqr
t(a)))/a^(3/2) + 2*((b*sqrt(c/x) + a)^(3/2)*b^2 + sqrt(b*sqrt(c/x) + a)*a*b
^2)/((b*sqrt(c/x) + a)^2*a - 2*(b*sqrt(c/x) + a)*a^2 + a^3))*c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a + b\sqrt{\frac{c}{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c/x)^(1/2))^(1/2),x)
```

```
[Out] int((a + b*(c/x)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sqrt{\frac{c}{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c/x)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sqrt(c/x)), x)
```

3.2984 $\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x}\,dx$

Optimal. Leaf size=51

$$4\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{\sqrt{a}}\right)-4\sqrt{a+b\sqrt{\frac{c}{x}}}$$

```
[Out] 4*arctanh((a+b*(c/x)^(1/2))^(1/2)/a^(1/2))*a^(1/2)-4*(a+b*(c/x)^(1/2))^(1/2
)
```

Rubi [A] time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {369, 266, 50, 63, 208}

$$4\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{\sqrt{a}}\right)-4\sqrt{a+b\sqrt{\frac{c}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[c/x]]/x,x]
```

```
[Out] -4*Sqrt[a + b*Sqrt[c/x]] + 4*Sqrt[a]*ArcTanh[Sqrt[a + b*Sqrt[c/x]]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
```

```
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x}\,dx = \text{Subst}\left(\int \frac{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{x}\,dx, \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= -\text{Subst}\left(2\,\text{Subst}\left(\int \frac{\sqrt{a+b\sqrt{c}\,x}}{x}\,dx, x, \frac{1}{\sqrt{x}}\right), \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= -4\sqrt{a+b\sqrt{\frac{c}{x}}} - \text{Subst}\left((2a)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+b\sqrt{c}\,x}}\,dx, x, \frac{1}{\sqrt{x}}\right), \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= -4\sqrt{a+b\sqrt{\frac{c}{x}}} - \text{Subst}\left(\frac{(4a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b\sqrt{c}}+\frac{x^2}{b\sqrt{c}}}\,dx, x, \sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}\right)}{b\sqrt{c}}, \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= -4\sqrt{a+b\sqrt{\frac{c}{x}}} + 4\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{\sqrt{a}}\right)$$

Mathematica [A] time = 0.05, size = 51, normalized size = 1.00

$$4\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{\sqrt{a}}\right) - 4\sqrt{a+b\sqrt{\frac{c}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[c/x]]/x,x]
```

```
[Out] -4*Sqrt[a + b*Sqrt[c/x]] + 4*Sqrt[a]*ArcTanh[Sqrt[a + b*Sqrt[c/x]]/Sqrt[a]]
```

fricas [A] time = 0.94, size = 110, normalized size = 2.16

$$\left[2\sqrt{a}\,\log\left(2\sqrt{b\sqrt{\frac{c}{x}}+a}\,\sqrt{a}\,x\sqrt{\frac{c}{x}}+2\,ax\sqrt{\frac{c}{x}}+bc\right) - 4\sqrt{b\sqrt{\frac{c}{x}}+a}, -4\sqrt{-a}\,\arctan\left(\frac{\sqrt{b\sqrt{\frac{c}{x}}+a}\,\sqrt{-a}}{a}\right) - 4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c/x)^(1/2))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [2*sqrt(a)*log(2*sqrt(b*sqrt(c/x) + a)*sqrt(a)*x*sqrt(c/x) + 2*a*x*sqrt(c/x
) + b*c) - 4*sqrt(b*sqrt(c/x) + a), -4*sqrt(-a)*arctan(sqrt(b*sqrt(c/x) + a
)*sqrt(-a)/a) - 4*sqrt(b*sqrt(c/x) + a)]
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c/x)^(1/2))^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, integration of abs or sign assumes constant sign by intervals (cor
rect if the argument is real):Check [abs(t_nostep)]Warning, choosing root o
f [1,0,%%%{-2,[1,1,2,0]%%%}+%%%{-2,[1,1,0,0]%%%}+%%%{-2,[0,2,1,1]%%%},0,%%%
{1,[2,2,4,0]%%%}+%%%{-2,[2,2,2,0]%%%}+%%%{1,[2,2,0,0]%%%}+%%%{2,[1,3,3,1]%%
%}+%%%{-2,[1,3,1,1]%%%}+%%%{1,[0,4,2,2]%%%}] at parameters values [-97,-82,
7,-27]Sign error (%%%{(-c*sqrt(a*c))*b,0%%%}+%%%{2*a*sqrt(b)*abs(c),1/2%%%}
+%%%{-2*a*sqrt(a*c),1%%%}+%%%{a^2*sqrt(b)*abs(c)/(b*c),3/2%%%}+%%%{-a^3*sqr
t(b)*abs(c)/(4*b^2*c^2),5/2%%%}+%%%{undef,7/2%%%})Limit: Max order reached
or unable to make series expansion Error: Bad Argument Value
```

maple [**B**] time = 0.02, size = 150, normalized size = 2.94

$$\frac{2\sqrt{a+\sqrt{\frac{c}{x}}\,b}\left(\sqrt{\frac{c}{x}}\,ab\,x^{\frac{3}{2}}\ln\left(\frac{2a\sqrt{x}+\sqrt{\frac{c}{x}}\,b\sqrt{x}+2\sqrt{ax+\sqrt{\frac{c}{x}}\,bx}\,\sqrt{a}}{2\sqrt{a}}\right)+2\sqrt{ax+\sqrt{\frac{c}{x}}\,bx}\,a^{\frac{3}{2}}x-2\left(ax+\sqrt{\frac{c}{x}}\,bx\right)^{\frac{3}{2}}\sqrt{a}\right)}{\sqrt{\left(a+\sqrt{\frac{c}{x}}\,b\right)x}\,\sqrt{\frac{c}{x}}\,\sqrt{a}\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(c/x)^(1/2)*b)^(1/2)/x,x)
```

```
[Out] 2*(a+(c/x)^(1/2)*b)^(1/2)*(ln(1/2*(2*a*x^(1/2)+(c/x)^(1/2)*b*x^(1/2)+2*(a*x
+(c/x)^(1/2)*b*x)^(1/2)*a^(1/2))/a^(1/2))*(c/x)^(1/2)*x^(3/2)*a*b+2*a^(3/2)
*(a*x+(c/x)^(1/2)*b*x)^(1/2)*x-2*(a*x+(c/x)^(1/2)*b*x)^(3/2)*a^(1/2))/x/((a
+(c/x)^(1/2)*b)*x)^(1/2)/b/(c/x)^(1/2)/a^(1/2)
```

maxima [**A**] time = 1.29, size = 61, normalized size = 1.20

$$-2\sqrt{a}\,\log\left(\frac{\sqrt{b\sqrt{\frac{c}{x}}+a}-\sqrt{a}}{\sqrt{b\sqrt{\frac{c}{x}}+a}+\sqrt{a}}\right)-4\sqrt{b\sqrt{\frac{c}{x}}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c/x)^(1/2))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -2*sqrt(a)*log((sqrt(b*sqrt(c/x) + a) - sqrt(a))/(sqrt(b*sqrt(c/x) + a) + s
qrt(a))) - 4*sqrt(b*sqrt(c/x) + a)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c/x)^(1/2))^(1/2)/x,x)
```

```
[Out] int((a + b*(c/x)^(1/2))^(1/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\sqrt{\frac{c}{x}}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c/x)**(1/2))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a + b*sqrt(c/x))/x, x)
```

3.2985 $\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^2}\,dx$

Optimal. Leaf size=56

$$\frac{4a\left(a+b\sqrt{\frac{c}{x}}\right)^{3/2}}{3b^2c}-\frac{4\left(a+b\sqrt{\frac{c}{x}}\right)^{5/2}}{5b^2c}$$

```
[Out] 4/3*a*(a+b*(c/x)^(1/2))^(3/2)/b^2/c-4/5*(a+b*(c/x)^(1/2))^(5/2)/b^2/c
```

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {369, 266, 43}

$$\frac{4a\left(a+b\sqrt{\frac{c}{x}}\right)^{3/2}}{3b^2c}-\frac{4\left(a+b\sqrt{\frac{c}{x}}\right)^{5/2}}{5b^2c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[c/x]]/x^2,x]
```

```
[Out] (4*a*(a + b*Sqrt[c/x])^(3/2))/(3*b^2*c) - (4*(a + b*Sqrt[c/x])^(5/2))/(5*b^
2*c)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^2}\,dx = \text{Subst}\left(\int \frac{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{x^2}\,dx, \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$
$$= -\text{Subst}\left(2\,\text{Subst}\left(\int x\sqrt{a+b\sqrt{c}\,x}\,dx, x, \frac{1}{\sqrt{x}}\right), \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$
$$= -\text{Subst}\left(2\,\text{Subst}\left(\int \left(-\frac{a\sqrt{a+b\sqrt{c}\,x}}{b\sqrt{c}} + \frac{\left(a+b\sqrt{c}\,x\right)^{3/2}}{b\sqrt{c}}\right)dx, x, \frac{1}{\sqrt{x}}\right), \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$
$$= \frac{4a\left(a+b\sqrt{\frac{c}{x}}\right)^{3/2}}{3b^2c} - \frac{4\left(a+b\sqrt{\frac{c}{x}}\right)^{5/2}}{5b^2c}$$

Mathematica [A] time = 0.04, size = 43, normalized size = 0.77

$$\frac{4\left(2a-3b\sqrt{\frac{c}{x}}\right)\left(a+b\sqrt{\frac{c}{x}}\right)^{3/2}}{15b^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[c/x]]/x^2,x]
[Out] (4*(2*a - 3*b*Sqrt[c/x])*(a + b*Sqrt[c/x])^(3/2))/(15*b^2*c)
```

fricas [A] time = 0.66, size = 48, normalized size = 0.86

$$-\frac{4\left(abx\sqrt{\frac{c}{x}} + 3\,b^2c - 2\,a^2x\right)\sqrt{b\sqrt{\frac{c}{x}} + a}}{15\,b^2cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c/x)^(1/2))^(1/2)/x^2,x, algorithm="fricas")
[Out] -4/15*(a*b*x*sqrt(c/x) + 3*b^2*c - 2*a^2*x)*sqrt(b*sqrt(c/x) + a)/(b^2*c*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c/x)^(1/2))^(1/2)/x^2,x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 0.02, size = 70, normalized size = 1.25

$$-\frac{4\sqrt{a+\sqrt{\frac{c}{x}}\,b}\,\left(ax+\sqrt{\frac{c}{x}}\,bx\right)^{\frac{3}{2}}\left(-2a+3\sqrt{\frac{c}{x}}\,b\right)}{15\sqrt{\left(a+\sqrt{\frac{c}{x}}\,b\right)x}\,b^2cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(c/x)^(1/2)*b)^(1/2)/x^2,x)
```

```
[Out] -4/15*(a+(c/x)^(1/2)*b)^(1/2)*(a*x+(c/x)^(1/2)*b*x)^(3/2)/c/x*(3*(c/x)^(1/2
)*b-2*a)/((a+(c/x)^(1/2)*b)*x)^(1/2)/b^2
```

maxima [A] time = 0.54, size = 43, normalized size = 0.77

$$-\frac{4\left(\frac{3\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{5}{2}}}{b^2}-\frac{5\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{3}{2}}a}{b^2}\right)}{15\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c/x)^(1/2))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -4/15*(3*(b*sqrt(c/x) + a)^(5/2)/b^2 - 5*(b*sqrt(c/x) + a)^(3/2)*a/b^2)/c
```

mupad [B] time = 1.39, size = 52, normalized size = 0.93

$$-\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}\;{}_2\mathrm{F}_1\left(-\frac{1}{2},2;\;3;\;-\frac{b\sqrt{\frac{c}{x}}}{a}\right)}{x\sqrt{\frac{b\sqrt{\frac{c}{x}}}{a}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c/x)^(1/2))^(1/2)/x^2,x)
```

```
[Out] -((a + b*(c/x)^(1/2))^(1/2)*hypergeom([-1/2, 2], 3, -(b*(c/x)^(1/2))/a))/(x
*((b*(c/x)^(1/2))/a + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c/x)**(1/2))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(a + b*sqrt(c/x))/x**2, x)
```

3.2986 $\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^3}\,dx$

Optimal. Leaf size=116

$$\frac{4a^3\left(a+b\sqrt{\frac{c}{x}}\right)^{3/2}}{3b^4c^2}-\frac{12a^2\left(a+b\sqrt{\frac{c}{x}}\right)^{5/2}}{5b^4c^2}-\frac{4\left(a+b\sqrt{\frac{c}{x}}\right)^{9/2}}{9b^4c^2}+\frac{12a\left(a+b\sqrt{\frac{c}{x}}\right)^{7/2}}{7b^4c^2}$$

```
[Out] 4/3*a^3*(a+b*(c/x)^(1/2))^(3/2)/b^4/c^2-12/5*a^2*(a+b*(c/x)^(1/2))^(5/2)/b^
4/c^2+12/7*a*(a+b*(c/x)^(1/2))^(7/2)/b^4/c^2-4/9*(a+b*(c/x)^(1/2))^(9/2)/b^
4/c^2
```

Rubi [A] time = 0.07, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {369, 266, 43}

$$-\frac{12a^2\left(a+b\sqrt{\frac{c}{x}}\right)^{5/2}}{5b^4c^2}+\frac{4a^3\left(a+b\sqrt{\frac{c}{x}}\right)^{3/2}}{3b^4c^2}-\frac{4\left(a+b\sqrt{\frac{c}{x}}\right)^{9/2}}{9b^4c^2}+\frac{12a\left(a+b\sqrt{\frac{c}{x}}\right)^{7/2}}{7b^4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[c/x]]/x^3,x]
```

```
[Out] (4*a^3*(a + b*Sqrt[c/x])^(3/2))/(3*b^4*c^2) - (12*a^2*(a + b*Sqrt[c/x])^(5/
2))/(5*b^4*c^2) + (12*a*(a + b*Sqrt[c/x])^(7/2))/(7*b^4*c^2) - (4*(a + b*Sq
rt[c/x])^(9/2))/(9*b^4*c^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^3}\,dx = \text{Subst}\left(\int \frac{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{x^3}\,dx, \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= -\text{Subst}\left(2\,\text{Subst}\left(\int x^3\sqrt{a+b\sqrt{c}\,x}\,dx, x, \frac{1}{\sqrt{x}}\right), \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= -\text{Subst}\left(2\,\text{Subst}\left(\int\left(-\frac{a^3\sqrt{a+b\sqrt{c}\,x}}{b^3c^{3/2}} + \frac{3a^2\left(a+b\sqrt{c}\,x\right)^{3/2}}{b^3c^{3/2}} - \frac{3a\left(a+b\sqrt{c}\,x\right)^{5/2}}{b^3c^{3/2}} + \frac{\left(a+b\sqrt{\right.}\right.\right.\right.$$

$$= \frac{4a^3\left(a+b\sqrt{\frac{c}{x}}\right)^{3/2}}{3b^4c^2} - \frac{12a^2\left(a+b\sqrt{\frac{c}{x}}\right)^{5/2}}{5b^4c^2} + \frac{12a\left(a+b\sqrt{\frac{c}{x}}\right)^{7/2}}{7b^4c^2} - \frac{4\left(a+b\sqrt{\frac{c}{x}}\right)^{9/2}}{9b^4c^2}$$

Mathematica [A] time = 0.05, size = 75, normalized size = 0.65

$$\frac{4\left(a+b\sqrt{\frac{c}{x}}\right)^{3/2}\left(16a^3x - 24a^2bx\sqrt{\frac{c}{x}} + 30ab^2c - 35b^3c\sqrt{\frac{c}{x}}\right)}{315b^4c^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[c/x]]/x^3,x]
```

```
[Out] (4*(a + b*Sqrt[c/x])^(3/2)*(30*a*b^2*c - 35*b^3*c*Sqrt[c/x] + 16*a^3*x - 24
*a^2*b*Sqrt[c/x]*x))/(315*b^4*c^2*x)
```

fricas [A] time = 0.91, size = 77, normalized size = 0.66

$$-\frac{4\left(35\,b^4c^2 - 6\,a^2b^2cx - 16\,a^4x^2 + \left(5\,ab^3cx + 8\,a^3bx^2\right)\sqrt{\frac{c}{x}}\right)\sqrt{b\sqrt{\frac{c}{x}} + a}}{315\,b^4c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c/x)^(1/2))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] -4/315*(35*b^4*c^2 - 6*a^2*b^2*c*x - 16*a^4*x^2 + (5*a*b^3*c*x + 8*a^3*b*x^
2)*sqrt(c/x))*sqrt(b*sqrt(c/x) + a)/(b^4*c^2*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c/x)^(1/2))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.02, size = 97, normalized size = 0.84

$$-\frac{4\sqrt{a+\sqrt{\frac{c}{x}}\,b}\,\left(ax+\sqrt{\frac{c}{x}}\,bx\right)^{\frac{3}{2}}\left(-16a^3x + 24\sqrt{\frac{c}{x}}\,a^2bx - 30a\,b^2c + 35\left(\frac{c}{x}\right)^{\frac{3}{2}}b^3x\right)}{315\sqrt{\left(a+\sqrt{\frac{c}{x}}\,b\right)x}\,b^4c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(c/x)^(1/2)*b)^(1/2)/x^3,x)
```

```
[Out] -4/315*(a+(c/x)^(1/2)*b)^(1/2)*(a*x+(c/x)^(1/2)*b*x)^(3/2)*(35*x*(c/x)^(3/2
)*b^3+24*x*(c/x)^(1/2)*a^2*b-16*a^3*x-30*a*b^2*c)/c^2/x^2/((a+(c/x)^(1/2)*b
)*x)^(1/2)/b^4
```

maxima [A] time = 0.45, size = 85, normalized size = 0.73

$$-\frac{4\left(\frac{35\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{9}{2}}}{b^4}-\frac{135\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{7}{2}}a}{b^4}+\frac{189\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{5}{2}}a^2}{b^4}-\frac{105\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{3}{2}}a^3}{b^4}\right)}{315\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c/x)^(1/2))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -4/315*(35*(b*sqrt(c/x) + a)^(9/2)/b^4 - 135*(b*sqrt(c/x) + a)^(7/2)*a/b^4
+ 189*(b*sqrt(c/x) + a)^(5/2)*a^2/b^4 - 105*(b*sqrt(c/x) + a)^(3/2)*a^3/b^4
)/c^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c/x)^(1/2))^(1/2)/x^3,x)
```

```
[Out] int((a + b*(c/x)^(1/2))^(1/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c/x)**(1/2))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(a + b*sqrt(c/x))/x**3, x)
```

3.2987 $\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^4}\,dx$

Optimal. Leaf size=174

$$\frac{4a^5\left(a+b\sqrt{\frac{c}{x}}\right)^{3/2}}{3b^6c^3}-\frac{4a^4\left(a+b\sqrt{\frac{c}{x}}\right)^{5/2}}{b^6c^3}+\frac{40a^3\left(a+b\sqrt{\frac{c}{x}}\right)^{7/2}}{7b^6c^3}-\frac{40a^2\left(a+b\sqrt{\frac{c}{x}}\right)^{9/2}}{9b^6c^3}-\frac{4\left(a+b\sqrt{\frac{c}{x}}\right)^{13/2}}{13b^6c^3}+\frac{20a\left(a+b\sqrt{\frac{c}{x}}\right.}{11b^6c^3}$$

```
[Out] 4/3*a^5*(a+b*(c/x)^(1/2))^(3/2)/b^6/c^3-4*a^4*(a+b*(c/x)^(1/2))^(5/2)/b^6/c
^3+40/7*a^3*(a+b*(c/x)^(1/2))^(7/2)/b^6/c^3-40/9*a^2*(a+b*(c/x)^(1/2))^(9/2
)/b^6/c^3+20/11*a*(a+b*(c/x)^(1/2))^(11/2)/b^6/c^3-4/13*(a+b*(c/x)^(1/2))^(
13/2)/b^6/c^3
```

Rubi [A] time = 0.11, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {369, 266, 43}

$$-\frac{40a^2\left(a+b\sqrt{\frac{c}{x}}\right)^{9/2}}{9b^6c^3}+\frac{40a^3\left(a+b\sqrt{\frac{c}{x}}\right)^{7/2}}{7b^6c^3}-\frac{4a^4\left(a+b\sqrt{\frac{c}{x}}\right)^{5/2}}{b^6c^3}+\frac{4a^5\left(a+b\sqrt{\frac{c}{x}}\right)^{3/2}}{3b^6c^3}-\frac{4\left(a+b\sqrt{\frac{c}{x}}\right)^{13/2}}{13b^6c^3}+\frac{20a\left(a+b\sqrt{\right.}}{11b^6c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[c/x]]/x^4,x]
```

```
[Out] (4*a^5*(a + b*Sqrt[c/x])^(3/2))/(3*b^6*c^3) - (4*a^4*(a + b*Sqrt[c/x])^(5/2
))/(b^6*c^3) + (40*a^3*(a + b*Sqrt[c/x])^(7/2))/(7*b^6*c^3) - (40*a^2*(a +
b*Sqrt[c/x])^(9/2))/(9*b^6*c^3) + (20*a*(a + b*Sqrt[c/x])^(11/2))/(11*b^6*c
^3) - (4*(a + b*Sqrt[c/x])^(13/2))/(13*b^6*c^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^4}\,dx = \text{Subst}\left(\int \frac{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}{x^4}\,dx, \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= -\text{Subst}\left(2\,\text{Subst}\left(\int x^5\sqrt{a+b\sqrt{c}\,x}\;dx, x, \frac{1}{\sqrt{x}}\right), \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= -\text{Subst}\left(2\,\text{Subst}\left(\int\left(-\frac{a^5\sqrt{a+b\sqrt{c}\,x}}{b^5c^{5/2}} + \frac{5a^4\left(a+b\sqrt{c}\,x\right)^{3/2}}{b^5c^{5/2}} - \frac{10a^3\left(a+b\sqrt{c}\,x\right)^{5/2}}{b^5c^{5/2}} + \frac{10a}{}\right.\right.\right.$$

$$= \frac{4a^5\left(a+b\sqrt{\frac{c}{x}}\right)^{3/2}}{3b^6c^3} - \frac{4a^4\left(a+b\sqrt{\frac{c}{x}}\right)^{5/2}}{b^6c^3} + \frac{40a^3\left(a+b\sqrt{\frac{c}{x}}\right)^{7/2}}{7b^6c^3} - \frac{40a^2\left(a+b\sqrt{\frac{c}{x}}\right)^{9/2}}{9b^6c^3} + \frac{20}{}$$

Mathematica [A] time = 0.06, size = 111, normalized size = 0.64

$$\frac{4\left(a+b\sqrt{\frac{c}{x}}\right)^{3/2}\left(256a^5x^2 - 384a^4bx^2\sqrt{\frac{c}{x}} + 480a^3b^2cx - 560a^2b^3cx\sqrt{\frac{c}{x}} + 630ab^4c^2 - 693b^5cx\left(\frac{c}{x}\right)^{3/2}\right)}{9009b^6c^3x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[c/x]]/x^4,x]
```

```
[Out] (4*(a + b*Sqrt[c/x])^(3/2)*(630*a*b^4*c^2 + 480*a^3*b^2*c*x - 560*a^2*b^3*c
*Sqrt[c/x]*x - 693*b^5*c*(c/x)^(3/2)*x + 256*a^5*x^2 - 384*a^4*b*Sqrt[c/x]*
x^2))/(9009*b^6*c^3*x^2)
```

fricas [A] time = 0.90, size = 105, normalized size = 0.60

$$-\frac{4\left(693\,b^6c^3 - 70\,a^2b^4c^2x - 96\,a^4b^2cx^2 - 256\,a^6x^3 + \left(63\,ab^5c^2x + 80\,a^3b^3cx^2 + 128\,a^5bx^3\right)\sqrt{\frac{c}{x}}\right)\sqrt{b\sqrt{\frac{c}{x}}+a}}{9009\,b^6c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c/x)^(1/2))^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -4/9009*(693*b^6*c^3 - 70*a^2*b^4*c^2*x - 96*a^4*b^2*c*x^2 - 256*a^6*x^3 +
(63*a*b^5*c^2*x + 80*a^3*b^3*c*x^2 + 128*a^5*b*x^3)*sqrt(c/x))*sqrt(b*sqrt(
c/x) + a)/(b^6*c^3*x^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c/x)^(1/2))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,0,0,%%%{-1,[1]%%%}] at parameters values [74
.7709350525]Warning, choosing root of [1,0,0,0,%%%{-1,[1]%%%}] at parameter
s values [47.3295757947]Warning, choosing root of [1,0,0,0,%%%{-1,[1]%%%}]
at parameters values [50.5901726987]Warning, choosing root of [1,0,0,0,%%%{
```

```
-1,[1]%%%}] at parameters values [91.0141688026]Warning, choosing root of [
1,0,0,0,%%%{-1,[1]%%%}] at parameters values [59.8656459874]Warning, choosi
ng root of [1,0,0,0,%%%{-1,[1]%%%}] at parameters values [25.8388736797]War
ning, choosing root of [1,0,0,0,%%%{-1,[1]%%%}] at parameters values [33.92
85577983]Warning, choosing root of [1,0,0,0,%%%{-1,[1]%%%}] at parameters v
alues [15.451549686]Warning, choosing root of [1,0,0,0,%%%{-1,[1]%%%}] at p
arameters values [8]Warning, choosing root of [1,0,0,0,%%%{-1,[1]%%%}] at p
arameters values [-64]Warning, choosing root of [1,0,0,0,%%%{-1,[1]%%%}] at
 parameters values [2]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.Non regular value [0] was dis
carded and replaced randomly by 0=[37]Algebraic extensions not allowed in a
 rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions not
 allowed in a rootofWarning, need to choose a branch for the root of a poly
nomial with parameters. This might be wrong.The choice was done assuming [b
]=[-56]Warning, need to choose a branch for the root of a polynomial with p
arameters. This might be wrong.The choice was done assuming [b]=[-99]Evalua
tion time: 5.9index.cc index_m operator + Error: Bad Argument Value
```

maple [A] time = 0.02, size = 133, normalized size = 0.76

$$-\frac{4\sqrt{a+\sqrt{\frac{c}{x}}\,b}\,\left(ax+\sqrt{\frac{c}{x}}\,bx\right)^{\frac{3}{2}}\left(-256a^5x^2+384\sqrt{\frac{c}{x}}\,a^4bx^2-480a^3b^2cx+560\left(\frac{c}{x}\right)^{\frac{3}{2}}a^2b^3x^2-630ab^4c^2+693\left(\frac{c}{x}\right.\right.}{9009\sqrt{\left(a+\sqrt{\frac{c}{x}}\,b\right)x}\,b^6c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(c/x)^(1/2)*b)^(1/2)/x^4,x)
```

```
[Out] -4/9009*(a+(c/x)^(1/2)*b)^(1/2)*(a*x+(c/x)^(1/2)*b*x)^(3/2)*(693*x^2*(c/x)^
(5/2)*b^5+560*x^2*(c/x)^(3/2)*a^2*b^3+384*x^2*(c/x)^(1/2)*a^4*b-256*a^5*x^2
-480*c*x*a^3*b^2-630*c^2*a*b^4)/c^3/x^3/((a+(c/x)^(1/2)*b)*x)^(1/2)/b^6
```

maxima [A] time = 0.59, size = 127, normalized size = 0.73

$$-\frac{4\left(\frac{693\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{13}{2}}}{b^6}-\frac{4095\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{11}{2}}a}{b^6}+\frac{10010\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{9}{2}}a^2}{b^6}-\frac{12870\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{7}{2}}a^3}{b^6}+\frac{9009\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{5}{2}}a^4}{b^6}-\frac{3003\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{3}{2}}a^5}{b^6}\right)}{9009\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c/x)^(1/2))^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] -4/9009*(693*(b*sqrt(c/x) + a)^(13/2)/b^6 - 4095*(b*sqrt(c/x) + a)^(11/2)*a
/b^6 + 10010*(b*sqrt(c/x) + a)^(9/2)*a^2/b^6 - 12870*(b*sqrt(c/x) + a)^(7/2
)*a^3/b^6 + 9009*(b*sqrt(c/x) + a)^(5/2)*a^4/b^6 - 3003*(b*sqrt(c/x) + a)^(
3/2)*a^5/b^6)/c^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c/x)^(1/2))^(1/2)/x^4,x)
```

```
[Out] int((a + b*(c/x)^(1/2))^(1/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c/x)**(1/2))**(1/2)/x**4,x)
[Out] Integral(sqrt(a + b*sqrt(c/x))/x**4, x)
```

3.2988 $\int \frac{x}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx$

Optimal. Leaf size=172

$$\frac{35b^4c^2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{\sqrt{a}}\right)}{32a^{9/2}}-\frac{35b^3c^2\sqrt{a+b\sqrt{\frac{c}{x}}}}{32a^4\sqrt{\frac{c}{x}}}+\frac{35b^2cx\sqrt{a+b\sqrt{\frac{c}{x}}}}{48a^3}-\frac{7bc^2\sqrt{a+b\sqrt{\frac{c}{x}}}}{12a^2\left(\frac{c}{x}\right)^{3/2}}+\frac{x^2\sqrt{a+b\sqrt{\frac{c}{x}}}}{2a}$$

```
[Out] 35/32*b^4*c^2*arctanh((a+b*(c/x)^(1/2))^(1/2)/a^(1/2))/a^(9/2)-7/12*b*c^2*(
a+b*(c/x)^(1/2))^(1/2)/a^2/(c/x)^(3/2)+35/48*b^2*c*x*(a+b*(c/x)^(1/2))^(1/2
)/a^3+1/2*x^2*(a+b*(c/x)^(1/2))^(1/2)/a-35/32*b^3*c^2*(a+b*(c/x)^(1/2))^(1/
2)/a^4/(c/x)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 175, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {369, 266, 51, 63, 208}

$$-\frac{35b^3c^2\sqrt{a+b\sqrt{\frac{c}{x}}}}{32a^4\sqrt{\frac{c}{x}}}+\frac{35b^4c^2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{\sqrt{a}}\right)}{32a^{9/2}}+\frac{35b^2cx\sqrt{a+b\sqrt{\frac{c}{x}}}}{48a^3}-\frac{7bx^3\left(\frac{c}{x}\right)^{3/2}\sqrt{a+b\sqrt{\frac{c}{x}}}}{12a^2c}+\frac{x^2\sqrt{a+b\sqrt{\frac{c}{x}}}}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*Sqrt[c/x]],x]
```

```
[Out] (-35*b^3*c^2*Sqrt[a + b*Sqrt[c/x]])/(32*a^4*Sqrt[c/x]) + (35*b^2*c*Sqrt[a +
 b*Sqrt[c/x]]*x)/(48*a^3) + (Sqrt[a + b*Sqrt[c/x]]*x^2)/(2*a) - (7*b*Sqrt[a
 + b*Sqrt[c/x]]*(c/x)^(3/2)*x^3)/(12*a^2*c) + (35*b^4*c^2*ArcTanh[Sqrt[a +
b*Sqrt[c/x]]/Sqrt[a]])/(32*a^(9/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx = \text{Subst}\left(\int \frac{x}{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}\,dx, \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= -\text{Subst}\left(2\,\text{Subst}\left(\int \frac{1}{x^5\sqrt{a+b\sqrt{c}\,x}}\,dx, x, \frac{1}{\sqrt{x}}\right), \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}\,x^2}{2a} + \text{Subst}\left(\frac{\left(7b\sqrt{c}\right)\text{Subst}\left(\int \frac{1}{x^4\sqrt{a+b\sqrt{c}\,x}}\,dx, x, \frac{1}{\sqrt{x}}\right)}{4a}, \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}\,x^2}{2a} - \frac{7b\sqrt{a+b\sqrt{\frac{c}{x}}}\,\left(\frac{c}{x}\right)^{3/2}x^3}{12a^2c} - \text{Subst}\left(\frac{\left(35b^2c\right)\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+b\sqrt{c}\,x}}\,dx, x, \frac{1}{\sqrt{x}}\right)}{24a^2}\right.$$

$$= \frac{35b^2c\sqrt{a+b\sqrt{\frac{c}{x}}}\,x}{48a^3} + \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}\,x^2}{2a} - \frac{7b\sqrt{a+b\sqrt{\frac{c}{x}}}\,\left(\frac{c}{x}\right)^{3/2}x^3}{12a^2c} + \text{Subst}\left(\frac{\left(35b^3c^{3/2}\right)\text{Sub}\right.$$

$$= -\frac{35b^3c^2\sqrt{a+b\sqrt{\frac{c}{x}}}}{32a^4\sqrt{\frac{c}{x}}} + \frac{35b^2c\sqrt{a+b\sqrt{\frac{c}{x}}}\,x}{48a^3} + \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}\,x^2}{2a} - \frac{7b\sqrt{a+b\sqrt{\frac{c}{x}}}\,\left(\frac{c}{x}\right)^{3/2}x^3}{12a^2c} -$$

$$= -\frac{35b^3c^2\sqrt{a+b\sqrt{\frac{c}{x}}}}{32a^4\sqrt{\frac{c}{x}}} + \frac{35b^2c\sqrt{a+b\sqrt{\frac{c}{x}}}\,x}{48a^3} + \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}\,x^2}{2a} - \frac{7b\sqrt{a+b\sqrt{\frac{c}{x}}}\,\left(\frac{c}{x}\right)^{3/2}x^3}{12a^2c} -$$

$$= -\frac{35b^3c^2\sqrt{a+b\sqrt{\frac{c}{x}}}}{32a^4\sqrt{\frac{c}{x}}} + \frac{35b^2c\sqrt{a+b\sqrt{\frac{c}{x}}}\,x}{48a^3} + \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}\,x^2}{2a} - \frac{7b\sqrt{a+b\sqrt{\frac{c}{x}}}\,\left(\frac{c}{x}\right)^{3/2}x^3}{12a^2c} +$$

Mathematica [A] time = 0.16, size = 126, normalized size = 0.73

$$\frac{35b^4c^2\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\right)}{32a^{9/2}}+\frac{48a^4x^2-8a^3bx^2\sqrt{\frac{c}{x}}+14a^2b^2cx-35ab^3cx\sqrt{\frac{c}{x}}-105b^4c^2}{96a^4\sqrt{a+b\sqrt{\frac{c}{x}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b*Sqrt[c/x]],x]
```

```
[Out] (-105*b^4*c^2 + 14*a^2*b^2*c*x - 35*a*b^3*c*Sqrt[c/x]*x + 48*a^4*x^2 - 8*a^
3*b*Sqrt[c/x]*x^2)/(96*a^4*Sqrt[a + b*Sqrt[c/x]]) + (35*b^4*c^2*ArcTanh[Sqr
t[a]/Sqrt[a + b*Sqrt[c/x]]])/(32*a^(9/2))
```

fricas [A] time = 0.96, size = 224, normalized size = 1.30

$$\left[\frac{105\sqrt{a}\,b^4c^2\log\left(2\sqrt{b\sqrt{\frac{c}{x}}+a}\,\sqrt{a}\,x\sqrt{\frac{c}{x}}+2\,ax\sqrt{\frac{c}{x}}+bc\right)+2\left(70\,a^2b^2cx+48\,a^4x^2-7\left(15\,ab^3cx+8\,a^3bx^2\right)\sqrt{\frac{c}{x}}\right)}{192\,a^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(c/x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/192*(105*sqrt(a)*b^4*c^2*log(2*sqrt(b*sqrt(c/x) + a)*sqrt(a)*x*sqrt(c/x)
 + 2*a*x*sqrt(c/x) + b*c) + 2*(70*a^2*b^2*c*x + 48*a^4*x^2 - 7*(15*a*b^3*c*
x + 8*a^3*b*x^2)*sqrt(c/x))*sqrt(b*sqrt(c/x) + a))/a^5, -1/96*(105*sqrt(-a)
*b^4*c^2*arctan(sqrt(b*sqrt(c/x) + a)*sqrt(-a)/a) - (70*a^2*b^2*c*x + 48*a^
4*x^2 - 7*(15*a*b^3*c*x + 8*a^3*b*x^2)*sqrt(c/x))*sqrt(b*sqrt(c/x) + a))/a^
5]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{b\sqrt{\frac{c}{x}}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(c/x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(b*sqrt(c/x) + a), x)
```

maple [B] time = 0.05, size = 298, normalized size = 1.73

$$-\frac{\sqrt{a+\sqrt{\frac{c}{x}}\,b}\left(-192a\,b^4c^2\ln\left(\frac{2a\sqrt{x}+\sqrt{\frac{c}{x}}\,b\sqrt{x}+2\sqrt{\left(a+\sqrt{\frac{c}{x}}\,b\right)x}\,\sqrt{a}}{2\sqrt{a}}\right)+87a\,b^4c^2\ln\left(\frac{2a\sqrt{x}+\sqrt{\frac{c}{x}}\,b\sqrt{x}+2\sqrt{ax+\sqrt{\frac{c}{x}}\,bx}\,\sqrt{a}}{2\sqrt{a}}\right)-348\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+(c/x)^(1/2)*b)^(1/2),x)
```

```
[Out] -1/192*(a+(c/x)^(1/2)*b)^(1/2)*x^(1/2)*(384*((a+(c/x)^(1/2)*b)*x)^(1/2)*a^(
3/2)*(c/x)^(3/2)*x^(3/2)*b^3-174*(a*x+(c/x)^(1/2)*b*x)^(1/2)*(c/x)^(3/2)*a^
(3/2)*b^3*x^(3/2)+208*(a*x+(c/x)^(1/2)*b*x)^(3/2)*(c/x)^(1/2)*a^(5/2)*b*x^(
1/2)-96*(a*x+(c/x)^(1/2)*b*x)^(3/2)*a^(7/2)*x^(1/2)-348*(a*x+(c/x)^(1/2)*b*
x)^(1/2)*a^(5/2)*b^2*c*x^(1/2)-192*ln(1/2*((c/x)^(1/2)*b*x^(1/2)+2*((a+(c/x
)^(1/2)*b)*x)^(1/2)*a^(1/2)+2*a*x^(1/2))/a^(1/2))*c^2*a*b^4+87*a*b^4*c^2*ln
(1/2*(2*a*x^(1/2)+(c/x)^(1/2)*b*x^(1/2)+2*(a*x+(c/x)^(1/2)*b*x)^(1/2)*a^(1/
2))/a^(1/2)))/((a+(c/x)^(1/2)*b)*x)^(1/2)/a^(11/2)
```

maxima [A] time = 1.34, size = 211, normalized size = 1.23

$$-\frac{1}{192}c^2\left(\frac{105\,b^4\log\left(\frac{\sqrt{b\sqrt{\frac{c}{x}}+a}-\sqrt{a}}{\sqrt{b\sqrt{\frac{c}{x}}+a}+\sqrt{a}}\right)}{a^{\frac{9}{2}}}+\frac{2\left(105\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{7}{2}}b^4-385\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{5}{2}}ab^4+511\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{3}{2}}a^2b^4-2\right.}{\left(b\sqrt{\frac{c}{x}}+a\right)^4a^4-4\left(b\sqrt{\frac{c}{x}}+a\right)^3a^5+6\left(b\sqrt{\frac{c}{x}}+a\right)^2a^6-4\left(b\sqrt{\frac{c}{x}}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(c/x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/192*c^2*(105*b^4*log((sqrt(b*sqrt(c/x) + a) - sqrt(a))/(sqrt(b*sqrt(c/x)
 + a) + sqrt(a)))/a^(9/2) + 2*(105*(b*sqrt(c/x) + a)^(7/2)*b^4 - 385*(b*sqr
t(c/x) + a)^(5/2)*a*b^4 + 511*(b*sqrt(c/x) + a)^(3/2)*a^2*b^4 - 279*sqrt(b*
sqrt(c/x) + a)*a^3*b^4)/((b*sqrt(c/x) + a)^4*a^4 - 4*(b*sqrt(c/x) + a)^3*a^
5 + 6*(b*sqrt(c/x) + a)^2*a^6 - 4*(b*sqrt(c/x) + a)*a^7 + a^8))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*(c/x)^(1/2))^(1/2),x)
```

```
[Out] int(x/(a + b*(c/x)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(c/x)**(1/2))**(1/2),x)
```

```
[Out] Integral(x/sqrt(a + b*sqrt(c/x)), x)
```

3.2989 $\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx$

Optimal. Leaf size=95

$$\frac{3b^2c\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3bc\sqrt{a+b\sqrt{\frac{c}{x}}}}{2a^2\sqrt{\frac{c}{x}}}+\frac{x\sqrt{a+b\sqrt{\frac{c}{x}}}}{a}$$

```
[Out] 3/2*b^2*c*arctanh((a+b*(c/x)^(1/2))^(1/2)/a^(1/2))/a^(5/2)+x*(a+b*(c/x)^(1/
2))^(1/2)/a-3/2*b*c*(a+b*(c/x)^(1/2))^(1/2)/a^2/(c/x)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {255, 190, 51, 63, 208}

$$\frac{3b^2c\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3bc\sqrt{a+b\sqrt{\frac{c}{x}}}}{2a^2\sqrt{\frac{c}{x}}}+\frac{x\sqrt{a+b\sqrt{\frac{c}{x}}}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sqrt[c/x]],x]
```

```
[Out] (-3*b*c*Sqrt[a + b*Sqrt[c/x]])/(2*a^2*Sqrt[c/x]) + (Sqrt[a + b*Sqrt[c/x]]*x
)/a + (3*b^2*c*ArcTanh[Sqrt[a + b*Sqrt[c/x]]/Sqrt[a]])/(2*a^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 255

```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_.), x_Symbol] :> With[{k = De
nominator[n]}, Subst[Int[(a + b*c^n*x^(n*q))^p, x], x^(1/k), (c*x^q)^(1/k)/
(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c, p, q}, x] && FractionQ[n]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}\,dx, \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= -\text{Subst}\left(2\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+b\sqrt{c}\,x}}\,dx, x, \frac{1}{\sqrt{x}}\right), \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}\,x}{a} + \text{Subst}\left(\frac{\left(3b\sqrt{c}\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+b\sqrt{c}\,x}}\,dx, x, \frac{1}{\sqrt{x}}\right)}{2a}, \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= -\frac{3bc\sqrt{a+b\sqrt{\frac{c}{x}}}}{2a^2\sqrt{\frac{c}{x}}} + \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}\,x}{a} - \text{Subst}\left(\frac{\left(3b^2c\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+b\sqrt{c}\,x}}\,dx, x, \frac{1}{\sqrt{x}}\right)}{4a^2}, \sqrt{x}, \sqrt{}\right.$$

$$= -\frac{3bc\sqrt{a+b\sqrt{\frac{c}{x}}}}{2a^2\sqrt{\frac{c}{x}}} + \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}\,x}{a} - \text{Subst}\left(\frac{\left(3b\sqrt{c}\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b\sqrt{c}}+\frac{x^2}{b\sqrt{c}}}\,dx, x, \sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}\right.\right.}{2a^2}\right.$$

$$= -\frac{3bc\sqrt{a+b\sqrt{\frac{c}{x}}}}{2a^2\sqrt{\frac{c}{x}}} + \frac{\sqrt{a+b\sqrt{\frac{c}{x}}}\,x}{a} + \frac{3b^2c\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{\sqrt{a}}\right)}{2a^{5/2}}$$

Mathematica [A] time = 0.10, size = 89, normalized size = 0.94

$$\frac{3b^2c\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\right)}{2a^{5/2}} + \frac{2a^2x - abx\sqrt{\frac{c}{x}} - 3b^2c}{2a^2\sqrt{a+b\sqrt{\frac{c}{x}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sqrt[c/x]],x]
```

```
[Out] (-3*b^2*c + 2*a^2*x - a*b*Sqrt[c/x]*x)/(2*a^2*Sqrt[a + b*Sqrt[c/x]]) + (3*b
^2*c*ArcTanh[Sqrt[a]/Sqrt[a + b*Sqrt[c/x]]])/(2*a^(5/2))
```

fricas [A] time = 0.97, size = 165, normalized size = 1.74

$$\left[\frac{3\sqrt{a}\,b^2c\log\left(2\sqrt{b\sqrt{\frac{c}{x}}+a}\,\sqrt{a}\,x\sqrt{\frac{c}{x}}+2\,ax\sqrt{\frac{c}{x}}+bc\right)-2\left(3\,abx\sqrt{\frac{c}{x}}-2\,a^2x\right)\sqrt{b\sqrt{\frac{c}{x}}+a}}{4\,a^3},-\frac{3\sqrt{-a}\,b^2c\arctan\left(\frac{\sqrt{}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(c/x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*sqrt(a)*b^2*c*log(2*sqrt(b*sqrt(c/x) + a)*sqrt(a)*x*sqrt(c/x) + 2*a
*x*sqrt(c/x) + b*c) - 2*(3*a*b*x*sqrt(c/x) - 2*a^2*x)*sqrt(b*sqrt(c/x) + a)
)/a^3, -1/2*(3*sqrt(-a)*b^2*c*arctan(sqrt(b*sqrt(c/x) + a)*sqrt(-a)/a) + (3
*a*b*x*sqrt(c/x) - 2*a^2*x)*sqrt(b*sqrt(c/x) + a))/a^3]
```

giac [A] time = 0.41, size = 139, normalized size = 1.46

$$\frac{\frac{3\,b^2c^2\log\left(c^2|b|\right)}{\sqrt{ac}\,a^2}-\frac{3\,b^2c^2\log\left(\left|-bc^2-2\sqrt{ac}\left(\sqrt{ac}\,\sqrt{cx}-\sqrt{ac^2x+\sqrt{cx}\,bc^2}\right)\right|\right)}{\sqrt{ac}\,a^2}-2\sqrt{ac^2x+\sqrt{cx}\,bc^2}\left(\frac{3\,b}{a^2}-\frac{2\sqrt{cx}}{ac}\right)}{4\sqrt{c}\,\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(c/x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*b^2*c^2*log(c^2*abs(b))/(sqrt(a*c)*a^2) - 3*b^2*c^2*log(abs(-b*c^2 -
 2*sqrt(a*c)*(sqrt(a*c)*sqrt(c*x) - sqrt(a*c^2*x + sqrt(c*x)*b*c^2))))/(sqr
t(a*c)*a^2) - 2*sqrt(a*c^2*x + sqrt(c*x)*b*c^2)*(3*b/a^2 - 2*sqrt(c*x)/(a*c
)))/(sqrt(c)*sgn(x))
```

maple [B] time = 0.04, size = 229, normalized size = 2.41

$$-\frac{\sqrt{a+\sqrt{\frac{c}{x}}\,b}\left(-4a\,b^2c\ln\left(\frac{2a\sqrt{x}+\sqrt{\frac{c}{x}}\,b\sqrt{x}+2\sqrt{\left(a+\sqrt{\frac{c}{x}}\,b\right)x}\,\sqrt{a}}{2\sqrt{a}}\right)+a\,b^2c\ln\left(\frac{2a\sqrt{x}+\sqrt{\frac{c}{x}}\,b\sqrt{x}+2\sqrt{ax+\sqrt{\frac{c}{x}}\,bx}\,\sqrt{a}}{2\sqrt{a}}\right)-4\sqrt{ax+\sqrt{}\right.}}{4\sqrt{\left(a+\sqrt{\frac{c}{x}}\,b\right)x}\,a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+(c/x)^(1/2)*b)^(1/2),x)
```

```
[Out] -1/4*(a+(c/x)^(1/2)*b)^(1/2)*x^(1/2)*(8*((a+(c/x)^(1/2)*b)*x)^(1/2)*a^(3/2)
*(c/x)^(1/2)*x^(1/2)*b-4*(a*x+(c/x)^(1/2)*b*x)^(1/2)*a^(5/2)*x^(1/2)-2*(a*x
+(c/x)^(1/2)*b*x)^(1/2)*(c/x)^(1/2)*a^(3/2)*b*x^(1/2)-4*c*a*ln(1/2*(2*a*x^(
1/2)+(c/x)^(1/2)*b*x^(1/2)+2*((a+(c/x)^(1/2)*b)*x)^(1/2)*a^(1/2))/a^(1/2))*
b^2+a*b^2*c*ln(1/2*(2*a*x^(1/2)+(c/x)^(1/2)*b*x^(1/2)+2*(a*x+(c/x)^(1/2)*b*
x)^(1/2)*a^(1/2))/a^(1/2)))/((a+(c/x)^(1/2)*b)*x)^(1/2)/a^(7/2)
```

maxima [A] time = 1.16, size = 131, normalized size = 1.38

$$-\frac{1}{4}\,c\left(\frac{3\,b^2\log\left(\frac{\sqrt{b\sqrt{\frac{c}{x}}+a}-\sqrt{a}}{\sqrt{b\sqrt{\frac{c}{x}}+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\left(3\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{3}{2}}b^2-5\sqrt{b\sqrt{\frac{c}{x}}+a}\,ab^2\right)}{\left(b\sqrt{\frac{c}{x}}+a\right)^2a^2-2\left(b\sqrt{\frac{c}{x}}+a\right)a^3+a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(c/x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*c*(3*b^2*log((sqrt(b*sqrt(c/x) + a) - sqrt(a))/(sqrt(b*sqrt(c/x) + a)
+ sqrt(a)))/a^(5/2) + 2*(3*(b*sqrt(c/x) + a)^(3/2)*b^2 - 5*sqrt(b*sqrt(c/x)
 + a)*a*b^2)/((b*sqrt(c/x) + a)^2*a^2 - 2*(b*sqrt(c/x) + a)*a^3 + a^4))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(c/x)^(1/2))^(1/2),x)
```

```
[Out] int(1/(a + b*(c/x)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(c/x)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sqrt(c/x)), x)
```

3.2990 $\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}\,x}}\,dx$

Optimal. Leaf size=31

$$\frac{4\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] 4*arctanh((a+b*(c/x)^(1/2))^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {369, 266, 63, 208}

$$\frac{4\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*Sqrt[c/x]]*x),x]
```

```
[Out] (4*ArcTanh[Sqrt[a + b*Sqrt[c/x]]/Sqrt[a]])/Sqrt[a]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}\,x}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}\,x}}\,dx, \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= -\text{Subst}\left(2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+b\sqrt{c}\,x}}\,dx, x, \frac{1}{\sqrt{x}}\right), \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= -\text{Subst}\left(\frac{4\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b\sqrt{c}}+\frac{x^2}{b\sqrt{c}}}\,dx, x, \sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}\right)}{b\sqrt{c}}, \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= \frac{4\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Mathematica [A] time = 0.05, size = 31, normalized size = 1.00

$$\frac{4\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*Sqrt[c/x]]*x),x]
```

```
[Out] (4*ArcTanh[Sqrt[a + b*Sqrt[c/x]]/Sqrt[a]])/Sqrt[a]
```

fricas [A] time = 0.96, size = 81, normalized size = 2.61

$$\left[\frac{2\,\log\left(2\sqrt{b\sqrt{\frac{c}{x}}+a}\,\sqrt{a}\,x\sqrt{\frac{c}{x}}+2\,ax\sqrt{\frac{c}{x}}+bc\right)}{\sqrt{a}}, -\frac{4\sqrt{-a}\,\arctan\left(\frac{\sqrt{b\sqrt{\frac{c}{x}}+a}\,\sqrt{-a}}{a}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c/x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] [2*log(2*sqrt(b*sqrt(c/x) + a)*sqrt(a)*x*sqrt(c/x) + 2*a*x*sqrt(c/x) + b*c)
/sqrt(a), -4*sqrt(-a)*arctan(sqrt(b*sqrt(c/x) + a)*sqrt(-a)/a)/a]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sqrt{\frac{c}{x}}+a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c/x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sqrt(c/x) + a)*x), x)
```

maple [B] time = 0.04, size = 200, normalized size = 6.45

$$\frac{\sqrt{a+\sqrt{\frac{c}{x}}\,b}\left(\sqrt{\frac{c}{x}}\,b\sqrt{x}\,\ln\left(\frac{2a\sqrt{x}+\sqrt{\frac{c}{x}}\,b\sqrt{x}+2\sqrt{\left(a+\sqrt{\frac{c}{x}}\,b\right)x}\,\sqrt{a}}{2\sqrt{a}}\right)+\sqrt{\frac{c}{x}}\,b\sqrt{x}\,\ln\left(\frac{2a\sqrt{x}+\sqrt{\frac{c}{x}}\,b\sqrt{x}+2\sqrt{ax+\sqrt{\frac{c}{x}}\,bx}\,\sqrt{a}}{2\sqrt{a}}\right)+2\sqrt{a}\right)}{\sqrt{\left(a+\sqrt{\frac{c}{x}}\,b\right)x}\,\sqrt{\frac{c}{x}}\,\sqrt{a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+(c/x)^(1/2)*b)^(1/2),x)
```

```
[Out] (a+(c/x)^(1/2)*b)^(1/2)*(b*(c/x)^(1/2)*x^(1/2)*ln(1/2*(2*a*x^(1/2)+(c/x)^(1
/2)*b*x^(1/2)+2*((a+(c/x)^(1/2)*b)*x)^(1/2)*a^(1/2))/a^(1/2))+b*(c/x)^(1/2)
*x^(1/2)*ln(1/2*(2*a*x^(1/2)+(c/x)^(1/2)*b*x^(1/2)+2*(a*x+(c/x)^(1/2)*b*x)^
(1/2)*a^(1/2))/a^(1/2))+2*(a*x+(c/x)^(1/2)*b*x)^(1/2)*a^(1/2)-2*((a+(c/x)^(
1/2)*b)*x)^(1/2)*a^(1/2))/((a+(c/x)^(1/2)*b)*x)^(1/2)/b/(c/x)^(1/2)/a^(1/2)
```

maxima [A] time = 1.11, size = 45, normalized size = 1.45

$$-\frac{2\log\left(\frac{\sqrt{b\sqrt{\frac{c}{x}}+a}-\sqrt{a}}{\sqrt{b\sqrt{\frac{c}{x}}+a}+\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c/x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*log((sqrt(b*sqrt(c/x) + a) - sqrt(a))/(sqrt(b*sqrt(c/x) + a) + sqrt(a)))
/sqrt(a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*(c/x)^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x*(a + b*(c/x)^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c/x)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*sqrt(c/x))), x)
```

3.2991 $\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}\, x^2}\, dx$

Optimal. Leaf size=54

$$\frac{4a\sqrt{a+b\sqrt{\frac{c}{x}}}}{b^2c} - \frac{4\left(a+b\sqrt{\frac{c}{x}}\right)^{3/2}}{3b^2c}$$

```
[Out] -4/3*(a+b*(c/x)^(1/2))^(3/2)/b^2/c+4*a*(a+b*(c/x)^(1/2))^(1/2)/b^2/c
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {369, 266, 43}

$$\frac{4a\sqrt{a+b\sqrt{\frac{c}{x}}}}{b^2c} - \frac{4\left(a+b\sqrt{\frac{c}{x}}\right)^{3/2}}{3b^2c}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*Sqrt[c/x]]*x^2),x]
```

```
[Out] (4*a*Sqrt[a + b*Sqrt[c/x]])/(b^2*c) - (4*(a + b*Sqrt[c/x])^(3/2))/(3*b^2*c)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}\,x^2}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}\,x^2}\,dx, \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$
$$= -\text{Subst}\left(2\,\text{Subst}\left(\int \frac{x}{\sqrt{a+b\sqrt{c}\,x}}\,dx, x, \frac{1}{\sqrt{x}}\right), \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$
$$= -\text{Subst}\left(2\,\text{Subst}\left(\int\left(-\frac{a}{b\sqrt{c}\sqrt{a+b\sqrt{c}\,x}}+\frac{\sqrt{a+b\sqrt{c}\,x}}{b\sqrt{c}}\right)dx, x, \frac{1}{\sqrt{x}}\right), \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$
$$= \frac{4a\sqrt{a+b\sqrt{\frac{c}{x}}}}{b^2c} - \frac{4\left(a+b\sqrt{\frac{c}{x}}\right)^{3/2}}{3b^2c}$$

Mathematica [A] time = 0.04, size = 42, normalized size = 0.78

$$-\frac{4\left(b\sqrt{\frac{c}{x}}-2a\right)\sqrt{a+b\sqrt{\frac{c}{x}}}}{3b^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*Sqrt[c/x]]*x^2),x]
```

```
[Out] (-4*(-2*a + b*Sqrt[c/x])*Sqrt[a + b*Sqrt[c/x]])/(3*b^2*c)
```

fricas [A] time = 0.74, size = 34, normalized size = 0.63

$$-\frac{4\sqrt{b\sqrt{\frac{c}{x}}+a}\left(b\sqrt{\frac{c}{x}}-2a\right)}{3b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(c/x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -4/3*sqrt(b*sqrt(c/x) + a)*(b*sqrt(c/x) - 2*a)/(b^2*c)
```

giac [A] time = 0.18, size = 60, normalized size = 1.11

$$-\frac{4\left(\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{3}{2}}b-3\sqrt{b\sqrt{\frac{c}{x}}+a}\,ab\right)\text{sgn}\left(\left(b\sqrt{\frac{c}{x}}+a\right)b-ab\right)}{3b^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(c/x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -4/3*((b*sqrt(c/x) + a)^(3/2)*b - 3*sqrt(b*sqrt(c/x) + a)*a*b)*sgn((b*sqrt(
c/x) + a)*b - a*b)/(b^3*c)
```

maple [C] time = 0.04, size = 274, normalized size = 5.07

$$-\frac{\sqrt{a+\sqrt{\frac{c}{x}}\,b}\left(-3\sqrt{\frac{c}{x}}\,a^2bx^2\ln\left(\frac{2a\sqrt{x}+\sqrt{\frac{c}{x}}\,b\sqrt{x}+2\sqrt{\left(a+\sqrt{\frac{c}{x}}\,b\right)x}\,\sqrt{a}}{2\sqrt{a}}\right)+3\sqrt{\frac{c}{x}}\,a^2bx^2\ln\left(\frac{2a\sqrt{x}+\sqrt{\frac{c}{x}}\,b\sqrt{x}+2\sqrt{ax+\sqrt{\frac{c}{x}}\,bx}\,\sqrt{a}}{2\sqrt{a}}\right)\right.}{3\sqrt{\left(a+\sqrt{\frac{c}{x}}\,b\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+(c/x)^(1/2)*b)^(1/2),x)
```

```
[Out] -1/3*(a+(c/x)^(1/2)*b)^(1/2)*(6*x^(3/2)*a^(5/2)*((a+(c/x)^(1/2)*b)*x)^(1/2)
+6*x^(3/2)*a^(5/2)*(a*x+(c/x)^(1/2)*b*x)^(1/2)-12*x^(1/2)*a^(3/2)*(a*x+(c/x
)^(1/2)*b*x)^(3/2)+4*x^(1/2)*a^(1/2)*(c/x)^(1/2)*(a*x+(c/x)^(1/2)*b*x)^(3/2
)*b+3*(c/x)^(1/2)*ln(1/2*(2*a*x^(1/2)+(c/x)^(1/2)*b*x^(1/2)+2*(a*x+(c/x)^(1
/2)*b*x)^(1/2)*a^(1/2))/a^(1/2))*x^2*a^2*b-3*(c/x)^(1/2)*ln(1/2*(2*a*x^(1/2
)+(c/x)^(1/2)*b*x^(1/2)+2*((a+(c/x)^(1/2)*b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^2
*a^2*b)/x^(5/2)/((a+(c/x)^(1/2)*b)*x)^(1/2)/b^3/(c/x)^(3/2)/a^(1/2)
```

maxima [A] time = 0.56, size = 42, normalized size = 0.78

$$-\frac{4\left(\frac{\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{3}{2}}}{b^2}-\frac{3\sqrt{b\sqrt{\frac{c}{x}}+a}\,a}{b^2}\right)}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(c/x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] -4/3*((b*sqrt(c/x) + a)^(3/2)/b^2 - 3*sqrt(b*sqrt(c/x) + a)*a/b^2)/c
```

mupad [B] time = 1.44, size = 52, normalized size = 0.96

$$-\frac{\sqrt{\frac{b\sqrt{\frac{c}{x}}}{a}+1}\;{}_2\mathrm{F}_1\left(\frac{1}{2},2;\;3;\;-\frac{b\sqrt{\frac{c}{x}}}{a}\right)}{x\sqrt{a+b\sqrt{\frac{c}{x}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*(c/x)^(1/2))^(1/2)),x)
```

```
[Out] -(((b*(c/x)^(1/2))/a + 1)^(1/2)*hypergeom([1/2, 2], 3, -(b*(c/x)^(1/2))/a))
/(x*(a + b*(c/x)^(1/2))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*(c/x)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a + b*sqrt(c/x))), x)
```

3.2992 $\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}\, x^3}\, dx$

Optimal. Leaf size=112

$$\frac{4a^3\sqrt{a+b\sqrt{\frac{c}{x}}}}{b^4c^2} - \frac{4a^2\left(a+b\sqrt{\frac{c}{x}}\right)^{3/2}}{b^4c^2} - \frac{4\left(a+b\sqrt{\frac{c}{x}}\right)^{7/2}}{7b^4c^2} + \frac{12a\left(a+b\sqrt{\frac{c}{x}}\right)^{5/2}}{5b^4c^2}$$

```
[Out] -4*a^2*(a+b*(c/x)^(1/2))^(3/2)/b^4/c^2+12/5*a*(a+b*(c/x)^(1/2))^(5/2)/b^4/c
^2-4/7*(a+b*(c/x)^(1/2))^(7/2)/b^4/c^2+4*a^3*(a+b*(c/x)^(1/2))^(1/2)/b^4/c^
2
```

Rubi [A] time = 0.07, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {369, 266, 43}

$$-\frac{4a^2\left(a+b\sqrt{\frac{c}{x}}\right)^{3/2}}{b^4c^2} + \frac{4a^3\sqrt{a+b\sqrt{\frac{c}{x}}}}{b^4c^2} - \frac{4\left(a+b\sqrt{\frac{c}{x}}\right)^{7/2}}{7b^4c^2} + \frac{12a\left(a+b\sqrt{\frac{c}{x}}\right)^{5/2}}{5b^4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*Sqrt[c/x]]*x^3),x]
```

```
[Out] (4*a^3*Sqrt[a + b*Sqrt[c/x]])/(b^4*c^2) - (4*a^2*(a + b*Sqrt[c/x])^(3/2))/(
b^4*c^2) + (12*a*(a + b*Sqrt[c/x])^(5/2))/(5*b^4*c^2) - (4*(a + b*Sqrt[c/x]
)^(7/2))/(7*b^4*c^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}\,x^3}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}\,x^3}\,dx, \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= -\text{Subst}\left(2\,\text{Subst}\left(\int \frac{x^3}{\sqrt{a+b\sqrt{c}\,x}}\,dx, x, \frac{1}{\sqrt{x}}\right), \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= -\text{Subst}\left(2\,\text{Subst}\left(\int \left(-\frac{a^3}{b^3c^{3/2}\sqrt{a+b\sqrt{c}\,x}} + \frac{3a^2\sqrt{a+b\sqrt{c}\,x}}{b^3c^{3/2}} - \frac{3a\left(a+b\sqrt{c}\,x\right)^{3/2}}{b^3c^{3/2}} + \frac{(a\right.\right.\right.$$

$$= \frac{4a^3\sqrt{a+b\sqrt{\frac{c}{x}}}}{b^4c^2} - \frac{4a^2\left(a+b\sqrt{\frac{c}{x}}\right)^{3/2}}{b^4c^2} + \frac{12a\left(a+b\sqrt{\frac{c}{x}}\right)^{5/2}}{5b^4c^2} - \frac{4\left(a+b\sqrt{\frac{c}{x}}\right)^{7/2}}{7b^4c^2}$$

Mathematica [A] time = 0.08, size = 75, normalized size = 0.67

$$\frac{4\sqrt{a+b\sqrt{\frac{c}{x}}}\left(16a^3x - 8a^2bx\sqrt{\frac{c}{x}} + 6ab^2c - 5b^3c\sqrt{\frac{c}{x}}\right)}{35b^4c^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*Sqrt[c/x]]*x^3),x]
```

```
[Out] (4*Sqrt[a + b*Sqrt[c/x]]*(6*a*b^2*c - 5*b^3*c*Sqrt[c/x] + 16*a^3*x - 8*a^2*
b*Sqrt[c/x]*x))/(35*b^4*c^2*x)
```

fricas [A] time = 1.08, size = 61, normalized size = 0.54

$$\frac{4\left(6\,ab^2c + 16\,a^3x - \left(5\,b^3c + 8\,a^2bx\right)\sqrt{\frac{c}{x}}\right)\sqrt{b\sqrt{\frac{c}{x}} + a}}{35\,b^4c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(c/x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 4/35*(6*a*b^2*c + 16*a^3*x - (5*b^3*c + 8*a^2*b*x)*sqrt(c/x))*sqrt(b*sqrt(c
/x) + a)/(b^4*c^2*x)
```

giac [A] time = 0.25, size = 152, normalized size = 1.36

$$-\frac{4\left(5\left(b\sqrt{\frac{c}{x}} + a\right)^{\frac{7}{2}}\text{sgn}\left(\left(b\sqrt{\frac{c}{x}} + a\right)b - ab\right) - 21\left(b\sqrt{\frac{c}{x}} + a\right)^{\frac{5}{2}}a\text{sgn}\left(\left(b\sqrt{\frac{c}{x}} + a\right)b - ab\right) + 35\left(b\sqrt{\frac{c}{x}} + a\right)^{\frac{3}{2}}a^2\text{sgn}\right.}{35\,b^4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(c/x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -4/35*(5*(b*sqrt(c/x) + a)^(7/2)*sgn((b*sqrt(c/x) + a)*b - a*b) - 21*(b*sqr
t(c/x) + a)^(5/2)*a*sgn((b*sqrt(c/x) + a)*b - a*b) + 35*(b*sqrt(c/x) + a)^(
3/2)*a^2*sgn((b*sqrt(c/x) + a)*b - a*b) - 35*sqrt(b*sqrt(c/x) + a)*a^3*sgn(
(b*sqrt(c/x) + a)*b - a*b))/(b^4*c^2)
```

maple [C] time = 0.05, size = 336, normalized size = 3.00

$$-\frac{\sqrt{a+\sqrt{\frac{c}{x}}\,b}\left(-35\sqrt{\frac{c}{x}}\,a^4 b\,x^3\ln\left(\frac{2a\sqrt{x}+\sqrt{\frac{c}{x}}\,b\sqrt{x}+2\sqrt{\left(a+\sqrt{\frac{c}{x}}\,b\right)x}\,\sqrt{a}}{2\sqrt{a}}\right)+35\sqrt{\frac{c}{x}}\,a^4 b\,x^3\ln\left(\frac{2a\sqrt{x}+\sqrt{\frac{c}{x}}\,b\sqrt{x}+2\sqrt{ax+\sqrt{\frac{c}{x}}\,bx}\,\sqrt{a}}{2\sqrt{a}}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+(c/x)^(1/2)*b)^(1/2),x)
```

```
[Out] -1/35*(a+(c/x)^(1/2)*b)^(1/2)*(70*x^(5/2)*(a*x+(c/x)^(1/2)*b*x)^(1/2)*a^(9/
2)+70*x^(5/2)*a^(9/2)*((a+(c/x)^(1/2)*b)*x)^(1/2)-140*x^(3/2)*(a*x+(c/x)^(1
/2)*b*x)^(3/2)*a^(7/2)+76*x^(3/2)*(a*x+(c/x)^(1/2)*b*x)^(3/2)*a^(5/2)*(c/x)
^(1/2)*b+20*x^(3/2)*(a*x+(c/x)^(1/2)*b*x)^(3/2)*a^(1/2)*(c/x)^(3/2)*b^3-44*
x^(1/2)*(a*x+(c/x)^(1/2)*b*x)^(3/2)*a^(3/2)*b^2*c-35*ln(1/2*(2*a*x^(1/2)+(c
/x)^(1/2)*b*x^(1/2)+2*((a+(c/x)^(1/2)*b)*x)^(1/2)*a^(1/2))/a^(1/2))*(c/x)^(
1/2)*x^3*a^4*b+35*ln(1/2*(2*a*x^(1/2)+(c/x)^(1/2)*b*x^(1/2)+2*(a*x+(c/x)^(1
/2)*b*x)^(1/2)*a^(1/2))/a^(1/2))*(c/x)^(1/2)*x^3*a^4*b)/x^(9/2)/((a+(c/x)^(
1/2)*b)*x)^(1/2)/b^5/(c/x)^(5/2)/a^(1/2)
```

maxima [A] time = 0.53, size = 85, normalized size = 0.76

$$-\frac{4\left(\frac{5\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{7}{2}}}{b^4}-\frac{21\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{5}{2}}a}{b^4}+\frac{35\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{3}{2}}a^2}{b^4}-\frac{35\sqrt{b\sqrt{\frac{c}{x}}+a}\,a^3}{b^4}\right)}{35\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(c/x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] -4/35*(5*(b*sqrt(c/x) + a)^(7/2)/b^4 - 21*(b*sqrt(c/x) + a)^(5/2)*a/b^4 + 3
5*(b*sqrt(c/x) + a)^(3/2)*a^2/b^4 - 35*sqrt(b*sqrt(c/x) + a)*a^3/b^4)/c^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*(c/x)^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x^3*(a + b*(c/x)^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*(c/x)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a + b*sqrt(c/x))), x)
```

3.2993 $\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}\, x^4}\, dx$

Optimal. Leaf size=172

$$\frac{4a^5\sqrt{a+b\sqrt{\frac{c}{x}}}}{b^6c^3}-\frac{20a^4\left(a+b\sqrt{\frac{c}{x}}\right)^{3/2}}{3b^6c^3}+\frac{8a^3\left(a+b\sqrt{\frac{c}{x}}\right)^{5/2}}{b^6c^3}-\frac{40a^2\left(a+b\sqrt{\frac{c}{x}}\right)^{7/2}}{7b^6c^3}-\frac{4\left(a+b\sqrt{\frac{c}{x}}\right)^{11/2}}{11b^6c^3}+\frac{20a\left(a+b\sqrt{\frac{c}{x}}\right.}{9b^6c^3}$$

```
[Out] -20/3*a^4*(a+b*(c/x)^(1/2))^(3/2)/b^6/c^3+8*a^3*(a+b*(c/x)^(1/2))^(5/2)/b^6
/c^3-40/7*a^2*(a+b*(c/x)^(1/2))^(7/2)/b^6/c^3+20/9*a*(a+b*(c/x)^(1/2))^(9/2
)/b^6/c^3-4/11*(a+b*(c/x)^(1/2))^(11/2)/b^6/c^3+4*a^5*(a+b*(c/x)^(1/2))^(1/
2)/b^6/c^3
```

Rubi [A] time = 0.10, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {369, 266, 43}

$$-\frac{40a^2\left(a+b\sqrt{\frac{c}{x}}\right)^{7/2}}{7b^6c^3}+\frac{8a^3\left(a+b\sqrt{\frac{c}{x}}\right)^{5/2}}{b^6c^3}-\frac{20a^4\left(a+b\sqrt{\frac{c}{x}}\right)^{3/2}}{3b^6c^3}+\frac{4a^5\sqrt{a+b\sqrt{\frac{c}{x}}}}{b^6c^3}-\frac{4\left(a+b\sqrt{\frac{c}{x}}\right)^{11/2}}{11b^6c^3}+\frac{20a\left(a+b.\right.}{9b^6c}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*Sqrt[c/x]]*x^4),x]
```

```
[Out] (4*a^5*Sqrt[a + b*Sqrt[c/x]])/(b^6*c^3) - (20*a^4*(a + b*Sqrt[c/x])^(3/2))/
(3*b^6*c^3) + (8*a^3*(a + b*Sqrt[c/x])^(5/2))/(b^6*c^3) - (40*a^2*(a + b*Sq
rt[c/x])^(7/2))/(7*b^6*c^3) + (20*a*(a + b*Sqrt[c/x])^(9/2))/(9*b^6*c^3) -
(4*(a + b*Sqrt[c/x])^(11/2))/(11*b^6*c^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}\,x^4}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}\,x^4}\,dx, \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= -\text{Subst}\left(2\,\text{Subst}\left(\int \frac{x^5}{\sqrt{a+b\sqrt{c}\,x}}\,dx, x, \frac{1}{\sqrt{x}}\right), \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= -\text{Subst}\left(2\,\text{Subst}\left(\int \left(-\frac{a^5}{b^5c^{5/2}\sqrt{a+b\sqrt{c}\,x}} + \frac{5a^4\sqrt{a+b\sqrt{c}\,x}}{b^5c^{5/2}} - \frac{10a^3\left(a+b\sqrt{c}\,x\right)^{3/2}}{b^5c^{5/2}} + \frac{10}{}\right.\right.\right.$$

$$= \frac{4a^5\sqrt{a+b\sqrt{\frac{c}{x}}}}{b^6c^3} - \frac{20a^4\left(a+b\sqrt{\frac{c}{x}}\right)^{3/2}}{3b^6c^3} + \frac{8a^3\left(a+b\sqrt{\frac{c}{x}}\right)^{5/2}}{b^6c^3} - \frac{40a^2\left(a+b\sqrt{\frac{c}{x}}\right)^{7/2}}{7b^6c^3} + \frac{20a\left(}{}$$

Mathematica [A] time = 0.09, size = 111, normalized size = 0.65

$$\frac{4\sqrt{a+b\sqrt{\frac{c}{x}}}\left(256a^5x^2 - 128a^4bx^2\sqrt{\frac{c}{x}} + 96a^3b^2cx - 80a^2b^3cx\sqrt{\frac{c}{x}} + 70ab^4c^2 - 63b^5cx\left(\frac{c}{x}\right)^{3/2}\right)}{693b^6c^3x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*Sqrt[c/x]]*x^4),x]
```

```
[Out] (4*Sqrt[a + b*Sqrt[c/x]]*(70*a*b^4*c^2 + 96*a^3*b^2*c*x - 80*a^2*b^3*c*Sqrt
[c/x]*x - 63*b^5*c*(c/x)^(3/2)*x + 256*a^5*x^2 - 128*a^4*b*Sqrt[c/x]*x^2))/
(693*b^6*c^3*x^2)
```

fricas [A] time = 0.91, size = 89, normalized size = 0.52

$$\frac{4\left(70\,ab^4c^2 + 96\,a^3b^2cx + 256\,a^5x^2 - \left(63\,b^5c^2 + 80\,a^2b^3cx + 128\,a^4bx^2\right)\sqrt{\frac{c}{x}}\right)\sqrt{b\sqrt{\frac{c}{x}} + a}}{693\,b^6c^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*(c/x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 4/693*(70*a*b^4*c^2 + 96*a^3*b^2*c*x + 256*a^5*x^2 - (63*b^5*c^2 + 80*a^2*b
^3*c*x + 128*a^4*b*x^2)*sqrt(c/x))*sqrt(b*sqrt(c/x) + a)/(b^6*c^3*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sqrt{\frac{c}{x}} + a}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*(c/x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sqrt(c/x) + a)*x^4), x)
```

maple [C] time = 0.06, size = 400, normalized size = 2.33

$$-\frac{\sqrt{a+\sqrt{\frac{c}{x}}\,b}\left(-693\sqrt{\frac{c}{x}}\,a^6 b\,x^4\ln\left(\frac{2a\sqrt{x}+\sqrt{\frac{c}{x}}\,b\sqrt{x}+2\sqrt{\left(a+\sqrt{\frac{c}{x}}\,b\right)x}\,\sqrt{a}}{2\sqrt{a}}\right)+693\sqrt{\frac{c}{x}}\,a^6 b\,x^4\ln\left(\frac{2a\sqrt{x}+\sqrt{\frac{c}{x}}\,b\sqrt{x}+2\sqrt{ax+\sqrt{}}\right.\right.}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a+(c/x)^(1/2)*b)^(1/2),x)
```

```
[Out] -1/693*(a+(c/x)^(1/2)*b)^(1/2)*(1386*x^(7/2)*(a*x+(c/x)^(1/2)*b*x)^(1/2)*a^
(13/2)+1386*x^(7/2)*((a+(c/x)^(1/2)*b)*x)^(1/2)*a^(13/2)+252*x^(5/2)*(a*x+(
c/x)^(1/2)*b*x)^(3/2)*(c/x)^(5/2)*a^(1/2)*b^5+852*x^(5/2)*(a*x+(c/x)^(1/2)*
b*x)^(3/2)*(c/x)^(3/2)*a^(5/2)*b^3+1748*x^(5/2)*(a*x+(c/x)^(1/2)*b*x)^(3/2)
*(c/x)^(1/2)*a^(9/2)*b-2772*x^(5/2)*(a*x+(c/x)^(1/2)*b*x)^(3/2)*a^(11/2)-12
36*x^(3/2)*(a*x+(c/x)^(1/2)*b*x)^(3/2)*a^(7/2)*b^2*c-532*x^(1/2)*(a*x+(c/x)
^(1/2)*b*x)^(3/2)*a^(3/2)*b^4*c^2-693*ln(1/2*(2*a*x^(1/2)+(c/x)^(1/2)*b*x^(
1/2)+2*((a+(c/x)^(1/2)*b)*x)^(1/2)*a^(1/2))/a^(1/2))*(c/x)^(1/2)*x^4*a^6*b+
693*ln(1/2*(2*a*x^(1/2)+(c/x)^(1/2)*b*x^(1/2)+2*(a*x+(c/x)^(1/2)*b*x)^(1/2)
*a^(1/2))/a^(1/2))*(c/x)^(1/2)*x^4*a^6*b)/x^(13/2)/((a+(c/x)^(1/2)*b)*x)^(1
/2)/b^7/(c/x)^(7/2)/a^(1/2)
```

maxima [A] time = 0.48, size = 127, normalized size = 0.74

$$-\frac{4\left(\frac{63\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{11}{2}}}{b^6}-\frac{385\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{9}{2}}a}{b^6}+\frac{990\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{7}{2}}a^2}{b^6}-\frac{1386\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{5}{2}}a^3}{b^6}+\frac{1155\left(b\sqrt{\frac{c}{x}}+a\right)^{\frac{3}{2}}a^4}{b^6}-\frac{693\sqrt{b\sqrt{\frac{c}{x}}+a}\,a^5}{b^6}\right)}{693\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*(c/x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] -4/693*(63*(b*sqrt(c/x) + a)^(11/2)/b^6 - 385*(b*sqrt(c/x) + a)^(9/2)*a/b^6
 + 990*(b*sqrt(c/x) + a)^(7/2)*a^2/b^6 - 1386*(b*sqrt(c/x) + a)^(5/2)*a^3/b
^6 + 1155*(b*sqrt(c/x) + a)^(3/2)*a^4/b^6 - 693*sqrt(b*sqrt(c/x) + a)*a^5/b
^6)/c^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*(c/x)^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x^4*(a + b*(c/x)^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a+b*(c/x)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(a + b*sqrt(c/x))), x)
```

3.2994 $\int \frac{1}{\sqrt{1+\sqrt{\frac{1}{x}}}}\,dx$

Optimal. Leaf size=58

$$\sqrt{\sqrt{\frac{1}{x}}+1}\,x-\frac{3\sqrt{\sqrt{\frac{1}{x}}+1}}{2\sqrt{\frac{1}{x}}}+\frac{3}{2}\tanh^{-1}\left(\sqrt{\sqrt{\frac{1}{x}}+1}\right)$$

```
[Out] 3/2*arctanh((1+(1/x)^(1/2))^(1/2))+x*(1+(1/x)^(1/2))^(1/2)-3/2*(1+(1/x)^(1/
2))^(1/2)/(1/x)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {255, 190, 51, 63, 207}

$$\sqrt{\sqrt{\frac{1}{x}}+1}\,x-\frac{3\sqrt{\sqrt{\frac{1}{x}}+1}}{2\sqrt{\frac{1}{x}}}+\frac{3}{2}\tanh^{-1}\left(\sqrt{\sqrt{\frac{1}{x}}+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + Sqrt[x^(-1)]],x]
```

```
[Out] (-3*Sqrt[1 + Sqrt[x^(-1)]])/(2*Sqrt[x^(-1)]) + Sqrt[1 + Sqrt[x^(-1)]]*x + (
3*ArcTanh[Sqrt[1 + Sqrt[x^(-1)]]])/2
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 255

```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_.), x_Symbol] :> With[{k = De
nominator[n]}, Subst[Int[(a + b*c^n*x^(n*q))^p, x], x^(1/k), (c*x^q)^(1/k)/
(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c, p, q}, x] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{1+\sqrt{\frac{1}{x}}}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{1}{\sqrt{x}}}}\,dx, \sqrt{x}, \frac{1}{\sqrt{\frac{1}{x}}}\right) \\
&= -\text{Subst}\left(2\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{1+x}}\,dx, x, \frac{1}{\sqrt{x}}\right), \sqrt{x}, \frac{1}{\sqrt{\frac{1}{x}}}\right) \\
&= \sqrt{1+\sqrt{\frac{1}{x}}}\,x + \text{Subst}\left(\frac{3}{2}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x}}\,dx, x, \frac{1}{\sqrt{x}}\right), \sqrt{x}, \frac{1}{\sqrt{\frac{1}{x}}}\right) \\
&= -\frac{3\sqrt{1+\sqrt{\frac{1}{x}}}}{2\sqrt{\frac{1}{x}}} + \sqrt{1+\sqrt{\frac{1}{x}}}\,x - \text{Subst}\left(\frac{3}{4}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \frac{1}{\sqrt{x}}\right), \sqrt{x}, \frac{1}{\sqrt{\frac{1}{x}}}\right) \\
&= -\frac{3\sqrt{1+\sqrt{\frac{1}{x}}}}{2\sqrt{\frac{1}{x}}} + \sqrt{1+\sqrt{\frac{1}{x}}}\,x - \text{Subst}\left(\frac{3}{2}\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\frac{1}{\sqrt{x}}}\right), \sqrt{x}, \frac{1}{\sqrt{\frac{1}{x}}}\right) \\
&= -\frac{3\sqrt{1+\sqrt{\frac{1}{x}}}}{2\sqrt{\frac{1}{x}}} + \sqrt{1+\sqrt{\frac{1}{x}}}\,x + \frac{3}{2}\tanh^{-1}\left(\sqrt{1+\sqrt{\frac{1}{x}}}\right)
\end{aligned}$$

Mathematica [A] time = 0.14, size = 70, normalized size = 1.21

$$\frac{1}{4}\left(2\left(2-3\sqrt{\frac{1}{x}}\right)\sqrt{\sqrt{\frac{1}{x}}+1}\,x - 3\log\left(1-\frac{1}{\sqrt{\sqrt{\frac{1}{x}}+1}}\right) + 3\log\left(\frac{1}{\sqrt{\sqrt{\frac{1}{x}}+1}}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + Sqrt[x^(-1)]],x]
```

```
[Out] (2*(2 - 3*Sqrt[x^(-1)])*Sqrt[1 + Sqrt[x^(-1)]]*x - 3*Log[1 - 1/Sqrt[1 + Sqr
t[x^(-1)]]] + 3*Log[1 + 1/Sqrt[1 + Sqrt[x^(-1)]]])/4
```

fricas [A] time = 0.73, size = 55, normalized size = 0.95

$$\frac{1}{2}\left(2x-3\sqrt{x}\right)\sqrt{\frac{x+\sqrt{x}}{x}} + \frac{3}{4}\log\left(\sqrt{\frac{x+\sqrt{x}}{x}}+1\right) - \frac{3}{4}\log\left(\sqrt{\frac{x+\sqrt{x}}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(1/x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*x - 3*sqrt(x))*sqrt((x + sqrt(x))/x) + 3/4*log(sqrt((x + sqrt(x))/x)
 + 1) - 3/4*log(sqrt((x + sqrt(x))/x) - 1)
```

giac [A] time = 0.24, size = 36, normalized size = 0.62

$$\frac{1}{2}\sqrt{x+\sqrt{x}}\left(2\sqrt{x}-3\right)-\frac{3}{4}\log\left(-2\sqrt{x+\sqrt{x}}+2\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(1/x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x + sqrt(x))*(2*sqrt(x) - 3) - 3/4*log(-2*sqrt(x + sqrt(x)) + 2*sq
rt(x) + 1)
```

maple [B] time = 0.02, size = 92, normalized size = 1.59

$$-\frac{\sqrt{1+\sqrt{\frac{1}{x}}}\left(-3\ln\left(\frac{\sqrt{\frac{1}{x}}\sqrt{x}}{2}+\sqrt{x}+\sqrt{\sqrt{\frac{1}{x}}x+x}\right)+6\sqrt{\sqrt{\frac{1}{x}}x+x}\sqrt{\frac{1}{x}}\sqrt{x}-4\sqrt{\sqrt{\frac{1}{x}}x+x}\sqrt{x}\right)\sqrt{x}}{4\sqrt{\left(1+\sqrt{\frac{1}{x}}\right)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+(1/x)^(1/2))^(1/2),x)
```

```
[Out] -1/4*(1+(1/x)^(1/2))^(1/2)*x^(1/2)*(6*((1/x)^(1/2)*x+x)^(1/2)*(1/x)^(1/2)*x
^(1/2)-4*((1/x)^(1/2)*x+x)^(1/2)*x^(1/2)-3*ln(1/2*(1/x)^(1/2)*x^(1/2)+x^(1/
2)+((1/x)^(1/2)*x+x)^(1/2)))/(x*(1+(1/x)^(1/2)))^(1/2)
```

maxima [A] time = 0.55, size = 62, normalized size = 1.07

$$-\frac{3\left(\frac{1}{\sqrt{x}}+1\right)^{\frac{3}{2}}-5\sqrt{\frac{1}{\sqrt{x}}+1}}{2\left(\left(\frac{1}{\sqrt{x}}+1\right)^{2}-\frac{2}{\sqrt{x}}-1\right)}+\frac{3}{4}\log\left(\sqrt{\frac{1}{\sqrt{x}}+1}+1\right)-\frac{3}{4}\log\left(\sqrt{\frac{1}{\sqrt{x}}+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(1/x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(3*(1/sqrt(x) + 1)^(3/2) - 5*sqrt(1/sqrt(x) + 1))/((1/sqrt(x) + 1)^2 -
 2/sqrt(x) - 1) + 3/4*log(sqrt(1/sqrt(x) + 1) + 1) - 3/4*log(sqrt(1/sqrt(x)
 + 1) - 1)
```

mupad [B] time = 1.88, size = 37, normalized size = 0.64

$$\frac{3\,\mathrm{atanh}\left(\sqrt{\sqrt{\frac{1}{x}}+1}\right)}{2}+\frac{5x\sqrt{\sqrt{\frac{1}{x}}+1}}{2}-\frac{3x\left(\sqrt{\frac{1}{x}}+1\right)^{3/2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1/x)^(1/2) + 1)^(1/2),x)
```

```
[Out] (3*atanh(((1/x)^(1/2) + 1)^(1/2)))/2 + (5*x*((1/x)^(1/2) + 1)^(1/2))/2 - (3
*x*((1/x)^(1/2) + 1)^(3/2))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{\sqrt{\frac{1}{x}}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(1/x)**(1/2))**(1/2),x)
[Out] Integral(1/sqrt(sqrt(1/x) + 1), x)
```

3.2995 $\int \sqrt{a+b\left(\frac{c}{x}\right)^{3/2}}\,(dx)^m\,dx$

Optimal. Leaf size=102

$$\frac{x(dx)^m\sqrt{a+\frac{bc^3}{x^3\left(\frac{c}{x}\right)^{3/2}}}\;{}_2F_1\left(-\frac{1}{2},-\frac{2}{3}(m+1);\frac{1}{3}(1-2m);-\frac{bc^3}{a\left(\frac{c}{x}\right)^{3/2}x^3}\right)}{(m+1)\sqrt{\frac{bc^3}{ax^3\left(\frac{c}{x}\right)^{3/2}}+1}}$$

```
[Out] x*(d*x)^m*hypergeom([-1/2, -2/3-2/3*m],[1/3-2/3*m],-b*c^3/a/(c/x)^(3/2)/x^3
)*(a+b*c^3/(c/x)^(3/2)/x^3)^(1/2)/(1+m)/(1+b*c^3/a/(c/x)^(3/2)/x^3)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {369, 343, 341, 339, 365, 364}

$$\frac{x(dx)^m\sqrt{a+\frac{bc^3}{x^3\left(\frac{c}{x}\right)^{3/2}}}\;{}_2F_1\left(-\frac{1}{2},-\frac{2}{3}(m+1);\frac{1}{3}(1-2m);-\frac{bc^3}{a\left(\frac{c}{x}\right)^{3/2}x^3}\right)}{(m+1)\sqrt{\frac{bc^3}{ax^3\left(\frac{c}{x}\right)^{3/2}}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*(c/x)^(3/2)]*(d*x)^m,x]
```

```
[Out] (Sqrt[a + (b*c^3)/((c/x)^(3/2)*x^3)]*x*(d*x)^m*Hypergeometric2F1[-1/2, (-2*
(1 + m))/3, (1 - 2*m)/3, -((b*c^3)/(a*(c/x)^(3/2)*x^3))])/((1 + m)*Sqrt[1 +
 (b*c^3)/(a*(c/x)^(3/2)*x^3)])
```

Rule 339

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Dist[((c
*x)^(m + 1)*(1/x)^(m + 1))/c, Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x
], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 343

```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(c^In
tPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a + b*x^n)^p, x], x] /;
 FreeQ[{a, b, c, m, p}, x] && FractionQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Rubi steps

$$\int \sqrt{a+b\left(\frac{c}{x}\right)^{3/2}}\,(dx)^m\,dx = \text{Subst}\left(\int \sqrt{a+\frac{bc^{3/2}}{x^{3/2}}}\,(dx)^m\,dx, \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= \text{Subst}\left((x^{-m}(dx)^m)\int \sqrt{a+\frac{bc^{3/2}}{x^{3/2}}}\,x^m\,dx, \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= \text{Subst}\left((2x^{-m}(dx)^m)\,\text{Subst}\left(\int \sqrt{a+\frac{bc^{3/2}}{x^3}}\,x^{-1+2(1+m)}\,dx, x, \sqrt{x}\right), \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= -\text{Subst}\left((2x^{-m}(dx)^m)\,\text{Subst}\left(\int x^{-1-2(1+m)}\sqrt{a+bc^{3/2}x^3}\,dx, x, \frac{1}{\sqrt{x}}\right), \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= -\text{Subst}\left(\frac{\left(2\sqrt{a+\frac{bc^{3/2}}{x^{3/2}}}\,x^{-m}(dx)^m\right)\text{Subst}\left(\int x^{-1-2(1+m)}\sqrt{1+\frac{bc^{3/2}x^3}{a}}\,dx, x, \frac{1}{\sqrt{x}}\right)}{\sqrt{1+\frac{bc^{3/2}}{ax^{3/2}}}}, \sqrt{x}\right.$$

$$= \frac{\sqrt{a+\frac{bc^3}{\left(\frac{c}{x}\right)^{3/2}x^3}}\,x(dx)^m\,{}_2F_1\left(-\frac{1}{2}, -\frac{2}{3}(1+m); \frac{1}{3}(1-2m); -\frac{bc^3}{a\left(\frac{c}{x}\right)^{3/2}x^3}\right)}{(1+m)\sqrt{1+\frac{bc^3}{a\left(\frac{c}{x}\right)^{3/2}x^3}}}$$

Mathematica [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\left(\frac{c}{x}\right)^{3/2}}\,(dx)^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*(c/x)^(3/2)]*(d*x)^m,x]
```

```
[Out] Integrate[Sqrt[a + b*(c/x)^(3/2)]*(d*x)^m, x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*(c/x)^(3/2))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   alglo
gextint: unimplemented
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\left(\frac{c}{x}\right)^{\frac{3}{2}} + a}\ (dx)^m\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*(c/x)^(3/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*(c/x)^(3/2) + a)*(d*x)^m, x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \sqrt{a + \left(\frac{c}{x}\right)^{\frac{3}{2}} b}\ (dx)^m\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*(c/x)^(3/2))^(1/2),x)
```

```
[Out] int((d*x)^m*(a+b*(c/x)^(3/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\left(\frac{c}{x}\right)^{\frac{3}{2}} + a}\ (dx)^m\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*(c/x)^(3/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*(c/x)^(3/2) + a)*(d*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m \sqrt{a + b\left(\frac{c}{x}\right)^{3/2}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*(c/x)^(3/2))^(1/2),x)
```

```
[Out] int((d*x)^m*(a + b*(c/x)^(3/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{a + b\left(\frac{c}{x}\right)^{\frac{3}{2}}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*(c/x)**(3/2))**(1/2),x)
```

```
[Out] Integral((d*x)**m*sqrt(a + b*(c/x)**(3/2)), x)
```

3.2996 $\int \sqrt{a+b\sqrt{\frac{c}{x}}}\,(dx)^m\,dx$

Optimal. Leaf size=60

$$\frac{4x^{m+1}\left(a+b\sqrt{\frac{c}{x}}\right)^{3/2}\,{}_2F_1\left(1,\frac{1}{2}(-4m-1);\frac{5}{2};\frac{a+b\sqrt{\frac{c}{x}}}{a}\right)}{3a}$$

```
[Out] 4/3*x^(1+m)*hypergeom([1, -1/2-2*m],[5/2],(a+b*(c/x)^(1/2))/a)*(a+b*(c/x)^(
1/2))^(3/2)/a
```

Rubi [A] time = 0.09, antiderivative size = 80, normalized size of antiderivative = 1.33, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {369, 343, 341, 339, 67, 65}

$$\frac{4b^2c(dx)^m\left(a+b\sqrt{\frac{c}{x}}\right)^{3/2}\left(-\frac{b\sqrt{\frac{c}{x}}}{a}\right)^{2m}\,{}_2F_1\left(\frac{3}{2},2m+3;\frac{5}{2};\frac{\sqrt{\frac{c}{x}}\,b}{a}+1\right)}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[c/x]]*(d*x)^m,x]
```

```
[Out] (4*b^2*c*(a + b*Sqrt[c/x])^(3/2)*(-((b*Sqrt[c/x])/a))^(2*m)*(d*x)^m*Hyperge
ometric2F1[3/2, 3 + 2*m, 5/2, 1 + (b*Sqrt[c/x])/a])/(3*a^3)
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0]
```

Rule 339

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Dist[((c
*x)^(m + 1)*(1/x)^(m + 1))/c, Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x
], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 343

```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(c^In
tPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a + b*x^n)^p, x], x] /;
 FreeQ[{a, b, c, m, p}, x] && FractionQ[n]
```

Rule 369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Rubi steps

$$\int \sqrt{a+b\sqrt{\frac{c}{x}}}\,(dx)^m\,dx = \text{Subst}\left(\int \sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}\,(dx)^m\,dx, \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= \text{Subst}\left((x^{-m}(dx)^m)\int \sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}\,x^m\,dx, \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= \text{Subst}\left((2x^{-m}(dx)^m)\,\text{Subst}\left(\int \sqrt{a+\frac{b\sqrt{c}}{x}}\,x^{-1+2(1+m)}\,dx, x, \sqrt{x}\right), \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= -\text{Subst}\left((2x^{-m}(dx)^m)\,\text{Subst}\left(\int x^{-1-2(1+m)}\sqrt{a+b\sqrt{c}\,x}\,dx, x, \frac{1}{\sqrt{x}}\right), \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= \text{Subst}\left(\frac{\left(2b^3c^{3/2}\left(-\frac{b\sqrt{c}}{a\sqrt{x}}\right)^{2m}(dx)^m\right)\text{Subst}\left(\int \left(-\frac{b\sqrt{c}\,x}{a}\right)^{-1-2(1+m)}\sqrt{a+b\sqrt{c}\,x}\,dx, x, \frac{1}{\sqrt{x}}\right)}{a^3}, \right.$$

$$= \frac{4b^2c\left(a+b\sqrt{\frac{c}{x}}\right)^{3/2}\left(-\frac{b\sqrt{\frac{c}{x}}}{a}\right)^{2m}(dx)^m\,{}_2F_1\left(\frac{3}{2}, 3+2m; \frac{5}{2}; 1+\frac{b\sqrt{\frac{c}{x}}}{a}\right)}{3a^3}$$

Mathematica [A] time = 0.10, size = 78, normalized size = 1.30

$$\frac{x(dx)^m\sqrt{a+b\sqrt{\frac{c}{x}}}\;{}_2F_1\left(-\frac{1}{2}, -2(m+1); -2m-1; -\frac{b\sqrt{\frac{c}{x}}}{a}\right)}{(m+1)\sqrt{\frac{b\sqrt{\frac{c}{x}}}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[c/x]]*(d*x)^m,x]
```

```
[Out] (Sqrt[a + b*Sqrt[c/x]]*x*(d*x)^m*Hypergeometric2F1[-1/2, -2*(1 + m), -1 - 2
*m, -((b*Sqrt[c/x])/a)])/((1 + m)*Sqrt[1 + (b*Sqrt[c/x])/a])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*(c/x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   alglo
gextint: unimplemented
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*(c/x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, integration of abs or sign assumes constant sign by intervals (cor
rect if the argument is real):Check [abs(t_nostep)]Unable to divide, perhap
s due to rounding error%%%{1,[0,1,1,0]%%%} / %%%{1,[0,0,0,1]%%%} Error: Bad
 Argument Value
```

maple [**F**] time = 0.24, size = 0, normalized size = 0.00

$$\int \sqrt{a + \sqrt{\frac{c}{x}}\ b}\ (dx)^m\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+(c/x)^(1/2)*b)^(1/2),x)
```

```
[Out] int((d*x)^m*(a+(c/x)^(1/2)*b)^(1/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sqrt{\frac{c}{x}} + a}\ (dx)^m\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*(c/x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sqrt(c/x) + a)*(d*x)^m, x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int (d\,x)^m \sqrt{a + b\sqrt{\frac{c}{x}}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*(c/x)^(1/2))^(1/2),x)
```

```
[Out] int((d*x)^m*(a + b*(c/x)^(1/2))^(1/2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{a + b\sqrt{\frac{c}{x}}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*(c/x)**(1/2))**(1/2),x)
```

```
[Out] Integral((d*x)**m*sqrt(a + b*sqrt(c/x)), x)
```

3.2997 $\int \sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}\,(dx)^m\,dx$

Optimal. Leaf size=78

$$-\frac{4ac(dx)^m\left(a+\frac{b}{\sqrt{\frac{c}{x}}}\right)^{3/2}\left(-\frac{b}{a\sqrt{\frac{c}{x}}}\right)^{-2m}{}_2F_1\left(\frac{3}{2},-2m-1;\frac{5}{2};\frac{b}{a\sqrt{\frac{c}{x}}}+1\right)}{3b^2}$$

```
[Out] -4/3*a*c*(d*x)^m*hypergeom([3/2, -1-2*m],[5/2],1+b/a/(c/x)^(1/2))*(a+b/(c/x
)^(1/2))^(3/2)/b^2/((-b/a/(c/x)^(1/2))^(2*m))
```

Rubi [A] time = 0.08, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {369, 343, 341, 67, 65}

$$-\frac{4ac(dx)^m\left(a+\frac{b}{\sqrt{\frac{c}{x}}}\right)^{3/2}\left(-\frac{b}{a\sqrt{\frac{c}{x}}}\right)^{-2m}{}_2F_1\left(\frac{3}{2},-2m-1;\frac{5}{2};\frac{b}{a\sqrt{\frac{c}{x}}}+1\right)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/Sqrt[c/x]]*(d*x)^m,x]
```

```
[Out] (-4*a*c*(a + b/Sqrt[c/x])^(3/2)*(d*x)^m*Hypergeometric2F1[3/2, -1 - 2*m, 5/
2, 1 + b/(a*Sqrt[c/x])])/(3*b^2*(-(b/(a*Sqrt[c/x])))^(2*m))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 343

```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(c^In
tPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a + b*x^n)^p, x], x] /;
 FreeQ[{a, b, c, m, p}, x] && FractionQ[n]
```

Rule 369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
```

```
d, m, p, q}, x] && FractionQ[n]
```

Rubi steps

$$\int \sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}\,(dx)^m\,dx = \text{Subst}\left(\int \sqrt{a+\frac{b\sqrt{x}}{\sqrt{c}}}\,(dx)^m\,dx, \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= \text{Subst}\left((x^{-m}(dx)^m)\int \sqrt{a+\frac{b\sqrt{x}}{\sqrt{c}}}\,x^m\,dx, \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= \text{Subst}\left((2x^{-m}(dx)^m)\,\text{Subst}\left(\int x^{-1+2(1+m)}\sqrt{a+\frac{bx}{\sqrt{c}}}\,dx, x, \sqrt{x}\right), \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= -\text{Subst}\left(\frac{\left(2a\sqrt{c}\left(-\frac{b\sqrt{x}}{a\sqrt{c}}\right)^{-2m}(dx)^m\right)\text{Subst}\left(\int \left(-\frac{bx}{a\sqrt{c}}\right)^{-1+2(1+m)}\sqrt{a+\frac{bx}{\sqrt{c}}}\,dx, x, \sqrt{x}\right)}{b},\right.$$

$$= -\frac{4ac\left(a+\frac{b}{\sqrt{\frac{c}{x}}}\right)^{3/2}\left(-\frac{b}{a\sqrt{\frac{c}{x}}}\right)^{-2m}(dx)^m\,{}_2F_1\left(\frac{3}{2}, -1-2m; \frac{5}{2}; 1+\frac{b}{a\sqrt{\frac{c}{x}}}\right)}{3b^2}$$

Mathematica [A] time = 0.12, size = 85, normalized size = 1.09

$$\frac{4x(dx)^m\sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}\;{}_2F_1\left(-\frac{1}{2}, -2m-\frac{5}{2}; -2m-\frac{3}{2}; -\frac{a\sqrt{\frac{c}{x}}}{b}\right)}{(4m+5)\sqrt{\frac{a\sqrt{\frac{c}{x}}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/Sqrt[c/x]]*(d*x)^m,x]
```

```
[Out] (4*Sqrt[a + b/Sqrt[c/x]]*x*(d*x)^m*Hypergeometric2F1[-1/2, -5/2 - 2*m, -3/2
 - 2*m, -((a*Sqrt[c/x])/b)])/((5 + 4*m)*Sqrt[1 + (a*Sqrt[c/x])/b])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b/(c/x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   alglo
gextint: unimplemented
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m\sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b/(c/x)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate((d*x)^m*sqrt(a + b/sqrt(c/x)), x)
```

maple **[F]** time = 0.23, size = 0, normalized size = 0.00

$$\int \sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}\ (dx)^m\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b/(c/x)^(1/2))^(1/2),x)
[Out] int((d*x)^m*(a+b/(c/x)^(1/2))^(1/2),x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b/(c/x)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate((d*x)^m*sqrt(a + b/sqrt(c/x)), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m \sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b/(c/x)^(1/2))^(1/2),x)
[Out] int((d*x)^m*(a + b/(c/x)^(1/2))^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b/(c/x)**(1/2))**(1/2),x)
[Out] Integral((d*x)**m*sqrt(a + b/sqrt(c/x)), x)
```

3.2998 $\int \sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{3/2}}}\,(dx)^m\,dx$

Optimal. Leaf size=102

$$\frac{x(dx)^m\sqrt{a+\frac{bx^3\left(\frac{c}{x}\right)^{3/2}}{c^3}}\;{}_2F_1\left(-\frac{1}{2},\frac{2(m+1)}{3};\frac{1}{3}(2m+5);-\frac{b\left(\frac{c}{x}\right)^{3/2}x^3}{ac^3}\right)}{(m+1)\sqrt{\frac{bx^3\left(\frac{c}{x}\right)^{3/2}}{ac^3}+1}}$$

```
[Out] x*(d*x)^m*hypergeom([-1/2, 2/3+2/3*m],[5/3+2/3*m],-b*(c/x)^(3/2)*x^3/a/c^3)
*(a+b*(c/x)^(3/2)*x^3/c^3)^(1/2)/(1+m)/(1+b*(c/x)^(3/2)*x^3/a/c^3)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {369, 343, 341, 365, 364}

$$\frac{x(dx)^m\sqrt{a+\frac{bx^3\left(\frac{c}{x}\right)^{3/2}}{c^3}}\;{}_2F_1\left(-\frac{1}{2},\frac{2(m+1)}{3};\frac{1}{3}(2m+5);-\frac{b\left(\frac{c}{x}\right)^{3/2}x^3}{ac^3}\right)}{(m+1)\sqrt{\frac{bx^3\left(\frac{c}{x}\right)^{3/2}}{ac^3}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/(c/x)^(3/2)]*(d*x)^m,x]
```

```
[Out] (x*(d*x)^m*Sqrt[a + (b*(c/x)^(3/2)*x^3)/c^3]*Hypergeometric2F1[-1/2, (2*(1
+ m))/3, (5 + 2*m)/3, -((b*(c/x)^(3/2)*x^3)/(a*c^3))])/((1 + m)*Sqrt[1 + (b
*(c/x)^(3/2)*x^3)/(a*c^3)])
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 343

```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(c^In
tPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a + b*x^n)^p, x], x] /;
 FreeQ[{a, b, c, m, p}, x] && FractionQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{3/2}}}\,(dx)^m\,dx &= \text{Subst}\left(\int (dx)^m \sqrt{a+\frac{bx^{3/2}}{c^{3/2}}}\,dx, \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right) \\
&= \text{Subst}\left((x^{-m}(dx)^m)\int x^m \sqrt{a+\frac{bx^{3/2}}{c^{3/2}}}\,dx, \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right) \\
&= \text{Subst}\left((2x^{-m}(dx)^m)\,\text{Subst}\left(\int x^{-1+2(1+m)} \sqrt{a+\frac{bx^3}{c^{3/2}}}\,dx, x, \sqrt{x}\right), \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right) \\
&= \text{Subst}\left(\frac{\left(2x^{-m}(dx)^m\sqrt{a+\frac{bx^{3/2}}{c^{3/2}}}\right)\text{Subst}\left(\int x^{-1+2(1+m)}\sqrt{1+\frac{bx^3}{ac^{3/2}}}\,dx, x, \sqrt{x}\right)}{\sqrt{1+\frac{bx^{3/2}}{ac^{3/2}}}}, \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right) \\
&= \frac{x(dx)^m\sqrt{a+\frac{b\left(\frac{c}{x}\right)^{3/2}x^3}{c^3}}\;{}_2F_1\left(-\frac{1}{2}, \frac{2(1+m)}{3}; \frac{1}{3}(5+2m); -\frac{b\left(\frac{c}{x}\right)^{3/2}x^3}{ac^3}\right)}{(1+m)\sqrt{1+\frac{b\left(\frac{c}{x}\right)^{3/2}x^3}{ac^3}}}
\end{aligned}$$

Mathematica [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{3/2}}}\,(dx)^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b/(c/x)^(3/2)]*(d*x)^m,x]
```

```
[Out] Integrate[Sqrt[a + b/(c/x)^(3/2)]*(d*x)^m, x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b/(c/x)^(3/2))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   alglo
gextint: unimplemented
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{\frac{3}{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b/(c/x)^(3/2))^(1/2),x, algorithm="giac")
[Out] integrate((d*x)^m*sqrt(a + b/(c/x)^(3/2)), x)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \sqrt{a + \frac{b}{\left(\frac{c}{x}\right)^{\frac{3}{2}}}}\ (dx)^m\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b/(c/x)^(3/2))^(1/2),x)
[Out] int((d*x)^m*(a+b/(c/x)^(3/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{a + \frac{b}{\left(\frac{c}{x}\right)^{\frac{3}{2}}}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b/(c/x)^(3/2))^(1/2),x, algorithm="maxima")
[Out] integrate((d*x)^m*sqrt(a + b/(c/x)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m \sqrt{a + \frac{b}{\left(\frac{c}{x}\right)^{3/2}}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b/(c/x)^(3/2))^(1/2),x)
[Out] int((d*x)^m*(a + b/(c/x)^(3/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{a + \frac{b}{\left(\frac{c}{x}\right)^{\frac{3}{2}}}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b/(c/x)**(3/2))**(1/2),x)
[Out] Integral((d*x)**m*sqrt(a + b/(c/x)**(3/2)), x)
```

3.2999 $\int \frac{(dx)^m}{\sqrt{a+b\left(\frac{c}{x}\right)^{3/2}}}\,dx$

Optimal. Leaf size=102

$$\frac{x(dx)^m\sqrt{\frac{bc^3}{ax^3\left(\frac{c}{x}\right)^{3/2}}+1}\;{}_2F_1\left(\frac{1}{2},-\frac{2}{3}(m+1);\frac{1}{3}(1-2m);-\frac{bc^3}{a\left(\frac{c}{x}\right)^{3/2}x^3}\right)}{(m+1)\sqrt{a+\frac{bc^3}{x^3\left(\frac{c}{x}\right)^{3/2}}}}$$

```
[Out] x*(d*x)^m*hypergeom([1/2, -2/3-2/3*m],[1/3-2/3*m],-b*c^3/a/(c/x)^(3/2)/x^3)
*(1+b*c^3/a/(c/x)^(3/2)/x^3)^(1/2)/(1+m)/(a+b*c^3/(c/x)^(3/2)/x^3)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {369, 343, 341, 339, 365, 364}

$$\frac{x(dx)^m\sqrt{\frac{bc^3}{ax^3\left(\frac{c}{x}\right)^{3/2}}+1}\;{}_2F_1\left(\frac{1}{2},-\frac{2}{3}(m+1);\frac{1}{3}(1-2m);-\frac{bc^3}{a\left(\frac{c}{x}\right)^{3/2}x^3}\right)}{(m+1)\sqrt{a+\frac{bc^3}{x^3\left(\frac{c}{x}\right)^{3/2}}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/Sqrt[a + b*(c/x)^(3/2)],x]
```

```
[Out] (Sqrt[1 + (b*c^3)/(a*(c/x)^(3/2)*x^3)]*x*(d*x)^m*Hypergeometric2F1[1/2, (-2
*(1 + m))/3, (1 - 2*m)/3, -((b*c^3)/(a*(c/x)^(3/2)*x^3))])/((1 + m)*Sqrt[a
+ (b*c^3)/((c/x)^(3/2)*x^3)])
```

Rule 339

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Dist[((c
*x)^(m + 1)*(1/x)^(m + 1))/c, Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x
], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 343

```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(c^In
tPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a + b*x^n)^p, x], x] /;
 FreeQ[{a, b, c, m, p}, x] && FractionQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Rubi steps

$$\int \frac{(dx)^m}{\sqrt{a+b\left(\frac{c}{x}\right)^{3/2}}}\,dx = \text{Subst}\left(\int \frac{(dx)^m}{\sqrt{a+\frac{bc^{3/2}}{x^{3/2}}}}\,dx, \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= \text{Subst}\left((x^{-m}(dx)^m)\int \frac{x^m}{\sqrt{a+\frac{bc^{3/2}}{x^{3/2}}}}\,dx, \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= \text{Subst}\left((2x^{-m}(dx)^m)\,\text{Subst}\left(\int \frac{x^{-1+2(1+m)}}{\sqrt{a+\frac{bc^{3/2}}{x^3}}}\,dx, x, \sqrt{x}\right), \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= -\text{Subst}\left((2x^{-m}(dx)^m)\,\text{Subst}\left(\int \frac{x^{-1-2(1+m)}}{\sqrt{a+bc^{3/2}x^3}}\,dx, x, \frac{1}{\sqrt{x}}\right), \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= -\text{Subst}\left(\frac{\left(2\sqrt{1+\frac{bc^{3/2}}{ax^{3/2}}}\,x^{-m}(dx)^m\right)\text{Subst}\left(\int \frac{x^{-1-2(1+m)}}{\sqrt{1+\frac{bc^{3/2}x^3}{a}}}\,dx, x, \frac{1}{\sqrt{x}}\right)}{\sqrt{a+\frac{bc^{3/2}}{x^{3/2}}}}, \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= \frac{\sqrt{1+\frac{bc^3}{a\left(\frac{c}{x}\right)^{3/2}x^3}}\,x(dx)^m\,{}_2F_1\left(\frac{1}{2}, -\frac{2}{3}(1+m); \frac{1}{3}(1-2m); -\frac{bc^3}{a\left(\frac{c}{x}\right)^{3/2}x^3}\right)}{(1+m)\sqrt{a+\frac{bc^3}{\left(\frac{c}{x}\right)^{3/2}x^3}}}$$

Mathematica [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{a+b\left(\frac{c}{x}\right)^{3/2}}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m/Sqrt[a + b*(c/x)^(3/2)],x]
```

```
[Out] Integrate[(d*x)^m/Sqrt[a + b*(c/x)^(3/2)], x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*(c/x)^(3/2))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   alglo
gextint: unimplemented
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{b\left(\frac{c}{x}\right)^{\frac{3}{2}}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*(c/x)^(3/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/sqrt(b*(c/x)^(3/2) + a), x)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{a+\left(\frac{c}{x}\right)^{\frac{3}{2}}b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+(c/x)^(3/2)*b)^(1/2),x)
```

```
[Out] int((d*x)^m/(a+(c/x)^(3/2)*b)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{b\left(\frac{c}{x}\right)^{\frac{3}{2}}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*(c/x)^(3/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/sqrt(b*(c/x)^(3/2) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x)^m}{\sqrt{a+b\left(\frac{c}{x}\right)^{3/2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*(c/x)^(3/2))^(1/2),x)
```

```
[Out] int((d*x)^m/(a + b*(c/x)^(3/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{a+b\left(\frac{c}{x}\right)^{\frac{3}{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*(c/x)**(3/2))**(1/2),x)
```

```
[Out] Integral((d*x)**m/sqrt(a + b*(c/x)**(3/2)), x)
```

3.3000 $\int \frac{(dx)^m}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx$

Optimal. Leaf size=58

$$\frac{4x^{m+1}\sqrt{a+b\sqrt{\frac{c}{x}}}\ {}_2F_1\left(1,\frac{1}{2}(-4m-3);\frac{3}{2};\frac{a+b\sqrt{\frac{c}{x}}}{a}\right)}{a}$$

```
[Out] 4*x^(1+m)*hypergeom([1, -3/2-2*m],[3/2],(a+b*(c/x)^(1/2))/a)*(a+b*(c/x)^(1/
2))^(1/2)/a
```

Rubi [A] time = 0.09, antiderivative size = 78, normalized size of antiderivative = 1.34, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {369, 343, 341, 339, 67, 65}

$$\frac{4b^2c(dx)^m\sqrt{a+b\sqrt{\frac{c}{x}}}\left(-\frac{b\sqrt{\frac{c}{x}}}{a}\right)^{2m}\ {}_2F_1\left(\frac{1}{2},2m+3;\frac{3}{2};\frac{\sqrt{\frac{c}{x}}b}{a}+1\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/Sqrt[a + b*Sqrt[c/x]],x]
```

```
[Out] (4*b^2*c*Sqrt[a + b*Sqrt[c/x]]*(-((b*Sqrt[c/x])/a))^(2*m)*(d*x)^m*Hypergeom
etric2F1[1/2, 3 + 2*m, 3/2, 1 + (b*Sqrt[c/x])/a])/a^3
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0]
```

Rule 339

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Dist[((c
*x)^(m + 1)*(1/x)^(m + 1))/c, Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x
], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 343

```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(c^In
tPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a + b*x^n)^p, x], x] /;
 FreeQ[{a, b, c, m, p}, x] && FractionQ[n]
```

Rule 369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Rubi steps

$$\int \frac{(dx)^m}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx = \text{Subst}\left(\int \frac{(dx)^m}{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}\,dx, \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= \text{Subst}\left((x^{-m}(dx)^m)\int \frac{x^m}{\sqrt{a+\frac{b\sqrt{c}}{\sqrt{x}}}}\,dx, \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= \text{Subst}\left((2x^{-m}(dx)^m)\,\text{Subst}\left(\int \frac{x^{-1+2(1+m)}}{\sqrt{a+\frac{b\sqrt{c}}{x}}}\,dx, x, \sqrt{x}\right), \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= -\text{Subst}\left((2x^{-m}(dx)^m)\,\text{Subst}\left(\int \frac{x^{-1-2(1+m)}}{\sqrt{a+b\sqrt{c}\,x}}\,dx, x, \frac{1}{\sqrt{x}}\right), \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= \text{Subst}\left(\frac{\left(2b^3c^{3/2}\left(-\frac{b\sqrt{c}}{a\sqrt{x}}\right)^{2m}(dx)^m\right)\text{Subst}\left(\int \frac{\left(-\frac{b\sqrt{c}\,x}{a}\right)^{-1-2(1+m)}}{\sqrt{a+b\sqrt{c}\,x}}\,dx, x, \frac{1}{\sqrt{x}}\right)}{a^3}, \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= \frac{4b^2c\sqrt{a+b\sqrt{\frac{c}{x}}}\left(-\frac{b\sqrt{\frac{c}{x}}}{a}\right)^{2m}(dx)^m\,{}_2F_1\left(\frac{1}{2}, 3+2m; \frac{3}{2}; 1+\frac{b\sqrt{\frac{c}{x}}}{a}\right)}{a^3}$$

Mathematica [A] time = 0.25, size = 96, normalized size = 1.66

$$\frac{4b^2c(dx)^m\left(1-\frac{a}{a+b\sqrt{\frac{c}{x}}}\right)^{2m}{}_2F_1\left(2m+\frac{5}{2}, 2m+3; 2m+\frac{7}{2}; \frac{a}{a+b\sqrt{\frac{c}{x}}}\right)}{(4m+5)\left(a+b\sqrt{\frac{c}{x}}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/Sqrt[a + b*Sqrt[c/x]],x]
```

```
[Out] (4*b^2*c*(1 - a/(a + b*Sqrt[c/x]))^(2*m)*(d*x)^m*Hypergeometric2F1[5/2 + 2*
m, 3 + 2*m, 7/2 + 2*m, a/(a + b*Sqrt[c/x])])/((5 + 4*m)*(a + b*Sqrt[c/x])^(
5/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*(c/x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   alglo
gextint: unimplemented
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*(c/x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{a+\sqrt{\frac{c}{x}}\,b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+(c/x)^(1/2)*b)^(1/2),x)
```

```
[Out] int((d*x)^m/(a+(c/x)^(1/2)*b)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{b\sqrt{\frac{c}{x}}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*(c/x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/sqrt(b*sqrt(c/x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(d\,x)^m}{\sqrt{a+b\,\sqrt{\frac{c}{x}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*(c/x)^(1/2))^(1/2),x)
```

```
[Out] int((d*x)^m/(a + b*(c/x)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{a+b\sqrt{\frac{c}{x}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*(c/x)**(1/2))**(1/2),x)
```

```
[Out] Integral((d*x)**m/sqrt(a + b*sqrt(c/x)), x)
```

3.3001 $\int \frac{(dx)^m}{\sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}}\,dx$

Optimal. Leaf size=76

$$-\frac{4ac(dx)^m\sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}\left(-\frac{b}{a\sqrt{\frac{c}{x}}}\right)^{-2m}{}_2F_1\left(\frac{1}{2},-2m-1;\frac{3}{2};\frac{b}{a\sqrt{\frac{c}{x}}}+1\right)}{b^2}$$

```
[Out] -4*a*c*(d*x)^m*hypergeom([1/2, -1-2*m],[3/2],1+b/a/(c/x)^(1/2))*(a+b/(c/x)^
(1/2))^(1/2)/b^2/((-b/a/(c/x)^(1/2))^(2*m))
```

Rubi [A] time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {369, 343, 341, 67, 65}

$$-\frac{4ac(dx)^m\sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}\left(-\frac{b}{a\sqrt{\frac{c}{x}}}\right)^{-2m}{}_2F_1\left(\frac{1}{2},-2m-1;\frac{3}{2};\frac{b}{a\sqrt{\frac{c}{x}}}+1\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/Sqrt[a + b/Sqrt[c/x]],x]
```

```
[Out] (-4*a*c*Sqrt[a + b/Sqrt[c/x]]*(d*x)^m*Hypergeometric2F1[1/2, -1 - 2*m, 3/2,
 1 + b/(a*Sqrt[c/x])])/(b^2*(-(b/(a*Sqrt[c/x])))^(2*m))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 343

```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(c^In
tPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a + b*x^n)^p, x], x] /;
 FreeQ[{a, b, c, m, p}, x] && FractionQ[n]
```

Rule 369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
```

```
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Rubi steps

$$\int \frac{(dx)^m}{\sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}}\,dx = \text{Subst}\left(\int \frac{(dx)^m}{\sqrt{a+\frac{b\sqrt{x}}{\sqrt{c}}}}\,dx, \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= \text{Subst}\left((x^{-m}(dx)^m)\int \frac{x^m}{\sqrt{a+\frac{b\sqrt{x}}{\sqrt{c}}}}\,dx, \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= \text{Subst}\left((2x^{-m}(dx)^m)\,\text{Subst}\left(\int \frac{x^{-1+2(1+m)}}{\sqrt{a+\frac{bx}{\sqrt{c}}}}\,dx, x, \sqrt{x}\right), \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= -\text{Subst}\left(\frac{\left(2a\sqrt{c}\left(-\frac{b\sqrt{x}}{a\sqrt{c}}\right)^{-2m}(dx)^m\right)\text{Subst}\left(\int \frac{\left(-\frac{bx}{a\sqrt{c}}\right)^{-1+2(1+m)}}{\sqrt{a+\frac{bx}{\sqrt{c}}}}\,dx, x, \sqrt{x}\right)}{b}, \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= -\frac{4ac\sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}\left(-\frac{b}{a\sqrt{\frac{c}{x}}}\right)^{-2m}(dx)^m\,{}_2F_1\left(\frac{1}{2}, -1-2m; \frac{3}{2}; 1+\frac{b}{a\sqrt{\frac{c}{x}}}\right)}{b^2}$$

Mathematica [A] time = 0.43, size = 116, normalized size = 1.53

$$\frac{a^2c(dx)^m\left(\frac{a\sqrt{\frac{c}{x}}}{a\sqrt{\frac{c}{x}}+b}\right)^{2m-\frac{1}{2}}{}_2F_1\left(2m+2, 2m+\frac{5}{2}; 2m+3; \frac{b}{\sqrt{\frac{c}{x}}\,a+b}\right)}{(m+1)\sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}\left(a\sqrt{\frac{c}{x}}+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/Sqrt[a + b/Sqrt[c/x]],x]
```

```
[Out] (a^2*c*((a*Sqrt[c/x])/(b + a*Sqrt[c/x]))^(-1/2 + 2*m)*(d*x)^m*Hypergeometri
c2F1[2 + 2*m, 5/2 + 2*m, 3 + 2*m, b/(b + a*Sqrt[c/x])])/((1 + m)*Sqrt[a + b
/Sqrt[c/x]]*(b + a*Sqrt[c/x])^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b/(c/x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   alglo
gextint: unimplemented
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b/(c/x)^(1/2))^(1/2),x, algorithm="giac")
[Out] sage0*x
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+1/(c/x)^(1/2)*b)^(1/2),x)
[Out] int((d*x)^m/(a+1/(c/x)^(1/2)*b)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b/(c/x)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate((d*x)^m/sqrt(a + b/sqrt(c/x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x)^m}{\sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b/(c/x)^(1/2))^(1/2),x)
[Out] int((d*x)^m/(a + b/(c/x)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{a+\frac{b}{\sqrt{\frac{c}{x}}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b/(c/x)**(1/2))**(1/2),x)
[Out] Integral((d*x)**m/sqrt(a + b/sqrt(c/x)), x)
```

3.3002 $\int \frac{(dx)^m}{\sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{3/2}}}}\,dx$

Optimal. Leaf size=102

$$\frac{x(dx)^m\sqrt{\frac{bx^3\left(\frac{c}{x}\right)^{3/2}}{ac^3}+1}\;{}_2F_1\left(\frac{1}{2},\frac{2(m+1)}{3};\frac{1}{3}(2m+5);-\frac{b\left(\frac{c}{x}\right)^{3/2}x^3}{ac^3}\right)}{(m+1)\sqrt{a+\frac{bx^3\left(\frac{c}{x}\right)^{3/2}}{c^3}}}$$

```
[Out] x*(d*x)^m*hypergeom([1/2, 2/3+2/3*m],[5/3+2/3*m],-b*(c/x)^(3/2)*x^3/a/c^3)*
(1+b*(c/x)^(3/2)*x^3/a/c^3)^(1/2)/(1+m)/(a+b*(c/x)^(3/2)*x^3/c^3)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {369, 343, 341, 365, 364}

$$\frac{x(dx)^m\sqrt{\frac{bx^3\left(\frac{c}{x}\right)^{3/2}}{ac^3}+1}\;{}_2F_1\left(\frac{1}{2},\frac{2(m+1)}{3};\frac{1}{3}(2m+5);-\frac{b\left(\frac{c}{x}\right)^{3/2}x^3}{ac^3}\right)}{(m+1)\sqrt{a+\frac{bx^3\left(\frac{c}{x}\right)^{3/2}}{c^3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/Sqrt[a + b/(c/x)^(3/2)],x]
```

```
[Out] (x*(d*x)^m*Sqrt[1 + (b*(c/x)^(3/2)*x^3)/(a*c^3)]*Hypergeometric2F1[1/2, (2*
(1 + m))/3, (5 + 2*m)/3, -((b*(c/x)^(3/2)*x^3)/(a*c^3))])/((1 + m)*Sqrt[a +
 (b*(c/x)^(3/2)*x^3)/c^3])
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 343

```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(c^In
tPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a + b*x^n)^p, x], x] /;
 FreeQ[{a, b, c, m, p}, x] && FractionQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> With[{k = Denominator[n]}, Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x
], x^(1/k), (c*x^q)^(1/k)/(c^(1/k)*(x^(1/k))^(q - 1))]] /; FreeQ[{a, b, c,
d, m, p, q}, x] && FractionQ[n]
```

Rubi steps

$$\int \frac{(dx)^m}{\sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{3/2}}}}\,dx = \text{Subst}\left(\int \frac{(dx)^m}{\sqrt{a+\frac{bx^{3/2}}{c^{3/2}}}}\,dx, \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= \text{Subst}\left((x^{-m}(dx)^m)\int \frac{x^m}{\sqrt{a+\frac{bx^{3/2}}{c^{3/2}}}}\,dx, \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= \text{Subst}\left((2x^{-m}(dx)^m)\,\text{Subst}\left(\int \frac{x^{-1+2(1+m)}}{\sqrt{a+\frac{bx^3}{c^{3/2}}}}\,dx, x, \sqrt{x}\right), \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= \text{Subst}\left(\frac{\left(2x^{-m}(dx)^m\sqrt{1+\frac{bx^{3/2}}{ac^{3/2}}}\right)\text{Subst}\left(\int \frac{x^{-1+2(1+m)}}{\sqrt{1+\frac{bx^3}{ac^{3/2}}}}\,dx, x, \sqrt{x}\right)}{\sqrt{a+\frac{bx^{3/2}}{c^{3/2}}}}, \sqrt{x}, \frac{\sqrt{\frac{c}{x}}\,x}{\sqrt{c}}\right)$$

$$= \frac{x(dx)^m\sqrt{1+\frac{b\left(\frac{c}{x}\right)^{3/2}x^3}{ac^3}}\;{}_2F_1\left(\frac{1}{2}, \frac{2(1+m)}{3}; \frac{1}{3}(5+2m); -\frac{b\left(\frac{c}{x}\right)^{3/2}x^3}{ac^3}\right)}{(1+m)\sqrt{a+\frac{b\left(\frac{c}{x}\right)^{3/2}x^3}{c^3}}}$$

Mathematica [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{3/2}}}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x)^m/Sqrt[a + b/(c/x)^(3/2)],x]
```

```
[Out] Integrate[(d*x)^m/Sqrt[a + b/(c/x)^(3/2)], x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b/(c/x)^(3/2))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   alglo
gextint: unimplemented
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{\frac{3}{2}}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b/(c/x)^(3/2))^(1/2),x, algorithm="giac")
[Out] integrate((d*x)^m/sqrt(a + b/(c/x)^(3/2)), x)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{\frac{3}{2}}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+1/(c/x)^(3/2)*b)^(1/2),x)
[Out] int((d*x)^m/(a+1/(c/x)^(3/2)*b)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{\frac{3}{2}}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b/(c/x)^(3/2))^(1/2),x, algorithm="maxima")
[Out] integrate((d*x)^m/sqrt(a + b/(c/x)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x)^m}{\sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{3/2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b/(c/x)^(3/2))^(1/2),x)
[Out] int((d*x)^m/(a + b/(c/x)^(3/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{a+\frac{b}{\left(\frac{c}{x}\right)^{\frac{3}{2}}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b/(c/x)**(3/2))**(1/2),x)
[Out] Integral((d*x)**m/sqrt(a + b/(c/x)**(3/2)), x)
```

3.3003 $\int \left(a + b\left(cx^n\right)^{\frac{1}{n}}\right) dx$

Optimal. Leaf size=19

$$ax + \frac{1}{2}bx\left(cx^n\right)^{\frac{1}{n}}$$

```
[Out] a*x+1/2*b*x*(c*x^n)^(1/n)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {15, 30}

$$ax + \frac{1}{2}bx\left(cx^n\right)^{\frac{1}{n}}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*(c*x^n)^n^(-1),x]
```

```
[Out] a*x + (b*x*(c*x^n)^n^(-1))/2
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \left(a + b\left(cx^n\right)^{\frac{1}{n}}\right) dx &= ax + b\int \left(cx^n\right)^{\frac{1}{n}}\, dx \\ &= ax + \frac{\left(b\left(cx^n\right)^{\frac{1}{n}}\right)\int x\, dx}{x} \\ &= ax + \frac{1}{2}bx\left(cx^n\right)^{\frac{1}{n}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$ax + \frac{1}{2}bx\left(cx^n\right)^{\frac{1}{n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*(c*x^n)^n^(-1),x]
```

```
[Out] a*x + (b*x*(c*x^n)^n^(-1))/2
```

fricas [A] time = 0.85, size = 15, normalized size = 0.79

$$\frac{1}{2}\, bc^{\left(\frac{1}{n}\right)}x^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*(c*x^n)^(1/n),x, algorithm="fricas")
[Out] 1/2*b*c^(1/n)*x^2 + a*x
```

giac [A] time = 0.16, size = 15, normalized size = 0.79

$$\frac{1}{2}\, bc^{\left(\frac{1}{n}\right)} x^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*(c*x^n)^(1/n),x, algorithm="giac")
[Out] 1/2*b*c^(1/n)*x^2 + a*x
```

maple [A] time = 0.02, size = 22, normalized size = 1.16

$$\frac{bx\, \mathrm{e}^{\frac{\ln\left(c\, \mathrm{e}^{n \ln(x)}\right)}{n}}}{2} + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*(c*x^n)^(1/n),x)
[Out] a*x+1/2*x*b*exp(1/n*ln(c*exp(n*ln(x))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$bc^{\left(\frac{1}{n}\right)} \int (x^n)^{\left(\frac{1}{n}\right)}\, dx + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*(c*x^n)^(1/n),x, algorithm="maxima")
[Out] b*c^(1/n)*integrate((x^n)^(1/n), x) + a*x
```

mupad [B] time = 1.20, size = 17, normalized size = 0.89

$$a\, x + \frac{b\, x\, (c\, x^n)^{1/n}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*(c*x^n)^(1/n),x)
[Out] a*x + (b*x*(c*x^n)^(1/n))/2
```

sympy [A] time = 0.24, size = 19, normalized size = 1.00

$$ax + \frac{bc^{\frac{1}{n}}\, x\, (x^n)^{\frac{1}{n}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*(c*x**n)**(1/n),x)
[Out] a*x + b*c**(1/n)*x*(x**n)**(1/n)/2
```

3.3004 $\int \left(a + b\left(cx^n\right)^{\frac{1}{n}}\right)^2 dx$

Optimal. Leaf size=34

$$\frac{x\left(cx^n\right)^{-1/n}\left(a + b\left(cx^n\right)^{\frac{1}{n}}\right)^3}{3b}$$

```
[Out] 1/3*x*(a+b*(c*x^n)^(1/n))^3/b/((c*x^n)^(1/n))
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {254, 32}

$$\frac{x\left(cx^n\right)^{-1/n}\left(a + b\left(cx^n\right)^{\frac{1}{n}}\right)^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c*x^n)^n^(-1))^2,x]
```

```
[Out] (x*(a + b*(c*x^n)^n^(-1))^3)/(3*b*(c*x^n)^n^(-1))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 254

```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_.), x_Symbol] :> Dist[x/(c*x^
q)^(1/q), Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\int \left(a + b\left(cx^n\right)^{\frac{1}{n}}\right)^2 dx = \left(x\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int (a + bx)^2\, dx, x, \left(cx^n\right)^{\frac{1}{n}}\right)$$
$$= \frac{x\left(cx^n\right)^{-1/n}\left(a + b\left(cx^n\right)^{\frac{1}{n}}\right)^3}{3b}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 1.12

$$a^2x + abx\left(cx^n\right)^{\frac{1}{n}} + \frac{1}{3}b^2x\left(cx^n\right)^{2/n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c*x^n)^n^(-1))^2,x]
```

```
[Out] a^2*x + a*b*x*(c*x^n)^n^(-1) + (b^2*x*(c*x^n)^(2/n))/3
```

fricas [A] time = 0.89, size = 32, normalized size = 0.94

$$\frac{1}{3}b^2c^{\frac{2}{n}}x^3 + abc^{\left(\frac{1}{n}\right)}x^2 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^n)^(1/n))^2,x, algorithm="fricas")
[Out] 1/3*b^2*c^(2/n)*x^3 + a*b*c^(1/n)*x^2 + a^2*x
```

giac [**A**] time = 0.19, size = 32, normalized size = 0.94

$$\frac{1}{3}\,b^2 c^{\frac{2}{n}} x^3 + abc^{\left(\frac{1}{n}\right)} x^2 + a^2 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^n)^(1/n))^2,x, algorithm="giac")
[Out] 1/3*b^2*c^(2/n)*x^3 + a*b*c^(1/n)*x^2 + a^2*x
```

maple [**F**] time = 0.21, size = 0, normalized size = 0.00

$$\int \left(b\,(c\,x^n)^{\frac{1}{n}} + a\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(c*x^n)^(1/n))^2,x)
[Out] int((a+b*(c*x^n)^(1/n))^2,x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$b^2 c^{\frac{2}{n}} \int (x^n)^{\frac{2}{n}}\,dx + 2\,abc^{\left(\frac{1}{n}\right)} \int (x^n)^{\left(\frac{1}{n}\right)}\,dx + a^2 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^n)^(1/n))^2,x, algorithm="maxima")
[Out] b^2*c^(2/n)*integrate((x^n)^(2/n), x) + 2*a*b*c^(1/n)*integrate((x^n)^(1/n)
, x) + a^2*x
```

mupad [**B**] time = 1.23, size = 36, normalized size = 1.06

$$a^2\,x + \frac{b^2\,x\,(c\,x^n)^{2/n}}{3} + a\,b\,x\,(c\,x^n)^{1/n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x^n)^(1/n))^2,x)
[Out] a^2*x + (b^2*x*(c*x^n)^(2/n))/3 + a*b*x*(c*x^n)^(1/n)
```

sympy [**A**] time = 0.43, size = 39, normalized size = 1.15

$$a^2 x + abc^{\frac{1}{n}} x\,(x^n)^{\frac{1}{n}} + \frac{b^2 c^{\frac{2}{n}} x\,(x^n)^{\frac{2}{n}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x**n)**(1/n))**2,x)
[Out] a**2*x + a*b*c**(1/n)*x*(x**n)**(1/n) + b**2*c**(2/n)*x*(x**n)**(2/n)/3
```

3.3005 $\int \left(a + b\left(cx^n\right)^{\frac{1}{n}}\right)^3 dx$

Optimal. Leaf size=34

$$\frac{x\left(cx^n\right)^{-1/n}\left(a + b\left(cx^n\right)^{\frac{1}{n}}\right)^4}{4b}$$

```
[Out] 1/4*x*(a+b*(c*x^n)^(1/n))^4/b/((c*x^n)^(1/n))
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {254, 32}

$$\frac{x\left(cx^n\right)^{-1/n}\left(a + b\left(cx^n\right)^{\frac{1}{n}}\right)^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c*x^n)^n^(-1))^3,x]
```

```
[Out] (x*(a + b*(c*x^n)^n^(-1))^4)/(4*b*(c*x^n)^n^(-1))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 254

```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_.), x_Symbol] :> Dist[x/(c*x^
q)^(1/q), Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\int \left(a + b\left(cx^n\right)^{\frac{1}{n}}\right)^3 dx = \left(x\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int (a + bx)^3\, dx, x, \left(cx^n\right)^{\frac{1}{n}}\right)$$
$$= \frac{x\left(cx^n\right)^{-1/n}\left(a + b\left(cx^n\right)^{\frac{1}{n}}\right)^4}{4b}$$

Mathematica [A] time = 0.04, size = 60, normalized size = 1.76

$$a^3x + \frac{3}{2}a^2bx\left(cx^n\right)^{\frac{1}{n}} + ab^2x\left(cx^n\right)^{2/n} + \frac{1}{4}b^3x\left(cx^n\right)^{3/n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c*x^n)^n^(-1))^3,x]
```

```
[Out] a^3*x + (3*a^2*b*x*(c*x^n)^n^(-1))/2 + a*b^2*x*(c*x^n)^(2/n) + (b^3*x*(c*x^
n)^(3/n))/4
```

fricas [A] time = 0.82, size = 50, normalized size = 1.47

$$\frac{1}{4}b^3c^{\frac{3}{n}}x^4 + ab^2c^{\frac{2}{n}}x^3 + \frac{3}{2}a^2bc^{\left(\frac{1}{n}\right)}x^2 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^n)^(1/n))^3,x, algorithm="fricas")
```

```
[Out] 1/4*b^3*c^(3/n)*x^4 + a*b^2*c^(2/n)*x^3 + 3/2*a^2*b*c^(1/n)*x^2 + a^3*x
```

giac [**A**] time = 0.21, size = 50, normalized size = 1.47

$$\frac{1}{4}\,b^3c^{\frac{3}{n}}x^4 + ab^2c^{\frac{2}{n}}x^3 + \frac{3}{2}\,a^2bc^{\left(\frac{1}{n}\right)}x^2 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^n)^(1/n))^3,x, algorithm="giac")
```

```
[Out] 1/4*b^3*c^(3/n)*x^4 + a*b^2*c^(2/n)*x^3 + 3/2*a^2*b*c^(1/n)*x^2 + a^3*x
```

maple [**F**] time = 0.20, size = 0, normalized size = 0.00

$$\int \left(b\,(c\,x^n)^{\frac{1}{n}} + a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*x^n)^(1/n)+a)^3,x)
```

```
[Out] int((b*(c*x^n)^(1/n)+a)^3,x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$b^3c^{\frac{3}{n}}\int (x^n)^{\frac{3}{n}}\,dx + 3\,ab^2c^{\frac{2}{n}}\int (x^n)^{\frac{2}{n}}\,dx + 3\,a^2bc^{\left(\frac{1}{n}\right)}\int (x^n)^{\left(\frac{1}{n}\right)}\,dx + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^n)^(1/n))^3,x, algorithm="maxima")
```

```
[Out] b^3*c^(3/n)*integrate((x^n)^(3/n), x) + 3*a*b^2*c^(2/n)*integrate((x^n)^(2/
n), x) + 3*a^2*b*c^(1/n)*integrate((x^n)^(1/n), x) + a^3*x
```

mupad [**B**] time = 1.27, size = 56, normalized size = 1.65

$$a^3\,x + \frac{b^3\,x\,(c\,x^n)^{3/n}}{4} + \frac{3\,a^2\,b\,x\,(c\,x^n)^{1/n}}{2} + a\,b^2\,x\,(c\,x^n)^{2/n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x^n)^(1/n))^3,x)
```

```
[Out] a^3*x + (b^3*x*(c*x^n)^(3/n))/4 + (3*a^2*b*x*(c*x^n)^(1/n))/2 + a*b^2*x*(c*
x^n)^(2/n)
```

sympy [**B**] time = 0.78, size = 63, normalized size = 1.85

$$a^3x + \frac{3a^2bc^{\frac{1}{n}}x\,(x^n)^{\frac{1}{n}}}{2} + ab^2c^{\frac{2}{n}}x\,(x^n)^{\frac{2}{n}} + \frac{b^3c^{\frac{3}{n}}x\,(x^n)^{\frac{3}{n}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x**n)**(1/n))**3,x)
```

```
[Out] a**3*x + 3*a**2*b*c**(1/n)*x*(x**n)**(1/n)/2 + a*b**2*c**(2/n)*x*(x**n)**(2
/n) + b**3*c**(3/n)*x*(x**n)**(3/n)/4
```

3.3006 $\int \frac{x^3}{a+b(cx^n)^{\frac{1}{n}}}\,dx$

Optimal. Leaf size=101

$$-\frac{a^3x^4\left(cx^n\right)^{-4/n}\log\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}{b^4}+\frac{a^2x^4\left(cx^n\right)^{-3/n}}{b^3}-\frac{ax^4\left(cx^n\right)^{-2/n}}{2b^2}+\frac{x^4\left(cx^n\right)^{-1/n}}{3b}$$

```
[Out] a^2*x^4/b^3/((c*x^n)^(3/n))-1/2*a*x^4/b^2/((c*x^n)^(2/n))+1/3*x^4/b/((c*x^n
)^(1/n))-a^3*x^4*ln(a+b*(c*x^n)^(1/n))/b^4/((c*x^n)^(4/n))
```

Rubi [A] time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {368, 43}

$$\frac{a^2x^4\left(cx^n\right)^{-3/n}}{b^3}-\frac{a^3x^4\left(cx^n\right)^{-4/n}\log\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}{b^4}-\frac{ax^4\left(cx^n\right)^{-2/n}}{2b^2}+\frac{x^4\left(cx^n\right)^{-1/n}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*(c*x^n)^n^(-1)),x]
```

```
[Out] (a^2*x^4)/(b^3*(c*x^n)^(3/n)) - (a*x^4)/(2*b^2*(c*x^n)^(2/n)) + x^4/(3*b*(c
*x^n)^n^(-1)) - (a^3*x^4*Log[a + b*(c*x^n)^n^(-1)])/(b^4*(c*x^n)^(4/n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{a+b\left(cx^n\right)^{\frac{1}{n}}}\,dx &= \left(x^4\left(cx^n\right)^{-4/n}\right)\text{Subst}\left(\int \frac{x^3}{a+bx}\,dx, x, \left(cx^n\right)^{\frac{1}{n}}\right)\\ &= \left(x^4\left(cx^n\right)^{-4/n}\right)\text{Subst}\left(\int\left(\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right)dx, x, \left(cx^n\right)^{\frac{1}{n}}\right)\\ &= \frac{a^2x^4\left(cx^n\right)^{-3/n}}{b^3}-\frac{ax^4\left(cx^n\right)^{-2/n}}{2b^2}+\frac{x^4\left(cx^n\right)^{-1/n}}{3b}-\frac{a^3x^4\left(cx^n\right)^{-4/n}\log\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}{b^4}\end{aligned}$$

Mathematica [A] time = 0.06, size = 87, normalized size = 0.86

$$\frac{x^4\left(cx^n\right)^{-4/n}\left(b\left(cx^n\right)^{\frac{1}{n}}\left(6a^2-3ab\left(cx^n\right)^{\frac{1}{n}}+2b^2\left(cx^n\right)^{2/n}\right)-6a^3\log\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)\right)}{6b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*(c*x^n)^n^(-1)),x]
```

```
[Out] (x^4*(b*(c*x^n)^n^(-1)*(6*a^2 - 3*a*b*(c*x^n)^n^(-1) + 2*b^2*(c*x^n)^(2/n))
 - 6*a^3*Log[a + b*(c*x^n)^n^(-1)]))/(6*b^4*(c*x^n)^(4/n))
```

fricas [A] time = 0.93, size = 74, normalized size = 0.73

$$\frac{2\,b^3 c^{\frac{3}{n}} x^3 - 3\,ab^2 c^{\frac{2}{n}} x^2 + 6\,a^2 b c^{\left(\frac{1}{n}\right)} x - 6\,a^3 \log\left(b c^{\left(\frac{1}{n}\right)} x + a\right)}{6\,b^4 c^{\frac{4}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(c*x^n)^(1/n)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*c^(3/n)*x^3 - 3*a*b^2*c^(2/n)*x^2 + 6*a^2*b*c^(1/n)*x - 6*a^3*lo
g(b*c^(1/n)*x + a))/(b^4*c^(4/n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(cx^n)^{\left(\frac{1}{n}\right)} b + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(c*x^n)^(1/n)),x, algorithm="giac")
```

```
[Out] integrate(x^3/((c*x^n)^(1/n)*b + a), x)
```

maple [C] time = 0.20, size = 387, normalized size = 3.83

$$-\frac{a^3 x^4 c^{-\frac{3}{n}} c^{-\frac{1}{n}} (x^n)^{-\frac{3}{n}} (x^n)^{-\frac{1}{n}} \mathrm{e}^{-\frac{2i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}\left(ic\,x^n\right)\right)\left(-\mathrm{csgn}\left(ix^n\right)+\mathrm{csgn}\left(ic\,x^n\right)\right)\mathrm{csgn}\left(ic\,x^n\right)}{n}} \ln\left(b\,c^{\frac{1}{n}} (x^n)^{\frac{1}{n}} \mathrm{e}^{\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}\left(ic\,x^n\right)\right)\left(-\mathrm{csgn}\left(ix^n\right)\right.}{2n}}\right.}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*(c*x^n)^(1/n)+a),x)
```

```
[Out] 1/3/((x^n)^(1/n))/(c^(1/n))*x^4*exp(-1/2*I*Pi*csgn(I*c*x^n)*(csgn(I*c)-csgn
(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n)/b-1/2/((x^n)^(1/n))^2/(c^(1/n))^
2*x^4*exp(-I*Pi*csgn(I*c*x^n)*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(
I*c*x^n))/n)/b^2*a+1/((x^n)^(1/n))^3/(c^(1/n))^3*x^4*exp(-3/2*I*Pi*csgn(I*c
*x^n)*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n)/b^3*a^2-ln(
b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*csgn(I*c*x^n)*(csgn(I*c)-csgn(I*c*x^n))*
(-csgn(I*x^n)+csgn(I*c*x^n))/n)+a)/(c^(1/n))/((x^n)^(1/n))*a^3*(x^n)^(-3/n)
*c^(-3/n)*x^4/b^4*exp(-2*I*Pi*csgn(I*c*x^n)*(csgn(I*c)-csgn(I*c*x^n))*(-csg
n(I*x^n)+csgn(I*c*x^n))/n)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(cx^n)^{\left(\frac{1}{n}\right)} b + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(c*x^n)^(1/n)),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((c*x^n)^(1/n)*b + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{a + b\,(c\,x^n)^{1/n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*(c*x^n)^(1/n)),x)
```

```
[Out] int(x^3/(a + b*(c*x^n)^(1/n)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{a + b\,(cx^n)^{\frac{1}{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*(c*x**n)**(1/n)),x)
```

```
[Out] Integral(x**3/(a + b*(c*x**n)**(1/n)), x)
```

3.3007 $\int \frac{x^2}{a+b(cx^n)^{\frac{1}{n}}}\,dx$

Optimal. Leaf size=77

$$\frac{a^2x^3\left(cx^n\right)^{-3/n}\log\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}{b^3}-\frac{ax^3\left(cx^n\right)^{-2/n}}{b^2}+\frac{x^3\left(cx^n\right)^{-1/n}}{2b}$$

```
[Out] -a*x^3/b^2/((c*x^n)^(2/n))+1/2*x^3/b/((c*x^n)^(1/n))+a^2*x^3*ln(a+b*(c*x^n)
^(1/n))/b^3/((c*x^n)^(3/n))
```

Rubi [A] time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {368, 43}

$$\frac{a^2x^3\left(cx^n\right)^{-3/n}\log\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}{b^3}-\frac{ax^3\left(cx^n\right)^{-2/n}}{b^2}+\frac{x^3\left(cx^n\right)^{-1/n}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*(c*x^n)^n^(-1)),x]
```

```
[Out] -((a*x^3)/(b^2*(c*x^n)^(2/n))) + x^3/(2*b*(c*x^n)^n^(-1)) + (a^2*x^3*Log[a
+ b*(c*x^n)^n^(-1)])/(b^3*(c*x^n)^(3/n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{a+b\left(cx^n\right)^{\frac{1}{n}}}\,dx &= \left(x^3\left(cx^n\right)^{-3/n}\right)\text{Subst}\left(\int \frac{x^2}{a+bx}\,dx, x, \left(cx^n\right)^{\frac{1}{n}}\right)\\ &= \left(x^3\left(cx^n\right)^{-3/n}\right)\text{Subst}\left(\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx, x, \left(cx^n\right)^{\frac{1}{n}}\right)\\ &= -\frac{ax^3\left(cx^n\right)^{-2/n}}{b^2}+\frac{x^3\left(cx^n\right)^{-1/n}}{2b}+\frac{a^2x^3\left(cx^n\right)^{-3/n}\log\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}{b^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 67, normalized size = 0.87

$$\frac{x^3\left(cx^n\right)^{-3/n}\left(2a^2\log\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)+b\left(cx^n\right)^{\frac{1}{n}}\left(b\left(cx^n\right)^{\frac{1}{n}}-2a\right)\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*(c*x^n)^n^(-1)),x]
```

```
[Out] (x^3*(b*(c*x^n)^n^(-1)*(-2*a + b*(c*x^n)^n^(-1)) + 2*a^2*Log[a + b*(c*x^n)^
n^(-1)]))/(2*b^3*(c*x^n)^(3/n))
```

fricas [A] time = 1.00, size = 55, normalized size = 0.71

$$\frac{b^2 c^{\frac{2}{n}} x^2 - 2\,abc^{\left(\frac{1}{n}\right)} x + 2\,a^2 \log\left(bc^{\left(\frac{1}{n}\right)} x + a\right)}{2\,b^3 c^{\frac{3}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(c*x^n)^(1/n)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*c^(2/n)*x^2 - 2*a*b*c^(1/n)*x + 2*a^2*log(b*c^(1/n)*x + a))/(b^3*c
^(3/n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(cx^n)^{\left(\frac{1}{n}\right)} b + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(c*x^n)^(1/n)),x, algorithm="giac")
```

```
[Out] integrate(x^2/((c*x^n)^(1/n)*b + a), x)
```

maple [C] time = 0.12, size = 310, normalized size = 4.03

$$\frac{a^2 x^3 c^{-\frac{2}{n}} c^{-\frac{1}{n}} (x^n)^{-\frac{2}{n}} (x^n)^{-\frac{1}{n}} \mathrm{e}^{-\frac{3i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}\left(ic\,x^n\right)\right)\left(-\mathrm{csgn}\left(ix^n\right)+\mathrm{csgn}\left(ic\,x^n\right)\right)\mathrm{csgn}\left(ic\,x^n\right)}{2n}} \ln\left(b\,c^{\frac{1}{n}} (x^n)^{\frac{1}{n}} \mathrm{e}^{\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}\left(ic\,x^n\right)\right)\left(-\mathrm{csgn}\left(ix^n\right)+\mathrm{csgn}\right.}{2n}}\right.}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*(c*x^n)^(1/n)+a),x)
```

```
[Out] 1/2/(c^(1/n))/((x^n)^(1/n))*x^3*exp(-1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-c
sgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))/b-1/(c^(1/n))^2/((x^n)^(1/n))^2*
x^3*exp(-I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn
(I*c*x^n))/b^2*a+ln(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*(csgn(I*c)-csgn(I*c*
x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))+a)/((x^n)^(1/n))/(c^(1/
n))*a^2*c^(-2/n)*(x^n)^(-2/n)/b^3*x^3*exp(-3/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n
))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(cx^n)^{\left(\frac{1}{n}\right)} b + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(c*x^n)^(1/n)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((c*x^n)^(1/n)*b + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{a + b\,(c\,x^n)^{1/n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*(c*x^n)^(1/n)),x)
```

```
[Out] int(x^2/(a + b*(c*x^n)^(1/n)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a + b\left(cx^n\right)^{\frac{1}{n}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*(c*x**n)**(1/n)),x)
```

```
[Out] Integral(x**2/(a + b*(c*x**n)**(1/n)), x)
```

3.3008 $\int \frac{x}{a+b(cx^n)^{\frac{1}{n}}}\,dx$

Optimal. Leaf size=53

$$\frac{x^2\left(cx^n\right)^{-1/n}}{b}-\frac{ax^2\left(cx^n\right)^{-2/n}\log\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}{b^2}$$

```
[Out] x^2/b/((c*x^n)^(1/n))-a*x^2*ln(a+b*(c*x^n)^(1/n))/b^2/((c*x^n)^(2/n))
```

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {368, 43}

$$\frac{x^2\left(cx^n\right)^{-1/n}}{b}-\frac{ax^2\left(cx^n\right)^{-2/n}\log\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*(c*x^n)^n^(-1)),x]
```

```
[Out] x^2/(b*(c*x^n)^n^(-1)) - (a*x^2*Log[a + b*(c*x^n)^n^(-1)])/(b^2*(c*x^n)^(2/
n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{a+b\left(cx^n\right)^{\frac{1}{n}}}\,dx &= \left(x^2\left(cx^n\right)^{-2/n}\right)\text{Subst}\left(\int \frac{x}{a+bx}\,dx, x, \left(cx^n\right)^{\frac{1}{n}}\right)\\ &= \left(x^2\left(cx^n\right)^{-2/n}\right)\text{Subst}\left(\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx, x, \left(cx^n\right)^{\frac{1}{n}}\right)\\ &= \frac{x^2\left(cx^n\right)^{-1/n}}{b}-\frac{ax^2\left(cx^n\right)^{-2/n}\log\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}{b^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.92

$$x^2\left(cx^n\right)^{-2/n}\left(\frac{\left(cx^n\right)^{\frac{1}{n}}}{b}-\frac{a\log\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}{b^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*(c*x^n)^n^(-1)),x]
```

```
[Out] (x^2*((c*x^n)^n^(-1)/b - (a*Log[a + b*(c*x^n)^n^(-1)])/b^2))/(c*x^n)^(2/n)
```

fricas [A] time = 0.78, size = 36, normalized size = 0.68

$$\frac{bc^{\left(\frac{1}{n}\right)}x - a\log\left(bc^{\left(\frac{1}{n}\right)}x + a\right)}{b^2c^{\frac{2}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(c*x^n)^(1/n)),x, algorithm="fricas")
```

```
[Out] (b*c^(1/n)*x - a*log(b*c^(1/n)*x + a))/(b^2*c^(2/n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(cx^n)^{\left(\frac{1}{n}\right)}\,b + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(c*x^n)^(1/n)),x, algorithm="giac")
```

```
[Out] integrate(x/((c*x^n)^(1/n)*b + a), x)
```

maple [C] time = 0.12, size = 233, normalized size = 4.40

$$-\frac{a\,x^2\left(c^{-\frac{1}{n}}\right)^2\left((x^n)^{-\frac{1}{n}}\right)^2 e^{-\frac{i\pi\left(\mathrm{csgn}\left(ic\right)-\mathrm{csgn}\left(ic\,x^n\right)\right)\left(-\mathrm{csgn}\left(ix^n\right)+\mathrm{csgn}\left(ic\,x^n\right)\right)\mathrm{csgn}\left(ic\,x^n\right)}{n}}\ln\left(b\,c^{\frac{1}{n}}\,(x^n)^{\frac{1}{n}}\,e^{\frac{i\pi\left(\mathrm{csgn}\left(ic\right)-\mathrm{csgn}\left(ic\,x^n\right)\right)\left(-\mathrm{csgn}\left(ix^n\right)+\mathrm{csgn}\left(ic\,x^n\right)\right)\mathrm{csgn}\left(ic\,x^n\right)}{2n}}\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*(c*x^n)^(1/n)+a),x)
```

```
[Out] 1/(c^(1/n))/((x^n)^(1/n))*x^2*exp(-1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csg
n(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))/b-ln(b*c^(1/n)*(x^n)^(1/n)*exp(1/2
*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n
))+a)/((x^n)^(1/n))/(c^(1/n))*a*c^(-1/n)*(x^n)^(-1/n)/b^2*x^2*exp(-I*Pi*(cs
gn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(cx^n)^{\left(\frac{1}{n}\right)}\,b + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(c*x^n)^(1/n)),x, algorithm="maxima")
```

```
[Out] integrate(x/((c*x^n)^(1/n)*b + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{a + b\,(c\,x^n)^{1/n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*(c*x^n)^(1/n)),x)
```

```
[Out] int(x/(a + b*(c*x^n)^(1/n)), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a + b\left(cx^{n}\right)^{\frac{1}{n}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(c*x**n)**(1/n)),x)
```

```
[Out] Integral(x/(a + b*(c*x**n)**(1/n)), x)
```

3.3009 $\int \frac{1}{a+b(cx^n)^{\frac{1}{n}}}\,dx$

Optimal. Leaf size=30

$$\frac{x\,(cx^n)^{-1/n}\log\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}{b}$$

```
[Out] x*ln(a+b*(c*x^n)^(1/n))/b/((c*x^n)^(1/n))
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {254, 31}

$$\frac{x\,(cx^n)^{-1/n}\log\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c*x^n)^n^(-1))^(-1),x]
```

```
[Out] (x*Log[a + b*(c*x^n)^n^(-1)])/(b*(c*x^n)^n^(-1))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 254

```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_.), x_Symbol] :> Dist[x/(c*x^q)^(1/q), Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\int \frac{1}{a+b\,(cx^n)^{\frac{1}{n}}}\,dx = \left(x\,(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, (cx^n)^{\frac{1}{n}}\right)$$
$$= \frac{x\,(cx^n)^{-1/n}\log\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}{b}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{x\,(cx^n)^{-1/n}\log\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c*x^n)^n^(-1))^(-1),x]
```

```
[Out] (x*Log[a + b*(c*x^n)^n^(-1)])/(b*(c*x^n)^n^(-1))
```

fricas [A] time = 0.87, size = 22, normalized size = 0.73

$$\frac{\log\left(bc^{\left(\frac{1}{n}\right)}x+a\right)}{bc^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(c*x^n)^(1/n)),x, algorithm="fricas")
```

```
[Out] log(b*c^(1/n)*x + a)/(b*c^(1/n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^n)^{\left(\frac{1}{n}\right)} b + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(c*x^n)^(1/n)),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^n)^(1/n)*b + a), x)
```

maple [C] time = 0.06, size = 139, normalized size = 4.63

$$\frac{x\, c^{-\frac{1}{n}} (x^n)^{-\frac{1}{n}}\, \mathrm{e}^{-\frac{i\pi\left(\operatorname{csgn}(ic)-\operatorname{csgn}\left(ic\,x^n\right)\right)\left(-\operatorname{csgn}\left(ix^n\right)+\operatorname{csgn}\left(ic\,x^n\right)\right)\operatorname{csgn}\left(ic\,x^n\right)}{2n}} \ln\left(b\, c^{\frac{1}{n}} (x^n)^{\frac{1}{n}}\, \mathrm{e}^{\frac{i\pi\left(\operatorname{csgn}(ic)-\operatorname{csgn}\left(ic\,x^n\right)\right)\left(-\operatorname{csgn}\left(ix^n\right)+\operatorname{csgn}\left(ic\,x^n\right)\right)\operatorname{csgn}\left(ic\,x^n\right)}{2n}} + \right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*(c*x^n)^(1/n)+a),x)
```

```
[Out] ln(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^
n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))+a)/(c^(1/n))/((x^n)^(1/n))*x*exp(-1/2*I*
Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))/
b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^n)^{\left(\frac{1}{n}\right)} b + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(c*x^n)^(1/n)),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^n)^(1/n)*b + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{a + b\,(c\,x^n)^{1/n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(c*x^n)^(1/n)),x)
```

```
[Out] int(1/(a + b*(c*x^n)^(1/n)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b\,(cx^n)^{\frac{1}{n}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(c*x**n)**(1/n)),x)
```

```
[Out] Integral(1/(a + b*(c*x**n)**(1/n)), x)
```

3.3010 $\int \frac{1}{x\left(a+b(cx^n)^{\frac{1}{n}}\right)}\,dx$

Optimal. Leaf size=26

$$\frac{\log(x)}{a}-\frac{\log\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}{a}$$

```
[Out] ln(x)/a-ln(a+b*(c*x^n)^(1/n))/a
```

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {368, 36, 29, 31}

$$\frac{\log(x)}{a}-\frac{\log\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*(c*x^n)^n^(-1))),x]
```

```
[Out] Log[x]/a - Log[a + b*(c*x^n)^n^(-1)]/a
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}\,dx &= \text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx, x, (cx^n)^{\frac{1}{n}}\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, (cx^n)^{\frac{1}{n}}\right)}{a}-\frac{b\,\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, (cx^n)^{\frac{1}{n}}\right)}{a}\\ &= \frac{\log(x)}{a}-\frac{\log\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.88

$$\frac{\log(x) - \log\left(a + b\,(cx^n)^{\frac{1}{n}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*(c*x^n)^n^(-1))),x]
[Out] (Log[x] - Log[a + b*(c*x^n)^n^(-1)])/a
```

fricas [A] time = 0.89, size = 21, normalized size = 0.81

$$-\frac{\log\left(bc^{\left(\frac{1}{n}\right)}x + a\right) - \log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x^n)^(1/n)),x, algorithm="fricas")
[Out] -(log(b*c^(1/n)*x + a) - log(x))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((cx^n)^{\left(\frac{1}{n}\right)}\,b + a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x^n)^(1/n)),x, algorithm="giac")
[Out] integrate(1/(((c*x^n)^(1/n)*b + a)*x), x)
```

maple [A] time = 0.02, size = 35, normalized size = 1.35

$$\frac{\ln\left((c\,x^n)^{\frac{1}{n}}\right)}{a} - \frac{\ln\left(b\,(c\,x^n)^{\frac{1}{n}} + a\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*(c*x^n)^(1/n)+a),x)
[Out] -ln(b*(c*x^n)^(1/n)+a)/a+1/a*ln((c*x^n)^(1/n))
```

maxima [A] time = 0.65, size = 40, normalized size = 1.54

$$\frac{\log(x)}{a} - \frac{\log\left(\frac{bc^{\left(\frac{1}{n}\right)}(x^n)^{\left(\frac{1}{n}\right)} + a}{bc^{\left(\frac{1}{n}\right)}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x^n)^(1/n)),x, algorithm="maxima")
[Out] log(x)/a - log((b*c^(1/n)*(x^n)^(1/n) + a)/(b*c^(1/n)))/a
```

mupad [B] time = 1.22, size = 24, normalized size = 0.92

$$-\frac{\ln\left(a + b\,(c\,x^n)^{1/n}\right) - \ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*(c*x^n)^(1/n))),x)
```

```
[Out] -(log(a + b*(c*x^n)^(1/n)) - log(x))/a
```

sympy [**A**] time = 1.55, size = 56, normalized size = 2.15

$$\begin{cases} \tilde{\infty} c^{-\frac{1}{n}} \left(x^{n}\right)^{-\frac{1}{n}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{c^{-\frac{1}{n}} \left(x^{n}\right)^{-\frac{1}{n}}}{b} & \text{for } a = 0 \\ \frac{\log\left(x\right)}{a} & \text{for } b = 0 \\ \frac{\log\left(x\right)}{a} - \frac{\log\left(\frac{a}{b} + c^{\frac{1}{n}} \left(x^{n}\right)^{\frac{1}{n}}\right)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x**n)**(1/n)),x)
```

```
[Out] Piecewise((zoo*c**(-1/n)*(x**n)**(-1/n), Eq(a, 0) & Eq(b, 0)), (-c**(-1/n)*
(x**n)**(-1/n)/b, Eq(a, 0)), (log(x)/a, Eq(b, 0)), (log(x)/a - log(a/b + c*
*(1/n)*(x**n)**(1/n))/a, True))
```

3.3011 $\int \frac{1}{x^2\left(a+b(cx^n)^{\frac{1}{n}}\right)}\,dx$

Optimal. Leaf size=60

$$-\frac{b\log(x)\,(cx^n)^{\frac{1}{n}}}{a^2x}+\frac{b\,(cx^n)^{\frac{1}{n}}\log\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}{a^2x}-\frac{1}{ax}$$

```
[Out] -1/a/x-b*(c*x^n)^(1/n)*ln(x)/a^2/x+b*(c*x^n)^(1/n)*ln(a+b*(c*x^n)^(1/n))/a^
2/x
```

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {368, 44}

$$-\frac{b\log(x)\,(cx^n)^{\frac{1}{n}}}{a^2x}+\frac{b\,(cx^n)^{\frac{1}{n}}\log\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}{a^2x}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*(c*x^n)^n^(-1))),x]
```

```
[Out] -(1/(a*x)) - (b*(c*x^n)^n^(-1)*Log[x])/(a^2*x) + (b*(c*x^n)^n^(-1)*Log[a +
b*(c*x^n)^n^(-1)])/(a^2*x)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}\,dx &= \frac{(cx^n)^{\frac{1}{n}}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)}\,dx, x, (cx^n)^{\frac{1}{n}}\right)}{x}\\ &= \frac{(cx^n)^{\frac{1}{n}}\,\text{Subst}\left(\int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx, x, (cx^n)^{\frac{1}{n}}\right)}{x}\\ &= -\frac{1}{ax}-\frac{b\,(cx^n)^{\frac{1}{n}}\log(x)}{a^2x}+\frac{b\,(cx^n)^{\frac{1}{n}}\log\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}{a^2x}\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.82

$$-\frac{-b\,(cx^n)^{\frac{1}{n}}\log\left(a+b\,(cx^n)^{\frac{1}{n}}\right)+a+b\log(x)\,(cx^n)^{\frac{1}{n}}}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*(c*x^n)^n^(-1))),x]
```

```
[Out] -((a + b*(c*x^n)^n^(-1)*Log[x] - b*(c*x^n)^n^(-1)*Log[a + b*(c*x^n)^n^(-1)]
)/(a^2*x))
```

fricas [A] time = 0.93, size = 41, normalized size = 0.68

$$\frac{bc^{\left(\frac{1}{n}\right)} x \log\left(bc^{\left(\frac{1}{n}\right)} x + a\right) - bc^{\left(\frac{1}{n}\right)} x \log(x) - a}{a^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(c*x^n)^(1/n)),x, algorithm="fricas")
```

```
[Out] (b*c^(1/n)*x*log(b*c^(1/n)*x + a) - b*c^(1/n)*x*log(x) - a)/(a^2*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((cx^n)^{\left(\frac{1}{n}\right)} b + a\right) x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(c*x^n)^(1/n)),x, algorithm="giac")
```

```
[Out] integrate(1/(((c*x^n)^(1/n)*b + a)*x^2), x)
```

maple [C] time = 0.16, size = 220, normalized size = 3.67

$$-\frac{b\, c^{\frac{1}{n}} (x^n)^{\frac{1}{n}}\, \mathrm{e}^{\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}\left(ic\, x^n\right)\right)\left(-\mathrm{csgn}\left(ix^n\right)+\mathrm{csgn}\left(ic\, x^n\right)\right)\mathrm{csgn}\left(ic\, x^n\right)}{2n}} \ln(x)}{a^2 x} + \frac{b\, c^{\frac{1}{n}} (x^n)^{\frac{1}{n}}\, \mathrm{e}^{\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}\left(ic\, x^n\right)\right)\left(-\mathrm{csgn}\left(ix^n\right)+\mathrm{csgn}\left(ic\, x^n\right)\right)\mathrm{csgn}\left(ic\, x^n\right)}{2n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*(c*x^n)^(1/n)+a),x)
```

```
[Out] ln(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^
n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))+a)*b/x*c^(1/n)*(x^n)^(1/n)/a^2*exp(1/2*I
*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))
-1/a/x-1/a^2*b/x*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))
*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))*ln(x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((cx^n)^{\left(\frac{1}{n}\right)} b + a\right) x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(c*x^n)^(1/n)),x, algorithm="maxima")
```

```
[Out] integrate(1/(((c*x^n)^(1/n)*b + a)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2 \left(a + b\,(c\,x^n)^{1/n}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*(c*x^n)^(1/n))),x)
```

```
[Out] int(1/(x^2*(a + b*(c*x^n)^(1/n))), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a + b\left(cx^n\right)^{\frac{1}{n}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*(c*x**n)**(1/n)),x)
```

```
[Out] Integral(1/(x**2*(a + b*(c*x**n)**(1/n))), x)
```

3.3012 $\int \frac{1}{x^3\left(a+b(cx^n)^{\frac{1}{n}}\right)}\,dx$

Optimal. Leaf size=87

$$\frac{b^2\log(x)\,(cx^n)^{2/n}}{a^3x^2}-\frac{b^2\,(cx^n)^{2/n}\log\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}{a^3x^2}+\frac{b\,(cx^n)^{\frac{1}{n}}}{a^2x^2}-\frac{1}{2ax^2}$$

```
[Out] -1/2/a/x^2+b*(c*x^n)^(1/n)/a^2/x^2+b^2*(c*x^n)^(2/n)*ln(x)/a^3/x^2-b^2*(c*x
^n)^(2/n)*ln(a+b*(c*x^n)^(1/n))/a^3/x^2
```

Rubi [A] time = 0.03, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {368, 44}

$$\frac{b^2\log(x)\,(cx^n)^{2/n}}{a^3x^2}-\frac{b^2\,(cx^n)^{2/n}\log\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}{a^3x^2}+\frac{b\,(cx^n)^{\frac{1}{n}}}{a^2x^2}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*(c*x^n)^n^(-1))),x]
```

```
[Out] -1/(2*a*x^2) + (b*(c*x^n)^n^(-1))/(a^2*x^2) + (b^2*(c*x^n)^(2/n)*Log[x])/(a
^3*x^2) - (b^2*(c*x^n)^(2/n)*Log[a + b*(c*x^n)^n^(-1)])/(a^3*x^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}\,dx &= \frac{(cx^n)^{2/n}\,\text{Subst}\left(\int \frac{1}{x^3(a+bx)}\,dx,x,(cx^n)^{\frac{1}{n}}\right)}{x^2}\\ &= \frac{(cx^n)^{2/n}\,\text{Subst}\left(\int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)dx,x,(cx^n)^{\frac{1}{n}}\right)}{x^2}\\ &= -\frac{1}{2ax^2}+\frac{b\,(cx^n)^{\frac{1}{n}}}{a^2x^2}+\frac{b^2\,(cx^n)^{2/n}\log(x)}{a^3x^2}-\frac{b^2\,(cx^n)^{2/n}\log\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}{a^3x^2}\end{aligned}$$

Mathematica [A] time = 0.05, size = 75, normalized size = 0.86

$$-\frac{a^2+2b^2\,(cx^n)^{2/n}\log\left(a+b\,(cx^n)^{\frac{1}{n}}\right)-2ab\,(cx^n)^{\frac{1}{n}}-2b^2\log(x)\,(cx^n)^{2/n}}{2a^3x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*(c*x^n)^n^(-1))),x]
```

```
[Out] -1/2*(a^2 - 2*a*b*(c*x^n)^n^(-1) - 2*b^2*(c*x^n)^(2/n)*Log[x] + 2*b^2*(c*x^
n)^(2/n)*Log[a + b*(c*x^n)^n^(-1)])/(a^3*x^2)
```

fricas [A] time = 0.87, size = 65, normalized size = 0.75

$$-\frac{2\,b^2c^{\frac{2}{n}}x^2\log\left(bc^{\left(\frac{1}{n}\right)}x+a\right)-2\,b^2c^{\frac{2}{n}}x^2\log(x)-2\,abc^{\left(\frac{1}{n}\right)}x+a^2}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(c*x^n)^(1/n)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*c^(2/n)*x^2*log(b*c^(1/n)*x + a) - 2*b^2*c^(2/n)*x^2*log(x) - 2
*a*b*c^(1/n)*x + a^2)/(a^3*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left((cx^n)^{\left(\frac{1}{n}\right)}b+a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(c*x^n)^(1/n)),x, algorithm="giac")
```

```
[Out] integrate(1/(((c*x^n)^(1/n)*b + a)*x^3), x)
```

maple [C] time = 0.18, size = 302, normalized size = 3.47

$$\frac{b^2c^{\frac{2}{n}}\,(x^n)^{\frac{2}{n}}\,\mathrm{e}^{\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}\left(ic\,x^n\right)\right)\left(-\mathrm{csgn}\left(ix^n\right)+\mathrm{csgn}\left(ic\,x^n\right)\right)\mathrm{csgn}\left(ic\,x^n\right)}{n}}\ln(x)}{a^3x^2}-\frac{b^2c^{\frac{2}{n}}\,(x^n)^{\frac{2}{n}}\,\mathrm{e}^{\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}\left(ic\,x^n\right)\right)\left(-\mathrm{csgn}\left(ix^n\right)+\mathrm{csgn}\left(ic\,x^n\right)\right)\mathrm{csgn}\left(ic\,x^n\right)}{n}}\,\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*(c*x^n)^(1/n)+a),x)
```

```
[Out] -ln(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x
^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))+a)*b^2/x^2*((x^n)^(1/n))^2*(c^(1/n))^2/
a^3*exp(I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(
I*c*x^n))-1/2/a/x^2+1/a^2*b/x^2*(x^n)^(1/n)*c^(1/n)*exp(1/2*I*Pi*(csgn(I*c)
-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))+1/a^3/x^2*(x^
n)^(2/n)*c^(2/n)*exp(I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*
x^n))/n*csgn(I*c*x^n))*b^2*ln(x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left((cx^n)^{\left(\frac{1}{n}\right)}b+a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(c*x^n)^(1/n)),x, algorithm="maxima")
```

```
[Out] integrate(1/(((c*x^n)^(1/n)*b + a)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3 \left(a + b \left(c\, x^n\right)^{1/n}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*(c*x^n)^(1/n))),x)
```

```
[Out] int(1/(x^3*(a + b*(c*x^n)^(1/n))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(a + b \left(cx^n\right)^{\frac{1}{n}}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*(c*x**n)**(1/n)),x)
```

```
[Out] Integral(1/(x**3*(a + b*(c*x**n)**(1/n))), x)
```

3.3013 $\int \frac{x^3}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^2}\,dx$

Optimal. Leaf size=114

$$\frac{a^3x^4\left(cx^n\right)^{-4/n}}{b^4\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}+\frac{3a^2x^4\left(cx^n\right)^{-4/n}\log\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}{b^4}-\frac{2ax^4\left(cx^n\right)^{-3/n}}{b^3}+\frac{x^4\left(cx^n\right)^{-2/n}}{2b^2}$$

```
[Out] -2*a*x^4/b^3/((c*x^n)^(3/n))+1/2*x^4/b^2/((c*x^n)^(2/n))+a^3*x^4/b^4/((c*x^
n)^(4/n))/(a+b*(c*x^n)^(1/n))+3*a^2*x^4*ln(a+b*(c*x^n)^(1/n))/b^4/((c*x^n)^
(4/n))
```

Rubi [A] time = 0.04, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {368, 43}

$$\frac{a^3x^4\left(cx^n\right)^{-4/n}}{b^4\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}+\frac{3a^2x^4\left(cx^n\right)^{-4/n}\log\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}{b^4}-\frac{2ax^4\left(cx^n\right)^{-3/n}}{b^3}+\frac{x^4\left(cx^n\right)^{-2/n}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*(c*x^n)^n^(-1))^2,x]
```

```
[Out] (-2*a*x^4)/(b^3*(c*x^n)^(3/n)) + x^4/(2*b^2*(c*x^n)^(2/n)) + (a^3*x^4)/(b^4
*(c*x^n)^(4/n)*(a + b*(c*x^n)^n^(-1))) + (3*a^2*x^4*Log[a + b*(c*x^n)^n^(-1
)])/(b^4*(c*x^n)^(4/n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx &= \left(x^4\left(cx^n\right)^{-4/n}\right)\text{Subst}\left(\int \frac{x^3}{(a+bx)^2}\,dx, x, \left(cx^n\right)^{\frac{1}{n}}\right)\\ &= \left(x^4\left(cx^n\right)^{-4/n}\right)\text{Subst}\left(\int \left(-\frac{2a}{b^3}+\frac{x}{b^2}-\frac{a^3}{b^3(a+bx)^2}+\frac{3a^2}{b^3(a+bx)}\right)dx, x, \left(cx^n\right)^{\frac{1}{n}}\right)\\ &= -\frac{2ax^4\left(cx^n\right)^{-3/n}}{b^3}+\frac{x^4\left(cx^n\right)^{-2/n}}{2b^2}+\frac{a^3x^4\left(cx^n\right)^{-4/n}}{b^4\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}+\frac{3a^2x^4\left(cx^n\right)^{-4/n}\log\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}{b^4}\end{aligned}$$

Mathematica [A] time = 0.08, size = 89, normalized size = 0.78

$$\frac{x^4 (cx^n)^{-4/n} \left(\frac{2a^3}{a+b(cx^n)^{\frac{1}{n}}} + 6a^2 \log\left(a + b (cx^n)^{\frac{1}{n}}\right) - 4ab (cx^n)^{\frac{1}{n}} + b^2 (cx^n)^{2/n} \right)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*(c*x^n)^n^(-1))^2,x]
```

```
[Out] (x^4*(-4*a*b*(c*x^n)^n^(-1) + b^2*(c*x^n)^(2/n) + (2*a^3)/(a + b*(c*x^n)^n^
(-1)) + 6*a^2*Log[a + b*(c*x^n)^n^(-1)]))/(2*b^4*(c*x^n)^(4/n))
```

fricas [A] time = 0.91, size = 105, normalized size = 0.92

$$\frac{b^3 c^{\frac{3}{n}} x^3 - 3\,ab^2 c^{\frac{2}{n}} x^2 - 4\,a^2 bc^{\left(\frac{1}{n}\right)} x + 2\,a^3 + 6\left(a^2 bc^{\left(\frac{1}{n}\right)} x + a^3\right) \log\left(bc^{\left(\frac{1}{n}\right)} x + a\right)}{2\left(b^5 c^{\frac{5}{n}} x + ab^4 c^{\frac{4}{n}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(c*x^n)^(1/n))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*c^(3/n)*x^3 - 3*a*b^2*c^(2/n)*x^2 - 4*a^2*b*c^(1/n)*x + 2*a^3 + 6*
(a^2*b*c^(1/n)*x + a^3)*log(b*c^(1/n)*x + a))/(b^5*c^(5/n)*x + a*b^4*c^(4/n
))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left((cx^n)^{\left(\frac{1}{n}\right)} b + a\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(c*x^n)^(1/n))^2,x, algorithm="giac")
```

```
[Out] integrate(x^3/((c*x^n)^(1/n)*b + a)^2, x)
```

maple [C] time = 0.13, size = 463, normalized size = 4.06

$$\frac{3a^2 x^4 c^{-\frac{3}{n}} c^{-\frac{1}{n}} (x^n)^{-\frac{3}{n}} (x^n)^{-\frac{1}{n}} \mathrm{e}^{-\frac{2i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}\left(ic\,x^n\right)\right)\left(-\mathrm{csgn}\left(ix^n\right)+\mathrm{csgn}\left(ic\,x^n\right)\right)\mathrm{csgn}\left(ic\,x^n\right)}{n}} \ln\left(b\, c^{\frac{1}{n}} (x^n)^{\frac{1}{n}} \mathrm{e}^{\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}\left(ic\,x^n\right)\right)\left(-\mathrm{csgn}\left(ix^n\right)\right.}{2n}} \right.}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*(c*x^n)^(1/n)+a)^2,x)
```

```
[Out] 1/a*x^4/(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csg
n(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))+a)-1/a/((x^n)^(1/n))/(c^(1/n))*x^4
*exp(-1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csg
n(I*c*x^n))/b+3/2/((x^n)^(1/n))^2/(c^(1/n))^2*x^4*exp(-I*Pi*(csgn(I*c)-csgn
(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))/b^2-3*a/((x^n)^(1/
n))^3/(c^(1/n))^3*x^4*exp(-3/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)
+csgn(I*c*x^n))/n*csgn(I*c*x^n))/b^3+3*a^2*ln(b*c^(1/n)*(x^n)^(1/n)*exp(1/2
*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n
))+a)/(c^(1/n))/((x^n)^(1/n))*(x^n)^(-3/n)*c^(-3/n)*x^4/b^4*exp(-2*I*Pi*(cs
gn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x^4}{abc^{\left(\frac{1}{n}\right)}(x^n)^{\left(\frac{1}{n}\right)}+a^2}-3\int\frac{x^3}{abc^{\left(\frac{1}{n}\right)}(x^n)^{\left(\frac{1}{n}\right)}+a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(c*x^n)^(1/n))^2,x, algorithm="maxima")
```

```
[Out] x^4/(a*b*c^(1/n)*(x^n)^(1/n) + a^2) - 3*integrate(x^3/(a*b*c^(1/n)*(x^n)^(1
/n) + a^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^3}{\left(a+b\,(c\,x^n)^{1/n}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*(c*x^n)^(1/n))^2,x)
```

```
[Out] int(x^3/(a + b*(c*x^n)^(1/n))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*(c*x**n)**(1/n))**2,x)
```

```
[Out] Integral(x**3/(a + b*(c*x**n)**(1/n))**2, x)
```

3.3014 $\int \frac{x^2}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^2}\,dx$

Optimal. Leaf size=90

$$-\frac{a^2x^3\left(cx^n\right)^{-3/n}}{b^3\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}-\frac{2ax^3\left(cx^n\right)^{-3/n}\log\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}{b^3}+\frac{x^3\left(cx^n\right)^{-2/n}}{b^2}$$

```
[Out] x^3/b^2/((c*x^n)^(2/n))-a^2*x^3/b^3/((c*x^n)^(3/n))/(a+b*(c*x^n)^(1/n))-2*a
*x^3*ln(a+b*(c*x^n)^(1/n))/b^3/((c*x^n)^(3/n))
```

Rubi [A] time = 0.03, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {368, 43}

$$-\frac{a^2x^3\left(cx^n\right)^{-3/n}}{b^3\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}-\frac{2ax^3\left(cx^n\right)^{-3/n}\log\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}{b^3}+\frac{x^3\left(cx^n\right)^{-2/n}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*(c*x^n)^n^(-1))^2,x]
```

```
[Out] x^3/(b^2*(c*x^n)^(2/n)) - (a^2*x^3)/(b^3*(c*x^n)^(3/n)*(a + b*(c*x^n)^n^(-1
))) - (2*a*x^3*Log[a + b*(c*x^n)^n^(-1)])/(b^3*(c*x^n)^(3/n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx &= \left(x^3\left(cx^n\right)^{-3/n}\right)\text{Subst}\left(\int \frac{x^2}{(a+bx)^2}\,dx, x, \left(cx^n\right)^{\frac{1}{n}}\right)\\ &= \left(x^3\left(cx^n\right)^{-3/n}\right)\text{Subst}\left(\int \left(\frac{1}{b^2}+\frac{a^2}{b^2(a+bx)^2}-\frac{2a}{b^2(a+bx)}\right)dx, x, \left(cx^n\right)^{\frac{1}{n}}\right)\\ &= \frac{x^3\left(cx^n\right)^{-2/n}}{b^2}-\frac{a^2x^3\left(cx^n\right)^{-3/n}}{b^3\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}-\frac{2ax^3\left(cx^n\right)^{-3/n}\log\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}{b^3}\end{aligned}$$

Mathematica [A] time = 0.07, size = 67, normalized size = 0.74

$$\frac{x^3 (cx^n)^{-3/n} \left(-\frac{a^2}{a+b(cx^n)^{\frac{1}{n}}} - 2a \log\left(a + b (cx^n)^{\frac{1}{n}}\right) + b (cx^n)^{\frac{1}{n}}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*(c*x^n)^n^(-1))^2,x]
```

```
[Out] (x^3*(b*(c*x^n)^n^(-1) - a^2/(a + b*(c*x^n)^n^(-1)) - 2*a*Log[a + b*(c*x^n)
^n^(-1)]))/(b^3*(c*x^n)^(3/n))
```

fricas [A] time = 0.96, size = 83, normalized size = 0.92

$$\frac{b^2 c^{\frac{2}{n}} x^2 + abc^{\left(\frac{1}{n}\right)} x - a^2 - 2\left(abc^{\left(\frac{1}{n}\right)} x + a^2\right) \log\left(bc^{\left(\frac{1}{n}\right)} x + a\right)}{b^4 c^{\frac{4}{n}} x + ab^3 c^{\frac{3}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(c*x^n)^(1/n))^2,x, algorithm="fricas")
```

```
[Out] (b^2*c^(2/n)*x^2 + a*b*c^(1/n)*x - a^2 - 2*(a*b*c^(1/n)*x + a^2)*log(b*c^(1
/n)*x + a))/(b^4*c^(4/n)*x + a*b^3*c^(3/n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left((cx^n)^{\left(\frac{1}{n}\right)} b + a\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(c*x^n)^(1/n))^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/((c*x^n)^(1/n)*b + a)^2, x)
```

maple [C] time = 0.13, size = 386, normalized size = 4.29

$$-\frac{2a\, x^3 c^{-\frac{2}{n}} c^{-\frac{1}{n}} (x^n)^{-\frac{2}{n}} (x^n)^{-\frac{1}{n}} \mathrm{e}^{-\frac{3i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}\left(ic\,x^n\right)\right)\left(-\mathrm{csgn}\left(ix^n\right)+\mathrm{csgn}\left(ic\,x^n\right)\right)\mathrm{csgn}\left(ic\,x^n\right)}{2n}} \ln\left(b\, c^{\frac{1}{n}} (x^n)^{\frac{1}{n}} \mathrm{e}^{\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}\left(ic\,x^n\right)\right)\left(-\mathrm{csgn}\left(ix^n\right)+\mathrm{cs}\right.}{2n}}\right.}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*(c*x^n)^(1/n)+a)^2,x)
```

```
[Out] 1/a*x^3/(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csg
n(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))+a)-1/a/(c^(1/n))/((x^n)^(1/n))*x^3
*exp(-1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csg
n(I*c*x^n))/b+2/(c^(1/n))^2/((x^n)^(1/n))^2*x^3*exp(-I*Pi*(csgn(I*c)-csgn(I
*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))/b^2-2*a*ln(b*c^(1/n)
*(x^n)^(1/n)*exp(1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*
x^n))/n*csgn(I*c*x^n))+a)/((x^n)^(1/n))/(c^(1/n))*c^(-2/n)*(x^n)^(-2/n)/b^3
*x^3*exp(-3/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n
*csgn(I*c*x^n))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x^3}{abc^{\left(\frac{1}{n}\right)}(x^n)^{\left(\frac{1}{n}\right)}+a^2}-2\int\frac{x^2}{abc^{\left(\frac{1}{n}\right)}(x^n)^{\left(\frac{1}{n}\right)}+a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(c*x^n)^(1/n))^2,x, algorithm="maxima")
```

```
[Out] x^3/(a*b*c^(1/n)*(x^n)^(1/n) + a^2) - 2*integrate(x^2/(a*b*c^(1/n)*(x^n)^(1
/n) + a^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^2}{\left(a+b\,(c\,x^n)^{1/n}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*(c*x^n)^(1/n))^2,x)
```

```
[Out] int(x^2/(a + b*(c*x^n)^(1/n))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*(c*x**n)**(1/n))**2,x)
```

```
[Out] Integral(x**2/(a + b*(c*x**n)**(1/n))**2, x)
```

3.3015 $\int \frac{x}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^2}\,dx$

Optimal. Leaf size=67

$$\frac{ax^2\left(cx^n\right)^{-2/n}}{b^2\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}+\frac{x^2\left(cx^n\right)^{-2/n}\log\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}{b^2}$$

```
[Out] a*x^2/b^2/((c*x^n)^(2/n))/(a+b*(c*x^n)^(1/n))+x^2*ln(a+b*(c*x^n)^(1/n))/b^2
/((c*x^n)^(2/n))
```

Rubi [A] time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {368, 43}

$$\frac{ax^2\left(cx^n\right)^{-2/n}}{b^2\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}+\frac{x^2\left(cx^n\right)^{-2/n}\log\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*(c*x^n)^n^(-1))^2,x]
```

```
[Out] (a*x^2)/(b^2*(c*x^n)^(2/n)*(a + b*(c*x^n)^n^(-1))) + (x^2*Log[a + b*(c*x^n)
^n^(-1)])/(b^2*(c*x^n)^(2/n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx &= \left(x^2\left(cx^n\right)^{-2/n}\right)\text{Subst}\left(\int \frac{x}{(a+bx)^2}\,dx, x, \left(cx^n\right)^{\frac{1}{n}}\right)\\ &= \left(x^2\left(cx^n\right)^{-2/n}\right)\text{Subst}\left(\int\left(-\frac{a}{b(a+bx)^2}+\frac{1}{b(a+bx)}\right)dx, x, \left(cx^n\right)^{\frac{1}{n}}\right)\\ &= \frac{ax^2\left(cx^n\right)^{-2/n}}{b^2\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}+\frac{x^2\left(cx^n\right)^{-2/n}\log\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}{b^2}\end{aligned}$$

Mathematica [A] time = 0.04, size = 50, normalized size = 0.75

$$\frac{x^2 (cx^n)^{-2/n} \left(\frac{a}{a+b(cx^n)^{\frac{1}{n}}} + \log\left(a + b\,(cx^n)^{\frac{1}{n}}\right)\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*(c*x^n)^n^(-1))^2,x]
```

```
[Out] (x^2*(a/(a + b*(c*x^n)^n^(-1)) + Log[a + b*(c*x^n)^n^(-1)]))/(b^2*(c*x^n)^(
2/n))
```

fricas [A] time = 0.92, size = 52, normalized size = 0.78

$$\frac{\left(bc^{\left(\frac{1}{n}\right)}x + a\right)\log\left(bc^{\left(\frac{1}{n}\right)}x + a\right) + a}{b^3 c^{\frac{3}{n}} x + ab^2 c^{\frac{2}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(c*x^n)^(1/n))^2,x, algorithm="fricas")
```

```
[Out] ((b*c^(1/n)*x + a)*log(b*c^(1/n)*x + a) + a)/(b^3*c^(3/n)*x + a*b^2*c^(2/n)
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left((cx^n)^{\left(\frac{1}{n}\right)} b + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(c*x^n)^(1/n))^2,x, algorithm="giac")
```

```
[Out] integrate(x/((c*x^n)^(1/n)*b + a)^2, x)
```

maple [C] time = 0.11, size = 310, normalized size = 4.63

$$\frac{x^2 \left(c^{-\frac{1}{n}}\right)^2 \left((x^n)^{-\frac{1}{n}}\right)^2 \mathrm{e}^{-\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}\left(ic\,x^n\right)\right)\left(-\mathrm{csgn}\left(ix^n\right)+\mathrm{csgn}\left(ic\,x^n\right)\right)\mathrm{csgn}\left(ic\,x^n\right)}{n}} \ln\left(b\,c^{\frac{1}{n}} (x^n)^{\frac{1}{n}} \mathrm{e}^{\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}\left(ic\,x^n\right)\right)\left(-\mathrm{csgn}\left(ix^n\right)+\mathrm{csgn}\left(ic\,x^n\right)\right)}{2n}}\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*(c*x^n)^(1/n)+a)^2,x)
```

```
[Out] 1/a*x^2/(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csg
n(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))+a)-1/a/(c^(1/n))/((x^n)^(1/n))*x^2
*exp(-1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csg
n(I*c*x^n))/b+ln(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n
))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))+a)/((x^n)^(1/n))/(c^(1/n))
*c^(-1/n)*(x^n)^(-1/n)/b^2*x^2*exp(-I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I
*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x^2}{abc^{\left(\frac{1}{n}\right)}(x^n)^{\left(\frac{1}{n}\right)} + a^2} - \int \frac{x}{abc^{\left(\frac{1}{n}\right)}(x^n)^{\left(\frac{1}{n}\right)} + a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(c*x^n)^(1/n))^2,x, algorithm="maxima")
```

```
[Out] x^2/(a*b*c^(1/n)*(x^n)^(1/n) + a^2) - integrate(x/(a*b*c^(1/n)*(x^n)^(1/n)
+ a^2), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\left(a + b\left(c\,x^n\right)^{1/n}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*(c*x^n)^(1/n))^2,x)
```

```
[Out] int(x/(a + b*(c*x^n)^(1/n))^2, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a + b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(c*x**n)**(1/n))**2,x)
```

```
[Out] Integral(x/(a + b*(c*x**n)**(1/n))**2, x)
```

3.3016 $\int \frac{1}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^2}\,dx$

Optimal. Leaf size=20

$$\frac{x}{a^2+ab\,(cx^n)^{\frac{1}{n}}}$$

```
[Out] x/(a^2+a*b*(c*x^n)^(1/n))
```

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.60, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {254, 32}

$$-\frac{x\,(cx^n)^{-1/n}}{b\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c*x^n)^n^(-1))^(-2),x]
```

```
[Out] -(x/(b*(c*x^n)^n^(-1)*(a + b*(c*x^n)^n^(-1))))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 254

```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_.), x_Symbol] :> Dist[x/(c*x^
q)^(1/q), Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\int \frac{1}{\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^2}\,dx = \left(x\,(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{1}{(a+bx)^2}\,dx, x, (cx^n)^{\frac{1}{n}}\right)$$

$$= -\frac{x\,(cx^n)^{-1/n}}{b\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 1.65

$$-\frac{x\,(cx^n)^{-1/n}}{ab+b^2\,(cx^n)^{\frac{1}{n}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c*x^n)^n^(-1))^(-2),x]
```

```
[Out] -(x/((c*x^n)^n^(-1)*(a*b + b^2*(c*x^n)^n^(-1))))
```

fricas [A] time = 0.86, size = 25, normalized size = 1.25

$$-\frac{1}{b^2 c^{\frac{2}{n}} x + abc^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(c*x^n)^(1/n))^2,x, algorithm="fricas")
```

```
[Out] -1/(b^2*c^(2/n)*x + a*b*c^(1/n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((cx^n)^{\left(\frac{1}{n}\right)} b + a\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(c*x^n)^(1/n))^2,x, algorithm="giac")
```

```
[Out] integrate(((c*x^n)^(1/n)*b + a)^(-2), x)
```

maple [C] time = 0.03, size = 74, normalized size = 3.70

$$\frac{x}{\left(b\, c^{\frac{1}{n}} (x^n)^{\frac{1}{n}}\, \mathrm{e}^{\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}(ic\,x^n)\right)\left(-\mathrm{csgn}(ix^n)+\mathrm{csgn}(ic\,x^n)\right)\mathrm{csgn}(ic\,x^n)}{2n}} + a\right) a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*(c*x^n)^(1/n)+a)^2,x)
```

```
[Out] 1/a*x/(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(
I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))+a)
```

maxima [A] time = 0.54, size = 23, normalized size = 1.15

$$\frac{x}{abc^{\left(\frac{1}{n}\right)}(x^n)^{\left(\frac{1}{n}\right)} + a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(c*x^n)^(1/n))^2,x, algorithm="maxima")
```

```
[Out] x/(a*b*c^(1/n)*(x^n)^(1/n) + a^2)
```

mupad [B] time = 1.19, size = 20, normalized size = 1.00

$$\frac{x}{a\left(a + b\,(c\,x^n)^{1/n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(c*x^n)^(1/n))^2,x)
```

```
[Out] x/(a*(a + b*(c*x^n)^(1/n)))
```

sympy [**A**] time = 169.22, size = 80, normalized size = 4.00

$$\begin{cases} \tilde{\infty} c^{-\frac{2}{n}} x \left(x^n\right)^{-\frac{2}{n}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{c^{-\frac{2}{n}} x \left(x^n\right)^{-\frac{2}{n}}}{b^2} & \text{for } a = 0 \\ \tilde{\infty} c^{\frac{2}{n}} x \left(x^n\right)^{\frac{2}{n}} & \text{for } b = -a c^{-\frac{1}{n}} \left(x^n\right)^{-\frac{1}{n}} \\ \frac{x}{a^2 + a b c^{\frac{1}{n}} \left(x^n\right)^{\frac{1}{n}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(c*x**n)**(1/n))**2,x)
```

```
[Out] Piecewise((zoo*c**(-2/n)*x*(x**n)**(-2/n), Eq(a, 0) & Eq(b, 0)), (-c**(-2/n
)*x*(x**n)**(-2/n)/b**2, Eq(a, 0)), (zoo*c**(2/n)*x*(x**n)**(2/n), Eq(b, -a
*c**(-1/n)*(x**n)**(-1/n))), (x/(a**2 + a*b*c**(1/n)*(x**n)**(1/n)), True))
```

3.3017 $\int \frac{1}{x\left(a+b(cx^n)^{\frac{1}{n}}\right)^2}\,dx$

Optimal. Leaf size=45

$$-\frac{\log\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}{a^2}+\frac{\log(x)}{a^2}+\frac{1}{a\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}$$

```
[Out] 1/a/(a+b*(c*x^n)^(1/n))+ln(x)/a^2-ln(a+b*(c*x^n)^(1/n))/a^2
```

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {368, 44}

$$-\frac{\log\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}{a^2}+\frac{\log(x)}{a^2}+\frac{1}{a\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*(c*x^n)^n^(-1))^2),x]
```

```
[Out] 1/(a*(a + b*(c*x^n)^n^(-1))) + Log[x]/a^2 - Log[a + b*(c*x^n)^n^(-1)]/a^2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^2}\,dx &= \text{Subst}\left(\int \frac{1}{x(a+bx)^2}\,dx, x, (cx^n)^{\frac{1}{n}}\right)\\ &= \text{Subst}\left(\int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right)dx, x, (cx^n)^{\frac{1}{n}}\right)\\ &= \frac{1}{a\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}+\frac{\log(x)}{a^2}-\frac{\log\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.04, size = 40, normalized size = 0.89

$$\frac{\frac{a}{a+b(cx^n)^{\frac{1}{n}}}-\log\left(a+b\,(cx^n)^{\frac{1}{n}}\right)+\log(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*(c*x^n)^n^(-1))^2),x]
```

```
[Out] (a/(a + b*(c*x^n)^n^(-1)) + Log[x] - Log[a + b*(c*x^n)^n^(-1)])/a^2
```

fricas [A] time = 0.93, size = 57, normalized size = 1.27

$$\frac{bc^{\left(\frac{1}{n}\right)}x\log(x) - \left(bc^{\left(\frac{1}{n}\right)}x + a\right)\log\left(bc^{\left(\frac{1}{n}\right)}x + a\right) + a\log(x) + a}{a^2bc^{\left(\frac{1}{n}\right)}x + a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x^n)^(1/n))^2,x, algorithm="fricas")
```

```
[Out] (b*c^(1/n)*x*log(x) - (b*c^(1/n)*x + a)*log(b*c^(1/n)*x + a) + a*log(x) + a
)/(a^2*b*c^(1/n)*x + a^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((cx^n)^{\left(\frac{1}{n}\right)}b + a\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x^n)^(1/n))^2,x, algorithm="giac")
```

```
[Out] integrate(1/(((c*x^n)^(1/n)*b + a)^2*x), x)
```

maple [A] time = 0.01, size = 54, normalized size = 1.20

$$\frac{1}{\left(b\,(c\,x^n)^{\frac{1}{n}} + a\right)a} + \frac{\ln\left((c\,x^n)^{\frac{1}{n}}\right)}{a^2} - \frac{\ln\left(b\,(c\,x^n)^{\frac{1}{n}} + a\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*(c*x^n)^(1/n)+a)^2,x)
```

```
[Out] -ln(b*(c*x^n)^(1/n)+a)/a^2+1/a/(b*(c*x^n)^(1/n)+a)+1/a^2*ln((c*x^n)^(1/n))
```

maxima [A] time = 0.53, size = 61, normalized size = 1.36

$$\frac{1}{abc^{\left(\frac{1}{n}\right)}(x^n)^{\left(\frac{1}{n}\right)} + a^2} + \frac{\log(x)}{a^2} - \frac{\log\left(\frac{bc^{\left(\frac{1}{n}\right)}(x^n)^{\left(\frac{1}{n}\right)} + a}{bc^{\left(\frac{1}{n}\right)}}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x^n)^(1/n))^2,x, algorithm="maxima")
```

```
[Out] 1/(a*b*c^(1/n)*(x^n)^(1/n) + a^2) + log(x)/a^2 - log((b*c^(1/n)*(x^n)^(1/n)
 + a)/(b*c^(1/n)))/a^2
```

mupad [B] time = 1.19, size = 44, normalized size = 0.98

$$\frac{\ln(x)}{a^2} - \frac{\ln\left(a + b\,(c\,x^n)^{1/n}\right)}{a^2} + \frac{1}{a^2 + a\,b\,(c\,x^n)^{1/n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*(c*x^n)^(1/n))^2),x)
[Out] log(x)/a^2 - log(a + b*(c*x^n)^(1/n))/a^2 + 1/(a^2 + a*b*(c*x^n)^(1/n))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x**n)**(1/n))**2,x)
[Out] Timed out
```

3.3018 $\int \frac{1}{x^2\left(a+b(cx^n)^{\frac{1}{n}}\right)^2}\,dx$

Optimal. Leaf size=94

$$-\frac{2b\log(x)\,(cx^n)^{\frac{1}{n}}}{a^3x}+\frac{2b\,(cx^n)^{\frac{1}{n}}\log\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}{a^3x}-\frac{b\,(cx^n)^{\frac{1}{n}}}{a^2x\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}-\frac{1}{a^2x}$$

```
[Out] -1/a^2/x-b*(c*x^n)^(1/n)/a^2/x/(a+b*(c*x^n)^(1/n))-2*b*(c*x^n)^(1/n)*ln(x)/
a^3/x+2*b*(c*x^n)^(1/n)*ln(a+b*(c*x^n)^(1/n))/a^3/x
```

Rubi [A] time = 0.04, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {368, 44}

$$-\frac{b\,(cx^n)^{\frac{1}{n}}}{a^2x\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}-\frac{2b\log(x)\,(cx^n)^{\frac{1}{n}}}{a^3x}+\frac{2b\,(cx^n)^{\frac{1}{n}}\log\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}{a^3x}-\frac{1}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*(c*x^n)^n^(-1))^2),x]
```

```
[Out] -(1/(a^2*x)) - (b*(c*x^n)^n^(-1))/(a^2*x*(a + b*(c*x^n)^n^(-1))) - (2*b*(c*
x^n)^n^(-1)*Log[x])/(a^3*x) + (2*b*(c*x^n)^n^(-1)*Log[a + b*(c*x^n)^n^(-1)]
)/(a^3*x)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^2}\,dx = \frac{(cx^n)^{\frac{1}{n}}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2}\,dx, x, (cx^n)^{\frac{1}{n}}\right)}{x}$$

$$= \frac{(cx^n)^{\frac{1}{n}}\,\text{Subst}\left(\int \left(\frac{1}{a^2x^2}-\frac{2b}{a^3x}+\frac{b^2}{a^2(a+bx)^2}+\frac{2b^2}{a^3(a+bx)}\right)dx, x, (cx^n)^{\frac{1}{n}}\right)}{x}$$

$$= -\frac{1}{a^2x}-\frac{b\,(cx^n)^{\frac{1}{n}}}{a^2x\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}-\frac{2b\,(cx^n)^{\frac{1}{n}}\log(x)}{a^3x}+\frac{2b\,(cx^n)^{\frac{1}{n}}\log\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}{a^3x}$$

Mathematica [A] time = 0.13, size = 71, normalized size = 0.76

$$-\frac{(cx^n)^{\frac{1}{n}}\left(a\left(\frac{b}{a+b(cx^n)^{\frac{1}{n}}}+(cx^n)^{-1/n}\right)-2b\log\left(a+b\,(cx^n)^{\frac{1}{n}}\right)+2b\log(x)\right)}{a^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*(c*x^n)^n^(-1))^2),x]
```

```
[Out] -(((c*x^n)^n^(-1)*(a*((c*x^n)^(-n^(-1)) + b/(a + b*(c*x^n)^n^(-1))) + 2*b*L
og[x] - 2*b*Log[a + b*(c*x^n)^n^(-1)]))/(a^3*x))
```

fricas [A] time = 0.93, size = 99, normalized size = 1.05

$$-\frac{2\,b^2c^{\frac{2}{n}}x^2\log(x)+a^2+2\left(abx\log(x)+abx\right)c^{\left(\frac{1}{n}\right)}-2\left(b^2c^{\frac{2}{n}}x^2+abc^{\left(\frac{1}{n}\right)}x\right)\log\left(bc^{\left(\frac{1}{n}\right)}x+a\right)}{a^3bc^{\left(\frac{1}{n}\right)}x^2+a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(c*x^n)^(1/n))^2,x, algorithm="fricas")
```

```
[Out] -(2*b^2*c^(2/n)*x^2*log(x) + a^2 + 2*(a*b*x*log(x) + a*b*x)*c^(1/n) - 2*(b^
2*c^(2/n)*x^2 + a*b*c^(1/n)*x)*log(b*c^(1/n)*x + a))/(a^3*b*c^(1/n)*x^2 + a
^4*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((cx^n)^{\left(\frac{1}{n}\right)}b+a\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(c*x^n)^(1/n))^2,x, algorithm="giac")
```

```
[Out] integrate(1/(((c*x^n)^(1/n)*b + a)^2*x^2), x)
```

maple [C] time = 0.14, size = 296, normalized size = 3.15

$$-\frac{2b\,c^{\frac{1}{n}}(x^n)^{\frac{1}{n}}\,\mathrm{e}^{\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}\left(ic\,x^n\right)\right)\left(-\mathrm{csgn}\left(ix^n\right)+\mathrm{csgn}\left(ic\,x^n\right)\right)\mathrm{csgn}\left(ic\,x^n\right)}{2n}}\ln(x)}{a^3x}+\frac{2b\,c^{\frac{1}{n}}(x^n)^{\frac{1}{n}}\,\mathrm{e}^{\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}\left(ic\,x^n\right)\right)\left(-\mathrm{csgn}\left(ix^n\right)+\mathrm{csgn}\left(ic\,x^n\right)\right)\mathrm{csgn}\left(ic\,x\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*(c*x^n)^(1/n)+a)^2,x)
```

```
[Out] 1/a/x/(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(
I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))+a)-2/a^3*b/x*c^(1/n)*(x^n)^(1/n)*exp
(1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c
*x^n))*ln(x)+2/a^3*b/x*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*(csgn(I*c)-csgn(I*c
*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))*ln(b*c^(1/n)*(x^n)^(1/
n)*exp(1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*cs
gn(I*c*x^n))+a)-2/a^2/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{abc^{\left(\frac{1}{n}\right)}x(x^n)^{\left(\frac{1}{n}\right)}+a^2x}+2\int\frac{1}{abc^{\left(\frac{1}{n}\right)}x^2(x^n)^{\left(\frac{1}{n}\right)}+a^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(c*x^n)^(1/n))^2,x, algorithm="maxima")
```

```
[Out] 1/(a*b*c^(1/n)*x*(x^n)^(1/n) + a^2*x) + 2*integrate(1/(a*b*c^(1/n)*x^2*(x^n
)^(1/n) + a^2*x^2), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2 \left(a + b \left(c\, x^n\right)^{1/n}\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*(c*x^n)^(1/n))^2),x)
```

```
[Out] int(1/(x^2*(a + b*(c*x^n)^(1/n))^2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(a + b \left(c x^n\right)^{\frac{1}{n}}\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*(c*x**n)**(1/n))**2,x)
```

```
[Out] Integral(1/(x**2*(a + b*(c*x**n)**(1/n))**2), x)
```

3.3019 $\int \frac{1}{x^3\left(a+b(cx^n)^{\frac{1}{n}}\right)^2}\,dx$

Optimal. Leaf size=125

$$\frac{3b^2\log(x)\,(cx^n)^{2/n}}{a^4x^2}-\frac{3b^2\,(cx^n)^{2/n}\log\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}{a^4x^2}+\frac{b^2\,(cx^n)^{2/n}}{a^3x^2\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}+\frac{2b\,(cx^n)^{\frac{1}{n}}}{a^3x^2}-\frac{1}{2a^2x^2}$$

```
[Out] -1/2/a^2/x^2+2*b*(c*x^n)^(1/n)/a^3/x^2+b^2*(c*x^n)^(2/n)/a^3/x^2/(a+b*(c*x^
n)^(1/n))+3*b^2*(c*x^n)^(2/n)*ln(x)/a^4/x^2-3*b^2*(c*x^n)^(2/n)*ln(a+b*(c*x
^n)^(1/n))/a^4/x^2
```

Rubi [A] time = 0.05, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {368, 44}

$$\frac{b^2\,(cx^n)^{2/n}}{a^3x^2\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}+\frac{3b^2\log(x)\,(cx^n)^{2/n}}{a^4x^2}-\frac{3b^2\,(cx^n)^{2/n}\log\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}{a^4x^2}+\frac{2b\,(cx^n)^{\frac{1}{n}}}{a^3x^2}-\frac{1}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*(c*x^n)^n^(-1))^2),x]
```

```
[Out] -1/(2*a^2*x^2) + (2*b*(c*x^n)^n^(-1))/(a^3*x^2) + (b^2*(c*x^n)^(2/n))/(a^3*
x^2*(a + b*(c*x^n)^n^(-1))) + (3*b^2*(c*x^n)^(2/n)*Log[x])/(a^4*x^2) - (3*b
^2*(c*x^n)^(2/n)*Log[a + b*(c*x^n)^n^(-1)])/(a^4*x^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^2}\,dx = \frac{(cx^n)^{2/n}\,\text{Subst}\left(\int \frac{1}{x^3(a+bx)^2}\,dx, x, (cx^n)^{\frac{1}{n}}\right)}{x^2}$$

$$= \frac{(cx^n)^{2/n}\,\text{Subst}\left(\int \left(\frac{1}{a^2x^3}-\frac{2b}{a^3x^2}+\frac{3b^2}{a^4x}-\frac{b^3}{a^3(a+bx)^2}-\frac{3b^3}{a^4(a+bx)}\right)dx, x, (cx^n)^{\frac{1}{n}}\right)}{x^2}$$

$$= -\frac{1}{2a^2x^2}+\frac{2b\,(cx^n)^{\frac{1}{n}}}{a^3x^2}+\frac{b^2\,(cx^n)^{2/n}}{a^3x^2\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}+\frac{3b^2\,(cx^n)^{2/n}\log(x)}{a^4x^2}-\frac{3b^2\,(cx^n)^{2/n}\log\left(a\right.}{a^4x^2}$$

Mathematica [A] time = 0.21, size = 99, normalized size = 0.79

$$\frac{(cx^n)^{2/n}\left(a\left(\frac{2b^2}{a+b(cx^n)^{\frac{1}{n}}}-a\,(cx^n)^{-2/n}+4b\,(cx^n)^{-1/n}\right)-6b^2\log\left(a+b\,(cx^n)^{\frac{1}{n}}\right)+6b^2\log(x)\right)}{2a^4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*(c*x^n)^n^(-1))^2),x]
```

```
[Out] ((c*x^n)^(2/n)*(a*(-(a/(c*x^n)^(2/n)) + (4*b)/(c*x^n)^n^(-1) + (2*b^2)/(a +
 b*(c*x^n)^n^(-1))) + 6*b^2*Log[x] - 6*b^2*Log[a + b*(c*x^n)^n^(-1)]))/(2*a
^4*x^2)
```

fricas [A] time = 0.92, size = 131, normalized size = 1.05

$$\frac{6\,b^3c^{\frac{3}{n}}x^3\log(x)+3\,a^2bc^{\left(\frac{1}{n}\right)}x-a^3+6\left(ab^2x^2\log(x)+ab^2x^2\right)c^{\frac{2}{n}}-6\left(b^3c^{\frac{3}{n}}x^3+ab^2c^{\frac{2}{n}}x^2\right)\log\left(bc^{\left(\frac{1}{n}\right)}x+a\right)}{2\left(a^4bc^{\left(\frac{1}{n}\right)}x^3+a^5x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(c*x^n)^(1/n))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*b^3*c^(3/n)*x^3*log(x) + 3*a^2*b*c^(1/n)*x - a^3 + 6*(a*b^2*x^2*log(
x) + a*b^2*x^2)*c^(2/n) - 6*(b^3*c^(3/n)*x^3 + a*b^2*c^(2/n)*x^2)*log(b*c^(
1/n)*x + a))/(a^4*b*c^(1/n)*x^3 + a^5*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((cx^n)^{\left(\frac{1}{n}\right)}b+a\right)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(c*x^n)^(1/n))^2,x, algorithm="giac")
```

```
[Out] integrate(1/(((c*x^n)^(1/n)*b + a)^2*x^3), x)
```

maple [C] time = 0.16, size = 379, normalized size = 3.03

$$\frac{3b^2c^{\frac{2}{n}}(x^n)^{\frac{2}{n}}\,\mathrm{e}^{\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}\left(ic\,x^n\right)\right)\left(-\mathrm{csgn}\left(ix^n\right)+\mathrm{csgn}\left(ic\,x^n\right)\right)\mathrm{csgn}\left(ic\,x^n\right)}{n}}\ln(x)}{a^4x^2}-\frac{3b^2c^{\frac{2}{n}}(x^n)^{\frac{2}{n}}\,\mathrm{e}^{\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}\left(ic\,x^n\right)\right)\left(-\mathrm{csgn}\left(ix^n\right)+\mathrm{csgn}\left(ic\,x^n\right)\right)\mathrm{csgn}}}{n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*(c*x^n)^(1/n)+a)^2,x)
```

```
[Out] 1/a/x^2/(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csg
n(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))+a)-3/a^4*ln(b*c^(1/n)*(x^n)^(1/n)*
exp(1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(
I*c*x^n))+a)*b^2/x^2*((x^n)^(1/n))^2*(c^(1/n))^2*exp(I*Pi*(csgn(I*c)-csgn(I
*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))-3/2/a^2/x^2+3/a^3*b/
x^2*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n
)+csgn(I*c*x^n))/n*csgn(I*c*x^n))+3/a^4*b^2/x^2*c^(2/n)*(x^n)^(2/n)*exp(I*P
i*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))*l
n(x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{abc^{\left(\frac{1}{n}\right)}x^2(x^n)^{\left(\frac{1}{n}\right)}+a^2x^2}+3\int\frac{1}{abc^{\left(\frac{1}{n}\right)}x^3(x^n)^{\left(\frac{1}{n}\right)}+a^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(c*x^n)^(1/n))^2,x, algorithm="maxima")
```

```
[Out] 1/(a*b*c^(1/n)*x^2*(x^n)^(1/n) + a^2*x^2) + 3*integrate(1/(a*b*c^(1/n)*x^3*
(x^n)^(1/n) + a^2*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{x^3\left(a+b\left(c\,x^n\right)^{1/n}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*(c*x^n)^(1/n))^2),x)
```

```
[Out] int(1/(x^3*(a + b*(c*x^n)^(1/n))^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^3\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*(c*x**n)**(1/n))**2,x)
```

```
[Out] Integral(1/(x**3*(a + b*(c*x**n)**(1/n))**2), x)
```

3.3020 $\int \frac{1}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^3}\,dx$

Optimal. Leaf size=34

$$-\frac{x\,(cx^n)^{-1/n}}{2b\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^2}$$

```
[Out] -1/2*x/b/((c*x^n)^(1/n))/(a+b*(c*x^n)^(1/n))^2
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {254, 32}

$$-\frac{x\,(cx^n)^{-1/n}}{2b\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c*x^n)^n^(-1))^(-3),x]
```

```
[Out] -x/(2*b*(c*x^n)^n^(-1)*(a + b*(c*x^n)^n^(-1))^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 254

```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_.), x_Symbol] :> Dist[x/(c*x^
q)^(1/q), Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\int \frac{1}{\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^3}\,dx = \left(x\,(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{1}{(a+bx)^3}\,dx, x, (cx^n)^{\frac{1}{n}}\right)$$

$$= -\frac{x\,(cx^n)^{-1/n}}{2b\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^2}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 1.00

$$-\frac{x\,(cx^n)^{-1/n}}{2b\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c*x^n)^n^(-1))^(-3),x]
```

```
[Out] -1/2*x/(b*(c*x^n)^n^(-1)*(a + b*(c*x^n)^n^(-1))^2)
```

fricas [A] time = 0.89, size = 43, normalized size = 1.26

$$-\frac{1}{2\left(b^3c^{\frac{3}{n}}x^2+2\,ab^2c^{\frac{2}{n}}x+a^2bc^{\left(\frac{1}{n}\right)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(c*x^n)^(1/n))^3,x, algorithm="fricas")
```

```
[Out] -1/2/(b^3*c^(3/n)*x^2 + 2*a*b^2*c^(2/n)*x + a^2*b*c^(1/n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((cx^n)^{\left(\frac{1}{n}\right)}b+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(c*x^n)^(1/n))^3,x, algorithm="giac")
```

```
[Out] integrate(((c*x^n)^(1/n)*b + a)^(-3), x)
```

maple [C] time = 0.03, size = 143, normalized size = 4.21

$$\frac{\left(b\,c^{\frac{1}{n}}(x^n)^{\frac{1}{n}}\,\mathrm{e}^{\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}\left(ic\,x^n\right)\right)\left(-\mathrm{csgn}\left(ix^n\right)+\mathrm{csgn}\left(ic\,x^n\right)\right)\mathrm{csgn}\left(ic\,x^n\right)}{2n}}+2a\right)x}{2\left(b\,c^{\frac{1}{n}}(x^n)^{\frac{1}{n}}\,\mathrm{e}^{\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}(ic\,x^n)\right)\left(-\mathrm{csgn}(ix^n)+\mathrm{csgn}(ic\,x^n)\right)\mathrm{csgn}(ic\,x^n)}{2n}}+a\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*(c*x^n)^(1/n)+a)^3,x)
```

```
[Out] 1/2*x*(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(
I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))+2*a)/a^2/(b*c^(1/n)*(x^n)^(1/n)*exp(
1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*
x^n))+a)^2
```

maxima [B] time = 0.56, size = 69, normalized size = 2.03

$$\frac{bc^{\left(\frac{1}{n}\right)}x(x^n)^{\left(\frac{1}{n}\right)}+2\,ax}{2\left(a^2b^2c^{\frac{2}{n}}(x^n)^{\frac{2}{n}}+2\,a^3bc^{\left(\frac{1}{n}\right)}(x^n)^{\left(\frac{1}{n}\right)}+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(c*x^n)^(1/n))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c^(1/n)*x*(x^n)^(1/n) + 2*a*x)/(a^2*b^2*c^(2/n)*(x^n)^(2/n) + 2*a^3*
b*c^(1/n)*(x^n)^(1/n) + a^4)
```

mupad [B] time = 1.20, size = 36, normalized size = 1.06

$$\frac{x\left(2\,a+b\,(c\,x^n)^{1/n}\right)}{2\,a^2\left(a+b\,(c\,x^n)^{1/n}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(c*x^n)^(1/n))^3,x)
```

```
[Out] (x*(2*a + b*(c*x^n)^(1/n)))/(2*a^2*(a + b*(c*x^n)^(1/n))^2)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(c*x**n)**(1/n))**3,x)
```

```
[Out] Timed out
```

3.3021 $\int \frac{x}{\left(1+(x^n)^{\frac{1}{n}}\right)^2}\,dx$

Optimal. Leaf size=48

$$\frac{x^2\,(x^n)^{-2/n}}{(x^n)^{\frac{1}{n}}+1}+x^2\,(x^n)^{-2/n}\log\left((x^n)^{\frac{1}{n}}+1\right)$$

```
[Out] x^2/((x^n)^(2/n))/(1+(x^n)^(1/n))+x^2*ln(1+(x^n)^(1/n))/((x^n)^(2/n))
```

Rubi [A] time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {368, 43}

$$\frac{x^2\,(x^n)^{-2/n}}{(x^n)^{\frac{1}{n}}+1}+x^2\,(x^n)^{-2/n}\log\left((x^n)^{\frac{1}{n}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + (x^n)^n^(-1))^2,x]
```

```
[Out] x^2/((x^n)^(2/n)*(1 + (x^n)^n^(-1))) + (x^2*Log[1 + (x^n)^n^(-1)])/(x^n)^(2
/n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(1+(x^n)^{\frac{1}{n}}\right)^2}\,dx &= \left(x^2\,(x^n)^{-2/n}\right)\text{Subst}\left(\int \frac{x}{(1+x)^2}\,dx, x, (x^n)^{\frac{1}{n}}\right)\\ &= \left(x^2\,(x^n)^{-2/n}\right)\text{Subst}\left(\int \left(-\frac{1}{(1+x)^2}+\frac{1}{1+x}\right)dx, x, (x^n)^{\frac{1}{n}}\right)\\ &= \frac{x^2\,(x^n)^{-2/n}}{1+(x^n)^{\frac{1}{n}}}+x^2\,(x^n)^{-2/n}\log\left(1+(x^n)^{\frac{1}{n}}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.73

$$x^2\,(x^n)^{-2/n}\left(\frac{1}{(x^n)^{\frac{1}{n}}+1}+\log\left((x^n)^{\frac{1}{n}}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + (x^n)^n^(-1))^2,x]
[Out] (x^2*((1 + (x^n)^n^(-1))^(-1) + Log[1 + (x^n)^n^(-1)]))/(x^n)^(2/n)
```

fricas [A] time = 0.97, size = 16, normalized size = 0.33

$$\frac{(x+1)\log(x+1)+1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+(x^n)^(1/n))^2,x, algorithm="fricas")
[Out] ((x + 1)*log(x + 1) + 1)/(x + 1)
```

giac [A] time = 0.17, size = 11, normalized size = 0.23

$$\frac{1}{x+1}+\log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+(x^n)^(1/n))^2,x, algorithm="giac")
[Out] 1/(x + 1) + log(abs(x + 1))
```

maple [A] time = 0.04, size = 63, normalized size = 1.31

$$x^2\left((x^n)^{-\frac{1}{n}}\right)^2\ln\left((x^n)^{\frac{1}{n}}+1\right)-x^2(x^n)^{-\frac{1}{n}}+\frac{x^2}{(x^n)^{\frac{1}{n}}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+(x^n)^(1/n))^2,x)
[Out] x^2/(1+(x^n)^(1/n))-1/((x^n)^(1/n))*x^2+x^2*(x^n)^(-1/n)/((x^n)^(1/n))*ln(1
+(x^n)^(1/n))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x^2}{(x^n)^{\left(\frac{1}{n}\right)}+1}-\int\frac{x}{(x^n)^{\left(\frac{1}{n}\right)}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+(x^n)^(1/n))^2,x, algorithm="maxima")
[Out] x^2/((x^n)^(1/n) + 1) - integrate(x/((x^n)^(1/n) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{x}{\left((x^n)^{1/n}+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^n)^(1/n) + 1)^2,x)
[Out] int(x/((x^n)^(1/n) + 1)^2, x)
```

sympy [**A**] time = 0.11, size = 19, normalized size = 0.40

$$\log\left((x^n)^{\frac{1}{n}}+1\right)+\frac{1}{(x^n)^{\frac{1}{n}}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+(x**n)**(1/n))**2,x)
[Out] log((x**n)**(1/n) + 1) + 1/((x**n)**(1/n) + 1)
```

3.3022 $\int x^3 \left(a + b\,(cx^n)^{\frac{1}{n}}\right)^p \, dx$

Optimal. Leaf size=171

$$-\frac{a^3x^4\,(cx^n)^{-4/n}\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^{p+1}}{b^4(p+1)}+\frac{3a^2x^4\,(cx^n)^{-4/n}\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^{p+2}}{b^4(p+2)}-\frac{3ax^4\,(cx^n)^{-4/n}\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^{p+3}}{b^4(p+3)}+\frac{x^4\,(cx^n}{}$$

```
[Out] -a^3*x^4*(a+b*(c*x^n)^(1/n))^(1+p)/b^4/(1+p)/((c*x^n)^(4/n))+3*a^2*x^4*(a+b
*(c*x^n)^(1/n))^(2+p)/b^4/(2+p)/((c*x^n)^(4/n))-3*a*x^4*(a+b*(c*x^n)^(1/n))
^(3+p)/b^4/(3+p)/((c*x^n)^(4/n))+x^4*(a+b*(c*x^n)^(1/n))^(4+p)/b^4/(4+p)/((
c*x^n)^(4/n))
```

Rubi [A] time = 0.06, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {368, 43}

$$-\frac{a^3x^4\,(cx^n)^{-4/n}\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^{p+1}}{b^4(p+1)}+\frac{3a^2x^4\,(cx^n)^{-4/n}\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^{p+2}}{b^4(p+2)}-\frac{3ax^4\,(cx^n)^{-4/n}\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^{p+3}}{b^4(p+3)}+\frac{x^4\,(cx^n}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*(c*x^n)^n^(-1))^p,x]
```

```
[Out] -((a^3*x^4*(a + b*(c*x^n)^n^(-1))^(1 + p))/(b^4*(1 + p)*(c*x^n)^(4/n))) + (
3*a^2*x^4*(a + b*(c*x^n)^n^(-1))^(2 + p))/(b^4*(2 + p)*(c*x^n)^(4/n)) - (3*
a*x^4*(a + b*(c*x^n)^n^(-1))^(3 + p))/(b^4*(3 + p)*(c*x^n)^(4/n)) + (x^4*(a
 + b*(c*x^n)^n^(-1))^(4 + p))/(b^4*(4 + p)*(c*x^n)^(4/n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\int x^3 \left(a + b\,(cx^n)^{\frac{1}{n}}\right)^p \, dx = \left(x^4\,(cx^n)^{-4/n}\right) \text{Subst}\left(\int x^3 (a+bx)^p \, dx, x, (cx^n)^{\frac{1}{n}}\right)$$

$$= \left(x^4\,(cx^n)^{-4/n}\right) \text{Subst}\left(\int \left(-\frac{a^3(a+bx)^p}{b^3}+\frac{3a^2(a+bx)^{1+p}}{b^3}-\frac{3a(a+bx)^{2+p}}{b^3}+\frac{(a+bx}{b^3}\right.\right.$$

$$= -\frac{a^3x^4\,(cx^n)^{-4/n}\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^{1+p}}{b^4(1+p)}+\frac{3a^2x^4\,(cx^n)^{-4/n}\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^{2+p}}{b^4(2+p)}-\frac{3ax^4\,(cx^n)^{-}}{}$$

Mathematica [A] time = 0.13, size = 113, normalized size = 0.66

$$\frac{x^4 (cx^n)^{-4/n} \left(a + b (cx^n)^{\frac{1}{n}}\right)^{p+1} \left(-\frac{a^3}{p+1} + \frac{3a^2\left(a+b(cx^n)^{\frac{1}{n}}\right)}{p+2} - \frac{3a\left(a+b(cx^n)^{\frac{1}{n}}\right)^2}{p+3} + \frac{\left(a+b(cx^n)^{\frac{1}{n}}\right)^3}{p+4}\right)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*(c*x^n)^n^(-1))^p,x]
```

```
[Out] (x^4*(a + b*(c*x^n)^n^(-1))^(1 + p)*(-(a^3/(1 + p)) + (3*a^2*(a + b*(c*x^n)
^n^(-1)))/(2 + p) - (3*a*(a + b*(c*x^n)^n^(-1))^2)/(3 + p) + (a + b*(c*x^n)
^n^(-1))^3/(4 + p)))/(b^4*(c*x^n)^(4/n))
```

fricas [A] time = 0.93, size = 183, normalized size = 1.07

$$\frac{\left(6\,a^3bc^{\left(\frac{1}{n}\right)}px + \left(b^4p^3 + 6\,b^4p^2 + 11\,b^4p + 6\,b^4\right)c^{\frac{4}{n}}x^4 + \left(ab^3p^3 + 3\,ab^3p^2 + 2\,ab^3p\right)c^{\frac{3}{n}}x^3 - 6\,a^4 - 3\left(a^2b^2p^2 + a^2b^2p\right)\right.}{\left(b^4p^4 + 10\,b^4p^3 + 35\,b^4p^2 + 50\,b^4p + 24\,b^4\right)c^{\frac{4}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(c*x^n)^(1/n))^p,x, algorithm="fricas")
```

```
[Out] (6*a^3*b*c^(1/n)*p*x + (b^4*p^3 + 6*b^4*p^2 + 11*b^4*p + 6*b^4)*c^(4/n)*x^4
 + (a*b^3*p^3 + 3*a*b^3*p^2 + 2*a*b^3*p)*c^(3/n)*x^3 - 6*a^4 - 3*(a^2*b^2*p
^2 + a^2*b^2*p)*c^(2/n)*x^2)*(b*c^(1/n)*x + a)^p/((b^4*p^4 + 10*b^4*p^3 + 3
5*b^4*p^2 + 50*b^4*p + 24*b^4)*c^(4/n))
```

giac [B] time = 0.47, size = 384, normalized size = 2.25

$$\frac{\left(bc^{\left(\frac{1}{n}\right)}x + a\right)^p b^4c^{\frac{4}{n}}p^3x^4 + \left(bc^{\left(\frac{1}{n}\right)}x + a\right)^p ab^3c^{\frac{3}{n}}p^3x^3 + 6\left(bc^{\left(\frac{1}{n}\right)}x + a\right)^p b^4c^{\frac{4}{n}}p^2x^4 + 3\left(bc^{\left(\frac{1}{n}\right)}x + a\right)^p ab^3c^{\frac{3}{n}}p^2x^3 + 11\left(b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(c*x^n)^(1/n))^p,x, algorithm="giac")
```

```
[Out] ((b*c^(1/n)*x + a)^p*b^4*c^(4/n)*p^3*x^4 + (b*c^(1/n)*x + a)^p*a*b^3*c^(3/n
)*p^3*x^3 + 6*(b*c^(1/n)*x + a)^p*b^4*c^(4/n)*p^2*x^4 + 3*(b*c^(1/n)*x + a)
^p*a*b^3*c^(3/n)*p^2*x^3 + 11*(b*c^(1/n)*x + a)^p*b^4*c^(4/n)*p*x^4 - 3*(b*
c^(1/n)*x + a)^p*a^2*b^2*c^(2/n)*p^2*x^2 + 2*(b*c^(1/n)*x + a)^p*a*b^3*c^(3
/n)*p*x^3 + 6*(b*c^(1/n)*x + a)^p*b^4*c^(4/n)*x^4 - 3*(b*c^(1/n)*x + a)^p*a
^2*b^2*c^(2/n)*p*x^2 + 6*(b*c^(1/n)*x + a)^p*a^3*b*c^(1/n)*p*x - 6*(b*c^(1/
n)*x + a)^p*a^4)/(b^4*c^(4/n)*p^4 + 10*b^4*c^(4/n)*p^3 + 35*b^4*c^(4/n)*p^2
 + 50*b^4*c^(4/n)*p + 24*b^4*c^(4/n))
```

maple [C] time = 0.37, size = 1127, normalized size = 6.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*(c*x^n)^(1/n)+a)^p,x)
```

```
[Out] (b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)
+csgn(I*c*x^n))/n*csgn(I*c*x^n))+a)^(p+1)/(c^(1/n))/((x^n)^(1/n))*x^4*exp(-
1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*
```

```
x^n))/b/(p+1)-3/b/(p+1)*(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*(csgn(I*c)-csgn
(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))+a)^(p+1)/(p+4)*x^4
/((x^n)^(1/n))/(c^(1/n))*exp(-1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x
^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))-3/b^2/(p+1)*(b*c^(1/n)*(x^n)^(1/n)*exp(
1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*
x^n))+a)^(p+1)/(p+4)/(p+3)*a/(c^(1/n))^2/((x^n)^(1/n))^2*x^4*exp(-I*Pi*(csg
n(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))*p-3/b^2
/(p+1)*(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn
(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))+a)^(p+1)/(p+4)/(p+3)*a/(c^(1/n))^2/
((x^n)^(1/n))^2*x^4*exp(-I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(
I*c*x^n))/n*csgn(I*c*x^n))-6/b^4/(p+1)*(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*
(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))+a)^
(p+1)/((x^n)^(1/n))^4*x^4/(c^(1/n))^4/(p+3)/(p+2)/(p+4)*a^3*exp(-2*I*Pi*(cs
gn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))+6/b^3/
(p+1)*(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(
I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))+a)^(p+1)/(p+4)*a^2/(p+2)/((x^n)^(1/n
))^3*x^4/(c^(1/n))^3/(p+3)*exp(-3/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I
*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))*p+6/b^3/(p+1)*(b*c^(1/n)*(x^n)^(1/n)*
exp(1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(
I*c*x^n))+a)^(p+1)/(p+4)*a^2/(p+2)/((x^n)^(1/n))^3*x^4/(c^(1/n))^3/(p+3)*ex
p(-3/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I
*c*x^n))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left((cx^n)^{\left(\frac{1}{n}\right)} b + a \right)^p x^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(c*x^n)^(1/n))^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*x^n)^(1/n)*b + a)^p*x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \left(a + b \, (c \, x^n)^{1/n} \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*(c*x^n)^(1/n))^p,x)
```

```
[Out] int(x^3*(a + b*(c*x^n)^(1/n))^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(a + b \, (cx^n)^{\frac{1}{n}} \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*(c*x**n)**(1/n))**p,x)
```

```
[Out] Integral(x**3*(a + b*(c*x**n)**(1/n))**p, x)
```

3.3023 $\int x^2 \left(a + b\left(cx^n\right)^{\frac{1}{n}}\right)^p \, dx$

Optimal. Leaf size=126

$$\frac{a^2x^3\left(cx^n\right)^{-3/n}\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^{p+1}}{b^3(p+1)} - \frac{2ax^3\left(cx^n\right)^{-3/n}\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^{p+2}}{b^3(p+2)} + \frac{x^3\left(cx^n\right)^{-3/n}\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^{p+3}}{b^3(p+3)}$$

```
[Out] a^2*x^3*(a+b*(c*x^n)^(1/n))^(1+p)/b^3/(1+p)/((c*x^n)^(3/n))-2*a*x^3*(a+b*(c
*x^n)^(1/n))^(2+p)/b^3/(2+p)/((c*x^n)^(3/n))+x^3*(a+b*(c*x^n)^(1/n))^(3+p)/
b^3/(3+p)/((c*x^n)^(3/n))
```

Rubi [A] time = 0.05, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {368, 43}

$$\frac{a^2x^3\left(cx^n\right)^{-3/n}\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^{p+1}}{b^3(p+1)} - \frac{2ax^3\left(cx^n\right)^{-3/n}\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^{p+2}}{b^3(p+2)} + \frac{x^3\left(cx^n\right)^{-3/n}\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^{p+3}}{b^3(p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*(c*x^n)^n^(-1))^p,x]
```

```
[Out] (a^2*x^3*(a + b*(c*x^n)^n^(-1))^(1 + p))/(b^3*(1 + p)*(c*x^n)^(3/n)) - (2*a
*x^3*(a + b*(c*x^n)^n^(-1))^(2 + p))/(b^3*(2 + p)*(c*x^n)^(3/n)) + (x^3*(a
+ b*(c*x^n)^n^(-1))^(3 + p))/(b^3*(3 + p)*(c*x^n)^(3/n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\begin{aligned}
\int x^2\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^p dx &= \left(x^3\left(cx^n\right)^{-3/n}\right)\text{Subst}\left(\int x^2(a+bx)^p\,dx, x, \left(cx^n\right)^{\frac{1}{n}}\right) \\
&= \left(x^3\left(cx^n\right)^{-3/n}\right)\text{Subst}\left(\int\left(\frac{a^2(a+bx)^p}{b^2} - \frac{2a(a+bx)^{1+p}}{b^2} + \frac{(a+bx)^{2+p}}{b^2}\right)dx, x, \left(cx^n\right)^{\frac{1}{n}}\right) \\
&= \frac{a^2x^3\left(cx^n\right)^{-3/n}\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^{1+p}}{b^3(1+p)} - \frac{2ax^3\left(cx^n\right)^{-3/n}\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^{2+p}}{b^3(2+p)} + \frac{x^3\left(cx^n\right)^{-3/n}\left(a+\right.}{b^3(3+}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 95, normalized size = 0.75

$$\frac{x^3\left(cx^n\right)^{-3/n}\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^{p+1}\left(2a^2 - 2ab(p+1)\left(cx^n\right)^{\frac{1}{n}} + b^2\left(p^2+3p+2\right)\left(cx^n\right)^{2/n}\right)}{b^3(p+1)(p+2)(p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*(c*x^n)^n^(-1))^p,x]
```

```
[Out] (x^3*(a + b*(c*x^n)^n^(-1))^(1 + p)*(2*a^2 - 2*a*b*(1 + p)*(c*x^n)^n^(-1) +
 b^2*(2 + 3*p + p^2)*(c*x^n)^(2/n)))/(b^3*(1 + p)*(2 + p)*(3 + p)*(c*x^n)^(
3/n))
```

fricas [A] time = 0.90, size = 129, normalized size = 1.02

$$-\frac{\left(2\,a^2bc^{\left(\frac{1}{n}\right)}px-\left(b^3p^2+3\,b^3p+2\,b^3\right)c^{\frac{3}{n}}x^3-\left(ab^2p^2+ab^2p\right)c^{\frac{2}{n}}x^2-2\,a^3\right)\left(bc^{\left(\frac{1}{n}\right)}x+a\right)^p}{\left(b^3p^3+6\,b^3p^2+11\,b^3p+6\,b^3\right)c^{\frac{3}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(c*x^n)^(1/n))^p,x, algorithm="fricas")
```

```
[Out] -(2*a^2*b*c^(1/n)*p*x - (b^3*p^2 + 3*b^3*p + 2*b^3)*c^(3/n)*x^3 - (a*b^2*p^
2 + a*b^2*p)*c^(2/n)*x^2 - 2*a^3)*(b*c^(1/n)*x + a)^p/((b^3*p^3 + 6*b^3*p^2
 + 11*b^3*p + 6*b^3)*c^(3/n))
```

giac [A] time = 0.46, size = 243, normalized size = 1.93

$$\frac{\left(bc^{\left(\frac{1}{n}\right)}x+a\right)^p b^3c^{\frac{3}{n}}p^2x^3+\left(bc^{\left(\frac{1}{n}\right)}x+a\right)^p ab^2c^{\frac{2}{n}}p^2x^2+3\left(bc^{\left(\frac{1}{n}\right)}x+a\right)^p b^3c^{\frac{3}{n}}px^3+\left(bc^{\left(\frac{1}{n}\right)}x+a\right)^p ab^2c^{\frac{2}{n}}px^2+2\left(bc^{\left(\frac{1}{n}\right)}\right.}{b^3c^{\frac{3}{n}}p^3+6\,b^3c^{\frac{3}{n}}p^2+11\,b^3c^{\frac{3}{n}}p+6\,b^3c^{\frac{3}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(c*x^n)^(1/n))^p,x, algorithm="giac")
```

```
[Out] ((b*c^(1/n)*x + a)^p*b^3*c^(3/n)*p^2*x^3 + (b*c^(1/n)*x + a)^p*a*b^2*c^(2/n
)*p^2*x^2 + 3*(b*c^(1/n)*x + a)^p*b^3*c^(3/n)*p*x^3 + (b*c^(1/n)*x + a)^p*a
*b^2*c^(2/n)*p*x^2 + 2*(b*c^(1/n)*x + a)^p*b^3*c^(3/n)*x^3 - 2*(b*c^(1/n)*x
 + a)^p*a^2*b*c^(1/n)*p*x + 2*(b*c^(1/n)*x + a)^p*a^3)/(b^3*c^(3/n)*p^3 + 6
*b^3*c^(3/n)*p^2 + 11*b^3*c^(3/n)*p + 6*b^3*c^(3/n))
```

maple [C] time = 0.31, size = 787, normalized size = 6.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*(c*x^n)^(1/n)+a)^p,x)
```

```
[Out] (b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)
+csgn(I*c*x^n))/n*csgn(I*c*x^n))+a)^(p+1)/(c^(1/n))/((x^n)^(1/n))*x^3*exp(-
1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*
x^n))/b/(p+1)-2/b/(p+1)*(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*(csgn(I*c)-csgn
(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))+a)^(p+1)/(p+3)*x^3
/((x^n)^(1/n))/(c^(1/n))*exp(-1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x
^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))-2/b^2/(p+1)*(b*c^(1/n)*(x^n)^(1/n)*exp(
1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*
x^n))+a)^(p+1)/(p+3)/(p+2)*a/((x^n)^(1/n))^2*x^3/(c^(1/n))^2*exp(-I*Pi*(csg
n(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))*p-2/b^2
/(p+1)*(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn
(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))+a)^(p+1)/(p+3)/(p+2)*a/((x^n)^(1/n)
)^2*x^3/(c^(1/n))^2*exp(-I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(
I*c*x^n))/n*csgn(I*c*x^n))+2/b^3/(p+1)*(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*
(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))+a)^
```

```
(p+1)/(p+3)*a^2/((x^n)^(1/n))^3*x^3/(c^(1/n))^3/(p+2)*exp(-3/2*I*Pi*(csgn(I
*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left((cx^n)^{\left(\frac{1}{n}\right)} b + a \right)^p x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(c*x^n)^(1/n))^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*x^n)^(1/n)*b + a)^p*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left(a + b \, (c \, x^n)^{1/n} \right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*(c*x^n)^(1/n))^p,x)
```

```
[Out] int(x^2*(a + b*(c*x^n)^(1/n))^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + b \, (cx^n)^{\frac{1}{n}} \right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*(c*x**n)**(1/n))**p,x)
```

```
[Out] Integral(x**2*(a + b*(c*x**n)**(1/n))**p, x)
```

3.3024 $\int x\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^p\,dx$

Optimal. Leaf size=83

$$\frac{x^2\left(cx^n\right)^{-2/n}\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^{p+2}}{b^2(p+2)}-\frac{ax^2\left(cx^n\right)^{-2/n}\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^{p+1}}{b^2(p+1)}$$

```
[Out] -a*x^2*(a+b*(c*x^n)^(1/n))^(1+p)/b^2/(1+p)/((c*x^n)^(2/n))+x^2*(a+b*(c*x^n)
^(1/n))^(2+p)/b^2/(2+p)/((c*x^n)^(2/n))
```

Rubi [A] time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {368, 43}

$$\frac{x^2\left(cx^n\right)^{-2/n}\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^{p+2}}{b^2(p+2)}-\frac{ax^2\left(cx^n\right)^{-2/n}\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^{p+1}}{b^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*(c*x^n)^n^(-1))^p,x]
```

```
[Out] -((a*x^2*(a + b*(c*x^n)^n^(-1))^(1 + p))/(b^2*(1 + p)*(c*x^n)^(2/n))) + (x^
2*(a + b*(c*x^n)^n^(-1))^(2 + p))/(b^2*(2 + p)*(c*x^n)^(2/n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\begin{aligned}\int x\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^p\,dx &= \left(x^2\left(cx^n\right)^{-2/n}\right)\text{Subst}\left(\int x(a+bx)^p\,dx,x,\left(cx^n\right)^{\frac{1}{n}}\right)\\ &= \left(x^2\left(cx^n\right)^{-2/n}\right)\text{Subst}\left(\int\left(-\frac{a(a+bx)^p}{b}+\frac{(a+bx)^{1+p}}{b}\right)dx,x,\left(cx^n\right)^{\frac{1}{n}}\right)\\ &= -\frac{ax^2\left(cx^n\right)^{-2/n}\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^{1+p}}{b^2(1+p)}+\frac{x^2\left(cx^n\right)^{-2/n}\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^{2+p}}{b^2(2+p)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 63, normalized size = 0.76

$$\frac{x^2\left(cx^n\right)^{-2/n}\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)^{p+1}\left(b(p+1)\left(cx^n\right)^{\frac{1}{n}}-a\right)}{b^2(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*(c*x^n)^n^(-1))^p,x]
```

```
[Out] (x^2*(a + b*(c*x^n)^n^(-1))^(1 + p)*(-a + b*(1 + p)*(c*x^n)^n^(-1)))/(b^2*(
1 + p)*(2 + p)*(c*x^n)^(2/n))
```

fricas [A] time = 0.92, size = 79, normalized size = 0.95

$$\frac{\left(abc^{\left(\frac{1}{n}\right)}px + \left(b^2p + b^2\right)c^{\frac{2}{n}}x^2 - a^2\right)\left(bc^{\left(\frac{1}{n}\right)}x + a\right)^p}{\left(b^2p^2 + 3\,b^2p + 2\,b^2\right)c^{\frac{2}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(c*x^n)^(1/n))^p,x, algorithm="fricas")
```

```
[Out] (a*b*c^(1/n)*p*x + (b^2*p + b^2)*c^(2/n)*x^2 - a^2)*(b*c^(1/n)*x + a)^p/((b
^2*p^2 + 3*b^2*p + 2*b^2)*c^(2/n))
```

giac [A] time = 0.46, size = 136, normalized size = 1.64

$$\frac{\left(bc^{\left(\frac{1}{n}\right)}x + a\right)^p b^2c^{\frac{2}{n}}px^2 + \left(bc^{\left(\frac{1}{n}\right)}x + a\right)^p abc^{\left(\frac{1}{n}\right)}px + \left(bc^{\left(\frac{1}{n}\right)}x + a\right)^p b^2c^{\frac{2}{n}}x^2 - \left(bc^{\left(\frac{1}{n}\right)}x + a\right)^p a^2}{b^2c^{\frac{2}{n}}p^2 + 3\,b^2c^{\frac{2}{n}}p + 2\,b^2c^{\frac{2}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(c*x^n)^(1/n))^p,x, algorithm="giac")
```

```
[Out] ((b*c^(1/n)*x + a)^p*b^2*c^(2/n)*p*x^2 + (b*c^(1/n)*x + a)^p*a*b*c^(1/n)*p*
x + (b*c^(1/n)*x + a)^p*b^2*c^(2/n)*x^2 - (b*c^(1/n)*x + a)^p*a^2)/(b^2*c^(
2/n)*p^2 + 3*b^2*c^(2/n)*p + 2*b^2*c^(2/n))
```

maple [C] time = 0.18, size = 304, normalized size = 3.66

$$\frac{x^2c^{-\frac{1}{n}}\left(x^n\right)^{-\frac{1}{n}}\left(b\,c^{\frac{1}{n}}\left(x^n\right)^{\frac{1}{n}}\mathrm{e}^{\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}\left(ic\,x^n\right)\right)\left(-\mathrm{csgn}\left(ix^n\right)+\mathrm{csgn}\left(ic\,x^n\right)\right)\mathrm{csgn}\left(ic\,x^n\right)}{2n}} + a\right)^{p+1}\mathrm{e}^{-\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}\left(ic\,x^n\right)\right)\left(-\mathrm{csgn}\left(ix^n\right)+\mathrm{csgn}\left(ic\,x^n\right)\right)\mathrm{csgn}(}{2n}}}{\left(p+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*(c*x^n)^(1/n)+a)^p,x)
```

```
[Out] (b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)
+csgn(I*c*x^n))/n*csgn(I*c*x^n))+a)^(p+1)/(c^(1/n))/((x^n)^(1/n))*x^2*exp(-
1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*
x^n))/b/(p+1)-1/b^2/(p+1)/(c^(1/n))^2/((x^n)^(1/n))^2*x^2*exp(-I*Pi*(csgn(I
*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))*(b*c^(1/n)
*(x^n)^(1/n)*exp(1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*
x^n))/n*csgn(I*c*x^n))+a)^(p+2)/(p+2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(cx^n\right)^{\left(\frac{1}{n}\right)}b + a\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(c*x^n)^(1/n))^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*x^n)^(1/n)*b + a)^p*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \left(a + b \left(c\, x^n\right)^{1/n}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*(c*x^n)^(1/n))^p,x)
```

```
[Out] int(x*(a + b*(c*x^n)^(1/n))^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \left(a + b \left(cx^n\right)^{\frac{1}{n}}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(c*x**n)**(1/n))**p,x)
```

```
[Out] Integral(x*(a + b*(c*x**n)**(1/n))**p, x)
```

3.3025 $\int \left(a + b\left(cx^n\right)^{\frac{1}{n}}\right)^p dx$

Optimal. Leaf size=38

$$\frac{x\left(cx^n\right)^{-1/n}\left(a + b\left(cx^n\right)^{\frac{1}{n}}\right)^{p+1}}{b(p+1)}$$

```
[Out] x*(a+b*(c*x^n)^(1/n))^(1+p)/b/(1+p)/((c*x^n)^(1/n))
```

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {254, 32}

$$\frac{x\left(cx^n\right)^{-1/n}\left(a + b\left(cx^n\right)^{\frac{1}{n}}\right)^{p+1}}{b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c*x^n)^n^(-1))^p,x]
```

```
[Out] (x*(a + b*(c*x^n)^n^(-1))^(1 + p))/(b*(1 + p)*(c*x^n)^n^(-1))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 254

```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_.), x_Symbol] :> Dist[x/(c*x^
q)^(1/q), Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\int \left(a + b\left(cx^n\right)^{\frac{1}{n}}\right)^p dx = \left(x\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int (a + bx)^p\, dx, x, \left(cx^n\right)^{\frac{1}{n}}\right)$$
$$= \frac{x\left(cx^n\right)^{-1/n}\left(a + b\left(cx^n\right)^{\frac{1}{n}}\right)^{1+p}}{b(1+p)}$$

Mathematica [A] time = 0.15, size = 64, normalized size = 1.68

$$\frac{x\left(a + b\left(cx^n\right)^{\frac{1}{n}}\right)^p\left(\frac{a\left(cx^n\right)^{-1/n}\left(1 - \left(\frac{b\left(cx^n\right)^{\frac{1}{n}}}{a} + 1\right)^{-p}\right)}{b} + 1\right)}{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c*x^n)^n^(-1))^p,x]
```

```
[Out] (x*(a + b*(c*x^n)^n^(-1))^p*(1 + (a*(1 - (1 + (b*(c*x^n)^n^(-1))/a)^(-p)))/
(b*(c*x^n)^n^(-1))))/(1 + p)
```

fricas [A] time = 0.90, size = 37, normalized size = 0.97

$$\frac{\left(bc^{\left(\frac{1}{n}\right)}x + a\right)\left(bc^{\left(\frac{1}{n}\right)}x + a\right)^{p}}{\left(bp + b\right)c^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^n)^(1/n))^p,x, algorithm="fricas")
```

```
[Out] (b*c^(1/n)*x + a)*(b*c^(1/n)*x + a)^p/((b*p + b)*c^(1/n))
```

giac [A] time = 0.31, size = 54, normalized size = 1.42

$$\frac{\left(bc^{\left(\frac{1}{n}\right)}x + a\right)^{p} bc^{\left(\frac{1}{n}\right)}x + \left(bc^{\left(\frac{1}{n}\right)}x + a\right)^{p} a}{bc^{\left(\frac{1}{n}\right)}p + bc^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^n)^(1/n))^p,x, algorithm="giac")
```

```
[Out] ((b*c^(1/n)*x + a)^p*b*c^(1/n)*x + (b*c^(1/n)*x + a)^p*a)/(b*c^(1/n)*p + b*
c^(1/n))
```

maple [C] time = 0.16, size = 147, normalized size = 3.87

$$\frac{x\,c^{-\frac{1}{n}}\left(x^{n}\right)^{-\frac{1}{n}}\left(b\,c^{\frac{1}{n}}\left(x^{n}\right)^{\frac{1}{n}}\mathrm{e}^{\frac{i\pi\left(\mathrm{csgn}\left(ic\right)-\mathrm{csgn}\left(ic\,x^{n}\right)\right)\left(-\mathrm{csgn}\left(ix^{n}\right)+\mathrm{csgn}\left(ic\,x^{n}\right)\right)\mathrm{csgn}\left(ic\,x^{n}\right)}{2n}}+a\right)^{p+1}\mathrm{e}^{-\frac{i\pi\left(\mathrm{csgn}\left(ic\right)-\mathrm{csgn}\left(ic\,x^{n}\right)\right)\left(-\mathrm{csgn}\left(ix^{n}\right)+\mathrm{csgn}\left(ic\,x^{n}\right)\right)\mathrm{cs}}}{\left(p+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*x^n)^(1/n)+a)^p,x)
```

```
[Out] (b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)
+csgn(I*c*x^n))/n*csgn(I*c*x^n))+a)^(p+1)/(c^(1/n))/((x^n)^(1/n))*x*exp(-1/
2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^
n))/b/(p+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left((cx^{n})^{\left(\frac{1}{n}\right)}b + a\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^n)^(1/n))^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*x^n)^(1/n)*b + a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \left(a + b\left(c\,x^{n}\right)^{1/n}\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x^n)^(1/n))^p,x)
```

```
[Out] int((a + b*(c*x^n)^(1/n))^p, x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\left(cx^{n}\right)^{\frac{1}{n}}\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x**n)**(1/n))**p,x)
```

```
[Out] Integral((a + b*(c*x**n)**(1/n))**p, x)
```

3.3026 $\int \frac{\left(a+b(cx^n)^{\frac{1}{n}}\right)^p}{x}\,dx$

Optimal. Leaf size=51

$$-\frac{\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b(cx^n)^{\frac{1}{n}}}{a}+1\right)}{a(p+1)}$$

```
[Out] -(a+b*(c*x^n)^(1/n))^(1+p)*hypergeom([1, 1+p],[2+p],1+b*(c*x^n)^(1/n)/a)/a/
(1+p)
```

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {368, 65}

$$-\frac{\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b(cx^n)^{\frac{1}{n}}}{a}+1\right)}{a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c*x^n)^n^(-1))^p/x,x]
```

```
[Out] -(((a + b*(c*x^n)^n^(-1))^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b
*(c*x^n)^n^(-1))/a])/(a*(1 + p)))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\int \frac{\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^p}{x}\,dx = \text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, (cx^n)^{\frac{1}{n}}\right)$$
$$= -\frac{\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^{1+p}\,{}_2F_1\left(1,1+p;2+p;1+\frac{b(cx^n)^{\frac{1}{n}}}{a}\right)}{a(1+p)}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 1.00

$$-\frac{\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b(cx^n)^{\frac{1}{n}}}{a}+1\right)}{a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c*x^n)^n^(-1))^p/x,x]
```

```
[Out] -(((a + b*(c*x^n)^n^(-1))^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b
*(c*x^n)^n^(-1))/a])/(a*(1 + p)))
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left((cx^n)^{\left(\frac{1}{n}\right)} b + a\right)^p}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^n)^(1/n))^p/x,x, algorithm="fricas")
```

```
[Out] integral(((c*x^n)^(1/n)*b + a)^p/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((cx^n)^{\left(\frac{1}{n}\right)} b + a\right)^p}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^n)^(1/n))^p/x,x, algorithm="giac")
```

```
[Out] integrate(((c*x^n)^(1/n)*b + a)^p/x, x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,(c\,x^n)^{\frac{1}{n}} + a\right)^p}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*x^n)^(1/n)+a)^p/x,x)
```

```
[Out] int((b*(c*x^n)^(1/n)+a)^p/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((cx^n)^{\left(\frac{1}{n}\right)} b + a\right)^p}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^n)^(1/n))^p/x,x, algorithm="maxima")
```

```
[Out] integrate(((c*x^n)^(1/n)*b + a)^p/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(a + b\,(c\,x^n)^{1/n}\right)^p}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x^n)^(1/n))^p/x,x)
```

```
[Out] int((a + b*(c*x^n)^(1/n))^p/x, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\left(cx^n\right)^{\frac{1}{n}}\right)^p}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x**n)**(1/n))**p/x,x)
```

```
[Out] Integral((a + b*(c*x**n)**(1/n))**p/x, x)
```

3.3027 $\int \frac{\left(a+b(cx^n)^{\frac{1}{n}}\right)^p}{x^2}\,dx$

Optimal. Leaf size=63

$$\frac{b\,(cx^n)^{\frac{1}{n}}\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^{p+1}\,{}_2F_1\left(2,p+1;p+2;\frac{b(cx^n)^{\frac{1}{n}}}{a}+1\right)}{a^2(p+1)x}$$

```
[Out] b*(c*x^n)^(1/n)*(a+b*(c*x^n)^(1/n))^(1+p)*hypergeom([2, 1+p],[2+p],1+b*(c*x
^n)^(1/n)/a)/a^2/(1+p)/x
```

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {368, 65}

$$\frac{b\,(cx^n)^{\frac{1}{n}}\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^{p+1}\,{}_2F_1\left(2,p+1;p+2;\frac{b(cx^n)^{\frac{1}{n}}}{a}+1\right)}{a^2(p+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c*x^n)^n^(-1))^p/x^2,x]
```

```
[Out] (b*(c*x^n)^n^(-1)*(a + b*(c*x^n)^n^(-1))^(1 + p)*Hypergeometric2F1[2, 1 + p
, 2 + p, 1 + (b*(c*x^n)^n^(-1))/a])/(a^2*(1 + p)*x)
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\int \frac{\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^p}{x^2}\,dx = \frac{(cx^n)^{\frac{1}{n}}\,\text{Subst}\left(\int \frac{(a+bx)^p}{x^2}\,dx,x,(cx^n)^{\frac{1}{n}}\right)}{x}$$

$$= \frac{b\,(cx^n)^{\frac{1}{n}}\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^{1+p}\,{}_2F_1\left(2,1+p;2+p;1+\frac{b(cx^n)^{\frac{1}{n}}}{a}\right)}{a^2(1+p)x}$$

Mathematica [A] time = 0.01, size = 63, normalized size = 1.00

$$\frac{b\,(cx^n)^{\frac{1}{n}}\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^{p+1}\,{}_2F_1\left(2,p+1;p+2;\frac{b(cx^n)^{\frac{1}{n}}}{a}+1\right)}{a^2(p+1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c*x^n)^n^(-1))^p/x^2,x]
```

```
[Out] (b*(c*x^n)^n^(-1)*(a + b*(c*x^n)^n^(-1))^(1 + p)*Hypergeometric2F1[2, 1 + p
, 2 + p, 1 + (b*(c*x^n)^n^(-1))/a])/(a^2*(1 + p)*x)
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left((cx^n)^{\left(\frac{1}{n}\right)}b + a\right)^p}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^n)^(1/n))^p/x^2,x, algorithm="fricas")
```

```
[Out] integral(((c*x^n)^(1/n)*b + a)^p/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((cx^n)^{\left(\frac{1}{n}\right)}b + a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^n)^(1/n))^p/x^2,x, algorithm="giac")
```

```
[Out] integrate(((c*x^n)^(1/n)*b + a)^p/x^2, x)
```

maple [F] time = 1.04, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,(c\,x^n)^{\frac{1}{n}} + a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*x^n)^(1/n)+a)^p/x^2,x)
```

```
[Out] int((b*(c*x^n)^(1/n)+a)^p/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((cx^n)^{\left(\frac{1}{n}\right)}b + a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^n)^(1/n))^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate(((c*x^n)^(1/n)*b + a)^p/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(a + b\,(c\,x^n)^{1/n}\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x^n)^(1/n))^p/x^2,x)
```

```
[Out] int((a + b*(c*x^n)^(1/n))^p/x^2, x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\left(cx^n\right)^{\frac{1}{n}}\right)^p}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x**n)**(1/n))**p/x**2,x)
```

```
[Out] Integral((a + b*(c*x**n)**(1/n))**p/x**2, x)
```

3.3028 $\int \left(a + b\left(cx^n\right)^{2/n}\right)^3 \, dx$

Optimal. Leaf size=62

$$a^3x + a^2bx\left(cx^n\right)^{2/n} + \frac{3}{5}ab^2x\left(cx^n\right)^{4/n} + \frac{1}{7}b^3x\left(cx^n\right)^{6/n}$$

```
[Out] a^3*x+a^2*b*x*(c*x^n)^(2/n)+3/5*a*b^2*x*(c*x^n)^(4/n)+1/7*b^3*x*(c*x^n)^(6/
n)
```

Rubi [A] time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {254, 194}

$$a^2bx\left(cx^n\right)^{2/n} + a^3x + \frac{3}{5}ab^2x\left(cx^n\right)^{4/n} + \frac{1}{7}b^3x\left(cx^n\right)^{6/n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c*x^n)^(2/n))^3,x]
```

```
[Out] a^3*x + a^2*b*x*(c*x^n)^(2/n) + (3*a*b^2*x*(c*x^n)^(4/n))/5 + (b^3*x*(c*x^n
)^(6/n))/7
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 254

```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_.), x_Symbol] :> Dist[x/(c*x^
q)^(1/q), Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\begin{aligned}
\int \left(a + b\left(cx^n\right)^{2/n}\right)^3 \, dx &= \left(x\left(cx^n\right)^{-1/n}\right) \text{Subst}\left(\int \left(a + bx^2\right)^3 \, dx, x, \left(cx^n\right)^{\frac{1}{n}}\right) \\
&= \left(x\left(cx^n\right)^{-1/n}\right) \text{Subst}\left(\int \left(a^3 + 3a^2bx^2 + 3ab^2x^4 + b^3x^6\right) \, dx, x, \left(cx^n\right)^{\frac{1}{n}}\right) \\
&= a^3x + a^2bx\left(cx^n\right)^{2/n} + \frac{3}{5}ab^2x\left(cx^n\right)^{4/n} + \frac{1}{7}b^3x\left(cx^n\right)^{6/n}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 62, normalized size = 1.00

$$a^3x + a^2bx\left(cx^n\right)^{2/n} + \frac{3}{5}ab^2x\left(cx^n\right)^{4/n} + \frac{1}{7}b^3x\left(cx^n\right)^{6/n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c*x^n)^(2/n))^3,x]
```

```
[Out] a^3*x + a^2*b*x*(c*x^n)^(2/n) + (3*a*b^2*x*(c*x^n)^(4/n))/5 + (b^3*x*(c*x^n
)^(6/n))/7
```

fricas [A] time = 0.70, size = 52, normalized size = 0.84

$$\frac{1}{7}b^3c^{\frac{6}{n}}x^7 + \frac{3}{5}ab^2c^{\frac{4}{n}}x^5 + a^2bc^{\frac{2}{n}}x^3 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^n)^(2/n))^3,x, algorithm="fricas")
```

```
[Out] 1/7*b^3*c^(6/n)*x^7 + 3/5*a*b^2*c^(4/n)*x^5 + a^2*b*c^(2/n)*x^3 + a^3*x
```

giac [A] time = 0.20, size = 52, normalized size = 0.84

$$\frac{1}{7} b^3 c^{\frac{6}{n}} x^7 + \frac{3}{5} a b^2 c^{\frac{4}{n}} x^5 + a^2 b c^{\frac{2}{n}} x^3 + a^3 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^n)^(2/n))^3,x, algorithm="giac")
```

```
[Out] 1/7*b^3*c^(6/n)*x^7 + 3/5*a*b^2*c^(4/n)*x^5 + a^2*b*c^(2/n)*x^3 + a^3*x
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \left(b \left(c\, x^n\right)^{\frac{2}{n}} + a\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(c*x^n)^(2/n))^3,x)
```

```
[Out] int((a+b*(c*x^n)^(2/n))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^3 c^{\frac{6}{n}} \int (x^n)^{\frac{6}{n}}\, dx + 3\, a b^2 c^{\frac{4}{n}} \int (x^n)^{\frac{4}{n}}\, dx + 3\, a^2 b c^{\frac{2}{n}} \int (x^n)^{\frac{2}{n}}\, dx + a^3 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^n)^(2/n))^3,x, algorithm="maxima")
```

```
[Out] b^3*c^(6/n)*integrate((x^n)^(6/n), x) + 3*a*b^2*c^(4/n)*integrate((x^n)^(4/
n), x) + 3*a^2*b*c^(2/n)*integrate((x^n)^(2/n), x) + a^3*x
```

mupad [B] time = 1.25, size = 58, normalized size = 0.94

$$a^3\, x + \frac{b^3\, x\, (c\, x^n)^{6/n}}{7} + a^2\, b\, x\, (c\, x^n)^{2/n} + \frac{3\, a\, b^2\, x\, (c\, x^n)^{4/n}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x^n)^(2/n))^3,x)
```

```
[Out] a^3*x + (b^3*x*(c*x^n)^(6/n))/7 + a^2*b*x*(c*x^n)^(2/n) + (3*a*b^2*x*(c*x^n
)^(4/n))/5
```

sympy [A] time = 0.99, size = 63, normalized size = 1.02

$$a^3 x + a^2 b c^{\frac{2}{n}} x\, (x^n)^{\frac{2}{n}} + \frac{3 a b^2 c^{\frac{4}{n}} x\, (x^n)^{\frac{4}{n}}}{5} + \frac{b^3 c^{\frac{6}{n}} x\, (x^n)^{\frac{6}{n}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x**n)**(2/n))**3,x)
```

```
[Out] a**3*x + a**2*b*c**(2/n)*x*(x**n)**(2/n) + 3*a*b**2*c**(4/n)*x*(x**n)**(4/n
)/5 + b**3*c**(6/n)*x*(x**n)**(6/n)/7
```

3.3029 $\int \left(a + b\left(cx^n\right)^{2/n}\right)^2 dx$

Optimal. Leaf size=43

$$a^2x + \frac{2}{3}abx\left(cx^n\right)^{2/n} + \frac{1}{5}b^2x\left(cx^n\right)^{4/n}$$

```
[Out] a^2*x+2/3*a*b*x*(c*x^n)^(2/n)+1/5*b^2*x*(c*x^n)^(4/n)
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {254, 194}

$$a^2x + \frac{2}{3}abx\left(cx^n\right)^{2/n} + \frac{1}{5}b^2x\left(cx^n\right)^{4/n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c*x^n)^(2/n))^2,x]
```

```
[Out] a^2*x + (2*a*b*x*(c*x^n)^(2/n))/3 + (b^2*x*(c*x^n)^(4/n))/5
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 254

```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_.), x_Symbol] :> Dist[x/(c*x^
q)^(1/q), Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\begin{aligned}
\int \left(a + b\left(cx^n\right)^{2/n}\right)^2 dx &= \left(x\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int \left(a + bx^2\right)^2 dx, x, \left(cx^n\right)^{\frac{1}{n}}\right) \\
&= \left(x\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int \left(a^2 + 2abx^2 + b^2x^4\right) dx, x, \left(cx^n\right)^{\frac{1}{n}}\right) \\
&= a^2x + \frac{2}{3}abx\left(cx^n\right)^{2/n} + \frac{1}{5}b^2x\left(cx^n\right)^{4/n}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 1.00

$$a^2x + \frac{2}{3}abx\left(cx^n\right)^{2/n} + \frac{1}{5}b^2x\left(cx^n\right)^{4/n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c*x^n)^(2/n))^2,x]
```

```
[Out] a^2*x + (2*a*b*x*(c*x^n)^(2/n))/3 + (b^2*x*(c*x^n)^(4/n))/5
```

fricas [A] time = 0.92, size = 35, normalized size = 0.81

$$\frac{1}{5}\,b^2c^{\frac{4}{n}}x^5 + \frac{2}{3}\,abc^{\frac{2}{n}}x^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^n)^(2/n))^2,x, algorithm="fricas")
```

```
[Out] 1/5*b^2*c^(4/n)*x^5 + 2/3*a*b*c^(2/n)*x^3 + a^2*x
```

giac [A] time = 0.19, size = 35, normalized size = 0.81

$$\frac{1}{5} b^2 c^{\frac{4}{n}} x^5 + \frac{2}{3} abc^{\frac{2}{n}} x^3 + a^2 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^n)^(2/n))^2,x, algorithm="giac")
```

```
[Out] 1/5*b^2*c^(4/n)*x^5 + 2/3*a*b*c^(2/n)*x^3 + a^2*x
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \left(b \left(c\, x^n\right)^{\frac{2}{n}} + a\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*x^n)^(2/n)+a)^2,x)
```

```
[Out] int((b*(c*x^n)^(2/n)+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^2 c^{\frac{4}{n}} \int (x^n)^{\frac{4}{n}}\, dx + 2\, abc^{\frac{2}{n}} \int (x^n)^{\frac{2}{n}}\, dx + a^2 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^n)^(2/n))^2,x, algorithm="maxima")
```

```
[Out] b^2*c^(4/n)*integrate((x^n)^(4/n), x) + 2*a*b*c^(2/n)*integrate((x^n)^(2/n)
, x) + a^2*x
```

mupad [B] time = 1.22, size = 39, normalized size = 0.91

$$a^2\, x + \frac{b^2\, x\, (c\, x^n)^{4/n}}{5} + \frac{2\, a\, b\, x\, (c\, x^n)^{2/n}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x^n)^(2/n))^2,x)
```

```
[Out] a^2*x + (b^2*x*(c*x^n)^(4/n))/5 + (2*a*b*x*(c*x^n)^(2/n))/3
```

sympy [A] time = 0.57, size = 42, normalized size = 0.98

$$a^2 x + \frac{2abc^{\frac{2}{n}} x\, (x^n)^{\frac{2}{n}}}{3} + \frac{b^2 c^{\frac{4}{n}} x\, (x^n)^{\frac{4}{n}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x**n)**(2/n))**2,x)
```

```
[Out] a**2*x + 2*a*b*c**(2/n)*x*(x**n)**(2/n)/3 + b**2*c**(4/n)*x*(x**n)**(4/n)/5
```

3.3030 $\int \left(a + b\left(cx^n\right)^{2/n}\right) dx$

Optimal. Leaf size=21

$$ax + \frac{1}{3}bx\left(cx^n\right)^{2/n}$$

```
[Out] a*x+1/3*b*x*(c*x^n)^(2/n)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {15, 30}

$$ax + \frac{1}{3}bx\left(cx^n\right)^{2/n}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*(c*x^n)^(2/n),x]
```

```
[Out] a*x + (b*x*(c*x^n)^(2/n))/3
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \left(a + b\left(cx^n\right)^{2/n}\right) dx &= ax + b\int \left(cx^n\right)^{2/n}\, dx \\ &= ax + \frac{\left(b\left(cx^n\right)^{2/n}\right)\int x^2\, dx}{x^2} \\ &= ax + \frac{1}{3}bx\left(cx^n\right)^{2/n}\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$ax + \frac{1}{3}bx\left(cx^n\right)^{2/n}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*(c*x^n)^(2/n),x]
```

```
[Out] a*x + (b*x*(c*x^n)^(2/n))/3
```

fricas [A] time = 0.88, size = 17, normalized size = 0.81

$$\frac{1}{3}\, bc^{\frac{2}{n}} x^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*(c*x^n)^(2/n),x, algorithm="fricas")
```

```
[Out] 1/3*b*c^(2/n)*x^3 + a*x
```

giac [A] time = 0.16, size = 17, normalized size = 0.81

$$\frac{1}{3} bc^{\frac{2}{n}} x^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*(c*x^n)^(2/n),x, algorithm="giac")
[Out] 1/3*b*c^(2/n)*x^3 + a*x
```

maple [A] time = 0.05, size = 23, normalized size = 1.10

$$\frac{bx\,\mathrm{e}^{\frac{2\ln\left(c\,\mathrm{e}^{n\ln(x)}\right)}{n}}}{3} + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*(c*x^n)^(2/n)+a,x)
[Out] a*x+1/3*x*b*exp(2/n*ln(c*exp(n*ln(x))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$bc^{\frac{2}{n}} \int (x^n)^{\frac{2}{n}}\,dx + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*(c*x^n)^(2/n),x, algorithm="maxima")
[Out] b*c^(2/n)*integrate((x^n)^(2/n), x) + a*x
```

mupad [B] time = 1.17, size = 19, normalized size = 0.90

$$a\,x + \frac{b\,x\,(c\,x^n)^{2/n}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*(c*x^n)^(2/n),x)
[Out] a*x + (b*x*(c*x^n)^(2/n))/3
```

sympy [A] time = 0.31, size = 19, normalized size = 0.90

$$ax + \frac{bc^{\frac{2}{n}} x\,(x^n)^{\frac{2}{n}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*(c*x**n)**(2/n),x)
[Out] a*x + b*c**(2/n)*x*(x**n)**(2/n)/3
```

3.3031 $\int \frac{1}{a+b(cx^n)^{2/n}}\,dx$

Optimal. Leaf size=44

$$\frac{x\,(cx^n)^{-1/n}\tan^{-1}\left(\frac{\sqrt{b}\,(cx^n)^{\frac{1}{n}}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

```
[Out] x*arctan((c*x^n)^(1/n)*b^(1/2)/a^(1/2))/((c*x^n)^(1/n))/a^(1/2)/b^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {254, 205}

$$\frac{x\,(cx^n)^{-1/n}\tan^{-1}\left(\frac{\sqrt{b}\,(cx^n)^{\frac{1}{n}}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c*x^n)^(2/n))^(-1),x]
```

```
[Out] (x*ArcTan[(Sqrt[b]*(c*x^n)^n^(-1))/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*(c*x^n)^n^(-1
))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 254

```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_.), x_Symbol] :> Dist[x/(c*x^
q)^(1/q), Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\int \frac{1}{a+b\,(cx^n)^{2/n}}\,dx = \left(x\,(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, (cx^n)^{\frac{1}{n}}\right)$$
$$= \frac{x\,(cx^n)^{-1/n}\tan^{-1}\left(\frac{\sqrt{b}\,(cx^n)^{\frac{1}{n}}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 1.00

$$\frac{x\,(cx^n)^{-1/n}\tan^{-1}\left(\frac{\sqrt{b}\,(cx^n)^{\frac{1}{n}}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c*x^n)^(2/n))^(-1),x]
```

```
[Out] (x*ArcTan[(Sqrt[b]*(c*x^n)^n^(-1))/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*(c*x^n)^n^(-1
))
```

fricas [A] time = 0.66, size = 127, normalized size = 2.89

$$\left[-\frac{\sqrt{-abc^{\frac{2}{n}}}\,\log\left(\frac{bc^{\frac{2}{n}}x^2-2\sqrt{-abc^{\frac{2}{n}}}\,x-a}{bc^{\frac{2}{n}}x^2+a}\right)}{2\,abc^{\frac{2}{n}}},\ \frac{\sqrt{abc^{\frac{2}{n}}}\,\arctan\left(\frac{\sqrt{abc^{\frac{2}{n}}}\,x}{a}\right)}{abc^{\frac{2}{n}}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(c*x^n)^(2/n)),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b*c^(2/n))*log((b*c^(2/n)*x^2 - 2*sqrt(-a*b*c^(2/n))*x - a)/(
b*c^(2/n)*x^2 + a))/(a*b*c^(2/n)), sqrt(a*b*c^(2/n))*arctan(sqrt(a*b*c^(2/n
))*x/a)/(a*b*c^(2/n))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^n)^{\frac{2}{n}}\,b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(c*x^n)^(2/n)),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^n)^(2/n)*b + a), x)
```

maple [C] time = 0.47, size = 222, normalized size = 5.05

$$\frac{\arctan\left(\frac{bc^{\frac{2}{n}}(x^n)^{\frac{2}{n}}\mathrm{e}^{\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}\left(ic\,x^n\right)\right)\left(-\mathrm{csgn}\left(ix^n\right)+\mathrm{csgn}\left(ic\,x^n\right)\right)\mathrm{csgn}\left(ic\,x^n\right)}{n}}}{\sqrt{\frac{abc^{\frac{2}{n}}(x^n)^{\frac{2}{n}}\mathrm{e}^{\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}\left(ic\,x^n\right)\right)\left(-\mathrm{csgn}\left(ix^n\right)+\mathrm{csgn}\left(ic\,x^n\right)\right)\mathrm{csgn}\left(ic\,x^n\right)}{n}}}{x^2}}\,x\right)}{\sqrt{\frac{abc^{\frac{2}{n}}(x^n)^{\frac{2}{n}}\mathrm{e}^{\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}\left(ic\,x^n\right)\right)\left(-\mathrm{csgn}\left(ix^n\right)+\mathrm{csgn}\left(ic\,x^n\right)\right)\mathrm{csgn}\left(ic\,x^n\right)}{n}}}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*(c*x^n)^(2/n)+a),x)
```

```
[Out] 1/(a*b/x^2*(x^n)^(2/n)*c^(2/n)*exp(I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*
x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n)))^(1/2)*arctan(b/x*(x^n)^(2/n)*c^(2/n)*
exp(I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*
x^n))/(a*b/x^2*(x^n)^(2/n)*c^(2/n)*exp(I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csg
n(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^n)^{\frac{2}{n}}\,b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(c*x^n)^(2/n)),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^n)^(2/n)*b + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{a + b\,(c\,x^n)^{2/n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(c*x^n)^(2/n)),x)
```

```
[Out] int(1/(a + b*(c*x^n)^(2/n)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b\,(cx^n)^{\frac{2}{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(c*x**n)**(2/n)),x)
```

```
[Out] Integral(1/(a + b*(c*x**n)**(2/n)), x)
```

3.3032 $\int \frac{1}{\left(a+b(cx^n)^{2/n}\right)^2}\,dx$

Optimal. Leaf size=73

$$\frac{x\,(cx^n)^{-1/n}\tan^{-1}\left(\frac{\sqrt{b}\,(cx^n)^{\frac{1}{n}}}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+b\,(cx^n)^{2/n}\right)}$$

```
[Out] 1/2*x/a/(a+b*(c*x^n)^(2/n))+1/2*x*arctan((c*x^n)^(1/n)*b^(1/2)/a^(1/2))/a^(
3/2)/((c*x^n)^(1/n))/b^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {254, 199, 205}

$$\frac{x\,(cx^n)^{-1/n}\tan^{-1}\left(\frac{\sqrt{b}\,(cx^n)^{\frac{1}{n}}}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+b\,(cx^n)^{2/n}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c*x^n)^(2/n))^(-2),x]
```

```
[Out] x/(2*a*(a + b*(c*x^n)^(2/n))) + (x*ArcTan[(Sqrt[b]*(c*x^n)^n^(-1))/Sqrt[a]]
)/(2*a^(3/2)*Sqrt[b]*(c*x^n)^n^(-1))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 254

```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_.), x_Symbol] :> Dist[x/(c*x^
q)^(1/q), Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{2/n}\right)^2}\,dx = \left(x\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^2}\,dx, x, \left(cx^n\right)^{\frac{1}{n}}\right)$$

$$= \frac{x}{2a\left(a+b\left(cx^n\right)^{2/n}\right)} + \frac{\left(x\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \left(cx^n\right)^{\frac{1}{n}}\right)}{2a}$$

$$= \frac{x}{2a\left(a+b\left(cx^n\right)^{2/n}\right)} + \frac{x\left(cx^n\right)^{-1/n}\tan^{-1}\left(\frac{\sqrt{b}\left(cx^n\right)^{\frac{1}{n}}}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}$$

Mathematica [A] time = 0.06, size = 73, normalized size = 1.00

$$\frac{x\left(cx^n\right)^{-1/n}\tan^{-1}\left(\frac{\sqrt{b}\left(cx^n\right)^{\frac{1}{n}}}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}} + \frac{x}{2a\left(a+b\left(cx^n\right)^{2/n}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c*x^n)^(2/n))^(-2),x]
```

```
[Out] x/(2*a*(a + b*(c*x^n)^(2/n))) + (x*ArcTan[(Sqrt[b]*(c*x^n)^n^(-1))/Sqrt[a]]
)/(2*a^(3/2)*Sqrt[b]*(c*x^n)^n^(-1))
```

fricas [A] time = 0.90, size = 218, normalized size = 2.99

$$\left[\frac{2\,abc^{\frac{2}{n}}x - \left(bc^{\frac{2}{n}}x^2+a\right)\sqrt{-abc^{\frac{2}{n}}}\,\log\left(\frac{bc^{\frac{2}{n}}x^2-2\sqrt{-abc^{\frac{2}{n}}}\,x-a}{bc^{\frac{2}{n}}x^2+a}\right)}{4\left(a^2b^2c^{\frac{4}{n}}x^2+a^3bc^{\frac{2}{n}}\right)}, \frac{abc^{\frac{2}{n}}x + \left(bc^{\frac{2}{n}}x^2+a\right)\sqrt{abc^{\frac{2}{n}}}\,\arctan\left(\frac{\sqrt{abc^{\frac{2}{n}}}\,x}{a}\right)}{2\left(a^2b^2c^{\frac{4}{n}}x^2+a^3bc^{\frac{2}{n}}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(c*x^n)^(2/n))^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b*c^(2/n)*x - (b*c^(2/n)*x^2 + a)*sqrt(-a*b*c^(2/n))*log((b*c^(2/
n)*x^2 - 2*sqrt(-a*b*c^(2/n))*x - a)/(b*c^(2/n)*x^2 + a)))/(a^2*b^2*c^(4/n)
*x^2 + a^3*b*c^(2/n)), 1/2*(a*b*c^(2/n)*x + (b*c^(2/n)*x^2 + a)*sqrt(a*b*c^
(2/n))*arctan(sqrt(a*b*c^(2/n))*x/a))/(a^2*b^2*c^(4/n)*x^2 + a^3*b*c^(2/n))
]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(cx^n\right)^{\frac{2}{n}}b+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(c*x^n)^(2/n))^2,x, algorithm="giac")
```

```
[Out] integrate(((c*x^n)^(2/n)*b + a)^(-2), x)
```

maple [C] time = 0.42, size = 305, normalized size = 4.18

$$\frac{x}{2\left(b\,c^{\frac{2}{n}}\,(x^n)^{\frac{2}{n}}\,e^{\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}(ic\,x^n)\right)\left(-\mathrm{csgn}(ix^n)+\mathrm{csgn}(ic\,x^n)\right)\mathrm{csgn}(ic\,x^n)}{n}}+a\right)a}+\frac{\arctan\left(\frac{b\,c^{\frac{2}{n}}\,(x^n)^{\frac{2}{n}}\,e^{\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}(ic\,x^n)\right)\left(-\mathrm{csgn}(ix^n)+\mathrm{csgn}(ic\,x^n)\right)}{n}}}{\sqrt{\frac{ab\,c^{\frac{2}{n}}\,(x^n)^{\frac{2}{n}}\,e^{\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}(ic\,x^n)\right)\left(-\mathrm{csgn}(ix^n)+\mathrm{csgn}(ic\,x^n)\right)}{n}}}{x^2}}}\right)}{2\sqrt{\frac{ab\,c^{\frac{2}{n}}\,(x^n)^{\frac{2}{n}}\,e^{\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}(ic\,x^n)\right)\left(-\mathrm{csgn}(ix^n)+\mathrm{csgn}(ic\,x^n)\right)\mathrm{csg}}{n}}}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*(c*x^n)^(2/n)+a)^2,x)
```

```
[Out] 1/2/a*x/(b*c^(2/n)*(x^n)^(2/n)*exp(I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*
x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))+a)+1/2/a/(a*b/x^2*c^(2/n)*(x^n)^(2/n)*
exp(I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*
x^n)))^(1/2)*arctan(1/(a*b/x^2*c^(2/n)*(x^n)^(2/n)*exp(I*Pi*(csgn(I*c)-csgn
(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n)))^(1/2)*b/x*c^(2/n)
*(x^n)^(2/n)*exp(I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n)
)/n*csgn(I*c*x^n)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x}{2\left(abc^{\frac{2}{n}}(x^n)^{\frac{2}{n}}+a^2\right)}+\int\frac{1}{2\left(abc^{\frac{2}{n}}(x^n)^{\frac{2}{n}}+a^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(c*x^n)^(2/n))^2,x, algorithm="maxima")
```

```
[Out] 1/2*x/(a*b*c^(2/n)*(x^n)^(2/n) + a^2) + integrate(1/2/(a*b*c^(2/n)*(x^n)^(2
/n) + a^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\left(a+b\,(c\,x^n)^{2/n}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(c*x^n)^(2/n))^2,x)
```

```
[Out] int(1/(a + b*(c*x^n)^(2/n))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(a+b\,(cx^n)^{\frac{2}{n}}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(c*x**n)**(2/n))**2,x)
```

```
[Out] Integral((a + b*(c*x**n)**(2/n))**(-2), x)
```

3.3033 $\int \frac{1}{\left(a+b(cx^n)^{2/n}\right)^3}\,dx$

Optimal. Leaf size=98

$$\frac{3x\,(cx^n)^{-1/n}\tan^{-1}\left(\frac{\sqrt{b}\,(cx^n)^{\frac{1}{n}}}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}+\frac{3x}{8a^2\left(a+b\,(cx^n)^{2/n}\right)}+\frac{x}{4a\left(a+b\,(cx^n)^{2/n}\right)^2}$$

```
[Out] 1/4*x/a/(a+b*(c*x^n)^(2/n))^2+3/8*x/a^2/(a+b*(c*x^n)^(2/n))+3/8*x*arctan((c
*x^n)^(1/n)*b^(1/2)/a^(1/2))/a^(5/2)/((c*x^n)^(1/n))/b^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {254, 199, 205}

$$\frac{3x}{8a^2\left(a+b\,(cx^n)^{2/n}\right)}+\frac{3x\,(cx^n)^{-1/n}\tan^{-1}\left(\frac{\sqrt{b}\,(cx^n)^{\frac{1}{n}}}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}+\frac{x}{4a\left(a+b\,(cx^n)^{2/n}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c*x^n)^(2/n))^(-3),x]
```

```
[Out] x/(4*a*(a + b*(c*x^n)^(2/n))^2) + (3*x)/(8*a^2*(a + b*(c*x^n)^(2/n))) + (3*
x*ArcTan[(Sqrt[b]*(c*x^n)^n^(-1))/Sqrt[a]])/(8*a^(5/2)*Sqrt[b]*(c*x^n)^n^(-
1))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 254

```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_.), x_Symbol] :> Dist[x/(c*x^
q)^(1/q), Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{2/n}\right)^3}\,dx = \left(x\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^3}\,dx, x, \left(cx^n\right)^{\frac{1}{n}}\right)$$

$$= \frac{x}{4a\left(a+b\left(cx^n\right)^{2/n}\right)^2} + \frac{\left(3x\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^2}\,dx, x, \left(cx^n\right)^{\frac{1}{n}}\right)}{4a}$$

$$= \frac{x}{4a\left(a+b\left(cx^n\right)^{2/n}\right)^2} + \frac{3x}{8a^2\left(a+b\left(cx^n\right)^{2/n}\right)} + \frac{\left(3x\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \left(cx^n\right)^{\frac{1}{n}}\right)}{8a^2}$$

$$= \frac{x}{4a\left(a+b\left(cx^n\right)^{2/n}\right)^2} + \frac{3x}{8a^2\left(a+b\left(cx^n\right)^{2/n}\right)} + \frac{3x\left(cx^n\right)^{-1/n}\tan^{-1}\left(\frac{\sqrt{b}\left(cx^n\right)^{\frac{1}{n}}}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}$$

Mathematica [A] time = 0.14, size = 91, normalized size = 0.93

$$\frac{x\left(\frac{\sqrt{a}\left(5a+3b\left(cx^n\right)^{2/n}\right)}{\left(a+b\left(cx^n\right)^{2/n}\right)^2} + \frac{3\left(cx^n\right)^{-1/n}\tan^{-1}\left(\frac{\sqrt{b}\left(cx^n\right)^{\frac{1}{n}}}{\sqrt{a}}\right)}{\sqrt{b}}\right)}{8a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c*x^n)^(2/n))^(-3),x]
```

```
[Out] (x*((Sqrt[a]*(5*a + 3*b*(c*x^n)^(2/n)))/(a + b*(c*x^n)^(2/n))^2 + (3*ArcTan
[(Sqrt[b]*(c*x^n)^n^(-1))/Sqrt[a]])/(Sqrt[b]*(c*x^n)^n^(-1))))/(8*a^(5/2))
```

fricas [A] time = 0.98, size = 328, normalized size = 3.35

$$\left[\frac{6\,ab^2c^{\frac{4}{n}}x^3 + 10\,a^2bc^{\frac{2}{n}}x - 3\left(b^2c^{\frac{4}{n}}x^4 + 2\,abc^{\frac{2}{n}}x^2 + a^2\right)\sqrt{-abc^{\frac{2}{n}}}\,\log\left(\frac{bc^{\frac{2}{n}}x^2 - 2\sqrt{-abc^{\frac{2}{n}}}\,x - a}{bc^{\frac{2}{n}}x^2 + a}\right)}{16\left(a^3b^3c^{\frac{6}{n}}x^4 + 2\,a^4b^2c^{\frac{4}{n}}x^2 + a^5bc^{\frac{2}{n}}\right)}, \frac{3\,ab^2c^{\frac{4}{n}}x^3 + 5\,a^2bc^{\frac{2}{n}}x + 3\right.$$

$$\left.8\left(a^3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(c*x^n)^(2/n))^3,x, algorithm="fricas")
```

```
[Out] [1/16*(6*a*b^2*c^(4/n)*x^3 + 10*a^2*b*c^(2/n)*x - 3*(b^2*c^(4/n)*x^4 + 2*a*
b*c^(2/n)*x^2 + a^2)*sqrt(-a*b*c^(2/n))*log((b*c^(2/n)*x^2 - 2*sqrt(-a*b*c^
(2/n))*x - a)/(b*c^(2/n)*x^2 + a)))/(a^3*b^3*c^(6/n)*x^4 + 2*a^4*b^2*c^(4/n
)*x^2 + a^5*b*c^(2/n)), 1/8*(3*a*b^2*c^(4/n)*x^3 + 5*a^2*b*c^(2/n)*x + 3*(b
^2*c^(4/n)*x^4 + 2*a*b*c^(2/n)*x^2 + a^2)*sqrt(a*b*c^(2/n))*arctan(sqrt(a*b
*c^(2/n))*x/a))/(a^3*b^3*c^(6/n)*x^4 + 2*a^4*b^2*c^(4/n)*x^2 + a^5*b*c^(2/n
))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(cx^n\right)^{\frac{2}{n}}b + a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(c*x^n)^(2/n))^3,x, algorithm="giac")
```

```
[Out] integrate(((c*x^n)^(2/n)*b + a)^(-3), x)
```

maple [C] time = 0.43, size = 378, normalized size = 3.86

$$\frac{\left(3b\,c^{\frac{2}{n}}\,(x^n)^{\frac{2}{n}}\,\mathrm{e}^{\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}\left(ic\,x^n\right)\right)\left(-\mathrm{csgn}\left(ix^n\right)+\mathrm{csgn}\left(ic\,x^n\right)\right)\mathrm{csgn}\left(ic\,x^n\right)}{n}}+5a\right)x}{8\left(b\,c^{\frac{2}{n}}\,(x^n)^{\frac{2}{n}}\,\mathrm{e}^{\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}\left(ic\,x^n\right)\right)\left(-\mathrm{csgn}\left(ix^n\right)+\mathrm{csgn}\left(ic\,x^n\right)\right)\mathrm{csgn}\left(ic\,x^n\right)}{n}}+a\right)^2a^2}+\frac{3\arctan\left(\frac{b\,c^{\frac{2}{n}}\,(x^n)^{\frac{2}{n}}\,\mathrm{e}^{\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}\left(ic\,x^n\right)\right)\left(-\mathrm{csgn}\left(ix^n\right)+c\right.}{n}}}{\sqrt{\frac{ab\,c^{\frac{2}{n}}\,(x^n)^{\frac{2}{n}}\,\mathrm{e}^{\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}(ic\,x^n)\right)\left(-\mathrm{csgn}(ix^n)+c\right.}{n}}}{x^2}}}\right.}{8\sqrt{\frac{ab\,c^{\frac{2}{n}}\,(x^n)^{\frac{2}{n}}\,\mathrm{e}^{\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}(ic\,x^n)\right)\left(-\mathrm{csgn}(ix^n)+\mathrm{csgn}(ic\right.}{n}}}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*(c*x^n)^(2/n)+a)^3,x)
```

```
[Out] 1/8*x*(3*b*c^(2/n)*(x^n)^(2/n)*exp(I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*
x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))+5*a)/a^2/(b*c^(2/n)*(x^n)^(2/n)*exp(I*
Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))+
a)^2+3/8/a^2/(a*b/x^2*c^(2/n)*(x^n)^(2/n)*exp(I*Pi*(csgn(I*c)-csgn(I*c*x^n)
)*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n)))^(1/2)*arctan(1/(a*b/x^2*c^
(2/n)*(x^n)^(2/n)*exp(I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c
*x^n))/n*csgn(I*c*x^n)))^(1/2)*b/x*c^(2/n)*(x^n)^(2/n)*exp(I*Pi*(csgn(I*c)-
csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3\,bc^{\frac{2}{n}}x(x^n)^{\frac{2}{n}}+5\,ax}{8\left(a^2b^2c^{\frac{4}{n}}(x^n)^{\frac{4}{n}}+2\,a^3bc^{\frac{2}{n}}(x^n)^{\frac{2}{n}}+a^4\right)}+3\int\frac{1}{8\left(a^2bc^{\frac{2}{n}}(x^n)^{\frac{2}{n}}+a^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(c*x^n)^(2/n))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*b*c^(2/n)*x*(x^n)^(2/n) + 5*a*x)/(a^2*b^2*c^(4/n)*(x^n)^(4/n) + 2*a^
3*b*c^(2/n)*(x^n)^(2/n) + a^4) + 3*integrate(1/8/(a^2*b*c^(2/n)*(x^n)^(2/n)
 + a^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\left(a+b\,(c\,x^n)^{2/n}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(c*x^n)^(2/n))^3,x)
```

```
[Out] int(1/(a + b*(c*x^n)^(2/n))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(a+b\,(cx^n)^{\frac{2}{n}}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(c*x**n)**(2/n))**3,x)
```

```
[Out] Integral((a + b*(c*x**n)**(2/n))**(-3), x)
```

3.3034 $\int \frac{1}{1+4\sqrt{x^4}}\,dx$

Optimal. Leaf size=22

$$\frac{x\tan^{-1}\left(2\sqrt[4]{x^4}\right)}{2\sqrt[4]{x^4}}$$

```
[Out] 1/2*x*arctan(2*(x^4)^(1/4))/(x^4)^(1/4)
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {254, 203}

$$\frac{x\tan^{-1}\left(2\sqrt[4]{x^4}\right)}{2\sqrt[4]{x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*Sqrt[x^4])^(-1),x]
```

```
[Out] (x*ArcTan[2*(x^4)^(1/4)])/(2*(x^4)^(1/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 254

```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_.), x_Symbol] :> Dist[x/(c*x^
q)^(1/q), Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+4\sqrt{x^4}}\,dx &= \frac{x\,\text{Subst}\left(\int \frac{1}{1+4x^2}\,dx, x, \sqrt[4]{x^4}\right)}{\sqrt[4]{x^4}}\\ &= \frac{x\tan^{-1}\left(2\sqrt[4]{x^4}\right)}{2\sqrt[4]{x^4}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{x\tan^{-1}\left(2\sqrt[4]{x^4}\right)}{2\sqrt[4]{x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*Sqrt[x^4])^(-1),x]
```

```
[Out] (x*ArcTan[2*(x^4)^(1/4)])/(2*(x^4)^(1/4))
```

fricas [A] time = 0.95, size = 6, normalized size = 0.27

$$\frac{1}{2}\arctan(2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+4*(x^4)^(1/2)),x, algorithm="fricas")
[Out] 1/2*arctan(2*x)
```

giac [A] time = 0.15, size = 6, normalized size = 0.27

$$\frac{1}{2}\arctan(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+4*(x^4)^(1/2)),x, algorithm="giac")
[Out] 1/2*arctan(2*x)
```

maple [A] time = 0.05, size = 29, normalized size = 1.32

$$\frac{\arctan\left(2\sqrt{\frac{\sqrt{x^4}}{x^2}}\,x\right)}{2\sqrt{\frac{\sqrt{x^4}}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+4*(x^4)^(1/2)),x)
[Out] 1/2/((x^4)^(1/2)/x^2)^(1/2)*arctan(2*((x^4)^(1/2)/x^2)^(1/2)*x)
```

maxima [A] time = 1.36, size = 6, normalized size = 0.27

$$\frac{1}{2}\arctan(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+4*(x^4)^(1/2)),x, algorithm="maxima")
[Out] 1/2*arctan(2*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{4\sqrt{x^4}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*(x^4)^(1/2) + 1),x)
[Out] int(1/(4*(x^4)^(1/2) + 1), x)
```

sympy [A] time = 0.14, size = 5, normalized size = 0.23

$$\frac{\operatorname{atan}(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+4*(x**4)**(1/2)),x)
[Out] atan(2*x)/2
```

3.3035 $\int \frac{1}{1-4\sqrt{x^4}}\,dx$

Optimal. Leaf size=22

$$\frac{x \tanh^{-1}\left(2\sqrt[4]{x^4}\right)}{2\sqrt[4]{x^4}}$$

```
[Out] 1/2*x*arctanh(2*(x^4)^(1/4))/(x^4)^(1/4)
```

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {254, 206}

$$\frac{x \tanh^{-1}\left(2\sqrt[4]{x^4}\right)}{2\sqrt[4]{x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 4*Sqrt[x^4])^(-1),x]
```

```
[Out] (x*ArcTanh[2*(x^4)^(1/4)])/(2*(x^4)^(1/4))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 254

```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_.), x_Symbol] :> Dist[x/(c*x^
q)^(1/q), Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1-4\sqrt{x^4}}\,dx &= \frac{x\,\text{Subst}\left(\int \frac{1}{1-4x^2}\,dx, x, \sqrt[4]{x^4}\right)}{\sqrt[4]{x^4}} \\ &= \frac{x \tanh^{-1}\left(2\sqrt[4]{x^4}\right)}{2\sqrt[4]{x^4}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$\frac{x \tanh^{-1}\left(2\sqrt[4]{x^4}\right)}{2\sqrt[4]{x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 4*Sqrt[x^4])^(-1),x]
```

```
[Out] (x*ArcTanh[2*(x^4)^(1/4)])/(2*(x^4)^(1/4))
```

fricas [A] time = 0.68, size = 17, normalized size = 0.77

$$\frac{1}{4}\log(2x+1) - \frac{1}{4}\log(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-4*(x^4)^(1/2)),x, algorithm="fricas")
[Out] 1/4*log(2*x + 1) - 1/4*log(2*x - 1)
```

giac [**A**] time = 0.15, size = 15, normalized size = 0.68

$$\frac{1}{4}\log\left(\left|x+\frac{1}{2}\right|\right)-\frac{1}{4}\log\left(\left|x-\frac{1}{2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-4*(x^4)^(1/2)),x, algorithm="giac")
[Out] 1/4*log(abs(x + 1/2)) - 1/4*log(abs(x - 1/2))
```

maple [**A**] time = 0.04, size = 29, normalized size = 1.32

$$\frac{\operatorname{arctanh}\left(2\sqrt{\frac{\sqrt{x^4}}{x^2}}\,x\right)}{2\sqrt{\frac{\sqrt{x^4}}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-4*(x^4)^(1/2)),x)
[Out] 1/2/((x^4)^(1/2)/x^2)^(1/2)*arctanh(2*((x^4)^(1/2)/x^2)^(1/2)*x)
```

maxima [**A**] time = 0.54, size = 17, normalized size = 0.77

$$\frac{1}{4}\log\left(2\,x+1\right)-\frac{1}{4}\log\left(2\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-4*(x^4)^(1/2)),x, algorithm="maxima")
[Out] 1/4*log(2*x + 1) - 1/4*log(2*x - 1)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.05

$$\int -\frac{1}{4\sqrt{x^4}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(4*(x^4)^(1/2) - 1),x)
[Out] int(-1/(4*(x^4)^(1/2) - 1), x)
```

sympy [**A**] time = 0.15, size = 15, normalized size = 0.68

$$-\frac{\log\left(x-\frac{1}{2}\right)}{4}+\frac{\log\left(x+\frac{1}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-4*(x**4)**(1/2)),x)
[Out] -log(x - 1/2)/4 + log(x + 1/2)/4
```

3.3036 $\int \frac{1}{1+4\sqrt[3]{x^6}}\,dx$

Optimal. Leaf size=22

$$\frac{x\tan^{-1}\left(2\sqrt[6]{x^6}\right)}{2\sqrt[6]{x^6}}$$

```
[Out] 1/2*x*arctan(2*(x^6)^(1/6))/(x^6)^(1/6)
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {254, 203}

$$\frac{x\tan^{-1}\left(2\sqrt[6]{x^6}\right)}{2\sqrt[6]{x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*(x^6)^(1/3))^(-1),x]
```

```
[Out] (x*ArcTan[2*(x^6)^(1/6)])/(2*(x^6)^(1/6))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 254

```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_.), x_Symbol] :> Dist[x/(c*x^
q)^(1/q), Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+4\sqrt[3]{x^6}}\,dx &= \frac{x\,\text{Subst}\left(\int \frac{1}{1+4x^2}\,dx, x, \sqrt[6]{x^6}\right)}{\sqrt[6]{x^6}}\\ &= \frac{x\tan^{-1}\left(2\sqrt[6]{x^6}\right)}{2\sqrt[6]{x^6}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 1.00

$$\frac{x\tan^{-1}\left(2\sqrt[6]{x^6}\right)}{2\sqrt[6]{x^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*(x^6)^(1/3))^(-1),x]
```

```
[Out] (x*ArcTan[2*(x^6)^(1/6)])/(2*(x^6)^(1/6))
```

fricas [A] time = 0.75, size = 6, normalized size = 0.27

$$\frac{1}{2}\arctan(2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+4*(x^6)^(1/3)),x, algorithm="fricas")
[Out] 1/2*arctan(2*x)
```

giac [A] time = 0.15, size = 6, normalized size = 0.27

$$\frac{1}{2}\arctan(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+4*(x^6)^(1/3)),x, algorithm="giac")
[Out] 1/2*arctan(2*x)
```

maple [A] time = 0.33, size = 17, normalized size = 0.77

$$\frac{x\arctan\left(2\left(x^6\right)^{\frac{1}{6}}\right)}{2\left(x^6\right)^{\frac{1}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+4*(x^6)^(1/3)),x)
[Out] 1/2*x*arctan(2*(x^6)^(1/6))/(x^6)^(1/6)
```

maxima [A] time = 1.25, size = 6, normalized size = 0.27

$$\frac{1}{2}\arctan(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+4*(x^6)^(1/3)),x, algorithm="maxima")
[Out] 1/2*arctan(2*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{4\left(x^6\right)^{1/3}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*(x^6)^(1/3) + 1),x)
[Out] int(1/(4*(x^6)^(1/3) + 1), x)
```

sympy [A] time = 0.13, size = 5, normalized size = 0.23

$$\frac{\operatorname{atan}(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+4*(x**6)**(1/3)),x)
[Out] atan(2*x)/2
```

3.3037 $\int \frac{1}{1-4\sqrt[3]{x^6}}\,dx$

Optimal. Leaf size=22

$$\frac{x\tanh^{-1}\left(2\sqrt[6]{x^6}\right)}{2\sqrt[6]{x^6}}$$

```
[Out] 1/2*x*arctanh(2*(x^6)^(1/6))/(x^6)^(1/6)
```

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {254, 206}

$$\frac{x\tanh^{-1}\left(2\sqrt[6]{x^6}\right)}{2\sqrt[6]{x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 4*(x^6)^(1/3))^(-1),x]
```

```
[Out] (x*ArcTanh[2*(x^6)^(1/6)])/(2*(x^6)^(1/6))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 254

```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_.), x_Symbol] :> Dist[x/(c*x^
q)^(1/q), Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1-4\sqrt[3]{x^6}}\,dx &= \frac{x\,\text{Subst}\left(\int \frac{1}{1-4x^2}\,dx, x, \sqrt[6]{x^6}\right)}{\sqrt[6]{x^6}}\\ &= \frac{x\tanh^{-1}\left(2\sqrt[6]{x^6}\right)}{2\sqrt[6]{x^6}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 1.00

$$\frac{x\tanh^{-1}\left(2\sqrt[6]{x^6}\right)}{2\sqrt[6]{x^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 4*(x^6)^(1/3))^(-1),x]
```

```
[Out] (x*ArcTanh[2*(x^6)^(1/6)])/(2*(x^6)^(1/6))
```

fricas [A] time = 0.91, size = 17, normalized size = 0.77

$$\frac{1}{4}\log(2x+1) - \frac{1}{4}\log(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-4*(x^6)^(1/3)),x, algorithm="fricas")
[Out] 1/4*log(2*x + 1) - 1/4*log(2*x - 1)
```

giac [**A**] time = 0.15, size = 15, normalized size = 0.68

$$\frac{1}{4}\log\left(\left|x+\frac{1}{2}\right|\right)-\frac{1}{4}\log\left(\left|x-\frac{1}{2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-4*(x^6)^(1/3)),x, algorithm="giac")
[Out] 1/4*log(abs(x + 1/2)) - 1/4*log(abs(x - 1/2))
```

maple [**A**] time = 0.32, size = 17, normalized size = 0.77

$$\frac{x\,\operatorname{arctanh}\left(2\left(x^6\right)^{\frac{1}{6}}\right)}{2\left(x^6\right)^{\frac{1}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-4*(x^6)^(1/3)),x)
[Out] 1/2*x*arctanh(2*(x^6)^(1/6))/(x^6)^(1/6)
```

maxima [**A**] time = 0.47, size = 17, normalized size = 0.77

$$\frac{1}{4}\log\left(2\,x+1\right)-\frac{1}{4}\log\left(2\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-4*(x^6)^(1/3)),x, algorithm="maxima")
[Out] 1/4*log(2*x + 1) - 1/4*log(2*x - 1)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.05

$$-\int\frac{1}{4\left(x^6\right)^{1/3}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(4*(x^6)^(1/3) - 1),x)
[Out] -int(1/(4*(x^6)^(1/3) - 1), x)
```

sympy [**A**] time = 0.13, size = 15, normalized size = 0.68

$$-\frac{\log\left(x-\frac{1}{2}\right)}{4}+\frac{\log\left(x+\frac{1}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-4*(x**6)**(1/3)),x)
[Out] -log(x - 1/2)/4 + log(x + 1/2)/4
```

3.3038 $\int \frac{1}{1+4\left(x^{2n}\right)^{\frac{1}{n}}}\,dx$

Optimal. Leaf size=34

$$\frac{1}{2}x\left(x^{2n}\right)^{-\frac{1}{2}/n}\tan^{-1}\left(2\left(x^{2n}\right)^{\frac{1}{2}/n}\right)$$

```
[Out] 1/2*x*arctan(2*(x^(2*n))^(1/2/n))/((x^(2*n))^(1/2/n))
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {254, 203}

$$\frac{1}{2}x\left(x^{2n}\right)^{-\frac{1}{2}/n}\tan^{-1}\left(2\left(x^{2n}\right)^{\frac{1}{2}/n}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*(x^(2*n))^n^(-1))^(-1),x]
```

```
[Out] (x*ArcTan[2*(x^(2*n))^(1/(2*n))])/(2*(x^(2*n))^(1/(2*n)))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 254

```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_.), x_Symbol] :> Dist[x/(c*x^
q)^(1/q), Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\int \frac{1}{1+4\left(x^{2n}\right)^{\frac{1}{n}}}\,dx = \left(x\left(x^{2n}\right)^{-\frac{1}{2}/n}\right)\text{Subst}\left(\int \frac{1}{1+4x^2}\,dx, x, \left(x^{2n}\right)^{\frac{1}{2}/n}\right)$$
$$= \frac{1}{2}x\left(x^{2n}\right)^{-\frac{1}{2}/n}\tan^{-1}\left(2\left(x^{2n}\right)^{\frac{1}{2}/n}\right)$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.00

$$\frac{1}{2}x\left(x^{2n}\right)^{-\frac{1}{2}/n}\tan^{-1}\left(2\left(x^{2n}\right)^{\frac{1}{2}/n}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*(x^(2*n))^n^(-1))^(-1),x]
```

```
[Out] (x*ArcTan[2*(x^(2*n))^(1/(2*n))])/(2*(x^(2*n))^(1/(2*n)))
```

fricas [A] time = 0.87, size = 6, normalized size = 0.18

$$\frac{1}{2}\arctan(2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+4*(x^(2*n))^(1/n)),x, algorithm="fricas")
[Out] 1/2*arctan(2*x)
```

giac [A] time = 0.17, size = 6, normalized size = 0.18

$$\frac{1}{2}\arctan(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+4*(x^(2*n))^(1/n)),x, algorithm="giac")
[Out] 1/2*arctan(2*x)
```

maple [A] time = 0.16, size = 42, normalized size = 1.24

$$\frac{x\left(x^{2n}\right)^{-\frac{1}{2n}}\arctan\left(2\left(x^{2n}\right)^{\frac{1}{n}}\left(x^{2n}\right)^{-\frac{1}{2n}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+4*(x^(2*n))^(1/n)),x)
[Out] 1/2*x/((x^(2*n))^(1/2/n))*arctan(2*(x^(2*n))^(1/n)/((x^(2*n))^(1/2/n)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{4\left(x^{2\,n}\right)^{\left(\frac{1}{n}\right)}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+4*(x^(2*n))^(1/n)),x, algorithm="maxima")
[Out] integrate(1/(4*(x^(2*n))^(1/n) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{4\left(x^{2\,n}\right)^{1/n}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*(x^(2*n))^(1/n) + 1),x)
[Out] int(1/(4*(x^(2*n))^(1/n) + 1), x)
```

sympy [A] time = 0.13, size = 5, normalized size = 0.15

$$\frac{\operatorname{atan}(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+4*(x**(2*n))**(1/n)),x)
[Out] atan(2*x)/2
```

3.3039 $\int \frac{1}{1-4\left(x^{2n}\right)^{\frac{1}{n}}}\,dx$

Optimal. Leaf size=34

$$\frac{1}{2}x\left(x^{2n}\right)^{-\frac{1}{2}/n}\tanh^{-1}\left(2\left(x^{2n}\right)^{\frac{1}{2}/n}\right)$$

```
[Out] 1/2*x*arctanh(2*(x^(2*n))^(1/2/n))/((x^(2*n))^(1/2/n))
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {254, 206}

$$\frac{1}{2}x\left(x^{2n}\right)^{-\frac{1}{2}/n}\tanh^{-1}\left(2\left(x^{2n}\right)^{\frac{1}{2}/n}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 4*(x^(2*n))^n^(-1))^(-1),x]
```

```
[Out] (x*ArcTanh[2*(x^(2*n))^(1/(2*n))])/(2*(x^(2*n))^(1/(2*n)))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 254

```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_.), x_Symbol] :> Dist[x/(c*x^
q)^(1/q), Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\int \frac{1}{1-4\left(x^{2n}\right)^{\frac{1}{n}}}\,dx = \left(x\left(x^{2n}\right)^{-\frac{1}{2}/n}\right)\text{Subst}\left(\int \frac{1}{1-4x^2}\,dx, x, \left(x^{2n}\right)^{\frac{1}{2}/n}\right)$$

$$= \frac{1}{2}x\left(x^{2n}\right)^{-\frac{1}{2}/n}\tanh^{-1}\left(2\left(x^{2n}\right)^{\frac{1}{2}/n}\right)$$

Mathematica [A] time = 0.00, size = 34, normalized size = 1.00

$$\frac{1}{2}x\left(x^{2n}\right)^{-\frac{1}{2}/n}\tanh^{-1}\left(2\left(x^{2n}\right)^{\frac{1}{2}/n}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 4*(x^(2*n))^n^(-1))^(-1),x]
```

```
[Out] (x*ArcTanh[2*(x^(2*n))^(1/(2*n))])/(2*(x^(2*n))^(1/(2*n)))
```

fricas [A] time = 0.88, size = 17, normalized size = 0.50

$$\frac{1}{4}\log(2x+1) - \frac{1}{4}\log(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-4*(x^(2*n))^(1/n)),x, algorithm="fricas")
[Out] 1/4*log(2*x + 1) - 1/4*log(2*x - 1)
```

giac [A] time = 0.15, size = 15, normalized size = 0.44

$$\frac{1}{4}\log\left(\left|x+\frac{1}{2}\right|\right)-\frac{1}{4}\log\left(\left|x-\frac{1}{2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-4*(x^(2*n))^(1/n)),x, algorithm="giac")
[Out] 1/4*log(abs(x + 1/2)) - 1/4*log(abs(x - 1/2))
```

maple [A] time = 0.13, size = 42, normalized size = 1.24

$$\frac{x\left(x^{2n}\right)^{-\frac{1}{2n}}\operatorname{arctanh}\left(2\left(x^{2n}\right)^{\frac{1}{n}}\left(x^{2n}\right)^{-\frac{1}{2n}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-4*(x^(2*n))^(1/n)),x)
[Out] 1/2*x/((x^(2*n))^(1/2/n))*arctanh(2*(x^(2*n))^(1/n)/((x^(2*n))^(1/2/n)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{1}{4\left(x^{2\,n}\right)^{\left(\frac{1}{n}\right)}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-4*(x^(2*n))^(1/n)),x, algorithm="maxima")
[Out] -integrate(1/(4*(x^(2*n))^(1/n) - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$-\int\frac{1}{4\left(x^{2\,n}\right)^{1/n}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(4*(x^(2*n))^(1/n) - 1),x)
[Out] -int(1/(4*(x^(2*n))^(1/n) - 1), x)
```

sympy [A] time = 0.14, size = 15, normalized size = 0.44

$$-\frac{\log\left(x-\frac{1}{2}\right)}{4}+\frac{\log\left(x+\frac{1}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-4*(x**(2*n))**(1/n)),x)
[Out] -log(x - 1/2)/4 + log(x + 1/2)/4
```

3.3040 $\int \left(a + b\left(cx^n\right)^{3/n}\right)^3 \, dx$

Optimal. Leaf size=65

$$a^3x + \frac{3}{4}a^2bx\left(cx^n\right)^{3/n} + \frac{3}{7}ab^2x\left(cx^n\right)^{6/n} + \frac{1}{10}b^3x\left(cx^n\right)^{9/n}$$

```
[Out] a^3*x+3/4*a^2*b*x*(c*x^n)^(3/n)+3/7*a*b^2*x*(c*x^n)^(6/n)+1/10*b^3*x*(c*x^n
)^(9/n)
```

Rubi [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {254, 194}

$$\frac{3}{4}a^2bx\left(cx^n\right)^{3/n} + a^3x + \frac{3}{7}ab^2x\left(cx^n\right)^{6/n} + \frac{1}{10}b^3x\left(cx^n\right)^{9/n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c*x^n)^(3/n))^3,x]
```

```
[Out] a^3*x + (3*a^2*b*x*(c*x^n)^(3/n))/4 + (3*a*b^2*x*(c*x^n)^(6/n))/7 + (b^3*x*
(c*x^n)^(9/n))/10
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 254

```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_.), x_Symbol] :> Dist[x/(c*x^
q)^(1/q), Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\begin{aligned}
\int \left(a + b\left(cx^n\right)^{3/n}\right)^3 \, dx &= \left(x\left(cx^n\right)^{-1/n}\right) \text{Subst}\left(\int \left(a + bx^3\right)^3 \, dx, x, \left(cx^n\right)^{\frac{1}{n}}\right) \\
&= \left(x\left(cx^n\right)^{-1/n}\right) \text{Subst}\left(\int \left(a^3 + 3a^2bx^3 + 3ab^2x^6 + b^3x^9\right) \, dx, x, \left(cx^n\right)^{\frac{1}{n}}\right) \\
&= a^3x + \frac{3}{4}a^2bx\left(cx^n\right)^{3/n} + \frac{3}{7}ab^2x\left(cx^n\right)^{6/n} + \frac{1}{10}b^3x\left(cx^n\right)^{9/n}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 65, normalized size = 1.00

$$a^3x + \frac{3}{4}a^2bx\left(cx^n\right)^{3/n} + \frac{3}{7}ab^2x\left(cx^n\right)^{6/n} + \frac{1}{10}b^3x\left(cx^n\right)^{9/n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c*x^n)^(3/n))^3,x]
```

```
[Out] a^3*x + (3*a^2*b*x*(c*x^n)^(3/n))/4 + (3*a*b^2*x*(c*x^n)^(6/n))/7 + (b^3*x*
(c*x^n)^(9/n))/10
```

fricas [A] time = 0.94, size = 53, normalized size = 0.82

$$\frac{1}{10}\,b^3c^{\frac{9}{n}}x^{10} + \frac{3}{7}\,ab^2c^{\frac{6}{n}}x^7 + \frac{3}{4}\,a^2bc^{\frac{3}{n}}x^4 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^n)^(3/n))^3,x, algorithm="fricas")
```

```
[Out] 1/10*b^3*c^(9/n)*x^10 + 3/7*a*b^2*c^(6/n)*x^7 + 3/4*a^2*b*c^(3/n)*x^4 + a^3
*x
```

giac [A] time = 0.40, size = 53, normalized size = 0.82

$$\frac{1}{10} b^3 c^{\frac{9}{n}} x^{10} + \frac{3}{7} a b^2 c^{\frac{6}{n}} x^7 + \frac{3}{4} a^2 b c^{\frac{3}{n}} x^4 + a^3 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^n)^(3/n))^3,x, algorithm="giac")
```

```
[Out] 1/10*b^3*c^(9/n)*x^10 + 3/7*a*b^2*c^(6/n)*x^7 + 3/4*a^2*b*c^(3/n)*x^4 + a^3
*x
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \left(b \left(c\, x^n\right)^{\frac{3}{n}} + a\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(c*x^n)^(3/n))^3,x)
```

```
[Out] int((a+b*(c*x^n)^(3/n))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^3 c^{\frac{9}{n}} \int (x^n)^{\frac{9}{n}}\, dx + 3\, a b^2 c^{\frac{6}{n}} \int (x^n)^{\frac{6}{n}}\, dx + 3\, a^2 b c^{\frac{3}{n}} \int (x^n)^{\frac{3}{n}}\, dx + a^3 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^n)^(3/n))^3,x, algorithm="maxima")
```

```
[Out] b^3*c^(9/n)*integrate((x^n)^(9/n), x) + 3*a*b^2*c^(6/n)*integrate((x^n)^(6/
n), x) + 3*a^2*b*c^(3/n)*integrate((x^n)^(3/n), x) + a^3*x
```

mupad [B] time = 1.25, size = 59, normalized size = 0.91

$$a^3\, x + \frac{b^3\, x\, \left(c\, x^n\right)^{9/n}}{10} + \frac{3\, a^2\, b\, x\, \left(c\, x^n\right)^{3/n}}{4} + \frac{3\, a\, b^2\, x\, \left(c\, x^n\right)^{6/n}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x^n)^(3/n))^3,x)
```

```
[Out] a^3*x + (b^3*x*(c*x^n)^(9/n))/10 + (3*a^2*b*x*(c*x^n)^(3/n))/4 + (3*a*b^2*x
*(c*x^n)^(6/n))/7
```

sympy [A] time = 1.06, size = 66, normalized size = 1.02

$$a^3 x + \frac{3 a^2 b c^{\frac{3}{n}} x\, (x^n)^{\frac{3}{n}}}{4} + \frac{3 a b^2 c^{\frac{6}{n}} x\, (x^n)^{\frac{6}{n}}}{7} + \frac{b^3 c^{\frac{9}{n}} x\, (x^n)^{\frac{9}{n}}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x**n)**(3/n))**3,x)
```

```
[Out] a**3*x + 3*a**2*b*c**(3/n)*x*(x**n)**(3/n)/4 + 3*a*b**2*c**(6/n)*x*(x**n)**
(6/n)/7 + b**3*c**(9/n)*x*(x**n)**(9/n)/10
```

3.3041 $\int \left(a + b\left(cx^n\right)^{3/n}\right)^2 dx$

Optimal. Leaf size=43

$$a^2x + \frac{1}{2}abx\left(cx^n\right)^{3/n} + \frac{1}{7}b^2x\left(cx^n\right)^{6/n}$$

```
[Out] a^2*x+1/2*a*b*x*(c*x^n)^(3/n)+1/7*b^2*x*(c*x^n)^(6/n)
```

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {254, 194}

$$a^2x + \frac{1}{2}abx\left(cx^n\right)^{3/n} + \frac{1}{7}b^2x\left(cx^n\right)^{6/n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c*x^n)^(3/n))^2,x]
```

```
[Out] a^2*x + (a*b*x*(c*x^n)^(3/n))/2 + (b^2*x*(c*x^n)^(6/n))/7
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 254

```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_.), x_Symbol] :> Dist[x/(c*x^
q)^(1/q), Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\begin{aligned}
\int \left(a + b\left(cx^n\right)^{3/n}\right)^2 dx &= \left(x\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int \left(a + bx^3\right)^2 dx, x, \left(cx^n\right)^{\frac{1}{n}}\right) \\
&= \left(x\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int \left(a^2 + 2abx^3 + b^2x^6\right) dx, x, \left(cx^n\right)^{\frac{1}{n}}\right) \\
&= a^2x + \frac{1}{2}abx\left(cx^n\right)^{3/n} + \frac{1}{7}b^2x\left(cx^n\right)^{6/n}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 1.00

$$a^2x + \frac{1}{2}abx\left(cx^n\right)^{3/n} + \frac{1}{7}b^2x\left(cx^n\right)^{6/n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c*x^n)^(3/n))^2,x]
```

```
[Out] a^2*x + (a*b*x*(c*x^n)^(3/n))/2 + (b^2*x*(c*x^n)^(6/n))/7
```

fricas [A] time = 0.88, size = 35, normalized size = 0.81

$$\frac{1}{7}b^2c^{\frac{6}{n}}x^7 + \frac{1}{2}abc^{\frac{3}{n}}x^4 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^n)^(3/n))^2,x, algorithm="fricas")
```

```
[Out] 1/7*b^2*c^(6/n)*x^7 + 1/2*a*b*c^(3/n)*x^4 + a^2*x
```

giac [A] time = 0.20, size = 35, normalized size = 0.81

$$\frac{1}{7}\,b^2c^{\frac{6}{n}}x^7 + \frac{1}{2}\,abc^{\frac{3}{n}}x^4 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^n)^(3/n))^2,x, algorithm="giac")
```

```
[Out] 1/7*b^2*c^(6/n)*x^7 + 1/2*a*b*c^(3/n)*x^4 + a^2*x
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \left(b\,(c\,x^n)^{\frac{3}{n}} + a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*x^n)^(3/n)+a)^2,x)
```

```
[Out] int((b*(c*x^n)^(3/n)+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^2c^{\frac{6}{n}}\int (x^n)^{\frac{6}{n}}\,dx + 2\,abc^{\frac{3}{n}}\int (x^n)^{\frac{3}{n}}\,dx + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^n)^(3/n))^2,x, algorithm="maxima")
```

```
[Out] b^2*c^(6/n)*integrate((x^n)^(6/n), x) + 2*a*b*c^(3/n)*integrate((x^n)^(3/n)
, x) + a^2*x
```

mupad [B] time = 1.22, size = 39, normalized size = 0.91

$$a^2\,x + \frac{b^2\,x\,(c\,x^n)^{6/n}}{7} + \frac{a\,b\,x\,(c\,x^n)^{3/n}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x^n)^(3/n))^2,x)
```

```
[Out] a^2*x + (b^2*x*(c*x^n)^(6/n))/7 + (a*b*x*(c*x^n)^(3/n))/2
```

sympy [A] time = 0.56, size = 41, normalized size = 0.95

$$a^2x + \frac{abc^{\frac{3}{n}}x\,(x^n)^{\frac{3}{n}}}{2} + \frac{b^2c^{\frac{6}{n}}x\,(x^n)^{\frac{6}{n}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x**n)**(3/n))**2,x)
```

```
[Out] a**2*x + a*b*c**(3/n)*x*(x**n)**(3/n)/2 + b**2*c**(6/n)*x*(x**n)**(6/n)/7
```

3.3042 $\int \left(a + b\left(cx^n\right)^{3/n}\right) dx$

Optimal. Leaf size=21

$$ax + \frac{1}{4}bx\left(cx^n\right)^{3/n}$$

```
[Out] a*x+1/4*b*x*(c*x^n)^(3/n)
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {15, 30}

$$ax + \frac{1}{4}bx\left(cx^n\right)^{3/n}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*(c*x^n)^(3/n),x]
```

```
[Out] a*x + (b*x*(c*x^n)^(3/n))/4
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \left(a + b\left(cx^n\right)^{3/n}\right) dx &= ax + b\int \left(cx^n\right)^{3/n}\, dx \\ &= ax + \frac{\left(b\left(cx^n\right)^{3/n}\right)\int x^3\, dx}{x^3} \\ &= ax + \frac{1}{4}bx\left(cx^n\right)^{3/n}\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$ax + \frac{1}{4}bx\left(cx^n\right)^{3/n}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*(c*x^n)^(3/n),x]
```

```
[Out] a*x + (b*x*(c*x^n)^(3/n))/4
```

fricas [A] time = 0.90, size = 17, normalized size = 0.81

$$\frac{1}{4}\, bc^{\frac{3}{n}} x^4 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*(c*x^n)^(3/n),x, algorithm="fricas")
```

```
[Out] 1/4*b*c^(3/n)*x^4 + a*x
```

giac [A] time = 0.17, size = 17, normalized size = 0.81

$$\frac{1}{4} bc^{\frac{3}{n}} x^4 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*(c*x^n)^(3/n),x, algorithm="giac")
[Out] 1/4*b*c^(3/n)*x^4 + a*x
```

maple [A] time = 0.05, size = 23, normalized size = 1.10

$$\frac{bx\, \mathrm{e}^{\frac{3 \ln\left(c\, \mathrm{e}^{n \ln(x)}\right)}{n}}}{4} + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*(c*x^n)^(3/n)+a,x)
[Out] a*x+1/4*x*b*exp(3/n*ln(c*exp(n*ln(x))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$bc^{\frac{3}{n}} \int (x^n)^{\frac{3}{n}}\, dx + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*(c*x^n)^(3/n),x, algorithm="maxima")
[Out] b*c^(3/n)*integrate((x^n)^(3/n), x) + a*x
```

mupad [B] time = 1.17, size = 19, normalized size = 0.90

$$a\, x + \frac{b\, x\, (c\, x^n)^{3/n}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*(c*x^n)^(3/n),x)
[Out] a*x + (b*x*(c*x^n)^(3/n))/4
```

sympy [A] time = 0.30, size = 19, normalized size = 0.90

$$ax + \frac{bc^{\frac{3}{n}} x\, (x^n)^{\frac{3}{n}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*(c*x**n)**(3/n),x)
[Out] a*x + b*c**(3/n)*x*(x**n)**(3/n)/4
```

3.3043 $\int \frac{1}{a+b(cx^n)^{3/n}}\,dx$

Optimal. Leaf size=183

$$-\frac{x\,(cx^n)^{-1/n}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(cx^n)^{\frac{1}{n}}+b^{2/3}\,(cx^n)^{2/n}\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{x\,(cx^n)^{-1/n}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(cx^n)^{\frac{1}{n}}\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{x\,(cx^n)^{-1/n}\tan^{-1}\left(\frac{\sqrt[3]{a}-}{}\right.}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}}$$

```
[Out] 1/3*x*ln(a^(1/3)+b^(1/3)*(c*x^n)^(1/n))/a^(2/3)/b^(1/3)/((c*x^n)^(1/n))-1/6
*x*ln(a^(2/3)-a^(1/3)*b^(1/3)*(c*x^n)^(1/n)+b^(2/3)*(c*x^n)^(2/n))/a^(2/3)/
b^(1/3)/((c*x^n)^(1/n))-1/3*x*arctan(1/3*(a^(1/3)-2*b^(1/3)*(c*x^n)^(1/n))/
a^(1/3)*3^(1/2))/a^(2/3)/b^(1/3)/((c*x^n)^(1/n))*3^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {254, 200, 31, 634, 617, 204, 628}

$$-\frac{x\,(cx^n)^{-1/n}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(cx^n)^{\frac{1}{n}}+b^{2/3}\,(cx^n)^{2/n}\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{x\,(cx^n)^{-1/n}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(cx^n)^{\frac{1}{n}}\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{x\,(cx^n)^{-1/n}\tan^{-1}\left(\frac{\sqrt[3]{a}-}{}\right.}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c*x^n)^(3/n))^(-1),x]
```

```
[Out] -((x*ArcTan[(a^(1/3) - 2*b^(1/3)*(c*x^n)^n^(-1))/(Sqrt[3]*a^(1/3))])/(Sqrt[
3]*a^(2/3)*b^(1/3)*(c*x^n)^n^(-1))) + (x*Log[a^(1/3) + b^(1/3)*(c*x^n)^n^(-
1)])/(3*a^(2/3)*b^(1/3)*(c*x^n)^n^(-1)) - (x*Log[a^(2/3) - a^(1/3)*b^(1/3)*
(c*x^n)^n^(-1) + b^(2/3)*(c*x^n)^(2/n)])/(6*a^(2/3)*b^(1/3)*(c*x^n)^n^(-1))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 254

```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_.), x_Symbol] :> Dist[x/(c*x^
q)^(1/q), Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{a+b\left(cx^n\right)^{3/n}}\,dx = \left(x\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, \left(cx^n\right)^{\frac{1}{n}}\right)$$

$$= \frac{\left(x\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, \left(cx^n\right)^{\frac{1}{n}}\right)}{3a^{2/3}} + \frac{\left(x\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\right.}{3a^{2/3}}$$

$$= \frac{x\left(cx^n\right)^{-1/n}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\left(cx^n\right)^{\frac{1}{n}}\right)}{3a^{2/3}\sqrt[3]{b}} + \frac{\left(x\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, \left(cx^n\right.\right.}{2\sqrt[3]{a}}$$

$$= \frac{x\left(cx^n\right)^{-1/n}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\left(cx^n\right)^{\frac{1}{n}}\right)}{3a^{2/3}\sqrt[3]{b}} - \frac{x\left(cx^n\right)^{-1/n}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\left(cx^n\right)^{\frac{1}{n}}+b^{2/3}\left(cx^n\right)^{2/n}\right.}{6a^{2/3}\sqrt[3]{b}}$$

$$= -\frac{x\left(cx^n\right)^{-1/n}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\left(cx^n\right)^{\frac{1}{n}}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}} + \frac{x\left(cx^n\right)^{-1/n}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\left(cx^n\right)^{\frac{1}{n}}\right)}{3a^{2/3}\sqrt[3]{b}} - \frac{x\left(cx^n\right)^{-1/n}\log\left(\right.}{}$$

Mathematica [A] time = 0.09, size = 133, normalized size = 0.73

$$-\frac{x\left(cx^n\right)^{-1/n}\left(\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\left(cx^n\right)^{\frac{1}{n}}+b^{2/3}\left(cx^n\right)^{2/n}\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\left(cx^n\right)^{\frac{1}{n}}\right)+2\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\left(cx^n\right)^{\frac{1}{n}}}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)}{6a^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c*x^n)^(3/n))^(-1),x]
```

```
[Out] -1/6*(x*(2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*(c*x^n)^n^(-1))/a^(1/3))/Sqrt[3]]
 - 2*Log[a^(1/3) + b^(1/3)*(c*x^n)^n^(-1)] + Log[a^(2/3) - a^(1/3)*b^(1/3)*
(c*x^n)^n^(-1) + b^(2/3)*(c*x^n)^(2/n)]))/(a^(2/3)*b^(1/3)*(c*x^n)^n^(-1))
```

fricas [A] time = 0.79, size = 549, normalized size = 3.00

$$\left[\frac{3\sqrt{\frac{1}{3}}\,abc^{\frac{3}{n}}\sqrt{-\frac{\left(a^2bc^{\frac{3}{n}}\right)^{\frac{1}{3}}}{bc^{\frac{3}{n}}}}\log\left(\frac{2\,abc^{\frac{3}{n}}x^3-3\left(a^2bc^{\frac{3}{n}}\right)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2\,abc^{\frac{3}{n}}x^2+\left(a^2bc^{\frac{3}{n}}\right)^{\frac{2}{3}}x-\left(a^2bc^{\frac{3}{n}}\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(a^2bc^{\frac{3}{n}}\right)^{\frac{1}{3}}}{bc^{\frac{3}{n}}}}}{bc^{\frac{3}{n}}x^3+a}\right)-\left(a^2bc^{\frac{3}{n}}\right)^{\frac{2}{3}}\log\left(\right.\right.}{6\,a^2bc^{\frac{3}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(c*x^n)^(3/n)),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*c^(3/n)*sqrt(-(a^2*b*c^(3/n))^(1/3)/(b*c^(3/n)))*log(
(2*a*b*c^(3/n)*x^3 - 3*(a^2*b*c^(3/n))^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b
*c^(3/n)*x^2 + (a^2*b*c^(3/n))^(2/3)*x - (a^2*b*c^(3/n))^(1/3)*a)*sqrt(-(a^
2*b*c^(3/n))^(1/3)/(b*c^(3/n))))/(b*c^(3/n)*x^3 + a)) - (a^2*b*c^(3/n))^(2/
3)*log(a*b*c^(3/n)*x^2 - (a^2*b*c^(3/n))^(2/3)*x + (a^2*b*c^(3/n))^(1/3)*a)
 + 2*(a^2*b*c^(3/n))^(2/3)*log(a*b*c^(3/n)*x + (a^2*b*c^(3/n))^(2/3)))/(a^2
*b*c^(3/n)), 1/6*(6*sqrt(1/3)*a*b*c^(3/n)*sqrt((a^2*b*c^(3/n))^(1/3)/(b*c^(
3/n)))*arctan(sqrt(1/3)*(2*(a^2*b*c^(3/n))^(2/3)*x - (a^2*b*c^(3/n))^(1/3)*
a)*sqrt((a^2*b*c^(3/n))^(1/3)/(b*c^(3/n)))/a^2) - (a^2*b*c^(3/n))^(2/3)*log
(a*b*c^(3/n)*x^2 - (a^2*b*c^(3/n))^(2/3)*x + (a^2*b*c^(3/n))^(1/3)*a) + 2*(
a^2*b*c^(3/n))^(2/3)*log(a*b*c^(3/n)*x + (a^2*b*c^(3/n))^(2/3)))/(a^2*b*c^(
3/n))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^n)^{\frac{3}{n}}\,b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(c*x^n)^(3/n)),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^n)^(3/n)*b + a), x)
```

maple [C] time = 0.44, size = 821, normalized size = 4.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*(c*x^n)^(3/n)+a),x)
```

```
[Out] 1/3/b/(c^(3/n))/((x^n)^(3/n))*x^3*exp(-3/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(
-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))/(a/b/(c^(3/n))/((x^n)^(3/n))*x
^3*exp(-3/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*c
sgn(I*c*x^n)))^(2/3)*ln(x+(a/b/(c^(3/n))/((x^n)^(3/n))*x^3*exp(-3/2*I*Pi*(c
sgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n)))^(1/3
))-1/6/b/(c^(3/n))/((x^n)^(3/n))*x^3*exp(-3/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n)
)*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))/(a/b/(c^(3/n))/((x^n)^(3/n)
```

```
)*x^3*exp(-3/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/
n*csgn(I*c*x^n)))^(2/3)*ln(x^2-(a/b/(c^(3/n))/((x^n)^(3/n))*x^3*exp(-3/2*I*
Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n)))
^(1/3)*x+(a/b/(c^(3/n))/((x^n)^(3/n))*x^3*exp(-3/2*I*Pi*(csgn(I*c)-csgn(I*c
*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n)))^(2/3))+1/3/b/(c^(3/n)
)/((x^n)^(3/n))*x^3*exp(-3/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+c
sgn(I*c*x^n))/n*csgn(I*c*x^n))/(a/b/(c^(3/n))/((x^n)^(3/n))*x^3*exp(-3/2*I*
Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n)))
^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b/(c^(3/n))/((x^n)^(3/n))*x^3*exp(-
3/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*
x^n)))^(1/3)*x-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^n)^{\frac{3}{n}}\, b + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(c*x^n)^(3/n)),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^n)^(3/n)*b + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{a + b\,(c\,x^n)^{3/n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(c*x^n)^(3/n)),x)
```

```
[Out] int(1/(a + b*(c*x^n)^(3/n)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b\,(cx^n)^{\frac{3}{n}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(c*x**n)**(3/n)),x)
```

```
[Out] Integral(1/(a + b*(c*x**n)**(3/n)), x)
```

3.3044 $\int \frac{1}{\left(a+b(cx^n)^{3/n}\right)^2}\,dx$

Optimal. Leaf size=210

$$-\frac{x\,(cx^n)^{-1/n}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(cx^n)^{\frac{1}{n}}+b^{2/3}\,(cx^n)^{2/n}\right)}{9a^{5/3}\sqrt[3]{b}}+\frac{2x\,(cx^n)^{-1/n}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(cx^n)^{\frac{1}{n}}\right)}{9a^{5/3}\sqrt[3]{b}}-\frac{2x\,(cx^n)^{-1/n}\tan^{-1}\left(\frac{\sqrt[3]{}}{}\right.}{3\sqrt{3}\,a^{5/3}\sqrt[3]{}}$$

```
[Out] 1/3*x/a/(a+b*(c*x^n)^(3/n))+2/9*x*ln(a^(1/3)+b^(1/3)*(c*x^n)^(1/n))/a^(5/3)
/b^(1/3)/((c*x^n)^(1/n))-1/9*x*ln(a^(2/3)-a^(1/3)*b^(1/3)*(c*x^n)^(1/n)+b^(
2/3)*(c*x^n)^(2/n))/a^(5/3)/b^(1/3)/((c*x^n)^(1/n))-2/9*x*arctan(1/3*(a^(1/
3)-2*b^(1/3)*(c*x^n)^(1/n))/a^(1/3)*3^(1/2))/a^(5/3)/b^(1/3)/((c*x^n)^(1/n)
)*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {254, 199, 200, 31, 634, 617, 204, 628}

$$-\frac{x\,(cx^n)^{-1/n}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(cx^n)^{\frac{1}{n}}+b^{2/3}\,(cx^n)^{2/n}\right)}{9a^{5/3}\sqrt[3]{b}}+\frac{2x\,(cx^n)^{-1/n}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(cx^n)^{\frac{1}{n}}\right)}{9a^{5/3}\sqrt[3]{b}}-\frac{2x\,(cx^n)^{-1/n}\tan^{-1}\left(\frac{\sqrt[3]{}}{}\right.}{3\sqrt{3}\,a^{5/3}\sqrt[3]{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c*x^n)^(3/n))^(-2),x]
```

```
[Out] x/(3*a*(a + b*(c*x^n)^(3/n))) - (2*x*ArcTan[(a^(1/3) - 2*b^(1/3)*(c*x^n)^n^
(-1))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(1/3)*(c*x^n)^n^(-1)) + (2*x
*Log[a^(1/3) + b^(1/3)*(c*x^n)^n^(-1)])/(9*a^(5/3)*b^(1/3)*(c*x^n)^n^(-1))
- (x*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c*x^n)^n^(-1) + b^(2/3)*(c*x^n)^(2/n)])
/(9*a^(5/3)*b^(1/3)*(c*x^n)^n^(-1))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 254

```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_.), x_Symbol] :> Dist[x/(c*x^
q)^(1/q), Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{3/n}\right)^2}\,dx = \left(x\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{1}{\left(a+bx^3\right)^2}\,dx, x, \left(cx^n\right)^{\frac{1}{n}}\right)$$

$$= \frac{x}{3a\left(a+b\left(cx^n\right)^{3/n}\right)} + \frac{\left(2x\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, \left(cx^n\right)^{\frac{1}{n}}\right)}{3a}$$

$$= \frac{x}{3a\left(a+b\left(cx^n\right)^{3/n}\right)} + \frac{\left(2x\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, \left(cx^n\right)^{\frac{1}{n}}\right)}{9a^{5/3}} + \frac{\left(2x\left(cx^n\right)^{-1/n}\right)}{}$$

$$= \frac{x}{3a\left(a+b\left(cx^n\right)^{3/n}\right)} + \frac{2x\left(cx^n\right)^{-1/n}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\left(cx^n\right)^{\frac{1}{n}}\right)}{9a^{5/3}\sqrt[3]{b}} + \frac{\left(x\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{}{a^{2/}}\right.}{3}$$

$$= \frac{x}{3a\left(a+b\left(cx^n\right)^{3/n}\right)} + \frac{2x\left(cx^n\right)^{-1/n}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\left(cx^n\right)^{\frac{1}{n}}\right)}{9a^{5/3}\sqrt[3]{b}} - \frac{x\left(cx^n\right)^{-1/n}\log\left(a^{2/3}-\sqrt[3]{a}\right.}{9a^{5/}}$$

$$= \frac{x}{3a\left(a+b\left(cx^n\right)^{3/n}\right)} - \frac{2x\left(cx^n\right)^{-1/n}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\left(cx^n\right)^{\frac{1}{n}}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{5/3}\sqrt[3]{b}} + \frac{2x\left(cx^n\right)^{-1/n}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c\right.}{9a^{5/3}\sqrt[3]{b}}$$

Mathematica [A] time = 0.34, size = 158, normalized size = 0.75

$$\frac{x\left(\frac{1}{a+b(cx^n)^{3/n}}-\frac{(cx^n)^{-1/n}\left(\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(cx^n)^{\frac{1}{n}}+b^{2/3}(cx^n)^{2/n}\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(cx^n)^{\frac{1}{n}}\right)+2\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,(cx^n)^{\frac{1}{n}}}{\sqrt[3]{a}}}{\sqrt{3}}\right)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c*x^n)^(3/n))^(-2),x]
```

```
[Out] (x*((a + b*(c*x^n)^(3/n))^(-1) - (2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*(c*x^n)^
n^(-1))/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3) + b^(1/3)*(c*x^n)^n^(-1)] + Log[a
^(2/3) - a^(1/3)*b^(1/3)*(c*x^n)^n^(-1) + b^(2/3)*(c*x^n)^(2/n)])/(3*a^(2/3
)*b^(1/3)*(c*x^n)^n^(-1))))/(3*a)
```

fricas [A] time = 1.04, size = 705, normalized size = 3.36

$$\left[\frac{3\,a^2bc^{\frac{3}{n}}x+3\sqrt{\frac{1}{3}}\left(ab^2c^{\frac{6}{n}}x^3+a^2bc^{\frac{3}{n}}\right)\sqrt{-\frac{\left(a^2bc^{\frac{3}{n}}\right)^{\frac{1}{3}}}{bc^{\frac{3}{n}}}}\log\left(\frac{2\,abc^{\frac{3}{n}}x^3-3\left(a^2bc^{\frac{3}{n}}\right)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2\,abc^{\frac{3}{n}}x^2+\left(a^2bc^{\frac{3}{n}}\right)^{\frac{2}{3}}x-\left(a^2bc^{\frac{3}{n}}\right)^{\frac{1}{3}}a\right)\sqrt{-}\right.}{bc^{\frac{3}{n}}x^3+a}\right.}{9\left(a^{3}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(c*x^n)^(3/n))^2,x, algorithm="fricas")
```

```
[Out] [1/9*(3*a^2*b*c^(3/n)*x + 3*sqrt(1/3)*(a*b^2*c^(6/n)*x^3 + a^2*b*c^(3/n))*s
qrt(-(a^2*b*c^(3/n))^(1/3)/(b*c^(3/n)))*log((2*a*b*c^(3/n)*x^3 - 3*(a^2*b*c
^(3/n))^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*c^(3/n)*x^2 + (a^2*b*c^(3/n))^
(2/3)*x - (a^2*b*c^(3/n))^(1/3)*a)*sqrt(-(a^2*b*c^(3/n))^(1/3)/(b*c^(3/n)))
)/(b*c^(3/n)*x^3 + a)) - (b*c^(3/n)*x^3 + a)*(a^2*b*c^(3/n))^(2/3)*log(a*b*
c^(3/n)*x^2 - (a^2*b*c^(3/n))^(2/3)*x + (a^2*b*c^(3/n))^(1/3)*a) + 2*(b*c^(
3/n)*x^3 + a)*(a^2*b*c^(3/n))^(2/3)*log(a*b*c^(3/n)*x + (a^2*b*c^(3/n))^(2/
3)))/(a^3*b^2*c^(6/n)*x^3 + a^4*b*c^(3/n)), 1/9*(3*a^2*b*c^(3/n)*x + 6*sqrt
(1/3)*(a*b^2*c^(6/n)*x^3 + a^2*b*c^(3/n))*sqrt((a^2*b*c^(3/n))^(1/3)/(b*c^(
3/n)))*arctan(sqrt(1/3)*(2*(a^2*b*c^(3/n))^(2/3)*x - (a^2*b*c^(3/n))^(1/3)*
a)*sqrt((a^2*b*c^(3/n))^(1/3)/(b*c^(3/n)))/a^2) - (b*c^(3/n)*x^3 + a)*(a^2*
b*c^(3/n))^(2/3)*log(a*b*c^(3/n)*x^2 - (a^2*b*c^(3/n))^(2/3)*x + (a^2*b*c^(
3/n))^(1/3)*a) + 2*(b*c^(3/n)*x^3 + a)*(a^2*b*c^(3/n))^(2/3)*log(a*b*c^(3/n
)*x + (a^2*b*c^(3/n))^(2/3)))/(a^3*b^2*c^(6/n)*x^3 + a^4*b*c^(3/n))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((cx^n)^{\frac{3}{n}}\,b+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(c*x^n)^(3/n))^2,x, algorithm="giac")
```

```
[Out] integrate(((c*x^n)^(3/n)*b + a)^(-2), x)
```

maple [C] time = 0.38, size = 908, normalized size = 4.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*(c*x^n)^(3/n)+a)^2,x)
```

```
[Out] 1/3/a*x/(b*(x^n)^(3/n)*c^(3/n)*exp(3/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csg
n(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))+a)+2/9/a/b/(c^(3/n))/((x^n)^(3/n))
*x^3*exp(-3/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n
*csgn(I*c*x^n))/(a/b/(c^(3/n))/((x^n)^(3/n))*x^3*exp(-3/2*I*Pi*(csgn(I*c)-c
sgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n)))^(2/3)*ln(x+(a/
b/(c^(3/n))/((x^n)^(3/n))*x^3*exp(-3/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csg
n(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n)))^(1/3))-1/9/a/b/(c^(3/n))/((x^n)^(
3/n))*x^3*exp(-3/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^
n))/n*csgn(I*c*x^n))/(a/b/(c^(3/n))/((x^n)^(3/n))*x^3*exp(-3/2*I*Pi*(csgn(I
*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n)))^(2/3)*ln(
x^2-(a/b/(c^(3/n))/((x^n)^(3/n))*x^3*exp(-3/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n)
)*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n)))^(1/3)*x+(a/b/(c^(3/n))/((x
^n)^(3/n))*x^3*exp(-3/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I
*c*x^n))/n*csgn(I*c*x^n)))^(2/3))+2/9/a/b/(c^(3/n))/((x^n)^(3/n))*x^3*exp(-
3/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*
x^n))/(a/b/(c^(3/n))/((x^n)^(3/n))*x^3*exp(-3/2*I*Pi*(csgn(I*c)-csgn(I*c*x^
n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n)))^(2/3)*3^(1/2)*arctan(1/3
*3^(1/2)*(2/(a/b/(c^(3/n))/((x^n)^(3/n))*x^3*exp(-3/2*I*Pi*(csgn(I*c)-csgn(
I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n)))^(1/3)*x-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x}{3\left(abc^{\frac{3}{n}}(x^n)^{\frac{3}{n}}+a^2\right)}+2\int\frac{1}{3\left(abc^{\frac{3}{n}}(x^n)^{\frac{3}{n}}+a^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(c*x^n)^(3/n))^2,x, algorithm="maxima")
```

```
[Out] 1/3*x/(a*b*c^(3/n)*(x^n)^(3/n) + a^2) + 2*integrate(1/3/(a*b*c^(3/n)*(x^n)^
(3/n) + a^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{1}{\left(a+b\,(c\,x^n)^{3/n}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(c*x^n)^(3/n))^2,x)
```

```
[Out] int(1/(a + b*(c*x^n)^(3/n))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(a+b\,(cx^n)^{\frac{3}{n}}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(c*x**n)**(3/n))**2,x)
[Out] Integral((a + b*(c*x**n)**(3/n))**(-2), x)
```

3.3045 $\int \frac{1}{\left(a+b(cx^n)^{3/n}\right)^3}\,dx$

Optimal. Leaf size=235

$$-\frac{5x\,(cx^n)^{-1/n}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(cx^n)^{\frac{1}{n}}+b^{2/3}\,(cx^n)^{2/n}\right)}{54a^{8/3}\sqrt[3]{b}}+\frac{5x\,(cx^n)^{-1/n}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(cx^n)^{\frac{1}{n}}\right)}{27a^{8/3}\sqrt[3]{b}}-\frac{5x\,(cx^n)^{-1/n}\tan^{-}}{9\sqrt{3}\,a}$$

```
[Out] 1/6*x/a/(a+b*(c*x^n)^(3/n))^2+5/18*x/a^2/(a+b*(c*x^n)^(3/n))+5/27*x*ln(a^(1
/3)+b^(1/3)*(c*x^n)^(1/n))/a^(8/3)/b^(1/3)/((c*x^n)^(1/n))-5/54*x*ln(a^(2/3
)-a^(1/3)*b^(1/3)*(c*x^n)^(1/n)+b^(2/3)*(c*x^n)^(2/n))/a^(8/3)/b^(1/3)/((c*
x^n)^(1/n))-5/27*x*arctan(1/3*(a^(1/3)-2*b^(1/3)*(c*x^n)^(1/n))/a^(1/3)*3^(
1/2))/a^(8/3)/b^(1/3)/((c*x^n)^(1/n))*3^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {254, 199, 200, 31, 634, 617, 204, 628}

$$-\frac{5x\,(cx^n)^{-1/n}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(cx^n)^{\frac{1}{n}}+b^{2/3}\,(cx^n)^{2/n}\right)}{54a^{8/3}\sqrt[3]{b}}+\frac{5x}{18a^2\left(a+b\,(cx^n)^{3/n}\right)}+\frac{5x\,(cx^n)^{-1/n}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(cx^n\right.}{27a^{8/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c*x^n)^(3/n))^(-3),x]
```

```
[Out] x/(6*a*(a + b*(c*x^n)^(3/n))^2) + (5*x)/(18*a^2*(a + b*(c*x^n)^(3/n))) - (5
*x*ArcTan[(a^(1/3) - 2*b^(1/3)*(c*x^n)^n^(-1))/(Sqrt[3]*a^(1/3))])/(9*Sqrt[
3]*a^(8/3)*b^(1/3)*(c*x^n)^n^(-1)) + (5*x*Log[a^(1/3) + b^(1/3)*(c*x^n)^n^(
-1)])/(27*a^(8/3)*b^(1/3)*(c*x^n)^n^(-1)) - (5*x*Log[a^(2/3) - a^(1/3)*b^(1
/3)*(c*x^n)^n^(-1) + b^(2/3)*(c*x^n)^(2/n)])/(54*a^(8/3)*b^(1/3)*(c*x^n)^n^
(-1))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 254

```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_.), x_Symbol] :> Dist[x/(c*x^
q)^(1/q), Subst[Int[(a + b*x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{
a, b, c, n, p, q}, x] && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(a+b\left(cx^n\right)^{3/n}\right)^3}\,dx = \left(x\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{1}{\left(a+bx^3\right)^3}\,dx, x, \left(cx^n\right)^{\frac{1}{n}}\right)$$

$$= \frac{x}{6a\left(a+b\left(cx^n\right)^{3/n}\right)^2} + \frac{\left(5x\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{1}{\left(a+bx^3\right)^2}\,dx, x, \left(cx^n\right)^{\frac{1}{n}}\right)}{6a}$$

$$= \frac{x}{6a\left(a+b\left(cx^n\right)^{3/n}\right)^2} + \frac{5x}{18a^2\left(a+b\left(cx^n\right)^{3/n}\right)} + \frac{\left(5x\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, (cx^{\text{n}}\right.}{9a^2}$$

$$= \frac{x}{6a\left(a+b\left(cx^n\right)^{3/n}\right)^2} + \frac{5x}{18a^2\left(a+b\left(cx^n\right)^{3/n}\right)} + \frac{\left(5x\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, \right.}{27a^{8/3}}$$

$$= \frac{x}{6a\left(a+b\left(cx^n\right)^{3/n}\right)^2} + \frac{5x}{18a^2\left(a+b\left(cx^n\right)^{3/n}\right)} + \frac{5x\left(cx^n\right)^{-1/n}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\left(cx^n\right)^{\frac{1}{n}}\right)}{27a^{8/3}\sqrt[3]{b}} + \frac{}{}$$

$$= \frac{x}{6a\left(a+b\left(cx^n\right)^{3/n}\right)^2} + \frac{5x}{18a^2\left(a+b\left(cx^n\right)^{3/n}\right)} + \frac{5x\left(cx^n\right)^{-1/n}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\left(cx^n\right)^{\frac{1}{n}}\right)}{27a^{8/3}\sqrt[3]{b}} - \frac{}{}$$

$$= \frac{x}{6a\left(a+b\left(cx^n\right)^{3/n}\right)^2} + \frac{5x}{18a^2\left(a+b\left(cx^n\right)^{3/n}\right)} - \frac{5x\left(cx^n\right)^{-1/n}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\left(cx^n\right)^{\frac{1}{n}}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{9\sqrt{3}\,a^{8/3}\sqrt[3]{b}} + \frac{5x\left(c\right.}{}$$

Mathematica [A] time = 0.23, size = 215, normalized size = 0.91

$$x\left(-\frac{5\left(cx^n\right)^{-1/n}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\left(cx^n\right)^{\frac{1}{n}}+b^{2/3}\left(cx^n\right)^{2/n}\right)}{\sqrt[3]{b}} + \frac{15a^{2/3}}{a+b\left(cx^n\right)^{3/n}} + \frac{9a^{5/3}}{\left(a+b\left(cx^n\right)^{3/n}\right)^2} + \frac{10\left(cx^n\right)^{-1/n}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\left(cx^n\right)^{\frac{1}{n}}\right)}{\sqrt[3]{b}} - \frac{10\sqrt{3}\left(cx^n\right)^{-1/}}{}\right)$$
$$\Big/\ 54a^{8/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c*x^n)^(3/n))^(-3),x]
```

```
[Out] (x*((9*a^(5/3))/(a + b*(c*x^n)^(3/n))^2 + (15*a^(2/3))/(a + b*(c*x^n)^(3/n)
) - (10*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*(c*x^n)^n^(-1))/a^(1/3))/Sqrt[3]])/(
b^(1/3)*(c*x^n)^n^(-1)) + (10*Log[a^(1/3) + b^(1/3)*(c*x^n)^n^(-1)])/(b^(1/
3)*(c*x^n)^n^(-1)) - (5*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c*x^n)^n^(-1) + b^(2
/3)*(c*x^n)^(2/n)])/(b^(1/3)*(c*x^n)^n^(-1))))/(54*a^(8/3))
```

fricas [B] time = 0.81, size = 885, normalized size = 3.77

$$\left[\frac{15\,a^2b^2c^{\frac{6}{n}}x^4+24\,a^3bc^{\frac{3}{n}}x+15\sqrt{\frac{1}{3}}\left(ab^3c^{\frac{9}{n}}x^6+2\,a^2b^2c^{\frac{6}{n}}x^3+a^3bc^{\frac{3}{n}}\right)\sqrt{-\frac{\left(a^2bc^{\frac{3}{n}}\right)^{\frac{1}{3}}}{bc^{\frac{3}{n}}}}\log\left(\frac{2\,abc^{\frac{3}{n}}x^3-3\left(a^2bc^{\frac{3}{n}}\right)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(c*x^n)^(3/n))^3,x, algorithm="fricas")
```

```
[Out] [1/54*(15*a^2*b^2*c^(6/n)*x^4 + 24*a^3*b*c^(3/n)*x + 15*sqrt(1/3)*(a*b^3*c^
(9/n)*x^6 + 2*a^2*b^2*c^(6/n)*x^3 + a^3*b*c^(3/n))*sqrt(-(a^2*b*c^(3/n))^(1
/3)/(b*c^(3/n)))*log((2*a*b*c^(3/n)*x^3 - 3*(a^2*b*c^(3/n))^(1/3)*a*x - a^2
 + 3*sqrt(1/3)*(2*a*b*c^(3/n)*x^2 + (a^2*b*c^(3/n))^(2/3)*x - (a^2*b*c^(3/n
))^(1/3)*a)*sqrt(-(a^2*b*c^(3/n))^(1/3)/(b*c^(3/n))))/(b*c^(3/n)*x^3 + a))
- 5*(b^2*c^(6/n)*x^6 + 2*a*b*c^(3/n)*x^3 + a^2)*(a^2*b*c^(3/n))^(2/3)*log(a
*b*c^(3/n)*x^2 - (a^2*b*c^(3/n))^(2/3)*x + (a^2*b*c^(3/n))^(1/3)*a) + 10*(b
^2*c^(6/n)*x^6 + 2*a*b*c^(3/n)*x^3 + a^2)*(a^2*b*c^(3/n))^(2/3)*log(a*b*c^(
3/n)*x + (a^2*b*c^(3/n))^(2/3)))/(a^4*b^3*c^(9/n)*x^6 + 2*a^5*b^2*c^(6/n)*x
^3 + a^6*b*c^(3/n)), 1/54*(15*a^2*b^2*c^(6/n)*x^4 + 24*a^3*b*c^(3/n)*x + 30
*sqrt(1/3)*(a*b^3*c^(9/n)*x^6 + 2*a^2*b^2*c^(6/n)*x^3 + a^3*b*c^(3/n))*sqrt
((a^2*b*c^(3/n))^(1/3)/(b*c^(3/n)))*arctan(sqrt(1/3)*(2*(a^2*b*c^(3/n))^(2/
3)*x - (a^2*b*c^(3/n))^(1/3)*a)*sqrt((a^2*b*c^(3/n))^(1/3)/(b*c^(3/n)))/a^2
) - 5*(b^2*c^(6/n)*x^6 + 2*a*b*c^(3/n)*x^3 + a^2)*(a^2*b*c^(3/n))^(2/3)*log
(a*b*c^(3/n)*x^2 - (a^2*b*c^(3/n))^(2/3)*x + (a^2*b*c^(3/n))^(1/3)*a) + 10*
(b^2*c^(6/n)*x^6 + 2*a*b*c^(3/n)*x^3 + a^2)*(a^2*b*c^(3/n))^(2/3)*log(a*b*c
^(3/n)*x + (a^2*b*c^(3/n))^(2/3)))/(a^4*b^3*c^(9/n)*x^6 + 2*a^5*b^2*c^(6/n)
*x^3 + a^6*b*c^(3/n))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((cx^n)^{\frac{3}{n}}\,b+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(c*x^n)^(3/n))^3,x, algorithm="giac")
```

```
[Out] integrate(((c*x^n)^(3/n)*b + a)^(-3), x)
```

maple [C] time = 0.40, size = 981, normalized size = 4.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*(c*x^n)^(3/n)+a)^3,x)
```

```
[Out] 1/18*x*(5*b*c^(3/n)*(x^n)^(3/n)*exp(3/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-cs
gn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))+8*a)/a^2/(b*c^(3/n)*(x^n)^(3/n)*e
xp(3/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I
*c*x^n))+a)^2+5/27/a^2/b/(c^(3/n))/((x^n)^(3/n))*x^3*exp(-3/2*I*Pi*(csgn(I*
c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))/(a/b/(c^(3/
n))/((x^n)^(3/n))*x^3*exp(-3/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)
+csgn(I*c*x^n))/n*csgn(I*c*x^n)))^(2/3)*ln(x+(a/b/(c^(3/n))/((x^n)^(3/n))*x
^3*exp(-3/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*c
sgn(I*c*x^n)))^(1/3))-5/54/a^2/b/(c^(3/n))/((x^n)^(3/n))*x^3*exp(-3/2*I*Pi*
(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))/(a/
b/(c^(3/n))/((x^n)^(3/n))*x^3*exp(-3/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csg
n(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n)))^(2/3)*ln(x^2-(a/b/(c^(3/n))/((x^n
)^(3/n))*x^3*exp(-3/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c
*x^n))/n*csgn(I*c*x^n)))^(1/3)*x+(a/b/(c^(3/n))/((x^n)^(3/n))*x^3*exp(-3/2*
I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n)
))^(2/3))+5/27/a^2/b/(c^(3/n))/((x^n)^(3/n))*x^3*exp(-3/2*I*Pi*(csgn(I*c)-c
sgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))/(a/b/(c^(3/n))/
((x^n)^(3/n))*x^3*exp(-3/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csg
n(I*c*x^n))/n*csgn(I*c*x^n)))^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b/(c^(
3/n))/((x^n)^(3/n))*x^3*exp(-3/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^
n)+csgn(I*c*x^n))/n*csgn(I*c*x^n)))^(1/3)*x-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{5\,bc^{\frac{3}{n}}x(x^n)^{\frac{3}{n}}+8\,ax}{18\left(a^2b^2c^{\frac{6}{n}}(x^n)^{\frac{6}{n}}+2\,a^3bc^{\frac{3}{n}}(x^n)^{\frac{3}{n}}+a^4\right)}+5\int\frac{1}{9\left(a^2bc^{\frac{3}{n}}(x^n)^{\frac{3}{n}}+a^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(c*x^n)^(3/n))^3,x, algorithm="maxima")
```

```
[Out] 1/18*(5*b*c^(3/n)*x*(x^n)^(3/n) + 8*a*x)/(a^2*b^2*c^(6/n)*(x^n)^(6/n) + 2*a
^3*b*c^(3/n)*(x^n)^(3/n) + a^4) + 5*integrate(1/9/(a^2*b*c^(3/n)*(x^n)^(3/n
) + a^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{1}{\left(a+b\,(c\,x^n)^{3/n}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(c*x^n)^(3/n))^3,x)
```

```
[Out] int(1/(a + b*(c*x^n)^(3/n))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(a+b\,(cx^n)^{\frac{3}{n}}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(c*x**n)**(3/n))**3,x)
```

```
[Out] Integral((a + b*(c*x**n)**(3/n))**(-3), x)
```

3.3046 $\int (dx)^m \left(a + b\left(cx^q\right)^n\right)^p \, dx$

Optimal. Leaf size=86

$$\frac{(dx)^{m+1}\left(a + b\left(cx^q\right)^n\right)^p \left(\frac{b\left(cx^q\right)^n}{a} + 1\right)^{-p} {}_2F_1\left(-p, \frac{m+1}{nq}; \frac{m+1}{nq} + 1; -\frac{b\left(cx^q\right)^n}{a}\right)}{d(m+1)}$$

```
[Out] (d*x)^(1+m)*(a+b*(c*x^q)^n)^p*hypergeom([-p, (1+m)/n/q],[1+(1+m)/n/q],-b*(c
*x^q)^n/a)/d/(1+m)/((1+b*(c*x^q)^n/a)^p)
```

Rubi [A] time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {370, 365, 364}

$$\frac{(dx)^{m+1}\left(a + b\left(cx^q\right)^n\right)^p \left(\frac{b\left(cx^q\right)^n}{a} + 1\right)^{-p} {}_2F_1\left(-p, \frac{m+1}{nq}; \frac{m+1}{nq} + 1; -\frac{b\left(cx^q\right)^n}{a}\right)}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*(c*x^q)^n)^p,x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*(c*x^q)^n)^p*Hypergeometric2F1[-p, (1 + m)/(n*q), 1 +
 (1 + m)/(n*q), -((b*(c*x^q)^n)/a)])/(d*(1 + m)*(1 + (b*(c*x^q)^n)/a)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x], x^(n*q), (c*x^q)^n/c^n]
/; FreeQ[{a, b, c, d, m, n, p, q}, x] &&  !RationalQ[n]
```

Rubi steps

$$\begin{aligned}
\int (dx)^m \left(a + b\left(cx^q\right)^n\right)^p \, dx &= \text{Subst}\left(\int (dx)^m \left(a + bc^n x^{nq}\right)^p \, dx, x^{nq}, c^{-n}\left(cx^q\right)^n\right) \\
&= \text{Subst}\left(\left(\left(a + bc^n x^{nq}\right)^p \left(1 + \frac{bc^n x^{nq}}{a}\right)^{-p}\right) \int (dx)^m \left(1 + \frac{bc^n x^{nq}}{a}\right)^p \, dx, x^{nq}, c^{-n}\left(cx^q\right)^n\right) \\
&= \frac{(dx)^{1+m}\left(a + b\left(cx^q\right)^n\right)^p \left(1 + \frac{b\left(cx^q\right)^n}{a}\right)^{-p} {}_2F_1\left(-p, \frac{1+m}{nq}; 1 + \frac{1+m}{nq}; -\frac{b\left(cx^q\right)^n}{a}\right)}{d(1+m)}
\end{aligned}$$

Mathematica [A] time = 0.23, size = 82, normalized size = 0.95

$$\frac{x(dx)^m \left(a + b\left(cx^q\right)^n\right)^p \left(\frac{b(cx^q)^n}{a} + 1\right)^{-p} {}_2F_1\left(-p, \frac{m+1}{nq}; \frac{m+1}{nq} + 1; -\frac{b(cx^q)^n}{a}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*(c*x^q)^n)^p,x]
```

```
[Out] (x*(d*x)^m*(a + b*(c*x^q)^n)^p*Hypergeometric2F1[-p, (1 + m)/(n*q), 1 + (1
+ m)/(n*q), -((b*(c*x^q)^n)/a)])/((1 + m)*(1 + (b*(c*x^q)^n)/a)^p)
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(cx^q\right)^n b + a\right)^p (dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*(c*x^q)^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*x^q)^n*b + a)^p*(d*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(cx^q\right)^n b + a\right)^p (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*(c*x^q)^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*x^q)^n*b + a)^p*(d*x)^m, x)
```

maple [F] time = 2.61, size = 0, normalized size = 0.00

$$\int (dx)^m \left(b\left(c\,x^q\right)^n + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*(c*x^q)^n)^p,x)
```

```
[Out] int((d*x)^m*(a+b*(c*x^q)^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(cx^q\right)^n b + a\right)^p (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*(c*x^q)^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*x^q)^n*b + a)^p*(d*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m \left(a + b\left(c\,x^q\right)^n\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*(c*x^q)^n)^p,x)
```

```
[Out] int((d*x)^m*(a + b*(c*x^q)^n)^p, x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a + b\left(cx^q\right)^n\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*(c*x**q)**n)**p,x)
[Out] Integral((d*x)**m*(a + b*(c*x**q)**n)**p, x)
```

3.3047 $\int x^2 \left(a + b\left(cx^q\right)^n\right)^p \, dx$

Optimal. Leaf size=73

$$\frac{1}{3}x^3\left(a + b\left(cx^q\right)^n\right)^p \left(\frac{b\left(cx^q\right)^n}{a} + 1\right)^{-p} {}_2F_1\left(-p, \frac{3}{nq}; 1 + \frac{3}{nq}; -\frac{b\left(cx^q\right)^n}{a}\right)$$

```
[Out] 1/3*x^3*(a+b*(c*x^q)^n)^p*hypergeom([-p, 3/n/q],[1+3/n/q],-b*(c*x^q)^n/a)/(
(1+b*(c*x^q)^n/a)^p)
```

Rubi [A] time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {370, 365, 364}

$$\frac{1}{3}x^3\left(a + b\left(cx^q\right)^n\right)^p \left(\frac{b\left(cx^q\right)^n}{a} + 1\right)^{-p} {}_2F_1\left(-p, \frac{3}{nq}; 1 + \frac{3}{nq}; -\frac{b\left(cx^q\right)^n}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*(c*x^q)^n)^p,x]
```

```
[Out] (x^3*(a + b*(c*x^q)^n)^p*Hypergeometric2F1[-p, 3/(n*q), 1 + 3/(n*q), -((b*(
c*x^q)^n)/a)])/(3*(1 + (b*(c*x^q)^n)/a)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x], x^(n*q), (c*x^q)^n/c^n]
/; FreeQ[{a, b, c, d, m, n, p, q}, x] &&  !RationalQ[n]
```

Rubi steps

$$\begin{aligned}
\int x^2\left(a + b\left(cx^q\right)^n\right)^p \, dx &= \text{Subst}\left(\int x^2\left(a + bc^n x^{nq}\right)^p \, dx, x^{nq}, c^{-n}\left(cx^q\right)^n\right) \\
&= \text{Subst}\left(\left(\left(a + bc^n x^{nq}\right)^p \left(1 + \frac{bc^n x^{nq}}{a}\right)^{-p}\right)\int x^2\left(1 + \frac{bc^n x^{nq}}{a}\right)^p \, dx, x^{nq}, c^{-n}\left(cx^q\right)^n\right) \\
&= \frac{1}{3}x^3\left(a + b\left(cx^q\right)^n\right)^p \left(1 + \frac{b\left(cx^q\right)^n}{a}\right)^{-p} {}_2F_1\left(-p, \frac{3}{nq}; 1 + \frac{3}{nq}; -\frac{b\left(cx^q\right)^n}{a}\right)
\end{aligned}$$

Mathematica [A] time = 0.10, size = 73, normalized size = 1.00

$$\frac{1}{3}x^3\left(a + b\left(cx^q\right)^n\right)^p \left(\frac{b\left(cx^q\right)^n}{a} + 1\right)^{-p} {}_2F_1\left(-p, \frac{3}{nq}; 1 + \frac{3}{nq}; -\frac{b\left(cx^q\right)^n}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*(c*x^q)^n)^p,x]
```

```
[Out] (x^3*(a + b*(c*x^q)^n)^p*Hypergeometric2F1[-p, 3/(n*q), 1 + 3/(n*q), -((b*(
c*x^q)^n)/a)])/(3*(1 + (b*(c*x^q)^n)/a)^p)
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\left((cx^q)^n b + a\right)^p x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(c*x^q)^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*x^q)^n*b + a)^p*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left((cx^q)^n b + a\right)^p x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(c*x^q)^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*x^q)^n*b + a)^p*x^2, x)
```

maple [F] time = 0.90, size = 0, normalized size = 0.00

$$\int x^2 \left(b \left(c\, x^q\right)^n + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*(c*x^q)^n+a)^p,x)
```

```
[Out] int(x^2*(b*(c*x^q)^n+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left((cx^q)^n b + a\right)^p x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(c*x^q)^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*x^q)^n*b + a)^p*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left(a + b \left(c\, x^q\right)^n\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*(c*x^q)^n)^p,x)
```

```
[Out] int(x^2*(a + b*(c*x^q)^n)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + b \left(cx^q\right)^n\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*(c*x**q)**n)**p,x)
```

```
[Out] Integral(x**2*(a + b*(c*x**q)**n)**p, x)
```

3.3048 $\int x\left(a+b\left(cx^q\right)^n\right)^p\,dx$

Optimal. Leaf size=73

$$\frac{1}{2}x^2\left(a+b\left(cx^q\right)^n\right)^p\left(\frac{b\left(cx^q\right)^n}{a}+1\right)^{-p}\,{}_2F_1\left(-p,\frac{2}{nq};1+\frac{2}{nq};-\frac{b\left(cx^q\right)^n}{a}\right)$$

```
[Out] 1/2*x^2*(a+b*(c*x^q)^n)^p*hypergeom([-p, 2/n/q],[1+2/n/q],-b*(c*x^q)^n/a)/(
(1+b*(c*x^q)^n/a)^p)
```

Rubi [A] time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {370, 365, 364}

$$\frac{1}{2}x^2\left(a+b\left(cx^q\right)^n\right)^p\left(\frac{b\left(cx^q\right)^n}{a}+1\right)^{-p}\,{}_2F_1\left(-p,\frac{2}{nq};1+\frac{2}{nq};-\frac{b\left(cx^q\right)^n}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*(c*x^q)^n)^p,x]
```

```
[Out] (x^2*(a + b*(c*x^q)^n)^p*Hypergeometric2F1[-p, 2/(n*q), 1 + 2/(n*q), -((b*(
c*x^q)^n)/a)])/(2*(1 + (b*(c*x^q)^n)/a)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x], x^(n*q), (c*x^q)^n/c^n]
/; FreeQ[{a, b, c, d, m, n, p, q}, x] &&  !RationalQ[n]
```

Rubi steps

$$\begin{aligned}\int x\left(a+b\left(cx^q\right)^n\right)^p\,dx &= \text{Subst}\left(\int x\left(a+bc^nx^{nq}\right)^p\,dx, x^{nq}, c^{-n}\left(cx^q\right)^n\right)\\ &= \text{Subst}\left(\left(\left(a+bc^nx^{nq}\right)^p\left(1+\frac{bc^nx^{nq}}{a}\right)^{-p}\right)\int x\left(1+\frac{bc^nx^{nq}}{a}\right)^p\,dx, x^{nq}, c^{-n}\left(cx^q\right)^n\right)\\ &= \frac{1}{2}x^2\left(a+b\left(cx^q\right)^n\right)^p\left(1+\frac{b\left(cx^q\right)^n}{a}\right)^{-p}\,{}_2F_1\left(-p,\frac{2}{nq};1+\frac{2}{nq};-\frac{b\left(cx^q\right)^n}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.09, size = 73, normalized size = 1.00

$$\frac{1}{2}x^2\left(a+b\left(cx^q\right)^n\right)^p\left(\frac{b\left(cx^q\right)^n}{a}+1\right)^{-p}\,{}_2F_1\left(-p,\frac{2}{nq};1+\frac{2}{nq};-\frac{b\left(cx^q\right)^n}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*(c*x^q)^n)^p,x]
```

```
[Out] (x^2*(a + b*(c*x^q)^n)^p*Hypergeometric2F1[-p, 2/(n*q), 1 + 2/(n*q), -((b*(
c*x^q)^n)/a)])/(2*(1 + (b*(c*x^q)^n)/a)^p)
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\left((cx^q)^n b + a\right)^p x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(c*x^q)^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*x^q)^n*b + a)^p*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left((cx^q)^n b + a\right)^p x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(c*x^q)^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*x^q)^n*b + a)^p*x, x)
```

maple [F] time = 0.82, size = 0, normalized size = 0.00

$$\int x \left(b \left(c\, x^q\right)^n + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*(c*x^q)^n+a)^p,x)
```

```
[Out] int(x*(b*(c*x^q)^n+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left((cx^q)^n b + a\right)^p x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(c*x^q)^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*x^q)^n*b + a)^p*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \left(a + b \left(c\, x^q\right)^n\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*(c*x^q)^n)^p,x)
```

```
[Out] int(x*(a + b*(c*x^q)^n)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \left(a + b \left(cx^q\right)^n\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(c*x**q)**n)**p,x)
```

```
[Out] Integral(x*(a + b*(c*x**q)**n)**p, x)
```

3.3049 $\int \left(a + b\left(cx^q\right)^n\right)^p \, dx$

Optimal. Leaf size=66

$$x\left(a + b\left(cx^q\right)^n\right)^p \left(\frac{b\left(cx^q\right)^n}{a} + 1\right)^{-p} {}_2F_1\left(-p, \frac{1}{nq}; 1 + \frac{1}{nq}; -\frac{b\left(cx^q\right)^n}{a}\right)$$

```
[Out] x*(a+b*(c*x^q)^n)^p*hypergeom([-p, 1/n/q],[1+1/n/q],-b*(c*x^q)^n/a)/((1+b*(
c*x^q)^n/a)^p)
```

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {256, 246, 245}

$$x\left(a + b\left(cx^q\right)^n\right)^p \left(\frac{b\left(cx^q\right)^n}{a} + 1\right)^{-p} {}_2F_1\left(-p, \frac{1}{nq}; 1 + \frac{1}{nq}; -\frac{b\left(cx^q\right)^n}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c*x^q)^n)^p,x]
```

```
[Out] (x*(a + b*(c*x^q)^n)^p*Hypergeometric2F1[-p, 1/(n*q), 1 + 1/(n*q), -((b*(c*
x^q)^n)/a)])/(1 + (b*(c*x^q)^n)/a)^p
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 256

```
Int[((a_) + (b_.)*((c_.)*(x_)^(q_.))^(n_))^(p_.), x_Symbol] :> Subst[Int[(a
 + b*c^n*x^(n*q))^p, x], x^(n*q), (c*x^q)^n/c^n] /; FreeQ[{a, b, c, n, p, q
}, x] &&  !RationalQ[n]
```

Rubi steps

$$\begin{aligned}
\int \left(a + b\left(cx^q\right)^n\right)^p \, dx &= \text{Subst}\left(\int \left(a + bc^n x^{nq}\right)^p \, dx, x^{nq}, c^{-n}\left(cx^q\right)^n\right) \\
&= \text{Subst}\left(\left(\left(a + bc^n x^{nq}\right)^p \left(1 + \frac{bc^n x^{nq}}{a}\right)^{-p}\right) \int \left(1 + \frac{bc^n x^{nq}}{a}\right)^p \, dx, x^{nq}, c^{-n}\left(cx^q\right)^n\right) \\
&= x\left(a + b\left(cx^q\right)^n\right)^p \left(1 + \frac{b\left(cx^q\right)^n}{a}\right)^{-p} {}_2F_1\left(-p, \frac{1}{nq}; 1 + \frac{1}{nq}; -\frac{b\left(cx^q\right)^n}{a}\right)
\end{aligned}$$

Mathematica [A] time = 0.07, size = 66, normalized size = 1.00

$$x\left(a + b\left(cx^q\right)^n\right)^p \left(\frac{b\left(cx^q\right)^n}{a} + 1\right)^{-p} {}_2F_1\left(-p, \frac{1}{nq}; 1 + \frac{1}{nq}; -\frac{b\left(cx^q\right)^n}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c*x^q)^n)^p,x]
```

```
[Out] (x*(a + b*(c*x^q)^n)^p*Hypergeometric2F1[-p, 1/(n*q), 1 + 1/(n*q), -((b*(c*
x^q)^n)/a)])/(1 + (b*(c*x^q)^n)/a)^p
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\left((cx^q)^n\, b + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^q)^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*x^q)^n*b + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left((cx^q)^n\, b + a\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^q)^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*x^q)^n*b + a)^p, x)
```

maple [F] time = 0.94, size = 0, normalized size = 0.00

$$\int \left(b\,(c\,x^q)^n + a\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*x^q)^n+a)^p,x)
```

```
[Out] int((b*(c*x^q)^n+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left((cx^q)^n\, b + a\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^q)^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*x^q)^n*b + a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(a + b\,(c\,x^q)^n\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x^q)^n)^p,x)
```

```
[Out] int((a + b*(c*x^q)^n)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\,(cx^q)^n\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x**q)**n)**p,x)
```

```
[Out] Integral((a + b*(c*x**q)**n)**p, x)
```

3.3050 $\int \frac{\left(a+b(cx^q)^n\right)^p}{x}\,dx$

Optimal. Leaf size=53

$$-\frac{\left(a+b\left(cx^q\right)^n\right)^{p+1}\ {}_2F_1\left(1,p+1;p+2;\frac{b(cx^q)^n}{a}+1\right)}{an(p+1)q}$$

```
[Out] -(a+b*(c*x^q)^n)^(1+p)*hypergeom([1, 1+p],[2+p],1+b*(c*x^q)^n/a)/a/n/(1+p)/
q
```

Rubi [A] time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {370, 266, 65}

$$-\frac{\left(a+b\left(cx^q\right)^n\right)^{p+1}\ {}_2F_1\left(1,p+1;p+2;\frac{b(cx^q)^n}{a}+1\right)}{an(p+1)q}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c*x^q)^n)^p/x,x]
```

```
[Out] -(((a + b*(c*x^q)^n)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*(c*x
^q)^n)/a])/(a*n*(1 + p)*q))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x], x^(n*q), (c*x^q)^n/c^n]
/; FreeQ[{a, b, c, d, m, n, p, q}, x] &&  !RationalQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\left(cx^q\right)^n\right)^p}{x}\,dx &= \text{Subst}\left(\int \frac{(a+bc^nx^{nq})^p}{x}\,dx, x^{nq}, c^{-n}\left(cx^q\right)^n\right)\\ &= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{(a+bc^nx)^p}{x}\,dx, x, x^{nq}\right)}{nq}, x^{nq}, c^{-n}\left(cx^q\right)^n\right)\\ &= -\frac{\left(a+b\left(cx^q\right)^n\right)^{1+p}\ {}_2F_1\left(1,1+p;2+p;1+\frac{b(cx^q)^n}{a}\right)}{an(1+p)q}\end{aligned}$$

Mathematica [A] time = 0.06, size = 53, normalized size = 1.00

$$-\frac{\left(a+b\left(cx^q\right)^n\right)^{p+1}\ {}_2F_1\left(1,p+1;p+2;\frac{b(cx^q)^n}{a}+1\right)}{an(p+1)q}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c*x^q)^n)^p/x,x]
```

```
[Out] -(((a + b*(c*x^q)^n)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*(c*x
^q)^n)/a])/(a*n*(1 + p)*q))
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\left(cx^q\right)^n b+a\right)^p}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^q)^n)^p/x,x, algorithm="fricas")
```

```
[Out] integral(((c*x^q)^n*b + a)^p/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(cx^q\right)^n b+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^q)^n)^p/x,x, algorithm="giac")
```

```
[Out] integrate(((c*x^q)^n*b + a)^p/x, x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\left(c\,x^q\right)^n+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*x^q)^n+a)^p/x,x)
```

```
[Out] int((b*(c*x^q)^n+a)^p/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(cx^q\right)^n b+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^q)^n)^p/x,x, algorithm="maxima")
```

```
[Out] integrate(((c*x^q)^n*b + a)^p/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(a+b\left(c\,x^q\right)^n\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x^q)^n)^p/x,x)
```

```
[Out] int((a + b*(c*x^q)^n)^p/x, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\left(cx^{q}\right)^{n}\right)^{p}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x**q)**n)**p/x,x)
```

```
[Out] Integral((a + b*(c*x**q)**n)**p/x, x)
```

3.3051 $\int \frac{\left(a+b(cx^q)^n\right)^p}{x^2}\,dx$

Optimal. Leaf size=71

$$-\frac{\left(a+b\left(cx^q\right)^n\right)^p\left(\frac{b(cx^q)^n}{a}+1\right)^{-p}\,{}_2F_1\left(-p,-\frac{1}{nq};1-\frac{1}{nq};-\frac{b(cx^q)^n}{a}\right)}{x}$$

```
[Out] -(a+b*(c*x^q)^n)^p*hypergeom([-p, -1/n/q],[1-1/n/q],-b*(c*x^q)^n/a)/x/((1+b
*(c*x^q)^n/a)^p)
```

Rubi [A] time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {370, 365, 364}

$$-\frac{\left(a+b\left(cx^q\right)^n\right)^p\left(\frac{b(cx^q)^n}{a}+1\right)^{-p}\,{}_2F_1\left(-p,-\frac{1}{nq};1-\frac{1}{nq};-\frac{b(cx^q)^n}{a}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c*x^q)^n)^p/x^2,x]
```

```
[Out] -(((a + b*(c*x^q)^n)^p*Hypergeometric2F1[-p, -(1/(n*q)), 1 - 1/(n*q), -((b*
(c*x^q)^n)/a)])/(x*(1 + (b*(c*x^q)^n)/a)^p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Subst[Int[(d*x)^m*(a + b*c^n*x^(n*q))^p, x], x^(n*q), (c*x^q)^n/c^n]
/; FreeQ[{a, b, c, d, m, n, p, q}, x] &&  !RationalQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\left(cx^q\right)^n\right)^p}{x^2}\,dx &= \text{Subst}\left(\int \frac{(a+bc^nx^{nq})^p}{x^2}\,dx, x^{nq}, c^{-n}\left(cx^q\right)^n\right)\\ &= \text{Subst}\left(\left((a+bc^nx^{nq})^p\left(1+\frac{bc^nx^{nq}}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bc^nx^{nq}}{a}\right)^p}{x^2}\,dx, x^{nq}, c^{-n}\left(cx^q\right)^n\right)\\ &= -\frac{\left(a+b\left(cx^q\right)^n\right)^p\left(1+\frac{b(cx^q)^n}{a}\right)^{-p}\,{}_2F_1\left(-p,-\frac{1}{nq};1-\frac{1}{nq};-\frac{b(cx^q)^n}{a}\right)}{x}\end{aligned}$$

Mathematica [A] time = 0.11, size = 71, normalized size = 1.00

$$-\frac{\left(a+b\left(cx^q\right)^n\right)^p\left(\frac{b\left(cx^q\right)^n}{a}+1\right)^{-p}\,{}_2F_1\left(-p,-\frac{1}{nq};1-\frac{1}{nq};-\frac{b\left(cx^q\right)^n}{a}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c*x^q)^n)^p/x^2,x]
```

```
[Out] -(((a + b*(c*x^q)^n)^p*Hypergeometric2F1[-p, -(1/(n*q)), 1 - 1/(n*q), -((b*
(c*x^q)^n)/a)])/(x*(1 + (b*(c*x^q)^n)/a)^p))
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\left(cx^q\right)^n b+a\right)^p}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^q)^n)^p/x^2,x, algorithm="fricas")
```

```
[Out] integral(((c*x^q)^n*b + a)^p/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(cx^q\right)^n b+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^q)^n)^p/x^2,x, algorithm="giac")
```

```
[Out] integrate(((c*x^q)^n*b + a)^p/x^2, x)
```

maple [F] time = 1.19, size = 0, normalized size = 0.00

$$\int \frac{\left(b\left(c\,x^q\right)^n+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*x^q)^n+a)^p/x^2,x)
```

```
[Out] int((b*(c*x^q)^n+a)^p/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(cx^q\right)^n b+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x^q)^n)^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate(((c*x^q)^n*b + a)^p/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\left(c\,x^q\right)^n\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x^q)^n)^p/x^2,x)
```

```
[Out] int((a + b*(c*x^q)^n)^p/x^2, x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\left(cx^q\right)^n\right)^p}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x**q)**n)**p/x**2,x)
```

```
[Out] Integral((a + b*(c*x**q)**n)**p/x**2, x)
```

3.3052 $\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^m\,dx$

Optimal. Leaf size=230

$$\frac{x^{m+1}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,F_1\left(-2(m+1);-\frac{1}{2},-\frac{1}{2};-2m-1;-\frac{2c\sqrt{\frac{d}{x}}}{\sqrt{d}\left(b\sqrt{d}-\sqrt{b^2d-4ac}\right)},-\frac{2c\sqrt{\frac{d}{x}}}{\sqrt{d}\left(\sqrt{d}\,b+\sqrt{b^2d-4ac}\right)}\right)}{(m+1)\sqrt{\frac{2c\sqrt{\frac{d}{x}}}{\sqrt{d}\left(b\sqrt{d}-\sqrt{b^2d-4ac}\right)}+1}\sqrt{\frac{2c\sqrt{\frac{d}{x}}}{\sqrt{d}\left(\sqrt{b^2d-4ac}+b\sqrt{d}\right)}+1}}$$

```
[Out] x^(1+m)*AppellF1(-2-2*m,-1/2,-1/2,-1-2*m,-2*c*(d/x)^(1/2)/d^(1/2)/(b*d^(1/2
)-(b^2*d-4*a*c)^(1/2)),-2*c*(d/x)^(1/2)/d^(1/2)/(b*d^(1/2)+(b^2*d-4*a*c)^(1
/2)))*(a+c/x+b*(d/x)^(1/2))^(1/2)/(1+m)/(1+2*c*(d/x)^(1/2)/d^(1/2)/(b*d^(1/
2)-(b^2*d-4*a*c)^(1/2)))^(1/2)/(1+2*c*(d/x)^(1/2)/d^(1/2)/(b*d^(1/2)+(b^2*d
-4*a*c)^(1/2)))^(1/2)
```

Rubi [A] time = 0.72, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1971, 1379, 759, 133}

$$\frac{x^{m+1}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,F_1\left(-2(m+1);-\frac{1}{2},-\frac{1}{2};-2m-1;-\frac{2c\sqrt{\frac{d}{x}}}{\sqrt{d}\left(b\sqrt{d}-\sqrt{b^2d-4ac}\right)},-\frac{2c\sqrt{\frac{d}{x}}}{\sqrt{d}\left(\sqrt{d}\,b+\sqrt{b^2d-4ac}\right)}\right)}{(m+1)\sqrt{\frac{2c\sqrt{\frac{d}{x}}}{\sqrt{d}\left(b\sqrt{d}-\sqrt{b^2d-4ac}\right)}+1}\sqrt{\frac{2c\sqrt{\frac{d}{x}}}{\sqrt{d}\left(\sqrt{b^2d-4ac}+b\sqrt{d}\right)}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[d/x] + c/x]*x^m,x]
```

```
[Out] (Sqrt[a + b*Sqrt[d/x] + c/x]*x^(1 + m)*AppellF1[-2*(1 + m), -1/2, -1/2, -1
- 2*m, (-2*c*Sqrt[d/x])/(Sqrt[d]*(b*Sqrt[d] - Sqrt[-4*a*c + b^2*d])), (-2*c
*Sqrt[d/x])/(Sqrt[d]*(b*Sqrt[d] + Sqrt[-4*a*c + b^2*d]))])/((1 + m)*Sqrt[1
+ (2*c*Sqrt[d/x])/(Sqrt[d]*(b*Sqrt[d] - Sqrt[-4*a*c + b^2*d]))]*Sqrt[1 + (2
*c*Sqrt[d/x])/(Sqrt[d]*(b*Sqrt[d] + Sqrt[-4*a*c + b^2*d]))])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 1379

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x
^(k*n) + c*x^(2*k*n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, m, p}, x]
```

```
&& EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && FractionQ[n]
```

Rule 1971

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*((d_.)/(x_))^(n_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> -Dist[d*(e*x)^m*(d/x)^m, Subst[Int[(a + b*x^n + (c*x^(2
*n))/d^(2*n))^p/x^(m + 2), x], x, d/x], x] /; FreeQ[{a, b, c, d, e, n, p},
x] && EqQ[n2, -2*n] &&  !IntegerQ[m] && IntegerQ[2*n]
```

Rubi steps

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^m\,dx = -\left(\left(d\left(\frac{d}{x}\right)^m x^m\right)\text{Subst}\left(\int x^{-2-m}\sqrt{a+b\sqrt{x}+\frac{cx}{d}}\,dx, x, \frac{d}{x}\right)\right)$$

$$= -\left(\left(2d\left(\frac{d}{x}\right)^m x^m\right)\text{Subst}\left(\int x^{-1+2(-1-m)}\sqrt{a+bx+\frac{cx^2}{d}}\,dx, x, \sqrt{\frac{d}{x}}\right)\right)$$

$$= -\frac{\left(2d\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\left(\frac{d}{x}\right)^m x^m\right)\text{Subst}\left(\int x^{-1+2(-1-m)}\sqrt{1+\frac{2cx}{\sqrt{d}\left(b\sqrt{d}-\sqrt{-4ac+b^2d}\right)}}\sqrt{1+}\right.}{\sqrt{1+\frac{2c\sqrt{\frac{d}{x}}}{d\left(b-\frac{\sqrt{-4ac+b^2d}}{\sqrt{d}}\right)}}\sqrt{1+\frac{2c\sqrt{\frac{d}{x}}}{d\left(b+\frac{\sqrt{-4ac+b^2d}}{\sqrt{d}}\right)}}}$$

$$= \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^{1+m}F_1\left(-2(1+m); -\frac{1}{2}, -\frac{1}{2}; -1-2m; -\frac{2c\sqrt{\frac{d}{x}}}{\sqrt{d}\left(b\sqrt{d}-\sqrt{-4ac+b^2d}\right)}, -\frac{2}{\sqrt{d}\left(b\sqrt{d}-\right.}\right.}{(1+m)\sqrt{1+\frac{2c\sqrt{\frac{d}{x}}}{\sqrt{d}\left(b\sqrt{d}-\sqrt{-4ac+b^2d}\right)}}\sqrt{1+\frac{2c\sqrt{\frac{d}{x}}}{\sqrt{d}\left(b\sqrt{d}+\sqrt{-4ac+b^2d}\right)}}}$$

Mathematica [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sqrt[d/x] + c/x]*x^m,x]
```

```
[Out] Integrate[Sqrt[a + b*Sqrt[d/x] + c/x]*x^m, x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+c/x+b*(d/x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   alglo
gextint: unimplemented
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sqrt{\frac{d}{x}}+a+\frac{c}{x}}\,x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+c/x+b*(d/x)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sqrt(d/x) + a + c/x)*x^m, x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \sqrt{a + \sqrt{\frac{d}{x}}\, b + \frac{c}{x}}\, x^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+c/x+b*(d/x)^(1/2))^(1/2),x)
[Out] int(x^m*(a+c/x+b*(d/x)^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sqrt{\frac{d}{x}} + a + \frac{c}{x}}\, x^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+c/x+b*(d/x)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sqrt(d/x) + a + c/x)*x^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^m \sqrt{a + \frac{c}{x} + b\sqrt{\frac{d}{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + c/x + b*(d/x)^(1/2))^(1/2),x)
[Out] int(x^m*(a + c/x + b*(d/x)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \sqrt{a + b\sqrt{\frac{d}{x}} + \frac{c}{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+c/x+b*(d/x)**(1/2))**(1/2),x)
[Out] Integral(x**m*sqrt(a + b*sqrt(d/x) + c/x), x)
```

3.3053 $\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^2\,dx$

Optimal. Leaf size=333

$$\frac{7bd^2\left(28ac-15b^2d\right)\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{480a^4\left(\frac{d}{x}\right)^{3/2}}-\frac{x^2\left(20ac-21b^2d\right)\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{80a^3}-\frac{3bd^3\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{10a^2\left(\frac{d}{x}\right)^{5/2}}+\frac{\left(4ac-b^2$$

```
[Out] 1/512*(-b^2*d+4*a*c)*(21*b^4*d^2-56*a*b^2*c*d+16*a^2*c^2)*arctanh(1/2*(2*a+
b*(d/x)^(1/2))/a^(1/2)/(a+c/x+b*(d/x)^(1/2))^(1/2))/a^(11/2)-3/10*b*d^3*(a+
c/x+b*(d/x)^(1/2))^(3/2)/a^2/(d/x)^(5/2)+7/480*b*d^2*(-15*b^2*d+28*a*c)*(a+
c/x+b*(d/x)^(1/2))^(3/2)/a^4/(d/x)^(3/2)-1/80*(-21*b^2*d+20*a*c)*x^2*(a+c/x
+b*(d/x)^(1/2))^(3/2)/a^3+1/3*x^3*(a+c/x+b*(d/x)^(1/2))^(3/2)/a+1/256*(21*b
^4*d^2-56*a*b^2*c*d+16*a^2*c^2)*x*(2*a+b*(d/x)^(1/2))*(a+c/x+b*(d/x)^(1/2))
^(1/2)/a^5
```

Rubi [A] time = 0.60, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1970, 1357, 744, 834, 806, 720, 724, 206}

$$\frac{x\left(16a^2c^2-56ab^2cd+21b^4d^2\right)\left(2a+b\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{256a^5}+\frac{\left(4ac-b^2d\right)\left(16a^2c^2-56ab^2cd+21b^4d^2\right)\tanh^{-1}\left($$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[d/x] + c/x]*x^2,x]
```

```
[Out] (-3*b*d^3*(a + b*Sqrt[d/x] + c/x)^(3/2))/(10*a^2*(d/x)^(5/2)) + (7*b*d^2*(2
8*a*c - 15*b^2*d)*(a + b*Sqrt[d/x] + c/x)^(3/2))/(480*a^4*(d/x)^(3/2)) + ((
16*a^2*c^2 - 56*a*b^2*c*d + 21*b^4*d^2)*(2*a + b*Sqrt[d/x])*Sqrt[a + b*Sqrt
[d/x] + c/x]*x)/(256*a^5) - ((20*a*c - 21*b^2*d)*(a + b*Sqrt[d/x] + c/x)^(3
/2)*x^2)/(80*a^3) + ((a + b*Sqrt[d/x] + c/x)^(3/2)*x^3)/(3*a) + ((4*a*c - b
^2*d)*(16*a^2*c^2 - 56*a*b^2*c*d + 21*b^4*d^2)*ArcTanh[(2*a + b*Sqrt[d/x])/
(2*Sqrt[a]*Sqrt[a + b*Sqrt[d/x] + c/x])])/(512*a^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1970

```
Int[(x_)^(m_.)*((a_) + (b_.)*((d_.)/(x_))^(n_) + (c_.)*(x_)^(n2_.))^(p_.),
x_Symbol] :> -Dist[d^(m + 1), Subst[Int[(a + b*x^n + (c*x^(2*n))/d^(2*n))^p
/x^(m + 2), x], x, d/x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n2, -2*n
] && IntegerQ[2*n] && IntegerQ[m]
```

Rubi steps

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^2\,dx = -\left(d^3\,\text{Subst}\left(\int \frac{\sqrt{a+b\sqrt{x}+\frac{cx}{d}}}{x^4}\,dx, x, \frac{d}{x}\right)\right)$$

$$= -\left(\left(2d^3\right)\text{Subst}\left(\int \frac{\sqrt{a+bx+\frac{cx^2}{d}}}{x^7}\,dx, x, \sqrt{\frac{d}{x}}\right)\right)$$

$$= \frac{\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}x^3}{3a} + \frac{d^3\,\text{Subst}\left(\int \frac{\left(\frac{9b}{2}+\frac{3cx}{d}\right)\sqrt{a+bx+\frac{cx^2}{d}}}{x^6}\,dx, x, \sqrt{\frac{d}{x}}\right)}{3a}$$

$$= -\frac{3bd^3\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{10a^2\left(\frac{d}{x}\right)^{5/2}} + \frac{\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}x^3}{3a} - \frac{d^3\,\text{Subst}\left(\int \frac{\left(\frac{3}{4}\left(21b^2-\frac{20ac}{d}\right)+\frac{9bcx}{d}\right)\sqrt{a+}}{x^5}\right.}{15a^2}$$

$$= -\frac{3bd^3\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{10a^2\left(\frac{d}{x}\right)^{5/2}} - \frac{\left(20ac-21b^2d\right)\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}x^2}{80a^3} + \frac{\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{3a}$$

$$= -\frac{3bd^3\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{10a^2\left(\frac{d}{x}\right)^{5/2}} + \frac{7bd^2\left(28ac-15b^2d\right)\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{480a^4\left(\frac{d}{x}\right)^{3/2}} - \frac{\left(20ac-21b^2d\right)}{}$$

$$= -\frac{3bd^3\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{10a^2\left(\frac{d}{x}\right)^{5/2}} + \frac{7bd^2\left(28ac-15b^2d\right)\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{480a^4\left(\frac{d}{x}\right)^{3/2}} + \frac{\left(16a^2c^2-56ab\right.}{}$$

$$= -\frac{3bd^3\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{10a^2\left(\frac{d}{x}\right)^{5/2}} + \frac{7bd^2\left(28ac-15b^2d\right)\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{480a^4\left(\frac{d}{x}\right)^{3/2}} + \frac{\left(16a^2c^2-56ab\right.}{}$$

$$= -\frac{3bd^3\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{10a^2\left(\frac{d}{x}\right)^{5/2}} + \frac{7bd^2\left(28ac-15b^2d\right)\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{480a^4\left(\frac{d}{x}\right)^{3/2}} + \frac{\left(16a^2c^2-56ab\right.}{}$$

Mathematica [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^2\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sqrt[d/x] + c/x]*x^2,x]
```

```
[Out] Integrate[Sqrt[a + b*Sqrt[d/x] + c/x]*x^2, x]
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+c/x+b*(d/x)^(1/2))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+c/x+b*(d/x)^(1/2))^(1/2),x, algorithm="giac")
[Out] Timed out
```

maple [**B**] time = 0.19, size = 655, normalized size = 1.97

$$\frac{\sqrt{\frac{ax+\sqrt{\frac{d}{x}}\,bx+c}{x}}\left(-315a\,b^6d^3\ln\left(\frac{2a\sqrt{x}+\sqrt{\frac{d}{x}}\,b\sqrt{x}+2\sqrt{ax+\sqrt{\frac{d}{x}}\,bx+c}\,\sqrt{a}}{2\sqrt{a}}\right)+2100a^2b^4c\,d^2\ln\left(\frac{2a\sqrt{x}+\sqrt{\frac{d}{x}}\,b\sqrt{x}+2\sqrt{ax+\sqrt{\frac{d}{x}}\,bx+c}}{2\sqrt{a}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+(d/x)^(1/2)*b+c/x)^(1/2),x)
[Out] 1/7680*((b*(d/x)^(1/2)*x+a*x+c)/x)^(1/2)*x^(1/2)*(630*(b*(d/x)^(1/2)*x+a*x+
c)^(1/2)*a^(3/2)*(d/x)^(5/2)*x^(5/2)*b^5-1680*(b*(d/x)^(1/2)*x+a*x+c)^(3/2)
*a^(5/2)*(d/x)^(3/2)*x^(3/2)*b^3-2304*(b*(d/x)^(1/2)*x+a*x+c)^(3/2)*a^(9/2)
*(d/x)^(1/2)*x^(3/2)*b+2560*x^(3/2)*(b*(d/x)^(1/2)*x+a*x+c)^(3/2)*a^(11/2)-
1680*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*a^(5/2)*(d/x)^(3/2)*x^(3/2)*b^3*c+3136*(
b*(d/x)^(1/2)*x+a*x+c)^(3/2)*a^(7/2)*(d/x)^(1/2)*x^(1/2)*b*c-1920*(b*(d/x)^
(1/2)*x+a*x+c)^(3/2)*a^(9/2)*x^(1/2)*c+2016*(b*(d/x)^(1/2)*x+a*x+c)^(3/2)*d
*a^(7/2)*x^(1/2)*b^2+480*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*a^(7/2)*(d/x)^(1/2)*
x^(1/2)*b*c^2+960*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*a^(9/2)*x^(1/2)*c^2-3360*(b
*(d/x)^(1/2)*x+a*x+c)^(1/2)*d*a^(7/2)*x^(1/2)*b^2*c+1260*(b*(d/x)^(1/2)*x+a
*x+c)^(1/2)*d^2*a^(5/2)*x^(1/2)*b^4-315*d^3*ln(1/2*(b*(d/x)^(1/2)*x^(1/2)+2
*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*a^(1/2)+2*a*x^(1/2))/a^(1/2))*a*b^6+2100*d^2
*ln(1/2*(b*(d/x)^(1/2)*x^(1/2)+2*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*a^(1/2)+2*a*
x^(1/2))/a^(1/2))*a^2*b^4*c-3600*d*ln(1/2*(b*(d/x)^(1/2)*x^(1/2)+2*(b*(d/x)
^(1/2)*x+a*x+c)^(1/2)*a^(1/2)+2*a*x^(1/2))/a^(1/2))*a^3*b^2*c^2+960*ln(1/2*
(b*(d/x)^(1/2)*x^(1/2)+2*(b*(d/x)^(1/2)*x+a*x+c)^(1/2)*a^(1/2)+2*a*x^(1/2))
/a^(1/2))*a^4*c^3)/(b*(d/x)^(1/2)*x+a*x+c)^(1/2)/a^(13/2)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sqrt{\frac{d}{x}}+a+\frac{c}{x}}\,x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+c/x+b*(d/x)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sqrt(d/x) + a + c/x)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \sqrt{a + \frac{c}{x} + b \sqrt{\frac{d}{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + c/x + b*(d/x)^(1/2))^(1/2),x)
[Out] int(x^2*(a + c/x + b*(d/x)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{a + b \sqrt{\frac{d}{x}} + \frac{c}{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+c/x+b*(d/x)**(1/2))**(1/2),x)
[Out] Integral(x**2*sqrt(a + b*sqrt(d/x) + c/x), x)
```

3.3054 $\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x\,dx$

Optimal. Leaf size=209

$$-\frac{\left(4ac-5b^2d\right)\left(4ac-b^2d\right)\tanh^{-1}\left(\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right)}{64a^{7/2}}-\frac{x\left(4ac-5b^2d\right)\left(2a+b\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{32a^3}-\frac{5bd^2\left(a+b\right.}{12a^2}$$

```
[Out] -1/64*(-5*b^2*d+4*a*c)*(-b^2*d+4*a*c)*arctanh(1/2*(2*a+b*(d/x)^(1/2))/a^(1/
2)/(a+c/x+b*(d/x)^(1/2))^(1/2))/a^(7/2)-5/12*b*d^2*(a+c/x+b*(d/x)^(1/2))^(3
/2)/a^2/(d/x)^(3/2)+1/2*x^2*(a+c/x+b*(d/x)^(1/2))^(3/2)/a-1/32*(-5*b^2*d+4*
a*c)*x*(2*a+b*(d/x)^(1/2))*(a+c/x+b*(d/x)^(1/2))^(1/2)/a^3
```

Rubi [A] time = 0.28, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1970, 1357, 744, 806, 720, 724, 206}

$$-\frac{x\left(4ac-5b^2d\right)\left(2a+b\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{32a^3}-\frac{\left(4ac-5b^2d\right)\left(4ac-b^2d\right)\tanh^{-1}\left(\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right)}{64a^{7/2}}-\frac{5bd^2\left(a+b\right.}{12a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[d/x] + c/x]*x,x]
```

```
[Out] (-5*b*d^2*(a + b*Sqrt[d/x] + c/x)^(3/2))/(12*a^2*(d/x)^(3/2)) - ((4*a*c - 5
*b^2*d)*(2*a + b*Sqrt[d/x])*Sqrt[a + b*Sqrt[d/x] + c/x]*x)/(32*a^3) + ((a +
 b*Sqrt[d/x] + c/x)^(3/2)*x^2)/(2*a) - ((4*a*c - 5*b^2*d)*(4*a*c - b^2*d)*A
rcTanh[(2*a + b*Sqrt[d/x])/(2*Sqrt[a]*Sqrt[a + b*Sqrt[d/x] + c/x])])/(64*a^
(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1970

```
Int[(x_)^(m_.)*((a_) + (b_.)*((d_.)/(x_))^(n_) + (c_.)*(x_)^(n2_.))^(p_.),
x_Symbol] :> -Dist[d^(m + 1), Subst[Int[(a + b*x^n + (c*x^(2*n))/d^(2*n))^p
/x^(m + 2), x], x, d/x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n2, -2*n
] && IntegerQ[2*n] && IntegerQ[m]
```

Rubi steps

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x\,dx = -\left(d^2\,\text{Subst}\left(\int \frac{\sqrt{a+b\sqrt{x}+\frac{cx}{d}}}{x^3}\,dx, x, \frac{d}{x}\right)\right)$$

$$= -\left(\left(2d^2\right)\text{Subst}\left(\int \frac{\sqrt{a+bx+\frac{cx^2}{d}}}{x^5}\,dx, x, \sqrt{\frac{d}{x}}\right)\right)$$

$$= \frac{\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}x^2}{2a} + \frac{d^2\,\text{Subst}\left(\int \frac{\left(\frac{5b}{2}+\frac{cx}{d}\right)\sqrt{a+bx+\frac{cx^2}{d}}}{x^4}\,dx, x, \sqrt{\frac{d}{x}}\right)}{2a}$$

$$= -\frac{5bd^2\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{12a^2\left(\frac{d}{x}\right)^{3/2}} + \frac{\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}x^2}{2a} + \frac{\left(d\left(4ac-5b^2d\right)\right)\text{Subst}\left(\int \frac{\sqrt{a+}}{}\right.}{8a^2}$$

$$= -\frac{5bd^2\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{12a^2\left(\frac{d}{x}\right)^{3/2}} - \frac{\left(4ac-5b^2d\right)\left(2a+b\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x}{32a^3} + \frac{\left(a+b\sqrt{\right.}}{}$$

$$= -\frac{5bd^2\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{12a^2\left(\frac{d}{x}\right)^{3/2}} - \frac{\left(4ac-5b^2d\right)\left(2a+b\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x}{32a^3} + \frac{\left(a+b\sqrt{\right.}}{}$$

$$= -\frac{5bd^2\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{12a^2\left(\frac{d}{x}\right)^{3/2}} - \frac{\left(4ac-5b^2d\right)\left(2a+b\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x}{32a^3} + \frac{\left(a+b\sqrt{\right.}}{}$$

Mathematica [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sqrt[d/x] + c/x]*x,x]
[Out] Integrate[Sqrt[a + b*Sqrt[d/x] + c/x]*x, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+c/x+b*(d/x)^(1/2))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+c/x+b*(d/x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.16, size = 398, normalized size = 1.90

$$\frac{\sqrt{\frac{ax+\sqrt{\frac{d}{x}}\,bx+c}{x}}\left(-15a\,b^4d^2\ln\left(\frac{2a\sqrt{x}+\sqrt{\frac{d}{x}}\,b\sqrt{x}+2\sqrt{ax+\sqrt{\frac{d}{x}}\,bx+c}\,\sqrt{a}}{2\sqrt{a}}\right)+72a^2b^2cd\ln\left(\frac{2a\sqrt{x}+\sqrt{\frac{d}{x}}\,b\sqrt{x}+2\sqrt{ax+\sqrt{\frac{d}{x}}\,bx+c}\,\sqrt{a}}{2\sqrt{a}}\right)-4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+(d/x)^(1/2)*b+c/x)^(1/2),x)
```

```
[Out] 1/192*((a*x+(d/x)^(1/2)*b*x+c)/x)^(1/2)*x^(1/2)*(30*(a*x+(d/x)^(1/2)*b*x+c)
^(1/2)*a^(3/2)*(d/x)^(3/2)*x^(3/2)*b^3+60*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*a^(
5/2)*d*x^(1/2)*b^2-15*ln(1/2*(2*a*x^(1/2)+(d/x)^(1/2)*b*x^(1/2)+2*(a*x+(d/x
)^(1/2)*b*x+c)^(1/2)*a^(1/2))/a^(1/2))*d^2*a*b^4+96*x^(1/2)*(a*x+(d/x)^(1/2
)*b*x+c)^(3/2)*a^(7/2)-80*(a*x+(d/x)^(1/2)*b*x+c)^(3/2)*a^(5/2)*(d/x)^(1/2)
*x^(1/2)*b-48*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*a^(7/2)*x^(1/2)*c-24*(a*x+(d/x)
^(1/2)*b*x+c)^(1/2)*a^(5/2)*(d/x)^(1/2)*x^(1/2)*b*c+72*ln(1/2*(2*a*x^(1/2)+
(d/x)^(1/2)*b*x^(1/2)+2*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*a^(1/2))/a^(1/2))*d*a
^2*b^2*c-48*ln(1/2*(2*a*x^(1/2)+(d/x)^(1/2)*b*x^(1/2)+2*(a*x+(d/x)^(1/2)*b*
x+c)^(1/2)*a^(1/2))/a^(1/2))*a^3*c^2)/(a*x+(d/x)^(1/2)*b*x+c)^(1/2)/a^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sqrt{\frac{d}{x}}+a+\frac{c}{x}}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+c/x+b*(d/x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sqrt(d/x) + a + c/x)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + c/x + b*(d/x)^(1/2))^(1/2),x)
```

```
[Out] int(x*(a + c/x + b*(d/x)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+c/x+b*(d/x)**(1/2))**(1/2),x)
```

```
[Out] Integral(x*sqrt(a + b*sqrt(d/x) + c/x), x)
```

3.3055 $\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,dx$

Optimal. Leaf size=113

$$\frac{\left(4ac-b^2d\right)\tanh^{-1}\left(\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right)}{4a^{3/2}}+\frac{x\left(2a+b\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{2a}$$

```
[Out] 1/4*(-b^2*d+4*a*c)*arctanh(1/2*(2*a+b*(d/x)^(1/2))/a^(1/2)/(a+c/x+b*(d/x)^(
1/2))^(1/2))/a^(3/2)+1/2*x*(2*a+b*(d/x)^(1/2))*(a+c/x+b*(d/x)^(1/2))^(1/2)/
a
```

Rubi [A] time = 0.12, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1969, 1357, 720, 724, 206}

$$\frac{\left(4ac-b^2d\right)\tanh^{-1}\left(\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right)}{4a^{3/2}}+\frac{x\left(2a+b\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[d/x] + c/x],x]
```

```
[Out] ((2*a + b*Sqrt[d/x])*Sqrt[a + b*Sqrt[d/x] + c/x]*x)/(2*a) + ((4*a*c - b^2*d
)*ArcTanh[(2*a + b*Sqrt[d/x])/(2*Sqrt[a]*Sqrt[a + b*Sqrt[d/x] + c/x])])/(4*
a^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
```

```
4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1969

```
Int[((a_.) + (b_.)*((d_.)/(x_))^(n_) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol]
:> -Dist[d, Subst[Int[(a + b*x^n + (c*x^(2*n))/d^(2*n))^p/x^2, x], x, d/x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n2, -2*n] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,dx &= -\left(d\,\text{Subst}\left(\int \frac{\sqrt{a+b\sqrt{x}+\frac{cx}{d}}}{x^2}\,dx, x, \frac{d}{x}\right)\right)\\
&= -\left((2d)\,\text{Subst}\left(\int \frac{\sqrt{a+bx+\frac{cx^2}{d}}}{x^3}\,dx, x, \sqrt{\frac{d}{x}}\right)\right)\\
&= \frac{\left(2a+b\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x}{2a} + \frac{\left(\left(b^2-\frac{4ac}{d}\right)d\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+\frac{cx^2}{d}}}\,dx, x, \sqrt{\frac{d}{x}}\right)}{4a}\\
&= \frac{\left(2a+b\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x}{2a} - \frac{\left(\left(b^2-\frac{4ac}{d}\right)d\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+b\sqrt{\frac{d}{x}}}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right)}{2a}\\
&= \frac{\left(2a+b\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x}{2a} + \frac{\left(4ac-b^2d\right)\tanh^{-1}\left(\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right)}{4a^{3/2}}
\end{aligned}$$

Mathematica [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sqrt[d/x] + c/x],x]
```

```
[Out] Integrate[Sqrt[a + b*Sqrt[d/x] + c/x], x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x+b*(d/x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.99, size = 213, normalized size = 1.88

$$\left(2\sqrt{ad^2x+\sqrt{dx}\,bd^2+cd^2}\left(\frac{bd}{a}+2\sqrt{dx}\right)+\frac{\left(b^2d^3-4acd^2\right)\log\left(\left|-bd^2-2\sqrt{ad}\left(\sqrt{ad}\sqrt{dx}-\sqrt{ad^2x+\sqrt{dx}\,bd^2+cd^2}\right)\right|\right)}{\sqrt{ad}\,a}-\frac{b^2d^3\log\left(\left|-bd^2+2\right.\right.\right.}{4\,d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x+b*(d/x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(a*d^2*x + sqrt(d*x)*b*d^2 + c*d^2)*(b*d/a + 2*sqrt(d*x)) + (b^2
*d^3 - 4*a*c*d^2)*log(abs(-b*d^2 - 2*sqrt(a*d)*(sqrt(a*d)*sqrt(d*x) - sqrt(
a*d^2*x + sqrt(d*x)*b*d^2 + c*d^2))))/(sqrt(a*d)*a) - (b^2*d^3*log(abs(-b*d
^2 + 2*sqrt(c*d^2)*sqrt(a*d))) - 4*a*c*d^2*log(abs(-b*d^2 + 2*sqrt(c*d^2)*s
qrt(a*d))) + 2*sqrt(c*d^2)*sqrt(a*d)*b*d)/(sqrt(a*d)*a))*sgn(x)/d^(3/2)
```

maple [B] time = 0.14, size = 213, normalized size = 1.88

$$\frac{\sqrt{\frac{ax+\sqrt{\frac{d}{x}}\,bx+c}{x}}\left(-a\,b^2d\ln\left(\frac{2a\sqrt{x}+\sqrt{\frac{d}{x}}\,b\sqrt{x}+2\sqrt{ax+\sqrt{\frac{d}{x}}\,bx+c}\,\sqrt{a}}{2\sqrt{a}}\right)+4a^2c\ln\left(\frac{2a\sqrt{x}+\sqrt{\frac{d}{x}}\,b\sqrt{x}+2\sqrt{ax+\sqrt{\frac{d}{x}}\,bx+c}\,\sqrt{a}}{2\sqrt{a}}\right)+4\sqrt{ax}$$

$$4\sqrt{ax+\sqrt{\frac{d}{x}}\,bx+c}\;a^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(d/x)^(1/2)*b+c/x)^(1/2),x)
```

```
[Out] 1/4*((a*x+(d/x)^(1/2)*b*x+c)/x)^(1/2)*x^(1/2)*(2*a^(3/2)*(a*x+(d/x)^(1/2)*b
*x+c)^(1/2)*(d/x)^(1/2)*x^(1/2)*b-ln(1/2*(2*a*x^(1/2)+(d/x)^(1/2)*b*x^(1/2)
+2*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*a^(1/2))/a^(1/2))*d*a*b^2+4*a^(5/2)*(a*x+(
d/x)^(1/2)*b*x+c)^(1/2)*x^(1/2)+4*ln(1/2*(2*a*x^(1/2)+(d/x)^(1/2)*b*x^(1/2)
+2*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*a^(1/2))/a^(1/2))*a^2*c)/a^(5/2)/(a*x+(d/x
)^(1/2)*b*x+c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sqrt{\frac{d}{x}}+a+\frac{c}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x+b*(d/x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sqrt(d/x) + a + c/x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c/x + b*(d/x)^(1/2))^(1/2),x)
```

```
[Out] int((a + c/x + b*(d/x)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x+b*(d/x)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sqrt(d/x) + c/x), x)
```

3.3056 $\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}+\frac{c}{x}}}}{x}\,dx$

Optimal. Leaf size=145

$$-2\sqrt{a+b\sqrt{\frac{d}{x}+\frac{c}{x}}}+2\sqrt{a}\,\tanh^{-1}\left(\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\sqrt{a+b\sqrt{\frac{d}{x}+\frac{c}{x}}}}\right)-\frac{b\sqrt{d}\,\tanh^{-1}\left(\frac{bd+2c\sqrt{\frac{d}{x}}}{2\sqrt{c}\,\sqrt{d}\sqrt{a+b\sqrt{\frac{d}{x}+\frac{c}{x}}}}\right)}{\sqrt{c}}$$

```
[Out] 2*arctanh(1/2*(2*a+b*(d/x)^(1/2))/a^(1/2)/(a+c/x+b*(d/x)^(1/2))^(1/2))*a^(1
/2)-b*arctanh(1/2*(b*d+2*c*(d/x)^(1/2))/c^(1/2)/d^(1/2)/(a+c/x+b*(d/x)^(1/2
))^(1/2))*d^(1/2)/c^(1/2)-2*(a+c/x+b*(d/x)^(1/2))^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1970, 1357, 734, 843, 621, 206, 724}

$$-2\sqrt{a+b\sqrt{\frac{d}{x}+\frac{c}{x}}}+2\sqrt{a}\,\tanh^{-1}\left(\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\sqrt{a+b\sqrt{\frac{d}{x}+\frac{c}{x}}}}\right)-\frac{b\sqrt{d}\,\tanh^{-1}\left(\frac{bd+2c\sqrt{\frac{d}{x}}}{2\sqrt{c}\,\sqrt{d}\sqrt{a+b\sqrt{\frac{d}{x}+\frac{c}{x}}}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[d/x] + c/x]/x,x]
```

```
[Out] -2*Sqrt[a + b*Sqrt[d/x] + c/x] + 2*Sqrt[a]*ArcTanh[(2*a + b*Sqrt[d/x])/(2*S
qrt[a]*Sqrt[a + b*Sqrt[d/x] + c/x])] - (b*Sqrt[d]*ArcTanh[(b*d + 2*c*Sqrt[d
/x])/(2*Sqrt[c]*Sqrt[d]*Sqrt[a + b*Sqrt[d/x] + c/x])])/Sqrt[c]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

```
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1970

```
Int[(x_)^(m_.)*((a_) + (b_.)*((d_.)/(x_))^(n_) + (c_.)*(x_)^(n2_.))^(p_.),
x_Symbol] :> -Dist[d^(m + 1), Subst[Int[(a + b*x^n + (c*x^(2*n))/d^(2*n))^p
/x^(m + 2), x], x, d/x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n2, -2*n
] && IntegerQ[2*n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x}\,dx = -\operatorname{Subst}\left(\int \frac{\sqrt{a+b\sqrt{x}+\frac{cx}{d}}}{x}\,dx, x, \frac{d}{x}\right)$$

$$= -\left(2\operatorname{Subst}\left(\int \frac{\sqrt{a+bx+\frac{cx^2}{d}}}{x}\,dx, x, \sqrt{\frac{d}{x}}\right)\right)$$

$$= -2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}} + \operatorname{Subst}\left(\int \frac{-2a-bx}{x\sqrt{a+bx+\frac{cx^2}{d}}}\,dx, x, \sqrt{\frac{d}{x}}\right)$$

$$= -2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}} - (2a)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx+\frac{cx^2}{d}}}\,dx, x, \sqrt{\frac{d}{x}}\right) - b\operatorname{Subst}\left(\int \frac{}{\sqrt{a}}\right.$$

$$= -2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}} + (4a)\operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+b\sqrt{\frac{d}{x}}}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right) - (2b)\operatorname{Subst}\left($$

$$= -2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}} + 2\sqrt{a}\,\tanh^{-1}\left(\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right) - \frac{b\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\left(b+\frac{2c}{}\right.\right.}{2\sqrt{c}\sqrt{a+b}\right.}}{\sqrt{c}}$$

Mathematica [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sqrt[d/x] + c/x]/x,x]
[Out] Integrate[Sqrt[a + b*Sqrt[d/x] + c/x]/x, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x+b*(d/x)^(1/2))^(1/2)/x,x, algorithm="fricas")
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x+b*(d/x)^(1/2))^(1/2)/x,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, integration of abs or sign assumes constant sign by intervals (cor
rect if the argument is real):Check [abs(t_nostep)]Sign error (%%%{(-d*sqrt
(a*d))*b+2*sqrt(c)*a*abs(d),0%%%}+%%%{(a*b*sqrt(c)*abs(d)-2*a*c*sqrt(a*d))/
c,1%%%}+%%%{(4*a^2*c*sqrt(c)*abs(d)-a*b^2*d*sqrt(c)*abs(d))/(4*c^2*d),2%%%}
+%%%{undef,3%%%})Limit: Max order reached or unable to make series expansio
n Error: Bad Argument Value
```

maple [B] time = 0.16, size = 237, normalized size = 1.63

$$\frac{\sqrt{\frac{ax+\sqrt{\frac{d}{x}}\,bx+c}{x}}\left(-\sqrt{\frac{d}{x}}\,a^{\frac{3}{2}}b\sqrt{c}\,x\ln\left(\frac{\sqrt{\frac{d}{x}}\,bx+2c+2\sqrt{ax+\sqrt{\frac{d}{x}}\,bx+c}\,\sqrt{c}}{\sqrt{x}}\right)+2a^2c\sqrt{x}\,\ln\left(\frac{2a\sqrt{x}+\sqrt{\frac{d}{x}}\,b\sqrt{x}+2\sqrt{ax+\sqrt{\frac{d}{x}}\,bx+c}\,\sqrt{a}}{2\sqrt{a}}\right)+2\right.}{\sqrt{ax+\sqrt{\frac{d}{x}}\,bx+c}\,a^{\frac{3}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(d/x)^(1/2)*b+c/x)^(1/2)/x,x)
[Out] ((a*x+(d/x)^(1/2)*b*x+c)/x)^(1/2)*(-a^(3/2)*c^(1/2)*ln((2*c+(d/x)^(1/2)*b*x
+2*c^(1/2)*(a*x+(d/x)^(1/2)*b*x+c)^(1/2))/x^(1/2))*(d/x)^(1/2)*x*b+2*(a*x+(
d/x)^(1/2)*b*x+c)^(1/2)*a^(3/2)*(d/x)^(1/2)*x*b+2*(a*x+(d/x)^(1/2)*b*x+c)^(
1/2)*a^(5/2)*x-2*(a*x+(d/x)^(1/2)*b*x+c)^(3/2)*a^(3/2)+2*ln(1/2*(2*a*x^(1/2
)+(d/x)^(1/2)*b*x^(1/2)+2*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*a^(1/2))/a^(1/2))*x
^(1/2)*a^2*c)/(a*x+(d/x)^(1/2)*b*x+c)^(1/2)/c/a^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sqrt{\frac{d}{x}}+a+\frac{c}{x}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x+b*(d/x)^(1/2))^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt(b*sqrt(d/x) + a + c/x)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c/x + b*(d/x)^(1/2))^(1/2)/x,x)
[Out] int((a + c/x + b*(d/x)^(1/2))^(1/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x+b*(d/x)**(1/2))**(1/2)/x,x)
[Out] Integral(sqrt(a + b*sqrt(d/x) + c/x)/x, x)
```

3.3057 $\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^2}\,dx$

Optimal. Leaf size=155

$$\frac{b\sqrt{d}\left(4ac-b^2d\right)\tanh^{-1}\left(\frac{bd+2c\sqrt{\frac{d}{x}}}{2\sqrt{c}\sqrt{d}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right)}{8c^{5/2}}+\frac{b\left(bd+2c\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{4c^2}-\frac{2\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{3c}$$

```
[Out] 1/8*b*(-b^2*d+4*a*c)*arctanh(1/2*(b*d+2*c*(d/x)^(1/2))/c^(1/2)/d^(1/2)/(a+c
/x+b*(d/x)^(1/2))^(1/2))*d^(1/2)/c^(5/2)-2/3*(a+c/x+b*(d/x)^(1/2))^(3/2)/c+
1/4*b*(b*d+2*c*(d/x)^(1/2))*(a+c/x+b*(d/x)^(1/2))^(1/2)/c^2
```

Rubi [A] time = 0.18, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1970, 1341, 640, 612, 621, 206}

$$\frac{b\sqrt{d}\left(4ac-b^2d\right)\tanh^{-1}\left(\frac{bd+2c\sqrt{\frac{d}{x}}}{2\sqrt{c}\sqrt{d}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right)}{8c^{5/2}}+\frac{b\left(bd+2c\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{4c^2}-\frac{2\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[d/x] + c/x]/x^2,x]
```

```
[Out] (b*(b*d + 2*c*Sqrt[d/x])*Sqrt[a + b*Sqrt[d/x] + c/x])/(4*c^2) - (2*(a + b*S
qrt[d/x] + c/x)^(3/2))/(3*c) + (b*Sqrt[d]*(4*a*c - b^2*d)*ArcTanh[(b*d + 2*
c*Sqrt[d/x])/(2*Sqrt[c]*Sqrt[d]*Sqrt[a + b*Sqrt[d/x] + c/x])])/(8*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1970

```
Int[(x_)^(m_.)*((a_) + (b_.)*((d_.)/(x_))^(n_) + (c_.)*(x_)^(n2_.))^(p_.),
x_Symbol] :> -Dist[d^(m + 1), Subst[Int[(a + b*x^n + (c*x^(2*n))/d^(2*n))^p
/x^(m + 2), x], x, d/x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n2, -2*n
] && IntegerQ[2*n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^2}\,dx = -\frac{\text{Subst}\left(\int \sqrt{a+b\sqrt{x}+\frac{cx}{d}}\,dx, x, \frac{d}{x}\right)}{d}$$

$$= -\frac{2\,\text{Subst}\left(\int x\sqrt{a+bx+\frac{cx^2}{d}}\,dx, x, \sqrt{\frac{d}{x}}\right)}{d}$$

$$= -\frac{2\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{3c} + \frac{b\,\text{Subst}\left(\int \sqrt{a+bx+\frac{cx^2}{d}}\,dx, x, \sqrt{\frac{d}{x}}\right)}{c}$$

$$= \frac{b\left(bd+2c\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{4c^2} - \frac{2\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{3c} + \frac{\left(b\left(4ac-b^2d\right)\right)\text{Subst}\left(\int -\right.}{8c^2}$$

$$= \frac{b\left(bd+2c\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{4c^2} - \frac{2\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{3c} + \frac{\left(b\left(4ac-b^2d\right)\right)\text{Subst}\left(\int \frac{4}{d}\right.}{4c^2}$$

$$= \frac{b\left(bd+2c\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{4c^2} - \frac{2\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{3c} + \frac{b\sqrt{d}\,\left(4ac-b^2d\right)\tanh^{-1}\left(\right.}{8c^{5/2}}$$

Mathematica [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sqrt[d/x] + c/x]/x^2,x]
```

```
[Out] Integrate[Sqrt[a + b*Sqrt[d/x] + c/x]/x^2, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x+b*(d/x)^(1/2))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x+b*(d/x)^(1/2))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.13, size = 331, normalized size = 2.14

$$\frac{\sqrt{\frac{ax+\sqrt{\frac{d}{x}}\,bx+c}{x}}\left(-3\left(\frac{d}{x}\right)^{\frac{3}{2}}b^3\sqrt{c}\,x^3\ln\left(\frac{\sqrt{\frac{d}{x}}\,bx+2c+2\sqrt{ax+\sqrt{\frac{d}{x}}\,bx+c}\,\sqrt{c}}{\sqrt{x}}\right)+12\sqrt{\frac{d}{x}}\,ab\,c^{\frac{3}{2}}x^2\ln\left(\frac{\sqrt{\frac{d}{x}}\,bx+2c+2\sqrt{ax+\sqrt{\frac{d}{x}}\,bx+c}\,\sqrt{c}}{\sqrt{x}}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(d/x)^(1/2)*b+c/x)^(1/2)/x^2,x)
```

```
[Out] 1/24*((a*x+(d/x)^(1/2)*b*x+c)/x)^(1/2)/x*(-3*ln(((d/x)^(1/2)*b*x+2*c+2*(a*x
+(d/x)^(1/2)*b*x+c)^(1/2)*c^(1/2))/x^(1/2))*c^(1/2)*(d/x)^(3/2)*x^3*b^3+6*(
a*x+(d/x)^(1/2)*b*x+c)^(1/2)*(d/x)^(3/2)*x^3*b^3+6*(a*x+(d/x)^(1/2)*b*x+c)^
(1/2)*a*d*x^2*b^2+12*ln(((d/x)^(1/2)*b*x+2*c+2*(a*x+(d/x)^(1/2)*b*x+c)^(1/2
)*c^(1/2))/x^(1/2))*c^(3/2)*a*(d/x)^(1/2)*x^2*b-6*(a*x+(d/x)^(1/2)*b*x+c)^(
3/2)*d*x*b^2-12*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*a*(d/x)^(1/2)*x^2*b*c+12*(a*x
+(d/x)^(1/2)*b*x+c)^(3/2)*(d/x)^(1/2)*x*b*c-16*(a*x+(d/x)^(1/2)*b*x+c)^(3/2
)*c^2)/(a*x+(d/x)^(1/2)*b*x+c)^(1/2)/c^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sqrt{\frac{d}{x}}+a+\frac{c}{x}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x+b*(d/x)^(1/2))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sqrt(d/x) + a + c/x)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c/x + b*(d/x)^(1/2))^(1/2)/x^2,x)
```

```
[Out] int((a + c/x + b*(d/x)^(1/2))^(1/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x+b*(d/x)**(1/2))**(1/2)/x**2,x)
[Out] Integral(sqrt(a + b*sqrt(d/x) + c/x)/x**2, x)
```

3.3058 $\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^3}\,dx$

Optimal. Leaf size=233

$$-\frac{b\sqrt{d}\left(12ac-7b^2d\right)\left(4ac-b^2d\right)\tanh^{-1}\left(\frac{bd+2c\sqrt{\frac{d}{x}}}{2\sqrt{c}\sqrt{d}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right)}{128c^{9/2}}-\frac{b\left(12ac-7b^2d\right)\left(bd+2c\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{64c^4}+\frac{\left(3$$

```
[Out] -1/128*b*(-7*b^2*d+12*a*c)*(-b^2*d+4*a*c)*arctanh(1/2*(b*d+2*c*(d/x)^(1/2))
/c^(1/2)/d^(1/2)/(a+c/x+b*(d/x)^(1/2))^(1/2))*d^(1/2)/c^(9/2)-2/5*(a+c/x+b*
(d/x)^(1/2))^(3/2)/c/x+1/120*(a+c/x+b*(d/x)^(1/2))^(3/2)*(32*a*c-35*b^2*d+4
2*b*c*(d/x)^(1/2))/c^3-1/64*b*(-7*b^2*d+12*a*c)*(b*d+2*c*(d/x)^(1/2))*(a+c/
x+b*(d/x)^(1/2))^(1/2)/c^4
```

Rubi [A] time = 0.31, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1970, 1357, 742, 779, 612, 621, 206}

$$\frac{\left(32ac-35b^2d+42bc\sqrt{\frac{d}{x}}\right)\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{120c^3}-\frac{b\left(12ac-7b^2d\right)\left(bd+2c\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{64c^4}-\frac{b\sqrt{d}\left(12ac-7b^2d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[d/x] + c/x]/x^3,x]
```

```
[Out] -(b*(12*a*c - 7*b^2*d)*(b*d + 2*c*Sqrt[d/x])*Sqrt[a + b*Sqrt[d/x] + c/x])/(
64*c^4) + ((32*a*c - 35*b^2*d + 42*b*c*Sqrt[d/x])*(a + b*Sqrt[d/x] + c/x)^(
3/2))/(120*c^3) - (2*(a + b*Sqrt[d/x] + c/x)^(3/2))/(5*c*x) - (b*Sqrt[d]*(1
2*a*c - 7*b^2*d)*(4*a*c - b^2*d)*ArcTanh[(b*d + 2*c*Sqrt[d/x])/(2*Sqrt[c]*S
qrt[d]*Sqrt[a + b*Sqrt[d/x] + c/x])])/(128*c^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
```

```
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1970

```
Int[(x_)^(m_.)*((a_) + (b_.)*((d_.)/(x_))^(n_) + (c_.)*(x_)^(n2_.))^(p_.),
x_Symbol] :> -Dist[d^(m + 1), Subst[Int[(a + b*x^n + (c*x^(2*n))/d^(2*n))^p
/x^(m + 2), x], x, d/x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n2, -2*n
] && IntegerQ[2*n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^3}\,dx = -\frac{\text{Subst}\left(\int x\sqrt{a+b\sqrt{x}+\frac{cx}{d}}\,dx, x, \frac{d}{x}\right)}{d^2}$$

$$= -\frac{2\,\text{Subst}\left(\int x^3\sqrt{a+bx+\frac{cx^2}{d}}\,dx, x, \sqrt{\frac{d}{x}}\right)}{d^2}$$

$$= -\frac{2\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{5cx} - \frac{2\,\text{Subst}\left(\int x\left(-2a-\frac{7bx}{2}\right)\sqrt{a+bx+\frac{cx^2}{d}}\,dx, x, \sqrt{\frac{d}{x}}\right)}{5cd}$$

$$= \frac{\left(32ac-7b\left(5bd-6c\sqrt{\frac{d}{x}}\right)\right)\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{120c^3} - \frac{2\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{5cx} - \frac{\left(b\left(12ac-7b^2d\right)\right.}{}$$

$$= -\frac{b\left(12ac-7b^2d\right)\left(bd+2c\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{64c^4} + \frac{\left(32ac-7b\left(5bd-6c\sqrt{\frac{d}{x}}\right)\right)\left(a+b\sqrt{\frac{d}{x}}\right.}{120c^3}$$

$$= -\frac{b\left(12ac-7b^2d\right)\left(bd+2c\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{64c^4} + \frac{\left(32ac-7b\left(5bd-6c\sqrt{\frac{d}{x}}\right)\right)\left(a+b\sqrt{\frac{d}{x}}\right.}{120c^3}$$

$$= -\frac{b\left(12ac-7b^2d\right)\left(bd+2c\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{64c^4} + \frac{\left(32ac-7b\left(5bd-6c\sqrt{\frac{d}{x}}\right)\right)\left(a+b\sqrt{\frac{d}{x}}\right.}{120c^3}$$

Mathematica [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sqrt[d/x] + c/x]/x^3,x]
[Out] Integrate[Sqrt[a + b*Sqrt[d/x] + c/x]/x^3, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x+b*(d/x)^(1/2))^(1/2)/x^3,x, algorithm="fricas")
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x+b*(d/x)^(1/2))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.14, size = 615, normalized size = 2.64

$$-\frac{\sqrt{\frac{ax+\sqrt{\frac{d}{x}}\,bx+c}{x}}\left(105\left(\frac{d}{x}\right)^{\frac{5}{2}}b^5\sqrt{c}\,x^5\ln\left(\frac{\sqrt{\frac{d}{x}}\,bx+2c+2\sqrt{ax+\sqrt{\frac{d}{x}}\,bx+c}\,\sqrt{c}}{\sqrt{x}}\right)-600\left(\frac{d}{x}\right)^{\frac{3}{2}}a\,b^3c^{\frac{3}{2}}x^4\ln\left(\frac{\sqrt{\frac{d}{x}}\,bx+2c+2\sqrt{ax+\sqrt{\frac{d}{x}}\,b}\right.\right.}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(d/x)^(1/2)*b+c/x)^(1/2)/x^3,x)
```

```
[Out] -1/1920*((a*x+(d/x)^(1/2)*b*x+c)/x)^(1/2)/x^2*(105*c^(1/2)*ln(((d/x)^(1/2)*
b*x+2*c+2*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*c^(1/2))/x^(1/2))*(d/x)^(5/2)*x^5*b
^5-210*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*(d/x)^(5/2)*x^5*b^5-600*a*c^(3/2)*ln((
(d/x)^(1/2)*b*x+2*c+2*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*c^(1/2))/x^(1/2))*(d/x)
^(3/2)*x^4*b^3-210*a*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*d^2*x^3*b^4+720*a^2*c^(5
/2)*ln(((d/x)^(1/2)*b*x+2*c+2*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*c^(1/2))/x^(1/2
))*(d/x)^(1/2)*x^3*b+780*a*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*(d/x)^(3/2)*x^4*b^
3*c+360*a^2*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*d*x^3*b^2*c+210*(a*x+(d/x)^(1/2)*
b*x+c)^(3/2)*d^2*x^2*b^4-420*(a*x+(d/x)^(1/2)*b*x+c)^(3/2)*(d/x)^(3/2)*x^3*
b^3*c-720*a^2*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*(d/x)^(1/2)*x^3*b*c^2-360*a*(a*
x+(d/x)^(1/2)*b*x+c)^(3/2)*d*x^2*b^2*c+720*a*(a*x+(d/x)^(1/2)*b*x+c)^(3/2)*
(d/x)^(1/2)*x^2*b*c^2+560*(a*x+(d/x)^(1/2)*b*x+c)^(3/2)*d*x*b^2*c^2-512*a*(
a*x+(d/x)^(1/2)*b*x+c)^(3/2)*x*c^3-672*(a*x+(d/x)^(1/2)*b*x+c)^(3/2)*(d/x)^
(1/2)*x*b*c^3+768*(a*x+(d/x)^(1/2)*b*x+c)^(3/2)*c^4)/(a*x+(d/x)^(1/2)*b*x+c
)^(1/2)/c^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sqrt{\frac{d}{x}}+a+\frac{c}{x}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x+b*(d/x)^(1/2))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sqrt(d/x) + a + c/x)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c/x + b*(d/x)^(1/2))^(1/2)/x^3,x)
```

```
[Out] int((a + c/x + b*(d/x)^(1/2))^(1/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x+b*(d/x)**(1/2))**(1/2)/x**3,x)
[Out] Integral(sqrt(a + b*sqrt(d/x) + c/x)/x**3, x)
```

3.3059 $\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^4}\,dx$

Optimal. Leaf size=371

$$\frac{b\sqrt{d}\left(4ac-b^2d\right)\left(80a^2c^2-120ab^2cd+33b^4d^2\right)\tanh^{-1}\left(\frac{bd+2c\sqrt{\frac{d}{x}}}{2\sqrt{c}\,\sqrt{d}\,\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right)}{1024c^{13/2}}+\frac{b\left(80a^2c^2-120ab^2cd+33b^4d^2\right)(}{512c}$$

```
[Out] 1/1024*b*(-b^2*d+4*a*c)*(33*b^4*d^2-120*a*b^2*c*d+80*a^2*c^2)*arctanh(1/2*(
b*d+2*c*(d/x)^(1/2))/c^(1/2)/d^(1/2)/(a+c/x+b*(d/x)^(1/2))^(1/2))*d^(1/2)/c
^(13/2)+11/42*b*(d/x)^(3/2)*(a+c/x+b*(d/x)^(1/2))^(3/2)/c^2/d-2/7*(a+c/x+b*
(d/x)^(1/2))^(3/2)/c/x^2+1/140*(-33*b^2*d+32*a*c)*(a+c/x+b*(d/x)^(1/2))^(3/
2)/c^3/x-1/6720*(a+c/x+b*(d/x)^(1/2))^(3/2)*(1024*a^2*c^2-3276*a*b^2*c*d+11
55*b^4*d^2+18*b*c*(-77*b^2*d+148*a*c)*(d/x)^(1/2))/c^5+1/512*b*(33*b^4*d^2-
120*a*b^2*c*d+80*a^2*c^2)*(b*d+2*c*(d/x)^(1/2))*(a+c/x+b*(d/x)^(1/2))^(1/2)
/c^6
```

Rubi [A] time = 0.66, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1970, 1357, 742, 832, 779, 612, 621, 206}

$$-\frac{\left(1024a^2c^2+18bc\sqrt{\frac{d}{x}}\left(148ac-77b^2d\right)-3276ab^2cd+1155b^4d^2\right)\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{6720c^5}+\frac{b\left(80a^2c^2-120ab^2cd+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[d/x] + c/x]/x^4,x]
```

```
[Out] (b*(80*a^2*c^2 - 120*a*b^2*c*d + 33*b^4*d^2)*(b*d + 2*c*Sqrt[d/x])*Sqrt[a +
 b*Sqrt[d/x] + c/x])/(512*c^6) - ((1024*a^2*c^2 - 3276*a*b^2*c*d + 1155*b^4
*d^2 + 18*b*c*(148*a*c - 77*b^2*d)*Sqrt[d/x])*(a + b*Sqrt[d/x] + c/x)^(3/2)
)/(6720*c^5) + (11*b*(a + b*Sqrt[d/x] + c/x)^(3/2)*(d/x)^(3/2))/(42*c^2*d)
 - (2*(a + b*Sqrt[d/x] + c/x)^(3/2))/(7*c*x^2) + ((32*a*c - 33*b^2*d)*(a + b
*Sqrt[d/x] + c/x)^(3/2))/(140*c^3*x) + (b*Sqrt[d]*(4*a*c - b^2*d)*(80*a^2*c
^2 - 120*a*b^2*c*d + 33*b^4*d^2)*ArcTanh[(b*d + 2*c*Sqrt[d/x])/(2*Sqrt[c]*S
qrt[d]*Sqrt[a + b*Sqrt[d/x] + c/x])])/(1024*c^(13/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1970

```
Int[(x_)^(m_.)*((a_) + (b_.)*((d_.)/(x_))^(n_) + (c_.)*(x_)^(n2_.))^(p_.),
x_Symbol] :> -Dist[d^(m + 1), Subst[Int[(a + b*x^n + (c*x^(2*n))/d^(2*n))^p
/x^(m + 2), x], x, d/x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n2, -2*n
] && IntegerQ[2*n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^4}\,dx = -\frac{\text{Subst}\left(\int x^2\sqrt{a+b\sqrt{x}+\frac{cx}{d}}\,dx,x,\frac{d}{x}\right)}{d^3}$$

$$= -\frac{2\,\text{Subst}\left(\int x^5\sqrt{a+bx+\frac{cx^2}{d}}\,dx,x,\sqrt{\frac{d}{x}}\right)}{d^3}$$

$$= -\frac{2\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{7cx^2} - \frac{2\,\text{Subst}\left(\int x^3\left(-4a-\frac{11bx}{2}\right)\sqrt{a+bx+\frac{cx^2}{d}}\,dx,x,\sqrt{\frac{d}{x}}\right)}{7cd^2}$$

$$= \frac{11b\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}\left(\frac{d}{x}\right)^{3/2}}{42c^2d} - \frac{2\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{7cx^2} - \frac{\text{Subst}\left(\int x^2\left(\frac{33ab}{2}-\frac{3\left(32ac-33b^2\right.\right.\right.}{4d}\right.}{21c}$$

$$= \frac{11b\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}\left(\frac{d}{x}\right)^{3/2}}{42c^2d} - \frac{2\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{7cx^2} + \frac{\left(32ac-33b^2d\right)\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right.}{140c^3x}$$

$$= -\frac{\left(1024a^2c^2-3276ab^2cd+1155b^4d^2+18bc\left(148ac-77b^2d\right)\sqrt{\frac{d}{x}}\right)\left(a+b\sqrt{\frac{d}{x}}+\frac{c}{x}\right)^{3/2}}{6720c^5}$$

$$= \frac{b\left(80a^2c^2-120ab^2cd+33b^4d^2\right)\left(bd+2c\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{512c^6} - \frac{\left(1024a^2c^2-3276a\right.}{}$$

$$= \frac{b\left(80a^2c^2-120ab^2cd+33b^4d^2\right)\left(bd+2c\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{512c^6} - \frac{\left(1024a^2c^2-3276a\right.}{}$$

$$= \frac{b\left(80a^2c^2-120ab^2cd+33b^4d^2\right)\left(bd+2c\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{512c^6} - \frac{\left(1024a^2c^2-3276a\right.}{}$$

Mathematica [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sqrt[d/x] + c/x]/x^4,x]
[Out] Integrate[Sqrt[a + b*Sqrt[d/x] + c/x]/x^4, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x+b*(d/x)^(1/2))^(1/2)/x^4,x, algorithm="fricas")
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x+b*(d/x)^(1/2))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.15, size = 979, normalized size = 2.64

$$\frac{\sqrt{\frac{ax+\sqrt{\frac{d}{x}}\,bx+c}{x}}\left(-3465\left(\frac{d}{x}\right)^{\frac{7}{2}}b^7\sqrt{c}\,x^7\ln\left(\frac{\sqrt{\frac{d}{x}}\,bx+2c+2\sqrt{ax+\sqrt{\frac{d}{x}}\,bx+c}\,\sqrt{c}}{\sqrt{x}}\right)+26460\left(\frac{d}{x}\right)^{\frac{5}{2}}a\,b^5c^{\frac{3}{2}}x^6\ln\left(\frac{\sqrt{\frac{d}{x}}\,bx+2c+2\sqrt{ax+\sqrt{\frac{d}{x}}}\right.\right.}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(d/x)^(1/2)*b+c/x)^(1/2)/x^4,x)
```

```
[Out] 1/107520*((a*x+(d/x)^(1/2)*b*x+c)/x)^(1/2)/x^3*(-16384*(a*x+(d/x)^(1/2)*b*x
+c)^(3/2)*a^2*x^2*c^4+24576*(a*x+(d/x)^(1/2)*b*x+c)^(3/2)*a*x*c^5+6930*(a*x
+(d/x)^(1/2)*b*x+c)^(1/2)*(d/x)^(7/2)*x^7*b^7-6930*(a*x+(d/x)^(1/2)*b*x+c)^
(3/2)*d^3*x^3*b^6-58800*a^2*c^(5/2)*ln(((d/x)^(1/2)*b*x+2*c+2*(a*x+(d/x)^(1
/2)*b*x+c)^(1/2)*c^(1/2))/x^(1/2))*(d/x)^(3/2)*x^5*b^3+33600*a^3*c^(7/2)*ln
(((d/x)^(1/2)*b*x+2*c+2*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*c^(1/2))/x^(1/2))*(d/
x)^(1/2)*x^4*b+67200*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*a^2*(d/x)^(3/2)*x^5*b^3*
c^2-50400*(a*x+(d/x)^(1/2)*b*x+c)^(3/2)*a*(d/x)^(3/2)*x^4*b^3*c^2-33600*(a*
x+(d/x)^(1/2)*b*x+c)^(1/2)*a^3*(d/x)^(1/2)*x^4*b*c^3+33600*(a*x+(d/x)^(1/2)
*b*x+c)^(3/2)*a^2*(d/x)^(1/2)*x^3*b*c^3-42624*(a*x+(d/x)^(1/2)*b*x+c)^(3/2)
*a*(d/x)^(1/2)*x^2*b*c^4+26460*a*c^(3/2)*ln(((d/x)^(1/2)*b*x+2*c+2*(a*x+(d/
x)^(1/2)*b*x+c)^(1/2)*c^(1/2))/x^(1/2))*(d/x)^(5/2)*x^6*b^5+52416*(a*x+(d/x
)^(1/2)*b*x+c)^(3/2)*a*d*x^2*b^2*c^3-25200*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*a^
2*d^2*x^4*b^4*c+25200*(a*x+(d/x)^(1/2)*b*x+c)^(3/2)*a*d^2*x^3*b^4*c+16800*(
a*x+(d/x)^(1/2)*b*x+c)^(1/2)*a^3*d*x^4*b^2*c^2-16800*(a*x+(d/x)^(1/2)*b*x+c
)^(3/2)*a^2*d*x^3*b^2*c^2-39060*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*a*(d/x)^(5/2)
*x^6*b^5*c+28160*(a*x+(d/x)^(1/2)*b*x+c)^(3/2)*(d/x)^(1/2)*x*b*c^5-30720*(a
*x+(d/x)^(1/2)*b*x+c)^(3/2)*c^6-3465*c^(1/2)*ln(((d/x)^(1/2)*b*x+2*c+2*(a*x
+(d/x)^(1/2)*b*x+c)^(1/2)*c^(1/2))/x^(1/2))*(d/x)^(7/2)*x^7*b^7+13860*(a*x+
(d/x)^(1/2)*b*x+c)^(3/2)*(d/x)^(5/2)*x^5*b^5*c+22176*(a*x+(d/x)^(1/2)*b*x+c
)^(3/2)*(d/x)^(3/2)*x^3*b^3*c^3+6930*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*a*d^3*x^
4*b^6-18480*(a*x+(d/x)^(1/2)*b*x+c)^(3/2)*d^2*x^2*b^4*c^2-25344*(a*x+(d/x)^
(1/2)*b*x+c)^(3/2)*d*x*b^2*c^4)/(a*x+(d/x)^(1/2)*b*x+c)^(1/2)/c^7
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sqrt{\frac{d}{x}}+a+\frac{c}{x}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x+b*(d/x)^(1/2))^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sqrt(d/x) + a + c/x)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c/x + b*(d/x)^(1/2))^(1/2)/x^4,x)
[Out] int((a + c/x + b*(d/x)^(1/2))^(1/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\sqrt{\frac{d}{x}} + \frac{c}{x}}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x+b*(d/x)**(1/2))**(1/2)/x**4,x)
[Out] Integral(sqrt(a + b*sqrt(d/x) + c/x)/x**4, x)
```

3.3060 $\int \frac{x^m}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$

Optimal. Leaf size=230

$$\frac{x^{m+1}\sqrt{\frac{2c\sqrt{\frac{d}{x}}}{\sqrt{d}\left(b\sqrt{d}-\sqrt{b^2d-4ac}\right)}+1}\sqrt{\frac{2c\sqrt{\frac{d}{x}}}{\sqrt{d}\left(\sqrt{b^2d-4ac}+b\sqrt{d}\right)}+1}\,F_1\left(-2(m+1);\frac{1}{2},\frac{1}{2};-2m-1;-\frac{2c\sqrt{\frac{d}{x}}}{\sqrt{d}\left(b\sqrt{d}-\sqrt{b^2d-4ac}\right)},-\frac{2c}{\sqrt{d}\left(\sqrt{d}\,b+\right.}\right.}{(m+1)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}$$

```
[Out] x^(1+m)*AppellF1(-2-2*m,1/2,1/2,-1-2*m,-2*c*(d/x)^(1/2)/d^(1/2)/(b*d^(1/2)-
(b^2*d-4*a*c)^(1/2)),-2*c*(d/x)^(1/2)/d^(1/2)/(b*d^(1/2)+(b^2*d-4*a*c)^(1/2
)))*(1+2*c*(d/x)^(1/2)/d^(1/2)/(b*d^(1/2)-(b^2*d-4*a*c)^(1/2)))^(1/2)*(1+2*
c*(d/x)^(1/2)/d^(1/2)/(b*d^(1/2)+(b^2*d-4*a*c)^(1/2)))^(1/2)/(1+m)/(a+c/x+b
*(d/x)^(1/2))^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1971, 1379, 759, 133}

$$\frac{x^{m+1}\sqrt{\frac{2c\sqrt{\frac{d}{x}}}{\sqrt{d}\left(b\sqrt{d}-\sqrt{b^2d-4ac}\right)}+1}\sqrt{\frac{2c\sqrt{\frac{d}{x}}}{\sqrt{d}\left(\sqrt{b^2d-4ac}+b\sqrt{d}\right)}+1}\,F_1\left(-2(m+1);\frac{1}{2},\frac{1}{2};-2m-1;-\frac{2c\sqrt{\frac{d}{x}}}{\sqrt{d}\left(b\sqrt{d}-\sqrt{b^2d-4ac}\right)},-\frac{2c}{\sqrt{d}\left(\sqrt{d}\,b+\right.}\right.}{(m+1)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/Sqrt[a + b*Sqrt[d/x] + c/x],x]
```

```
[Out] (Sqrt[1 + (2*c*Sqrt[d/x])/(Sqrt[d]*(b*Sqrt[d] - Sqrt[-4*a*c + b^2*d]))]*Sqr
t[1 + (2*c*Sqrt[d/x])/(Sqrt[d]*(b*Sqrt[d] + Sqrt[-4*a*c + b^2*d]))]*x^(1 +
m)*AppellF1[-2*(1 + m), 1/2, 1/2, -1 - 2*m, (-2*c*Sqrt[d/x])/(Sqrt[d]*(b*Sq
rt[d] - Sqrt[-4*a*c + b^2*d])), (-2*c*Sqrt[d/x])/(Sqrt[d]*(b*Sqrt[d] + Sqrt
[-4*a*c + b^2*d]))])/((1 + m)*Sqrt[a + b*Sqrt[d/x] + c/x])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 1379

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x
^(k*n) + c*x^(2*k*n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, m, p}, x]
```

```
&& EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && FractionQ[n]
```

Rule 1971

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*((d_.)/(x_))^(n_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> -Dist[d*(e*x)^m*(d/x)^m, Subst[Int[(a + b*x^n + (c*x^(2
*n))/d^(2*n))^p/x^(m + 2), x], x, d/x], x] /; FreeQ[{a, b, c, d, e, n, p},
x] && EqQ[n2, -2*n] &&  !IntegerQ[m] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{x^m}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx = -\left(\left(d\left(\frac{d}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2-m}}{\sqrt{a+b\sqrt{x}+\frac{cx}{d}}}\,dx, x, \frac{d}{x}\right)\right)$$

$$= -\left(\left(2d\left(\frac{d}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-1+2(-1-m)}}{\sqrt{a+bx+\frac{cx^2}{d}}}\,dx, x, \sqrt{\frac{d}{x}}\right)\right)$$

$$= -\frac{\left(2d\sqrt{1+\frac{2c\sqrt{\frac{d}{x}}}{d\left(b-\frac{\sqrt{-4ac+b^2d}}{\sqrt{d}}\right)}}\sqrt{1+\frac{2c\sqrt{\frac{d}{x}}}{d\left(b+\frac{\sqrt{-4ac+b^2d}}{\sqrt{d}}\right)}}\left(\frac{d}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-1-}}{\sqrt{1+\frac{2cx}{\sqrt{d}\left(b\sqrt{d}-\sqrt{-4ac+b^2d}\right.}}}\right.}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}$$

$$= \frac{\sqrt{1+\frac{2c\sqrt{\frac{d}{x}}}{\sqrt{d}\left(b\sqrt{d}-\sqrt{-4ac+b^2d}\right)}}\sqrt{1+\frac{2c\sqrt{\frac{d}{x}}}{\sqrt{d}\left(b\sqrt{d}+\sqrt{-4ac+b^2d}\right)}}\,x^{1+m}F_1\left(-2(1+m);\frac{1}{2},\frac{1}{2};-1-2m\right.}{(1+m)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}$$

Mathematica [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/Sqrt[a + b*Sqrt[d/x] + c/x],x]
```

```
[Out] Integrate[x^m/Sqrt[a + b*Sqrt[d/x] + c/x], x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+c/x+b*(d/x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   alglo
gextint: unimplemented
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+c/x+b*(d/x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument ValueWarning, int
egration of abs or sign assumes constant sign by intervals (correct if the
argument is real):Check [abs(t_nostep)]Warning, integration of abs or sign
assumes constant sign by intervals (correct if the argument is real):Check
[abs(t_nostep)]Evaluation time: 0.51Done
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{a + \sqrt{\frac{d}{x}}\, b + \frac{c}{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a+(d/x)^(1/2)*b+c/x)^(1/2),x)
```

```
[Out] int(x^m/(a+(d/x)^(1/2)*b+c/x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{b\sqrt{\frac{d}{x}} + a + \frac{c}{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a+c/x+b*(d/x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/sqrt(b*sqrt(d/x) + a + c/x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^m}{\sqrt{a + \frac{c}{x} + b\sqrt{\frac{d}{x}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + c/x + b*(d/x)^(1/2))^(1/2),x)
```

```
[Out] int(x^m/(a + c/x + b*(d/x)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{a + b\sqrt{\frac{d}{x}} + \frac{c}{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a+c/x+b*(d/x)**(1/2))**(1/2),x)
```

```
[Out] Integral(x**m/sqrt(a + b*sqrt(d/x) + c/x), x)
```

3.3061 $\int \frac{x^2}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$

Optimal. Leaf size=386

$$\frac{bd^2\left(156ac-77b^2d\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{160a^4\left(\frac{d}{x}\right)^{3/2}}-\frac{x^2\left(100ac-99b^2d\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{240a^3}-\frac{11bd^3\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{30a^2\left(\frac{d}{x}\right)^{5/2}}-\frac{7bd\left(528a$$

```
[Out] -1/512*(-231*b^6*d^3+1260*a*b^4*c*d^2-1680*a^2*b^2*c^2*d+320*a^3*c^3)*arcta
nh(1/2*(2*a+b*(d/x)^(1/2))/a^(1/2)/(a+c/x+b*(d/x)^(1/2))^(1/2))/a^(13/2)-11
/30*b*d^3*(a+c/x+b*(d/x)^(1/2))^(1/2)/a^2/(d/x)^(5/2)+1/160*b*d^2*(-77*b^2*
d+156*a*c)*(a+c/x+b*(d/x)^(1/2))^(1/2)/a^4/(d/x)^(3/2)+1/640*(385*b^4*d^2-1
176*a*b^2*c*d+400*a^2*c^2)*x*(a+c/x+b*(d/x)^(1/2))^(1/2)/a^5-1/240*(-99*b^2
*d+100*a*c)*x^2*(a+c/x+b*(d/x)^(1/2))^(1/2)/a^3+1/3*x^3*(a+c/x+b*(d/x)^(1/2
))^(1/2)/a-7/1280*b*d*(165*b^4*d^2-680*a*b^2*c*d+528*a^2*c^2)*(a+c/x+b*(d/x
)^(1/2))^(1/2)/a^6/(d/x)^(1/2)
```

Rubi [A] time = 0.73, antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1970, 1357, 744, 834, 806, 724, 206}

$$-\frac{7bd\left(528a^2c^2-680ab^2cd+165b^4d^2\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{1280a^6\sqrt{\frac{d}{x}}}+\frac{x\left(400a^2c^2-1176ab^2cd+385b^4d^2\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{640a^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*Sqrt[d/x] + c/x],x]
```

```
[Out] (-11*b*d^3*Sqrt[a + b*Sqrt[d/x] + c/x])/(30*a^2*(d/x)^(5/2)) + (b*d^2*(156*
a*c - 77*b^2*d)*Sqrt[a + b*Sqrt[d/x] + c/x])/(160*a^4*(d/x)^(3/2)) - (7*b*d
*(528*a^2*c^2 - 680*a*b^2*c*d + 165*b^4*d^2)*Sqrt[a + b*Sqrt[d/x] + c/x])/(
1280*a^6*Sqrt[d/x]) + ((400*a^2*c^2 - 1176*a*b^2*c*d + 385*b^4*d^2)*Sqrt[a
+ b*Sqrt[d/x] + c/x]*x)/(640*a^5) - ((100*a*c - 99*b^2*d)*Sqrt[a + b*Sqrt[d
/x] + c/x]*x^2)/(240*a^3) + (Sqrt[a + b*Sqrt[d/x] + c/x]*x^3)/(3*a) - ((320
*a^3*c^3 - 1680*a^2*b^2*c^2*d + 1260*a*b^4*c*d^2 - 231*b^6*d^3)*ArcTanh[(2*
a + b*Sqrt[d/x])/(2*Sqrt[a]*Sqrt[a + b*Sqrt[d/x] + c/x])])/(512*a^(13/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
 + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1970

```
Int[(x_)^(m_.)*((a_) + (b_.)*((d_.)/(x_))^(n_) + (c_.)*(x_)^(n2_.))^(p_.),
x_Symbol] :> -Dist[d^(m + 1), Subst[Int[(a + b*x^n + (c*x^(2*n))/d^(2*n))^p
/x^(m + 2), x], x, d/x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n2, -2*n
] && IntegerQ[2*n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx = -\left(d^3\,\text{Subst}\left(\int \frac{1}{x^4\sqrt{a+b\sqrt{x}+\frac{cx}{d}}}\,dx, x, \frac{d}{x}\right)\right)$$

$$= -\left(\left(2d^3\right)\text{Subst}\left(\int \frac{1}{x^7\sqrt{a+bx+\frac{cx^2}{d}}}\,dx, x, \sqrt{\frac{d}{x}}\right)\right)$$

$$= \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^3}{3a} + \frac{d^3\,\text{Subst}\left(\int \frac{\frac{11b}{2}+\frac{5cx}{d}}{x^6\sqrt{a+bx+\frac{cx^2}{d}}}\,dx, x, \sqrt{\frac{d}{x}}\right)}{3a}$$

$$= -\frac{11bd^3\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{30a^2\left(\frac{d}{x}\right)^{5/2}} + \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^3}{3a} - \frac{d^3\,\text{Subst}\left(\int \frac{\frac{1}{4}\left(99b^2-\frac{100ac}{d}\right)+\frac{22bcx}{d}}{x^5\sqrt{a+bx+\frac{cx^2}{d}}}\,dx, x,\right.}{15a^2}$$

$$= -\frac{11bd^3\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{30a^2\left(\frac{d}{x}\right)^{5/2}} - \frac{\left(100ac-99b^2d\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^2}{240a^3} + \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^3}{3a}$$

$$= -\frac{11bd^3\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{30a^2\left(\frac{d}{x}\right)^{5/2}} + \frac{bd^2\left(156ac-77b^2d\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{160a^4\left(\frac{d}{x}\right)^{3/2}} - \frac{\left(100ac-99b^2d\right)}{240}$$

$$= -\frac{11bd^3\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{30a^2\left(\frac{d}{x}\right)^{5/2}} + \frac{bd^2\left(156ac-77b^2d\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{160a^4\left(\frac{d}{x}\right)^{3/2}} + \frac{\left(400a^2c^2-1176a\right.}{}$$

$$= -\frac{11bd^3\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{30a^2\left(\frac{d}{x}\right)^{5/2}} + \frac{bd^2\left(156ac-77b^2d\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{160a^4\left(\frac{d}{x}\right)^{3/2}} - \frac{7bd\left(528a^2c^2-68\right.}{}$$

$$= -\frac{11bd^3\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{30a^2\left(\frac{d}{x}\right)^{5/2}} + \frac{bd^2\left(156ac-77b^2d\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{160a^4\left(\frac{d}{x}\right)^{3/2}} - \frac{7bd\left(528a^2c^2-68\right.}{}$$

$$= -\frac{11bd^3\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{30a^2\left(\frac{d}{x}\right)^{5/2}} + \frac{bd^2\left(156ac-77b^2d\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{160a^4\left(\frac{d}{x}\right)^{3/2}} - \frac{7bd\left(528a^2c^2-68\right.}{}$$

Mathematica [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2/Sqrt[a + b*Sqrt[d/x] + c/x],x]
```

```
[Out] Integrate[x^2/Sqrt[a + b*Sqrt[d/x] + c/x], x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+c/x+b*(d/x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+c/x+b*(d/x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument ValueWarning, int
egration of abs or sign assumes constant sign by intervals (correct if the
argument is real):Check [abs(t_nostep)]Warning, integration of abs or sign
assumes constant sign by intervals (correct if the argument is real):Check
[abs(t_nostep)]Evaluation time: 0.53Done
```

maple [A] time = 0.16, size = 655, normalized size = 1.70

$$\frac{\sqrt{\frac{ax+\sqrt{\frac{d}{x}}\,bx+c}{x}}\left(3465a\,b^6d^3\ln\left(\frac{2a\sqrt{x}+\sqrt{\frac{d}{x}}\,b\sqrt{x}+2\sqrt{ax+\sqrt{\frac{d}{x}}\,bx+c}\,\sqrt{a}}{2\sqrt{a}}\right)-18900a^2b^4c\,d^2\ln\left(\frac{2a\sqrt{x}+\sqrt{\frac{d}{x}}\,b\sqrt{x}+2\sqrt{ax+\sqrt{\frac{d}{x}}\,bx+c}\,\sqrt{}}{2\sqrt{a}}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+(d/x)^(1/2)*b+c/x)^(1/2),x)
```

```
[Out] 1/7680*((a*x+(d/x)^(1/2)*b*x+c)/x)^(1/2)*x^(1/2)*(2560*x^(5/2)*(a*x+(d/x)^(
1/2)*b*x+c)^(1/2)*a^(13/2)-2816*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*a^(11/2)*(d/x
)^(1/2)*x^(5/2)*b-3696*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*a^(7/2)*(d/x)^(3/2)*x^
(5/2)*b^3-6930*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*(d/x)^(5/2)*a^(3/2)*b^5*x^(5/2
)-3200*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*a^(11/2)*x^(3/2)*c+7488*(a*x+(d/x)^(1/
2)*b*x+c)^(1/2)*a^(9/2)*(d/x)^(1/2)*x^(3/2)*b*c+3168*(a*x+(d/x)^(1/2)*b*x+c
)^(1/2)*d*a^(9/2)*x^(3/2)*b^2+28560*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*(d/x)^(3/
2)*a^(5/2)*b^3*c*x^(3/2)+4800*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*a^(9/2)*c^2*x^(
1/2)-22176*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*(d/x)^(1/2)*a^(7/2)*b*c^2*x^(1/2)-
14112*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*a^(7/2)*b^2*c*d*x^(1/2)+4620*(a*x+(d/x)
^(1/2)*b*x+c)^(1/2)*a^(5/2)*b^4*d^2*x^(1/2)+3465*a*b^6*d^3*ln(1/2*(2*a*x^(1
```

```
/2)+(d/x)^(1/2)*b*x^(1/2)+2*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*a^(1/2))/a^(1/2))
-18900*a^2*b^4*c*d^2*ln(1/2*(2*a*x^(1/2)+(d/x)^(1/2)*b*x^(1/2)+2*(a*x+(d/x)
^(1/2)*b*x+c)^(1/2)*a^(1/2))/a^(1/2))+25200*a^3*b^2*c^2*d*ln(1/2*(2*a*x^(1/
2)+(d/x)^(1/2)*b*x^(1/2)+2*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*a^(1/2))/a^(1/2))-
4800*a^4*c^3*ln(1/2*(2*a*x^(1/2)+(d/x)^(1/2)*b*x^(1/2)+2*(a*x+(d/x)^(1/2)*b
*x+c)^(1/2)*a^(1/2))/a^(1/2)))/(a*x+(d/x)^(1/2)*b*x+c)^(1/2)/a^(15/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{b\sqrt{\frac{d}{x}} + a + \frac{c}{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+c/x+b*(d/x)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/sqrt(b*sqrt(d/x) + a + c/x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{a + \frac{c}{x} + b\sqrt{\frac{d}{x}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + c/x + b*(d/x)^(1/2))^(1/2),x)
[Out] int(x^2/(a + c/x + b*(d/x)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a + b\sqrt{\frac{d}{x}} + \frac{c}{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+c/x+b*(d/x)**(1/2))**(1/2),x)
[Out] Integral(x**2/sqrt(a + b*sqrt(d/x) + c/x), x)
```

3.3062 $\int \frac{x}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$

Optimal. Leaf size=248

$$\frac{5bd\left(44ac-21b^2d\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{96a^4\sqrt{\frac{d}{x}}}-\frac{x\left(36ac-35b^2d\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{48a^3}-\frac{7bd^2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{12a^2\left(\frac{d}{x}\right)^{3/2}}+\frac{\left(48a^2c^2-120ab$$

```
[Out] 1/64*(35*b^4*d^2-120*a*b^2*c*d+48*a^2*c^2)*arctanh(1/2*(2*a+b*(d/x)^(1/2))/
a^(1/2)/(a+c/x+b*(d/x)^(1/2))^(1/2))/a^(9/2)-7/12*b*d^2*(a+c/x+b*(d/x)^(1/2
))^(1/2)/a^2/(d/x)^(3/2)-1/48*(-35*b^2*d+36*a*c)*x*(a+c/x+b*(d/x)^(1/2))^(1
/2)/a^3+1/2*x^2*(a+c/x+b*(d/x)^(1/2))^(1/2)/a+5/96*b*d*(-21*b^2*d+44*a*c)*(
a+c/x+b*(d/x)^(1/2))^(1/2)/a^4/(d/x)^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1970, 1357, 744, 834, 806, 724, 206}

$$\frac{\left(48a^2c^2-120ab^2cd+35b^4d^2\right)\tanh^{-1}\left(\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right)}{64a^{9/2}}+\frac{5bd\left(44ac-21b^2d\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{96a^4\sqrt{\frac{d}{x}}}-\frac{x\left(36ac-35b^2d\right)}{48a}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*Sqrt[d/x] + c/x],x]
```

```
[Out] (-7*b*d^2*Sqrt[a + b*Sqrt[d/x] + c/x])/(12*a^2*(d/x)^(3/2)) + (5*b*d*(44*a*
c - 21*b^2*d)*Sqrt[a + b*Sqrt[d/x] + c/x])/(96*a^4*Sqrt[d/x]) - ((36*a*c -
35*b^2*d)*Sqrt[a + b*Sqrt[d/x] + c/x]*x)/(48*a^3) + (Sqrt[a + b*Sqrt[d/x] +
 c/x]*x^2)/(2*a) + ((48*a^2*c^2 - 120*a*b^2*c*d + 35*b^4*d^2)*ArcTanh[(2*a
+ b*Sqrt[d/x])/(2*Sqrt[a]*Sqrt[a + b*Sqrt[d/x] + c/x])])/(64*a^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
```

```
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
 + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1970

```
Int[(x_)^(m_.)*((a_) + (b_.)*((d_.)/(x_))^(n_) + (c_.)*(x_)^(n2_.))^(p_.),
x_Symbol] :> -Dist[d^(m + 1), Subst[Int[(a + b*x^n + (c*x^(2*n))/d^(2*n))^p
/x^(m + 2), x], x, d/x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n2, -2*n
] && IntegerQ[2*n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx = -\left(d^2\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+b\sqrt{x}+\frac{cx}{d}}}\,dx, x, \frac{d}{x}\right)\right)$$

$$= -\left(\left(2d^2\right)\text{Subst}\left(\int \frac{1}{x^5\sqrt{a+bx+\frac{cx^2}{d}}}\,dx, x, \sqrt{\frac{d}{x}}\right)\right)$$

$$= \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^2}{2a} + \frac{d^2\,\text{Subst}\left(\int \frac{\frac{7b}{2}+\frac{3cx}{d}}{x^4\sqrt{a+bx+\frac{cx^2}{d}}}\,dx, x, \sqrt{\frac{d}{x}}\right)}{2a}$$

$$= -\frac{7bd^2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{12a^2\left(\frac{d}{x}\right)^{3/2}} + \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^2}{2a} - \frac{d^2\,\text{Subst}\left(\int \frac{\frac{1}{4}\left(35b^2-\frac{36ac}{d}\right)+\frac{7bcx}{d}}{x^3\sqrt{a+bx+\frac{cx^2}{d}}}\,dx, x, \sqrt{\frac{d}{x}}\right)}{6a^2}$$

$$= -\frac{7bd^2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{12a^2\left(\frac{d}{x}\right)^{3/2}} - \frac{\left(36ac-35b^2d\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x}{48a^3} + \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^2}{2a} + \frac{d^2\,\text{S}}{}$$

$$= -\frac{7bd^2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{12a^2\left(\frac{d}{x}\right)^{3/2}} + \frac{5bd\left(44ac-21b^2d\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{96a^4\sqrt{\frac{d}{x}}} - \frac{\left(36ac-35b^2d\right)\sqrt{a+b}}{48a^3}$$

$$= -\frac{7bd^2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{12a^2\left(\frac{d}{x}\right)^{3/2}} + \frac{5bd\left(44ac-21b^2d\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{96a^4\sqrt{\frac{d}{x}}} - \frac{\left(36ac-35b^2d\right)\sqrt{a+b}}{48a^3}$$

$$= -\frac{7bd^2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{12a^2\left(\frac{d}{x}\right)^{3/2}} + \frac{5bd\left(44ac-21b^2d\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{96a^4\sqrt{\frac{d}{x}}} - \frac{\left(36ac-35b^2d\right)\sqrt{a+b}}{48a^3}$$

Mathematica [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/Sqrt[a + b*Sqrt[d/x] + c/x],x]
[Out] Integrate[x/Sqrt[a + b*Sqrt[d/x] + c/x], x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+c/x+b*(d/x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+c/x+b*(d/x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument ValueWarning, int
egration of abs or sign assumes constant sign by intervals (correct if the
argument is real):Check [abs(t_nostep)]Warning, integration of abs or sign
assumes constant sign by intervals (correct if the argument is real):Check
[abs(t_nostep)]Evaluation time: 0.47Done
```

maple [A] time = 0.14, size = 398, normalized size = 1.60

$$\frac{\sqrt{\frac{ax+\sqrt{\frac{d}{x}}\,bx+c}{x}}\left(105a\,b^4d^2\ln\left(\frac{2a\sqrt{x}+\sqrt{\frac{d}{x}}\,b\sqrt{x}+2\sqrt{ax+\sqrt{\frac{d}{x}}\,bx+c}\,\sqrt{a}}{2\sqrt{a}}\right)-360a^2b^2cd\ln\left(\frac{2a\sqrt{x}+\sqrt{\frac{d}{x}}\,b\sqrt{x}+2\sqrt{ax+\sqrt{\frac{d}{x}}\,bx+c}\,\sqrt{a}}{2\sqrt{a}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+(d/x)^(1/2)*b+c/x)^(1/2),x)
```

```
[Out] 1/192*((a*x+(d/x)^(1/2)*b*x+c)/x)^(1/2)*x^(1/2)*(96*x^(3/2)*(a*x+(d/x)^(1/2
)*b*x+c)^(1/2)*a^(9/2)-112*a^(7/2)*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*(d/x)^(1/2
)*x^(3/2)*b-210*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*(d/x)^(3/2)*a^(3/2)*b^3*x^(3/
2)+140*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*a^(5/2)*b^2*d*x^(1/2)+105*a*b^4*d^2*ln
(1/2*(2*a*x^(1/2)+(d/x)^(1/2)*b*x^(1/2)+2*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*a^(
1/2))/a^(1/2))-144*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*a^(7/2)*c*x^(1/2)+440*(a*x
+(d/x)^(1/2)*b*x+c)^(1/2)*(d/x)^(1/2)*a^(5/2)*b*c*x^(1/2)-360*a^2*b^2*c*d*l
n(1/2*(2*a*x^(1/2)+(d/x)^(1/2)*b*x^(1/2)+2*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*a^
(1/2))/a^(1/2))+144*a^3*c^2*ln(1/2*(2*a*x^(1/2)+(d/x)^(1/2)*b*x^(1/2)+2*(a*
x+(d/x)^(1/2)*b*x+c)^(1/2)*a^(1/2))/a^(1/2)))/(a*x+(d/x)^(1/2)*b*x+c)^(1/2)
/a^(11/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{b\sqrt{\frac{d}{x}}+a+\frac{c}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+c/x+b*(d/x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(b*sqrt(d/x) + a + c/x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + c/x + b*(d/x)^(1/2))^(1/2),x)
```

```
[Out] int(x/(a + c/x + b*(d/x)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a + b\sqrt{\frac{d}{x}} + \frac{c}{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+c/x+b*(d/x)**(1/2))**(1/2),x)
```

```
[Out] Integral(x/sqrt(a + b*sqrt(d/x) + c/x), x)
```

3.3063 $\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$

Optimal. Leaf size=135

$$-\frac{\left(4ac-3b^2d\right)\tanh^{-1}\left(\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right)}{4a^{5/2}}-\frac{3bd\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{2a^2\sqrt{\frac{d}{x}}}+\frac{x\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{a}$$

```
[Out] -1/4*(-3*b^2*d+4*a*c)*arctanh(1/2*(2*a+b*(d/x)^(1/2))/a^(1/2)/(a+c/x+b*(d/x
)^(1/2))^(1/2))/a^(5/2)+x*(a+c/x+b*(d/x)^(1/2))^(1/2)/a-3/2*b*d*(a+c/x+b*(d
/x)^(1/2))^(1/2)/a^2/(d/x)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1969, 1357, 744, 806, 724, 206}

$$-\frac{\left(4ac-3b^2d\right)\tanh^{-1}\left(\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right)}{4a^{5/2}}-\frac{3bd\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{2a^2\sqrt{\frac{d}{x}}}+\frac{x\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sqrt[d/x] + c/x],x]
```

```
[Out] (-3*b*d*Sqrt[a + b*Sqrt[d/x] + c/x])/(2*a^2*Sqrt[d/x]) + (Sqrt[a + b*Sqrt[d
/x] + c/x]*x)/a - ((4*a*c - 3*b^2*d)*ArcTanh[(2*a + b*Sqrt[d/x])/(2*Sqrt[a]
*Sqrt[a + b*Sqrt[d/x] + c/x])])/(4*a^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1969

```
Int[((a_.) + (b_.)*((d_.)/(x_))^(n_) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol]
:> -Dist[d, Subst[Int[(a + b*x^n + (c*x^(2*n))/d^(2*n))^p/x^2, x], x, d/x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n2, -2*n] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx = -\left(d\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+b\sqrt{x}+\frac{cx}{d}}}\,dx, x, \frac{d}{x}\right)\right)$$

$$= -\left((2d)\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx+\frac{cx^2}{d}}}\,dx, x, \sqrt{\frac{d}{x}}\right)\right)$$

$$= \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x}{a} + \frac{d\,\text{Subst}\left(\int \frac{\frac{3b}{2}+\frac{cx}{d}}{x^2\sqrt{a+bx+\frac{cx^2}{d}}}\,dx, x, \sqrt{\frac{d}{x}}\right)}{a}$$

$$= -\frac{3bd\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{2a^2\sqrt{\frac{d}{x}}} + \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x}{a} + \frac{\left(4ac-3b^2d\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+\frac{cx^2}{d}}}\,dx, x, \sqrt{}\right.}{4a^2}$$

$$= -\frac{3bd\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{2a^2\sqrt{\frac{d}{x}}} + \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x}{a} - \frac{\left(4ac-3b^2d\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+b\sqrt{}}{\sqrt{a+b\sqrt{\frac{d}{x}}}}\right.}{2a^2}$$

$$= -\frac{3bd\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{2a^2\sqrt{\frac{d}{x}}} + \frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x}{a} - \frac{\left(4ac-3b^2d\right)\tanh^{-1}\left(\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right)}{4a^{5/2}}$$

Mathematica [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/Sqrt[a + b*Sqrt[d/x] + c/x],x]
```

```
[Out] Integrate[1/Sqrt[a + b*Sqrt[d/x] + c/x], x]
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x+b*(d/x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [**A**] time = 0.59, size = 218, normalized size = 1.61

$$-\frac{2\sqrt{ad^2x+\sqrt{dx}\,bd^2+cd^2}\left(\frac{3b}{a^2}-\frac{2\sqrt{dx}}{ad}\right)+\frac{\left(3b^2d^2-4acd\right)\log\left(\left|-bd^2-2\sqrt{ad}\left(\sqrt{ad}\,\sqrt{dx}-\sqrt{ad^2x+\sqrt{dx}\,bd^2+cd^2}\right)\right|\right)}{\sqrt{ad}\,a^2}-\frac{3b^2d^2\log\left(\left|-b\right.\right.}{4\sqrt{d}\,\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x+b*(d/x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(2*sqrt(a*d^2*x + sqrt(d*x)*b*d^2 + c*d^2)*(3*b/a^2 - 2*sqrt(d*x)/(a*d
)) + (3*b^2*d^2 - 4*a*c*d)*log(abs(-b*d^2 - 2*sqrt(a*d)*(sqrt(a*d)*sqrt(d*x
) - sqrt(a*d^2*x + sqrt(d*x)*b*d^2 + c*d^2))))/(sqrt(a*d)*a^2) - (3*b^2*d^2
*log(abs(-b*d^2 + 2*sqrt(c*d^2)*sqrt(a*d))) - 4*a*c*d*log(abs(-b*d^2 + 2*sq
rt(c*d^2)*sqrt(a*d))) + 6*sqrt(c*d^2)*sqrt(a*d)*b)/(sqrt(a*d)*a^2))/(sqrt(d
)*sgn(x))
```

maple [**A**] time = 0.14, size = 213, normalized size = 1.58

$$-\frac{\sqrt{\frac{ax+\sqrt{\frac{d}{x}}\,bx+c}{x}}\left(-3ab^2d\ln\left(\frac{2a\sqrt{x}+\sqrt{\frac{d}{x}}\,b\sqrt{x}+2\sqrt{ax+\sqrt{\frac{d}{x}}\,bx+c}\,\sqrt{a}}{2\sqrt{a}}\right)+4a^2c\ln\left(\frac{2a\sqrt{x}+\sqrt{\frac{d}{x}}\,b\sqrt{x}+2\sqrt{ax+\sqrt{\frac{d}{x}}\,bx+c}\,\sqrt{a}}{2\sqrt{a}}\right)-4\sqrt{}}{4\sqrt{ax+\sqrt{\frac{d}{x}}\,bx+c}\,a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+(d/x)^(1/2)*b+c/x)^(1/2),x)
```

```
[Out] -1/4*((a*x+(d/x)^(1/2)*b*x+c)/x)^(1/2)*x^(1/2)*(6*(a*x+(d/x)^(1/2)*b*x+c)^(
1/2)*(d/x)^(1/2)*a^(3/2)*b*x^(1/2)-4*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*a^(5/2)*
x^(1/2)-3*a*b^2*d*ln(1/2*(2*a*x^(1/2)+(d/x)^(1/2)*b*x^(1/2)+2*(a*x+(d/x)^(1
/2)*b*x+c)^(1/2)*a^(1/2))/a^(1/2))+4*a^2*c*ln(1/2*(2*a*x^(1/2)+(d/x)^(1/2)*
b*x^(1/2)+2*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*a^(1/2))/a^(1/2)))/(a*x+(d/x)^(1/
2)*b*x+c)^(1/2)/a^(7/2)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sqrt{\frac{d}{x}}+a+\frac{c}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x+b*(d/x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sqrt(d/x) + a + c/x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c/x + b*(d/x)^(1/2))^(1/2),x)
[Out] int(1/(a + c/x + b*(d/x)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x+b*(d/x)**(1/2))**(1/2),x)
[Out] Integral(1/sqrt(a + b*sqrt(d/x) + c/x), x)
```

3.3064 $\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}x}}\,dx$

Optimal. Leaf size=54

$$\frac{2\tanh^{-1}\left(\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right)}{\sqrt{a}}$$

```
[Out] 2*arctanh(1/2*(2*a+b*(d/x)^(1/2))/a^(1/2)/(a+c/x+b*(d/x)^(1/2))^(1/2))/a^(1
/2)
```

Rubi [A] time = 0.10, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1970, 1357, 724, 206}

$$\frac{2\tanh^{-1}\left(\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*Sqrt[d/x] + c/x]*x),x]
```

```
[Out] (2*ArcTanh[(2*a + b*Sqrt[d/x])/(2*Sqrt[a]*Sqrt[a + b*Sqrt[d/x] + c/x])])/Sq
rt[a]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1970

```
Int[(x_)^(m_.)*((a_) + (b_.)*((d_.)/(x_))^(n_) + (c_.)*(x_)^(n2_.))^(p_.),
x_Symbol] :> -Dist[d^(m + 1), Subst[Int[(a + b*x^n + (c*x^(2*n))/d^(2*n))^p
/x^(m + 2), x], x, d/x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n2, -2*n
] && IntegerQ[2*n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x}\,dx = -\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+b\sqrt{x}+\frac{cx}{d}}}\,dx, x, \frac{d}{x}\right)$$
$$= -\left(2\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx+\frac{cx^2}{d}}}\,dx, x, \sqrt{\frac{d}{x}}\right)\right)$$
$$= 4\operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+b\sqrt{\frac{d}{x}}}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right)$$
$$= \frac{2\tanh^{-1}\left(\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right)}{\sqrt{a}}$$

Mathematica [A] time = 0.39, size = 54, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*Sqrt[d/x] + c/x]*x),x]
```

```
[Out] (2*ArcTanh[(2*a + b*Sqrt[d/x])/(2*Sqrt[a]*Sqrt[a + b*Sqrt[d/x] + c/x])])/Sq
rt[a]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+c/x+b*(d/x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+c/x+b*(d/x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument ValueWarning, int
egration of abs or sign assumes constant sign by intervals (correct if the
argument is real):Check [abs(t_nostep)]Warning, integration of abs or sign
assumes constant sign by intervals (correct if the argument is real):Check
[abs(t_nostep)]Evaluation time: 0.42Done
```

maple [B] time = 0.14, size = 94, normalized size = 1.74

$$\frac{2\sqrt{\frac{ax+\sqrt{\frac{d}{x}}\,bx+c}{x}}\,\sqrt{x}\,\ln\left(\frac{2a\sqrt{x}+\sqrt{\frac{d}{x}}\,b\sqrt{x}+2\sqrt{ax+\sqrt{\frac{d}{x}}\,bx+c}\,\sqrt{a}}{2\sqrt{a}}\right)}{\sqrt{ax+\sqrt{\frac{d}{x}}\,bx+c}\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+(d/x)^(1/2)*b+c/x)^(1/2),x)
```

```
[Out] 2*((a*x+(d/x)^(1/2)*b*x+c)/x)^(1/2)*x^(1/2)/(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*l
n(1/2*(2*a*x^(1/2)+(d/x)^(1/2)*b*x^(1/2)+2*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*a^
(1/2))/a^(1/2))/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sqrt{\frac{d}{x}}+a+\frac{c}{x}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+c/x+b*(d/x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sqrt(d/x) + a + c/x)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + c/x + b*(d/x)^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x*(a + c/x + b*(d/x)^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+c/x+b*(d/x)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*sqrt(d/x) + c/x)), x)
```

3.3065 $\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^2}\,dx$

Optimal. Leaf size=93

$$\frac{b\sqrt{d}\,\tanh^{-1}\left(\frac{bd+2c\sqrt{\frac{d}{x}}}{2\sqrt{c}\,\sqrt{d}\,\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right)}{c^{3/2}}-\frac{2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{c}$$

```
[Out] b*arctanh(1/2*(b*d+2*c*(d/x)^(1/2))/c^(1/2)/d^(1/2)/(a+c/x+b*(d/x)^(1/2))^(
1/2))*d^(1/2)/c^(3/2)-2*(a+c/x+b*(d/x)^(1/2))^(1/2)/c
```

Rubi [A] time = 0.12, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1970, 1341, 640, 621, 206}

$$\frac{b\sqrt{d}\,\tanh^{-1}\left(\frac{bd+2c\sqrt{\frac{d}{x}}}{2\sqrt{c}\,\sqrt{d}\,\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right)}{c^{3/2}}-\frac{2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*Sqrt[d/x] + c/x]*x^2),x]
```

```
[Out] (-2*Sqrt[a + b*Sqrt[d/x] + c/x])/c + (b*Sqrt[d]*ArcTanh[(b*d + 2*c*Sqrt[d/x
])/(2*Sqrt[c]*Sqrt[d]*Sqrt[a + b*Sqrt[d/x] + c/x])])/c^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1341

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1970

```
Int[(x_)^(m_.)*((a_) + (b_.)*((d_.)/(x_))^(n_) + (c_.)*(x_)^(n2_.))^(p_.),
x_Symbol] :> -Dist[d^(m + 1), Subst[Int[(a + b*x^n + (c*x^(2*n))/d^(2*n))^p
```

```
/x^(m + 2), x], x, d/x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n2, -2*n
] && IntegerQ[2*n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b\sqrt{x}+\frac{cx}{d}}}\,dx, x, \frac{d}{x}\right)}{d}$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{x}{\sqrt{a+bx+\frac{cx^2}{d}}}\,dx, x, \sqrt{\frac{d}{x}}\right)}{d}$$

$$= -\frac{2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{c} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+\frac{cx^2}{d}}}\,dx, x, \sqrt{\frac{d}{x}}\right)}{c}$$

$$= -\frac{2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{c} + \frac{(2b)\,\text{Subst}\left(\int \frac{1}{\frac{4c}{d}-x^2}\,dx, x, \frac{b+\frac{2c\sqrt{\frac{d}{x}}}{d}}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right)}{c}$$

$$= -\frac{2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{c} + \frac{b\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\left(b+\frac{2c\sqrt{\frac{d}{x}}}{d}\right)}{2\sqrt{c}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right)}{c^{3/2}}$$

Mathematica [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sqrt[a + b*Sqrt[d/x] + c/x]*x^2),x]
```

```
[Out] Integrate[1/(Sqrt[a + b*Sqrt[d/x] + c/x]*x^2), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+c/x+b*(d/x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+c/x+b*(d/x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, checking for positivity of a root depending of parameters might return
wrong sign: -dWarning, checking for positivity of a root depending of param
eters might return wrong sign: d-1/d/abs(d)*d^2*2*(1/d/c*sqrt(a*d^2+b*d^2*s
qrt(d/x)+c*d*d/x)+2*b*d/4/c/sqrt(c*d)*ln(abs(2*sqrt(c*d)*(sqrt(a*d^2+b*d^2*
sqrt(d/x)+c*d*d/x)-sqrt(c*d)*sqrt(d/x))-d^2*b)))
```

maple [A] time = 0.14, size = 120, normalized size = 1.29

$$-\frac{\sqrt{\frac{ax+\sqrt{\frac{d}{x}}\,bx+c}{x}}\left(-\sqrt{\frac{d}{x}}\,bcx\ln\left(\frac{\sqrt{\frac{d}{x}}\,bx+2c+2\sqrt{ax+\sqrt{\frac{d}{x}}\,bx+c}\,\sqrt{c}}{\sqrt{x}}\right)+2\sqrt{ax+\sqrt{\frac{d}{x}}\,bx+c}\,c^{\frac{3}{2}}\right)}{\sqrt{ax+\sqrt{\frac{d}{x}}\,bx+c}\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+(d/x)^(1/2)*b+c/x)^(1/2),x)
```

```
[Out] -((a*x+(d/x)^(1/2)*b*x+c)/x)^(1/2)*(-b*(d/x)^(1/2)*x*ln(((d/x)^(1/2)*b*x+2*
c+2*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*c^(1/2))/x^(1/2))*c+2*(a*x+(d/x)^(1/2)*b*
x+c)^(1/2)*c^(3/2))/(a*x+(d/x)^(1/2)*b*x+c)^(1/2)/c^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sqrt{\frac{d}{x}}+a+\frac{c}{x}}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+c/x+b*(d/x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sqrt(d/x) + a + c/x)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + c/x + b*(d/x)^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x^2*(a + c/x + b*(d/x)^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+c/x+b*(d/x)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a + b*sqrt(d/x) + c/x)), x)
```

3.3066 $\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^3}\,dx$

Optimal. Leaf size=165

$$-\frac{b\sqrt{d}\left(12ac-5b^2d\right)\tanh^{-1}\left(\frac{bd+2c\sqrt{\frac{d}{x}}}{2\sqrt{c}\sqrt{d}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right)}{8c^{7/2}}+\frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\left(16ac-15b^2d+10bc\sqrt{\frac{d}{x}}\right)}{12c^3}-\frac{2\sqrt{a+b\sqrt{\frac{d}{x}}+}}{3cx}$$

```
[Out] -1/8*b*(-5*b^2*d+12*a*c)*arctanh(1/2*(b*d+2*c*(d/x)^(1/2))/c^(1/2)/d^(1/2)/
(a+c/x+b*(d/x)^(1/2))^(1/2))*d^(1/2)/c^(7/2)-2/3*(a+c/x+b*(d/x)^(1/2))^(1/2
)/c/x+1/12*(16*a*c-15*b^2*d+10*b*c*(d/x)^(1/2))*(a+c/x+b*(d/x)^(1/2))^(1/2)
/c^3
```

Rubi [A] time = 0.23, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1970, 1357, 742, 779, 621, 206}

$$\frac{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\left(16ac-15b^2d+10bc\sqrt{\frac{d}{x}}\right)}{12c^3}-\frac{b\sqrt{d}\left(12ac-5b^2d\right)\tanh^{-1}\left(\frac{bd+2c\sqrt{\frac{d}{x}}}{2\sqrt{c}\sqrt{d}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right)}{8c^{7/2}}-\frac{2\sqrt{a+b\sqrt{\frac{d}{x}}+}}{3cx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*Sqrt[d/x] + c/x]*x^3),x]
```

```
[Out] ((16*a*c - 15*b^2*d + 10*b*c*Sqrt[d/x])*Sqrt[a + b*Sqrt[d/x] + c/x])/(12*c^
3) - (2*Sqrt[a + b*Sqrt[d/x] + c/x])/(3*c*x) - (b*Sqrt[d]*(12*a*c - 5*b^2*d
)*ArcTanh[(b*d + 2*c*Sqrt[d/x])/(2*Sqrt[c]*Sqrt[d]*Sqrt[a + b*Sqrt[d/x] + c
/x])])/(8*c^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
```

```
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1970

```
Int[(x_)^(m_.)*((a_) + (b_.)*((d_.)/(x_))^(n_) + (c_.)*(x_)^(n2_.))^(p_.),
x_Symbol] :> -Dist[d^(m + 1), Subst[Int[(a + b*x^n + (c*x^(2*n))/d^(2*n))^p
/x^(m + 2), x], x, d/x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n2, -2*n
] && IntegerQ[2*n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x}{\sqrt{a+b\sqrt{x}+\frac{cx}{d}}}\,dx, x, \frac{d}{x}\right)}{d^2}$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{x^3}{\sqrt{a+bx+\frac{cx^2}{d}}}\,dx, x, \sqrt{\frac{d}{x}}\right)}{d^2}$$

$$= -\frac{2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{3cx} - \frac{2\,\text{Subst}\left(\int \frac{x\left(-2a-\frac{5bx}{2}\right)}{\sqrt{a+bx+\frac{cx^2}{d}}}\,dx, x, \sqrt{\frac{d}{x}}\right)}{3cd}$$

$$= \frac{\left(16ac-5b\left(3bd-2c\sqrt{\frac{d}{x}}\right)\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{12c^3} - \frac{2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{3cx} - \frac{\left(b\left(12ac-5b^2d\right)\right)}{}$$

$$= \frac{\left(16ac-5b\left(3bd-2c\sqrt{\frac{d}{x}}\right)\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{12c^3} - \frac{2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{3cx} - \frac{\left(b\left(12ac-5b^2d\right)\right)}{}$$

$$= \frac{\left(16ac-5b\left(3bd-2c\sqrt{\frac{d}{x}}\right)\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{12c^3} - \frac{2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{3cx} - \frac{b\sqrt{d}\,\left(12ac-5b^2d\right.}{}$$

Mathematica [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sqrt[a + b*Sqrt[d/x] + c/x]*x^3),x]
```

```
[Out] Integrate[1/(Sqrt[a + b*Sqrt[d/x] + c/x]*x^3), x]
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+c/x+b*(d/x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+c/x+b*(d/x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, checking for positivity of a root depending of parameters might return
wrong sign: -dWarning, checking for positivity of a root depending of param
eters might return wrong sign: d-1/d/d/abs(d)*d^2*2*(2*((16*c^2/96/d/c^3*sq
rt(d/x)-20*d*c*b/96/d/c^3)*sqrt(d/x)+(30*d^2*b^2-32*d*c*a)/96/d/c^3)*sqrt(a
*d^2+b*d^2*sqrt(d/x)+c*d*d/x)+2*(-12*a*b*c*d^2+5*b^3*d^3)/32/c^3/sqrt(c*d)*
ln(abs(2*sqrt(c*d)*(sqrt(a*d^2+b*d^2*sqrt(d/x)+c*d*d/x)-sqrt(c*d)*sqrt(d/x)
)-d^2*b)))
```

maple [**A**] time = 0.16, size = 267, normalized size = 1.62

$$-\frac{\sqrt{\frac{ax+\sqrt{\frac{d}{x}}\,bx+c}{x}}\left(-15\left(\frac{d}{x}\right)^{\frac{3}{2}}b^3c\,x^3\ln\left(\frac{\sqrt{\frac{d}{x}}\,bx+2c+2\sqrt{ax+\sqrt{\frac{d}{x}}\,bx+c}\,\sqrt{c}}{\sqrt{x}}\right)+36\sqrt{\frac{d}{x}}\,ab\,c^2x^2\ln\left(\frac{\sqrt{\frac{d}{x}}\,bx+2c+2\sqrt{ax+\sqrt{\frac{d}{x}}\,bx+c}\,\sqrt{c}}{\sqrt{x}}\right.\right.}{24\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+(d/x)^(1/2)*b+c/x)^(1/2),x)
```

```
[Out] -1/24*((a*x+(d/x)^(1/2)*b*x+c)/x)^(1/2)/x*(-15*ln(((d/x)^(1/2)*b*x+2*c+2*(a
*x+(d/x)^(1/2)*b*x+c)^(1/2)*c^(1/2))/x^(1/2))*(d/x)^(3/2)*x^3*b^3*c+30*c^(3
/2)*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*d*x*b^2+36*ln(((d/x)^(1/2)*b*x+2*c+2*(a*x
+(d/x)^(1/2)*b*x+c)^(1/2)*c^(1/2))/x^(1/2))*(d/x)^(1/2)*x^2*a*b*c^2-20*c^(5
/2)*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*(d/x)^(1/2)*x*b-32*c^(5/2)*(a*x+(d/x)^(1/
2)*b*x+c)^(1/2)*x*a+16*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*c^(7/2))/(a*x+(d/x)^(1
/2)*b*x+c)^(1/2)/c^(9/2)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sqrt{\frac{d}{x}}+a+\frac{c}{x}x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+c/x+b*(d/x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sqrt(d/x) + a + c/x)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3 \sqrt{a + \frac{c}{x} + b \sqrt{\frac{d}{x}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + c/x + b*(d/x)^(1/2))^(1/2)),x)
[Out] int(1/(x^3*(a + c/x + b*(d/x)^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \sqrt{a + b \sqrt{\frac{d}{x}} + \frac{c}{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+c/x+b*(d/x)**(1/2))**(1/2),x)
[Out] Integral(1/(x**3*sqrt(a + b*sqrt(d/x) + c/x)), x)
```

3.3067 $\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^4}\,dx$

Optimal. Leaf size=289

$$\frac{b\sqrt{d}\left(240a^2c^2-280ab^2cd+63b^4d^2\right)\tanh^{-1}\left(\frac{bd+2c\sqrt{\frac{d}{x}}}{2\sqrt{c}\sqrt{d}\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}\right)}{128c^{11/2}}-\frac{\left(1024a^2c^2+14bc\sqrt{\frac{d}{x}}\left(92ac-45b^2d\right)-294\right.}{960c^5}$$

```
[Out] 1/128*b*(63*b^4*d^2-280*a*b^2*c*d+240*a^2*c^2)*arctanh(1/2*(b*d+2*c*(d/x)^(
1/2))/c^(1/2)/d^(1/2)/(a+c/x+b*(d/x)^(1/2))^(1/2))*d^(1/2)/c^(11/2)+9/20*b*
(d/x)^(3/2)*(a+c/x+b*(d/x)^(1/2))^(1/2)/c^2/d-2/5*(a+c/x+b*(d/x)^(1/2))^(1/
2)/c/x^2+1/120*(-63*b^2*d+64*a*c)*(a+c/x+b*(d/x)^(1/2))^(1/2)/c^3/x-1/960*(
1024*a^2*c^2-2940*a*b^2*c*d+945*b^4*d^2+14*b*c*(-45*b^2*d+92*a*c)*(d/x)^(1/
2))*(a+c/x+b*(d/x)^(1/2))^(1/2)/c^5
```

Rubi [A] time = 0.54, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1970, 1357, 742, 832, 779, 621, 206}

$$-\frac{\left(1024a^2c^2+14bc\sqrt{\frac{d}{x}}\left(92ac-45b^2d\right)-2940ab^2cd+945b^4d^2\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{960c^5}+\frac{b\sqrt{d}\left(240a^2c^2-280ab^2cd+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*Sqrt[d/x] + c/x]*x^4),x]
```

```
[Out] -((1024*a^2*c^2 - 2940*a*b^2*c*d + 945*b^4*d^2 + 14*b*c*(92*a*c - 45*b^2*d)
*Sqrt[d/x])*Sqrt[a + b*Sqrt[d/x] + c/x])/(960*c^5) + (9*b*Sqrt[a + b*Sqrt[d
/x] + c/x]*(d/x)^(3/2))/(20*c^2*d) - (2*Sqrt[a + b*Sqrt[d/x] + c/x])/(5*c*x
^2) + ((64*a*c - 63*b^2*d)*Sqrt[a + b*Sqrt[d/x] + c/x])/(120*c^3*x) + (b*Sq
rt[d]*(240*a^2*c^2 - 280*a*b^2*c*d + 63*b^4*d^2)*ArcTanh[(b*d + 2*c*Sqrt[d/
x])/(2*Sqrt[c]*Sqrt[d]*Sqrt[a + b*Sqrt[d/x] + c/x])])/(128*c^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1970

```
Int[(x_)^(m_.)*((a_) + (b_.)*((d_.)/(x_))^(n_) + (c_.)*(x_)^(n2_.))^(p_.),
x_Symbol] :> -Dist[d^(m + 1), Subst[Int[(a + b*x^n + (c*x^(2*n))/d^(2*n))^p
/x^(m + 2), x], x, d/x], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n2, -2*n
] && IntegerQ[2*n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^4}\,dx = -\frac{\text{Subst}\left(\int \frac{x^2}{\sqrt{a+b\sqrt{x}+\frac{cx}{d}}}\,dx, x, \frac{d}{x}\right)}{d^3}$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{x^5}{\sqrt{a+bx+\frac{cx^2}{d}}}\,dx, x, \sqrt{\frac{d}{x}}\right)}{d^3}$$

$$= -\frac{2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{5cx^2} - \frac{2\,\text{Subst}\left(\int \frac{x^3\left(-4a-\frac{9bx}{2}\right)}{\sqrt{a+bx+\frac{cx^2}{d}}}\,dx, x, \sqrt{\frac{d}{x}}\right)}{5cd^2}$$

$$= \frac{9b\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\left(\frac{d}{x}\right)^{3/2}}{20c^2d} - \frac{2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{5cx^2} - \frac{\text{Subst}\left(\int \frac{x^2\left(\frac{27ab}{2}-\frac{\left(64ac-63b^2d\right)x}{4d}\right)}{\sqrt{a+bx+\frac{cx^2}{d}}}\,dx, x\right.}{10c^2d}$$

$$= \frac{9b\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\left(\frac{d}{x}\right)^{3/2}}{20c^2d} - \frac{2\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{5cx^2} + \frac{\left(64ac-63b^2d\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{120c^3x}$$

$$= -\frac{\left(1024a^2c^2-2940ab^2cd+945b^4d^2+14bc\left(92ac-45b^2d\right)\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{960c^5}$$

$$= -\frac{\left(1024a^2c^2-2940ab^2cd+945b^4d^2+14bc\left(92ac-45b^2d\right)\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{960c^5}$$

$$= -\frac{\left(1024a^2c^2-2940ab^2cd+945b^4d^2+14bc\left(92ac-45b^2d\right)\sqrt{\frac{d}{x}}\right)\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}}{960c^5}$$

Mathematica [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{d}{x}}+\frac{c}{x}}\,x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sqrt[a + b*Sqrt[d/x] + c/x]*x^4),x]
[Out] Integrate[1/(Sqrt[a + b*Sqrt[d/x] + c/x]*x^4), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+c/x+b*(d/x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+c/x+b*(d/x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument ValueWarning, int
egration of abs or sign assumes constant sign by intervals (correct if the
argument is real):Check [abs(t_nostep)]Warning, integration of abs or sign
assumes constant sign by intervals (correct if the argument is real):Check
[abs(t_nostep)]Evaluation time: 0.5Done
```

maple [A] time = 0.16, size = 487, normalized size = 1.69

$$\frac{\sqrt{\frac{ax+\sqrt{\frac{d}{x}}\,bx+c}{x}}\left(945\left(\frac{d}{x}\right)^{\frac{5}{2}}b^5c\,x^5\ln\left(\frac{\sqrt{\frac{d}{x}}\,bx+2c+2\sqrt{ax+\sqrt{\frac{d}{x}}\,bx+c}\,\sqrt{c}}{\sqrt{x}}\right)-4200\left(\frac{d}{x}\right)^{\frac{3}{2}}a\,b^3c^2x^4\ln\left(\frac{\sqrt{\frac{d}{x}}\,bx+2c+2\sqrt{ax+\sqrt{\frac{d}{x}}\,bx+c}\,\sqrt{c}}{\sqrt{x}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a+(d/x)^(1/2)*b+c/x)^(1/2),x)
```

```
[Out] 1/1920*((a*x+(d/x)^(1/2)*b*x+c)/x)^(1/2)*(945*ln(((d/x)^(1/2)*b*x+2*c+2*(a*
x+(d/x)^(1/2)*b*x+c)^(1/2)*c^(1/2))/x^(1/2))*(d/x)^(5/2)*x^5*b^5*c-4200*ln(
((d/x)^(1/2)*b*x+2*c+2*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*c^(1/2))/x^(1/2))*(d/x
)^(3/2)*x^4*a*b^3*c^2+1260*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*c^(5/2)*(d/x)^(3/2
)*x^3*b^3-1890*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*c^(3/2)*d^2*x^2*b^4-2576*(a*x+
(d/x)^(1/2)*b*x+c)^(1/2)*c^(7/2)*(d/x)^(1/2)*x^2*a*b+3600*ln(((d/x)^(1/2)*b
*x+2*c+2*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*c^(1/2))/x^(1/2))*(d/x)^(1/2)*x^3*a^
2*b*c^3+864*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*c^(9/2)*(d/x)^(1/2)*x*b-1008*(a*x
+(d/x)^(1/2)*b*x+c)^(1/2)*c^(7/2)*d*x*b^2+5880*(a*x+(d/x)^(1/2)*b*x+c)^(1/2
)*c^(5/2)*d*x^2*a*b^2-768*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*c^(11/2)+1024*(a*x+
(d/x)^(1/2)*b*x+c)^(1/2)*c^(9/2)*x*a-2048*(a*x+(d/x)^(1/2)*b*x+c)^(1/2)*c^(
7/2)*x^2*a^2)/x^2/(a*x+(d/x)^(1/2)*b*x+c)^(1/2)/c^(13/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sqrt{\frac{d}{x}}+a+\frac{c}{x}}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+c/x+b*(d/x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sqrt(d/x) + a + c/x)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + c/x + b*(d/x)^(1/2))^(1/2)),x)
[Out] int(1/(x^4*(a + c/x + b*(d/x)^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt{a + b\sqrt{\frac{d}{x}} + \frac{c}{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a+c/x+b*(d/x)**(1/2))**(1/2),x)
[Out] Integral(1/(x**4*sqrt(a + b*sqrt(d/x) + c/x)), x)
```

3.3068 $\int \sqrt{\sqrt{\frac{1}{x}}+\frac{1}{x}}\,dx$

Optimal. Leaf size=26

$$\frac{4\left(\sqrt{\frac{1}{x}}+\frac{1}{x}\right)^{3/2}}{3\left(\frac{1}{x}\right)^{3/2}}$$

```
[Out] 4/3*(1/x+(1/x)^(1/2))^(3/2)/(1/x)^(3/2)
```

Rubi [A] time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1966, 2014}

$$\frac{4\left(\sqrt{\frac{1}{x}}+\frac{1}{x}\right)^{3/2}}{3\left(\frac{1}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sqrt[x^(-1)] + x^(-1)],x]
```

```
[Out] (4*(Sqrt[x^(-1)] + x^(-1))^(3/2))/(3*(x^(-1))^(3/2))
```

Rule 1966

```
Int[((a_.) + (c_.)*((d_.)/(x_))^(n2_.) + (b_.)*((d_.)/(x_))^(n_))^(p_.), x_
Symbol] :> -Dist[d, Subst[Int[(a + b*x^n + c*x^(2*n))^p/x^2, x], x, d/x], x
] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n2, 2*n]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \sqrt{\sqrt{\frac{1}{x}}+\frac{1}{x}}\,dx = -\text{Subst}\left(\int \frac{\sqrt{\sqrt{x}+x}}{x^2}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{4\left(\sqrt{\frac{1}{x}}+\frac{1}{x}\right)^{3/2}}{3\left(\frac{1}{x}\right)^{3/2}}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 1.00

$$\frac{4\left(\sqrt{\frac{1}{x}}+\frac{1}{x}\right)^{3/2}}{3\left(\frac{1}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sqrt[x^(-1)] + x^(-1)],x]
[Out] (4*(Sqrt[x^(-1)] + x^(-1))^(3/2))/(3*(x^(-1))^(3/2))
```

fricas [A] time = 0.93, size = 18, normalized size = 0.69

$$\frac{4}{3}\left(x+\sqrt{x}\right)\sqrt{\frac{\sqrt{x}+1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x+(1/x)^(1/2))^(1/2),x, algorithm="fricas")
[Out] 4/3*(x + sqrt(x))*sqrt((sqrt(x) + 1)/x)
```

giac [A] time = 0.17, size = 11, normalized size = 0.42

$$\frac{4}{3}\left(\sqrt{x}+1\right)^{\frac{3}{2}}-\frac{4}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x+(1/x)^(1/2))^(1/2),x, algorithm="giac")
[Out] 4/3*(sqrt(x) + 1)^(3/2) - 4/3
```

maple [A] time = 0.11, size = 32, normalized size = 1.23

$$\frac{4\sqrt{\frac{\sqrt{\frac{1}{x}}\,x+1}{x}}\,\left(\sqrt{\frac{1}{x}}\,x+1\right)}{3\sqrt{\frac{1}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x+(1/x)^(1/2))^(1/2),x)
[Out] 4/3*(((1/x)^(1/2)*x+1)/x)^(1/2)*((1/x)^(1/2)*x+1)/(1/x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{1}{\sqrt{x}}+\frac{1}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x+(1/x)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(1/sqrt(x) + 1/x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{\sqrt{\frac{1}{x}}+\frac{1}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/x)^(1/2) + 1/x)^(1/2),x)
[Out] int(((1/x)^(1/2) + 1/x)^(1/2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{\frac{1}{x}} + \frac{1}{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x+(1/x)**(1/2))**(1/2),x)
[Out] Integral(sqrt(sqrt(1/x) + 1/x), x)
```

3.3069 $\int \sqrt{2+\sqrt{\frac{1}{x}}+\frac{1}{x}}\,dx$

Optimal. Leaf size=75

$$\frac{1}{4}\left(\sqrt{\frac{1}{x}}+4\right)\sqrt{\sqrt{\frac{1}{x}}+\frac{1}{x}+2}\,x+\frac{7\tanh^{-1}\left(\frac{\sqrt{\frac{1}{x}}+4}{2\sqrt{2}\sqrt{\sqrt{\frac{1}{x}}+\frac{1}{x}+2}}\right)}{8\sqrt{2}}$$

```
[Out] 7/16*arctanh(1/4*(4+(1/x)^(1/2))*2^(1/2)/(2+1/x+(1/x)^(1/2))^(1/2))*2^(1/2)
+1/4*x*(4+(1/x)^(1/2))*(2+1/x+(1/x)^(1/2))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1966, 1357, 720, 724, 206}

$$\frac{1}{4}\left(\sqrt{\frac{1}{x}}+4\right)\sqrt{\sqrt{\frac{1}{x}}+\frac{1}{x}+2}\,x+\frac{7\tanh^{-1}\left(\frac{\sqrt{\frac{1}{x}}+4}{2\sqrt{2}\sqrt{\sqrt{\frac{1}{x}}+\frac{1}{x}+2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + Sqrt[x^(-1)] + x^(-1)],x]
```

```
[Out] ((4 + Sqrt[x^(-1)])*Sqrt[2 + Sqrt[x^(-1)] + x^(-1)]*x)/4 + (7*ArcTanh[(4 +
Sqrt[x^(-1)])/(2*Sqrt[2]*Sqrt[2 + Sqrt[x^(-1)] + x^(-1)])])/(8*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1966

```
Int[((a_.) + (c_.)*((d_.)/(x_))^(n2_.) + (b_.)*((d_.)/(x_))^(n_))^(p_.), x_
Symbol] :> -Dist[d, Subst[Int[(a + b*x^n + c*x^(2*n))^p/x^2, x], x, d/x], x
] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \sqrt{2+\sqrt{\frac{1}{x}}+\frac{1}{x}}\,dx = -\text{Subst}\left(\int \frac{\sqrt{2+\sqrt{x}+x}}{x^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\left(2\,\text{Subst}\left(\int \frac{\sqrt{2+x+x^2}}{x^3}\,dx, x, \sqrt{\frac{1}{x}}\right)\right)$$

$$= \frac{1}{4}\left(4+\sqrt{\frac{1}{x}}\right)\sqrt{2+\sqrt{\frac{1}{x}}+\frac{1}{x}}\,x - \frac{7}{8}\,\text{Subst}\left(\int \frac{1}{x\sqrt{2+x+x^2}}\,dx, x, \sqrt{\frac{1}{x}}\right)$$

$$= \frac{1}{4}\left(4+\sqrt{\frac{1}{x}}\right)\sqrt{2+\sqrt{\frac{1}{x}}+\frac{1}{x}}\,x + \frac{7}{4}\,\text{Subst}\left(\int \frac{1}{8-x^2}\,dx, x, \frac{4+\sqrt{\frac{1}{x}}}{\sqrt{2+\sqrt{\frac{1}{x}}+\frac{1}{x}}}\right)$$

$$= \frac{1}{4}\left(4+\sqrt{\frac{1}{x}}\right)\sqrt{2+\sqrt{\frac{1}{x}}+\frac{1}{x}}\,x + \frac{7\tanh^{-1}\left(\frac{4+\sqrt{\frac{1}{x}}}{2\sqrt{2}\sqrt{2+\sqrt{\frac{1}{x}}+\frac{1}{x}}}\right)}{8\sqrt{2}}$$

Mathematica [A] time = 0.08, size = 75, normalized size = 1.00

$$\frac{1}{16}\left(4\left(\sqrt{\frac{1}{x}}+4\right)\sqrt{\sqrt{\frac{1}{x}}+\frac{1}{x}+2}\,x + 7\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{\frac{1}{x}}+4}{2\sqrt{2}\sqrt{\sqrt{\frac{1}{x}}+\frac{1}{x}+2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + Sqrt[x^(-1)] + x^(-1)],x]
```

```
[Out] (4*(4 + Sqrt[x^(-1)])*Sqrt[2 + Sqrt[x^(-1)] + x^(-1)]*x + 7*Sqrt[2]*ArcTanh
[(4 + Sqrt[x^(-1)])/(2*Sqrt[2]*Sqrt[2 + Sqrt[x^(-1)] + x^(-1)])])/16
```

fricas [B] time = 3.69, size = 94, normalized size = 1.25

$$\frac{1}{4}\left(4x+\sqrt{x}\right)\sqrt{\frac{2x+\sqrt{x}+1}{x}} + \frac{7}{64}\sqrt{2}\,\log\left(-2048x^2 - 64\left(32x+9\right)\sqrt{x} - 8\left(3\sqrt{2}\left(32x+3\right)\sqrt{x} + 4\sqrt{2}\left(32x^2+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+1/x+(1/x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*x + sqrt(x))*sqrt((2*x + sqrt(x) + 1)/x) + 7/64*sqrt(2)*log(-2048*x^
2 - 64*(32*x + 9)*sqrt(x) - 8*(3*sqrt(2)*(32*x + 3)*sqrt(x) + 4*sqrt(2)*(32
*x^2 + 13*x))*sqrt((2*x + sqrt(x) + 1)/x) - 1664*x - 113)
```

giac [A] time = 0.20, size = 74, normalized size = 0.99

$$-\frac{1}{16}\sqrt{2}\left(2\sqrt{2}-7\,\log\left(2\sqrt{2}-1\right)\right)+\frac{1}{4}\sqrt{2x+\sqrt{x}+1}\left(4\sqrt{x}+1\right)-\frac{7}{16}\sqrt{2}\,\log\left(-2\sqrt{2}\left(\sqrt{2}\sqrt{x}-\sqrt{2x+\sqrt{x}+1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+1/x+(1/x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*(2*sqrt(2) - 7*log(2*sqrt(2) - 1)) + 1/4*sqrt(2*x + sqrt(x) +
 1)*(4*sqrt(x) + 1) - 7/16*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*sqrt(x) - sqrt(2
*x + sqrt(x) + 1)) - 1)
```

maple [B] time = 0.11, size = 123, normalized size = 1.64

$$\frac{\sqrt{\frac{\sqrt{\frac{1}{x}}\,x+2x+1}{x}}\left(7\sqrt{2}\,\ln\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{x}}\,\sqrt{x}}{4}+\sqrt{2}\,\sqrt{x}+\sqrt{\sqrt{\frac{1}{x}}\,x+2x+1}\right)+16\sqrt{\sqrt{\frac{1}{x}}\,x+2x+1}\,\sqrt{x}+4\sqrt{\sqrt{\frac{1}{x}}\,x+2x}\right.}{16\sqrt{\sqrt{\frac{1}{x}}\,x+2x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+1/x+(1/x)^(1/2))^(1/2),x)
```

```
[Out] 1/16*(((1/x)^(1/2)*x+2*x+1)/x)^(1/2)*x^(1/2)*(7*2^(1/2)*ln(1/4*2^(1/2)*(1/x
)^(1/2)*x^(1/2)+2^(1/2)*x^(1/2)+((1/x)^(1/2)*x+2*x+1)^(1/2))+16*((1/x)^(1/2
)*x+2*x+1)^(1/2)*x^(1/2)+4*((1/x)^(1/2)*x+2*x+1)^(1/2)*(1/x)^(1/2)*x^(1/2))
/((1/x)^(1/2)*x+2*x+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{1}{\sqrt{x}}+\frac{1}{x}+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+1/x+(1/x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(1/sqrt(x) + 1/x + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\sqrt{\frac{1}{x}}+\frac{1}{x}+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/x)^(1/2) + 1/x + 2)^(1/2),x)
```

```
[Out] int(((1/x)^(1/2) + 1/x + 2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{\frac{1}{x}}+2+\frac{1}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+1/x+(1/x)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(sqrt(1/x) + 2 + 1/x), x)
```

3.3070 $\int (cx^n)^{\frac{1}{n}} \left(a + b\,(cx^n)^{\frac{1}{n}}\right)^p \, dx$

Optimal. Leaf size=79

$$\frac{x\,(cx^n)^{-1/n}\left(a + b\,(cx^n)^{\frac{1}{n}}\right)^{p+2}}{b^2(p+2)} - \frac{ax\,(cx^n)^{-1/n}\left(a + b\,(cx^n)^{\frac{1}{n}}\right)^{p+1}}{b^2(p+1)}$$

```
[Out] -a*x*(a+b*(c*x^n)^(1/n))^(1+p)/b^2/(1+p)/((c*x^n)^(1/n))+x*(a+b*(c*x^n)^(1/
n))^(2+p)/b^2/(2+p)/((c*x^n)^(1/n))
```

Rubi [A] time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {15, 368, 43}

$$\frac{x\,(cx^n)^{-1/n}\left(a + b\,(cx^n)^{\frac{1}{n}}\right)^{p+2}}{b^2(p+2)} - \frac{ax\,(cx^n)^{-1/n}\left(a + b\,(cx^n)^{\frac{1}{n}}\right)^{p+1}}{b^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^n)^n^(-1)*(a + b*(c*x^n)^n^(-1))^p,x]
```

```
[Out] -((a*x*(a + b*(c*x^n)^n^(-1))^(1 + p))/(b^2*(1 + p)*(c*x^n)^n^(-1))) + (x*(
a + b*(c*x^n)^n^(-1))^(2 + p))/(b^2*(2 + p)*(c*x^n)^n^(-1))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\begin{aligned}
\int (cx^n)^{\frac{1}{n}} \left(a + b\,(cx^n)^{\frac{1}{n}}\right)^p \, dx &= \frac{(cx^n)^{\frac{1}{n}} \int x \left(a + b\,(cx^n)^{\frac{1}{n}}\right)^p \, dx}{x} \\
&= \left(x\,(cx^n)^{-1/n}\right) \text{Subst}\left(\int x(a+bx)^p \, dx, x, (cx^n)^{\frac{1}{n}}\right) \\
&= \left(x\,(cx^n)^{-1/n}\right) \text{Subst}\left(\int \left(-\frac{a(a+bx)^p}{b} + \frac{(a+bx)^{1+p}}{b}\right) dx, x, (cx^n)^{\frac{1}{n}}\right) \\
&= -\frac{ax\,(cx^n)^{-1/n}\left(a + b\,(cx^n)^{\frac{1}{n}}\right)^{1+p}}{b^2(1+p)} + \frac{x\,(cx^n)^{-1/n}\left(a + b\,(cx^n)^{\frac{1}{n}}\right)^{2+p}}{b^2(2+p)}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 61, normalized size = 0.77

$$\frac{x\,(cx^n)^{-1/n}\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^{p+1}\left(b(p+1)\,(cx^n)^{\frac{1}{n}}-a\right)}{b^2(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^n)^n^(-1)*(a + b*(c*x^n)^n^(-1))^p,x]
```

```
[Out] (x*(a + b*(c*x^n)^n^(-1))^(1 + p)*(-a + b*(1 + p)*(c*x^n)^n^(-1)))/(b^2*(1
+ p)*(2 + p)*(c*x^n)^n^(-1))
```

fricas [A] time = 0.88, size = 77, normalized size = 0.97

$$\frac{\left(abc^{\left(\frac{1}{n}\right)}px+\left(b^2p+b^2\right)c^{\frac{2}{n}}x^2-a^2\right)\left(bc^{\left(\frac{1}{n}\right)}x+a\right)^p}{\left(b^2p^2+3\,b^2p+2\,b^2\right)c^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^n)^(1/n)*(a+b*(c*x^n)^(1/n))^p,x, algorithm="fricas")
```

```
[Out] (a*b*c^(1/n)*p*x + (b^2*p + b^2)*c^(2/n)*x^2 - a^2)*(b*c^(1/n)*x + a)^p/((b
^2*p^2 + 3*b^2*p + 2*b^2)*c^(1/n))
```

giac [A] time = 0.50, size = 130, normalized size = 1.65

$$\frac{\left(bc^{\left(\frac{1}{n}\right)}x+a\right)^p b^2c^{\frac{2}{n}}px^2+\left(bc^{\left(\frac{1}{n}\right)}x+a\right)^p abc^{\left(\frac{1}{n}\right)}px+\left(bc^{\left(\frac{1}{n}\right)}x+a\right)^p b^2c^{\frac{2}{n}}x^2-\left(bc^{\left(\frac{1}{n}\right)}x+a\right)^p a^2}{b^2c^{\left(\frac{1}{n}\right)}p^2+3\,b^2c^{\left(\frac{1}{n}\right)}p+2\,b^2c^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^n)^(1/n)*(a+b*(c*x^n)^(1/n))^p,x, algorithm="giac")
```

```
[Out] ((b*c^(1/n)*x + a)^p*b^2*c^(2/n)*p*x^2 + (b*c^(1/n)*x + a)^p*a*b*c^(1/n)*p*
x + (b*c^(1/n)*x + a)^p*b^2*c^(2/n)*x^2 - (b*c^(1/n)*x + a)^p*a^2)/(b^2*c^(
1/n)*p^2 + 3*b^2*c^(1/n)*p + 2*b^2*c^(1/n))
```

maple [C] time = 0.82, size = 377, normalized size = 4.77

$$-\frac{a^2x\,c^{-\frac{1}{n}}(x^n)^{-\frac{1}{n}}\left(b\,c^{\frac{1}{n}}(x^n)^{\frac{1}{n}}\,\mathrm{e}^{\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}\left(ic\,x^n\right)\right)\left(-\mathrm{csgn}\left(ix^n\right)+\mathrm{csgn}\left(ic\,x^n\right)\right)\mathrm{csgn}\left(ic\,x^n\right)}{2n}}+a\right)^p\mathrm{e}^{-\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}\left(ic\,x^n\right)\right)\left(-\mathrm{csgn}\left(ix^n\right)+\mathrm{csgn}\left(ic\,x^n\right)\right)}{2n}}}{\left(p+2\right)\left(p+1\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^n)^(1/n)*(b*(c*x^n)^(1/n)+a)^p,x)
```

```
[Out] a*p/b/(p+2)/(p+1)*x*(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*(csgn(I*c)-csgn(I*c
*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))+a)^p+(b*c^(1/n)*(x^n)^
(1/n)*exp(1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n
*csgn(I*c*x^n))+a)^p/(p+2)*(x^n)^(1/n)*x*c^(1/n)*exp(1/2*I*Pi*(csgn(I*c)-cs
gn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))-(b*c^(1/n)*(x^n)
^(1/n)*exp(1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/
n*csgn(I*c*x^n))+a)^p*a^2/(p+2)/(p+1)/b^2/((x^n)^(1/n))*x/(c^(1/n))*exp(-1/
2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^
n))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left((cx^n)^{\left(\frac{1}{n}\right)} b + a \right)^p (cx^n)^{\left(\frac{1}{n}\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^n)^(1/n)*(a+b*(c*x^n)^(1/n))^p,x, algorithm="maxima")
[Out] integrate(((c*x^n)^(1/n)*b + a)^p*(c*x^n)^(1/n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x^n)^{1/n} \left(a + b\,(c\,x^n)^{1/n} \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^n)^(1/n)*(a + b*(c*x^n)^(1/n))^p,x)
[Out] int((c*x^n)^(1/n)*(a + b*(c*x^n)^(1/n))^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx^n)^{\frac{1}{n}} \left(a + b\,(cx^n)^{\frac{1}{n}} \right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**n)**(1/n)*(a+b*(c*x**n)**(1/n))**p,x)
[Out] Integral((c*x**n)**(1/n)*(a + b*(c*x**n)**(1/n))**p, x)
```

3.3071 $\int (cx^n)^{\frac{1}{n}} \left(a + b\,(cx^n)^{\frac{1}{n}}\right)^3 dx$

Optimal. Leaf size=70

$$\frac{x\,(cx^n)^{-1/n}\left(a + b\,(cx^n)^{\frac{1}{n}}\right)^5}{5b^2} - \frac{ax\,(cx^n)^{-1/n}\left(a + b\,(cx^n)^{\frac{1}{n}}\right)^4}{4b^2}$$

```
[Out] -1/4*a*x*(a+b*(c*x^n)^(1/n))^4/b^2/((c*x^n)^(1/n))+1/5*x*(a+b*(c*x^n)^(1/n)
)^5/b^2/((c*x^n)^(1/n))
```

Rubi [A] time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {15, 368, 43}

$$\frac{x\,(cx^n)^{-1/n}\left(a + b\,(cx^n)^{\frac{1}{n}}\right)^5}{5b^2} - \frac{ax\,(cx^n)^{-1/n}\left(a + b\,(cx^n)^{\frac{1}{n}}\right)^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^n)^n^(-1)*(a + b*(c*x^n)^n^(-1))^3,x]
```

```
[Out] -(a*x*(a + b*(c*x^n)^n^(-1))^4)/(4*b^2*(c*x^n)^n^(-1)) + (x*(a + b*(c*x^n)^
n^(-1))^5)/(5*b^2*(c*x^n)^n^(-1))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\begin{aligned}
\int (cx^n)^{\frac{1}{n}}\left(a + b\,(cx^n)^{\frac{1}{n}}\right)^3 dx &= \frac{(cx^n)^{\frac{1}{n}}\int x\left(a + b\,(cx^n)^{\frac{1}{n}}\right)^3 dx}{x} \\
&= \left(x\,(cx^n)^{-1/n}\right)\text{Subst}\left(\int x(a + bx)^3\,dx, x, (cx^n)^{\frac{1}{n}}\right) \\
&= \left(x\,(cx^n)^{-1/n}\right)\text{Subst}\left(\int\left(-\frac{a(a + bx)^3}{b} + \frac{(a + bx)^4}{b}\right)dx, x, (cx^n)^{\frac{1}{n}}\right) \\
&= -\frac{ax\,(cx^n)^{-1/n}\left(a + b\,(cx^n)^{\frac{1}{n}}\right)^4}{4b^2} + \frac{x\,(cx^n)^{-1/n}\left(a + b\,(cx^n)^{\frac{1}{n}}\right)^5}{5b^2}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 48, normalized size = 0.69

$$-\frac{x\,(cx^n)^{-1/n}\left(a-4b\,(cx^n)^{\frac{1}{n}}\right)\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^4}{20b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^n)^n^(-1)*(a + b*(c*x^n)^n^(-1))^3,x]
```

```
[Out] -1/20*(x*(a - 4*b*(c*x^n)^n^(-1))*(a + b*(c*x^n)^n^(-1))^4)/(b^2*(c*x^n)^n^
(-1))
```

fricas [A] time = 0.47, size = 60, normalized size = 0.86

$$\frac{1}{5}b^3c^{\frac{4}{n}}x^5+\frac{3}{4}ab^2c^{\frac{3}{n}}x^4+a^2bc^{\frac{2}{n}}x^3+\frac{1}{2}a^3c^{\left(\frac{1}{n}\right)}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^n)^(1/n)*(a+b*(c*x^n)^(1/n))^3,x, algorithm="fricas")
```

```
[Out] 1/5*b^3*c^(4/n)*x^5 + 3/4*a*b^2*c^(3/n)*x^4 + a^2*b*c^(2/n)*x^3 + 1/2*a^3*c
^(1/n)*x^2
```

giac [A] time = 0.27, size = 60, normalized size = 0.86

$$\frac{1}{5}b^3c^{\frac{4}{n}}x^5+\frac{3}{4}ab^2c^{\frac{3}{n}}x^4+a^2bc^{\frac{2}{n}}x^3+\frac{1}{2}a^3c^{\left(\frac{1}{n}\right)}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^n)^(1/n)*(a+b*(c*x^n)^(1/n))^3,x, algorithm="giac")
```

```
[Out] 1/5*b^3*c^(4/n)*x^5 + 3/4*a*b^2*c^(3/n)*x^4 + a^2*b*c^(2/n)*x^3 + 1/2*a^3*c
^(1/n)*x^2
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int\left(b\,(c\,x^n)^{\frac{1}{n}}+a\right)^3(c\,x^n)^{\frac{1}{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^n)^(1/n)*(b*(c*x^n)^(1/n)+a)^3,x)
```

```
[Out] int((c*x^n)^(1/n)*(b*(c*x^n)^(1/n)+a)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left((cx^n)^{\left(\frac{1}{n}\right)}b+a\right)^3(cx^n)^{\left(\frac{1}{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^n)^(1/n)*(a+b*(c*x^n)^(1/n))^3,x, algorithm="maxima")
```

```
[Out] integrate(((c*x^n)^(1/n)*b + a)^3*(c*x^n)^(1/n), x)
```

mupad [B] time = 1.26, size = 68, normalized size = 0.97

$$\frac{b^3\,x\,(c\,x^n)^{4/n}}{5}+\frac{a^3\,x\,(c\,x^n)^{1/n}}{2}+a^2\,b\,x\,(c\,x^n)^{2/n}+\frac{3\,a\,b^2\,x\,(c\,x^n)^{3/n}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^n)^(1/n)*(a + b*(c*x^n)^(1/n))^3,x)
```

```
[Out] (b^3*x*(c*x^n)^(4/n))/5 + (a^3*x*(c*x^n)^(1/n))/2 + a^2*b*x*(c*x^n)^(2/n) +
 (3*a*b^2*x*(c*x^n)^(3/n))/4
```

sympy [**A**] time = 1.84, size = 76, normalized size = 1.09

$$\frac{a^3 c^{\frac{1}{n}} x \left(x^n\right)^{\frac{1}{n}}}{2} + a^2 b c^{\frac{2}{n}} x \left(x^n\right)^{\frac{2}{n}} + \frac{3 a b^2 c^{\frac{3}{n}} x \left(x^n\right)^{\frac{3}{n}}}{4} + \frac{b^3 c^{\frac{4}{n}} x \left(x^n\right)^{\frac{4}{n}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**n)**(1/n)*(a+b*(c*x**n)**(1/n))**3,x)
```

```
[Out] a**3*c**(1/n)*x*(x**n)**(1/n)/2 + a**2*b*c**(2/n)*x*(x**n)**(2/n) + 3*a*b**
2*c**(3/n)*x*(x**n)**(3/n)/4 + b**3*c**(4/n)*x*(x**n)**(4/n)/5
```

3.3072 $\int (cx^n)^{\frac{1}{n}} \left(a + b\,(cx^n)^{\frac{1}{n}}\right)^2 dx$

Optimal. Leaf size=55

$$\frac{1}{2}a^2x\,(cx^n)^{\frac{1}{n}} + \frac{2}{3}abx\,(cx^n)^{2/n} + \frac{1}{4}b^2x\,(cx^n)^{3/n}$$

```
[Out] 1/2*a^2*x*(c*x^n)^(1/n)+2/3*a*b*x*(c*x^n)^(2/n)+1/4*b^2*x*(c*x^n)^(3/n)
```

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {15, 368, 43}

$$\frac{1}{2}a^2x\,(cx^n)^{\frac{1}{n}} + \frac{2}{3}abx\,(cx^n)^{2/n} + \frac{1}{4}b^2x\,(cx^n)^{3/n}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^n)^n^(-1)*(a + b*(c*x^n)^n^(-1))^2,x]
```

```
[Out] (a^2*x*(c*x^n)^n^(-1))/2 + (2*a*b*x*(c*x^n)^(2/n))/3 + (b^2*x*(c*x^n)^(3/n)
)/4
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\begin{aligned}
\int (cx^n)^{\frac{1}{n}} \left(a + b\,(cx^n)^{\frac{1}{n}}\right)^2 dx &= \frac{(cx^n)^{\frac{1}{n}} \int x \left(a + b\,(cx^n)^{\frac{1}{n}}\right)^2 dx}{x} \\
&= \left(x\,(cx^n)^{-1/n}\right) \text{Subst}\left(\int x(a + bx)^2\,dx, x, (cx^n)^{\frac{1}{n}}\right) \\
&= \left(x\,(cx^n)^{-1/n}\right) \text{Subst}\left(\int \left(a^2x + 2abx^2 + b^2x^3\right)\,dx, x, (cx^n)^{\frac{1}{n}}\right) \\
&= \frac{1}{2}a^2x\,(cx^n)^{\frac{1}{n}} + \frac{2}{3}abx\,(cx^n)^{2/n} + \frac{1}{4}b^2x\,(cx^n)^{3/n}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.89

$$\frac{1}{12}x\,(cx^n)^{\frac{1}{n}} \left(6a^2 + 8ab\,(cx^n)^{\frac{1}{n}} + 3b^2\,(cx^n)^{2/n}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^n)^n^(-1)*(a + b*(c*x^n)^n^(-1))^2,x]
[Out] (x*(c*x^n)^n^(-1)*(6*a^2 + 8*a*b*(c*x^n)^n^(-1) + 3*b^2*(c*x^n)^(2/n)))/12
```

fricas [A] time = 0.44, size = 43, normalized size = 0.78

$$\frac{1}{4}\,b^2c^{\frac{3}{n}}x^4 + \frac{2}{3}\,abc^{\frac{2}{n}}x^3 + \frac{1}{2}\,a^2c^{\left(\frac{1}{n}\right)}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^n)^(1/n)*(a+b*(c*x^n)^(1/n))^2,x, algorithm="fricas")
[Out] 1/4*b^2*c^(3/n)*x^4 + 2/3*a*b*c^(2/n)*x^3 + 1/2*a^2*c^(1/n)*x^2
```

giac [A] time = 0.23, size = 43, normalized size = 0.78

$$\frac{1}{4}\,b^2c^{\frac{3}{n}}x^4 + \frac{2}{3}\,abc^{\frac{2}{n}}x^3 + \frac{1}{2}\,a^2c^{\left(\frac{1}{n}\right)}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^n)^(1/n)*(a+b*(c*x^n)^(1/n))^2,x, algorithm="giac")
[Out] 1/4*b^2*c^(3/n)*x^4 + 2/3*a*b*c^(2/n)*x^3 + 1/2*a^2*c^(1/n)*x^2
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \left(b\,(c\,x^n)^{\frac{1}{n}} + a\right)^2 (c\,x^n)^{\frac{1}{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^n)^(1/n)*(b*(c*x^n)^(1/n)+a)^2,x)
[Out] int((c*x^n)^(1/n)*(b*(c*x^n)^(1/n)+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left((cx^n)^{\left(\frac{1}{n}\right)} b + a\right)^2 (cx^n)^{\left(\frac{1}{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^n)^(1/n)*(a+b*(c*x^n)^(1/n))^2,x, algorithm="maxima")
[Out] integrate(((c*x^n)^(1/n)*b + a)^2*(c*x^n)^(1/n), x)
```

mupad [B] time = 1.22, size = 47, normalized size = 0.85

$$\frac{x\,(c\,x^n)^{1/n}\left(3\,b^2\,(c\,x^n)^{2/n} + 6\,a^2 + 8\,a\,b\,(c\,x^n)^{1/n}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^n)^(1/n)*(a + b*(c*x^n)^(1/n))^2,x)
[Out] (x*(c*x^n)^(1/n)*(3*b^2*(c*x^n)^(2/n) + 6*a^2 + 8*a*b*(c*x^n)^(1/n)))/12
```

sympy [A] time = 1.00, size = 56, normalized size = 1.02

$$\frac{a^2c^{\frac{1}{n}}x\,(x^n)^{\frac{1}{n}}}{2} + \frac{2abc^{\frac{2}{n}}x\,(x^n)^{\frac{2}{n}}}{3} + \frac{b^2c^{\frac{3}{n}}x\,(x^n)^{\frac{3}{n}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**n)**(1/n)*(a+b*(c*x**n)**(1/n))**2,x)
```

```
[Out] a**2*c**(1/n)*x*(x**n)**(1/n)/2 + 2*a*b*c**(2/n)*x*(x**n)**(2/n)/3 + b**2*c
**(3/n)*x*(x**n)**(3/n)/4
```

3.3073 $\int (cx^n)^{\frac{1}{n}} \left(a + b\,(cx^n)^{\frac{1}{n}}\right) dx$

Optimal. Leaf size=33

$$\frac{1}{2}ax\,(cx^n)^{\frac{1}{n}} + \frac{1}{3}bx\,(cx^n)^{2/n}$$

```
[Out] 1/2*a*x*(c*x^n)^(1/n)+1/3*b*x*(c*x^n)^(2/n)
```

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {15, 14, 30}

$$\frac{1}{2}ax\,(cx^n)^{\frac{1}{n}} + \frac{1}{3}bx\,(cx^n)^{2/n}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^n)^n^(-1)*(a + b*(c*x^n)^n^(-1)),x]
```

```
[Out] (a*x*(c*x^n)^n^(-1))/2 + (b*x*(c*x^n)^(2/n))/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (cx^n)^{\frac{1}{n}} \left(a + b\,(cx^n)^{\frac{1}{n}}\right) dx &= \frac{(cx^n)^{\frac{1}{n}} \int x \left(a + b\,(cx^n)^{\frac{1}{n}}\right) dx}{x} \\
&= \frac{(cx^n)^{\frac{1}{n}} \int \left(ax + bx\,(cx^n)^{\frac{1}{n}}\right) dx}{x} \\
&= \frac{1}{2}ax\,(cx^n)^{\frac{1}{n}} + \frac{\left(b\,(cx^n)^{\frac{1}{n}}\right) \int x\,(cx^n)^{\frac{1}{n}}\, dx}{x} \\
&= \frac{1}{2}ax\,(cx^n)^{\frac{1}{n}} + \frac{\left(b\,(cx^n)^{2/n}\right) \int x^2\, dx}{x^2} \\
&= \frac{1}{2}ax\,(cx^n)^{\frac{1}{n}} + \frac{1}{3}bx\,(cx^n)^{2/n}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.91

$$\frac{1}{6}x\,(cx^n)^{\frac{1}{n}} \left(3a + 2b\,(cx^n)^{\frac{1}{n}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^n)^n^(-1)*(a + b*(c*x^n)^n^(-1)),x]
[Out] (x*(c*x^n)^n^(-1)*(3*a + 2*b*(c*x^n)^n^(-1)))/6
```

fricas [A] time = 0.42, size = 25, normalized size = 0.76

$$\frac{1}{3}\,bc^{\frac{2}{n}}x^3 + \frac{1}{2}\,ac^{\left(\frac{1}{n}\right)}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^n)^(1/n)*(a+b*(c*x^n)^(1/n)),x, algorithm="fricas")
[Out] 1/3*b*c^(2/n)*x^3 + 1/2*a*c^(1/n)*x^2
```

giac [A] time = 0.19, size = 25, normalized size = 0.76

$$\frac{1}{3}\,bc^{\frac{2}{n}}x^3 + \frac{1}{2}\,ac^{\left(\frac{1}{n}\right)}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^n)^(1/n)*(a+b*(c*x^n)^(1/n)),x, algorithm="giac")
[Out] 1/3*b*c^(2/n)*x^3 + 1/2*a*c^(1/n)*x^2
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \left(b\,(c\,x^n)^{\frac{1}{n}} + a\right)(c\,x^n)^{\frac{1}{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^n)^(1/n)*(b*(c*x^n)^(1/n)+a),x)
[Out] int((c*x^n)^(1/n)*(b*(c*x^n)^(1/n)+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left((cx^n)^{\left(\frac{1}{n}\right)}\,b + a\right)(cx^n)^{\left(\frac{1}{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^n)^(1/n)*(a+b*(c*x^n)^(1/n)),x, algorithm="maxima")
[Out] integrate(((c*x^n)^(1/n)*b + a)*(c*x^n)^(1/n), x)
```

mupad [B] time = 1.18, size = 28, normalized size = 0.85

$$\frac{x\,(c\,x^n)^{1/n}\,\left(3\,a + 2\,b\,(c\,x^n)^{1/n}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^n)^(1/n)*(a + b*(c*x^n)^(1/n)),x)
[Out] (x*(c*x^n)^(1/n)*(3*a + 2*b*(c*x^n)^(1/n)))/6
```

sympy [A] time = 0.50, size = 32, normalized size = 0.97

$$\frac{ac^{\frac{1}{n}}x\,(x^n)^{\frac{1}{n}}}{2} + \frac{bc^{\frac{2}{n}}x\,(x^n)^{\frac{2}{n}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**n)**(1/n)*(a+b*(c*x**n)**(1/n)),x)
[Out] a*c**(1/n)*x*(x**n)**(1/n)/2 + b*c**(2/n)*x*(x**n)**(2/n)/3
```

3.3074 $\int \frac{(cx^n)^{\frac{1}{n}}}{a+b(cx^n)^{\frac{1}{n}}}\,dx$

Optimal. Leaf size=38

$$\frac{x}{b}-\frac{ax\left(cx^n\right)^{-1/n}\log\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}{b^2}$$

```
[Out] x/b-a*x*ln(a+b*(c*x^n)^(1/n))/b^2/((c*x^n)^(1/n))
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {15, 368, 43}

$$\frac{x}{b}-\frac{ax\left(cx^n\right)^{-1/n}\log\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^n)^n^(-1)/(a + b*(c*x^n)^n^(-1)),x]
```

```
[Out] x/b - (a*x*Log[a + b*(c*x^n)^n^(-1)])/(b^2*(c*x^n)^n^(-1))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^n)^{\frac{1}{n}}}{a+b\left(cx^n\right)^{\frac{1}{n}}}\,dx &= \frac{(cx^n)^{\frac{1}{n}}\int \frac{x}{a+b(cx^n)^{\frac{1}{n}}}\,dx}{x}\\
&= \left(x\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{x}{a+bx}\,dx, x, (cx^n)^{\frac{1}{n}}\right)\\
&= \left(x\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)\,dx, x, (cx^n)^{\frac{1}{n}}\right)\\
&= \frac{x}{b}-\frac{ax\left(cx^n\right)^{-1/n}\log\left(a+b\left(cx^n\right)^{\frac{1}{n}}\right)}{b^2}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 35, normalized size = 0.92

$$\frac{x\left(b - a\,(cx^n)^{-1/n}\log\left(a + b\,(cx^n)^{\frac{1}{n}}\right)\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^n)^n^(-1)/(a + b*(c*x^n)^n^(-1)),x]
[Out] (x*(b - (a*Log[a + b*(c*x^n)^n^(-1)])/(c*x^n)^n^(-1)))/b^2
```

fricas [A] time = 0.43, size = 34, normalized size = 0.89

$$\frac{bc^{\left(\frac{1}{n}\right)}x - a\log\left(bc^{\left(\frac{1}{n}\right)}x + a\right)}{b^2c^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^n)^(1/n)/(a+b*(c*x^n)^(1/n)),x, algorithm="fricas")
[Out] (b*c^(1/n)*x - a*log(b*c^(1/n)*x + a))/(b^2*c^(1/n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^n)^{\left(\frac{1}{n}\right)}}{(cx^n)^{\left(\frac{1}{n}\right)}b + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^n)^(1/n)/(a+b*(c*x^n)^(1/n)),x, algorithm="giac")
[Out] integrate((c*x^n)^(1/n)/((c*x^n)^(1/n)*b + a), x)
```

maple [C] time = 0.19, size = 147, normalized size = 3.87

$$-\frac{ax\,c^{-\frac{1}{n}}(x^n)^{-\frac{1}{n}}\,\mathrm{e}^{-\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}\left(ic\,x^n\right)\right)\left(-\mathrm{csgn}\left(ix^n\right)+\mathrm{csgn}\left(ic\,x^n\right)\right)\mathrm{csgn}\left(ic\,x^n\right)}{2n}}\ln\left(b\,c^{\frac{1}{n}}(x^n)^{\frac{1}{n}}\,\mathrm{e}^{\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}\left(ic\,x^n\right)\right)\left(-\mathrm{csgn}\left(ix^n\right)+\mathrm{csgn}\left(ic\,x^n\right)\right)\mathrm{csgn}\left(i\right.}{2n}}\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^n)^(1/n)/(b*(c*x^n)^(1/n)+a),x)
[Out] 1/b*x-a/b^2*ln(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))
*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))+a)/(c^(1/n))/((x^n)^(1/n))*x
*exp(-1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csg
n(I*c*x^n))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-a\int \frac{1}{b^2c^{\left(\frac{1}{n}\right)}(x^n)^{\left(\frac{1}{n}\right)} + ab}\,dx + \frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^n)^(1/n)/(a+b*(c*x^n)^(1/n)),x, algorithm="maxima")
[Out] -a*integrate(1/(b^2*c^(1/n)*(x^n)^(1/n) + a*b), x) + x/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(c\,x^n)^{1/n}}{a + b\,(c\,x^n)^{1/n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^n)^(1/n)/(a + b*(c*x^n)^(1/n)),x)
[Out] int((c*x^n)^(1/n)/(a + b*(c*x^n)^(1/n)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^n)^{\frac{1}{n}}}{a + b\,(cx^n)^{\frac{1}{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**n)**(1/n)/(a+b*(c*x**n)**(1/n)),x)
[Out] Integral((c*x**n)**(1/n)/(a + b*(c*x**n)**(1/n)), x)
```

3.3075 $\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^2}\,dx$

Optimal. Leaf size=63

$$\frac{ax\,(cx^n)^{-1/n}}{b^2\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}+\frac{x\,(cx^n)^{-1/n}\log\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}{b^2}$$

```
[Out] a*x/b^2/((c*x^n)^(1/n))/(a+b*(c*x^n)^(1/n))+x*ln(a+b*(c*x^n)^(1/n))/b^2/((c
*x^n)^(1/n))
```

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {15, 368, 43}

$$\frac{ax\,(cx^n)^{-1/n}}{b^2\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}+\frac{x\,(cx^n)^{-1/n}\log\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^n)^n^(-1)/(a + b*(c*x^n)^n^(-1))^2,x]
```

```
[Out] (a*x)/(b^2*(c*x^n)^n^(-1)*(a + b*(c*x^n)^n^(-1))) + (x*Log[a + b*(c*x^n)^n^
(-1)])/(b^2*(c*x^n)^n^(-1))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^2}\,dx = \frac{(cx^n)^{\frac{1}{n}} \int \frac{x}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^2}\,dx}{x}$$

$$= \left(x\,(cx^n)^{-1/n}\right) \text{Subst}\left(\int \frac{x}{(a+bx)^2}\,dx, x, (cx^n)^{\frac{1}{n}}\right)$$

$$= \left(x\,(cx^n)^{-1/n}\right) \text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^2} + \frac{1}{b(a+bx)}\right)\,dx, x, (cx^n)^{\frac{1}{n}}\right)$$

$$= \frac{ax\,(cx^n)^{-1/n}}{b^2\left(a+b\,(cx^n)^{\frac{1}{n}}\right)} + \frac{x\,(cx^n)^{-1/n}\log\left(a+b\,(cx^n)^{\frac{1}{n}}\right)}{b^2}$$

Mathematica [A] time = 0.04, size = 48, normalized size = 0.76

$$\frac{x\,(cx^n)^{-1/n}\left(\frac{a}{a+b(cx^n)^{\frac{1}{n}}} + \log\left(a+b\,(cx^n)^{\frac{1}{n}}\right)\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^n)^n^(-1)/(a + b*(c*x^n)^n^(-1))^2,x]
```

```
[Out] (x*(a/(a + b*(c*x^n)^n^(-1)) + Log[a + b*(c*x^n)^n^(-1)]))/(b^2*(c*x^n)^n^(
-1))
```

fricas [A] time = 0.42, size = 50, normalized size = 0.79

$$\frac{\left(bc^{\left(\frac{1}{n}\right)}x + a\right)\log\left(bc^{\left(\frac{1}{n}\right)}x + a\right) + a}{b^3c^{\frac{2}{n}}x + ab^2c^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^n)^(1/n)/(a+b*(c*x^n)^(1/n))^2,x, algorithm="fricas")
```

```
[Out] ((b*c^(1/n)*x + a)*log(b*c^(1/n)*x + a) + a)/(b^3*c^(2/n)*x + a*b^2*c^(1/n)
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^n)^{\left(\frac{1}{n}\right)}}{\left((cx^n)^{\left(\frac{1}{n}\right)}b + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^n)^(1/n)/(a+b*(c*x^n)^(1/n))^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^n)^(1/n)/((c*x^n)^(1/n)*b + a)^2, x)
```

maple [C] time = 0.16, size = 214, normalized size = 3.40

$$\frac{x\,c^{-\frac{1}{n}}(x^n)^{-\frac{1}{n}}\,\mathrm{e}^{-\frac{i\pi\left(\text{csgn}(ic)-\text{csgn}\left(ic\,x^n\right)\right)\left(-\text{csgn}\left(ix^n\right)+\text{csgn}\left(ic\,x^n\right)\right)\text{csgn}\left(ic\,x^n\right)}{2n}}\ln\left(b\,c^{\frac{1}{n}}(x^n)^{\frac{1}{n}}\,\mathrm{e}^{\frac{i\pi\left(\text{csgn}(ic)-\text{csgn}\left(ic\,x^n\right)\right)\left(-\text{csgn}\left(ix^n\right)+\text{csgn}\left(ic\,x^n\right)\right)\text{csgn}\left(ic\,x^n\right)}{2n}} + \right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^n)^(1/n)/(b*(c*x^n)^(1/n)+a)^2,x)
```

```
[Out] -1/b*x/(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn
(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))+a)+1/b^2*ln(b*c^(1/n)*(x^n)^(1/n)*e
xp(1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I
*c*x^n))+a)/((x^n)^(1/n))/(c^(1/n))*x*exp(-1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n
))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{x}{b^2 c^{\left(\frac{1}{n}\right)} (x^n)^{\left(\frac{1}{n}\right)} + ab} + \int \frac{1}{b^2 c^{\left(\frac{1}{n}\right)} (x^n)^{\left(\frac{1}{n}\right)} + ab}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^n)^(1/n)/(a+b*(c*x^n)^(1/n))^2,x, algorithm="maxima")
```

```
[Out] -x/(b^2*c^(1/n)*(x^n)^(1/n) + a*b) + integrate(1/(b^2*c^(1/n)*(x^n)^(1/n) +
 a*b), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c\,x^n)^{1/n}}{\left(a + b\,(c\,x^n)^{1/n}\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^n)^(1/n)/(a + b*(c*x^n)^(1/n))^2,x)
```

```
[Out] int((c*x^n)^(1/n)/(a + b*(c*x^n)^(1/n))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a + b\,(cx^n)^{\frac{1}{n}}\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**n)**(1/n)/(a+b*(c*x**n)**(1/n))**2,x)
```

```
[Out] Integral((c*x**n)**(1/n)/(a + b*(c*x**n)**(1/n))**2, x)
```

3.3076 $\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^3}\,dx$

Optimal. Leaf size=32

$$\frac{x\,(cx^n)^{\frac{1}{n}}}{2a\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^2}$$

```
[Out] 1/2*x*(c*x^n)^(1/n)/a/(a+b*(c*x^n)^(1/n))^2
```

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {15, 368, 37}

$$\frac{x\,(cx^n)^{\frac{1}{n}}}{2a\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^n)^n^(-1)/(a + b*(c*x^n)^n^(-1))^3,x]
```

```
[Out] (x*(c*x^n)^n^(-1))/(2*a*(a + b*(c*x^n)^n^(-1))^2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^3}\,dx = \frac{(cx^n)^{\frac{1}{n}}\int \frac{x}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^3}\,dx}{x}$$

$$= \left(x\,(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x}{(a+bx)^3}\,dx, x, (cx^n)^{\frac{1}{n}}\right)$$

$$= \frac{x\,(cx^n)^{\frac{1}{n}}}{2a\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^2}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 1.00

$$\frac{x\,(cx^n)^{\frac{1}{n}}}{2a\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^n)^n^(-1)/(a + b*(c*x^n)^n^(-1))^3,x]
```

```
[Out] (x*(c*x^n)^n^(-1))/(2*a*(a + b*(c*x^n)^n^(-1))^2)
```

fricas [A] time = 0.42, size = 56, normalized size = 1.75

$$-\frac{2\,bc^{\left(\frac{1}{n}\right)}x + a}{2\left(b^4c^{\frac{3}{n}}x^2 + 2\,ab^3c^{\frac{2}{n}}x + a^2b^2c^{\left(\frac{1}{n}\right)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^n)^(1/n)/(a+b*(c*x^n)^(1/n))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*c^(1/n)*x + a)/(b^4*c^(3/n)*x^2 + 2*a*b^3*c^(2/n)*x + a^2*b^2*c^(
1/n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^n)^{\left(\frac{1}{n}\right)}}{\left((cx^n)^{\left(\frac{1}{n}\right)}b + a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^n)^(1/n)/(a+b*(c*x^n)^(1/n))^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^n)^(1/n)/((c*x^n)^(1/n)*b + a)^3, x)
```

maple [C] time = 0.07, size = 137, normalized size = 4.28

$$\frac{x\,c^{\frac{1}{n}}\,(x^n)^{\frac{1}{n}}\,\mathrm{e}^{\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}\left(ic\,x^n\right)\right)\left(-\mathrm{csgn}\left(ix^n\right)+\mathrm{csgn}\left(ic\,x^n\right)\right)\mathrm{csgn}\left(ic\,x^n\right)}{2n}}}{2\left(b\,c^{\frac{1}{n}}\,(x^n)^{\frac{1}{n}}\,\mathrm{e}^{\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}\left(ic\,x^n\right)\right)\left(-\mathrm{csgn}\left(ix^n\right)+\mathrm{csgn}\left(ic\,x^n\right)\right)\mathrm{csgn}\left(ic\,x^n\right)}{2n}}+a\right)^2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^n)^(1/n)/(b*(c*x^n)^(1/n)+a)^3,x)
```

```
[Out] 1/2*x*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x
^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))/a/(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*(
csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))+a)^2
```

maxima [A] time = 0.67, size = 60, normalized size = 1.88

$$\frac{c^{\left(\frac{1}{n}\right)} x (x^n)^{\left(\frac{1}{n}\right)}}{2\left(a b^2 c^{\frac{2}{n}} (x^n)^{\frac{2}{n}} + 2\,a^2 b c^{\left(\frac{1}{n}\right)} (x^n)^{\left(\frac{1}{n}\right)} + a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^n)^(1/n)/(a+b*(c*x^n)^(1/n))^3,x, algorithm="maxima")
```

```
[Out] 1/2*c^(1/n)*x*(x^n)^(1/n)/(a*b^2*c^(2/n)*(x^n)^(2/n) + 2*a^2*b*c^(1/n)*(x^n
)^(1/n) + a^3)
```

mupad [B] time = 1.18, size = 30, normalized size = 0.94

$$\frac{x\,(c\,x^n)^{1/n}}{2\,a\left(a + b\,(c\,x^n)^{1/n}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^n)^(1/n)/(a + b*(c*x^n)^(1/n))^3,x)
```

```
[Out] (x*(c*x^n)^(1/n))/(2*a*(a + b*(c*x^n)^(1/n))^2)
```

sympy [A] time = 178.51, size = 160, normalized size = 5.00

$$\begin{cases} \tilde{\infty} c^{-\frac{2}{n}} x\,(x^n)^{-\frac{2}{n}} & \text{for } a = 0 \wedge b = 0 \\ -\dfrac{x}{b^3\left(2\cdot 0^n \tilde{\infty}^n (0^n)^{\frac{2}{n}} (x^n)^{\frac{2}{n}} - (0^n)^{\frac{2}{n}} (x^n)^{\frac{2}{n}}\right)} & \text{for } a = 0 \wedge c = 0^n \\ -\dfrac{c^{-\frac{2}{n}} x (x^n)^{-\frac{2}{n}}}{b^3} & \text{for } a = 0 \\ \tilde{\infty} c^{\frac{1}{n}} x\,(x^n)^{\frac{1}{n}} & \text{for } a = -b c^{\frac{1}{n}} (x^n)^{\frac{1}{n}} \\ \dfrac{c^{\frac{1}{n}} x (x^n)^{\frac{1}{n}}}{2a^3 + 4a^2 b c^{\frac{1}{n}} (x^n)^{\frac{1}{n}} + 2a b^2 c^{\frac{2}{n}} (x^n)^{\frac{2}{n}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**n)**(1/n)/(a+b*(c*x**n)**(1/n))**3,x)
```

```
[Out] Piecewise((zoo*c**(-2/n)*x*(x**n)**(-2/n), Eq(a, 0) & Eq(b, 0)), (-x/(b**3*
(2*0**n*zoo**n*(0**n)**(2/n)*(x**n)**(2/n) - (0**n)**(2/n)*(x**n)**(2/n))),
 Eq(a, 0) & Eq(c, 0**n)), (-c**(-2/n)*x*(x**n)**(-2/n)/b**3, Eq(a, 0)), (zo
o*c**(1/n)*x*(x**n)**(1/n), Eq(a, -b*c**(1/n)*(x**n)**(1/n))), (c**(1/n)*x*
(x**n)**(1/n)/(2*a**3 + 4*a**2*b*c**(1/n)*(x**n)**(1/n) + 2*a*b**2*c**(2/n)
*(x**n)**(2/n)), True))
```

3.3077 $\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^4}\,dx$

Optimal. Leaf size=70

$$\frac{ax\,(cx^n)^{-1/n}}{3b^2\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^3}-\frac{x\,(cx^n)^{-1/n}}{2b^2\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^2}$$

```
[Out] 1/3*a*x/b^2/((c*x^n)^(1/n))/(a+b*(c*x^n)^(1/n))^3-1/2*x/b^2/((c*x^n)^(1/n))
/(a+b*(c*x^n)^(1/n))^2
```

Rubi [A] time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {15, 368, 43}

$$\frac{ax\,(cx^n)^{-1/n}}{3b^2\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^3}-\frac{x\,(cx^n)^{-1/n}}{2b^2\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^n)^n^(-1)/(a + b*(c*x^n)^n^(-1))^4,x]
```

```
[Out] (a*x)/(3*b^2*(c*x^n)^n^(-1)*(a + b*(c*x^n)^n^(-1))^3) - x/(2*b^2*(c*x^n)^n^
(-1)*(a + b*(c*x^n)^n^(-1))^2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^4}\,dx = \frac{(cx^n)^{\frac{1}{n}}\int \frac{x}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^4}\,dx}{x}$$

$$= \left(x\,(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x}{(a+bx)^4}\,dx, x, (cx^n)^{\frac{1}{n}}\right)$$

$$= \left(x\,(cx^n)^{-1/n}\right)\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^4}+\frac{1}{b(a+bx)^3}\right)\,dx, x, (cx^n)^{\frac{1}{n}}\right)$$

$$= \frac{ax\,(cx^n)^{-1/n}}{3b^2\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^3} - \frac{x\,(cx^n)^{-1/n}}{2b^2\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^2}$$

Mathematica [A] time = 0.05, size = 48, normalized size = 0.69

$$-\frac{x\,(cx^n)^{-1/n}\left(a+3b\,(cx^n)^{\frac{1}{n}}\right)}{6b^2\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^n)^n^(-1)/(a + b*(c*x^n)^n^(-1))^4,x]
```

```
[Out] -1/6*(x*(a + 3*b*(c*x^n)^n^(-1)))/(b^2*(c*x^n)^n^(-1)*(a + b*(c*x^n)^n^(-1)
)^3)
```

fricas [A] time = 0.44, size = 74, normalized size = 1.06

$$-\frac{3\,bc^{\left(\frac{1}{n}\right)}x + a}{6\left(b^5c^{\frac{4}{n}}x^3 + 3\,ab^4c^{\frac{3}{n}}x^2 + 3\,a^2b^3c^{\frac{2}{n}}x + a^3b^2c^{\left(\frac{1}{n}\right)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^n)^(1/n)/(a+b*(c*x^n)^(1/n))^4,x, algorithm="fricas")
```

```
[Out] -1/6*(3*b*c^(1/n)*x + a)/(b^5*c^(4/n)*x^3 + 3*a*b^4*c^(3/n)*x^2 + 3*a^2*b^3
*c^(2/n)*x + a^3*b^2*c^(1/n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^n)^{\left(\frac{1}{n}\right)}}{\left((cx^n)^{\left(\frac{1}{n}\right)}b + a\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^n)^(1/n)/(a+b*(c*x^n)^(1/n))^4,x, algorithm="giac")
```

```
[Out] integrate((c*x^n)^(1/n)/((c*x^n)^(1/n)*b + a)^4, x)
```

maple [C] time = 0.19, size = 205, normalized size = 2.93

$$\frac{\left(b\,c^{\frac{1}{n}}\,(x^n)^{\frac{1}{n}}\,\mathrm{e}^{\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}\left(ic\,x^n\right)\right)\left(-\mathrm{csgn}\left(ix^n\right)+\mathrm{csgn}\left(ic\,x^n\right)\right)\mathrm{csgn}\left(ic\,x^n\right)}{n}} + 3a\,\mathrm{e}^{\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}\left(ic\,x^n\right)\right)\left(-\mathrm{csgn}\left(ix^n\right)+\mathrm{csgn}\left(ic\,x^n\right)\right)\mathrm{csgn}\left(ic\,x^n\right)}{2n}}\right)x\,c^{\frac{1}{n}}\,(x^n)^{\frac{1}{n}}}{6\left(b\,c^{\frac{1}{n}}\,(x^n)^{\frac{1}{n}}\,\mathrm{e}^{\frac{i\pi\left(\mathrm{csgn}(ic)-\mathrm{csgn}\left(ic\,x^n\right)\right)\left(-\mathrm{csgn}\left(ix^n\right)+\mathrm{csgn}\left(ic\,x^n\right)\right)\mathrm{csgn}\left(ic\,x^n\right)}{2n}} + a\right)^3 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^n)^(1/n)/(b*(c*x^n)^(1/n)+a)^4,x)
```

```
[Out] 1/6*x*(x^n)^(1/n)*c^(1/n)/(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*(csgn(I*c)-cs
gn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))+a)^3/a^2*((x^n)^
(1/n)*c^(1/n)*b*exp(I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x
^n))/n*csgn(I*c*x^n))+3*a*exp(1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x
^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n)))
```

maxima [A] time = 0.78, size = 109, normalized size = 1.56

$$\frac{bc^{\frac{2}{n}}x(x^n)^{\frac{2}{n}} + 3\,ac^{\left(\frac{1}{n}\right)}x(x^n)^{\left(\frac{1}{n}\right)}}{6\left(a^2b^3c^{\frac{3}{n}}(x^n)^{\frac{3}{n}} + 3\,a^3b^2c^{\frac{2}{n}}(x^n)^{\frac{2}{n}} + 3\,a^4bc^{\left(\frac{1}{n}\right)}(x^n)^{\left(\frac{1}{n}\right)} + a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^n)^(1/n)/(a+b*(c*x^n)^(1/n))^4,x, algorithm="maxima")
```

```
[Out] 1/6*(b*c^(2/n)*x*(x^n)^(2/n) + 3*a*c^(1/n)*x*(x^n)^(1/n))/(a^2*b^3*c^(3/n)*
(x^n)^(3/n) + 3*a^3*b^2*c^(2/n)*(x^n)^(2/n) + 3*a^4*b*c^(1/n)*(x^n)^(1/n) +
 a^5)
```

mupad [B] time = 1.18, size = 127, normalized size = 1.81

$$\frac{x}{6\,a^2\,b\,\left(a + b\,(c\,x^n)^{1/n}\right)} - \frac{x}{3\,b\,\left(b^3\,(c\,x^n)^{3/n} + a^3 + 3\,a\,b^2\,(c\,x^n)^{2/n} + 3\,a^2\,b\,(c\,x^n)^{1/n}\right)} + \frac{x}{6\,a\,b\,\left(b^2\,(c\,x^n)^{2/n} + a^2 + 2\,a\,b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^n)^(1/n)/(a + b*(c*x^n)^(1/n))^4,x)
```

```
[Out] x/(6*a^2*b*(a + b*(c*x^n)^(1/n))) - x/(3*b*(b^3*(c*x^n)^(3/n) + a^3 + 3*a*b
^2*(c*x^n)^(2/n) + 3*a^2*b*(c*x^n)^(1/n))) + x/(6*a*b*(b^2*(c*x^n)^(2/n) +
a^2 + 2*a*b*(c*x^n)^(1/n)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**n)**(1/n)/(a+b*(c*x**n)**(1/n))**4,x)
```

```
[Out] Timed out
```

3.3078 $\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^5}\,dx$

Optimal. Leaf size=70

$$\frac{ax\,(cx^n)^{-1/n}}{4b^2\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^4}-\frac{x\,(cx^n)^{-1/n}}{3b^2\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^3}$$

```
[Out] 1/4*a*x/b^2/((c*x^n)^(1/n))/(a+b*(c*x^n)^(1/n))^4-1/3*x/b^2/((c*x^n)^(1/n))
/(a+b*(c*x^n)^(1/n))^3
```

Rubi [A] time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {15, 368, 43}

$$\frac{ax\,(cx^n)^{-1/n}}{4b^2\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^4}-\frac{x\,(cx^n)^{-1/n}}{3b^2\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^n)^n^(-1)/(a + b*(c*x^n)^n^(-1))^5,x]
```

```
[Out] (a*x)/(4*b^2*(c*x^n)^n^(-1)*(a + b*(c*x^n)^n^(-1))^4) - x/(3*b^2*(c*x^n)^n^
(-1)*(a + b*(c*x^n)^n^(-1))^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 368

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_)^(q_))^(n_))^(p_.), x_Symbo
l] :> Dist[(d*x)^(m + 1)/(d*((c*x^q)^(1/q))^(m + 1)), Subst[Int[x^m*(a + b*
x^(n*q))^p, x], x, (c*x^q)^(1/q)], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && IntegerQ[n*q] && NeQ[x, (c*x^q)^(1/q)]
```

Rubi steps

$$\int \frac{(cx^n)^{\frac{1}{n}}}{\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^5}\,dx = \frac{(cx^n)^{\frac{1}{n}}\int \frac{x}{\left(a+b(cx^n)^{\frac{1}{n}}\right)^5}\,dx}{x}$$

$$= \left(x\,(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x}{(a+bx)^5}\,dx, x, (cx^n)^{\frac{1}{n}}\right)$$

$$= \left(x\,(cx^n)^{-1/n}\right)\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^5}+\frac{1}{b(a+bx)^4}\right)\,dx, x, (cx^n)^{\frac{1}{n}}\right)$$

$$= \frac{ax\,(cx^n)^{-1/n}}{4b^2\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^4} - \frac{x\,(cx^n)^{-1/n}}{3b^2\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^3}$$

Mathematica [A] time = 0.03, size = 48, normalized size = 0.69

$$-\frac{x\,(cx^n)^{-1/n}\left(a+4b\,(cx^n)^{\frac{1}{n}}\right)}{12b^2\left(a+b\,(cx^n)^{\frac{1}{n}}\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^n)^n^(-1)/(a + b*(c*x^n)^n^(-1))^5,x]
```

```
[Out] -1/12*(x*(a + 4*b*(c*x^n)^n^(-1)))/(b^2*(c*x^n)^n^(-1)*(a + b*(c*x^n)^n^(-1
))^4)
```

fricas [A] time = 0.42, size = 92, normalized size = 1.31

$$-\frac{4\,bc^{\left(\frac{1}{n}\right)}x + a}{12\left(b^6c^{\frac{5}{n}}x^4 + 4\,ab^5c^{\frac{4}{n}}x^3 + 6\,a^2b^4c^{\frac{3}{n}}x^2 + 4\,a^3b^3c^{\frac{2}{n}}x + a^4b^2c^{\left(\frac{1}{n}\right)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^n)^(1/n)/(a+b*(c*x^n)^(1/n))^5,x, algorithm="fricas")
```

```
[Out] -1/12*(4*b*c^(1/n)*x + a)/(b^6*c^(5/n)*x^4 + 4*a*b^5*c^(4/n)*x^3 + 6*a^2*b^
4*c^(3/n)*x^2 + 4*a^3*b^3*c^(2/n)*x + a^4*b^2*c^(1/n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^n)^{\left(\frac{1}{n}\right)}}{\left((cx^n)^{\left(\frac{1}{n}\right)}b + a\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^n)^(1/n)/(a+b*(c*x^n)^(1/n))^5,x, algorithm="giac")
```

```
[Out] integrate((c*x^n)^(1/n)/((c*x^n)^(1/n)*b + a)^5, x)
```

maple [C] time = 0.18, size = 279, normalized size = 3.99

$$\left(4ab\,c^{\frac{1}{n}}\,(x^n)^{\frac{1}{n}}\,\mathrm{e}^{\frac{i\pi\left(\text{csgn}(ic)-\text{csgn}\left(ic\,x^n\right)\right)\left(-\text{csgn}\left(ix^n\right)+\text{csgn}\left(ic\,x^n\right)\right)\text{csgn}\left(ic\,x^n\right)}{n}} + b^2c^{\frac{2}{n}}\,(x^n)^{\frac{2}{n}}\,\mathrm{e}^{\frac{3i\pi\left(\text{csgn}(ic)-\text{csgn}\left(ic\,x^n\right)\right)\left(-\text{csgn}\left(ix^n\right)+\text{csgn}\left(ic\,x^n\right)\right)\text{csgn}\left(ic\,x^n\right)}{2n}}\right)$$

$$12\left(b\,c^{\frac{1}{n}}\,(x^n)^{\frac{1}{n}}\,\mathrm{e}^{\frac{i\pi\left(\text{csgn}(ic)-\text{csgn}\left(ic\,x^n\right)\right)\left(-\text{csgn}\left(ix^n\right)+\text{csgn}\left(ic\,x^n\right)\right)\text{csgn}\left(ic\,x^n\right)}{2n}} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^n)^(1/n)/(b*(c*x^n)^(1/n)+a)^5,x)
```

```
[Out] 1/12*x*(x^n)^(1/n)*c^(1/n)/a^3/(b*c^(1/n)*(x^n)^(1/n)*exp(1/2*I*Pi*(csgn(I*
c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))+a)^4*(((x^n
)^(1/n))^2*(c^(1/n))^2*b^2*exp(3/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*
x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))+4*c^(1/n)*(x^n)^(1/n)*a*b*exp(I*Pi*(cs
gn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(I*c*x^n))+6*a^2*
exp(1/2*I*Pi*(csgn(I*c)-csgn(I*c*x^n))*(-csgn(I*x^n)+csgn(I*c*x^n))/n*csgn(
I*c*x^n)))
```

maxima [B] time = 0.65, size = 158, normalized size = 2.26

$$\frac{b^2 c^{\frac{3}{n}} x (x^n)^{\frac{3}{n}} + 4\,a b c^{\frac{2}{n}} x (x^n)^{\frac{2}{n}} + 6\,a^2 c^{\left(\frac{1}{n}\right)} x (x^n)^{\left(\frac{1}{n}\right)}}{12\left(a^3 b^4 c^{\frac{4}{n}} (x^n)^{\frac{4}{n}} + 4\,a^4 b^3 c^{\frac{3}{n}} (x^n)^{\frac{3}{n}} + 6\,a^5 b^2 c^{\frac{2}{n}} (x^n)^{\frac{2}{n}} + 4\,a^6 b c^{\left(\frac{1}{n}\right)} (x^n)^{\left(\frac{1}{n}\right)} + a^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^n)^(1/n)/(a+b*(c*x^n)^(1/n))^5,x, algorithm="maxima")
```

```
[Out] 1/12*(b^2*c^(3/n)*x*(x^n)^(3/n) + 4*a*b*c^(2/n)*x*(x^n)^(2/n) + 6*a^2*c^(1/
n)*x*(x^n)^(1/n))/(a^3*b^4*c^(4/n)*(x^n)^(4/n) + 4*a^4*b^3*c^(3/n)*(x^n)^(3
/n) + 6*a^5*b^2*c^(2/n)*(x^n)^(2/n) + 4*a^6*b*c^(1/n)*(x^n)^(1/n) + a^7)
```

mupad [B] time = 1.20, size = 208, normalized size = 2.97

$$\frac{x}{12\,a\,b\left(b^3\,(c\,x^n)^{3/n} + a^3 + 3\,a\,b^2\,(c\,x^n)^{2/n} + 3\,a^2\,b\,(c\,x^n)^{1/n}\right)} - \frac{x}{4\,b\left(b^4\,(c\,x^n)^{4/n} + a^4 + 4\,a\,b^3\,(c\,x^n)^{3/n} + 6\,a^2\,b^2\,(c\,x^n)^{2/}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^n)^(1/n)/(a + b*(c*x^n)^(1/n))^5,x)
```

```
[Out] x/(12*a*b*(b^3*(c*x^n)^(3/n) + a^3 + 3*a*b^2*(c*x^n)^(2/n) + 3*a^2*b*(c*x^n
)^(1/n))) - x/(4*b*(b^4*(c*x^n)^(4/n) + a^4 + 4*a*b^3*(c*x^n)^(3/n) + 6*a^2
*b^2*(c*x^n)^(2/n) + 4*a^3*b*(c*x^n)^(1/n))) + x/(12*a^3*b*(a + b*(c*x^n)^(
1/n))) + x/(12*a^2*b*(b^2*(c*x^n)^(2/n) + a^2 + 2*a*b*(c*x^n)^(1/n)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**n)**(1/n)/(a+b*(c*x**n)**(1/n))**5,x)
```

```
[Out] Timed out
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
        max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#               Port of original Maple grading function by
#               Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#               added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
    )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
    ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
    type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
    expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
    ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```